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Multi-Fidelity Design Optimization of Installed Aero-Engines With Non-Axisymmetric Exhausts

Larger ultra-high bypass ratio (UHBR) aero-engines introduce an aerodynamic integration challenge. In close-coupled, podded underwing configurations, the aerodynamic interference between the propulsion system and the airframe could penalize the aircraft net vehicle force (NVF) and erode some of the novel cycle benefits and fuel burn reduction. Non-axisymmetric designs of the bypass nozzle can improve the performance of the aircraft by mitigating some of the penalizing effects induced by the integration of the powerplant. However, due to the prohibitive computational cost of the design methods, only lower-fidelity design approaches have been feasible in an industrial time-scale. This work develops a relatively low-cost multi-fidelity design optimization methodology for non-axisymmetric exhausts where the effects of the propulsion system installation are considered. The methodology combines inviscid and viscous aerodynamic data to formulate multi-fidelity surrogate models which drive a genetic algorithm (GA) optimization. The method enabled the incorporation of the viscosity effects in the optimization process at a reasonable computational cost and led to better designs relative to a methodology based only on lower-fidelity data. Overall, the optimization of non-axisymmetric exhausts can benefit the net vehicle force of the complete engine–aircraft system in cruise by up to 0.9% of the engine standard net thrust which can reduce fuel burn by a similar amount. The optimization with multi-fidelity surrogate models reduced the computational time by a factor of four relative to a method based only on viscous aerodynamic data. [DOI: 10.1115/1.4069036]

Keywords: non-axisymmetric exhausts, propulsion system integration, computational fluid dynamics, multi-fidelity design optimization methods, response surface model, neural networks

1 Introduction

1.1 Background. The next generation of civil turbofan aero-engines are expected to operate with higher bypass ratios (BPR) and lower fan pressure ratios [1] to increase propulsive efficiency and reduce fuel consumption [2]. While current in-service turbofans are designed with a BPR > 10 [3], ultra-high bypass ratio (UHBR) engines may operate with BPR > 15 [3,4]. Higher BPR can be accommodated with larger fan diameters. However, this is likely to increase the dimensions and mass of the powerplant. In addition, larger aero-engines may require close-coupled installations with the

wing to meet ground clearance requirements [5]. In such arrangements, the aerodynamic interference between propulsion system and airframe could penalize the aircraft cruise performance by 0.7% of the nominal thrust relative to a conventional installation position [4]. It is therefore imperative to consider the effects of engine–aircraft integration in the design of the propulsion system and airframe.

1.2 Non-Axisymmetric Exhaust Systems. Non-axisymmetric exhaust systems could improve the integration of UHBR aero-engines with the airframe. These configurations can potentially mitigate the adverse aerodynamic effects of propulsion integration and improve aerodynamic performance at aircraft level [6–9]. Otter et al. [6] proposed a design method for non-axisymmetric separate-jet nozzles. The methodology was based on intuitive class-shape

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transformation functions [10,11] for geometric control and was applied to the design of dropped, scarfed, and noncircular exhaust shapes as well as non-axisymmetric after-bodies. For installed configurations of the propulsion system [6], non-axisymmetric designs led to benefits of approximately $\approx 0.6\%$ in cruise fuel consumption relative to an axisymmetric baseline. Although this research set the basis for non-axisymmetric exhaust design, the process did not consider a systematic exploration of the design space and optimization. Bagy et al. [7] developed an optimization method for non-axisymmetric exhausts based on the combination of three different noncircular nozzle shapes. The three base nozzle designs comprised ovalized, extended, and shortened bypass configurations. The optimization successfully mitigated some of the adverse flow features at the nozzle exit plane and improved the net propulsive force by 0.09%. However, the study neglected the effects of the propulsion system integration as the aero-engine was assessed in isolation. Although the methodology proved useful for fine tuning of an industrial geometry, it lacked the geometric control of a parametric approach, required to study the design space in early stages of the design. Hueso-Rebassa et al. [8,9] developed an aerodynamic shape optimization methodology for non-axisymmetric exhausts systems based on response surface models (RSMs). The research targeted the optimization of separate-jet exhausts of an UHBR turbofan fully integrated with the airframe. Inviscid and compressible computational fluid dynamics (CFD) data were used to sample the design space and generate Kriging RSMs which guided a genetic algorithm (GA) optimization. The method identified designs with benefits up to 0.7% on net vehicle force (NVF) when evaluated with viscous and turbulent CFD methods. In addition, the RSMs were used to derive tradeoff between aircraft performance, exhaust performance, and thrust vectoring. However, the effects of viscosity were not modeled in the design process.

1.3 Multi-fidelity Methods for Aerodynamic Design. For coupled powerplant and airframe configurations, even the acquisition of enough data to build reliable surrogate models is infeasible in an industrial time-scale. For this reason, design methodologies are mostly limited to simplified configurations or lower-fidelity aerodynamics. The use of RSMs provides a way to characterize the design space and enables the global optimization of the system with a reduced number of CFD evaluations. However, the design capabilities are limited by the computational time of viscous CFD simulations. In this context, multi-fidelity response surface modeling gains interest.

A multi-fidelity response surface model (MFRSM) is an RSM created with two sets of data of lower and higher-fidelity, respectively. The low-fidelity data provide an approximation of the gradients of the design space, while the higher-fidelity data are used to correct the lower-fidelity prediction [12]. With this approach, multiple fidelity aerodynamic data can be combined to create RSMs with a reduced number of high-fidelity samples. This can improve the accuracy of the prediction model at a fraction of the computational cost relative to RSMs formulated only with higher-fidelity data [13]. Some examples of multi-fidelity RSMs are Co-Kriging [14,15], variable complexity models (VCM) [12,16], and multi-fidelity artificial neural networks (MFANN) [12,17–19]. Kaya et al. [14] studied the application of Co-Kriging to the prediction of the aerodynamic lift and drag polar characteristic of an unmanned air vehicle. The model was constructed with 31 Euler CFD samples as the lower-fidelity dataset and up to seven Reynolds-averaged Navier–Stokes (RANS) CFD points as higher-fidelity. The method could approximate the high-fidelity functions with root-mean-square deviations (RMSD) of 0.006, 0.0017, and 0.0008 for the lift, drag, and pitching moment coefficient, respectively. However, the RSM was not used in an optimization framework. Tejero et al. [15] combined Co-Kriging with dimensionality reduction methods to optimize aero-engine nacelles. The method successfully projected the 32-dimensional design space of 3D nacelles into a reduced design space characterized by two degrees-of-freedom (DOF), which enabled the optimization of three-

dimensional nacelles on installed arrangements with the airframe. Zhang et al. [18] applied multi-fidelity artificial neural networks for the minimization of the drag of the DLR-F4 wing-body configuration with 30 degrees-of-freedom. RANS with boundary layer resolution and wall functions were used as higher and lower-fidelity CFD methods, respectively. The optimization with MFANN was compared with another process based on a single-fidelity artificial neural network (ANN) that was trained only with higher-fidelity data. For the same number of higher-fidelity data points, the drag was reduced by 14% with the multi-fidelity optimization, while only a 3.4% benefit could be achieved with single-fidelity method, relative to a baseline configuration. Sánchez-Moreno et al. [19] developed a nacelle optimization workflow based on MFANN. The method targeted the multipoint optimization of 2D-axisymmetric nacelles for UHBR aero-engines. Multi-fidelity surrogate models were constructed with Euler and RANS CFD solutions as lower- and higher-fidelity data, respectively, and were integrated into a gradient-free optimization process. The multi-fidelity optimization method converged to a similar design space as an equivalent CFD in-the-loop approach but reduced the computational cost of the optimization by 92%. He et al. [12] compared Co-Kriging, VCM, and MFANN for the prediction of the lift (C_L) and drag (C_D) coefficients of the NACA0012 airfoil at a range of Mach numbers and angles of attack. The multi-fidelity database comprised Euler and RANS CFD. In the research, MFANN were proven more accurate with a reduction in RMSD of 1 order of magnitude for both C_L and C_D relative to the other methodologies.

Overall, multi-fidelity methods provide a way to increase the fidelity of low-order prediction models. These can drive design processes at a fraction of the cost compared to methods based only on higher-fidelity data.

1.4 Overview and Scope. Previous research on the design of non-axisymmetric exhausts [8,9] utilized an optimization method based only on inviscid aerodynamic data. However, the omission of the viscous terms may affect the optimum combination of design parameters. Multi-fidelity methods provide a way to increase the fidelity of the prediction methods at a relatively low-cost. These may unlock better regions of the non-axisymmetric exhaust design space including the effects of aircraft integration.

This work develops a multi-fidelity design optimization methodology for non-axisymmetric exhaust systems of large civil turbofans. A large lower-fidelity aerodynamic dataset is combined with fewer higher-fidelity samples. This enables the formulation of higher-fidelity prediction models at a reduced computational cost relative to a design methodology based only on higher-fidelity data. The multi-fidelity prediction methods are VCM and MFANN. These use a combination of inviscid and viscous aerodynamic data and guide an optimization process based on genetic algorithms. The novelty of this research is the exploration of the aircraft-installed non-axisymmetric exhaust design space at a greater degree of aerodynamic fidelity. Databases of exhaust designs are assessed with inviscid and viscous computational methods. This enables back-to-back comparisons between different degrees of accuracy, which leads to the formulation of higher-fidelity models and the potential reduction of computational cost of aerodynamic design methods.

