

CRANFIELD UNIVERSITY

NICOLÒ STRONATI

EFFECTS OF CLOSE APPROACH DYNAMICS ON
ASTEROID ORBIT DETERMINATION

FACULTY OF ENGINEERING AND APPLIED SCIENCES

PhD

Supervisors:

Dr Marta Ceccaroni

Dr Jennifer Kingston

April 2025

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Abstract

Asteroid orbit determination is the process of fitting astrometric observations to estimate osculating orbital elements using least squares methods. The degree of quality of the fit is assessed through residuals, which are influenced by factors such as observation accuracy, the applied weighting scheme, and the adopted dynamical model. Reliable asteroid orbits are critical for planetary defence, particularly in assessing impact risks with Earth. An analysis of the orbital database maintained by the Near-Earth Object Coordination Centre (NEOCC) of the European Space Agency (ESA) highlighted a particular subset of orbits with anomalously high residuals, here defined *broken orbits*. These anomalies may lead to the hypothesis of the presence of unmodelled impulsive events within the observational arc, potentially caused by close encounters with asteroids not included in the dynamical model. The aim of this research is to investigate whether flybys between two asteroids, when not properly accounted for in the dynamical model, contribute to such discrepancies, and whether extending the dynamical model to include additional perturbing asteroids could improve orbit determination. To this aim, a statistical characterisation of the NEOCC residual database is initially performed, followed by identification of non-dynamical and dynamical factors influencing the orbit determination process. The former focuses on revising the weighting scheme by identifying observer-dependent biases linked to asteroid dynamical classes. The latter include improved mass estimates for known perturbers and the study of the effect of a flyby with an unmodelled object on the residuals. This is performed using numerical simulations of asteroid-asteroid flybys, orbital propagation and generation of synthetic observations. Results demonstrate that the unmodelled dynamics of flybys between asteroids shall be discarded as the cause of the

broken orbits residual anomalies. However, the dynamical model can still be improved through proposed refinements in mass estimation and weighting schemes.

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List of main abbreviations

ADES	Astrometric Data Exchange Standard
CA	Close Approach
CE	Close Encounter
CNEOS	Centre for Near-Earth Object Studies
CRTBP	Circular Restricted Three-Body Problem
Dec	Declination
ESA	European Space Agency
ESRIN	European Space Research Institute
FO	find_orb
JPL	Jet Propulsion Laboratory
KM	Keplerian Map
LOV	Line of Variations
LPE	Lagrange Planetary Equations
MBA	Main-Belt Asteroid
MD	Mass Determination
MOID	Minimum Orbit Intersection Distance
MPC	Minor Planet Center
MPCORB	MPC Orbit Database
NASA	National Aeronautics and Space Administration
NEA	Near-Earth Asteroid
NEO	Near-Earth Object
NEOCC	Near-Earth Object Coordination Centre
(O-C)	Observed Minus Computed
OD	Orbit Determination
PC	Patched Conics
PHA	Potentially Hazardous Asteroid
RA	Right Ascension
RAAN	Right Ascension of the Ascending Node
RMS	Root Mean Square

$\Delta(\text{RMS})$	Change in RMS residuals
SOI	Sphere of Influence
SRP	Solar Radiation Pressure
SSODNET	Small Solar System Object Data Network
T-P	Tisserand–Poincaré
TNO	Trans-Neptunian Object
YORP	Yarkovsky–O’Keefe–Radzievskii–Paddack Effect

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Me taking part in the celebrations for the 10th anniversary of the NEOCC in Castel Gandolfo (Rome) in May 2023.

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Quale Marte? Io vengo dalle Marche

(Fabrizio Tarducci)

Chapter 1

Introduction

Since the dawn of humanity, we have been called by the grand spectacle of the sky. From the very beginning, we have been captivated by celestial bodies like stars, planets, and occasionally, comets. These astronomical objects, visible yet so intangible and mysterious to our prehistoric ancestors, have been interconnected with the development of human knowledge. Just think of astrology or the biblical references to stars as guides from Heaven. How much significance did the ancients place on the visible sky? Yet, little did they know that among the vast array of perfectly visible objects in the sky, there were others, perfectly invisible, that, unlike their visible but too far counterparts, have covered an actual role for life on Earth. In both creation and destruction. Though invisible, they were the true silent contributors, bringing water and essential molecules to our planet [1, 2, 3], and responsible for catastrophic events, such as the extinction of the dinosaurs. In ancient times, they were still unknown, but today we are well aware of what asteroids are.

Asteroids, or minor planets, are rocky objects that orbit the Sun but are not classified as planets or comets due to their smaller sizes or lack of activity respectively¹. As of today The Minor Planet Center (MPC), which serves as the central registry for small-

¹However, some asteroids exhibit a form of activity, meaning they display an observable tail or coma of debris or an outgassing stream. This has made the comet-asteroid continuum less sharp, thanks to improved observational techniques that have made it possible to detect small debris streams around asteroids previously thought to be inactive. Additionally, activity has been observed due to non-gravitational accelerations associated with material sublimation. Active asteroids exist, including the biggest asteroid in the Solar System, (1) Ceres. Activity can arise from the sublimation of ice, impacts, thermal fracturing, other processes, or a combination of these factors. Impacts generate a debris tail but can also expose an icy core, leading to more sustained sublimation. These reasons, along with new discoveries, have contributed also to a reassessment of the dynamical classification of asteroids and comets. Traditionally, this classification relied on the Tisserand parameter with respect to Jupiter, but recent findings reinforce the idea that a sharp distinction between comets and asteroids does not apply; rather, they should be viewed as part of a continuum. For a more extensive review on the topic, one can refer to [4].

body discoveries, lists more than 1.4 million asteroids, catalogued using over 475 million ground- and space-based observations. Their sizes range from small rubble piles, with diameters as small as 2 meters (e.g., 2015 TC25), to dwarf planets like (1) Ceres, which measures over 900 km in diameter.

Asteroids are typically categorised into three main dynamical classes based on the distance of their orbits from the Sun: Near-Earth Asteroids (NEAs), Main Belt Asteroids (MBAs), and Trans-Neptunian Objects (TNOs). NEAs have perihelion distances less than 1.3 au, MBAs are located in the main asteroid belt with perihelion distances ranging from 1.78 to 5 au, and TNOs lie beyond Neptune, with perihelion distances greater than 30 au.

