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A Nonlinear, Unsteady, Aerodynamic Model for Insect-like Flapping Wings in the Hover with Micro Air Vehicle Applications

PhD Thesis

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Abstract

The essence of this PhD thesis is the analytical, aerodynamic modelling, as opposed to using grid-based methods such as CFD, of insect-like flapping wings in the hover for micro air vehicle applications. A key feature of such flapping-wing flows is their unsteadiness and the formation of a leading-edge vortex in addition to the conventional wake shed from the trailing edge. What ensues is a complex interaction between the shed wakes which, in part, determines the forces and moments on the wing. In an attempt to describe such a flow, two coupled, nonlinear, wake integral equations are derived and these form the foundation upon which the rest of the work stands. The model so developed is unsteady and inviscid in nature and essentially two-dimensional. It is converted to a 'quasi-three-dimensional' model using a blade-element-type analogy but with radial chords. The governing equations developed in the study are exact but do not have a closed form. Solutions are, therefore, found by numerical methods and implemented in FORTRAN.

The model is validated against existing experimental data and remarkable agreement is found both in terms of flow field representation and force prediction. The importance of including the effect of the leading-edge vortex for such problems is also established. The model is then used for a parametric study to analyse the effects of various wing geometry and wing kinematics parameters. From these results, a preferred wing design for a flapping-wing MAV is proposed which is the ultimate aim of this work.

The results from the unsteady, aerodynamic model are also compared with earlier work in the PhD using a simple quasi-steady model and good agreement is found in terms of the relative merits of the various wing parameters, thereby establishing the usefulness of using such simple models for initial design studies.

To my parents

for always being there for me

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These are the only few lines in this thesis where I can drop the technical prose and esoteric jargon and write casually. And I shall do just that! I owe thanks to the following persons who have contributed in one way or another to my work.

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¹À la ‘Batman & Robin’.

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In conclusion, All Praise Belongs To Allah, Lord Of The Worlds!

Preface

This PhD thesis arose as part of a wider study at Cranfield University, RMCS Shrivenham into insect-like flapping flight for micro air vehicle applications. It was integrated with an experimental project at Cambridge University and post-doctoral work by a research assistant at Shrivenham. The entire project was jointly funded by the EPSRC (under grant number GR/M78472/01) and DERA (now Dstl).

The central novelty of this work is the theoretical development of an analytical, inviscid, nonlinear and essentially two-dimensional unsteady aerodynamic model of insect-like flapping wings in the hover (Chapter 4), its numerical implementation in FORTRAN (Chapter 5) and its use for a parametric study of the effects of wing geometry and kinematics on lift, drag and lift-to-torque ratio (Chapter 6). Preliminary work on the subject involved a simple quasi-steady model that provided useful insight into the relative merits of various wing parameters (Chapter 3). A more detailed outline of the thesis is given at the end of this Preface.

The overall aim of this work is to develop a model for determining performance (lift, thrust and aerodynamic moment) characteristics for an insect-like flapping wing in the hover. With this information, wing-design guidelines for a flapping-wing MAV can be proposed.

Contributions

The main contributions of this thesis are:

- A quasi-steady aerodynamic method for computing the 3-D lift (and lift distribution) on a flapping wing is developed using a modified form of the analysis used by Glauert for 3-D wings but based on the downwash of a rotary wing with radially-varying incident velocity (§ 3.2.1–3.2.2).
- The above quasi-steady method is used to compare wing designs for insect-like flapping wings in terms of kinematics and planform shape (§ 3.2.3).
- Identification of the need for radial chords (§ 3.2.4) and the presence of radial cross-flow planes (§ 4.1.2) in flapping-wing applications.
- Development of a circulation-based 2-D unsteady flapping-wing model (§ 4.2.2) that combines the quasi-steady component associated with the unsteady motion of the flapping wing (pp. 109ff) and the unsteady effect of both shed wakes from the leading and trailing edges (pp. 113ff).
- Endowing the model with a capability to handle thick and cambered aerofoil profiles without any modification (p. 101).
- Derivation of the nonlinear form of the Wagner wake integral equation incorporating the effects of both the leading-edge vortex and trailing-edge wake (§ 4.2.2, pp. 113ff).
- Definition of a quasi-3-D model to represent the combined effect of the flapping-wing system by incorporating radial chords, radial cross-flow planes, cylindrical wakes and a blade-element-type method (§ 4.2.4).
- A method for computing the forces and moments on a flapping wing based on the method of impulses and including the effect of the leading-edge vortex, and their

decomposition into the constituent components (§ 4.2.3).

- The conversion of the aforementioned unsteady flapping-wing model to a modular FORTRAN code (pp. 160ff), including the definition of a representative length specific to the wing planform and kinematics (p. 157) and the introduction of a limit to prevent artificially high circulations (pp. 162ff).
- Numerical experiments to establish the independence of solutions on numerical, code-related parameters (§ 5.2.1).
- Validation of the model against available force and flow-visualisation data (§ 5.2.2).
- Identification of a spanwise phase lead-lag between wing sections and the dependence of flow on 'age' defined in terms of chord lengths travelled (§ 6.1.1).
- Detailed description of the origin of the forces generated during insect-like flapping motion from the physics of the flow and their decomposition into constituent elements (§ 6.1.2–6.1.3).
- Establishment of the importance of including the effect of the leading edge vortex in any modelling framework aiming to represent the insect-like flapping-wing problem (§ 6.1.4).
- By means of the FORTRAN code implementation, the unsteady flapping-wing model is used to conduct a parametric study to investigate the effects of various wing shape and wing kinematics on the aerodynamic performance of flapping wings (§ 6.2).
- Insect-like flapping kinematics based on Lissajous curves have been devised that allow variations in durations of stroke reversals and the phases with constant sweeping velocity, and variations of phase differences between translational and rotational motion (pp. 231ff).
- Wing design specifications (in terms of wing kinematics and wing shape parameters) for a preferred flapping-wing micro air vehicle are proposed (§ 7.1.1).

Thesis Outline

This PhD thesis is divided into two main parts — the main report and the appendices.

The main report is divided into seven chapters that are arranged as follows:

In Chapter 1, the concept of micro air vehicles (MAVs) is introduced and the motivation behind the current work established. This is followed by a brief discourse on existing work on MAVs, finally leading to the requirement for flapping-wing MAVs. The chapter concludes with a discussion of the aims and objectives of the present PhD.

Chapter 2 presents a review of the literature pertaining to flapping wings. It is divided broadly into two parts: insect-like flapping flight and the associated aerodynamics. The first section details what insect-like flapping actually entails, followed by a brief description of the unsteady aerodynamics pertaining to such a flight regime and then a discussion of the commonly observed lift mechanisms. Literature in this field is primarily from the biological community.

The second part of Chapter 2 considers some of the fundamental aerodynamics relating to insect-like flapping. The discussion is divided into three sections. The first section describes some of the classical works on unsteady aerodynamics. This is followed by a section on delta wing aerodynamics due to the similarity between their leading-edge vortices and those observed on insect wings. Finally, existing work pertaining directly to flapping-wing flight is reviewed in the last section.

Some of the preliminary work in this PhD involved using a quasi-steady model to provide initial design guidelines for insect-like flapping-wing MAVs. This study is described in Chapter 3. The chapter begins with a brief look at the classical momentum theory for propellers before delving into a more complex blade-element-type method modified for flapping wings. The work presented in the latter section is completely original and is complemented with full details of calculations in Appendix B.

Chapter 4 describes the core of this thesis, an analytical, inviscid, nonlinear and

essentially two-dimensional unsteady aerodynamic model of insect-like flapping wings in the hover. It is broadly divided into two parts. The first part describes the modelling methodology and identifies the relevant features that need to be modelled. It also lays out the assumptions used in the subsequent analysis and includes a brief description of the complex variable upon which most of the analysis is based.

The second part of Chapter 4 describes in detail the derivations of the governing equations for the problem. This section is complemented by Appendix C where full details of the derivations can be found. It is in this section of Chapter 4 that the two novel, coupled, nonlinear integral equations are derived. The latter parts of the chapter discuss the computation of forces and moments. The chapter concludes with a description of how the 2-D modelling framework is converted to a quasi-3-D model.

In Chapter 5, the analytical equations described in the preceding chapter are converted to forms that are amenable to numerical solution. The first part of the chapter begins with a review of current vortex methods (which are used extensively in the present work for solving the wake integral equations from the previous chapter). This is followed by a discussion of the method of solution and its numerical implementation in FORTRAN. Some code-related numerical parameters are also identified.

The second part of Chapter 5 presents the methods used to validate the model. It begins with some numerical experiments, aimed at establishing the independence of the solution from numerical parameters. This is then followed by validation of the model against existing experimental data. Here, both flow visualisation and force prediction results are compared.

Chapter 6 is divided into two parts. The first part describes the forces observed on the basis of the physics of the flow. Later on, the importance of modelling the leading-edge vortex is established with a brief discourse on the limitations of using the current inviscid, theoretical model.

The second part of Chapter 6 is where the aim of the present work — the development of wing-design guidelines — is addressed. A parametric study is presented that analyses the effects of wing kinematics and wing geometry on lift, thrust and lift-to-torque ratio characteristics of insect-like flapping. Several parameters relating to the shape and dynamics of the flapping wing are identified.

Chapter 7 is the final chapter in the main report part of the thesis. It summarises the main findings from both the simple quasi-steady treatment in Chapter 3 and the more complex and accurate study in Chapter 6 with a brief comparison of the results of the two approaches. On the basis of these findings, a preferred flapping-wing MAV design is proposed. The chapter concludes with suggestions for further refinements.

There are four appendices to complement the main report. Appendix A contains useful mathematics to accompany the analyses in Chapters 3 and 4. Appendix B gives full details of the calculations presented in Chapter 3 while Appendix C similarly gives full details of the derivations in Chapter 4. Finally, Appendix D gives details of selected results from Chapter 6.

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List of Symbols

Latin Alphabet

<i>Symbol</i>	<i>Meaning</i>	<i>Reference/ Definition</i>
a	location of pitch axis w.r.t. mid-chord	Figure 4.6
a_0	2-D lift-curve slope, rad^{-1}	Equation 3.6
A	area swept by actuator disk, m^2	Equation 3.1
A_n	Fourier constant	Equation 3.7
A_i	quasi-steady components of circulation ($i = 1 \dots 11$), m^2/s	Equation 4.13
AR	aspect ratio	Equation 5.24
b	aerofoil semi-chord, m	Equation 2.5
c	wing section chord length, m	Equation C.5
c_i	chord length at wing section i , m	
c_r	chord length at radial distance r , m	Equation 3.6
C_L	coefficient of lift	$L/\frac{1}{2}\rho U_\infty^2 S$
C_D	coefficient of drag	$D/\frac{1}{2}\rho U_\infty^2 S$
D	drag force parallel to incident velocity, N	
D_i	(induced) drag parallel to incident velocity, N	Equation 3.10
$\mathcal{D}$	horizontal drag force orthogonal to gravity, N	Equation 4.48
f	flapping frequency, Hz	
$\mathcal{F}$	complex force, N	Equation 4.34
$\mathbf{F}$	force vector, N	Equation 4.30
h	heave displacement, m	Figure 4.6
$\dot{h}$	heave velocity, m/s	Figure 4.6

Latin alphabet (continued)

<i>Symbol</i>	<i>Meaning</i>	<i>Reference/ Definition</i>
$\mathcal{I}$	complex impulse, Ns	p. 123
$\Im$	imaginary part of	
$\mathbf{I}$	impulse vector, Ns	Equation 4.30
$\mathbf{I}_m$	moment of impulse vector, Nms	Equation 4.31
l	lunge displacement, m	Figure 4.6
$\dot{l}$	lunge velocity, m/s	Figure 4.6
L	lift force perpendicular to incident velocity,	
$\mathcal{L}$	vertically upwards lift force parallel to gravity, N	Equation 4.48
M	moment, Nm	Equation 2.6
$\mathcal{M}$	complex moment, Nm	Equation 4.40
$\mathbf{M}$	moment vector, Nm	Equation 4.31
$\mathcal{M}$	moment about wing pitch axis, Nm	p. 151
n_{foil}	aerofoil discretisation	
n_{sec}	number of wing sections	
n_{steps}	number of time-steps	
$\mathcal{O}$	of the order of	
p	pressure, Pa	Equation 2.2
P	(induced) power, W	Equation 3.2
$\mathbf{q}$	velocity vector, m/s	Equation 2.2
q	complex velocity, m/s	Equation 4.5
Q_i or Q	(induced) torque, Nm	Equation 3.11
Q	torque at centre of wing rotation, Nm	p. 151
r	radial distance from centre, m	Equation 3.3
r_c	vortex-core radius, m	Figure 5.1
R	radius of circle in $\mathcal{Z}$ -plane, m	p. 101
R	wing length, m	Equation 3.1
$\Re$	real part of	
Re	Reynolds number	p. 29

Latin alphabet (continued)

<i>Symbol</i>	<i>Meaning</i>	<i>Reference/ Definition</i>
s	aerofoil semi-chord, m (Wagner notation)	
S	wing area, m ²	
St	Strouhal number	p. 29
t	time, s	
$\mathcal{T}$	horizontal thrust force orthogonal to gravity and in the direction of motion, N	p. 151
u	x -component of velocity, m/s	Equation 4.10
U_∞ or U	freestream velocity, m/s	
v	y -component of velocity, m/s	Equation 4.10
v_r	radial component of velocity, m/s	Equation 4.6
v_θ	circumferential component of velocity, m/s	Equation 4.6
v_x	x -component of velocity, m/s	Equation 4.6
v_y	y -component of velocity, m/s	Equation 4.6
w	induced velocity, m/s	Equation 3.1
w	downwash velocity, m/s	Equation 3.5
W	weight, kg	Equation 3.1
x	real axis coordinate in circle ($\mathcal{Z}$) plane, m	
x	abscissa in Cartesian coordinate system, m	
y	imaginary axis coordinate in circle ($\mathcal{Z}$) plane, m	
y	ordinate in Cartesian coordinate system, m	
$\mathcal{Z}$	complex coordinates in circle plane, m	$x + iy$

Greek Symbols

<i>Symbol</i>	<i>Meaning</i>	<i>Reference/ Definition</i>
α	angle of attack, rad	Figure 4.6
$\dot{\alpha}$	pitch rate, rad/s	Figure 4.6
β	stroke-plane angle, rad	Figure 2.1
γ	vorticity, m/s	Equation 4.3d
Γ	circulation, m ² /s	Equation 4.3e
δ_{ij}	distance between <i>i</i> th and <i>j</i> th elements, m/s	Equation 5.12
ϵ	(complex) shape parameter	Equation 4.2
ζ	complex coordinates in physical plane, m	$\xi + i\eta$
η	imaginary axis coordinate in physical (ζ) plane, m	
θ	inclination angle, rad	Figure 2.3
θ	dummy parameter along wing, rad	Equation 3.3
θ	angle along circle in Z -plane	Equation 4.4
ϑ	Lissajous phase angle, rad	Equation 6.1
Θ	inclination amplitude, rad	Equation 6.1
i	complex i	$\sqrt{-1}$
λ	representative length, m	Equation 5.23
μ	absolute viscosity, Ns/m ²	
ν	kinematic viscosity, m ² /s	
ξ	real axis coordinate in physical (ζ) plane, m	
π	circle constant	≈ 3.1416
ω	dummy variable	Equation A.10
ρ	fluid density, kg/m ³	
σ	aerofoil camber parameter, m	Equation 4.2
ς	dummy variable	Equation 4.11
τ	aerofoil thickness parameter, m	Equation 4.2
ϕ	stroke angle, rad	Figure 2.3
φ	velocity potential, m ² /s	Equation 4.4
Φ	stroke amplitude, rad	Figure 2.3

Greek symbols (continued)

<i>Symbol</i>	<i>Meaning</i>	<i>Reference/ Definition</i>
ψ	stream function, m ² /s	Equation 4.4
Ω	angular velocity, rad/s	Equation 3.4
Ω	complex potential, m ² /s	Equation 4.4

Subscripts/Superscripts/Notation

<i>Symbol</i>	<i>Meaning</i>	<i>Reference/ Definition</i>
$\mathcal{A}_0$	quasi-steady component	
$\mathcal{A}_1$	apparent-mass component	
$\mathcal{A}_2$	wake-induced (and LEV-induced) component	
$\mathcal{A}_3$	wake (and LEV) component	
$\mathcal{A}_a$	aerofoil	
$\mathcal{A}_i$	induced	
$\mathcal{A}_{lev}$	leading-edge vortex	
$\mathcal{A}_w$	trailing-edge wake	
$\mathcal{A}_{wake}$	trailing-edge wake	
$\tilde{\mathcal{A}}$	rotating coordinate system	Figure 4.6
$\hat{\mathcal{A}}$	translating coordinate system	Figure 4.6
$\bar{\mathcal{A}}$	inertial coordinate system	Figure 4.6
$\bar{\mathcal{A}}$	mean value of	
$\overline{\mathcal{A}}$	complex conjugate of	
$\mathbf{A}$	vector	
$\dot{\mathcal{A}}$	first derivative w.r.t. time, s ⁻¹	
$\ddot{\mathcal{A}}$	second derivative w.r.t. time, s ⁻²	

where $\mathcal{A}$ is a generic variable, e.g. $\gamma, \Gamma, L, \mathcal{T}$, etc.

Abbreviations/Acronyms

<i>Short Form</i>	<i>Long Form</i>
2-D	two-dimensional, planar
3-D	three-dimensional, spatial
FMAV	flapping-wing micro air vehicle
CFD	computational fluid dynamics
LEV	leading-edge vortex
MAV	micro air vehicle
RANS	Reynolds-averaged Navier-Stokes
rpm	revolutions per minute
UAV	unmanned air (or aerial) vehicle
VTOL	vertical take-off & landing
w.r.t.	with respect to

Chapter 1

Introduction

In this chapter, the concept of micro air vehicles (MAVs) is introduced (§ 1.1). The requirements and specifications for such vehicles are laid out followed by a brief look at existing work on MAVs (§ 1.2). The chapter then goes on to describe the need for a unique type of MAV — the flapping-wing micro air vehicle (FMAV) based on insect-like flapping (§ 1.3). The benefits of such a design are presented. Finally, from the aforementioned discussion, aims and objectives of the current PhD study are derived (§ 1.4).

1.1 Motivation and Background

The need for small, intelligent reconnaissance robots that are capable of discreetly penetrating into confined spaces such as shafts and tunnels, and manoeuvring in them, is the motivation behind this PhD project. Agile flight inside buildings, stairwells, shafts and tunnels is also of significant military and civilian value.

This project is part of a wider study by Cranfield University at RMCS, Shrivenham aimed at paving the way for the development of autonomous, information-gathering robots, and attracts both civilian and military (defence) funding. The motivation for micro air vehicle research at the university was developed in parallel with DARPA’s view (Żbikowski, 1999a,b). Whereas the present PhD project is primarily theoretical, a sister project at Cambridge University has been involved with experimental aspects (Nolan, 2004) and a third project (also at Shrivenham) is concerned with developing a CFD approach to low Reynolds number flapping-wing aerodynamics.

1.1.1 Micro Air Vehicles (MAVs)

The concept of a hand-held (c. 6 in) flying vehicle originated with the launch of DARPA’s micro air vehicle (MAV) program. With particular applicability to a fixed-wing vehicle, DARPA came up with the following arbitrary definition of a MAV (McMichael & Francis, 1997)

<i>Parameter</i>	<i>Specification</i>
Size	15 cm (6 in) maximum dimension
Weight	50 g or so
Payload	20 g or less
Endurance	20–60 min
Range	10 km

These definitions were for guidance purposes and have since been extended to include greater size (~ 9 in) and higher weight (~ 100 g), DARPA themselves having fallen victim to these arbitrary definitions (see *Wasp* and *Hornet* on p. 7). The choice of the word *micro* is to distinguish it from *mini* (as applied to mini-UAVS, such as the *Sentry* (Goebel, 2001), which are at least an order of magnitude bigger) and does not imply vehicles of μm -scales.

1.1.2 Applications of MAVs

Although the original idea behind MAVs was predominantly for military use (McMichael & Francis, 1997), several other applications exist. They show particular promise in so-called D^3 — dull, dirty and dangerous — environments (Żbikowski *et al.*, 2000). Some useful applications are:

Reconnaissance Operations for tactical short-range surveillance (e.g. to enable an operator on one side of an obstacle to monitor enemy movements on the other side) or for spying purposes.

Law Enforcement to assist in anti-terrorist operations, bomb disposal and for reconnaissance in hostage situations.

Combatting Urban Warfare by means of military operations in urban terrain (MOUT).

Rescue Operations in search of life in the rubble of an earthquake or bombing aftermath.

Remote Sensing as biological, chemical and nuclear reconnaissance robots in pipes, shafts and tunnels, contaminated areas, nuclear reactors, machine rooms on ships, for utility inspection and leak detection, or other places deemed unsafe for human presence.

Space Exploration has recently been added to the list of applications. After the recent

successes of the Mars Rovers especially, NASA has speculated about sending unmanned aircraft, and even MAVs, to Mars (NASA, 2004).

Other Applications such as non-attainments areas where either human presence or remote human assistance is not feasible.

1.1.3 Requirements for MAVs

Current surveillance assets (e.g. satellites, manned aircraft, UAVs) possess virtually no capabilities of information-gathering *inside* buildings. Also, due to their small size (and hence, limited energy storage), MAVs are unlikely to be very useful for precise outdoor flying, especially in adverse weather conditions. Our focus is, therefore, on indoor flight which leads to the requirement of a distinct flight envelope:

Low Speed is required to manoeuvre in confined spaces and to avoid collision with walls and other obstacles (e.g. furniture, debris, etc.).

Vertical Take-off and Landing (VTOL) Capability is necessary for launch and touch-down without any special ground preparation. It is also mandatory for flight up (or down) a stairwell or a vertical shaft.

Hovering Capability is required for precise observation. It is also necessary for stopping to assess an environment or devise an alternative trajectory.

Zero (or Small) Acoustic Signature is needed for silent penetration into hostile environments (e.g. in urban warfare).

Minimal Interference due to the presence of walls, floors and ceilings is compulsory for a MAV to remain airborne and avoid losing control in confined spaces.

High Agility and Manoeuvrability is essential for taking evasive actions necessary when manoeuvring in confined spaces.

High Lift is mandatory for a vehicle of this size because payload forms a significant fraction of vehicle weight.

High Efficiency is crucial because the small size of MAVs allows only limited energy storage.

A further requirement that is not of aerodynamic concern is autonomous flight to enable mission completion without the assistance of a human telepilot.

1.2 Existing Work on MAVs

In 1992, the US Defense Advanced Research Projects Agency (DARPA) conducted a workshop entitled 'Future Technology-Driven Revolutions In Military Operations' (Hundley & Gritton, 1992), aimed at briefing the RAND National Defense Research Institute. At this presentation, the concept of so-called *micro-drones* was discussed. After a number of 'paper studies' into this micro-drone concept, DARPA launched a four-year \$35 million programme in 1997 aimed at innovating in the field of MAVs, or *micro aerial vehicles* as they were initially called (DARPA, 2000).

Industry responded to the call resulting in a number of prototypes. Some of the leading research from these earlier works is presented below.

A Lockheed Martin company by the name of Sanders in Nashua, NH (now part of BAE SYSTEMS) has been developing the *MicroSTAR* (Figure 1.1). It is a fixed-wing MAV, designed to have a take-off weight of 85 g. Being a fixed-wing aircraft, it is restricted to forward flight.

Lutronix Corporation in Del Mar, CA are also involved in this research. Their designs (and prototypes) are of the rotary-wing type (Figure 1.2), and are capable of both hovering and forward flight. Their debut vehicle was called the *Kolibri* and weighed in at a hefty 320 g, with an endurance of about 30 min (DARPA, 2000). Second-generation (and now

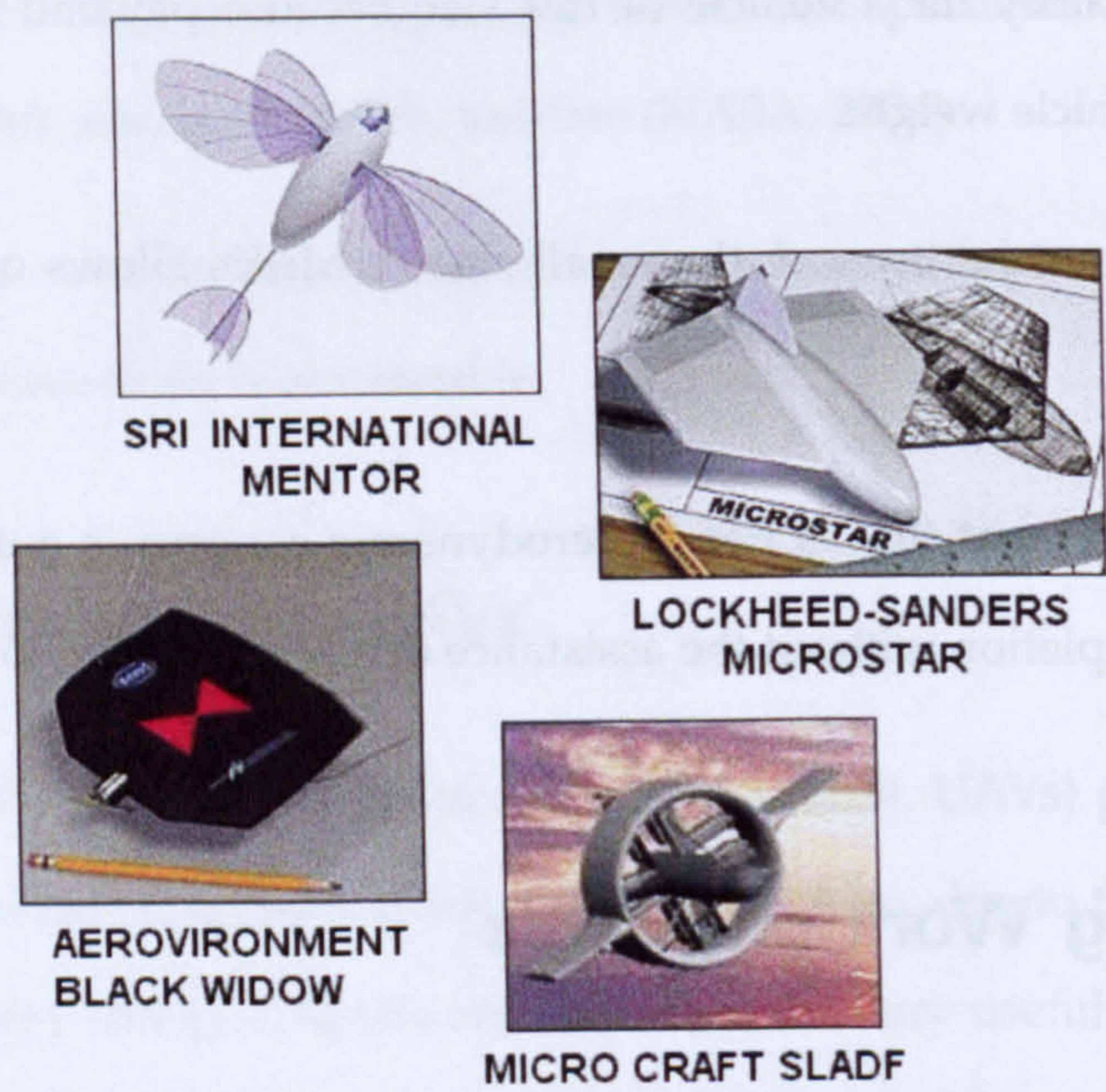


Figure 1.1: Examples of MAVs [from Goebel (2001)]

third-generation) models are of ducted-fan design (Lutronix Corp., 2001).

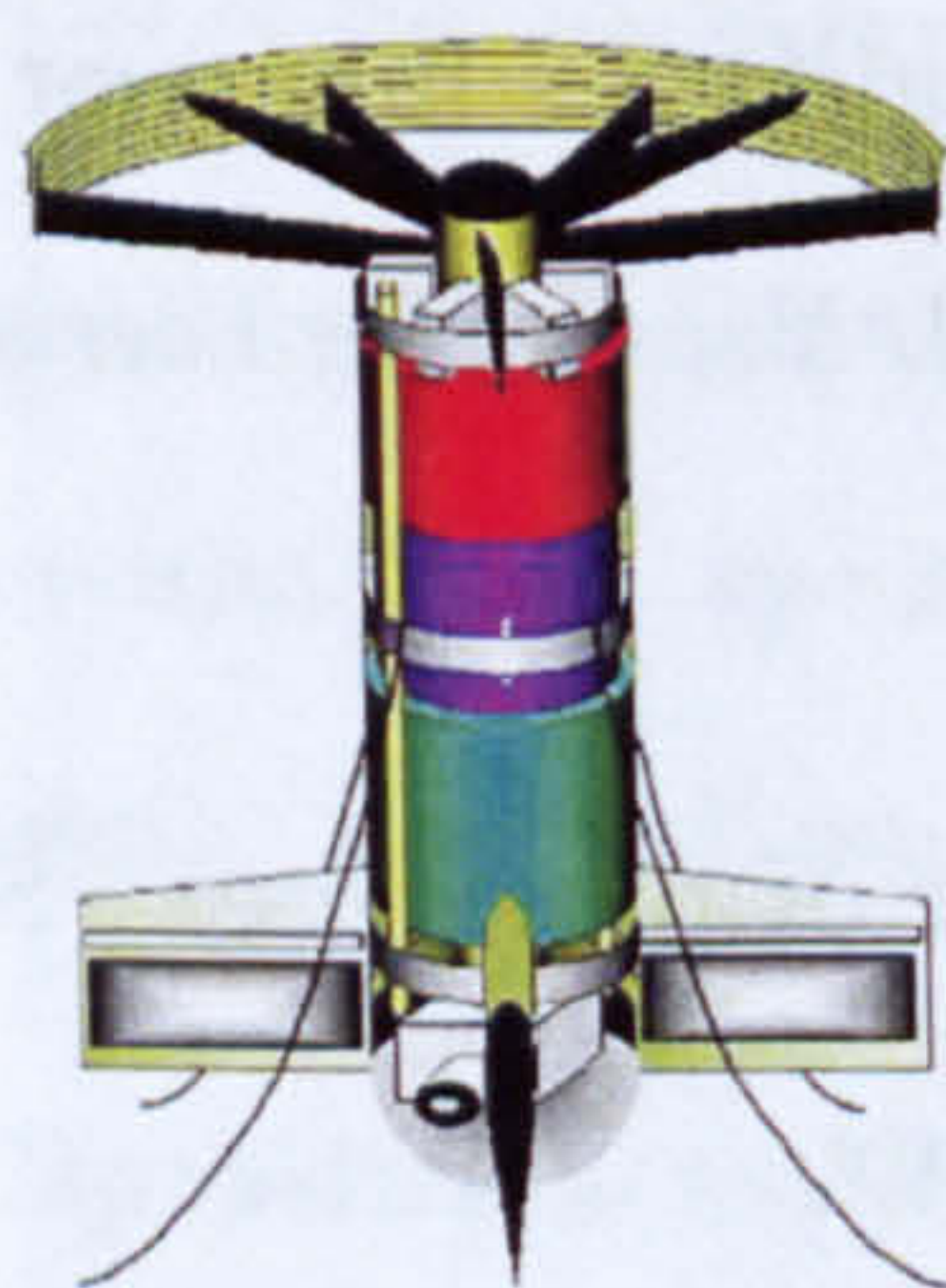


Figure 1.2: Lutronix Kolibri

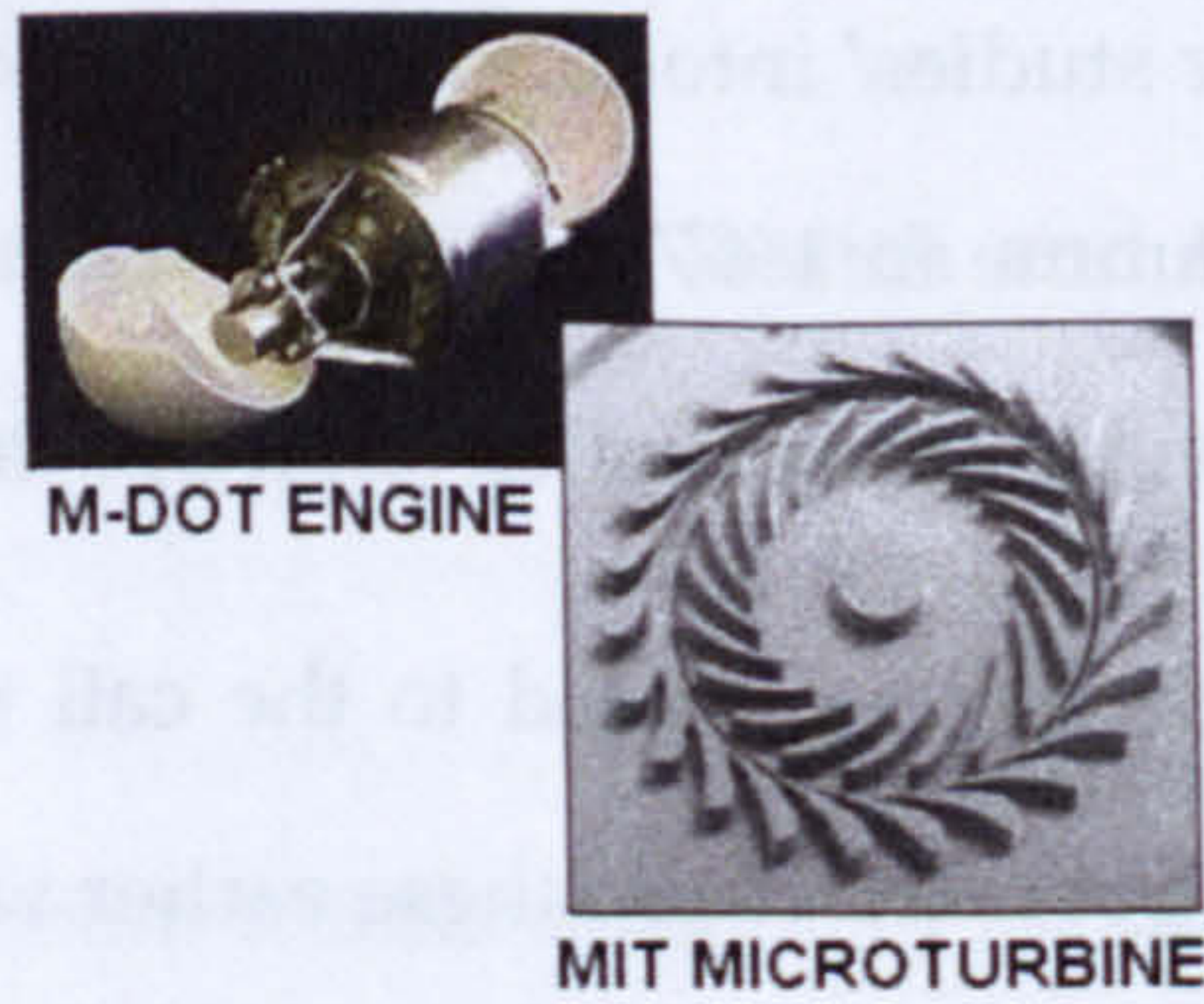


Figure 1.3: Microturbines

AeroVironment Inc. in Simi Valley, CA have developed a fixed wing MAV, called *Black Widow* (Figure 1.1). It requires a skilled radio control pilot and has an endurance of about 20 min in flight (DARPA, 2000).

Another company by the name of Micro Craft Technology in San Diego, CA, is also engaged with DARPA’s so-called TTO, or Tactical Technology Office (DARPA, 2000). They

have been developing a fixed-rotary-wing combination — a vehicle that flies like a fixed wing aircraft, but can hover and land vertically (Figure 1.1).

In 2000, the UK defence laboratories DERA (now split into Dstl and QinetiQ) demonstrated a tiny powerplant for MAVs called the *Microjet*. The tiny, cylindrical jet had dimensions 13 mm×5 mm, weighed less than 2 g and gave an impressive thrust-to-weight ratio $\mathcal{O}(75:1)$. The jet was fuelled by hydrogen peroxide and kerosene and DERA claimed to have achieved continuous operation up to 1 hour.

Much more recently, Seiko Epson Corporation of Japan unveiled their μ FR (Micro Flying Robot, see Figure 1.4) which is a tiny helicopter with contra-rotating rotor blades (Epson, 2003). Unfortunately, power requirements are high and the robot has to be tethered to a power supply.



Figure 1.4: Epson μ FR



Figure 1.5: DARPA Wasp

DARPA meanwhile have concentrated their efforts on flying-wing configurations. Their earlier MAV the *Wasp* (Figure 1.5) spanned 13 in and weighed in at 120 g (DARPA, 2002). A later improvement (in terms of power) was the *Hornet* which spans 15 in and weighs 6 oz (170 g) (DARPA, 2003). It would appear that the emphasis in their exercises has been on powerplant efficiency rather than size.

Research on MAVs is also taking place in academic circles, as opposed to the commercial or military examples cited above. At Massachusetts Institute of Technology (MIT),

for example, work is underway on tiny jet engines for MAVs whose turbines revolve at phenomenal speeds typically $\mathcal{O}(10^6 \text{ rpm})$ (Figure 1.3),(DARPA, 2000).

At the Georgia Tech Research Institute in Atlanta, GA, work on a reciprocating chemical muscle has been carried out (Michelson *et al.*, 1997) albeit with a debatable degree of success.

Caltech (California Institute of Technology in Pasadena, CA) has been developing a *Microbat* — a tiny flapping-wing vehicle (Figure 1.6). To date, flight durations of only 5–20 s have been achieved, the limiting factor being the power plant. Similarly, the University of California at Berkeley (UCB) has been working on a *Micromechanical Flying Insect (MFI)* since 1998. The design appears to resemble a ‘hypothetical insect’ with short, stubby wings (Figure 1.7). Last year they demonstrated a mere $500\mu\text{N}$ of lift from either wing. Remote power still appears to be a main hurdle but with the leaps in miniaturisation of mobile (cellular) phone batteries, a solution may be close.



Figure 1.6: Caltech Microbat concept

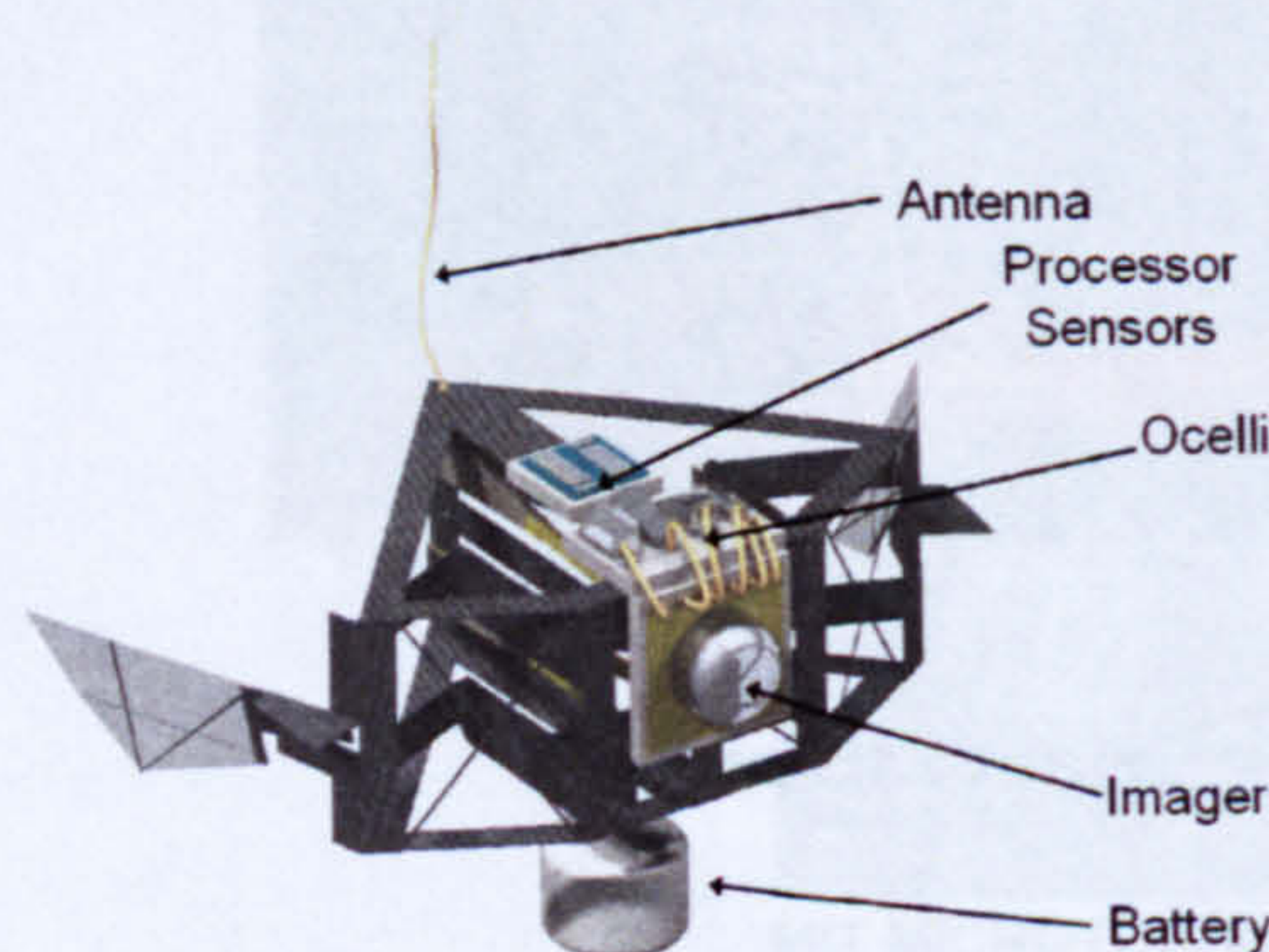


Figure 1.7: UCB MFI

Here in the UK, research in this field is also being conducted. Cranfield University at RMCS Shrivenham is home to this PhD project and is a UK-leader in insect-like flapping-wing MAV research. At the Cranfield University site in Cranfield, Bedfordshire, work is being carried out on piezoelectric microactuators for smart MAVs (Wilson *et al.*, 2001). Work on artificial muscles is being conducted at the Imperial College of Science,

Technology & Medicine in London too (Kucernak, 2001).

1.3 Flapping-Wing Micro Air Vehicles (FMAVs)

To meet the distinct flight envelope described in § 1.1.3 above for a MAV, an obvious approach is to scale down conventional designs.

Lighter-than-air craft such as balloons and dirigibles offer good power-to-weight ratios since little power is required to keep them airborne. However, they have significant drawbacks in terms of manoeuvrability and size.

Ground-effect machines (GEMs) such as hovercraft are proven technology and are reasonably fast at manoeuvring. Unfortunately, they too fall short of meeting the MAV requirements because they rely on the presence of a solid boundary (such as the ground) to remain airborne and so would be very inefficient at climbing through shafts.

The only other conventional designs are winged aircraft — fixed-wing/forward thrust aircraft and rotorcraft.

Fixed-wing aircraft have significant drawbacks in that they require high speeds for adequate lift generation and use bank-to-turn manoeuvres to change direction (and hence preclude tight turns which are inevitably required in confined spaces). They are also not capable of hovering or vertical flight. Specific power requirements are also high, of the order of 150 W/kg or 15 W for a 100 g vehicle (Żbikowski *et al.*, 2003).

Rotorcraft offer better agility than fixed-wing aircraft and VTOL capability. They do, however, suffer from wall-proximity effects (due to recirculation problems) and have

significant acoustic signatures. Their power requirements are also higher than fixed-wing aircraft of the same weight, typically by a factor of about 2.

Therefore, without pre-judging the feasibility of scaling down the above conventional designs, the conclusion is that the resulting aircraft would not satisfy the flight envelope specifications for MAVs.

1.3.1 Insect flapping flight

The *proven* solution is provided in nature by insect flapping flight which has been present for over 300 million years (Dudley, 2000, p. 262). Fossil records show that the basic morphology of insects has not changed much over this period and so it can safely be surmised that their flight performance has been consistent for a very long time.

Insect flapping flight satisfies the MAV requirements remarkably well. Insects

- fly at low speeds, typically < 10 mph;
- are extremely manoeuvrable at low speeds in all directions;
- can take-off and land vertically and can even fly backwards;
- are inaudible if they flap below 20 Hz;
- have high power efficiencies, of the order of 30 W/kg (Dudley & Srygley, 1994); and
- many are capable of sustained hover.

Further, a preliminary study by Woods *et al.* (2001) compares the effectiveness of the three possibilities for low speed flight at the scale of MAVs — fixed, rotary and flapping wings (see Figure 1.8). As the figure clearly shows, a flapping-wing MAV is the most effective at the speeds that are of interest for indoor flight, especially hover.

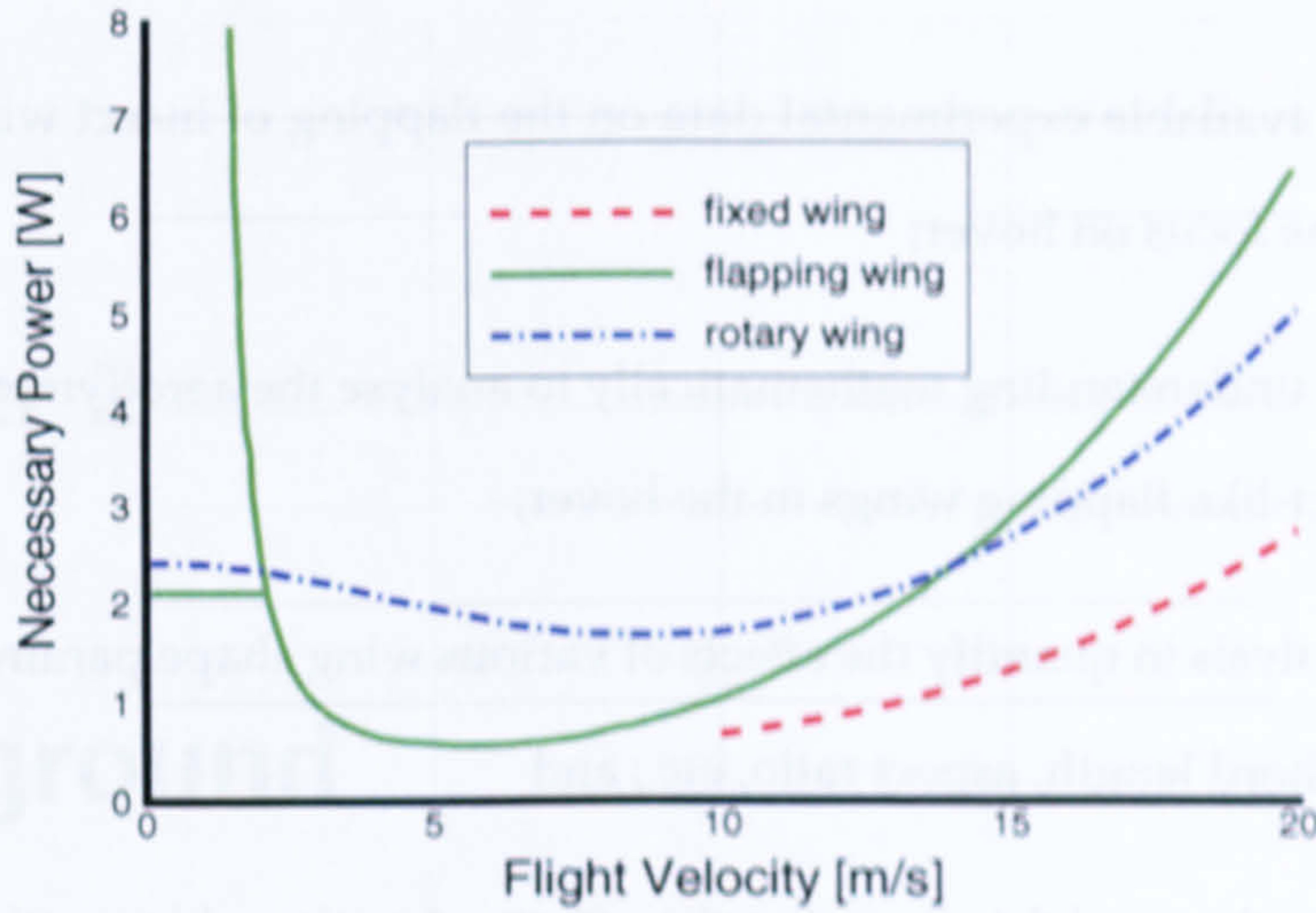


Figure 1.8: Power efficiency of flapping wings vs. fixed and rotary wings for a 50 g MAV [adapted from Woods et al. (2001)]

The discussion above clearly shows that the scales are tipped in favour of a MAV that uses insect-like flapping wings for flight. This indeed is our reasoning and forms the basis of the motivation behind this work.

1.4 Aims and Objectives

Having drawn inspiration from insects and their flight, this work aims to analyse insect-like flapping flight with a view to finding the aerodynamic effects of various wing parameters in the hover. This understanding will enable the development of an engineering model which will then be used to satisfy the wider objective of designing wings (and their kinematics) for a man-made flapping-wing micro air vehicle or FMAV.

The overall aim is to use this insight into insect-like flapping flight and its quantitative analysis to develop an engineering tool that can be used to iterate relatively quickly through various wing designs in an attempt to find the optimum wing specifications for a FMAV in the hover.

The objectives of the study can, therefore, be summarised as

- to study the available experimental data on the flapping of insect wings and their flight with the focus on hover;
- to apply this understanding mathematically to analyse the aerodynamic characteristics of insect-like flapping wings in the hover;
- to use the analysis to quantify the effects of various wing shape parameters, such as wing span, chord length, aspect ratio, etc.; and
- to further use the model to examine the effects of various kinematic parameters, such as stroke length, flapping frequency, etc.

Hover is considered mainly because it is the simplest case. The complication of including a freestream flow (in forward or climbing flight) is avoided. Further, as noted by Weis-Fogh (1972), hover is also the most costly in terms of power requirement. Therefore, a vehicle designed to hover is likely to have sufficient power for forward flight.

Chapter 2

Background

A review of literature pertaining to flapping wings is presented in this chapter. The material reviewed is broadly divided into two parts — insect flight and the associated aerodynamics.

In the first part on insect flight (§ 2.1), insect flight is introduced and the kinematics of a generic insect are presented (§ 2.1.1–2.1.3). A number of plausible unsteady aerodynamic phenomena relating to insect flapping flight are then identified, the most important being the leading-edge vortex (§ 2.1.4). This is followed by a discussion on the mechanisms that insects use for lift (§ 2.1.5) and the section concludes with a brief discourse on real insect wings (§ 2.1.6). The bulk of the material in this section is sourced from work by biologists.

The second part of the chapter (§ 2.2) is concerned with fundamentals of flapping-wing aerodynamics and is subdivided further into three — unsteady aerodynamics, delta wing aerodynamics and flapping-wing aerodynamics. The work in this section is predominantly from work by aerodynamicists. The first section reviews classical unsteady aerodynamic techniques and identifies elements that may be applicable to insect flight aerodynamics (§ 2.2.2).

Due to the similarity between the leading-edge vortex found on insect wings and

that found on slender delta wings, the second section is dedicated to a review of literature pertaining to the latter (§ 2.2.3). The problem is explained and existing methods of solution are presented. The third section summarizes aerodynamics-related work specifically relating to insect-like flapping wings (§ 2.2.4). A review of both the theoretical and experimental work is presented in this section.

2.1 Insect Flight

2.1.1 Introduction

The flight of insects has long beguiled man. During the early part of the 20th century, research into this area of science had already begun, although the focus was more on the flight of birds. Indeed, the Wright brothers studied the flight of birds when designing their first gliders (Heppenheimer, 2003, p. 89).

Hoff (1919) was probably the first to attempt to analyse insect flight. He based his explanation on previous data and tried to bring insect flight into the realm of conventional aerodynamics. His analysis was based on the forward flight of insects and did not account for the flapping motion of their wings. Demoll (1919) was quick to refute Hoff's work and proved that in certain cases, the required force coefficients were excessively high. Hoff had, however, pointed out the similarity between insect flapping flight and a spinning propeller, arguing that the analysis had to hold from momentum considerations.

Experimental work, as would be expected, had been lacking in this flight regime mainly due to the size of insects and the instruments available for analysis. Nevertheless, as early as 1934, Magnan was able to make a detailed investigation into insect flapping-wing flight. However, he still remained unable to explain his results theoretically (Magnan, 1934).

2.1.2 Quasi-Steady Aerodynamics

In 1951, Osborne explored the problem of the high lift coefficients required for the flight of insects and postulated that a special mechanism was required for insect flight that was not present in the conventional aerodynamic phenomena then known (Osborne, 1951). He used a *quasi-steady aerodynamics* approach to model insect flight — the forces on the insect wing at any point in time were assumed to be the steady-state values that would

be achieved by the wing at the same velocity and angle of attack. His investigation was based on data for 25 insects from the US National Museum and from Magnan’s earlier work (Magnan, 1934).

In principle, Osborne used the actuator disk theory of propellers to calculate the forces generated by the flapping motion of insects, as well as the mechanical power expended in the process. He also showed that, for an ideal fluid, the power required for flapping the wings was equal to the rate of change of kinetic energy passing into the slipstream. By use of a blade-element type quasi-steady analysis he concluded that in reality most insects derive more of their forces for flight from the downstroke. He also argued that in order to minimise the power required for flight, the wing tips must follow a figure-of-eight (Figure 2.1).

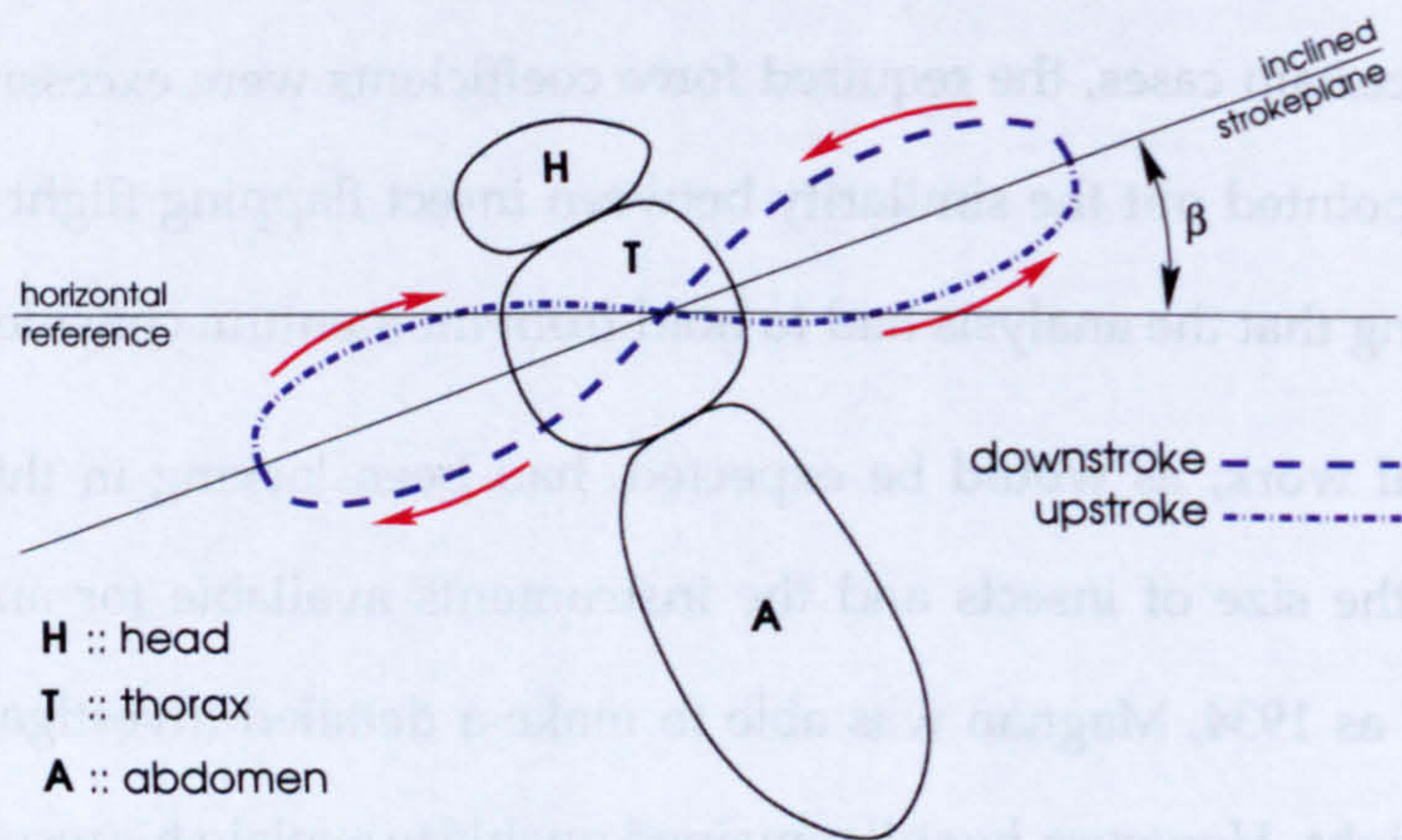


Figure 2.1: Wing tip path of an insect tracing a figure-of-eight profile

Osborne’s analysis yielded very large values for mean lift coefficient $\overline{C_L}$ and mean drag coefficient $\overline{C_D}$ ($\mathcal{O}(5)$ and $\mathcal{O}(2)$ respectively, lift and drag being defined with respect to the mean stroke plane) which were attributed to the enormous accelerations involved because of the distinct correlation he found between $\sqrt{C_L^2 + C_D^2}$ and advance ratio¹ of the insect flight. A recent CFD study by Wang (2003) has also shown that a flapping wing is likely to use both lift and drag to hover.

¹Advance ratio is the ratio of flapping velocity to forward flight velocity.

In 1956, Weis-Fogh & Jensen quantitatively analysed data on wing motion and kinematics available at the time (Weis-Fogh & Jensen, 1956). In most cases, fast forward flight was considered and the quasi-steady assumption appeared to hold for the reason that, as flight velocity increased, unsteady effects reduced in comparison to quasi-steady effects.

Magnan was probably the first to realise the necessity for non-steady effects for insect flight when he 'declared' that insect flight was impossible: 'Tout d'abord poussé par ce qui fait en aviation, j'ai appliqué aux insectes les lois de la resistance de l'air, et je suis arrivé avec M. Sainte-Lague à cette conclusion que leur vol est impossible.' (Magnan, 1934).

Weis-Fogh continued research in this area (Weis-Fogh, 1972, 1973). He devised a method for generating 'quick' estimates for the forces and coefficients on the wings of flapping animals, and found that average coefficients of lift $\overline{C_L}$ as high as 2.0 in birds and 1.3 in insects were required for hover (Weis-Fogh, 1973, Table 5). Because this method was based on a quasi-steady approach, he concluded that unsteady aerodynamic effects were responsible for the high forces required.

Dudley & Ellington (1990) found that for bumblebees, the quasi-steady estimates of lift and power requirements fail at all flight speeds. In a study by Wakeling & Ellington (1997b), the free flight of the dragonfly *Sympetrum sanguineum* and the damselfly *Calopteryx splendens* was analysed, and the required mean lift coefficient $\overline{C_L}$ was 'reverse-engineered' using a quasi-steady approach. In each case, $\overline{C_L}$ was found to exceed the maximum lift coefficient possible in steady flow C_{Lmax} .

Support for quasi-steady methods has continued to fall especially in describing the hovering flight of insects, and the need to include unsteady aerodynamic effects is essential. It should be noted that despite so much evidence in favour of unsteady aerodynamic phenomena for insect flight, the quasi-steady approach cannot be discounted altogether, especially at high forward speeds. At these speeds, the wings move longer distances and quasi-steady aerodynamics are able to account for the forces quite reasonably (Brodsky,

1991; Willmott & Ellington, 1997b; Sane & Dickinson, 2002).

Hovering is a design requirement for flapping-wing micro air vehicles (§ 1.1.3) and since it is also probably the most energy-demanding type of locomotion in animals (Weis-Fogh, 1972), the hovering mode of flight of insects is of utmost importance. For this reason, this is the mode considered in the current work (the underlying assumption being that a vehicle capable of hover would, from a power-required point of view, be capable of other flight modes).

2.1.3 The Wing Stroke

The availability of high-speed photography has enabled reasonably good descriptions of the kinematics of insect wings. The overall motion is similar to the sculling motion of the oars on a rowboat. Insects generally have either one or two pairs of wings. The description that follows is for a ‘generic’ two-winged insect.

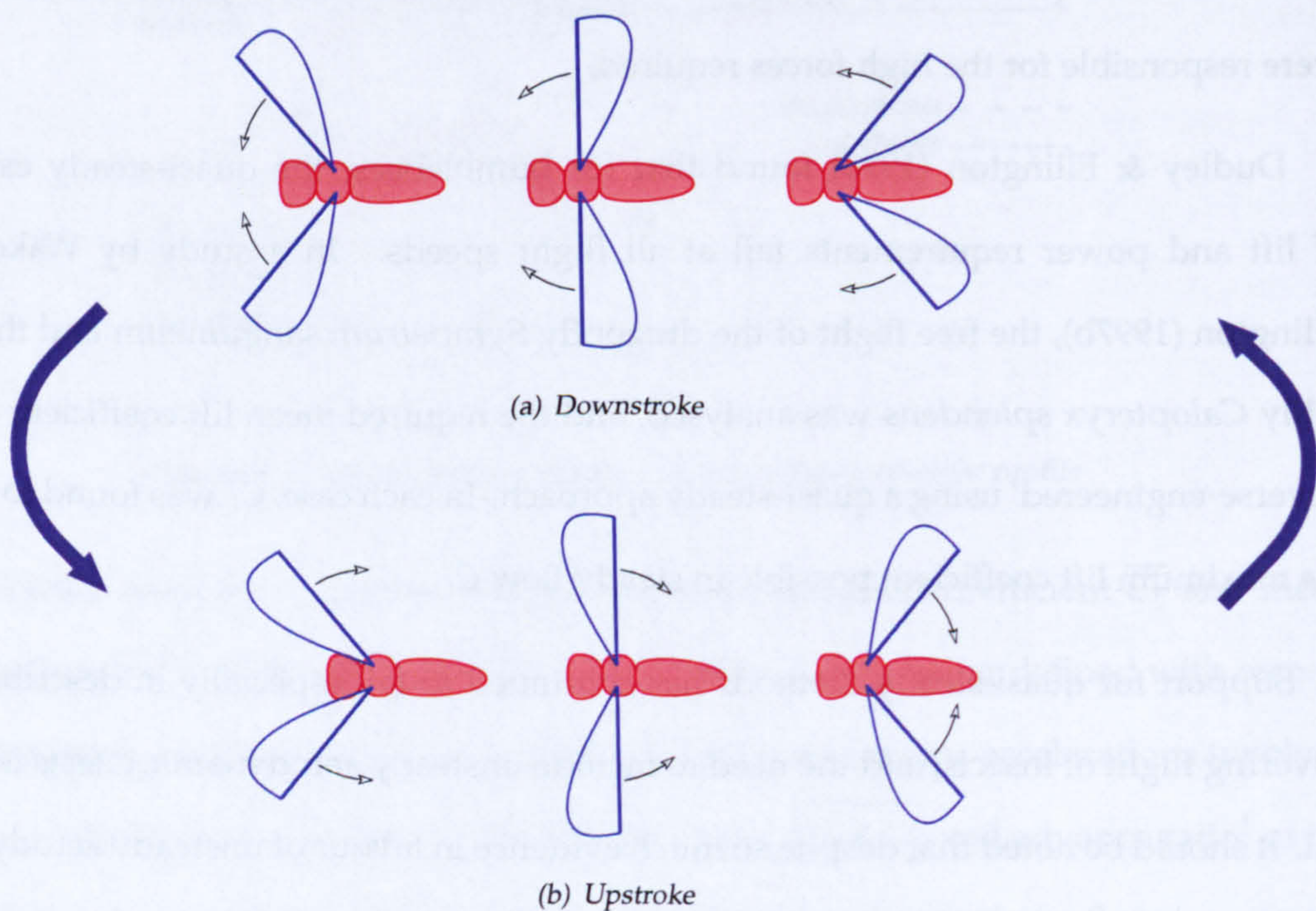


Figure 2.2: Schematic of flapping-wing halfstrokes. Note that the leading edge (thick line) always leads

The wing motion can be divided broadly into two phases — *translational* and *rotational*.

The translational phase consists of two halfstrokes — the *downstroke* and the *upstroke* (see Figure 2.2). The downstroke refers to the motion of the wing from its rearmost position to its foremost position, relative to the body. The upstroke describes the return cycle. At either end of the halfstrokes, the rotational phases come into play — the wing rotates rapidly and reverses direction. The resulting path traced out by the wing tip (relative to the body) is similar to a figure-of-eight on a spherical surface (see Figure 2.1).

The wing flaps back and forth in a roughly constant plane called the *stroke plane* (analogous to the tip-path-plane of rotorcraft). The stroke plane is inclined to the horizontal at a *stroke-plane angle* β (Figure 2.3). At any time, the wing tip may be inclined with respect to the stroke plane at an *elevation angle* θ . The angle swept by the wing during a halfstroke is the *stroke amplitude* Φ . The *stroke angle* is ϕ_{min} at the start of the upstroke and ϕ_{max} at the end of it.

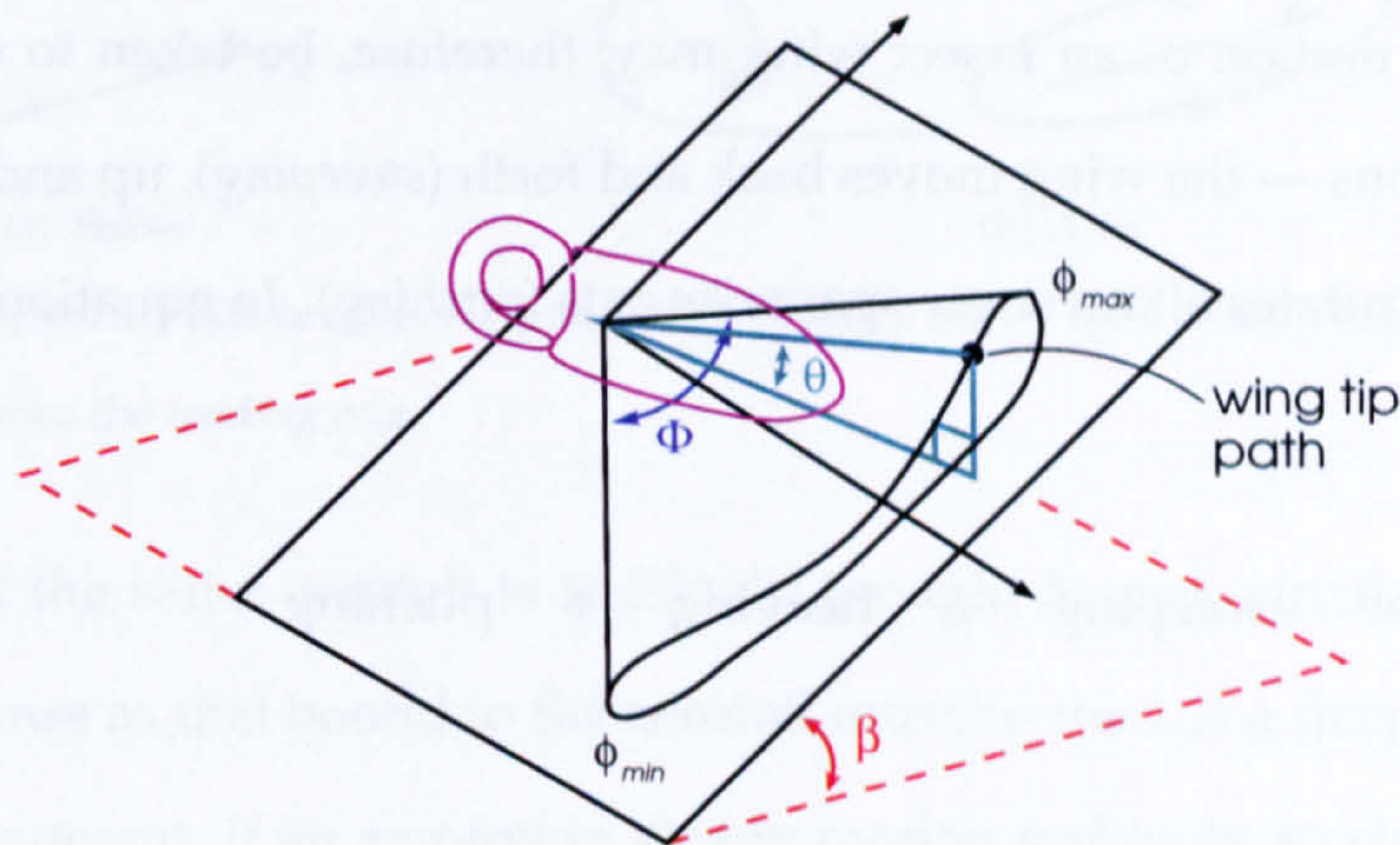


Figure 2.3: The stroke plane of an insect. Φ is stroke amplitude, θ is inclination and β is stroke-plane angle [after Willmott & Ellington (1997a)]

During a halfstroke, the wing accelerates to a roughly constant speed around the middle of the halfstroke, before slowing down to rest at the end of it. The angle of attack also changes during the halfstroke, increasing gradually as the halfstroke proceeds (Ellington, 1984c). On average, the angle of attack during a halfstroke is about 35° at the 70%-span position (ibid.).

At the end of the downstroke, *supination* occurs — the wing rapidly rotates (through an angle of about 120°), the morphological under surface of the wing becoming the upper surface — in preparation for the subsequent upstroke. This rotation ensures that the leading edge of the wing always leads. The upstroke proceeds in much the same way as the downstroke, and at the end of it, the wing again rotates to start the downstroke and repeat the cycle. The latter rotation is termed *pronation*. Both supination and pronation are *stroke reversals* and typical speeds are of the order of hundreds of radians per second. For the fruit fly *Drosophila melanogaster*, for example, estimated values lie between 3×10^5 and 5×10^5 degrees per second (Dickinson, 1994). Supination and pronation typically last about 10–20% of a cycle, and are each of approximately equal duration. For example, pronation and supination together last about 10% of the cycle for crane flies, 15% for ladybirds and hoverflies, and 20% for drone flies (Ellington, 1984c, §3.6.3a).

The flapping motion of an insect wing may, therefore, be taken to consist of three component motions — the wing moves back and forth (*sweeping*), up and down (*heaving* or *plunging*), and rotates about some spanwise axis (*pitching*). In equation form, this may be written as

$$\text{flapping} \equiv \text{sweeping} + \text{heaving} + \text{pitching} \quad (2.1)$$

2.1.4 Unsteady Aerodynamics

Before proceeding further, it is useful to consider some unsteady aerodynamic phenomena that the authors of previous work have alluded to in explaining the mechanisms for lift generation in insects and other flapping animals.

The motion of a flapping wing differs from conventional aeroplane and rotorcraft wings in that it starts from rest, accelerates to some roughly constant speed and then decelerates to rest again. Stroke reversal occurs and the cycle is repeated. The angle of attack of the wings also changes during each cycle and the overall flapping motion

suggests the presence of unsteady aerodynamic phenomena associated with this mode of flight. The extraordinarily high lift forces achieved also point in the same direction. Some basic concepts of unsteady aerodynamics are, therefore, considered here.

Starting Vortex

When an aerofoil (or wing for that matter) is started from rest, it develops circulation around itself and generates lift in much the same way as in steady flow. By Kelvin's theorem, the total circulation in a control volume made up of the same particles must remain constant (see Lamb, 1932, § 33, p. 36). Hence, if circulation is 'created' around the aerofoil, circulation of opposite sense must be shed into the wake. This is done in the form of a *starting vortex* (see Figure 2.4).

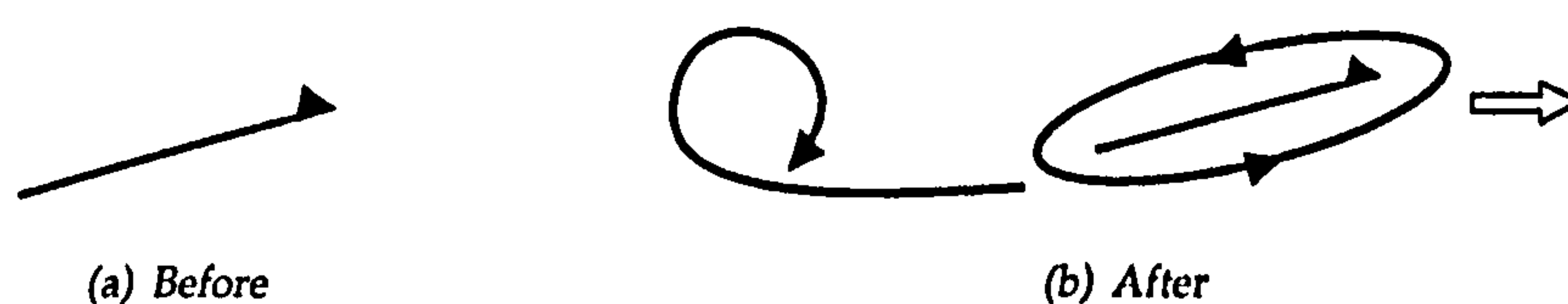


Figure 2.4: Starting vortex before and after motion. The block arrow indicates direction of motion and the solid triangle signifies the leading edge

Similarly, if the same aerofoil is suddenly brought to rest, circulation of the same strength and sense as that bound to the aerofoil must be shed as a *stopping vortex*. Again by the same argument, if an aerofoil in steady motion suddenly accelerates, circulation around it increases and equal and opposite circulation must be shed into the wake in the form of a shed vortex. Conversely, a decelerating aerofoil also sheds vorticity into the wake, equal in magnitude to the reduction in circulation and opposite in direction to the accelerating case.

It is worth noting, especially for flapping wings, the two types of vorticity associated with a wing in motion — *trailing vorticity* which is concentrated around the wing tip and trails downstream roughly perpendicular to the trailing edge of the wing, and *shed vorticity* which is shed roughly parallel to the trailing edge and also convects downstream. For a

rectangular wing, these vortices are essentially orthogonal. Trailing vorticity arises from the pressure difference between the upper and lower surfaces of the wing (tip vorticity), and shed vorticity is due to wing accelerations (or decelerations). Fundamentally, both 'types' of vorticity are needed in 3-D flow to satisfy Kelvin's (see above) and Helmholtz's (see Kuethe & Chow, 1998, p. 60) laws.

Wagner Effect

As an aerofoil accelerates from rest, it sheds a starting vortex. The circulation around the aerofoil grows towards the steady-state value over a number of chords. It reaches only about 90% of the steady-state value after about 6 chords of travel (Dickinson, 1996, p. 544). The vorticity of the starting vortex is of the opposite sense to the circulation around the aerofoil. This slows the growth of circulation around the aerofoil because the effect of the starting vortex is to change the velocity around the aerofoil so as to reduce the overall (bound) circulation around it. This is termed the *Wagner effect*, after the German aerodynamicist² who predicted it (see also Figure 2.13).

In conventional aircraft, this effect is insignificant because take-offs are several hundred chord lengths down from the initial rest position. By contrast, an insect wing starts and stops every halfstroke, and the distance travelled in each is typically of the order of about 3–5 chord lengths (see Table 2.1) so that the Wagner effect is significant.

Dynamic (or Delayed) Stall

When a moving aerofoil or wing is rapidly pitched up to an angle past the steady-state stall angle, it continues to produce greater lift momentarily before separation occurs and a catastrophic drop in lift ensues. This is a transient aerodynamic effect and is termed *dynamic* or *delayed stall* (see McCroskey, 1981). In insect flight, dynamic stall could offer a

²Herbert Wagner

plausible explanation for the high lifts that can be achieved because the insect wings only travel a few chord lengths before stopping and reversing stroke.

Apparent Mass

As a body moves through air, it encounters forces due to the acceleration of the surrounding air (von Kármán & Sears, 1938). These forces arise from the mass of air that is set in motion due to the body's movement, and hence, are called *apparent-mass*³ forces. From Theodorsen's calculations on the unsteady motion of an aerofoil the apparent-mass contribution to lift L_1 is given by

$$L_1 = \rho\pi b^2 (\ddot{h} + U_\infty \dot{\alpha} - ba\ddot{\alpha})$$

where $\ddot{h}$ is the plunging acceleration, $\dot{\alpha}$ is the pitch angular velocity, $\ddot{\alpha}$ is the pitch angular acceleration, U_∞ is (constant) freestream velocity and a is the position of the pitching axis from the mid-chord point as a fraction of b (the semi-chord length $b = c/2$) (Theodorsen, 1935). The quantity $\rho\pi b^2$ is the mass of the cylinder of air of unit length formed with the chord as diameter, and denotes the mass of air set in motion when the aerofoil moves perpendicular to the chord.

For flapping insect wings, $\ddot{h}$ and $\ddot{\alpha}$ are periodic functions with a mean value of zero. Therefore, they will have no *net* effect on the mean lift. On the basis of the symmetry of the wing kinematics, Ellington argued that the contribution due to the $V\dot{\alpha}$ term must also be small, if not zero (Ellington, 1984e, §4.4). Hence, the effect of the wing's apparent mass on *mean* lift is negligible. A similar effect is noted for drag. However, work must be done to accelerate and decelerate the apparent mass during each cycle, and this results in power expenditure that must be accounted for.

³Also known as *added mass*.

Wake Capture

Wake capture occurs when a wing translates back into its own wake (Figure 2.5). Dickinson is probably the most active proponent of this mechanism of unsteady lift generation and postulates his theory on the basis of his experiments (Dickinson, 1994; Dickinson *et al.*, 1999; Sane & Dickinson, 2001).

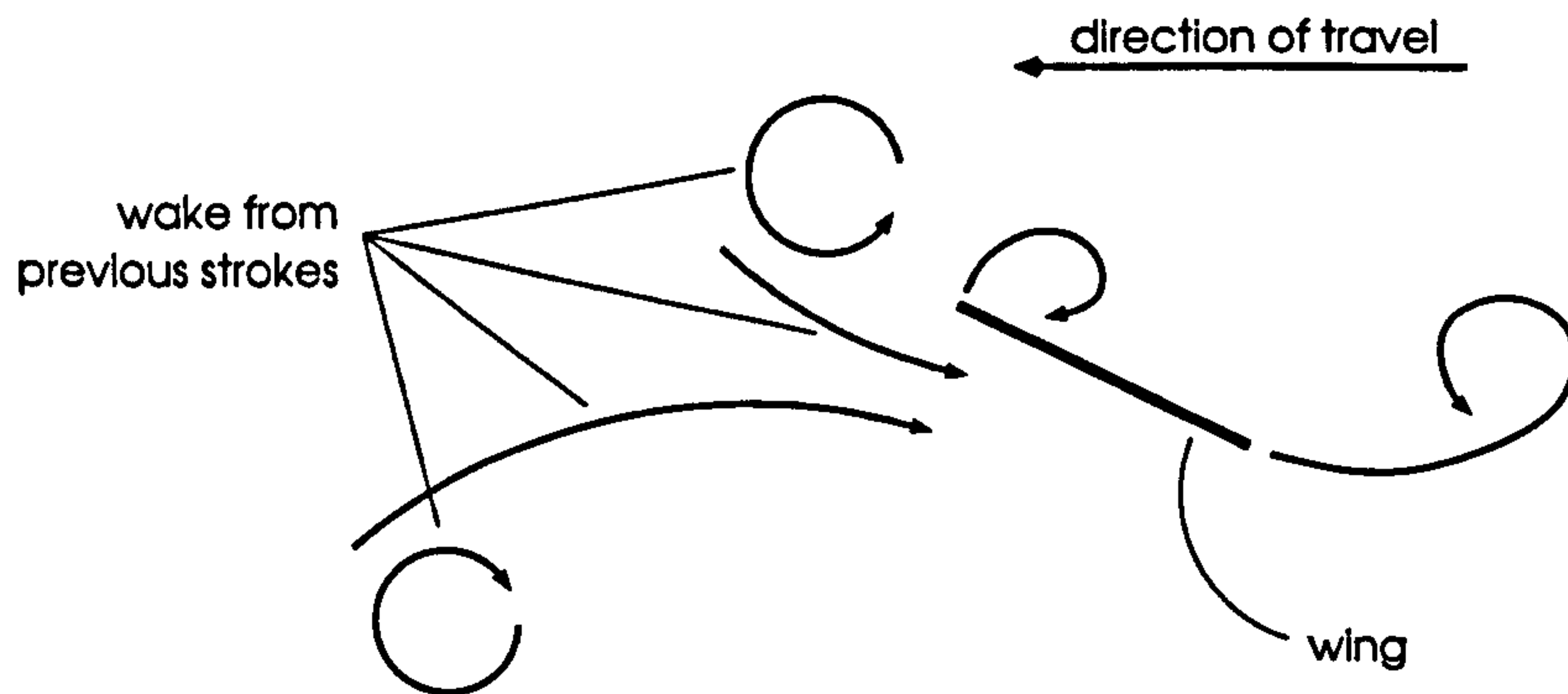


Figure 2.5: Wake capture showing the wing moving back into its own shed wake from previous strokes [after Dickinson (2001)]

In his experiments on 2-D wings, Dickinson used an aluminium wing section immersed in a sucrose solution and measured the lift and drag forces on the wing (Dickinson, 1994). His experiments were intended to match the motion of the wings of the fruit fly *Drosophila*, with a Reynolds number Re (based on chord) of 236. The 2-D wing was moved in a downstroke direction, rotated rapidly and then moved back through its previous wake. Large forces were detected and transient lift coefficients approaching 4 were achieved. Neither translational nor rotational mechanisms could explain this behaviour.

In a later experiment, Dickinson *et al.* (1999) modelled the 3-D motion of scaled-up replicas of the wings of *Drosophila*. Once again, substantial forces were reported that could not be accounted for by either translational or rotational mechanisms. With the help of flow visualisation, these were shown to be due to wake capture. A further experiment was carried out to test this theory. After translation, the wings were impulsively stopped, and it was found that the wings continued to produce forces for some time. This was

because the shed wake flowed over the wing owing to its own momentum, inducing circulation around the wing and, hence, producing lift.

The wake behind any flying object is energy lost to the surroundings in the form of momentum and heat. Wake capture is, therefore, a method to recover some of this lost energy and utilise it usefully for flight. The importance of wake capture was also suggested by Grodnitsky & Morozov (1993) who proposed that insects and birds have special mechanisms whereby they extract energy back from their near vortex wake. A similar view was expressed by Ennos who speculated that in flies, the kinematics were helped by the aerodynamics (Ennos, 1989a).

2.1.5 Lift Mechanisms

It has now been established that insects exploit the aforementioned unsteady aerodynamic phenomena to derive their high-lift characteristics. The process whereby they do this are termed the *lift mechanisms* and this section discusses some of them.

A number of lift mechanisms are observed in insect flapping flight and can be broadly classified into two categories:

Translational where the generation of forces is primarily due to the sweeping motion of the wings, and

Rotational where pitching or rotational motions dominate force-production, especially at stroke reversals.

Translational Mechanisms

The translational phase of the wingbeat is what first 'legitimised' use of quasi-steady models for analysing the flapping flight of insects and birds. In comparison to wing rotation (pitching), it is temporally the longer phase of the wingbeat lasting about 80% of

the cycle (Ellington, 1984c, p. 68).

The short distances travelled by the wing during a halfstroke, and the rapid accelerations and decelerations involved strongly suggest the presence of unsteady effects. Table 2.1 below shows mean distances travelled by wings during a halfstroke for some insects. The phenomenon of *dynamic stall* would, therefore, appear to be quite applicable in this case. The ‘delay’ before the separated flow becomes well-developed could be exploited to temporarily produce greater lift than would be possible in steady flow while the short wingstrokes would ensure that the lift-breakdown state was not achieved.

Table 2.1: Mean distance travelled during halfstrokes in non-dimensionalised mean chords ($= \frac{1}{4}\Phi R$) [adapted from Ellington (1984d, Table 1)]

horizontal stroke plane		inclined stroke plane	
<i>Apis mellifera</i>	3.8	<i>Aeschna juncea</i>	2.5
<i>Bombus hortorum</i>	3.4	<i>Episyrphus balteatus</i>	2.4
<i>Eristalis tenax</i>	3.3		
<i>Tipula paludosa</i>	5.9		

The impulsive start of aerofoils at low Reynolds numbers was investigated by Dickinson & Götz. They tested at $Re \sim 75\text{--}230$ (based on mean chord) — the realm of the chalcid wasp *Encarsia formosa* (Dickinson & Götz, 1993). A trace of their flow-visualisation results is shown in Figure 2.6. About one chord length after the impulsive start, a leading-edge bubble forms, rapidly increasing in size and circulation (and hence, lift) around the aerofoil, (Figure 2.6(a)). A starting vortex is shed at the same time. The bubble continues to grow in size and in accordance with Kelvin’s law, a second starting vortex is generated at the trailing edge after about 3–4 chords of travel (Figure 2.6(b)). The leading-edge vortex eventually breaks away and is shed into the wake (Figure 2.6(c)).

The process repeats and a second smaller lift peak is achieved after about 5–6 chords of travel (Ellington, 1995). Although Dickinson & Götz’s experiment ended after 7.5 chords

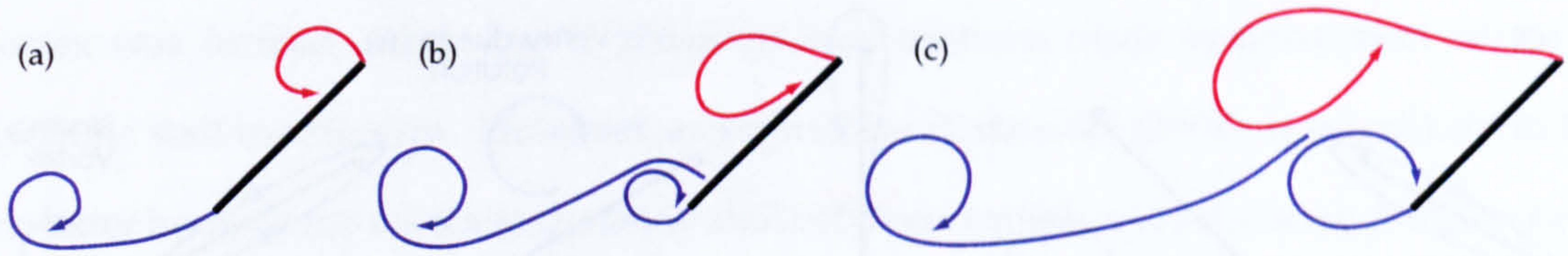


Figure 2.6: Impulsive start of an aerofoil at low Re and high angle of attack showing separation from leading (red) and trailing (green) edges. [adapted from Dickinson (1996, Figure 3)]

of travel, a long chain of alternating vortices — a von Kármán street⁴ — was shed and the lift on the aerofoil oscillated. Delayed stall could be *the* mechanism for insects that have short stroke amplitudes, such as dragonflies and hoverflies, because stall is not reached during their brief wingbeats (Ellington, 1995). However, as will be seen later, evidence opposing delayed stall as a lift-augmentation mechanism is very convincing.

The Wagner effect also comes into play during the translational phase of the wingbeat. It is not a lifting mechanism as such but is highly influential, especially since it delays the growth of circulation (and hence, lift) in subsequent motion. Dickinson (1996) described a more acute form of the phenomenon — the so-called *cumulative Wagner effect* (see Figure 2.7). At the end of a halfstroke (Figure 2.7(a)), a stopping vortex is shed (Figure 2.7(b)). The wing then rapidly rotates in preparation for the subsequent halfstroke and generates circulation around itself (Figure 2.7(c)), shedding vorticity into the wake in accordance with Kelvin’s law (Figure 2.7(d)) of the same sense as the stopping vortex of the previous stroke. At the start of the next halfstroke, a starting vortex is shed (Figure 2.7(e)), and this is again of the same sense as the previous stopping vortex. These three vortices may coalesce, giving a cumulative Wagner effect during subsequent translation and delay the growth of lift. This effect will, however, depend upon the pause between the halfstrokes, the speed of rotation and the axis of rotation during stroke reversal (Dickinson, 1994).

Dickinson & Götz (1996) argued that the cumulative Wagner effect might be so influ-

⁴The terminology *Bénard-von Kármán* vortex street also exists instead of the more common *von Kármán* street in recognition of the historical precedence (1910) of the papers of French physicist Henri Bénard on the subject (Guyon *et al.*, 2001).

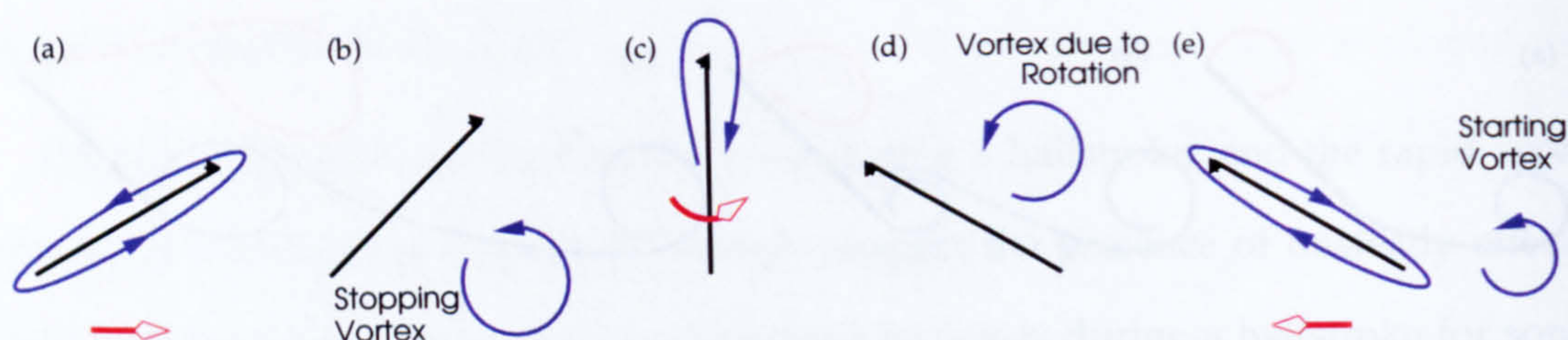


Figure 2.7: Cumulative Wagner effect. Each shed vortex is shown only once for clarity [after Dickinson (1994, Figure 3a)]

ential to circulation-based lift production in some cases that it would be more profitable to have only one lift-producing stroke. The other halfstroke would serve as a recovery stroke, helping to distance the wing from the large inhibitive vortex (or vortices). As will be seen later, the presence of a 'mirror' wing close by (as in a clap and fling, see later) helps to reduce this effect (the shed vortex of one wing adds to the bound circulation of the other due to symmetry).

Leading-Edge Vortex (LEV) Perhaps the most common (and certainly very important) lift-augmentation phenomenon occurring during the translational phase of the wingbeat of an insect is the *leading-edge vortex (LEV)* on the upper surface of the wing. Its presence was first observed in the 1970s (e.g. Martin & Carpenter (1977); Maxworthy (1979)) but it was not until recently that it received proper recognition as a plausible mechanism for the enhanced lift in insects (e.g. Brodsky, 1991; Dickinson & Götz, 1993; Sunada *et al.*, 1993; Ellington *et al.*, 1996).

The leading-edge vortex appears at the start of the downstroke and persists till the end. It has a three-dimensional structure, spiralling near-conically from the base of the wing to the tip. This spanwise transport of vorticity was first observed by Maxworthy (1979). At the wingtip, it coalesces with the tip vortex and trails into the wake. The swirl and spanwise velocities of the vortex add to the existing high flow velocity over the wing, further reducing pressure there and enhancing lift. At the end of the halfstroke, the vortex is shed as the wing rapidly rotates. Willmott *et al.* (1997) speculated that the leading-edge

vortex was formed either due to the rotational motions prior to translation or via a dynamic stall mechanism. However, as argued by Żbikowski (2002), it is unlikely to be the latter because the vortex in dynamic stall convects rapidly over the wing (McCroskey, 1981); it is not bound as is the case for the leading-edge vortex observed in insect flight.

Nature of the Leading-Edge Vortex In many insects, especially larger ones like the hawkmoth *Manduca sexta*, the leading-edge vortex is the mechanism largely responsible for lift augmentation. Ellington designed a 10-scale model of the hawkmoth — the ‘flapper’ — which was subsequently used to study the nature of the leading-edge vortex, taking care to match both Reynolds number⁵ Re and Strouhal number⁶ St (Ellington *et al.*, 1996).

The investigation showed that the leading-edge vortex was a conical spiral, enlarging as it moved along the wing. Both swirl and axial (spanwise) velocities were large and comparable to the mean wing tip velocity. Vortex breakdown was observed in the distal (outboard) regions of the wing (beyond the 60%-span position) and the vortex appeared to lift off the wing surface at about 87%-span. By this point, however, the tip vortex had joined in, keeping the leading-edge vortex more or less attached. The mean lift produced during the downstroke was found to be equivalent to about 1.5 times the weight of the hawkmoth — proof that it was a lift-augmentation mechanism.

In a later study by Willmott *et al.* (1997) but this time with a real hawkmoth, a similar leading-edge vortex was observed whose size increased with increasing forward speed. The vortex was also found to be much more stable than in a 2-D dynamic stall, where the vortex breaks away as soon as the wing translates (McCroskey, 1981; Dickinson & Götz, 1993). This stability was attributed to the spanwise flow which was probably pressure-driven (due to the radial velocity gradient along the wing).

Van den Berg & Ellington (1997a) suggested that a similarity existed between the

⁵ $Re = \rho V l / \mu$, where ρ is fluid density, l is wing span, V is a characteristic velocity, and μ is fluid viscosity.

⁶ $St = \omega l / V$, where ω is the flapping frequency, l is wing span, and V is a characteristic velocity.

leading-edge vortex observed on insect wings and that found on slender delta wings, which would suggest the existence of similar vortex-stabilising mechanisms. In view of this, Birch & Dickinson (2001) suggested that spanwise transport of the leading-edge vorticity served to remove energy from it, and hence, limited its growth and shedding (see also Rossow (1978); Riddle *et al.* (1999)).

Whereas the leading-edge vortex may be common to most insects (Brodsky, 1991; Grodnitsky & Morozov, 1993; Willmott *et al.*, 1997; Birch & Dickinson, 2001), its spanwise flow characteristic is not. Recent experiments by Birch & Dickinson (2001) and Sane & Dickinson (2001) on a dynamically-scaled mechanical model of the tiny fruit fly *Drosophila melanogaster* have revealed that despite the presence of a strong leading-edge vortex, no spanwise spiralling was observed (unlike for the hawkmoth above). Their experiments were at very low Reynolds number ($Re < 300$), suggesting that the characteristics of the leading-edge vortex in insects are Reynolds number-dependent.

Rotational Mechanisms

The rotational mechanisms in insect flight take place during the stroke reversals at either end of the halfstrokes. Since more of the forces are usually generated during the downstroke, rotational mechanisms are especially useful during pronation. The rotational motions are usually of much shorter duration compared to the translational motions, typically 10–15% of a complete cycle (Ellington, 1984c). A number of mechanisms have been proposed, but due to the vast diversity found in nature no general mechanism covers all insects. Nevertheless, inspiration for FMAVs is drawn from each of these postulations and given below are some of the more relevant mechanisms.

Clap and Fling The *clap and fling* is one of the novel mechanisms suggested by Weis-Fogh for the augmented lift found in insects (Weis-Fogh, 1973, pp. 213–218). He proposed the mechanism for the tiny chalcid wasp *Encarsia formosa* (Figure 2.8). Since its introduction,

the clap-and-fling mechanism has been the subject of numerous theoretical analyses (e.g. Lighthill, 1973; Edwards & Cheng, 1982) and experimental investigations (e.g. Bennett, 1977; Maxworthy, 1979; Spedding & Maxworthy, 1986; Ennos, 1989b). It is generally more common in small insects.

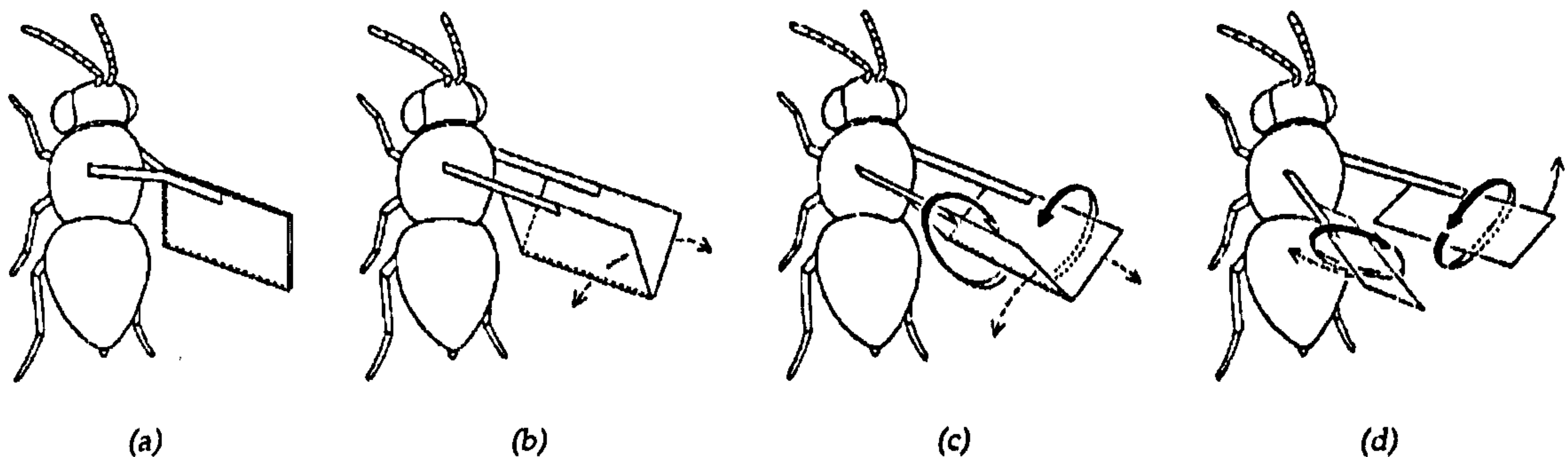


Figure 2.8: Clap and fling mechanism [adapted from Ellington (1995, Figure3)]

In this mechanism, the insect claps its wings dorsally (above the body) so that they touch as in Figure 2.8(a), their longitudinal (spanwise) axes being horizontal. The wings fling open about their trailing edges, Figure 2.8(b). As the fling continues, air rushes into the V-shaped gap, creating circulation of opposite sense on each wing, Figure 2.8(c). This fling gives the wings circulation (and hence, lift) even before the wings translate. As the wing rotation proceeds, the wings also pull apart from each other, Figure 2.8(d) — this is the start of translation (see also Figure 2.9).

As translation begins, circulation is created around the wings in the form of a bound vortex, adding to the circulation from the clap and fling. By Kelvin's law, a starting vortex of the opposite sense must be shed. Due to the symmetry of the wings, the shed vortex of one wing forms part of the bound vortex of the other so that no starting vortices are left behind. In this way, the clap-and-fling mechanism offers a method for overcoming the Wagner effect so that lift may grow more rapidly.

The 2-D nature of the vortices generated during a fling, however, renders them unstable and promotes their shedding in subsequent translation. Maxworthy (1979) used a 3-D mechanical flapping model for his experiments and found that there was a strong

spanwise transport of vorticity towards the wing tip which tended to stabilise the vortices, in much the same way as the leading-edge vortices on a highly swept delta wing. This was one of the first indications of the leading-edge vortex. Ennos suggested that by varying the extent of the clap and fling, an insect is able to alter the direction of force, and hence, manoeuvre quickly (Ennos, 1989b, p. 79).

Clap and Peel Ellington noted a variation to Weis-Fogh’s clap-and-fling mechanism in *Lepidoptera* (butterflies and moths) and the fruit fly *Drosophila* — he observed that the wings *peel* open like two pieces of paper as opposed to a fling (Ellington, 1984c, § 3.7.2). This is the *clap-and-peel* mechanism (see Figure 2.9).

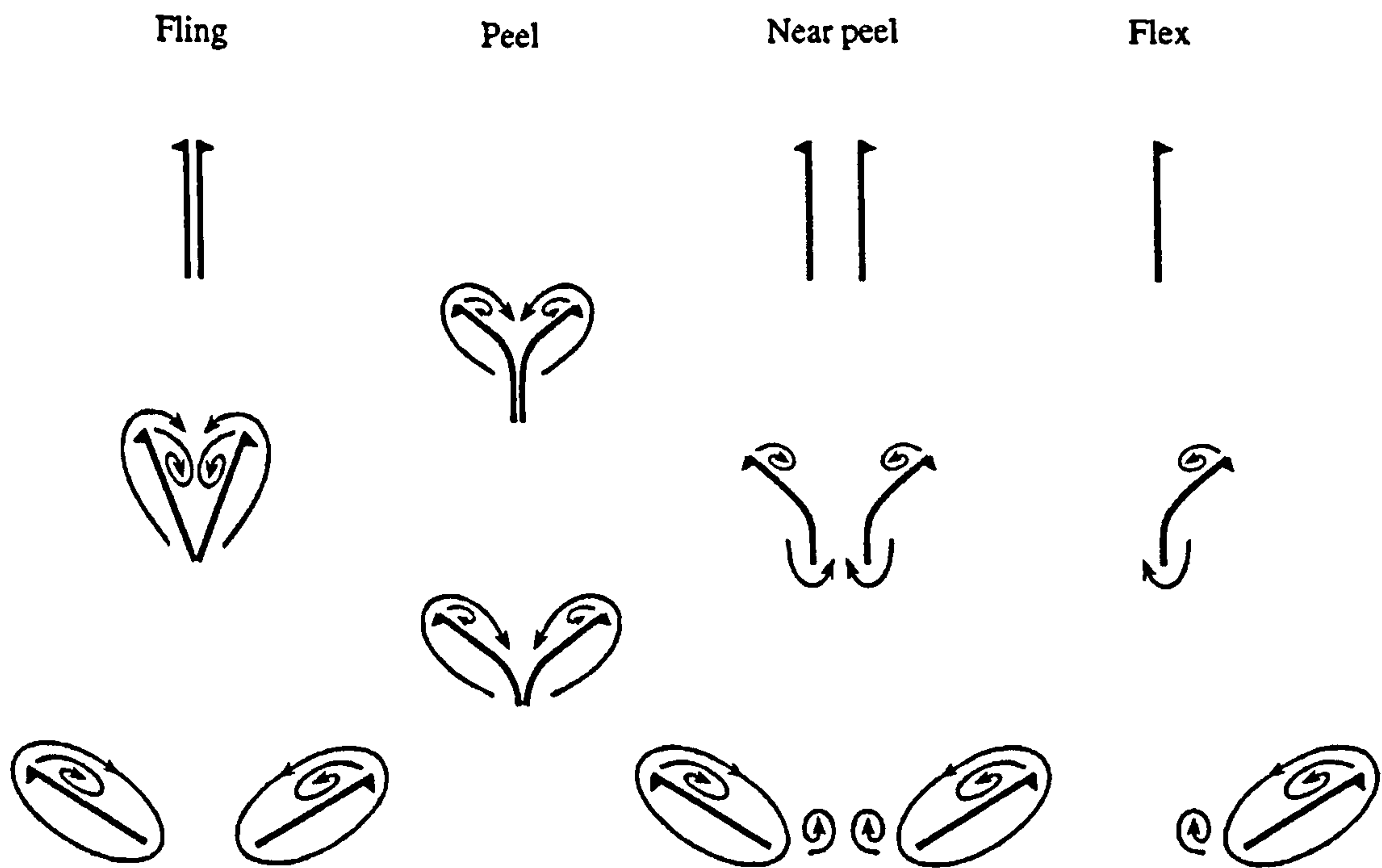


Figure 2.9: Variations of rotational mechanisms. The solid triangle signifies the leading edge [from Ellington (1995, Figure 4, p. 116)]

As can be seen from the figure, the two wings start off curved along their chords and unzip rapidly from leading edge to trailing edge. Ellington (1984d) showed that the peel created 2.6 times more lift than a fling of the same duration, noting that his estimate of

circulation tied in very well with the experimental values observed by Maxworthy (1979).

Near Fling and Peel Despite the advantages the clap-and-fling and the clap-and-peel mechanisms have to offer, the majority of insects do not use this method mainly due to the physical wear and tear associated with the repeated clapping of the wings. Instead, a *near fling* or *near peel* is more common (Figure 2.9). The near fling or peel is less effective than a complete fling or peel because air is 'lost' from the trailing edges which in turn tends to reduce the shear past the leading edges, and hence, circulation (Ellington, 1995).

Clap Before the execution of a clap and fling or a clap and peel, the wings of the insect are brought together in a *clap*, for example, in butterflies. The clap is essentially the opposite of a peel — the leading edges of the wing come together and the wings 'zip' all the way down to the trailing edge, closing the gap. Bocharova-Messner & Aksyuk (1981) suggested that the downward-directed jet from the 'tunnel' produced by this clap of the two wings was of importance to the flight of butterflies. The jet produced an upward force that enhanced the lifting capabilities of the wings.

Ellington analysed the clap in a similar way to the peel for *Encarsia* (Ellington, 1984d, §6.3) and found that the mean force from the jet produced due to the clap of the wings was almost equal to that required for weight support.

Isolated Rotation If the distance between the wings for a near fling or peel is increased to the point that the interference effect of one wing on the other is insignificant, the rotational mechanism is an *isolated rotation*. This is the most widely-occurring method employed by insects. It is more common in supination, although it is also observed in pronation (Ellington, 1995, p. 117). The idea of an isolated rotation was first proposed by Weis-Fogh as his second novel mechanism for insect flight (Weis-Fogh, 1973, pp. 218–222) where he described it for the hoverfly *Syrphinae* and called it a *flip*.

Ellington later rejected this as a plausible mechanism (Ellington, 1984d, §6.4), arguing that the vortex (and hence, lift) created during the first part of the flip would be cancelled out by the reverse action in the latter part, and hence, produce zero net lift. Instead, he described a *flex* mechanism for isolated rotation (Figure 2.9).

If the wing were to rotate about the trailing edge, then isolated rotation would be similar to the near fling or peel except for the separation distance between the wings. In reality, however, wings rotate about a spanwise axis that is roughly at the quarter-chord point (Ellington, 1984c). If the wing moved as a rigid flat plate, the trailing edge would move faster than the leading edge and produce a net downward force. Ellington instead suggested that the trailing edge of the wing momentarily comes to rest and the leading edge peels the wing away into the subsequent halfstroke. The wing resumes motion once the flexion action reaches the trailing edge.

Dickinson *et al.* (1999) conducted a series of experiments in which the effect of the axis of rotation on the total rotational circulation was measured and found that circulation decreased as the axis of rotation was moved towards the trailing edge, changing sign (i.e. becoming negative) around the $\frac{3}{4}$ -chord point.

Before proceeding, it should be mentioned that all of the above considerations have been predominantly qualitative (and not quantitative), owing to the fact that most work on insect flight is from the biological community.

2.1.6 Insect Wings

A project concerned with wing-design guidelines for flapping wings would be incomplete without some discussion of existing flapping wings — insect wings in nature. This section outlines some of the aerodynamic and aeroelastic properties that have been observed in such wings.

A dragonfly wing structure and material is typical of most insects, and is shown in Figure 2.10 below. The wing has an encircling marginal vein that runs along the periphery of the wing, giving it mechanical strength and stiffening, and a complex network of smaller veins that stretch amongst themselves a thin membrane (Kesel, 2001).

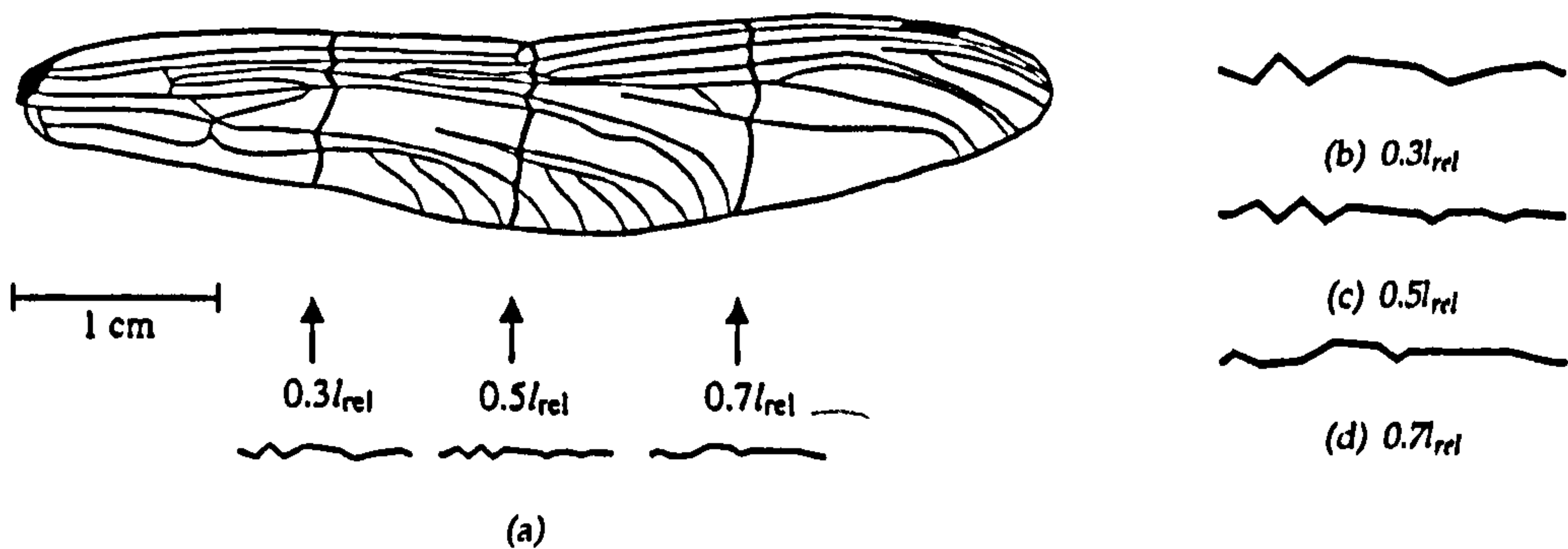


Figure 2.10: Profile of a dragonfly wing, wing base being at the shaded left-most point (a). Cross-sections of the wing are shown for three spanwise positions (b), (c) and (d), where l_{rel} is wing length [adapted from Kesel (2000, Figures 1 & 2, p. 3126)]

As sub-figures (b), (c) and (d) in Figure 2.10 show, a cross-section through the wing reveals a corrugated profile. Negative pressures are found in all the profile valleys, irrespective of side (Kesel, 2000). Kesel found that a cambered aerofoil achieved a higher lift-to-drag ratio than a corrugated wing but argued that the latter was preferred because a cambered one alone would collapse under the loads experienced in flapping flight. Upon closer inspection, the leading edge of dragonfly wings were found to have sharp, serrated and triangular leading-edge veins (Kesel, 2001). These generate turbulence in the surrounding flow and increase lift by delaying laminar separation (Okamoto *et al.*, 1996).

A comprehensive study by Okamoto *et al.* (1996) on dragonfly wings at low Reynolds numbers ($Re \sim 10^3-10^4$) investigated the effect on lift and drag of camber, thickness, sharpness of the leading edge and surface roughness of the wings. They found that thinner wings gave higher maximum lift coefficients C_{Lmax} , lower drag coefficients C_D and slightly smaller lift-curve slopes ($dC_L/d\alpha$). A positive camber (upward convex) was found to increase each of C_{Lmax} , minimum drag coefficient C_{D0} and $dC_L/d\alpha$. A negative

camber, however, only increased C_{D_0} and $dC_L/d\alpha$. A triangular camber ('kinked' aerofoil) was found to deteriorate in aerodynamic characteristics as the kink was moved aft. The effect of increasing leading-edge sharpness was to markedly increase C_{Lmax} while C_{D_0} and $dC_L/d\alpha$ remained almost unaltered. Increasing surface roughness was also found to increase C_{Lmax} and maximum lift-to-drag ratio $(L/D)_{max}$.

Another important wing property is its moment of area. Weis-Fogh (1973) showed that the mean lift by quasi-steady analysis was proportional to the second moment of wing area and that profile power was proportional to third moment of wing area⁷. Sunada *et al.* (1998) investigated the effect of wing corrugation on its structural properties and found that corrugation increased the second moment of area and that the lowest natural frequency of torsional deformation was over two times greater than the wing beat frequency. Hence, resonance is not an issue in flapping-wing flight.

Norberg (1972) found that the centre of mass of insect wings was aft of the torsional axis. He also found that the centre of pressure for insect wings was generally behind the torsional axis too. To reduce the energy expenditure in accelerating the wing and high bending moments, more of the wing mass is concentrated near the wing base. In this way, the outboard portions of the wing are lighter, and reduce stress at the wing base (Ennos, 1989a).

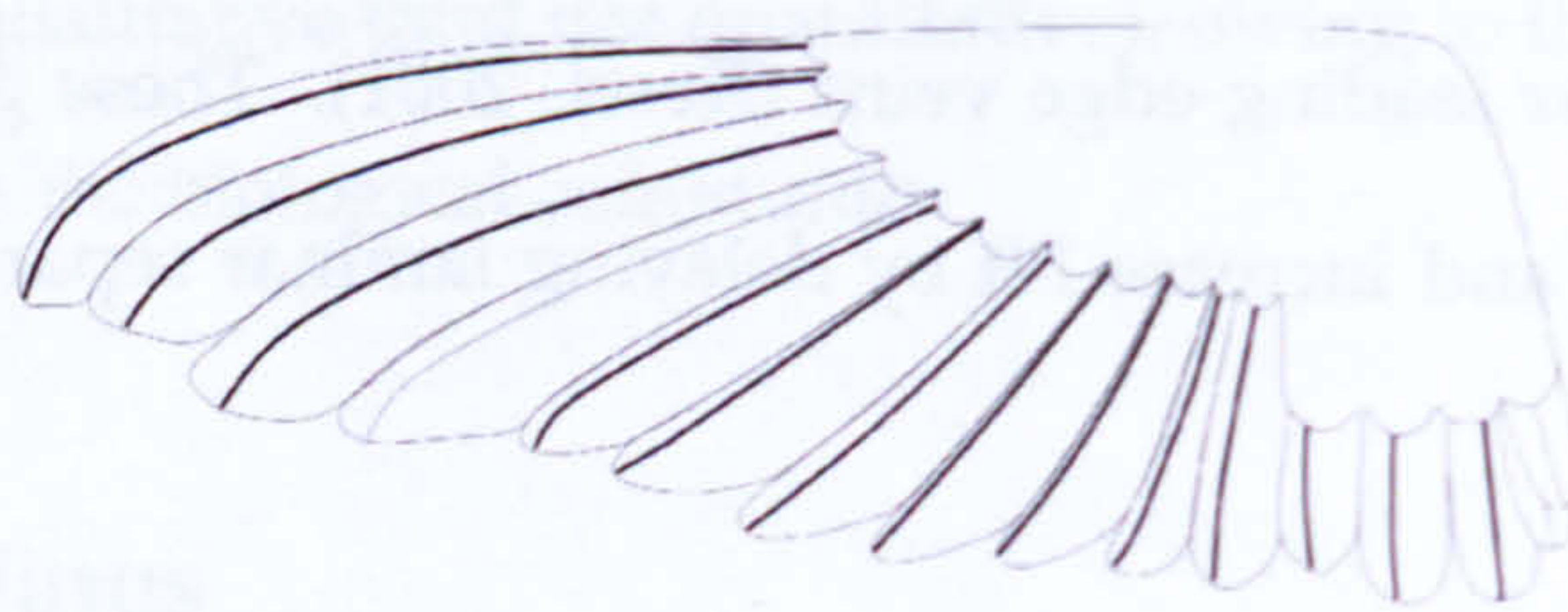


Figure 2.11: Hummingbird wing [adapted from Chai (1997, fig. 1)]

The wing of a hummingbird⁸ (Figure 2.11) is currently being studied at Cambridge

⁷The n th moment of wing area is given by $S_n = 2 \int_0^R c_r r^n dr$ where c_r is the wing chord at spanwise position r (Ellington, 1984b, §3.1, Eqn. 6).

⁸The hummingbird is considered an 'honorary insect' from an aerodynamic perspective.

University in an experimental project (Nolan, 2004). The presence of 3–5 spanwise-running ‘tubes’ on the wing surface has been observed (Ellington, priv. comm.), probably to introduce turbulence into the flow. The presence of turbulence in the flow (at $Re \sim 11000\text{--}15000$) has been found to increase the lift coefficient and reduce drag coefficient (Okamoto *et al.*, 1996).

Insect wings essentially move as shown schematically in Figure 2.12. The so-called tergal plate (upward convex) moves up and down (contraction and extension respectively) within the thorax of the insect by means of the vertical tergo-sternal muscles (contraction) and the longitudinal dorsal muscles (extension) respectively, and this in turn flaps the wings (Snodgrass, 1993, pp. 233-234). The wing angle of attack and warping is most probably controlled from the wing base (Żbikowski *et al.*, 2000). In insects, the wing is twisted along its length with the wing base typically at an angle of attack $10\text{--}20^\circ$ higher than at the tip (Ellington, 1984c, p. 42).

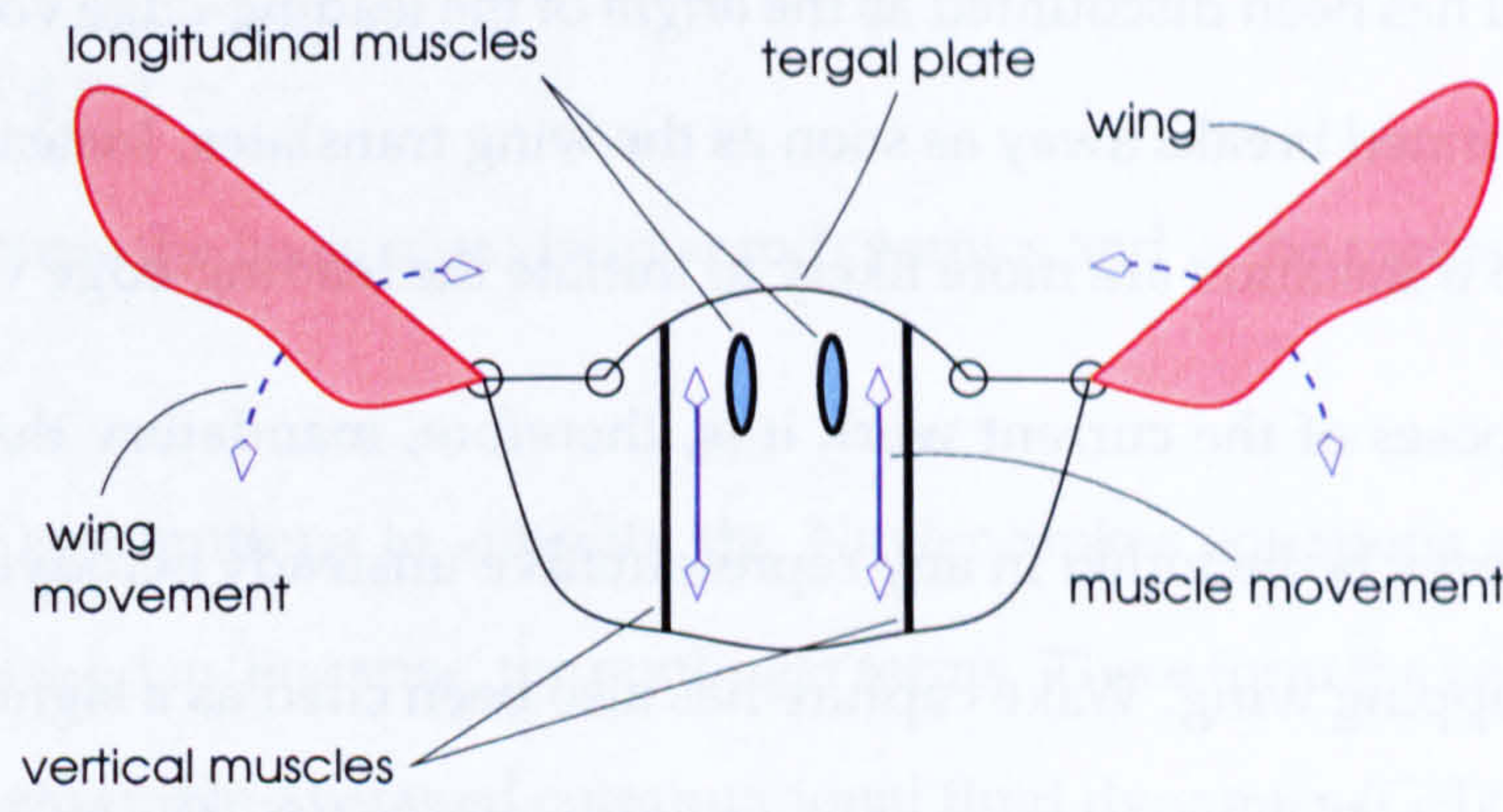


Figure 2.12: Schematic of insect wing movement [adapted from Snodgrass (1993, fig. 131)]

A good qualitative discourse on insects, their wings and their flight can be found in Dudley (1992) and Snodgrass (1993). A good pictorial overview is given by Brackenbury (1995).

2.1.7 Summary

From the discussion thus far it can clearly be seen that unsteady aerodynamic effects are responsible for the large forces generated by insects during flight. These forces originate from the flapping motion of the wings and are attributed to various phenomena. Of the mechanisms proposed, the most prominent is the leading-edge vortex.

The leading-edge vortex is a swirling structure extending from the root to the tip of a flapping wing, remaining attached through most of the halfstroke and shedding at stroke reversal. Its swirl component of velocity adds to the velocity of chordwise flow over the wing's upper surface, further reducing pressure there and augmenting lift. At Reynolds numbers pertaining to hawkmoth flight, spanwise transport of vorticity in the leading-edge vortex is observed, but at lower Reynolds numbers (e.g. for a fruit fly), little or no spanwise motion is detected.

Delayed stall has been discounted as the *origin* of the leading-edge vortex because the vortex thus generated breaks away as soon as the wing translates. Instead, the rotational motions prior to translation are more likely to initiate the leading-edge vortex.

For the purposes of the current work it is, therefore, mandatory that the effect of a leading-edge vortex be included in any representative unsteady aerodynamic model for an insect-like flapping wing. Wake capture has also been cited as a significant effect and should be included. Because power requirement is greatest for the hovering mode, it is essential that this mode of flapping-wing flight be considered in the design of a FMAV.

2.2 Fundamentals of Flapping Flight Aerodynamics

2.2.1 Introduction

All fluid motion is governed by the so-called *Navier-Stokes equations*⁹ which for Newtonian fluids are given by

$$\rho \left(\frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} \cdot \nabla \mathbf{q} \right) = \rho \mathbf{f} - \nabla p + \mu \nabla^2 \mathbf{q} + \frac{\mu}{3} \nabla (\nabla \cdot \mathbf{q}) \quad (2.2)$$

where $\mathbf{q}$ is fluid velocity, ρ is fluid density, p is pressure, $\mathbf{f}$ is body force, μ is viscosity and ∇ is the gradient operator¹⁰. The number of exact solutions to the Navier-Stokes equations is small due to the nonlinear nature of the differential equations involved. However, in many cases some assumptions can be made that simplify the equations and ease solution.

One such case occurs when the flow is assumed to be incompressible and inviscid, so that Equation 2.2 reduces to the *Euler equation*¹¹

$$\frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} \cdot \nabla \mathbf{q} = \mathbf{f} - \frac{\nabla p}{\rho} \quad (2.3)$$

This equation forms the basis of inviscid aerodynamics and is the mainstay of potential-flow methods.

Where such assumptions to simplify the Navier-Stokes equations are not possible, empirical fits are used to 'linearise' the nonlinear terms. These form the basis of turbulence models used in ensemble-averaged computational fluid dynamics (CFD).

For solving flow problems, the equations above are used together with the *continuity equation*, which for incompressible flow is given by

$$\nabla \cdot \mathbf{q} = 0 \quad (2.4)$$

⁹After Claude-Louis Navier (1785–1836) and George Stokes (1819–1903) who arrived at them independently. Historically, they were also known as the *Navier-Poisson equations* (Trkal, 1994) after Siméon Denis Poisson (1781–1840) but that terminology is now obsolete.

¹⁰Also known as the 'del' operator and defined in Cartesian coordinates as $\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$

¹¹After Leonhard Euler (1707–1783).

Deducing from the previous section, it can be seen that flapping-wing flight is an unsteady problem that involves severe manoeuvres and leading-edge separation. To analyse this problem, unsteady aerodynamic methods must be employed together with methods for representing the leading-edge vortex successfully. The aim of this section is to review related work in this area. The discussion below comprises three parts. The first part (§ 2.2.2) reviews some of the unsteady aerodynamics methods available. Second, in § 2.2.3, the leading-edge vortices on slender delta wings are considered due to their similarity to those found on insects. Finally, in § 2.2.4, studies relating directly to insect-like flapping-wing flight are surveyed.

2.2.2 Unsteady Aerodynamics

Velocity Potential Approach

The concept of unsteady aerodynamics (also known as non-stationary flow) first received major attention when the problem of wing flutter was encountered early in the 20th century. Notable early works on the topic were by Theodorsen (1935) and Garrick (1937). Although Glauert (1929) was probably the first to consider the airloads on an oscillating aerofoil, pioneering work in this field is usually attributed to Theodorsen. He used potential-flow methods to derive relations for the unsteady forces and moments experienced by an aerofoil executing simple harmonic motion in pitch and heave (plunge).

By enforcing the Kutta condition at the trailing edge and by suitably relating the change in circulation around the aerofoil to that shed in the wake, Theodorsen was able to derive closed-form equations for lift and moment involving the so-called *Theodorsen function* $\mathcal{H}$. Thus, lift was given by

$$L = \pi \rho U_\infty^2 b \left(\frac{b}{U_\infty^2} \ddot{h} + \frac{b}{U_\infty} \dot{\alpha} - \frac{b^2}{U_\infty^2} a \ddot{\alpha} \right) + 2\pi \rho U_\infty^2 b \left[\frac{\ddot{h}}{U_\infty} + \alpha + \frac{b\dot{\alpha}}{U_\infty} \left(\frac{1}{2} - a \right) \right] \mathcal{H}(k) \quad (2.5)$$

and moment by

$$M = -\rho b^2 \left[\pi \left(\frac{1}{2} - a \right) U_\infty b \dot{\alpha} + \pi b^2 \left(\frac{1}{8} + a^2 \right) \ddot{\alpha} - a \pi b \ddot{h} \right] \\ + 2\rho U_\infty b^2 \pi \left(a + \frac{1}{2} \right) \left[U_\infty \alpha + \dot{h} + b \left(\frac{1}{2} - a \right) \dot{\alpha} \right] \mathcal{T}_l(k) \quad (2.6)$$

where a is location of pitch axis relative to mid-chord (measured here in semi-chords), b is aerofoil semi-chord, α is angle of attack, h is heave position, U_∞ is freestream velocity and ρ is fluid density. The $\mathcal{T}_l(k)$ term is the complex-valued Theodorsen function and is itself expressed in terms of Hankel functions with reduced frequency k as argument¹².

The first set of terms in Equations 2.5 and 2.6 are *apparent-mass* effects (resulting from the acceleration of the surrounding fluid) and the second set represent the *circulatory* effect following from the ‘creation’ of circulation around the aerofoil. Theodorsen’s analysis assumed a planar wake and used small-angle approximations. A detailed account of the derivation can be found in Bisplinghoff *et al.* (1955). More recently, van der Wall & Leishman (1994) extended Theodorsen’s analysis to include a time-varying incident velocity.

Garrick (1937) combined the velocity-potential-based approach of Theodorsen with a method for aerofoils in non-uniform motion outlined by von Kármán & Burgers (1935) to compute the horizontal forces generated by an aerofoil oscillating in a uniform stream.

Earlier on, Wagner (1925) considered another important problem in unsteady aerodynamics — the impulsive start of a thin aerofoil. He derived the so-called *Wagner function* which estimates the lift generated by such an aerofoil. The problem is akin to the case of a step change in angle of attack of an aerofoil in a constant freestream. The Wagner function does not have a convenient analytic form due to the complex nature of the integral equation involved. Simple algebraic and exponential relations, therefore, have been

¹²Reduced frequency k is used to characterise the ‘unsteadiness’ of the flow and is defined as $k = \frac{\omega b}{U_\infty}$ where ω is oscillation angular frequency. It is related to the Strouhal number (see Footnote 6 on p. 29).

proposed in an attempt to approximate the function. Reasonably good fits were found by Garrick (1938) and Jones (1940) who suggested

$$W(s) \approx \frac{s + 2}{s + 4} \quad \text{and} \quad W(s) \approx 1.0 - 0.165e^{-0.0455s} - 0.335e^{-0.3s}$$

respectively, where s is the distance travelled by the aerofoil relative to the fluid in semi-chords. It can easily be noticed that at the start of impulsive motion ($s = 0$), lift generated is equal to *one-half* of the final steady-state value for that flight condition (see Figure 2.13).

This is one of the fundamental results of Wagner’s analysis.