2 Methodology

The multi-fidelity optimization methodology (Fig. 1) is applied to the design of aero-engine non-axisymmetric exhaust systems coupled with the airframe in a podded underwing configuration. Three-dimensional non-axisymmetric exhaust geometries are defined parametrically and coupled with the propulsion system and airframe (step 1, Fig. 1). Installed propulsion system configurations are assessed with computational methods of various levels of fidelity (step 2, Fig. 1). The purpose of the computational methods is to compile a multi-fidelity database of exhaust designs (step 3, Fig. 1) which is used to generate multi-fidelity response surface models (step 4, Fig. 1). The response surface models guide an

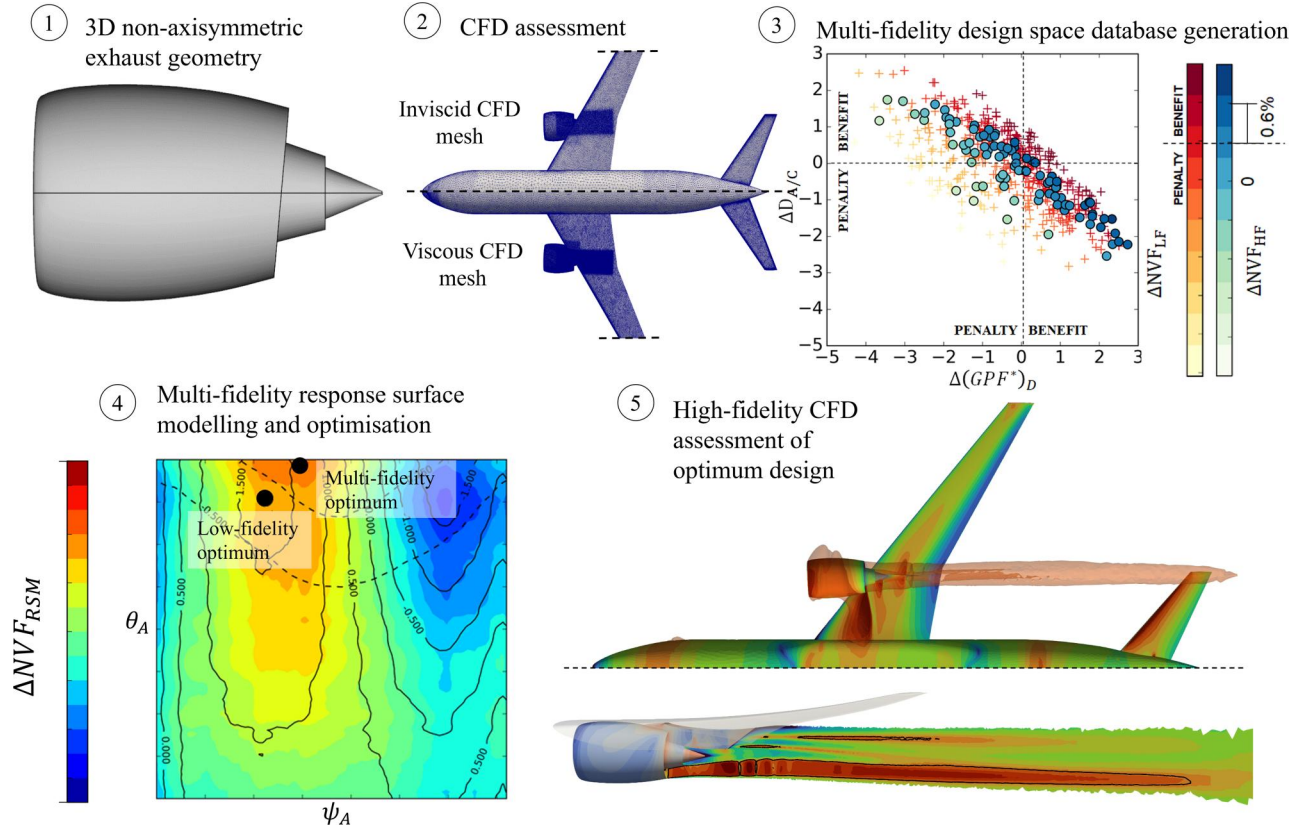


Fig. 1 Flow diagram of the multi-fidelity design methodology for installed propulsion systems with non-axisymmetric exhausts

optimization process based on genetic algorithms. Finally, the optimum designs are assessed with a viscous CFD approach based on validated methods (step 5, Fig. 1).

2.1 Baseline Propulsion System and Airframe. The baseline propulsion system is a modern UHBR aero-engine with a bypass ratio of $\dot{m}_{\text{Bypass}}/\dot{m}_{\text{Core}} > 15$ and a nominal standard net thrust of $F_N = 60 \text{ kN}$ [4]. The powerplant has a compact non-axisymmetric nacelle with $L_{\text{nac}}/R_{hi} = 3.1$ and $R_{TE}/R_{hi} = 0.91$ [20] and an optimized axisymmetric exhaust [21] (Fig. 2(b)). The propulsion system is installed as a podded underwing configuration of the NASA Common Research Model (CRM) [22–25] with a pylon [8] (Fig. 2(a)). The propulsion system is installed with pitch ($\phi_{\text{pitch}} = 1.75 \text{ deg}$) and toe ($\phi_{\text{toe}} = 2.25 \text{ deg}$) angles [25] in a close-coupled arrangement with the CRM wing. In this configuration, the nacelle is overlapped with the wing by 3% ($dX/C_{\text{ref}} = 0.03$) and 7.5% down ($dZ/C_{\text{ref}} = 0.075$) [8,9].

The nominal midcruise operating conditions are a flight Mach of $M_{\infty} = 0.85$ at an altitude of 10,668 m and a constant aircraft lift

coefficient of $C_L^{A/C} = 0.5$ [4,8]. The propulsion system operates at design point with a mass flow capture ratio of $\text{MFCR} = 0.7$, a fan nozzle pressure ratio of $\text{FNPR} = P_{BP}/p_{\infty} \approx 2.2$, and a core nozzle pressure ratio of $\text{CNPR} = P_{CR}/p_{\infty} \approx 1.8$ [4,8].

2.2 Geometry Parametrization and Design Space. Non-axisymmetric exhaust configurations are designed with perturbations of the fan cowl trailing edge relative to the axisymmetric baseline configuration [6,8]. Exhaust geometries are parametrized with intuitive class-shape transformation [10,11]. These enable the control of the axial position and radius distributions of the fan cowl trailing edge azimuthally (Figs. 2(b) and 3). The distribution of axial position of the fan cowl trailing edge is controlled with a shear transformation. Exhaust shear is parametrized with 2DOF, the shear angle (θ_A), and the azimuthal position with maximum change (ψ_A) (Fig. 3(a)). With these two design parameters, the shear can be applied at any azimuthal location of the bypass nozzle. The parametrization of trailing edge radius enables localized changes of radius at specific azimuthal locations (Fig. 3(b)). This is controlled with three design variables, the azimuthal location where the change is centered (ψ_R), the radius ratio with respect to the unperturbed design (R/R_{ref}), and the azimuthal extend of the perturbation ($\Delta\psi_R$). A sixth DOF, the mean after-body length (L_{AFTB}) controls the length of the core cowl (Fig. 2).

Modifications of the fan cowl trailing edge may affect the bypass exit area. For this reason, an automatic area matching process adjusts the radius of the core cowl at the bypass nozzle exit to ensure the same bypass exit area as the baseline configuration. The geometric definition of the nacelle aerolines is scaled as a function of the local chord to maintain the same nondimensional design. More detailed explanations about the parametrization and control of non-axisymmetric exhaust geometries can be found in previous works [8,9].

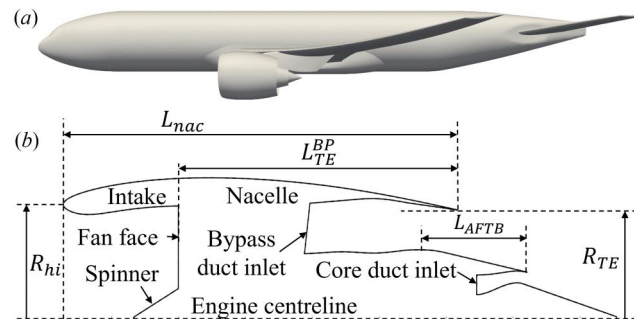


Fig. 2 (a) Propulsion system podded underwing of the CRM and (b) schematic of the engine housing components

2.3 Computational Assessment. Computational fluid dynamics methods of different fidelities are used to assess aircraft-installed

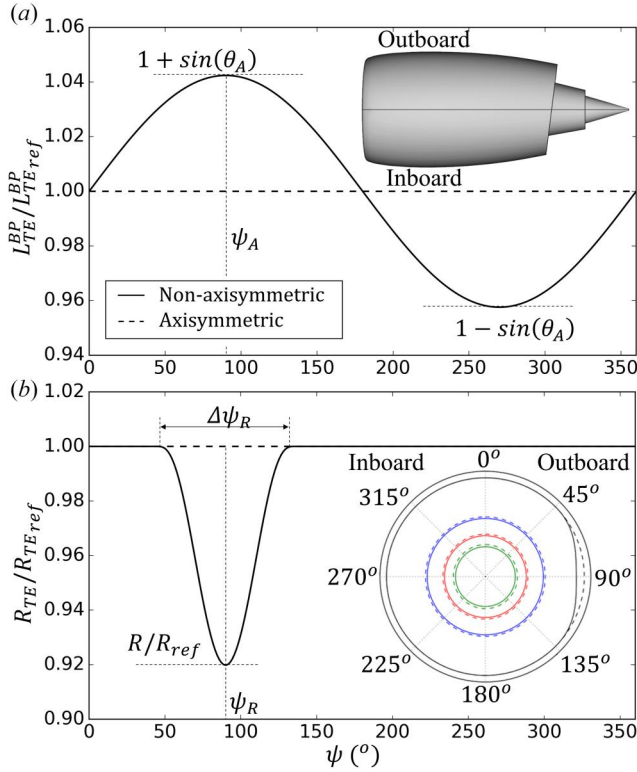


Fig. 3 (a) Shear perturbation and (b) parametrization of bypass trailing edge radius

configurations of the propulsion system. The methods comprise compressible, inviscid methods as the lower-fidelity approach and compressible, turbulent models as the higher-fidelity methodology. The lower-fidelity CFD is based on the solution of the compressible Euler equations with a pressure-based, steady-state algorithm [26]. The higher-fidelity CFD is based on the Favre-averaged Navier–Stokes equations. The method uses a density-based, implicit, and steady-state solver [26] as well as second-order convective schemes and the $k - \omega$ shear stress transport turbulence model [27]. The gas properties of air are defined as a function of static temperature. The dynamic viscosity of the fluid is calculated with the Sutherland’s law [28]. The specific heat at constant pressure ($C_p = f(T)$) is computed with a nine-order piecewise polynomial, and the thermal conductivity is calculated with kinetic theory [26].

