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Pressure Fluctuations in Shear flow  
turbulence. Part I.

Hodgson, T H



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PRESSURE FLUCTUATIONS IN SHEAR FLOW TURBULENCE

by

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### Abstract.

An investigation of the fluctuating pressure in turbulent shear flows at low Mach numbers is described. The treatment is both theoretical and experimental although, in the latter case, present limitations of pressure transducer design have restricted the experiments to surface measurements in wall shear flows over rigid plane boundaries.

The turbulent flat-plate boundary layer is studied in detail. Calculations from existing turbulent velocity data show that the ratio of the root-mean-square wall fluctuating pressure to free stream dynamic pressure is approximately twice the local skin-friction coefficient  $C_f$ . The dependence on  $C_f$  has been verified for the Reynolds number range  $3 \times 10^5$  to  $8 \times 10^6$ . The factor of proportionality was 2.2.

Although part of the test programme was performed in a low-noise wind-tunnel, it was not possible to determine completely the pressure power spectral density from laboratory measurements. This problem has been resolved by full-scale flight measurements on the wing of a high performance glider. The power spectral density of the surface pressure fluctuations was found to vary as the square of the frequency at low frequencies, a result in agreement with analysis of the turbulence-mean shear contribution to the wall pressure.

Confirmation of the results of the boundary layer experiments

has been obtained from measurements in a second wall shear flow, the wall jet.

As well as root-mean-square values, the space-time correlation and spatial correlation of the pressure field have been measured using a specially constructed correlator. These measurements were used in an investigation of the source terms of the noise generated aerodynamically by turbulent shear flow at low Mach numbers over a plane boundary.

### Foreword.

This thesis describes work performed in the Department of Aerodynamics, The College of Aeronautics, Cranfield, between April 1957 and September 1961. The work was supported by a Ministry of Civil Aviation Contract.

The author is greatly indebted to Mr. G.M. Lilley, who supervised the thesis, for his advice and encouragement and to Dr. J.F. Williams for helpful discussions.

The author also acknowledges the valuable assistance of Mr. S.H. Lilley and his laboratory staff who supervised the wind-tunnel control gear and constructed apparatus, the helpful comments of Mr. E.C. Sills on the design of the electronic circuits, and the co-operation of Mr. M.A. Perry and the many members of the Department of Flight who assisted in the glider flight-test work.

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# Principal Notation

|                               |                                                                                            |
|-------------------------------|--------------------------------------------------------------------------------------------|
| $A$                           | $\frac{\partial p}{\partial t}$ correlation area                                           |
| $a_o$                         | speed of sound                                                                             |
| $C_f$                         | skin friction coefficient $(= \frac{\tau_w}{\frac{1}{2} \rho U_1^2})$                      |
| $d$                           | pressure transducer diameter                                                               |
| $\bar{e}_{ij}$                | mean shear                                                                                 |
| $f$                           | frequency                                                                                  |
| $f(r)$                        | longitudinal velocity correlation coefficient in isotropic turbulence                      |
| $G, G_o, G_i, G_+, G_-$       | Green's functions                                                                          |
| $g$                           | mean shear-turbulent intensity product, mean value $g_o$                                   |
| $H$                           | boundary layer form parameter $\delta_1/\delta_2$                                          |
| $i$                           | $\sqrt{-1}$                                                                                |
| $i, j, l, m$                  | tensor notation suffices                                                                   |
| $i(x)$                        | sound intensity per unit volume (area) of turbulence                                       |
| $k$                           | two-dimensional wave-number in the plane $k_1 k_3$                                         |
| $L, L_1, L_2, \ell_1, \ell_2$ | scales of turbulence                                                                       |
| $M$                           | Mach number                                                                                |
| $P_i$                         | dipole strength per unit area                                                              |
| $P(\xi), \bar{P}(\xi)$        | pressure covariance and correlation coefficient respectively                               |
| $\bar{P}(\tau)$               | pressure autocorrelation coefficient                                                       |
| $\bar{P}(\xi, \tau)$          | space-time pressure correlation coefficient                                                |
| $\bar{P}_{f_o}(\xi, \tau)$    | space-time correlation coefficient in filter band centre-frequency $f_o$                   |
| $P_v, P_c$                    | sound power output per unit volume of turbulence and per unit cylinder length respectively |

|                                                                  |                                                                             |
|------------------------------------------------------------------|-----------------------------------------------------------------------------|
| $p$                                                              | pressure (generally denotes fluctuating value)                              |
| $p_{f_0}$                                                        | fluctuating pressure in filter band centre frequency $f_0$                  |
| $p_{ij}$                                                         | stress tensor                                                               |
| $\overline{p^2}(\omega)$                                         | measured pressure power spectral density                                    |
| $q$                                                              | source strength                                                             |
| $R$                                                              | radius                                                                      |
| $R_{ij}(\underline{r}), \bar{R}_{ij}(\underline{r})$             | velocity covariance and correlation coefficient respectively                |
| $\bar{R}_{ij}(\underline{r})$                                    | velocity correlation coefficient for homogeneous field                      |
| $\bar{R}_{ij}(\tau)$                                             | velocity autocorrelation                                                    |
| $\bar{R}_{ij}(\underline{r}, \tau)$                              | space-time velocity correlation coefficient                                 |
| $\underline{r} = \underline{r}' - \underline{r}, \underline{r}'$ | velocity correlation separation in fixed and moving axes respectively       |
| $S$                                                              | area                                                                        |
| $T$                                                              | integral time scale in moving frame of reference                            |
| $T_{ij}$                                                         | quadrupole strength per unit volume                                         |
| $t, t^*$                                                         | time, and retarded time                                                     |
| $U$                                                              | mean velocity component parallel to plane boundary                          |
| $U_e$                                                            | external mean velocity                                                      |
| $U_0$                                                            | averaged convection speed                                                   |
| $U_m$                                                            | peak mean velocity in wall jet flow.                                        |
| $U_\tau$                                                         | shear velocity $(= \sqrt{\frac{\tau_w}{\rho_0}})$                           |
| $u_i$                                                            | velocity component in $x_i$ direction (generally denotes fluctuating value) |
| $\bar{u}_i$                                                      | velocity component of homogeneous turbulent field                           |
| $\underline{x}$                                                  | two-dimensional vector in plane $x_1, x_2$                                  |
| $\underline{x}, \underline{x}'$                                  | usually denote field point coordinates                                      |

|                                                               |                                                                                                              |
|---------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|
| $y, y', z, z'$                                                | usually denote source point coordinates                                                                      |
| $\bar{w}$                                                     | Fourier coefficient of pressure                                                                              |
| $Y_{\frac{1}{2}}, Y_m$                                        | two-dimensional wall jet half-width, and inner-region thickness                                              |
| $Z_{\frac{1}{2}}, Z_m$                                        | radial wall jet half-width, and inner-region thickness                                                       |
| $\bar{w}_i$                                                   | Fourier coefficient of velocity component $u_i$                                                              |
| $\alpha$                                                      | anisotropy factor or wall jet velocity distribution parameter                                                |
| $\delta$                                                      | boundary layer thickness or Dirac delta function                                                             |
| $\delta_0$                                                    | boundary layer ordinate at which $U_1 - U = U_\tau$                                                          |
| $\delta_1, \delta_2$                                          | boundary layer displacement and momentum thicknesses                                                         |
| $\delta_{ij}$                                                 | unit diagonal tensor; $\delta_{ij} = 1, i = j$ and $\delta_{ij} = 0, i \neq j$                               |
| $\underline{k}$                                               | three-dimensional wave-number                                                                                |
| $\nu$                                                         | kinematic viscosity                                                                                          |
| $\underline{\xi} = \underline{x}' - \underline{x}$            | pressure correlation separation in fixed frame of reference                                                  |
| $\rho, \rho_0$                                                | density, and ambient density                                                                                 |
| $\tau, \tau_m$                                                | time delay, and optimum time delay                                                                           |
| $\tau_w$                                                      | wall stress                                                                                                  |
| $\omega$                                                      | circular frequency                                                                                           |
| $\omega_0$                                                    | reciprocal of integral time scale                                                                            |
| $\omega_d$                                                    | dominant frequency of sound radiation                                                                        |
| $\Phi_{ij}(\underline{x}_2, \underline{x}'_2; \underline{k})$ | two-dimensional velocity spectrum function                                                                   |
| $\Phi_{ij}(\underline{k})$                                    | isotropic three-dimensional velocity spectrum function                                                       |
| $\Pi(\underline{x}_2; \underline{k})$                         | two-dimensional wave-number pressure spectrum function, value at the wall often written $\Pi(\underline{k})$ |
| $\Pi(\omega)$                                                 | frequency pressure spectrum function                                                                         |

|                  |                                                                               |
|------------------|-------------------------------------------------------------------------------|
| $\Pi(k, \omega)$ | pressure spectrum function in frequency and two-dimensional wave-number space |
| $f( )$           | denotes 'in fixed frame of reference'                                         |
| $m( )$           | denotes 'in moving frame of reference'                                        |
| $\overline{( )}$ | denotes time or ensemble average value                                        |
| $( )^*$          | denotes complex conjugate                                                     |

1. Introduction.

For many years now, there has been a great deal of research into the mechanism of turbulence from measurements of the velocity fluctuations in turbulent flows. With the progress in electronic engineering an acceptable transducer, the hot-wire, for measuring the rapid velocity fluctuations has been developed. The design of hot-wire anemometers has now reached an advanced state.

The microphone, a transducer for converting acoustic pressures into electrical signals, was used in acoustics as early as 1876. However, the measurement of the fluctuating pressures that occur in problems of fluid mechanics appears to have been neglected.

It is only recently that great attention has been focussed on the study of fluctuating pressures in turbulent flows. This has arisen because of two major aeronautical-engineering problems of the day. Firstly, the acoustic noise that is produced by the turbulent motion in the efflux of jet-engines is very intense, and secondly turbulent boundary layer flow over aircraft skins can lead to panel vibration. At high speeds these vibrations may cause structural fatigue and may also produce considerable noise in the aircraft cabin. Both of these phenomena can be directly related to the fluctuating pressure field within the turbulent flow.

This fluctuating pressure field, like the velocity, defies an exact mathematical description because of the non-linearity of the dynamical processes involved. Even so, much theoretical work

has been done on the velocity field in idealised models such as homogeneous and isotropic turbulence, see Batchelor (1953) for instance. For the more complicated inhomogeneous and anisotropic cases, to which turbulent shear flows belong, one is forced to rely mainly on experimental measurements of the fluctuating velocity and pressure.

However, the relatively slow development of very small pressure transducers with good response and sensitivity has meant that it is still extremely difficult to measure the fluctuating pressures within a turbulent flow. When present transducers are placed in a flow, large disturbances are produced because of separated flow around the transducer. The disturbances are usually of the same frequencies as the fluctuations to be measured and are greater in amplitude. Streamlining the transducer shape or connecting a hypodermic probe-tube usually result in a resonant frequency response with limited bandwidth.

The only experimental determinations of the fluctuating pressure within a turbulent flow have been described by Strasberg and Cooper (1956) and Kobashi (1956) who used probe-type transducers in the wake of a circular cylinder. There, the fluctuations to be measured were of relatively low frequency and periodic. It may be noted that because of this lack of experimental data the contribution of the pressure term in evaluations of the turbulent energy balance has so far been determined by difference (Townsend 1956).

Fortunately the design of small microphones with good response and sensitivity for acoustic work has reached a stage where it is possible to use them as flush-mounted pressure transducers in a smooth wall. Hence the obvious starting point for a study of the fluctuating pressure field in turbulent shear flows is to investigate firstly the surface pressure fluctuations in wall shear flows.

This thesis describes such an investigation, where for simplicity in both the experimental and theoretical work the wall is rigid and plane.

#### 1.1. Summary of previous work.

An equivalent treatment of the fluctuating pressure field for the case of isotropic turbulence with no mean velocity gradients has been described independently by Heisenberg (1948), Obukhov (1949) and Batchelor (1951). As mentioned previously, an exact description of the velocity field has not, at the moment, been obtained. It was therefore necessary to make a hypothesis about the statistical quantities which specify the turbulent velocity field. It was assumed that the distribution of the joint probability function for the turbulent velocities at any two points was normal, in which case fourth order velocity correlations could be expressed in terms of those of second order. Batchelor pointed out that although this assumption is consistent with the well-known experimental result that the distribution of the velocity fluctuations at one point in grid

turbulence is closely normal, the assumption cannot be exactly true since it can be shown that the effects of the non-linear terms in the equations of motion are then neglected. Measurements by Stewart (1951) showed that in grid turbulence this hypothesis of normality of the joint probability distribution for two points was a reasonable one.

Batchelor computed the two-point pressure covariance scalar

$$P(\xi) = \overline{p(\underline{x}) p(\underline{x} + \underline{\xi})} \quad (1.1)$$

where  $p(\underline{x})$  is the fluctuating pressure at  $\underline{x}$ ,  $\underline{\xi}$  is the separation vector between the two points and the bar denotes a mean value, using Heisenberg's form at high Reynolds number for the velocity correlation scalar  $f(r)$  in isotropic turbulence. The pressure covariance was always positive and decreased monotonically to zero at large separation.

The root-mean-square value of the fluctuating pressure was found to be

$$\sqrt{\overline{p^2}} = 0.58 \sqrt{\overline{u^2}} \quad (1.2)$$

where  $\overline{u^2}$  is the mean-square value of the fluctuating velocity.

Uberoi (1953) has calculated the root-mean-square pressure from measured velocity correlations in grid turbulence and obtained the constant in equation (1.2) to be approximately 0.65 over a wide Reynolds number range. The results of an extensive experimental

determination of the fourth order velocity correlations were compared with the normality hypothesis and showed good agreement. There is so far no experimental confirmation of the result (1.2) due to the difficulties mentioned previously. In the present problem of wall shear flow it is necessary to use this same normality hypothesis to calculate the 'turbulence-turbulence' contribution to the wall fluctuating pressure (see Appendix B).

The first attempts to calculate the pressure field in a turbulent shear flow, that is turbulence in which an appreciable mean velocity gradient exists, were made by Kraichnan (1956a, 1956b). He showed that for the case of incompressible turbulent flow over a flat plate the contribution to the wall fluctuating pressure from the effect of mean shear was

$$\sqrt{p^2} \approx 6\tau_w \quad (1.3)$$

where  $\tau_w$  is the local wall shear stress. That is

$$\sqrt{p^2} / \frac{1}{2}\rho_0 U_i^2 \approx 6 C_f \quad (1.4)$$

where  $\rho$  is the density and  $U_i$  is the external mean velocity. This result was obtained using the turbulence measurements of Laufer (1955) for pipe flow. Kraichnan fitted the two-point velocity correlations by taking the turbulence to be isotropic with correlation scalar

$$f(r) = e^{-r/L} \quad (1.5a)$$

$$\text{or} \quad f(r) = e^{-r^2/L^2} \quad (1.5b)$$

where  $\mathbf{r}$  is the separation vector between the two points and  $L$  is a measure of the integral scale of the turbulence.

An important inference from Kraichnan's work was that the contribution of the turbulence itself to the surface pressure, that is from the terms not containing the mean velocity gradient as in Batchelor's result for isotropic turbulence equation (1.1), should be small compared with the mean shear contribution equation (1.3). For this turbulence mean-shear interaction the one-dimensional spatial power spectral density  $\Pi(k_1)$  was found to vary as  $k_1^{-2}$  at low  $k_1$ , where  $k_1$  is the wave-number in the streamwise direction. Two results follow immediately if the spectrum has this form.

Firstly, it can be shown, by a Fourier transform, that the longitudinal spatial correlation of the wall pressure must take significant negative values as compared with the always positive form for isotropic turbulence with no mean velocity gradient. In fact, the area under the spatial correlation for the turbulence-mean shear contribution in incompressible flow must be zero.

Secondly, from the fact that the pressure field may be regarded as being convected past the fixed measuring position at some mean velocity, the power spectral density  $\Pi(\omega)$  of the time fluctuations in pressure measured at the fixed stations must have the same form

as  $\Pi(k_1)$  and so should vary as  $\omega^2$  at low frequency, where  $\omega$  is the angular frequency.

Kraichnan also treated the case of a free shear flow with constant mean shear superimposed on isotropic turbulence. Lilley (1958) has calculated the pressure correlations for this idealised model and shown the existence of a negative loop in the streamwise direction. Kraichnan also demonstrated that departure from isotropy of the turbulence resulted in a lower mean-square value of the fluctuating pressure for a given mean-square kinetic energy of the turbulence.

The availability of more complete experimental data on the velocity field in wall shear flows has enabled calculations in the present work to be made in greater detail. However, the author has employed the methods of Batchelor and Kraichnan extensively.

When the present research programme was started the only measurements of surface pressure fluctuations in wall shear flows that had been published were by Willmarth (1956). Many investigations for the turbulent boundary layer case have been reported since.

Willmarth (1956) found from measurements on the wall of a pipe that the ratio

$$\sqrt{\bar{p}^2}/\frac{1}{2}\rho U_1^2 \approx 0.0035 \quad (1.6)$$

over the Reynolds number range  $1.5$  to  $20 \times 10^6$  and the Mach number range  $0.2$  to  $0.8$ . The transducers used in this early investigation

had a poor performance and were not accurately calibrated. In later work Willmarth (1959), using improved instrumentation, found that

$$\sqrt{\overline{p^2}} / \frac{1}{2} \rho_0 U_1^2 \approx 0.006 \quad (1.7)$$

for the same Reynolds number and Mach number range as before and that the power spectral density  $\Pi(\omega)$  decreased rapidly with increase of frequency at high frequencies. Willmarth was not able to obtain accurate information on the spectrum shape at low frequencies even after a most painstaking effort to reduce the intensity of a noisy air supply. The measurements appeared to be scattered about a mean curve which tended to a constant value at low frequency.

Although Willmarth did not discuss the possible effects of varying wall stress and Mach number in his experiments, his work has been of great value. His later results were the first to demonstrate the effects of the lack of resolving power of a relatively large transducer in boundary layer pressure measurements. From space-time correlations using two transducers separated in the streamwise direction, he showed that the pressure field could be regarded as being convected at an average convection speed  $U_0$ , say, where

$$U_0 = 0.82U_1 \quad (1.8)$$

Tack et alii (1961), Richards (1961), and Laufer (1961) at supersonic speeds, as well as the present work, have all confirmed this result.

The magnitude of the experimental difficulties involved in measurements of the fluctuating pressure is demonstrated on examining the results so far published. From wind tunnel measurements Lane-Smith\* (1959) found that  $\sqrt{p^2}/\frac{1}{2}\rho_0 U_1^2 = 0.0005$  and Harrison (1958) obtained the value 0.0095 for this ratio. Willmarth referred to unpublished measurements by Corcos in a pipe of  $\sqrt{p^2}/\frac{1}{2}\rho_0 U_1^2 = 0.0045$ . From full-scale flight tests on a jet aircraft Mull and Algranti (1956) obtained the value 0.0013.

