

CRANFIELD UNIVERSITY

KHANGAMLUNG KAMEI

STRUCTURAL DYNAMICS AND CRACK PROPAGATION
BEHAVIOUR UNDER UNIFORM AND NON-UNIFORM
TEMPERATURE CONDITIONS

SCHOOL OF AEROSPACE, TRANSPORT AND
MANUFACTURING
PhD in Manufacturing

PhD
Academic Year: 2019 - 2022

Supervisor: Dr Muhammad Khan
Associate Supervisor: Prof Andrew Starr
February 2022

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ABSTRACT

The robustness and stability of machinery depend on structural integrity. This stability is, however, compromised by aging, wear and tear, overloads, and environmental factors. A study of vibration and fatigue crack growth for structural health monitoring is one of the core research areas in recent times. The research is yet to input sufficient explanations about the dynamic behaviour of the structure under distributed temperature. The structural dynamics can be influenced by material microstructure, temperature distribution, and duration of exposure to the thermal environment. The applied temperature can cause significant variations in the modal response. The existing studies are limited concerning temperature change and compel extensive investigation in a crack and uncracked condition. In this research, the structural dynamics and fatigue crack propagation behaviour when subjected to thermal and mechanical loads have been studied. It investigates the modal parameters of uncracked and various cracked specimens under uniform and non-uniform temperature conditions. An analytical model considering the effective length of the beam is developed to analyse the modal response of the beam. Then, the model is modified to enumerate the modal behaviour of the beam in the presence of crack. The model is validated by experimental and numerical approaches. The experimental evaluation is conducted by considering three heating rates to attain the required temperatures. In the first case, ramping at $2^{\circ}\text{C}/\text{min}$ is assumed as a slow heating rate. While ramping at $5^{\circ}\text{C}/\text{min}$ and $8^{\circ}\text{C}/\text{min}$ are assumed as moderate and rapid heating rates respectively. The heating rates are considered to compare the structural response changes. A small variation on modal parameters is noticed for different heating rates and when the applied uniform temperatures are changed to non-uniform temperatures, especially at elevated temperatures. This signifies heating at different rates has a slight effect while measuring the dynamic response of any mechanical system. The results showed that changes in modal parameters of the beam are associated with the change in temperatures and heating rate.

Furthermore, this research substantiates the fatigue crack propagation behaviour of pre-seeded cracks. The propagated crack depths are measured based on pixels contains in the crack. It is found that propagated crack depends on applied temperatures and associated mass. The appearance of double crack fronts and multiple cracks are observed. The multiple crack appearance seems due to the selection of pre-seeded crack shapes. Hence, the real crack and pre-seeded crack are distinct and need careful consideration in crack propagation evaluation.

Keywords:

Modal Parameters; Fatigue Crack; Crack Depth Measurement; Fracture Mechanics; Vibration Analysis; Temperature Distribution

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The key contributions and publications from this thesis are given below.

1. Kamei, K.; Khan, M.A. Investigating the structural dynamics and crack propagation behavior under uniform and non-uniform temperature conditions. *Materials*. **2021**, *14*, doi:10.3390/ma14227071.
2. Kamei, K.; Khan, M.A.; Khan, K.A. Characterising modal behaviour of a cantilever beam at different heating rates for isothermal conditions. *Appl. Sci.* **2021**, *11*, doi:10.3390/app11104375.
3. Kamei, K.; Khan, M.A. Current challenges in modelling vibrational fatigue and fracture of structures: a review. *J. Brazilian Soc. Mech. Sci. Eng.* **2021**, *43*, doi:10.1007/s40430-020-02777-6.
4. Fleet, T.; Kamei, K.; He, F.; Khan, M.A.; Khan, K.A.; Starr, A. A machine learning approach to model interdependencies between dynamic response and crack propagation. *Sensors*. **2020**, *20*, 1–13, doi:10.3390/s20236847.
5. Kamei, K.; Khan, M.A. Influence of Temperature on Fatigue Crack Growth and Structural Dynamics. *SSRN Electron. J.* **2020**, 1–7, doi:10.2139/ssrn.3717712.
6. Kamei, K.; Khan, M.A. Design of an experiment to analyze modal parameters in a crack and without crack conditions in the presence of thermal and mechanical vibration. *The 9th International Conference on Fracture Fatigue and Wear*. **2021** (waiting for the publication)

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LIST OF ABBREVIATIONS

Al	Aluminium
BC	Before Christ
CNC	Computer Numerical Control
CNT	Carbon Nano Tubes
CT	Computer Tomography
DAQ	Data Acquisition
DMA	Dynamic Mechanical Analyzer
DSIF	Dynamic Stress Intensity Factor
FE	Finite Element
FG	Functional Graded
FGM	Functional Graded Material
NI	National Instrument
P-FGM	Power-Law Function
PID	Proportional Integral Derivative
S-FGM	Sigmoid Function
SHPB	Split Hopkinson Pressure Bar
SIF	Stress Intensity Factor
SMA	Shape Memory Alloy
TMA	Thermal-Mechanical Analyzer
TSIF	Thermal Stress Intensity Factor
Al	Aluminium
BC	Before Christ
CNC	Computer Numerical Control
CNT	Carbon Nano Tubes
CT	Computer Tomography
DAQ	Data Acquisition
DMA	Dynamic Mechanical Analyzer
DSIF	Dynamic Stress Intensity Factor
FE	Finite Element
FG	Functional Graded
FGM	Functional Graded Material
NI	National Instrument
P-FGM	Power Law Function
PID	Proportional Integral Derivative

S-FGM	Sigmoid Function
SHPB	Split Hopkinson Pressure Bar
SIF	Stress Intensity Factor
SMA	Shape Memory Alloy
TMA	Thermal Mechanical Analyzer
TSIF	Thermal Stress Intensity Factor

NOMENCLATURE

a_f	Final crack length
a_o	Initial crack length
A	Cross-sectional area
α	Coefficient of thermal expansion
B	Beam width
C	Material constant
D	Fatigue damage
$\frac{da}{dN}$	Crack growth rate
D_g	Gaussian damage
D_{ng}	Non-Gaussian damage
E	Young modulus
$E(T)$	Elastic modulus at Temperature
E_o	Modulus at 20 °C
ε	Strain
f	Frequency
F	Vibration force
h	Beam thickness
Hz	Hertz
I	Moment of inertia
k	Thermal conductivity
K_{sta}	Static stress intensity factor
K_{total}	Total stress intensity factor
Ku	Kurtosis
$\Delta K_{I,th}$	Threshold value of stress intensity factor
ΔK	Stress intensity range
L	Beam length
l_c	Critical length
L_e	Effective length
M	Moment
m	Mass
N	Fatigue life/ number of cycles

n_t	Number of cycles per unit time
P	Inertia force
Q	Deformation energy
ΔQ	Changes in the deformation energy
q	Heat flow
q''	Heat flux
$S(x, t)$	Shear force
Sk	Skewness
T	Temperature
t	Time
t_c	Crack depth
u	Displacement
$U(x)$	Mode function
$V(t)$	Time function
χ_t	Stiffness of the crack beam
λ_{ng}	Coefficient of non-Gaussianity
σ^*	Threshold stress
σ_n	Normal stress
τ_{ns}	Shear stress
ω_n	Frequency of uncracked beam
ω_{nc}	Frequency of crack beam
μ	Mean value
ρ	Mass density
σ	Stress
φ	Proportional constant
ω	Angular frequency

1 INTRODUCTION

1.1 Research background

Engineering structures and components experience fatigue and failure during their service time. This failure is due to many reasons like wear and tear, cycles of loads, working environments, crack occurrence and propagation, etc. Researchers considered that early detection is the solution to prevent catastrophic damage because catastrophic failure is a cost to bear as the machinery is expensive and not easily replaceable rather be repaired.

Vibration analysis is one of the non-destructive investigation techniques for structural health monitoring. Generally, the vibration of the components impacts the durability and reliability of the system. Traditionally, vibration-based inspections on structures were performed in the absence of a thermal environment. In recent years, the research is focused to include temperature load in structural health analysis. This is vital, because most of the engineering structures, particularly, in industries such as automotive, aircraft, power plants, etc. operate in thermal environments, in addition to the mechanical loads. The system response may vary depending on the working condition especially during the heating and temperature exposure duration.

Vibration fatigue damage is a concern in engineering applications, particularly for metal structures. The design phase of a structure considers factors that can prevent or delay fatigue and fracture failures and increase its working life. A study on crack propagation behaviour is one approach to reduce and prevent the occurrences of fracture failure of the structure. It has many challenges due to the changing needs and applications. One of the main causes for crack propagation is the applied fluctuating loads, and temperature change. The fatigue life is dependent on the cycle history of loading magnitude, the residual stress, and properties of the material, manufacturing defects, design geometry, and environmental factors. The complexity of the crack propagation evaluation arises due to the complex loading and operation conditions. This research

intends to investigate the modal parameters and crack propagation behaviour when it is subjected to thermal and mechanical loads.

1.2 Research problem

Vibration-based inspection on mechanical structures for structural health monitoring is widely studied. However, many engineering practical applications, such as automotive, aircraft, power plants, etc. operate in a thermal environment in addition to the mechanical loads. Especially, during the heating and cooling process of the thermal systems, the vibration response may vary significantly. Published research methods are infeasible to capture the addition of thermal disturbances. The rate of heating and temperature exposure duration may vastly affect the system response, particularly during crack growth. Under the coupled loads, the dynamic response of the structure is subjective by the mechanical and thermal loads. The temperature change in the structure fluctuates the vibration centre and eigen frequencies. The vibration response in such conditions is the combined effects of thermal, loading mode, material properties, and boundary conditions. The temperature change alters the fundamental frequency of the system. The mechanical structures which operate in cyclic loads for a long duration generate heat on the system apart from the applied thermal loads. This change in global temperature affects the intensity of stress concentration, which consequently modifies the structural dynamic response.

The thermal effect on the dynamic response of structure has a great concern in engineering applications. Structure exposed to reasonable high temperatures can modify the material properties and impact crack propagation. This modification influences the overall deformation of the structure. The temperature changes impact the amount of stress concentration thereby diminishing the global stiffness. Moreover, thermomechanical vibration of structure relating to crack propagation and modal properties are still inadequate. Even though there are various studies on structure dynamics, the existing research has not considered the heating duration and non-uniform temperatures conditions which could potentially affect the structural dynamics.

In this research, the modal parameters and crack propagation behaviour of a cantilever beam subjected to thermal and mechanical loads were evaluated. It investigates the heating rate effect on modal parameters for both with crack and without crack beam under uniform and non-uniform temperature conditions. This beam was selected because many aircraft wings and turbine blades operate akin to a cantilever beam. This study substantiates the fatigue crack propagation behaviour of pre-seeded cracks under thermal and mechanical loads.

1.3 Research objectives

This research aims to analyse the influence of temperature distribution on structural dynamics and crack propagation behaviour. The research has the following objectives.

1. To develop an analytical model to investigate the effects of temperature distribution on the modal response of a beam structure
2. To establish the changes in the analytical model for a beam with a crack damage
3. Experimental validation of the analytical model under uniform and variable temperature distribution across the structure
4. Experimentally analyse the crack propagation behaviour and study the modal response of the beam during crack propagation.

1.4 Thesis structure

The thesis proceeds with a literature review on structural dynamics and crack propagation behaviour under thermomechanical loads. This comprehensive review encompassed the current challenges in modelling vibrational fatigue and fracture of structures at elevated temperatures. The research methodology is presented in chapter 3. Chapter 4 discussed the analytical formulation to estimate the modal parameters subjected to thermal and mechanical loads. This formulation considered the effective length of the beam due to thermal expansion along with the accelerometer mass in a fixed-free boundary condition. A linear temperature distribution approximation method is also

presented. The experimental method and numerical analysis are presented in chapter 5. It describes the performed tests in uniform and non-uniform temperature conditions. The results and discussion section are presented in chapter 6. This chapter discussed the thermal properties of Al 2024-T3, thermal and elastic strain intensity analysis, heating rate, and exposure time as well as the impact of temperature on modal parameters. The comparison of the modal behaviours of a beam under uniform and non-uniform temperatures conditions is also reported. The fatigue crack propagation behaviour, when subjected to thermal and mechanical loads, is presented in chapter 7. The last chapter 8 provides the conclusions of the thesis.

2 LITERATURE REVIEW

In the engineering world, crack occurrence and propagation in the material components and the structures are serious issues to address. Failure in early detection has consequences leading to catastrophic damage. The catastrophic failure is a cost to bear as the machinery is expensive and not easily replaceable rather be repaired. For this reason, researchers have performed extensive research to find solutions and minimize crack growth.

Engineering materials especially metals develop cracks and fractures during their service time. Most of the applications, such as automotive, aircraft, power plants, etc., operate under high temperatures. Such structures and components of the machine develop crack more prevalent under the combined effect of thermal and mechanical loads and experiences fatigue [1–3]. Although the knowledge of fracture and fatigue was confirmed in 1754 BC [4], the scientific society faced many challenges while considering the continuously changing needs of the people. Several review articles published related to vibration fatigue and fracture: Thermal loadings fracture of shape memory alloys [5], Multiaxial vibration of linear and non-linear systems [6], Variable amplitude fatigue [7], Interpretive technical history of fracture mechanics [4], Role of dynamic response parameters in damage prediction [8], Cumulative fatigue damage mechanisms and quantifying parameters [9], Stability and significance of residual stress during fatigue [10]. The existing reviews showed that the researchers somehow managed to explain the details about the micro mechanics of fatigue and fracture and the macroscopic behaviour of structure distinctly separate. However, these details are insufficient to address a variety of modelling issues that may arise if the aforementioned mechanics and behaviour are correlated.

These challenges are still unaddressed especially when the loads are no more singular and quasi-static in nature. The couple loads dynamic in nature involve mechanical and thermal external disturbances and this duality in time domain creates complexity in analytical mathematics to solve the real phenomenon on structures with exactness.

This review presents the significant efforts of the past and reviews the modelling challenges on vibration and crack growth. It provides an overview of the research complexities in the area of vibration fatigue and fracture modelling. This review organises the previous research work under five prominent themes: Assessing fatigue damage accurately under thermomechanical vibrational loads, complications in fatigue and fracture life estimation, intricacy in fatigue crack propagation, quantification of cracks, and the stochastic response of the structure under thermal environment. In the conclusion, new directions of work are suggested that still require comprehensive research efforts to bridge the existing gap in the current academic domain.

2.1 Assessing fatigue damage accurately under thermomechanical vibrational loads

Thermomechanical vibrations have been concerned for engineering applications. In the growing advancement of technology, these applications are exposed to mechanical loads under thermal conditions and have tendered scientists and engineers to investigate the dynamic behaviour of structures and their components [11]. The vibration of the components influences durability and reliability. Generally, vibration is analysed in four domains and each domain offers identifiable data on the operational conditions and characteristics of the vibration [12]. Researchers apply different theories to study the characteristics of vibration: higher-order shear deformation theory [13–17], nonlocal elastic theory [18,19], modified couple stress theory [20], generalised differential quadrature method [21], Bubnov-Galerkin method [22], thermal-energy method [23], finite element analysis [24,25] among many others. Most of these methods are however purely analysing the vibrational characteristic behaviour of the structure without concerning the fatigue damage. Assessment of fatigue damage under such conditions is still a concern and challenge. Further the current investigations on thermomechanical vibration are more focused on different materials: materials like graphene sheet [26,27], functional graded Nano-plate [28], composite beams [29], and nano-composite plate [30].

Primarily, it is on applications-based studies rather than emphasizing accurate measurement of fatigue.

Warminska et al. [31] demonstrated that structural response is directed by the level of thermal loading and distribution. A large increase in temperature can shift the vibration centre and affects the fundamental frequency. Yang and Shen [32] studied similarly confirmed the vibration response as a result of thermal effect, material composition, boundary condition, and loading mode. A free-vibration analysis in cylindrical bodies highlighted the frequency vibration depends on mode, thickness, and curvature of structure [33]. Shen and Wang [34] and Ghayesh et al. [35] investigated nonlinear vibration. They noticed the increase in temperature minimized the natural frequency but has a marginal effect on the nonlinear to linear frequency ratio. Julien et al. [36] and Kitipornchai et al. [37] examined probabilistic random vibration. They concluded that the linear frequency dispersion is an influence of temperature change. Rezaee et al. [38] studied vibration analysis of Nano-wires and Ubertini [39] and Bao et al. [40] studied vibrations of bridge structures. A few of the articles reported fatigue due to vibration [41–43] predominantly based on applications. They suggested the fatigue assessment in the studied applications is on a weak foundation due to a lack of analytical relations and arguments especially when the structures operated under thermomechanical loads.

Modal analysis is a common method to predict structural health based on experimentally measured vibrations [44]. This method is more often used for extracting defects and faults from real structural applications, which are too complex to model analytically and numerically. For example, modal analysis of exhaust manifold in hot condition [45], the turbine blade at complex thermo-mechanical loads [46], high-speed machine-like servo-hydraulic [47], jet engine fan [48], extracting full-field dynamic strain subjected to arbitrary excitations on a wind turbine rotor [49], etc. Modal analysis can also determine the dynamic stress intensity factor [50]. Similarly, it is also utilized in different approaches. Jezequel [51] used a hybrid method of modal synthesis. The approach is acknowledged for assisting in the research of structure remodelling. Yam et al.

[52] experimentally studied and suggested that modal strain analysis can be effectively utilized to study structural dynamics and fatigue life estimation. Haider and Nayfeh [53] investigated the modal interaction in the non-linear forced vibrations of a heated plate. Lamb and Rouillard [54] stated that system fatigue characteristics can be obtained by monitoring its modal parameters. Braghin et al. [55] modal control technique presented modal coordinate in fatigue analysis for the very first time. However, modal analysis is sometime constrained due to the difficulties to understand how the mechanical parameters of the system are modified, and hence guiding to disinformation.

Thermomechanical vibration examinations are important to establish failure possibilities and crack propagation before catastrophic failure. In past, researchers used the system dynamic response and predicted the possible faults, defect, and their conceivable locations [11]. Mathan and Prased [56] studied the dynamic response of the piping system at various temperatures using finite element analysis. They performed FE simulation with the help of modal and harmonic analysis. Experiments and numerical simulations were also performed to study the dynamic and acoustic response of a clamped rectangular plate and a composite structure [54, 55]. The flexural responses of a thermoelastic thin beam were also studied under thermal and mechanical load [59]. Shen et al. [60] studied Reissner-Mindlin plates resting on elastic foundations. Besides the mentioned, researchers evaluated the vibration modes using video measurements [61]. It is well established that the academics studied vibration behaviour of structures of different sizes, thicknesses and layers i.e. sandwich plates [62–64]. Furthermore, the study of vibration in finite element method becomes familiar among the researchers due to its cost-effectiveness and ease of evaluating the data. Nevertheless, the data obtained from experiments are always far from an agreement with the simulation results due to practical compromises [65].

2.2 Complications in fatigue and fracture life estimation

Fatigue is a failure phenomenon in materials that involves cyclic loads with developing cracks [66]. Fatigue normally occurs when a repeated or fluctuating

load is applied, which may either or both tensile, compressive, or torsional. There are three stages in fatigue life [66,67]: crack initiation, crack propagation, and failure stage. The crack initiates where there are discontinuities in the structure of the material like pre-existed crack, void, corrosion, stress concentration, etc'. The development of the discontinuity actually strengthens the structure through plastic deformation but lost its ability after reaching the deformation limit and thereafter developed minute crack [65]. The slow and steady growth of a crack threatens structural integrity. The final stage of crack development is fatigue failure. The fatigue life is dependent upon the load history, residual stresses, properties of the material, manufacturing defects, grain size, design geometry, environmental conditions, etc. Studies on thermomechanical fatigue [68–75] are limited to some applications and need comprehensive assessment in particular for fatigue life estimation. The following section discusses the fatigue life estimation methods to highlight the unaddressed challenges.

2.2.1 Life estimation methods

Knowing the life of the component before being used is the most important aspect to avoid fatigue or catastrophic failure. There are several methods and techniques to predict fatigue life. Approaches like frequency-domain method [76,77], modal approach [43,50,74], accumulative approach [78], thermal fracture model [79], continuum mechanics model [80] and intrinsic dissipation model [81] are predominant. The foremost and well-known method was formulated by Paris and Erdogan [82] given in Equation (2-1).

$$\frac{da}{dN} = C \Delta K^m \quad (2-1)$$

Where ΔK the stress intensity range, C and m is are the material constants. The fatigue life is then determined by integration as in Equation (2-2).

$$N = \int_{a_o}^{a_f} \frac{da}{C \Delta K^m} \quad (2-2)$$

Damir et al. [83] proved that modal analysis are a great potential for fatigue evaluation and characterisation of the mechanical components. This investigation determined the capability of experimental modal analysis as a non-destructive tool. Braccesi et al. [76] used the modal frequency domain approach to evaluate the virtual fatigue life in a non-linear system. They considered the random loads to estimate the component life considering the dynamic behaviour and state of stress. The fatigue damage due to the stress time history was acquired by multiplying the Gaussian damage (D_g). However, in real situation damage is non-Gaussian. The use of corrective coefficient of non-Gaussianity, λ_{ng} to estimate the non-Gaussian damage, D_{ng} is given by Equation (2-3).

$$D_{ng} = \lambda_{ng} D_g \quad (2-3)$$

The functional relation for the corrective coefficient was developed in the form of Equation (2-4).

$$\lambda_{ng} = f(\mu, \sigma, Sk, Ku, m) \quad (2-4)$$

Where m is the slope of the material's from Wohler's curve, which is expressed in the form of $S_f = k^{-1} N^{\frac{-1}{m}}$, where S_f is the fatigue strength (stress amplitude) and N is the number of cycles. σ represents the standard deviation, μ is the mean value, Sk is skewness and Ku is kurtosis, is an index of concentration of the probability density function area around the mean. The Gaussian damage curve in given Equation (2-5) was used to analyse the fatigue damage.

$$\lambda_{ng} = e^{\frac{m^2}{\pi} \left(\frac{(Ku - 3)}{5} - \frac{Sk^2}{4} \right)} \quad (2-5)$$

In other papers of Braccesi [77,84], a modal approach was used to evaluate the frequency domain stress recovery and fatigue damage of mechanical components. The stress tensor of the model was obtained from the power spectral density matrix. In Dirlik's method, the fatigue damage D was calculated by the relation of Equation (2-6).

$$D = \frac{T \cdot n_t}{C} \int_0^\infty (\Delta\sigma)^k \cdot f_\sigma(\Delta\sigma) d(\Delta\sigma) \quad (2-6)$$

Where T is the duration of the random signal application, n_t is the number of cycles per unit time, $(\Delta\sigma)^k$ the probability density function of the stress range, $f_\sigma(\Delta\sigma)$ is frequency domain criteria. The other method of calculating fatigue damage is the Bands method given by the relation of Equation (2-7).

$$D = \frac{K \cdot \alpha^{\frac{1}{\beta}} \cdot f_{ref}}{(\int G_{ref}(f) df)^{1/2\beta}} \quad (2-7)$$

Where, the functions K parameters α and β are from the Wöhler' curve, f_{ref} is reference frequency and $G_{ref}(f)$ is the ratio function.

In non-local-based fatigue life estimation, the dispersal of stress/strain in the locality of the crack tip is estimated [85]. The non-local damage parameter is the integration of all local damage parameters in the material domain. The local parameters are however estimated from the classical mechanics without considering the material homogeneity. The non-local concept considered the complexity of the geometry structure by analysing the influence of the inhomogeneity stress field [86]. This approach is termed as critical distance method. The local parameter is estimated by a point, a line, an area, or volume. The local parameter is calculated by:

$$\sigma_{eff}(t, r_o) = \frac{1}{\hat{w}(r_o)} \int_A \sigma_n(t, r) w(r - r_o) dA \quad (2-8)$$

$$\hat{w}(r_o) = \int_A w(r - r_o) dA \quad (2-9)$$

Where t is the time, r is the unit vector and r_o is the reference point in the crack tip and A is the critical plane area. The weight function is presented in Equation (2-10).

$$w(r) = \exp\left(-\left(\frac{2r}{l_c}\right)^2\right) \quad (2-10)$$

Where, $r = |r - r_o|$ and l_c is the critical length.

In the point method, the fatigue failure occurs because of the presence of stress/strain to some critical distance from the crack tip. It is considered non-local since the selected point is not from the crack-tip region. The fatigue life is calculated by reducing the stress/strain into equivalent stress/strain parameters. The critical distance is calculated [85] by Equation (2-11).

$$D_c = \frac{1}{\pi} \left(\frac{\Delta K_{I,th}}{\Delta \sigma_{af}} \right)^2 \quad (2-11)$$

Where, $\Delta K_{I,th}$ is a range of threshold value of stress intensity factor, $\Delta \sigma_{af}$ is the range of fatigue limit.

The line method assumes that fatigue failure is due to the presence of stress over the line. The average of the stress is calculated as:

$$\sigma_{af} = \frac{1}{D} \int_0^D \sigma_y w(x) dx \quad (2-12)$$

Where D is the distance from the notch point and $w(x)$ is the weight function.

The area method assumes that failure is due to the damage parameter over some area. The average failure function is calculated by:

$$\bar{R}_\sigma = \frac{1}{d_0^2} \int_A R_\sigma(\sigma_n, \tau_{ns}) dA \quad (2-13)$$

Where, $\bar{R}_\sigma$ is the average failure function, σ_n, τ_{ns} are normal and shear stresses on the physical plane, the critical distance d_0 in area, method is calculated by Equation (2-14).

$$d_0 = \frac{2}{\pi} \left(\frac{K_{I,th}}{\sigma_{af}} \right)^2 \quad (2-14)$$

The volumetric method assumes that fatigue failure is due to the damage parameter over a critical volume. Saintier et al. [87] proposed a non-local energy-based fatigue life calculation subjected to multiaxial variable amplitude

loading. They anticipated two hypotheses of volumetric damage parameters. The volumetric damage parameters are expressed as:

$$\bar{\omega}_{g_{eq,dam}}(C_i) = \frac{1}{V^*(C_i)} \int_{v^*(C_i)} W_{g_{eq,dam}} dV \quad (2-15)$$

Where, $\bar{\omega}_{g_{eq,dam}}$ is the non-local damage parameter, V^* is the volume influencing fatigue crack initiation, C_i is the potential critical point and $W_{g_{eq,dam}}$ is damaging strain energy density. The final energy damage parameters S-N curve equation is given as:

$$\bar{\omega}_{g_{eq,dam}}(N) = \frac{(C/N^\gamma)^2 - (\sigma^*)^2}{E} \quad (2-16)$$

Where E is the young modulus, σ^* is threshold stress, N is the number of cycles, C , and γ are material constant.

Another technique of evaluating fatigue damage is the accumulative approach. This technique is known as the Palmgren-Miner rule which is a linear damage accumulation rule. It states that failure occurs when the fatigue damage D is equal to unity.

$$D = \sum \frac{n_i}{N_i} = 1 \quad (2-17)$$

Palmgren-Miner rule is, however, not suitable for non-linear damage. Proso et al. [41] proposed the damage-accumulation correction rule for structural non-linearity. The corrected damage accumulation is estimated by the following relation:

$$D_c = \sum_i D_i \cdot R^{n_i} = 1 \quad (2-18)$$

Where R is the constant correction factor and n is the rate of non-linearity which is obtained experimentally. The fatigue damage is calculated by Equation (2-19).

$$D = 1 = \sum \frac{n_i}{N_i} D_i = \sum \frac{t_i}{T_i} \quad (2-19)$$

Where, t_i and T_i are respectively the time of excitation and fatigue lifetime. Several factors contribute to non-linear damage mechanisms. There are several factors, which are responsible for non-linear damage mechanisms. The effects of overloading and geometrical discontinuities are predominately studied in the existing literature [88–90]. The non-linear nature of fatigue damage due to overloading and underloading creates irregular local stress amplitude near the crack tip. However, damage evolution due to overload is also dependent on the nature of the microstructure which can be improved by the heat treatment processes [89]. The geometrical discontinuities affect the fatigue life due to stress concentration. These discontinuities are inevitable in the structures mainly due to design requirements. However, researchers do relate their influence with microstructural voids, corrosion pits, riveting joints, wear and tear [91–93]. Therefore, these discontinuities can lead to a non-linear damage progression.

There are other approaches to fatigue life estimation modelling found in articles: Thermal fracture modelling method for functionally grade plate [79], continuum mechanics modelling for fracture analysis [80], intrinsic dissipation modelling for high-cycle fatigue life prediction [78]. Susmel and Taylor [94] attempted to use a critical distance method to estimate the finite life of notch components based on the modified Wohler curve method under variable amplitude loading. In similar work using a critical plane approach, Gates and Fatemi [95] studied the multi-axial amplitude fatigue life of un-notch specimens. The shortcoming of the critical distance method is averaging the local parameters which may theoretically not be precisely measured. Other researchers studied the lifetime by simulation method. Riedler et al. [96] simulated the lifetime of thermo-mechanical loaded components which investigated the transferability of the simulation models to the real components. Khan et al. [97,98] studied the remaining life prediction method based on the natural frequency. A high cycle and low cycle fatigue life prediction model was also presented for a Nickel-

based single crystal blade [99]. Researchers applied the thermal-fatigue life assessment on real applications [25,42,100–103]. Kamaya and Kawakubo [104] studied the effect of loading sequence on fatigue life. They found fatigue life is longer in the case of high-low loading than low-high loading sequences. Besides, the study found crack mouth opening effect is the main cause in reducing fatigue life whereas stress intensification, damage interaction, and damage sequence effect demonstrated less impact on fatigue life. Several life extension techniques have been proposed in the literature. They have made a fair impact on the existing fatigue life estimation approaches. Ayatollahi et al. [105] studied the crack growth retardation technique by introducing a drilling hole along the crack flanks. Their numerical analysis results revealed that the presence of holes around the crack tip reduces the stress concentration and the stress intensity factor. In other papers of the same authors [106,107], crack growth retardation was applied with the help of a stop hole at the crack tip. These investigations demonstrated an extension in the overall fatigue life. All these studies specify despite numerous concepts of fatigue life estimation, it is hypothetically demanding to locate the perfect explanation and amplification.

2.3 Intricacy in fatigue crack propagation

A fatigue crack growth curve is a graphical representation of the stress intensity factor range and cracks growth rate [63-64] as shown in Figure 2-1. The region I describe the crack initiation which is termed as near-threshold zone below which the fatigue crack does not happen. Region II is the intermediary zone where there is linear and steady crack propagation occurs appropriated for Paris and Walker models. The abnormal fatigue crack growth occurs in the region III due to the phenomenon of plasticity that leads it to the catastrophic failure. The crack growth rate rises rapidly as the maximum stress intensity factor reaches the fracture toughness, as shown in Figure 2-1. However, the crack growth rate may not be same for all the materials.

The simplest form of fatigue crack growth analysis is those which are subjected to constant amplitude loading because no loading history has to be considered [66]. The complexity of the crack growth evaluation arises due to the variable

amplitude loading where the existing loading interaction effects. Chowdhury et al. [66] proposed a framework to select the fatigue crack growth method for life cycle assessment. Their paper discussed the advantages and highlighted the model's limitations of the four fatigue crack models namely: Paris-Erdogan model, Walker model, Forman model, and ASGRO model. Zang et al. [108] explored the judgment criterion of the dominant factor of creep fatigue. They considered the interaction of creep fatigue at different intensity factor range at different temperature with dwell time under various loading condition. Their study highlighted that both time-dependent (creep-fatigue) and cycle-dependent (fatigue) involves in the crack growth process as shown in Equation (2-20). The crack growth rates of fatigue, creep, and creep-fatigue interaction is shown in Figure 2-2 (a-e). The crack growth rates were compared for 0-s with 90-s dwell time at temperatures 650°C, 670°C, 690°C, 710°C, and 750°C. Dwell time refers to the time required to generate the trapezoidal waveform by imposing a dwell time at the peak load. The graphs indicated that the fatigue item dominated the crack growth rates at lower temperatures and the creep item has

$$\frac{da}{dN} = \left(\frac{da}{dN}\right)_{\text{cycle-dependent}} + \left(\frac{da}{dN}\right)_{\text{time-dependent}} \quad (2-20)$$

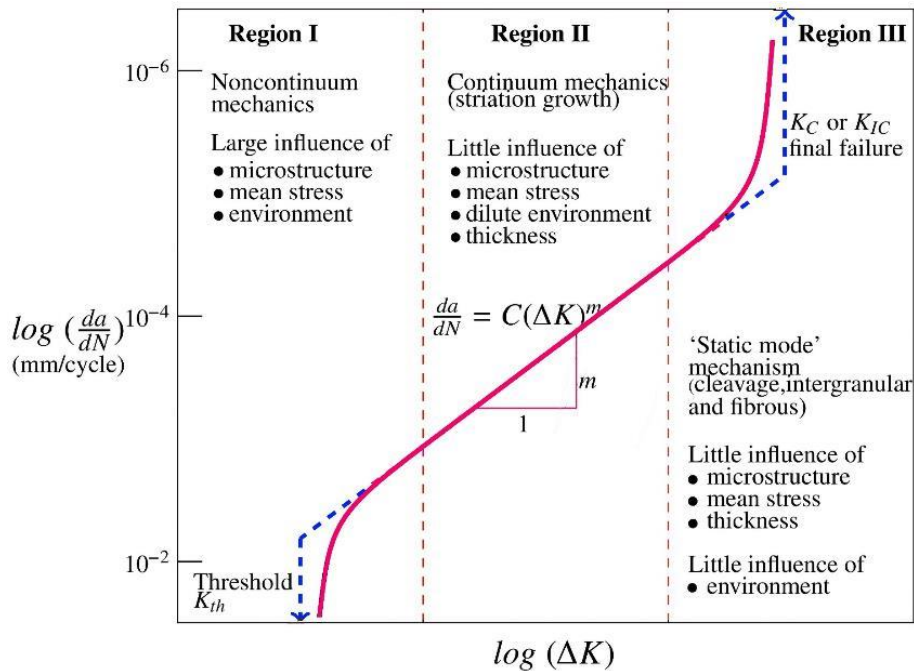


Figure 2-1 Fatigue crack growth curve [66]

little influence in the whole stress intensity range. However, the creep item dominates the crack growth at higher temperatures. Crack growth tests are normally performed in isothermal situations at different temperatures to illustrate the material behaviour [109]. Fatigue crack growth rate increases with the temperature rise. Lu et al. [110] studied the hold-time effect of elevated temperature crack growth behaviour of strengthening super

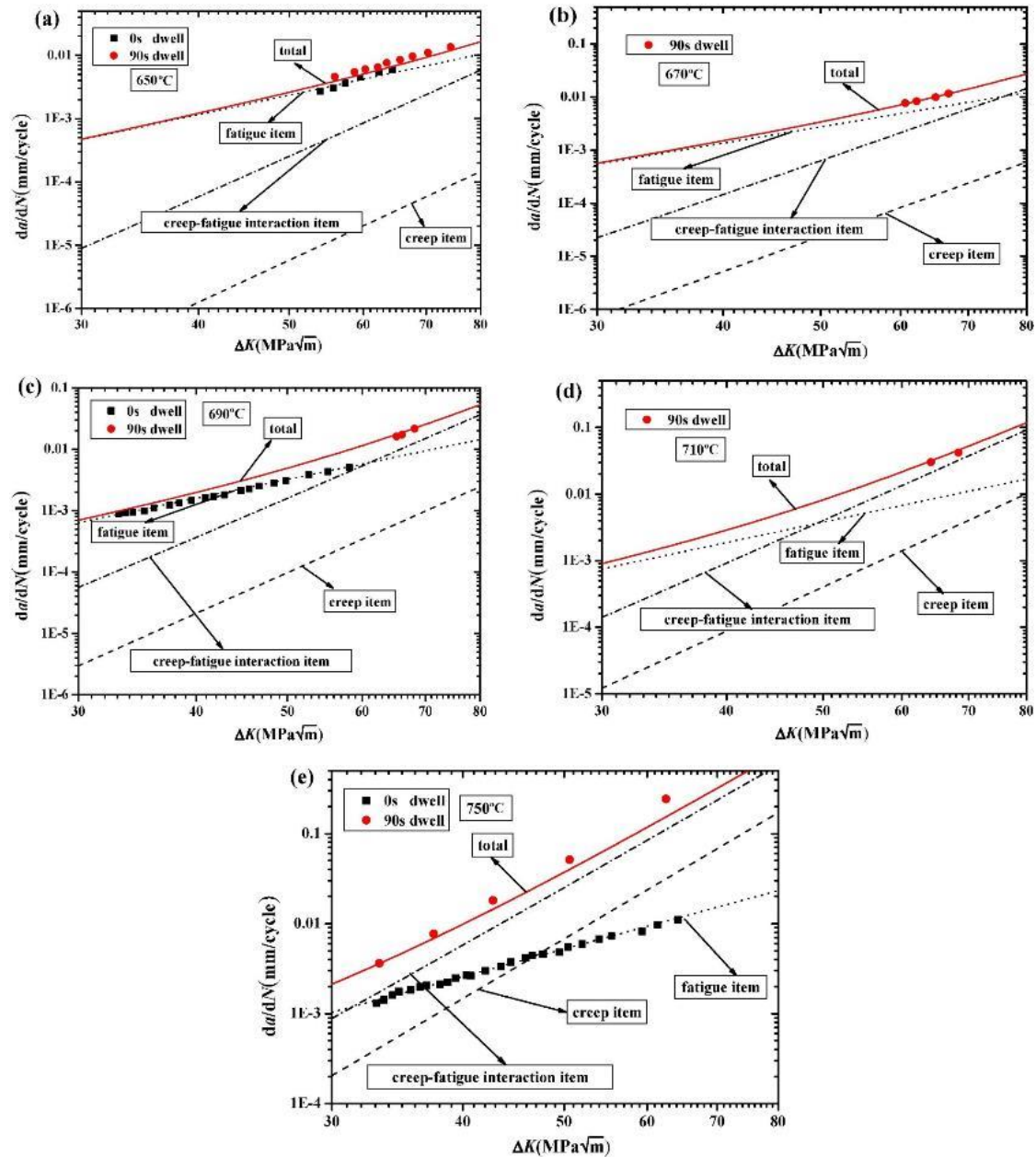


Figure 2-2 Crack growth of creep-fatigue interaction at a (a) 650°C (b) 670°C (c) 690°C, (d) 710°C (e) 750°C. with 90 sec dwell time [108]

alloys. This research applied the fracture mechanics parameters to correlate the crack behaviour at a different temperature. The result showed that the time-dependent creep damage mechanisms control the cracking path. Narashimhachary et al. [111] found that there is a difference in the stress intensity solutions for a rectangular edge-cracked plate emphasizing the importance of using correct boundary conditions. Bouvard et al. [112] formulated a phenomenological model to predict the crack growth in crystal super alloys at elevated temperatures. Fatigue and creep-fatigue tests were taken out to inspect the consequences of time on crack growth rates. A different form of loading conditions was performed to examine the crack growth process for (a) without hold time (b) hold time, (c), and (d) at overloads as shown in Figure 2-3.

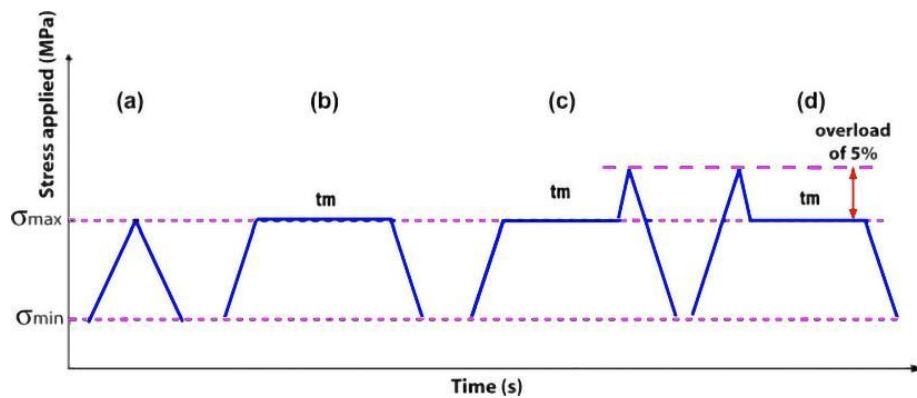


Figure 2-3 The shape of analysed cycles[112]

Remarkably, the two regimes in Figure 2-4 are also viewed from the fracture surface from scanning electron micrographs as shown in Figure 2-5. The four zones are the pre-crack, short crack regime, long crack regime, and failure zone. Sankararaman et al. [113] presented a method to quantify the uncertainty in fatigue crack growth prognosis which is applied to structures with complex geometry and subjected to variable amplitude multi-axial loading. The model modified the Paris law to include the retardation effects which is shown in Figure 2-6. The sources of the uncertainty mentioned the physical variability like loading, initial flaw size, and data uncertainty. They also discussed model uncertainty parameters like crack growth law, calculation of stress intensity factor, etc.

