

Nonlinear behaviour in flexible, large-scale space structures: dynamics and control

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ABSTRACT

Large-scale structures are becoming increasingly common for space applications, driven by advances in in-orbit manufacturing and construction, lightweight materials, and robotics. While these structures have the potential to be key enablers for the next generation of space application—see, for example, recent interest in space-based solar power—their ability to deliver such goals depends on the stability and controllability of their dynamics. Large-scale space structures, which often have architectures at the kilometer length scale, face several important dynamical challenges. In this work, we propose a novel control strategy for flexible satellites based on a Dual Unscented Kalman Filter strategy. The methodology is outlined and then applied to a smaller satellite, which is assumed to be linear. A key aim of this research is the accurate identification of the moment of inertia of the system, which we demonstrate the methodology can achieve to within 1% (or significantly closer), even when the initial assumptions are inaccurate. Finally, we discuss the implications for larger structures, and how the work will be expanded in the future.

Keywords: Nonlinear Vibrations, Control, Attitude and Orbit Dynamics, Flexible Satellites, Kalman Filters

INTRODUCTION

The identification and modeling of the mass and structural properties of spacecraft represents one of the key challenges for the design of accurate and reliable attitude and orbit control systems (AOCS) [1]. It is possible for the spacecraft mass distribution to differ from computational predictions as a result of material imperfections and floating components, such as wires and heat pipes, which are not easily predictable with mathematical models. The determination of mass and structural parameters of the spacecraft during the on-ground development can be achieved only with a certain degree of accuracy, as external factors, such as the gravity and the air friction, can strongly affect the measurements [2]. As such, the accurate prediction of the moment of inertia of the system—a vital ability for robust control strategies—remains a key challenge for satellites with flexible solar panels, or similar appendages. Furthermore, the mass and inertia configuration of the spacecraft is liable to change during the mission due to deployable elements [3] or propellant consumption during orbital maneuver [4].

Recent missions and developing concepts, such as the Vasant [5] and INMARSAT-4 [6] are also proposing to utilize extremely large and lightweight structures for their appendages, such as solar arrays and telecommunication antennas, adding additional complexity and challenges for the AOCS design. These adaptations have the potential to greatly increase the geometric nonlinearity of the flexible structure, so it is important that the control strategy can (a) robustly control the coupled dynamics of a smaller, linear satellite, and (b) be easily adaptable to include nonlinear equations of motion. While there is a wealth of literature on AOCS through time domain-based system identification [7], including the use of various Kalman filters (e.g., [8]), the application of Dual Unscented Kalman Filters (DUKF) [9] has not been previously considered. The DUKF method builds upon the unscented version of the Kalman filter (UKF), which is based upon the use of sigma points that probabilistically capture the full system behaviour, by introducing one UKF for the unobserved system state, and another for the observed signal. Therefore, the DUKF algorithm simultaneously estimates both the mass and distribution, and kinematic state, of the satellite. While [8] has attempted to incorporate structural nonlinearity into AOCS, this approach requires a linearization at each time step, which may hinder its ability to fully capture the nonlinear dynamics.

METHODOLOGY

The coordinate frames for the satellite, including its position relative to Earth, are presented in Fig. 1. In the Earth Centered Inertial (ECI) reference frame [10], the deformed positions of the satellite, $\mathbf{x}$, are defined by:

$$\mathbf{x} = \mathbf{x}_0 + \mathbf{T}(\mathbf{d} + \mathbf{w}(\mathbf{d}, t)),$$

where $\mathbf{x}_0$ denotes the satellite origin, and $\mathbf{T}$ is used to rotate the coordinates of both the flexible position, $\mathbf{d}$, and time-dependent deformation, $\mathbf{w}(\mathbf{d}, t)$, from the Body Reference Frame (BRF) to the ECI reference frame. This methodology assumes that:

- The flexible components of the model can be resolved using Finite Element (FE) software, resulting in a set of N_n modes.
- The deformation, $\mathbf{w}$, can be accurately approximated through the Galerkin decomposition $\mathbf{w}(\mathbf{d}, t) = \sum_{k=1}^{N_n} A_k(t) \phi_k(\mathbf{d})$, where $\phi_k(\mathbf{d})$ denotes the k^{th} mode shape of the structure, and $A_k(t)$ is its time-varying scale factor.
- The time derivative of $\mathbf{T}$ can be expressed with respect to the skew of the angular velocity matrix, $\boldsymbol{\omega}$ [10].