More recent work by Mull and Algranti showed that  $\sqrt{p^2}/\frac{1}{2}\rho_0 U_1^2 = 0.005$  on the fuselage and 0.001 on the wing for the Reynolds number range  $0.4$  to  $1.8 \times 10^7$  and Mach number range 0.3 to 0.9. When corrected for the effects of transducer size, these flight results were in reasonable agreement with Willmarth's result, equation (1.7). Measurements in a water-tunnel have been reported by Skudryk and Haddle (1960). They found  $\sqrt{p^2}/\frac{1}{2}\rho_0 U_1^2 = 0.0024$ .

All the spectra measurements of pressure so far published have suffered in varying degrees from the presence of spurious interference such as vibration and the noise from wind-tunnel fans and jet-engines. This has prevented any definite conclusions being made about the spectrum shape at low frequencies.

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\* Dr. Gerrard of Manchester University, in a private communication to the author, has now confirmed that this result was in error due to transducer difficulties.

Attention has been drawn to the variation of the wall fluctuating pressure with the wall stress by Lane-Smith (1959) and Hodgson and Lilley (1960). In view of Kraichnan's work it is surprising how often in the published experiments this variation has not been discussed or wall stress measurements have not been presented. In fact, a large amount of the scatter in the root-mean-square pressure values quoted could be attributed to the different wall stress values for each experiment. If this variation is allowed for, together with correction for transducer size, the published measurements suggest that

$$\sqrt{\bar{p}^2} / \frac{1}{2} \rho_0 U_1^2 \approx 1.5 - 3 C_f \quad (1.9)$$

although it is necessary to keep in mind compressibility effects for the higher subsonic Mach number results. Measurements by Williams (1960) for the Mach number range 1.2 to 1.6 and by Kistler (described by Laufer 1961) for the range  $M = 1.2$  to 6 have shown that  $\sqrt{\bar{p}^2} / \frac{1}{2} \rho_0 U_1^2 \approx 4 C_f$  at  $M = 1.2$  rising to a value of  $6 C_f$  at  $M = 5$ .

## 1.2 Scope of the present work

The present research has been undertaken with the object of gaining a greater knowledge of the surface pressure fluctuations in wall shear flows. All the cases considered were low Mach number flows. The main purposes were:-

1. To measure the magnitude of the surface pressure

fluctuations for turbulent boundary layer flow and to investigate the dependence on the wall stress.

2. To determine the form of the power spectral density of the pressure fluctuations.

3. From measurements of the spatial correlations and space-time correlations of the wall pressure, to investigate the source terms of the aerodynamic noise radiated away from the turbulent boundary layer.

4. To augment the experimental investigations with improved calculations of the root-mean-square pressure, spectrum and spatial correlation from the more detailed experimental knowledge of the fluctuating velocity field now available. In these calculations the fluid has been regarded as incompressible.

Great emphasis has been placed on accurate, reliable instrumentation. It was discovered, however, that the experimental difficulties could not be completely overcome in an ordinary type of wind-tunnel or even in a tunnel of exceptionally low extraneous noise level. Eventually measurements were taken on the wing of a streamlined glider in flight in order to overcome this difficulty.

Although the major interest was in turbulent boundary layer flow, it was necessary to seek verification of the ideas formulated in that part of the work from a study of a second type of wall shear flow, the wall jet. This shear flow was found to be an excellent model for an experimental investigation of the surface fluctuating

pressure. Some preliminary pressure measurements in the wall jet have been reported by Hodgson and Lilley (1960).

Finally, in order to avoid possible confusion in the theoretical treatment and in the discussion of the experimental results it must be stressed that, in general, we are dealing with the problem of pressure fluctuations within an essentially incompressible flow. Some workers have referred to this part of the fluctuating pressure field as 'pseudo-sound', only a small part of the total energy being radiated away as sound. The situation has become confused because the noise produced in aircraft cabins by the vibrations of the aircraft skin, caused by the surface boundary-layer pressure fluctuations, has been called 'boundary-layer noise'. It is preferred to regard boundary layer noise as the energy radiated away from the flow to the far field. This latter problem has been treated theoretically by Lighthill (1952, 1954), Curle (1955), Phillips (1956) and Doak (1960). Theoretical calculations of cabin noise have been made by Ribner (1956), Corcos and Liepmann (1956) and Kraichnan (1957), also some measurements have been reported by McLeod and Jordan (1958). Cabin noise, a structural problem of flexible surfaces subject to a random loading, will not be treated in the present work.

## 2. Theory

### 2.1 The Green's function solution for the fluctuating pressure.

In many instances where the pressure fluctuations are significant the fluctuations in the fluid density are significant also. However, for the moment, we will be more concerned with the pressure fluctuations inside the field of turbulent shear flow, which in this problem will be an essentially 'low-speed' flow, than with the noise in the far field radiated away from the turbulent flow. Hence we may legitimately assume the fluid density is constant.

The equations of continuity and motion are therefore respectively:-

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2.1)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = - \frac{1}{\rho_0} \frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i \quad (2.2)$$

$$i, j = 1, 2, 3.$$

with an orthogonal set of axes  $x_1, x_2, x_3$  and where  $p$  is the pressure,  $u_i$  is the velocity component in the  $x_i$  direction,  $\rho_0$  is the density and  $\nu$  is the kinematic viscosity. (Repeated suffices imply summation.)

If we take the divergence of equation (2.2) and use equation (2.1) we obtain the following equation for the pressure:-

$$\nabla^2 p(\mathbf{x}, t) = - \rho_0 \frac{\partial^2}{\partial x_i \partial x_j} u_i u_j(\mathbf{x}, t) = - q(\mathbf{x}, t) \quad (2.3)$$

where  $q(x,t)$  is written for the source function. Several deductions from this equation may be made immediately.

In the case of inviscid steady flow the pressure at a point follows at once from the dynamic pressure at that point. Equation (2.3) shows that the fluctuating pressure at a point in a turbulent flow, since it obeys an equation of the Poisson type, is governed by fluctuations in velocity throughout the entire flow, with suitable weighting about the field point, and not just at the field point.

Also equation (2.3) contains no time derivatives and so the time variation of pressure can be said to 'follow' the time variation of the velocity fluctuations. Therefore to obtain the time variation of the pressure either requires the time description of the velocity field or else a relation between the time and spatial variation of the pressure field.

It is emphasized again that we are solving for the pressure fluctuations within an incompressible flow. Had we wished at this stage to estimate the noise radiated away from the turbulent flow, equation (2.3) must be replaced by a wave-equation for the compressible pressure field. Hence, conclusions about the noise radiated using the following calculations cannot be drawn directly. However, as will be shown in section 7, use of the present calculations and measurements of the wall fluctuating pressure (which in the experiments is the true compressible pressure field although the flow Mach number may be small) interpreted in conjunction with the theory of aerodynamic

noise will allow some inferences regarding the radiated noise to be made.

Since the turbulent boundary layer is the most relevant case of a turbulent wall shear flow, it will be treated in detail.

The boundary layer is taken to be two-dimensional flowing over the plane  $x_2 = 0$ , with zero external mean pressure gradient. We will denote the free-stream velocity by  $U_1$  in the  $x_1$  direction with the flow extending to infinity in the  $x_2$  direction normal to the plane.

Writing

$$u_1 = U + u_1 \quad u_2 = u_2 \text{ and } u_3 = u_3$$

where now  $u_i$  denotes the fluctuating velocity components and  $U = U(x_2)$  is the mean velocity component in the  $x_1$  direction, equation (2.3) may be written:

$$\nabla^2 p = -\rho \frac{\partial u}{\partial x_1} \frac{\partial U}{\partial x_2} - \rho \frac{\partial^2}{\partial x_i \partial x_j} (\overline{u_i u_j} - \overline{u_i} \overline{u_j}) \quad (2.4)$$

after subtracting the mean pressure so that  $p$  now denotes the fluctuating pressure, and the effects of other mean velocity gradient terms are assumed small being of  $O(1)$  under the usual boundary layer approximations.

The fluctuating pressure thus depends on two types of term, one a product of a spatial derivative of a fluctuating velocity and the mean shear  $\frac{\partial U}{\partial x_2}$ , and the other a product of two spatial derivatives of fluctuating velocities. With the usual boundary layer approximation that the derivative of a mean quantity is usually much larger than the derivative of a fluctuating quantity, the 'turbulence-mean

shear' term should be much greater than the 'turbulence-turbulence' term. Hence the pressure fluctuations in a shear flow are expected to be greater than in the zero mean-velocity gradient case, where only 'turbulence-turbulence' terms contribute, because of the amplifying effect of the mean shear.

Equation (2.4) is then

$$\nabla^2 p = -\rho_0 \bar{e}_{12} \frac{\partial u_2}{\partial x_1} \quad (2.5)$$

where  $\bar{e}_{12}$  is the mean shear  $\frac{\partial U}{\partial x_2}$ .

To neglect the 'turbulence-turbulence' terms without further investigation could however lead to error for there are altogether nine such terms, although by continuity the effects of each are not necessarily additive. Furthermore, although their magnitude may be small, their power spectral density could be significant especially at low frequencies or wave-numbers.

It is shown in Appendix B that although the form of the power spectral density  $\Pi(k_1)$  for the 'turbulence-turbulence' terms is 'flat' at low  $k_1$  the 'turbulence-turbulence' contribution does not effect the overall spectrum over the dominant wave-number range. This is because the 'turbulence-turbulence' contribution to the mean-square pressure is only about 5% of the total, probably less. Results of the experimental programme, to be described later, have shown that the neglect of the 'turbulence-turbulence' terms is justified.

We will now attempt to solve equation (2.5) as exactly as possible. The solution will be obtained by use of an appropriate Green's function. This has the advantage that the fluctuating pressure is given in terms of the mean shear variation and the two-point correlations of the velocity field. The solution can then be interpreted from a physical point of view rather easier than a Fourier-transform solution which is used later in the calculations.

If  $G(\underline{x}, \underline{y})$  is the Green's function which satisfies

$$\nabla^2 G(\underline{x}, \underline{y}) = -\delta(\underline{x} - \underline{y}) \quad (2.6)$$

where  $\delta(\underline{x})$  is the Dirac delta function, then the solution of equation (2.5) is

$$p(\underline{x}, t) = \int_V q(\underline{y}, t) G(\underline{x}, \underline{y}) d\underline{y} + \int_S \left( G \frac{\partial p(\underline{y}, t)}{\partial n} - p(\underline{y}, t) \frac{\partial G}{\partial n} \right) dS(\underline{y}) \quad (2.7)$$

where  $V$  is the total volume occupied by the flow,  $S$  is the area of the plane over which the fluid flows and  $n$  is the unit normal towards the surface from the volume.

Now the Green's function  $G_0$  for the unbounded problem can be shown to be

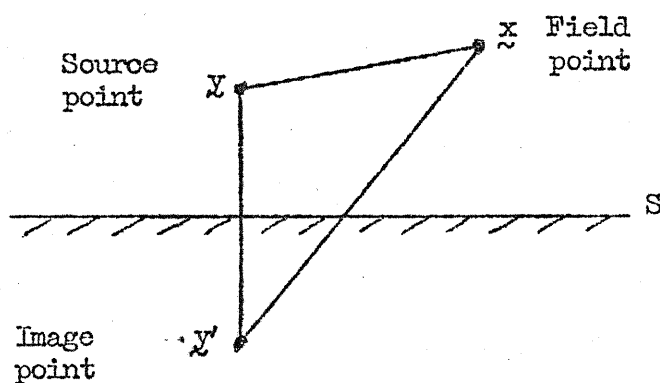
$$G_0(\underline{x}, \underline{y}) = \frac{1}{4\pi |\underline{x} - \underline{y}|} \quad (2.8)$$

Writing  $G_-$  for the Green's function which vanishes on the plane  $S$  and  $G_+$  for the Green's function such that  $\frac{\partial G_+}{\partial n} = 0$  on  $S$ , then

$$\left. \begin{aligned} G_- &= G_0 - G_i \\ G_+ &= G_0 + G_i \end{aligned} \right\} \quad (2.9)$$

where  $G_i$  is the Green's function for the image point  $x'$  in the boundary, that is

$$G_i(x, x') = \frac{1}{4\pi |x - x'|} \quad (2.10)$$



Hence we can find three equivalent forms of the solution for the pressure in equation (2.7) according to which Green's function  $G_0$ ,  $G_-$  or  $G_+$  is chosen. The particular choice must depend on what is known about  $p$  or  $\frac{\partial p}{\partial n}$  on  $S$ . But since we are solving for the pressure  $p$  on  $S$  it is convenient to choose  $G_+$ . Equation (2.7) then becomes

$$p(x, t) = \int_V q G_+ dV + \int_S G_+ \frac{\partial p}{\partial n} dS \quad (2.11)$$

But on  $S$  where  $x = x' = 0$  we have

$$\left. \begin{aligned} G_0 &= G_i \\ \frac{\partial G_0}{\partial n} &= - \frac{\partial G_i}{\partial n} \end{aligned} \right\} \quad (2.12)$$

and so equation (2.12) can be written

$$p(\mathbf{x}, t) = \int_V q(G_0 + G_1) dV + 2 \int_S G_0 \frac{\partial p}{\partial n} dS \quad (2.13)$$

Townsend (1956) has estimated that for the turbulent boundary layer

$$\overline{\left(\frac{\partial p}{\partial x_2}\right)^2}_{x_2=0} \approx 3.4 \times 10^{-4} \frac{\tau_w^3}{\rho_0 \nu^2} \quad (2.14)$$

and from Klebanoff's measurements (1954) shown in figure 2,  $\overline{u^2} \approx U_\tau^2$ , where  $U_\tau$  is the shear velocity  $\sqrt{\tau_w / \rho_0}$ . If  $\ell_1$  and  $\ell_2$  are typical lengths over which the fluctuating pressure and velocity are correlated respectively, then the contributions to the pressure from the volume and surface integrals in equation (2.13) are in the ratio

$$100 \ell_1^2 : \ell_2^2$$

Now we would expect  $\ell_2$  to be less than  $\ell_1$  and certainly not large compared with  $\ell_1$ , because of the effect of weighting of the Green's function (experiment has confirmed they are of the same order) so that the surface integral in equation (2.13) can be neglected in agreement with the approximation made by Kraichnan (1956b).

We may therefore write equation (2.13) as

$$p(\mathbf{x}) = \frac{\rho_0}{2\pi} \int_V \left( \frac{1}{|\mathbf{x}-\mathbf{y}|} + \frac{1}{|\mathbf{x}-\mathbf{y}'|} \right) \bar{e}_{12}(\mathbf{y}) \frac{\partial u_2}{\partial x_1}(\mathbf{y}) d\mathbf{y} \quad (2.15)$$

where the time variable  $t$  has been omitted for writing convenience and the source function  $q$  is now written in full.

On the boundary  $\underline{x} = (x_1, 0, x_3)$  the fluctuating wall pressure is given by

$$p(x_1, 0, x_3) = \frac{\rho_0}{\pi} \int_{y_2 > 0} \frac{\bar{e}_{12}(\underline{y}) \frac{\partial u_2}{\partial x_1}(\underline{y})}{|\underline{x} - \underline{y}|} dy \quad (2.16)$$

The pressure covariance between the wall pressures at the points  $\underline{x}$  and  $\underline{x}'$  respectively is

$$P(\underline{x}; \underline{x}')_{\substack{x_2 = 0 \\ x'_2 = 0}} = \frac{\rho_0^2}{\pi^2} \iint_{\substack{y_2 > 0 \\ y'_2 > 0}} \frac{\bar{e}_{12}(\underline{y}) \bar{e}_{12}(\underline{y}')}{|\underline{x} - \underline{y}| |\underline{x}' - \underline{y}'|} \frac{\partial^2}{\partial y_1 \partial y'_1} R_{22}(\underline{y}; \underline{y}') d\underline{y} d\underline{y}' \quad (2.17)$$

where  $R_{22}(\underline{y}; \underline{y}')$  is the two-point lateral velocity covariance, that is

$$R_{22}(\underline{y}; \underline{y}') = \overline{u_2(\underline{y}) u_2(\underline{y}')} \quad (2.18)$$

It should be noted that in equation (2.17) the pressures and velocities are evaluated at the same times in a fixed co-ordinate system. Later it will be necessary to introduce time delay and use a moving frame of reference.

The great difficulty in evaluating equation (2.17) is that of specifying the velocity field, that is  $R_{22}$  in this case. This difficulty, although common to all turbulence problems, is made worse by the fact that the turbulence in shear flows is both inhomogeneous

and anisotropic. Usually an artificial model for the turbulence must be adopted whereby the velocity field is described in terms of the simpler forms for the isotropic case.

The presence of the  $\frac{1}{|\tilde{x} - \tilde{y}|}$  weighting function in equation (2.16) shows that the pressure at the wall must depend essentially on local velocity fluctuations around it. Now the scale of the velocity fluctuations is of order the boundary-layer thickness  $\delta$ . It follows that the turbulence can certainly be assumed homogeneous in the  $x_1$  and  $x_3$  directions since the boundary-layer growth for zero external pressure gradient is so slow. This means that  $\bar{e}_{12}$  and  $\bar{u}_2$  are functions of the normal direction  $y_2$  only and also that the wall pressure covariance  $P$  is a function of the separation distance  $\tilde{x}' - \tilde{x} = \xi$ . Equation (2.17) may therefore be rewritten as

$$P(\xi_1, 0, \xi_3)_{\tilde{x}_2 = 0} = \frac{\rho_0^2}{\pi^2} \int_{y_2 > 0} \int_{y_2' > 0} \frac{\sqrt{\bar{u}_2^2(y_2)} \sqrt{\bar{u}_2^2(y_2')} \bar{e}_{12}(y_2) \bar{e}_{12}(y_2')}{|\tilde{x} - \tilde{y}| |\tilde{x}' - \tilde{y}'|} \cdot \frac{\partial^2}{\partial y_1 \partial y_1'} \bar{R}_{22}(\tilde{y}; \tilde{y}') \quad (2.19)$$

where  $\bar{R}_{22}$  is the non-dimensional lateral velocity correlation coefficient, that is

$$\bar{R}_{22}(\tilde{y}; \tilde{y}') = \frac{\bar{u}_2(\tilde{y}) \bar{u}_2(\tilde{y}')}{\sqrt{\bar{u}_2^2(\tilde{y})} \sqrt{\bar{u}_2^2(\tilde{y}')}} \quad (2.20)$$

If  $\bar{R}_{22}$  and its derivatives with respect to  $y_1$  and  $y_1'$  vanish

at infinity two integrations by parts of equation (2.19) lead to

$$P(\xi_1, 0, \xi_3)_{x_2=0} = \frac{E_0^2}{\pi^2} \int_{-\infty}^{\infty} dy_1 \int_0^{\infty} dy_2 \int_{-\infty}^{\infty} dy_3 \int_{-\infty}^{\infty} dr_1 \int_{-\infty}^{\infty} dr_2 \int_{-\infty}^{\infty} dr_3 \bar{R}_{22}(y_2; r) \cdot \sqrt{u_2^2(y_2)} \sqrt{u_2^2(y_2+r_2)} \bar{e}_{12}(y_2) \bar{e}_{12}(y_2+r_2) \frac{\partial}{\partial y_1} \frac{1}{|\underline{x}-\underline{y}|} \frac{\partial}{\partial r_1} \frac{1}{|\underline{x}'-\underline{y}-\underline{r}|} \quad (2.21)$$

where  $\underline{r} = \underline{y}' - \underline{y}$  is the separation between the two points in the velocity field.

