

State Dependent Regional Pole Assignment Controller Design for a 3-DOF Helicopter Model

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Abstract—The proposed method produces a state feedback law to assign all closed-loop poles of nonlinear systems in a specified disk. For linear systems, a state feedback control law can be easily designed to keep all closed-loop poles inside a specified disk since the locations of the poles are unique. However, its application to nonlinear systems is not so simple since the locations of the closed loop poles change due to the state varying or uncertain dynamics of the nonlinear systems. Therefore, this paper introduces a new pole placement method, named as State Dependent Regional Pole Assignment, for nonlinear systems. This proposed method produces a state dependent feedback control law, enabling the closed-loop system poles to be placed in a specified disk to achieve the desired control performance characteristics. The effectiveness of the method is tested on the 3 DOF Helicopter experimental setup. To verify its effectiveness, the experimental results of the nonlinear method are compared with those of the linear method.

I. INTRODUCTION

Pole assignment method using a state feedback control law is a popular and well-studied controller design approach for linear systems [1]–[5]. It is a well-known fact that shifting the prescribed region further left in the complex plane produces better control performance characteristics in terms of higher stability margin and lower settling time. Therefore, the feedback controller is designed so that the closed-loop poles of a linear system remain in a pre-specified and bounded region located in the left-half of the complex plane to meet both the stability and transient response requirements.

The defined region for pole assignment can be formed in different geometries such as disk, sector, ellipse, etc. The disk-shaped region is a widely-used geometry in literature because is describable by a radius and centre [6]–[10]. These design parameters must be selected appropriately to enable all the eigenvalues to be located inside the region and thereby satisfy the desired control performances.

The main problem in pole placement is to develop a control law shifting all the poles inside the defined region. In literature, several methods have been introduced to deal with this problem. The Linear Matrix Inequality (LMI) approach offers effective tools to improve the robustness of the closed-loop system for regional pole assignment [11]–[14].

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In a study [15], both H_2/H_∞ and guaranteed cost control method with pole placement constraints in a LMI region were considered and their performance were compared in simulations in terms of the attitude accuracy of a satellite orbiting around the Earth. In [16], another pole assignment method was introduced that extends a single LMI region to multiple LMI regions to control an unmanned aerial vehicle over a wide range of flight conditions. In some research, PID controller was utilized together with regional pole placement. A gain scheduled PID controller using robust pole placement was developed in [17] to ensure the stability of uncertain discrete-time linear parameter varying systems. Another study introduced a robust PI-D controller using LMI regions and H_2 quadratic cost function to guarantee robustness of uncertain linear systems [18].

An alternative method to tackle the regional pole placement control problem is to design a state feedback control law obtained by solving a Riccati Equation. In this approach, a special Riccati Equation is derived to locate the poles inside the desired region by only selecting the weighting matrices of the Riccati Equation [2], [19]. Furuta and Kim [2] also examined the optimality and robustness properties of their Riccati Equation based control approach, named D-pole assignment, for both continuous and discrete linear systems. They also clearly presented the technical issues related to the D-pole assignment approach and the proofs of their approaches.

In this study, a State Dependent Regional Pole Assignment method for nonlinear systems is introduced to design a state feedback control law allowing the closed-loop system poles of nonlinear systems in a prescribed region. This study extends the regional pole placement method for linear systems proposed in [2] to nonlinear system. To the best of the author's knowledge, this is the first time that a regional pole placement method using a state dependent Riccati Equation is proposed to ensure the desired stability and control performance for nonlinear systems. The effectiveness of the proposed method is validated through the experiments conducted on a 3-DOF Helicopter experimental setup; a commercial platform produced by Quanser specifically to test developed control algorithms in real time [20]–[22]. The proposed control approach is separately designed for different disk-shaped regions, each formed by different radius and center location. To verify its performance, it is compared against the traditional linear algorithm.

The structure of the paper is as follows; Section II explains linear and nonlinear D-Pole Assignment Algorithm. Section III presents the model of the 3-DOF Helicopter experimental

setup and its controller design. Experimental results are introduced in Section IV. Finally, the conclusions are given in Section V.