Studying asteroids is considered fundamental to understanding the formation of the Solar System [5], as they are close analogues to the objects in the protoplanetary disk around stars where planetary systems form. Their size, geology, and weak gravity have preserved features from the early stages of Solar System formation, highlighting their scientific significance. Additionally, asteroids are of growing interest for their mineralogical properties, particularly for mining rare materials and space-based resource utilisation, including water supply for deep space missions [6]. Another key driver of asteroid research is the potential collision risk posed by objects orbiting the Solar System on trajectories close enough to Earth to be considered a threat, with varying degrees of catastrophic consequences. As of 2025, over 37 000 NEAs have been discovered, with nearly half of these detections occurring in the past decade. As of today, it is estimated that between 87% and 95% of Near-Earth Objects (NEOs) larger than one kilometre in diameter have been identified. Of these NEAs, more than 2000 have been designated as potentially hazardous asteroids (PHAs)² due to their potential for very close approaches to Earth.

In light of these aspects, which may pose real risks to the survivability of our planet, the necessity for the international research community to study these objects and their dynamics becomes evident. The primary goal is to achieve an increasingly detailed understanding of their trajectory evolution, mass, and composition. These factors are of critical importance not only for assessing impact risks with Earth, determining impact probabilities, and identifying potential impact corridors and locations, but also for planetary defence missions aimed at deflecting or neutralising impactors [7, 8, 9].

Concerns about asteroid impacts on Earth have attracted the attention of governments and space agencies, leading to dedicated planetary defence campaigns. Since the 1990s, efforts have focused on sky surveys to identify more asteroids, determine their orbits,

²<https://www.minorplanetcenter.net/mpc/summary>

and assess their impact probabilities as a priority. Pioneering in this field was NASA's NEO Observation Program, which began in the early 1970s by assembling and developing telescopes for asteroid and comet observations. Subsequent survey efforts, along with advancements in telescope and detection technology, led to a significant push in the early 1990s when the U.S. government set NASA the objective of conducting the Spaceguard program [10], with the objective of discovering 90% of all NEOs with a diameter of at least one kilometre within ten years, a goal that was finally achieved by the end of 2010. A key event that further emphasised the importance of planetary defence was the 1994 impact of comet P/Shoemaker-Levy 9 (D/1993 F2) on Jupiter. On the U.S. side, another directive in 2005 set the goal of discovering at least 90% of all NEOs larger than 140 metres by 2020. NASA's asteroid survey data and information management are handled by the Planetary Defence Coordination Office (PDCO), established in 2016, and the NASA and JPL Center for Near-Earth Object Studies (CNEOS). Since 2002, CNEOS has operated the automated collision monitoring system Sentry, which tracks asteroids and assesses potential future Earth impacts. For a more extensive review of NASA's NEO observation efforts over the years, see [11].

On the European side, the ESA Space Situational Awareness Programme (now called the Space Safety Programme) has been active since 2009, in cooperation with the University of Pisa and NEODyS, starting in 1999 with CLOMON, one of the first automated systems designed to predict potential asteroid impacts by analysing NEO orbits [12], and CLOMON2 in 2002 [13]. Eventually, ESA's NEO Coordination Centre (NEOCC) was established in May 2013 at ESA's European Space Research Institute (ESRIN) facilities in Frascati. NEOCC serves as ESA's office coordinating Planetary Defence Office (PDO) activities and is responsible for tracking NEOs, with the mission of 'protecting our planet, humanity, and assets in space and on Earth from dangers originating in space' [14]. Despite significant progress, further advancements in NEO impact monitoring are still needed [15]. Recently, the NEOCC team developed an autonomous impact monitoring system called Aegis [16], designed to operate independently of the previous CLOMON2 and Sentry systems. For a review of their similarities and differences of pre-Aegis systems, see [17].

Among its services, the NEOCC also manages a public database where it publishes information on the orbits and dynamic properties of all detected asteroids, laying the groundwork for its broader objectives in impact risk assessment for planetary defence. As mentioned earlier, the NEOCC is still in the process of enhancing its capabilities and contributing to global planetary defence efforts through expanded observation campaigns [18, 19] and improvements in dynamical analyses. Potential developments in NEOCC's OD

pipeline play a central role in this work.

NEOCC database structure

Before examining how this thesis can contribute to the NEOCC pipeline, it is necessary to analyse the contents of its database, specifically, the most relevant dynamical features at this stage, to establish a foundation for the following discussion³.

- For each asteroid, a .rwo file stores all available *observations* in a format suitable for orbit computation. These observations, or astrometric points, describe the object's position in the sky at a given time, expressed in right ascension (RA) and declination (Dec), as measured from a specific station. The station is responsible for performing the observation and communicating it to the MPC. Each file contains a list of RA/Dec observations, along with the epoch of the observation, the accuracy of the positional and temporal measurements, the residuals of the data, and other relevant information describing the astrometric point.
- The *orbital parameters* of the asteroid are also provided. Orbit determination (OD) is performed by fitting the observations of an object with the appropriate dynamical model to derive its orbital parameters, using a least-squares method solved through a differential corrections algorithm [20]. While OD will be discussed in more detail in later chapters (also see [20] and [16] for a mathematical and algorithmic description of its implementation) a basic understanding is sufficient at this stage. OD is subject to errors, with the uncertainty in the results primarily depending on two factors: the *quality of the observational data* and the *dynamical model* used for integration. The former is discussed in Chapter 2: astrometric errors arise from both systematic and random sources [21] and can be mitigated through proper data reduction, weighting schemes, or outlier rejection techniques [22, 23]. The dynamical model accounts for all relevant perturbations beyond simple Keplerian motion

³For a more detailed explanation of the dynamical data provided by the NEOCC and the .rwo file format, refer to https://neo.ssa.esa.int/search?p_p_id=com_liferay_portal_search_web_search_results_portlet_SearchResultsPortlet_INSTANCE_uLgscvzhB2DW&p_p_lifecycle=0&p_p_state=maximized&p_p_mode=view&_com_liferay_portal_search_web_search_results_portlet_SearchResultsPortlet_INSTANCE_uLgscvzhB2DW_mvcPath=%2Fview_content.jsp&_com_liferay_portal_search_web_search_results_portlet_SearchResultsPortlet_INSTANCE_uLgscvzhB2DW_redirect=%2Fsearch%3Fq%3Drwo&_com_liferay_portal_search_web_search_results_portlet_SearchResultsPortlet_INSTANCE_uLgscvzhB2DW_assetEntryId=9571445&_com_liferay_portal_search_web_search_results_portlet_SearchResultsPortlet_INSTANCE_uLgscvzhB2DW_type=content&p_l_back_url=%2Fsearch%3Fq%3Drwo&p_l_back_url_title=Search.

around the Sun. This includes gravitational influences from planets and other asteroids, as well as non-gravitational effects such as solar radiation pressure (SRP), the Yarkovsky effect, and the Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect.