Yao et al. [114] studied the effect of thermal-induced damage on dynamic fracture toughness. The notch specimen of two mortars was heat treated and tested with a split Hopkinson pressure bar (SHPB). X-ray computer tomography (CT) was also used to quantify the thermally induced micro-crack and changes

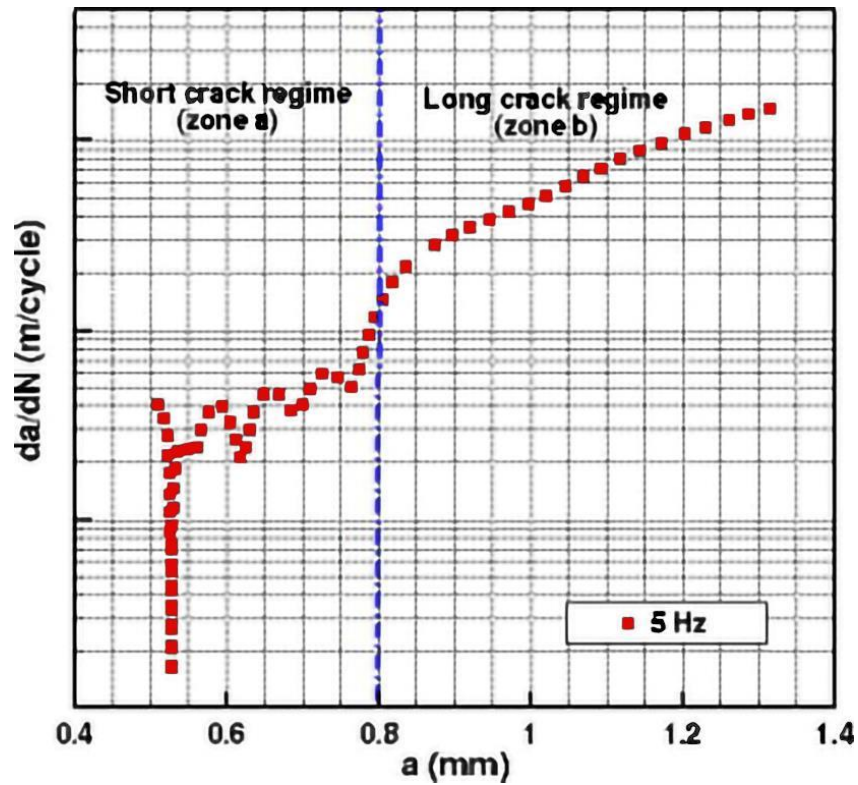


Figure 2-4 Crack growth rate evolution as a function of crack length increase at a frequency of 5Hz for single crystal super alloy [112]

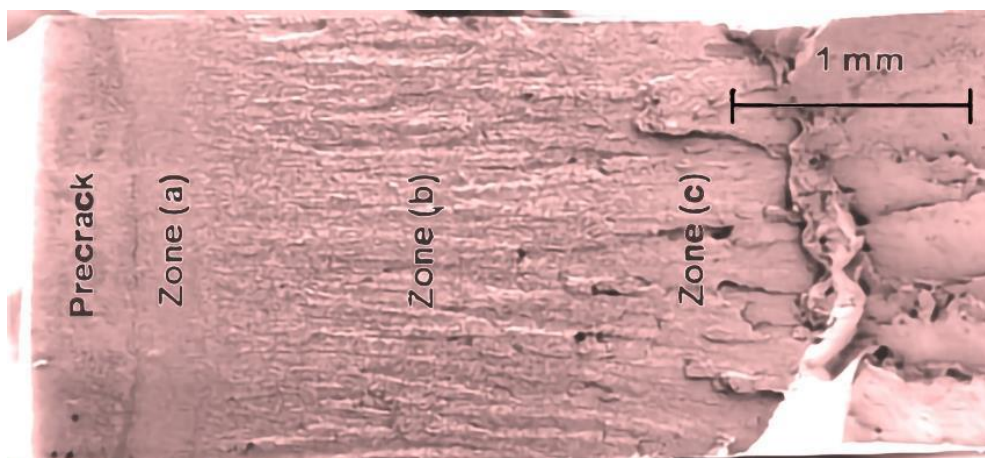


Figure 2-5 Crack scanning electron micrographs of the fracture surface from the specimen tested at 5Hz for single crystal super alloy [112]

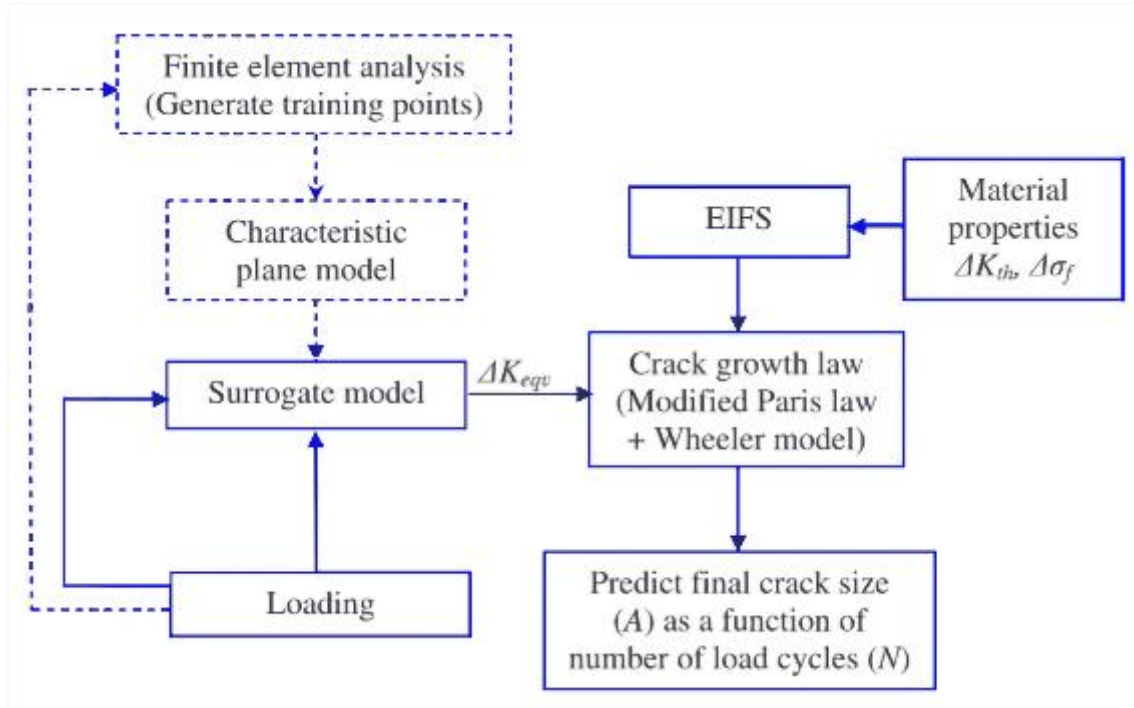


Figure 2-6 Crack propagation analysis [113]

in chemical in terms of CT value. They observed that there was a decrease in fracture toughness due to the heat treatment. They found fracture damage is a combined effect of micro-crack and deterioration in the binding property of mortars. Kidane and Shukla [115] also investigated the effect of a thermal and loading rate on the fracture toughness of Ti/TiB using a three-point bend specimen. Unlike Yao et al [114], they used the modified SHPB apparatus with an induction coil as a heating system. The fracture toughness was found to be sensitive to temperature as shown in Figure 2-7. Fracture initiation toughness was noticed to be strain rate sensitive and higher for dynamic loading as compared to the quasi-static conditions. Verlesen and Peirs [116] investigated the fracture behaviour of Ti6Al14V under quasi-static and high strain rate conditions. The fracture behaviour was illustrated from the Johnson-Cook damage initiation criterion combined with energy-based ductile damage law. The process of void nucleation and crack growth was unexpectedly culminated by strain localization. The strain rate effect was experienced caused by thermal softening which support the strain localization.

In past studies, fatigue crack growth was tested under the isothermal situation. Lansinger et al. [109] attempted to test the crack growth rate in the combined condition of thermal and mechanical loading. The tests used a temperature loading frequency of 1 Hz with cooling rates of 3 to 5°C/min. They discovered that the combined test was in good agreement with the isothermal crack propagation at the same loading frequency. Estimation of fatigue crack growth was also performed in complex loading conditions [78]. You and Lee [117] evaluated the crack behaviour of steel in mixed mode I and mode II loadings. Their studies revealed the accelerated crack growth by the addition of cyclic mode II. The result of mixed-mode crack growth was affected by the collaboration of plastic deformation zone, load ratio, and increase in contact area on the rough crack surface.

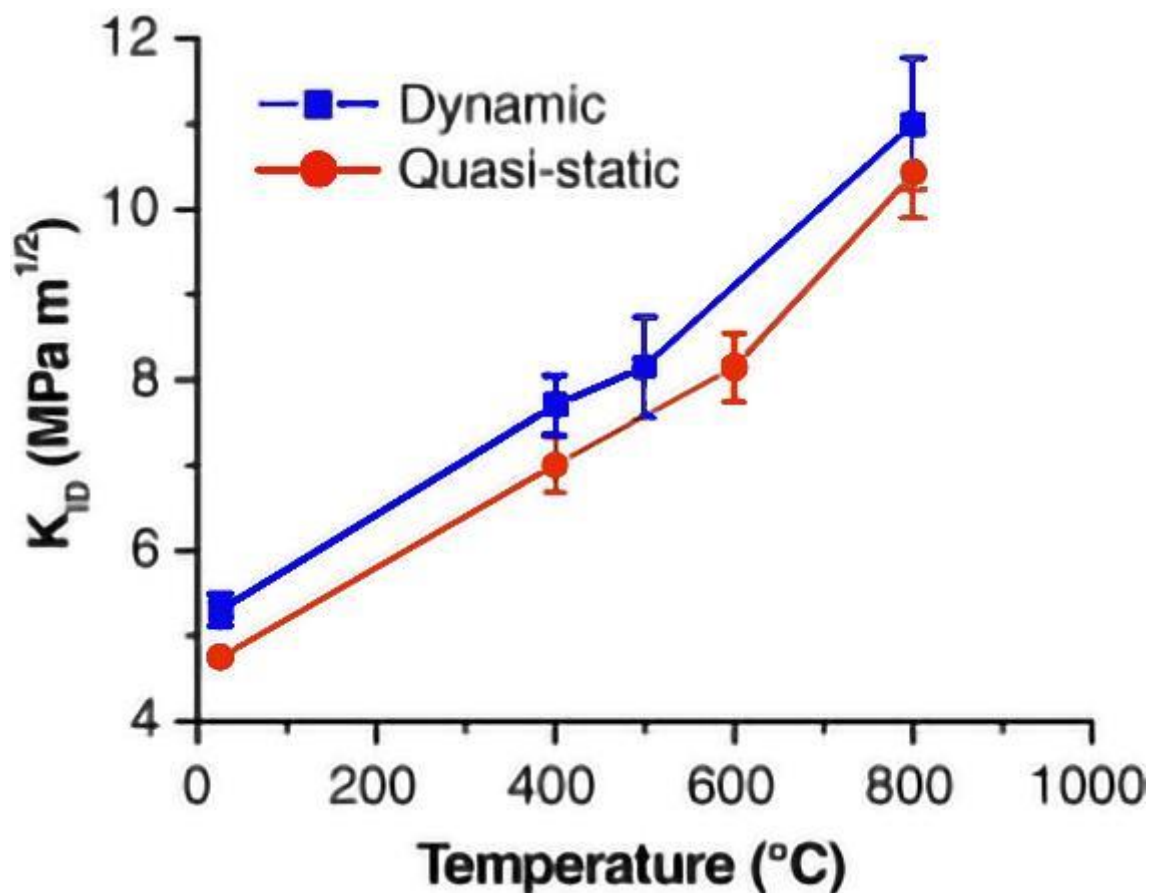


Figure 2-7 Effect of temperature on the fracture initiation toughness of FGM graded in the crack direction under quasi-static and dynamic loading [115]

Fatigue crack growth rate at elevated temperature was studied extensively in the recent past. These studies concluded that crack propagation is caused by several factors like environmental, stress-assisted grain boundary, oxygen embrittlement [118], stress intensity factor with the effect of frequency, load ratio, and temperature [119,120]. Wen et al. [121] analysed the finite deformation under small creep conditions. The crack growth rate was discovered to be more sensitive to the finite element size under cyclic loading than monotonic loading conditions. Tvergaard and Needleman [122] studied the brittle-ductile transition in dynamic crack growth by considering thermal softening and ductile failure. They established the transition from cleavage crack at a lower temperature to ductile cracked growth at a higher temperature. Nishioka et al. [123] investigated the effect of specimen size on dynamic crack propagation of double cantilever beams. The propagating crack was observed through high-speed photography. The investigation reported the values of crack arrest toughness were larger for a shorter specimen than a longer specimen. Thus, it can be concluded that the complexities in estimating crack growth are due to the inconsistent crack propagation pattern.

2.4 Quantification of cracks

Quantification of cracks is the measurement of crack magnitude. There is no such single equation that satisfies different crack problems and there concerning boundary conditions. However, in most cases, the researcher attempted to quantify cracks by calculating the state of the stress/strain field, residual stress, and displacement data. In many situations, the stress intensity factor (SIF) is used to predict the stress state near the crack tip [124–127]. In dynamic loading, SIF is reinstated by the dynamic stress intensity factor (DSIF) [128–135]. In thermal loading conditions, several works of literature [125,136–150] used the thermal stress intensity factor (TSIF) to determine the thermal stress state. Kim et al. [124] stated that the stress intensity factor due to the thermal loads is the function not only to immediate boundary conditions but also of the preceding history of the boundary temperature. The stress intensity factor is nonetheless dependent on the state of stress/strain, displacement and

temperature fields which are the potential factors to quantify cracks. Henceforth quantifying cracks is perplexed especially in complex geometrical structures. Consequently, the study of cracks includes classification of remote stress, orientations of the obtained cracks, and respective material properties.

Defining the stress intensity factor is vital to estimate accurate crack propagation. Different stress intensity factors are defined in the literature based on boundary conditions. Tran et al. [151] conducted a modal analysis to compute the stress intensity factor. Modal stress intensity factor was extracted from stationary crack and computed the dynamic stress intensity factor using the extended finite element method. In similar work, Galenne et al. [152] offered a modal approach to linear fracture mechanics. Modal stress intensity factor is extracted from either the crack-tip displacement method or by energy method. The general equation for energy release rate G for the crack in dynamic loading was given as [151,152].

$$G = -\frac{d}{dl} \left(W^{el}(u) + W^{kin}(\dot{u}) - P(u) \right) \quad (2-21)$$

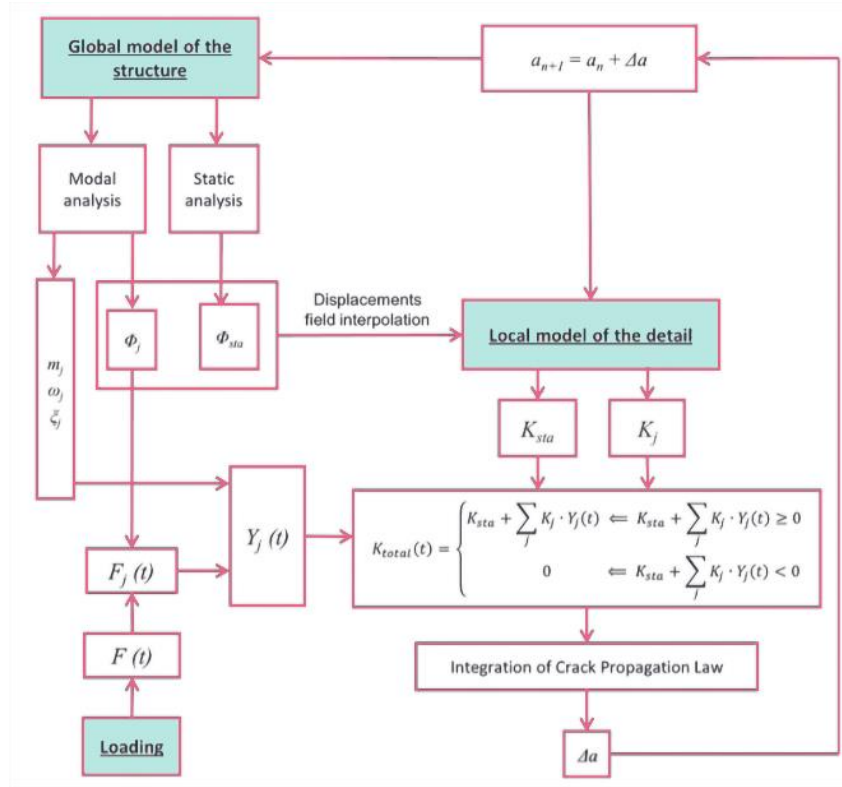
Where, W^{el} is the elastic energy, W^{kin} is the kinetic energy, P is the external power forces, u is the displacement and $\dot{u}$ is the time derivative. Ni and Zhang [153] defined the dynamic stress intensity factor under the ultrasonic loading. They claimed to have the equivalent with the effective stress intensity factor values as in traditional fatigue tests. Calcada et al. [67]·[154] developed an efficient crack analysis using modal stress intensity factor, and their research methodology flow chart is shown in Figure 2-8. The total stress intensity factor for dynamic loading is given in Equation (2-22):

$$K_{total} = K_{sta} + K_{dyn} \quad (2-22)$$

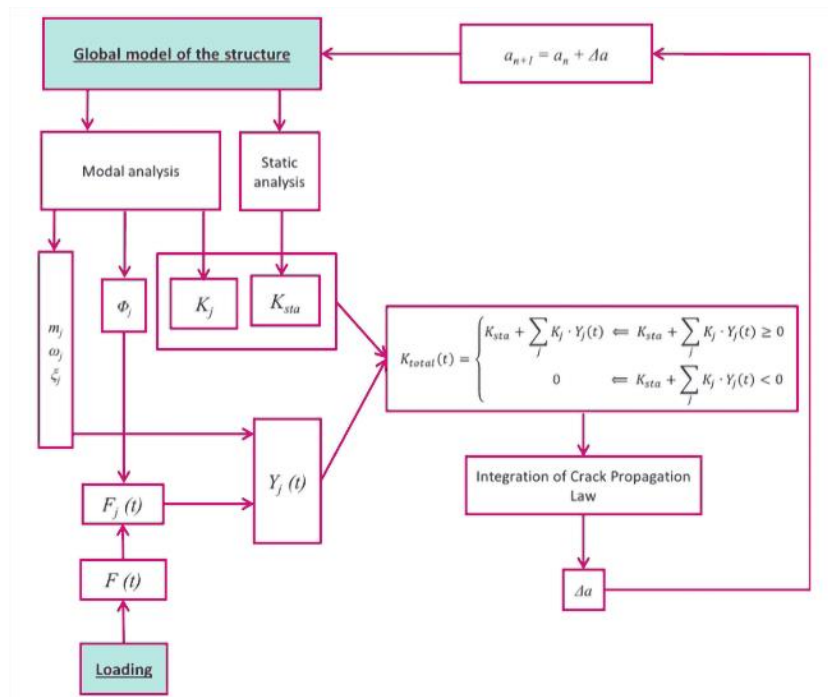
Where, $K_{sta}(t) = C_n \sqrt{\pi a} \cdot \sigma_{sta}$

$$K_{dyn}(t) = C_n \sqrt{\pi a} \cdot \sigma_{dyn}$$

The total value of K through time is expressed as Equation (2-23)



(a) With sub modelling



(b) Without sub modelling

Figure 2-8 Flow chart for the application of the proposed methodology [67]

$$K_{total}(t) = \begin{cases} K_{sta} + \sum_j K_j \cdot Y_j(t) & \text{if } K_{sta} + \sum_j K_j \cdot Y_j(t) \geq 0 \\ 0 & \text{if } K_{sta} + \sum_j K_j \cdot Y_j(t) \leq 0 \end{cases} \quad (2-23)$$

Kabayashi and Ramulu [133] evaluated DSIF for unsymmetrical dynamics. Rokach and Łabędzki [130] proposed a method to determine the dynamic stress intensity factor for a four-point bend impact test. Wang et al. [131] established a dynamic stress intensity factor applying the interaction integral method. The integral method was executed without crack tip enrichment for both homogeneous and non-homogeneous material. The relationship between interaction integral and stress intensity factor is given in the equation below [131,155]:

$$I = \frac{2}{E_{tip}^*} (K_I K_I^{aux} + K_{II} K_{II}^{aux}) \quad (2-24)$$

$$\text{Where, } E_{tip}^* = \begin{cases} E_{tip} & \text{for plane stress} \\ \frac{E_{tip}}{1-\nu_{tip}^2} & \text{for plane strain} \end{cases}$$

Moreover, Huang et al. [125, 153] used the domain-independent interaction integral method to evaluate the DSIF for crack with nonhomogeneous materials. The relationship is given in Equation (2-25)

$$I = \frac{2}{E_{tip}^* \cosh^2(\pi \epsilon_{tip})} (K_I K_I^{aux} + K_{II} K_{II}^{aux}) \quad (2-25)$$

Dynamic stress intensity factor is also derived through the finite element and J-integral methods [132]. A formula was developed from the path independent J-integral by taking the effect of inertia. Aloui et al. [129] studied the dynamic vibration behaviour of a cracked plate subjected to sinusoidal loading and analysed the stress field and dynamic stress intensity factor. Rokach [134] numerical investigation on the accuracy in the determination of dynamic stress intensity. Wen et al. [157] studied the dynamic behaviour of stationary cracks

using dual boundary methods and determined the dynamic stress intensity factor through crack opening displacement.

Evaluation of crack in the thermal environment is vastly studied. Han and wang [158] investigated the thermal shock resistance of multiple cracking in functionally graded material (FGM). Thermal shock resistance was determined based on the stress-based and fracture mechanics-based criteria. Zhang et al. [155] also examined the thermal shock resistance of FGM in a mixed-mode crack. The modelled both strength and fracture theories pondering the failure criteria of maximum local tensile stress, maximum thermal stress intensity, and maximum hoops stress criterion. The transient temperature field for the finite element equation is expressed in Equation (2-26).

$$[K_1] + [K_2]\{T(x, y, t)\} + [K_3]\{\dot{T}(x, y, t)\} = \{P(x, y, t)\} \quad (2-26)$$

Where, $[K_1]$ is the heat conduction matrix, $[K_2]$ is the convection matrix and $[K_3]$ is the heat capacity matrix. Petrova and Schmauder [142] developed mathematical modelling and evaluated the TSIF of FGM for an interface crack. The parametric analysis showed the dependence of the TSIF at the crack tip location and its orientation. The stress intensity factor under thermal stress was analysed for the interfacial crack of bimaterial [121, 135, 42]. The transient SIFs are observed to be of greater value than in the transient state. Nemat and Noda [143] analysed TSIF for functionally gradient material. They observed that TSIF decreased dramatically for selected nonhomogeneous parameters. Transient thermal stress intensity factor was investigated for a hollow cylinder with a circumferential crack [149,150]. Xue et al. [149] evaluated based on the heat conduction model. It was found that the thermal stress at the crack tip is different for different parameter values. Zhang and Li [150] studied the generalised fractional heat conduction and found a striking difference in thermal stress intensity factor with the Fourier heat conduction model.

Different approaches were used for calculating the thermal stress intensity factor for a cracked cylinder [125,137,139,140,145]. Zhong and Huang [136] observed that TSIF did not only depend on applied thermal loading but also

dependent on mechanical loading and physical properties of the crack interior. Tanigawa and Komatsubara [148] evaluated the thermal stress of a rectangular plate. The fracture behaviour of the crack was examined with a non-uniform heat source. Their study explained the variation of stress intensity depending on the kind of parameters like mechanical boundary conditions, the thickness of the plate, the orientation of the crack, surface friction, etc. Lee et al. [141,146] studied the TSIF for interface crack under uniform heat flow. The finding showed the stress intensity factor was strongly dependent on material properties. Analysis of thermal stress intensity factor was also studied using different methods and approaches [124,147,160]. Assessment of mixed-mode stress intensity factor under the thermal and mechanical loading [161].

Le and Gardin [162] used the analytical approach based on stress intensity factor and weight function to study the crack shape and kinetics of crack propagation under the thermal cyclic loadings. The result showed the crack decelerated at the deepest point and accelerated at the surface due to the compression state in the centre and tension near the surface. Sankraraman et al. [163] examined the influence of an initial flaw size of the crack. The stress intensity factor is expressed in terms of the initial flaw size of crack and loadings. The size of the crack opening was however dependent on the geometry [164]. Another type of crack is a breathing-fatigue crack as discussed by Yan et al. [165]. The researcher proposed the procedure to identify the prevailing breathing crack and computed the crack quantitatively.

Crack is also analysed in different schemes. Huan Chen [166] considered a partially insulated crack. Using the hyperbolic heat conduction theory as per Equation (2-27), thermal stresses and thermal effects on the crack were examined. They expressed that the conduction parameters, thermal conductivity, and geometry size have a greater influence on the stress of the crack surface and temperature gradient.

$$a \cdot \nabla^2 T = \frac{\partial T}{\partial t} + \tau \frac{\partial^2 T}{\partial t^2} \quad (2-27)$$

Where α is thermal diffusivity and ∇^2 is Laplace differential operator. Ekhlakov et al. [167] presented thermoelastic crack analysis in FGM. They observed that the influence of the thermomechanical loading in DSIF was much lower than the material gradation. Moreover, DSIF is dependent on the crack angle and orientation with corresponds to the material gradation. Lee et al. [168] and Zhang et al. [169] similarly analysed the exponential gradation of FGM to develop the thermomechanical stress and displacement.

Nicholas et al. [170] estimated the crack propagation as the function of the phase angle between load and temperature. Their research acknowledged the contribution of load position and time effects. John et al. [171] experimentally worked out TSIF generated by temperature gradient around the crack and hence confirmed the thermal load induces crack growth. Park et al. [172] in a similar way categorized the thermal fatigue of an underfill cracking by applying the thermal stress and calculated the TSIF using the crack length correlated to life cycles. K.S Kim and R.H. Van Stone [173] studied the utility of integral parameters for crack propagation under thermo-mechanical loading. Their research reported that the parameters are path-independent for deformation. Chu et al. [174] examined the dynamic crack path in a brittle material. Their study aimed to evaluate the transient heat transfer, dynamic stress evolution under thermo-mechanical couple load. Their experiment in disk specimen showed that the crack branching is within the large heating radius however it goes outside the heating area in case of a smaller radius. Maletta et al. [175] analysed the crack-tip thermal and mechanical hysteresis. They tested shape memory alloys subjected to fatigue loads. Using infrared thermography, they reported a sudden rise in surface temperature at the initial stage. The SIF versus load curves appeared a non-linear hysteretic behaviour reflecting the complex stress-strain hysteretic behaviour of SMAs. Merhy et al. [176] studied the crack growth characterisation of aluminium alloy. The contribution of parameters like temperature, frequency and SIF on crack growth were analysed. Crack growth increased at high temperature with a load ratio on decreasing frequencies. The temperature effect on crack growth was found dependent on the applied frequencies.

2.5 Stochastic response of the structure under thermo-mechanical environment

A structure exposed to cyclic loads for a reasonable time may change its global temperature. The temperature changes impact the amount of stress moderation which in turn alters the deformation of the structures. Khan et al. [177] studied the outcomes of heat generation in viscoelastic material under cyclic loading. The cyclic mechanical loading was applied in a polymeric beam with or without heat conduction. They found that the higher stress point produces more heat which hastened the stress relaxation and triggered a thermal gradient in the polymer beam. In the case of the heat-conduction, a uniform temperature preceded the instability of the beam by diminishing the overall stiffness as shown in Figure 2-9.

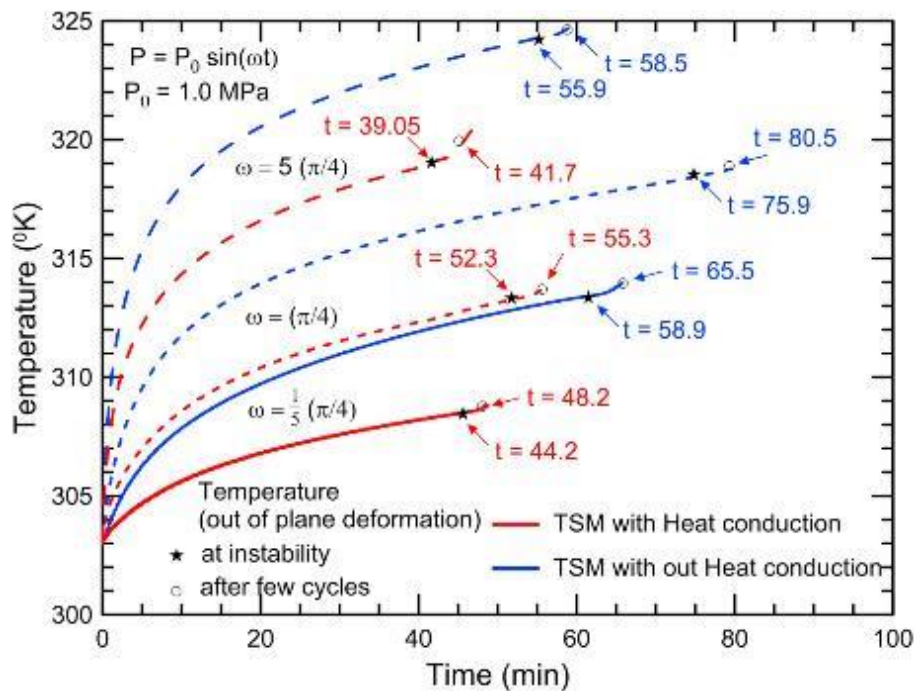


Figure 2-9 Temperature increase at point 1 of the simply supported at 1.0 MPa with/without heat conduction at different frequencies [177]

El-Kafrawy [178] and Bovsunovsky and Surace [179] confirmed that the presence of a crack in beam or frame reduces the modal frequencies. S. Hasbroucq et al. [180] studied the elastic-plastic response under the thermomechanical loading where the elastic properties vary with the

temperature. The result showed that the residual stress and strain fields were time-dependent. Kidane and Shukla [181] experimentally investigated the dynamic behaviour of Ti/TiB functionally graded material exposed to thermo-mechanical loading. The effect of the temperature on dynamic response was studied by infrared spot heaters. The stress-strain curve at a different temperature is illustrated in Figure 2-10. At high temperature, the materials displayed thermal relaxing and caused a reduction in compressive strength and increase in failure strain.

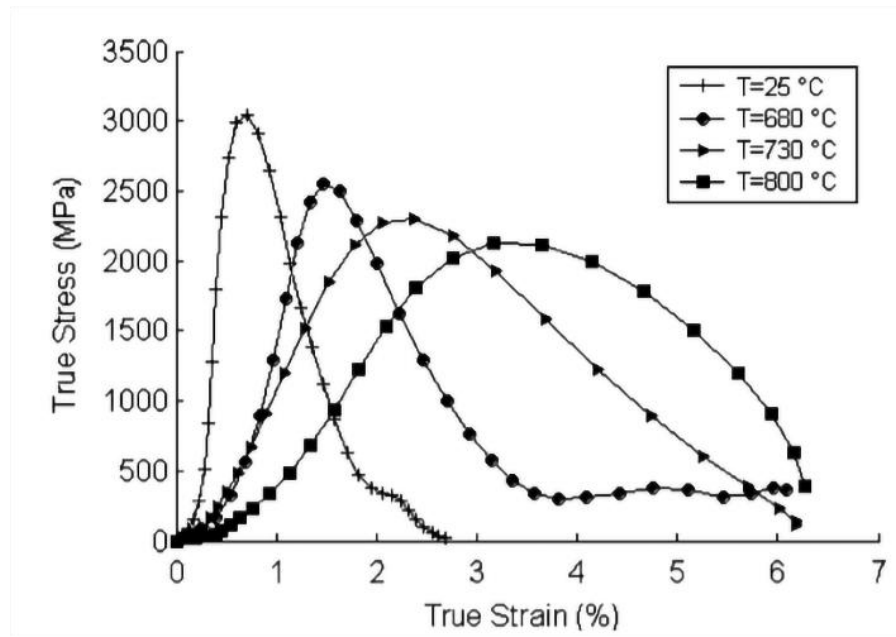


Figure 2-10 True stress-strain curve as a function of temperature [181]

Thermal effect on vibrating structure has been widely investigated [15,21,28,35,64,182–188]. Cao et al. [185] investigated numerically the thermal vibration and thermal contraction of single-wall Carbon Nano Tubes (CNT). They explained that the effect of thermal vibration is more distinct than quasi-static thermal expansion which causes the thermal contraction in both the radial and axial direction when the temperature is well below 800K. Zhang et al. [188] also investigated the thermal effect on the beam. Their work focuses on the high-frequency response in thermal exposure using the energy flow analysis method. The study highlighted the increase in vibration response of the beam with the increase in temperature. In supplement, two temperature-dependent

materials properties, elastic modulus, and thermal expansion coefficient were considered to verify the thermal effect as shown in Figure 2-11.

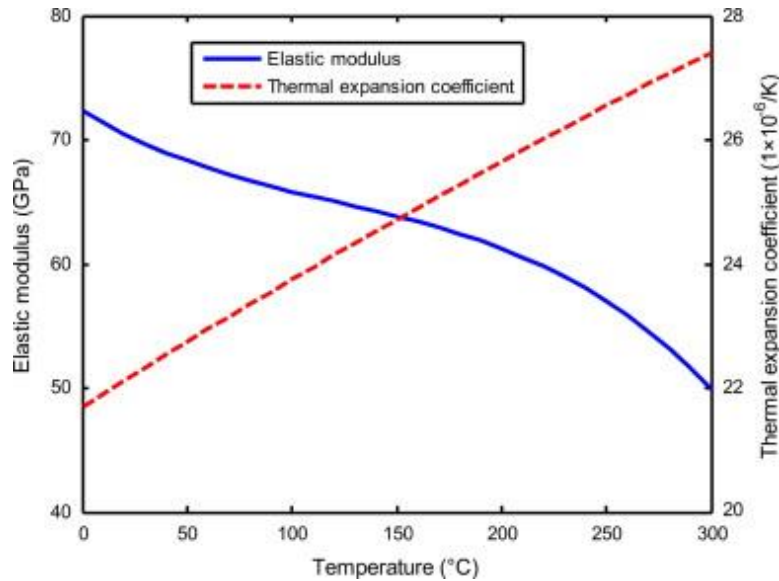


Figure 2-11 Temperature-dependent material properties for aluminium alloy[188]

Ebrahimi et al. [182] analysed the thermal effect on the vibration behaviour of functionally graded beams with porosities. The influence of porosity volume fraction, material profile, temperature, and boundary condition on frequencies was examined. The increase in porosity fraction increased the dimensionless frequencies. But at elevated temperatures, the increase in gradient-index value decreased the frequency. The natural frequencies decreased with an increase in temperature and moved to zero at a critical temperature. However, the frequencies increased afterward.

In another paper, Ebrahimi and Salari [183] studied the vibration behaviour of functional graded (FG) nanobeams under the effects of in-plane thermal loading. The parametric study showed that changes in temperature, mode shape, and boundary conditions influenced the fundamental frequencies. Liu et al. [189] investigated the effect of temperature-dependent material properties. They focused on thermal shock fracture in the principle of non-Fourier heat conduction. The thermal stress intensity factor was found to be vastly receptive to the temperature-dependent material properties. The variations of TSIF with shock time are presented in Figure 2-12. Cui and Hu [184] studied on Euler-

Bernoulli beam under uniform heating. They observed that there is a small influence of temperature-dependent material properties on the natural frequency at low temperature but developed greater influence at high temperature.

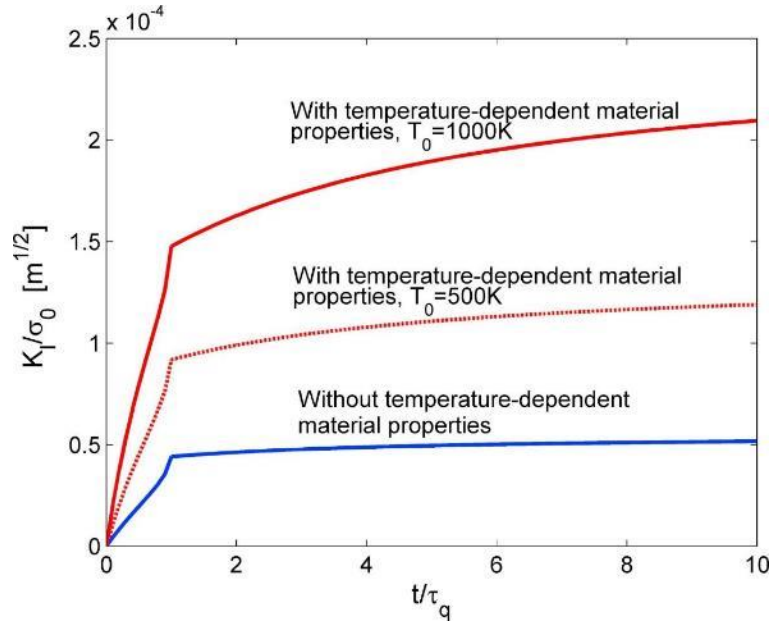


Figure 2-12 TSIF for the surface crack of depth $d/l=1$ [189]

Fazzolari [190] also analysed the modal characteristic of FG with temperature-dependence materials under thermal loading. They analysed the use of the higher-order equivalent single-plate theories to study the two-volume fractions: power-law function (P-FGM) and sigmoid function (S-FGM). The declining rate of dimensionless frequency of temperature-dependent FGM was found to be more than temperature-independent FGM at elevated temperatures. Silva et al. [191] performed the modal analysis at elevated temperatures. They studied the effect of non-uniform temperature distribution on heated rectangular plates. The resonant frequency and displacement data were extracted by the pulse laser-digital image correlation method. The resonant frequencies were noticed to be higher in transverse heating while longitudinal heating did not have much effect. Finite element analysis suggested the results were an after-effect of the disparity in the curvature of the plate at elevated temperature in a steady state. In related studies, Kawamura et al. [192] performed numerical studies on nonhomogeneous material properties of a rectangular plate subjected to a

varying temperature at one surface and other at zero temperature change. They observed the amplitude of deflection decreased with the decrease in young modulus.

2.6 Conclusion

This review emphasises the existing research complexities in the area of fatigue and fracture modelling. Assessing fatigue damage accurately under thermomechanical vibration loads is still a challenge. Most of the available research on thermomechanical vibrations is more or less focused on application-based studies. Researchers applied different theories to evaluate the characteristic of complex geometry. However, modelling of thermomechanical vibrations becomes multifaceted when the geometry is complicated. Thus, the available research data for the fatigue damage assessment is still lacking analytical reasoning and proper explanations concerning accurate fatigue measurement.

Complications in fatigue and fracture life estimation are another challenging obligation. The difficulties of fatigue life estimation are due to the inhibiting variables. Fatigue life is dependent on the loading histories, loading sequences, loading amplitude, and frequency, operation environment, physical properties, etc. Different materials correspond to different fatigue life patterns allowing existing life estimation methods inadequate. Several methods and techniques are available to predict fatigue life. Nevertheless, the challenges are the precise measurement considering those variables which change independently according to the working conditions.

Crack propagation is not a new topic but still requires comprehensive research to describe its governing mathematics with experimental justifications. Crack growths behave in a pseudo manner, especially in thermal conditions. Many investigations have been complied about the nature of crack growth but fail to conclude with valid explanations. Some references are presented in the above review, where the crack growth rate for TiAl-based alloys, 18% Cr ferritic stainless steel, and 18Cr–2Mo ferritic stainless steel increases with the increase in temperature. Yet, there is no convincing reason why and how the crack will

behave for different working conditions. It is learned that fracture mechanics parameters correlated the crack behaviour at different temperatures. In theory, crack growth and fracture behaviour at elevated temperatures is still to address.

The thermal effect on the dynamic response of structure has a great concern in engineering applications. Structures exposed to reasonable high temperatures can modify the material properties and increase crack propagation. This modification influences the overall deformation of the structure. The temperature changes impact the amount of stress concentration thereby diminishing the global stiffness. Several studies were presented for structural vibration in the above review. Moreover, thermomechanical vibration of structure relating to crack propagation and modal properties are still inadequate. It is well known that the presence of crack reduces the dynamics response of a structure. The existing studies are yet to input sufficient explanations about the dynamic behaviour of the structure under distributed temperature across the structure. The structural stiffness can be influenced by material microstructure, temperature distribution, and time of exposure. Different temperature ranges can cause significant variations in the material properties. Existing studies are limited concerning temperature change and compel extensive investigation in crack and uncrack conditions. The impact of thermal properties on structural response under varying temperatures needs to be investigated. Temperature-dependent properties like elastic modulus, thermal expansion, and thermal conductivity change to temperature. The relationship of temperature-dependent properties with structural response in crack and uncrack conditions has not been established yet. There could be considerable changes in structural response due to varying temperature-dependent properties. Dissimilar temperature distributions across the structure may also have diverse dynamic response trends. Subsequently, the modal properties will change corresponding to the changes in the temperature.

3 RESEARCH METHODOLOGY

Vibration analysis is a universally used non-destructive technique for structural health monitoring. In recent times, vibration assessments are implemented in the thermal environment. This is essential because a majority of the engineering applications are exposed to a thermal setting. Considering this circumstance, this research focused on investigating the structural dynamics and crack propagation behaviour due to thermal and mechanical loads.

This research has four objectives. The first and second objectives are analytical-based modelling. The third objective is an experimental and numerical validation method. The last objective is an experimental analysis of fatigue crack propagation. Thus, this research has been divided into three sections as discussed below.

3.1 Analytical formulation

The study of the structural dynamics of cantilever beams is familiar, because many engineering applications, particularly aircraft and turbines, operate akin to a cantilever beam. In this thesis, the Bernoulli–Euler beam theory with a uniform cross-section in fixed-free boundary conditions is considered. The axial force is used as vibrational force. This modelling incorporated the accelerometer mass mounted at the free end of the beam. Practically, the original length of the beam changes due to the applied temperature. Therefore, the effective length of the beam, which is the original length with its thermal expansion, is considered for this analysis. The beam is assumed in one-dimensional longitudinal expansion only because the beam width and thickness are insignificant compared to its length. This model accesses the structural dynamics for both types of temperatures i.e uniform and non-uniform temperature distribution.

The equation of the beam is developed based on Bernoulli–Euler beam hypotheses. Thus, this modelling assumed that the section of the beam remains perpendicular to the neutral axis of the beam after the beam deforms. This convention is effective for bending beam which does not undergo significant shear or torsional stresses. The second postulation is the beam deform angle or

deflection is relatively small. The solution of the equation is found by the separation of the dependent variable method. The modal parameters of the beam are obtained by solving the vibration equation.

The fundamental frequency of the beam is dependent on the types of loadings. Especially, the frequency changes during thermal loading. This is due to the changes in elastic modulus concerning the temperature changes. This modelling used the approximate relationship for calculating the elastic modulus for different temperatures. The changes in the fundamental frequency for a crack beam are calculated based on changes in deformation energy due to the crack formation.

3.2 Experimental and numerical approaches

Aluminium alloy 2024 is widely used in the aerospace and military sector due to its excellent fracture toughness and fatigue resistance. This research experimental specimen is prepared from aluminium 2024-T3. The thermal properties of the specimen are measured from the lab equipment. The elastic modulus was measured using a dynamic mechanical analyser. The thermal expansion was measured using a thermal-mechanical analyser. The thermal conductivity was obtained from thermal diffusivity. The specimens were manufactured with accurate dimensions using a computer numerical control (CNC) machine.

The experiment was designed to analyse the modal characteristic of the beam. There are three types of specimens considered in this evaluation, namely: without crack, with crack, and propagating crack. The crack specimens are subdivided into three different crack depths. The beam responses are evaluated at three heating rates at 2, 5, and 8 °C/min. This experimental arrangement is selected to examine the modal response of the beam under uniform and non-uniform temperature conditions at a maximum temperature of 200°C.

The numerical simulation to analyse the modal parameters was conducted using the ANSYS©2019 R2 workbench. The specimen geometry was drawn from the build-in design modeler. The modal analysis module was utilized to

evaluate the modal frequency of the beam for healthy and pre-seeded crack specimens separately. The harmonic module was used to analyse the modal amplitude of the beam.

