

WEAK COMPRESSION WAVES IN RELAXING GASES

by

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ABSTRACT

Studies have been made of the structure of weak compression (shock) waves in relaxing gases. These studies have been primarily concerned with the development of separate theoretical and experimental techniques. These techniques are not inter-related in general except in the overall context of vibrational relaxation.

The theoretical studies have been concerned with the influence on the structure of weak normal shock waves of translational non-equilibrium, bimodal relaxation and second order unimodal relaxation. The affect of translational non-equilibrium on the relaxation process has been studied by forming an asymptotic expansion in the ratio of the viscous length to the relaxation length. The perturbation scheme was singular and required the application of the method of matched asymptotic expansions. Bimodal relaxation has been studied by forming an asymptotic expansion in the ratio of the energy of the secondary vibrational mode to the total vibrational energy.

The addition of a second order term to the rate equation describing the behaviour of a single vibrational mode has also been studied by forming an asymptotic expansion. In this case, the perturbation parameter was the ratio of the two relaxation times concerned.

The experimental studies have been concerned with the production and study of weak normal shock waves in the Cranfield Institute of Technology 2" shock tube. A time resolved quantitative schlieren system has been used for the study of the weak normal shock waves. This particular system had been developed previously for this purpose, and further developments and refinements have been made to it.

Experimental studies have been made with the schlieren system of the structure of strong incident shock waves in carbon dioxide. The vibrational relaxation time of carbon dioxide determined in this way for translational temperatures from 300 K to 1200 K has been found to be in reasonable agreement with measurements made elsewhere.

A technique has been developed for the production of weak incident shock waves in the shock tube, which involved the positioning of a perforated plate in the channel of the shock tube. The vibrational relaxation time of carbon dioxide determined in this way for translational temperatures of approximately 300^o K has been found to be in good agreement with measurements made elsewhere. Good agreement has also been obtained between the experimentally measured density gradient profiles and theoretical profiles. The curvature of the shock waves obscured the transition from a fully dispersed to a partly dispersed shock wave.

Further improvements and refinements have been made to a technique which had been developed previously for the propagation of weak normal shock waves through the reflected shock region of the shock tube flow. This technique was used to study the behaviour of fully dispersed shock waves at high temperatures. The relaxation time of carbon dioxide determined in this way for temperatures from 300^o K to 600^o K has been found to be in good agreement with measurements made elsewhere.

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1. GENERAL INTRODUCTION.

1.1. Vibrational Energy

The excitation and de-excitation of the vibrational modes of gas molecules is but one aspect of the real dynamic behaviour of gases. All gases exhibit real gas behaviour; only the magnitude of the effects vary. It is fortuitous that at normal temperatures and pressures the major constituents of air, namely oxygen and nitrogen, exhibit negligible real gas effects. At high ambient temperatures or in high velocity flight, where large amounts of the kinetic energy of the flow are converted into thermal energy in the vicinity of the vehicle, these real gas effects cease to be negligible and must be taken into account in any rigorous evaluation of the aerodynamics of the flow concerned. The particular real gas phenomenon of interest here is the manner and rate at which energy is re-distributed amongst the vibrational modes of a polyatomic gas molecule in a non-equilibrium situation. The studies which have been conducted have been concerned with general aspects of vibrational behaviour rather than the evaluation of the properties of any particular gas.

In general, the rotational behaviour of a polyatomic gas cannot be simply described. However, as carbon dioxide belongs to the class of linear polyatomic molecules, that is, its nuclei lie on average along a straight line, the rotational behaviour can be described by a classically excited rigid rotator model.

The vibrational behaviour of carbon dioxide can be described by either a truncated harmonic oscillator or an anharmonic oscillator; depending on the accuracy required and the vibrational energy levels. The carbon dioxide molecule comprises two oxygen atoms and a carbon atom connected together by chemical bonds, as indicated in figure 1.1. It can be seen that the molecule has two bending modes, a symmetric stretching mode and an asymmetric stretching mode.

Each of these modes may be described by either of the above models. However, the bending modes of the carbon dioxide molecule are in Fermi resonance with the symmetric stretching mode, so, for dynamic considerations, these modes can be considered to be acting together.

The simple harmonic oscillator does not take account of the real behaviour of the atoms either at large separation distances; approaching dissociation, or very small separation distances, when the atoms should collide. Nevertheless, the truncated harmonic oscillator does provide an excellent description of the behaviour of the molecule for low vibrational energy levels.

The potential energy curves for an harmonic oscillator and an anharmonic oscillator are shown in figure 1.2. The anharmonic curve is due to Morse. It can be seen that the anharmonic model takes account of the approach to dissociation and the limiting minimum separation distance of the atoms.

For the purposes of calculating the vibrational energies in the present studies, the tabulated data of Hilsenrath et al (Ref.1.1.) has been fitted with polynomial curves.

The description given by the truncated harmonic oscillator model of the energy content of a vibrational mode is as follows:

$$\epsilon_{\text{vib}} = \frac{R_g \theta_{\text{vib}}}{\exp\left(\frac{\theta_{\text{vib}}}{T}\right) - 1} \quad 1.1.$$

where θ_{vib} is the characteristic vibrational temperature of the appropriate mode. For carbon dioxide, these temperature are,

$$\begin{aligned} \theta_{\text{vib}} \quad (\text{bending}) &= 959^\circ\text{K} \\ \theta_{\text{vib}} \quad (\text{symmetric stretch}) &= 1920^\circ\text{K} \\ \theta_{\text{vib}} \quad (\text{asymmetric stretch}) &= 3380^\circ\text{K} \end{aligned}$$

The quantum description of the dynamic behaviour of gas molecules given by so called 'Master Equation' (page 298 of Ref 1.2.) is considerably simplified for the case of a simple harmonic oscillator. This was first noted by Landau and Teller (Ref. 1.3.) and the following equation for a collection of simple harmonic oscillators was first given by Bethe and Teller (Ref. 1.4.)

$$\frac{d}{dt} \epsilon_{\text{vib}} = - \frac{1}{\tau} \left\{ \epsilon_{\text{vib}} - \epsilon_{\text{vib}} (\text{equil.}) \right\} \quad 1.2.$$

where τ is the relaxation time, or time constant, of the mode and ϵ_{vib} , its energy. It can be seen that the equation states that the rate of change of energy in the mode is proportional to its departure from equilibrium.

It can be shown from a consideration of the transition probabilities that the relaxation time can be approximated by the following relation (Page 315 of Ref. 1.2.).

As a result of this solution, most relaxation time data is presented on the Landau-Teller plot of
in (p) versus $T^{-1/3}$

1.2. The Study of Vibrational Behaviour

The vibrational behaviour of a gas molecule can be studied by subjecting the molecule to a known input of energy and observing the response of the vibrational modes; directly or indirectly. There are many forms of input which can be used, but the two most commonly used are a sinusoidal and a step input of energy. In the case of the sinusoidal input, the characteristics of the vibrational modes can be determined from the attenuation and phase shift in the response of the vibrational modes. In the case of the step input, the transient response of the vibrational modes is observed.

The earlier studies of vibrational relaxation made use of the first of these methods, Pierce (Ref. 1.5). Ultrasonic sinusoidal waves were driven through the gas by means of a piezo-electric crystal, which was made to oscillate at a chosen frequency. The waves were reflected from a flat plate positioned a short distance from the crystal to form a system of standing waves between the crystal and the plate. The attenuation and speed of propagation of these waves were determined for a range of frequencies.

The form of the result which was obtained is shown in Figure 1.3. The frequency of maximum attenuation per wavelength was the inverse of the relaxation time. The majority of the results which were obtained in this way were restricted to near room temperature conditions and only recently have relaxation times been determined in this way for elevated temperatures, Carnevale (Ref. 1.6).

The step input of energy to the gas molecule is obtained by way of the shock wave. However, the shock wave is only one feature of supersonic flows. As the strength of a disturbance in a gas is increased it becomes possible to distinguish between compressions and expansions. This is of great value in this context, because it enables the excitation and de-excitation of vibrational modes to be studied separately. One, two and three dimensional flows can be generated, but it is advantageous for practical and analytical purposes to keep the flow field as simple as possible and to use the minimum number of dimensions required for a stable flow situation. A steady compression, or shock wave, can be generated in a one-dimensional flow, whereas an expansion is essentially unsteady and requires a minimum of two dimensions for stability.

The form of the shock waves which can be produced in a vibrationally relaxing gas have been categorised by Lighthill (Ref. 1.7) into partly dispersed and fully dispersed shock waves. Sketches of typical velocity profiles for these types waves are presented in Figure 1.4. A partly dispersed shock wave is defined as one in which the speed of the gas flowing into the wave is greater than the speed of sound of the gas calculated with all of the vibrational modes frozen. It can be seen from Figure 1.4 that the partly dispersed shock wave consists of a thin diffusion resisted shock front followed by a broad relaxation region. This characteristic of the partly dispersed shock wave arises from the fact that the vibrational modes of a gas molecule require several thousands of molecular collisions in order to equilibrate, whereas the translational and rotational modes require only a few collisions. It can be assumed for most studies of vibrational relaxation that the vibrational modes remain frozen through the diffusion resisted shock front and that they equilibrate with the translational and rotational modes in the relaxation region.

If the velocity of the gas flowing into the shock wave is less than the speed of sound of the gas calculated with the vibrational modes frozen, then the diffusion resisted shock front ceases to exist and the wave becomes fully dispersed as shown in Figure 1.4. The structure of the fully dispersed shock wave is determined solely by the balance between the convective steepening of the wave and the relaxation of the vibrational modes. The thermodynamic and velocity gradients within the fully dispersed shock wave are so small that diffusive effects can be neglected.

The expansion is inherently more unstable than the shock wave in that there is no convective steepening present, and the expansion continues to grow with distance or time from its point of origin. The structure of the expansion is therefore very sensitive to disturbances at its point of origin. The expansion is also far less sensitive to vibrational relaxation than the shock wave. Consider the sketch of an unsteady expansion which is shown in Figure 1.5. For a given upstream velocity, u_1 , the downstream velocity, u_2 , will depend upon the degree to which the vibrational modes equilibrate with the translational and rotational modes.

The two extremes of frozen and equilibrium flow are shown. The velocity profile must lie between these two extremes regardless of the vibrational relaxation times. Clearly, the profile is very insensitive to the relaxation time concerned.

Shock waves and both steady and unsteady expansions can be produced and studied in gases at high temperatures by the use of the shock tube. The strong incident shock wave has provided a means of studying many gas-dynamic rate processes, not least of which has been vibrational relaxation. Wedges have been placed in the flow of hot gas behind the incident shock wave to produce two-dimensional shock waves and expansion fans, and multiple diaphragm techniques have been used for the production of unsteady expansions. The reflected shock wave has also been used for the study of rate processes.

Many techniques have also been developed for the observation and measurement of these rate processes.

The details of these techniques for the production and study of rate processes in the shock tube will not be entered into here. The relevant aspects of these are reviewed and discussed in the appropriate sections herein.

1.3 Present Studies

The present studies have been primarily concerned with the gas dynamics of flows in relaxing gases and not with the determination of the molecular properties of the gases.

The two main aims of the present studies were the extension of the basic theories governing the dynamics of relaxing gases and the investigation of experimental techniques for the production and study of relaxing gas flows.

The theoretical and experimental studies are not necessarily related, except in the overall context of vibrationally relaxing gas flows. However, where applicable, and bearing in mind the limitations of both the theoretical and the experimental studies, comparisons have been made between the results of the various studies.

In addition, an essential part of the assessment of any experimental results is the determination of relaxation times for comparison with the results of other researchers.

The studies have been restricted to vibrational behaviour in normal shock waves. The theoretical studies have been further restricted to weak normal shock waves in order to obtain closed form analytical solutions, but these solutions will also be qualitatively correct for strong shock waves. Normal shock waves have been produced experimentally in carbon dioxide in the Cranfield Institute of Technology 2" shock tube.

An assumption which is normally made with regard to the shock wave in a vibrationally relaxing gas is that transport effects can be neglected in the analysis of the relaxation process. It is assumed that the thermodynamic gradients are so small in the relaxation resisted parts of the wave as to render diffusion negligible, and that the diffusion resisted front of the partly dispersed shock wave is negligibly thin. An analysis has therefore been developed which takes into account the first order effects of diffusion. This has made use of a perturbation scheme based on the ratio of the viscous length to the relaxation length of the gas, which is a small quantity for most gases. This analysis together with some theoretical profiles is presented in Section 2.2.

Previous experimental analyses of the behaviour of gases with more than one active vibrational mode have assumed that the vibrational modes behave as a single mode having the total vibrational energy of the molecule and governed by a single rate equation. Theoretical analyses have been based on the calculation of special cases, see Clarke and Rodgers (Ref. 1.8), and closed form analytical solutions restricted to limiting cases of infinite relaxation times, Becker (Ref. 1.9) for example.

It has been observed that the vibrational behaviour of certain gases is dominated by one of the vibrational modes. This arises because the amount of energy stored in the dominant mode is considerably in excess of that in the other modes. A bimodal analysis has therefore been developed using a perturbation scheme based on the ratio of the energy in the secondary vibrational modes to the total vibrational energy. This allows the relaxation times to be arbitrarily chosen. A first order correction to the unimodal behaviour has been developed in this way. This analysis is presented in Section 2.3.

A further assumption which has been made use of extensively in vibrational studies is that the behaviour of the vibrational modes can be described by the linear rate law derived by Bethe and Teller (Ref. 1.4). The experimental findings of Zienkiewicz and Johannesen (Ref. 1.10) had indicated that this rate law may not have been applicable. A first order correction to this behaviour has been derived by assuming that the linear rate law is the first term of an expansion of a more complex function and determining the second term. This analysis is presented in Section 2.4.

A technique was required for the observation and measurement of the normal shock waves produced in the shock tube. At the outset of the work, a Mach Zehnder interferometer and a time resolved schlieren system were available for use with the shock tube. A review was made of other available techniques and it was concluded that the time resolved schlieren system would be the most applicable to the present studies. The reasons leading to this choice, a description of the schlieren system, an analysis of its performance, and the developments which have been made to it are described in Section 3.1.

Extensive use has been made of the relaxation regions of strong incident shock waves for the study of vibrational relaxation, and many measurements have been made of the relaxation time of carbon dioxide in this way. The performance of the schlieren system was evaluated by applying it to the study of such relaxation regions and comparing the results with those of other researchers. A better understanding of the problems associated with the study of strong incident shock waves was obtained, and techniques for the improvement of the observation and measurement of these regions have been investigated. These studies are presented in Section 3.2.

At the outset of this work, doubts existed regarding the behaviour of the relaxation regions behind the strong incident shock waves, because, as noted above, the results obtained by Zienkiewicz and Johannesen (Ref. 1.10) indicated that the relaxation time might depend on the departure from equilibrium within the relaxation regions. The departures from equilibrium within fully dispersed shock waves are inherently less than those in the relaxation regions of strong incident shock waves. Therefore, the vibrational behaviour should be much closer to that given by the 'heat bath' theory of Bethe and Teller (Ref. 1.4).

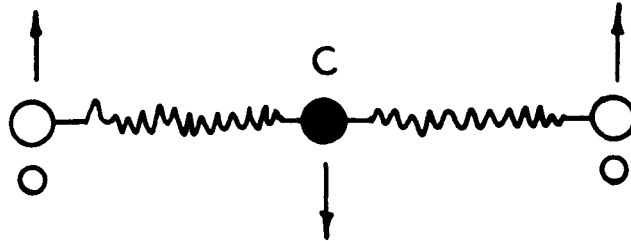
The fully dispersed shock waves also had the advantage that their structures were inherently more sensitive to vibrational behaviour, while changes in the relaxation time(s) through them could be neglected. A major part of the present experimental studies has therefore been concerned with the production and study of fully dispersed shock waves propagating through regions of elevated temperatures.

As a preliminary to these studies, weak incident shock waves have been produced and studied. These waves have been used to study weak partly dispersed shock waves and fully dispersed shock waves, including the transition between the two. The small change in the translational temperature through these waves has allowed the relaxation time to be taken as a constant through them. This has enabled direct comparisons to be made between theoretical and experimental profiles without the added complication of variations in the relaxation time through them. A double diaphragm technique has been developed for this aspect of the experimental studies. The development of this and the results which have been obtained with it are presented in Section 3.3.

A technique had been developed previously for the propagation of fully dispersed shock waves through gases at elevated temperatures (Ref. 1.11). This technique involved the use of a porous end wall in the channel of the shock tube. As the incident shock wave reflected off the end wall of the shock tube, a small portion of the wave travelled into the porous end face to reflect from the back face and re-emerge into the reflected shock region as a fully dispersed shock wave some time later. The technique has been further developed and improved, although no major advance has been made with it. This work is presented in Section 3.4.

A description of the Cranfield Institute of Technology 2" shock tube, the improvements which have been made to it, and a discussion of real shock tube behaviour are given in Appendix 2. An error analysis for the schlieren system and the particular experiments is given in Appendix 3.

Conclusions have been reached regarding the applicability of the theoretical and experimental techniques which have been investigated. These, and recommendations for future work are presented in Section 4.

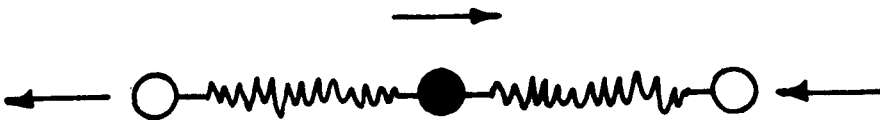


Bending modes (2). $\theta_{\text{vib.}} = 959^{\circ}\text{K}$



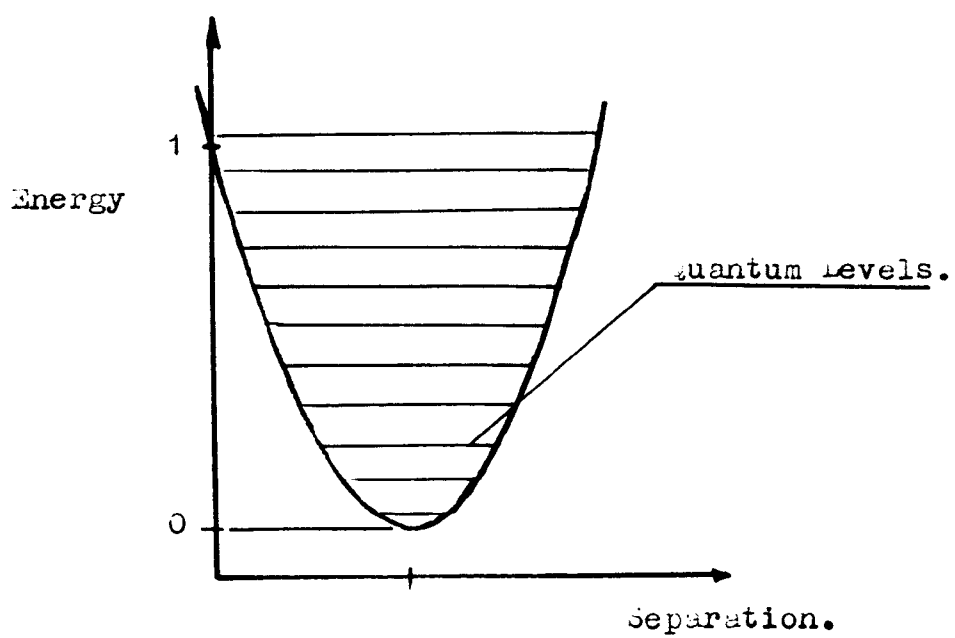
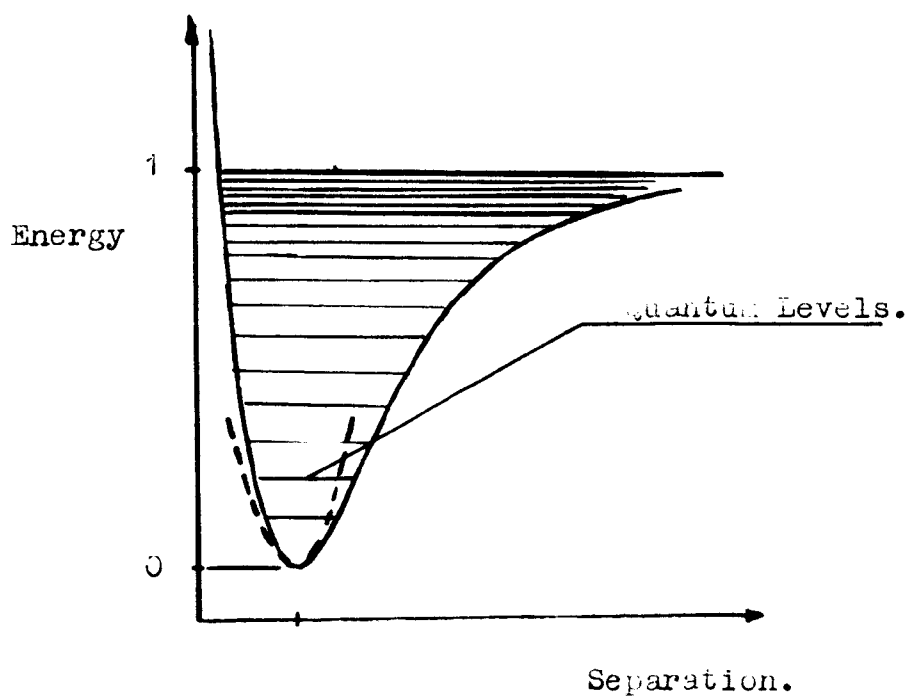
Symmetric stretching mode.

$\theta_{\text{vib.}} = 1920^{\circ}\text{K}$

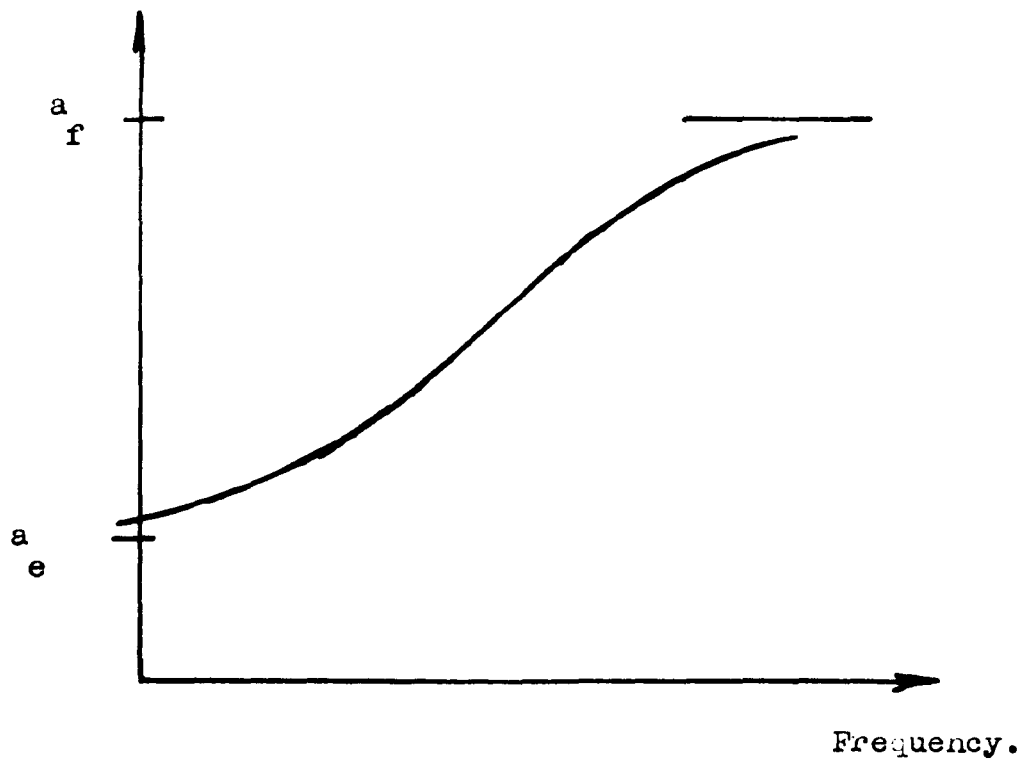
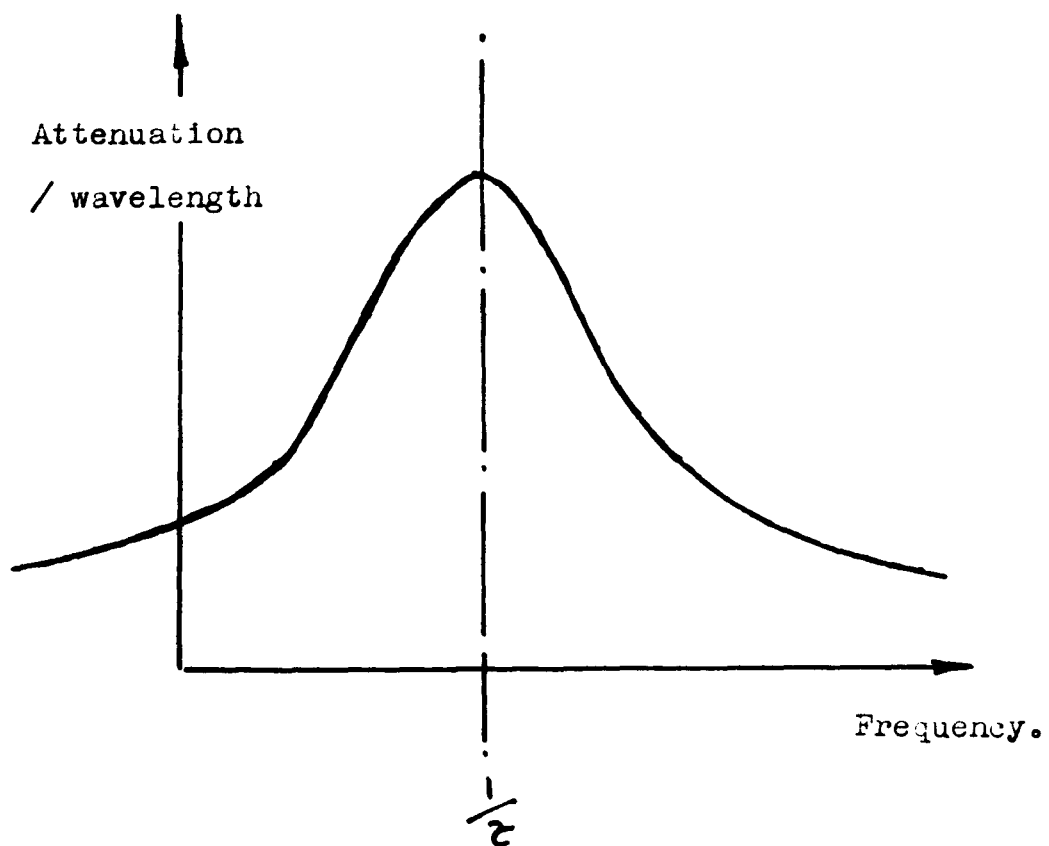


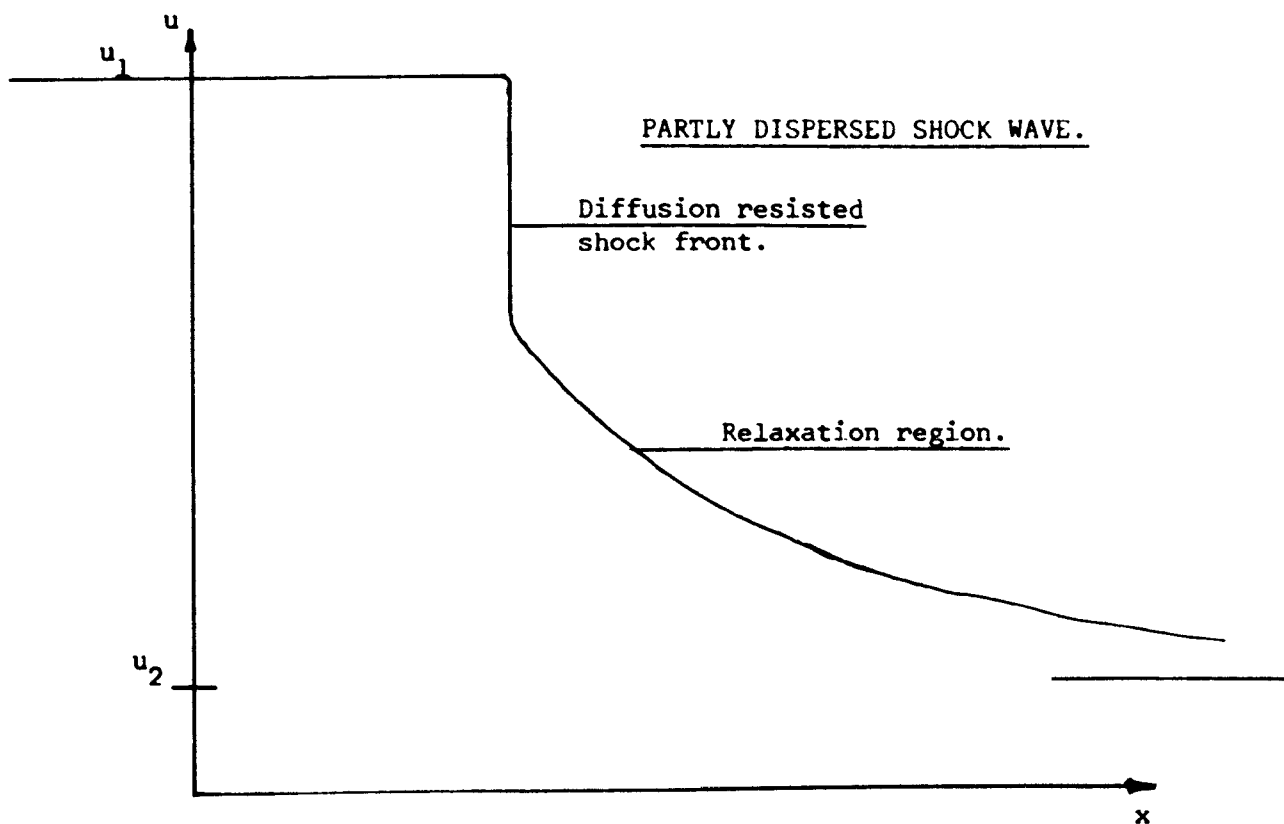
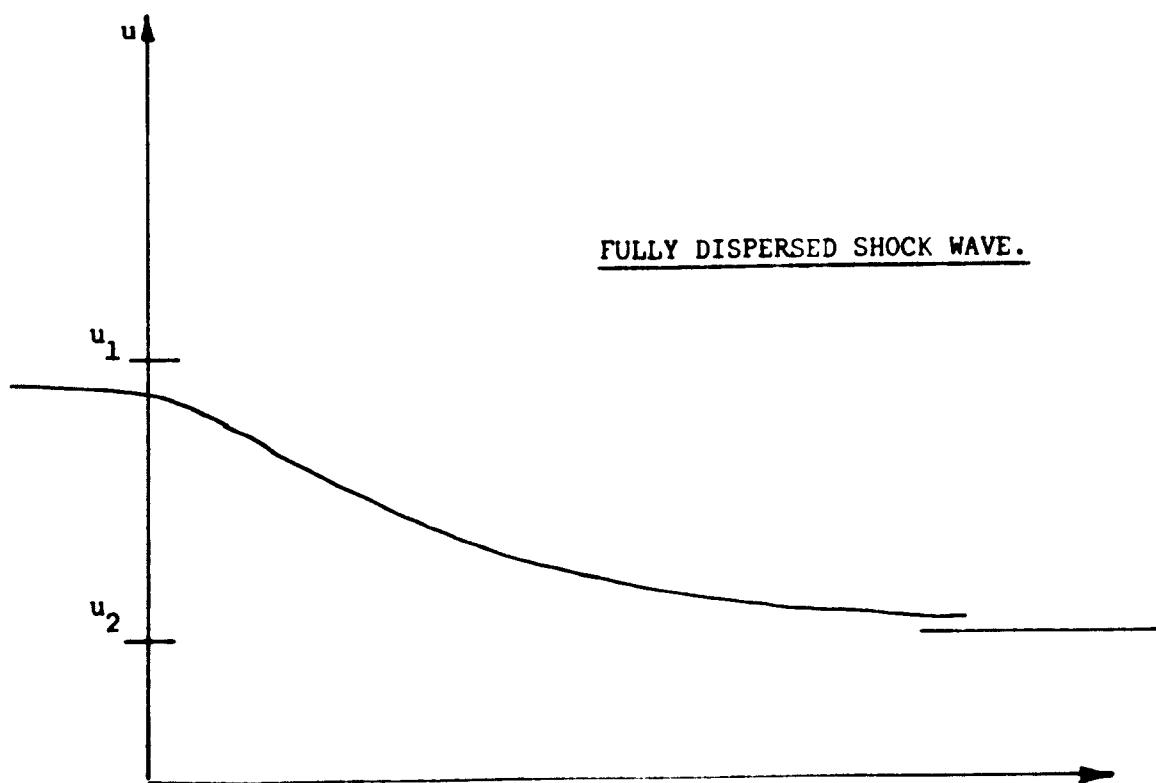
Asymmetric stretching mode.

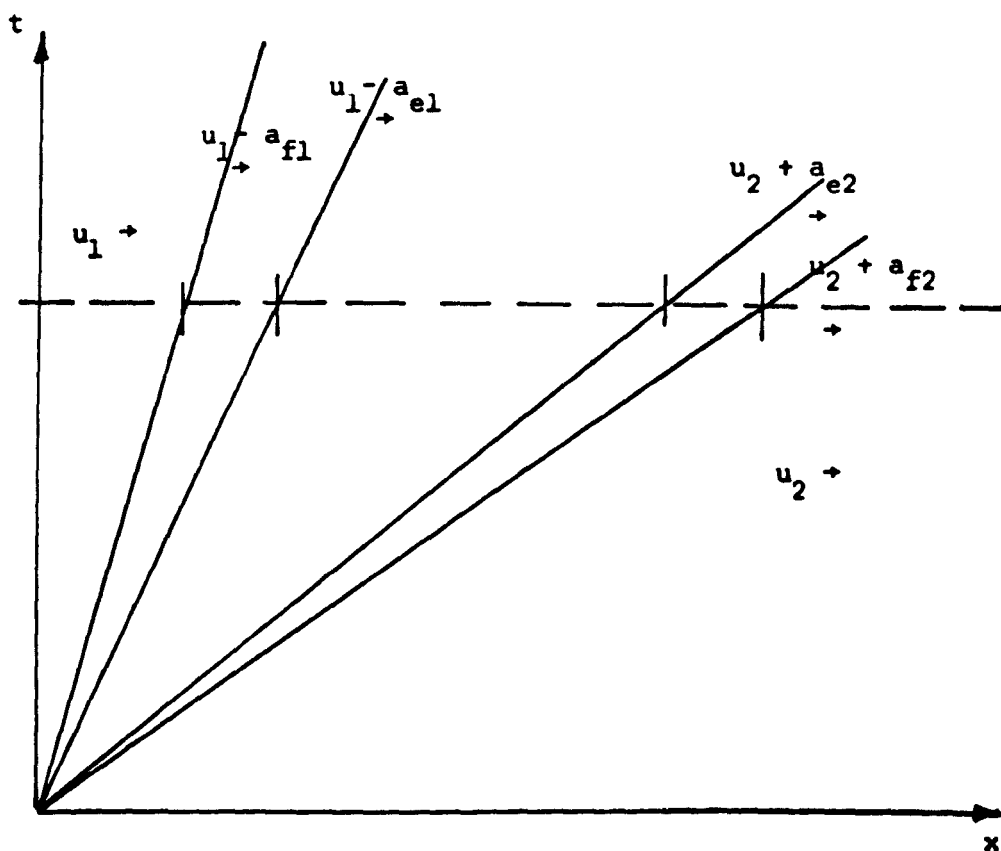
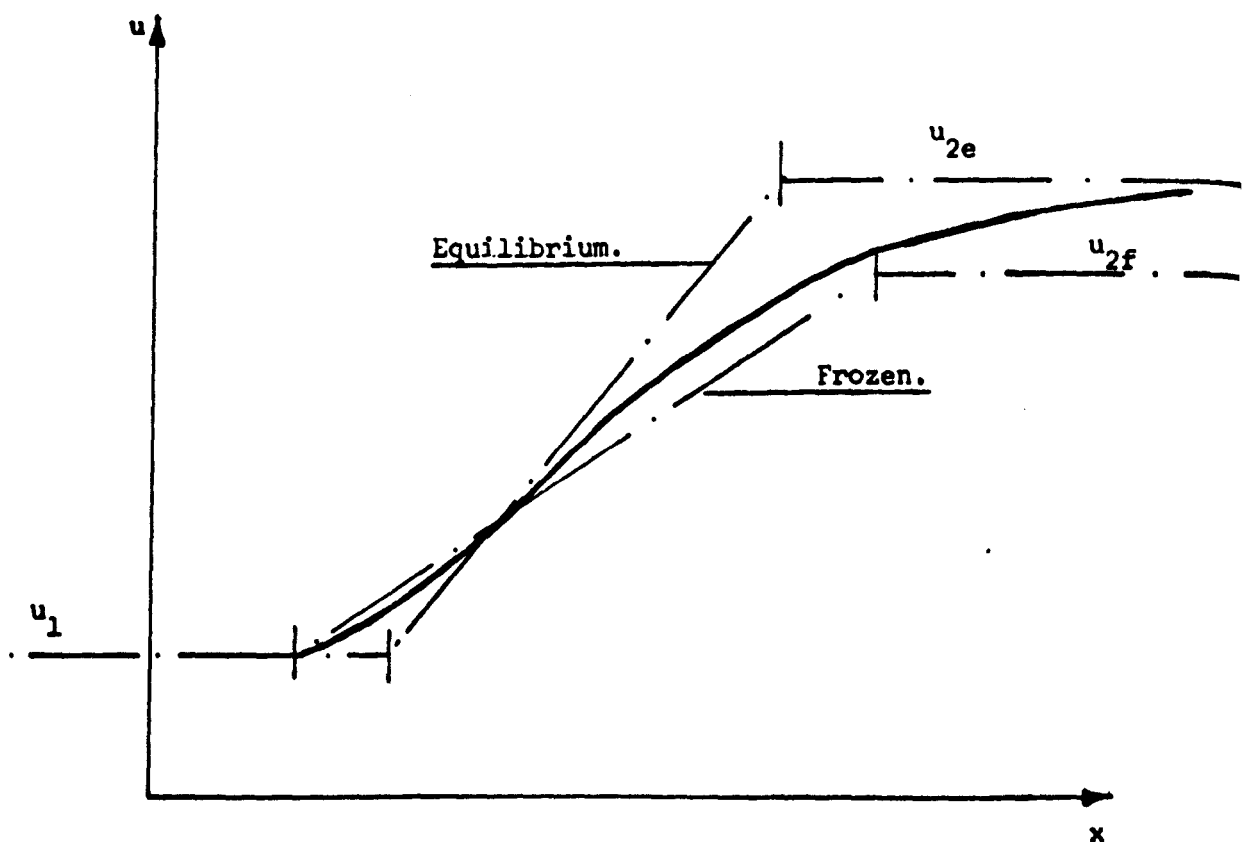
$\theta_{\text{vib.}} = 3380^{\circ}\text{K}$



Speed of wave
propagation.







THE UNSTEADY EXPANSION FAN IN A RELAXING GAS.

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2. THEORETICAL STUDIES.

SUMMARY

Theoretical studies have been made of certain aspects of vibrational relaxation in weak normal shock waves.

The effect of translational non-equilibrium on vibrational relaxation has been studied by forming an asymptotic expansion in the ratio of the viscous length to the relaxation length. The perturbation scheme was singular under certain conditions and required the application of the method of matched asymptotic expansions.

Inviscid bimodal vibrational relaxation has been studied by forming an asymptotic expansion in the ratio of the energy of the secondary vibrational mode to the total vibrational energy. In this way, no restriction had to be placed on the relaxation times involved in order to obtain an analytical solution.

The addition of a second order term to the rate equation describing the behaviour of a single active vibrational mode has also been studied.

NOTATION.

a	Speed of sound.
C	Specific heat.
D	Diffusion coefficient.
e	Energy.
F	Total momentum.
h	Enthalpy.
H	Total enthalpy.
k	Constant.
M	Mach number.
n	Number.
$\dot{N}$	Number rate of production of molecular species.
p	Pressure.
p_{11}	X-wise component of stress tensor, p_{xy} .
q_1	X-wise component of energy flux vector, q_x .
Q	Mass flow.
R_g	Particular gas constant.
R	Rate term.
s	Entropy.
t	Time.
T	Temperature.
u	Velocity.
v, V	Non-dimensionalised velocity.
x	Co-ordinate.
z, Z	Non-dimensionalised co-ordinate.
α	Constant.
β	Non-dimensionalised specific heat.
ϵ	Perturbation parameter.
κ	Equivalent bulk viscosity.
μ	Coefficient of viscosity.
v	Vibrational mode.
ρ	Density.

NOTATION CONT'D.

γ	Ratio of specific heats.
θ, Θ	Non-dimensionalised temperature.
σ	Perturbation parameter.
τ	Relaxation time.
δ	Perturbation parameter.
Γ	Constant.

SUBSCRIPTS.

s	Downstream state.
t	Translational mode.
r	Rotational mode.
1	Combined translational and rotational modes.
2	First vibrational mode.
3	Second vibrational mode.
f	Frozen state.
e	Equilibrium state.
∞	Upstream state.
a	Molecular species.
v	Vibrational modes.

2.1 Introduction

Three aspects of the structure of shock waves in relaxing gases have been investigated theoretically. The approach taken has been to develop extensions to the basic theory of fully and partly dispersed shock waves by way of perturbation techniques.

The first extension to the basic theory was concerned with the effects of viscosity, heat conduction, rotational relaxation and mass diffusion of the vibrational energy on the profiles of shock waves in vibrationally relaxing gases. In the theory of fully and partly dispersed shock waves, as presented by Lighthill (Ref. 2.1) and Clarke and McChesney (Ref. 2.2), all of these effects are neglected. In the case of the partly dispersed shock wave they are assumed to be either confined within a negligibly thin diffusion resisted shock front or negligible in the relaxation resisted parts of the wave. In the case of the fully dispersed shock wave these effects are assumed to be negligible throughout the wave.

These are very reasonable assumptions, because the translational and rotational modes adjust very quickly compared to the vibrational modes. For example, the rotational relaxation time of carbon dioxide, has been determined by ultrasonic techniques; Ener, Gabrysh & Hubbard (Ref. 2.3), to be of the order of 2.3×10^{-3} microseconds at NTP, whereas the relaxation time of the vibrational modes is of the order of 7 microseconds at NTP. Thus, the thickness of the diffusion resisted shock front is of the order of 10^{-4} times that of the vibrational relaxation region, and the translational and rotational modes adjust so rapidly on the scale of the vibrational relaxation region that they can be taken as being in equilibrium throughout it. These assumptions are also borne out by measurements of the thicknesses of diffusion resisted shock waves in carbon dioxide made by Anderson and Hubbard (Ref. 2.4), who found the thickness of a shock wave of Mach 1.23 to be 5×10^{-5} cms. at NTP. This compares to vibrational relaxation regions having thicknesses of 0.5 cms. at NTP, Zienkiewicz and Johannesen (Ref. 2.5) and Simpson et al (Ref. 2.6) and many others.

Most experimental studies have been concerned with either the diffusion resisted shock front or the vibrational relaxation region. It is of interest to question the nature of the interaction between translational and rotational non-equilibrium and vibrational non-equilibrium at positions in the wave where they have the same order of effect on the shock profile. Such a region must exist between the downstream end of the diffusion resisted shock front and the upstream end of the vibrational relaxation

region. In this region the thickness of the diffusion resisted shock front would constitute 1% of the width of the region and there would be a 1% change in the thermodynamic state of the gas through the region due to vibrational relaxation. Clearly, any attempt to study this region would be fraught with difficulties.

As the strength of the shock wave is reduced, a stage is reached when the diffusion resisted shock front disappears and the wave becomes fully dispersed, as discussed in Part 1. However, the width of the diffusion resisted shock front increases as its strength decreases, so the possibility exists that it could become of the same thickness as the vibrational relaxation region.

It is clear that, if such a region could be found, it would be in a weak shock wave in the vicinity of the transition from a fully dispersed to a partly dispersed shock wave. The extension to the theory which has been developed has therefore been restricted to weak shock waves. This is also convenient mathematically, because several approximations can be made which simplify the solution and enable an analytical solution to be obtained without significantly changing the essential physics of the flow. Nevertheless, if necessary, the results could be qualitatively applied to strong shock waves also.

The requisite conservation and rate equations and their derivation from the equations of change together with the approximations which have been made are presented in Appendix 1.

As noted above, the effects of translational and rotational non-equilibrium on the scale of the vibrational relaxation region are very small. This fact appears in the non-dimensional equations derived in Appendix 1 through the small parameter, ϵ .

This parameter has the appearance of the inverse of a Reynolds number based on the characteristic length of the vibrational relaxation region.

A perturbation scheme has been developed with this parameter. The first term of the solution, with $\epsilon = 0$, yields the partly dispersed/fully dispersed solution, as would be expected. This solution is presented in Section 2.2.1.

The presence of the diffusion resisted shock front appears as a singularity at the upstream end of the partly dispersed shock wave in this solution. It is found that the perturbation scheme must be developed in three separate flow regimes. These are: the fully dispersed shock wave where the departure of the wave strength from that required for transition is greater than the perturbation parameter; the partly dispersed shock wave where the departure of the wave strength from that required for transition is also greater than the perturbation parameter; and the region of the transition wave where the departure of the wave strength from that required for transition is of the same order as the perturbation parameter.

The solution for the partly dispersed shock wave is given in Section 2.2.2. The region of the singularity is investigated by the application of the method of matched asymptotic expansions due to Van Dyke (Ref. 2.7.) The variables are scaled by functions of the perturbation parameters so as to transform the singularity to a region of order unity. The first order solution for this "inner" region, with $\epsilon = 0$, is that for a diffusion resisted shock wave with rotational relaxation and constant thermodynamic properties.

The equations which are obtained in this case are not amenable to analytical solution. An analytical solution has been obtained by the use of an equivalent bulk viscosity description for the rotational behaviour. This is acceptable because of the weak nature of the wave, Anderson and Hornig (Ref. 2.4), but does preclude any consideration of separate rotational/vibrational coupling.

According to the theory of matched asymptotic expansions, the "inner" solution must "match" the "outer" solution for the relaxation region.

In general, the first term of the "inner" solution should also provide information regarding the scaling of the second term of the "outer" solution. In this particular case, the first requirement is met, but the exponential nature of the solution for the diffusion resisted shock wave at its upstream and downstream limits precluded any information regarding the next term of the "outer" expansion. The solution for the vibrational relaxation region was therefore continued as an asymptotic power series in ϵ .

The separate contributions from viscous diffusion and heat conduction, rotational relaxation; via the bulk viscosity, and mass diffusion of the vibrational energy could be identified in the next term of this outer expansion. There was no evidence in this term of any conditions which might produce the desired region, because the solution was dominated by the small value of the perturbation parameter. The second term of the expansion in the "inner" region was found in a similar manner. This was also of order ϵ ; for matching with the "outer" solution. This did not yield any further information and the partly dispersed wave solution was left at this stage.

The first term of the expansion for the fully dispersed shock wave possessed no singularities and was therefore continued as an asymptotic power series. This solution is presented in Section 2.2.3. As with the partly dispersed shock wave, the contributions of translational and rotational non-equilibrium remained of the same order as the perturbation parameter throughout the wave.

The last flow regime to be considered was that for the transition from a fully dispersed shock wave to a partly dispersed shock wave. This solution was developed as an asymptotic expansion about the exact transition as well as in the perturbation parameter ϵ , because the behaviour depended on the relative magnitudes of the departure from transition, measured in terms of the wave strength, and the size of the perturbation parameter ϵ .

The first term in the asymptotic expansion was therefore that for the exact transition. This was found to contain a singularity in the velocity derivative at the upstream end of the wave.

The equations were transformed into velocity derivative/velocity space, and a solution was sought for the region of the singularity using the method of matched asymptotic expansions. This solution is given in Section 2.2.4.

It was found from the requirements for "matching" of the "inner" and "outer" expansions that the region of singularity had to be scaled by the perturbation parameter ϵ , raised to the one half power. Taking this to have a numerical value of the order of 10^{-4} indicated that the effects of translational and rotational non-equilibrium were of the order of 1% of those due to vibrational relaxation at the exact transition. The first term of the "inner" expansion was derived and theoretical profiles obtained.

These results indicated that it may be possible to find a region in which the diffusion resisted shock front constitutes 10% of the thickness of the vibrational relaxation region and in which the vibrational relaxation causes a 10% change in the thermodynamic state of the gas.

This shows that it should be possible to study the combined effects of translational and rotational non-equilibrium and vibrational non-equilibrium. However, the experimental studies of part 3 have shown that it would not be possible to study such behaviour with the existing equipment. Nevertheless, an experiment could be designed for a larger, possibly lower density shock tube in which it should be possible to observe such a region.

The second extension to the theory of fully and partly dispersed shock waves was concerned with bimodal vibrational relaxation. Bimodal analyses have been applied to the results of experimental measurements, Zienkiewicz and Johannesen (Ref 2.8) and Witterman (Ref. 2.9) for example, in order to determine the relative rates of relaxation of the separate modes of vibration in linear tri-atomic molecules. Numerical solutions have been investigated, Becker (Ref. 2.10) for example, and limiting cases of infinite relaxation times have been studied, Clarke and Rodgers (Ref. 2.11). The present studies have been concerned with the determination of any measureable features of shock waves which were related to bimodal behaviour.

An analytical solution was required in which the effects of the bimodal behaviour could be readily identified in terms of the thermodynamic state of the gas. The solution had to contain the essential physics of the flow and yet not necessarily be exact.

As with the first extension to the theory, the approach taken has been to develop a perturbation solution around the basic solution for the fully dispersed and partly dispersed shock waves. In many simple polyatomic gases the vibrational behaviour is dominated by one of the vibrational modes at certain temperatures. The combined bending and symmetric stretching modes of carbon dioxide compared to the asymmetric stretching mode is an example of this. A perturbation scheme has therefore been developed on the basis of the ratio of the energy in the secondary vibrational mode to the total vibrational energy.

The solution to this problem is given in Section 2.3. Inviscid vibrational relaxation is considered and linear rate equations are used to describe the vibrational behaviour. The first correction term to the unimodal behaviour has been derived.

An interesting feature of the solution was that a singularity occurred if the ratio of the rate of energy transfer between the modes to the rate of transfer between translation and the secondary mode became of the same order as the perturbation parameter. This result can be viewed in two ways. If the two vibrational modes are not in resonance or near resonance, the perturbation can be treated as an asymptotic power series to give the first order correction if to bimodal behaviour directly. However, if the modes are in near resonance, a solution could be sought in the region of the singularity in order to obtain a solution for the near resonance case. There are obviously other ways of obtaining this latter form of solution by a perturbation scheme based on the fact that the modes are near to resonance, which is probably more direct.

In the present studies the solution for the case where the modes are not in near resonance has been derived.

The last extension to the basic theory of fully and partly dispersed shock waves to be investigated was that of unimodal vibrational relaxation governed by a non-linear rate equation.

Linear rate equations are normally used in the analysis of vibrational relaxation, but the results of Zienkiewicz and Johannesen indicated that the linear rate equation might not be applicable in strong shock waves. This has since been shown not to be the case by Simpson et al. However, at the outset of this work this was not known.

A non-linear equation has been derived by assuming that the linear rate equation is the first term in an asymptotic expansion of a more complex function about an equilibrium state. This equation has been applied to an inviscid gas with a single vibrationally relaxing mode. A perturbation solution has been obtained by assuming that the correction to the linear rate equation was small compared to the linear term. A solution could have been obtained more directly, but in this way, comparisons could be made between this extension to the theory and those mentioned above if required. The solution to this problem is given in Section 2.4.

Theoretical profiles have been computed for this solution which show that even if the results of Zienkiewicz and Johannesen could be explained by a non-linear rate equation, the effects on a fully dispersed shock wave would be negligible. Thus, the use of fully dispersed shock waves for the study of vibrational relaxation would overcome any uncertainties associated with the departures from equilibrium in the strong incident shock wave.

The solutions to the above extensions to the theory of fully and partly dispersed shock waves is given in the following sections.

2.2. Viscous Vibrational Relaxation Using A Single First Order Rate Equation.

2.2.1. The Basic Solution.

Applying the upstream boundary conditions to equations A.1.37 and A.1.39 we have,

$$v^2 + \frac{\theta_t}{\gamma_e M_\infty^2} - \epsilon v \frac{dv}{dz} = v \left(1 + \frac{1}{\gamma_e M_\infty^2} \right) \quad 2.1.$$

and,

$$\begin{aligned} \frac{\gamma_t}{\gamma_t - 1} \theta_t + \beta_r \theta_r + \beta_2 \theta_2 + \frac{\gamma_e M_\infty^2 v^2}{2} \\ = \frac{\gamma_t}{\gamma_t - 1} + \beta_r + \beta_2 + \frac{\gamma_e M_\infty^2}{2} \end{aligned} \quad 2.2.$$

where we have put $v=2$.

The appropriate rate equations for this problem are equations A.1.40 and A.1.41, that is,

$$\begin{aligned} \epsilon v \frac{d}{dz}(\theta_r + \epsilon \frac{d\theta_r}{dz}) &= R(\theta_t - \theta_r) \\ &+ \epsilon \frac{\tau_2}{\tau_{r2}} \frac{\beta_2}{\beta_r}(\theta_2 - \theta_r) \end{aligned} \quad 2.3.$$

and,

$$\begin{aligned} v \frac{d}{dz}(\theta_2 + \epsilon \frac{d\theta_2}{dz}) &= (\theta_t - \theta_2) \\ &+ \frac{\tau_2}{\tau_{r2}}(\theta_r - \theta_2) \end{aligned} \quad 2.4.$$

Equations 2.1, and 2.2 can be rewritten to express θ_t , θ_r , and θ_2 in terms of v . These relationships can then be differentiated as required and substituted into equations 2.3, and 2.4 to give a fifth order non-linear ordinary differential equation in $v(z)$. Such an equation is not amenable to analytical solution, so an approximate solution will be found by forming an asymptotic expansion in ϵ , which is a small quantity for most gases.

We shall seek a solution to the above equations for $v(z)$ of the form,

$$\begin{aligned} v(z; \epsilon) \sim & \Delta_1(\epsilon) \cdot v^{(1)}(z) + \Delta_2(\epsilon) \cdot v^{(2)}(z) \\ & + o\{\Delta_2(\epsilon)\} \end{aligned} \quad 2.5.$$

as $\epsilon \rightarrow 0$, and where $\Delta_2(\epsilon) = o\{\Delta_1(\epsilon)\}$.

Writing similar expansions for the thermodynamic variables, substituting them into the above equations and expanding to order $\Delta_1(\epsilon)$, it is clear that the first value of $\Delta_1(\epsilon)$ which will yield a non-trivial solution is given by $\Delta_1(\epsilon) = O(1)$.

So, putting $\Delta_1(\epsilon) = 1$ without loss of generality and expanding to order unity, the above equations become,

from 2.1 ,

$$v^{(1)2} \frac{\theta_t^{(1)}}{\gamma_e M_\infty^2} = v^{(1)} \left(1 + \frac{1}{\gamma_e M_\infty^2} \right)$$

from equation 2.2 , which is unchanged,

$$\begin{aligned} \frac{\gamma_t}{\gamma_t - 1} \theta_t^{(1)} + \beta_r \theta_r^{(1)} + \beta_2 \theta_2^{(1)} + \frac{\gamma_e M_\infty^2}{2} v^{(1)2} \\ = \frac{\gamma_t}{\gamma_t - 1} + \beta_r + \beta_2 + \frac{\gamma_e M_\infty^2}{2} \end{aligned}$$

from equation 2.3 ,

$$\theta_t^{(1)} = \theta_r^{(1)}$$

and from equation 2.4 ,

$$v^{(1)} \frac{d\theta^{(1)}}{dz} = (\theta_t^{(1)} - \theta_2^{(1)}) + \frac{\tau_2}{\tau_{r2}} (\theta_r^{(1)} - \theta_2^{(1)})$$

Solving these equations we find that $v^{(1)}$ must satisfy the following equation,

$$v^{(1)} \left\{ v^{(1)} - v_a \right\} \frac{dv^{(1)}}{dz} = \alpha \left\{ v^{(1)} - 1 \right\} \left\{ v^{(1)} - v_s \right\} \quad 2.6.$$

where,

$$\alpha = -\frac{1}{2} \frac{\gamma - 1}{\gamma + 1} \frac{\gamma_e + 1}{\gamma_e - 1} \left(1 + \frac{\tau_2}{\tau_{r2}} \right),$$

and,

$$v_a = \frac{\gamma}{2\gamma_e} \frac{\gamma_e + 1}{\gamma + 1} (v_s + 1)$$

This solution has been derived and discussed in detail elsewhere, by Lighthill (Ref. 2.1.) and Clarke (Ref. 2.2.) for example, so only the pertinent features of the solution will be discussed here.

Firstly, it will be noted that $dv^{(1)}/dz$ becomes singular if $v^{(1)}$ passes through v_a , providing $v_a \neq 1$, or v_s . Now, $v^{(1)}$ falls from the upstream value of unity to the downstream value of v_s , and it can be shown that $v_a > v_s$. Thus, $dv^{(1)}/dz$ is regular for $v_a \geq 1$, and singular for $v_a < 1$.

Clearly, v_a has great significance with regard to the form of the wave, so we shall consider its physical significance.

It can be shown from the overall shock ratios that,

$$v_a = \frac{\gamma}{\gamma + 1} \left(1 + \frac{1}{\gamma_e M_\infty^2} \right)$$

Putting $v_a = 1$ and re-arranging this expression we have,

$$M_\infty^2 = \frac{\gamma}{\gamma_e} = \frac{a_{f\infty}}{a_{e\infty}}$$

where $a_{f\infty}$ is the upstream frozen sound speed, that is, the speed of sound when only the translational and rotational modes are active in this context, ie.

$$a_{f\infty} = (\gamma R_g T_\infty)^{\frac{1}{2}}$$

where $\gamma = 1.4$.

Thus, the condition $v_a = 1$ occurs when the Mach number of the flow into the wave is equal to the ratio of the frozen to equilibrium sound speeds. The solutions defined by $v_a > 1$ have been designated fully dispersed and partly dispersed shock waves respectively. From the above definition of v_a it can be seen that these designations are equivalent to,

$$M_\infty < \frac{a_{f\infty}}{a_{e\infty}}$$

That is, the flow into a fully dispersed shock wave is subsonic with respect to the frozen sound speed, while the flow into a partly dispersed shock wave is supersonic with respect to the frozen sound speed.

A sketch of the behaviour of $v^{(1)}(z)$ for $v_a > 1$ is shown in figure 2.1.a. It can be seen that $v^{(1)}$ falls monotonically from the upstream value of unity to the downstream value of v_s .

It has been shown from equation 2.6 that $dv^{(1)}/dz$ is regular in this case; however, we must inquire if the higher derivatives which appear in the general solution are also regular, that is, such terms as,

$$\epsilon \frac{d^2 v^{(1)}}{dz^2}, \quad \epsilon^2 \frac{d^3 v^{(1)}}{dz^3}, \quad \text{etc.}$$

It can be shown that these terms do cease to be regular when both $v_a - 1$ and $v^{(1)} - 1$ are $O(\epsilon)$, so that the cases $|v_a - 1| = o(1)$ and $|v_a - 1| = O(1)$ must be considered separately.

When $|v_a - 1| = O(1)$ the perturbation scheme is regular and the asymptotic expansion, equation 2.5 can be continued to determine $\Delta_2(\epsilon)$ and $v^{(2)}$ directly. When $|v_a - 1| = o(1)$, however, we must first investigate the region of the singularity.

The behaviour of $v^{(1)}(z)$ for $v_a < 1$ is shown in figure 2.1.b. It can be seen that in this case the continuous solution of equation 2.6 can only satisfy one of the boundary conditions. The only physically acceptable solution is for v to fall discontinuously from the upstream value of unity to $2v_a - 1$ and thence to decrease monotonically to the downstream value of v_s . The solution to this problem will be found by stretching the variables to examine the discontinuous region and matching the resulting 'inner' expansion to the original 'outer' expansion in the manner of matched asymptotic expansions, Van Dyke (Ref. 2.7.).

In general both v and z should be stretched; however, the change in v is $O(1)$ throughout, except when $|v_a - 1| = o(1)$, so that in this case only the independent variable, z , need be stretched, and the case when $|v_a - 1| = o(1)$ will be considered separately.

Thus, we shall treat the problem in three separate parts, namely,

$$v_a > 1 ; |v_a - 1| = O(1)$$

$$v_a < 1 ; |v_a - 1| = O(1)$$

$$|v_a - 1| = o(1) .$$

The last of these cases includes both fully dispersed and partly dispersed shock waves for $|v_a - 1| = o(1)$.

Let us now return to the solution of equation 2.6. Rewriting this equation in partial fractions we have,

$$\left\{ -A \frac{1}{v^{(1)} - 1} + B \frac{1}{v^{(1)} - 1} - 1 \right\} \frac{dv^{(1)}}{dz} = -a \quad 2.7.$$

where,

$$A = \frac{1 - v_a}{1 - v_s} \quad \& \quad B = \frac{v_s(v_s - v_a)}{1 - v_s}$$

Integrating this equation in the regions of continuity we have,

$$v^{(1)} + A \ln(v^{(1)} - 1) - B \ln(v^{(1)} - v_s) = az + k_1 \quad 2.8.$$

In the case of the fully dispersed shock wave the constant of integration can be arbitrarily chosen and will be left unspecified for the present.

For the partly dispersed shock wave the solution on each side of the discontinuity must be considered separately. It is convenient to set $z = 0$ at the discontinuity, so that, for $z < 0$, $v^{(1)} = 1$, and as $z \rightarrow 0$ on the downstream side of the discontinuity $v^{(1)} \rightarrow 2v_a - 1$.

We therefore have,

$$v^{(1)} = 1 \text{ for } z < 0 \quad 2.9.$$

and,

$$\begin{aligned} v^{(1)} - (2v_a - 1) + A \ln \frac{(v^{(1)} - 1)}{2(v_a - 1)} - B \ln \frac{(v^{(1)} - v_s)}{(2v_a - 1 - v_s)} \\ = az \text{ for } z > 0 \end{aligned} \quad 2.10.$$

We shall now proceed to consider the above three cases separately.

2.2.2. The Partly Dispersed Shock Wave. $|v_a - 1| = O(1)$.

In this section a solution will be obtained for the discontinuous region of the partly dispersed shock wave, which has been set at $z = 0$. This will be achieved by enlarging the region to order unity by stretching the variables by appropriate functions of ϵ . Attention has been restricted to waves through which the total change in v is $O(1)$, so that only the independent variable, z , need be stretched. That is, the following inner variables will be defined,

$$V = v, \quad \theta = \theta, \quad \& \quad Z = z/\sigma(\epsilon) \quad 2.11.$$

where $\sigma(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 0$.

The general equations, 2.1 , 2.2 , 2.3 and 2.4 become after scaling,

from equation 2.1 ,

$$V^2 + \frac{\theta_t}{\gamma_e M_\infty^2} - \frac{\epsilon V}{\sigma} \frac{dV}{dZ} = V(1 + \frac{1}{\gamma_e M_\infty^2}) \quad 2.12.$$

from equation 2.2 ,

$$\begin{aligned} \frac{\gamma_t}{\gamma_t - 1} \theta_t + \beta_r \theta_r + \beta_2 \theta_2 + \frac{\gamma_e M_\infty^2}{2} V^2 \\ = \frac{\gamma_t}{\gamma_t - 1} + \beta_r + \beta_2 + \frac{\gamma_e M_\infty^2}{2} \end{aligned} \quad 2.13.$$

from equation 2.3 ,

$$\begin{aligned} \frac{\epsilon}{\sigma} V \frac{d}{dZ} (\theta_r + \frac{\epsilon}{\sigma} \frac{d\theta}{dZ} r) = \\ R(\theta_t - \theta_r) + \frac{\tau_2}{\tau_{r2}} \frac{\beta_2}{\beta_r} (\theta_2 - \theta_r) \end{aligned} \quad 2.14.$$

and from equation 2.4,

$$\frac{\epsilon}{\sigma} v \frac{d}{dz} (\theta_2 + \frac{\epsilon}{\sigma} \frac{d\theta_2}{dz}) = (\theta_t - \theta_2) + \frac{\tau_2}{\tau_{r2}} (\theta_r - \theta_2) \quad 2.15.$$

An asymptotic expansion must again be formed in ϵ in this inner region, that is, we shall write,

$$v(z; \epsilon) \sim v^{(1)}(z) + \delta_2(\epsilon) \cdot v^{(2)}(z) + o\{\delta_2(\epsilon)\}. \quad 2.16.$$

as $\epsilon \rightarrow 0$, and where $\delta_2(\epsilon) = o(1)$.

Writing similar expansions for the other thermodynamic variables, θ_t , θ_r , & θ_2 , substituting them into the above equations and expanding to order $\sigma(\epsilon)$, it is found that the only value of $\sigma(\epsilon)$ which yields a non-trivial solution that matches with the outer solution is $\sigma(\epsilon) = O(\epsilon)$. So, without loss of generality we can put $\sigma(\epsilon) = \epsilon$ in the above equations and expand to order unity. The equations become,

from equation 2.12 ,

$$v^{(1)2} + \frac{\theta_t^{(1)}}{\gamma_e M_\infty^2} - v^{(1)} \frac{dv^{(1)}}{dz} = v^{(1)} (1 + \frac{1}{\gamma_e M_\infty^2})$$

from equation 2.13 ,

$$\begin{aligned} \frac{\gamma_t}{\gamma_t - 1} \theta_t^{(1)} + \beta_r \theta_r^{(1)} + \beta_2 \theta_2^{(1)} + \frac{\gamma_e M_\infty^2}{2} v^{(1)2} \\ = \frac{\gamma_t}{\gamma_t - 1} + \beta_r + \beta_2 + \frac{\gamma_e M_\infty^2}{2} \end{aligned}$$

from equation 2.14 ,

$$v^{(1)} \frac{d}{dz} (\theta_r^{(1)} + \frac{d\theta_r^{(1)}}{dz}) = R(\theta_t^{(1)} - \theta_r^{(1)})$$

and from equation 2.15 ,

$$v^{(1)} \frac{d}{dz} (\theta_2^{(1)} + \frac{d\theta_2^{(1)}}{dz}) = 0$$

The only solution of the last of these equations which will match with the first outer solution for $z < 0$ is,

$$\theta_2^{(1)} = 1 \quad 2.17.$$

Thus, the above equations reduce to those of the diffusion resisted shock wave with rotational relaxation, viz.,

$$v^{(1)2} + \frac{\theta_t^{(1)}}{\gamma_e M_\infty^2} - v^{(1)} \frac{dv^{(1)}}{dz} = v^{(1)} (1 + \frac{1}{\gamma_e M_\infty^2}) \quad 2.18.$$

and,

$$\begin{aligned} \frac{\gamma_t}{\gamma_t - 1} \theta_t^{(1)} + \beta_r \theta_r^{(1)} + \frac{\gamma_e M_\infty^2}{2} v^{(1)2} \\ = \frac{\gamma_t}{\gamma_t - 1} + \beta_r + \frac{\gamma_e M_\infty^2}{2} \end{aligned} \quad 2.19.$$

and finally,

$$v^{(1)} \frac{d}{dz} (\theta_r^{(1)} + \frac{d\theta_r^{(1)}}{dz}) = R(\theta_t^{(1)} - \theta_r^{(1)}) \quad 2.20.$$

Eliminating $\theta_t^{(1)}$ and $\theta_r^{(1)}$ in favour of $v^{(1)}$ in the above equations yields a third order non-linear differential equation in $v^{(1)}(z)$, which is not amenable to analytical solution. The use of an equivalent bulk viscosity model to describe the rotational behaviour would appear to be an acceptable way of further simplifying the above equations in order to obtain an analytical solution for $v^{(1)}(z)$, for the following reasons:

a. The necessary restriction of the theory to weak waves, which has already been imposed.

b. The already approximate nature of the description of the rotational behaviour by a simple linear rate law.

c. The knowledge that the rotational mode rapidly equilibrates with the translational mode in a non-equilibrium situation.

d. The use of a bulk viscosity model has been shown experimentally and theoretically to be justified in the case of weak waves, Anderson and Hornig (Ref. 2.4.).

Accordingly, an equivalent bulk viscosity, κ , will be defined as,

$$\kappa = \frac{P_g R C_r \tau_r}{(C_r + C v_t)^2} \quad 2.21.$$

and $\theta_r^{(1)}$ put equal to $\theta_t^{(1)}$.

Applying the bulk viscosity model, equations 2.18 , 2.19 and 2.20 reduce to,

$$v^{(1)2} + \frac{\theta_t^{(1)}}{\gamma_e M_\infty^2} - (1 + \frac{\kappa}{\mu}) v^{(1)} \frac{dv^{(1)}}{dz} = v^{(1)} (1 + \frac{1}{\gamma_e M_\infty^2}) \quad 2.22.$$

and,

$$\frac{\gamma}{\gamma - 1} \theta_t^{(1)} + \frac{\gamma_e M_\infty^2}{2} v^{(1)2} = \frac{\gamma}{\gamma - 1} + \frac{\gamma_e M_\infty^2}{2} \quad 2.23.$$

$\theta_t^{(1)}$ can be eliminated in favour of $v^{(1)}$ to give,

$$(1 + \frac{\kappa}{\mu}) v^{(1)} \frac{dv^{(1)}}{dz} = \frac{\gamma + 1}{2\gamma} (v^{(1)} - 1)(v^{(1)} - 2v_a + 1) \quad 2.24.$$

It will be recalled that μ has been taken as constant, so that this equation can be integrated directly to give,

$$\begin{aligned} \ln(v^{(1)} - 1) - (2v_a - 1) \ln(v^{(1)} - 2v_a + 1) \\ = (1 - v_a) \frac{\gamma + 1}{\gamma} (1 + \frac{\kappa}{\mu})^{-1} z + K \end{aligned} \quad 2.25.$$

It can be seen that the constant of integration, K , is independent of the first outer solution.

Thus, the behaviour of the solution in the inner region has been determined to order unity, and as might have been expected it is the solution for a diffusion resisted shock wave with a Prandtl number of $\frac{2}{3}$ and with constant properties.

This solution must now be investigated to determine if it provides any information regarding the next term of the outer expansion. Rewriting equation 2.25 in outer coordinates we have,

$$\begin{aligned} \ln(V^{(1)} - 1) - (2v_a - 1) \cdot \ln(V^{(1)} - 2v_a + 1) \\ = \frac{(1 - v_a)}{(1 + \frac{2K}{\mu})} \frac{\gamma + 1}{\gamma} \frac{z}{\epsilon} + K \end{aligned} \quad 2.26.$$

It can be seen that for $z < 0$, $V^{(1)} - 1$ is an exponentially small quantity as $\epsilon \rightarrow 0$. The limit as $\epsilon \rightarrow 0$ for $z > 0$ is not so obvious, so, putting $V^{(1)} - 2v_a + 1 = \Delta$ and letting $\epsilon \rightarrow 0$, equation 2.26 can be expanded for small Δ as,

$$\begin{aligned} \ln(\Delta + 2v_a - 2) - (2v_a - 1) \cdot \ln(\Delta) \\ = \frac{(1 - v_a)}{(1 + \frac{2K}{\mu})} \frac{\gamma + 1}{\gamma} \frac{z}{\epsilon} + K \end{aligned}$$

The dominant term on the left hand side of this equation for $\Delta \rightarrow 0$ is,

$$- (2v_a - 1) \cdot \ln(\Delta)$$

so that the equation can be reduced to,

$$\Delta = \left\{ \exp\left\{-\frac{(1-v_a)}{(1+\frac{3K}{4\mu})} \frac{\gamma+1}{\gamma} \frac{z}{\epsilon} + K\right\} \left\{\frac{1}{(2v_a-1)}\right\} \right\}$$

and it can be seen that Δ is an exponentially small quantity as $\epsilon \rightarrow 0$, $z > 0$.

Thus, the outer representation of the first term of the inner expansion to any order of ϵ is,

$$v^{(1)}(\pm \infty) + \text{E.S.T.} \quad 2.27.$$

The lack of coupling between the inner and outer solutions enables the second term of the outer expansion to be evaluated from the full equations, 2.1, 2.2, 2.3 and 2.4, whilst retaining the equivalent bulk viscosity model for the first term of the inner expansion. However, the thermodynamic gradients in the regions of the wave governed by the outer solutions are much smaller than those in the diffusion resisted part of the wave. Consequently, the rotational mode will be nearer to equilibrium with the translational mode and the bulk viscosity description of the behaviour of the rotational mode will be valid in the outer region even for strong shock waves.

Accordingly, the bulk viscosity model will be applied to the full equations, as follows,

$$v^2 + \frac{\theta_t}{\gamma_e M_\infty^2} - \epsilon(1 + \frac{3K}{4\mu})v \frac{dv}{dz} = v(1 + \frac{1}{\gamma_e M_\infty^2}) \quad 2.28.$$

$$\frac{\gamma}{\gamma-1} \theta_t + \beta_2 \theta_2 + \frac{\gamma_e M_\infty^2}{2} v^2 = \frac{\gamma_e}{\gamma_e-1} + \frac{\gamma_e M_\infty^2}{2} \quad 2.29.$$

and,

$$v \frac{d}{dz} \left(\theta_2 + \epsilon \frac{d\theta_2}{dz} \right) = (1 + \frac{\tau_2}{\tau_{r2}})(\theta_t - \theta_2) \quad 2.30.$$

The form of the equations has been radically altered, but the outer and inner asymptotic sequences given by equations 2.5 and 2.17 still apply, as do the solutions for $v^{(1)}(z)$ and $V^{(1)}(z)$ given by equations 2.6 and 2.24 respectively.

The second term of the outer expansion can now be evaluated. As no information has been provided by the first term of the inner expansion, we will proceed by means of a simple asymptotic power series in ϵ , i.e. $\Delta_2(\epsilon) = \epsilon$. After extracting the solution for $v^{(1)}(z)$ the above equations become,

$$2v^{(1)}v^{(2)} + \frac{\theta_t^{(2)}}{\gamma_e M_\infty^2} - (1 + \frac{\beta_k}{\beta_u})v^{(1)} \frac{dv^{(1)}}{dz} = v^{(2)}(1 + \frac{1}{\gamma_e M_\infty^2}) \quad 2.31.$$

and,

$$\frac{\gamma}{\gamma - 1} \theta_t^{(2)} + \beta_2 \theta_2 + \gamma_e M_\infty^2 v^{(1)} v^{(2)} = 0 \quad 2.32.$$

and,

$$v^{(1)} \frac{d\theta_2^{(2)}}{dz^2} + v^{(2)} \frac{d\theta_2^{(1)}}{dz^2} + v^{(1)} \frac{d^2 \theta_2^{(1)}}{dz^2} = (1 + \frac{\tau_2}{\tau_{r2}})(\theta_t^{(2)} - \theta_2^{(2)}) \quad 2.33.$$

The following requisite relationships can be found from the solution for $v^{(1)}(z)$,

$$\frac{d\theta_2^{(1)}}{dz^2} = \frac{\gamma + 1}{\gamma - 1} \frac{\gamma_e M_\infty^2}{\beta_2} (v^{(1)} - v_a) \frac{dv^{(1)}}{dz} \quad 2.34.$$

and,

$$\frac{d^2 \theta_2^{(1)}}{dz^2} = \frac{\gamma + 1}{\gamma - 1} \frac{\gamma_e M_\infty^2}{\beta_2} \left\{ \left(\frac{dv^{(1)}}{dz} \right)^2 + (v^{(1)} - v_a) \frac{d^2 v^{(1)}}{dz^2} \right\} \quad 2.35.$$

These relationships, together with equation

2.6 can now be substituted into equations 2.31, 2.32 and 2.33 in order to obtain a solution for $v^{(2)}(z)$ in terms of $v^{(1)}(z)$. After substituting and differentiating as appropriate, a linear first order differential equation in $v^{(2)}(v^{(1)})$ is obtained. The integrating factor for this equation is found to be $dz/dv^{(1)}$, so that the equation becomes,

$$\begin{aligned} \frac{d}{dv^{(1)}} \left\{ v^{(2)} \frac{dz}{dv^{(1)}} \right\} = \\ \frac{\gamma}{\gamma + 1} \frac{dz}{dv^{(1)}} \frac{1}{v^{(1)} - v_a} \left\{ - \frac{v_s + 1}{v_a} (1 + \frac{\beta K}{\mu}) + (1 + \frac{\beta K}{\mu}) \frac{d}{dv^{(1)}} (v^{(1)} \frac{dv^{(1)}}{dz}) \right. \\ \left. - \frac{\gamma + 1}{\gamma} \frac{d}{dv^{(1)}} (v^{(1)} - v_a) \frac{dv^{(1)}}{dz} \right\} \end{aligned} \quad 2.36.$$

This equation can be split into partial fractions and integrated to give,

$$\begin{aligned} v^{(2)} = & \frac{\gamma}{\gamma + 1} (1 + \frac{\beta K}{\mu}) \frac{dv^{(1)}}{dz} \left\{ - \frac{1}{v_a} \frac{(2v_a - 1) - v_s}{(1 - v_s)(v_a - 1)} \cdot \ln(1 - v^{(1)}) \right. \\ & + \frac{v_s^2 (2v_a - 1) - v_s}{v_a (1 - v_s)(v_a - v_s)} \cdot \ln(v^{(1)} - v_s) \\ & + \left(v_a \cdot \frac{(2v_a - 1) - v_s}{(v_a - 1)(v_a - v_s)} - 1 \right) \cdot \ln(v_a - v^{(1)}) + \frac{v_a}{v^{(1)} - v_a} \left. \right\} \\ & - \frac{dv^{(1)}}{dz} \left\{ \ln(1 - v^{(1)}) + \ln(v^{(1)} - v_s) - \ln v^{(1)} \right\} \\ & + k_2 \frac{dv^{(1)}}{dz} \end{aligned} \quad 2.37.$$

It can be seen that the constant of the integration cannot be evaluated from the boundary conditions, because $dv^{(1)}/dz$ is zero for $z \rightarrow -\infty$ and for $z \rightarrow +\infty$. It will be noted however that for $z < 0$, $v^{(2)} = 0$, which is the solution for the second term of the outer expansion for $z < 0$. The constant of the integration must be evaluated for $z > 0$ by matching with the inner expansion, or by the application of an overall conservation law. It has been shown that the inner expansion provides no information for matching with this outer solution so the latter course of action must be adopted.

To this end, the divergence of the conservation equations 2.28 and 2.29, and the vibrational rate equation, 2.30, will be integrated across the discontinuity at $z = 0$. This will be done by integrating from $z = -e$ to $z = +e$, expanding the equations for $\epsilon \rightarrow 0$ and finally letting $e \rightarrow 0$. Applying this procedure to the above equations and expanding them to order unity we have,

$$\left[v^{(1)2} + \frac{\theta_t^{(1)}}{\gamma_e M_\infty^2} - \frac{\gamma + 1}{\gamma} v_a v^{(1)} \right]_{-e}^{+e} = 0 \quad 2.38.$$

and,

$$\left[\frac{\gamma}{\gamma - 1} \theta_t^{(1)} + \beta_2 \theta_2^{(1)} + \frac{\gamma_e M_\infty^2}{2} v^{(1)2} \right]_{-e}^{+e} = 0 \quad 2.39.$$

and,

$$\left[\theta_2^{(1)} \right]_{-e}^{+e} = \left(1 + \frac{\tau_2}{\tau_{r2}} \right) \int_{-e}^{+e} \frac{\theta_t^{(1)} - \theta_2^{(1)}}{v^{(1)}} dz \quad 2.40.$$

which are the equations for the $v^{(1)}(z)$ solution. The integrand of the last of these equations is regular and the integral tends to zero as $e \rightarrow 0$, and the equation becomes,

$$\theta_2^{(1)}(-0) = \theta_2^{(1)}(+0) \quad 2.41.$$

where (± 0) means that the variable is evaluated at $z = \pm 0$.

Equation 2.41 shows that to order unity the vibrational temperature remains frozen across the discontinuity at its upstream value, and yields the result which has already been used, namely that $v^{(1)}(+0) = 2v_a - 1$.

Applying the aforementioned procedure to the full equations once more, expanding to order ϵ , and extracting the above first order solution, the equations become,

$$\left[2v^{(1)}v^{(2)} + \frac{\theta_t^{(2)}}{\gamma_e M_\infty^2} - (1 + \frac{3\kappa}{4\mu})v^{(1)} \frac{dv^{(1)}}{dz} - \frac{\gamma + 1}{\gamma} v_a v^{(2)} \right]_{-e}^{+e} = 0 \quad 2.42.$$

$$\left[\frac{\gamma}{\gamma + 1} \theta_t^{(2)} + \beta_2 \theta_2^{(2)} + \gamma_e M_\infty^2 v^{(1)} v^{(2)} \right]_{-e}^{+e} = 0 \quad 2.43.$$

$$\left[\theta_2^{(2)} + \frac{d\theta_2^{(1)}}{dz} \right]_{-e}^{+e} = (1 + \frac{\tau_2}{\tau_{r2}}) \int_{-e}^{+e} \left\{ \frac{\theta_t^{(2)} - \theta_2^{(2)}}{v^{(1)}} - \frac{v^{(2)} \theta_t^{(1)} - \theta_2^{(1)}}{v^{(1)}} \right\} dz \quad 2.44.$$

From the upstream boundary conditions,

$$v^{(2)}(-0) = \theta_t^{(2)}(-0) = \theta_2^{(2)}(-0) = 0 \quad 2.45.$$

so that the above equations become,

$$2v^{(1)}(+0)v^{(2)}(+0) + \frac{\theta_t^{(2)}(+0)}{\gamma_e M_\infty^2} - (1 + \frac{3\kappa}{4\mu})v^{(1)}(+0) \frac{dv^{(1)}}{dz} (+0) - \frac{\gamma + 1}{\gamma} v_a v^{(2)}(+0) = 0 \quad 2.46.$$

$$\frac{\gamma}{\gamma - 1} \theta_t^{(2)}(+0) + \beta_2 \theta_2^{(2)}(+0) + \gamma_e M_\infty^2 v^{(1)}(+0)v^{(2)}(+0) = 0$$

2.47.

$$\theta_2^{(2)}(+0) + \frac{d\theta_2^{(1)}}{dz} (+0) = 0 \quad 2.48.$$

Equation 2.48 results from the fact that the integrand of equation 2.44 is regular and the integral tends to zero as $\epsilon \rightarrow 0$. The above equations can now be solved to give,

$$v^{(2)}(+0) = (1 + \frac{\kappa}{4\mu}) \frac{\gamma}{\gamma + 1} \left(\frac{v_a}{v_a - 1} + 1 \right) \frac{dv^{(1)}}{dz} (+0) - \frac{dv^{(1)}}{dz} (+0)$$

2.49.

The constant of integration in equation 2.37 can now be evaluated for $z > 0$ by comparing the above equation with equation 2.37 evaluated at $v^{(1)} = 2v_a - 1$. The solution for the second term of the outer expansion therefore becomes,

$$v^{(2)}(z) = 0 \quad \text{for } z < 0 \quad 2.50.$$

and,

$$\begin{aligned} v^{(2)} = & \frac{\gamma}{\gamma + 1} \frac{dv^{(1)}}{dz} \left\{ - \left(\frac{1}{v_a} \frac{(2v_a - 1) - v_s}{(1 - v_s)(v_a - 1)} \right) \cdot \ln \frac{1 - v^{(1)}}{2(1 - v_a)} \right. \\ & + \left(\frac{v_s^2}{v_a} \frac{(2v_a - 1) - v_s}{(1 - v_s)(v_a - v_s)} \right) \cdot \ln \frac{v^{(1)} - v_s}{(2v_a - 1) - v_s} \\ & + \left(v_a \frac{(2v_a - 1) - v_s}{(v_a - 1)(v_a - v_s)} - 1 \right) \cdot \ln \frac{v_a - v^{(1)}}{1 - v_a} \\ & \left. + \frac{v_a}{v^{(1)} - v_a} + 1 \right\} \\ & + \frac{\gamma}{\gamma + 1} \frac{\kappa}{4\mu} \frac{dv^{(1)}}{dz} \left\{ - \frac{1}{v_a} \frac{(2v_a - 1) - v_s}{(1 - v_s)(v_a - 1)} \cdot \ln \frac{1 - v^{(1)}}{2(1 - v_a)} \right. \\ & + \frac{v_s^2}{v_a} \frac{(2v_a - 1) - v_s}{(1 - v_s)(v_a - v_s)} \cdot \ln \frac{v^{(1)} - v_s}{(2v_a - 1) - v_s} \end{aligned}$$

$$\begin{aligned}
& + \left(v_a \frac{(2v_a - 1) - v_s}{(v_a - 1)(v_a - v_s)} - 1 \right) \ln \frac{v_a - v^{(1)}}{1 - v_a} \\
& + \frac{v_a}{v^{(1)} - v_a} + 1 \Big\} \\
& - \frac{dv^{(1)}}{dz} \left\{ 1 + \ln \frac{1 - v^{(1)}}{2(1 - v_a)} + \ln \frac{v^{(1)} - v_s}{(2v_a - 1) - v_s} \right. \\
& \left. - \ln \frac{v^{(1)}}{2v_a - 1} \right\} \quad \text{for } z > 0 \quad 2.51.
\end{aligned}$$

The first term in parentheses is the contribution from viscous diffusion, the second term from rotational relaxation and the third term from mass diffusion of the vibrational energy.

This completes the solution for the second term of the outer expansion, and we shall now return to the inner expansion. The outer asymptotic sequence to order ϵ is,

$$v(z; \epsilon) \sim v^{(1)}(z) + \epsilon v^{(2)}(z) \quad 2.52.$$

where $v^{(1)}(z)$ and $v^{(2)}(z)$ are $O(1)$.