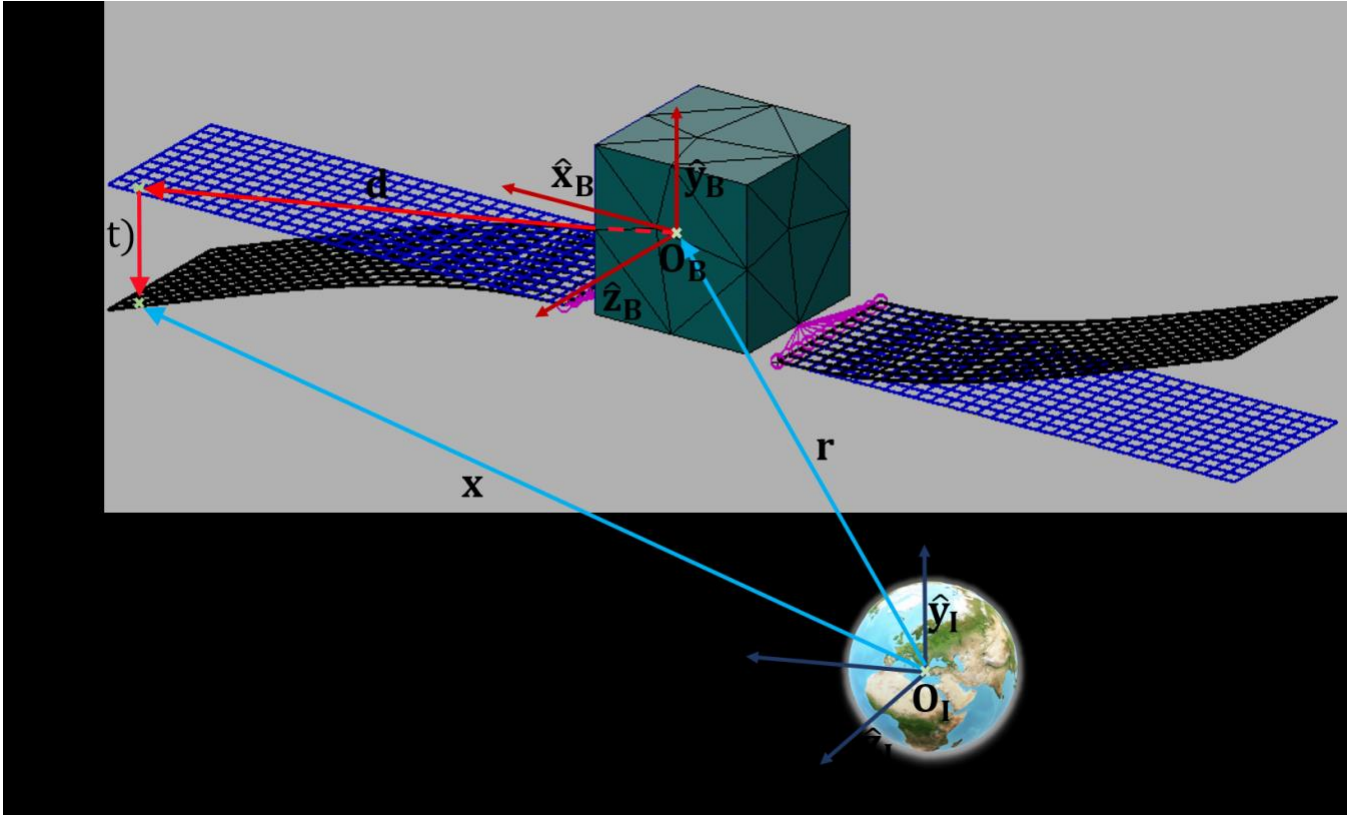


Figure 1: Schematic of the satellite and flexible appendage coordinate systems.

