

A New Heuristic Approach to Rotorcraft System Identification

Linghai Lu (corresponding author, l.lu@cranfield.ac.uk)

*Senior Lecturer,
Cranfield University,
Cranfield, United Kingdom*

Dheeraj Agarwal
*Lecturer,
University of Liverpool,
Liverpool, United Kingdom*

Gareth D. Padfield
*Emeritus Professor,
University of Liverpool,
Liverpool, United Kingdom*

Mark D. White
*Professor,
University of Liverpool,
Liverpool, United Kingdom*

Neil Cameron
*Senior Simulation Engineer
Vertical Aerospace
Bristol, UK*

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Abstract

High-fidelity rotorcraft flight simulation relies on the availability of a quality flight model that further demands a good level of understanding of the complexities arising from aerodynamic couplings and interference effects. This paper explores rotorcraft flight dynamics in the low-speed regime where such complexities abound and presents a new heuristic approach in the time domain to aid identification of nonlinear dynamics and fidelity assessment. The approach identifies flight model parameters ‘additively’, based on their contribution to the local dynamic response of the system, in contrast with conventional approaches where parameter values are identified to minimize errors over a whole maneuver. In these early investigations, identified low-order, rigid-body, linear models show good comparison with flight test data. The approach is extended to explore nonlinearities attributed to so-called maneuver wake distortion and wake skew effects emerging in larger maneuvers. The results show a good correlation for the proposed nonlinear model structure, demonstrated by its capability to capture the time response and variations of the stability and control derivatives with response magnitude.

Nomenclature

A, B	=	state and control matrices in the state-space form [-]
g	=	acceleration due to gravity [ft/s ²]
I_{xz}	=	product of inertia about the helicopter x and z axes [slug·ft ²]
K_{X_a}	=	gain between rotor swashplate and pilot's stick deflection [rad/inch]
L, M, N	=	external aerodynamic moments about the x -, y - and z -axes [ft·lbf]
L_v, M_q etc.	=	moment stability derivatives [rad/(s·ft), 1/s etc.]
L_{X_a}, M_{X_b} , etc.	=	moment control derivatives [rad/(s ² ·inch) etc.]
M_d, M_k, N_d	=	number of analysis windows, data point in an analysis window, and data points, respectively [-]
nk	=	number of input samples before the input affects the output
N_t	=	number of data points up to the time point t_{x_n} [-]
p, q, r	=	angular rates about fuselage x -, y -, z -axes [deg/s, rad/s]
$\dot{p}, \dot{q}$	=	pitch and roll accelerations [deg/s ² , rad/s ²]
T	=	local time-period for identifying x_i^{th} derivative [s]
t_{x_n}, T_m	=	time point and analysis window period, respectively, [s]
u	=	control input vector [-]
u, v, w	=	surge, sway, and heave velocities along fuselage x -, y -, z -axes [ft/s]
$\dot{u}, \dot{v}$	=	rates of change of translational velocities [ft/s ²]
x_i	=	i^{th} state/control variable [-]
X_a, X_b	=	pilot lateral and longitudinal cyclic stick inputs [inch]
X_c, X_p	=	pilot collective and pedal inputs [inch]
X_v, X_r etc.	=	X force stability derivatives [1/s, ft/(s·rad) etc.]
Y_p, Y_v , etc.	=	Y force stability derivatives [1/s, ft/(s·rad) etc.]
Y_{x_i}, Y_{X_j} , etc.	=	stability and control derivatives
Y_v, Y_p, Z_w etc.	=	Y and Z force stability derivatives [1/s, ft/(s·rad) etc.]
z_t	=	measured dependent variable [-]
β_{1s}	=	lateral cyclic flapping angle [deg, rad]

Δt	=	sampling interval [s]
γ	=	rotor Lock number [-]
θ_{lc}	=	lateral cyclic pitch angle (equivalent to swashplate angle) [deg, rad]
θ, ϕ	=	fuselage Euler pitch and roll angles [deg, rad]
ρ	=	correlation coefficient [-]
τ_f	=	rotor flap time constant [s]
τ	=	effective time delay in each control channel [s]
$\tau_{x_{at}}$	=	transport delay in the lateral channel [s]
v	=	measurement noise [-]
χ	=	wake skew angle [deg]
ω_{lp}	=	lateral phugoid natural frequency
Ω	=	rotor speed [rad/s]

Symbols

$\cdot$	=	derivative with respect to time
$\hat{\cdot}$	=	identified value
$\sim$	=	variable scaled with control input amplitude
$-$	=	variables in the Laplace plane

Acronyms

ARX	=	Autoregressive with Extra Input
ASRA	=	Advanced Systems Research Aircraft
ASysID	=	Additive System IDentification
B412	=	Bell 412
CS	=	Certification Specification
DoF(s)	=	Degree(s)-of-Freedom
EE	=	Equation Error
F-B412	=	Non-linear FLIGHTLAB Bell 412 model
FSI	=	Finite State Inflow
FSTD(H)	=	Flight Simulation Training Devices (Helicopter)

FT	=	Flight Test
LF-B412	=	Linear FLIGHTLAB Bell 412 model
MWD	=	Maneuver Wake Distortion
NRC	=	National Research Council of Canada
RWI	=	Rotor Wake Interference
SysID	=	System IDentification
SWR	=	StepWise Regression

I. Introduction

The ability to replicate real-world flight behavior in a virtual environment is key to achieving sufficiently high flight-model fidelity to enable pilots to train to operate aircraft safely and designers to evaluate and optimize concepts with confidence. System IDentification (SysID) can be described as a rational and systematic approach for supporting the development and validation of such high-fidelity aircraft flight dynamic models using flight test (FT) data [1–3]. SysID can be undertaken in either the frequency domain [1,2,4] or time domain [1,3,5]. In the former, identification of a model is usually performed using control frequency sweeps, while the latter typically uses multi-step control inputs such as doublets, 3211/2311s etc. The approach has been used extensively for fixed-wing aircraft [1,6], unmanned aircraft [7–9], and rotorcraft [2,10–13] for performance and handling qualities evaluation, control law development, aircraft dynamic loads analysis, dynamic inflow modelling, and the creation of models for use in piloted simulation environments. SysID has been applied as an effective approach for informing these tuning processes. The NATO AVT-296 activities have documented various methods for updating flight models with flight data, together with simulator assessment methodologies [14].

Important to the success of SysID is the availability of good-quality (i.e. minimal measurement and process errors and noise) FT data. However, there are challenges in obtaining suitable data for rotorcraft in hover and low-speed flight regimes, as the test data often exhibit low signal-to-(process) noise ratios and the rotorcraft model can feature strong nonlinearities. Furthermore, the helicopter in hover requires secondary corrective inputs from a pilot to prevent large deviations from the trim condition. This leads to input/output correlation problems and the characteristics of bare-airframe rotorcraft responses at hover and low-speed pose challenges in the use of traditional SysID approaches.

In previous work by the authors [15], a SysID technique was developed and applied to improve the fidelity of a FLIGHTLAB [16] Bell 412 (F-B412) simulation model. The identified model was compared with FT data over a range of forward flight conditions. The FT data were from trials with the National Research Council of Canada (NRC) Advanced Systems Research Aircraft (ASRA) Bell 412 (B412) helicopter. A group of SysID candidate derivatives were used to improve the off-axis responses of the model. The work continues in the current project [17] aimed at developing a physics-based toolset for flight-model fidelity enhancement, using a heuristic approach to SysID in the time domain. While conventional SysID time and frequency domain methods are adequate for capturing linear, instantaneous dynamics, the motivation behind our new approach is to identify parameters in more complex model structures representative of the nonlinear, hysteretic nature of rotor wake dynamics. We refer to this method as Additive System IDentification (ASysID), with the model parameters identified additively based on their physical contribution to the local dynamic response of the system. One or more candidate parameters in the proposed model structure are identified using the primary response characteristics of the rotorcraft; others

are then identified in an additive manner. Preliminary work is reported in this paper, including application to the creation of extended linear and non-linear models to capture and describe the responses to classical multi-step control inputs from a nominal hover condition.

The paper is structured as follows. Potential nonlinear behavior in rotorcraft dynamics is discussed in Section II. Detailed developments of the ASysID approach are presented in Section III. The analyses using both simulation and FT data at hover are conducted in Sections IV and V, respectively, to demonstrate its effectiveness. In Section VI, the approach is applied to explore nonlinearities excited by large amplitude control inputs, particularly related to so-called Maneuver Wake Distortion (MWD) and wake skew effects. Finally, the paper ends with conclusions drawn from the work to date.

II. Exploration of Nonlinearities in the Low-Speed Flight Regime

High-fidelity modelling is considered a challenging task in the low-speed regime due to the strong rotor-wake effects. In hover, the horizontal tail of the B412 is immersed in the rotor wake and, as forward speed increases, the effects are unsteady but also hysteretic, i.e. the velocity field at the tail is dependent on the rotor loading at previous times, along with wake-decay dynamics. The consequent motion of the helicopter is also likely related to the rotor and empennage loads in a nonlinear manner. Modelling and identification of these effects in a way that enables the stability and response to small perturbations to be derived is the challenge that this paper addresses. The following illustrates nonlinearities from these aspects: MWD and interference, the evolution of rotational and translational velocities, and effects of wake skew angle.

A. Nonlinearities Associated with MWD and Wake Interference

In the following, F-B412 refers to the nonlinear FLIGHTLAB simulation of the ASRA, while LF-B412 refers to a linear version, derived from numerical perturbation of the F-B412 or through SysID. Figure 1 compares responses, with those from FT, for the baseline F-B412 [17] with different FLIGHTLAB modelling options implemented, and tailored to the ASRA; MWD [18] and rotor-wake-interference (RWI [19]) on the fuselage and empennage. Responses are shown for a lateral cyclic input at hover along with the Certification Specifications (CS) – Flight Simulation Training Devices (Helicopter), (CS-FSTD(H)) proof-of-match boundaries [20], the same for both on- and off-axis. The F-B412 main rotor includes the Peters-He finite-state-inflow (FSI) model [21]. The FLIGHTLAB MWD effect [16] is implemented as an augmentation to the FSI and is a function of tip-path-plane dynamics, varying in strength with advance ratio, and lagged relative to the tip-path-plane motion to account for air mass dynamics, as with the FSI itself.

The default parameters in the MWD dynamics are based on the FLIGHTLAB Generic Rotorcraft configuration (similar to a UH-60 helicopter) but have been tuned to be representative of a Bell-412 helicopter, in terms of responses, for the present study [17]. The principal impact of the MWD inflow is to reverse the off-axis response (e.g. pitch from roll). The rotor blades descend into, or rise out of, the rotor wake during pitch and roll maneuvers, leading to radial incidence changes that counter the linear aerodynamic effects.

The model on-axis responses generally correlate well with FT, judged against the CS-FSTD(H) tolerances. Figure 1 confirms that the FSI model, without (w/o) MWD, predicts the off-axis pitch response from a lateral cyclic input to be of opposite sign to that from the FT data, as discussed by Peters and He in Ref. [21]; this is reversed when MWD is included.

The sway velocity (v) has a good initial match and MWD preserves this through the maneuver. The surge velocity (u) departs from the FT after 2sec, but the translational velocity perturbations remain relatively small ($< 8\text{ft/s}$). With RWI included, the yaw rate (r) from lateral cyclic is improved, and pitch rate (q) slightly improved after 2.5sec.

These preliminary results show the complexity of both MWD and RWI effects in hover, which will be even more complicated in larger amplitude maneuvers. Our research is addressing both these areas, but the main objective of this paper is to present the new concept of additive SysID to explore nonlinearities in the low-speed regime, with application to MWD effects in the first instance. Therefore, the scope of investigation in this paper will focus on the F-B412 model version with FSI and MWD engaged but RWI disabled. It is acknowledged that the restriction to MWD effects will impact the results and the extension to RWI effects will be presented in a future paper.