Rewriting this sequence in inner coordinates and taking the inner limit with Z fixed, we have,

$$v(z; \epsilon) \sim v^{(1)}(\pm 0) + \epsilon Z \frac{dv^{(1)}}{dz}(\pm 0) + \epsilon v^{(2)}(\pm 0) \quad 2.53.$$

For $z < 0$ this becomes,

$$v(z; \epsilon) \sim 1 \quad 2.54.$$

from which it is concluded that as $Z \rightarrow -\infty$, $v^{(2)} \rightarrow 0$ for matching.

For $z > 0$ equation 2.53 becomes,

$$v(z; \epsilon) \sim 2v_a - 1 + \epsilon \, 2\alpha(Z - 1) \frac{(2v_a - 1 - v_s)}{2v_a - 1} + \epsilon \left(1 + \frac{\beta\kappa}{\mu}\right) \frac{\gamma}{\gamma + 1} 2\alpha \frac{(2v_a - 1 - v_s)}{v_a - 1} \quad 2.55.$$

It can be seen from this equation that the second term of the inner expansion must be $O(\epsilon)$ for order matching with this outer limit. Accordingly we put $\delta_2 = \epsilon$ in the inner expansion. The full equations using a bulk viscosity model for the behaviour of the rotational mode become in inner variables,

$$v^2 + \frac{\theta_t}{M} - \left(1 + \frac{\beta\kappa}{\mu}\right) v \frac{dv}{dz} = v \left(1 + \frac{1}{\gamma_e M^2}\right) \quad 2.56.$$

$$\frac{\gamma}{\gamma - 1} \theta_t + \beta_2 \theta_2 + \frac{\gamma_e M^2}{2} v^2 = \frac{\gamma_e}{\gamma_e - 1} + \frac{\gamma_e M^2}{2} \quad 2.57.$$

$$v \frac{d}{dz} (\theta_2 + \frac{d\theta_2}{dz}) = \epsilon \left(1 + \frac{\tau_2}{\tau_{r2}}\right) (\theta_t - \theta_2) \quad 2.58.$$

Substituting the inner expansion with $\delta_2 = \epsilon$ into the above equations and extracting the solution for $v^{(1)}(z)$ we have to order ϵ ,

$$2v^{(1)}v^{(2)} + \frac{\theta_t^{(2)}}{\gamma_e M^2} - \left(1 + \frac{\beta\kappa}{\mu}\right) (v^{(1)} \frac{dv^{(2)}}{dz} + v^{(2)} \frac{dv^{(1)}}{dz}) = v^{(2)} \left(1 + \frac{1}{\gamma_e M^2}\right) \quad 2.59.$$

$$\frac{\gamma}{\gamma - 1} \theta_t^{(2)} + \beta_2 \theta_2^{(2)} + \gamma_e M^2 v^{(1)} v^{(2)} = 0 \quad 2.60.$$

$$v^{(1)} \frac{d}{dz} (\theta_2^{(2)} + \frac{d\theta_2^{(2)}}{dz}) = \left(1 + \frac{\tau_2}{\tau_{r2}}\right) (\theta_t^{(1)} - 1) \quad 2.61.$$

The last of these equations can be integrated directly to give,

$$\theta_2^{(2)} + \frac{d\theta_2^{(2)}}{dZ} = -\left(1 + \frac{\tau_2}{\tau_{r2}}\right) \left(1 + \frac{\beta_2}{\beta_\mu}\right) \gamma_e \frac{M^2 \gamma - 1}{\gamma - 1} \left\{ v^{(1)} - 1 + 2v_a \ln \frac{v^{(1)} - 2v_a + 1}{2(1 - v_a)} \right\} \quad 2.62.$$

The constant of the integration has been evaluated by matching with the outer solution for $z < 0$. Equation 2.62 can be integrated once more in principle by the use of the integrating factor $\exp Z$. The solution for $\theta_2^{(2)}$ then becomes,

$$\theta_2^{(2)} = -\left(1 + \frac{\tau_2}{\tau_{r2}}\right) \gamma_e \frac{M^2 \gamma - 1}{\gamma - 1} \exp(-Z) \int_{-\infty}^Z \left\{ \hat{v}^{(1)} - 1 + 2v_a \ln \frac{\hat{v}^{(1)} - 2v_a + 1}{2(1 - v_a)} \right\} \exp(\hat{Z}) d\hat{Z} \quad 2.63.$$

$\hat{v}^{(1)}$ and $\hat{Z}$ are dummy variables which bear the same relation to each other as $v^{(1)}$ and Z . The constant of the integration has been evaluated from the requirement that the solution remains bounded for $z < 0$. As we can write $\theta_2^{(2)}(v^{(1)})$, equations 2.59 and 2.60 can be written,

$$-\frac{d}{dv^{(1)}} \left\{ v^{(2)} \frac{dZ}{dv^{(1)}} \right\} = \left\{ 1 + \frac{\beta_2}{\beta_\mu} \right\}^{-1} \frac{\gamma - 1}{\gamma} \frac{\beta_2}{\gamma_e M^2} \frac{\theta_2^{(2)}}{v^{(1)}} \left\{ \frac{dZ}{dv^{(1)}} \right\}^2 \quad 2.64.$$

and $v^{(2)}(Z)$ has the solution,

$$-v^{(2)} = \left(1 + \frac{\beta_2}{\beta_\mu}\right)^{-1} \frac{\gamma - 1}{\gamma} \frac{\beta_2}{\gamma_e M^2} \frac{dv^{(1)}}{dZ} \int_{-\infty}^Z \frac{\hat{\theta}_2^{(2)}}{\hat{v}^{(1)}} \left\{ \frac{d\hat{Z}}{d\hat{v}^{(1)}} \right\} d\hat{Z} + k_3 \frac{dv^{(1)}}{dZ} \quad 2.65.$$

$\hat{\theta}_2^{(2)}$, $\hat{v}^{(1)}$ and $\hat{Z}$ are dummy variables which bear the same relation to each other as $\theta_2^{(2)}$, $v^{(1)}$ and Z . Equations 2.63 and 2.65 must be integrated numerically because their analytical solution is intractable unless further approximations are resorted to. No attempt has therefore been made to evaluate the constant of this last integration analytically.

This completes the solution of viscous vibrational relaxation for the case $v_a < 1$ and $|1 - v_a| = O(1)$ to first order in ϵ .

These solutions for the partly dispersed shock wave have been computed for a shock Mach number of 1.2. It was assumed that the upstream state of the gas was at NTP, i.e. $\gamma_0 = 1.293$.

The theoretical profiles obtained are presented in figures 2.2 and 2.3 for the first and second order terms respectively. It can be seen that the solutions match in terms of the velocity change. However, it will be noted from equation 2.25 that the position of the first inner solution cannot be fixed along the Z axis from the matching requirements. For convenience, we have set $v = v_a$ at $Z = 0$.

This freedom of choice with regard to the first inner solution indicates a lack of coupling between the inner and outer solutions. The reason for this can be seen if the solutions are plotted on a velocity gradient/ velocity plane, as shown in figure 2.4. It would appear from this that a more rigorous solution would be obtained by the determination of an intermediate expansion at $v = 2v_a - 1$, as indicated. This has not been pursued here, but it is clear that a future development of the analysis should examine this region.

2.2.3. The Fully Dispersed Shock Wave. $|v - 1| = 0(1)$.

In this case the perturbation scheme is regular and we can proceed directly to the evaluation of the next term in the original asymptotic expansion, $\Delta_2(\epsilon)v^{(2)}(z)$, in equation 2.5. The expansion will be continued as an asymptotic power series in ϵ , so that $\Delta_2(\epsilon) = \epsilon$.

A bulk viscosity model for the behaviour of the rotational mode will be used to be consistent with the solution for the partly dispersed shock wave given in the preceding section. The resulting basic equations are exactly those given by equations 2.28, 2.29 and 2.30, as follows:

$$v^2 + \frac{\theta_t}{\gamma_e M_\infty^2} - \epsilon(1 + \frac{\beta_K}{\beta_\mu})v \frac{dv}{dz} = v(1 + \frac{1}{\gamma_e M_\infty^2}) \quad 2.66.$$

$$\frac{\gamma}{\gamma - 1} \theta_t + \beta_2 \theta_2 + \frac{\gamma_e M_\infty^2}{2} v^2 = \frac{\gamma_e}{\gamma_e - 1} + \frac{\gamma_e M_\infty^2}{2} \quad 2.67.$$

$$v \frac{d}{dz} (\theta_2 + \epsilon \frac{d\theta_2}{dz}) = (1 + \frac{\tau_2}{\tau_{r2}})(\theta_t - \theta_2) \quad 2.68.$$

Substituting the original asymptotic expansion, equation 2.5, with $\Delta_2(\epsilon) = \epsilon$, into the above equations and extracting the solution for $v^{(1)}(z)$ we have,

$$2v^{(1)}v^{(2)} + \frac{\theta_t}{\gamma_e M_\infty^2} - (1 + \frac{\beta_K}{\beta_\mu})v^{(1)} \frac{dv^{(1)}}{dz} = v^{(2)}(1 + \frac{1}{\gamma_e M_\infty^2}) \quad 2.69.$$

$$\frac{\gamma}{\gamma - 1} \theta_t^{(2)} + \beta_2 \theta_2^{(2)} + \gamma_e M_\infty^2 v^{(1)}v^{(2)} = 0 \quad 2.70.$$

$$v^{(1)} \frac{d\theta_2^{(2)}}{dz} + v^{(2)} \frac{d\theta_2^{(2)}}{dz} + v^{(1)} \frac{d^2 \theta_2^{(1)}}{dz^2} = (1 + \frac{\tau_2}{\tau_{r2}})(\theta_t^{(2)} - \theta_2^{(2)}) \quad 2.71.$$

Using equations 2.6 , 2.34 and 2.35, the above equations reduce to a first order linear differential equation in $v^{(2)}(v^{(1)})$. This equation can be integrated by the use of the integrating factor $dz/dv^{(1)}$ to give,

$$\begin{aligned} v^{(2)} = & \frac{dv^{(1)}}{dz} \frac{\gamma}{\gamma + 1} (1 + \frac{3\kappa}{4\mu}) \left\{ - \frac{(2v_a - 1) - v_s}{v_a(1 - v_s)(v_a - 1)} \cdot \ln(1 - v^{(1)}) \right. \\ & + \frac{v_s^2((2v_a - 1) - v_s)}{v_a(1 - v_s)(v_a - v_s)} \cdot \ln(v^{(1)} - v_s) \\ & + \left(\frac{v_a((2v_a - 1) - v_s)}{(v_a - 1)(v_a - v_s)} - 1 \right) \cdot \ln(v_a - v^{(1)}) \\ & \left. + \frac{v_a}{v^{(1)} - v_a} \right\} \\ & - \frac{dv^{(1)}}{dz} \left\{ \ln(1 - v^{(1)}) + \ln(v^{(1)} - v_s) - \ln(v^{(1)}) \right\} \\ & + k_4 \frac{dv^{(1)}}{dz} \quad 2.72. \end{aligned}$$

As with the first term for the fully dispersed shock wave, the constant of integration is arbitrary and will be left unspecified for the present. This completes the solution of viscous vibrational relaxation for the case $v_a > 1$ and $|1 - v_a| = O(1)$, to first order in ϵ . This is a straightforward perturbation, and no profiles have been computed.

2.2.4. The Transition From A Fully Dispersed To A Partly Dispersed Shock Wave. $|v_a - 1| = o(1)$.

In this section the analysis of viscous vibrational relaxation will be completed by considering the transition from a fully dispersed to a partly dispersed shock wave. In terms of the total range of waves strengths considered this constitutes a very small part, but it is instructive to inquire into the precise nature of the transition.

We shall write,

$$v_a = 1 + \delta \quad 2.73.$$

where δ is $o(1)$ and is positive or negative according as to whether the wave is fully or partly dispersed.

It has already been shown that the downstream velocity, v_s , is related to v_a by,

$$v_s = 2 \frac{\gamma_e}{\gamma} \frac{\gamma + 1}{\gamma_e + 1} v_a - 1 \quad 2.74.$$

so that in this case v_s becomes,

$$v_s = 2 \frac{\gamma_e}{\gamma} \frac{\gamma + 1}{\gamma_e + 1} - 1 + \delta \cdot 2 \frac{\gamma_e}{\gamma} \frac{\gamma + 1}{\gamma_e + 1}$$

or, putting,

$$v_b = 2 \frac{\gamma_e}{\gamma} \frac{\gamma + 1}{\gamma_e + 1} - 1 \quad 2.75.$$

we have,

$$v_s = v_b + \delta (v_b + 1) \quad 2.76.$$

It can also be shown that,

$$\frac{1}{\gamma M^2} = \frac{1}{\gamma} + \delta \frac{\gamma + 1}{\gamma} \quad 2.77.$$

Consider now the solution for $v^{(1)}$ given by equation 2.6, which is common to both the fully dispersed and the partly dispersed shock wave solutions. Substituting equations 2.73 and 2.76 this equation becomes,

$$v^{(1)} \left\{ v^{(1)} - (1 + \delta) \right\} \frac{dv^{(1)}}{dz} = (v^{(1)} - 1) \left\{ v^{(1)} - (v_b + \delta(v_b + 1)) \right\} \quad 2.78.$$

The solution of equation 2.6 for $v_a < 1$ has been shown to be a discontinuous fall in $v^{(1)}$ from the upstream value of unity to $2v_a - 1$, followed by a monotonic decrease to the downstream value of v_s . In this case therefore the velocity falls discontinuously from the upstream value of unity to $1 + 2\delta$, and thence monotonically to $v_b + \delta(v_b + 1)$. In the limit as $\delta \rightarrow 0$ the discontinuity disappears and the solution for $v^{(1)}$ becomes,

$$\frac{dv^{(1)}}{dz} = 0 \quad \text{for } z < 0 \quad 2.79.$$

and,

$$\frac{dv^{(1)}}{dz} = \alpha \frac{(v^{(1)} - v_b)}{v^{(1)}} \quad \text{for } z > 0 \quad 2.80.$$

where as before $v^{(1)} = 1$ at $z = -0$, and $v^{(1)} = 1 + 2\delta$ at $z = +0$.

Equations 2.79 and 2.80 can be integrated directly to give upon application of the boundary conditions,

$$v^{(1)} = 1 \quad \text{for } z < 0 \quad 2.81.$$

and,

$$v^{(1)} - 1 + v_b \ln \frac{v^{(1)} - v_b}{1 - v_b} = az \quad \text{for } z > 0 \quad 2.82.$$

The behaviour in this case is as sketched in figure 2.5. It can be seen from the foregoing set of equations that the first term of the original outer expansion is valid for $\delta = o(1)$, and the effect of δ is postponed until higher order terms. Similarly, the first term of the inner expansion in the original partly dispersed shock wave solution given by equation 2.24 becomes,

$$(1 + \frac{\mu}{\gamma}) v^{(1)} \frac{dv^{(1)}}{dz} = \frac{\gamma + 1}{2\gamma} (v^{(1)} - 1)(v^{(1)} - 1 - 2\delta) \quad 2.83.$$

In this case $v^{(1)}$ decreases monotonically from the upstream value of unity to $1 + 2\delta$. When $\delta \rightarrow 0$ equation 2.83 has the trivial solution,

$$v^{(1)} = 1 \quad \text{for all } z \quad 2.84.$$

and an alternative scaling must be sought.

It can be seen from equations 2.79 and 2.80 that the discontinuity has been transferred from the velocity to its derivative, and an inner expansion must now be provided to describe the behaviour within that discontinuity. For convenience the equations will be transferred to a velocity derivative/velocity plane, that is, the following new variables will be defined,

$$\phi = 1 - v \quad \& \quad p = d\phi/dz$$

and the equations rewritten in them. In order to incorporate the requirement that $\delta = o(1)$, the analysis must be started from the basic equations in the form given by equations A.1.37, A.1.39 and A.1.41, together with a bulk viscosity description of the behaviour of the rotational mode.

Making the appropriate substitutions and applying the upstream boundary conditions equation A.1.37 becomes,

$$\hat{v}(\hat{v} - 2) + \theta_t \left(\frac{1}{\gamma} + \delta \frac{\gamma + 1}{\gamma} \right) + \epsilon \left(1 + \frac{\beta K}{\mu} \right) (1 - \hat{v}) p = \hat{v} \frac{\gamma + 1}{\gamma} (1 + \delta) \quad 2.85.$$

similarly from equation A.1.39,

$$\frac{\gamma}{\gamma - 1} \theta_t \left(\frac{1}{\gamma} + \delta \frac{\gamma + 1}{\gamma} \right) + \beta_2 \theta_2 \left(\frac{1}{\gamma} + \delta \frac{\gamma + 1}{\gamma} \right) + \frac{\hat{v}}{2} (\hat{v} - 2) = 0 \quad 2.86.$$

and from equation A.1.41,

$$(1 - \hat{v}) p \frac{d}{d\hat{v}} (\theta_2 + \epsilon p \frac{d\theta_2}{d\hat{v}}) = (1 + \frac{\tau_2}{\tau_{r2}}) (\theta_t - \theta_2) \quad 2.87.$$

where we have put $\hat{\theta}_t = \theta_t - 1$, & $\hat{\theta}_2 = \theta_2 - 1$.

Differentiating and substituting as appropriate the following second order differential equation in $p(\hat{v})$ is obtained,

$$\epsilon^2 A \frac{d^2 p}{d\hat{v}^2} + \epsilon^2 B \left(\frac{dp}{d\hat{v}} \right)^2 + \epsilon C \frac{dp}{d\hat{v}} = D \quad 2.88.$$

where,

$$A = (1 - \hat{v})^2 p^2 \frac{\gamma}{\gamma - 1} (1 + \frac{\beta K}{\mu})$$

$$B = (1 - \hat{v})^2 p \frac{\gamma}{\gamma - 1} (1 + \frac{\beta K}{\mu})$$

$$C = (1 - \hat{v})^2 p \frac{\gamma}{\gamma - 1} (1 + \frac{\beta K}{\mu}) - 3\epsilon (1 - \hat{v}) p^2 \frac{\gamma}{\gamma - 1} (1 + \frac{\beta K}{\mu})$$

$$- (1 - \hat{v})^2 p \frac{\gamma + 1}{\gamma - 1} + (1 - \hat{v}) p \frac{\gamma + 1}{\gamma - 1} (1 + \delta)$$

$$\begin{aligned}
\text{and, } D &= (1 + \frac{\tau_2}{\tau_{r2}}) \left\{ \frac{\gamma_e}{\gamma_e - 1} (-\frac{\gamma + 1}{\gamma} (1 + \delta) \hat{v} - \hat{v}(\hat{v} - 2)) \right. \\
&\quad \left. - \epsilon (1 + \frac{3K}{4\mu}) (1 - \hat{v}) p + \frac{\hat{v}}{2} (\hat{v} - 2) \right\} - (1 - \hat{v}) p \frac{\gamma + 1}{\gamma - 1} (\delta) \\
&\quad - (1 - \hat{v}) p \frac{\gamma + 1}{\gamma - 1} \hat{v} + \epsilon (1 - \hat{v}) p^2 \frac{\gamma}{\gamma - 1} (1 + \frac{3K}{4\mu}) \\
&\quad - \epsilon (1 - \hat{v}) p^2 \frac{\gamma + 1}{\gamma - 1}
\end{aligned}$$

An approximate solution to equation 2.88 will be found by forming an asymptotic expansion in ϵ , that is, we shall seek a solution for $\hat{v}(z)$ of the form,

$$\hat{v}(z; \delta; \epsilon) \sim \hat{v}^{(\alpha)}(z; \delta) + \delta_\beta(\epsilon) \cdot \hat{v}^{(\beta)}(z; \delta) + o\{\delta_\beta(\epsilon)\} \quad 2.89.$$

as $\epsilon \rightarrow 0$, and where $\delta_\beta(\epsilon) = o(1)$.

Differentiating equation 2.89 with respect to z we have,

$$\begin{aligned}
p(z; \delta; \epsilon) &\sim p^{(\alpha)}(z; \delta) + \delta_\beta(\epsilon) \cdot p^{(\beta)}(z; \delta) \\
&\quad + o\{\delta_\beta(\epsilon)\} \quad 2.90.
\end{aligned}$$

Substituting these equations into equation 2.88, and expanding to order unity it becomes,

$$\frac{d\hat{v}^{(\alpha)}}{dz} = \frac{\alpha(\hat{v}^{(\alpha)} + v_b - 1)}{(1 - \hat{v}^{(\alpha)})} \quad \text{for } z > 0 \quad 2.91.$$

and,

$$\frac{d\hat{v}^{(\alpha)}}{dz} = 0 \quad \text{for } z < 0 \quad 2.92.$$

Integrating these equations we have,

from equation 2.91 ,

$$\hat{v}^{(\alpha)} - 1 + v_b \cdot \ln \frac{\hat{v}^{(\alpha)} - v_b}{1 - v_b} = \alpha z \quad \text{for } z > 0 \quad 2.93.$$

and from equation 2.92 ,

$$\hat{v}^{(\alpha)} = 1 \quad \text{for } z < 0 \quad 2.94.$$

The behaviour of $d\hat{v}^{(\alpha)}/dz(z)$ is sketched in figure 2.5. It can be seen that an inner expansion is required in the region of the discontinuity, and the variables must be scaled in such a way as to become of order unity in this inner region. It will be noted that the derivative of the velocity $d\hat{v}^{(\alpha)}/dz$, must be of order unity in both the inner and outer regions. Thus, v and z must be scaled by the same function of ϵ in order to preserve this condition. Accordingly, the following inner variables will be defined,

$$\Delta(\epsilon)Z = z, \Delta(\epsilon)\hat{V} = \hat{v}, \text{ \& } P = p \quad 2.95.$$

Equation 2.88 becomes in inner variables,

$$\begin{aligned} \epsilon^2 \Delta^{-2}(\epsilon) a \frac{d^2 P}{d\hat{V}^2} + \epsilon^2 \Delta^{-2}(\epsilon) b \left(\frac{dP}{d\hat{V}} \right)^2 \\ + \epsilon \Delta^{-1}(\epsilon) c \frac{dP}{d\hat{V}} = d \end{aligned} \quad 2.96.$$

where,

$$a = (1 - \Delta(\epsilon)\hat{V})^2 P^2 \frac{\gamma}{\gamma - 1} \left(1 + \frac{\beta K}{\mu} \right)$$

$$b = (1 - \Delta(\epsilon)\hat{V})^2 P \frac{\gamma}{\gamma - 1} \left(1 + \frac{\beta K}{\mu} \right)$$

$$c = (1 - \Delta(\epsilon)\hat{V})^2 P \frac{\gamma}{\gamma - 1} \left(1 + \frac{\beta K}{\mu} \right) - 3\epsilon (1 - \Delta(\epsilon)\hat{V}) P^2 \frac{\gamma}{\gamma - 1} \left(1 + \frac{\beta K}{\mu} \right)$$

$$- (1 - \Delta(\epsilon)\hat{V})^2 P \frac{\gamma + 1}{\gamma - 1} + (1 - \Delta(\epsilon)\hat{V}) P \frac{\gamma + 1}{\gamma - 1} (1 + \delta)$$

$$\begin{aligned}
\text{and, } d = (1 + \frac{\tau_2}{\tau_{r2}}) \{ & \frac{\gamma_e}{\gamma_e - 1} - (\frac{\gamma + 1}{\gamma}(1 + \delta)\Delta(\epsilon)\hat{V} - \Delta(\epsilon)\hat{V}(\Delta(\epsilon)\hat{V} - 2) \\
& - \epsilon(1 + \frac{3\kappa}{4\mu})(1 - \Delta(\epsilon)\hat{V})P\} + \frac{1}{2}\Delta(\epsilon)\hat{V}(\Delta(\epsilon)\hat{V} - 2) \} \\
& - (1 - \Delta(\epsilon)\hat{V})P \frac{\gamma + 1}{\gamma - 1} \delta - (1 - \hat{V})P \frac{\gamma + 1}{\gamma - 1} \Delta(\epsilon)\hat{V} \\
& + \epsilon(1 - \Delta(\epsilon)\hat{V})P^2 \frac{\gamma}{\gamma - 1}(1 + \frac{3\kappa}{4\mu}) - \epsilon(1 - \Delta(\epsilon)\hat{V})P^2 \frac{\gamma + 1}{\gamma - 1}
\end{aligned}$$

As with the outer solution, an asymptotic expansion will be formed in ϵ in order to obtain an approximate solution for $\hat{V}(Z)$, that is, we shall write,

$$\begin{aligned}
\hat{V}(Z; \delta; \epsilon) \sim \hat{V}^{(\alpha)}(Z; \delta) + \Delta_\beta(\epsilon) \cdot \hat{V}^{(\beta)}(Z; \delta) \\
+ o\{\Delta_\beta(\epsilon)\}
\end{aligned} \tag{2.97}$$

as $\epsilon \rightarrow 0$, and where $\Delta_\beta(\epsilon) = o(1)$.

Differentiating equation 2.97 with respect to Z we have,

$$\begin{aligned}
P(Z; \delta; \epsilon) \sim P^{(\alpha)}(Z; \delta) + \Delta_\beta(\epsilon) \cdot P^{(\beta)}(Z; \delta) \\
+ o\{\Delta_\beta(\epsilon)\}
\end{aligned} \tag{2.98}$$

When these equations are substituted into equation 2.96 and it expanded to the lowest order in ϵ it is found that the relative magnitudes of ϵ and δ must be considered. So, taking the lowest order terms of each we have, after multiplying through by $\Delta^2(\epsilon)$,

$$\begin{aligned}
\epsilon^2 P^{(\alpha)^2} \frac{\gamma}{\gamma - 1} (1 + \frac{3\kappa}{4\mu}) \frac{d^2 P^{(\alpha)}}{d\hat{V}^{(\alpha)^2}} + \epsilon^2 P^{(\alpha)} \frac{\gamma}{\gamma - 1} (1 + \frac{3\kappa}{4\mu}) (\frac{dP^{(\alpha)}}{d\hat{V}^{(\alpha)}})^2 \\
+ \epsilon \Delta(\epsilon) P^{(\alpha)} \frac{\gamma}{\gamma - 1} (1 + \frac{3\kappa}{4\mu}) \frac{dP^{(\alpha)}}{d\hat{V}^{(\alpha)}} = \Delta^3(\epsilon) (1 + \frac{\tau_2}{\tau_{r2}}) \{ \frac{\gamma_e}{\gamma_e - 1} \frac{\gamma - 1}{\gamma} \hat{V}^{(\alpha)} - \hat{V}^{(\alpha)} \} \\
- \delta \Delta^2(\epsilon) P^{(\alpha)} \frac{\gamma + 1}{\gamma - 1} - \Delta^3(\epsilon) P^{(\alpha)} \frac{\gamma + 1}{\gamma - 1} \hat{V}^{(\alpha)}
\end{aligned} \tag{2.99}$$

It will be recalled from equations 2.91 and 2.92 that the first term of the outer expansion did not contain a term in δ . Consequently, the first term of the inner expansion cannot contain a term in δ for it to match with the outer solution. In this case, the only scaling factor which produces a solution to match with the outer solution is $\epsilon^{\frac{1}{2}}$. Therefore, substituting $\Delta(\epsilon) = \epsilon^{\frac{1}{2}}$ and $\delta = 0$ into equation 2.99 and expanding to order $(\epsilon)^{3/2}$ we have,

$$-\frac{\gamma}{\gamma+1} \frac{P(\alpha)}{\hat{V}(\alpha)} \left(1 + \frac{3\kappa}{4\mu}\right) \frac{dP(\alpha)}{d\hat{V}(\alpha)} = P(\alpha) - \alpha(v_b - 1) \quad 2.100.$$

This equation can be integrated directly by separation of variables to give,

$$-\frac{2\gamma}{\gamma+1} \left(1 + \frac{3\kappa}{4\mu}\right) \left\{ \frac{d\hat{V}(\alpha)}{dZ} + \alpha(v_b - 1) \ln \frac{\frac{d\hat{V}(\alpha)}{dZ} - \alpha(v_b - 1)}{-\alpha(v_b - 1)} \right\} = \hat{V}(\alpha)^2 \quad 2.101.$$

where the constant of the integration has been evaluated by matching with the outer solution, that is, as $\hat{V}(\alpha) \rightarrow 0$, $d\hat{V}(\alpha)/dZ \rightarrow 0$, and as $\hat{V}(\alpha) \rightarrow +\infty$, $d\hat{V}(\alpha)/dZ \rightarrow \alpha(v_b - 1)$, which match with the outer solutions given by equations 2.91 and 2.92,

Equation 2.100 must be solved numerically, and the resulting behaviour of $d\hat{V}(\alpha)/dZ(Z)$ is sketched in figure 2.5 .

The first term of the inner expansion provides no information about the next term of the outer expansion, which can therefore proceed as a simple asymptotic power series in ϵ , and the next term of the inner expansion found by matching. This course of action will not be pursued and the solution will be terminated at this stage.

The particular solution for $V_a = 1$ has been computed for an upstream state of NTP, ie $\delta_e = 1.293$. The theoretical velocity gradient profile which is obtained is presented in figure 2.6. This solution was integrated to yield the velocity profile shown in figure 2.7.

A future analysis of this problem should include an investigation of the cases where $\delta \neq 0$, which have not been pursued here.

The value of the perturbation parameter ϵ , at NTP is of the order of 10^{-4} for carbon dioxide. It can be seen from the theoretical profiles which have been presented that the effects of diffusion will be of the order of 1% of the total velocity change through the relaxation resisted parts of the waves, or contained within a diffusion resisted shock front of the order of 1% of the thickness of the relaxation resisted parts of the waves.

This indicates that conditions could be found where the contributions of diffusion and relaxation of the vibrational mode are of the same order and where they would contribute a 10% change in the distance through the wave.

It should therefore be possible to design an experiment in which diffusion and vibrational relaxation can be seen occurring at the same position in the wave and at levels which can be measured.

This completes the solution of viscous vibrational relaxation which has been investigated here.

2.3. Inviscid Bimodal Vibrational Relaxation.

In this section a solution will be obtained for the one-dimensional flow of a gas with two coupled vibrational modes. Viscosity, heat conduction and mass diffusion will be neglected, and it will be assumed that the rotational mode is in equilibrium with the translational mode. That is, the conservation equations to be used are those given by equations A.1.37 and A.1.39 with $\epsilon = 0$ and $\theta_t = \theta_r$. The behaviour of the vibrational modes will be described by equations A.1.42 and A.1.43.

By substituting and differentiating as appropriate these equations can be combined to form a second order non-linear differential equation in $v(z)$. This equation is not amenable to exact analytical solution and a perturbation scheme must be found to provide an approximate solution.

Over the temperature range of interest, the energy of the primary vibrational mode of carbon dioxide (bending) is much greater than that of the secondary vibrational modes (stretching). A reasonable perturbation scheme can thus be formed by characterising the modal energies by temperatures and using the ratio of the specific heat(s) of the secondary mode(s) to the total vibrational specific heat as the perturbation parameter.

The perturbation parameter, σ , will be defined as,

$$\sigma = C_3/C_v \quad 2.102.$$

where,

$$C_v = C_3 + C_2 \quad 2.103.$$

C_2 is the specific heat of the primary vibrational mode and C_3 is the specific heat of the secondary vibrational mode.

From the above equations it follows that,

$$\frac{C_2}{C_v} = 1 - \sigma \quad 2.104.$$

It will be noted that by defining σ in this way, γ_e does not become a function of σ , and unnecessary complications are obviated.

The conservation equations and rate equations become,

$$v^2 + \frac{\theta_t}{\gamma_e M_\infty^2} = \frac{Fv}{Qu_\infty} \quad 2.105.$$

and,

$$\frac{\gamma}{\gamma - 1} \theta_t + \beta_2 \theta_2 + \beta_3 \theta_3 + \frac{\gamma_e M_\infty^2}{2} v^2 = \frac{H\gamma_e}{a_{e\infty}^2} \quad 2.106.$$

and,

$$(1 - \sigma)v \frac{d\theta_2}{dz} = (1 - \sigma)(\theta_t - \theta_2) + \sigma \frac{\tau_2}{\tau_{23}} (\theta_3 - \theta_2) \quad 2.107.$$

and,

$$v \frac{d\theta_3}{dz} = \frac{\tau_2}{\tau_3} (\theta_t - \theta_3) + \frac{\tau_2}{\tau_{23}} (\theta_2 - \theta_3) \quad 2.108.$$

Applying the upstream boundary conditions to equations 2.105. & 2.106 they become,

$$v^2 + \frac{\theta_t}{\gamma_e M_\infty^2} = v \left(1 + \frac{1}{\gamma_e M_\infty^2} \right) \quad 2.109.$$

and,

$$\frac{\gamma}{\gamma - 1} \theta_t + \beta_v \left((1 - \sigma)\theta_2 + \sigma\theta_3 \right) + \frac{\gamma_e M_\infty^2}{2} v^2 = \frac{\gamma}{\gamma_e - 1} + \frac{\gamma_e M_\infty^2}{2} \quad 2.110.$$

A solution for $v(z)$ of the form,

$$v(z; \sigma) \sim \lambda_a(\sigma) v^{(a)}(z) + \lambda_b(\sigma) v^{(b)}(z) + o\{\lambda_b(\sigma)\} \quad 2.111.$$

as $\sigma \rightarrow 0$, will be sought, where $\lambda_b(\sigma) = o\{\lambda_a(\sigma)\}$.

It can be seen from an inspection of equation 2.107 that the expansion breaks down when $\tau_{23}/\tau_2 = o(1)$, and the solution must be restricted to the case $\tau_{23}/\tau_2 \neq o(1)$. This is not unduly restrictive, because it only excludes the case when the second mode equilibrates so quickly with the primary mode that they can be regarded as being in equilibrium and behaving as a single mode.

When equation 2.111 is substituted into equations 2.107, 2.108 and 2.109, and they are expanded to order $\lambda_a(\sigma)$ it is found that the first value of $\lambda_a(\sigma)$ to yield a non-trivial solution is $\lambda_a(\sigma) = O(1)$. So, putting $\lambda_a(\sigma) = 1$ without loss of generality the equations become,

$$v^{(a)} \frac{d\theta_2^{(a)}}{dz} = \theta_t^{(a)} - \theta_2^{(a)} \quad 2.112.$$

and,

$$v^{(a)} \frac{d\theta_3^{(a)}}{dz} = \frac{\tau_2}{\tau_3} (\theta_t^{(a)} - \theta_3^{(a)}) + \frac{\tau_2}{\tau_{23}} (\theta_2^{(a)} - \theta_3^{(a)}) \quad 2.113.$$

and,

$$v^{(a)2} + \frac{\theta_t^{(a)}}{\gamma_e M_\infty^2} = v^{(a)} \left(1 + \frac{1}{\gamma_e M_\infty^2}\right) \quad 2.114.$$

and,

$$\frac{\gamma}{\gamma - 1} \theta_t^{(a)} + \beta_v \theta_2^{(a)} + \frac{\gamma_e M_\infty^2}{2} v^{(a)2} = \frac{\gamma_e}{\gamma_e - 1} + \frac{\gamma_e M_\infty^2}{2} \quad 2.115.$$

Solving these equations, it is found that $v(z)$ must satisfy the following equation,

$$v^{(a)}(v^{(a)} - v_a) \frac{dv^{(a)}}{dz} = \alpha'(v^{(a)} - 1)(v^{(a)} - v_s) \quad 2.116.$$

$$\text{where } \alpha' = -\frac{1}{2} \frac{\gamma - 1}{\gamma + 1} \frac{\gamma_e + 1}{\gamma_e - 1}, \text{ \& } v_a = 2 \frac{\gamma}{\gamma_e} \frac{\gamma_e + 1}{\gamma + 1} (v_s + 1)$$

This is exactly the result obtained in section 2.2.1, except that α' has a slightly different meaning. The remarks which were made therein also apply here, however, this section will only be concerned with the regions where the solutions are continuous. Integrating equation 2.116 in these regions of continuity we have,

$$v^{(a)} - A \ln(v^{(a)} - 1) - B \ln(v^{(a)} - v_s) = \alpha' z + k_3 \quad 2.117.$$

where,

$$A = \frac{1 - v_a}{1 - v_s} \quad \text{and } B = \frac{v_s(v_s - v_a)}{1 - v_s}.$$

The constant of the integration will only be evaluated for the partly dispersed shock wave. So, putting $v^{(a)} = 2v_a - 1$, at $z = +0$, equation 2.117 becomes for the partly dispersed shock wave,

$$\begin{aligned} v^{(a)} - (2v_a - 1) - A \ln \frac{(v^{(a)} - 1)}{2(v_a - 1)} - B \ln \frac{(v^{(a)} - v_s)}{2v_a - 1 - v_s} \\ = \alpha' z \quad \text{for } z > 0 \end{aligned} \quad 2.118.$$

and,

$$v^{(a)} = 1 \quad \text{for } z < 0 \quad 2.119.$$

It can be seen that the behaviour of the secondary mode does not enter in to this first term. However, its behaviour must be determined in order that the perturbation scheme be continued.

Equation 2.113 can be rewritten as,

$$v^{(a)} \frac{d\theta_3^{(a)}}{dz} + \left(\frac{\tau_2}{\tau_3} + \frac{\tau_2}{\tau_{23}} \right) \theta_3^{(a)} = \frac{\tau_2}{\tau_3} \theta_t^{(a)} + \frac{\tau_2}{\tau_{23}} \theta_2^{(a)} \quad 2.120.$$

The integrating factor for this linear differential equation is,

$$\exp \left\{ \left(\frac{\tau_2}{\tau_3} + \frac{\tau_2}{\tau_{23}} \right) \int_{-\infty}^z \frac{1}{\hat{v}^{(a)}} dz \right\}$$

and the solution for $\theta_3^{(a)}$ is,

$$\theta_3^{(a)} = \exp \left\{ - \left(\frac{\tau_2}{\tau_3} + \frac{\tau_2}{\tau_{23}} \right) \int_{-\infty}^z \frac{1}{\hat{v}^{(a)}} dz \right\}.$$

$$\int_{-\infty}^z \left(\frac{\tau_2}{\tau_3} \hat{\theta}_t^{(a)} + \frac{\tau_2}{\tau_{23}} \hat{\theta}_2^{(a)} \right) \frac{1}{\hat{v}^{(a)}} \exp \left\{ \left(\frac{\tau_2}{\tau_3} + \frac{\tau_2}{\tau_{23}} \right) \int_{-\infty}^z \frac{1}{\hat{v}^{(a)}} dz \right\} d\hat{z}$$

2.121.

where $\hat{v}^{(a)}$, $\hat{z}$, $\hat{\theta}$, $\hat{v}^{(a)}$, $\hat{z}$ and $\hat{\theta}$ are dummy variables, each set of which are related to each other in the same way as $v^{(a)}$, z , etc. ,

The constant of the integration has been evaluated from the boundedness requirement of the downstream boundary condition. This equation will be made use of in determining the next term of the expansion.

The first term of this expansion is uniformly valid in the regions of interest, so the expansion can be continued directly as an asymptotic power series in σ . Substituting equation 2.111 with $\lambda_b(\sigma) = \sigma$ into equations 2.107, 2.108, 2.109 and 2.110 and extracting the $v^{(a)}$ solution we have,

$$v^{(a)} \frac{d\theta_2^{(b)}}{dz} + v^{(b)} \frac{d\theta_2^{(a)}}{dz} - v^{(a)} \frac{d\theta_2^{(a)}}{dz} =$$

$$(\theta_t^{(b)} - \theta_2^{(b)}) - (\theta_t^{(a)} - \theta_2^{(a)}) + \frac{\tau_2}{\tau_{23}} (\theta_2^{(a)} - \theta_3^{(a)}) \quad 2.122.$$

and,

$$v^{(a)} \frac{d\theta_3^{(b)}}{dz} + v^{(b)} \frac{d\theta_3^{(a)}}{dz} = \frac{\tau_2}{\tau_3} (\theta_t^{(b)} - \theta_3^{(b)}) + \frac{\tau_2}{\tau_{23}} (\theta_2^{(b)} - \theta_3^{(b)})$$

2.123.

and,

$$2v^{(a)}v^{(b)} + \frac{\theta_t^{(b)}}{\gamma_e M_\infty^2} = v^{(b)} \left(1 - \frac{1}{\gamma_e M_\infty^2}\right)$$

2.124.

and,

$$\frac{\gamma}{\gamma - 1} \theta_t^{(b)} + \beta_v \theta_2^{(b)} + \beta_v (\theta_3^{(a)} - \theta_2^{(a)}) + \gamma_e M_\infty^2 v^{(a)} v^{(b)} = 0$$

2.125.

It can be shown from these equations that $v^{(b)}$ must obey the following first order linear differential equation,

$$\frac{dv^{(b)}}{dz} - \frac{d}{dv^{(a)}} \left(\frac{dv^{(a)}}{dz} \right) v^{(b)} = \frac{\gamma - 1}{\gamma_e M_\infty^2 \gamma + 1} \frac{1}{v^{(a)} (v^{(a)} - v_a)} \left\{ \left(\frac{\tau_2}{\tau_3} - 1 \right) (\theta_t^{(a)} - \theta_3^{(a)}) + 2 \frac{\tau_2}{\tau_{23}} (\theta_2^{(a)} - \theta_3^{(a)}) \right\}$$

2.126.

The integrating factor for this equation is $dz/dv^{(a)}$, so that $v^{(b)}$ has the solution,

$$v^{(b)} = \frac{\beta_v \gamma - 1}{\gamma_e M_\infty^2 \gamma + 1} \frac{dv^{(a)}}{dz} \int_1^{v^{(a)}} \frac{(dz/d\hat{v}^{(a)})^2}{\hat{v}^{(a)} (\hat{v}^{(a)} - v_a)} \cdot \left\{ \left(\frac{\tau_2}{\tau_3} - 1 \right) (\hat{\theta}_t^{(a)} - \hat{\theta}_3^{(a)}) + 2 \frac{\tau_2}{\tau_{23}} (\hat{\theta}_2^{(a)} - \hat{\theta}_3^{(a)}) \right\} d\hat{v}^{(a)} + k_4 \frac{dv^{(a)}}{dz}$$

2.127.

where $\hat{v}^{(a)}$ and $\hat{z}$ are dummy variables related to each other in the same way as $v^{(a)}$ and z .

Clearly, many forms of theoretical profiles could be computed from this solution, depending on the relative magnitudes of τ_2 , τ_3 and τ_{23} . As there was no information available regarding the magnitude of these relaxation times for carbon dioxide, no theoretical profiles have been computed. It was not considered to be worth while to compute the limiting cases of infinite relaxation times as this had been done elsewhere for the unperturbed solutions. This may have provided a useful check on the accuracy of the perturbation scheme, but this was not considered to be relevant to the present studies. This solution will be applicable at such time that bimodal behaviour can be observed in weak compression waves.

2.4. Inviscid Vibrational Relaxation Using A Second Order Rate Equation.

In this section a solution will be obtained for the one-dimensional flow of a gas with a single active vibrational mode whose behaviour is governed by a second order rate equation, that is, equation A.1.44. Viscosity, heat conduction and mass diffusion will be neglected, and it will be assumed that the rotational mode is in equilibrium with the translational mode. The conservation equations to be used are therefore those given by equations A.1.37 and A.1.39 with $\epsilon = 0$, and $\theta_r = \theta_t$.

Applying the upstream boundary conditions to equations A.1.37 and A.1.39 with $\epsilon = 0$ they become,

$$v^2 + \frac{\theta_t}{\gamma_e M_\infty^2} = v(1 + \frac{1}{\gamma_e M_\infty^2}) \quad 2.128.$$

and,

$$\frac{\gamma}{\gamma - 1} \theta_t + \beta_2 \theta_2 + \frac{\gamma_e M_\infty^2}{2} v^2 = \frac{\gamma_e}{\gamma_e - 1} + \frac{\gamma_e M_\infty^2}{2} \quad 2.129.$$

These equations can be substituted into the rate equation, A.1.44, and a solution obtained directly, however, the form of the solution is so complex as to disguise the essential features. Equation A.1.44 can be rewritten using the Binomial theorem as,

$$\frac{v}{(\theta_t - \theta_2)} \frac{d\theta_2}{dz} - \frac{\tau_2}{\tau_2'} \frac{v}{2} \frac{d\theta_2}{dz} = 1 \quad 2.130.$$

where it is assumed that $\tau_2' \gg \tau_2(\theta_t - \theta_2)$.

Substituting equations 2.128 and 2.129 into equation 2.130 we have,

$$\frac{v(v - v_a)}{(v - 1)(v - v_s)} \frac{dv}{dz} + \Gamma v(v - v_a) \frac{dv}{dz} = \alpha' \quad 2.131.$$

where $\alpha' = -\frac{1}{2} \frac{\gamma_e + 1}{\gamma_e - 1} \frac{\gamma - 1}{\gamma + 1}$, & $\Gamma = \frac{1}{4} \frac{\gamma_e M_\infty^2}{2} \frac{\gamma_e + 1}{\gamma_e - 1} \frac{\tau_2}{\tau_2'}$.

The first term on the left hand side of equation 2.131 is the basic solution obtained in section 2.2.1, except that α' has a different meaning. The second order term in the rate equation thus appears as a correction to this basic solution in a similar manner to the other solutions herein.

Integrating equation 2.131 in the regions of continuity in v we have, for the partly dispersed shock wave,

$$v = 1 \quad \text{for } z < 0 \quad 2.132.$$

and,

$$v - (2v_a - 1) + A \ln \frac{(v - 1)}{2(v_a - 1)} - B \ln \frac{(v - v_s)}{(2v_a - 1 - v_s)} + \Gamma \frac{v^2}{3} (v - \frac{3}{2} v_a) - \Gamma \frac{(2v_a - 1)^2 (v_a - 2)}{6} = \alpha' z \quad \text{for } z > 0$$

$$2.133.$$

and for the fully dispersed shock wave,

$$v + A \ln(v - 1) - B \ln(v - v_s) + \Gamma \frac{v^2}{3} (v - \frac{3}{2} v_a) = \alpha' z + k_5 \quad 2.134.$$

where,

$$A = \frac{1 - v_a}{1 - v_s}, \quad \& \quad B = \frac{v_s (v_s - v_a)}{1 - v_s}$$

and for the partly dispersed shock wave $v = 2v_a - 1$ at $z = 0$.

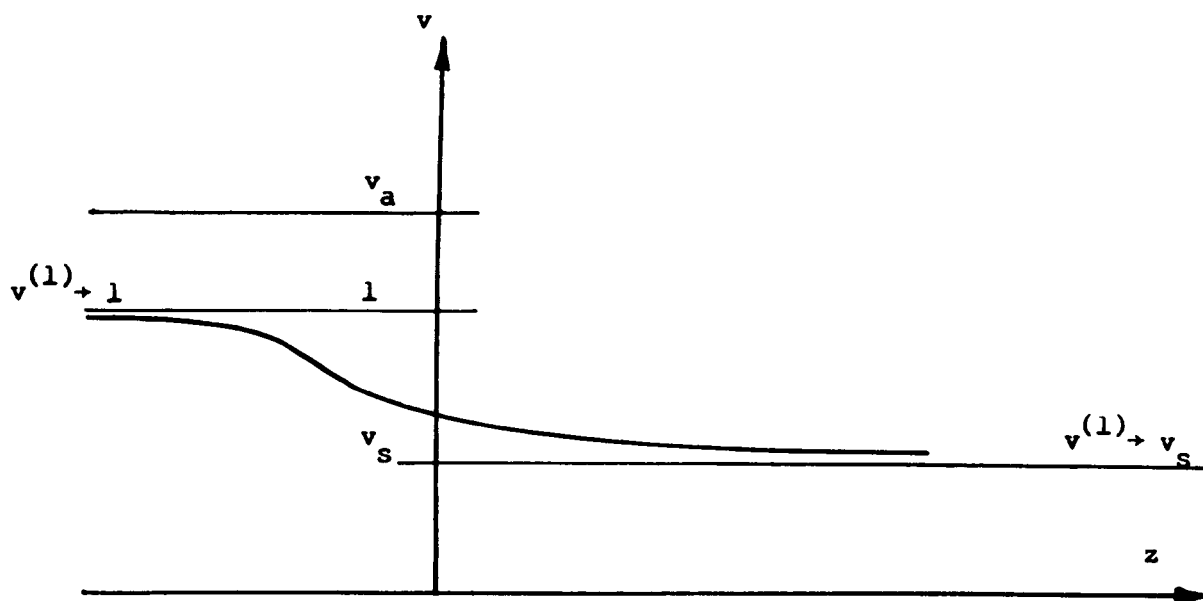
If we write,

$$\frac{\rho}{\rho_1} (z; \tau_2/\tau_2') \sim \frac{\rho^{(1)}}{\rho_1} (z) + \tau_2/\tau_2' \frac{\rho^{(2)}}{\rho_1} (z) ,$$

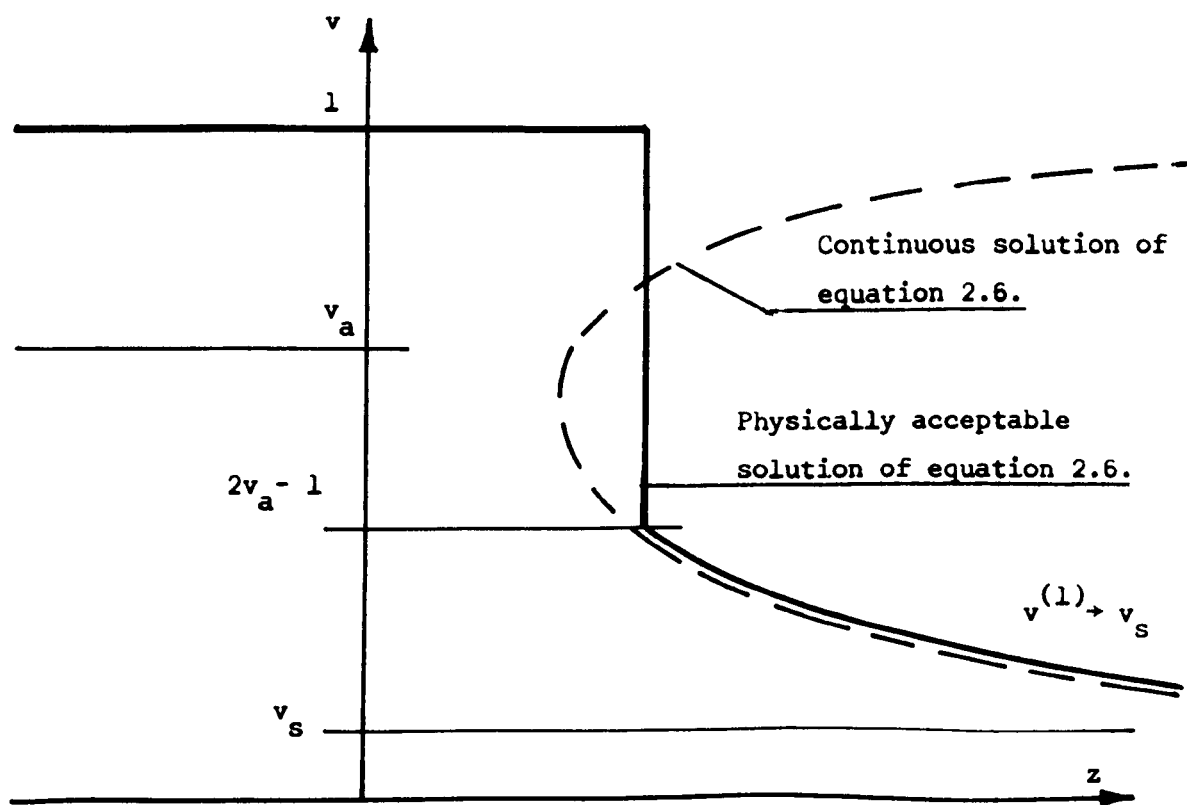
then the first and second order terms can be identified in terms of the basic solution and the correction term of equation 2.133. In this way, theoretical profiles have been computed for $\rho^{(1)}$ and for $\rho^{(2)}$. These computations have been performed for shock Mach numbers of 1.03, 1.04, 1.05 and 3.0. The resulting profiles are presented in figures 2.8, 2.9 and 2.10.

The difference between the first and second order solutions for the $M = 3.0$ case were compared to the dependence on departure from equilibrium observed by Johannesen for that Mach number (Ref. 2.5). It was found that, for the non-correlation of the Johannesen data to be explained in terms of a second order rate equation, the magnitude of the ratio of the relaxation times would have to be of the order of 0.3, as shown in figure 2.10.

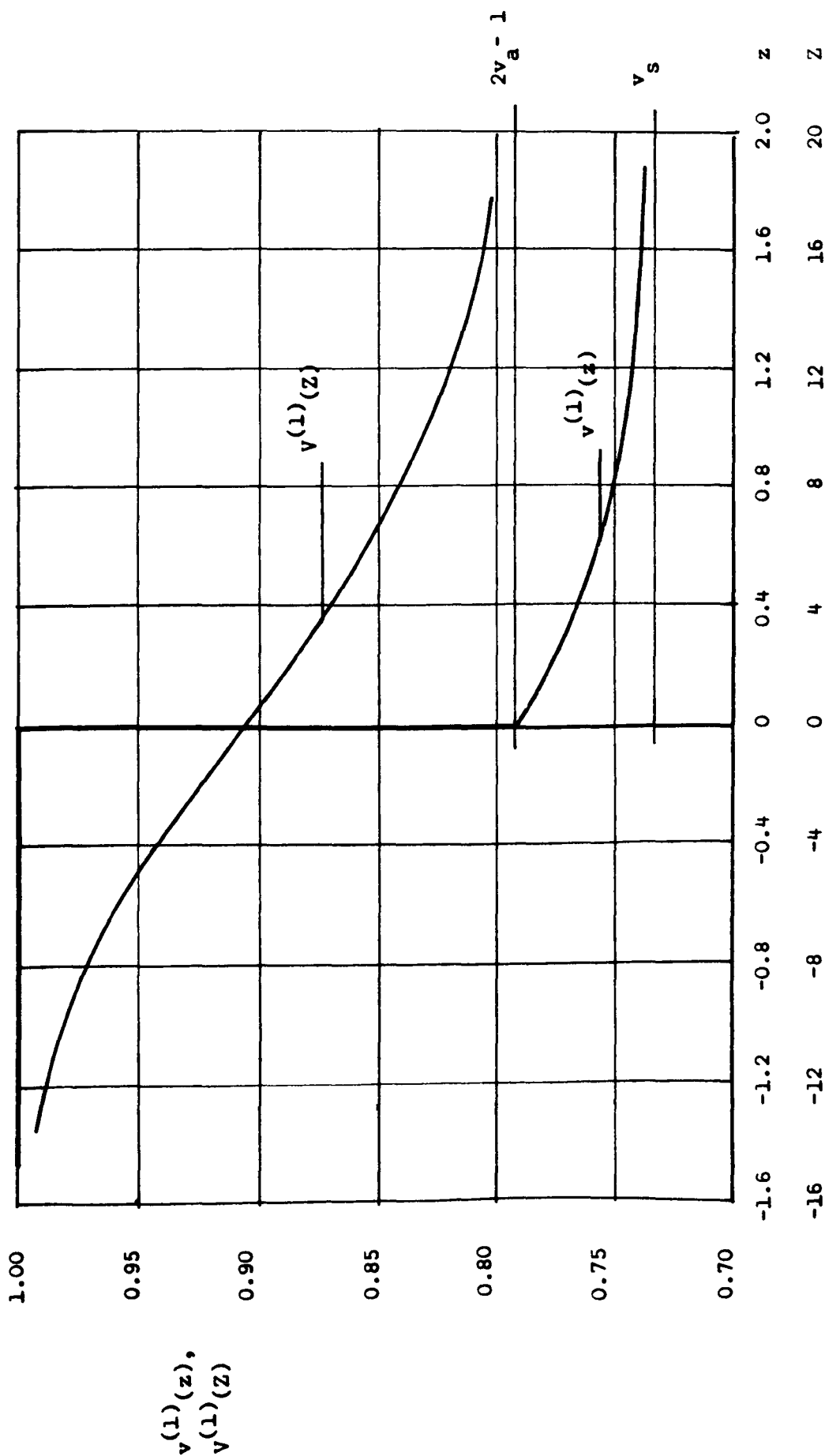
It can be seen from figures 2.8 and 2.9 that, if the ratio of the relaxation times was of this order, the correction to the velocity profiles for the weak normal shock waves would be negligible.



A. $v_a > 1$.

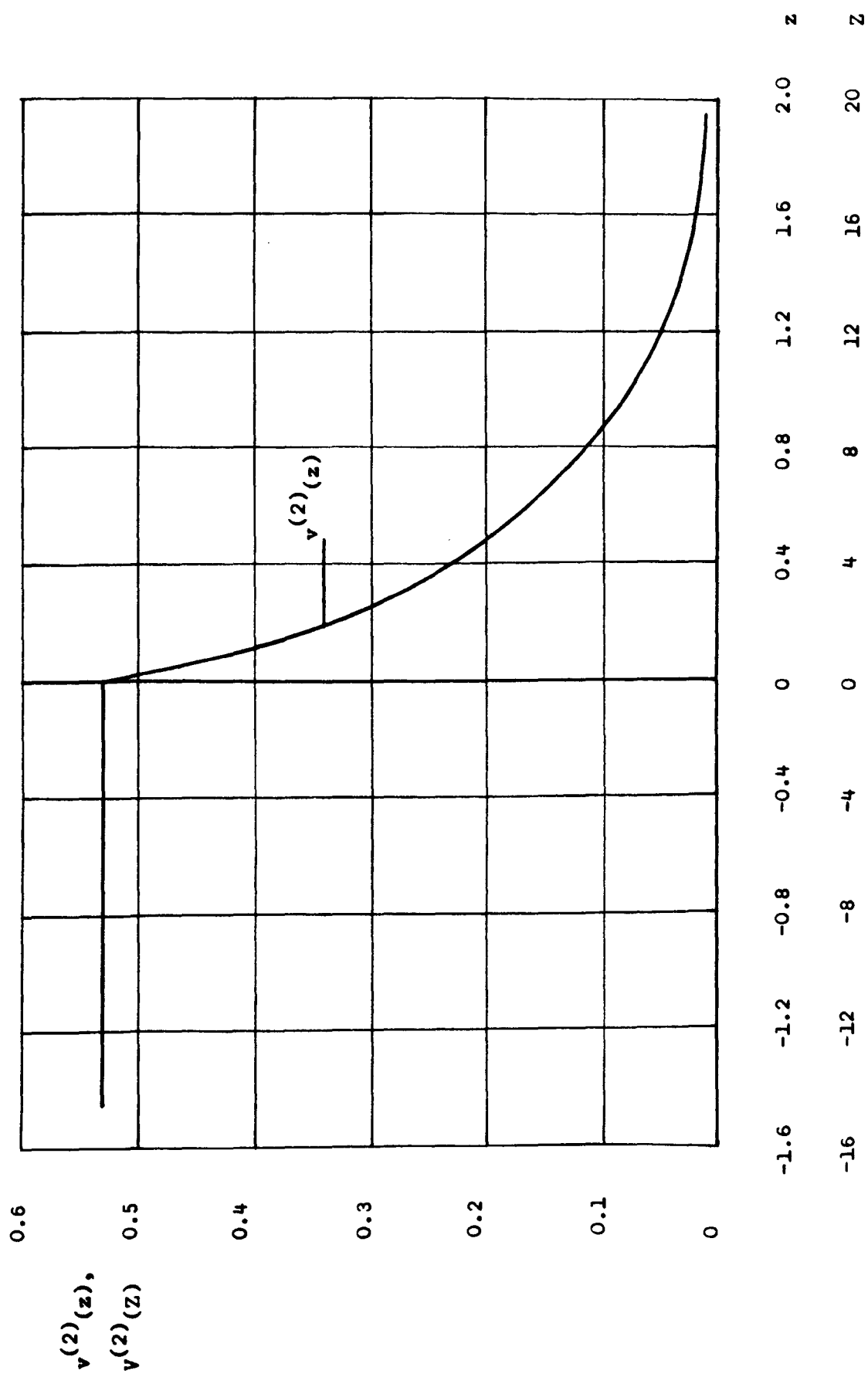


B. $v_a < 1$.



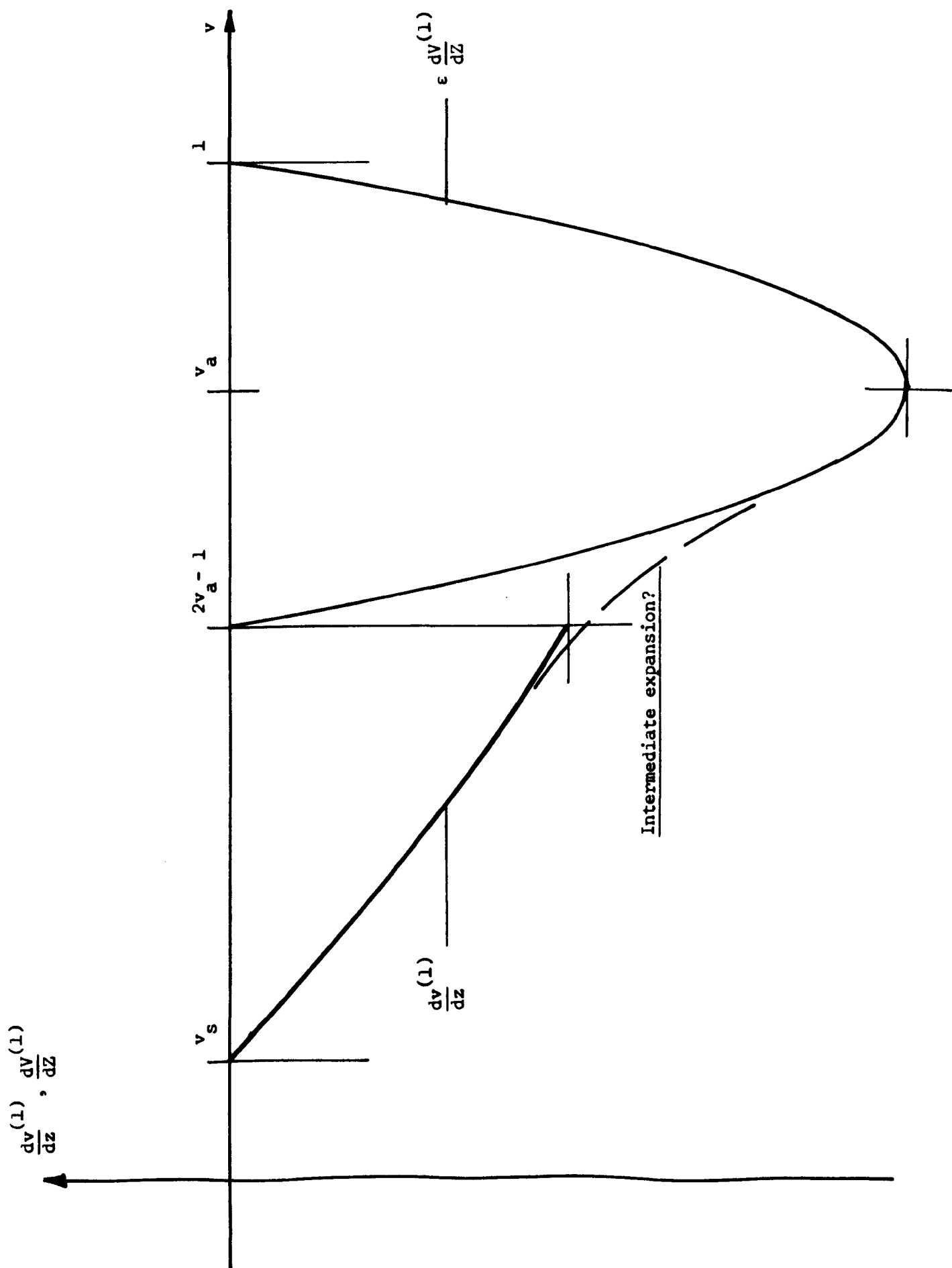
FIRST INNER AND OUTER SOLUTIONS FOR A PARTLY DISPERSED SHOCK WAVE OF MACH NUMBER 1.2.

FIGURE: 2.2.

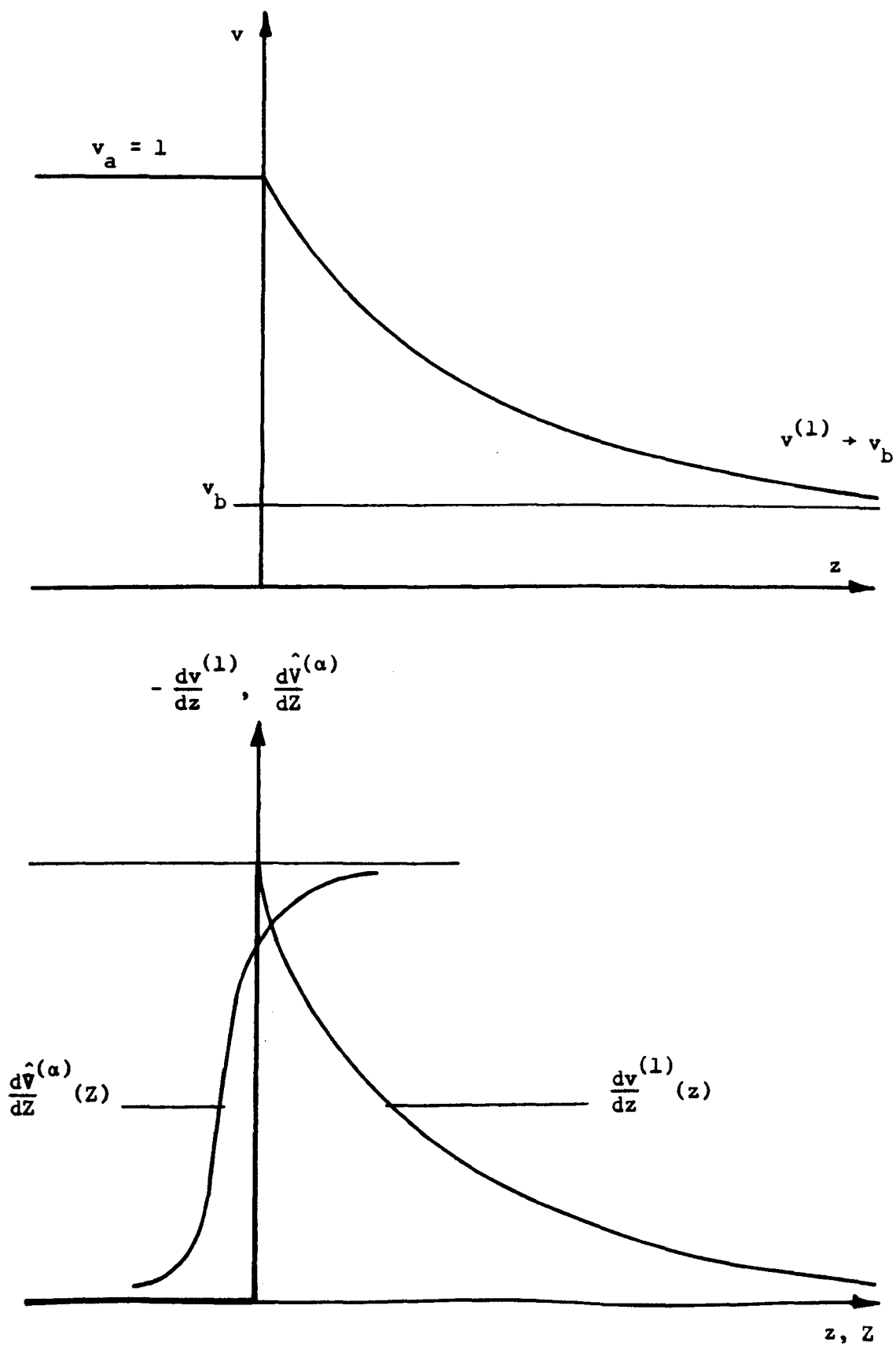


SECOND INNER AND OUTER SOLUTIONS FOR A PARTLY
DISPERSED SHOCK WAVE HAVING A MACH NUMBER OF 1.2.

FIGURE: 2.3.

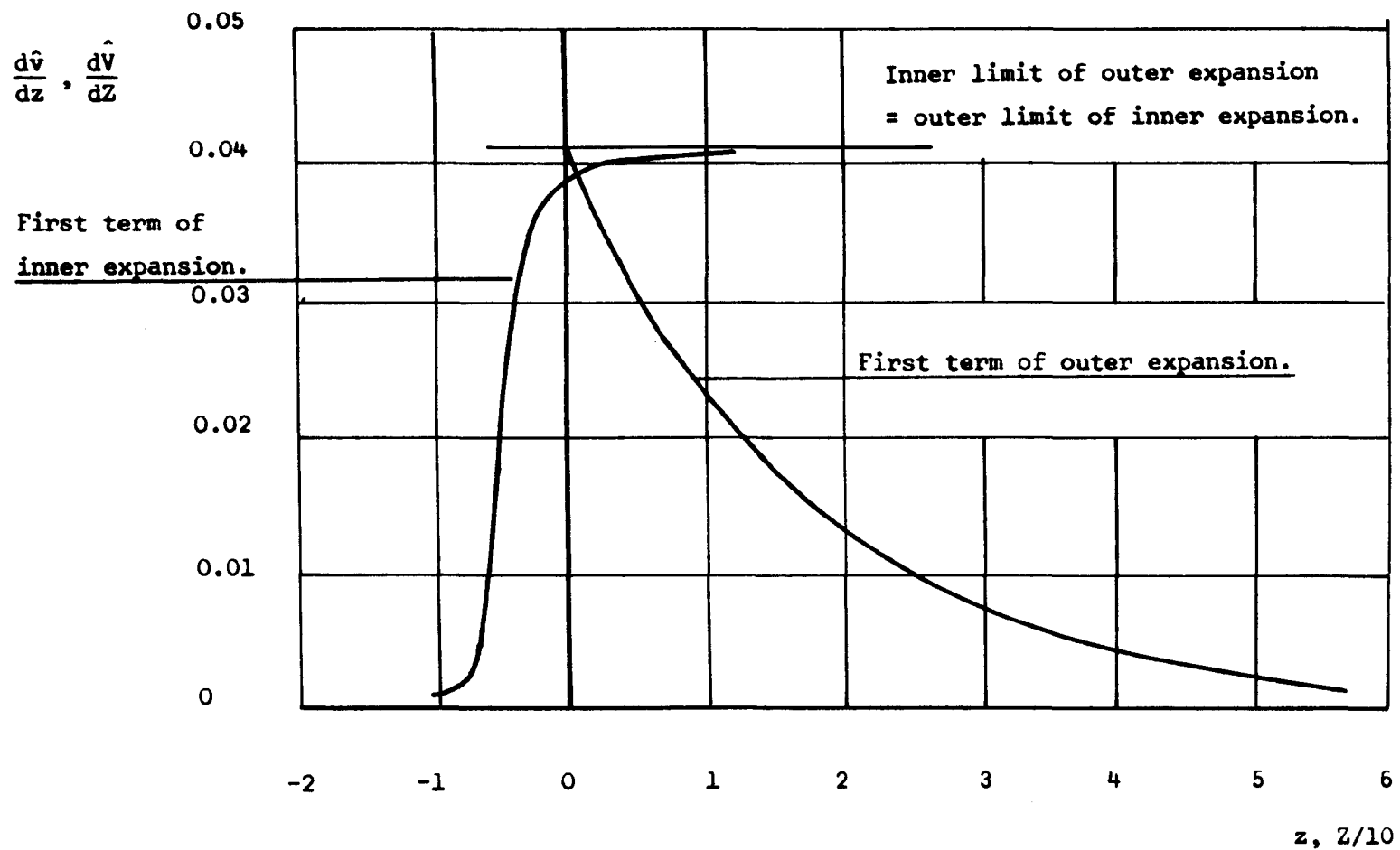


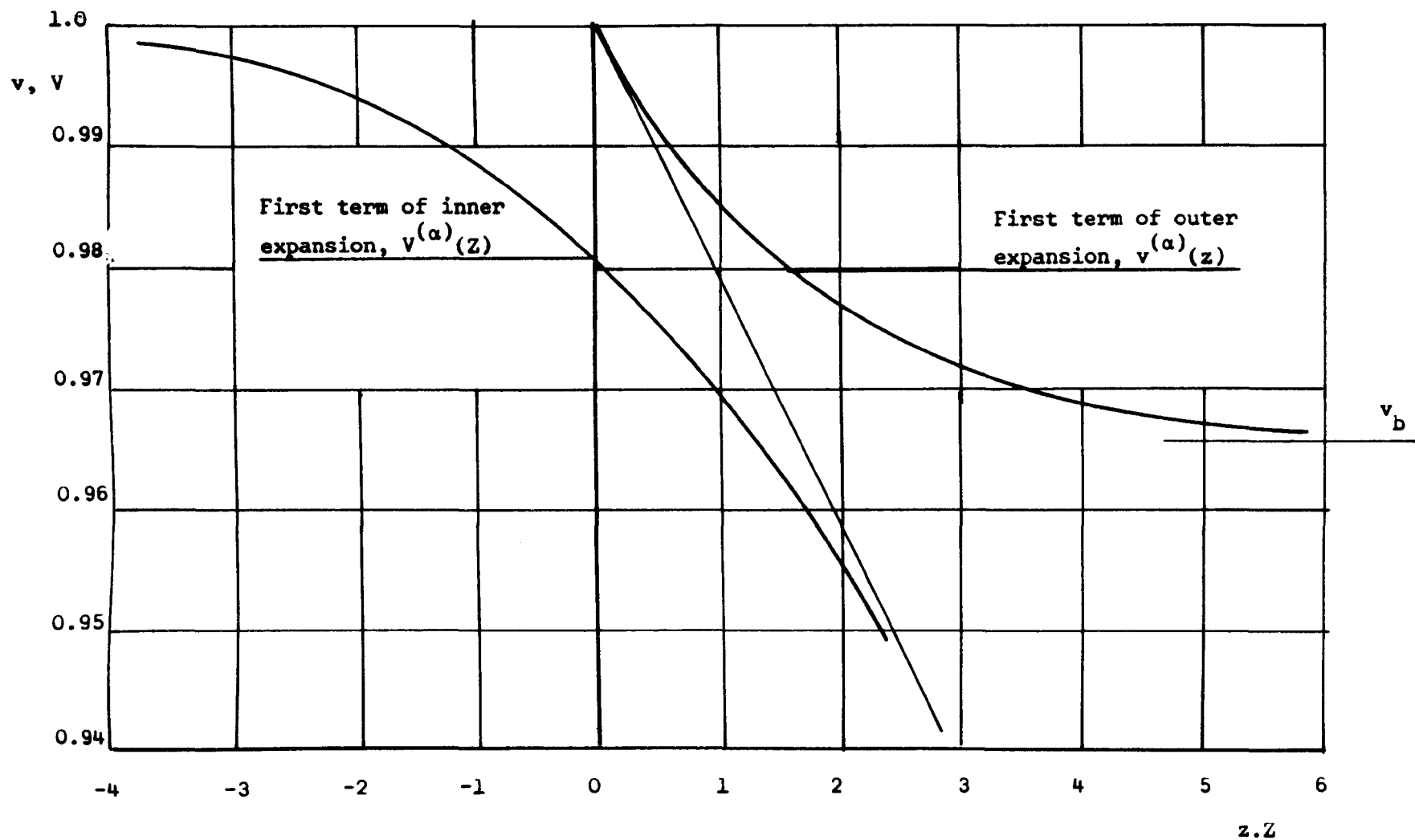
VELOCITY GRADIENT / VELOCITY PRESENTATION OF THE
PARTLY DISPERSED SHOCK WAVE,

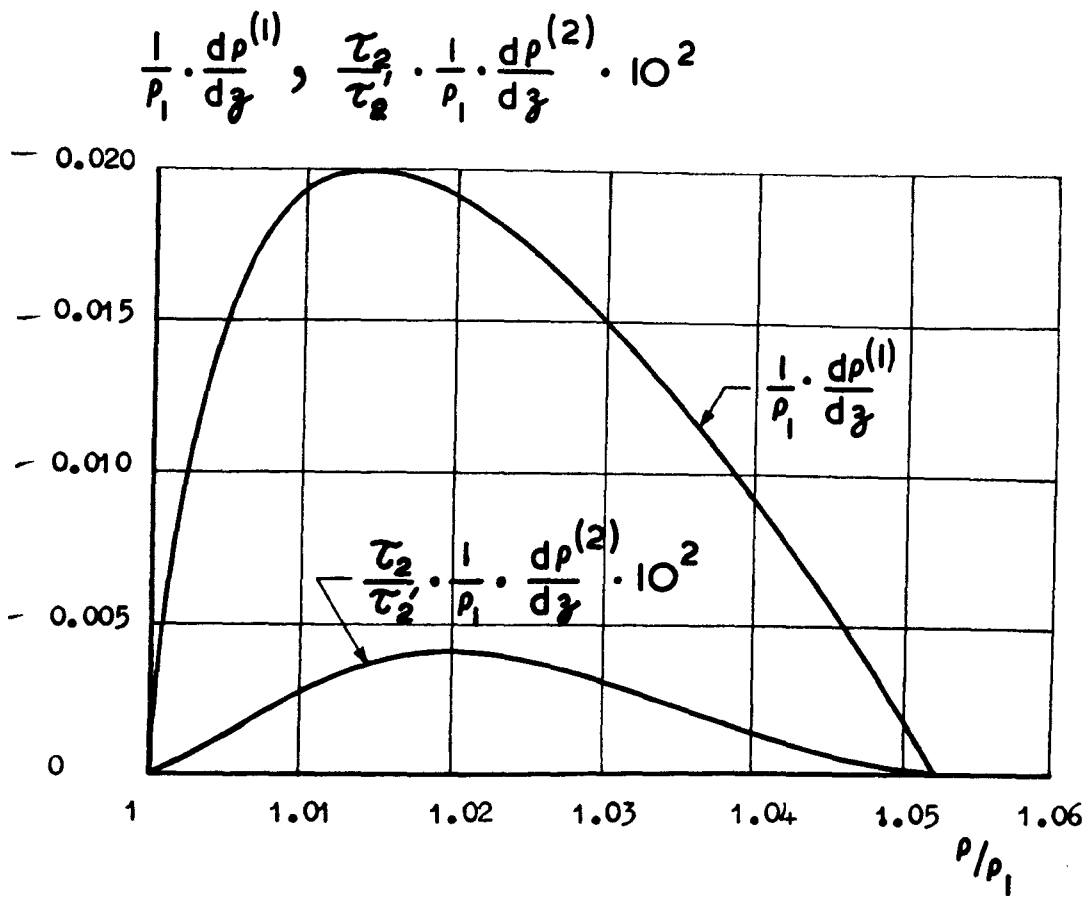
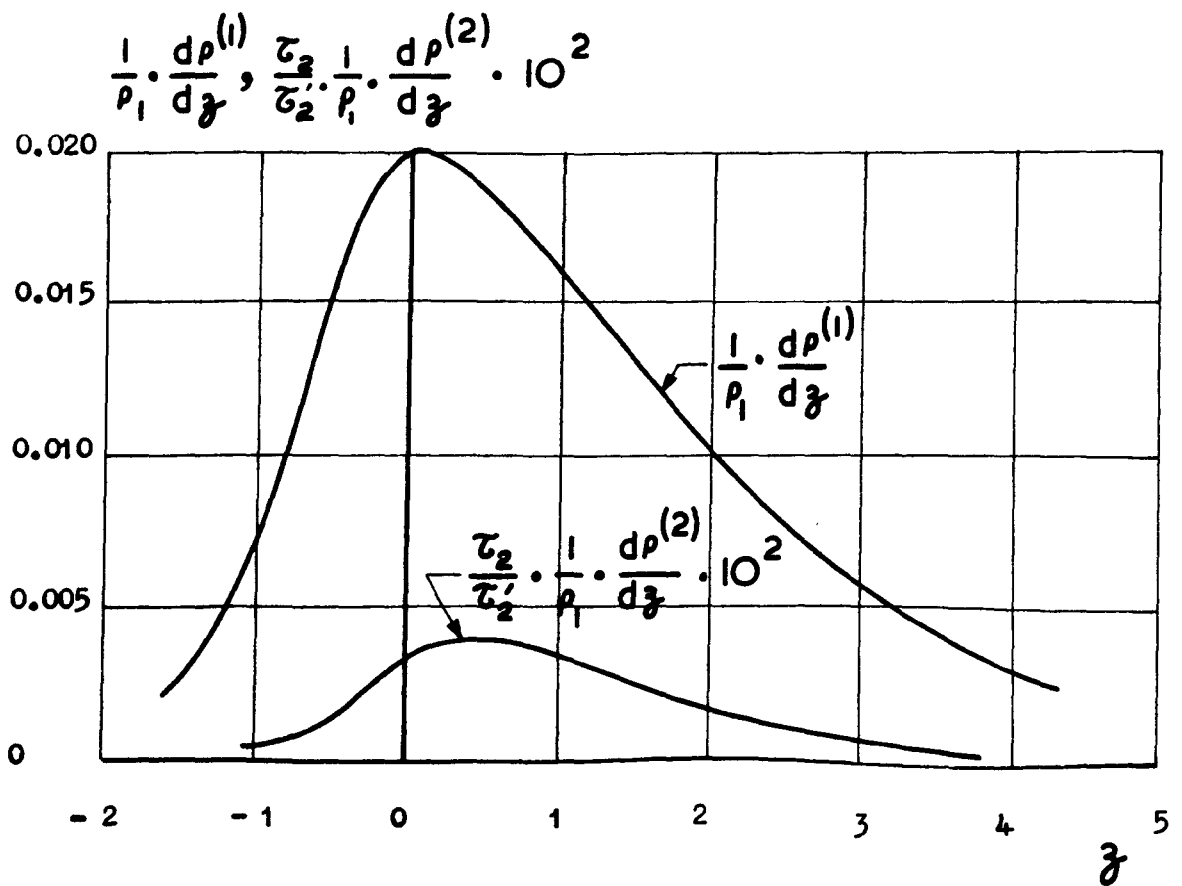


VELOCITY AND VELOCITY GRADIENT PROFILES OF THE
TRANSITION FROM A FULLY TO A PARTLY DISPERSED SHOCK.

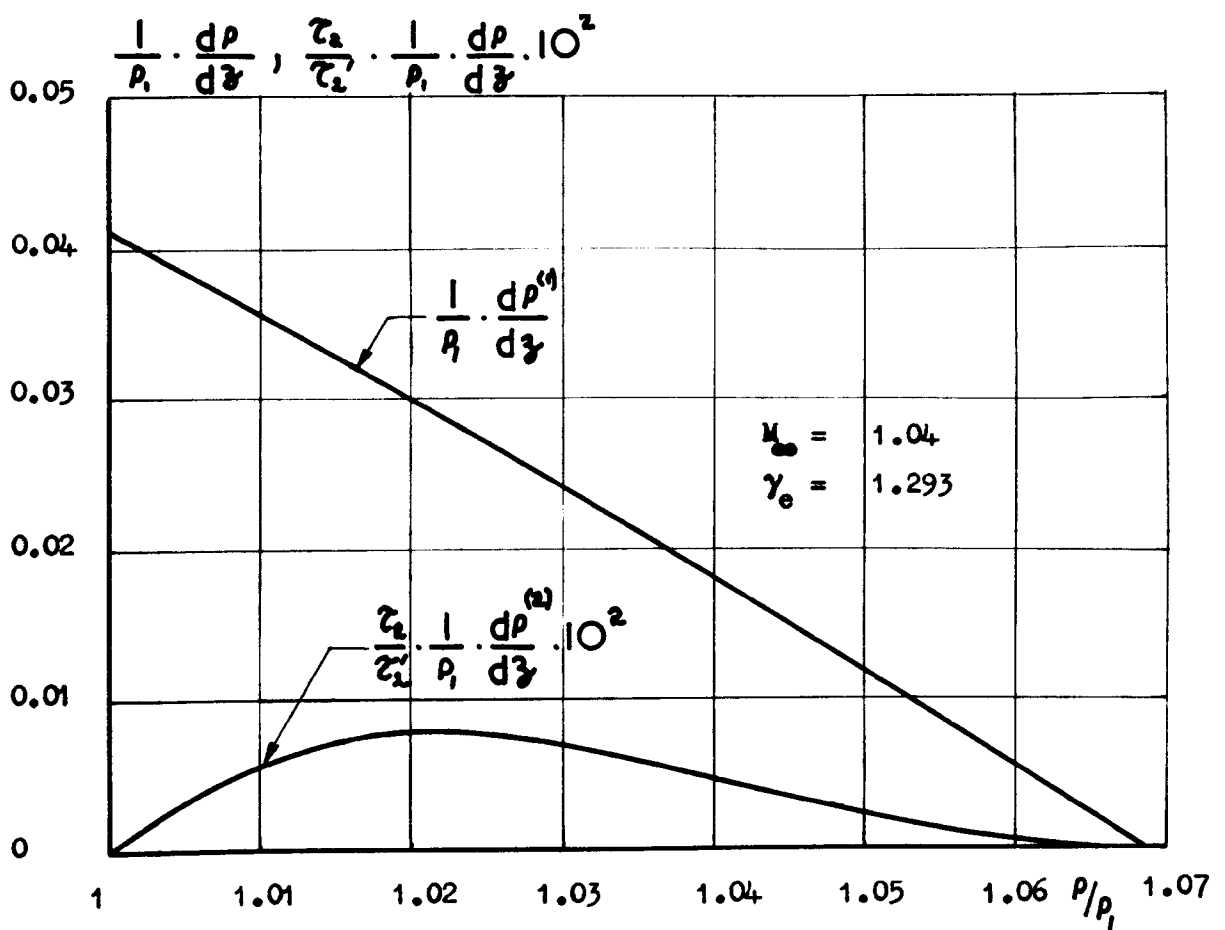
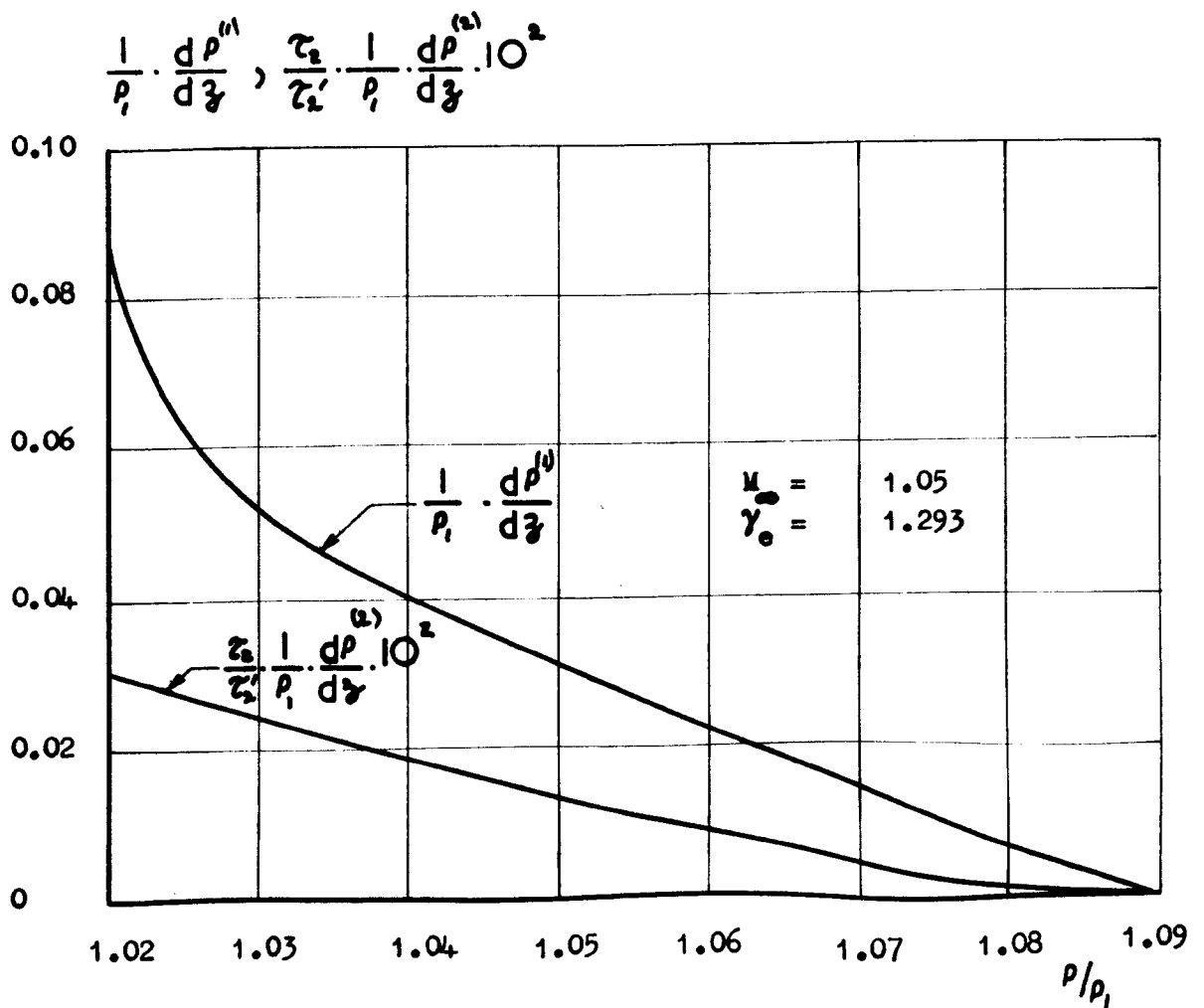
FIGURE: 2.5.