3.3 Crack propagation analysis

The thermal effect on the dynamic response of structure has a great interest in engineering applications. Structure exposure to temperatures modified the material properties and impact structural dynamics. This modification influences the overall deformation of the structure. The changes in temperature influence stress concentration thereby diminishing the global stiffness. Moreover, thermomechanical vibration of structure relating to crack propagation and modal properties are still inadequate. Crack propagation analysis is one of the fundamental research areas in structural dynamics and fracture mechanics. It is because the appearance of crack on structure changes the structural response. The crack initiation may occur due to cyclic loads, overloading, and operating conditions. Moreover, the crack growth may not be the same for all materials. In this research, the crack propagation behaviour of aluminium 2024-T3 is studied under uniform and non-uniform temperature distribution. In this study, different pre-seeded crack depths have been selected to analyse the crack propagation. The pre-seeded rectangular crack shape was adopted for all the experimental specimens. However, different crack depths were selected separately for uniform and non-uniform temperatures. The crack propagation crack depths were captured using a digital microscope. Then, the crack depths were measured based on the pixel contained in the crack. The corresponding structural responses for different crack depths are measured during crack propagation.

4 ANALYTICAL FORMULATION TO ESTIMATE THE MODAL PARAMETERS

The assessment of modal response on mechanical structures for structural health monitoring is a common practice. However, most of the engineering applications operate in a thermal environment in addition to the mechanical loads. Especially, during the heating and cooling process of the thermal systems, the modal response may vary significantly. Mechanical structures are commonly subjected to combine thermal and mechanical loads. The structure may experience different structural dynamics under a thermal setting depending on the nature of the applications. In this analytical analysis, the effect of temperature distribution on modal parameters is investigated.

4.1 Beam equation to estimate the modal response

The study of the structural dynamics of cantilever beams is familiar, because many engineering applications, particularly aircraft and turbines, operate akin to a cantilever beam. In this formulation, the Bernoulli–Euler beam theory with a uniform cross-section in fixed-free boundary conditions is considered as shown in Figure 4-1. The vibration force $F(x, t)$ and the accelerometer with its mass m is mounted at the free end of the beam. Generally, the original length of the beam changes due to the applied temperature. Therefore, the effective length of the beam L_e , which is the original length with its thermal expansion, is considered for this analysis. The beam is assumed in one-dimensional longitudinal expansion only because the beam width B and thickness h are insignificant compared to its length. The inertia force in the beam element can be expressed using the free-body diagram as:

$$F(x, t) = (\rho A + m) dx \frac{\partial^2 u}{\partial t^2}(x, t) \quad (4-1)$$

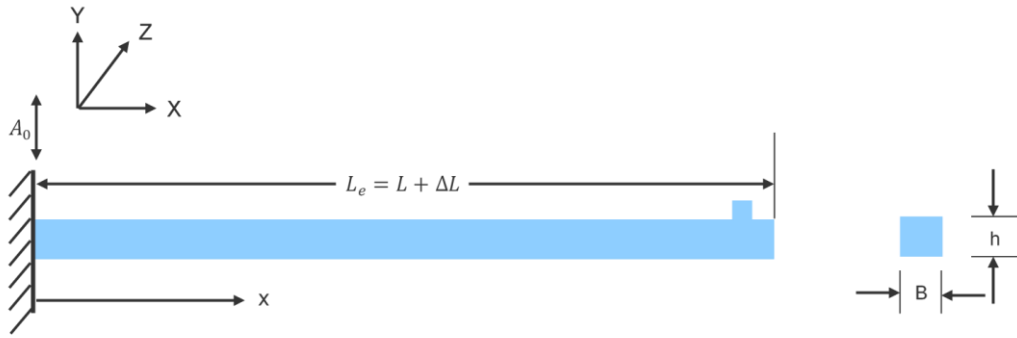
where u represents the transverse displacement, ρ is the mass density of the beam, A is the cross-sectional area, and m is accelerometer mass. The shear force equation for the beam element can be written as:

$$(S(x, t) + dS(x, t)) - S(x, t) + F(x, t)dx = (\rho A + m)dx \frac{\partial^2 u}{\partial t^2}(x, t) \quad (4-2)$$

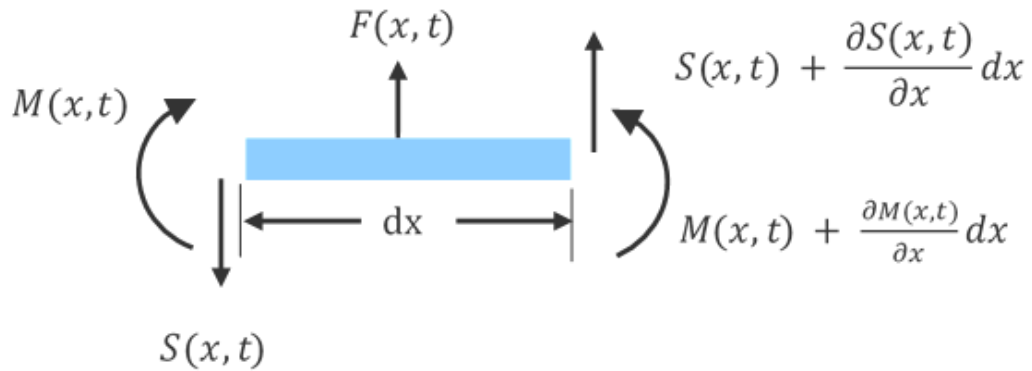
Where $S(x, t)$ is the shear force and $dS = \frac{\partial S}{\partial x} dx$. Similarly, the moment equation for the beam can be written as:

$$(M(x, t) + dM(x, t)) - M(x, t) + (S(x, t) + dS(x, t))dx + F(x, t)dx \frac{dx}{2} = 0 \quad (4-3)$$

where $dM = \frac{\partial M}{\partial x} dx$. The shear force and moment equations reduce to Equations (4-4) and (4-5), respectively:



(a)



(b)

Figure 4-1 (a) Cantilever beam; (b) free body diagram for beam element.

$$\frac{\partial S}{\partial x}(x, t) + F(x, t) = (\rho A + m) \frac{\partial^2 u}{\partial t^2}(x, t) \quad (4-4)$$

$$\frac{\partial M}{\partial x}(x, t) + S(x, t) = 0 \quad (4-5)$$

$$-\frac{\partial M}{\partial x}(x, t) = S(x, t) \quad (4-6)$$

Putting Equation (4-6) into Equation (4-4), the force equation can be written in terms of displacement and transverse force as:

$$-\frac{\partial^2 M}{\partial x^2}(x, t) + F(x, t) = (\rho A + m) \frac{\partial^2 u}{\partial t^2}(x, t) \quad (4-7)$$

Using the Euler-Bernoulli beam theory, the relation between bending movement and bending displacement can be expressed as:

$$M(x, t) = EI(x) \frac{\partial^2 u}{\partial x^2}(x, t) \quad (4-8)$$

where E is the elastic modulus and I is the moment of inertia of the beam. Therefore, the shear force can be expressed in terms of displacement as:

$$S(x, t) = -\frac{\partial}{\partial x} \left[EI(x) \frac{\partial^2 u}{\partial x^2}(x, t) \right] \quad (4-9)$$

And the equation of motion of the vibrational beam can be written as:

$$-EI(x) \frac{\partial^4 u}{\partial x^4}(x, t) + F(x, t) = (\rho A + m) \frac{\partial^2 u}{\partial t^2}(x, t) \quad (4-10)$$

$$F(x, t) = EI(x) \frac{\partial^4 u}{\partial x^4}(x, t) + (\rho A + m) \frac{\partial^2 u}{\partial t^2}(x, t) \quad (4-11)$$

When the beam has uniform cross-section, the equation of the vibrational beam can be transformed as:

$$F(x, t) = EI \frac{\partial^4 u}{\partial x^4}(x, t) + (\rho A + m) \frac{\partial^2 u}{\partial t^2}(x, t) \quad (4-12)$$

For free bending vibration, the applied force is zero, therefore the equation of the beam is reduced to Equation (4-13)

$$EI \frac{\partial^4 u}{\partial x^4}(x, t) + (\rho A + m) \frac{\partial^2 u}{\partial t^2}(x, t) = 0 \quad (4-13)$$

$$\frac{EI}{(\rho A + m)} \left(\frac{\partial^4 u(x, t)}{\partial x^4} \right) + \frac{\partial^2 u(x, t)}{\partial t^2} = 0 \quad (4-14)$$

4.2 Solution method

The solution of Equation (4-14) can be obtained through separation of variable method expressed as:

$$u(x, t) = U(x)V(t) \quad (4-15)$$

Where $U(x)$ is the mode function and $V(t)$ is the time function. Inserting Equation (4-15) into Equation (4-14), the free vibration equation of the beam can be deduced in the form of a separable variable method as Equations (4-16)-(4-18)

$$\frac{EI}{(\rho A + m)} \left(\frac{\partial^4 U(x)V(t)}{\partial x^4} \right) + \frac{\partial^2 U(x)V(t)}{\partial t^2} = 0 \quad (4-16)$$

$$\frac{EI}{(\rho A + m)} V(t) \left(\frac{d^4 U(x)}{dx^4} \right) + U(x) \frac{d^2 V(t)}{dt^2} = 0 \quad (4-17)$$

$$-\frac{EI}{(\rho A + m)} \frac{\left(\frac{d^4 U(x)}{dx^4} \right)}{U(x)} = \frac{d^2 V(t)}{dt^2} \quad (4-18)$$

The equation represents the acceleration, therefore Equation (4-18) is constant and can be written as:

$$-\frac{EI}{(\rho A + m)} \frac{\left(\frac{d^4 U(x)}{dx^4} \right)}{U(x)} = \frac{d^2 V(t)}{dt^2} = -\omega^2 \quad (4-19)$$

Separating the time and spatial variables respectively, we have:

$$\frac{d^2 V(t)}{dt^2} + \omega^2 V(t) = 0 \quad (4-20)$$

$$\left(\frac{d^4 U(x)}{dx^4}\right) - \omega^2 \frac{(\rho A + m)U(x)}{EI} = 0 \quad (4-21)$$

$$\left(\frac{d^4 U(x)}{dx^4}\right) - \beta^4 U(x) = 0 \quad (4-22)$$

The solution of Equation (4-20) can be written as:

$$V(t) = B_1 \sin \omega t + B_2 \cos \omega t \quad (4-23)$$

The solution of Equation (4-22) is satisfied if:

$$\beta^4 = \frac{(\rho A + m)}{EI} \omega^2 \quad (4-24)$$

The solution for vibration mode, Equation (4-22) is assumed as:

$$U(x) = A_0 \sin(\beta x) + A_1 \cos(\beta x) + A_2 \sinh(\beta x) + A_3 \cosh(\beta x) \quad (4-25)$$

where $A_0, A_1, A_2,$ and A_3 are constants that can be obtained by applying the boundary conditions. The boundary conditions of the fixed-free beam are:

$$\left. \begin{array}{l} U(x) = 0 \text{ (zero displacement)} \\ \frac{dU(x)}{dx} = 0 \text{ (zero slope)} \end{array} \right| \text{ at } x = 0$$

And

$$\left. \begin{array}{l} \frac{d^2 U(x)}{dx^2} = 0 \text{ (zero bending moment)} \\ \frac{d^3 U}{dx^3}(x) = 0 \text{ (zero shear force)} \end{array} \right| \text{ at } x = L$$

Applying the first boundary conditions, we get:

$$A_1 = -A_3 \quad (4-26)$$

$$A_0 = -A_2 \quad (4-27)$$

Therefore, the harmonic solution can be rewritten using Equations (4-26) and (4-27) as:

$$U(x) = A_0\{\sin(\beta x) - \sinh(\beta x)\} + A_1\{\cos(\beta x) - \cosh(\beta x)\} \quad (4-28)$$

$$\frac{d^2 U(x)}{dx^2} = -\beta^2[A_0\{\sin(\beta x) + \sinh(\beta x)\} + A_1\{\cos(\beta x) + \cosh(\beta x)\}] \quad (4-29)$$

$$\frac{d^3 U(x)}{dx^3} = -\beta^3[A_0\{\cos(\beta x) + \cosh(\beta x)\} - A_1\{\sin(\beta x) - \sinh(\beta x)\}] \quad (4-30)$$

Since the bending moment and shear force is zero at $x = L$, we have from the second boundary condition:

$$\begin{aligned} \frac{d^2 U(L_e)}{dx^2} &= -\beta^2[A_0\{\sin(\beta L_e) + \sinh(\beta L_e)\} + A_1\{\cos(\beta L_e) + \cosh(\beta L_e)\}] \\ &= 0 \end{aligned} \quad (4-31)$$

$$A_1 = -A_0 \left[\frac{\sin(\beta L_e) + \sinh(\beta L_e)}{\cos(\beta L_e) + \cosh(\beta L_e)} \right] \quad (4-32)$$

$$[A_0\{\sin(\beta L_e) + \sinh(\beta L_e)\} + A_1\{\cos(\beta L_e) + \cosh(\beta L_e)\}] = 0 \quad (4-33)$$

$$\frac{d^3 U(x)}{dx^3} = -\beta^3[A_0\{\cos(\beta x) + \cosh(\beta x)\} - A_1\{\sin(\beta x) - \sinh(\beta x)\}] = 0 \quad (4-34)$$

$$[A_0\{\cos(\beta L_e) + \cosh(\beta L_e)\} - A_1\{\sin(\beta L_e) - \sinh(\beta L_e)\}] = 0 \quad (4-35)$$

Thus, we get the transverse displacement as:

$$\begin{aligned} u(x, t) &= A_0 \sin(\omega t) \left[\{\sin(\beta x) - \sinh(\beta x)\} - \left\{ \frac{\sin(\beta L_e) + \sinh(\beta L_e)}{\cos(\beta L_e) + \cosh(\beta L_e)} \right\} * \right. \\ &\quad \left. \{\cos(\beta x) - \cosh(\beta x)\} \right] \end{aligned} \quad (4-36)$$

Forming Equations (4-33) and (4-35) into matrix format, we have:

$$\begin{bmatrix} \sin(\beta L_e) + \sinh(\beta L_e) & \cos(\beta L_e) + \cosh(\beta L_e) \\ \cos(\beta L_e) + \cosh(\beta L_e) & -\sin(\beta L_e) + \sinh(\beta L_e) \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (4-37)$$

Solving Equation (4-37), we obtained the characteristic Equation (4-38). The subscript is added because multiple roots satisfy the equation.

$$\cos(\beta_n L_e) \cosh(\beta_n L_e) = -1 \quad (4-38)$$

The roots of the Equation (4-38) are:

$$\beta_n L_e = 1.87510 \text{ for } n = 1$$

$$\beta_n L_e = 4.69409 \text{ for } n = 2$$

Therefore, the circular frequency of the beam can be expressed by rearranging Equation (4-24) as:

$$\omega^2 = \beta^4 \frac{EI}{(\rho A + m)} \quad (4-39)$$

$$\omega = (\beta_n L_e)^2 \sqrt{\frac{EI}{(\rho A + m)L_e^4}} \quad (4-40)$$

$$f = \frac{\omega}{2\pi} = \frac{(\beta_n L_e)^2}{2\pi} \sqrt{\frac{EI}{(\rho A + m)L_e^4}} \quad (4-41)$$

4.3 Effect of temperature on structural dynamics of beam

The modulus E is a temperature-dependent property that varies with temperature. The modulus decreases with increasing thermal energy along the beam. This decrease is assumed as being linear with temperature. The approximate relationship to estimate the modulus [193] is given in Equation (4-42):

$$E(T) = E_o \left(1 - \varphi \frac{T}{T_o} \right) \quad (4-42)$$

where $\varphi = 0.3$ is the proportional constant, T_o is the melting temperature, and T is the temperature at the measured value of E , and E_o is the modulus at 20 °C. When the beam is long, such that it has different temperature zones across the beam length, the effective or the equivalent modulus of the beam can be calculated by taking the average of the modulus. In this case, the effective or the equivalent modulus of the beam can be calculated by taking the average of the modulus as Equation (4-43).

$$E_e = \frac{\sum \left(E_o \left(1 - \varphi \frac{T_n}{T_o} \right) \right)}{n} \quad (4-43)$$

Where n is the number of temperature zone in the beam. Considering of E_e is more applicable when the cross-section area of the beam is non-uniform because the temperature distribution will be different. Hence, the fundamental frequency of the beam can be written as:

$$f = \frac{\omega}{2\pi} = \frac{(\beta_n L_e)^2}{2\pi} \sqrt{\frac{E(T)I}{(\rho A + m)L_e^4}} \quad (4-44)$$

The bending moment equation of the beam considering the temperature-dependent elastic modulus can be written as:

$$M = E(T)I \frac{d^2 U}{dx^2} \quad (4-45)$$

$$M = -E(T)I\beta^2 A_0 \left[\{ \sin(\beta L_e) + \sinh(\beta L_e) \} - \left\{ \frac{\sin(\beta L_e) + \sinh(\beta L_e)}{\cos(\beta L_e) + \cosh(\beta L_e)} \right\} \{ \cos(\beta L_e) + \cosh(\beta L_e) \} \right] \quad (4-46)$$

where A_0 is the load amplitude at time t . The total deformation energy (Q) due to the beam deformation is derived from the direct bending moment equation.

$$Q = \int_0^{L_e} \frac{(M)^2}{2E(T)I} dx \quad (4-47)$$

$$Q = \int_0^{L_e} \left[\frac{(E(T)I\beta^4 A_0 [\{ \sin(\beta x) + \sinh(\beta x) \}^2 - C^2 \{ \cos(\beta x) + \cosh(\beta x) \}])^2}{2E(T)I} \right] dx \quad (4-48)$$

$$Q = \frac{E(T)I}{2} A_0^2 (\beta)^4 (Q_1^2 - C^2 Q_2^2) \quad (4-49)$$

Where $C = \left\{ \frac{\sin(\beta L_e) + \sinh(\beta L_e)}{\cos(\beta L_e) + \cosh(\beta L_e)} \right\}$

$$Q_1 = \int_0^{L_e} \{\sin(\beta x) + \sinh(\beta x)\}^2 dx \quad (4-50)$$

$$Q_1 = \left(\frac{L_e}{2} - \frac{\sin 2(\beta L_e)}{4\beta} \right) + \left(\frac{L_e}{2} - \frac{\sinh 2(\beta L_e)}{4\beta} \right) - \frac{\cos(\beta L_e) \sinh(\beta L_e)}{\beta} + \frac{\sin(\beta L_e) \cosh(\beta L_e)}{\beta} \quad (4-51)$$

$$Q_2 = \int_0^{L_e} \{\cos(\beta x) + \cosh(\beta x)\}^2 dx \quad (4-52)$$

$$Q_2 = \left(\frac{L_e}{2} + \frac{\sin 2(\beta L_e)}{4\beta} \right) + \left(\frac{L_e}{2} + \frac{\sinh 2(\beta L_e)}{4\beta} \right) + \left(\frac{\sin(\beta L_e) \cosh(\beta L_e)}{\beta} + \frac{\cos(\beta L_e) \sinh(\beta L_e)}{\beta} \right) \quad (4-53)$$

The strain of the beam is dependent on applied stress and temperature. Therefore, we can express strain as a function of stress and temperature as Equation (4-54).

$$\varepsilon_x = f(\sigma, T) \quad (4-54)$$

$$d\varepsilon_x = \left(\frac{\partial \varepsilon_x}{\partial \sigma} \right)_T d\sigma + \left(\frac{\partial \varepsilon_x}{\partial T} \right)_\sigma dT \quad (4-55)$$

where $E = \left(\frac{\partial \sigma}{\partial \varepsilon_x} \right)_T$ and $\alpha = \frac{1}{L} \frac{dL}{dT} = \left(\frac{\partial x}{\partial T} \right)_\sigma$. While $\left(\frac{\partial \varepsilon_x}{\partial \sigma} \right)_T$ and $\left(\frac{\partial \varepsilon_x}{\partial T} \right)_\sigma$ are variation in strain due to stresses and temperature respectively. The direct strain of the beam can be found by integrating Equation (4-56)

$$d\varepsilon_x = \frac{d\sigma}{E} + \alpha dT \quad (4-56)$$

4.4 Equation of dynamic response of the beam under the crack condition

Changes in the deformation energy and natural frequencies due to crack can be computed from the bending moment and flexural rigidity [194] as:

$$\Delta Q = \frac{M^2}{2\chi_t} \quad (4-57)$$

where χ_t is the stiffness of the crack beam represented by Equation (4-58).

$$\chi_t = \frac{Bh^2E(T)}{72\pi F\left(\frac{t_c}{H}\right)} \quad (4-58)$$

where $F\left(\frac{t_c}{H}\right) = 0.638\left(\frac{t_c}{H}\right)^2 - 1.035\left(\frac{t_c}{H}\right)^3 + 3.720\left(\frac{t_c}{H}\right)^4 - 5.177\left(\frac{t_c}{H}\right)^5 + 7.553\left(\frac{t_c}{H}\right)^6 - 7.332\left(\frac{t_c}{H}\right)^7 + 2.491\left(\frac{t_c}{H}\right)^8$ is the crack function and t_c is the crack depth. Change in the natural frequency due to the crack on the beam can be computed as:

$$\Delta\omega_{nc} = \frac{\Delta Q}{2Q}\omega_n \quad (4-59)$$

Therefore, the new natural frequency of the crack beam can be computed from Equation (4-61).

$$\omega_{nc} = \omega_n - \Delta\omega_{nc} \quad (4-60)$$

$$\omega_{nc} = \left(1 - \frac{\Delta Q}{2Q}\right)\omega_n \quad (4-61)$$

The modal amplitude of the cantilever beam was analyzed using Equation (4-36). The amplitude is dependent on the respective fundamental frequency of the without crack and with a crack beam. Therefore, the fundamental frequency of the without crack specimen was obtained from Equations (4-44) and (4-61) for a crack beam.

4.5 Linear temperature distribution approximation

Aircraft wings and turbine blades operate akin to a cantilever beam, and the temperature distribution is not uniform, but instead resembles a type of linear temperature distribution. This beam can be treated as one-dimensional because the thickness of the blades is thin as compared to its length. In such a case, one-dimensional law can be used. It expressed the heat conduction equation

based on Fourier's law of the beam in Equation (4-62). Figure 4-2 shows the heat flow by conduction. We can write the equilibrium energy as Equation (4-63).

$$q = -kA \frac{\partial T}{\partial x} \quad (4-62)$$

$$q - \left(q + \frac{\partial q(x)}{\partial x} dx \right) = 0 \quad (4-63)$$

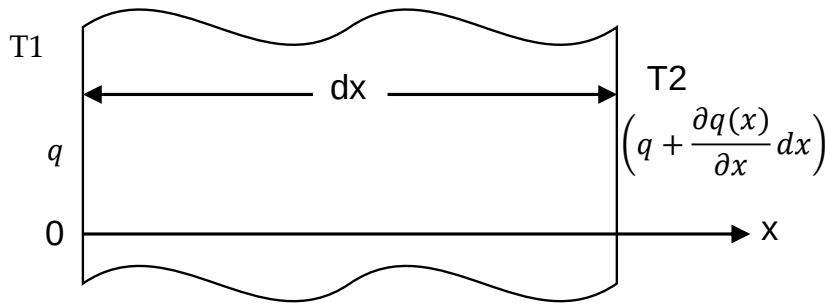


Figure 4-2 Heat flow by conduction

By solving the equation, we can find that $\left(-kA \frac{\partial^2 T}{\partial x^2} \right) = 0$. If the cross-section area and thermal conductivity are constant, we find that $\left(\frac{\partial^2 T}{\partial x^2} \right) = 0$. Integrating, we get $\frac{\partial T}{\partial x} = C_1$, Integrating again, we obtained:

$$T = C_1 x + C_2 \quad (4-64)$$

where C_1 and C_2 are integration constants, which can be found from the given boundary conditions as $T(0) = T_1$ at $x = 0$ and $T(L_e) = T_2$ at $x = L_e$

Hence, $T_1 = C_2$ and $T_2 = C_1 L_e + T_1$

Therefore, the linear temperature distribution at any point on the beam can be found as:

$$T(x) = \left(\frac{T_2 - T_1}{L_e} \right) x + T_1 \quad (4-65)$$

And the heat flux across the beam can be written as:

$$q'' = -k \left(\frac{T_2 - T_1}{L_e} \right) \quad (4-66)$$

Where k is the thermal conductivity, $q'' = \frac{Ph}{A}$, P is the power (in Watt) supplied to heat the beam, h is the thickness of the beam, A is the cross-section area of the beam.

4.6 Conclusion

The structural dynamic is influenced by temperature and mechanical loads. This analytical model investigates the effects of temperature distribution on the modal response of a beam structure. The model can investigate the modal parameters of an uncracked and cracked beam under uniform and non-uniform temperature conditions. The model considers the effective length of the beam to investigate the modal response of the beam. The effective length of the beam is the original length with its thermal expansion due to the applied temperature. Then, the model is modified to estimate the modal behaviour of the beam at crack presence. This modelling used the approximate relationship for calculating the elastic modulus for different temperatures. The changes in the fundamental frequency for a crack beam are calculated based on changes in deformation energy due to the crack formation. The list of input values to run the analytical model are mass of accelerometer 2 gram, the beam length 200mm, width 25mm, thickness 3.15, poisson ratio 0.33, temperature values 25, 50, 100, 150, 200, temperature profile 25–27°C, 33–50°C, 80–100°C, 131–150°C, 185–200°C, Thermal expansion values for different temperature, elastic modulus values for different temperature, modulus-temperature relationship $E(T) = E_o \left(1 - \varphi \frac{T}{T_o} \right)$, density of material 2.78g/cc, proportional constant $\varphi = 0.3$, and melting temperature 570 °C

5 EXPERIMENTAL AND NUMERICAL APPROACHES

The analytical model presented in the previous chapter is validated by the experimental and numerical approaches as discussed below. In this chapter, the details of the experimental method and experimental setup are presented. The experiments are designed to analyse the modal characteristic of the beam under uniform and non-uniform temperature conditions. The selection of the experimental specimen and material properties are discussed. The sample preparation and its geometry dimensions and data acquisition process are elaborated.

5.1 Materials of the experimental specimen

Aluminium alloy 2024 is widely used in the aerospace and military sector due to its excellent fracture toughness and fatigue resistance. This research experimental specimen is prepared from aluminium 2024-T3. The chemical property of the material is given in Table 5-1.

Table 5-1 Chemical composition of Al 2024-T3

Chemical	Si	Fe	Mn	Mg	Cr	Zn	Ti	V	Other
Actual (wt %)	0.07	0.18	4.4	0.58	1.3	0.01	0.13	0.02	0.02

5.1.1 Measuring thermal properties of Al 2024-T3

Aluminium alloy 2024-T3 is selected for this research. The thermal properties of Al 2024-T3 were tested for various temperatures in the dynamic mechanical analyzer (DMA) and thermal-mechanical analyzer (TMA). The macro-expansion probe was used for TMA results; however, a single cantilever clamp was used for elastic modulus DMA results. The thermal conductivity is calculated from the relation of thermal diffusivity as given in Equation (5-1). The thermal diffusivity is extracted from [195].

$$K = \frac{k}{\rho c} \quad (5-1)$$

Where K is the thermal diffusivity, k is the thermal conductivity, ρ is the density and c is the specific heat of the aluminium alloy.

5.2 Experimental specimen preparation and its geometry

Experimental specimens are classified into three types of specimens: without crack, with crack, and propagating cracks. The crack specimens were classified as 0.25, 0.5, and 1 mm, each with a rectangular shape. This predefined crack depth was selected to test the beam at various crack conditions under thermal and mechanical loads. The crack location is the same for all the experiment specimens, which is 4.5 mm away from the fixed point. The fixed point starts from the fillet region of the geometry as indicated in the specimen drawing. This crack location was considered because of the maximum stress concentration in the fillet region of the geometry. The geometry and its dimensions are shown in Figure 5-1 and the sample in Figure 5-2. The specimens were manufactured to have accurate dimensions using a computer numerical control (CNC) machine.

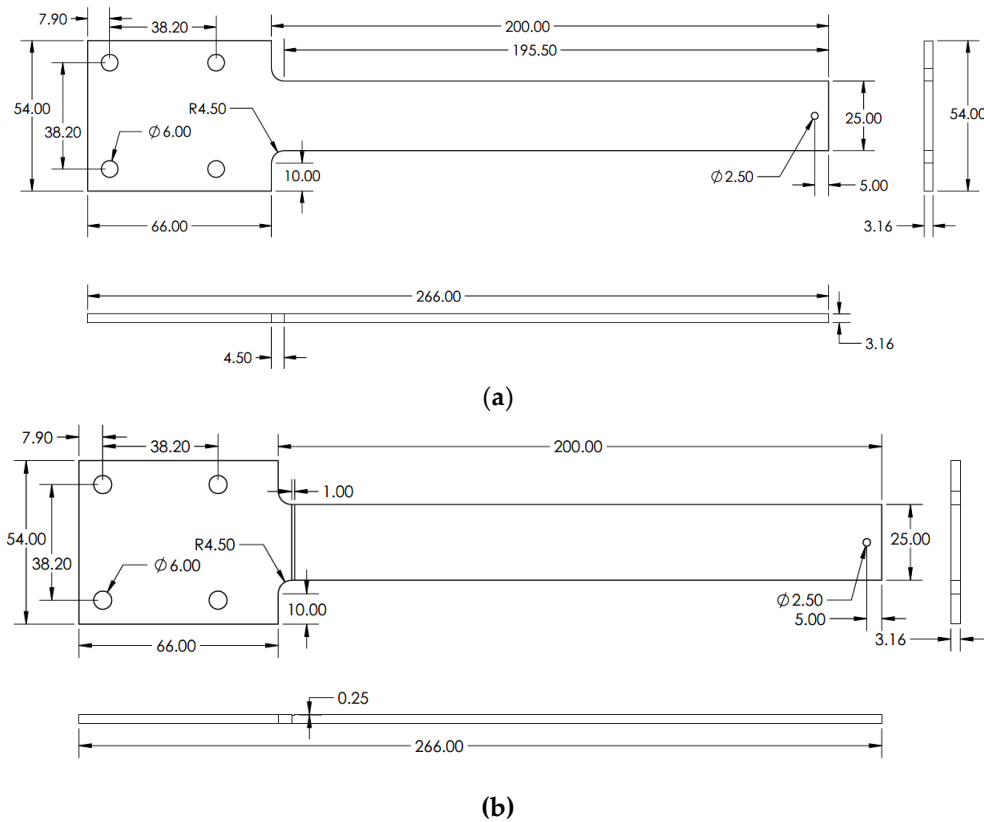


Figure 5-1 Specimen geometry dimensions in mm (a) without crack (b) with crack.

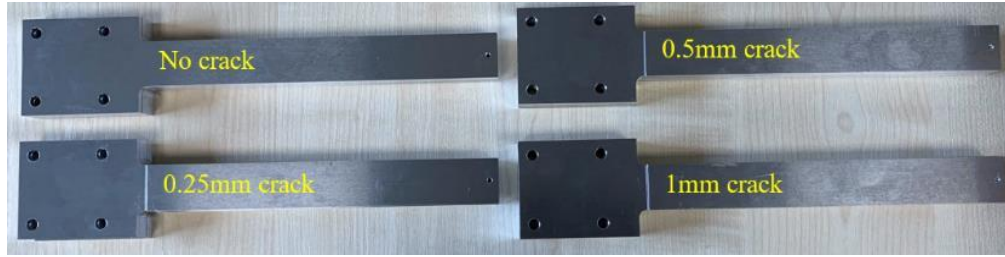


Figure 5-2 Specimen samples

5.3 Experimental scheme

The detail of this research experimental design is presented in Figure 5-3. This experimental arrangement was considered to access the modal response of the beam under uniform and non-uniform temperatures at a maximum temperature of 200 °C. The experiment was designed to analyze the modal characteristic of the predefined crack and without crack beam under uniform temperatures in the range of 25°C-200°C. A beam subjected to three heating rates, i.e., 2°C/min, 5°C/min, and 8°C/min were applied to analyze the modal behavior. In the first case, the beam was heated slowly at the rate of 2°C/min to reach the desired temperature. In the second case, the temperature was increased at a moderate rate of 5°C/min to reach the required temperature for the experiments. In the third case, it was allowed to increase rapidly at the rate of 8°C/min for the same isothermal temperature conditions. The structural response was monitored with the help of an accelerometer mounted on the specimen. These rates were chosen to test the influence of heating duration on modal parameters.

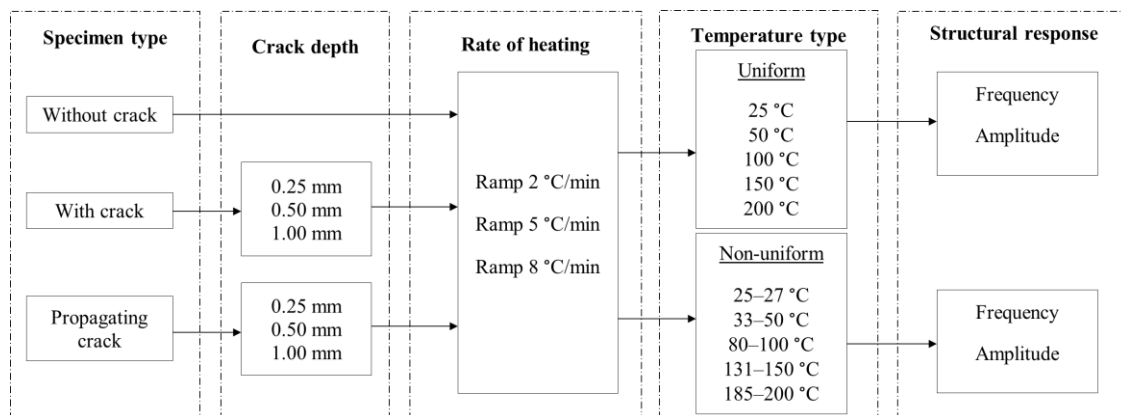


Figure 5-3 Detail of the experiments.

5.4 Experimental set-up

The experimental setup is divided into three units, the layout is shown in Figure 5-4 and the details of the experimental setup are shown in Figure 5-5.

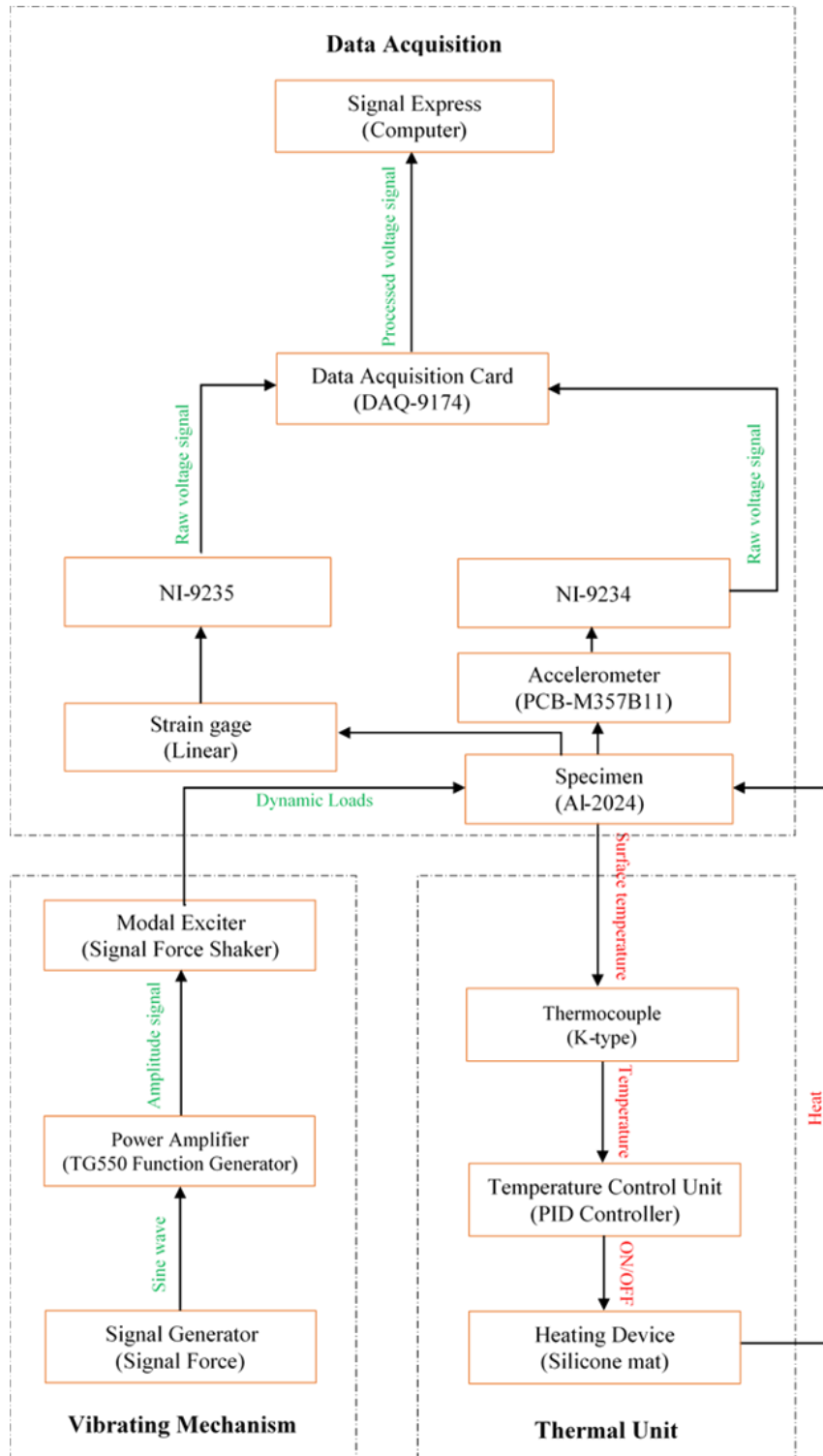


Figure 5-4 Experimental layout

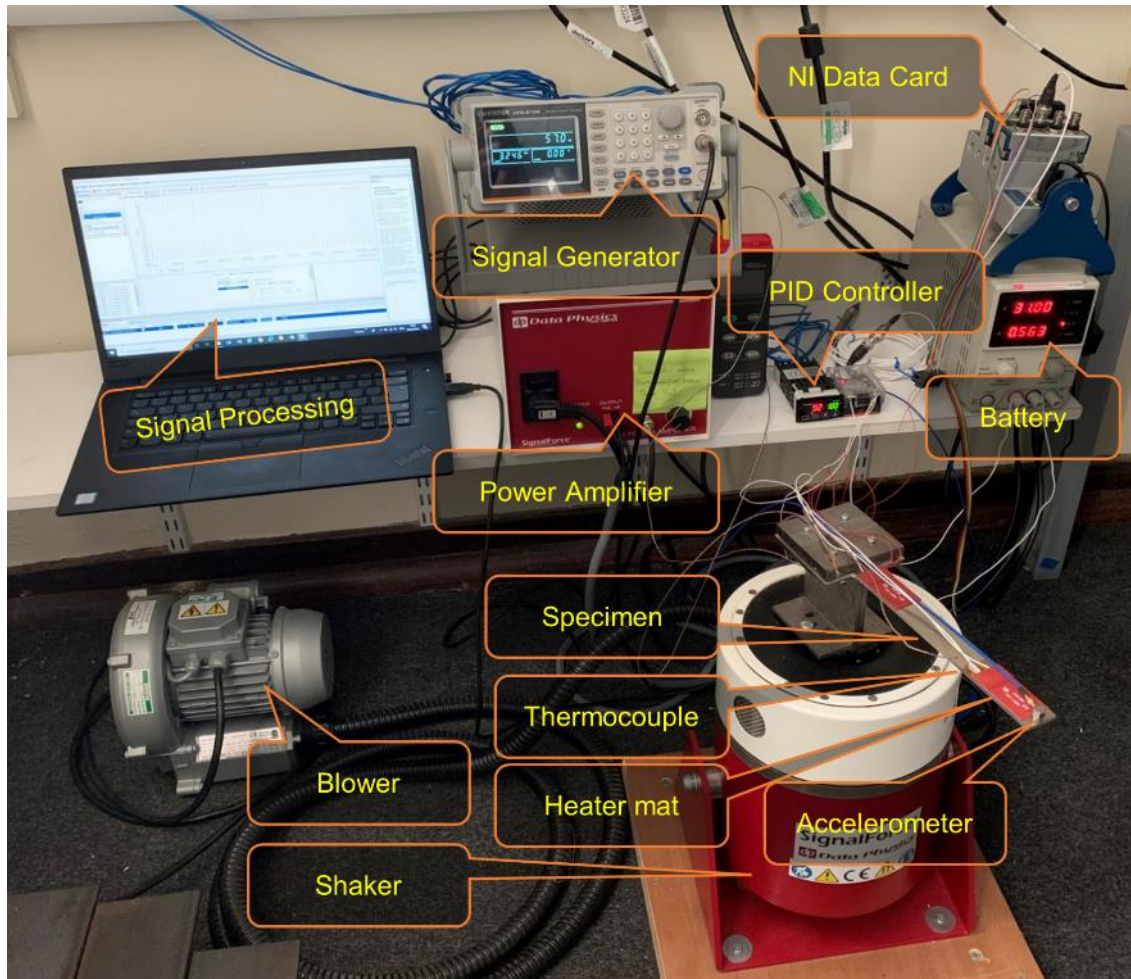


Figure 5-5 Experiment set-up.

The first unit is the vibrating mechanism which consists of a signal generator, power amplifier, and mechanical shaker. The function of the power amplifier is to amplify the input sine wave from the signal generator which is then input to the shaker. The shaker provides the vibration to the specimen with a fixed amplitude of 2mm against amplified voltage 3.2V. The second unit is the thermal unit comprising a thermal couple, PID temperature controller, and thermal mat. The K-type thermal couple is used to measure the specimen surface temperature. The function of the PID temperature controller is to control the heat supply to the specimen through the heating device thermal mat. The uniform temperature was achieved by placing a thermal mat at both ends while the non-uniform temperature was achieved by supplying heat at only one end as shown in Figure 5-5. The third unit is the data acquisition which consists of an accelerometer, data acquisition card, and National Instrument signal

express. The strain gage is attached near the fixed end, while the accelerometer is mounted at the free end of the specimen to measure the modal amplitude. National Instrument DAQ card (i.e., NI 9234 and NI 9235 with DAQ-9174) and signal express are used to analyse the dynamic response of the system.

5.5 Signal processing and data analysis

The modal response of the beam was measured with the help of an accelerometer. The acquired acceleration and time data by accelerometer were exported to National Instrument Signal Express. Then, the acquired data were saved in text format and imported to MATLAB for processing the data. The natural frequency of the specimen beam was determined using Equation (5-2)

$$f = \frac{1}{t_p - t_q} \quad (5-2)$$

Where, t_p and t_q are the times between the two wave cycles. The modal amplitude of the specimen beam was computed by using Equation (5-3).

$$U = \frac{a_{max}}{(2\pi f)^2} \quad (5-3)$$

Where, a_{max} is the maximum acceleration amplitude.