The boundary conditions represent the operating conditions of the aircraft and propulsion system. The fan face is treated as mass flow pressure outlet that enforces the desired MFCR. Both bypass and core nozzle duct inlets are modeled as total pressure inlets according to the nozzle pressure ratios. The inflow conditions are set as pressure far field with $M_\infty = 0.85$ and static conditions of pressure and temperature representative of the target altitude, calculated with the International Standard Atmosphere [29]. For the viscous method, all walls are nonslip, while these are slip for the inviscid calculations.

The computational grids are hybrid, with layers of triangular prisms in the boundary layer region and tetrahedra elsewhere. The grid is refined with volumetric density boxes at the regions of interest such as the exhaust jet, the wing, the pylon, and the ducts. For the viscous simulations, this approach ensures a $y^+ < 1$ over the whole geometry and results in 115×10^6 cells with a grid convergence index (GCI) [30] of $GCI_{NPF} = 0.34\%$ and $GCI_{D_{A/F}} = 1.57\%$ for net propulsive force (NPF) and airframe drag ($D_{A/F}$), respectively [8]. The same computational approach with hybrid grids was validated against the Double Stream Flow Reference Nozzle [31] for exhaust flows and against the NASA Common Research Model [22,25] for the prediction of installation drag

(Goulos et al. [4]). For the Double Stream Flow Reference Nozzle, the RMSD between CFD and experimental results was of 0.033% for the velocity coefficient across the FNPR range (1.4 – 2.8) with an 80×10^6 cell mesh. For the CRM at design conditions ($C_L = 0.5$), the installation drag was captured within two drag counts of the experimental results.

The selection of the inviscid CFD mesh was based on the tradeoff between accuracy and computational performance [8]. The objective was to identify a suitable mesh that would capture the general trends of the design space of non-axisymmetric exhausts at the lowest computational cost possible. First, two grids of 80×10^6 cells (fine) and 25×10^6 cells (coarse) were assessed. The fine mesh had a grid convergence index in NPF and $D_{A/F}$ of $GCI_{NPF} = 0.2\%$ and $GCI_{D_{A/F}} = 2.8\%$, respectively. However, the fine mesh was computationally expensive and was not appropriate as a low-cost lower-fidelity method. For this reason, fine and coarse grids were used to assess 96 different non-axisymmetric exhaust designs. It was found that the coarse mesh could predict the variability of the performance metrics in the design space with a root-mean-square deviation in NPF and $D_{A/F}$ of $RMSD_{NPF} = 0.0012F_N$ and $RMSD_{D_{A/F}} = 0.0008F_N$ relative to the converged fine mesh, respectively [8]. With the coarse mesh, the computational time required to assess one design was reduced by a factor of approximately 36 relative to the viscous method and was considered appropriate for lower-fidelity datasets.

2.4 Thrust and Drag Accounting System. A modified near-field method for thrust and drag bookkeeping [32] is applied to extract the metrics of interest from the CFD solutions. The method was applied to installed aero-engine configurations, and it is detailed in the work of Goulos et al. [4]. For this work, it is more convenient to use a 3D vectorial notation in the aerodynamic reference frame ($\hat{i}_D, \hat{i}_S, \hat{i}_L$), where all the forces are composed by drag (D), lift (L), and side (S) terms. The forces that act on the propulsion system and airframe are represented in Fig. 4(a). All wall forces are evaluated through numerical integration of gauge static pressure ($p - p_\infty$) and shear-stresses (τ) along the surfaces. These are defined as ϕ in the drag domain and as θ the thrust domain (Fig. 4(a)). Gauge stream forces (F_G) and mass flows ($\dot{m}$) are also calculated with numerical integration over the fan face (2), bypass (13), and core (7) inlet boundaries (Fig. 4(a)).

The performance of the exhaust system is quantified with the modified velocity coefficient ($(C_v^*)_D$, Eq. (1)) and discharge coefficients (C_d , Eq. (3)). The $(C_v^*)_D$ is calculated with the gauge stream forces at nozzle inlets ($F_{G13} + F_{G7}$) and wall forces over the walls of the thrust domain ($\theta_{BP} + \theta_{CR} + \theta_{CC} + \theta_{plug} + \theta_{pyl}$,

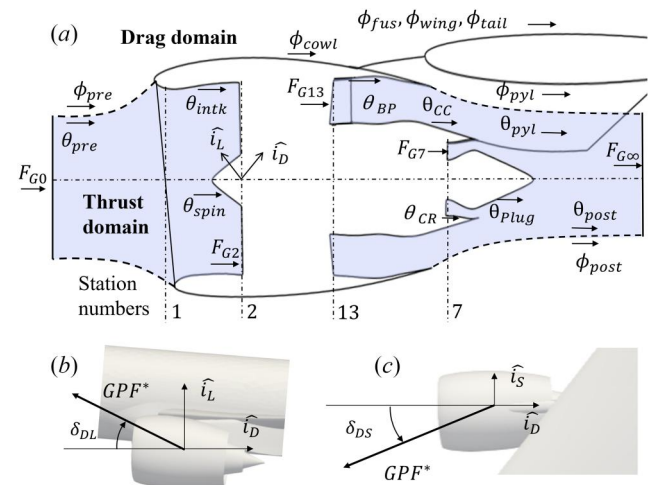


Fig. 4 (a) Schematic of forces that act on the propulsion system and airframe, (b) schematic of the vertical thrust vector angle, and (c) schematic of the lateral thrust vector angle

Fig. 4(a)) normalized with the ideal propulsive force evaluated with CFD-computed nozzle mass flows ($\dot{m}_{\text{CFD}}$) and ideal velocities (v_{id}). The modified gross propulsive force ($(\text{GPF}^*)_D$, Eq. (2)) is the $(C_v^*)_D$ scaled with an ideal propulsive force calculated with the engine cycle mass flows ($\dot{m}_{13\text{cycle}} v_{13\text{id}} + \dot{m}_{7\text{cycle}} v_{7\text{id}}$) to ensure continuity between the intake and exhaust [4]. The discharge coefficients are the CFD-computed mass flows ($\dot{m}_{\text{CFD}}$) normalized with the ideal mass flow per unit area ($(\dot{m}/A)_{\text{id}}$) and the nozzle throat area (A_{th})

$$(C_v^*)_D = \frac{(F_{G13} + F_{G7})_D - (\theta_{\text{BP}} + \theta_{\text{CR}} + \theta_{\text{CC}} + \theta_{\text{plug}} + \theta_{\text{pyl}})_D}{\dot{m}_{13\text{CFD}} v_{13\text{id}} + \dot{m}_{7\text{CFD}} v_{7\text{id}}} \quad (1)$$

$$(\text{GPF}^*)_D = (C_v^*)_D (\dot{m}_{13\text{cycle}} v_{13\text{id}} + \dot{m}_{7\text{cycle}} v_{7\text{id}}) \quad (2)$$

$$C_d = \frac{\dot{m}_{\text{CFD}}}{(\dot{m}/A)_{\text{id}} A_{\text{th}}} \quad (3)$$

The modified nacelle drag (D_{nac}^*) is calculated with the pre-entry force (ϕ_{pre}) and the wall forces exerted on the cowl (ϕ_{cowl}) and on the region of the pylon outside the exhaust jet (ϕ_{pyl}) (Eq. (4)). ϕ_{pre} results from the force balance within the inlet control volume (Eq. (5)), and it includes the gauge stream force on the fan face (F_{G2}), the intake momentum ($F_{G0} = \dot{m}_0 V_\infty$), and the wall forces over the intake (θ_{intk}) and spinner (θ_{spin}). The airframe drag ($D_{\text{A/F}}$, Eq. (6)) includes all the drag components of the wing (ϕ_{wing}), fuselage (ϕ_{fus}), and tail (ϕ_{tail}). The aircraft drag ($D_{\text{A/C}}$, Eq. (7)) is the combination of the modified nacelle and airframe drag terms ($D_{\text{A/F}}$ and D_{nac}^*)

$$D_{\text{nac}}^* = (\phi_{\text{pre}} + \phi_{\text{cowl}} + \phi_{\text{pyl}})_D \quad (4)$$

$$(\phi_{\text{pre}})_D = F_{G2} - F_{G0} - (\theta_{\text{intk}} + \theta_{\text{spin}})_D \quad (5)$$

$$D_{\text{A/F}} = (\phi_{\text{wing}} + \phi_{\text{fus}} + \phi_{\text{tail}})_D \quad (6)$$