II. STATE DEPENDENT RICCATI EQUATION CONTROLLER

Consider a nonlinear system taking a form of

$$\dot{x}(t) = f(x) + B(x)u(t), \quad x(0) = x_0 \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector. $f(x)$ is a continuously differentiable vector-valued function, and the origin is an equilibrium point of the system (1) with $f(0) = 0$ when $u(t) = 0$. Under the assumptions that $f(0) = 0$ and $f(x)$ is continuously differentiable [23], $f(x)$ can be factorized as

$$f(x) = A(x)x$$

and then the nonlinear system (1) becomes

$$\dot{x}(t) = A(x)x + B(x)u(t), \quad x(0) = x_0 \quad (2)$$

where $A(x)$ and $B(x)$ are the State Dependent Coefficient (SDC) matrices and $\{A(x), B(x)\}$ are assumed to be pointwise controllable, thereby stabilizable pair for all x . Then, the state-feedback control law, i.e. $u = -K(x)x$, is applied to achieve the desired control performance and stability margin. $K(x)$ can be computed by

$$K(x) = R^{-1}(x)B^T(x)P(x)$$

where $P(x)$ is a positive definite matrix obtained by solving a differential state-dependent Riccati Equation given as

$$P(x)A(x) + A^T(x)P(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \quad (3)$$

where $Q(x)$ is a pointwise semi-positive definite weighting matrix and $R(x)$ is a pointwise positive definite weighting matrix.

The success of the SDRE controller highly depends on the proper selection of the weighting matrices due to the fact that the different pair of $Q(x)$ and $R(x)$ corresponds to the different solutions of the Riccati Equation (3). Since no generally accepted method yet exists to determine the weighting matrices, their selection is a painful and time-consuming process in the consideration of fulfilling the control performance and stability requirements.

To address this problem, the following section introduces a novel regional pole placement approach. A state-feedback law for the nonlinear systems is developed to ensure that the closed-loop system poles of the nonlinear systems can be kept in a desired region. Thus, the desired control objectives can be accomplished.

III. STATE DEPENDENT REGIONAL POLE ASSIGNMENT METHOD FOR NONLINEAR SYSTEMS

In this proposed approach, the state-feedback law is tailored so that it can keep the closed-loop system poles of the nonlinear systems (2) in a bounded disk described by the following definition

Definition 1: A bounded disk D is a closed circular region with the centre at α constrained on the negative real axis and the radius of r , as shown in Fig. 1.

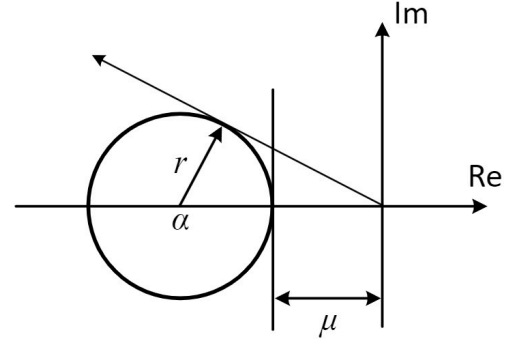


Fig. 1. Disk D in the left of the complex plane. μ denotes the stability margin.

The disk parameters (α and r) must be selected in accordance with the desired control objectives in terms of the stability margin (μ in Fig. 1) and the time response characteristics such as maximum overshoot and settling time. To meet this need, this study proposes a state dependent regional pole placement method that constrains the poles of the closed-loop nonlinear system to lie within a desired disk. This approach, also inspired by the previous study [2], designs a feedback law based on solving a modified Riccati equation. To pave the way for this new approach, the general assumption accepted for the implementation of SDRE controller is also made here and given as follows:

Axiom 1 (see [23]): In nonlinear case, $A(x)$ and $B(x)$ in (2) are computed at each time instant t_k to produce a linear-like system comprising of $A(x(t_k)) = A(x_k) = A_k$ and $B(x(t_k)) = B(x_k) = B_k$.