If these assumptions hold true, as they do in this case, the velocity of the flexible appendages can be written as

$$\dot{\mathbf{x}} = \dot{\mathbf{x}}_0 + \mathbf{T} \cdot \text{skew}(\boldsymbol{\omega}) \left(\mathbf{d} + \sum_{k=1}^{N_n} A_k(t) \phi_k(\mathbf{d}) \right) + \mathbf{T} \left(\sum_{k=1}^{N_n} \dot{A}_k(t) \phi_k(\mathbf{d}) \right).$$

The dynamic model for the system is derived from its Lagrangian, $\mathcal{L} = \mathcal{T} - \mathcal{V}$, where $\mathcal{T}$ and $\mathcal{V}$ denote the kinetic and potential energy of the system, respectively. These energies can be calculated as

$$\mathcal{T} = \frac{1}{2} \int_{m_b} \dot{\mathbf{x}}_b^T \dot{\mathbf{x}}_b dm_b + \frac{1}{2} \int_{m_f} \dot{\mathbf{x}}_f^T \dot{\mathbf{x}}_f dm_f,$$

$$\mathcal{V} = \frac{1}{2} M \sum_{k=1}^{N_n} A_k(t)^2 \omega_k^2 - \frac{\mu}{|\mathbf{x}_0|} \left[m - \frac{\tilde{\mathbf{p}} \hat{\mathbf{k}}}{|\mathbf{x}_0|} + \frac{1}{|\mathbf{x}_0|^2} [\text{tr}(\mathbf{I}) - \mathbf{I} - 3(\hat{\mathbf{k}}^T \tilde{\mathbf{I}} \hat{\mathbf{k}})] \right].$$

By incorporating the velocity and energy expressions the Lagrangian expression, the governing equations for the flexible body under gravitational forces are written as

$$m\ddot{\mathbf{x}}_0 = \mathbf{F}_G,$$

$$\tilde{\mathbf{I}}\dot{\boldsymbol{\omega}} + \sum_{m=1}^{N_m} [\bar{\mathbf{I}}_1^m \boldsymbol{\omega} \dot{A}_m + \bar{\mathbf{I}}_m \ddot{A}_m] + \boldsymbol{\omega} \times \left[\tilde{\mathbf{I}}\boldsymbol{\omega} + \sum_{m=1}^{N_m} \bar{\mathbf{I}}_m \dot{A}_m \right] = \mathbf{C}_G,$$

$$\boldsymbol{\omega} \Gamma + \sum_{m=1}^{N_m} [\ddot{A}_m + 2\omega_m \xi \dot{A}_m + \omega_m^2] - \frac{1}{2} \boldsymbol{\omega}^T \mathbf{I}_1 \boldsymbol{\omega} - \boldsymbol{\omega}^T \left(\sum_{m=1}^{N_m} \mathbf{I}_{2,m} A_m \right) \boldsymbol{\omega} + 2\boldsymbol{\omega} \sum_{m=1}^{N_m} \Xi_m \dot{A}_m = \mathbf{Q},$$

where

$$\mathbf{F}_G = -\frac{\mu}{|\mathbf{x}_0|^2} \left[m + \frac{1}{|\mathbf{x}_0|} [\tilde{\mathbf{p}} - \hat{\mathbf{k}}^T \hat{\mathbf{k}} \tilde{\mathbf{p}}] + \frac{3}{2|\mathbf{x}_0|^2} [\text{tr}(\mathbf{I}) \hat{\mathbf{k}} + 2\mathbf{I} \hat{\mathbf{k}} - 5(\hat{\mathbf{k}}^T \tilde{\mathbf{I}} \hat{\mathbf{k}})] \right],$$

$$\mathbf{C}_G = -\frac{\mu}{|\mathbf{x}_0|^2} \left[\tilde{\mathbf{p}} \times \hat{\mathbf{k}} - \frac{\hat{\mathbf{k}} \times \tilde{\mathbf{I}} \hat{\mathbf{k}}}{|\mathbf{x}_0|} \right],$$

$$\mathbf{Q} = -\frac{\mu}{|\mathbf{x}_0|^2} \left[\Lambda \hat{\mathbf{k}} - \frac{\text{tr}(\tilde{\mathbf{I}}_1)}{2|\mathbf{x}_0|} + \frac{\hat{\mathbf{k}} \times \tilde{\mathbf{I}}_1 \hat{\mathbf{k}}}{2|\mathbf{x}_0|^2} \right].$$