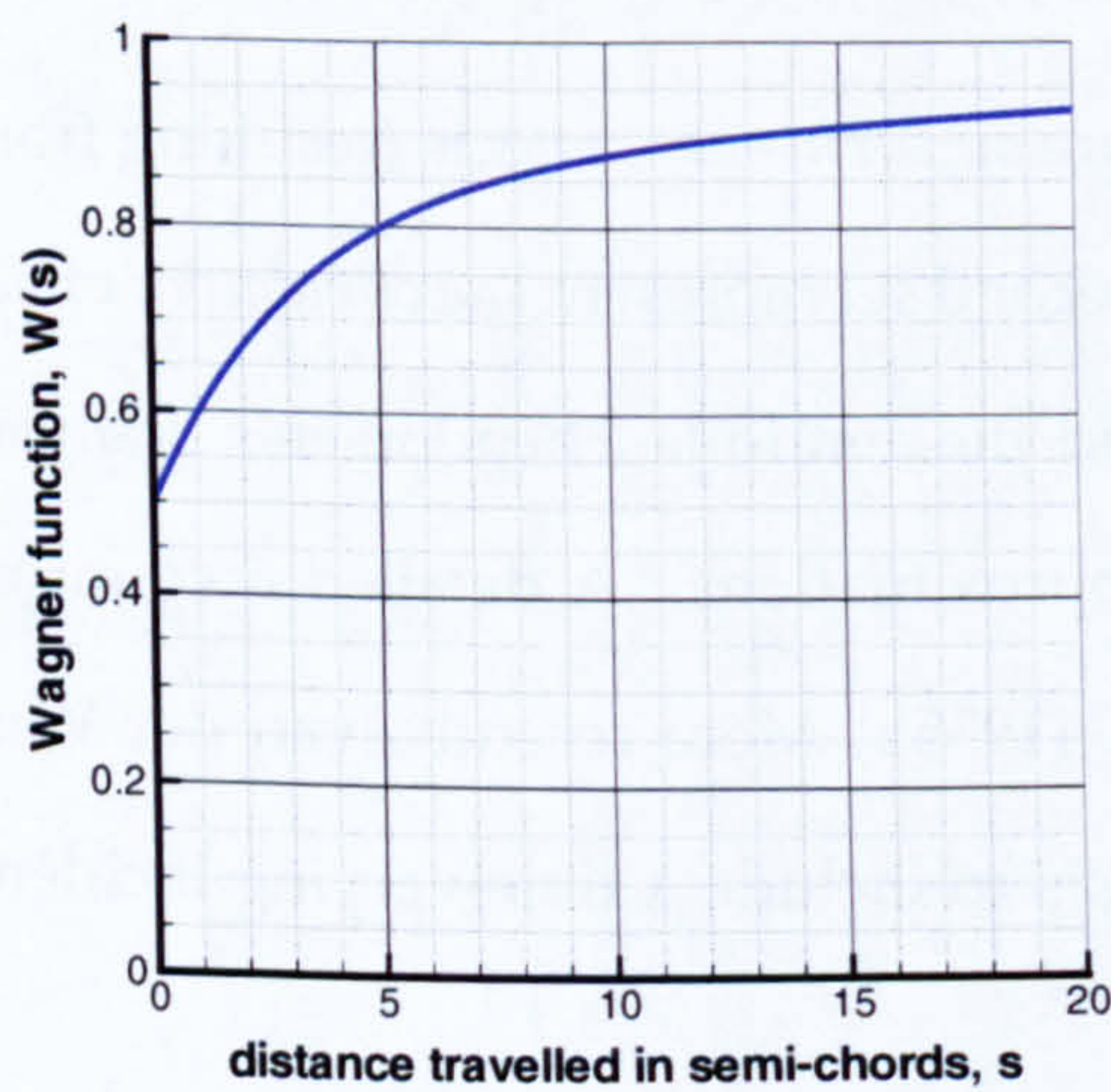


Figure 2.13: Wagner function

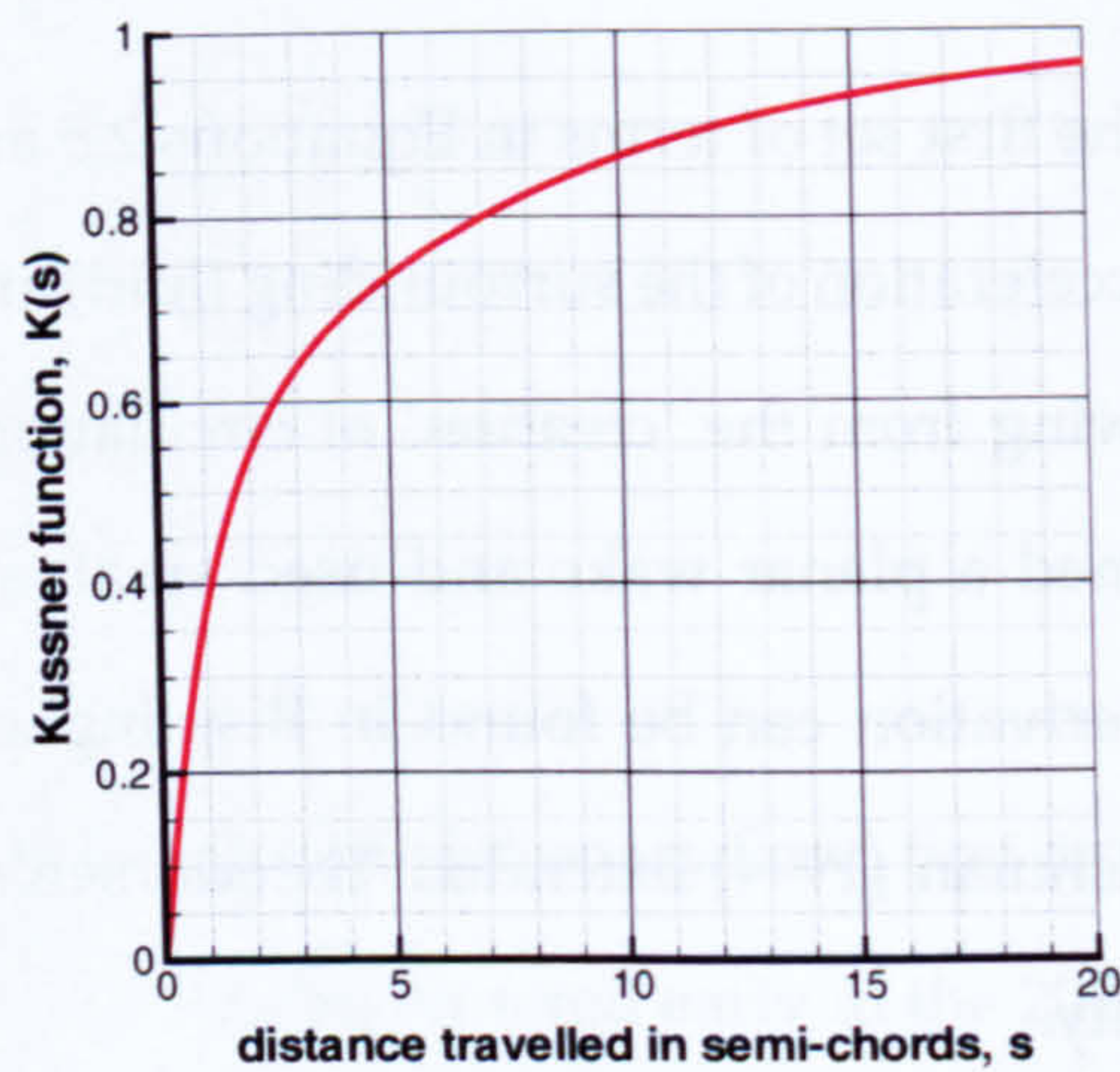


Figure 2.14: Küssner function

Küssner (1936) considered a similar problem — the transient flow associated with a sharp-edged gust. In a manner not very dissimilar to Wagner, he derived the *Küssner function* (which was later rectified for an error by von Kármán & Sears (1938)) that represented the growth in lift as a wing passed through the gust. In contrast to the Wagner function, however, the Küssner function predicts that the additional lift due to the gust grows from *zero* to its full steady-state value (see Figure 2.14). Again, due to the complexity of the equations and lack of closure, exponential fits have been suggested (see e.g. Bisplinghoff *et al.*, 1955).

The effect of a returning wake is an omnipresent problem in rotor flows — the wake from an ‘upstream’ rotor or even the previous wake of the rotor in question interacts with

the rotor and affects its lifting characteristics. The problem was considered by Loewy (1957) and Jones (1958). Loewy modelled the wing and wake as done by Theodorsen but with additional wakes from previous cycles simulated by a series of 2-D vortex sheets *below* the wing, each extending to $\pm \infty$. He developed the so-called *Loewy function* which was an extension of Theodorsen's function and which reduced to the latter if the wakes below the wing were moved far away down. Jones used a similar methodology and found that the most important factor was the number of wake cycles per revolution of the rotor.

In analysing the problem of a flapping wing, the problem of impulsive starts, heaving and sweeping wings and returning wakes is commonly encountered (see § 2.1.3). Therefore, due to the linearity of the models described above, it should be possible to consolidate them effectively to model the flapping-wing problem. As suggested by Leishman (2000), the nature of an impulsive start and a sharp-edged gust makes the problem amenable to solution by indicial methods (and employing the Duhamel integral; see von Kármán & Biot (1940, p. 403)). Indeed, this approach was successfully implemented by Pedersen (2003).

Circulation Approach

All the above analysis used a *velocity-potential approach* to tackle the problems associated with unsteady flow, which brought with it a lot of complexity. So much so that Morris remarked that the general formulae obtained thus far had become so complex that they failed to convey the underlying physical significance (Morris, 1937, 1938). In view of this, von Kármán & Sears (1938) introduced the *circulation approach*. They represented the surface of a thin, flat-plate aerofoil by a thin vortex sheet — a system of vortices with a *continuous* distribution of vorticity (this property differentiates this method from a panel method; see e.g. Katz & Plotkin (2001, ch. 9)).

When an aerofoil begins motion, it develops around it a bound circulation to satisfy

the smooth-flow Kutta condition at the trailing edge. At the same instance, a starting vortex of opposite sense is shed into the wake in view of Kelvin's law (see Lamb, 1932, § 33, p. 36) that the total circulation in a system must remain constant. The bound and starting vortices constitute a 'vortex-pair' with an ever-increasing distance separating them. The approach used by von Kármán & Sears (1938) is based on the idea that the momentum per unit span of the system can be expressed by the sum of the momentum of the vortex-pairs that constitute the system.

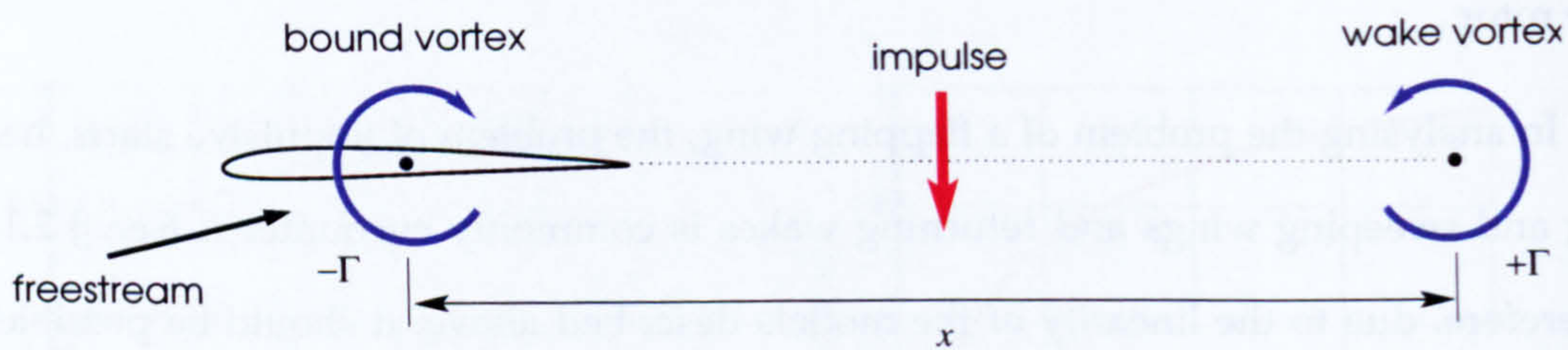


Figure 2.15: 2-D vortex ring [after von Kármán & Sears (1938)]

If each vortex pair is considered to be part of a '2-D vortex ring' so that only the bound and wake vortices are in view, then the distance between the two constitutes the 'area' of the ring. The vortex-pair induce a downward velocity between them giving rise to an impulse (see Figure 2.15). Assuming a flat wake so that the distance between a vortex-pair of circulation Γ_i is x_i , their impulse¹³ is equal to $\rho \Gamma_i x_i$. Because the impulse acts halfway between the vortex-pair, the moment of impulse is $\frac{\rho}{2} \Gamma_i x_i^2$. The lift and moment of the entire system can then be evaluated from the sum of the time rates of change of the respective impulses, thus

$$L = \rho \frac{d}{dt} \sum_i \Gamma_i x_i \quad \text{and} \quad M = \frac{\rho}{2} \frac{d}{dt} \sum_i \Gamma_i x_i^2$$

An important consequence of aerofoil motion is the presence of wake vorticity and its influence on the bound vorticity of the aerofoil. Therefore, total bound circulation Γ_a

¹³For a 3-D vortex ring, the impulse of a vortex ring has magnitude $\rho \Gamma A$ where ρ is fluid density, Γ is the circulation of the vortex ring and A is the enclosed area (Milne-Thomson, 1966, § 10.21). Also, von Kármán & Sears (1938) did not use the normal convention and have a negative sign in their version of impulse.

around the aerofoil can be expressed as the sum (superposition) of two contributions (thereby expressing linearity of the theory) thus

$$\Gamma_a(t) = \Gamma_0(t) + \Gamma_1(t)$$

where Γ_0 is the quasi-steady (wake-free) component and Γ_1 is the part induced by the wake. From Kelvin's law, the total circulation of the system must remain constant so that

$$\Gamma = \Gamma_a(t) + \Gamma_w(t) = \text{constant}$$

where Γ_w is total circulation in the wake. By using the above relations and by enforcing the Kutta condition at the trailing edge, von Kármán & Sears (1938) arrived at the following simple formula for unsteady lift:

$$L(t) = L_0(t) + L_1(t) + L_2(t) \quad (2.7)$$

where

$$\begin{aligned} L_0(t) &= -\rho U \Gamma_0(t) \\ L_1(t) &= \rho \frac{d}{dt} \int_{-b}^{+b} x \gamma_0(x) dx \\ L_2(t) &= -\rho U \int_{+b}^{\infty} \frac{\gamma_w(x)}{\sqrt{x^2 - b^2}} dx \end{aligned}$$

where $\pm b$ refer to the limits of the aerofoil (equal to a semi-chord), γ_0 is wake-free bound vorticity and γ_w is wake vorticity. Here, $L_0(t)$ gives the quasi-steady contribution to lift (the negative sign arises because positive circulation is counterclockwise). $L_1(t)$ is the non-circulatory apparent-mass¹⁴ contribution due to the inertia of the fluid accelerated by the aerofoil motion. Finally, the $L_2(t)$ -term gives the wake-induced contribution corresponding to the Wagner function (Wagner, 1925) and is responsible for the lift-deficiency characteristics of the Wagner effect (see § 2.1.4 above).

¹⁴Also referred to as 'added mass'. The terminology 'virtual mass' also exists but more accurately refers to the sum of the actual mass of the wing and the apparent mass (Massey, 1998, p. 412).

The expression for moment was a similar three-part equation. Note that in unsteady motion, both the distribution of bound vorticity around the aerofoil and its wake counterpart vary with time (von Kármán & Sears, 1938; Sears, 1940). Hence the time-dependency in the above equations.

Using this methodology, von Kármán & Sears were able to deduce shortcut versions of the derivations of Wagner (1925), Glauert (1929), Theodorsen (1935) and Küssner (1936) with relative simplicity. More recently, McCune *et al.* (1990) extended the linear theory of von Kármán & Sears to include effects of roll-up and a nonlinear deforming wake whilst still preserving the transparency of the original work (see also Scott, 1987; Lam, 1989). In doing so, a nonlinear term was introduced so that the lift formula now became

$$L(t) = L_0(t) + L_1(t) + L_2(t) + \tilde{L}(t) \quad (2.8)$$

where the first three terms on the right-hand side are similar to those in Equation 2.7. The fourth term $\tilde{L}(t)$, however, refers to a nonlinear correction which vanishes if the assumptions of linear theory are restored. This nonlinear generalisation enabled Tavares & McCune (1993) to apply a similar approach to the problem of a delta wing in severe unsteady manoeuvre (see also Tavares, 1990; Lee, 1991).

As done by Pedersen (2003) with the velocity-potential approach, the circulation method described above may also be used to model a flapping wing and its nonlinear wake. This indeed is the cornerstone of the current work (see Chapter 4), following the suggestion of Żbikowski (2002).

2.2.3 Delta Wing Aerodynamics

The similarity between the leading-edge vortices found on insect flapping wings and the leading-edge vortices found on low-aspect-ratio delta wings was pointed out by van den Berg & Ellington (1997a). Since then, many authors have pointed out this similarity (e.g. Żbikowski, 2002) as has been affirmed by the discussion on insect flight above. It is,

therefore, pertinent to the problem at hand to consider this flow regime¹⁵. Moreover, the overwhelming amount of information on delta wings may help make up for some of the paucity of information on the leading-edge vortices on insect wings.

Nature and Characteristics of Delta Wing Leading-Edge Vortices

Most modern fighter aircraft have thin, low-aspect-ratio (or slender) delta wings because of their benefit in flight at high Mach numbers¹⁶. At low, subsonic speeds (such as during take-off and landing), such wings are unable to produce significant lift in the 'conventional' manner. Instead, they rely on *vortex lift* to sustain flight (see next section).

At moderate to high angles of attack, the highly swept leading edges on thin delta wings cause the separation of a free shear layer from the leading edge¹⁷. The latter develops into the *leading-edge vortex* which rolls in a spiral manner along the leading edge towards the aft sections of the wing where it finally merges with the tip vortex and trails into the wake (see Figure 2.16(a)).

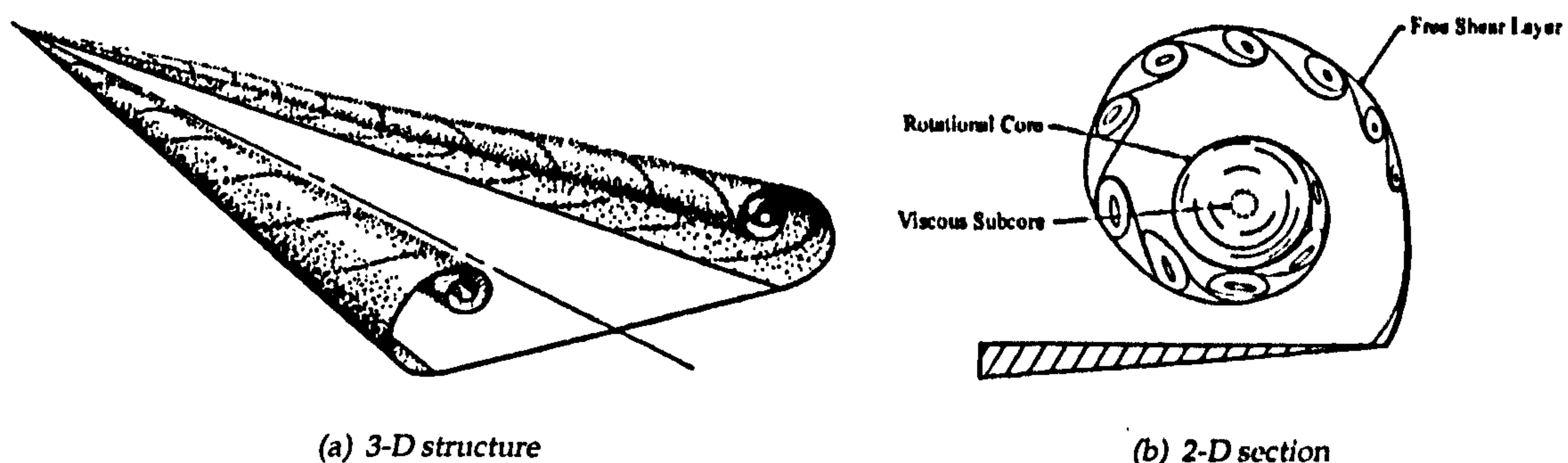


Figure 2.16: Leading edge vortex on a delta wing [from Nelson & Pelletier (2003)]

¹⁵Some have doubted the presence of a leading-edge vortex on delta wings at Reynolds numbers pertaining to insect flight. However, flow-visualisation work by Prenel *et al.* has refuted this claim. They have shown leading-edge vortices on delta wings in the range $480 < Re < 1920$ (Prenel *et al.*, 1997).

¹⁶Such wing shapes are required to ensure that the entire wing remains within the Mach cone in supersonic flight (see e.g. Houghton & Carpenter, 1993, § 7.4).

¹⁷The incidence angle at which separation occurs may be predicted from the expression for leading-edge thrust for a given planform (Traub, 1994).

As Figure 2.16(b) shows, the leading-edge vortex consists of three main regions: a free shear layer, a rotational core and a viscous subcore (Earnshaw, 1961; Lee, 1991). The high vorticity fed through the free shear layer causes it to roll up under its own influence into a relatively compact core of concentrated vorticity. The outer region of this core rotates as would a free potential vortex, but at its centre, viscosity effects prevent a singularity and the vortex core behaves as a solid cylinder (similar to a Rankine-type vortex). Low-speed experiments have shown that the free shear layer is in fact not smooth but consists of tiny vortices travelling along the layer towards the vortex core (Gad-el-Hak & Blackwelder, 1985, see also Figure 2.16(b)).

Axial velocities in the viscous subcore can reach speeds of up to three times the freestream value (Earnshaw, 1961). The swirl (circumferential) velocities in the shear layer are also similarly high. Lee & Ho (1989) noted that a leading-edge vortex becomes stationary when the convection of vorticity along the core axis balances the vorticity generation from the boundary layer of the leading-edge vortex, and that the swirl (helix) angle is an indicator of this balance.

The proximity of the vortex core to the wing surface and its high vorticity creates a region of significantly low pressure there. This suction peak is mainly responsible for the augmented lift observed — the so-called vortex lift. Depending on its strength, the velocity induced by the leading-edge vortex itself may cause separation in regions of the wing outboard of itself, giving rise to a so-called secondary vortex. In rare cases, even a tertiary vortex has been observed. As the leading-edge vortex moves downstream, it continuously feeds vorticity through the shear layer into the core (Hoeijmakers, 1991). The latter grows in strength and dimension, giving the overall structure its conical shape.

The benefits of vortex lift are, however, limited by the onset of *vortex breakdown* (or *vortex bursting*)¹⁸ — the flow condition where the cross-sectional area of the leading-edge vortex suddenly increases together with a reduction in axial and circumferential velocities

¹⁸Good reviews on the subject can be found in Leibovich (1984) and Délery (1994).

and an increase in turbulence levels — at high angles of attack ($\alpha > 30^\circ$ for low-sweep-angle wings). For wings with high sweep, $\alpha > 45^\circ$ is possible before vortex breakdown begins. It occurs at high values of swirl velocities in the leading-edge vortex (Bossel, 1968)¹⁹.

Vortex breakdown occurs in the trailing wake anyway but becomes of concern once it progresses upstream onto the wing where it manifests itself as a significant reduction in lift-curve slope. Although the exact mechanism behind it is still elusive (Délery, 2001; Nelson & Pelletier, 2003), it is widely accepted that vortex breakdown is promoted by increasing the angle of attack (Werlé, 1960; Hall, 1961, 1966). When the leeward flow completely separates from the wing, stall occurs.

Before moving on, it is noteworthy that delta-wing-type leading-edge vortices also occur in the form of *corner vortices* on roof tops. Notable works in the field are by Williams & Baker (1997), Banks *et al.* (2000) and Banks & Meroney (2001a,b,c).

Another class of leading-edge vortices are the Kasper-type formed by causing separation at the leading edge and then trapping the vortex (Kasper, 1974). Kasper's configurations made use of a nose flap and split trailing-edge flaps to trap vortices. Kasper claimed that high lift and low drag were achievable using this method and that the resulting vortices were constant in size and stable. Later, wind-tunnel experiments by Kruppa refuted this claim by showing that not only was the vortex not stable but also that the lift and drag performance were nowhere near that claimed by Kasper (Kruppa, 1977).

The benefits of vortex trapping have also been studied theoretically. Saffman & Sheffield (1977) considered the flow over a wing with an attached free vortex and found a significant increase in lift due to the vortex. Mourtos & Brooks (1996) extended Saffman

¹⁹In fact, Rusak & Lamb (1999) found that vortex breakdown occurs when the ratio of the maximum circumferential (swirl) velocity to the maximum axial velocity was about 0.58, and that this value was more or less independent of angle of attack and location along the wing.

& Sheffield's model by having a vortex-sink combination instead and still reported augmented lift. Huang & Chow (1982) found that to maintain a stable free vortex on a Joukowski aerofoil, a thick wing and small angle of attack were necessary.

Meanwhile, Rossow (1978, 1992, 1994) theoretically analysed the problem of a trapped vortex between parallel fences with suction and found that lift coefficients of the order of 10 could be achieved. A more recent study by Riddle *et al.* (1999) experimentally verified Rossow's theory and found that a spanwise vortex could be stabilised using suction at either end through parallel fences.

Modelling the Leading-Edge Vortex

The dawn of supersonic flight following World War II and the use of delta wings soon proved that the methods available then (notably Jones's slender-wing theory (Jones, 1946)) for predicting lift characteristics of such planforms were inadequate. The wings operated at angles of attack much higher than the stall angles for most aircraft wings of the time and potential methods failed to give satisfactory results.

The lift generated by a slender delta wing at moderate angles of attack is much higher than that predicted by conventional methods. The reason for this is that the lift for such wings is made up of two components — *potential* and *vortex* lift (see Figure 2.17). Potential lift refers to the component that is produced due to the attached flow on the wing and can be approximated using potential-flow methods. The lift produced from the separation that leads to the creation of the leading-edge vortex constitutes the vortex lift, and makes a substantial contribution to the overall lift.

Leading-Edge Suction Analogy Method The leading-edge-suction analogy is due to Polhamus (1966) who suggested the revolutionary method to account for the augmented lift observed on delta wings in an attempt to bypass the difficulties associated with accurately determining the strength, shape and position of the leading-edge vortex.

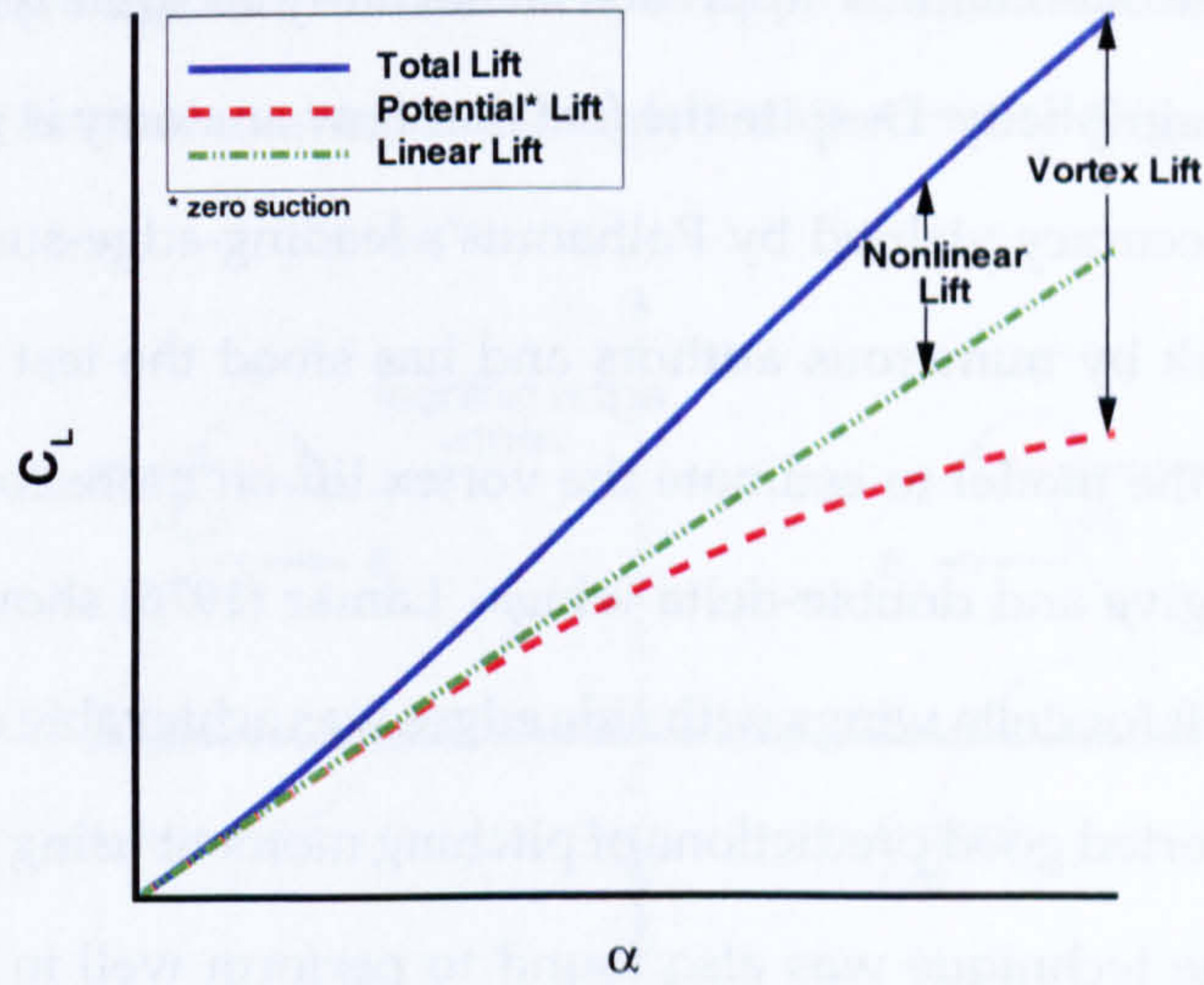


Figure 2.17: Contributions of various components in Polhamus' leading-edge-suction analogy model

Polhamus proposed that if the lift on a slender delta wing at moderate angles of attack was decoupled into potential and vortex components, then the latter could be predicted from the leading-edge suction associated with the potential flow over the leading edge. He showed that if the nonlinearities in the linear potential-flow method were included together with the loss of lift due to loss of leading-edge suction, then a very accurate estimate of the total lift could be found by summing the individual potential- and vortex-lift components. In his notation, the potential- and vortex-lift coefficient components were given by

$$C_{L_p} = K_p \sin \alpha \cos^2 \alpha \quad (2.9a)$$

$$C_{L_v} = K_v \sin^2 \alpha \cos \alpha \quad (2.9b)$$

respectively, where K_p and K_v are constants for a given slender delta configuration and α is angle of attack. K_p designates the lift-curve slope for the wing at zero angle of attack and K_v is directly related to the coefficient for leading-edge suction. The value of K_p depends solely on planform and is determined theoretically by any suitable lifting-surface method. The sum of Equations 2.9 gives the total lift coefficient for a given α .

The main advantage of Polhamus' approach is that fairly accurate results are obtained with relative ease and simplicity. Despite the fact that flow anatomy is generally ignored in the approach, the accuracy yielded by Polhamus's leading-edge-suction analogy has prompted further work by numerous authors and has stood the test of time. Bradley *et al.* (1973) extended the model to estimate the vortex lift on more complex planforms such as rectangular, ogive and double-delta wings. Lamar (1976) showed that accurate predictions of vortex lift for delta wings with side edges was achievable using the method. Purvis (1981) even reported good predictions of pitching moment using the leading-edge-suction approach. The technique was also found to perform well in supersonic flight regimes (Polhamus, 1971). The approach continues to remain a good engineering tool for rapidly iterating through various wing designs for delta wing and related configurations (Lamar, 1998).

Vortex Methods The difficulty in modelling the complex three-dimensional flows associated with the leading-edge vortex on a delta wing can be circumvented by reducing the problem to two dimensions by considering cross-flow planes. The wing can be divided into successive chordwise sections and the slender-body assumption made. This is the basis of vortex methods which can broadly be subdivided into two — *two-vortex* and *multi-discrete-vortex* methods.

Two-Vortex Model This model was pioneered by Edwards (1954)²⁰ who represented the spiralling vortex sheets of the leading-edge vortex on a delta wing by an inviscid model. Brown & Michael (1954) showed that the model could also be applied to flows at supersonic Mach numbers. Cheng (1954) used a similar approach to model the vortex pair behind a moving flat plate.

The model consists of two concentrated symmetrical vortices (representing the vortex

²⁰Actually, Legendre (1953) was probably one of the first to introduce this technique but Edwards (1954) modified it to give it its current accepted form.

cores of the leading-edge vortex) that lie above the wing and are connected to it via two feeding sheets (or branch cuts) of vanishingly small vorticity (Figure 2.18).

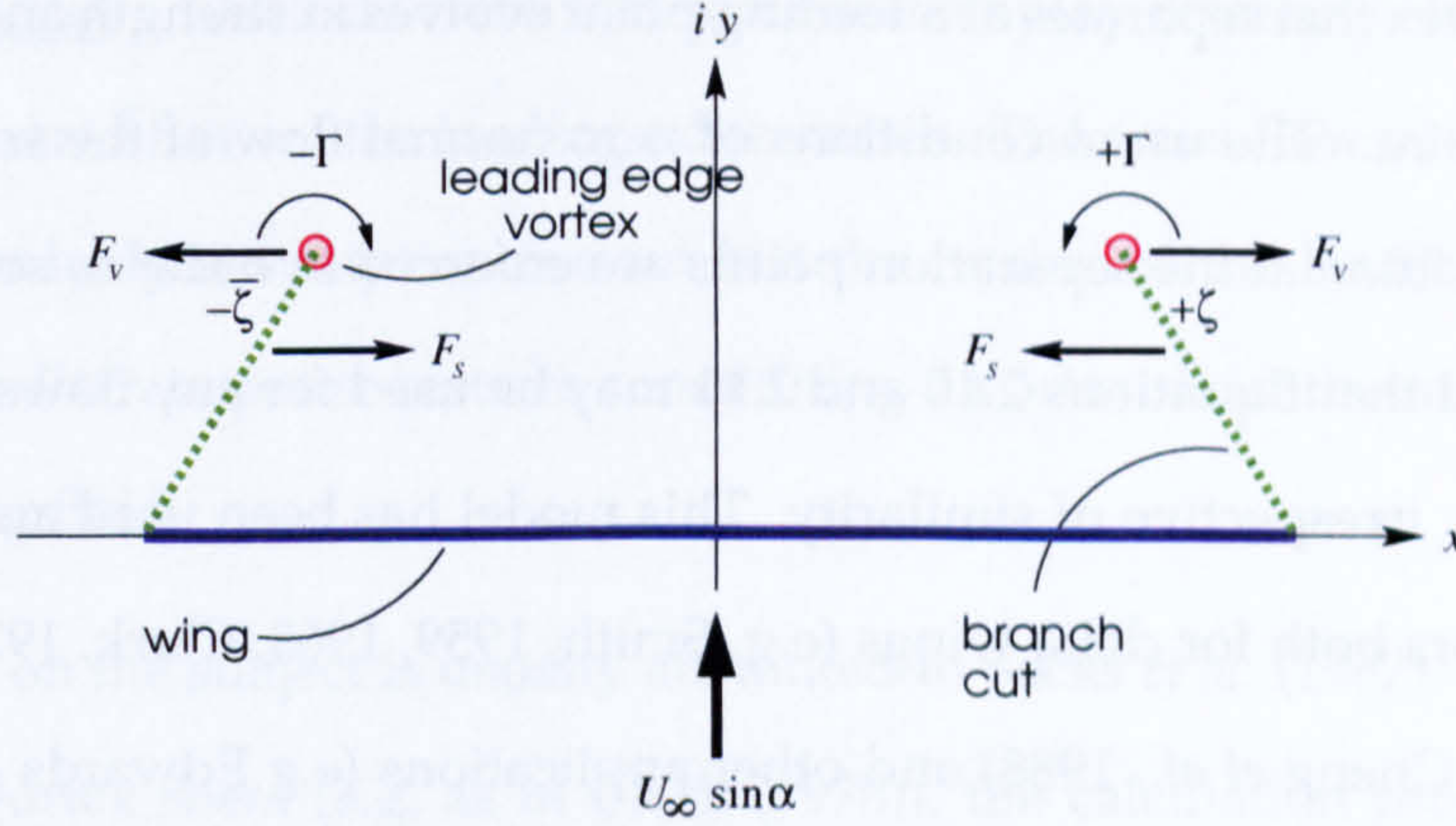


Figure 2.18: Two-vortex model for the leading-edge vortex

In Legendre's original model, the two vortices were force-free and allowed to move with the fluid. The presence of the branch cuts in the current case, however, restricts this motion and a Kutta-Joukowski force F_v acts on the vortices, thus

$$F_v = -i\rho\Gamma\left(\overline{W} - \frac{d\zeta_1}{dt}\right) \quad (2.10a)$$

where ρ is fluid density, Γ is vortex circulation, $\overline{W}$ is the complex freestream component of velocity, ζ_1 is the current position of the vortex and F_v is a complex-valued force. This Joukowski force is balanced by the force across the branch cut²¹ which is given by

$$F_s = i\rho(\zeta_1 - \zeta_0) \frac{d\Gamma}{dt} \quad (2.10b)$$

where ζ_0 is the point of separation (leading edge). By requiring no net force on the vortex and connecting sheet system, the sum of Equations 2.10 is equated to zero to yield

$$\overline{W} = \frac{1}{\Gamma} \frac{d\Gamma}{dt} (\zeta_1 - \zeta_0) + \frac{d\zeta_1}{dt} \quad (2.11)$$

²¹An artefact of a branch cut (or connecting sheet) is that a difference in pressure exists across it which gives rise to a force. This property differentiates it from a vortex sheet across which pressure is continuous and no Joukowski force acts (see p. 135).

A detailed derivation of the above equation can be found in Bryson (1959) where the symmetric vortex separation on circular cylinders and cones was considered. The technique tracks how a vortex that separates at a feeding point evolves in strength and distance from its separation point. The usual conditions of zero normal flow at the solid boundaries and a Kutta condition at the separation points are enforced in order to solve the system. Rott (1956) noted that Equations 2.10 and 2.11 may be used for any flows involving vorticity generation, irrespective of similarity. This model has been used and improved by numerous authors both for delta wings (e.g. Smith, 1959, 1968; Clark, 1978; Peace, 1983; Nathman, 1984; Cheng *et al.*, 1988) and other applications (e.g. Edwards & Cheng, 1982; Graham, 1983).

Mangler & Smith (1959) introduced a spiral vortex sheet of distributed vorticity that originated at the leading edge and rolled in a spiral manner to a concentrated vortex at its centre. The shape of this vortex sheet was determined by making the sheet a stream surface so that it sustained no pressure. Later, by combining this method with that of Edwards (1954), Smith (1959) presented a model that consisted of a spiralling vortex sheet that ended in a branch cut to a vortex filament at the centre. The stream surface boundary condition was satisfied on the vortex sheet and the force balance condition across the cut. The underlying assumption throughout was that of conical flow.

Smith later presented an improvement on this method where he showed that the final position of the vortex core did not vary much with the number of coils of the outer vortex sheet (Smith, 1968). Subsequently, Clark (1978) extended the method to model non-conical flow past slender delta wings and got reasonable results. Peace (1983) implemented a similar method and showed good results for more complex wing planform shapes such as double-delta and lenticular configurations.

Before proceeding it should be pointed out that although equilibrium is maintained in the model above by enforcing a net-zero-force condition across the branch-cut-vortex system, an unbalanced moment still remains (Bryson, 1959). This was the main objection

raised by Gainer (2002) who used a two-vortex model but without a branch cut²².

Multi-Vortex Models Multi-discrete-vortex models employ discrete point vortices to represent the separated flow of the leading-edge vortex. The cross-flow plane methodology and the slender-body assumption remain but the 'wake' of the leading-edge vortex is represented by discrete point vortices constituting a vortex sheet that moves freely with the surrounding fluid.

Early work on the subject is usually attributed to Sacks *et al.* (1967). Instead of using a continuous vortex sheet (e.g. as in Clark (1978)), the calculation procedure involved marching downstream in small, regular steps and releasing a new discrete vortex filament at each step in order to maintain smooth flow at the leading edges. The subsequent positions of these vortex filaments as they evolved were followed downstream and in this way were used to represent the roll-up of the leading-edge vortex sheet. Hoeijmakers & Bennekers (1978) presented a variation on the method whereby each discrete point vortex was replaced by a vortex-sink combination instead to depict the axial (downstream) flow in the leading-edge vortex structure.

Clark (1978) remarked that this slender-body method had a drawback in that the representation of the flow in regions close to the apex would be inadequate because of the overall limitations on the number of vortices that could be handled. Despite this, the approach has yielded satisfactory results for various slender-wing and wing-body configurations (see e.g. Cheng *et al.*, 1988).

Further reassurance is provided by the fact that very good implementations of this approach can be found in the works of Tavares (1990) and Lee (1991) who used the technique to successfully model the flow around slender delta wings in severe unsteady manoeuvres. One of the advantages of this approach is that three-dimensional flows can still be computed by using the quasi-two-dimensional analogy of cross-flow planes and

²²Gainer used a different set of boundary conditions to 'close' the system.

that the method is grid-free.

Another class of multi-vortex models are vortex lattices models. These are full-blown 3-D models and, therefore, come with a huge computational overhead. However, they have been implemented with some success. A good discussion on the subject can be found in Hoeijmakers (1991). The fact that they are not grid-free, however, makes them less attractive.

2.2.4 Flapping Wing Aerodynamics

Whereas insects and their flight have been widely discussed, aerodynamic analysis of their flapping-wing flight has received relatively less attention. Research in this field evolved from early experimental work, followed by analytical studies and has finally reverted to more experimental work. Until recently, most of the literature on the subject came from biologists but this is changing as it is slowly receiving increasing attention from aerodynamicists (the author included). This section aims to describe some of the fluid-mechanics-related work on insect flight aerodynamics.

Following earlier work in this area by Osborne (1951, noted earlier), the first major contribution is usually attributed to Pringle (1975). Both of these works were based on quasi-steady models — the instantaneous forces on the flapping wing were assumed to be equivalent to those for steady flight at the same instantaneous velocity and angle of attack.

Following on from the elucidative work of Weis-Fogh (1972, 1973) on insect flight mechanisms and a light quasi-steady treatment (actuator-disk momentum theory and blade-element analysis) of some aspects of their flight, research into the aerodynamics of insect flapping flight was rejuvenated by Ellington's seminal work in the 1980s (Ellington, 1984a,b,c,d,e,f). Although his work still did not step out very far from quasi-steady models (momentum and blade-element methods), Ellington strongly speculated about

the presence of unsteady mechanisms and his work led to further study in this area. The nature of the 'unsteadiness' was still unclear.

Earlier, Lighthill (1973) used aerodynamic theory to analyse Weis-Fogh's 'clap-and-fling' mechanism for insects. He applied conformal transformation techniques to derive the lift force generated by a wing pair flinging open after a clap, again employing a quasi-steady approach. Later, Edwards & Cheng (1982) revisited the problem and refined the analysis to include a vortex-shedding model similar to that used in their earlier work on delta wings (Edwards, 1954; Cheng, 1954).

Another approach for modelling insect flapping flight that is found in the literature is the method of vortex rings (Rayner, 1979; Ellington, 1984e). Drawing inspiration from the experimental observations of Magnan and Ellington, Rayner represented the wake of a hovering insect as a chain of small-cored coaxial vortex rings (one produced for each halfstroke; see also Grodnitsky & Morozov (1993)) that stacked one upon the other²³ (see also Brackenbury (1995, p. 139)). The approach bypassed the need to determine lift and drag coefficients and ignored the mechanism by which the vortex wake was generated. Rayner concluded that circular vortex rings could efficiently represent wakes shed by hovering insects. Sunada & Ellington devised a similar method by representing each vortex ring as a grid of small vortex rings and estimating induced power from the added mass of the vortex rings. They found good agreement with the Rankine-Froude momentum theory (Sunada & Ellington, 2001).

Wang has been tackling the problem of insect-flight aerodynamics from a CFD point of view (Wang, 2000a). Using these methods, she claims that optimal flapping flight frequency was inversely proportional to wing dimensions (Wang, 2000b) and that insects, and dragonflies in particular, are able to use both lift and drag to hover by means of an inclined stroke plane (Wang, 2003). A similar study by Pullin & Wang (2004) employing Navier-Stokes calculations for solving the flow associated with an accelerating flat plate

²³A similar pattern of vortex rings is observed in the wake of fish (Dickinson, 1996).

revealed that maximum vortex lift was obtained for a wing at an incidence angle of $\arccos(\sqrt{3/8}) \approx 52.2^\circ$.

Dickinson has also made a contribution to the understanding of insect-like flapping flight albeit from a biologist's perspective. In 1994, he presented results from his 2-D experiments on flapping-wing motion (Dickinson, 1994). He investigated the effects on lift of several kinematics parameters and found that lift generally increased for forward wing pitch axes (notably 0.2-chord). He also found that although force-generation was higher for faster stroke reversals, this trend did not continue as rotational speeds increased probably due to the opposing effects of shed vortex strength and its proximity to the wing²⁴.

Later 3-D experiments by Sane & Dickinson using a *Drosophila*-type wing revealed similar findings (Sane & Dickinson, 2002). Mean force generated decreased as the wing pitch axis was moved rearward towards the trailing edge. They also found that as angular velocity increased, force initially decreased to a minimum and then increased almost linearly. The similarity between the 2-D and 3-D results provided proof that results for a 3-D flow regime could indeed be derived from an extension of 2-D results (using, for example, blade-element theory) at least for the order of Reynolds number in question ($Re \approx 200 - 300$ based on mean chord and tip velocity). The absence of spanwise flow, however, prompted Birch & Dickinson (2001) to conclude that although a spiral vortex was present on a hawkmoth (Ellington *et al.*, 1996, where spanwise flow was clearly observed), the precise flow structure of a leading-edge vortex depended critically on Reynolds number.

Gustafson & Sethian presented a very elucidative review of flow-visualisation results of the vortex flows associated with flapping wings and presented results for a 2-D insect-like hovering mode (Gustafson & Sethian, 1991, ch. 6). By citing the work of Robinson &

²⁴Although bound and shed vortex strength increase with increasing rotational speeds, the shed vortex has less time to get away from the wing during fast stroke reversals and hence hinders lift growth more.

Wissler (1988), they postulated the near-independence of force coefficients with Reynolds number. As will be seen later in Chapter 6, their results were found to be in good agreement with the solutions from the current work. Leishman (priv. comm.) has also reported similar flow-visualisation results.

Meanwhile, Azuma (1992) in his work provided a broad analysis of insect flight, alluding to various quasi-steady approaches including momentum and blade-element methods. He also suggested unsteady velocity-potential methods such as the works of Theodorsen (1935, see § 2.2.2 above) but did not provide any results. His work also included the flight of birds. More recently, in his review of progress in the aerodynamics of micro air vehicles, Mueller (2001) dedicated a section to flapping flight and examined some of the recent and ongoing work in the field, notably studies pertaining to low Reynolds number flows, unsteady aerodynamics, leading-edge vortices and aeroelasticity.

Following on from Garrick (1937)'s early work, Jones, Platzer *et al* have investigated the propulsion properties of an aerofoil in heaving oscillation — the so-called *Knoller-Betz* effect²⁵ — both theoretically by means of panel methods and experimentally (Jones *et al.*, 1996; Jones & Platzer, 1997, 1999, 2000; Jones *et al.*, 2001). Although initially beneficial, they reported diminishing returns with increasing forward speed as thrust was outweighed by profile drag.

Analytical treatments relating to flapping wings are relatively scarce. Żbikowski proposed two different approaches to analytical modelling of the insect-like flapping-wing problem — the velocity-potential and circulation approaches (Żbikowski, 2002). As mentioned earlier, Pedersen (2003) used the velocity-potential approach to devise a method for predicting the forces on a flapping wing for a given kinematics and found particularly good agreement for lift forces. His model used a combination of the approaches of Wagner (1925), Theodorsen (1935) and Loewy (1957) for attached flow together with a

²⁵After the independent works of Knoller (1909) and Betz (1912) on the subject that predicted it.

form of the leading-edge-suction analogy of Polhamus (1966) for vortex lift. The present PhD work substantially develops and significantly advances the ideas of Żbikowski on the circulation approach.

Until recently, the closest case to flapping wings considered was the problem of wings in severe unsteady manoeuvres (e.g. Scott, 1987; van der Wall & Leishman, 1994). Benson (1989) and Lam (1989) presented a particularly comprehensive technique based on the classical circulation approach of von Kármán & Sears (1938) but by dropping a number of linearising assumptions in their analyses. They considered the case of 2-D flat plate wing executing severe unsteady motion. The wing was represented as a sheet of bound vorticity with a nonlinear wake emanating from the trailing edge in the form of a free vortex sheet. The model was inviscid and governed by Laplace's equation. A time-marching algorithm was used to track the wake evolution (which was represented using discrete point vortices) as it convected over time. The approach yielded a nonlinear form of the Wagner function in the form of an integral equation (a nonlinear form of the $L_2(t)$ -term in Equation 2.7 above).

The approach employed by Tavares (1990) and Lee (1991) for slender delta wings reduced the 3-D vortical flow to a pseudo-2-D problem in cross-planes. This enabled use of a method similar to Benson's and Lam's model but with an extension in the form of separation from *both* edges. A more complicated analogue of the Wagner function was derived by incorporating the effects of both 'wakes' and enforcing stagnation at both edges. A similar time-marching algorithm was employed to track the solution. Tavares reported good results for delta wings in unsteady manoeuvres.

Minotti (2002) produced a circulation-based analysis for insect-like flapping wings. In his approach, the problem was divided into the trailing-edge wake and the leading-edge vortex, and their respective influences summed. The change in bound circulation with time was computed using Graham's method (Graham, 1983)²⁶. The leading-edge vortex

²⁶Graham (1983) used an approach similar to Edwards (1954) to investigate the variation of bound circula-

was represented by a single ‘fixed’ point vortex at a position that gave best agreement with the experimental data of Sane & Dickinson (2001)²⁷.

The recent work of Yongliang *et al.* (2003) was devised on a strategy similar to the circulation approach proposed by Żbikowski (2002) albeit with a few differences. They reported reasonable agreement between their 2-D approach and experimental data from Dickinson & Götz (1993). Their method of extension to 3-D was, however, left unclear and any details shelved for a later paper (yet unpublished). Jones (2003), on the other hand, considered the leading-edge separation problem for 2-D flows. He modelled the inviscid flow past a moving flat plate in an impulsive start with separation from both edges. He also presented rigorous proof that the traditional steady Kutta condition had an exact parallel in the unsteady case.

In the meantime, qualitative work on insect flight has continued alongside in the form of experiments by the likes of Martin & Carpenter (1977), Maxworthy (1979), Spedding & Maxworthy (1986), Ennos (1989a,b), Grodnitsky & Morozov (1992, 1993), Ellington *et al.* (1996), Okamoto *et al.* (1996), van den Berg & Ellington (1997a,b), Dickinson (1994, 2001), Dickinson & Götz (1993, 1996), Dickinson *et al.* (1999), Sane & Dickinson (2001, 2002) and Nolan (2004).

As described earlier in § 1.1, the experimental work of Nolan (2004) on flapping-flight aerodynamics has been involved with the investigation of vortex lift on insect wings. At Shrivenham, a flapping-wing mechanism has been developed (Żbikowski *et al.*, 2005) and has been subjected to a PIV treatment for measuring flow properties (Ansari *et al.*, 2003a,b). Later experiments will also involve force measurements.

tion with time by modelling the starting vortex as a point vortex connected to the trailing edge via a branch cut and tracking the evolution.

²⁷The placement of the leading-edge vortex was somewhat arbitrary in that in certain instances (around stroke reversals), it was placed much closer to the trailing edge than the leading edge, depending on a specially-defined incidence velocity.

2.2.5 Summary

Analytical work pertaining to unsteady aerofoil flows has historically involved separation only from the trailing edge with the smooth-flow Kutta-Joukowski condition enforced there. A number of flows have been investigated in this way, such as impulsive starts, gusts and wake reentry. In insect-like flapping-wing problems, leading-edge separation in the form of a leading-edge vortex (similar to the vortical structure observed on slender delta wings) is also encountered. Of the methods found in the literature for modelling the latter, the Polhamus leading-edge-suction analogy offers quick and reasonably accurate results. By combining the methods for unsteady aerofoil flows with leading-edge separation methods, therefore, it may be possible to model insect-like flapping-wing flight. This was done with success by Pedersen (2003).

The unsteady aerodynamic methods referred to in the last paragraph employ the velocity-potential approach which brings with it a lot of complexity. An alternative method is the circulation approach whereby the aerofoil is represented as a continuous distribution of vorticity and so is the wake. In classical approaches, a (linear) wake emanates from the trailing edge where the Kutta-Joukowski condition is enforced. To mimic insect-like flapping, a wake would need to be shed from both the leading and trailing edges. A parallel can again be seen in delta-wing flows where separation is observed on both leading edges and circulation-based approaches have been used for solving the flow. Also, due to the severity of insect-wing kinematics, the wake is likely to be nonlinear. Therefore, by suitably modifying and combining unsteady aerodynamics and leading-edge separation approaches it may be possible to devise a flapping-wing model. This is the methodology developed and applied in the current work.

Chapter 3

Quasi-Steady Analysis

This chapter presents some of the early work done in this PhD project with regards to wing design for flapping-wing MAVS. In this chapter, the flapping-wing problem is simplified to a quasi-steady one and analysed. The work here is complemented with more detailed calculations in Appendix B.

The chapter begins with a look at existing propeller actuator disk (§ 3.1) and blade-element methods. These are then modified (§ 3.2) by incorporating a Glauert-type 3-D analysis and applying them to a propeller-type problem but with insect-like kinematics.

In the later sections of this chapter (§ 3.2.3), the results from this analysis are used to investigate the effects of various flapping-wing parameters such as aspect ratio and stroke amplitude. In this way, favourable wing shape and wing kinematics parameters are identified. The chapter concludes with a brief discussion of the underlying assumptions.

3.1 Introduction

The aim of this project is to develop wing-design guidelines for flapping-wing MAVs; the simplest analysis available is to apply the very successful propeller theory with modifications for flapping-wing flight. As stated previously, the use of propeller theory for insect flight had been realised quite early on in aerodynamic research into this area (see e.g. Osborne, 1951). However, differences exist between propellers and flapping wings, and this calls for a *modified* propeller theory.

In the analysis that follows, only the case for hover has been considered, mainly for simplicity of the analysis. Another advantage of considering hover is that it is the most demanding in terms of power requirement. Therefore, a vehicle capable of hover would be expected to be capable of forward flight over a certain speed range.

3.1.1 Rankine-Froude Momentum Theory

Insects sweep their wings in a stroke plane analogous to the tip-path-plane of rotorcraft. Therefore, the actuator-disk model can be extended to insect-like flapping wings but with a slight difference — a *partial* actuator-disk model (see Figure 3.1).

The classical *Rankine-Froude momentum theory* can then be applied to model some aspects of the flapping-wing system. The wing stroke plane acts as a ‘disk’ across which pressure is imparted regularly in much the same way as a rotor (or propeller). The pressure difference across the disk generates forces that support the weight. The *modus operandi* — the mechanism by which the forces are generated — is ignored in this approach.

For reasons discussed earlier, only the case of hovering flight is considered here. The velocity induced in the far wake downstream is found to be

$$w = 2 \sqrt{\frac{W}{2\rho R^2 \Phi \cos \beta}} = 2 \sqrt{\frac{1}{2\rho \cos \beta} \left(\frac{W}{A} \right)} \quad (3.1)$$

where W is weight, ρ is air density, R is wing length, Φ is stroke amplitude, β is stroke-plane

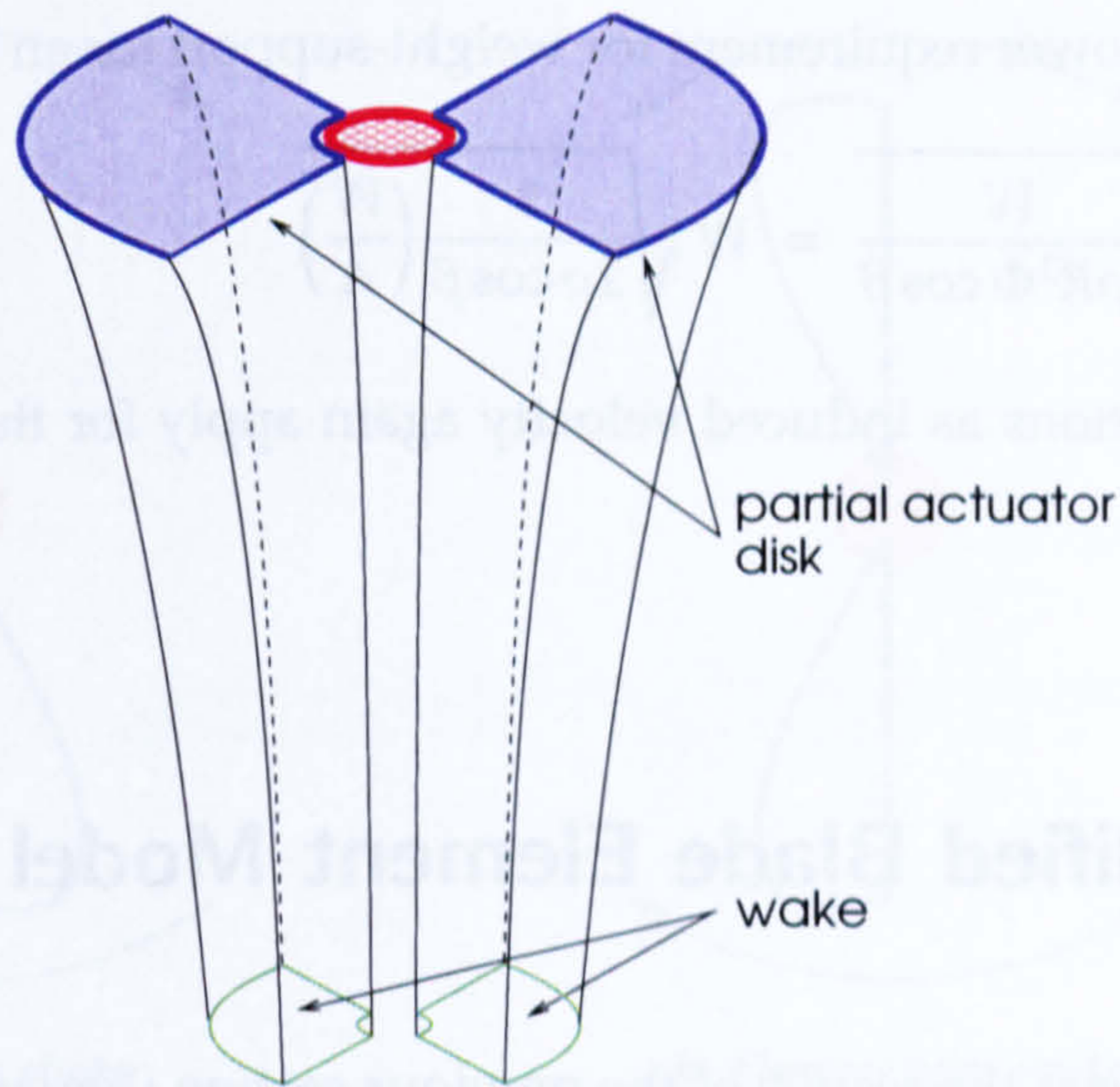


Figure 3.1: Partial actuator disk for flapping wings

angle and A is disk area (full details of the derivations can be found in Appendix B.1). For maximum power economy (maximum thrust per unit power), this induced velocity must be kept to a minimum (Stepniewski & Keys, 1984, ch. II, § 2). In hover, therefore, it is advantageous to have a low *disk loading* W/A and to keep the stroke plane horizontal. A low disk loading corresponds to longer wings and larger sweeping area. Most insects maintain a horizontal stroke plane in the hover.

As mentioned earlier (§1.1.1), DARPA arbitrarily defined MAVs as hand-held flying machines that must fit into a 6 in-square before launch. Hence, R is restricted, and so only Φ and β can be exploited for power economy for a given weight. However, as observed in nature, flapping wings lend themselves to folding. In this way, each folded wing can be about 4–6 in (10–15 cm) long and still meet the design requirement. When extended, these wings can stretch beyond the 6 in-box. So there is a bit of flexibility in the choice of R too.

A preliminary estimate of induced power is also yielded by the Rankine-Froude theory.

The minimum power requirement for weight-support for an insect (or FMAV) in hover is

$$P = W \sqrt{\frac{W}{2\rho R^2 \Phi \cos \beta}} = W \sqrt{\frac{1}{2\rho \cos \beta} \left(\frac{W}{A}\right)} \quad (3.2)$$

The same conditions as induced velocity again apply for the minimum power requirement.

3.2 Modified Blade Element Model

As can be seen from the results of the previous section (Equations 3.1 and 3.2), the simple momentum approach is limited as a tool for wing design as it singles out disk loading as the only important parameter. It makes no attempt to describe any dependence on, for example, planform distribution of the wings. A more powerful and flexible method is, therefore, employed in the form of a *modified blade element method*.

The work in this section is complemented by more detailed calculations in Appendix B.2.

3.2.1 Model

In blade element theory, the wing is divided into thin strips and subjected to a quasi-steady treatment. The overall effect (i.e. forces and moments generated) is obtained by integrating over the entire wing.

One of the results from the simple momentum theory is that it predicts a constant induced velocity (downwash) across the entire actuator disk (Equation 3.1). In reality, this is not the case and a spanwise variation in downwash exists. The particular shape of the downwash distribution depends, amongst other things, on wing shape and kinematics.

In the model used here, the flapping wing is modelled as a propeller but with insect-like wing kinematics. Propeller motion differs from that of a flapping wing in terms

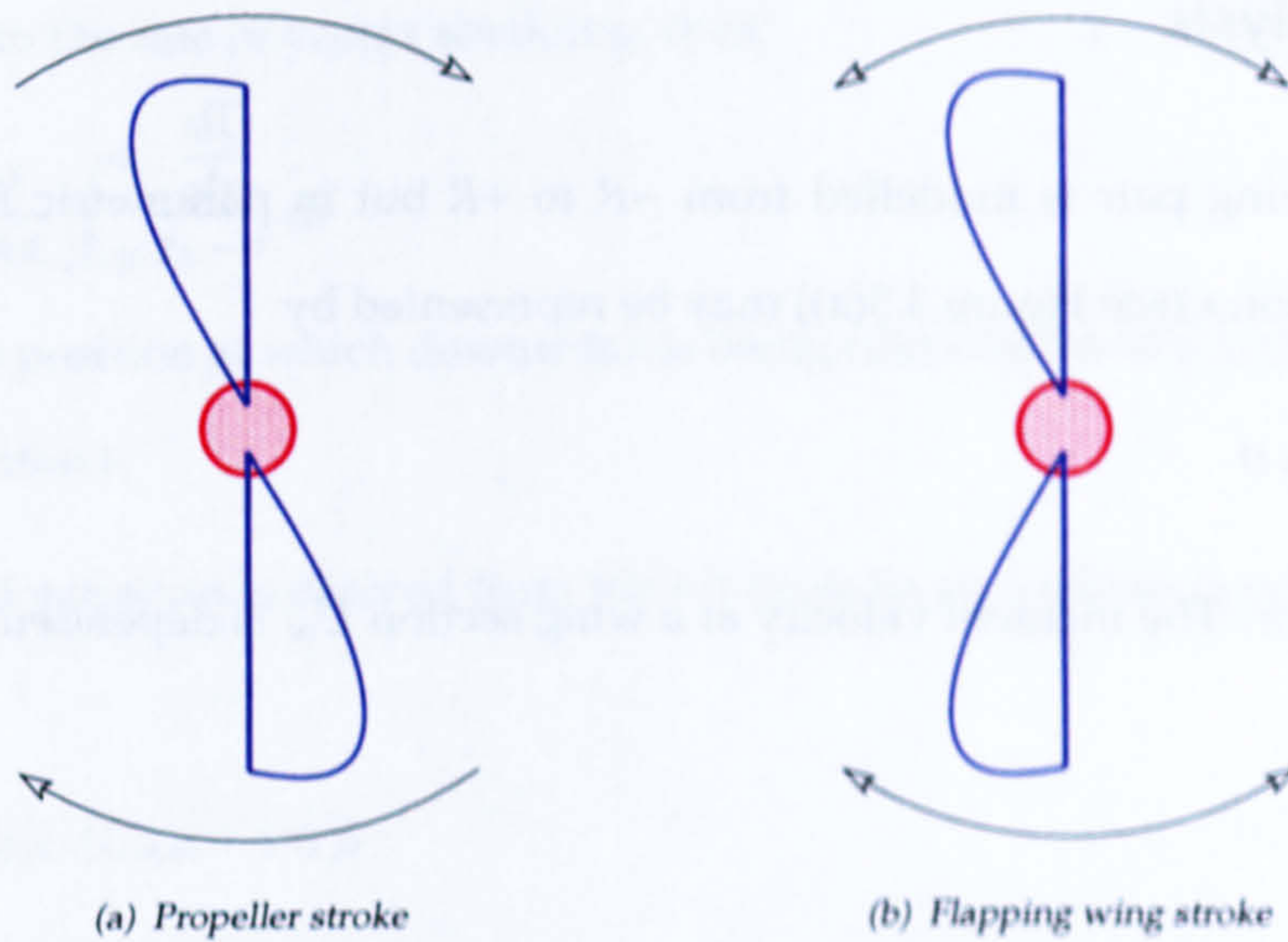


Figure 3.2: Comparison of propeller and flapping-wing strokes

of symmetry (see Figure 3.2). The motion of the two-bladed propeller arrangement (Figure 3.2(a)) is antisymmetrical whereas the flapping-wing system (Figure 3.2(b)) is symmetrical. Their lift distributions, however, are symmetrical¹ (assuming identical wing shape and kinematics for each wing). The latter is the basis for using a propeller to model the wing.

A further difference exists between propellers and flapping wings in terms of their angular positions with respect to each other. Whereas the ‘wings’ of the propeller are always a fixed angle apart (Figure 3.2(a)), in a flapping-wing arrangement the angle between the wing pair starts from a small value, increases to a maximum and then reduces to a similar small value (Figure 3.2(b); see also Figure 2.2). This variation in angular separation is ignored in the current methodology on the basis that any supplementary effects on the wings when they first part at the start of a halfstroke are balanced out by opposite effects when they come together again at the end of the halfstroke. The main benefit of this assumption is, of course, simplification of the problem.

¹In the case of the propeller, the lift distribution is also axisymmetrical.

3.2.2 Analysis

The flapping wing pair is modelled from $-R$ to $+R$ but in parametric form so that a spanwise position r (see Figure 3.3(a)) may be represented by

$$r = -R \cos \theta \tag{3.3}$$

where $\theta = 0 \dots \pi$. The incident velocity at a wing section U_∞ is dependent on the radial position, thus

$$U_\infty = \Omega r \tag{3.4}$$

for hover, where Ω is the angular velocity. The stroke plane of the flapping-wing system is inclined to the horizontal at an angle β (Figure 3.3(b)).

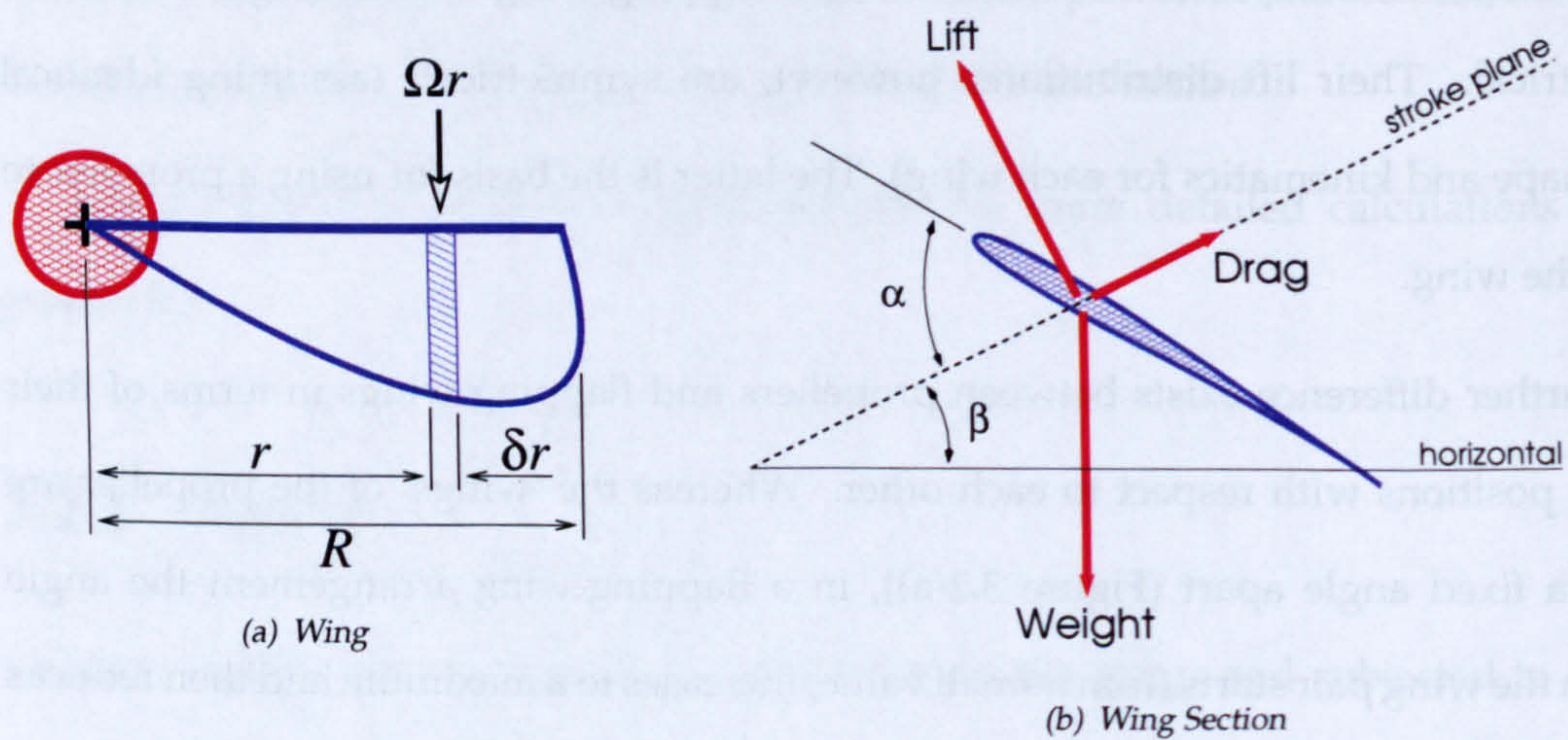


Figure 3.3: Blade element analysis

Note that incident velocity U_∞ is along the stroke plane. By comparison, it is horizontal in Figure B.2(b) in Appendix B where the rotor-wing system has been rotated through the stroke-plane angle β . Also note that lift is defined as the force normal to the stroke plane and drag as the orthogonal force to lift, parallel to the stroke plane.

By using the method proposed by Glauert (1959), it is possible to estimate the downwash at any spanwise position r . The method depends on two equations. The first relates

downwash w to the rate of vortex shedding, thus²

$$w(r_1) = \frac{1}{4\pi} \int_{-R}^R \frac{\frac{d\Gamma}{dr}}{r_1 - r} dr \quad (3.5)$$

where r_1 is the position at which downwash is being calculated and Γ is the circulation at spanwise position r .

The second equation is derived from the lift formula and relates downwash to circulation Γ , thus

$$\Gamma(r) = \frac{1}{2} a_0 c_r (U_\infty \alpha - w(r)) \quad (3.6)$$

where a_0 is the two-dimensional lift-curve slope³, α is angle of attack and c_r is chord length at spanwise distance r . From this point onwards, all derivations presented are original.

By writing Γ as a Fourier series in θ such that

$$\Gamma(\theta) = -4 \Omega R^2 \cos \theta \sum_{n=1}^{\infty} A_n \sin n\theta \quad (3.7)$$

and using the two relations above (Equations 3.5 and 3.6), it is possible to compute a unique set of constants A_n for a given wing⁴ by judicious use of the Glauert integrals (see Appendix A.1). Details of the derivations in the analysis can be found in Appendix B.2.

Main Results from Analysis

The expression for downwash at a general point θ is then found to be

$$w(\theta) = -\Omega R \left[\sum_{n=1}^{\infty} \left(n A_n \frac{\cos \theta \sin n\theta}{\sin \theta} \right) + \sum_{n=1}^{\infty} (A_n \cos n\theta) \right] \quad (3.8)$$

The total lift L of the flapping-wing system and its (induced) drag D_i can be found by integrating

$$L = \int_{-R}^R \rho U_\infty \Gamma dr \quad \text{and} \quad D_i = \int_{-R}^R \rho w \Gamma dr$$

²Note that here downwash $w(r_1)$ is positive downwards.

³ a_0 is taken equal to 2π throughout the analysis.

⁴Due to the symmetrical nature of the lift distribution, only odd values of n occur.

respectively, where ρ is air density. For lift, the expression thus obtained is

$$L = \frac{1}{2} \rho \pi \Omega^2 R^4 (A_1 + A_3) \quad (3.9)$$

and for induced drag is⁵

$$D_i = \rho \pi \Omega^2 R^4 \left[\sum_{n=1}^{\infty} n A_n^2 + \sum_{n=3}^{\infty} (n-1) A_{n-2} A_n \right] \quad (n \text{ odd}) \quad (3.10)$$

A more useful quantity than drag in propeller-type problems is (induced) torque Q_i which is evaluated as

$$Q_i = \int_{-R}^R \rho w \Gamma r \, dr \quad (3.11)$$

Unlike lift and drag, the integration of the expression for total torque does not yield a closed expression so it must be computed numerically⁶. For comparison purposes, more useful than torque is the lift-to-torque ratio

$$\frac{L}{Q}$$

Note that the subscript i has been dropped from Q for clarity. It may further be noted that for a nonzero stroke-plane angle β , the drag actually contributes to the vertical 'lifting' force due to an upward component, thus

$$W = L \cos \beta + D_i \sin \beta \quad (3.12)$$

where W is weight supported.

3.2.3 Results

The analysis in the previous section can be used to compare various wing shapes and kinematics for flapping-wing applications. Before comparisons can be made, however,

⁵Drag is nonzero because an artefact of the analysis is that drag is evaluated as Total Drag = $2 \times$ Drag of either wing

⁶In fact, the integral for torque actually yields a zero sum because the positive drag on each wing (see Footnote 5) gives a net zero moment instead of a couple. Torque must, therefore, be computed as Total Torque = $2 \times$ Torque on either wing

it is necessary to have a ‘default’ set of parameters for wing shape and wing kinematics. Keeping in mind the DARPA guidelines for MAVs (§ 1.1.1) and ‘typical’ insect data, the values shown in Table 3.1 were chosen.

Table 3.1: ‘Default’ values for wing shape and wing kinematics comparisons

Parameter	Value
Flapping Frequency, f	20 Hz
Stroke Amplitude, Φ	120°
Stroke Plane Angle, β	0°
Wing Length, R	125 mm (=0.125 m)
Aspect Ratio, AR	7.5
Wing Area, S	$8.3333 \times 10^{-3} \text{ m}^2$

By varying only one parameter at a time, a comparison of various wing planform shapes can be made. The wing shapes compared were rectangular, elliptical, reverse elliptical, triangular, reverse triangular and full elliptical (see Figure 3.4). In each of the comparisons that follows, the values of the parameters given in Table 3.1 (e.g. wing length, aspect ratio, etc.) were kept at the same values for each planform shape.

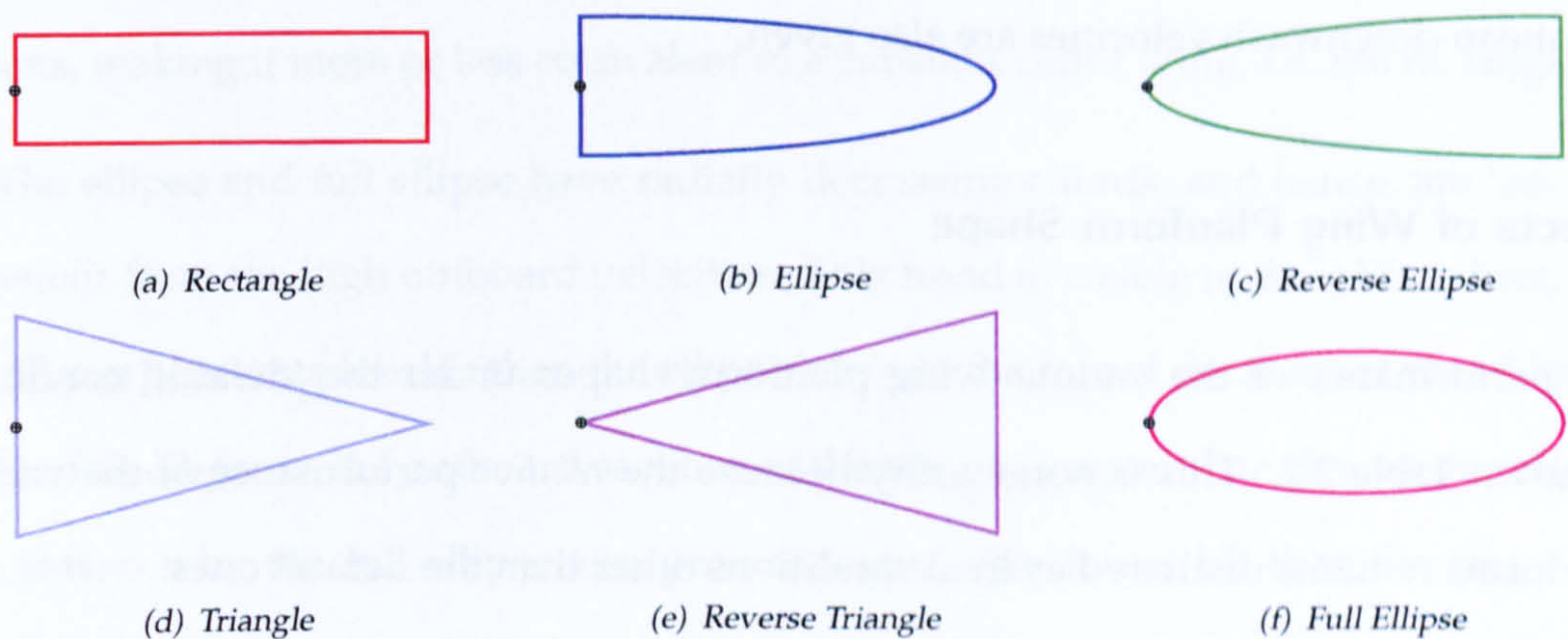


Figure 3.4: Wing planform shapes considered (only one wing is shown). The marker indicates point of rotation

A fixed ‘shape’ of wing kinematics was used, based on the flat figure-of-eight kine-

matics provided by Dickinson (priv. comm.). The pitching and sweeping characteristics for one halfstroke of the kinematics are shown in Figure 3.5.

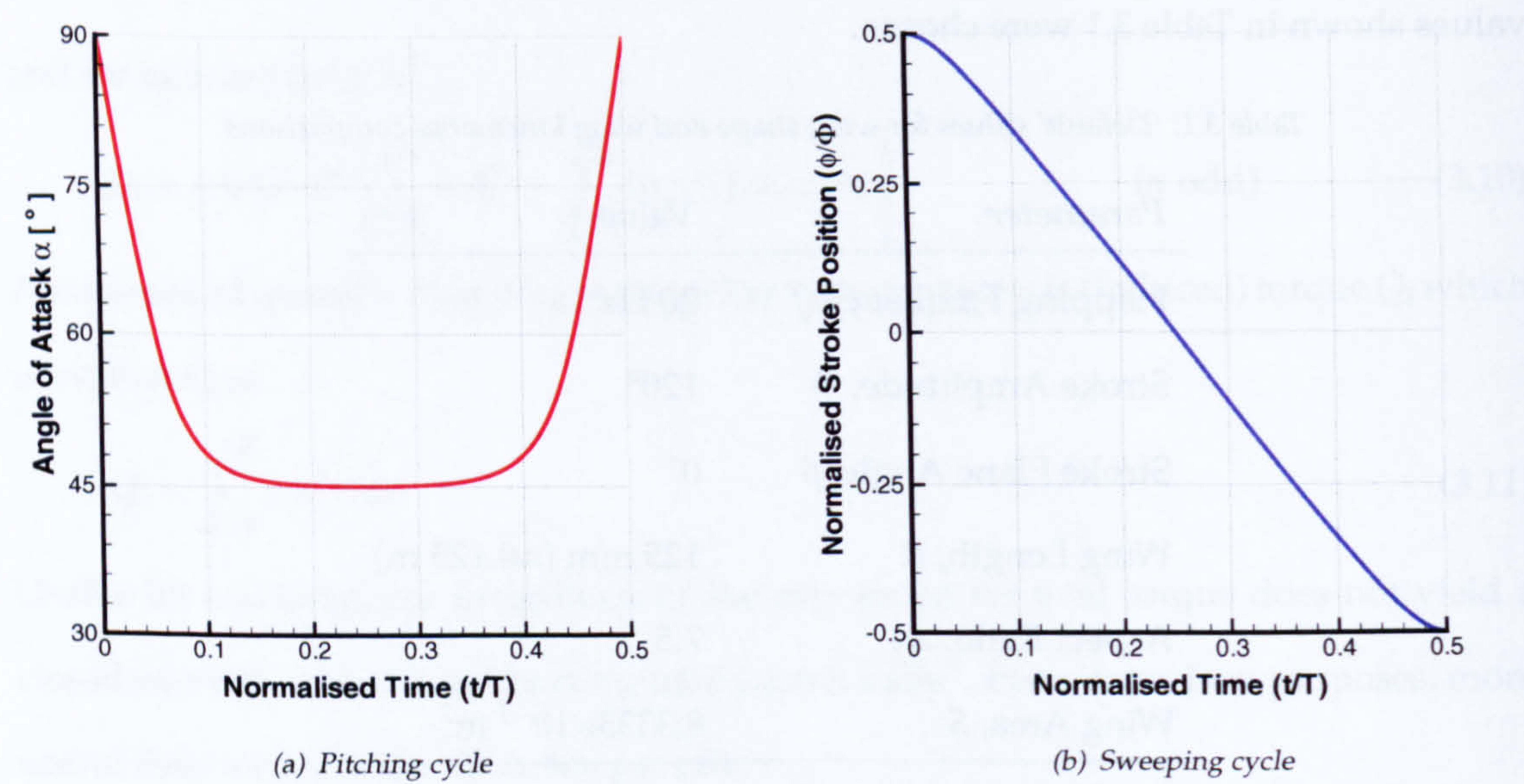


Figure 3.5: Wing kinematics for one halfstroke

The quasi-steady model developed was implemented numerically by means of a FORTRAN routine and then used to compare the various wing parameters, kinematics and shapes discussed above. The results from this study are now presented.

The results broadly categorise into plots of mean lift and mean lift-to-torque ratio. Further plots showing spanwise distributions of mean circulation, 3-D-lift-to-2-D-lift ratio and mean downwash velocities are also given.

Effects of Wing Planform Shape

The performance of the various wing planform shapes under the 'default' conditions is shown Table 3.2. This is noteworthy because the *relative* performance of the various planforms remains unaltered even at conditions other than the default ones.

Lift Characteristics From the table, it can be seen that the trend in lift performance consists of a remarkably high value, an unusually low value and a band in between for

Table 3.2: Performance of wing planforms under ‘default’ conditions[†]

Wing Planform	Mean Lift (N)	Mean Torque (Nm)	Mean Lift-to-Torque Ratio [‡]
Rectangle	0.580264	0.019323	1.0010
Ellipse	0.515758	0.013198	1.3026
Reverse Ellipse	0.632138	0.023591	0.8932
Triangle	0.578814	0.015444	1.2493
Reverse Triangle	0.889917	0.046476	0.6383
Full Ellipse	0.369119	0.007215	1.7054

[†]Time-averaged values over one halfstroke are used for computing mean values for lift, torque and lift-to-torque ratio.
[‡] Lift-to-torque ratio is nondimensionalised with mean chord.

the remaining planforms. The reverse triangle performs best in terms of lift because it has the greatest area outboard where velocities are greatest (due to the radially increasing flapping velocity). Understandably, the second highest is the reverse ellipse due to the similarity it shares with the reverse triangle in the outboard sections of the wing (both wings grow in chord length in the outboard direction; cf. Figures 3.4(c) and 3.4(e)).

The rectangular and triangular wings are next with very similar values for lift. Remembering that all wings have the same area, the longer chord lengths in the inboard sections of the triangular wing exactly compensate for reduced chord in its outboard regions, making it more or less equivalent to a constant chord wing, i.e. the rectangle.

The ellipse and full ellipse have radially decreasing chords, and hence, are less able to benefit from the high outboard velocities. This trend is visible in their lift values. The full ellipse generates less lift than the ellipse because the rate of decrease of chord length for the full ellipse in the outboard sections of the wing is greater than that for the ellipse. The reason why the full ellipse also produces markedly lower lift than the rest of the planforms is because it is the only wing that tapers in both root and tip directions⁷.

⁷In inboard regions where flow velocities are low, a high chord length is beneficial for lift. In outboard regions, larger chords harness more lift from the high velocities. The full ellipse, therefore, loses out in both

Torque Characteristics The trend for torque characteristics is similar to the lift characteristics — the wings producing high lift also give high torque. This has two implications — that lift is related to (induced) drag and that the high-drag regions are further outboard so as to exert greater moments.

The trend can be better understood by considering the circulation distribution along the wingspans for the various planforms (see Figure 3.6). Circulation generally increases in the spanwise direction as flow speeds increase but decreases with proximity to the wing tip. It is also proportional to local chord length (see Equation 3.6). The plots are, therefore, generally skewed towards the tip, more so for wings that end ‘abruptly’ (rectangle, reverse ellipse and reverse triangle) because the effects of longer chord lengths in the outboard regions offsets the effect of proximity to the tip⁸. The reverse triangle wing has the largest chords in the tip region, and therefore, the greatest torque. By contrast, planforms whose chord lengths gradually decrease to zero at the wing tip (ellipse, triangle and full ellipse) show a gentler decline in circulation in the tip regions.

Because torque is directly proportional to circulation and distance from the point of rotation (Equation 3.11), the wings with nonzero tip chords produce greater torque. Such wings also have larger tip vortices as can be inferred from their steeper decline in circulation. Wing sections with higher values of $\frac{d\Gamma}{dr}$ have greater downwash, and therefore, greater induced drag (and hence, torque) (see Equations 3.5 and 3.11).

The trend in downwash distribution is worthy of mention (Figure 3.7). Downwash generally increases towards the tip due to proximity to the tip vortex and higher velocities there (see Equation 3.8) and is greater for wings with nonzero tip chords for reasons explained above. The triangle planform, however, exhibits a rather unusual downwash distribution. Because its tip vortex is relatively weak, its spanwise chord distribution

areas.

⁸For this reason, the comparatively long chord lengths of the inboard sections of the triangle planform shift the bulk of its graph away from the wing tip.

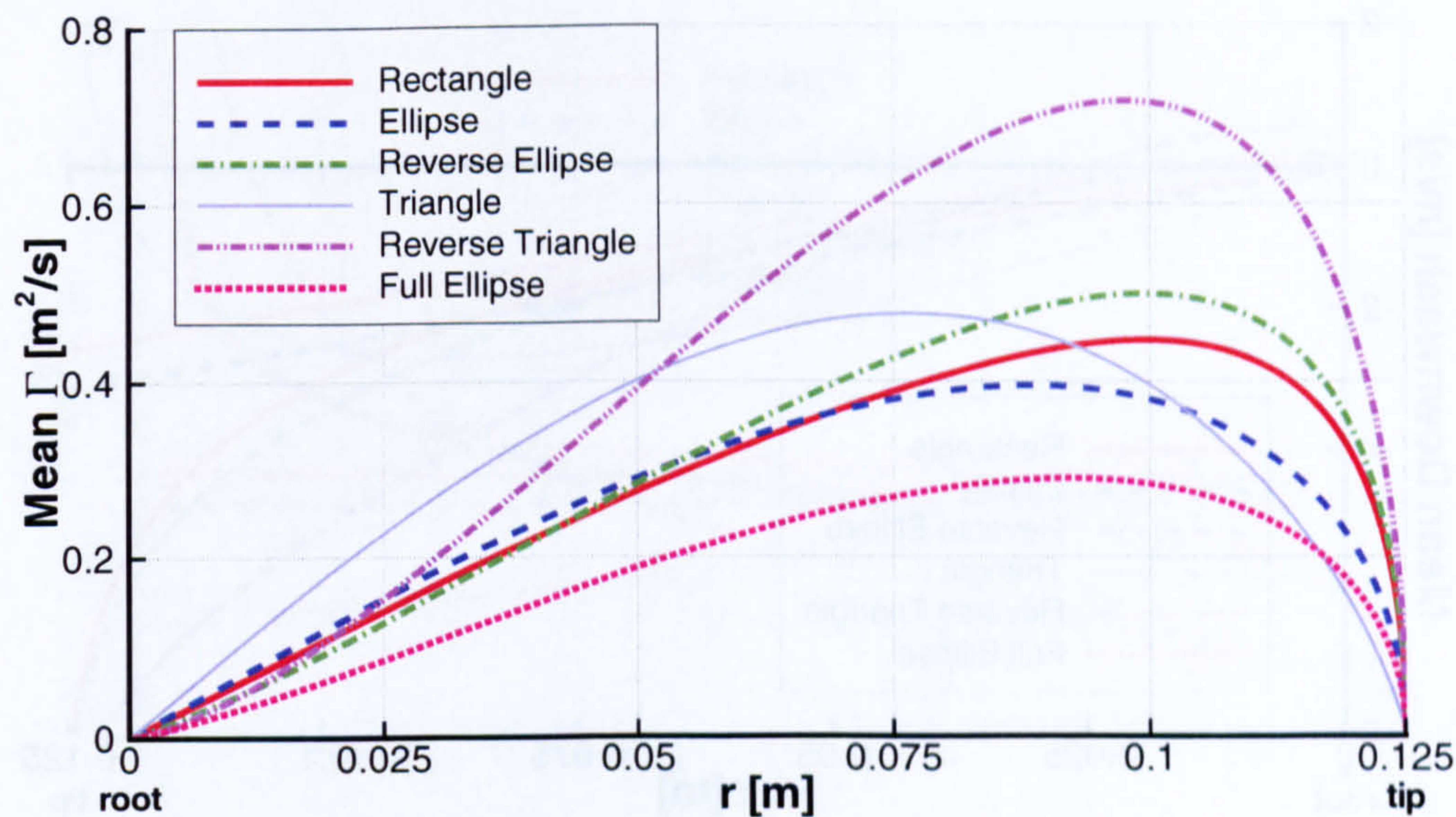


Figure 3.6: Spanwise distribution of circulation for wings

ensures that upwash induced by inboard sections outweighs the downwash from the weak tip vortex resulting in net upwash in the tip regions.

The effect of upwash is to increase local angle of attack at the wing section, and hence, lift. Because most wings have downwash induced across their span, 3-D lift is generally lower than the equivalent 2-D lift (see Figure 3.8). By the same argument, at sections where there is net upwash, the lift generated is greater than the 2-D equivalent as the graph shows.

Some interesting patterns are worthy of note in Figure 3.8. For wings with nonzero tip chords, lift ratio drops to 0 at the wing tip, whereas for wings with nonzero root chords, lift ratio approaches a constant value (< 1) at the wing root. By contrast, wings with zero root chords have high lift ratios ($\gg 1$) at the wing root, whereas for wings with zero tip chords (with the exception of the triangle), lift ratio approaches a constant value (< 1) at the wing tip.

Interestingly, the ellipse planform shape maintains a constant lift ratio of about 65% throughout. In fixed-wing applications, an elliptical planform gives constant downwash

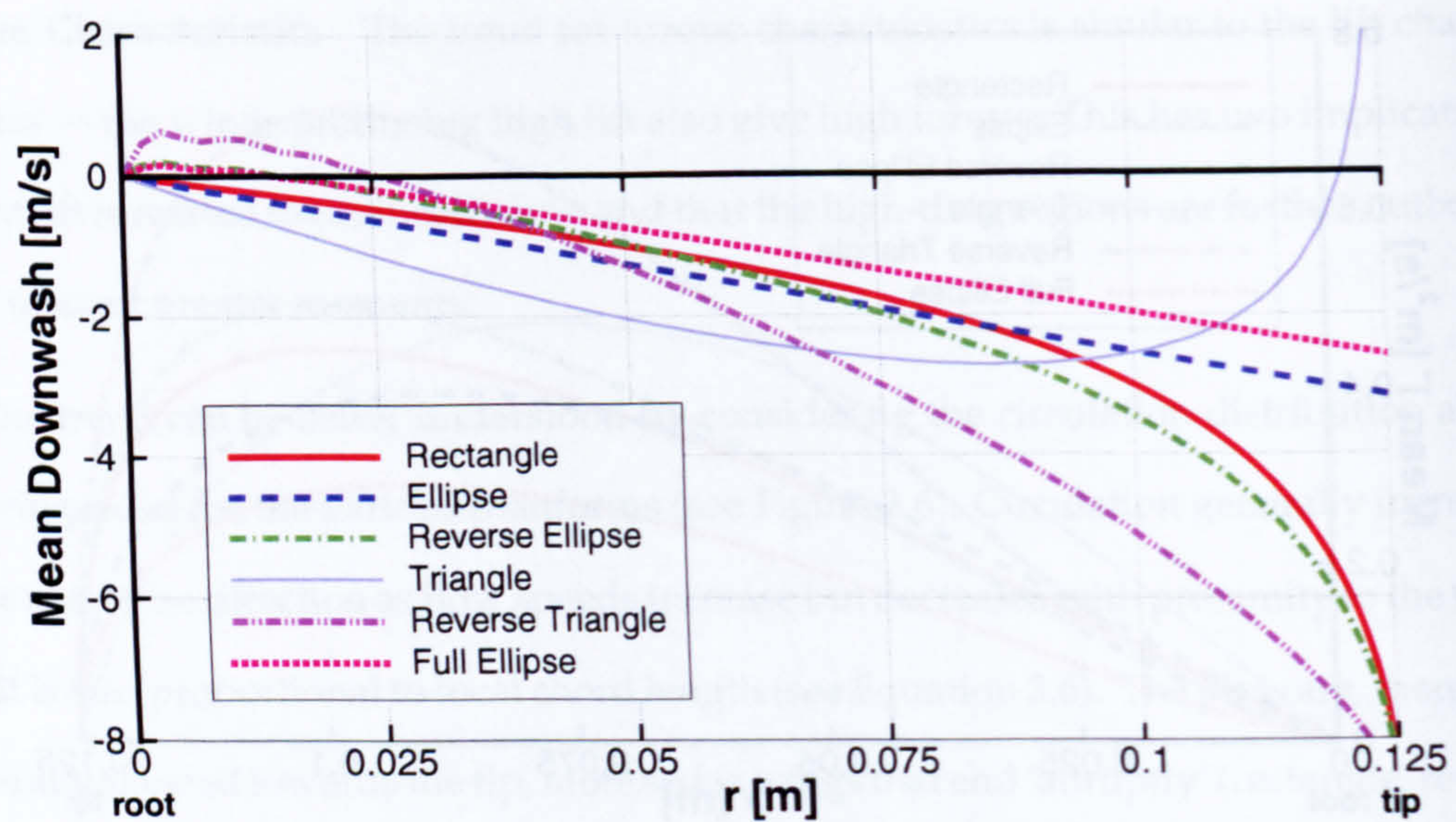


Figure 3.7: Spanwise distribution of downwash velocity for wings

across the span (and minimum induced drag) but in the current application this is not possible due to the radially increasing velocities. Instead, the ellipse planform exhibits a constant spanwise 3-D-lift-to-2-D-lift ratio⁹.

A design guideline that can be drawn from Figure 3.8 is that wings with nonzero tip chords are rather inefficient in the tip regions where flow velocities are highest. This is because their wings end ‘abruptly’, leading to stronger tip vortices and downwash, and hence lower lift. Such wings are, therefore, unable to reap fully the potential benefit that tip regions have to offer without suffering a drag penalty.

In rotary-wing applications, minimum induced power comes from constant downwash. If the current model is ‘reverse-engineered’, it is possible to derive a wing planform shape that gives constant downwash. A typical shape of such a planform is shown in Figure 3.9 below. In rotary wings, wing twist is usually incorporated so that a near-constant-chord wing results.

⁹In the current setup, the full ellipse actually gives the best minimum drag characteristics.

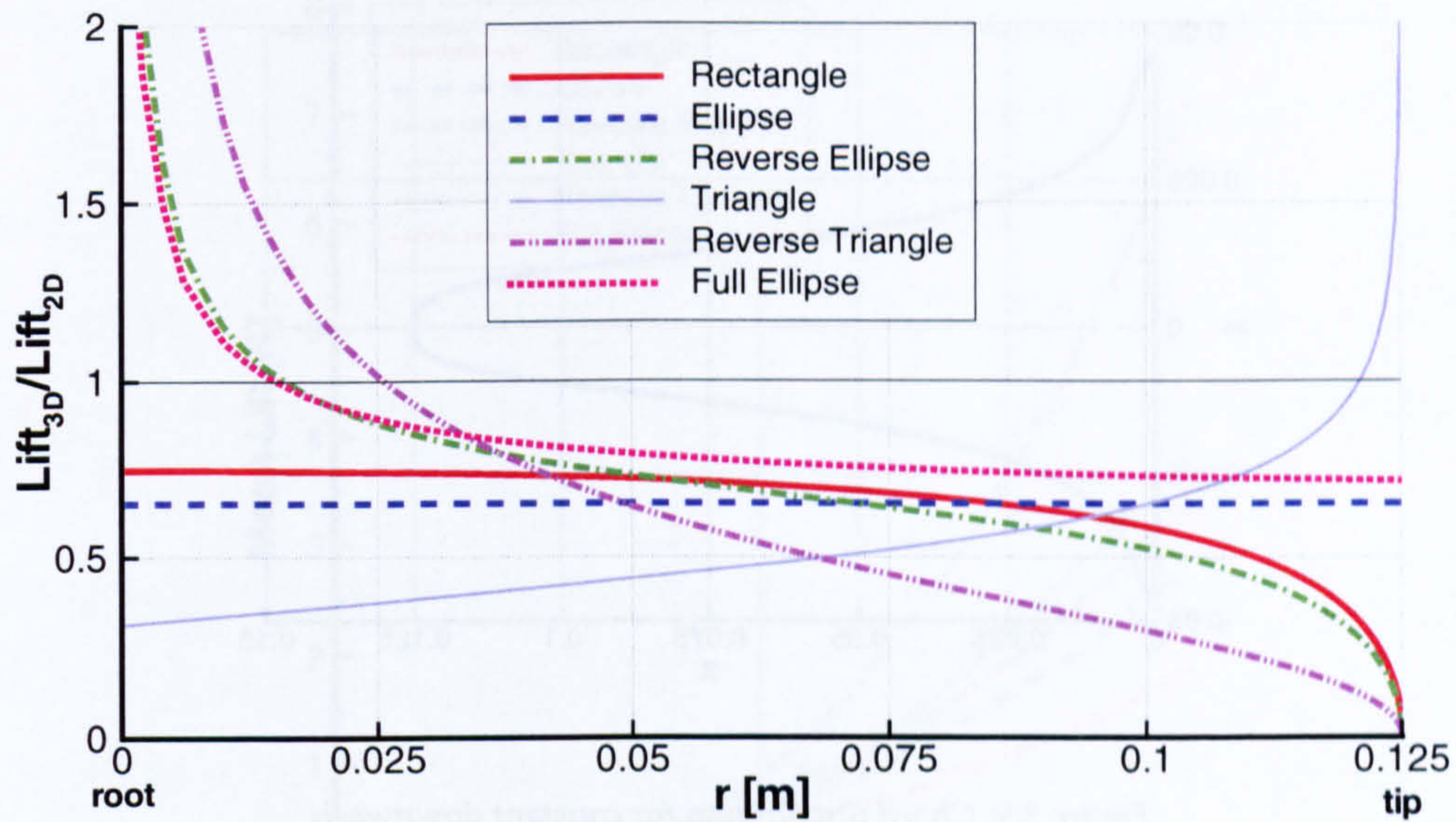


Figure 3.8: Spanwise distribution of 3-D-lift-to-2-D-lift for wings

Lift-to-Torque Ratio Lift-to-torque ratio L/Q is representative of the power required to maintain a certain value of lift and is a key design parameter for propellers. It is, therefore, desirable to have the highest possible value of L/Q for maximum power efficiency. Because the trends for both lift and torque characteristics are similar with regards to planform shapes, the trend in terms of lift-to-torque ratio is not immediately obvious.

In fact, the trend is the exact opposite of the lift and torque trends with the full ellipse performing best. The wing planforms with zero tip chords give better lift-to-torque ratios than the rest because they have weaker tip vortices, and hence, less induced drag. Therefore, from a minimum-power perspective, the most favourable wing shape for a flapping-wing system would be a full ellipse. From a lift viewpoint, however, the full ellipse performs markedly worse than the others. Therefore, some other wing shape parameters would need to be adjusted (such as wing length or wing area; see next section), if possible, to get the desired performance.

A true comparison of the relative merits of the various wing planforms can only be made by comparing their lift and lift-to-torque ratios simultaneously. This is discussed

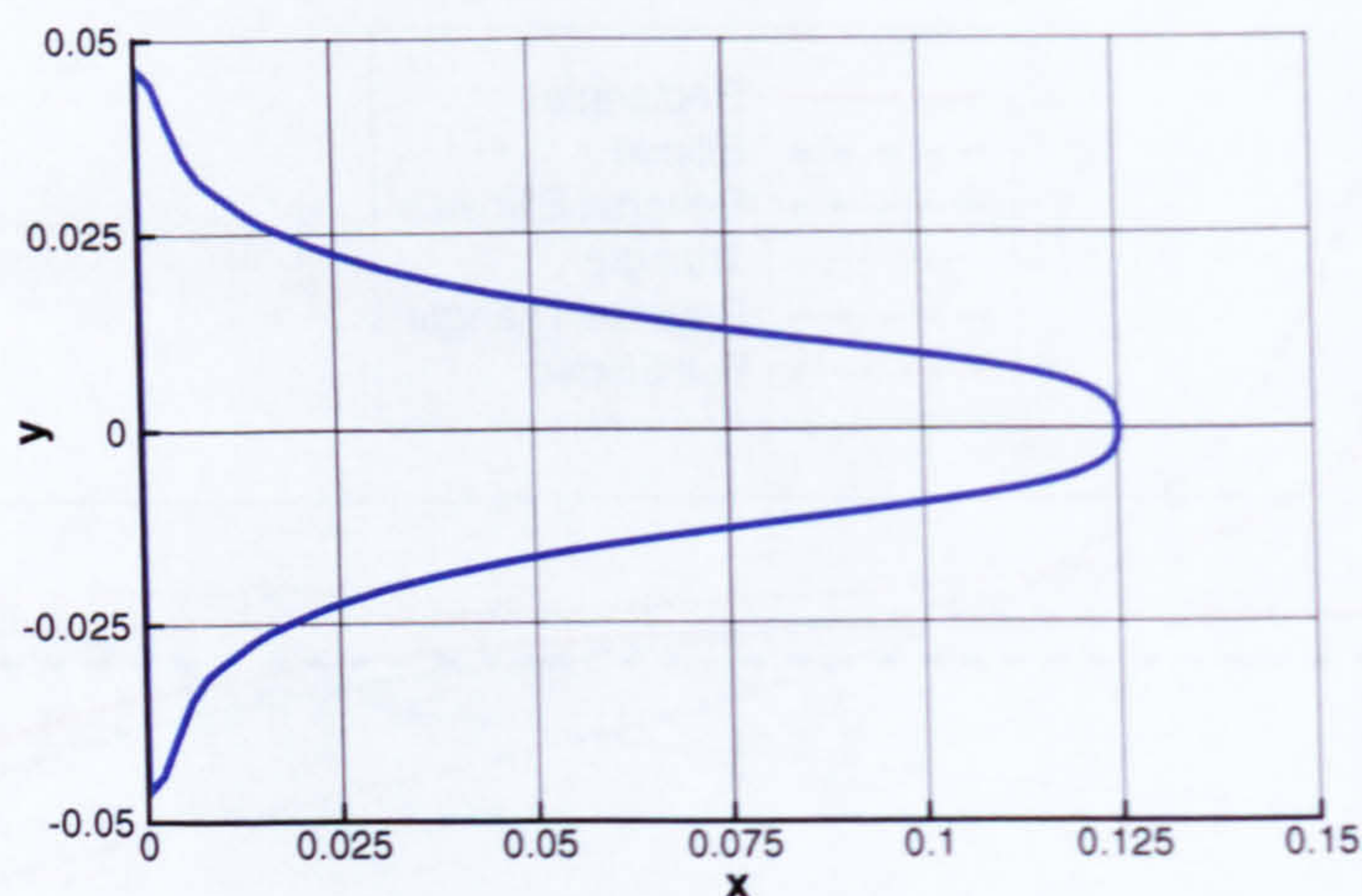


Figure 3.9: Chord distribution for constant downwash

later on p. 88.

Effects of Wing Shape Parameters

In the following results, stroke-plane angle $\beta = 0$ so mean lift is equal to the mean upward force generated for weight support (see Equation 3.12). Therefore, only plots of mean lift are shown as they are sufficient. It may also be noted that due to the computational overhead and visualisation difficulty, only one parameter is varied at a time.

Wing Length R The variation of mean lift with wing length R is plotted in Figure 3.10. Although wing area is kept constant throughout, mean lift L increases with R because for longer wings more of the wing is exposed to higher velocities. Lift force is a function of the square of velocity, hence the parabolic shape of the plots. The trend in lift values for the various planforms is the same as that for the default conditions discussed above.

However, extending wing length comes with a cost, as shown by the lift-to-torque ratio graph of Figure 3.11. As wing length increases, so does the moment arm, and hence the torque, signifying increasing power requirements. Therefore, the plots show a

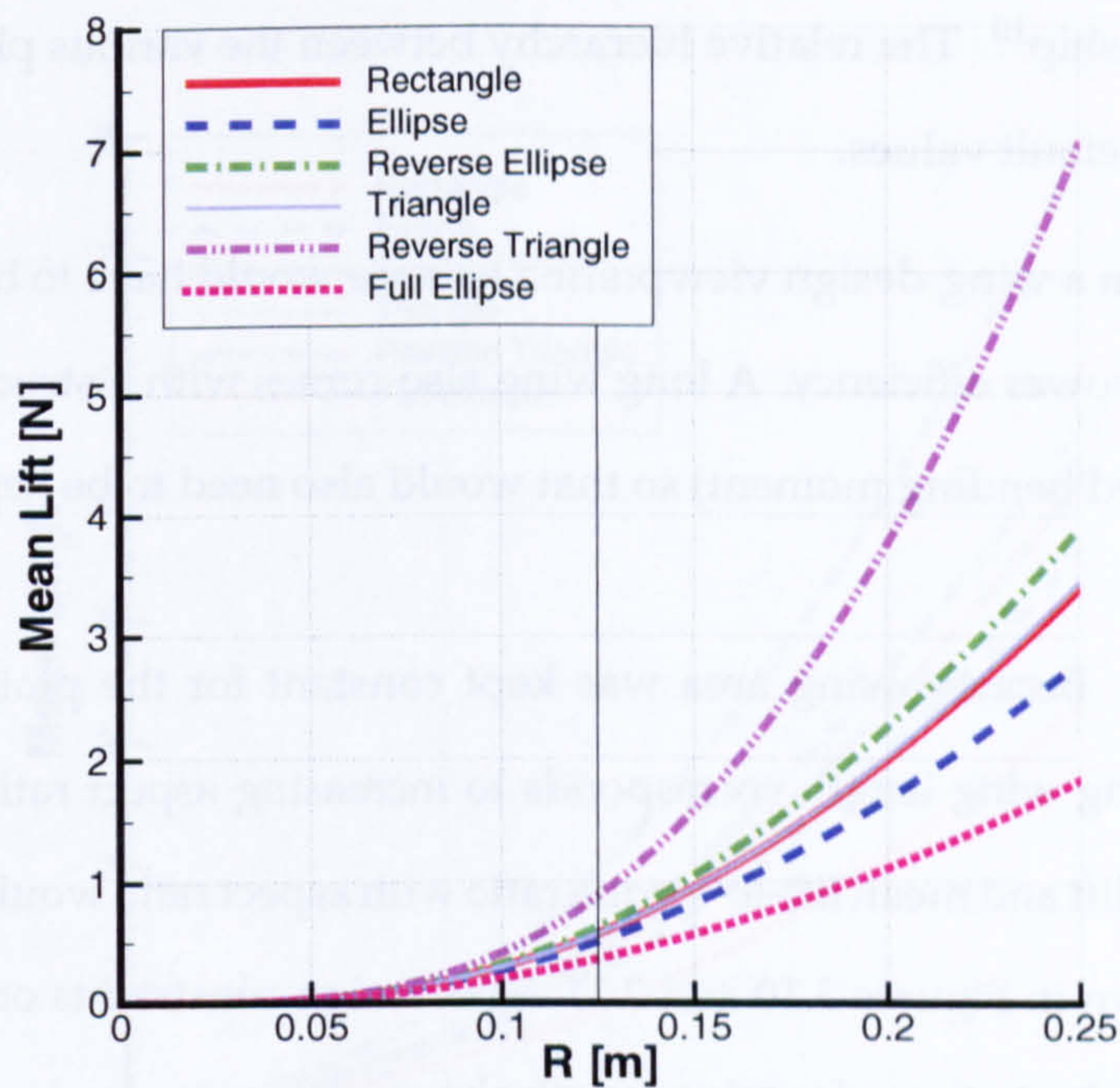


Figure 3.10: Variation of mean lift with wing length R (at constant wing area). Trend is similar for variation with aspect ratio $\mathcal{AR}$. The vertical line marks the ‘default’ conditions

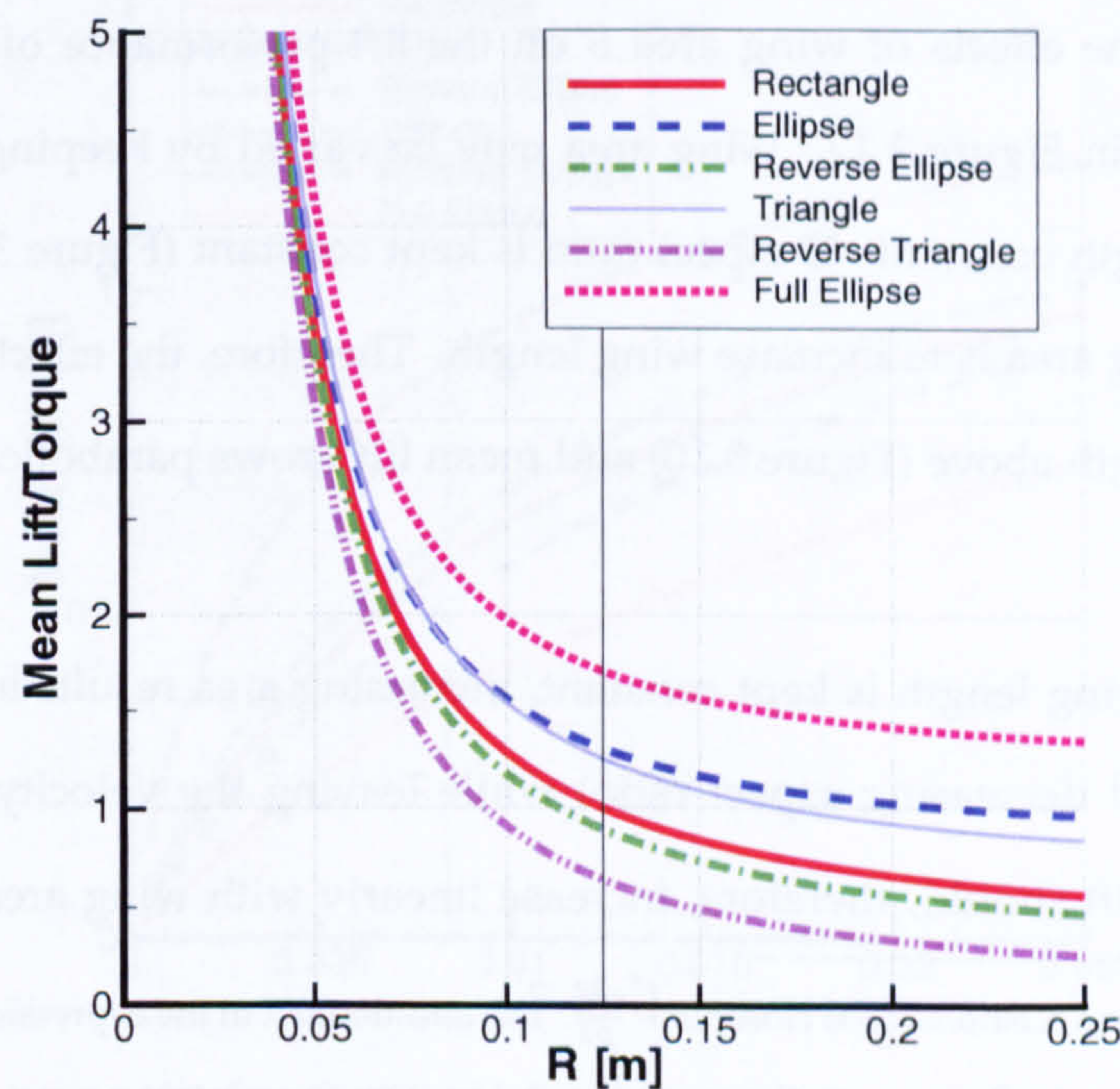


Figure 3.11: Variation of mean lift-to-torque ratio with wing length R (at constant wing area). Trend is similar for variation with aspect ratio $\mathcal{AR}$. L/Q is nondimensionalised by mean chord. The vertical line marks the ‘default’ conditions

reciprocal relationship¹⁰. The relative hierarchy between the various planforms remains preserved at the default values.

Therefore, from a wing-design viewpoint a balance would have to be struck between wing length and power efficiency. A long wing also comes with a structural implication (owing to increased bending moment) so that would also need to be kept in perspective.

Aspect Ratio $\mathcal{R}$ Because wing area was kept constant for the plots in Figures 3.10 and 3.11, increasing wing length corresponds to increasing aspect ratio $\mathcal{R}$. Hence, the variation of mean lift and mean lift-to-torque ratio with aspect ratio would follow a similar trend to that shown in Figures 3.10 and 3.11. The design constraints on aspect ratio are, therefore, also similar to those for wing length above.

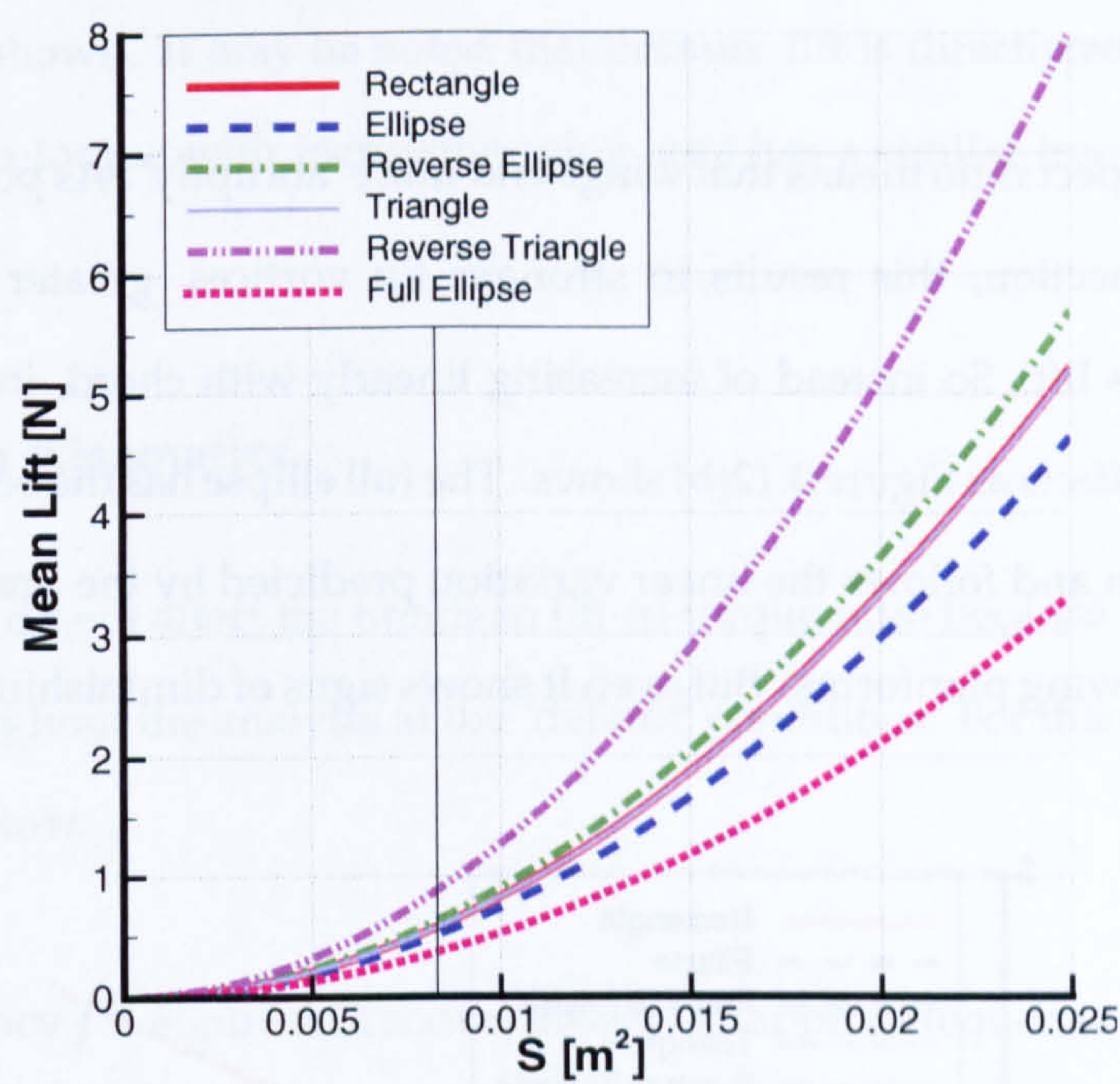
So, whereas large aspect ratios are beneficial in fixed-wing aircraft from a drag perspective, this advantage does not carry over for flapping-wing aircraft.

Wing Area S The effects of wing area S on the lift performance of a flapping-wing aircraft is shown in Figure 3.12. Wing area may be varied by keeping either of aspect ratio or wing length constant. If aspect ratio is kept constant (Figure 3.12(a)), the effect of increasing wing area is to increase wing length. Therefore, the effect is similar to that seen for wing length above (Figure 3.10) and mean lift grows parabolically as the square of wing area.

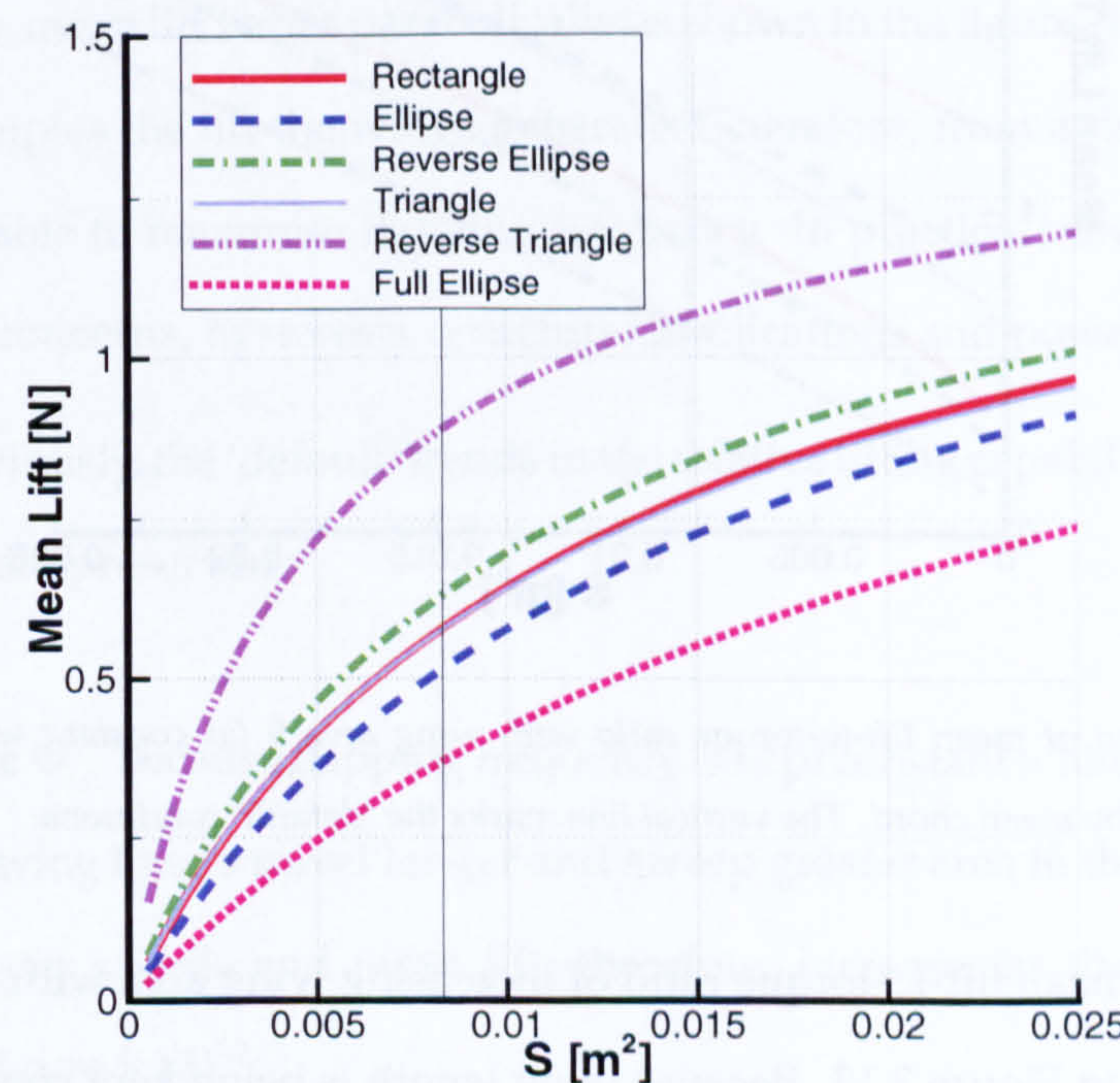
If, however, wing length is kept constant, increasing area results in increasing local chord length (and decreasing aspect ratio) while leaving the velocity distribution unchanged. Mean lift should, therefore, increase linearly with wing area¹¹. However, as

¹⁰Actually, the inverse relationship is closer to $\sim \frac{1}{R^2}$. The additional R in the expression is an artefact of the nondimensionalisation. As R increases, the mean chord decreases to maintain a constant wing area. Hence L/Q varies as $\left(\frac{1}{R}\right)^2$.

¹¹Recall that the general expression for lift for a spanwise section may be written as $\delta L = \frac{1}{2}\rho U_\infty^2 c \delta r$, where



(a) With aspect ratio kept constant



(b) With wing length kept constant

Figure 3.12: Variation of mean lift with wing area S . The vertical line marks the ‘default’ conditions

shown by the plots in Figure 3.12(b), this is not the case. The reason for this can be explained as follows.

Decreasing aspect ratio means that wings end more 'abruptly'. As pointed out already in the previous section, this results in stronger tip vortices, greater downwash, and consequently, less lift. So instead of increasing linearly with chord, increases in lift are subdued by 3-D effects as Figure 3.12(b) shows. The full ellipse has the best combination of chord distribution and follows the linear variation predicted by the preceding argument best amongst the wing planforms. But even it shows signs of diminishing returns as wing area increases.

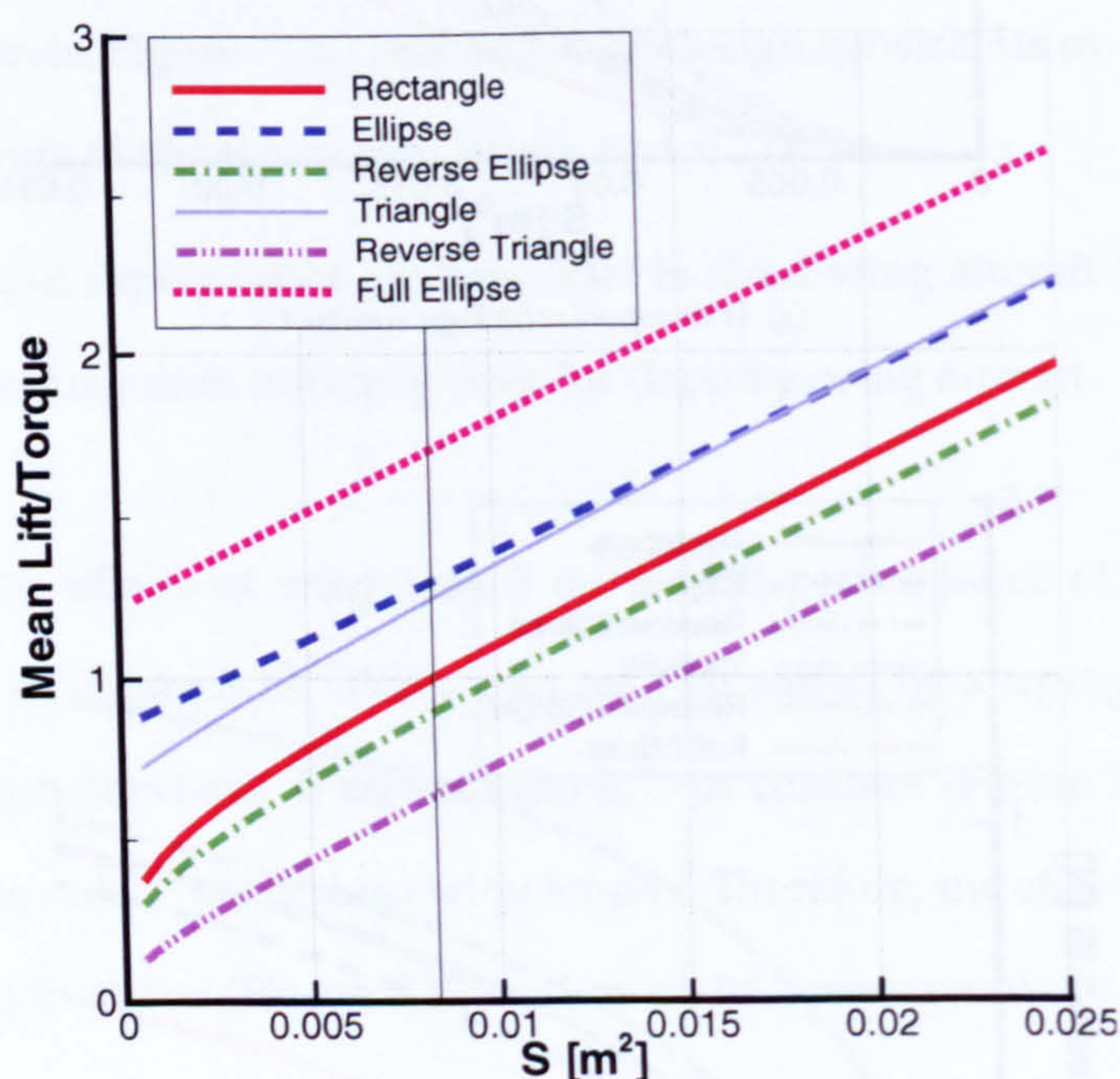


Figure 3.13: Variation of mean lift-to-torque ratio with wing area S (at constant wing length). L/Q is nondimensionalised by mean chord. The vertical line marks the 'default' conditions

The effect on mean lift-to-torque ratio of increasing wing area with wing length constant is as shown in Figure 3.13. Because wing length is being kept constant, the lift and torque vary only with chord length. The ratio of lift-to-torque L/Q is, therefore, constant. However, because L/Q is nondimensionalised using chord and because chord is increasing, c is local chord length and the rest of the symbols have their usual meanings.

ing linearly with area, the relationship between mean lift-to-torque ratio with wing area is also linear, as shown. It may be noted that because lift is directly related to drag, the variation of mean torque with increasing wing area has a similar trend to that seen for mean lift above.

Effects of Wing Kinematics

Wing kinematics do not affect the trends in lift-to-torque ratio because wing shape is left unchanged throughout the analysis at the 'default' conditions. For this reason, L/Q plots are not shown below.

Flapping Frequency f Figure 3.14 shows the effect flapping frequency f has on the mean lift generated through a halfstroke. As all other parameters are kept unchanged (such as stroke amplitude), increasing f increases flapping velocities. Because lift varies as the square of velocity, mean lift varies parabolically as shown in the figure. Doubling flapping frequency quadruples the lift the wings generate. Therefore, from a design perspective, it is highly desirable to maximise flapping frequency. In practice, however, this may be limited by noise concerns, hysteresis, structural implications and power requirements.

As noted previously, the 'default' trends in the relative lifting capabilities of the various wing planforms are preserved.

Stroke Amplitude Φ Because flapping frequency is kept constant while stroke amplitude is increased, the wing has to travel longer and sweep greater area in the same time. This translates to greater speeds and mean lift, therefore, increases as the square of stroke amplitude (see Figure 3.15)¹².

As noted for flapping frequency above, doubling stroke amplitude quadruples mean

¹²Whereas amplitude is normally taken to mean the maximum deviation from the mean, in the current context, stroke amplitude refers to the maximum arc swept by the flapping wings.

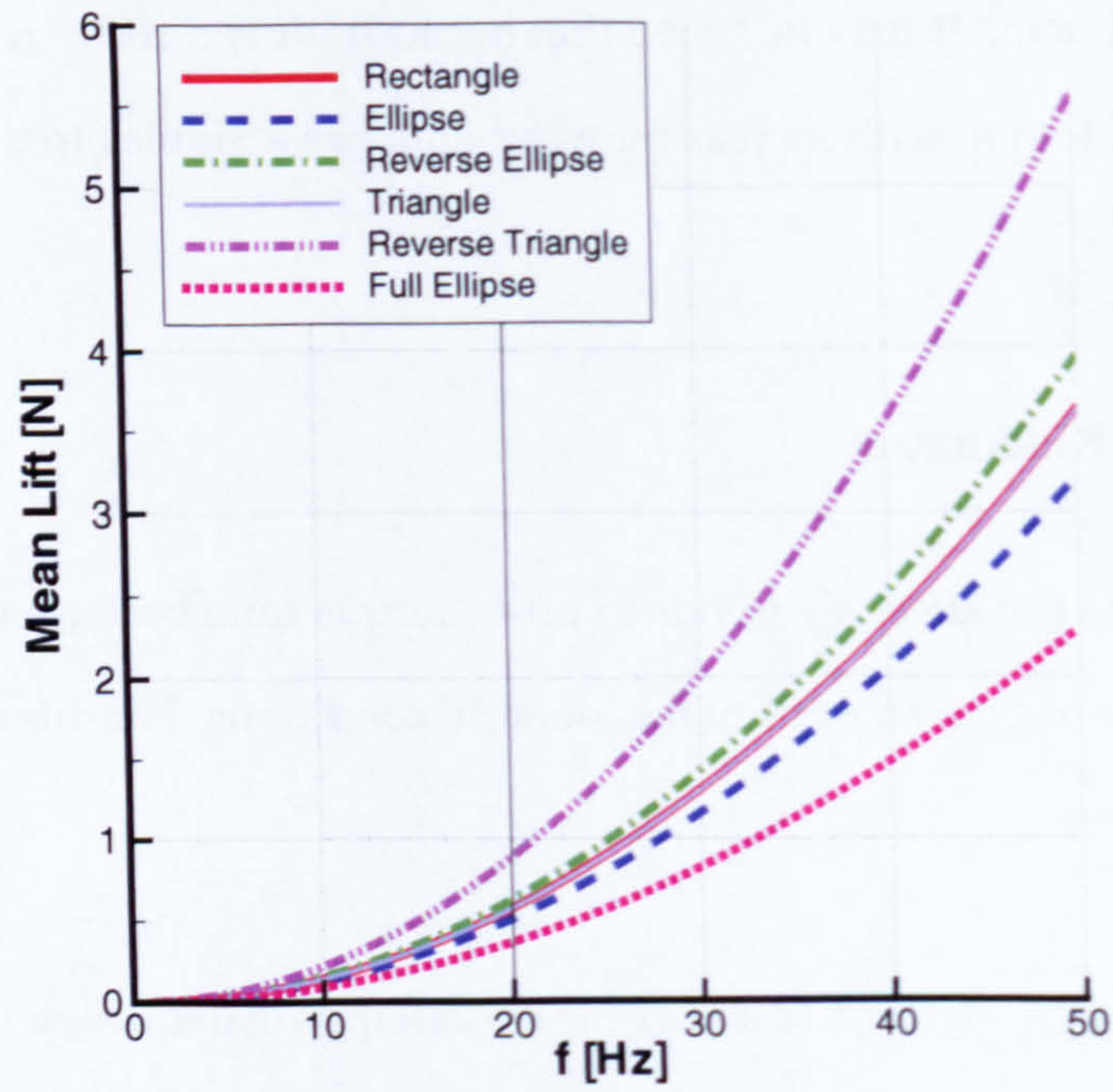


Figure 3.14: Variation of mean lift with flapping frequency f . The vertical line marks the 'default' conditions

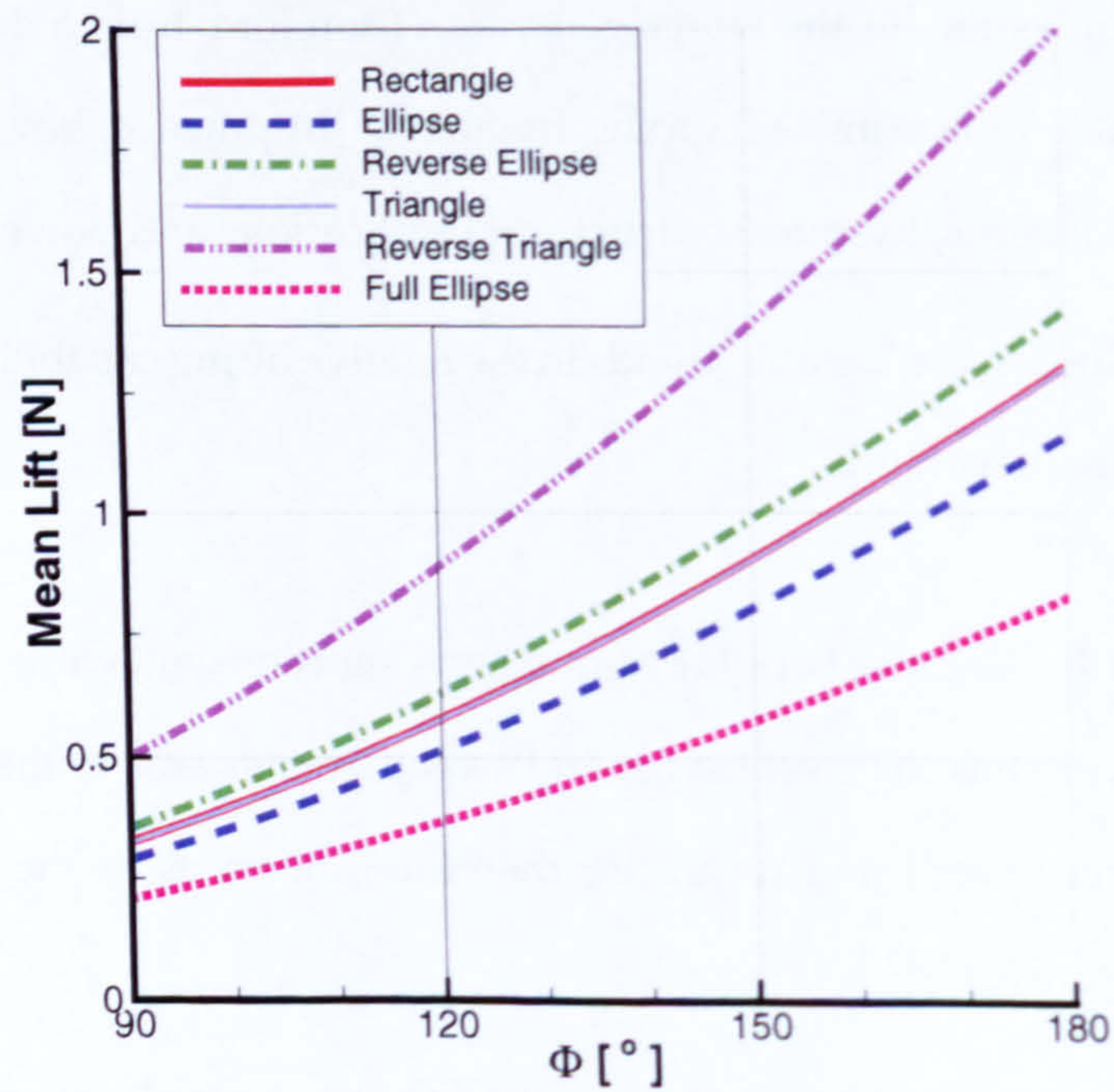


Figure 3.15: Variation of mean lift with stroke amplitude Φ . The vertical line marks the 'default' conditions

lift generated. Increasing stroke amplitude is, therefore, highly desirable but may be limited by the associated high power requirements (recalling that because L/Q remains unchanged, torque increases with increasing stroke amplitude), hysteresis and other structural concerns.