The integrations over  $y_1$  and  $y_3$  have been performed in Appendix A. From equation A.14.

$$P(\xi_1, 0, \xi_3)_{x_2=0} = \frac{2\rho_0^2}{\pi} \frac{\partial}{\partial \xi_1} \int_0^{\infty} dy_2 \int_{-\infty}^{\infty} dr_2 \int_{-\infty}^{\infty} dr_1 \int_{-\infty}^{\infty} dr_3 \bar{R}_{22}(y_2; \underline{r}) \frac{\sqrt{u_2^2(y_2)} \sqrt{u_2^2(y_2+r_2)} \bar{e}_{12}(y_2) \bar{e}_{12}(y_2+r_2) (\xi_1 - r_1)}{\sqrt{(2y_2+r_2)^2 + (\xi_1 - r_1)^2 + (\xi_3 - r_3)^2} \left[ 2y_2+r_2 + \sqrt{(2y_2+r_2)^2 + (\xi_1 - r_1)^2 + (\xi_3 - r_3)^2} \right]} \quad (2.22)$$

which is the important result of this section.

Equation (2.22) shows that the covariance of the fluctuating wall pressure depends on the variation of the mean shear, the lateral turbulent intensity and the two point lateral velocity correlation coefficient  $\bar{R}_{22}(y_2; \underline{r})$ , which are all functions of the co-ordinate  $y_2$ , modified by a rather complicated weighting function. Equation (2.22) is an exact result for the turbulence-mean shear contribution to the wall pressure covariance with the assumptions of homogeneity in the  $x_1$  and  $x_3$  directions and that the surface integral in equation

(2.13) is very small.

## 2.2 The root-mean-square wall pressure

If the differentiation with respect to  $\xi_1$  is performed in equation (2.22), then putting  $\xi_1 = \xi_3 = 0$  an expression for the mean-square pressure at the wall results.

$$\begin{aligned} \bar{p}^2 = P(0, 0, 0) = \frac{2\rho_0^2}{\pi} \int_0^\infty dy_2 \int_{-y_2}^\infty dr_2 \int_{-\infty}^\infty dr_1 \int_{-\infty}^\infty dr_3 \bar{R}_{22}(y_2; \mathbf{r}) \\ \cdot \frac{\sqrt{u_2^2(y_2)} \sqrt{u_2^2(y_2 + r_2)} \bar{e}_{12}(y_2) \bar{e}_{12}(y_2 + r_2)}{\sqrt{(2y_2 + r_2)^2 + r_1^2 + r_3^2} \left[ 2y_2 + r_2 + \sqrt{(2y_2 + r_2)^2 + r_1^2 + r_3^2} \right]} \\ \cdot \left[ 1 - \frac{r_1^2 \left( 2 + \frac{2y_2 + r_2}{\sqrt{(2y_2 + r_2)^2 + r_1^2 + r_3^2}} \right)}{\sqrt{(2y_2 + r_2)^2 + r_1^2 + r_3^2} \left[ 2y_2 + r_2 + \sqrt{(2y_2 + r_2)^2 + r_1^2 + r_3^2} \right]} \right] \end{aligned} \quad (2.23)$$

It remains to specify the mean shear, turbulent intensity and velocity correlation. These quantities are taken from existing measurements.

### a) The mean shear,

Townsend (1956) has described in detail the variation of mean velocity across the boundary layer. Next to the wall is a very thin region where viscous forces predominate. Over this viscous region the mean velocity profile is described by

$$\frac{U}{U_T} = \frac{U_T y_2}{\nu} \quad (2.24)$$

for  $\frac{U_\tau y_2}{\nu} < 5$ , where  $U_\tau$  is the friction velocity  $\sqrt{\frac{\tau_w}{\rho_0}}$

It is convenient to describe the boundary layer profile (other than the thin viscous layer) by the non-dimensional forms given by Townsend (1956), that is

$$K \frac{(U_1 - U)}{U_\tau} = - \ln(y_2/\delta_0) + 1.4 \quad (2.25)$$

where  $K$  is the Von-Karman constant = 0.41 and  $\delta_0$  is the value of  $y_2$  at which  $U_1 - U = U_\tau$ . This logarithmic law describes the measured profiles of many experimenters over the range  $0.01 < y/\delta_0 < 0.3$ . Over the region  $y/\delta_0 > 0.3$  the profile is closely fitted by

$$K \frac{(U_1 - U)}{U_\tau} = 4(1 - \operatorname{erf} 1.13 y_2/\delta_0) \quad (2.26)$$

where  $\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-s^2) ds$  is the error function.

So for the region described by equation (2.25) the mean shear is

$$\bar{e}_{12}(y_2) = \frac{1}{K} \frac{U_\tau}{y_2} \quad (2.27)$$

and from equation (2.26) for the region  $y_2/\delta_0 > 0.3$

$$\bar{e}_{12}(y_2) = \frac{8.94}{K} U_\tau \exp - \left( \frac{1.13 y_2}{\delta_0} \right)^2 \quad (2.28)$$

b) The turbulent intensity  $\sqrt{\overline{u^2}}_2$

The turbulent intensities in the two-dimensional boundary layer on a flat plate with zero external pressure gradient have been measured by Klebanoff (1954). Klebanoff's results have been

reproduced in fig. 2. The intensity  $\sqrt{u_2^2}$  is proportional to the friction velocity  $U_\tau$ .

The product of the mean shear, given by equations (2.25) and (2.26), and the turbulent intensity  $\sqrt{u_2^2}$  is plotted non-dimensionally against  $y_2/\delta_0$  in fig. 1. A logarithmic scale for the abscissa has been chosen only for convenience. It is incorrect to deduce at this stage that because this curve peaks very close to the wall at  $y_2/\delta_0 = 0.003$  that the major contribution to the wall pressure comes from around this region. The variations of  $\bar{R}_{22}(y_2; \underline{r})$  and the weighting function with  $y_2$  have still to be taken into account.

c) The lateral velocity correlation coefficient  $\bar{R}_{22}(y_2; \underline{r})$

As was mentioned earlier the inherent difficulty of the problem is now encountered in that because of the inhomogeneity and anisotropy of the flow there is no simple description of the velocity field, in this case  $\bar{R}_{22}(y_2; \underline{r})$ .

Grant (1958) has measured the velocity correlation coefficients  $\bar{R}_{11}(\underline{r})$ ,  $\bar{R}_{22}(\underline{r})$  and  $\bar{R}_{33}(\underline{r})$  at various values of  $y_2/\delta_0$  for the turbulent boundary layer. The measurements of  $\bar{R}_{22}(\underline{r})$  are reproduced in fig. 4.

The strong anisotropy results from the large scale motion of the turbulence, that is from the motion of the big eddies described by Townsend (1956). From Grant's measurements Hodgson and Lilley (1960) have given a relatively simple description of the big eddies in the inner region of the boundary layer that agrees with the

physical representation given by Townsend (1957). However, an exact analytical form that describes the variation of the velocity correlation coefficient across the whole layer has at the moment not been obtained.

Although this variation of  $\bar{R}_{22}(y_2; r)$  with  $y_2$  is so difficult to describe, Townsend (1956) has pointed out that over the constant stress region the variation should be of the form

$$\bar{R}_{22}(y_2; r) = \bar{R}_{22}(r/y_2) \quad (2.29)$$

To a first approximation and for simplicity in the analysis we will take

$$\bar{R}_{22}(r) = e^{-r^2/L^2} \quad (2.30)$$

where  $L$  is a measure of the integral scale.

It may be noted here that the form (2.30) for the velocity correlation coefficient, which is used again in later calculations, is not a good fit to the measured velocity correlations of Grant nor is its transform a good fit to the spectra measured by Klebanoff (reproduced in figure 3). A much better fit is  $e^{-r/L}$  but with this form the analysis is more difficult. Examination of Klebanoff's one-dimensional wave-number spectra of  $\bar{u}_2^2$  shows that a reasonable fit with the form  $e^{-r^2/L^2}$  is obtained if the scale  $L$  is chosen to be twice the value given by the better form  $e^{-r/L}$ .

In view of equation (2.29) and from the measurements of Townsend (1956) and Grant (1958) we will take

$$L = y_2 \quad (2.31)$$

This is a reasonable approximation over the region extending out to  $y_2/\delta = 0.6$ . For values of  $y_2$  greater than this the approximation (2.31) overestimates the scale but, as will be shown, the contribution from this outer region is negligible.

With the form

$$\bar{R}_{22}(y_2; r) = e^{-r^2/y_2^2} \quad (2.32)$$

the integrations over  $r_1$  and  $r_2$  in equation (2.23) can be performed to give

$$\begin{aligned} \bar{p}^2 = & 2\rho_0^2 \int_0^\infty dy_2 \int_{-y_2}^\infty dr_2 \sqrt{u_2^2(y_2)} \sqrt{u_2^2(y_2+r_2)} \bar{e}_{12}(y_2) \bar{e}_{12}(y_2+r_2) \\ & \cdot \left\{ 1 - \sqrt{r} \exp\left[\left(2+r^2/y_2\right)^2\right] \left[1 - \operatorname{erf}\left(2+r_2/y_2\right)\right] \left[2+r_2/y_2\right] \right\} \exp\left(-\frac{r_2}{y_2}\right)^2 \end{aligned} \quad (2.33)$$

Using the product of mean shear and turbulent intensity variation of fig. 1, equation (2.33) has been evaluated numerically to give the root-mean-square pressure as

$$\frac{\sqrt{\bar{p}^2}}{\frac{1}{2}\rho_0 U_1^2} = 2.56 C_f \quad (2.34)$$

The analytical complication of equation (2.23) and the difficulty in specifying  $\bar{R}_{22}(y_2; r)$  has prevented an exact calculation of the mean square pressure being made. Nevertheless it is possible to demonstrate, from the Green's function solution equation (2.23), an important property of the pressure field suggested by the average

convection speed measurements referred to in the introduction and to be described later.

It is expected that all parts of the fluctuating velocity field across the layer contribute in some proportion to the fluctuating pressure at the wall. The space-time correlation measurements show that the pressure field is on the average being convected past the fixed measuring station at a speed of approximately  $0.8 U_1$ . This velocity corresponds to a region  $y_2/\delta \approx 0.25$  in the boundary layer and so it may be deduced that there are considerable contributions to the wall pressure by the fluctuating velocity field from regions at quite large distances from the wall. This is clearly verified on examination of equation (2.23).

In equation (2.23) for the mean-square wall pressure the  $y_2$  and  $r_2$  integrations are such that the integral is essentially the square of the 'mean-shear times the turbulent intensity' curve of fig. 1 multiplied by the velocity correlation coefficient  $e^{-r_2^2/L^2}$  (where  $L$  is zero at the wall and large at the outer edge of the layer) and also by a weighting function of the form  $1/(2y_2 + r_2)^2$ .

Since the  $r_2$  integration is over the range  $-y_2$  to infinity the weighting function has a maximum value of  $1/y_2$  at the lower limit  $r_2 = -y_2$ . The velocity correlation coefficient has a maximum value of unity at  $r_2 = 0$ , but gives a contribution over the largest range of  $r_2$  when  $L$  is large. The combination of these facts means that near the wall, where although the mean shear is

largest and the weighting function is large, the turbulent intensity is zero and the scale  $L$  is zero so that the contribution to the wall pressure is negligible. At the edge of the layer, although the scale is large, the intensity and mean shear are zero and the weighting function is small. The contribution to the wall pressure thus falls very rapidly to zero towards the outer part of the layer.

The numerical integration of equation (2.33) showed clearly that the major contribution to the wall pressure came from the region  $y_2/\delta = 0.02$  to  $0.4$ . There was a negligible contribution above  $y_2/\delta = 0.6$ .

The result of equation (2.34) should actually be reduced by a small factor to allow for anisotropy of the velocity correlation  $\bar{R}_{22}(r)$  in the  $r_3$  direction. The result is in reasonable agreement with the experimental value of  $2.2 C_f$ . Further refinement of the calculation is not justified in view of the inadequate representation of  $\bar{R}_{22}(y_2; r)$ .

Due in part to the analytical difficulties but mainly to the difficulty in describing the fluctuating velocity field, it is concluded that great emphasis must be placed on experimental investigations. The calculations from the Green's function solution have shown that  $\sqrt{\overline{v^2}}/\frac{1}{2}\rho U_1^2$  is of order  $2 C_f$  and have demonstrated the effect of the weighting of the velocity fluctuations away from the wall.

## 2.5 The pressure covariance

For the reasons just mentioned the solving of equation (2.22) for the pressure covariance is too difficult. In section 5 it will be shown that a Fourier transform solution for the pressure yields a simpler form for the pressure covariance which will enable an approximate calculation to be made.

Before passing on to the experimental investigation it is worth noting at this stage a very powerful and most important boundary condition that exists on the form of the pressure covariance in incompressible flow over plane boundaries. This condition was first demonstrated by Phillips (1954) and is reproduced here.

The continuity condition equation (2.1) for the incompressibility of the fluid at the point  $(x)$  is multiplied by the fluctuating pressure  $p(x')$  at the point  $x'$  and the mean value taken then

$$\frac{\partial}{\partial x_i} \overline{u_i(x)p(x')} = 0 \quad (2.35)$$

The covariance of the pressure at  $x'$  with quantities at  $x$  falls to zero faster than  $\frac{1}{|x' - x|^2}$  then integration of (2.35) over the plane  $x_1, x_3$  gives

$$\frac{\partial}{\partial x_2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{u_2(x)p(x')} dx_1 dx_3 = 0 \quad (2.36)$$

Integrating equation (2.36) over  $x_2$  we have

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{p(\underline{x}') u_2(\underline{x})} dx_1 dx_3 = \phi(\underline{x}') \quad (2.37)$$

But since the incompressible fluid extends to infinity the turbulent intensity vanishes at infinity and so at  $x_2 = \infty^*$ ,  $\overline{p(\underline{x}') u_2(\underline{x})} = 0$  for all  $x_1, x_3$  and all finite  $\underline{x}'$ , so  $\phi(\underline{x}') = 0$  and

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{p(\underline{x}') u_2(\underline{x})} dx_1 dx_3 = 0 \quad (2.38)$$

Several corollaries of this result follow:

$$\left. \begin{aligned} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{p(\underline{x}') \frac{\partial u_2}{\partial x_2}(\underline{x})} dx_1 dx_3 &= 0 \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{p(\underline{x}') \frac{\partial^2 u_2}{\partial x_2^2}(\underline{x})} dx_1 dx_3 &= 0 \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{p(\underline{x}') \frac{\partial u_2}{\partial t}(\underline{x})} dx_1 dx_3 &= 0 \end{aligned} \right\} \quad (2.39)$$

If now the equation of motion (2.2) in the  $x_2$  direction is

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\* It must be stressed that this is an incompressible boundary condition. When the fluid is compressible  $u_2(\underline{x})$  at large  $x_2$  will be the perturbation velocity of the sound waves generated 'aero-dynamically' by the turbulence. The quantity  $\overline{p(\underline{x}') u_2(\underline{x})}$  will now be finite, being the correlation of the sound field with the field of turbulence producing it.

multiplied by  $p(\underline{x}')$  and the mean value taken we have

$$-\frac{1}{\rho_0} \frac{\partial p}{\partial x_2} \overline{p(\underline{x})p(\underline{x}')} = \overline{p(\underline{x}') \frac{\partial u_2}{\partial t}(\underline{x})} + \frac{\partial}{\partial x_1} \overline{p(\underline{x}') u_1(\underline{x}) u_2(\underline{x})} - \nu \nabla^2 \overline{p(\underline{x}') u_2(\underline{x})} \quad (2.40)$$

By integrating equation (2.40) over the  $x_1, x_3$  plane, which is also the plane boundary  $x_2 = 0$ , and using equation (2.39), and since as before  $\phi(\underline{x}') = 0$  at infinity and also  $u_2 = 0$  on the wall, it follows that at the wall

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{p(\underline{x}') p(\underline{x})} dx_1 dx_3 = 0 \quad (2.41)$$

that is

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{P(\xi_1, 0, \xi_3)} d\xi_1 d\xi_3 = 0 \quad (2.42)$$

The same result has been obtained by Kraichnan (1956b).

A direct consequence of equation (2.42) is that the pressure covariance must take significant negative values.

An alternative form of equation (2.42) may be obtained in terms of the power spectral density of the wall pressure in wave-number space. If the two-dimensional Fourier transform of the pressure covariance is

$$(\underline{x}_2 = 0; k_1, k_3) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{P(\xi_1, 0, \xi_3)}_{x_2=0} e^{i \underline{\xi} \cdot \underline{k}} d\xi_1 d\xi_3 \quad (2.43)$$

where  $\underline{k}$  is the two-dimensional wave-number  $(k_1, k_3)$  then

$$\Pi(0; \underline{k})_{\underline{k}=0} = 0 \quad (2.44)$$

that is the two-dimensional power spectral density of the wall pressure fluctuations in wave-number space must vanish at zero wave number  $k$ . It will be shown in section 7 that this result is of great significance in the estimation of the radiated noise using wall pressure measurements. In particular one of the subsidiary results obtained by Phillips in a similar way to equation (2.41) is important, namely that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial p(x)}{\partial t} \frac{\partial p(x')}{\partial t} dx_1 dx_3 = 0 \quad (2.45)$$

on a plane boundary.

### 3. Experimental Technique and Measuring Equipment.

Before describing the pressure transducers and measuring technique employed in the experiments, it is worth noting the order of magnitude of the fluctuating wall pressure to be measured relative to ambient sound pressure levels that were likely to be encountered; for, like the ear, the transducer could not differentiate between the pressure fluctuations due to essentially local turbulent velocity fluctuations and those of sound radiated from extraneous sources far away.

From section 2,  $\sqrt{p^2}/\frac{1}{2}\rho U_1^2 \approx 2 C_f$  for the wall pressure fluctuations in a turbulent boundary layer; so for a skin-friction coefficient  $C_f = 0.003$  say, which was a typical value for the Reynolds number range of the present experiments,  $\sqrt{p^2}/\frac{1}{2}\rho U_1^2 \approx 0.006$ . For a wind-tunnel speed of 100 feet/second, say, the root-mean-square wall pressure is therefore approximately 0.07 lbs/sq. foot, that is 15  $\mu$ b. (1 microbar = 1 dyne/sq. cm.) Expressed as a ratio relative to the usual sound intensity datum of 0.0002  $\mu$ b, the turbulent pressure fluctuation level is 98 decibels.\*

Unfortunately this intensity was not much greater than the extraneous background noise level in the usual fan-driven wind-tunnel. The overall 'signal to noise' ratio in this type of

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\* Intensity in decibels =  $20 \log_{10} \frac{\sqrt{p^2}}{\text{datum}}$

tunnel is therefore likely to be small. Furthermore the power spectral density of noise in a fan-driven tunnel usually has significant value at low frequencies of order 20-200 c/s. Even if the energy contained by these low frequencies is small, it can still prevent reliable spectrum measurements of the wall pressure being made, that is the 'signal to noise' ratio at low frequencies in filtered frequency bands can be less than unity although the overall 'signal to noise' ratio is acceptable.