- The *observed-minus-computed (O-C) residuals* of the least-squares solution are reported for each asteroid, both as residuals for individual observations and as the sigma-weighted root mean square (RMS) residuals of the global solution. This value indicates how well the osculating orbit fits the observational data. The RMS residual can be used as a measure of the solution’s quality: in general, smaller residuals suggest a more accurate orbit, while larger residuals indicate a poorer fit.

As of March 2024, the RMS values across the database vary significantly, ranging from as low as 0.025 for asteroid 2020 KX55 to 22.85 for 2014 QY71. A systematic analysis of the RMS of all orbits in the database revealed some cases worth investigating further, both to gain a better understanding of asteroid dynamics and to identify possible systematic errors in the OD process or limitations in the model that should be considered. This work focuses on the analysis of these cases, hereafter referred to as *broken orbits* (see Section below).

1.1 Broken Orbits

This research focusses on a particular family of asteroid orbits with residual anomalies. These orbits were noted while working on orbits with high residuals reported by the NEOCC team in their database. Roughly speaking, these can be defined as orbits with a *good quality*⁴ of their observations, i.e., of the data used for the orbit fitting, but with high residuals when considering the whole set of observations; however, they also show a significantly lower RMS, hence an improved orbit fitting, when the observations are split in two sets, before and after an hypothetical dynamic event happening at a certain epoch. Figure 1.1 illustrates this behaviour for asteroid (446816). The residuals for the pre and post orbit determinations are plotted against the last observation included in each solution. For the pre orbit, each point represents the residual from an orbit fit using all observations up to that year, while for the post orbit, it represents the residual from using observations from that year onward. The data are obtained through the open source OD

⁴For a better understanding of the concept of *good quality observation* one can refer to Section 1.4 or Ch. 2

software *Find_Orb*⁵.

As fewer observations are included, the residuals decrease, suggesting a better fit. This implies that excluding certain time periods improves the OD. Interestingly, both pre and post solutions reach a common minimum between 2011 and 2015, indicating that this may be the best point to *break* the orbit. A meaningful pre-orbit can be determined from 2000 to 2011, while the post-orbit is best constrained from 2015 to 2022. However, determining the exact epoch at which to break these orbits is not straightforward and depends on several factors, such that a preliminary analysis has led to the conclusion that an optimal criterion for breaking the orbit can not be found with the current knowledge. Figure 1.2 provides a visual comparison of the residual distribution for each case, highlighting this improvement.

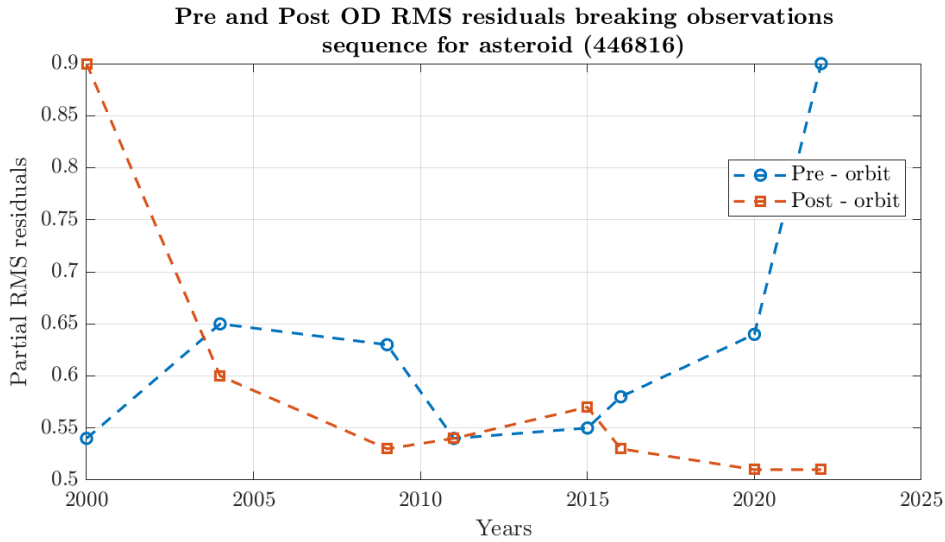
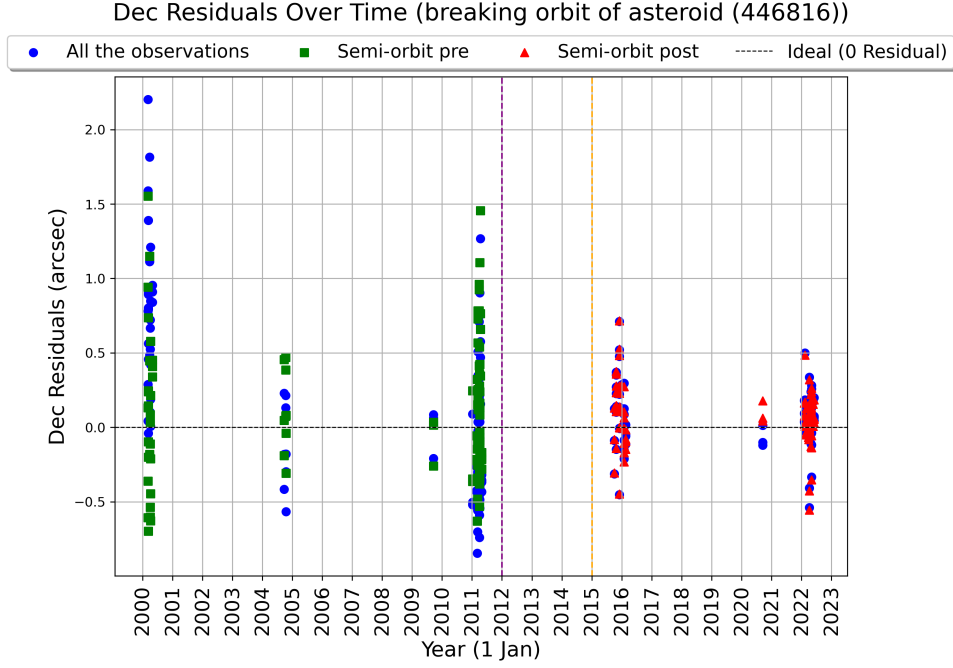