Stroke Plane Angle β Investigating the effect of stroke-plane angle β requires that the measure be changed from mean lift to mean weight supported because the wing continues to perform as before irrespective of stroke-plane angle. As noted in Equation 3.11, both lift and induced drag contribute to weight support for nonzero β . The corresponding variation of weight with stroke-plane angle is, therefore, as shown in Figure 3.16.

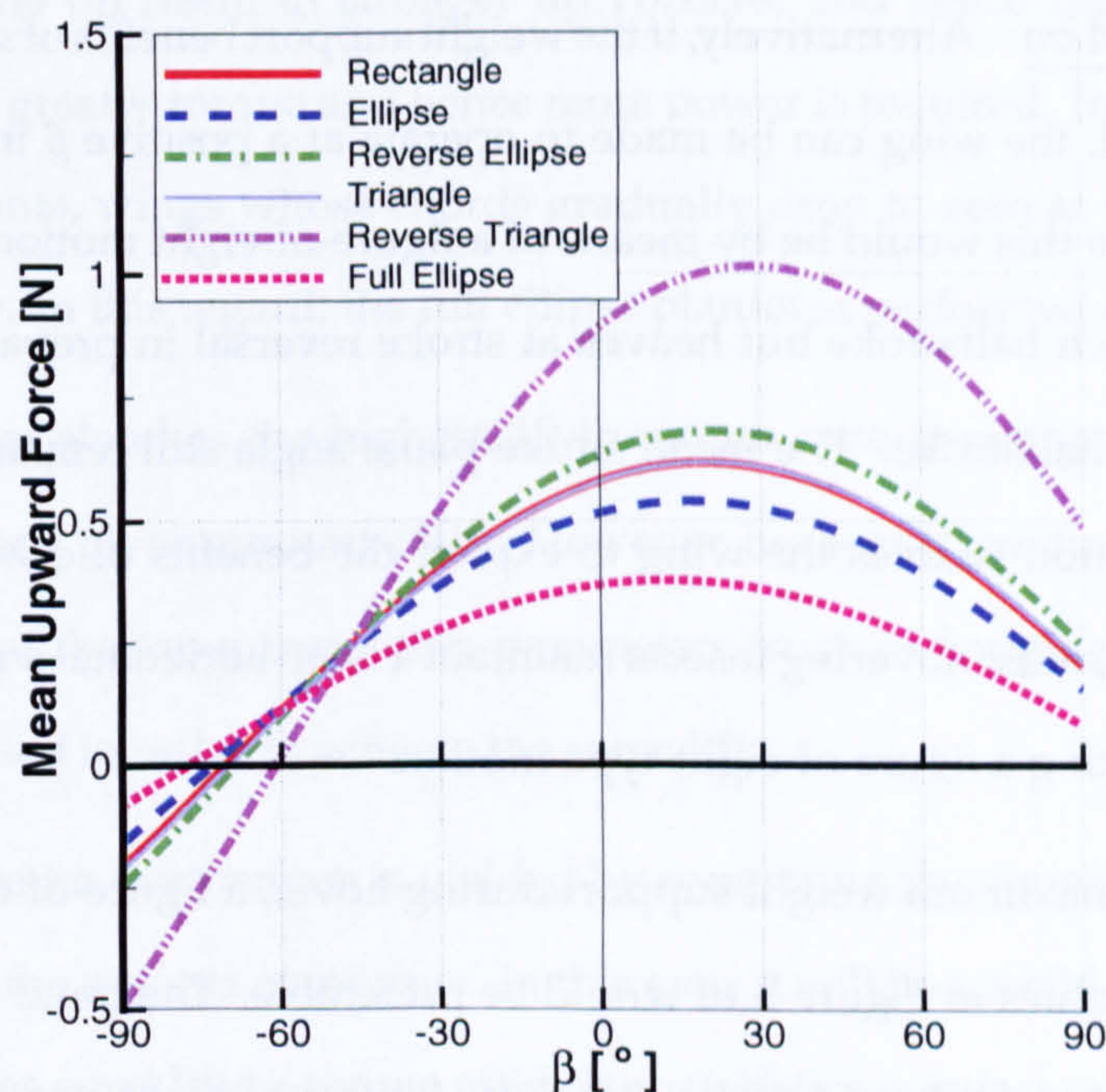


Figure 3.16: Variation of mean lift with stroke-plane angle β . The vertical line marks the 'default' conditions

Each of the plots shows a peak in maximum weight supported at a positive value of stroke-plane angle. This corresponds to the downstroke (see Figure 3.3(b)). By contrast, a negative stroke-plane angle refers to the upstroke. Because lift is the dominant term in the weight support equation (Equation 3.11), the trend in maxima is similar to the lift characteristics. The reverse triangle planform plot peaks at $\beta \approx 27.6^\circ$, followed by reverse

ellipse, rectangle and triangle all at $\beta \approx 20.2^\circ$. Next is the ellipse at $\beta \approx 16.5^\circ$ followed finally by the full ellipse at $\beta \approx 12.9^\circ$.

If a constant stroke-plane angle is maintained through hover, a positive β through the downstroke would automatically correspond to a negative β in the upstroke. Not only would more lift would be produced in the former than the latter but also the unbalanced drag forces would destabilise the hover. In order to produce the same lift in both halfstrokes at constant stroke-plane angle, therefore, the ideal value of β would be zero — a horizontal stroke plane. The weight supported would, however, be lower than the maximum possible.

Figure-of-Eight Motion Alternatively, if the weight support benefits of stroke-plane angle are to be exploited, the wing can be made to operate at a positive β in *both* halfstrokes. The way to achieve this would be by means of a figure-of-eight motion, where the wing plunges during each halfstroke but heaves at stroke reversal in preparation for plunge in the subsequent halfstroke. The mean stroke-plane angle still remains at zero but the figure-of-eight motion enables the wing to exploit the benefits of downstrokes in both halfstrokes. In fact, most hovering insects maintain a near-horizontal stroke plane during hover while executing a figure-of-eight-type motion.

Therefore, for maximum weight support during hover, a figure-of-eight-type motion at the maximum values in Figure 3.16 would be preferable. The trend is that wings that produce more lift would tend to have 'thicker' figures-of-eight owing to their higher values of β for maximum weight support. This analysis has also explained the reason behind the figure-of-eight motion observed in insects.

3.2.4 Conclusions

The above analysis has highlighted a number of useful wing parameters affecting the design of flapping wings. The analysis shows that wings with radially increasing chord lengths tend to produce greater lift because more wing area is exposed to the higher velocities in the tip region. Of the six planforms considered, the reverse triangle, therefore, performed best in this regard. By contrast, the full ellipse performed worst due to contributing factors — in the root region, its narrow chord subdued the production of lift and in the tip region less chord was available to benefit from the high velocities there.

However, high-lift wings come with a drag penalty. More ‘abrupt’ changes in chord length towards the tip result in stronger tip vortices, and hence higher drag. This in turn translates to greater torque and hence more power is required. In order to minimise power requirements, wings whose chords gradually drop to zero at the tip provide the best performance. In this regard, the full ellipse planform performed best.

The full ellipse also has the highest lift-to-torque ratio meaning that it is the most power-efficient for a given amount of lift. However, because it produces less lift than the other planforms of the same area, other parameters (such as flapping frequency) would need to be increased in order to achieve the same lift.

Therefore, a better comparison is yielded by comparing the variation of lift-to-torque ratio with lift for the various planforms. In this way, it will be possible to see if one planform that promises good lift-to-torque ratio characteristics is in fact capable of producing the required lift in the first place. The relevant graphs are shown in Figure 3.17 below.

It can be seen that the order of wing planforms seen in the plot of lift-to-torque ratio with wing area (Figure 3.13) is preserved. However, it can also be seen that the full ellipse planform offers the smallest range of lift values so that if a high value of lift were required, the full ellipse is the least likely to deliver. Therefore, a plot of L/Q vs. L is crucial to wing design.

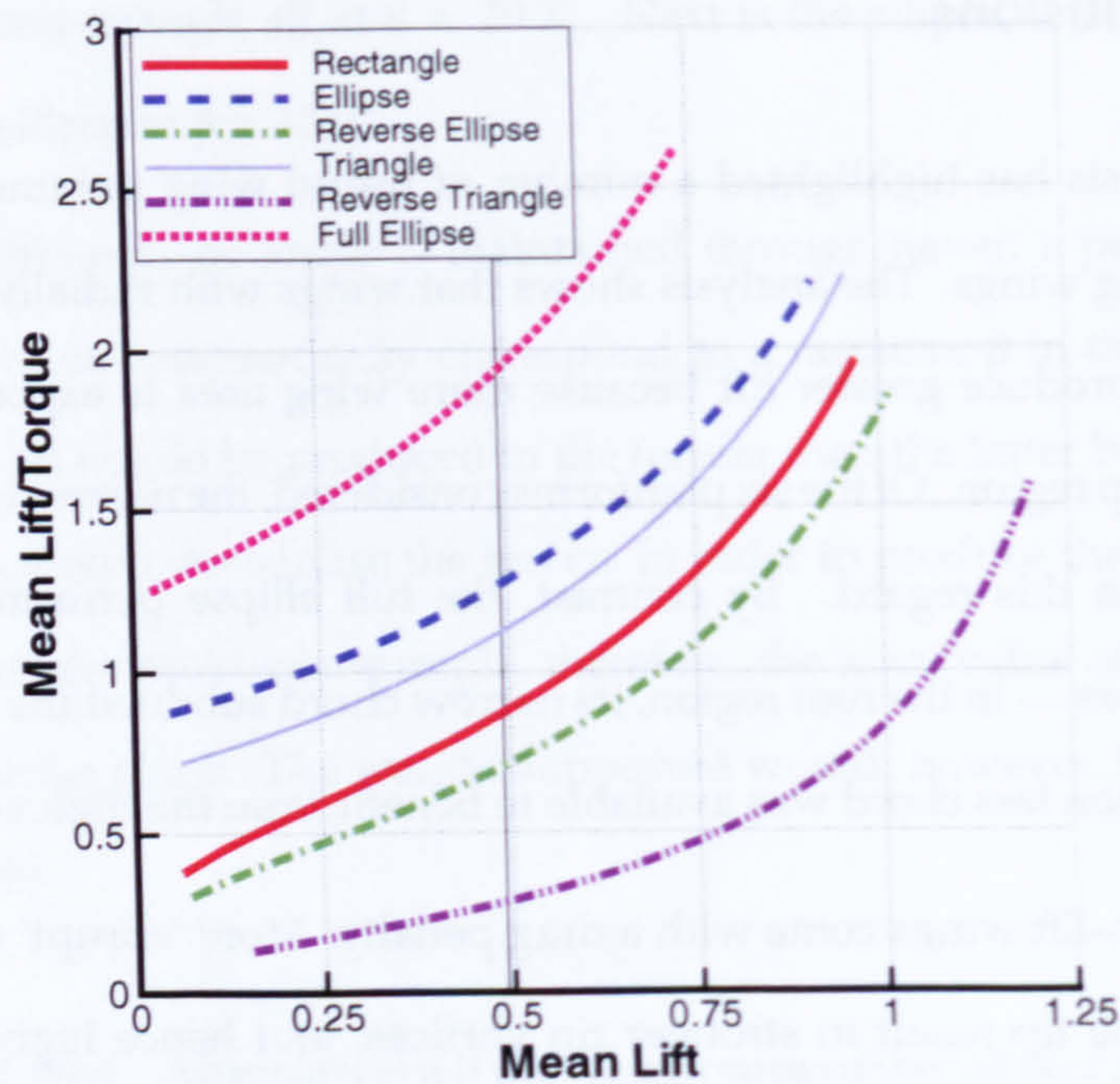


Figure 3.17: Variation of mean lift-to-torque ratio with mean lift. From left to right along the curves, wing area is increasing with wing length constant (decreasing aspect ratio). The vertical line represents the condition for a MAV (from § 1.1.1)

The dependence of mean lift on wing shape parameters such as wing length and aspect ratio indicates that there is benefit in maximising these parameters where possible. However, long wings imply large moment arms which mean greater torque and power requirement. Therefore, a balance must be struck to optimise for power efficiency. Generally, lift produced scales up with the square of wing length and aspect ratio. A similar trend exists for wing area because the latter is inextricably linked to wing length and aspect ratio.

A further parameter that was not considered but may offer more insight into optimising wing shapes for flapping wings is wing twist (or washout). It may be possible to use it to increase lift performance without as much of a drag (torque) penalty.

Wing kinematics also greatly influence lift performance while maintaining a constant lift-to-torque ratio because the wing shape remains unchanged. Lift increases as the square of both flapping frequency and stroke amplitude. The reason for this is that each

corresponds to an increase in velocity.

The analysis of stroke-plane angle has shown the benefit and reason behind the figure-of-eight kinematics found in insects. In order to benefit from the best weight support, it is necessary to have 'two downstrokes' per cycle, and this is achievable through a figure-of-eight-type wing movement. This also ensures a net horizontal stroke plane through a cycle which is essential for hover. Wings with better lift performance tend to have 'thicker' figures-of-eight.

Assumptions Having made the above analysis, it is important to remember the assumptions leading to the results. The analysis was essentially based on the quasi-steady definition of lift, and therefore, may not correctly account for all the forces generated. One significant simplification of the approach is that lift is assumed to be zero at the end of each halfstroke where the wing sweeping velocity is momentarily zero. It is, however, known that rotational (pitching) motions of the wing continue to produce forces even at stroke reversals (Dickinson, 1994; Sane & Dickinson, 2002).

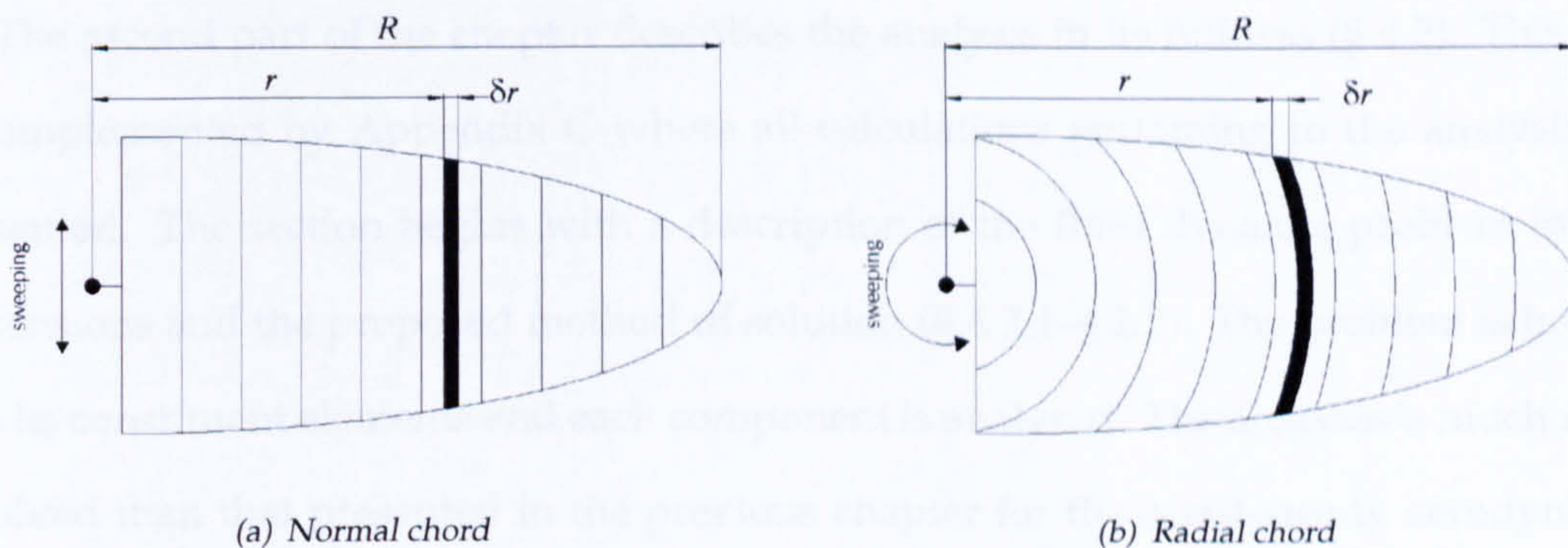


Figure 3.18: Normal and radial chords

For wings of high aspect ratio, such as helicopter rotors, the ratio of blade area to disk area (the so-called *solidity*) is very small (about 7.5% for a rotor with blades of aspect ratio 17) and it is reasonable to assume that the blade elements are straight (i.e. normal to the rotor spar; see Figure 3.18(a)). In the case of flapping wings such as those of insects, solidity is much higher (about 39% for the hawkmoth used by Willmott & Ellington

(1997b)) and the straight chord assumption is less tenable. Instead, a *radial chord* would make more sense (Figure 3.18(b)) and should be used. Although this is omitted in the current approach, it has been included in the more complex analysis later (Chapter 4).

The effect of the leading-edge vortex which has been observed in insect-like flapping wings (Ellington *et al.*, 1996; Birch & Dickinson, 2001) was not included in the above analysis. Nevertheless, the analysis has provided a valuable stepping stone for a flapping-wing design.

Chapter 4

Methodology and Analysis

This chapter is broadly divided into two parts. The first part (§ 4.1) lays the framework for the theoretical model developed in the second part. It identifies characteristics pertaining to the flapping-wing problem and details the assumptions made (§ 4.1.1–4.1.2). A brief look is also taken at the conformal transformation and the complex variable which together form the basis of the current modelling approach (§ 4.1.3–4.1.4).

The second part of the chapter describes the analysis in its fullness (§ 4.2). This part is complemented by Appendix C where all calculations pertaining to the analysis are presented. The section begins with a description of the fluid dynamic problem in two dimensions and the proposed method of solution (§ 4.2.1–4.2.2). The problem is broken into its constituent elements and each component is analysed. The analysis is much more involved than that presented in the previous chapter for the quasi-steady aerodynamic modelling. Two novel equations, presented for the first time in this PhD work, are derived in the process. The end results are expressions for vorticity and circulation.

In the later sections of the chapter, a discussion of the computation of forces and moments from the preceding analysis is given (§ 4.2.3). This is then converted from the original vector notation to complex number notation. This is followed by a brief discussion of the constituent elements making up the forces and moments. Finally, a

description of how the method is converted to a 'quasi-3-D' model is presented (§ 4.2.4).

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4.1 Methodology

4.1.1 Introduction

In developing an engineering model, the ultimate goal is to explain or predict the behaviour of some physical phenomenon. The process begins with constructing a mathematical model that captures the essential features of the problem without delving into overwhelming detail. This is followed by a process of analysis where every possible tool is tried (and even some new tools developed) in order to understand the behaviour of the model as thoroughly as possible. Finally, the results are interpreted and validated against real-world facts to better understand the workings of the observed physical phenomenon. This is the approach taken in the current work.

As noted by Thwaites, any new theory must take into account as far as possible what appear to be the most important physical characteristics (Thwaites, 1960, p. 512). Thwaites further noted that for wing aerodynamics the positions of the separation lines, in particular, must be taken into account. Therefore, those properties and flow features that need to be represented in a model for insect-like flapping wings must first be identified.

From the review in Chapter 2, the following conclusions can be drawn. The flow associated with insect-like flapping comprises two components — *attached* and *separated* flow. The attached flow on the aerofoil refers to all flow characteristics associated with the *freestream* flow on the aerofoil as well as the effects of *unsteady motions* (sweeping, heaving and pitching). In insect-like flapping wings, flow separation is usually observed at both leading and trailing edges due to the high angles of attack and the severity¹ of the kinematics. These ‘wakes’ affect the forces (and moments) generated by the wing due to their interaction with the wing in the form of inhibitory effects (e.g. the Wagner effect). The back and forth motion of the wings also gives rise to effects associated with wake

¹Severity implies rapidity and complexity in this case.

reentry. The *leading-edge vortex* is bound to the wing for most of the duration of each halfstroke and flow remains more or less attached in all other regions of the wing. The *trailing-edge wake* leaves smoothly from the trailing edge (except perhaps during stroke reversals when the smooth-flow condition may not hold). All of the above features must be included in any representative model.

With this flow 'picture' in mind, Żbikowski (2002) proposed a circulation-based approach for solving the flapping-wing problem. By using the principle of linear superposition, the problem may be decoupled into attached and separated flow, and the overall effect obtained by summing the individual contributions using the principle of linear superposition². It is based on the original approach of von Kármán & Sears (1938) together with the nonlinear extensions proposed by McCune *et al.* (1990) and McCune & Tavares (1993). This is broadly the approach used in the current work albeit with further significant extensions.

4.1.2 Model

In the following investigation, the flow around an insect-like flapping wing is considered.

Physical Aspects

The problem is essentially reduced to an array of 2-D wing (aerofoil) sections extending from the root of the wing to the tip. A blade-element-type method is used whereby flow in each 2-D cross-plane along the wing is analysed and the combined effect obtained by integrating for all wing sections. Because the motion is rotational, the velocities and

²Equations of fluid motion are generally not linear. However, there exist solutions that are linearly superposable. One such class of flows was defined by Beltrami where the local velocity and vorticity are parallel (Beltrami, 1985). Such flows have since come to be known as *Beltrami flows*. Trkal studied Beltrami flows and showed that special cases existed where viscosity could be incorporated (Trkal, 1994). Therefore, the principle of linear superposition may even be extended to certain viscous flows.

distances travelled increase radially from root to tip. Two physical properties that are a consequence of this rotational motion are taken into account and described below.

Radial Chord An insect-like flapping wing bears resemblance to a rotorcraft wing in that velocities increase radially from root to tip. For wings of high aspect ratio, such as helicopter rotors, the ratio of blade area to disk area (the so-called *solidity*) is very small (about 7.5% for a rotor with blades of aspect ratio 17) and it is reasonable to assume that the blade elements are straight (i.e. normal to the rotor spar). In the case of flapping wings such as those of insects, solidity is much higher (about 39% for the hawkmoth used by Wakeling & Ellington (1997a) and 55% for the fruit fly scaled-wing used by Birch & Dickinson (2001)) and the straight-chord assumption is less tenable. Instead, the concept of a *radial chord* is more suitable where each section of the wing chord still 'sees' a normal incident velocity (Figure 4.1; see also Figure 3.18).

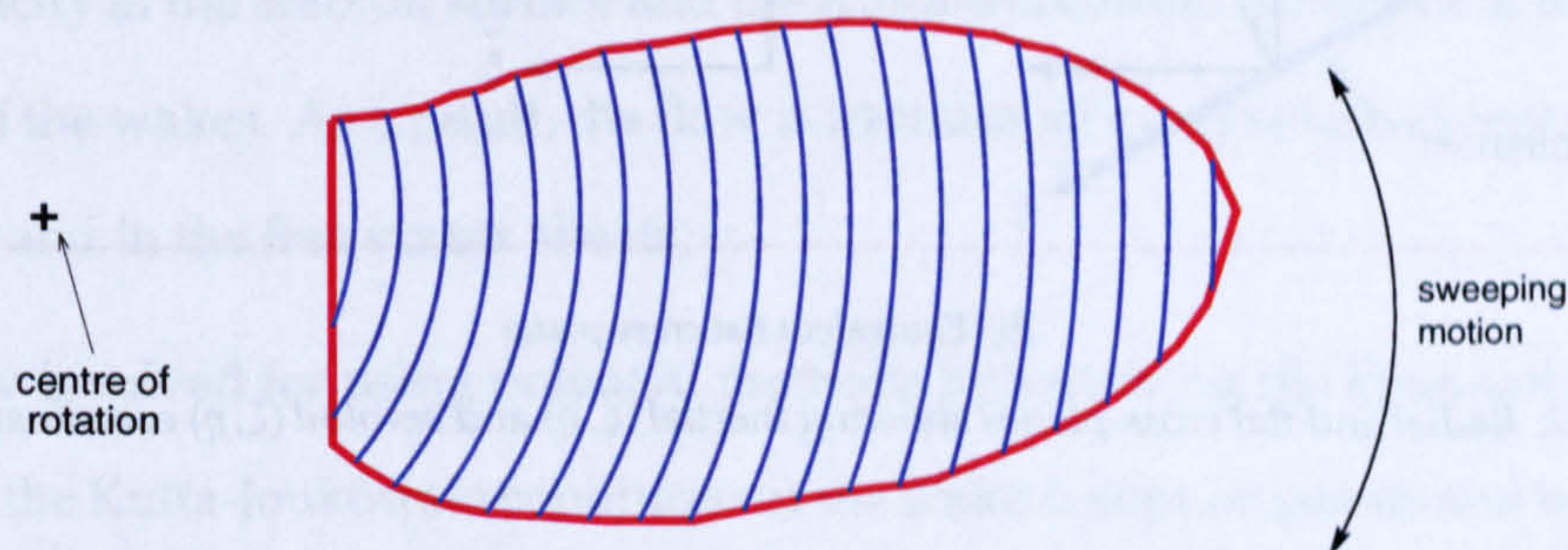
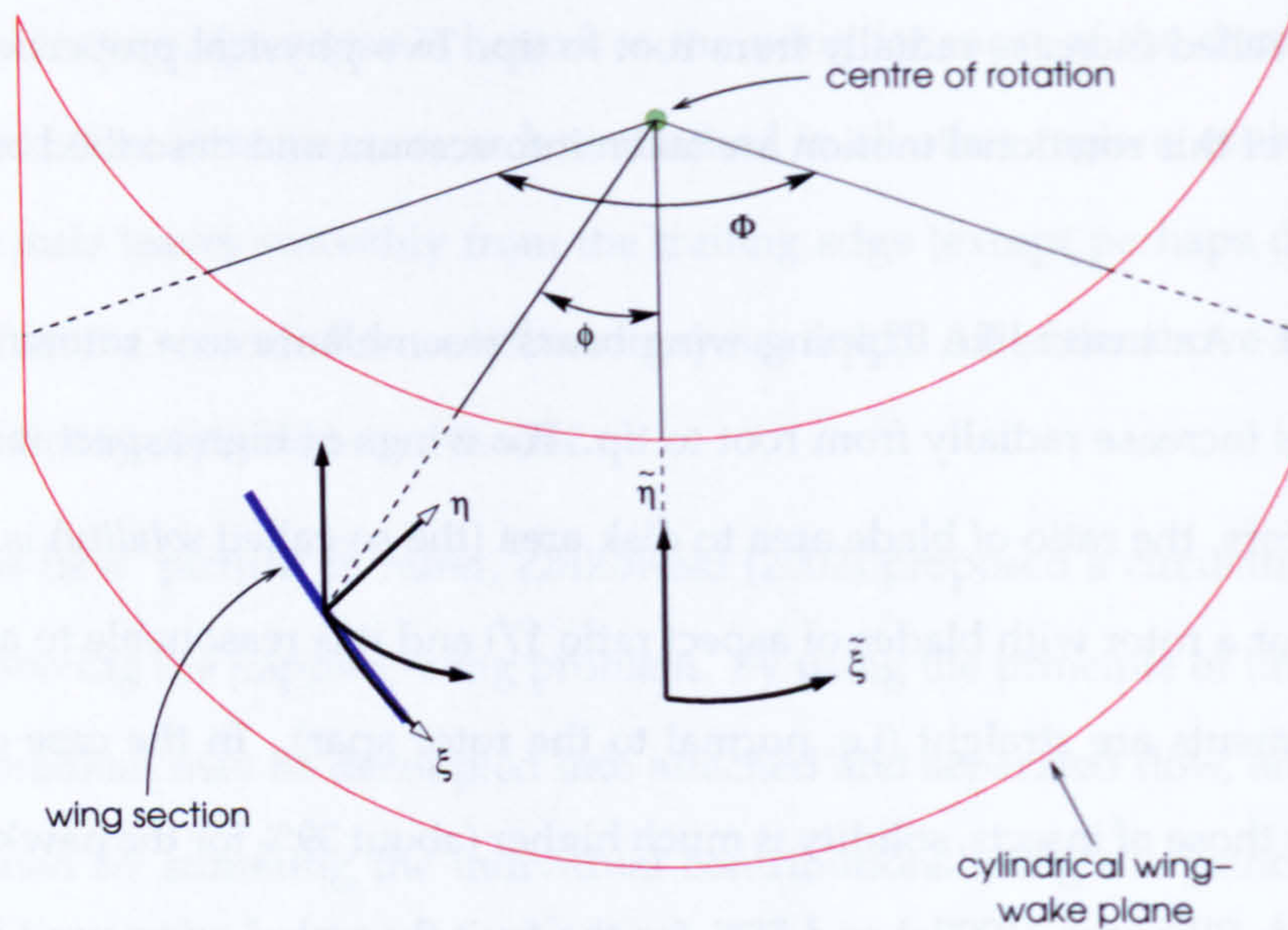
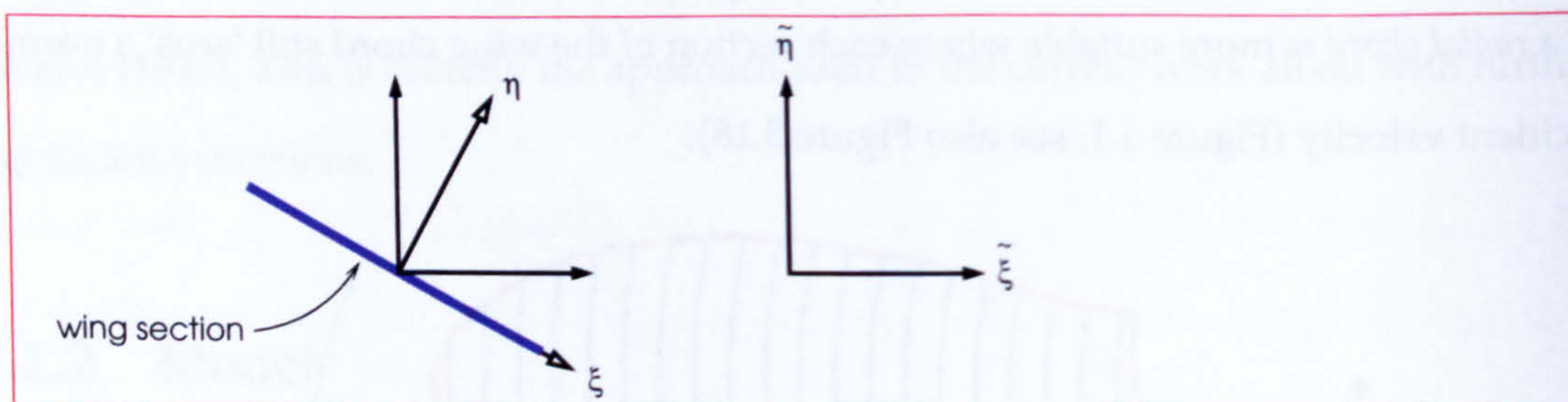


Figure 4.1: Radial chord representation of a fruit fly wing

In the analysis that follows, all radial chords are computed with the wing lying flat as if at zero angle of attack α in a stroke plane at zero stroke-plane angle β . In reality, radial chords can be very complex, depending on instantaneous angle of attack and local incident velocity (which includes a spherical component due to the nature of the wing motion). For reasons of simplicity, however, the latter have been ignored so one set of radial chords is used for a given problem.



(a) Actual radial cross-plane



(b) Equivalent flat cross-plane

Figure 4.2: Radial and flat cross-planes showing inertial $(\tilde{\xi}, \tilde{\eta})$ and aerofoil (ξ, η) coordinate systems

Radial and Flat Cross-Planes A consequence of combining a blade-element method with the concept of radial chords discussed above results in consecutive wing sections that lie in cylindrical cross-planes (Figure 4.2(a)). In the approach used here, each cylindrical cross-plane is then ‘unwrapped’ into an equivalent flat cross-plane (Figure 4.2(b)) where the flow is then solved as a 2-D problem. This concept is met in differential geometry problems where the radial cross-plane would be referred to as a developable surface³.

³A developable (or ruled) surface in differential geometry is a surface formed by a motion of a straight line. Therefore, a cylindrical surface (as used here) is developable but a spherical one is not (Lipschutz, 1969).

Aerodynamic Aspects

There exist a large number of fluid flows that are affected by viscosity to the first order and yet their motions do not show signs of restrictive amounts of viscous dissipation. Such flows may be treated as special cases of potential or irrotational flows, allowing them to be solved by well-known analytical methods (e.g. Hele-Shaw flows). Brown & Michael (1954) lend support to this view and this assumption is made here.

In each 2-D section, the aerofoil is represented by a continuous distribution of bound vorticity⁴ and the zero-through-flow condition is enforced, i.e. the local flow velocity always remains tangential to the aerofoil surface. Two wakes are shed in the form of free vortex sheets — one each emanating from the leading and trailing edges — which are also continuous distributions of vorticity. The flow is assumed to be entirely inviscid but viscosity is introduced indirectly in the form of flow separation and shear flow leading to bound vorticity at the aerofoil surface and the Kutta-Joukowski condition at the points of inception of the wakes. As a result, the flow is irrotational everywhere except at the solid boundaries and in the free vortex sheets.

The flow is solved for using potential methods by satisfying the kinematic boundary conditions, the Kutta-Joukowski conditions at the wake-inception points and by requiring that the total circulation in a control volume enclosing the system must remain constant (Kelvin's law; see Lamb (1932, § 33, p. 36)). More is said about these in the detailed analysis later (see § 4.2).

The exact nature of the leading-edge vortex found on insect flapping wings remains elusive. Whereas Ellington *et al.* (1996) reported spanwise velocity in the leading-edge vortex, Birch & Dickinson (2001) observed no such entrainment. Because the only val-

⁴This property differentiates the method from panel methods where the aerofoil is divided into panels each with two *distinct* points — one for vortex position and one for collocation point (see e.g. Katz & Plotkin, 2001, ch. 9).

idation data available were provided by Dickinson (priv. comm.), no spanwise flow is considered here. Therefore, no interaction is assumed to exist between adjacent wing sections. Glauert (1959) supported this view in his blade-element theory for propellers⁵, although he did not completely discount the possibility of radial phenomena in the tip regions.

An extension of the previous assumption (no interference between adjacent sections) is that 3-D tip vortex effects are also ignored. The performance of the entire wing is obtained simply by summing the individual contributions of the constituent 2-D wing sections. The validity of this assumption was established *a posteriori* as the results were found to compare well with real data. It would appear that the loss of lift (and thrust) due to 2-D vortex breakdown in the outboard sections of the wing is equivalent to the drop in lift (and thrust) that would be experienced in those regions due to tip-vortex effects.

Assumptions

While care has been taken to maintain generality and to keep simplifications to a minimum, a number of simplifying assumptions had to be made. These are summarised

⁵The basis for this assumption given by Glauert is quoted verbatim below (Glauert, 1959, pp. 211–212):

“Consider the action of the blade elements δr at radial distance r when the remainder of each airscrew blade is inoperative. The trailing vortices which spring from the ends of the element lie on the surfaces of two circular cylinders of radii r and $r + \delta r$ respectively, and the vorticity may be resolved into two components: (a) one having its axis parallel to the airscrew axis and (b) the other being circumferential and similar to a succession of vortex rings.

“The first component of the vorticity acts as the roller bearings between the rotating shell of air bounded by the cylindrical surfaces and the general mass of air. Now the general mass of air cannot acquire any circulation about the axis and hence the rotation due to the torque of the blade elements is confined to the region between the two cylindrical surfaces. Hence also the rotational interference due to the vortex system is experienced only by those blade elements which gave rise to the vorticity. A similar argument can be applied to the second component of the vorticity and thus the independence of the blade elements at different radial distances from the axis of the airscrew is established.”

below.

- The wing is a thin, flat plate (zero thickness and camber).
- The wing is rigid (there is no flexion irrespective of severity of kinematics).
- The flow is entirely inviscid although viscosity is indirectly introduced in the form of the Kutta-Joukowski condition and flow separation at the sharp edges.
- The flow is irrotational (zero vorticity and incompressible) everywhere except at solid boundaries and in the distribution of wake vorticity.
- Radial chords are used to describe each wing section because, as a result of the rotational motion of the wing, each wing section sees a 'curved' oncoming velocity.
- Each radial chord is computed with the wing lying flat (in reality, radial chords are spherical in nature and vary with incidence and incident velocity).
- As a result of the radial chords, the wake is enclosed within cylindrical surfaces (whereas the wing motion is actually spherical and the wake is likely to have a much more complex form).
- No interaction is assumed between adjacent wing sections so that the combined effect is obtained simply by integrating along the span.
- As a consequence of the previous assumption, any spanwise flow (as in, for example, the leading-edge vortex) is not modelled.
- Any 3-D tip-vortex effects are ignored.
- No interference effects of the mirror wing are considered so that the combined effect is obtained simply by doubling the results for one wing.
- Only the hover mode of flight is considered.

- Wing twist is ignored and the wing is assumed to have a common pitch axis for all wing sections.
- Laminar flow is assumed throughout on account of the low Reynolds numbers encountered in insect (and likely FMAV) flows.
- The effects of attached, separated and shed flow are linearly superposable (on the basis of the linearity of the Laplace equation).

4.1.3 Conformal Transformation

Conformal transformation⁶ is an invertible continuous mapping preserving angles so that a pair of continuous lines in one domain intersecting at an angle will map into another pair of lines in the co-domain intersecting at the same angle. In this way, a point in one plane is mapped uniquely to a point in another using a holomorphic function and the velocity potential at the corresponding points in the two planes is the same. The preservation of angles is essential so that the zero-through-flow condition (see p. 110), which requires that fluid flow be tangential to the aerofoil surface at all times, can be enforced properly in the circle plane where it is being computed. The method is based on the *Riemann mapping theorem* for simply connected domains (see e.g. Keener, 1988, p. 239).

The crux of the current approach is the conformal mapping of a circle into a Joukowski-type aerofoil. In this way, a complicated problem in physical space is transformed into a simpler problem in transformed space. The simple case of flow around a 2-D circular cylinder (or circle, in the Z -plane) is mapped into that around a Joukowski-type aerofoil (in the ζ -plane) using the following transformation

$$\zeta = Z + \frac{(1 - \epsilon) R^2}{Z} + \frac{\epsilon R^3}{2Z^2} \quad (4.1)$$

⁶Also called *conformal mapping*. 'Conformal' refers to a mapping where form is preserved.

where

$$\epsilon = \frac{\tau - i\sigma}{R} \quad (4.2)$$

and where R is the radius of the circle, and τ and σ are thickness and camber parameters respectively. This form of conformal transformation was developed by McCune (see Benson (1989) and Lam (1989)) based on the rules presented by Karamcheti (1966, ch. 14) whereby a whole range of Joukowski-type aerofoils could be defined while still maintaining the same corresponding circle in the Z -plane. Sample transformations are shown in Figure 4.3.

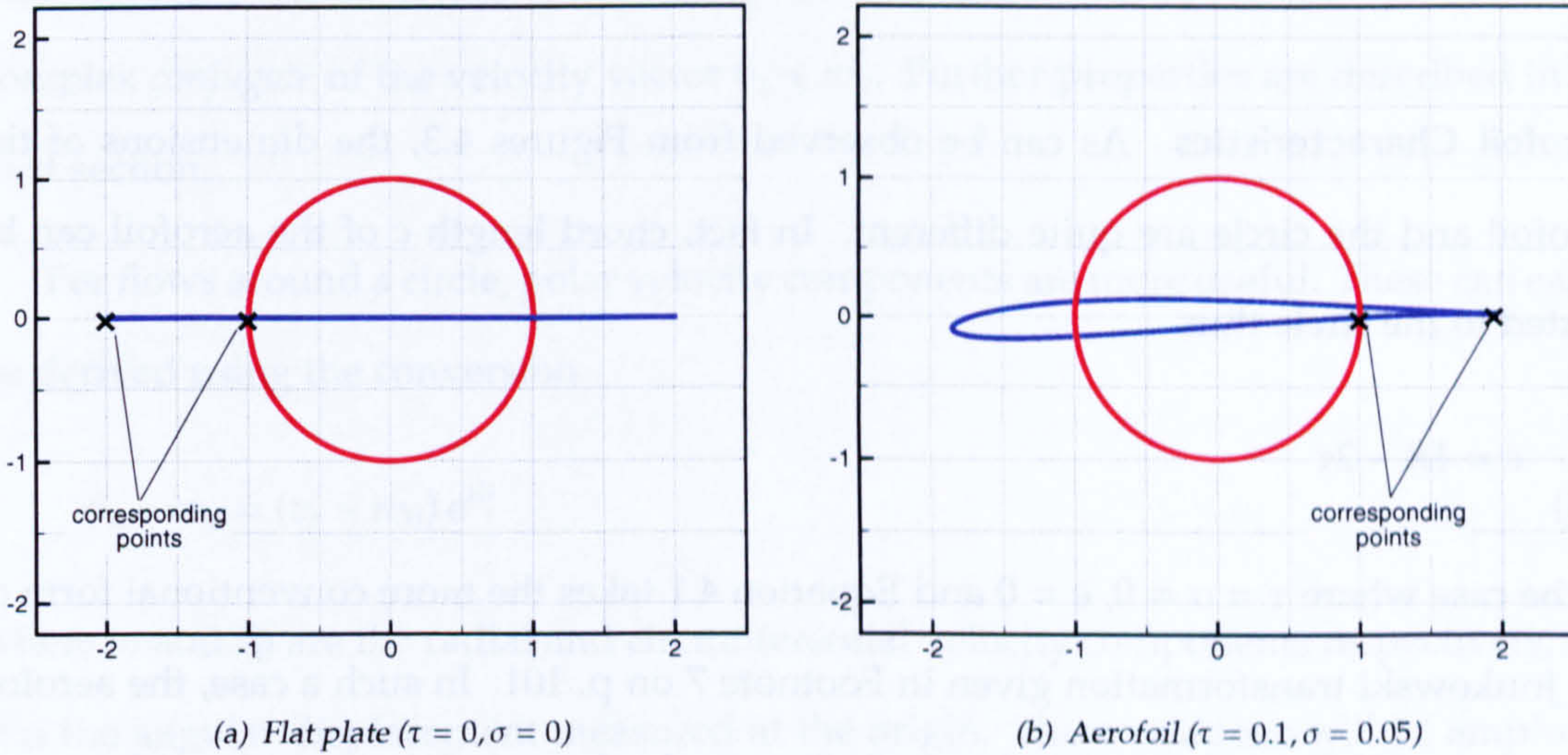


Figure 4.3: Conformal transformations

The advantage of using this transformation is that the trailing edge is always at the point $Z = R$ and that the circle is always centred at the origin. The latter enables use of *conjugate function theory* (see Appendix A.3) to calculate tangential or radial velocity components from each other (which will be essential as we will see later)⁷. Another advantage of this transformation is that a horizontal freestream velocity remains horizontal throughout — a rotation through the angle of attack is not required.

⁷For a conventional Joukowski transformation of the form $\zeta = Z + \frac{R^2}{Z}$, the centre of the circle is not necessarily at the origin in the Z -plane, and hence, conjugate function theory cannot always be applied.

The usual laws of conformal transformation are observed so that the following relationships in particular hold for an aerofoil in the physical (ζ) plane transformed into a circle in the transformed (Z) plane.

$$\zeta = f(Z) \quad (f : \text{mapping function}) \quad (4.3a)$$

$$\Omega(\zeta) = \Omega(Z) \quad (\Omega : \text{complex potential}) \quad (4.3b)$$

$$q(\zeta) = \frac{q(Z)}{\frac{d\zeta}{dZ}} \quad (q : \text{complex velocity}) \quad (4.3c)$$

$$\gamma(\zeta) = \frac{\gamma(Z)}{\frac{d\zeta}{dZ}} \quad (\gamma : \text{vorticity}) \quad (4.3d)$$

$$\Gamma(\zeta) = \Gamma(Z) \quad (\Gamma : \text{circulation}) \quad (4.3e)$$

Aerofoil Characteristics As can be observed from Figures 4.3, the dimensions of the aerofoil and the circle are quite different. In fact, chord length c of the aerofoil can be related to the circle thus

$$c = 4R - 2\tau$$

In the case where $\tau = \sigma = 0$, $\epsilon = 0$ and Equation 4.1 takes the more conventional form of the Joukowski transformation given in Footnote 7 on p. 101. In such a case, the aerofoil reduces to a flat plate with chord $c = 4R$. It may also be noted that τ and σ only *control* thickness and camber respectively, and are not the maximum thickness and maximum camber of the aerofoil *per se*. These can, however, easily be derived (see Appendix C.1.1).

4.1.4 Complex Variable

In the subsequent analysis, extensive use is made of the complex variable and its properties. It is, therefore, beneficial to familiarise with the conventions and basics. A detailed account can be found in Karamcheti (1966, ch. 15).

Two-dimensional potential flow can be described conveniently by means of the com-

plex variable. This approach is based on the Cauchy-Riemann⁸ equations which state the equivalence of *velocity potential* φ and *stream function* ψ in describing potential flows⁹. The complex potential Ω is described as

$$\Omega(Z) = \varphi + i\psi \quad (4.4)$$

from which the complex velocity q at a point $Z = x + iy$ can be found thus

$$q(Z) = \frac{d\Omega}{dZ} = v_x - iv_y \quad (4.5)$$

where (x, y) give position in Cartesian coordinates and (v_x, v_y) describe the respective velocity components at the point. Note that complex velocity described thus is the complex *conjugate* of the velocity vector $v_x + iv_y$. Further properties are described in the next section.

For flows around a circle, polar velocity components are more useful. These can easily be derived using the conversion

$$v_x - iv_y = (v_r - iv_\theta) e^{i\theta} \quad (4.6)$$

where v_r and v_θ are the radial and circumferential velocity components respectively, and θ is the angular displacement measured at the origin. These relations will be employed later.

⁸After Augustin Louis baron de Cauchy (1789–1857) and Georg Friedrich Bernhard Riemann (1826–1866).

⁹This equivalence is represented in a rectangular $(x-y)$ coordinate system as

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y} \quad \text{and} \quad \frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

4.2 Analysis

4.2.1 Introduction

For an aerofoil in unsteady motion, a shear layer of distributed vorticity is shed continuously from the trailing edge. The shear layer develops as a result of enforcing the Kutta-Joukowski condition¹⁰ there. Each time the aerofoil moves with respect to the surrounding fluid the bound vorticity around the aerofoil changes, and for an infinitesimal length of time the normally finite velocity at the trailing edge is disturbed. In an attempt to make the velocity finite there (thereby satisfying the Kutta-Joukowski condition), fluid flows towards the trailing edge from the upper and lower surfaces with different velocities and, therefore, a line of velocity discontinuity — a *vortex sheet* — is formed extending over the trailing edge into the fluid. A vortex sheet is a line (or a surface in three-dimensional flow) across which the components of tangential velocity are discontinuous but pressure and normal velocity are continuous. Although not necessarily so, a vortex sheet separates regions of irrotational fluid.

Based on the assumption that the mean velocity of flow from the trailing edge is finite and nonzero, Giesing (1969) was able to infer that the vortex sheet was shed parallel to either the upper or lower surface of the trailing edge, depending on the direction of shed vorticity. Maskell (1972) later came to the same conclusion. Mook & Dong (1994) applied the Giesing-Maskell model and showed that the surface of separation could be deduced from the rate of change of bound circulation $\frac{d\Gamma}{dt}$. When $\frac{d\Gamma}{dt} > 0$, the streamline along the lower surface leaves the aerofoil smoothly. Conversely, when $\frac{d\Gamma}{dt} < 0$, the flow relative to the aerofoil comes smoothly off the upper surface. In the steady-state case of $\frac{d\Gamma}{dt} = 0$, the

¹⁰After Martin Wilhelm Kutta (1867–1944) and Nikolai Egorovich Joukowski (1847–1921). Also known simply as the Kutta condition. In fact, there are two distinct conditions — the *Joukowski conjecture* which requires smooth flow at the trailing edge and the *Kutta condition* which indicates that no pressure discontinuity exists at the trailing edge (Vaz *et al.*, 2003). Hence the Kutta-Joukowski condition.

stagnation streamline bisects the trailing-edge angle.

Poling & Telionis (1986, 1987) later corroborated these results through experiments. They released dye of different colour from the upper and lower surfaces of the trailing edge of a pitching aerofoil and noted that the behaviour of the flow was as described above.

In the analysis that follows, the aerofoil is represented by a continuous distribution of bound vortices that move with the aerofoil. The wakes that emanate at both leading and trailing edges are similarly continuous distributions of vortices but are free and move with the fluid. The advantage of conformal transformation is exploited so that all calculations are performed in the circle plane for simplicity. As necessary, the relevant elements are then converted into the physical plane where they have more meaning.

4.2.2 Two-dimensional Unsteady Analysis

Analysis of the flow around a flapping-wing aerofoil is described by means of a 2-D potential-flow method, incorporating observations from insect flight and relying heavily on the complex variable and conformal transformations discussed above¹¹. Smith *et al.* (1996) have noted that even flows with Reynolds numbers as low as $Re \approx 10000$ are inertia-dominated so that viscous effects should not be directly important in most of the flow field and, hence, potential methods can be employed. Reynolds numbers for FMAV flows are likely to be in this region so potential methods are used here. All the analysis in this section is complemented with full details of the workings in Appendix C.

The overall aim of this work is to develop a model for determining performance (lift, thrust and moment) characteristics for an insect-like flapping wing in the hover. With

¹¹The analysis presented in this section is based on an approach similar to that used by Benson (1989) and Lam (1989) albeit with several extensions and modifications. All calculations presented were made from first principles and errors in the original works of Benson and Lam have also been corrected.

this information, wing-design guidelines for a flapping-wing MAV will be proposed.

The flow around a flapping wing is treated essentially as two-dimensional so that potential-flow methods can be used. By assuming that the flow is inviscid and incompressible, the Navier-Stokes equations reduce to the Euler equation (see Equation 2.3). Further, the flow is assumed to be irrotational everywhere (except at solid boundaries and discontinuities in the wake). So in essence, the flow is solved for using Laplace's equation for 2-D flows, which in complex number notation requires that

$$\nabla^2(\varphi + i\psi) = 0 \quad (4.7)$$

where φ is velocity potential and ψ is stream function. The nature of Laplace's equation means that the principle of linear superposition can be applied. This implies that the effects of various components contributing to the fluid motion may be computed separately and the overall effect obtained by taking their sum. This is the cornerstone of the current approach.

Although viscosity is generally ignored, its effects are included indirectly in the form of the Kutta-Joukowski condition, separation at the leading and trailing edges, and in the formation and shedding of vortices.

In our approach, the linearity of Laplace's equation is exploited to describe the flow associated with an insect-like flapping wing. The problem is subdivided into wake-free and wake-induced components (see Figure 4.4). The former subdivides further into effects due to *freestream* velocity and those due to the *unsteady motion* of the flapping wing itself. Since these effects exclude any wake contribution, they are collectively termed the *quasi-steady* components.

The wake-induced components also further subdivide into the contributions of the *leading-edge vortex* and the *trailing-edge wake*. These components induce the remaining forces and moments on the wing and are henceforth referred to as the *unsteady* components. The unsteady nature of the whole problem makes each of the component

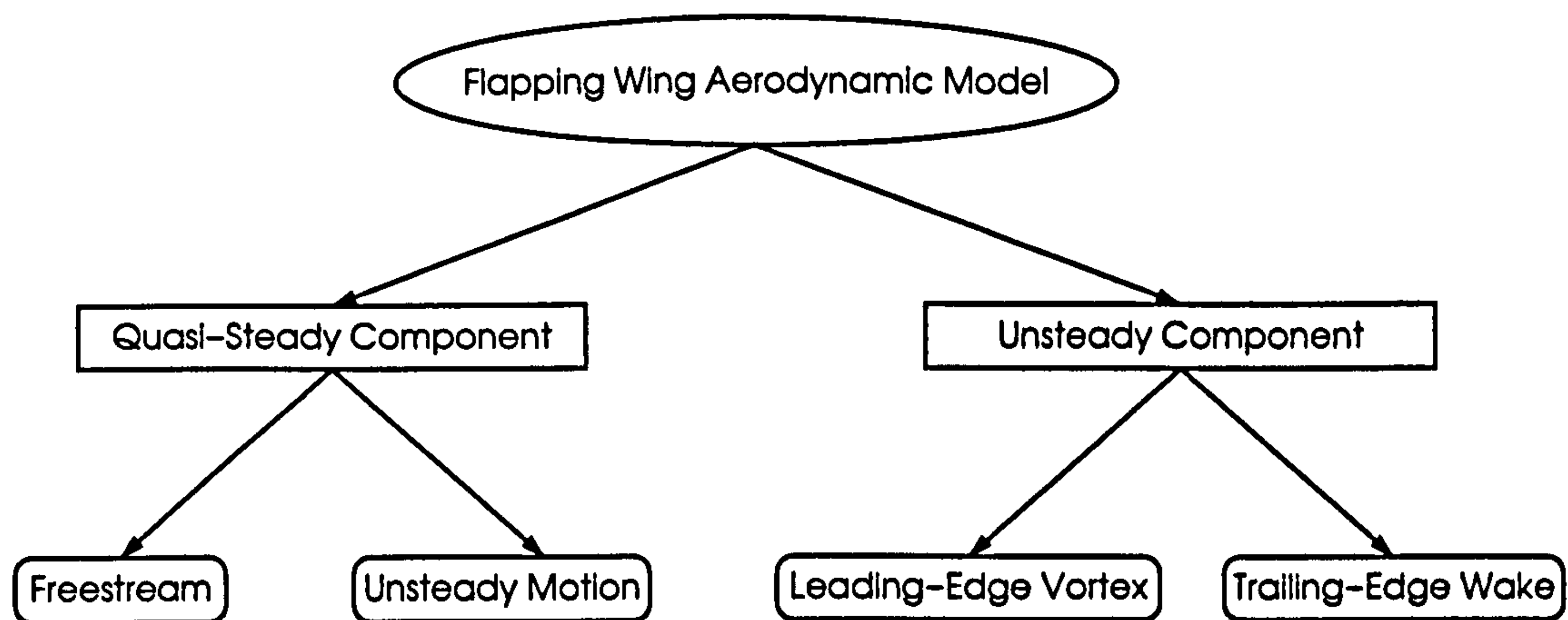


Figure 4.4: Flapping wing aerodynamic model components. ‘Quasi-steady’ components refer to the wake-free contributions and ‘unsteady’ components refer to the wake-induced contribution

contributions functions of time.

Each of these components is considered below. The vorticity and circulation developed by the aerofoil (and in the wake, where applicable) as a result of each of the components is calculated. These are later used to compute forces and moments. Dependence on time t is assumed throughout although it has been omitted from the notation on several occasions for reasons of clarity. As noted earlier, all analysis is performed in the circle (Z) plane for simplicity.

Conventions and Coordinate Systems Before proceeding, a quick note on some of the conventions and coordinates is appropriate.

Conventions In the analysis that follows, circulation (and vorticity) is taken positive counterclockwise ($\cup$), in the direction of increasing angle along a circle. Moments are also taken positive counterclockwise.

Conventional rectangular coordinates are used so that the abscissa and ordinate point left to right and bottom to top respectively, and are orthogonal with respect to each other (see Figure 4.6). By default, freestream flow is from left to right. Consequently, positive

angle of attack and pitch rate are taken positive clockwise. Due to the unsteady flapping motion, the aerofoil may either point leftwards or rightwards depending on the instantaneous kinematics. When the aerofoil points rightwards, angle of attack exceeds 90° . The aerofoil has three degrees of freedom — lunge (horizontal perturbations), heave (vertical perturbations) and pitch (angular perturbations). Positive lunge is in the direction of the freestream flow and positive heave is upwards.

Coordinate Systems Four different Lagrangian coordinate systems¹² are defined — *aerofoil*, *rotating*, *translating* and *inertial* (see Figure 4.6). The aerofoil coordinate system (ζ) is fixed at the centre¹³ of the aerofoil and moves with it. The rotating coordinate system ($\check{\zeta}$) also moves with the aerofoil but is centred at the pitch axis of the aerofoil and is oriented with the aerofoil. When the pitch axis is at the centre of the aerofoil, the aerofoil and rotating axes coincide exactly.

The translating coordinate system ($\hat{\zeta}$) shares its origin with the rotating axes system but differs in that it always remains horizontal. When the aerofoil is horizontal and the pitch axis is at its centre, the translating and aerofoil axes coincide. The inertial coordinate system ($\tilde{\zeta}$) remains fixed with respect to the Earth. For the wing sections, the origins of their respective inertial systems lie along the same radial line from root to tip, coinciding with the $\phi = 0$ line (ϕ being sweep angle; see Figure 4.2).

The rotating coordinate system is used to describe the rotational motion of the aerofoil whereas the translating coordinate system describes its translational motion. Both these coordinate systems move with the aerofoil and are hence termed *aerofoil-inertia* systems.

¹²In a Lagrangian system, the position of a fluid particle is followed for time t . By contrast, in an Eulerian system, changes in fluid properties at a fixed point in space are monitored for time t . Incidentally, both Lagrangian and Eulerian systems are due to Leonhard Euler (1707–1783).

¹³This is the geometric centre of the aerofoil and can be found by mapping the centre of the circle in the Z -plane into the ζ -plane.

All coordinate systems are rectangular so that position can be expressed in Argand diagram notation thus

$$\zeta = \xi + i\eta$$

Quasi-Steady Component

Details of the full derivations are given in Appendix C.2.

Free Stream Component The case for an aerofoil at an angle of attack α in a freestream flow is considered (Figure 4.5).

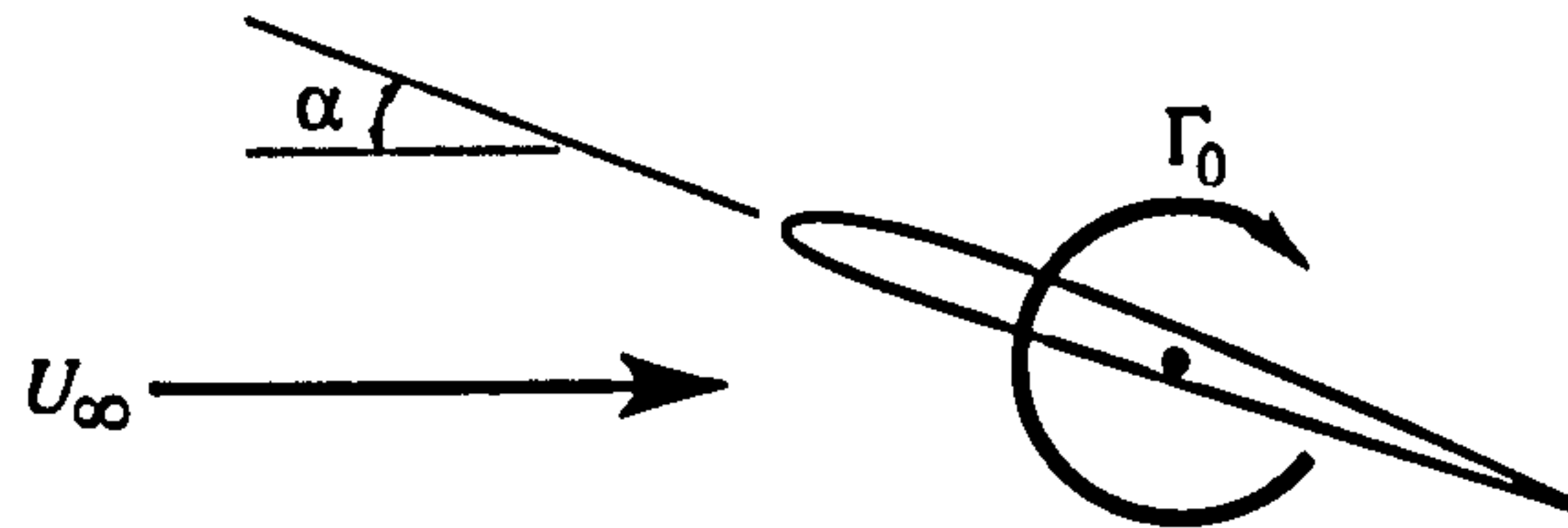


Figure 4.5: Wing at an angle of attack α in a constant freestream U_∞

Using the circle theorem (Milne-Thomson, 1966, § 5.2), the complex potential of freestream flow of velocity U_∞ around the corresponding circle is

$$\Omega(Z) = U_\infty \left(Ze^{-i\alpha} + \frac{R^2 e^{i\alpha}}{Z} \right)$$

which differentiates to give the complex velocity, thus

$$q(Z) = \frac{d\Omega}{dZ} = U_\infty \left(e^{-i\alpha} - \frac{R^2 e^{i\alpha}}{Z^2} \right)$$

By using the substitution $Z = Re^{i\theta}$ on the circle (where R is radius and θ is angular displacement about its centre) and converting to polar coordinates using Equation 4.6, the circumferential (tangential) velocity can be obtained. For smooth flow at the trailing edge (i.e. at $\theta = 0$), the Kutta-Joukowski condition must be enforced there by adding a circulation Γ_k at the origin so that the circumferential velocity v_θ becomes

$$v_\theta(\theta)|_f = -2U_\infty \sin(\theta - \alpha) + \frac{\Gamma_k}{2\pi R}$$

where the subscript f refers to the freestream component, $\Gamma_k = -4\pi R U_\infty \sin \alpha$ and where α and hence Γ_k can be time-varying. Assuming that no flow exists within the circle, the circumferential velocity at any point on the circle is equivalent to the vorticity there, thus

$$\gamma_0(\theta, t)|_f = -2U_\infty [\sin \theta - \alpha(t) + \sin \alpha(t)] \quad (4.8)$$

from which the circulation is obtained by integration as

$$\Gamma_0(t)|_f = -4\pi R U_\infty \sin \alpha(t) \quad (4.9)$$

As discussed previously (Equation 4.3e), circulation has the same value in both circle and aerofoil planes. The negative sign is a consequence of the convention used — positive circulation is taken counterclockwise (in the sense of increasing θ).

Unsteady Motion Component The unsteady motion of the flapping-wing aerofoil disturbs the fluid around it. Because the wing is represented by a series of vortices, their strengths have to be adjusted so that the solid boundary condition is upheld. Therefore, in order to determine the bound vorticity (and circulation), the kinematic boundary condition must first be derived. This yields velocity components at the aerofoil surface which can then be used to derive expressions for vorticity (and circulation) as done for the freestream component above.

Kinematic Boundary Condition The usual boundary condition for flow around an aerofoil is the zero-through-flow condition, i.e. that fluid does not penetrate the aerofoil surface. In other words, the normal velocity at the surface must be zero, i.e. $v_n = 0$. Before such a condition can be derived, however, the instantaneous position of the aerofoil must be determined.

The 2-D aerofoil has three degrees of freedom (see Figure 4.6) — pitch ($\alpha, \dot{\alpha}$), lunge ($l, \dot{l}$) and heave ($h, \dot{h}$). To enforce the zero-through-flow boundary condition at the surface of the aerofoil, the fluid flow there must be parallel to the surface. In other words, the

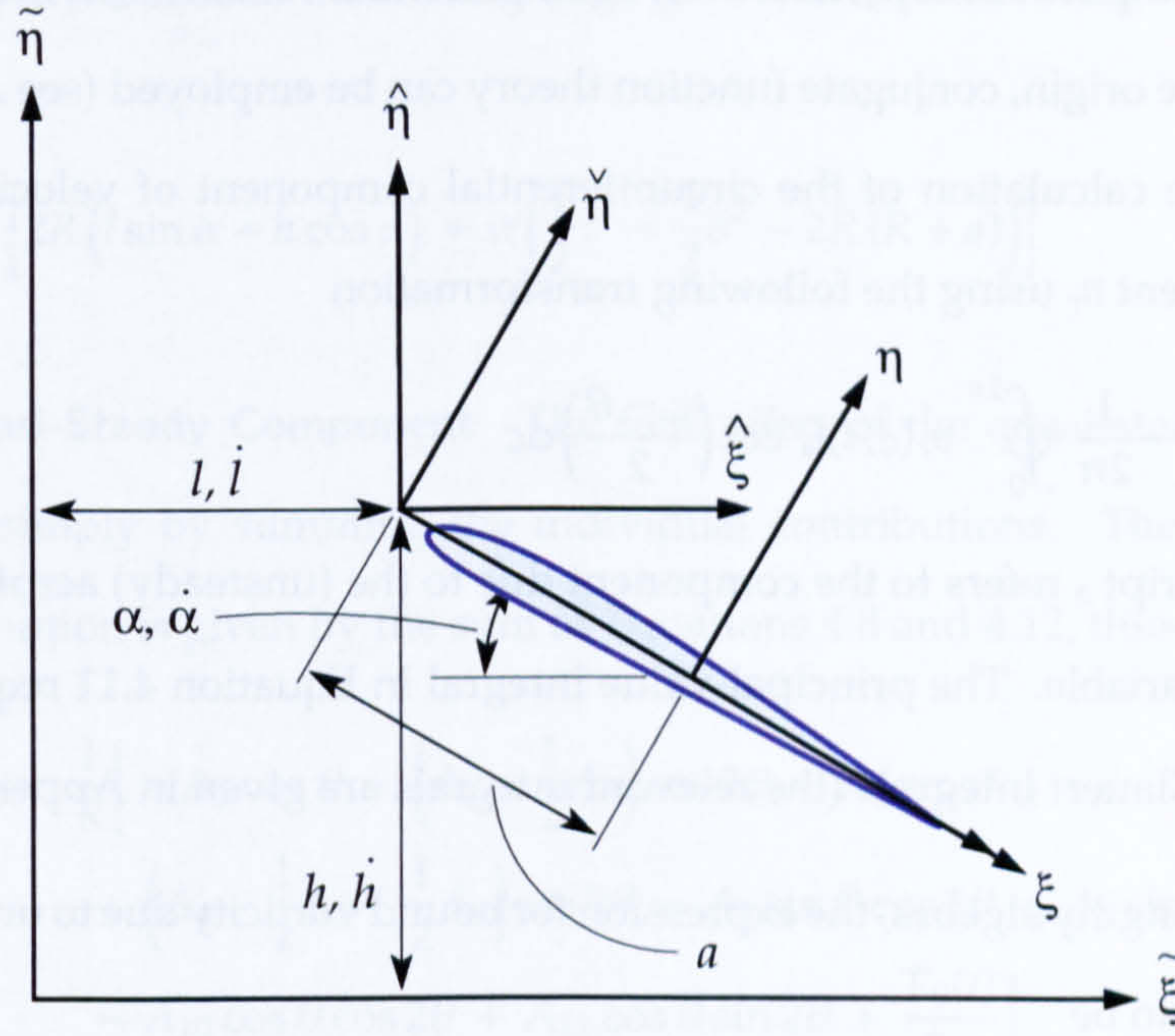


Figure 4.6: Coordinate systems

fluid velocity $(\tilde{u}, \tilde{v})$ and aerofoil kinematic velocity $(\tilde{u}_a, \tilde{v}_a)$ must be equal at the aerofoil surface. This condition is satisfied when

$$u = \dot{l} \cos \alpha - \dot{h} \sin \alpha + \eta \dot{\alpha} \quad (4.10a)$$

$$v = \dot{l} \sin \alpha + \dot{h} \cos \alpha - (\xi + a) \dot{\alpha} \quad (4.10b)$$

where (u, v) are the orthogonal velocity components at (ξ, η) on the aerofoil and a is the ‘offset’ distance of the pitch axis from the aerofoil centre. Full details of the analysis are given in Appendix C.2.2.

Vorticity and Circulation Since calculation of circulation is easier in the circle plane, the kinematic boundary conditions for zero-through-flow derived in the physical plane need to be transformed into the circle plane (using Equation 4.3c) and converted to polar coordinates (with Equation 4.6). In this way, an expression for radial velocity v_r is obtained.

In order to compute vorticity, however, circumferential velocity v_θ is required. Because the circle is at the origin, conjugate function theory can be employed (see Appendix A.3). This enables the calculation of the circumferential component of velocity v_θ from the normal component v_r using the following transformation

$$v_\theta(\theta, t)|_a = -\frac{1}{2\pi} \oint_0^{2\pi} v_r(\varsigma, t)|_a \cot\left(\frac{\varsigma - \theta}{2}\right) d\varsigma \quad (4.11)$$

where the subscript a refers to the component due to the (unsteady) aerofoil motion and ς is a dummy variable. The principal-value integral in Equation 4.11 requires judicious solution using Glauert integrals (the relevant integrals are given in Appendix A.1).

After some lengthy algebra, the expression for bound vorticity due to unsteady aerofoil motion is found to be

$$\begin{aligned} \gamma_0(\theta)|_a = \frac{1}{R} & \left[-A_1 \cos \theta - \left(A_2 + \frac{1}{2}A_7\right) \cos 2\theta + A_3 \sin \theta \right. \\ & + \left(A_4 - \frac{1}{2}A_5 + \frac{1}{2}A_6\right) \sin 2\theta - A_8 \sin \theta \cos 2\theta + A_9 \sin \theta \sin 2\theta \\ & \left. - A_{10} \cos \theta \cos 2\theta + A_{11} \cos \theta \sin 2\theta + \frac{\Gamma_0|_a}{2\pi} \right] \end{aligned} \quad (4.12)$$

where

$$\left. \begin{aligned} A_1 &= -\sigma(\dot{l} \cos \alpha - \dot{h} \sin \alpha) + (2R - \tau)(\dot{l} \sin \alpha + \dot{h} \cos \alpha) + a(\tau - 2R)\dot{\alpha} \\ A_2 &= \sigma(\dot{l} \cos \alpha - \dot{h} \sin \alpha) + \tau(\dot{l} \sin \alpha + \dot{h} \cos \alpha) - a\tau\dot{\alpha} \\ A_3 &= \tau(\dot{l} \cos \alpha - \dot{h} \sin \alpha) - \sigma(\dot{l} \sin \alpha + \dot{h} \cos \alpha) + a\sigma\dot{\alpha} \\ A_4 &= -\tau(\dot{l} \cos \alpha - \dot{h} \sin \alpha) + \sigma(\dot{l} \sin \alpha + \dot{h} \cos \alpha) - a\sigma\dot{\alpha} \\ A_5 &= -2R\sigma\dot{\alpha} \\ A_6 &= 2R\sigma\dot{\alpha} \\ A_7 &= 4R(\tau - R)\dot{\alpha} \\ A_8 &= R\sigma\dot{\alpha} \\ A_9 &= -\frac{1}{2}(\tau^2 + \sigma^2 + 2R\tau)\dot{\alpha} \\ A_{10} &= \frac{1}{2}(\tau^2 + \sigma^2 - 4R\tau)\dot{\alpha} \\ A_{11} &= -2R\sigma\dot{\alpha} \end{aligned} \right\} \quad (4.13)$$

By setting vorticity to zero at the trailing edge ($\theta = 0$) in Equation 4.12 bound circulation is evaluated as

$$\Gamma_0|_a = 2\pi \left[2R \left(\dot{l} \sin \alpha + \dot{h} \cos \alpha \right) + \dot{\alpha} \left(\frac{1}{2} \tau^2 + \frac{1}{2} \sigma^2 - 2R(R+a) \right) \right] \quad (4.14)$$

Combined Quasi-Steady Component The total effect of the quasi-steady components can be found simply by summing the individual contributions. The total wake-free vorticity distribution is given by the sum of Equations 4.8 and 4.12, thus

$$\begin{aligned} \gamma_0(\theta, t) = & \frac{1}{R} \left[-A_1 \cos \theta - \left(A_2 + \frac{1}{2} A_7 \right) \cos 2\theta + A_3 \sin \theta \right. \\ & + \left(A_4 - \frac{1}{2} A_5 + \frac{1}{2} A_6 \right) \sin 2\theta - A_8 \sin \theta \cos 2\theta + A_9 \sin \theta \sin 2\theta \\ & - A_{10} \cos \theta \cos 2\theta + A_{11} \cos \theta \sin 2\theta + \frac{\Gamma_0(t)}{2\pi} \Big] \\ & - 2U_\infty \sin(\theta - \alpha(t)) \end{aligned} \quad (4.15)$$

where the symbols $A_i, i = 1 \dots 11$ are as in Equation 4.13 above and where time-dependence has been shown for accuracy. Similarly, by summing Equations 4.9 and 4.14, the total quasi-steady wake-free bound circulation is found to be

$$\Gamma_0(t) = 2\pi \left[2R \left(\left(\dot{l}(t) - U_\infty \right) \sin \alpha(t) + \dot{h}(t) \cos \alpha(t) \right) + \dot{\alpha}(t) \left(\frac{1}{2} \tau^2 + \frac{1}{2} \sigma^2 - 2R(R+a) \right) \right] \quad (4.16)$$

Unsteady Component

The unsteady component in the flapping-wing model (see Figure 4.4) arises from effects induced by the two wakes — the leading-edge vortex (denoted by $_{lev}$) and the trailing-edge wake (denoted by $_{wake}$). Full details of workings can be found in Appendix C.3.

Due to the severity of the manoeuvres during flapping, flow is likely to separate from both the leading and trailing edges of the aerofoil. Therefore, in addition to the conventional wake shed from the trailing edge, a wake is also shed from the leading edge in the form of the leading-edge vortex. The fact that vortex sheets emerge from these points on the aerofoil means that the loading there must be zero, there being none across

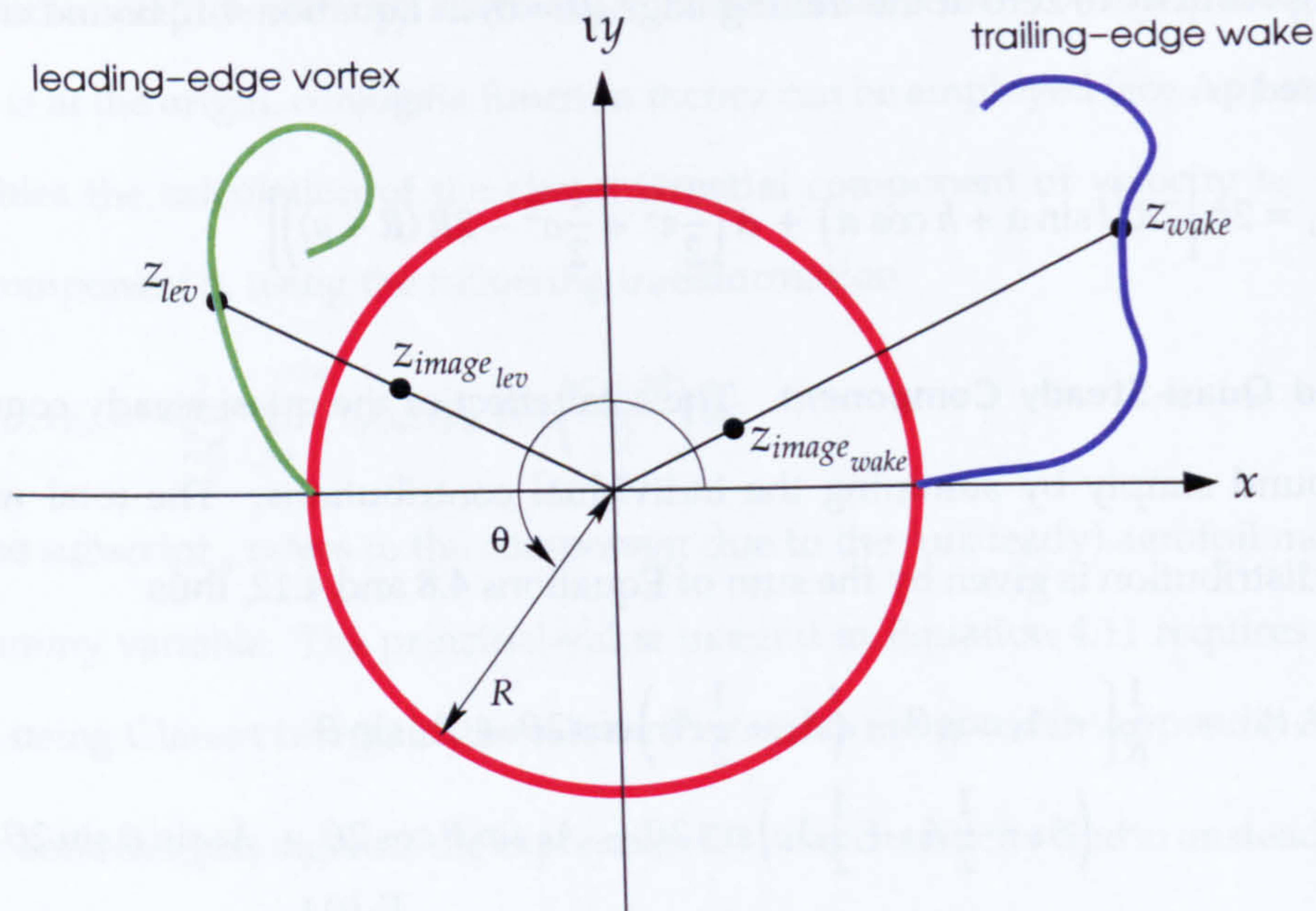


Figure 4.7: Wake systems

the vortex sheets. Therefore, the Kutta-Joukowski condition must be satisfied at these points.

The method of images (see e.g. Milne-Thomson, 1966, § 5.1) forms the basis for the analysis in this section. For simplicity, all calculations are performed again in the circle plane. The wakes emanating from the leading and trailing edges of the aerofoil are, therefore, mapped into the circle (Z) plane and may be represented by the schematic in Figure 4.7. The leading-edge vortices enter the fluid at the leading edge ($\theta = \pi$ on the equivalent circle in the Z -plane) and the trailing-edge vortices make their entry at $\theta = 0$, where θ is measured in the conventional sense. The circle corresponding to the aerofoil has radius R .

For a vortex of strength $d\Gamma_{wake} (\equiv \gamma_{wake} dZ)$ at a point Z_{wake} in the wake, there must exist an (imaginary) image vortex of strength $d\Gamma_{image_{wake}}$ at the inverse square point¹⁴ in

¹⁴If two points lie along the same line $e^{i\theta}$, one point outside the circle of radius R at $r_{out} e^{i\theta}$ and the other inside it at $r_{in} e^{i\theta}$, then the two points lie at inverse square points if $r_{in} \cdot r_{out} = R^2$. See also Milne-Thomson (1966, § 5.1).

the circle to satisfy the zero-through-flow boundary condition at the surface. The strength and position respectively of this image vortex are given by

$$d\Gamma_{image_{wake}} = -d\Gamma_{wake} \quad \text{and} \quad Z_{image_{wake}} = \frac{R^2}{\overline{Z_{wake}}}$$

where $\overline{Z_{wake}}$ refers to the complex conjugate of Z_{wake} . By writing the complex potential of the system $\Omega(Z)$, the velocity at an arbitrary point on the circle $Z (= Re^{i\theta})$ can be calculated by differentiation w.r.t. Z , thus

$$\begin{aligned} \Omega(Z) &= -i \frac{\delta\Gamma_{wake}}{2\pi} \ln(Z - Z_{wake}) + i \frac{\delta\Gamma_{wake}}{2\pi} \ln\left(Z - \frac{R^2}{\overline{Z_{wake}}}\right) \\ \Rightarrow q(Z) &= i \frac{\delta\Gamma_{wake}}{2\pi} \left(\frac{1}{Re^{i\theta} - \overline{Z_{wake}}} - \frac{\overline{Z_{wake}}}{R^2 - Re^{i\theta} \overline{Z_{wake}}} \right) \end{aligned}$$

Again, using Equation 4.6, the circumferential velocity is calculated to be¹⁵

$$v_\theta = -\frac{\delta\Gamma_{wake}}{2\pi R} \Re\left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}}\right)$$

The circumferential velocity induced by the entire wake can then be obtained by integration, thus

$$v_\theta(\theta, t)|_{wake} = -\frac{1}{2\pi R} \oint_{wake} \Re\left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}}\right) \gamma_{wake} dZ_{wake}$$

A similar expression results for the leading-edge vortex so that the combined effect of the two wakes is

$$\begin{aligned} v_\theta(\theta, t)|_{wake+lev} &= -\frac{1}{2\pi R} \oint_{wake} \Re\left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}}\right) \gamma_{wake} dZ_{wake} \\ &\quad - \frac{1}{2\pi R} \oint_{lev} \Re\left(\frac{Z_{lev} + Re^{i\theta}}{Z_{lev} - Re^{i\theta}}\right) \gamma_{lev} dZ_{lev} \end{aligned}$$

To solve for the two unknown vorticities in the above equation (γ_{wake} and γ_{lev}), two simultaneous ‘constraint’ equations are required. The equations are provided by satisfying the Kutta-Joukowski condition at the trailing edge of the aerofoil by the addition of circulation and by similarly requiring stagnation at the leading edge. These are derived in turn below.

¹⁵Equation 4.6 rearranges to give $v_\theta = -\Im(q(Z)) \cos \theta - \Re(q(Z)) \sin \theta$, where $\Re$ and $\Im$ denote ‘real part of’ and ‘imaginary part of’ respectively.

First Constraint Equation — Kutta-Joukowski Condition at Trailing Edge In order to enforce the Kutta-Joukowski condition, a circulation Γ_k must be added (as done previously for the quasi-steady components) so that the circumferential velocity becomes

$$v_\theta = -\frac{1}{2\pi R} \oint_{wake} \Re\left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}}\right) \gamma_{wake} dZ_{wake} - \frac{1}{2\pi R} \oint_{lev} \Re\left(\frac{Z_{lev} + Re^{i\theta}}{Z_{lev} - Re^{i\theta}}\right) \gamma_{lev} dZ_{lev} + \frac{\Gamma_k}{2\pi R} \quad (4.17)$$

To ensure smooth flow at the trailing edge ($\theta = 0$), the Kutta-Joukowski condition ($v_\theta = 0$) is enforced there to give

$$\Gamma_k = \oint_{wake} \Re\left(\frac{Z_{wake} + R}{Z_{wake} - R}\right) \gamma_{wake} dZ_{wake} + \oint_{lev} \Re\left(\frac{Z_{lev} + R}{Z_{lev} - R}\right) \gamma_{lev} dZ_{lev} \quad (4.18)$$

Remembering from the method of images that for each vortex outside the circle there exists a vortex of equal but opposite strength inside, Kelvin's law can be applied. Assuming that the initial circulation of the aerofoil-wake-LEV system was zero, we have

$$\Gamma_0(t) + \Gamma_{wake}(t) + \Gamma_{image_{wake}}(t) + \Gamma_{lev}(t) + \Gamma_{image_{lev}} + \Gamma_k(t) = 0$$

from which the following simple result is obtained

$$\Gamma_k(t) = -\Gamma_0(t)$$

Substituting the above into Equation 4.18 gives the first constraint equation

$$\boxed{\Gamma_0(t) = -\left[\oint_{wake} \Re\left(\frac{Z_{wake} + R}{Z_{wake} - R}\right) \gamma_{wake} dZ_{wake} + \oint_{lev} \Re\left(\frac{Z_{lev} + R}{Z_{lev} - R}\right) \gamma_{lev} dZ_{lev} \right]} \quad (4.19)$$

This is the general wake integral equation for the flapping-wing problem and is also the circle plane rendering of a generalised, *nonlinear* form of the original Wagner equation. In his work, Wagner presented a simple, linear form of this equation (Wagner, 1925). Later, von Kármán & Sears obtained the same linear form but using the circulation approach (von Kármán & Sears, 1938, Eqn. 38)). More recently, McCune *et al.* (1990) generalised the form of the equation derived by von Kármán & Sears to include the effect of a nonlinear deforming wake (see also Lam, 1989).

The equation presented here (Equation 4.19) is a further generalisation that is still nonlinear but now includes the effect of a second wake — the leading-edge vortex. This form of the equation has never been presented before and is one of the novel equations derived in this work.

Second Constraint Equation — Stagnation at Leading Edge Having enforced the Kutta-Joukowski condition at the trailing edge, the velocity distribution on the circle can be found by summing the wake-free (Equation 4.15, remembering that velocity and vorticity are interchangeable, i.e. $v_\theta \equiv \gamma_\theta$) and wake-induced contributions (Equation 4.17) thus

$$v_\theta = \overbrace{A + \frac{\Gamma_0}{2\pi R}}^{\text{wake-free}} + \underbrace{\left(-\frac{\Gamma_0}{2\pi R}\right) + B}_{\text{wake-induced}} = A + B \quad (4.20)$$

where

$$A = \frac{1}{R} \left[-A_1 \cos \theta - \left(A_2 + \frac{1}{2}A_7\right) \cos 2\theta + A_3 \sin \theta + \left(A_4 - \frac{1}{2}A_5 + \frac{1}{2}A_6\right) \sin 2\theta \right. \\ \left. - A_8 \sin \theta \cos 2\theta + A_9 \sin \theta \sin 2\theta - A_{10} \cos \theta \cos 2\theta + A_{11} \cos \theta \sin 2\theta \right] \\ - 2U_\infty \sin(\theta - \alpha)$$

and

$$B = -\frac{1}{2\pi R} \left[\oint_{lev} \Re\left(\frac{z_{lev} + Re^{i\theta}}{z_{lev} - Re^{i\theta}}\right) \gamma_{lev} dz_{lev} + \oint_w \Re\left(\frac{z_w + Re^{i\theta}}{z_w - Re^{i\theta}}\right) \gamma_w dz_w \right]$$

and where U_∞ is freestream velocity, α is angle of attack and the symbols $A_i, i = 1 \dots 11$ are as defined in Equation 4.13.

The leading-edge vortex sheet emanates from the leading edge so that a streamline originating there encloses the leading-edge vortex and reattaches (on most occasions) on the aerofoil further downstream. This implies that there is no load on the wing itself

along the leading edge, there being none across the vortex sheets, and hence, it must be a stagnation point ($v_\theta = 0$). In the circle plane, the leading edge occurs at $\theta = \pi$ so that

$$A|_{\theta=\pi} = \frac{1}{R} \left[A_1 - \left(A_2 + \frac{1}{2} A_7 \right) + A_{10} \right] - 2U_\infty \sin \alpha(t) \quad (4.22a)$$

and

$$B|_{\theta=\pi} = -\frac{1}{2\pi R} \left[\oint_{wake} \Re \left(\frac{Z_{wake} - R}{Z_{wake} + R} \right) \gamma_{wake} dZ_{wake} + \oint_{lev} \Re \left(\frac{Z_{lev} - R}{Z_{lev} + R} \right) \gamma_{lev} dZ_{lev} \right] \quad (4.22b)$$

Substituting Equations 4.22 into Equation 4.20, equating to zero and rearranging gives

$$\boxed{\begin{aligned} & \frac{1}{R} \left[A_1 - \left(A_2 + \frac{1}{2} A_7 \right) + A_{10} \right] - 2U_\infty \sin \alpha(t) \\ &= \frac{1}{2\pi R} \left[\oint_{wake} \Re \left(\frac{Z_{wake} - R}{Z_{wake} + R} \right) \gamma_{wake} dZ_{wake} + \oint_{lev} \Re \left(\frac{Z_{lev} - R}{Z_{lev} + R} \right) \gamma_{lev} dZ_{lev} \right] \end{aligned}} \quad (4.23)$$

This is the second constraint equation. It expresses the condition for stagnation at the leading edge and is based on the nonlinear wake integral (derived earlier; see Equation 4.19). This is the second novel equation derived in the current form for the first time.

Together, Equations 4.19 and 4.23 form the cornerstone of the current nonlinear, unsteady, inviscid approach. They are two simultaneous equations that are then used to solve for γ_{wake} and γ_{lev} at any point in time t .

Combined Effect of Leading-Edge Vortex and Trailing-Edge Wake By assuming that there is no flow inside the circle (or aerofoil for that matter), the surface velocity directly gives the vorticity distribution. By substituting $\Gamma_k(t) = -\Gamma_0(t)$ into Equation 4.17, the vorticity distribution induced by the leading-edge vortex and the trailing-edge wake vortices on the circle becomes

$$\gamma_1(\theta, t)|_{wake+lev} = -\frac{\Gamma_0(t)}{2\pi R} - \frac{1}{2\pi R} \left[\oint_{wake} \Re \left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}} \right) \gamma_{wake} dZ_{wake} + \oint_{lev} \Re \left(\frac{Z_{lev} + Re^{i\theta}}{Z_{lev} - Re^{i\theta}} \right) \gamma_{lev} dZ_{lev} \right] \quad (4.24)$$

This expression gives the additional bound vorticity around the circle induced by the effects of the two shed wakes.

In the absence of leading-edge separation, only the classical Kutta-Joukowski condition at the trailing edge is satisfied and Equation 4.24 reduces to

$$\gamma_1(\theta, t)|_{wake} = -\frac{\Gamma_0(t)}{2\pi R} - \frac{1}{2\pi R} \oint_{wake} \Re\left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}}\right) \gamma_{wake} dZ_{wake} \quad (4.25)$$

Note that in this case, the stagnation condition at the leading edge is not enforced so the vorticity distribution around the aerofoil may be quite different from the case of twin separation. The corresponding nonlinear Wagner integral equation reduces to the result presented by Lam (1989), thus

$$\Gamma_0(t) = - \oint_{wake} \Re\left(\frac{Z_{wake} + R}{Z_{wake} - R}\right) \gamma_{wake} dZ_{wake} \quad (4.26)$$

It may be noted that the forms of the Wagner integral equation derived thus far have all been in the circle plane. The form in the aerofoil plane can be obtained by inverting Z from the mapping in Equation 4.1 into ζ . This is nontrivial and not necessary¹⁶.

A note on the Wagner integral equation is in order. The linear form of the function derived by von Kármán & Sears (1938) (in the aerofoil plane) was

$$\Gamma_0(t) = - \int_1^\infty \sqrt{\frac{x+1}{x-1}} \gamma(x) dx$$

where the variable x ranges from 1 at the trailing edge to infinity. The wake was assumed to be linear in that it laid flat on the x -axis in an x - y Cartesian system. As suggested by Leishman (2000, § 8.10), the indicial nature of Wagner's equation lends to solution using Duhamel's integral where a step change (in angle of attack or velocity) would be represented by a Dirac-delta function (see von Kármán & Biot, 1940, pp. 403–405).

For insect-like flapping wings, the trailing wake is unlikely to be linear. In fact, it is not so. The problem is further complicated by the presence of a second separation in

¹⁶This is because vorticity (in the circle plane) can be converted easily to circulation (which is the same in both circle and aerofoil planes) using $\Gamma = \oint \gamma dZ$. See also Equation 4.3e.

the form of the leading-edge vortex which spends considerable time above the aerofoil. A Duhamel-integral-type representation is, therefore, going to be very complicated¹⁷. What is obvious though, is that a suitable form for the Duhamel integral for flapping wings would have to be kinematics-dependent if it is to capture the nonlinearity with any accuracy.

4.2.3 Forces and Moments

For a body moving in an incompressible and irrotational flow, two classical methods for calculating the forces and moments on it are found in literature. The more common approach is to use the unsteady form of the Bernoulli equation¹⁸

$$\frac{\partial \varphi}{\partial t} + \frac{q^2}{2} + \frac{p}{\rho} + \Pi = f(t) \quad (\text{a function of time}) \quad (4.27)$$

where φ is velocity potential, q is velocity, p is pressure, ρ is density and Π is the potential due to extraneous forces on the body ($= gz$ along a streamline, g and z being acceleration due to gravity and height respectively). This equation is a direct result of the Euler equation. The computation of the $\frac{\partial \varphi}{\partial t}$ term makes the approach cumbersome especially for the flapping wing where perturbations are large and a freestream velocity does not necessarily exist¹⁹. This approach has, therefore, not been adopted here.

The second method for computing forces and moments is the momentum-based method of vortex pairs used by von Kármán & Sears (1938). The idea is that for every shed vortex, there exists an equal and opposite vortex on the wing and the two

¹⁷Perhaps a stepping stone to writing a suitable kernel for the Duhamel integral would be to define it for the vortex wake of a delta wing and for the nonlinear trailing-edge wake, and then to combine the two.

¹⁸After Daniel Bernoulli (1700–1782) who published his work *Hydrodynamica* (1738). Actually, it was Leonhard Euler (1707–1783) who in 1755 derived the mathematical form of the equation that we have today.

¹⁹For steady flows, Blasius's theorem can also be used to determine the forces (and moments) on a body in a fluid (Blasius, 1910, p. 10). In unsteady flows, however, due to the presence of discontinuities in the wake (from shed vortices) the approach cannot be applied (Panton, 1996, p. 498).

constitute a vortex-pair (see Figure 2.15). The momentum per unit span of the system can be expressed by the sum of the momentum of the vortex-pairs that constitute the system from which force and moment data can then be extracted.

The method was first suggested by Lord Kelvin²⁰ and has since been discussed by many authors (e.g. Lamb, 1932; Karamcheti, 1966; Wu, 1981). Thomson (1910) showed that for a system of vortices, it is possible to relate the impulse and moment of impulse to the force and moment respectively — the so-called *Kelvin's Impulse Theorem*. It may be noted that although the formulation of the above two approaches are quite different, they are both derived from the same Euler equations.

The second method of vortex pairs is the approach that has been adopted here for the present work. The main advantage is the relative simplicity with which the forces and moments can be computed. The disadvantage, however, is that pressure distributions on the aerofoil surface cannot be computed directly. For the purposes of this study, however, the method is sufficient.

The calculation of the forces and moments presented below is in terms of vectors. This is followed by a section dedicated to conversion to complex number notation which ties up well with the preceding complex-variable-based unsteady aerodynamic analysis. Full details of the derivations presented below can be found in Appendix C.5.

Vector Form of Force and Moment Equations

Kelvin's theorem showed that the *external* force F_e (in an inertial coordinate system) applied to a body in a fluid to produce a prescribed motion is given by

$$\mathbf{F}_e = \frac{d}{dt} \left(\mathbf{L}_v + \rho \iiint_S \boldsymbol{\varphi} \mathbf{n} dS \right) \quad (4.28)$$

²⁰William Thomson (1824–1907) was knighted and became Sir William Thomson. Later, he was elevated to the House of Lords where he dropped his original name and took up Lord Kelvin. His seminal work (Thomson, 1910) was published posthumously.

where L_v is the linear momentum of the body, ρ is the fluid density, S is the boundary surface enclosing the body and $\mathbf{n}$ is the unit normal vector on the boundary pointing out of the fluid (i.e. *into* the body). Similarly, the external moment M_e (taken about the origin of the inertial coordinate system) that must be exerted on the body is given by

$$M_e = \frac{d}{dt} \left(L_\omega + \rho \iiint_S \varphi (\mathbf{r} \wedge \mathbf{n}) dS \right) \quad (4.29)$$

where $\mathbf{r}$ is the vector distance from the origin of the coordinate system with respect to which the flow is stationary at infinity and L_ω is the angular momentum of the body in this reference frame. Because this approach is momentum-based, all derivations are made in an inertial coordinate system where Newton's 2nd law of motion is valid.

It is important to note, as pointed out by Sears (1960), that F_e and M_e are not the resultants of fluid pressures that we usually calculate (as from Bernoulli's equation) but are the external force and moment respectively required to overcome both the inertia of the body and that of the fluid. It is the latter that manifests itself in the form of fluid pressures on the body. So, Equations 4.28 and 4.29 include the resultants of the pressures but with the sign reversed²¹. The force and moment exerted on the body by the fluid are, therefore, given by the right hand members of the right-hand side of Equations 4.28 and 4.29.

The force on the body in the inertial reference frame $\tilde{\mathbf{F}}$ is then equal to the time rate of change of the impulse of the vortex system $\tilde{\mathbf{I}}$, i.e.

$$\tilde{\mathbf{F}} = \frac{d\tilde{\mathbf{I}}}{dt} \quad \text{where} \quad \tilde{\mathbf{I}} = \rho \iiint_S \varphi \mathbf{n} dS \quad (4.30)$$

and where $\tilde{\mathbf{I}}$ is the Kelvin impulse. Similarly, the moment of the system about the origin is given by the time rate of change of the moment of impulse of the system $\tilde{\mathbf{I}}_m$, i.e.

$$\tilde{\mathbf{M}} = \frac{d\tilde{\mathbf{I}}_m}{dt} \quad \text{where} \quad \tilde{\mathbf{I}}_m = \rho \iiint_S \varphi (\tilde{\mathbf{r}} \wedge \mathbf{n}) dS \quad (4.31)$$

²¹Incidentally, the quantity $-\rho \varphi$ is the impulsive pressure.

It is, however, more useful to have forces and moments in body-fixed coordinates, and all the more so for a flapping wing. Converting from the inertial ($\sim$) to the body ($\hat{\sim}$) coordinate system (see Figure 4.6) gives for force (Karamcheti, 1966)

$$\hat{\mathbf{F}} = \frac{d\hat{\mathbf{I}}}{dt} \quad (4.32)$$

and for moment

$$\hat{\mathbf{M}} = \frac{d\hat{\mathbf{I}}_m}{dt} + \tilde{\mathbf{U}} \wedge \hat{\mathbf{I}} \quad (4.33)$$

where $\tilde{\mathbf{U}}$ is the translational velocity of the body-fixed system relative to the inertial reference system.

Conversion to Complex Number Form

Due to extensive use of the complex variable in the preceding analysis, it is appropriate to convert the force and moment formulae presented above to complex plane coordinate systems. In this way, force and moment information can be extracted immediately from the calculations without major modifications.

In complex coordinate system notation (ζ -plane), impulse is given by

$$\mathcal{I} = \mathcal{I}_\xi + i\mathcal{I}_\eta$$

and force is

$$\mathcal{F} = \mathcal{F}_\xi + i\mathcal{F}_\eta \quad (4.34)$$

where the orthogonal axes are defined by the ξ - η Cartesian system²². Moment is not a vector quantity and is given simply by the real number $\mathcal{M}$. It is taken positive counter-clockwise (same sense as positive circulation).

²²Alternative notations suggest $\mathcal{F} = \mathcal{F}_\xi - i\mathcal{F}_\eta$ where clockwise circulation is taken as positive (e.g. Robinson & Laurmann, 1956, § 2.3, p. 86). In our case, positive circulation is counterclockwise.

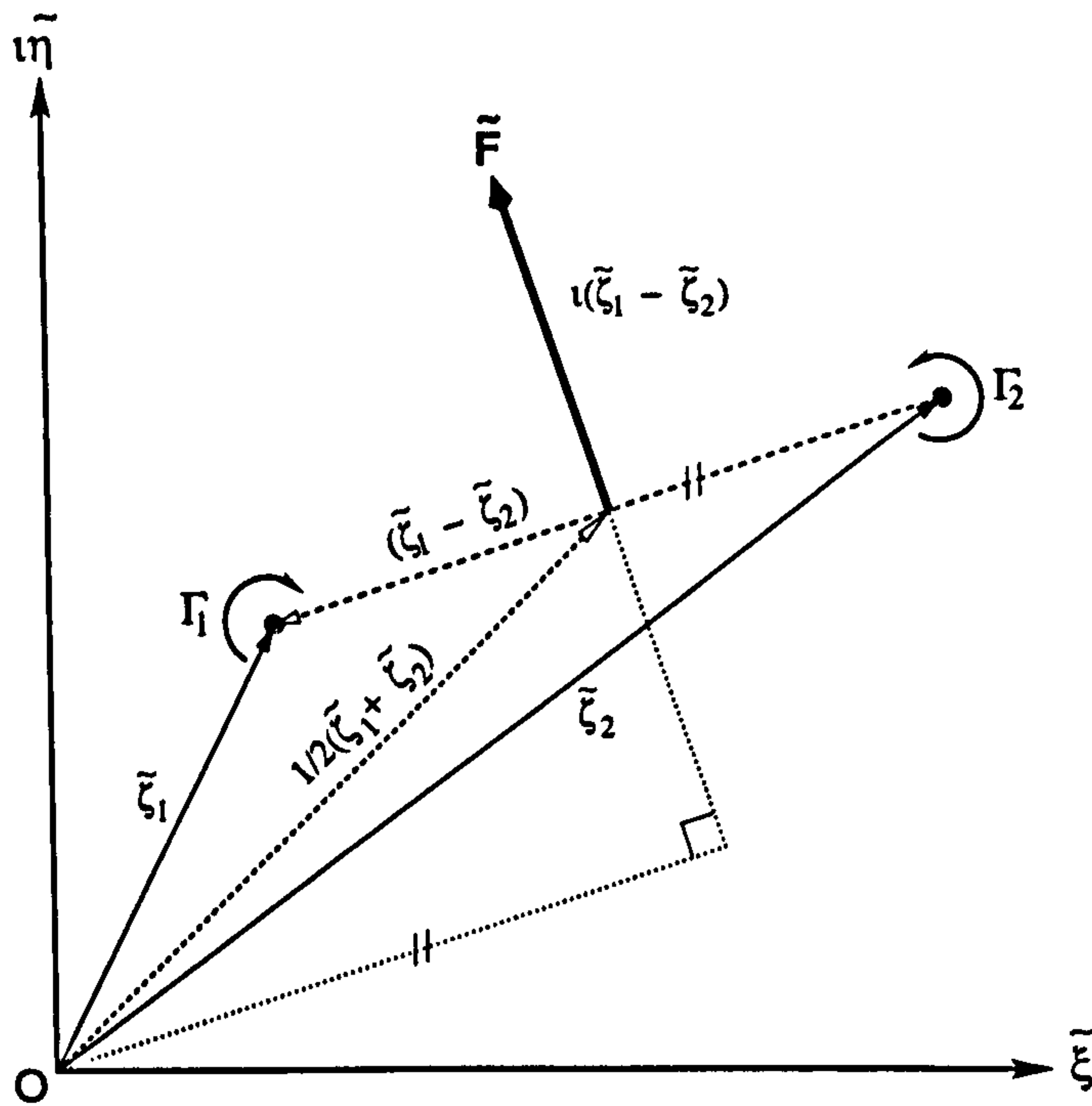


Figure 4.8: Impulse of vortex pairs

For a vortex pair of strength Γ and separated by a distance l , the absolute value of impulse is given by $|\mathcal{I}| = \rho\Gamma l$ and acts perpendicular to the 'plane' of the vortices (see Durand (1935, p. 325)). Now consider two vortices of strengths Γ_1 and Γ_2 at positions $\tilde{\zeta}_1$ and $\tilde{\zeta}_2$ respectively, that constitute a vortex pair²³ in complex space (see Figure 4.8). The impulse of the vortex pair acts halfway between the two vortices at $\frac{1}{2}(\tilde{\zeta}_1 - \tilde{\zeta}_2)$ and in a direction perpendicular to the line joining the two vortices, i.e. in the direction of $i(\tilde{\zeta}_1 - \tilde{\zeta}_2)$. The complex impulse is, therefore, given by

$$\begin{aligned}\tilde{\mathcal{I}} &= \rho\Gamma_1 l \times \text{direction} = \rho\Gamma_1 |\tilde{\zeta}_1 - \tilde{\zeta}_2| \times \frac{i(\tilde{\zeta}_1 - \tilde{\zeta}_2)}{|\tilde{\zeta}_1 - \tilde{\zeta}_2|} \\ &= i\rho\Gamma_1(\tilde{\zeta}_1 - \tilde{\zeta}_2)\end{aligned}$$

$$\therefore \tilde{\mathcal{I}} = i\rho(\Gamma_1\tilde{\zeta}_1 + \Gamma_2\tilde{\zeta}_2) \quad (\because \Gamma_1 = -\Gamma_2) \quad (4.35)$$

Calculation of the moment of impulse follows similarly

²³In the sense described by von Kármán & Sears (1938, p. 380). See also Figure 2.15.

$$\begin{aligned}
 \tilde{\mathcal{I}}_m &= \rho \Gamma_1 l \times \text{perpendicular distance} = \rho \Gamma_1 |\tilde{\zeta}_1 - \tilde{\zeta}_2| \times \frac{1}{2} \frac{(|\tilde{\zeta}_1|^2 - |\tilde{\zeta}_2|^2)}{|\tilde{\zeta}_1 - \tilde{\zeta}_2|} \\
 &= \frac{1}{2} \rho \Gamma_1 (|\tilde{\zeta}_1|^2 - |\tilde{\zeta}_2|^2) \\
 \therefore \tilde{\mathcal{I}}_m &= \frac{\rho}{2} (\Gamma_1 |\tilde{\zeta}_1|^2 + \Gamma_2 |\tilde{\zeta}_2|^2) \quad (\because \Gamma_1 = -\Gamma_2) \quad (4.36)
 \end{aligned}$$

From Equations 4.35 and 4.36, it can be seen that the total impulse and moment of impulse respectively of the entire system are given by

$$\tilde{\mathcal{I}} = \rho \sum_{j=1}^{\infty} \Gamma_j \tilde{\zeta}_j \quad (4.37)$$

$$\tilde{\mathcal{I}}_m = \frac{\rho}{2} \sum_{j=1}^{\infty} \Gamma_j |\tilde{\zeta}_j|^2 \quad (4.38)$$

If all shed vortices were to lie on the real axis (and there were no leading-edge vortices), Equations 4.37 and 4.38 would reduce to the form given in the treatment of von Kármán & Sears (1938) for a flat (linear) wake.

Following on from Equations 4.30 and 4.31, the force and moment about the origin are found by taking the derivatives with respect to time of Equations 4.37 and 4.38, thus

$$\tilde{\mathcal{F}} = \rho \frac{d}{dt} \sum_{j=1}^{\infty} \Gamma_j \tilde{\zeta}_j = \rho \frac{d}{dt} \int \tilde{\zeta} \gamma(\tilde{\zeta}) d\tilde{\zeta} \quad (4.39)$$

$$\tilde{\mathcal{M}} = \frac{\rho}{2} \frac{d}{dt} \sum_{j=1}^{\infty} \Gamma_j |\tilde{\zeta}_j|^2 = \frac{\rho}{2} \frac{d}{dt} \int |\tilde{\zeta}|^2 \gamma(\tilde{\zeta}) d\tilde{\zeta} \quad (4.40)$$

Since the wakes are free vortex sheets, they are incapable of supporting Kutta-Joukowski forces. Any forces on the system are sustained only by the bound vortices, i.e. the aerofoil surface. Therefore, computing the forces and moments on the entire system of vortices is equivalent to evaluating forces and moments respectively on the wing, the free vortex sheets being unable to support any loads.

As mentioned already, it is more useful to have forces and moments in a coordinate system that moves with the aerofoil, in this case the aerofoil-inertia system (ξ - η) system in

Figure 4.6). Using Equations 4.32 and 4.33, we get

$$\mathcal{F} = \frac{d\hat{\mathcal{T}}}{dt} \quad (4.41)$$

and

$$\hat{\mathcal{M}} = \frac{d\hat{\mathcal{T}}_m}{dt} - \Im(U_0 \hat{\mathcal{T}}) \quad (4.42)$$

where

$$\hat{\mathcal{T}} = i\rho \sum_{j=1}^{\infty} \Gamma_j \hat{\zeta}_j \quad (4.43a)$$

$$\hat{\mathcal{T}}_m = \frac{\rho}{2} \sum_{j=1}^{\infty} \Gamma_j |\hat{\zeta}_j|^2 \quad (4.43b)$$

$$U_0 = (-U_{\infty} + i) + ih \quad (4.43c)$$

and U_0 is the translational velocity of the moving system.

Calculation of Forces and Moments

A better understanding of the origin of the forces and moments in the current unsteady aerodynamic model for flapping flight can be achieved by decoupling them into their constituent components. This idea of decomposing the forces and moments goes back to the original work of von Kármán & Sears (1938) (see Equation 2.7, p. 45). Later, McCune *et al.* (1990) extended the approach to incorporate the effects of the nonlinear extensions into the forces and moments, thereby introducing a further constituent — the deficit due to the wake. In the present work, the constituent elements have been further modified from the form given by McCune *et al.* to include the effects of the leading-edge vortices. Such a decomposition is again a novel aspect of this work.

The vorticity in the aerofoil-wake system γ can be decoupled into the aerofoil-bound component γ_a and the shed component γ_s . The bound vorticity term can further be divided into a wake-free component γ_0 and a wake-induced component γ_1 . The shed

wake can also be divided into a trailing-edge-wake component γ_{wake} and a leading-edge-vortex component γ_{lev} , thus

$$\gamma = \gamma_a + \gamma_s = \gamma_0 + \gamma_1 + \gamma_{wake} + \gamma_{lev}$$

Therefore, from Equation 4.41 force can be decoupled as

$$\hat{\mathcal{F}} = i\rho \left[\frac{d}{dt} \int_a \hat{\zeta} \gamma_0(\hat{\zeta}) d\hat{\zeta} + \frac{d}{dt} \int_a \hat{\zeta} \gamma_1(\hat{\zeta}) d\hat{\zeta} + \frac{d}{dt} \int_{wake} \hat{\zeta} \gamma_{wake}(\hat{\zeta}) d\hat{\zeta} + \frac{d}{dt} \int_{lev} \hat{\zeta} \gamma_{lev}(\hat{\zeta}) d\hat{\zeta} \right] \quad (4.44)$$

where the subscript a means the integral is taken around aerofoil, and similarly the subscripts $wake$ and lev mean the integral is taken over the respective shed wakes.

For steady motion (aerofoil at a small constant angle of attack in a uniform freestream), the first term above vanishes because the wake-free bound vorticity does not change with time, i.e. $\frac{d\gamma_0}{dt} = 0$. The second term also vanishes because the starting vortex is so far away as to have no significant effect on the bound vorticity, i.e. $\gamma_1 \approx 0$. In the absence of a leading-edge vortex (which is reasonable for steady flow at small angle of attack), the only term remaining in Equation 4.44 is the trailing-edge-wake term. For steady flow, this refers to the starting vortex (of circulation Γ_{wake}). Since this is the only wake vortex, Kelvin's law requires that the bound circulation Γ_0 be equal in strength but of opposite sense. At any time t , the starting vortex is located at $\hat{\zeta}_{wake} = (U_\infty t - l) - ih$ so that

$$\begin{aligned} i\rho \frac{d}{dt} \int_{wake} \gamma_{wake}(\hat{\zeta}) \hat{\zeta} d\hat{\zeta} &= i\rho \Gamma_w \frac{d}{dt} [(U_\infty t - l) - ih] = i\rho \Gamma_w [(U_\infty - l) - ih] \\ &= i\rho \overline{U_0} \Gamma_{wake} \end{aligned} \quad (4.45)$$

where $\overline{U_0}$ refers to the complex conjugate of the incident velocity U_0 . This equation yields the classical quasi-steady Kutta-Joukowski lift per unit span because of the equivalence of the bound and starting vortices, i.e. $i\rho \overline{U_0} \Gamma_{wake} = -i\rho \overline{U_0} \Gamma_0$.

Forces Force may, therefore, be decoupled as

$$\hat{\mathcal{F}}(t) = \hat{\mathcal{F}}_0(t) + \hat{\mathcal{F}}_1(t) + \hat{\mathcal{F}}_2(t) + \hat{\mathcal{F}}_3(t) \quad (4.46)$$

where

$$\hat{\mathcal{F}}_0(t) = -i\rho\overline{U_0}\Gamma_0 \quad (4.47a)$$

$$\hat{\mathcal{F}}_1(t) = i\rho\frac{d}{dt}\int_a\gamma_0(\hat{\zeta})\hat{\zeta}d\hat{\zeta} \quad (4.47b)$$

$$\hat{\mathcal{F}}_2(t) = i\rho\frac{d}{dt}\int_a\gamma_1(\hat{\zeta})\hat{\zeta}d\hat{\zeta} \quad (4.47c)$$

$$\hat{\mathcal{F}}_3(t) = i\rho\overline{U_0}\Gamma_0 + \frac{d}{dt}\int_{wake}\gamma_{wake}(\hat{\zeta})\hat{\zeta}d\hat{\zeta} + \frac{d}{dt}\int_{lev}\gamma_{lev}(\hat{\zeta})\hat{\zeta}d\hat{\zeta} \quad (4.47d)$$

Force $\hat{\mathcal{F}}$ is complex-valued and may be written as $\hat{\mathcal{F}} = \hat{\mathcal{F}}_H + i\hat{\mathcal{F}}_V$ where $\hat{\mathcal{F}}_H$ and $\hat{\mathcal{F}}_V$ refer to the horizontal and vertical components respectively. For flapping-wing dynamics, the severity of the manoeuvres makes it difficult to define a mean direction for lift and drag. Therefore, the aerofoil-inertia ($\hat{\xi}$ - $\hat{\eta}$) system is used to define lift $\mathcal{L}$ and drag $\mathcal{D}$ as the vertical and horizontal forces (with respect to gravity) respectively, i.e.

$$\hat{\mathcal{F}} = \mathcal{D} + i\mathcal{L} \quad (4.48)$$

From Equation 4.46, lift and drag may then be written respectively as

$$\mathcal{L}(t) = \mathcal{L}_0(t) + \mathcal{L}_1(t) + \mathcal{L}_2(t) + \mathcal{L}_3(t) \quad (4.49)$$

$$\mathcal{D}(t) = \mathcal{D}_0(t) + \mathcal{D}_1(t) + \mathcal{D}_2(t) + \mathcal{D}_3(t) \quad (4.50)$$

In Equation 4.49, $\mathcal{L}_0$ refers to the quasi-steady contribution to lift — the so-called Kutta-Joukowski lift — that would be achieved in the absence of any wake.

The $\mathcal{L}_1$ -term refers to the lift contribution due to apparent-mass effects. In order to allow the wing to pass, fluid must move aside ahead of it and close up after it. This requires kinetic energy which is constant for a wing in uniform flow at constant angle of attack, hence there is no net force. However, during wing accelerations there are changes in kinetic energy imparted to the fluid and this manifests in the form of extra or ‘added’ forces that are termed added or apparent mass. A similar effect would be observed when a wing enters a region of nonuniform flow, e.g. during wake reentry. Depending on

the exact nature of the disturbance, apparent mass could produce either beneficial or detrimental effects.

The $\mathcal{L}_2$ -term in Equation 4.49 represents the lift induced by the presence of the leading-edge vortex and the trailing-edge wake. It is responsible for the Wagner function due to its resistance to any change in flow conditions. In its classical form, it inhibits the growth of bound circulation, and is hence referred to as the lift-deficiency term. The third term ($\mathcal{L}_3$) is of a similar form to that given by Lam (1989) but with an additional component due to the leading-edge vortex added. It represents the lift deficit experienced in the presence of the two wakes.

Apparent-mass forces ($\mathcal{L}_1$) arise from impulsive pressures and are, therefore, referred to as *non-circulatory* — they can exist in the absence of circulation (e.g. symmetric flow about a flat plate executing heave oscillations in stationary flow without separation). The rest of the components in Equation 4.49 ($\mathcal{L}_0, \mathcal{L}_2, \mathcal{L}_3$) are described as *circulatory* since they rely on influencing the bound circulation to affect lift.

The first three terms in Equation 4.49 have analogues in the classical work of von Kármán & Sears (1938) albeit without any nonlinear terms. The fourth term has an analogue in the nonlinear extension described by McCune *et al.* (1990) but still without the effect of the leading-edge vortex incorporated. The latter is included in the current work along the lines of the approach for delta wings developed in McCune & Tavares (1993), albeit with appropriate modifications.

Generally, for small-angle perturbations with only trailing-edge separation, the drag component of force is small and usually ignored (except probably in Knoller-Betz investigations; see p. 59). However, in flapping-wing applications with leading-edge separation, this component of force is as significant as lift (Birch & Dickinson, 2001; Wang, 2003). The current approach, therefore, provides a novel insight into the drag analogues of the circulatory and non-circulatory lift components described above (Equation 4.50).

Moments In a manner similar to that used for forces above, moments may also be decoupled as

$$\begin{aligned} \hat{\mathcal{M}} = & \frac{\rho}{2} \left[\frac{d}{dt} \int_a |\hat{\zeta}|^2 \gamma_0(\hat{\zeta}) d\hat{\zeta} + \frac{d}{dt} \int_a |\hat{\zeta}|^2 \gamma_1(\hat{\zeta}) d\hat{\zeta} \right. \\ & \left. + \frac{d}{dt} \int_{wake} |\hat{\zeta}|^2 \gamma_{wake}(\hat{\zeta}) d\hat{\zeta} + \frac{d}{dt} \int_{lev} |\hat{\zeta}|^2 \gamma_{lev}(\hat{\zeta}) d\hat{\zeta} \right] \\ & - \Im \left[\rho U_0 \left(\int_a \hat{\zeta} \gamma_0(\hat{\zeta}) d\hat{\zeta} + \int_a \hat{\zeta} \gamma_1(\hat{\zeta}) d\hat{\zeta} \right. \right. \\ & \left. \left. + \int_{wake} \hat{\zeta} \gamma_{wake}(\hat{\zeta}) d\hat{\zeta} + \int_{lev} \hat{\zeta} \gamma_{lev}(\hat{\zeta}) d\hat{\zeta} \right) \right] \quad (4.51) \end{aligned}$$

and the constituent component moments are analogously

$$\hat{\mathcal{M}}(t) = \hat{\mathcal{M}}_0(t) + \hat{\mathcal{M}}_1(t) + \hat{\mathcal{M}}_2(t) + \hat{\mathcal{M}}_3(t) \quad (4.52)$$

where

$$\hat{\mathcal{M}}_0 = -\Im \left[\rho U_0 \int_a \hat{\zeta} \gamma_0(\hat{\zeta}) d\hat{\zeta} \right] \quad (4.53a)$$

$$\hat{\mathcal{M}}_1 = \frac{\rho}{2} \frac{d}{dt} \int_a |\hat{\zeta}|^2 \gamma_0(\hat{\zeta}) d\hat{\zeta} \quad (4.53b)$$

$$\hat{\mathcal{M}}_2 = \frac{\rho}{2} \frac{d}{dt} \int_a |\hat{\zeta}|^2 \gamma_1(\hat{\zeta}) d\hat{\zeta} - \Im \left[\rho U_0 \int_a \hat{\zeta} \gamma_1(\hat{\zeta}) d\hat{\zeta} \right] \quad (4.53c)$$

$$\begin{aligned} \hat{\mathcal{M}}_3 = & \frac{\rho}{2} \frac{d}{dt} \int_{wake} |\hat{\zeta}|^2 \gamma_{wake}(\hat{\zeta}) d\hat{\zeta} - \Im \left[\rho U_0 \int_a \hat{\zeta} \gamma_{wake}(\hat{\zeta}) d\hat{\zeta} \right] \\ & + \frac{\rho}{2} \frac{d}{dt} \int_{wake} |\hat{\zeta}|^2 \gamma_{lev}(\hat{\zeta}) d\hat{\zeta} - \Im \left[\rho U_0 \int_a \hat{\zeta} \gamma_{lev}(\hat{\zeta}) d\hat{\zeta} \right] \quad (4.53d) \end{aligned}$$

where $\Im$ denotes the ‘imaginary part of’ and U_0 is the incident velocity at the origin of the aerofoil-inertia system.

4.2.4 Extension to Three Dimensions

The approach is extended to three dimensions by using a simple blade-element-type method — the contributions of the individual wing sections are summed to give the total value for the entire wing. The framework is, therefore, essentially a ‘quasi-3-D’ model with the following assumptions made in the process:

- that there is no mutual interference between adjacent wing sections;
- that there are no 3-D tip vortex effects;
- the effect of a mirror wing on the other side of the FMAV is ignored; and
- that there is no spanwise transport of vorticity in the leading-edge vortex.

Defence for the last assumption is based on the fact that the only ‘real’ data available to validate against at the time of the current work were provided by Dickinson (priv. comm.) who had observed no spanwise entrainment in his experiments on a scaled-up model wing of the fruit fly *Drosophila* (Birch & Dickinson, 2001; Sane & Dickinson, 2001).

Dickinson’s experiments were carried out on a *single* scaled-up fruit fly wing (dubbed *Robofly*) flapping in a tank of mineral oil (of density 880 kg/m^3) at a frequency of about 0.17 Hz and sweeping a half-circle arc ($\Phi = 180^\circ$; corresponding to a Reynolds number of 160). These conditions are used in the validation cases for the current model. A second data set that was also available for comparison at the time of this work was from the indicial model of Pedersen (2003) for flapping wings (which itself was also validated against Dickinson’s data).

Chapter 5

Implementation and Validation

This chapter describes how the analysis in the preceding chapter is converted into numerical schemes in order to solve the nonlinear equations derived. It is broadly divided into two parts. The first part (§ 5.1) begins with a short review of vortex methods which form the backbone of the numerical schemes (§ 5.1.1). This is followed by a section on method of solution (§ 5.1.2). The latter describes how the equations derived in the previous chapter are converted to forms that are amenable to numerical calculation. A description of the calculation procedure (and the associated computer program) is presented and a number of pertinent numerical parameters are identified. The section ends with a look at some judicious modifications that were used to overcome numerical instabilities.

The second part of the chapter deals with the validation of the theoretical model developed (§ 5.2). It begins with a number of numerical experiments (§ 5.2.1) that were conducted to determine the best possible values of the numerical parameters identified in the previous section. This is finally followed by a section where results from the current model are compared with data from experiments for validation purposes (§ 5.2.2).

5.1 Implementation

5.1.1 Vortex Methods

Introduction

All real flows feature some form of rotational motion developed in regions adjacent to flow surfaces or in the downstream wake of bodies. Another feature of real flows is separation, usually originating at sharp corners or produced as a result of adverse pressure gradients. Vortex methods attempt to model these flows by discretisation of the distributed vorticity or separated shear layers by a finite number of small discrete vortex elements.

Discrete Point Vortices

Historically, Rosenhead (1931) was probably the first to study the unsteady motion of two-dimensional vortex sheets. He replaced the vortex sheet by a (small) finite number of *discrete point vortices*¹ and monitored their evolution under their mutual influence. Using numerical computation he studied the Kelvin-Helmholtz instability² of vortex sheets — the process whereby the sheets periodically break up into sections along their length which themselves roll up into stable vortex cores.

Since Rosenhead, discrete vortex methods have been used by large numbers of researchers to model the wakes behind bodies. Some have doubted the validity of the method (e.g. Moore, 1971) arguing that there is a fundamental difference between vortex sheets and an assembly of vortices. But the method has stood the test of time and has been used successfully to make it one of the best approaches for vortex dynamics in inviscid flows.

¹These are an extension of Helmholtz point vortices (von Helmholtz, 1858) and represent the simplest vortex methods in an irrotational and two-dimensional flow field.

²Also known simply as Helmholtz instability after Hermann von Helmholtz (1821–1894).

One of the main advantages of the technique is that it lends itself very well to computational methods, and with the advent of fast microcomputers yields quick and reasonably accurate results.

Before proceeding further, it may be noted that one of the major difficulties encountered in solving the Navier-Stokes equations using finite-difference or finite-element (conventional CFD) methods is the number of grid points. The number of grid points required to obtain a solution scales up with increasing Reynolds number, typically $\mathcal{O}(Re^{9/4})$ (Sumer & Fredsøe, 1997, §5.2). It is, therefore, of great interest to develop a grid-free (or almost grid-free) numerical method like the discrete vortex method.

Evolution and Convection of Vortex Sheets

The discrete point vortices comprising a vortex sheet each induce a circular vortex flow around them. A velocity field exists around each vortex due to the combined velocities induced by all neighbouring vortices. In accordance with the Kutta-Joukowski law, therefore, the vortices must be subject to a force thus

$$l = \rho U \Gamma \quad (5.1)$$

where ρ is fluid density, U is the relative velocity at the point vortex, Γ is circulation and l is force per unit length on the vortex acting normal to U . A discrete point vortex in a vortex sheet is free so that the Kutta-Joukowski force on it must be zero. Therefore, the flow velocity relative to the vortex at the vortex position must be zero. In other words, a free vortex must move at the *Kirchhoff velocity*³ — the speed of the flow in the place of the vortex if the latter were nonexistent (Betz, 1932)⁴.

The complex conjugate velocity q induced at ζ by a two-dimensional vortex sheet C of

³After Gustav Robert Kirchhoff (1824–1887).

⁴Lamb (1932) gives a similar argument for vortex convection but based on mass and inertia — the so-called *limit solution of vanishing mass* (see Ting & Klein, 1991, pp. 42–43).

strength $\gamma(s, t) = \frac{\partial \Gamma}{\partial s}$ is given by the Rott-Birkhoff nonlinear integro-differential equation⁵ (Rott, 1956; Birkhoff, 1962)

$$q(\zeta) = u - iv = -\frac{i}{2\pi} \oint_C \frac{\gamma(s', t) ds'}{\zeta - \zeta(s', t)} + (U_E - iV_E) \quad (5.2a)$$

where (U_E, V_E) are the components of the external irrotational velocity field evaluated at ζ and ' (the 'prime' symbol) refers to all vortices other than the one in question. If circulation Γ is chosen as the Lagrangian variable then Equation 5.2a may be written as

$$q(\zeta) = -\frac{i}{2\pi} \oint_C \frac{d\Gamma'}{\zeta(\Gamma, t) - \zeta(\Gamma', t)} + (U_E - iV_E) \quad (5.2b)$$

Although Equations 5.2 satisfy the continuity of pressure across the vortex sheet, they do not ensure analytically tractable solutions and numerical methods must be resorted to.

Thus, the (complex) velocity induced by a vortex system may be discretised as

$$q = -\frac{i}{2\pi} \oint \frac{\gamma}{\zeta} d\zeta \approx -\frac{i}{2\pi} \sum \frac{\gamma}{\zeta} d\zeta = -\frac{i}{2\pi} \sum \frac{\delta\Gamma}{\zeta} \quad (5.3)$$

Equation 5.3 essentially represents the Kirchhoff velocity with which all free vortices are convected. A number of vortex convection schemes are found in the literature. The most common is the simple *forward Euler* scheme, thus

$$\zeta(t + \delta t) = \zeta(t) + q(t) \delta t \quad (5.4)$$

where δt represents a small increment in time t .

Despite its simplicity and crudeness, Chorin & Bernard (1973) argue that it is reasonable to use a straightforward Euler scheme because it is more economical in computing time and does not introduce errors of larger order than those originating from the approximation used in Equation 5.3. The forward Euler scheme is essentially a trapezoidal quadrature scheme for the integral in Equation 5.2 where the discrete point vortices are markers for the intervals.

⁵This equation satisfies the Euler equation exactly (Bene *et al.*, 2000).

A number of other convection schemes have been suggested, such as (Sarpkaya, 1975; Xu, 1999)

$$\zeta(t + \delta t) = \zeta(t - \delta t) + 2q(t) \delta t \quad (5.5a)$$

$$\zeta(t + \delta t) = \zeta(t) + \frac{1}{2} [3q(t) - q(t - \delta t)] \delta t \quad (5.5b)$$

The latter schemes are better suited to flows that have a dominant freestream flow with small perturbations. For a flapping wing where perturbations dominate and a freestream flow may not exist, Equations 5.5 are not appropriate because they depend on ‘historical’ values. The convection schemes shown in Equations 5.4–5.5 are not specific to fluid mechanics but are commonly used in the solution of initial-value problems (see e.g. Stocer & Bulirsch, 2002, §7.2).

A more appropriate approach is to divide each time-step into sub-time-steps as suggested by Sarpkaya (1975) and Lewis (1991), thus

$$\zeta(t + \frac{1}{n}\delta t) = \zeta(t) + \frac{1}{n} q(t) \delta t \quad (5.6)$$

and evaluate using the Euler scheme n times per time-step. This is particularly useful in regions where flow curvature or angular displacement are high (such as stroke reversals). A practical reason for using this technique is that if δt were too large, a vortex close to the wing could move far enough to penetrate the impermeable wing surface before being checked by the image vortex (Cheng *et al.*, 1988). The method was also implemented successfully by for delta wings by Tavares (1990) and Lee (1991).

Lewis (1991) argued that a consistent convection scheme should trace back the original convection path if the problem was run backwards in time, i.e. replacing δt by $-\delta t$. He showed that good consistency was obtained either by decreasing the time-step δt or by increasing the number of sub-time-steps n . Sarpkaya (1975) suggested a value of 5 for n . For a flapping wing, for example, sub-time-steps may be increased during the more extreme phases of the wingstroke cycle.

Desingularisation Schemes

As the distance between a pair of vortices diminishes, the error of the Euler scheme increases because induced velocities become very large (cf. Equations 5.2 and 5.4). Real vortices are not concentrated singularities of infinite vorticity. This was the main objection raised by Moore (1971) who cited that using more point vortices would only make matters worse in terms of Kelvin-Helmholtz instabilities.

The velocity distribution of an inviscid potential vortex is asymptotic at its centre, tending to infinity there (see Figure 5.1). In reality, the presence of viscosity prevents the occurrence of infinite velocities. In fact, viscosity is responsible for the formation of vortices in the first place and its effects extend to their evolution and decay.

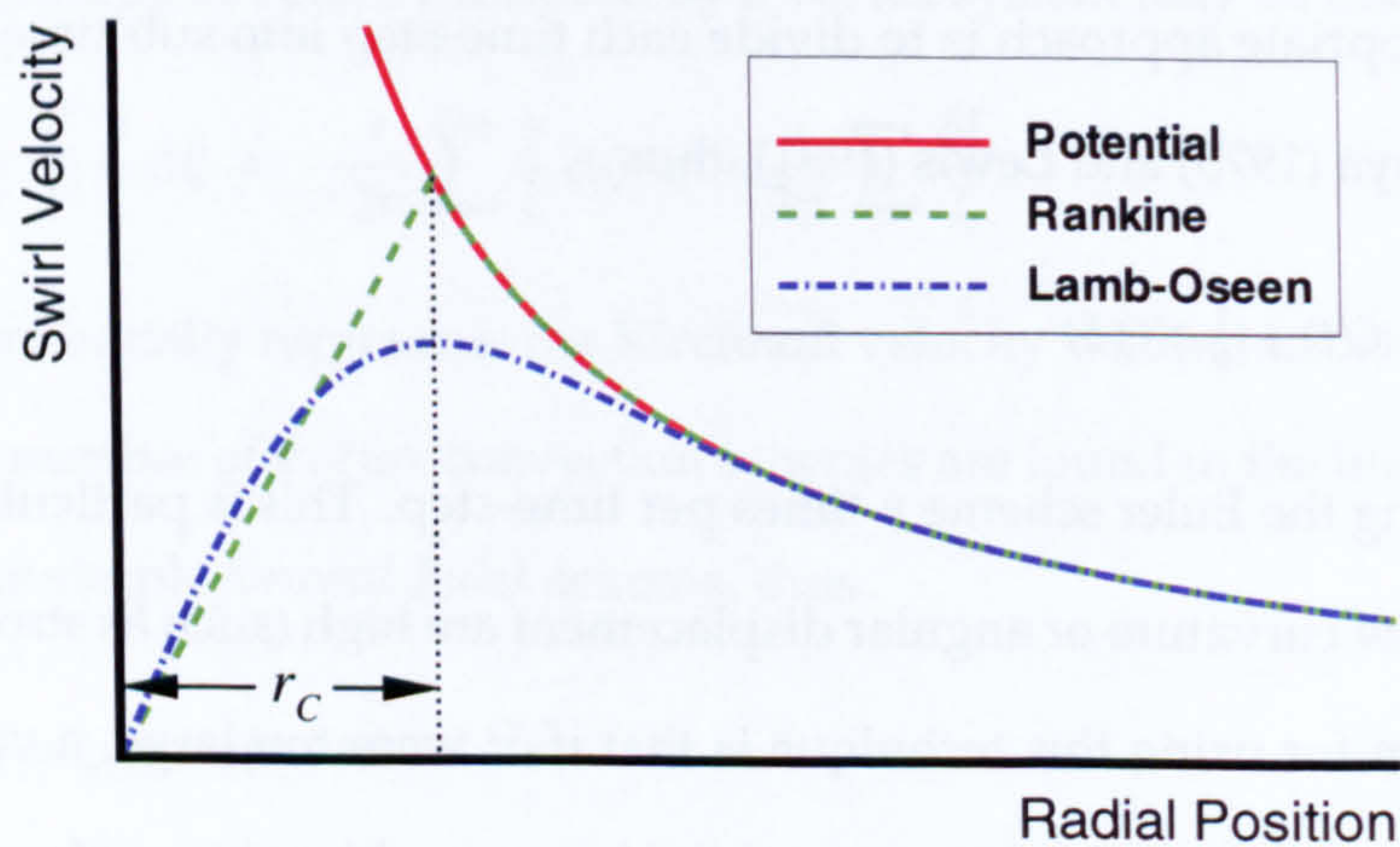


Figure 5.1: Vortex models (r_c is vortex-core radius)

A common desingularisation scheme is to use the Rankine⁶ model — all vorticity is confined to a core region that rotates like a solid body surrounded by a region behaving as a free potential vortex⁷ (see Figure 5.1). The tangential velocity distribution v_θ at a

⁶After William Rankine (1820–1872).
⁷This is a *vortex blob* method where the vorticity of a point vortex is distributed over a small but finite circular area (Chorin & Bernard, 1973).

radial distance r from the centre of a Rankine vortex is given by

$$v_{\theta}(r) = \begin{cases} \frac{\Gamma}{2\pi} \frac{r}{r_c^2} & 0 \leq r \leq r_c \\ \frac{\Gamma}{2\pi r} & r > r_c \end{cases} \quad (5.7)$$

where r_c is the radius of the vortex core. As can be seen, an artificial discontinuity exists at $r = r_c$. Ling *et al.* (1987, p. 124) gave a method for estimating this initial radius of separated vortices. They argued that because each discrete vortex comes from boundary-layer separation, it is reasonable to consider that it has a nonzero initial radius related to the boundary-layer dimensions at separation.