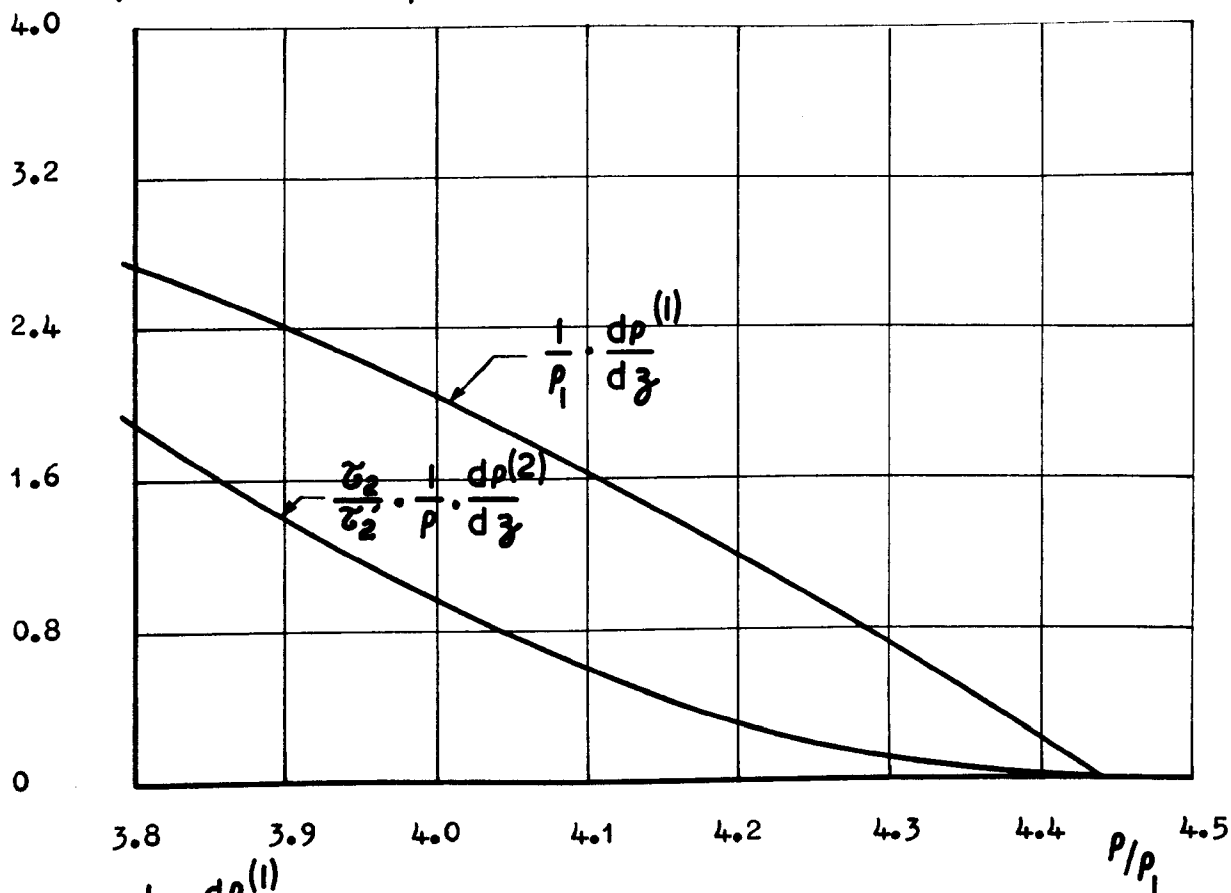
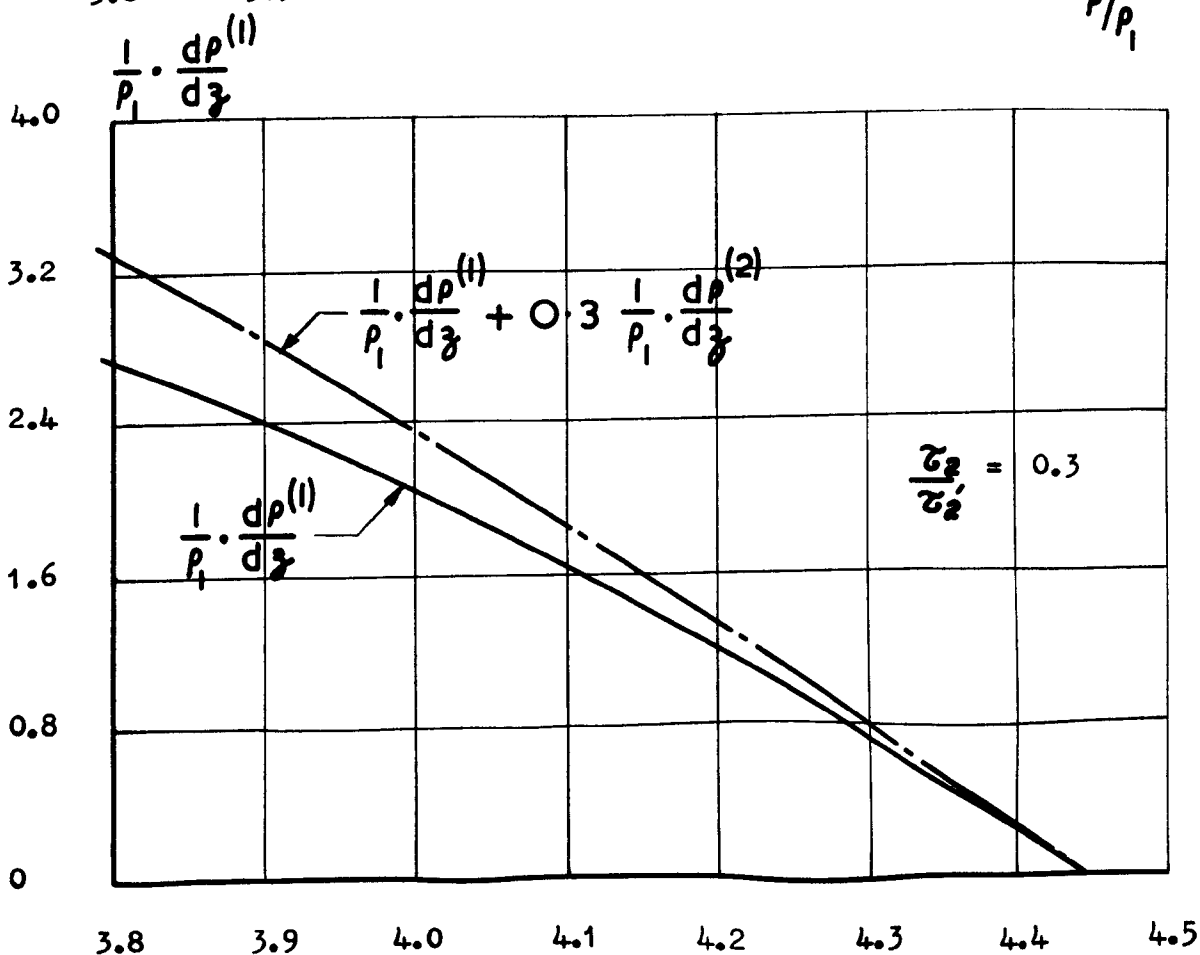
$M_\infty = 1.03 \quad \gamma_e = 1.293$



PARTLY DISPERSED SHOCK WAVES WITH A SECOND ORDER RATE EQUATION.

FIGURE: 2.9.

$$\frac{1}{\rho_1} \cdot \frac{d\rho^{(1)}}{dz}, \quad \frac{\tau_2}{\tau_2'} \cdot \frac{1}{\rho_1} \cdot \frac{d\rho^{(2)}}{dz}$$



$$M_\infty = 3.0 \quad \gamma_0 = 1.293$$

PARTLY DISPERSED SHOCK WAVES WITH A SECOND ORDER RATE EQUATION.

FIGURE: 2.10.

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3.1. THE SCHLIEREN SYSTEM.

SUMMARY

A quantitative schlieren system has been used to study the normal shock waves which have been produced in the shock tube. This particular system was developed for this purpose in an earlier study, and further developments and refinements have been made to it.

The principle of operation of the system is described, and the relevant optical theory is outlined. The physical layout of the system is described, and consideration is given to its practical limitations with regard to the spatial and temporal resolution and the calibration. The method of setting up the system for optimum performance is also described together with the developments and refinements which have been made.

Conclusions are drawn regarding the suitability of the system for this application and future developments are discussed.

NOTATION.

A	Aperture.
b	Height of sampling slit.
C	Speed of light
C_0	Speed of light in a vacuum,
c_s	Cathode luminous sensitivity of photomultiplier.
D	Beam divergence.
F	Focal length.
f	Bandwidth.
G	Photomultiplier gain.
h	Height of source slit image at knife edge.
I	Intensity.
k	Width of sampling slit in screen.
L	Width of working section.
M	Mirror.
n	Refractive index.
R	Light ray.
R_a	Anode resistance of photomultiplier.
t	Time.
W	Wave front.
X	Coordinate.
Y	Coordinate.
β	Gladstone-Dale constant.
δ	Displacement of source image at knife edge.
ξ	Coordinate.
η	Coordinate.

NOTATION Cont'd.

ϕ Angle of light rays leaving working section.

ρ Density.

ρ_0 Reference density.

3.1.1. Introduction

Many measuring techniques have been applied to the study of vibrational relaxation processes in shock waves. These techniques may be categorised into those which observe the behaviour of the gas at a macro-scoptic level, that is, by the measurement of the translational state of the gas or by flow visualisation techniques, and those which observe the gas at a microscopic level; by monitoring the behaviour of particular vibrational modes for example.

One of the simplest means of determining the translational state of a gas is to measure its static or stagnation pressure. Unfortunately, neither of these measurements are suited to the study of relaxation processes in shock waves produced in a shock tube. The relaxation regions of partly dispersed shock waves are essentially isentropic. Consequently, the change in the stagnation pressure in the flow, which is measure of the entropy produced, would not yield any information regarding the relaxation process.

The static pressure change through the shock wave could be measured with a wall mounted pressure transducer. However, the boundary layer on the walls of the shock tube would distort the wall pressure history to an unacceptable extent.

In addition, the physical size of a commercially available pressure transducer, typically 3 mm. diameter, does not give adequate spatial resolution for the thickness of the relaxation regions that can be produced in the shock tube which is to be used for the present studies. Nevertheless, the static pressure change across shock waves produced in the shock tube does provide a useful means of determining the shock strength where other, more accurate techniques, cannot be made use of.

End wall pressure history measurements have been used to study the Vibrational relaxation of gases. Baganoff (Ref.3.1.) has made measurements of the vibrational relaxation of carbon dioxide with this technique, and the results obtained were in good agreement with those obtained by other more universally applied techniques such as Mach Zehnder interferometry. More recently, Hanson and Baganoff (Ref.3.2.) have made similar measurements in nitrogen with similar results. This technique is most suitable for use in shock tubes having internal dimensions sufficiently large to ensure that corner reflections at the end face do not interfere with the measurement times which could be obtained in the shock tube used for the present studies. This technique could not have been applied to the study of the fully dispersed shock waves produced in the reflected shock region.

The translational temperature of a gas can be determined from measurements of the heat transfer rates to stagnation point or wall mounted probes. The stagnation point probe would not be suitable because it would measure the stagnational enthalpy of the flow, which is constant through the relaxation regions. As with the wall mounted pressure transducers, the boundary layer would severely distort the heat transfer history at the wall. In addition, the absolute change in temperature through the relaxation region can be very small.

Spectral Line Reversal is one of the preferred techniques for the determination of the vibrational temperature of a gas. The technique requires the gas to be seeded with an element, such as Sodium, which is one of the more commonly used. Holbeche and Woodley (Ref. 3.3.) have used Sodium line reversal for relaxation measurements in nitrogen, carbon monoxide and oxygen. Hall and Russo (Ref. 3.4.) have also applied the technique to relaxation studies in nitrogen and carbon monoxide. The only objection to the use of this technique for the present studies was the amount of time which would have to be devoted to its development compared to other more readily available techniques.

Other optical techniques such as infra-red emission and ultra-violet spectroscopy [#] have been developed for the study of the behaviour of particular vibrational modes in a relaxing gas. Hodgson and Hine (Ref. 3.5.) have recently made measurements of the relaxation time of the asymmetric stretching mode of carbon dioxide using an infra-red emission technique. In these studies, the gas samples were not optically thin and the results had to be corrected to allow for re-absorption of energy. Earlier work using infra-red emission was conducted by Hooker and Millikan (Ref. 3.6.) on carbon monoxide, Camac (Ref. 3.7.), Borrell (Ref. 3.8.) and Weaner, Roach and Smith (Ref. 3.9.) for example. This technique was not considered suitable for the present studies because part of the work was concerned with the study of the translational state of the gas as well as the vibrational states.

Extensive use has been made of density measuring techniques for the study of vibrational relaxation. Interferometry has been most extensively used, although the laser schlieren technique has been made increasing use of in recent years. These techniques share the attraction for the present studies that they respond to both the translational behaviour of the gas and the vibrational behaviour, because of the sensitivity of gas density to the vibrational state.

[#]C.F. Reference 3.27.

In addition, these techniques are optical and do not introduce disturbances into the flow. They are affected by the boundary layers on the walls of the shock tube, but not to the same extent as wall pressure and heat transfer measurements.

Many researchers have made use of the Mach Zehnder interferometer, for example Zienkiewicz and Johannesen (Ref. 3.10.) Griffith (Ref. 3.11.) and Simpson (Ref. 3.12.). A Mach Zehnder interferometer was available for the present studies, but its sensitivity was considered to be inadequate for the study of the weak shock waves. The sensitivity is determined by the extent to which the fringe shifts can be resolved. Simpson has reported obtaining a resolution of 0.05 of a fringe shift defined by fluctuations in an equilibrium flow in the shock tube. The interferometer which was available for the studies had a resolution which was a factor of three worse than that quoted by Simpson, but this was determined by optical imperfections in the system. The interferometer could have been developed to achieve a higher performance, but its inherent insensitivity for the study of weak shock waves was considered to be a major disadvantage for the present studies. It is noted that the interferometer is one of the best techniques available for the determination of shock strengths from their total density change; Zienkiewicz and Johannesen (Ref. 3.13.).

The schlieren technique appeared to offer greater scope in terms of its range and sensitivity than the interferometer. Witteman (Ref. 3.14.) was one of the first researchers to draw attention to the use of the schlieren technique in this type of role and to demonstrate the use of a time resolved quantitative schlieren system in which the output is proportional to the spatial integral of the density gradients in the flow field. The range of the system was increased to encompass the gradients in the diffusion resisted shock fronts of the shock waves by the use of an inclined knife edge. Daen and de Boer (Ref. 3.15.) have applied this technique to the study of vibrational relaxation in several gases. The oscillograms presented by them do not exhibit the sharp rise normally associated with the output from the diffusion resisted shock front and the relaxation times were found to be a factor of four greater than those obtained by other researchers, but this was attributed to impurities in the gases studied by other researchers. An alternative explanation might have been that the particular arrangement introduced significant errors into the output signal from the system. de Boer (Ref. 3.16.) has analysed the optics of the integrating schlieren system and shown that the spatial resolution in the results obtained by Witteman could be accounted for by optical imperfections.

The resolving power of the arrangement used by Witteman was calculated by de Boer to be 0.18 mm. compared to a measured value of 0.15 mm.. Similarly, the resolving power of the arrangement in the form used by Daen and de Boer was calculated by de Boer to be 0.3 mm. compared to measured values between 0.2 mm. and 0.3 mm. in helium and between 0.3 mm. and 0.4 mm. in argon. It was concluded that the resolution could be explained in terms of the optical limitations of the system and it has since been developed to give a resolving power of 0.9 mm., although no reports have been found of results obtained with the improved arrangement. In his original work Witteman calculated the affect of shock curvature on the apparent spatial resolution of the system using the analysis of Hartunian (Ref. 3.17.). He showed that shock curvature could only account for between 10% and 50% of the apparent resolving power.

In 1965, Kiefer and Lutz (Ref. 3.18.) drew attention to the use of a laser schlieren technique. In their arrangement, the system operated in a non-integrating mode and the spatial resolution of the system was defined by the narrow beam from the laser. A spatial resolution of 1.25 mm., defined at the e^{-2} points of the gaussian distribution of the laser beam, was obtained together with a temporal resolution, determined by the electronics of 0.22 microseconds.

Applications of this technique to the study of vibrational relaxation in oxygen by Kiefer and Lutz (Ref. 3.19.) yielded results which agreed closely with those obtained by White and Millikan (Ref. 3.20.) using an interferometric technique. Kiefer and Lutz have also applied the technique to the study of vibrational relaxation in deuterium (Ref. 3.21.) and hydrogen (Ref. 3.22.). In these studies the resolution of the system has been improved to 0.8 mm. by the use of a converging telescope in the light beam between the light source and the shock tube. This was the best resolution which could be obtained in this way, because further convergence caused the mean diameter of the beam to increase towards the latter stages of its transit across the shock tube flow.

The use of the laser schlieren technique is very attractive for the study of vibrational relaxation, because of its range, sensitivity and inherent simplicity. It is also inherently more sensitive to the rate processes associated with vibrational relaxation than other density measuring techniques, because it responds to the gradient of the density.

A non-integrating schlieren system using a conventional light source has many of the attributes of the laser schlieren, but a more complex optical system has to be used. The conventional arrangement has the advantage over the laser schlieren that the light source can be interchanged with a spark source for schlieren photography. This facility was found to be particularly useful in the studies of fully dispersed shock waves propagating through the reflected shock region.

A decision was made at the outset of an earlier study by Scott (Ref. 3.23.) to adopt the use of a conventional non-integrating schlieren system for relaxation studies. This system has been developed and refined for the present studies. This section presents an analysis of the optics of this system and a consideration of possible errors in the system. Particular problems were encountered in the calibration of this system and the way in which these were overcome is also described.

3.1.2. PRINCIPLE OF OPERATION

The schlieren system utilises the fact that light rays passing through a translucent substance are turned towards the regions of higher density of that substance.

In this application the system is arranged to produce an image of the working section of the shock tube on a screen. The intensity of this image is related to the density gradients in the gas flow within the shock tube.

A conventional schlieren layout is shown schematically in figure 3.1.1. The light source is focussed with a thin lens onto the source slit which is an adjustable rectangular aperture with the longest side lying in the horizontal plane. This source slit is placed at the focus of the first concave parabolic mirror, M_1 , which collimates the light beam before it passes through the working section. After passing through the working section the light beam is reflected by a second concave parabolic mirror, M_2 , to produce a focussed image of the source slit at the focus of this mirror. A knife edge is placed at this point to partially obscure the image of the source slit. The part of the beam which passes over the knife edge produces a focussed image of the working section on the screen.

Density gradients in the flow within the shock tube cause the rays of light passing through the working section to be refracted, to emerge at a slight angle to the incident beam. This produces a displacement of these rays at the knife edge, causing more or less rays to pass over the knife edge. In this way, the intensity of image of that part of the working section on the screen is either increased or decreased accordingly. The system only responds to density gradients normal to the knife edge, because displacements of the image of the source slit parallel to the knife edge do not change the intensity of the image.

In this particular application the intensity of the image is sampled by a small slit in the screen which allows part of the light beam to pass through to a photomultiplier placed behind the screen. The photomultiplier converts the intensity of the incident light into an electrical output. All of the shock waves to be studied are convected through the working section of the shock tube, so that a time resolved electrical output can be obtained which can be related to the spatial variation of the density gradients within these shock waves.

3.1.3. The Theory Of Operation.

We are concerned with the behaviour of light rays within the working section of the shock tube at position Y_1, X_1 , as shown in figure 3.2.A. Light enters the working section from the left hand side travelling parallel to the Y axis.

Consider the elemental behaviour of the light rays at the point Y_1, X_1 . It will be assumed that the speed of light in the gas, C , is a function of η only. R_1 and R_2 are two light rays and W_1 & W_2 are the positions of a wave front at times t and $t + \Delta t$ respectively.

The time taken for the light wave to travel along the light ray R_1 must be equal to the time taken for the light wave to travel along R_2 , that is,

$$\Delta t = \frac{\Delta \xi}{c} = \frac{\Delta \xi + (\partial \xi / \partial \eta) \cdot \Delta \eta}{c + (\partial / \partial \eta) \cdot \Delta \eta} \quad 3.1.1.$$

Or,

$$\Delta \xi \cdot \frac{\partial c}{\partial \eta} = c \cdot \frac{\partial \xi}{\partial \eta} \quad 3.1.2.$$

It can be seen from the figure that the angle $\Delta \phi$ is given by,

$$\Delta \phi = \frac{(\partial \xi / \partial \eta) \cdot \Delta \eta}{\Delta \eta + (\partial \eta / \partial \xi) \cdot \Delta \xi} \quad 3.1.3.$$

Or, neglecting second order terms,

$$\Delta \phi = \frac{\partial \xi}{\partial \eta} \quad 3.1.4.$$

Combining equations 3.1.2 and 3.1.4, we have,

$$\frac{\Delta \phi}{\Delta \xi} = \frac{1}{c} \cdot \frac{\partial c}{\partial \eta} \quad 3.1.5.$$

Or, taking the limit as $\Delta \xi \rightarrow 0$, we have,

$$\phi = \int_0^{\xi} \frac{1}{c} \cdot \frac{dc}{d\eta} \cdot d\tilde{\xi} \quad 3.1.6.$$

If the light beam enters the working section normal to the flow, i.e. $\phi = 0$ at $Y = 0$, equation 3.1.6 becomes,

$$\phi = \int_0^Y \frac{1}{c} \cdot \frac{dc}{dx} \cdot d\tilde{Y} \quad 3.1.7.$$

If only two dimensional flows are considered, ie. $dC/dX(X)$, equation 3.1.7 reduces to,

$$\phi = \frac{1}{C} \cdot \frac{dC}{dX} \cdot Y \quad 3.1.8.$$

The speed of light in a substance is normally referred to the speed of light in an absolute vacuum, and the ratio of these two speeds is the refractive index of the substance, that is,

$$n = C_0 / C \quad 3.1.9.$$

where n is the refractive index of the substance and C_0 is the speed of light in an absolute vacuum.

It has been shown from quantum considerations, and confirmed experimentally for a wide range of substances that the refractive index of a substance is related to its density by the Lorentz - Lorentz law (Ref. 3.24.), viz.

$$\frac{(n-1)(n+1)}{(n^2+2) \cdot \rho} = \text{const.} \quad 3.1.10.$$

where ρ is the density of the substance. For a gas $n-1$ is typically 10^{-3} to 10^{-4} , so that the ratio $(n+1)/(n^2+2)$ can be assumed constant for all practical purposes. This equation then reduces to the Gladstone - Dale law,

$$(n-1)/\rho = \beta/\rho_0 \quad 3.1.11.$$

$$d_4 = 10^{-3} \text{ cms.}$$

It can similarly be shown that,

$$d_3 = b. (1 - 1/n_{\text{glass}}) \cdot \delta / F_2 \quad 3.1.32$$

So that in this case,

$$d_3 = 4 \cdot 10^{-4} \text{ cms.}$$

It can be seen from the above calculations that provided the displacement at the edge of the working section window of $3 \cdot 10^{-3}$ cms. ($d_2 + d_4$) is within the required spatial resolution of the system shadowgraph effects can be neglected. Similarly, provided the apparent displacement of the ray at the centreline of the working section of $4 \cdot 10^{-4}$ cms. ($d_1 + d_3$) is within the required spatial resolution of the system it can be assumed that the image on the screen is exact from this standpoint.

3.1.5.3. Spherical Aberration.

It was assumed in the preceeding theory that the concave mirrors were parabolic. In practice these mirrors are spherical and they only become identical to parabolic mirrors in the limit as the apertures tend to zero.

For a finite aperture the focal length of a spherical mirror is a function of the radial position of the point of reflection. In the limit as the aperture tends to zero the focal length tends to half the radius of curvature of the mirror. If this is taken as the position of the knife edge then a ray of light originating at a distance P from the axis of the mirror will be displaced a distance δ_1 at the knife edge, and a distance δ_2 at the screen. This will cause the calibration of the system to be a function of the radial point of reflection and allow light to enter the sampling slit in the screen from a distance δ_2 away from the correct position.

It can be shown from the geometry of figure 3.1.9 that,

$$P/R = \sin \theta ,$$

$$F_2 = R - (R^2 - P^2)^{\frac{1}{2}} + P / \tan 2 \theta ,$$

and,

$$\delta_1 = ((R/2) - F_2) \cdot \tan 2 \theta .$$

From which it can be shown that,

$$\begin{aligned} \delta_1 &= R (1 - (P^2/R^2))^{\frac{1}{2}} \cdot \tan 2 (\sin^{-1}(P/R)) \\ &- \left\{ P + (R/2) \cdot \tan 2 (\sin^{-1} (P/R)) \right\} 3.1.33. \end{aligned}$$

For the present arrangement $P < 1$ cms. and $R = 200$ cms., so that,

$$\delta_1(\text{max.}) = 2.5 \cdot 10^{-5} \text{ cms.}$$

and,

$$\delta_2(\text{max.}) = 2 \cdot \delta_1(\text{max.}) = 5 \cdot 10^{-5} \text{ cms.}$$

3.1.5.4. Depth Of Focus.

The depth of focus of a schlieren system is determined by the size of the image of the source slit. In this case only the height of the image has to be considered. It has been shown (Ref. 3.23.) that the diameter of the circles of confusion is given by,

$$d = \left\{ L / (U - F_2) \right\} \cdot (h/2) \quad 3.1.34.$$

where U is the distance from the centre of the working section to the concave mirror, M_2 , and d is the diameter of the circles of confusion at the edges of the working section.

For the present arrangement $U = 200$ cms., so that,

$$d = 2 \cdot 10^{-3} \text{ cms.}$$

3.1.5.5. Beam Divergence And Alignment.

It was assumed in the preceeding theory that the incident light beam was properly collimated and entered the working section of the shock tube normal to its axis. In practice the collimation and alignment depend upon the above mentioned spherical aberrations and the accuracy with which the system can be set up. This setting is achieved by reflecting the light beam from the surfaces of the working section windows to produce an image of the source slit adjacent to itself.

When a focussed image of the source slit is obtained the source slit is at the focus of the concave mirror, M_1 , and the light beam is collimated. It can be shown that the total divergence of the beam in passing through the working section is related to the distance between the source slit and the focus of the concave mirror by,

$$D = \frac{A_1}{2 F_1} \cdot e_1 \cdot L \quad 3.1.35.$$

where D is the total divergence, A_1 is the aperture of the mirror, and e_1 is the distance between the source slit and the focus of the mirror.

For the present arrangement $A_1 < 1$ cms. and $e_1 = \pm 0.5$ cms., so that the maximum divergence over the width of the working section was,

$$D_{\max.} = \pm 1.25 \cdot 10^{-4} \text{ cms.}$$

When the image of the source slit returns through itself the beam is aligned normal to the windows of the working section and hence normal to the axis of the shock tube. It can be shown that the total alignment error over the width of the working section is related to distance between the image and the source slit by,

$$E = (e_2 \cdot L) / B \quad 3.1.36.$$

where E is the total alignment error, B is the distance from the source slit to the working section, and e_2 is the distance between the source slit and its image.

In practice the four images were not coincident because the windows of the working section were neither perfectly flat nor exactly parallel to each other. The images were spread over 0.5 cms. . The centre of these images could be aligned with the source slit to within ± 0.1 cms., and $B = 400$ cms., so that the maximum error was,

$$E_{\max.} = \pm 1.25 \cdot 10^{-3} \text{ cms.}$$

3.1.5.6. Bandwidth.

For a given optical and electronic system the signal to noise ratio of the main photomultiplier is proportional to the height of the sampling slit and inversely proportional to the bandwidth of the system, that is,

$$S / N \quad \sim \quad b / f \qquad 3.1.37.$$

where b is the height of the sampling slit and f is the bandwidth of the system.

This expression demonstrates the compromise which has to be made between the signal to noise ratio, which must be a maximum, the height of the sampling slit (and hence the spatial resolution) , which must be a minimum, and the bandwidth, which must be a maximum.

For these experiments the bandwidth of the system was set at 50 MHz.. The spatial resolution and hence the size of the sampling slit was determined from the above considerations, and the resulting noise level accepted, which for most of the traces obtained constituted 2 % of the signal.

3.1.6. Setting Up The System.

It is clear from the calculations of the preceding section that great care must be exercised in the setting up of the schlieren system to ensure that an adequate spatial resolution is obtained. The initial setting of the system described herein was conducted in the conventional manner, employing the reflected crosswire technique for beam alignment and divergence, and the visual setting of the foci of images. The final settings were conducted and quantified by means of optical/electronic techniques as will be discussed in this section.

3.1.6.1. The Measurement Of The Intensity Distribution Of The Source Image.

The image of the source slit was scanned by replacing the knife edge with a sampling slit, and placing a photomultiplier behind this slit. The sampling slit was traversed across the image of the source slit by driving the vibrator on a reduced mains voltage at a frequency of 50 Hz.. A typical output from the photomultiplier is shown in figure 3.1.7. In the top trace the output is displayed against a time base, whilst in the bottom trace the output is displayed against the output from the position transducer which was monitoring the position of the sampling slit. The hysteresis was caused by the position transducer which was an electromagnetic device

Ideally, if the source slit had been uniformly illuminated, these outputs would have appeared as discontinuous square wave functions. In practice diffraction, astigmatism, focussing errors, and malalignment produce a finite rise and fall. The object of this setting was to maximise the output and hence the sensitivity while retaining an undistorted output, and to maximise the linear range of the system by minimising the rise and fall of the output.

This display was used to ensure that the source slit had parallel edges which were free from distortions, to set the source slit parallel to the knife edge, and to optimise the relative positions of the light source, lens and source slit for maximum undistorted output.

3.1.6.2. The Optimisation Of The Spatial Resolution.

A simple device has been incorporated into the system to determine some of the aforementioned factors which affect the spatial resolution of the system. A sketch of the device is shown in figure 3.1.10. It can be seen that the device consists of a blade mounted in an alignment block which is traversed through the working section by means of the motor driven cam.

The alignment block was 15 cms. long and was machined to give a running clearance in the shock tube of $5 \cdot 10^{-3}$ cms.

The blade was aligned to the sides of the block to within $\pm 10^{-3}$ cms. over the width of the sampling slit of 2 cms. . So, to within $\pm 1.5 \cdot 10^{-3}$ cms. over the width of the sampling slit, the blade represented the normal to the axis of the shock tube for the purposes of checking the spatial resolution of the system.

This chopper arrangement was traversed through the working section at a frequency of 0.5 Hz. . The speed of the chopper was determined from the time taken for it to traverse both of the light probes, which were a known distance apart. Knowing the speed of the chopper, the time taken for the blade to traverse the sampling slit could be related to the spatial resolution of the system.

An example of the type of output which was obtained when the system was set up visually is shown in figure 3.1.11. It can be seen that with the sampling slit set at 0.008 cms. the measured spatial resolution was 0.013 cms. . There is also evidence of diffraction fringes on the upper part of the trace. The screen configuration for this case is also shown in the figure. It was found that the two slits were too far from the axis of the system for them both to be optimised for spatial resolution.

These two slits were replaced by a single slit and the two light probes for the determination of the shock wave velocity.

The optimised output with this configuration is shown in figure 3.1.12. It can be seen that the slit was formed by double knife edges to prevent light entering the slit obliquely.

A measured spatial resolution of 0.007 cms. was obtained with the sampling slit set at 0.008 cms. . It will also be noted that there was no evidence of diffraction fringes.

As an example of the usefulness of this technique the effect of the focussing of the working section image is shown in figure 3-1.13. The marked degree to which the output changes can be clearly seen, but these changes would not have been discerned with the naked eye.

This technique could not evaluate the effects of depth of focus, beam alignment and beam divergence. This could have been done by using a double blade and is an improvement which should be born in mind for the future development of the system.

These tests have shown that without taking into account those effects mentioned in the last paragraph the system can be adjusted to give a spatial resolution of $7 \cdot 10^{-3}$ cms. The malalignment and the divergence of the beam could account for a further $1.3 \cdot 10^{-3}$ cms., and the depth of focus a further $2 \cdot 10^{-3}$ cms. . This latter figure is the diameter of the circles of confusion which would be obtained at the edges of the working section, and a more representative figure would be the mean value of $1 \cdot 10^{-3}$ cms. . Thus, the total spatial resolution of the system would have been $9.3 \cdot 10^{-3}$ cms. .

The only other effect of any significance which could further reduce the effective spatial resolution of the system was the shadowgraph effect. Again this effect has been over-calculated, because it was assumed in section 3.1.5.2 that the edge of the working section was in focus. However, it has been shown with the above technique that the centreline of the working section could be focussed to within 0.5 cms. along the axis of the light beam. A more representative figure for the shadowgraph effect would therefore be 0.75×10^{-3} cms. It is therefore reasonable to conclude that the schlieren system should have a spatial resolution of better than 10^{-2} cms.

Other researchers such as Kiefer and Lutz (Ref. 3.19.) have checked the spatial resolution of their schlieren systems by using them to observe incident shock waves propagating through monatomic gases such as argon and helium. This technique does guarantee that the resolution is no worse than the apparent thickness of the shock wave, but a true measure of the resolution is not obtained because the curvature of the shock wave detracts from the apparent spatial resolution of the system.

The schlieren system used in the present studies has been applied to the observation of the diffusion resisted shock front of incident shock waves propagating through carbon dioxide.

The spatial resolution of the system has been determined from the risetime of the signal as the front of the shock wave passes the observation position. Figure 3.1.14 shows the results of two such tests applied to the schlieren system in its final form. It can be seen that the risetime of the signal was less than 0.3 microseconds. As the system had a bandwidth of the order of 50 MHz., the majority of this risetime was attributable to the spatial resolution of the system. If this was so, then the system had a resolution of better than 0.02 cms.

It can also be seen that the thickness of the diffusion resisted shock front was of the order of 0.1 cms., which is consistent with the value of 0.06 cms. quoted by Kiefer and Lutz for Oxygen (Ref. 3.19.)

3.1.7. Calibration.

The schlieren system has been calibrated by assuming that the theory developed above holds and calibrating the photomultiplier outputs in terms of the knife edge displacement. This assumes that displacements of the knife edge relative to the light beam are equivalent to displacements of the light beam relative to the knife edge.

This equivalence is guaranteed if the disturbed light rays return to the same position on the screen as the undisturbed light rays. It has been shown in the preceding sections that for this application those displacements which do occur do so within the required spatial resolution and therefore do not affect the calibration.

This type of calibration is normally conducted statically. The knife edge is displaced by a measured amount and the resulting change in the photomultiplier output is measured. In this application this was not possible and a device was developed for dynamically calibrating the system.

The whole of the system was mounted on a single stand, which was isolated from the floor by springs to prevent 50 to 100 Hz. vibrations from reaching the optical components and causing fluctuations in the photomultiplier outputs. The complete arrangement had a natural frequency of 2 Hz. . This arrangement reduced the fluctuations on the photomultiplier outputs to an acceptable level for measurement, but they were still large enough to prevent a static calibration from being made.

The system which was developed to overcome this problem is shown schematically in figure 3.1.15. The knife edge was mounted on a vibrator which was driven from reduced mains voltage at a frequency of 50 Hz. . The amplitude of the knife edge displacement was such that it completely traversed the source slit image. The position of the knife edge was monitored by a position transducer as shown.

Several different types of position transducer were investigated and the best performance was obtained with a Linear Variable Differential Transformer type 100 MS-L manufactured by Electro Mechanisms Ltd. This transducer had a maximum displacement of 0.25 cms. and a claimed linearity of $\pm 1\%$ over the full range. The primary excitation and demodulation were performed by a Tektronix 'Q' strain gauge unit. The complete arrangement had a bandwidth of 2.5 KHz. . This output was used to drive the X plates of a Tektronix oscilloscope, whilst the output from the main photomultiplier was used to drive the Y plates of the oscilloscope. The resulting display was as indicated in the sketch in figure 3.1.15.

The L.V.D.T. was calibrated statically by switching the vibrator off and displacing the knife edge with a micrometer head. This displacement was related to the output from the L.V.D.T. measured with a digital voltmeter or with the above oscilloscope. A typical calibration curve of the latter type is shown in figure 3.1.16. A linearity of 0.8 % was obtained over a range of 0.06 cms. after much care had been taken in setting up the transducer. It was thought that the reduction in the linearity was caused by the proximity of the large vibrator as can be seen in figure 3.1.6. This linearity was however acceptable for the present application.

A typical knife edge calibration is shown in figure 3.1.17. The rather broad trace arises from the hysteresis of the L.V.D.T., as does the slight non-linearity.

This calibration can be related to the density gradients in the flow by the application of the above theory. This part of the calibration requires a knowledge of the Gladstone Dale constant for the gas under study, which was carbon dioxide. From Allen (Ref. 3.26.) the refractive index of carbon dioxide at normal temperature and pressure is given by,

$$n - 1 = A \left(1 + \frac{B}{\lambda^2} \right)$$

where λ is the wavelength of the light source in Å,
 $A = 4.39 \times 10^{-4}$ and $B = 6.4 \times 10^5$.

The light source used for the present studies was a mercury vapour arc lamp, the light from which was passed through an optical filter centered at 5461 Å and having a half height spectral width of 50 Å.

3.1.8. The Suitability Of The Schlieren System For This Application And Future Developments.

The schlieren system was chosen for this application because it was considered to offer certain advantages over other available techniques. The results which have been obtained with the schlieren system in the present studies have confirmed this.

As far as can be ascertained from the results obtained herein, the calibration of the schlieren system as determined above appears to be correct. However, scatter in the results of several percent due to other causes has rendered an exact check on the calibration impossible. It can be said that no evidence has been found to suggest that the calibration of the system was incorrect, and to this extent the above analytical findings have been confirmed.

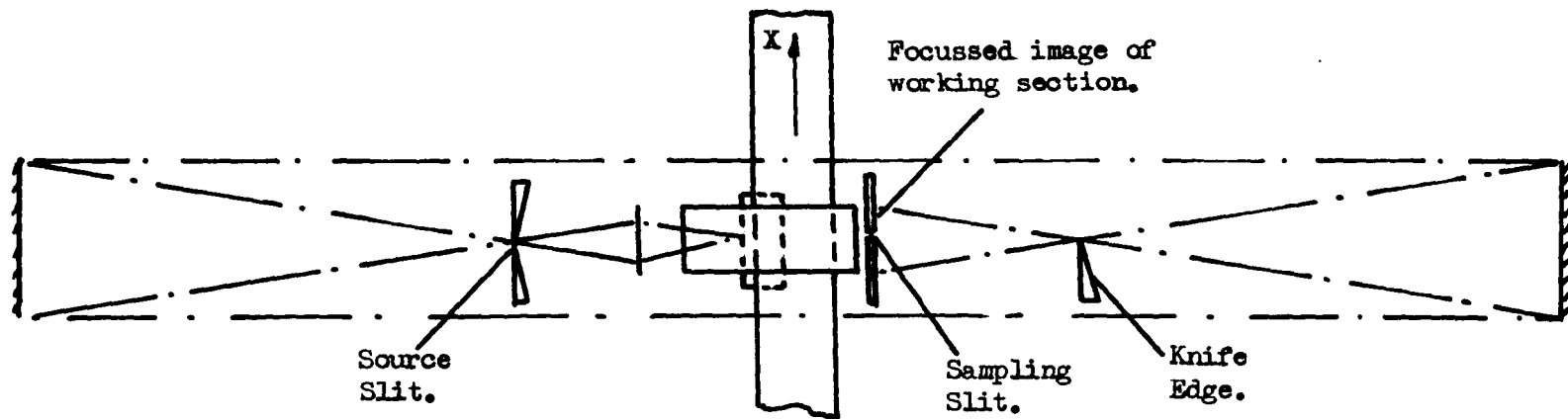
If a more accurate check on the calibration of the system is required, it is recommended that a study should be made of fully dispersed incident shock waves (see section 3.3.) and the strength of the waves determined by other means such as pressure measurement or interferometry. The calculated density change can then be compared with the integral of the schlieren output in order to determine the calibration.

The measured spatial resolution of the system was found to be better than 0.02 cms. compared to a value of 0.01 cms. arrived at from a theoretical analysis and from simple tests. Other results obtained in section 3.4. confirm that the spatial resolution was better than 0.03 cms.

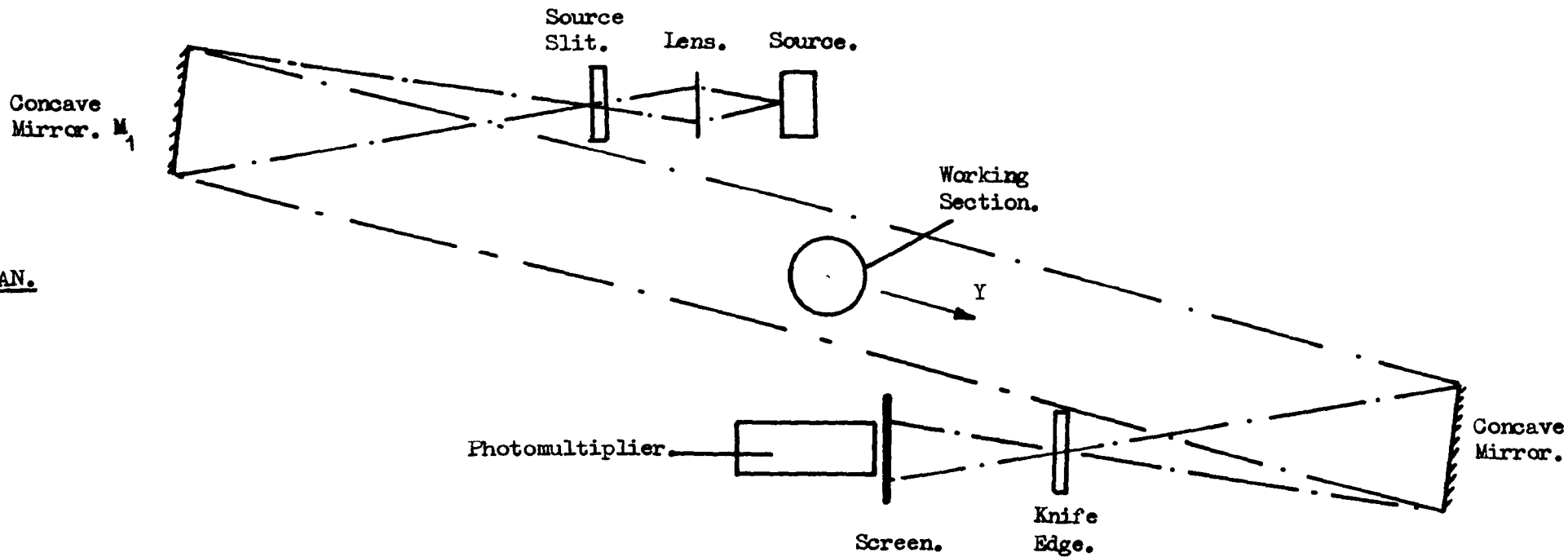
Considerable effort has been devoted to linearising the output from the system. This is not in general essential, for, if the calibration characteristic is known, the traces can be adjusted accordingly. However, the data reduction is simplified considerably if the calibration is known to be linear.

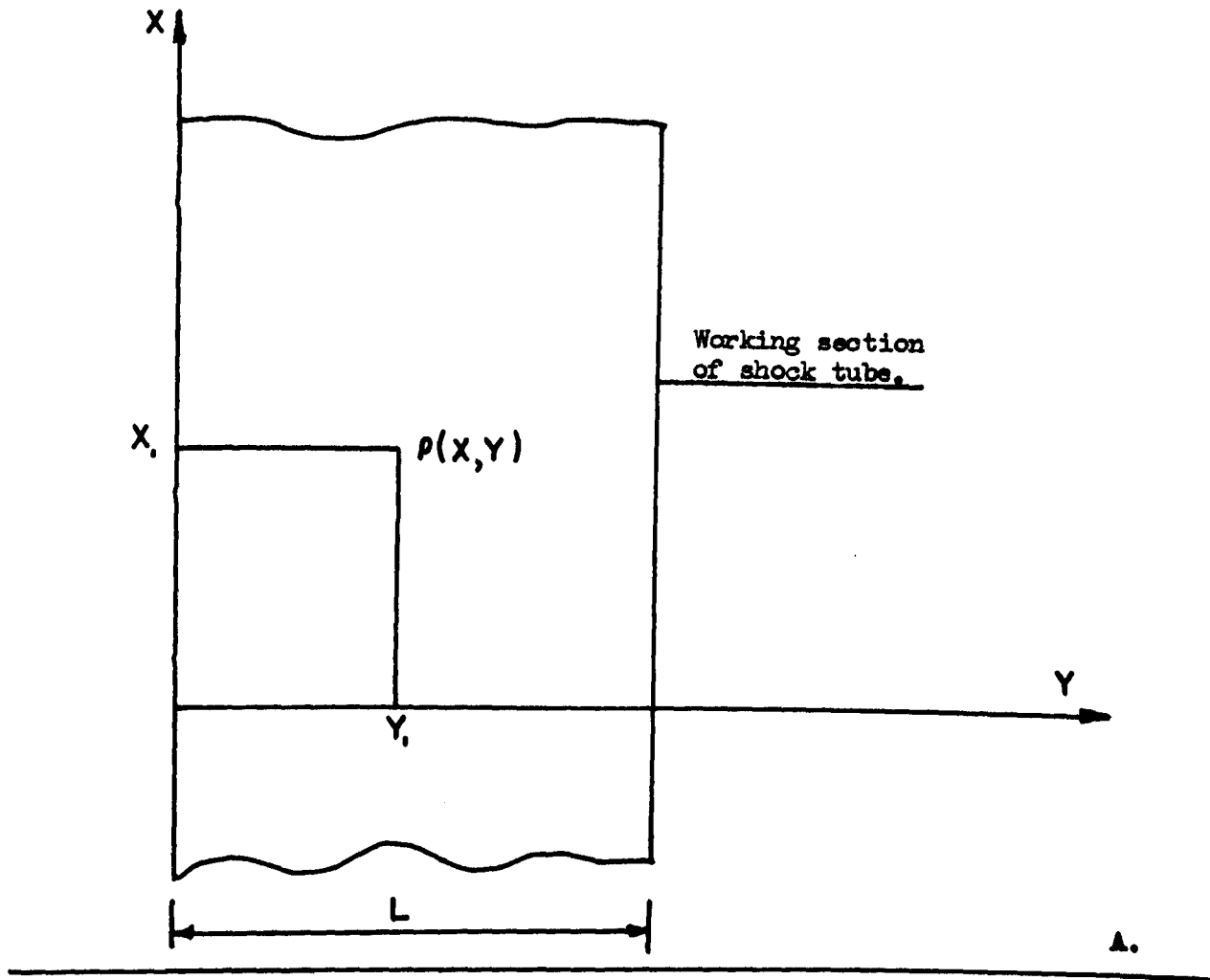
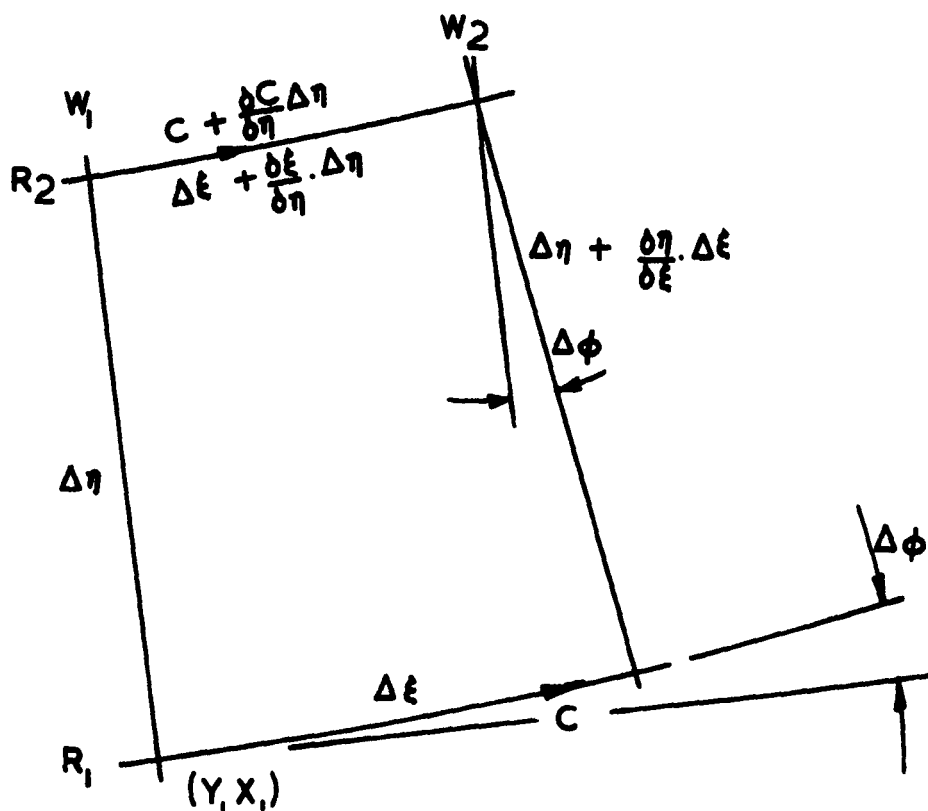
As will be discussed in section 3.2., the electronic form of the output is amenable to electronic data reduction, which would be highly impractical with a non-linear calibration. Of particular interest in this respect is the possibility of integrating the output from the schlieren system electronically to give the total density change through the shock waves. This permits a density gradient / density presentation to be used for the data, which removes one of the main sources of error in the experiments.

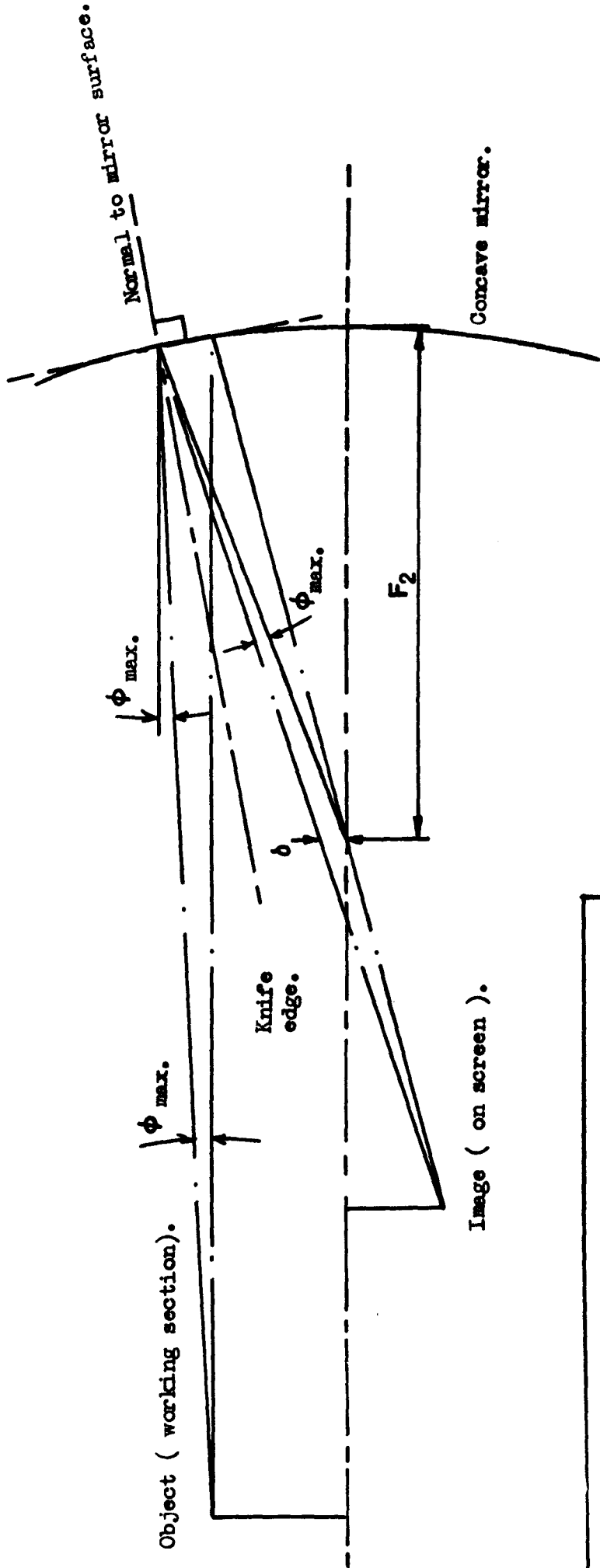
ELEVATION.



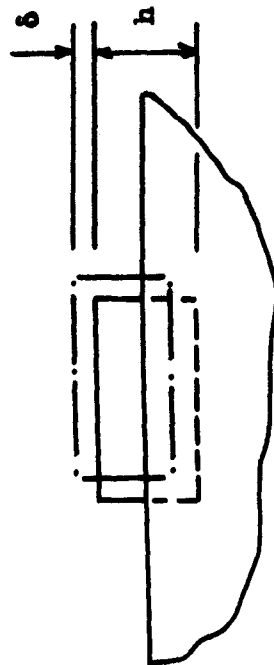
PLAN.



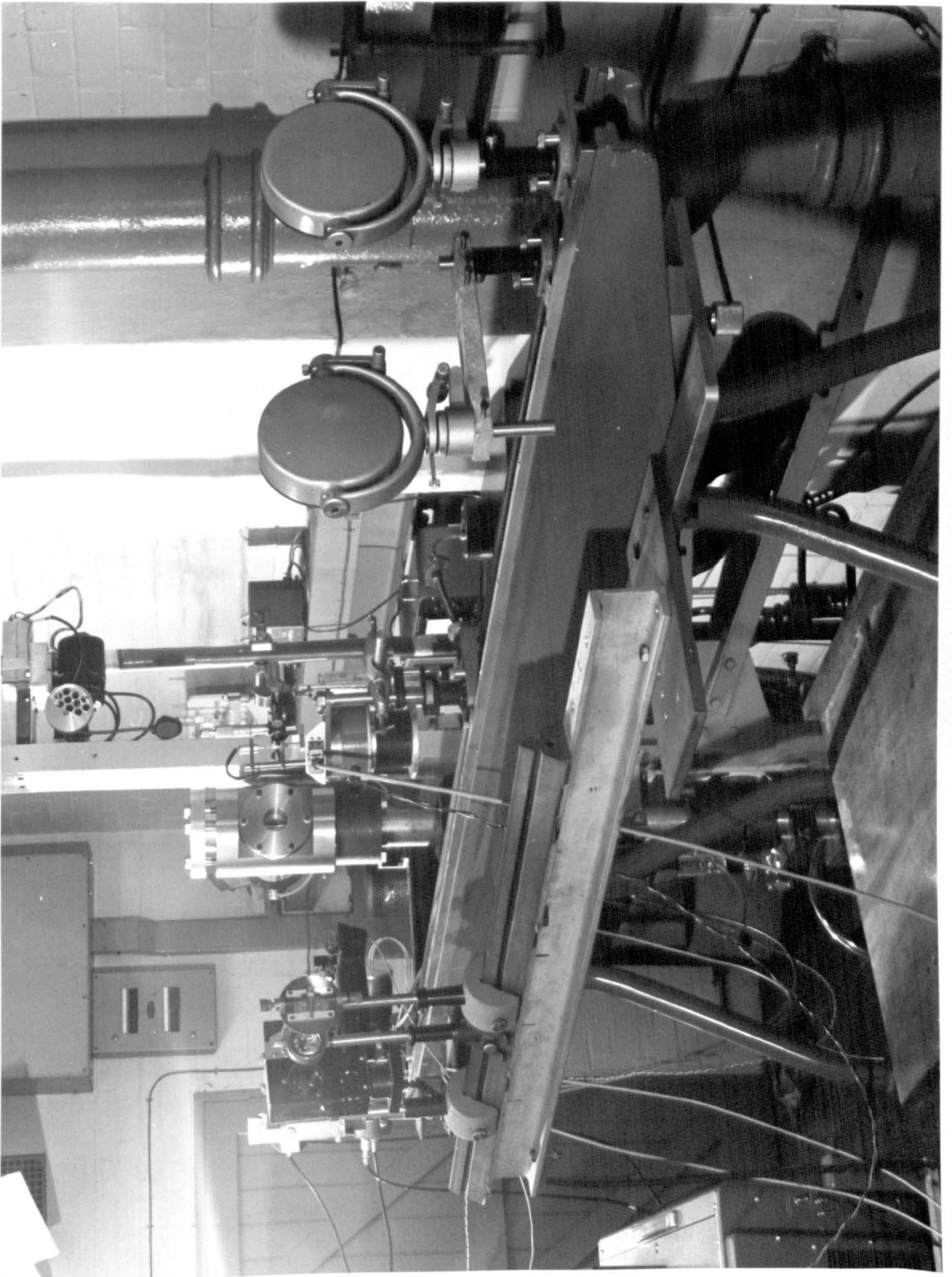




A.

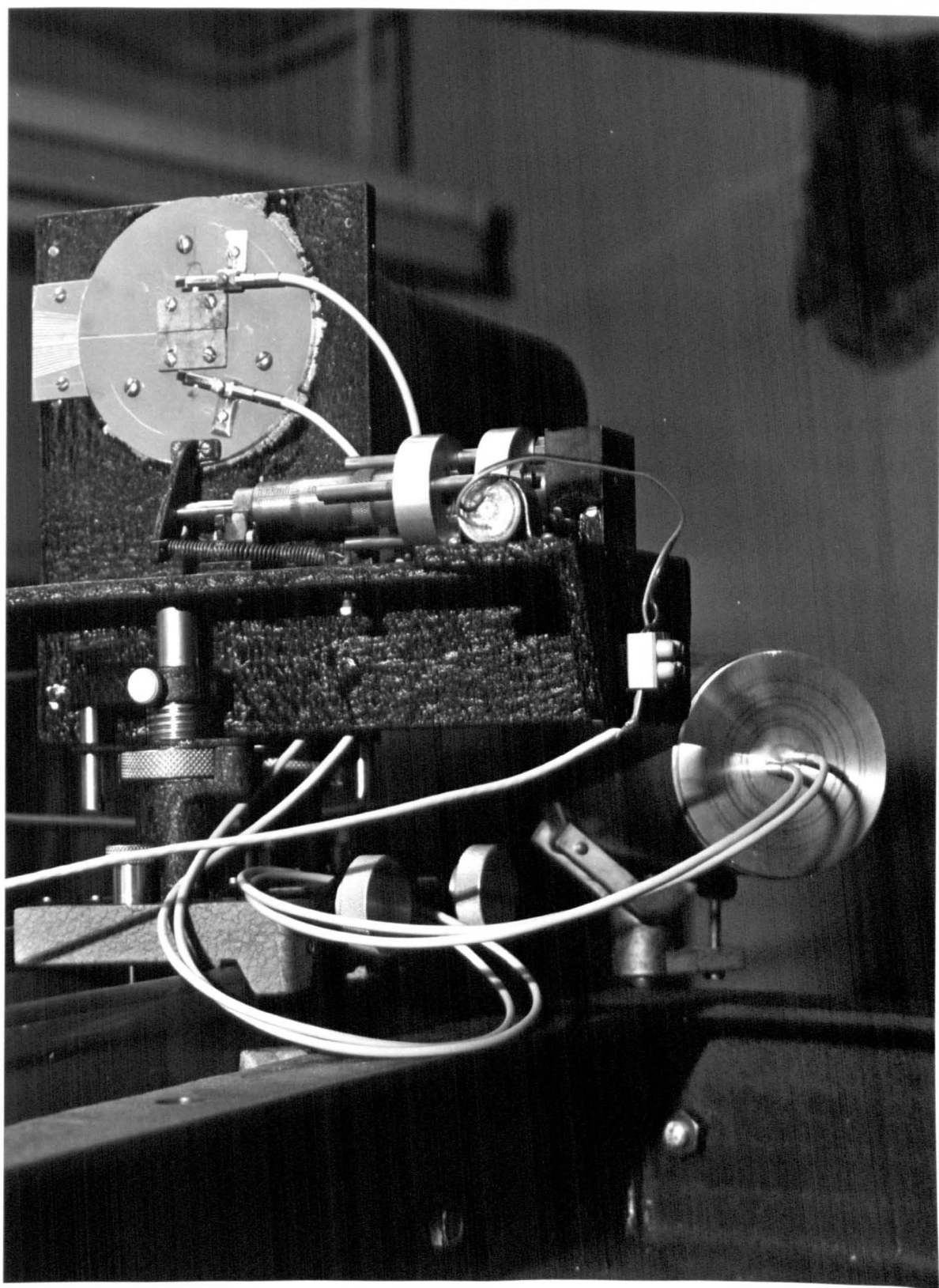


B.



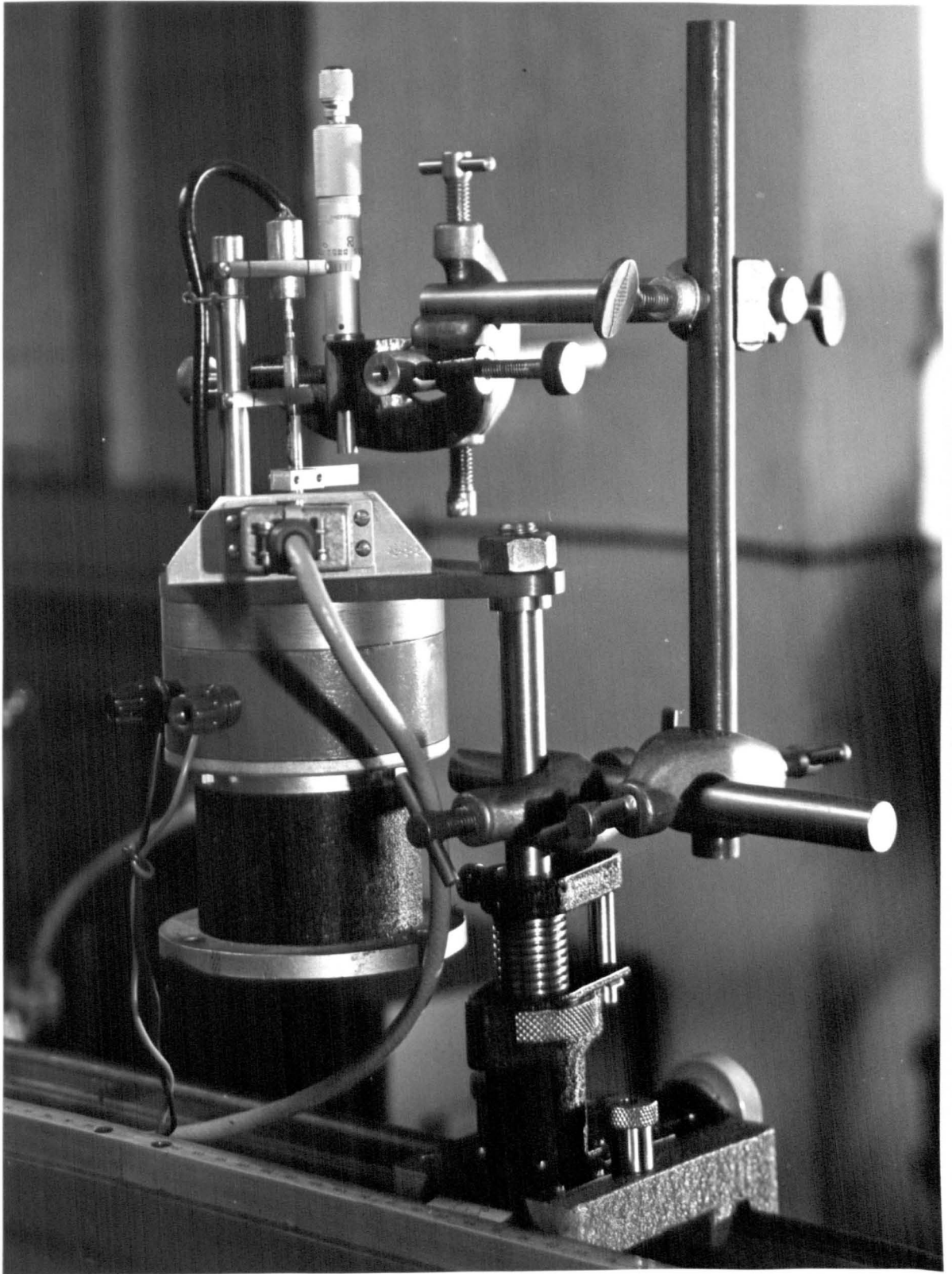
THE SCHLIEREN SYSTEM

FIGURE: 3.1.4.



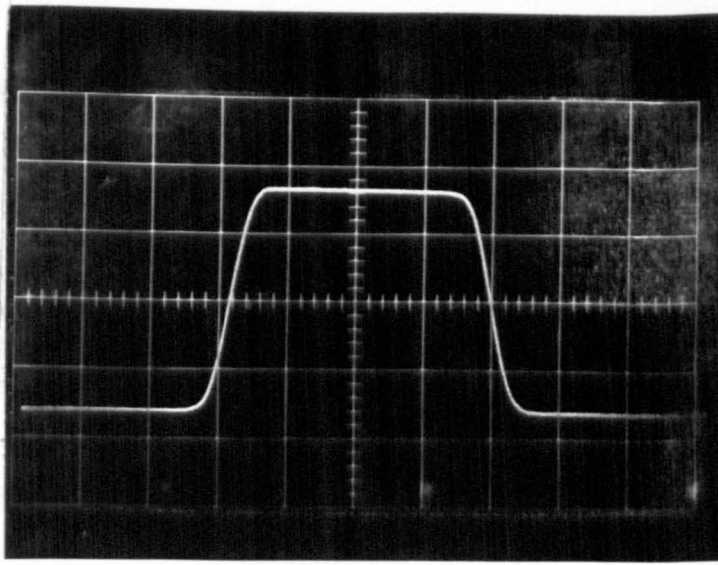
THE SCHLIEREN SCREEN ARRANGEMENT

FIGURE : 3.1.5.



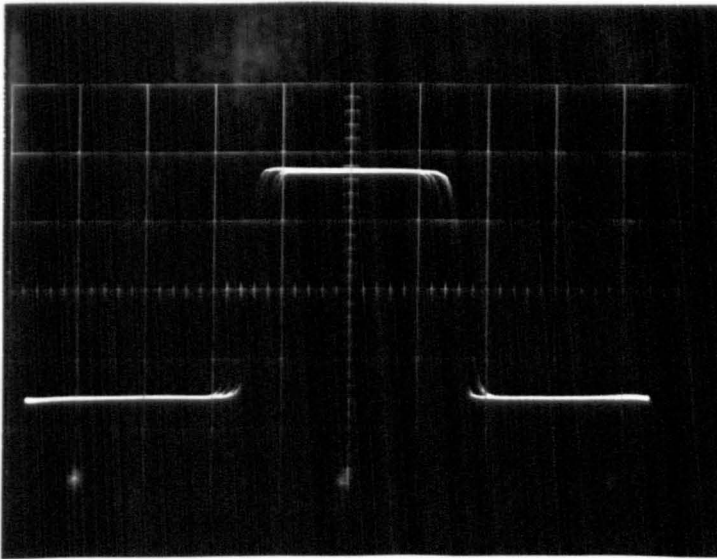
THE KNIFE EDGE ARRANGEMENT

FIGURE : 3.1.6.



↑ Photomultiplier
output 5mv/cm.

→ 0.5ms/cm.

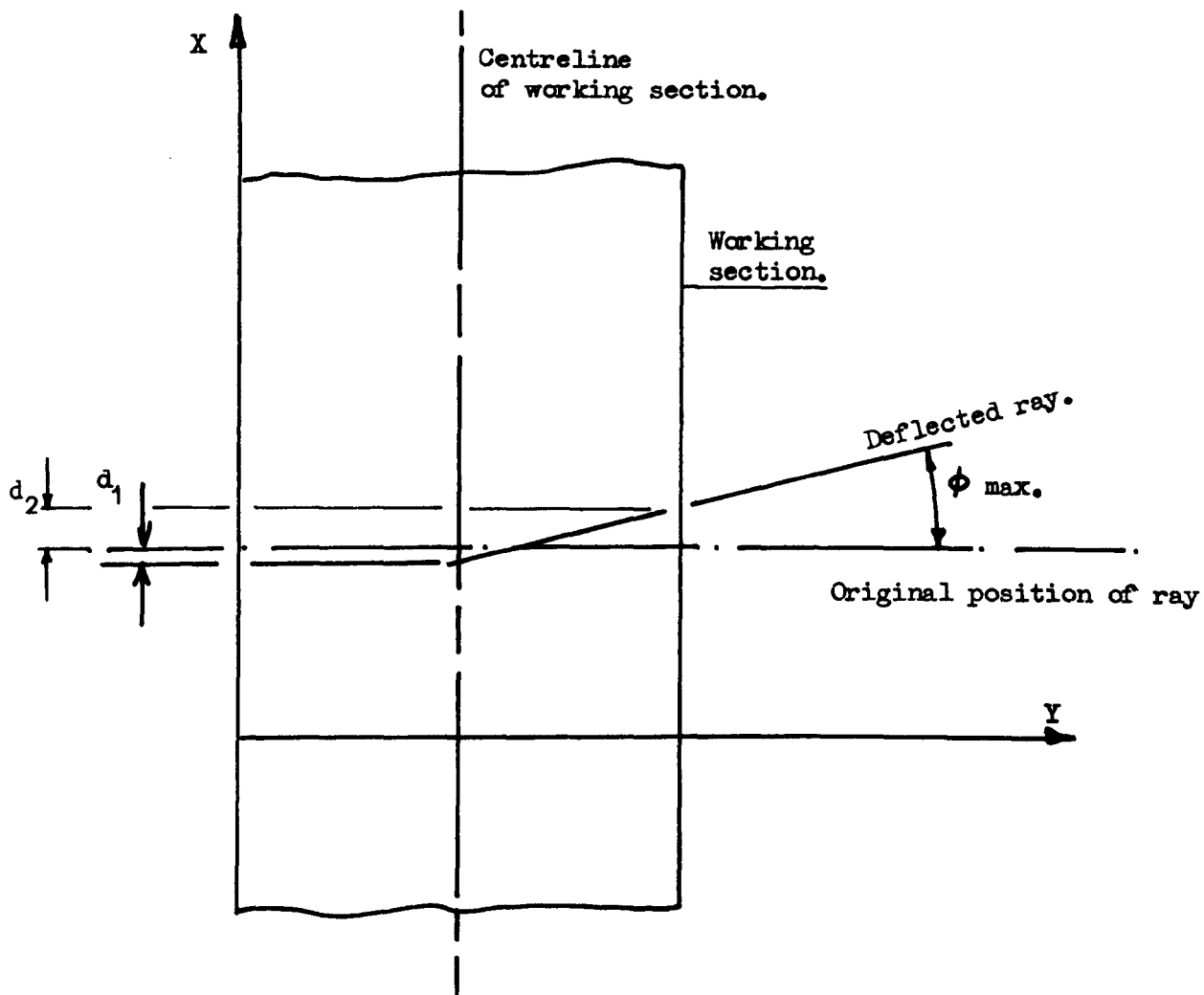
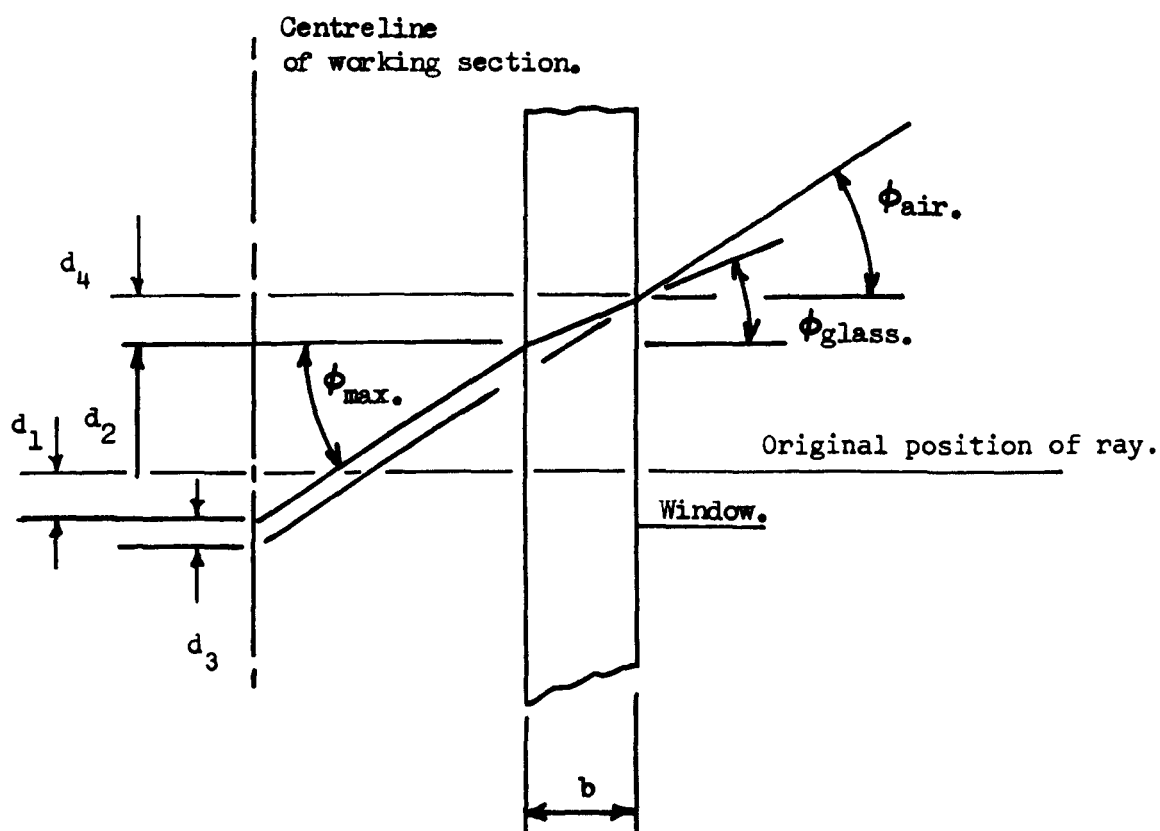


↑ Photomultiplier
output 5mv/cm.

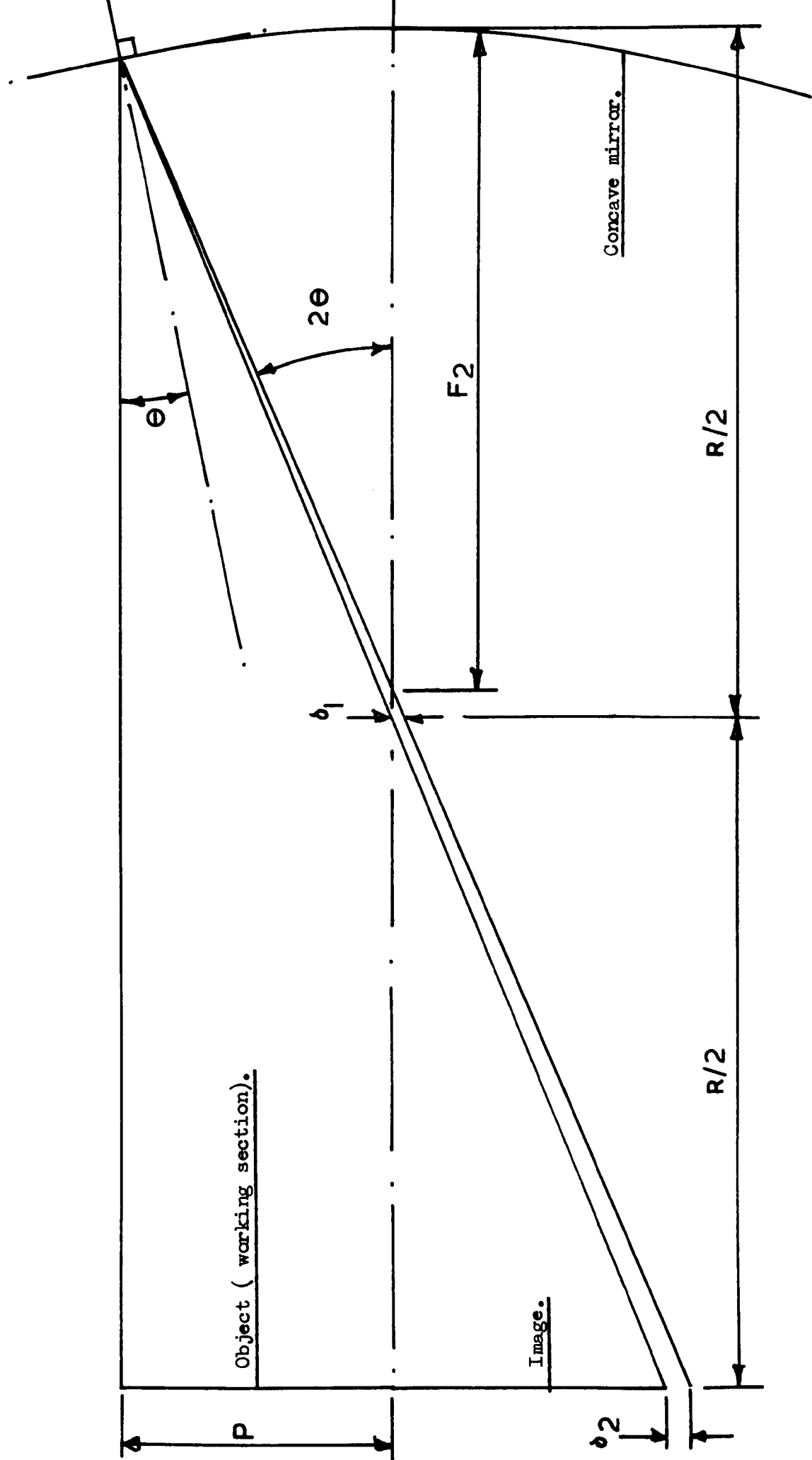
→ L.V.D.T output 1 v/cm.

Source slit = 0.046 cms.

Sampling slit = 0.008 cms.

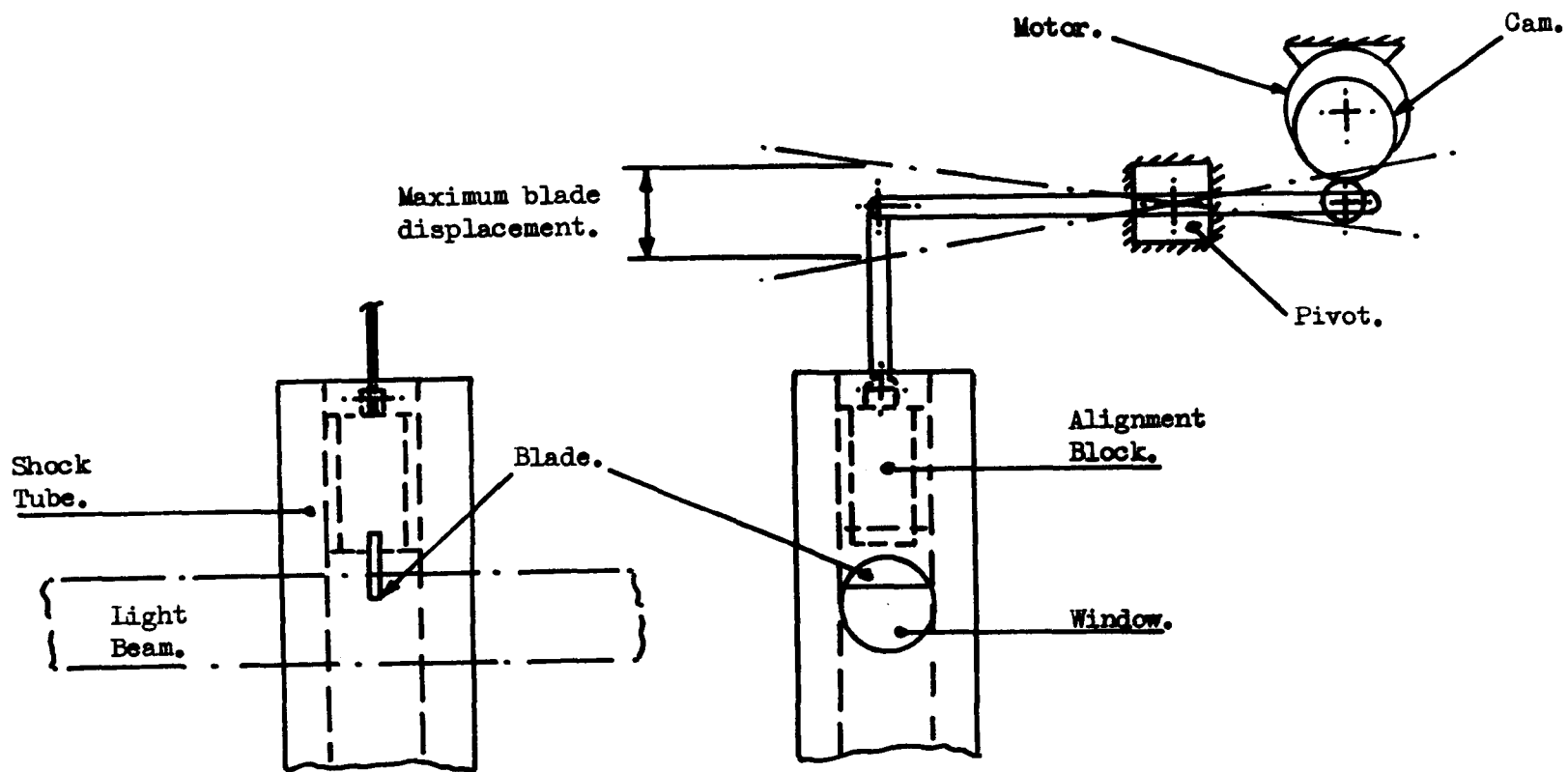


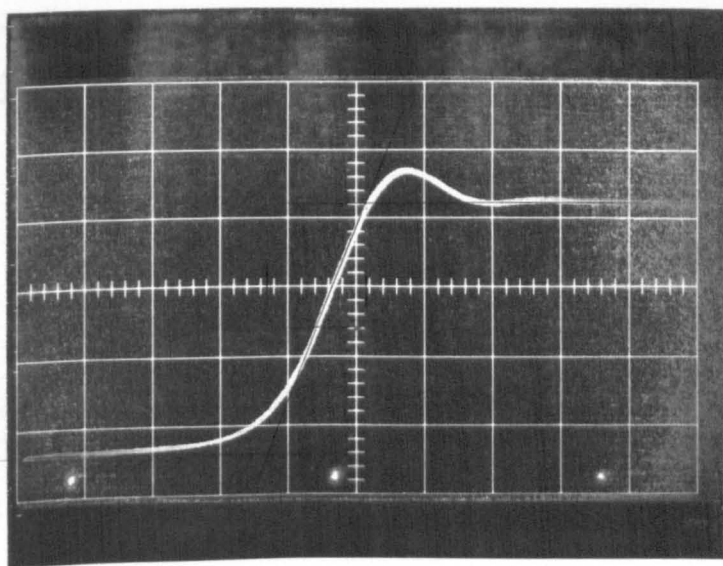
APPARENT POSITION OF RAY DUE TO REFRACTION.