Lemma 1 (see [2]): Let the following matrix equation be defined as

$$\alpha A_k^* P_k + \alpha P_k A_k - A_k^* P_k A_k - (\alpha^2 - r^2) P_k = Q_k \quad (4)$$

where Q_k is a semi-positive definite matrix and $*$ denotes the conjugate transpose. Let the eigenvalues of $A(x)$ at each time instant t_k define as $\lambda(A_k)$. Then $\lambda(A_k) \forall t_k$ remain inside the specified disk if and only if a positive definite matrix $P_k \forall t_k$ exists.

Proof: Define λ_k and v_k as the eigenvalue and the eigenvector of A_k , respectively. Then,

$$A_k v_k = \lambda_k v_k, \quad v_k^* A_k^* = \bar{\lambda}_k v_k^* \quad (5)$$

By using v_k , v_k^* and (4), the following equation can be obtained

$$v^* \{ \alpha A_k^* P_k + \alpha P_k A_k - A_k^* P_k A_k - (\alpha^2 - r^2) P_k \} v = v^* Q_k v \quad (6)$$

Substituting (5) into (6) yields

$$(-\alpha(\bar{\lambda}_k + \lambda_k) + \bar{\lambda}_k \lambda_k + (\alpha^2 - r^2)) v_k^* P_k v_k = -v_k^* Q_k v_k \quad (7)$$

Given $\lambda_k = \sigma_k + i\omega_k$, (7) becomes

$$(-2\alpha\sigma_k + \sigma_k^2 + \omega_k^2 + \alpha^2 - r^2) v_k^* P_k v_k = -v_k^* Q_k v_k \quad (8)$$

and (8) can be rewritten as

$$((\sigma_k - \alpha)^2 + \omega_k^2 - r^2) v_k^* P_k v_k = -v_k^* Q_k v_k \quad (9)$$

Since $P_k > 0$ and $Q_k > 0$, this yields

$$(\sigma_k - \alpha)^2 + \omega_k^2 - r^2 < 0 \quad (10)$$

This means that $\lambda(A_k)$ stays inside the disk D at each time instant t_k . Accordingly, $\lambda(A(x))$ also stay inside the disk D for all the time range. ■

Following Lemma 1, the closed-loop system matrix $(A_k - B_k K_k)$ has its eigenvalues in a desired disk at each time instant t_k if the following theorem is satisfied.

Theorem 1: Consider the following matrix equation

$$\begin{aligned} & -\alpha(A_k - B_k K_k)^* P_k - \alpha P_k (A_k - B_k K_k) \\ & + (A_k - B_k K_k)^* P_k (A_k - B_k K_k) \\ & + (\alpha^2 - r^2) P_k = -Q_k \end{aligned} \quad (11)$$

where Q_k is a positive definite. If a positive definite P_k is obtained, the eigenvalues of the closed-loop matrix $(A_k - B_k K_k)$ are located in a desired disk at each time instant t_k .

To find the proposed state-feedback law, the following theorem is presented.

Theorem 2: The feedback control law

$$u_k = -(r^2 R + B_k^\top P_k B_k)^{-1} B_k^\top P_k (A_k - \alpha I) x_k \quad (12)$$

locate all eigenvalues inside a desired disk if P_k is positive definite symmetric solution of the Riccati equation given as,

$$\begin{aligned} P_k = & \frac{(A_k - \alpha I)^\top}{r} P_k \frac{(A_k - \alpha I)}{r} + H_k^\top H_k \\ & - \frac{(A_k - \alpha I)^\top}{r} P_k B_k (r^2 R + B_k^\top P_k B_k)^{-1} \times \\ & B_k^\top P_k (A_k - \alpha I) r \end{aligned} \quad (13)$$

where the weighting matrix $R > 0$ is selected arbitrarily, and H_k is a matrix such that the pair (A_k, H_k) is observable.