A second model (which is probably the most common) is the Lamb-Oseen vortex (see Figure 5.1) which is based on an exact Navier-Stokes solution of the one-dimensional⁸ laminar flow around a single viscous vortex in an unbounded incompressible domain (Oseen, 1911; Lamb, 1932). The model assumes a Gaussian vorticity distribution so that the circumferential velocity is given by

$$v_{\theta}(r) = \frac{\Gamma}{2\pi r} \left[1 - e^{(-r^2/4\nu t)} \right] \quad (5.8)$$

where ν is kinematic viscosity and t is time. The Lamb-Oseen vortex model not only removes the singularity at the vortex centre but also includes a measure for vortex decay through *viscous diffusion*. Careful examination of Equation 5.8 shows that the vortex-core radius grows as

$$r_c(t) = \sqrt{4\alpha\nu t} \quad (5.9)$$

where α is the Oseen parameter ($\alpha = 1.25643$) (Sarpkaya, 1989; Bhagwat & Leishman, 2002). In other words, the Lamb-Oseen vortex depends on vortex decay for the core radius to grow, suggesting an initial vortex-core radius of zero.

To get around this problem, Squire (1965) suggested an ‘offset’ time t_0 to give a finite core radius at time $t = 0$. Kuwahara & Takami (1973) argued that the induced velocities

⁸The axial and radial velocities are zero.

of a two-dimensional vortex given by the Lamb-Oseen derivation (Equation 5.8) were not strictly compatible with the Navier-Stokes equations whose nonlinearity does not permit the superposition of the vortex field. Therefore, ν in Equation 5.8 should not be taken as the real viscosity but instead as an ‘artificial viscosity’. Serrin (1972) lent support to this ‘concept’ of a virtual or eddy viscosity, especially in modelling large-scale phenomena such as tornadoes where turbulence was also involved.

Although a whole range of ‘desingularisation’ schemes are found in the literature, the vortex blob family of vortices are by far the commonest and most successfully implemented. Even Rosenhead (1930, as explained by Sarpkaya (1989)) realised the need for desingularisation and proposed the following simple vortex blob representation

$$v_{\theta}(r) = \frac{\Gamma}{2\pi} \frac{r}{r_c^2 + r^2} \quad (5.10)$$

Another desingularising scheme based on the Lamb-Oseen model was proposed by Vatis-tas *et al.* (1991), thus⁹

$$v_{\theta}(r) = \frac{\Gamma}{2\pi} \frac{r}{\sqrt{r_c^4 + r^4}} \quad (5.11)$$

Kornev (1999) presented a clever technique that built on the fact that vortex strength was finite, so that even if distance to the vortex centre δ reduced the product of δ and vortex strength must remain finite.

Vortex Amalgamation

One way of limiting the effects of the unrealistically high induced velocities on vortices in close proximity is to amalgamate them, i.e. merge two or more vortices into one representative vortex. This method is usually driven by the need to reduce computing (CPU) time and also has the added advantage of reducing the tendency of (discrete

⁹Vatistas *et al.* actually gave a more generic form. The version quoted above is of order 2 and shows good agreement with the Lamb-Oseen vortex.

point) vortices to orbit each other. Vortex amalgamation is also a smoothing scheme. A good discourse on the rules for merging vortices can be found in Ting & Klein (1991, pp. 147–148).

Two vortices are merged when the distance between their vortex centres falls below a certain value (Spallart, 1988; Sarpkaya, 1989; Ting & Klein, 1991). (By the same argument, when the distance between a consecutive pair of vortices grows beyond a certain value, a third vortex may be introduced at an intermediate position to represent the vortex sheet better (Mook & Dong, 1994).)

Ting & Klein (1991) proposed that if the vortex-core sizes of adjacent vortices i and j are r_{c_i} and r_{c_j} respectively then merging begins when the distance between them δ_{ij} is

$$\frac{\delta_{ij}}{r_{c_i} + r_{c_j}} = 2 \quad (5.12)$$

and only if the vortices are of like sign ($\Gamma_i \Gamma_j > 0$) (Mook & Dong, 1994). Spallart (1988) offered a slightly different criterion — vortex merging begins when

$$\left| \frac{\Gamma_i \Gamma_j}{\Gamma_i + \Gamma_j} \right| \delta_{ij} < \epsilon \quad (5.13)$$

where ϵ is a small number.

Two main rules are observed in vortex amalgamation. First is to enforce conservation of vorticity (circulation), i.e.

$$\Gamma = \sum_{i=1}^n \Gamma_i \quad (5.14)$$

where n and Γ_i are the number and circulation of vortices being amalgamated respectively and Γ is the circulation of the final amalgamated vortex. Second is to conserve linear momentum by ensuring that the impulse of circulation is kept constant through the process, i.e.

$$i\Gamma\zeta = i \sum_{i=1}^n \Gamma_i \zeta_i \quad (5.15)$$

where ζ is the position of the amalgamated vortex (in complex coordinates) and ζ_i similarly represents the individual vortex positions before amalgamation. The amalgamated vortex is, therefore, placed at the combined centre of circulation, i.e.

$$\zeta = \frac{\sum \Gamma_i \zeta_i}{\sum \Gamma_i} \quad (5.16)$$

where $i = 1 \dots n$. Sarpkaya (1989) noted that although total circulation and linear momentum is conserved in the amalgamation process, the distribution of second moment of vorticity decreases and the velocity field before and after amalgamation are different. However, once vortices are far away from the body, vortex amalgamation is of little consequence, as shown by Walter & Larsen (1997) for bluff bodies.

Other Vortex Method Parameters

Vortex methods are too vast a subject to cover in any great detail here. However, it should be mentioned that numerous schemes and associated judicious modifications have been devised and utilised in an effort to best capture what is observed in nature. Two parameters that are critical to successful solution of flow problems using vortex methods are now described.

Vortex Placement The placement of the latest vortex released from an edge of flow separation can be critical in regions where the streamlines are rapidly curving. The classical method is to place the newly-shed vortex at one-half of the displacement of the trailing edge (in the case of trailing-edge separation) since the previous time-step (e.g. Scott, 1987; Lam, 1989). Katz & Plotkin argue that the discrete nature of the point vortex released underestimates the induced velocity when compared with the continuous wake vortex sheet result. Therefore, the latest vortex should be placed closer to the shedding edge, within the range $0.2-0.3\delta\zeta$ where $\delta\zeta$ is the displacement of the edge since the last time-step (Katz & Plotkin, 2001, § 13.8.2).

Tavares (1990) and Lee (1991) suggested a similar scheme basing their argument on the variation of a 'characteristic variable'¹⁰ with time over a time-step. They used the following scheme

$$\zeta_t = \zeta_{edge} + \frac{1}{3} (\zeta_{edge} - \zeta_{t-1}) \quad (5.17)$$

The consensus appears to be to place the latest shed vortex closer to the shedding point than to the previous vortex.

Platzer (priv. comm.) suggests that the location of the latest shed vortex should be determined more accurately by locating the position that gives zero pressure difference across the sheet connecting it to the vortex-shedding edge.

Time-step Size The size of time-step is critical in accurately determining the convection of a vortex wake. It is desirable that the time-step be kept to a value that prevents large position changes from occurring. Time-steps must be small enough so as to follow the true streamlines accurately in regions of the flow where the radii of curvature of the streamlines are small (Sarpkaya, 1975). Small time-steps are particularly essential for vortices close to a solid boundary where the zero-through-flow boundary condition requires that flow does not penetrate the boundary but cannot be enforced during the convection phase of solution (Cheng *et al.*, 1988).

Dalton & Wang (1988) suggest that time-step control can be accomplished by allowing only small angular rotations of the line connecting two vortex centres. In their approach, the strength of all vortices were known at startup and a (variable) time-step size was chosen by ensuring that a maximum permitted angular displacement was maintained.

In our case, however, vortex circulations are unknown at the start and a constant time-step is required for the calculation of forces and moments. Hence, this approach of

¹⁰In their approach for delta wings, Tavares and Lee devised a 'characteristic variable' to track the trajectory of a vortex shed in one cross-plane as it evolved downstream in subsequent cross-planes.

estimating time-step size is not tractable. However, a similar time-step scheme may be drawn from the maximum permissible wing movement during a time-step.

Rosenhead (1931) advised that the choice of time-step size should be such that the direction of motion of a vortex before and after any time-step does not differ greatly.

5.1.2 Method of Solution

Introduction

The current approach is formulated as an initial-value problem, in that initial conditions are known and the flow is solved for all subsequent times using a time-marching algorithm.

In the unsteady motion of an aerofoil, the kinematic boundary condition induces velocities over the aerofoil surface. These upset the Kutta-Joukowski condition that the trailing edge must be a stagnation point because nature abhors the infinite velocity that must exist there if the flow did not stagnate. A similar condition holds for the leading edge where separation is also observed in insect-like flapping wings. The Kutta-Joukowski condition is, therefore, enforced by the 'creation' of bound circulation.

In the current formulation, a pair of vortices are shed at every time-step, one each from the leading and trailing edges in accordance with Kelvin's law and to satisfy the Kutta-Joukowski conditions there. Over the duration of the time-step, these vortices convect with the local Kirchhoff velocity (see p. 135), so that at the next time-step the Kutta-Joukowski condition is disturbed once again. In addition, the kinematics of the wing and the associated boundary condition may have changed so that the velocity distribution on the wing is further altered. The Kutta-Joukowski condition is restored by shedding another pair of vortices from the respective edges. This process repeats over time and wakes from both separation edges evolve. A spin-off of the method is that a time-history

of the evolution of the wake is automatically generated which is useful in demonstrating the physics of the flow.

Discretisation of Equations for Numerical Solution

Some of the equations derived in the previous chapter (§ 4.2) are highly nonlinear and do not have closed-form solutions. Therefore, they must be solved using numerical schemes. Before numerical methods can be applied, however, the equations (as necessary) must be converted to forms that are amenable to numerical computation.

The equations for quasi-steady (wake-free) circulation are straightforward and can be computed directly from their derived forms. The relevant equations (see p. 113) are reproduced below:

$$\begin{aligned} \gamma_0(\theta, t) = \frac{1}{R} \bigg[& -A_1 \cos \theta - \left(A_2 + \frac{1}{2}A_7\right) \cos 2\theta + A_3 \sin \theta \\ & + \left(A_4 - \frac{1}{2}A_5 + \frac{1}{2}A_6\right) \sin 2\theta - A_8 \sin \theta \cos 2\theta + A_9 \sin \theta \sin 2\theta \\ & - A_{10} \cos \theta \cos 2\theta + A_{11} \cos \theta \sin 2\theta + \frac{\Gamma_0(t)}{2\pi} \bigg] - 2U_\infty \sin(\theta - \alpha(t)) \end{aligned} \quad (4.15)$$

$$\Gamma_0(t) = 2\pi \left[2R \left((l(t) - U_\infty) \sin \alpha(t) + h(t) \cos \alpha(t) \right) + \dot{\alpha}(t) \left(\frac{1}{2}\tau^2 + \frac{1}{2}\sigma^2 - 2R(R + a) \right) \right] \quad (4.16)$$

where the symbols $A_i, i = 1 \dots 11$ are as in Equation 4.13. Elemental vorticity from the above equation can be converted to elemental circulation (which is the same in both circle and physical planes; see Equation 4.3e) using the relation

$$\delta\Gamma = \delta\gamma_Z \delta Z \quad (= \delta\gamma_\zeta \delta\zeta) \quad (5.18)$$

remembering that Z and ζ correspond to the circle and physical planes respectively.

The expression for wake-induced vorticity (Equation 4.24) is a little more complex and is discretised thus

$$\gamma_1(\theta, t)|_{wake+lev} = -\frac{\Gamma_0(t)}{2\pi R} - \frac{1}{2\pi R} \left[\sum_{i=1}^{n_{wake}} \Re \left(\frac{Z_{wake_i} + Re^{i\theta}}{Z_{wake_i} - Re^{i\theta}} \right) \delta\Gamma_{wake_i} + \sum_{i=1}^{n_{lev}} \Re \left(\frac{Z_{lev_i} + Re^{i\theta}}{Z_{lev_i} - Re^{i\theta}} \right) \delta\Gamma_{lev_i} \right] \quad (5.19)$$

where the subscripts *wake* and *lev* refer to the trailing-edge wake and leading-edge vortex respectively, and *n* refers to the number of shed vortices. Equation 5.18 is again used to convert vorticity to circulation. Before Equation 5.19 can be evaluated, however, all circulations $\delta\Gamma_{wake_i}$ and $\delta\Gamma_{lev_i}$ must be known. This is true for all circulations except the latest shed vortices $\delta\Gamma_{wake_n}$ and $\delta\Gamma_{lev_n}$. The latter are computed by simultaneously solving Equations 4.19 and 4.23 whose discretised forms are

$$\Gamma_0(t) = - \left[\sum_{i=1}^{n_{wake}} \Re \left(\frac{Z_{wake_i} + R}{Z_{wake_i} - R} \right) \delta\Gamma_{wake_i} + \sum_{i=1}^{n_{lev}} \Re \left(\frac{Z_{lev_i} + R}{Z_{lev_i} - R} \right) \delta\Gamma_{lev_i} \right]$$

and

$$\begin{aligned} \frac{1}{R} \left[A_1 - \left(A_2 + \frac{1}{2} A_7 \right) + A_{10} \right] - 2U_\infty \sin \alpha(t) \\ = \frac{1}{2\pi R} \left[\sum_{i=1}^{n_{wake}} \Re \left(\frac{Z_{wake_i} - R}{Z_{wake_i} + R} \right) \delta\Gamma_{wake_i} + \sum_{i=1}^{n_{lev}} \Re \left(\frac{Z_{lev_i} - R}{Z_{lev_i} + R} \right) \delta\Gamma_{lev_i} \right] \end{aligned}$$

respectively, where again the symbols A_i are as in Equation 4.13 and n_{wake} and n_{lev} refer to the total number of vortices shed in the trailing-edge wake and the leading-edge vortex respectively. In order to solve for the latest wake and LEV circulations, the equations can be rearranged as follows:

$$\Re \left(\frac{Z_{wake_n} + R}{Z_{wake_n} - R} \right) \delta\Gamma_{wake_n} + \Re \left(\frac{Z_{lev_n} + R}{Z_{lev_n} - R} \right) \delta\Gamma_{lev_n} = -\Gamma_0(t) - f_1(wake) - g_1(lev) \quad (5.20)$$

and

$$\Re \left(\frac{Z_{wake_n} - R}{Z_{wake_n} + R} \right) \delta\Gamma_{wake_n} + \Re \left(\frac{Z_{lev_n} - R}{Z_{lev_n} + R} \right) \delta\Gamma_{lev_n} = 2\pi R \Upsilon_0(t) - f_2(wake) - g_2(lev) \quad (5.21)$$

respectively, where

$$f_1(wake) = \sum_{i=1}^{n-1} \Re \left(\frac{Z_{wake_i} + R}{Z_{wake_i} - R} \right) \delta\Gamma_{wake_i} \quad \text{and} \quad g_1(lev) = \sum_{i=1}^{n-1} \Re \left(\frac{Z_{lev_i} + R}{Z_{lev_i} - R} \right) \delta\Gamma_{lev_i},$$

$$f_2(wake) = \sum_{i=1}^{n-1} \Re \left(\frac{Z_{wake_i} - R}{Z_{wake_i} + R} \right) \delta \Gamma_{wake_i} \quad \text{and} \quad g_2(lev) = \sum_{i=1}^{n-1} \Re \left(\frac{Z_{lev_i} - R}{Z_{lev_i} + R} \right) \delta \Gamma_{lev_i},$$

$$\Upsilon_0(t) = \frac{1}{R} \left[A_1 - \left(A_2 + \frac{1}{2} A_7 \right) + A_{10} \right] + 2U_\infty \sin \alpha(t),$$

and where $i = n$ refers to the latest shed vortex and $i = 1 \dots n - 1$ to all previous shed vortices, the subscripts $wake$ and lev having been dropped from n for clarity. .

Equations 5.20 and 5.21 are two linear simultaneous equations of the form $Ax + By = C$ which can be solved together for the circulations of the latest shed vortices $\delta \Gamma_{wake_n}$ and $\delta \Gamma_{lev_n}$. Equations 5.20 and 5.21 still have two ‘unknowns’ — the positions of the latest shed vortices (Z_{wake_n} and Z_{lev_n}). These are estimated from the movement of the shedding edge and the position of the previous shed vortex (see later).

Calculation Procedure

The calculation process, which is shown in Figure 5.2, was implemented by means of a FORTRAN routine (see later). The basic procedure that is followed is outlined below.

1. The computation is started by loading the various variables, reading in simulation parameters and initialising them to the desired values.
2. The wing planform to be solved for is divided into the required number of wing sections (blade elements) using the radial chord concept introduced earlier.
3. Because all wing sections are on the same wing, they share common global kinematics in terms of sweep parameters $(\phi, \dot{\phi})$, inclination parameters $(\theta, \dot{\theta})$ and pitch parameters $(\alpha, \dot{\alpha})$. These are computed next.
4. The program then descends into calculations for each spanwise section:
 - (a) First, the shape and coordinates of the aerofoil section are computed.
 - (b) Then, the specific kinematics for the wing section are deduced from the global kinematics computed earlier.

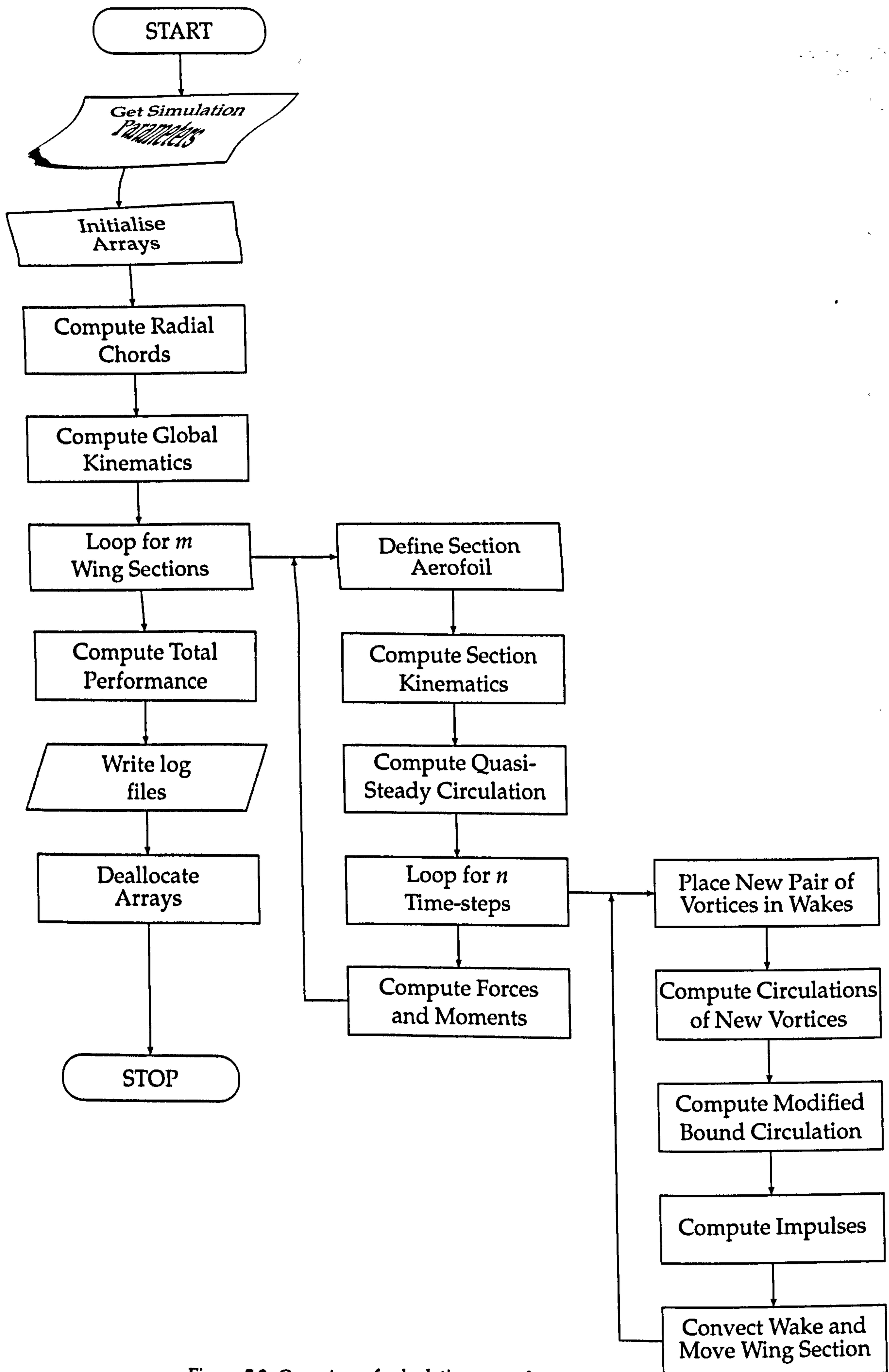


Figure 5.2: Overview of calculation procedure

- (c) With the information deduced so far, the quasi-steady (wake-free) vorticity and circulations can be computed. This is done next using Equations 4.15 and 4.16 (p. 113).

The sub-steps above are repeated for each wing section.

5. Now that all the basic and quasi-steady parameters for each wing section have been computed, the time-marching algorithm for the unsteady flapping-wing problem is invoked. All computations are carried out in the body-fixed coordinate system (in the ζ - or Z -plane, as appropriate). At each time-step:

- (a) The latest trailing-edge and leading-edge vortices are shed and positioned appropriately in the wake.
- (b) The circulations of these latest wake and LEV vortices are then computed by simultaneously solving Equations 5.20 and 5.21 (p. 146).
- (c) The presence of the new vortices upsets the zero-through-flow boundary condition at the aerofoil surface so this is corrected by adding additional bound vorticity. This is computed in this step using Equation 5.19 (p. 146).
- (d) The circulation and positions of all bound and shed vortices are used to compute the impulse and moment of impulse (for later force and moment calculation) in the translating aerofoil-inertia system.
- (e) Now that all computations for the current time-step have been completed and all necessary variables determined, the entire wake system is convected and reoriented in preparation for the subsequent time-step, accounting for any change in pitch angle (α) and wing location (l, h).

The above sub-steps are performed for each wing section at each time-step.

6. Once time-marching has been completed, forces and moments can be computed from the impulses and moments of impulse respectively that were computed earlier. This is done for each wing section.

7. The performance of the entire wing is obtained summing the contributions of the individual wing sections.
8. Finally, data and log files for the simulation are written to disk, all arrays are deallocated to free up memory and the simulation is terminated.

Some Aspects of the Calculation Procedure

A number of aspects of the calculation need further detailing. This is done below.

Conventions

Kinematics The convention for the 3-D motion of the insect-like flapping wing is based on the nomenclature introduced in Figure 2.3 (p. 19). The sweep angle at any point is defined by the angular displacement ϕ from the mean position. ϕ is generally positive at the start of the downstroke and negative at the end of it (see Figure 5.3). The reverse is true for the upstroke. The time rate of change of sweep is defined by the angular velocity $\dot{\phi}$ and the stroke amplitude is denoted by Φ . In the wing section 'cross-plane', sweep translates into lunge so that

$$\phi \mapsto l \quad \text{and} \quad \dot{\phi} \mapsto \dot{l}$$

where l and $\dot{l}$ are lunge distance and lunge velocity respectively.

In a similar manner, the inclination angle above the stroke plane is defined by θ and the time rate of change of inclination by the angular velocity $\dot{\theta}$ (Figure 5.3). Inclination displacements translate into heave (or plunge) displacements in the wing section cross-planes, thus

$$\theta \mapsto h \quad \text{and} \quad \dot{\theta} \mapsto \dot{h}$$

where h and $\dot{h}$ are heave distance and heave velocity respectively.

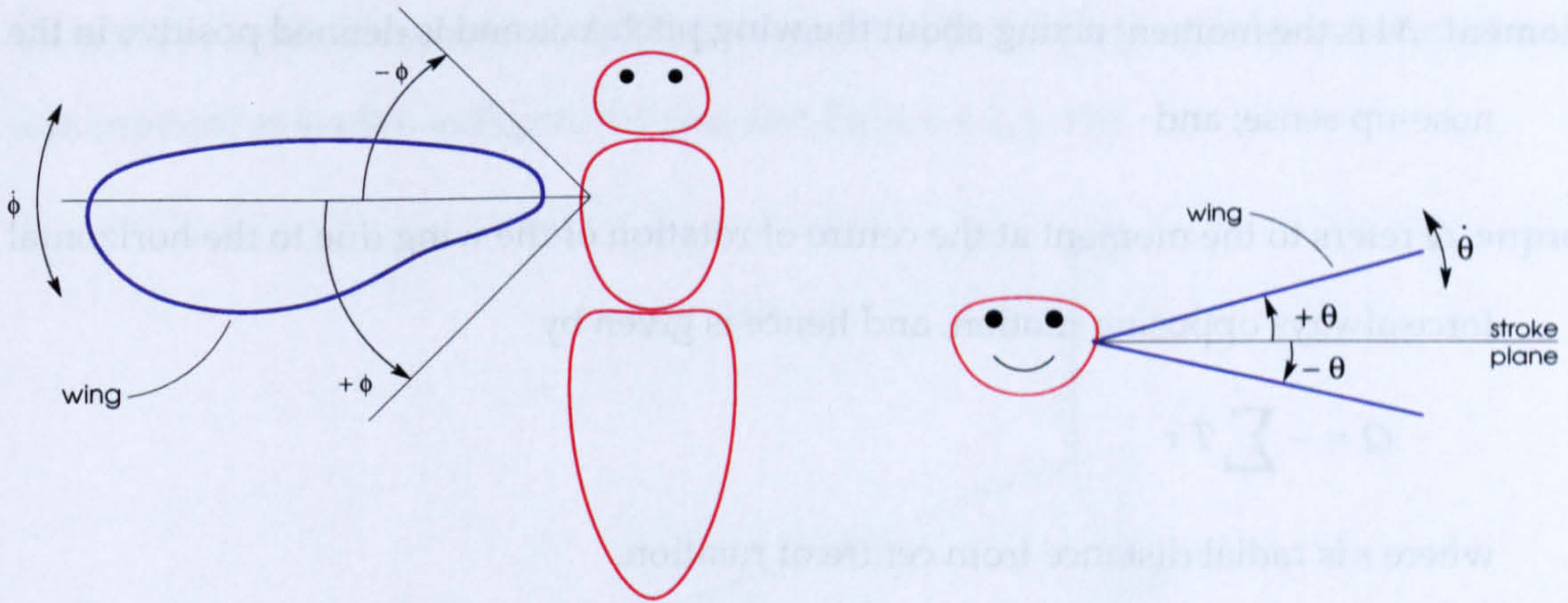


Figure 5.3: Definition of sweep (left) and inclination (right) parameters. Only port wing is shown

Pitch is defined as the angular displacement about the pitch axis (which is common to all wing sections). Therefore, pitch angles and pitch rates are the same for all wing sections.

Due to the extensive use of calculations in the circle ($\mathcal{Z}$) plane, it is important to distinguish coordinates in that plane from those in the physical (ζ) plane. This distinction is expressed by using different notation for the orthogonal components, thus

$$\mathcal{Z} = x + iy \quad \text{and} \quad \zeta = \xi + i\eta$$

Forces and Moments Forces and moments are computed in the translational aerofoil-inertia system ($\hat{\zeta}$ -system in Figure 4.6) where the origin coincides with the wing pitch axis. In all the analysis that follows, the following terminology is used (see also Figure 4.6).

lift $\mathcal{L}$ is defined as the vertical force opposing gravity, acting in the direction of the positive $\hat{\eta}$ -axis;

thrust $\mathcal{T}$ is defined as the horizontal force that acts parallel to the $\hat{\xi}$ -axis but always acts in the direction of motion of the origin of the translational coordinate system ($\hat{\zeta}$);

drag $\mathcal{D}$ is defined as the horizontal force acting in the direction of the positive $\hat{\xi}$ -axis (and does not change direction unlike thrust $\mathcal{T}$);

moment M is the moment acting about the wing pitch axis and is defined positive in the nose-up sense; and

torque Q refers to the moment at the centre of rotation of the wing due to the horizontal force always opposing motion, and hence is given by

$$Q = - \sum \mathcal{T} r$$

where r is radial distance from centre of rotation.

All vortex positions are converted to the $\hat{\zeta}$ -system for computing the impulses and moments of impulse.

Aerofoil Discretisation The aerofoil shape in the physical (ζ) plane is derived from a circle in the Z -plane using the Joukowski-type transformation of Equation 4.1. The aerofoil coordinates are equally spaced on the circle $Z = Re^{i\theta}$ in the circle plane. The advantage of this is that, upon transformation into the physical plane, the resulting aerofoil has much tighter spacing in the leading- and trailing-edge regions than elsewhere. This is useful for accurately enforcing the Kutta-Joukowski condition at the two separation edges.

Kinematics Based on the assumption that all spanwise wing sections share the same pitch axis, global kinematics for the whole vehicle in terms of sweep parameters $(\phi, \dot{\phi})$, inclination parameters $(\theta, \dot{\theta})$ and pitch parameters $(\alpha, \dot{\alpha})$ can be derived. Recalling the assumption of cylindrical wakes (p. 95), these can then be converted into local kinematics for each wing section, thus

$$\begin{aligned} l &= \phi r & \text{and} & & \dot{l} &= \dot{\phi} r \\ h &= r \tan \theta & \text{and} & & \dot{h} &= r \dot{\theta} \cos \theta \end{aligned}$$

where r is radial position of wing section. Angle of attack α and pitch rate $\dot{\alpha}$ remain unchanged. Note that the heave motion is defined as the projection of the radial motion

on to the cylindrical cross-plane of the wing section (in keeping with the cylindrical wake assumption) as shown in Figure 5.4 (see also Figure 4.2, p. 96).

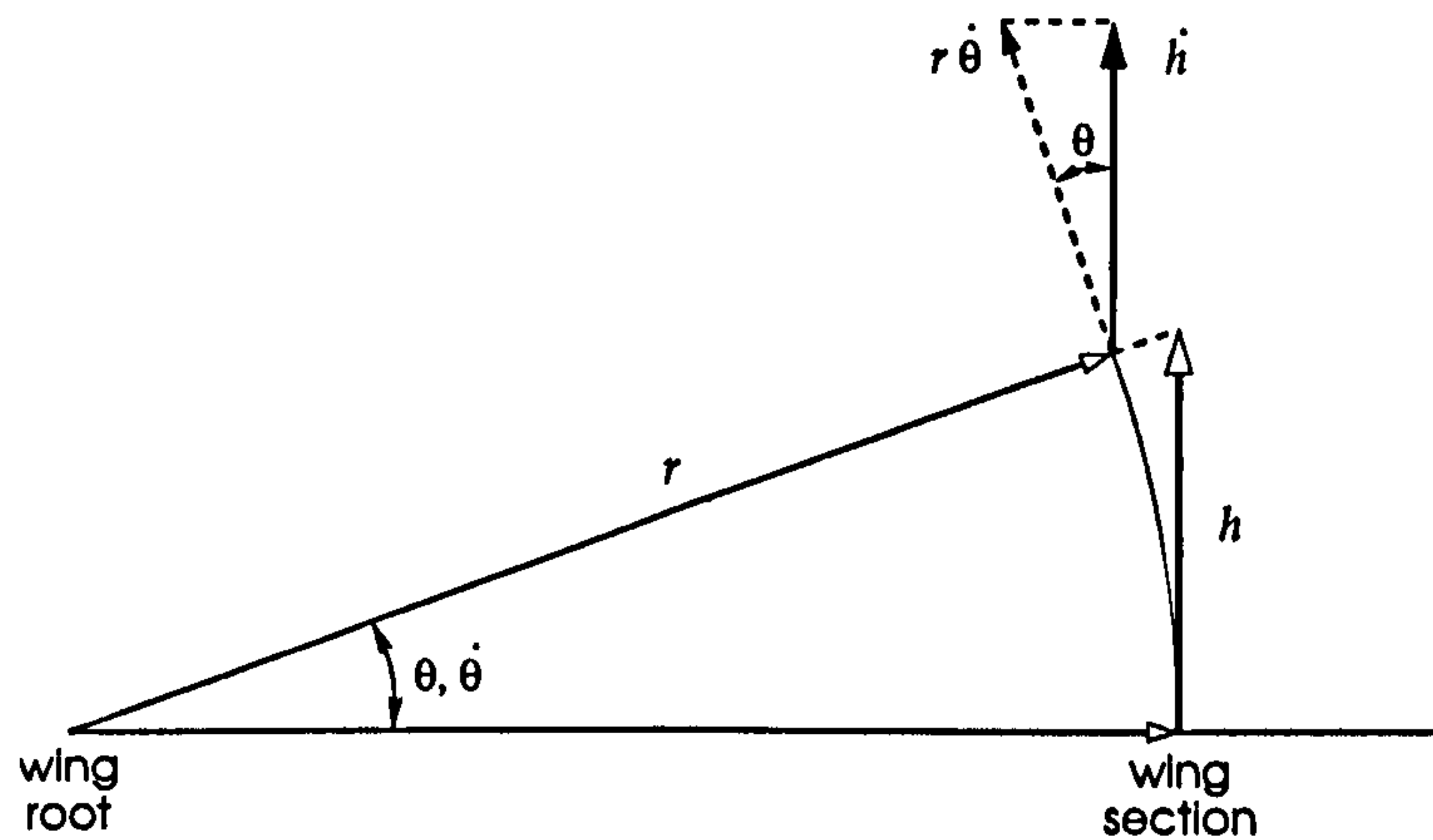


Figure 5.4: Conversion from global to local kinematics

Vortex Placement Traditionally, the latest shed vortex is positioned along the path of the shedding edge (leading or trailing edge) since the last time-step (see e.g. Katz & Plotkin, 2001, § 13.8.2). The approach used here is similar but the latest vortex is instead placed at one-third of the distance from the shedding edge to the previous vortex (Equation 5.17; see also Figure 5.5).

The use of the $\frac{1}{3}$ -distance can be explained as follows. The discrete point vortices that are shed are an approximation to the vortex sheet emanating from the shedding edge. Therefore, each shed vortex actually represents a vortex element of length, say $\delta\zeta$, and is placed approximately in the middle of it. In this way, the centre of the latest shed vortex is distant $\frac{1}{2}\delta\zeta$ from the shedding edge (see Figure 5.5(a)).

At the next time-step, the last vortex has convected with the flow and the aerofoil has moved some distance. Assuming that time-step size is small enough, the distance between the shedding edge and the previous vortex element is not very dissimilar to $\delta\zeta$. The latest vortex is placed in the middle of this latest vortex element (see Figure 5.5(b)). Again assuming that time-step is reasonably small, the orientations of the last two vortex

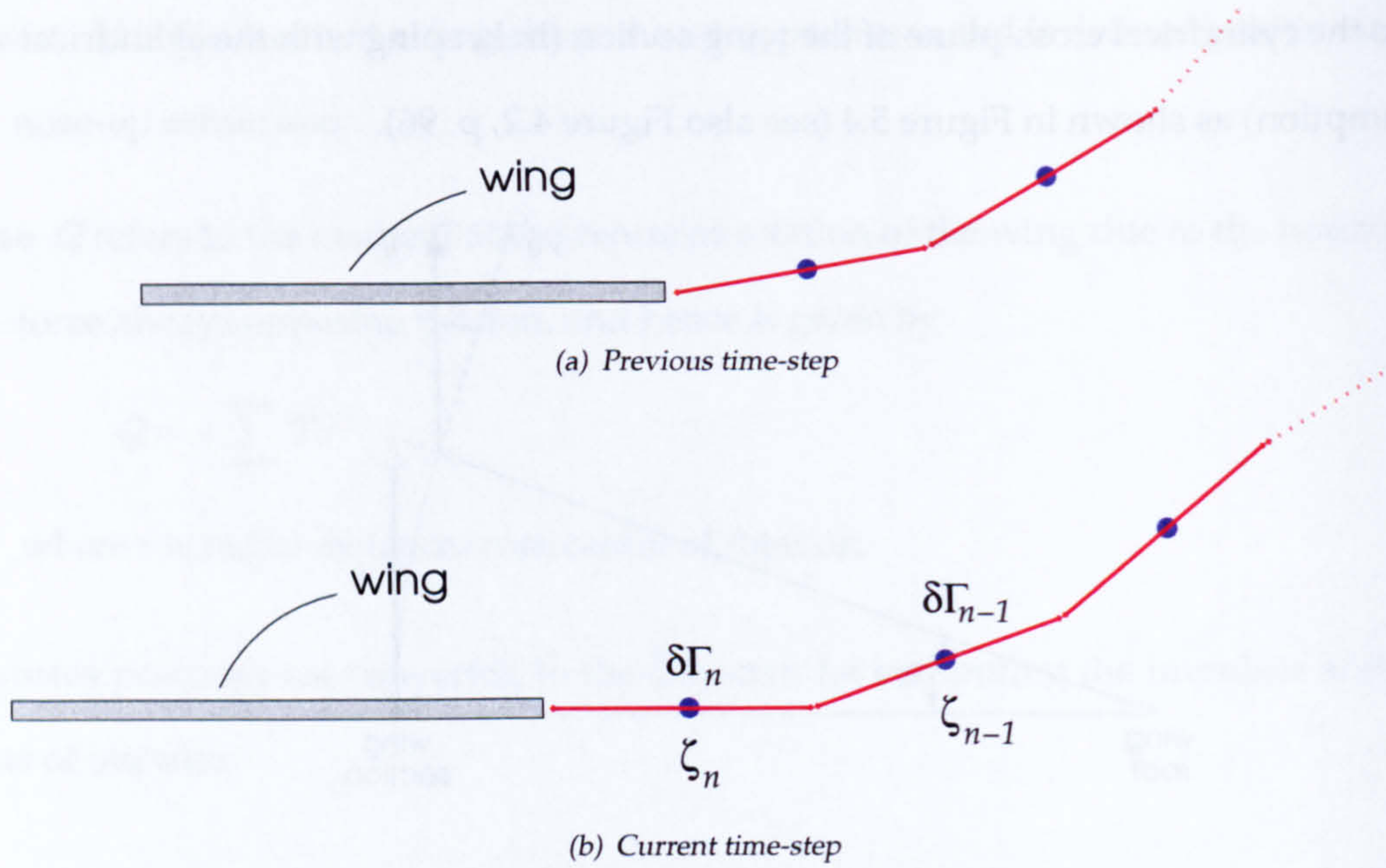


Figure 5.5: Vortex placement (wake deformations are exaggerated for clarity)

elements are very similar so that the position of the latest vortex may reasonably be represented by the vector drawn from the shedding edge to the previous vortex but only one-third the distance, i.e.

$$\zeta_t = \zeta_{edge} + \frac{1}{3} (\zeta_{edge} - \zeta_{t-1}) \quad (5.17)$$

The advantage of the current approach over the traditional one is that account is taken not only of the wing motion since the last time-step but also the convection of the previous shed vortex, giving an overall more accurate depiction of the flow.

As regards the first shed vortex, its position is determined from the local velocity at the shedding edge. If this complex velocity is q and time-step size is δt , then the first vortex is placed at

$$\zeta_t = \frac{1}{2} \bar{q} \delta t$$

where $\bar{q}$ refers to the complex conjugate of q . As the solution progresses, the accuracy of this initial placement is of negligible consequence (provided time-step size is reasonably small).

Wake Convection At the end of each time-step, the whole problem is converted to the aerofoil coordinate system (ζ) and the wakes from each of the leading and trailing edges are convected. Since all the shed vortices are free, they move with the fluid at the local Kirchhoff velocity. This (complex-valued) velocity is described by the Rott-Birkhoff equation (described earlier; see Equation 5.2) and can be decoupled as follows

$$q(\zeta) = \overline{U_0} - \frac{1}{2\pi} \oint_{\text{aerofoil}} \frac{\gamma_0 + \gamma_1}{\zeta - \zeta_a} d\zeta - \frac{1}{2\pi} \oint_{\text{wake}} \frac{\gamma_{\text{wake}}}{\zeta - \zeta_{\text{wake}}} d\zeta - \frac{1}{2\pi} \oint_{\text{lev}} \frac{\gamma_{\text{lev}}}{\zeta - \zeta_{\text{lev}}} d\zeta$$

where $\overline{U_0}$ is the complex conjugate of the incident velocity at the wing corrected for wing orientation α , thus

$$\overline{U_0} = ((U_\infty - i) + i\dot{h})e^{-i\alpha}$$

In the current work, only the hover mode of flight is considered so that U_∞ in the above equation is zero. The local velocity at a point ζ in the wake is calculated using the discrete form of the Rott-Birkhoff equation, thus

$$q(\zeta) = \overline{U_0} - \frac{1}{2\pi} \sum_{i=1}^{n_{\text{foil}}} \frac{\delta\Gamma_{0i} + \delta\Gamma_{1i}}{\zeta - \zeta_i} - \frac{1}{2\pi} \sum_{i=1}^{n_{\text{wake}}} \frac{\delta\Gamma_{\text{wake}_i}}{\zeta - \zeta_i} - \frac{1}{2\pi} \sum_{i=1}^{n_{\text{lev}}} \frac{\delta\Gamma_{\text{lev}_i}}{\zeta - \zeta_i} \quad (5.22)$$

One consequence of computing local velocity using this method is that the self-induced velocity of the vortices are ignored¹¹. The convection itself is implemented by means of a simple forward Euler scheme (Equation 5.4), together with sub-time-steps (Equation 5.6) as necessary. For the duration of the halfstroke that the aerofoil angle of attack remains more or less unchanged, the simple Euler method is employed. During stroke reversals, however, the severe manoeuvres warrant the need for finer resolution and sub-time-steps are used (see next section).

All convection is implemented in the body-fixed coordinate system. Therefore, at the end of the convection routine, the problem is 'prepared' for the subsequent time-step

¹¹Fink & Soh (1978) argued that the Rott-Birkhoff integral must be solved as a Cauchy principal value problem and showed that an error proportional to the logarithm of the ratio of the distances between adjacent vortices was introduced by ignoring self-induced velocities. Sarpkaya (1989), however, remained unconvinced of the benefits of using such a scheme.

by correctly orientating the wake for the new angle of attack (if different). This is done by converting to the translating coordinate system ($\hat{\zeta}$) using the current angle of attack, rotating and then reverting back to the body-fixed system (ζ) using the angle of attack at the next time-step, i.e.

$$\zeta_{new} = (\zeta_{old} + a) e^{i\delta\alpha} - a$$

where $\delta\alpha = \alpha_{new} - \alpha_{old}$ and a is the offset distance between the aerofoil centre and the pitch axis (see Figure 4.6, p. 111). Convection may also be effected in the inertial coordinate system instead and yields very similar results.

Sub-time-steps The sub-time-stepping scheme utilised in the current work is based on Equation 5.6 with n set to either 1, 2 or 4 depending on the severity of the kinematics. The latter was quantified by determining the *normal* kinematic velocity at the trailing edge and then computing the ratio of the distance that would be travelled in a time-step at that velocity to the chord length of the wing section. So if the kinematic velocity at the trailing edge were $u - iv$ in the aerofoil-fixed coordinate system, then this ratio would be defined as $\frac{|v| \delta t}{c}$ where δt is time-step size and c is wing section chord. The values of this ratio that were used to determine the number of sub-time-steps for a particular time-step are shown in Figure 5.6, together with values for a typical halfstroke.

By defining the ratio in this way, the number of sub-time-steps may be different for each wing section, depending on both chord length and local velocity. The sub-stepping procedure was implemented by dividing each time-step into the required number of sub-steps and convecting the wakes for each sub-time-step. In doing so, the wakes were repositioned at the end of each sub-time-step and the induced velocities computed again using the discrete form of the Rott-Birkhoff equation. As with the main convection scheme, all sub-time-step convection routines were implemented in the body-fixed coordinate system.

Lewis (1991) claimed that the inclusion of sub-time-steps allows for better flow-

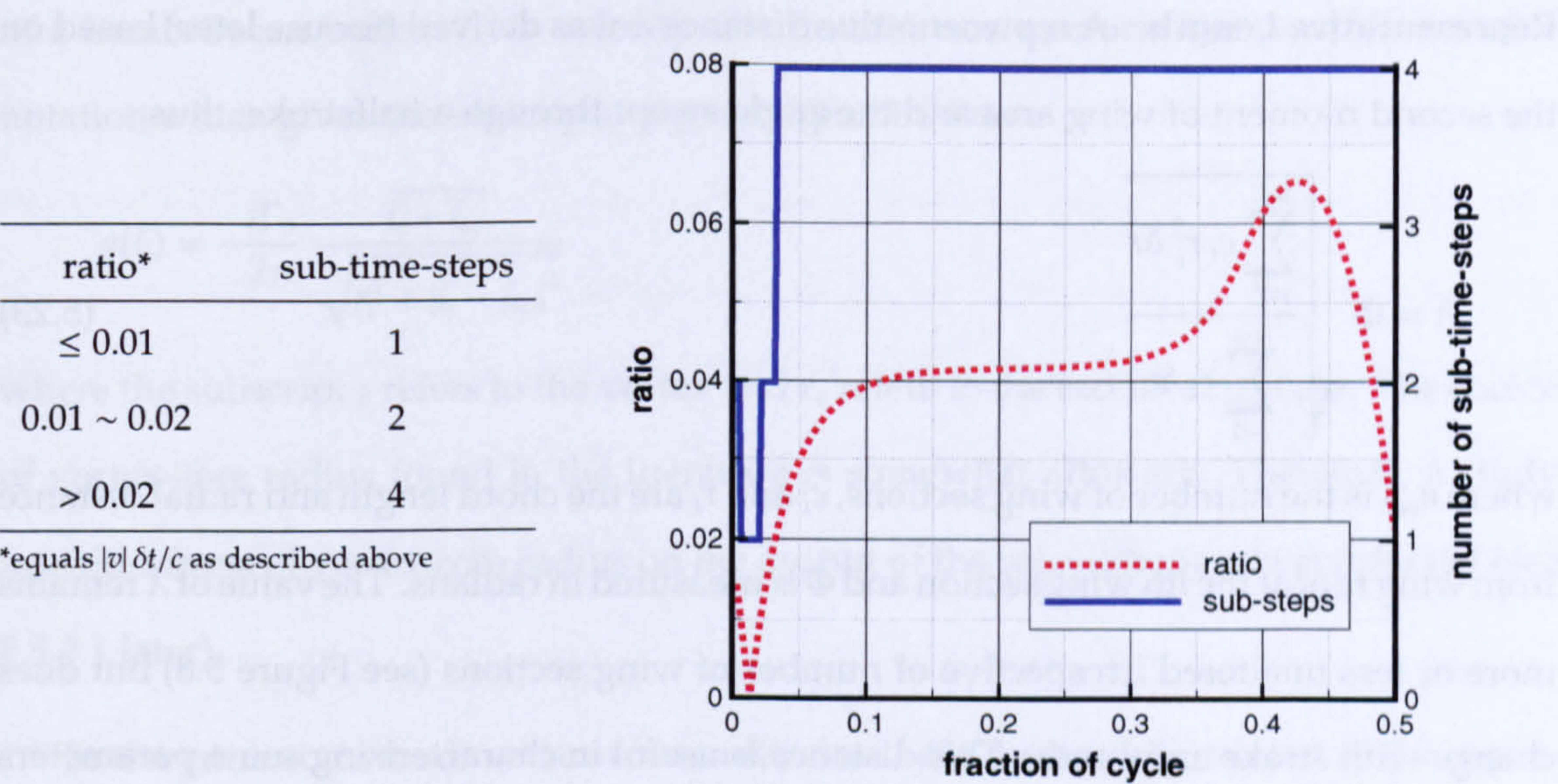


Figure 5.6: Variation of number of sub-time-steps for a typical wing section with the ratio of distance that would be travelled at the normal velocity at the trailing edge during a time-step to chord length.

tracking, especially in regions of high flow curvature as shown in Figure 5.7. A similar sub-time-stepping scheme was successfully implemented by Tavares (1990) for manoeuvring delta wings.

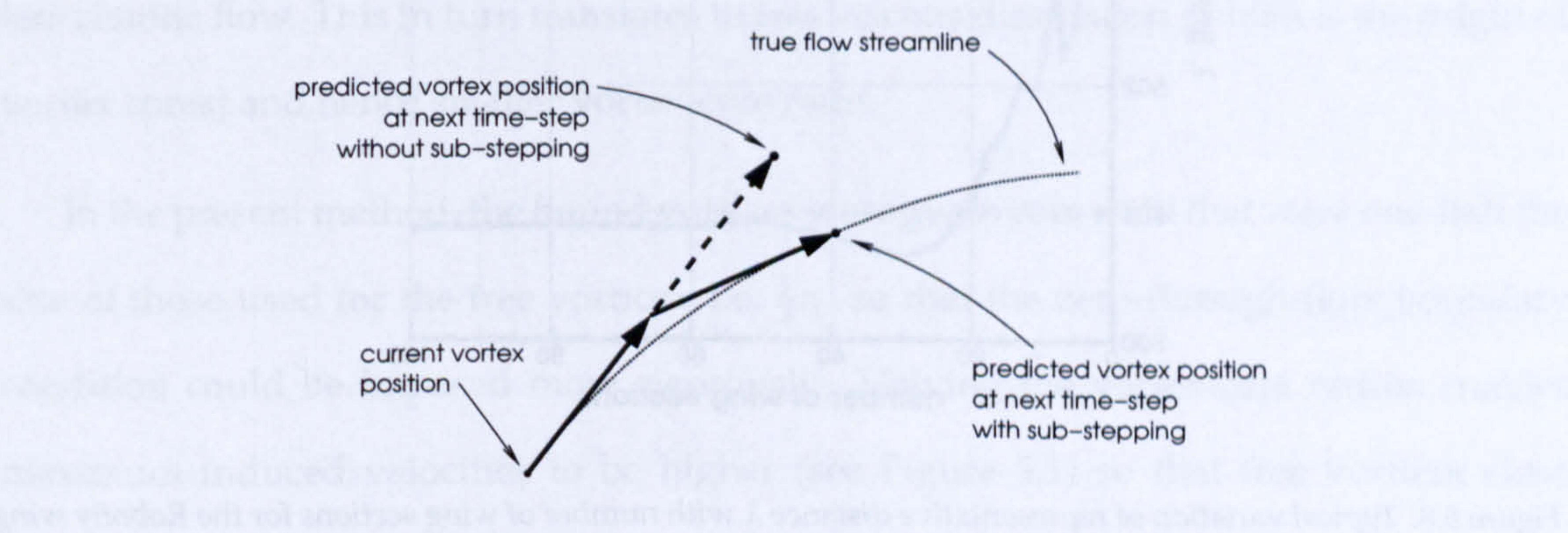


Figure 5.7: Advantage of using sub-time-steps in regions of high flow curvature

Representative Length A representative distance λ was derived (for use later) based on the second moment of wing area and the angle swept through a halfstroke, thus

$$\lambda = \Phi \sqrt{\frac{\sum_{i=1}^{n_{sec}} c_i r_i^2 \delta r}{\sum_{i=1}^{n_{sec}} c_i \delta r}} \quad (5.23)$$

where n_{sec} is the number of wing sections, c_i and r_i are the chord length and radial distance from wing root at the i th wing section and Φ is measured in radians. The value of λ remains more or less unaltered irrespective of number of wing sections (see Figure 5.8) but does change with stroke amplitude. This distance is useful in characterising some parameters such as vortex-core sizes (see later).

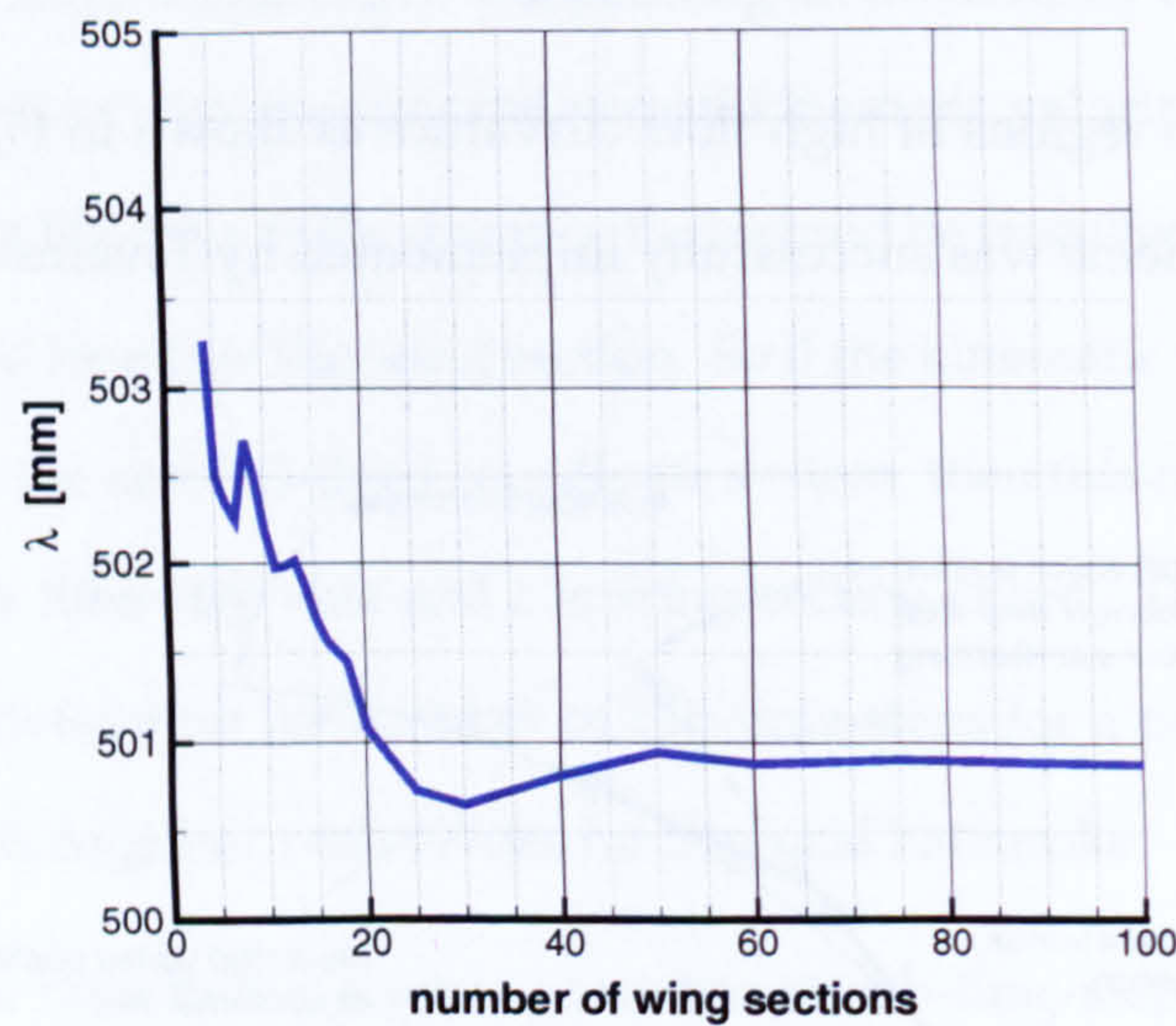


Figure 5.8: Typical variation of representative distance λ with number of wing sections for the Robofly wing from Dickinson's data

Desingularisation Scheme and Vortex Core Size The current model is inviscid and in doing so makes use of potential vortices. These vortices have a singularity at their centres where they give unrealistically high induced velocities. In order to remove this singularity, a desingularisation scheme was employed. The method due to Vatisstas *et al.* (Equation 5.11) was chosen because the velocity distribution then closely resembles that

of a Lamb-Oseen vortex. This velocity distribution was converted to complex variable notation which gives the velocity at a general point ζ thus

$$q(\zeta) = -\frac{i\Gamma_v}{2\pi} \frac{\overline{\zeta - \zeta_v}}{\sqrt{r_c^4 + |\zeta - \zeta_v|^4}}$$

where the subscript v refers to the vortex and r_c refers to the radius of its core. The choice of vortex-core radius found in the literature is somewhat arbitrary. Therefore, a study into the effects of vortex-core radius on the output of the calculations was conducted (see § 5.2.1 later).

In the current method, vortex-core radius has been necessarily related to the representative length λ (see above) so that the same vortex-core radius is used for all wing sections and also because it is then related to a quantity that has been derived from the contributions of all wing sections. A second reason for doing this is that the core radius would then change with stroke amplitude (see Equation 5.23). This is in line with the fact that shorter strokes mean slower strokes for the same flapping frequency, and hence less chaotic flow. This in turn translates to less viscous dissipation (which is the origin of vortex cores) and hence smaller vortex-core radii.

In the present method, the bound vortices were given core radii that were one-half the size of those used for the free vortices, i.e. $\frac{1}{2}r_c$, so that the zero-through-flow boundary condition could be imposed more rigorously. Halving the vortex-core radius enables maximum induced velocities to be higher (see Figure 5.1) so that free vortices close to a solid boundary are more likely to be ‘swayed’ tangentially along the surface than through it. This is essential so that flow does not penetrate any solid boundary during the convection phase of solution when the impermeability condition cannot be enforced.

One advantage of using the Vatisas desingularisation scheme over the Rankine vortex is that it is a continuous function. This offers a saving because an IF-THEN-ELSE construct is avoided by computing induced velocity directly. For the Rankine vortex, it must first be determined whether the point in question lies outside the vortex-core radius (where

the potential-vortex model is used to compute velocity) or inside (where the solid-body-rotation model is used). Hence, an IF-THEN-ELSE construct must be used which is an extra operation per vortex per time-step.

Vortex Amalgamation To save computing time, a vortex amalgamation scheme was employed to merge vortices and, hence, reduce the number of vortices to operate on. A pair of vortices were merged only when all of the following conditions were satisfied:

- the vortices were more than 2.5 chord lengths from the aerofoil centre;
- the vortices were of the same sign; and
- the vortices were close enough.

The last condition was evaluated by using Spallart's criterion for merging vortices (Equation 5.13). The dependence of the method on the small number ϵ was studied (and is presented in § 5.2.1). The usual conditions of conserving circulation (Equation 5.14) and impulse (Equation 5.15) of the vortices were observed. Vortex amalgamation was implemented every 10 time-steps.

FORTRAN Code

The aforementioned computational schemes were implemented by means of a FORTRAN program. The choice of using FORTRAN as the programming language was mainly due to its intrinsic capability of handling complex numbers (in contrast to C or C++, for example, where no such support exists). This made for easy conversion of the complex variable-based analysis in the current work into computable code. The advantage of using FORTRAN over an interpretive language capable of handling complex numbers (such as MATLAB) was speed — the difference in speeds was as much as 50-fold.

The code itself was written strictly to the FORTRAN 95 standard so that a number of new

features could be used while avoiding sloppy coding. Compiler optimisations were used to maximise speed and these were verified against non-optimised code. All computations were performed in double precision because of the extensive use of difference formulae in the computation of forces and moments. Although the code compiled for single-precision calculations ran approximately 2.5 times faster, numerical inaccuracy was introduced due to cumulative truncation errors.

A new feature of FORTRAN 95 is the concept of *modules* which are small sub-programs that are accessible to the rest of the program. Use was made of these to compute and store arrays that were common to all wing sections such as global kinematics and constants (e.g. π (circle constant), fluid density, number of time-steps, etc.).

Another powerful feature of FORTRAN 95 that was exploited was the concept of *derived data types*. These are 'superarrays' that themselves can contain as many arrays as required and can be of various types (i.e. strings, integers, real or complex numbers). One superarray was allotted to each wing section so that data for each wing section could be kept 'private'. Use of these superarrays meant that only a limited number of subroutines were required that could be accessed by each wing section. In doing so, therefore, extensive use was made of pointers and dynamically allocated arrays.

The size of the compiled program was quite small (of the order of 2.0 MB). Memory requirements, however, were determined at runtime, depending on the number of wing sections and time-steps. Typically, 10 MB were required for 16000 section-time-steps (e.g. 8 wing sections with 2000 time-steps each). The program was compiled using the Intel Fortran Compiler v. 8.0 on Linux workstations running RedHat 9.0. The program was also run on the same machines, which were single-processor Dell Precision 360 workstations with 2.66 GHz Intel Pentium 4 processors. A typical run with 6400 section-time-steps took about 10 min to complete.

The bottleneck for the program was the convection subroutine where approximately $4n^2$ velocity computations were made at each time step (where n is the number of time-

steps)¹². Simulation times, however, scaled roughly as $n^{2.5}$. This is because of the different number of sub-time-steps used by each wing section depending on the radial distance from the rotation centre (wing root), aerofoil chord length and pitch-axis location. Computation time also depended on the number of stroke reversals in the simulation (because the number of sub-time-steps was greater during those phases of the simulations) and also on whether vortex amalgamation was employed.

By far the most notable advantage of the code structure is its modularity. This was possible due to the explicitness (modular nature) of the theory it was based on. In this way, all subroutines stemmed from the main program and the calculations in each small subprogram unit were passed smoothly between subroutines. If a subroutine needed modification, it was replaced by another without the rest of the program requiring any change.

Numerical Modifications

Due to the nature of the numerical implementations and the limitations of the inviscid potential method on which they were based, situations arose where the behaviour of the wakes (and the associated forces and moments) gave rise to unrealistically high deformations or changes. In reality, such colossal variations are limited or dampened by viscosity. Therefore, it was necessary to introduce artificially such limitations as necessary.

Kutta-Joukowski Condition Relaxation During the more extreme phases of insect-like flapping flight (such as stroke reversals), it was found that the newly-shed vortices acquired circulations that could be as much as three orders of magnitude higher than the previous shed vortex, all in an effort to satisfy the Kutta-Joukowski condition at the vortex-shedding edge. In such cases, the subsequent vortex that was shed was also of similarly large strength but of opposite sense to ensure stagnation at the shedding edge,

¹²At each time-step, the effect of $2n$ vortices on the $2n$ vortices, hence $4n^2$ computations in all.

and the process continued for a number of time-steps.

These un-physically strong vortices produced colossal deformations of the wake due to their unrealistically high convection velocities. In this way, all free vortices were chaotically dispersed in the fluid, and in some cases convected tens of chord lengths from the aerofoil in one time-step. This itself resulted in further ‘strong’ vortices. The situation occurred more commonly during wake reentry when the wing’s leading and/or trailing edge passed in close proximity to previously shed vortices. The leading-edge vortices were, therefore, more susceptible to such large increases in circulation of shed vortices than those shed off the trailing edge because the leading edge was more likely to travel back into its own wake during stroke reversals. Due to some numerical dependencies, the problem was found to be intermittent and related to the choice of time-step size. This whole behaviour is likely to be an artefact of the numerical implementation.

It was, therefore, necessary to curb the circulation strengths of the shed vortices in such situations, thereby temporarily not satisfying the Kutta-Joukowski condition for that particular time-step. In reality, viscosity is likely to introduce similar limitations.

During stroke reversals, the velocity vector of the flow at the aerofoil trailing edge, for example, is closer to perpendicular to the aerofoil surface than it is parallel to it. In such cases, the smooth-flow Kutta-Joukowski condition is of dubious validity anyway and is likely to be temporarily relaxed¹³. Circulation strengths of shed vortices were, therefore, curbed by imposing a circulation limit that was defined simply as the ratio of circulations of consecutive shed vortices $\frac{\delta\Gamma_n}{\delta\Gamma_{n-1}}$.

During most of the computation, this ratio of circulation did not drift far from 1. It was necessary not to set this limit too high so that if imposition of the Kutta-Joukowski condition required two consecutive vortex strengths to be curbed, then the vortex strength

¹³In fact, Crighton (1985) questioned the validity of the Kutta-Joukowski condition in unsteady flows. See also the discussion on p. 104.

was not much more than 5 times the original uncurbed vortex¹⁴. A brief study was conducted to investigate the effect of this limit on the computation and it was found that it was affected by the number of time-steps and that the computation was generally corrupted for values of circulation limit above 4 (depending on time-step size; see Figure 5.9). Therefore, a circulation limit of 2.5 was conservatively chosen.

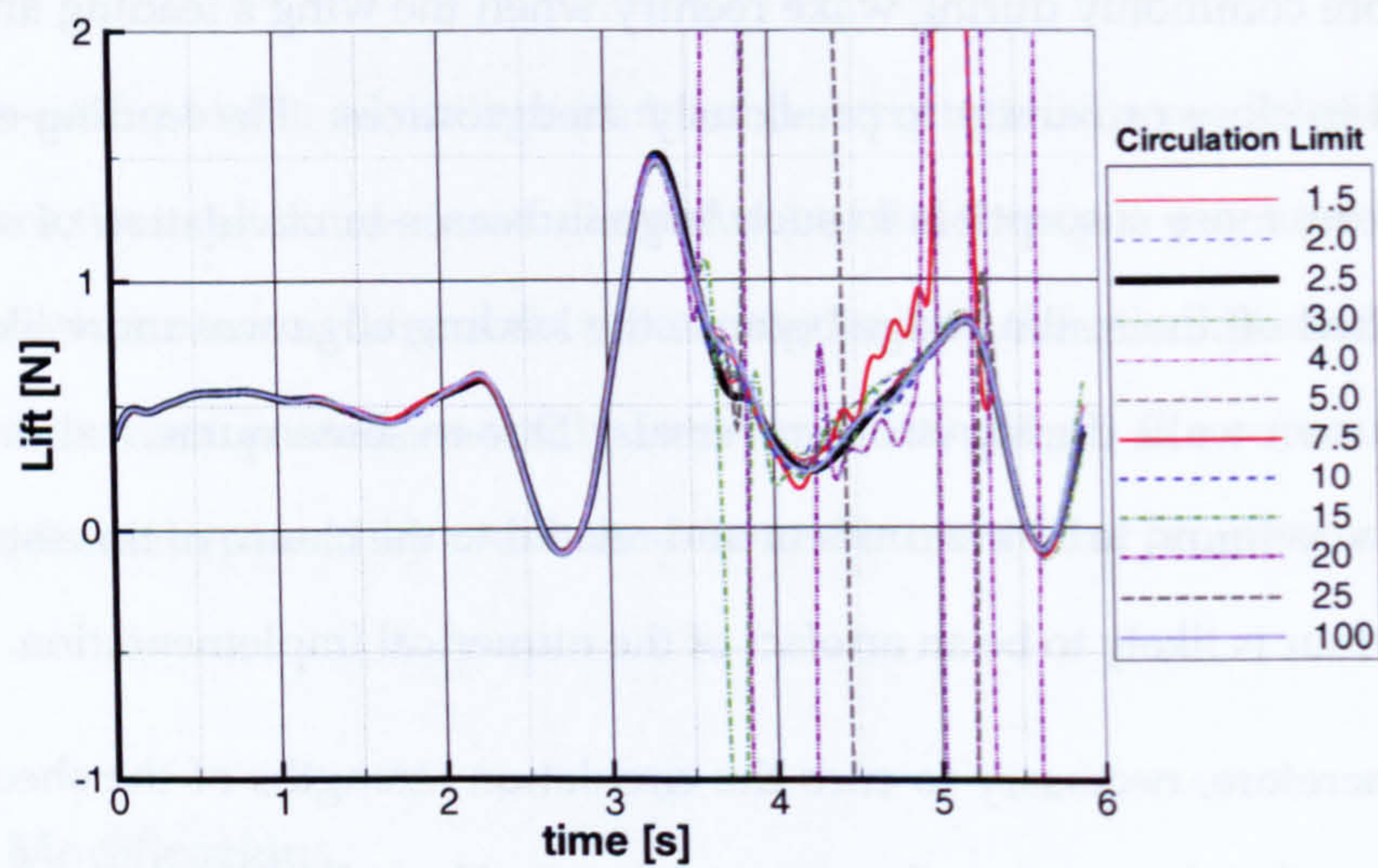


Figure 5.9: Effect of circulation limit on lift. In this typical simulation, the computation corrupted for values of circulation limit above 5.0

A recent CFD study by Sun & Boyd (2004) has shown that for a flat plate at a Reynolds number $Re = 135$, the slip velocity on the aerofoil surface especially near the leading and trailing edges was significant¹⁵. Therefore, the concept of relaxing the Kutta-Joukowski condition (as proposed above) may not be that unrealistic in these flow regimes.

¹⁴If this circulation limit were set to 5 or 10, then two time-steps later vortex strength could potentially be 25 or 100 times larger respectively than the former (which would be unrealistic and corrupt the rest of the computation).

¹⁵Sun & Boyd showed that for a flat plate at 0° incidence, the slip velocity at the leading edge was as high as 20% of the freestream velocity and that it increased over the entire aerofoil with decreasing Reynolds number. They also found that in the range $Re = 1 \sim 200$, flow was compressible.

Smoothing Another artefact of the numerical simulations was the appearance of spikes (both positive and negative) in the force and moment data each time the wing entered its own wake, and especially during stroke reversals. This is an artefact of the method used for computing forces and moments (Kelvin’s impulse method). As a wing traverses through a wake, the free vortices part to either side of it so that some vortex positions that were in ‘positive’ locations with respect to the translational aerofoil-inertia axis in one time step were suddenly at ‘negative’ positions, and vice versa.

Symbolically, a vortex position might change from $+\hat{\zeta}$ to $-(\hat{\zeta} + \delta\hat{\zeta})$ through a time-step instead of to $+(\hat{\zeta} + \delta\hat{\zeta})$ where $\delta\hat{\zeta}$ is a small change in $\hat{\zeta}$. Assuming that the vortex strength is $\delta\Gamma$, then the rate of change of impulse for the two cases (using Equations 4.41–4.43) are $i\delta\Gamma \frac{-(2\hat{\zeta} + \delta\hat{\zeta})}{\delta t}$ and $i\delta\Gamma \frac{\delta\hat{\zeta}}{\delta t}$ respectively where δt is time-step size. Clearly, the former will give rise to a much larger (and artificial) spike than the latter.

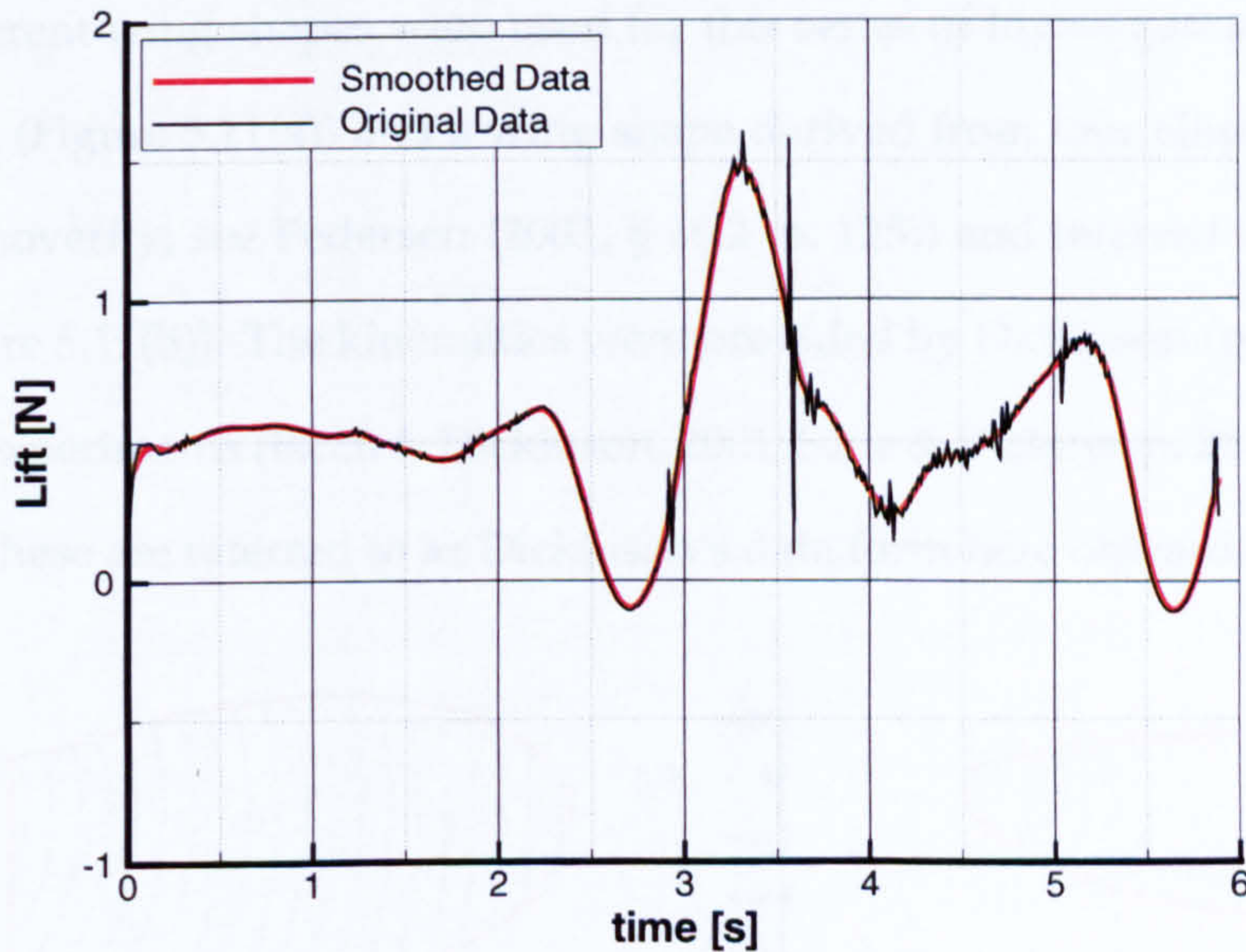
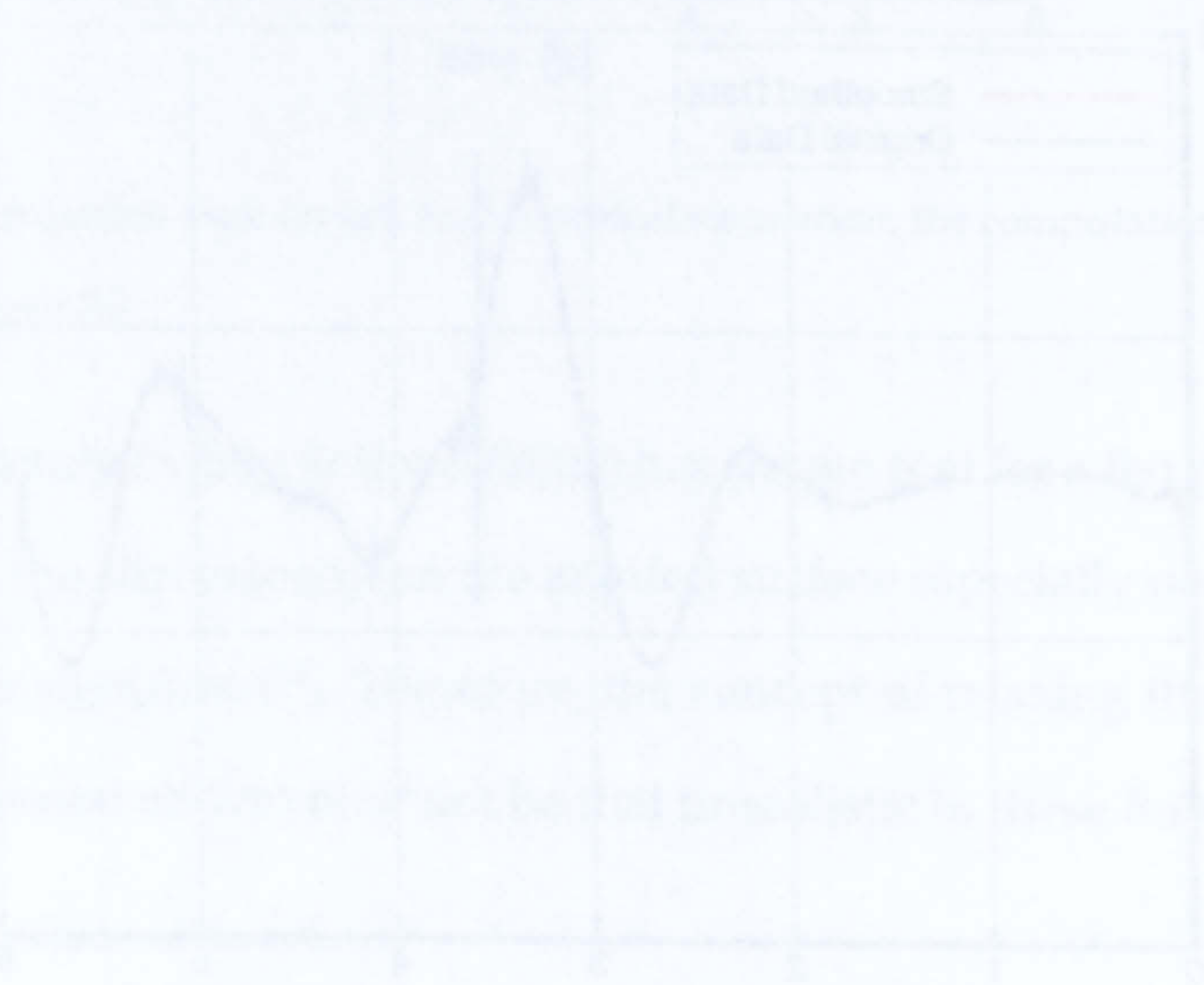


Figure 5.10: Effect of smoothing on lift using a simple least-squares approach

Such spikes increased as the number of cycles increased due to the increasing number of encounters with previously shed wakes, and in doing so increasingly hid the underlying actual force (and moment) data. It was, therefore, necessary to remove such spikes. This

was achieved by using a smoothing technique and this was implemented by means of a simple least-squares method. As Figure 5.10 shows, the basis for using a smoothing scheme is very convincing.

As will be shown later (§ 5.2.1), one of the effects of increasing the number of wing sections in the computation is also to smooth force and moment data. This offers further support for using such a scheme.



5.2 Validation

This section starts off with a look at the characteristics of numerical solutions before considering the suitability of the current inviscid model and validating it against some real data.

5.2.1 Numerical Experiments

The analytical approach used in the current work relies in its implementation on the use of numerical techniques. Therefore, it is inevitable that some sort of dependence on numerical parameters exists. The aim of this section is to describe these characteristics of numerical solutions with a view to determining the best values for the numerical parameters. This was achieved by running a number of *numerical experiments*.

Two different wing shapes were used for this series of investigations — Dickinson's *Robofly* wing (Figure 5.11(a)) and a wing shape derived from four elliptical arcs (similar to that of a hoverfly; see Pedersen (2003, § 16.2, p. 125)) and referred to as *generic* from hereon (Figure 5.11(b)). The kinematics were provided by Dickinson (priv. comm.) from his Robofly experiments (Birch & Dickinson, 2001; Sane & Dickinson, 2002), together with force data. These are referred to as Dickinson's data from here onwards.

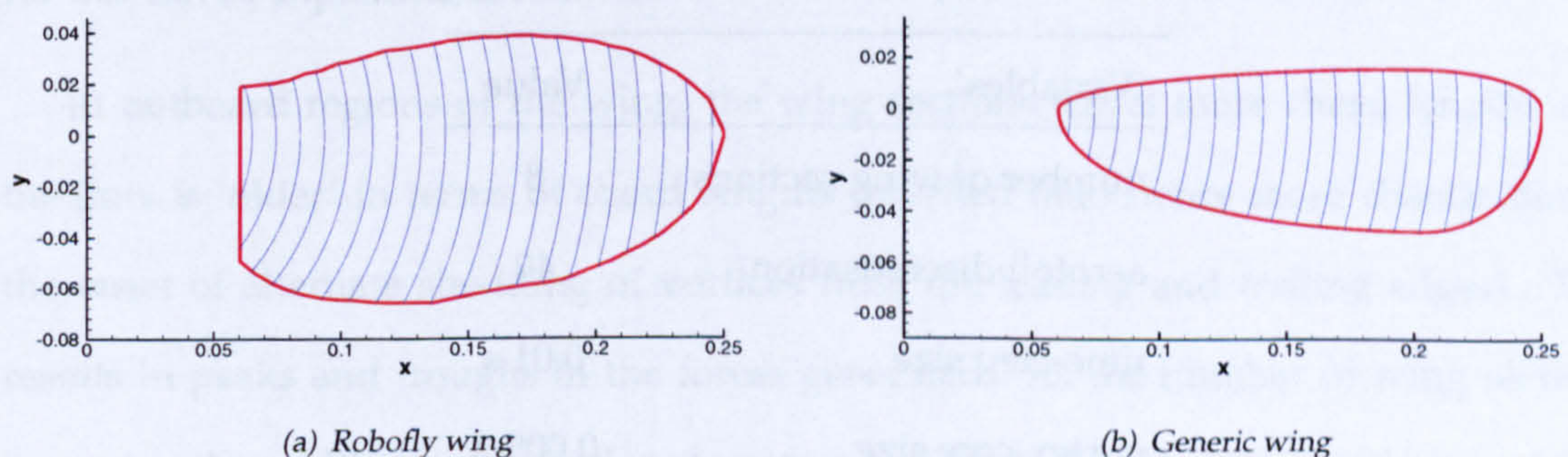


Figure 5.11: Wing planforms used for numerical experiments

Characteristics of Dickinson's data are that only one wing (of a wing pair) was considered. The original experiment was conducted in a tank of mineral oil (of density

880 kg/m³) at a frequency of about 0.17 Hz and sweeping a half-circle arc ($\Phi = 180^\circ$). The two wing shapes considered for the numerical experiments shared all of the above kinematics and differed only in their planform shape. These differences are shown in Table 5.1 below.

Table 5.1: Properties of Robofly and generic wing planform shapes

	wing area (mm ²)	aspect ratio
Robofly	16065	4.49
generic	9450	7.64

In Table 5.1, aspect ratio AR is defined as

$$AR = \frac{4R^2}{S} \tag{5.24}$$

where R is wing length and S is wing area (of both wings of a wing pair). Aspect ratio, therefore, only describes (and depends on) the shape of the wing(s). It is unaffected by the point of attachment of the wing¹⁶.

The effects of a number of parameters were investigated. In doing so, all other parameters were kept constant at some ‘base’ values while the parameter in question was varied through a whole range of values. These base values are given in Table 5.2 below.

Table 5.2: Base values of parameters used in numerical experiments

‘Variables’	Value
number of wing sections	8
aerofoil discretisation	40
time-step size	0.01 s
vortex-core size	0.005 λ

¹⁶In conventional aircraft applications, aspect ratio is defined differently in that it is based on wing-tip-to-wing-tip span and includes wing area buried in the fuselage.

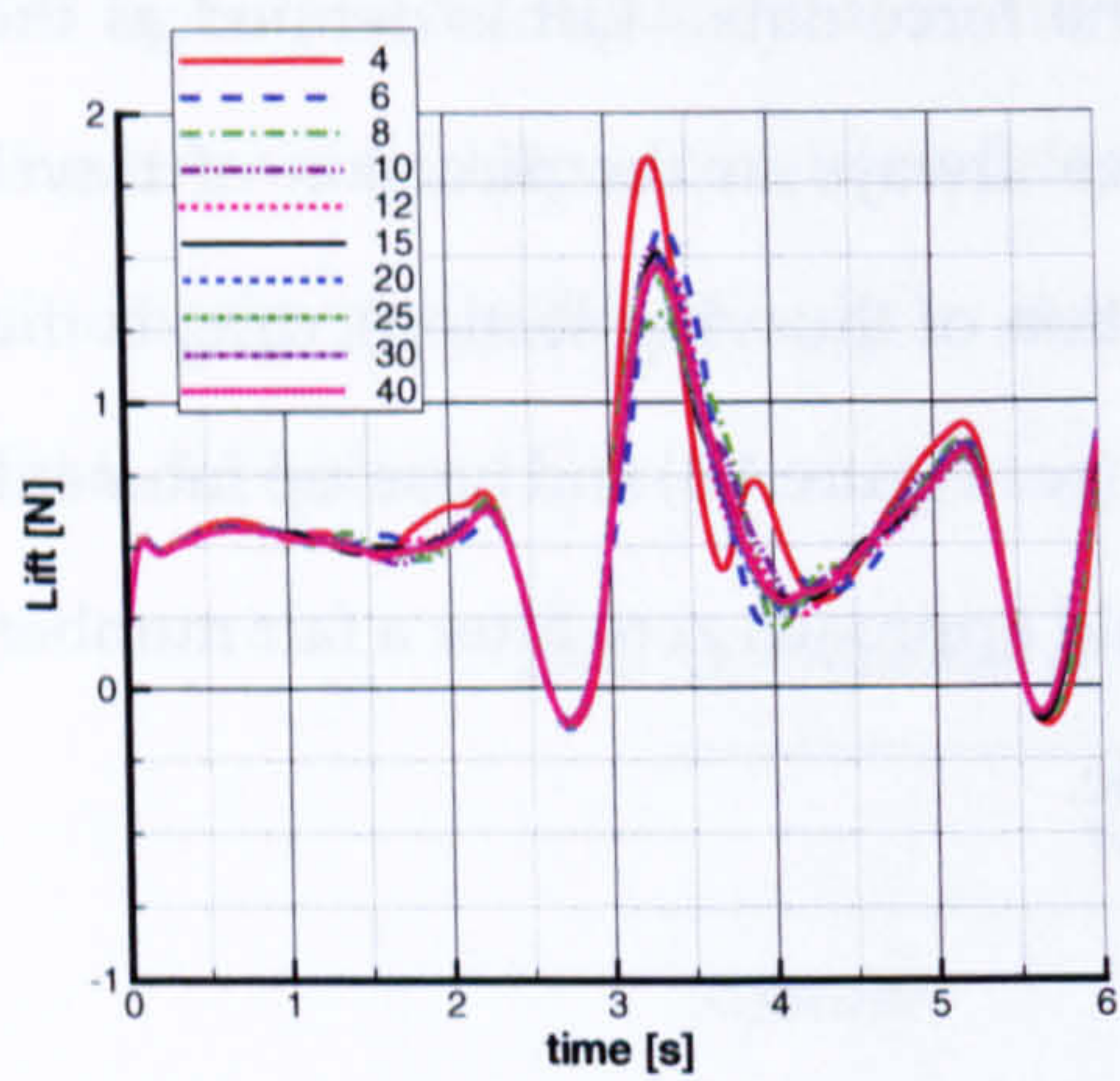
The convention mentioned earlier is used for the force data. Lift is defined as the vertical upward force, thrust is the horizontal force always in the direction of travel (this direction is determined by the *translational* motion of the wing section), drag is the horizontal force in the direction of the positive ξ -axis (see Figure 4.6) and nose-up moment is defined as positive. For hover, the drag force should approach zero after a fair number of cycles when the forces become reasonably periodic.

Effect of Number of Wing Sections

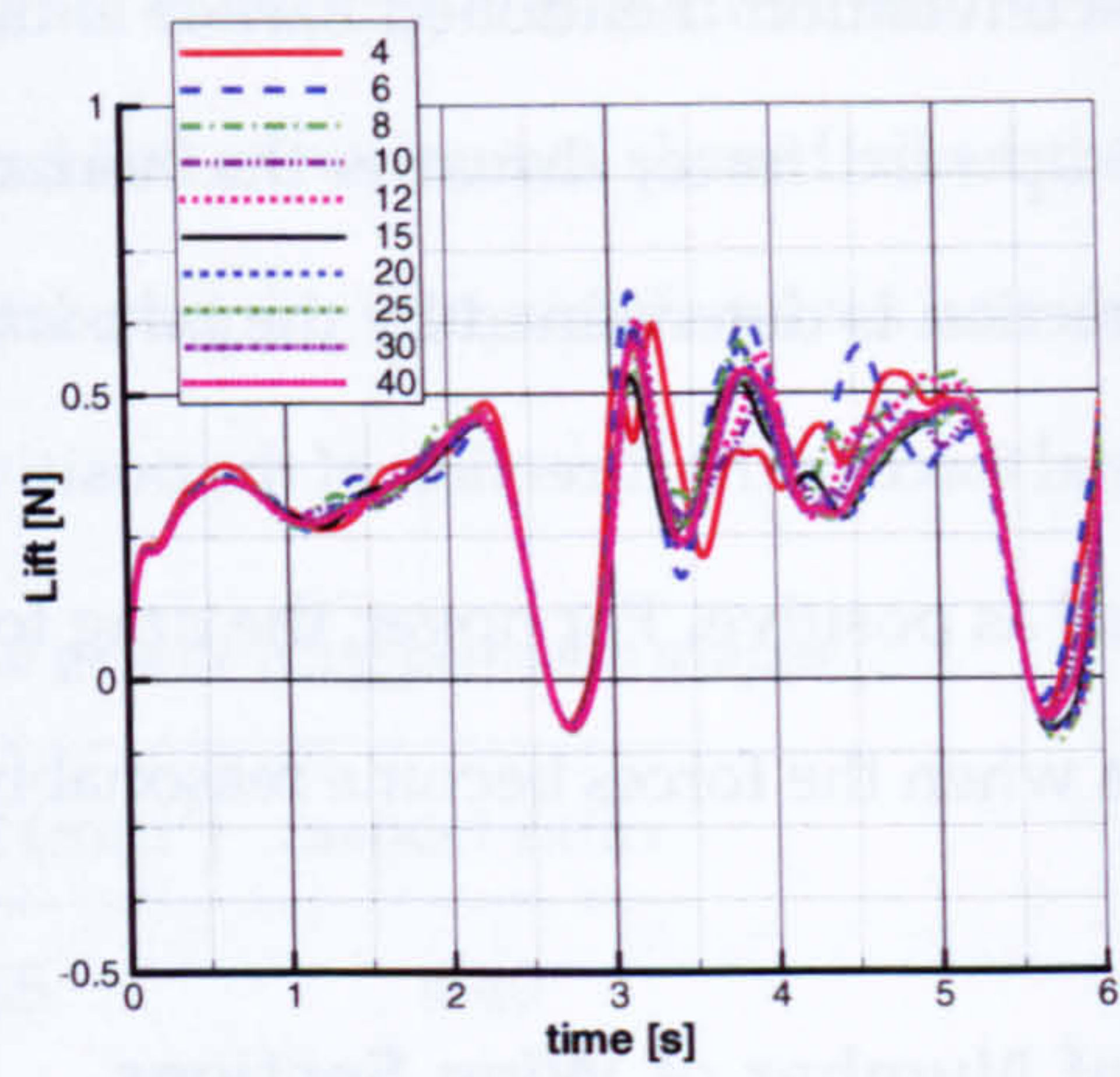
The wing was radially divided into a number of sections n_{sec} and computations were performed in each of the cylindrical sections thus generated. The overall effect was achieved by summing the individual contributions. A higher number of wing sections would give better resolution but comes at the cost of computing time and power. Hence, it was necessary to determine what minimum number of wing sections could reasonably reproduce the results at high resolution.

Therefore, the variation in lift and thrust for both the Robofly and generic wing planforms was investigated and the results are plotted in Figure 5.12. Whereas the trends for all cases considered show very similar force distributions over time, it is easily observed that smaller number of wing sections give ‘peakier’ force histories. The reason for this can be explained as follows.

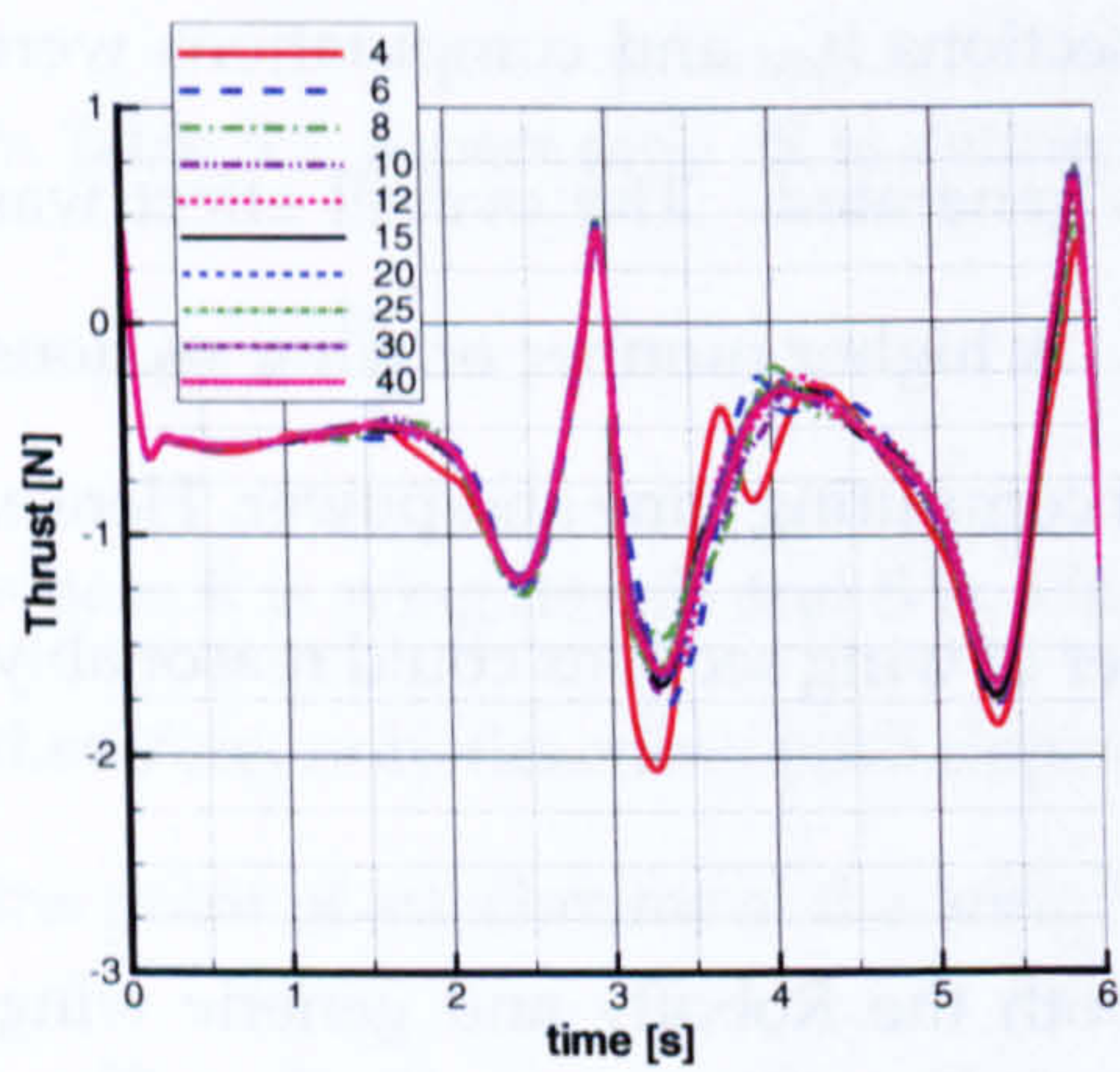
In outboard regions of the wing, the wing sections travel more chord lengths and the flow is ‘older’ in terms of chord lengths travelled (and hence more chaotic due to the onset of alternate shedding of vortices from the leading and trailing edges). This results in peaks and troughs in the forces generated. As the number of wing sections increases, the width of each section decreases so that each 2-D section makes a smaller contribution to the overall lift and thrust. For a small number of wing sections, each wing section makes a substantial contribution to the overall forces and the peaky nature of the outboard forces filters through to the combined result for the wing. In the results for



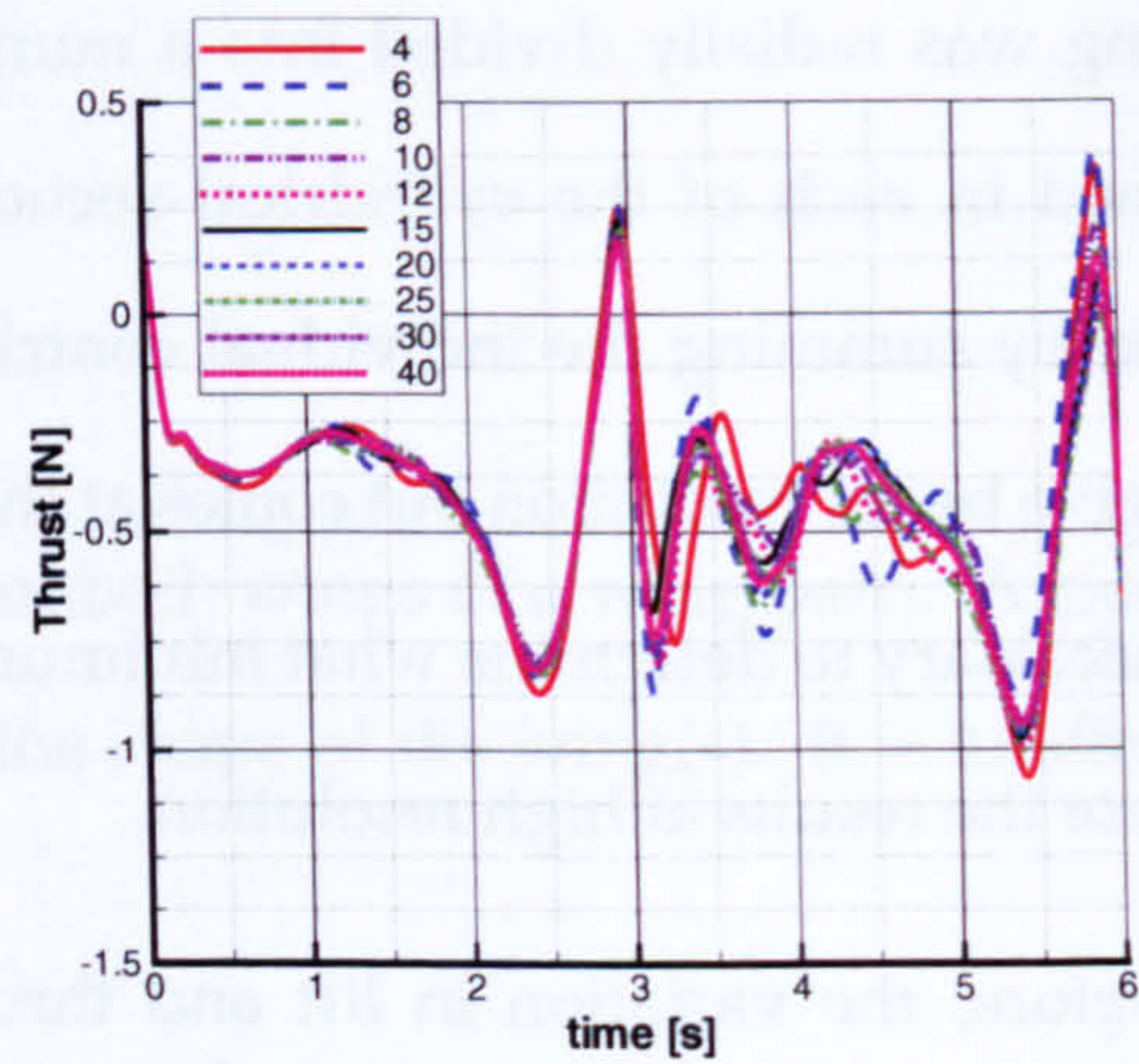
(a) Effect on lift



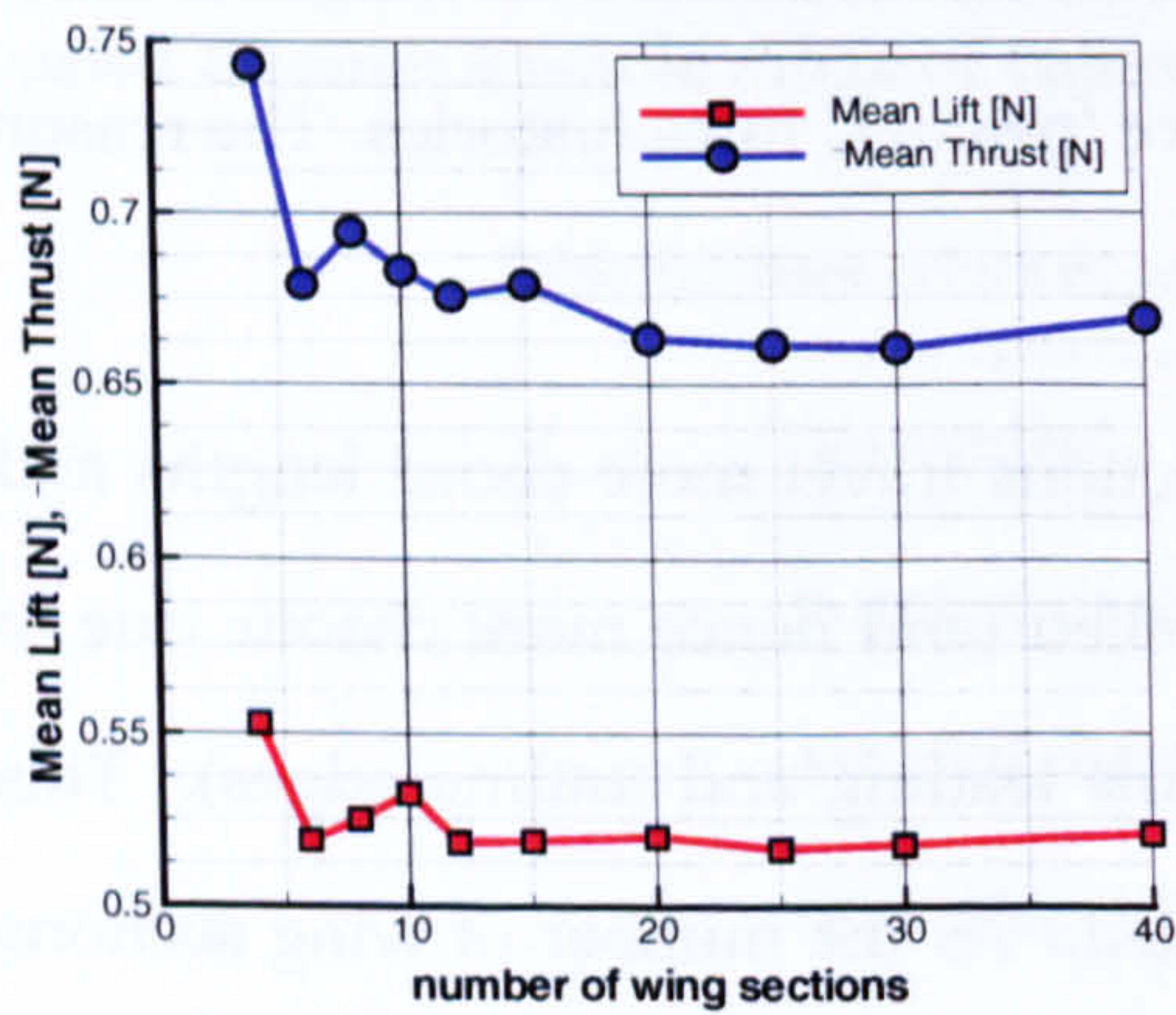
(d) Effect on lift



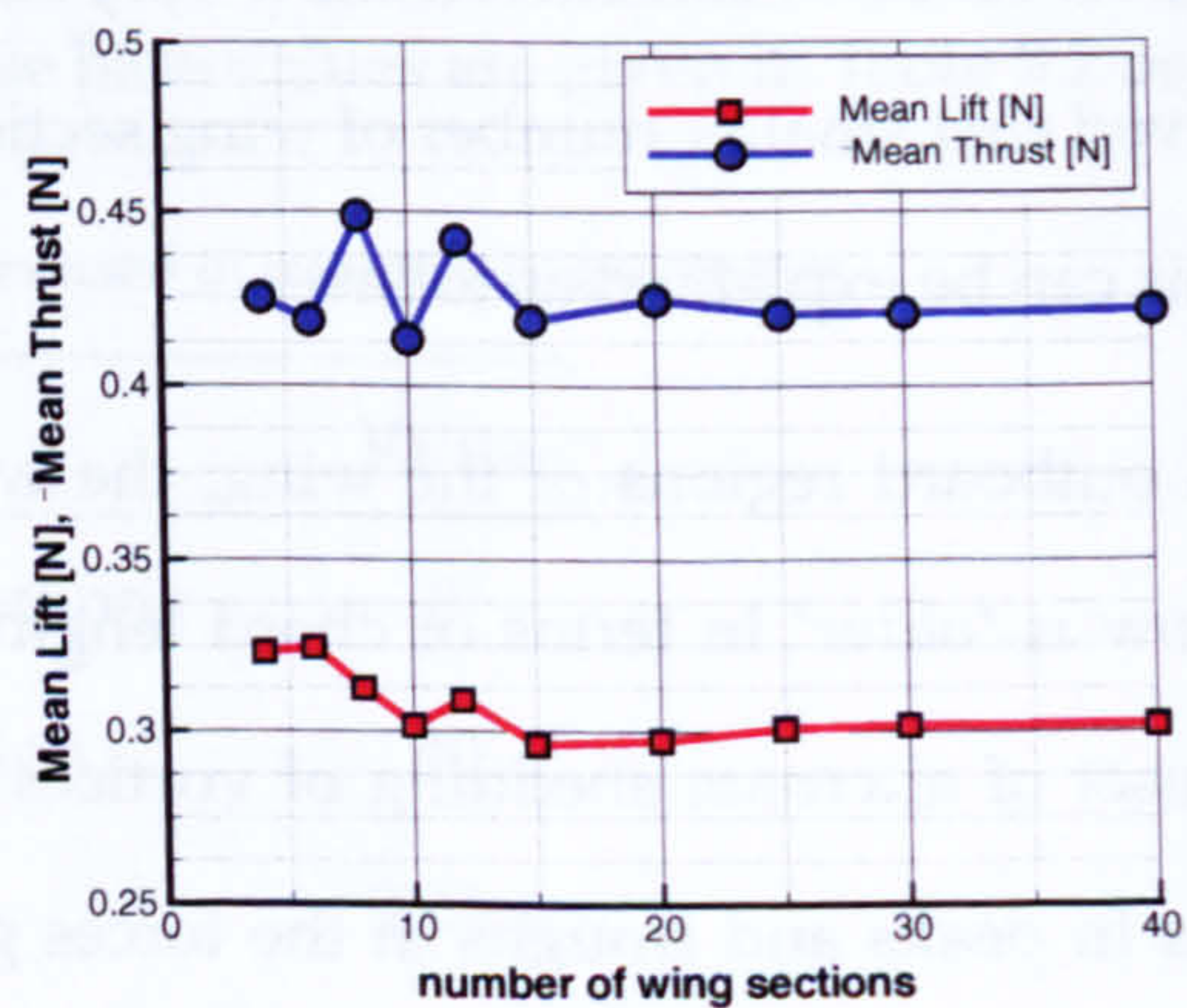
(b) Effect on thrust



(e) Effect on thrust



(c) Effect on mean lift and mean thrust



(f) Effect on mean lift and mean thrust

Figure 5.12: Effect of number of wing sections n_{sec} on computed lift and thrust for Robofly (left) and generic (right) wing planforms through one cycle (downstroke and upstroke). The plot legends show number of wing sections n_{sec}

the Robofly (Figures 5.12(a)–5.12(c)), values of $n_{sec} \geq 8$ give reasonably consistent results. For the generic planform (Figures 5.12(d)–5.12(f)), this condition is similarly achieved also for $n_{sec} \geq 8$. It may be observed that for the case of $n_{sec} = 4$ in particular, the forces generated show more undulations than the other cases (cf. Figures 5.12(a) & 5.12(b) and Figures 5.12(d) & 5.12(e)), resulting from the greater influence of the outboard regions where flow, for the most part, breaks down early.

From Figures 5.12(c) and 5.12(f), therefore, it may be inferred that the mean forces converge for $n_{sec} \geq 20$. However, this required timescales that were prohibitive for the parametric study conducted later (see § 6.2, pp. 223ff). Therefore, a value of $n_{sec} = 10$ was chosen for subsequent simulations as a compromise. The percentage error of using this value relative to the ‘converged’ values was always less than 2.5%.

One artefact of increasing number of wing sections (not shown) is its smoothing effect on the force distributions. Due to the lead-lag in terms of chord lengths travelled between wing sections along the span, force spikes in some sections were found to counterbalance similar but opposite spikes in other sections. This observation bolsters support for using smoothing schemes, their effect being synonymous with increasing the number of wing sections (see also the discussion on pp. 165ff).

Effect of Aerofoil Discretisation

Aerofoil discretisation n_{foil} refers to the number of elements that the aerofoil is divided into. This division is carried out in the circle plane so that, upon transformation to the physical plane, the aerofoil has a tighter packing of bound vortices in the regions of the leading and trailing edges, where the Kutta-Joukowski condition can then be enforced more accurately. Higher discretisation also means that the zero-through-flow boundary condition can be imposed more rigorously, reducing the chances of free vortices from accidentally penetrating the wing during the convection phases of the computation.

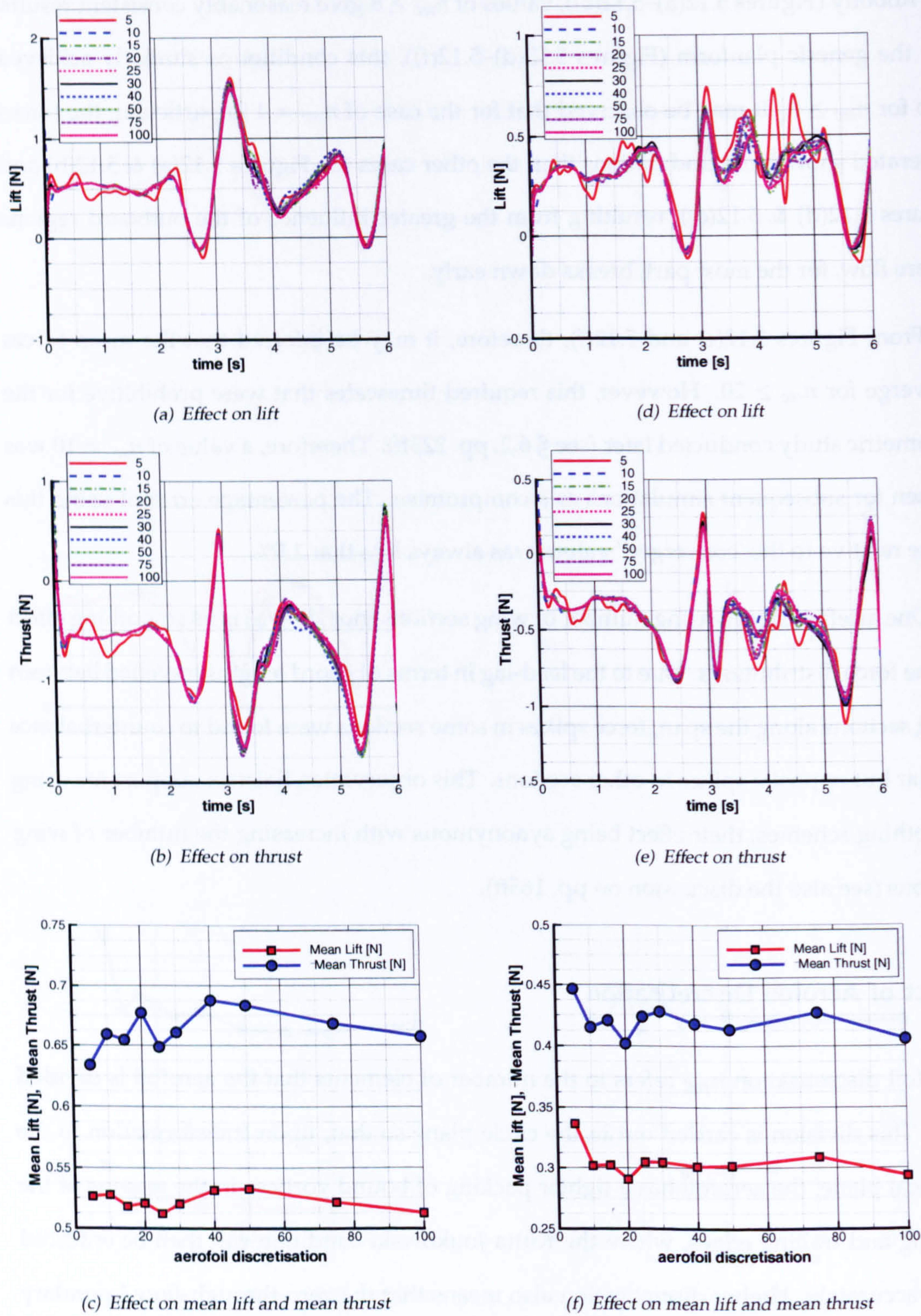


Figure 5.13: Effect of aerofoil discretisation on computed lift and thrust for Robofly (left) and generic (right) wing planforms through one cycle (downstroke and upstroke). The plot legends show aerofoil discretisation n_{foil}

The influence of aerofoil discretisation on lift and thrust for the Robofly and generic wings are shown in Figure 5.13. It is immediately obvious that there is little influence on the overall result. Higher discretisation, however, shows a smoothing effect on the force time-histories. Again, for very small values of n_{foil} , the forces generated exhibit more peaks and troughs, especially for the generic wing planform. From the trend described above, a value of $n_{foil} = 50$ was chosen for the detailed analysis later (Chapter 6).

Effect of Time-step Size

Although time-step size is bound to influence the solution, it is not a universal parameter in that it shows different trends for the same wing shape at different flapping frequencies. A faster wing moves further in the same time than a slower wing, and therefore, demands higher time-resolution to yield accurate results.

A different parameter was, therefore, defined that would circumvent the limitation of the time-step — the mean angle swept by the wing per time-step. The trend for the Robofly and generic wings are shown in Figure 5.14.

For the cases investigated, the simulations were run for one cycle (downstroke and upstroke) which swept two arcs of 180° each as per Dickinson's kinematics, so that the mean angle per time-step was computed thus

$$\frac{\text{angle swept}}{\text{number of time-steps}} = \frac{2 \times 180}{n_{steps}}$$

where n_{steps} is number of time-steps for the entire simulation. The trend seen from Figure 5.14 is that the force data generally gets less peaky with decreasing time-step size (or mean angle per time-step). Whereas the cases for small time-steps are almost synchronous in phase, larger time-step sizes exhibit a phase lag especially for thrust (Figures 5.14(b) and 5.14(e)).

Ideally, it is desirable to keep time-step size to a minimum so that the true 'continuous' nature of the flow is captured. However, due to computational limitations this is not

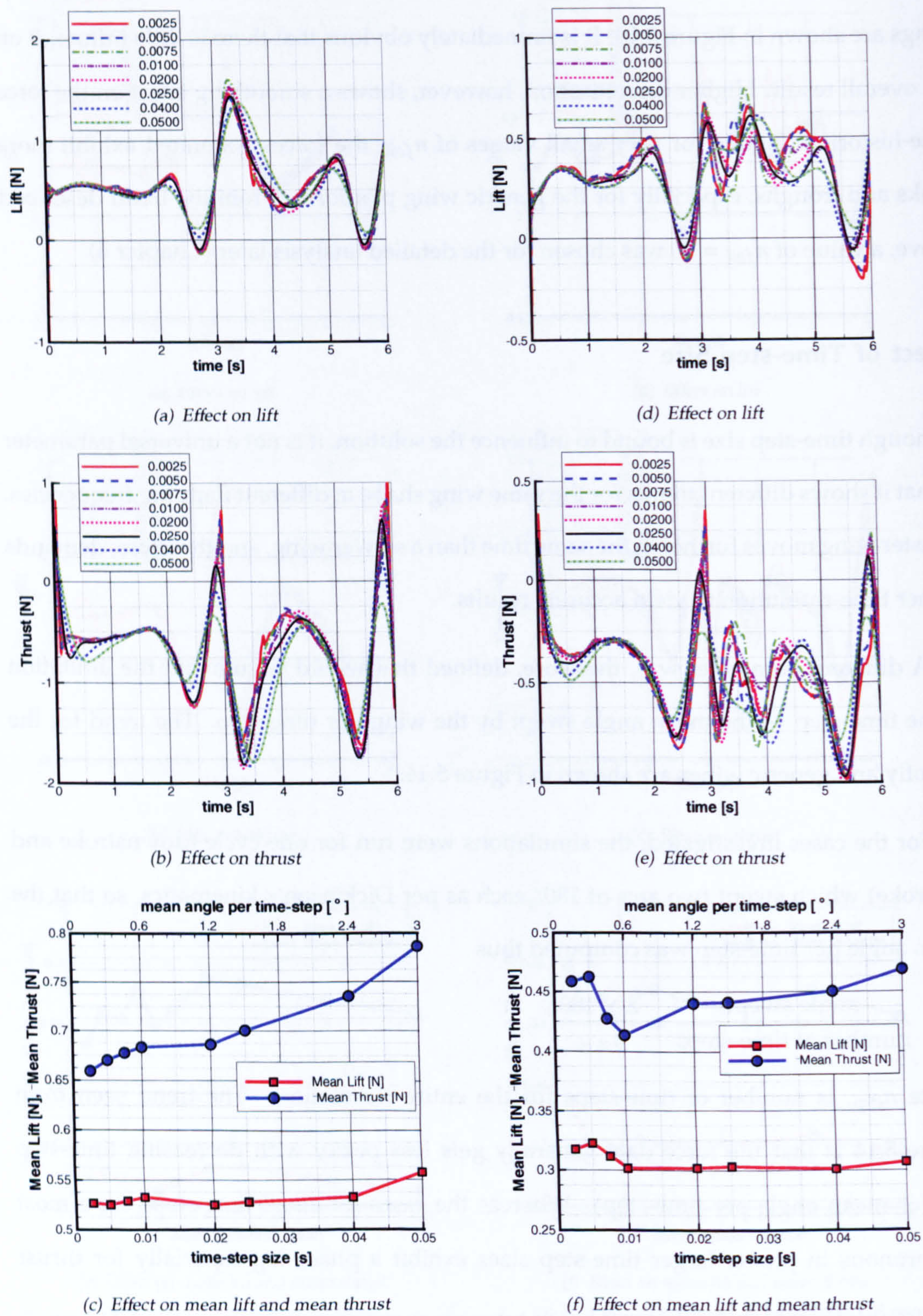


Figure 5.14: Effect of time-step size on computed lift and thrust for Robofly (left) and generic (right) wing planforms through one cycle (downstroke and upstroke). The plot legends show time-step size

possible and larger values of time-step size are used. If too large a value of time-step size is used, results become unreliable. Careful consideration of the plots in Figures 5.14(c) and 5.14(f) shows that a 'turning point'¹⁷ occurs around a time-step size of 0.01 s (mean angle per time-step of 0.6°). This value is assumed to separate the 'bad' values associated with large time-step sizes and 'good' values from use of small time-step sizes. Hence, a mean value of 0.6° per time-step is chosen as the criterion for determining time-step size in the detailed analysis later.

Effect of Vortex-Core Size

The aim of using a vortex core is to remove the singularity that would otherwise exist at the centres of the vortices. The model used in the current work employed the Vatistas vortex (Equation 5.11) so that each vortex exhibited solid-body rotation near its centre and a constant vortex-core size was used throughout the simulation.

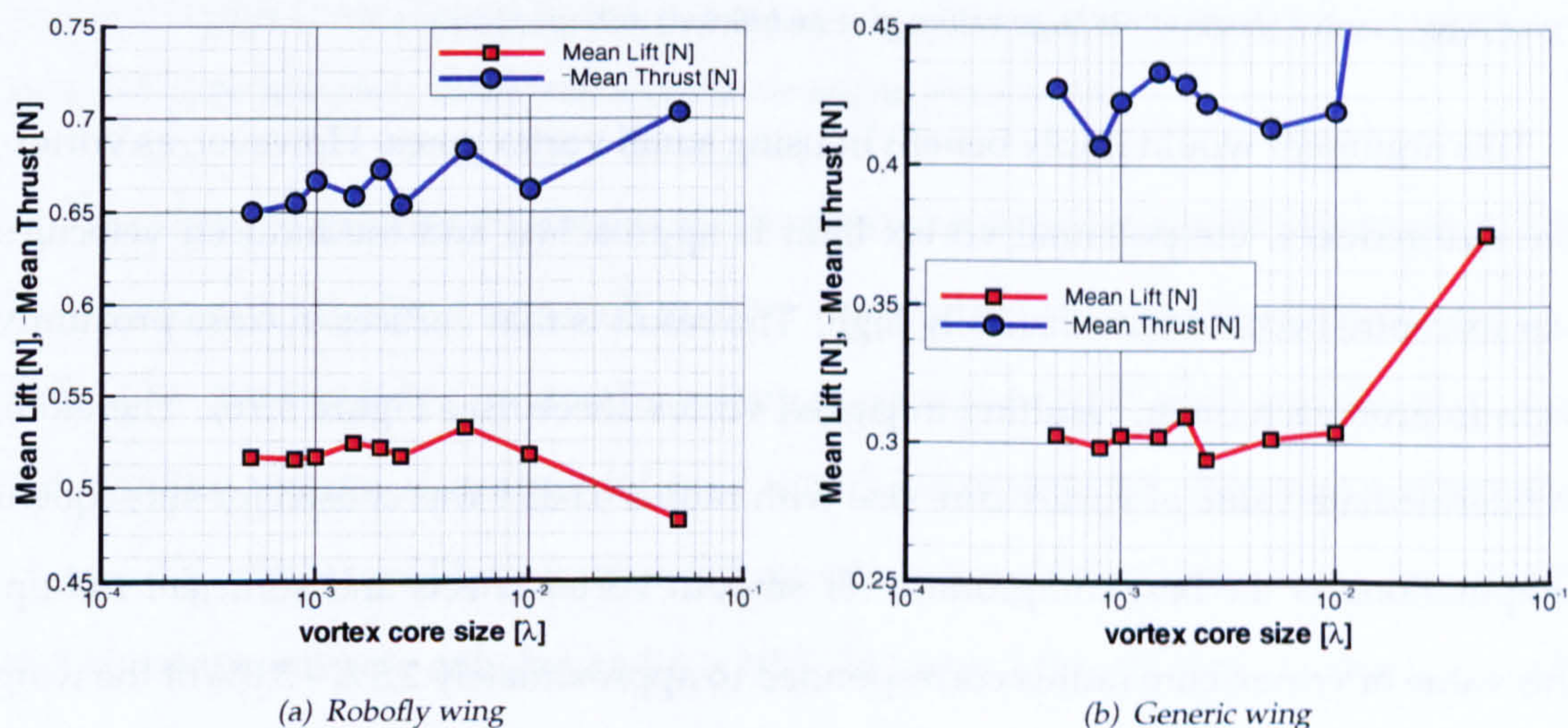


Figure 5.15: Effect of vortex-core size on mean lift and thrust through one cycle for the Robofly (left) and generic (right) wing planforms. λ has units of length (m)

The size of the vortex core was kept the same for all wing sections and was based on a fraction of the representative length λ described earlier (Equation 5.23). Figure 5.15 shows

¹⁷Not in the mathematical sense.

the effect of vortex-core size on mean lift and thrust for the two planforms investigated. Vortex core size appears to have negligible effect on the overall performance except for very large values where (especially for the generic planform) the force data begin to diverge from the 'norm'. This is mainly due to insufficient roll-up (see Figure 5.16).

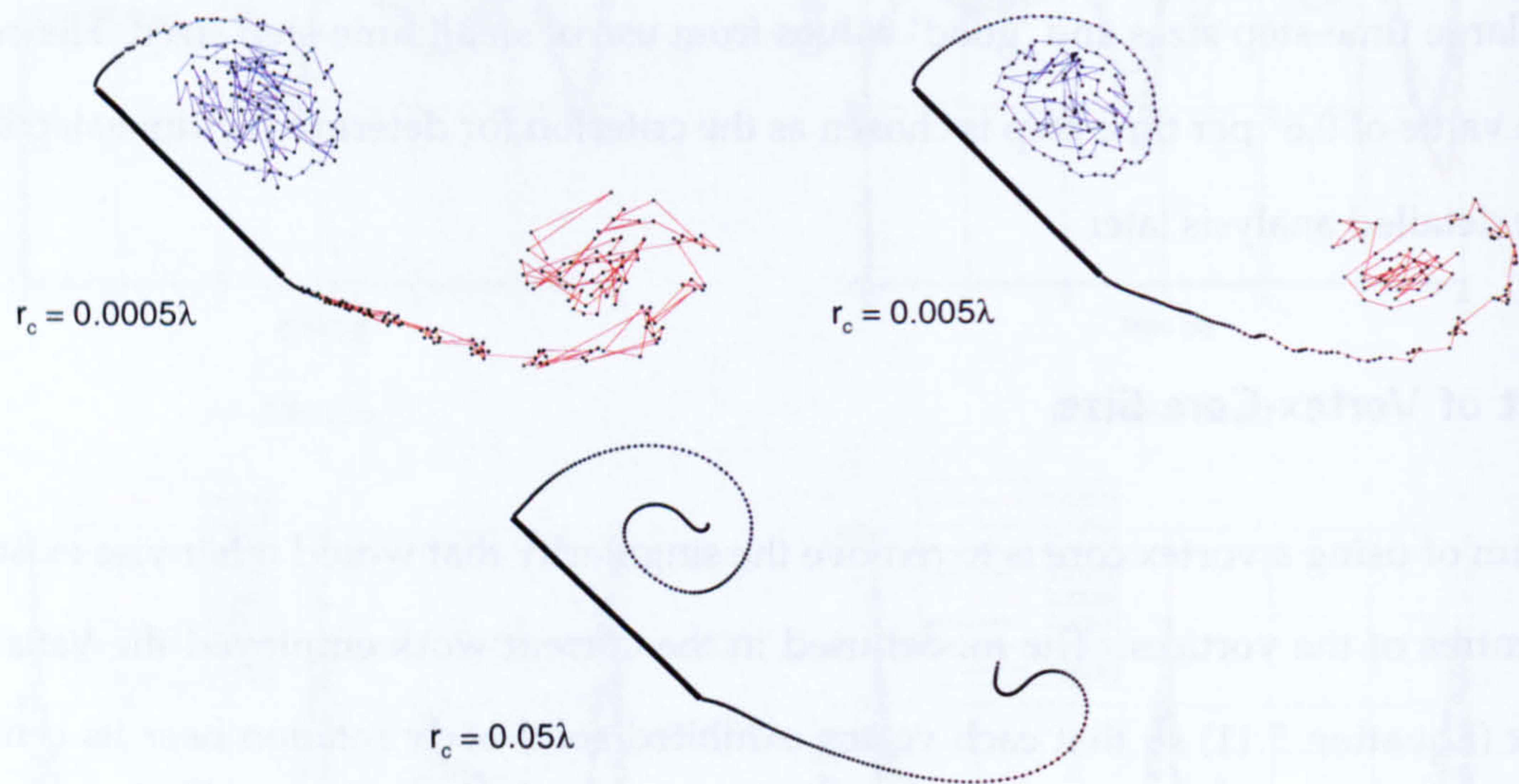


Figure 5.16: Effect of vortex-core size on roll-up for wing at mid-stroke. Small values of vortex-core radius r_c give jagged vortex sheets while large values give insufficient roll-up

This argument would imply benefit in using small vortex cores. However, as vortex-core size reduces, the potential vortex limit is approached and the induced velocities near its centre become unrealistically high. The result is that vortices in close proximity begin to orbit each other, resulting in jagged vortex sheets (see Figure 5.16). Therefore, an intermediate value of vortex-core size with radius 0.005λ was chosen for subsequent computations as the best compromise for smooth vortex sheets and sufficient roll-up. This value of vortex-core radius corresponded to approximately 2.5% ~ 5.0% of the wing section chords for the two wing planforms considered.

Effect of Vortex Amalgamation

Vortex amalgamation is the process whereby free vortices of like sign in close proximity are merged together in an effort to reduce the number of vortices and aid speedy computation.

This benefit in computing time, however, comes at the cost of accuracy of the calculations and of flow representation. Before such a process could be implemented it was, therefore, necessary to determine the limit to which this process could be exploited before adverse effects on the accuracy of the calculation surfaced. The results from this investigation are shown in Figure 5.17.

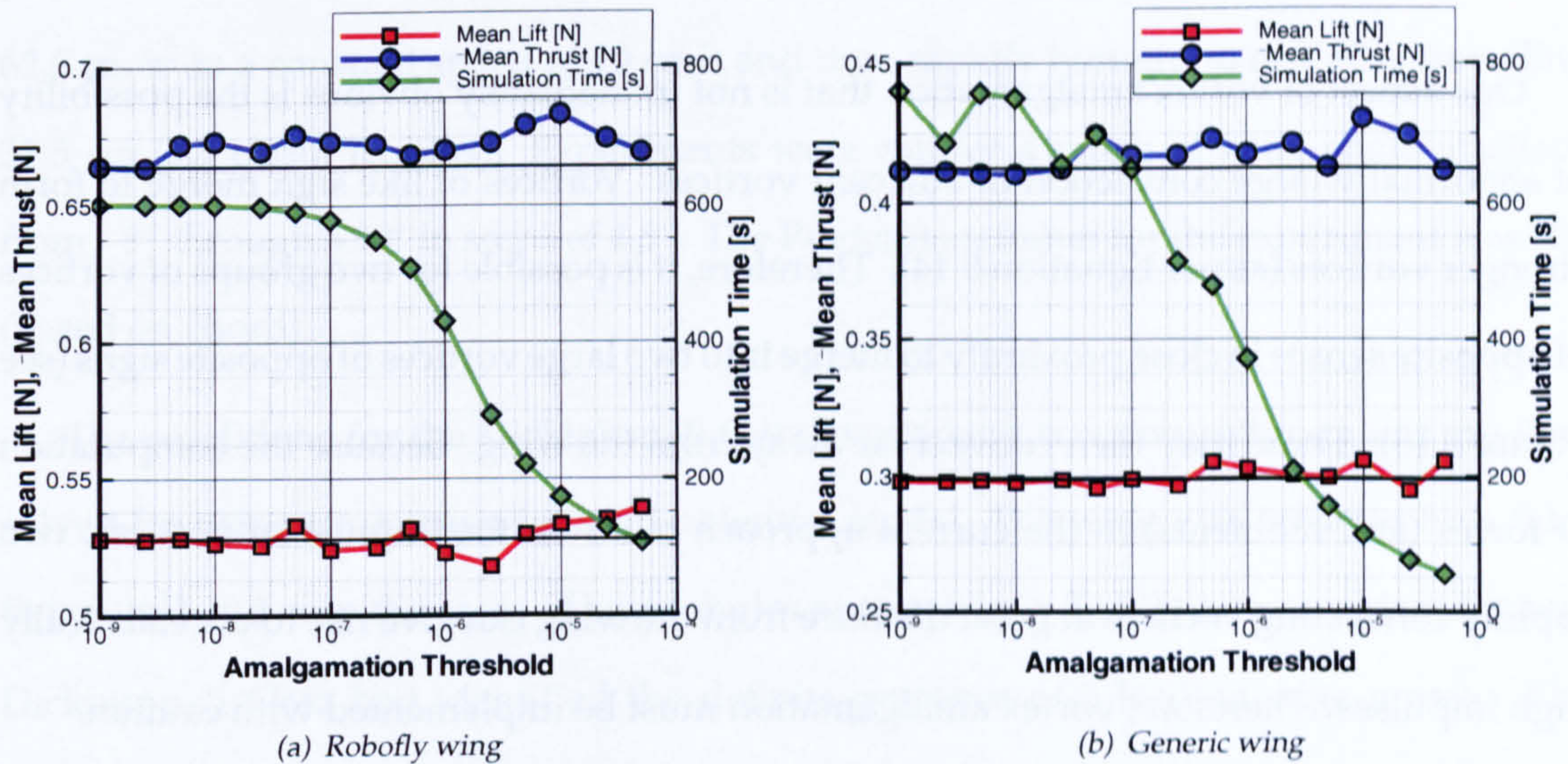


Figure 5.17: Effect of vortex amalgamation threshold ϵ on mean lift and thrust through one cycle. The solid black horizontal lines show values without vortex amalgamation invoked

The criterion used for the amalgamation process was that suggested by Spallart (1988, see Equation 5.13) where the small number ϵ is referred to as the *amalgamation threshold* here. The driving factor for amalgamation is savings in CPU time and the plots show that this only became significant for $\epsilon > 5 \times 10^{-7}$. As regards the effect on accuracy of the computation (measured here in terms of deviation from the amalgamation-free results), this became noticeable only beyond $\epsilon > 10^{-6}$. In view of this finding, a value of $\epsilon = 10^{-6}$ was used for the amalgamation threshold in the detailed analysis later (Chapter 6).