B. Nonlinearities Associated with Translational Velocities

In this section, we consider the classical 6 Degree(s)-of-Freedom (DoF) linear model and compare the responses with the corresponding nonlinear model (initially F-B412 without MWD) for different amplitudes of control inputs, to illustrate the impact of velocity-based nonlinearities.

Figure 2 shows responses in hover for three different magnitudes of lateral cyclic 2311 input: 0.25, 0.50, and 0.75in. To highlight the comparison, all responses are scaled to the lowest input size. For a linear model, the scaled responses for the three different input sizes would have the same magnitude, a situation that is unlikely to hold for the nonlinear model. In particular, the scaled surge ($\tilde{u}$) and sway velocities ($\tilde{v}$) reflect nonlinear behavior as the amplitude of the input increases. The scaled pitch and yaw rates ($\tilde{q}$, $\tilde{r}$) show strong

nonlinear variations after around 5sec. The attitude and translational velocity perturbations are closely linked to these rates of course. These differences will have a large impact on the relative contribution of terms such as $L_q q$, for example, to the overall roll moment. Values of these derivatives, identified for small amplitude motions, will likely distort the relative contributions from other states as the maneuver progresses. Similar results are found for pitch maneuvers.

C. Nonlinearities Associated with Wake Skew Angle and MWD

Zhao [18] and Pitt and Peters [22] included the wake skew as an important variable to model the dynamic wake distortion phenomenon. The predicted wake skew angle, χ , in the F-B412 varies during a roll maneuver from hover as shown in Fig. 3, in which four levels of lateral cyclic pulse input were applied. The skew effects behave differently with and without MWD, and are increasingly nonlinear as control magnitude increases. For example, the contributions from wake skewing to the rotor moments will impact the translational velocity derivatives L_u and M_v [23]. Wake skewing contributes to L_u through the strong gradient in the longitudinal downwash component along the rotor wind-axis. This will lead to lateral flapping, requiring lateral control compensation by the pilot when performing the longitudinal maneuver, such as the acceleration-deceleration. Similar results, due to lateral downwash gradient effects on M_v , can be expected for roll maneuvers. A more advanced model structure will be required to capture the cross-couplings with high fidelity, and this will be explored in the paper. The paper continues with a description of the heuristic approach to model structure identification using ASysID.

III. Introduction to ASysID Development

The comparisons in Figs. 1 and 2 provide some insight into the fidelity and response behavior of the model and inform the ASysID development in this Section in two ways: the variation of the relative significance of each motion response as the maneuver progresses, and when and how the translational velocities play their roles and interact with angular motions. In the ASysID method, the model parameters are identified in an additive manner, based on their physical contribution to the temporal dynamic response of the system, i.e. over a defined time range [17]. The ASysID approach is outlined in Fig. 4.

A. Assumed Model Structure

The general 6DoF rigid-body linear model used for identification in the time domain can be written as,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}(t - \tau) \quad (1)$$

in which $\mathbf{x} = [u \ v \ w \ p \ q \ r \ \phi \ \theta]^T$ and $\mathbf{u} = [X_a \ X_b \ X_c \ X_p]^T$. u , v , and w are the aircraft perturbation translational velocities in the body-frame, p , q , and r are aircraft perturbation rotational velocities in the body-frame, and ϕ and θ are Euler roll and pitch angles. The vector $\mathbf{u}$ consists of the four controls for a conventional rotorcraft: lateral cyclic stick (X_a), longitudinal cyclic stick (X_b), collective (X_c), and pedal (X_p). The effective time delay (τ) is included to account for the control system/actuator delays/lags and unmodelled higher-order dynamics [2, 23], which can be different for each control axis. The identification of higher-order models of the Bell 412 accounting for flapping dynamics in the frequency domain has been reported in Ref. [12].

The ASysID approach adopted in this investigation is based on Equation-Error (EE) analysis, often the first step in model-structure identification. One general motion state equation of Eq. (1) used for the EE analysis can be described as,

$$\dot{x} = \sum_{i=1}^6 Y_{x_i} x_i + \sum_{j=a,b,c,p} Y_{X_j} X_j(t - \tau_{X_j}) \quad (2)$$

in which Y_{x_i} are stability derivatives and Y_{X_j} are control derivatives. These are the unknown parameters in Fig. 4 dedicated to identification. The measurement equation can be expressed as,

$$z(k) = \sum_{i=1}^6 Y_{x_i} x_i(k) + \sum_{j=a,b,c,p} Y_{X_j} X_j(t - \tau_{X_j})(k) + v(k) \quad k = 1, 2, \dots, N_d \quad (3)$$

v is the measurement noise and N_d is the number of data points. It is acknowledged that the measurement noise can lead to biased estimates in the EE analysis [1]. However, these can be minimized by measurement compatibility checks and data smoothing [3, 24].

The linearized roll equation can be written in the form of Eq. (4) to illustrate the ASysID approach,

$$\dot{p} = L_u u + L_w w + L_q q + L_v v + L_p p + L_r r + L_{X_a} X_a(t - \tau_{X_a}) \quad (4)$$

where L represents the rolling moment normalized by inertia (i.e. roll acceleration). The angular acceleration $\dot{p}$ is not normally available from FT but obtained through a smoothed numerical differentiation of the measured roll rate.

B. Time-Delay Estimation

After establishing the database, the first step in the ASysID approach is to estimate the time delay, which is a well-known issue for SysID in the time domain. Techniques for estimating this ‘equivalent’ time delay are widely available, such as the linear backwards-extrapolation in time to the trim from the point of the maximum slope after the initial sharp-edge input [3]; but they suffer from difficulty in implementation and sensitivity to data processing. Another option would be to adopt the frequency-domain approach based on the

phase-slope information, which also requires the analyst's judgement as to an appropriate frequency range to measure the slope [2]; but this is beyond the scope of ASysID in the time domain. In this paper, the first-order regressing flap mode is used for estimating a major contribution to τ in the longitudinal and lateral channels in the time domain. The first-order heave and yaw rate response models are used in the collective and pedal control channels, respectively. The procedure of identifying the equivalent time delay τ_{X_a} in the rolling motion is illustrated in the following.

The regressing flap roll mode can be approximated as follows [2],

$$\tau_f \dot{\beta}_{1s} = -\beta_{1s} + \tau_f p + \theta_{1c} \quad (5)$$

where τ_f is the rotor flap time constant, θ_{1c} is the lateral cyclic pitch and β_{1s} the lateral flapping. The roll response to the pilot stick input (X_a), can be written in transfer function form for coupled roll-flap dynamics,

$$\frac{\bar{p}}{\bar{X}_a}(s) = \frac{K_{X_a} L_{\beta_{1s}} e^{-\tau_{X_{at}} s}}{\tau_f s^2 + s - \tau_f L_{\beta_{1s}}} \quad (6)$$

in which the bar ‘ $\bar{\cdot}$ ’ represents variables in the Laplace plane. $L_{\beta_{1s}}$ is the effective flap stiffness (negative), K_{X_a} represents the linkage/actuation between X_a and the rotor swashplate, and $\tau_{X_{at}}$ is the transport delay. The equivalent time delay is then,

$$\tau_{X_a} = \tau_f + \tau_{X_{at}} \quad (7)$$

Eq. (6) can be converted to the ‘Autoregressive with Extra Input’ model structure (ARX) [25] in Eq. (8) with the back differentiation in Eq. (9),

$$p(t) + a_1 p(t - \Delta t) + a_2 p(t - 2\Delta t) = b_1 X_a(t - nk \cdot \Delta t) \quad (8)$$

$$\dot{p}(t) = \frac{p(t) - p(t - \Delta t)}{\Delta t} \quad \text{and} \quad \ddot{p}(t) = \frac{p(t) - 2p(t - \Delta t) + p(t - 2\Delta t)}{\Delta t^2} \quad (9)$$

in which Δt is the sampling interval and nk is the number of input samples that occur before the input affects the output, or $\tau_{X_{at}} = nk \cdot \Delta t$. It is acknowledged that this may pose a limitation if Δt cannot be described as an integer number of sampling interval (nk), hence slightly reducing the accuracy [1]. After substituting the parameters of Eq. (6) into Eq. (8) and further simplification, we can write the relationship,

$$\tau_f = \frac{\Delta t}{a_1 / a_2 - 2} \quad (10)$$

The least-squares estimation algorithm is used to estimate the parameters in Eq. (8) and hence the equivalent time delay in Eq. (7) is obtained.

C. Derivative Estimation

After obtaining an appropriate value for the time delay, the overall strategy is to estimate the derivatives in Eq. (4) additively, based on the activated timing point at which the corresponding physical dynamics are dominant. These steps involve determining the timing points and the sequence of variables and associated derivatives for identification. A suitable approach is based on a combination of physical insight and justification based on statistical metrics. The ASysID approach relies to a degree on physical expectations in determining the sequence of derivative candidates, supported by the following general guidelines. Firstly, with a step-type input, the control derivatives Y_{X_j} (e.g., L_{X_a} in Eq. (4)) will always be the first group of derivatives chosen for identification. This is physically understandable; for example, after application of the lateral stick input X_a , the rotor disk re-orientates rapidly, producing a roll moment and leading to the fuselage roll acceleration. Secondly, the angular rate derivatives form the next group of candidates, as the fuselage motion builds up following the rotor disk tilting. These derivatives play strong roles during

this period, particularly the primary-damping effects L_p , M_q , and N_r , that are dominant for a certain period before cross-damping derivatives become significant, e.g. L_q . Finally, the translational velocity derivatives form the third group as these motions normally build up at a slower rate, taking time to make their impact.

These three derivative categorizations are generally applicable, but the focus is always on the physical importance of the variable of interest over others for a certain window of time. For example, the heave damping Z_w should be the second derivative in the collective channel, since the heave response is physically more dominant and responds rapidly to collective input. During the estimation procedure, it can be difficult to determine a candidate from the candidate pools by relying only on the contribution of a single derivative and motion in a time window. The partial correlation coefficient between a candidate's measurements and the dependent variables (e.g., $\hat{p}$ in Eq. (4)) is evaluated, as used in StepWise Regression (SWR) methods [26]. The correlation values between a variable x_i (or multiple) and the measured dependent variable z_t (defined in Eq. (11)) are calculated in Eq. (12) to indicate a level of linear dependence between the two variables.

$$z_t(k) = z(k) - \left(\sum \hat{Y}_{x_j} x_j(k) + \sum \hat{Y}_{X_j} X_{jm}(k) \right) - \sum \hat{Y}_{x_n} x_n(k) \quad k = 1, 2, \dots, N_t \quad (11)$$

The correlation coefficient is defined as,

$$\rho_{x_i z_t} = \frac{S_{x_i z_t}}{\sqrt{S_{x_i x_i} S_{z_t z_t}}} \quad (12)$$

in which,

$$S_{x_i z_t} = \sum_{k=1}^{N_t} [x_i(k) - \bar{x}_i][z_t(k) - \bar{z}_t]$$

$$S_{x_i x_i} = \sum_{k=1}^{N_t} [x_i(k) - \bar{x}_i]^2 \text{ and } S_{z_t z_t} = \sum_{k=1}^{N_t} [z_t(k) - \bar{z}_t]^2 \quad (13)$$

and N_t is the number of data points up to the time point t_{x_n} . The variables with a correlation value larger than the recommended threshold 0.6 in Ref. [26] are considered in a time window. Finally, the variables with the most dominant physical dynamics and large partial correlation coefficients are chosen for identification in the following step.