5.6 Numerical analysis method

The numerical simulation to analyze the modal parameters was conducted using the ANSYS©2019 R2 workbench. The geometry of the specimen was drawn from the build-in design modeler as shown in Figure 5-6. The model was built in a fixed-free boundary condition. The sinusoidal load with an amplitude of 2 mm was enforced as a mechanical load at the fixed end. The thermal loads were provided in a steady-state condition. The thermal properties of the material, such as the elastic modulus, coefficient of thermal expansion, and thermal conductivity were incorporated into the model at respective temperatures.

The modal analysis module was utilized to evaluate the modal frequency of the beam for healthy and pre-seeded crack specimens separately. The harmonic module was used to analyze the modal amplitude of the beam. The meshed density of this study was 3 mm, with 8833 nodes and 1728 elements. This evaluation of modal parameters was conducted separately for with crack and without crack conditions at various temperatures.

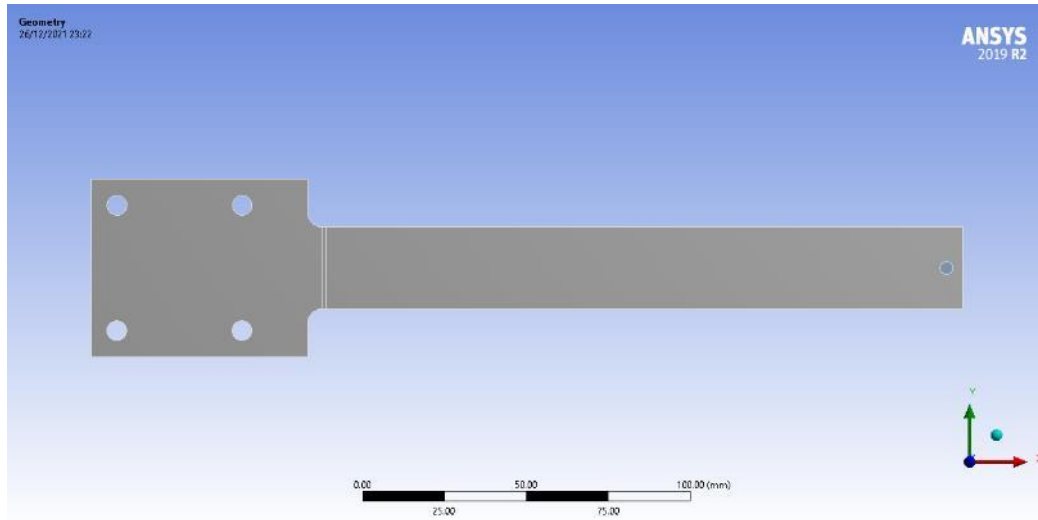


Figure 5-6 ANSYS model geometry

6 RESULTS AND DISCUSSIONS

In this chapter, the analytical, numerical, and experimental results are discussed. The first part of the discussion is about the thermal properties of the material. It is important because the material properties are temperature-dependent. The heating rate and heating duration which could impact the structural response are explained. The comparison of analytical, numerical, and experimental results is presented. The impact of temperature on modal parameters in a crack and without crack under uniform temperature and non-uniform temperature conditions is discussed.

6.1 Thermal properties of Al 2024-T3

The thermal properties of the material at different temperatures are given in Table 6-1. The elastic modulus is sensitive to temperatures, as shown in Figure 6-1. The modulus decreases linearly with the temperature rise. The orientation

Table 6-1 Thermal properties of Al 2024-T3 at different temperatures [195]

Temperature (°C)	Elastic modulus (E) GPa	Thermal expansion (α) K ⁻¹	Thermal conductivity (k) W/mm-°C
25	73.4	21.32×10^{-6}	0.1136
50	72.7	21.56×10^{-6}	0.1180
100	69.3	21.82×10^{-6}	0.1260
150	68.5	25.74×10^{-6}	0.1287
200	65.2	27.18×10^{-6}	0.1309

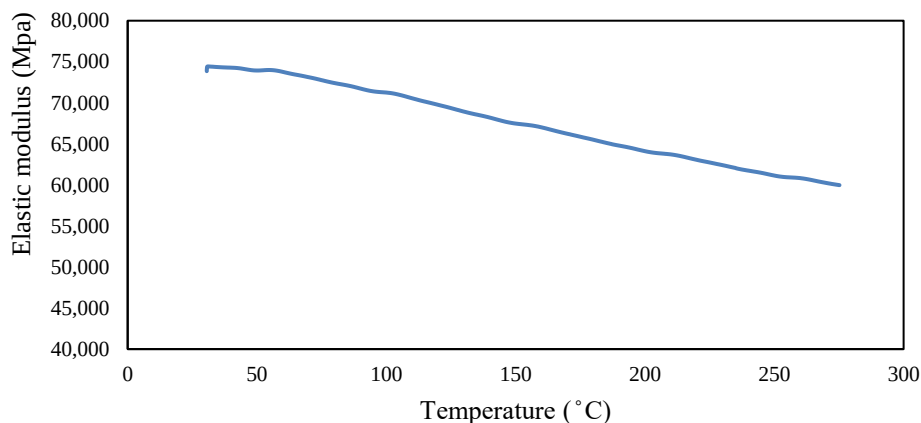


Figure 6-1 Elastic modulus of Al 2024-T3.

of crack for measuring the thermal expansion is shown in Figure 6-2. The coefficient of thermal expansion of healthy specimen is shown in Figure 6-3, crack specimens of 0.25 mm, 0.5 mm, and 1 mm crack depth are respectively shown in Figure 6-4, Figure 6-5, and Figure 6-6. The graphs show that thermal expansion increases linearly with the temperature rise. It also shows that changes in temperature and crack depths have small changes in dimension per degree Celsius as depicted in the figures. This suggests that the coefficient of thermal expansion varies per degree Celsius and also concerning crack depth. The thermal expansions were measured without crack and with crack conditions. It was noticed that the coefficient of thermal expansion is increased from 0.039 to 0.097 $\mu\text{m}/^\circ\text{C}$ in the case of without cracks. However, in 0.25 mm crack depth, it increases from 0.041 to 0.096 $\mu\text{m}/^\circ\text{C}$ while in 0.5 mm it rises from 0.0071 to 0.083957 $\mu\text{m}/^\circ\text{C}$ and 0.1 mm crack depths start from -0.031 to 0.073 $\mu\text{m}/^\circ\text{C}$. Therefore, the coefficient of thermal expansion is not uniform in all cases. Comparatively, the greater temperature has higher thermal expansion per degree Celsius than lower temperature. This indicates the thermal expansion increases linearly with an increase in temperature, but with different rate and cracks conditions.

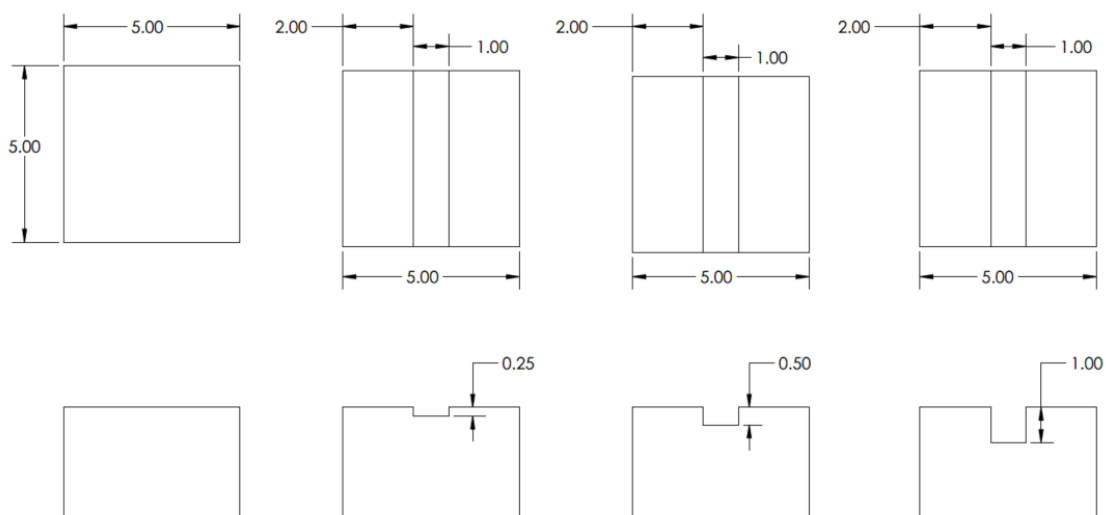


Figure 6-2 Orientation of crack for measuring the thermal expansion

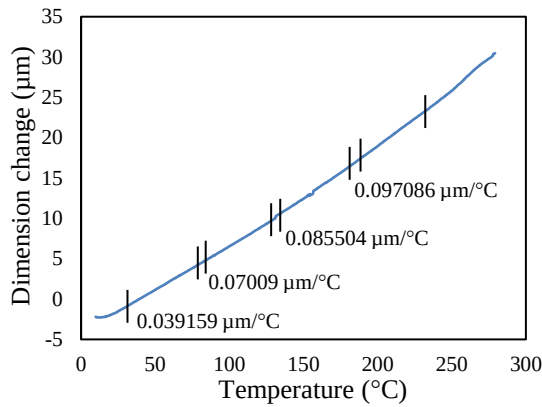


Figure 6-3 Thermal expansion of healthy specimen with 3.15 mm thickness

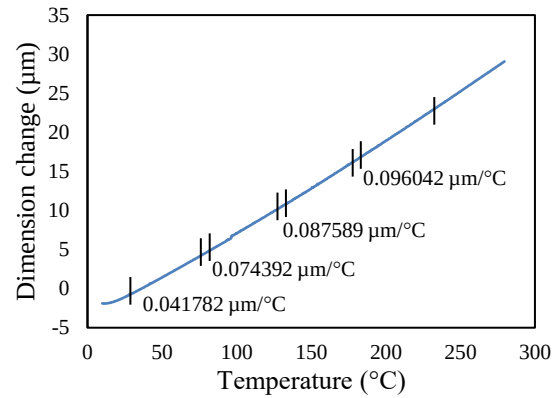


Figure 6-4 Thermal expansion of 0.25mm crack depth specimen with 3.15 mm thickness

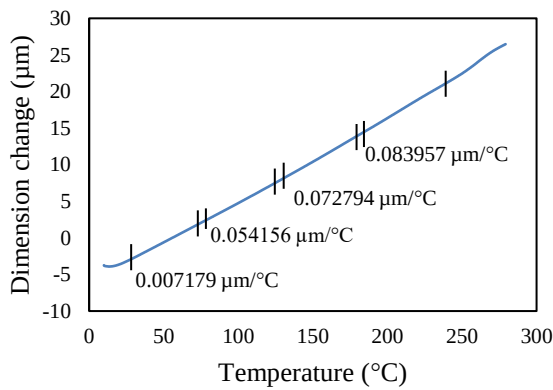


Figure 6-5 Thermal expansion of 0.5mm crack depth specimen with 3.15 mm thickness

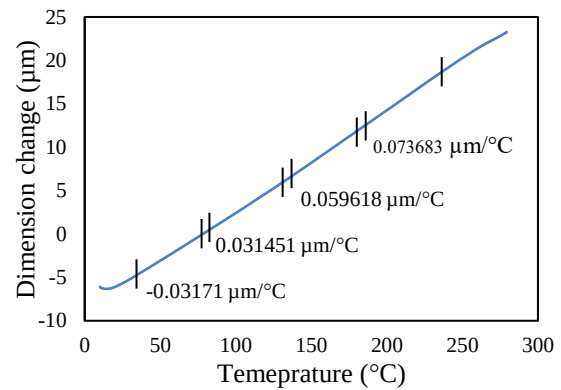


Figure 6-6 Thermal expansion of 1 mm crack depth specimen with 3.15 mm thickness

6.2 Heat rate and exposure time

In this research, three cases of heating rate were considered to analyse the effect of temperature exposure time on modal parameters. In the first case, the beam temperature was allowed to increase in a slow and steady-state at the rate of 2°C/min. This enabled the beam to expose to temperature for a longer duration to reach the test temperatures. In the second case, the temperature was increased at the rate of 5°C/min. This heating rate was considered as a moderate rate to reach the required temperature for the experiments. In the

third case, the beam temperature was allowed to increase rapidly at the rate of $8^{\circ}\text{C}/\text{min}$. This controlled rate of heating the beam was achieved by using a PID temperature controller. Therefore, the temperature along the beam was controlled to meet the experimental requirements. Figure 6-7 (a) (b) (c) (d) presents the time required for all the three cases for the temperature of 50°C , 100°C , 150°C , and 200°C respectively. The graphs show that the same temperature was achieved at different times. This means the increase in temperature at the rate of $2^{\circ}\text{C}/\text{min}$ takes more time to heat the beam for all isothermal conditions. The heating at the rate of $2^{\circ}\text{C}/\text{min}$ (i.e., slow heat rate) may adversely affect the material properties. It takes more time to reach the required temperature as compared to medium and fast heating rates. Therefore, during a slow heating rate, the beam is exposed to heat for a longer duration and hence tends to influence the material properties more especially like reducing the stiffness.

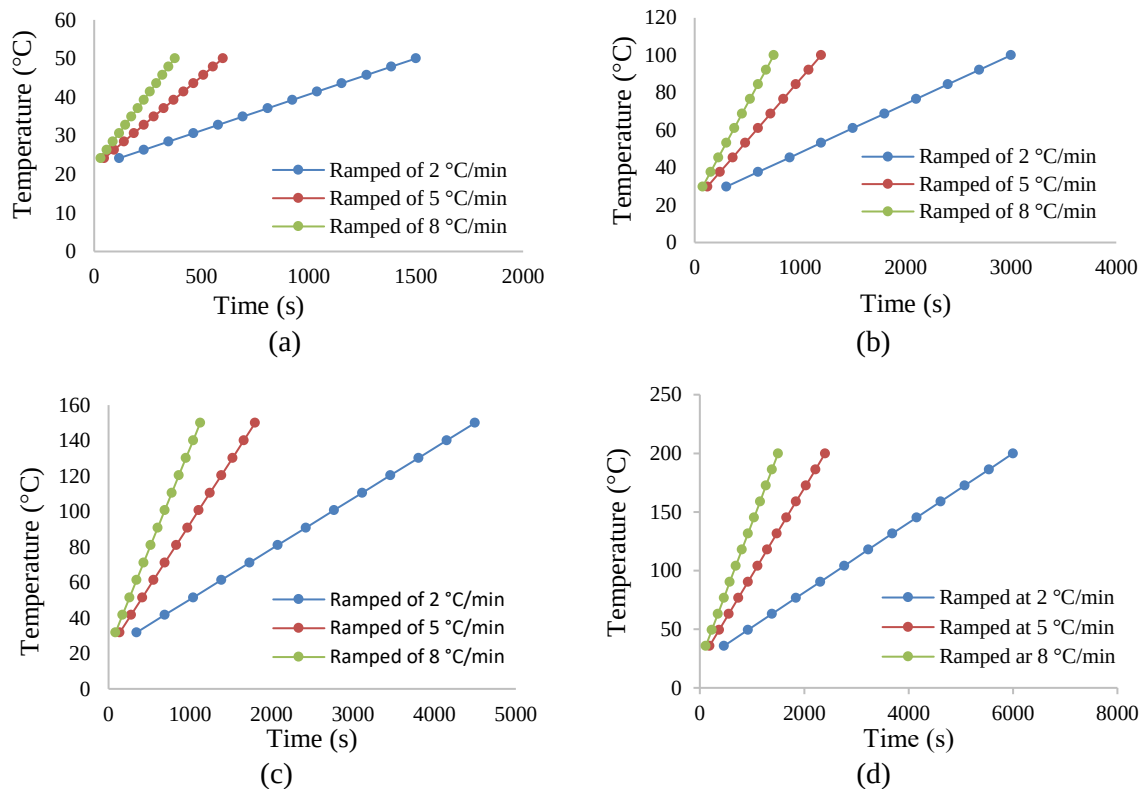
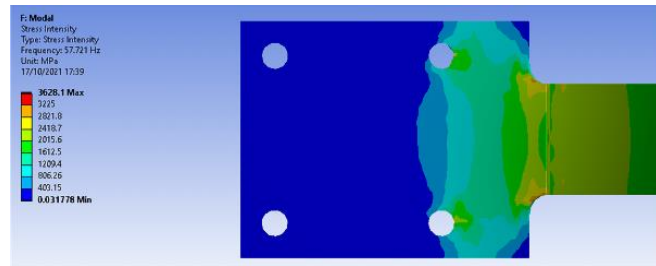


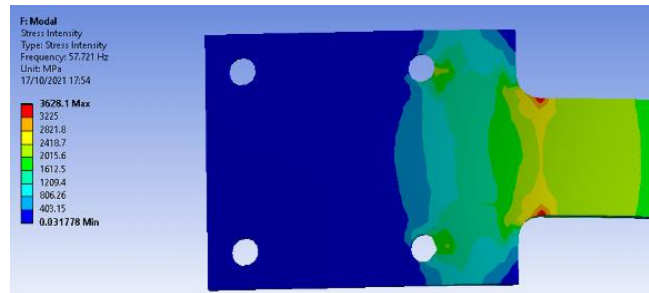
Figure 6-7 Temperature exposure duration for (a) 50°C (b) 100°C (c) 150°C and (d) 200°C

6.3 Thermal and elastic strain intensity analysis

The applied temperature is one of the factors that affect the modal response of the beam. In this study, two types of temperatures, uniform and non-uniform temperature loads were employed to evaluate the effect on modal parameters and stress and strain intensity. The evaluations were executed separately for both temperatures and different cracked and uncracked specimens. In uniform temperatures, the temperature across the beam was maintained at the same temperatures. However, in non-uniform temperatures, the two ends of the beam were maintained at different temperatures, as described in the details of the experiments. The numerical analysis showed that the temperatures were linearly distributed across the beam in the case of non-uniform temperatures.



(a)



(b)

Figure 6-8 Stress intensity of 0.25 mm crack depth at 100–180 °C. (a) Top view, (b) Bottom view.

The heat flow was from the high-temperature region to the low-temperature zone. In this study, all the higher temperatures were maintained at the free end, while lower temperatures were maintained at the fixed end. It was noticed that heat flux was concentrated more at the crack region, which was the fillet zone of the beam.

The assessment of stress and strain intensity was conducted for all types of specimens in both uniform and non-uniform temperatures distribution. It was observed that there was a stress concentration in the crack zone than in any other area of the beam. Moreover, the stress and strain intensity were more concentrated at the back of the crack region than in the crack area, as shown in Figure 6-8. It was also noticed that the stress concentration for the uncracked specimen was in the fillet region of the specimen. It was seen that stress and strain were produced more when the temperature increases. There was an augmentation of stress and strain intensity in the crack tip region when the crack depth increases. Hence, crack depth and applied thermal influence the stress and strain intensity.

6.4 Impact of temperature on modal parameters in without seeded crack

The natural frequency of the beam is determined by impact test separately for each specimen. The impact test was carried out three times. The differences in measurement was ± 0.13 Hertz. The frequency value used in this research is obtained by taking the average frequency value. Similarly, the temperature measurement was conducted three times and the differences in measurement was ± 2 °C. The temperature value used in this research is obtained by taking the average value of the three measurements. The impact test response to determine natural frequency for healthy specimen is shown in Figure 6-9 and modal amplitude in Figure 6-10. The temperature along the beam was heated at different temperature rates to investigate the dynamics response accordingly. The comparisons of experimental results with the numerical and analytical results are presented. Figure 6-11 presents the comparison of natural frequency for different heating rates for the specific temperature. The graph suggests that the beam natural frequency was decreased with an increase in temperature. This implies temperature on the beam surface deteriorated the beam stiffness to a certain level. Moreover, ramping of temperature at different rates had some effect on the beam natural frequency.

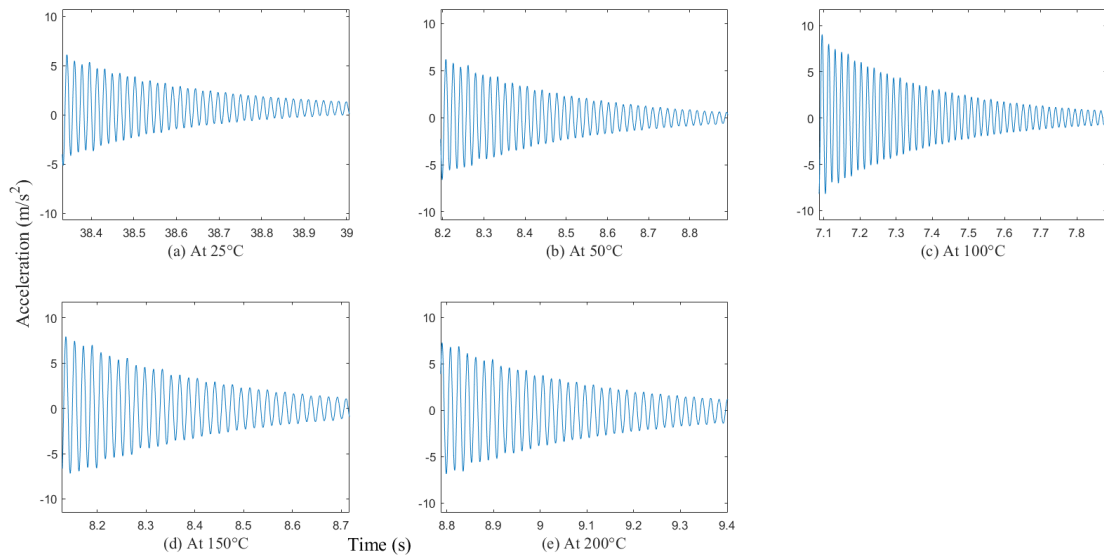


Figure 6-9 Determined the natural frequency of healthy specimen

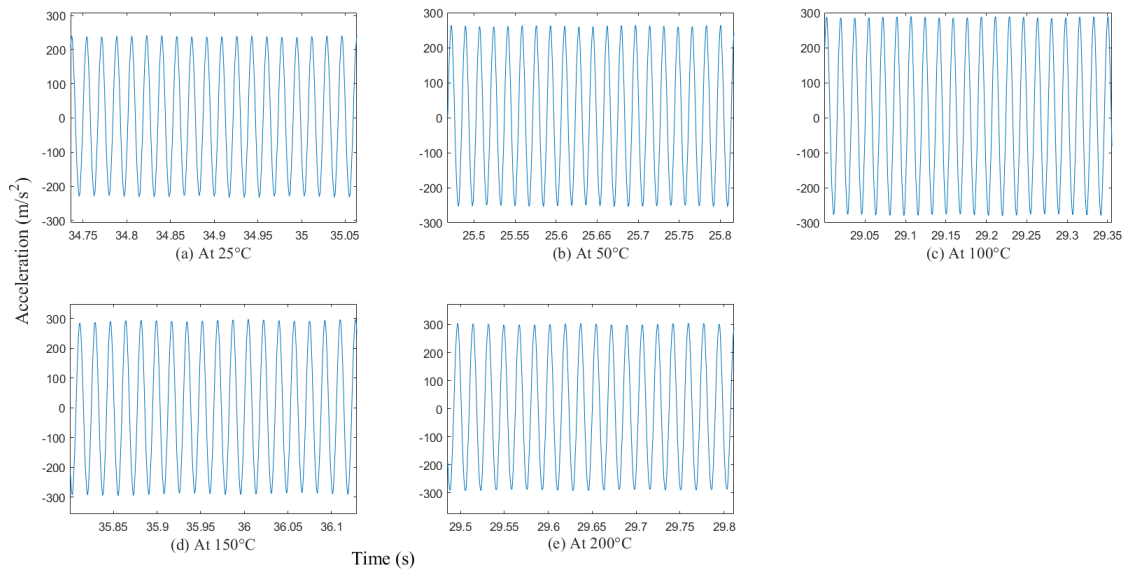


Figure 6-10 Determined the modal acceleration amplitude of healthy specimen

Ramping of temperature at the rate of 2°C/min had more impact on natural frequency by diminishing its frequency. However, ramping at 5°C/min and 8°C/min had little influence compared to ramping at 2°C/min. This effect of ramping temperature was more significant for higher temperatures as shown in Figure 6-11. This means that the heating at 2°C/min took more time to reach the desired temperature as shown in Figure 6-7, thereby accumulating the heat on the beam for a longer duration might have softened the material properties.

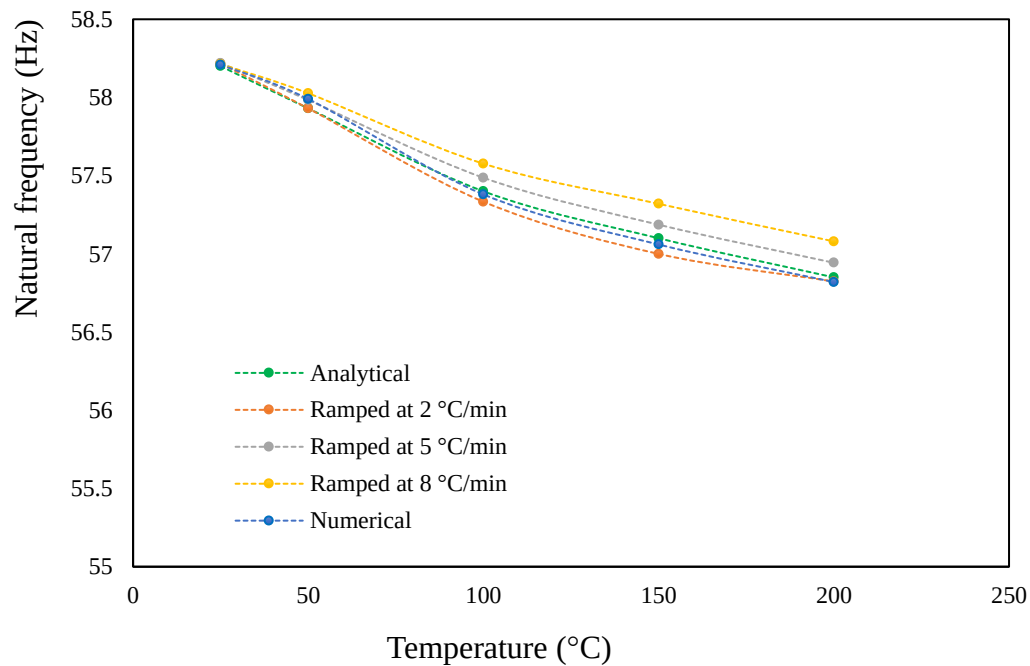


Figure 6-11 Natural frequency for uncracked beam

This could be explained as changes in interatomic bond length especially at higher temperatures as suggested in [185]. Nevertheless, the experimental results of 2°C/min cases are very close to the numerical and analytical results for all the temperature values. The difference in frequency for the different ramping and numerical approaches are presented in Table 6-2. Figure 6-12 illustrates that the amplitude decayed as the temperature increased. The drop of amplitude was associated with beam fundamental frequency. Therefore, the modal amplitude displayed a heating effect similar to the fundamental frequency. The heating rate at 2°C/min showed lower vibration amplitude as compared to the others. This indicates structures went through a more elastic behaviour at higher heating rates. This influence of heating rate was quite visible at high-temperature values. The difference of modal amplitude for different ramping and numerical with the analytical is presented in Table 6-3.

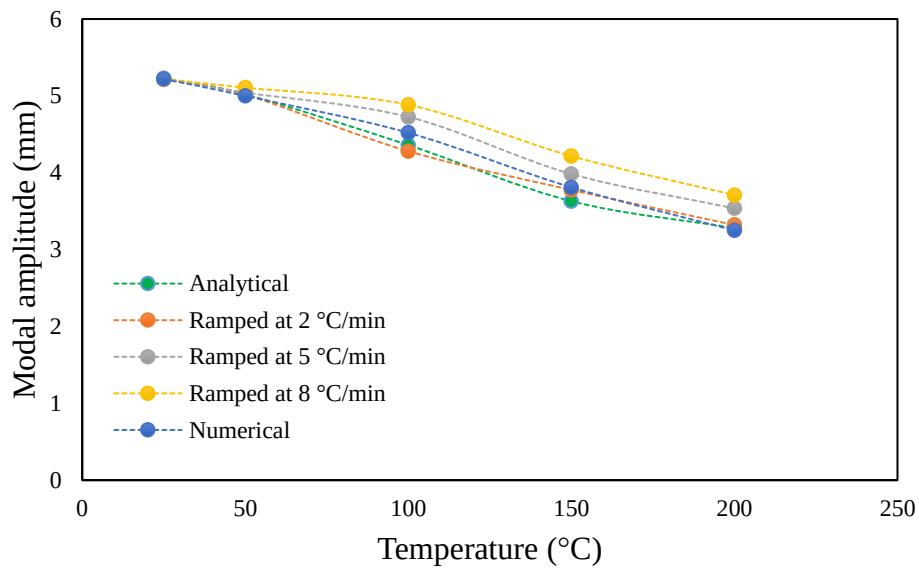


Figure 6-12 Modal amplitude for uncracked beam

Table 6-2 Difference of natural frequency in undamaged under uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
25	0.017	0.020	0.013	0.010
50	0.000	0.053	0.097	0.060
100	-0.067	0.087	0.177	-0.020
150	-0.100	0.087	0.220	-0.040
200	-0.027	0.093	0.230	-0.030

Table 6-3 Difference of amplitude in undamaged under uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
25	-0.017	-0.013	-0.010	-0.010
50	-0.003	0.023	0.087	-0.020
100	-0.080	0.367	0.523	0.160
150	0.147	0.353	0.587	0.180
200	0.040	0.257	0.430	-0.030

6.5 Impact of temperature on modal parameters in seeded crack

The impact of temperature for three crack depths 0.25mm, 0.50mm, and 1mm were chosen for this assessment. The experimental procedures are the same for all the crack specimens as those without cracks. The natural frequency for 0.25mm, 0.5mm and 1mm crack depths are shown in Figure 6-13, Figure 6-14, and Figure 6-15 respectively. All the graphs show a similar trend of decreasing frequency with the increase in temperature. Moreover, the trend of decaying is lesser at lower temperatures while deep trends were observed at higher temperatures. This suggested that the material properties were deteriorated at higher temperatures, meaning less elastic. These changes in the material properties were associated with the stiffness of the material at different temperatures.

The rate of heating the specimens had little impact on the fundamental frequency. This research aims to verify this by ramping at 2°C/min, 5°C/min, and 8°C/min for all three crack depths. Thus, the effect of heating rates was compared for all the specimens as shown in the respective figures. It is seen that the analytical and numerical results were comparable to the ramped at 2°C/min. The frequency difference in 0.25mm from the analytical is presented in Table 6-4. The frequency difference in 0.5mm and 1mm from the analytical is presented in Appendix A. Though, ramping effect is minimal at lower temperatures and more noticeable at higher temperatures. Therefore, this signifies that the rate of heating can make a slight effect on the dynamic response of any mechanical system.

Table 6-4 Difference of natural frequency in 0.25mm under uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
25	-0.053	-0.030	-0.017	0.059
50	-0.010	0.013	0.057	-0.008
100	0.083	0.210	0.247	-0.029
150	-0.117	0.130	0.233	-0.109
200	-0.067	0.253	0.363	-0.090

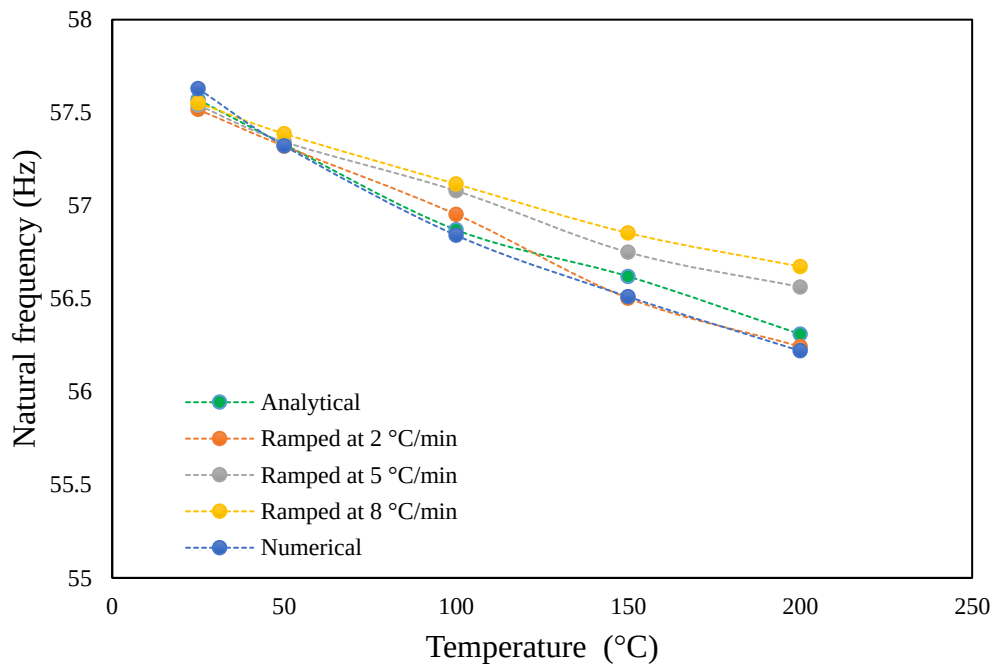


Figure 6-13 Natural frequency for 0.25mm crack depth

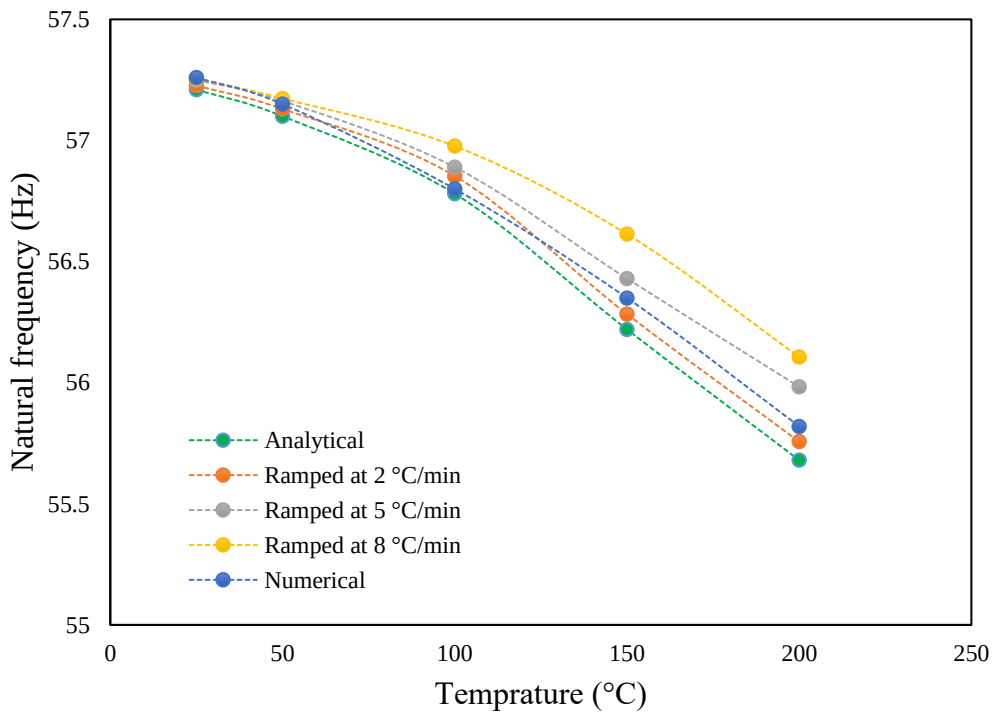


Figure 6-14 Natural frequency for 0.50mm crack depth

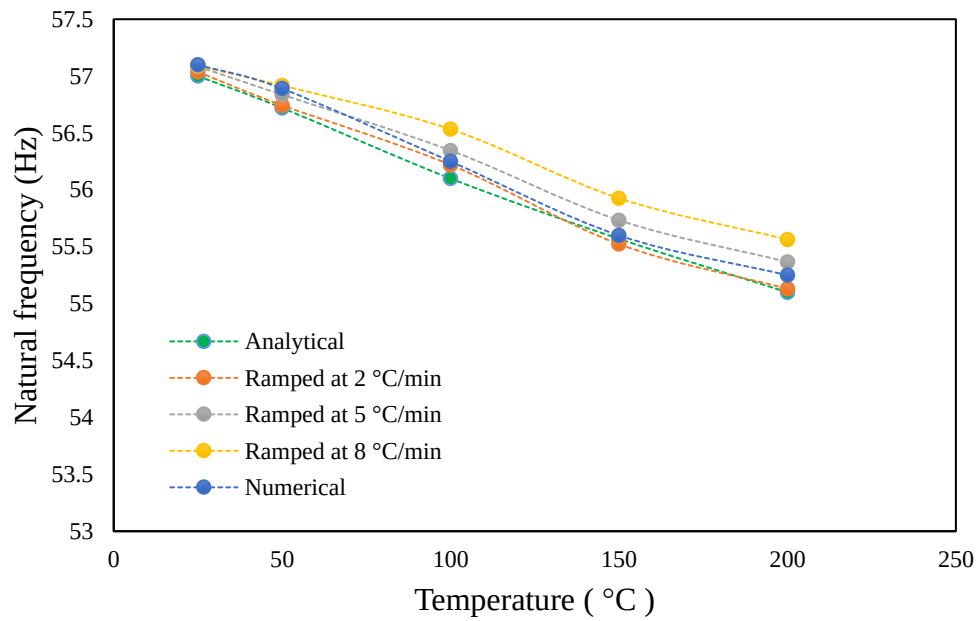


Figure 6-15 Natural frequency for 1mm crack beam

The effect of temperature on modal amplitude is plotted for all the crack depths. The modal behaviour of 0.25mm, 0.5mm and 1mm crack depths are shown in Figure 6-16, Figure 6-17, and Figure 6-18 respectively. It was noticed that modal amplitude decreases with the increase in crack depth. The decrease in amplitude was also associated with a temperature rise.

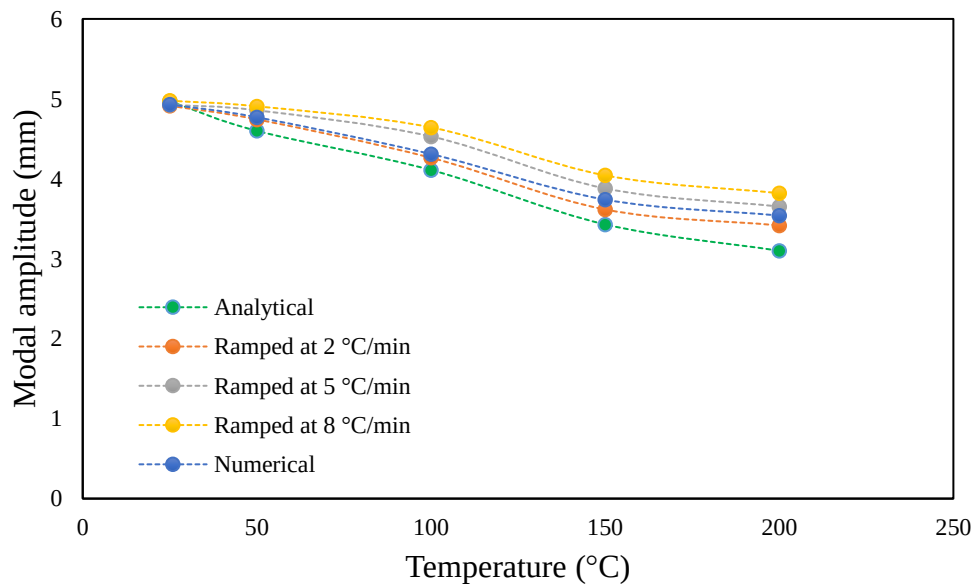


Figure 6-16 Modal amplitude for 0.25mm crack depth

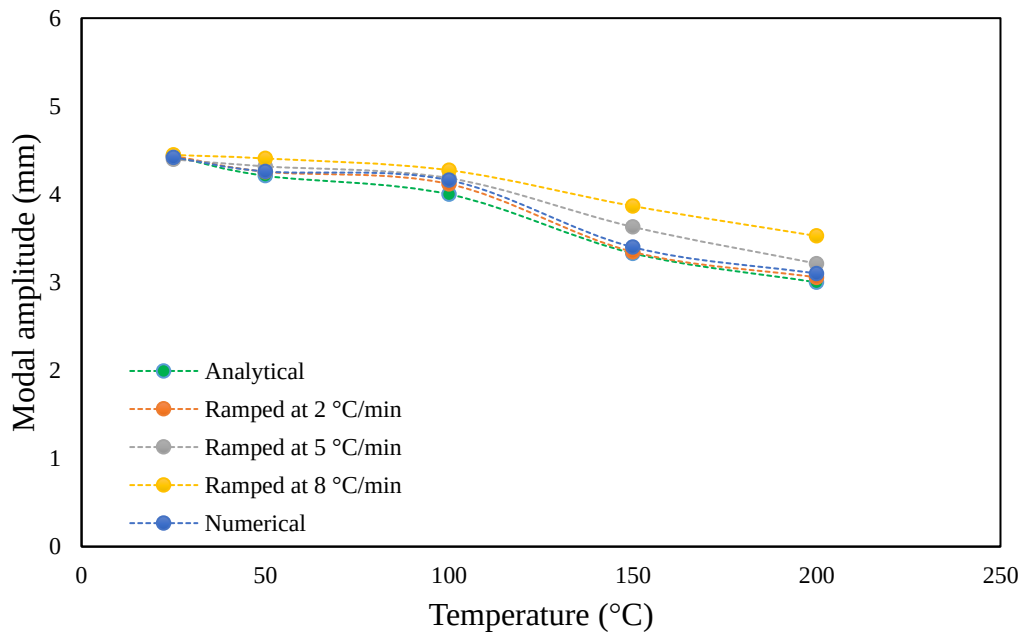


Figure 6-17 Modal amplitude for 0.50mm crack depth

Comparatively, the declining trend of amplitude behaviour for 1mm crack depth was more linear than 0.25mm and 0.5mm crack depth. This implied the increase in crack depth reduced the beam stiffness. Thus, this evidence can be utilized for determining crack depth in structuring health monitoring. It was observed vibration amplitude was different on all heating rates. The comparison of modal amplitude for different crack depth at different temperatures is presented in Table 6-5. The effect of heating rate on amplitude is obvious for higher temperatures. The vibration behaviour of 0.25mm crack depth was observed linear till 100°C. A nonlinear response for all higher temperature ranges was also observed. Similarly, 0.5mm crack depth showed a small drop in vibrating amplitude from 25°C to 100°C, and then it dropped abruptly. This implied the specimen loses its stiffness greatly when the temperature goes above 100°C. Moreover, 1mm crack depth showed a linear response at all values of temperatures. Thus, 1mm crack depth was a special case where its stiffness played a role in vibrating behaviour and less with the temperature as shown in Figure 6-18.