It may be noted that for this chosen value of ϵ , computing times were reduced by about one-third for the Robofly wing and one-half for the generic wing planform¹⁸.

¹⁸The difference in computation time for the two wings executing identical kinematics arises from the number of sub-time-steps used. Because the criterion for the latter depends on the number of chord lengths

Although dependent on circulation strength, this amalgamation threshold corresponded to a physical separation of about 1–3 vortex-core radii between vortex centres.



Figure 5.18: Vortex amalgamation problems

One aspect of vortex amalgamation that is not immediately obvious is the possibility of abnormally large convection of adjacent vortices. Vortices of like sign merge to form stronger vortices (as per Equation 5.14). Therefore, it is possible for two groups of vortices of opposite senses in close proximity to merge into two large vortices of opposite signs (see Figure 5.18). These may then convect far away from the wing. Because the computation of forces (and moments) in the current approach relies on the impulse of vortices, two rapidly convecting vortices at great distance from the wing can give rise to unrealistically high impulses. Therefore, vortex amalgamation must be implemented with caution.

5.2.2 Validation of the Model

The model developed in the current work was tested and validated against real-world data in order to assess its validity and limitations. Data from two sets of experiments were used for this model-verification process. The first set was adapted from Dickinson & Götz (1993) and the second was provided by Dickinson (priv. comm.). A third comparison was made with the results from the indicial model of Pedersen (2003) but this is presented in the discussion in the next chapter (§ 6.1.5, pp. 214ff).

travelled by the trailing edge per time-step, different planform shapes require different number of sub-time-steps per time-step.

Flow Visualisation Comparison with Dickinson & Götz (1993)

The original experiment was conducted by Dickinson & Götz in a glass aquarium filled with a 54% sucrose solution. A rectangular wing with 5 cm chord and 15 cm span was traversed in a straight line and at constant angle of attack through this solution between a pair of baffles to limit any 3-D tip flow. The wing was impulsively started, accelerated at 62.5 cm/s^2 to a constant speed of 10 cm/s and then rapidly brought to rest after travelling 37.5 cm (7.5 chord lengths). Experiments were run for a range of wing angle of attack from -9° through $+90^\circ$ in steps of 4.5° . The Reynolds number for the experiment was 192 (based on chord).

The conditions for the Dickinson & Götz experiment were reproduced and the flow solved for using the current inviscid potential model. The wing was replaced by a thin, flat plate¹⁹ and only the cases of large angles of attack ($\alpha \geq 22.5^\circ$) were considered where Dickinson & Götz had identified the definite presence of a leading-edge vortex. The kinematics for the experiment are shown in Figure 5.19 below.

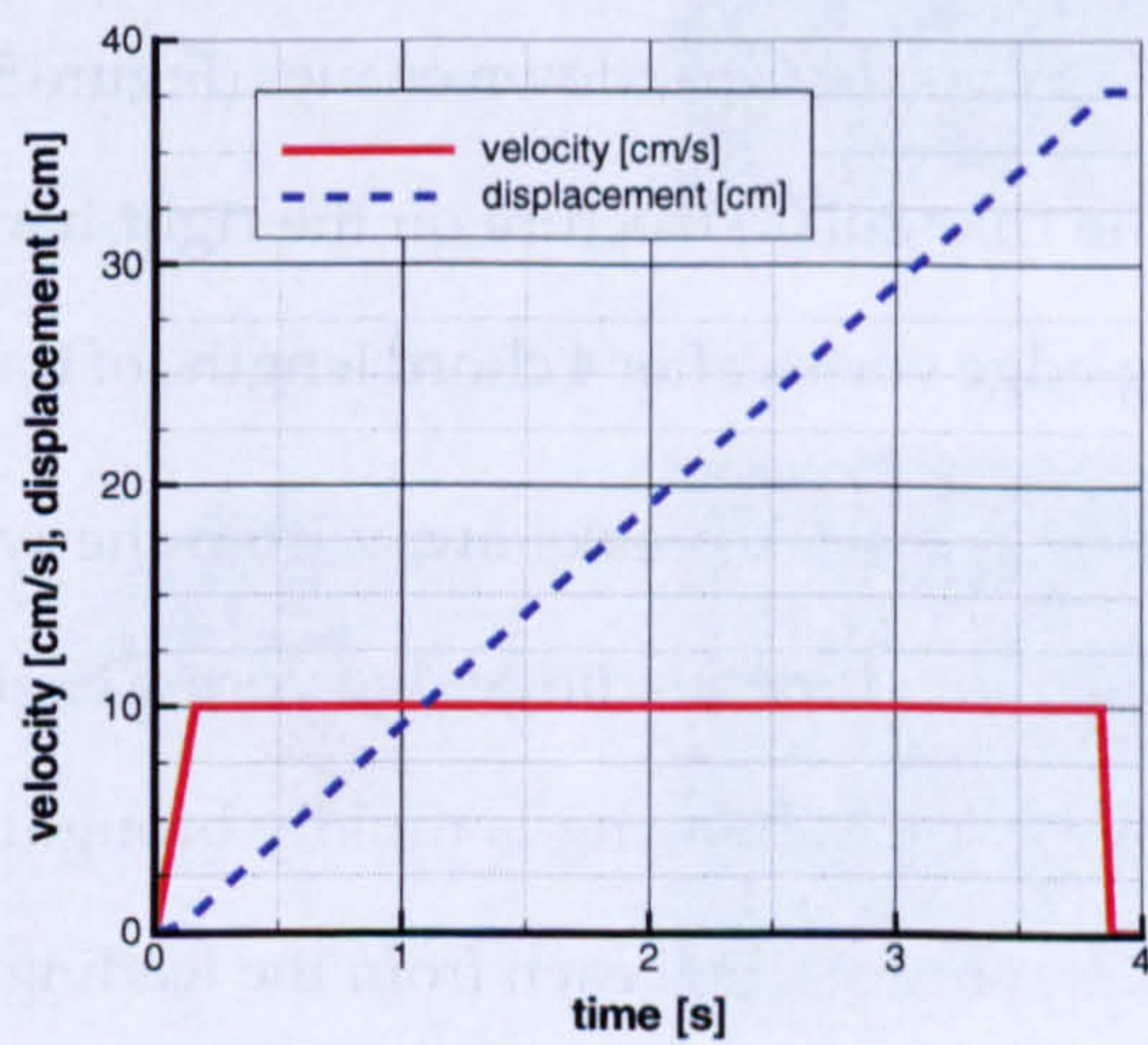


Figure 5.19: Translational motion kinematics used by Dickinson & Götz (1993)

Dickinson & Götz presented flow-visualisation photographs for the case of angle of

¹⁹The wing used in the Dickinson & Götz experiment was 1 mm thick giving it a thickness-to-chord ratio of 2% making the thin wing assumption a valid one.

attack of 45° . These are compared with the results from the current model in Figure 5.20 below. There is remarkable agreement between the practical, viscous experiment and the inviscid theoretical prediction, despite the very low Reynolds number.

The flow-visualisation photographs were for fixed instances in time ranging from 1 through 4 chord lengths of travel. After one chord length of travel, flow has begun to separate from both leading and trailing edges (Figure 5.20(a)). Flow from the leading edge has rolled up very tightly due to the influence of the bound vorticity on the wing. The trailing-edge vortex has also rolled up albeit less than the leading-edge vortex being 'freer' than the latter. As the wing moves, roll-up continues in both vortices but the leading-edge vortex also grows significantly in size, lifting off slowly from the wing surface (Figure 5.20(b)).

At this point, the original trailing-edge vortex has reached a near-constant size but continues to roll up. Due to its growing size and strength, the leading-edge vortex causes flow from the trailing edge to roll up into a second trailing-edge vortex that spills forwards on to the wing surface (Figure 5.20(c)). This roll-up eventually leads to the distortion of the leading-edge vortex and finally vortex breakdown ensues (Figure 5.20(d)). The theoretical prediction even captures the triangular structure on the right-hand side of the rolled-up end of the distorted leading-edge vortex after 4 chord lengths of travel (see Figure 5.20(d)).

Eventually, the leading-edge vortex breaks away from the wing (not shown) and a new one begins to form. As this second leading-edge vortex begins interaction with the existing second trailing-edge vortex, the wing is rapidly brought to a halt. The result is the shedding of two stopping vortices, one each from the leading and trailing edges (see Figure 6.15, p. 219).

Dickinson & Götz (1993) also presented force data for the experiment. This is discussed together with the force results from the theoretical model in the next chapter (§ 6.1.6, pp. 216).

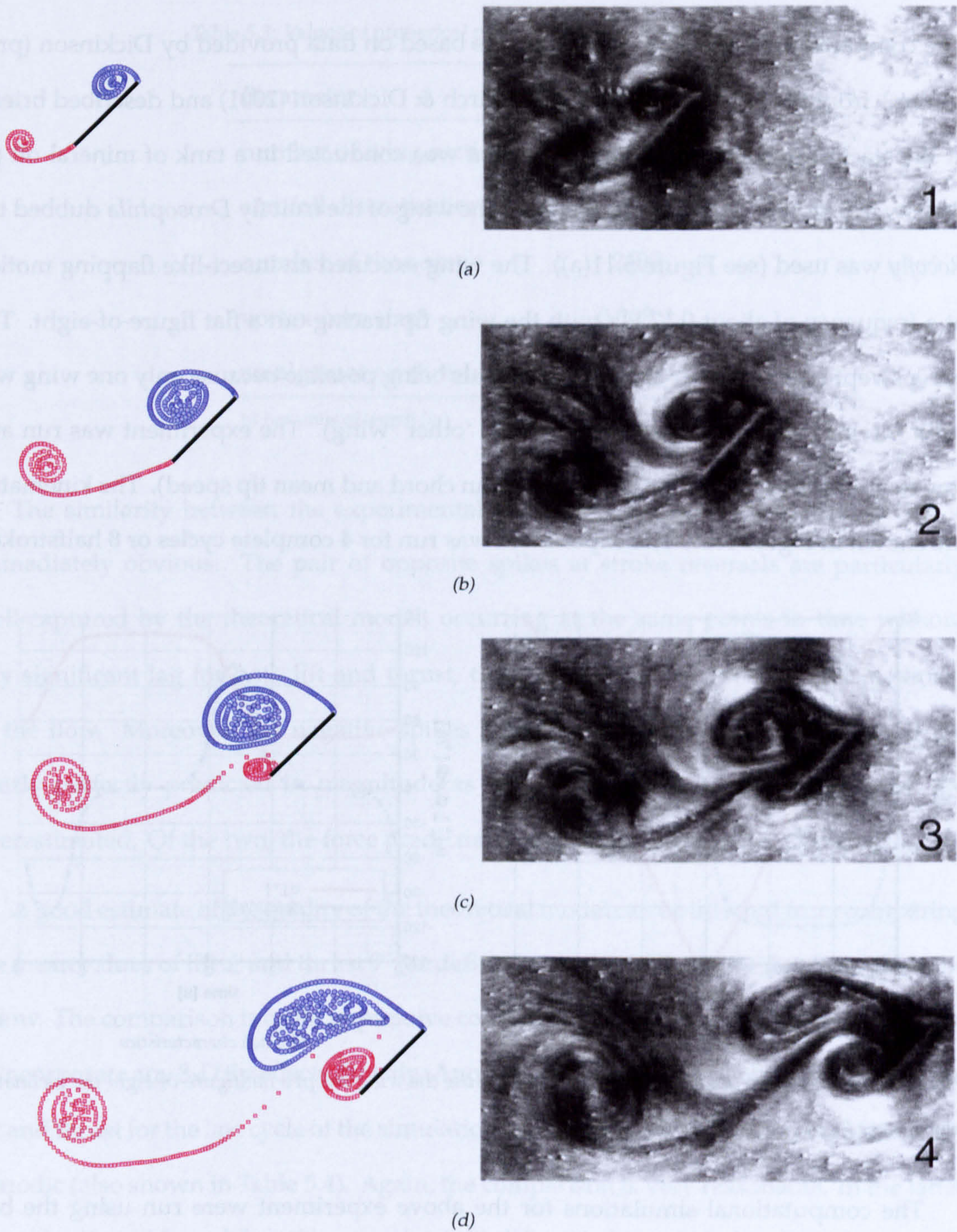


Figure 5.20: Comparison of flow-visualisation results from the current inviscid theoretical model (left) with the Dickinson & Götzt experiment at $Re = 192$ (right). The numbers 1 through 4 refer to the number of chord lengths travelled since impulsive start

Force Comparison with Dickinson (priv. comm.)

The comparison presented in this section is based on data provided by Dickinson (priv. comm.) from the setup that was used in Birch & Dickinson (2001) and described briefly at the start of this chapter. The experiment was conducted in a tank of mineral oil (of density 880 kg/m^3). A scaled-up model of the wing of the fruit fly *Drosophila* dubbed the *Robofly* was used (see Figure 5.11(a)). The wing executed an insect-like flapping motion at a frequency of about 0.17 Hz with the wing tip tracing out a flat figure-of-eight. The wing swept a semicircular arc ($\Phi = 180^\circ$), this being possible because only one wing was used (no mechanical interference from the 'other' wing). The experiment was run at a low Reynolds number of 160 (based on mean chord and mean tip speed). The kinematics are shown in Figure 5.21. The experiment was run for 4 complete cycles or 8 halfstrokes.

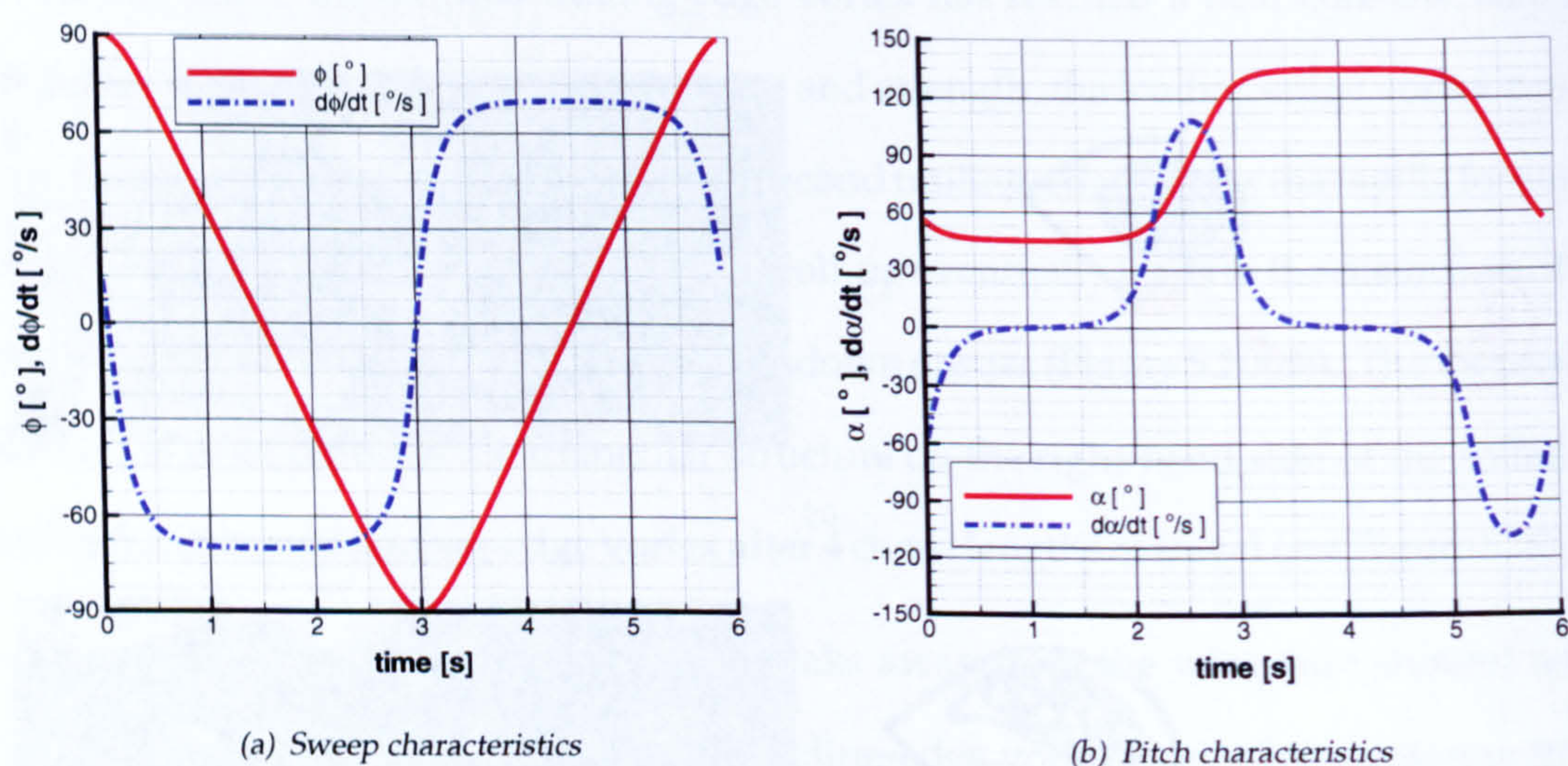


Figure 5.21: Kinematics of Dickinson's Robofly. Because the wing swept a flat figure-of-eight the inclination parameters ($\theta, \dot{\theta}$) were zero and are not shown

The computational simulations for the above experiment were run using the best combination of the parameters inferred from the numerical experiments described earlier. These are given in Table 5.3. This benchmark simulation was one of the longest (due to 24000 section-time-steps with as many as 4 sub-time-steps per time-step) and took about 11 hours to complete. The results from the simulation are compared with the experimental

data in Figure 5.22.

Table 5.3: Values of numerical parameters used in validation

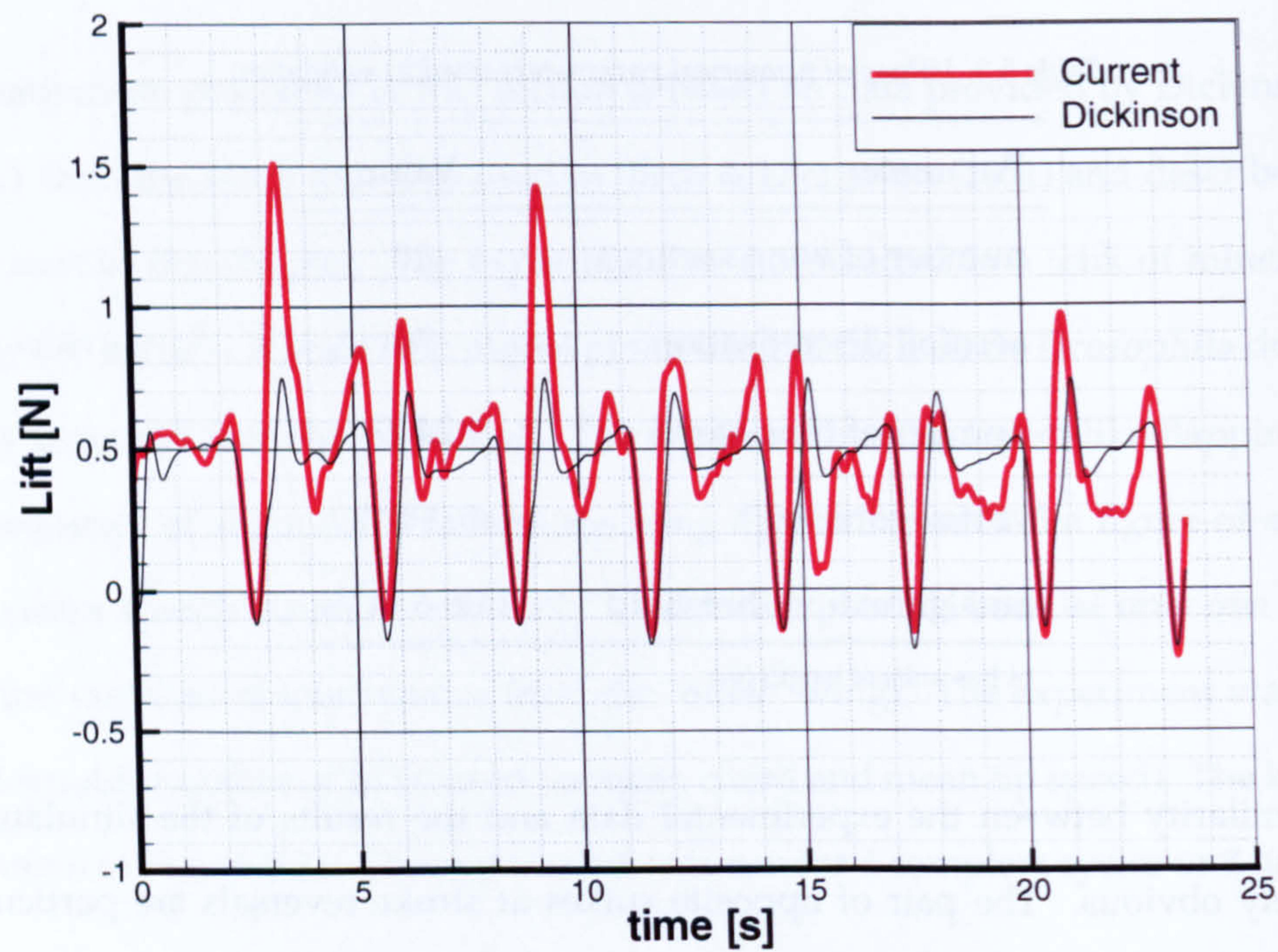
Parameter	Value
number of wing sections	10
aerofoil discretisation	50
number of time-steps	2400
vortex-core size*	0.005λ
amalgamation threshold	$1.0\text{e-}6 \text{ m}^3/\text{s}$

* λ has units of length (m)

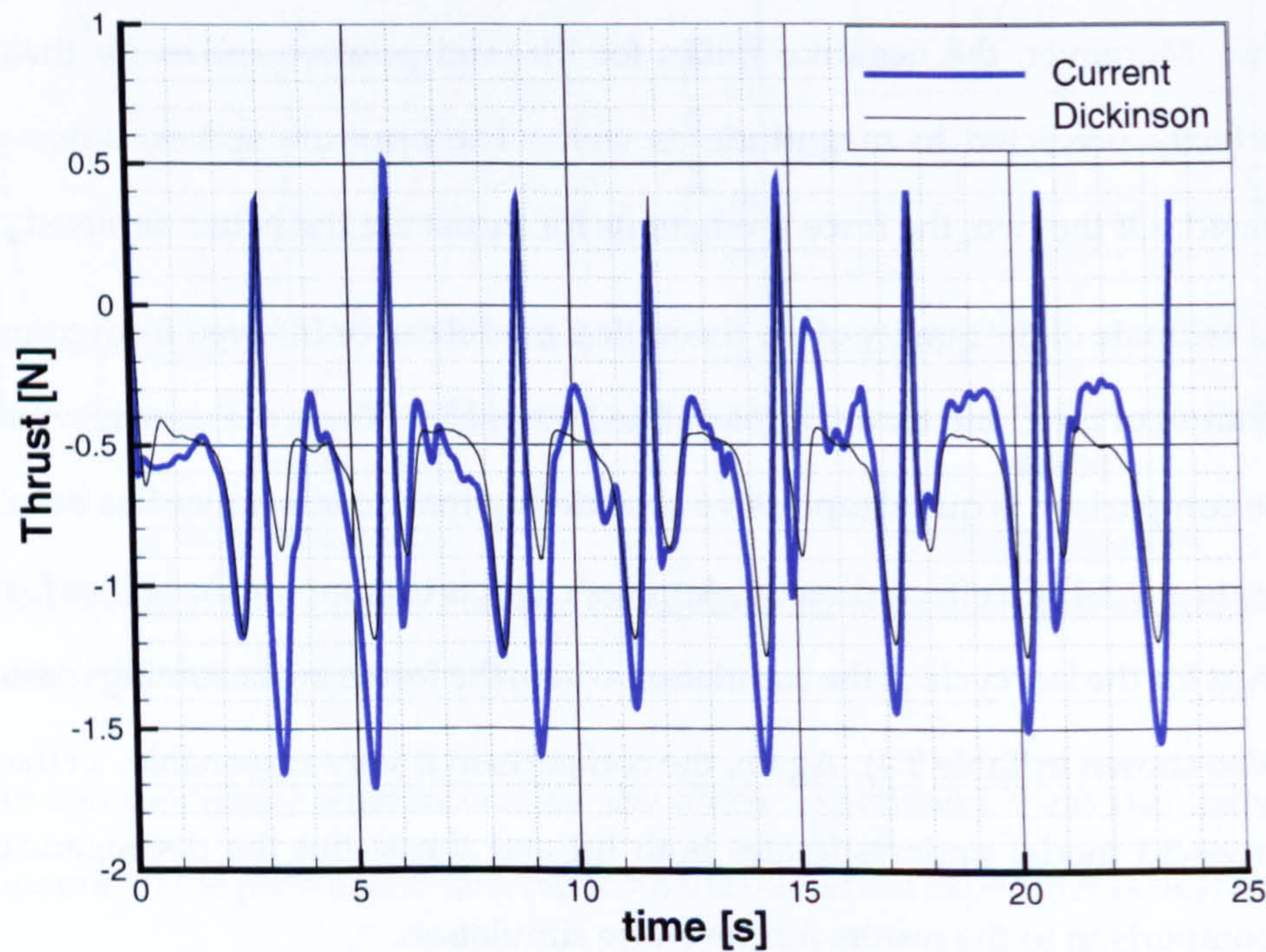
The similarity between the experimental data and the results of the simulation is immediately obvious. The pair of opposite spikes at stroke reversals are particularly well-captured by the theoretical model, occurring at the same points in time without any significant lag for both lift and thrust, thus accounting very well for unsteadiness of the flow. Moreover, the negative spikes for lift (and positive spikes for thrust) are nearly perfectly predicted in magnitude as well. The opposite spikes, however, are overestimated. Of the two, the force predictions for thrust are the better depicted.

A good estimate of the quality of the theoretical model can be inferred from comparing the mean values of lift $\mathcal{L}$ and thrust $\mathcal{T}$ (as defined on p. 151). These are shown in Table 5.4 below. The comparison is quite impressive considering that no allowance has been made to incorporate any 3-D tip effects directly. Another check is to compare the values for mean lift and thrust for the last cycle of the simulation where the forces are becoming reasonably periodic (also shown in Table 5.4). Again, the comparison is very reasonable. In the latter case, the inviscid model underestimates both lift and thrust but the discrepancies are halved in comparison to the results for the entire simulation.

Another check that can be made is to see if the drag force $\mathcal{D}$ is close to zero once the data became reasonably periodic. This is a necessary condition for hover. From



(a) Lift comparison



(b) Thrust comparison

Figure 5.22: Comparison of force data for Robofly data from Dickinson (priv. comm.) with results from the current theoretical simulation

Table 5.4: Comparison between experimental and theoretical values for mean lift and mean thrust

	<u>Entire Simulation</u>		<u>Last Cycle</u>	
	Lift [N]	Thrust [N]	Lift [N]	Thrust [N]
Dickinson	0.4034	−0.6011	0.4081	−0.6106
Current	0.4568	−0.6445	0.3805	−0.5904
discrepancy [%]	13	7.2	6.8	3.3

Dickinson’s data, mean $\mathcal{D}$ for the last cycle is 0.0055 N which makes it 0.9% of mean $\mathcal{T}$ for the same period. For the theoretical model, the respective values are 6.29×10^{-4} N and a ratio of about 0.1% — a better condition for hover. This also means that the forces are becoming reasonably periodic after 4 cycles.

A comparison of Figures 5.22(a) and 5.22(b) shows that the bandwidth of the force-measuring equipment used by Dickinson may have been insufficient in the region of the forces generated. The spikes are always better captured in one direction (negative spikes for lift and positive spikes for thrust) than those in the other direction. Dickinson himself does not report on this characteristic of the experimental setup. Of course, it is also possible that those peaks predicted by the theoretical model and not observed to the same extent in the experiments are in fact damped by viscosity and other effects. What is noteworthy, however, is that irrespective of magnitude, the positions of all peaks are quite accurately predicted by the theoretical model. This implies that the physics of the flow are well-captured by the theoretical model. More will be said about the latter in the more detailed discussion in the following chapter (see § 6.1.3).

Chapter 6

Results & Discussion

This chapter presents the main findings of the study and is divided into two main parts. The first part (§ 6.1) starts off with a look at the validation results presented at the end of the previous chapter but with a view to explaining the observed phenomena on the basis of the physics of the flow. First, the performance of the entire wing is considered (§ 6.1.1) in an effort to describe briefly the relative contributions of each wing section. This is followed by a more detailed account in which the component forces responsible for lift and thrust are considered (§ 6.1.2). A discourse where the physics of the flow are used to explain the forces observed is then presented (§ 6.1.3).

Thereafter, the importance of the leading-edge vortex to any insect-like flapping-wing model is established (§ 6.1.4) by comparing results with and without a leading-edge vortex. In the following section (§ 6.1.5) results from the current circulation-based model are compared with the velocity-potential-based model of Pedersen (2003). Finally, a brief look is taken at the possible limitations of the current inviscid approach (§ 6.1.6).

The second part of the chapter (§ 6.2) presents results from a parametric study that was conducted to investigate the effects of various flapping-wing parameters. These provide insight into wing design for a flapping-wing MAV — the ultimate aim of this PhD. This part is broadly subdivided into two. The first section is concerned with investigating the

effects of various wing kinematics-related parameters (§ 6.2.1), such as flapping frequency and stroke amplitude.

The second section (§ 6.2.2) shows how a flapping-wing's aerodynamic performance is affected by wing-shape-related parameters such as aspect ratio, wing area and location of pitch axis. The section concludes with a comparison of 13 different wing planform shapes based on lift, lift-to-torque ratio and wing loading.

6.1 Physics of Flow

This section is concerned with the validation results presented at the end of the previous chapter with the aim of describing the observed force characteristics from both the flow-visualisation results as well as the force data.

6.1.1 Whole Wing Performance

The current approach features a ‘quasi-3-D’ model whereby the performance of the entire wing is obtained by summing the contributions of the individual wing sections. Inboard sections make a smaller contribution than outboard sections mainly due to the slower velocities there. However, the maximum contribution does not necessarily come from the outermost section because the forces (and moments) generated also depend on local chord length and the pitch-axis location. For the Robofly wing used for validation in the previous chapter, almost half of the total contribution comes from sections 7, 8 and 9 (out of 10), the biggest contribution coming from section 8 (see Figure 6.1).

The force-time-history for each wing section can be better shown as a contour or a surface plot where the relative magnitude of the contributions of each section is more apparent. These are shown for lift $\mathcal{L}$ and thrust $\mathcal{T}$ in Figures 6.2 and 6.3 respectively.

One feature that can be inferred from Figures 6.2 and 6.3 is the phase difference between wing sections. The physics of the flow during a halfstroke is strongly dependent on the number of chord lengths travelled. Flow in the outboard regions is more developed and ‘older’ in terms of chord lengths travelled. The latter then leads to a phase lead-lag between sections. An important observation made during the current study is that flow behaviour is very strongly linked to the ‘age’ of the flow in terms of number of chord lengths travelled. It was probably for this reason that Wagner (1925) (and later von Kármán & Sears (1938)) in his seminal work used semi-chords travelled as the independent

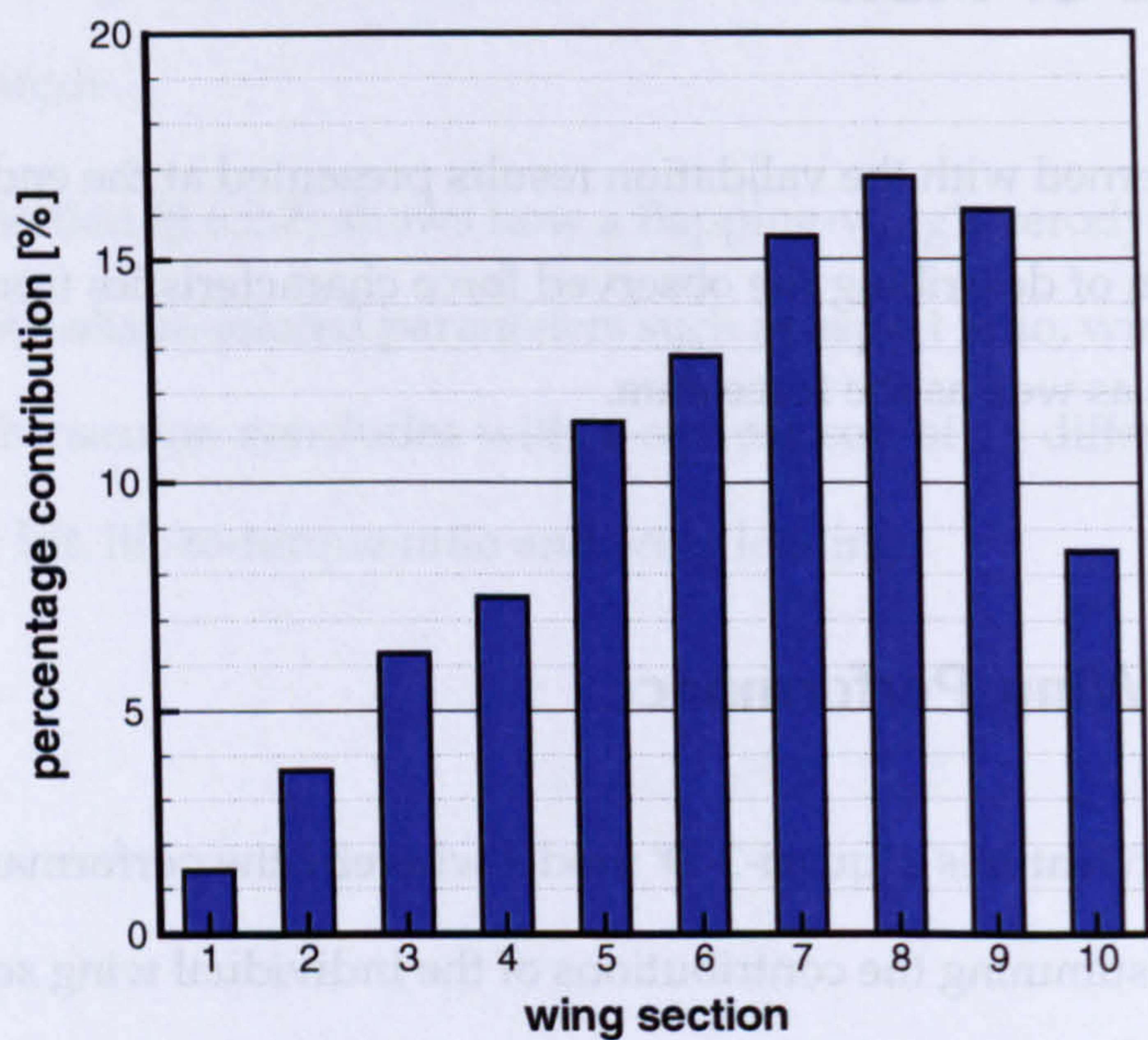


Figure 6.1: Contribution of individual wing sections to total lift. The plot for thrust is identical

variable instead of time. This shows that a reduced-frequency-type parameter is still relevant in this form of unsteady flow.

A consequence of this phase lead-lag is that (for the Robofly wing especially) flow in the outboard sections, having travelled more chord lengths, is more prone to vortex breakdown. A similar observation was made by Ellington *et al.* (1996) and van den Berg & Ellington (1997a) from experiments on Ellington’s ‘flapper’. They reported that the vortex breakdown caused the leading-edge vortex to ‘lift off’ the wing surface in these regions of the wing. In extreme cases, therefore, the outermost sections feature a von Kármán-type vortex system composed of alternately shed leading- and trailing-edge vortices (see Figure 6.4). This leads to a rather messy force history. However, the messy nature of the forces involved also makes them relatively small for the flapping velocities involved so that the contribution of the outermost wing sections on the whole wing is small. Thrust (drag) contributions are, however, significant.

In the current quasi-3-D model, no 3-D tip vortex effects are included. Comparison

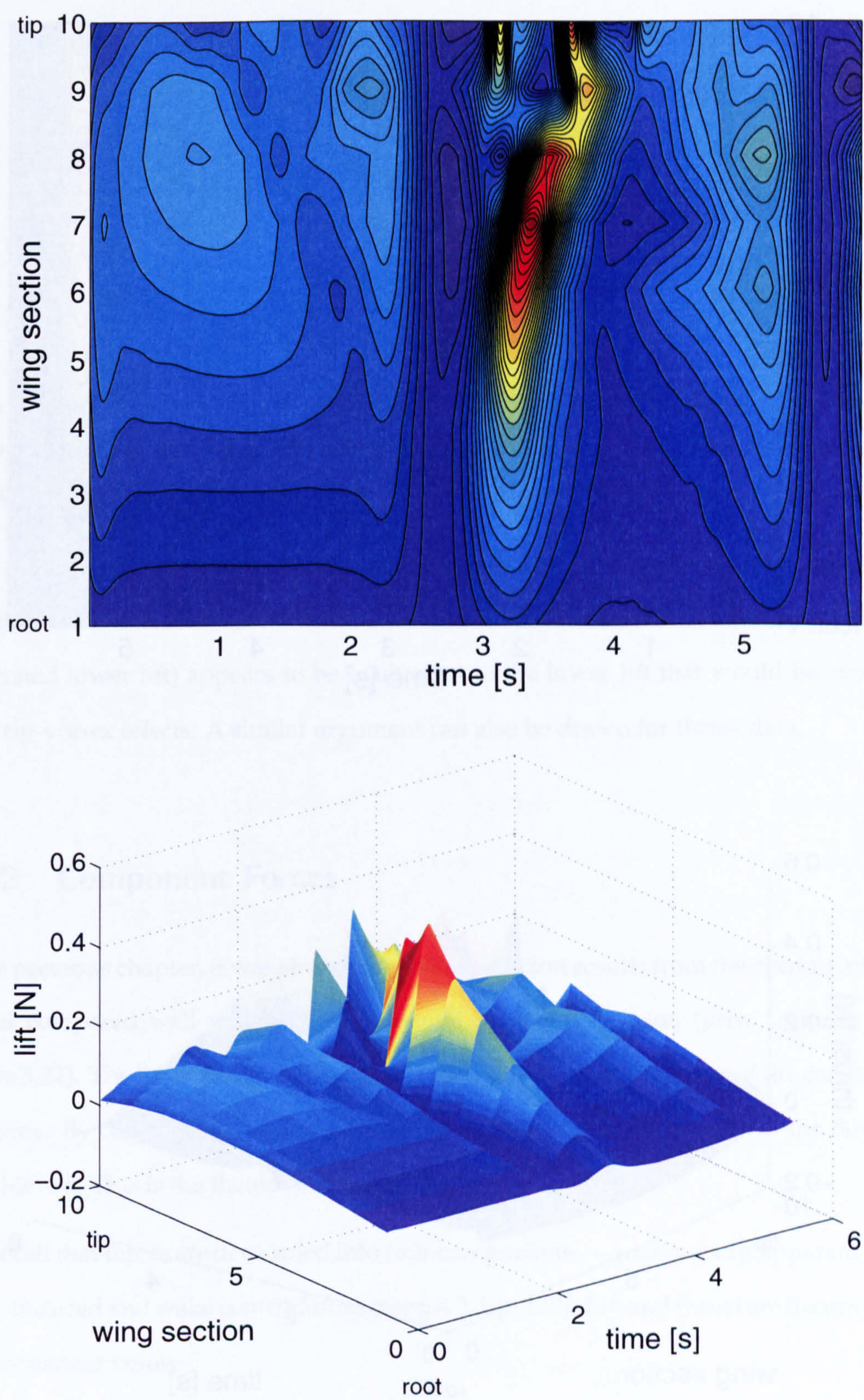


Figure 6.2: Contour (top) and surface (bottom) plots showing time history of lift along Robofly wing for one complete cycle

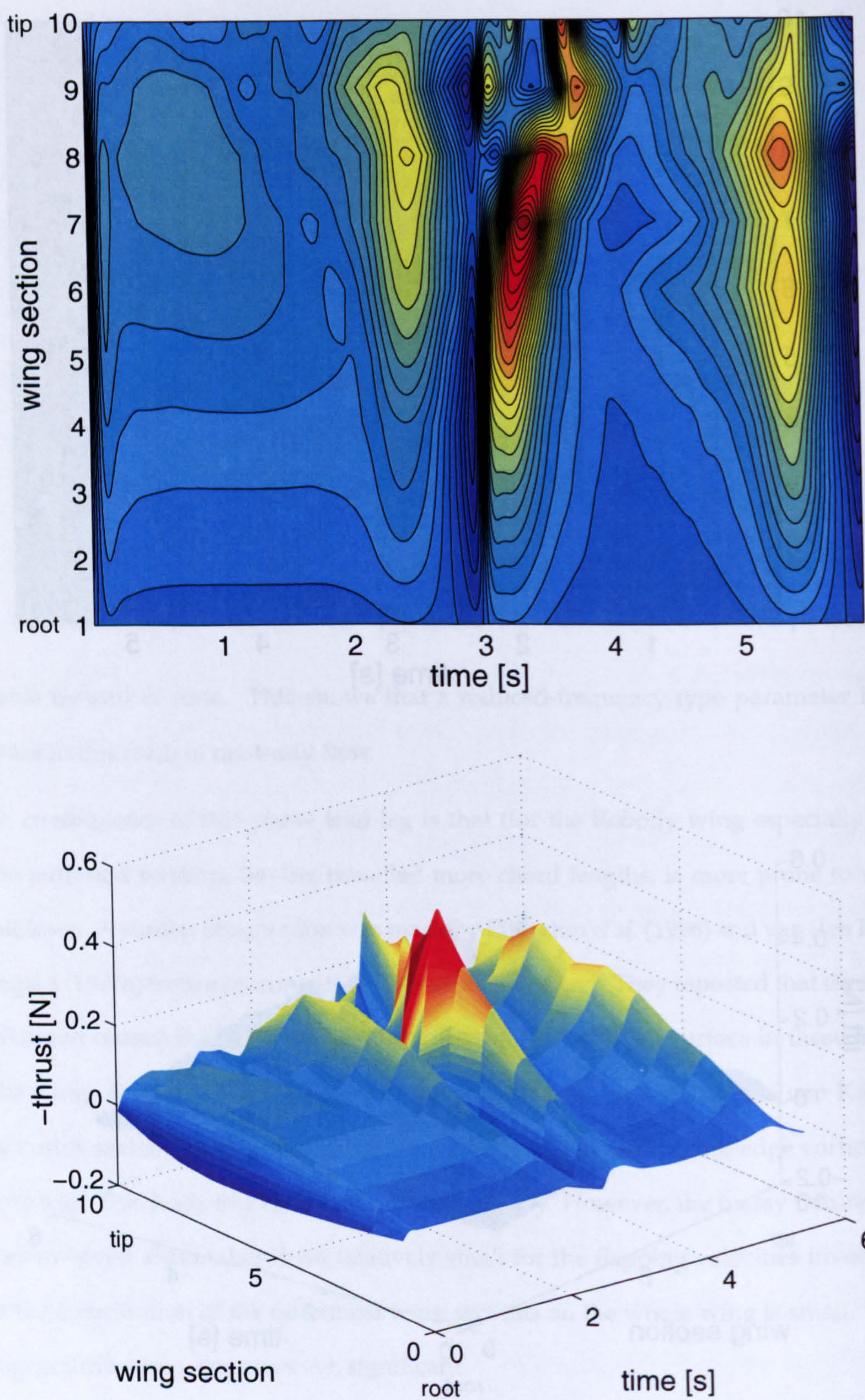


Figure 6.3: Contour (top) and surface (bottom) plots showing time history of thrust along Robofly wing for one complete cycle

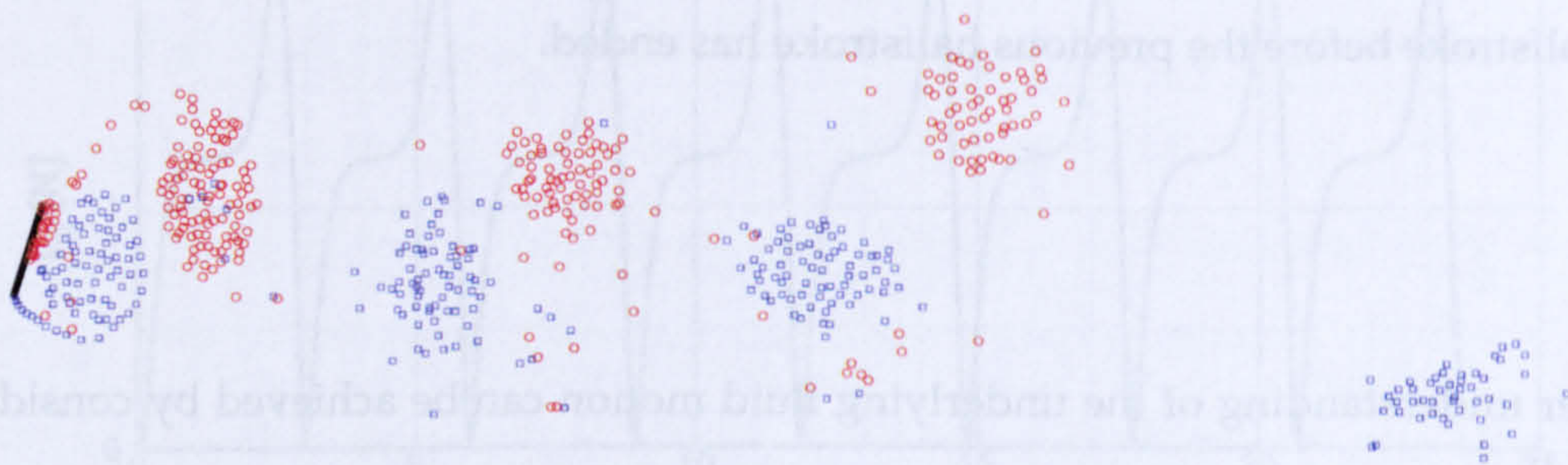


Figure 6.4: Von Kármán-type vortex system in outboard wing sections where local chord length is small and flapping velocities are high. The discrete vortices are colour-coded: red circles for leading-edge vortices and blue squares for trailing-edge vortices

with experimental data (see § 5.2.2), however, has shown remarkably good agreement. The reason for this is that vortex breakdown in outboard sections of the wing (and the associated lower lift) appears to be equivalent to the lower lift that would be expected from tip-vortex effects. A similar argument can also be drawn for thrust data.

6.1.2 Component Forces

In the previous chapter, it was shown that the simulation results from the current inviscid model compared well with the experimental data of Dickinson (priv. comm.) (see Figure 5.22). The lift and thrust time-histories shown are in fact the sum of the constituent elements. By considering these elements, better insight into the physics of the flow can be achieved. This is the theme of this section.

Recall that forces are decoupled into four components — quasi-steady, apparent mass, wake-induced and wake contributions (see § 4.2.3, p. 127). Lift and thrust are decomposed in this manner below.

Recall also that lift $\mathcal{L}$ here refers to the vertically upward force opposing gravity and flapping frequency is about 0.17 Hz so that each halfstroke lasts about 2.94 s. Also,

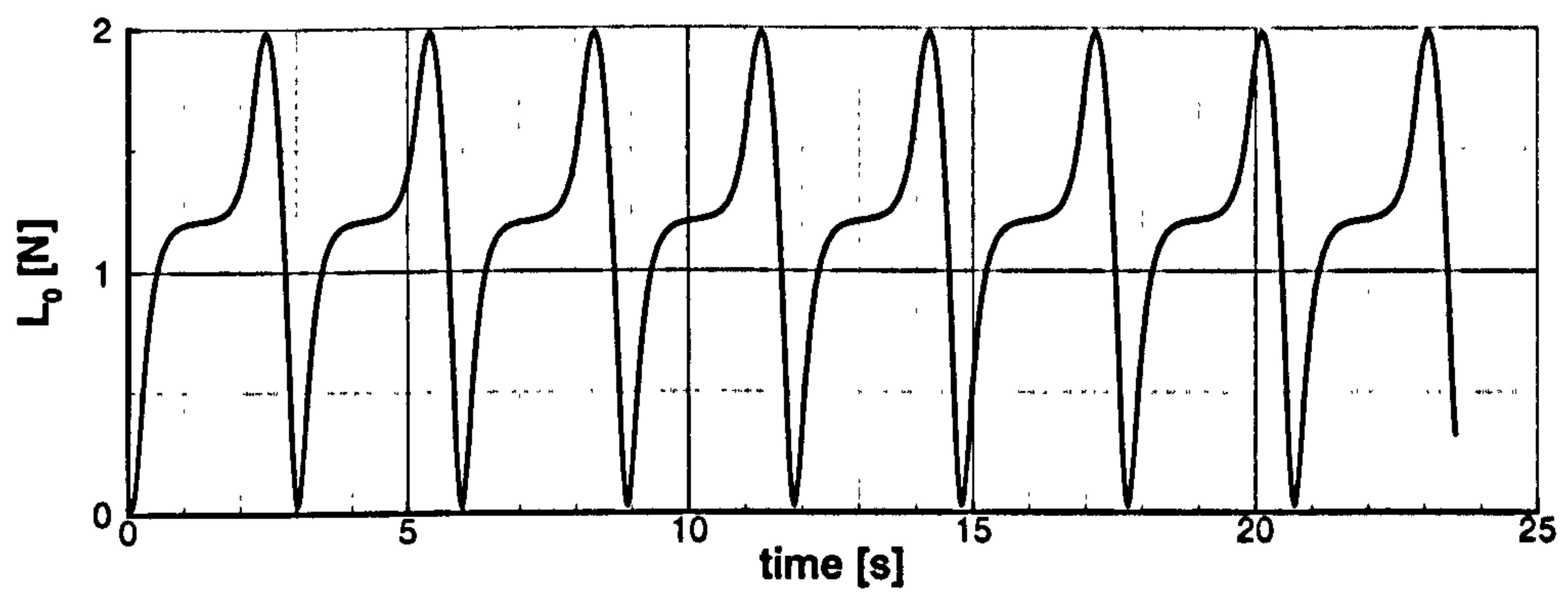
rotation is advanced in Dickinson's kinematics, i.e. the wing is correctly orientated for the next halfstroke before the previous halfstroke has ended.

Lift

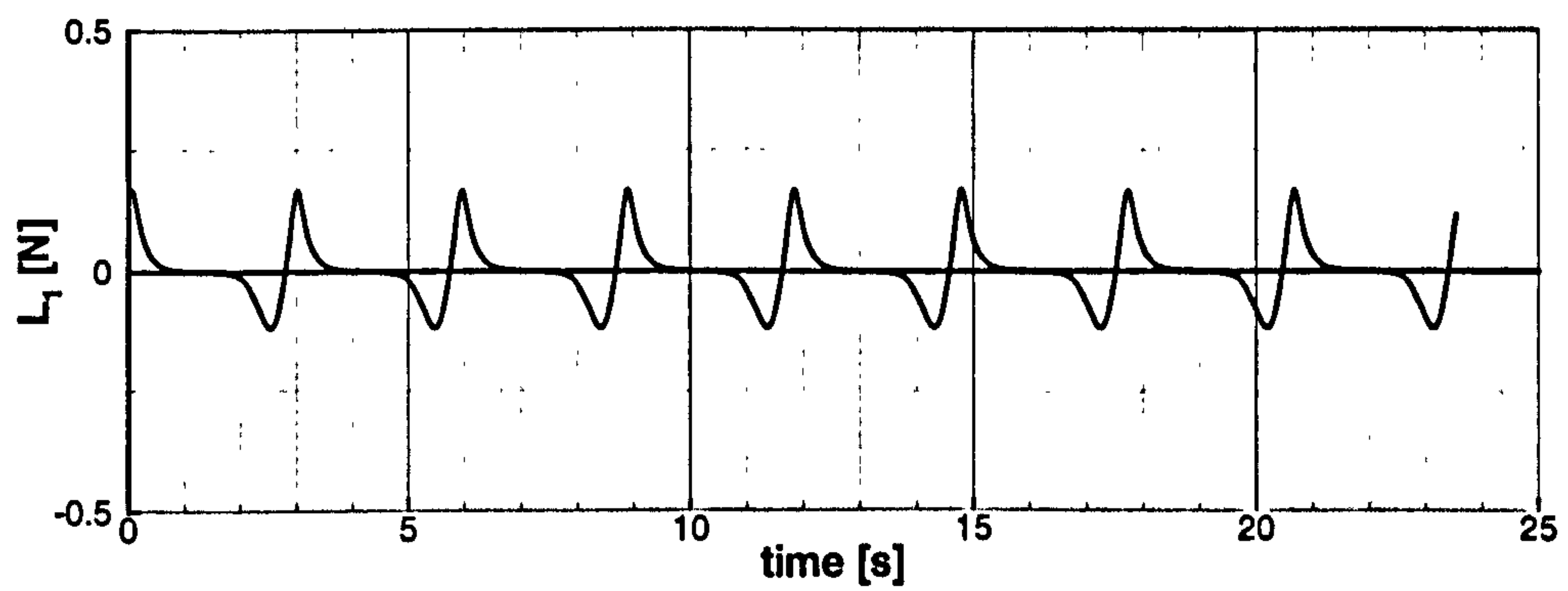
A better understanding of the underlying fluid motion can be achieved by considering the component contributions of the lift components. These are shown in Figure 6.5. Recall that $\mathcal{L}_0$ is the quasi-steady (Kutta-Joukowski) lift, $\mathcal{L}_1$ is the apparent-mass contribution, $\mathcal{L}_2$ is the contribution of the extra bound circulation 'created' due to the presence of the wake and $\mathcal{L}_3$ is the contribution of the wake (both leading-edge vortex and trailing-edge wake) itself.

Quasi-Steady Component $\mathcal{L}_0$ Although wing rotation (about the pitch axis) is advanced in Dickinson's kinematics, part of the stroke reversal nevertheless still occurs in the subsequent halfstroke. Therefore at the start of a halfstroke, the wing is still pitching nose-down in order to assume the correct position for the following halfstroke. Reducing angle of attack corresponds to decreasing bound circulation, and hence, the Kutta-Joukowski lift. At the same time, the wing is beginning translational acceleration which increases the Kutta-Joukowski lift. Hence, the combined effect is that $\mathcal{L}_0$ drops low at the start of the halfstroke but then rises steeply as soon as wing pitching (stroke reversal) has stopped or reduced significantly (see Figure 6.5(a)).

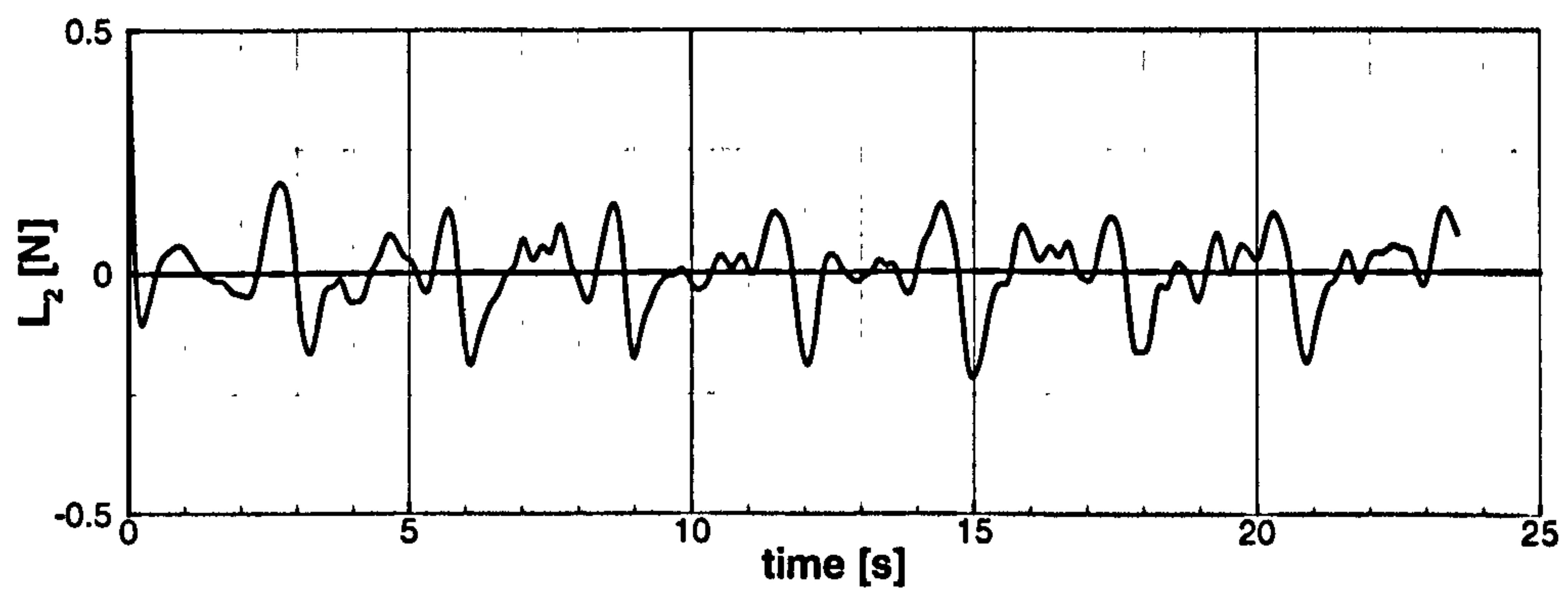
During the translational phase of the halfstroke, angle of attack is constant and so is translational velocity. Hence, the quasi-steady lift reaches a constant value during mid-stroke (Figure 6.5(a)). As the wing comes to the end of the halfstroke, stroke reversal begins in preparation for the subsequent halfstroke. The wing angle of attack increases which results in an increase in quasi-steady lift $\mathcal{L}_0$. However, at about the same time, the translational velocity approaches zero, reducing quasi-steady lift as it does so. Therefore, the steep increase in $\mathcal{L}_0$ due to the increasing angle of attack is immediately followed by a



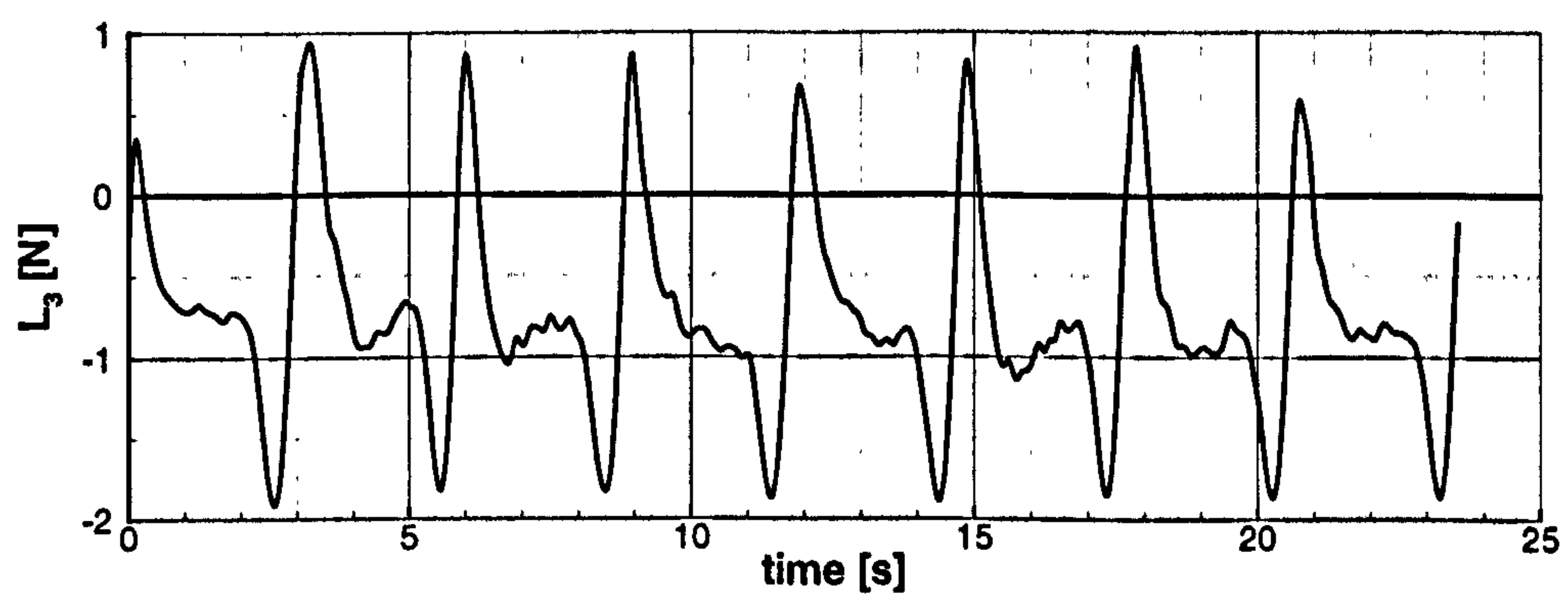
(a)



(b)



(c)



(d)

Figure 6.5: Lift components. Notice that the scales are different for the various components

sharp decrease due to near-zero translational velocity (Figure 6.5(a)). The above process repeats for all halfstrokes.

Apparent Mass Component $\mathcal{L}_1$ The apparent-mass contribution $\mathcal{L}_1$ is a non-circulatory effect, i.e. it is not affected by circulation unlike the other three components. Therefore, it can be inferred by considering the forces arising from the impulsive pressures (see Figure 6.6). Impulsive pressures arise due to inertial accelerations of the fluid surrounding the wing. Therefore, during mid-stroke when both translational and rotational (pitching) velocities are constant, the apparent-mass contribution is zero (see Figure 6.5(b)).

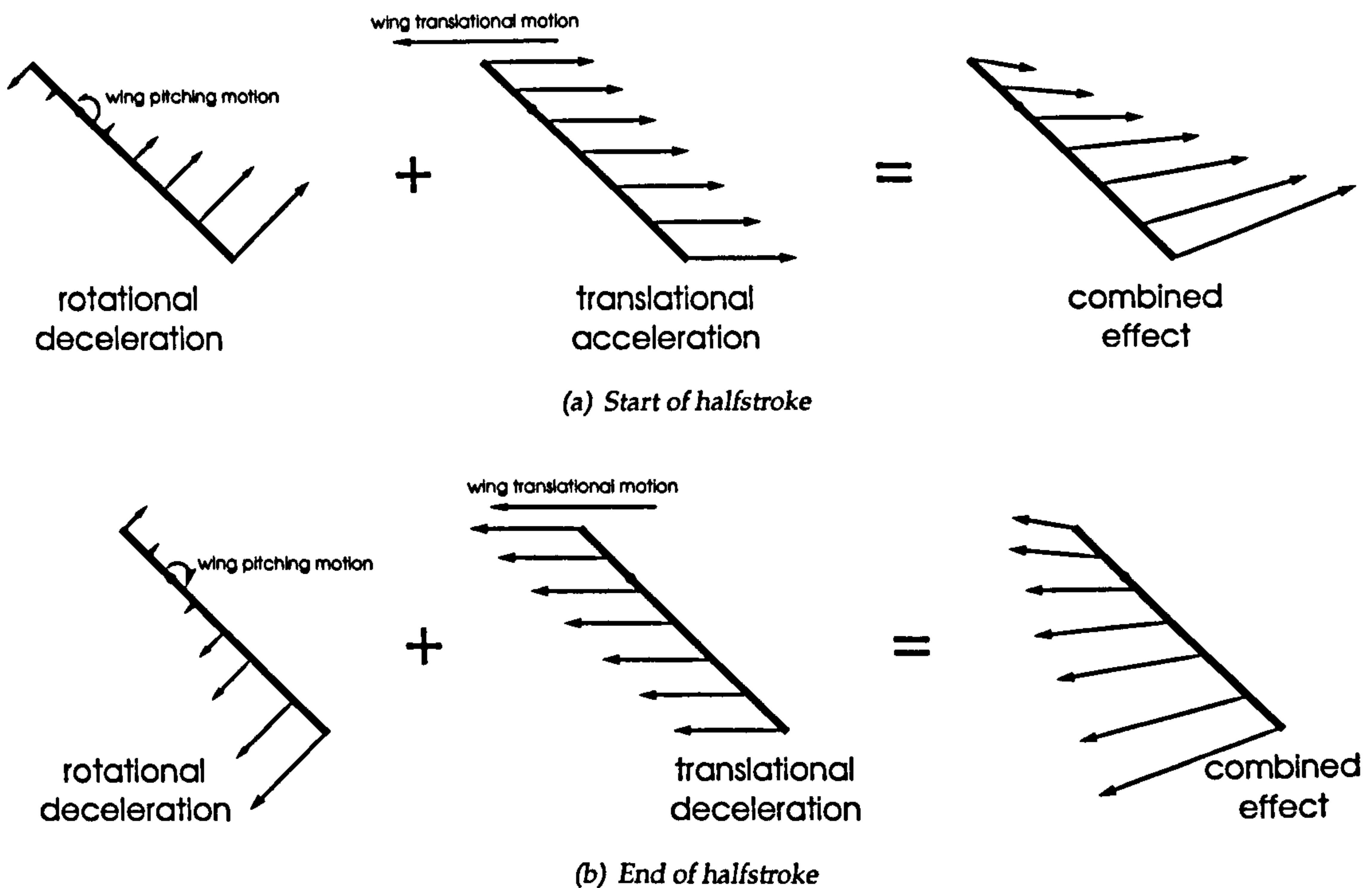


Figure 6.6: Source of apparent-mass forces in terms of impulsive pressure forces. Note that sense of pitching deceleration is reversed at end of halfstroke

At the start of the halfstroke, the rotation due to stroke reversal is coming to an end. This angular deceleration of the wing gives rise to impulsive pressures (and hence forces) that resist this rate of change. These forces have different magnitudes along the wing, depending on the location of the pitch axis (see Figure 6.6(a)). At the same time, the wing begins to accelerate translationally and impulsive forces oppose this motion. (In

Dickinson's kinematics, there is no heaving motion. Hence, the apparent-mass forces due to translational accelerations are horizontal, as shown in Figure 6.6). The overall effect of the two accelerations is that the resultant lift is positive (Figure 6.5(b)).

At the end of halfstroke, the reverse happens (see Figure 6.6(b)). The wing slows to a halt which gives rise to apparent-mass forces that resist this deceleration and act in the direction of positive thrust. Therefore, apparent-mass forces favour further translational motion in this case (because there is no heaving or plunging). At the same time, the stroke-reversal motion that has already begun is slowing down as it approaches the maximum angular velocity. This angular deceleration gives rise to impulsive forces opposite to that generated at the start of the halfstroke (the direction being opposite because the pitching motion is of the opposite sense even though they are both decelerating; see Figure 6.6(b)). The combined effect of the two is a decrease in $\mathcal{L}_1$ at the end of stroke reversal (Figure 6.5(b)).

Wake-Induced Circulation Component $\mathcal{L}_2$ and Wake Circulation Component $\mathcal{L}_3$ According to Kelvin's law, the total circulation in a system must remain constant. Therefore, for any bound circulation created due to quasi-steady effects an equal amount of vorticity must be shed into the wake. In the current model, this is done in the form of two wakes — one shed from the leading edge and the other from the trailing edge. The lift due to the wake $\mathcal{L}_3$, therefore, always lags behind the quasi-steady effect. Also, because quasi-steady bound circulation and circulation shed into the wake originate from the same control volume of fluid, forces due to $\mathcal{L}_3$ are more or less a mirror image of $\mathcal{L}_0$ (cf. Figure 6.5(a) and Figure 6.5(d)).

The presence of the wake upsets the zero-through-flow boundary condition at the wing surface as well as the Kutta-Joukowski condition at the vortex-shedding edges. Therefore, additional bound circulation is 'created' due to the presence of the wake which contributes to the lift on the wing $\mathcal{L}_2$. Therefore, a slight lag exists between $\mathcal{L}_2$ and $\mathcal{L}_3$

similar to that between $\mathcal{L}_3$ and $\mathcal{L}_0$, the latter leading in both cases. Also, $\mathcal{L}_2$ and $\mathcal{L}_3$ are roughly mirror images of each other albeit of different magnitudes (cf. Figure 6.5(c) and Figure 6.5(d)).

Thrust

Recall that thrust $\mathcal{T}$ refers to the horizontal force (perpendicular to gravity) in the direction of translational motion. Thrust can, similarly to lift, be decomposed into four components — a quasi-steady component $\mathcal{T}_0$, an apparent-mass contribution $\mathcal{T}_1$, a wake-induced bound-circulation element $\mathcal{T}_2$ and a wake-induced part $\mathcal{T}_3$. These are each described in detail below with reference to Figure 6.7 below.

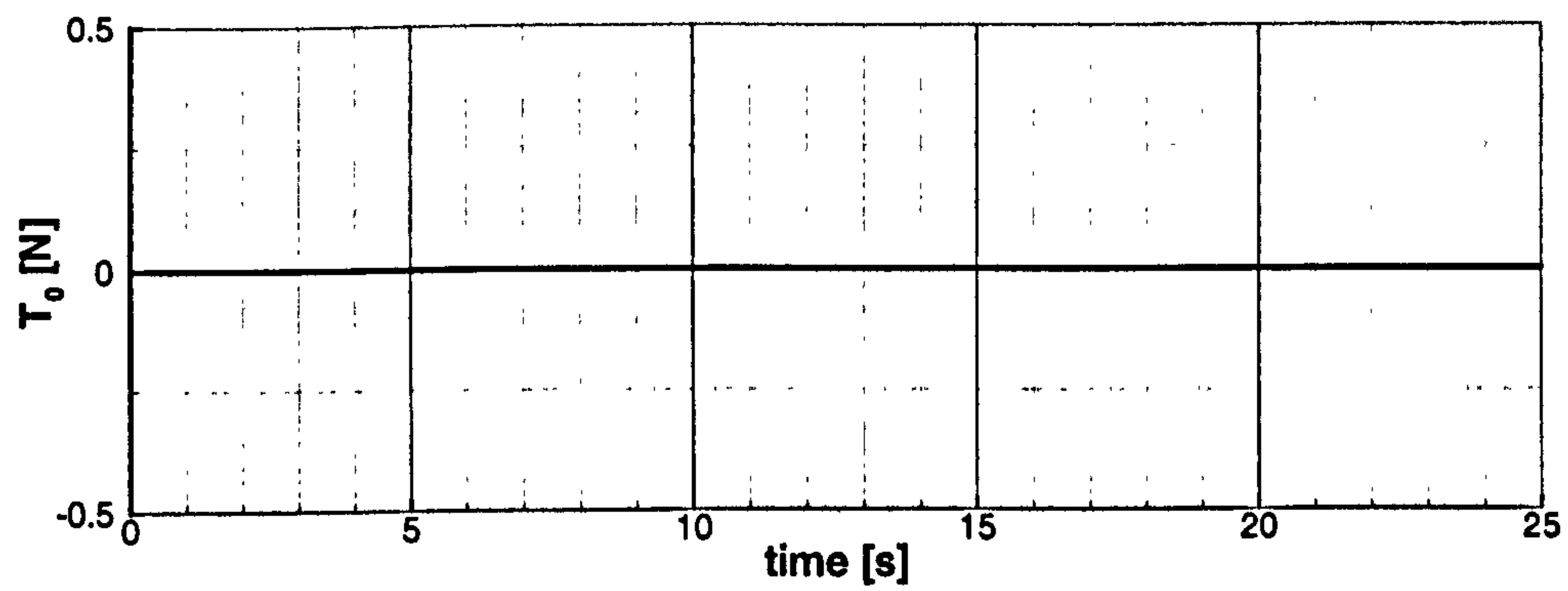
Quasi-Steady Component $\mathcal{T}_0$ Since there is no heave or plunge in Dickinson's kinematics, all incident translational velocity is horizontal. Therefore, in accordance with d'Alembert's paradox¹, the quasi-steady drag contribution (and hence thrust $\mathcal{T}_0$) is zero (Figure 6.7(a)).

Apparent Mass Component $\mathcal{T}_1$ From the earlier discussion on lift due to apparent-mass effects $\mathcal{L}_1$ (p. 196), a similar analysis can be drawn for thrust due to apparent-mass effects $\mathcal{T}_1$. Consider again the scenario depicted in Figure 6.6.

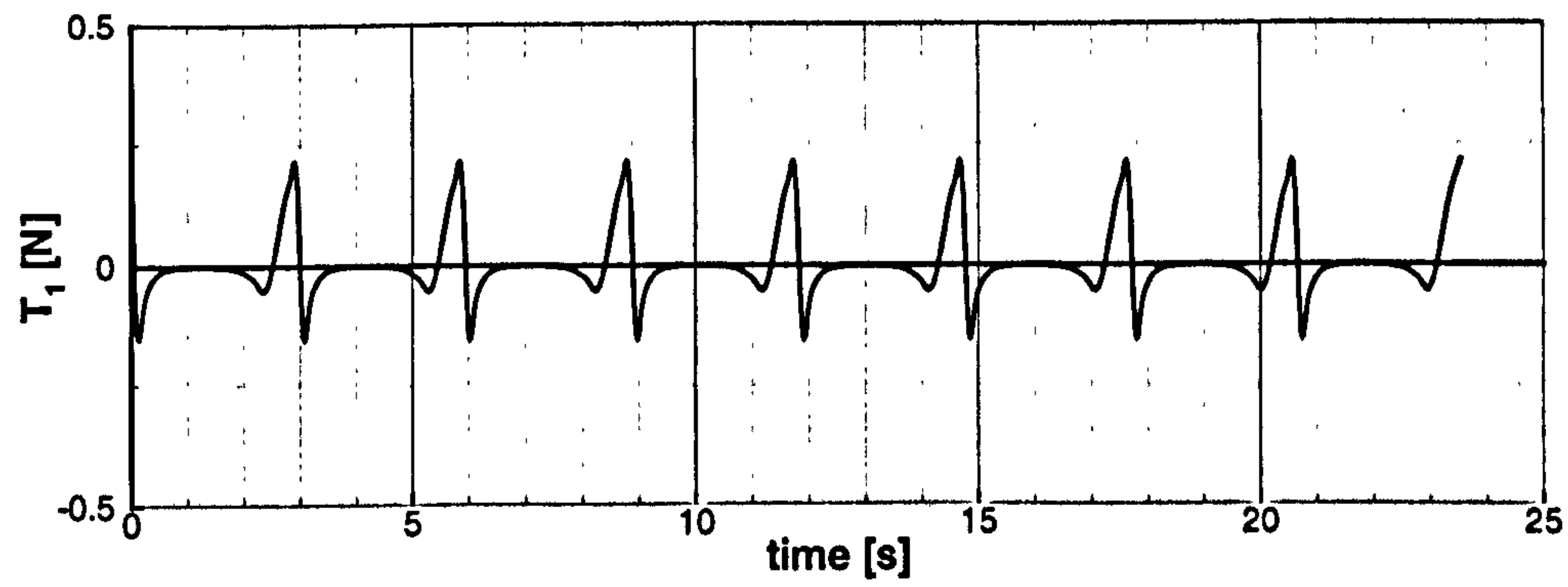
At the start of the halfstroke, the accelerations due to the translational motion of the wing and those due to the rotational motion (pitching) give an overall negative thrust component (Figure 6.6(a)). Hence, the dip in $\mathcal{T}_1$ at the start of the halfstrokes (Figure 6.7(b)). At the end of the halfstroke, the impulsive forces are more or less reversed (Figure 6.6(b)) and hence the peak in $\mathcal{T}_1$.

Just before the end of the halfstroke, a negative dip in $\mathcal{T}_1$ can be seen (see Figure 6.7(b)). This is a consequence of the advanced rotation. When stroke reversal first begins, the

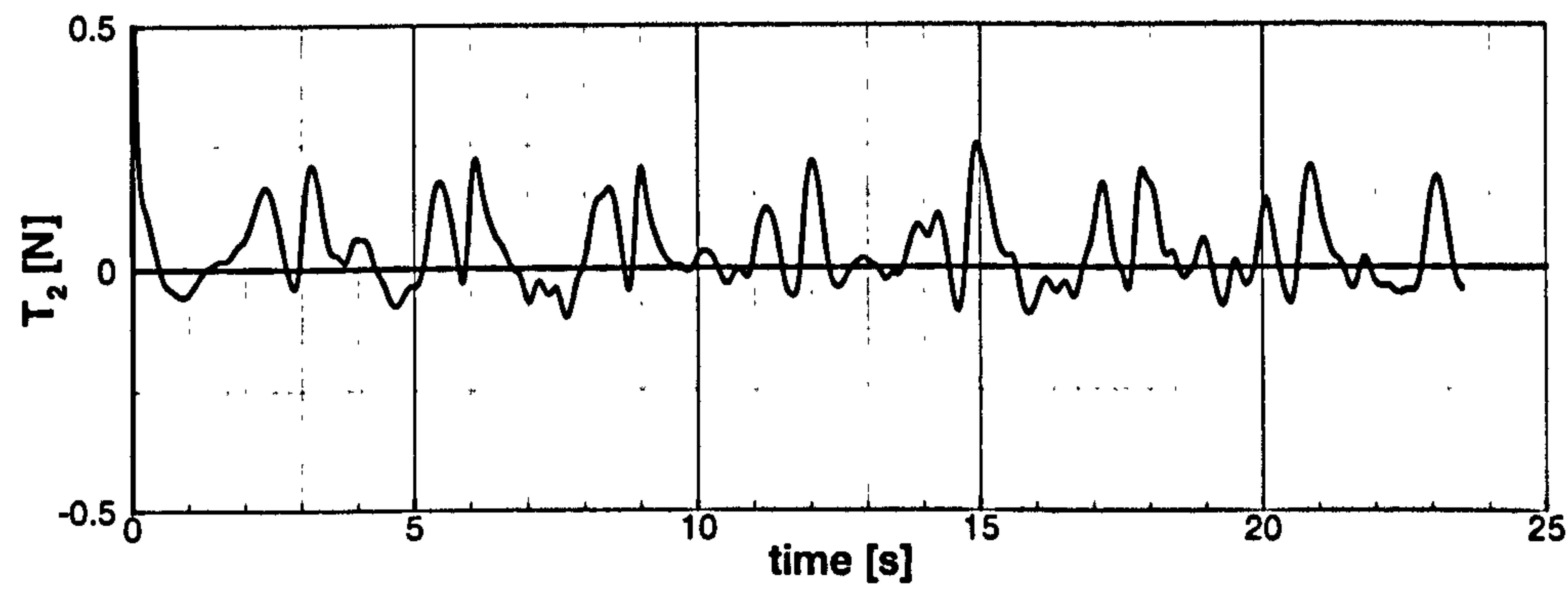
¹After Jean Le Rond d'Alembert (1717–1783) who first identified this 'discrepancy'.



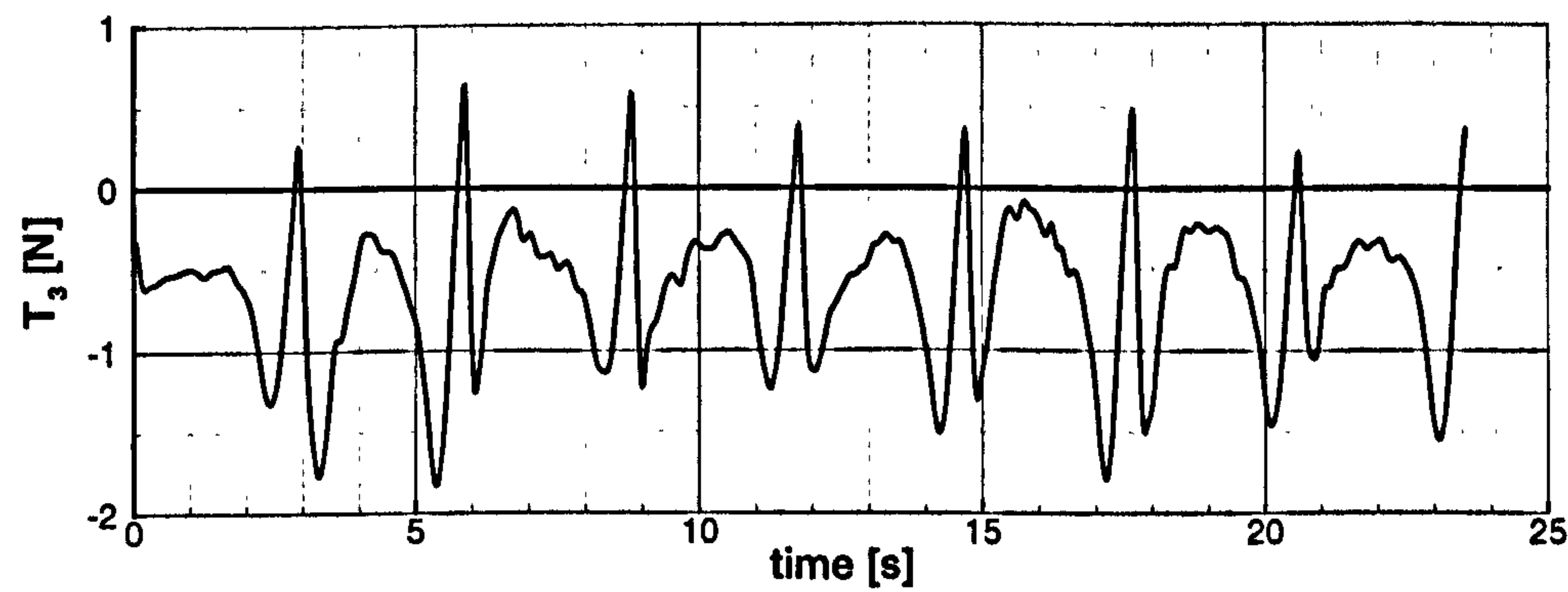
(a)



(b)



(c)



(d)

Figure 6.7: Thrust components. Notice that the scales are different for the various components

impulsive forces due to rotation are opposite to those shown in Figure 6.6(b) as the wing accelerates to the required pitch angular velocity. Because translational deceleration has yet to begin, pitching motion contributes most of the impulsive forces. These are similar to those in Figure 6.6(a), and hence, produce a similar thrust signature to the latter albeit subdued.

Wake-Induced Circulation Component $\mathcal{T}_2$ and Wake Circulation Component $\mathcal{T}_3$ As discussed in terms of lift above (p. 197), a wake is shed in the form of the leading-edge vortex and the trailing-edge wake while satisfying Kelvin's theorem. In addition to the lift component of this shed wake, a thrust component $\mathcal{T}_3$ is inevitably generated. Similarly, the wake-induced bound circulation created to correct for the wake effects also affects the thrust on the system by contributing a component $\mathcal{T}_2$.

As observed for lift above, $\mathcal{T}_2$ lags slightly behind $\mathcal{T}_3$. Also, the time-histories of $\mathcal{T}_2$ and $\mathcal{T}_3$ show that they are roughly mirror images of each other but of different magnitudes (cf. Figure 6.7(c) and Figure 6.7(d)).

It may be noted that (as found in steady flow) the bulk of the thrust (or more appropriately, drag) is produced by the wake (Figure 6.7(d)).

Circulation

Most of the forces described above are circulation-based. Therefore, before proceeding further, it is useful to consider the component circulation histories. These are shown for a typical wing section (section 2 of 10 for the Robofly wing executing Dickinson's kinematics) through one complete cycle (two halfstrokes) in Figure 6.8 below.

As would be expected, the quasi-steady bound circulation Γ_0 closely resembles the quasi-steady lift $\mathcal{L}_0$ (cf. Figure 6.5(a)). Due to the convention used, positive circulation is taken counterclockwise. Hence, after the first stroke reversal when the wing moves from

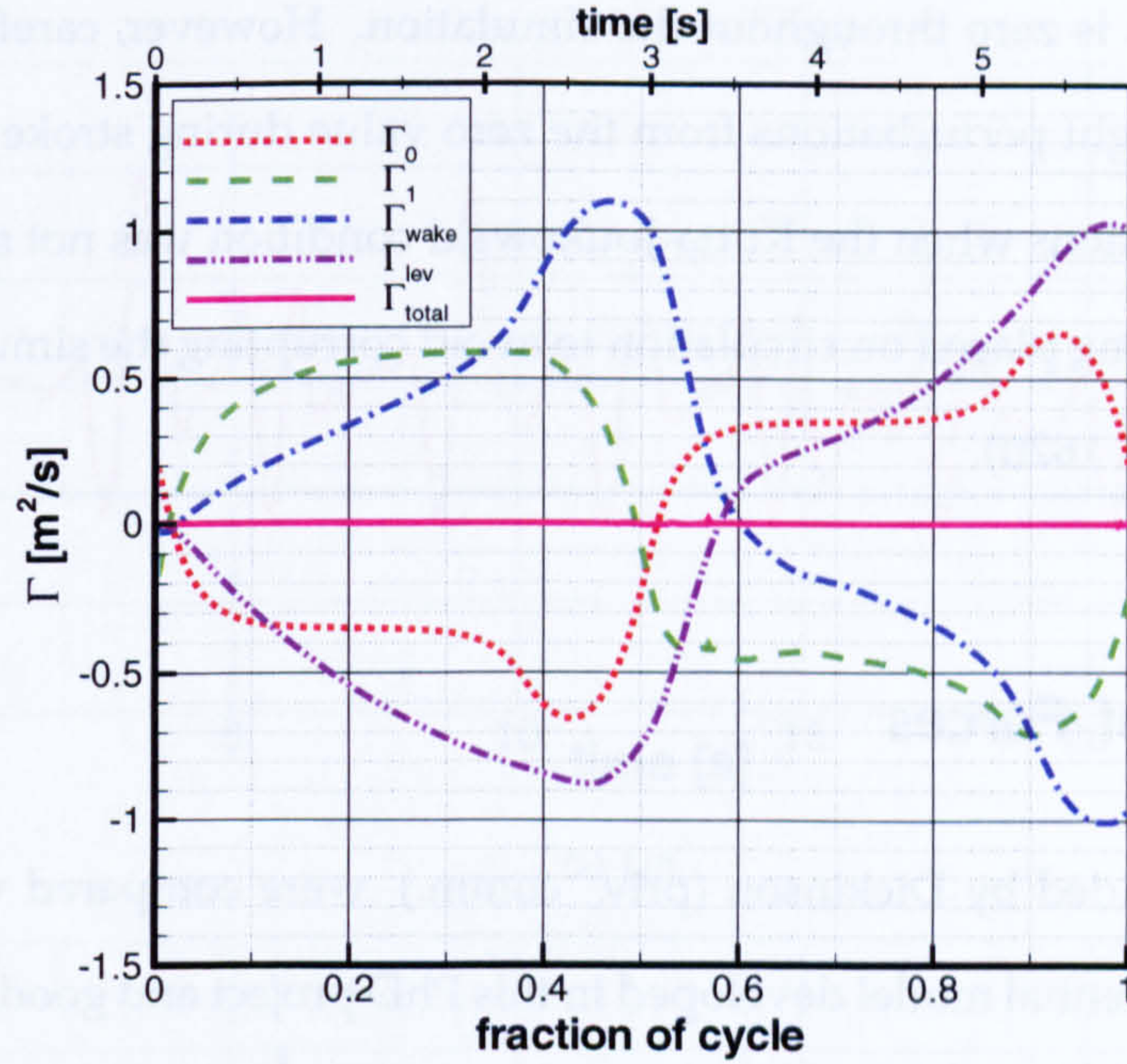


Figure 6.8: Circulation histories for a typical wing section for one flapping cycle. For the trailing-edge wake (denoted wake) and leading-edge vortex (denoted lev), the figure shows total circulation of shed vortices

left to right, Γ_0 is positive.

The circulation shed into the wake in the form of the leading-edge vortex Γ_{lev} and the trailing-edge wake Γ_{wake} are generally of opposite sense to each other and are more or less mirror images. This can be better understood by considering the more extreme case of a flat plate (wing) normal to a freestream. Vortices shed from each edge would roll up in opposite senses and hence have opposite vorticity. Figure 6.8 also shows the lag between the quasi-steady bound circulation and wake circulation (compare, for example, the positions of the peaks at the end of the first halfstroke).

The remaining circulation component is that induced by the wake on the aerofoil Γ_1 . The circulation shed in the leading-edge vortex Γ_{lev} and trailing-edge wake Γ_{wake} more or less balance each other out. The quasi-steady circulation Γ_0 , therefore, remains unbalanced and is balanced by the wake-induced bound circulation Γ_1 . For this reason, the latter generally opposes Γ_0 and is of similar magnitude (see Figure 6.8).

A plot of total circulation Γ_{total} is also shown in Figure 6.8 as a check. As required

by Kelvin's law, this is zero throughout the simulation. However, careful examination of the plot shows slight perturbations from the zero value during stroke reversals. This represents the conditions when the Kutta-Joukowski condition was not satisfied exactly because of a limit being placed on circulation to avoid corrupting the simulation (see also the discussion on pp. 162ff).

6.1.3 Origin of Forces

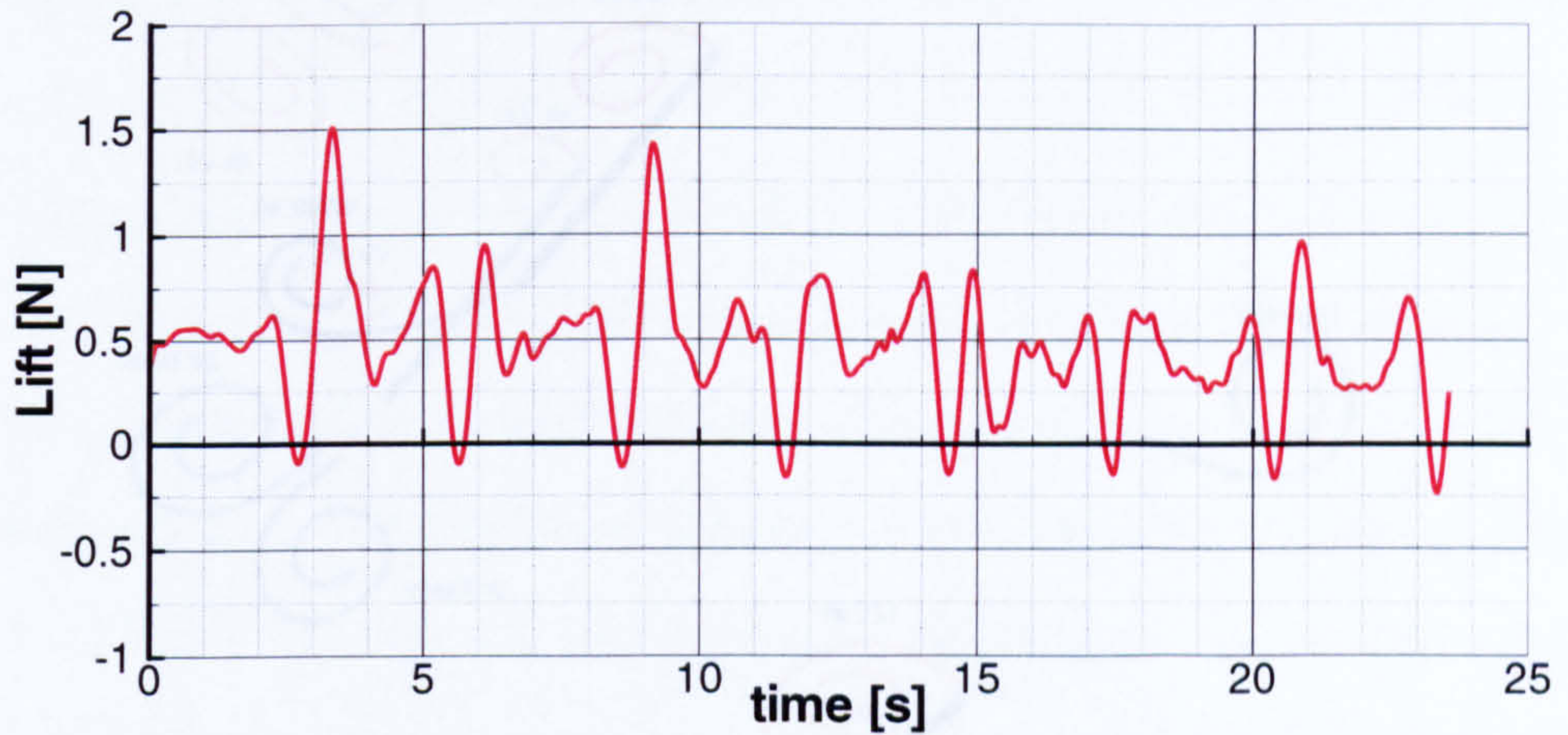
The force data provided by Dickinson (priv. comm.) were compared with the results from the inviscid potential model developed in this PhD project and good agreement was found (see Figure 5.22, p. 184). The lift $\mathcal{L}$ and thrust $\mathcal{T}$ histories result from the particular flow dynamics of the experiment. This section seeks to describe the latter and relate them to the observed forces.

Recall that $\mathcal{L}$ and $\mathcal{T}$ refer to the vertically upward force opposing gravity and the horizontal force in the direction of motion respectively. Data provided by Dickinson (priv. comm.) were limited to these two components only. Flapping frequency was about 0.17 Hz so that each halfstroke lasted about 2.94 s. Consider now the results from the simulation model (reproduced below for completeness in Figure 6.9).

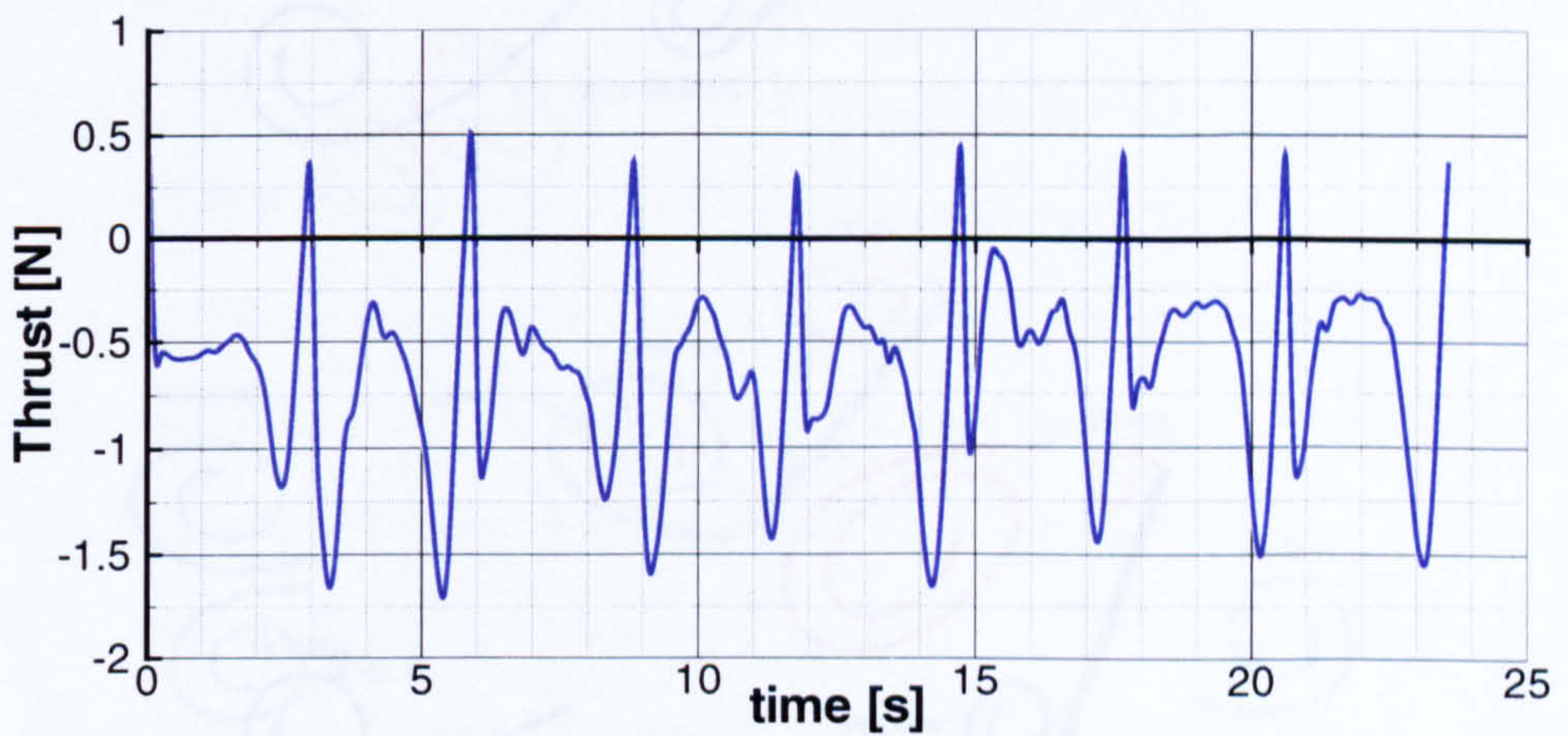
Lift

The following features are prominent from the lift history (Figure 6.9(a)):

- For the first halfstroke, lift grows rapidly from zero, forming a small peak.
- Thereafter, for the rest of the halfstroke the lift remains more or less constant, gradually rising to a second small peak just before stroke reversal.
- At stroke reversal, a steep negative peak is formed, momentarily giving negative lift.



(a) Lift



(b) Thrust

Figure 6.9: Lift (a) and Thrust (b) time histories for simulated Robofly wing executing Dickinson's kinematics

- The negative peak is immediately followed by positive spike at the start of the following halfstroke.
- After the spike, the lift then grows steadily through the translational phase of the halfstroke, peaking again towards the end.
- Very much like the previous halfstroke, this is followed by a sharp negative peak and then a steep positive spike, and the process repeats.

Since most of the lift contribution comes from wing section 8 (out of 10), in the description that follows the flow field described refers to that in this section and is shown

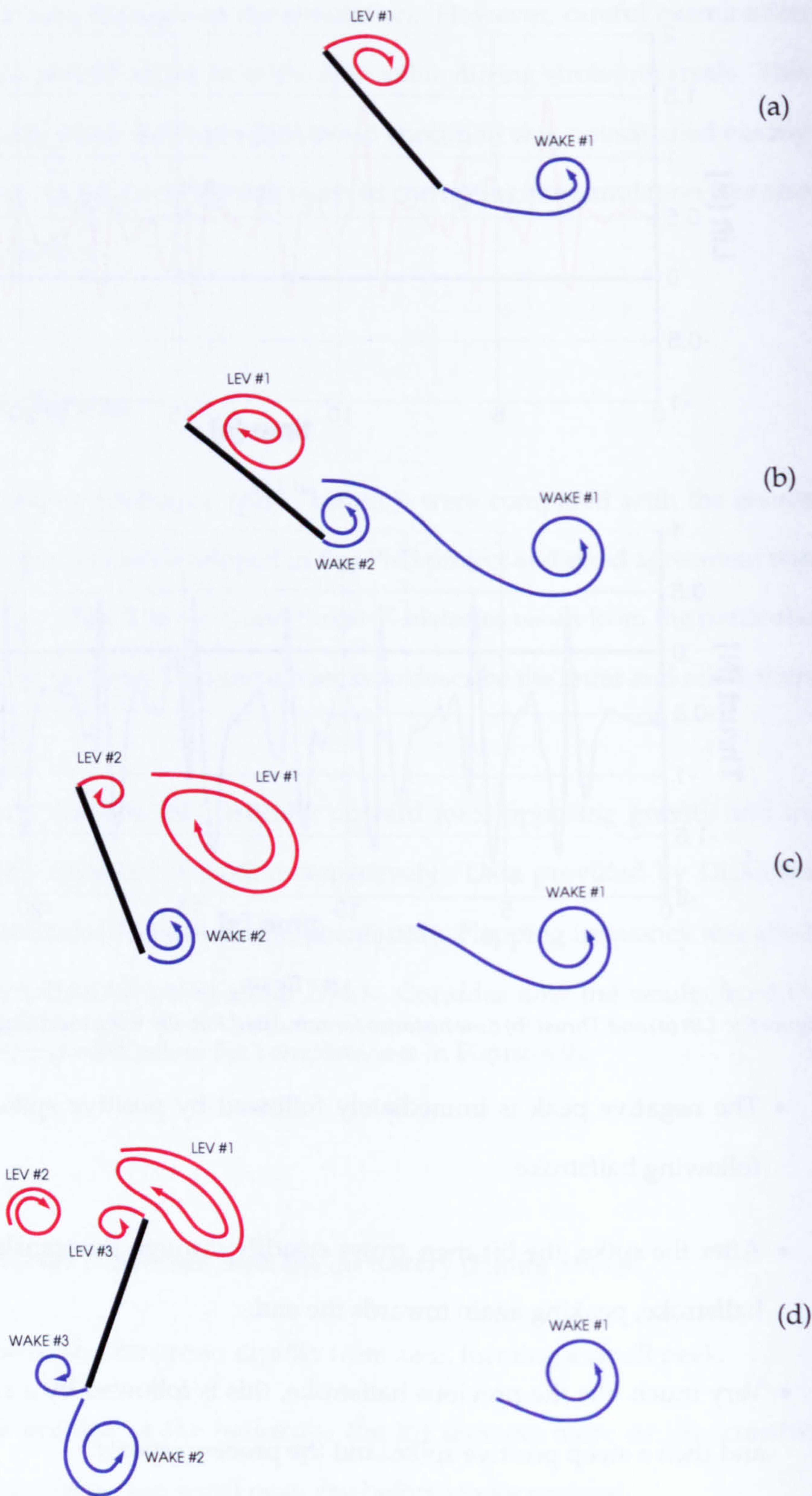


Figure 6.10: Simplified version of flow field at different times through a flapping cycle. Colour scheme is red for leading-edge vortex and blue for trailing-edge wake

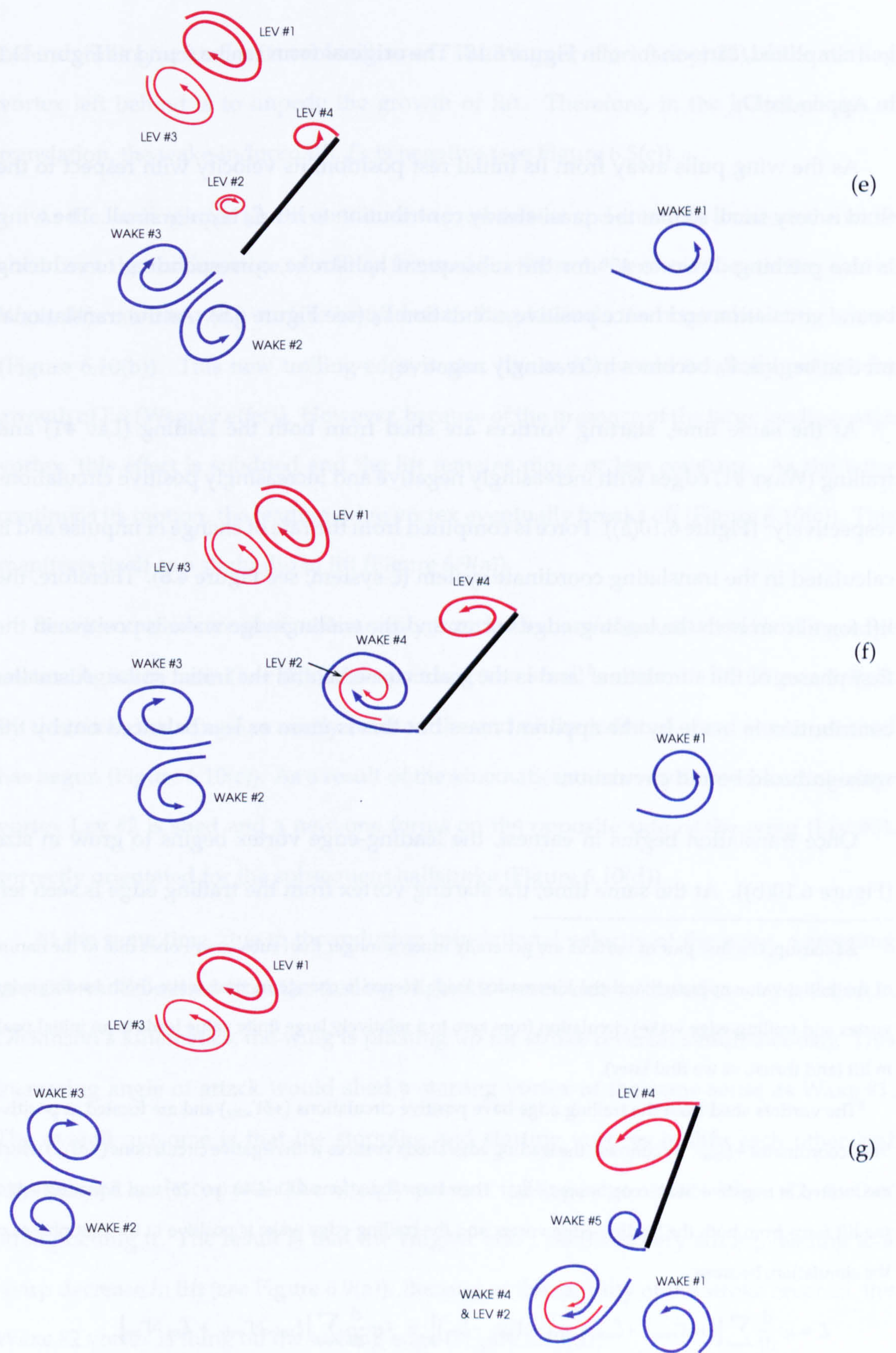


Figure 6.10: Continued. Colour scheme is red for leading-edge vortex and blue for trailing-edge wake

in a simplified, cartoon form in Figure 6.10. The original form can be found as Figure D.1 in Appendix D.

As the wing pulls away from its initial rest position, its velocity with respect to the fluid is very small so that the quasi-steady contribution to lift $\mathcal{L}_0$ is quite small. The wing is also pitching down to 45° for the subsequent halfstroke, corresponding to reducing bound circulation and hence positive circulation Γ_0 (see Figure 6.8). As the translational motion begins, Γ_0 becomes increasingly negative.

At the same time, starting vortices are shed from both the leading (LEV #1) and trailing (WAKE #1) edges with increasingly negative and increasingly positive circulations respectively² (Figure 6.10(a)). Force is computed from the rate of change of impulse and is calculated in the translating coordinate system ($\hat{\zeta}$ -system; see Figure 4.6). Therefore, the lift force from both the leading-edge vortex and the trailing-edge wake is positive in the first phases of the simulation³ and is the main cause behind the initial spike. A smaller contribution is made by the apparent mass but this is more or less balanced out by the wake-induced bound circulation.

Once translation begins in earnest, the leading-edge vortex begins to grow in size (Figure 6.10(b)). At the same time, the starting vortex from the trailing edge is seen left

²At startup, the first pair of vortices are generally much stronger than subsequent ones due to the nature of the initial-value approach and the kinematics used. Hence, a change in shed wake (both leading-edge vortex and trailing-edge wake) circulation from zero to a relatively large finite value leads to an initial peak in lift (and thrust, as we find later).

³The vortices shed from the trailing edge have positive circulations ($+\delta\Gamma_{wake}$) and are located at positive ‘real’ coordinates $+\hat{\xi}_{wake}$. By contrast, the leading edge sheds vortices with negative circulations ($-\delta\Gamma_{lev}$) which are located at negative ‘real’ coordinates $-\hat{\xi}_{lev}$. Then from Equations 4.41–4.43 (p. 126) and Equation 4.47d, the lift force from both the leading-edge vortex and the trailing-edge wake is positive in the first phases of the simulation, because

$$\mathcal{L} = \rho \frac{d}{dt} \sum \left[\left(+\delta\Gamma_{wake} \cdot +\hat{\xi}_{wake} \right) + \left(-\delta\Gamma_{lev} \cdot -\hat{\xi}_{lev} \right) \right] = +\rho \frac{d}{dt} \sum \left[\hat{\xi}_{wake} \delta\Gamma_{wake} + \hat{\xi}_{lev} \delta\Gamma_{lev} \right]$$

behind, rolling up under its own influence. The Wagner effect (see p. 22) of the starting vortex left behind is to impede the growth of lift. Therefore, in the initial phases of translation, the wake-induced lift $\mathcal{L}_2$ is negative (see Figure 6.5(c)).

As the leading-edge vortex continues to grow in size and strength, it causes the wake shed from the trailing edge to roll up forward (in the same sense as the starting vortex WAKE #1) onto the wing surface instead of trailing towards the original starting vortex (Figure 6.10(b)). This new trailing-edge vortex (WAKE #2) would normally restrict the growth of lift (Wagner effect). However, because of the presence of the large leading-edge vortex, this effect is subdued and the lift remains more or less constant. As the wing continues its motion, the leading-edge vortex eventually breaks off (Figure 6.10(c)). This manifests itself as a slight dip in lift (Figure 6.9(a)).

Because vortices are shed continuously from both sharp edges, a second leading-edge vortex begins to form (LEV #2), corresponding to a small increase in lift (Figure 6.9(a)). By this time, the wing has nearly reached the end of the halfstroke and stroke reversal has begun (Figure 6.10(c)). As a result of the kinematics, the newly-formed leading-edge vortex LEV #2 is shed and a new one forms on the opposite side of the wing (LEV #3), correctly orientated for the subsequent halfstroke (Figure 6.10(d)).

At the same time, due to the reducing translational velocity of the wing, a stopping vortex would be shed from the trailing edge. However, due to the advanced rotation in Dickinson's kinematics, the wing is pitching up for stroke reversal simultaneously. This increasing angle of attack would shed a starting vortex of the same sense as WAKE #1. The overall outcome is that the stopping and starting vortices nullify each other and any balance coalesces with the already-present trailing-edge vortex WAKE #2, further strengthening it. The result is that the Wagner effect becomes very strong, leading to a sharp decrease in lift (see Figure 6.9(a)). Because of the rapidity of the stroke reversal, the WAKE #2 vortex is flung off the trailing edge (Figure 6.10(d)).

The shedding of WAKE #2 vortex, however, does not immediately produce high lift because the wing has begun motion in the opposite direction so that the upper surface of the wing has switched sides. The translational motion gives rise to a starting vortex WAKE #3 of the opposite sense to WAKE #1 and WAKE #2 but of the correct sense to impede the growth of lift. This further ensures that lift drops, becoming negative momentarily.

The continuing stroke reversal means that the wing is pitching nose-down towards a 45° angle of attack. This produces a stopping vortex that partially neutralises the WAKE #3 vortex. Due to the severity of the stroke reversal, both trailing-edge vortices WAKE #2 and WAKE #3 are flung away from the wing (Figure 6.10(e)). Additionally, their opposite senses ensure that they mutually convect away from the wing. This suddenly removes the resistance to lift growth.

At the same time, the leading edge of the wing passes under the first large leading-edge vortex LEV #1. This causes the newly-formed leading-edge vortex LEV #3 to break off and the two, being of opposite senses, mutually convect behind the wing (Figure 6.10(e)). Both of these effects are responsible for the high positive peak in lift following stroke reversal (Figure 6.9(a)).

Subsequently, as the wing moves, a new vortex each is shed from the leading (LEV #4) and trailing (WAKE #4) edges. Because of its proximity to the latter and because of the induced velocity field, the previously shed vortex LEV #2 coalesces with the latest trailing-edge vortex WAKE #4 (Figure 6.10(f)). The effect of these three vortices is to suddenly but temporarily produce a drop in wing lift (Figure 6.9(a)).

As the wing continues to translate, the trailing-edge vortex WAKE #4 becomes large and breaks off, allowing lift to rise gradually. It joins the very first starting vortex WAKE #1 and because of their opposite signs, the pair start to mutually convect away from the wing, further increasing lift. In the meantime, the leading-edge vortex also becomes large and increases in strength, thereby increasing lift because of the low pressures it creates on the wing's upper surface. This is manifested in the form of a peak just before stroke

reversal.

As the stroke reversal begins, a new vortex (WAKE #5) begins to form at the trailing edge which is further strengthened by the starting vortex due to nose-up pitching (Figure 6.10(g)). A scenario similar to that experienced at the same point during the previous halfstroke results and a rapid drop in lift is seen. From here on, the process more or less repeats itself periodically, albeit becoming more complicated as the number of shed vortices increases.

After the first halfstroke, the flow field becomes 'dirty' or 'messy' as a result of previous wakes littering the path of the wing. They all affect the forces on the wing which become increasingly complicated and even more difficult to explain. Numerous small and large vortex structures fill the domain, each affecting the performance of the wing in different ways and this can be seen in how, in the four complete cycles considered, each halfstroke shows a different force history (Figure 6.9(a)).

Thrust

During the translational phases of the cycle, the lift and thrust forces are almost identical albeit of opposite magnitudes because the wing is inclined at an angle of attack of 45° and the forces generated are pressure-based (a result of the inviscid assumption). In fact, lift and thrust characteristics for the Robofly wing executing Dickinson's kinematics only differ towards the end of the halfstrokes. The relatively small peak observed for lift occurring around stroke reversal is much more pronounced in the thrust history (cf. Figure 6.9(a) and Figure 6.9(b)).

The latter corresponds to the scenario depicted in Figure 6.10(c) where the leading-edge vortex LEV #1 has just broken off and is left behind. Whereas leaving the leading-edge vortex behind has little effect on lift (because ξ remains more or less unchanged), it has a

more severe effect on thrust⁴. Therefore, if the vortex LEV #1 of strength $-\Gamma_{lev}$ located at the ‘imaginary’ coordinate $\hat{\eta}_{lev}$ is left behind, thrust decreases roughly as $-\Gamma_{lev} \frac{d}{dt} (\hat{\eta}_{lev})$. The effect of the trailing-edge vortex WAKE #2 is minimal because its imaginary-axis coordinate $\hat{\eta}_{wake}$ does not vary much as it moves off the wing. Instead, its real-axis coordinate increases rapidly which is manifested in the lift history (described in the previous section).

6.1.4 Importance of the Leading-Edge Vortex

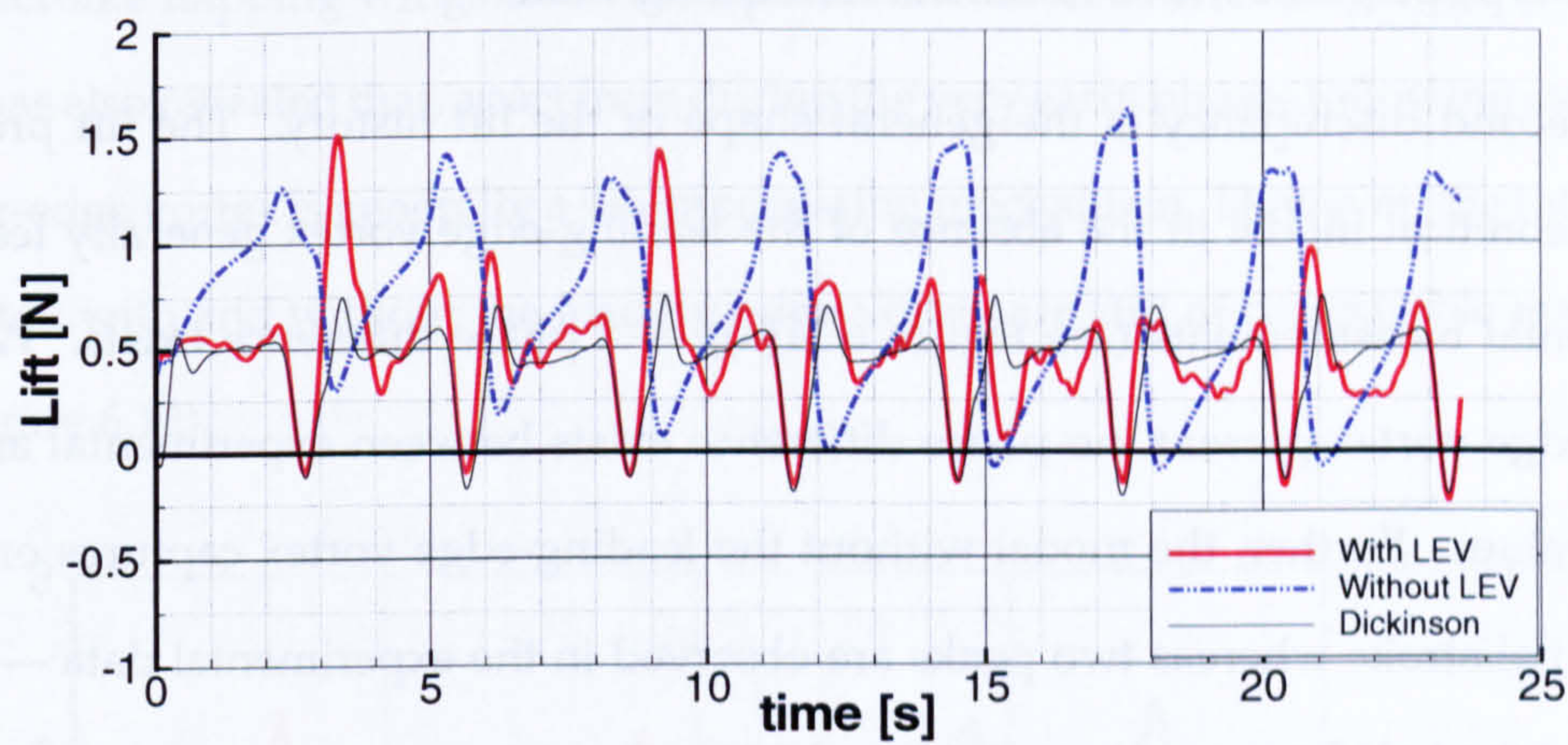
In many engineering applications, the simplest approach is usually not very different from the best approach. Therefore, in view of this, the performance of the wing in the absence of the leading-edge vortex, i.e. simple attached flow, was investigated and the results from the study are shown in Figure 6.11 below. It must be remembered that in this simpler case, in addition to the absence of a leading-edge vortex, the condition for stagnation at the leading edge is not upheld.

Two discrepancies are immediately obvious from the lift plot (Figure 6.11(a)). One is that lift is massively overestimated. Over the entire four-cycle simulation, the mean lift predicted by the theoretical model for the Robofly in the absence of the leading-edge vortex comes to almost double that obtained by Dickinson (priv. comm.) in his experiments. The main resistance to the growth of lift is offered by vortices shed from the trailing-edge wake. Unlike the leading-edge vortices considered earlier, these do not spend much time on the wing surface. Hence, the Wagner effect is much more diminished and lift rises towards the quasi-steady values much quicker. The whole case is essentially

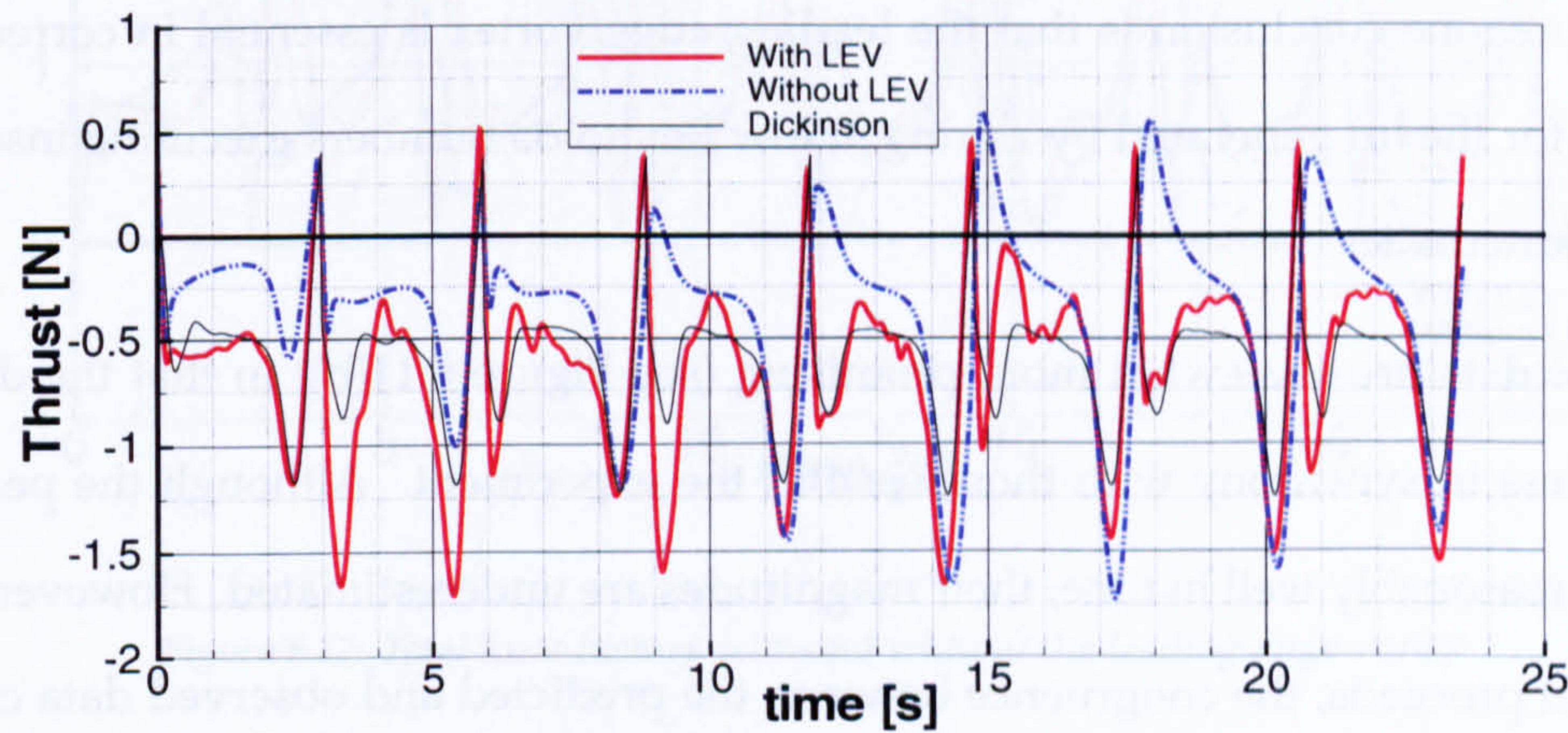
⁴ From Equations 4.41–4.43 (p. 126) and Equation 4.47d, lift and thrust are related such that

$$\mathcal{L} \propto \frac{d}{dt} \sum \xi \delta \Gamma \quad \text{and} \quad \mathcal{T} \propto \frac{d}{dt} \sum \hat{\eta} \delta \Gamma$$

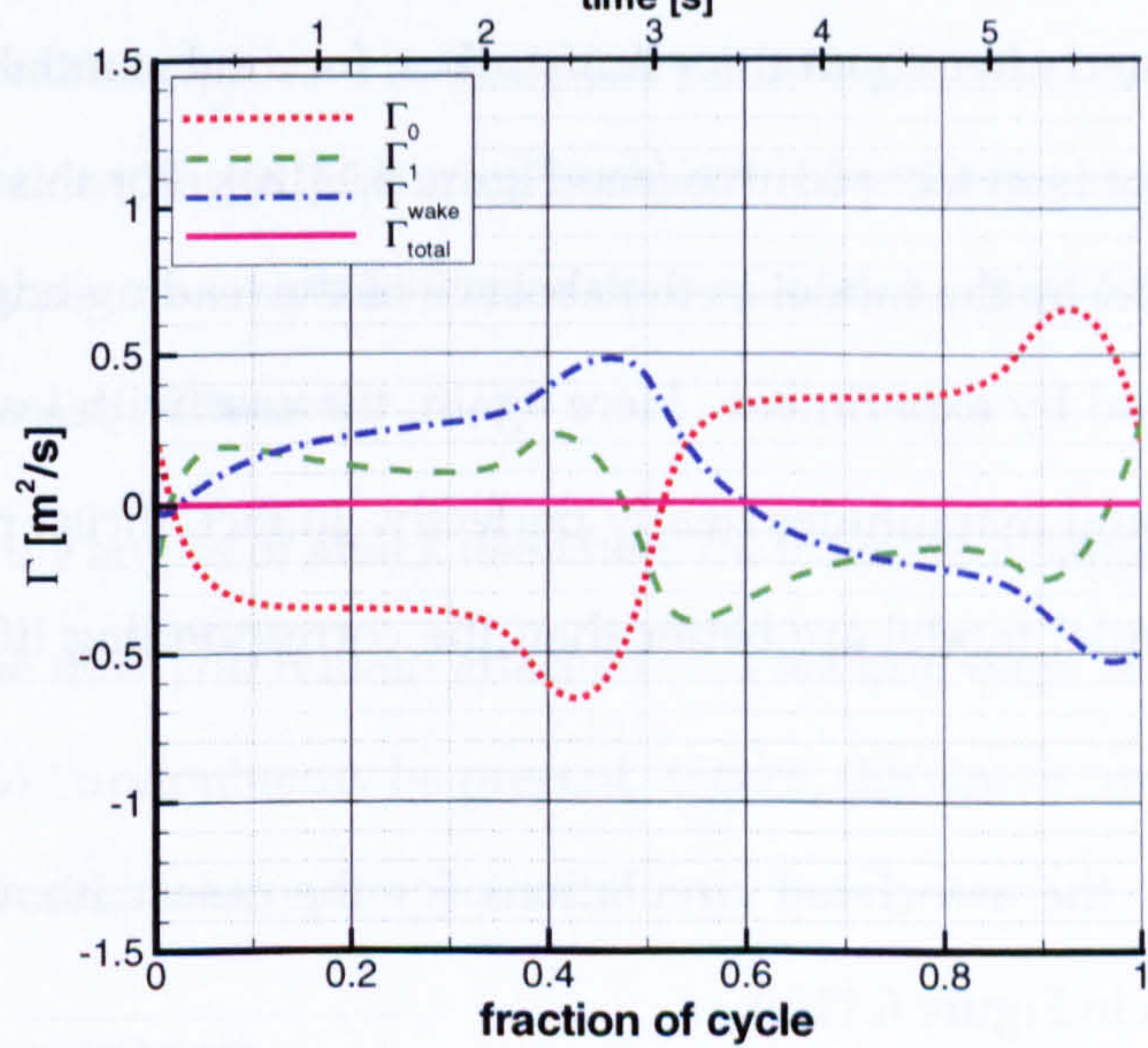
where $\zeta = \xi + i\eta$ represents the location of the vortex. For a vortex left behind in the wake of a highly pitched aerofoil, its $\hat{\eta}$ coordinate increases rapidly and, hence, so does its effect on thrust. By the same argument, greater effect on lift is achieved by a vortex rolling off along the wing (greater change in ξ).



(a) Lift



(b) Thrust
time [s]



(c) Circulation (analogous to and on the same scale as Figure 6.8)

Figure 6.11: Force and circulation histories in the absence of a leading-edge vortex

a Wagner-type problem with a nonlinear deforming wake.

The second discrepancy is the general shape of the lift history. The lift predicted by the theoretical model in the absence of the leading-edge vortex generally leads the experimental measurements (see Figure 6.11(a)) — a phase difference exists. With the leading-edge vortex present, no phase difference exists between experimental and theoretical values. Further, the model without the leading-edge vortex captures only one peak per halfstroke whereas two peaks are observed in the experimental data — one at the start of the halfstroke and one at the end.

The foregone conclusion is that the leading-edge vortex is essential in correctly accounting for the lift generated by a wing at low Reynolds numbers executing insect-like flapping kinematics.

Thrust data are somewhat more promising (see Figure 6.11(b)) in that the data are more or less in synchrony with those seen in the experiment. Although the peaks are captured reasonably well in time, their magnitudes are underestimated. However, as the simulation proceeds, the congruence between the predicted and observed data changes dramatically — the second smaller negative peak following stroke reversal becomes increasingly positive and after about three halfstrokes, for a substantial part of the thrust history, the thrust force is in fact positive (see Figure 6.11(b)). For this reason, the mean value for thrust yielded by the model in the absence of the leading-edge vortex is of the order of half that found by experiment. Here again, the case with leading-edge vortex predicts the position and magnitudes nearly perfectly. In fact, thrust predictions by the current inviscid potential model are better than the corresponding lift predictions (see Table 5.4).

For completeness, the associated circulations for the case without a leading-edge vortex are also shown in Figure 6.11(c).

This brief investigation has, therefore, proved beyond doubt that any suitable model

for insect-like flapping wings must incorporate the effect of a leading-edge vortex. The study has also revealed that, apart from during the very early phases following startup, the leading-edge vortex is generally a lift-suppressing mechanism. However, the total forces⁵ generated with and without the leading-edge vortex are still of comparable magnitudes (see Figure 6.12).

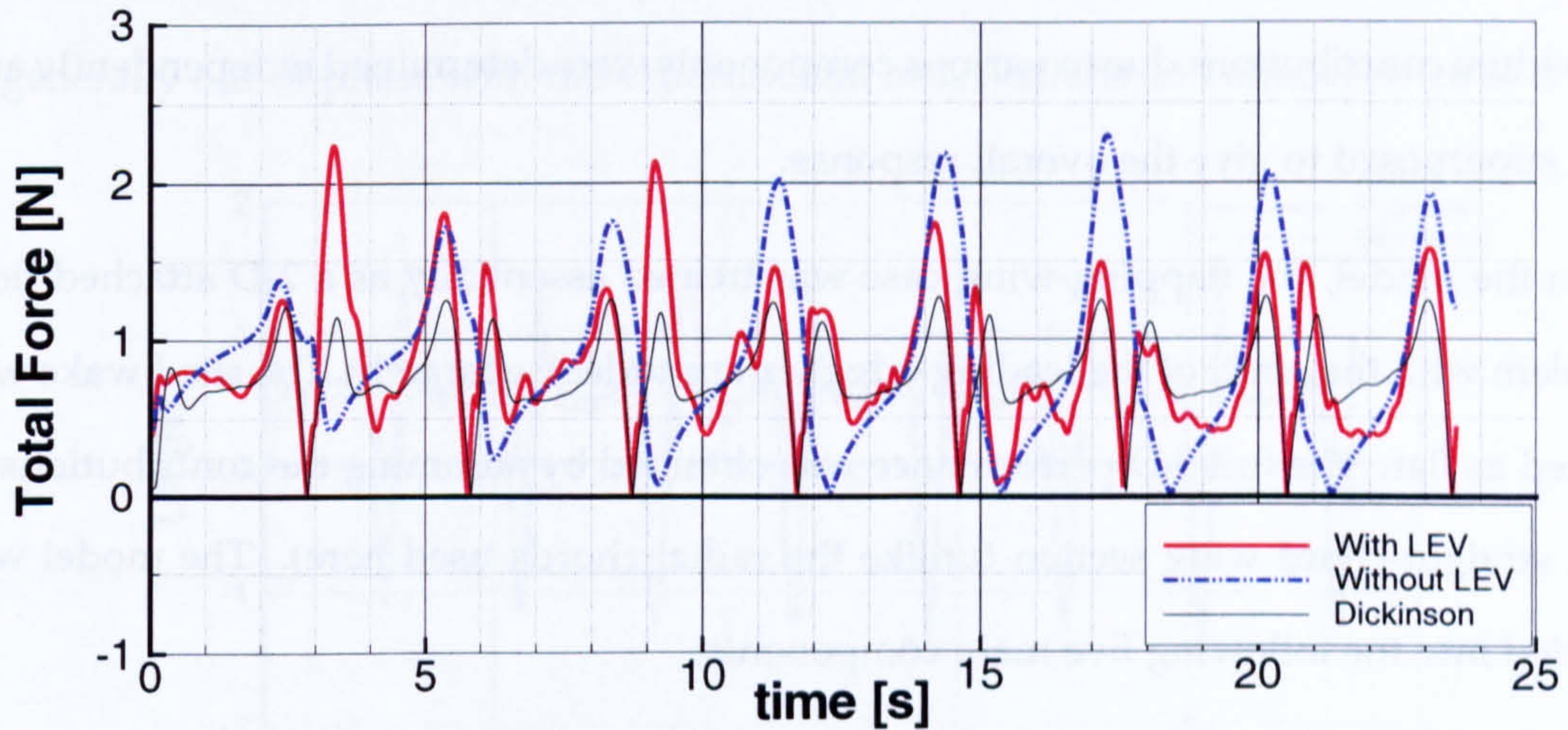


Figure 6.12: Total force history with and without the leading-edge vortex

Again the agreement between theory and experiment is much better with the leading-edge vortex present than without it. The peak following stroke reversal is essentially not captured when the leading-edge vortex is absent. The discrepancy in mean total force over the entire simulation is about 10% for the case with leading-edge vortex present and about 15% for the case without it.