Proof: Substituting (12) into (13) produces

$$\begin{aligned} & -\alpha(A_k - B_k K_k)^\top P_k - \alpha P_k (A_k - B_k K_k) \\ & + (A_k - B_k K_k)^\top P_k (A_k - B_k K_k) \\ & + (\alpha^2 - r^2) P_k = -r^2 (K_k^\top R K_k + H_k^\top H_k) \end{aligned} \quad (14)$$

where

$$K_k = (r^2 R + B_k^\top P_k B_k)^{-1} B_k^\top P_k (A_k - \alpha I) \quad (15)$$

and we can write $Q_k = r^2 (K_k^\top R K_k + H_k^\top H_k)$. Finally, using Lemma 1 and Theorem 1, it is proven that the eigenvalues of $A_k - B_k K_k$ locate in the desired disk D at each time instant t_k . ■

IV. 3-DOF HELICOPTER DYNAMICS AND CONTROLLER DESIGN

The 3-DOF helicopter experimental setup of Quanser Inc. is widely used to test the developed control algorithms in real-time. Test setup is free to rotate around three mutually orthogonal axes called elevation, travel and pitch axes, as seen in Fig. 2. θ , ϕ and ψ denote the rotations around the elevation, pitch and travel axes. Two DC motors, named as Back Motor and Front Motor, generate the necessary forces for rotating the helicopter body. To perform the rotation around the elevation axis, a collective force that is the sum of two equal forces produced by the motors in the same direction is applied to the helicopter body, whereas for the rotation around the travel axis, a cyclic force is needed. It is worth reminding that the system that involves only and three degrees of freedom results in an underactuated system.

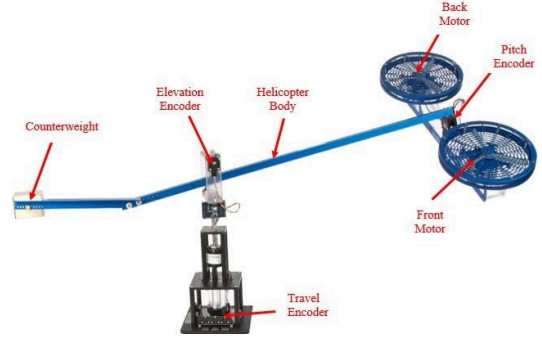


Fig. 2. 3-DOF Helicopter Motions [24]

The nonlinear model [25] can be expressed by

$$\begin{aligned} \ddot{\theta} &= -d_1 \dot{\theta} - d_2 \sin(\theta) + d_3 \tau_{coll} \cos(-\phi) \\ \ddot{\phi} &= -b_1 (\dot{\phi}) + b_2 \sin(\phi) - b_3 \tau_{cyc} \\ \ddot{\psi} &= -a_1 \dot{\psi} + a_2 (\gamma \tau_{coll} + 1) \sin(-\phi) \\ \dot{\tau}_{cyc} &= -c_1 \tau_{cyc} + 0.5 c_2 (V_b - V_f) \\ \dot{\tau}_{coll} &= -e_1 \tau_{coll} + 0.5 e_2 (V_b + V_f) \end{aligned} \quad (16)$$

where τ_{cyc} , τ_{coll} , V_b and V_f denote the cyclic thrust, the collective thrust, the back motor voltage and the front motor voltage, respectively. The second order differential equations represent the Euler dynamics of the rotations while the first order differential equation represent motor dynamics. The free body diagram and parameters are seen in Fig.3 and Table I.

Let the state vector be

$$[x_1 \ x_2 \ \dots \ x_8]^\top = [\theta \ \phi \ \psi \ \dot{\theta} \ \dot{\phi} \ \dot{\psi} \ \tau_{cyc} \ \tau_{coll}]^\top \quad (17)$$

and the control inputs generated by the back and front motors generate the control inputs be

$$[u_1 \ u_2]^\top = [V_f \ V_b]^\top \quad (18)$$

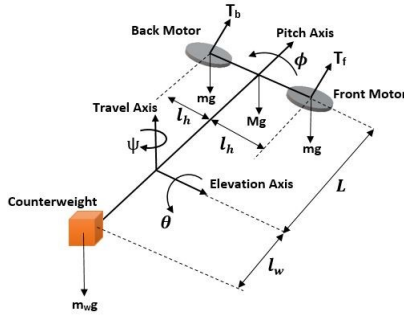


Fig. 3. Free-body diagram of 3-DOF Helicopter [22]

TABLE I
PARAMETER VALUES OF THE HELICOPTER MODEL [25]