Hence in all turbulent pressure fluctuation measurements great care must be exercised owing to the possible interference of noise and vibrations from fans, motors, compressors, diffusers, engines or any other extraneous sources. If the noise cannot be reduced to a small level then an attempt must be made to allow for it in the experiments.

The recognised unit of pressure measurement used in acoustics is the decibel, a logarithmic unit. Most acoustic apparatus is calibrated in integers of this unit because an accuracy of one decibel (11%) is usually sufficient. However, the variation of skin-friction coefficient  $C_f$  in the present experiments was only from a value 0.0025 to 0.0035. The decibel, as a unit of measurement, was considered to be too coarse to detect variations of pressure intensity with skin-friction coefficient and so in all measurements a linear scale calibration was preferred.

The overall calibration accuracy of the apparatus used in measuring the wall fluctuating pressures was of order 1-2%. Calibrations were checked frequently throughout the experiments. The maximum possible bandwidth of equipment, without sacrificing good signal to electrical noise ratio, was always demanded. Corrections were made for frequency response characteristics which varied over the operating range.

### 3.1 The Pressure transducers

In acoustic pressure measurement the wavelength of sound is usually greater than the diameter of the microphone. For instance at a frequency of 1 kc/s the wavelength  $\frac{\text{speed of sound}(1100\text{ft/sec})}{\text{frequency}}$  = 1.1 feet. Only at the higher audible frequencies is the wavelength comparable with the microphone size, in which case pressure 'doubling' or cancellation can occur according to whether the sound waves are normal to or graze the diaphragm. But the wavelength of the wall pressure fluctuations is of order  $\frac{\text{mean flow velocity}}{\text{frequency}}$  and can be of order 0.1 in. which may be small relative to the size of the pressure transducer diaphragm. When this occurs cancellation of the pressure signals take place on the transducer face. Hence in order to obtain the maximum resolving power the transducer should be as small as possible. Unfortunately small diameter usually means low sensitivity.

Two types of pressure transducer were used in the experiments,

both were of commercial manufacture and are shown in figure 5. Most of the measurements were taken with two ammonium di-hydrogen phosphate crystal transducers type M213 (Massa Company, U.S.A.). The M213, as supplied, had a nominal diameter of 0.24 in. with the diaphragm retained by a ring such that the transducer face was recessed about 0.020 in.

It was discovered in early experiments that although this recess made little difference to the overall root-mean-square levels it did augment the low frequency spectral measurements. This was probably due to some type of secondary flow in the recess.\* Fortunately the manufacturers were able to machine the retaining ring away although the possibility of damage to the diaphragm through careless handling was enhanced. Figure 6 shows the final flush mounting that was attained.

The capacity of the crystal was 12pF. For extended low-frequency response the transducer was operated into a high input impedance circuit, in this case a special cathode-follower type M114B. The transducer sensitivity at the output of the cathode-follower was  $2.35\mu\text{V}/\mu\text{b}$  and the useful frequency response extended from 40 c/s to 50 kc/s (see Section 3.2).

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\* This insensitivity of the overall level to sufficiently small imperfections in the flush mounting of the transducer was further confirmation that there was negligible contribution to the wall pressure fluctuations from the viscous layer next to the wall as described in section 2.

In common with all piezo-electric crystal transducers the M213 was a very sensitive accelerometer and so it was essential to prevent vibration of the transducer. The method of mounting is shown in figure 7. The crystal was held in a heavy brass holder and a certain amount of isolation from high-frequency vibration resulted from the use of soft O-rings in the mounting. Experience showed that by far the best cure for cases where vibration was likely to cause trouble was to make the experimental rig as massive as possible rather than attempt to isolate the transducer completely from vibrations of the wall. With this principle in mind, and always keeping a very careful check on the possible occurrence of sinusoidal signals due to vibration, no extraneous signals that could be attributed to this cause were encountered.

The circuit of the M114B cathode-follower is reproduced in figure 10a. The input impedance was 200 Megohms with a flat frequency response from 3 c/s to 100 kc/s. The M213 transducer, when operated into the M114B cathode-follower, would respond to pressures of 5 atmospheres before non-linearity occurred. The cathode-follower was battery driven for low hum and low electrical noise. For the input short-circuited the electrical noise-level referred to the input of the cathode-follower was  $5\mu\text{V}$ . Below 300 c/s the electrical noise-level due to the transducer was  $25\mu\text{V}$ . At 1 kc/s the threshold was  $0.01\mu\text{b/c/s}$  and improved at the rate

of 6 db/octave. The M114B cathode-follower, also shown in figure 5, was of extremely rugged construction being shock-mounted inside a heavy steel case. The cathode-follower was connected to the transducer through a short length of low noise 'Microdot' flexible cable and was held in a heavy rubber-padded clamp stand.

As a reliable check on the influence of vibration on the M213 readings and also in order to discover the influence of transducer size on resolving power a condenser transducer type 4133 (Bruel and Kjaer, Denmark) was used, see figure 5. The transducer had a nominal diameter of 0.5 in. and was almost insensitive to vibration. The transducer was integral with its cathode-follower, the circuit of the latter is reproduced in figure 10b. The input impedance was 700 Megohms and the sensitivity at the cathode-follower output was  $0.93\text{mV}/\mu\text{b}$  with a useful frequency response from 1 c/s to 15 kc/s (see section 3.2)

The method of mounting that was used is shown in figure 9. The diaphragm of the transducer was louvred around the edge, however no spurious effects due to the secondary flow across the louvred edge were ever detected. The transducer could therefore be regarded as flush with the wall (see figure 30 for the mounting in the glider).

The BK4133 transducer could also be fitted with a small bore probe-tube. The transducer of Harrison (1958) used this type of

construction. The great advantage the probe-tube offers is good resolving power, provided that the Helmholtz resonances (which usually result in a poor response) can be suppressed. One method is to use a short probe-tube containing a small wad of porous damping material. However the same remarks made earlier in the introduction on the use of probe-type transducers in free shear flows must apply. There is again no convincing argument for their use in wall shear flows since secondary flows across the end of the probe might still occur. In fact, the use of a probe tube of 0.125 in. dia. with the BK4133 transducer was abandoned after spurious effects, similar to those found with the recessed M213 transducer, were experienced.

While awaiting modification of the M213 transducer, a transducer was constructed in the laboratory based on the design of Willmarth (1958). A slightly simpler form of construction was employed as shown in figure 8. As in Willmarth's transducer two crystals, 0.125 in. dia. of lead zirconate material (Brush Crystal Co.), were glued together using conducting silver paint (Johnson-Mathney). It was found essential to retain a small air gap around the crystals and to employ a diaphragm, 'Sellotape' being a convenient material for the latter. The air gap presented a high acoustic impedance to fluctuations reaching the sides of the crystals and so prevented a non-uniform frequency response. The sensitivity was  $2.5\mu\text{V}/\mu\text{b}$  and the frequency response was within  $\pm 30\%$

from 20 c/s - 100 kc/s. Several sets of results were obtained using these transducers which were in good agreement with the modified M213 readings. Since their calibration was inclined to vary and their response was not as level as the M213 transducer they were not used again in the experimental programme. The method of construction employed does however offer a relatively cheap transducer of wide bandwidth for pressure measurements in less exacting problems.

### 3.2 Transducer calibration

The BK4133 transducer was supplied with an accurate calibration response curve and is reproduced in figure 11. In the case of the M213, allowance had to be made for the attenuation through the connecting cable and cathode-follower since the calibration curve supplied by the manufacturers was the open-circuit transducer sensitivity. The BK4133 was calibrated by the manufacturers only a few weeks before the research programme was started and so the calibration was taken as a standard. The M213 was then calibrated against the BK4133. Sound from a 15 watt loudspeaker of frequency range 30 c/s - 5 kc/s was used. The M213 was placed about 12 in. from the loudspeaker and then replaced by the BK4133. The frequency range of calibration was restricted because of falling loudspeaker response. The calibration readings obtained for the M213 are shown in figure 11. The manufacturers open-circuit response curve has

then been superimposed at the calibrated low-frequency level. Finally, at the end of the programme, the BK4133 calibration was checked and found to be in agreement with another of the same type used in the Aerodynamics Division of the National Physical Laboratory.

Lastly, it should be noted that the M213 and BK4133 transducers were designed by the manufacturers for essentially acoustic work. The acoustic calibration curve for the M213 in figure 11 shows the effect of pressure doubling that occurred at 40 kc/s for normal incidence and shows a flat response for grazing incidence. The free-field response curve for the BK4133 was based on sound at random incidence. The calibration required for turbulent pressure fluctuation measurement is the so-called pressure response, where wave reinforcement effects are absent. This is shown in figure 11 for the BK4133 where it is seen that the makers' pressure response curve falls rapidly, after the frequency of 5 kc/s, as compared with the acoustic response. Similarly, due to the smaller diameter, the pressure response for the M213 would be expected to depart rapidly from the acoustic calibration at around 30 kc/s - 40 kc/s. At frequencies below 5-10 kc/s however the acoustic response was an accurate and convenient method of calibrating the pressure transducers against each other. Some workers have calibrated their transducers in a shock tube although the transient response is notoriously insensitive to

quite large departures from a level frequency response. Since the frequencies to be measured were never higher than 20 kc/s and in view of the excellent calibration curves supplied by the manufacturers over this range, it was felt that shock-tube calibration was unnecessary. (Recently the M213 transducers have been used in a shock-tube for pressure measurements and showed excellent transient characteristics).

Thus from the calibration curves the usable frequency response for turbulent pressure fluctuation measurement was from 50 c/s to around 50 kc/s for the M213 transducer and 1 c/s to 15 kc/s for the BK4133 transducer.

### 3.3. Mean-square pressure and spectral measurements.

The layout of the equipment for pressure mean-square and spectrum measurements is shown in figure 12. The electrical signals from the M213 or BK4133 cathode-followers were first amplified by a battery driven preamplifier with a gain of about 15 in order to maintain the excellent low noise and low hum characteristics of the cathode-followers. The circuit diagram is given in figure 10c. Two accurate attenuators were provided, the 50db attenuator at the input was necessary to prevent overloading when using the more sensitive BK4133 transducer. The output stage also contained a variable high-pass filter with 12db/octave slope and turn-over frequencies of 30 c/s, 60 c/s,

120 c/s and 240 c/s in case low-frequency spurious signals of large amplitude were present in the wide-band signal.

Final amplification of the signal was by a Solartron laboratory amplifier type AWS51A. The total gain of this amplifier was  $5 \times 10^4$  and the circuit incorporated an accurate attenuator of 20db at the input and a main attenuator of 60db in 1db steps. This amplifier was provided with two 600ohm external filter sockets to which was connected a set of one-third-octave filters with frequency range 40 c/s - 16 kc/s, manufactured by the Standard Telephones and Cable Company.

Electronic integration of the output signal to obtain the mean-square was preferred to the tedious and less accurate averaging by eye of the unsteady position of the pointer on the output meter of the Solartron amplifier. This was accomplished by using a Dawe true RMS valve voltmeter. This instrument, besides incorporating an accurate squaring circuit, produced a DC voltage of - 0.2V full-scale which was directly proportional to the mean-square signal. This voltage was integrated using a specially built electronic integrator described fully in Appendix C.

The overall frequency response of the cathode-follower plus amplifying chain was level from 20 c/s to at least 100 kc/s. The overall gain and response of the system was measured by injecting sinusoidal signals across an insert resistor in the M114B cathode

follower when using the M213 transducer, and in the case of the BK4133 by injecting the sinusoidal signals into the cathode-follower after first removing the transducer capsule.

The usual precautions when measuring random signals were always taken. Switching in and out alternately the numerous attenuators that were provided throughout the amplifier chain and observing the output signal guarded against 'clipping' of the signal. Checks were continually made to ensure that the integrating time was sufficient. Usually a time of 10 seconds was adequate, increasing the time making no difference to the mean-square reading, though in certain of the very unsteady low frequency measurements it was necessary to increase the integrating time to 100 seconds.

For the cross-power spectral density measurements two sets of one-third octave filters (Bruel and Kjaer, Denmark) of identical pattern, type 2107, were used. It was essential to employ two sets with the same filter response to avoid possible phase differences between signals because of the large phase distortion introduced in the filtering process.

### 3.4 Storage of data and time-delay

In order to provide storage of data for processing at a later date and also to provide time delay between signals from two transducers required for space-time correlation measurements,

a special correlator using a magnetic tape-recorder was built. The correlator bandwidth was DC-100 kc/s, covered in two ranges, with a maximum delay time of 25 milliseconds and a resolution of less than 1 microsecond. The construction of the correlator was one of the major tasks of the experimental work and is described in Appendix C.

### 3.5 Mean velocity profiles and skin friction measurement

The mean velocity profiles were measured with a 1 mm. dia. pitot-tube, flattened at the tip. The usual Young and Maas correction was applied near the wall.

The skin friction coefficient was determined from the mean velocity measurements using the well known 'law of the wall'

$$\frac{U}{U_\tau} = A \log_{10} \frac{U_\tau y_2}{\nu} + B$$

The values of the constants A and B chosen for the two-dimensional boundary-layer on a flat plate are due to Coles (1954) namely:

$$A = 5.75 \text{ and } B = 5.1$$

The constants A and B as found by Coles were average values obtained from experiments emphasizing the higher Reynolds numbers. At lower Reynolds numbers the constants A and B are known to be slowly varying functions of Reynolds number. In order to obtain consistent estimates of the skin friction coefficient Coles' values for the constants have been used throughout.

A Preston tube, using the flat-plate calibration of Bradshaw and Gregory (1961), was used to check the measurements of skin friction coefficient as obtained from the mean velocity profile measurements.

### 3.6 Hot-wire apparatus

The hot-wire anemometer is now a well-known device for measuring turbulent velocities and certainly at low speeds the techniques are quite standard. The equipment used in the present work will therefore only be briefly described.

For measuring the longitudinal and lateral turbulent velocity components, improved versions of the constant temperature anemometer described by Laurence and Landes (1953) were used. The amplifier bandwidth was DC-100 kc/s (-3db point). Full connection and operating details using two similar sets have been described by Sandborn and Slogar (1955).

The spatial correlation measurements of the longitudinal and lateral velocity components were made using two sets of constant current apparatus (Shapiro-Edwards, Pasadena, U.S.A.). The amplifier bandwidth (uncompensated) was 1 c/s - 1 Mc/s.

Tungsten wires of 0.0002 in. dia. were used. The ends of the wires were copper-plated in a jig using the method described by Strehlow (1949). A length of 0.080 in. was left unplated. The wires were soldered using silver solder and 'Coraline' flux to

fine gauge sewing needles.

The cross-wires for the lateral velocity measurements were angled at  $45^{\circ}$ . Strehlow's electroplating method was found to be very satisfactory since wire length was controlled to two or three thousandths of an inch for a batch of wires made at any one time. This allowed very good matching of the wires in the cross-wire probes.

All of the turbulence measurements presented were made in wall jet shear flows. For the intensity measurements the hot-wire probe was mounted on a micrometer traversing gear attached to the under-side of the wall. For the spatial correlation measurements the two probes were attached to a 'right-hand' and a 'left-hand' XYZ traversing gear (Prior and Co.) A great advantage of the wall jet was the ease with which the traversing gear could be mounted above the shear flow (see figure 41a).

#### 4. Turbulent boundary layer experiments.

##### 4.1 Boundary layer on a flat plate at zero incidence

A natural choice of test facility for the initial experiments was a flat plate at zero incidence since experimental data on the mean and turbulent velocities in the two-dimensional flat plate boundary layer were extensive and also an acceptable change in the skin friction coefficient along the plate could be obtained at usual laboratory Reynolds numbers.

##### 4.1.1 The test facility

A heavy tuftol flat plate of 1.5 in. thickness and 3 feet chord was mounted horizontally in the College of Aeronautics return-flow open-jet tunnel of  $3\frac{1}{2}$  ft. x  $2\frac{1}{2}$  ft. elliptical cross-section (see figure 13a). The leading edge of the plate was elliptical in shape and was of 8:1 axis ratio. Initially the trailing edge of the plate was left bluff but pressure fluctuations which were nearly sinusoidal, corresponding to the usual Strouhal number of  $\approx 0.2$  for the shedding frequency of vortices from bluff bodies, were sensed at the measuring stations on the plate and swamped the boundary layer pressure measurements. Indeed these signals were first attributed to vibration of the rig, however the sinusoidal disturbances vanished completely after ensuring that the flow left the trailing-edge smoothly by the addition of a sharp trailing-edge section.

The plate was mounted on a heavy steel framework and was fitted with side plates to prevent strong three-dimensional flow effects. The under-side of the plate served as the working surface and the transducer cathode-followers, which protruded above the plate, were protected from buffetting or from causing disturbances in the flow by enclosing them in a streamlined section. When using the M213 crystal transducer it was necessary to suspend the cathode-follower from a cantilever arm above the plate in order to avoid microphony caused by rig vibration. The cantilever arm and the girder framework rested on rubber pads which acted as insulation from vibrations of the laboratory floor caused by the fan-motor. The overall weight of the rig was nearly one ton and, provided that measurements were taken when reciprocating compressors also stationed in the laboratory were not operating, no trouble from rig vibration was experienced.

The maximum tunnel speed was 130 ft./sec.; the speed chosen for the tests was 67 ft./sec. since this represented a reasonable optimum between obtaining large signals due to the turbulent pressure fluctuations in the boundary layer and excessive tunnel noise. At this chosen speed natural transition occurred at approximately 21 in. from the leading edge of the plate. To realise a turbulent boundary layer over the whole chord of the plate a

0.030 in. diameter transition wire was attached 1.5 in. from the leading-edge. Fluctuating pressure measurements were taken at the three measuring stations shown in figure 13b, that is at 6 in., 18 in., and 30 in. from the leading-edge.

The mean pressure gradient which existed along the plate was small enough to neglect since the contribution to the wall pressure fluctuations of the ensuing mean velocity gradient terms would still be small compared with the mean shear.

#### 4.1.2. Discussion of results

The mean velocity profiles for the artificially-thickened turbulent boundary layer on the flat plate are plotted logarithmically in figure 14, which shows that the value of  $C_f$  varied from 0.0044 to 0.0033 over the measuring stations.