In practice, at the angles of attack used here for insect-like flapping wings ($\alpha > 45^\circ$), it is unlikely that the flow will remain attached — a leading-edge vortex (or some form of separated flow) will undoubtedly be present. Hence, the above comparison is only valid from a theoretical comparison perspective.

⁵Total Force = $\sqrt{\text{Lift}^2 + \text{Thrust}^2}$

6.1.5 Comparison with the Flapping Wing Model of Pedersen (2003)

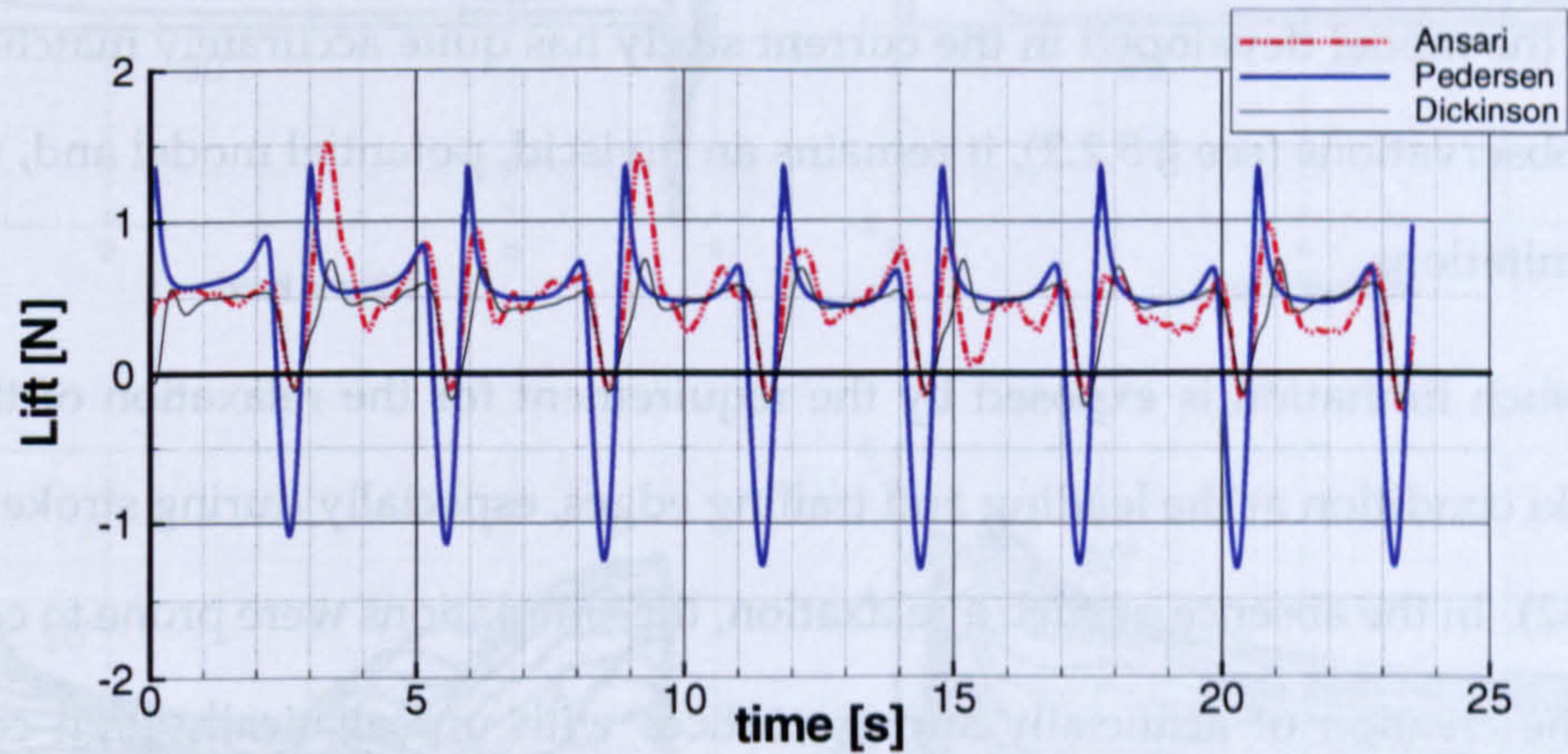
Pedersen (2003) presented a velocity-potential-based approach for modelling the aerodynamic performance of insect-like flapping wings and validated it against the same set of data provided by Dickinson (priv. comm.) as used here. The model was modular so that individual contributions due to various components were determined independently and then superposed to give the overall response.

In the model, the flapping-wing case was treated essentially as a 2-D attached flow problem with the effect of the leading-edge vortex added separately. The shed wake was treated as flat. The full 3-D performance was obtained by summing the contributions of each straight-chord wing section (unlike the radial chords used here). The model was divided into the following five main components:

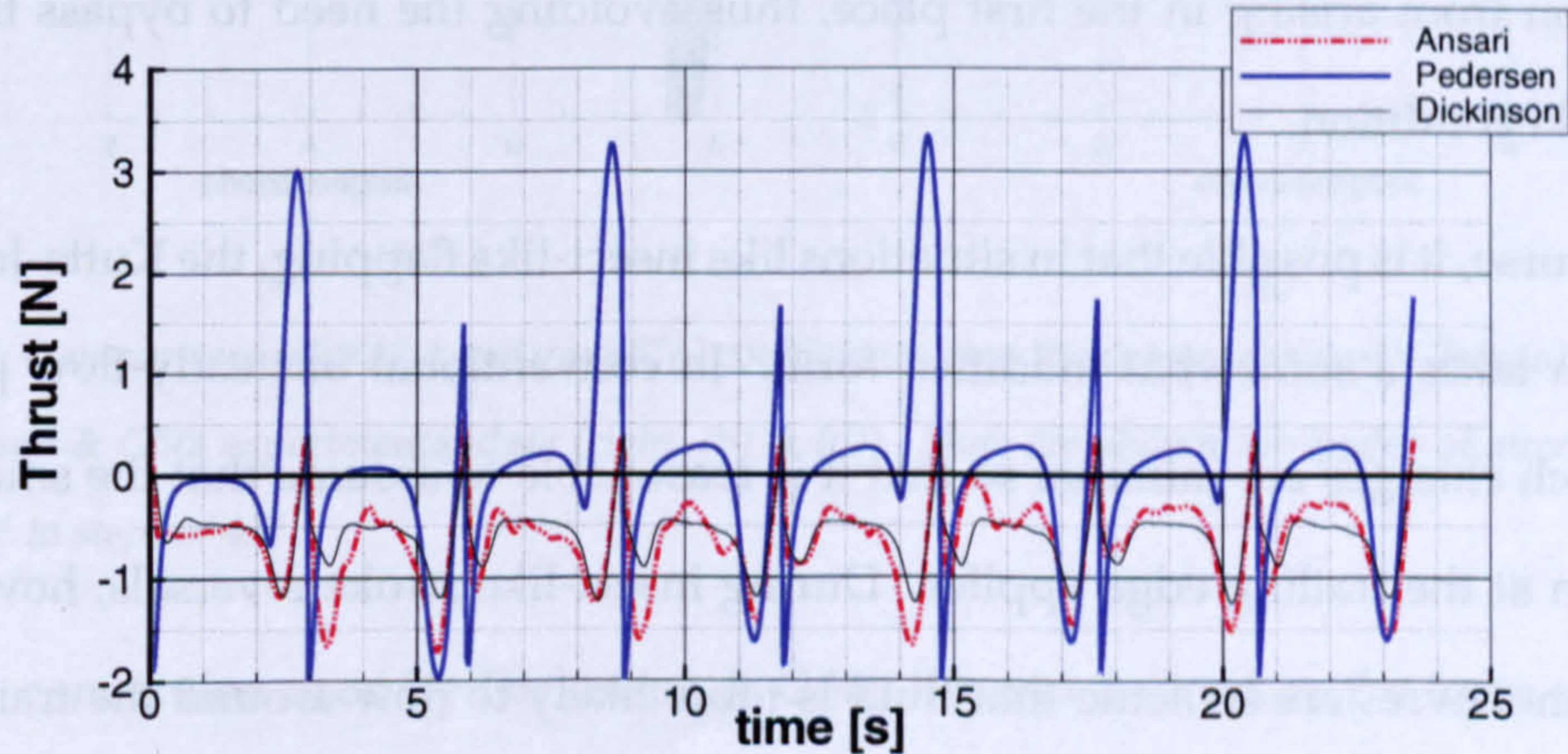
- quasi-steady
- apparent mass
- Wagner
- Küssner
- Polhamus

The quasi-steady component was analogous to the wake-free contribution derived in the current work from the Kutta-Joukowski circulation Γ_0 . Apparent mass was again similar to the equivalent component derived in the current work, although Pedersen applied a 'Dirichlet' correction to reduce the extremity of the peaks. The Wagner component referred to the classical Wagner response of the aerofoil to changes in angle of attack and translational velocity as the wing flapped. This was computed by means of the Duhamel integral (see von Kármán & Biot (1940, p. 403)). The Küssner component referred to the 'gust' response of the aerofoil to the shed wake from previous flapping cycles. Finally, the Polhamus analogy (see p. 50) was used to add the effect of the leading-edge vortex. Indicial methods were used extensively. Full details can be found in Pedersen (2003).

Despite the simplicity of the method, Pedersen (2003) was able to get reasonably good comparison for lift as shown in Figure 6.13(a) below. In the regions of the plot where the wing motion is mainly translational, good agreement is found between Pedersen’s and Dickinson’s results (and indeed the current work, marked ‘Ansari’). The negative spike towards the end of each halfstroke is also captured well but only in time — the magnitude is overestimated considerably. The positive spike at the start of the halfstrokes, however, is generally out-of-phase with the experimental observations and almost twice as large.



(a) Lift



(b) Thrust

Figure 6.13: Lift and thrust comparison with the indicial-Polhamus model of Pedersen (2003)

Pedersen also gave a comparison for thrust (which he described as drag) but remarked that it was unfair to compare as the agreement was poor. His results are shown in

Figure 6.13(b) for completeness.

The scale used for comparison with Pedersen's work in Figure 6.13 also show how well the predictions from the current work match the experimental observations of Dickinson (priv. comm.).

6.1.6 Limitations of the Current Model

Whereas the model developed in the current study has quite accurately matched experimental observations (see § 5.2.2), it remains an inviscid, potential model and, therefore, has its limitations.

One such limitation is exposed by the requirement for the relaxation of the Kutta-Joukowski condition at the leading and trailing edges, especially during stroke reversals (see p. 162). In the absence of such a relaxation, the simulations were prone to corruption due to the creation of artificially strong vortices with unrealistically high convection velocities. In reality, it is likely that viscosity (and viscous dissipation) prevent such a situation from arising in the first place, thus avoiding the need to bypass the Kutta-Joukowski condition.

Of course, it is possible that in situations like insect-like flapping, the Kutta-Joukowski condition takes a somewhat modified form. In conventional unsteady-flow problems, wing-pitch changes are minimal so that it is reasonable to assume that the smooth-flow condition at the trailing edge applies. During insect-like stroke reversals, however, the pitch manoeuvres are so acute that fluid is more likely to flow around the trailing edge rather than along it. In such cases, the validity of Kutta-Joukowski condition in the conventional sense is, therefore, somewhat dubious (see also Footnote 13 on p. 163). This would provide support for the temporarily relaxed Kutta-Joukowski condition used here. However, experimental evidence is required before such a claim can be substantiated.

Another limitation is brought to light by comparison with force data from the exper-

iments of Dickinson & Götz (1993). Although flow-visualisation results for the instants shown by Dickinson & Götz match almost perfectly (Figure 5.20, p. 181), comparison of force data shows some discrepancy (see Figure 6.14)⁶.

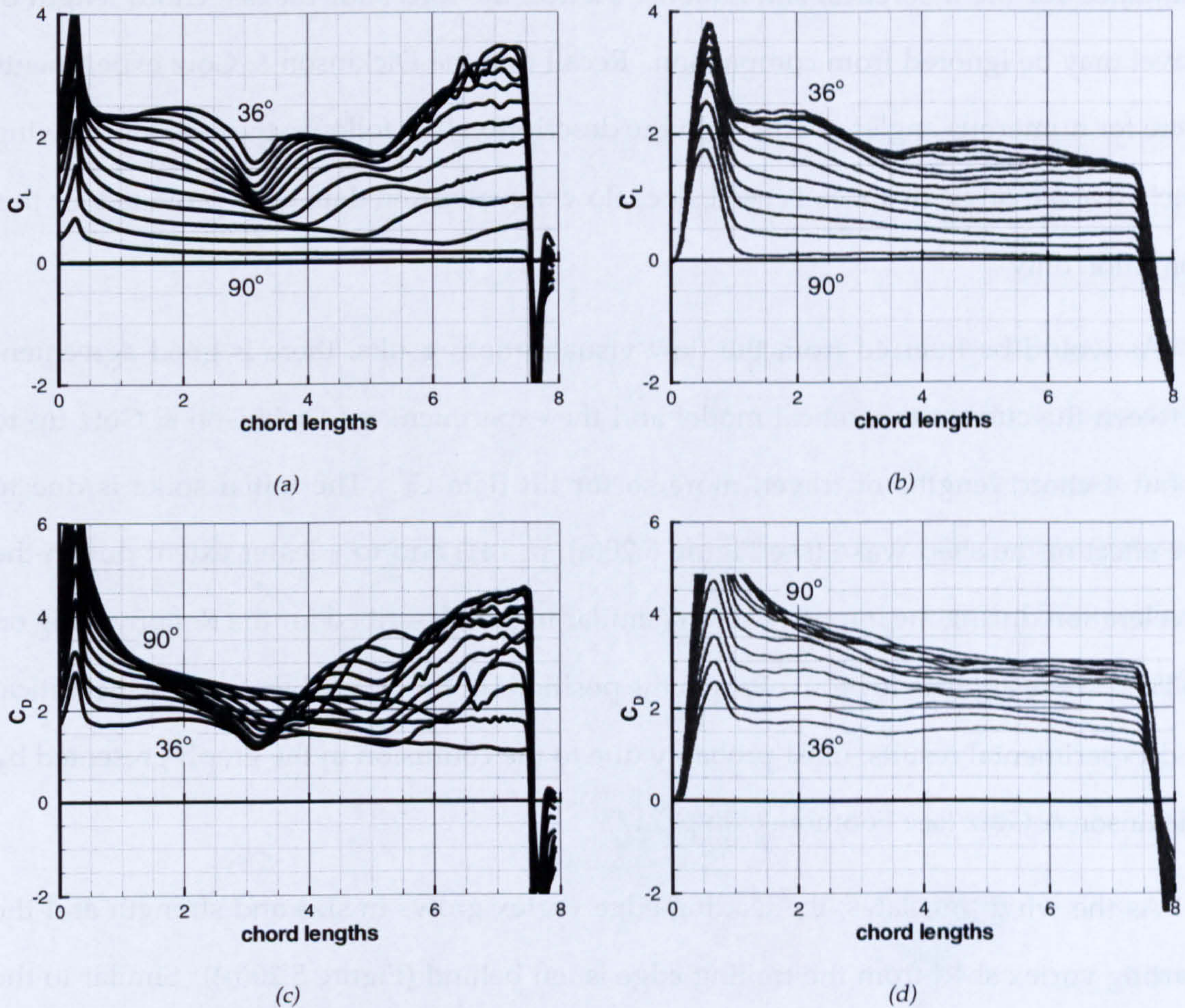


Figure 6.14: Comparison of lift (C_L) and drag (C_D) coefficients from the current approach (left, (a) & (c)) with the Dickinson & Götz experimental data (right, (b) & (d)). Plots are shown for angles of attack from 36° through 90° in steps of 4.5°

In the convention used by Dickinson & Götz, lift was defined as the force normal to the translational motion and drag as the force parallel to it. The coefficients of lift C_L and drag C_D were based on chord length (5 cm) and the constant translational velocity

⁶ The right-hand side figures (Figures 6.14(b) and (d)) are reproduced from Dickinson & Götz (1993) where the scale on the horizontal axis was questionable because it continued to show increase in chord lengths travelled even after motion had stopped. The results from the current work show time on the horizontal axis in terms of chord lengths that would be traversed at the maximum speed (10 cm/s).

(10 cm/s). The Reynolds number for the experiment ($Re = 192$) was similarly based on the same values. The wing was stopped rapidly after translating 7.5 chord lengths. Dickinson & Götz provided no information on the stopping kinematics so they were estimated for the theoretical simulations. Hence, the forces for the last chord length of travel may be ignored from comparison. Recall that the Dickinson & Götz experiments were for numerous angles of attack. In the description that follows, the case for the wing pitched at 45° angle of attack is used since flow-visualisation data were provided for this condition only.

As would be inferred from the flow-visualisation results, there is good agreement between the current theoretical model and the experiments of Dickinson & Götz up to about 4 chord lengths of travel, more so for lift data C_L . The initial spike is due to the effect of the shed wake (see Figure 5.20(a), p. 181) and to a lesser extent due to the acceleration during the impulsive start (similar to that described for the Robofly wing on p. 197). There appears to be an offset in the positions of the spikes between the theoretical and experimental results, most probably due to the confusion in the graph presented by Dickinson & Götz (see Footnote 6 on p. 217).

As the wing translates, the leading-edge vortex grows in size and strength and the starting vortex shed from the trailing edge is left behind (Figure 5.20(b)). Similar to the Robofly wing (discussed earlier), this prevents a classical Wagner response where the lift continues to increase towards the quasi-steady value. Instead, the leading-edge vortex keeps lift in check so that it remains more or less constant after the initial spike.

As the leading-edge vortex grows, its size and strength force flow from the trailing edge to roll-up forward on to the wing surface in the form of a second starting vortex (Figure 5.20(c)). This is manifested by the decline in lift that ensues after about 2.5 chord lengths of travel. This new trailing-edge wake vortex continues to grow in size, reducing lift further. After about 3.5 chord lengths of travel, the leading-edge vortex becomes strong enough to compensate for the effect of the trailing-edge wake vortex and lift begins to

rise. Up to this point, the experimental data are in agreement with the estimates from the current model. From hereon, however, the experimental data show a decrease in lift that continues until the end of the experiment. Since flow-visualisation images were not available for the experiment beyond this point, it is difficult to infer the reason for this steady decline.

The theoretical lift data, however, show a different trend. The rising strength of the trailing-edge vortex distorts the leading-edge vortex (Figure 5.20(d)) which eventually breaks off producing a second trough in lift (see Figure 6.14(a)). Soon after, the growing trailing-edge vortex also breaks off, and joins the leading-edge vortex in the wake. In the mean time, a new leading-edge vortex forms that continues to grow until the end of the experiment. This is reflected in the growing C_L until the experiment concludes. When the wing finally stops, leading- and trailing-edge stopping vortices are shed as shown in Figure 6.15 below. The sudden deceleration to a halt results in a negative spike (similar to that observed upon startup). This again is captured well by the inviscid, potential model.

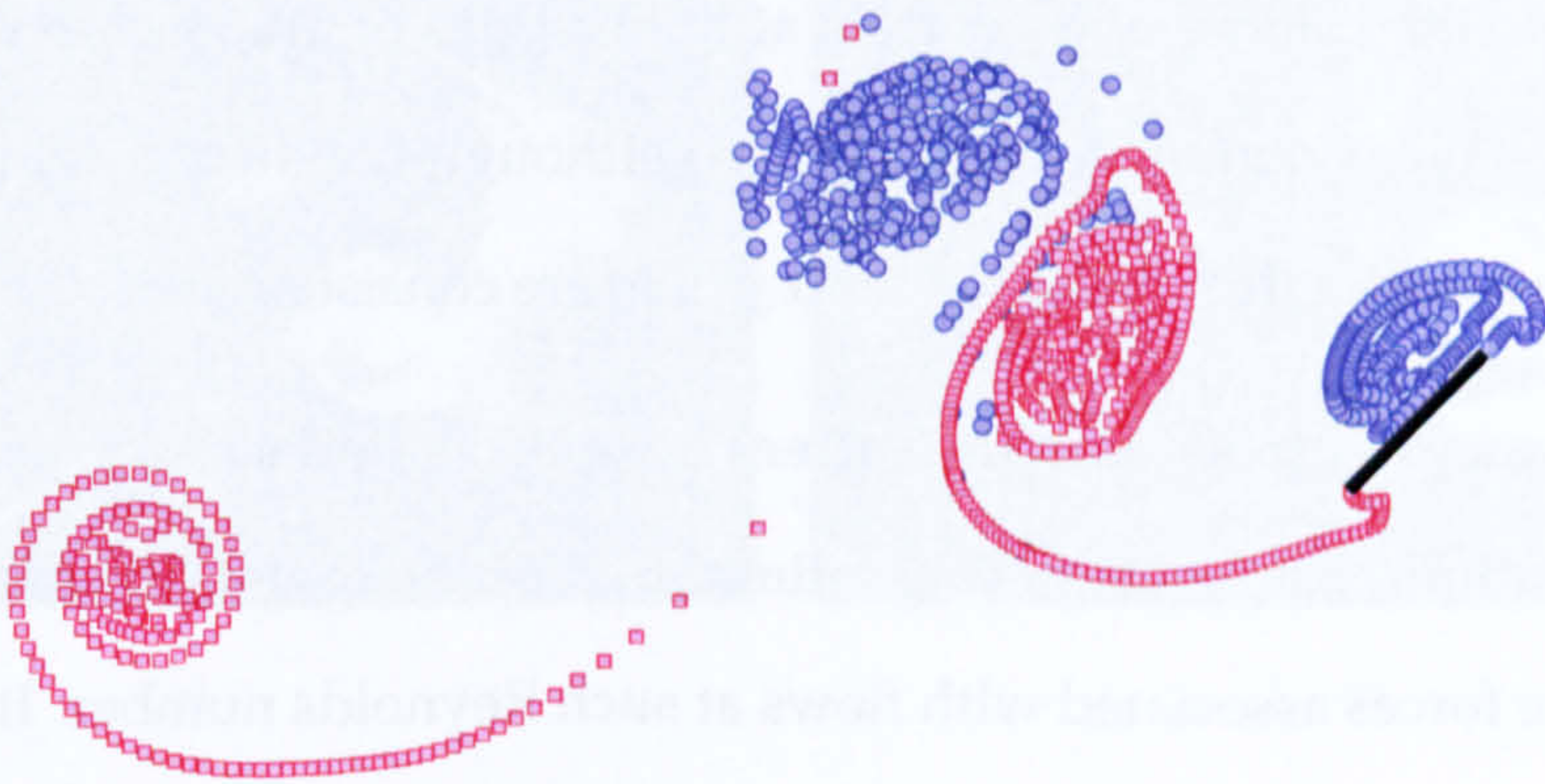


Figure 6.15: Flow field for theoretical model when wing stops

The reason why C_L continues to rise unchecked after about 5 chords of travel is because of the absence of a trailing-edge vortex. Therefore, it would appear that both leading- and trailing-edge vortex 'structures' are required to maintain a more or less constant lift coefficient.

One feature that is captured well by the theoretical model is that maximum lift (over the duration of the simulation) is achieved at an angle of attack of 36° . For larger angles thereafter, lift drops continuously approaching zero at 90° angle of attack — the normal-flat-plate condition⁷. Also, as angle of attack increases, the theoretical model shows that shedding of the first leading-edge vortex is increasingly delayed (Figure 6.14(a)).

Results for drag exhibit greater discrepancy. After the initial peak, the Dickinson & Götz experiment show that drag reduces gradually with chord lengths travelled before terminating in a negative spike (thrust) when the wing is finally brought to rest. Although the initial peak is captured relatively well by the current theoretical model, the drag force for the remainder of the simulation is in poor agreement with experiment. For the first 4 chord lengths of travel, predicted drag (for most pitch angles) is generally less (by about 25%) than that measured by Dickinson & Götz. Thereafter, a rise in drag (similar to that seen for lift) is observed that continues until the wing finally stops.

As explained earlier for the Robofly, the effect of leaving behind the shed leading-edge vortex (and later, the trailing-edge vortex) at high angles of attack corresponds to large changes in thrust (see Footnote 4 on p. 210). So although the theoretical model may not accurately be predicting thrust, the values it yields are consistent with the flow pattern.

The discrepancy in lift between the current theoretical model and experiment beyond the 4-chord position could be exposing a limitation of the inviscid model in accurately representing the forces associated with flows at such Reynolds number. It would appear that after shedding the first pair of (leading- and trailing-edge) vortices, the second pair somehow take much longer to shed.

Inconsistencies in the measurements of Dickinson & Götz cannot be discounted altogether either. The fact that the results for the Robofly (which were taken almost 10 years

⁷This is in contrast to the recent work by Pullin & Wang (2004) who found from their viscous CFD study that maximum lift was achieved at an angle of attack of about 52° . Results from the Dickinson & Götz experiment, therefore, call into question their findings.

later and at a similar Reynolds number of $Re = 160$) have compared much better could imply that the quality of measurements have improved.

This quality of the Dickinson & Götz measurements is further thrown into question when comparison is made with flow-visualisation experiments of Peter Freymouth (Gustafson & Sethian, 1991, ch. 4). Freymouth used smoke to visualise the flow around an impulsively started aerofoil at an angle of attack of 50° and flow Reynolds number of 1000 (based on chord). Some results are shown in Figure 6.16.

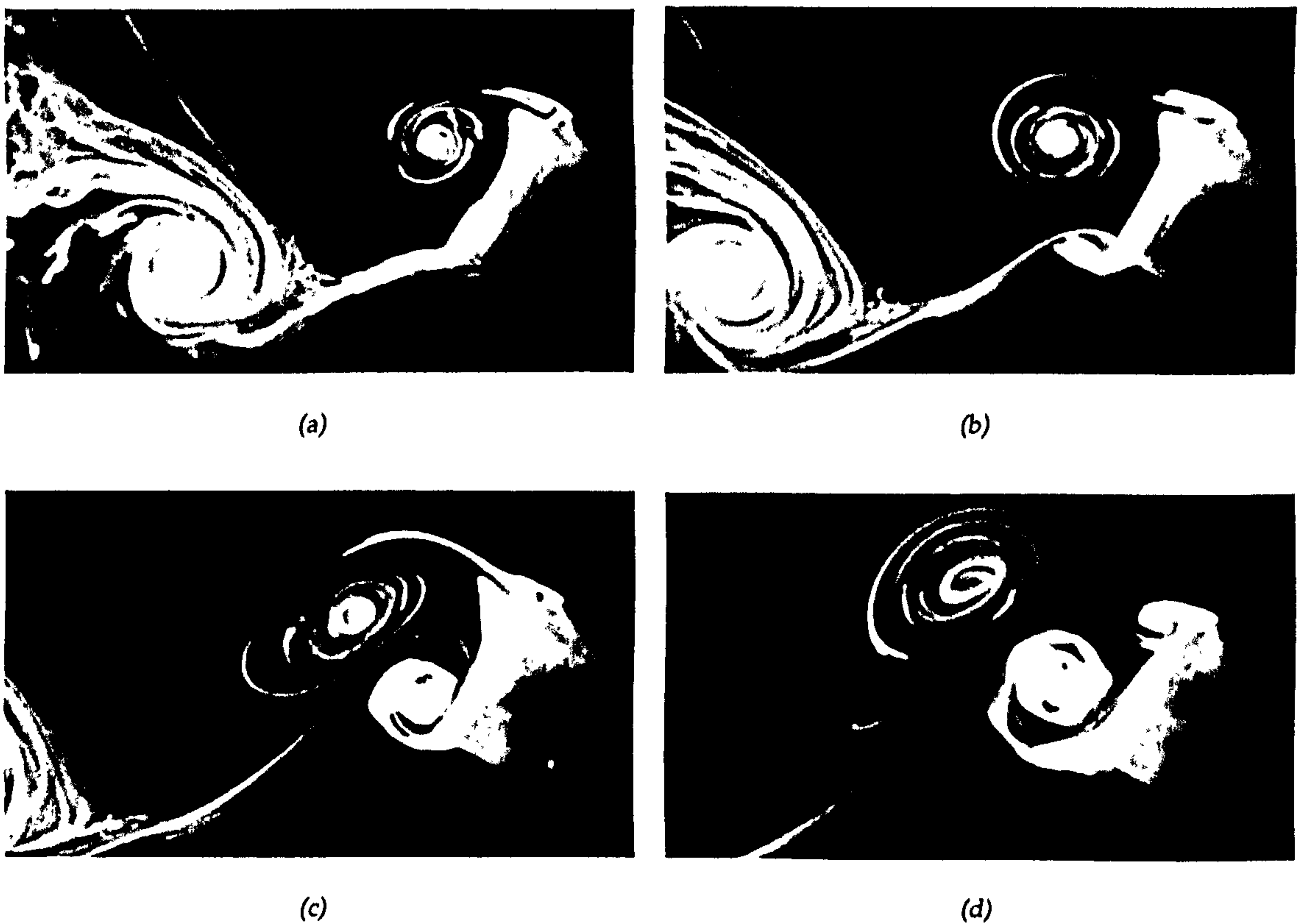


Figure 6.16: Smoke flow visualisation showing an impulsively started aerofoil at 50° angle of attack ($Re = 1000$) [adapted from Gustafson & Sethian (1991, fig. 4.5)]. The wing travels approximately 0.7 chord lengths between successive frames

The first three frames (Figures 6.16(a)–(c)) are more or less consistent with the results of Dickinson & Götz. The fourth frame shows that a second leading-edge vortex is formed after the first one grows large and breaks off (Figure 6.16(d)), consistent with the prediction from the current inviscid model. A similarity can also be seen between

Figure 6.15 and Figure 6.16(d) in the roll-up and mutual convection of the shed leading- and trailing-edge vortex 'pair'. Since lift data from the theoretical model were consistent for the frames shown in the Dickinson & Götz experiment, the formation of a second leading-edge vortex (proven by Freymouth's experiments) would be consistent with the subsequent rise in lift shown in Figure 6.14(a).

Therefore, it can be concluded that the current theoretical model predicts with sufficient accuracy (both in size and location) the flow field associated with a wing at a high angle of attack. Force data (and especially drag) may be dubious in some cases but this can only be said with certainty once more data are available for comparison. For the purposes of this PhD work, the current model provides a reasonable platform for fulfilling the aims of the study — to develop wing-design guidelines for a flapping-wing micro air vehicle. This is achieved by means of the parametric study presented in the next section.

6.2 Parametric Study

The aim of the current work is to determine wing-design guidelines for a FMAV. It was, therefore, necessary to carry out a parametric study to investigate the effects of various pertinent parameters and, thus, deduce the best combination for maximum lifting capability while minimising torque. This is the theme of the current section.

In order to carry out the parametric study, the relevant parameters were first identified. These subdivide broadly into those related to wing *kinematics* and those to wing *shape*. Figure 6.17 summarises the extent of the parametric study.

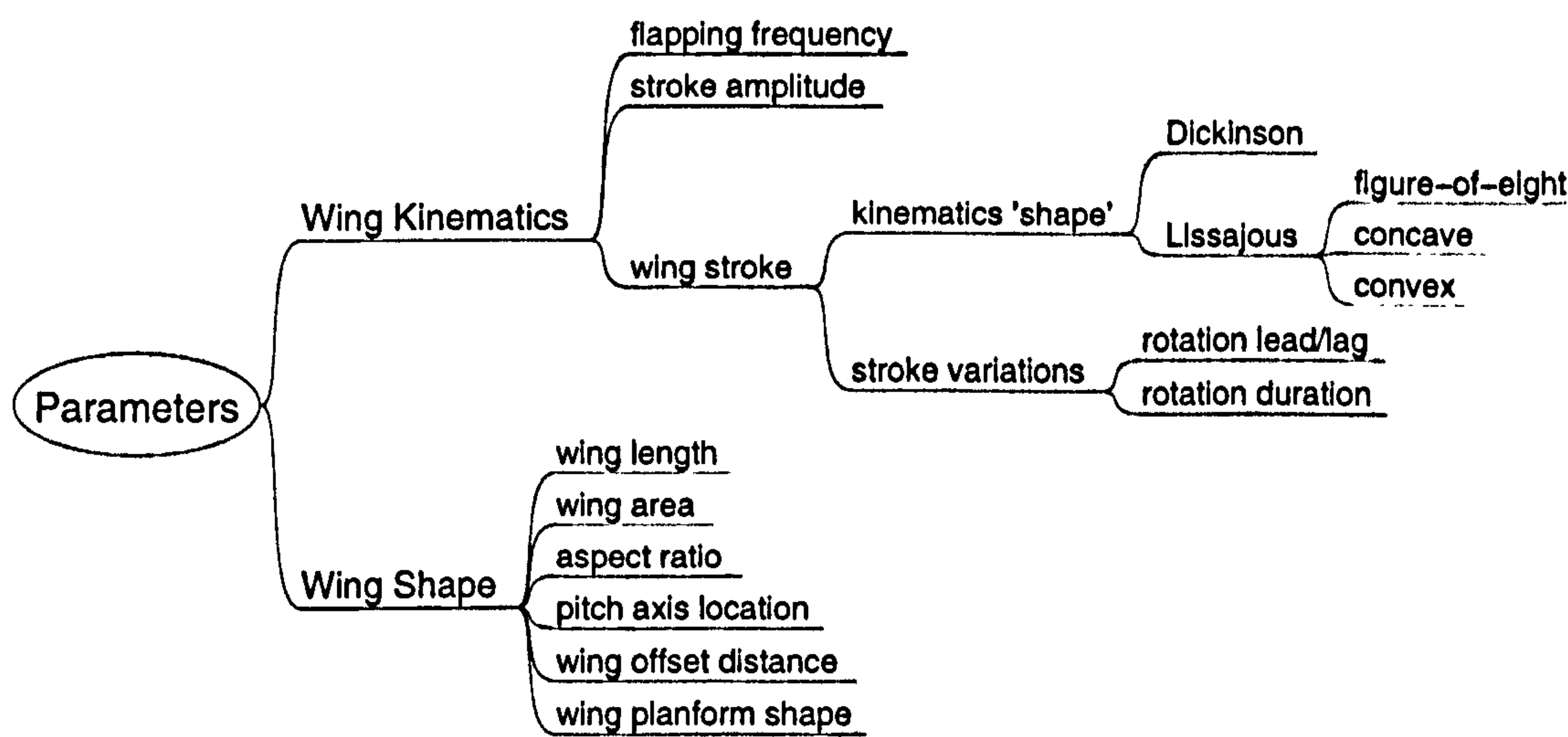


Figure 6.17: Overview of parametric study

Wing Kinematics These include the following:

- flapping frequency** This refers to the frequency of flapping f .
- stroke amplitude** This refers to the length of each stroke (i.e. angle swept through each halfstroke Φ).
- wing stroke** This describes the variations in the properties of the wing stroke itself and subdivides into:
 - kinematics 'shape'** Two types of wing motion were considered — kinematics

from Dickinson (priv. comm., referred to as Dickinson kinematics from hereon) and Lissajous-type⁸ kinematics (figure-of-eight, concave arc and convex arc).

stroke variations These include such variations as duration of rotation phases (pronation and supination) and advanced or delayed rotation.

Wing Shape These parameters subdivide as follows:

wing length This refers to the length R of each of the flapping wings comprising the FMAV.

wing area This refers to the planform area of the 'wing pair' S .

aspect ratio This refers to the aspect ratio of the 'wing pair' and is computed as $\frac{4R^2}{S}$.

pitch-axis location This refers to the position of the pitch axis along the chord length and is measured with respect to the longest chord length along the wing planform.

wing offset distance This refers to the distance of the physical wing root from the point of rotation at the centre of the FMAV.

wing planform shape This refers to the actual shape of the wing planform, i.e. whether rectangular, elliptical, etc.

The aim of this section is to investigate the effects of various flapping-wing-related parameters with a view to identifying the best combination for a FMAV. The aim is to describe the conditions for a flapping-wing-based vehicle that would fit the design requirements described earlier (in § 1.1.1).

In the investigation that follows, simulations are performed on only one of the wings comprising the wing-pair of a flapping-wing vehicle. The sum effect is obtained simply

⁸After Jules Antoine Lissajous (1822–1880).

by doubling the contribution of the wing. In doing so, symmetry is assumed and the effect of the 'other wing' is ignored.

The 'experiments' were run in air assuming standard atmospheric conditions (air density $\rho = 1.225 \text{ kg/m}^3$). The wing sections were assumed to be thin and flat with no flexion. The concept of radial chords (see p. 95) was used to derive wing section chord lengths along the wing. The radial cross-plane analogy (see p. 95) was used to describe the various '2-D' planes for each wing section.

All simulations were run as batch jobs with almost no user intervention. The number of time-steps was calculated automatically from the wing shape and wing kinematics.

Recall that lift $\mathcal{L}$ refers to the force acting vertically upwards and opposing gravity. Thrust $\mathcal{T}$ refers to the force component orthogonal to lift and always acting in the direction of translational motion of the wing. In the discourse that follows, higher lift or thrust refers to lift or thrust of greater magnitude respectively, and not to more positive values.

Since the aim of the current work is to devise wing-design guidelines for a flapping-wing micro air vehicle, some method of comparing various wing kinematics and wing shapes is needed. Due to the nature of the flapping wing (and the associated continuously varying forces), a good measure is the mean lift $\mathcal{L}$ (and mean thrust $\mathcal{T}$). From a wing-design perspective, an even better measure is the mean lift-to-torque ratio $\mathcal{L}/Q$ as it gives an indication of power required. Another good measure for comparing wing planform shapes is the lift-to-drag ratio. This is analogous to the lift-to-thrust ratio $\mathcal{L}/\mathcal{T}$ considered here. In the discourse that follows, the above measures are used for comparison. It may be noted that thrust $\mathcal{T}$ is used interchangeably with drag.

6.2.1 Effects of Wing Kinematics

The study of the effects of various kinematics-related parameters was carried out using scaled-down versions of the wings used for the numerical experiments in Chapter 5. Because the investigation in this section relates to kinematics characteristics, wing size is of little consequence. Of greater importance is the manner in which the flapping wing moves.

The behaviours of lift, thrust, torque and related parameters were investigated. Owing to the computational limitations on the total number of simulations that could be run, only one parameter was varied at a time. The remaining parameters were maintained at the values shown in Table 6.1 below. For the relevant simulation constants, the numerical values quoted in Table 5.3 (p. 183) earlier were used.

Table 6.1: Simulation constants for wing kinematics study

flapping frequency	2 Hz
simulation time	1.0 s
stroke amplitude	maximum possible (see Figure 6.20)
wing length	125 mm
kinematics	Dickinson (see Figure 5.21, p. 182)

For the study, two wing planform shapes were used so that the independence of wing kinematics effects from wing shape could be established. The wing planforms used were scaled down versions of Robofly and generic wings encountered earlier (see Figure 5.11, p. 167).

Effect of Flapping Frequency

The effect of flapping frequency on both scaled-down Robofly and generic wing planforms was studied. These simulations were some of the lengthiest ones owing to the number of time-steps (and hence shed vortices) that had to be handled. As flapping frequency

increased so did the number of stroke reversals (and hence sub-time-steps) in the same simulation time. Also, due to the different shapes of the planforms, the Robofly wing swept an arc of 101.7° and required 160 time-steps per halfstroke while the generic planform swept an arc of 143.5° and required 240 time-steps for the same duration. The simulations were, therefore, limited to maximum frequencies of 15 Hz and 10 Hz respectively.

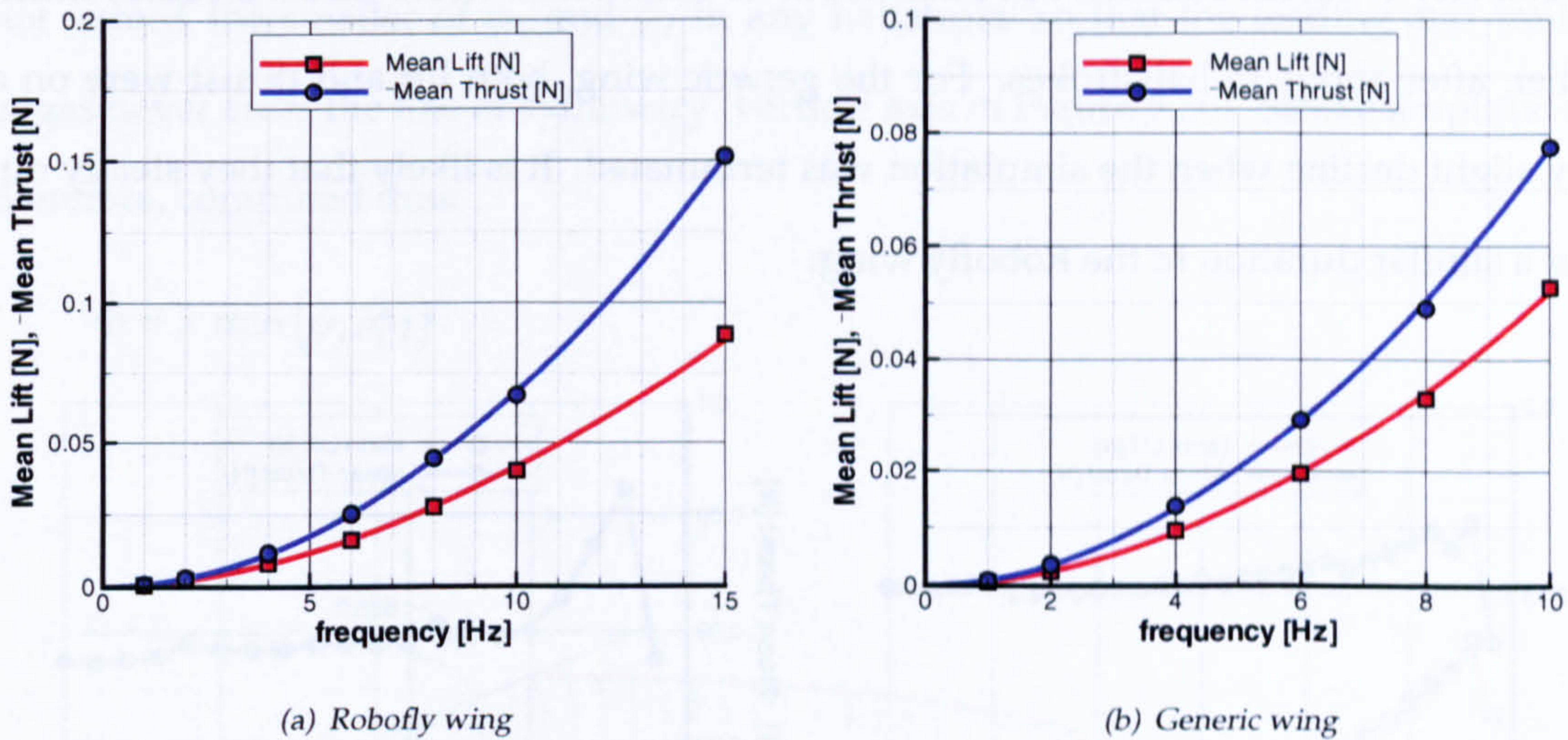


Figure 6.18: Effect of flapping frequency on lift and thrust

The results for the study are shown in Figure 6.18 where a power law fit is drawn to connect the individual test points. As can be seen, there is minimal deviation from the line of best fit. Lift $\mathcal{L}$ and thrust $\mathcal{T}$ scale up roughly with the square of flapping frequency f . This is because aerodynamic forces roughly scale with dynamic pressure, i.e. with the square of velocity. Flapping frequency is directly proportional to the sweeping velocity, and hence, the forces quadruple when frequency doubles.

Therefore, higher flapping frequencies f are beneficial because they increase lift without any change in lift-to-torque ratio (the latter depends mainly on wing planform which is unchanged). However, an upper limit might be set on f , provided by the requirement to keep it inaudible. In practice, there will also be the added noise of the mechanism exciting the wing into flapping motion. Another practical constraint is hysteresis: as

flapping frequency increases, hysteresis may lead to increasing power losses.

Another piece of information that can be extracted from the study on flapping frequency is the limit beyond which the forces become reasonably periodic. A simple determination of this condition can be made by computing the mean forces at the end of each halfstroke and identifying the point when changes in mean lift and thrust become minimal. The resulting graphs are plotted in Figure 6.19 below. It can be seen that for the Robofly wing lift steadies out after about 25 halfstrokes while thrust steadies much earlier, after about 15 halfstrokes. For the generic wing, both lift and thrust were on a very slight decline when the simulation was terminated. It is likely that they steady out after a similar duration to the Robofly wing.

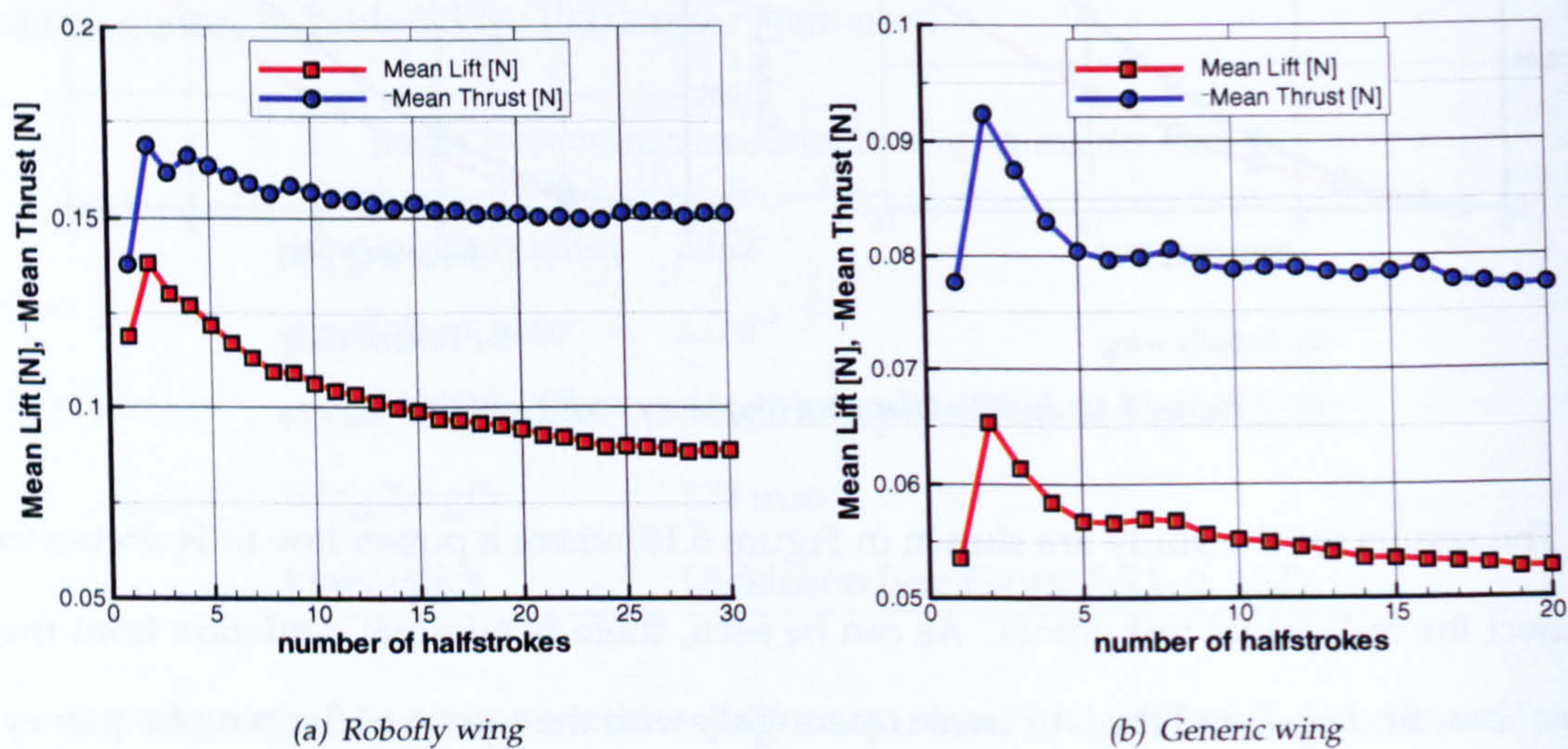


Figure 6.19: Time history of mean lift and thrust

Therefore, it may be inferred that after about 20 halfstrokes (or 10 cycles) a simulation will give a reasonable approximation of the forces on the system should the motion continue perpetually. However, owing to limitations in computational power and the associated prohibitive timescales, it was not possible to continue simulations for this number of flapping cycles. Instead, in all the remaining simulations 4 halfstrokes were used. Whereas this does not give the final ‘steady’ values for hover, it allows for the comparison of various kinematic and shape characteristics.

Effect of Stroke Amplitude

The stroke amplitude was computed from the maximum angle that could be swept by the wing within a semicircle. Consider Figure 6.20 below. For one halfstroke, the maximum angle that can be swept is $\phi_1 + \phi_2$. At the end of the halfstroke, stroke reversal causes the angles ϕ_1 and ϕ_2 to be interchanged and the situation in Figure 6.20 is reversed. Therefore, in order to avoid collision with the ‘other wing’, it is necessary that the sweep *extents* do not exceed the smaller of ϕ_1 and ϕ_2 in any halfstroke so that the leading and trailing edges never cross the line of symmetry (vertical axis in Figure 6.20). Stroke amplitude is, therefore, computed thus

$$\Phi = 2 \min(\phi_1, \phi_2)$$

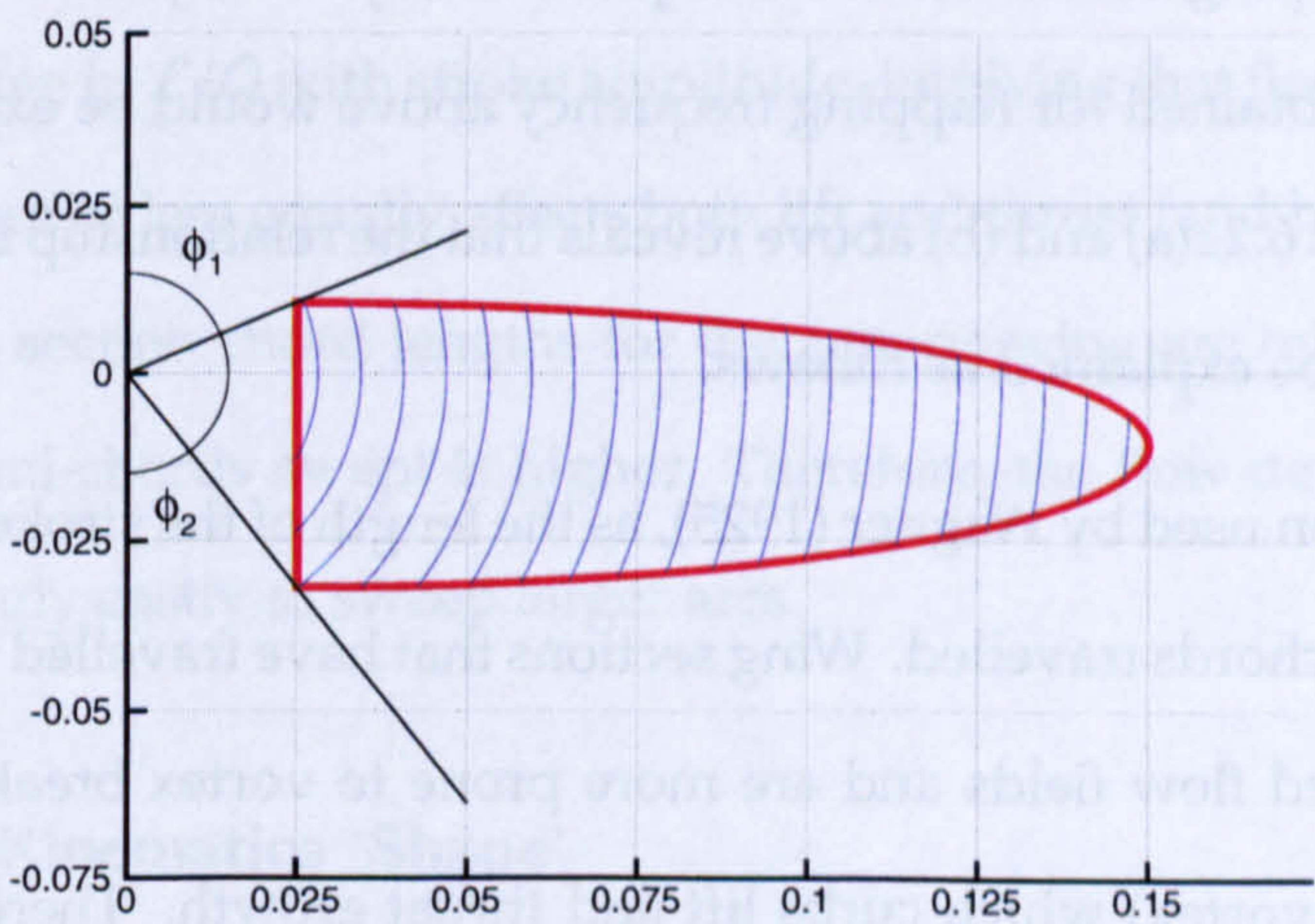
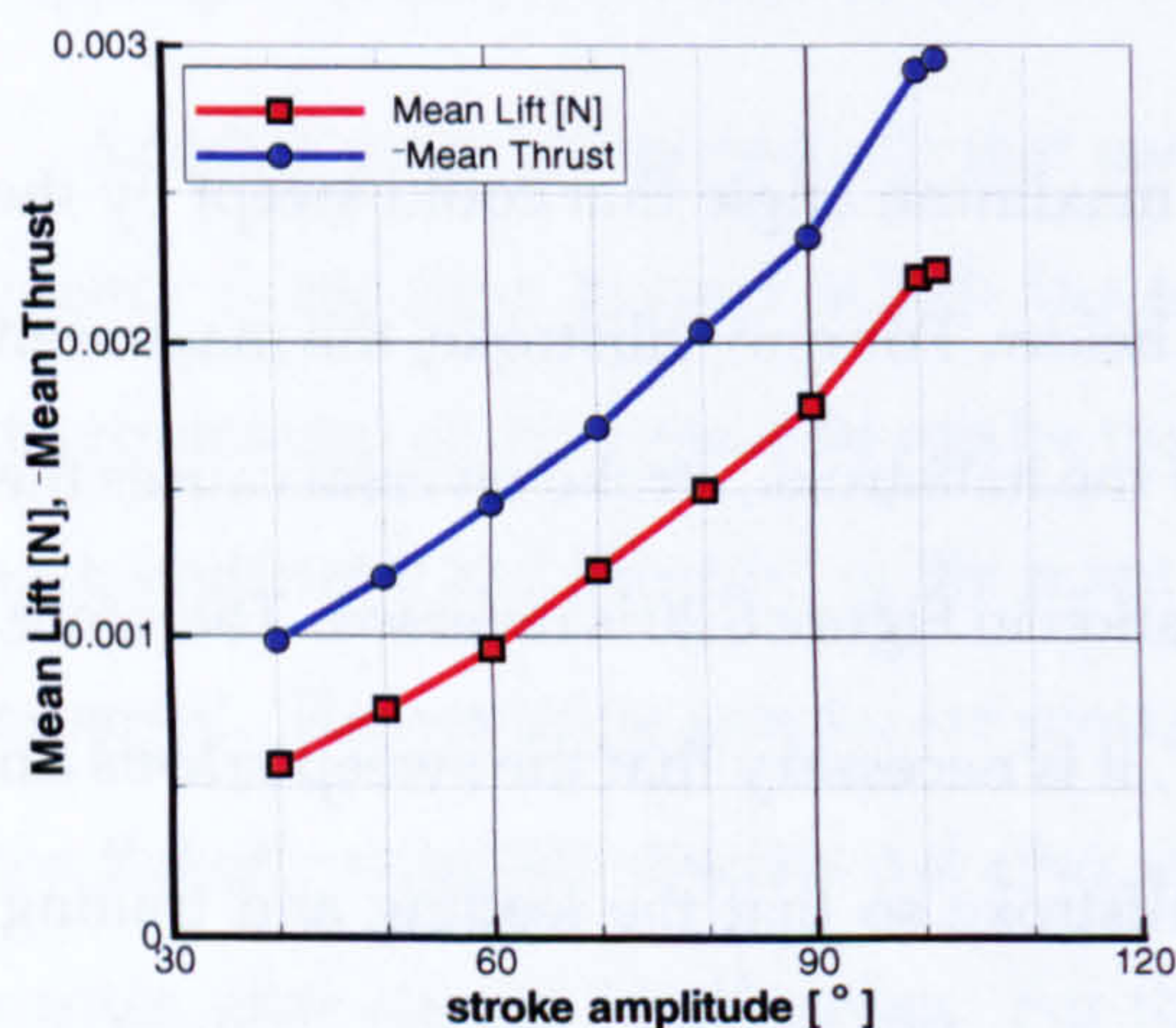


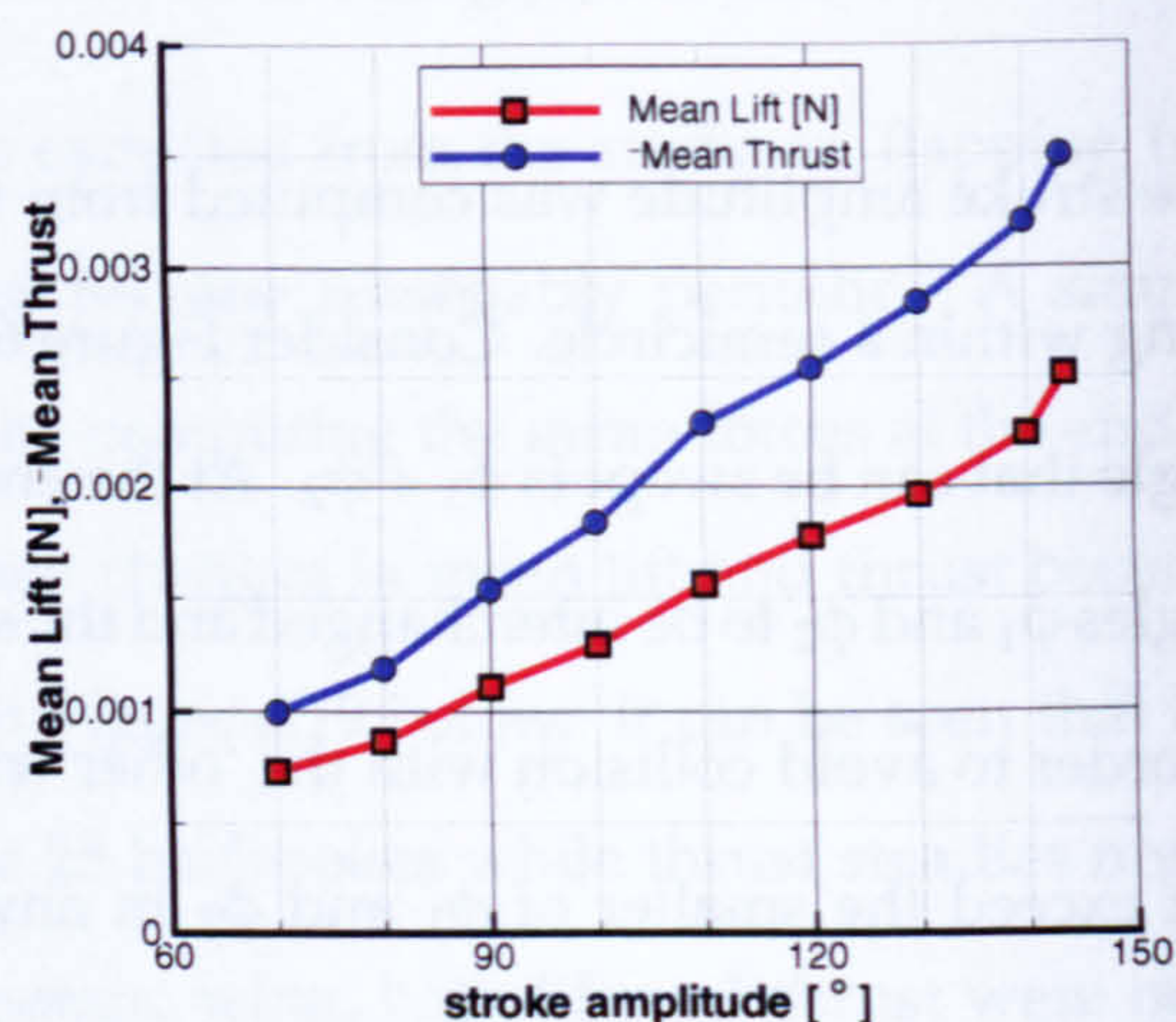
Figure 6.20: Calculating stroke amplitude from sweep angles.

Throughout all the simulations, the stroke amplitude Φ was taken as the maximum angle that could be swept by the flapping wing without ever crossing the 180° arc boundary in order to allow for the other wing of the wing-pair. For this investigation on the effect of stroke amplitude, the latter was varied from small values up to the maximum possible described above. The results from the study are shown in Figure 6.21 below.

The simulations were all run at the same frequency and for the same duration. There-



(a) Robofly wing



(b) Generic wing

Figure 6.21: Effect of stroke amplitude on lift and thrust

fore, increasing stroke amplitude required that a greater arc be swept in the same time, implying higher flapping velocities. From a quasi-steady aerodynamic perspective, a result similar to that obtained for flapping frequency above would be expected. However, inspection of Figures 6.21(a) and (b) above reveals that the relationship is more linear than quadratic. This can be explained as follows.

Using the notation used by Wagner (1925), as the length of the stroke increases so does the number of semi-chords travelled. Wing sections that have travelled more semi-chords have more developed flow fields and are more prone to vortex breakdown (especially in the leading-edge vortex) which curbs lift and thrust growth. Therefore, the effect of increasing velocity accompanying increasing stroke amplitude is partly offset by the flow being more developed (and partially breaking down).

An examination of the lift-to-torque ratio $\mathcal{L}/Q$ for the simulations explains this phenomenon further (see Figure 6.22). For the Robofly wing (Figure 6.22(a)), there is a general increase in $\mathcal{L}/Q$ with stroke amplitude, peaking at about 90° . This condition represents the situation where the stroke is long enough so as to enable high lift and yet short enough to avoid major flow breakdown (and the associated loss of lift).

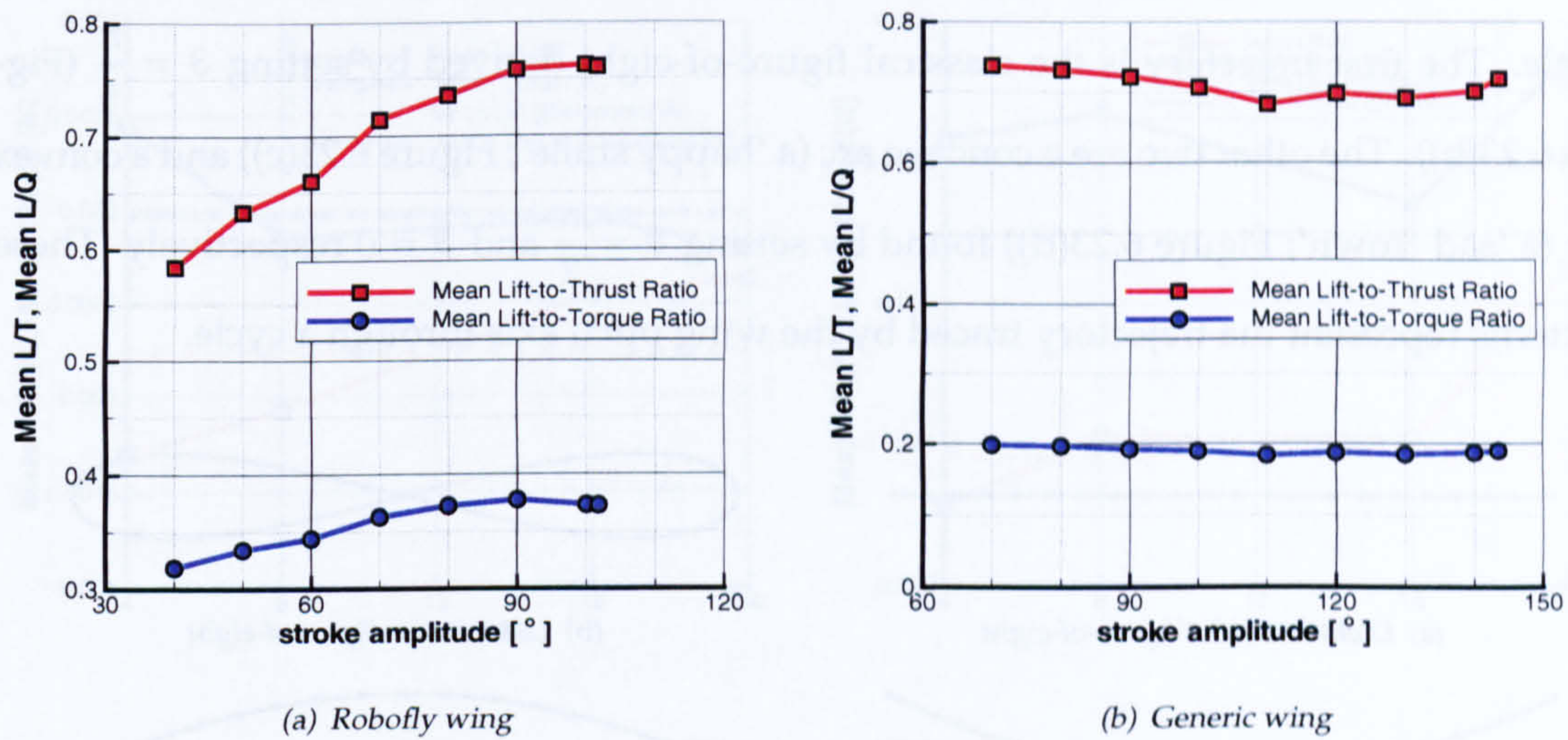


Figure 6.22: Effect of stroke amplitude on mean lift-to-thrust ratios ($\mathcal{L}/\mathcal{T}$) and mean lift-to-torque ratios ($\mathcal{L}/Q$). $\mathcal{L}/Q$ is non-dimensionalised by mean chord

The results for the generic wing (Figure 6.22(b)) show a slightly different behaviour. There is little change in $\mathcal{L}/Q$ with stroke amplitude, implying that flow breakdown begins earlier on and more or less equally affects both lift and thrust (and hence torque). This is because the wing section chord lengths for the generic wing are relatively short so that the number of semi-chords swept is higher. Therefore, the flow develops more quickly and it is increasingly costly to sweep larger arcs.

Effect of Wing Kinematics ‘Shape’

In this section, the effect of the kinematics ‘shape’ is investigated. Four different types of kinematics shapes are considered (see Figure 6.23). The first set of kinematics (Figure 6.23(a)) is a flat figure-of-eight provided by Dickinson (priv. comm., see also Figure 5.21, p. 182). The remaining three are derivatives of the Lissajous family of curves which have the general form

$$\phi = \frac{\Phi}{2} \cos(2\pi ft + \vartheta) \quad (6.1a)$$

$$\theta = \frac{\Theta}{2} \cos(4\pi ft) \quad (6.1b)$$

where Φ is stroke amplitude, Θ is inclination amplitude, f is frequency and ϑ is a phase angle. The first trajectory is the classical figure-of-eight derived by setting $\vartheta = \frac{\pi}{4}$ (Figure 6.23(b)). The other two are a concave arc (a 'happy smile'; Figure 6.23(c)) and a convex arc (a 'sad frown'; Figure 6.23(d)) found by setting $\vartheta = \frac{\pi}{2}$ and $\vartheta = 0$ respectively. These patterns represent the trajectory traced by the wing pitch axis through a cycle.

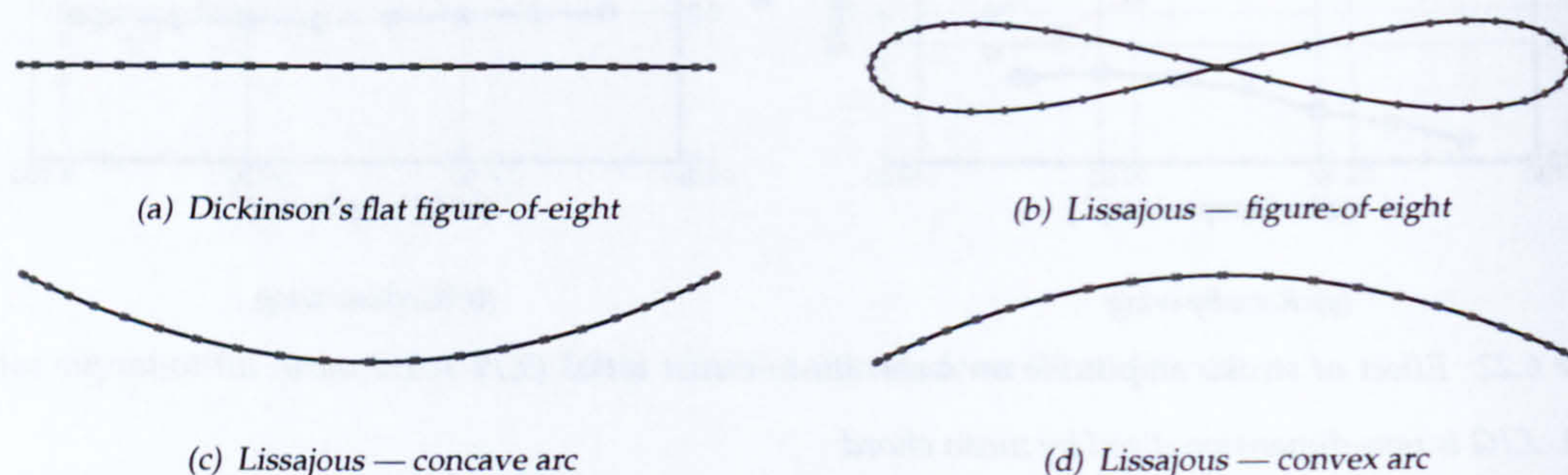


Figure 6.23: Kinematics 'shapes' considered. The dots indicate positions at regular time intervals

A 'Lissajous ratio' was defined as $\frac{\Phi}{\Theta}$, describing the 'thickness' of the Lissajous curve (smaller ratios describing fatter curves). When this ratio was set to ∞ a flat figure-of-eight resulted, irrespective of phase angle ϑ . The main comparison made was between Dickinson's kinematics and a Lissajous figure-of-eight. As can be appreciated from Figure 6.23(b), a figure-of-eight may be traced in two directions — one starting above the mean position, moving downward through a halfstroke and ending at the mean position (as shown in Figure 2.1, p. 16). The other direction is the exact opposite — starting below the mean position, moving upward through most of the halfstroke and ending again at the mean position. The former method is used in the subsequent analysis unless stated otherwise.

The results for lift and thrust at different Lissajous ratios for figure-of-eight-type motion are shown in Figure 6.24. Note that the 'Flat' kinematics referred to in Figure 6.24 (and subsequently) are similar to Dickinson's flat figure-of-eight kinematics but differ temporally due to a different variation in sweep angle ϕ with time (cf. Figures 5.21 and 6.33; see also Equations 6.1).

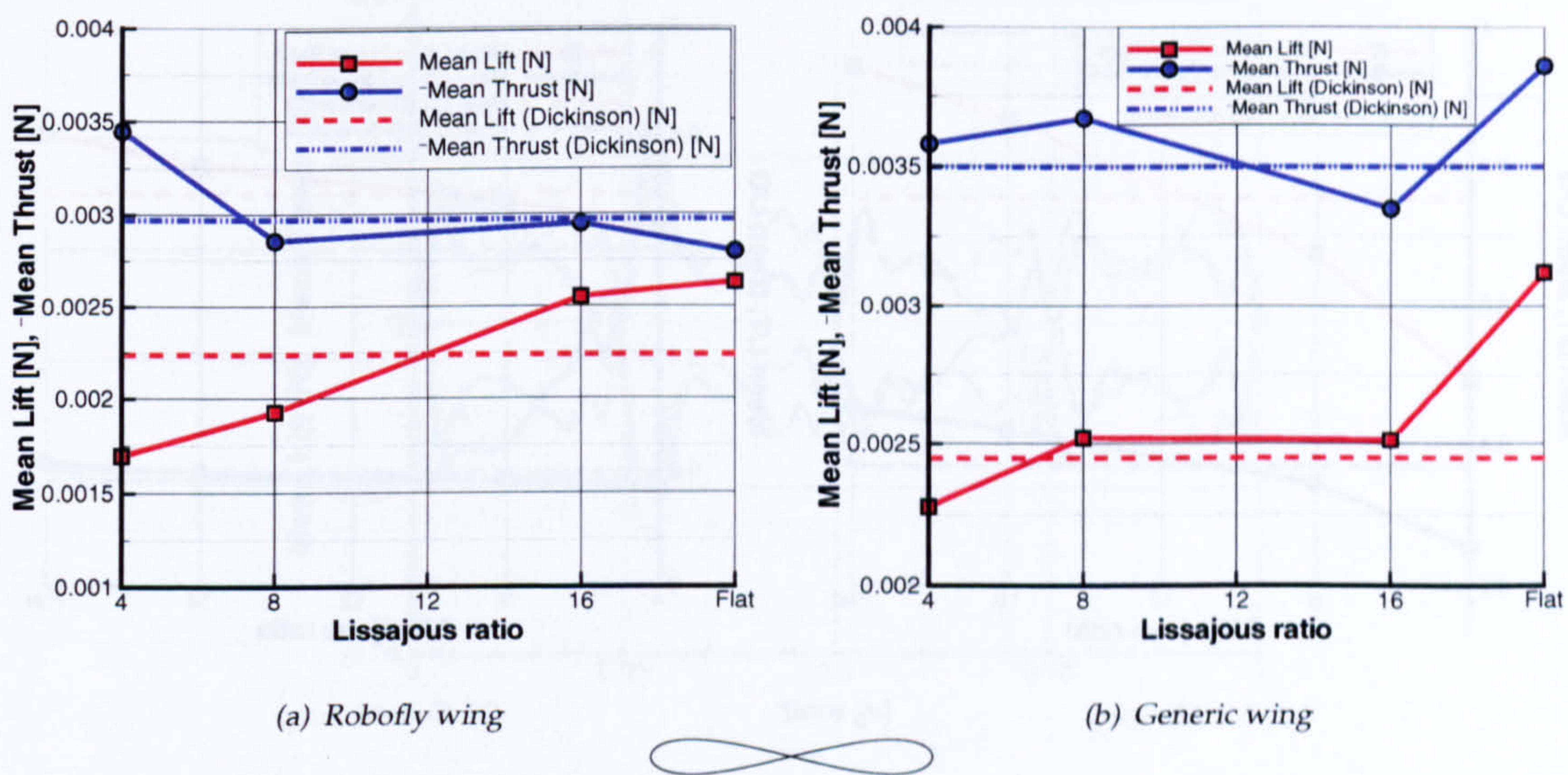


Figure 6.24: Comparison of Dickinson’s kinematics with Lissajous figure-of-eight kinematics using mean lift and mean thrust

For the Robofly wing, there is a general increase in mean lift $\mathcal{L}$ and a general decline in mean thrust $\mathcal{T}$ as the flapping trajectory gets flatter. This would imply improving mean lift-to-torque ratios $\mathcal{L}/Q$ (and lift-to-thrust ratios $\mathcal{L}/\mathcal{T}$) and is borne out by the respective plots in Figure 6.25 below. Fatter figures-of-eight trace longer routes so that the wing travels more chords (for the same stroke amplitude Φ). In doing so, vortex breakdown occurs sooner (resulting in lower lift and higher thrust) as the leading-edge vortex first breaks off, followed by a similar break off from the trailing edge. If the motion were continued for long enough, a von Kármán vortex street would result.

For the Lissajous figure-of-eight kinematics, the wing moves from a higher position to a lower position during a halfstroke. Shed wake moves downward due to the downwash generated by the ‘lifting’ wing. Therefore, in moving downwards the wing is likely to meet previously-shed wake which affects and generally impedes wing lift. By this argument, a wing performing the opposite Lissajous figure-of-eight-type motion moving from a lower position to a higher position would produce more lift and thrust as it better avoids previously shed wake. This was indeed found to be the case in a small study

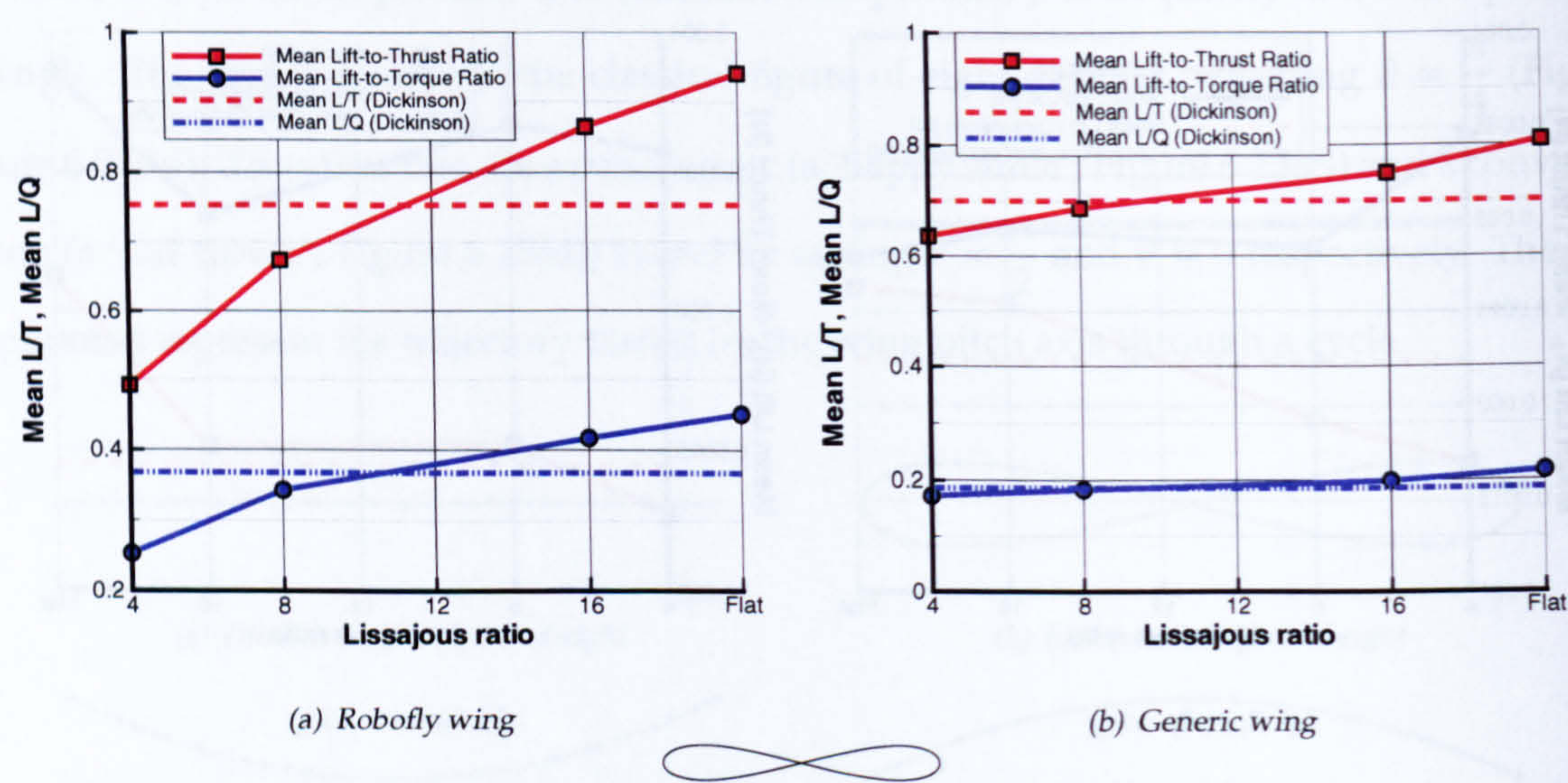


Figure 6.25: Comparison of Dickinson's kinematics with Lissajous figure-of-eight kinematics using mean $\mathcal{L}/\mathcal{T}$ and mean $\mathcal{L}/Q$. $\mathcal{L}/Q$ is non-dimensionalised by mean chord

conducted to this effect. Lift and thrust time histories from this study are shown in Figure 6.26. For the general case used, mean lift and thrust for the first case (higher to lower) were found to be 1.8064 mN and -1.9871 mN respectively. For the second case, these forces were 2.3195 mN and -2.5867 mN respectively. Interestingly, the lift-to-thrust ratios (and lift-to-torque ratios) for the two variations on the figure-of-eight are almost equal. It may, therefore, be inferred that a flapping motion that starts lower but ends higher through a halfstroke is more beneficial in hover.

The mean-lift and mean-thrust characteristics for the other two Lissajous motions are shown for the Robofly wing only in Figures 6.27 and 6.28 below. Unlike for the figure-of-eight motion, lift is generally higher for all values of Lissajous ratio than the equivalent that would be produced using Dickinson's kinematics (Figures 6.27(a) and 6.28(a)). For the concave-arc kinematics, this is also generally the case for thrust so that the mean $\mathcal{L}/\mathcal{T}$ and mean $\mathcal{L}/Q$ plots are similar to those for the figure-of-eight motion (cf. Figures 6.25(a) and 6.27(b)).

The convex-arc Lissajous kinematics shows particular promise (see Figure 6.28). Lift is

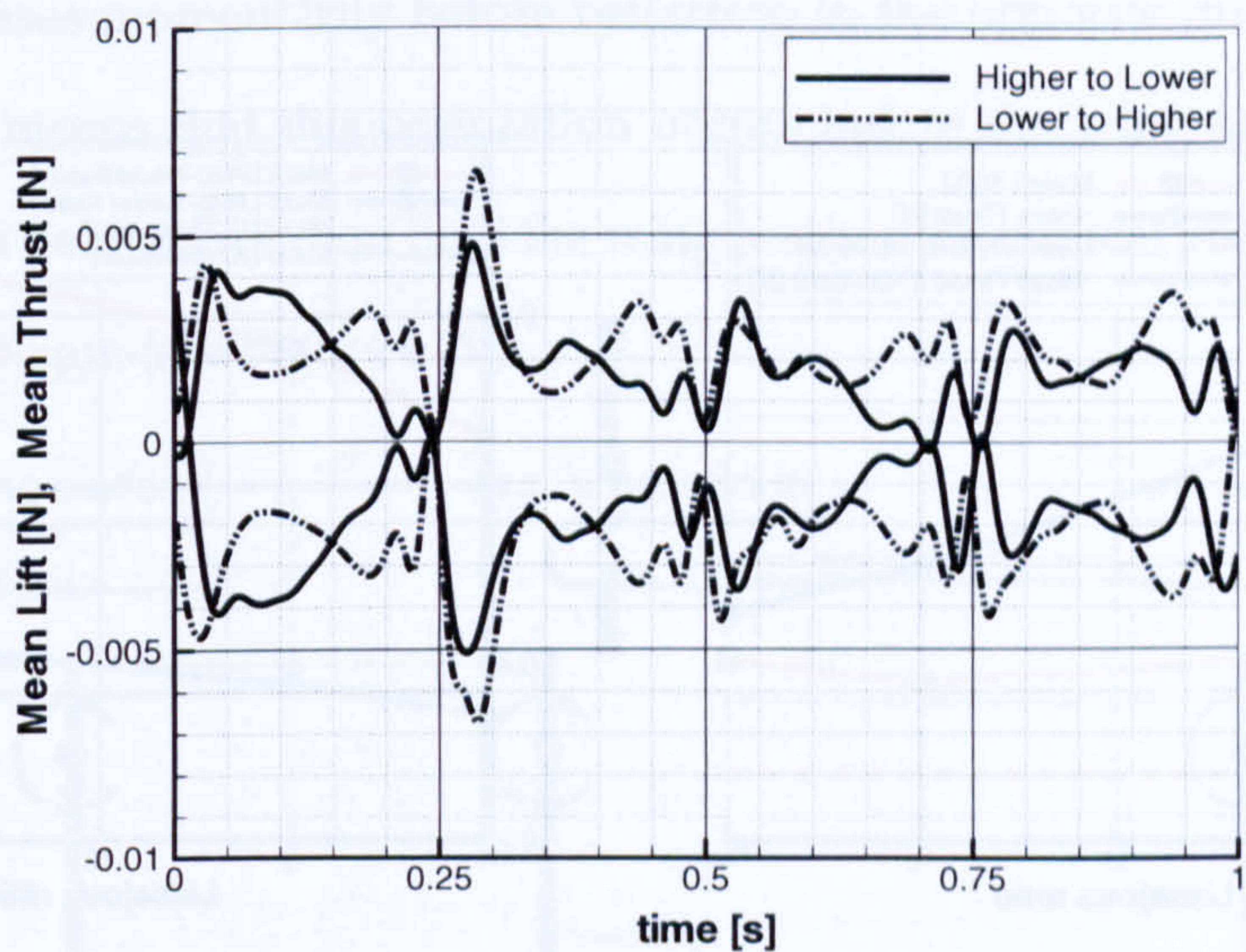


Figure 6.26: Comparison of effect of direction of motion in a Lissajous figure-of-eight. The upper pair of curves in the plot are lift and the lower pair are thrust

generally higher than that produced by the equivalent Dickinson kinematics while thrust is generally much lower. The resulting mean lift-to-torque ratio is up to about 50% greater than that for Dickinson’s kinematics. Mean $\mathcal{L}/Q$ is greatest for a thick convex arc. The wing starts from a low position, rises to a peak at mid-stroke and then returns to a similar low position at the end of the halfstroke.

Effect of Rotation Lead

The rotational phases of insect-like flapping motion may not be synchronised with the translational phases, so that stroke reversals are not necessarily symmetrical about the translational phases of the cycle, i.e. a phase lead or lag may exist between the two. The study presented in this section shows the effect of advanced or delayed rotation on the lift, thrust and related characteristics.

The kinematics are based on those provided by Dickinson (priv. comm.) but with phase lead/lag introduced. A lead or lag of 0% refers to the symmetric condition where the wing is vertical ($\alpha = 90^\circ$) at the end of the halfstroke when the translational motion

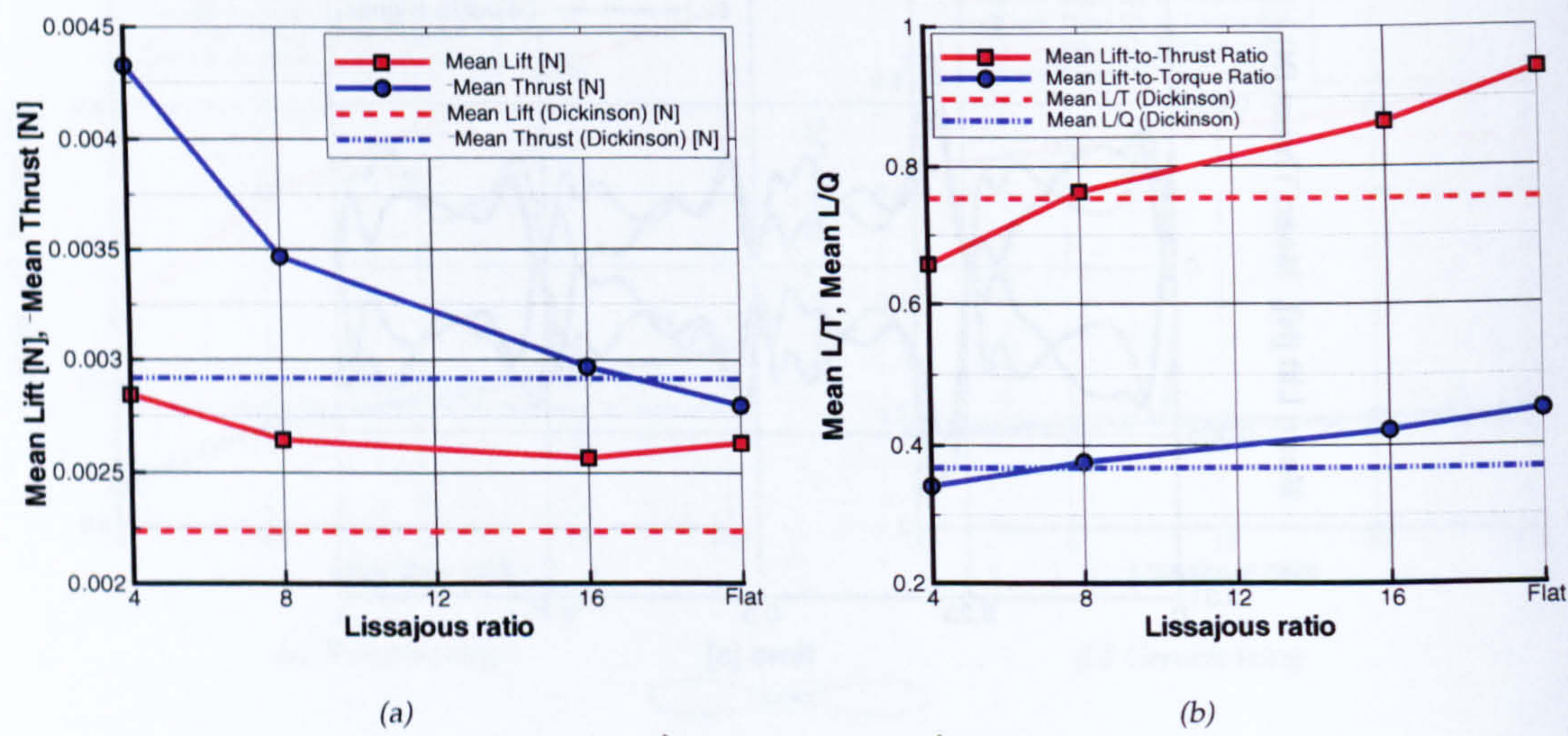


Figure 6.27: Comparison of Dickinson's kinematics with Lissajous concave-arc kinematics for the Robofly wing. $\mathcal{L}/Q$ is non-dimensionalised by mean chord. Note that the kinematics at the 'Flat' ratios are identical to those for the 'Flat' figure-of-eight in Figures 6.24(a) and 6.25(a)

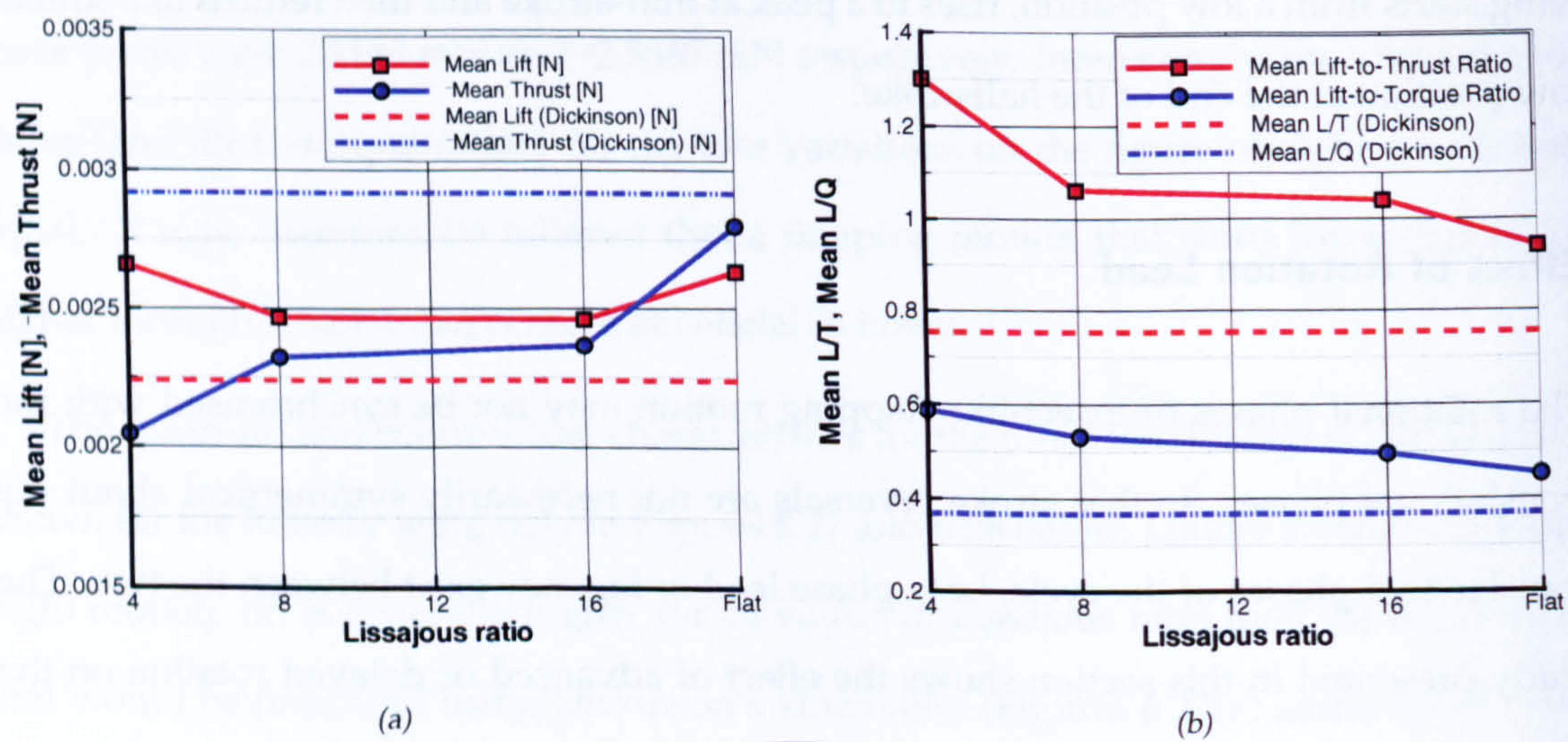


Figure 6.28: Comparison of Dickinson's kinematics with Lissajous convex-arc kinematics for the Robofly wing. $\mathcal{L}/Q$ is non-dimensionalised by mean chord. Note that the kinematics at the 'Flat' ratios are identical to those for the 'Flat' figure-of-eight in Figures 6.24(a) and 6.25(a)

of the wing ceases momentarily before restarting in the opposite direction. Advanced rotation (lead) means that this orientation occurs before the wing has come to a stop. Conversely, in a delayed rotation (lag) the wing reaches the vertical position after the next halfstroke has begun (see Figure 6.29).

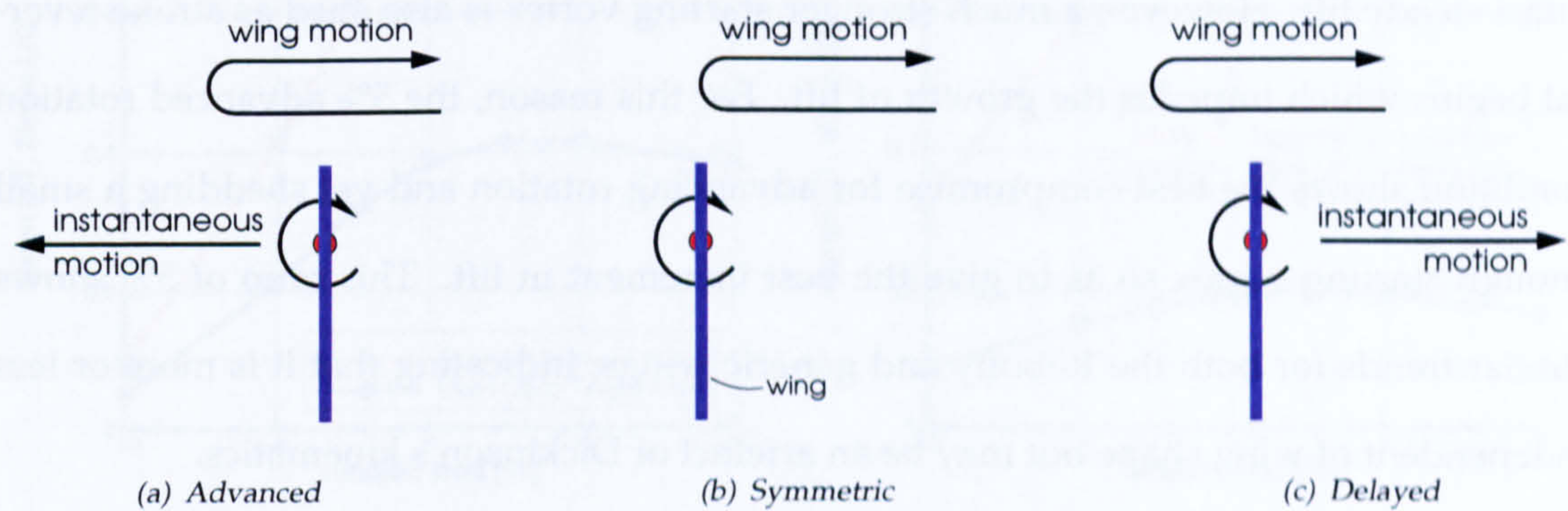


Figure 6.29: Lead-lag in rotational motion determined by direction of wing motion when wing is vertical

Results from the study are shown Figures 6.30. It is immediately obvious that best lift characteristics are obtained if stroke reversals have a 5% lead on the translational motion.

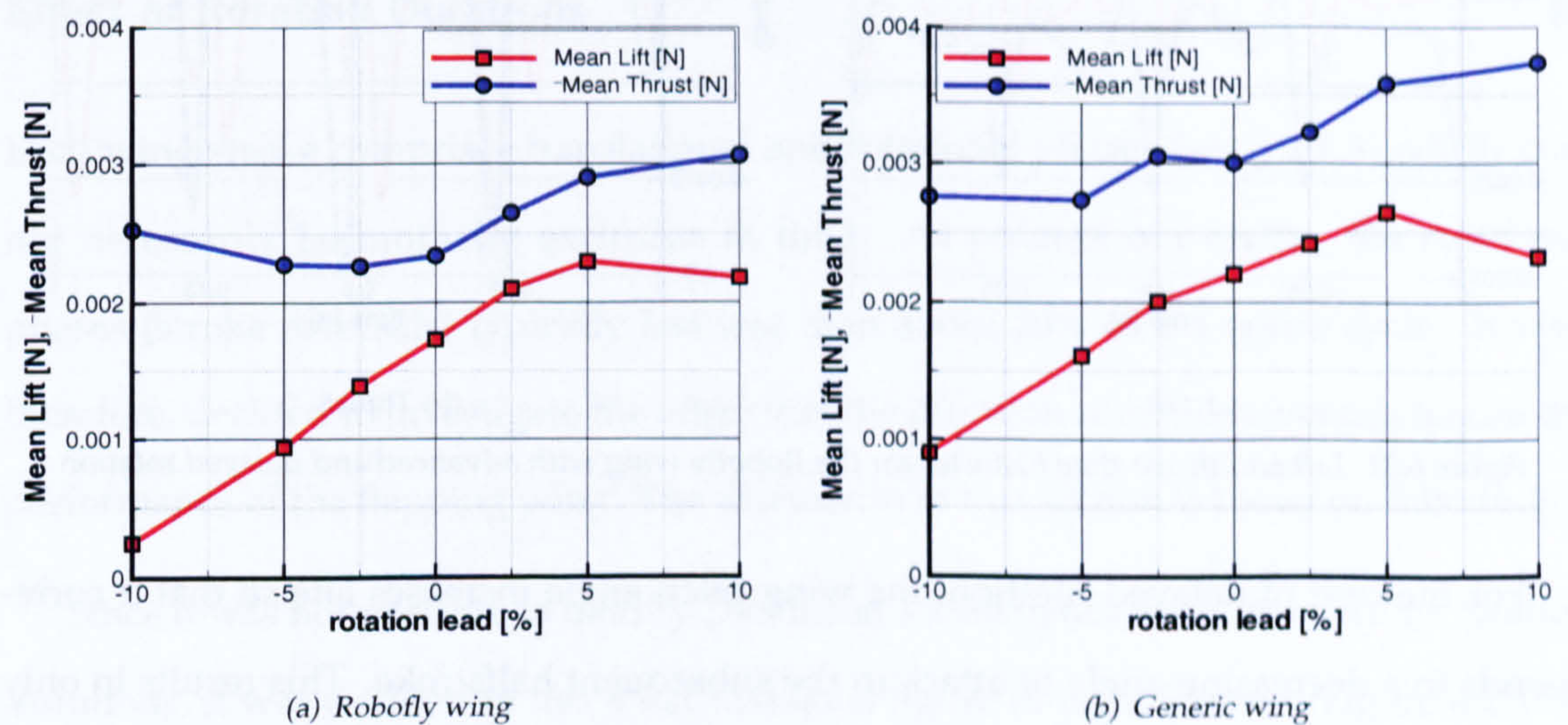


Figure 6.30: Effect of rotation lead on lift and thrust. Negative values of lead are lag

This behaviour can be understood by considering the lift time-history shown in Figure 6.31 below. For the case of advanced rotation, the wing rotates to a high angle of attack towards the end of a halfstroke while the translational velocity has not reduced significantly. This results in an increase in the quasi-steady component of lift which con-

tributes an increase to the total lift (Figure 6.31(a)). There are, however, limited returns in advancing rotation due to the shedding of a starting vortex. Consider, for example, the case where rotation is advanced by 10% (Figure 6.30). Stroke reversal begins much earlier than in the 5% case so that the translational velocity is much higher, and hence, so is the quasi-steady lift. However, a much stronger starting vortex is also shed as stroke reversal begins which impedes the growth of lift. For this reason, the 5% advanced rotation condition shows the best compromise for advancing rotation and yet shedding a small enough starting vortex so as to give the best increment in lift. This ratio of 5% shows similar trends for both the Robofly and generic wings, indicating that it is more or less independent of wing shape but may be an artefact of Dickinson's kinematics.

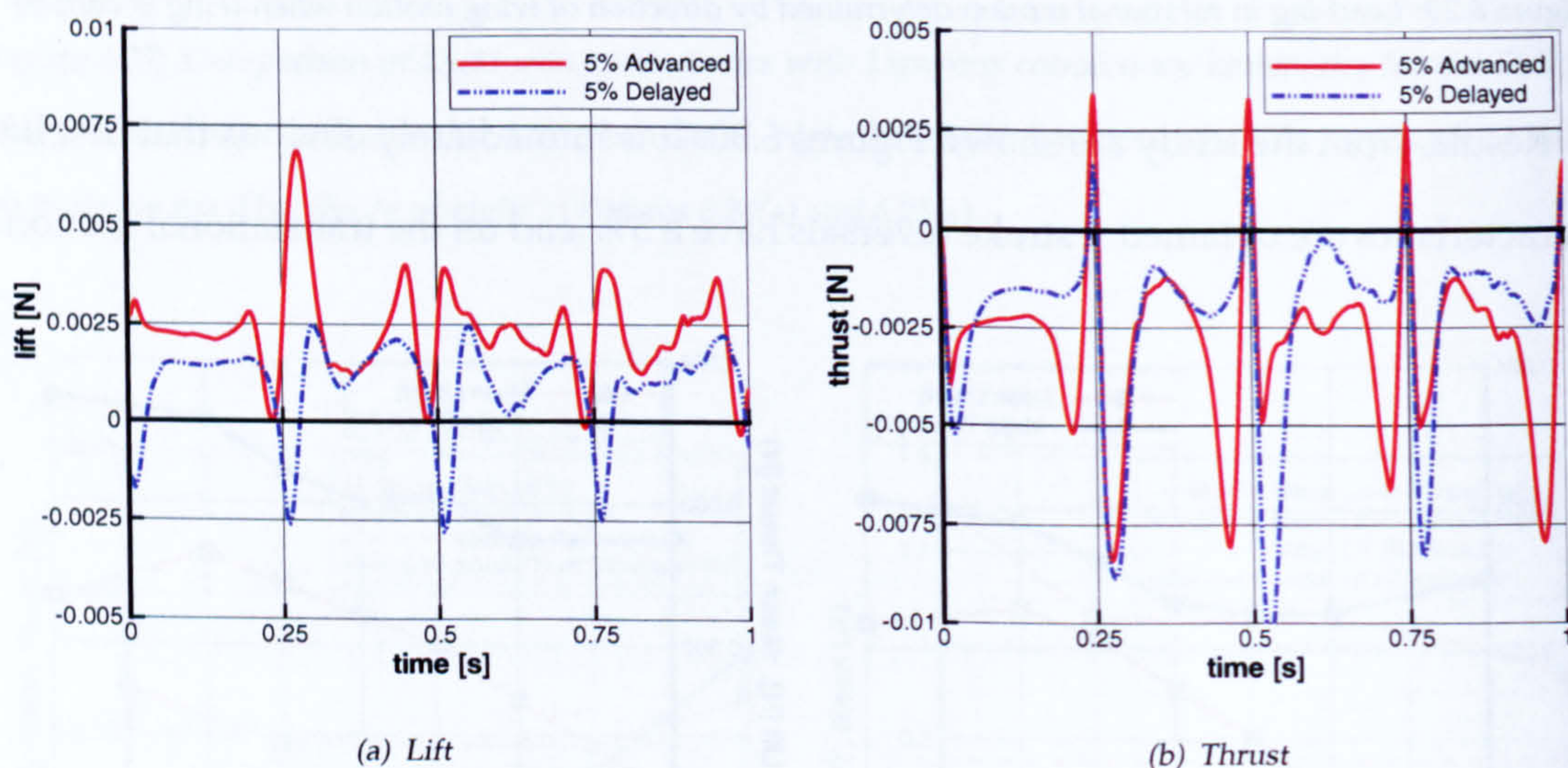


Figure 6.31: Lift and thrust time histories for the Robofly wing with advanced and delayed rotation

For the case of delayed rotation, the wing pitch angle increases late so that it corresponds to a decreasing angle of attack in the subsequent halfstroke. This results in only a small peak in lift towards the end of each halfstroke and a much bigger negative spike at the start of the next halfstroke, giving an overall smaller lift (Figure 6.31(a)).

The effect of rotation lead on lift-to-torque ratio $\mathcal{L}/Q$ characteristics is shown for both the Robofly and the generic wing planforms in Figure 6.32 below. Although they peak at an advanced rotation of 2.5%, the value of $\mathcal{L}/Q$ is insignificantly lower at the 5% condition.

Therefore, for Dickinson’s kinematics the best aerodynamic characteristics are achieved with rotation lead of about 5%⁹.

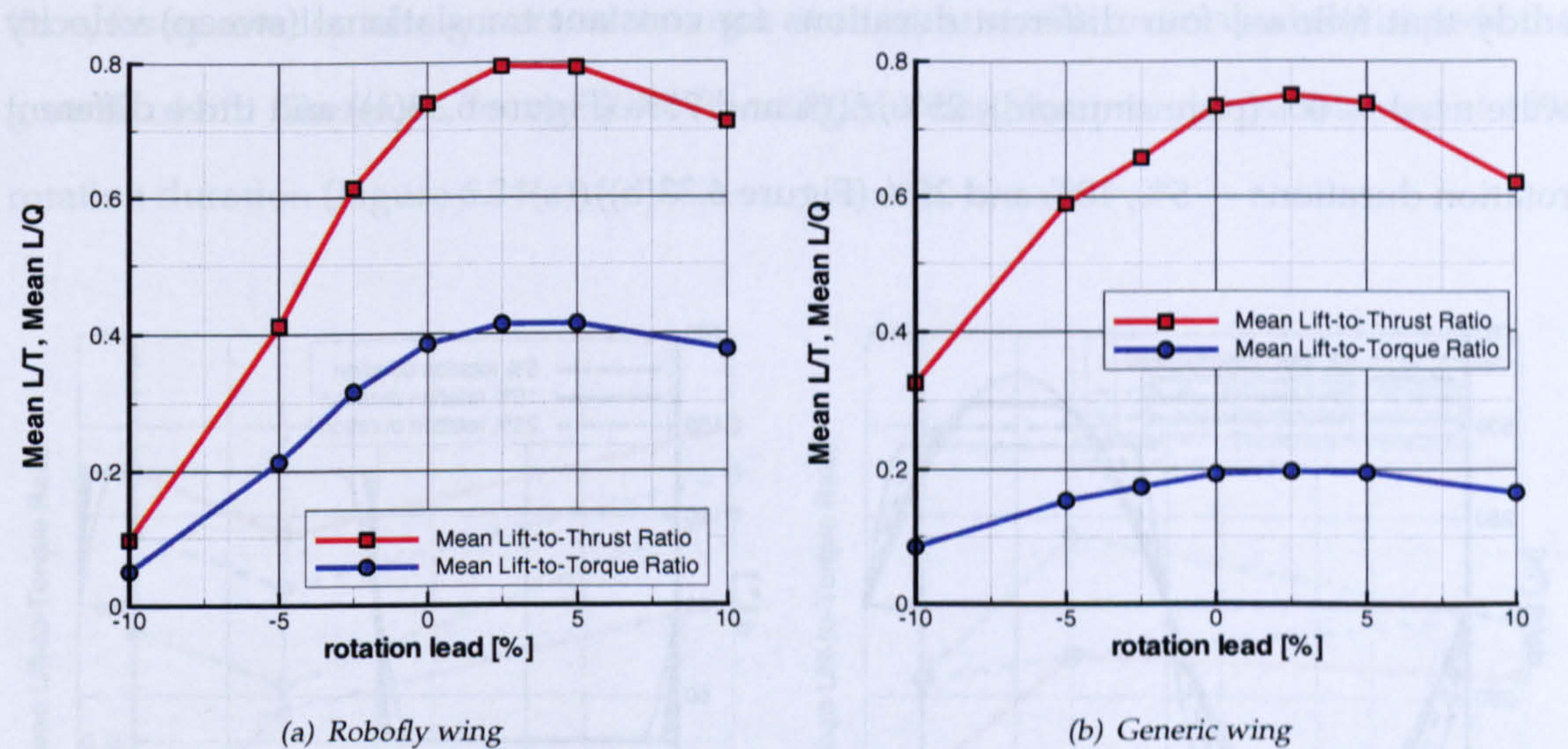


Figure 6.32: Effect of rotation lead on lift-to-thrust and lift-to-torque ratios. $\mathcal{L}/Q$ is non-dimensionalised by mean chord

Effect of Rotation Durations

Each wing stroke comprises translational and rotational phases (see § 2.1.3) which may not necessarily be mutually exclusive in time. As pointed out earlier, the rotational phases (stroke reversals) typically last less than about 20% of the entire cycle. It was, therefore, decided to investigate the effect that the duration of stroke reversals has on the performance of the flapping wing. The discussion in this section is based on this study.

Since it was not possible to modify Dickinson’s kinematics easily for various rotation durations, it was decided to use a flat Lissajous figure-of-eight instead (Figure 6.23(b)). In doing so, not only could the duration of the rotational phases be altered but also the duration of the translational phases with constant velocity. The latter is explained as follows.

For a Lissajous-type motion, the flapping velocity is constantly varying sinusoidally.

⁹In fact, in the original form of Dickinson’s kinematics, rotation had a 6% lead on translation.

However, it is possible to modify the translational phases such that periods with constant velocity are introduced, and in doing so produce Dickinson-like kinematics. In the study that follows, four different durations for constant translational (sweep) velocity were used — 0% (pure sinusoid), 25%, 50% and 75% (Figure 6.33(a)) and three different rotation durations — 5%, 10% and 25% (Figure 6.33(b)).

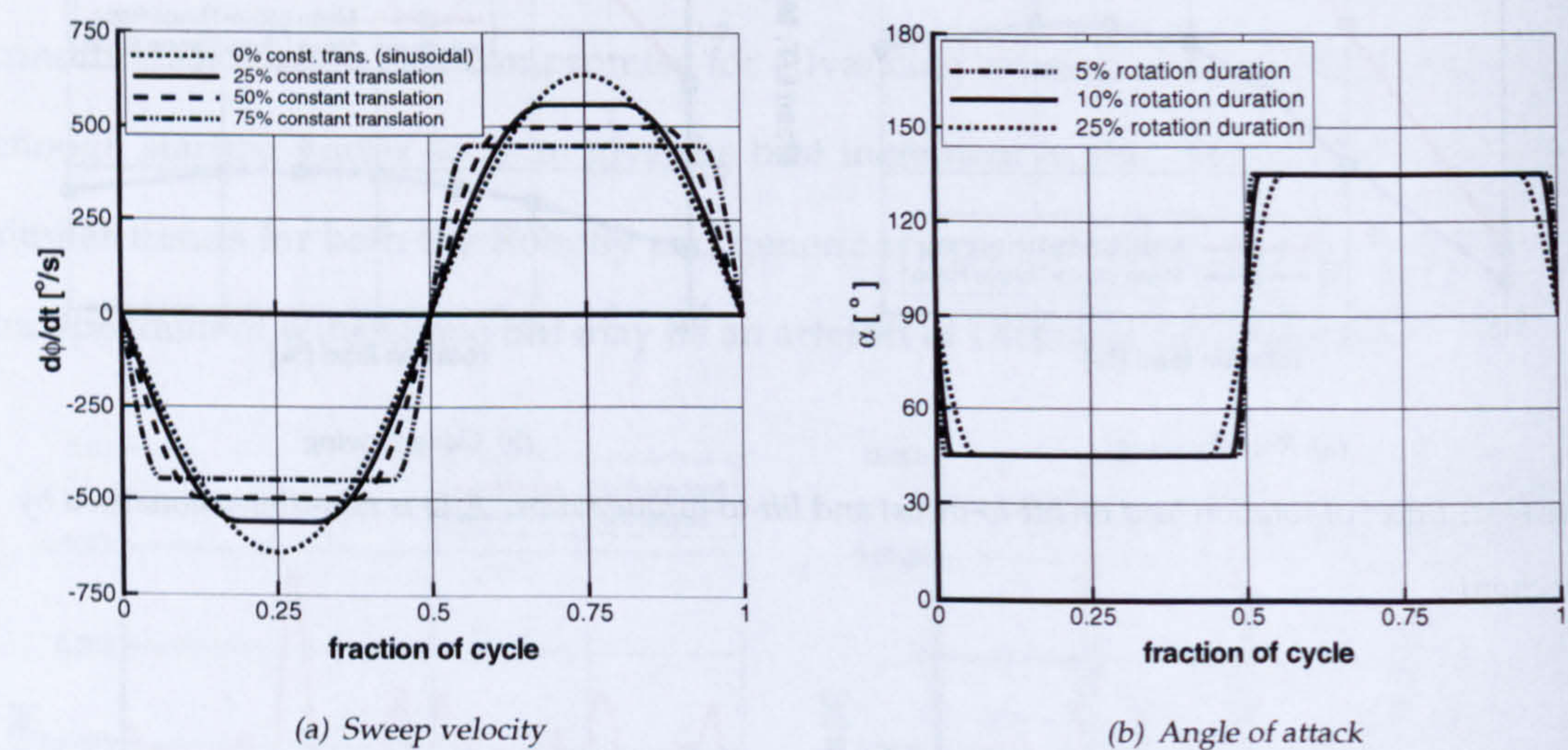


Figure 6.33: Translational and rotational durations used for comparison

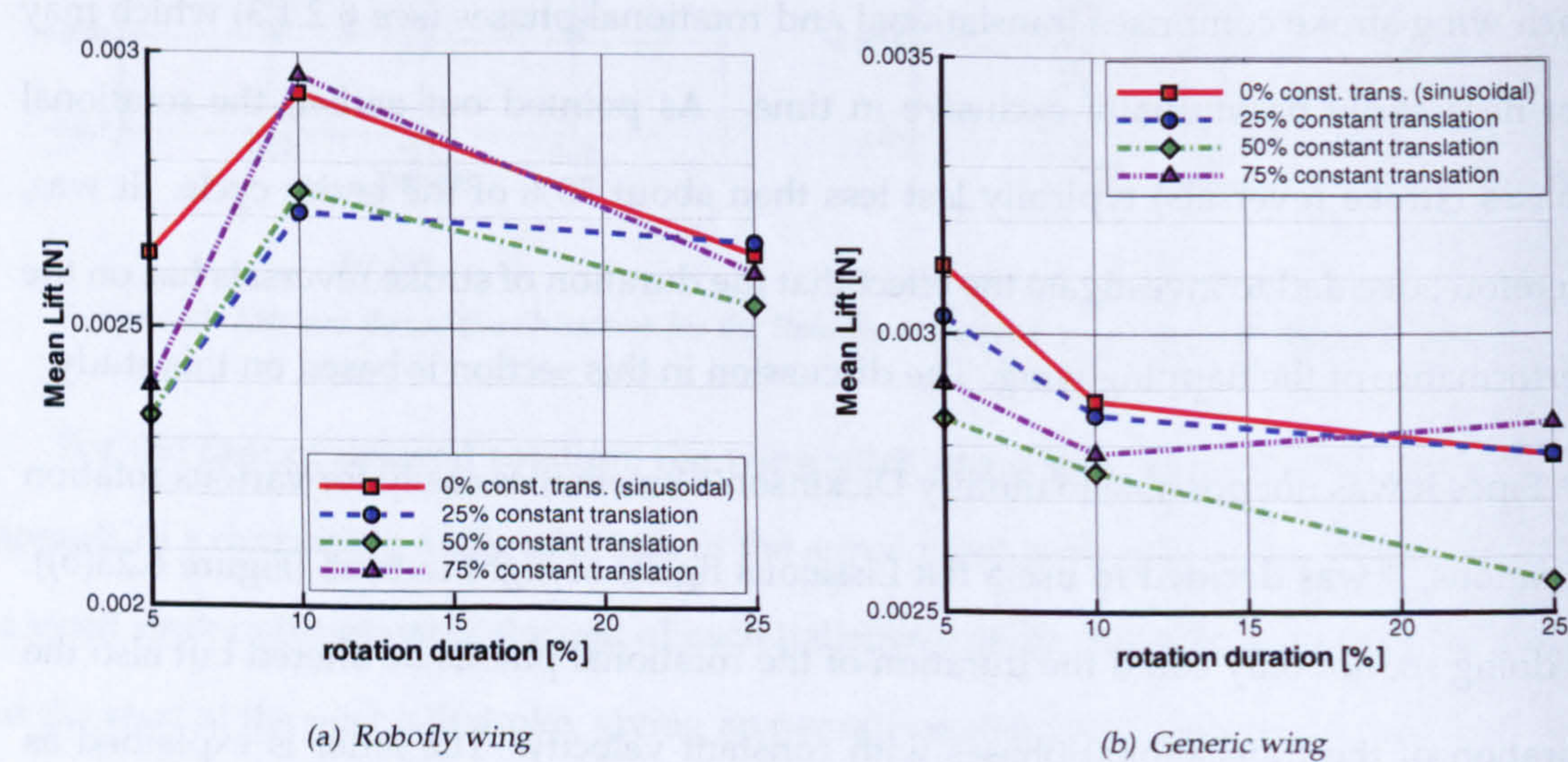


Figure 6.34: Effect of rotation duration on mean lift

Results for mean lift as a function of different rotation durations are shown for both the Robofly and generic wings in Figure 6.34. Two opposing effects are in play. For wings

with faster stroke reversals the starting and stopping vortices generated during stroke reversal are generally ejected further, reducing their impeding effect on lift. At the same time, faster rotation also generates stronger vortices at stroke reversal which impede the growth of lift. Therefore, for the Robofly wing, the best compromise is obtained at 10% rotation duration (Figure 6.34(a)).

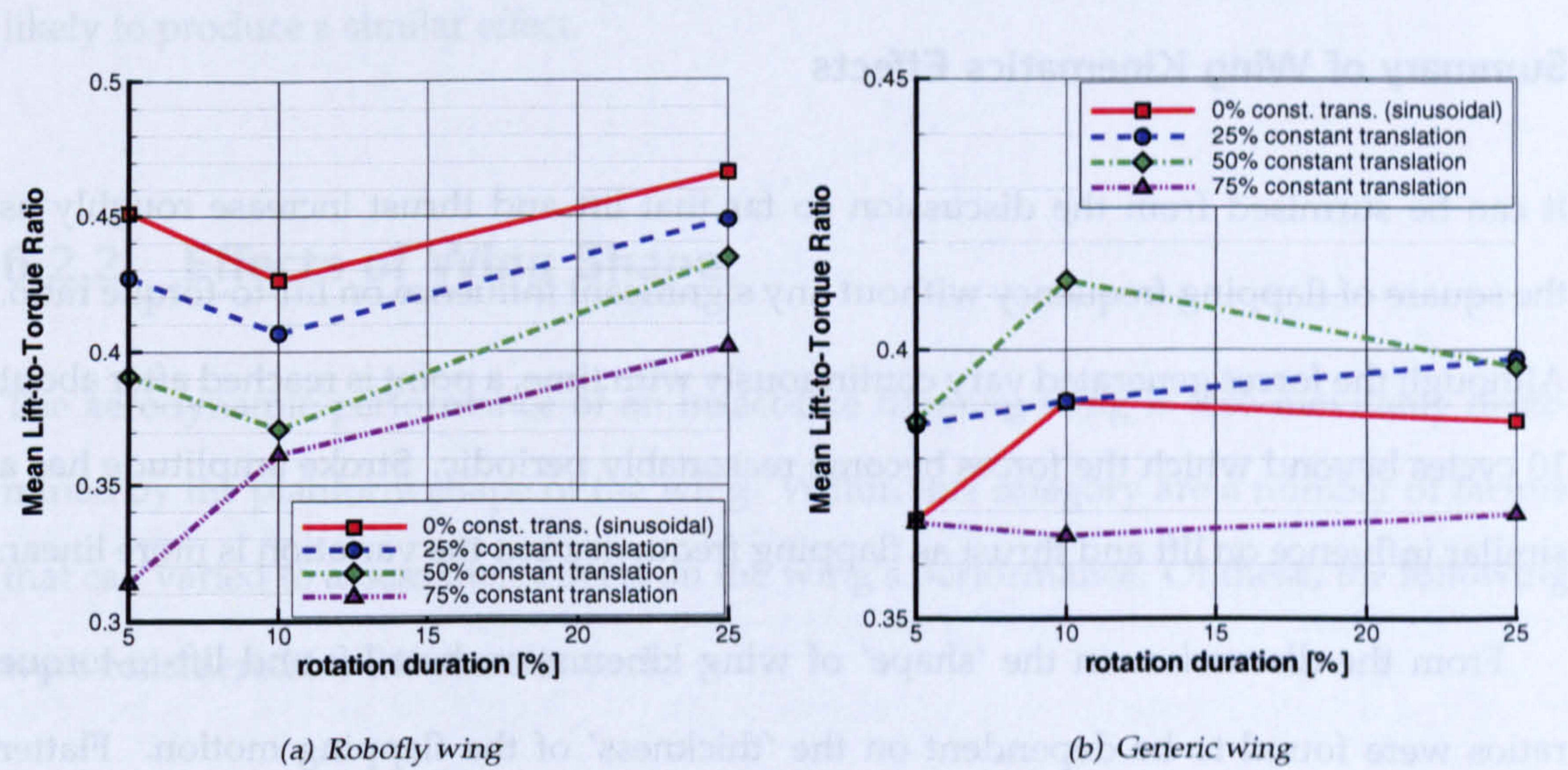


Figure 6.35: Effect of rotation duration on mean lift-to-torque ratio. $\mathcal{L}/Q$ is non-dimensionalised by mean chord

For the generic wing, rotation is more or less about the quarter-chord point whereas for the Robofly the wing pitch axis is much closer to the mid-chord point (see Figure 5.11). Therefore, vortex ejection is more severe for the generic wing. The impeding effect of the strong vortices generated during stroke reversals is better avoided and, therefore, the generic wing shows best lift performance for fast rotations (Figure 6.34(b)).

As discussed earlier, the leaving behind of vortices generally comes with an increased drag (or thrust) penalty (see Footnote 4 on p. 210). Therefore, despite the fact that fast stroke reversals are more beneficial for the generic wing, the best compromise in terms of lift-to-torque ratio $\mathcal{L}/Q$ is achieved for a rotation duration of about 10% (Figure 6.35(b)). For the Robofly wing, best $\mathcal{L}/Q$ occurs for the case of 25% rotation duration (Figure 6.35(a)).

Figures 6.34 and 6.35 also show that the effects of the duration of constant translational

velocity appear to be rather inconclusive in that no immediate pattern can be identified. For the Robofly wing, the mean $\mathcal{L}/Q$ ratio generally improves as the sweep velocity becomes more sinusoidal but for the generic wing no consistent effect can be seen in $\mathcal{L}/Q$ across the range of rotation durations studied.

Summary of Wing Kinematics Effects

It can be surmised from the discussion so far that lift and thrust increase roughly as the square of flapping frequency without any significant influence on lift-to-torque ratio. Although the forces generated vary continuously with time, a point is reached after about 10 cycles beyond which the forces become reasonably periodic. Stroke amplitude has a similar influence on lift and thrust as flapping frequency but the variation is more linear.

From the discussion on the 'shape' of wing kinematics, best lift and lift-to-torque ratios were found to be dependent on the 'thickness' of the flapping motion. Flatter kinematics were generally found to give better performance. The exception to the rule was the upward convex Lissajous kinematics which gave improved performance for fatter kinematics.

It was found that advancing the wing pitch rotation generally increases lift (and lift-to-torque ratio), peaking when rotation leads translation by about 5%. For larger values of rotation lead, however, the effect of the starting vortex shed during rotation outweighed the benefit of increasing angle of attack. This conclusion is drawn for Dickinson's kinematics.

The effect of rotation duration on the aerodynamic performance of a wing was found to be dependent on the position of the pitch axis. Faster rotation seemed to favour wings that pitch about axes closer to the leading edge. Also, kinematics with long phases of constant sweeping velocity were found to perform worse than those with more sinusoidal kinematics.

Finally, for both the Robofly and generic wings used for the preceding parametric study, the lift-to-thrust ratio plots were generally steeper than the corresponding lift-to-torque ratio plots. This would imply that outboard sections made smaller contributions to the forces generated. Although this was achieved in the current quasi-3-D model by means of early vortex breakdown in outboard regions, in reality 3-D tip vortex effects are likely to produce a similar effect.

6.2.2 Effects of Wing Shape

The aerodynamic performance of an insect-like flapping wing is also inevitably determined by the planform shape of the wing. Within this category are a number of factors that can be varied to assess their effects on the wing's performance. Of these, the following were considered for the current work:

- pitch-axis location
- wing length
- aspect ratio
- wing area
- wing offset distance
- wing planform shape

Consider Figure 6.36 below. *Pitch axis location* refers to the position of the spanwise rotational axis of wing defined along the wing section with the largest chord length. The pitch axis itself is a line drawn from the centre of rotation radially outwards. *Wing length* R refers to the length of one of the wing pair, excluding the distance from the centre of rotation of the wing¹⁰. The latter is termed the *wing offset distance*. *Wing area* S refers to the planform area of each wing and *aspect ratio* AR is computed from wing length and wing area thus

$$AR = \frac{4R^2}{2S}$$

¹⁰Note, therefore, that R is not the tip radius or semi-span of the wing pair.

Note that AR here is based on the total length and area of both wings and not on the total span, i.e. the effect of offset distance is excluded. Finally, *wing planform shape* refers to the actual shape of the wing, for example, rectangular, elliptical, etc.

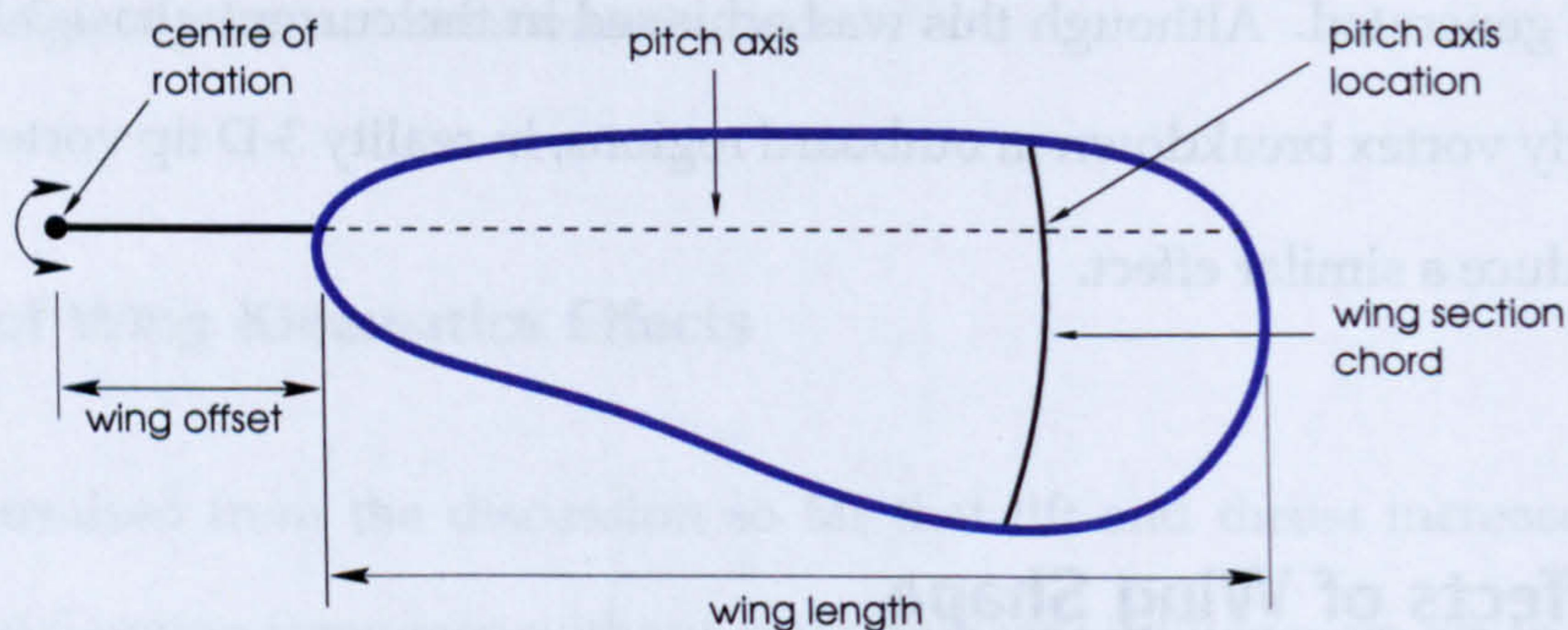


Figure 6.36: Wing shape parameters

For this study, a number of planform shapes were required so that a useful comparison could be made. For purposes of comparison, it was decided to include the planforms investigated in the preliminary quasi-steady work (see Figure 3.4, p. 71). A total of 13 planforms were considered for the investigation and are shown in Figure 6.37 below.

Each of the wing planforms was investigated for its dependence on the shape parameters noted on p. 243. The results from the investigation are now discussed below. For purposes of clarity, the resulting graphs have been subdivided into two groups. The first group comprises the 6 wing sections considered earlier in Chapter 3 (rectangle, triangle, reverse triangle, ellipse, reverse ellipse and full ellipse) and are referred to as the *symmetric* group due to the symmetry of the wing planforms about the spanwise axis through their mid-chords. The second group comprises the 7 additional planforms considered here (semi-ellipse, inverted semi-ellipse, reverse semi-ellipse, inverted reverse semi-ellipse, half ellipse, inverted half ellipse and generic). These are termed the *asymmetric* wings because of the absence of symmetry about any spanwise axis.