SPHERICAL ABERRATION.

FIGURE: 3.1.9.





5mv/cm.

2ms/cm.

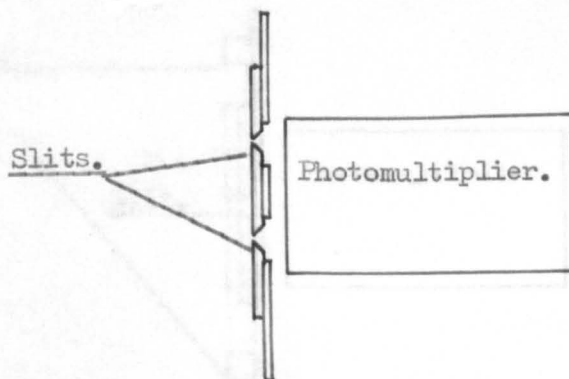
1 cycle of chopper = 2 secs.

Chopper displacement = 4.6 cms.

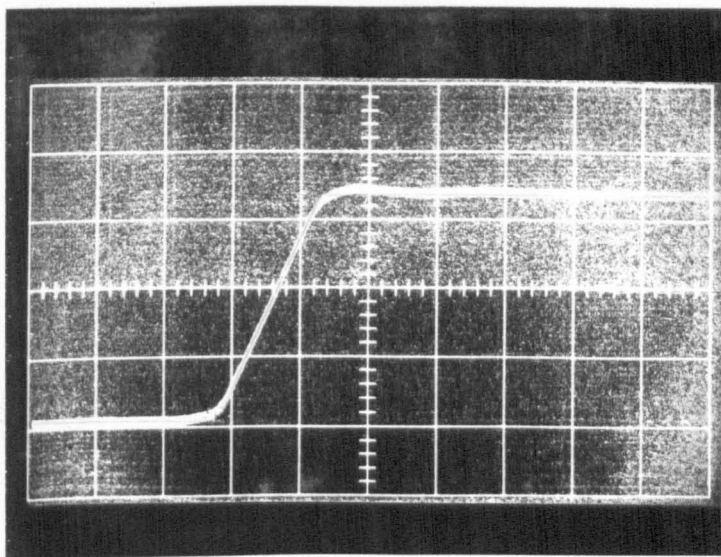
Maximum slope rise time = 3ms.

Resolution based on rise time = 0.013 cms.

Screen slit set at 0.008 cms.



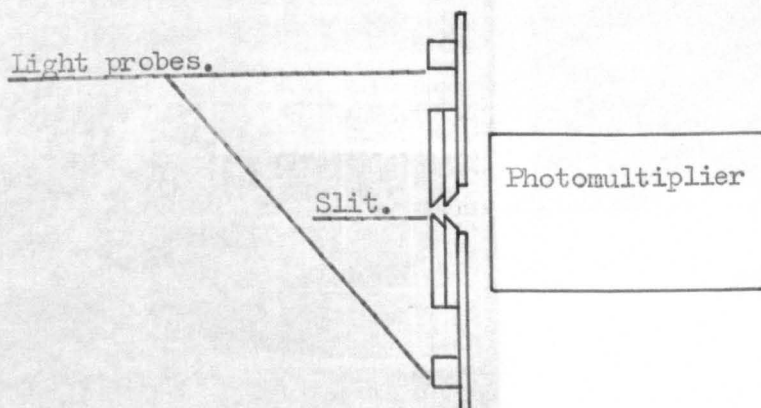
Screen arrangement.



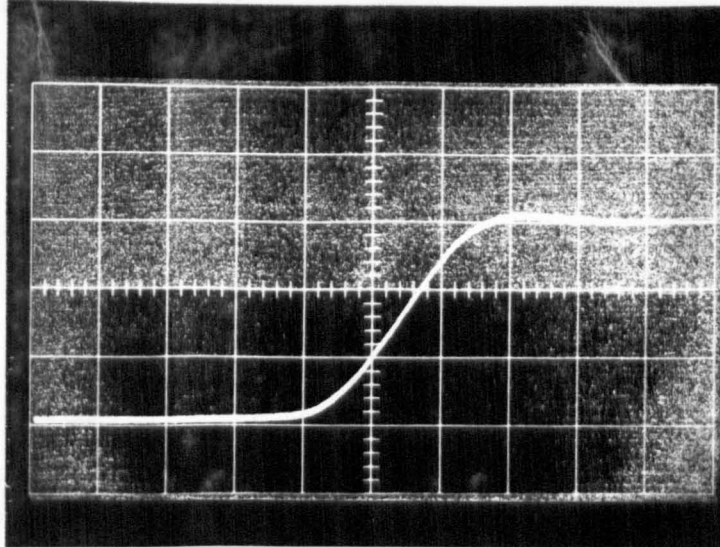
→
1ms/cm.

Resolution based on maximum slope
rise time = 0.007cms.

Screen slit set at 0.008cms.

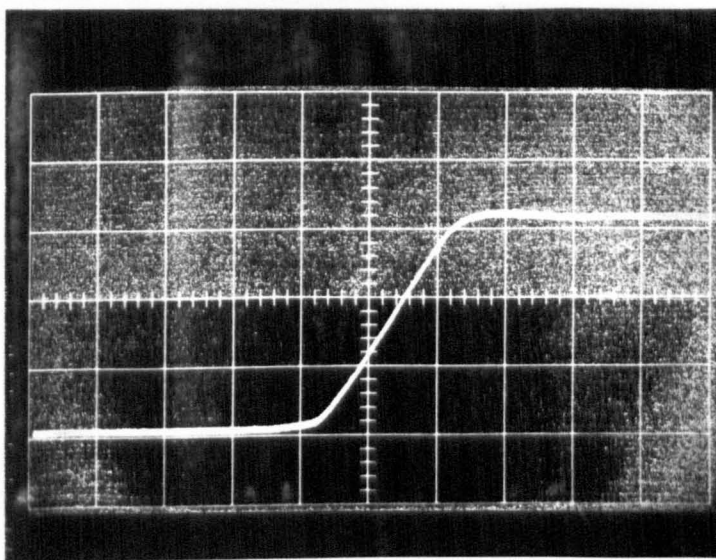


Screen arrangement.



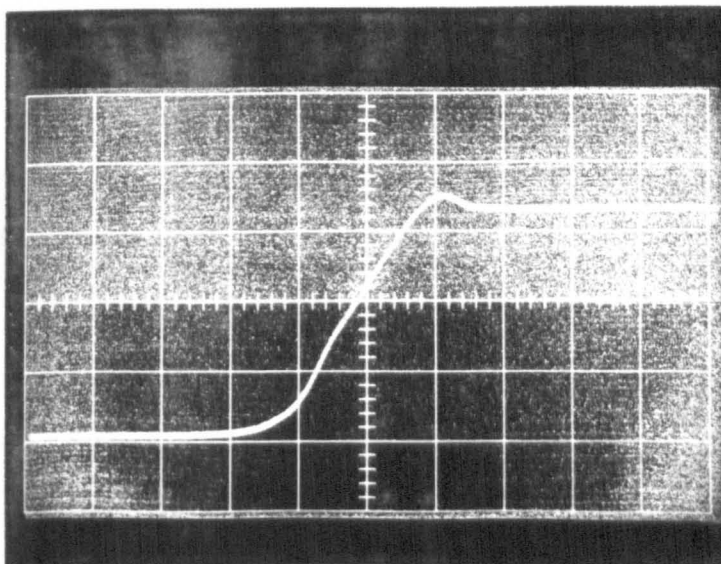
Screen at + 0.5 cms.
from best focus.

1ms/cm. →



Screen set at
best focus.

1ms/cm. →



Screen at - 0.5 cms.
from best focus.

RUN 3312

$N_2 \rightarrow CO_2$

$M_1 = 2.44$

1 cm trace

$= 0.33 \text{ cm}$ true

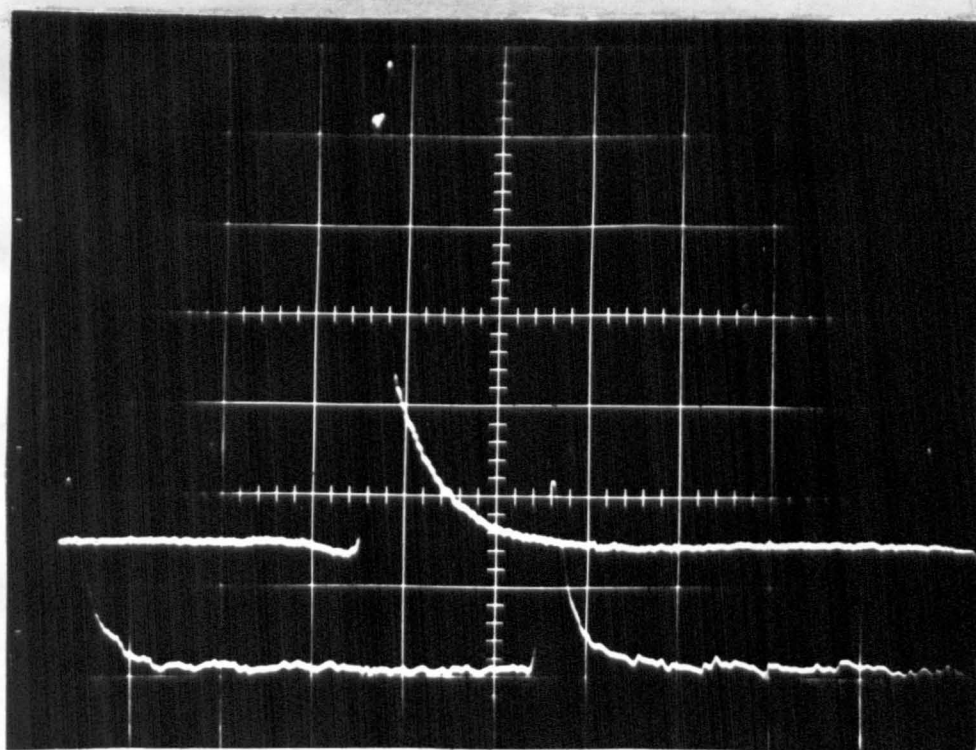
Spatial
Resolution

$< 2 \times 10^{-2} \text{ cm}$

Shock

Thickness

$\approx 10^{-1} \text{ cm}$



Spus/cm $\rightarrow$

RUN 3313

$N_2 \rightarrow CO_2$

$M_1 = 2.29$

1 cm trace

$= 0.31 \text{ cm}$ true

Spatial

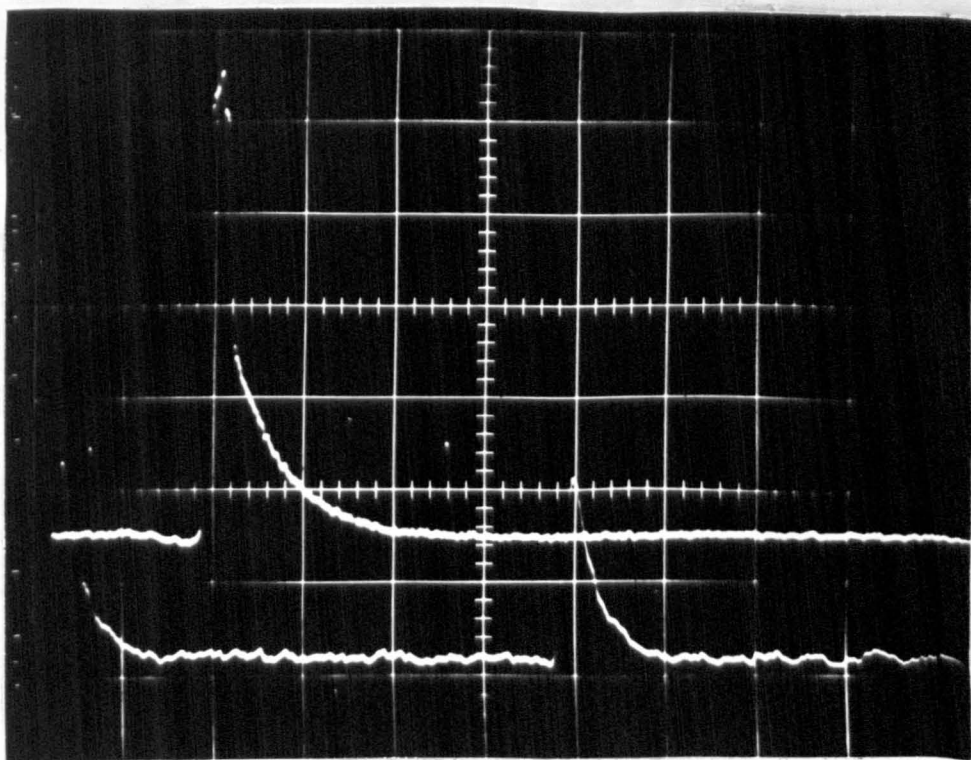
Resolution

$< 2 \times 10^{-2} \text{ cm}$

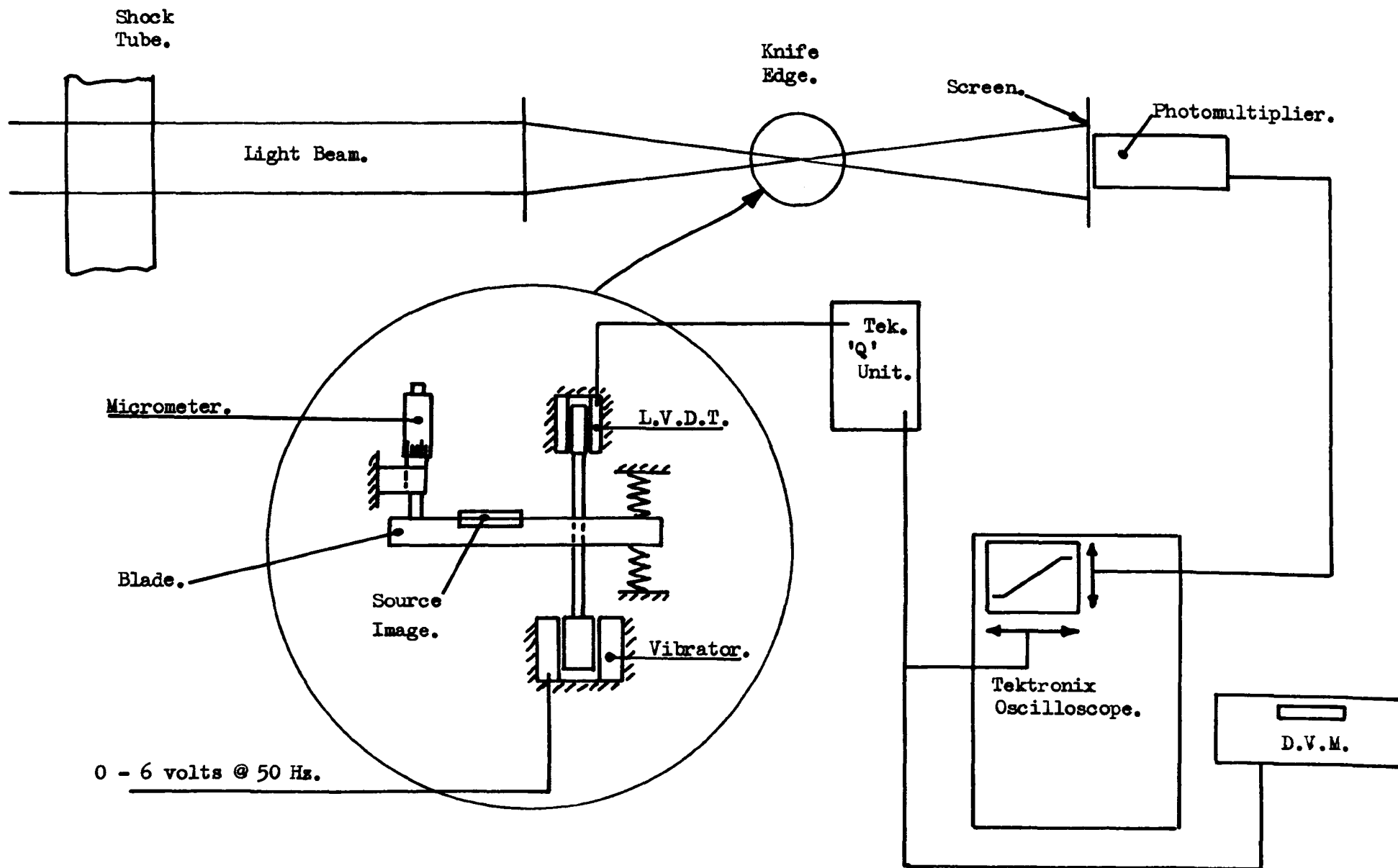
Shock

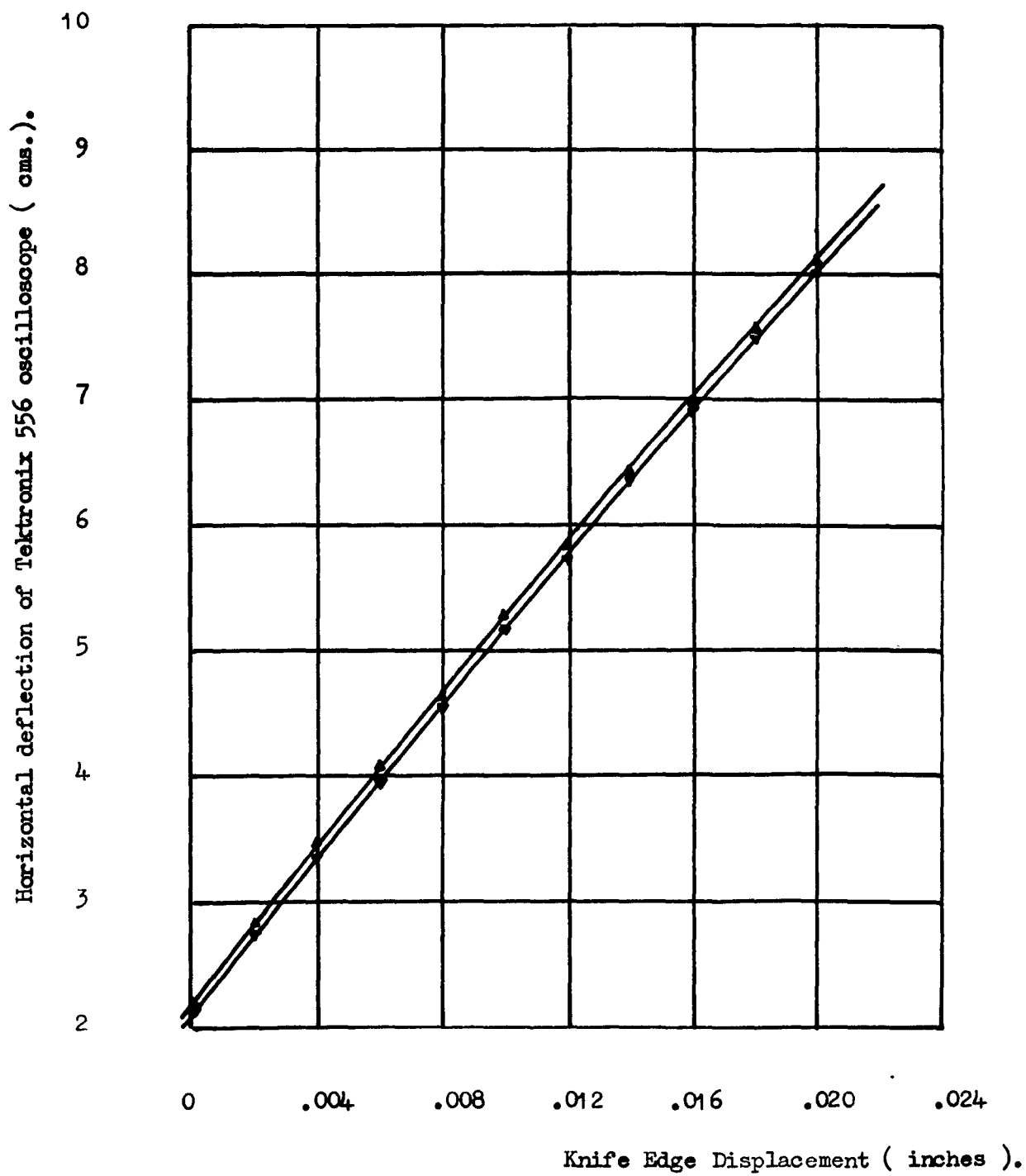
Thickness

$\approx 10^{-1} \text{ cm}$

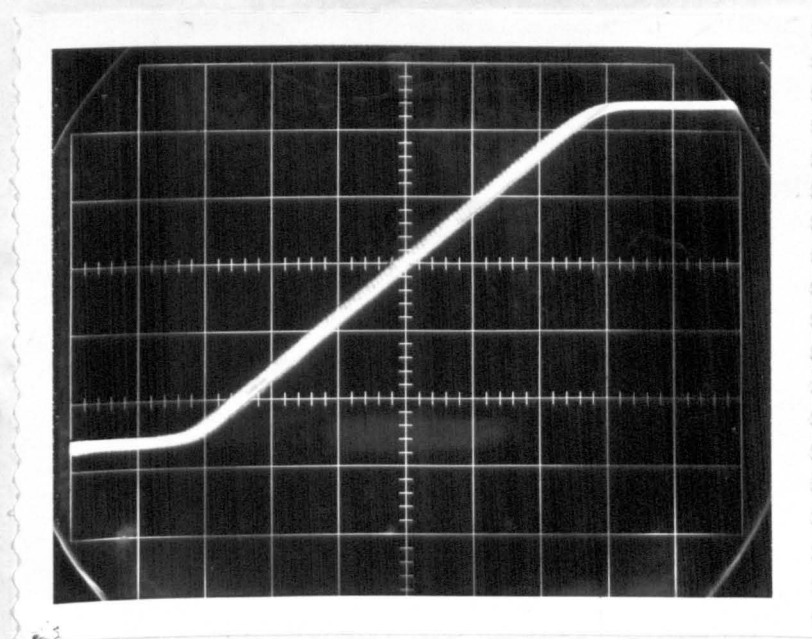


Spus/cm $\rightarrow$





Linearity = $\pm .8 \%$



L.V.D.T. output 1v/cm.

Photomultiplier
output 20mv/cm.

From L.V.D.T. calibration:

1 cm. horizontal = 0.00845 cms. at knife edge.

∴ 1mv. from photomultiplier = 0.000515 cms.

at knife edge.

3.2. STRONG INCIDENT SHOCK WAVES.

SUMMARY

Studies have been made of the structure of strong incident shock waves in carbon dioxide. The shock waves were produced in the Cranfield Institute of Technology 2" shock tube described in appendix 2, and the structure of these shock waves was studied with the time resolved schlieren system described in section 3.1.

The vibrational relaxation time of carbon dioxide has been determined in this way for translational temperatures from 300^o K to 1200^o K. The degree of agreement which was obtained between these results and those of other researchers has been found to be highly dependant on the method of analysis and data reduction.

Comparison of experimental density gradient profiles with theoretical profiles calculated from existing relaxation time data confirms the assessment of the performance of the schlieren system presented in section 3.1.

A means of improving the accuracy of the measurements by the use of electronic data reduction is proposed.

3.2.1. Introduction.

The study of the structure of strong incident shock waves produced in a shock tube is a commonly used technique for the evaluation of vibrational relaxation in gases. These waves consist of a thin diffusion resisted shock front followed by a broad relaxation region through which the vibrational modes equilibrate with the translational modes. Relaxation regions are typically 10^4 times as thick as the diffusion resisted shock fronts and the latter can be regarded as discontinuities on the scale of the relaxation regions.

The change in the vibrational state of the gas through the diffusion resisted shock front can be neglected, and it can be assumed that the vibrational modes remain frozen at their upstream state. Equally, the effects of viscosity, heat conduction and mass diffusion can be neglected in the relaxation region.

These are the partly dispersed shock waves which have been referred to in part 2. The structure, which consists of a discontinuity followed by a relaxation region, is given by the so called 'basic solution' of part 2.

The change in the thermodynamic state of the gas through the relaxation region is essentially exponential in form and the relaxation times of the vibrational modes are related to the local characteristic lengths of the regions. This characteristic length can be evaluated from the ratio of the departure from the downstream equilibrium of a thermodynamic variable to its local spatial derivative, that is, if the density is the particular thermodynamic variable in question,

$$\hat{x} = \frac{(\rho_s - \rho)}{d\rho/dx} \quad 3.2.1.$$

where ρ_s is the downstream equilibrium value of the density and $\hat{x}$ is the required characteristic length. It can be shown that this characteristic length is directly related to the relaxation time, as follows,

$$\frac{P}{P_0} \cdot \tau = \frac{\hat{x}}{W_s} \cdot f\left(M_s, \frac{\rho_s - P}{\rho_s - P_0}\right) \cdot \frac{P_1}{P_0} \quad 3.2.2.$$

where ρ_c is the density at the upstream end of the relaxation region, that is, behind the diffusion resisted shock front, τ , is the relaxation time and M_s & W_s are the Mach number and velocity respectively of the incident shock wave. It is implicit in the relation that only one vibrational mode is considered or that a number of vibrational modes combine in such a way as to act as a single mode.

If the characteristic length, $\hat{x}$, can be determined experimentally, it is a simple matter to compute the function, f , from the initial conditions in the shock tube, the shock mach number and the appropriate position in the relaxation region measured in terms of the density, ρ , and hence to compute the relaxation time, τ .

Many researchers have made use of the incident shock wave in this way and the most recent have made use of the above relation in some form or other to determine the behaviour of the relaxation time within the relaxation region.

Smiley, Winkler and Slawsky (Ref. 3.28) were one of the first groups of researchers to study vibrational relaxation behind incident shock waves in carbon dioxide. The studies were made with a Mach Zehnder interferometer. Smiley, Winkler (Ref. 3.29) have published results for relaxation times measured in carbon dioxide at temperatures from 400 to 1500°K. The physical characteristics of the shock tube used were such that the number of fringe shifts obtained in the relaxation regions were small and the relaxation region was very thin. This limited the accuracy of the results to $\pm 20\%$. The relaxation times obtained are in reasonable agreement with more recent data at the lower temperatures, but they are significantly shorter at higher temperatures.

An approximate form of analysis was used, which gave an overall relaxation time for each region. This was calculated from the distance over which the density changed by $1/e$ of its equilibrium value. It is stated that the total change in density across the shock waves were consistent with all of the vibrational modes being in equilibrium downstream of the relaxation region.

Griffith, Brickl and Blackman (Ref. 3.30) were also one of the earlier groups of researchers to study vibrational relaxation in this way. They also used a Mach Zehnder interferometer to monitor the density changes in the relaxation regions. Their method of analysis

differed from that of Smiley and Winkler in that the relaxation time was determined from the mean slope of a plot of the logarithm of the density change versus distance through the relaxation region. The relaxation times obtained were of the same order as those obtained by Smiley and Winkler at the lower temperatures, but were significantly longer at the higher temperatures. The data was much closer to that which has been obtained more recently.

Johannesen (Ref. 3.31) suggested an approach to the detailed analysis of the relaxation regions which made use of the Rayleigh-line equations. This technique removed the inaccuracies resulting from previous approximate analyses and provided a simple means of determining the behaviour of the relaxation time through the relaxation region.

In a companion article, Blythe (Ref. 3.32) reviewed the techniques which had been used for the analysis of relaxation regions. The accuracies of these techniques were compared with a solution obtained from a numerical integration of the relaxation equation and with the Rayleigh-line technique proposed by Johannesen. The assumption of a constant relaxation time through the region was the main source of error in the previous techniques. This accounted for errors of 15 to 20% in the Smiley and Winkler results and for errors of 10% in the Griffith, Brickl and Blackman results. It was suggested that the Rayleigh-line approach should be adopted in order to provide a simple means of removing these errors.

Daen and de Boer (Ref. 3.15) have studied vibrational relaxation behind incident shock waves with their integrating schlieren instrument (see Section 3.1).

Particular attention was paid to the assessment of the performance of the instrument and to the effects of shock curvature. The apparent thicknesses of shock waves produced in argon and helium and studied with the instrument could not be accounted for by instrument errors. The apparent thicknesses that would be apparent according to the Hartunian theory for shock curvature. (see Appendix 2).

The carbon dioxide results which were obtained with the instrument were analysed by measuring the time taken for the output to change from the initial zero to 95% of the final value. These times were then converted to $1/e$ distances for the exponentials. The relaxation times obtained were substantially longer than those obtained by both Smiley and Winkler and Griffith, Brickl and Blackman. The water vapour content of the test gas in the experiments was known to be 10 to 15 ppm and it was suggested that the differences could be

accounted for by the presence of water vapour in the test gases used by the other researchers, and not to errors in the integrating schlieren instrument.

Wittman (Ref. 3.33.) has also obtained relaxation time data for carbon dioxide with an integrating schlieren instrument; apparently with the same equipment as was used by Daen and de Boer.

The relaxation times were determined in this case from the initial slope of the density profile at $t = 0$. That is, immediately behind the diffusion resisted shock front. The relaxation times obtained by this method were significantly shorter than those obtained by Daen and de Boer, but it would appear that the difference is attributable to the method of analysis. The Wittman results lie much closer to those which were obtained by Griffith et alia and to those which have been obtained more recently. Wittman also analysed the distribution of vibrational energies in the relaxation regions and showed that the valency modes followed the bending modes very closely.

Johannesen, Zienkiewicz, Blythe and Gerrard (Ref.3.34.) have also studied the relaxation regions behind incident shock waves in carbon dioxide with a Mach Zehnder interferometer. They applied the Rayleigh-line approach to the analysis of the density profiles so obtained. It was found that the data correlated linearly onto a plot of the logarithm of the density change through the relaxation region versus distance through the region. This correlation applied over the whole of the region within the experimental errors. By analysing the data in this way, it was possible to plot the behaviour of the relaxation time within the relaxation region on the Landau-Teller plot.

It was concluded from this data that the relaxation time was not a single valued function of the transitional temperature of the gas and that it was in some way dependant on the departure from equilibrium within the relaxation region. The relaxation times obtained showed a similar overall trend to those of Griffith et al, but the absolute values were a factor of two longer. The total density changes across the shock waves were consistent with all of the vibrational modes being in equilibrium downstream of the shock wave.

Possible sources of error in their density measurements were considered by Zienkiewicz, Johannesen and Gerrard (Ref. 3.10.) An attempt was made to explain the difference between their findings, which indicated complete vibrational equilibrium downstream of the shock wave, with those of Griffith et al, which indicated that only the bending modes had equilibrated. It was shown that an impurity level equivalent to 15% of nitrogen would be required to explain the differences, and this possibility was discounted. Particular attention was given to the effects of the group refractive index on the brightest fringe

shift when white light is used in the interferometer. It was shown that this had negligible effect on the difference between the results of the two groups of researchers. The question was left unresolved.

The apparent dependence of the relaxation time on the departure from equilibrium within the relaxation regions was investigated by Zienkiewicz and Johannesen (Ref. 3.13.). They showed that a bimodal model of the relaxation behaviour could lead towards relaxation times which were single valued functions of the translational temperature. However, the results were inconclusive.

In the same paper, Zienkiewicz and Johannesen reported on a similar analysis which they had made of the relaxation data obtained by Blackman in oxygen (Ref. 3.35.). It was found that this gas exhibited a similar behaviour although it could not be explained in terms of a bimodal model. The apparent dependence on the departure from equilibrium was less with oxygen, but this was partially attributable to the smaller change in the transitional temperature through the relaxation region.

Following the work of Zienkiewicz and Johannesen, Kiefer and Lutz (Ref. 3.19.) extended a previously developed numerical technique (Ref. 3.21.) to obtain an exact solution of the relaxation equation from density data. Studies were conducted with a laser schlieren system on oxygen to obtain results for comparison with those presented by Zienkiewicz and Johannesen.

Their analysis was based on a piecewise integration of the relaxation equation through the relaxation region. This was an iterative technique which was started with initial estimates of the relaxation time obtained from measurements made by White and Millikan (Ref. 3.20). The iterations were continued until the input and output values for the relaxation times agreed within 1%. The results obtained in this way indicated that the relaxation time of oxygen was a single-valued function of the translational temperature within the experimental errors. The technique relied upon a knowledge of the start of the relaxation region. However, the results were shown to be insensitive to the choice of this position and it was assumed to occur at the centre of the initial spike from the diffusion resisted shock front.

They also showed that the forcing of a linear fit to the plot of the logarithm of density change versus distance through the relaxation region masked a slight curvature in the plot which was critical in the determination of the relaxation time behaviour. It was also concluded that the measurement of density as made by Zienkiewicz and Johannesen was too insensitive to test for variations in the relaxation time through the relaxation regions.

Simpson, Bridgeman and Chandler (Ref. 3.12.) have also studied the vibrational relaxation of carbon dioxide behind incident shock waves. Their measurements were made with a Mach Zehnder interferometer

in the same manner as those made by Zienkiewicz and Johannesen. They noted the existence of systematic deviations in the density profiles from the exact exponentials assumed by Zienkiewicz and Johannesen. Accordingly, the data was fitted with a quadratic term in product with the exponential term in order to allow for these deviations. The relaxation times obtained in this way for carbon dioxide were found to be single-valued functions of the translational temperature within the experimental errors. The total density changes across the shock waves were consistent with all of the vibrational modes being in equilibrium downstream of the shock waves.

At the outset of the present experiments, no detailed studies of relaxation regions in carbon dioxide using a schlieren system had been reported, although since, Simpson, Chandler and Strawson (Ref.3.36.) have published results of such experiments. As Kiefer and Lutz (Ref.3.19.) had commented on the insensitivity of the Mach Zehnder interferometer to detailed relaxational behaviour, it was considered that the application of the schlieren system to carbon dioxide studies might make a useful contribution to the knowledge of the subject.

The integrating schlieren instrument had been applied by Daen and de Boer and by Witteman to carbon dioxide studies. However, the output of their instrument was proportional to the density change and not to the density gradient. In addition, the sensitivity of the instrument was determined by its response to the diffusion resisted shock front. This instrument was, therefore, no more sensitive to vibrational relaxation than the Mach Zehnder interferometer.

The application of the schlieren system to the study of relaxation regions in carbon dioxide would also provide a useful check on the performance of the schlieren system prior to its application to other studies. There were detailed differences between the relaxation times obtained by various researchers, but these had little impact on the density profiles which were observed. Therefore a gross check on the performance of the schlieren system could be made.

The application of the schlieren system would also lead to a better understanding of the problems associated with the study and analysis of the relaxation regions, and in turn might enable possible improvements to be considered and investigated experimentally.

3.2.2. First Series of Experiments

The time resolved quantitative schlieren system which is described in section 3.1. herein had been partially developed for this type of application at the outset of these experiments and these experiments were conducted with the schlieren system in its original form. The vacuum system had not been modified at the time of these experiments and the ultimate vacuum which could be obtained was 10^{-3} torr. It was therefore considered that the results from these experiments would only serve to evaluate the performance of the schlieren system and provide relative information regarding the relaxation times, and that the absolute values of the relaxation times obtained would not be reliable.

The screen of the schlieren system contained two sampling slits one larger than the other. The largest slit was used for the analysis of the wave profiles, whilst the two slits together acted as timing markers from which the shock velocity could be determined.

The oscillogram which is shown in figure 3.2.2.A is typical of the results obtained in those experiments. Both of the traces are of the same output from the photomultiplier behind the two sampling slits. The upper trace was taken on a faster time base for the analysis of the wave profile whilst the lower trace shows both outputs for the determination of the shock velocity.

Both of the traces were triggered by the arrival of the shock wave at the first slit, the output from which is shown on the upper trace. This method of shock velocity determination was necessitated because the timing section was far upstream of the working section in the cylindrical part of the shock tube. (see Appendix 2.).

It has been shown in section 3.1 that the output from the schlieren system was proportional to the density gradient in the flow, and hence the amplitude of the upper trace was proportional to the density gradient. However, as there was no free run on the trace prior to the arrival of the signal to be analysed the absolute amplitude relative to the zero density gradient was not known. It was assumed for the purposes of this analysis that the trace had returned to zero density gradient towards the right hand side of the trace.

This series of experiments were analysed by plotting the amplitude of the trace versus oscilloscope time on a log-linear plot, as shown in figure 3.2.3. for run 2559. This method followed that of Zienkiewicz and Johannesen, except that the density gradient is used here whereas Zienkiewicz and Johannesen analysed the density change through the relaxation region. The data presented in figure 3.2.3. has been non-dimensionalised by the total density change through the relaxation region.

It can be seen from Figure 3.2.3 that the data presented correlated linearly on this plot, and it could be concluded from such presentations that, within experimental errors, the density gradient was exponential in form through the relaxation region. It will be noted that this form of curve fitting gives an undue weighting to the near equilibrium data and Simpson et al (ref. 3.12) have commented on the sensitivity of the relaxation times to the near equilibrium data.

It was assumed that the characteristic length of the region was constant throughout. The constant, A , was calculated from the slope of the graph in Figure 3.2.3. The conditions behind the shock wave were obtained from a measurement of its velocity and the initial conditions in the shock tube channel. Values of the constant, A were then plotted versus $T_2^{-1/2}$ in the same manner as presented by Zienkiewicz and Johannesen for each of the experiments. The results are shown in Figure 3.2.4., where they are compared with the same mean line through Zienkiewicz and Johannesen's data.

The influence of the undue weighting which was given to the near equilibrium data was not assessed, but it was considered that an alternative method analysis should be investigated. To this end, the density gradient data was integrated to give the density change through the relaxation region. It was necessary to calibrate the schlieren system to place the data in the correct position in the relaxation region; in terms of density change. For this, a conventional calibration was made of the schlieren system, which was known to be within 10% and therefore adequate for these purposes.

The density gradient was plotted against the density change as shown in Figure 3.2.5. for run 2559. It is clear that the constant, A , can be obtained directly from such a plot as the mean slope of the curve, as indicated by the straight solid line. In this plot the near equilibrium data is given its proper weighting, and an alternative fit can be made to the data as indicated by the dotted line. It can be seen that both of the fittings shown suffer a zero error, which leads to an infinite or zero value for the relaxation time at equilibrium; depending on how the data is fitted. This error was attributed to the wrong choice of equilibrium; depending on how the data is fitted. This error was attributed to the wrong choice of equilibrium state for the traces and the data was consequently adjusted to give no zero error.

The relaxation time of carbon dioxide was calculated for each of the experimental points shown in Figure 3.2.4. using the relation given by equation 3.2.2. It will be recalled that for an exponential density gradient in the relaxation region, the characteristic length, x , is the inverse of the constant A .

The dependence of the function, f , given in equation 3.2.2. on the Mach number of the incident shock wave is shown in figure 3.2.6. The curves shown are for six points equally spaced in terms of density change through the relaxation region. The curve 'A' is the upstream 'frozen' end of the relaxation region whilst the curve 'f' is the downstream 'equilibrium' end of the region. The behaviour of the translational temperature raised to the one third power is shown in figure 3.2.7. for each of the six points through the relaxation region. It can be seen that great care must be exercised in the use of the curves for the calculation of the relaxation times in order to preserve the detailed differences which appear in the data in the form shown in figure 3.2.5.

The relaxation times obtained in this way for the first series of experiments are shown in figure 3.2.8. where they are compared with the Simpson data.

The dashed lines indicate that the data has been extrapolated linearly towards the frozen end of the region. It can be seen that there was a large amount of scatter in the results. This was not considered surprising in view of the limitations of the experimental set-up. This point is particularly brought out in figure 3.2.5. in which the dashed curve is that which is obtained if the data is made to fit the relaxation time measurements to small changes in the experimental profiles.

3.2.3. Second Series of Experiments

A second series of experiments were initiated when the schlieren system and the vacuum system had been improved as far as was considered necessary. The schlieren system was brought to the final configuration described in section 3.1., and had a spatial resolution of better than 0.02cms. and a bandwidth of 50 MHz. The vacuum system was in the final configuration described in appendix 2 and the system had an overall impurity level of less than 30 p.p.m. of water vapour.

From experience gained in the first series of experiments, greater care was exercised in the choice of initial channel pressures and diaphragms in order to optimise the spatial resolution of the wave profiles and the signal to noise ratio of the system. The schlieren system was also calibrated each day during these experiments in the manner described in section 3.1. using the vibrating knife edge, so that the absolute value of the density and its gradient were more accurately known.

The screen of the schlieren system in its final configuration contained only one sampling slit which allowed light to pass through to the main photomultiplier for the analysis of the wave profile.

The two light probes were mounted on the front of the screen, which conveyed the light to a second photomultiplier. These probes were primarily required for the determination of the velocity of weak shock waves in other experiments as a gross check on the incident shock velocity. The primary incident shock velocity measurement was made by timing the passage of the shock wave with wall mounted thermometers upstream of the working section. These measurements were corrected according to the calibration discussed in appendix 2.

The oscillogram shown in figure 3.2.2.B is typical of the results which were obtained in this second series of experiments. The upper trace is the output from the main photomultiplier behind the sampling slit, whilst the lower trace is the output from the photomultiplier connected to the light probes. Both of these traces were triggered by the passage of the image of the incident shock wave over the first light probe. In this way, a free run appeared on the output from the main photomultiplier prior to the arrival of the image of the incident shock wave at the main sampling slit.

It can be seen from figure 3.2.2.B that there was a precursor dip in the trace ahead of the output from the incident shock wave. This could not be attributed to radiation, because a narrow bandwidth filter centred at $5461\text{\AA}$ was positioned in the light beam between the working section and the screen. The most probable cause is believed to be a reflection of the light beam off the curved front of the incident shock wave. The position of the trace ahead of the precursor dip was taken to be zero density gradient reference for the experiments.

As mentioned above, the schlieren system was more rigorously calibrated for these experiments, and the amplitudes of the traces were readily converted into density gradients. The time resolved outputs were converted to spatial variations and integrated numerically to give the density changes through the relaxation regions. This procedure was conducted on a computer using point by point measurements taken off the oscillograms.

This data was plotted as density gradient versus density change, as shown in the upper part of figure 3.2.9. for run 3148. Any zero errors which occurred were corrected for this type of plot and the required ratio, $(P_2 - P) / (dP/dx)$, evaluated directly.

The behaviour of this ratio for run 3148 is shown in the lower part of figure 3.2.9. It can be seen that the scatter in the ratio increases as the downstream equilibrium is approached, as would be expected from the decreasing amplitude of the trace there, Simpson (Ref. 3.12.). The ratio appears to remain nominally constant as the downstream equilibrium is approached, although it will be recalled that this has been enforced to a certain extent by the requirement that there be no zero error.

The relaxation times were calculated in the manner described in the first series of experiments, except that instead of the ratio being taken as a constant through the relaxation region, the variation was taken into account and point values were taken of the ratio through the region. The relaxation times obtained in this way for the second series of experiments are shown in figure 3.2.10. It can be seen that even with the incorporation of the improvements the scatter has only been slightly reduced and there is no tendency for the data to lie along the Simpson curve.

As a further check on these experiments, theoretical curves were plotted from the relaxation time data published by Zienkiewicz and Johannesen and by Simpson et al. Figure 3.2.9. presents curves obtained in this way for run 3148. It can be seen that slight adjustment of the zero positions could cause all of the three curves to agree over most of the relaxation region.

Figures 3.2.11., 3.2.12., and 3.2.13., present comparisons of measured profiles with theoretical profiles obtained from the Simpson data for a range of incident shock Mach numbers. It can be seen that the data can be made to agree over most of the relaxation region by adjusting the choice of the start of the relaxation region.

If the fitting shown is correct, then there can be no residual density gradient downstream of the shock wave, because the zero gradient datum was taken upstream of the shock wave. This implies that the difference between the present data and that due to Simpson could be accounted for by the manner in which equilibrium is approached rather than the equilibrium values of the traces. This does lead to the possibility that the near equilibrium behaviour is affected by the separate relaxation of the asymmetric stretching mode. Unfortunately, the present experimental arrangement is not sufficiently refined to prove or disprove this.

It can be seen from figures 3.11, 3.12 and 3.13 that the apparent thicknesses of the diffusion resisted shock fronts ranged from 0.02 to 0.15 cms.; depending on the incident shock Mach number. The values are of the same order as those obtained by other researchers. The apparent thickness is attributable to the curvature of the incident shock wave; as discussed in appendix 2.

3.2.4. Electronic Data Reduction

One of the prime difficulties in the deeper understanding of the strong incident shock wave in the relaxing gas is the essentially exponential nature of the relaxation region. The data reduction which is involved in reducing the measurements to useable form precludes any accurate and detailed assessment of the behaviour in the relaxation region. The weakest link in the reduction of the data to the form presented in figures 3.2.5. or 3.2.9. is the measurement of the traces and the subsequent numerical integration. It is considered that substantial improvements could be made by conducting this measurement and computation electronically, as will now be described.

The output from the time resolved quantitative schlieren system is in an ideal form for electronic analysis in that it is a real time analogue signal. It is a simple matter to pass the output through an operational amplifier with the required feedback for time integration over the desired period. As the velocity of the incident shock wave is nominally constant, the output from the integrator would be proportional to the density change, and hence essentially exponential in form. However, by driving the Y plates of an oscilloscope with the direct output and the X plates with the integral a display of the form shown in figures 3.2.5. and 3.2.9. is obtained.

An investigation was conducted into the possibilities of using this type of data reduction. The operational amplifier employed was a Tektronix 'O' unit, which was equipped with controls for the direct setting of the feedback elements to give the required period of integration. A Tektronix 536 oscilloscope was used for the display.

An electronic simulation was made of the output from the schlieren system, an example of which is shown in figure 3.2.14.A. It can be seen that the signal consisted of a delta function followed by an exponential decay, which was essentially the form of the output from the schlieren system. The resulting display obtained on the oscilloscope is shown in figure 3.2.14.B. The finite response of the system distorted the initial delta function, but the subsequent trace is linear indicating that the simulated input was truly exponential and the system was integrating correctly. The display shown is for a longer time constant than was obtained in the shock tube experiments. A range of time constants were investigated and it was found that the system did not have a sufficiently fast rise time for the shock tube measurements to yield accurate results. In addition, the large variation in the velocity of the spot across the screen of the oscilloscope gave rise to a variation in the intensity of the trace. This occurred to such an extent that part of the trace was too bright whilst part of the trace was too dim for recording with Polaroid film. The Z modulation of the oscilloscope was modified to take into account the variation in the velocity of the spot, but there was not sufficient improvement for any accurate measurements to be made.

It was decided that the system should be applied to the reduction of shock tube data to evaluate its performance, but it realised that the data would not be sufficiently accurate for any relaxation time information to be obtained. Two typical outputs which were obtained from shock tube tests are shown in figure 3.2.15. The region to the left hand side of the trace was the diffusion resisted shock front, and the bright spot on the right hand side is the downstream equilibrium state. It can be seen that not only must the intensity of the trace be better controlled, but the duration of the intensity must be finely adjusted so that the upstream and downstream equilibrium points do not appear too bright and thereby reduce the resolution.

The curvature of the trace is very evident and is consistent with the relaxation time data presented by Simpson et al. The left hand side of the trace shows the effects of the limited bandwidth of the equipment, but the majority of the trace is a true representation of the actual profile.

These tests have shown that the technique of electronic data reduction could be applied to the study of the relaxation regions of strong shock waves in order to obtain more detailed information regarding the relaxation process. This technique warrants further development, but, as the incident shock wave was not the central part of this study, it was not pursued here.

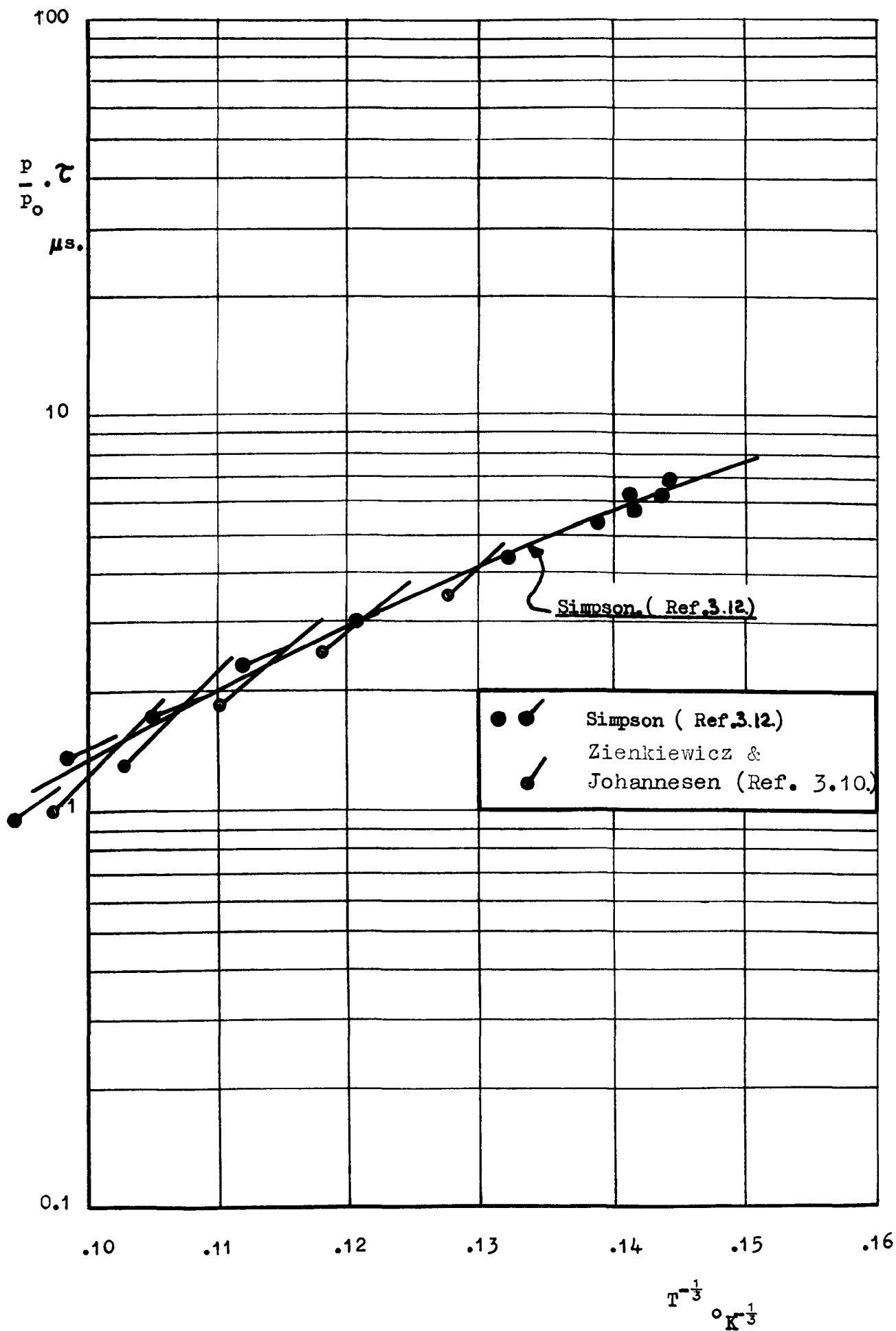
3.2.5. CONCLUDING REMARKS

This investigation has demonstrated the applicability of the conventional schlieren system to the study of vibrational relaxation in strong incident shock waves produced in the shock tube. The performance of the schlieren system so far has indicated that with further development it has the potential to provide further detailed information about the relaxation regions of strong incident shock waves. It has the advantage over the laser schlieren that it can be arranged to give an output which is directly proportional to density gradient. This reduces the amount of data reduction required for each experiment and is well suited to the proposed electronic data reduction, because the latter relies on the data being proportional to the density gradients in order to perform the electronic integration.

The precursor dip is a source of doubt in the performance of the schlieren system. It has been noted that a similar dip appears in the data obtained by Simpson, Chandler and Strawson (Ref. 3.36) in argon. This aspect has not been investigated here, but it should receive further attention before the technique is applied to more accurate studies of strong incident shock waves.

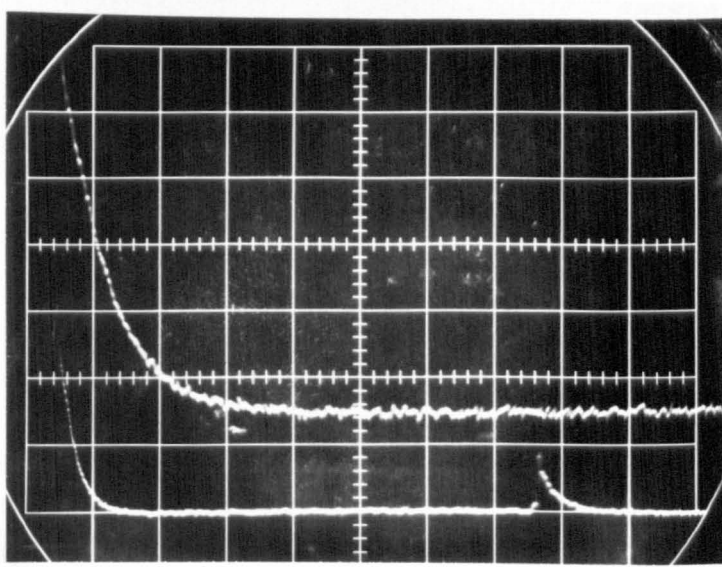
The relaxation time for carbon dioxide has been obtained for temperatures from 300°K to 1200°K from the study of the strong incident shock waves to an accuracy of $\pm 30\%$ by direct data reduction. However, it has also been shown that if the near equilibrium behaviour is ignored the relaxation time data can be made to correlate on to the curve obtained by Simpson et al, except for the near frozen data which is affected by the curvature of the shock wave. Accepting the form of curve fitting proposed by Simpson et al, and limiting measurements to the central part of the relaxation region, it can be concluded that the relaxation time is a single valued function of the translational temperature, as determined by Simpson, Bridgeman and Chandler. (Ref 3.12.).

Since the completion of the present experimental studies, Simpson Chandler and Strawson (Ref. 3.36.) have published the results of similar studies using a laser schlieren technique, which confirmed their earlier findings. They have also demonstrated conclusively that the relaxation time does not depend on the departure from equilibrium within the relaxation region by studying waves propagating through mixtures of argon and carbon dioxide.



THE LANDAU-TELLER PLOT FOR CO₂

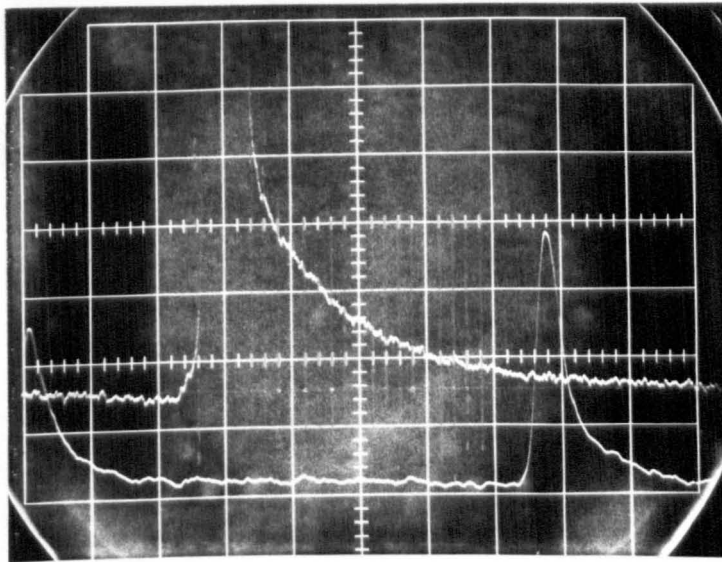
FIGURE: 3.2.1.



A. Run 2559

Upper trace: output from lower slit $2\mu\text{s}/\text{cm}$.

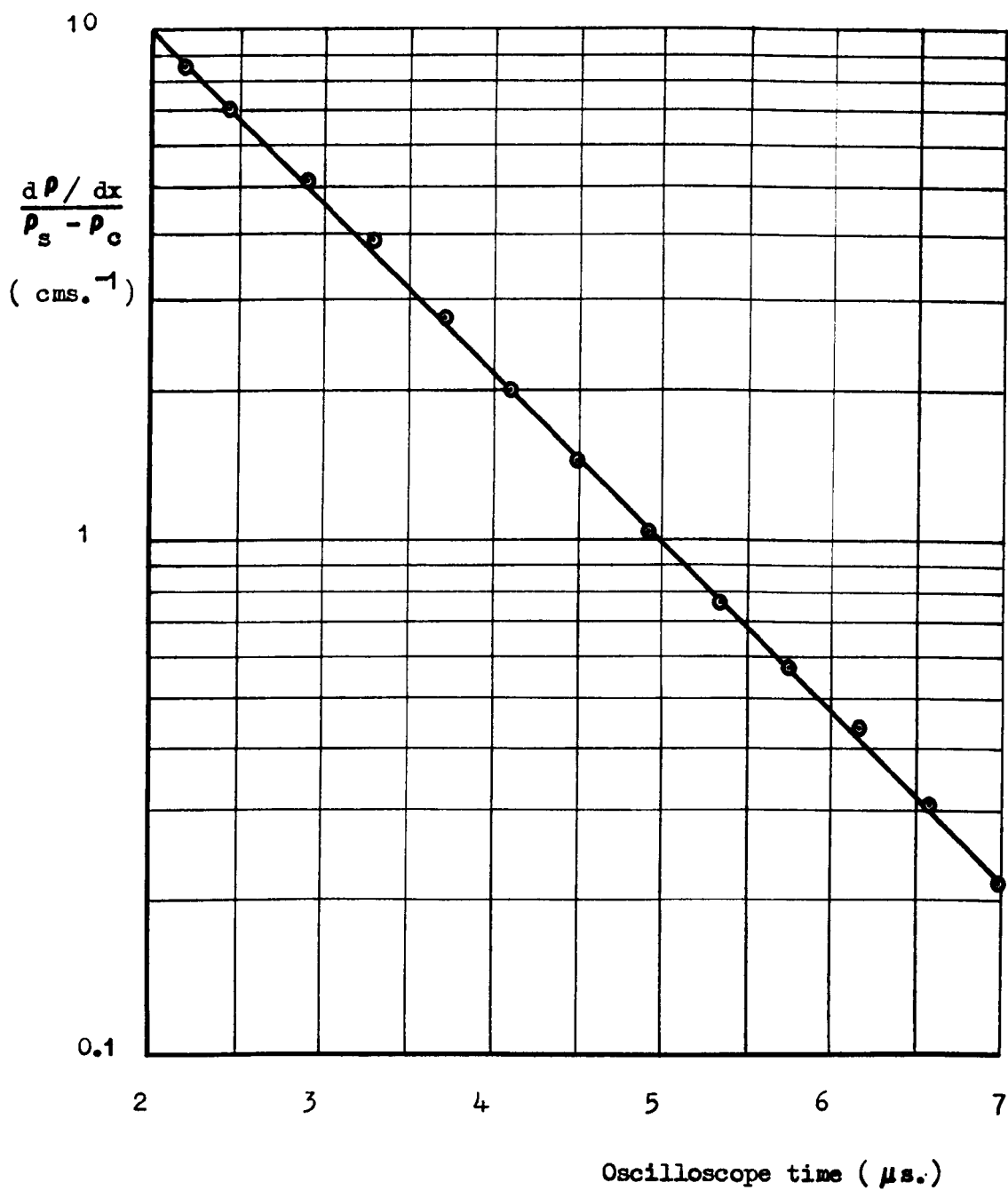
Lower trace: outputs from both slits for shock velocity measurement $5\mu\text{s}/\text{cm}$.

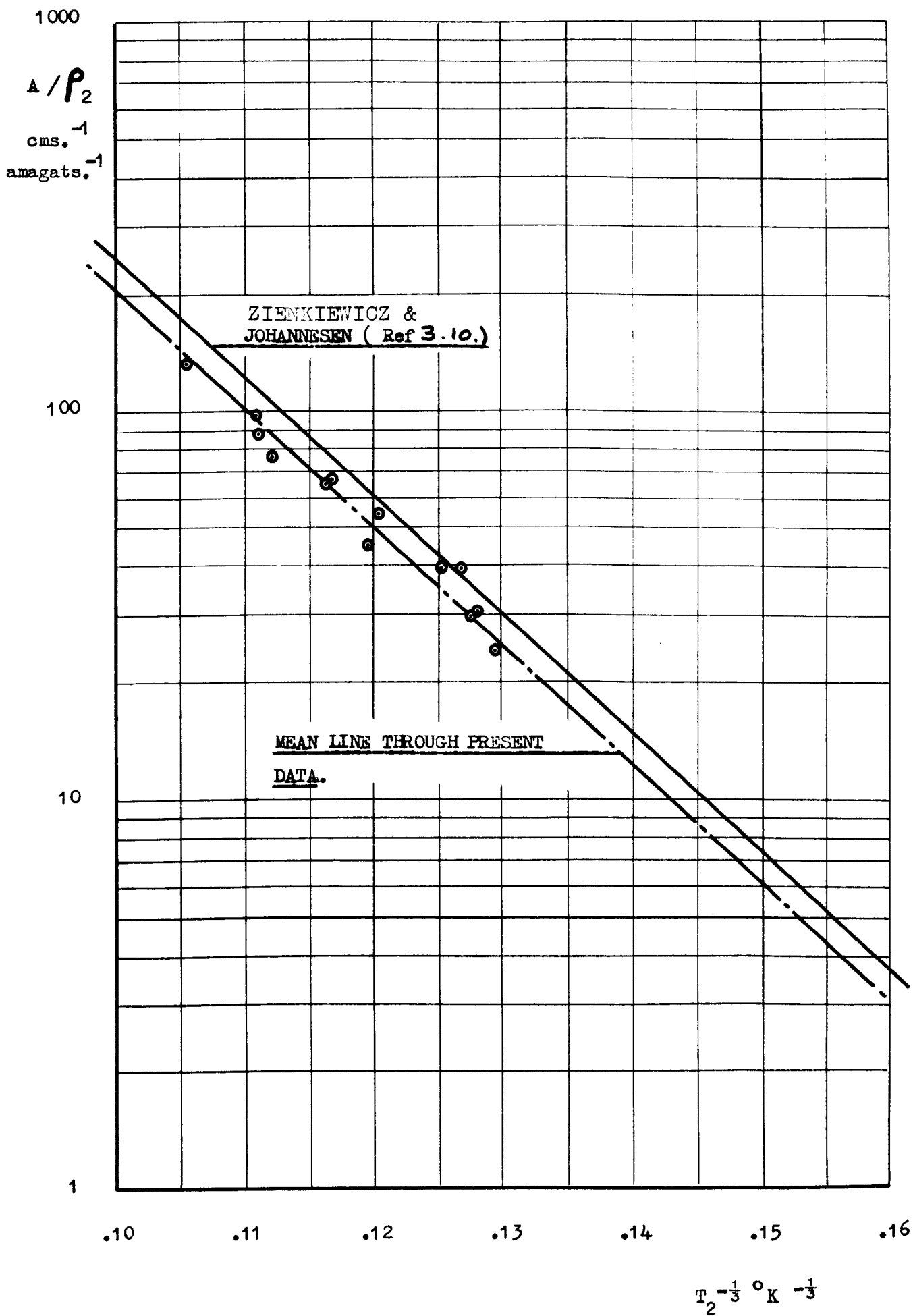


B. Run 3148

Upper trace: output from sampling slit $2\mu\text{s}/\text{cm}$.

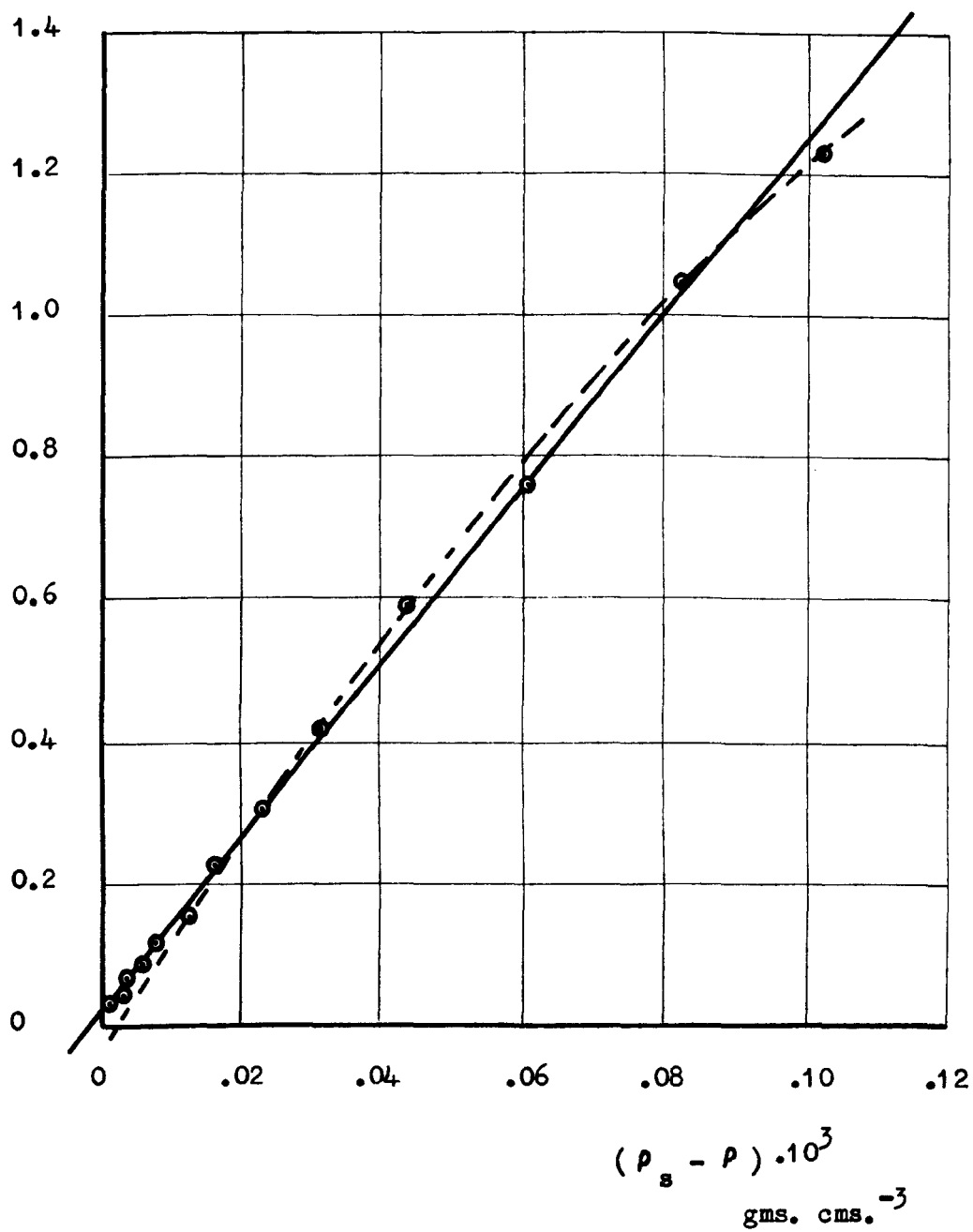
Lower trace: output from shock velocity measuring probes $5\mu\text{s}/\text{cm}$.

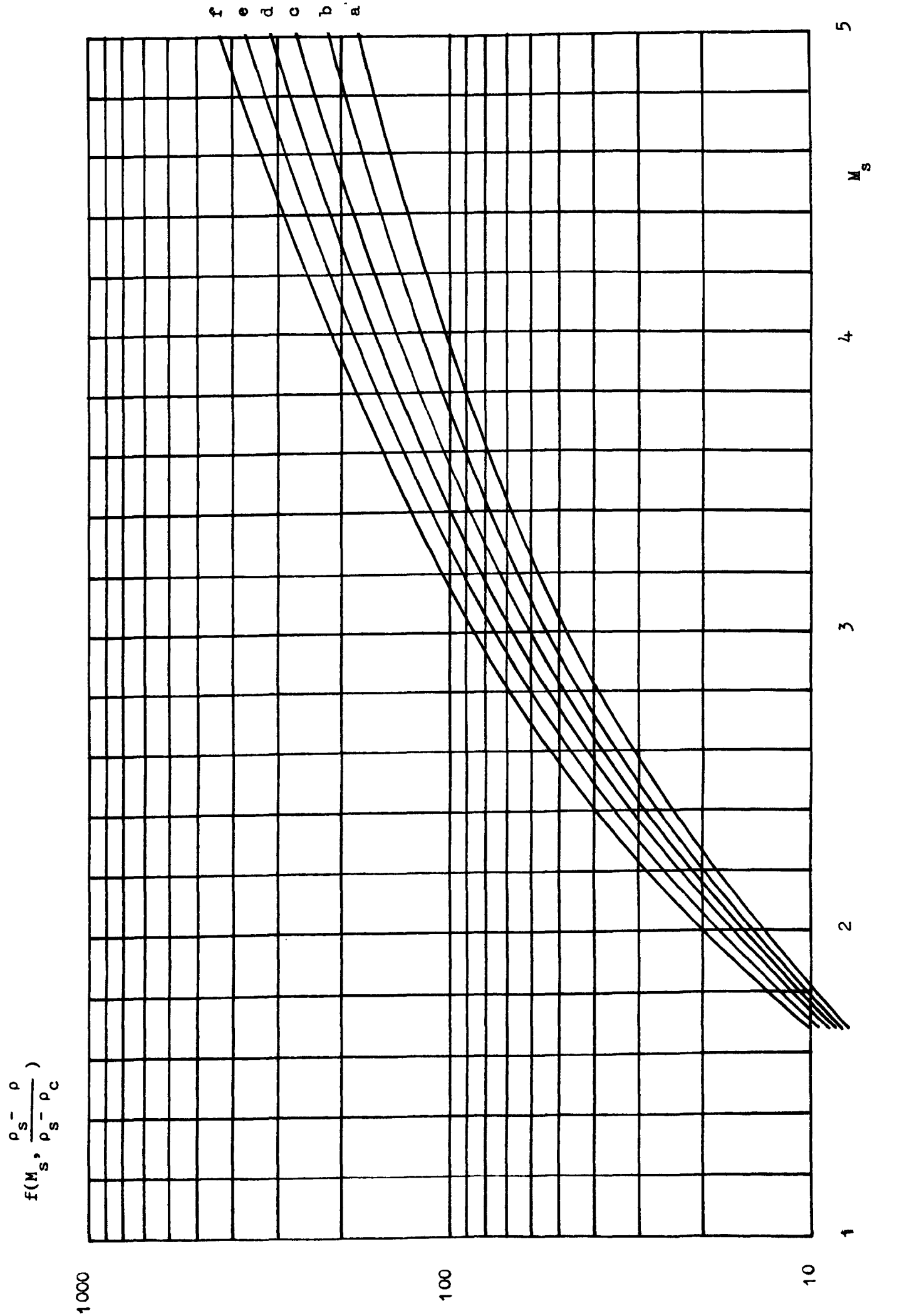




$d\rho/dx \cdot 10^3$

gms. cms.⁻⁴

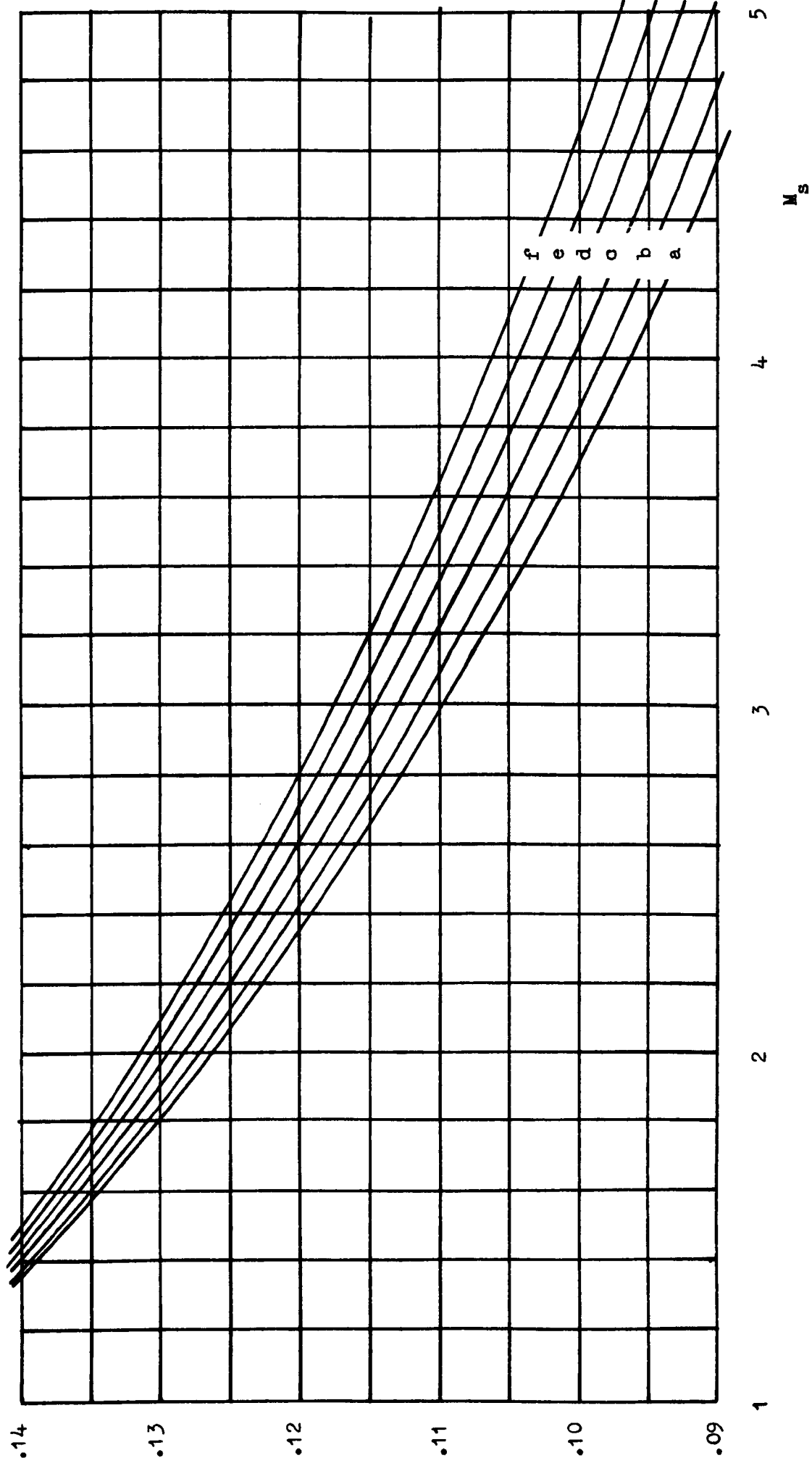




CONVERSION FUNCTION FOR RELAXATION TIME COMPUTATION.

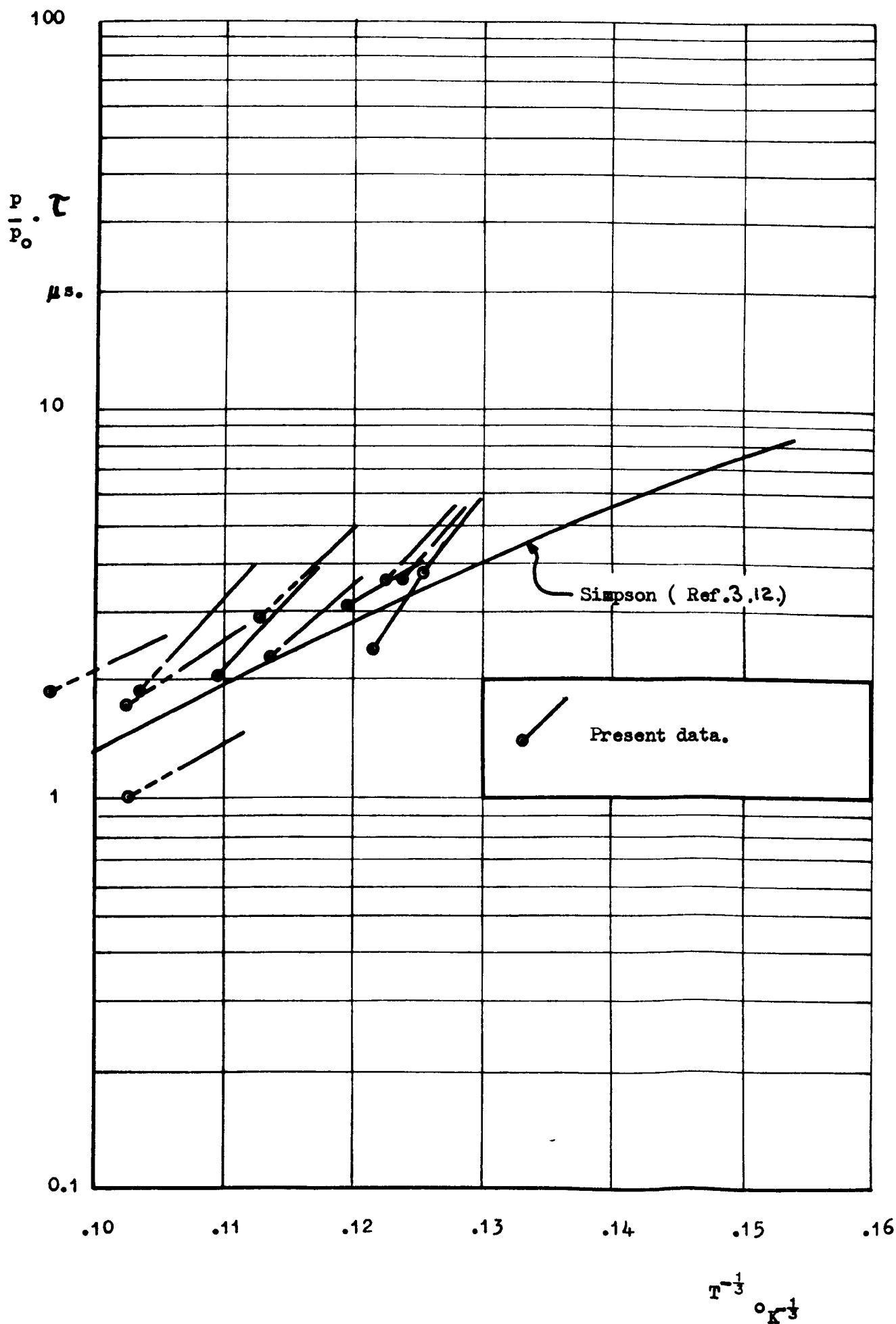
FIGURE: 3.2.6.

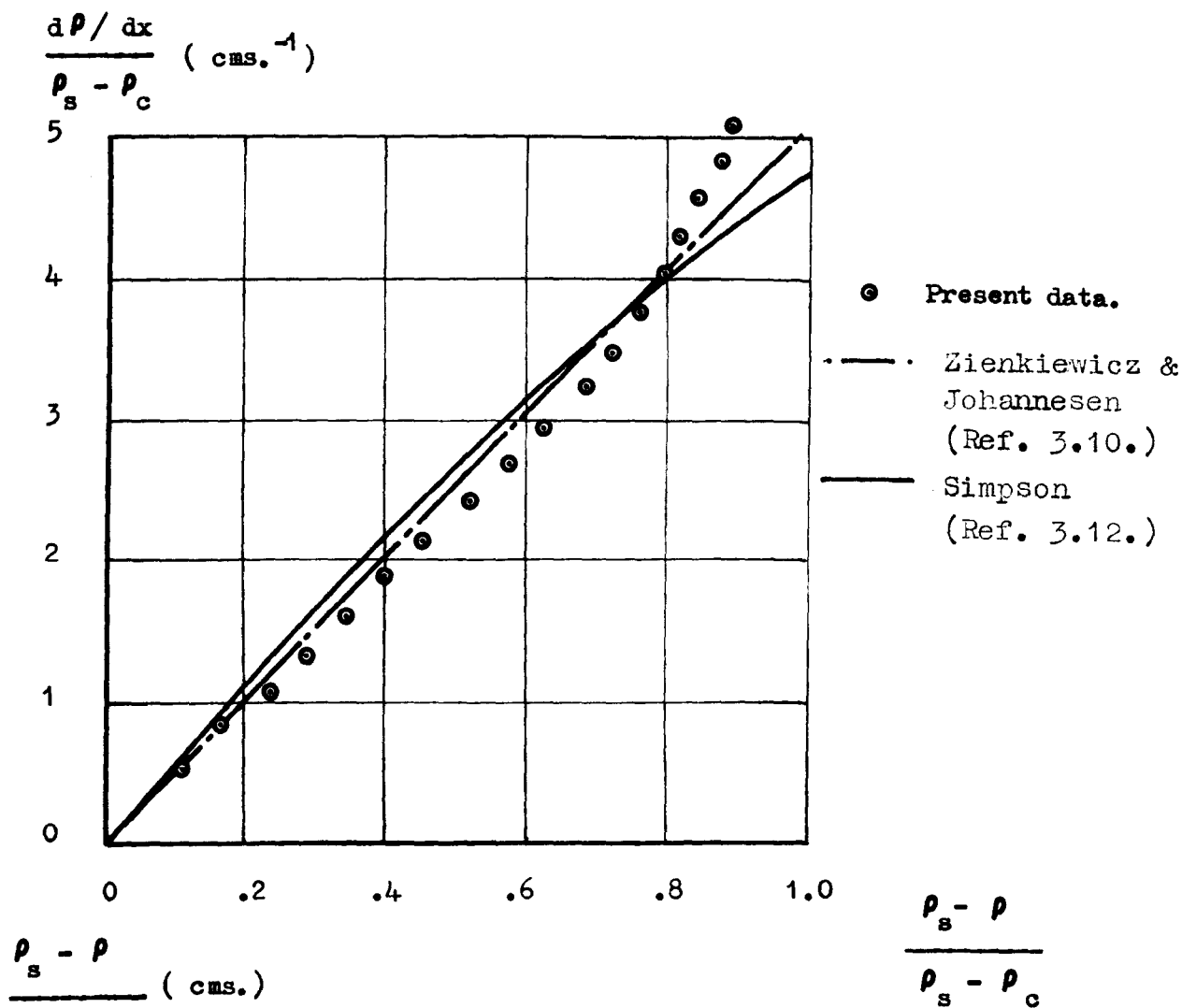
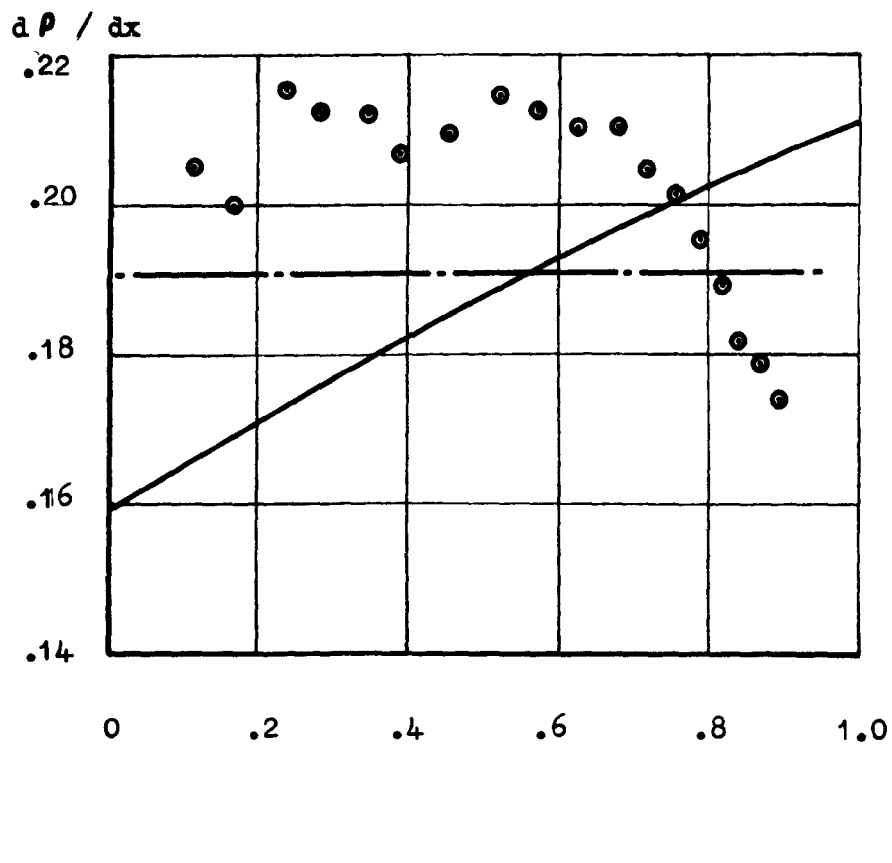
$T^{-\frac{1}{3}}$
 $\alpha_K^{-\frac{1}{3}}$

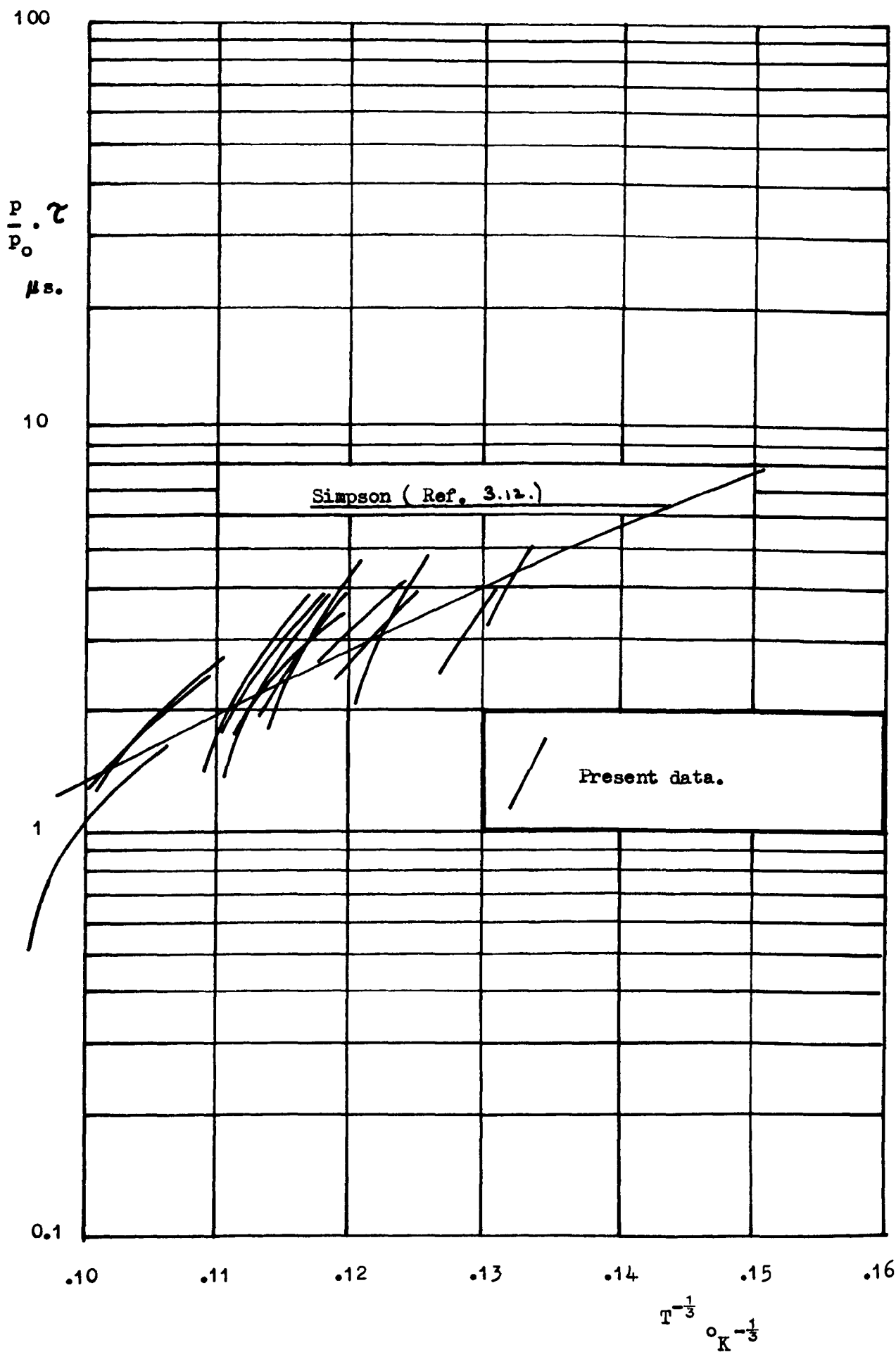


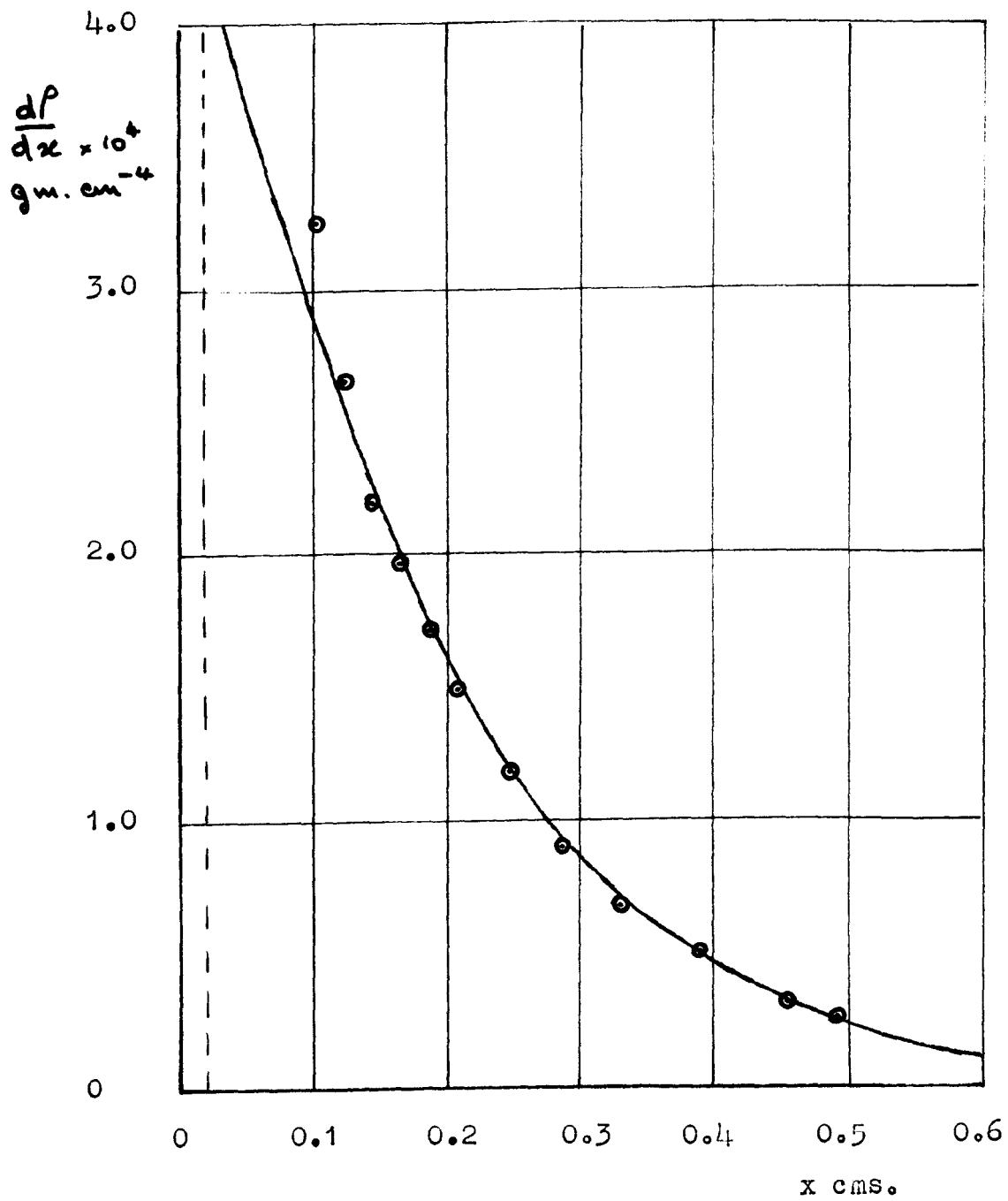
BEHAVIOUR OF TEMPERATURE THROUGH RELAXATION REGION.