Table 6-5 Comparison of modal amplitude for different crack depth at different temperatures

0.25 mm crack depth						0.5 mm crack depth					1 mm crack depth				
Temp	Analytical	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical	Analytical	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical	Analytical	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
25	4.97	4.91	4.93	4.98	4.93	4.43	4.44	4.40	4.45	4.42	3.72	3.72	3.79	3.80	3.77
50	4.60	4.74	4.86	4.91	4.77	4.21	4.26	4.32	4.41	4.26	3.51	3.55	3.68	3.69	3.59
100	4.11	4.26	4.53	4.64	4.31	4.00	4.12	4.18	4.27	4.16	3.23	3.17	3.41	3.53	3.33
150	3.43	3.62	3.88	4.04	3.74	3.33	3.35	3.63	3.87	3.4	3.00	2.92	3.21	3.38	3.16
200	3.10	3.42	3.65	3.82	3.54	3.00	3.06	3.21	3.53	3.1	2.76	2.65	2.92	3.11	2.89

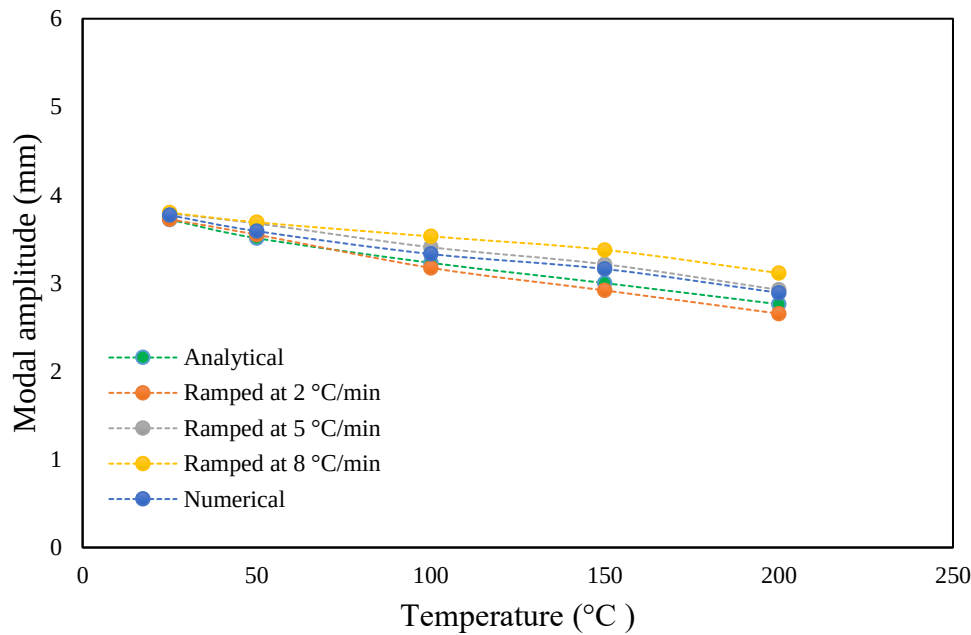


Figure 6-18 Modal amplitude for 1mm crack depth

And heating at 2°C/min, 5°C/min and 8°C/min appeared similar response but heating at 5°C/min and 8°C/min are small in amplitude drop compared to heating at 2°C/min. This indicated ramping at 2°C/min has accumulated more heat on the beam as it takes a longer duration to heat the required temperature and through this process softens the material properties. Nevertheless, ramped at 2°C/min is in good agreement with the analytical and numerical results. This meant heating up at a lower rate is more accurate for theoretical and experimental vibration testing. The amplitude difference for different heating rates from the analytical model is presented in Appendix B.

6.6 Modal behaviour of a beam under non-uniform temperature

This research work studies the modal behaviour of the cantilever beam when subjected to thermal and mechanical loads. The evaluation of the modal parameters was implemented in uniform and non-uniform temperature distribution conditions. The modal behaviour of the beam was examined at three heating rates, as discussed above. The modal characteristic of the beam in uniform temperature was presented previously. This section analysed the beam response under non-uniform temperature conditions. The natural

frequencies of the healthy and crack specimens were evaluated. The decrease in modal parameters with the temperature rise was seen in the case of non-uniform temperature. The decreasing trend of fundamental frequency for the healthy specimen is shown in Figure 6-19. The natural frequencies of crack specimens with crack depths of 0.25, 0.5, and 1 mm are presented in Figure 6-20, Figure 6-21, and Figure 6-22 respectively. It was seen that heating rates impacted the modal frequencies, as depicted in the figures. Moreover, this heating effect was negligible at low temperatures but considerable at high temperatures. The change in fundamental frequencies for crack depths was also visible. The frequencies decrease when the crack depth increases.

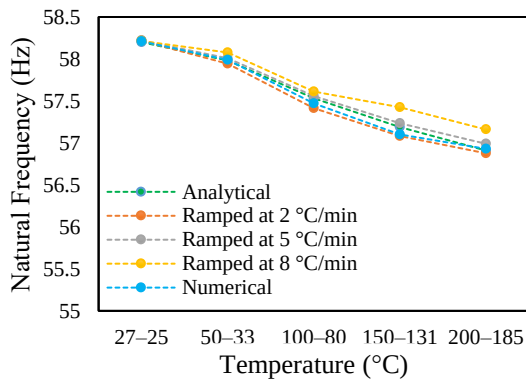


Figure 6-19 Natural frequency of healthy specimen in non-uniform temperature

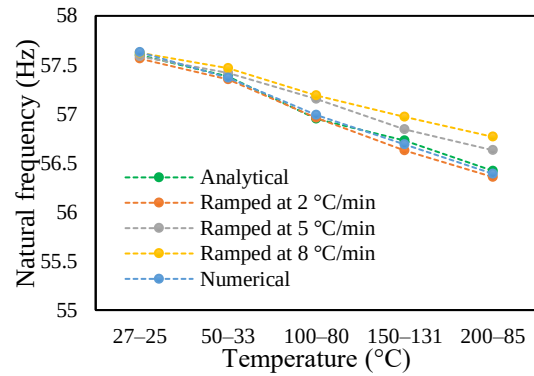


Figure 6-20 Natural frequency of 0.25 mm crack depth in non-uniform temperature

This dropped in frequencies was steadier at low temperatures and crack depths. However, this tends to progress rapidly when the crack depth and temperature rise. However, this frequency behaviour is limited, as it becomes steadier once it reaches certain crack depths, as represented in Figure 6-21 and Figure 6-22. The modal amplitudes of the beam when subjected to thermal and mechanical loads for uncracked samples and samples with crack depths of 0.25, 0.5, and 1 mm are presented in Figure 6-23, Figure 6-24, Figure 6-25, and Figure 6-26 respectively.

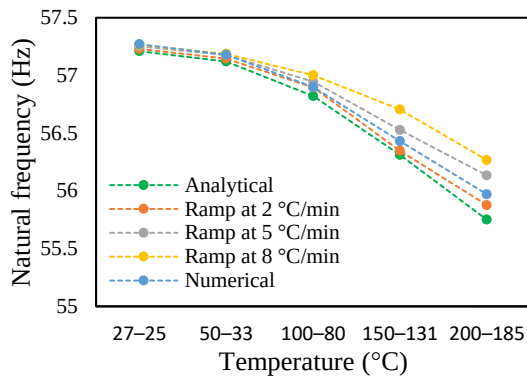


Figure 6-21 Natural frequency of 0.5 mm crack depth in non-uniform temperature

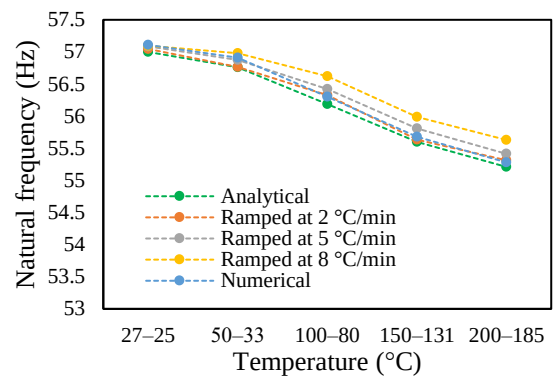


Figure 6-22 Natural frequency of 1 mm crack depth in non-uniform temperature

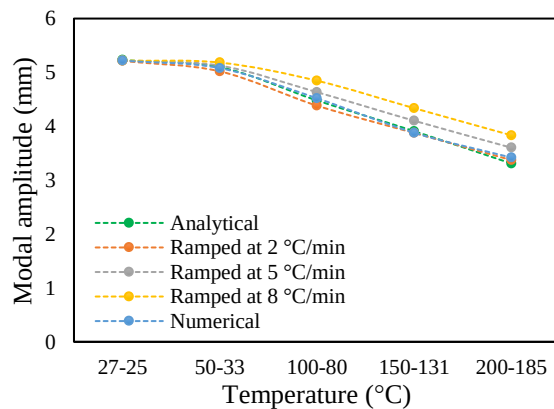


Figure 6-23 Modal amplitude of healthy specimen in non-uniform temperature

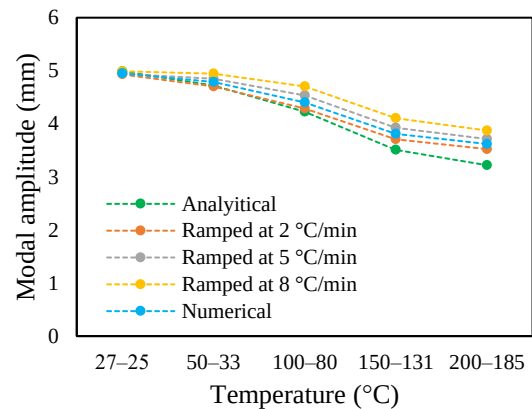


Figure 6-24 Modal amplitude of 0.25mm crack depth in non-uniform temperature

The amplitude drop corresponded to the frequencies. It decreases with the decrease in frequencies as the temperatures rise. Similarly, the heating effect affected it. The ramp in temperature of 2 °C/min showed the lowest amplitude compared to the ramp of 5 °C/min and 8 °C/min. This signifies that heating at a lower rate accommodates more heat into the beam, and thereby the beam becomes less stiff. However, this effect was insignificant at low temperatures and was more considerable at high temperatures.

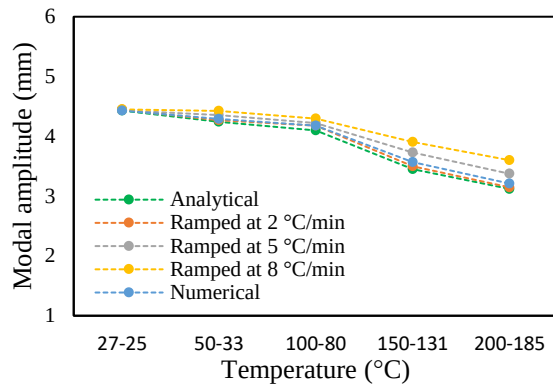


Figure 6-25 Modal amplitude of 0.5mm crack depth in non-uniform temperature

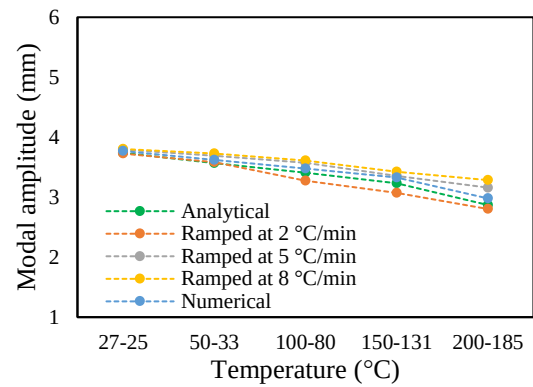


Figure 6-26 Modal amplitude of 1mm crack depth in non-uniform temperature

6.7 Comparison of modal behaviour of a beam under uniform and non-uniform temperature

In this study, the modal parameters of the cantilever beam were compared for both uniform and non-uniform temperatures. The results show that the modal parameters vary for both the temperatures conditions. It was noticed that the modal parameters were higher under non-uniform temperatures than under uniform temperatures. Even though there was a small variation in their responses, it was still noticeable. This small difference is due to the small temperature ranges. Their dissimilarities were minimal at low temperatures and visible at higher temperatures. The comparison of the fundamental frequencies for healthy and various crack depths specimens is shown in Figure 6-27 to Figure 6-30.

Similarly, the modal amplitude of the cantilever beam varies for both temperatures. The results showed that the amplitude differences were obvious only at high temperatures. The variations in the amplitude are due to the differences in the frequencies at the respective temperature range. The modal amplitude of uncracked and cracked beams is presented in Figure 6-31 to Figure 6-34. The modal parameters of the cantilever beam were examined with the changes in thermal properties due to the changes in temperatures.

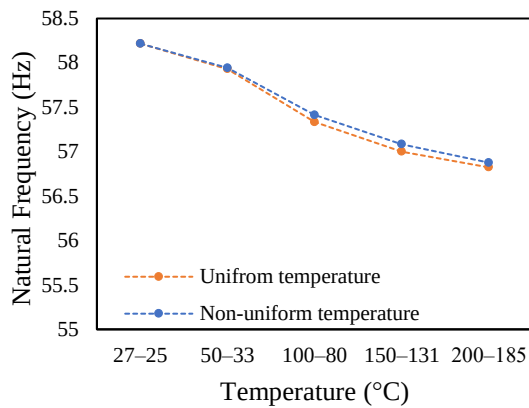


Figure 6-27 Comparison of the natural frequency of undamaged specimen ramped at 2 °C/min

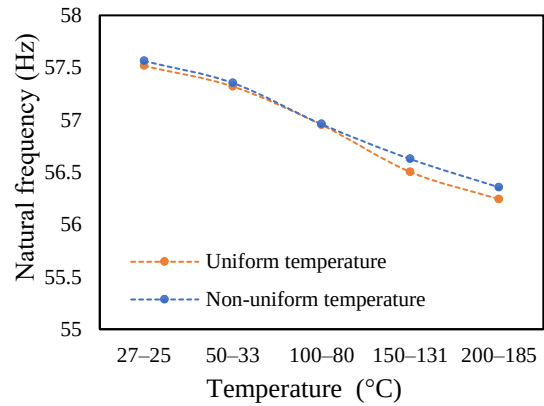


Figure 6-28 Comparison of the natural frequency of 0.25 mm crack depth ramped at 2 °C/min

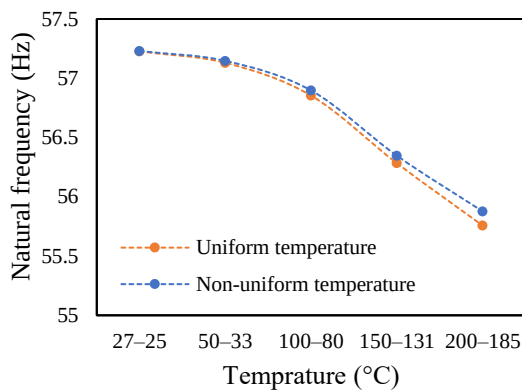


Figure 6-29 Comparison of the natural frequency of 0.5 mm crack depth ramped at 2 °C/min

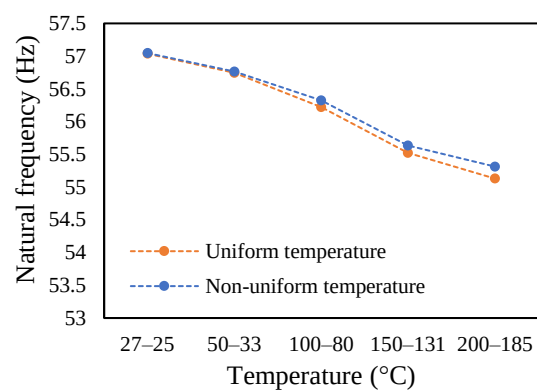


Figure 6-30 Comparison of the natural frequency of 1 mm crack depth ramped at 2 °C/min

The illustration of the thermal properties concerning temperatures is shown in Figure 6-35. As is seen from the graph, the elastic modulus decreases with an increase in temperature, while the coefficient of thermal expansion increases linearly with the elevation in temperature. The corresponding changes in natural frequencies with the changes in storage modulus are presented in Figure 6-36. It is seen that the fundamental frequencies of the cantilever beam are related to the changes in the modulus under given temperatures. However, the beam frequencies and thermal expansion are contrasting. The beam response decreases while thermal expansion increases when the temperatures rise, as

shown in Figure 6-37. Thus, the modal characteristic of the cantilever beam is dependent on the nature of the changes in thermal properties concerning temperatures.

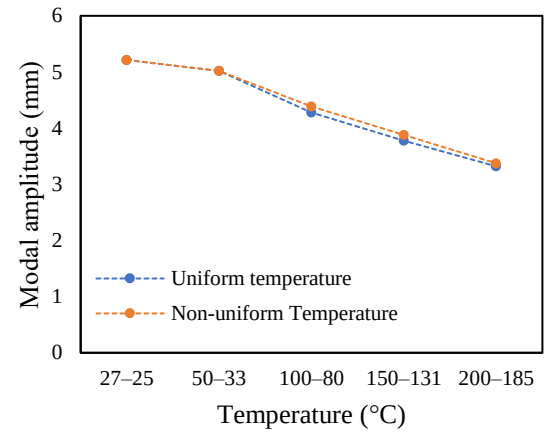


Figure 6-31 Comparison of modal amplitude of undamaged specimen ramped at 2 °C/min

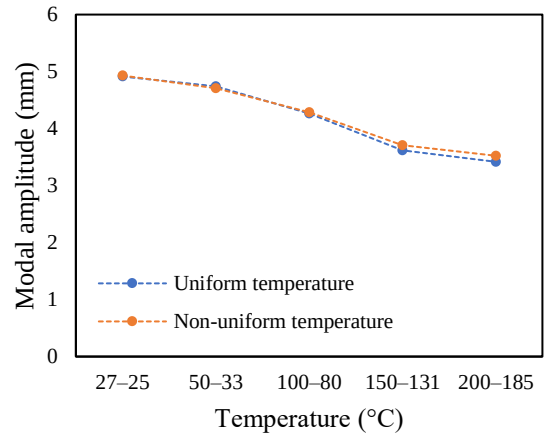


Figure 6-32 Comparison of modal amplitude of 0.25 mm crack depth ramped at 2 °C/min

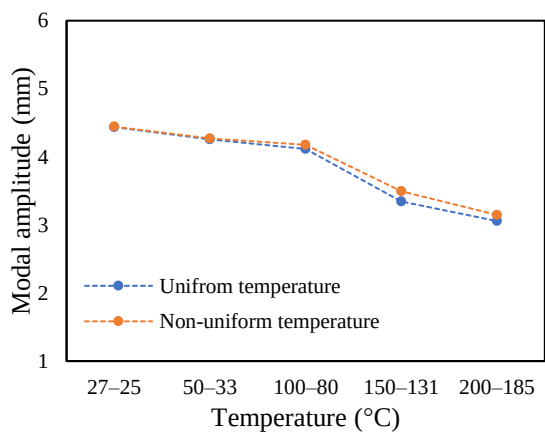


Figure 6-33 Comparison of modal amplitude of 0.5 mm crack depth ramped at 2 °C/min

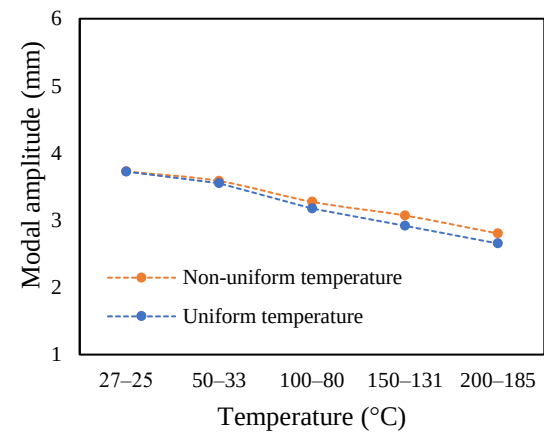


Figure 6-34 Comparison of modal amplitude of 1 mm crack depth ramped at 2 °C/min

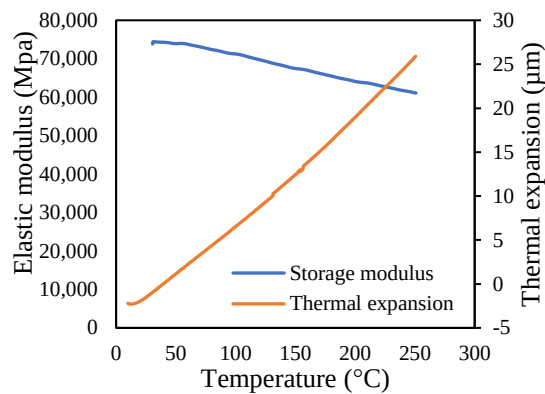


Figure 6-35 Thermal properties at different temperatures

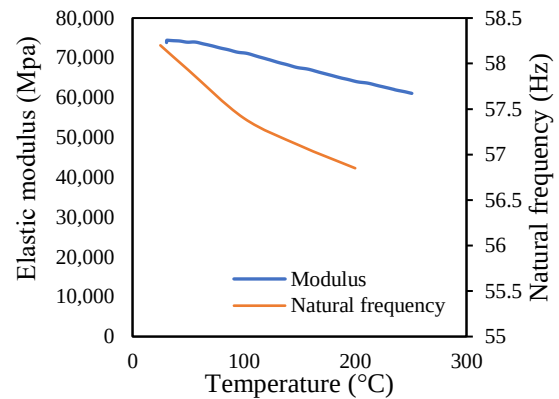


Figure 6-36 Modulus and frequency at different temperatures

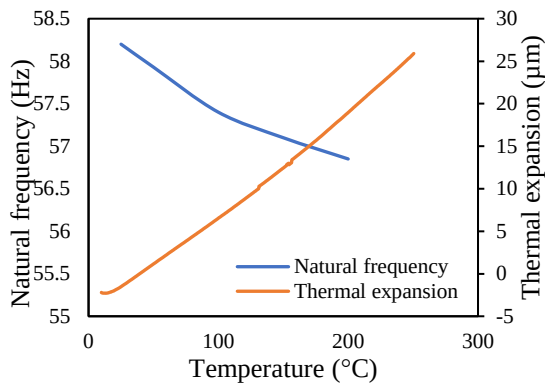


Figure 6-37 Thermal expansion and modal frequency at different temperatures

6.8 Conclusion

The influence of temperature on structural response is an issue in engineering applications. This study looked at the modal characteristics of a cantilever beam under uniform and non-uniform temperature circumstances. The findings revealed that the applied temperature and heating rate had an effect on the overall structural stability. The experiment was conducted for three different category as slow heating at 2 °C/min, moderate heating at 5 °C/min, and fast heating at 8 °C/min to analyse the modal response for both with crack and without crack specimens. Different heating rates were found to have slight effect on modal characteristics. The effect of heating rates on dynamic

responsiveness is mild at low temperatures but considerable at high temperatures. The modal parameters of different heating rates were compared to the analytical and numerical results. It was found that the results of heating at 2 °C/min appeared close to the analytical and numerical results. This signifies heating at a lower rate is more accurate for any experimental structural analysis. This also revealed that heating at different rates can have a slight effect during the evaluation of modal parameters of structure. The modal frequency and amplitude of the cantilever beam decrease as temperature rises in both crack and non-crack. This is because increase in temperature decrease the elastic modulus. Variations in the modal characteristics of the cantilever beam were found to be related to changes in applied temperatures and heating rate. However, slow heating takes longer to attain the requisite temperatures and have the potential to make the beam less rigid, especially at higher temperatures.

7 FATIGUE CRACK PROPAGATION BEHAVIOUR UNDER THERMAL AND MECHANICAL LOADS

In the previous chapter, it is seen that structure exposure to temperatures modified the material properties and impact structural dynamics. This modification influences the overall deformation of the structure. The changes in temperature influence stress concentration and increase in temperature reduced the global stiffness. The effect of temperature distribution on structure relating to crack propagation and its modal properties is still to address. Crack propagation analysis is one of the fundamental research areas in structural dynamics and fracture mechanics. It is because the appearance of crack on structure changes the structural response. The crack initiation occurs due to cyclic loads, overloading, operating conditions, etc. Moreover, the crack growth may not be the same for all materials. In this thesis, the crack propagation behaviour of aluminium 2024-T3 is studied under uniform and non-uniform temperature distribution. In this study, different pre-seeded crack depths were selected to analyse the crack propagation. The pre-seeded rectangular crack shape has opted for all the experimental specimens. This crack propagation analysis used the analytical model which was discussed earlier to measure the modal parameters. The corresponding structural responses for different crack depths are measured during crack propagation.

7.1 Selection of pre-seeded crack specimen

In this study, different types of pre-seeded crack specimens were selected to analyse crack propagation. Crack depths of 0.25, 0.5, and 1 mm were considered for the uniform temperature distribution between 25 and 200 °C, while crack depths of 1, 1.3, and 1.5 mm were preferred for the non-uniform temperature distribution. The differences in crack depth for both temperature types were considered, because lower crack depth takes a longer time to propagate, which becomes a problem regarding maintaining the non-uniform temperature. Therefore, greater crack depths were considered under non-uniform analysis.

7.2 Crack depth measurement method

Crack depth measurement means determining the crack depth during crack propagation. In this research, the propagated crack depth was captured using a Dino-Lite Digital Microscope as shown in Figure 7-1. Then, crack depths were determined based on pixels contained in the picture captured from the specimen. The process of analysing the number of pixels in the crack specimen is processed using Microsoft paint. The first step is to take the picture of the crack specimen placing a ruler on it. The purpose of placing a ruler is to determine the number of pixels contained in 1 mm. So, 'Scaled pixel' is the number of pixels contained in a 1 mm ruler. The next step is to determine the total pixel contained in the thickness of the specimen. It can be denoted that 'Total pixel' is the total number of pixels contained in the specimen thickness. Therefore, the total number of pixels contained in the 1mm can be found if the specimen thickness is known. The last step is to find the crack depth pixel. The crack depth pixel is the number of pixels present in the crack. Hence, crack depth can be found by using the number of pixels contained in the crack. These procedures for measuring crack depth were repeated until total damage was achieved.

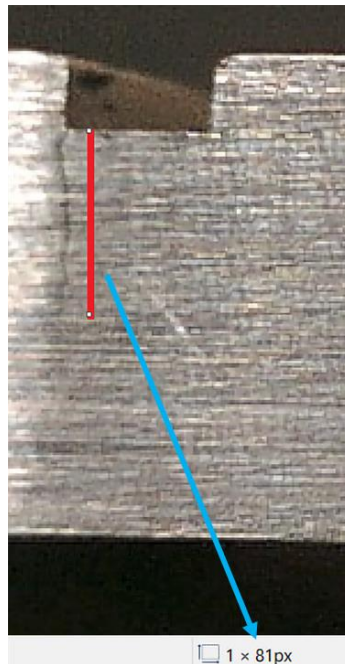


Figure 7-1 Counting the pixel in Microsoft paint.

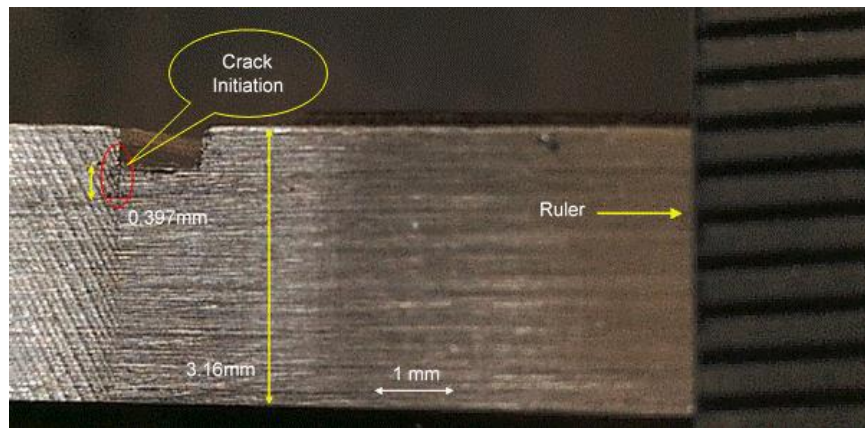
7.3 Fatigue crack propagation evaluation

Fatigue crack propagation evaluation was performed for various pre-seeded crack depths. The analysis was performed separately for uniform temperatures and non-uniform temperatures. The dynamic response of the beam was recorded during the crack propagation. The crack depth measurement and its modal parameters for a 0.25 mm pre-seeded crack depth subjected to isothermal and mechanical loads are given in Table 7-1. 'Total pixel' is the total number of pixels contains in the specimen thickness. 'Scaled pixel' is the number of pixels contained in a 1 mm ruler. The crack depth pixel is the number of pixels present in the crack. The crack depth is found by using the number of pixels contained in the crack. This procedure for measuring crack depth was repeated until total damage was achieved. Several experiments were conducted for the same condition to verify repeatability. This method of fatigue crack propagation evaluation was conducted in all specimens and is given in Appendix C. It was observed that crack propagation was different for different temperatures. The corresponding natural frequency and amplitude were recorded during the crack propagation. It showed that the experimental results were comparable to the analytical results, although there was a small variation. The results of crack propagation analysis show that a constant crack depth cannot be obtained for different temperatures. The propagation was not identical. The appearance of the first crack consumed much of the time in the whole propagation leading to failure. Additionally, the crack initiation was dependent on the pre-seeded crack depth. It was noticed that a deeper crack depth tends to appear to initiate cracking easily. Once the crack appeared, it was easier to accelerate the crack growth. It observed that crack propagations were irregular, and they varied according to applied temperatures and pre-seeded cracks. The propagated crack depth measurement and its sequence for a pre-seeded crack depth of 0.5 mm at 25 °C are shown in Figure 7-2. The others propagated crack depth measurement and its sequence was given in Appendix D. It was observed that crack initiation was quicker at lower temperatures than at elevated temperatures. This is due to the plasticity at lower temperatures and more ductility at higher temperatures. It was also

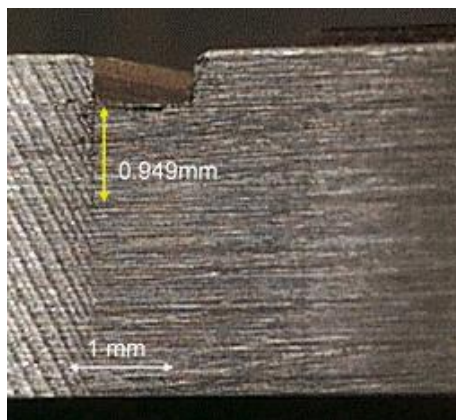
Table 7-1 Crack propagation at an isothermal temperature of 50 °C for pre-seeded 0.25 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.4047	26	203	64	64.2405	0.0156	16.3	16.23	18.11	18.13
0.8786	57	205	65	64.8734	0.0154	15.11	14.97	17.93	17.85
0.9511	62	206	65	65.1899	0.0153	14.74	14.62	17.26	17.14
1.6874	110	206	65	65.1899	0.0153	12.81	12.65	15.38	15.21
2.1996	142	204	65	64.5570	0.0155	10.73	10.61	11.81	11.69
2.6667	173	205	64	64.8734	0.0154	6.71	6.68	5.16	5.13
2.8812	186	204	65	64.5570	0.0155	5.21	5.1	4.1	3.97

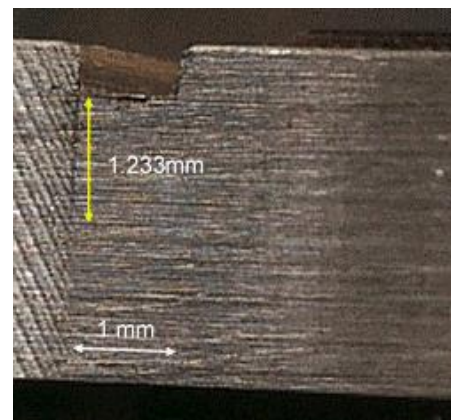
noticed that sometimes, a propagated crack develops multiple crack fronts within the pre-seeded crack. This is more frequent during the lower-temperature experiments. Moreover, the fatigue crack propagation was quicker when there was a single crack in the same direction. However, the propagation appeared sluggish whenever there was a dual crack front from the opposite end. Nevertheless, the speed of fatigue crack propagation was dependent on applied temperatures, attached end mass, and its alignment.



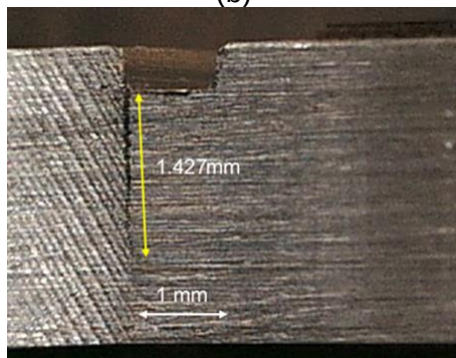
(a)



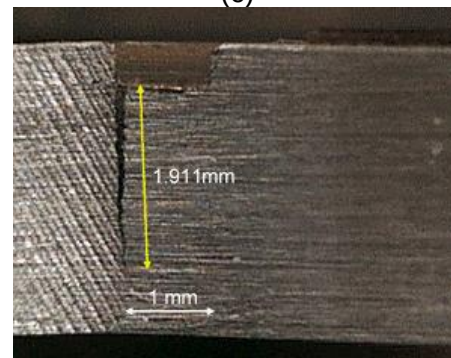
(b)



(c)



(d)



(e)

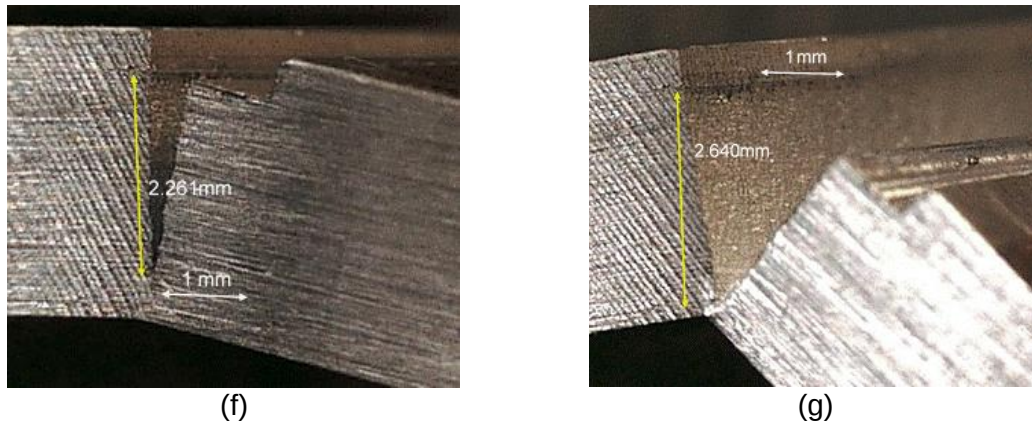


Figure 7-2 Sequence of crack propagation of pre-seeded 0.5 mm crack depth at room temperature (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture, (f) 6th capture, (g) 7th capture

The evaluation of fatigue crack propagation at different temperatures showed a non-uniform crack pattern. This seems to be due to the selected shape of the crack. The occurrence of the first crack from the pre-seeded crack was not identical, even under the same boundary conditions. In this research, a rectangular-shaped pre-seeded crack was considered, which sometimes generated double crack front, as shown in Figure 7-3. (a) showed the top view of the propagated double crack (b) showed the bottom view of the same double crack, (c) is the front view and (d) showed the final breaking of the double crack. This propagation accelerates at lower temperatures than at elevated temperatures. This happened due to the growth of plasticity at the crack tip. Therefore, the modal parameters of the beam were not identical in the case of propagated crack. The actual crack and pre-seeded crack might be the problem. Hence, crack propagation evaluation on a pre-seeded crack, especially a rectangular crack, was not viable. This suggested crack propagation analysis should be based on a real crack, or something resembling to it.

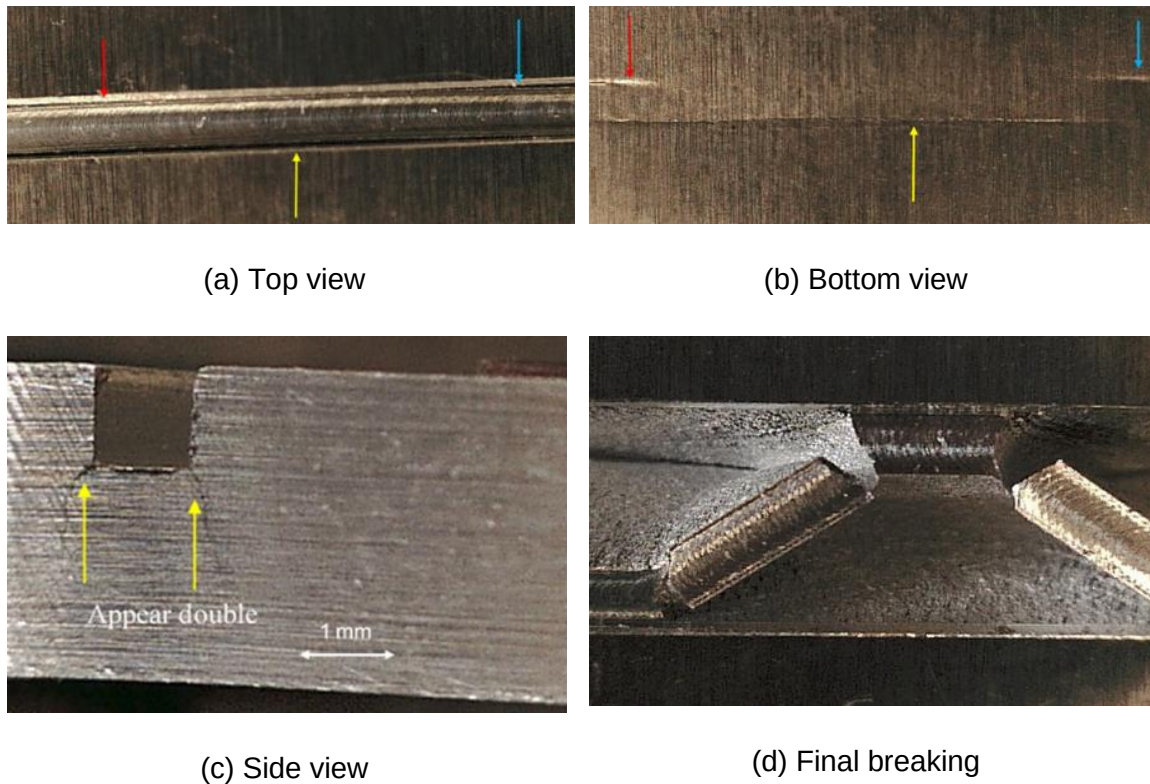
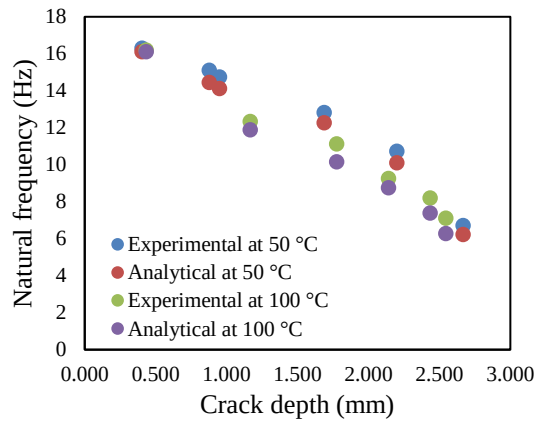


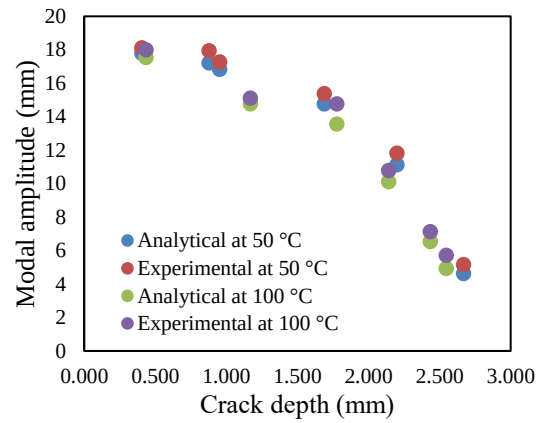
Figure 7-3 Double crack appearance in pre-seeded 1mm crack depth

7.4 Modal behaviour of beam during fatigue crack propagation

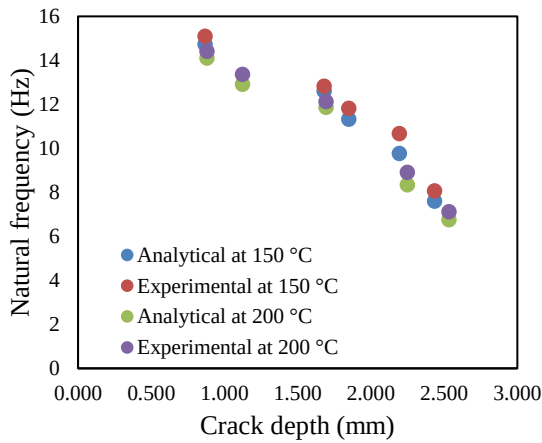
The modal behaviour of the beam was studied during fatigue crack propagation. This study observed that crack propagations were irregular, and they varied due to the applied temperatures and selected cracks depths. The modal amplitudes are dependent on their fundamental frequencies; therefore, they vary with the change in frequency. In this modal analysis, the modal parameters during crack propagation were compared with the analytical model. It showed the analytical model is in good agreement with the experimental results. The comparison of modal parameters under non-uniform and uniform temperatures is presented below. The results show that modal parameters under non-uniform temperatures are more linear in nature than those of uniform temperatures. This suggests that the applied temperatures affect the modal parameters of the beam. Figure 7-4 illustrates a comparison of the modal parameters at isothermal temperatures for a pre-seeded crack depth of 0.25 mm.