$$D_{\text{A/C}} = D_{\text{A/F}} + D_{\text{nac}}^* \quad (7)$$

The performance of the complete aircraft is reported in terms of the aircraft NVF. The NVF is defined as the global balance between propulsive and aerodynamic forces (Eq. (8)) on the drag direction ($\hat{i}_D$), and it is the main metric of interest for installed configurations. It comprises the modified gross propulsive force ($(\text{GPF}^*)_D$), intake momentum (F_{G0}), and aircraft drag ($D_{\text{A/C}}$). For this work, all metrics are expressed at the CRM design conditions with aircraft lift coefficient of $C_L^{\text{A/C}} = 0.5$. In addition, force metrics (F) are expressed as a benefit or penalty (ΔF) relative to a baseline configuration (F_{ref}) and normalized with the nominal net thrust (F_N , Eq. (9)). With this definition, positive ΔF indicate performance benefits

$$\text{NVF} = (\text{GPF}^*)_D - F_{G0} - D_{\text{A/C}} \quad (8)$$

$$\Delta F = \frac{F - F_{\text{ref}}}{F_N} \quad (9)$$

2.4.1 Thrust Vector Angles and Decomposition of the Modified Gross Propulsive Force. Other metrics of interest for exhaust design include the vertical (δ_{DL} , Eq. (10)) and lateral (δ_{DS} , Eq. (11)) thrust vector angles. The thrust vector angles are defined as the angles of the GPF^* vector (Figs. 4(b) and 4(c))

$$\delta_{\text{DL}} = \text{atan} \left(\frac{(\text{GPF}^*)_L}{(\text{GPF}^*)_D} \right) \quad (10)$$

$$\delta_{\text{DS}} = \text{atan} \left(\frac{(\text{GPF}^*)_S}{(\text{GPF}^*)_D} \right) \quad (11)$$

The change in gross propulsive force relative to a baseline configuration ($(\Delta \text{GPF}^*)_D$) is decomposed into the contributions related to changes in the vector magnitude ($(\Delta \text{GPF}^*_{\text{mag}})_D$) and thrust vectoring ($(\Delta \text{GPF}^*_{\text{vec}})_D$, Eq. (12)). $(\Delta \text{GPF}^*_{\text{mag}})_D$ represents a variation in $(\text{GPF}^*)_D$ related to a change in the module of the GPF^* vector, while the thrust vectoring term ($(\Delta \text{GPF}^*_{\text{vec}})_D$) accounts for the force component redistribution due to changes in thrust angles. The thrust vectoring term is further split into the vertical ($\Delta \text{GPF}^*_{\delta_{\text{DL}}}$) and lateral ($\Delta \text{GPF}^*_{\delta_{\text{DS}}}$) components (Eq. (13)). The vertical vectoring effect represents a variation of $(\Delta \text{GPF}^*)_D$ originated from a change of δ_{DL} , while the lateral vectoring effect is related to changes in δ_{DS} . The full vectorial decomposition of the modified gross propulsive force is detailed in previous works [8]

$$(\Delta \text{GPF}^*)_D = (\Delta \text{GPF}^*_{\text{mag}})_D + (\Delta \text{GPF}^*_{\text{vec}})_D \quad (12)$$

$$(\Delta \text{GPF}^*_{\text{vec}})_D = \Delta \text{GPF}^*_{\delta_{\text{DL}}} + \Delta \text{GPF}^*_{\delta_{\text{DS}}} \quad (13)$$

2.5 Design of Experiment. A database of non-axisymmetric exhaust configurations was compiled with the Latin hypercube sampling method [33]. The design space was sampled with a ratio of samples per degree-of-freedom of $n\text{LF}/n\text{DOF} = 80$ [8]. This resulted into 480 different non-axisymmetric exhaust configurations. All 480 designs were evaluated with the inviscid methodology. Of this database, a subselection of 92 designs were assessed with the higher-fidelity viscous approach. This resulted into a high-to-low-fidelity ratio of $n\text{HF}/n\text{LF} = 0.2$. The selection of samples for viscous evaluation was also based on Latin hypercube sampling. However, every sample was brought to the closest neighbor in the full 480 design database. This enabled a back-to-back comparison in terms of the effect of the CFD fidelity.

2.6 Response Surface Modeling. Single and multiple fidelity response surface models were considered. These depended on the aerodynamic data used to formulate the models. The single-fidelity prediction model is a Gaussian process regression (also known as Kriging) [34]. The multi-fidelity RSMs comprise incremental VCM [12,16] and multi-fidelity artificial neural networks [12,17–19].

2.6.1 Single-Fidelity Kriging Response Surface Models. The Gaussian process regression (Kriging) was used to create single-fidelity response surface models. Kriging methods have been used in previous works for the optimization of 2D-axisymmetric exhausts [21] and for the development of low-cost design methodologies for installed propulsion systems using inviscid methods [8,9]. Detailed mathematical formulations of the Gaussian regression process can be found in the literature [35]. A RSM based on Kriging (RSM_{krig} or $\tilde{y}$) approximates the value of a true function ($y(x)$) through a combination of regression ($\mu(x)$) and correlation functions ($z(x)$) (Eq. (14)). The method can be tuned through the selection of regression function, correlation function, and nugget size (ϵ)

$$\tilde{y}(x) = \mu(x) + z(x) \quad (14)$$

Previous works on the design of non-axisymmetric exhaust systems [8,9] identified the optimum combination of hyperparameters using a leave-one-out cross validation methodology [36]. For an exhaust database of 480 non-axisymmetric designs, the best response of the model was with a quadratic regression function, an absolute exponential correlation function, and a nugget size of $\epsilon = 1 \times 10^{-7}$. With these parameters, the RSM prediction for ΔNVF had a coefficient of determination (R^2) of $R^2 = 0.96$, a linear slope of $m = 0.945$, and a root-mean-square deviation of $\text{RMSD} = 0.002F_N$.

2.6.2 Multi-fidelity Variable Complexity Models. Variable complexity models are methods based on the correction of a low-fidelity function with higher-fidelity data. In the incremental formulation of VCM [12,16], the multi-fidelity prediction function ($\tilde{y}_1(x)$) is calculated with the low-fidelity prediction model ($\tilde{y}_2(x)$)

and an incremental function ($\beta(x)$) (Eq. (15)). The incremental function is a regression model ($g(x)$) calculated from the difference between the high ($f_1(x)$) and low-fidelity ($f_2(x)$) data (Eq. (16))

$$\tilde{y}_1(x) = \tilde{y}_2(x) + \beta(x) \quad (15)$$

$$\beta(x) = g(f_1(x) - f_2(x)) \quad (16)$$

The key to construct multi-fidelity surrogate models with VCM relies on the approximation of the low-fidelity ($\tilde{y}_2$, Eq. (17)) and the incremental functions ($\beta(x)$, Eq. (18)). Single-fidelity RSMs based on Kriging were used for that purpose. The hyperparameters of both sub RSMs (correlation function, regression function, and nugget size) were tuned to obtain the best possible response

$$\tilde{y}_2(x) = \text{RSM}(f_2(x)) \quad (17)$$

$$\beta(x) = \text{RSM}(f_1(x) - f_2(x)) \quad (18)$$

2.6.3 Multi-fidelity Artificial Neural Networks. A multi-fidelity artificial neural network is an ANN composed of two independent nets [12,17,19], the lower-fidelity (LF-ANN) and the higher-fidelity (HF-ANN) networks (Fig. 5). The first network (LF-ANN) models the low-fidelity response of the system (R_{LF}) given the design parameters as inputs (DOF1, ..., DOF n). The second (HF-ANN) is composed by two subnetworks and aims to correct the low-fidelity response with higher-fidelity data. The purpose of the two subnetworks is to capture the linear and nonlinear correlations between the lower and higher-fidelity models. The high-fidelity network takes the design parameters (DOF1, ..., DOF n) as inputs together with the prediction of the low-fidelity network (R_{LF}). The output of the complete network is the multi-fidelity prediction (R_{MF}).

Each of the subnetworks that comprise the MFANN is a feedforward ANN where all neurons are organized in layers. Each neuron of one layer is connected to all the neurons of the previous and following layers where the first layer is the input layer, the last is the output layer, and the layers in between are called hidden layers. The output of a neuron (k) is a linear combination of the outputs of all neurons in the previous layer (x_i) scaled by a weighting coefficient (w_{ki}) and with a constant bias term (b_k) (Eq. (19)). After that, an activation function ($f(x)$) is applied to v_k , and the final output of the neuron (y_k) is obtained (Eq. (20)). Typical activation functions include hyperbolic tangent (tanh), sigmoid, exponential, rectified linear unit (ReLU), and linear functions, among others

$$v_k = \sum_{i=1}^n w_{ki}x_i + b_k \quad (19)$$

$$y_k = f(v_k) \quad (20)$$

The training step of a neural network consists in the determination of weights and biases of all the neurons that minimize a loss function for a particular combination of hidden layers, number of neurons, and activation function. The root-mean-square error is the selected loss function. The minimization of the function is done with the adaptive moment estimation optimizer (ADAM) due to its ability to handle large datasets with high computational efficiency [37]. For the MFANN, the training process is composed by two independent steps. First, the low-fidelity ANN is trained with lower-fidelity data. The lower-fidelity ANN is independent from the rest of the network and can be trained independently. Once a prediction model for the LF-ANN is obtained, it is used as an extra input for the HF-ANN, which is trained with higher-fidelity data. For each design problem, an analysis of several combinations of hyperparameters is used to find the best response from the network. The different multi-fidelity response surface models have been verified with an independent exhaust database. The verification of the response surface models is presented in Sec. 3.2.