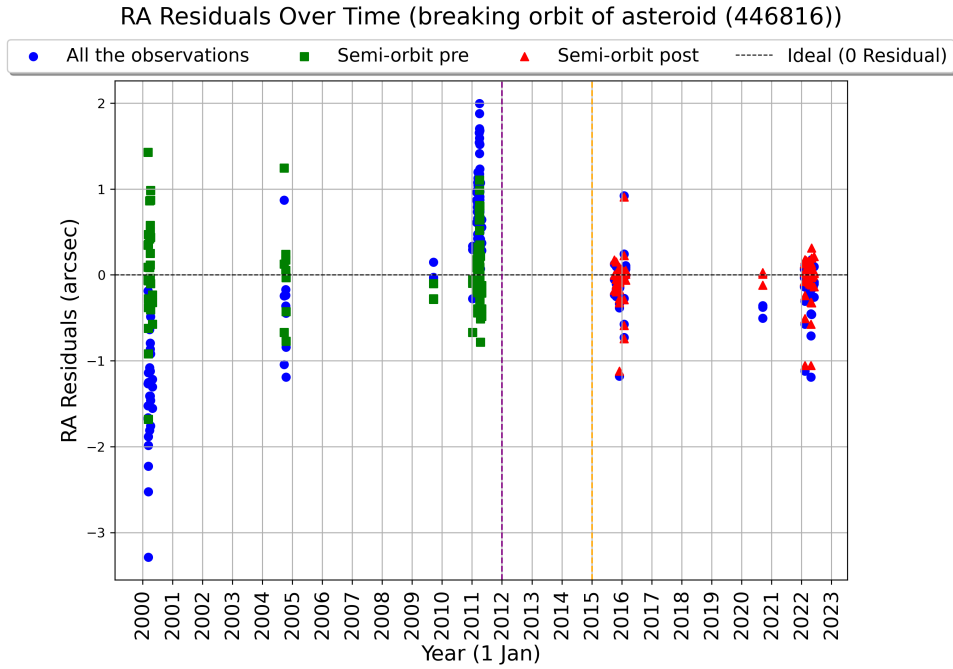
Figure 1.1: RMS residual of the OD for asteroid (446816) considering just observations from 2000 to the year on the x-axis for the pre orbit (blue line), or from the year on the x-axis to 2022 for the post orbit (red line).

The x-axis represents the timeline of observations, while the y-axis shows the residuals in arcseconds. Blue points correspond to residuals from using all observations together, that would lead to an RMS of 0.9. Green points show residuals from fitting only the 2000–2011 data, while red points correspond to the 2015–2022 fit. The improved OD is visible in how the green and red points are more concentrated around the zero-residual

⁵*Find_Orb* is the software used for OD throughout this project. It solves the orbit, providing results in terms of orbital parameters with associated uncertainties and residuals. It allows for the selection of the dynamical model, including the choice of gravitational perturbors based on the BC-405 model [24]. Other features used in this work include the detection of close encounters among asteroids, the determination of the mass of the perturbing asteroid during a close encounter, the generation of synthetic astrometric observations, and the computation of ephemerides. More details on the software, its features, and its implementation can be found at: https://www.projectpluto.com/find_orb.htm.



(a) Dec residuals



(b) RA residuals

Figure 1.2: Residuals in arcsec of the RA and Dec observations of asteroid (446816) obtained with 3 different OD strategies: blue dots, considering all the observations; green dots, considering the observations between 2000 and 2011; red dots, considering the observations from 2015 to 2022. The x-axis represents the 1 January of the years of observations. The purple and yellow vertical lines represent, respectively, when the set of observations has been split for the different ODs.

line compared to the full dataset.

This behaviour suggests that these orbits benefit from excluding observations within a certain time window. Similar to a broken bone, they appear *fractured*, opening to the hypothesis that something might have happened during this period that is not captured by the dynamical model. The operative explanation examined in this project is that an unmodeled flyby, possibly involving an asteroid not included in the current dynamical model, occurred and caused a deflection in the orbital trajectory. Since this perturbation is not included in the model, the least-squares method struggles to reconcile all observations into a single consistent solution, resulting in high residuals when the entire dataset is used. By splitting the observations, the possible disturbance would be removed from the model, allowing for a more accurate OD. It's a matter of putting a cast on the broken orbit.

1.2 Aims and Objectives

The overall aim of this work is to get a deeper understanding of the typical change in residuals caused by a close approach between two asteroids, and to understand if the inclusion of asteroidal close passages in the dynamical model might improve the OD of the *broken orbits*.

The thesis will thus aim at quantifying the maximal impact on the quality of the orbit determination of an hypothetical, unmodeled flyby with an object not included in the dynamical model. The main research question can hence be summarised as *what is the effect, in terms of maximum $\Delta(RMS)$, of improving asteroid close approach dynamics in the OD process?*

These effects will then be compared with the typical residuals of the broken orbits in the NEOCC database, to understand if these anomalies can be explained by missing perturbing bodies in the dynamical model. Operatively, this fulfils the need of the partner in the project to assess whether this improvement would significantly lower the number of *broken orbits* in their database and possibly explain their nature.

To answer this research question, the following *research objectives* have been identified:

1. *Select orbits whose residuals can be improved by enhancing the dynamic model, using residuals as a metric for the quality of orbit.* This will be achieved by successive

pruning of the orbits in the database, namely:

A) Prune out orbits that are already well determined:

- Performing a statistical characterisation of residuals in NEOCC orbits database;
- Evaluating a meaningful threshold for high/low residuals.

Orbits with residuals above the threshold need improvements in the OD process (Ch. 1).

B) Among the remaining, prune out orbits whose high residuals are due to poor data quality:

- Defining relevant, specific and measurable criteria (e.g., observation arc length, observation epoch, observers) to evaluate the quality of astrometric observations;
- Establishing a quality threshold for observations.

Residuals of orbits with observations quality below the threshold will not improve by changing their dynamical model (Ch. 1).

C) Identify other possible non-dynamical factors contributing to high residuals and estimate their relevance (Ch. 2).

2. *Estimating a realistic maximal Δ (RMS), obtainable by improving the current dynamical model for asteroid close approaches by:*

I) *Refining mass estimate and uncertainties of the perturbing asteroids already in the model*, by assessing how flybys with an object already considered in the model impacts residuals (Ch. 3, 4).

II) *Including more perturbing asteroids in the dynamical model:*

- A) Perform an analysis of actual past close approaches between asteroids, to characterise realistic close approach geometry (Ch. 3);
- B) Estimate the worse realistic close approach geometry: the configuration, within the range established above, that maximises gravitational interaction (Ch. 3, 5);
- C) Evaluate the maximal delta residuals obtainable by adding in the dynamical model a perturbing asteroid not yet included (Ch. 5).

3. *Determine if dynamical refinements identified can significantly reduce the number of broken orbits in database*, by performing a statistical analysis on broken orbits residuals and compare it to the maximum delta residuals obtained by dynamical improvements (Ch. 6).

1.3 Thesis structure

The outcome of the thesis, and the delivery towards the final objective, is presented as a collection of four papers. A summary of the thesis structure is outlined below, and also represented in Figure 1.3.