Parameter	a_1	a_2	b_1	b_2	b_3	c_1	c_2
Value	0.2517	0.2105	0.3290	1.5664	16.200	7.3200	1.000
Parameter	d_1	d_2	d_3	e_1	e_2	γ	
Value	0.1011	0.5040	1.3400	6.1600	1.000	4.000	

in terms of back and front motor voltage. Substituting (18) and (17) into Equation (16) yields

$$\begin{aligned}
 \dot{x}_1 &= x_4 \\
 \dot{x}_2 &= x_5 \\
 \dot{x}_3 &= x_6 \\
 \dot{x}_4 &= -d_1 x_4 - d_2 \sin(x_1) + d_3 x_8 \cos(-x_2) \\
 \dot{x}_5 &= -b_1 x_5 + b_2 \sin(-x_2) - b_3 x_7 \\
 \dot{x}_6 &= -a_1 x_6 + a_2 (\gamma x_8 + 1) \sin(-x_2) \\
 \dot{x}_7 &= -c_1 x_7 + 0.5 c_2 (u_2 - u_1) \\
 \dot{x}_8 &= -e_1 x_8 + 0.5 e_2 (u_1 + u_2)
 \end{aligned} \tag{19}$$

Now, (19) can be rewritten by using SDC factorization and the following SDC matrices can be used

$$A(x) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -d_2 L(x) & 0 & 0 & -d_1 & 0 & 0 & 0 & N(x) \\ 0 & b_2 M(x) & 0 & 0 & -b_1 & 0 & -b_3 & 0 \\ 0 & H(x) M(x) & 0 & 0 & 0 & -a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_1 \end{bmatrix}$$

and

$$B(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -0.5 c_2 & 0.5 c_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 e_2 & 0.5 e_2 \end{bmatrix}^T$$

where $M(x) = \sin(-x_2)x_2^{-1}$, $L(x) = \sin(x_1)x_1^{-1}$, $N(x) = d_3 \sin(-x_2)$ and $H(x) = a_2(\gamma x_8 + 1)$ to derive the suboptimal control law. In this study, the algorithm is designed in order to track the given step commands in terms of elevation and travel rotations. In this reference tracking problem, the helicopter body is left free to roll around the pitch axis due to the underactuated characteristic of the test equipment but the

travel motion is bounded within some range of ϕ to ensure the stability of the system. For zero steady state error in the reference tracking, the system matrices and output matrix must be augmented as

$$\tilde{A}(e) = \begin{bmatrix} A(x) & 0 \\ -C & 0 \end{bmatrix} \text{ and } \tilde{B}(e) = \begin{bmatrix} B(x) \\ 0 \end{bmatrix}$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

which leads to the nonlinear error dynamics given by

$$\dot{e} = \tilde{A}(e)e + \tilde{B}(e)u$$

In the next section, the experimental results are presented by comparing the control performance of the proposed method for nonlinear system to that of the method developed by Furuta and Kim [2] for linear systems.

V. EXPERIMENTAL RESULTS

The state dependent regional pole algorithm for nonlinear models and the regional pole assignment algorithm for linear models are tested on the 3-DOF helicopter experimental setup. To achieve a fair comparison, the same desired trajectory and the controller parameters, Q and R , are used in all experimental trials. Q and R are arbitrarily selected to be

$$Q = \text{diag}([40 \ 1 \ 40 \ 40 \ 1 \ 68 \ 1 \ 1 \ 50 \ 50])$$

and $R = \text{diag}([1 \ 1])$. The initial states of each experimental trial are set to be $\theta_0 = -15^\circ$, $\phi_0 = 0^\circ$ and $\psi_0 = 0^\circ$. Each trial lasts for 80 s. The desired elevation command is $\theta_d = 0^\circ$ during the whole execution time and the desired travel command ψ_d is set to be 0° for the first 25 s, then 30° for the following 35 s and 15° for the final 20 s. The linear case and the nonlinear case are compared with each other when the disk with the radius of 2 is centred at $\alpha = -2.5$.