The steps in ASysID are conducted using increasing amounts of dynamic response history in a time-window approach. As noted above, the ordering of derivative candidates is primarily determined by the sequence in which different dynamics are activated and their correlation values. Here, the analysis window T_m starts at the initial time point of the maneuver and increases up to T , a period during which $Y_{x_n} x_n$ is physically significant, this requires the analyst's judgment and a good level of understanding of the physical system. A least-squares parameter estimation approach [27] is used to identify the value $\hat{Y}_{x_{nm}}$ of Y_{x_n} for the n^{th} variable x_n in the analysis window T_m , by fitting the updated model with measured data, excluding the contributions from the already identified stability and control derivatives ($\sum \hat{Y}_{x_j} x_j + \sum \hat{Y}_{X_j} X_j$) as the time window increases.

$$\hat{Y}_{x_{nm}} = (\mathbf{X}_{nm}^T \mathbf{X}_{nm})^{-1} \mathbf{X}_{nm}^T \mathbf{z}_{nm} \quad m = 1, 2, \dots, M_d \quad (14)$$

M_d is the number of analysis windows of period T_m in time T . A larger M_d value means a smaller window size that can better reveal the variation of the derivative value against the time increase, at the expense of an increased computation load. The term $\mathbf{z}_{nm}$ is defined as,

$$z_{nm}(k) = z_m(k) - \left(\sum \hat{Y}_{x_j} x_{jm}(k) + \sum \hat{Y}_{X_j} X_{jm}(k) \right) \quad (15)$$

and the final value of Y_{x_n} is determined by the minimum fit error,

$$FitError = \min \left\{ \frac{1}{T_m} \frac{\sum_1^{M_k} (z_{nm}(k) - \hat{Y}_{x_{nm}} x_{nm}(k))^2}{\sum_1^{M_k} (z_m(k))^2} \right\}_{m=1,2,\dots,M; k=1,2,\dots,M_k} \quad (16)$$

in which M_k is the data point in the analysis window T_m . Once $\hat{Y}_{x_n}$ is identified at the time point t_{x_n} , its value remains fixed for the remainder of the maneuver. This assumes that the identified parameters will not change as conditions vary in the remaining part of the maneuver. The ASysID process advances in steps, as more and more motion variables x_i are added, until the complete maneuver is modelled, or no further improvement can be achieved.

Several metrics can be used to determine the significance of a variable, such as the widely-used F - and T -test statistics metrics as used in SWR. The ASysID method uses the ratio (as a percentage) of the standard error $s(\hat{Y}_{x_i})$ to the estimated parameter value ($\hat{Y}_{x_i}$) to provide a level of significance. The standard error is derived from the square root of the corresponding diagonal element in the least-squares covariance matrix [1]. A 30% standard error is used as a rough guideline for parameter retention, hence a derivative with a standard error $> 30\%$ would be deleted from the process. This threshold contrasts with the 20% Cramér–Rao bound in Ref. [2] or the F -test ratio threshold value of 20% in SWR [26].

The time-window feature of the ASysID approach clearly differentiates it from the classical SysID methods in both time and frequency domains in terms of either EE (e.g., SWR) or Output-Error analyses [2]. These conventional approaches require the complete time history to be processed in one piece, so the identified derivatives are optimized for the whole maneuver. The rationale behind the development of ASysID is that a large part of

the physics in the force and moment contributions to a rotorcraft's motion should be identifiable at the periods when they are strongest. Having identified a force or moment contribution, it can be fixed and not distorted later, perhaps to compensate for a mismatch occurring from an incorrect model structure. This assumes that linear, instantaneous approximations to the motion are valid of course. However, the method can also be applied to capture nonlinear dynamics, as will be explored later in this paper.

Compared with SWR, the order of candidate derivatives determined in ASysID is largely based on the physical response sequence, instead of the level of the input-output correlation, although the strength of correlation is also a factor in ASysID parameter selection. ASysID is also different from recursive estimation methods [25,28] that are widely used for real-time SysID, in that the latter conduct SysID based on the data point by point when they become available. These recursive estimation methods have no mechanism applied to judge the sequence of derivatives to be identified based on the physical information in the way that ASysID does. Moreover, the number of derivatives is typically fixed during the recursive SysID process but, in the ASysID method, this is variable.

IV. Application of ASysID to F-B412 without MWD

To demonstrate the effectiveness of the ASysID method, we use the F-B412 responses, initially without MWD, at hover, compared with the perturbation model in Fig. 5, based on Eq. (4). The heave response (w) is ignored due to its very small amplitude following the lateral cyclic input. The linear responses are included in a comparison to highlight the level of nonlinearities.

The first step is to identify the equivalent time delay for the higher-order dynamics as modelled in Eq. (5). The identified τ_f value of about 0.1sec is shown to have converged after about 0.3sec in Fig. 6. The circles in the middle plot represent each segment (from zero to the current timing point) used by ASysID to derive each corresponding parameter value shown as squares in the lower plot in Fig. 6. Simple rotor theory [23] gives this as $16/\gamma\Omega$, where γ is the rotor Lock number (≈ 6.2) and Ω is the rotorspeed ($\approx 34\text{rad/sec}$), hence a delay of 0.08sec. In the Fig. 6 example, the control inputs are applied at the swashplate so that $\tau_{x_{at}} = 0.0$; there is no transport time delay introduced in the nonlinear model.

As discussed above, the control derivative L_{x_a} is the first derivative chosen for identification; Fig. 5 shows the roll rate increasing after the control is applied. L_{x_a} (and subsequent derivatives) is determined by the minimum value of the associated fit-error metric in Eq. (16).

ASysID then progresses through the $\dot{p}$ time history adding one, or more, variables at a time, to reduce the fit error; the final results are summarized in Table 1. The minimum fit-error value usually occurs at the same time as the derivative converges locally to a ‘steady’ value. The time window used for identifying L_{x_a} should be as short as possible to reduce the level of the upcoming roll rate contribution. Therefore, a short-time window is chosen for ASysID as shown in Fig. 7 and the L_{x_a} value is determined to be 0.905, within the window between 1.02 and 1.08sec.

Figure 5 shows that there is negligible surge velocity (u) and very little sway velocity ($v \approx 2\text{ft/s}$) up to 2sec. Meanwhile, both pitch (q) and yaw rates (r) are less than 1.0deg/s at 2sec. The roll rate (p) is dominant between 1.1 and 2.5sec. which can be physically

understood in terms of the fuselage motion building up rapidly following the rotor disk tilting. Taking these factors into consideration, L_p is selected as the second derivative for identification.

After 2.1sec, both the sway velocity induced from lateral disc tilt and the coupled pitch rate are increasing. Although the former increases rapidly after 2.1sec, it contributes to $\dot{p}$ over a similar time as q . This makes it difficult to choose the order of these two variables. However, correlation coefficients (ρ) for these two variables with $\dot{p}$ (using Eq. (11)), after excluding L_{x_a} and p contributions, are close, as indicated in Fig. 8, during the time period (1.5 – 4.0sec). Their corresponding ρ values with $\dot{p}$ are 0.73 at $t = 2.1$ sec, larger than the recommended threshold 0.6 [26], indicating a strong dependence between the two variables and $\dot{p}$. Therefore, L_v and L_q are combined for the identification in the next window.

L_w is the fifth derivative chosen for identification due to the heave velocity beginning to contribute at around 3.8sec. However, the quality of this estimate is poor, as reflected by the large standard error ($s = 44\%$) in Table 1, due to the very small amplitude of heave response. Therefore, it is removed from the model structure in Eq. (4). L_u is chosen as the sixth derivative as its influence appears before the yaw rate, with L_r as the final derivative to be identified. The yaw rate builds up early in the maneuver, as shown in Fig. 5, due to the roll-yaw coupling excited by $\dot{p}$ with a non-zero product of inertia, I_{xz} [17]. However, it is relatively small until the sway velocity builds up at about 3sec. These factors are likely to degrade the quality of the identification of L_r that has two primary physical sources in the F-B412; one from the tail rotor, the other from the inertia coupling. The perturbation results in Table 1 suggest that the inertia coupling is slightly stronger in the real aircraft due to its larger L_r value.

The derivatives computed using ASysID are compared with those from SWR (with the same time delays estimated in ASysID) and the small-perturbation values from the F-B412 linearization process. The numbers in parentheses correspond to the order in which a variable is selected for identification. As shown in Table 1, the results from ASysID and SWR are broadly in agreement, although the derivative selection order is quite different. The key derivatives, L_{x_a} and L_p , for both ASysID and SWR are also close to the perturbation values. Both approaches also achieve similar cross-coupling derivative L_q , dihedral L_v , and the translational velocity derivative L_u , although these are quite different from the perturbation values, suggesting possible nonlinear influences. The sensitivity of the fit error to individual moment contributions is quantified by the standard error given in Table 1 as a percentage of the corresponding derivative value. The fit-error variation as the derivatives in the ASysID are added into the model structure is illustrated in Fig. 9. The time on the x -axis corresponds to the instants when the contributions from the derivatives are chosen.

As shown in Fig. 9, the primary derivative L_p reduces the fit error in the early stage of the maneuver, which is then reduced further as the dihedral effect, L_v , is added. L_u also reduces the fit error but mainly in the later stages of the maneuver. The responses with the three derivative sets in Table 1 are compared with $\dot{p}$ of the F-B412 in Fig. 10; the oscillations likely stem from higher-order dynamics, including the (2/rev) lead-lag motion. The summation values of the difference between the responses and $\dot{p}$ of the F-B412 have been included in Fig. 10 to indicate the level of fit. The 6DoF model structure described in Eq. (4) is not able to capture these oscillations that are considered process noise in the SysID. However, they certainly will increase the fit error. In fit terms, both the ASysID

and SWR results are superior to those from the perturbation model, and it may be that the translational velocities have an impact on these differences. Moreover, the fit of ASysID is very close to the SWR results.

V. Application of ASysID to ASRA Flight Test Data

In this section, ASysID is applied to the B-412 FT data at hover, following kinematic consistency analysis [13] and is extended from the one-DoF roll dynamics in Eq. (4) to the 6DoF state-space model in Eq. (1).

A. Test Aircraft and Flight Test Data

Flight test campaigns were performed on the ASRA to support Liverpool's control law design [29] and simulation fidelity research [30]. ASRA is specially configured with onboard research equipment for the development and testing of advanced flight control systems and modern cockpit technologies. Specialized research equipment installed in the aircraft includes a fly-by-wire control system (mechanical back-up with safety pilot), force-feel system, and data-acquisition system with graphical display capability. Open-loop responses to control steps, pulses, 2311s and frequency sweep control inputs have been measured throughout the flight envelope. This paper focuses on bare-airframe flight dynamics at hover with multi-step 2311 inputs.

Angular accelerations ($\dot{p}$, $\dot{q}$, and $\dot{r}$) used in the EE method are derived through smoothed numerical differentiation of the rate measurements [1]. Translational accelerations and velocities have been derived from the accelerometer measurements. A low-pass filter with 3Hz cut-off frequency has been implemented to smooth the FT data for both inputs and output responses.

B. Estimation Results

The stability and control (perturbation) derivatives of the F-B412 model trimmed at 5kts, in line with the flight test condition, for roll and pitch motions are compared in Table 2 with the derivatives computed using the ASysID and SWR approaches. All control derivatives are chosen as the first candidate in ASysID. The 6DoF stability and control derivatives and time delays from the ASysID method are given in Table A1 for reference, including a comparison with LF-B412 values, derived from the F-B412, with and without MWD included. Some discussion on the derivative values is included in Appendix A. The SWR results are not included in the full set. This paper aims to provide a new approach to rotorcraft SysID, not attempting to surpass those well-established conventional SysID methodologies for model-structure estimation.