As was done for the parametric study on wing kinematics (§ 6.2.1), the computational limitations meant that only one parameter could be varied at a time. The remaining

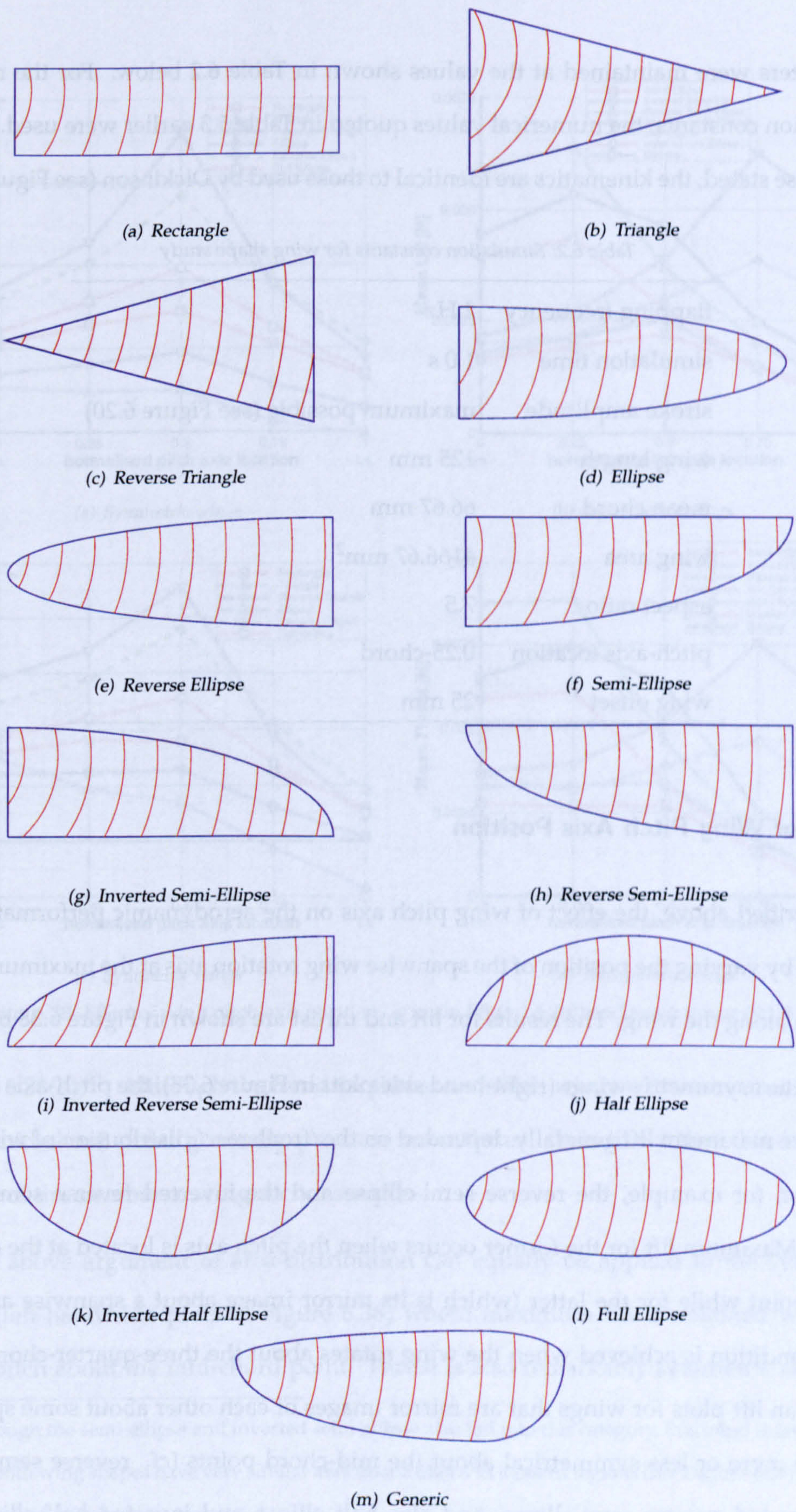


Figure 6.37: Wing planforms considered for parametric study. The centre of rotation (not shown) is to the left in each case

parameters were maintained at the values shown in Table 6.2 below. For the relevant simulation constants, the numerical values quoted in Table 5.3 earlier were used. Unless otherwise stated, the kinematics are identical to those used by Dickinson (see Figure 5.21).

Table 6.2: Simulation constants for wing shape study

flapping frequency	2 Hz
simulation time	1.0 s
stroke amplitude	maximum possible (see Figure 6.20)
wing length	125 mm
mean chord	66.67 mm
wing area	4166.67 mm ²
aspect ratio	7.5
pitch-axis location	0.25-chord
wing offset	25 mm

Effect of Wing Pitch Axis Position

As described above, the effect of wing pitch axis on the aerodynamic performance was studied by varying the position of the spanwise wing rotation axis at the maximum chord location along the wing. The results for lift and thrust are shown in Figure 6.38 below.

For the asymmetric wings (right-hand side plots in Figure 6.38), the pitch-axis location that gave maximum lift generally depended on the ‘front-rear’ distribution of wing area. Consider, for example, the reverse semi-ellipse and the inverted reverse semi-ellipse wings. Maximum lift for the former occurs when the pitch axis is located at the quarter-chord point while for the latter (which is its mirror image about a spanwise axis), the same condition is achieved when the wing rotates about the three-quarter-chord point. The mean lift plots for wings that are mirror images of each other about some spanwise axis are more or less symmetrical about the mid-chord points (cf. reverse semi-ellipse and inverted reverse semi-ellipse, and also half ellipse and inverted half ellipse; see

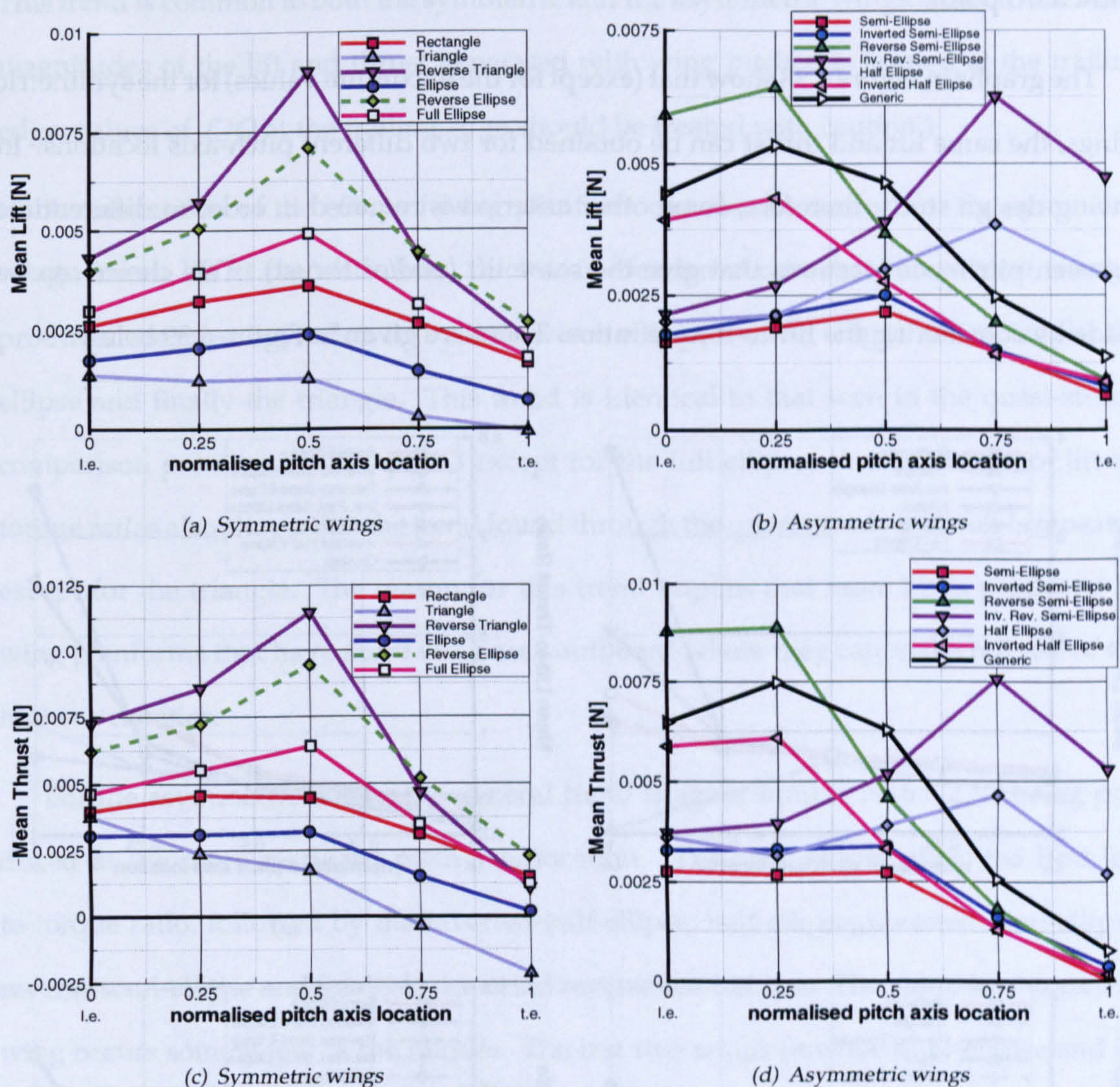


Figure 6.38: Effect of wing pitch-axis position on mean lift ((a) & (b)) and mean thrust ((c) & (d))

Figures 6.38(b))¹¹. Although the trend for thrust is similar, wings with more area forward of the mid-chord point generally produce more thrust. It is likely that this is just an artefact of the number of wing sections used.

The above argument of area-distribution can equally be applied to the symmetric wings (left-hand side plots in Figure 6.38) where maximum lift is obtained when the wings pitch about the mid-chord point. Thrust is also remarkably symmetric about the

¹¹Although the semi-ellipse and inverted semi-ellipse also fall into this category, this trend is less marked because both wing shapes have very similar area distributions in inboard regions (see Figures 6.37(f) and (g)) where most of the forces are generated.

mid-chord point.

The graphs in Figure 6.38 show that (except for the maximum values) for the symmetric wings, the same lift and thrust can be obtained for two different pitch-axis locations. In a wing-design study, therefore, some other criterion is required in order to differentiate between pitch-axis locations that give the same lift (and/or thrust). This choice can be made by considering the lift-to-torque ratios. These are given in Figure 6.39 below.

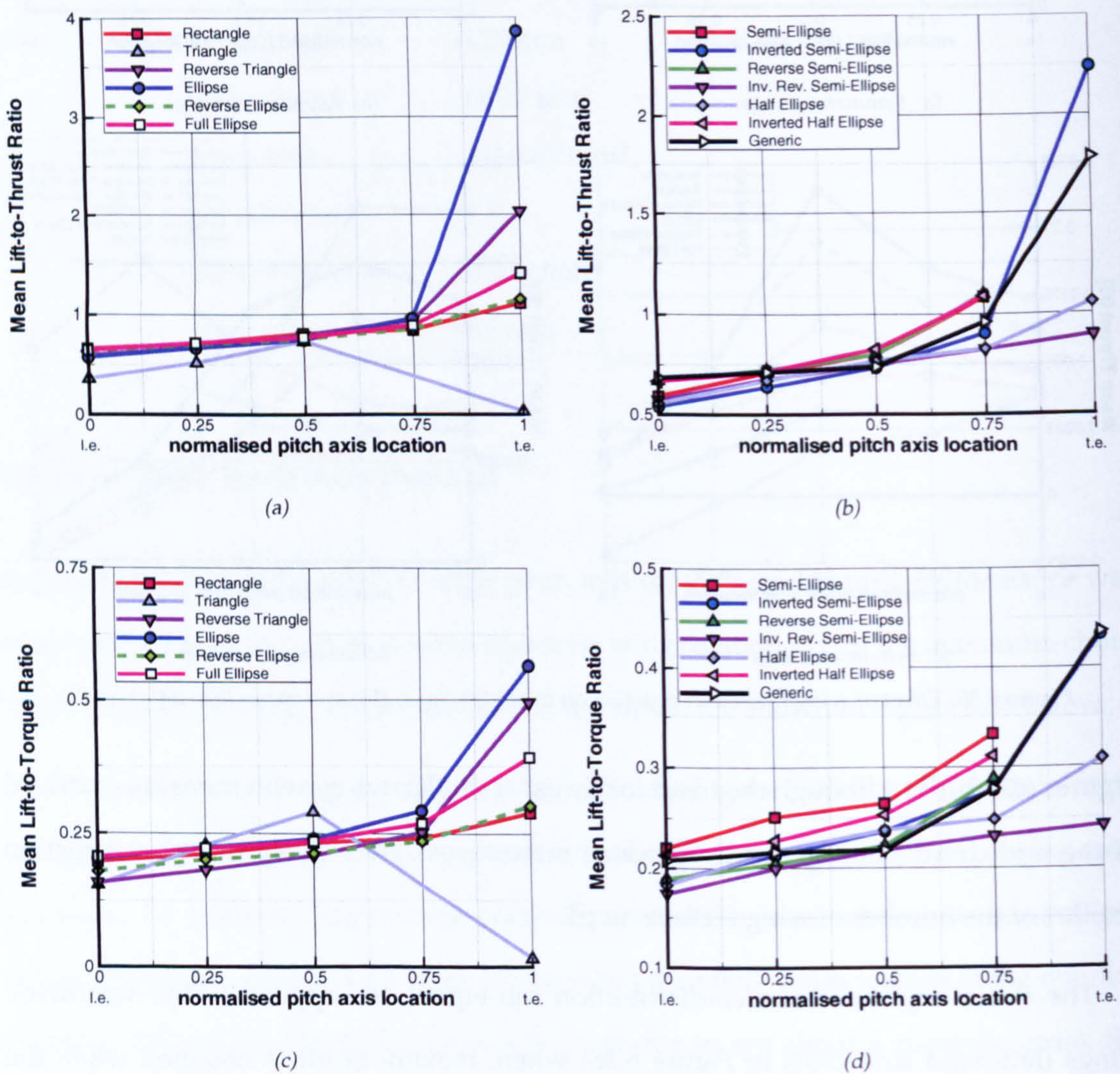


Figure 6.39: Effect of wing pitch-axis position on mean lift-to-thrust ratio ((a) & (b)) and mean lift-to-torque ratio ((c) & (d)). Some values have been omitted to preserve axis scales. $\mathcal{L}/Q$ is non-dimensionalised by mean chord

The trend in mean $\mathcal{L}/Q$ shows that rearward pitch axes are generally more beneficial.

This trend is common to both the symmetric and the asymmetric wings. (Due to the small magnitudes of the lift and thrust generated with wing pitch axis located at the trailing edge, values of $\mathcal{L}/Q$ at the trailing edge should be treated with caution.)

A comparison of the effect of wing planform can also be made. For the symmetric wings, mean lift at the mid-chord pitch-axis location shows that the reverse triangle produces the most lift, followed by the reverse ellipse, the full ellipse, the rectangle, the ellipse and finally the triangle. This trend is identical to that seen in the quasi-steady comparison presented in Chapter 3 except for the full ellipse. A comparison of lift-to-torque ratios also shows that the trend found through the quasi-steady analysis is repeated except for the triangle. The reason for this trend implies that more lift is produced by wing planforms that have more wing area outboard where they can make full use of the higher velocities.

For the asymmetric wings, the general trend is again similar, better $\mathcal{L}/Q$ being produced at the more rearward pitch-axis location. The semi-ellipse gives the best lift-to-torque ratio, followed by the inverted half ellipse, half ellipse, inverted semi-ellipse, reverse semi-ellipse and finally by inverted reverse semi-ellipse. The value for the generic wing occurs somewhere in the middle. The last two wings (reverse semi-ellipse and inverted reverse semi-ellipse) have large chord lengths in outboard sections. Whereas this is good from a lift perspective, these wing planforms also generate high drag (or thrust). Hence, they have the lowest $\mathcal{L}/Q$. Apart from the generic wing planform, the remaining four planforms have similar chord distributions with near-zero chords in outboard regions. Most of the lift and thrust are generated inboard (see the section on wing loading on p. 262), and hence, they have better $\mathcal{L}/Q$ characteristics.

Effect of Wing Length

The effect of wing length was investigated by varying wing length R while keeping wing area S constant. In doing so, the aspect ratio AR was changing. Therefore, longer wings

were thinner while shorter wings were stubbier. Wing offset distance was kept constant at 25 mm and the wing pitch axis was maintained at the quarter-chord point throughout. The results from the study are given in Figures 6.40 and 6.41 below.

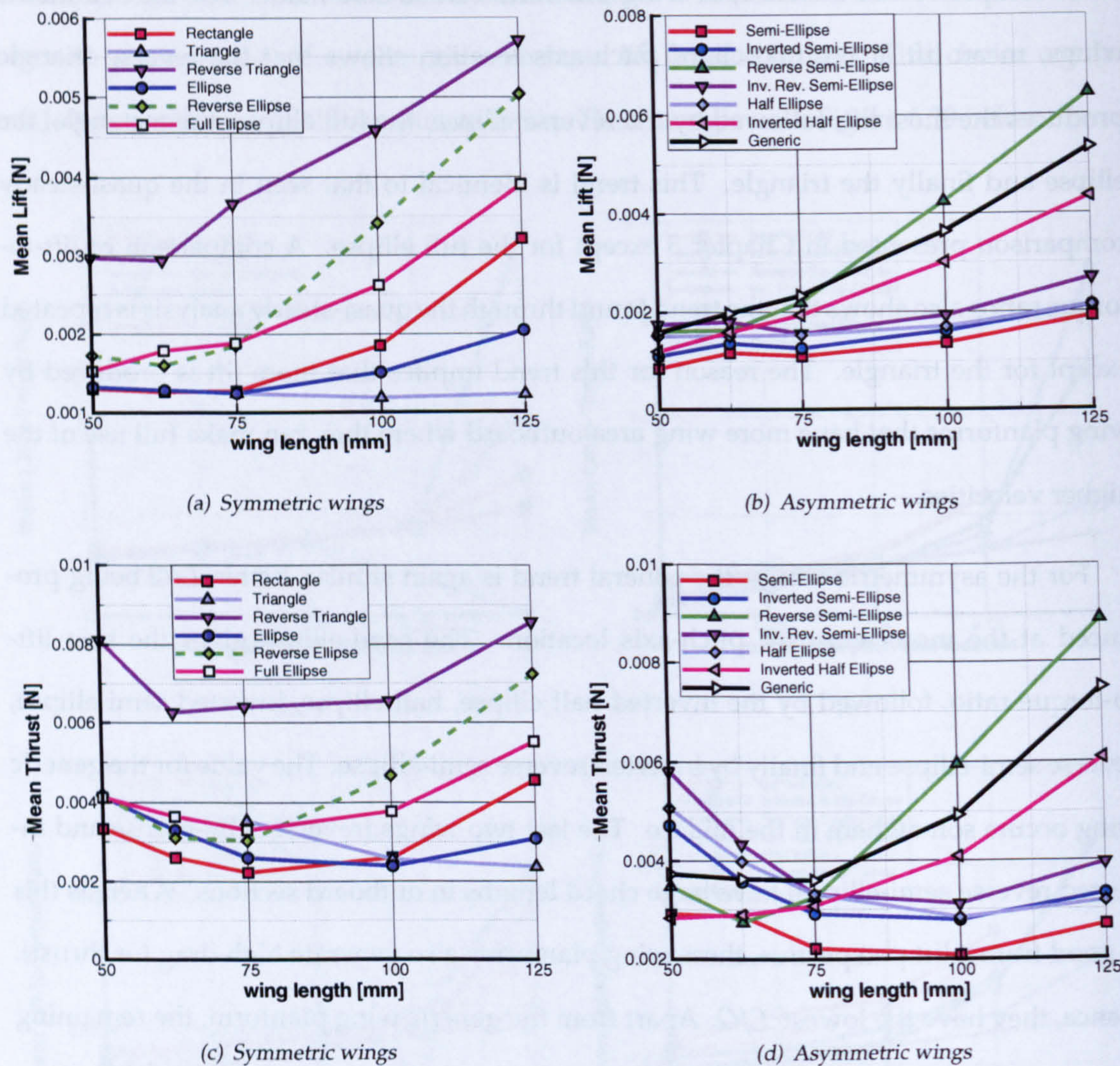


Figure 6.40: Effect of wing length on mean lift ((a) & (b)) and mean thrust ((c) & (d))

Consider the lift characteristics shown in Figures 6.40(a) and (b). From a quasi-steady perspective, doubling wing length would quadruple lift¹². The general trend

¹²Consider the case of a rectangular wing with chord c . The total lift on the wing with wing length R is

$$L_R = \frac{1}{2} \rho C_L \int_0^R (\omega r)^2 c \, dr = \frac{1}{2} \rho C_L \omega^2 c \frac{R^3}{3}$$

where ρ is air density and ω is sweeping angular velocity. If the wing length were now doubled to $2R$, the

observed here, however, is that lift is more or less linearly related to the wing length. This observation results from the fact that as wing length is increasing, chord lengths are decreasing while being exposed to increasing flow velocities. This results in more wing sections which are 'old' in terms of chord lengths travelled. As described earlier, such 'older' flows are more prone to vortex breakdown and hence lower lift. Therefore, as wing length increases, not only does the quasi-steady component of lift $\mathcal{L}_0$ increase but also the hindering wake $\mathcal{L}_3$ and wake-induced $\mathcal{L}_2$ components. Hence, the relationship between wing length is more linear than quadratic.

Another noteworthy feature from the plots is that for wing lengths in the range 50–75 mm, most wing planforms show almost no variation in lift. For this range, the increase in lift due to the increasing velocities is balanced out by the increase in vortex breakdown and the associated drop in lift. However, as wing length increases further, the effect of velocity becomes much more dominating and lift begins to rise.

Lift-to-thrust ratios (Figures 6.41(a) and (b)) appear to plateau beyond a wing length of 100 mm, implying that any further increases in wing length generate proportionate amounts of lift and drag (or thrust). Whereas lift-to-thrust ratios increase with wing length, the lift-to-torque ratios show a general decline beyond a wing length of 75 mm (Figures 6.41(c) and (d)). This implies that as wing length increases, the outboard sections are increasingly responsible for most of the drag on the wing which results in greater torque, and hence, lower $\mathcal{L}/Q$.

For most of the wing shapes considered, lift-to-torque ratio peaks for wing lengths in the range of 62.5–75.0 mm. The lift force $\mathcal{L}$ in this region is, however, quite low. Therefore,

chord length drops to $\frac{c}{2}$ in order to keep wing area constant. The new lift then becomes

$$L_{2R} = \frac{1}{2} \rho C_L \int_0^{2R} (\omega r)^2 \frac{c}{2} dr = 4 \cdot \frac{1}{2} \rho C_L \omega^2 c \frac{R^3}{3}$$

Therefore, doubling wing length quadruples lift.

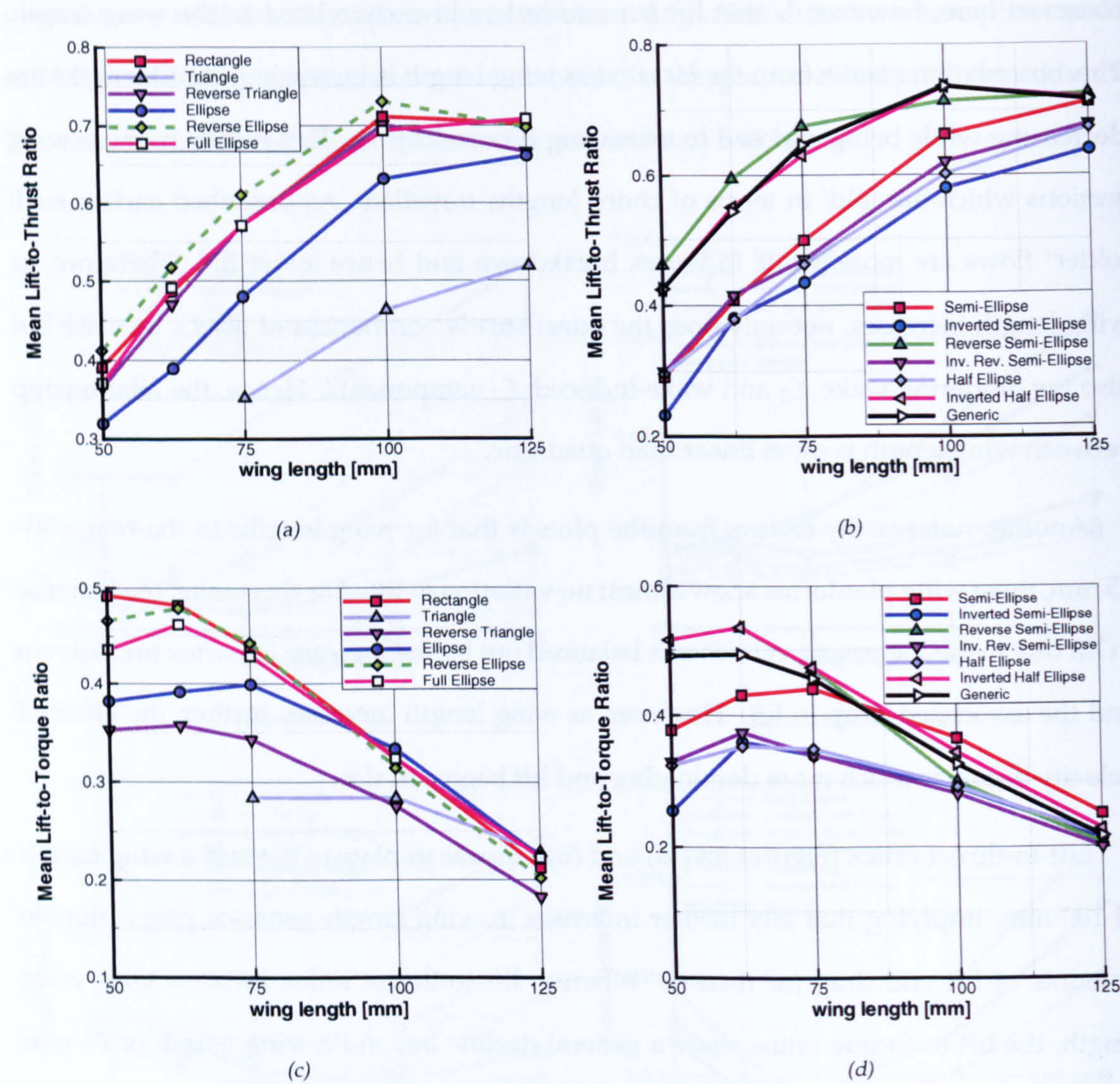


Figure 6.41: Effect of wing length on mean lift-to-thrust ratio ((a) & (b)) and mean lift-to-torque ratio ((c) & (d)). Some values have been omitted to preserve axis scales. $\mathcal{L}/Q$ is non-dimensionalised by mean chord

these high $\mathcal{L}/Q$ ratios are unlikely to be useful. A more useful plot for wing design would, therefore, be a graph of $\mathcal{L}/Q$ against $\mathcal{L}$. This is shown later (see Figure 6.48) where it is discussed in more detail.

Effect of Wing Aspect Ratio

As in the preceding study, the effect of varying aspect ratio¹³ is investigated by keeping wing area constant. In doing so, therefore, wing length increases. The results from this study are consequently very similar to those in the preceding section, the only difference being that they capture a different range of wing lengths. The results are shown in Figures 6.42 and 6.43 below.

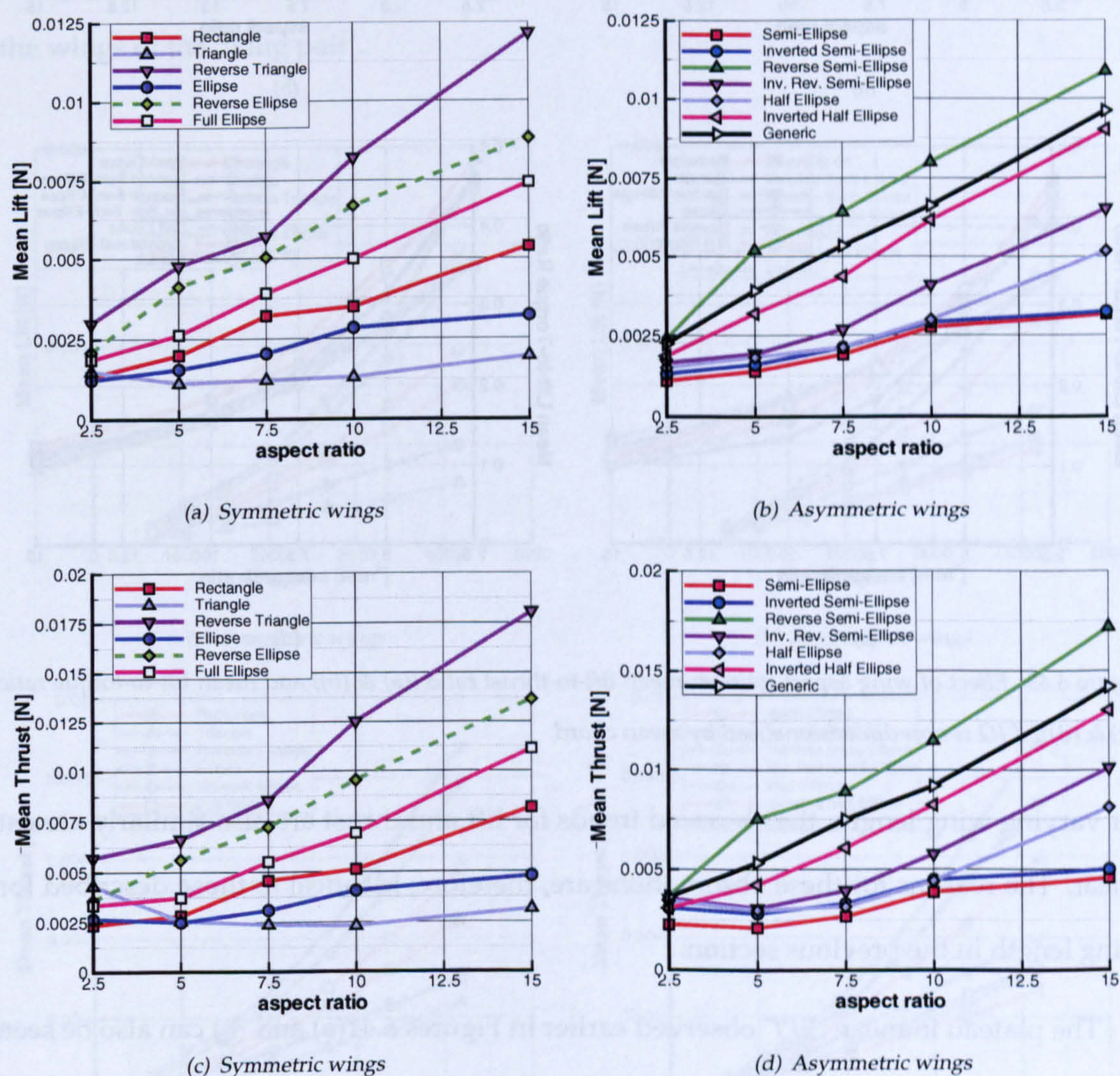


Figure 6.42: Effect of wing aspect ratio on mean lift ((a) & (b)) and mean thrust ((c) & (d))

Because these simulations with varying aspect ratio are similar to the previous ones

¹³The definition of aspect ratio used here is as given in Equation 5.24 (p. 168).

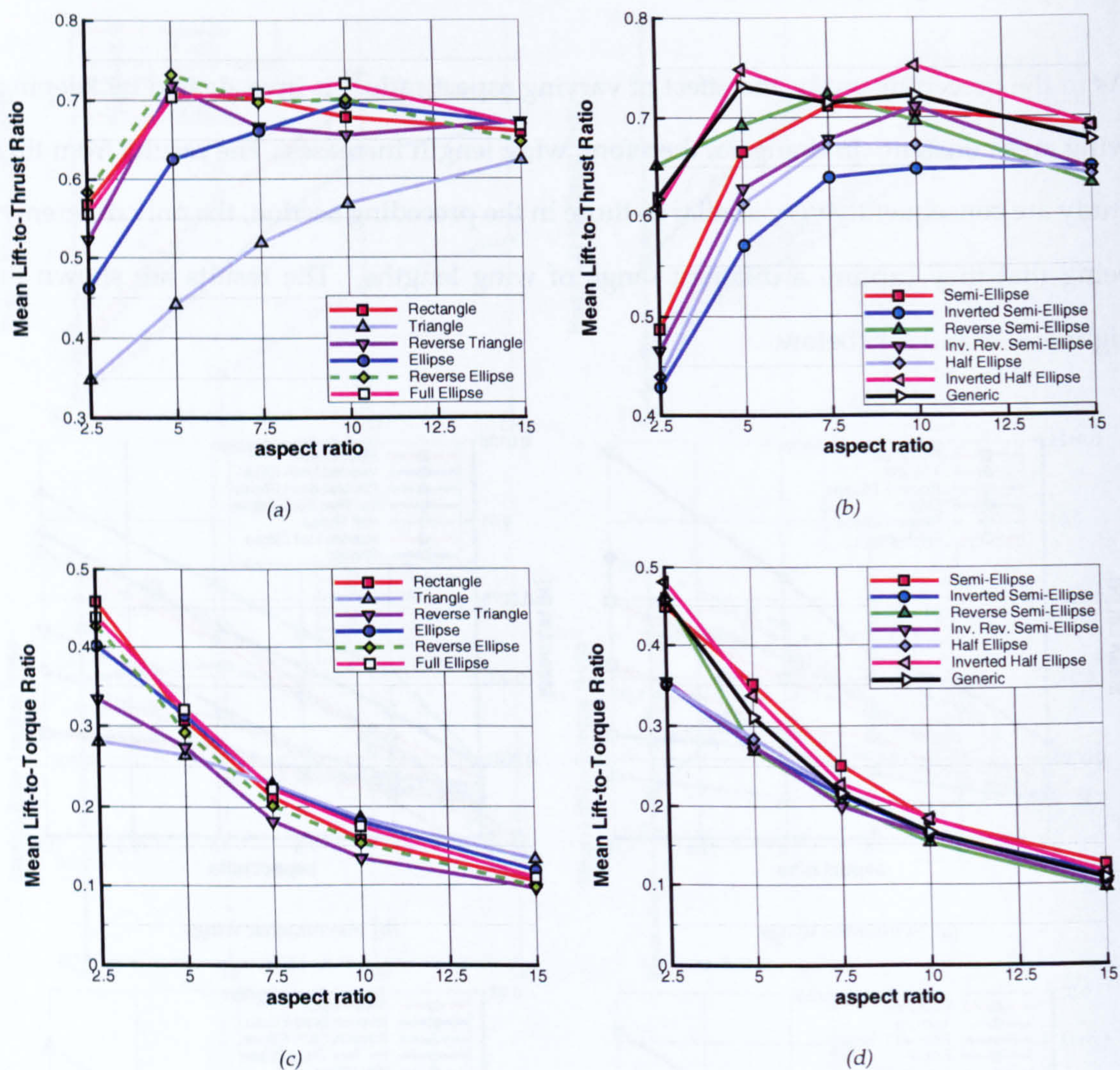


Figure 6.43: Effect of wing aspect ratio on mean lift-to-thrust ratio ((a) & (b)) and mean lift-to-torque ratio ((c) & (d)). $\mathcal{L}/Q$ is non-dimensionalised by mean chord

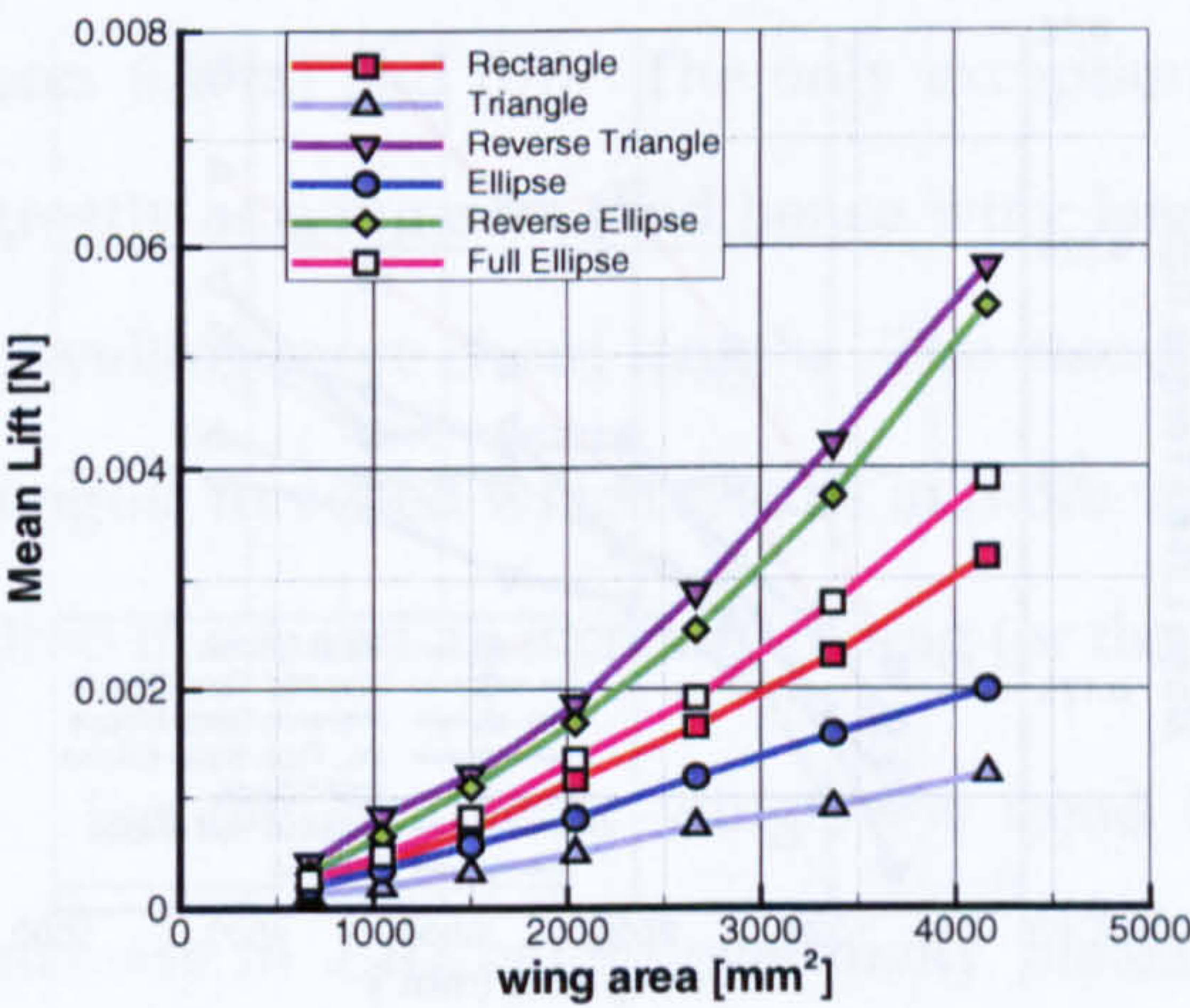
for varying wing length, the observed trends for lift and thrust are also similarly almost linear. The reasons for these observations are, therefore, identical to those described for wing length in the previous section.

The plateau in mean $\mathcal{L}/\mathcal{T}$ observed earlier in Figures 6.41(a) and (b) can also be seen in Figures 6.43(a) and (b) albeit shifted to the left because wing lengths are greater in the latter cases. Further, the flattening of the plots for $\mathcal{L}/Q$ shown in Figures 6.43(c) and (d) show that the benefit of increasing aspect ratio (and for that matter, wing length) is met with diminishing returns — the drag penalty would eventually outweigh the increase in

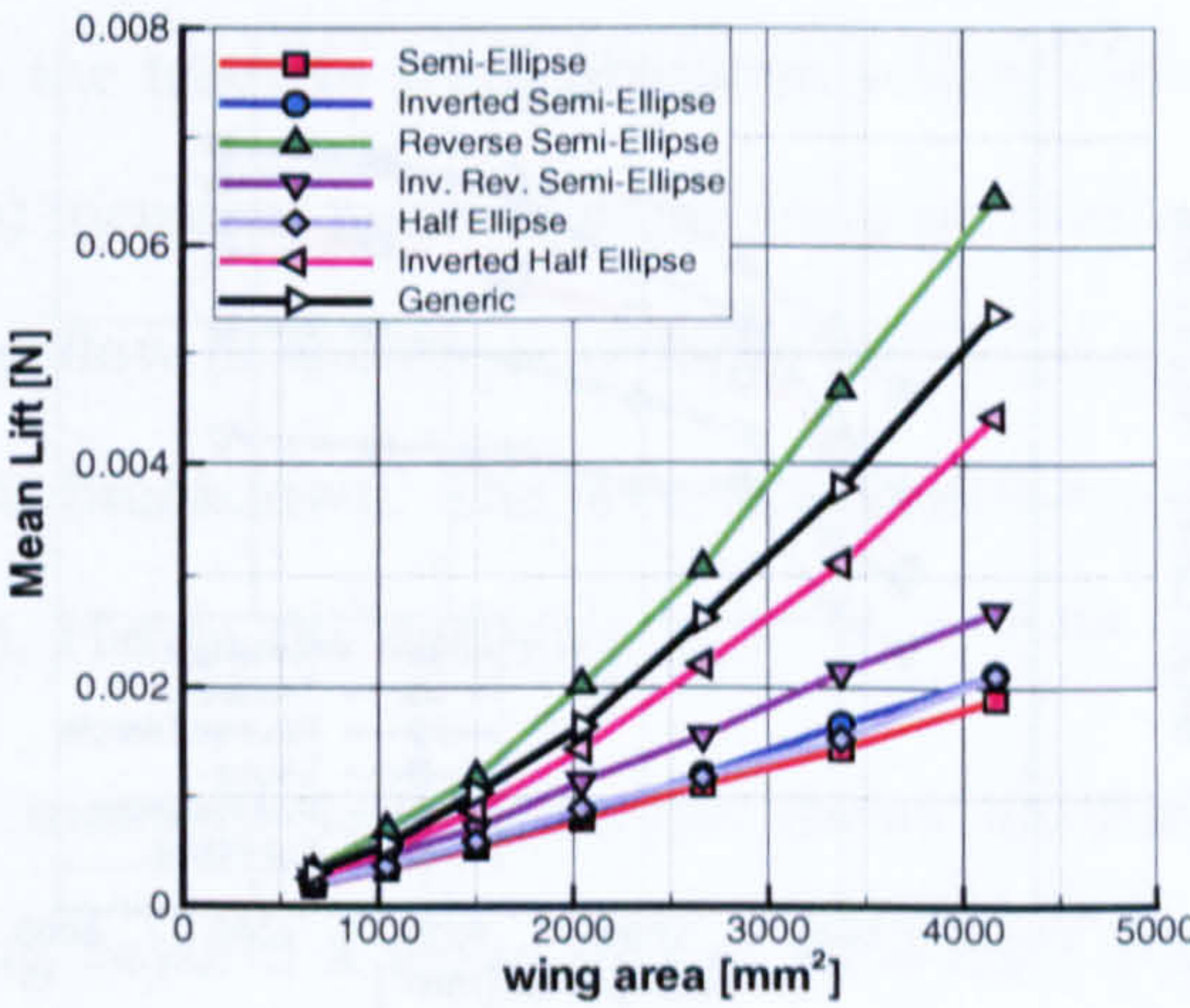
lift.

Effect of Wing Area

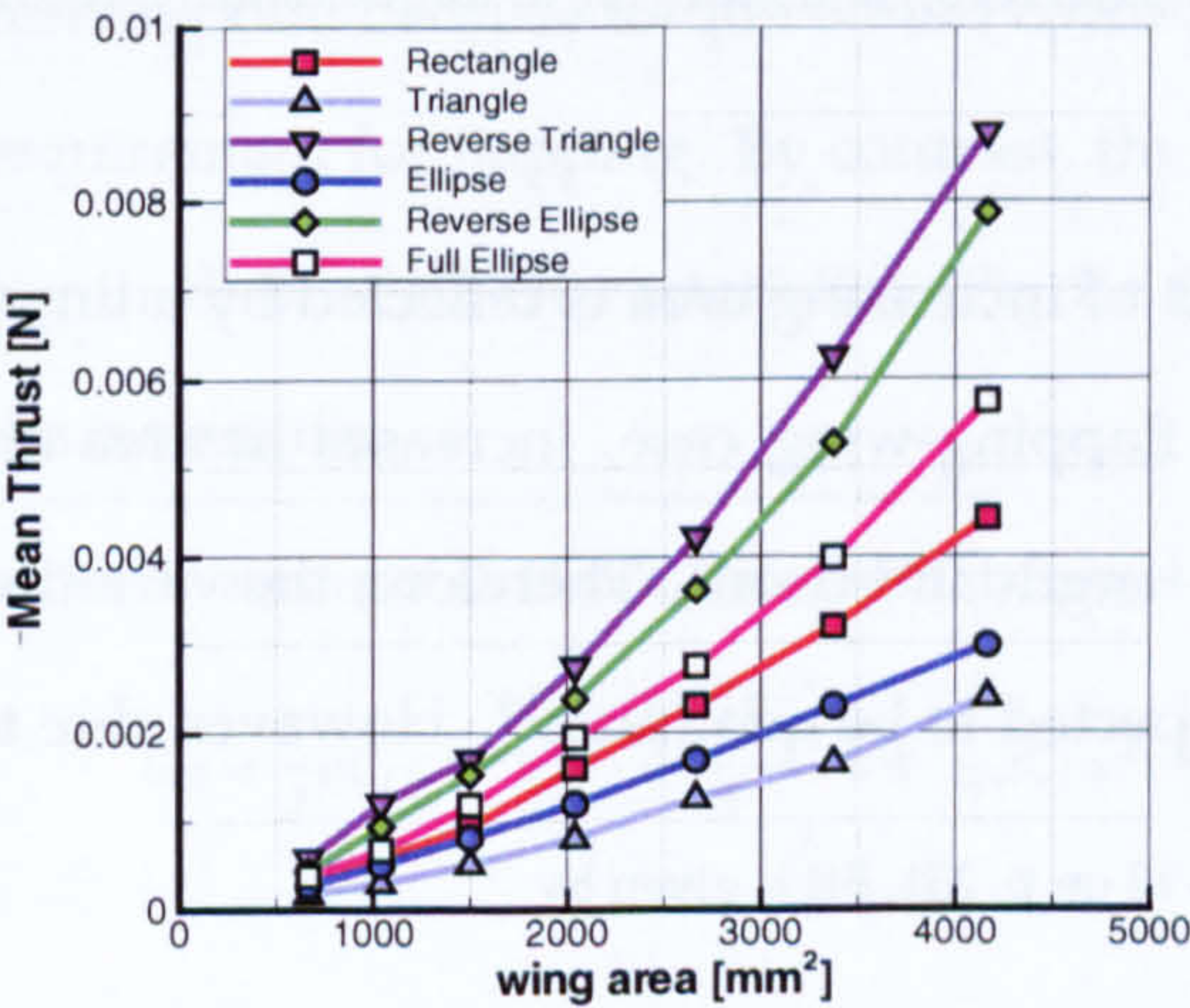
In order to investigate the effect of wing area on wing performance, aspect ratio was kept constant so that the ‘shape’ of the wing remained unchanged. Variations in wing area were achieved by varying wing length and chord. The results from this study are shown in Figures 6.44 and 6.45 below. Wing area here refers to the planform area of only one of the wings of the ‘wing pair’.



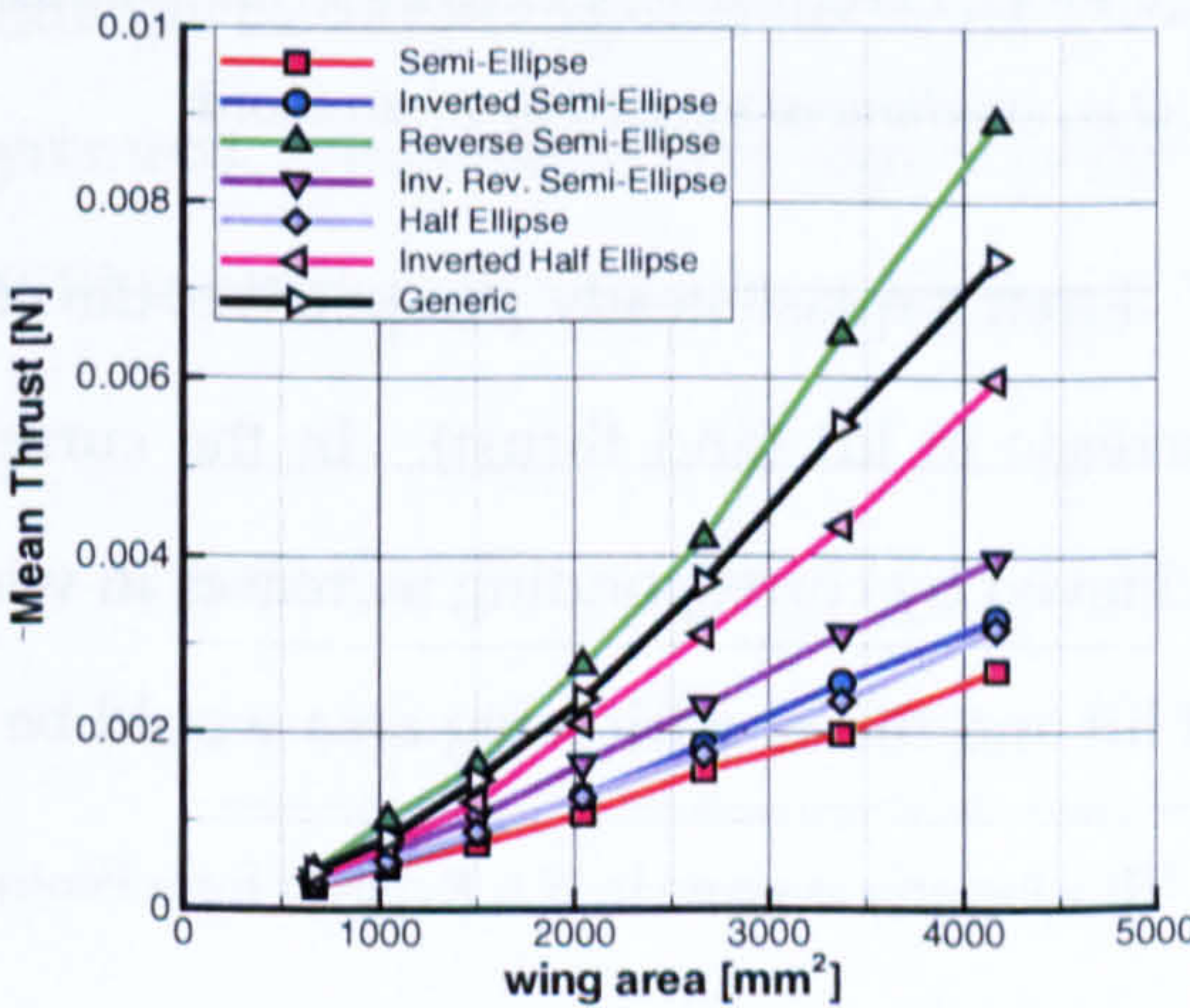
(a) Symmetric wings



(b) Asymmetric wings



(c) Symmetric wings



(d) Asymmetric wings

Figure 6.44: Effect of wing area on mean lift ((a) & (b)) and mean thrust ((c) & (d))

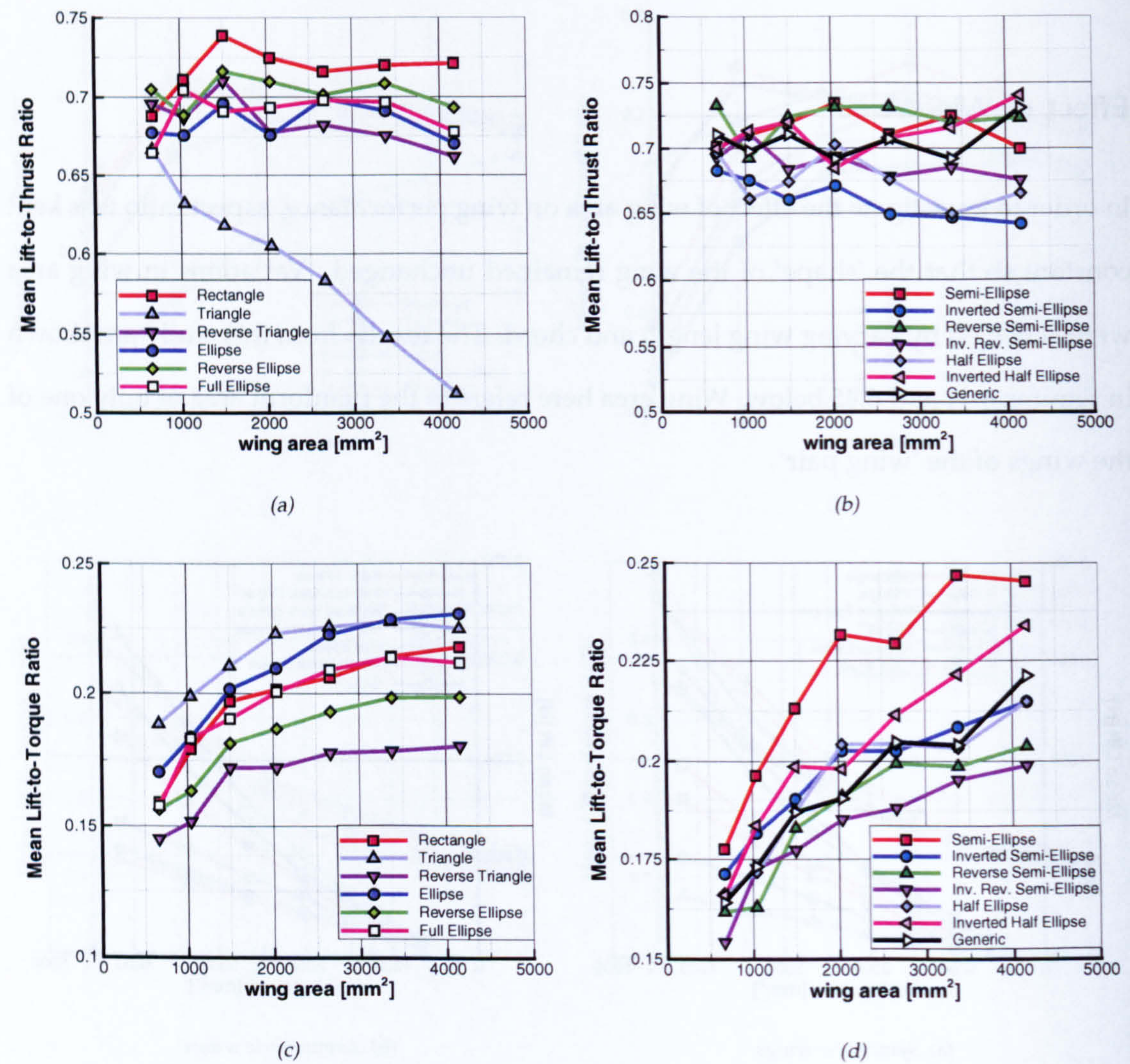


Figure 6.45: Effect of wing area on mean lift-to-thrust ratio ((a) & (b)) and mean lift-to-torque ratio ((c) & (d)). $\mathcal{L}/Q$ is non-dimensionalised by mean chord

From a quasi-steady perspective, the effect of increasing area is reflected by a linear increase in lift (and thrust). In the current flapping-wing case, increases in area are achieved by corresponding increases in wing length and chord. Therefore, the variation of lift and thrust with wing area would be expected to be quadratic¹⁴. However, due to

¹⁴If wing area is given by $S = Rc$, then from Footnote 12 on p. 251, lift is given by

$$L_S = \frac{1}{2} \rho C_L \omega^2 c \frac{R^3}{3}$$

If wing area is now doubled, wing length and chord increase as $2S = \sqrt{2}R \cdot \sqrt{2}c$. Lift varies linearly with c

the increasing breakdown of flow in the outboard regions as wing length increases, the actual relationship between wing area and lift is not quite as expected.

Unlike the variation with wing length, this relationship between wing area and lift appears to be more quadratic than linear. This is because wing length R and wing area S are related as $S \propto R^2$ (because aspect ratio is constant) so that variations in wing area are achieved by smaller variations in wing length. Hence, flow physics do not change as much with variations in wing area¹⁵.

Because the 'shape' of the wing planforms is preserved by keeping aspect ratio constant throughout, the lift-to-thrust ratio is more or less constant throughout (Figures 6.45(a) and (b)). The only exception is the triangle wing planform which suffers greatly as wing area (and hence wing length) increases because more wing sections are travelling more chord lengths. The associated flow is, therefore, older in terms of chord lengths travelled which results in more vortex breakdown. The latter is manifested as a drop in lift and an increase in drag (or thrust). Hence, the declining $\mathcal{L}/\mathcal{T}$.

For the symmetric wings, the trend for mean lift-to-torque ratio shows an initial increase in $\mathcal{L}/Q$ before eventually plateauing beyond a wing area of 2000 mm² (Figure 6.45(c)). Therefore, it is not beneficial to continue increasing wing area beyond a certain point because torque would correspondingly be increasing, and hence the power requirement for flapping. By contrast, the asymmetric wings show little sign of plateauing in their $\mathcal{L}/Q$ characteristics (Figure 6.45(d)) even though they show similar $\mathcal{L}/\mathcal{T}$ characteristics.

but as the square of R . Hence, the new lift becomes

$$L_{2S} = \frac{1}{2} \rho C_L \omega^2 \cdot \sqrt{2}c \cdot \frac{(\sqrt{2}R)^3}{3} = 4 \cdot \frac{1}{2} \rho C_L \omega^2 c \frac{R^3}{3}$$

¹⁵If the graphs in Figure 6.44 were plotted against the square root of wing area, a similar trend to that shown in Figure 6.40 would result.

Effect of Wing Offset

The aim behind this exercise was to assess the relative merit of increasing wing offset distance while staying within the 150 mm box for a MAV. Consider Figure 6.36. Wing offset distance was varied in the range 25–75 mm and the wing length was then adjusted accordingly ensuring that the wing tip did not extend further than 150 mm from the wing's centre of rotation, i.e.

$$\text{wing length} = 150 \text{ mm} - \text{wing offset}$$

To enable a fair comparison, wing area was kept constant so that aspect ratio changed with wing offset distance.

The effect of increasing wing offset distance was, therefore, to move the wing further away so that a bigger arc could be swept. However, the length of this arc was limited by the increasing chord length required to keep wing area constant. The results from the study are shown in Figures 6.46 and 6.47 below.

Lift and thrust increase almost linearly with increasing wing offset distance (Figure 6.46). This results from the fact that as the wing offset distance increases, more wing area is pushed further outboard where flow velocities are higher¹⁶.

Whereas increasing wing length (see above) resulted in increasingly more outboard sections becoming prone to vortex breakdown, the effect of increasing wing offset distance

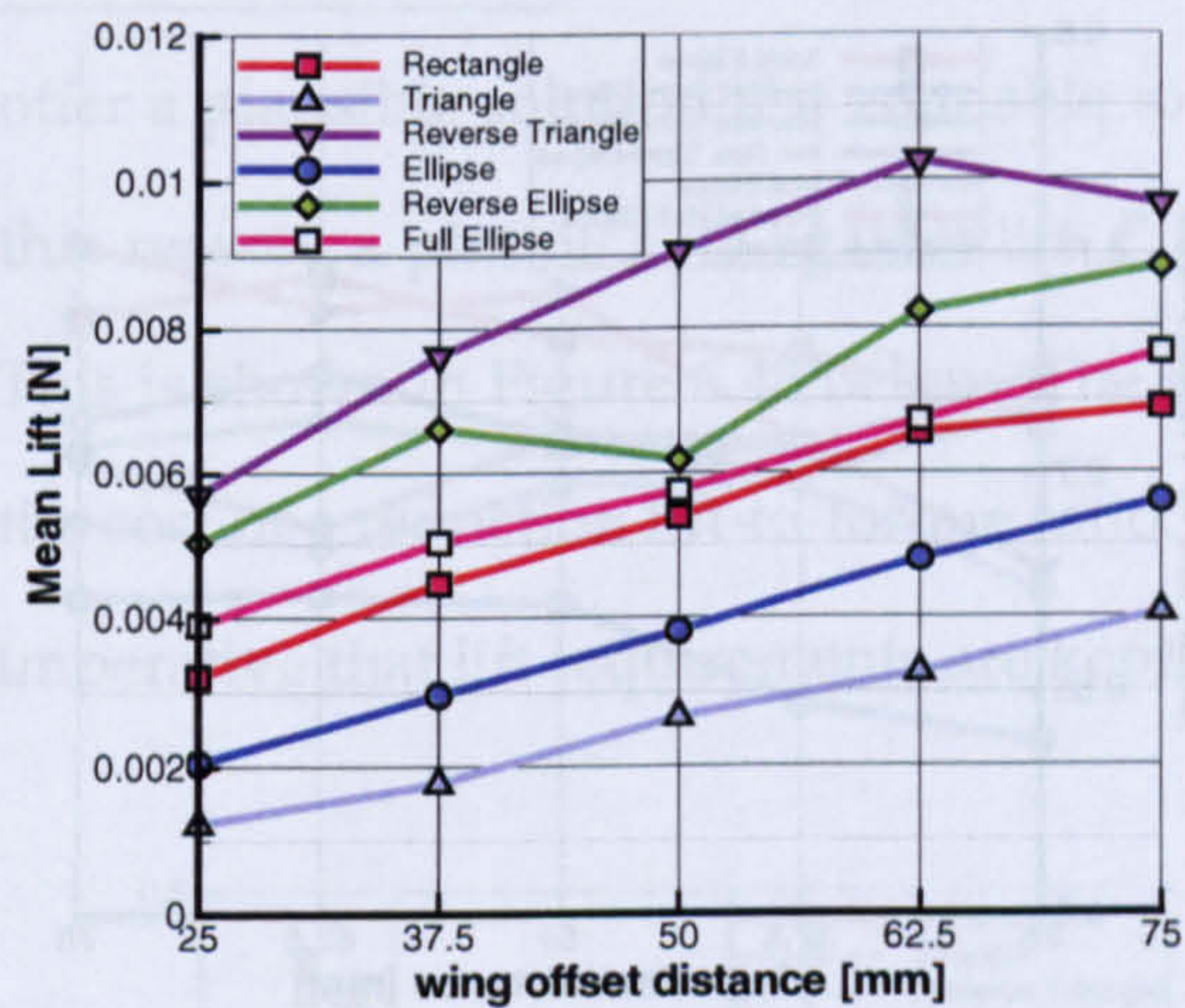
¹⁶This may be explained by the following simple analysis. Assume that the constant wing area to be maintained is S and that mean chord is c . If wing offset distance is d and wing extent (radial distance from centre of rotation to wing tip) is l , then mean chord may be written, thus

$$c = \frac{S}{l-d} \quad (\text{wing length } R = l - d)$$

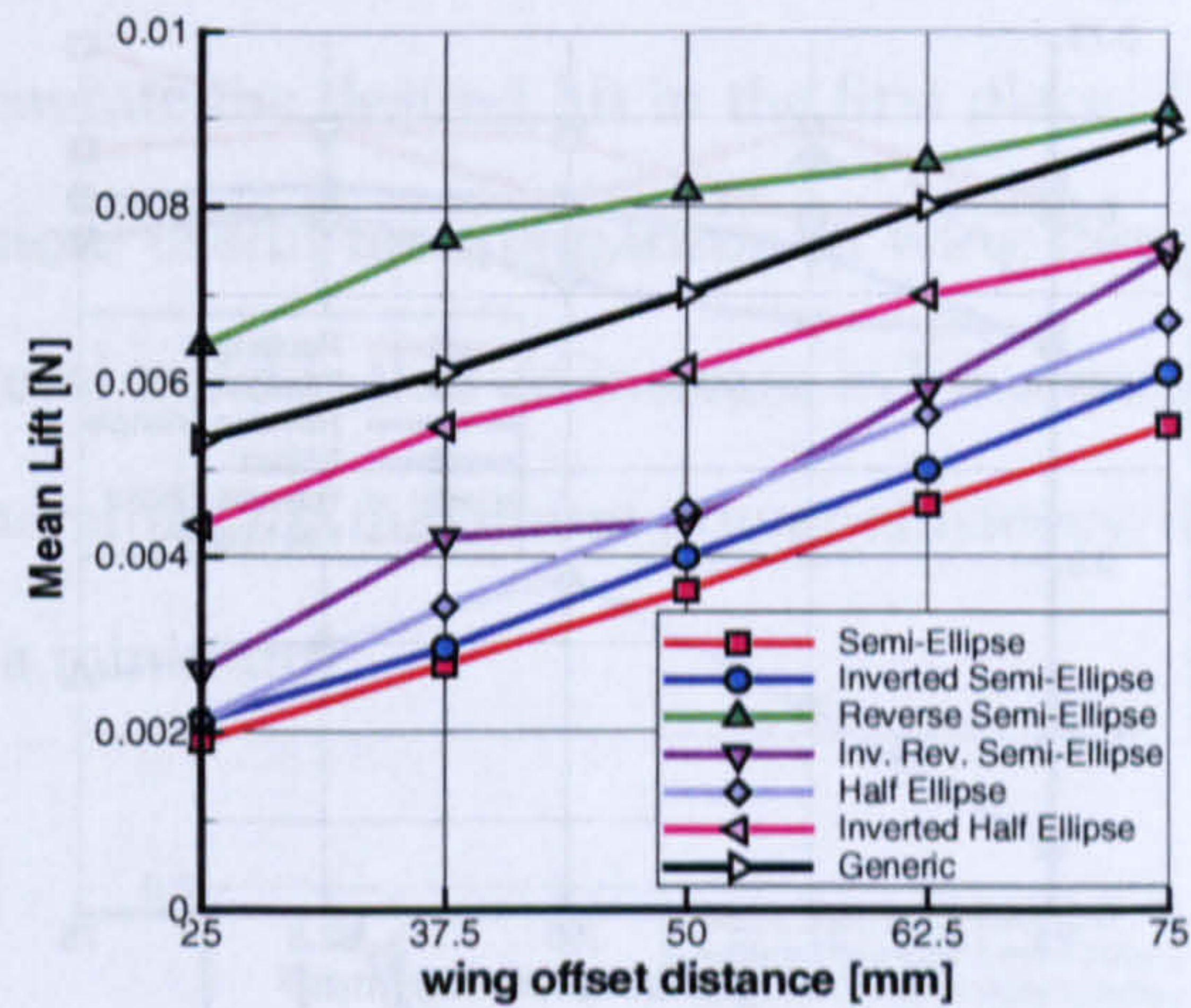
For a rectangular wing, therefore, lift is

$$\mathcal{L} = \frac{1}{2} \rho C_L \int_d^l (\omega r)^2 \frac{S}{l-d} dr = \frac{1}{2} \rho \omega^2 C_L \frac{S}{l-d} \left(\frac{l^3}{3} - \frac{d^3}{3} \right)$$

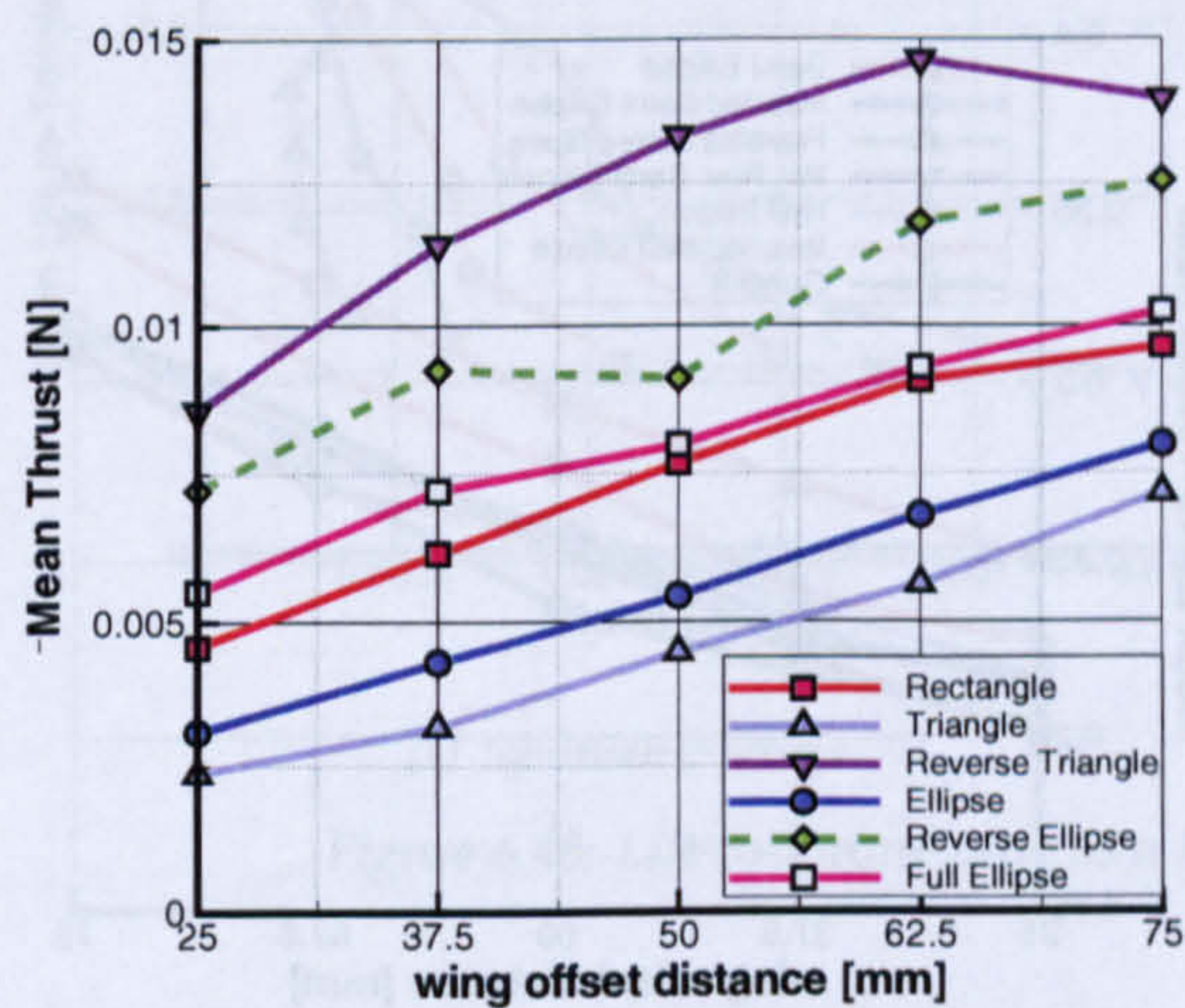
where ω is flapping angular velocity, ρ is air density and C_L is lift coefficient. The above expression yields an almost linear relationship between $\mathcal{L}$ and d in the region of interest ($0 < d < l$).



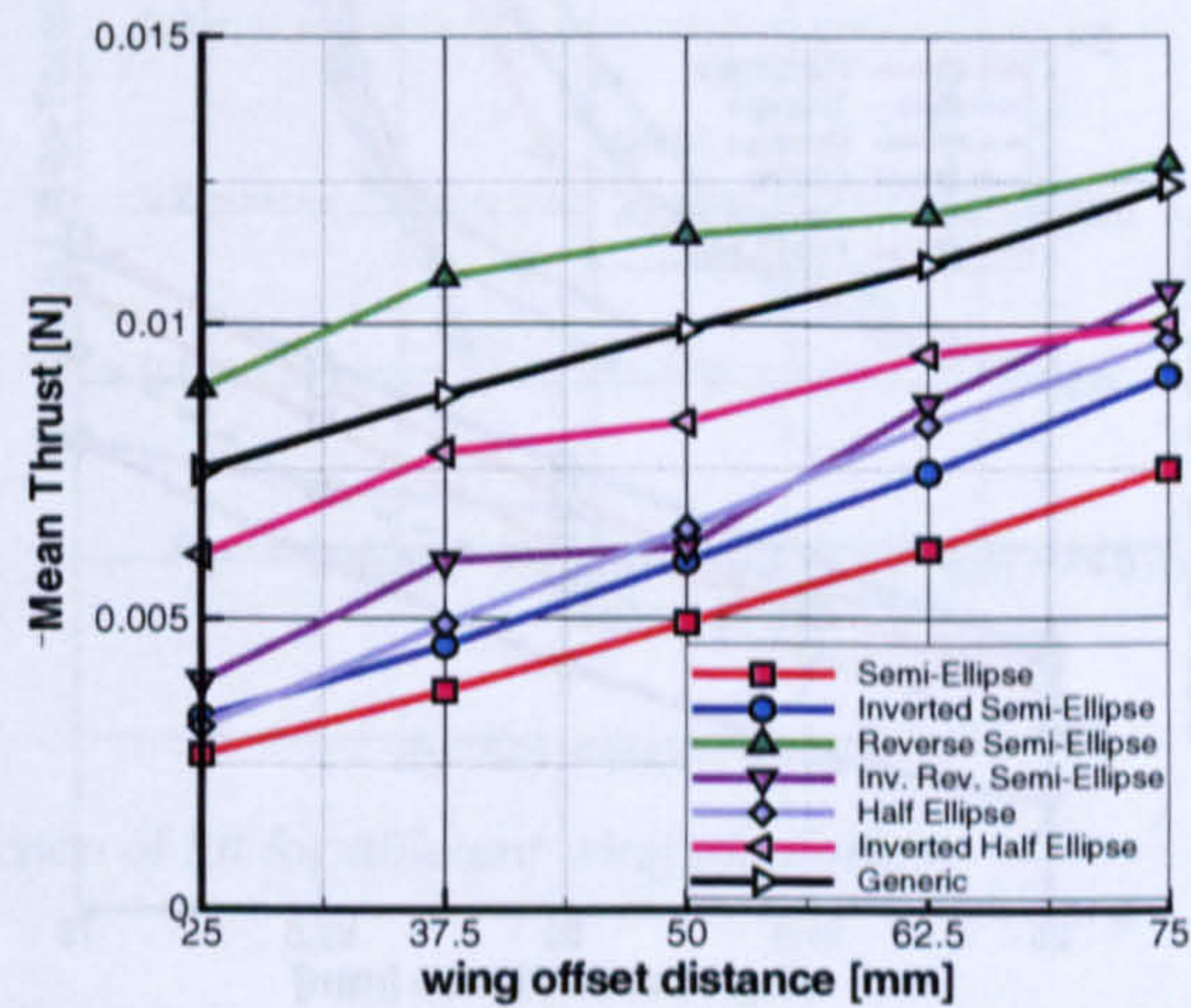
(a) Symmetric wings



(b) Asymmetric wings



(c) Symmetric wings



(d) Asymmetric wings

Figure 6.46: Effect of wing offset on mean lift ((a) & (b)) and mean thrust ((c) & (d))

is to make the flow ‘younger’ in terms of chord lengths travelled because as wing offset distance increases, chord lengths increase. Therefore, the flow becomes increasingly stable with growing offset distance. This is manifested in improving lift-to-thrust and lift-to-torque ratios (Figure 6.47).

Effect of Wing Planform Shape

Lift This section takes a brief look at the relative merits of the wing planforms. In the previous sections, it has been established that from a lift perspective the following

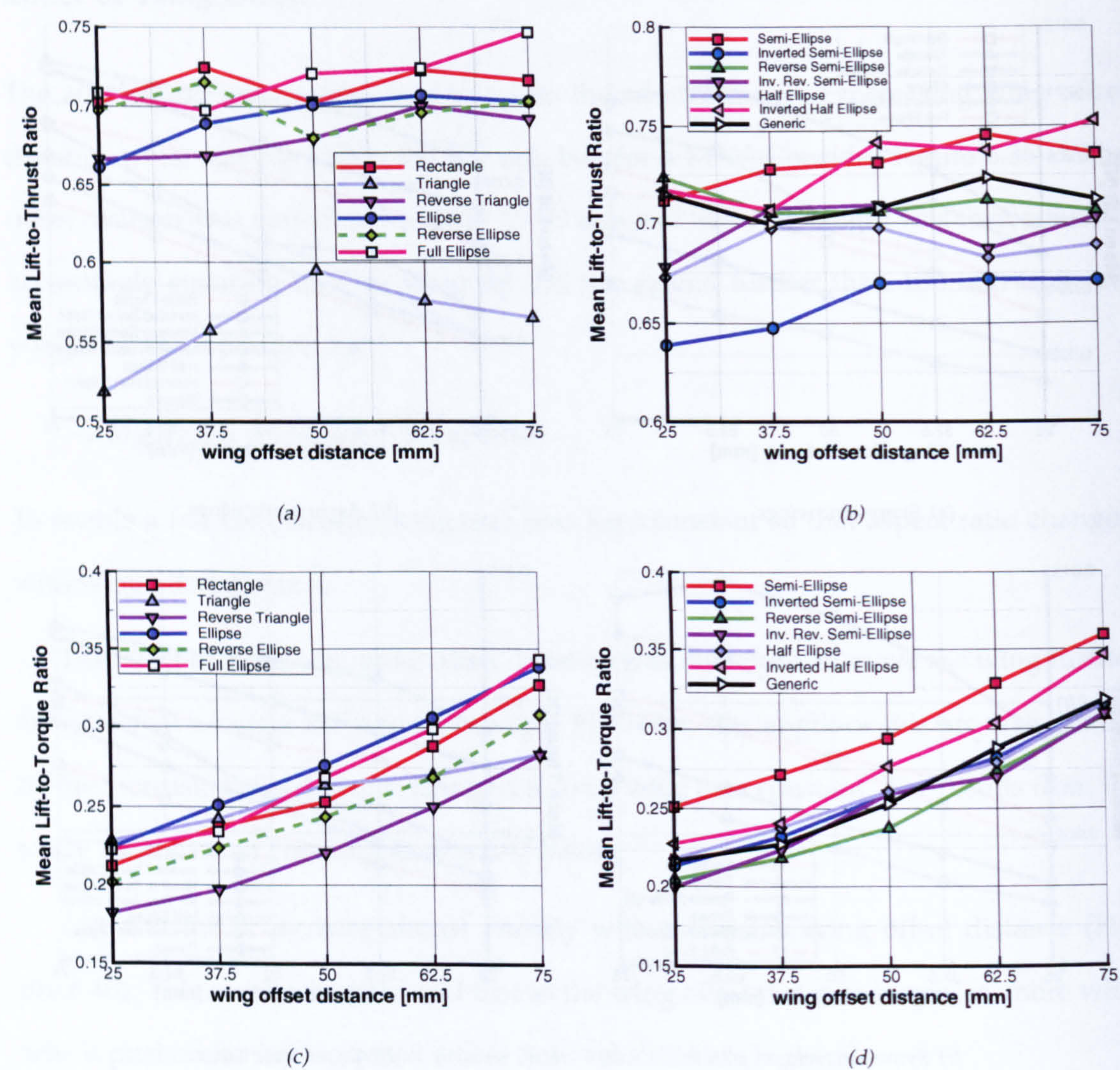


Figure 6.47: Effect of wing offset on mean lift-to-thrust ratio ((a) & (b)) and mean lift-to-torque ratio ((c) & (d)). L/Q is non-dimensionalised by mean chord

describes wing planforms in order of decreasing lift

symmetrical reverse triangle > reverse ellipse > full ellipse > rectangle > ellipse > triangle

asymmetrical reverse semi-ellipse > generic > inverted half ellipse > inverted reverse semi-ellipse > half ellipse > inverted semi-ellipse > semi-ellipse

However, as mentioned earlier, a more useful graph in wing design is the plot of lift-to-torque ratio against lift. This is shown in Figure 6.48 for the range of aspect ratios considered above.

Lift-to-Torque Ratio While a wing with a high lift-to-torque $\mathcal{L}/Q$ is desirable, it may not offer a plausible solution if it is unable to generate the desired lift in the first place. For this reason, a plot of $\mathcal{L}/Q$ against lift $\mathcal{L}$ is more useful for comparison in wing design. This is shown in Figure 6.48 below. The general trend is that an increase in lift comes at the cost of a reduction lift-to-torque ratio. Therefore, for maximum power efficiency, it is imperative that lift requirements are kept to a minimum.

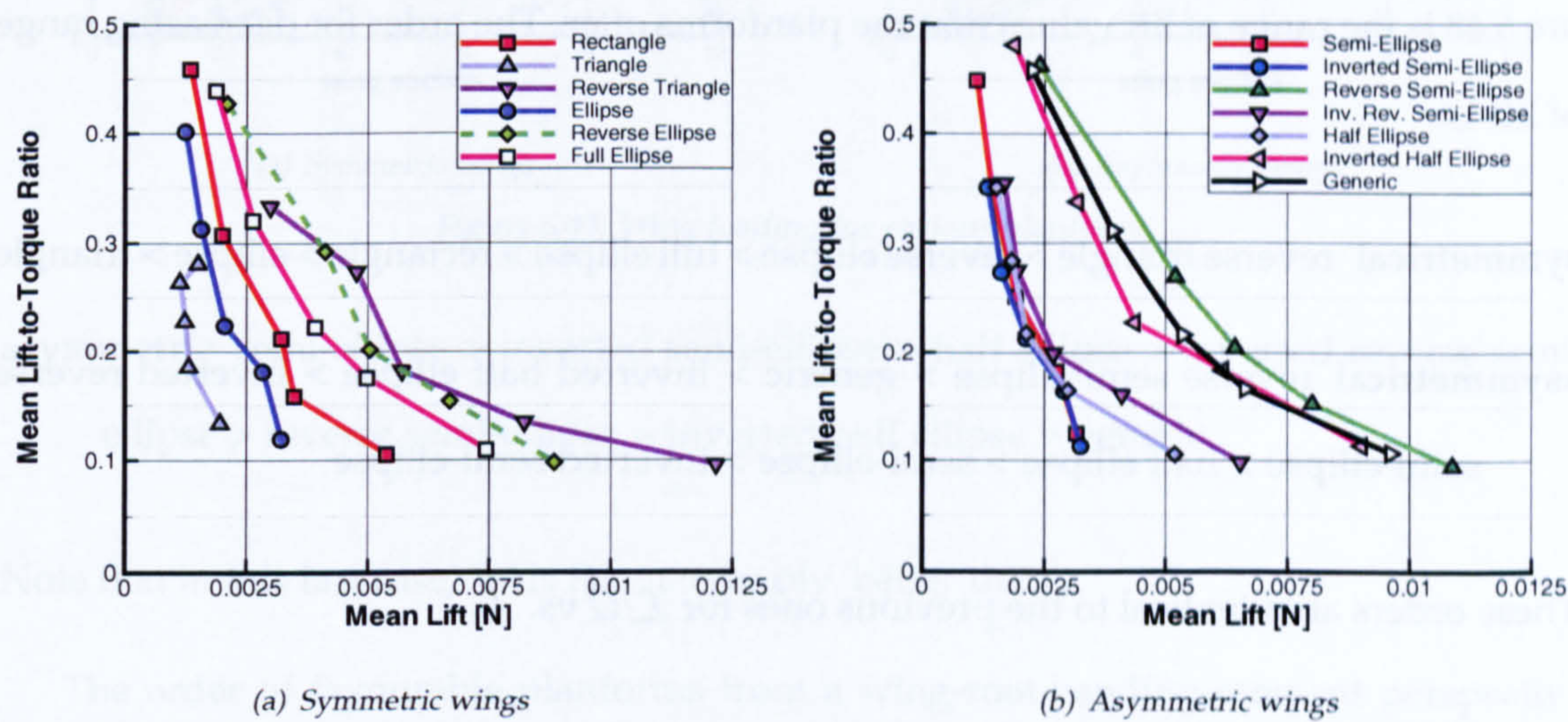


Figure 6.48: Lift-to-torque ratio as a function of lift for different wing planforms

Consider, for example, the design requirement for 0.0025 N of lift from a wing planform with minimum torque. This condition is best satisfied by the reverse ellipse from the symmetric wings (Figure 6.48(a)) and by the reverse semi-ellipse from the asymmetric planforms (Figure 6.48(b)). Figure 6.48(b) also shows that wings planforms with straight (or almost straight) leading edges and more wing area outboard show better $\mathcal{L}/Q$ vs. $\mathcal{L}$ characteristics.

From a lift-to-torque ratio perspective, therefore, the order of wing planforms with decreasing $\mathcal{L}/Q$ as a function of $\mathcal{L}$ is

symmetrical reverse triangle > reverse ellipse > full ellipse > rectangle > ellipse > triangle

asymmetrical reverse semi-ellipse > generic > inverted half ellipse > inverted reverse

semi-ellipse > half ellipse > semi-ellipse > inverted semi-ellipse

For the symmetric wings, the order is the same for both descending $\mathcal{L}$ and descending $\mathcal{L}/Q$. Except for the last pair of places interchanged, the order of descending $\mathcal{L}$ and descending $\mathcal{L}/Q$ is also the same for the asymmetric wings.

Range of Lift The second important inference that can be drawn from the plots in Figure 6.48 is the range of lift values that the planforms offer. The order for decreasing range of lift is

symmetrical reverse triangle > reverse ellipse > full ellipse > rectangle > ellipse > triangle

asymmetrical reverse semi-ellipse > generic > inverted half ellipse > inverted reverse semi-ellipse > half ellipse > semi-ellipse > inverted semi-ellipse

These orders are identical to the previous ones for $\mathcal{L}/Q$ vs. $\mathcal{L}$.

Wing Loading and Bending Moment Another requirement that is critical to wing design is wing loading and more so for insect-like flapping wings that are very flexible. It is generally desirable to keep the bending moment at the wing root at a minimum. This is achieved by limiting the level of forces generated outboard. From the quasi-3-D unsteady model developed in the current work, it is possible to derive wing-loading data. These are shown below in Figure 6.49.

A better comparison can be made by considering the root bending moment¹⁷ due to the wing loading (see Figure 6.50). The order for deteriorating wing bending moment is, therefore,

symmetric triangle > ellipse > rectangle > reverse ellipse > full ellipse > reverse triangle

¹⁷Root bending moment is taken at the centre of rotation of the wing (see Figure 6.36).

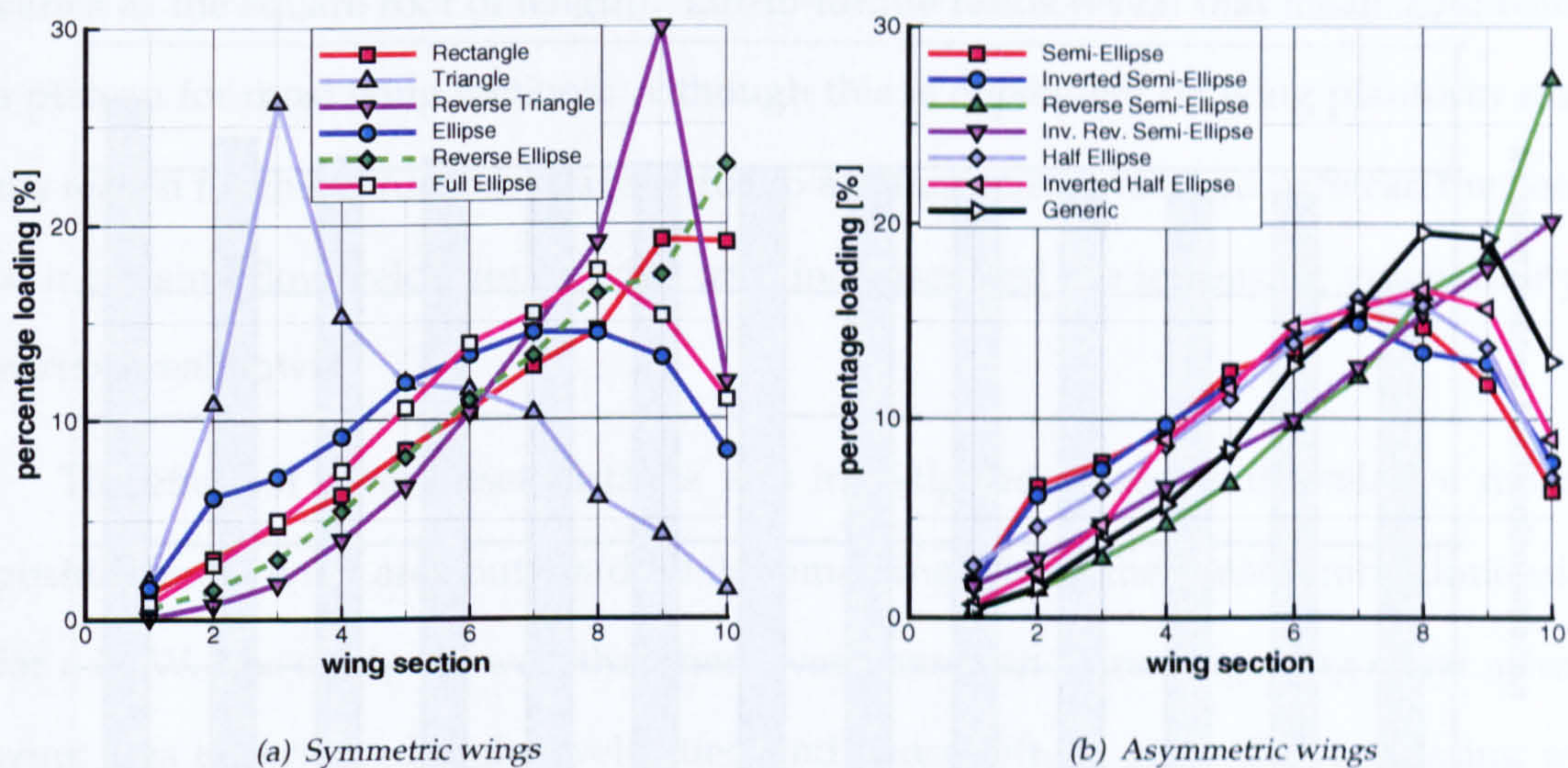


Figure 6.49: Wing loading for various planforms

asymmetric semi-ellipse > inverted semi-ellipse > half ellipse > inverted reverse semi-ellipse > reverse semi-ellipse > inverted half ellipse > generic

Note that in this last case, ‘>’ is taken to imply ‘better than’.

The order of favourable planforms from a wing-root-bending-moment perspective is more or less the opposite of the order from a lift or lift-to-torque-ratio perspective. Therefore, a compromise would have to be struck between the two to give the most suitable planform for an insect-like flapping-wing MAV.

For completeness, the time histories of lift, thrust, lift-to-thrust ratio and lift-to-torque ratio for all the wings are given respectively in Figures D.2–D.5 in Appendix D.

Summary of Wing Shape Effects

A number of important inferences can be drawn from the preceding study. It has been found that the location of the wing pitch axis has great influence on the wing’s aerodynamic performance. Greatest lift is generally generated with wing rotation about a pitch axis close to the centre of area in the front-rear direction. However, lift-to-torque ratios

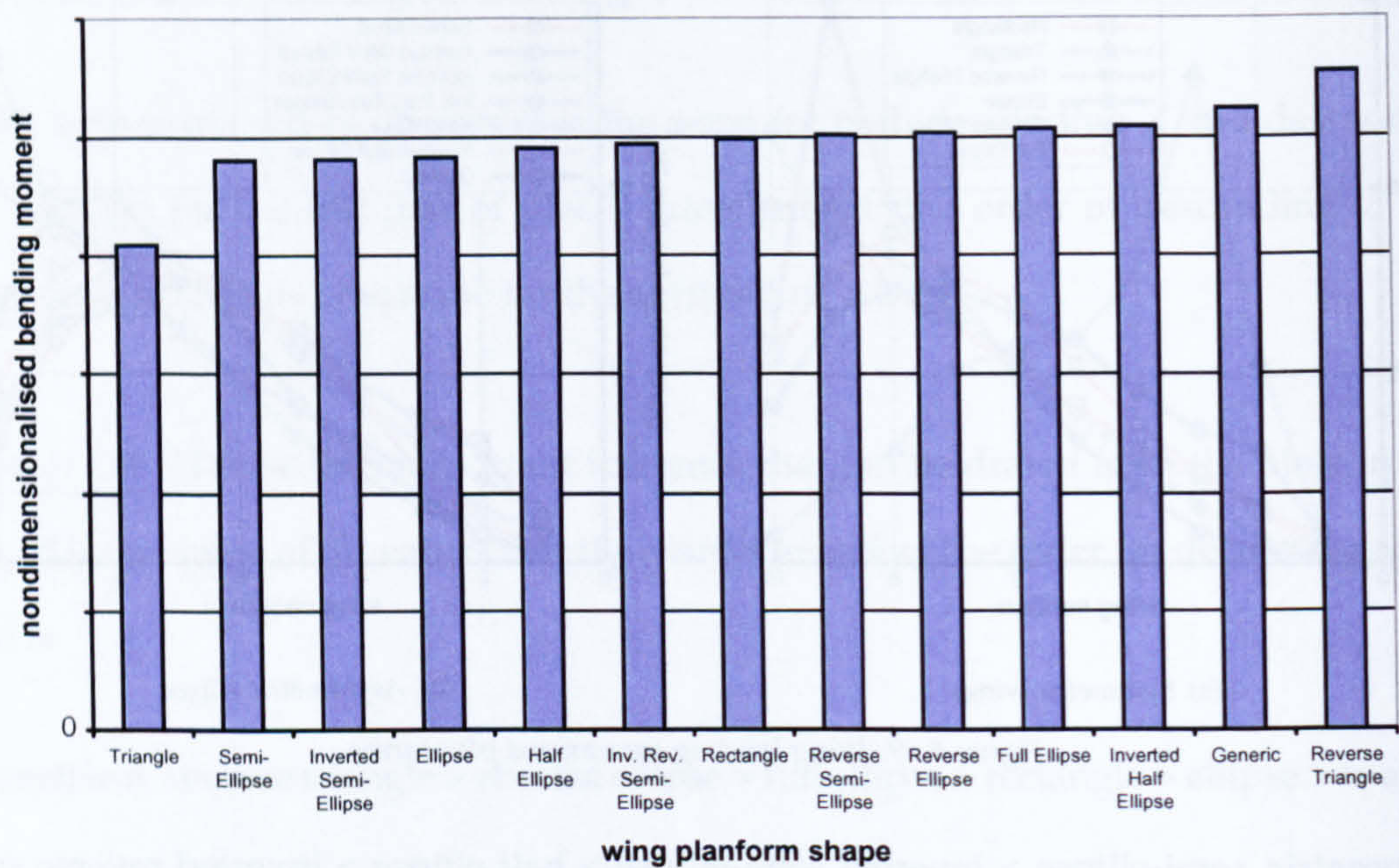


Figure 6.50: Order of increasing wing-root bending moment for various planforms. The bending moments were computed from the integral of the product of ‘percentage’ wing loading (from Figure 6.49) and radial distance from wing root. The scale on the vertical axis has, therefore, been omitted but the relative magnitudes are accurately represented

suggest that better $\mathcal{L}/Q$ is obtained for rearward pitch axes, irrespective of planform shape.

As would be expected, lift and thrust increase with wing length and aspect ratio. However, unlike what quasi-steady aerodynamics would suggest the increase is more linear than quadratic. This is due to the vortex breakdown that ensues in outboard regions as the wing gets longer. The latter causes reduction in lift and increase in drag (and hence thrust). For this reason, lift-to-torque ratios generally decline as wing length and aspect ratio increase. For some planforms, there exists a range within which no increase in lift is noted for increasing wing length. This is due to the balance between the opposing effects of increasing wing length and increasing vortex breakdown.

Wing area increases result in corresponding increases in lift and thrust. The effect is somewhat similar to increasing wing length but at a smaller scale (because wing area

varies as the square root of length). Lift-to-torque ratios reveal that mean $\mathcal{L}/Q$ reaches a plateau for most wing sections. Although this is dependent on wing planform shape, the reason for this behaviour is again due to a balance being reached between the benefit of increasing flow velocities as wing area increases and the increasing vulnerability to vortex breakdown.

The effect of wing offset distance was investigated to assess the relative merit of pushing more wing area outboard while remaining within the constraining dimensions for a MAV. The study showed that there was benefit in increasing wing offset as more wing area experienced higher velocities, and hence, lift. A spin-off of increasing wing offset distance is that the associated increase in chord lengths makes the flow ‘younger’ in terms of chord lengths travelled. Hence, there is less vortex breakdown in outboard regions and lift-to-torque ratios are better.

Finally, the 13 wing planforms were compared with respect to each other. From both lift and lift-to-torque-ratio perspectives, the reverse triangle and reverse ellipse planforms performed best from the group of symmetric wings while the reverse semi-ellipse and generic wings fared best from among the asymmetric planforms. The overall best performer from both groups was the reverse semi-ellipse planform. Ansari *et al.* (2003c) came to the same conclusion from their RANS CFD study on rotating propellers at low Reynolds numbers. The reason for this observation is that more wing area is situated outboard where flow velocities (and hence forces) are higher. Further, wings with straight (or almost straight) leading edges performed better in terms of lift and lift-to-torque. Again, Ansari *et al.* (2003c) came to a similar conclusion from their RANS CFD study.

From a wing-loading perspective, however, the order of planforms performing best is more or less reversed. The triangle planform wing performs best because it generates most of its forces inboard, and hence, has the smallest wing-root bending moment. At the same time, the triangular wing also has the smallest range of lift to offer (in the $\mathcal{L}/Q$

vs. $\mathcal{L}$ plot) which makes it unsuitable for, say, a wide range of payload weights. In practice, therefore, a compromise would have to be struck between the good $\mathcal{L}$ and $\mathcal{L}/Q$ characteristics and small root bending moments.

Chapter 7

Conclusions and Recommendations

This is the final chapter of the thesis; it is broadly divided into two parts. The first part (§ 7.1) presents a summary of the main findings from this study. This includes a brief summary of the two main approaches used — the simple quasi-steady model and the much more complex unsteady model. This is followed by a discussion of results from the two approaches. The effects of both wing shape and wing kinematics are discussed. The part ends with specifications for a practical flapping-wing micro air vehicle (§ 7.1.1), based on the findings from the parametric study in Chapter 6.

The second part (§ 7.2) presents some refinements and improvements that may be applied to the current implementation of the flapping-wing model. Improvements are suggested for the quasi-steady and the unsteady model with recommendations for further work as necessary. The part concludes with a brief consideration of how the numerical implementation (via the FORTRAN code) may be improved.

7.1 Conclusions

In this thesis, a quasi-steady aerodynamic model for insect-like flapping wings in hover has been implemented (Chapter 3). This was based on a modified form of the blade-element method used for rotors. The effect of tip vortices was included using an analysis similar to that used by Glauert (1959) but modified for rotary flow with spanwise-varying incident velocity.

This was followed by a much more detailed and thorough unsteady aerodynamic analysis that was inviscid, nonlinear and essentially two-dimensional (Chapter 4). The modelling framework included the effect of the leading-edge vortex (commonly observed on insect wings during flapping) in addition to the wake shed from the trailing edge. The model was extended to three dimensions using a blade-element-type analogy but by incorporating radial chords and radial cross-planes to give a so-called quasi-3-D model. Not only were the forces remarkably well-predicted by this model but so too were the physics of the flow.

From the quasi-steady aerodynamic treatment in Chapter 3, a number of criteria pertinent to flapping-wing design have been brought to light. It has been shown that wings with radially-increasing chord lengths (such as the reverse-triangle planform) produce high lift because more wing area is exposed to the higher velocities in the outboard regions. However, such high-lift wings also come with a high-drag penalty because wings that end ‘abruptly’ at the tip are associated with stronger tip vortices, and hence drag. It is, therefore, desirable to have a wing with a high lift-to-torque ratio. Although the full ellipse planform performed best in this regard, it was incapable of producing significant lift. The above would, therefore, suggest that a better wing design would be one where chord length increased in the outboard direction but then reduced gradually to zero at the wing tip.

Apart from planform shapes, wing kinematics were also found to be critical to the

performance of a flapping wing. In fact, since kinematics do not affect the lift-to-torque-ratio characteristics of a wing (being independent of wing shape), it is more desirable to optimise these. The quasi-steady model has shown that lift increases as the square of flapping frequency and stroke amplitude, each being directly proportional to velocity. The analysis has also provided insight into the merits (in terms of weight support) of using a figure-of-eight-type wing motion.

More dependable conclusions can be drawn from the results of the nonlinear, unsteady, quasi-3-D aerodynamic model developed in Chapter 4 due to its much better prediction of the forces on insect-like flapping wings (as validated against existing experimental data). Two novel equations were derived in the process to provide the boundary conditions necessary for 'closure' in the analysis. The first (Equation 4.19) was a generalised form of the wake integral equation originally derived by Wagner (1925) but modified here to include the effect of a nonlinear deforming wake and the effect of the leading-edge vortex. The second equation (Equation 4.23) resulted from consideration of the wake integral equation above, relating it to the velocity distribution on the aerofoil surface and then enforcing stagnation at the leading edge.

Validation of the model against experimental data provided by Dickinson (priv. comm.) showed particularly good agreement (Figure 5.21). It would appear that the reduction in lift and increase in drag in the outboard regions of the wings due to 3-D tip vortex effects are more or less equivalent to those arising from vortex breakdown instead. The discrepancy in the prediction of mean lift was as low as about 7% while for mean thrust the prediction was even better to within 4%. Lift-to-thrust ratios for the validation simulations were of the order of 1.5 mainly because the forces are essentially pressure-based and the angles of attack were never below 45° (so that the thrust or drag component was never less than the lift component). A more remarkable observation was the agreement in terms of the positions in time of all force peaks. These were all very accurately predicted by the theoretical model, implying that the physics of the flow

(especially the unsteadiness) are well-captured by the model. This was indeed the case as verified by the excellent correspondence of flow-field visualisation between theory and experiment (Figure 5.20).

Further, the requirement for a leading-edge vortex to be included in any model aimed at representing the insect-like flapping-wing problem has been established (§ 6.1.4). It is shown that in the absence of the leading-edge vortex, lift and thrust data from the theoretical model are not only out by a factor of about two (for the particular kinematics) but that lift especially is generally out of phase with the experimental values. It has also been found that the leading-edge vortex generally causes a reduction in lift when compared to the LEV-free solution.

Another noteworthy feature that can be inferred from the work is that in order to produce a near-constant lift (and thrust) force, both leading- and trailing-edge vortices must be shed. Of course, this requires that the angle of attack is sufficiently large to cause leading-edge separation. If, however, only a trailing-edge wake is shed then the classical Wagner response is observed with exponentially increasing lift.

Numerical implementation and solution of the model has revealed that relaxation of the Kutta-Joukowski condition at both leading and trailing edges is required, especially during stroke reversals (p. 162). Whereas, this could be exposing a limitation of the discrete-vortex method used here, it has brought into question the suitability of enforcing the Kutta-Joukowski condition in the conventional sense for such highly-skewed, unsteady flows. Another feature brought to light by this analysis and the numerical simulations is the dependence of flow characteristics on the number of chord lengths travelled instead of on time. This shows that a reduced-frequency-type parameter is still relevant in this form of unsteady flow.

As required by the aims of this PhD, a parametric study was conducted to investigate the effects of wing-kinematics and wing-shape parameters on a FMAV. From the discussion of the effect of wing kinematics (§ 6.2.1) it was found that lift and thrust increased

approximately as the square of flapping frequency and that lift-to-torque ratio was more or less unaffected. This is precisely the result that was obtained from the quasi-steady analysis described earlier. It was also found that the forces became reasonably periodic after about 10 cycles. The effect of stroke amplitude was similar to that for flapping frequency albeit less quadratic owing to the onset of vortex breakdown in outboard sections as the flow got older in terms of chord lengths travelled.

The effect of advancing stroke reversals was found to increase lift and lift-to-torque ratio, peaking when rotation led translation by about 5%. The effect of the duration of rotation was found to be dependent on the location of the pitch axis. Faster rotation achieved best results for wings with pitch axes closest to the leading edge because the stopping-starting vortices shed at stroke reversal were flung furthest away in those cases.

With regards to wing-shape parameters, it has been found that the position of the pitch axis of a flapping wing has great influence on its aerodynamic performance. Greatest lift was generated with rotation about a pitch axis close to the centre of area in the front-rear direction, whereas lift-to-torque ratios generally improved as the pitch axis was moved rearward along the wing section.

As would be expected, lift and thrust increase with wing length and aspect ratio but, unlike in the quasi-steady cases, this relationship was more linear than quadratic. This resulted from more wing area being involved in vortex breakdown as wing spans increased. For this reason, lift-to-torque ratios generally declined with increasing wing length and aspect ratio.

The effect of increasing wing-offset distance was to increase lift as more area was pushed outboard where flow velocities were higher. However, the real benefit in increasing wing-offset distance (and hence increasing chord lengths) was that the flow became 'younger' in terms of chord lengths travelled. This implied less vortex breakdown and improved lift-to-torque ratios.

In order to investigate the effects of wing planform shape, a number of simple geometric shapes were studied and compared to one another. It was found that wings with more area outboard performed best in terms of lift and lift-to-torque ratios, the reverse semi-ellipse planform being most outstanding. For best power efficiency per unit lift, plots of lift-to-torque ratio against lift are used. Here again, the reverse semi-ellipse performed best. However, when the bending moments due to wing loading were considered, the triangle planform fared best as it generated most of its forces inboard. The performance of the reverse semi-ellipse was average amongst the planforms considered while the reverse triangle planform was noticeably the worst.

The studies described above have shown that reasonable agreement can be found between the results from the quasi-steady analysis and the more detailed unsteady aerodynamic model. Therefore, a quasi-steady model can be used for initial design studies.

A quick note on the *values* of lift-to-torque ratios from the two studies, however, is befitting. The values generated in Chapter 3 are roughly an order of magnitude bigger than those shown in Chapter 6. The reason for this is that in the former, drag (and hence torque) was comprised only of the induced drag component. This is generally much less than the pressure drag component used in Chapter 6. In fact, in the latter cases, drag was generally greater than lift by about 50%.

Whereas the above comparisons represent a very thorough investigation, it remains a Monte Carlo-type simulation where only one parameter was varied at a time. Therefore, it is possible that the optimal combination of the various parameters has been missed due to the limited number of simulations that could be run. An n -dimensional comparison (where n is the number of parameters) is required for a true comparison but this would require a near-prohibitive number of simulations.

The overall conclusions are that a quasi-3-D, nonlinear, unsteady, inviscid model is capable of reproducing well the flow and associated forces for insect-like flapping

wings at low Reynolds numbers ($Re \lesssim 200$). However, any such model must include a representation of the leading-edge vortex. Flow-visualisation comparisons at higher $Re = 1000$ have also shown good agreement implying that potential-flow methods are capable of simulating low- Re flows provided the lines (or points) of separation have been accurately defined.

7.1.1 Wing Design for a Flapping Wing MAV

Using the preceding quasi-3-D unsteady analysis and the results from the parametric study, it is possible to propose the specifications for a suitable flapping-wing micro air vehicle conforming to the DARPA requirements and constraints (see p. 2). Before proceeding, it may be noted that using flapping wings means that an MAV that conforms to DARPA's requirements can have a wing span almost twice the maximum dimension permitted. This is enabled by the fact that the flapping wings may be stowed in a box conforming to the 150 mm-limit but extended to 150 mm prior to flight, giving an overall span of the order of 300 mm.

From the discussion above, best lift and lift-to-torque ratio properties were attributed to the reverse semi-ellipse wing (see p. 259). This wing has the added advantage that its bending moment at the wing root is relatively low (see Figure 6.50). This wing planform is, therefore, chosen for a potential FMAV.

For this wing planform shape, the best lift characteristics are achieved for a wing-pitch-axis location at 0.25-chord (Figure 6.38). To have better lift-to-torque ratios, use is made of wing offset (Figure 6.47). A conservatively low value of 37.5 mm is used so as not to reduce wing aspect ratio too much. Wing area is kept at the values used for the simulations (4166.67 mm²).

One of the cases shown in Figures 6.46 and 6.47 (pp. 259ff) in fact matches the specifications described so far. The lift force generated by such a wing would, however, need

to be modified for the conditions above. Under standard atmosphere conditions, the lift generated by the wing was 7.6384 mN. This was for a single wing with the maximum possible stroke amplitude of 145.2° , a flapping frequency of 2 Hz and executing Dickinson's kinematics. In practice, sweeping this maximum value of stroke amplitude is unlikely due to the presence of the driving mechanisms. Therefore, a value of $\Phi = 120^\circ$ is suggested. This requires that the lift value be adjusted to reflect the smaller stroke amplitude.

From Figure 6.21, lift varies roughly as $\Phi^{1.55}$ for the Robofly wing and as $\Phi^{1.7}$ for the generic wing where Φ is stroke amplitude. So it may conservatively be assumed that lift varies as $\Phi^{1.6}$ for the reverse semi-ellipse planform proposed here¹. Therefore, the lift for a stroke amplitude of 120° is approximately 5.63 mN.

Ignoring interference effects from the 'other' wing, the force generated by a pair of such flapping wings is found simply by doubling the value for one wing to give 11. mN.

It was seen from the discussion of kinematics 'shape' that the best lift and lift-to-torque ratio characteristics were achieved by the convex-arc Lissajous kinematics (see Figure 6.28). The best performance amongst these was from a Lissajous ratio of 4 where the mean lift was 1.189 times that produced using the Dickinson kinematics. These kinematics are adopted here for the proposed FMAV. The predicted lift for both wings, therefore, becomes 13.4 mN.

DARPA guidelines require that the minimum weight supported be 50 g (= 0.4905 N). This is achieved by increasing the flapping frequency. From the study in § 6.2.1 (p. 226), lift was found to vary as the square of flapping frequency. Remembering that the lift value derived so far is for a flapping frequency of 2 Hz, in order to get the DARPA minimum lift, flapping frequency will need to be increased to about 12.1 Hz. Assuming that the only source of noise is the flapping wings, this frequency is safely below the

¹For the values used here, the choice of power between 1.55 and 1.7 does not alter the end result by more than 3%

human-audible range². In practice, the mechanism driving the flapping wing would also make a contribution to noise.

The final specifications for a flapping-wing micro air vehicle, optimised using the results from the unsteady aerodynamic model and conforming to the original DARPA guidelines, are shown in Table 7.1 below.

Table 7.1: Wing parameters for an ‘optimised’ FMAV (in air)

wing planform shape	reverse semi-ellipse (Figure 6.37(h))
kinematics	convex arc (Figure 6.23(d))
Lissajous ratio	4
flapping frequency	~ 12.1 Hz
stroke amplitude	120°
wing offset	37.5 mm
wing length	112.5 mm
wing area	8333 mm ² (both wings)
mean chord	66.67 mm
aspect ratio	6.075
pitch-axis location	0.25-chord

²The range of frequencies audible to the average human ear is 20 Hz–20 kHz.

7.2 Recommendations for Further Work

Recommendations for further work can broadly be divided into theory and code refinements. The former subdivides into improvements that can be made on the quasi-steady model and those that can be made on the more advanced nonlinear, unsteady, quasi-3-D model. These are detailed in this order below.

7.2.1 Quasi-Steady Model

The quasi-steady model can be improved by using a more complex form of incident velocity in Equation 3.6 that includes the effect of heave rates $\dot{h}$ as well as pitch rates $\dot{\alpha}$. Using thin aerofoil theory (see e.g. Leishman, 2000, p. 488), Equation 3.6 would be modified to

$$\Gamma(r) = \frac{1}{2}a_0c_r \left[U_\infty \alpha - \dot{h} + c_r \left(a - \frac{1}{4} \right) \dot{\alpha} - w(r) \right]$$

where a_0 is the two-dimensional lift-curve slope, α is angle of attack, c_r is chord length at spanwise distance r , a is the distance the pitch axis is behind the leading edge as fraction of the local chord length c_r (the actual distance is $a c_r$) and $w(r)$ is the downwash induced at spanwise position r . Apart from providing better results, this approach may also give good insight into the analysis of the quasi-steady terms in the unsteady model. It may also be used as a first approximation for modelling the tip vortex proposed below for the unsteady model (p. 278).

In addition, this method would allow for comparison of the asymmetric planform shapes considered in Chapter 6 which are as yet indistinguishable in the quasi-steady model due to the absence of pitch rates and locations in the analysis. The method may further be used to compare the various wing kinematics tested in § 6.2.1.

Comparison is probably the limit, however, to which the method can be employed as a tool for flapping-wing flight. The quasi-steady model is generally unsuitable for

accurately modelling the flapping-wing problem as it fails to capture or represent many of the pertinent flow features. Therefore, it would be less useful to continue to improve the model too far as its accuracy would always be limited.

7.2.2 Unsteady Model

The unsteady aerodynamic model has proven to give a remarkably accurate representation of the flow field and forces associated with insect-like flapping wings. However, there is still scope for improvement.

Leading-Edge Vortex

In the current form of the inviscid model, the leading-edge vortex remains restricted to within its radial cross-plane and in doing so quite accurately predicts experimental results at similar low Reynolds numbers ($Re \lesssim 300$). At higher Reynolds numbers ($Re \approx 5000$), however, a definite spanwise transport of vorticity has been reported (see e.g. Ellington *et al.*, 1996; van den Berg & Ellington, 1997a). The current model would need to be modified to accommodate such a flow feature.

Tip Vortex

The present quasi-3-D model is essentially two-dimensional. No tip-vortex effects have been incorporated. Comparison with experiment has, however, yielded reasonable agreement due to vortex-breakdown effects compensating for 3-D tip-vortex effects. As the model is improved and grows in complexity, it will be necessary to include a method that directly accounts for tip-vortex effects.

The vortex model proposed by Strickland *et al.* (1979) for the Darrieus turbine (a vertical-axis wind turbine) might be a useful starting point. Strickland *et al.* divided each

wing into sections that each shed a pair of trailing vortices whose strength was determined by the change in bound-vortex strength across the wing section (see Figure 7.1). The trailing vortices were shed as filaments from the trailing edges at the two extremities of the wing section. At every time-step, a shed-vortex filament was released that closed the loop. The shed-vortex filaments convected with the fluid and their motion was determined by computing the induced velocities at their ends. This was done using the full 3-D form of the Biot-Savart law. Therefore, the filaments stretched, translated and rotated as a function of time.

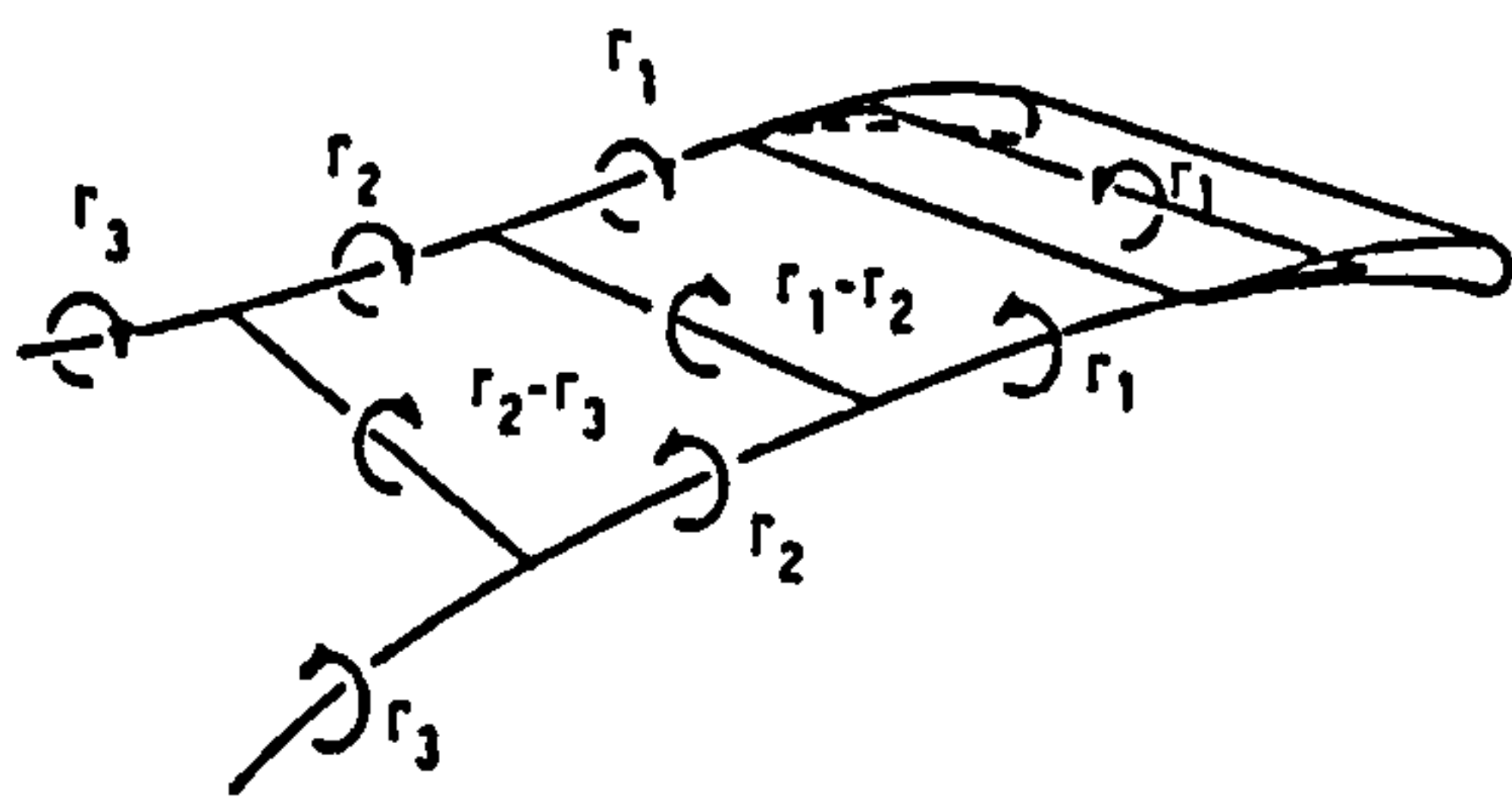


Figure 7.1: Vortex system for a single blade element [from Strickland et al. (1979, fig. 1)]

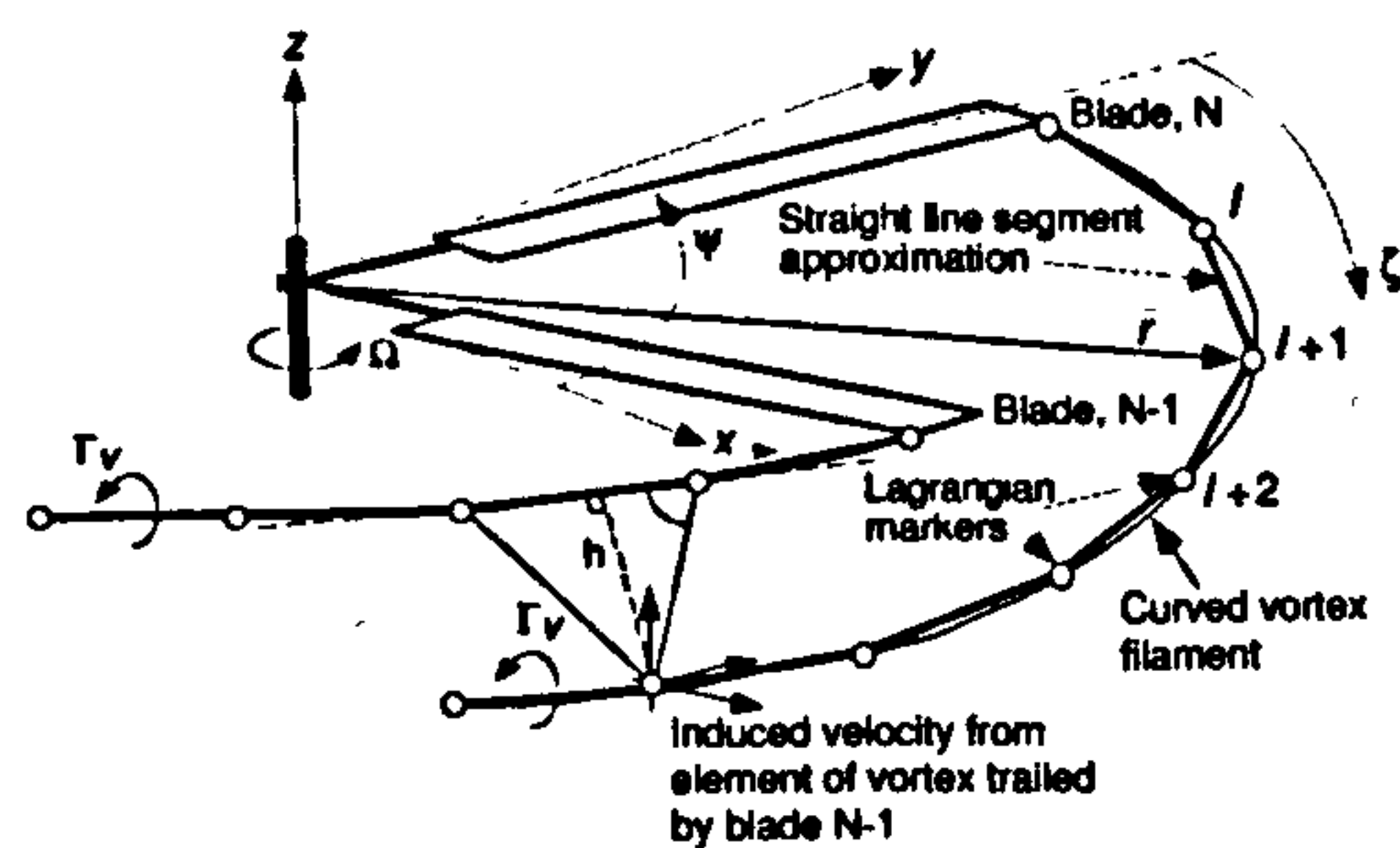


Figure 7.2: Free vortex filament representation of the tip vortex [from Ananthan et al. (2002, fig. 2)]

A similar, more recent method for computing tip vortex effects is that proposed by Leishman *et al.* (Bhagwat & Leishman, 2001; Leishman *et al.*, 2002; Ananthan *et al.*, 2002). They represent the tip vortex (whose strength is determined from wing loading) in terms of straight-line free-vortex filaments that are released from the wing tip at each time-step (see Figure 7.2). Each end of the filaments has a Lagrangian marker that determines its convection in much the same way as described for the approach of Strickland *et al.* above. The method used by Leishman *et al.* is much simpler, and hence more attractive, to implement in that only one stream of vortex filaments needs to be tracked.

Pressure Distribution

The calculation of forces (and moments) in the current model has been achieved relatively simply by means of Kelvin's impulse theorem. Whereas this has been suitable and accurate for the scope of this work, one disadvantage of the method is that pressure distributions on the wing cannot be determined. The latter require use of the unsteady form of Bernoulli's equation. However, as remarked by Platzer (priv. comm.), this comes with great complexity but would be necessary for any accurate aeroelastic analysis of the insect-like flapping wing³.

In the meantime, a quick determination of the centre of pressure may be implemented by taking pitching-moment measurements at two different positions along the chord and inferring the centre of lift from that.

Thick wings

Although in all implementations of the unsteady model presented, a thin flat-plate aerofoil has been used, the model is ready to be used for thick and cambered aerofoils. It would, therefore, be interesting to investigate the effect that wing-section thickness has on the performance of the flapping wing. Caution must, however, be exercised because as wings get thicker, flow around them becomes less likely to separate at the leading edge (Platzer, priv. comm.). When flow does indeed separate, then separation is not necessarily always from the exact same point in the leading-edge region. In such cases, a Navier-Stokes treatment may be required to predict the separation point.

A further wing parameter that was not tested in the current model was the effect of wing twist on insect-like flapping. Because of the rapidity of especially the stroke

³In RANS CFD methods, this problem is obviated as pressures result directly from the calculation method. However, use of such methods for insect-like flapping wings is likely to run into meshing problems, amongst others.

reversals, wing deformations in practice are very substantial. For this reason, washout (or wing twist) is less likely to have much effect on the performance of the flapping wing. Nevertheless, it may still be a likely contender for a rigid flapping-wing analysis.

Experimental Data

The paucity of experimental data available for comparison and validation is a limitation. More force measurements and flow-visualisation experiments on the insect-like flapping-wing problem are, therefore, highly desirable. In addition, most of the experimental work in the area to date has come from the biological community. Perhaps an engineering perspective would provide further insight into the more technical aspects of the problem.

Due to the diversity of insects and their various flapping and lifting mechanisms, it is of great interest to investigate the dependence of the flow and the associated forces on Reynolds number. It is likely that in addition to the changes in the behaviour of the leading-edge vortex (described above), there may be other salient features that have yet been ignored or missed. This is where a contribution from the aerodynamics/fluid mechanics community would be highly valuable.

Forward Flight

Of course, once the hovering flight of insect-like flapping wings has been reasonably understood, the next logical step would be to consider the forward flight of such wings. This is likely to bring with it complications in terms of representation of the wake. The radial cross-plane analogy may become untenable and a new method which is more truly 3-D will have to be devised.

7.2.3 Numerical Implementation

The numerical implementation of the current nonlinear, unsteady, inviscid model has provided useful insight into the nature of the flapping-wing problem, such as highlighting the requirement for relaxing the Kutta-Joukowski condition during stroke reversals. A number of modifications and improvements can, however, be implemented in order to solve the problem better and perhaps provide further insight.

One simple refinement that can be made to the numerical model is to experiment further with the effects of various numerical parameters (as done in § 5.2.1). An interesting prospect would be to see if a characteristic parameter pertaining to the flow is easily identifiable (unlike the characteristic length in the current work that had to be defined manually and *a priori*). The extraction of a reduced-frequency-type parameter for insect-like flapping would be another important result.

Another relatively simple modification to the existing FORTRAN code for the model would be to shed leading-edge vortices only as required. Normally, a new leading-edge vortex is shed at each time-step. However, situations may arise (such as low angles of attack) when leading-edge separation may not occur. In those cases, only a trailing-edge vortex would need to be shed. Therefore, incorporating such a feature into the program may be useful.

One of the main bottlenecks of the current implementation of the model is the large number of vortices that are generated. Therefore, a method whereby a vortex is only shed from the shedding edge once the Kutta-Joukowski condition has been disturbed beyond a certain limit could offer a solution. This method would, however, have to be rigorously tested for any fundamental flaws before being accepted.

The computation of forces and moments is achieved by differentiating the respective impulses. Differentiation is already a coarsening process and the use of finite difference methods (used in the FORTRAN code) only worsens the problem. This was one of the

reasons for employing smoothing schemes. The latter, however, have the disadvantage of flattening peaks and being unreliable at the ends of the data sets. Therefore, it would be useful to employ a differentiation scheme (perhaps by way of a Kalman filter) that would produce better data in the first place and partially obviate the need for smoothing.

To enable an aeroelastic treatment of the flapping wing, it would be necessary that forces be known at each time-step in the simulation. In the current form of the implementation, forces and moments are computed at the end of the simulation so that central difference formulae may be used. To compute the instantaneous forces, the simple backward-difference Runge-Kutta formula would normally be used. These would lead to erroneous results especially since, in the current implementation, even the central difference formulae results require significant amounts of smoothing. Hence, a better differentiation scheme will need to be devised.

7.3 Summary

In this work, new aerodynamic models have been developed for hovering FMAVs using insect-like wing kinematics and validated with very good agreement in terms of forces and flow field representation. Parametric studies have been made of wing kinematics and wing shapes, and favourable design features have been identified. On this basis, a preferred FMAV wing design has been proposed. In addition, further refinements have been recommended.

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Appendices

Appendix A

Some Useful Mathematics

A.1 Glauert-type Integrals

Glauert (1959) solved a number of principal value integrals. The selection below have been used in this text and have been adapted from Karamcheti (1966).

$$\oint_0^\pi \frac{\cos n\phi}{\cos \theta - \cos \phi} d\phi = -\frac{\pi \sin n\theta}{\sin \theta} \quad (\text{A.1})$$

$$\oint_0^\pi \frac{\sin n\phi \sin \phi}{\cos \theta - \cos \phi} d\phi = \pi \cos n\theta \quad (\text{A.2})$$

$$\oint_0^\pi \frac{\cos n\phi \cos \phi}{\cos \theta - \cos \phi} d\phi = \pi \cos n\theta - \frac{\pi \sin (n+1)\theta}{\sin \theta} \quad (\text{A.3})$$

$$\oint_0^\pi \frac{\cos^2 n\phi}{\cos \theta - \cos \phi} d\phi = -\frac{\pi \sin 2n\theta}{2 \sin \theta} \quad (\text{A.4})$$

$$\oint_0^\pi \frac{\sin^2 n\phi}{\cos \theta - \cos \phi} d\phi = \frac{\pi \sin 2n\theta}{2 \sin \theta} \quad (\text{A.5})$$

$$\oint_0^{2\pi} \frac{\sin n\phi}{\cos \theta - \cos \phi} d\phi = 0 \quad \text{if } n = 1, 2 \quad (\text{A.6})$$

$$\oint_0^{2\pi} \frac{\sin n\phi \cos m\phi}{\cos \theta - \cos \phi} d\phi = 0 \quad \text{if} \quad \begin{cases} n = 1, m = 2 \\ n = 2, m = 1 \\ n = m \end{cases} \quad (\text{A.7})$$

$$\oint_0^{2\pi} \frac{\sin^n \phi}{\cos \theta - \cos \phi} d\phi = 0 \quad \text{if} \quad n = 1, 3, 5 \quad (\text{A.8})$$

$$\oint_0^{\pi} \frac{\sin \phi}{\cos \theta - \cos \phi} d\phi = 2 \sum_{n=1}^{\infty} \frac{\cos^n \theta}{n} \quad n \text{ even} \quad (\text{A.9})$$

A.2 Coordinate Systems

See Table A.1 below.

Table A.1: Conversion between coordinate systems

From ↓ To →	Aerofoil	Rotating	Translating	Inertial
Aerofoil		$\zeta = \zeta + a$	$\zeta = (\zeta + a)e^{-i\alpha}$	$\bar{\zeta} = (\zeta + a)e^{-i\alpha} + (l + ih)$
		$\xi = \xi + a$	$\xi = \eta \sin \alpha + (\xi + a) \cos \alpha$	$\bar{\xi} = \eta \sin \alpha + (\xi + a) \cos \alpha + l$
		$\tilde{\eta} = \eta$	$\hat{\eta} = \eta \cos \alpha - (\xi + a) \sin \alpha$	$\bar{\eta} = \eta \cos \alpha - (\xi + a) \sin \alpha + h$
Rotating		$\zeta = \bar{\zeta} - a$	$\hat{\zeta} = \bar{\zeta}e^{-i\alpha}$	$\bar{\zeta} = \bar{\zeta}e^{-i\alpha} + (l + ih)$
		$\xi = \bar{\xi} - a$	$\xi = \tilde{\eta} \sin \alpha + \bar{\xi} \cos \alpha$	$\bar{\xi} = \tilde{\eta} \sin \alpha + \bar{\xi} \cos \alpha + l$
		$\eta = \tilde{\eta}$	$\hat{\eta} = \tilde{\eta} \cos \alpha - \bar{\xi} \sin \alpha$	$\bar{\eta} = \tilde{\eta} \cos \alpha - \bar{\xi} \sin \alpha + h$
Translating		$\zeta = \bar{\zeta}e^{i\alpha}$		$\bar{\zeta} = \bar{\zeta} + (l + ih)$
		$\xi = \bar{\xi} \cos \alpha - \hat{\eta} \sin \alpha - a$		$\bar{\xi} = \bar{\xi} + l$
		$\eta = \bar{\xi} \sin \alpha + \hat{\eta} \cos \alpha$		$\bar{\eta} = \bar{\eta} + h$
Inertial		$\zeta = [\bar{\zeta} - (l + ih)]e^{i\alpha} - a$	$\hat{\zeta} = \bar{\zeta} - (l + ih)$	
		$\xi = (\bar{\xi} - l) \cos \alpha - (\bar{\eta} - h) \sin \alpha - a$	$\xi = \bar{\xi} - l$	
		$\eta = (\bar{\xi} - l) \sin \alpha + (\bar{\eta} - h) \cos \alpha$	$\hat{\eta} = \bar{\eta} - h$	

A.3 Conjugate Function Theory

This section has been adapted from Benson (1989) who originally extracted it from Karamcheti (1966).

If a single-valued function, say $v_r(R, \phi)$, is known for points on a circle $\omega = Re^{i\phi}$ (centred at the origin) then it is possible to determine the single-valued continuous potential function $v_r(r, \theta)$ in the external region $Z = re^{i\theta}$ using Poisson's integrals. This is the basis of *conjugate function theory*, which essentially allows the imaginary part of the boundary values of a complex function on a circle to be determined in terms of the real part, and vice versa.