Fig. 4 gives the time responses of the system for the linear and nonlinear cases while moving in elevation axis. From Fig. 4, it can be seen that the nonlinear state dependent algorithm is able to achieve shorter settling time and smoother response than the algorithm for the linear case. At 25 s and 60 s, the desired travel commands are applied and therefore the responses are disturbed at these time instants.

Fig. 5 illustrates the tracking control response in the travel axis. Fig. 5 also reveals the same result as Fig. 4 that the nonlinear state-dependent algorithm produces a shorter settling time and smoother time response. In the nonlinear case, the time response reaches the steady state after 10 s. However, the steady state error converges to zero after 17 s under the control of the classical pole assignment approach.

Fig. 6 and Fig. 7 show the locations of the closed loop system poles for the linear and nonlinear cases, respectively. The augmented system in Eq. 51 has a total of ten poles. The dashed circle indicates the boundary of the desired circle-shaped region centred at -2.5 and with the radius of 2 for each case. In Fig. 6, the locations of the poles remain fixed throughout the entire experiment for the linear case. Fig. 7 shows how the poles of the system are shifted

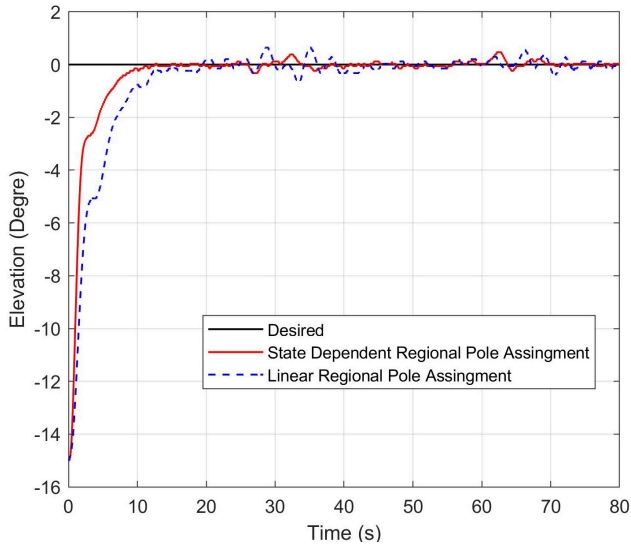


Fig. 4. Elevation responses of the linear and nonlinear models

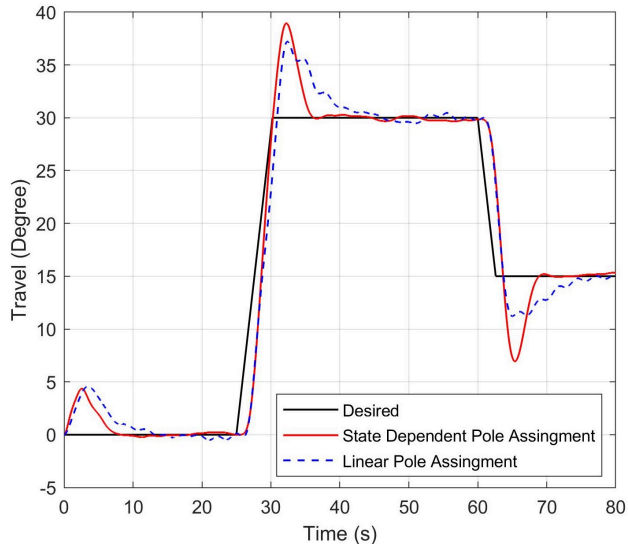


Fig. 5. Travel responses of linear and nonlinear model

inside the desired region at each time step. Since the system matrices are dependent on the system states, the closed-loop eigenvalues are placed at different locations at each time step.

The effect of the disk parameters on the control performance is also investigated in the state dependent pole assignment method. To examine the effects of the centre location, three experiments are conducted where the disk radius is set to be constant and the disk centre is moved away from the complex plane to the left. Fig. 8 and Fig. 9 give the time responses of the system where the disk centre α is selected to be -2, -2.3 and -1.8 and the disk radius is kept constant at 1.5 for each disk centre. From Fig. 8 and Fig. 9, it can be seen that moving further away the disk center to the left on the complex plane results in shorter settling time.