The SWR ordering in Table 2 is the final sequence after multiple interactive steps. Several points can be drawn from the derivative table. Firstly, considering the first two steps in the ASysID process, there is a reasonable agreement for the key (on-axis) damping and control derivatives, except M_q from SWR which is about double the ASysID result and a large standard deviation. M_r is removed from both approaches, due to the weakly-excited yaw motion impacting the pitch acceleration. Moreover, the ASysID results are close to those derived independently using the frequency-domain approach as previously reported in [12]. Secondly, there are larger differences among the off-axis derivatives, e.g., L_q and M_p , that are both predicted with an opposite sign by the LF-B412 w/o MWD, compared with the ASysID estimates. The large contribution of the MWD effect to the cross-damping derivative L_q , reduced from -1.754 (no MWD) to -1.019 (with MWD), will be further investigated later in the paper. The ASysID M_p value (-0.267) is reasonably close to the

value predicted by the LF-B412 with MWD (-0.190), demonstrating that wake distortion effects have a significant impact on the cross-damping (note M_p w/o MWD = +0.272), as discussed in Ref. [31]. Finally, L_u and M_v are estimated to be smaller and larger, respectively, than LF-B412 predictions possibly reflecting nonlinearity associated with contributions from wake skewing as introduced in the previous section (Fig. 3). These comparisons deserve more attention before conclusions can be drawn and are investigated in the next section.

Fig. 11 compares the 6Dof eigenvalues from the ASysID and LF-B412 models. The natural frequency of the lateral phugoid mode from FT is close to the value reported in Ref. [12] but higher than the LF-B412 prediction, in line with the well-known difficulty of predicting this mode through modelling and simulation [23]. According to Ref. [23], as for M_u in the longitudinal phugoid, L_v has a strong influence on the lateral phugoid natural frequency at low speed as shown below,

$$\omega_{lp}^2 \approx g \frac{L_v}{L_p} \quad (17)$$

The difference in ω_{lp} is likely due to the FT value of L_v (-0.046) being more than double the LF-B412 value (-0.019).

The impact of MWD on the roll and pitch damping is related to the roll and pitch subsidence locations in Fig. 11. From Table 2, MWD appears to increase both LF-B412 derivatives, but while the roll mode increases in strength, the pitch mode decreases; the ASysID results lie between the two. As noted above, the FLIGHTLAB MWD model effectively distorts the influence coefficients in the FSI model, with baseline parameters tuned for the FLIGHTLAB Generic Rotorcraft model. Further exploration of the parameter

tunings in both roll and pitch axes is a topic in ongoing research. As shown in Fig. 12, the on-axis control and damping derivatives for each case capture most of the fit error in the ASysID model from FT, which reduces further as the dihedral derivative, L_v , or the speed stability derivative, M_u , enter the model structures. The remaining derivatives make only small contributions to fit errors. This is reflected in Fig. 13 and Fig. 14 where the significance level of each component contributes to the overall $\dot{p}$ and $\dot{q}$ responses is illustrated by reconstructing the dynamics using the identified components.

In accordance with Fig. 12a, the reconstructed roll response in Fig. 13 has reached a good agreement with FT after only L_{x_a} and L_p components are introduced. The fit is improved when the dihedral stability derivative L_v makes its contribution after 1.5sec, and is further improved when L_u plays its role at 4.8sec. The latter also significantly improves the fit after 5sec. The moment $L_q q$ has a minor contribution to $\dot{p}$ after 1.0sec. Similar findings for the $\dot{q}$ response are shown in Fig. 14.

Both these figures, effectively motion signatures that reconstruct the dynamics using the identified derivatives one by one, reveal contributions from the various aircraft motions to the total acceleration. Furthermore, the quality of the derivatives in Table 2 can be illustrated by comparing the reconstructed $\dot{p}$ and $\dot{q}$ responses using Eq. (1), as shown in Fig. 15. Different FT cases than those used for SysID are implemented for this reconstruction. As can be seen, the ASysID model achieves the best fit with FT data for both roll and pitch responses.

These comparisons not only highlight the accuracy of the identified ASysID model, but also demonstrate reasonably good fidelity of the F-B412 with MWD in hover, when viewed in motion signature form. The validation results of the ASysID process for roll and pitch

control inputs are shown in Fig. 16 for different test cases than those used in the identification of the 6DoF derivatives from Table A1. Both the ASysID model and F-B412 predict good on-axis responses. For the lateral input in Fig. 16 (left column), the ASysID model shows a better fit in the yaw rate and sway velocity, while the F-B412 appears to perform better in the surge and heave rates. For the longitudinal input in Fig. 16 (right column), both models predict the coupled roll rate, although the larger cross-control derivative excites a greater response in the F-B412. These results demonstrate reasonably good fidelity of the F-B412, as well as the 6DoF ASysID model. Notable exceptions are the pitch/surge responses from longitudinal cyclic input, distorting the consequent roll and yaw responses.

The results of the ASysID analysis presented above are encouraging. Physical effects are captured as they occur and then fixed, not requiring further adjustments to potentially compensate for modelling deficiencies encapsulated in average derivatives. The cross-damping derivatives of the F-B412 with MWD are closer to FT, particularly for M_p . However, L_q with MWD is still quite different from the estimated FT value; we hypothesize that the linear model fails to capture the nonlinear evolution as the maneuver progresses. The potential nonlinearities associated with MWD for L_q will now be explored.

VI. Exploring Nonlinearities with ASysID

To examine nonlinear effects arising in large amplitude motions excited by different levels of control input, the scaled lateral step responses of the F-B412 w/o and with MWD, with four levels of pulse input size are shown in Fig. 17 and the corresponding wake skew angle shown previously in Fig. 3.

For this analysis, the maximum input size is determined by the criteria that the corresponding roll attitude change is approximately 30deg. Such a large pulse might be applied for example during quickness-testing for handling qualities assessment. In this study, the large inputs are purposely applied to excite any nonlinearities that cannot be captured accurately by conventional linear analysis, only strictly valid within a small region around the trim condition. The large-input size is expected to violate this condition and, therefore, enable exploration of any nonlinearities in the flight model, as reflected, e.g., by the significant difference in the q and u responses.

The following linear model structure is adopted for the initial investigation:

$$\dot{p} = L_{X_a} X_a + L_p p + L_q q + L_v v + L_u u \quad (18)$$

Compared with Eq. (4), L_w and L_r are ignored due to their minor contributions at hover, reflected in the small fit-error change in Fig. 12. Moreover, from a physical perspective, the MWD effects on L_q are dominant earlier ($< 3\text{sec}$) than L_u effects associated with wake skewing ($> 3\text{sec}$), as shown in Fig. 3; therefore, L_u is retained. ASysID is applied to the F-B412 with and w/o MWD with the four input sizes; the estimated derivatives are shown in Fig. 18.

The ASysID derivatives for the F-B412 w/o MWD are relatively constant with different levels of a control step size. Similar results can be found with MWD, except L_q and L_u show much larger variations, indicating strong sensitivity of these aerodynamic effects to amplitude as the motion develops [18]. A natural step to introduce some nonlinearity in an aerodynamic parameter is to consider a polynomial structure with higher-order terms in the Taylor series expansion [26]. Hence, to capture the nonlinearity attributed to the MWD effect, L_q and L_u are expanded and the following model structure is proposed:

$$\dot{p} = L_{X_a} X_a + L_p p + L_v v + (L_{q_0} + L_{q_1} q + L_{q_2} q^2) q + (L_{u_0} + L_{u_1} u) u \quad (19)$$

in which a quadratic Taylor-series expansion is adopted for L_q and a first-order expansion for L_u . The subscripts indicate differentiation with respect to the first and second variables in the function. Eq. (19) resembles the approach by Zhao [18] to account for the nonlinear wake distortion behavior of the rotor inflows using a vortex tube model in which the contributions of the wake curvature are modelled up to the third order of q and those of wake skew effects up to the second order. After fixing L_{X_a} (0.90), L_p (-2.52) and L_v (-0.020), from Fig. 18, using the responses in Fig. 17, ASysID is first applied to estimate $L_{q_0} = -0.486$; $L_{q_1} = 7.166$ and $L_{q_2} = 115$. These values are then fixed, and the remaining two parameters estimated: $L_{u_0} = 0.008$; $L_{u_1} = -0.0005$. The predictability of the nonlinear model (Eq. (19)) is demonstrated for the three levels of doublet input shown in Fig. 19; the responses of the nonlinear model reach a good agreement with the F-B412. The fit of the model without the nonlinear terms slightly degrades after 2sec, for around 1.5sec and then degrades further for the latter part of responses. The fit also deteriorates as the input size increases.

These results provide a degree of validation of this model structure and associated identified coefficients for capturing the key nonlinear characters of MWD and wake skew. It is argued that, as the response amplitude builds up, the MWD effect reduces in magnitude as the wake spreads and skews downstream, evident in both the short- and long-term q responses of the F-B412 with MWD shown in Fig. 17. Moreover, the dynamic coupling between wake curvature and wake skew, found to be an important contributor to the wake distortion [18], is not reflected in Eq. (19). The inclusion of this coupling may result in improved predictability, but these hypotheses need to be put to a stronger test than

presented in this paper, and are the subject of the ongoing research. The continuing research will investigate ASysID-tailored control inputs that essentially provide larger window sizes where information-rich dynamics enable stronger identification of coefficients in the motion signatures.

Conclusions

The paper reports progress in the development of a new heuristic approach to the exploration of rotorcraft simulation fidelity assessment, referred to as Additive System IDentification (ASysID), for particular application to low-speed maneuvers. In the present investigation, flight test data have been used to assess the fidelity of a nonlinear model of a Bell 412 helicopter. From the analysis undertaken, the following conclusions can be drawn.

1. The ASysID approach has proved effective at identifying the primary motion and control derivatives following control inputs, confirmed by the resulting good matches with the StepWise Regression approach using both flight simulation and flight test data.
2. The fidelity of a baseline nonlinear simulation model is shown to be reasonably good with maneuver wake distortion and wake skewing rotor inflow effects included, compared with flight test and the derived ASysID model; confirming earlier discoveries that wake distortions reverse the cross-coupling effects and the associated cross-damping derivatives, L_q and M_p .
3. Motion Signatures are shown to be useful for revealing the contributions of the various motions and controls to the overall forces and moments. Good levels of comparison with flight test data can be achieved with combinations of motions of a

simulation model with a good fidelity level, and the motion signature differences have been used to explore fidelity deficiencies in the simulation model.

4. Nonlinearities associated with maneuver wake distortion and wake skew effects excited by large control inputs have shown a significant effect on estimates of the derivatives L_q and L_u . A nonlinear model structure has been introduced to capture these effects and the initial results demonstrate good predictability.

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Appendix- Complete 6DOF Results Estimated from Application of ASysID to Flight Test Data

As discussed in the main body of the paper, the primary pitch and roll damping and control sensitivity match well between ASysID, SWR, and perturbation method, typically within about 10%. The other ‘controlled’ axes damping derivatives show poorer comparisons, with yaw and heave being 26% and 36% higher in flight while the associated control sensitivities are within 10%. The much smaller estimates of the time delays in these axes are likely to be related to this mismatch and it is noted that the dynamics of the engine-drivetrain and rotor speed governor system are not included in the 6DoF model structure.

The static stability derivatives, M_u and L_v , are both estimated to be much larger in flight than predicted by theory. While aerodynamic loads on the fuselage and empennage will contribute to both these, the moments due to rotor flapping are expected to dominate at low

speed. The amount of flapping in response to perturbations in surge and sway velocities depends on the rotor stiffness of course, but also the distribution of loading along with the blades [23], influenced by blade twist and inflow. For the longitudinal maneuver, the ‘ u ’ state is picked up at about 2.5sec but the ensuing response of the ASysID model then degrades compared with flight, with little impact on the excellent match between flight and SysID model for pitch rate.