2.7 Optimization Methods. Single- and multi-fidelity RSMs were the objective functions in an optimization process. The chosen optimization algorithm was a single-objective GA [38] as in other nacelle [20] and exhaust [8,9,21] optimization studies. Optimizations were configured with 100 generations, an initial population of 50 times the number of design parameters ($N_s/N_{DOF} = 50$) and a population size per generation of 25 times the number of DOF ($N_g/N_{DOF} = 25$) [8]. The optimization targeted the maximization of $\Delta N_{VF_{RSM}}$ while ensuring both the bypass and core discharge coefficients are maintained within certain limits. The maximum penalties allowed for discharge coefficients are $(C_{d_{ref}}^{BP} - C_{d_{ref}}^{BP})/C_{d_{ref}}^{BP} > -1.0\%$ and $(C_{d_{ref}}^{CR} - C_{d_{ref}}^{CR})/C_{d_{ref}}^{CR} > -6.0\%$. Penalties beyond these values would compromise the performance of the new system once scaled to match the cycle mass flow requirements [8]. Once the optimization algorithm identified a candidate non-axisymmetric exhaust design, this was assessed with the higher-fidelity viscous and turbulent CFD method.

3 Results and Discussion

3.1 Multi-fidelity Assessment of Non-axisymmetric Exhausts. The differences between inviscid and viscous CFD methods were quantified for installed non-axisymmetric exhausts to

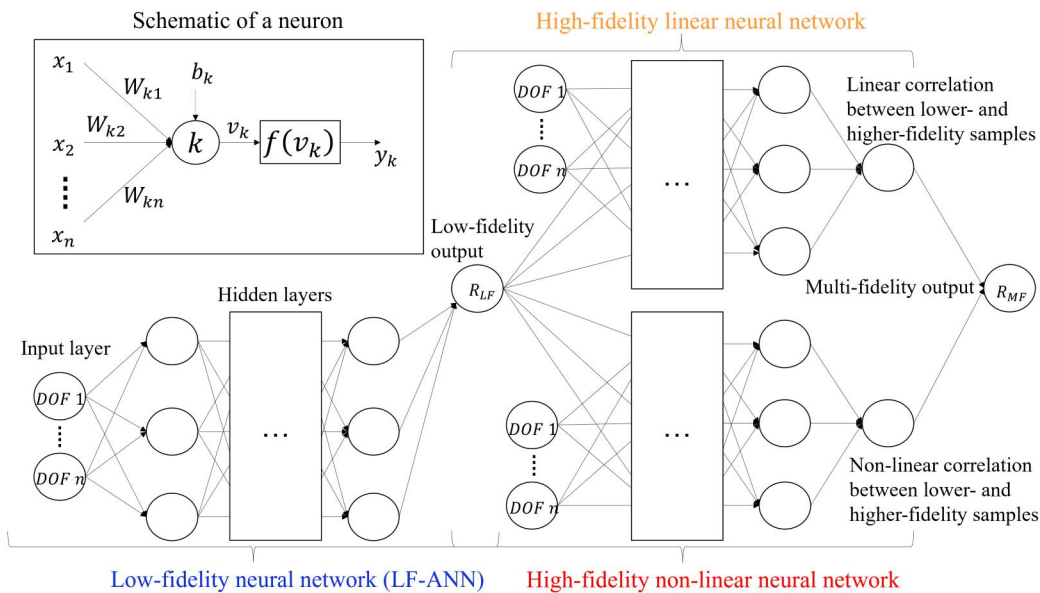


Fig. 5 Schematics of the multi-fidelity neural network architecture for response surface modeling

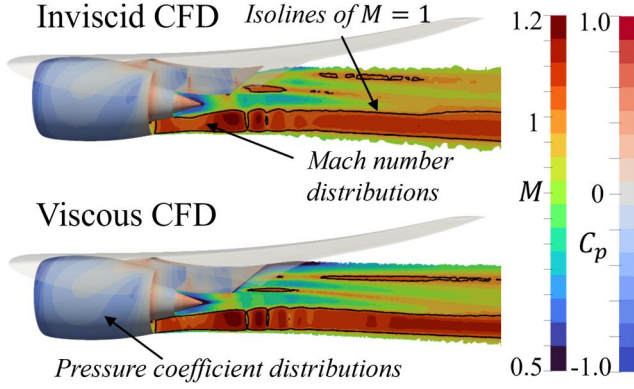


Fig. 6 Comparison of inviscid and viscous CFD results for the baseline configuration. The results are at the same aircraft lift coefficient ($C_{L_{A/C}}=0.5$).

establish a back-to-back comparison in terms of overall flow mechanisms and design space exploration results. The objective was to verify the inviscid CFD approach for the exploration of the non-axisymmetric exhaust design space and to gauge the applicability of the method as the lower-fidelity approach in the multi-fidelity design framework.

In terms of aerodynamic mechanisms, the inviscid CFD captured the main characteristics of installed propulsion systems (Fig. 6). These encompass the inboard shock-wave between the nacelle, wing, and fuselage; static pressure distributions over the airframe and nacelle; and Mach number distributions within the exhaust jet (Fig. 6).

To quantify the effect of the CFD fidelity on the design space of non-axisymmetric exhausts, 96 different separate-jet configurations were assessed with both methodologies. The NVF results for both computational methodologies (Figs. 7(a) and 7(b)) indicate that a 1% change of NVF with the inviscid CFD method equates to approximately 0.6% when assessed with the viscous approach ($\Delta NVF_{HF} = 0.621\Delta NVF_{LF} + 0.015$, $R^2 = 0.83$, Fig. 7(b)). Although the standard deviation (σ) of the inviscid solutions is significant ($\sigma = 0.3\%$ of F_N), the scatter is reduced for designs that

benefit the NVF ($\Delta NVF > 0$). Therefore, both computational methodologies would guide a design optimization process to similar regions of the design space.

Designs with greater levels of ΔNVF are located at the regions of the design space where the modified gross propulsive force and modified nacelle drag are beneficial ($(\Delta GPF^*)_D > 0$ and $\Delta D_{nac}^* > 0$, Figs. 7(c) and 7(d)), while the airframe drag is penalized ($\Delta D_{A/F} < 0$, Fig. 7(e)), relative to the baseline. Although the total drag of the system ($D_{A/C} = D_{nac}^* + D_{A/F}$) increases relative to the axisymmetric baseline design, overall the NVF is improved because the benefits in $(\Delta GPF^*)_D$ counter the penalizing effects [8,9]. This result is consistent for both numerical methodologies.

The correlation between numerical methods indicates a strong linearity for $(\Delta GPF^*)_D$ with a coefficient of determination of $R^2 = 0.94$, a slope of ($\zeta = 1.045$), and a bias of 0.27% of F_N (Fig. 7(c)). As expected, the discrepancies between results of different CFD fidelities are greater for the drag metrics (ΔD_{nac}^* and $\Delta D_{A/F}$, Figs. 7(d) and 7(e)). However, the scatter is reduced for designs that improve the NVF. Designs that penalize the NVF are configured with a bypass nozzle that is sheared toward the outboard side, where the nacelle is longer at the inboard [8]. At the inboard side, the flow velocities are greater due to the preshock expansion. This, combined with the increased wetted area of outboard-sheared designs, magnifies the importance of the viscosity effects and explains the discrepancy between CFD methods. Overall, these results indicate that inviscid CFD methods can provide a notable approximation of the exhaust performance ($(\Delta GPF^*)_D$). Moreover, although the method is less accurate for the prediction of drag metrics, these are reasonably well approximated for designs that improve NVF. The differences in prediction accuracy of inviscid methods for thrust and drag metrics indicate that the optimum tradeoff could change when the effects of viscosity are included. For this reason, although inviscid methods could potentially lead an optimization process to beneficial regions of the design space, viscous data are required to identify the optimum.

Non-axisymmetric exhaust systems benefit the aircraft performance primarily through reductions of lateral and vertical thrust vector angles relative to the baseline axisymmetric configuration [8]. The inviscid CFD approach approximates the design space characteristics of the vertical thrust angle ($\Delta \delta_{DL}$, Fig. 7(f)) better than the lateral one ($\Delta \delta_{DS}$, Fig. 7(g)). The correlation between CFD

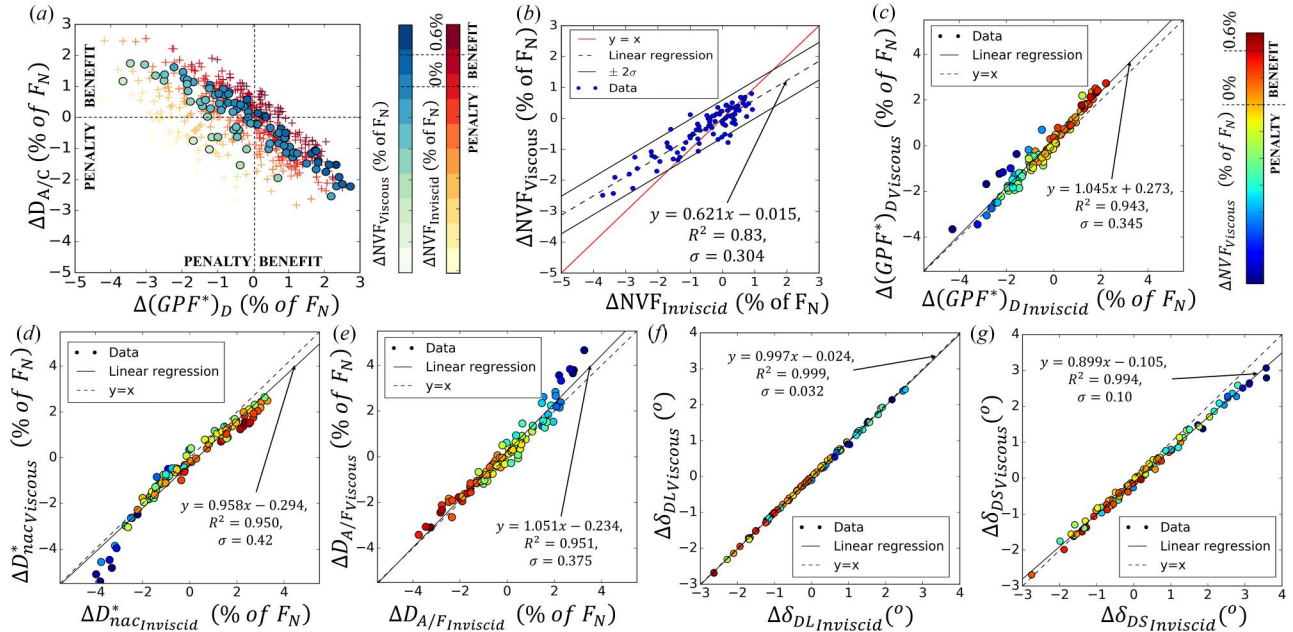


Fig. 7 Multi-fidelity design space exploration. (a) Inviscid and viscous CFD data distribution and correlations between higher and lower-fidelity data of (b) ΔNVF , (c) $(\Delta GPF^*)_D$, (d) ΔD_{nac}^* , (e) $\Delta D_{A/F}$, (f) $\Delta \delta_{DL}$, and (g) $\Delta \delta_{DS}$. The data points are colored with the ΔNVF calculated with RANS (c)–(g). (Color version online.)