Initial spectral measurements of the wall pressure fluctuations showed that spurious large amplitude low frequency signals were present. Since these low frequency fluctuations were of the same order as those measured outside the tunnel (equivalent to a sound pressure level of  $\approx 90\text{db}$ ) it was deduced that they were mainly radiated noise from the fan and motor. As mentioned earlier in the introduction it is this extraneous noise, from the fan, motor and bearings etc., which makes the measurement of boundary layer pressure fluctuations over the full spectrum range in the usual fan-driven tunnel so difficult. However, useful

information was obtained from the experiment in the following simple and what is regarded as original way.

The incompressible velocity fluctuations and hence the local pressure fluctuations in a laminar boundary layer well before transition can be regarded as negligible. When the transition wire was removed so that flow over the forward measuring station 1 was completely laminar the signal measured by the pressure transducer was, therefore, either the radiated pressure fluctuations of the background extraneous noise in the tunnel or in the case of the M213 transducer possible acceleration pick-up from vibrations of the rig. A legitimate assumption was that the causes of the extraneous disturbances did not interact with the turbulent pressure fluctuations in the boundary layer so that the correlation between them was zero. The true power spectral density of the turbulent pressure fluctuations at the forward station 1 was therefore obtained by subtracting the mean-square signal for the laminar case from the mean-square signal with the transition wire attached. Also, since the wavelengths of the acoustic noise were of the same order as the plate size, it was further assumed that the acoustic noise correlated perfectly over the plate. The laminar signals measured at the forward station 1 were therefore taken as representative of the extraneous noise at the other measuring stations 2 and 3 and so the subtractive method was repeated to obtain the turbulent power spectral density at these stations. When using the M213

transducer because the turbulent signal levels were so low, part of the extraneous signal was electrical noise, of the transducer and cathode-follower.

The overall signal to 'noise' <sup>\*</sup> ratio was never more than 6 decibels. In filtered bands the signal to 'noise' ratio at the lower frequencies was sometimes less than unity, increasing to 10 decibels over the dominant energy containing frequency range. The special integrator described in Appendix C, with an integrating time of 100 seconds, enabled the subtractive method to be used for the intermittently occurring low-frequency components with reasonable accuracy.

The resulting corrected spectra at the three measuring stations are plotted non-dimensionally against the Strouhal number  $\frac{\omega \delta_1}{U_1}$  in figure 15 (where  $\omega$  is the circular frequency in radians/sec. and  $\delta_1$  is the boundary layer displacement thickness). The spectra show large scatter at low Strouhal numbers because of the poor signal to 'noise' ratio referred to above. Although not conclusive the spectra do appear to fall at low frequencies. The scatter of the results at low frequencies has negligible effect on the mean-square levels which were obtained by integrating the spectra of figure 15, that is

$$\frac{\overline{p^2}}{(\frac{1}{2}\rho U_1^2)^2} = \int_0^\infty \frac{\overline{p^2(\omega)} U_1}{(\frac{1}{2}\rho U_1^2)^2 \delta_1} d\left(\frac{\omega \delta_1}{U_1}\right) \quad (4.1)$$

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\* By 'noise' here we mean extraneous sound plus electrical noise.

where  $\overline{p^2}(\omega)$  is the mean-square reading in a filter band divided by the bandwidth of the filter.

During this initial part of the test programme Willmarth (1959) published measurements which showed the importance of the ratio of the transducer diameter  $d$  to the boundary layer thickness in determining the resolving power of the transducer. The variation of the non-dimensional root-mean-square pressure fluctuations with  $d/\delta$ , plotted in figure 16 shows this effect clearly. Furthermore the results also appear to demonstrate that there is a dependence on the skin friction coefficient  $C_f$ , the rms pressure decreasing with decreasing  $C_f$  for a given  $d/\delta$ , ratio. It is interesting to note that had only one transducer size been used at each measuring station, the effects of increasing rms pressure level due to the increased resolving power of the transducer as the boundary layer thickened would just about have balanced the decreasing effect of decreasing  $C_f$ , that is  $\sqrt{\overline{p^2}}/2\rho U^2$ , would have been measured as nearly constant. This fact is confirmed by the flight measurements with one transducer by Mull and Algranti (1960) on an aircraft fuselage, who found this ratio was a constant ( $\approx 0.005$ ) over the Reynolds number range  $0.76$  to  $1.5 \times 10^6$ . Hence when measuring boundary layer pressure fluctuations it is important to take account of the boundary layer thickness and the skin friction coefficient. Failure to have noted both of these effects may be the reason why some research workers have concluded that the non-dimensional rms

value of the wall pressure fluctuations was independent of Reynolds number.

Due in part to the large low-frequency content of the background noise level and to the relatively large transducer size which prevented close transducer spacing it was not possible to make detailed measurements of the spatial correlation of the wall pressure fluctuations. Nevertheless it was possible to measure the longitudinal space-time correlation coefficient of the wall pressure fluctuations

$$\overline{p(x_1, 0, 0; \tau)} \overline{p(x_2, 0, 0; \tau)} = \frac{\overline{p(x_1, 0, 0; \tau) p(x_1 + \xi_1, 0, 0; \tau + \tau)}}{\sqrt{\overline{p^2}(x_1, 0, 0; \tau)} \sqrt{\overline{p^2}(x_1 + \xi_1, 0, 0; \tau)}} \quad (4.2)$$

using two M213 transducers separated a distance  $\xi_1$  in the stream-wise direction and the time correlator described in appendix C. In forming the correlation coefficient of equation (4.2) it was necessary to filter the pressure signals with a 250 c/s high-pass filter incorporated in the preamplifiers to remove most of the extraneous low frequency components but still retaining the dominant energy containing frequency range. The results plotted in figure 17 show that the correlation coefficient for a given spacing reaches a maximum at a time delay  $\tau_m$  such that  $\xi_1/\tau_m \approx 0.81 U_1$ . An interpretation of this result is that on the average the pressure field may be regarded as being convected past the fixed measuring station at a speed of  $0.81 U_1$ . Further discussion of the space-time correlation is deferred until later. It should

be noted that the large negative loop in the autocorrelation of the wall pressure fluctuations in figure 17 (which is the space-time correlation for zero  $\xi_1$  spacing) is due to the presence of the high-pass filter.

It was also interesting to discover that at large transverse separation the spatial correlation of the wall pressure fluctuations was nearly constant with value  $\approx 0.16$ . The mean-square ratio of the background acoustic noise level in the tunnel to the total signal level also had this value and verified the assumption made in the data reduction that the extraneous noise present in the tunnel, except for the effects of reflection and refraction, should have correlated exactly on the plate because of the large wavelengths involved. It is suggested that this may be the probable reason why the transverse correlation of the wall pressure measured by Harrison (1958) in a wind-tunnel boundary layer (reproduced in figure 38 for later comparison) tended to a constant value of 0.2. The transverse correlation must, of course, tend to zero at large separation so a measurement of the transverse correlation was a reliable check on the presence of extraneous acoustic noise in the tunnel.

In conclusion it may be stated that although extraneous tunnel noise prevented reliable low-frequency spectral measurements and spatial correlation measurements of the turbulent boundary layer pressure fluctuations from being made, nevertheless valuable

experience with the flat-plate facility was obtained, in particular, the magnitude of accuracy required and the care needed to make accurate fluctuating pressure measurements, the degree of flushness required for the transducer mountings in the surface (referred to earlier in section 3.1), the importance of the resolving power of the transducer and the probable dependence on the skin friction coefficient.

#### 4.2 Boundary layer on $2\frac{1}{2}$ in. x $2\frac{1}{2}$ in. tunnel wall.

The flat-plate boundary layer measurements clearly demonstrated that if reliable low-frequency measurements of the fluctuating pressure were to be obtained a special facility with low acoustic noise level was needed. Since the major part of the extraneous noise in the flat-plate experiment was caused by the wind-tunnel fan it was preferable that the special facility should not be fan driven; a blow-down tunnel was therefore an obvious choice.

Willmarth (1958) used this type of facility which was continuously driven from a high pressure air supply but, as pointed out in the introduction, Willmarth's low frequency measurements were still restricted because the ambient noise level of the air supply was very high (119 decibels). Complete sound absorption of an air supply around frequencies of 100 c/s and below is very difficult. It was therefore decided to make a short-term modification to the College of Aeronautics  $2\frac{1}{2}$ in. x  $2\frac{1}{2}$ in.

supersonic intermittent vacuum tunnel for subsonic operation, using the same principles of Willmarth's design.

#### 4.2.1. The test facility.

The layout of the modified  $2\frac{1}{2}$ in. x  $2\frac{1}{2}$ in. tunnel is shown in figures 18a and 18b. Originally the air supply was drawn from a large capacity inflatable canvas bag filled with dry air. The air entered the tunnel working-section through a duct 60 feet in length and a contraction of 200:1 area ratio. Since the large contraction would act as an acoustic horn amplifying any extraneous sound present in the long entry duct, the contraction was replaced by a relatively short contraction of 21:1 area ratio designed from calculations by Lilley (1945). The contraction was constructed from laminated plywood formed on an accurate female jig. The highest Mach number chosen for subsonic operation was 0.4 and it was found advantageous, for a low noise level flow, to draw air directly from the atmosphere through a smooth entry and short settling length, containing two gauzes and a honeycomb, ahead of the contraction.

One of the glass windows of the original tunnel was replaced by a smooth tufnol block of 1.5 in. thickness and was used as the working surface. The tunnel diffuser was nearly 30 feet long and slowly expanded to a 1.5 feet square cross-section duct which then entered two vacuum tanks of 3600 cu.ft. capacity. The

tanks were exhausted by two reciprocating pumps which operated at 300 rpm with a capacity of 450 cu.ft/min. (for an inlet pressure of 1 atmosphere). The tunnel was started by opening a manually operated quick-acting valve in the diffuser entry.

Subsonic flow in the working section was controlled and stabilised by a sonic throat just downstream of the working section. The throat was two-dimensional and consisted of two circular-arc wooden blocks. A steel wedge of  $5^{\circ}$  semi-angle was placed in the throat to stabilise the normal shock wave.

No noise from the diffuser and tanks should, therefore, have been able to travel upstream into the working section. The vacuum tanks and most of the long diffuser were outside the laboratory hence the laboratory wall provided further insulation from diffuser and tank noise. The working section was insulated from vibrations induced by the flow in the tanks, diffuser and throat by a short parallel section. This was fabricated from smooth sheet rubber and was inserted immediately downstream of the working section. The tunnel, with its foundations set in concrete, was of sufficiently massive construction that no troubles from vibration were experienced.

Tests were performed at two speeds which were 116 ft./sec. and 456 ft./sec. at entry to the working section. As in Willmarth's facility there was a small flow acceleration along the working section. However, the longitudinal mean velocity gradient term was still small compared with the mean shear and so its influence on the wall pressure

fluctuations could be neglected. Static pressure holes were available along the top of the working section for the boundary layer profile measurements. Fluctuating pressure measurements were taken at the four measuring stations shown in figure 18b.

It was found that the capacity of the exhausting pumps was sufficient to run the tunnel continuously and this was done for the boundary layer profile measurements. However, although the pumps produced little noise in the tunnel working section (being situated in a different part of the laboratory) they were switched off and the tunnel was run intermittently for the fluctuating pressure investigations in order to ensure that the noise from the pumps interfered in no way with the experiments.

#### 4.2.2. Discussion of results

The boundary layer in the working section was found to be fully turbulent; the turbulent velocity profiles on the working surface centre-line are plotted logarithmically in figures 19a and 20a, where it is seen that the skin friction coefficient varied from 0.0036 to 0.0023 for the lower and higher tunnel speeds respectively.

The corner flows in the square cross-section tunnel would produce strong three-dimensional effects on the boundary layer, especially had the boundary layer flow completely filled the tunnel. However, since the boundary layer thickness was not

too large compared to the tunnel dimensions it would be expected that the boundary layer properties, as measured along a wall centre-line, would be affected only slightly by the corner flows.

The spectra and mean-square measurements.

Because natural transition occurred within the contraction it was not possible to employ the laminar boundary-layer technique used in the flat-plate experiment to obtain an estimate of the extraneous noise level in the tunnel. Instead, the background level was measured outside the tunnel near the tunnel entry. For the lower speed case the sound pressure level was 83 decibels and for the higher speed case 91 decibels. These levels meant that (in the worst case) the overall signal level of the turbulent wall pressure fluctuations to the external background noise level was 20 decibels and 28 decibels respectively for the two test speeds. In filtered bands this ratio was not less than 18 decibels in the lower frequency bands, increasing to at least 30 decibels at the higher frequencies for the lower speed case, and 26 decibels and 30 decibels respectively for the higher speed case.

The measured spectra are plotted non-dimensionally in figures 19b and 20b, where it is seen that for the low-speed case the spectral measurements were very scattered with large peaks at low Strouhal numbers and even in the higher speed case, where the signal to 'noise' ratio was apparently much better, the scatter was

still large at low frequencies although the spectra did appear to fall with decreasing frequency.

The detection and elimination of the causes of low frequency extraneous disturbances is a long and laborious business. The presented spectra results were obtained only after a considerable amount of time and patience had been expended. The exact reasons for the large scatter in the low frequency readings were not discovered, although considerable evidence as to the possible causes was provided from the experiments.

For instance, it was discovered that taking the gauzes out of the settling length or adding more gauzes made little difference to the low-frequency wall pressure measurements. Yet when the honeycomb was removed the higher-speed spectral measurements increased by 700% below Strouhal numbers of 0.02, producing spectra very like the results for the lower-speed case (fig. 19b). One suggestion for explaining this phenomenon would lie in the large eddy disturbances in the tunnel produced at entry, for it is these large 'entry eddies' which are broken-up by the honeycomb. The mechanism involved in these low-frequency pressure fluctuations might be described by the interaction of the large-scale velocity disturbances in the main stream with the velocity fluctuations in the boundary layer, that is through the 'turbulence-turbulence' terms. These terms involve sums and differences of the fluctuating velocity spectra and could give rise to the spurious low-frequency

readings although the detailed calculation of Appendix B for the turbulence-turbulence terms suggests that at low frequencies the power spectral density due to the boundary layer turbulence alone is well below the magnitudes measured in the tunnel.

A further suggestion is that the disturbances were acoustic in origin. As with Willmarth the acoustic noise level in the working section was not known. It was very probable that the level inside the tunnel working section was greater than outside due to the contraction acting as a horn and amplifying the sound, though as mentioned previously the relatively small 21:1 contraction ratio was chosen particularly to keep this effect small and the small dimensions of the tunnel should have limited such horn effects to higher frequencies. There is also the noise radiated from the boundary layer itself to be considered. From the discussion in section 7 of the measurements of Wilson (1959) the noise from the boundary layer is not likely to be radiated with an acoustic efficiency greater than  $0.3 \times 10^{-6.3}$ . For the lower-speed case, taking the working-section surface area as 3 sq.ft., this noise level in the tunnel should not be greater than 48 decibels and for the higher-speed case, 83 decibels. Furthermore, the dominant frequency range should occur near a Strouhal number  $\omega \delta / U_1 \approx 0.3$  which is well above the spectral readings which exhibited large scatter. It is therefore difficult to attribute the low-frequency measurements to sound from the

boundary layer in view of the fact that the sound intensity levels were three orders of magnitude below the turbulent pressure levels. For the same reason, interaction of the sound on the turbulence in the boundary layer must also have been negligible.

Space-time correlation and transverse correlation measurements, to be discussed in the following sections, did suggest that the low-frequency readings were the result of both turbulent disturbances and noise in the tunnel.

Although the spectral measurements plotted non-dimensionally did not collapse on a single curve at low Strouhal numbers, for Strouhal numbers greater than 0.1 the results were very similar to those obtained by Willmarth and showed quite clearly the effect of resolving power of the transducer on the high-frequency part of the spectra. As in the flat-plate experiments the scatter at low frequencies made negligible difference to the overall mean-square values and so these have been obtained by integration of the measured spectra. The non-dimensional root-mean-square wall pressure for the two test speeds is plotted against  $d/\delta$ , in figure 21. Again there is a noticeable change at a given  $d/\delta$  ratio for change in the skin friction coefficient  $C_f$ .

All further measurements in the  $2\frac{1}{2}$ in. x  $2\frac{1}{2}$ in. tunnel that are described were taken at the higher speed only, because of the improved signal to 'noise' ratio over the lower-speed case.

The space-time correlation of the wall pressure.

The space-time correlation coefficient of the wall pressure fluctuations  $\bar{P}(\xi, 0, 0; \tau)$ , defined in equation (4.2), was measured in the same way as the flat-plate experiments, that is using two M213 transducers and the special time correlator. The results for separation  $\xi$ , varying from 1.0 in. - 9.0 in. are plotted in figure 22.

The space-time correlation coefficient describes the way in which the longitudinal spatial distribution of the fluctuating pressure field is related to the time fluctuations of the field as seen by an observer in a fixed frame of reference in the wall. As before, the fact that for a given separation  $\xi$ , the time correlation curves reach a maximum at a time delay  $\tau_m$  say, known as the optimum time delay, such that  $\xi/\tau_m \approx 0.8 U_1$  may be interpreted as a measure of the average convection speed  $U_c$  of the pressure field as seen by the fixed observer.\* The slight increase in the measured convection speed with increase of separation distance was probably due to boundary layer growth and acceleration down the tunnel. It is important to realise that  $U_c$  is an average convection speed; the turbulence in the boundary layer is moving past the fixed measuring station at all speeds associated

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Other definitions of the average convection speed which, in general, differ only slightly from the above are discussed by Williams, J. (1960).

with the variation of mean velocity across the layer and, since it was shown in section 2 that the wall pressure fluctuations depend on a weighted integral of the fluctuating velocity field across the layer, so this convection speed  $U_0$  is a weighted average of the varying convection speeds of the turbulence across the layer.

This point is further demonstrated in figure 23 where the space-time correlation coefficient  $\bar{P}_{f_0}(\xi_1, 0, 0; \tau)$  measured in third-octave filter bands with centre frequencies 500 c/s, 2 kc/s, and 5 kc/s have been plotted for two separation distances, that is

$$\bar{P}_{f_0}(\xi_1, 0, 0; \tau)_{x_2=0} = \frac{P_{f_0}(x_1, 0, 0; t) P_{f_0}(x_1 + \xi_1, 0, 0; t + \tau)}{\sqrt{P_{f_0}^2(x_1, 0, 0; t)} \sqrt{P_{f_0}^2(x_1 + \xi_1, 0, 0; t)}} \quad (4.3)$$

where  $p_{f_0}(x_1, 0, 0; t)$  is the time varying pressure signal measured at station  $x = (x_1, 0, 0)$  in the filter band of centre frequency  $f_0$ . The correlation curves are sinusoidal in shape due to the filtering process but their amplitude rises to a maximum at the optimum time delay then decreases, due to the 'non-frozen' nature of the convected pressure field. The average convection speed was again determined from the optimum time-delay and the results from figure 23 and measurements at other frequencies are plotted against the Strouhal number  $\frac{\omega \delta}{U_1}$  in figure 25 from which it may be seen that there is no apparent change of average convection

speed with Strouhal number and is in agreement with the measurements of Harrison (1958) and Bull (1960). This was to be expected for the reasons mentioned in the previous paragraph, for contributions to the wall pressure fluctuations at any one frequency, as observed at the fixed measuring station, must come from a wide range of eddy sizes in the turbulence as it moves past the fixed station at mean speeds from zero at the wall to  $U_1$  at the outer edge of the layer although, as demonstrated in section 2, the integration across the layer for the wall pressure is weighted around the region  $0.02 < \frac{y}{\delta} < 0.5$ .