The adverse yaw N_p is estimated to be 80% higher than predicted by theory and both estimate and prediction are much higher than the yaw damping. At low speed, strong contributions to N_p and L_r come from the inertia coupling due to the product of inertia I_{xz} . The value used in F-B412 is an estimate with a moderate level of uncertainty. The relatively high values and mismatches of the roll-yaw cross-control derivatives also suggest inertia couplings effects.

Most of the X and Y force derivatives make only minor contributions to the fit of the translational accelerations. Exceptions are, of course, Y_v and X_u , although it is noted that the drag derivative X_u could not contribute to the ASysID model. The much large value of Y_v estimated from flight is related to the dihedral mismatch discussed above but could also be attributed to the errors in the modelling of the fuselage side force. In the F-B412 the aero tables are derived from wind-tunnel tests on a similar, but different, fuselage shape [17].

These discrepancies are all worthy of further investigation in the ongoing developments of the ASysID methodology being undertaken in the rotorcraft simulation fidelity research by the team of authors.

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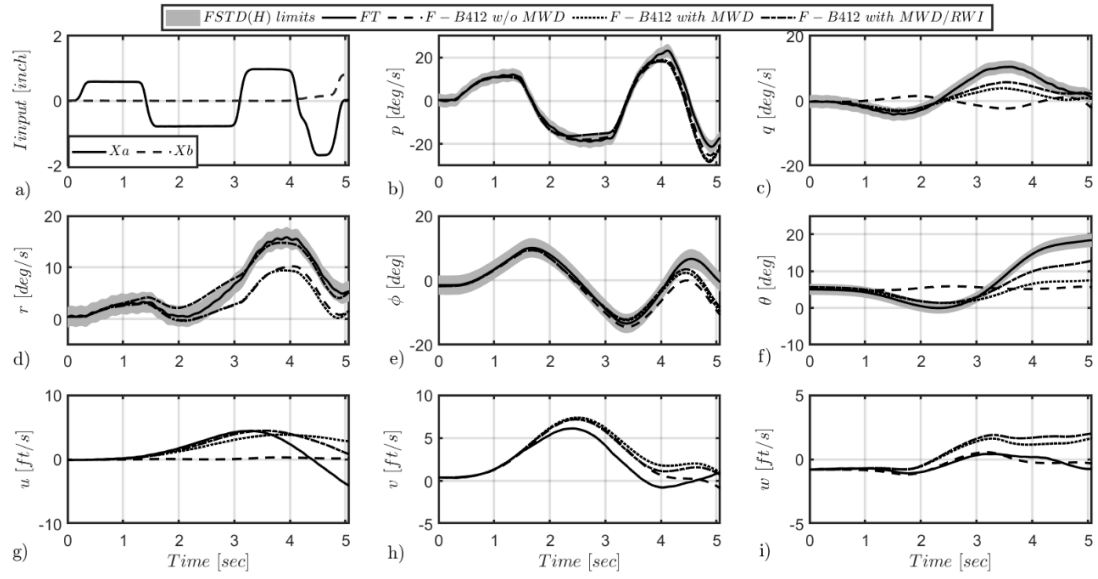


Fig. 1. Comparison of F-B412 responses at hover: without (w/o) both MWD and RWI, with MWD but w/o RWI, with both MWD and RWI.

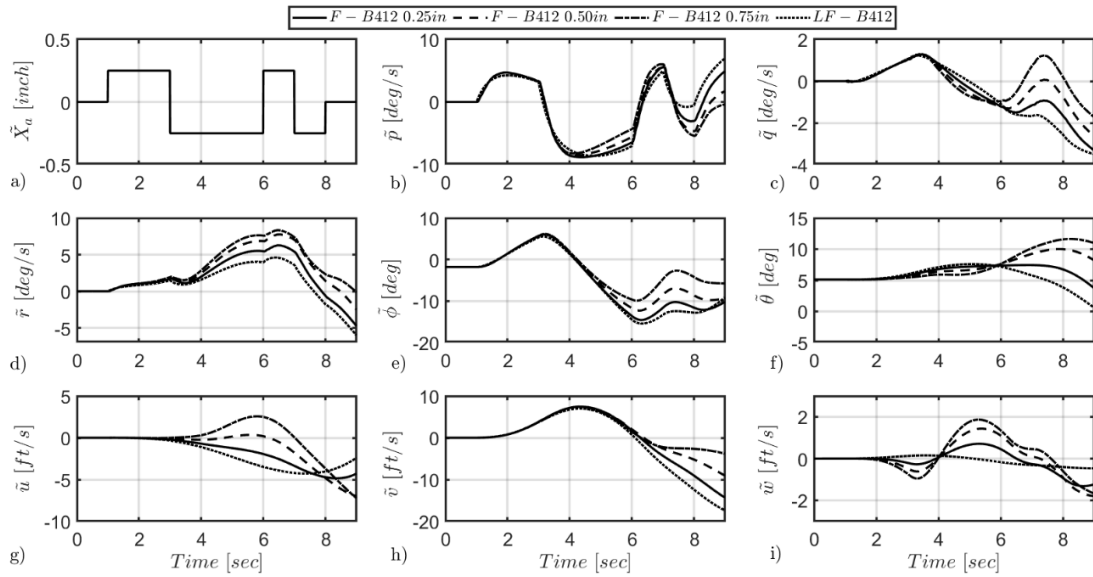


Fig. 2. Comparison of the LF-B412 and F-B412 (w/o MWD) responses at hover for different lateral inputs scaled to the lowest size input.

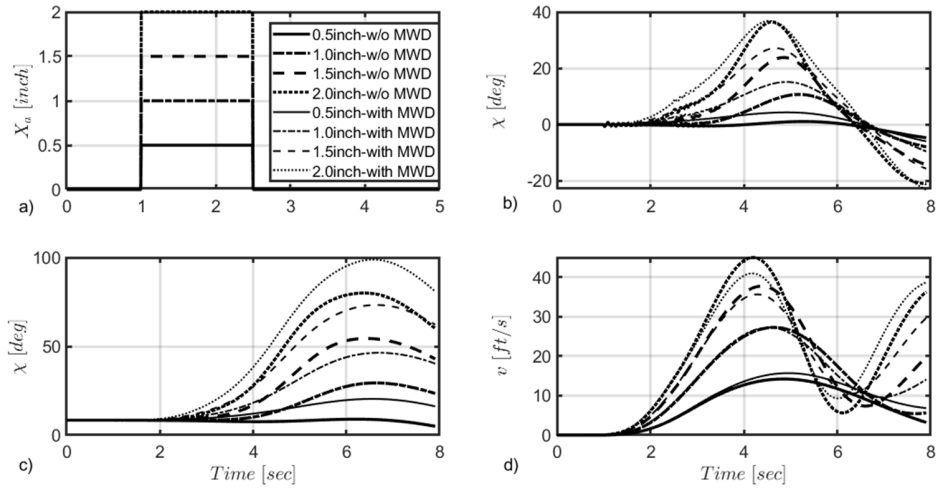


Fig. 3 Wake skew angle responses with different level of lateral cyclic step input (5kts)

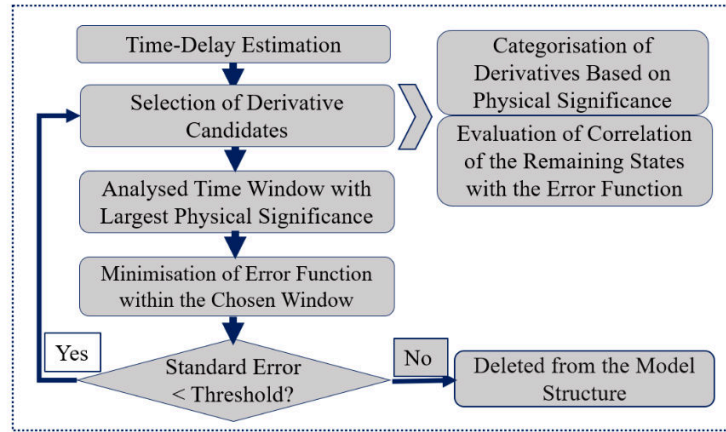


Fig. 4 Schematic of the ASysID approach

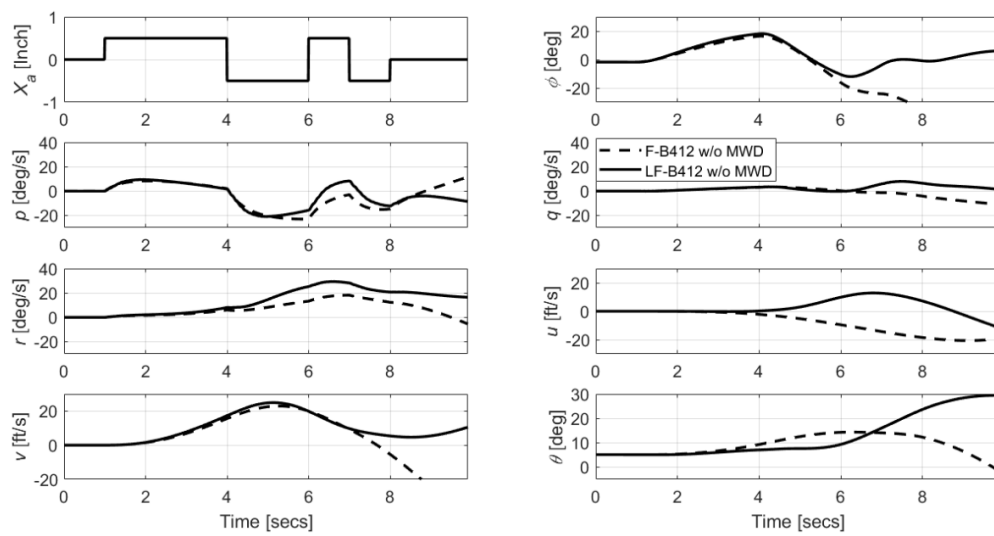


Fig. 5 Responses of F-B412 (w/o MWD) at hover with a 3211 lateral stick input.

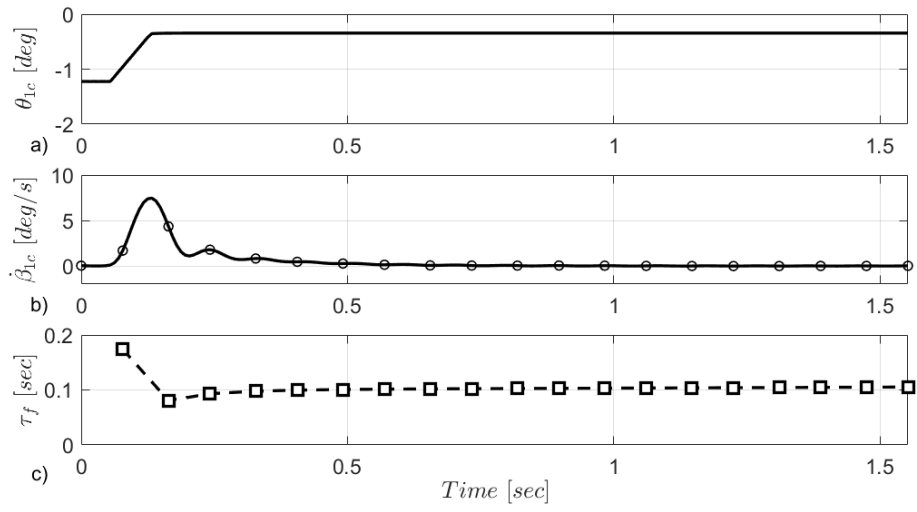


Fig. 6 ASysID results for τ_f using a step input.