Fig. 10 and Fig. 11 present the effects of a change in the radius of the disk on the transient response characteristics. In these experiments, the disks are located at the same

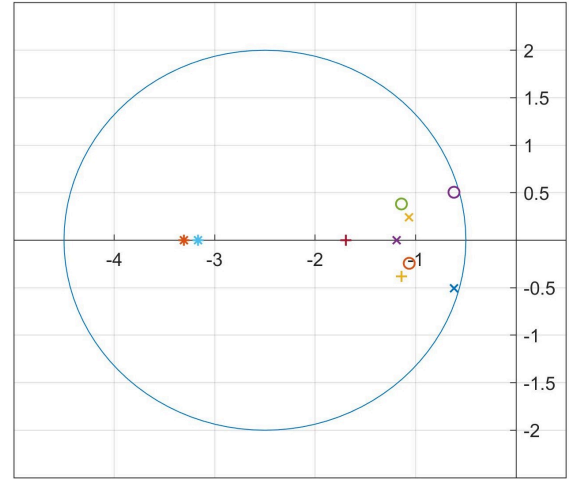


Fig. 6. Linear algorithm pole locations

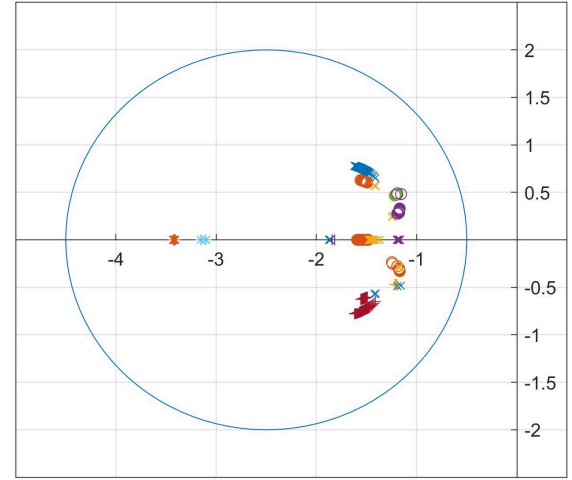


Fig. 7. State dependent pole locations

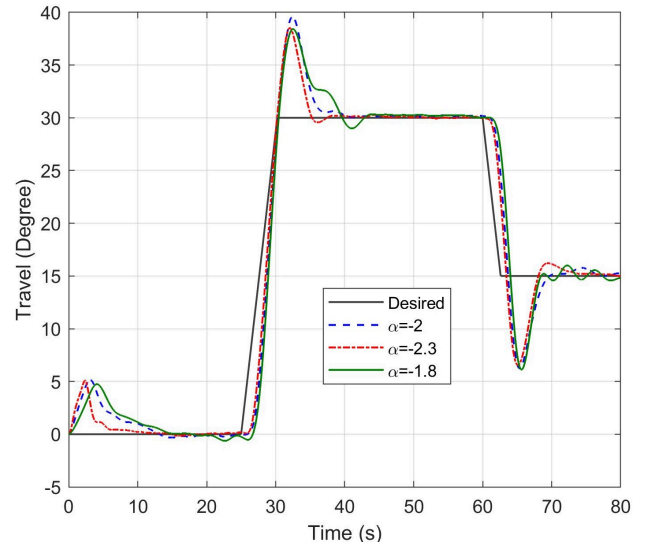


Fig. 8. Travel response for different disk center

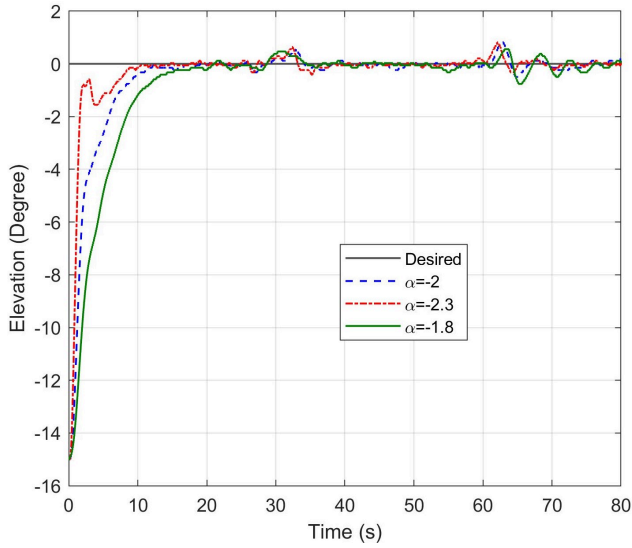


Fig. 9. Elevation response for different disk center

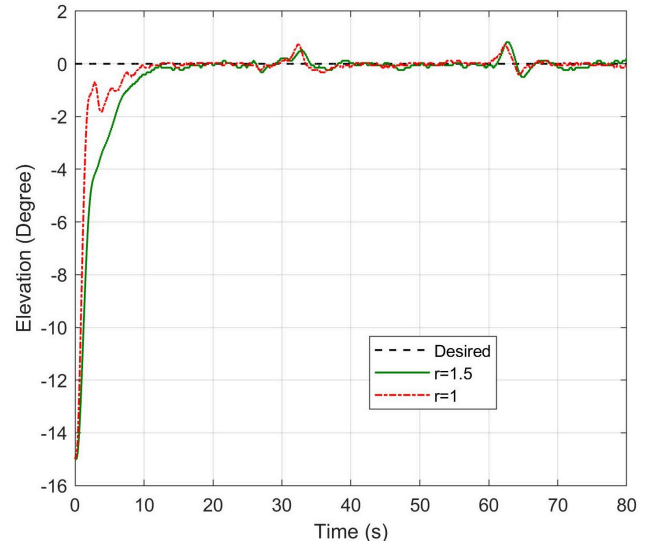


Fig. 11. Elevation response for different disk radius

centre point but they have different radii. As shown in these figures, smaller radius produces shorter settling time and faster system response. In addition, the stability margin of the system is increased as the radius of the disk is reduced since the point on the desired circle, which intersects the real axis and is closest to the imaginary axis of the complex plane, is shifted away to the left.

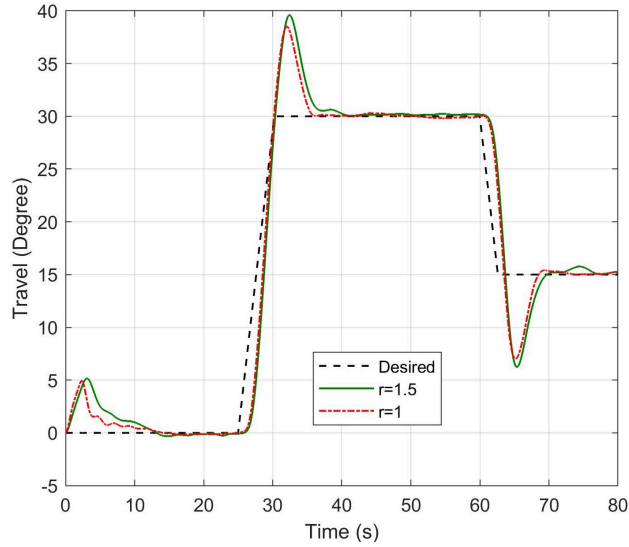


Fig. 10. Travel response for different disk radius

VI. CONCLUSIONS

State Dependent Regional Pole Assignment based controller is designed and applied to a 3-DOF Helicopter Experimental Setup. The method is implemented using different disk parameters. In addition, the proposed method is compared with Linear Regional Pole Assignment. The test results reveal that State Dependent Form is more effective for settling time. Because, system matrices and dynamics of the

system change with time. On the other hand, State Dependent Regional Pole Assignment need more computational loads because the algorithm is solved at each time step. Disk parameters are also examined. If the radius of the disk is kept constant and its centre is shifted to the left from the virtual axes, or if the centre is kept constant and the radius is reduced, the system responses seem to improve. As a result, state dependent pole assignment method may be used for nonlinear systems to obtain better behaviour and control action. In the future works, pole margin and disk parameters based on state will be studied.

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