Table 1 Summary of the datasets used to train and verify the multi-fidelity response surface models

Dataset	Use	CFD fidelity	Samples
DS1-LF	Training	Inviscid	480
DS1-HF	Training	Viscous	96
DS1-test	Verification	Viscous	24

fidelities indicates that 1 deg change in $\Delta\delta_{DL}$ is equivalent for both numerical methodologies ($\Delta\delta_{DLHF} = 0.997\delta_{DL-LF} - 0.024$, $\sigma = 0.03$ deg). However, the inviscid prediction is less accurate for the lateral thrust angle. For $\Delta\delta_{DS}$, 1 deg change calculated with the inviscid method would result into a variation of 0.9 deg when computed with the viscous CFD ($\Delta\delta_{DSHF} = 0.899\delta_{DS-LF} - 0.105$, $\sigma = 0.1$ deg). The differences in the accuracy of the inviscid CFD for the prediction of both thrust vector angles indicate that the optimal tradeoff between lateral and vertical vectoring effects could also be different in optimum designs if the more expensive viscous CFD method is used.

In conclusion, the inviscid CFD methodology can capture the main aerodynamic mechanisms and design space trends of installed non-axisymmetric exhausts. The lower-fidelity approach performs better at regions of the design space that benefit the NVF. In addition, these results confirm that inviscid CFD based methods can reproduce the effects of engine–airframe integration, as postulated by Smith et al. [39] and adopted in previous works for non-axisymmetric exhaust design [8,9]. However, the different predictive accuracy of the inviscid approach for thrust and drag forces, as well as for vertical and lateral thrust vector angles, indicates that the optimal tradeoff of performance metrics may be different in optimum designs if the viscous CFD is considered. For this reason, a multi-fidelity design methodology could unlock better non-axisymmetric exhaust configurations relative to a method based only on inviscid CFD. Nevertheless, inviscid and compressible CFD methods are sufficiently accurate to be used as a low-fidelity aerodynamic model in a multi-fidelity design methodology for installed non-axisymmetric exhausts.

3.2 Verification of the Multi-fidelity Response Surface Models. Inviscid and viscous aerodynamic data are blended using MFRSM. However, prior to the optimization of exhaust systems with these models, the RSMs need to be verified. The RSMs were verified for the prediction of the net vehicle force (ΔNVF) with an independent database of aircraft installed propulsion systems with non-axisymmetric exhaust designs.

Multi-fidelity RSMs were trained with the complete higher- and lower-fidelity databases of installed exhaust designs (Table 1). These encompassed 480 designs assessed with the inviscid CFD approach (lower-fidelity dataset, DS1-LF) and 96 (20%) assessed

with the viscous, turbulent methodology (higher-fidelity dataset, DS1-HF). To assess the performance of the RSMs, an independent database of 24 samples evaluated with the higher-fidelity method was compiled (DS1-test, Table 1). The response surface models were used to predict the performance of the independent dataset (DS1-test).

The hyperparameters of the response surface models were tuned to minimize the prediction error for the independent dataset. For the VCM method, the low-fidelity model and incremental correction function (β) were configured with quadratic regression and absolute exponential correlation functions as well as a nugget size of $\epsilon = 1 \times 10^{-7}$. These parameters minimized the prediction error and are consistent with previous works [8,9]. For the MFANN, different combinations of hidden layers, neurons per layer, and activation functions were tested with a full factorial approach. The best performance was obtained when the low-fidelity neural network was configured with five hidden layers, 50 neurons per layer, and the ReLU activation function; the linear subnetwork was configured with five hidden layers, 35 neurons per layer, and the linear activation function; and the nonlinear subnetwork was configured with five hidden layers, 60 neurons per layer, and the ReLU activation function. In addition to the multi-fidelity RSMs, single-fidelity Kriging and ANN models were trained only with the higher-fidelity aerodynamic data (DS1-HF). The single-fidelity Kriging model was configured with the same hyperparameters of the low-fidelity Kriging model used in VCM. The single-fidelity ANN was configured with five hidden layers, 80 neurons per layer, and the ReLU activation function. This was the optimum combination of hyperparameters for this model.

The accuracy of the RSMs was quantified in terms of coefficient of determination (R^2), linear regression slope (ζ), intercept, and standard deviation (σ) between RSM predictions and viscous CFD results. The improved performance of multi-fidelity surrogate models relative to single fidelity ones is clear from the results of the analysis (Fig. 8). The single-fidelity Kriging and ANN models trained with just higher-fidelity data could not approximate the main characteristics of the independent dataset. The single-fidelity Kriging model predicted the ΔNVF of the independent dataset with $R^2 = 0.140$, $\zeta = 0.314$, and $\sigma = 0.568\%$ of F_N (Fig. 8(a)), while the single-fidelity ANN prediction had a $R^2 = 0.124$, $\zeta = 0.385$, and $\sigma = 0.746\%$ of F_N (Fig. 8(b)). These results indicate that a ratio of high-fidelity samples per degree-of-freedom (n_{HF}/n_{DOF}) of 16 was not sufficient to create reliable single-fidelity surrogate models with Kriging and neural networks. However, both MFRSM improved the prediction and captured the variability of DS1-test (Figs. 8(c) and 8(d)). The VCM method approximated the ΔNVF with $R^2 = 0.905$, $\zeta = 0.862$, and $\sigma = 0.205\%$ of F_N . The approach with neural networks (MFANN) was better and predicted the independent dataset with $R^2 = 0.975$, $\zeta = 0.868$, and $\sigma = 0.101\%$ of F_N .

In conclusion, it is demonstrated that the combination of lower and higher-fidelity data can improve the predictive capabilities of

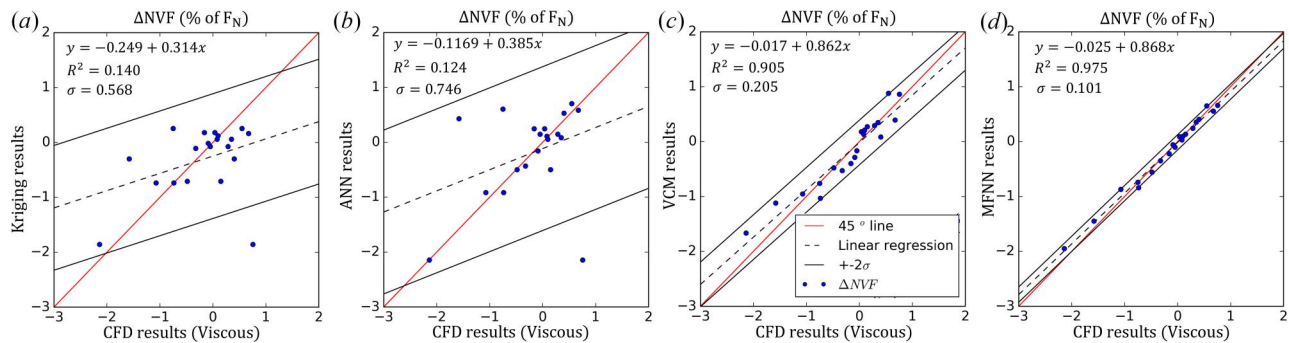


Fig. 8 Comparison of the prediction of the different RSMs with the independent dataset (DS1-test). The figure includes (a) the results of the Kriging RSM trained with high-fidelity data, (b) the results of the ANN trained with high-fidelity data, (c) the results of VCM, and (d) the results of MFANN for ΔNVF .