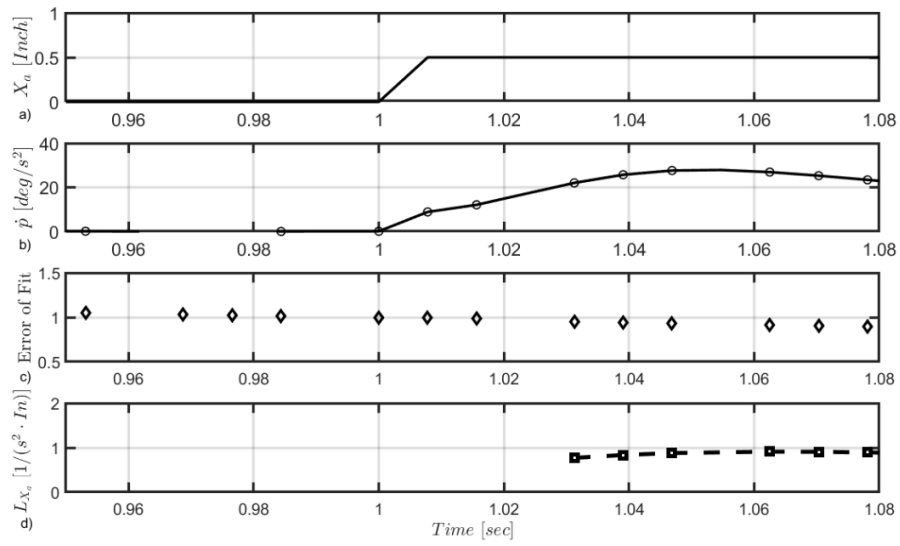


Fig. 7 Results for L_{X_a} identification using the ASysID approach.

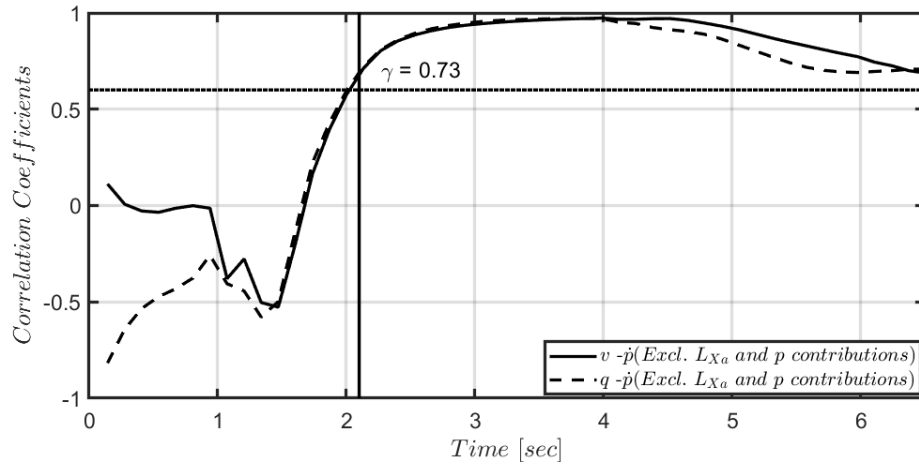


Fig. 8 Correlation coefficient comparison between $v\text{-}\dot{p}$ and $q\text{-}\dot{p}$ (L_{x_a} and p contributions excluded in $\dot{p}$)

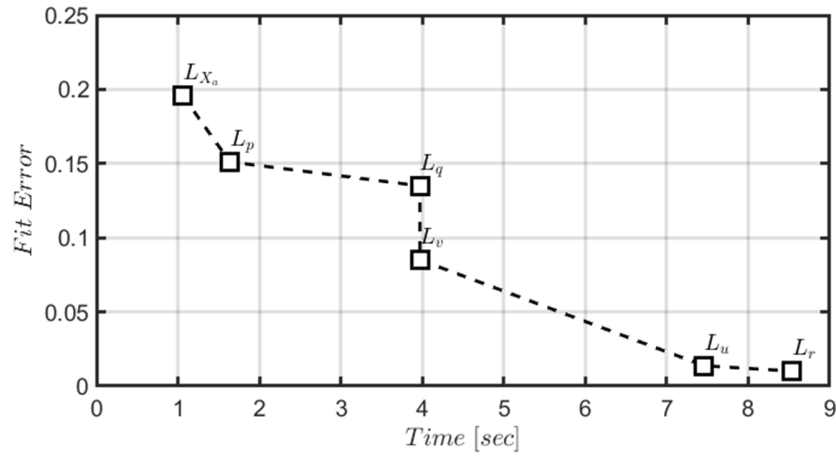


Fig. 9 Fit-error variation as the different contributions to the dynamics are added in the ASysID process ($\dot{p}$ response).

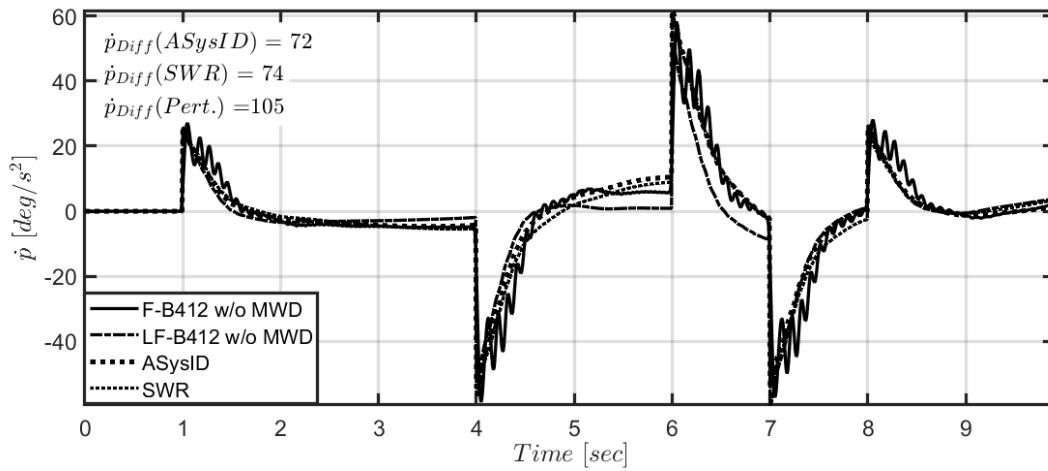


Fig. 10 Comparison of $\dot{p}$ with linear models derived with ASysID, SWR, and perturbation analyses.

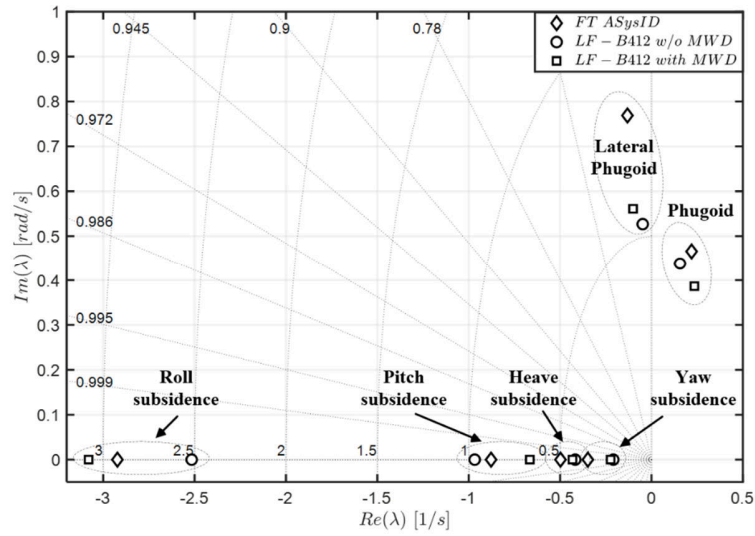


Fig. 11 Comparison of eigenvalues from ASysID and LF-B412.

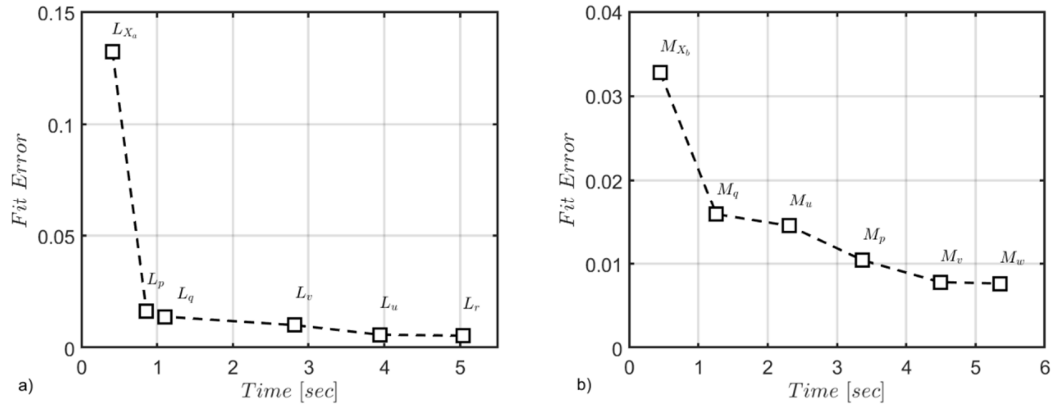


Fig. 12 Fit-error variation as the different contributions are added ((a), $\dot{p}$ and (b), $\dot{q}$).

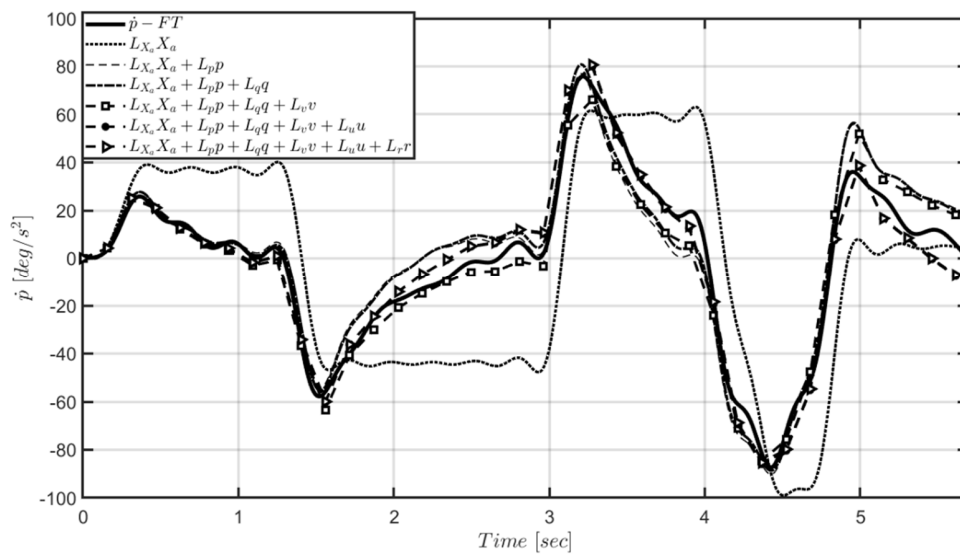


Fig. 13 Reconstructing the dynamics using the identified derivatives ($\dot{p}$ response).