It was at first hoped that the measurements of the convection speed for the very low frequency signals might have supplied definite evidence as to the source of origin of these fluctuations. For instance, had these fluctuations been extraneous sound from the diffuser say, a convection speed near the speed of sound might have been measured. Therefore, since the convection speeds measured were around  $0.8 U_1$ , a considerable part of the low-frequency signals was probably the result of boundary layer or tunnel velocity disturbances rather than diffuser noise; but it should also be noted that although the sound radiated from the boundary layer has been estimated as small, nevertheless this near-field noise pattern would also be expected to be convected at a speed of the same order as the convection speed of the pressure field producing the noise, that is of order  $U_1$ .

The space-time correlation curves of figure 22 for the overall

pressure signals will now be examined more closely. The locus of the time correlation maxima is the autocorrelation of the time fluctuations of the pressure that would be measured in a frame of reference moving with the averaged convection speed  $U_0$ . These time fluctuations may be regarded as the 'true' time fluctuations of the pressure field, whereas the time fluctuations seen by a fixed observer are dominated by the spatial variations in the pressure field as it is swept past. From figure 22 it is seen that the autocorrelation in the convected frame of reference involves times much longer than those of the autocorrelation in fixed axes (i.e. the curve for  $\xi_1=0$ ). That is the frequencies as measured in the fixed frame are an order of magnitude higher than in the convected frame. When the 'true' time fluctuations are neglected the flow pattern is sometimes described as 'frozen'.

These conclusions regarding the convected pressure field are exactly those drawn by many workers from measurements of the velocity field, see for instance the space-time velocity correlation measurements of Favre et al. (1958) for grid turbulence reproduced in figure 39. The assumption that the time fluctuations in the fixed frame could be very nearly identified with the spatial fluctuations was first put forward by Taylor (1938) for grid turbulence.

It should be noted that the relation between the spatial and

time fluctuations is not quite so simple, for from figure 22 it can be seen that not only do the time correlation maxima decrease in amplitude with increasing separation  $\xi_1$ , but the widths of the time correlation curves also increase. This broadening of the time correlation curves with increasing spatial separation  $\xi_1$  may be interpreted as showing that the fluctuations that correlate at large separations are of low frequency and vice-versa, that is the smaller eddies are decaying more rapidly than the larger eddies, as is well known for many turbulent flows (see Townsend 1956). Willmarth (1959b) attempted to use the simple 'frozen' pattern hypothesis to correct the measured spectra of the wall pressure fluctuations for the finite resolving power of the transducers. It was this complicated inter-relationship between the spatial and time fluctuations (also coupled with the complicated weighting of the contributions across the layer) which prevented Willmarth from performing this calculation accurately.

An alternative method of presenting the same space-time relation, and which has been measured by Harrison (1958), is obtained by considering the time Fourier transform  $P(\xi_1, 0, 0; \omega)$  of the longitudinal space-time correlation given by

$$P(\xi_1, 0, 0; \tau)_{x_2=0} = \int_{-\infty}^{\infty} P(\xi_1, 0, 0; \omega)_{x_2=0} e^{-i\omega\tau} d\omega \quad (4.4)$$

where  $P(\xi_1, 0, 0; \omega)$  is in fact the cross power spectral density of the wall pressure fluctuations at two points  $\xi_1$  apart and, as defined, is a complex quantity.

Harrison showed that the real and imaginary parts of  $P(\xi_1, 0, 0; \omega)$  could be obtained by correlating the two pressure signals in filter bands with zero time delay for the real part and then with  $\pi/2$  phase difference, that is with a time delay  $\frac{1}{4f_0}$  where  $f_0$  is the centre frequency of the filter, to obtain the imaginary part. These quantities, when non-dimensionalised, correspond exactly to the space-time correlation coefficient in a filter band defined in equation (4.3) and plotted in figure 23, but now with time-delay  $\tau = 0$  or  $\tau = \frac{1}{4f_0}$ . These two quantities  $\bar{P}_{f_0}(\xi_1, 0, 0; 0)$  and  $\bar{P}_{f_0}(\xi_1, 0, 0; \frac{1}{4f_0})$  have been plotted in figure 24 against the parameter  $f_0 \xi_1 / U_c$  with  $U_c = 0.8 U_1$ , as in Harrison's case.

If the pressure field was convected downstream as a 'frozen' pattern then the cosine and sine curves obtained in figure 24 for the real and imaginary parts of the cross-spectral density would continue to oscillate with unit amplitude (although as pointed out by Harrison the curves must first be corrected for the effect of the finite bandwidth of the filters, this correction factor is also shown in figure 24). However, the curves again demonstrate that the pattern is not completely frozen since the decay of the curves with increase of the parameter  $\frac{f_0 \xi_1}{U_c}$  may be regarded as a measure

of the spatial decay of the pressure field with frequency. This spatial decay is another representation of the broadening of the time correlation curves in figure 22 and the same conclusion may be drawn, namely that on the average the low-frequency fluctuations of the pressure field tend to correlate over larger distances than the higher frequencies.

The measured cross-spectral densities also corroborate the convection speed measurements in filter bands since the curves repeat over the interval  $\frac{f \delta \xi_1}{U_0} = 1$  where, with the assumed convection speed of  $U_0 = 0.8 U_1$  to non-dimensionalise the measurements, a reasonable collapse over the whole frequency range was obtained.

The relationship between space and time appears to be just as difficult to determine from the cross-spectral density measurements as from the space-time correlation curves, probably more so because of the greater experimental difficulties involved in measuring the former.

#### The spatial correlation of the wall pressure.

It was not possible to obtain the spatial pressure correlation coefficient

$$\bar{P}(\xi_1, 0, \xi_3; 0)_{x_2=0} = \frac{p(x_1, 0, 0; t) p(x_1 + \xi_1, 0, \xi_3; t)}{\sqrt{p^2(x_1, 0, 0; t)} \sqrt{p^2(x_1 + \xi_1, 0, \xi_3; t)}} \quad (4.5)$$

for separation distances less than  $\delta_1$  because of the relatively large size of the M213 transducers used in the measurements as compared with the boundary-layer thickness. However, some quite

useful information from the larger separation measurements can be obtained.

Considering first the lateral spatial coefficient  $\bar{P}(0,0,\xi_3;0)$ , the two points that were measured are plotted in figure 38 together with the measurements of Harrison (1958). It can be inferred from the plots that the transverse correlation falls monotonically to zero in a distance of order  $16\delta_1$ , that is about twice the boundary layer thickness. The two values measured were not as high as Harrison's but did seem to suggest that the correlation coefficient was tending to a constant value of about 0.1 and, as explained in the flat-plate experiments, this result may be regarded as showing that there were spurious disturbances present that correlated over large distances, although these two transverse measurements should be interpreted rather tentatively because they may have been subject to three-dimensional flow effects due to the small transverse dimensions of the tunnel.

It may be remembered that the existence of an important boundary condition on the spatial correlation was shown in section 2.3; namely that the integral of the surface pressure correlation should be zero, implying negative loops in the spatial correlation. If the correct deduction from the transverse correlation measurements is that there were extraneous disturbances correlating with value of order 10% over large distances then clearly the presence of these disturbances must have prevented

accurate measurement of possible negative loops in the pressure correlation.

Longitudinal spatial correlation measurements could only be obtained at three points and are plotted in figure 26 and do show a negative loop in the spatial correlation. Had the transverse correlation been zero at large separation there is good reason to suppose that this loop in the longitudinal correlation would have been more negative.

With the convected hypothesis put forward in the last section the autocorrelation of the pressure fluctuations, as measured by one fixed transducer, should be the same as the longitudinal spatial correlation measured with two transducers, that is

$$\bar{P}(\xi_1, 0, 0; 0) = \bar{P}(0, 0, 0; r) \quad (4.6)$$

with  $\xi_1 = U_c r$ . The autocorrelation at the fixed transducer station has been plotted in figure 26 for comparison with the longitudinal spatial correlation. Although the spatial results were so limited, there is fair agreement.

There is no doubt from the measurements that there is a marked difference between the transverse and longitudinal correlations of the wall pressure fluctuations. The differences are exactly as predicted by the calculations for the turbulence mean-shear interaction to be described in section 5. In particular, the presence of a large negative loop in the longitudinal spatial

correlation, and hence the autocorrelation, such that the integral under them vanishes is not only required to satisfy the boundary condition but will be shown to lead to a power spectral density which falls with decreasing frequency.

To summarise this discussion of the  $2\frac{1}{2}$ in. x  $2\frac{1}{2}$ in. tunnel boundary layer investigations it may be stated that reliable root-mean-square wall pressure measurements were obtained and the convective properties of the fluctuating pressure field were clearly demonstrated. However, the presence of low frequency disturbances in the tunnel, the source of which could not be completely identified, interfered with both spectra and spatial correlation measurements.

#### 4.3 Boundary layer on the wing of a glider in flight.

It is clear from the two wind-tunnel investigations just described and from the investigations of other workers in both wind tunnels and water-tunnels and on powered aircraft in flight, that the fluctuating pressure measurements, in particular the power spectral density, were severely limited by interference from such causes as fan noise, motor or engine noise, vibration and tunnel flow disturbances, the magnitude of which were often unknown so that adequate correction for them could not be made.

It was therefore decided to continue the boundary layer investigations on the wing of a glider in flight. The advantages over a wind-tunnel for fluctuating pressure measurements are

immediately apparent, namely the air flow around the glider has relatively low turbulence level with an ambient noise field of very low intensity. Of course, in common with all flight test work, the required information must either be recorded by instruments carried in flight or, as employed in the system to be described, be telemetered to ground.

#### 4.3.1 Description of the glider

The author was fortunate in having the use of an Elliott 'Olympia Eon' glider for the experiments. A view of the glider is given in figure 27a; the aerodynamic design is typical of high performance sail-planes being extremely 'clean' with no wing-supporting struts. This was an essential design requirement for the tests since the high intensity sound field (Aeolian tones) produced as a result of vortex shedding from struts would have interfered with the pressure measurements.

The BK4133 transducer was used for the surface fluctuating pressure measurements and was ideally suited for the purpose ~~because of its~~ because of its high sensitivity, freedom from vibration, and small calibration change with ambient temperature and pressure changes. Although the transducer was relatively large, thus limiting the tests to high  $d/\delta$ , ratio, this was no disadvantage since one of the prime purposes of the experiments was to investigate the low-frequency part of the pressure power-spectral density.

The transducer was flush mounted in the wing upper-surface (see figure 30) at a 50% chord position. This was convenient because the surface profile at this station had small curvature and also the mounting position was just behind the wing-spar box which then carried the transducer power and signal cable into the fuselage. In order to avoid flow disturbances from the fuselage and aileron the transducer was mounted nearly midway between them in the position shown in figure 27b.

The original canvas wing-covering was removed over a 3 feet spanwise length around the measuring station and was replaced by a painted and polished plywood section. The same method of mounting shown in figure 9 was used. The Gottingen 549 aerofoil-section of the wing was a high performance type with a pressure distribution favourable to the maintenance of laminar flow on the wing upper-surface. In order to ensure a well-developed turbulent boundary layer over the measuring station on the upper surface a 0.10 in. dia. transition wire, 3 feet long, was placed at 15% chord (a similar wire was also placed on the port wing).

The boundary layer on the lower surface of the wing was expected to be laminar owing to the favourable pressure gradient and so provision was made for mounting the transducer at 50% chord. Following the same arguments given previously in the flat-plate tests the fluctuating pressure measurements at the lower station provided a knowledge of the extraneous noise and flow disturbances

around the glider.

Boundary layer profiles were obtained from a multi-tube pitot comb which also contained two static tubes (see figure 30, though it should be noted that the comb was not in place during the fluctuating pressure measurements). The comb was screwed to the plywood surface and the manometer tubes carried directly back to the trailing edge and along to the fuselage, being attached with 'Sallotape'. The cockpit space was extremely limited but removal of the artificial horizon meter allowed a multi-tube manometer measuring 12 in. x 14 in. to be screwed to the instrument panel. The manometric fluid was red tinted water. The pilot photographed the manometer holding a 35 mm camera one-handed. The camera was hung from the pilot's neck by a short strap and the camera was pre-set focussed before take-off. The manometer was mounted at right-angles to the glider datum-line and from a knowledge of the glider sinking-speed the attitude of the manometer could be calculated and hence the correction for the manometer slope was made. The flight speeds chosen for the test were 40 knots ASI and 80 knots ASI corresponding to lift coefficients  $C_L \approx 0.3$  and 0.1 respectively. The manometer slope correction for these two cases was small.

#### 4.3.2. The telemetry system.

The transducer signals were telemetred to the ground using the system shown in figure 28a. The signal from the transducer was first amplified by a low-noise transistorised preamplifier, the circuit diagram for which is reproduced in figure 29. This preamplifier incorporated an accurate attenuator, placed before the first stage to prevent overloading of the system.

The telemetry system worked on the frequency-modulation principle and had the advantage of high amplitude accuracy and a flat frequency response. The transmitter carrier frequency was 465 Mc/s and used pseudo-pulse modulation. The sub-carrier frequency was 145 kc/s with  $\pm 15$  kc/s frequency deviation for a signal of  $\pm 1$  volt from the preamplifier.

The polarising voltage of 200 volts for the transducer was supplied from dry batteries and the HT voltages for the cathode-follower and transmitter were supplied from a transistorised static-inverter operated by 12 volt non-spill accumulators which also supplied the valve heaters.

The batteries were mounted in the wing centre-section close to the glider centre of gravity and the transmitter, inverter and a quarter-wavelength unipole aerial were mounted in the side of the fuselage (see figure 31).

The telemetry receiver was housed in a caravan which carried a 7 turn helical aerial of 12 decibels gain and  $45^\circ$  beam-width.

The aerial could be rotated for easy tracking of the transmitter in the glider. The line-of-sight range was at least 50,000 yards.

The system was calibrated overall by injecting a sinusoidal signal of known voltage to the cathode-follower and measuring the output signal from the receiver. The overall frequency-response of the system is shown in figure 28b.

#### 4.3.3 Test Technique

The glider was towed to an altitude of 5,000 feet by an Auster 'Aiglet' aircraft. Communication was maintained by radio with the towing-aircraft pilot who gave the time of release to ground control. The towing aircraft then lost height immediately and landed to avoid interference from engine noise. The glider was not equipped with radio and so a marker button which changed the sub-carrier frequency was provided on the cockpit instrument panel.

The procedure for the fluctuating pressure measurements was as follows. After release, the glider pilot executed three test runs at a speed of 40 knots A.S.I. and then three runs at 80 knots A.S.I. giving a marker at the beginning and end of each run. The duration of each run was 10 seconds and allowed the operator in the caravan to record a sample of the test run, 2 seconds long, on the tape-recorder of the time correlator. The glider pilot

recorded the altitude, rate of descent and speed for each test run.

The procedure for the boundary layer profile measurements was very similar except of course that contact with the ground was not necessary, the glider pilot executing the test runs and photographing the manometer at his own convenience.

The author was very fortunate to have the assistance of extremely competent pilots, well-briefed in flight test techniques, to fly the glider and towing aircraft. After two flights to cure minor faults in the telemetry system the test-programme was completed in a further six flights. The weather conditions prevailing at the time of the flights were clear and calm (wind speed  $< 15$  knots).

#### 4.3.4. Discussion of results.

The mean velocity profiles for the upper-surface station are plotted in figure 33 and show that the boundary-layer was fully turbulent at both test speeds with boundary layer thicknesses of order 1 inch. The scatter exhibited in the mean-velocity measurements was due entirely to the inaccuracies in reading the photographs of the small manometer mounted in the cockpit, nevertheless, the logarithmic plots of the profiles in figure 32 were of sufficient accuracy to enable the skin-friction coefficient to be determined from the 'law of the wall' method. This law is known to hold for

cases of adverse pressure gradient and the value of  $C_f = 0.0025$  estimated from figure 32 is also in agreement with the Ludweig-Tillmann skin-friction formula for cases of adverse pressure gradient, that is

$$C_f = 0.246 R_{\delta_2}^{-0.268} 10^{-0.678H} \quad (4.6)$$

where  $R_{\delta_2}$  is the Reynolds number based on the boundary-layer momentum thickness  $\delta_2$  and  $H$  is the form parameter  $\delta_1/\delta_2$

With regard to the magnitude of the mean pressure gradient and its effect on the fluctuating pressure measurements it is convenient to refer to the Gottingen wind-tunnel measurements for the Gottingen 549 aerofoil-section (see Riegels 1961). The chosen transducer position at 50% chord was just aft of the region where the mean pressure gradient on the upper surface, corresponding to the flight lift coefficients, was zero. The attachment of the transition wire at 15% chord on the upper surface with the resulting turbulent boundary layer should have made negligible change to the pressure distribution as given by the Gottingen results and so it was concluded that the mean pressure gradient at the measuring station was relatively small. This was confirmed by the measured values for  $H$  of 1.49 which were not much greater than the value of 1.38 given by Coles (1954) for the ideal two-dimensional boundary layer with zero mean pressure gradient at the same Reynolds number based on momentum thickness. Of course,

this difference in form parameter, though small, was associated with a much larger rate of boundary layer growth, but with only a small effect on  $C_f$  and hence on the mean shear.

With these facts in mind and with the use of the usual boundary layer approximations, it was argued that the contributions to the measured values of the surface fluctuating pressure, for the turbulent boundary layer at the upper-surface mid-chord position, from mean velocity gradient terms other than the mean shear  $\partial U_1 / \partial x_2$  were small.

Lastly, in view of the large aspect ratio of the wing, it was probable that three-dimensional effects on the fluctuating pressure measurements, due to cross flow, were negligible. The mean-velocity profile at the lower-surface mid-chord station for the 40 knots ASI speed case is also plotted in figure 33 and shows that the boundary layer was laminar.

The measured power-spectral densities of the upper-surface pressure fluctuations at the mid-chord station are plotted in figure 34, which shows clearly that the spectrum of the pressure falls approximately as the square of the frequency at low frequencies. This is the first of two major experimental results that were obtained.

The root-mean-square values of the fluctuating pressure were obtained by integration of the spectral curves and are also given in figure 34. The root-mean-square value of the fluctuating pressure on the lower surface for the lower-speed test, that is

with laminar flow at the mid-chord measuring station, was approximately one fifteenth of the signal measured on the upper surface.

This ratio was maintained in filter bands and confirmed the validity of the signals measured on the upper surface. Thus the excellent signal to 'noise' ratio attained in the experiments, including the low-frequency part of the pressure spectrum, proved that the original surmise was correct, namely that a glider would be an ideal test facility for low-speed turbulent pressure fluctuation measurements. A check on the extraneous disturbance level for the higher speed case was not possible because boundary layer transition to turbulent flow had already taken place upstream of the mid-chord lower-surface measuring station.