Table 2 Summary of optimum non-axisymmetric exhaust systems and the different RSMs used in the optimization

Design	Method	Data type	Target	Constraint 1	Constraint 2
LF-1	Kriging RSM	Inviscid	ΔNVF^{MAX}	$\Delta C_d^{\text{BP}} \geq -1\%$	$\Delta C_d^{\text{CR}} \geq -6\%$
VCM-1	VCM	Viscous and inviscid	ΔNVF^{MAX}	$\Delta C_d^{\text{BP}} \geq -1\%$	$\Delta C_d^{\text{CR}} \geq -6\%$
MFANN-1	MFANN	Viscous and inviscid	ΔNVF^{MAX}	$\Delta C_d^{\text{BP}} \geq -1\%$	$\Delta C_d^{\text{CR}} \geq -6\%$

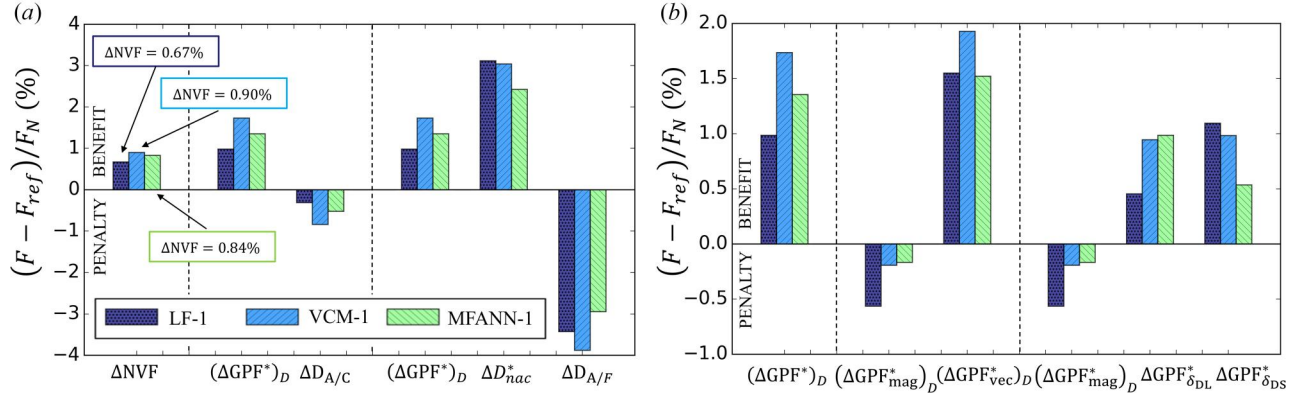


Fig. 9 (a) General decomposition of ΔNVF and (b) decomposition of the modified gross propulsive force (ΔGPF^*). These results are based on viscous computations.

Table 3 Thrust vector angles (δ) of the different optimum designs

Design	δ_{DL} (deg)	δ_{DS} (deg)	ϕ_{pitch} (deg)	ϕ_{toe} (deg)
Baseline	5.02	4.96	1.75	2.25
LF-1	4.23	2.66	1.75	2.25
VCM-1	3.17	2.98	1.75	2.25
MFANN-1	3.07	4.00	1.75	2.25

These results are based on viscous and compressible CFD.

response surface models for installed non-axisymmetric exhausts at a given computational cost. These results enable the optimization of installed propulsion systems with the effects of viscosity included.

3.3 Multi-fidelity Optimization of Non-axisymmetric Exhausts. Both multi-fidelity approaches (VCM and MFANN) were used to drive the optimization of non-axisymmetric exhausts in installed configurations of the propulsion system. The main objective was the comparison of the optima identified with the multi-fidelity methodologies relative to the optimum design that resulted from the optimization of a Kriging RSM constructed with lower-fidelity inviscid data [8,9]. Single-fidelity Kriging and ANN models trained with the higher-fidelity data were not used to drive the optimization because these could not approximate the validation dataset reliably (Sec. 3.2).

Three candidate designs were identified from the optimization of ΔNVF with each RSM (Table 2). The designs that resulted from the optimizations were named LF-1, VCM-1, and MFANN-1, obtained from the maximization of ΔNVF with the lower-fidelity Kriging, multi-fidelity variable complexity model, and multi-fidelity artificial neural network RSMs, respectively. The three designs were assessed with the higher-fidelity CFD method to calculate the performance metrics. Therefore, all the results presented in this section are based on the same standard of turbulent CFD simulations.

The multi-fidelity optimization of installed non-axisymmetric exhausts identified designs with greater benefit in net vehicle force (ΔNVF) relative to the lower-fidelity optimum (LF-1). The optimum designs obtained with VCM (VCM-1) and MFANN (MFANN-1)

improve the NVF by $\Delta NVF = 0.0090F_N$ and $\Delta NVF = 0.0084F_N$, respectively. This is approximately 0.2% of F_N better than for LF-1 ($\Delta NVF = 0.0067F_N$) [8]. Although the lower-fidelity design method based on inviscid data could find optimum designs with substantial benefit in net vehicle force, the addition of higher-fidelity viscous data in the process enabled the identification of better configurations. The general split of ΔNVF into the constituent metrics is similar for the three designs (Fig. 9(a)). However, the magnitude of the force components are different. VCM-1 and MFANN-1 further improve the exhaust performance through an increased $(\Delta GPF^*)_D$ relative to LF-1. However, both designs exhibit greater penalties in aircraft drag ($D_{A/C}$). The improvement of gross propulsive force offsets the drag penalties and improves the aircraft NVF.

The decomposition of GPF^* in terms of the change in vector magnitude ($(\Delta GPF_{\text{mag}}^*)_D$) and vectoring ($(\Delta GPF_{\text{vec}}^*)_D$) indicates that the benefit arises from different mechanisms depending on the design (Fig. 9(b)). For the low-fidelity optimum design (LF-1), the benefit is dominated by reductions on the lateral thrust vector angle ($\delta_{\text{DS}} = 2.66$ deg, Table 3). However, for VCM-1, the benefit in $(GPF^*)_D$ is governed by an increased gross propulsive force magnitude ($(\Delta GPF_{\text{mag}}^*)_D = -0.0019F_N$) and a better alignment of the thrust vector with the drag axis ($(\Delta GPF_{\text{vec}}^*)_D = 0.0193F_N$) relative to LF-1 (Fig. 9(b)). Both metrics are approximately $0.004F_N$ greater compared to LF-1. The additional benefit in thrust vectoring arises from the vertical component. While there are substantial reductions of lateral thrust angle relative to the baseline configurations ($\Delta \delta_{\text{DS}}$), these are similar to LF-1. However, the vertical thrust vector angle ($\Delta \delta_{\text{DL}}$) is approximately 1 deg lower for VCM-1 relative to LF-1 (Table 3). For MFANN-1, the increased gross propulsive force relative to LF-1 is only related to an improvement of $(\Delta GPF_{\text{mag}}^*)_D$ as the thrust vectoring term remains similar. However, the lateral and vertical contributions of the thrust vectoring have changed compared to LF-1. The vectoring term of MFANN-1 is governed by the vertical component with a vertical thrust vector angles of $\delta_{\text{DL}} = 3.07$ deg. On the other hand, the lateral thrust angle is $\delta_{\text{DS}} = 4.00$ deg, which is greater relative to LF-1 and VCM-1 (Table 3 and Figs. 9(c) and 9(d)).

The two multi-fidelity designs (VCM-1 and MFANN-1) feature inboard sheared bypass nozzles with a reduction of trailing edge radius at the inboard side. Although the general non-axisymmetric configuration is similar to LF-1, it differs in the values of the design

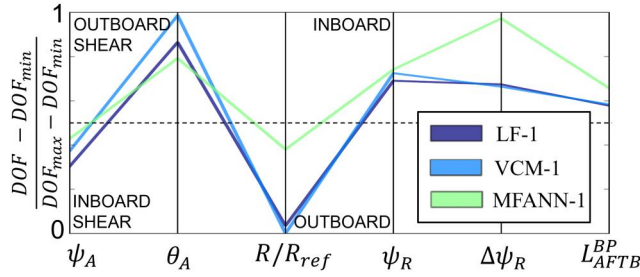


Fig. 10 Parallel coordinate chart of the different optimum non-axisymmetric exhaust designs

variables (Fig. 10). The shear perturbation is applied at the inboard side (ψ_A), but closer to the bottom aeroline relative to LF-1. Previous research [8] demonstrated that thrust vectoring was dominated by the shear perturbation. For this reason, values of ψ_A that are closer to $\psi_A = 180$ deg deflect the thrust vector upwards and reduce the vertical thrust angle (δ_{DL}). Moreover, the shear angle (θ_A) of VCM-1 is greater relative to LF-1 and MFANN-1, which may explain the increased benefit on propulsive force related to thrust vectoring ($(\Delta GPF_{vec}^*)_D$). The perturbation of the bypass trailing edge radius (ψ_R , R/R_{ref} , and $\Delta\psi_R$) is similar for VCM-1 and LF-1. The two designs have a 5% reduction in radius at the inboard side but VCM-1 centers the perturbation at slightly greater azimuthal position (ψ_R).

While MFANN-1 is configured with a similar value of ψ_R relative to VCM-1, R/R_{ref} and $\Delta\psi_R$ are greater.

Overall, the incorporation of viscous data through multi-fidelity response surface models in the optimization process led to designs with $NVF = 0.009F_N$ greater than the axisymmetric baseline configuration. This is approximately a 30% increase in the improvement of F_N relative to the optimum design obtained with the inviscid lower-fidelity approach. As anticipated during the design space analysis, the tradeoffs of the different performance metrics that lead to optimum configurations are different for lower-fidelity and multi-fidelity optimum designs. The use of multi-fidelity models has unveiled new mechanisms by which the NVF could be further improved with non-axisymmetric exhausts. In particular, the multi-fidelity method identified designs with reduced vertical thrust angles relative to the lower-fidelity optimization method.

An estimate of the computational time required for the designs is presented in Table 4. The computational cost considers the time required to compute all the CFD as well as geometry generation and meshing. It is expressed relative to the computational time required to assess one design with the inviscid method (t). In addition, the table includes the computational time of an hypothetical exploration of the design space where all 480 designs are assessed with the viscous CFD approach. Although this was not calculated, it is included in terms of computational cost to provide a reference. Relative to a viscous CFD exploration of the design space with 480 designs, the multi-fidelity optimization methods reduced the

Table 4 Summary of the computational time required to design exhaust systems with the low-fidelity method based on inviscid data (LF-1) and multi-fidelity methods based on VCM (VCM-1) and on MFANN (MFANN-1)

Design method	Kriging	VCM	MFANN	Viscous exploration
Training data	Inviscid	Viscous and inviscid	Viscous and inviscid	—
Design	LF-1	VCM-1	MFANN-1	—
ΔNVF (% of F_N)	0.70%	0.90%	0.84%	—
Computational cost	516t	3972t	3972t	17316t

The table includes the computational cost required to generate geometries, meshes, and compute CFD. The time unit (t) is the computational time required for one inviscid CFD assessment.