FIGURE: 3.2.7.





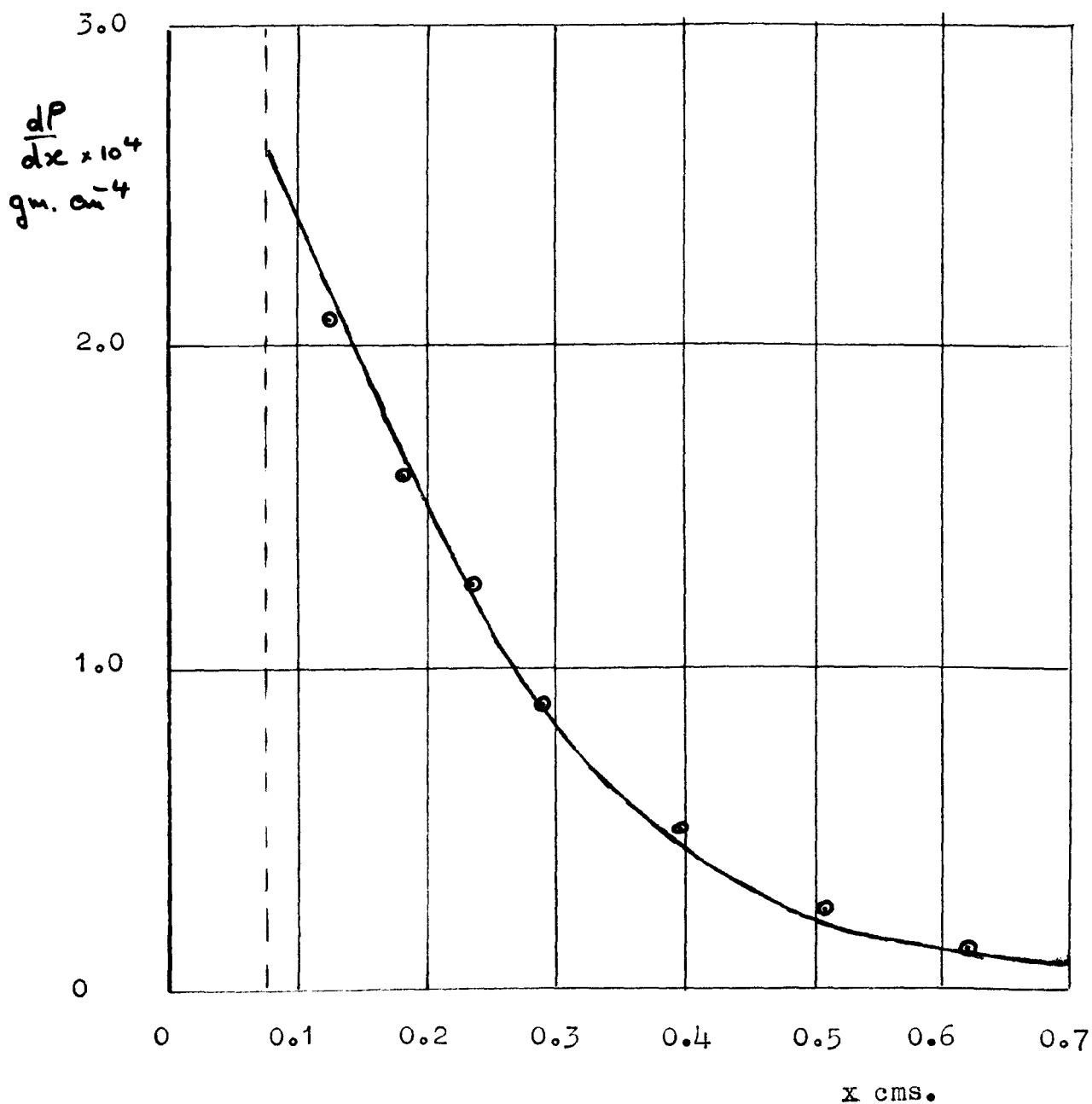




RUN 3168 $M_S = 1.91$

COMPARISON OF EXPERIMENTAL DENSITY GRADIENT
DATA WITH THEORETICAL PROFILE

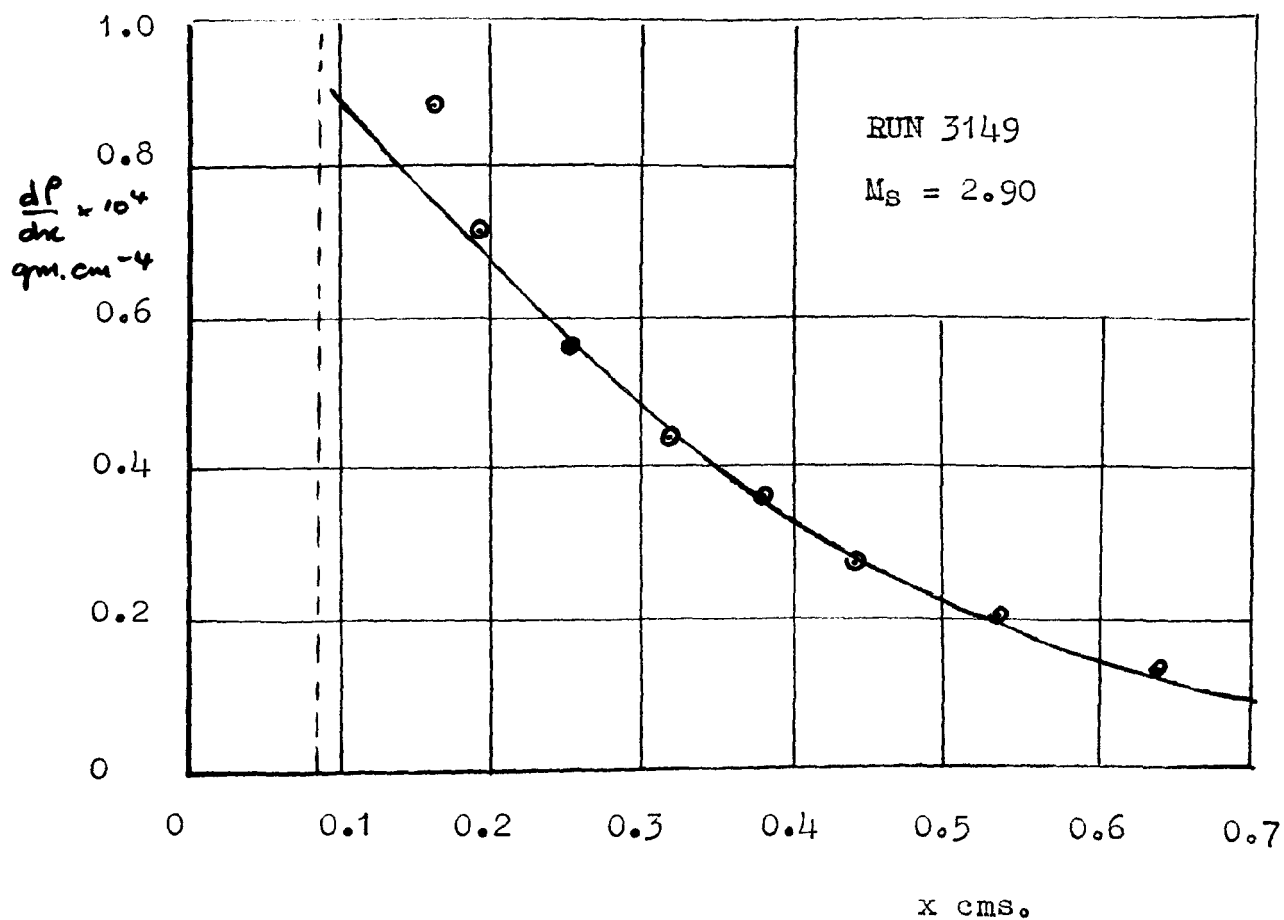
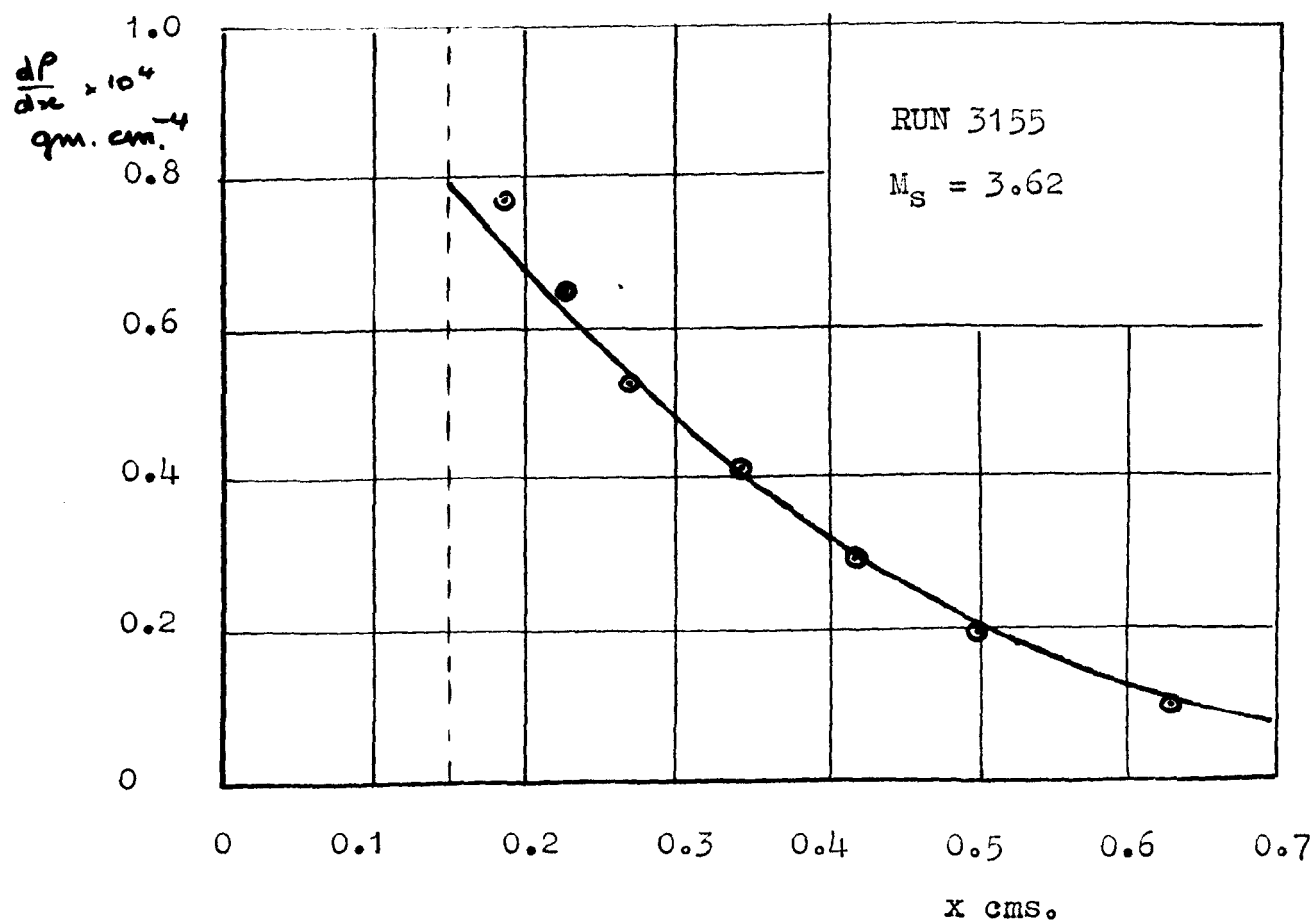
FIGURE.
3.2.11.



RUN 3146 $M_s = 2.55$

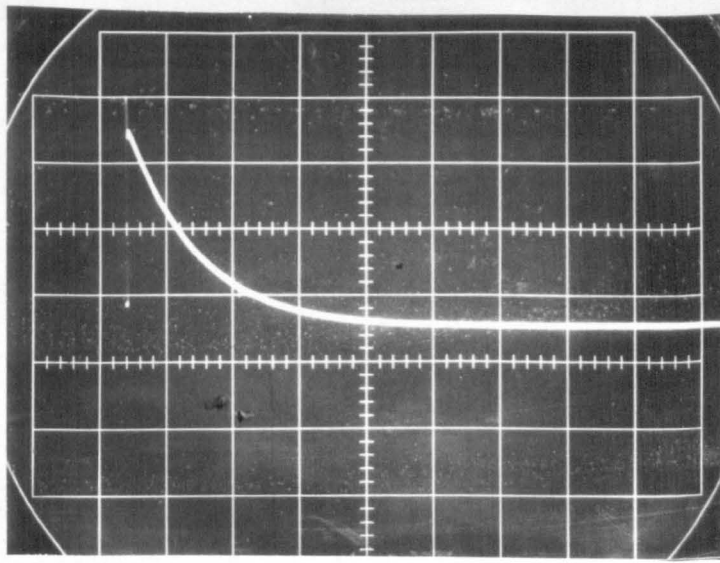
COMPARISON OF EXPERIMENTAL DENSITY GRADIENT
DATA WITH THEORETICAL CURVE.

FIGURE.
3.2.12.



COMPARISON OF EXPERIMENTAL DENSITY GRADIENT
 DATA WITH THEORETICAL CURVES

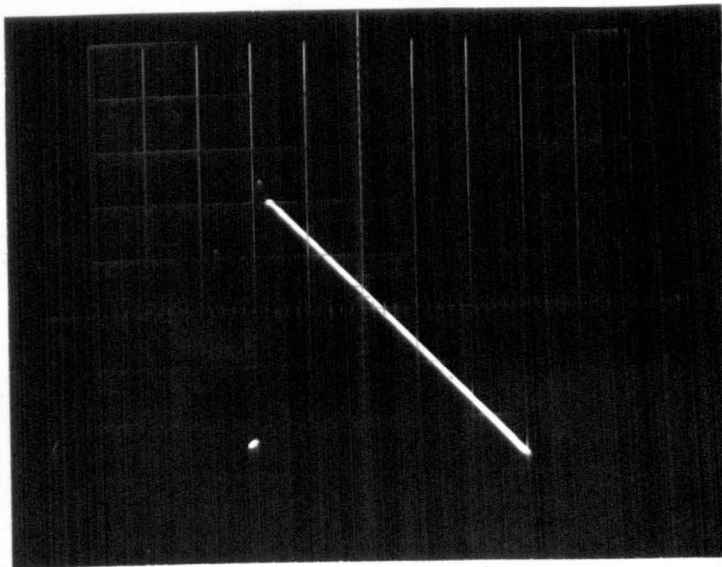
FIGURE. 3.2.13.



S 50mv/cm.

1 ms/cm.

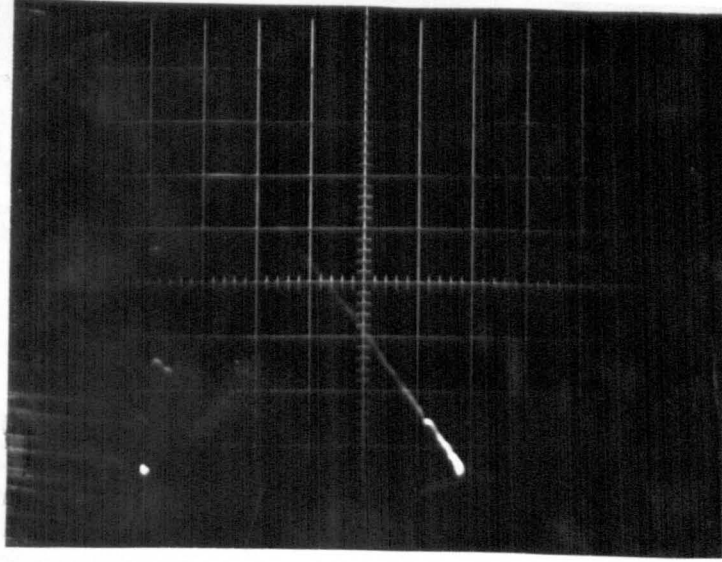
A. Electronic simulation(S) of schlieren output.



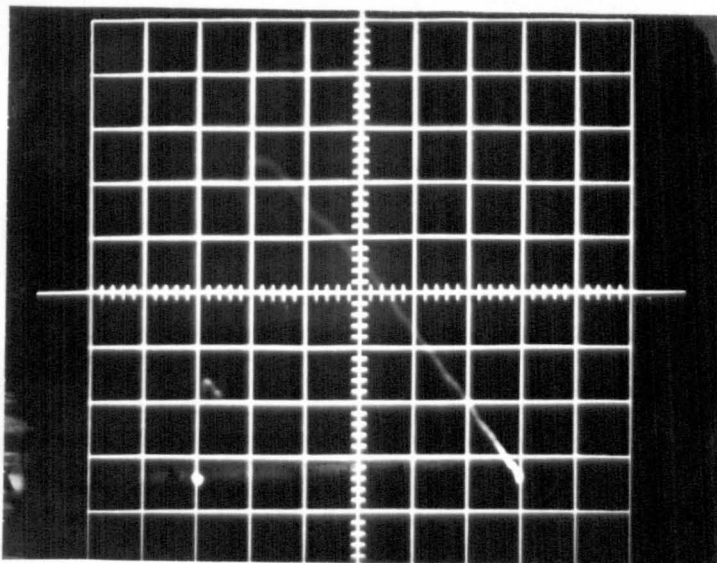
S 50mv/cm.

$\int S dt$ 500 mv/cm.

B. X-Y plot of simulated output versus time integral of simulated output.



A. Run 2581



B. Run 2580

3.3. WEAK INCIDENT SHOCK WAVES.

SUMMARY

Studies have been made of the structure of weak incident shock waves in carbon dioxide. A technique has been developed for the production of the weak incident shock waves, which involved the positioning of a perforated plate in the channel of the shock tube.

The shock waves were produced in the Cranfield Institute of Technology 2" shock tube described in appendix 2, and the structure of these waves was studied with the time resolved schlieren system described in section 3.1.

The vibrational relaxation time of carbon dioxide has been determined in this way for translational temperatures of approximately 300°K, and has been found to be in good agreement with measurements made elsewhere.

Comparison of the experimentally measured density gradient profiles with theoretical profiles calculated from the basic solution of part 2 have shown good agreement. The regions of maximum density gradient in the fully dispersed shock waves and the diffusion resisted shock fronts of the partly dispersed shock waves departed from the theoretical profiles. This has been attributed to the curvature of the shock waves as discussed in section 3.1 for the strong incident shock waves.

The suitability of the use of weak normal shock waves for the study of vibrational relaxation has been discussed, and conclusions have been reached regarding the optimum wave strength for this type of study.

It has been concluded that the suitability of the time resolved schlieren system for this type of application has been confirmed.

3.3.1. Introduction.

An essential feature of the strong incident shock waves discussed in the preceding section was that they were partly dispersed; the structure of these waves consisted of negligibly thin diffusion resisted shock fronts followed by broad relaxation regions. As the strength of this type of wave is reduced, a stage is reached when the diffusion resisted shock front disappears and the structure of the shock wave becomes solely determined by relaxation of the vibrational mode(s). These are the fully dispersed shock waves discussed in part 2 herein.

The existence of fully dispersed shock waves was first noted by Bethe and Teller (Ref. 3.38.) in 1941. Lighthill (Ref. 3.39.) derived an approximate theoretical solution for the distribution of flow parameters through fully dispersed shock waves. The theoretical predictions of Bethe and Teller were investigated experimentally by Griffith and Kenny (Ref. 3.37.) with the intention of demonstrating the existence of these waves. They produced fully dispersed shock waves in the form of incident shock waves in a shock tube. Carbon dioxide was used as the test gas because of its large vibrational energy content at room temperature.

The density profiles through the fully dispersed shock waves were studied with a Mach Zehnder interferometer and the total fringe shifts through the waves were used to determine the wave strengths. Good agreement was obtained between the experimental profiles and theoretical profiles calculated from the Lighthill approximation. Relaxation times of the order of 5 microseconds at NTP were obtained in these experiments. These times were shorter than expected and the difference was attributed to impurities in the test gas.

The objectives of the present investigation were to confirm these earlier findings with the schlieren system, to further evaluate the performance of the schlieren system in this type of application and to investigate the transition from a fully dispersed incident shock wave to a partly dispersed incident shock wave. The latter was conducted in conjunction with the theoretical results of part 2 for the transition.

Figure 3.3.1 shows the variation of the density gradient with density change, $\Delta\rho$, for a range of weak incident shock waves. The reason for this choice of presentation will be discussed later. The profiles shown were computed for a fixed equilibrium ratio of the specific heats of 1.293, which is nominally the room temperature value for carbon dioxide.

The diffusion resisted shock fronts of the partly dispersed shock wave profiles shown in figure 3.3.1 have not been included. Ideally, with a negligibly thin diffusion resisted shock front observed by a schlieren system having a finite resolution only the relaxation region should be observed, as shown.

The transition from a fully dispersed to a partly dispersed shock wave can be seen to occur at an incident shock Mach number slightly above 1.04, which, in terms of normal shock tube operation is a very weak shock wave. It will also be noted that there is a radical change in the profiles from a Mach number of 1.01 to 1.07, which is a change of only 6% in the shock velocity.

These then are the types of waves to be produced and studied with the schlieren system. It had been demonstrated previously (Ref. 3.23.) that the schlieren system was ideally suited to the study of weak shock waves. A reliable method of producing weak incident shock waves was not available and had to be developed.

3.3.2. Methods Of Weak Shock Wave Production.

In principle, it should be possible to produce very weak incident shock waves by the use of very weak diaphragms. In practice, it has been found to be very difficult for the pressure differentials required.

Weak diaphragms had been developed previously by the author (Ref. 3.23) for this type of application, where they proved to be adequate. These were manufactured from 0.0025 cms. thick aluminium household cooking foil and had a bursting pressure of 300 torr.

By etching these diaphragms in a cruciform pattern with a 2% solution of hydrofluoric acid, the bursting pressures could be reduced to as little as 50 torr. An example of the type of output which was obtained from the schlieren system of weak incident shock waves produced in this way is shown in run 2576 of figure 3.3.2. The screen of the schlieren system was in its original form for these tests, and contained two sampling slits which allowed the light to pass through to a single photomultiplier. The upper trace shows the output from the photomultiplier as the image of the incident shock wave was convected across both of the sampling slits. The lower trace was delayed to show only the output from the second slit.

The diaphragm which was used had been etched for a period of 20 minutes giving a bursting pressure of nominally 50 torr. It can be seen from the oscillogram that it was possible to produce incident shock waves in this way which were sufficiently weak to be fully dispersed. However, there were several disadvantages with this technique.

Firstly, the diaphragms did not petal cleanly, so there was no guarantee that the wave was properly formed prior to reaching the working section. Secondly, the bursting of the diaphragms was unreliable in that the bursting pressures varied widely and premature bursting occurred. Often, pinholing occurred causing a leakage of driver gas into the channel. It will also be noted that the initial pressure in the channel had to be very high for a sufficiently weak wave to be produced. This restricted the widths of the waves and hence reduced the effective spatial resolution of the measuring system. It was concluded that an alternative technique should be sought.

One alternative technique which was investigated consisted of placing an orifice plate behind the diaphragm. The intention was that it would partially convert the unsteady expansion into a steady one, and hence reduce the strength of the incident shock wave for a given diaphragm by choking the flow.

An example of the type of output which was obtained with the schlieren system of this type of wave is shown in run 2447 of figure 3.3.2. The oscillogram shows the outputs from both of the sampling slits. The orifice plate contained a 1 cm. diameter hole, and it reduced the Mach number of the incident shock wave from 1.6 to 1.1, as shown. It can be seen that the wave profile had the appearance of that of a fully dispersed shock wave, whereas the Mach number of the shock wave indicated that it was partly dispersed. This was attributed to incomplete formation of the incident shock wave and it was concluded that this technique was unacceptable.

In view of the success which had been achieved by the author with the slotted end face technique (Ref. 3.23.) it was decided that a similar technique should be investigated for this application. A perforated plate was therefore positioned in the channel of the shock tube between the diaphragm and the working section, as shown in figure 3.3.3.

The principle of operation was that the incident shock wave reflected off the plate allowing part of the wave to pass through to emerge as spherical waves which then coalesced into a plane normal shock wave prior to their arrival at the working section. The position of the plate was arranged so that none of the wave process upstream of the plate arrived at the working section during the test time.

A range of perforated plates were manufactured. These had holes drilled at 0.5 cms. x 0.5 cms. spacings over the whole of the cross-section of the shock tube, giving a total of approximately 80 holes. The sizes of the holes were chosen to give a range of open to total area ratios from 2% to 60%. Calculations showed that a reasonable position for the plate was half way between the diaphragm and the working section. A second diaphragm station was therefore fitted at this position to enable the plates to be readily interchanged.

A series of tests were initially conducted with air as the test gas in order to determine the performance of the plates in terms of transmitted to incident shock strengths. These tests were conducted at an incident shock Mach number of nominally 1.74, and the results which were obtained are presented in figure 3.3.4. This form of presentation was chosen because it enabled data for different gases to be correlated. M_t is the transmitted shock Mach number and M_s is the incident shock Mach number.

These tests were repeated with carbon dioxide as the test gas. The incident shock Mach number in this case was nominally 1.44 and these results are also presented in figure 3.3.4. The two sets of data correlate reasonably well and it may therefore be possible to use this form of presentation to predict the behaviour of other gases. No theoretical analysis of the performance of these plates has been attempted, but it is considered that a quasi-one-dimensional approach would produce some useful information. The reflected shock region behind the plate could be regarded as being the reservoir for the steady flow through a throat formed by the plate. It would probably be necessary to include some loss factor because of the discontinuous nature of the throat. However, for the present requirements such an approach would be academic.

It can be seen from figure 3.3.4 that a transmitted shock Mach number of 1.035 could be obtained in carbon dioxide by using the smallest hole size (2%). It was concluded that, if the technique could produce well formed transmitted shock waves, it would be well suited to this application, and the range of hole sizes would be adequate.

A series of tests were conducted to assess the degree of formation of the transmitted shock wave using the time resolved schlieren system. Typical examples of the outputs which were obtained are shown in figure 3.3.5. These results were obtained with the schlieren system in its improved configuration (see section 3.1). The upper traces are outputs from the main photomultiplier placed behind the single sampling slit. The lower traces are the outputs from the second photomultiplier which was monitoring the light probes.

It can be seen that well formed waves were produced with this technique, and, at much lower channel pressures than hitherto possible. The transition from a fully dispersed to a partly dispersed shock wave can be clearly seen in these oscillograms. The diffusion resisted shock front appears from its apparent thickness to be curved and therefore consistent with the results of the strong incident shock waves. It was therefore concluded that this technique would be suitable for the production of well formed, weak, normal shock waves.

This technique also offered other advantages over the other techniques. The strength of the transmitted shock wave was determined primarily by the area ratio of the plate and was therefore insensitive to changes in the diaphragm bursting pressure. In addition, the incident shock wave provided a sufficiently strong and reliable triggering pulse for the electronics which had not been available with other techniques.

We shall revert to the use of the term incident shock wave to mean the transmitted shock wave in the following parts of this section, because the original incident shock wave ceases to have any significance in that context.

3.3.3. Method Of Analysis.

The manner in which the wave profiles were analysed will be discussed prior to continuing with a description of some of the more rigorous studies which have been made.

The overall shock ratios and the ratios across the diffusion resisted shock front were calculated. These are presented graphically in figures 3.3.6 to 3.3.8, where 'frozen' refers to the ratios across the diffusion resisted shock front and 'equilibrium' refers to the overall ratios. The regime of the weak incident shock wave was arbitrarily chosen to be waves having Mach numbers up to 1.4. For this reason, the data has been terminated at that Mach number.

It can be seen from figure 3.3.6 that the frozen and equilibrium translational temperatures cross at an incident shock Mach number of approximately 1.2. Consequently, the total change in the equilibrium ratio of the specific heats is only 0.15% at an incident shock Mach number of 1.4. It was concluded that for the purposes of analysis it would be sufficiently accurate to take a mean value of the equilibrium ratio of the specific heats through the relaxation resisted parts of the shock waves. The relaxation time of carbon dioxide only changes by 5 % through the relaxation region of a shock wave of Mach number 1.4 and can also be taken as constant for these purposes. These approximations allow the fully dispersed and the partly dispersed solutions of part 2 to be used directly for the computation of theoretical profiles for comparison with experimentally measured profiles.

Thus, as the relaxation time can be taken as a constant through the wave, a closed form solution of the equations can be used. The widths of the shock waves will be proportional to the relaxation times evaluated at the mean translational temperatures of the relaxation regions.

A difficulty was encountered in the determination of the strength of the shock waves. At the higher Mach numbers, approaching 1.4, there was no difficulty. In that case, the strength of the waves could be determined from their velocity by direct measurement, either at the timing station or at the light probes. However, as the strength of the wave decreases, the velocity asymptotes towards the speed of sound of the gas and cannot be used for accurate determination of wave strengths.

An alternative means of determining the wave strength was available in the case of the fully dispersed shock waves. The schlieren system had been calibrated, and, as the profiles were continuous, the outputs could be integrated to give the total density change across the waves and hence the wave strengths.

This left a range of wave strengths just above the transition which were not continuous and yet not strong enough for the velocities to be used with any accuracy for the determination of their strengths. Additional difficulties were involved, as can be seen from figure 3.3.8. There is considerable doubt as to what the schlieren system will measure. On the one hand it could measure the change in density through the relaxation region alone. In this case the strength of the wave could be determined from a comparison of the integral of the output from the schlieren system with the theoretical density change across the relaxation region. However, even if it could be guaranteed that this was the case, the change in the density across the relaxation region is insensitive to the wave strength over this range.

As was mentioned in the introduction, the wave profiles were analysed by plotting them in a density gradient/density plane, and comparing them with theoretical profiles in that plane. Figure 3.3.9 shows a typical fully dispersed shock wave result in this plane. The output from the schlieren system was calibrated and numerically integrated with a computer to give result as shown. It can be seen that both the density gradient and the density have been non-dimensionalised by the free stream density, ρ_1 . In addition, the density gradient has been non-dimensionalised by the free stream velocity, u . Thus, the only difference between the experimental and theoretical profile was the relaxation time, τ .

The total density change across the shock wave was used to determine its strength and hence the Mach number using a mean value for the equilibrium ratio of the specific heats. The theoretical profile was then calculated from the basic solution given in part 2. The ratio of the theoretical to the experimental profile was taken at a number of points in the shock wave for fixed values of density, the result of which was the relaxation time at each point through the wave. The mean value of the relaxation time was then calculated and the experimental profile re-non-dimensionalised to compare the theoretical and experimental profiles in detail.

The procedure in the case of the partly dispersed shock waves was similar except that the strength of the waves was determined from their velocities.

3.3.4. Experimental Results.

Figure 3.3.10 shows a range of typical schlieren outputs of the waves produced by this technique. These results were obtained with the schlieren system in its final configuration. That is, with a spatial resolution of better than 0.02 cms. and a temporal resolution of better than 0.1 μ s. . The vacuum system had also been improved and the new working section incorporated.

The upper traces of the oscillograms are the outputs from the main photomultiplier which was monitoring the sampling slit. The lower traces are the outputs from the second photomultiplier, which was monitoring the light probes, and were used for the determination of the velocities of the shock waves.

It can be seen that the profiles remained continuous well above the wave strength required for the waves to become partly dispersed. This confirmed that a broadening of the diffusion resisted shock front had occurred as had been indicated in the preliminary tests. Unfortunately, this curvature of the waves masked the details of the transition.

The curvature of the waves fortuitously aided the analysis of the wave profiles, because it was clear that the schlieren system had responded to the total change in the density gradients across the weak partly dispersed shock waves. This enabled the integrals of the schlieren outputs to be taken as the total density change across the waves, and hence the strengths of the waves to be readily calculated.

Results such as these were analysed in the manner described above. The relaxation times obtained in this way for carbon dioxide are presented in figure 3.3.11.

Figure 3.3.11 shows that the relaxation times for carbon dioxide from these experiments were in good agreement with those obtained by Simpson (Ref. 3.12.), and it can be seen from the data presented by Simpson that the results were also in close agreement with those from ultrasonic measurements.

The results for the weak incident shock wave show that there was greater scatter in the fully dispersed shock wave data (at the lower temperature end of the results) than in the partly dispersed shock wave data. This was attributed to the high degree of sensitivity of the profiles of the fully dispersed shock waves to the wave strengths.

Having obtained the relaxation times and non-dimensionalised the experimental data accordingly, they were compared directly with theoretical profiles on plots of density gradient versus true spatial position through the wave. Figure 3.3.12 shows two typical fully dispersed shock wave profiles. It can be seen that the theoretical and experimental profiles agreed very closely except near to the maximum density gradient. The reduction of the experimental output near to the maximum density gradient could be accounted for by the curvature of the shock wave, as discussed in the preceding section. This effect was more pronounced in the case of the partly dispersed shock wave profiles which are shown in figures 3.3.13 and 3.3.14. It can be seen that the agreement was again very good except near to the front of the shock wave.

The apparent thicknesses of the diffusion resisted shock fronts of these partly dispersed shock waves were of the order of 0.1 cm. (1 unit in the non-dimensionalised variable, z , was approximately 0.2 cm.). This confirmed that these weak incident shock waves had been broadened by shock curvature, as had the strong incident shock waves of the preceding section. This demonstrated that the formation of the transmitted shock waves studied in this case was as good as that of the original strong incident shock waves.

3.3.5. Discussion Of Results And Concluding Remarks.

The relaxation times determined in these experiments were in good agreement with those determined elsewhere. However, it was noted that the scatter in the relaxation times was greater for the fully dispersed results than for the partly dispersed results. The measured density gradient profiles were in good agreement with the theoretical profiles except for the transition from fully dispersed to partly dispersed shock waves.

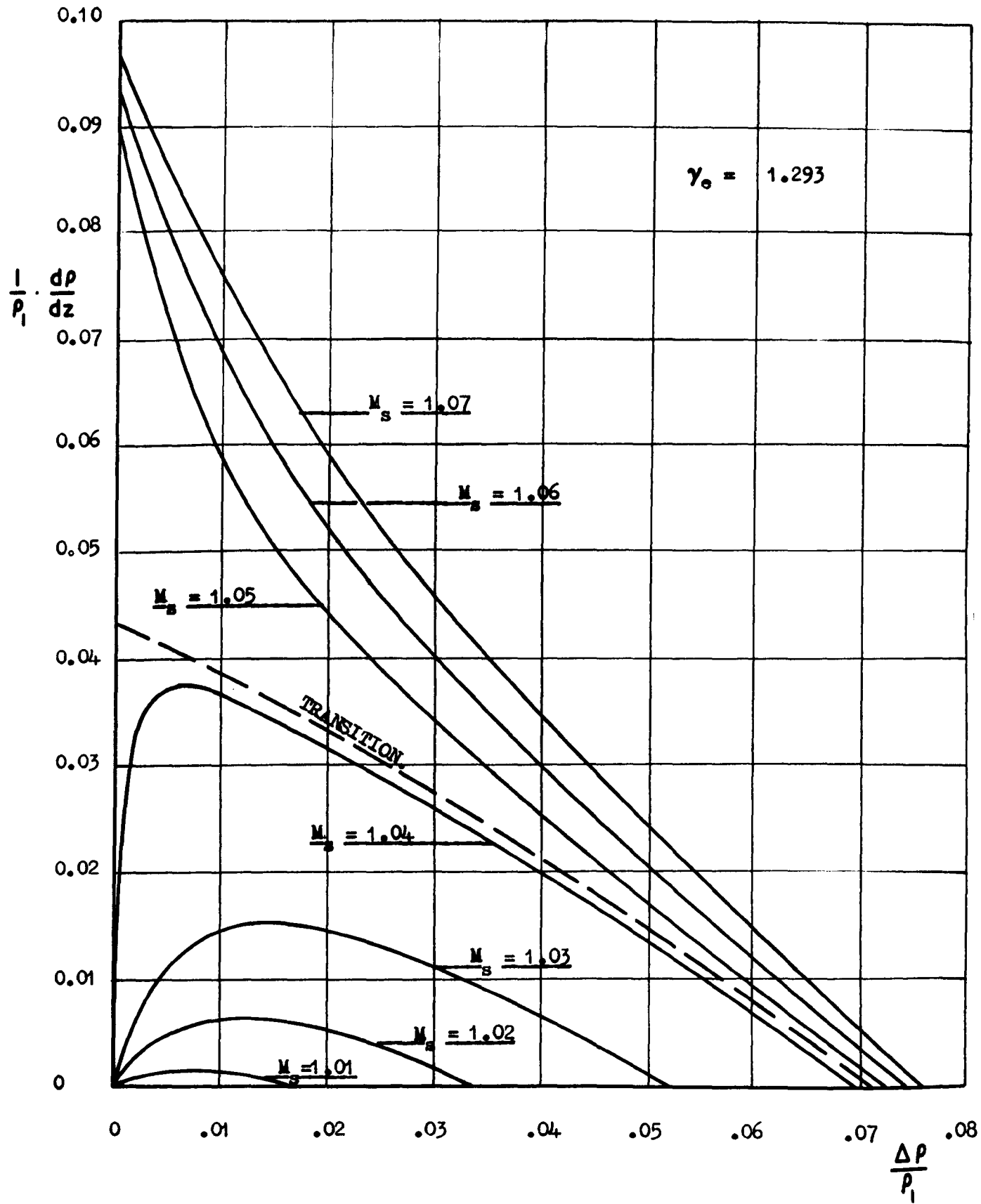
It was unfortunate that the actual transition was obscured by the curvature of the shock wave. However, the theoretical results of part 2 indicated that, in any event, the schlieren system would not have monitored the transition.

It was originally anticipated that the fully dispersed shock waves would be more sensitive to vibrational relaxation than the essentially exponential relaxation regions of the strong incident shock waves. This has been confirmed to a large extent, but some of the benefits have been offset by the sensitivity of the wave profile to its strength, as can be seen from figure 3.3.1. The larger scatter in the fully dispersed shock wave results is attributed to this fact.

Consider the profiles of the weak partly dispersed shock waves shown in figure 3.3.5. It can be seen that the slope of the profiles, and hence the relaxation time, is insensitive to changes in the wave strength. In addition, the translational temperature remains almost constant through the relaxation region. It is concluded from this that the weak partly dispersed shock wave would be more suitable for the study of vibrational behaviour, and the scatter in the relaxation times would be less than with the fully dispersed shock wave.

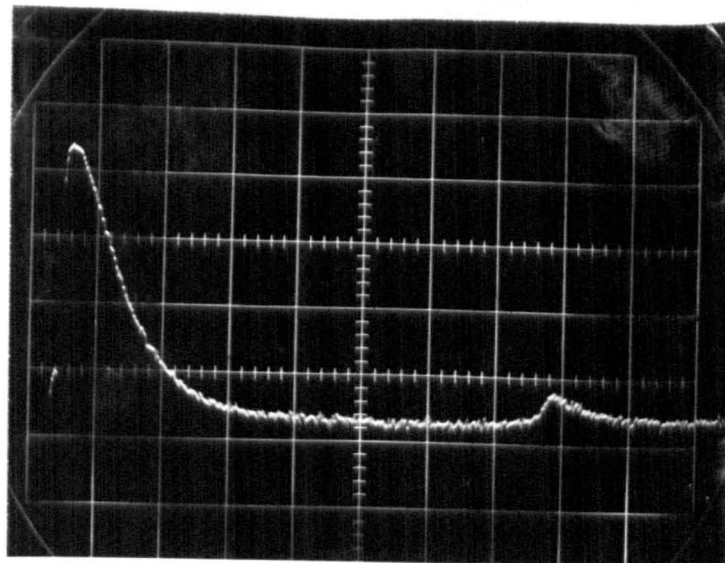
These studies were as accurate as could be expected in the small (2" diam.) shock tube which was used. They have confirmed the suitability of the schlieren system for this application and have given no cause to doubt the calibration of the schlieren system. The results which have been obtained provided a useful guide in the development of techniques for the propagation of weak normal shock waves through regions of elevated temperature, which will be discussed in the following section.

Note: $\Delta\rho$ is density change through the relaxation resisted parts of the wave, that is, throughout fully dispersed shock waves and through the relaxation regions of partly dispersed shock waves.



THEORETICAL DENSITY GRADIENT PROFILES THROUGH
RELAXATION RESISTED PARTS OF SHOCK WAVES.

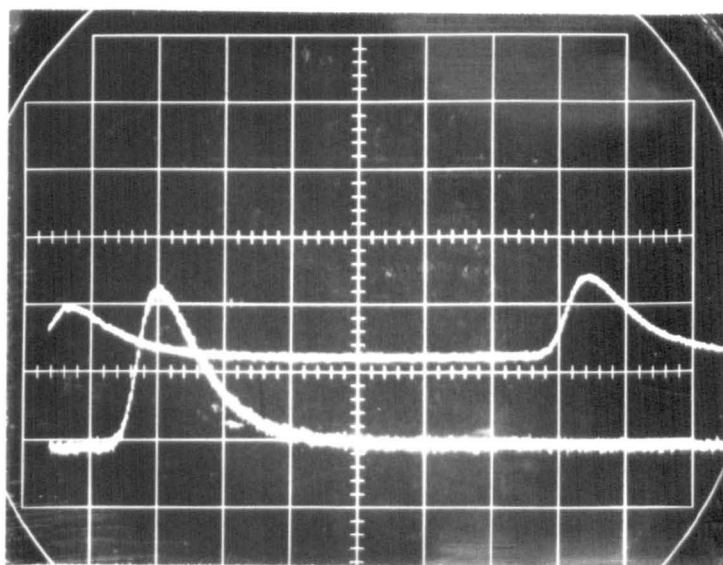
FIGURE: 3.3.1.



2.5 mv / cm.

10 μ s/cm.

Run 2447 $M_s = 1.10$ $P_i = 500$ torr.

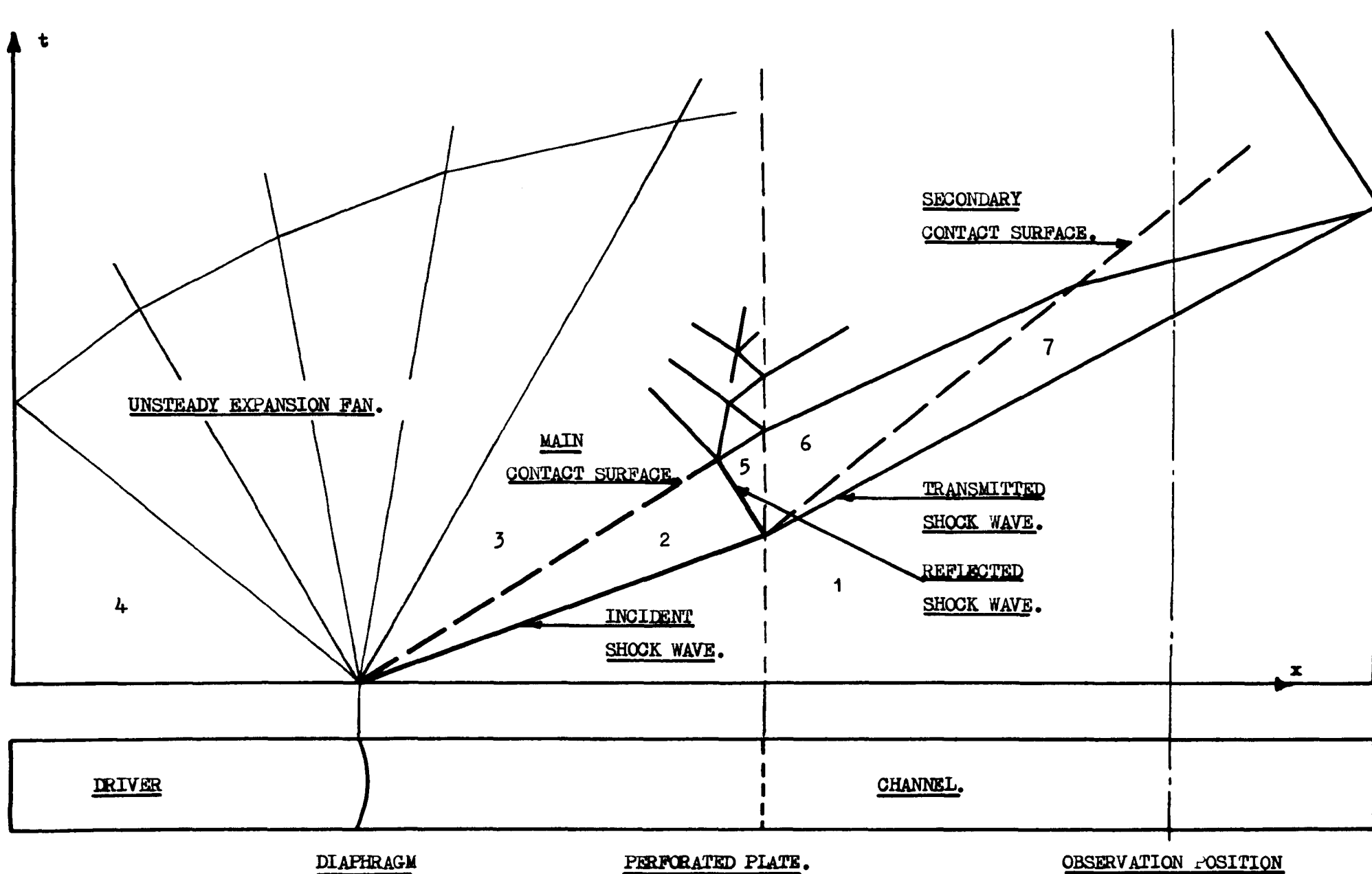


1 mv / cm.

0.5 mv / cm.

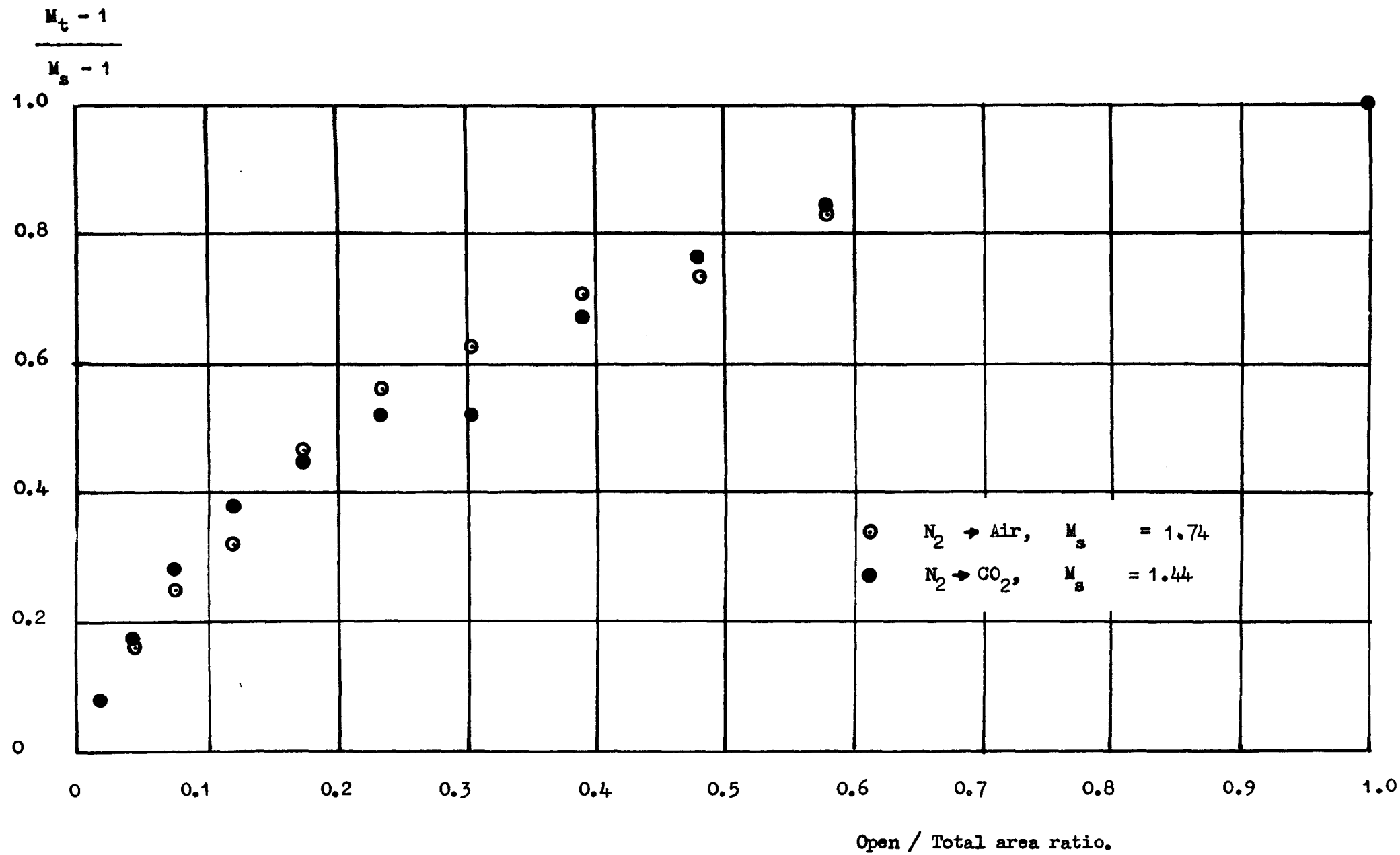
10 μ s/cm.

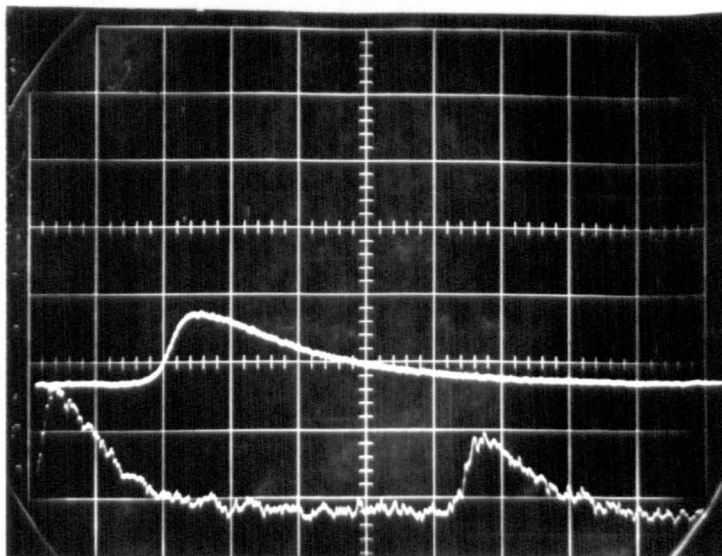
Run 2576 $M_s = 1.04$ $P_i = 1400$ torr.



CORRELATION OF PERFORATED PLATE PERFORMANCE.

FIGURE: 3.3.4



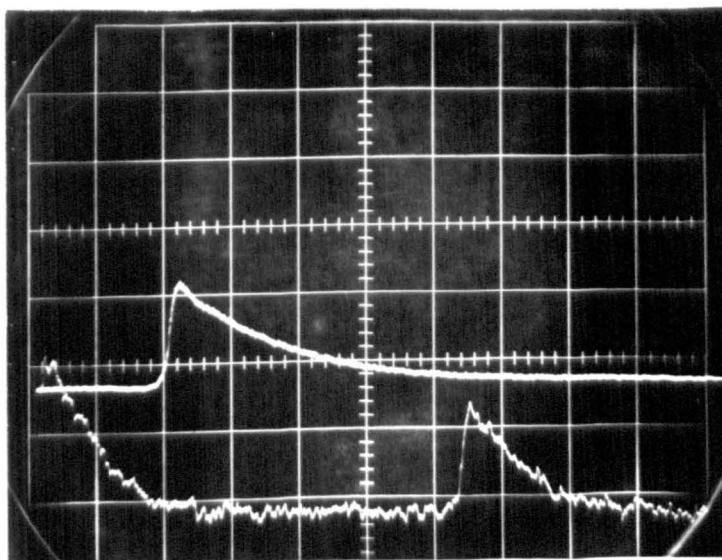


Run 3266

$M_t = 1.035$

$P_i = 300$ torr.

Upper traces $10 \mu\text{s/cm}$.

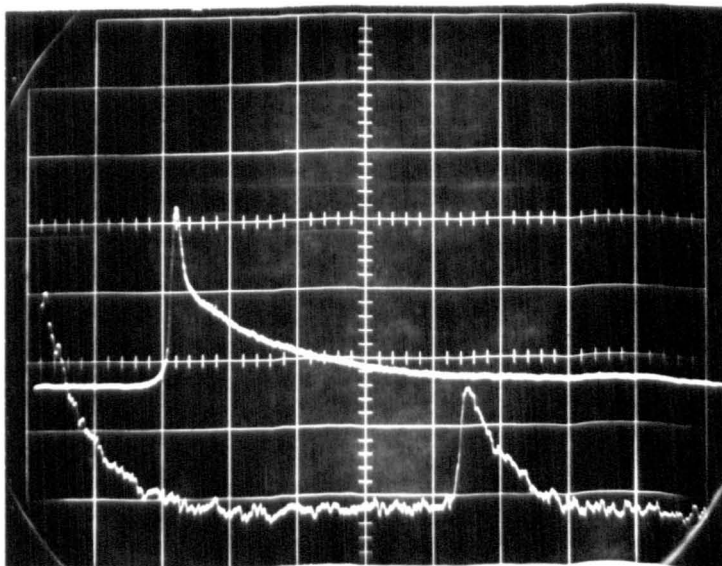


Run 3268

$M_t = 1.045$

$P_i = 100$ torr.

Lower traces $20 \mu\text{s/cm}$.



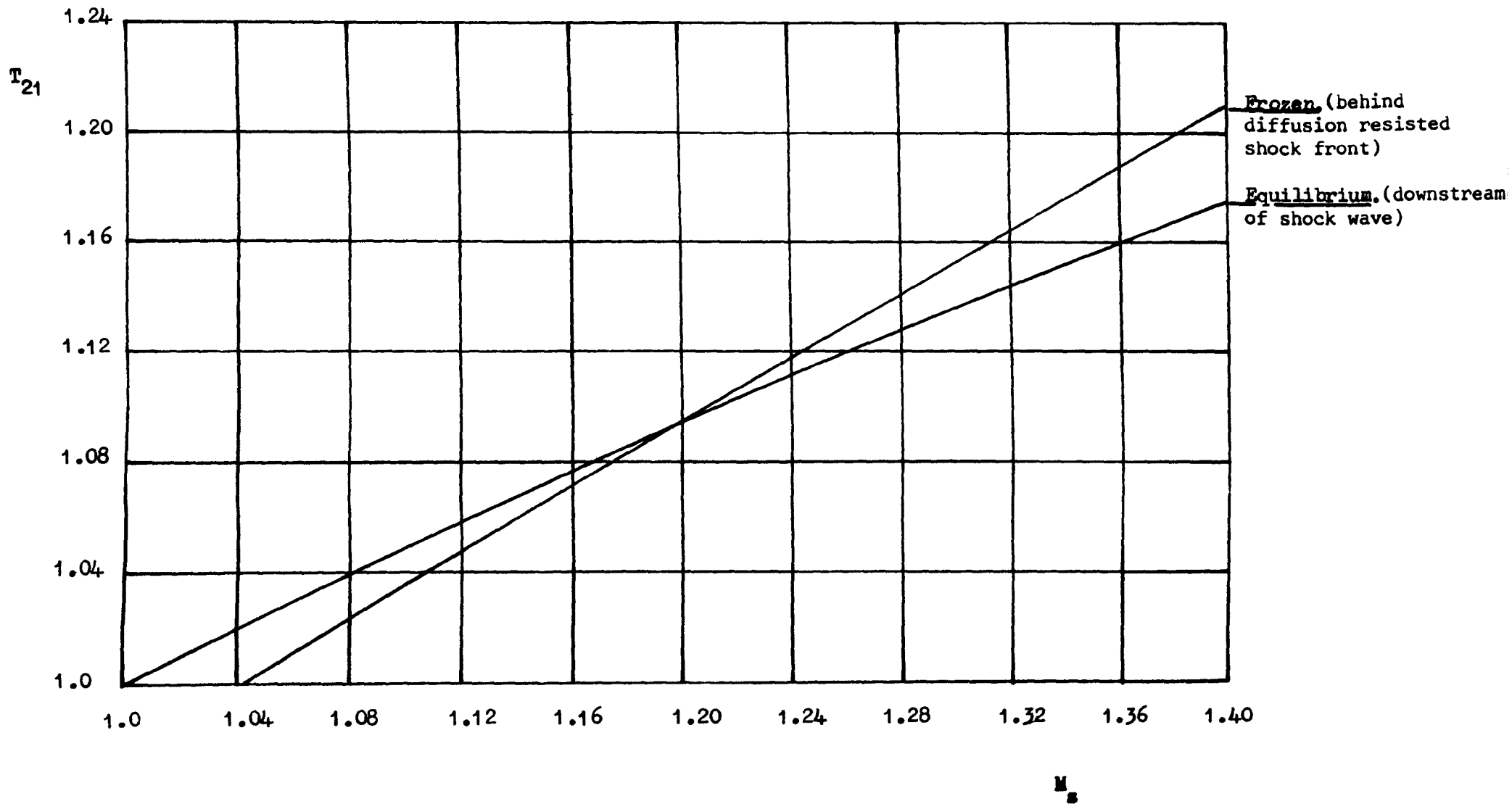
Run 3267

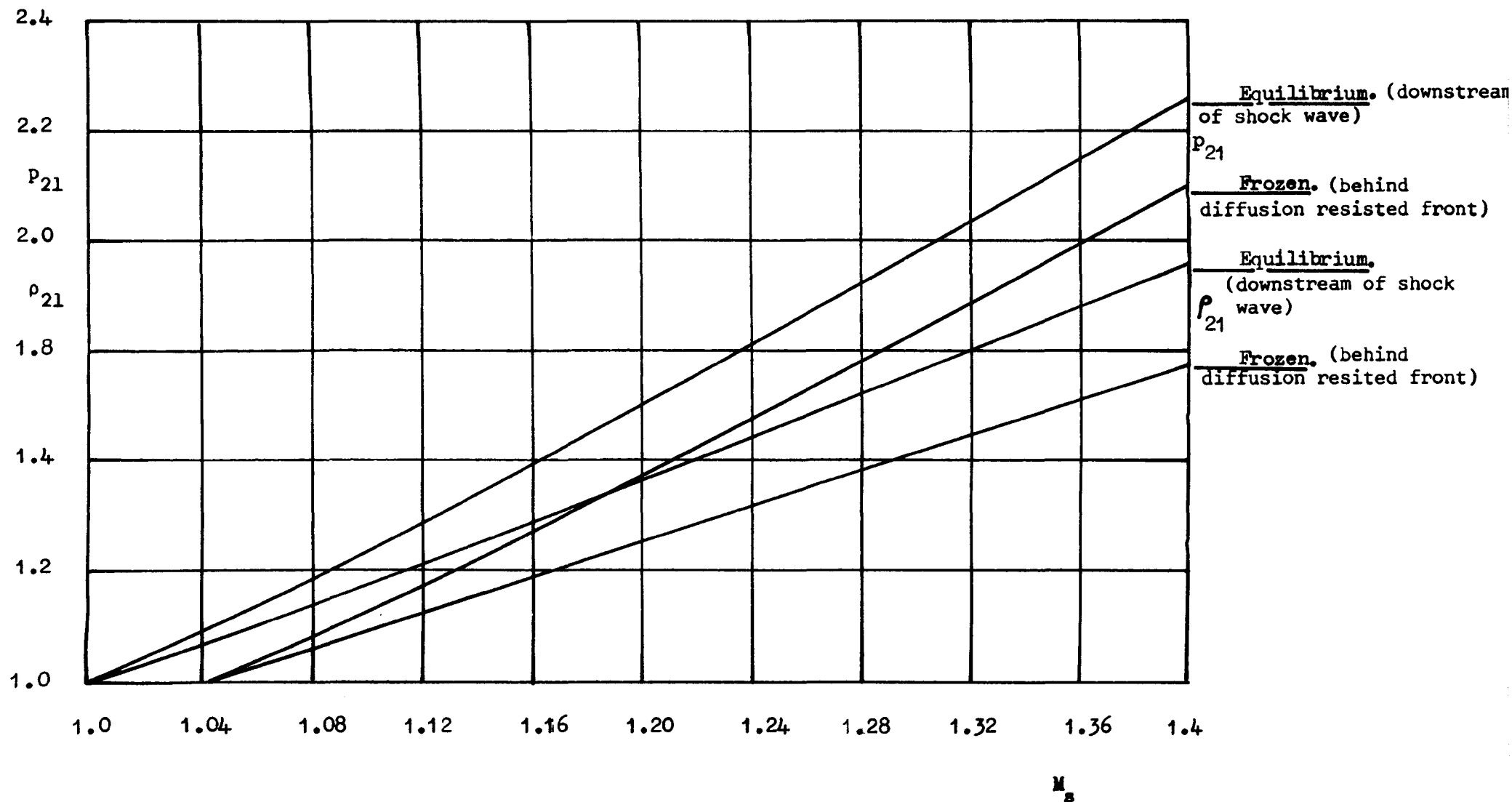
$M_t = 1.055$

$P_i = 75$ torr.

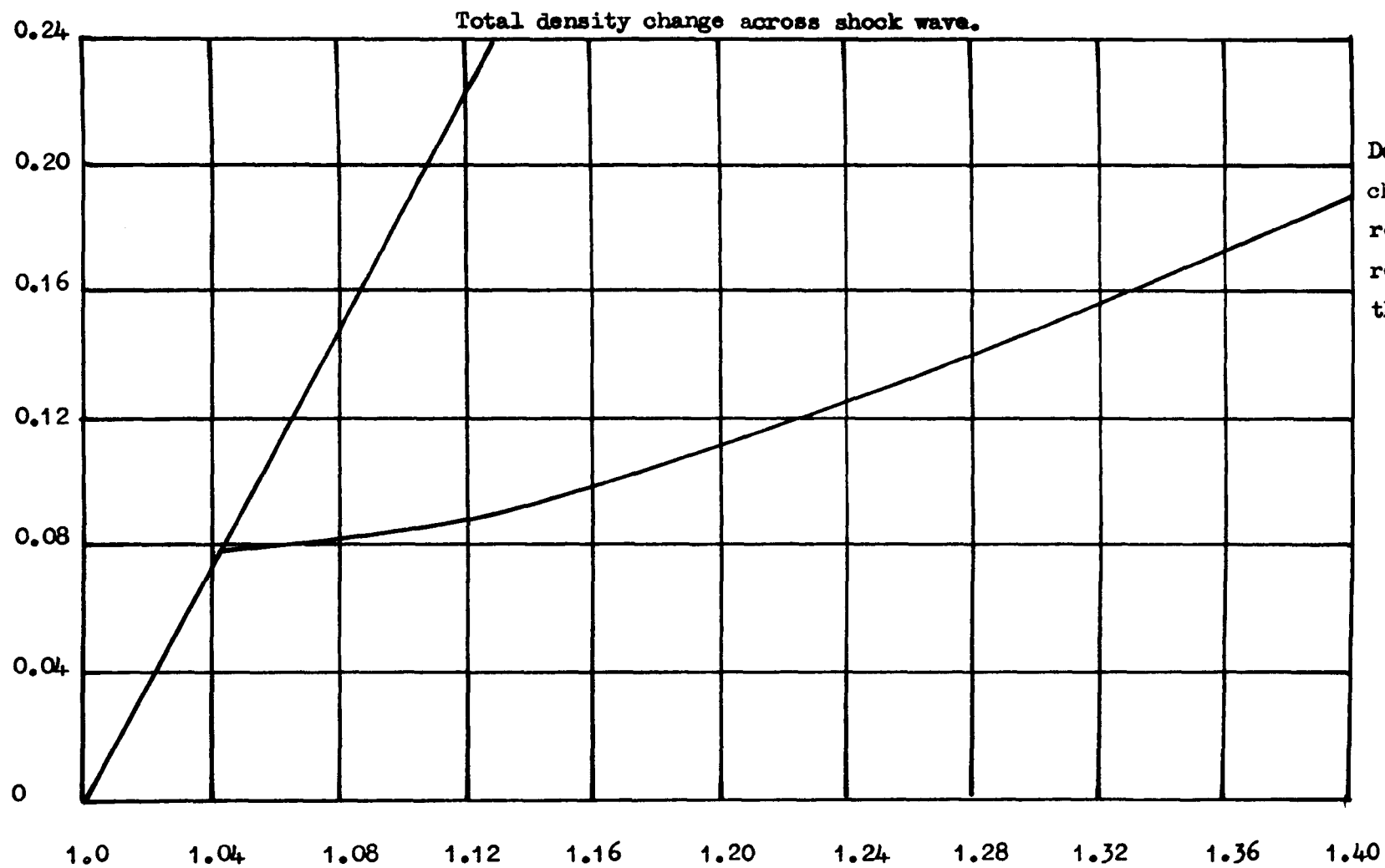
TEMPERATURE RATIOS ACROSS WEAK INCIDENT SHOCK WAVES.

FIGURE: 3.3.6.





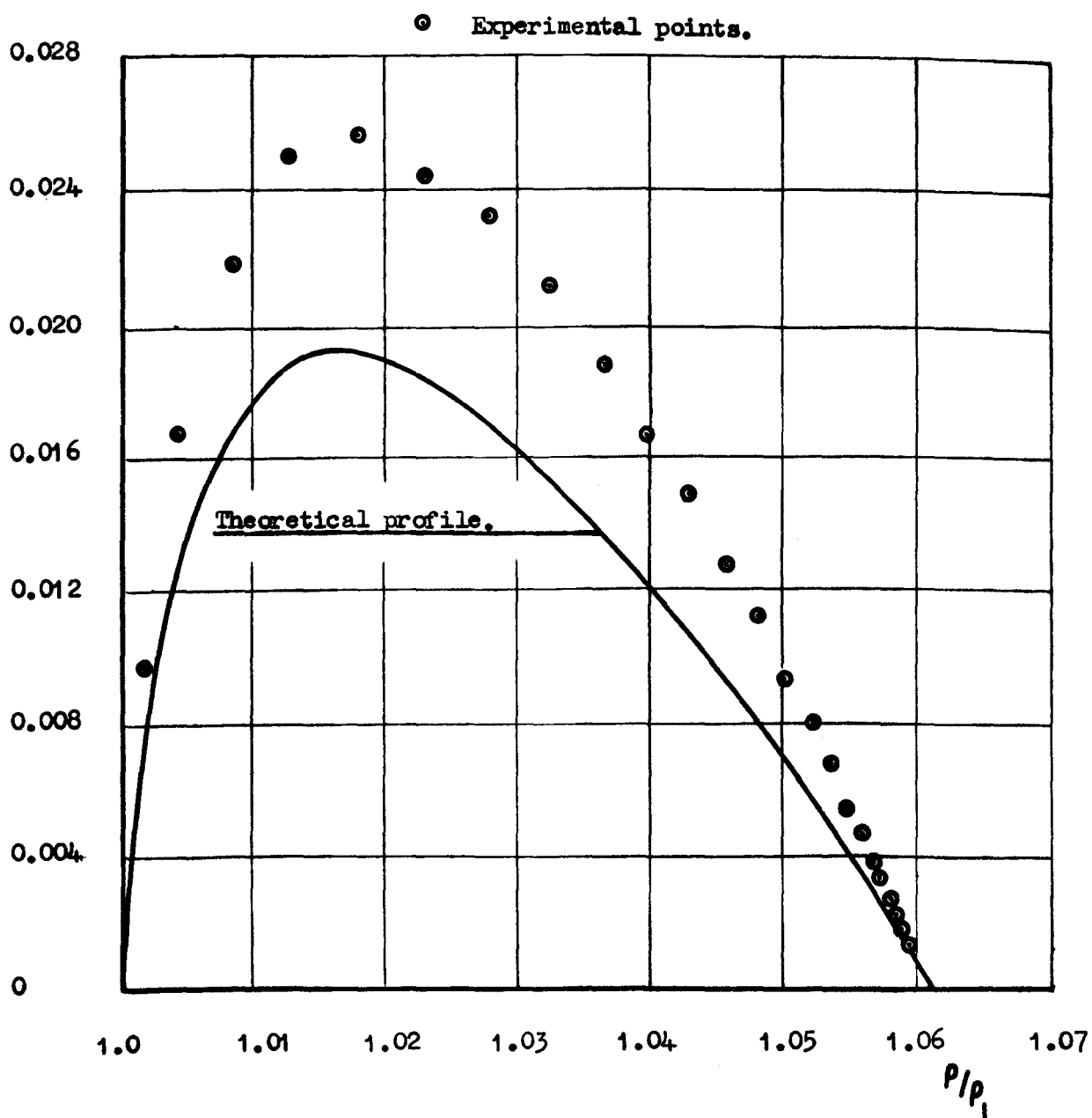
$\Delta P/P_1$

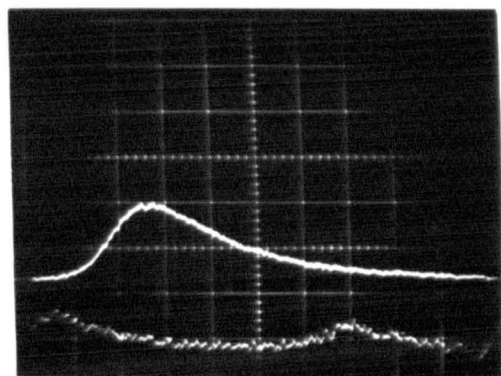


M_s

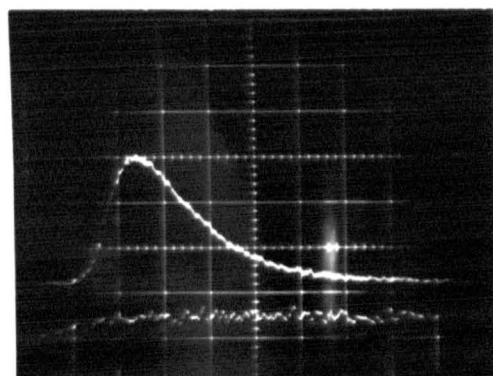
$$\frac{1}{P_1} \frac{dP}{dz} \frac{10^{-5}}{\tau} \quad (\text{From experiment.})$$

$$\frac{1}{P_1} \frac{dP}{dz} \quad (\text{From theory.})$$

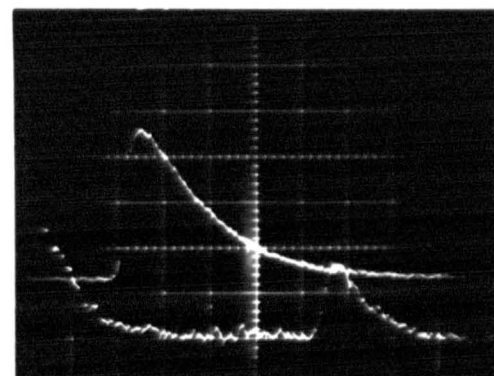




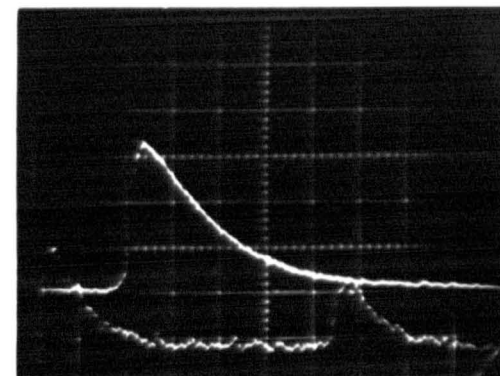
RUN 3262
 $M_s = 1.035$ $P_1 = 575$ torr.
 TIME BASES: UPPER $10\mu s/cm$.
 LOWER $20\mu s/cm$.



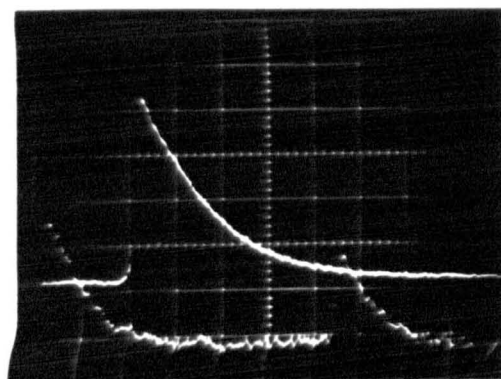
RUN 3256
 $M_s = 1.0335$ $P_1 = 750$ torr.
 TIME BASES: UPPER $10\mu s/cm$.
 LOWER $20\mu s/cm$.



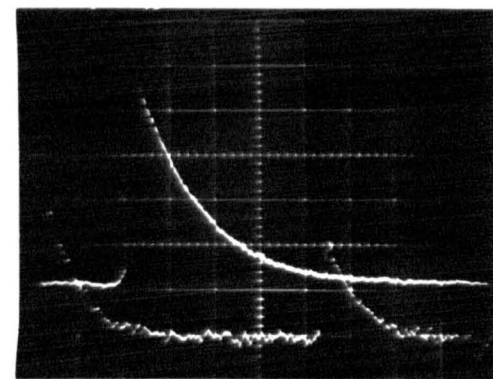
RUN 3260
 $M_s = 1.041$ $P_1 = 600$ torr.
 TIME BASES: UPPER $10\mu s/cm$.
 LOWER $20\mu s/cm$.



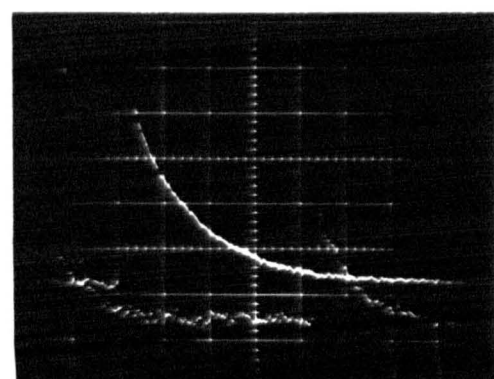
RUN 3264
 $M_s = 1.044$ $P_1 = 560$ torr.
 TIME BASES: UPPER $10\mu s/cm$.
 LOWER $20\mu s/cm$.



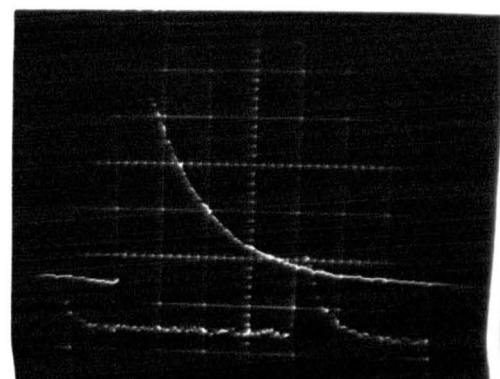
RUN 3263
 $M_s = 1.044$ $P_1 = 575$ torr.
 TIME BASES: UPPER $10\mu s/cm$.
 LOWER $20\mu s/cm$.



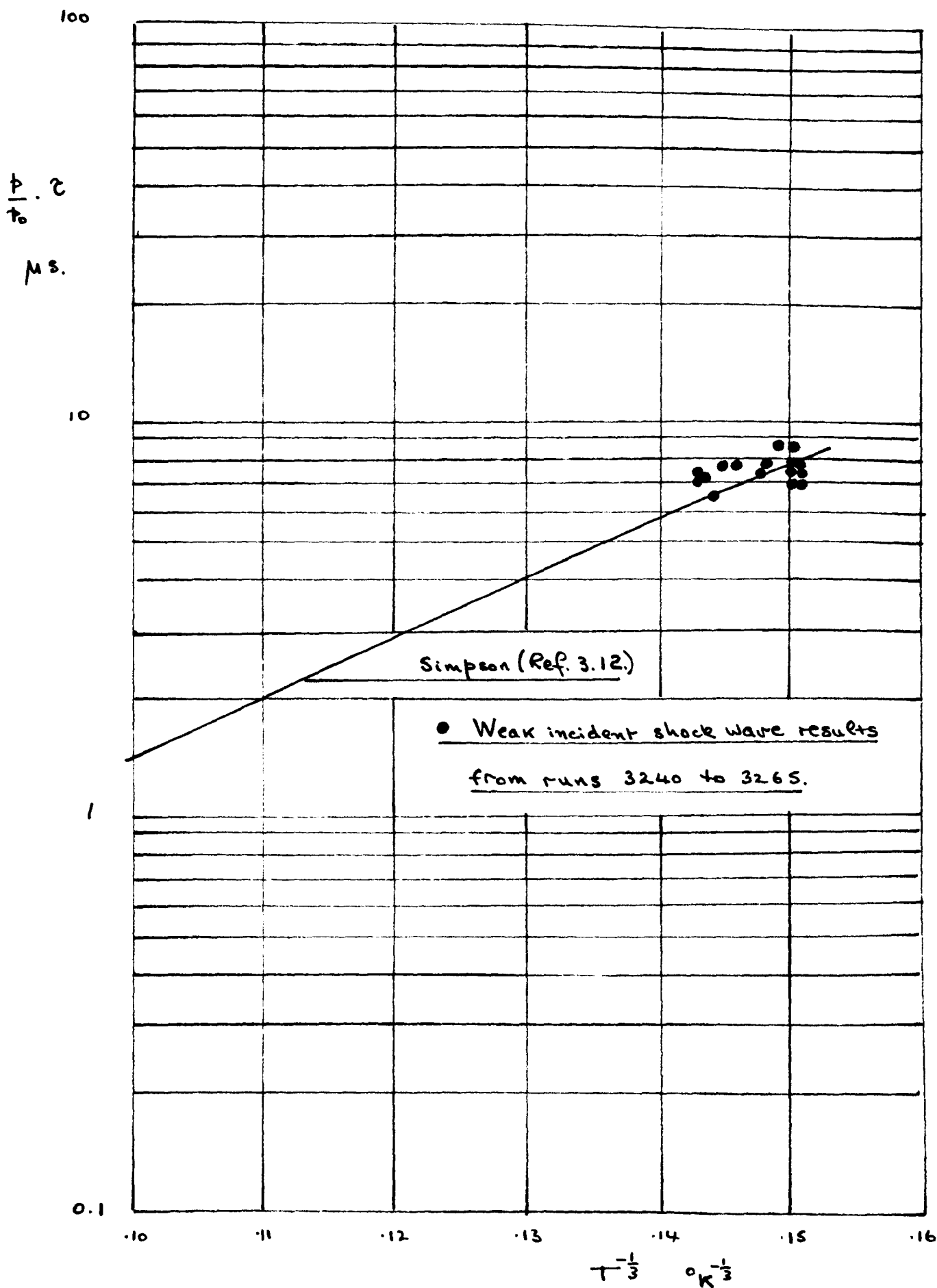
RUN 3265
 $M_s = 1.050$ $P_1 = 560$ torr.
 TIME BASES: UPPER $10\mu s/cm$.
 LOWER $20\mu s/cm$.



RUN 3257
 $M_s = 1.072$ $P_1 = 500$ torr.
 TIME BASES: UPPER $10\mu s/cm$.
 LOWER $20\mu s/cm$.



RUN 3248
 $M_s = 1.138$ $P_1 = 500$ torr.
 TIME BASES: UPPER $5\mu s/cm$.
 LOWER $20\mu s/cm$.