The measured autocorrelation of the turbulent pressure fluctuations on the upper-surface for the lower-speed case, corresponding to  $d/\delta_1 = 2.93$ , is plotted in figure 37 and is to be discussed again later in section 5. For the present the existence of the large negative loop in the autocorrelation may be noted, which is consistent with the falling power spectral density at low frequencies. It follows from the convection hypothesis that the longitudinal correlation of the wall pressure has the same form as the autocorrelation and so must also have this large negative loop.

#### 4.4. Boundary layer on 20in. x 10in. tunnel wall.

As in Willmarth's work it is convenient to obtain the true root-mean-square value of the fluctuating pressure, that is the value that would be measured with a transducer of unlimited resolving power, by extrapolating the graphs of rms pressure against  $d/\delta_1$  to zero transducer diameter. Finally, in order to obtain a value at a  $d/\delta_1$  ratio as small as possible with the M213 transducer, measurements of the wall fluctuating pressure were made in the College of Aeronautics 20in. x 10in. tunnel where it was possible to realise a very thick boundary layer and hence a low  $d/\delta_1$  ratio.

##### 4.4.1 The test facility

The 20in. x 10in. tunnel is a closed circuit fan-driven tunnel with a working-section of 12 feet length slowly increasing in cross-section to allow for boundary layer growth. Measurements were taken at two stations, 6in. and 8 feet downstream of the working-section entry. The boundary layer was fully turbulent at both measuring stations for the chosen test speed of 126 feet/sec. Although the tunnel was considerably quieter than the open working-section tunnel used in the flat-plate tests it was still necessary to filter the low-frequency part of the fluctuating pressure signal, originating from tunnel noise and vibration, with a 250 c/s high-pass filter which was incorporated in the battery powered preamplifier.

### Results

The turbulent mean-velocity profiles are plotted logarithmically in figure 35a and show that the skin friction coefficient changed from 0.0035 to 0.0025 over the two measuring stations. At the rear station the boundary layer thickness was of order 2 inches which gave a  $d/\delta$  ratio of 0.72 when using the M213 transducer.

The power-spectral densities obtained for the filtered signals are shown in figure 35b and as before the root-mean-square values of the fluctuating pressure, given on the spectrum plots, were obtained by integration. The neglect of the contribution from Strouhal numbers less than 0.1 made negligible difference to the results.

#### 4.5 Concluding discussion of the boundary layer measurements.

A detailed discussion of the root-mean-square fluctuating pressure measurements has been purposely deferred until now so that the results from all four boundary layer tests described could be assessed after allowing for the variation of skin friction coefficient between experiments. The non-dimensional rms fluctuating pressure  $\sqrt{\overline{p^2}} / \frac{1}{2} \rho U_\infty^2 C_f$  has been plotted against  $d/\delta$ , in figure 36 and shows quite clearly that besides the variation of rms pressure with the transducer diameter to boundary layer thickness ratio, there was a definite collapse of the results

which indicated a dependence on  $C_f$ ,

The true root-mean-square value of the wall fluctuating pressure obtained by extrapolating the curve of figure 36 to zero  $d/\delta_1$  ratio gave

$$\sqrt{\overline{p^2}} / \frac{1}{2} \rho U_1^2 \approx 2.2 C_f \quad (4.7)$$

and is the second of the two important experimental results described in this thesis. It should be noted that this result applies only to low Mach number flows and as shown in figure 36 compares favourably with the extrapolated result found by Willmarth (1959) that  $\sqrt{\overline{p^2}} / \frac{1}{2} \rho U_1^2 = 0.006$  at a flow Mach number of 0.33. A calculation from Willmarth's mean-velocity profiles gave  $C_f = 0.003$  so that the value of  $\sqrt{\overline{p^2}} / \frac{1}{2} \rho U_1^2$  was  $2 C_f$ .

The result in equation (4.7) might at first sight be taken as conflicting with Willmarth's other result that the value of  $\sqrt{\overline{p^2}} / \frac{1}{2} \rho U_1^2$  was again 0.006 measured at a flow Mach number of 0.67, for the value  $C_f$  was 0.0022 and so  $\sqrt{\overline{p^2}} / \frac{1}{2} \rho U_1^2$  was then  $\approx 2.7 C_f$ . However, as already stated in the introduction, the recent measurements at high speeds of Williams.D. (1960) and Kistler (quoted by Laufer 1961) have shown that the ratio  $\sqrt{\overline{p^2}} / \frac{1}{2} \rho U_1^2 C_f$  rises with increasing Mach number to a value as high as 6 at high supersonic speeds.

For this reason it is difficult to compare the present low-speed result of equation (4.7) with the high sub-sonic flight

measurements of Mull and Algranti(1960) since the effects of Reynolds number and Mach number would need to be separated. Taking the lowest Mach number result of Mull and Algranti they found that  $\sqrt{p^2}/\frac{1}{2}\rho U_1^2 \approx 0.005$  for  $M = 0.4$  and a  $d/\delta_1$  ratio of 6.25 and a Reynolds number of  $0.84 \times 10^6$ . Extrapolating their result to zero transducer diameter and calculating  $C_f \approx 0.003$  from their measured velocity profiles gives  $\sqrt{p^2}/\frac{1}{2}\rho U_1^2 C_f \approx 2.3$  which is in good agreement with the present results.

It may therefore be concluded from the present work and the high-speed measurements of other workers that when interpreting wall pressure measurements in the turbulent boundary layer due account must be taken of the transducer diameter, the wall stress and the Mach number.

In the next section the correlation and spectral measurements, especially those obtained on the glider tests, will be compared with calculations of these quantities.

5. Calculations of the wall pressure spectrum and the spatial pressure correlation coefficient in the turbulent boundary layer.

In section 2.2 expressions were derived for the mean-square wall pressure and the spatial pressure covariance. To obtain a numerical value for the mean-square pressure certain approximations were necessary. In the present section similar approximations will be made in order to derive numerical values for both the spatial wall pressure correlation coefficient and the wall pressure spectrum. If, at times, the approximations are not completely justified they do, at least, enable the principles and methods involved in the calculations to be demonstrated and do give results in good agreement with experiment.

5.1 The pressure spectrum from the Green's function solution.

If we put  $\xi_3 = 0$  in equation (2.22) for the spatial wall pressure covariance, then the turbulence-mean shear contribution to the longitudinal covariance of the wall fluctuating pressure at two points  $\xi_1$  apart taken at times  $t$  and  $t+\tau$  is given by

$$\begin{aligned}
 P(\xi_1, 0, 0; \tau)_{x_2=0} &= \overline{p(x_1, 0, 0; t) p(x_1 + \xi_1, 0, 0; t + \tau)} \\
 &= \frac{2\rho_0^2}{\pi} \frac{\partial}{\partial \xi_1} \int_0^\infty dy_2 \int_{-y_2}^\infty dr_2 \int_{-\infty}^\infty dr_1 \int_{-\infty}^\infty dr_3 \bar{R}_{22}(y_2; \vec{r}; \tau) \\
 &\quad \cdot \frac{g(y_2) g(y_2 + r_2) (\xi_1 - r_1)}{\sqrt{(2y_2 + r_2)^2 + (\xi_1 - r_1)^2 + r_3^2} \sqrt{(2y_2 + r_2)^2 + (\xi_1 - r_1)^2 + r_3^2}}
 \end{aligned}
 \tag{5.1}$$

where  $\bar{R}_{22}(y_2; \underline{r}; \tau)$  is now the space-time correlation coefficient of the lateral fluctuating velocity component and varies with  $y_2$ , and  $g(y_2)$  has been written for the mean shear-turbulent intensity product  $\bar{e}_{12}(y_2) \cdot \sqrt{u_2^2}(y_2)$

As mentioned in appendix A, Hodgson and Lilley (1960) approximated to the integral in equation (5.1) by first taking the mean shear-turbulent intensity product  $g(y_2)$  as constant with some average value  $g_0$  say, together with  $\bar{R}_{22}(y_2; \underline{r}; \tau)$  independent of  $y_2$  with some representative value  $\bar{R}_{22}(\underline{r}, \tau)$  say. If the  $y_2$  and  $r_2$  integrations are now reversed, since

$$\int_0^\infty dy_2 \int_{-y_2}^\infty dr_2 \equiv \int_0^\infty dr_2 \int_0^\infty dy_2 + \int_{-\infty}^0 dr_2 \int_{-r_2}^\infty dy_2$$

the  $y_2$  integration may then be performed and gives

$$P(\xi_1, 0, 0; \tau)_{x_2=0} = \frac{2\rho_0^2 g_0^2}{\pi} \frac{\partial}{\partial \xi_1} \int_{-\infty}^\infty dr_1 \int_0^\infty dr_2 \int_{-\infty}^\infty dr_3 \bar{R}_{22}(\underline{r}; \tau) \cdot \frac{(\xi_1 - r_1)}{[r_2 + \sqrt{(\xi_1 - r_1)^2 + r_2^2 + r_3^2}]} \quad (5.2)$$

Secondly, the  $y_2$  integration was further simplified by approximating the denominator of the integral in equation (5.2) so that

$$P(\xi_1, 0, 0; \tau)_{x_2=0} = \frac{\rho_0^2 g_0^2}{\pi} \frac{\partial}{\partial \xi_1} \int_{-\infty}^\infty dr_1 \int_0^\infty dr_2 \int_{-\infty}^\infty dr_3 \frac{\bar{R}_{22}(\underline{r}, \tau)(\xi_1 - r_1)}{|\xi_1 - \underline{r}|} \quad (5.3)$$

Now performing the  $\xi_1$  differentiation and putting  $\xi_1 = 0$ , we obtain the autocorrelation of the wall fluctuating pressure

$$P(0,0,0;\tau)_{x_2=0} = \overline{p(x_1,0,0;t)p(x_1,0,0;t+\tau)}$$

$$= \frac{\rho_0^2 g_0^2}{\pi} \int_{-\infty}^{\infty} dr_1 \int_0^{\infty} dr_2 \int_{-\infty}^{\infty} dr_3 \frac{\bar{R}_{22}(\underline{r};\tau)(r_2^2+r_3^2)}{r^3} \quad (5.4)$$

where  $r = |\underline{r}| = (r_1^2 + r_2^2 + r_3^2)^{1/2}$

Although the approximations leading to equation (5.4) have not been justified a priori, nevertheless the magnitude of the mean-square pressure obtained by integrating (5.4) is of the right order. For, if the turbulence is taken to be isotropic then the lateral velocity correlation coefficient can be shown to be

$$\bar{R}_{22}(\underline{r}) = f + \frac{r^2}{2} \frac{df}{dr} - \frac{r^2}{2r} \frac{df}{dr} \quad (5.5)$$

where  $f(r)$  is the two-point longitudinal velocity correlation scalar (see Batchelor 1953). Evaluation of the mean-square pressure  $P(0,0,0;0)$  by substituting (5.5) into equation (5.4), putting  $\tau = 0$  and integrating over angles yields

$$\overline{p^2} = \frac{8a}{15} \rho_0^2 g_0^2 \int_0^{\infty} r f dr \quad (5.6)$$

where  $\alpha$  is a factor less than unity to allow for anisotropy of the turbulence. As mentioned in appendix A the equation (5.6) is identical with that found by Kraichnan (1956a) for a free shear flow with constant mean-shear and superimposed isotropic turbulence. The velocity spectra measurements of Klebanoff (1954) and the spatial correlation measurements of Grant (1958), reproduced in figures 3 and 4, are well described if the correlation scalar is taken as

$$f = e^{-r/L} \quad (5.7)$$

where  $L = \int_0^\infty f dr$  is the longitudinal integral scale of the turbulence. Then equation (5.6) becomes

$$\overline{p^2} = \frac{8\alpha}{15} \rho_0^2 g_0^2 L^2 \quad (5.8)$$

If, from figure 1, we take the average value of the mean shear-turbulent intensity product  $g_0$  over the main contributing region  $0.02 < y_2/\delta_0 < 0.6$  as  $0.2 U_1^2/\delta_0 \times 10^2$  and if, from Klebanoff's spectra measurements in figure 3, we take the scale  $L = 1.5\delta_1$ , then the non-dimensional root-mean-square is

$$\sqrt{\overline{p^2}} / \frac{1}{2} \rho_0 U_1^2 \approx 4\sqrt{\alpha} C_f \quad (5.9)$$

Kraichnan (1956a) has calculated the anisotropy factor  $\alpha$  for the simplified model of initially isotropic turbulence in the presence

of a mean shear but with elongated scale in the streamwise direction. From the complete two-point velocity correlation measurements of Grant (1958) the longitudinal scale of boundary-layer turbulence is of order three times the scale in the other two directions giving a value of  $\alpha$  in the range 0.3 to 0.5. Thus the root-mean-square pressure as given by equation (5.9), is

$$\sqrt{p^2} / \frac{1}{2} \rho U_1^2 \approx 2.3 - 2.8 C_f \quad (5.10)$$

which agrees closely with the more accurate calculation of section 2.2, where it was found that  $\sqrt{p^2} / \frac{1}{2} \rho U_1^2$  was equal to  $2.5 C_f$ . This agreement is, to some extent, a justification of the step leading to equation (5.4).

In order to proceed further with the calculations of the pressure autocorrelation given in equation (5.4) and hence find the frequency spectrum, we now need to specify a representative value across the layer of the space-time lateral velocity correlation coefficient  $\bar{R}_{22}(r; \tau)$ , (where it is emphasized that so far we have referred all quantities to a set of axes fixed relative to the boundary surface).

To ascertain a suitable form for  $\bar{R}_{22}(x, \tau)$  we will refer to the measurements of  $\bar{R}_{11}(x, \tau)$  obtained by Favre (1958) and Favre et al. (1957, 1958) for both boundary layer and grid turbulence. The measurements for the latter case are shown in figure 39. The

boundary layer measurements were very similar except, of course, that the average convection speed must vary across the layer. They found that, to good approximation, the local averaged convection speed of the fluctuating velocity pattern for most of the boundary layer ( $0.03 < y_2/\delta < 0.8$ ) was the local mean-velocity  $U(y_2)$ .\*

From this we deduce that the turbulence has a convection speed, which is equal to the local mean velocity, and hence it is convenient to consider the fluctuations of the velocity by referring them to a frame of reference moving with this convection speed.

Let us first consider a field of homogeneous turbulence in a frame of reference moving with the constant mean velocity  $U_c$ , say. Denote the space-time two-point velocity correlation coefficient that would be measured in this moving frame by  $\bar{R}_{ij}(x', \tau)$  where  $x' = (r_1', r_2', r_3')$  is the separation vector between the two points in the moving co-ordinate system. Following Hodgson and Lilley (1960), as an approximation to this moving-frame space-time correlation it will be assumed that the spatial and the time fluctuations as seen in this moving frame may be

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\* The fact that the turbulence is convected on the average with the local mean velocity is only likely to be true where the turbulent mixing process across the layer has relatively small scale. Barratt (1960) and Bradshaw (1962) have shown that in the outer parts of the mixing region of a turbulent axial jet the averaged convection speed of the turbulence, as defined from space-time correlation measurements, can vary between 0.7 and 1.5 times the local mean velocity.

separated, that is

$${}^m\bar{R}_{ij}(\underline{x}', \tau) = {}^m\bar{R}_{ij}(\underline{x}') {}^m\bar{R}_{ij}(\tau) \quad (5.11)$$

where  ${}^m\bar{R}_{ij}(\underline{x}')$  is the spatial correlation coefficient and  ${}^m\bar{R}_{ij}(\tau)$  is the autocorrelation coefficient, both measured in the moving frame.

In the stationary frame ( $\underline{x}$ ), in which the turbulence is observed to be convected past at a speed  $U_0$  in the direction  $\underline{r}_1$ , the space-time correlation coefficient will be

$${}^f\bar{R}_{ij}(\underline{x}; \tau) = \bar{R}_{ij}(\underline{r}_1 - U_0\tau, \underline{r}_2, \underline{r}_3) {}^m\bar{R}_{ij}(\tau) \quad (5.12)$$

because

$$(\underline{r}', \underline{r}_2', \underline{r}_3'; \tau) \equiv (\underline{r}_1 - U_0\tau, \underline{r}_2, \underline{r}_3; \tau) \quad (5.13)$$

and the spatial structure of the turbulence is the same in either frame of reference.

This expression describes the space-time correlation curves with the form of figure 39 for, if in equation (5.12) we substitute the optimum time delay  $\tau_m = \underline{r}_1/U_0$ , then

$$\begin{aligned} {}^f\bar{R}_{ij}(\underline{x}; \tau) &= \bar{R}_{ij}(0, 0, 0) {}^m\bar{R}_{ij}(\tau) \\ &= {}^m\bar{R}_{ij}(\tau) \end{aligned}$$

that is, the locus of the fixed-frame correlation coefficient at the optimum time delay is the autocorrelation in the moving frame.

If we now assume the turbulence is isotropic with velocity

correlation scalar  $f = e^{-r^2/L^2}$  then from equation (5.5), and describing the autocorrelation of the velocity in moving axes as

$$\bar{R}_{22}(\tau) = e^{-\tau^2/T^2} \quad (5.14)$$

where  $T$  is an integral time scale in the moving frame, it follows from equation (5.11) that the space-time velocity correlation coefficient in the moving frame is

$$\bar{R}_{22}(\mathbf{r}', \tau) = e^{-r'^2/L^2} \left[ 1 - \frac{(r'_1 + r'_2)^2}{L^2} \right] e^{-\tau^2/T^2} \quad (5.15)$$

and so from equation (5.13) the space-time correlation coefficient of the lateral velocity component in the fixed frame is

$$\bar{R}_{22}(\mathbf{r}; \tau) = e^{-(r_2^2 + r_3^2)/L^2} e^{-(r_1 - U_c \tau)^2/L^2} \left[ 1 - \frac{r_3^2}{L^2} - \frac{(r_1 - U_c \tau)^2}{L^2} \right] e^{-\tau^2/T^2} \quad (5.16)$$

Since in obtaining equation (5.4) we have simplified the  $y_2$  integration across the boundary layer by substituting constant values for the varying mean-shear, turbulent intensity and scale representative of the main contributing part of the layer, so in equation (5.16) we will take the representative value of  $U_c(y_2)$  (which as we have seen is also  $U(y_2)$ ) as  $0.8U_1$ , the measured convection speed of the fluctuating pressure field. Substituting (5.16) into equation (5.4) and writing

$$\bar{r}_1 = \frac{r_1}{L}; \quad \bar{\tau} = \frac{U_c \tau}{L}; \quad \bar{T} = \frac{U_c T}{L}$$

the pressure autocorrelation in the fixed frame is given by

$$P(o, o, o; \tau)_{x_2=0} = \frac{4\rho_o^2}{\pi} g_o^2 L^2 e^{-\frac{\bar{r}^2}{T^2}} \int_0^\infty d\bar{r}_1 \int_0^\infty d\bar{r}_2 \int_0^\infty d\bar{r}_3 \frac{\frac{\bar{r}_2^2 + \bar{r}_3^2}{\bar{r}^2} - 1 - \bar{r}_3^2 - (\bar{r}_1 - \bar{r})^2}{e^{-\frac{\bar{r}_2^2}{T^2} - \frac{\bar{r}_3^2}{T^2} - (\bar{r}_1 - \bar{r})^2}} \quad (5.17)$$

Now, since the 'true' time fluctuations of the turbulence as observed in the moving frame are so slow compared with the higher frequency fluctuations observed in the fixed frame produced by the spatial pattern being converted past, so the  $e^{-\frac{\bar{r}^2}{T^2}}$  factor in equation (5.17) can be ignored.\*

The power spectral density of the wall-pressure time fluctuations as measured in the fixed frame is obtained by taking the Fourier transform of the pressure autocorrelation, namely

$$\Pi(o, o, o; \omega) = \Pi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(o, o, o; \tau)_{x_2=0} e^{-i\omega\tau} d\tau \quad (5.18)$$

(where  $\Pi(\omega)$  as defined here is half the mean-square pressure per rad./sec.  $\overline{p^2}(\omega)$  measured from the filter analysis and used in the experimental spectra measurements).