These expressions are defined in terms of the following mass dependence parameters:

$$m = \int_V \rho dV, \quad \mathbf{p} = \int_V \mathbf{d} \rho dV, \quad \tilde{\mathbf{p}} = \mathbf{p} + \sum_{m=1}^{N_m} \mathbf{\Gamma}_m A_m, \quad \mathbf{\Gamma}_k = \int_V \mathbf{d} \times \phi_k \rho dV, \quad \Lambda_k = \int_V \phi_k \rho dV,$$

$$\tilde{\mathbf{I}} = \int_V (\mathbf{d}^T \mathbf{d} - \mathbf{d} \mathbf{d}^T) \rho dV = \mathbf{I} + \sum_{m=1}^{N_m} \mathbf{I}_{1,m} A_m + \sum_{m,k=1}^{N_m} \mathbf{I}_{2,m,k} A_m A_k, \quad \tilde{\mathbf{I}}_{1,k} = \mathbf{I}_1 + 2 \sum_{m=1}^{N_m} \mathbf{I}_{2,m,k} A_m,$$

$$\Xi_{m,k} = \int_V \phi_m \times \phi_k \rho dV.$$

$\mathbf{I}$, $\mathbf{I}_1$, and $\mathbf{I}_2$ denote the moment of inertia, and its first and second variations, respectively. Further details on this system of equations can be found in [10].

As mentioned above, the structural properties and characteristics are found via FE software. In this study, the structural components of the satellite model are modelled using the MSC Nastran FE software package. The central rigid body is discretized using 3D tetrahedral elements (CTETRA), and the two flexible solar panels are modelled using quadrilateral plate elements (CQUAD4). The rigid connections that affix the two flexible appendages are added as massless Rigid Body Elements (RBE2), which fixes the translational and rotational displacements of the panel boundary nodes to those of the rigid body. The first five natural frequencies are 2.30, 14.32, 14.50, 41.38, and 45.58 Hz.

To facilitate the application of *dual* unscented Kalman filters, it is necessary to monitor the system in operation. Accelerometers are selected for the measurement of the vibrations of the flexible appendages, and, given the cost of both the sensors and the additional launch mass, this study aims to find a balance between sensor coverage and cost. To this end, we apply the Structural Health Monitoring (SHM)-inspired methodology defined in [11], which combines the Modal Assurance Criterion with an improved adaptive genetic algorithm (IAGA) to ascertain the optimal accelerometer positions. The maximum fitness value was calculated for a broad range of sensor number (presented in Fig. 2), and we concluded that the use of five sensors would balance the measurement accuracy with sensor cost. The positions of the five sensors are displayed in Fig. 3, along with the stationary points of modes 3, 4, and 6. It can be observed that the IAGA places the accelerometers away from the stationary points and at the edge of the solar panel, which is consistent with finding the areas of greatest displacement. The split between the two edges and spacing between the stationary points will also allow maximal capture of the modal displacements.