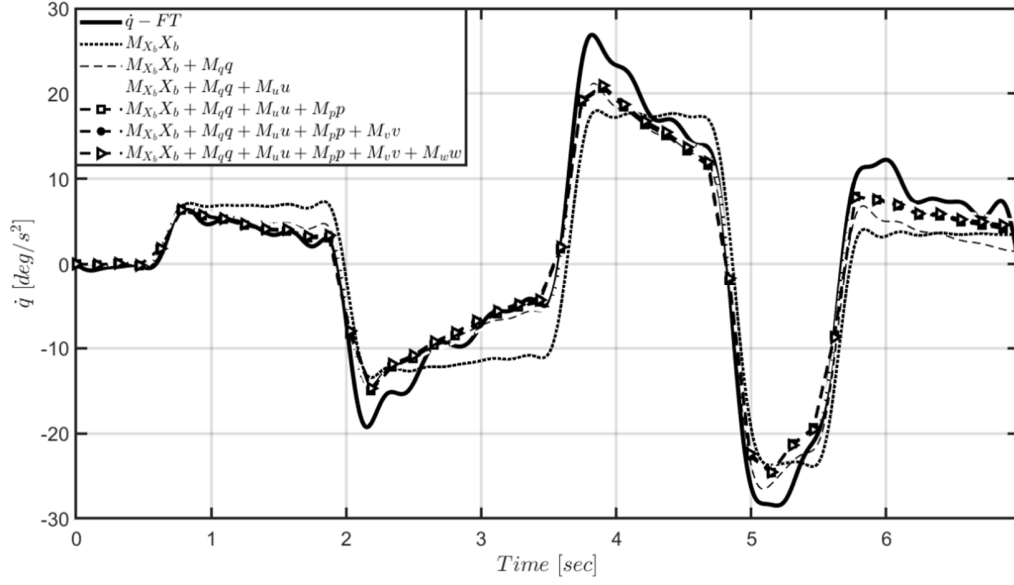


Fig. 14 Reconstructing the dynamics using the identified derivatives ($\dot{q}$ response).

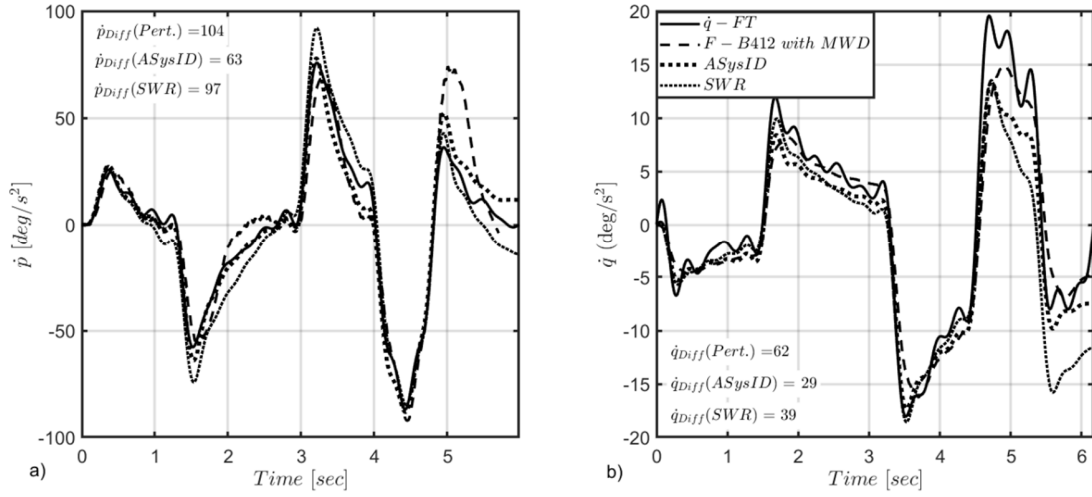


Fig. 15 Comparison of fit among FT, F-B412 with MWD, SWR and ASysID ((a), $\dot{p}$ and (b), $\dot{q}$).

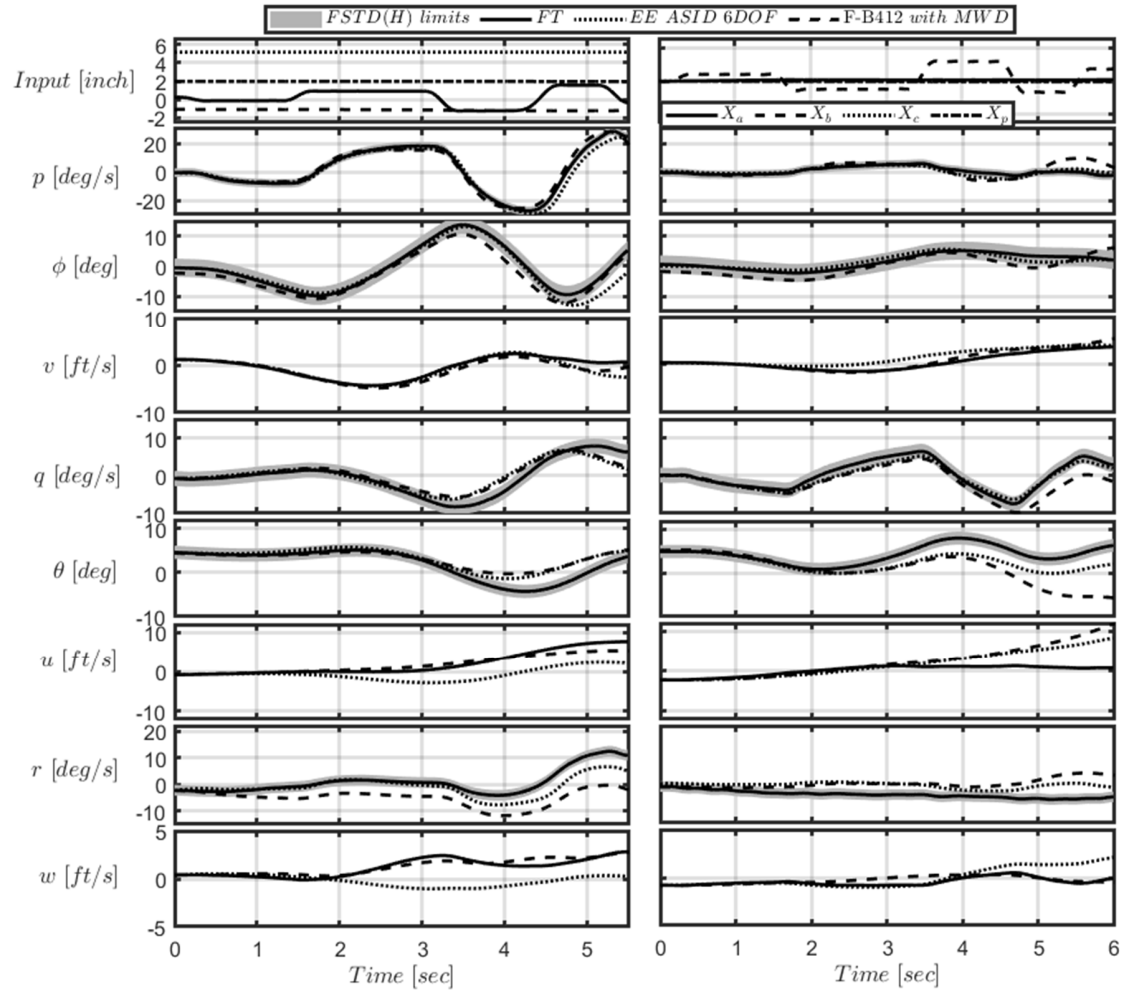


Fig. 16 Validation study for the ASysID 6DoF linear model, FT and F-B412 with MWD (lateral cyclic input (left), longitudinal cyclic input (right)).

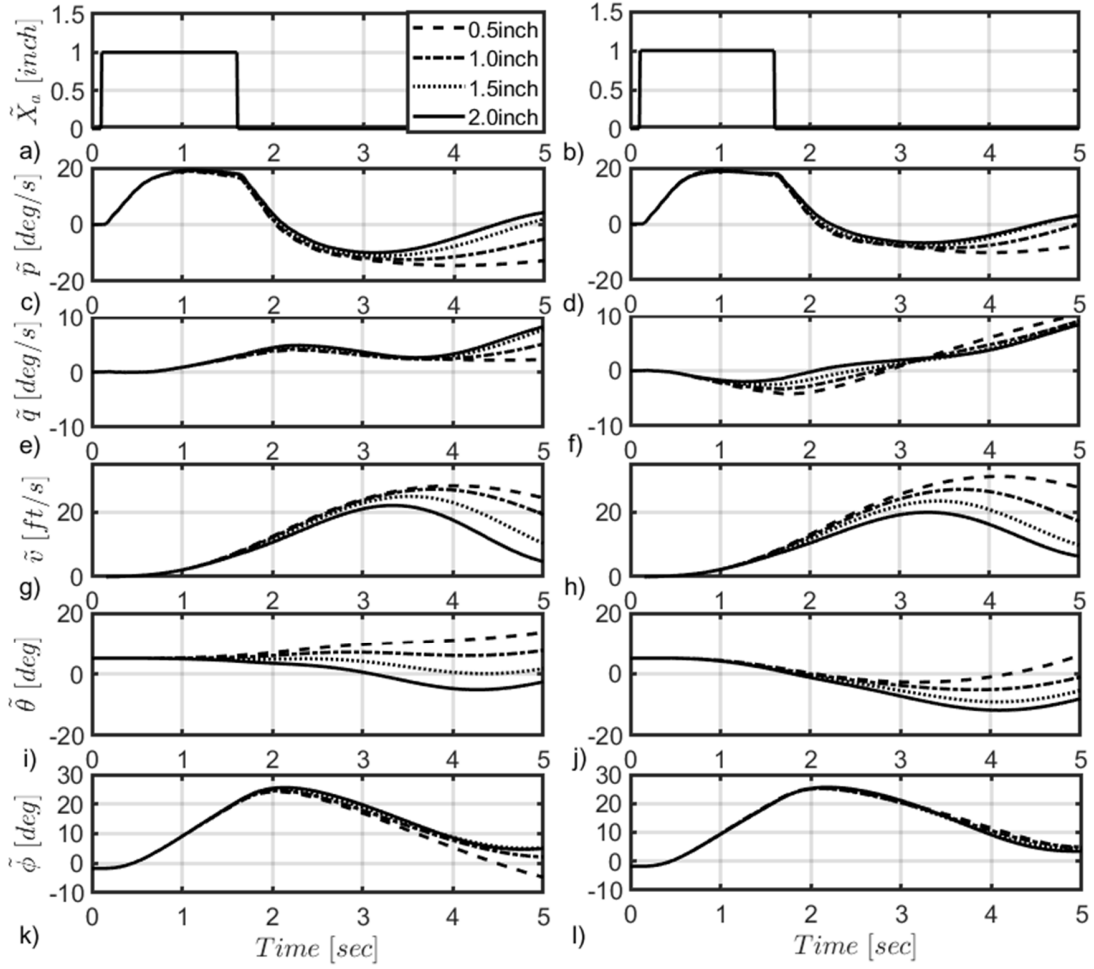


Fig. 17 Scaled response of F-B412 for different levels of lateral control input (pulse width 1.5sec)
Left: w/o MWD, Right: with MWD.

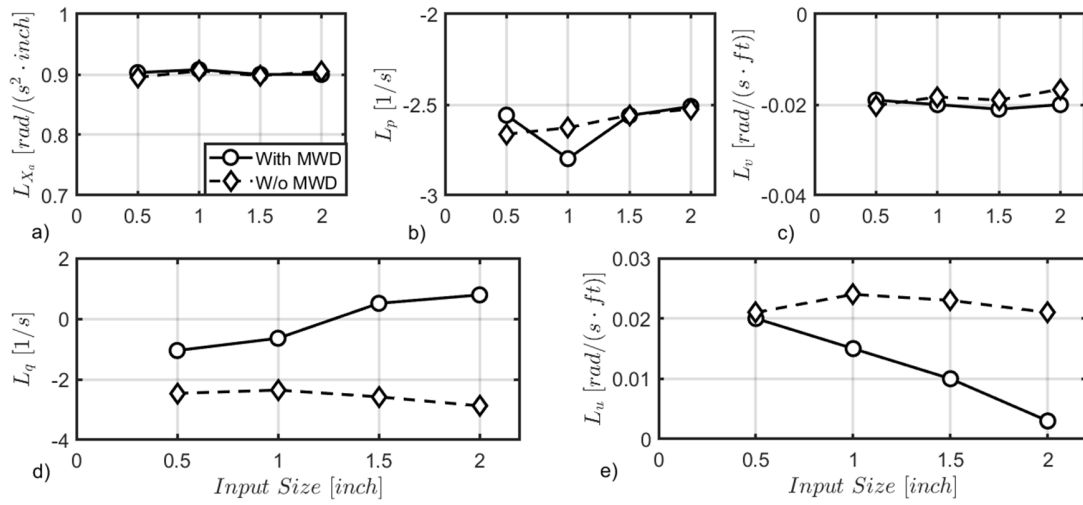


Fig. 18 Derivative variations vs step input size using ASysID.