Hodgson and Lilley (1961) have evaluated the transform (5.18)

\*

As mentioned earlier this is the well-known hypothesis due to Taylor (1938). If the true time fluctuations are neglected, (that is a 'frozen' pattern) we have from equation (5.12)

$$f\bar{R}_{ij}(o, o, o; \tau) = \bar{R}_{ij}(r_1 = U_o\tau, o, o)$$

Thus the autocorrelation in fixed axes is the same as the spatial correlation in fixed axes with  $r_1 = U_o\tau$ .

of equation (5.21) and obtained

$$\Pi(\omega) = \frac{\rho_o^2 L^3 g_o^2}{4 \pi U_c} \left[ \frac{\bar{\omega}^2}{4} e^{-\bar{\omega}^2/4} - \frac{\bar{\omega}^2}{4} (1 - \bar{\omega}^2/4) \text{Ei}(-\bar{\omega}^2/4) \right] \quad (5.19)$$

where  $\omega$  is the frequency in radians/second,  $\bar{\omega} = \frac{\omega L}{U_c}$  and  $\text{Ei}(-x) = -\int_x^\infty e^{-u} \frac{du}{u}$  is the exponential integral.

After choosing a representative value of the scale  $L$  as  $3\delta_1$  from Klebanoff's measurements as shown in figure 3, the power spectral density given by equation (5.19) has been plotted non-dimensionally against the Strouhal number  $\frac{\omega \delta_1}{U_1}$  in figure 40, where the ordinate has been appropriately divided by  $C_f^2$ . The spectrum level was fixed by taking the experimental value of  $\sqrt{p^2}/\frac{1}{2}\rho_o U_1^2$  as  $2.2 C_f$ . The calculated spectrum curve, when compared with the glider measurements, falls off much more rapidly at the higher frequencies because of the very fast fall-off with frequency of the assumed  $e^{-r^2/L^2}$  velocity correlation form. However, at low frequencies the two curves are in good agreement, showing quite clearly that the spectrum level falls off as the square of the frequency with decreasing frequency.

The turbulence-turbulence contribution to the spectrum, evaluated in appendix B, is also plotted in figure 40. The agreement between the calculated turbulence-mean shear contribution and the glider measurements at low frequencies corroborates the initial assumption made in section 2 that the contribution of the turbulence-turbulence

terms to the wall fluctuating pressure was probably small.

It should be noted that in the above calculation of the pressure spectrum the variations in the mean shear, the turbulent intensity and the scale, have been replaced by suitable average values. A more exact calculation could have been performed taking account of the variations in these quantities, but in this case the spectrum function could only be found using a numerical method. However, in the next section an alternative method is given which allows, more easily for the variation in some of the flow quantities across the layer.

## 5.2 The pressure spectrum and the spatial correlation from the Fourier transform solution.

The Fourier transform solution for the wall fluctuating pressure has been treated in detail by Kraichnan (1956b) and so the analysis will only be given briefly here except that Fourier-Stieltjes integrals are used following Batchelor (1953) and Moyal (1952).

The turbulence is again assumed to be homogeneous in the  $x_1$  and  $x_3$  directions. Let

$$p(x_2; X) = \iint e^{i k \cdot X} dW(x_2; k) \quad (5.20)$$

and

$$u_2(x_2; X) = \iint e^{i k \cdot X} dZ_2(x_2; k) \quad (5.21)$$

be the two-dimensional spatial Fourier-Stieltjes transforms of the

respectively pressure  $p(x, t)$  and the lateral velocity component  $u_2(x, t)$ , where  $x = (x_1, x_3)$ , and  $k = (k_1, k_3)$  is the two-dimensional wave-number in the plane. (The time variable is omitted for convenience).

The two-dimensional transform of the Poisson equation (2.3) for the fluctuating pressure due to the turbulence-mean shear contribution is then

$$\frac{d^2}{dx_2^2} - (k_1^2 + k_3^2) \quad dW(x_2; k) = -2i \rho_0 \bar{e}_{12}(x_2) k_1 \quad dZ_2(x_2; k) \quad (5.23)$$

The solution of this equation for the fluctuating pressure, with the boundary conditions  $p = 0$  as  $x_2 \rightarrow \infty$  and  $\frac{\partial p}{\partial x_2} = 0$  at  $x_2 = 0$ , has been given by both Kraichnan (1956b) and Lilley (1960). The two-dimensional transform of the fluctuating pressure at the wall is

$$dW(0; k) = \frac{2i \rho_0 k_1}{k} \int_0^\infty e^{-k x_2'} \bar{e}_{12}(x_2') dZ_2(x_2'; k) dx_2' \quad (5.24)$$

where  $k = \sqrt{k_1^2 + k_3^2}$ , and so multiplying equation (5.24) by its complex conjugate and taking the ensemble average

$$\overline{dW^*(0; k) dW(0; k')} = \frac{4 \rho_0^2 k_1 k_1'}{k k'} \int_0^\infty \int_0^\infty e^{-k x_2'' - k' x_2''} \bar{e}_{12}(x_2') \bar{e}_{12}(x_2'') \cdot dZ_2(x_2'; k) dZ_2(x_2''; k') dx_2' dx_2'' \quad (5.25)$$

where the asterisk denotes the complex conjugate. Now, by homogeneity in the  $x_1$  and  $x_3$  directions,  $\overline{dW^*(k) dW(k')}$  can be shown to vanish

everywhere except for a discontinuous jump at the point  $k = k'$   
(see Batchelor 1953 and Moyal 1952) and so can be written

$$\overline{dW^*(0; k) dW(0; k')} = \Pi(0; k) \delta(k - k') dk dk' \quad (5.26)$$

and also

$$\overline{dZ_2^*(x_2'; k) dZ_2(x_2''; k')} = \Phi_{22}(x_2', x_2''; k) \delta(k - k') dk dk' \quad (5.27)$$

where  $\Pi(0; k)$  and  $\Phi_{22}(x_2', x_2''; k)$  are the two-dimensional wave-number spectrum functions of the fluctuating wall pressure and the lateral velocity component respectively and  $\delta(k)$  is the Dirac delta function.

On substituting (5.26) and (5.27) in equation (5.25) and performing the integration with respect to  $k'$  we obtain

$$\begin{aligned} \Pi(0; k) &= \frac{4\rho_0^2 k_1^2}{k^2} \int_0^\infty \int_0^\infty e^{-k(x_2' + x_2'')} \bar{e}_{12}(x_2') \bar{e}_{12}(x_2'') \\ &\quad : \Phi_{22}(x_2', x_2''; k) dx_2' dx_2'' \end{aligned} \quad (5.28)$$

To proceed further the spectrum function  $\Phi_{22}$  for the lateral velocity component must be specified. The same difficulty arises as previously in that  $\Phi_{22}$  varies with distance from the wall. We could transform the correlation function  $R_{22}(\mathbf{r}/x_2)$  given by Townsend (1956) which was used in section 2. However, for analytical simplicity, it is convenient to use Kraichnan's (1956b) idealised 'mirror-flow' model for the turbulence, which allows an expression for  $\Phi_{22}$  to be obtained in terms of the well-known homogeneous turbulence results.

If  $\bar{x}^*$  is the mirror image with respect to the plane  $x_2 = 0$  of the coordinate  $\bar{x}$ , that is

$$\bar{x}_1^* = \bar{x}_1; \bar{x}_2^* = -\bar{x}_2; \bar{x}_3^* = \bar{x}_3 \quad (5.29)$$

then Kraichnan expressed the boundary layer turbulent velocity field as

$$u_1(\bar{x}, t) = \frac{1}{\sqrt{2}} [\bar{u}_1(\bar{x}, t) + \bar{u}_1(\bar{x}^*, t)] \quad (5.30a)$$

$$u_2(\bar{x}, t) = \frac{1}{\sqrt{2}} [\bar{u}_2(\bar{x}, t) - \bar{u}_2(\bar{x}^*, t)] \quad (5.30b)$$

$$u_3(\bar{x}, t) = \frac{1}{\sqrt{2}} [\bar{u}_3(\bar{x}, t) + \bar{u}_3(\bar{x}^*, t)] \quad (5.30c)$$

where  $\bar{u}_i(\bar{x}, t)$  is written for the homogeneous turbulent field. It is easily shown that  $u(\bar{x}, t)$  satisfies the equation of continuity (2.1) provided  $\bar{u}(\bar{x}, t)$  satisfies it. Furthermore, the boundary conditions on the wall

$$u_2 = 0; \frac{\partial^2 u_2}{\partial x_2^2} = 0; \frac{\partial p}{\partial x_2} = 0$$

are all satisfied. A physical drawback of the model is that  $u_1$  and  $u_3$  do not vanish at the wall, their mean-square value rising to twice the mean-square value of the original homogeneous components, also that the turbulence extends indefinitely away from the wall with finite intensity at infinity. Kraichnan argued quite correctly that except for the region very close to the wall (which, from the remarks

of section (2.2), makes negligible contribution to the wall pressure) and the finite intensity at infinity the model was a reasonable description of boundary layer turbulence (see for instance Klebanoff's intensity measurements in figure 2). For the present problem it is the lateral velocity component which is of prime interest and since the weighting function (now appearing as the  $e^{-k x_2}$  factor in the integrand of equation 5.24) falls rapidly to zero at large distances from the wall plus the fact that the mean shear is zero for  $\delta \leq x_2 < \infty$ , so the finite intensity of the turbulence near the edge of the boundary layer makes little difference to the final results.

From equation (5.30b) it follows that the lateral velocity covariance is given by

$$R_{22}(x'_2, x''_2; r_1, r_3) = \bar{R}_{22}(x'_2 - x''_2, r_1, r_3) - \bar{R}_{22}(x'_2 + x''_2, r_1, r_3) \quad (5.31)$$

where  $\bar{R}_{22}$  is the lateral velocity covariance for the homogeneous model. and  $r_1, r_3$  have been written for  $x'_1 - x''_1, x'_3 - x''_3$  respectively.

The two-dimensional transform of equation (5.31) in wave-number space  $k$  is then

$$\Phi_{22}(x'_2, x''_2, k) = \bar{\Phi}_{22}(x'_2 - x''_2; k) - \bar{\Phi}_{22}(x'_2 + x''_2; k) \quad (5.32)$$

Kraichnan approximated to the mean-shear variation with  $x_2$  by the form

$$\bar{e}_{12}(x_2) = \bar{e}_{12}(0) e^{-\beta x_2} \quad (5.33)$$

and using equations (5.32) and (5.33) in (5.33) obtained

$$\bar{\Pi}(0; \underline{k}) = \frac{4\rho_0^2 k^2 \bar{e}_{12}^2(0)}{k^2} \int_0^\infty \left[ \frac{1}{(k+\beta)} - x_2 \right] e^{-(k+\beta)x_2} \bar{\Phi}_{22}(x_2; \underline{k}) dx_2 \quad (5.34)$$

An acceptable fit to the boundary layer mean shear variation is obtained if in equation (5.33) we put

$$\bar{e}_{12}(0) = 3.7 U_T / \delta_1 \quad \text{and} \quad \beta = 0.31 / \delta_1 \quad (\text{see Lilley 1960})$$

If, as a further approximation the homogeneous field is taken as isotropic then the two-dimensional transform  $\bar{\Phi}_{22}(x_2; \underline{k})$  is given by

$$\bar{\Phi}_{22}(x_2; \underline{k}) = \int_{-\infty}^{\infty} \bar{\Phi}_{22}(\underline{k}) e^{ix_2 \kappa_2} d\kappa_2 \quad (5.35)$$

where  $\underline{k} = (\underline{k}, \kappa_2)$ , and  $\bar{\Phi}_{22}(\underline{k})$  is now the usual three-dimensional isotropic turbulence spectrum function defined by Batchelor (1953) that is

$$\bar{\Phi}_{ij}(\underline{k}) = \frac{E(k)}{4\pi k^4} (\kappa^2 \delta_{ij} - \kappa_i \kappa_j) \quad (5.36)$$

where  $\kappa^2 = \kappa_1^2 + \kappa_2^2 + \kappa_3^2$  and the scalar function  $E(k)$  is given by

$$E(k) = \frac{1}{\pi} \int_0^\infty \overline{u^2} f(r) \kappa^2 r^2 \left( \frac{\sin \kappa r}{\kappa r} - \cos \kappa r \right) dr \quad (5.37)$$

If  $f = e^{-r/L}$ , which as was noted in section 2.2 is a closer fit to measured velocity correlations and spectra than the previously assumed form  $f = e^{-r^2/L^2}$ , then the values of  $E(k)$  and  $\bar{\Phi}_{22}(\underline{k})$  can be evaluated

from equations (5.36) and (5.37). Using these forms and the mean shear variation given above, the integral in equation (5.34) has been evaluated and the following one-dimensional spectrum functions

$$\Pi(k_1)_{x_2=0} = \int_{-\infty}^{\infty} \Pi(0;k) dk_3 \quad (5.38)$$

and

$$\Pi(k_3)_{x_2=0} = \int_{-\infty}^{\infty} \Pi(0;k) dk_1 \quad (5.39)$$

obtained numerically with  $L = 1.5\delta_1$  (see figure 3).

By the convected hypothesis, deduced from the space-time correlation measurements of the fluctuating pressure field at the wall described in section (4.2), the time fluctuations observed at the fixed wall measuring station are dominated by the effect of the spatially varying pressure field sweeping past at speed  $U_c = 0.8U_1$ . Hence we may write

$$\Pi(\omega)_{x_2=0} = \Pi(k_1)_{x_2=0} \quad (5.40)$$

where  $\omega = U_c k_1$ . The resulting spectrum, for  $\sqrt{P^2}/\frac{1}{2}\rho U_1^2 = 2.2 C_f$ , is plotted against the Strouhal number  $\omega \delta_1 / U_1$  in figure 4.0 where a comparison is also made with the glider measurement and the calculated spectrum from the Green's function solution (see equation 5.19). The two calculated spectra are very similar in form except that the preferred form for  $f(r)$  in the latter calculation has resulted in closer agreement with experiment at higher frequencies.

Now the longitudinal and transverse spatial correlations of the wall pressure are given by

$$P(\xi_1, 0, 0)_{x_2=0} = \int_{-\infty}^{\infty} \Pi(k_1) e^{i\xi_1 k_1} dk_1 \quad (5.41)$$

and

$$P(0, 0, \xi_3)_{x_2=0} = \int_{-\infty}^{\infty} \Pi(k_3) e^{i\xi_3 k_3} dk_3 \quad (5.42)$$

These Fourier transforms have been obtained numerically from the previously calculated one-dimensional spectra  $\Pi(k_1)$  and  $\Pi(k_3)$ , by using a stand transform programme on a Pegasus digital computer. The resulting spatial correlation coefficients are plotted in figures 37 and 38.

Looking first at the longitudinal correlation, since by the convected hypothesis

$$\bar{P}(\xi_1, 0, 0; 0)_{x_2=0} \equiv \bar{P}(0, 0, 0; \tau) \quad (5.43)$$

where  $\xi_1 = U_c \tau$ , the measured autocorrelation in the glider experiment may be compared with the calculated longitudinal spatial correlation. The agreement is good and the negative loop as calculated is reproduced in the experiments. The autocorrelation coefficient measured in the  $2\frac{1}{2}$ in. x  $2\frac{1}{2}$ in. tunnel tests has also been plotted for comparison and shows how the presence of the spurious low-frequency signals effected the correlation shape.

The calculated transverse correlation coefficient was always positive, decreasing monotonically to zero in a length of order 1.5 - 2δ. The form of the curve corroborates the remarks made earlier that Harrison's (1958) transverse measurements and the present 2½in. x 2½in. tunnel measurements, which tended to a constant value at large ξ<sub>3</sub> separation, were in error because of extraneous disturbances in the tunnels.

The most important fact that has emerged from the calculations and the measurements is the connection between the spectrum Π(ω) the spatial correlation Π(o;k) and the boundary condition given in section (2.3). This condition was that

$$\Pi(o;k)_{k=0} = 0 \quad (5.44)$$

which is equivalent to

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(\xi_1, 0, \xi_3) d\xi_1 d\xi_3 = 0 \quad (5.45)$$

that is, the integral of the surface pressure correlation must vanish. However, for the turbulence-mean shear contribution, the condition (5.45) is satisfied because

$$\Pi(o;k)_{k_1=0} = 0 \quad (5.46)$$

which, from the inverse transform corresponding to equ.(5.41), implies that

$$\int_{-\infty}^{\infty} P(\xi_1, 0, 0) d\xi_1 = 0 \quad (5.47)$$

or by the convected hypothesis

$$\Pi(\omega)_{\omega=0} = 0 \quad (5.48)$$

and so  $\int_{-\infty}^{\infty} P(0,0,0;\tau) d\tau = 0 \quad (5.49)$

Thus, from equations (5.47) and (5.49), the integrals under both the longitudinal spatial correlation and the auto-correlation of the wall pressure, for the turbulence-mean shear contribution, must vanish; from equations (5.46) and (5.48) this is equivalent to both the one-dimensional wave-number spectrum  $\Pi(k_1)$  and the frequency spectrum  $\Pi(\omega)$  falling to zero at  $k_1 = 0$  and  $\omega = 0$  respectively.

It has been shown in Appendix B that the turbulence-turbulence contribution satisfies the boundary condition (5.45) as a result of weak negative loops in the longitudinal and transverse correlations although the frequency spectrum, already referred to in the previous calculation and plotted in figure 4.0, has finite power spectral density at  $\omega = 0$  but with value nearly two orders of magnitude lower than the peak value of the turbulence-mean shear contribution. It has been assumed throughout the present work, following Kraichnan, that the turbulence-turbulence term was small. Indeed, the calculations of Appendix B, and certainly the way in which the measured glider spectra fall at low frequencies (with the resulting negative loop in

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the autocorrelation) and the absence of a negative loop in the measured transverse spatial correlation, all corroborate this fact.