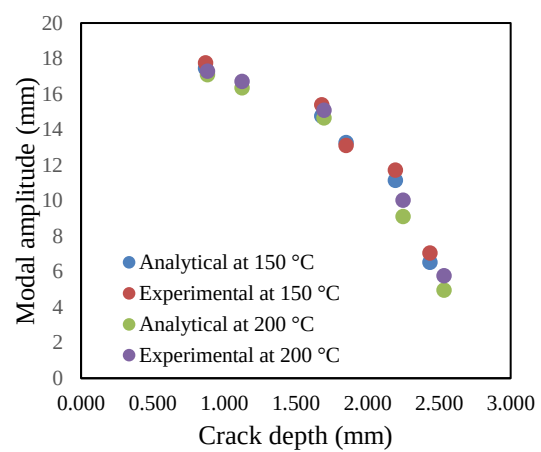
(a)



(b)



(c)



(d)

Figure 7-4 Comparison of modal parameters of a propagated crack in the pre-seeded crack depth of 0.25mm (a) 50 °C and 100 °C (b) 50 °C and 100 °C (c) 150 °C and 200 °C (d) 150 °C and 200 °C

Figure 7-4 (a) presented the natural frequency concerning the crack depth and Figure 7-4 (b) is the respective modal amplitude at the same temperature. However, Figure 7-4 (c) and Figure 7-4 (d) showed the modal behaviour at a higher temperature. It is noticed that modal amplitude is the product of its frequency. The amplitude decreases with the decrease in frequency. It is evident from this illustration that higher temperatures have a lower frequency. Furthermore, the fatigue crack propagations are not the same and the total fracture of the specimen was asymmetrical for different temperatures. Similarly,

the modal behaviour of pre-seeded crack depth of 0.5 mm exhibited a similar trend as shown in Figure 7-5, Figure 7-6, Figure 7-7, and Figure 7-8.

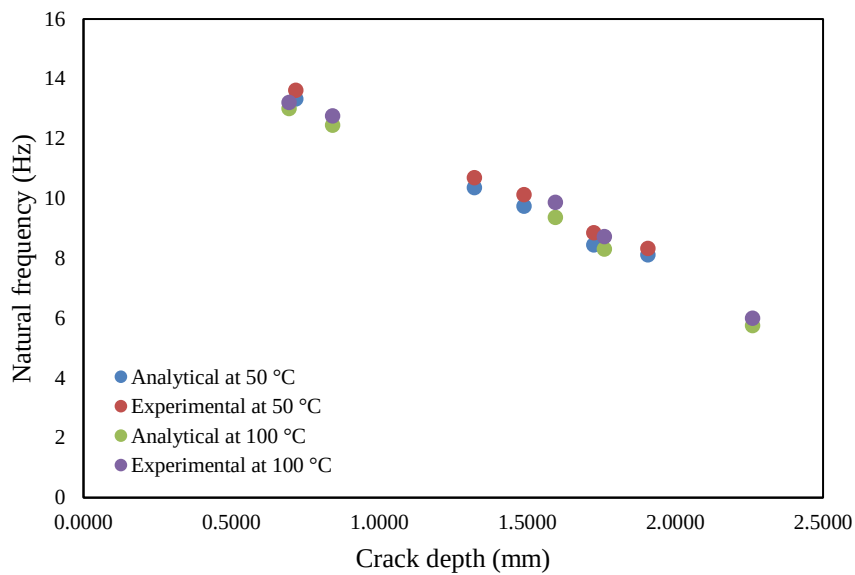


Figure 7-5 Comparison of the natural frequency of propagated crack in the pre-seeded crack depth of 0.5mm at 50 °C and 100 °C

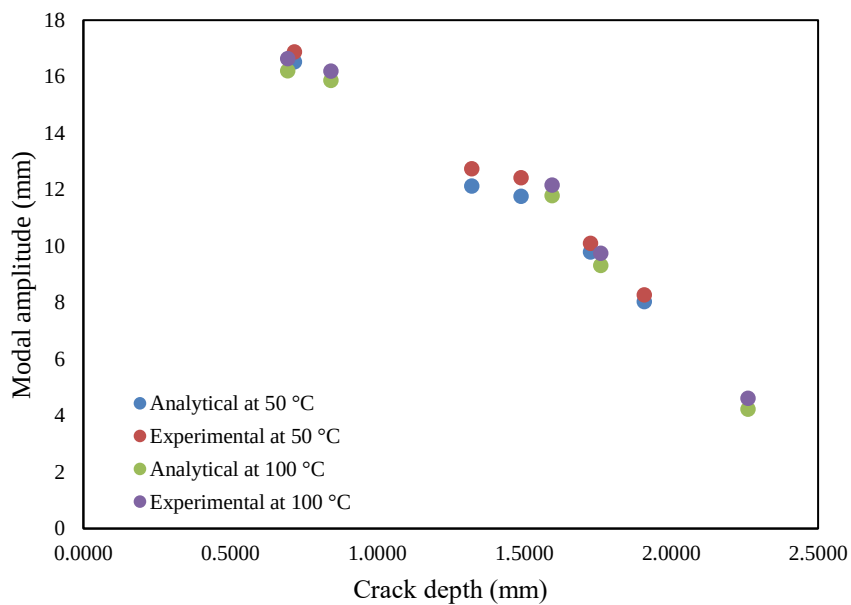


Figure 7-6 Comparison of modal amplitude of propagated crack in the pre-seeded crack depth of 0.5mm at 50 °C and 100 °C

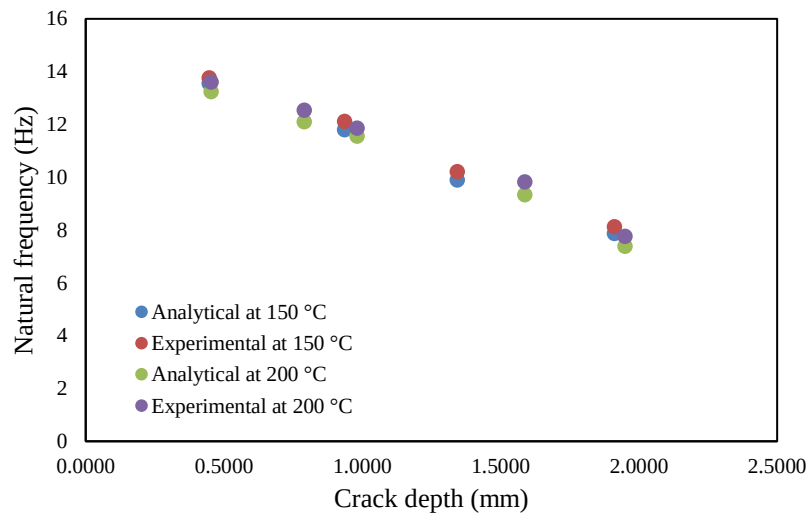


Figure 7-7 Comparison of the natural frequency of propagated crack in the pre-seeded crack depth of 0.5mm at 150 °C and 200 °C

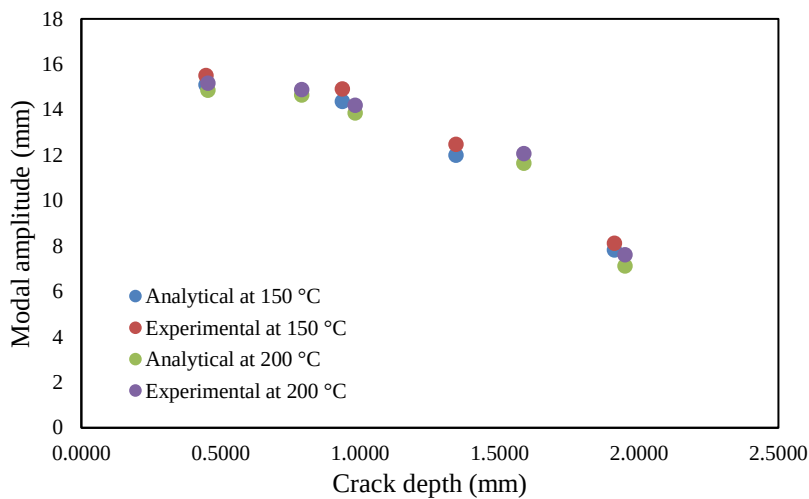


Figure 7-8 Comparison of modal amplitude of propagated crack in the pre-seeded crack depth of 0.5mm at 150 °C and 200 °C

However, the modal behaviour is more linear in nature than in 0.25mm pre-seeded crack depth. These results disclosed that crack growths are not identical but vary for different temperatures and selected crack depths. The modal behaviour of the beam under non-uniform temperatures was evaluated. Figure 7-9 showed the modal characteristics of the pre-seeded crack depth of 1 mm. The modal characteristic was compared between two types of temperature i.e., 50–85 °C and 131–150 °C. The modal responses showed a similar trend of decreasing the modal parameters. However, the crack growths are not the

same. The crack initiation appearance was easily noticeable for the higher temperatures range, but it appeared at greater crack length in the case of lower temperatures as shown in the figures. Similarly, the modal characteristics showed a similar pattern in the case of 1.3 mm pre-seeded crack depth as shown in Figure 7-10 and Figure 7-11. The crack growths exhibit a kind of linear form throughout the propagation. The crack appearance and total damage or

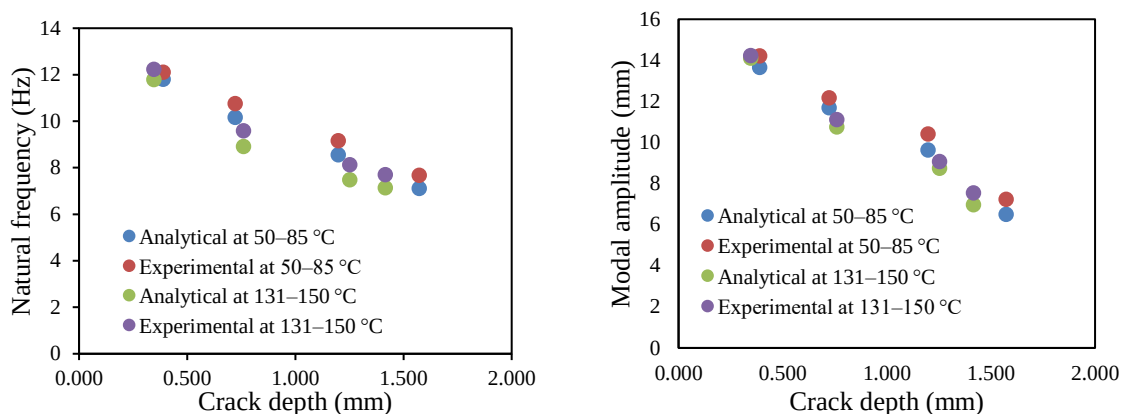


Figure 7-9 Comparison of modal parameters of propagated crack in pre-seeded crack depth of 1mm for 50–85°C and 131–150°C

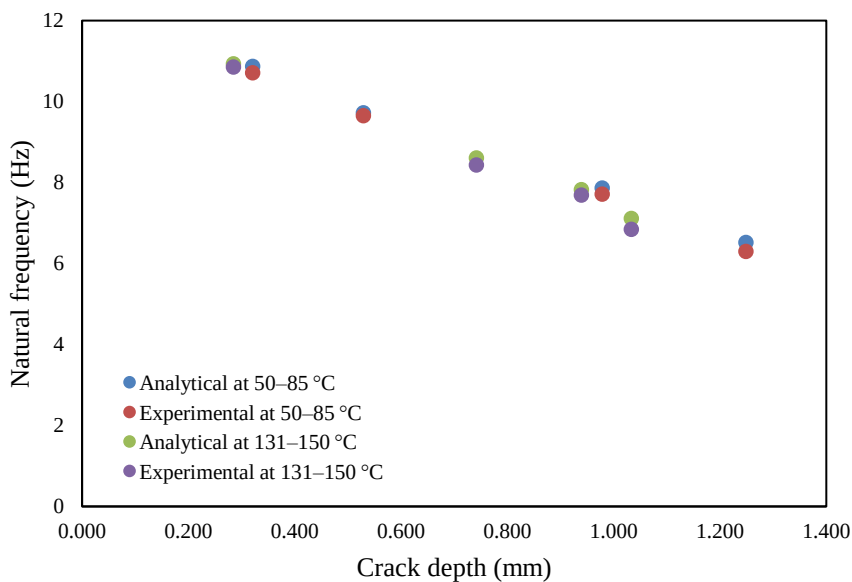


Figure 7-10 Comparison of natural frequency of propagated crack in pre-seeded crack depth of 1.3 mm at 50–85°C and 131–150°C

failures were not alike in all experiments. This seems to be due to the selected crack shape or crack depth. Moreover, the results of the modal characteristics analysis of beam during crack propagation indicate that the proposed analytical model for evaluation of modal behaviours is in good agreement with the experimental results. Therefore, the model is validated for characterising the modal behaviour of the beam during crack growth at different temperatures.

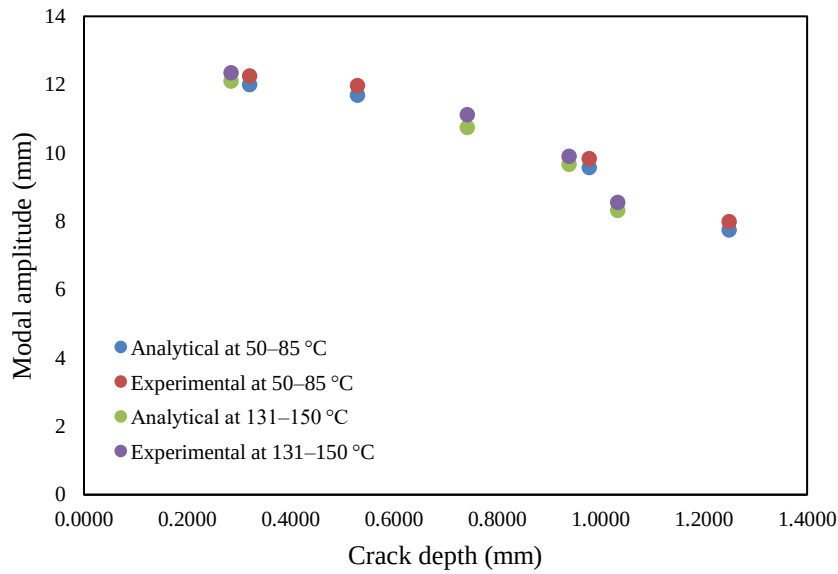


Figure 7-11 Comparison of modal amplitude of propagated crack in the pre-seeded crack depth of 1.3 mm at 50–85°C and 131–150°C

7.5 Conclusion

In this crack propagation analysis, different pre-seeded cracks were selected to evaluate the crack growth. The crack propagation was analysed in both uniform and non-uniform temperature conditions. Crack depths of 0.25, 0.5, and 1 mm were considered for the uniform temperature distribution while crack depths of 1, 1.3, and 1.5 mm were chosen for the non-uniform temperature distribution. The differences in crack depth for both temperature types were considered, because lower crack depth takes a longer time to propagate, which becomes a problem regarding maintaining the non-uniform temperature. Therefore, greater crack depths were considered under non-uniform analysis. The propagated crack depth was captured by a digital microscope. The crack depth was

determined by counting the number of pixels present in it. In this study, two types of evaluation were conducted. The first analysis was the fatigue crack propagation evaluation. In this analysis, the crack was allowed to propagate until failure. The crack depths were measured during the crack growth and evaluated the crack growth pattern under different temperatures. The second analysis was the evaluation of the modal parameter during crack propagation. The results of crack propagation analysis showed that a constant crack depth cannot be obtained for different temperatures. The propagation was not identical. It sometimes appeared double cracks and consumed time leading to failure. The appearance of multiple cracks could be due to the selected shape of the pre-seeded crack. The modal analysis during crack propagation showed that modal parameters under non-uniform temperatures are more linear in nature than those of uniform temperatures. This suggests that the applied temperatures affect the modal parameters of the beam. In this modal behaviour analysis, the modal parameters during crack propagation were compared with the analytical model. It showed that the analytical model is in good agreement with the experimental results.

8 CONCLUSIONS

Modal analysis is vital in engineering applications. The applied thermal and type of mechanical loads affect the overall structural integrity. This research investigates the structural dynamics and cracks propagation behaviour under uniform and non-uniform temperature conditions. An analytical model based on the Bernoulli–Euler beam theory was developed to analyse the structural dynamics and its modal parameters. The model was validated by experimental and numerical methods. The crack propagation behaviour was experimentally evaluated at the thermal setting. The research aims to study the impact of applied thermal load and the heating rate on modal dynamics and crack propagation behaviour. The evaluation showed that there was a variation in modal responses when changing the applied temperature, although the difference was relatively small but significant at high temperatures. The cantilever beam's modal characteristics were studied for both uniform and non-uniform temperatures. The results reveal that the modal parameters change depending on the temperature. The modal parameters were found to be greater at non-uniform temperatures than at uniform temperatures. Even though there was small variation in their responses, it was apparent. The small temperature ranges account for the little variation. Their differences were minor at low temperatures but apparent at higher temperatures. Similarly, the cantilever beam's modal amplitude fluctuates with temperature. The results shown that amplitude differences were mostly visible at high temperatures. The changes in amplitude are caused by variances in frequency at each temperature range. The modal parameters were different for different heating rates, especially at elevated temperatures. The effect of ramping temperature is negligible at low temperatures and increasingly obvious at high temperatures. As a result, the rate of heating has a little influence on the dynamic behaviour of the beam. The experimental analysis at slow heating rates provided more precise results with the analytical results. The results demonstrated that the modal parameters change with respect to temperature. This is due to the changes in thermal properties of the material especially modulus and thermal expansion at elevated temperature. The modulus decreases while thermal expansion increases

linearly with the rise in temperature. Both the factors are responsible for the changes in modal parameters. However, the effect of modulus is more significant than the thermal expansion.

The research also demonstrated the fatigue crack propagation behaviour of a cantilever beam under uniform and non-uniform temperatures conditions. The results of crack propagation exhibited nonlinear crack growth at both high and low temperatures. It appears double crack fronts and multiple cracks on the pre-seeded crack surface. However, this may be due to the selected pre-seeded crack shape. A pre-seeded rectangular crack shape was considered for this research. Hence, fatigue crack propagation evaluation on a pre-seeded crack, especially a rectangular crack, was not viable. It is worthwhile to choose a real crack for crack propagation analysis in the future.

8.1 Achievement of objectives

Objective 1: To develop an analytical model to investigate the effects of temperature distribution on the modal response of a beam structure.

An analytical model was established based on Bernoulli–Euler beam theory. This modelling incorporated the thermal expansion due to the applied thermal loads. The modelling used the approximate relationship for calculating the elastic modulus for different temperatures. Therefore, this model was able to capture the thermal effect on modal characteristic of the beam at different temperatures.

Objective 2: To establish the changes in the model for a beam with crack damage.

The model was effectively modified to accommodate the crack damage in estimating modal parameters of the beam. The modal response was measured based on changes in deformation energy due to the crack formation.

Objective 3: Experimental validation of the analytical model under uniform and variable temperature distribution across the structure.

This objective is achieved by designing proper experimental scheme and testing procedures. The analytical model was validated with the experimental data separately for uniform and non-uniform temperature distribution.

Objective 4: Experimentally analyse the crack propagation behaviour and study the modal response of the beam during crack propagation. The author achieved this objective by analysing the crack propagation behaviour at different temperatures. The propagation evaluation was conducted at different pre-seeded crack depths for uniform and non-uniform temperature

8.2 Novel contribution

8.2.1 Analytical modelling to study the modal characteristic of the beam at different temperatures

Generally, the assessment of modal response on structures was performed in absence of a thermal environment. Therefore, those research approaches are infeasible to capture the addition of thermal disturbances. Most of the practices are however purely analysing the vibrational characteristic behaviour of the structure without concerning the fatigue damage. This research is focused to include temperature load in structural health analysis. An analytical model was established based on Bernoulli–Euler beam theory. This model can be able to analyse the modal characteristic of the beam at different temperatures. The effect of temperature was incorporated by considering the effective length of the beam due to the thermal expansion. This modelling used an approximate relationship for calculating the elastic modulus for different temperatures. The changes in the modal response for a crack beam are calculated based on changes in deformation energy due to the crack formation. From this analysis, it was found that the modal characteristic is different for different temperatures. Therefore, the modal parameters vary at a standard treatment and elevated temperatures.

8.2.2 An experimental design technique to capture the modal response of a beam

The experimental methods which are reviewed in the previous chapter applied temperature without considering the heating duration. This research experimentally studied the modal characteristic of a cantilever beam at three heating rates at different temperatures condition. It was assumed as slow heating at 2°C/min, moderate heating at 5°C/min, and fast heating at 8°C/min to analyse the modal response for both cracks and without a crack specimen. It was observed that different heating rates have little influence on modal parameters. This impact of heating rates on dynamic response is negligible at lower temperatures and appeared significant at elevated temperatures. It was found that the results of heating at 2°C/min appeared close to the analytical and numerical results. This signifies heating at a lower rate is more accurate for any experimental analysis. This also suggested that heating at different rates can make a slight effect while measuring the dynamic response of any mechanical system. The modal frequency and amplitude of the cantilever beam decrease with an increase in temperature in both crack and without a crack specimen. The results showed that modal parameters of the cantilever beam are associated with the change in temperatures. This is due to the changes in the interatomic bond length of the material as discussed above. This experimental analysis suggested that a slow heating rate provides more accurate results to the analytical results. Therefore, slow heating is appropriate for experimental evaluation.

8.2.3 Assessment of fatigue crack propagation behaviour at thermal setting

This research anticipates assessing the crack propagation behaviour in a thermal setting. The evaluation of fatigue crack propagation at different temperatures showed a non-linear crack growth. The crack initiation from the pre-seeded crack surface was not identical in all experiments even though it applied the same boundary conditions. This seems to be due to the selected pre-seeded crack depth shape. In this analysis, a pre-seeded crack of rectangular shape was considered which sometimes generated multiple cracks.

It also appears a double crack front. The fatigue crack propagation was quicker when there is a single crack in the same direction. However, it appeared a sluggish propagation whenever there were dual crack fronts from the opposite end. Moreover, the speed of fatigue crack propagation was also dependent on the attached end mass and its alignment. The propagated crack accelerates at lower temperatures than at elevated temperatures. This happened due to the formation of plasticity at low temperatures while more elasticity at higher temperatures. Therefore, the modal parameters of the beam were not identical in the case of propagated crack.

8.3 Recommendation for the future study

The structural dynamics and crack propagation behaviour under uniform and non-uniform temperatures were investigated in this study. This research completed and met the research objectives. However, this research has various scopes for future research. The research can be extended in the future study as described below:

Bernoulli–Euler beam hypothesis served as the basis for the analytical model. As a result, once the beam deforms, the section of the beam is expected to stay perpendicular to the neutral axis. For bending beams that do not experience large shear or torsional loads, this convention works well. It was also taken into account that the deflection angle or deform angle of the beam was rather modest. Future research can enhance this aspect of the constraint.

The experimental scheme was designed for this research. The crack depths, specimen dimensions, rates of heating, and temperature range was selected for the convenience of a lab experiment. In the future, this experimental procedure can be carried out and tested in real physical structures.

One of the limitations of this research was choosing the maximum temperature as 200°C which is well below the phase transition temperature of Al 2024. Therefore, the modal response may behave differently at higher temperatures. This part of the research can be enhanced by encompassing at high temperatures.

The pre-seeded crack of rectangular shape was selected for this research in crack propagation analysis. This crack shape generates multiple cracks on the pre-seeded crack surface during the crack initiation and fails to fracture. Therefore, a suitable crack shape like V-shape or real crack for crack propagation assessment is recommended in future research.

Last but not least the author would like to recommend comprehensive research on crack growth rate retardation techniques. Several factors contributed to crack growth mechanisms. The researcher does relate the influence of microstructural voids, corrosions, riveting joins etcetera in damage processes. In general, future research should direct toward enriching fatigue life by reducing the crack growth rate.

REFERENCES

1. Uematsu, Y.; Akita, M.; Nakajima, M.; Tokaji, K. Effect of temperature on high cycle fatigue behaviour in 18Cr-2Mo ferritic stainless steel. *Int. J. Fatigue* **2008**, *30*, 642–648, doi:10.1016/j.ijfatigue.2007.05.004.
2. Ghonem, H.; Wen, Y.; Zheng, D.; Thompson, M.; Linsey, G. Effects of temperature and frequency on fatigue crack growth in Ti- β 21S monolithic laminate. *Mater. Sci. Eng. A* **1993**, *161*, 45–53, doi:10.1016/0921-5093(93)90474-S.
3. Feng, R.C.; Rui, Z.Y.; Zuo, Y.; Yan, C.F.; Zhang, G.T. Influence of temperature on fatigue crack propagation in TiAl alloys. *Strength Mater.* **2014**, *46*, 417–421, doi:10.1007/s11223-014-9563-5.
4. Antolovich, S.D.; Saxena, A.; Gerberich, W.W. Fracture mechanics – An interpretive technical history. *Mech. Res. Commun.* **2018**, *91*, 46–86, doi:10.1016/j.mechrescom.2018.03.003.
5. Baxevanis, T.; Lagoudas, D.C. Fracture mechanics of shape memory alloys: review and perspectives. *Int. J. Fract.* **2015**, *191*, 191–213, doi:10.1007/s10704-015-9999-z.
6. Habtour, E.; Connon, W.S.; Pohland, M.F.; Stanton, S.C.; Paulus, M.; Dasgupta, A. Review of Response and Damage of Linear and Nonlinear Systems under Multiaxial Vibration. *Shock Vib.* **2014**, *2014*, 1–21, doi:10.1155/2014/294271.
7. Sunder, R.; Daniewicz, S.R.; Dean, S.W. Unraveling the Science of Variable Amplitude Fatigue. *J. ASTM Int.* **2012**, *9*, 1–32, doi:10.1520/JAI103940.
8. Zai, B.A.; Khan, M.; Khan, K.A.; Mansoor, A.; Shah, A.; Shahzad, M. The role of dynamic response parameters in damage prediction. *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.* **2019**, *233*, 4620–4636, doi:10.1177/0954406219841083.

9. Yang, L.; Fatemi, A. Cumulative Fatigue Damage Mechanisms and Quantifying Parameters: A Literature Review. *J. Test. Eval.* **1998**, *26*, 89–100, doi:10.1520/JTE11978J.
10. McClung, R.C. A literature survey on the stability and significance of residual stresses during fatigue. *Fatigue Fract. Eng. Mater. Struct.* **2007**, *30*, 173–205, doi:10.1111/j.1460-2695.2007.01102.x.
11. Narasimha, M.; Kuttan, A.; Kadoli, R. Thermally Induced Vibration of a Simply Supported Beam Using Finite Element Method. *Int. J. Eng. Sci. Technol.* **2010**, *2*, 7874–7879.
12. Larizza, P. Measurement, Testing, and Diagnosis for Micro-manufacturing Systems. In *Micromanufacturing Engineering and Technology*; Elsevier, 2015; pp. 675–704.
13. Ebrahimi, F.; Jafari, A. A four-variable refined shear-deformation beam theory for thermo-mechanical vibration analysis of temperature-dependent FGM beams with porosities. *Mech. Adv. Mater. Struct.* **2018**, *25*, 212–224, doi:10.1080/15376494.2016.1255820.
14. Ebrahimi, F.; Salari, E. Thermo-mechanical vibration analysis of a single-walled carbon nanotube embedded in an elastic medium based on higher-order shear deformation beam theory. *J. Mech. Sci. Technol.* **2015**, *29*, 3797–3803, doi:10.1007/s12206-015-0826-2.
15. Ebrahimi, F.; Farazmandnia, N. Thermo-mechanical vibration analysis of sandwich beams with functionally graded carbon nanotube-reinforced composite face sheets based on a higher-order shear deformation beam theory. *Mech. Adv. Mater. Struct.* **2017**, *24*, 820–829, doi:10.1080/15376494.2016.1196786.
16. Ebrahimi, F.; Barati, M.R. Vibration analysis of parabolic shear-deformable piezoelectrically actuated nanoscale beams incorporating thermal effects. *Mech. Adv. Mater. Struct.* **2018**, *25*, 917–929, doi:10.1080/15376494.2017.1323141.

17. Ebrahimi, F.; Barati, M.R.; Zenkour, A.M. A new nonlocal elasticity theory with graded nonlocality for thermo-mechanical vibration of FG nanobeams via a nonlocal third-order shear deformation theory. *Mech. Adv. Mater. Struct.* **2018**, *25*, 512–522, doi:10.1080/15376494.2017.1285458.
18. Hosseini, S.A.H.; Rahmani, O. Thermomechanical vibration of curved functionally graded nanobeam based on nonlocal elasticity. *J. Therm. Stress.* **2016**, *39*, 1252–1267, doi:10.1080/01495739.2016.1215731.
19. Ebrahimi, F.; Salari, E. Nonlocal thermo-mechanical vibration analysis of functionally graded nanobeams in thermal environment. *Acta Astronaut.* **2015**, *113*, 29–50, doi:10.1016/j.actaastro.2015.03.031.
20. Mohammadimehr, M.; Atifeh, S.J.; Roustaei Navi, B. Stress and free vibration analysis of piezoelectric hollow circular FG-SWBNNTs reinforced nanocomposite plate based on modified couple stress theory subjected to thermo-mechanical loadings. *J. Vib. Control* **2018**, *24*, 3471–3486, doi:10.1177/1077546317706887.
21. Ghadiri, M.; Shafiei, N.; Alavi, H. Thermo-mechanical vibration of orthotropic cantilever and propped cantilever nanoplate using generalized differential quadrature method. *Mech. Adv. Mater. Struct.* **2017**, *24*, 636–646, doi:10.1080/15376494.2016.1196770.
22. Mustapha, K.B.; Zhong, Z.W. The thermo-mechanical vibration of a single-walled carbon nanotube studied using the Bubnov – Galerkin method. *Phys. E Low-dimensional Syst. Nanostructures* **2010**, *43*, 375–381, doi:10.1016/j.physe.2010.08.012.
23. Hao, Z.; Xu, Y.; Durgam, S.K. A thermal-energy method for calculating thermoelastic damping in micromechanical resonators. *J. Sound Vib.* **2009**, *322*, 870–882, doi:10.1016/j.jsv.2008.12.005.
24. Zare, J.; Hashemi, S.J.; Rashed, G. Finite Element Analysis of Drillstring Lateral Vibration. *J. Eng. Appl. Sci.* **2011**, *6*, 64–70, doi:10.3923/jeasci.2011.64.70.

25. Chan, T.H.T.; Guo, L.; Li, Z.X. Finite element modelling for fatigue stress analysis of large suspension bridges. *J. Sound Vib.* **2003**, *261*, 443–464, doi:10.1016/S0022-460X(02)01086-6.
26. Mohammadi, M.; Moradi, A.; Ghayour, M.; Farajpour, A. Exact solution for thermo-mechanical vibration of orthotropic mono-layer graphene sheet embedded in an elastic medium. *Lat. Am. J. Solids Struct.* **2014**, *11*, 437–458.
27. Mohammadi, M.; Farajpour, A.; Goodarzi, M.; Dinari, F. Thermo-mechanical vibration analysis of annular and circular graphene sheet embedded in an elastic medium. *Lat. Am. J. Solids Struct.* **2014**, *11*, 659–682, doi:10.1590/S1679-78252014000400007.
28. Goodarzi, M.; Mohammadi, M.; Khooran, M.; Saadi, F. Thermo-Mechanical Vibration Analysis of FG Circular and Annular Nanoplate Based on the Visco-Pasternak Foundation. *J. Solid Mech. Vol.* **2016**, *8*, 788–805.
29. Ebrahimi, F.; Dabbagh, A. On thermo-mechanical vibration analysis of multi-scale hybrid composite beams. *J. Vib. Control* **2019**, *25*, 933–945, doi:10.1177/1077546318806800.
30. Safaei, B.; Moradi-Dastjerdi, R.; Qin, Z.; Chu, F. Frequency-dependent forced vibration analysis of nanocomposite sandwich plate under thermo-mechanical loads. *Compos. Part B Eng.* **2019**, *161*, 44–54, doi:10.1016/j.compositesb.2018.10.049.
31. Warminska, A.; Manoach, E.; Warminski, J.; Samborski, S. Regular and chaotic oscillations of a Timoshenko beam subjected to mechanical and thermal loadings. *Contin. Mech. Thermodyn.* **2015**, *27*, 719–737, doi:10.1007/s00161-014-0381-6.
32. Yang, J.; Shen, H.-S. Vibration characteristics and transient response of shear-deformable functionally graded plates in thermal environments. *J. Sound Vib.* **2002**, *255*, 579–602, doi:10.1006/jsvi.2001.4161.

33. Tornabene, F. Free vibration analysis of functionally graded conical , cylindrical shell and annular plate structures with a four-parameter power-law distribution. *Comput. Methods Appl. Mech. Eng.* **2009**, *198*, 2911–2935, doi:10.1016/j.cma.2009.04.011.
34. Shen, H.; Wang, Z. Nonlinear vibration of hybrid laminated plates resting on elastic foundations in thermal environments. *Appl. Math. Model.* **2012**, *36*, 6275–6290, doi:10.1016/j.apm.2012.02.001.
35. Ghayesh, M.H.; Kazemirad, S.; Darabi, M.A.; Woo, P. Thermo-mechanical nonlinear vibration analysis of a spring-mass-beam system. *Arch. Appl. Mech.* **2012**, *82*, 317–331, doi:10.1007/s00419-011-0558-4.
36. Julien, B.; Bertrand, F.; Thierry, Y. Probabilistic Random Vibration Fatigue. *Procedia Eng.* **2013**, *66*, 522–529, doi:10.1016/j.proeng.2013.12.104.
37. Kitipornchai, S.; Yang, J.; Liew, K.M. Random vibration of the functionally graded laminates in thermal environments. *Comput. Methods Appl. Mech. Eng.* **2006**, *195*, 1075–1095, doi:10.1016/j.cma.2005.01.016.
38. SoltanRezaee, M.; Afrashi, M.; Rahmanian, S. Vibration analysis of thermoelastic nano-wires under Coulomb and dispersion forces. *Int. J. Mech. Sci.* **2018**, *142–143*, 33–43, doi:10.1016/j.ijmecsci.2018.04.034.
39. Ubertini, F. Effects of cables damage on vertical and torsional eigenproperties of suspension bridges. *J. Sound Vib.* **2014**, *333*, 2404–2421, doi:10.1016/j.jsv.2014.01.027.
40. Bao, T.; Andrew Swartz, R.; Vitton, S.; Sun, Y.; Zhang, C.; Liu, Z. Critical insights for advanced bridge scour detection using the natural frequency. *J. Sound Vib.* **2017**, *386*, 116–133, doi:10.1016/j.jsv.2016.06.039.
41. Proso, U.; Slavič, J.; Boltežar, M. Vibration-fatigue damage accumulation for structural dynamics with non-linearities. *Int. J. Mech. Sci.* **2016**, *106*, 72–77, doi:10.1016/j.ijmecsci.2015.12.005.

42. Kong, Y.; Abdullah, S.; Schramm, D.; Omar, M.; Haris, S. Vibration Fatigue Analysis of Carbon Steel Coil Spring under Various Road Excitations. *Metals (Basel)*. **2018**, 8, 617, doi:10.3390/met8080617.
43. Rani, S.; Agrawal, A.K.; Rastogi, V. Vibration analysis for detecting failure mode and crack location in first stage gas turbine blade. *J. Mech. Sci. Technol.* **2019**, 33, 1–10, doi:10.1007/s12206-018-1201-x.
44. Snoeys, R.; Sas, P.; Heylen, W.; Van der Auweraer, H. Trends in experimental modal analysis. *Mech. Syst. Signal Process.* **1987**, 1, 5–27, doi:10.1016/0888-3270(87)90080-X.
45. Rajadurai, S.; Prasad, M.G.; Kavin, R.; Sundaravadivelu, M. Modal Analysis for Exhaust Manifold in Hot Condition, Is There a Need? In Proceedings of the International Conference on Automotive Materials & Manufacturing 2014; 2014.
46. Witek, L.; Stachowicz, F. Modal Analysis of the Turbine Blade at Complex Thermomechanical Loads. *Strength Mater.* **2016**, 48, 474–480, doi:10.1007/s11223-016-9788-6.
47. Zhu, D.; Rajan, S.D.; Mobasher, B.; Peled, A.; Mignolet, M. Modal Analysis of a Servo-Hydraulic High Speed Machine and its Application to Dynamic Tensile Testing at an Intermediate Strain Rate. *Exp. Mech.* **2011**, 51, 1347–1363, doi:10.1007/s11340-010-9443-2.
48. Hollkamp, J.J.; Gordon, R.W. Modal test experiences with a jet engine fan model. *J. Sound Vib.* **2001**, 248, 151–165, doi:10.1006/jsvi.2001.3758.
49. Baqersad, J.; Niezrecki, C.; Avitabile, P. Extracting full-field dynamic strain on a wind turbine rotor subjected to arbitrary excitations using 3D point tracking and a modal expansion technique. *J. Sound Vib.* **2015**, 352, 16–29, doi:10.1016/j.jsv.2015.04.026.
50. Rokach, I. Modal approach for processing one- and three-point bend test data for dsif-time diagram determination part i-theory. *Fatigue Fract. Eng. Mater. Struct.* **1998**, 21, 1007–1114, doi:10.1046/j.1460-

2695.1998.00087.x.

51. Jezequel, L. A hybrid method of modal synthesis using vibration tests. *J. Sound Vib.* **1985**, *100*, 191–210, doi:10.1016/0022-460X(85)90415-8.
52. A, L.H.Y.; Leung, T.P.; Li, D.B.; Xue, K.Z. Theoretical and experimental study of modal strain analysis. *J. Sound Vib.* **1996**, *191*, 251–260.
53. Arafat, H.N.; Nayfeh, A.H. Modal interactions in the vibrations of a heated annular plate. *Int. J. Non. Linear. Mech.* **2004**, *39*, 1671–1685, doi:10.1016/j.ijnonlinmec.2004.02.015.
54. Lamb, M.; Rouillard, V. Assessing the Influence of Fourier Analysis Parameters on Short-Time Modal Parameter Extraction. *J. Vib. Acoust.* **2012**, *134*, 1–12, doi:10.1115/1.4005654.
55. Braghin, F.; Cinquemani, S.; Resta, F. A new approach to the synthesis of modal control laws in active structural vibration control. *J. Vib. Control* **2013**, *19*, 163–182, doi:10.1177/1077546311430246.
56. Mathan, G.; Prasad, N.S. Study of dynamic response of piping system with gasketed flanged joints using finite element analysis. *Int. J. Press. Vessel. Pip.* **2012**, *89*, 28–32, doi:10.1016/j.ijpvp.2011.09.002.
57. Geng, Q.; Li, H.; Li, Y. Dynamic and acoustic response of a clamped rectangular plate in thermal environments: Experiment and numerical simulation. *J. Acoust. Soc. Am.* **2014**, *135*, 2674–2682, doi:10.1121/1.4870483.
58. Huang, X.; Zhou, K.; Tian, Y.; Hua, H. Linear thermal-acoustic responses of a composite panel in progressive wave tube based on wavenumber-frequency analysis. *J. Sound Vib.* **2019**, *443*, 1–24, doi:10.1016/j.jsv.2018.11.011.
59. Sharma, J.N.; Kaur, R. Flexural response of thermoelastic thin beam resonators due to thermal and mechanical loads. *Int. J. Mech. Sci.* **2015**, *101–102*, 170–179, doi:10.1016/j.ijmecsci.2015.07.014.

60. Shen, H.-S.; Yang, J.; Zhang, L. Free and forced vibration of reissner–mindlin plates with free edges resting on elastic foundations. *J. Sound Vib.* **2001**, *244*, 299–320, doi:10.1006/jsvi.2000.3501.
61. Yang, Y.; Dorn, C.; Mancini, T.; Talken, Z.; Nagarajaiah, S.; Kenyon, G.; Farrar, C.; Mascareñas, D. Blind identification of full-field vibration modes of output-only structures from uniformly-sampled, possibly temporally-aliased (sub-Nyquist), video measurements. *J. Sound Vib.* **2017**, *390*, 232–256, doi:10.1016/j.jsv.2016.11.034.
62. Ebrahimi, F.; Barati, M.R. Vibration analysis of size-dependent flexoelectric nanoplates incorporating surface and thermal effects. *Mech. Adv. Mater. Struct.* **2018**, *25*, 611–621, doi:10.1080/15376494.2017.1285464.
63. Wauer, J.; Suherman, S. Thickness vibrations of a piezo-semiconducting plate layer. *Int. J. Eng. Sci.* **1997**, *35*, 1387–1404, doi:10.1016/S0020-7225(97)00060-8.
64. Pradeep, V.; Ganesan, N. Thermal buckling and vibration behavior of multi-layer rectangular viscoelastic sandwich plates. *J. Sound Vib.* **2008**, *310*, 169–183, doi:10.1016/j.jsv.2007.07.083.
65. Zai, B.A.; Khan, M.A.; Khan, K.A.; Mansoor, A. A novel approach for damage quantification using the dynamic response of a metallic beam under thermo-mechanical loads. *J. Sound Vib.* **2020**, *469*, 115134, doi:10.1016/j.jsv.2019.115134.
66. Chowdhury, S.; Deeb, M.; Zabel, V. Effects of parameter estimation techniques and uncertainty on the selection of fatigue crack growth model. *Structures* **2019**, *19*, 128–142, doi:10.1016/j.istruc.2018.11.018.
67. Albuquerque, C.; Castro, P.M.S.T. De; Calçada, R. Efficient crack analysis of dynamically loaded structures using a modal superposition of stress intensity factors. *Eng. Fract. Mech.* **2012**, *93*, 75–91, doi:10.1016/j.engfracmech.2012.06.009.