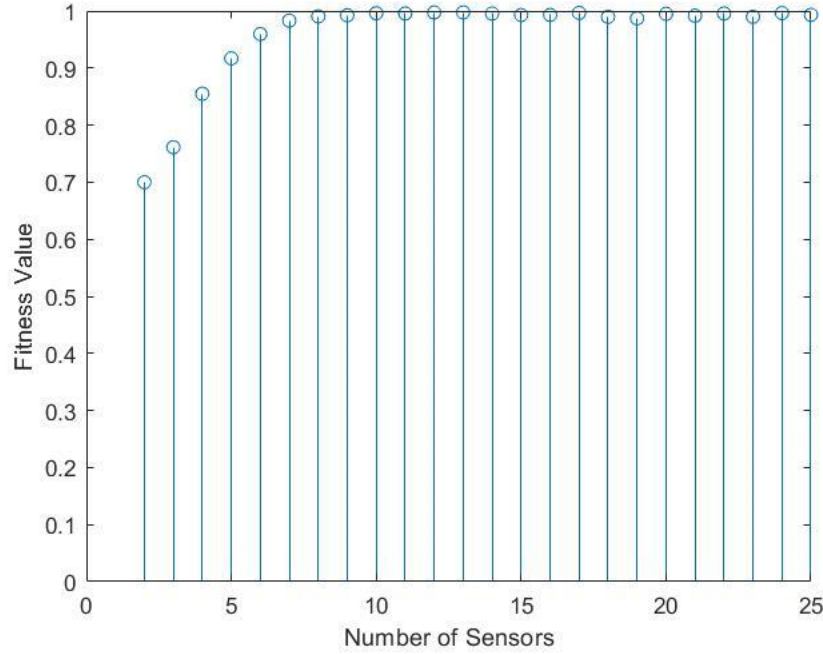


Figure 2: Maximum fitness values achieved for each number of sensors.

The estimation of the states involves the nonlinear dynamic system in discrete times as follows:

$$x_{k+1} = F(x_k, v_k),$$

$$y_k = H(x_k, n_k),$$

where x_k is the unobserved state vector at each iteration k , y_k is the observed signal, v_{k-1} is the process noise, and n_{k-1} is the observation noise. The functions F and H are used for the nonlinear mapping of measurements and states, respectively.

The DUKF strategy aims to determine a nonlinear mapping, $G: x_k \rightarrow y_k$, that maps the observed state to the observed signal. This function is assumed to be parameterisable by some vector, θ , so that it may be written as $y_k = G(x_k, \theta)$. This allows the new state space representation to be written as

$$\theta_k = \theta_{k-1} + u_k,$$

$$y_k = G(x_k, \theta_k) + e_k,$$

Here, the parameters, θ , are dependent on the system state, and represent a stationary process that is driven by the process noise, u_k . The process error is denoted e_k .

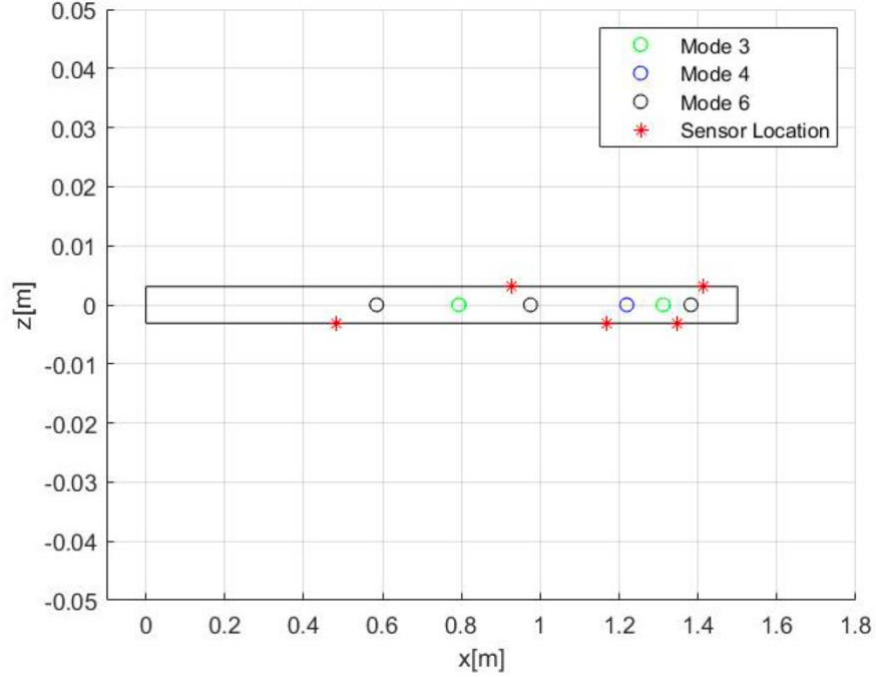


Figure 3: Final positions of the five accelerometers deployed on the flexible appendage.