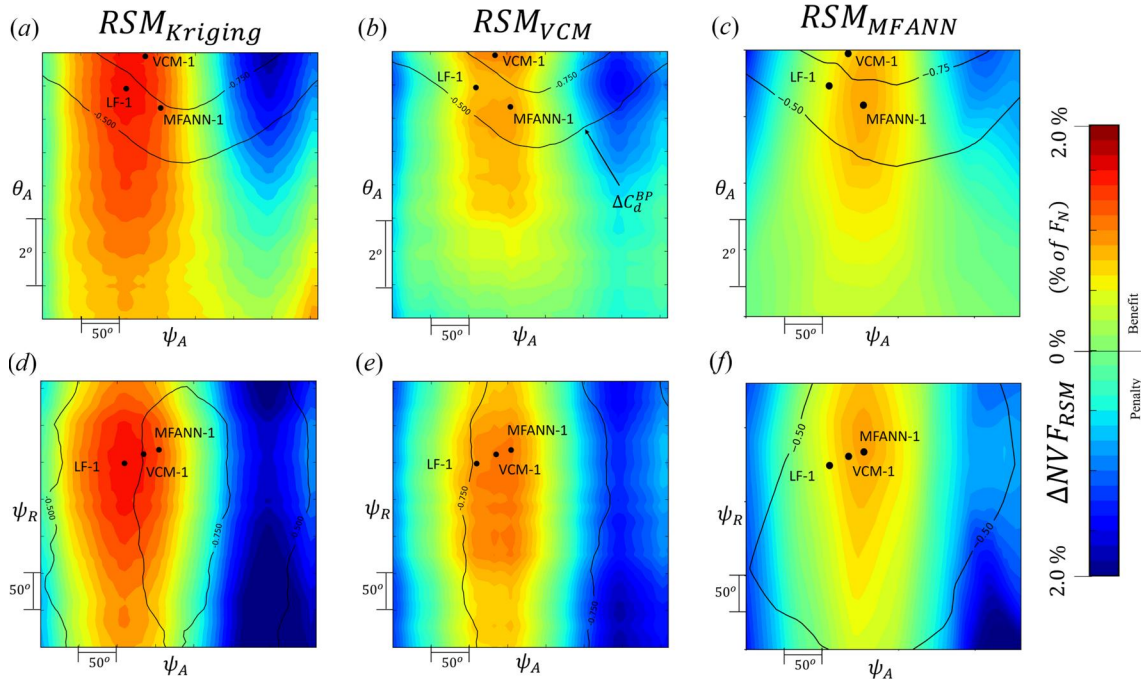


Fig. 11 Sensitivity maps of net vehicle force (ΔNVF) with the shear perturbation parameters (ψ_A , θ_A) and azimuthal location of the perturbation of radius (ψ_R). Positive values are benefits in ΔNVF relative to the baseline. The figure includes the low-fidelity RSM (left), VCM (center), and MFANN (right). The optimum designs obtained with each method are illustrated.

computational time by a factor of 4. However, this is still eight times greater relative to the lower-fidelity method (LF-1). Therefore, the multi-fidelity method has enabled the incorporation of viscous effects on the design methodology at a 25% of the cost relative to an exploration of the design space with viscous CFD. The extra 0.2% in cruise fuel reduction of VCM-1 relative to LF-1 could only be unveiled once lower- and higher-fidelity aerodynamic data were combined and added in the design process. In conclusion, multi-fidelity methods provide a way to increase the computational fidelity of prediction models at early stages of the design process at a reduced computational cost.

3.4 Design Space Sensitivities. One of the main advantages of RSM based methods relative to other optimization approaches such as CFD in-the-loop is that these models enable the visualization of the design space. With RSMs, the sensitivities of the design space and the tradeoffs between design parameters and performance metrics can be derived. This is useful to guide aerodynamicists at early stages of design projects. In this context, the lower-fidelity Kriging, the VCM, and the MFANN RSMs are interrogated in the vicinity of the optimum designs. The design variables are altered in pairs for the respective RSMs, where the unperturbed degrees-of-freedom remain constant at the value of each optimum design. This allows the derivation of design sensitivity maps and enables a qualitative comparison of the responses of the different surrogate models (Fig. 11).

In general, the three RSMs ($RSM_{Kriging}$, RSM_{VCM} , and RSM_{MFANN}) have similar sensitivities near the respective optimum designs. The overall trends of the design space are consistent between models but the absolute values are different. The shear parametrization, which is characterized by the shear angle (θ_A) and azimuthal position (ψ_A) (Fig. 11), dominates the NVF. For θ_A and ψ_A , multi-fidelity RSMs (RSM_{VCM} and RSM_{MFANN}) have a reduced peak-to-peak variation of ΔNVF_{RSM} relative to the lower-fidelity RSM ($RSM_{Kriging}$). In addition, the optimum region of the design space is at greater values of θ_A for both multi-fidelity RSMs (RSM_{VCM} and RSM_{MFANN}) relative to $RSM_{Kriging}$ (Figs. 11(a)–11(c)). While VCM identifies the optimum design close to the upper bound of θ_A , the optimum of the multi-fidelity neural network approach is at a reduced value (Figs. 11(a)–11(c)). The optimum combination of the shear and radius perturbations is defined by the pair ψ_A and ψ_R (Figs. 11(g)–11(i)). The region encompassed by these two parameters changes for the multi-fidelity RSMs relative to $RSM_{Kriging}$. For RSM_{VCM} , the optimum region encompasses a wider range of ψ_R . However, for RSM_{MFANN} , the region is reduced in extend and is centered toward greater values of ψ_R .

Although the multi-fidelity method with neural networks (MFANN) was more accurate to reproduce the performance of the independent dataset than VCM (Fig. 8), it fails to capture the trends of the design space near the boundaries. The reason is that VCM is based on Kriging, and therefore it represents a parametric and mathematical model defined across the complete design space [40]. However, MFANN are nonparametric and use local models over the different regions [40]. For this reason, as there are fewer data available near the bounds, MFANN methods perform worse in the region. In this case, the optimum design was found over the bounds of the design space, but for wider design spaces where the optimum does not lie over the bounds, MFANN are expected to perform better.

4 Conclusions

Future ultra-high bypass ratio aero-engines are likely to require a more close-coupled integration with the airframe. This may increase the aerodynamic interference between powerplant and wing which could negatively impact the aircraft fuel consumption.

The main contribution of this research is the development of a methodology that enables a faster aerodynamic design of complex systems with low-order models that are compiled with a mixture of fidelities. Inviscid and viscous aerodynamic data were combined to formulate multi-fidelity response surface models. These guided a

design optimization process that incorporated the nonlinear effects of viscosity. The novelty is the exploration of the aircraft-installed non-axisymmetric exhaust design space with a greater degree of aerodynamic fidelity. Non-axisymmetric exhaust systems were optimized with the effects of aircraft integration using the developed multi-fidelity framework. The impact is the identification of new candidate non-axisymmetric exhaust designs with an aircraft net vehicle force 0.9% of F_N greater than the axisymmetric baseline configuration. This is approximately a 30% increase relative to the optimum design obtained with a lower-fidelity state-of-art method based on inviscid aerodynamic data and is achieved at 25% of the computational cost relative to a design exploration using viscous computations, which makes it viable for an industrial design process.

Multi-fidelity design methods are required to extract the maximum benefits of non-axisymmetric exhausts. In addition, these provide a way to incorporate the viscosity effects into the design optimization process of complex systems with sparse populated viscous data.

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Data Availability Statement

Due to commercial confidentiality agreements, the supporting data are not available.

Nomenclature

C_L and C_D	= lift and drag coefficient
C_{ref}	= reference chord length (m)
C_v and C_d	= velocity and discharge coefficients
dX and dZ	= nacelle trailing edge axial and vertical offset from wing leading edge (m)
F_{G0}	= intake momentum (N)
F_N	= standard nominal net thrust (N)
$\hat{i}_D, \hat{i}_S, \hat{i}_L$	= aerodynamic axes reference frame unit vectors
L and D	= lift and drag forces (N)
L_{AFTB}	= after-body length (m)
$\dot{m}$	= mass flow rate (kg/s)
M_∞	= free-stream Mach number
N_s and N_g	= initial population and population per generation, respectively
P and p	= stagnation pressure and static pressure (Pa)
R/R_{ref}	= radius ratio of the radius perturbation
R^2	= coefficient of determination
$\hat{y}$	= prediction model of function y
y^+	= nondimensional wall distance

Greek Symbols

δ_{DL} and δ_{DS}	= vertical and lateral thrust vector angles (rad)
$\Delta\psi_R$	= azimuthal extend of the radius perturbation (rad)
ϵ	= nugget size
ζ	= linear regression slope
θ_A	= shear angle (rad)
σ	= standard deviation
ϕ_{pitch} and ϕ_{toe}	= installation pitch and toe angles (rad)
ψ	= azimuthal coordinate (rad)
ψ_A and ψ_R	= azimuthal position of the shear and radius perturbations (rad)

Subscripts and Superscripts

A/C and A/F	= referring to the whole aircraft and airframe only
BP and CR	= referring to the bypass and core nozzles
LF, HF, and MF	= relative to low-fidelity, high-fidelity, and multi-fidelity data
mag and vect	= referring to magnitude and vectoring components of the propulsive force

TE, hi, and nac = relative to the trailing edge, to the highlight plane, and to the nacelle

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