68. Kim, K.; Lee, Y.S. Modal characteristics and fatigue strength of compressor blades. *J. Mech. Sci. Technol.* **2014**, *28*, 1421–1429, doi:10.1007/s12206-014-0129-z.
69. Kim, J.W.; Choi, M.R.; Kim, Y.J. Fracture Behavior of Aged CF8A Austenite Cast Stainless Steel Under Dynamic and Cyclic Loading Conditions. In Proceedings of the Volume 3B: Design and Analysis; American Society of Mechanical Engineers: Waikoloa, Hawaii, USA, 2017; pp. 1–7.
70. Senapati, U.; Dhage, Y.; Sawant, V.; Saraf, M.R.; Dodds, C. Accelerated Test Method for Validation of Vehicle Components Subjected to Fatigue Failure. In Proceedings of the SAE Technical Paper; 2007; pp. 1–7.
71. Zhao, J.; Wang, S. Analysis for Fatigue Failure Causes on a Large-scale Reciprocating Compressor Vibration by Torsional Vibration. In Proceedings of the Procedia Engineering; 2014; Vol. 74, pp. 170–174.
72. Lu, Y.; Xiang, P.; Dong, P.; Zhang, X.; Zeng, J. Analysis of the effects of vibration modes on fatigue damage in high-speed train bogie frames. *Eng. Fail. Anal.* **2018**, *89*, 222–241, doi:10.1016/j.engfailanal.2018.02.025.
73. Maier, H.J.; Christ, H. Modeling of cyclic stress – strain behavior and damage mechanisms under thermomechanical fatigue conditions. *International J. Fatigue* **1997**, *19*, 267–274.
74. Česnik, M.; Slavič, J.; Boltežar, M. Uninterrupted and accelerated vibrational fatigue testing with simultaneous monitoring of the natural frequency and damping. *J. Sound Vib.* **2012**, *331*, 5370–5382, doi:10.1016/j.jsv.2012.06.022.
75. Sasaki, K.; Takahashi, T. Low cycle thermal fatigue and microstructural change of AC2B-T6 aluminum alloy. *Int. J. Fatigue* **2006**, *28*, 203–210, doi:10.1016/j.ijfatigue.2005.06.025.
76. Braccesi, C.; Cianetti, F.; Lori, G.; Pioli, D. The frequency domain

- approach in virtual fatigue estimation of non-linear systems : The problem of non-Gaussian states of stress. *Int. J. Fatigue* **2009**, 31, 766–775, doi:10.1016/j.ijfatigue.2008.03.007.
77. Braccesi, C.; Cianetti, F.; Tomassini, L. Validation of a New Method for Frequency Domain Dynamic Simulation and Damage Evaluation of Mechanical Components Modelled with Modal Approach. *Procedia Eng.* **2015**, 101, 493–500, doi:10.1016/j.proeng.2015.02.059.
 78. Kraemer, K.M.; Mueller, F.; Kontermann, C.; Oechsner, M. Estimation of fatigue crack growth under complex loading using an accumulative approach. *Mater. High Temp.* **2017**, 34, 350–361, doi:10.1080/09603409.2017.1369665.
 79. Guo, L.; Noda, N.; Wu, L. Thermal fracture model for a functionally graded plate with a crack normal to the surfaces and arbitrary thermomechanical properties. *Compos. Sci. Technol.* **2008**, 68, 1034–1041, doi:10.1016/j.compscitech.2007.07.003.
 80. Oller, S.; Salomón, O.; Oñate, E. A continuum mechanics model for mechanical fatigue analysis. *Comput. Mater. Sci.* **2005**, 32, 175–195, doi:10.1016/j.commatsci.2004.08.001.
 81. Guo, Q.; Zaïri, F.; Guo, X. An intrinsic dissipation model for high-cycle fatigue life prediction. *Int. J. Mech. Sci.* **2018**, 140, 163–171, doi:10.1016/j.ijmecsci.2018.02.047.
 82. Paris, P.; Erdogan, F. A critical analysis of crack propagation laws. *J. Fluids Eng. Trans. ASME* **1963**, 85, 528–533, doi:10.1115/1.3656900.
 83. Damir, A.; Elkhatab, A.; Nassef, G. Prediction of fatigue life using modal analysis for grey and ductile cast iron. *Int. J. Fatigue* **2007**, 29, 499–507, doi:10.1016/j.ijfatigue.2006.05.004.
 84. Braccesi, C.; Cianetti, F.; Tomassini, L. An innovative modal approach for frequency domain stress recovery and fatigue damage evaluation. *Int. J. Fatigue* **2016**, 91, 382–396, doi:10.1016/j.ijfatigue.2016.02.028.

85. Karolczuk, A. Non-local area approach to fatigue life evaluation under combined reversed bending and torsion. *Int. J. Fatigue* **2008**, *30*, 1985–1996, doi:10.1016/j.ijfatigue.2008.01.007.
86. Karolczuk, A.; Blacha, Ł. Fatigue life estimation under variable amplitude bending using the non-local damage parameter and multisurface plasticity model. *Int. J. Fatigue* **2011**, *33*, 1376–1383, doi:10.1016/j.ijfatigue.2011.05.003.
87. Saintier, N.; Palin-luc, T.; Bénabes, J.; Cocheteux, F. Non-local energy based fatigue life calculation method under multiaxial variable amplitude loadings. *Int. J. Fatigue* **2013**, *54*, 68–83, doi:10.1016/j.ijfatigue.2012.12.013.
88. Savkin, A.N.; Sunder, R.; Andronik, A. V.; Sedov, A.A. Effect of Overload on the Near-Threshold Fatigue Crack Growth Rate in a 2024-T3 Aluminum Alloy: II. Fatigue Crack Growth Simulation for Calculating the Fatigue Life under Alternating Loading. *Russ. Metall.* **2019**, *2019*, 542–547, doi:10.1134/S0036029519050100.
89. Iacoviello, F.; Di Cocco, V.; Bellini, C. Overload effects on fatigue cracks in a ferritized ductile cast iron. *Int. J. Fatigue* **2019**, *127*, 376–381, doi:10.1016/j.ijfatigue.2019.06.028.
90. Ordoñez, J.H.; Ambriz, R.R.; García, C.; Plascencia, G.; Jaramillo, D. Overloading effect on the fatigue strength in resistance spot welding joints of a DP980 steel. *Int. J. Fatigue* **2019**, *121*, 163–171, doi:10.1016/j.ijfatigue.2018.12.026.
91. Rahmat, M.A.; Ibrahim, R.N.; Oskouei, R.H. A study on the combined effect of notch and fretting on the fatigue life behaviour of Al 7075-T6. *Mater. Des.* **2014**, *60*, 136–145, doi:10.1016/j.matdes.2014.03.059.
92. Farhad, F.; Zhang, X.; Smyth-Boyle, D. Fatigue behaviour of corrosion pits in X65 steel pipelines. *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.* **2019**, *233*, 1771–1782, doi:10.1177/0954406218776338.

93. Robitaille, B.; Provencher, P.R.; St-Georges, L.; Brochu, M. Mechanical properties of 2024-T3 AlClad aluminum FSW lap joints and impact of surface preparation. *Int. J. Fatigue* **2021**, *143*, doi:10.1016/j.ijfatigue.2020.105979.
94. Susmel, L.; Taylor, D. A critical distance / plane method to estimate finite life of notched components under variable amplitude uniaxial / multiaxial fatigue loading. *Int. J. Fatigue* **2012**, *38*, 7–24, doi:10.1016/j.ijfatigue.2011.11.015.
95. Gates, N.R.; Fatemi, A. Multiaxial variable amplitude fatigue life analysis using the critical plane approach, Part I: Un-notched specimen experiments and life estimations. *Int. J. Fatigue* **2017**, *105*, 283–295, doi:10.1016/j.ijfatigue.2017.09.008.
96. Riedler, M.; Leitner, H.; Prillhofer, B.; Winter, G.; Eichlseder, W. Lifetime simulation of thermo-mechanically loaded components. *Meccanica* **2007**, *42*, 47–59, doi:10.1007/s11012-006-9020-z.
97. Khan, M.A.; Khan, S.Z.; Sohail, W.; Khan, H.; Sohaib, M.; Nisar, S. Mechanical fatigue in aluminium at elevated temperature and remaining life prediction based on natural frequency evolution. *Fatigue Fract. Eng. Mater. Struct.* **2015**, *38*, 897–903, doi:10.1111/ffe.12287.
98. Khan, M.A.; Khan, K.A.; Khan, S.Z.; Nisar, S.; Starr, A. Fracture life estimation of Al-1050 thin beams using empirical data and a numerical approach. *Insight - Non-Destructive Test. Cond. Monit.* **2018**, *60*, 363–368, doi:10.1784/insi.2018.60.7.363.
99. Hou, N.; Wen, Z.; Yu, Q.; Yue, Z. Application of a combined high and low cycle fatigue life model on life prediction of SC blade. *Int. J. Fatigue* **2009**, *31*, 616–619, doi:10.1016/j.ijfatigue.2008.03.021.
100. Wei, Z.; Lin, S.; Luo, L.; Yang, F.; Konson, D. A thermal-fatigue life assessment procedure for components under combined temperature and load cycling. In Proceedings of the SAE Technical Papers; 2013; Vol. 2.

101. Metzger, M.; Leidenfrost, M.; Werner, E.; Riedel, H.; Seifert, T. Lifetime Prediction of EN-GJV 450 Cast Iron Cylinder Heads under Combined Thermo-Mechanical and High Cycle Fatigue Loading. *SAE Int. J. Engines* **2014**, *7*, 1073–1083, doi:10.4271/2014-01-9047.
102. Kurna, S.; Soman, N. Life Estimation of Vehicle Sub-Systems Using Vibrational Fatigue. In Proceedings of the Symposium on International Automotive Technology 2019; 2019; pp. 1–11.
103. Dhar, D.; Sharan, A.M.; Rao, J.S. Transient Stress Analysis and Fatigue Life Estimation of Turbine Blades. *J. Vib. Acoust.* **2004**, *126*, 485–495, doi:10.1115/1.1804996.
104. Kamaya, M.; Kawakubo, M. Loading sequence effect on fatigue life of Type 316 stainless steel. *Int. J. Fatigue* **2015**, *81*, 10–20, doi:10.1016/j.ijfatigue.2015.07.009.
105. Ayatollahi, M.R.; Razavi, S.M.J.; Chamani, H.R. A numerical study on the effect of symmetric crack flank holes on fatigue life extension of a SENT specimen. *Fatigue Fract. Eng. Mater. Struct.* **2014**, *37*, 1153–1164, doi:10.1111/ffe.12199.
106. Razavi, S.M.J.; Ayatollahi, M.R.; Sommitsch, C.; Moser, C. Retardation of fatigue crack growth in high strength steel S690 using a modified stop-hole technique. *Eng. Fract. Mech.* **2017**, *169*, 226–237, doi:10.1016/j.engfracmech.2016.11.013.
107. Ayatollahi, M.R.; Razavi, S.M.J.; Yahya, M.Y. Mixed mode fatigue crack initiation and growth in a CT specimen repaired by stop hole technique. *Eng. Fract. Mech.* **2015**, *145*, 115–127, doi:10.1016/j.engfracmech.2015.03.027.
108. Zhang, M.; Zhang, Y.; Liu, H.; Zou, Q. Judgment criterion of the dominant factor of creep-fatigue crack growth in a nickel-based superalloy at elevated temperature. *Int. J. Fatigue* **2019**, *118*, 176–184, doi:10.1016/j.ijfatigue.2018.09.007.

109. Lansinger, J.; Hansson, T.; Clevfors, O. Fatigue crack growth under combined thermal cycling and mechanical loading. *Int. J. Fatigue* **2007**, *29*, 1383–1390, doi:10.1016/j.ijfatigue.2006.10.024.
110. Lu, Y.L.; Liaw, P.K.; Sun, Y.; Wang, G.Y.; Thompson, S.A.; Blust, J.W.; Browning, P.F.; Bhattacharya, A.K.; Aurrecoechea, J.M.; Klarstrom, D.L. Hold-time effect on the elevated-temperature crack growth behavior of solid-solution-strengthened superalloys. *Acta Mater.* **2007**, *55*, 767–775, doi:10.1016/j.actamat.2006.06.044.
111. Narasimhachary, S.B.; Bhachu, K.S.; Shinde, S.R.; Gravett, P.W.; Newman, J.C. A single edge notch specimen for fatigue, creep-fatigue and thermo-mechanical fatigue crack growth testing. *Eng. Fract. Mech.* **2018**, *199*, 760–772, doi:10.1016/j.engfracmech.2017.08.011.
112. Bouvard, J.L.; Gallerneau, F.; Paulmier, P.; Chaboche, J.L. A phenomenological model to predict the crack growth in single crystal superalloys at high temperature. *Int. J. Fatigue* **2012**, *38*, 130–143, doi:10.1016/j.ijfatigue.2011.12.011.
113. Sankararaman, S.; Ling, Y.; Mahadevan, S. Uncertainty quantification and model validation of fatigue crack growth prediction. *Eng. Fract. Mech.* **2011**, *78*, 1487–1504, doi:10.1016/j.engfracmech.2011.02.017.
114. Yao, W.; Xu, Y.; Liu, H.-W.; Xia, K. Quantification of thermally induced damage and its effect on dynamic fracture toughness of two mortars. *Eng. Fract. Mech.* **2017**, *169*, 74–88, doi:10.1016/j.engfracmech.2016.11.018.
115. Kidane, A.; Shukla, A. Quasi-static and dynamic fracture initiation toughness of Ti / TiB layered functionally graded material under thermo-mechanical loading. *Eng. Fract. Mech.* **2010**, *77*, 479–491, doi:10.1016/j.engfracmech.2009.10.006.
116. Verleysen, P.; Peirs, J. Quasi-static and high strain rate fracture behaviour of Ti6Al4V. *Int. J. Impact Eng.* **2017**, *108*, 370–388, doi:10.1016/j.ijimpeng.2017.03.001.

117. You, B.; Lee, S. Fatigue crack growth behaviour of sm45c steel under mixed-mode i and ii loading. *Fatigue Fract. Eng. Mater. Struct. Fract. Eng. Mater. Struct.* **1998**, *21*, 1037–1048, doi:10.1046/j.1460-2695.1998.00103.x.
118. Ma, L.; Roy, S.K.; Hasan, M.H.; Pal, J.; Chatterjee, S. Time-Dependent Fatigue Crack Propagation Behavior of Two Solid-Solution-Strengthened Ni-Based Superalloys—INCONEL 617 and HAYNES 230. *Metall. Mater. Trans. A* **2012**, *43*, 491–504, doi:10.1007/s11661-011-0877-7.
119. Tong; Byrne Effects of frequency on fatigue crack growth at elevated temperature. *Fatigue Fract. Eng. Mater. Struct.* **1999**, *22*, 185–193, doi:10.1046/j.1460-2695.1999.00160.x.
120. Makhoulouf, K.; Jones, J. Effects of temperature and frequency on fatigue crack growth in 18% Cr ferritic stainless steel. *Int. J. Fatigue* **1993**, *15*, 163–171, doi:10.1016/0142-1123(93)90173-N.
121. Wen, J.-F.; Srivastava, A.; Benzerga, A.; Tu, S.-T.; Needleman, A. Creep crack growth by grain boundary cavitation under monotonic and cyclic loading. *J. Mech. Phys. Solids* **2017**, *108*, 68–84, doi:10.1016/j.jmps.2017.07.018.
122. Tvergaard, V.; Needleman, A. An analysis of the brittle ductile transition in dynamic crack growth. *Int. J. Fract.* **1993**, *59*, 53–67, doi:10.1007/BF00032217.
123. Toshihisa, N.; Tatsuyuki, M.; Hidetoshi, U.; Keigo, S. Specimen size effects on dynamic crack propagation and arrest in DCB specimens. *Eng. Fract. Mech.* **1991**, *39*, 757–767, doi:10.1016/0013-7944(91)90224-O.
124. Yong-Wan, K.; Jae-Han, L.; Bong, Y. An analysis of stress intensity factor for thermal transient problems based on green's function. *Eng. Fract. Mech.* **1994**, *49*, 393–403, doi:10.1016/0013-7944(94)90267-4.
125. Jayadevan, K.R. Critical stress intensity factors for cracked hollow pipes under transient thermal loads. *J. Therm. Stress.* **2002**, *25*, 951–968,

doi:10.1080/01495730290074414.

126. Mahmoud, M.A. Stress intensity factors for single and double edge cracks in a simple beam subject to a moving load. *Int. J. Fract.* **2001**, *111*, 151–161, doi:10.1023/A:101228840.
127. Lam, K.Y.; Tay, T.E.; Yuan, W.G. Stress intensity factors of cracks in finite plates subjected to thermal loads. *Eng. Fract. Mech.* **1992**, *43*, 641–650, doi:10.1016/0013-7944(92)90205-S.
128. Huang, K.; Guo, L.; Yu, H.; Jia, P.; Kitamura, T. A domain-independent interaction integral method for evaluating the dynamic stress intensity factors of an interface crack in nonhomogeneous materials. *Int. J. Solids Struct.* **2016**, *100–101*, 547–557, doi:10.1016/j.ijsolstr.2016.09.027.
129. Aloui, A.; Hamrouni, K.; Fakhfakh, T.; Haddar, M. Analytical and Numerical Solution of the Stress Field and the Dynamic Stress Intensity Factors in a Cracked Plate under Sinusoidal Loading. *J. Fail. Anal. Prev.* **2008**, *8*, 551–556, doi:10.1007/s11668-008-9172-3.
130. Rokach, I. V; Łabędzki, P. Determination of the Dynamic Stress Intensity Factor for the Four-Point Bend Impact Test. *Int. J. Fract.* **2009**, *160*, 93–100, doi:10.1007/s10704-009-9404-x.
131. Wang, Z.; Ma, L.; Yu, H.; Wu, L. Dynamic stress intensity factors for homogeneous and non-homogeneous materials using the interaction integral method. *Eng. Fract. Mech.* **2014**, *128*, 8–21, doi:10.1016/j.engfracmech.2014.06.002.
132. Kishimoto, K.; Aoki, S.; Sakata, M. Dynamic stress intensity factors using J-integral and finite element method. *Eng. Fract. Mech.* **1980**, *13*, 387–394, doi:10.1016/0013-7944(80)90067-3.
133. Kobayashi, A.S.; Ramulu, M. Dynamic stress-intensity factors for unsymmetric dynamic isochromatics. *Exp. Mech.* **1981**, *21*, 41–48, doi:10.1007/BF02325929.

134. Rokach, I.V. On the numerical evaluation of the anvil force for accurate dynamic stress intensity factor determination. *Eng. Fract. Mech.* **2003**, 70, 2059–2074, doi:10.1016/S0013-7944(02)00256-4.
135. Nishioka, T.; Atluri, S.N. Path-independent integrals, energy release rates, and general solutions of near-tip fields in mixed-mode dynamic fracture mechanics. *Eng. Fract. Mech.* **1983**, 18, 1–22, doi:10.1016/0013-7944(83)90091-7.
136. Zhong, X.; Huang, Q. Thermal Stress Intensity Factor for an Opening Crack in Thermomagnetoelastic Solids. *J. Therm. Stress.* **2014**, 37, 928–946, doi:10.1080/01495739.2014.912940.
137. Shahani, A.R.; Nabavi, S.M. Transient thermal stress intensity factors for an internal longitudinal semi-elliptical crack in a thick-walled cylinder. *Eng. Fract. Mech.* **2007**, 74, 2585–2602, doi:10.1016/j.engfracmech.2006.11.018.
138. Simon, B.N.; Prasath, R.G.R.; Ramesh, K. Transient thermal stress intensity factors of bimaterial interface cracks using refined three-fringe photoelasticity. *J. strain Anal.* **2009**, 44, 427–438, doi:10.1243/03093247JSA506.
139. Nabavi, S.M.; Ghajar, R. Analysis of thermal stress intensity factors for cracked cylinders using weight function method. *Int. J. Eng. Sci.* **2010**, 48, 1811–1823, doi:10.1016/j.ijengsci.2010.08.006.
140. Ghajar, R.; Nabavi, S.M. Closed-form thermal stress intensity factors for an internal circumferential crack in a thick-walled cylinder. *Fatigue Fract. Eng. Mater. Struct.* **2010**, 33, 504–512, doi:10.1111/j.1460-2695.2010.01459.x.
141. Kang Yong Lee; Chang Won Shul Determination of thermal stress intensity factors for an interface crack under vertical uniform heat flow. *Eng. Fract. Mech.* **1991**, 40, 1067–1074, doi:10.1016/0013-7944(91)90171-V.

142. Petrova, V.; Schmauder, S. Mathematical modelling and thermal stress intensity factors evaluation for an interface crack in the presence of a system of cracks in functionally graded / homogeneous bimetals. *Comput. Mater. Sci.* **2012**, *52*, 171–177, doi:10.1016/j.commatsci.2011.02.028.
143. Nemat-Alla, M.; Noda, N. Thermal stress intensity factor for functionally gradient half space with an edge crack under thermal load. *Arch. Appl. Mech.* **1996**, *66*, 569–580, doi:10.1007/BF00808145.
144. You, J.H.; Bolt, H. Thermal stress intensity factor of interfacial cracks of a plasma facing component under high heat flux loading. *Fusion Eng. Des.* **2003**, *65*, 483–492, doi:10.1016/S0920-3796(03)00051-6.
145. Nabavi, S.M.; Shahani, A.R. Thermal stress intensity factors for a cracked cylinder under transient thermal loading. *Int. J. Press. Vessel. Pip.* **2009**, *86*, 153–163, doi:10.1016/j.ijpvp.2008.11.024.
146. Lee, K.Y.; Park, S. Thermal stress intensity factors for partially insulated interface crack under uniform heat flow. *Eng. Fract. Mech.* **1995**, *50*, 475–482, doi:10.1016/0013-7944(94)00243-B.
147. Leung, A.Y.T.; Xu, X.; Zhou, Z. Hamiltonian Approach to Analytical Thermal Stress Intensity Factors—Part 2 Thermal Stress Intensity Factor. *J. Therm. Stress.* **2010**, *33*, 279–301, doi:10.1080/01495730903543108.
148. Tanigawa, Y.; Komatsubara, Y. Thermal stress analysis of a rectangular plate and its thermal stress intensity for compressive stress field. *J. Therm. Stress.* **2007**, *5739*, 517–842, doi:10.1080/01495739708956117.
149. Xue, Z.-N.; Chen, Z.-T.; Tian, X.-G. Transient thermal stress analysis for a circumferentially cracked hollow cylinder based on memory-dependent heat conduction model. *Theor. Appl. Fract. Mech.* **2018**, *96*, 123–133, doi:10.1016/j.tafmec.2018.04.008.
150. Zhang, X.-Y.; Li, X.-F. Transient thermal stress intensity factors for a circumferential crack in a hollow cylinder based on generalized fractional

- heat conduction. *Int. J. Therm. Sci.* **2017**, *121*, 336–347, doi:10.1016/j.ijthermalsci.2017.07.015.
151. Tran, V.; Geniaut, S.; Galenne, E.; Nistor, I. A modal analysis for computation of stress intensity factors under dynamic loading conditions at low frequency using eXtended Finite Element Method. *Eng. Fract. Mech.* **2013**, *98*, 122–136, doi:10.1016/j.engfracmech.2012.12.005.
 152. Galenne, E.; Andrieux, S.; Ratier, L. A modal approach to linear fracture mechanics for dynamic loading at low frequency. *J. Sound Vib.* **2007**, *299*, 283–297, doi:10.1016/j.jsv.2006.07.013.
 153. Ni, J.; Zhang, X. Determination of stress intensity factor in ultrasonic fatigue loading by means of dynamic modal analysis and three-dimensional finite element calculation. *Eng. Fract. Mech.* **1995**, *52*, 1079–1086, doi:10.1016/0013-7944(95)00050-6.
 154. Horas, C.S.; Alencar, G.; De Jesus, A.M.P.; Calçada, R. Development of an efficient approach for fatigue crack initiation and propagation analysis of bridge critical details using the modal superposition technique. *Eng. Fail. Anal.* **2018**, *89*, 118–137, doi:10.1016/j.engfailanal.2018.03.009.
 155. Zhang, Y.; Guo, L.; Wang, X.; Shen, R.; Huang, K. Thermal shock resistance of functionally graded materials with mixed-mode cracks. *Int. J. Solids Struct.* **2019**, *164*, 202–211, doi:10.1016/j.ijsolstr.2019.01.012.
 156. Huang, K.; Guo, L.; Yu, H. Investigation on mixed-mode dynamic stress intensity factors of an interface crack in bi-materials with an inclusion. *Compos. Struct.* **2018**, *202*, 491–499, doi:10.1016/j.compstruct.2018.02.078.
 157. Wen, P.H.; Aliabadi, M.H.; Young, A. Dual boundary element methods for three-dimensional dynamic crack problems. *J. Strain Anal. Eng. Des.* **1999**, *34*, 373–394, doi:10.1177/030932479903400601.
 158. Han, J.; Wang, B. Thermal shock resistance enhancement of functionally graded materials by multiple cracking. *Acta Mater.* **2006**, *54*, 963–973,

doi:10.1016/j.actamat.2005.10.036.

159. Nomura, Y.; Ikeda, T.; Miyazaki, N. Stress intensity factor analysis at an interfacial corner between anisotropic bimaterials under thermal stress. *Eng. Fract. Mech.* **2009**, *76*, 221–235, doi:10.1016/j.engfracmech.2008.09.008.
160. Kang Yong Lee; Kwan-Bo, S. Thermal shock stress intensity factor by bueckner's weight function method. *Eng. Fract. Mech.* **1990**, *37*, 799–804, doi:10.1016/0013-7944(90)90077-T.
161. Menouillard, T. Thermo-mechanical stress intensity factors evaluation. *Nucl. Eng. Des.* **2010**, *240*, 2579–2588, doi:10.1016/j.nucengdes.2010.06.034.
162. Le, H.N.; Gardin, C. Analytical prediction of crack propagation under thermal cyclic loading inducing a thermal gradient in the specimen thickness – Comparison with experiments and numerical approach. *Eng. Fract. Mech.* **2011**, *78*, 638–652, doi:10.1016/j.engfracmech.2010.11.015.
163. Sankararaman, S.; Ling, Y.; Mahadevan, S. Statistical inference of equivalent initial flaw size with complicated structural geometry and multi-axial variable amplitude loading. *Int. J. Fatigue* **2010**, *32*, 1689–1700, doi:10.1016/j.ijfatigue.2010.03.012.
164. Magi, F.; Di Maio, D.; Sever, I. Validation of initial crack propagation under vibration fatigue by Finite Element analysis. *Int. J. Fatigue* **2017**, *104*, 183–194, doi:10.1016/j.ijfatigue.2017.07.003.
165. Yan, G.; De Stefano, A.; Matta, E.; Feng, R. A novel approach to detecting breathing-fatigue cracks based on dynamic characteristics. *J. Sound Vib.* **2013**, *332*, 407–422, doi:10.1016/j.jsv.2012.09.008.
166. Hu, K.; Chen, Z. Thermoelastic analysis of a partially insulated crack in a strip under thermal impact loading using the hyperbolic heat conduction theory. *Int. J. Eng. Sci.* **2012**, *51*, 144–160, doi:10.1016/j.ijengsci.2011.10.009.

167. Ekhlakov, A.; Khay, O.; Zhang, C.; Sladek, J.; Sladek, V.; Gao, X.W. Thermoelastic crack analysis in functionally graded materials and structures by a BEM. *Fatigue Fract. Eng. Mater. Struct.* **2012**, *35*, 742–766, doi:10.1111/j.1460-2695.2011.01657.x.
168. Lee, K.H.; Chalivendra, V.B.; Shukla, A. Dynamic Crack-Tip Stress and Displacement Fields Under Thermomechanical Loading in Functionally Graded Materials. *J. Appl. Mech.* **2008**, *75*, 1–7, doi:10.1115/1.2932093.
169. Zhang, H.H.; Liu, S.M.; Han, S.Y.; Fan, L.F. Modeling of 2D cracked FGMs under thermo-mechanical loadings with the numerical manifold method. *Int. J. Mech. Sci.* **2018**, *148*, 103–117, doi:10.1016/j.ijmecsci.2018.08.029.
170. Nicholas, T.; Heil, M.L.; Haritos, G.K. Predicting crack growth under thermo-mechanical cycling. *Int. J. Fract.* **1989**, *41*, 157–176, doi:10.1007/BF00018655.
171. John, R.; Hartman, G.A.; Gallagher, J.P. Crack growth induced by thermal-mechanical loading. *Exp. Mech.* **1992**, *32*, 102–108, doi:10.1007/BF02324720.
172. Park, S.; Feger, C.; Nnebe, I. Underfill acceleration factor based on thermal fatigue crack growth rate. In Proceedings of the 2010 Proceedings 60th Electronic Components and Technology Conference (ECTC); IEEE, 2010; pp. 490–495.
173. Kim, K.S.; Van Stone, R.H. Crack growth under thermo-mechanical and temperature gradient loads. *Eng. Fract. Mech.* **1997**, *58*, 133–147, doi:10.1016/S0013-7944(97)00065-9.
174. Chu, D.; Li, X.; Liu, Z. Study the dynamic crack path in brittle material under thermal shock loading by phase field modeling. *Int. J. Fract.* **2017**, *208*, 115–130, doi:10.1007/s10704-017-0220-4.
175. Maletta, C.; Bruno, L.; Corigliano, P.; Crupi, V.; Guglielmino, E. Crack-tip thermal and mechanical hysteresis in Shape Memory Alloys under fatigue

- loading. *Mater. Sci. Eng. A* **2014**, 616, 281–287, doi:10.1016/j.msea.2014.08.007.
176. Merhy, E.; Rémy, L.; Maitournam, H.; Augustins, L. Crack growth characterisation of A356-T7 aluminum alloy under thermo-mechanical fatigue loading. *Eng. Fract. Mech.* **2013**, 110, 99–112, doi:10.1016/j.engfracmech.2013.03.019.
 177. Khan, K.A.; Muliana, A.H.; Rajagopal, K.R.; Wineman, A. On viscoelastic beams undergoing cyclic loading: Determining the onset of structural instabilities. *Int. J. Non. Linear. Mech.* **2018**, 99, 40–50, doi:10.1016/j.ijnonlinmec.2017.10.021.
 178. El-Kafrawy, A. Crack detection by modal analysis in 3D beams based on FEM. *Int. J. Mech. Mater. Des.* **2011**, 7, 265–282, doi:10.1007/s10999-011-9164-4.
 179. Bovsunovsky, A.P.; Surace, C. Considerations regarding superharmonic vibrations of a cracked beam and the variation in damping caused by the presence of the crack. *J. Sound Vib.* **2005**, 288, 865–886, doi:10.1016/j.jsv.2005.01.038.
 180. Hasbroucq, S.; Oueslati, A.; Saxcé, G. De Inelastic responses of a two-bar system with temperature-dependent elastic modulus under cyclic thermomechanical loadings. *Int. J. Solids Struct.* **2010**, 47, 1924–1932, doi:10.1016/j.ijsolstr.2010.03.032.
 181. Kidane, A.; Shukla, A. Dynamic constitutive behavior of Ti/TiB FGM under thermo-mechanical loading. *J. Mater. Sci.* **2008**, 43, 2771–2777, doi:10.1007/s10853-008-2520-6.
 182. Ebrahimi, F.; Ghasemi, F.; Salari, E. Investigating thermal effects on vibration behavior of temperature-dependent compositionally graded Euler beams with porosities. *Meccanica* **2016**, 51, 223–249, doi:10.1007/s11012-015-0208-y.
 183. Ebrahimi, F.; Salari, E.; Hosseini, S.A.H. In-plane thermal loading effects

- on vibrational characteristics of functionally graded nanobeams. *Meccanica* **2016**, 51, 951–977, doi:10.1007/s11012-015-0248-3.
184. Cui, D.F.; Hu, H.Y. Thermal buckling and natural vibration of the beam with an axial stick–slip–stop boundary. *J. Sound Vib.* **2014**, 333, 2271–2282, doi:10.1016/j.jsv.2013.11.042.
 185. Cao, G.; Chen, X.; Kysar, J.W. Thermal vibration and apparent thermal contraction of single-walled carbon nanotubes. *J. Mech. Phys. Solids* **2006**, 54, 1206–1236, doi:10.1016/j.jmps.2005.12.003.
 186. Ebrahimi, F.; Jafari, A. Thermo-mechanical vibration analysis of temperature-dependent porous FG beams based on Timoshenko beam theory. *Struct. Eng. Mech.* **2016**, 59, 343–371, doi:10.12989/sem.2016.59.2.343.
 187. Ebrahimi, F.; Salari, E. Thermo-mechanical vibration analysis of nonlocal temperature-dependent FG nanobeams with various boundary conditions. *Compos. Part B Eng.* **2015**, 78, 272–290, doi:10.1016/j.compositesb.2015.03.068.
 188. Zhang, W.; Chen, H.; Zhu, D.; Kong, X. The thermal effects on high-frequency vibration of beams using energy flow analysis. *J. Sound Vib.* **2014**, 333, 2588–2600, doi:10.1016/j.jsv.2013.12.020.
 189. Liu, X.-F.; Chang, D.-M.; Wang, B.-L.; Cai, L.-R. Effect of temperature-dependency of material properties on thermal shock fracture of solids associated with non-Fourier heat conduction. *Theor. Appl. Fract. Mech.* **2018**, 93, 195–201, doi:10.1016/j.tafmec.2017.08.002.
 190. Fazzolari, F.A. Modal characteristics of P- and S-FGM plates with temperature-dependent materials in thermal environment. *J. Therm. Stress.* **2016**, 39, 854–873, doi:10.1080/01495739.2016.1189772.
 191. Santos Silva, A.C.; Sebastian, C.M.; Lambros, J.; Patterson, E.A. High temperature modal analysis of a non-uniformly heated rectangular plate: Experiments and simulations. *J. Sound Vib.* **2019**, 443, 397–410,

doi:10.1016/j.jsv.2018.11.041.

192. Kawamura, R.; Tanigawa, Y.; Kusuki, S.; Hamamura, H. Fundamental thermo-elasticity equations for thermally induced flexural vibration problems for inhomogeneous plates and thermo-elastic dynamical responses to a sinusoidally varying surface temperature. *J. Eng. Math.* **2008**, *61*, 143–160, doi:10.1007/s10665-007-9190-2.
193. Courtney, T.H. *Mechanical behavior of materials*; Waveland Press, Inc., 2000; ISBN 9781577664253.
194. Akbarzadeh Khorshidi, M.; Soltani, D. Diagnosis of Type, Location and Size of Cracks by Using Generalized Differential Quadrature and Rayleigh Quotient Methods. *J. Theor. Appl. Mech.* **2013**, *43*, 61–70, doi:10.2478/jtam-2013-0006.
195. Barron, R.F.; Barron, B.R. *Design for Thermal Stresses*; John Wiley & Sons, Inc: Hoboken, New Jersey, 2012; ISBN 9780470627693.

APPENDICES

Appendix A Difference of natural frequency from the analytical model

Table A-1 Difference of natural frequency in 0.5mm under uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
25	0.017	0.040	0.047	0.050
50	0.030	0.063	0.073	0.050
100	0.073	0.110	0.197	0.020
150	0.063	0.210	0.393	0.130
200	0.077	0.303	0.427	0.140

Table A-2 Difference of natural frequency in 1mm under uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
25	0.037	0.083	0.090	0.100
50	0.023	0.117	0.197	0.170
100	0.120	0.247	0.433	0.150
150	-0.047	0.163	0.357	0.030
200	0.030	0.267	0.463	0.150

Table A-3 Difference of natural frequency in undamaged under non-uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
27–25	0.017	0.020	0.013	0.010
50–33	-0.037	0.027	0.097	0.010
100–80	-0.117	0.030	0.083	-0.060
150–131	-0.107	0.043	0.237	-0.090
200–185	-0.033	0.080	0.250	0.020

Table A-4 Difference of natural frequency in 0.25mm under non-uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
27–25	-0.037	-0.013	0.023	0.030
50–33	-0.027	0.037	0.087	-0.010
100–80	0.010	0.203	0.240	0.040
150–131	-0.103	0.113	0.240	-0.040
200–185	-0.063	-0.063	0.347	-0.030

Table A-5 Difference of natural frequency in 0.5mm under non-uniform temperature

Temperature °C	Ramp 2°C /min	Ramp 5°C/Min	Ramp 8°C/Min	Numerical
27–25	0.020	0.043	0.053	0.060
50–33	0.027	0.057	0.067	0.060
100–80	0.077	0.127	0.180	0.080
150–131	0.037	0.217	0.393	0.120
200–185	0.127	0.383	0.517	0.220

Table A-6 Difference of natural frequency in 1mm under non-uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
27–25	0.047	0.087	0.097	0.110
50–33	0.007	0.113	0.220	0.150
100–80	0.137	0.233	0.433	0.110
150–131	0.037	0.207	0.390	0.080
200–185	0.103	0.203	0.420	0.070

Appendix B Difference of modal amplitude from the analytical model

Table B-1 Difference of amplitude in 0.25mm under uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
25	-0.057	-0.043	0.007	-0.040
50	0.143	0.257	0.307	0.170
100	0.153	0.420	0.533	0.200
150	0.187	0.450	0.613	0.310
200	0.317	0.553	0.553	0.440

Table B-2 Difference of amplitude in 0.5mm under uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
25	0.007	-0.033	0.017	-0.010
50	0.047	0.107	0.197	0.050
100	0.117	0.183	0.273	0.160
150	0.017	0.300	0.537	0.070
200	0.057	0.213	0.527	0.100

Table B-3 Difference of amplitude in 1mm under uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
25	0.003	0.070	0.080	0.050
50	0.040	0.167	0.180	0.080
100	-0.057	0.177	0.300	0.100
150	-0.083	0.213	0.377	0.160
200	-0.107	0.163	0.353	0.130

Table B-4 Difference of amplitude in 0.25mm under non-uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
27–25	-0.060	-0.047	0.000	-0.040
50–33	-0.013	0.127	0.223	0.070
100–80	0.057	0.303	0.477	0.170
150–131	0.197	0.413	0.597	0.300
200–185	0.303	0.493	0.653	0.400

Table B-5 Difference of amplitude in 0.5mm under non-uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
27–25	0.013	0.003	0.020	0.000
50–33	0.030	0.117	0.187	0.050
100–80	0.077	0.123	0.197	0.080
150–131	0.047	0.277	0.457	0.120
200–185	0.027	0.253	0.480	0.090

Table B-6 Difference of amplitude in 0.5mm under non-uniform temperature

Temperature °C	Ramp 2°C/min	Ramp 5°C/min	Ramp 8°C/min	Numerical
27–25	0.047	0.087	0.097	0.110
50–33	0.007	0.113	0.220	0.150
100–80	0.137	0.233	0.433	0.110
150–131	0.037	0.207	0.390	0.080
200–185	0.103	0.203	0.420	0.070

Appendix C Crack propagation crack depth and modal parameters

Table C-1 Crack propagation at a uniform temperature of 100 °C for pre-seeded 0.25 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.434	29	211	67	66.7722	0.0150	16.2	16.1	18	17.54
1.168	78	211	68	66.7722	0.0150	12.34	11.89	15.11	14.76
1.776	118	210	67	66.4557	0.0150	11.12	10.16	14.76	13.56
2.142	143	211	69	66.7722	0.0150	9.25	8.75	10.79	10.12
2.434	161	209	68	66.1392	0.0151	8.19	7.38	7.12	6.53
2.546	170	211	67	66.7722	0.0150	7.1	6.27	5.71	4.92

Table C-2 Crack propagation at a uniform temperature of 150 °C for pre-seeded 0.25 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.867	56	204	66	64.5570	0.0155	15.1	14.74	17.76	17.46
1.680	109	205	67	64.8734	0.0154	12.83	12.61	15.41	14.78
1.850	120	205	66	64.8734	0.0154	11.83	11.33	13.11	12.68
2.194	143	206	67	65.1899	0.0153	10.68	10.12	11.72	11.31
2.436	158	205	67	64.8734	0.0154	8.07	7.61	7.05	6.54

Table C-3 Crack propagation at a uniform temperature of 200 °C for pre-seeded 0.25 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.881	58	208	66	65.8228	0.0152	14.41	14.11	17.29	17.1
1.124	74	208	67	65.8228	0.0152	13.37	12.92	16.72	16.36
1.694	111	207	67	65.5063	0.0153	12.13	11.87	15.09	14.67
2.248	148	208	67	65.8228	0.0152	8.91	8.35	10.04	9.12
2.534	166	207	68	65.5063	0.0153	7.13	6.76	5.78	4.96

Table C-4 Crack propagation at a uniform temperature of 100 °C for pre-seeded 0.5 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.6955	46	209	67	66.1392	0.0151	13.21	13	16.64	16.21
0.8427	56	210	68	66.4557	0.0150	12.76	12.45	16.19	15.87
1.5950	106	210	68	66.4557	0.0150	9.87	9.37	12.16	11.79
1.7606	117	210	68	66.4557	0.0150	8.72	8.31	9.75	9.31
2.2614	151	211	69	66.7722	0.0150	6	5.75	4.61	4.23

Table C-5 Crack propagation at a uniform temperature of 150 °C for pre-seeded 0.5 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.4470	29	205	65	64.8734	0.0154	13.76	13.55	15.52	15.11
0.9357	61	206	66	65.1899	0.0153	12.11	11.8	14.93	14.37
1.3434	88	207	66	65.5063	0.0153	10.21	9.89	12.48	12
1.9114	124	205	65	64.8734	0.0154	8.12	7.87	8.13	7.83

Table C-6 Crack propagation at a uniform temperature of 200 °C for pre-seeded 0.5 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.4536	30	209	66	66.1392	0.0151	13.61	13.24	15.18	14.87
0.7900	52	208	67	65.8228	0.0152	12.54	12.1	14.89	14.65
0.9817	64	206	66	65.1899	0.0153	11.86	11.56	14.2	13.86
1.5876	105	209	67	66.1392	0.0151	9.82	9.34	12.08	11.65
1.9504	129	209	66	66.1392	0.0151	7.76	7.38	7.63	7.13

Table C-7 Crack propagation at a uniform temperature of 50 °C for pre-seeded 1 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.4182	27	204	65	64.5570	0.0155	12.12	11.97	14.93	14.53
0.9590	61	201	64	63.6076	0.0157	10.15	9.89	12.35	11.96
1.1208	72	203	65	64.2405	0.0156	9.74	9.36	11.87	11.42
1.4321	92	203	65	64.2405	0.0156	8.07	7.67	7.69	7.32

Table C-8 Crack propagation at a uniform temperature of 100 °C for pre-seeded 1 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.5817	37	201	62	63.6076	0.0157	11.95	11.43	14.46	14.11
0.8604	55	202	64	63.9241	0.0156	10.58	10.1	12.63	12.21
1.1986	77	203	64	64.2405	0.0156	9.24	8.86	11.15	10.79
1.4166	91	203	64	64.2405	0.0156	8.12	7.88	7.75	7.24
1.6420	106	204	65	64.5570	0.0155	7.87	7.35	7.18	6.62

Table C-9 Crack propagation at a uniform temperature of 150 °C for pre-seeded 1 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.4710	31	208	66	65.8228	0.0152	12.07	11.89	14.86	14.54
0.7444	49	208	66	65.8228	0.0152	11.53	11.23	14.26	13.98
1.0533	69	207	65	65.5063	0.0153	9.83	9.56	11.98	11.67
1.5952	105	208	65	65.8228	0.0152	7.88	7.48	7.18	6.69
1.7841	118	209	66	66.1392	0.0151	5.13	4.76	4.18	3.77

Table C-10 Crack propagation at a uniform temperature of 200 °C for pre-seeded 1 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.4427	29	207	64	65.5063	0.0153	12.11	11.94	14.89	14.35
0.7409	49	209	65	66.1392	0.0151	11.5	11	14.21	13.76
1.1850	78	208	65	65.8228	0.0152	9.26	8.91	11.17	10.81
1.5496	102	208	66	65.8228	0.0152	7.95	7.51	7.26	6.72
1.8166	119	207	66	65.5063	0.0153	5.71	5.39	4.43	4

Table C-11 Crack propagation at a non-uniform temperature of 50-85 °C for pre-seeded 1.3 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.389	26	211	67	66.7722	0.0150	12.11	11.81	14.21	13.65
0.722	48	210	66	66.4557	0.0150	10.76	10.17	12.18	11.7
1.198	80	211	66	66.7722	0.0150	9.16	8.56	10.42	9.63
1.573	105	211	67	66.7722	0.0150	7.68	7.11	7.23	6.51