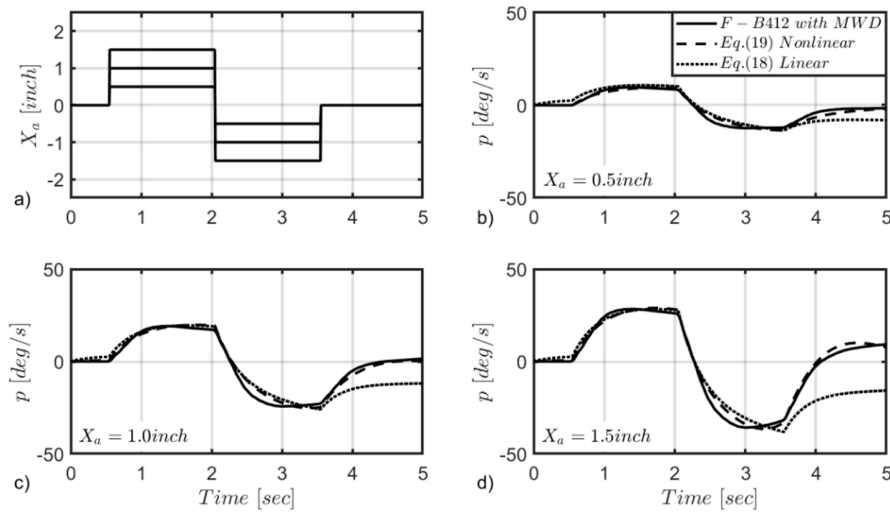


Fig. 19 Response of F-B412 (with MWD), compared with the modified $\dot{p}$ using Eq. (19) (Doublet input).

Table 1 Derivative comparisons between ASysID, SWR, and perturbation method (LF-B412) obtained from F-B412 (w/o MWD) at hover

Derivatives	ASysID		SWR		LF-B412 w/o MWD
	$\hat{Y}_{x_i}$	$ s(\hat{Y}_{x_i})/\hat{Y}_{x_i} $	$\hat{Y}_{x_i}$	$ s(\hat{Y}_{x_i})/\hat{Y}_{x_i} $	
L_{X_a}	0.905 (1)	10.6%	0.902 (3)	5.4%	0.916
L_p	-2.947 (2)	2.2%	-2.813 (4)	8.1%	-2.642
L_q	-2.145 (3)	8.2%	-3.838 (1)	15.5%	-1.636
L_v	-0.032 (3)	5.0%	-0.031 (5)	12.7%	-0.020
L_w	0.0 [‡] (5)	0.0 [‡]	0.0 [‡]	0.0 [‡]	-0.006
L_u	0.031 (6)	1.6%	0.033 (2)	31.5%	0.003
L_r	0.322 (7)	3.8%	0.0 [‡]	0.0 [‡]	-0.079
τ_{X_a}	0.10				

[‡]Deleted in the model structure

Table 2 Comparison of FT ASysID with SWR and pert. values (w/o and with MWD, 5kts) for roll and pitch

Derivatives	ASysID		SWR		Pert. Values	
	$\hat{Y}_{x_i}$	$ s(\hat{Y}_{x_i})/\hat{Y}_{x_i} $	$\hat{Y}_{x_i}$	$ s(\hat{Y}_{x_i})/\hat{Y}_{x_i} $	w/o MWD	with MWD
L_{X_a}	1.027 (1)	4.5%	1.022 (1)	7.3%	0.918	0.918
L_p	-2.778 (2)	2.7%	-2.628 (2)	19.3%	-2.673	-2.951
L_q	0.366 (3)	27.2%	1.747 (3)	20.0%	-1.754	-1.019
L_v	-0.046 (4)	4.1%	-0.082 (5)	33.3%	-0.019	-0.020
L_u	0.011 (5)	24.5%	0.042 (6)	32.0%	0.021	0.021
L_w	0.0 [‡]	0.0 [‡]	-0.015 (4)	7.3%	-0.003	-0.002
L_r	0.089 (6)	22.9%	0.0 [‡]	0.0 [‡]	-0.081	-0.106
M_{X_b}	-0.220 (1)	1.6%	-0.216 (1)	10.1%	-0.232	-0.232
M_q	-0.523 (2)	3.4%	-1.079 (2)	38.9%	-0.475	-0.488
M_u	0.008 (3)	11.3%	0.0 [‡]	0.0 [‡]	0.003	0.003
M_p	-0.267 (4)	6.0%	0.495 (4)	32.4%	0.272	-0.190
M_v	0.005 (5)	12.0%	0.0 [‡]	0.0 [‡]	0.003	0.003
M_w	-0.005 (6)	6.0%	0.008 (3)	33.6%	0.000	0.000
M_r	0.0 [‡]	0.0 [‡]	0.0 [‡]	0.0 [‡]	0.030	-0.009

[‡]Deleted in the model structure

Table A1 Comparison of 6DoF FT ASysID with LF-B412 (w/o and with MWD) at 5kts

	$\hat{Y}_{x_i}$	$\left \frac{s(\hat{Y}_{x_i})}{\hat{Y}_{x_i}} \right $	Time point (sec)	LF- B412	LF- B412 MWD		$\hat{Y}_{x_i}$	$\left \frac{s(\hat{Y}_{x_i})}{\hat{Y}_{x_i}} \right $	Time point (sec)	LF- B412	LF- B412 MWD
X_u	0.0 [‡]	0.0 [‡]	0.0 [‡]	-0.012	-0.012	N_q	-0.295 (4)	10.6%	4.37	-0.282	-0.153
X_w	-0.174 (5)	21.6%	4.82	0.032	0.031	N_v	0.0 [‡]	0.0 [‡]	5.09	0.005	0.005
X_q	5.846 (2)	3.0%	1.25	0.616	0.632	N_p	-0.846 (3)	5.2%	1.99	-0.460	-0.486
X_v	0.347 (4)	7.1%	3.57	-0.010	-0.010	N_r	-0.416 (2)	1.9%	1.46	-0.330	-0.330
X_p	-0.477 (3)	5.1%	1.34	-1.396	0.420	X_{X_h}	1.277 (1)	5.9%	0.47	1.131	1.131
X_r	0.0 [‡]	0.0 [‡]	0.0 [‡]	-0.356	-0.198	X_{X_n}	0.343 (1)	3.6%	0.58	0.144	0.144
Z_u	0.035 (4)	29.1%	6.60	-0.047	-0.046	X_{X_c}	0.624 (1)	5.2%	0.55	0.789	0.777
Z_w	-0.511 (2)	21.2%	0.61	-0.375	-0.369	X_{X_p}	0.0 [‡]	0.0 [‡]	0.0 [‡]	0.002	-0.002
Z_q	0.0 [‡]	0.0 [‡]	0.0 [‡]	8.701	8.825	Z_{X_b}	1.051 (1)	15.6%	0.34	0.188	0.187
Z_v	0.385 (3)	12.6%	3.34	-0.007	-0.007	Z_{X_n}	0.0 [‡]	0.0 [‡]	0.0 [‡]	0.024	0.024
Z_p	0.0 [‡]	0.0 [‡]	0.0 [‡]	1.093	-1.058	Z_{X_c}	-8.591 (1)	1.9%	0.35	-8.981	-8.851
Z_r	0.0 [‡]	0.0 [‡]	0.0 [‡]	2.131	1.945	Z_{X_p}	0.337 (1)	5.2%	0.39	-0.018	-0.017
M_u	0.008 (3)	11.3%	2.31	0.003	0.003	M_{X_b}	-0.220 (1)	1.6%	0.45	-0.232	-0.232
M_w	-0.005 (6)	6.0%	5.35	0.000	0.000	M_{X_a}	-0.044 (1)	10.5%	0.45	-0.035	-0.035
M_q	-0.523 (2)	3.4%	1.26	-0.475	-0.488	M_{X_c}	0.014 (1)	12.1%	0.43	0.009	0.009
M_v	0.005 (5)	12.0%	4.49	0.003	0.003	M_{X_p}	-0.050 (1)	10.0%	0.46	-0.008	-0.008
M_p	-0.267 (4)	6.0%	3.37	0.272	-0.190	Y_{X_b}	-0.933 (1)	2.2%	0.41	-0.236	-0.235
M_r	0.0 [‡]	0.0 [‡]	0.0 [‡]	0.030	-0.009	Y_{X_a}	1.108 (1)	8.6%	0.38	0.746	0.746
Y_u	0.0 [‡]	0.0 [‡]	3.87	0.014	0.014	Y_{X_c}	0.296 (1)	8.1%	0.38	-0.177	-0.175
Y_w	0.0 [‡]	0.0 [‡]	0.0 [‡]	-0.005	-0.004	Y_{X_p}	0.0 [‡]	0.0 [‡]	0.0 [‡]	0.885	0.873
Y_q	-1.457 (5)	13.1%	3.72	-1.347	-0.918	L_{X_b}	-0.224 (1)	1.7%	0.43	-0.364	-0.364
Y_v	-0.209 (3)	3.1%	3.01	-0.031	-0.031	L_{X_n}	1.027 (1)	4.5%	0.41	0.918	0.918
Y_p	-0.381 (2)	3.9%	1.03	-0.647	-0.719	L_{X_c}	-0.056 (1)	8.4%	0.40	-0.051	-0.050
Y_r	-1.429 (4)	14.9%	3.41	-8.080	-8.170	L_{X_p}	0.144 (1)	4.2%	0.41	0.276	0.273
L_u	0.011 (5)	24.5%	3.94	0.021	0.021	N_{X_h}	0.0 [‡]	0.0 [‡]	0.41	-0.064	-0.064
L_w	0.0 [‡]	0.0 [‡]	0.0 [‡]	-0.003	-0.002	N_{X_a}	0.282 (1)	1.4%	0.32	0.160	0.160
L_q	0.366 (3)	27.2%	1.10	-1.754	-1.019	N_{X_c}	0.328 (1)	0.7%	0.33	0.319	0.318
L_v	-0.046 (4)	4.1%	2.82	-0.019	-0.020	N_{X_p}	-0.465 (1)	0.9%	0.94	-0.501	-0.494
L_p	-2.778 (2)	2.7%	0.86	-2.673	-2.951	τ_{X_b}	0.14	--	--	--	--
L_r	0.089 (6)	22.9%	5.04	-0.081	-0.106	τ_{X_a}	0.14	--	--	--	--
N_u	-0.007 (5)	14.3%	6.80	0.002	0.002	τ_{X_c}	0.05	--	--	--	--
N_w	0.0 [‡]	0.0 [‡]	0.0 [‡]	0.002	0.002	τ_{X_p}	0.06	--	--	--	--

[‡]Deleted in the model structure



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A new heuristic approach to rotorcraft system identification

Lu, Linghai; Agarwal, Dheeraj; Padfield, Gareth D.; White, Mark D.; Cameron, Neil

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