An overview of the DUKF algorithm used to find G is presented in Fig. 4. It is divided into three key blocks: initialization, prediction, and correction. The UKF methodology assumes that there exists a set of points—named the *sigma points*—in the state space with probability density function similar to that of the system errors, with the implicit assumption that a probability function can be more readily approximated than an arbitrary nonlinear function. The random variable $\hat{x}$, with dimension L , is introduced, so that it may be propagated through the nonlinear function, G .

The selection of the sigma points depends on the initialization parameters, and these are chosen so that their mean and covariance are $\bar{x} = \bar{\hat{x}}$ and $P_{\hat{x}}$, respectively. The filter is initialized with the known values of the initial state of the system, x_0 , and its covariance, $P_0 = P(x_0)$. The scalar parameter $\lambda = \alpha^2(L + \kappa) - L$ is introduced, with α determining the dispersion of the sigma points around $\bar{x}$ (typically between 10^{-4} and 1), and λ a secondary scaling parameter that is typically set to 0 or $3 - L$ [12].

Then, the construction of the $2L + 1$ sigma vectors, X_i (and corresponding weights, W_i), are defined by [13]

$$X_i = \bar{x} \pm \left(\sqrt{(L + \lambda)P_x} \right)_i, \quad \text{for } i = 0, \dots, L,$$

$$W_0^m = \frac{\lambda}{L + \lambda}, \quad W_0^c = \frac{\lambda}{L + \lambda} + (1 - \alpha^2 + \beta), \quad W_i^{m,c} = \frac{1}{2(L + \lambda)}, \quad \text{for } i = 0, \dots, L.$$

Here, $\left(\sqrt{(L + \lambda)P_x} \right)_i$ denotes the i^{th} row of the matrix square root, and the superscripts m and c represent the weightings for the mean and covariance, respectively. The parameter β is introduced to incorporate prior information regarding the distribution of $\bar{x}$; for Gaussian distributions, the optimal value is $\beta = 2$. Details on how these vectors are used to update the system during the DUKF algorithm are presented in Fig. 4.