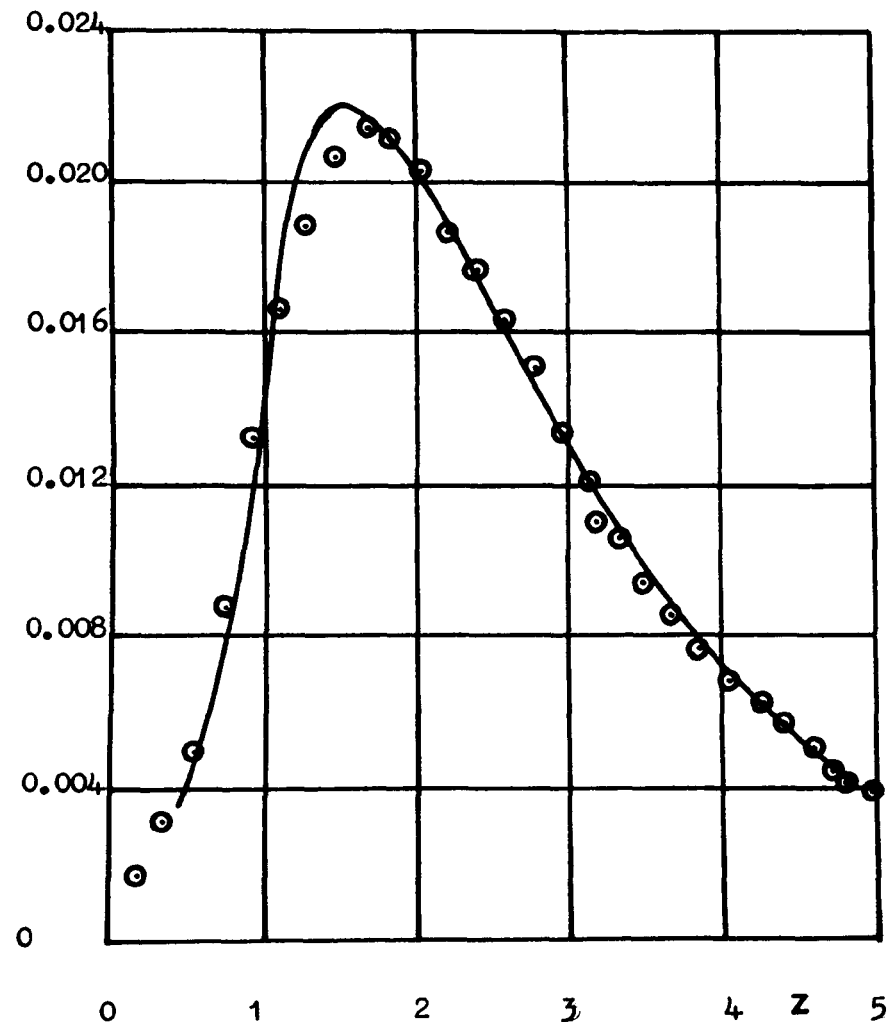


RELAXATION DATA FROM WEAK
INCIDENT SHOCK WAVES.

FIGURE 3.3.11.

$$\frac{1}{\rho} \frac{d\rho}{dz}$$

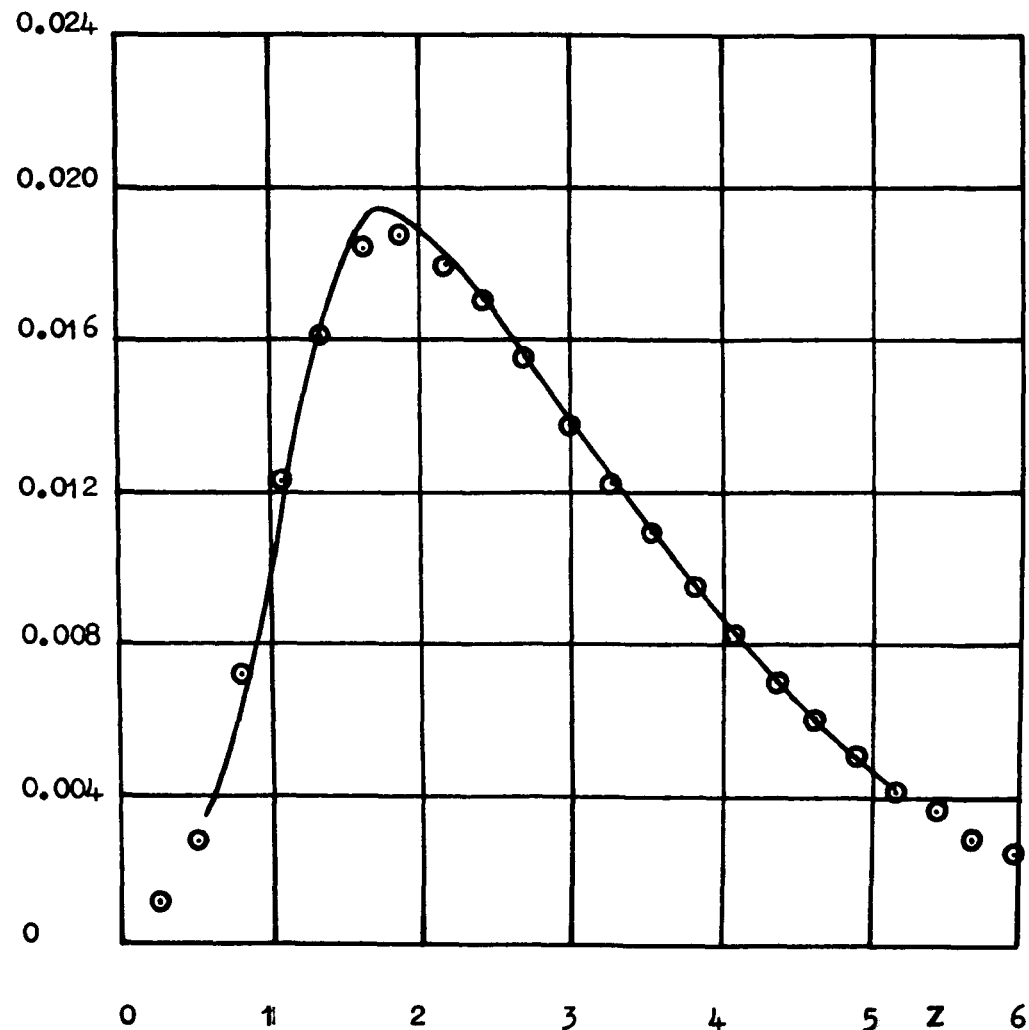
Run 3262 $M_s = 1.035$
 $p_1 = 575$ Torr.



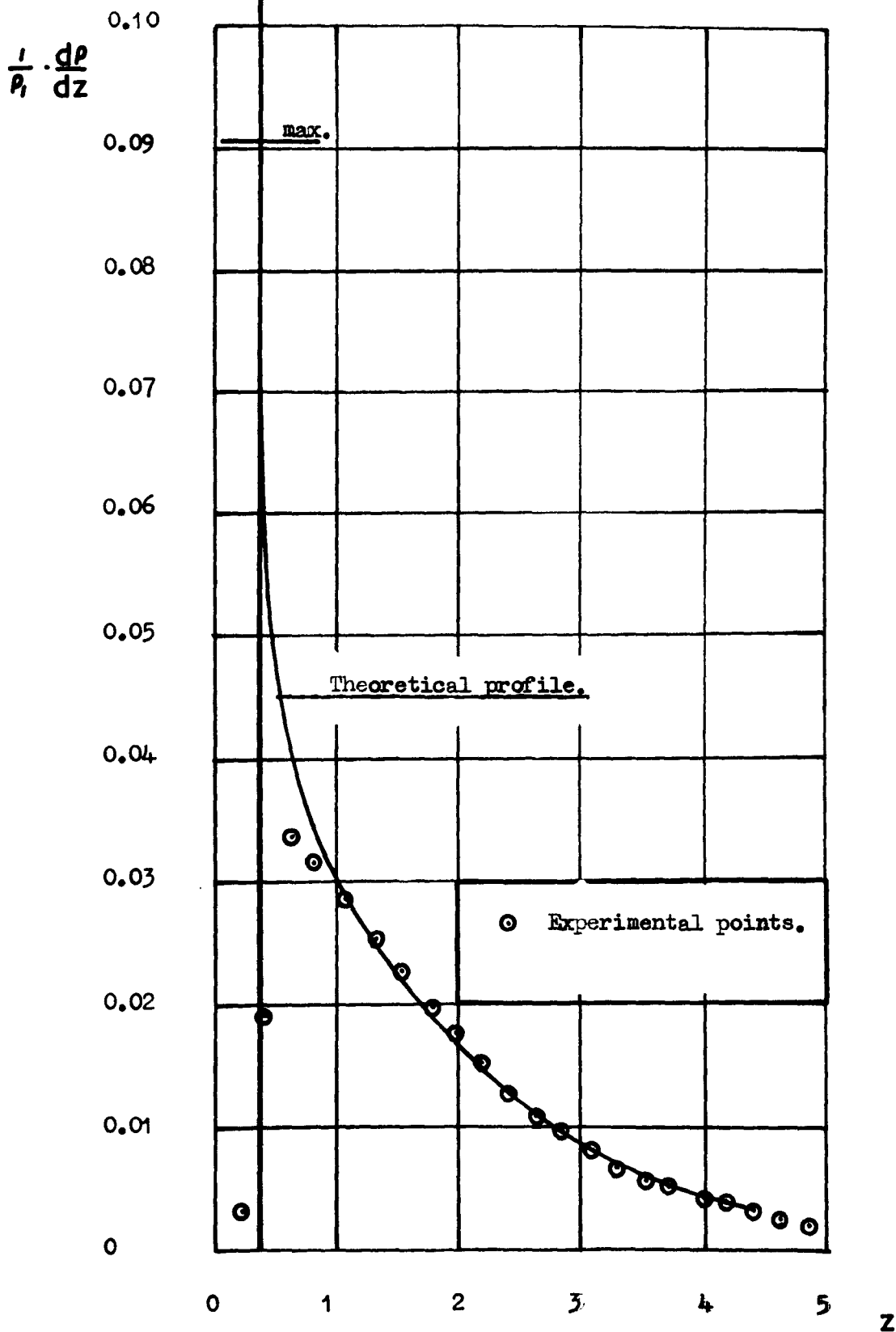
SOLID LINES ARE THEORETICAL PROFILES.

$$\frac{1}{\rho} \frac{d\rho}{dz}$$

Run 3256 $M_s = 1.0335$
 $p_1 = 750$ Torr.



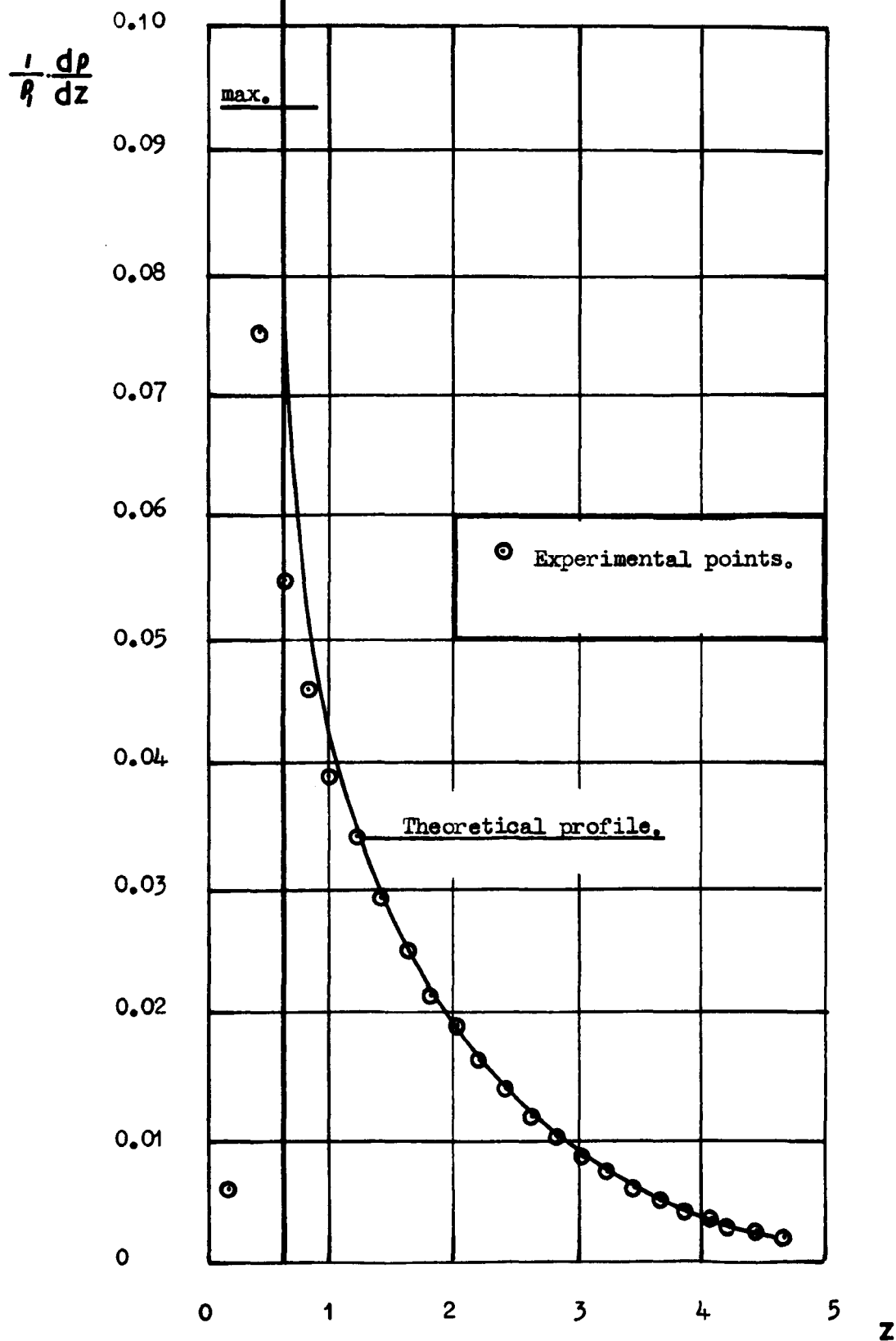
⊙ EXPERIMENTAL POINTS.



Run 3260 $M_s = 1.042$
 $p_1 = 600$ Torr.

COMPARISON OF EXPERIMENTAL PROFILE WITH THEORETICAL PROFILE FOR A PARTLY DISPERSED SHOCK WAVE.

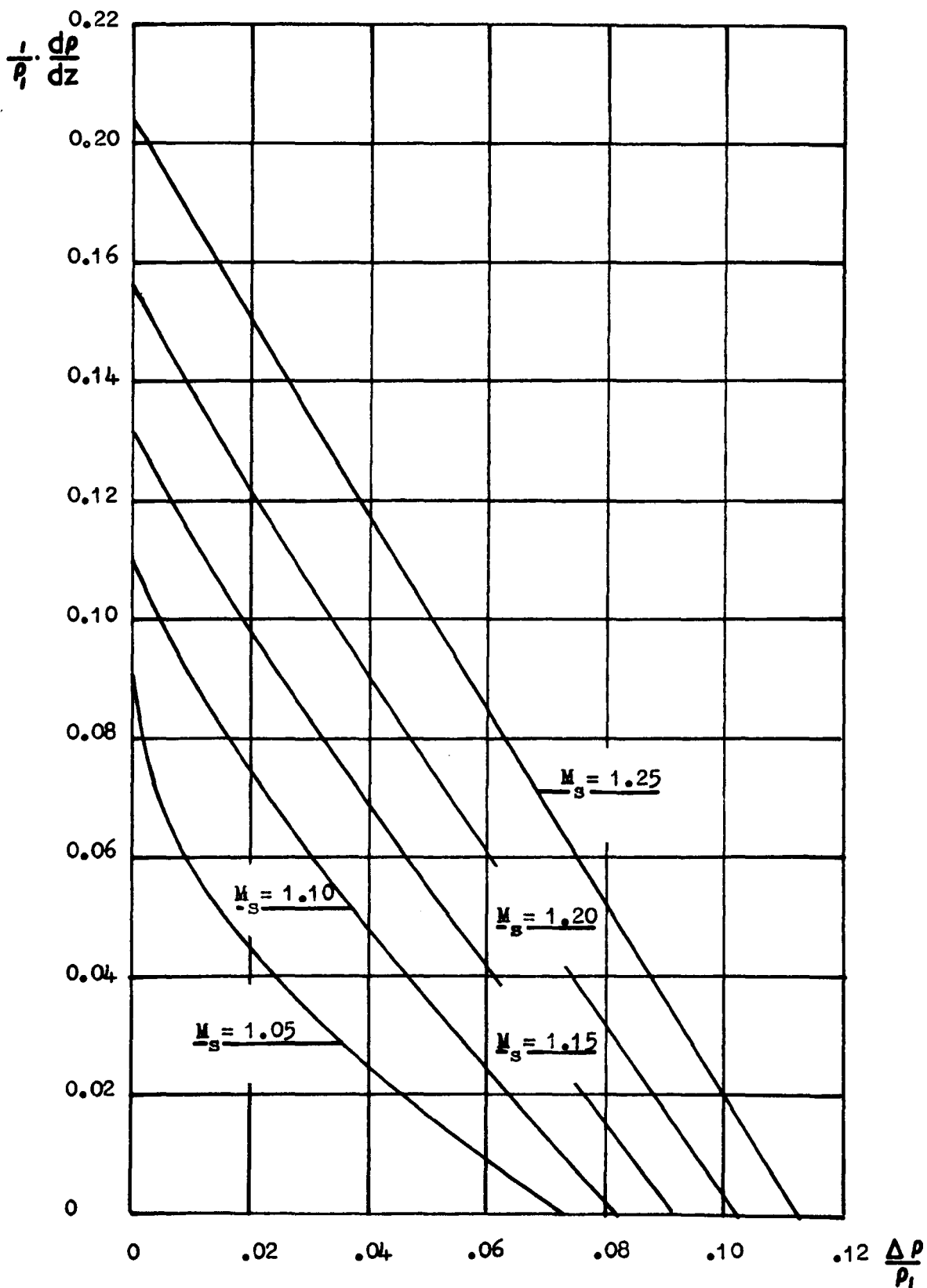
FIGURE: 3.3.13.



Run 3265 $M_s = 1.05$
 $P_1 = 560$ Torr.

COMPARISON OF EXPERIMENTAL PROFILE WITH THEORETICAL PROFILE FOR A PARTLY DISPERSED SHOCK WAVE.

FIGURE: 3.3.14.



THEORETICAL DENSITY GRADIENT PROFILES THROUGH THE
RELAXATION REGIONS OF WEAK INCIDENT SHOCK WAVES.

FIGURE: 3.3.15.

3.4. THE PROPAGATION OF WEAK NORMAL SHOCK WAVES
THROUGH GASES AT ELEVATED TEMPERATURES.

SUMMARY

Further developments and refinements have been made to a technique which had been developed previously for the propagation of weak normal shock waves through gases at elevated temperatures. This technique involved the use of a porous end face in the channel of the shock tube. The design of the end face was improved and a better understanding was obtained of the weak shock wave formation process and of the reflected shock region into which the weak shock waves were propagated.

Fully dispersed shock waves were produced in carbon dioxide with this technique. These were studied with the time resolved schlieren system described in section 3.1 herein. The relaxation time for carbon dioxide for temperatures from 300°K to 600°K was found to be in reasonable agreement with measurements made elsewhere. The maximum temperature limitation imposed by the bifurcation of the reflected shock wave was investigated, and it was concluded that no further improvements could be made in the present shock tube.

A comparison of experimentally measured density gradient profiles with theoretical profiles indicated reasonable agreement. A comparison of these with the results of section 3.2 indicated that the weak shock wave formation was almost as good as that of the weak incident shock wave. It was discovered that the degree of formation had been fortuitous, and that a shock wave in a monatomic gas such as argon would not have formed into a stable form in the time available.

It was concluded that the technique had been successful for the evaluation of relaxation times in a near equilibrium situation, but that the profiles could not be used for any detailed investigation of vibrational behaviour.

3.4.1. Introduction.

It was evident from the results of the preceding section that the weak normal shock wave offered certain advantages over the strong incident shock wave for the study of vibrational relaxation in gases.

The main advantage was that the departure from equilibrium of the vibrational mode(s) which occurred in the weak shock wave was substantially less than that which occurred in the strong partly dispersed shock wave. The behaviour of the vibrational mode(s) within the weak normal shock wave should therefore be much closer to the 'Heat Bath' behaviour predicted theoretically by Landau and Teller (Ref.3.40.) than in the strong partly dispersed shock wave.

The change in the relaxation time through the weak shock wave and its effect on the shape of the profile could be neglected; the relaxation time being determinable from the overall width of the shock wave. The fully dispersed shock wave had the particular advantage that its profile was inherently more sensitive to detailed relaxation phenomena than the essentially exponential behaviour of the relaxation region of the partly dispersed shock wave.

It was therefore concluded that the development of a technique for the propagation of weak normal shock waves through gases at elevated temperatures should be investigated. The weak incident shock wave of the preceding section was an example of a possible means of achieving this, in that the initial channel gas could have been heated prior to the firing of the shock tube.

However, to achieve the required degree of heating with any certainty and reliability would be difficult and prohibitively expensive. Therefore, the use of the weak incident shock wave must be restricted to the evaluation of room temperature data,

The shock heating of the test gas in the channel of a shock tube is in itself an efficient method of heating the gas. An alternative to the steady state heating of the test gas would therefore be to shock heat the gas and then to pass the weak normal shock wave through the region of heated gas. This method of heating is essentially transient and the propagation of the two shock waves would have to be closely coordinated. There are two regions of the shock heated gas in a shock tube which are suitable for this purpose. These are the regions behind the incident and reflected shock waves. Such techniques had been developed previously by the author, and this section is primarily concerned with the developments which have been made to one these techniques.

3.4.2. Shock Heating And Propagation Techniques.

An attempt had been made by the author to make use of the region behind the incident shock wave with what has come to be called the 'double diaphragm' technique. This experiment was initially designed by Goy (Ref.3.41.). It consisted of placing a second weak diaphragm in the channel of the shock tube, which supported a small pressure differential; the downstream pressure was greater than that upstream.

The principle of operation was that the incident shock wave which was propagating through the test gas would be attenuated by the pressure differential across the weak diaphragm causing a weak shock wave to be reflected back into the heated gas behind the incident shock wave. It was assumed that the diaphragm would play no part in the interaction process.

It was found after considerable investigation and development that the technique could not be made to work in the Cranfield Institute of Technology 2" shock tube. The main reasons for this were that the diaphragm created a larger disturbance than the pressure differential which it supported, and there was not sufficient running time available behind the incident shock wave for a stable shock wave to be formed. This technique was therefore abandoned in favour of a technique which made use of the region behind the reflected shock wave.

This second technique consisted of fitting a porous end face in the end of the channel of the shock tube. A wave diagram of the associated wave process is shown in figure 3.4.1. The principle of operation of this technique was that as the incident shock wave reflected off the front face of the porous end face a small percentage of the shock wave would be transmitted into the end face. This shock wave would be reflected off the bottom of the porous end face to re-emerge into the reflected shock region of the main shock tube flow as a weak normal shock wave. The main question to be answered was whether it would be possible to manufacture a porous end face which would generate weak normal shock waves in the time and distance available behind the reflected shock wave.

An end face was developed which consisted of laminations of steel gauge plate and steel shims to form rectangular slots spanning the width of the working section of the shock tube. A picture of the assembled end face is shown in figure 3.4.2. For these tests, it was necessary to use the square working section, which in turn necessitated the incorporation of a transition section between the cylindrical shock tube and the square cross section of the working section. It was realised that the presence of the transition section would create disturbances, but it was considered that these would be tolerable for an initial evaluation of the technique.

Limited success was obtained with this technique. Fully dispersed shock waves were produced in carbon dioxide and the vibrational relaxation times obtained were in reasonable agreement with measurements made by Simpson (Ref. 3. 12.) and Zienkiewicz and Johannesen (Ref. 3. 10.). However, it was not possible to evaluate the degree of formation of the weak shock wave or to investigate alternative end face designs in the time available.

The results which were obtained were restricted to a maximum translational temperature of 600⁰ K, because of the occurrence of turbulence in the reflected shock region. This resulted from the bifurcation of the reflected shock wave due to its interaction with the boundary layer on the walls of the shock tube behind the incident shock wave. Bifurcation of reflected shock waves is discussed in Appendix 2.

It was concluded that the technique showed sufficient promise for it to be actively pursued and developed. The main areas requiring investigation and development were considered to be the following:

1. Studies of alternative end face designs.
2. An evaluation of the reflected shock region with particular reference to carbon dioxide.
3. An evaluation of the degree of formation of the waves.
4. The use of fully dispersed shock waves produced by this technique for the detailed study of vibrational behaviour.

3.4.3. End Face Design.

It was clear from the original tests that the waves which emerged from the slotted end face were cylindrical in form. It was inferred from the density gradient profiles of the weak waves which were produced that they had been well formed, although no spark pictures were taken of the cylindrical waves to confirm this.

A series of spark schlieren pictures were therefore taken of the waves produced by the slotted end face. Some examples of the pictures which were obtained with air as the test gas are presented in figure 3.4.3. Air was used as the test gas in this context because the waves were more distinct than in carbon dioxide, as will be discussed in a later part of this section.

All of the spark pictures presented in figure 3.4.3. were taken at the same initial channel pressure of 200 torr and nominal shock Mach number of 1.72, but with progressively longer delay times after the passage of the incident shock wave. The delay time increases from run 3010 through 3016, 3011, 3012 to 3013. The front of the end face was approximately one shock tube diameter (5 cms.) from the top of the working section windows seen in the pictures.

The reflected shock wave can just be seen entering the top of the working section window in run 3010, and approximately half way across the window in run 3016. A complex wave pattern existed between the reflected shock wave and the end face, as can be seen in the spark picture of run 3011. This pattern was not present in the tests with carbon dioxide, as will be discussed later.

The weak waves which had emerged from the end face can be seen in the top of the spark picture of run 3012. The white vertical line was a fault in the photograph and not a feature of the flow. These waves can be seen again in run 3013. It can be seen from these last two pictures that the wave formation left a lot to be desired, although it must be remembered that these pictures were taken in air and the waves produced in carbon dioxide appeared to have coalesced completely.

One of the main reasons for the choice of the slotted end face was that it provided the best compromise between ease of adjustment, machining tolerance and internal finish (in the slots) available at that time. This will be appreciated from details of the slotted end face shown in figure 3.4.4.

A disadvantage of this design was that the plates were not restrained at their outer ends. Great care had to be exercised in the assembly of the gauge plates to ensure that they did not warp and hence produce non-uniform widths of slots. It was not possible to improve upon this arrangement because it would have necessitated substantial modifications to the working section which were considered to be unwarranted at that time.

The slotted end face was ideally suited to the square working section, and the latter could have been modified to retain the ends of the gauge plates. However, for reasons of minimum flow disturbance and ultimate purity of the gas, a cylindrical working section was preferred. The slotted end face could have been modified for use in a cylindrical working section, but it would have been far more difficult to adjust slot widths and depths in that case. Also, the above spark schlieren pictures indicated that there was a need to improve the distribution of porosity of the end face.

A circular holed end face had the obvious advantage of flexibility of the distribution of porosity, which could be made more uniform than was possible with the slotted end face. The use of a circular holed end face had been dismissed previously because of the inherent difficulties involved in obtaining good internal finish in the holes. It was considered that it would be worth while investigating the possibility of the use of an holed end face.

A circular holed end face was manufactured to fit into the square working section so that spark schlieren pictures could be taken of the ensuing wave process for its assessment. The end face which was manufactured is shown in figure 3.4.5. It was the end face in centre of the picture; the other end faces will be discussed later.

This square end face was manufactured from a solid brass block. $3/32$ " diameter holes were drilled and reamed into the block at a spacing of 0.2×0.2 ", giving a total of 64 holes over the end face. It was not possible to drill these holes deeper than 2" and consequently deeper holes were achieved by assembling blocks together as shown in figure 3.4.5. This end face was mounted in the same block as the slotted end face shown in figure 3.4.4.

Spark schlieren pictures were taken of the wave process ensuing from this end face. Four different configurations of hole depth and distance between the windows and the end face were investigated. The dimensions of these configurations are given in figure 3.4.6 and the resulting spark pictures in figures 3.4.7 and 3.4.8. These spark schlieren pictures will be discussed in more detail in the later parts of this section.

Of particular interest here are the runs which were made with air as the test gas, ie. pictures C and D of this series.

The upper pictures show the reflected shock waves passing the windows and the lower pictures show the weak shock waves which had emerged from the holes. The oscillograms were of pressure histories obtained on the side wall of the shock tube adjacent to the working section windows as shown in figure 3.4.6.

The spark schlieren pictures have not reproduced very well, but it can be seen from a comparison with figure 3.4.3. that the waves appeared to have coalesced to a far greater extent than those from the slotted end face. This arrangement was still not entirely satisfactory, as it was clear that the internal finish of the holes left a lot to be desired. However, it was generally concluded that the cylindrical working section should be employed together with an end face containing a large number of circular holes.

In seeking a means of improving the internal finish of the holes, it was discovered that hyperdermic tubing had the desired properties of good internal finish and high tolerance. Accordingly, a range of end faces were manufactured from hyperdermic tubing. A sample of these are shown in figure 3.4.5, from which it can be seen that the hyperdermic tubing was sandwiched between two plates to form the end face.

The fitting at the top of the photograph in figure 3.4.5 was used to locate the end faces in the end of the cylindrical working section. The rod running through the fitting, and upon which the end faces were mounted, was adjustable to enable the distance between the working section windows and the end faces to be varied. This rod was locked in position and sealed by means of an 'O' ring.

The results obtained with these end faces, which will be discussed later, indicated that the performance of this type of design was well within the limitations imposed by other aspects of the experiments. It was concluded that no further improvements were justifiable at that stage.

3.4.4. The Reflected Shock Region

It was not possible to make a detailed study of the reflected shock region in the earlier work (Ref. 3.23), because of lack of available time. It was assumed for those purposes that the weak shock waves propagated through an ideal reflected shock region, the conditions in which could be calculated from the incident shock velocity by using the normal shock relations. It was therefore essential for a detailed study to be made of the reflected shock region in carbon dioxide before proceeding with the relaxation measurements.

The main reasons for departures from ideal conditions behind the reflected shock wave have been discussed in Appendix 2. These are:

The initial acceleration of the incident shock wave, boundary layer attenuation of the incident shock wave, and bifurcation of the reflected shock wave.

To these has to be added non-uniformities due to the outflow into the porous end wall.

The distance between the diaphragm and the end wall of the channel was the main parameter over which control could be exercised to achieve a compromise between the first three factors listed above.

A series of tests were conducted to evaluate the affect of the distance between the diaphragm and the working section on the reflected shock conditions. Wall surface temperatures and pressures were measured in the working section for two different diaphragm positions, and over a range of incident shock Mach numbers. A range of results are presented in Figure 3.4.9. The upper diaphragm results (on the left hand side) were obtained at 60 diameters from the diaphragm and the lower diaphragm results (on the right hand side) were obtained at 110 diameters from the diaphragm. The latter position was that used in the previous experiments. The working section was fixed and two diaphragms locations were used for these studies (see Appendix 2).

The P traces are wall pressure histories, and the T traces are the wall temperature histories. It can be seen that the results have been compared at the same values of initial channel pressure to show the affect on the attenuation of the incident shock wave. The extra attenuation due to the increased channel length was of the order of 5%. The pressure traces show that the running time was curtailed by the arrival of the reflected head of the unsteady expansion in the case of the lower diaphragm results.

The first disturbance from the reflected shock/contact surface interaction curtailed the running time in the case of the upper diaphragm position.

The temperature traces indicated that there were large fluctuations in the gas temperature and a marked degree of cooling soon after the reflection of the incident shock wave. This was attributed to the flow of slightly colder gas through the bifurcated edges of the reflected shock wave. The explanation for this, Davies (Ref. 3.42), was that the gas was decelerated less in passing through the oblique shock waves than through the normal part of the reflected shock wave. Hence, less kinetic energy was converted into thermal energy in the flow through the oblique shock waves. The affect that this had on the reflected shock conditions increased as the reflected shock wave moved further from the end wall.

It was difficult to apply the wall pressure measurements in any quantitative manner to the flow through which the weak shock waves were travelling, because of the presence of the bifurcation. It was concluded that the original decision to accept the reflected shock conditions as being ideal was the best which could be made in the circumstances. The calculations were improved slightly by taking account of the flow into the end face.

This was done by assuming that the conditions behind, rather than ahead of, the weak shock waves were those given by the ideal theory for the reflected shock conditions.

The turbulence in the reflected shock region also warranted closer inspection, because the results of Strehlow and Cohen (Ref. 3.43) indicated that the flow adjacent to the end face became stationary and wave free after the reflected shock wave had moved away from the immediate vicinity of the end face. A series of spark schlieren photographs have been taken and these are presented in Figure 3.4.10. The photographs were obtained with the diaphragm in the upper position.

The reflected shock wave can just be seen entering the top of the picture in run 3020, and approximately half way across the picture of run 3021. In the latter case, the bifurcation of the reflected shock wave can be seen together with the turbulent flow of gas through it. (The end face was approximately one shock tube diameter away from the centreline of the working section windows). The turbulence adjacent to the end wall can be seen in the picture of run 3022. As the delay was increased, the local turbulence appeared to increase.

The upper temperature limit had been set by the incident shock Mach number at which the density gradients in the turbulence became of the same order as those in the waves being studied. Therefore, as the turbulence did not decay with time, the temperature limit could not be raised by studying the wave at longer times after the reflection of the incident shock wave. The maximum temperature was therefore limited, as with the previous experiments, to 600°K.

There appears to be no way of raising this limit in pure carbon dioxide. The onset of bifurcation does occur at higher incident shock Mach numbers in other gases, so the technique might be more applicable to them. Carbon dioxide could be mixed with argon for example, which does not cause bifurcation over the requisite shock Mach number range. In this way, it should be possible to either reduce the effects of bifurcation and/or raise the incident shock Mach at which bifurcation occurs.

It is also of interest to note that Mark (Ref. 3.44) showed theoretically that bifurcation should cease above a certain incident shock Mach number, typically seven. If this occurs in practice, then it may be possible to apply the technique at much higher translational temperatures.

3.4.5. The Formation Of The Weak Shock Waves.

In the earlier studies, the degree of formation of the weak shock waves was inferred from the agreement between the relaxation times obtained with them and measurements made elsewhere. This could have been fortuitous, as there was reason to believe that the relaxation times obtained in this way ~~need~~ not have agreed with those from the strong incident shock wave.

This was investigated by studying the formation of weak shock waves emerging from one of the hyperderic tubed end faces in the cylindrical working section. The end face was held in place by a fitting which facilitated the movement of the end face relative to the working section windows (see appendix 2). The distance between the end face and the windows could be varied between 0 and 3 inches.

A series of experiments were performed to investigate the formation of the weak shock waves using argon as the test gas. These experiments were conducted at an initial channel pressure of 100 torr, and at an incident shock Mach number of nominally 1.8 . This was considered to be representative of the conditions under which relaxation time measurements had been made. The time resolved quantitative schlieren system in its final configuration was applied to the study of these waves.

A range of the results which were obtained in this way are presented in figure 3.4.4. It can be seen that the shock waves emerged from the end face with a half height width of approximately 5 μ s. in time. As the wave moved further from the end face, the maximum density gradient increased whilst the width of the wave decreased, indicating that the wave was coalescing.

For a monatomic gas such as argon, the width of the shock wave should have been negligible on this scale. However, it was clear from the results shown that the wave was still coalescing after 3 inches. It might be concluded from this that the technique was unsuccessful.

These tests were repeated with carbon dioxide as the test gas. The results are presented in figure 3.4.12. It can be seen from these results that the formation of the weak shock waves in carbon dioxide had been fortuitous. The profile of the wave emerging from the end face was very close to its final equilibrium state. The profile appeared reasonably stable after having travelled 1.5 inches, although the traces were distorted by the turbulence from the reflected shock wave.

A distance/ time graph was plotted from these results for each test gas, as shown in figure 3.4.13. It can be seen that both of the gases appeared to be travelling at a constant velocity within the experimental error. It had been hoped that this form of presentation would indicate the degree of formation of the wave. However, the velocity of the waves were so close to the speed of sound that even the large changes in the profiles of the argon shock waves were not reflected in this plot.

The half height widths of the profiles have also been plotted versus distance from the end face, as shown in figure 3.4.14. It can be seen from an extrapolation of the argon results that the wave would have properly coalesced after travelling a distance of approximately 11 cms. . This was equivalent to 55 diameters based on the spacings of the holes in the end face, which was consistent with normal shock tube behaviour.

The carbon dioxide results confirmed the visual assessment that the wave was formed almost immediately into its final equilibrium profile.

Consider the wave diagram of the shock wave process associated with this end face shown in figure 3.4.15. The shock wave process inside the holes has been shown as being the same as that in the main part of the shock tube. This was not quite correct, as will be discussed later, but it is sufficiently accurate for the present purposes.

Having determined the formation distance required for the weak shock waves, it was necessary to optimise the end face geometry. The weak shock wave travelled at a higher velocity than the reflected shock wave and therefore overtook it. Consequently, the length of the end face and the distance between the windows and the end face had to be chosen such that the best compromise was reached between the time allowed for formation, the time to over take the reflected shock wave and the time of arrival of the first disturbance from the reflected shock wave / contact surface interaction.

A series of experiments were conducted to optimise the geometry. These experiments were conducted in the square working section to enable spark schlieren pictures to be taken of the full width of the flow. The square holed end face was used for this work, and the range of configurations have already been presented in figure 3.4.6. The spark schlieren pictures which were obtained in this way are presented in figures 3.4.7 and 3.4.8, together with side wall pressure histories in the working section. It will be recalled that the delays for the upper pictures were chosen to show the reflected shock waves whilst the delays for the lower pictures were chosen to show the weak shock waves.

It was found that the most reasonable compromise was obtained by having the distance between the end face and the windows equal to the depth of the holes in the end face. The final depth of the holes was set at 2 inches.

It was concluded from all of the above results that the formation of the weak normal shock waves had indeed been fortuitous. Nevertheless, the waves were well formed and useable for this application, but it was noted that the situation would have to be reviewed for other gases. In the presence of the disturbances from the bifurcation of the reflected shock wave, a better assessment of the weak wave formation could not be made.

It was of interest to note that the argon results (see figure 3.4.14) indicated that the spatial resolution of the schlieren system must have been better than 0.03 cms..

3.4.6. Fully Dispersed Shock Waves.

It was decided that the earlier experiments which had been made with this technique for the evaluation of the relaxation time of carbon dioxide using fully dispersed shock waves should be repeated. It was realised that the upper temperature limitation could not be raised, but it was considered that better low temperature data would be obtained with the incorporated improvements.

One the main improvements was in the method of determining the strengths of the fully dispersed shock waves. Previously, the strengths of the fully dispersed shock waves had been determined from the shape of the density gradient profiles of the waves. The sensitivity of the profile shape to the wave strength can be seen from figure 3.4.1. This technique was found to be unsatisfactory because of the distortion of the profile due to broadening.

The strengths of the fully dispersed shock waves studied in the present experiments were determined from the total density change across them. The output from the time resolved quantitative schlieren system was calibrated and integrated numerically on a computer to give the density profile and the total density change of each wave. The conditions behind the fully dispersed shock waves were calculated from the initial conditions in the channel and the incident shock velocity. It was assumed that the conditions behind the fully dispersed shock wave were those given by the ideal theory for the reflected shock region (see section 3.4.4). This normal shock data is presented in figure 3.4.6 for the incident shock wave and 3.4.7 for the reflected shock wave for a carbon dioxide test gas.

The methods of data reduction and analysis used for these experiments were similar to those used for the weak incident shock waves of the preceding section. The experimentally measured density gradient profiles were non-dimensionalised by the upstream density and the inverse of the upstream velocity. These were then plotted versus the density change through the waves non-dimensionalised by the upstream density. Theoretical profiles were calculated and plotted in a similar manner.

The ratio of the theoretical density gradient to the experimentally measured gradient was determined for a number of points equally spaced in terms of density through the wave. This ratio gave the relaxation time at each point through the wave. Any variation in the relaxation time through the wave was attributed to experimental error, and a mean value of the relaxation time was determined for each wave. The experimental profiles were then re-non-dimensionalised by this mean value of the relaxation time for direct comparison with the theoretical profiles.

The experiments were conducted with an hyperdermic tubed end face containing 76 x 2.38 mm. diameter holes. Figure 3.4.18 shows a range of outputs from the time resolved quantitative schlieren system of the waves produced by this end face in carbon dioxide. The incident shock Mach numbers range from 1.08 to 2.0.

The upper traces of the oscillograms are the outputs from the main photomultiplier placed behind the single sampling slit; it will be recalled that the schlieren system was in its final configuration. These traces were triggered by the incident shock waves passing the first of the light probes, and they have been delayed in order to expand them to the desired time base.

It can be seen that the fully dispersed shock waves were not well formed for incident shock Mach numbers below 1.2, which set a lower limit for the technique in terms of the translational temperature. However, the weak incident shock waves catered adequately for the lower temperatures. As the Mach number of the incident shock waves approached two, the profiles became very distorted. This was caused by the bifurcation of the reflected shock wave discussed above, which set an upper temperature limitation on the results of approximately 600 K.

The upper oscillograms of figure 3.4.19 are of similar outputs, but with the upper traces not delayed and displayed on longer time bases. These recordings were taken to assist in the assessment of the flow into which the fully dispersed waves were propagating. The incident shock waves can be seen as negative going spikes occurring within the first half centimetre of the traces. The reflected shock waves can be seen as the first positive going spikes occurring after approximately four centimetres from the start of the traces. The fully dispersed shock waves can be seen as the last positive going spikes occurring approximately eight centimetres after the start of the traces. A particular point of interest was the rising output following the incident shock wave in each trace. This was the negative density gradient which had been observed in section 3.2.

The lower oscillograms of figure 3.4.19 (runs 3294, 3292, 3289 & 3288) show the outputs from the light probes of the fully dispersed shock waves (upper traces) and of the incident shock waves (lower traces). As with the incident shock waves, the traces of the fully dispersed shock waves were used for the determination of their velocities. This information was used to convert the time resolved schlieren outputs of figure 3.4.18 into spatial variations.

The results shown in figure 3.4.18 and others were analysed in the manner described and the relaxation times for carbon dioxide evaluated. These results are presented in the Landau-Teller plot of figure 3.4.20. Also shown are the results of the previous series of experiments (Ref. 3.23), together with the results of Simpson (Ref. 3.12.) and Zienkiewicz and Johannesen (Ref. 3.10.).

The scatter in the present results did not appear to have been greatly improved over the results obtained previously. However, comparison with the results from the weak incident shock waves in carbon dioxide indicated that the scatter was peculiar to fully dispersed shock waves rather than this particular method of production. This was related to the method used for the determination of the wave strengths, as discussed in section 3.3.

It was concluded that further improvements in the means of determining the strengths of the fully dispersed shock waves were required. The total pressure or density change measured directly might be more suitable, although this was by no means clear. It may be that the strengths of fully dispersed shock waves cannot be determined with sufficient accuracy for this type of work.

Figures 3.4.21 and 3.4.22 show a comparison of the non-dimensionalised experimental density gradient profiles with the theoretical profiles calculated from the basic solution of part 1 herein for two of the above runs. The agreement was similar to that obtained with the weak incident shock waves of the preceding section. It was reasonable to conclude from this that, for a limited temperature range, this technique was as successful as could be expected in the present shock tube.

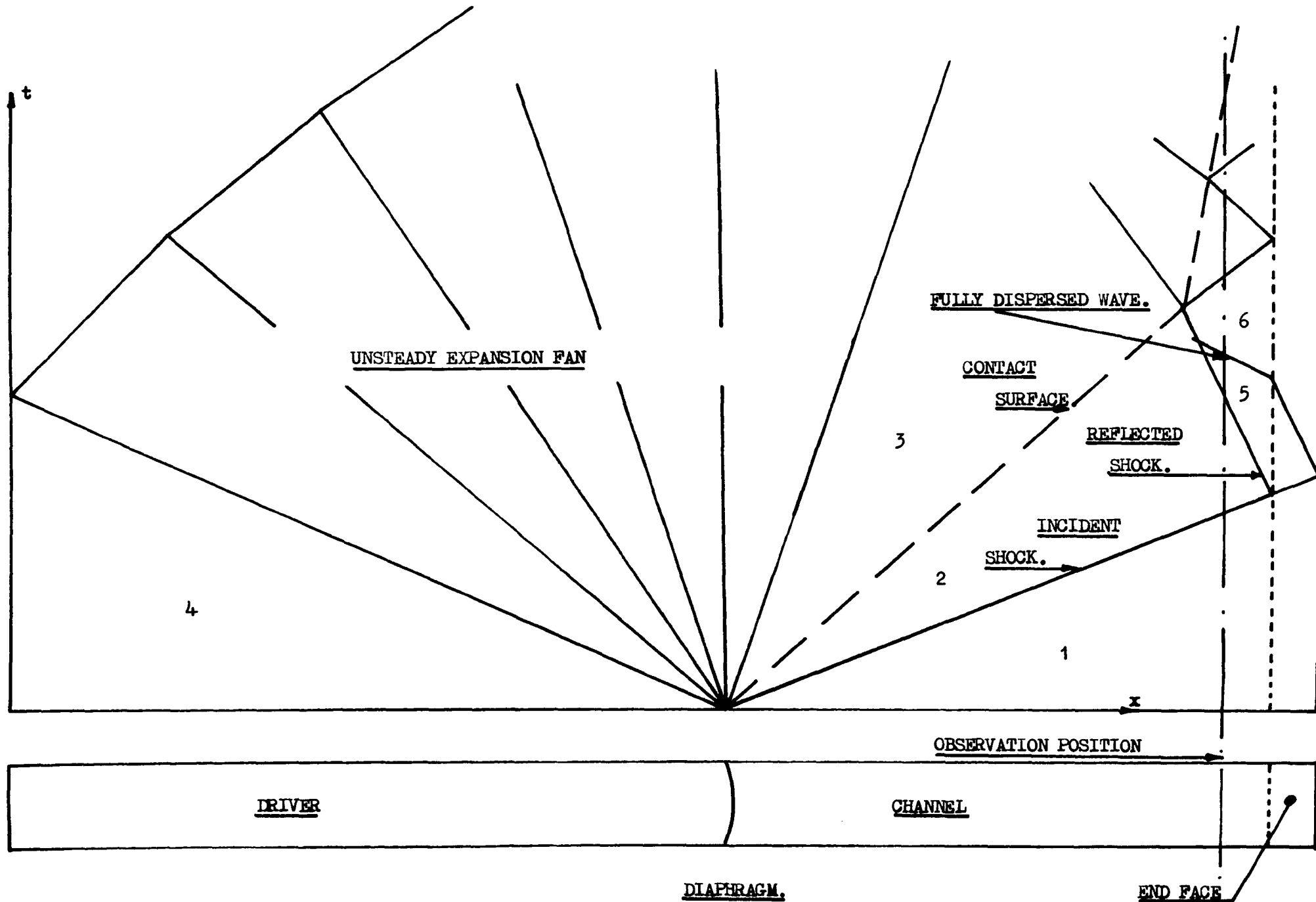
3.4.7. Concluding Remarks.

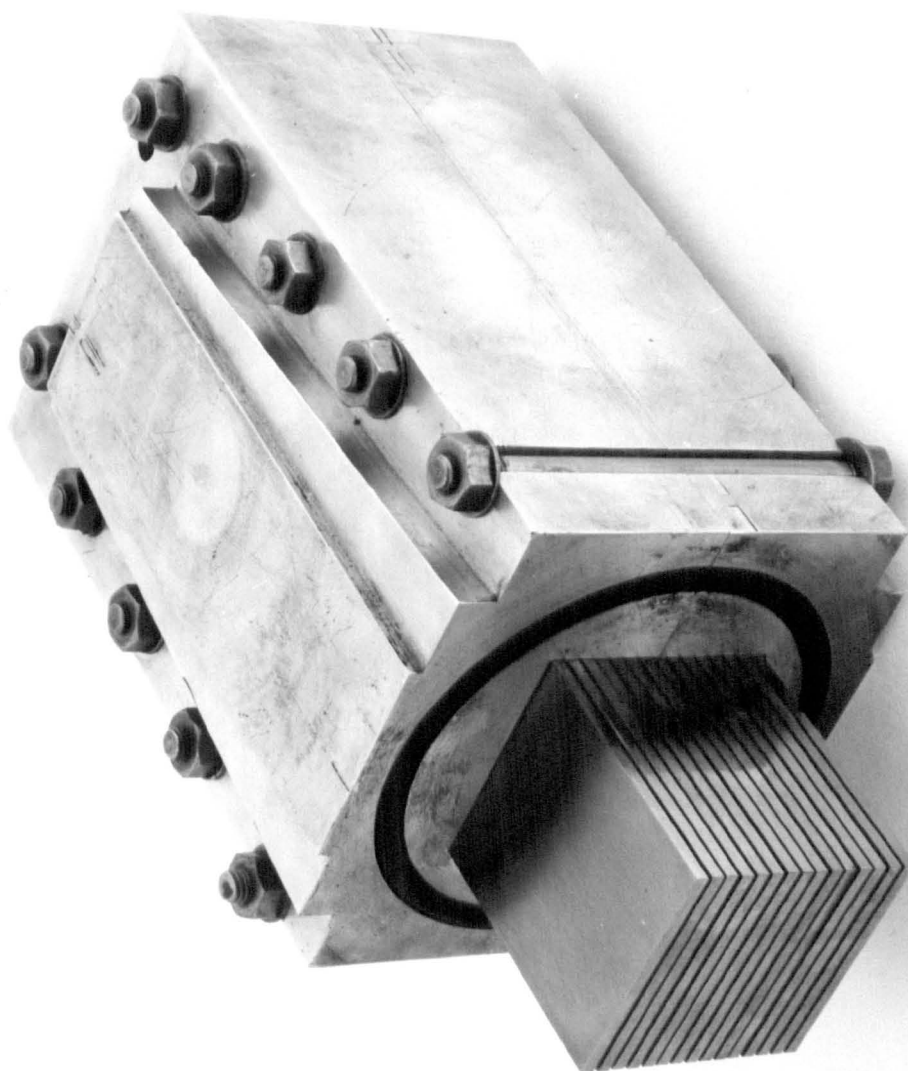
This investigation has yielded an increased knowledge of the wave processes associated with the porous end face technique, and it has been possible to make certain improvements to the design of the end face. Unfortunately, the upper temperature limitation has not been elevated. Nevertheless, the porous end face technique is still the most promising of the techniques available for the production of weak normal shock waves.

It is of interest to compare the agreement between the theoretical and experimental profiles of this section with those of the preceding section. It can be seen that the fully dispersed shock waves were as well formed as they were in the case of the weak incident shock waves. It was concluded from this that there would be little point in trying to improve the wave formation process in the present shock tube.

The results for argon showed that the maximum formation distance for the weak shock waves was of the order of 55 diameters based on the spacings of the holes. This was consistent with normal shock tube behaviour, and will therefore be a useful figure for use in any future design.

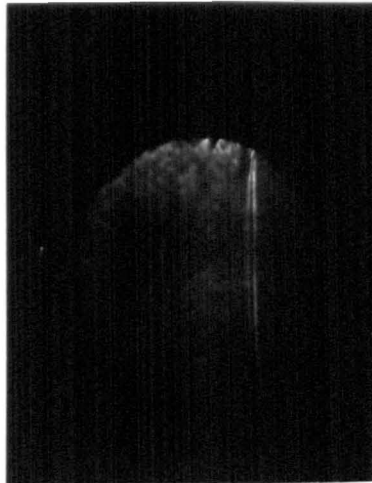
The exact mechanism of the wave processes within the holes on the end face has not been rigorously investigated. However, the times of arrival of the weak shock waves at the observation station after the passage of the incident shock wave were approximately 10% greater than those given by the ideal model shown in figure 3.4.15. This was therefore considered to be a reasonable figure to take in any future design.



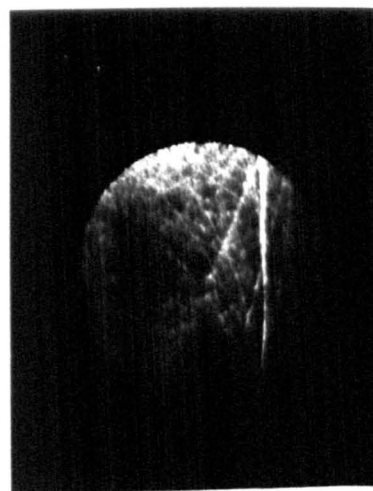


THE 'SLOTTED' END FACE - ASSEMBLED

FIGURE : 3.4.2.



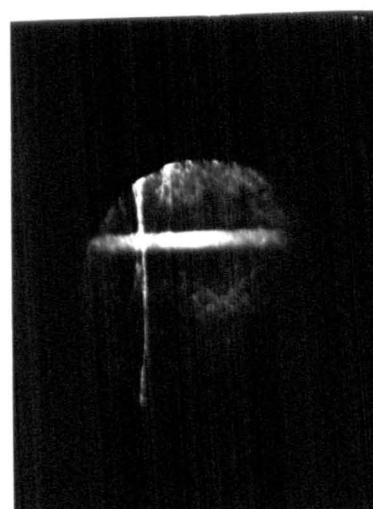
RUN 3011



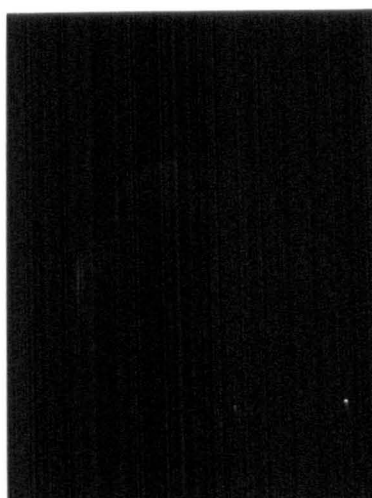
RUN 3013



RUN 3016



RUN 3012



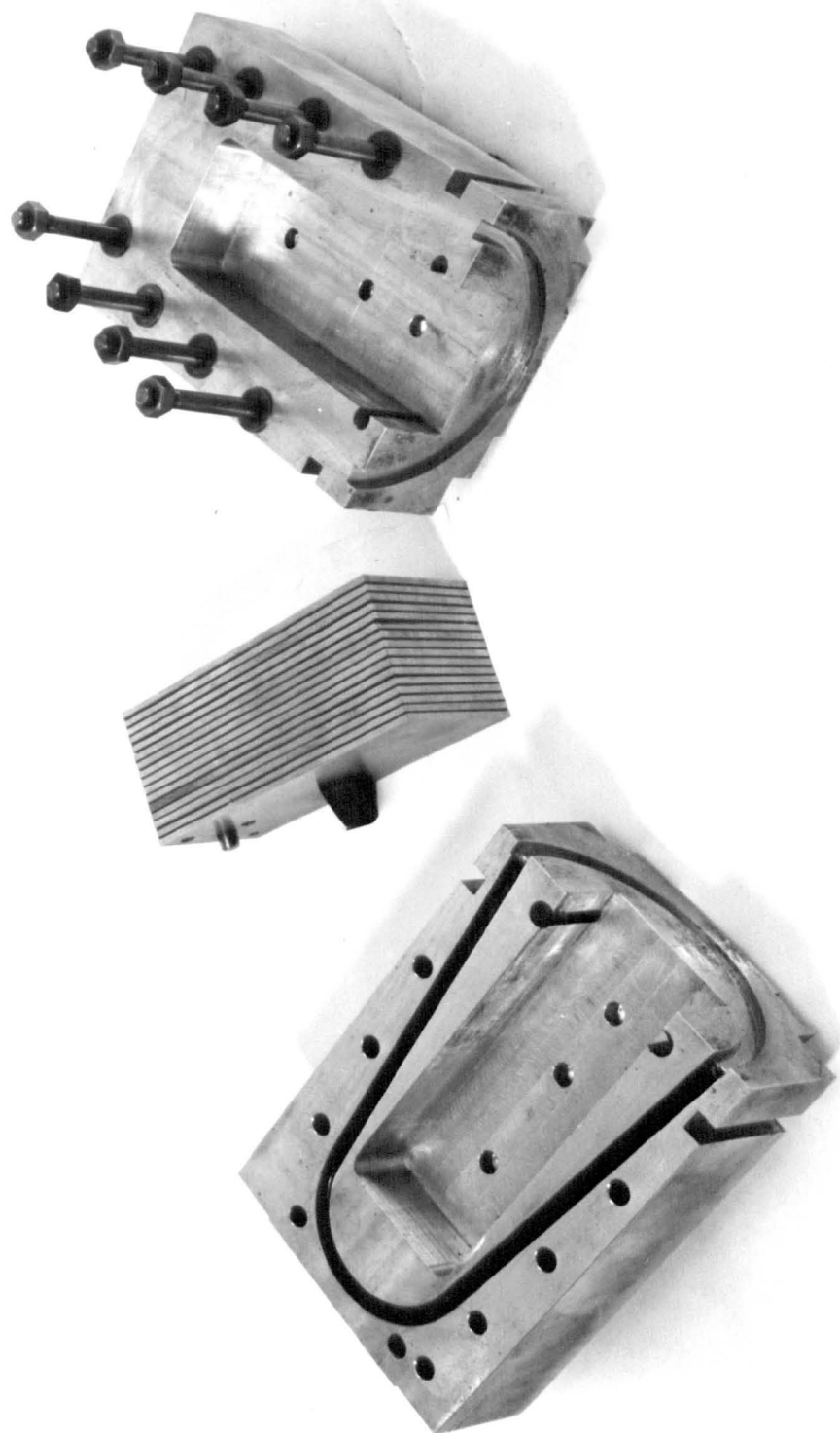
RUN 3010

$$M_s = 1.720$$

$$P_1 = 200 \text{ torr.}$$

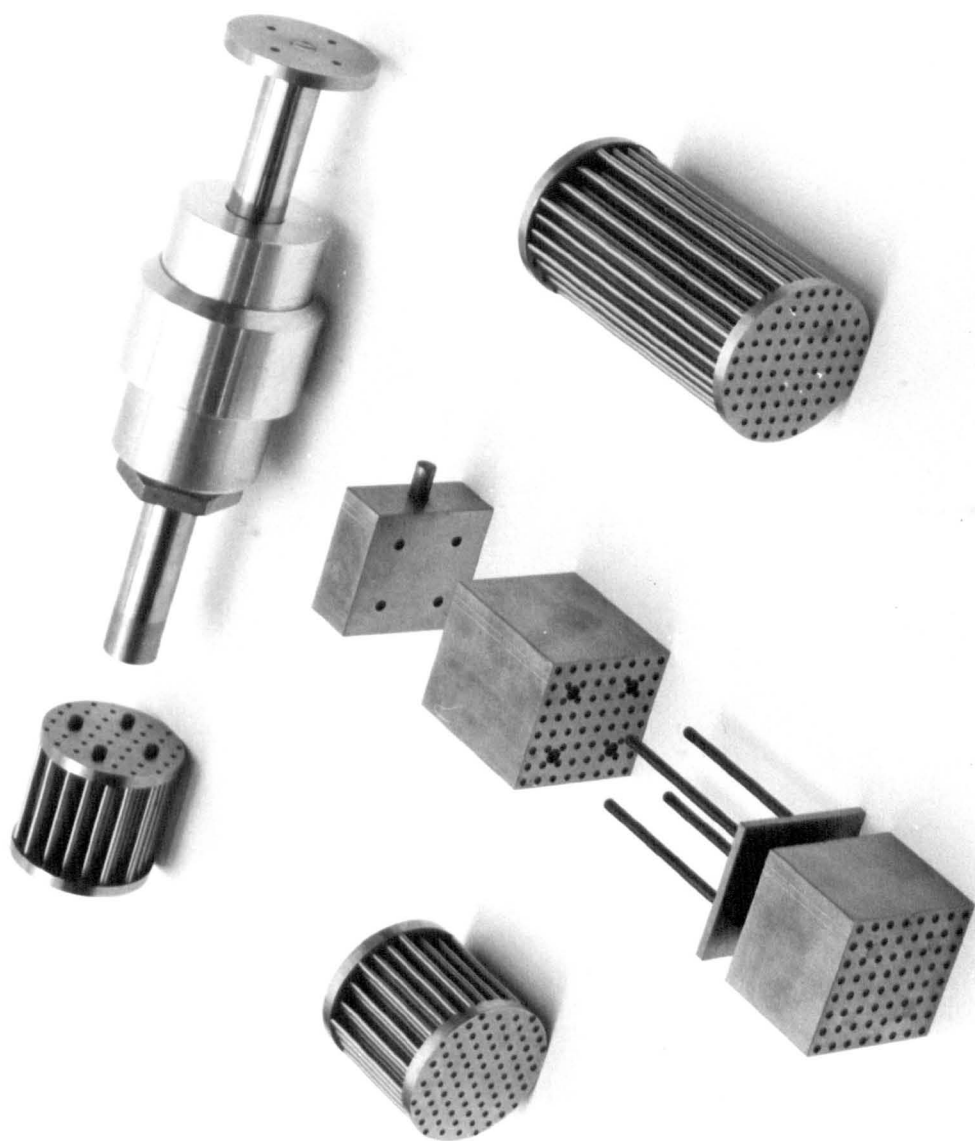
SPARK SCHLIEREN PICTURES OF REFLECTION PROCESS
IN AIR WITH 15×0.0381 cms. SLOTTED END FACE.

FIGURE 3.4.3.



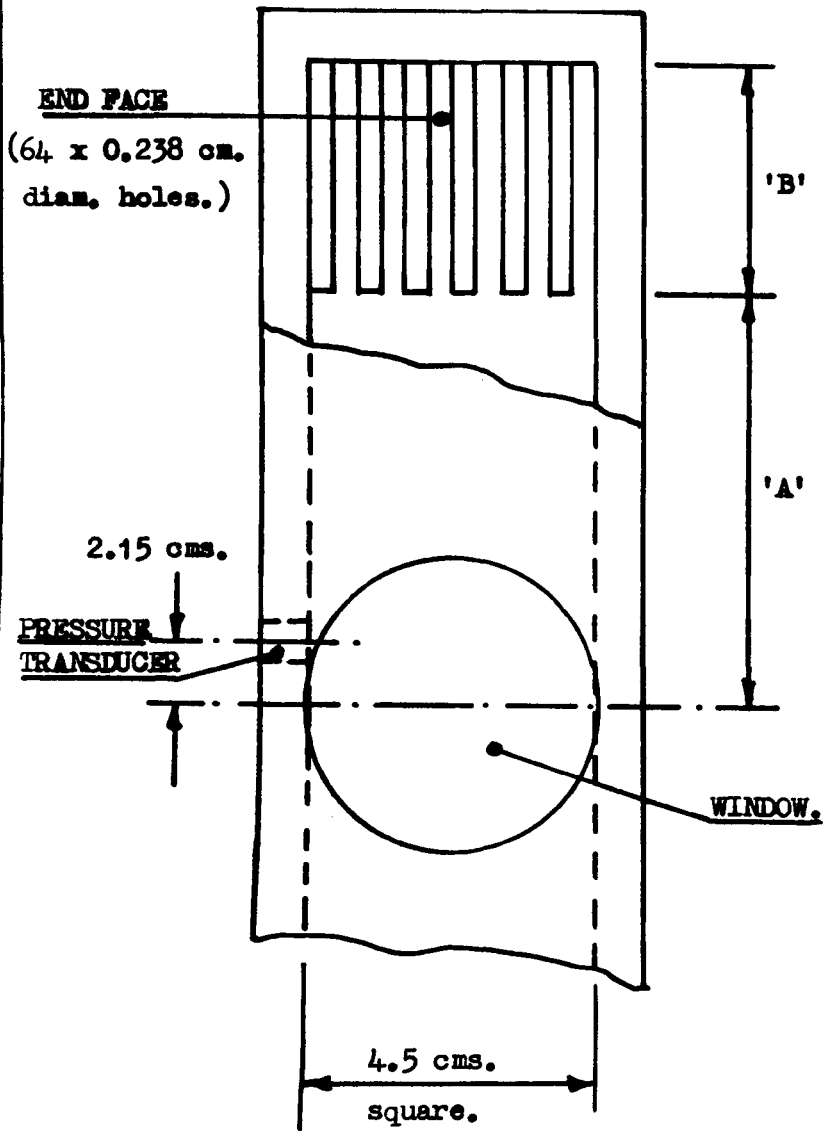
THE 'SLOTTED' END FACE — DETAILS

FIGURE: 3.4.4

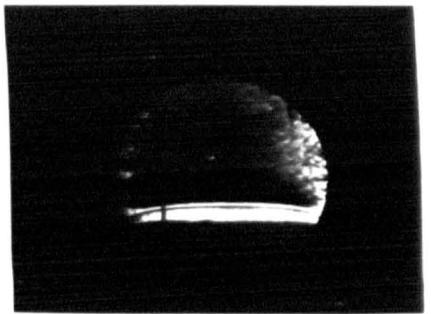
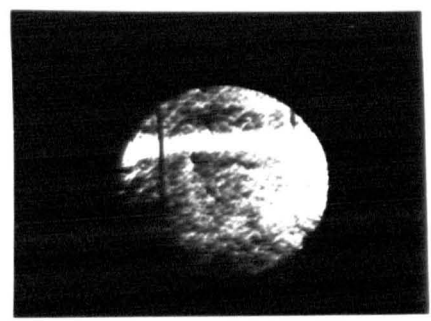
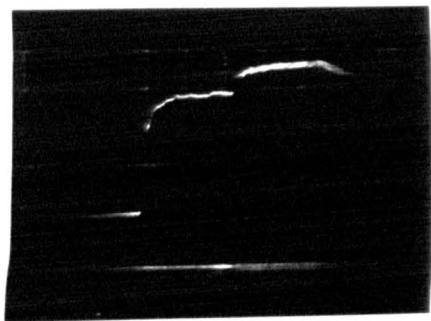


THE 'HOLED' END FACES

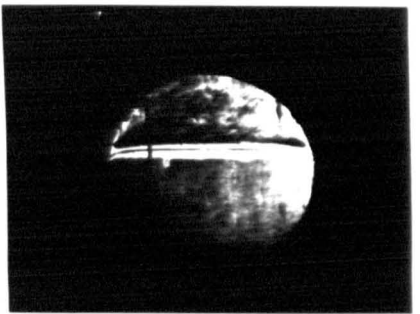
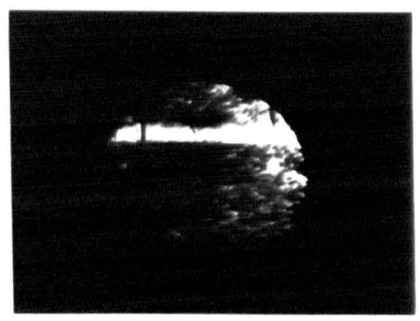
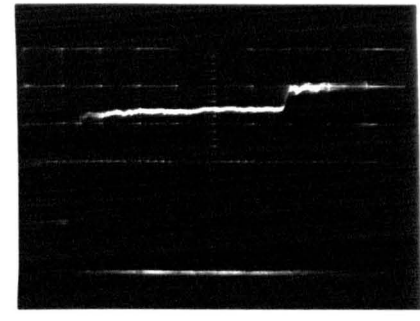
FIGURE: 3.4.5.



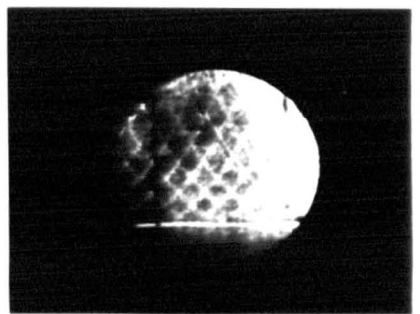
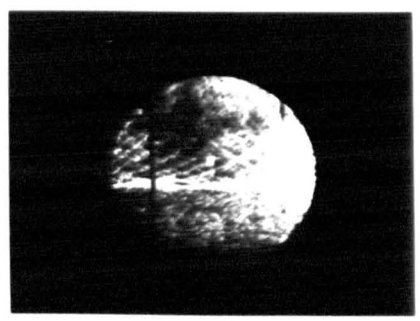
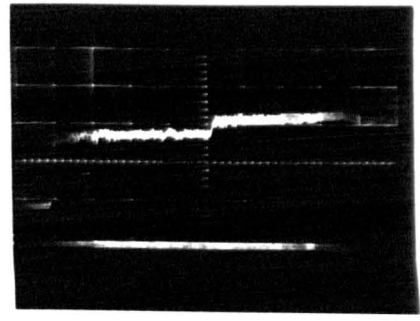
DIMN. (cms.)	CONFIGURATION			
	I	II	III	IV
A	10.15	5.80	10.15	5.80
B	8.89	8.89	2.81	2.81



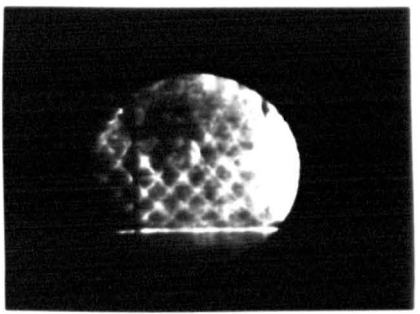
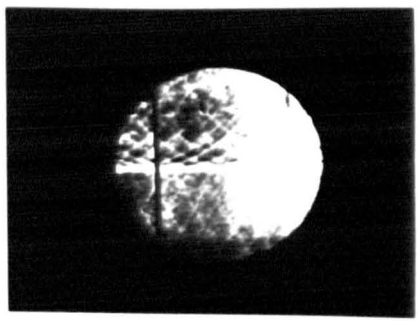
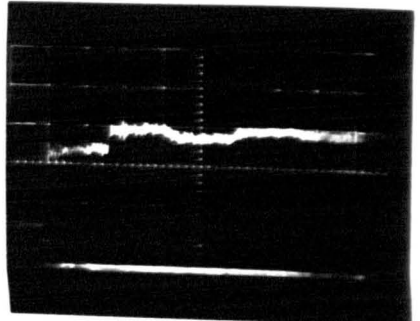
A. $\longrightarrow 200 \mu\text{s/cm.}$
 $M_s = 1.78 \quad P_1 = 200 \text{ torr.}$
 $N_2 \rightarrow CO_2$ CONFIGURATION I
 TOP PICTURE DELAYED 0.73 ms.
 BOTTOM PICTURE DELAYED 1.10 ms.



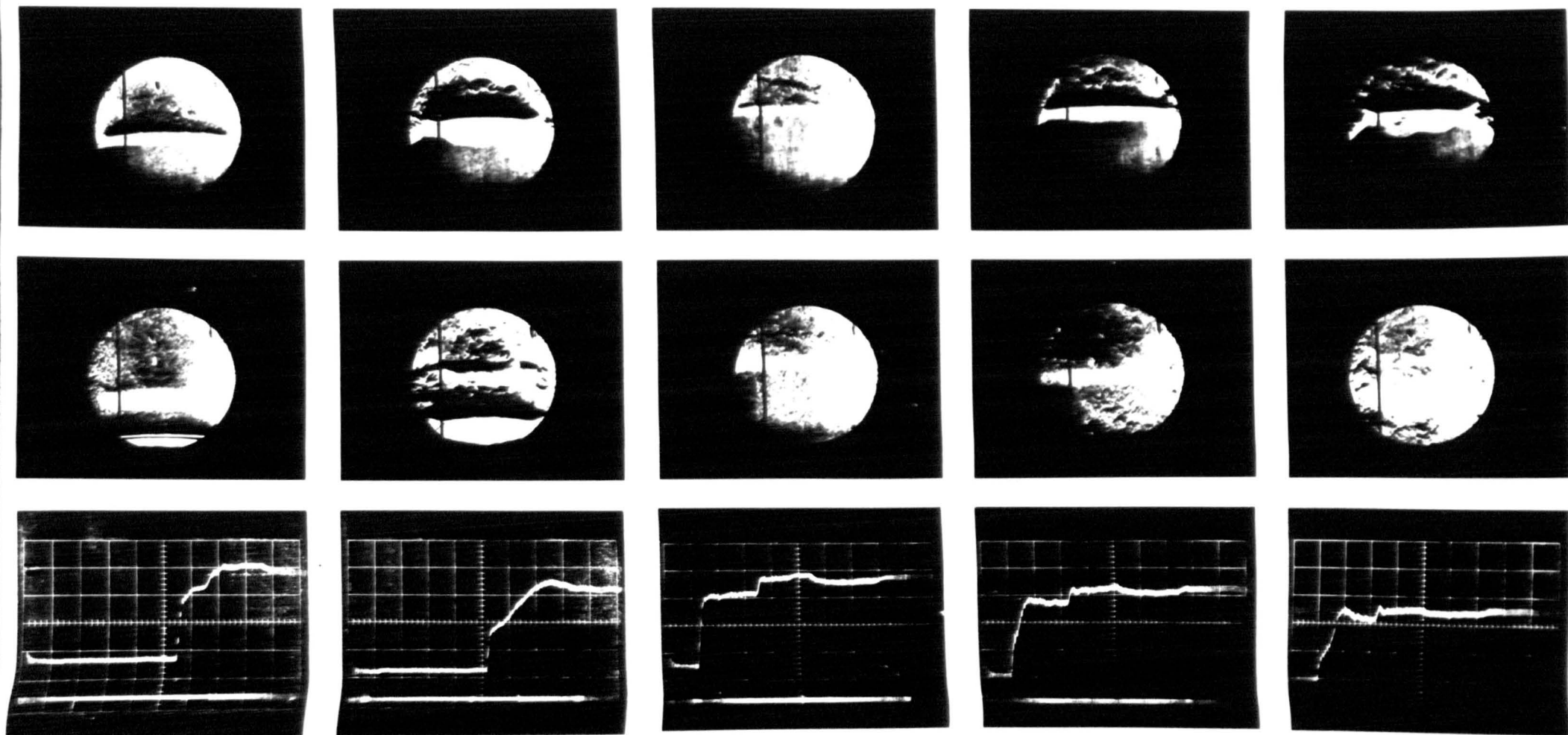
B. $\longrightarrow 200 \mu\text{s/cm.}$
 $M_s = 1.83 \quad P_1 = 200 \text{ torr.}$
 $N_2 \rightarrow CO_2$ CONFIGURATION II
 TOP PICTURE DELAYED 0.2 ms.
 BOTTOM PICTURE DELAYED 0.74 ms.



C. $\longrightarrow 200 \mu\text{s/cm.}$
 $M_s = 1.66 \quad P_1 = 200 \text{ torr.}$
 $N_2 \rightarrow \text{AIR}$ CONFIGURATION II
 TOP PICTURE DELAYED 0.2 ms.
 BOTTOM PICTURE DELAYED 0.57 ms.



D. $\longrightarrow 200 \mu\text{s/cm.}$
 $M_s = 1.68 \quad P_1 = 200 \text{ torr.}$
 $N_2 \rightarrow \text{AIR}$ CONFIGURATION IV
 TOP PICTURE DELAYED 0.2 ms.
 BOTTOM PICTURE DELAYED 0.3 ms.



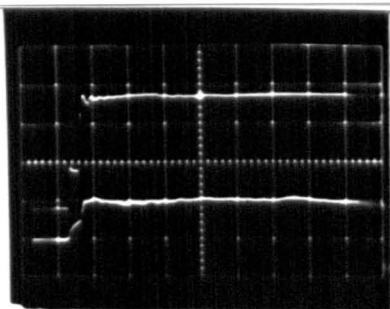
E. $\longrightarrow 200\mu\text{s/cm.}$
 $M_s = 1.75$ $P_1 = 200\text{ torr.}$
 $\text{N}_2 \rightarrow \text{CO}_2$ CONFIGURATION III
 TOP PICTURE DELAYED 0.7 ms.
 BOTTOM PICTURE DELAYED 0.78 ms.

F. $\longrightarrow 200\mu\text{s/cm.}$
 $M_s = 2.04$ $P_1 = 100\text{ torr.}$
 $\text{N}_2 \rightarrow \text{CO}_2$ CONFIGURATION III
 TOP PICTURE DELAYED 0.65 ms.
 BOTTOM PICTURE DELAYED 0.73 ms.

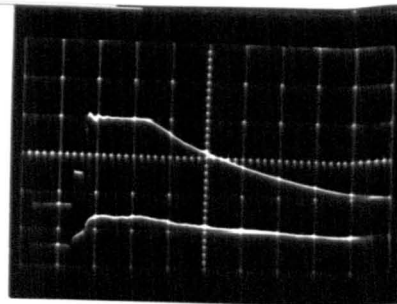
G. $\longrightarrow 200\mu\text{s/cm.}$
 $M_s = 1.76$ $P_1 = 200\text{ torr.}$
 $\text{N}_2 \rightarrow \text{CO}_2$ CONFIGURATION IV
 TOP PICTURE DELAYED 0.2 ms.
 BOTTOM PICTURE DELAYED 0.4 ms.

H. $\longrightarrow 200\mu\text{s/cm.}$
 $M_s = 2.02$ $P_1 = 100\text{ torr.}$
 $\text{N}_2 \rightarrow \text{CO}_2$ CONFIGURATION IV
 TOP PICTURE DELAYED 0.2 ms.
 BOTTOM PICTURE DELAYED 0.2 ms.

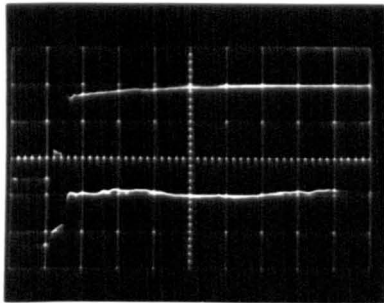
I. $\longrightarrow 200\mu\text{s/cm.}$
 $M_s = 2.33$ $P_1 = 50\text{ torr.}$
 $\text{N}_2 \rightarrow \text{CO}_2$ CONFIGURATION IV
 TOP PICTURE DELAYED 0.2 ms.
 BOTTOM PICTURE DELAYED 0.4 ms.



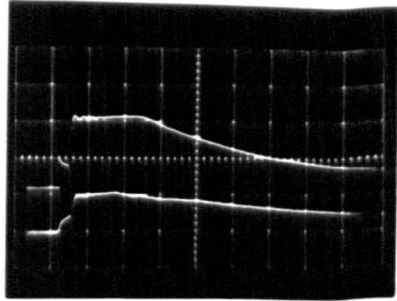
RUN 3061
 $M_s = 1.58$
 $P_i = 400 \text{ torr.}$



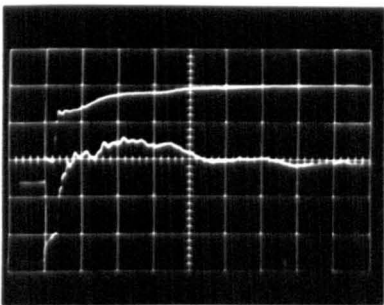
RUN 3060
 $M_s = 1.53$
 $P_i = 400 \text{ torr.}$



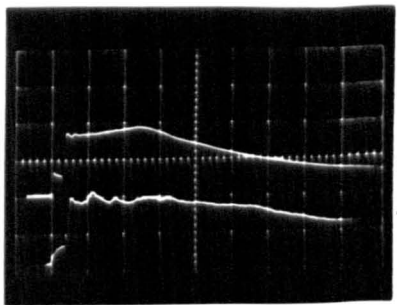
RUN 3056
 $M_s = 1.76$
 $P_i = 200 \text{ torr.}$



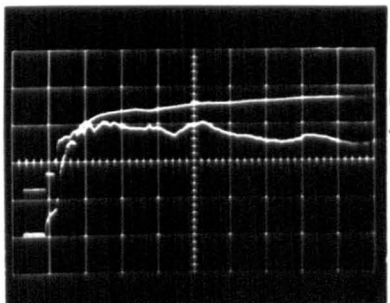
RUN 3054
 $M_s = 1.72$
 $P_i = 200 \text{ torr.}$



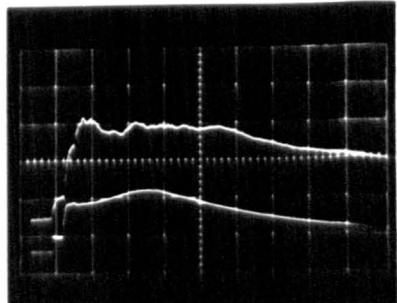
RUN 3052
 $M_s = 2.05$
 $P_i = 100 \text{ torr.}$



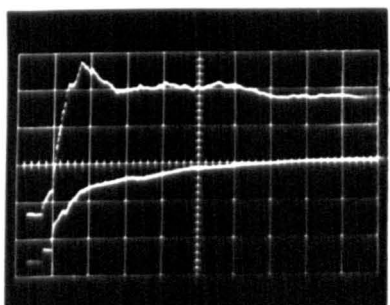
RUN 3053
 $M_s = 1.96$
 $P_i = 100 \text{ torr.}$



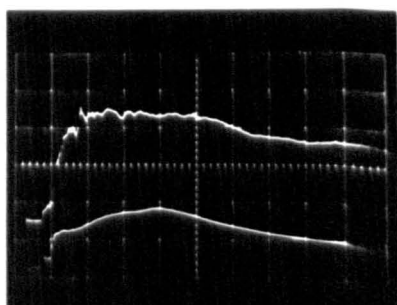
RUN 3055
 $M_s = 2.23$
 $P_i = 50 \text{ torr.}$



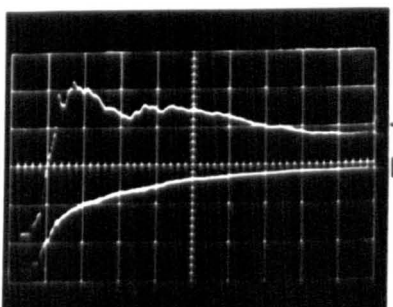
RUN 3063
 $M_s = 2.22$
 $P_i = 50 \text{ torr.}$



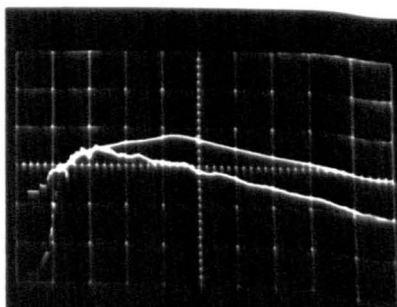
RUN 3562
 $M_s = 2.65$
 $P_i = 25 \text{ torr.}$



RUN 3058
 $M_s = 2.54$
 $P_i = 25 \text{ torr.}$



RUN 3057
 $M_s = 3.08$
 $P_i = 10 \text{ torr.}$



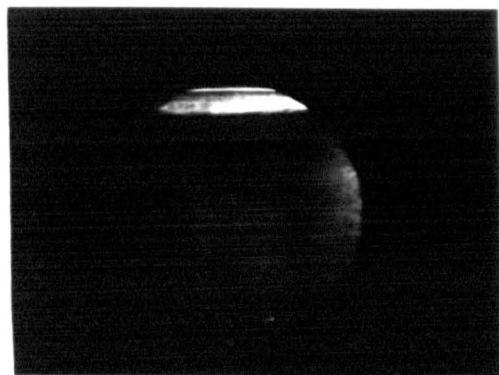
RUN 3059
 $M_s = 2.96$
 $P_i = 10 \text{ torr.}$

UPPER DIAPHRAGM POSITION (2 m/s/cm)

LOWER DIAPHRAGM POSITION (2 m/s/cm)

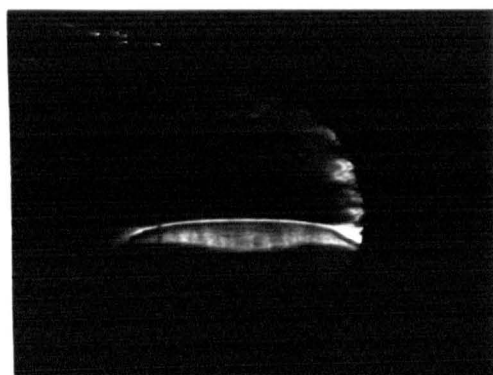
COMPARISON OF REFLECTED SHOCK PERFORMANCE USING UPPER AND LOWER DIAPHRAGM POSITIONS.

FIGURE: 3.4.9.