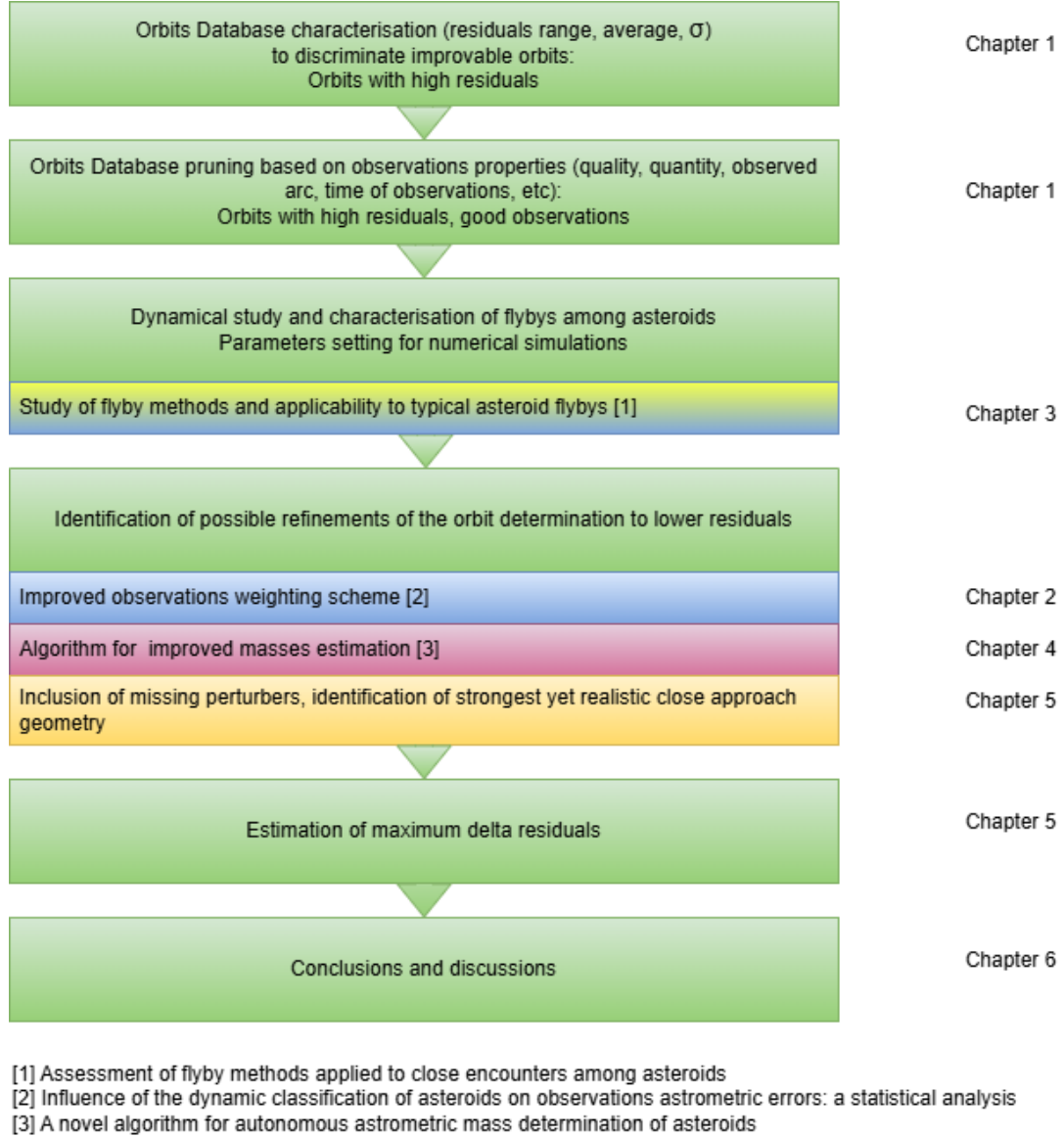


Figure 1.3: Summary of the thesis structure. The green blocks represent the actions to be taken, while the coloured boxes provide a breakdown of these actions, each linked to a chapter. The papers, whether published or submitted, that correspond to the chapters are indicated in the footnotes.

Ch. 1 aims to provide the general context for the importance of studying asteroid dynamics, highlighting their relevance in solar system evolution studies and planetary

defence. The structure of the NEOCC orbit database and the relevant data are then briefly introduced, as starting point for the research.

In Sec. 1.1, the concept of *broken orbits* and their existence in NEOCC database is introduced, outlining the need for more in-depth studies on asteroid dynamics to improve asteroid OD pipelines. The NEOCC database is later statistically characterised in Sec. 1.4, where the orbital residual range and distribution is analysed. This section aims at providing a measurable threshold for defining a *high residual* orbit, and identify them in the database, while in Sec. 1.4.1 these *outlier* orbits are pruned to identify which ones have high RMS due to observation properties, and generally non-dynamical features and discard them. The *broken orbits* will fall in the remaining orbits with high residuals, but good observation quality.

The identification of possible refinements in OD pipeline to lower residuals is investigated in three main ways. A strategy to resolve the poor observation quality is by applying suitable individual weights to the observations. An improvement to the currently employed weighting scheme is studied and proposed in Ch. 2. In this Chapter, a statistical analysis on the observation residuals on the NEOCC database is performed for the most productive observatories, assessing also their astrometric quality.

The mechanism of flybys among asteroids are then investigated, as a source of trajectory disturbance that, if unmodeled, could lead to an increase in residuals. In Ch. 3, methods for describing flybys are reviewed and their application to asteroid-asteroid flybys is evaluated. This chapter serves also as a description of these flybys, providing their typical dynamic and geometric parameters, and serves as a basis for numerical simulation setups of asteroid close encounters. Eventually, from this chapter one can conclude that flybys among asteroids take place and they have an influence on the RMS residuals if not properly accounted in the dynamic model.

After having assessed the flyby mechanisms, the dynamic model can be improved to reduce residuals by either refining the masses of existing perturbing asteroids or expanding the model to include additional perturbing bodies. Refining asteroid masses has great importance in solar system dynamics and for the overall improvement of the OD fitting. It is treated in Ch. 4, where a method for automated asteroid mass determination (MD) based on OD is presented. Upon the detection of a flyby, the mass of the an asteroid can, under certain conditions, be determined from the deflection it exerts on another asteroid.

Ch. 5 studies the effect on the orbit residuals of an extended dynamic model, including smaller perturbing bodies than those already included. It follows the process of:

1. Identifying the realistic flyby scenarios for maximising the $\Delta(\text{RMS})$, using the outcomes from Ch. 3.
2. Numerical simulations of asteroid-asteroid flybys under these conditions and generation of synthetic observations.
3. Estimation through OD in *Find_Orb* of the residuals and maximum $\Delta(\text{RMS})$ obtained in these cases.