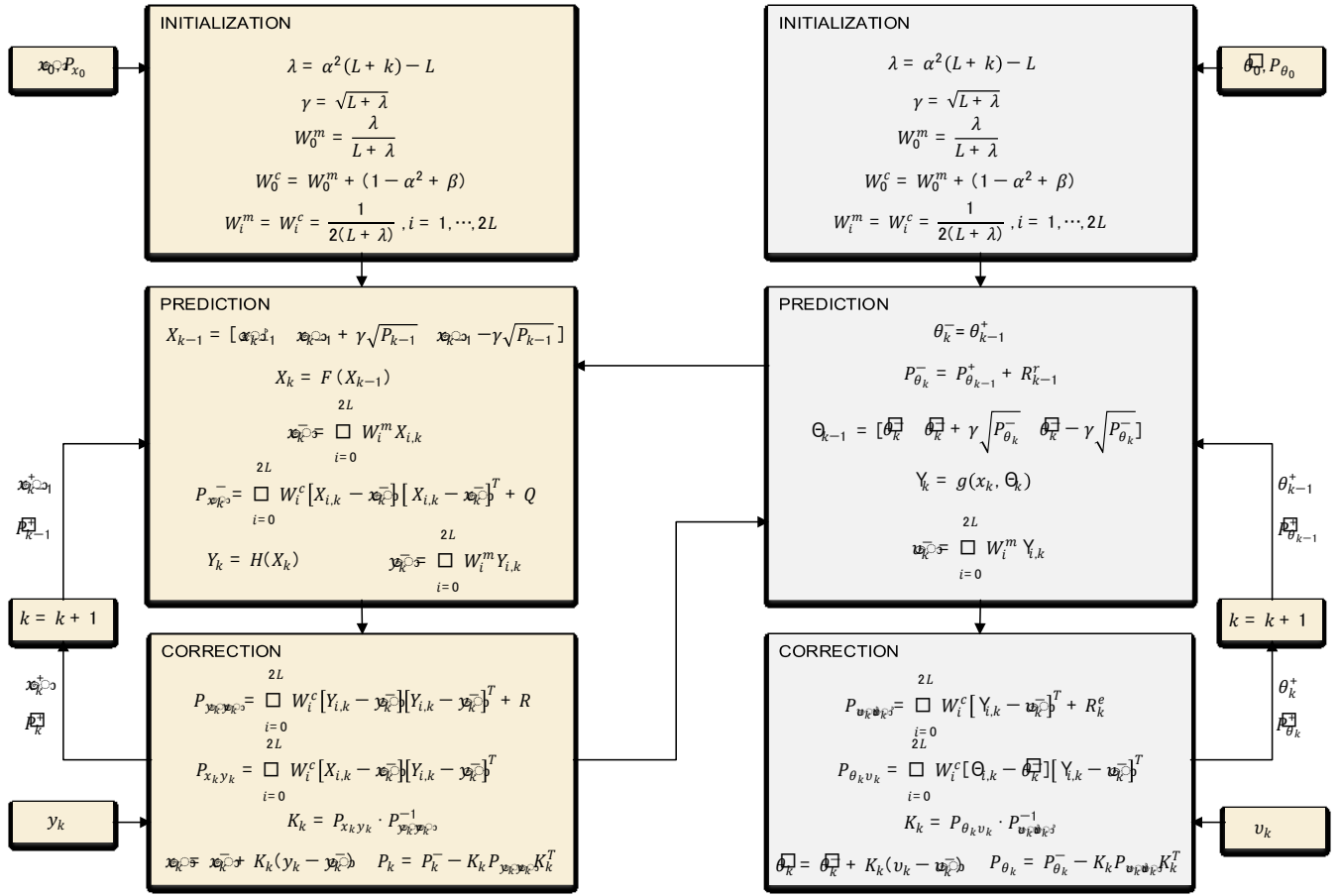


Figure 4: Schematic outlining the steps of the DUKF algorithm.

RESULTS

To assess the effectiveness of this methodology, it is necessary to first assess the ability of the methodology to capture the system dynamics, and then its ability to correctly identify the moment of inertia. This capability is investigated by applying the DUKF algorithm to a 60 s simulation of the coupled dynamics. The system is perturbed by a series of three-dimensional torques, as presented in Fig. 5(a). Representative time histories for the system attitude and vibrations are shown Fig. 5(b,c), in which it can be observed that the DUKF methodology is able to accurately capture the system dynamics. It must be noted that some elements of the higher-frequency vibration response are effectively averaged, but the reason for this will be explored in future work. Finally, Fig. 5(d) presents the percentage error in the DUKF prediction for the moment of inertia (about the x axis). As outlined above, an initial error of 40% was fed to the algorithm, so that its ability to update the prediction based on real-time system identification could be explored. As can be observed in Fig. 5(d), this error is very quickly corrected by the DUKF algorithm, and the error is reduced to less than 1% in under 5 seconds and remains that way until the end of the simulation. There is some minor deviation from the true value of I_{xx} after approximately 30 s, which is likely attributable to the lack of torques in this particular axis after this point; this error is likely to reduce further in operation as the system continuously adapts to its environment.