RUN 3020

DELAY = 3.16 ms



RUN 3021

DELAY = 3.26 ms



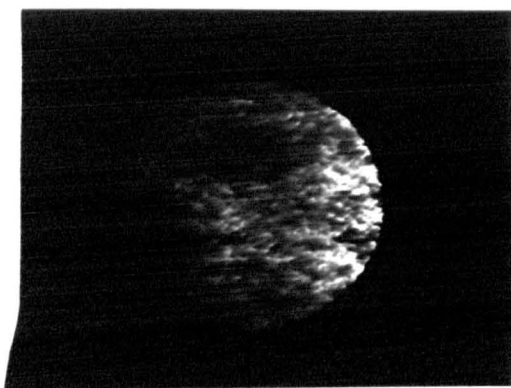
RUN 3022

DELAY = 3.36 ms



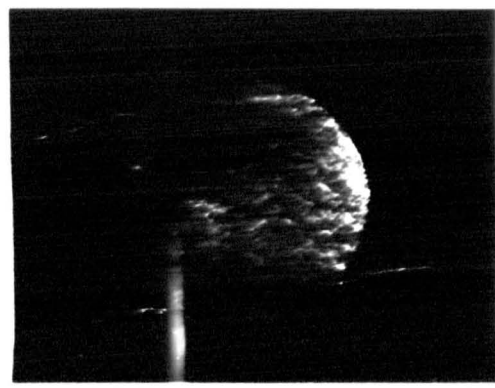
RUN 3023

DELAY = 3.46 ms



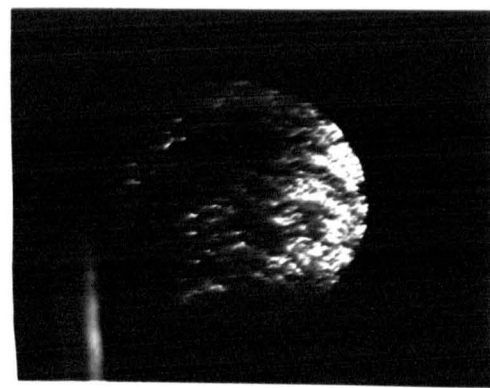
RUN 3024

DELAY = 3.56 ms



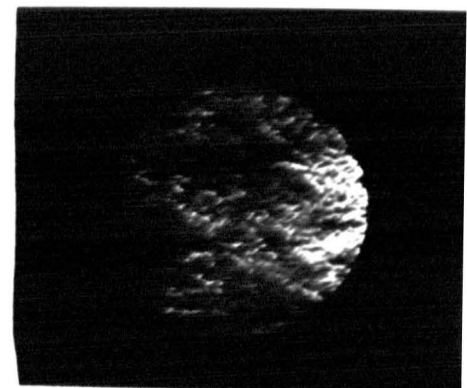
RUN 3025

DELAY = 3.76 ms



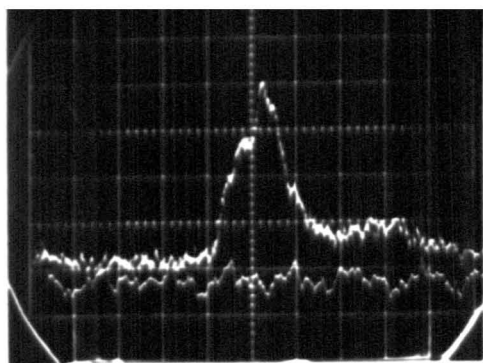
RUN 3026

DELAY = 4.50 ms

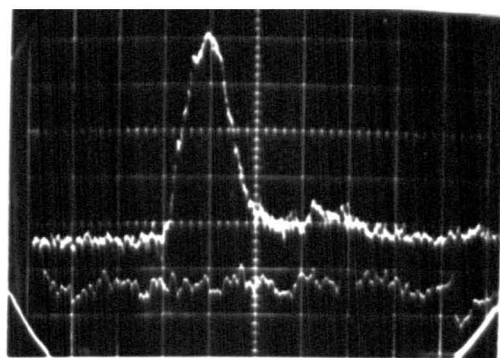


RUN 3027

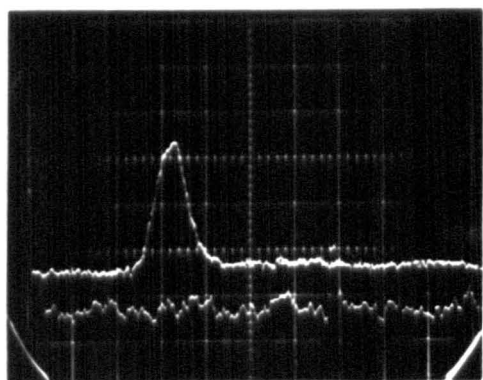
DELAY = 5.50 ms



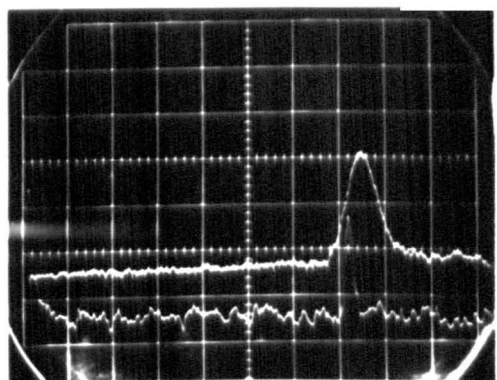
RUN 3178 $5 \mu\text{s/cm.}$
 5 mv/cm.
 $0.5'' (1.77 \text{ cm.})$



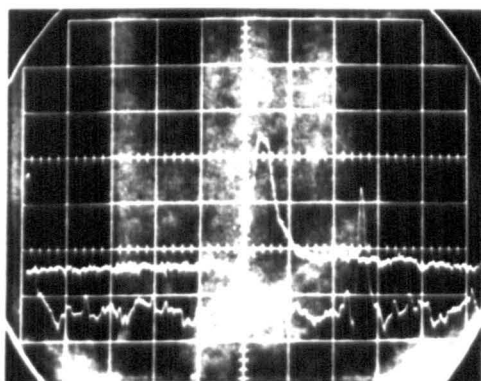
RUN 3181 $5 \mu\text{s/cm.}$
 5 mv/cm.
 $1.0'' (2.54 \text{ cm.})$



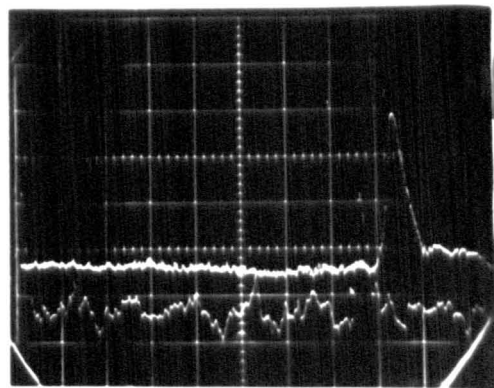
RUN 3183 $5 \mu\text{s/cm.}$
 10 mv/cm.
 $1.5'' (4.31 \text{ cm.})$



RUN 3188 $5 \mu\text{s/cm.}$
 10 mv/cm.
 $2.0'' (5.08 \text{ cm.})$

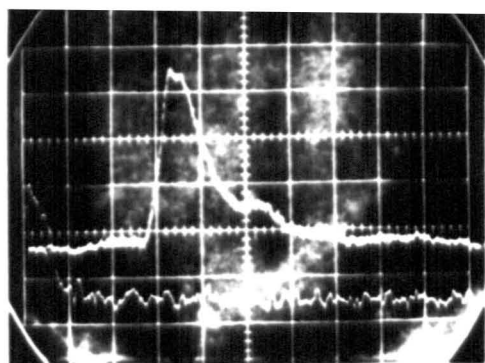


RUN 3189 $5 \mu\text{s/cm.}$
 10 mv/cm.
 $2.5'' (6.85 \text{ cm.})$



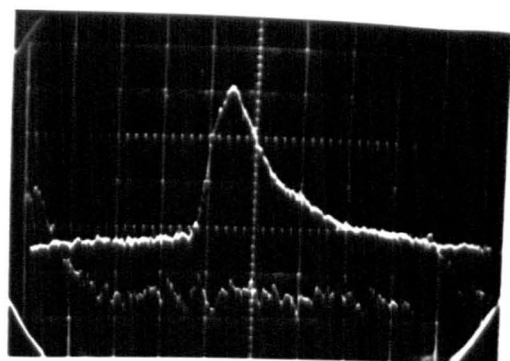
RUN 3193 $5 \mu\text{s/cm.}$
 10 mv/cm.
 $3.0'' (7.62 \text{ cm.})$

NOTE: DISTANCES ARE FROM FRONT OF END FACE TO SLIT.



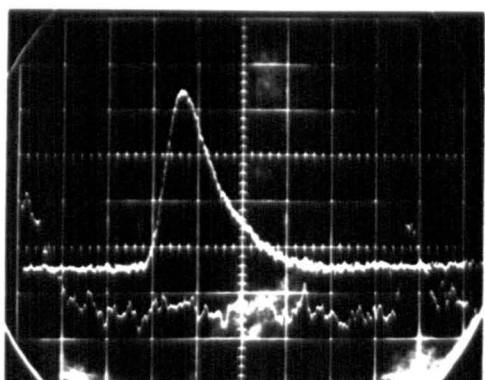
RUN 3198

0.5" (1.27 cm.)



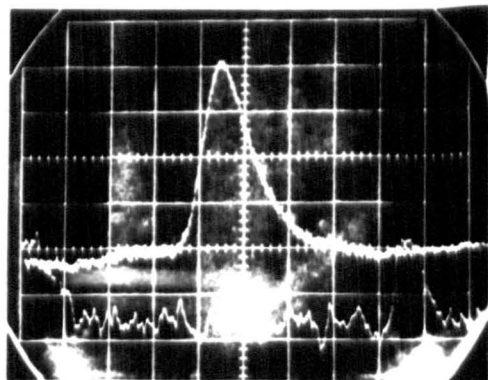
RUN 3199

1.0" (2.54 cm.)



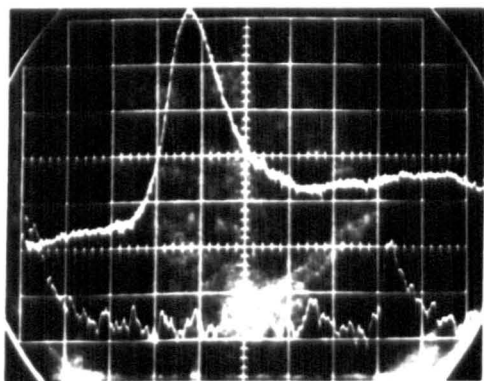
RUN 3200

1.5" (3.81 cm.)



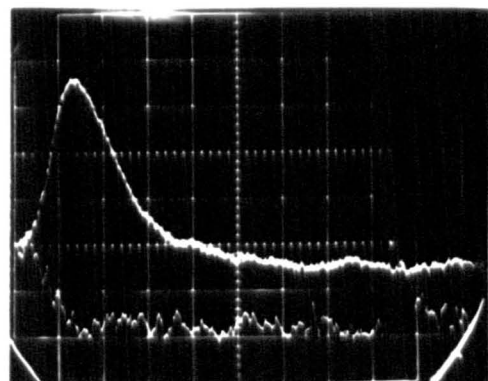
RUN 3203

2.0" (5.08 cm.)



RUN 3205

2.5" (6.35 cm.)



RUN 3207

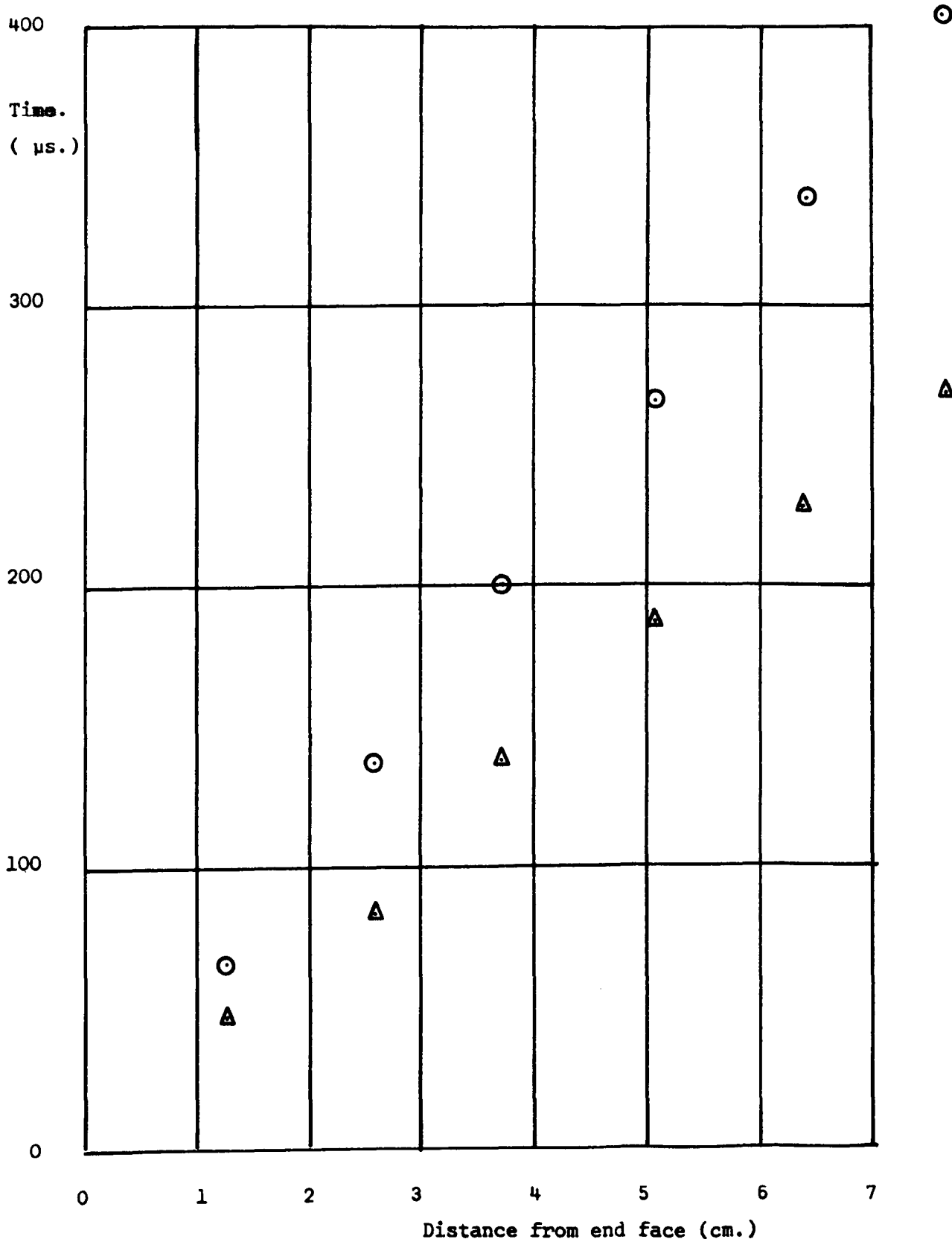
3.0" (7.62 cm.)

NOTE : DISTANCES ARE FROM FRONT OF END FACE TO SLIT.

SENSITIVITY = 10 mv/cm. TIME BASES: 10 μ s/cm.

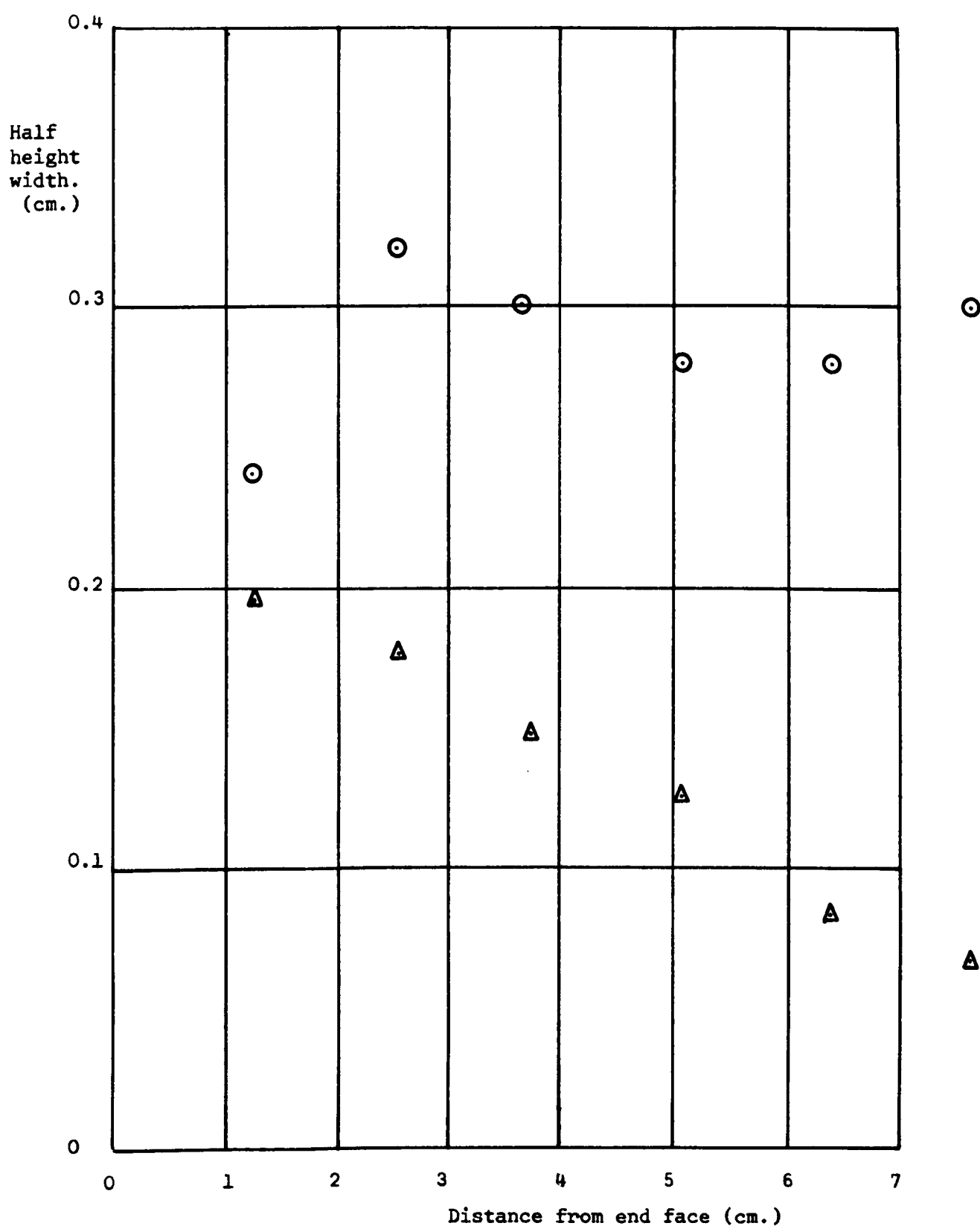
TIME RESOLVED SCHLIEREN OUTPUTS OF WEAK
REFLECTED WAVE IN CO_2 , $M_s = 1.75$, $P_1 = 100$ torr.

FIGURE: 3.4.12.



⊙ Carbon dioxide.

Δ Argon.

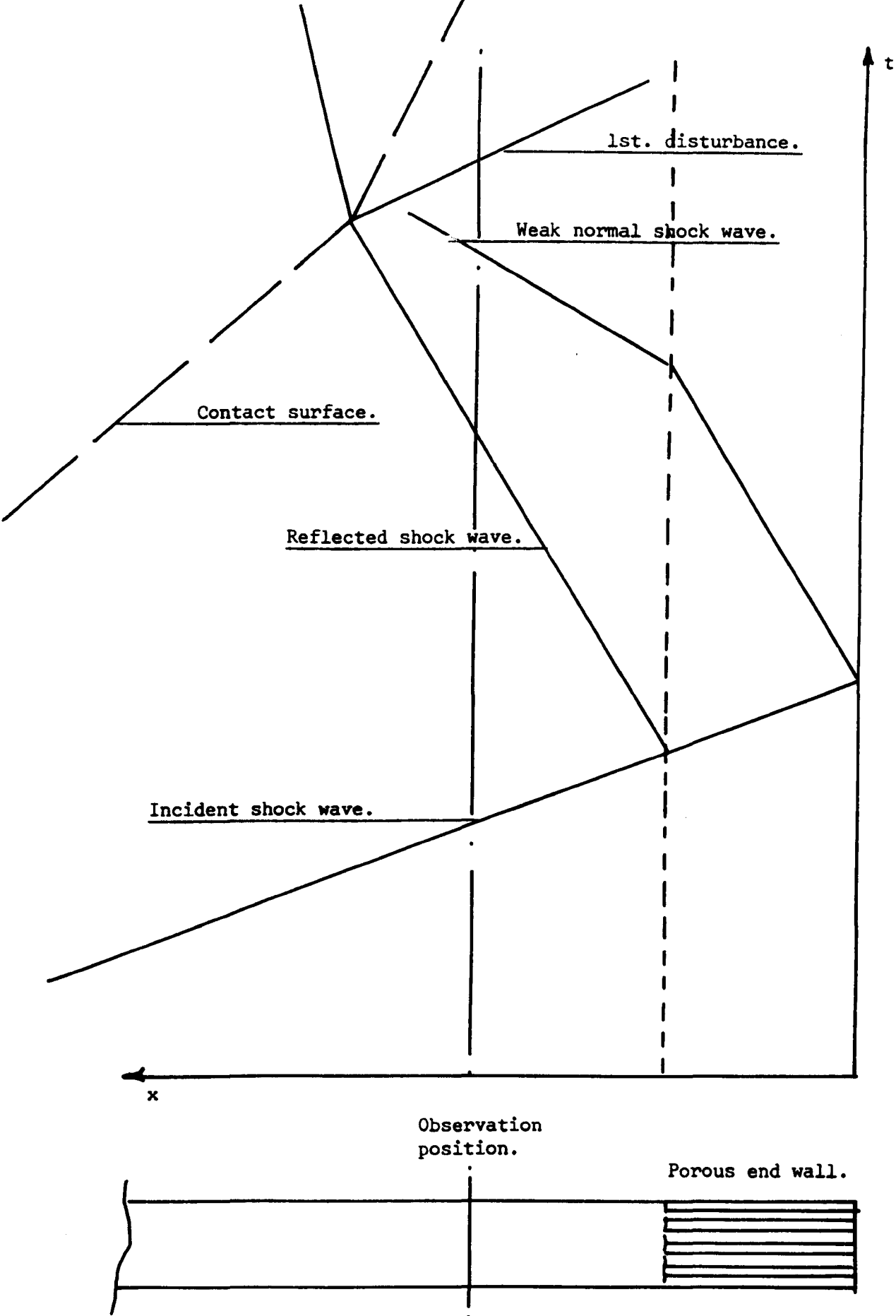


⊙ Carbon dioxide.

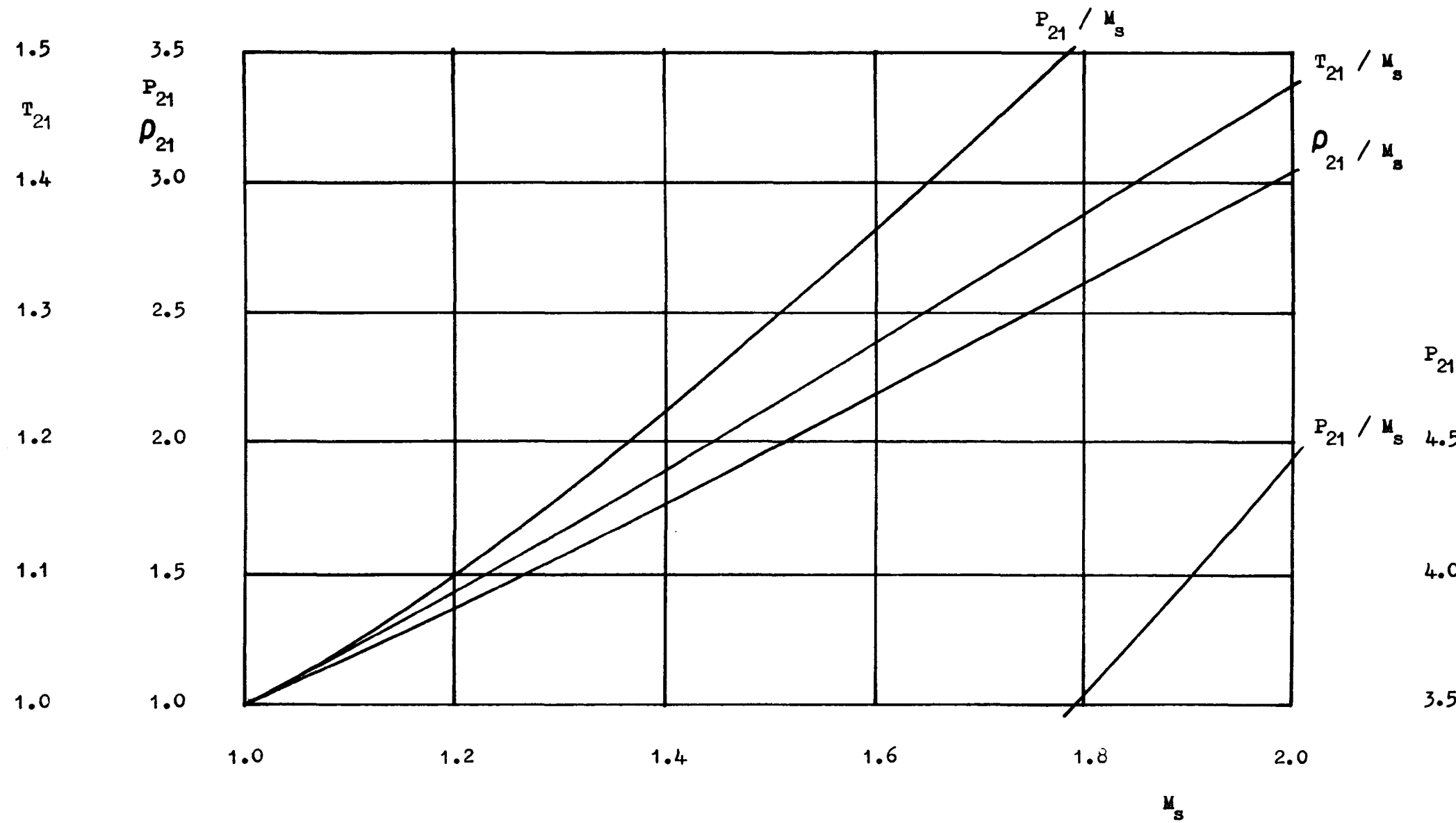
△ Argon.

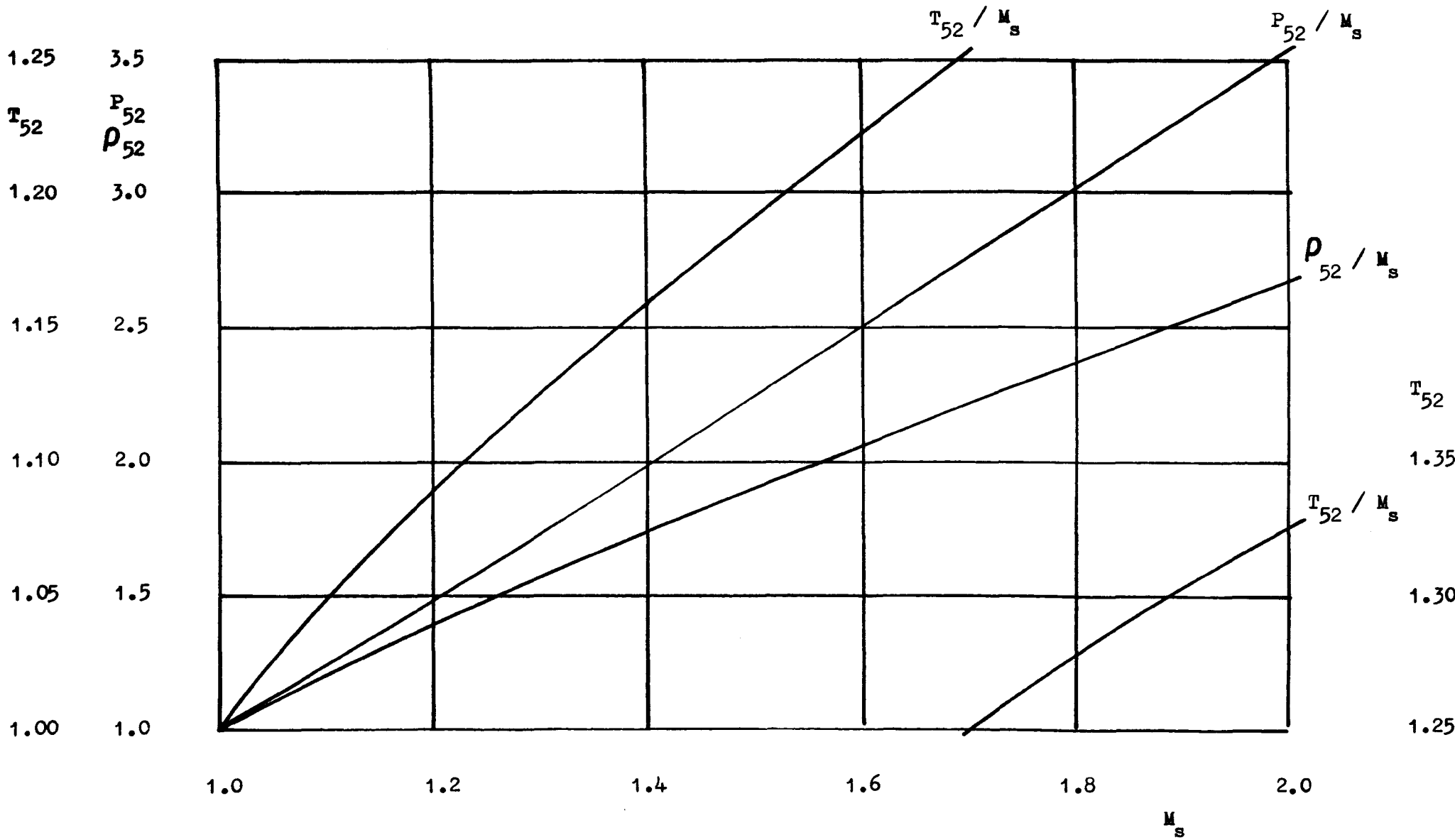
VARIATION OF HALF HEIGHT WIDTHS OF WEAK NORMAL SHOCK WAVES WITH DISTANCE FROM END FACE.

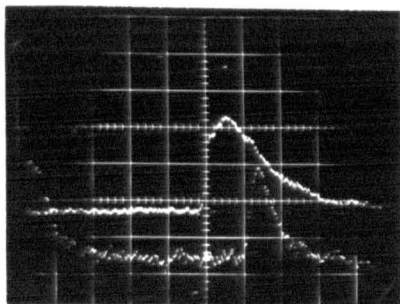
FIGURE: 3.4.14.



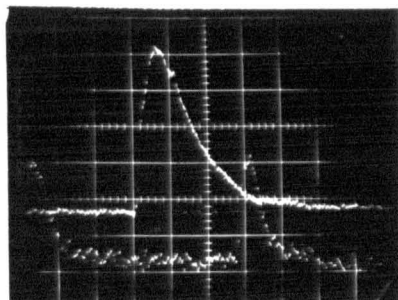
WAVE DIAGRAM OF WAVE PROCESS ASSOCIATED WITH WEAK
NORMAL SHOCK PROPAGATION.



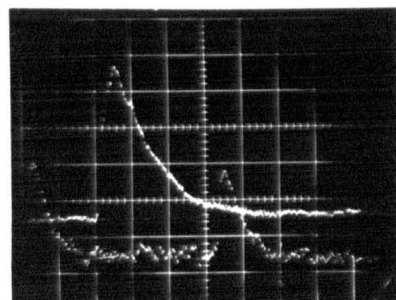




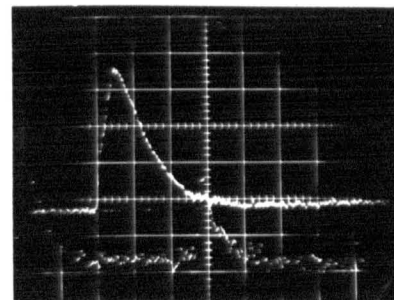
RUN 3271
 $M_s = 1.082$ $P_i = 500$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $20 \mu\text{s/cm.}$



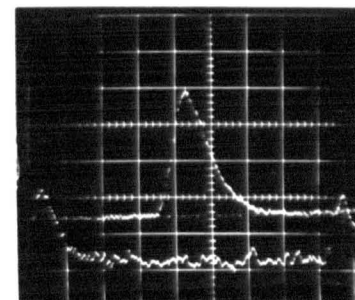
RUN 3272
 $M_s = 1.114$ $P_i = 500$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $20 \mu\text{s/cm.}$



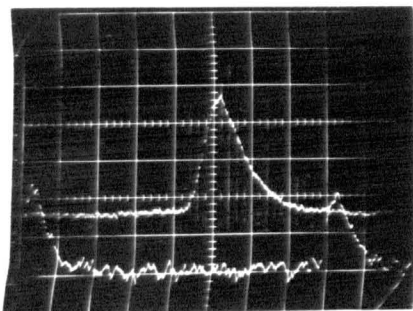
RUN 3273
 $M_s = 1.251$ $P_i = 250$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $20 \mu\text{s/cm.}$



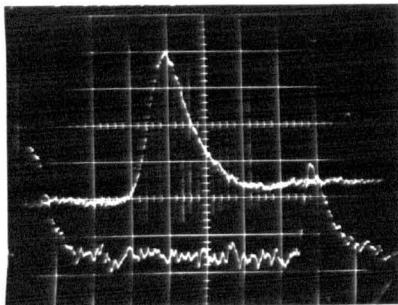
RUN 3274
 $M_s = 1.425$ $P_i = 150$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $20 \mu\text{s/cm.}$



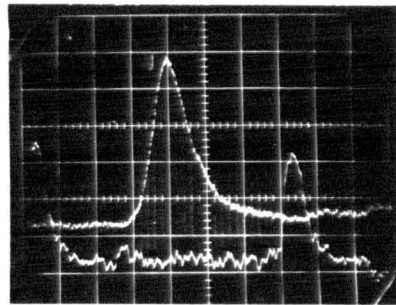
RUN 3284
 $M_s = 1.556$ $P_i = 150$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $10 \mu\text{s/cm.}$



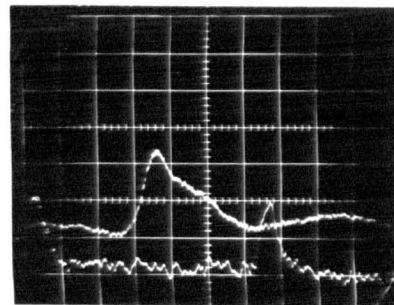
RUN 3285
 $M_s = 1.625$ $P_i = 125$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $10 \mu\text{s/cm.}$



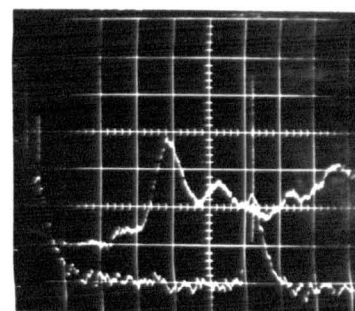
RUN 3276
 $M_s = 1.742$ $P_i = 100$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $10 \mu\text{s/cm.}$



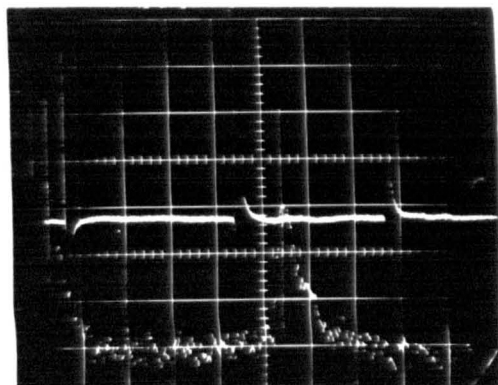
RUN 3277
 $M_s = 1.881$ $P_i = 100$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $10 \mu\text{s/cm.}$



RUN 3282
 $M_s = 2.040$ $P_i = 75$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $10 \mu\text{s/cm.}$



RUN 3281
 $M_s = 2.21$ $P_i = 50$ torr.
UPPER TRACE: $10 \mu\text{s/cm.}$
LOWER TRACE: $10 \mu\text{s/cm.}$

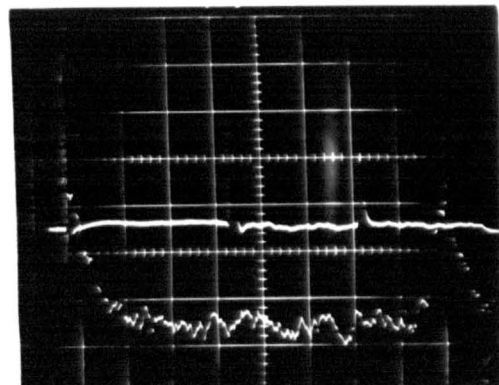


RUN 3295

$M_s = 1.271$ $P_1 = 250$ torr.

UPPER TRACE: $100 \mu\text{s/cm}$.

LOWER TRACE: $20 \mu\text{s/cm}$.

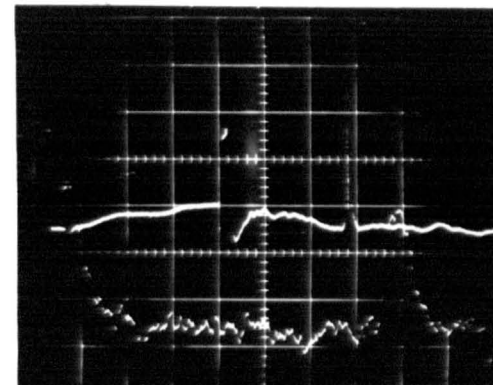


RUN 3291

$M_s = 1.553$ $P_1 = 150$ torr.

UPPER TRACE: $100 \mu\text{s/cm}$.

LOWER TRACE: $10 \mu\text{s/cm}$.

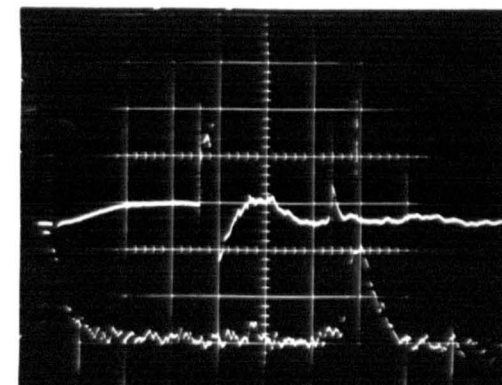


RUN 3290

$M_s = 1.818$ $P_1 = 100$ torr.

UPPER TRACE: $100 \mu\text{s/cm}$.

LOWER TRACE: $10 \mu\text{s/cm}$.

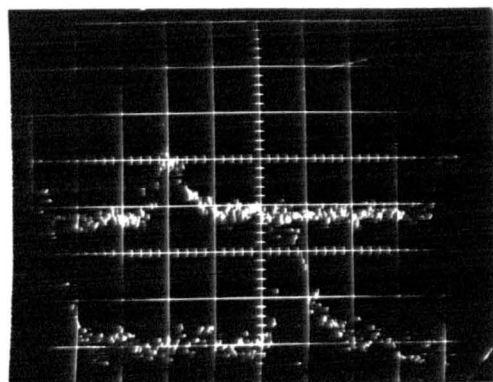


RUN 3286

$M_s = 1.949$ $P_1 = 75$ torr.

UPPER TRACE: $100 \mu\text{s/cm}$.

LOWER TRACE: $10 \mu\text{s/cm}$.

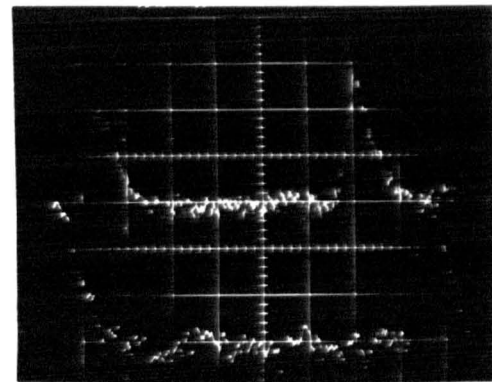


RUN 3294

$M_s = 1.247$ $P_1 = 250$ torr.

UPPER TRACE: $20 \mu\text{s/cm}$.

LOWER TRACE: $20 \mu\text{s/cm}$.

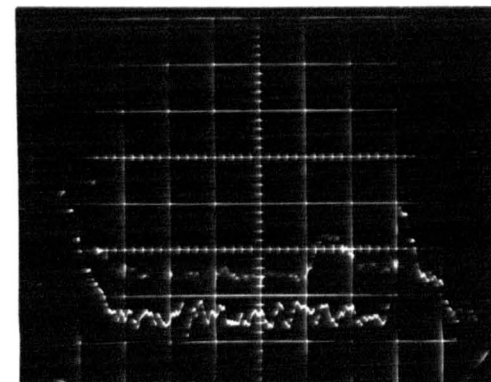


RUN 3292

$M_s = 1.577$ $P_1 = 150$ torr.

UPPER TRACE: $20 \mu\text{s/cm}$.

LOWER TRACE: $10 \mu\text{s/cm}$.

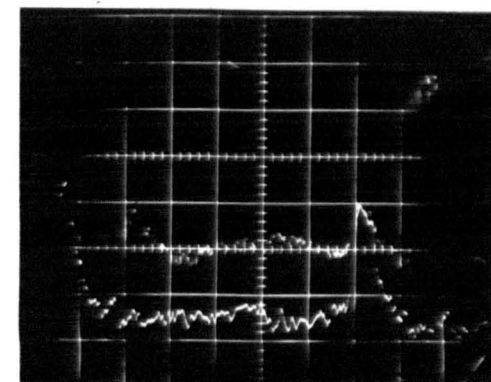


RUN 3289

$M_s = 1.752$ $P_1 = 100$ torr.

UPPER TRACE: $20 \mu\text{s/cm}$.

LOWER TRACE: $10 \mu\text{s/cm}$.

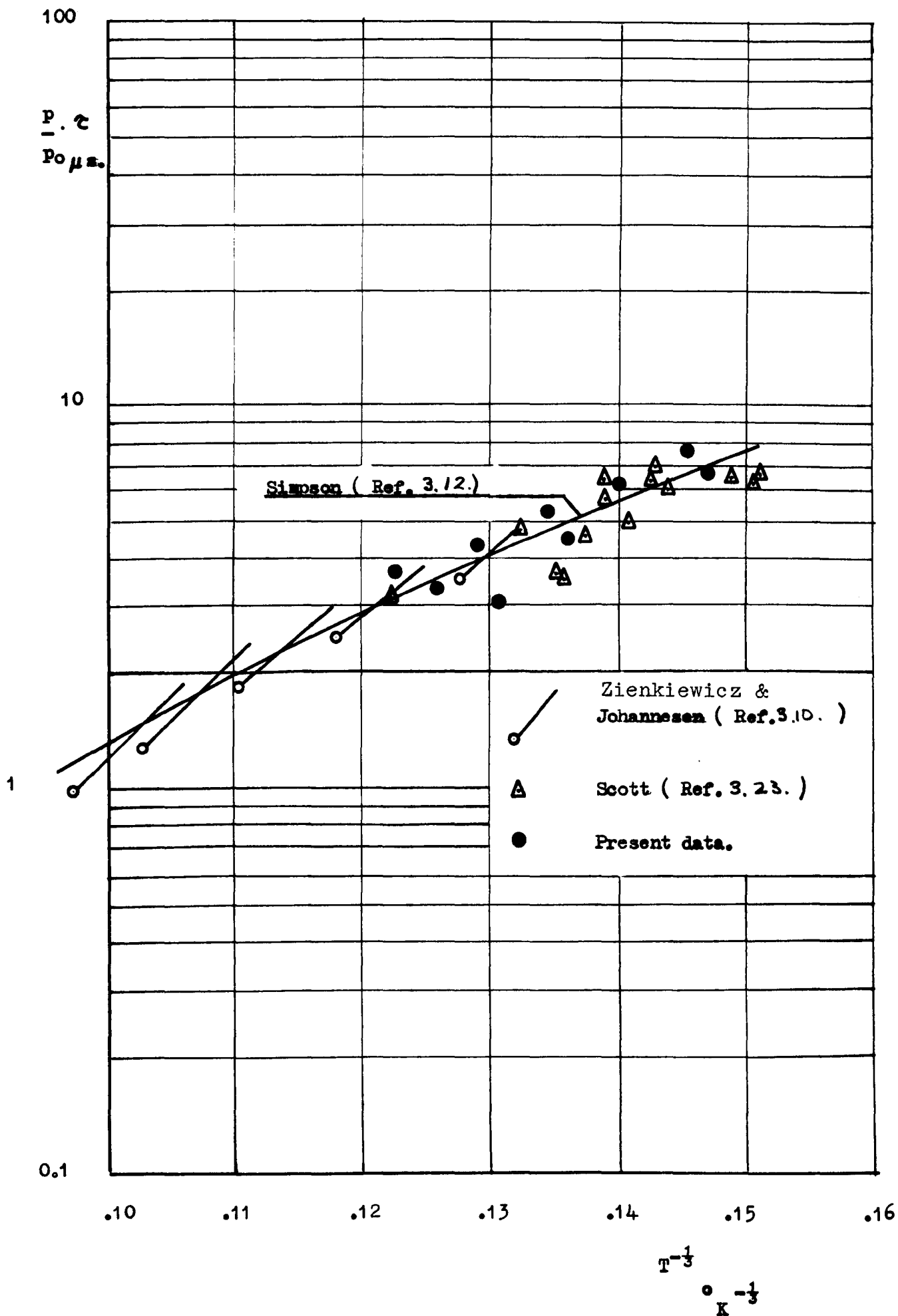


RUN 3288

$M_s = 2.035$ $P_1 = 75$ torr.

UPPER TRACE: $20 \mu\text{s/cm}$.

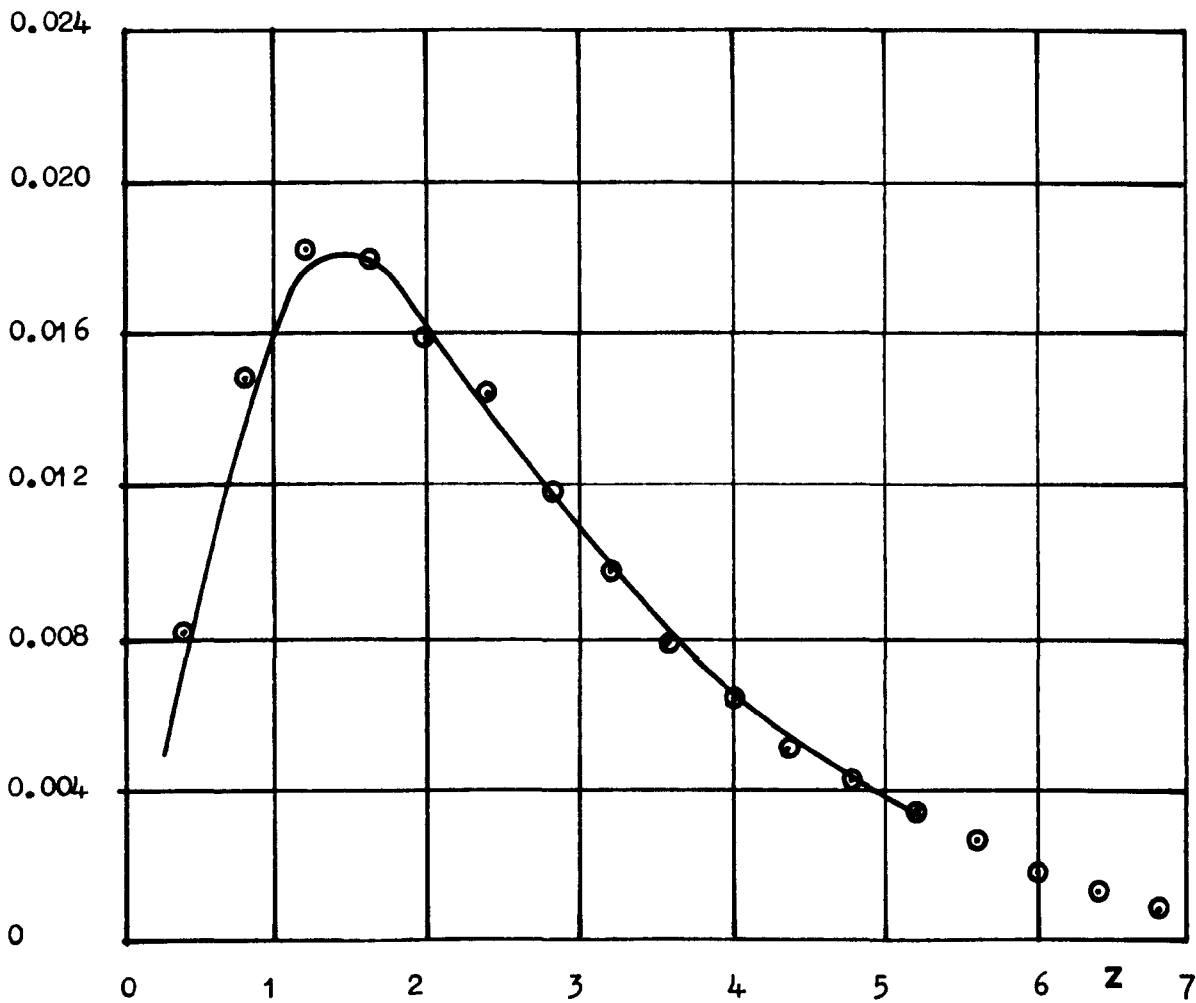
LOWER TRACE: $10 \mu\text{s/cm}$.



Solid line is theoretical profile.

$$\frac{1}{\rho_1} \frac{d\rho}{dz}$$

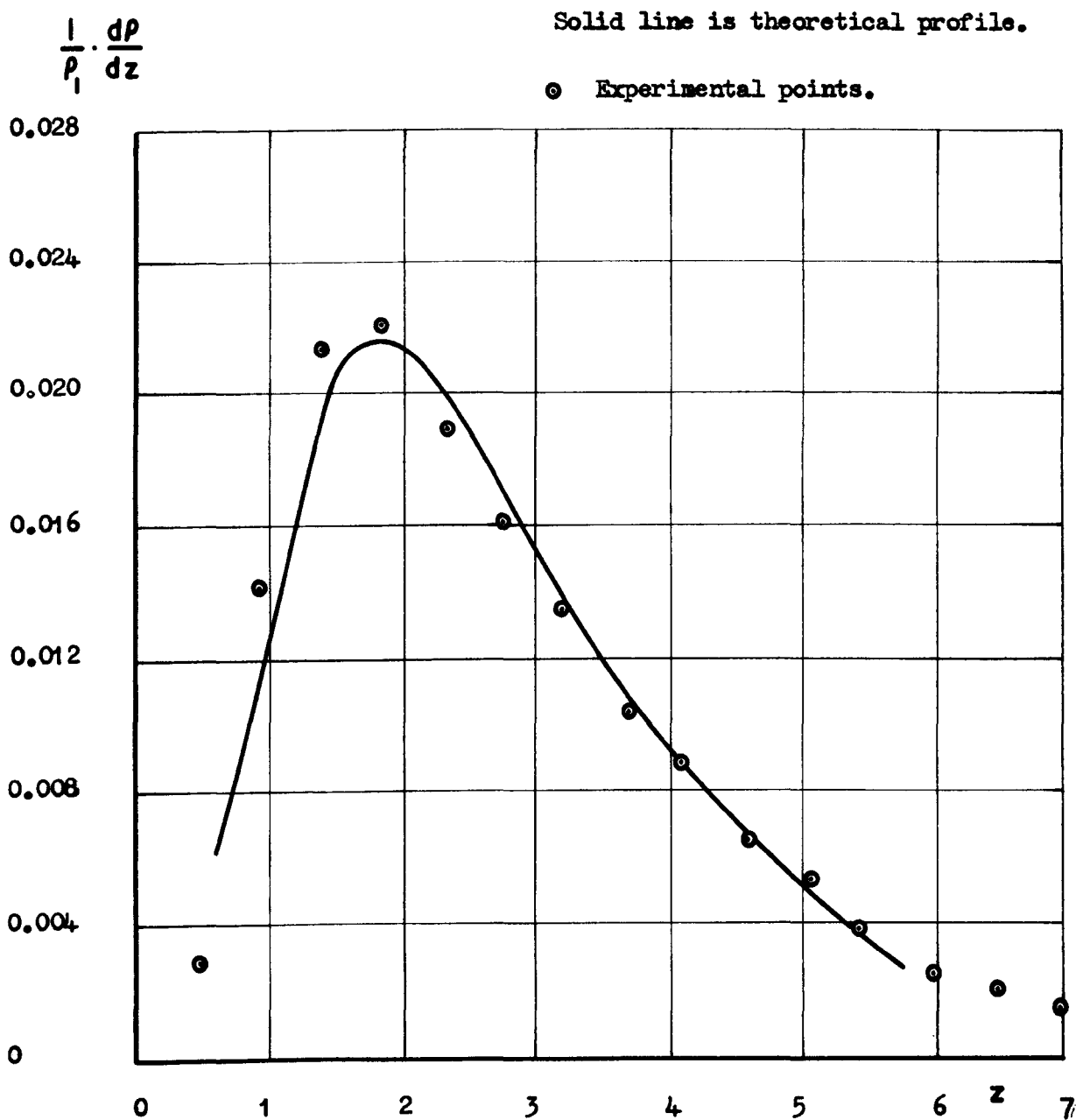
⊙ Experimental points.



RUN 3272. M = 1.034.

COMPARISON OF THEORETICAL AND EXPERIMENTAL
PROFILES OF FULLY DISPERSED SHOCK WAVE.

FIGURE: 3.4.21.



RUN 3274 M = 1.040 .

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4. CONCLUSIONS AND RECOMMENDATIONS
FOR FUTURE WORK.

4. CONCLUSIONS AND RECOMMENDATIONS
FOR FUTURE WORK.

This work has been a qualified success in that it has not been possible to develop the topics to a stage where a substantial contribution can be made to the knowledge of vibrational behaviour in gas molecules. However, the techniques have been developed to a stage where they can be applied to studies in a large low density shock tube, which will overcome many of the limitations that have been imposed on the present experiments. Such a shock tube is currently under development at the Cranfield Institute of Technology.

The three extensions to the theory which have been investigated have been developed to a stage where comparisons can be made between theoretical and experimental profiles when the latter become available.

The study of viscous vibrational relaxation has confirmed the earlier order of magnitude estimates of the effects of diffusion on the relaxation process. That is, the effects of diffusion are either negligible within the relaxation resisted parts of the shock waves or contained within a negligibly thin viscous shock front in the particular case of the partly dispersed shock wave.

The weakest aspect of this extension to the theory was the use of a bulk viscosity model for the behaviour of the rotational mode. Any future study of this topic should include the introduction of a more rigorous description of the rotational behaviour. An alternative approach to the development of the perturbation scheme should also be investigated in this case. The present approach, which has been based on the application of the method of matched asymptotic expansions in the velocity/distance plane, has proved to be adequate for these purposes. However, the lack of coupling between the inner and outer solutions gives cause for concern. A future scheme might be based on a solution for the velocity derivative/velocity plane, and an intermediate asymptotic expansion sought for the region between the upstream limit of the relaxation region and the downstream limit of the diffusion resisted shock front. This intermediate region can be studied experimentally in a low density shock tube.

The theoretical study of the transition from a fully dispersed to a partly dispersed shock wave has shown that the effects of diffusion are greatest in terms of the order of the scaling parameter at the exact transition.

The width of the diffusion resisted shock front was still less than 1% of the thickness of the relaxation resisted parts of the shock wave in carbon dioxide. Only the exact transition was investigated for the present purposes, but any future study should include the investigation of wave strengths slightly above and slightly below the exact transition, if only for the sake of completeness. Here again, the experimental study of the transition would be facilitated by the use of a low density shock tube.

The study of bimodal vibrational relaxation could not be taken further because of the lack of information regarding the relaxation times involved, although there is scope for computations based on order of magnitude estimates of these relaxation times, which might prove to be useful in a future study.

The perturbation scheme used for the present study was slightly restrictive, in that the case where a rapid transfer of energy occurred between the two vibrational modes has to be excluded from the solution because of its singular nature.

This is not unduly restrictive because, in this situation, the two modes will be in resonance or near resonance and can be treated as a single combined mode. Carbon dioxide is an example of this in that the bending modes are in resonance with the symmetric stretching mode. In this case, the two modes in the bimodal model would be the combined bending and symmetric stretching modes and the asymmetric stretching mode. However, it should be borne in mind that this extension to the theory in common with the others is intended for more general application and not restricted to carbon dioxide. Nevertheless, the near resonance situation is of interest, and a solution for this would ensue from a study of the singularity. An inspection of the equations indicates that the method of matched asymptotic expansions may be applicable to this problem.

At the outset of this work, doubts existed regarding relaxation time measurements obtained from strong incident shock waves, because of the large departures from equilibrium. The analysis of second order unimodal vibration served to demonstrate that, even if this was true, there would be negligible effects in shock waves having Mach numbers less than 1.1 in carbon dioxide.

It is noted that since the completion of this work Simpson et al (Ref. 4.1) have shown that the relaxation time does not depend on the departure from equilibrium within the relaxation regions of strong incident shock waves.

The theoretical profiles obtained in this case could be compared to the experimental profiles of strong incident shock waves on an order of magnitude basis only, because the assumptions of constant relaxation time etc. rendered the theory applicable to weak waves only. However, there is ample scope for computed solutions in which the variations in these parameters can be allowed.

The experimental studies of the strong incident shock wave have demonstrated the usefulness of the conventional schlieren system for the study of vibrational relaxation. The relaxation times obtained from the strong incident shock waves are in reasonable agreement with results obtained elsewhere and the centre portion of the experimental density gradient profiles agree closely with theoretical curves calculated from the relaxation time data of Simpson et al (Ref. 4.1).

The departure from the Simpson curve at the frozen end of the relaxation region is attributable to the curvature of the shock wave, but the precise source of the departure at the equilibrium end of the region cannot be identified.

Any future investigation of the strong incident shock wave with the conventional schlieren system should make use of the electronic data reduction which has been proposed. This will enable a more detailed study to be made of the relaxation region by reducing the errors incurred in the manual data reduction. The results indicate that this technique would not be of use in the present shock tube because of its small diameter and the extent of the non-uniformities. However, if the strong incident shock wave can be generated in a larger shock tube and only the core of the flow is studied, then the affect of these non-uniformities will be reduced permitting a more detailed study of the relaxation process.

The foregoing remarks apply equally well to the study of the weak incident shock wave. The results from these experiments showed that the perforated plate provided an excellent means of producing weak normal shock waves which were as well formed as the original incident shock waves.

Unfortunately, the exact transition from a fully dispersed shock wave to a partly dispersed shock wave was obscured by the curvature of the shock wave. However, the theoretical results indicate that it would not have been possible to have observed the transition with the time resolved schlieren system in any case, because of its inadequate spatial resolution.

The comparison of the experimental profiles with the theoretical profiles for the basic fully dispersed/partly dispersed shock wave solutions showed that the agreement was good and that the shock waves were aswell formed as could be expected in the small diameter shock tube. These results also provided a guide for the results which could be expected with the waves produced by the porous end face technique.

The porous end face technique has been subjected to further extensive study. Considerable knowledge has been gained about the technique, although little improvement has been made to the results which have been obtained with it. The discovery that the wave formation had been fortuitous was regarded as something of a setback.

The results obtained with the holed end face indicated that it should be possible to produce well formed shock waves by this technique in other gases as well as carbon dioxide if mixtures of gases are studied in order to reduce the bifurcation of the reflected shock wave.

It was intended that this technique should provide relaxation timedata for small departures from equilibrium at elevated temperatures. This requirements has been met elsewhere with high temperature ultrasonic measurements, Carnevale et al (Ref. 4.2), but the present technique may still be capable of providing information regarding wave structure which cannotbe obtained from ultrasonic measurements.

The relaxation times which have been obtained do not contribute much to the knowledge of vibrational behaviour in carbon dioxide. However, there was general agreement with results obtained elsewhere, thereby confirming the applicability of this technique to the study of vibrational behaviour.

All of the experiments have confirmed the usefulness of the schlieren system for the study of vibrational relaxation. The main limitation on the performance of the schlieren system was its spatial resolution.

It is concluded that the experimental and theoretical techniques which have been developed should be applied to the study of vibrational relaxation in gases by way of shock waves in a large, low density shock tube, such as that under development at the Cranfield Institute of Technology. Every advantage should be taken of that facility to overcome the limitations which have been imposed on the present experiments.

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APPENDIX 1

THE EQUATIONS GOVERNING THE ONE-

DIMENSIONAL FLOW OF A RELAXING GAS

SUMMARY

The equations governing the one-dimensional flow of a relaxing gas are developed from the conservation of mass, momentum and energy. The rate equations appropriate to the relevant aspects of vibrational relaxation are also developed from the conservation of chemical species. The appropriate boundary conditions for equilibrium at the upstream and downstream limits of the flow are applied to the equations and they are non-dimensionalised by the upstream conditions.

NOTATION.

a	Speed of sound.
C	Specific heat.
D	Diffusion coefficient.
e	Energy.
F	Total momentum.
h	Enthalpy.
H	Total enthalpy.
k	Constant.
M	Mach number.
n	Number.
$\dot{N}$	Number rate of production of molecular species.
p	Pressure.
p_{11}	X-wise component of stress tensor, p_{ij} .
q_1	X-wise component of energy flux vector, q_i .
Q	Mass flow.
R_g	Particular gas constant.
R	Rate term.
s	Entropy.
t	Time.
T	Temperature.
u	Velocity.
v, V	Non-dimensionalised velocity.
x	Co-ordinate.
z, Z	Non-dimensionalised co-ordinate.
α	Constant.
β	Non-dimensionalised specific heat.
c	Perturbation parameter.
κ	Equivalent bulk viscosity.
μ	Coefficient of viscosity.
v	Vibrational mode.
ρ	Density.

NOTATION CONT'D.

γ	Ratio of specific heats.
θ, θ	Non-dimensionalised temperature.
σ	Perturbation parameter.
τ	Relaxation time.
δ	Perturbation parameter.
Γ	Constant.

SUBSCRIPTS.

s	Downstream state.
t	Translational mode.
r	Rotational mode.
1	Combined translational and rotational modes.
2	First vibrational mode.
3	Second vibrational mode.
f	Frozen state.
e	Equilibrium state.
u	Upstream state.
α	Molecular species.
v	Vibrational modes.

A.1.1. Introduction

The equations governing the one-dimensional flow of a relaxing gas are well known and their derivation can be found in many references, Clarke and McChesney (Ref. A.1.1) and Vincenti and Kruger (Ref. A.1.6) for example.

The equations have been developed from the equations of change derived from the Boltzmann equation. It has been assumed that the translational and rotational modes are classically excited and that the gas is both thermally and calorifically perfect. Viscosity, heat conduction and mass diffusion of the vibrational energy has been included in the model and, because of the dilute nature of the gas, bulk viscosity has been neglected. The rotational behaviour has been neglected. The rotational behaviour has been described by a linear rate equation and coupling between rotation and vibration has been included.

The energy equation has been simplified by assuming that both the Prandtl number and the Schmidt number were constant through the region of interest and that they were numerically equal to a value of $\frac{2}{3}$.

Rate equations have been derived from the species continuity equation for each of the extensions to the theory considered in Part 2. These were as follows:

1. A rate equation describing a single vibrational mode including mass diffusion of the vibrational energy and rotational-vibrational coupling.
2. Two linear rate equations describing the bimodal relaxation of two vibrational modes.
3. A rate equation describing the behaviour of a single vibrational mode with the addition of a second order correction term.

The equations have been non-dimensionalised and the overall shock ratios have been determined.

A.1.2. Basic Equations.

Consider the steady one-dimensional flow of a pure, viscous, compressible, and heat conducting gas with active vibrational modes. It will be assumed that the translational and rotational modes are classically excited, and that the equations of change can be derived from the Boltzmann equation, as for example in reference A.1.1 .

For the case of steady one-dimensional flow these equations of change can be integrated to give,

the conservation of mass,

$$\rho u = Q \quad \text{A.1.1.}$$

the conservation of momentum,

$$\rho u^2 + p_{11} = F \quad \text{A.1.2.}$$

and the conservation of energy,

$$e_t + e_r + \sum_{v=2}^n e_v + \frac{1}{2}u^2 + \frac{q_1}{Q} + \frac{p_{11}}{\rho} = H \quad \text{A.1.3.}$$

where e_t , e_r and e_v are the translational, rotational and vibrational energies per unit mass respectively.

The x-wise component of the stress tensor, p_{11} , is given by,

$$p_{11} = p - \frac{4}{3}\mu \frac{du}{dx} \quad \text{A.1.4.}$$

The gas flows considered will be sufficiently dilute for bulk viscosity to be neglected, and an equivalent bulk viscosity description of the behaviour of the rotational and

vibrational modes will not be required because this description will be provided by rate equations.

The x-wise component of the energy flux vector, q_1 , for a pure gas is,

$$q_1 = - \lambda_t \frac{dT_t}{dx} - \rho D \frac{de_r}{dx} - \rho D \sum_{v=2}^n \frac{de_v}{dx} \quad A.1.5.$$

Only the mass diffusion of the internal mode energies has been considered, as other diffusive affects are negligible. The self diffusion coefficient, D , describes the inter-diffusion of molecules having rotational and vibrational modes at different quantum levels. The molecules are all of the same mass, so the use of a single diffusion coefficient is permissible providing the cross sections are the same for all quantum levels, and the number of inelastic molecular collisions is small in comparison with the total number (Ref.A.1.2). These assumptions are valid for the behaviour of the vibrational modes, but in the case of the behaviour of the rotational mode it must be tacitly assumed that its departure from equilibrium is sufficiently small for the above description to be a reasonable approximation to the true behaviour.

Substituting equations A.1.4 and A.1.5 into equations A.1.2 and A.1.3, we have,

$$\rho u^2 + p - \frac{4}{3} \mu \frac{du}{dx} = F \quad A.1.6.$$

and,

$$e_t + e_r + \sum_{v=2}^n e_v + \frac{1}{2} u^2 + \frac{p}{\rho} - \frac{4}{3} \mu \frac{du}{dx} - \frac{1}{Q} \left\{ \lambda_t \frac{dT_t}{dx} + \rho D \frac{de_r}{dx} + \rho D \frac{d}{dx} \left(\sum_{v=2}^n e_v \right) \right\} = H \quad A.1.7.$$

It will be assumed that the gas is thermally and calorifically perfect, so that the caloric equation of state

is

$$h_t = C_{p_t} T_t \quad \text{A. 1.8.}$$

where C_{p_t} and C_{v_t} are the translational specific heats at constant pressure and volume respectively, and are assumed constant, and where $C_{p_t} - C_{v_t} = R_g$, the gas constant for the particular gas.

For a pure gas the thermal equation of state is simply,

$$p = \rho R_g T_t \quad \text{A. 1.9.}$$

Making use of equations A.1.8 and A.1.9, equation A.1.7 can be written,

$$C_{p_t} T_t + e_r + \sum_{v=2}^n e_v + \frac{1}{2} u^2 - \frac{\mu}{Q} \left\{ \frac{\lambda_t}{\mu} \frac{dT_t}{dx} + \frac{\rho}{\mu} \frac{de}{dx} \right. \\ \left. + \frac{\rho}{\mu} \frac{d}{dx} \left(\sum_{v=2}^n e_v \right) + \frac{4}{3} u \frac{du}{dx} \right\} = H \quad \text{A. 1.10.}$$

Two well known parameters appear in this equation, namely the Prandtl number and the Schmidt number, defined respectively as,

$$\text{Prandtl number} = \frac{\mu C_{p_t}}{\lambda_t} \quad \text{A. 1.11.}$$

$$\text{Schmidt number} = \frac{\mu}{\rho D}$$

It will be assumed that both of these parameters are constant throughout the region of interest and numerically equal to $\frac{2}{3}$, which is a reasonable assumption for most simple gases in that it enables the equations to be simplified whilst retaining the essential physics.

The boundary conditions for this one-dimensional flow are simple boundedness conditions, that is, that the flow must be in equilibrium at $x = \pm \infty$, viz.,

$$\frac{d^n}{dx^n} \rightarrow 0 \text{ as } x \rightarrow \pm \infty \text{ for all } n \geq 1 \quad \text{A.1.12.}$$

These limits will be identified in terms of the thermodynamic variables by the subscripts ∞ , for $x \rightarrow -\infty$, and s , for $x \rightarrow +\infty$, respectively. To give direction to the flow it will be assumed that $u_\infty > u_s$.

If we write,

$$\hat{x} = \int_0^x \frac{1}{\mu} \frac{Q}{u} d\hat{x}$$

and take μ as a constant, equation A.1.10 becomes,

$$\begin{aligned} C_p T_t + e_r + \sum_{v=2}^n e_v + \frac{1}{2} u^2 - \frac{d}{d\hat{x}} \{ C_p T_t + e_r \\ + \sum_{v=2}^n e_v + \frac{1}{2} u^2 \} = H \end{aligned}$$

It will be noted that the assumption that μ is constant restricts the theory to weak waves, through which the change in μ can be neglected.

Applying the above boundary conditions to the integral of this equation we have,

$$C_p T_t + e_r + \sum_{v=2}^n e_v + \frac{1}{2} u^2 = H \quad \text{A.1.13.}$$

The statement of the problem is completed by a description of the behaviour of the rotational and vibrational modes. This description is provided by the rate equations, as will be discussed in the following section.

A.1.3. The Rate Equation.

The conservation of mass, equation A.1.1, can be shown to originate from a consideration of species continuity (Ref. A.1.1), but its derivation is so commonplace as to be accepted from the outset. However, in order to consider the behaviour of the internal modes of the molecule, we must return to the species continuity, which for the case of steady flow is,

$$\frac{d}{dx} \{ n_{\alpha} (u + u_{\alpha}) \} = \dot{N}_{\alpha} \quad \text{A.1.14.}$$

where n_{α} is the number of molecules of species α , u_{α} is the diffusion velocity of the particular species, and $\dot{N}_{\alpha}$ is the number rate of production of species α per unit volume due to reactions. We are considering a pure gas, so that the term 'species' refers to molecules in a particular internal quantum state.

We can write (Ref. A.1.1.)

$$n_{\alpha} u_{\alpha} = nD \frac{d}{dx} (n_{\alpha}/n)$$

where it is assumed that the diffusion coefficient, D , is the same for all quantum levels, which is reasonable in this case. Substituting this expression into equation A.1.14, multiplying through by the particular internal energy, ϵ_{α} , possessed by a molecule having an internal quantum state α , and summing over all quantum levels, we have,

$$\frac{d}{dx} \left(e + \rho \frac{D}{Q} \frac{de}{dx} \right) = \frac{1}{Q} \sum_{\alpha} \dot{N}_{\alpha} \epsilon_{\alpha} \quad \text{A.1.15.}$$

This equation can be applied to any internal mode, so we can write,

$$\frac{d}{dx} \left(e_r + \rho \frac{D}{Q} \frac{de_r}{dx} \right) = \frac{1}{Q} \sum_{\alpha} \dot{N}_{\alpha r} \epsilon_{\alpha r} \quad \text{A.1.16.}$$

for the rotational mode, and,

$$\frac{d}{dx} \left(e_v + \rho \frac{D}{Q} \frac{de_v}{dx} \right) = \frac{1}{Q} \sum_{\alpha} \dot{N}_{\alpha v} \epsilon_{\alpha v} \quad A.1.17.$$

for the v th vibrational mode. Here again, only the latter of these equations is valid in any strict sense, because their derivation is based on the assumption that inelastic collisions are infrequent. For want of a better description of the rotational behaviour, it is assumed that its departure from equilibrium is so small as to render equation A.1.16 a reasonable approximation to the true behaviour.

In accordance with the assumption of a perfect gas, it will be assumed that the rotational and vibrational mode energies can be characterised by temperatures, that is,

$$e_r = C_r T_r \quad \text{and} \quad e_v = C_v T_v \quad A.1.18.$$

Thus, writing,

$$R_r = \frac{1}{\rho C_r} \sum_{\alpha} \dot{N}_{\alpha r} \epsilon_{\alpha r} \quad \text{and} \quad R_v = \frac{1}{\rho C_v} \sum_{\alpha} \dot{N}_{\alpha v} \epsilon_{\alpha v}$$

equations A.1.16, and A.1.17, become,

$$u \frac{d}{dx} \left(T_r + \rho \frac{D}{Q} \frac{dT_r}{dx} \right) = R_r \quad A.1.19.$$

and,

$$u \frac{d}{dx} \left(T_v + \rho \frac{D}{Q} \frac{dT_v}{dx} \right) = R_v \quad A.1.20.$$

The simplest and most commonly used form for the rate term, R , expresses a linear dependence on the departure from equilibrium of the particular mode concerned (Ref A.1.3), that is,

$$R = \frac{1}{\tau} \{T(\text{EQUILIBRIUM}) - T\} \quad A.1.21.$$

where τ is the characteristic time for the particular energy transfer process concerned, and is known as the relaxation time, T is the characteristic temperature of the particular mode concerned and $T(\text{EQUILIBRIUM})$ is the equilibrium value of that temperature.

Energy can be transferred between any of the active modes, that is: translation to vibration, translation to rotation, rotation to vibration, or between vibrational modes. It will be assumed that each of these interchanges of energy can be expressed in the form of a linear combination of terms like equation A.1.21, where the equilibrium temperature of one mode is the temperature of the other mode concerned. For the rotational mode we have, therefore, from equation A.1.19,

$$u \frac{d}{dx} \left(T_r + \rho \frac{D}{Q} \frac{dT_r}{dx} \right) = \frac{1}{\tau_r} (T_t - T_r) + \sum_v \frac{1}{\tau_{vr}} (T_v - T_r) \quad \text{A.1.22.}$$

and from equation A.1.20, for the v th vibrational mode,

$$u \frac{d}{dx} \left(T_v + \rho \frac{D}{Q} \frac{dT_v}{dx} \right) = \frac{1}{\tau_v} (T_t - T_v) + \frac{1}{\tau_{rv}} (T_r - T_v) + \text{intervibrational}$$

coupling terms.

A.1.23.

where τ_r , τ_v , τ_{vr} , & τ_{rv} , are the relaxation times which characterise each of the energy transfer processes. In general the τ_{vr} are not equal to the τ_{rv} .

This problem will be investigated for the case of a single vibrational mode. This mode will be identified by the subscript 2, that is, $v = 2$. (This notation enables subscript 1 to be reserved for the combined translational and rotational modes for later use.)

It will be assumed that the rotational/vibrational transfer occurs independently of the translational/rotational transfer so that,

$$\tau_{2r} = \tau_{r2} \frac{C_r}{C_2}$$

in which case the rate equations become,

$$u \frac{d}{dx} (T_r + \rho \frac{D}{Q} \frac{dT_r}{dx}) = \frac{1}{\tau_r} (T_t - T_r) + \frac{1}{\tau_{r2}} \frac{C_2}{C_r} (T_2 - T_r) \quad A.1.24.$$

and,

$$u \frac{d}{dx} (T_2 + \rho \frac{D}{Q} \frac{dT_2}{dx}) = \frac{1}{\tau_2} (T_t - T_2) + \frac{1}{\tau_{r2}} (T_r - T_2) \quad A.1.25.$$

Two further forms of the rate term for the vibrational modes will be considered. The first of these is the case of bimodal vibrational relaxation in which there is an exchange of energy between the two vibrational modes which is independent of the translational and rotational modes. It will be assumed for this case that the gas is non-heatconducting and inviscid, and that the rotational mode is in equilibrium with the translational mode.

The rate equations in this case are,

$$u \frac{dT_2}{dx} = \frac{1}{\tau_2} (T_t - T_2) + \frac{1}{\tau_{23}} \frac{C_3}{C_2} (T_3 - T_2) \quad A.1.26.$$

and,

$$u \frac{dT_3}{dx} = \frac{1}{\tau_3} (T_t - T_3) + \frac{1}{\tau_{23}} (T_2 - T_3) \quad A.1.27.$$

Two special cases of bimodal vibrational relaxation have been investigated elsewhere, namely series and parallel relaxation, in which $\tau_3 \rightarrow \infty$, and $\tau_{23} \rightarrow \infty$, respectively. Both of

these cases have been investigated in some generality by Becker (Ref.A.1.4), and the case of parallel relaxation has been investigated in depth by Clarke and Rodgers (Ref.A.1.5). The object here will be to investigate the general case as expressed by equations A.1.26 and A.1.27 .

The third form of the rate term to be investigated involves the addition of a second order term to the linear term given by equation A.1.21 for the case when $v = 2$. The gas will again be assumed inviscid and non-heatconducting and the rotational mode will be assumed to be in equilibrium with the translational mode.

It can be shown (Ref.A.1.6) that the linear rate term is the first term in a Taylor series expansion of a more general rate function about a suitable equilibrium state in the flow. Two possible equilibrium states about which the rate term might be expanded are: A real equilibrium state, for example as $x \rightarrow \pm \infty$, or a fictitious equilibrium state obtained with p and T_t fixed at their local values within the flow. In both of these cases the first order rate term is of the form,

$$R_v \sim \left\{ \frac{\partial R_v}{\partial T_v} \right\}_{\text{EQUIL.}}^{-1} \cdot (T_v(\text{EQUIL.}) - T_v) + \text{H.O.} \quad \text{A.1.27. a.}$$

where $\left\{ \frac{\partial R_v}{\partial T_v} \right\}_{\text{EQUIL.}}^{-1}$ is the relaxation time mentioned above. If a real equilibrium is chosen then the relaxation time is evaluated at the reference state, ie. as $x \rightarrow \pm \infty$, and must be constant through the flow region. In the case of a fictitious equilibrium the coefficient is evaluated at the local values of p and T_t , so that it can in general vary through the flow region. For this reason the second form of definition is used more frequently for experimental analysis where the pressure and temperature changes are so large that the dependence of the relaxation time on them has to be taken into account. For analytical solutions it is sometimes more convenient to take the relaxation time as a constant through the flow field, as will be done here, and in which case both definitions of the equilibrium state yield the same first order term.

The second order term of the Taylor series expansion can be found in a similar manner. Expanding the rate term to second order about the local equilibrium state we have,

$$R_v \sim \left\{ \frac{\partial R_v}{\partial T_v} \right\}_{\text{EQUIL.}}^* (T_v \text{ EQUIL.} - T_v) + \frac{1}{2} \left\{ \frac{\partial^2 R_v}{\partial T_v^2} \right\}_{\text{EQUIL.}}^* (T_v \text{ (EQUIL.)} - T_v)^2 \quad \text{A.1.28.}$$

The asterisk in the above equation indicates that a local equilibrium has been used. However, for analytical simplicity it will be assumed that the change in these coefficients through the flow region can be neglected.

Writing,

$$\left\{ \partial R_v / \partial T_v \right\}_{\text{EQUIL.}}^* = 1/\tau_v \quad \text{A.1.29.}$$

and,

$$\left\{ \partial^2 R_v / \partial T_v^2 \right\}_{\text{EQUIL.}}^* = 1/\tau_v' T_{\infty} \quad \text{A.1.30.}$$

equation 2.3.12 becomes,

$$R_v \sim \frac{1}{\tau_v} \{ T_v \text{ (EQUIL.)} - T_v \} \left\{ 1 + \frac{\tau_v}{\tau_v'} \frac{1}{2T_{\infty}} (T_v \text{ (EQUIL.)} - T_v) \right\} \quad \text{A.1.31.}$$

Substituting this rate term into the inviscid rate equation with $v = 2$ we have,

$$u \frac{dT}{dx} = \frac{1}{\tau_2} (T_t - T_2) \left\{ 1 + \frac{\tau_2}{\tau_2'} \frac{1}{2T_{\infty}} (T_t - T_2) \right\} \quad \text{A.1.32.}$$

A.1.4. Non-dimensionalisation.

The variables involved will be non-dimensionalised as follows:

The temperatures will be non-dimensionalised with respect to the upstream temperature, T_∞ , that is,

$$\theta_t = T_t/T_\infty, \theta_v = T_v/T_\infty, \theta_r = T_r/T_\infty. \quad \text{A.1.33.}$$

The velocities will be non-dimensionalised with respect to the upstream velocity, u_∞ , that is,

$$v = u/u_\infty \quad \text{A.1.34.}$$

The independent variable, x , will be non-dimensionalised with respect to the relaxation length of the first vibrational mode, that is,

$$z = x/u_\infty \tau_2 \quad \text{A.1.35.}$$

The relaxation length of the first mode is chosen as the scale of the problem because we shall be primarily concerned with the behaviour of the flow on that scale.

The Mach number of the flow into the wave will be defined as,

$$M_\infty = u_\infty/a_{e\infty} \quad \text{A.1.36.}$$

where $a_{e\infty} (= \sqrt{\gamma_e R_g T_\infty})$ is the upstream equilibrium sound speed, that is, with all of the vibrational modes in equilibrium with the translational and rotational modes, and $\gamma_e (= C_p/C_v)$ is the ratio of the specific heats of the gas.

Non-dimensionalising equation A.1.6 in this manner we have, after some simplification,

$$v^2 + \frac{\theta_t}{\gamma_e} M_\infty^2 - \epsilon v \frac{dv}{dz} = \frac{Fv}{Qu_\infty} \quad A.1.37.$$

where,

$$\epsilon = \left\{ \frac{3\rho_\infty}{4\mu} u_\infty (u_\infty \tau_2) \right\}^{-1} \quad A.1.38.$$

It can be seen that ϵ^{-1} has the appearance of a Reynolds number based on the relaxation length of the first vibrational mode, $u_\infty \tau_2$.

Non-dimensionalising equation A.1.13 we have,

$$\frac{\gamma_t}{\gamma_t - 1} \theta_t + \beta_r \theta_r + \sum_{v=2}^n \beta_v \theta_v + \frac{\gamma_e}{2} M_\infty^2 v^2 = \frac{H\gamma_e}{a_{e\infty}^2} \quad A.1.39.$$

where $\gamma_t (=C_{p_t}/C_{v_t})$ is the ratio of the specific heats when only the translational mode is excited, $\beta_r = C_r/R_g$ and $\beta_v = C_v/R_g$.

The rate equations can be non-dimensionalised to give:

from equations A.1.24, and A.1.25,

$$\epsilon v \frac{d}{dz} \left(\theta_r + \epsilon \frac{d\theta_r}{dz} \right) = R(\theta_t - \theta_r) + \epsilon \frac{\tau_2}{\tau_{r2}} \frac{C_2}{C_r} (\theta_2 - \theta_r) \quad A.1.40.$$

and,

$$v \frac{d}{dz} (\theta_2 + \frac{d\theta_2}{dz}) = (\theta_t - \theta_2) + \frac{\tau_2}{\tau_{r2}} (\theta_r - \theta_2) \quad A.1.41.$$

where,
$$R = \frac{4\mu}{3\rho u_\infty} \cdot \frac{1}{u_\infty \tau_r} .$$

It will be noted that R has the appearance of a Reynolds number based on the rotational relaxation length.

From equations A.1.26 and A.1.27, we have,

$$v \frac{d\theta_2}{dz} = (\theta_t - \theta_2) + \frac{\tau_2}{\tau_{23}} \frac{C_3}{C_2} (\theta_3 - \theta_2) \quad A.1.42.$$

and,

$$v \frac{d\theta_3}{dz} = \frac{\tau_2}{\tau_3} (\theta_t - \theta_3) + \frac{\tau_2}{\tau_{23}} (\theta_2 - \theta_3) \quad A.1.43.$$

From equation A.1.32 we have,

$$v \frac{d\theta_2}{dz} = (\theta_t - \theta_2) \left\{ 1 + \frac{\tau_2}{\tau_2'} \frac{1}{2} (\theta_t - \theta_2) \right\} \quad A.1.44.$$

The boundary conditions become,

$$\frac{d^n}{dz^n} \rightarrow 0 \text{ as } z \rightarrow \pm \infty \text{ for } n \geq 1. \quad A.1.45.$$

Thus, equations A.1.37 and A.1.39 taken with the appropriate rate equations and the above boundary conditions complete the extensions of the theory to be investigated.

A.1.5. Overall Shock Ratios.

It will be useful to consider the relationship between the upstream and downstream limits of the flow.

Applying the upstream and downstream boundary conditions simultaneously to equations A.1.37 and A.1.39, we obtain,

$$v_s + \frac{\theta_s}{v_s \gamma_e M_\infty^2} = 1 + \frac{1}{\gamma_e M_\infty^2} \quad \text{A.1.46.}$$

and,

$$\begin{aligned} \beta_r \theta_s + \frac{\gamma_t}{\gamma_t - 1} \theta_s + \sum_{v=2}^n \beta_v \theta_s + \frac{\gamma_e M_\infty^2}{2} v_s^2 \\ = \beta_r + \frac{\gamma_t}{\gamma_t - 1} + \sum_{v=2}^n \beta_v + \frac{\gamma_e M_\infty^2}{2} \end{aligned} \quad \text{A.1.47.}$$

Eliminating θ_s between these two equations we have,

$$v_s = \frac{1}{\gamma_e + 1} \left(\gamma_e - 1 + \frac{2}{M_\infty^2} \right) \quad \text{A.1.48.}$$

which we shall make use of later.

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APPENDIX 2.

THE SHOCK TUBE AND ANCILLARY EQUIPMENT.

SUMMARY

The modifications which have been made to the Cranfield Institute of Technology 2" shock tube for this work are outlined.

A.2.1. Introduction

All of the experimental aspects of this work were conducted in the College of Aeronautics 2" diameter shock tube. The shock tube and its mode of operation are described herein. The developments which have been made to the shock tube and the ancillary equipment for the present experiments are also described.

A.2.2. Principle of Operation

The shock tube consists of a straight length of pipe divided in two parts by a diaphragm. One part of the tube, the channel, is filled with the gas to be tested, while the other side, the driver, is filled with a gas under pressure until the diaphragm ruptures. This causes a shock wave to propagate through the test gas. A wave diagram, which is a time/distance plot, of this incident shock wave and the subsequent wave processes is shown in Figure A.2.1.

The incident shock wave is balanced by an unsteady expansion fan propagating into the undisturbed driver gas. This reduces the pressure of the driver gas and increases its velocity to match the conditions behind the incident shock wave, that is, across the contact surface.

In general the temperature and density across the contact surface are not matched. It is assumed that the incident shock wave, the contact surface and the unsteady expansion fan are formed instantaneously.

The incident shock wave and the head of the unsteady expansion fan are reflected from the corresponding ends of the shock tube leaving the gas stationary adjacent to the end walls. If the end of the channel is plane the gas between the reflected shock wave and the end wall will be stationary and uniform.

The conditions throughout the wave process can be calculated theoretically from the geometry of the shock tube and the initial conditions in the driver and channel. For maximum accuracy in calculations of the conditions in the shock wave processes it is preferable to compute them from the initial conditions in the channel and a knowledge of the incident shock velocity, which can be measured directly. Normal shock relationships are used to compute the conditions behind both the incident and reflected shock waves.

It can be seen that the flow processes in the shock tube are essentially unsteady and it is necessary to know the duration of steady uniform conditions behind the incident and reflected shock waves. It can be seen from Figure A.2.1 that the duration of uniform conditions, or running time, behind the shock wave can be limited by the arrival of the contact surface, the reflected shock wave, or the reflected head of the unsteady expansion fan. The duration of uniform conditions behind the reflected shock wave can be limited by the arrival of the disturbance resulting from the reflected shock wave/contact surface interaction, the tail of the unsteady expansion, or the head of the unsteady expansion.

For the particular shock tube used in these experiments, the duration of uniform conditions behind the incident shock wave was found to be limited by the arrival of the contact surface. The duration of uniform conditions behind the reflected shock wave was limited by the arrival of a disturbance at the end face resulting from the interaction of the reflected shock wave with the contact surface. For the experiments conducted in this shock tube, these running times were measured directly.

It will be noted that the flow in the shock tube is constant area one-dimensional flow and in principle, ideal for the production of one-dimensional high temperature supersonic flows for the study of shock wave structure.