Table C-12 Crack propagation at a non-uniform temperature of 131-150 °C for pre-seeded 1.3 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.346	23	210	66	66.4557	0.0150	12.23	11.79	14.24	14.11
0.760	51	212	67	67.0886	0.0149	9.59	8.91	11.11	10.76
1.252	84	212	67	67.0886	0.0149	8.13	7.48	9.08	8.75
1.416	95	212	66	67.0886	0.0149	7.7	7.14	7.54	6.97

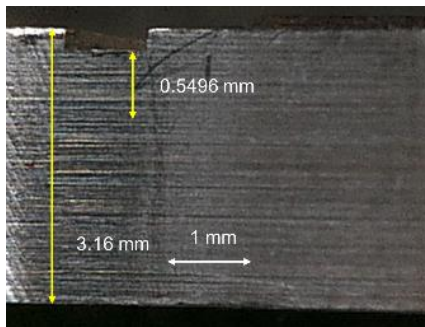
Table C-13 Crack propagation at a non-uniform temperature of 50-85 °C for pre-seeded 1.5 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.3206	21	207	65	65.5063	0.0153	10.87	10.71	12.25	12
0.5292	35	209	66	66.1392	0.0151	9.72	9.65	11.97	11.69
0.9781	65	210	66	66.4557	0.0150	7.86	7.71	9.84	9.57
1.2490	83	210	66	66.4557	0.0150	6.52	6.3	7.99	7.75

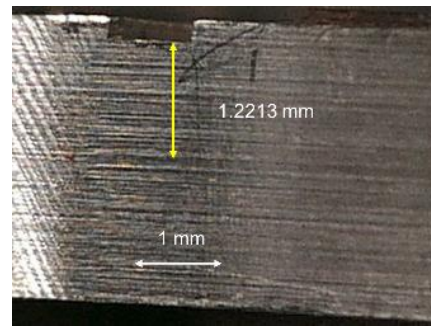
Table C-14 Crack propagation at a non-uniform temperature of 131-150 °C for pre-seeded 1.5 mm crack depth

Crack Depth (mm)	Crack Depth Pixel	Total Pixel	Scale Pixel	1 mm = Pixel	1 Pixel = mm	Natural Frequency (Hz)		Modal Amplitude (mm)	
						Experiment	Analytical	Experiment	Analytical
0.2845	19	211	65	66.7722	0.0150	10.93	10.85	12.35	12.1
0.7418	50	213	66	67.4051	0.0148	8.61	8.43	11.12	10.75
0.9391	63	212	66	67.0886	0.0149	7.82	7.69	9.9	9.67
1.0334	69	211	66	66.7722	0.0150	7.11	6.84	8.55	8.32

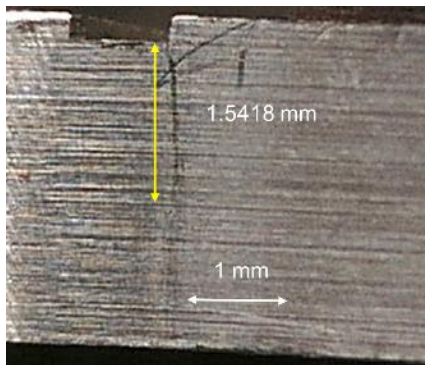
Appendix D Crack propagation evaluation



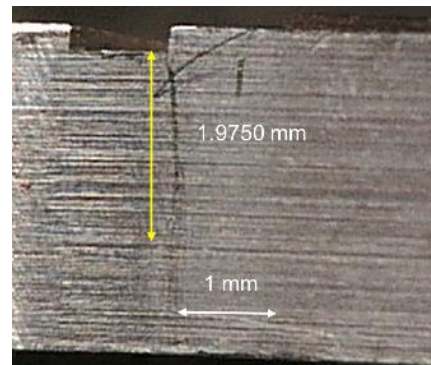
(a)



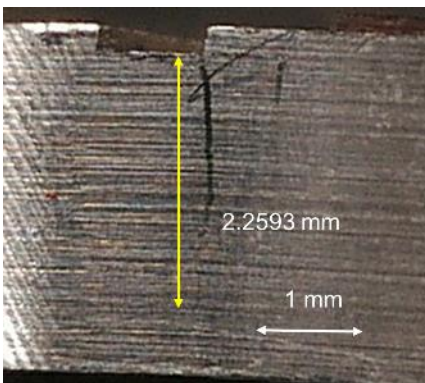
(b)



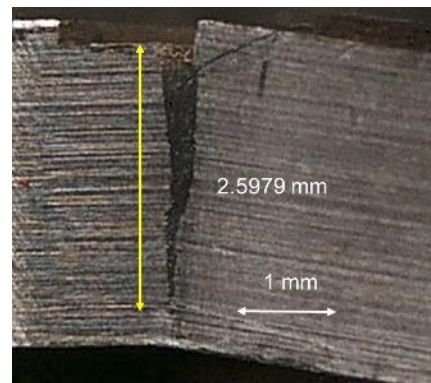
(c)



(d)

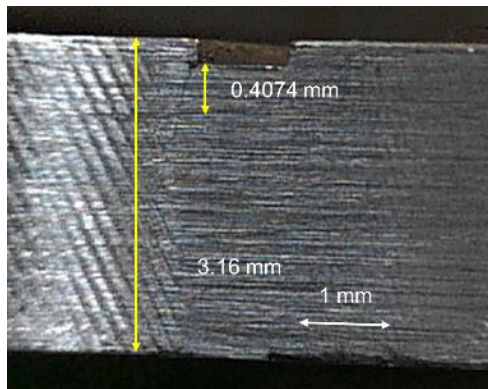


(e)

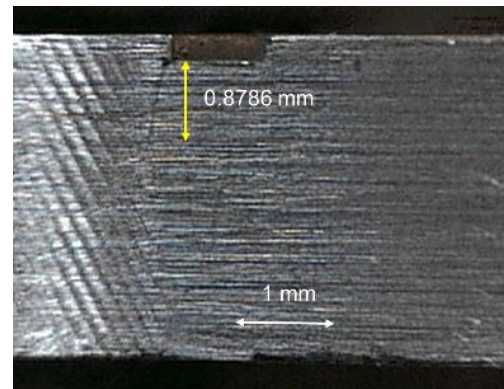


(f)

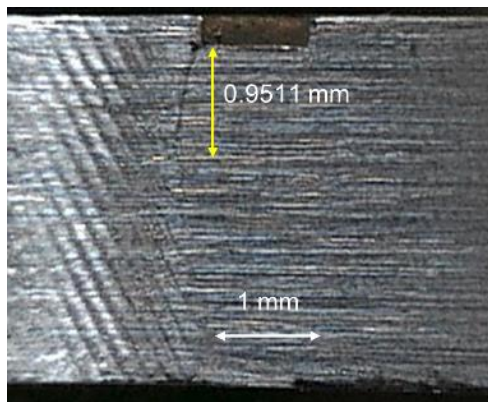
Figure D-1 Sequence of crack propagation of pre-seeded 0.25 mm crack depth at room temperature (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture, (f) 6th capture



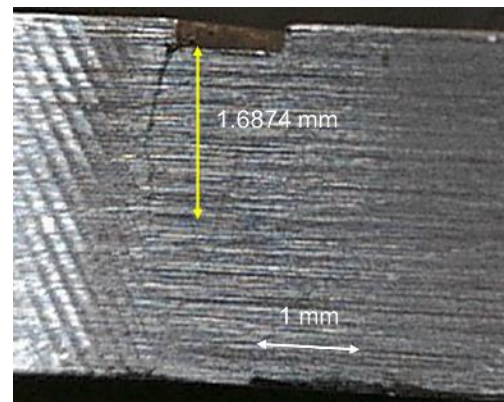
(a)



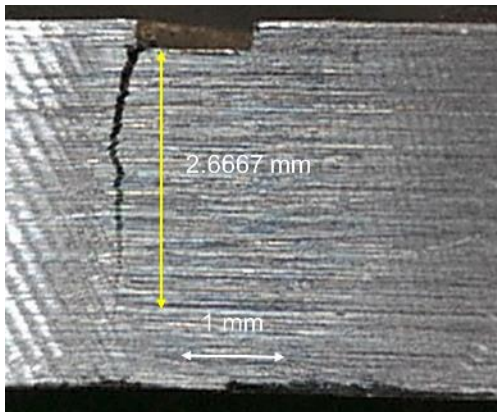
(b)



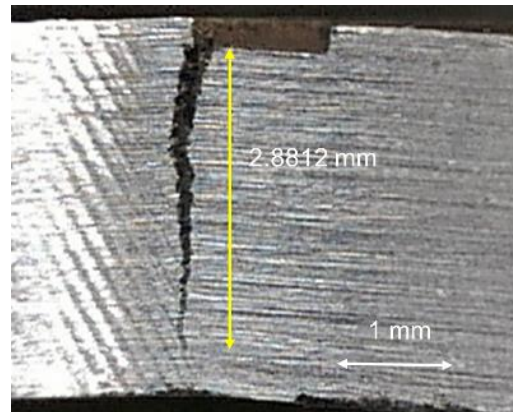
(c)



(d)

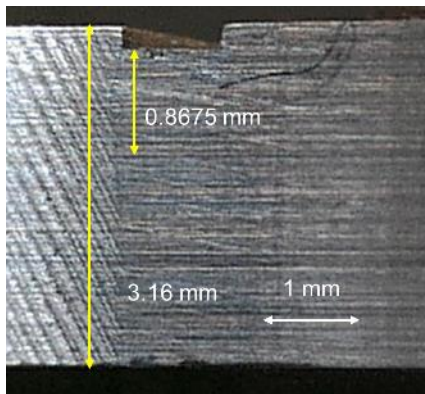


(e)

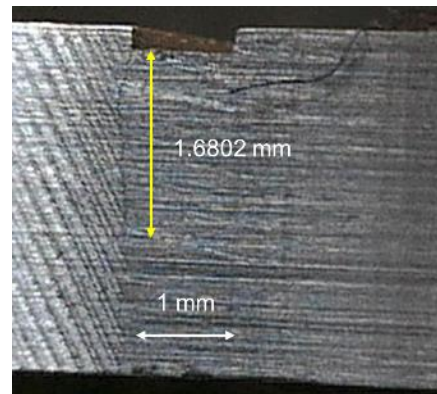


(f)

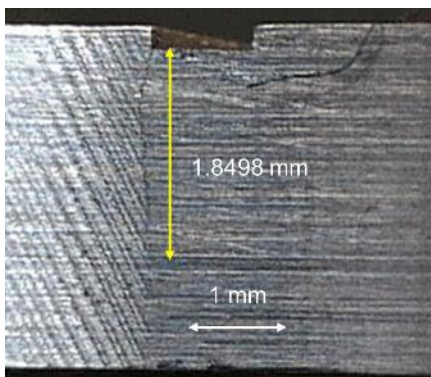
Figure D-2 Sequence of crack propagation of pre-seeded 0.25 mm crack depth at 50°C (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture, (f) 6th capture



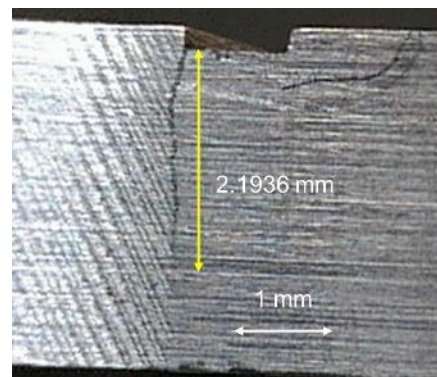
(a)



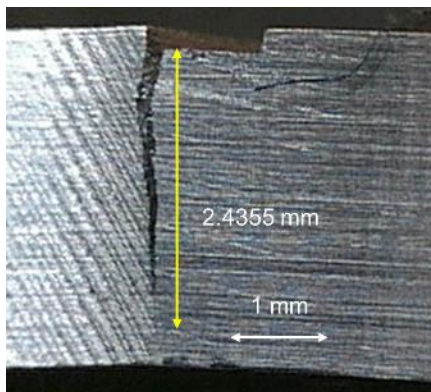
(b)



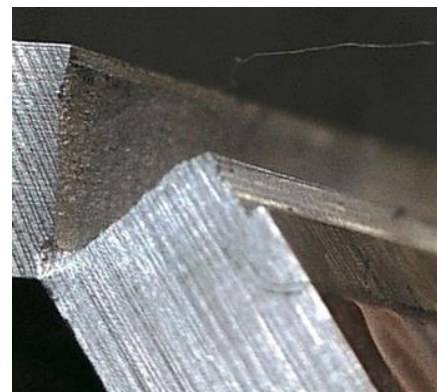
(c)



(d)

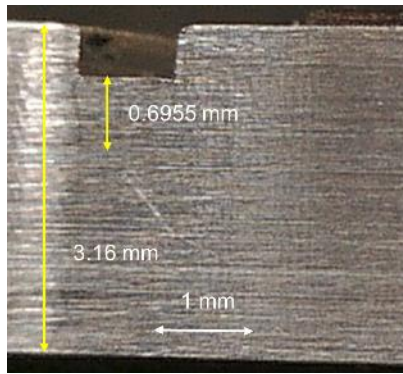


(e)

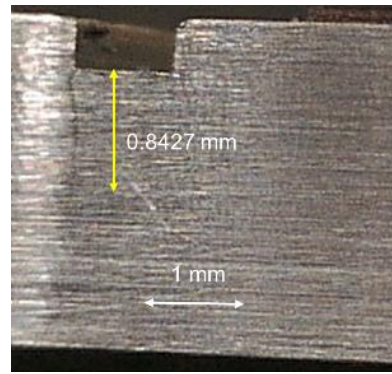


(f)

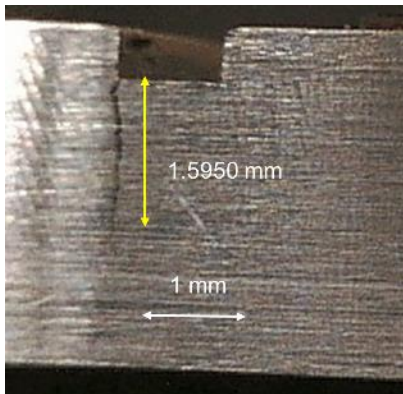
Figure D-3 Sequence of crack propagation of pre-seeded 0.25 mm crack depth at 150°C (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture, (f) 6th capture



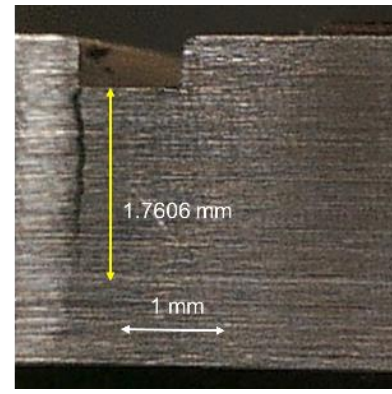
(a)



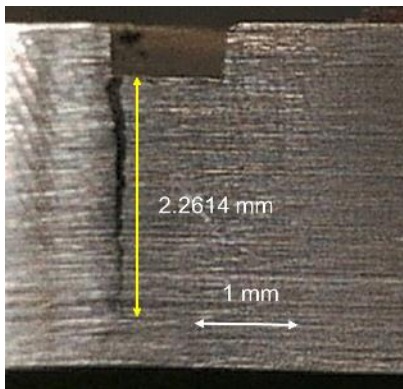
(b)



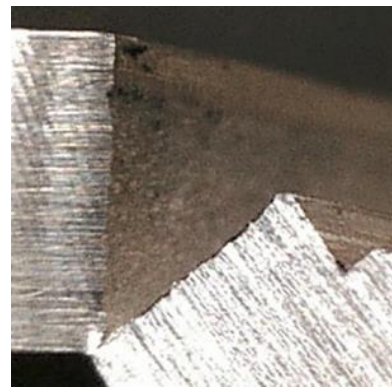
(c)



(d)

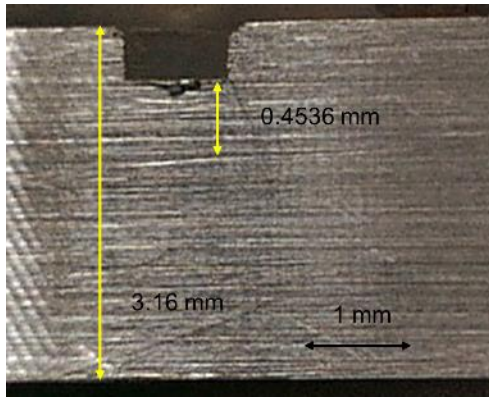


(e)

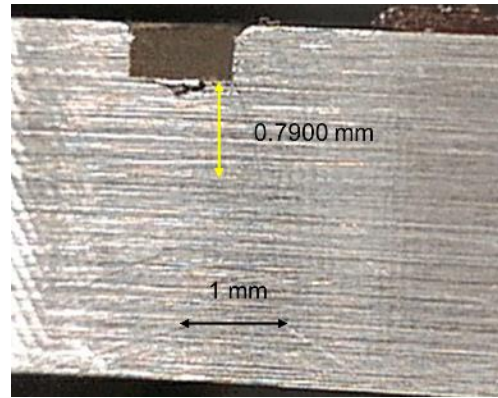


(f)

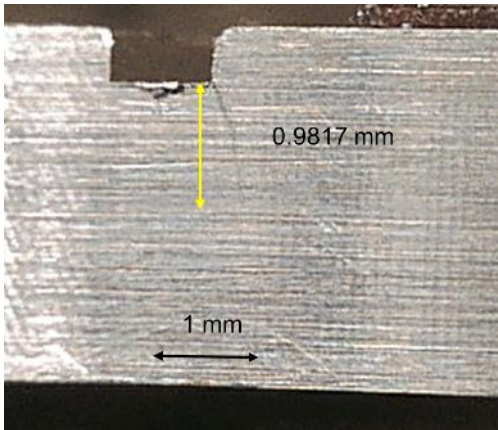
Figure D-4 Sequence of crack propagation of pre-seeded 0.5 mm crack depth at 100°C (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture, (f) 6th capture



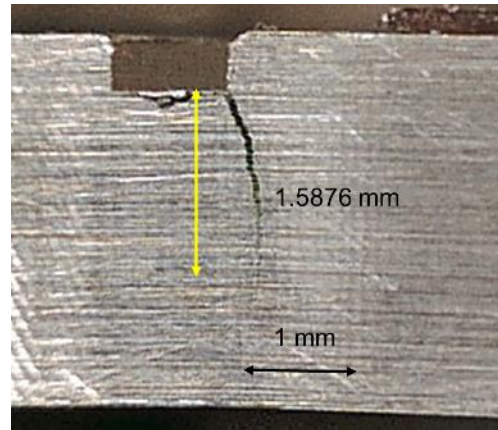
(a)



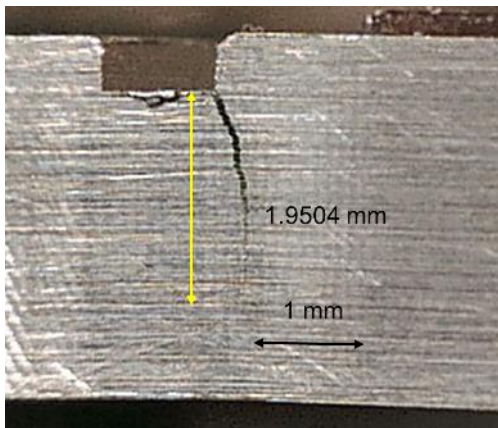
(b)



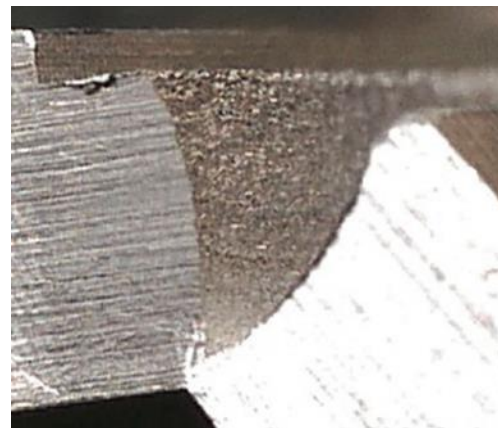
(c)



(d)

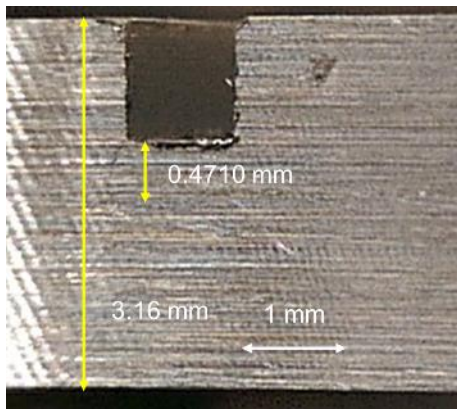


(e)

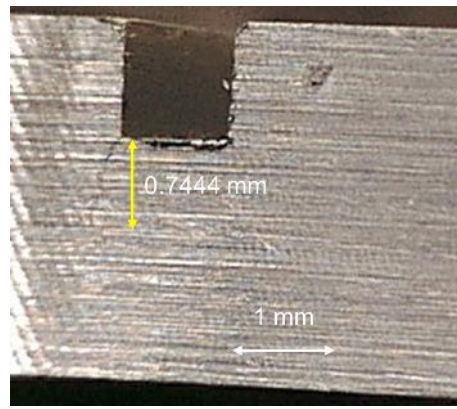


(f)

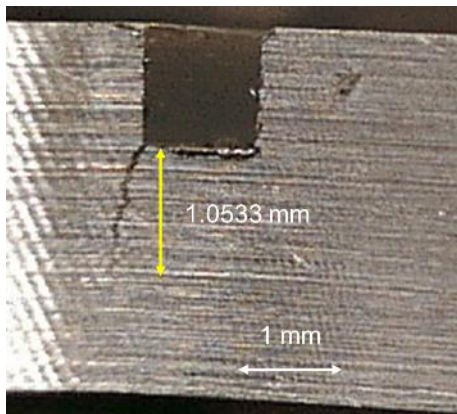
Figure D-5 Sequence of crack propagation of pre-seeded 0.5 mm crack depth at 200°C (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture, (f) 6th capture



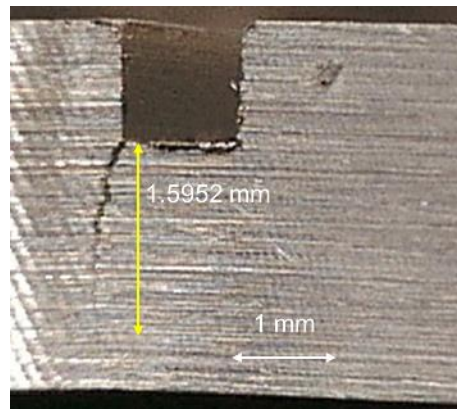
(a)



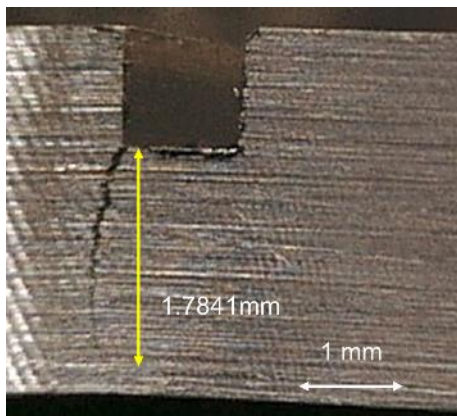
(b)



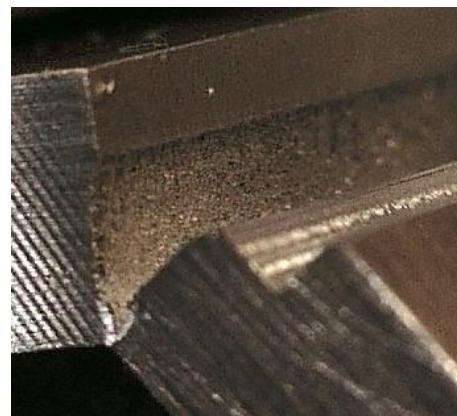
(c)



(d)

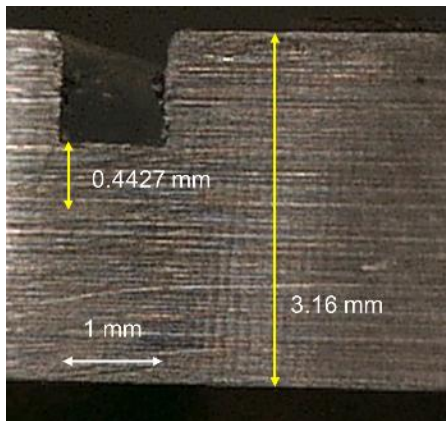


(e)

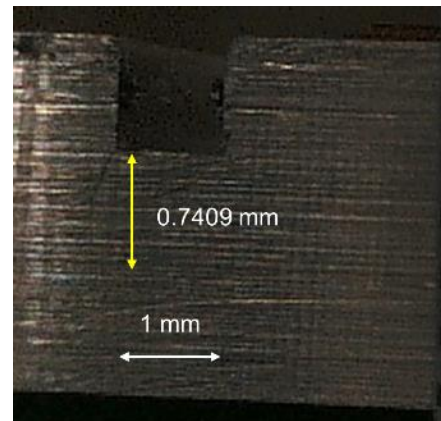


(f)

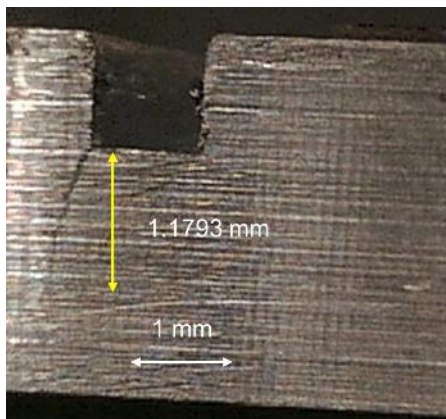
Figure D-6 Sequence of crack propagation of pre-seeded 1 mm crack depth at 150°C (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture, (f) 6th capture



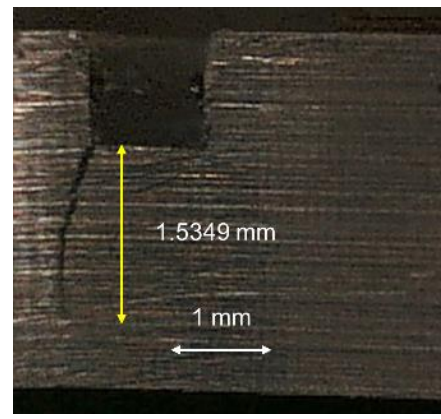
(a)



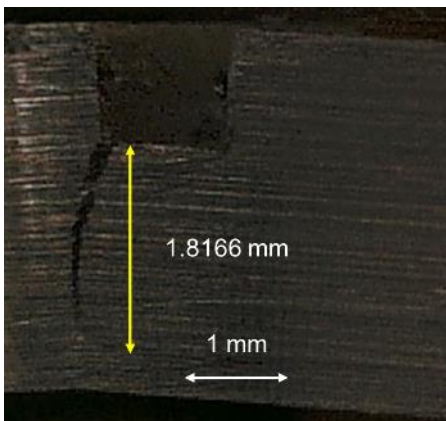
(b)



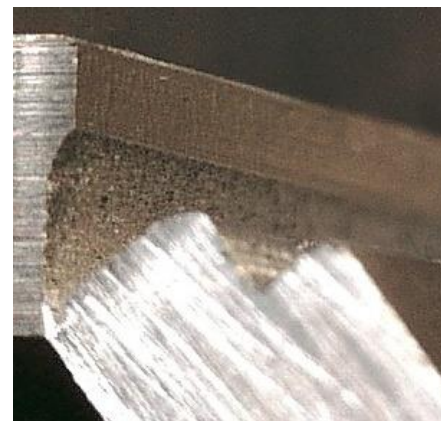
(c)



(d)

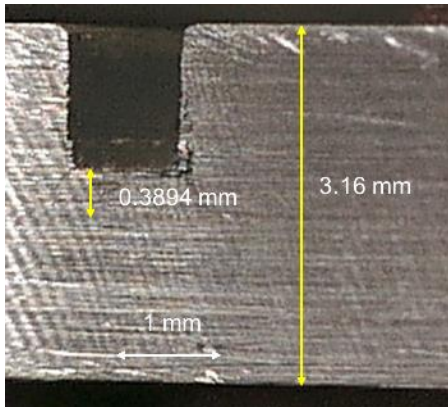


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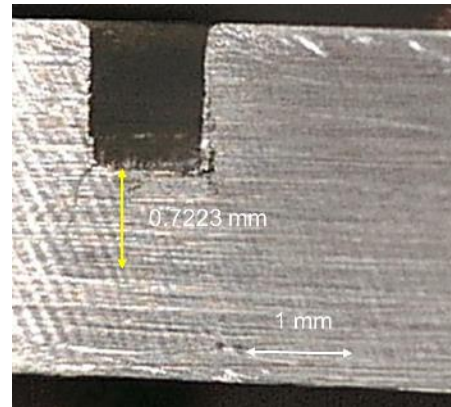


(f)

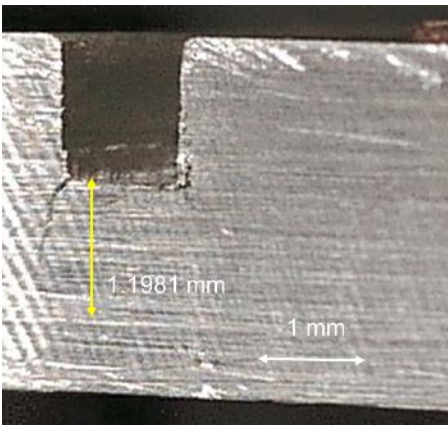
Figure D-7 Sequence of crack propagation of pre-seeded 1 mm crack depth at 200°C (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture, (f) 6th capture



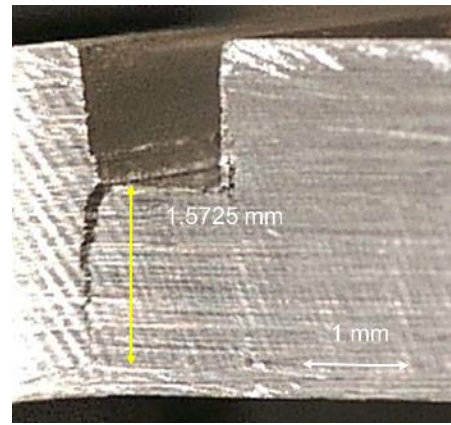
(a)



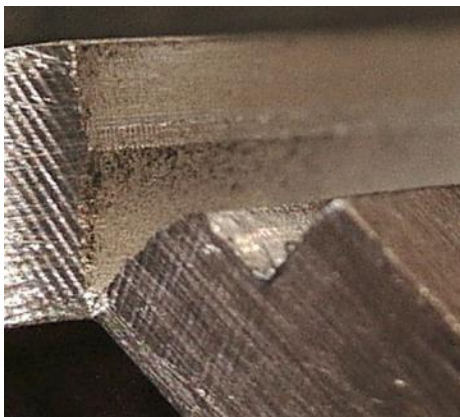
(b)



(c)

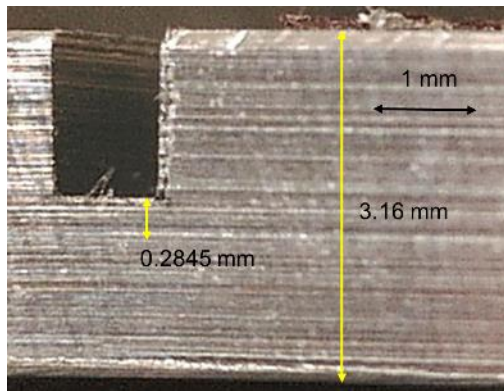


(d)

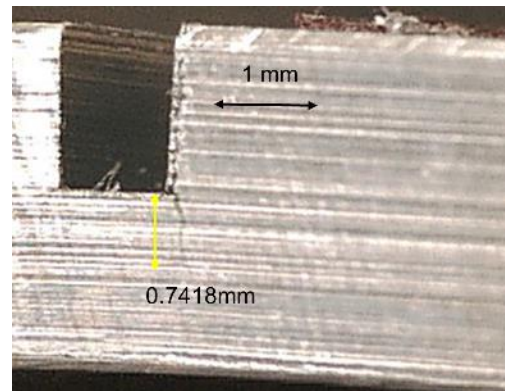


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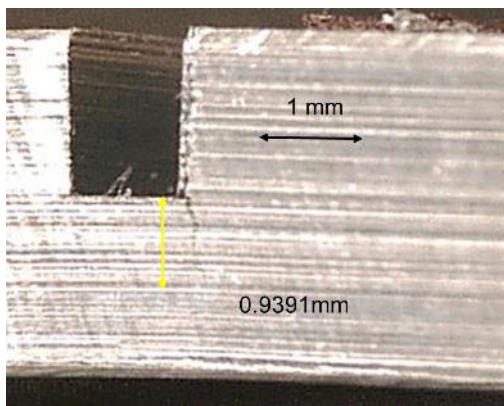
Figure D-8 Sequence of crack propagation of pre-seeded 1 mm crack depth at 50-85°C (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture



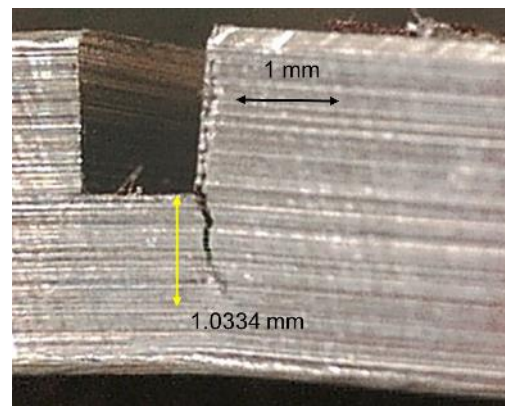
(a)



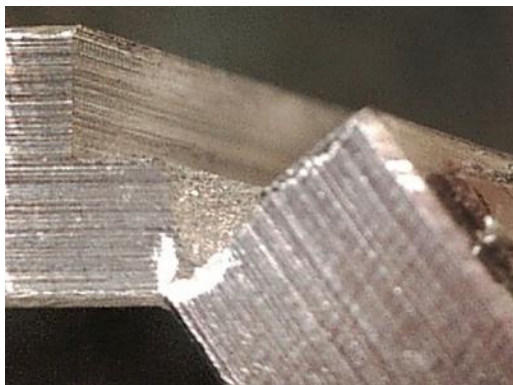
(b)



(c)



(d)



(e)

Figure D-9 Sequence of crack propagation of pre-seeded 1.5 mm crack depth at 131-150°C (a) 1st capture at crack initiation, (b) 2nd capture, (c) 3rd capture, (d) 4th capture, (e) 5th capture

Appendix E Specification of the equipment's

Table E-1 Strain gage: OMEGA/SGD Series

Geometry	Linear strain gage with one measurement grid measuring strain in one direction
Model	SGD-10/350-LY13
High Temperature Range	-75 to 200°C
Measuring grid lengths	17.7 mm
Nominal Resistance	350 Ω

Table E-2 Silicone Heater Mat Specifications

Mat Shape	Rectangle
Power Rating	1.25 W
Rectangular Mat Size	25 x 50mm
Peak Temperature	+300°C
Supply Voltage	12 V dc
Mounting	Adhesive

Table E-3 PCB Miniature accelerometer Model: M357B11

Sensitivity:	(± 10 %) 3.0 pC/g
Frequency Range:	(+5 %)12 kHz
Weight:	2 gm
Temperature Range	-71 to +260 °C
Mounting	Thread

Appendix F MATLAB code

```
% characteristic equation
eqn = @(beta, L) cos(beta .* L) .* cosh(beta .* L) + 1;
% define values for L
L = 1:4;
% call fsolve
sol = fsolve(@(beta)eqn(beta,L),[1 1 1 1])
% test the quality of the result - all results should be near zero
test = eqn(sol,L)
% plot results
plot(L, sol)
xlabel('L')
ylabel('beta')

%Calculation of frequency and amplitude
L = 200;
matx = 1:33:200;
matx = matx(:);
B = 25;
h = 3.15;
v = 0.33;
V = L*B*h;
A = L*B;
I = 1/2*B*h^3;
P = 1.25*4;
hro = 0.00278;
u_0 = 2;
Phi = 0.3;
T = [200 150 100 50 25];
T_0 = 570;
T1 = 200;
T2 = 24;
E_0 = 74;
E1 = E_0*(1-Phi*T./T_0);
n = 5;
En = mean(E1);
En1 = 70.83552;
En2 = 71.72807;
En3 = 72.53945;
En4 = 73.0263;
Ee = [69.9105, 70.83552, 71.72807, 72.53945, 73.0263];
St= 100;
AL = 0.0247/1000;
ex = St./En + AL*(T2-T);
ex1 = St./En + AL*(T2-T1);
AL1 = ex1./(T1-T2);
Le = L + ex;
T_x = ((24-T)./Le).*matx + T;
dT = T-T2;
bl = 1.875104;
beta = 1.875104/200;
```

```

m = 2;
f = ((b1)^2/2*pi)*(sqrt(I.*E1*10^9./((hro*A+m).*Le.^4)));
f1 = ((b1)^2/2*pi)*(sqrt(I.*Ee*10^9./((hro*A*m).*Le.^4)));
A_0 = 2;
A1 = ((sin(beta*Le)+sin(beta*Le))/(cos(beta*Le)+cosh(beta*Le)));
Y = A_0*sin(f)*((sin(b1)-sinh(b1))-A1*(cos(b1)-cosh(b1)));
Y1=A_0*sin(f1)*((sin(b1)-sinh(b1))-A1*(cos(b1)-cosh(b1)));
C = (sin(beta*L)+ sinh(beta*L))/(cos(beta*L)+cosh(beta*L));
x = 1;
M = -E1*I*beta^2*A_0*((sin(beta*x)+sinh(beta*x)-
C*(cos(beta*x)+cosh(beta*x))));
Q1 = ((Le/2)-sin(2*(beta.*Le))/(4*beta))+((Le/2)-
sinh(2*(beta.*Le))/(4*beta))-
(cos(beta.*Le).*sinh(beta.*Le)/beta)+(sin(beta.*Le).*cosh(beta.*Le)/beta;
Q2 =
((Le/2)+sin(2*(beta.*Le))/(4*beta))+((Le/2)+sinh(2*(beta.*Le))/(4*beta))+
(sin(beta.*Le).*cosh(beta.*Le)/beta)+(cos(beta.*Le).*sinh(beta.*Le)/beta;
Q = E1.*I/2.*A_0^2*beta^4.*(Q1-C^2.*Q2);
t_c1 = 0.25;
D1 = t_c1/h;
F1 = 0.638*D1^2 - 1.035*D1^3 + 3.720*D1^4 - 5.177*D1^5 + 7.553*D1^6 -
7.332*D1^7 + 2.491*D1^8;
chi1 =B*h^2.*E1./(72.*pi.*F1);
dQ1 = M.^2./(2.*chi1);
t_c2 = 0.5;
D2 = t_c2/h;
F2 = 0.638*D2^2 - 1.035*D2^3 + 3.720*D2^4 - 5.177*D2^5 + 7.553*D2^6 -
7.332*D2^7 + 2.491*D2^8;
chi2 =B*h^2.*E1./(72.*pi.*F2);
dQ2 = M.^2./(2.*chi2);
t_c3 = 1;
D3 = t_c3/h;
F3 = 0.638*D3^2 - 1.035*D3^3 + 3.720*D3^4 - 5.177*D3^5 + 7.553*D3^6 -
7.332*D3^7 + 2.491*D3^8;
chi3 =B*h^2.*E1./(72.*pi.*F3);
dQ3 = M.^2./(2.*chi3);
wnc = (1-dQ1./(2*Q)).*f;
wnc1 = (1-dQ2./(2*Q)).*f;
wnc2 = (1-dQ3./(2*Q)).*f;
Y2 =A_0*sin(wnc)*((sin(b1)-sinh(b1))-A1*(cos(b1)-cosh(b1)));
Y3=A_0*sin(wnc1)*((sin(b1)-sinh(b1))-A1*(cos(b1)-cosh(b1)));
Y4 =A_0*sin(wnc2)*((sin(b1)-sinh(b1))-A1*(cos(b1)-cosh(b1)));

%Experimental data extraction
%Import amp
data = table2array(Amp25)
acc = data(:,2)
time = data(:,1)
plot(time,acc)
data1 = table2array(Amp50)
acc1 = data1(:,2)
time1 = data1(:,1)

```

```

plot(time1,acc1)
data2 = table2array(Amp100)
acc2 = data2(:,2)
time2 = data2(:,1)
plot(time2,acc2)
data3 = table2array(Amp150)
acc3 = data3(:,2)
time3 = data3(:,1)
plot(time3,acc3)
data4 = table2array(Amp200)
acc4 = data4(:,2)
time4 = data4(:,1)
plot(time4,acc4)
subplot(2,3,1); plot(time, acc)
subplot(2,3,2); plot(time1, acc1)
subplot(2,3,3); plot(time2, acc2)
subplot(2,3,4); plot(time3,acc3)
subplot(2,3,5); plot(time4,acc4)
han=axes('visible', 'off')
han.Title.Visible='on'
han.XLabel.Visible='on'
han.YLabel.Visible='on'
title('Crack depth of 0.25mm under uniform temperature')
ylabel(han,'Acceleration m/s^2')
xlabel(han,'Time (s)')
%Import Temp
Data10 = table2array(T25)
Acc10 = data10(:,2)
Time10 = data10(:,1)
plot(time10,acc10)
data11 = table2array(T50)
acc11 = data11(:,2)
time11 = data11(:,1)
plot(time11,acc11)
data22 = table2array(T100)
acc22 = data22(:,2)
time22 = data22(:,1)
plot(time22,acc22)
data33 = table2array(T150)
acc33 = data33(:,2)
time33 = data33(:,1)
plot(time33,acc33)
data44 = table2array(T200)
acc44 = data44(:,2)
time44 = data44(:,1)
plot(time44,acc44)
subplot(2,3,1); plot(time10, acc10)
subplot(2,3,2); plot(time11, acc11)
subplot(2,3,3); plot(time22, acc22)
subplot(2,3,4); plot(time33,acc33)
subplot(2,3,5); plot(time44,acc44)
han=axes('visible', 'off')

```

```
han.Title.Visible='on'  
han.XLabel.Visible='on'  
han.YLabel.Visible='on'  
title('Crack depth of 0.25mm under uniform temperature')  
ylabel(han, 'Acceleration m/s^2')  
xlabel(han, 'Time (s)')
```