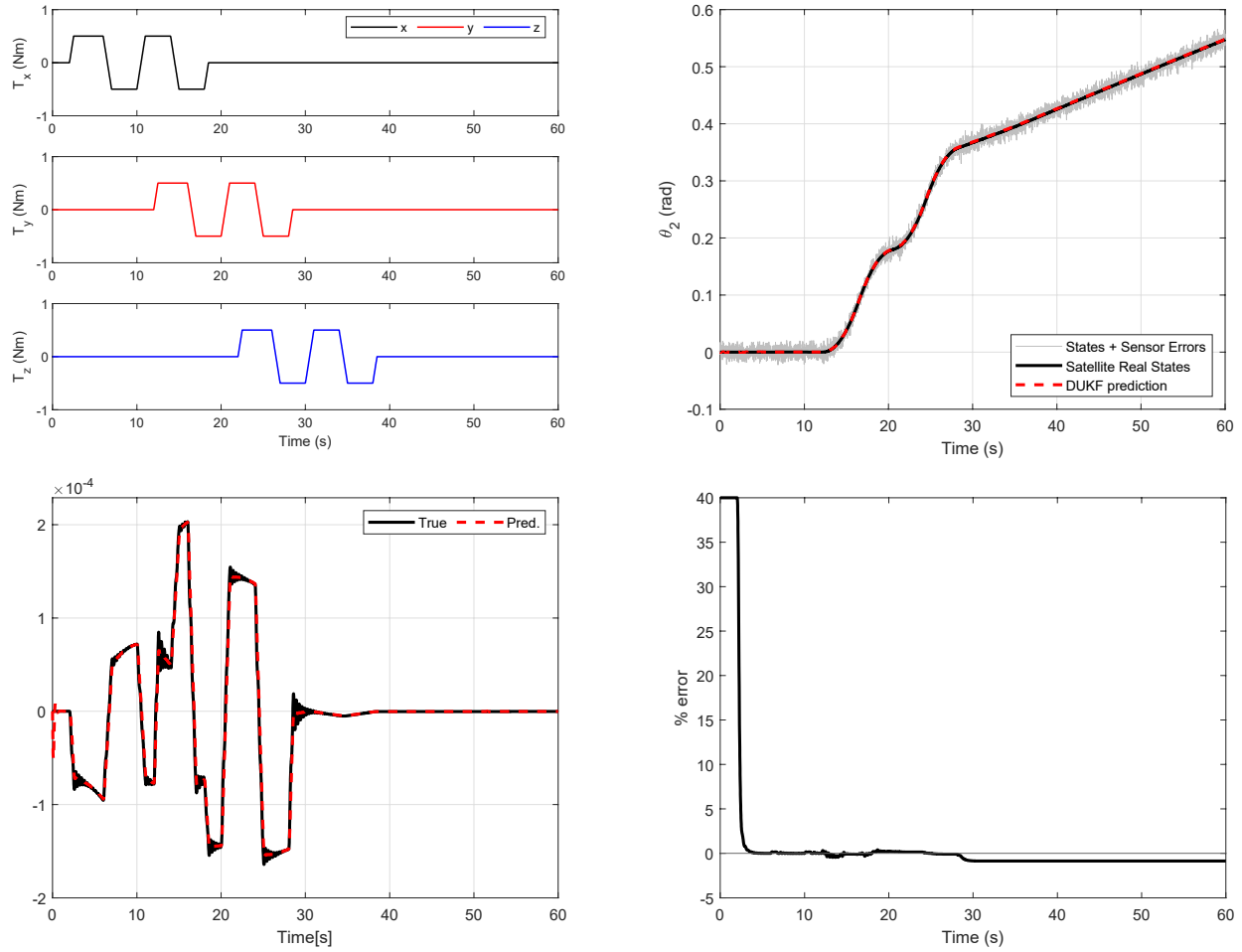


Figure 5: Time histories: (a) three-dimensional torque applied to the satellite, (b) attitude dynamics, (c) first modal displacement of the flexible appendage, (d) percentage error of the moment of inertia.

CONCLUSIONS AND FUTURE WORK

This paper presents a novel AOCS strategy based on the Dual Unscented Kalman Filter algorithm. The *dual* aspect of the methodology pertains to the fact that two UKFs are applied in parallel, one for the unobserved state vector and one for the observed sensor signals. The sensor locations were optimized using an IAGA, which successfully located points on the flexible appendages at which the largest displacements will occur. The DUKF was applied to a small satellite, for which it was demonstrated that the attitude and vibrational dynamics are accurately predicted. Further, we showed that the strategy was able to correct the moment of inertia from an initial error of 40% to one that was less than 1%. This successful demonstration for a small-scale satellite will provide the basis for future work exploring the application of the DUKF methodology for much larger flexible appendages. Given the need to reduce the mass of space structures to save of launch costs, these appendages will also be inherently slender, which will dramatically increase their geometric nonlinearity. As such, it may be necessary to update the system of equations and DUKF structure to approximate and capture such behaviour.

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