A.2.3. Description of the Facility

A photograph of the shock tube control room is shown in Figure A.2.2. The shock tube was mounted vertically and can be seen emerging from the floor of the control room in the bottom of the picture.

The dimensions of the shock tube are shown in Figure A.2.3. It can be seen that there were two diaphragm positions, the use of which will be discussed later. These diaphragm positions and the driver of the shock tube were below the floor level of the control room.

A photograph of a working section which was used at an early stage in this work is shown in Figure A.2.4. This working section was square in cross-section while the remainder of the shock tube was circular, so a circular to square transition block was incorporated at the upstream end of the working section.

Just below the working section shown in Figure A.2.4. can be seen the timing section in which two thin film thermometers were positioned to sense the passage of the incident shock wave. The velocity of the incident shock wave was obtained from the time taken for it to pass the two thermometers. This velocity was used to compute the incident shock Mach number and hence the conditions throughout the subsequent wave processes in the shock tube.

The gas inlet valve through which the shock tube was evacuated and refilled with the appropriate test gas can be seen below the timing section in Figure A.2.4. The shock tube was evacuated with a Leybold D2 rotary pump giving an ultimate vacuum of 10^{-3} torr and a combined leak and degassing rate of 2×10^{-3} torr per minute.

The vacuum system was redesigned for the final studies to incorporate a small oil diffusion pump connected directly to the channel of the shock tube via a new gas inlet valve designed specifically to accommodate the diffusion pump. The parts of this valve are shown in Figure A.2.5. and it is shown in its assembled form in Figure A.2.6. The original valve was pneumatically controlled and the vacuum system was exposed to wet compressed air during part of the valve operation. The new valve was designed to be manually operated in order to overcome this problem. The component on the left hand side of the picture in Figure A.2.6. formed part of the shock tube, while the vacuum pumps were connected to the large hole in the side of the assembly. It can be seen from these pictures that the valve consisted of a piston controlled by the leadscrew and hand wheel. The end of the piston was shaped so that it would fit flush with the bore of the shock tube when the valve was closed.

The valve was vacuum sealed with 'O' rings throughout. All of the fixed seals used butted 'O' rings. The seal line which caused greatest concern was that around the sliding joint between the piston and the sleeve. Two 'O' rings were fitted in grooves in the piston, and they can be seen on the right hand side of the leadscrew in Figure A.2.5. The region between these 'O' rings was connected directly to the rotary vacuum pump to maintain the region at a moderate vacuum. An 'O' ring was also fitted to the back of the piston to form a butted seal when the valve was fully opened.

The valve is shown fitted to the shock tube in Figure A.2.7. An Edwards EO2 3" oil diffusion pump can be seen connected to the valve via a Tee piece. The original Leybold D2 rotary pump was retained as a roughing and backing pump, and can also be seen in this picture. An ultimate vacuum of 2×10^{-5} torr and a combined leak and degassing rate of 2×10^{-4} torr per minute. The whole of the vacuum and gas inlet system is shown schematically in Figure A.2.8.

The vacuum pressures were measured with an Alpha particle gauge during the roughing period and with an Ionisation gauge for the determination of the ultimate vacuum. The initial channel pressures were measured with a Texas Instruments pressure galvanometer giving a digital readout of the channel pressure to an accuracy of 10^{-1} torr. For pressures below 100 torr a small tank having one tenth of the volume of the channel was first filled with the test gas and its pressure measured with the pressure galvanometer. The gas was then allowed to expand into the channel of the shock tube to give the required initial channel pressure. In this way, the accuracy of the Texas Instruments pressure galvanometer could be used to give an accuracy in the initial pressure of better than 0.1% over the range of initial channel pressures used in these experiments.

Doubts existed regarding the uniformity of the flow in the square working section, because of the transition from a circular cross section to a square cross section which was necessitated in the original arrangement of the working section.

It was decided that a new working section should be designed and incorporated for the final series of experiments. The working section was made from stainless steel for reasons of ultimate vacuum and high purity. A photograph of the new working section is shown in Figure A.2.9. The original windows from the square working section were used for reasons of economy and availability. The faces of the window mountings were machined back from the bore of the shock tube towards the windows at an angle of 2° to the axis of the shock tube, thus producing the elliptical window aperture which can be seen in the picture.

Experiments were conducted to relate the initial temperature of the test gas to the temperature of the shock tube walls. It was found, that if the channel was filled over a period of the order of several minutes, the gas acquired the same temperature as the channel to within 0.2° . The initial temperature of the test gas could therefore be measured indirectly by a mercury thermometer strapped to the outside walls of the shock tube channel to an accuracy of better than 0.1% .

A.2.4. Real Shock Tube Behaviour

The ideal shock tube behaviour described in Section A,2.2. is not generally obtained in practice due to the presence of the diaphragm in the flow and to 'real gas effects'. These latter effects are predominantly viscous, although other forms of diffusion and dispersion can also modify the flow in the shock tube.

The formation of the incident shock wave takes a finite time, which is dominated by the diaphragm opening time. Compression and expansion waves can continue to propagate between the diaphragm station and the incident shock wave long after the initial rupture of the diaphragm. These waves subject the incident shock wave to a prolonged period of acceleration and cause non-uniformities in the flow behind the incident shock wave.

Many researchers have investigated the influence of the diaphragm opening process on the formation of the incident shock wave. Rose and Nelson (Ref. A.2.1.) were two of the earlier researchers to investigate the nature of the region behind the incident shock wave at different distances from the diaphragm position.

A rise in density with distance behind the incident shock wave was found which increased as measurements were made closer to the diaphragm position. However, at 60 to 70 diameters from the diaphragm the rise in density between the incident shock wave and the contact surface was found to be less than 4% .

One of the more recent and more relevant studies was conducted by Simpson (Ref. A.2.2.). He compared the performance obtained with several diaphragms made from different materials of different thicknesses and having different score depths. The results showed that the time taken for the incident shock wave to accelerate to a uniform velocity was proportional to the product of the incident shock velocity and the diaphragm opening time. The results also showed that the minimum opening time was obtained when the score depth was a minimum consistent with 'clean' petalling.

Obvious modern materials for diaphragms are Melinex and Mylar plastics because of their inherently low inertia for a given bursting pressure. Unfortunately, it was not possible to obtain repeatable bursting pressures at the desired levels with these materials and their use was consequently abandoned. Aluminium alloy was a more suitable material and was readily available for the present experiments.

The material used for the diaphragms ranged in thickness from 26 standard wire gauge to 30 standard wire gauge, except for those diaphragms which were used for the production of the weak incident shock waves. The diaphragms were scored with a sharp edged roller in a cruciform pattern to a depth which was sufficient to ensure 'clean' petalling. A typical repeatability of $\pm 5\%$ in the bursting pressure was obtained with these diaphragms at a nominal bursting pressure of 3 atmospheres. The repeatability might have been improved by the incorporation of a diaphragm bursting device, but this was neither practical nor desirable.

Figure A.2.10 presents some measurements which were made of the variation of the incident shock velocity with position along the shock tube in the vicinity of the working section. It can be seen that the incident shock wave was still accelerating as it approached the observation position. According to the data presented by Simpson, these results indicated that the diaphragms had opening times of the order of 100 microseconds. It can be seen from Figure A.2.10 that the incident shock velocity increased by 1.5% over the 60 cms. upstream of the observation position.

As mentioned above, this acceleration of the incident shock wave gives rise to non-uniformities in the flow behind the shock wave. However, as the region of interest behind the incident shock wave was of the order of 1 cm. wide, the change in conditions due to the acceleration of the incident shock wave would be negligible compared to other errors.

Viscosity is neglected in the ideal theory of the shock tube behaviour. This is equivalent to assuming that there is a velocity discontinuity at the walls of the shock tube, and the gas velocity has its free stream value there. In practice the velocity must be zero at the walls and the free stream velocity must decrease to zero through a boundary layer. This boundary layer grows with distance behind the incident shock wave in the manner shown in Figure A.2.11. The boundary layer affects the shock tube flow in several ways.

Under certain conditions the boundary layer, which will be laminar immediately behind the incident shock wave, becomes turbulent some distance downstream of the incident shock wave.

If this transition occurs ahead of the contact region, the turbulent eddies will further broaden an already broadened contact region. (The contact region is also broadened under the action of diffusion induced by the large changes in density and temperature across it). The result is a broad undefined region of mixed driver and channel gases. This situation is further aggravated by turbulent eddies generated by the driver gas passing through the diaphragm position.

The boundary layer also attenuates the incident shock wave and accelerates the contact region thereby reducing the available running time behind the incident shock wave. The boundary layer grows with distance behind the incident shock wave and eventually engulfs the whole of the shock tube flow to produce fully developed pipe flow.

Hooker (Ref. A.2.3). has investigated the intermixing of driver and channel gas in the contact region by using a spectroscopic technique to determine the time of arrival of the driver gas at the observation position after the passage of the incident shock wave.

The length of the contact region was found to be insensitive to the initial channel pressure and the diaphragm opening time.

Henshall (Ref. A.2.4). has shown that the acceleration of the contact region and the attenuation of the incident shock wave can be related theoretically to the presence of the boundary layer. He showed that the contact region velocity exceeded the ideal particle velocity and that it approached the velocity of the incident shock wave. He also determined the growth and thickness of the boundary layer using a laminar incompressible model. This model was based on a mass flow balance between the inflow through the shock wave and the outflow to the boundary layer.

Mirels (Ref. A.2.5). and Trimpi and Cohen (Ref. A.2.6). applied more rigorous models to the boundary layer analysis, which took into account the essential unsteadiness of the flow and the compressibility and thermal conductivity of the gas. The theories assumed that the ratios of the specific heats were constant so the results were quantitatively correct for weak waves only. Mirels restricted his attention to laminar boundary layers, whereas Trimpi and Cohen extended the theory to cater for both laminar and turbulent boundary layers.

Calculations based on the work of Trimpi and Cohen indicated that the attenuation of incident shock waves in carbon dioxide would be much less than that in other gases. Experimental results have also been presented to support this aspect of the theory.

Spence and Woods (Ref. A.2.7) have also derived solutions for laminar and turbulent boundary layers and have accounted for the attenuation of the incident shock wave and the acceleration of the contact region. They have also determined the distribution of flow variables behind the incident shock wave.

Although many researchers have investigated various aspects of the flow behind the incident shock wave; both experimentally and theoretically, a comprehensive and systematic study has not yet been made (page 75 of Ref. A.2.8). It appears that the most reliable model is obtained by assuming that the velocity of the gas increases behind the incident shock wave from the ideal particle velocity to a velocity equal to the shock velocity at the contact region. However, for the present study, this limiting situation has not been reached and it is considered that such an approach would be inapplicable.

If this limiting case had occurred, a steady state treatment could be applied to the flow immediately behind the incident shock wave to determine the effect of the boundary layer growth on the uniformity of the relaxation region. Mirels (Ref. A.2.9) has treated such a flow as a subsonic compressible flow in an expanding channel with co-ordinates moving with the incident shock wave. The area of this pseudo duct changed in a manner described by,

$$1 - \sqrt{\frac{x}{L}}$$

where x was the distance behind the shock front and L was the distance downstream of the shock wave at which the boundary layer filled the entire cross section of the shock tube, i.e. at closure.

The distribution of the flow variables behind the shock front is very sensitive to the degree of attenuation of the shock wave. If the shock wave is not attenuated, theories predict that the thermodynamic variables and the velocity of the gas will all increase with distance downstream of the shock wave. However, if the incident shock wave is attenuated by only 10%, the downstream of the shock while the velocity will continue to increase.

The boundary layer also caused the incident shock wave to become curved in the manner indicated in Figure A.2.12. Duff and Young (Ref. A.2.10) have made measurements of the profile of incident shock waves in a low density shock tube. The measurements were made with piezoelectric pressure transducers mounted on stations across the shock tube. The shock waves were found to be spherical in profile and had an apparent thickness given by,

$$0.84 \sqrt{L_1 R}$$

where L_1 is the mean free path ahead of the shock wave and R is the radius of the shock tube cross section. The thickness was found to be independent of the shock wave velocity.

Hartunian (Ref. A.2.11) has analysed the shock curvature in terms of the displacement flow resulting from the presence of a boundary layer. He used a laminar boundary layer model with potential flow in the core between the boundary layers and a linearised oblique shock model for the incident shock wave adjacent to the shock tube wall. The analysis produced a result for the shock profile of the form,

$$x = q \sqrt{\frac{y}{p_1}} \times 10^{-2}$$

where x is the distance measure along the shock tube, y is the distance measured across the shock tube, p_1 is the initial channel pressure and q is a function of the incident shock Mach number. For x and y in mm. and p_1 in mm. Hg., q has values at an incident shock Mach number of 2 of 26 in air and 30 in argon; no figures have been calculated for carbon dioxide.

de Boer (Ref. A.2.12) has extended the theory developed by Hartunian to both laminar and turbulent boundary layers in viscous compressible flow. These results of these studies were compared with measurements made in dissociating oxygen and good agreement was obtained. The refinement to the theory indicated that the Hartunian result underpredicted the apparent shock thickness by 25%.

Lin and Fyfe (Ref. A.2.13) have investigated the slope of the incident shock wave to assess its effect in comparison with the effects of curvature. They have shown that contribution to the apparent thickness from sloping of the shock wave is negligible compared to the contribution from shock curvature.

The presence of the boundary layer also generates non-uniformities in the reflected shock region and reduces the running time available behind the reflected shock wave.

Part of the reflected shock wave adjacent to the side walls moves back into the boundary layer. Under certain circumstances the total pressure in the boundary layer exceeds that in the central core and causes the reflected shock wave to propagate at a higher velocity in the boundary layer than in the core. This leads to bifurcation of the reflected shock region as shown in Figure A.2.13. The boundary thickens and can separate from the walls under these conditions and the influence of the boundary layer can extend well into the core of the flow. This process continues as the reflected shock wave progresses down the shock tube until the shock tube is filled with turbulent flow behind the reflected shock wave.

Strehlow and Cohen (Ref. A.2.14) have studied the reflected shock region using a schlieren technique. They discovered that the reflected shock wave accelerated until a steady velocity is attained. When the reflected shock wave has travelled some distance, the bifurcation ceases and the rear of the turbulent region decelerates.

Further studies by Strehlow and Cohen (Ref. A.2.15) indicated that the bifurcation left a region adjacent to the end wall which was stationary and wave free.

The extent of the bifurcation is highly dependent on the particular gas being studied. Bifurcation occurs in carbon dioxide for shock Mach numbers above 1.3, whereas it is non-existent in argon and helium.

Mark (Ref. A.2.16) has analysed the bifurcation process and explained the underlying physics. He has shown that the bifurcation should cease above a certain Mach number, although it has not been possible to find any reports of experimental confirmation of this.

An assessment of the non-uniformities in the flow behind the incident shock waves produced in the present experiments has been made by observing the density gradients in the flow behind the relaxation region. Figure A.2.14 shows a typical results. It can be seen that the trace exhibits a slight second derivative of density behind the relaxation region. If this is attributable to non-uniformities caused by the presence of the boundary layer, the density must have been decreasing with distance downstream of the shock wave. It can be seen from the error estimate presented that this source of error could produce an error of $\pm 2\%$ in the total density change through the relaxation region.

A.2.5. Gas Purity

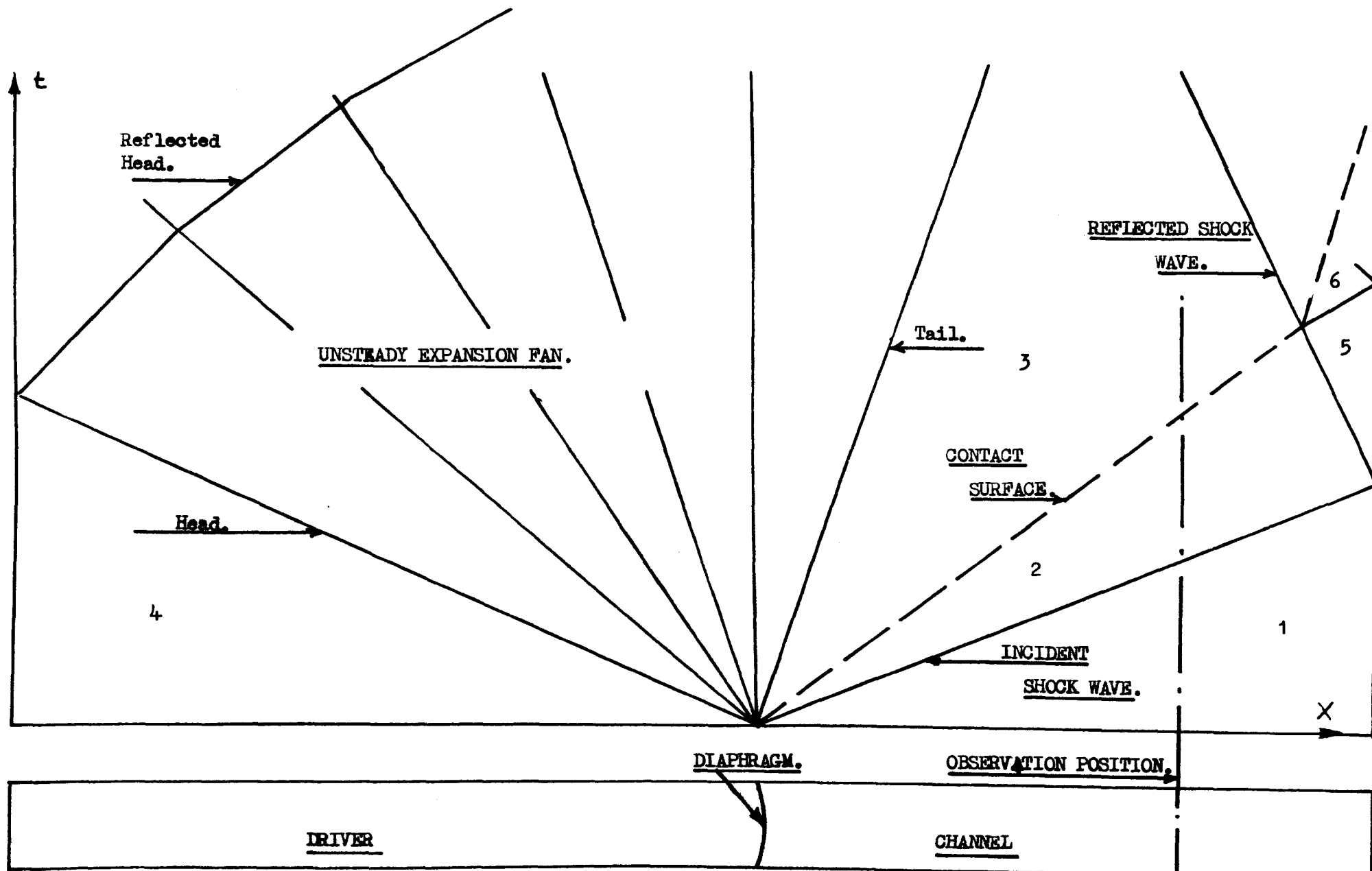
Collisions between water vapour molecules and carbon dioxide molecules are of the order of 300 times more efficient than collisions amongst carbon dioxide molecules alone for small concentrations of water vapour. For the effect of the water vapour on the relaxation time measurements in carbon dioxide to be less than 1%, its content must be less than 30 parts per million (PPM).

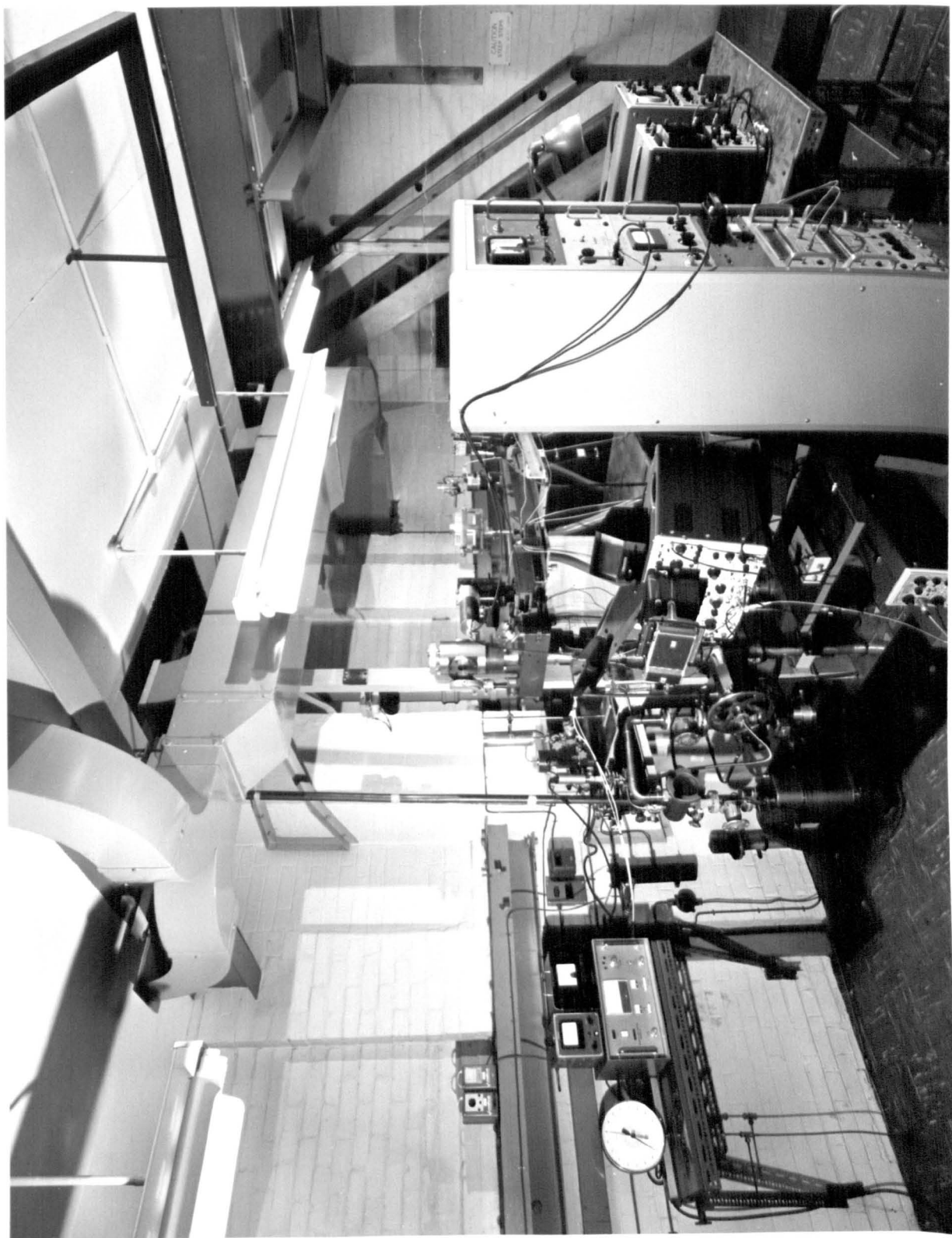
The carbon dioxide was analytic grade as supplied by the Distillers Company. This had a guaranteed water vapour content of less than 25 PPM, and was passed into the shock tube over a phosphorous pentoxide water vapour trap. The water vapour content was not measured after it had passed over the trap, but it was reasonable to assume that it was less than 20 PPM.

The impurity level of the test gas is also determined by the residual impurity in the shock tube channel prior to filling with the carbon dioxide. That is, by the ultimate vacuum which can be attained and by the combined leak and degassing rate of the system.

Figure A.2.15. shows the combined effects of the gas impurity level and the residual impurity level. The two curves show the performance obtained with the initial configuration and with the final configuration.

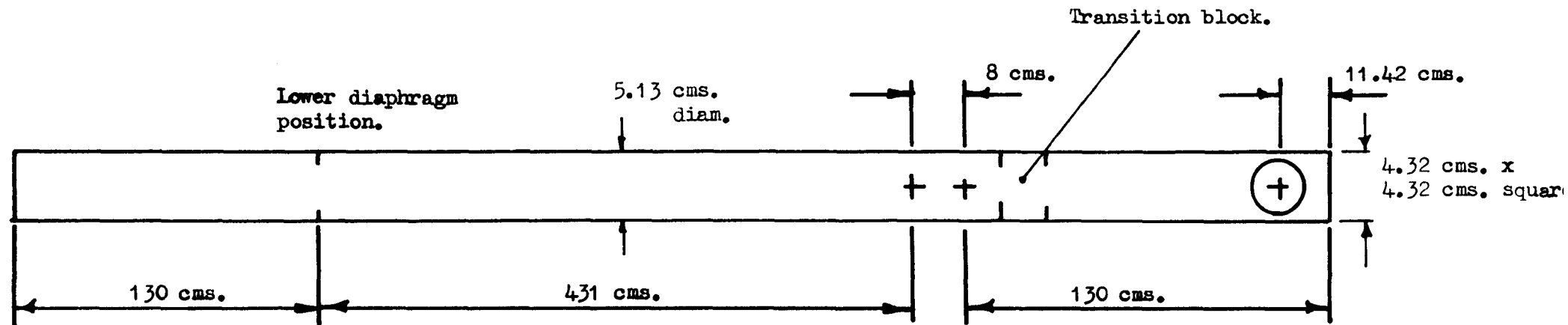
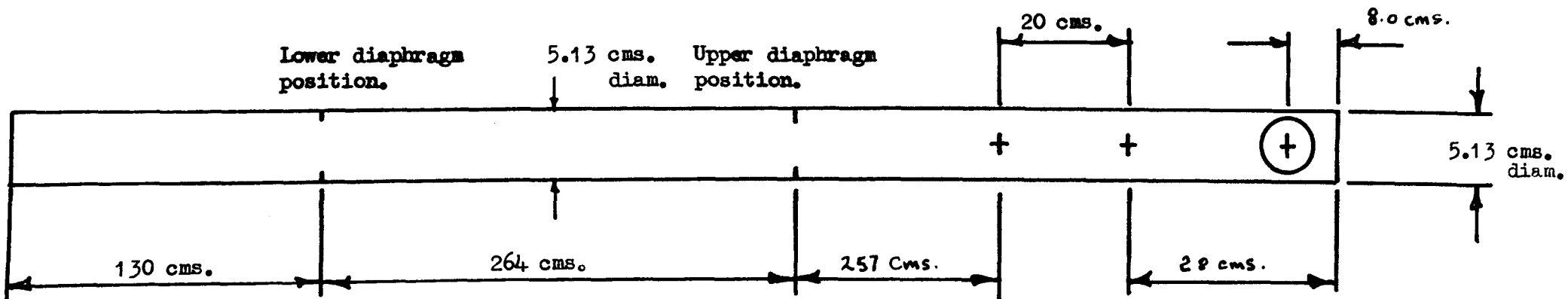
For a given incident shock Mach number, the initial channel pressure is determined by the spatial resolution and sensitivity of the optical system. These require that the initial pressure decreases with increasing shock Mach number. The ranges of the initial channel pressures required in the various experiments are also shown in Figure A.2.15. It can be seen that the initial configuration did not have a low enough impurity level for most of the experiments, but the level in the final configuration was adequate. Even so, the upper limit of the strong incident shock experiments was determined by the requirement to keep the impurity level below 30 PPM.

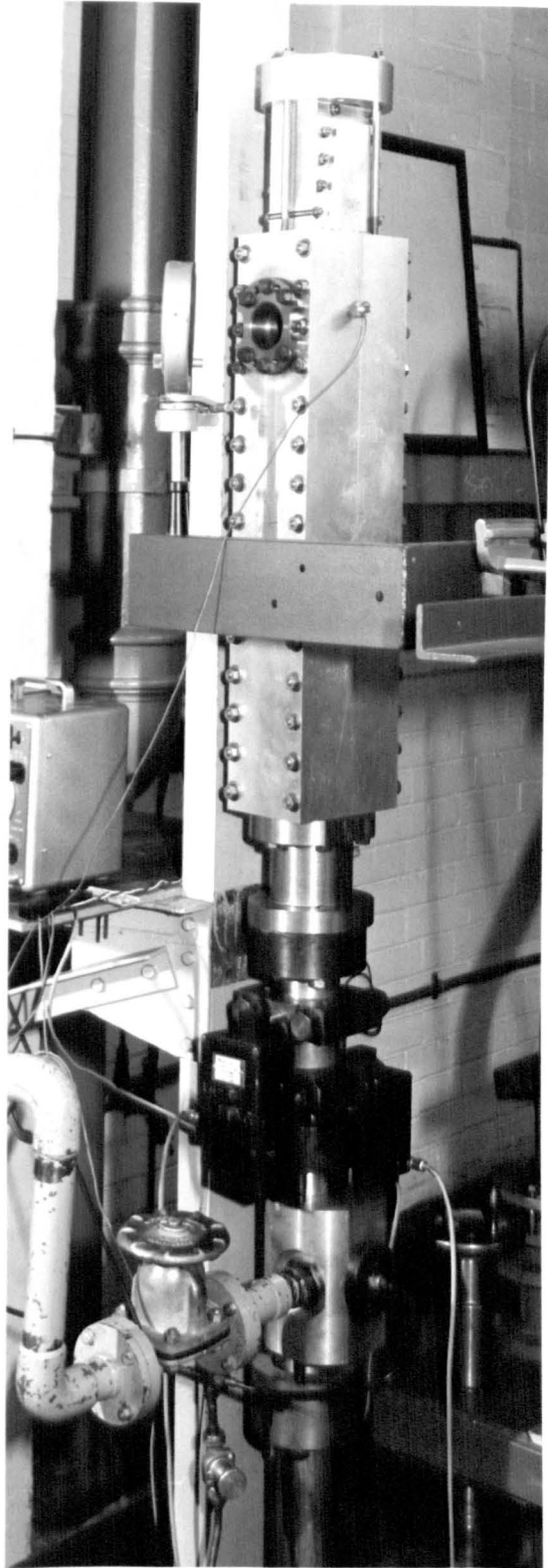




THE CONTROL ROOM

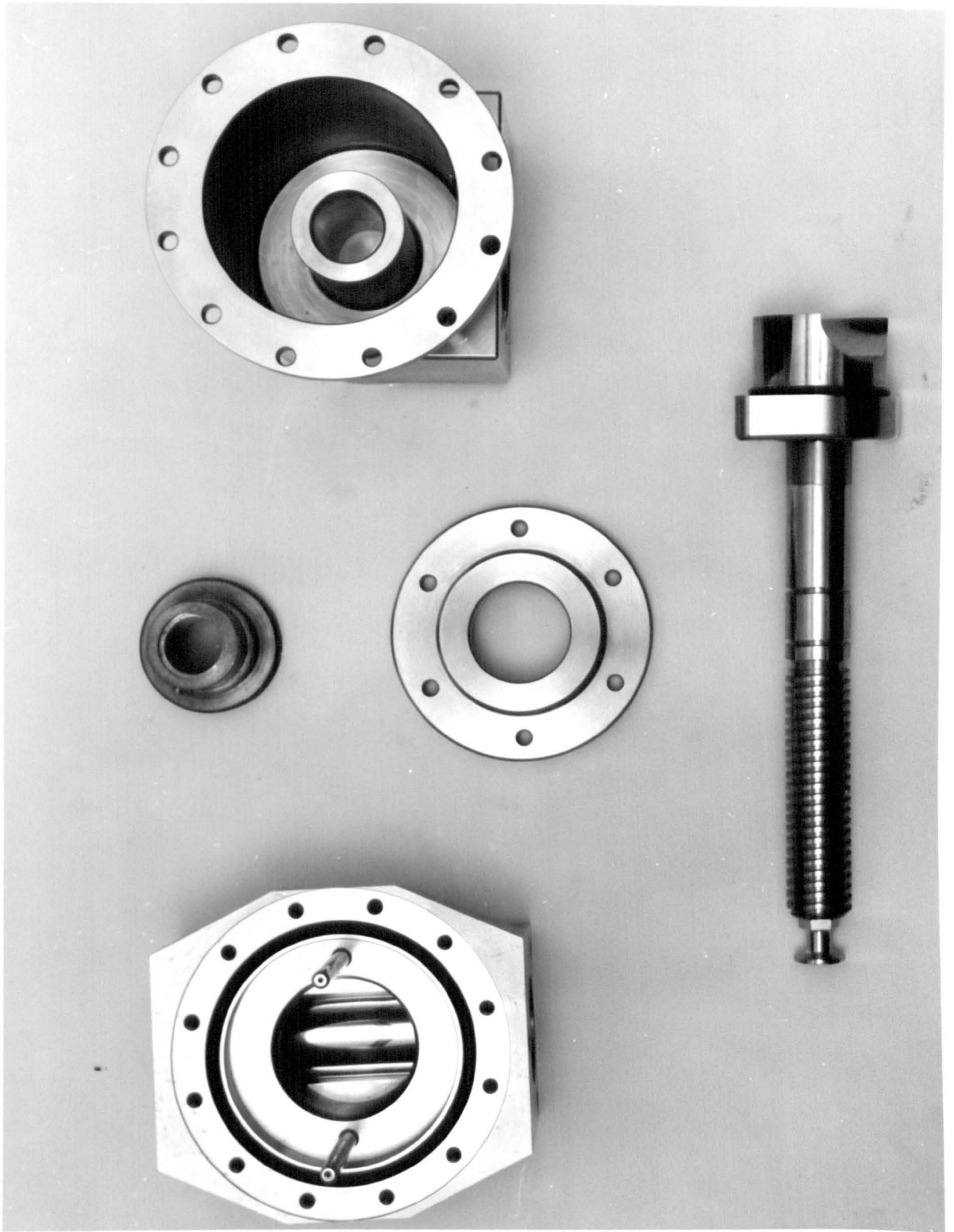
FIGURE : A.2.2.

Original Configuration.Present Configuration.



THE SQUARE WORKING SECTION

FIGURE: A.2.4.



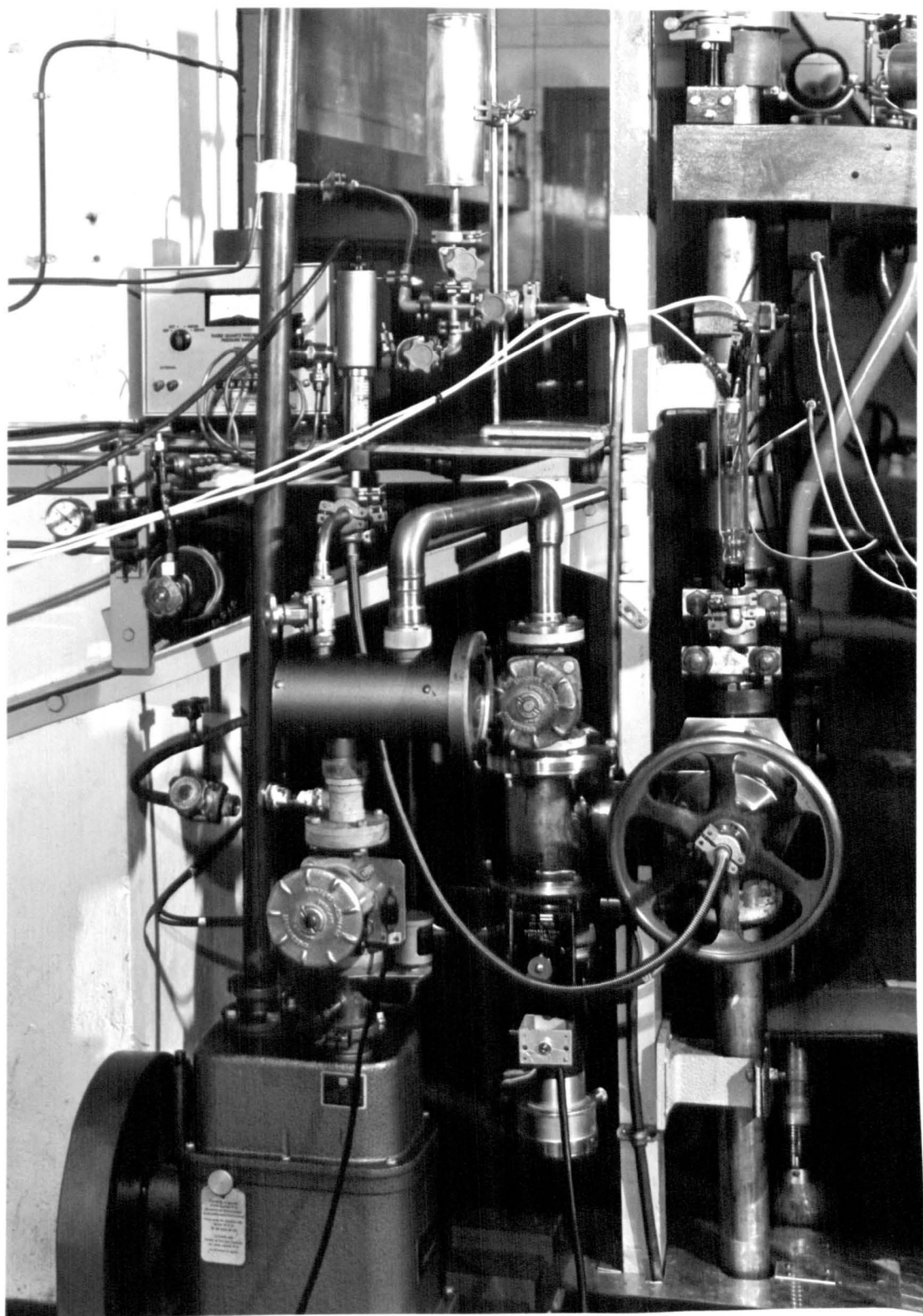
THE GAS INLET VALVE — DETAILS

FIGURE : A.2.5.



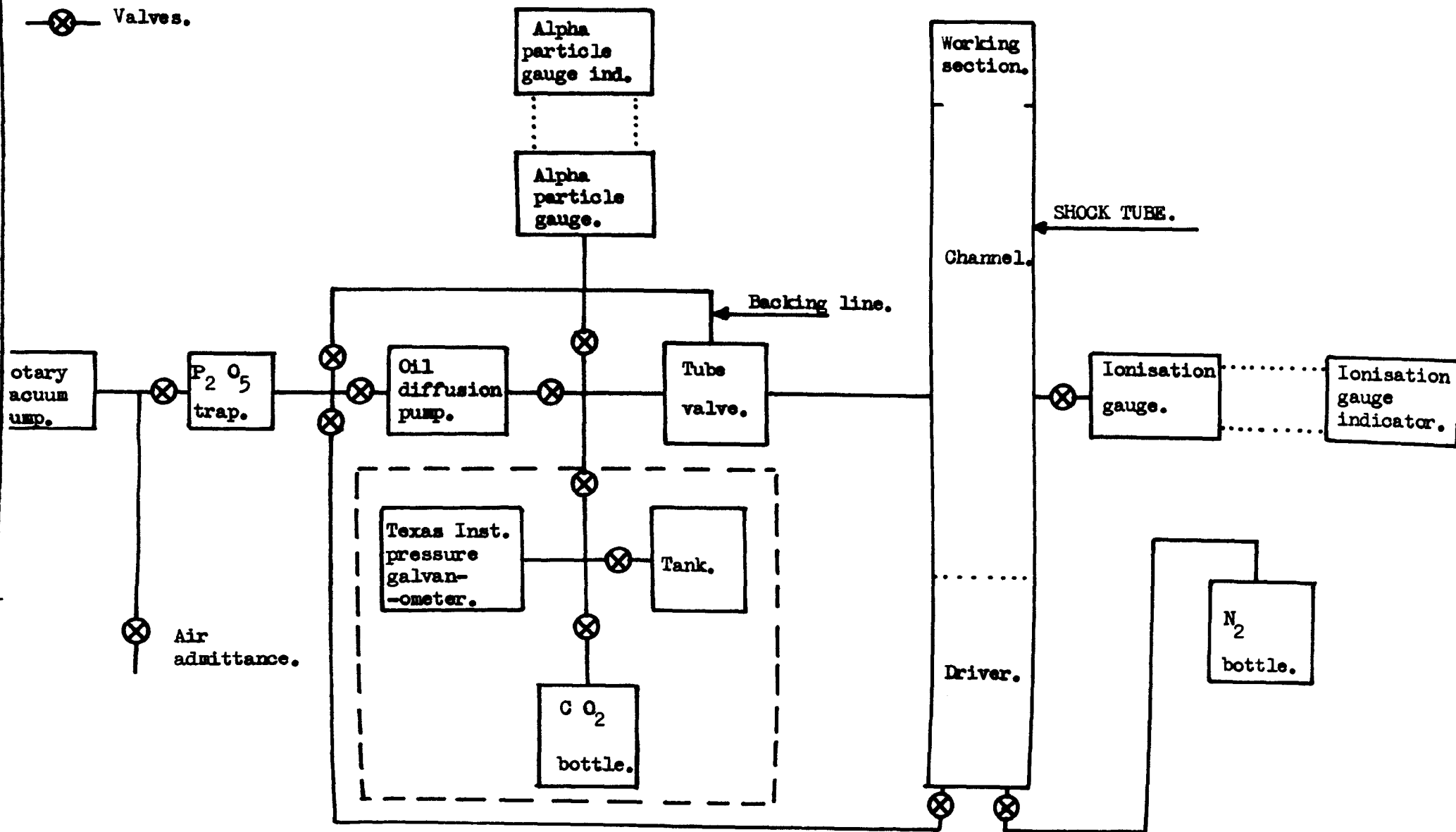
THE GAS INLET VALVE — PART ASSEMBLED

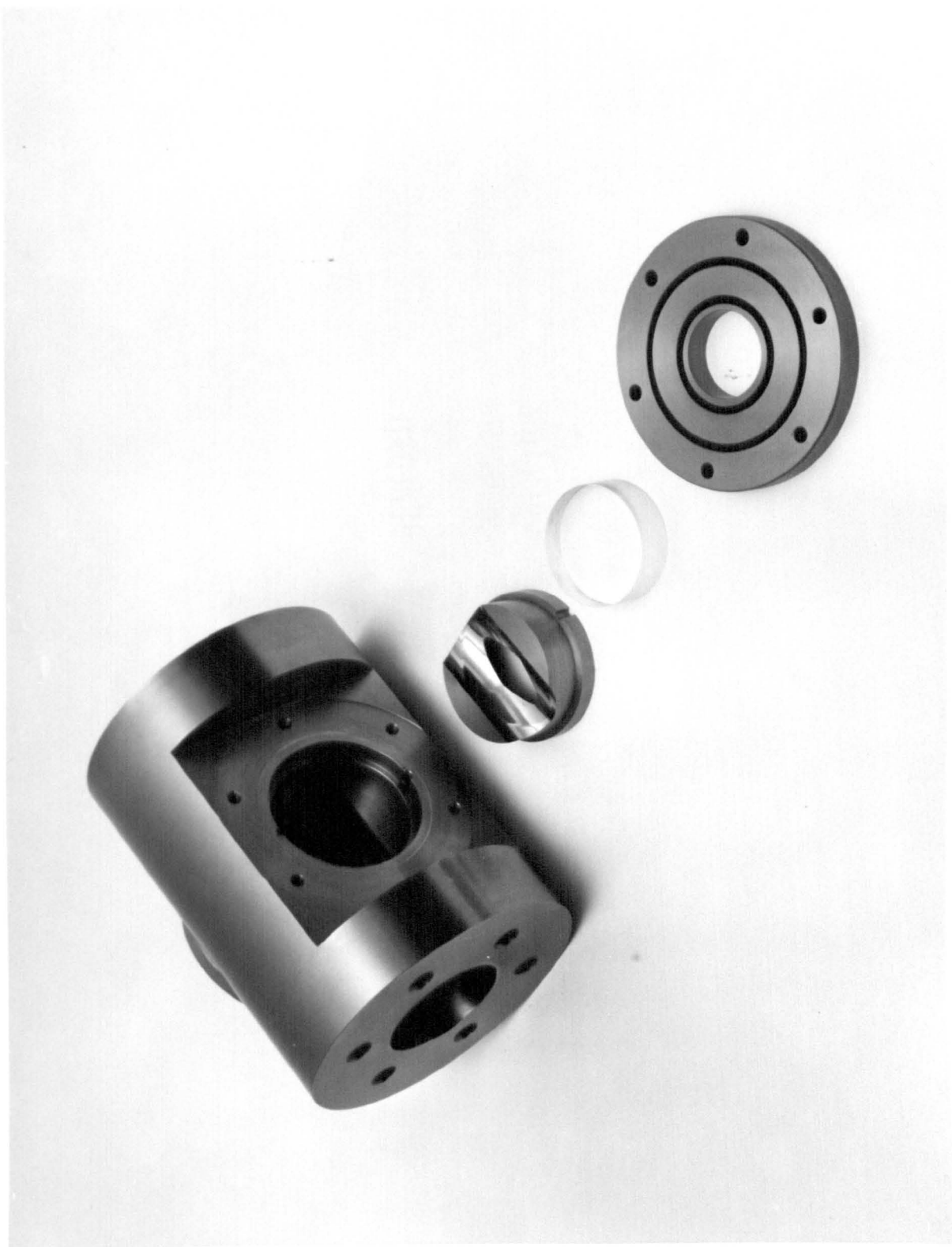
FIGURE: A.2.6.



THE VACUUM AND GAS INLET SYSTEM

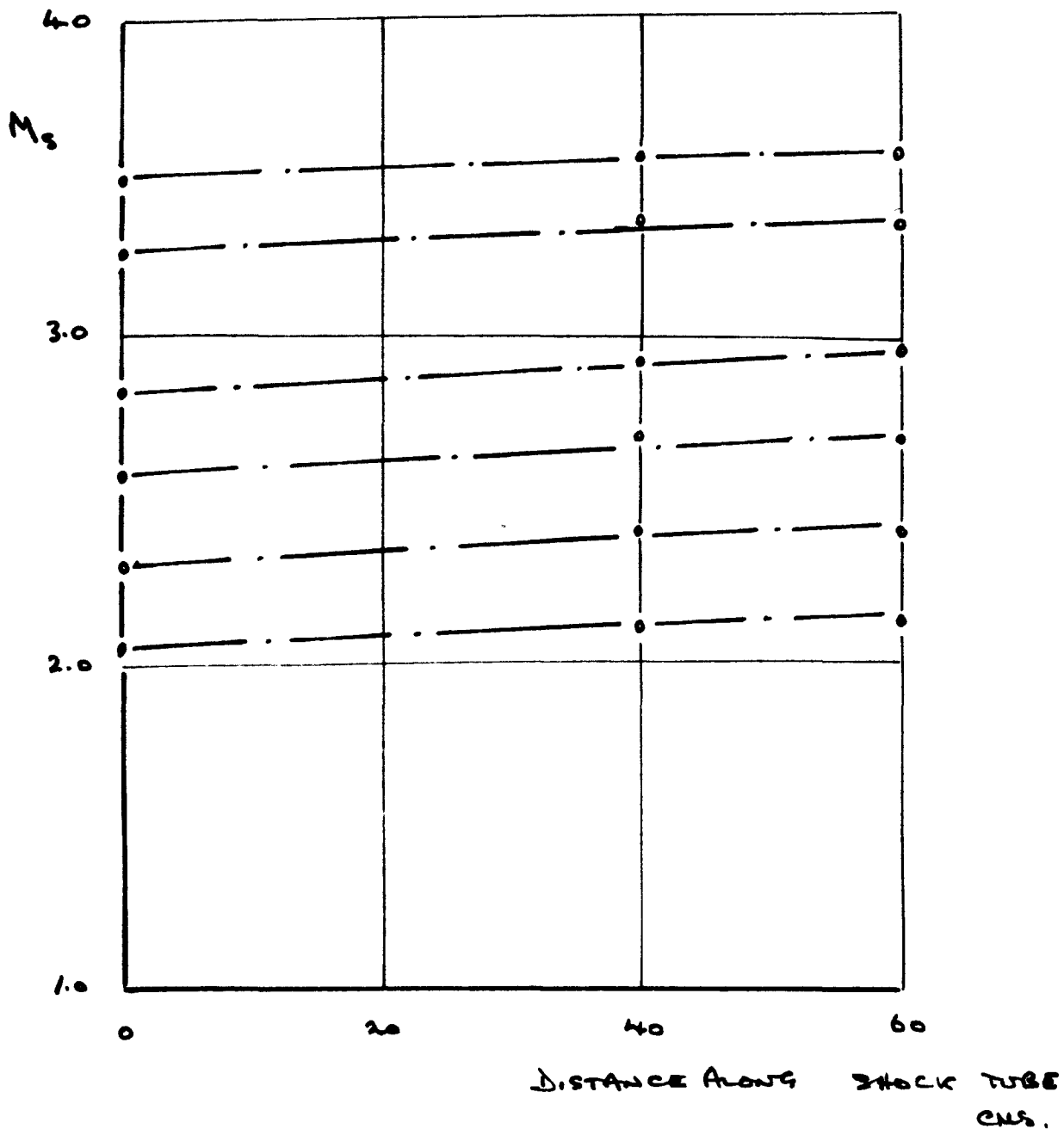
FIGURE : A.2.7.





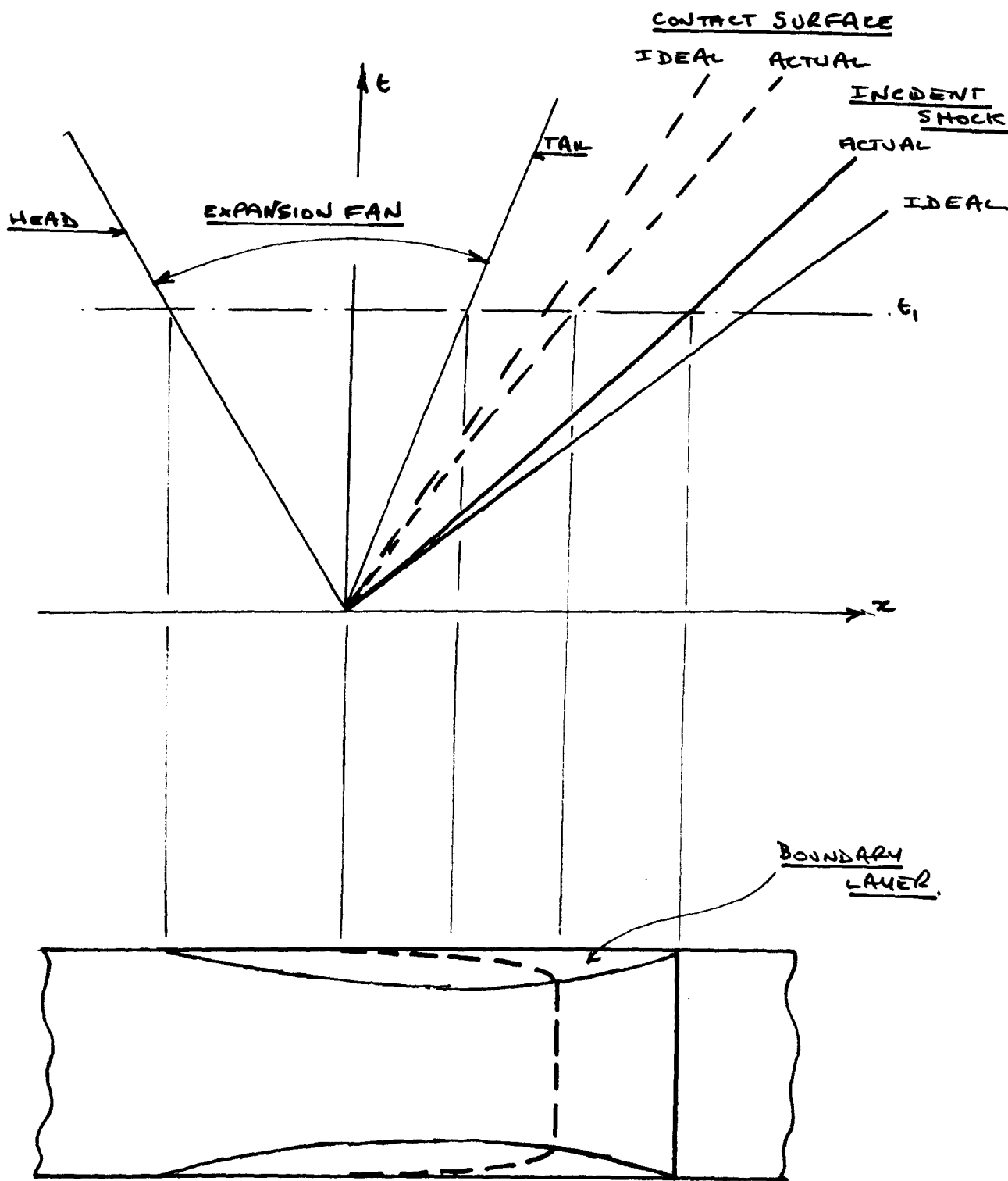
THE NEW WORKING SECTION

FIGURE : A.2. 9.

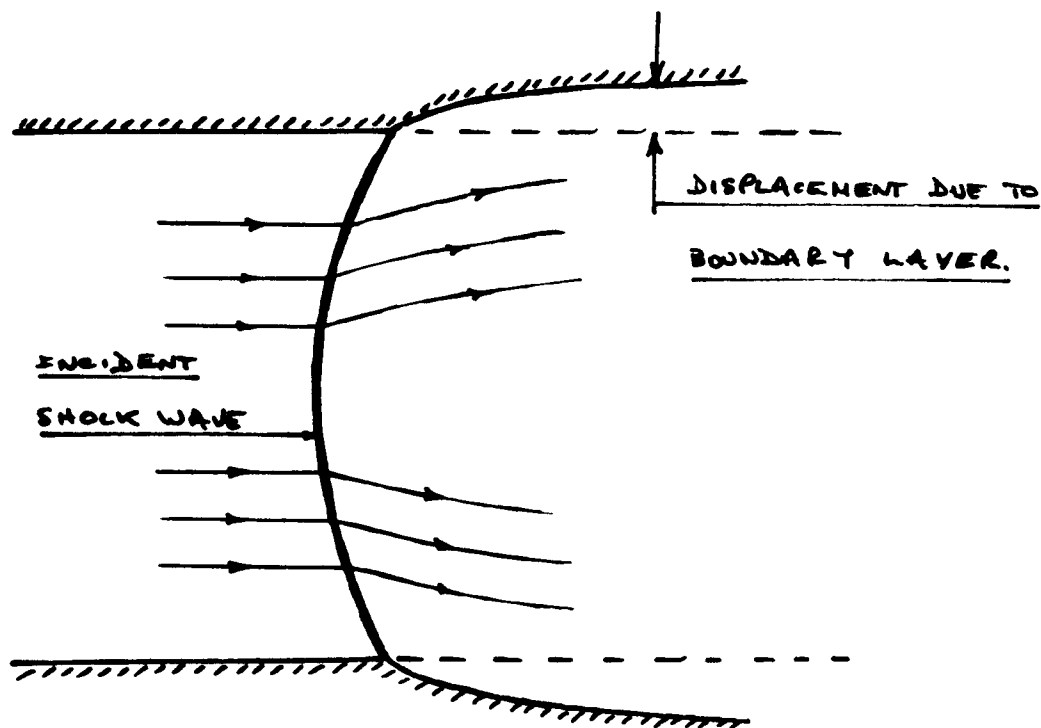


BEHAVIOUR OF SHOCK MACH NUMBER IN THE VICINITY OF THE WORKING SECTION.

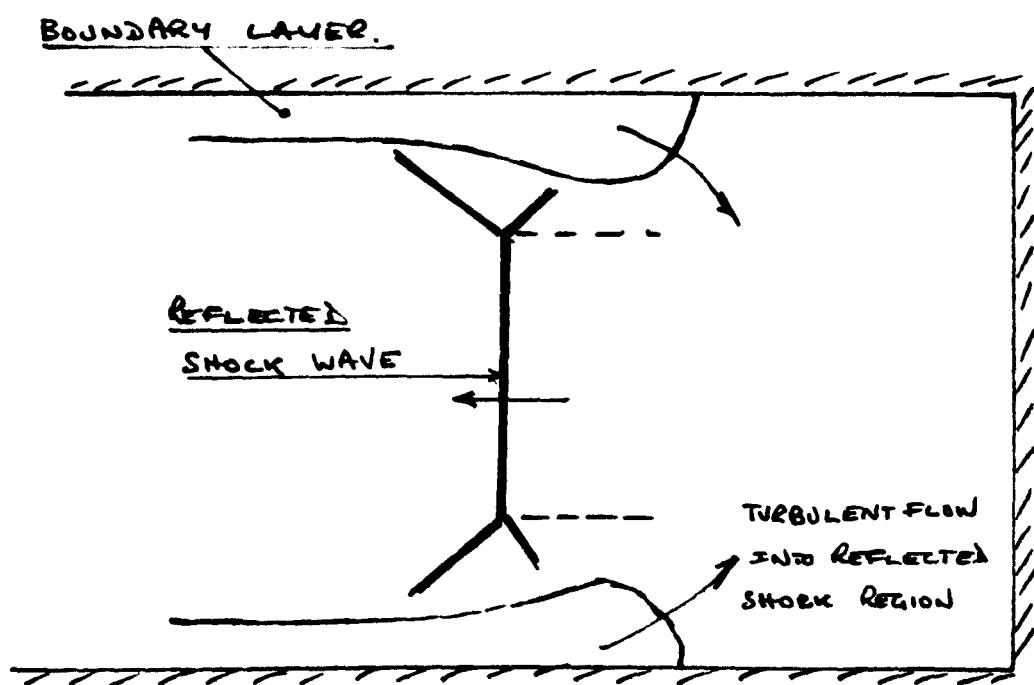
FIGURE A.2.10.



SHOCK TUBE FLOW AT $t = t_1$

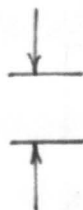
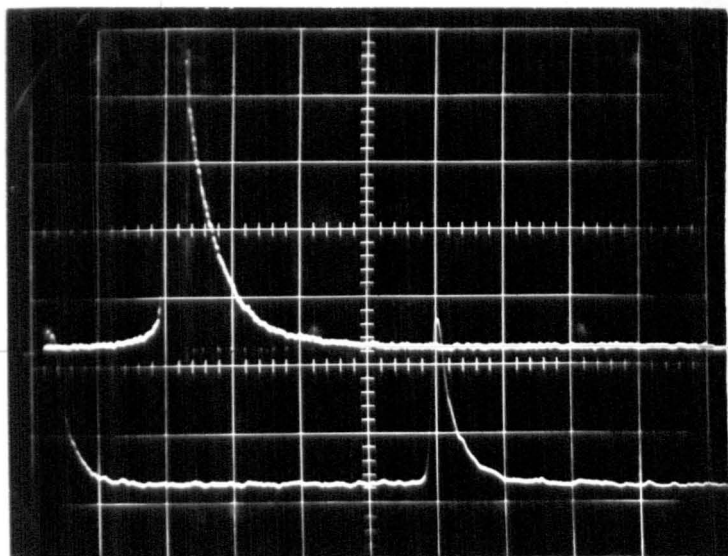
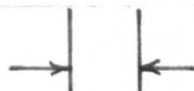


INCIDENT SHOCK CURVATURE DUE TO BOUNDARY LAYER FIGURE A.2.12.



BIFURCATION OF THE REFLECTED SHOCK WAVE FIGURE A.2.13.

0.7×10^{-4}
gm. cm.⁻⁴



0.289 cm.

Run 3316

$M_s = 2.28$

$$\rho_s - \rho_a = 1.072 \times 10^{-4} \text{ gm. cms.}^{-3}$$

Density gradient is flat within:

$$\pm 0.02 \times 10^{-4} \text{ gm. cms.}^{-4}$$

Integral of error over 1 cm. true is:

$$\pm 0.02 \times 10^{-4} \text{ gm. cms.}^{-3}$$

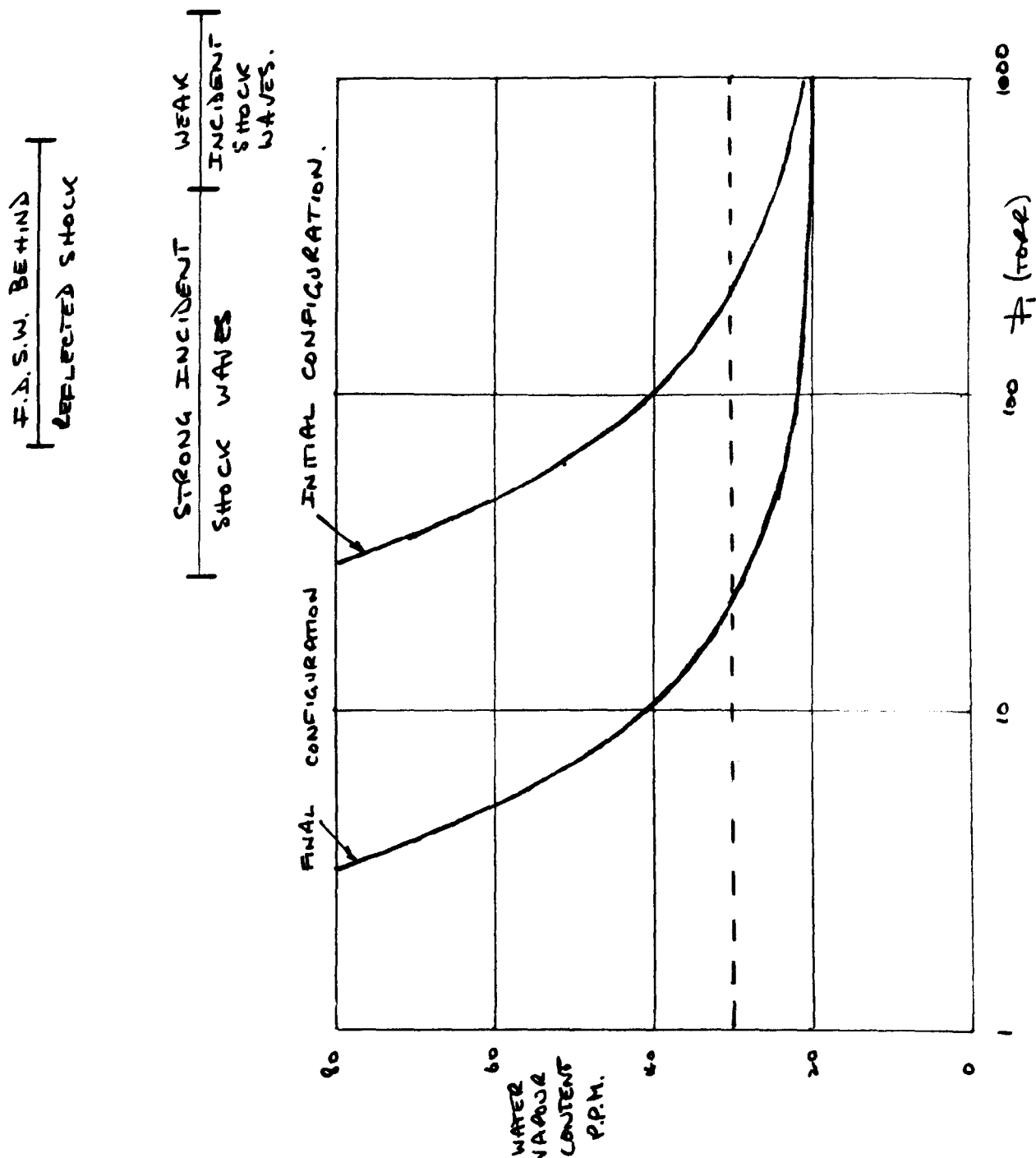
Error in terms of total density change
through relaxation region is:

$$\pm 2 \%$$

Example of non-uniformity in flow behind incident

shock wave in carbon dioxide.

Figure A.2.14.



WATER VAPOUR CONTENT FOR
VARIOUS EXPERIMENTS

FIGURE A.2.15.

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APPENDIX 3

ERROR ANALYSIS

An assessment is presented of the major factors which have determined the errors in the present experiments.

The error assessment has been broken down into the following main areas,

1. The determination of the calibration of the schlieren system.
2. The determination of density gradient profiles from oscillogram measurements.
3. The determination of the conditions behind the incident and reflected shock waves.
4. The determination of relaxation times from the strong incident shock waves.
5. The determination of the Mach number of a fully dispersed shock wave.

These areas will now be considered separately.

A simple block diagram of the schlieren calibration is shown in Figure A.3.1. The complexity of this calibration is regarded as one of the weak points of the present arrangement.

The calibration of the schlieren system affected the measurements in two ways. In the case of the strong partly dispersed shock waves, the accuracy of the calibration had little impact on the relaxation time results. Any inaccuracy caused a shortening or lengthening of the data from each experiment on the Landau-Teller plot. That is, it caused the data to be rotated about the low temperature equilibrium end. Any non-linearity in the calibration caused the orientation of the results on the Landau-Teller and a displacement of the data from a common line. The affect was of the same order as that due to inaccuracies. In the case of the fully dispersed shock waves, the inaccuracies and non-linearities had a different impact. The main impact was via the determination of the wave strength from the total density change through it, which was calculated from the integral of the schlieren output. Non-linearities had little impact on the relaxation times determined from fully dispersed shock waves providing the local mean value of the calibration was used.

The overall error in the knife edge calibration is estimated to be $\pm 2\%$, including inaccuracies and non-linearities.

The Gladstone-Dale constant was taken to be an order better than this, and was neglected in the assessment. Other sources of error were embodied in the equation relating the displacement of the source image at the knife edge to the density gradients in the flow. The two main factors to be considered here are the reduction in the effective path length through the shock tube flow due to the presence of the boundary layer and non-uniformities in the flow, particularly shock curvature, also due to the boundary layer.

An assessment of the reduction in effective path length can be obtained from Figure A.2.14. of Appendix 2. The extent of the non-uniformity can be seen to be of the order of $\pm 2\%$, and this is taken as a measure of the possible reduction in path length. The affect of shock curvature appears to dominate a region in the partly dispersed shock waves which is of the same order as the apparent shock thickness. If this area is neglected, any other errors can be taken as being included in the above 2%.

It is concluded that, providing care is exercised in the analysis of experimental data, the schlieren calibration errors can be taken as $\pm 4\%$.

Figure A.3.2. presents a block diagram of the determination of density gradient profiles from oscillogram measurements. A specified figure for the accuracy and linearity of an oscillogram is typically of the order of 3%. However, with careful setting up, errors of less than 1% can be obtained, depending on the noise level of the traces. It can be seen from Figure 3.2.2. of Section 3.2. that the noise level was of the order of 5% in some cases. As the data was effectively smoothed during the analysis, most of this random error was removed leaving a possible systematic error of the order of $\pm 2\%$, including oscillogram errors. It can be seen from Figure A.3.2 that the shock velocity also entered in at this stage. As will be discussed below, this contributed an error of less than 1%.

It is concluded that errors of up to $\pm 7\%$ could be present in the density gradient profiles obtained in these experiments. However, most of these errors were systematic scaling errors rather than random errors and non-linearities.

Figure A.3.3 presents a block diagram of the computations required for the determination of the conditions behind the incident and reflected shock waves. As noted in Appendix 2, the errors in the measurement of the initial channel pressure, p_1 , and the initial channel temperature, T_1 , are negligible on the scale of the other errors which are being considered.

The incident shock velocity has been determined in a combination of ways as indicated; by measurements with wall thermometers upstream of the working section, by measuring the delay between its passage at the last thermometer and its arrival at the working section, and by the time taken for it to traverse two light probes in the schlieren screen. A best estimate was obtained from these measurements taking into account the degree of attenuation as indicated in Figure A.2.10 of Appendix 2. The error in the shock velocity was estimated to be less than 1%.

The conditions behind the incident and reflected shock waves are calculated using the enthalpy data of Hilsenrath et al and no errors were attributed to this source. The computations were essentially iterative, but they were taken to accuracies outside the levels being considered in this estimate.

It would appear from the results of other workers that it would be reasonable to take the departure from ideal conditions behind the incident and reflected shock waves to be of the order of 2% and 4% respectively for the range of incident shock Mach numbers considered. However, it should be noted that the conditions into which the waves were travelling behind the reflected shock wave were highly suspect and the errors could be much greater.

A block diagram showing the determination of relaxation times from strong incident shock waves is presented in Figure A.3.4. As shown, the density gradient profile is integrated in order to obtain the presentation of $d\rho/dx$ versus $\rho_2 - \rho$ for analysis. This integration can introduce large errors into the measurements, as discussed in Section 3.2. This will be discussed separately later. However, it will be noted that this source of error could be removed by the use of electronic means of integration. The data was smoothed on this plot, so many of the random errors should have been removed. The conversion to characteristic time from characteristic length using U_1 cancels out the errors introduced by u_1 in Figure A.3.2.

As the amplitude of the density gradient trace does not affect the characteristic time, the main sources of error are the noise on the traces and the non-ideal conditions behind the shock wave. It is estimated that the resultant error in the characteristic time, taking into account the smoothing of the data and excluding the integration error, is $\pm 3\%$.

The only other factor having a major impact on the accuracy of the relaxation time is the incident shock Mach number. This affects the Landau-Teller presentation through the correction function, f , and the translational temperature raised to the one third power. The combined effects result in errors which are of the order of 5 to 8 times the original error in the Mach number. Accordingly, an error of $\pm 8\%$ is attributed to this source.

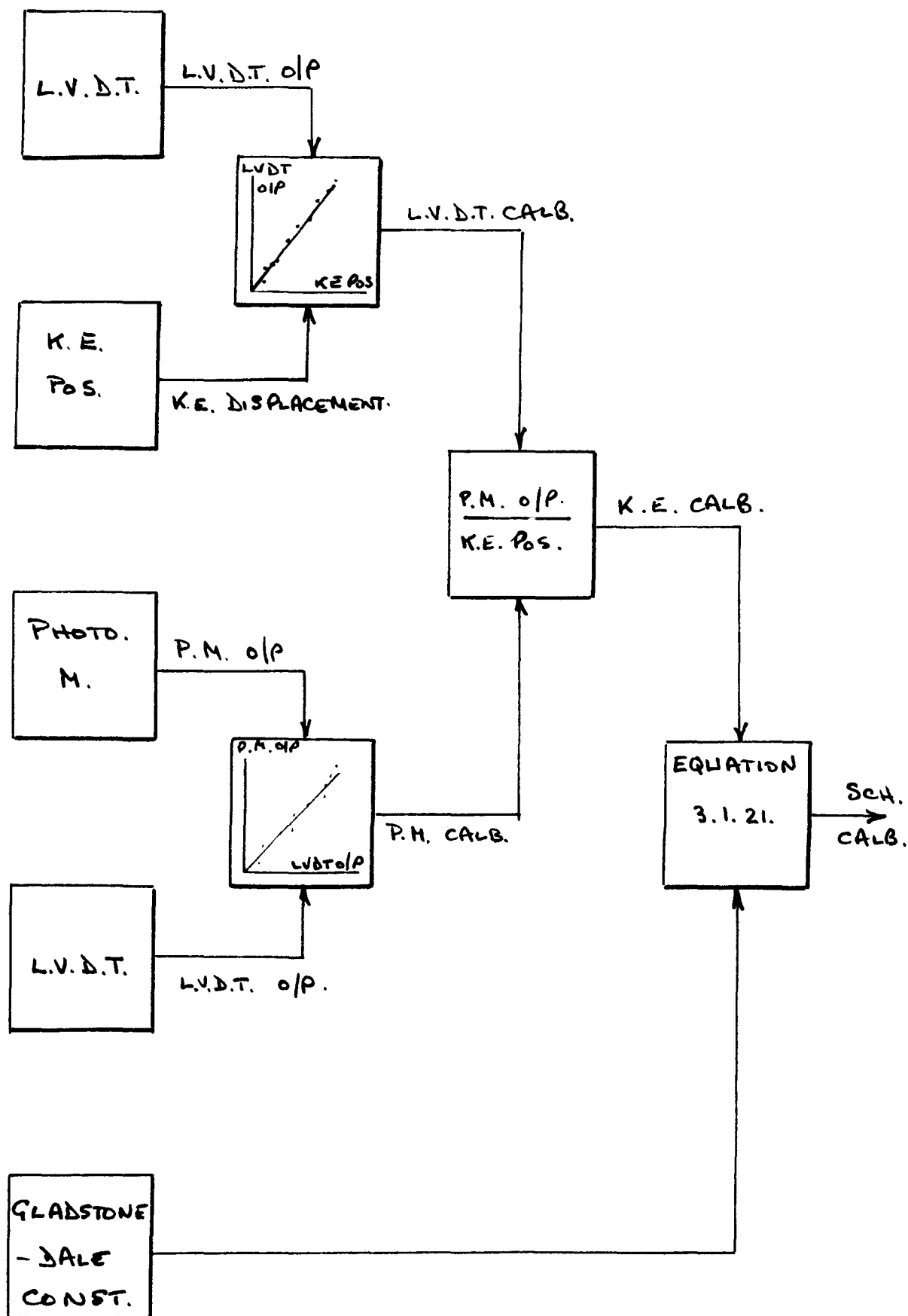
The total error in the relaxation times from this analysis is of the $\pm 12\%$. However, this does not account for the departure from the Simpson curve, see Figure 3.2.10, or for the amount of scatter. It is concluded that the errors must be attributable to the method of integration of the density gradient data.

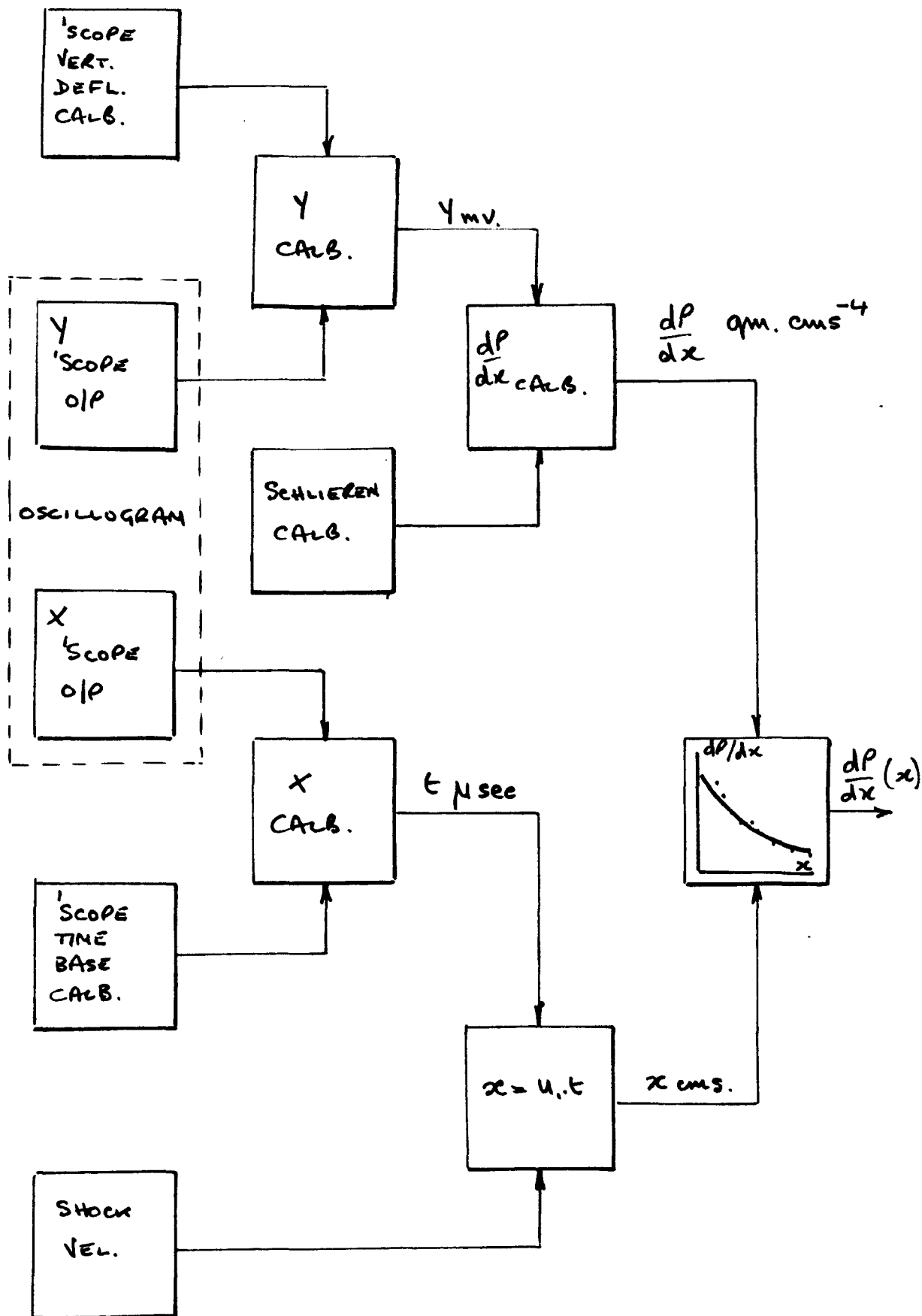
This conclusion is borne out by Figures 3.2.11 to 3.2.13 of Section 3.2, which show good agreement between the theoretical curve using the Simpson data and the measured density gradient profiles. The agreement is much better than would be expected on the basis of the error analysis. However, as was noted, these were mainly scaling errors and they could be masked by the freedom of choice in the start of the relaxation region.

Figure A.3.5 shows the method by which the strengths of the fully dispersed shock waves were determined. It can be seen that the density gradient profiles were integrated to give the total density change across the waves, which could be related to the Mach number of the waves. The accuracy of the Mach number determined in this way is determined by the accuracy of the density gradient profile, which is $\pm 8\%$. However, as a large portion of this was random error a more representative figure is $\pm 5\%$. It can be seen from Figure 3.3.1 of Section 3.3 that this will be reflected as the same order of error in the density gradient and hence in the characteristic time.

It can be seen from Figure 3.3.8 of Section 3.3 that the error of $\pm 5\%$ in the total density change is reflected as an error of $\pm 0.2\%$ in the shock Mach number. However, it can be seen from Figure 3.2.6 of Section 3.2 that the conversion function steepens as the wave strength decreases. The error in the relaxation time due to the $\pm 5\%$ error in the density change will therefore be $\pm 2\%$. The total error on the Landau-Teller plot should therefore be of the order of $\pm 10\%$. Figure 3.3.11 of Section 3.3 shows that this estimate is reasonably consistent with the results obtained.

In view of the uncertainties associated with the reflected shock region, an analysis for that part of the studies will not be considered. It is noted that the scatter in experimental results is between 10% and 20%.

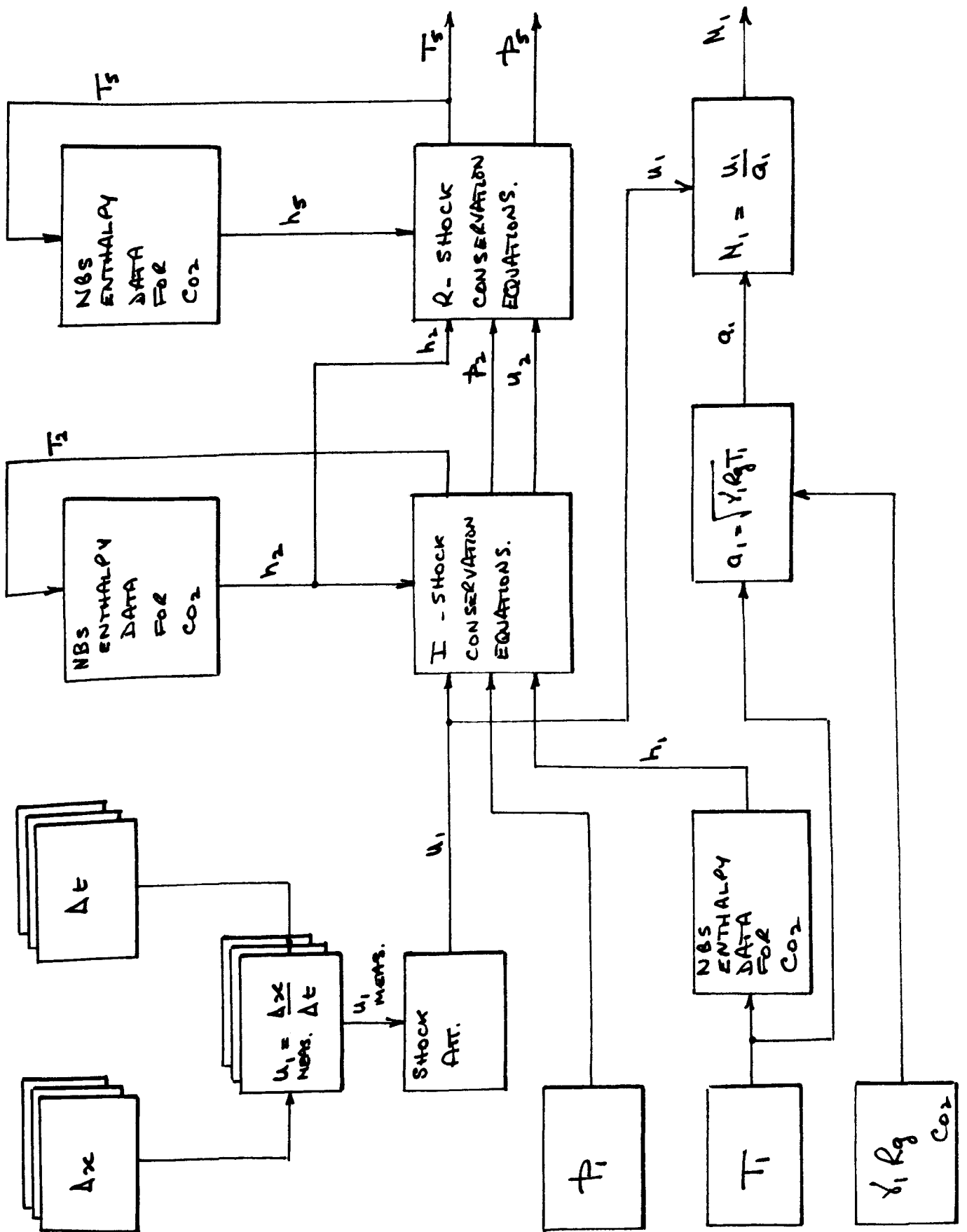




DETERMINATION OF DENSITY GRADIENT PROFILE
FROM OSCILLOGRAM.

FIGURE.

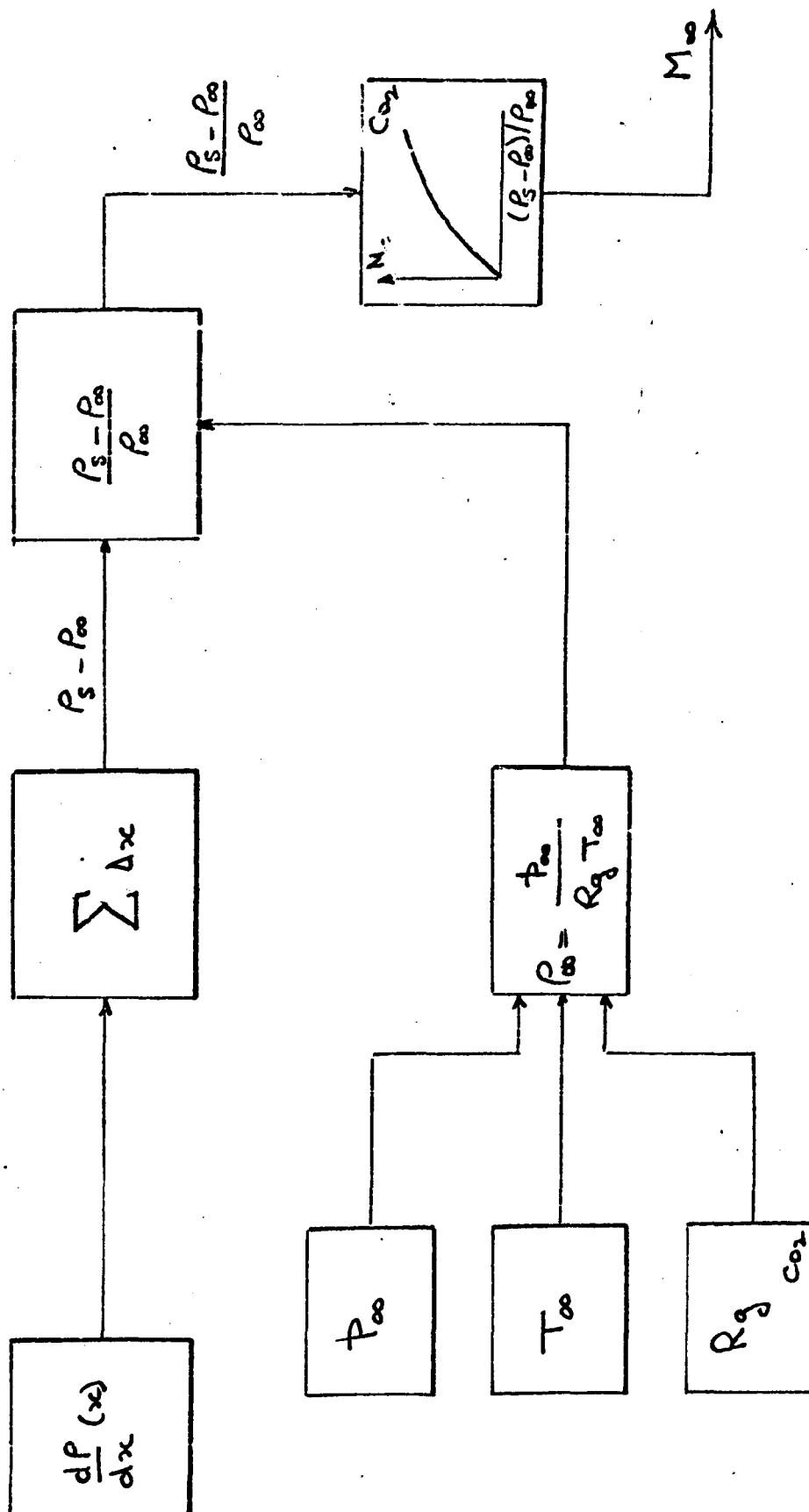
A.3.2.



DETERMINATION OF CONDITIONS BEHIND INCIDENT AND REFLECTED SHOCK WAVES.

FIGURE.

A.3.3.



DETERMINATION OF THE MACH NUMBER OF A FULLY DISPERSED SHOCK WAVE.

FIGURE.

A.3.5.