Overall discussion of the results and comparison with the threshold for high residual orbit introduced in Sec. 1.4 is presented in Ch. 6, together with the limitations and future developments of the project.

1.3.1 Main achievements and contribution to knowledge

This work aims at improving the process of asteroid OD through:

- A first systematic statistical characterisation of the NEOCC orbital residual database (Ch. 1).
- Non-dynamical improvements:
 - Proving that the astrometric precision of some observers are influenced by the dynamical class of the observed asteroid, an effect not accounted in current weighting schemes (Ch. 2).
- Dynamical improvements:
 - Determining the masses of two asteroids for the first time using the deflection method. Additionally, achieving the best mass precision in the literature for four other asteroids (Ch. 4).
 - Showing that expanding the BC-405 dynamical model for asteroids used in *Find_Orb* does not reduce residuals in asteroid flybys. To investigate this, numerical libraries for asteroid flyby simulations and synthetic astrometric observations were developed. The study also identifies the optimal distribution of observations within the observation window to maximise $\Delta(\text{RMS})$ after a flyby (Ch. 5).

- A study of asteroid-asteroid flybys, presenting the first analysis of such close encounters and generating a dataset describing these events in dynamical and geometrical terms. The applicability of flyby theories, such as patched conics and the Keplerian map, is tested, and their validity ranges are identified (Ch. 3).

1.4 Statistical characterisation of orbital database residuals

Differences in RMS residual levels among asteroid orbits in the NEOCC orbital database have already been introduced. This section presents a detailed analysis of the typical RMS residuals achievable with current observational techniques and data quality. It also identifies orbits with unusually high residuals and explores the factors contributing to these deviations.

Figure 1.4 presents the distribution of RMS residuals for all orbits, with a zoomed-in version of this distribution, omitting orbits with $RMS > 5$ for visual clarity, in Figure 1.5. These distributions, however, exclude two cases that should be treated as clear outliers even before beginning the analysis. In fact, the lowest value recorded is 0.0 for asteroid 2020 CD3, while the highest value is 1 632 570 for asteroid 2015 HX69. The first case is the result of failed OD solution, as this object was a temporarily captured mini-moon of Earth, and its highly chaotic orbital evolution prevents successful Keplerian integration, even when accounting for Earth's perturbations. The second case presents an extreme value due to a wrong identification issue, which is the case when observations of a different asteroid are mistakenly assigned to the wrong object, leading to an inconsistent OD with poor residuals.

From these plots, a simil-Gaussian trend is visible, as expected for experimental data subject to random observational errors [20] and in agreement with the Central Limit Theorem [25]. However, there is also a slight deviation from the Gaussian distribution on the right tail, which makes the data distribution resemble a Poissonian trend. This deviation is attributed to systematic errors introduced by astrometric measurements, the dynamical model, or even the OD process itself.

The parameters of the superimposed Gaussian curve, derived from the data, indicate an average RMS residual of $\overline{RMS} = 0.575$ with a standard deviation of $\sigma = 0.088$. To identify orbits with high residuals, a 3σ outlier rejection criterion is applied, according to

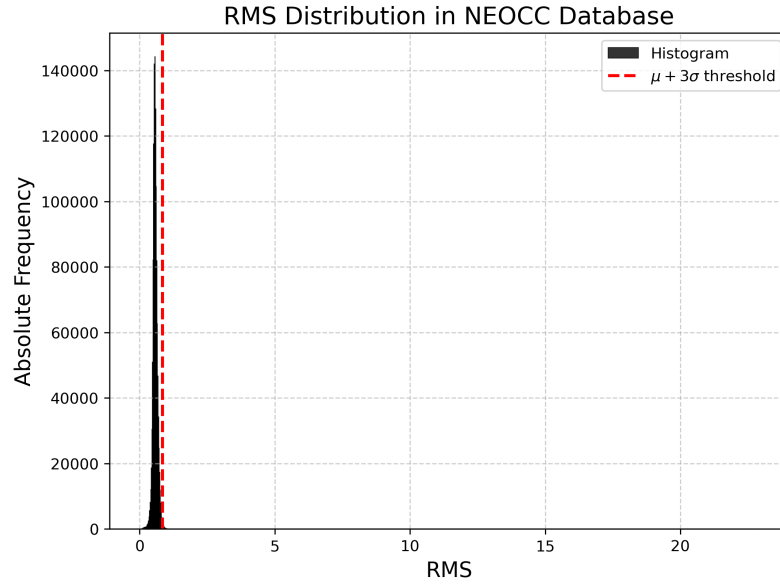
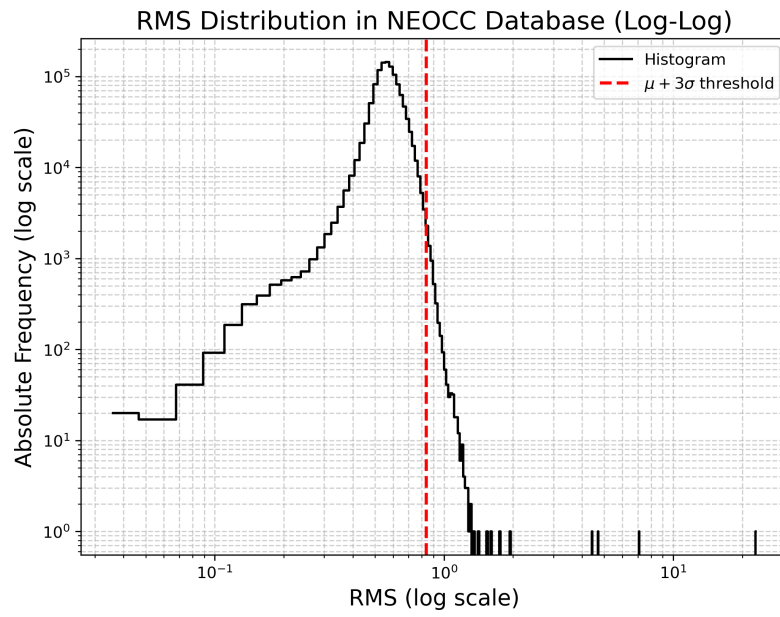
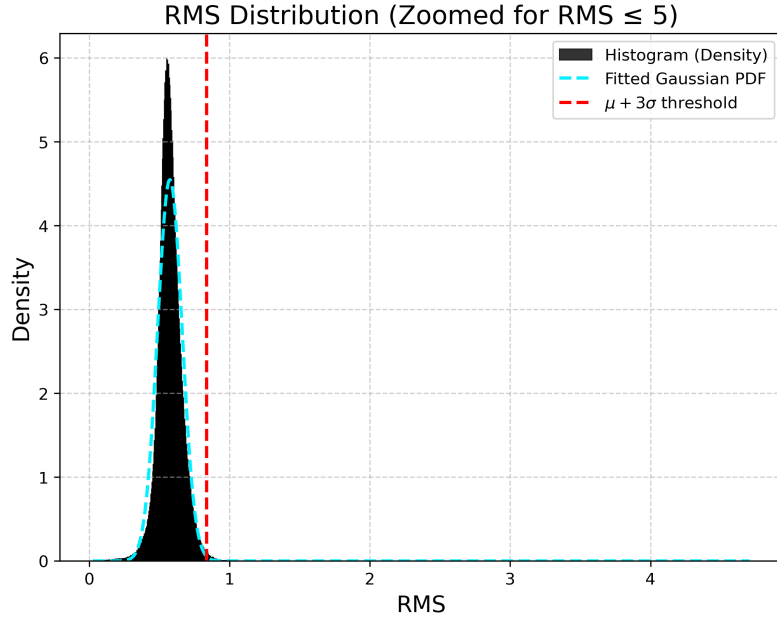
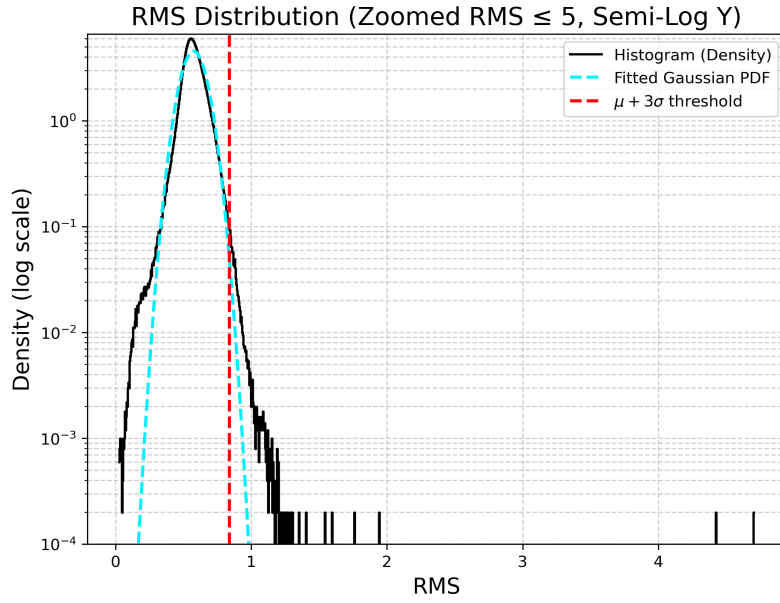