In equation form, these may be written thus

$$v_r(r, \theta) + iv_\theta(r, \theta) = V(Z) = \frac{1}{2\pi} \oint_C V(\omega) \frac{Z + \omega}{Z - \omega} d\omega \quad (\text{A.10})$$

where the subscript C indicates the integral is taken over the circumference of the circle ω . The equation shows that the value $V(Z)$ at a point in the external region $Z = re^{i\theta}$ is expressed in terms of known values $V(\omega)$ along the circumference $\omega = Re^{i\phi}$. On the surface of the circle ($r = R$), the integrand in the expression above becomes

$$\frac{Z + \omega}{Z - \omega} = \frac{e^{i\theta} + e^{i\phi}}{e^{i\theta} - e^{i\phi}} = i \cot\left(\frac{\phi - \theta}{2}\right)$$

so that Equation A.10 becomes

$$v_r(R, \theta) + iv_\theta(R, \theta) = -\frac{i}{2\pi} \oint_0^{2\pi} [v_r(R, \phi) + iv_\theta(R, \phi)] \cot\left(\frac{\phi - \theta}{2}\right) d\phi \quad (\text{A.11})$$

The real and imaginary parts in the equation above, which are conjugate harmonic functions, can be separated thus

$$v_r(R, \theta) = \frac{1}{2\pi} \oint_0^{2\pi} v_\theta(R, \phi) \cot\left(\frac{\phi - \theta}{2}\right) d\phi + v_{r_0} \quad (\text{A.12})$$

$$v_\theta(R, \theta) = -\frac{1}{2\pi} \oint_0^{2\pi} v_r(R, \phi) \cot\left(\frac{\phi - \theta}{2}\right) d\phi + v_{\theta_0} \quad (\text{A.13})$$

where v_{r_0} and v_{θ_0} are constants proportional to v_r and v_θ at the centre of the circle respectively. In fluid dynamics applications, v_{r_0} is related to the source (or sink) at the boundary while v_{θ_0} is related to the vortex at the boundary.

$$v_{r_0} = \frac{1}{2\pi} \oint_0^{2\pi} v_\theta(\phi) d\phi = \frac{M}{2\pi R} \quad (\text{A.14})$$

$$v_{\theta_0} = \frac{1}{2\pi} \oint_0^{2\pi} v_r(\phi) d\phi = \frac{\Gamma}{2\pi R} \quad (\text{A.15})$$

where M and Γ denote the source (or sink) and circulation (positive if counterclockwise) strengths at the circle boundary respectively. Since no fluid is being created or destroyed at the boundary, $M = 0$. The equations of conjugate function theory are, therefore

$$v_r(R, \theta) = \frac{1}{2\pi} \oint_0^{2\pi} v_\theta(R, \phi) \cot\left(\frac{\phi - \theta}{2}\right) d\phi \quad (\text{A.16})$$

$$v_\theta(R, \theta) = -\frac{1}{2\pi} \oint_0^{2\pi} v_r(R, \phi) \cot\left(\frac{\phi - \theta}{2}\right) d\phi + \frac{\Gamma}{2\pi R} \quad (\text{A.17})$$

Appendix B

Quasi-Steady Aerodynamic Modelling

This appendix contains details of the derivations used in the quasi-steady aerodynamic model presented in Chapter 3.

The bulk of the work in § B.1 is available in most text books. However, all analysis after Equation B.14 is original work. All work in § B.2 is original apart from Equations B.19 and B.21 which were taken from Glauert (1959).

B.1 Modified Momentum Theory

The Rankine-Froude theory¹ was developed as a momentum approach for propellers and later applied to rotors. The fundamental concept is that the rotor is idealised as an infinitesimally thin *actuator disk* across which a pressure difference exists responsible for thrust. For a rotor in hover, the assumptions of the theory are that the induced velocity is vertical and that the momentum flux of the far wake must equal the weight of the craft.

Consider Figure B.1(a). All numerical subscripts refer to station number, so that p_0 is the pressure at station 0, w_1 is the downward velocity at station 1, $\dot{m}_3$ is the massflow rate

¹After William Rankine (1820-1872) and William Froude (1810-1879).

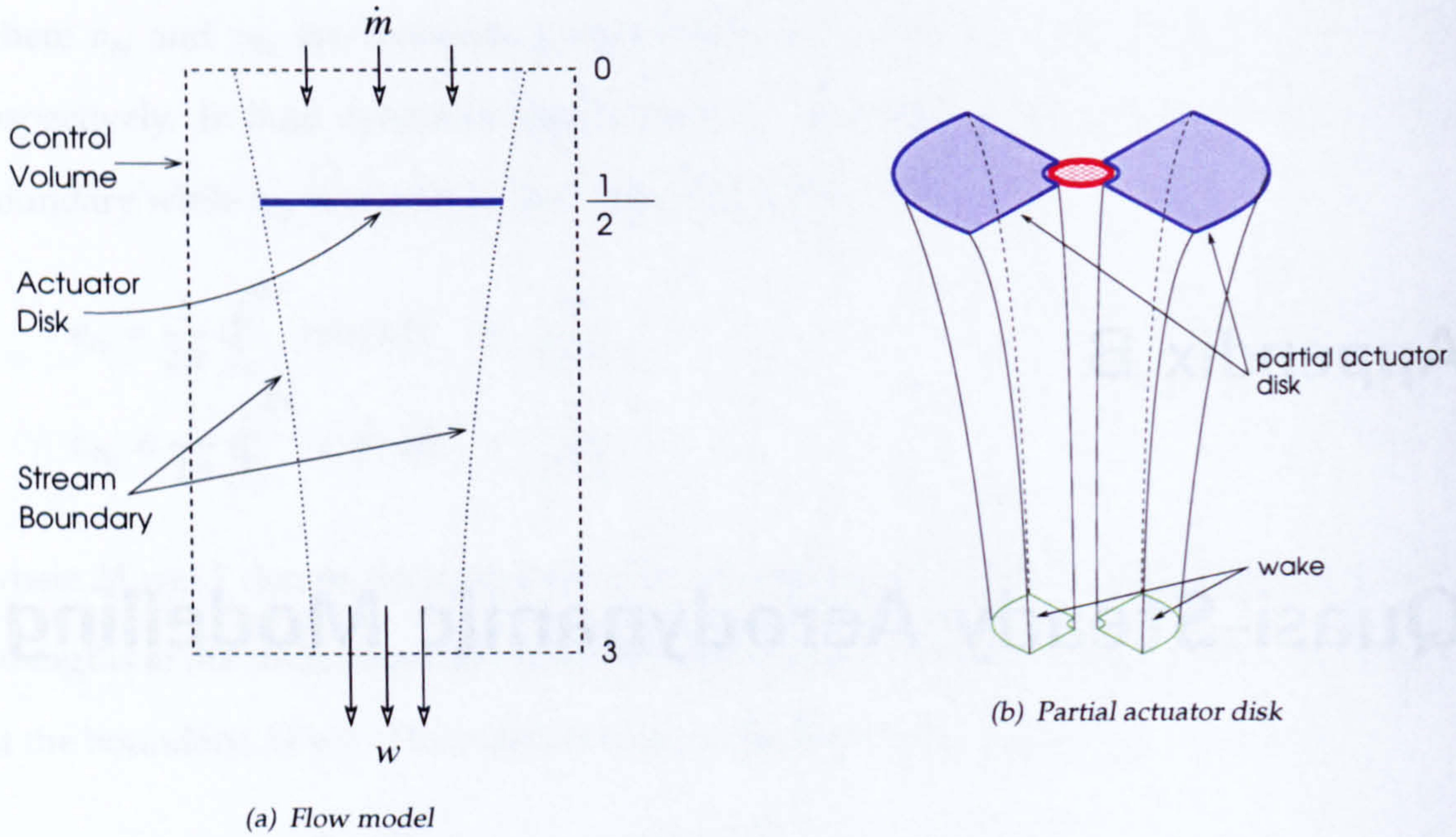


Figure B.1: Momentum theory

at station 3, etc.

From continuity considerations, the mass-flow rate across the actuator disk $\dot{m}$ is

$$\begin{aligned}\dot{m} &= \rho A w_1 = \rho A w_2 \\ \Rightarrow w_1 &= w_2\end{aligned}\tag{B.1}$$

The static pressures far above and far below the disk are the same, that is

$$p_0 = p_3 \quad (= p_\infty)\tag{B.2}$$

The total head of pressure above and below the actuator disk can be computed from Bernoulli's equation for steady flow and neglecting the gravity term (Milne-Thomson, 1966, §2.1) thus

$$p_0 + \frac{1}{2}\rho w_0^2 = p_1 + \frac{1}{2}\rho w_1^2 \quad \text{and} \quad p_2 + \frac{1}{2}\rho w_2^2 = p_3 + \frac{1}{2}\rho w_3^2$$

Adding these two equations and substituting from Equations B.1 and B.2 gives for hover

$$\begin{aligned}p_0 + \frac{1}{2}\rho w_0^2 + p_2 + \frac{1}{2}\rho w_2^2 &= p_1 + \frac{1}{2}\rho w_1^2 + p_3 + \frac{1}{2}\rho w_3^2 \\ \Rightarrow p_2 - p_1 &= \frac{1}{2}\rho w_3^2 \quad (= \Delta p)\end{aligned}\tag{B.3}$$

where Δp is the pressure difference across the actuator disk. This pressure difference is the source of the thrust T of the actuator disk, and for the craft (or insect) in hover, it must equal its weight W .

$$T = W = A \Delta p \quad (\text{B.4})$$

Based on the laws of classical mechanics, the Rankine-Froude theory states that the thrust of the propeller is equal to the momentum flux in the far wake, or

$$W = \dot{m}_3 w_3 \quad (\text{B.5})$$

but from continuity $\dot{m}_3 = \dot{m}_1$ so that

$$W = \rho A w_1 \cdot w_3 \quad (\text{B.6})$$

Combining Equations B.4 and B.6 gives

$$\begin{aligned} A \Delta p &= \rho A w_1 \cdot w_3 \\ \Rightarrow \Delta p &= \rho w_1 \cdot w_3 \end{aligned} \quad (\text{B.7})$$

Substituting Equation B.3 into B.7 gives

$$\begin{aligned} \rho w_1 \cdot w_3 &= \frac{1}{2} \rho w_3 \cdot w_3 \\ \Rightarrow w_1 &= \frac{1}{2} w_3 \end{aligned} \quad (\text{B.8})$$

Substituting Equation B.6 into Equation B.8 gives

$$\begin{aligned} W &= \rho A w_1 \cdot 2w_1 = 2\rho A w_1^2 \\ \Rightarrow w_1 &= \sqrt{\frac{W}{2\rho A}} \quad (= w_2) \end{aligned} \quad (\text{B.9})$$

If the velocity in the far wake is w ($=w_3$), then from Equation B.8

$$w = 2 \sqrt{\frac{W}{2\rho A}} \quad (\text{B.10})$$

From Equation B.5, the thrust due to the actuator disk may be rewritten as

$$T = \dot{m}w \quad (\text{B.11})$$

where $\dot{m}$ is mass flow rate through the actuator disk and w is the axial velocity far downstream (in the slipstream). The power imparted by the actuator disk to the flow can then computed from the difference in the rate of change of kinetic energy far downstream and far upstream, which for hover is

$$P = \frac{1}{2}\dot{m}(w^2 - 0^2) = \frac{1}{2}\dot{m}w^2 \quad (\text{B.12})$$

Substitution from Equation B.11 gives

$$P = \frac{1}{2}Tw \quad (\text{B.13})$$

Remembering that $T = W$, substituting from Equation B.10 into B.13 then yields

$$P = W \sqrt{\frac{W}{2\rho A}} \quad (\text{B.14})$$

In the case of an insect, the actuator disk is not completely circular (see Figure B.1(b)). Rather, each wing sweeps out an angle Φ (the stroke amplitude), covering an area $\frac{1}{2}A$ where $A = R^2\Phi$ and where R is wing length. Also, the stroke plane may be inclined at an angle β with the horizontal. Because the Rankine-Froude theory considers only vertical velocities, the area on which downward momentum is imparted is $A \cos \beta$.

Equations B.10 and B.14 then become

$$w = 2 \sqrt{\frac{W}{2\rho R^2\Phi \cos \beta}} \quad (\text{B.15})$$

and

$$P = W \sqrt{\frac{W}{2\rho R^2\Phi \cos \beta}} \quad (\text{B.16})$$

From Equation B.16, thrust due to the actuator disk is

$$T = \frac{w^2 \rho R^2 \Phi \cos \beta}{2} \quad (\text{B.17})$$

Equations B.15 and B.16 are the important results from this theory. It is common to calculate the velocity induced at the actuator disk. If this is denoted as w_i , then from Equations B.8 and B.15

$$w_i = \sqrt{\frac{W}{2\rho R^2 \Phi \cos \beta}} \quad (\text{B.18})$$

B.2 Modified Blade Element Method

In this approach, the rotor blade is divided into thin slices, or *blade elements* (see Figure B.2). The main assumption is that each blade-element section is two-dimensional and subjected to a quasi-steady treatment. The performance of the entire rotor is then evaluated by integrating over all the blade elements along the rotor span. Because of the similarity in the movement of insect (or FMAV) wings this theory, at least in principle, can be applied to them.

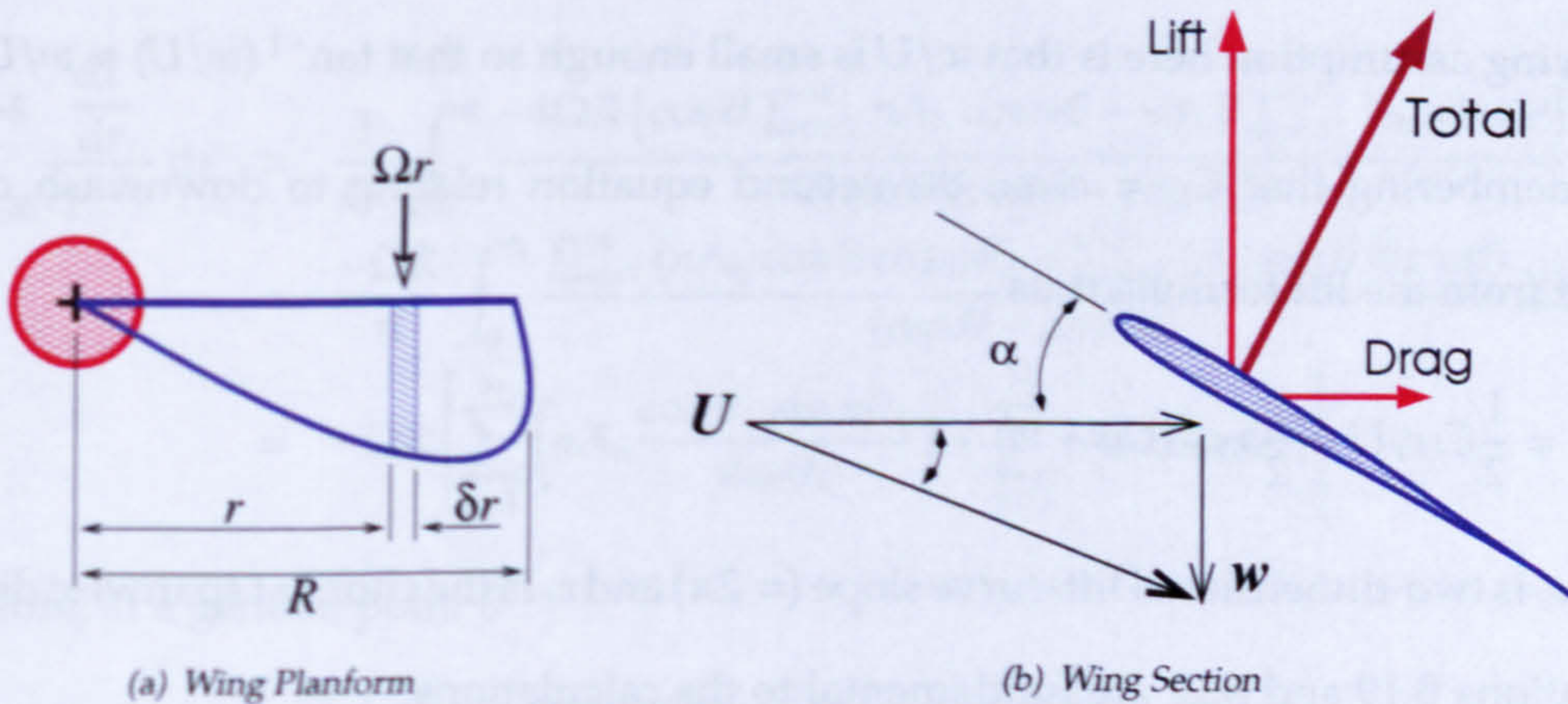


Figure B.2: Blade element analysis

The theory was first suggested by Drzwiecki in 1892, and then in 1909 with reference to aircraft propellers (Leishman, 2000, §3.1). As already stated, the theory assumes the blade elements to be 2-D aerofoils with no mutual influence on adjacent sections (Glauert, 1959, p. 212). In the current modified form, a 'realistic' downwash distribution is computed so that the wings show 3-D effects.

B.2.1 Downwash Distribution

The downwash distribution of a propeller (and in this case, an FMAV or insect wing) can be calculated by making use of relations used for three-dimensional wings. Two main relations are used in the calculation (see Glauert, 1959).

The first equation relates downwash to the rate of vortex shedding, thus

$$w(r_1) = \frac{1}{4\pi} \int_{-R}^R \frac{\frac{d\Gamma}{dr}}{r_1 - r} dr \quad (\text{B.19})$$

where r_1 is position at which downwash is being calculated, r is a general point spanwise along the wing and Γ is the circulation at this spanwise position r . Note that $w(r_1)$ is positive downwards. The induced downwash results in a modified angle of attack

$$\alpha_0 = \alpha - \frac{w}{U} \quad (\text{B.20})$$

where α_0 is the equivalent 2-D angle of attack and U is incident velocity at infinity. The underlying assumption here is that w/U is small enough so that $\tan^{-1}(w/U) \approx w/U$.

Remembering that $C_L = a_0 \alpha_0$, the second equation relating to downwash can be derived from the lift formula thus

$$\Gamma = \frac{1}{2} C_L c_r U = \frac{1}{2} a_0 c_r (U \alpha - w) \quad (\text{B.21})$$

where a_0 is two-dimensional lift-curve slope ($= 2\pi$) and c_r is the chord at spanwise distance r . Equations B.19 and B.21 are fundamental to the calculations.

If the propeller (or insect wing) spans from $-R$ to $+R$ then the position at any spanwise position may be represented parametrically by

$$r = -R \cos \theta \quad (\text{B.22})$$

where $\theta = 0 \dots \pi$. From this point onwards, all derivations presented are original.

Similar to that used by Glauert (1959), a Fourier series may then be used to represent

the circulation distribution along the span²

$$\Gamma = 4R \cdot \Omega r \cdot \sum_{n=1}^{\infty} A_n \sin n\theta = -4\Omega R^2 \cos \theta \sum_{n=1}^{\infty} A_n \sin n\theta \quad (\text{B.23})$$

(Note that this equation conveniently gives $\Gamma = 0$ at $r = 0$ (or $\theta = \frac{\pi}{2}$), and thereby the zero velocity at the hub of the propeller.) From Equation B.22

$$\frac{dr}{d\theta} = R \sin \theta \quad (\text{B.24})$$

and from Equation B.23

$$\frac{d\Gamma}{d\theta} = -4\Omega R^2 \left[\cos \theta \sum_{n=1}^{\infty} (nA_n \cos n\theta) - \sin \theta \sum_{n=1}^{\infty} (A_n \sin n\theta) \right] \quad (\text{B.25})$$

so that

$$\frac{d\Gamma}{dr} = \frac{d\Gamma}{d\theta} \cdot \frac{d\theta}{dr} = \frac{-4\Omega R}{\sin \theta} \left[\cos \theta \sum_{n=1}^{\infty} (nA_n \cos n\theta) - \sin \theta \sum_{n=1}^{\infty} (A_n \sin n\theta) \right] \quad (\text{B.26})$$

Using Equations B.22, B.24 and B.26 and the Glauert integrals³, the right-hand side of Equation B.19 can now be written as

$$\begin{aligned} \frac{1}{4\pi} \int_{-R}^R \frac{\frac{d\Gamma}{dr}}{r_1 - r} dr &= \frac{1}{4\pi} \int_0^\pi \frac{-4\Omega R \left(\cos \theta \sum_{n=1}^{\infty} nA_n \cos n\theta - \sin \theta \sum_{n=1}^{\infty} A_n \sin n\theta \right)}{R \sin \theta (\cos \theta - \cos \theta_1)} R \sin \theta d\theta \\ &= \frac{-\Omega R}{\pi} \int_0^\pi \frac{\sum_{n=1}^{\infty} (nA_n \cos \theta \cos n\theta) - \sum_{n=1}^{\infty} (A_n \sin \theta \sin n\theta)}{(\cos \theta - \cos \theta_1)} d\theta \\ &= -\Omega R \left[\sum_{n=1}^{\infty} \left(nA_n \frac{\cos \theta_1 \sin n\theta_1}{\sin \theta_1} \right) + \sum_{n=1}^{\infty} (A_n \cos n\theta_1) \right] \end{aligned}$$

Therefore, at a general point θ

$$w(\theta) = -\Omega R \left[\sum_{n=1}^{\infty} \left(nA_n \frac{\cos \theta \sin n\theta}{\sin \theta} \right) + \sum_{n=1}^{\infty} (A_n \cos n\theta) \right] \quad (\text{B.27})$$

²The distribution of circulation Γ is a function of spanwise distance r and incident freestream velocity $U = \Omega r$ (where Ω is time-varying angular velocity). Both of these dependencies are incorporated in the Fourier series expression for Γ .

³The relevant Glauert integrals are

$$\oint_0^\pi \frac{\sin n\phi \sin \phi}{\cos \theta - \cos \phi} d\phi = \pi \cos n\theta \quad \text{and} \quad \oint_0^\pi \frac{\cos n\phi \cos \phi}{\cos \theta - \cos \phi} d\phi = -\pi \frac{\cos \theta \sin n\theta}{\sin \theta}$$

Remembering that for a flapping wing, the incident velocity is of the form $U = \Omega r$ and the definition of r from Equation B.22, Equation B.21 can be re-written as

$$\Gamma = -\frac{1}{2}a_0c_\theta(\Omega R\alpha \cos \theta + w) \quad (\text{B.28})$$

where c_θ is the parametric equivalent of c_r . Substituting from Equations B.23 and B.27 into Equation B.28 gives

$$\begin{aligned} -4\Omega R^2 \sum_{n=1}^{\infty} (A_n \cos \theta \sin n\theta) \\ = -\frac{1}{2}a_0c_\theta \left(\Omega R\alpha \cos \theta - \Omega R \left[\sum_{n=1}^{\infty} \left(nA_n \frac{\sin n\theta}{\tan \theta} \right) + \sum_{n=1}^{\infty} (A_n \cos n\theta) \right] \right) \end{aligned}$$

Multiplying throughout by $\sin \theta$ and rearranging finally gives

$$\begin{aligned} a_0\alpha c_\theta \sin \theta \cos \theta = \\ 8R \sum_{n=1}^{\infty} (A_n \sin \theta \cos \theta \sin n\theta) + a_0c_\theta \left[\sum_{n=1}^{\infty} (nA_n \cos \theta \sin n\theta) + \sum_{n=1}^{\infty} (A_n \sin \theta \cos n\theta) \right] \end{aligned} \quad (\text{B.29})$$

Notice that although oncoming freestream velocity Ωr is absent in the above equation it has been accounted for in the relations leading to Equation B.27.

Due to the symmetrical distribution of downwash, only odd values of n occur and a reasonably good approximation can be found by using the first few values of n , typically A_1, A_3, A_5, A_7, A_9 . Simultaneous equations are required to solve for these and can be accomplished by solving Equation B.29 at various values of θ . Again, due to the symmetric nature of the problem, simultaneous equations can be formed with $0 < \theta < \frac{\pi}{2}$. Using these values of A_n , the downwash along the wing can then be evaluated from Equation B.27.

B.2.2 Forces and Moments

From the analysis above, it is possible to estimate the lift, induced drag and torque for the system.

Lift The lift for the propeller system is given by

$$L = \int_{-R}^R \rho U \Gamma dr \quad (\text{B.30})$$

In terms of parametric position θ , the integrands (from Equations B.22 and B.23) are given by

$$U = \Omega r = -\Omega R \cos \theta$$

$$r = -R \cos \theta \implies dr = R \sin \theta d\theta$$

$$\Gamma = -4 \Omega R^2 \cos \theta \sum_{n=1}^{\infty} A_n \sin n\theta$$

Equation B.30, therefore, becomes

$$\begin{aligned} L &= \int_0^{\pi} \rho \cdot \Omega R \cos \theta \cdot 4 \Omega R^2 \cos \theta \sum_{n=1}^{\infty} A_n \sin n\theta \cdot R \sin \theta d\theta \\ &= 4 \rho \Omega^2 R^4 \int_0^{\pi} \sin \theta \cos^2 \theta \sum_{n=1}^{\infty} A_n \sin n\theta d\theta \\ &= 4 \rho \Omega^2 R^4 \frac{(A_1 + A_3) \pi}{8} \end{aligned}$$

which simplifies to

$$L = \frac{1}{2} \rho \pi \Omega^2 R^4 (A_1 + A_3) \quad (\text{B.31})$$

Drag Similarly, induced drag for the whole propeller system is given by

$$D_i = \int_{-R}^R \rho w \Gamma dr \quad (\text{B.32})$$

Using the substitutions above and Equation B.27 then gives

$$\begin{aligned} D_i &= \int_0^{\pi} \rho \cdot \Omega R \left[\sum_{n=1}^{\infty} \left(n A_n \frac{\cos \theta \sin n\theta}{\sin \theta} \right) + \sum_{n=1}^{\infty} (A_n \cos n\theta) \right] \cdot 4 \Omega R^2 \cos \theta \sum_{n=1}^{\infty} A_n \sin n\theta \cdot R \sin \theta d\theta \\ &= 4 \rho \Omega^2 R^4 \int_0^{\pi} \sin \theta \cos \theta \left[\sum_{n=1}^{\infty} \left(n A_n \frac{\cos \theta \sin n\theta}{\sin \theta} \right) + \sum_{n=1}^{\infty} (A_n \cos n\theta) \right] \sum_{n=1}^{\infty} A_n \sin n\theta d\theta \\ &= 4 \rho \Omega^2 R^4 \cdot \frac{\pi}{4} \left[\sum_{n=1}^{\infty} n A_n^2 + \sum_{n=3}^{\infty} (n-1) A_{n-2} A_n \right] \end{aligned}$$

which finally reduces to⁴

$$D_i = \rho \pi \Omega^2 R^4 \left[\sum_{n=1}^{\infty} n A_n^2 + \sum_{n=3}^{\infty} (n-1) A_{n-2} A_n \right] \quad (n \text{ odd}) \quad (\text{B.33})$$

Note that only odd values of n are required⁵.

Torque A more important quantity in rotor-type problems is (induced) torque Q_i and is derived from induced drag, thus

$$Q_i = \int_{-R}^R \rho w \Gamma r dr \quad (\text{B.34})$$

Unlike lift and drag, the integration of the expression for total torque does not yield a closed expression so it must be computed numerically⁶. For purposes of comparison, another useful quantity is lift-to-torque ratio $\frac{L}{Q}$. Note that the subscript i has been dropped for clarity.

⁴Drag is nonzero because an artefact of the analysis is that drag is evaluated as Total Drag = $2 \times$ Drag of either wing

⁵Typical values of the sum are $A_1^2 + 3A_3^2 + 5A_5^2 + \dots + 2A_1A_3 + 4A_3A_5 + 6A_5A_7 + \dots$

⁶In fact, the integral for torque actually yields a zero sum because the positive drag on each wing (see Footnote 4 on p. 324) gives a net zero moment instead of a couple. Torque must, therefore, be computed as Total Torque = $2 \times$ Torque on either wing

Appendix C

Unsteady Aerodynamic Modelling

This appendix contains details of the derivations used in the 2-D, unsteady aerodynamic modelling presented in Chapter 4. All work is original and derived from first principles unless stated otherwise.

The work in § C.1 has been adapted from Benson (1989). The earlier sections of § C.2 are based on the work of Lam (1989) albeit with several corrections, modifications and extensions, and all formulae have been derived from first principles. § C.3 contains original work, including the extension to include the effect of the leading-edge vortex. It also contains the derivation two novel constraint equations for the flapping-wing problem (Equations C.62 and C.68). Finally, some of the work in § C.5 is again similar to Lam but with significant modifications and extensions.

C.1 Conformal Mapping

The crux of the approach is the conformal mapping of a circle into a Joukowski-type aerofoil. The essence of using this approach is that a complicated problem in physical space is transformed into a simpler problem in transformed space.

In our approach, the simple case of flow around a 2-D cylinder (in the Z -plane) is mapped into that around a Joukowski-type aerofoil (in the ζ -plane) using the following transformation

$$\zeta = Z + \frac{(1 - \epsilon)R^2}{Z} + \frac{\epsilon R^3}{2Z^2} \quad (\text{C.1})$$

where

$$\epsilon = \frac{\tau - i\sigma}{R} \quad (\text{C.2})$$

and where τ and σ are thickness and camber parameters respectively, and R is the radius of the circle. Sample transformations are shown in Figure 4.3.

The advantage of using this transformation is that the trailing edge is always at the point $Z = R$ and that the circle is always centred at the origin. The latter enables use of *conjugate function theory* (see Appendix A.3) to calculate the tangential or the radial velocity component from the other (which will be essential as seen later in § C.2.2)¹. Another advantage of this transformation is that the freestream velocity remains horizontal throughout — a rotation through the angle of attack is not required.

It is important to note that τ and σ only control thickness and camber respectively, and are not the maximum thickness and camber of the aerofoil *per se*. These can, however, easily be derived as shown below.

C.1.1 Aerofoil Characteristics

If an aerofoil with complex coordinates $\zeta = \xi + i\eta$ corresponds to the circle $Z = R e^{i\theta}$, then by Equation C.1 the aerofoil coordinates are

$$\xi = 2R \cos \theta + \sigma \left(\sin \theta - \frac{1}{2} \sin 2\theta \right) - \tau \left(\cos \theta - \frac{1}{2} \cos 2\theta \right) \quad (\text{C.3})$$

$$\eta = \tau \left(\sin \theta - \frac{1}{2} \sin 2\theta \right) + \sigma \left(\cos \theta - \frac{1}{2} \cos 2\theta \right) \quad (\text{C.4})$$

¹For a conventional Joukowski transformation of the form $\zeta = Z + \frac{R^2}{Z}$, the centre of the circle is not necessarily at the origin in the Z -plane, and hence, conjugate function theory cannot be applied.

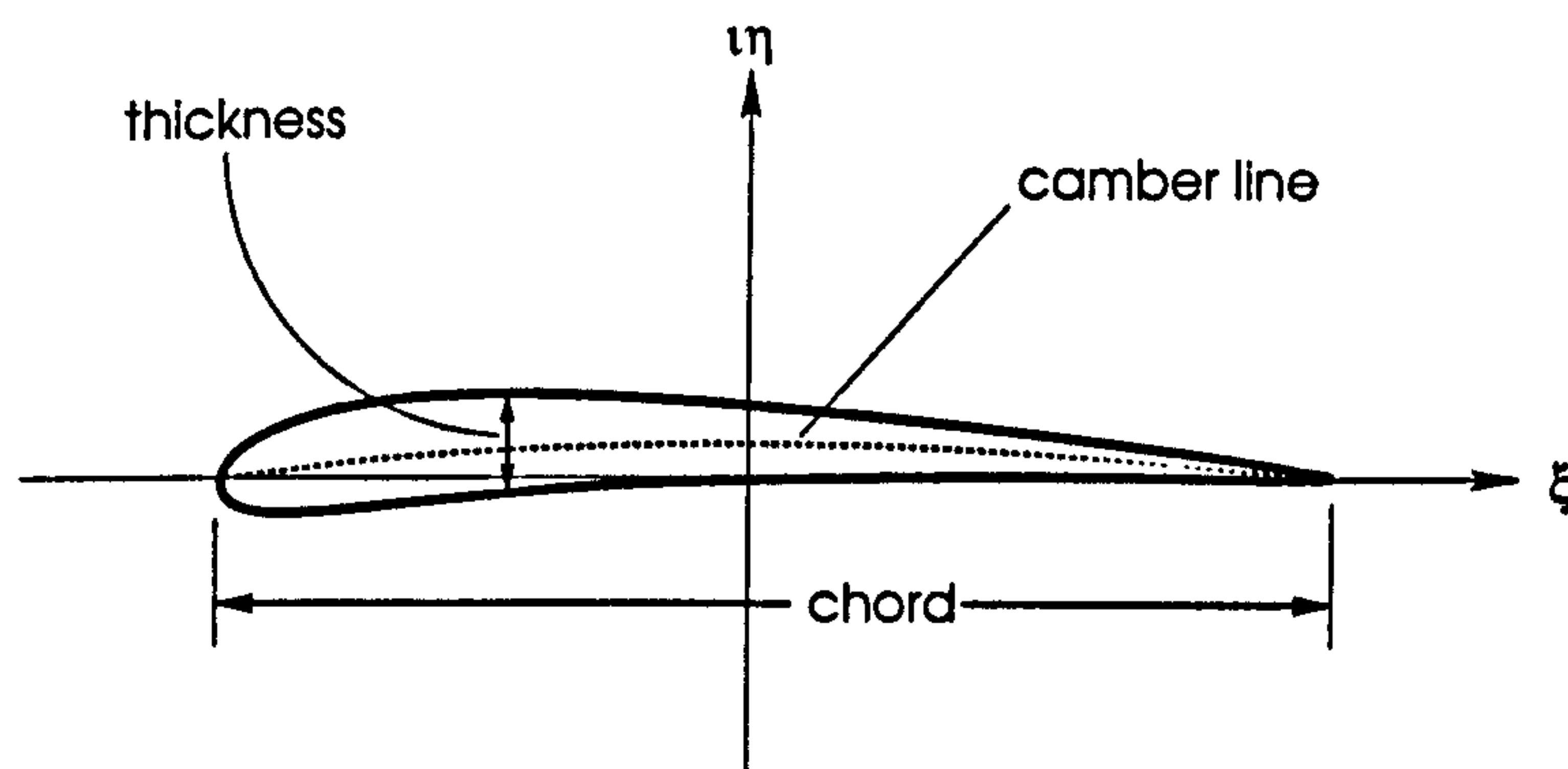


Figure C.1: Aerofoil characteristics

Chord The chord length c is given by the line joining the leading and trailing edges (see Figure C.1) and is equal to

$$c = |\xi(\pi) - \xi(0)| = 4R - 2\tau \quad (\text{C.5})$$

Thickness The thickness of the aerofoil $\bar{\tau}$ at any point is given by

$$\bar{\tau} = \eta(\theta) - \eta(-\theta) = 2\tau \left(\sin \theta - \frac{1}{2} \sin 2\theta \right) \quad (\text{C.6})$$

The maximum thickness of the aerofoil occurs at $\theta = \frac{2}{3}\pi$ where

$$\bar{\tau}_{max} = \frac{3\sqrt{3}}{2}\tau \quad (\text{C.7})$$

Camber The camber-line $\bar{\sigma}$ is taken as the line halfway between the upper and lower surfaces of an aerofoil, i.e.

$$\bar{\sigma} = \frac{1}{2}(\eta(\theta) - \eta(-\theta)) = \sigma \left(\cos \theta - \frac{1}{2} \cos 2\theta \right) \quad (\text{C.8})$$

The maximum positive camber occurs at $\theta = \frac{1}{3}\pi$ where

$$\bar{\sigma}_{max} = \frac{3}{4}\sigma \quad (\text{C.9})$$

It can easily be seen that Equations C.5, C.6 and C.8 reduce to the case of a flat plate for $\tau = \sigma = 0$. In such a case, $\epsilon = 0$ and Equation C.1 takes the more conventional form of the Joukowski-transformation given in footnote 1 on p. 326.

C.2 Wake-Free Circulation

This is the quasi-steady circulation component that is produced irrespective of the presence of a wake. It comprises a component due to the freestream velocity (§ C.2.1) and a component due to the unsteady motion of the aerofoil (§ C.2.2).

C.2.1 Freestream Component

The circulation due to the effect of the freestream can be simply calculated using the circle theorem (Milne-Thomson, 1966, § 5.2). The complex potential of the flow around a circular cylinder in a constant freestream of velocity U_∞ inclined at an angle α is given by

$$\Omega(Z) = U_\infty \left(Z e^{-i\alpha} + \frac{R^2 e^{i\alpha}}{Z} \right) \quad (\text{C.10})$$

which differentiates to give the complex velocity

$$q(Z) = \frac{d\Omega}{dZ} = U_\infty \left(e^{-i\alpha} - \frac{R^2 e^{i\alpha}}{Z^2} \right) \quad (\text{C.11})$$

Substituting for $Z (= R e^{i\theta})$ (on the circle) gives

$$q(Z) \Big|_{Z=R e^{i\theta}} = 2U_\infty \sin \theta \sin(\theta - \alpha) + i[2U_\infty \cos \theta \sin(\theta - \alpha)] = v_x - iv_y \quad (\text{C.12})$$

where v_x and v_y are the velocity components in the real and imaginary axes directions respectively. It can be shown that

$$v_x - iv_y = (v_r - iv_\theta) e^{-i\theta} \quad (\text{C.13})$$

where v_r and v_θ are the radial and circumferential (tangential) components of velocity respectively. Using this relation (and comparing real and imaginary parts), velocity can be transformed from rectangular to polar coordinates as

$$v_r = v_x \cos \theta + v_y \sin \theta \quad (\text{C.14})$$

$$v_\theta = v_y \cos \theta - v_x \sin \theta \quad (\text{C.15})$$

and vice versa². Comparing Equations C.14 and C.15 with Equation C.12 gives

$$v_r(\theta, t)|_f = 0 \quad (\text{C.18})$$

$$v_\theta(\theta, t)|_f = -2U_\infty \sin(\theta - \alpha(t)) \quad (\text{C.19})$$

where the subscript f refers to the freestream component. To produce smooth flow at the trailing edge ($\theta = 0$), the Kutta-Joukowski condition requires that there be zero pressure difference across the trailing edge. In other words, the velocity there must be finite so that $v_\theta(0, t)|_f = 0$. The tangential velocity, therefore, becomes

$$v_\theta(\theta, t)|_f = -2U_\infty \sin(\theta - \alpha(t)) + \frac{\Gamma_k(t)}{2\pi R} \quad (\text{C.20})$$

where $\Gamma_k(t) = -4\pi R U_\infty \sin \alpha(t)$. Assuming that the velocity within the circle is zero, the tangential velocity at any point on the circle is equivalent to the vorticity there so that

$$\gamma_0(\theta, t)|_f = v_\theta(\theta, t)|_f = -2U_\infty [\sin(\theta - \alpha(t)) + \sin \alpha(t)] \quad (\text{C.21})$$

and hence circulation is

$$\Gamma_0(t)|_f = \int_0^{2\pi} \gamma_0(\theta, t)|_f R d\theta = \Gamma_k(t) = -4\pi R U_\infty \sin \alpha(t) \quad (\text{C.22})$$

Since circulation does not depend on which plane it was computed in — whether circle ($\mathcal{Z}$) or aerofoil (ζ) plane — Equation C.22 gives the bound circulation around the aerofoil due to the presence of an inclined freestream.

C.2.2 Unsteady Motion Component

In order to determine the bound circulation around the aerofoil due to the unsteady (flapping) motion of the aerofoil, the kinematic boundary condition at the aerofoil surface

²Similarly, the inverse transformation yields

$$v_x = v_r \cos \theta - v_\theta \sin \theta \quad (\text{C.16})$$

$$v_y = v_r \sin \theta + v_\theta \cos \theta \quad (\text{C.17})$$

must first be derived. From this, the velocity components at the aerofoil surface can be computed. These, in turn, yield the expression for vorticity, and hence, circulation.

Aerofoil Kinematic Boundary Condition

The usual boundary condition for flow around an aerofoil is the zero-through-flow condition, i.e. fluid cannot penetrate the aerofoil surface. This implies that the normal velocity at the aerofoil surface is zero, i.e. $v_n = 0$ where the subscript n denotes the normal to the surface of the aerofoil. Before deriving the boundary condition, the instantaneous position of the aerofoil surface must be determined. The aerofoil has three degrees of freedom — pitch ($\alpha, \dot{\alpha}$), lunge ($l, \dot{l}$) and heave ($h, \dot{h}$) where the dot ($\dot{}$) implies a time rate of change. See Figure C.2 (reproduced from Figure 4.6) below.

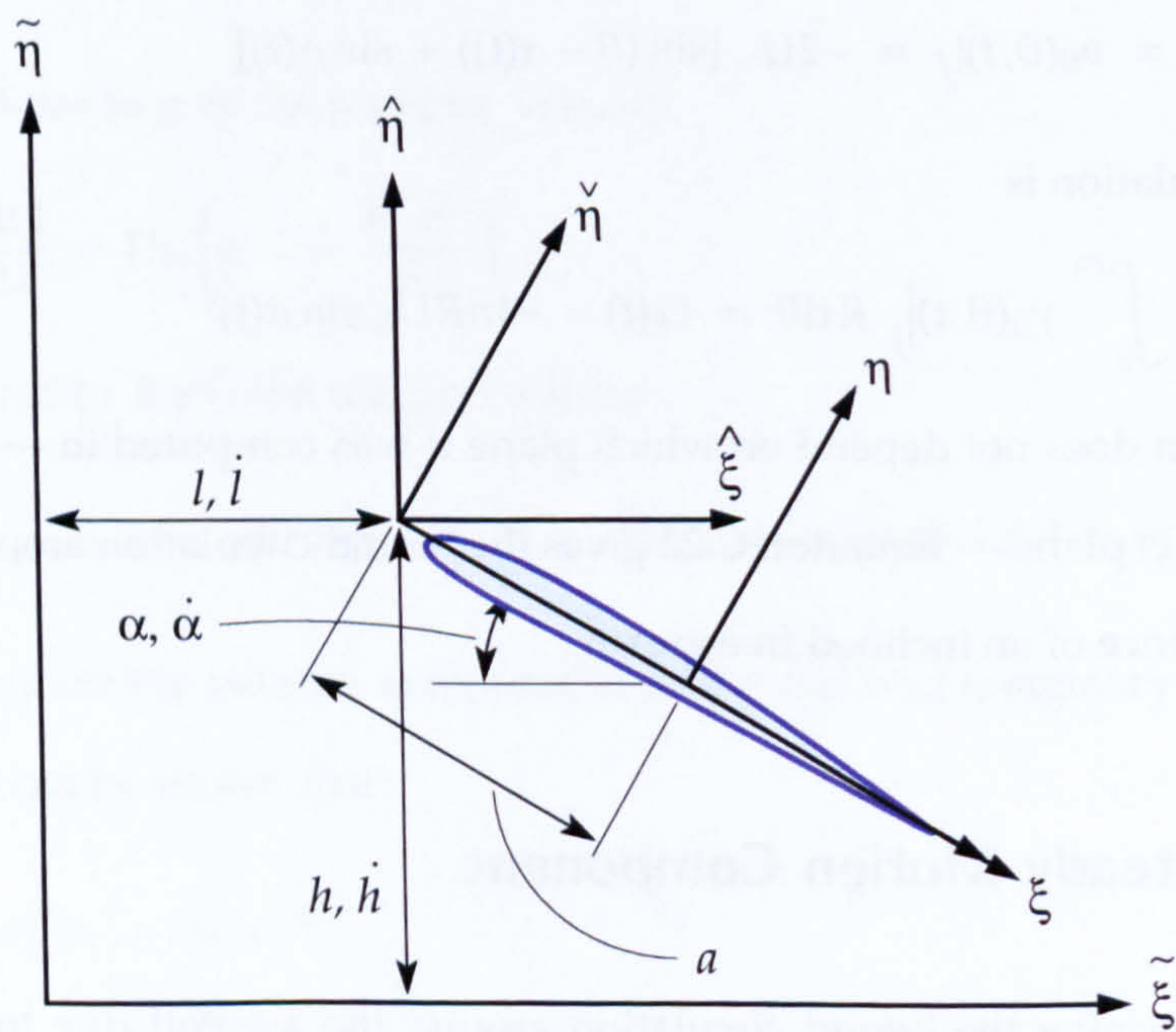


Figure C.2: Coordinate systems

Before proceeding, it is useful to describe the various Lagrangian coordinate systems. Figure C.2 depicts four coordinate systems — inertial ($\tilde{\xi}$), translating ($\hat{\eta}$), rotating ($\check{\eta}$) and aerofoil (η). The translating and rotating coordinate systems are aerofoil-inertia systems

because they move with the aerofoil.

By simple geometry, it can be seen that a point on the aerofoil at (ξ_a, η_a) in the aerofoil-fixed system is seen in the inertial system at $(\tilde{\xi}_a, \tilde{\eta}_a)$ such that

$$\zeta_a = (\tilde{\zeta}_a - (l + ih))e^{i\alpha} - a \quad (C.23)$$

where $\zeta_a = \xi_a + i\eta_a$, $\tilde{\zeta}_a = \tilde{\xi}_a + i\tilde{\eta}_a$, α is the (geometrical) angle of attack and a is the 'offset' distance between the point of rotation and the centre of the aerofoil (also see Table A.1 on p. 312). The real and imaginary parts of Equation C.23 can be separated to yield two equations, thus

$$\xi_a = (\tilde{\xi}_a - l) \cos \alpha - (\tilde{\eta}_a - h) \sin \alpha - a \quad (C.24)$$

$$\eta_a = (\tilde{\xi}_a - l) \sin \alpha + (\tilde{\eta}_a - h) \cos \alpha \quad (C.25)$$

Similarly, the kinematic velocity in the aerofoil (u_a, v_a) and inertial $(\tilde{u}_a, \tilde{v}_a)$ systems can be related thus

$$u_a = (\tilde{u}_a - \dot{l}) \cos \alpha - (\tilde{v}_a - \dot{h}) \sin \alpha - \eta_a \dot{\alpha} \quad (C.26)$$

$$v_a = (\tilde{u}_a - \dot{l}) \sin \alpha + (\tilde{v}_a - \dot{h}) \cos \alpha - (\xi_a + a) \dot{\alpha} \quad (C.27)$$

where $\dot{l}$ is lunge velocity, $\dot{h}$ is heave (plunge) velocity and $\dot{\alpha}$ is pitch rate (Figure C.2).

To enforce the zero-through-flow boundary condition at the surface of the aerofoil, the fluid flow there must be parallel to the surface. In other words, the fluid velocity $(\tilde{u}, \tilde{v})$ and aerofoil kinematic velocity $(\tilde{u}_a, \tilde{v}_a)$ must be equal at the aerofoil surface. In complex number notation, this is given by

$$\tilde{u}_a - i\tilde{v}_a = \tilde{u} - i\tilde{v} \quad (C.28)$$

The no-slip boundary condition on the solid-fluid interface cannot be enforced because viscosity has been neglected.

Now, fluid flow occurs because a gradient in velocity potential exists, i.e. $\nabla\Phi = (\tilde{u}, \tilde{v})$. It is important to realise that the gradient $\nabla\Phi$ has the same magnitude in the aerofoil

coordinate system, so that in the latter reference system $\nabla\Phi = (u, v)$. Fluid velocity in the inertial and aerofoil coordinate systems can, therefore, be related thus

$$\tilde{u} = v \sin \alpha + u \cos \alpha \quad (\text{C.29})$$

$$\tilde{v} = v \cos \alpha - u \sin \alpha \quad (\text{C.30})$$

Equating Equations C.29 and C.30 to Equations C.26 and C.27 at the aerofoil surface respectively and solving simultaneously finally yields

$$u = \dot{l} \cos \alpha - \dot{h} \sin \alpha + \eta \dot{\alpha} \quad (\text{C.31})$$

$$v = \dot{l} \sin \alpha + \dot{h} \cos \alpha - (\xi + a) \dot{\alpha} \quad (\text{C.32})$$

Equations C.31 and C.32 are the zero-through-flow boundary conditions and must not be confused as conversions between coordinate systems³.

³These boundary conditions can also be derived in another way. If $\mathbf{N}(\xi, \eta, t) = 0$ defines the aerofoil surface throughout its motion, then the zero-through-flow boundary condition for the fluid flow can be defined in terms of the substantial derivative of $\mathbf{N}$ as

$$\left. \frac{D\mathbf{N}}{Dt} \right|_a = 0 \quad (\text{C.33})$$

where the subscript $_a$ indicates that the derivative is taken at the aerofoil surface. Equation C.33 denotes that the dot product of fluid velocity and surface velocity at the aerofoil surface is zero, i.e. the two are parallel there. Although, this kinematic boundary condition is defined in the aerofoil-fixed system, the substantial derivative must be taken in the inertial ($\tilde{\cdot}$) system.

At the aerofoil surface $\mathbf{N} = \xi + i\eta$, so that the boundary condition requires that

$$\left. \frac{D\xi}{Dt} \right|_a + i \left. \frac{D\eta}{Dt} \right|_a = 0 \quad (\text{C.34})$$

Since both ξ and η are real, the real and imaginary parts of Equation C.34 must both vanish at the aerofoil surface. The two parts are, therefore, considered separately.

Remember that the substantial derivative is defined as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \tilde{u} \frac{\partial}{\partial \tilde{\xi}} + \tilde{v} \frac{\partial}{\partial \tilde{\eta}} \quad (\text{C.35})$$

Circulation

Since circulation is the same in both the aerofoil and circle planes, it will be determined in the latter (Z -plane) because it is simpler.

Now that the kinematic boundary condition for the unsteady motion of the aerofoil has been found (Equations C.31 and C.32), the circumferential (tangential) velocity on the corresponding circle needs to be derived. This in turn will yield vorticity which will then be used to compute the bound circulation around the aerofoil due to its unsteady motion.

The complex velocity at a point in the aerofoil (ζ -plane) is given by

$$q(Z) = u - iv \quad (C.36)$$

By combining Equations 4.3c, C.13 and C.32, the velocity in the circle plane can be written in terms of that in the aerofoil plane thus

$$v_r - iv_\theta = (u - iv) e^{i\theta} \frac{d\zeta}{dZ} \quad (C.37)$$

By using the substitution $e^{i\theta} \frac{d\zeta}{dZ} = A + iB$, Equation C.37 becomes

$$\begin{aligned} v_r - iv_\theta &= (u - iv)(A + iB) \\ &= (Au + Bv) + i(Bu - Av) \end{aligned} \quad (C.38)$$

Substituting for ξ_a from Equation C.24, the real part of Equation C.34 gives

$$0 = -\dot{l} \cos \alpha + \dot{h} \sin \alpha - \underbrace{\left((\tilde{\xi}_a - l) \sin \alpha - (\tilde{\eta} - h) \cos \alpha \right) \dot{\alpha}}_{-\eta_a \dot{\alpha}} + \underbrace{\tilde{u} \cos \alpha - \tilde{v} \sin \alpha}_u$$

which is equivalent to Equation C.31. Similarly, substituting for η_a from Equation C.25, the imaginary part of Equation C.34 yields

$$0 = -\dot{l} \sin \alpha - \dot{h} \cos \alpha + \underbrace{\left((\tilde{\xi}_a - l) \cos \alpha - (\tilde{\eta} - h) \sin \alpha \right) \dot{\alpha}}_{\xi_a \dot{\alpha}} + \underbrace{\tilde{u} \sin \alpha + \tilde{v} \cos \alpha}_v$$

which again is equivalent to Equation C.32.

where

$$A = \frac{1}{R} [\tau (\cos \theta - \cos 2\theta) - \sigma (\sin \theta - \sin 2\theta)] \quad (C.39)$$

$$B = \frac{1}{R} [2R \sin \theta - \tau (\sin \theta - \sin 2\theta) - \sigma (\cos \theta - \cos 2\theta)] \quad (C.40)$$

The zero-through-flow boundary condition stated earlier applies only to the component of velocity normal to the aerofoil surface — it makes no attempt to describe the tangential component. The values for u and v deduced above (Equations C.31 and C.32 respectively), therefore, only correspond to the component of velocity normal to the surface of the circle, i.e. v_r . Since u and v do not carry information about the component of velocity tangential to the surface, only the real parts of Equation C.38 can be used, thus

$$v_r = Au + Bv \quad (C.41)$$

It is important to make the above distinction. Substituting from Equations C.3, C.4, C.31, C.32, C.39 and C.40 into Equation C.41 gives

$$\begin{aligned} R v_r = & 2R [\sin \theta (\dot{l} \sin \alpha + \dot{h} \cos \alpha)] \\ & + \tau [-\sin \theta (\dot{l} \sin \alpha + \dot{h} \cos \alpha) + \sin 2\theta (\dot{l} \sin \alpha + \dot{h} \cos \alpha) \\ & + \cos \theta (\dot{l} \cos \alpha - \dot{h} \sin \alpha) - \cos 2\theta (\dot{l} \cos \alpha - \dot{h} \sin \alpha)] \\ & + \sigma [-\sin \theta (\dot{l} \cos \alpha - \dot{h} \sin \alpha) + \sin 2\theta (\dot{l} \cos \alpha - \dot{h} \sin \alpha) \\ & - \cos \theta (\dot{l} \sin \alpha + \dot{h} \cos \alpha) + \cos 2\theta (\dot{l} \sin \alpha + \dot{h} \cos \alpha)] \\ & + \dot{\alpha} \left[(-2aR + a\tau) \sin \theta - a\tau \sin 2\theta + a\sigma \cos \theta - a\sigma \cos 2\theta - 2R\sigma \sin^2 \theta \right. \\ & + 2R\sigma \cos^2 \theta + (4R\tau - 4R^2) \sin \theta \cos \theta + R\sigma \sin \theta \sin 2\theta - 2R\sigma \cos \theta \cos 2\theta \\ & \left. + \left(-\frac{1}{2}\tau^2 - \frac{1}{2}\sigma^2 - R\tau \right) \sin \theta \sin 2\theta + \left(\frac{1}{2}\tau^2 + \frac{1}{2}\sigma^2 - 2R\tau \right) \cos \theta \sin 2\theta \right] \quad (C.42) \end{aligned}$$

Rearranging Equation C.42 gives

$$\begin{aligned}
 v_r(\theta, t)|_a = & \frac{1}{R} \left(A_1 \sin \theta + A_2 \sin 2\theta + A_3 \cos \theta + A_4 \cos 2\theta \right. \\
 & + A_5 \sin^2 \theta + A_6 \cos^2 \theta + A_7 \sin \theta \cos \theta + A_8 \sin \theta \sin 2\theta \\
 & \left. + A_9 \sin \theta \cos 2\theta + A_{10} \cos \theta \sin 2\theta + A_{11} \cos \theta \cos 2\theta \right) \quad (C.43)
 \end{aligned}$$

where

$$\left. \begin{aligned}
 A_1 &= -\sigma(\dot{l} \cos \alpha - \dot{h} \sin \alpha) + (2R - \tau)(\dot{l} \sin \alpha + \dot{h} \cos \alpha) + a(\tau - 2R)\dot{\alpha} \\
 A_2 &= \sigma(\dot{l} \cos \alpha - \dot{h} \sin \alpha) + \tau(\dot{l} \sin \alpha + \dot{h} \cos \alpha) - a\tau\dot{\alpha} \\
 A_3 &= \tau(\dot{l} \cos \alpha - \dot{h} \sin \alpha) - \sigma(\dot{l} \sin \alpha + \dot{h} \cos \alpha) + a\sigma\dot{\alpha} \\
 A_4 &= -\tau(\dot{l} \cos \alpha - \dot{h} \sin \alpha) + \sigma(\dot{l} \sin \alpha + \dot{h} \cos \alpha) - a\sigma\dot{\alpha} \\
 A_5 &= -2R\sigma\dot{\alpha} \\
 A_6 &= 2R\sigma\dot{\alpha} \\
 A_7 &= 4R(\tau - R)\dot{\alpha} \\
 A_8 &= R\sigma\dot{\alpha} \\
 A_9 &= -\frac{1}{2}(\tau^2 + \sigma^2 + 2R\tau)\dot{\alpha} \\
 A_{10} &= \frac{1}{2}(\tau^2 + \sigma^2 - 4R\tau)\dot{\alpha} \\
 A_{11} &= -2R\sigma\dot{\alpha}
 \end{aligned} \right\} \quad (C.44)$$

and the subscript a indicates contribution due to aerofoil motion. To calculate vorticity (and hence, circulation) on the surface of the circle, the circumferential velocity v_θ must be known. This can be evaluated using the radial velocity (Eqn. C.43) and applying the ‘conjugate function theory’ (see Appendix A.3). The transformation equation is

$$v_\theta(\theta, t)|_a = -\frac{1}{2\pi} \oint_0^{2\pi} v_r(\phi, t)|_a \cot\left(\frac{\phi - \theta}{2}\right) d\phi \quad (C.45)$$

The right-hand side of Equation C.45 features a Glauert-type principal value integral⁴

⁴The kernel expands as

$$\cot\left(\frac{\phi - \theta}{2}\right) = \frac{\sin \theta + \sin \phi}{\cos \theta - \cos \phi}$$

Common Glauert-type integrals are given in Appendix A.1.

and evaluates as

$$\oint_0^{2\pi} v_r(\phi, t)|_a \left(\frac{\sin \theta + \sin \phi}{\cos \theta - \cos \phi} \right) d\phi = \frac{2\pi}{R} \left(A_1 \cos \theta + A_2 \cos 2\theta - A_3 \sin \theta \right. \\ \left. - A_4 \sin 2\theta + A_5 \sin \theta \cos \theta - A_6 \sin \theta \cos \theta \right. \\ \left. + A_7 \sin \theta \cos \theta + A_8 \sin \theta \cos 2\theta \right. \\ \left. - A_9 \sin \theta \cos 2\theta + A_{10} \cos \theta \cos 2\theta \right. \\ \left. - A_{11} \cos \theta \sin \theta \right)$$

The circumferential velocity can then be readily calculated to be

$$v_\theta(\theta, t)|_a = \frac{1}{R} \left[-A_1 \cos \theta - \left(A_2 + \frac{1}{2}A_7 \right) \cos 2\theta + A_3 \sin \theta \right. \\ \left. + \left(A_4 - \frac{1}{2}A_5 + \frac{1}{2}A_6 \right) \sin 2\theta - A_8 \sin \theta \cos 2\theta \right. \\ \left. + A_9 \sin \theta \sin 2\theta - A_{10} \cos \theta \cos 2\theta + A_{11} \cos \theta \sin 2\theta \right] \quad (C.46)$$

where the symbols $A_i, i = 1 \dots 11$ are as in Equation C.44 above. Again, assuming there is no flow within the aerofoil, the circumferential velocity is equivalent to the local vorticity. In order to have a finite velocity at the trailing edge, the Kutta-Joukowski condition needs to be enforced there. Hence, the vorticity distribution is given by

$$\gamma_0(\theta, t)|_a = \frac{1}{R} \left[-A_1 \cos \theta - \left(A_2 + \frac{1}{2}A_7 \right) \cos 2\theta + A_3 \sin \theta \right. \\ \left. + \left(A_4 - \frac{1}{2}A_5 + \frac{1}{2}A_6 \right) \sin 2\theta - A_8 \sin \theta \cos 2\theta + A_9 \sin \theta \sin 2\theta \right. \\ \left. - A_{10} \cos \theta \cos 2\theta + A_{11} \cos \theta \sin 2\theta + \frac{\Gamma_0(t)|_a}{2\pi} \right] \quad (C.47)$$

By setting $\theta = 0$ (trailing edge) in the above equation, the circulation due to the unsteady motion of the aerofoil is found to be

$$\Gamma_0(t)|_a = 2\pi \left[2R(l \sin \alpha + h \cos \alpha) + a \left(\frac{1}{2}\tau^2 + \frac{1}{2}\sigma^2 - 2R(R + a) \right) \right] \quad (C.48)$$

C.2.3 Total Wake-Free Effect

The total vorticity induced by the wake-free effects can be computed by combining the freestream (Equation C.21) with that due to the aerofoil motion (Equation C.47) to give

$$\begin{aligned} \gamma_0(\theta, t)|_a = & \frac{1}{R} \left[-A_1 \cos \theta - \left(A_2 + \frac{1}{2} A_7 \right) \cos 2\theta + A_3 \sin \theta \right. \\ & + \left(A_4 - \frac{1}{2} A_5 + \frac{1}{2} A_6 \right) \sin 2\theta - A_8 \sin \theta \cos 2\theta + A_9 \sin \theta \sin 2\theta \\ & - A_{10} \cos \theta \cos 2\theta + A_{11} \cos \theta \sin 2\theta + \frac{\Gamma_0(t)|_a}{2\pi} \Big] \\ & - 2U_\infty \sin(\theta - \alpha) \end{aligned} \quad (\text{C.49})$$

Total wake-free bound circulation can be evaluated in a similar way by combining Equations C.22 and C.48 to give

$$\Gamma_0(t)|_a = 2\pi \left[2R \left((l - U_\infty) \sin \alpha + h \cos \alpha \right) + \alpha \left(\frac{1}{2} \tau^2 + \frac{1}{2} \sigma^2 - 2R(R + a) \right) \right] \quad (\text{C.50})$$

C.3 Wake-Induced Circulation

The presence of the leading-edge vortex (denoted by lev) and the wake vortices affects the vorticity (and hence, circulation) distribution over the aerofoil. Once again, the calculation of their effect is simpler to compute in the circle (Z) plane.

Due to the severity of the manoeuvres during flapping, flow is likely to separate from both the leading and trailing edges of the aerofoil. Therefore, in addition to the conventional wake shed from the trailing edge, a wake is also shed from the leading edge — the leading-edge vortex. The fact that vortex sheets emerge from these points on the aerofoil means that the loading there must be zero, there being none across the vortex sheets. Therefore, the Kutta-Joukowski condition must be satisfied at these points.

The analysis relies on the circle theorem (Milne-Thomson, 1966, § 5.1) and the method of images. See Figure C.3 below (reproduced from Figure 4.7). To distinguish between

the two wakes, the leading and trailing edges are said to shed LEVs and wake vortices respectively. Two cases are considered here — one with both leading- and trailing-edge separation and the other one with a trailing-edge separation only.

It may be noted that in the analysis that follows, ‘LEV’ refers to the entire leading-edge-vortex structure so it is not tautological to refer to ‘LEV vortices’. Individual vortices are denoted leading-edge vortices. Also, where reference is made to ‘wake vortices’ or a singular ‘wake’, the trailing-edge wake is implied.

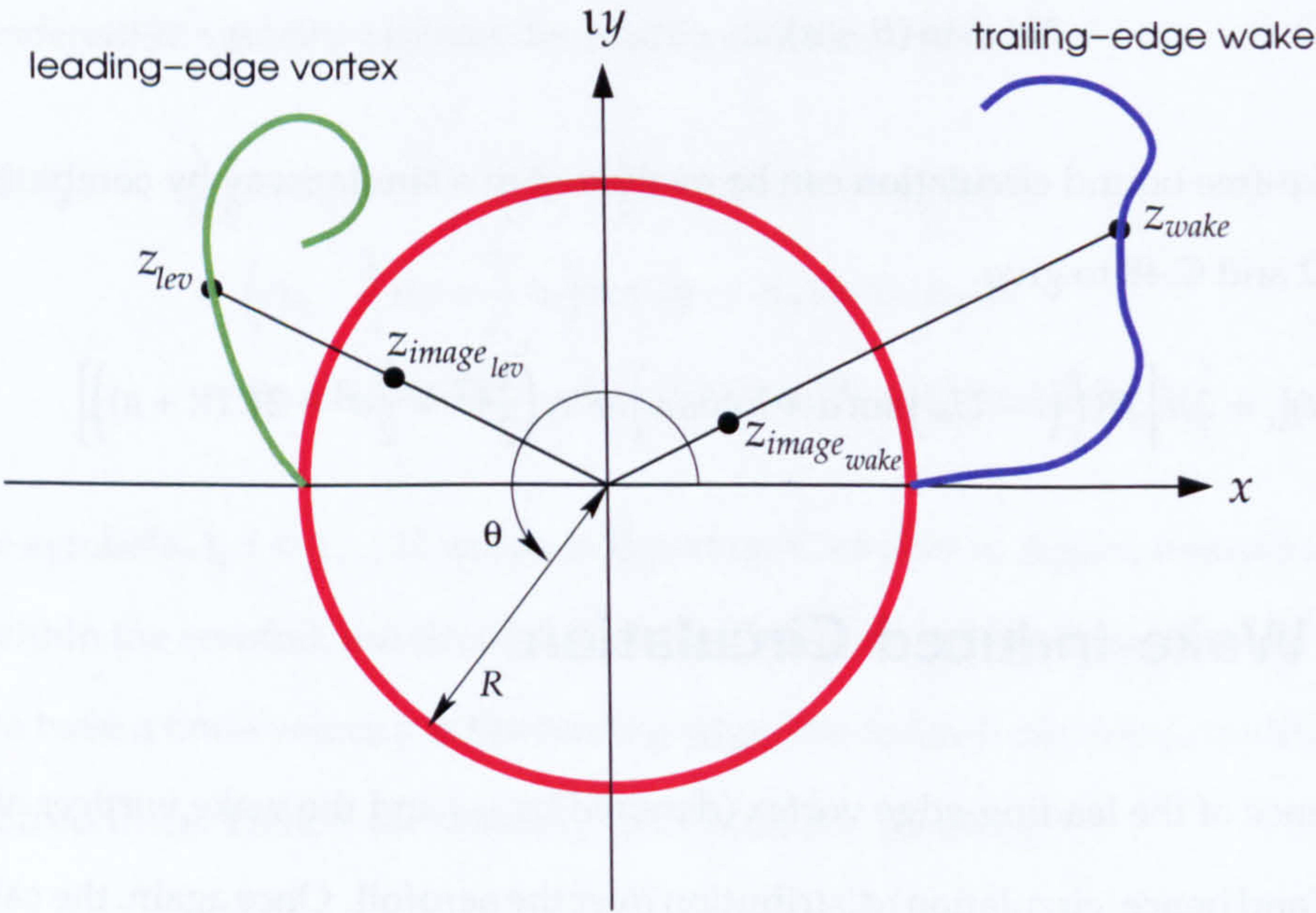


Figure C.3: Wake systems

C.3.1 Case I: Leading- and Trailing-Edge Separation

Consider a wake vortex at point z_{wake} in the wake with a circulation $\delta\Gamma_{wake} (\equiv \gamma_{wake}dZ)$. By the circle theorem, an (imaginary) image vortex of strength $\delta\Gamma_{image_{wake}}$ must exist inside the circle in order to satisfy the zero-through-flow boundary condition at the surface. The

strength and location of this image vortex are given by

$$\delta\Gamma_{image_{wake}} = -\delta\Gamma_{wake} \quad (C.51)$$

$$Z_{image_{wake}} = \frac{R^2}{\overline{Z_{wake}}} \quad (C.52)$$

where R is the radius of the circle and $\overline{Z_{wake}}$ refers to the complex conjugate of Z_{wake} . The velocity induced by the vortex pair at an arbitrary point on the circle $Z (= Re^{i\theta})$ can be calculated from their combined complex potential thus

$$\Omega(Z) = -i\frac{\delta\Gamma_{wake}}{2\pi} \ln(Z - Z_{wake}) + i\frac{\delta\Gamma_{wake}}{2\pi} \ln\left(Z - \frac{R^2}{\overline{Z_{wake}}}\right) \quad (C.53)$$

$$\Rightarrow q(Z) = i\frac{\delta\Gamma_{wake}}{2\pi} \left(\frac{1}{Re^{i\theta} - \overline{Z_{wake}}} - \frac{\overline{Z_{wake}}}{R^2 - Re^{i\theta}\overline{Z_{wake}}} \right) \quad (C.54)$$

Substituting from Equation C.15 for circumferential velocity v_θ and some algebra eventually gives

$$\begin{aligned} v_\theta &= -\Im(q(Z)) \cos \theta - \Re(q(Z)) \sin \theta \\ &= -\frac{\delta\Gamma_{wake}}{2\pi R} \Re\left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}}\right) \end{aligned}$$

where $\Re$ and $\Im$ denote 'real part of' and 'imaginary part of' respectively. The circumferential velocity induced by the entire wake is then obtained as an integral

$$v_\theta(\theta, t)|_{wake} = -\frac{1}{2\pi R} \oint_{wake} \Re\left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}}\right) \gamma_{wake} dZ_{wake} \quad (C.55)$$

A similar analysis for the leading-edge vortex yields

$$v_\theta(\theta, t)|_{lev} = -\frac{1}{2\pi R} \oint_{lev} \Re\left(\frac{Z_{lev} + Re^{i\theta}}{Z_{lev} - Re^{i\theta}}\right) \gamma_{lev} dZ_{lev} \quad (C.56)$$

The combined contribution of the LEV and wake vortices is, therefore,

$$\begin{aligned} v_\theta(\theta, t)|_{wake+lev} &= -\frac{1}{2\pi R} \oint_{wake} \Re\left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}}\right) \gamma_{wake} dZ_{wake} \\ &\quad - \frac{1}{2\pi R} \oint_{lev} \Re\left(\frac{Z_{lev} + Re^{i\theta}}{Z_{lev} - Re^{i\theta}}\right) \gamma_{lev} dZ_{lev} \quad (C.57) \end{aligned}$$

To solve for the two unknown vorticities in the above equation (γ_{wake} and γ_{lev}), two simultaneous 'constraint' equations are required. The equations are provided by satisfying the Kutta-Joukowski condition at the trailing edge of the aerofoil by the addition of circulation and by requiring smooth flow at the leading edge. These are deduced in turn below.

First Constraint Equation — Kutta-Joukowski Condition at Trailing Edge

In order to enforce the Kutta condition, a circulation Γ_k must be added so that from Equation C.57 the circumferential velocity becomes

$$v_\theta = -\frac{1}{2\pi R} \oint_{wake} \Re\left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}}\right) \gamma_{wake} dZ_{wake} - \frac{1}{2\pi R} \oint_{lev} \Re\left(\frac{Z_{lev} + Re^{i\theta}}{Z_{lev} - Re^{i\theta}}\right) \gamma_{lev} dZ_{lev} + \frac{\Gamma_k}{2\pi R} \quad (C.58)$$

To ensure smooth flow at the trailing edge ($\theta = 0$), the Kutta-Joukowski condition ($v_\theta = 0$) is enforced there to give

$$\Gamma_k = \oint_{wake} \Re\left(\frac{Z_{wake} + R}{Z_{wake} - R}\right) \gamma_{wake} dZ_{wake} + \oint_{lev} \Re\left(\frac{Z_{lev} + R}{Z_{lev} - R}\right) \gamma_{lev} dZ_{lev} \quad (C.59)$$

Remembering from the method of images that for each vortex outside the circle there exists a vortex of equal but opposite strength inside, Kelvin's law can be applied. Assuming that the initial circulation of the aerofoil-wake-LEV system was zero, we have

$$\Gamma_0(t) + \Gamma_{wake}(t) + \Gamma_{image_{wake}}(t) + \Gamma_{lev}(t) + \Gamma_{image_{lev}} + \Gamma_k(t) = 0 \quad (C.60)$$

from which the following simple result is obtained

$$\Gamma_k(t) = -\Gamma_0(t) \quad (C.61)$$

Substituting from Equation C.60 into Equation C.59

$$\Gamma_0(t) = -\left[\oint_{wake} \Re\left(\frac{Z_{wake} + R}{Z_{wake} - R}\right) \gamma_{wake} dZ_{wake} + \oint_{lev} \Re\left(\frac{Z_{lev} + R}{Z_{lev} - R}\right) \gamma_{lev} dZ_{lev} \right] \quad (C.62)$$

This is the first constraint equation. It is the general wake integral equation and is also a *nonlinear* form of the well-known *Wagner equation* (in the sense expressed by von Kármán & Sears (1938)) but given here in the circle plane.

Second Constraint Equation — Stagnation at Leading Edge

Having enforced the Kutta-Joukowski condition at the trailing edge, the velocity distribution on the circle can be found by summing the wake-free (Equation C.49) and wake-induced contributions (Equation C.57) thus

$$v_\theta = \overbrace{A + \frac{\Gamma_0}{2\pi R}}^{\text{wake-free}} + \underbrace{\left(-\frac{\Gamma_0}{2\pi R}\right) + B}_{\text{wake-induced}} = A + B \quad (\text{C.63})$$

where

$$\begin{aligned} A = \frac{1}{R} & \left[-A_1 \cos \theta - \left(A_2 + \frac{1}{2}A_7\right) \cos 2\theta + A_3 \sin \theta + \left(A_4 - \frac{1}{2}A_5 + \frac{1}{2}A_6\right) \sin 2\theta \right. \\ & \left. - A_8 \sin \theta \cos 2\theta + A_9 \sin \theta \sin 2\theta - A_{10} \cos \theta \cos 2\theta + A_{11} \cos \theta \sin 2\theta \right] \\ & - 2U_\infty \sin(\theta - \alpha) \end{aligned} \quad (\text{C.64})$$

and

$$B = -\frac{1}{2\pi R} \left[\oint_{lev} \Re\left(\frac{z_{lev} + Re^{i\theta}}{z_{lev} - Re^{i\theta}}\right) \gamma_{lev} dz_{lev} + \oint_w \Re\left(\frac{z_w + Re^{i\theta}}{z_w - Re^{i\theta}}\right) \gamma_w dz_w \right] \quad (\text{C.65})$$

where U_∞ is freestream velocity, α is angle of attack and the symbols $A_i, i = 1 \dots 11$ are as defined in Equation C.44.

The leading-edge vortex emanates from the leading edge so that a streamline originating there encloses the leading-edge vortex and (usually) reattaches again on the aerofoil further downstream. This implies that the leading edge of the aerofoil must be a stag-

nation point⁵ ($v_\theta = 0$ there) and another stagnation point occurs where the leading-edge streamline meets the aerofoil again, if at all. At the leading edge, $\theta = \pi$ so that

$$A|_{\theta=\pi} = \frac{1}{R} \left[A_1 - \left(A_2 + \frac{1}{2} A_7 \right) + A_{10} \right] - 2U_\infty \sin \alpha(t) \quad (C.66)$$

and

$$B|_{\theta=\pi} = -\frac{1}{2\pi R} \left[\oint_{wake} \Re \left(\frac{Z_{wake} - R}{Z_{wake} + R} \right) \gamma_{wake} dZ_{wake} + \oint_{lev} \Re \left(\frac{Z_{lev} - R}{Z_{lev} + R} \right) \gamma_{lev} dZ_{lev} \right] \quad (C.67)$$

Substituting Equations C.66 and C.67 into Equation C.63, equating to zero and rearranging gives

$$\begin{aligned} & \frac{1}{R} \left[A_1 - \left(A_2 + \frac{1}{2} A_7 \right) + A_{10} \right] - 2U_\infty \sin \alpha(t) \\ &= \frac{1}{2\pi R} \left[\oint_{wake} \Re \left(\frac{Z_{wake} - R}{Z_{wake} + R} \right) \gamma_{wake} dZ_{wake} + \oint_{lev} \Re \left(\frac{Z_{lev} - R}{Z_{lev} + R} \right) \gamma_{lev} dZ_{lev} \right] \end{aligned} \quad (C.68)$$

This is the second constraint equation. Equations C.62 and C.68 are the two simultaneous equations required to solve for γ_{wake} and γ_{lev} .

Again, because there is no flow inside the aerofoil, the surface velocity directly gives the vorticity distribution. By substituting $\Gamma_k(t) = -\Gamma_0(t)$ in Equation C.58, the vorticity distribution induced by the LEV and wake vortices on the circle is

$$\begin{aligned} \gamma_1(\theta, t)|_{wake+lev} = & -\frac{\Gamma_0(t)}{2\pi R} - \frac{1}{2\pi R} \left[\oint_{wake} \Re \left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}} \right) \gamma_{wake} dZ_{wake} \right. \\ & \left. + \frac{1}{2\pi R} \oint_{lev} \Re \left(\frac{Z_{lev} + Re^{i\theta}}{Z_{lev} - Re^{i\theta}} \right) \gamma_{lev} dZ_{lev} \right] \end{aligned} \quad (C.69)$$

C.3.2 Case II: Trailing-Edge Separation Only

In the case of wake separating solely from the trailing edge, only one ‘constraint’ equation is required, namely, the Kutta-Joukowski condition at the trailing edge (Equation C.62).

⁵The argument given by Thwaites is that the loading at a point of separation is zero, there being none across the vortex sheet emanating there, which implies zero velocity from the Kutta-Joukowski law (Thwaites, 1960, p. 514).

The nonlinear Wagner equation, therefore, becomes

$$\Gamma_0(t) = - \oint_{wake} \Re \left(\frac{Z_{wake} + R}{Z_{wake} - R} \right) \gamma_{wake} dZ_{wake} \quad (C.70)$$

The wake-induced bound circulation is similar to Equation C.69 but with the leading-edge vortex term removed so that

$$\gamma_1(\theta, t)|_{wake} = - \frac{\Gamma_0(t)}{2\pi R} - \frac{1}{2\pi R} \oint_{wake} \Re \left(\frac{Z_{wake} + Re^{i\theta}}{Z_{wake} - Re^{i\theta}} \right) \gamma_{wake} dZ_{wake} \quad (C.71)$$

C.4 Complex Potential

A consequence of the above approach is that the complex potential for the whole aerofoil-wake-LEV system presented above results automatically from the component velocity fields. The expressions for circle plane are given below.

Combined Wake-Free Complex Potential

$$\begin{aligned} \Omega(Z) = & U_{\infty} \left(Ze^{-i\alpha} + \frac{R^2 e^{i\alpha}}{Z} \right) \\ & + \left[- \left(A_3 + \frac{1}{2}A_8 + \frac{1}{2}A_{11} \right) - i \left(A_1 - \frac{1}{2}A_9 + \frac{1}{2}A_{10} \right) \right] \left(\frac{R}{Z} \right) \\ & + \left[- \left(\frac{1}{2}A_4 - \frac{1}{4}A_5 + \frac{1}{4}A_6 \right) - i \left(\frac{1}{2}A_2 + \frac{1}{4}A_7 \right) \right] \left(\frac{R^2}{Z^2} \right) \\ & + \left[\left(\frac{1}{6}A_8 - \frac{1}{6}A_{11} \right) - i \left(\frac{1}{6}A_9 + \frac{1}{6}A_{10} \right) \right] \left(\frac{R^3}{Z^3} \right) \\ & + \left(\frac{1}{2}A_5 + \frac{1}{2}A_6 \right) \ln(Z) \\ & - \frac{i\Gamma_0}{2\pi} \ln(Z) \end{aligned}$$

Combined Wake and LEV Effect

$$\begin{aligned}
 \Omega(Z) = & \frac{i\Gamma_0}{2\pi} \ln(Z) \\
 & - \frac{i}{2\pi} \sum_{wake} \delta\Gamma_{wake} \ln(Z - Z_{wake}) \\
 & + \frac{i}{2\pi} \sum_{wake} \delta\Gamma_{wake} \ln\left(Z - \frac{R^2}{Z_{wake}}\right) \\
 & - \frac{i}{2\pi} \sum_{lev} \delta\Gamma_{lev} \ln(Z - Z_{lev}) \\
 & + \frac{i}{2\pi} \sum_{lev} \delta\Gamma_{lev} \ln\left(Z - \frac{R^2}{Z_{lev}}\right) \\
 & + K
 \end{aligned}$$

where

$$K = \frac{i}{2\pi} (\Gamma_{wake} \ln |Z_{wake}|^2 + \Gamma_{lev} \ln |Z_{lev}|^2) - \frac{1}{2} (\Gamma_{wake} + \Gamma_{lev})$$

Total Complex Potential of Aerofoil-Wake-LEV System

$$\begin{aligned}
 \Omega(Z) = & U_\infty \left(Ze^{-i\alpha} + \frac{R^2 e^{i\alpha}}{Z} \right) \\
 & + \left[-\left(A_3 + \frac{1}{2}A_8 + \frac{1}{2}A_{11}\right) - i\left(A_1 - \frac{1}{2}A_9 + \frac{1}{2}A_{10}\right) \right] \left(\frac{R}{Z}\right) \\
 & + \left[-\left(\frac{1}{2}A_4 - \frac{1}{4}A_5 + \frac{1}{4}A_6\right) - i\left(\frac{1}{2}A_2 + \frac{1}{4}A_7\right) \right] \left(\frac{R^2}{Z^2}\right) \\
 & + \left[\left(\frac{1}{6}A_8 - \frac{1}{6}A_{11}\right) - i\left(\frac{1}{6}A_9 + \frac{1}{6}A_{10}\right) \right] \left(\frac{R^3}{Z^3}\right) \\
 & + \left(\frac{1}{2}A_5 + \frac{1}{2}A_6\right) \ln(Z) \\
 & - \frac{i}{2\pi} \sum_{wake} \delta\Gamma_{wake} \ln(Z - Z_{wake}) \\
 & + \frac{i}{2\pi} \sum_{wake} \delta\Gamma_{wake} \ln\left(Z - \frac{R^2}{Z_{wake}}\right) \\
 & - \frac{i}{2\pi} \sum_{lev} \delta\Gamma_{lev} \ln(Z - Z_{lev}) \\
 & + \frac{i}{2\pi} \sum_{lev} \delta\Gamma_{lev} \ln\left(Z - \frac{R^2}{Z_{lev}}\right) \\
 & + K
 \end{aligned}$$

Notice that the quasi-steady circulation term Γ_0 vanishes from the above expression for total velocity potential, signifying that the circulation created due to the aerofoil motion is balanced by an equal amount shed into the wake. This is in keeping with Kelvin's law that total circulation in a system is conserved. Also notice the presence of source (or sink) terms which result from enforcing the zero-through-flow boundary condition.

C.5 Calculation of Forces and Moments

The approach suggested by Lord Thomson (1910) is used here. Thomson showed that for a system of vortices, impulse and moment of impulse could be related to the force and moment respectively — the so-called *Kelvin's Impulse Theorem*. This method is used in the current approach for its simplicity and quickness.

Kelvin's theorem showed that the *external* force F_e (in an inertial coordinate system) applied to a body in a fluid to produce a prescribed motion is given by

$$F_e = \frac{d}{dt} \left(L_v + \rho \iiint_S \varphi \mathbf{n} dS \right) \quad (C.72)$$

where L_v is the linear momentum of the body, ρ is the fluid density, S is the boundary surface enclosing the body and $\mathbf{n}$ is the unit normal vector on the boundary pointing out of the fluid (i.e. *into* the body). Similarly, the external moment M_e taken about the origin of the inertial coordinate system that must be exerted on the body is given by

$$M_e = \frac{d}{dt} \left(L_\omega + \rho \iiint_S \varphi (\mathbf{r} \wedge \mathbf{n}) dS \right) \quad (C.73)$$

where $\mathbf{r}$ is the vector distance from the origin of the coordinate system with respect to which the flow is stationary at infinity and L_ω is the angular momentum of the body in this reference frame. Because this approach is momentum-based, all derivations are made in an inertial coordinate system where Newton's 2nd law of motion is valid.

It is important to note, as pointed out by Sears (1960), that F_e and M_e are not the resultants of fluid pressures that we usually calculate (as from Bernoulli's equation) but

are the external force and moment respectively required to overcome both the inertia of the body and that of the fluid. It is the latter that manifests itself in the form of fluid pressures on the body. So, Equations C.72 and C.73 include the resultants of the pressures but with the sign reversed⁶. The force and moment exerted on the body by the fluid are, therefore, given by the right members of the right-hand side of Equations C.72 and C.73.

Therefore, the force on the body in the inertial reference frame $\tilde{\mathbf{F}}$ is equal to the time rate of change of the impulse of the vortex system $\tilde{\mathbf{I}}$, i.e.

$$\tilde{\mathbf{F}} = \frac{d\tilde{\mathbf{I}}}{dt} \quad (\text{C.74})$$

where $\tilde{\mathbf{I}}$ — the Kelvin Impulse — is given by

$$\tilde{\mathbf{I}} = \rho \oint \oint_S \varphi \mathbf{n} dS \quad (\text{C.75})$$

Similarly, the moment of the system about the origin is given by the time rate of change of the moment of impulse of the system $\tilde{\mathbf{I}}_m$, i.e.

$$\tilde{\mathbf{M}} = \frac{d\tilde{\mathbf{I}}_m}{dt} \quad (\text{C.76})$$

where $\tilde{\mathbf{I}}_m$ is

$$\tilde{\mathbf{I}}_m = \rho \oint \oint_S \varphi (\tilde{\mathbf{r}} \wedge \mathbf{n}) dS \quad (\text{C.77})$$

It is, however, more useful to have forces and moments in body-fixed coordinates, and more so for a flapping wing. Converting from the inertial ($\tilde{\cdot}$) to the body ($\hat{\cdot}$) system gives for force (Karamcheti, 1966)

$$\hat{\mathbf{F}} = \frac{d\hat{\mathbf{I}}}{dt} \quad (\text{C.78})$$

where

$$\hat{\mathbf{I}} = \rho \oint \oint_S \varphi \mathbf{n} dS \quad (\text{C.79})$$

⁶Incidentally, the quantity $-\rho \varphi$ is the impulsive pressure.

Similarly, for moment

$$\hat{\mathbf{M}} = \frac{d\hat{\mathbf{I}}_m}{dt} + \tilde{\mathbf{U}} \wedge \hat{\mathbf{I}} \quad (\text{C.80})$$

where

$$\tilde{\mathbf{I}}_m = \rho \oint \oint_S \varphi (\hat{\mathbf{r}} \wedge \mathbf{n}) dS \quad (\text{C.81})$$

and $\tilde{\mathbf{U}}$ is the translational velocity of the body-fixed system relative to the inertial reference system.

C.5.1 Conversion to Complex Plane Coordinate Systems

In complex coordinate system notation (ζ -plane), impulse is given by

$$\mathcal{I} = \mathcal{I}_\xi + i\mathcal{I}_\eta \quad (\text{C.82})$$

and force by

$$\mathcal{F} = \mathcal{F}_\xi + i\mathcal{F}_\eta \quad (\text{C.83})$$

For a vortex pair of strength Γ and separated by a distance l , the impulse is given by $|\mathcal{I}| = \rho\Gamma l$ and acts perpendicularly to the 'plane' of the vortices (see Durand (1935, p. 325)). Now consider two vortices of strengths Γ_1 and Γ_2 and positions ζ_1 and ζ_2 respectively, that constitute a vortex pair in complex space (see Figure C.4 below, reproduced from Figure 4.8). The impulse of the vortex pair acts halfway between the two vortices at $\frac{1}{2}(\zeta_1 + \zeta_2)$ and in a direction perpendicular to the line joining the two vortices, i.e. in the direction of $i(\zeta_1 - \zeta_2)$.

The complex impulse is, therefore, given by

$$\begin{aligned} \tilde{\mathcal{I}} &= \rho\Gamma_1 l \times \text{direction} = \rho\Gamma_1 |\zeta_1 - \zeta_2| \times \frac{i(\zeta_1 - \zeta_2)}{|\zeta_1 - \zeta_2|} \\ &= i\rho\Gamma_1 (\zeta_1 - \zeta_2) \\ \therefore \tilde{\mathcal{I}} &= i\rho(\Gamma_1\zeta_1 + \Gamma_2\zeta_2) \quad (\because \Gamma_1 = -\Gamma_2) \end{aligned} \quad (\text{C.84})$$

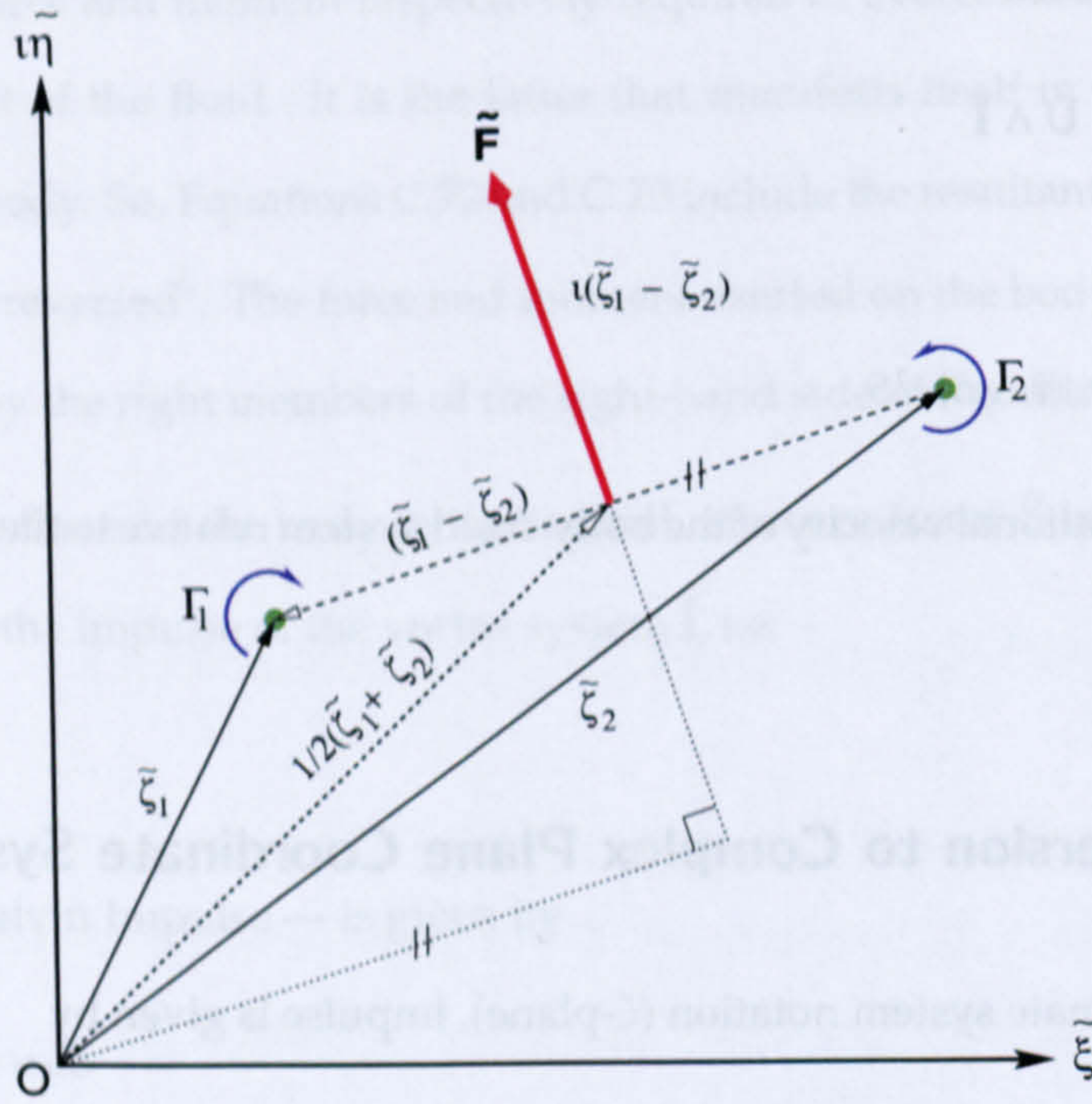


Figure C.4: Impulse of vortex pairs

Calculation of the moment of impulse follows similarly

$$\begin{aligned}
 \tilde{\mathcal{I}}_m &= \rho \Gamma_1 l \times \text{perpendicular distance} = \rho \Gamma_1 |\tilde{\zeta}_1 - \tilde{\zeta}_2| \times \frac{1}{2} \frac{(|\tilde{\zeta}_1|^2 - |\tilde{\zeta}_2|^2)}{|\tilde{\zeta}_1 - \tilde{\zeta}_2|} \\
 &= \frac{1}{2} \rho \Gamma_1 (|\tilde{\zeta}_1|^2 - |\tilde{\zeta}_2|^2) \\
 \therefore \tilde{\mathcal{I}}_m &= \frac{\rho}{2} (\Gamma_1 |\tilde{\zeta}_1|^2 + \Gamma_2 |\tilde{\zeta}_2|^2) \quad (\because \Gamma_1 = -\Gamma_2) \quad (C.85)
 \end{aligned}$$

From Equations C.83 and C.84, it can be seen that the total impulse and moment of impulse of the entire system respectively are given by

$$\tilde{\mathcal{I}} = i\rho \sum_{j=1}^{\infty} \Gamma_j \tilde{\zeta}_j \quad (C.86)$$

$$\tilde{\mathcal{I}}_m = \frac{\rho}{2} \sum_{j=1}^{\infty} \Gamma_j |\tilde{\zeta}_j|^2 \quad (C.87)$$

Following on from Equations C.74 and C.76, the force and moment about the origin are

found by taking the derivatives with respect to time of Equations C.85 and C.86, thus

$$\tilde{\mathcal{F}} = i\rho \frac{d}{dt} \sum_{j=1}^{\infty} \Gamma_j \tilde{\zeta}_j = i\rho \frac{d}{dt} \int \tilde{\zeta} \gamma(\tilde{\zeta}) d\tilde{\zeta} \quad (\text{C.88})$$

$$\tilde{\mathcal{M}} = \frac{\rho}{2} \frac{d}{dt} \sum_{j=1}^{\infty} \Gamma_j |\tilde{\zeta}_j|^2 = \frac{\rho}{2} \frac{d}{dt} \int |\tilde{\zeta}|^2 \gamma(\tilde{\zeta}) d\tilde{\zeta} \quad (\text{C.89})$$

As mentioned already, it is more useful to have forces and moments in a coordinate system that moves with the aerofoil, in this case the aerofoil-inertia system (see Figure C.2). Using Equations C.78 and C.80, we get

$$\hat{\mathcal{F}} = \frac{d\hat{\mathcal{I}}}{dt} \quad (\text{C.90})$$

$$\hat{\mathcal{M}} = \frac{d\hat{\mathcal{I}}_m}{dt} - \Im(U_0 \hat{\mathcal{I}}) \quad (\text{C.91})$$

where

$$\hat{\mathcal{I}} = i\rho \sum_{j=1}^{\infty} \Gamma_j \hat{\zeta}_j \quad (\text{C.92})$$

$$\hat{\mathcal{I}}_m = \frac{\rho}{2} \sum_{j=1}^{\infty} \Gamma_j |\hat{\zeta}_j|^2 \quad (\text{C.93})$$

$$U_0 = (-U_{\infty} + i\dot{h}) + i\dot{\alpha} \quad (\text{C.94})$$

where U_0 is the translational velocity of the origin of the aerofoil-inertia system.

Appendix D

Details of Selected Results

D.1 Flow Field in Section 08 of 10 of the Robofly Wing Executing Dickinson's Kinematics

During each simulation, flow field data for each wing section could be extracted. The sequences shown in Figure D.1 below are for the 8th wing section (out of 10) of the Robofly wing executing Dickinson's kinematics and correspond to the discussion on pp. 202ff. A cartoon form of the sequences is sketched in Figure 6.10.

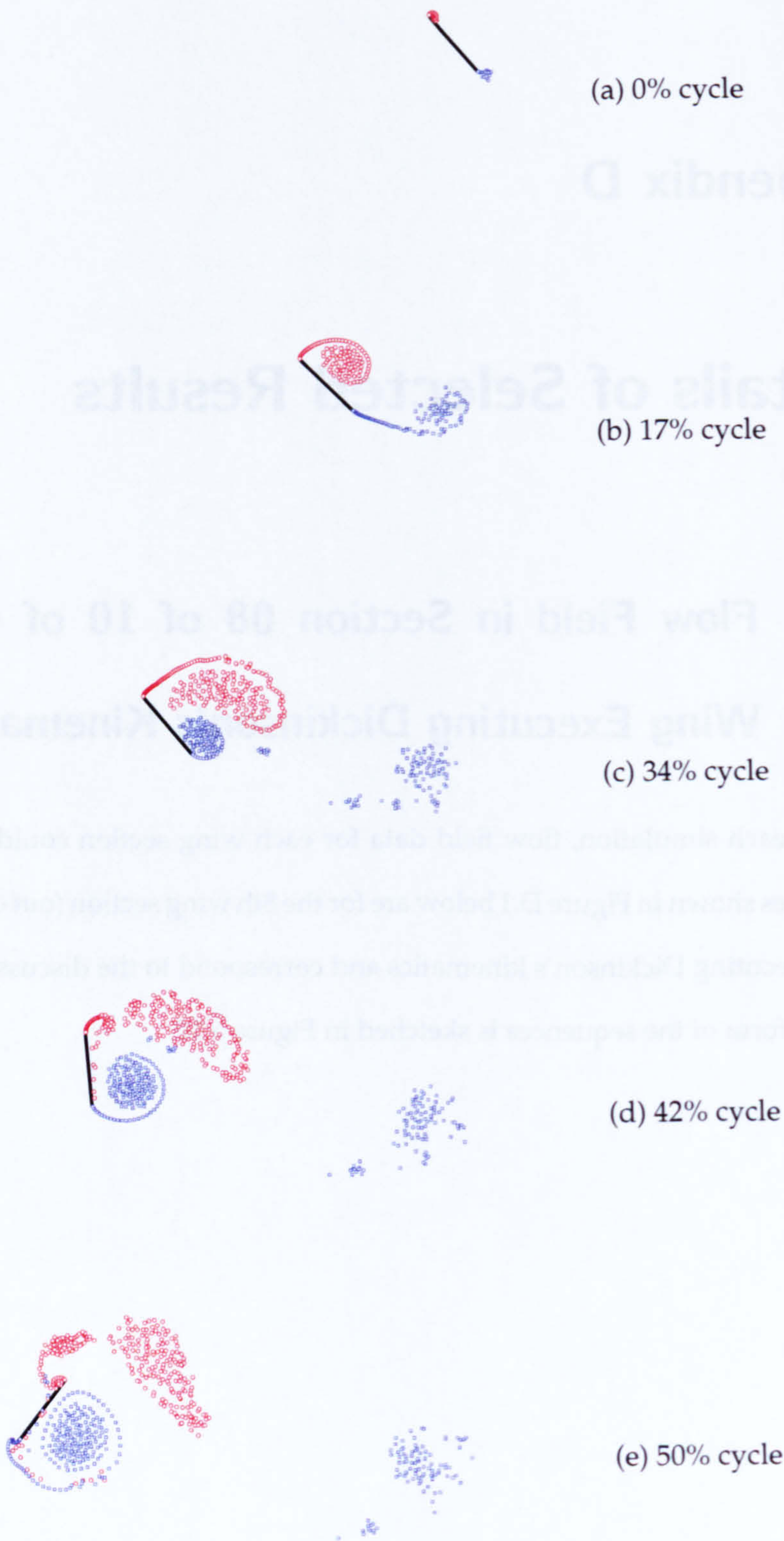


Figure D.1: Flow field at different times through one complete cycle. Colour scheme is red for leading-edge vortices and blue for trailing-edge vortices

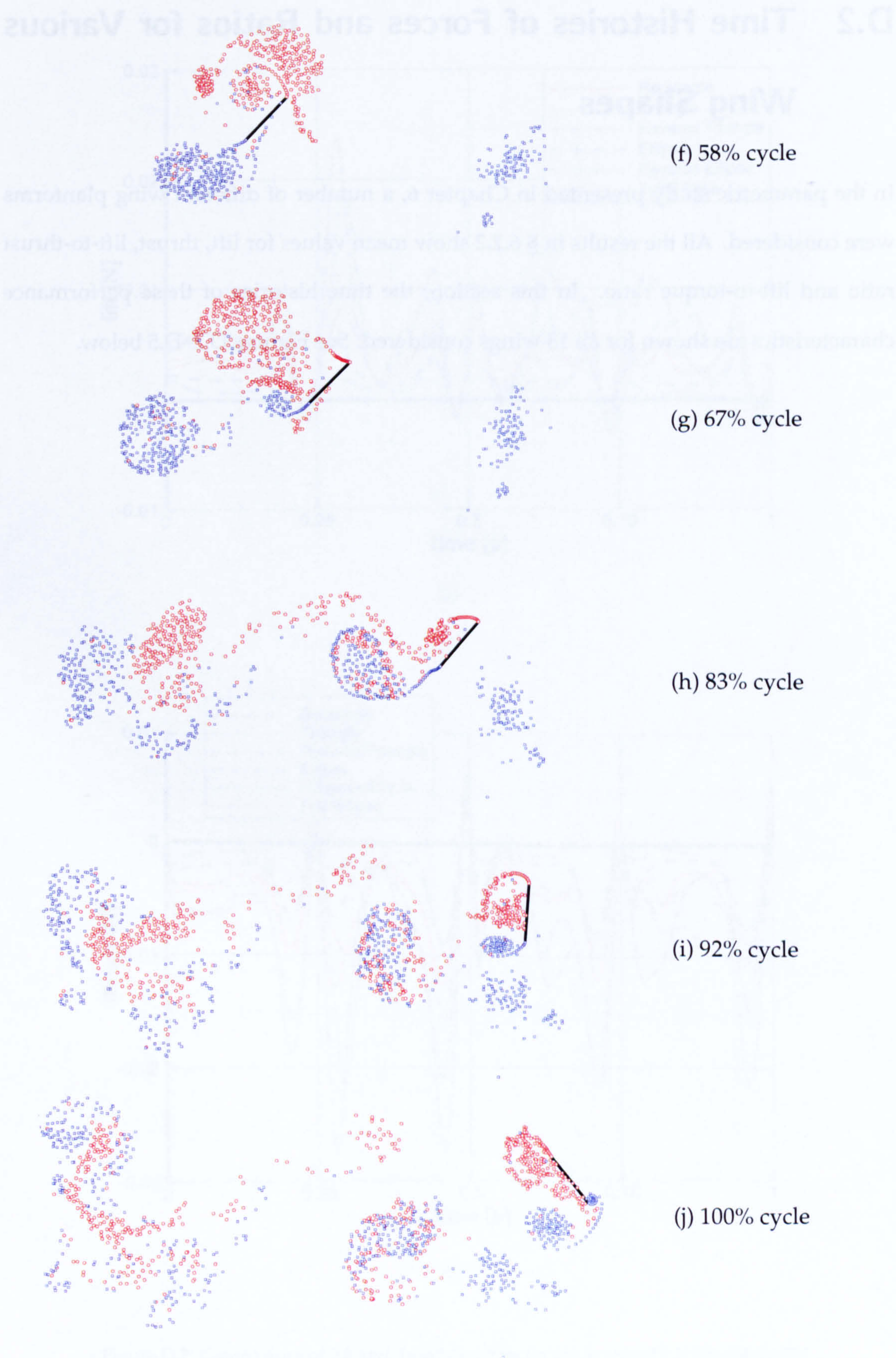
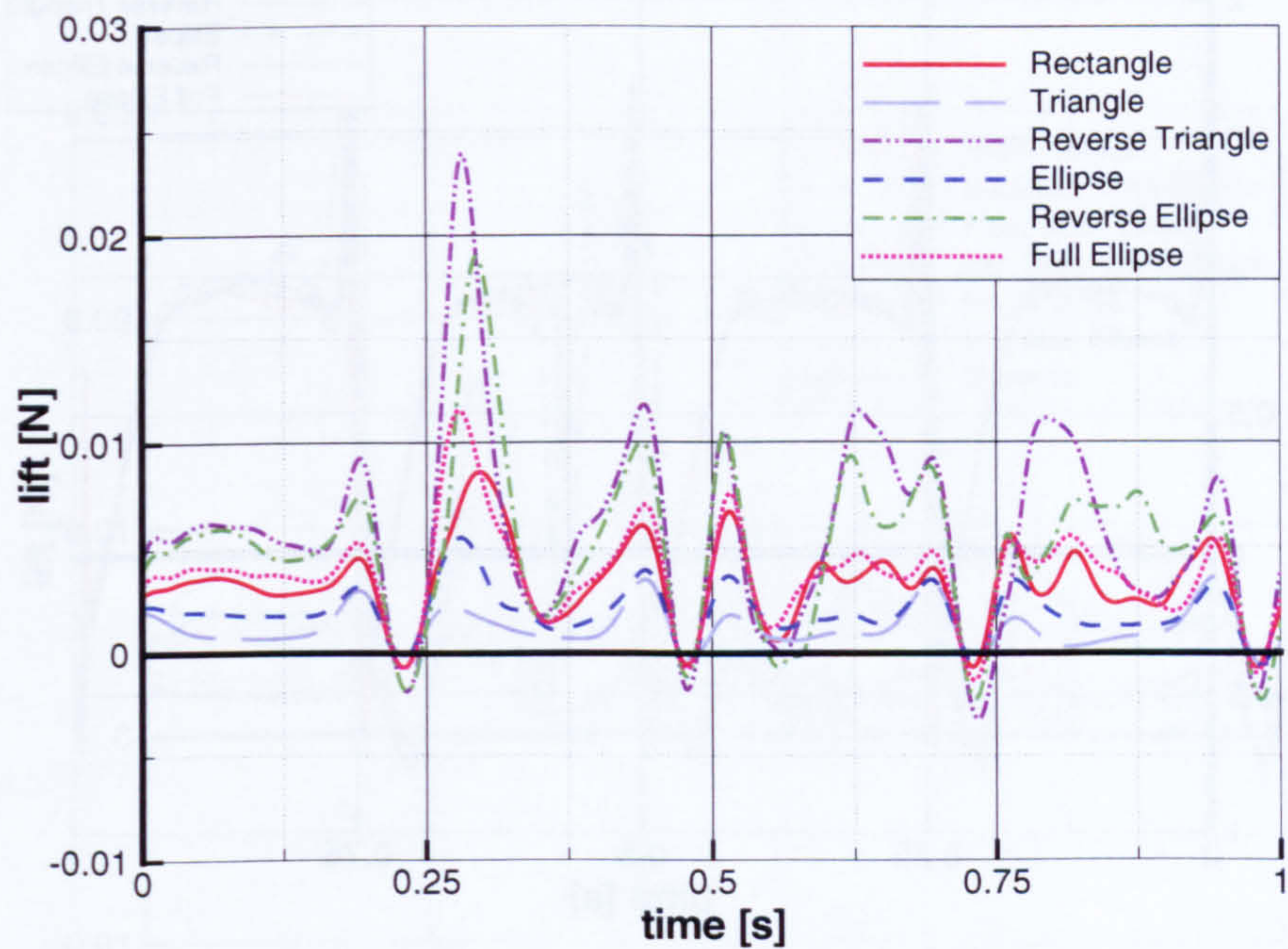


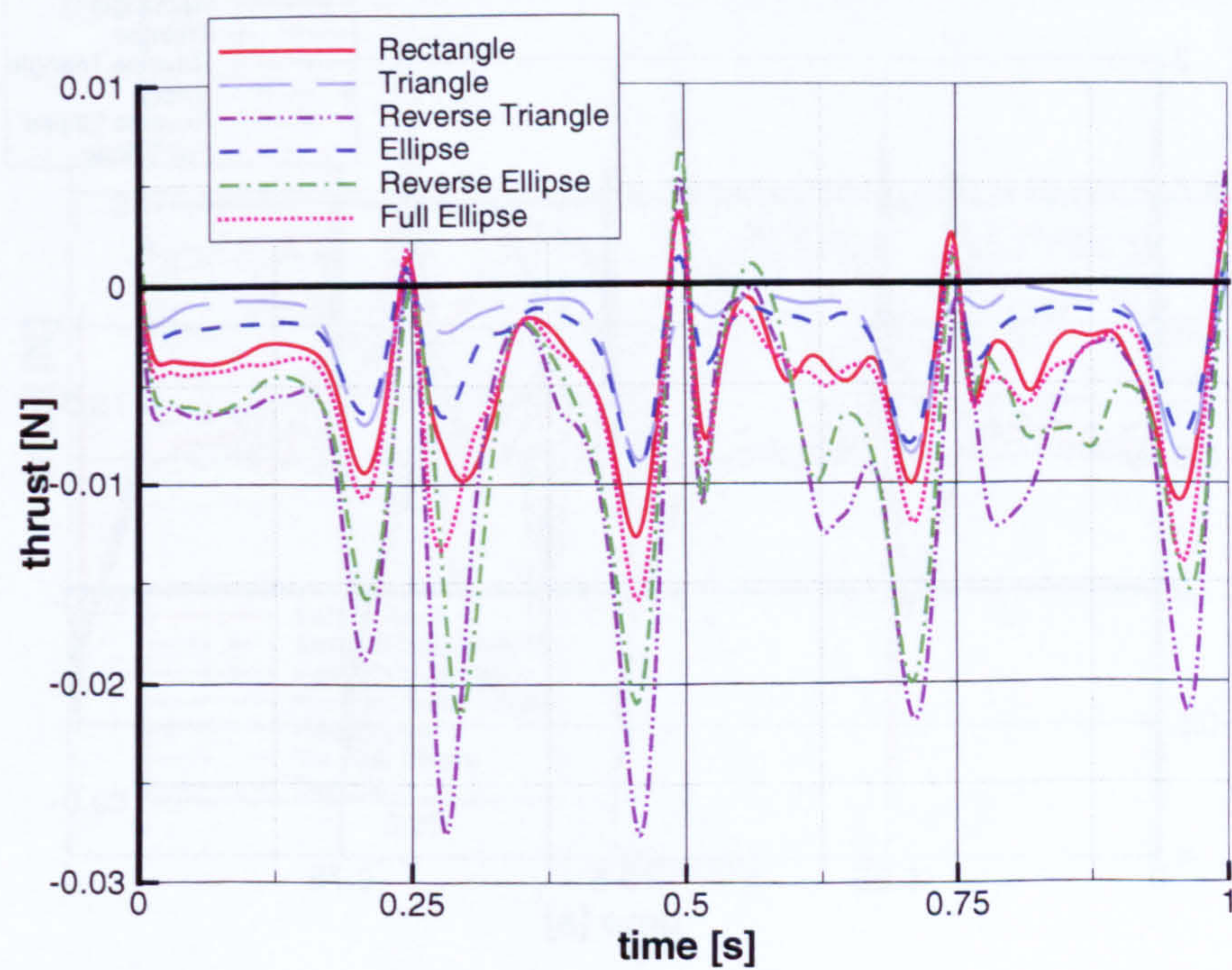
Figure D.1: Continued. Colour scheme is red for leading-edge vortices and blue for trailing-edge vortices

D.2 Time Histories of Forces and Ratios for Various Wing Shapes

In the parametric study presented in Chapter 6, a number of different wing planforms were considered. All the results in § 6.2.2 show mean values for lift, thrust, lift-to-thrust ratio and lift-to-torque ratio. In this section, the time histories of these performance characteristics are shown for all 13 wings considered. See Figures D.2–D.5 below.



(a)



(b)

Figure D.2: Comparison of lift and thrust histories for the symmetric wing planforms

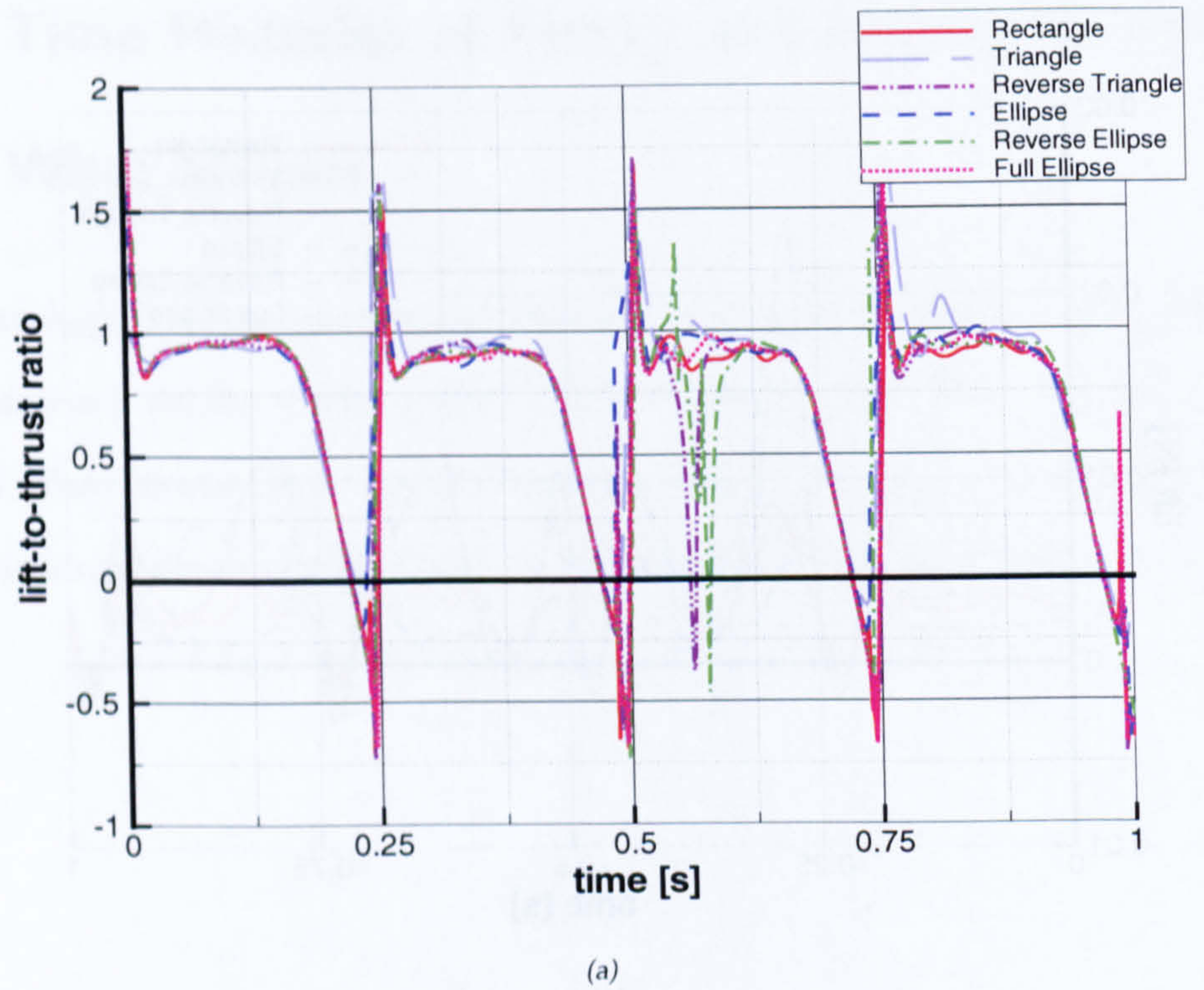
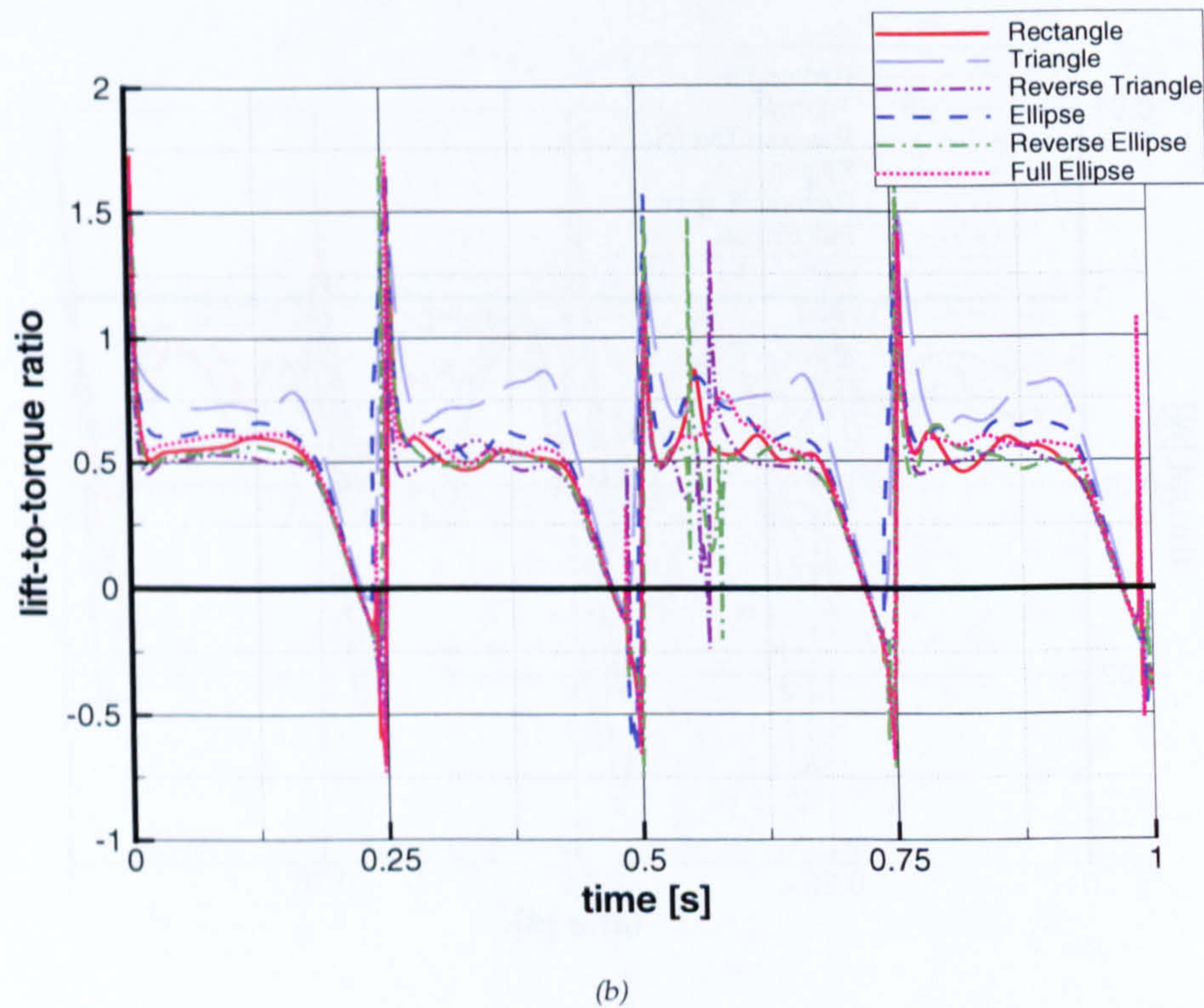
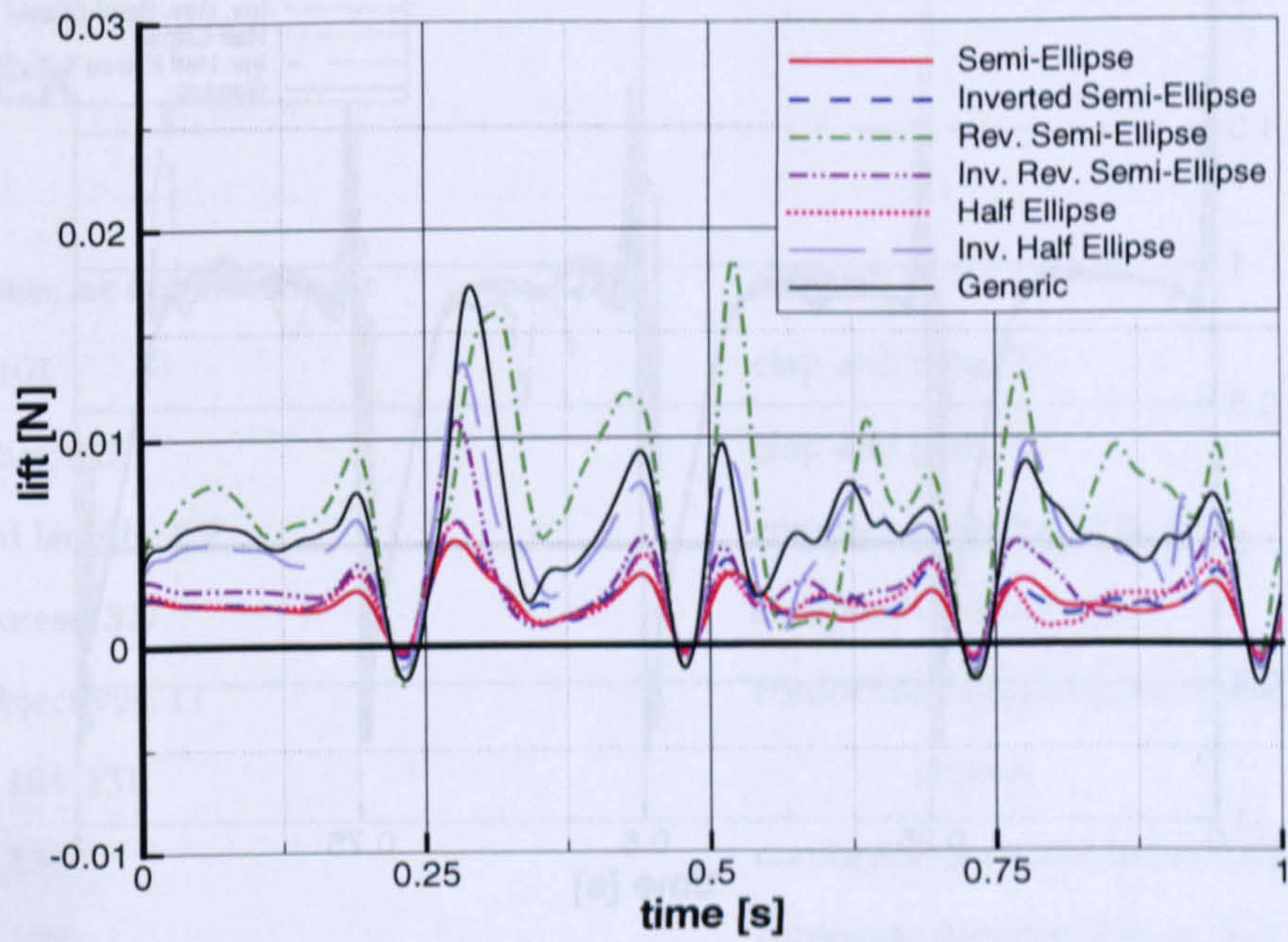
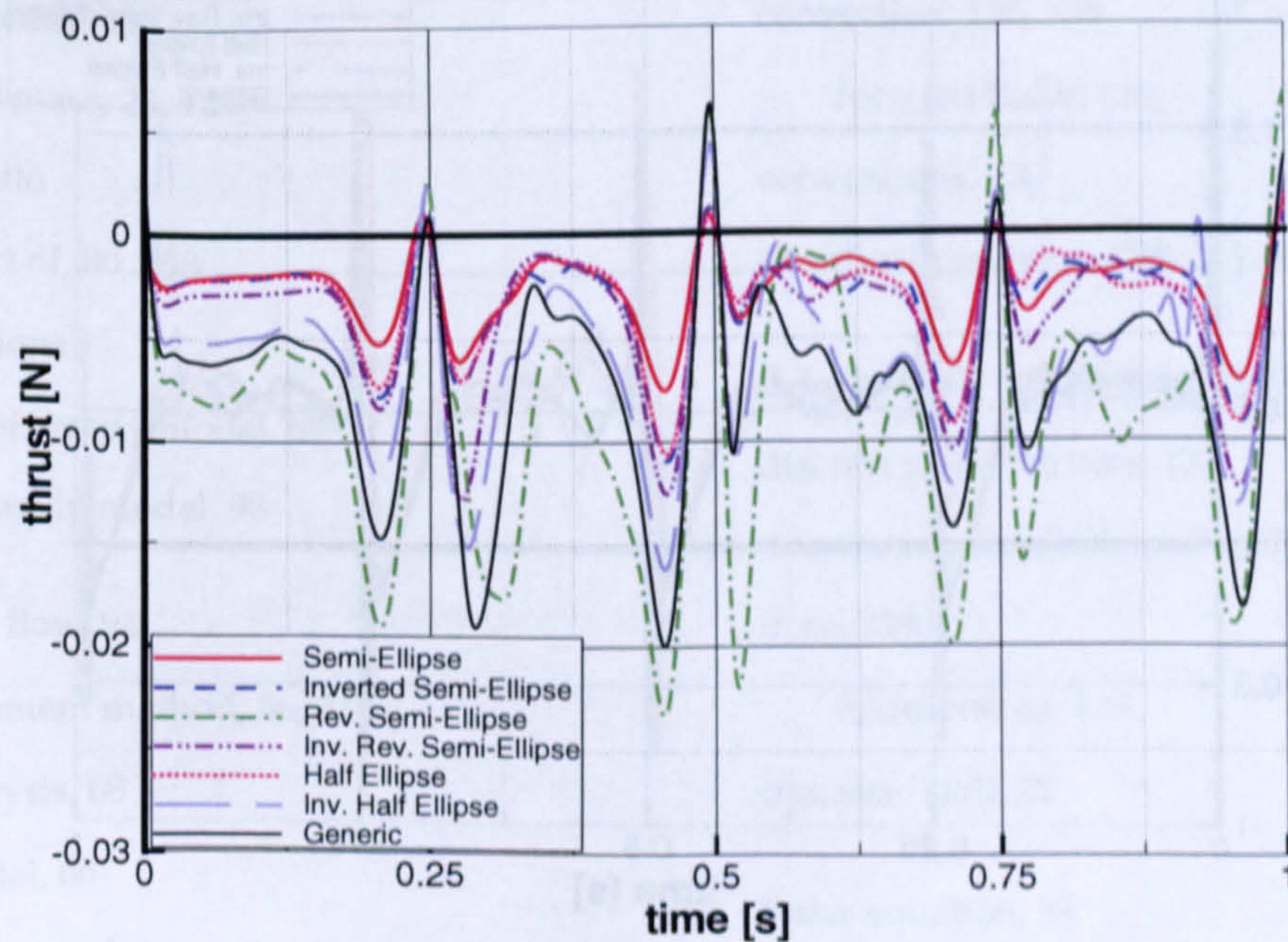


Figure D.3: Comparison of lift-to-thrust ratio and lift-to-torque ratio histories for the symmetric wing plan-forms. Lift-to-torque ratio is non-dimensionalised using mean chord

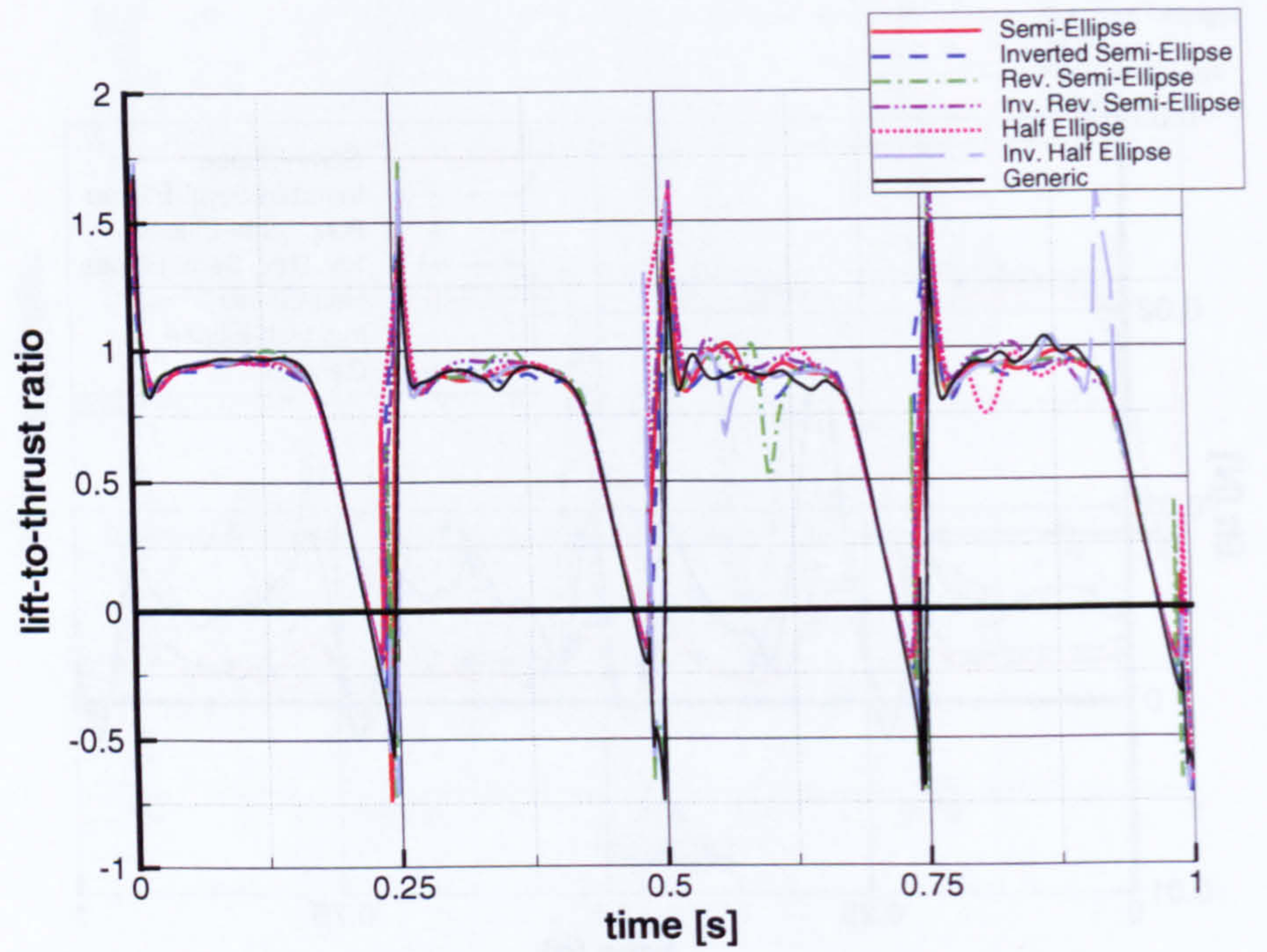


(a)

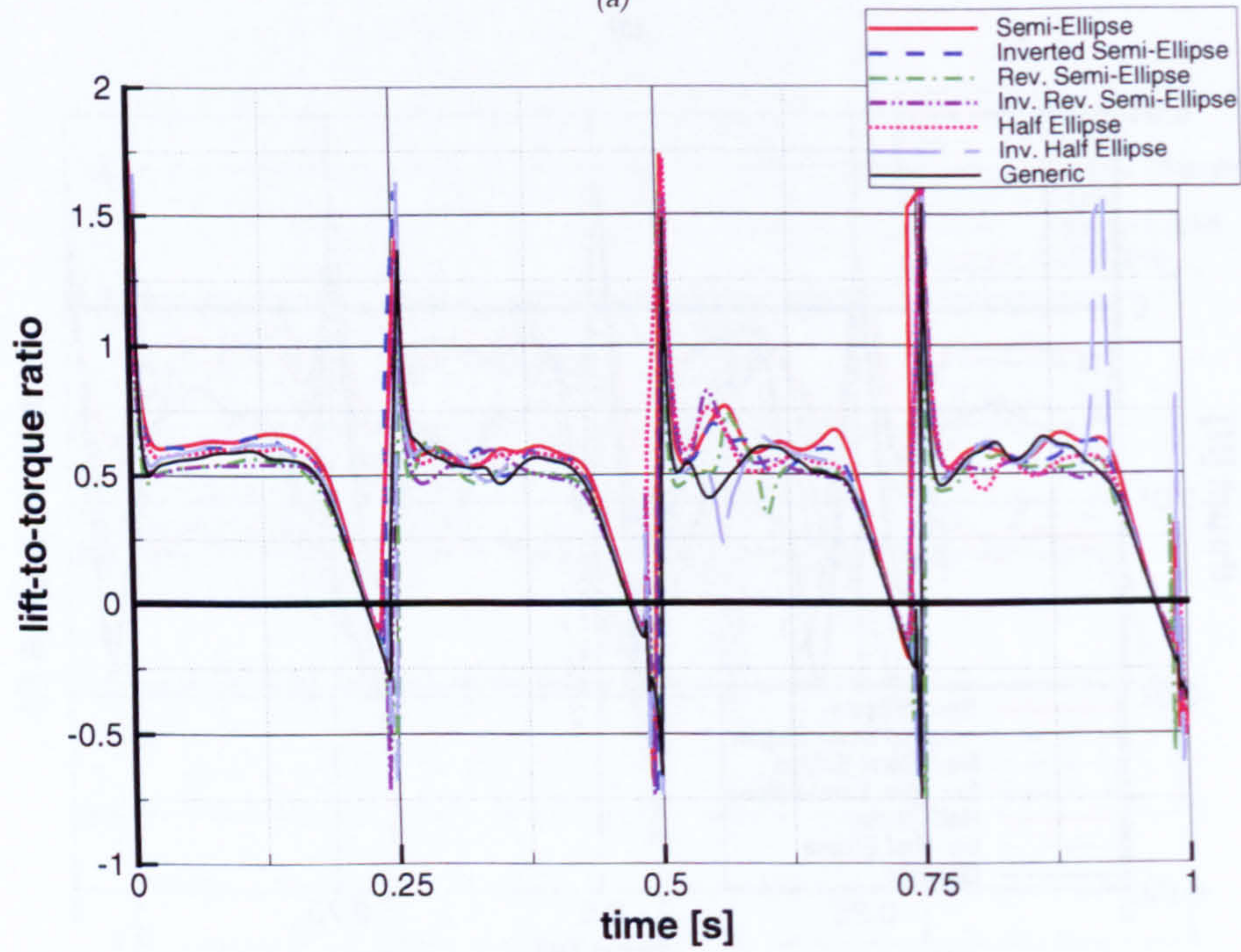


(b)

Figure D.4: Comparison of lift and thrust histories for the asymmetric wing planforms



(a)



(b)

Figure D.5: Comparison of lift-to-thrust ratio and lift-to-torque ratio histories for the asymmetric wing planforms. Lift-to-torque ratio is non-dimensionalised using mean chord

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