Figure 1.4: Histogram of RMS values in the NEOCC database. The red dashed line represents the 3σ threshold, used to identify outliers. Panel (b) shows the same distribution in a log-log scale.



(a)



(b)

Figure 1.5: Histogram of RMS values below 5, with a fitted Gaussian probability density function (blue dashed line). The red dashed line marks the 3σ threshold. Panel (b) shows the same distribution with y-axis in logarithmic scale.

which any data exceeding $\mu + 3\sigma$ are considered outliers. In a normal distribution, this threshold would exclude approximately 0.135% of the data in the right tail. However, due to systematic biases, a larger proportion of data is excluded in this case. The number of outliers beyond this threshold is 5 322, representing 0.457% of the total dataset.

Table 1.1: Summary of Gaussian residual analysis and outlier rejection for asteroid orbit RMS values.

Parameter	Value
Mean RMS residual ($\overline{RMS}$)	0.575
Standard deviation (σ)	0.088
Outlier threshold ($\mu + 3\sigma$)	0.839
Outliers in the right tail	5322
Expected outlier proportion (Normal Distribution, right tail)	0.135%
Actual outlier proportion	0.457%

Since the focus is on orbits with high residuals, only right-tail outliers are considered. This analysis establishes a baseline for RMS residuals in the NEOCC database, with a typical value around 0.575 and a threshold for high residuals at $\mu + 3\sigma = 0.838$. Orbits exceeding this threshold require further investigation.

1.4.1 Observations quality pruning

After identifying the 5 322 outlier orbits, the next step is to isolate those with high residuals due to dynamical effects rather than other factors. Orbits where high residuals likely result from systematic errors in astrometric data or the OD process are filtered out. This *pruning* procedure follows empirically derived criteria and is summarised in Fig. 1.6.

1. Orbits with fewer than three oppositions, i.e., periods when the asteroid is opposite the Sun relative to Earth, are excluded. Asteroids with low oppositions have limited observational coverage over time which may lead to higher uncertainties in their orbits. 1 382 asteroids are removed.
2. For a similar reason, orbits with fewer than 20 total observations are excluded, as generally more data are required to refine their residuals and covariance. This criterion removes 67 asteroids from the dataset.

3. Orbits where the most recent observation was taken more than 10 years ago (i.e., predating 2015) are also filtered out. Such asteroids may be lost, making it difficult to assess if they can be part of the *broken* orbits category. Additionally, outdated observations may be the primary cause of high residuals, since they are likely to have been taken with older instruments and procedures, as outlined in Ch. 2. This criterion removes 10 asteroids. These cases are still worth investigating to determine whether they were lost due to catastrophic collisions or other dynamical phenomena.
4. A further filter is based on observer reliability. Previous studies [22] indicate that different observers have different levels of observation accuracy. We consider nine observatories to be reliable [26], as listed below:

- G96 Mt. Lemmon Survey
- F51 Pan-STARRS 1, Haleakala
- F52 Pan-STARRS 2, Haleakala
- I41 Palomar Mountain/ZTF
- V06 Catalina Sky Survey-Kuiper
- 703 Catalina Sky Survey
- I52 Steward Observatory, Mt. Lemmon Station
- T05 ATLAS-HKO, Haleakala
- T08 ATLAS-MLO, Mauna Loa

If less than half of an asteroid's observations come from these reliable sources, the high residuals are considered to be more likely due to observational errors rather than dynamical effects. 45 asteroids are thus removed from the dataset.

5. Additionally, asteroids with observations obtained from radar-based surveys are omitted. These observation sources, such as WISE or TESS, usually introduce significant observational errors. The number of orbits affected by this criterion is 1 030.
6. Asteroids that present a case of assigned identification are disregarded. In fact, when the identification is wrong, it leads to erroneous OD solutions, as seen in the case of 2020 CD3. The number of affected orbits is 162.

After applying these pruning criteria, 2626 asteroids remain without outstanding observational issues, 92 of which are numbered. Broken orbits such as (446816) lie in this pool, although, as already said, identifying them automatically is not easy.

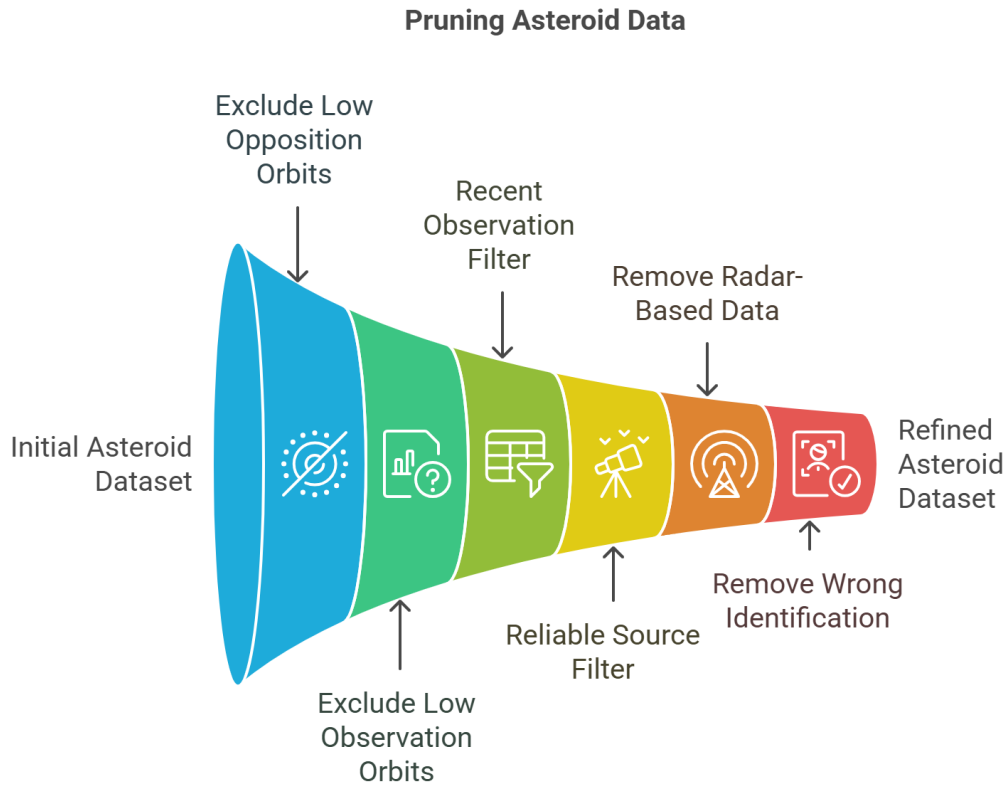


Figure 1.6: Schematic representation of the pruning criteria for the high residuals asteroid orbit dataset.

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Chapter 2

Statistical analysis of asteroid observation astrometric errors, and astrometric performance of the most productive asteroid observatories

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Foreword

This chapter presents and analyses the asteroid observations available in the NEOCC database, focusing on astrometric performance in terms of observational precision. It evaluates the precision achieved by major observatories and examines the current weighting system for observations. The analysis considers how precision depends on factors such as epoch, target magnitude, and the astrometric catalogue used. Finally, the influence of the asteroid’s dynamical class on observational accuracy is assessed, leading to insights that may lead to improvements to the existing weighting scheme.

Abstract

The ephemerides of minor planets are computed on the basis of astrometric observations. The asteroid orbit determination process requires these observations to be properly weighted to take into account the expected accuracy of the data. If not directly provided by the observers, the weights are, in general, computed after a station-specific statistical analysis on the observation residuals, where the influence of external factors such as epoch of observation, magnitude and employed catalogue has been proven. In this paper we perform a statistical analysis on observation residuals of the major surveys taking into account a new factor, i.e., the dynamical classification of asteroids, to understand if the observation quality may have a dependency on the different type of observed object. If an influence is actually found, then it will be possible to develop a new weighting system based on these results. The weights will be easily applicable once one knows the asteroid orbit. In particular, four stations have been found having different quality depending on whether they are observing Near-Earth Asteroids (NEAs) or Main Belt Asteroids (MBAs). Moreover, the cross-correlation between the dynamic classification and epoch, magnitude, catalogue is investigated, as well as the influence of these factors on observational quality.

Keywords: Asteroids – Astrometry – Catalogues – Methods: statistical – Surveys

2.1 Introduction

More than 1.2 million asteroids have currently been discovered thanks to increasing dedicated observation campaigns and technical improvements in the equipments such as CCD cameras and automatic surveillance pipelines. Moreover, it is thanks to the available observations for each of the detected objects that it is possible to define their orbits and trajectories, leading to accurate predictions of the positions of minor planets in future epochs.

The asteroid Orbit Determination (OD) process, indeed, integrates all the available astrometric points through numerical algorithms to compute the minor planets orbits. In particular, the trajectory is obtained solving a least-square problem in the osculating orbital elements, in which the proper dynamic model is considered and each observation is

weighted to take into account the expected accuracy of the data.

The reason for the need of a weighting system is the presence of errors in measures, and how they are made and processed, that lead to uncertainties in the results of the OD process. These are in general due to factors such as the quality of astrometric catalogues, employed for the determination of the asteroids' positions in the sky, the stations responsible for carrying out the measurement or physical parameters of the observing objects.

The errors introduced by the catalogues have been significantly reduced thanks to the introduction of more recent and accurate reference star catalogues such as 2MASS and Gaia DR2. Moreover, methods to debias old observations reduced with old and less reliable catalogues, referring them to the new ones, have been proposed and applied [1, 2], leading to a subsequent general improvement of previous observations too.

If it is possible to say that the errors due to the star catalogues are, at the state of the art, under control, thanks to the quality of the most recent ones, the same can not be said for the uncertainties introduced by the observing stations themselves. These are responsible for the quality of the instruments such as telescopes, the seeing of the station, the resolutions both spatial and temporal, the adopted working procedures, the number of observations performed per night, and the errors due to human operators. Moreover, also other factors can create more or less biases and decrease the accuracy of the measure such as the magnitude and the motion rate of the object.

It has been shown that the distribution of these errors, either of random or systematic origin, does not follow a Gaussian distribution in agreement with the Central Limit Theorem [3]. Furthermore, for most observations performed before the introduction of the ADES astrometric format in 2015, information on the accuracy adopted in time and angular quantities are in general not available.

Due to this lack of a-priori information on the measurement quality, a weighting system based on the expected accuracy of the single observation has been introduced to account for the quality of the integrated data. Indeed each observation, in the solution of the least-square method, has its own weight, which is based on the overall performance of the single observatories [4, 1]. These weights are computed taking as a parameter the Root Mean Square (RMS) of the Observed-Computed (O-C) residuals of the observations made earlier by the same station. It is clear how the computation of these weights requires the statistical treatment of a high amount of data, i.e. observations residuals, as per the probabilistic approach to the matter.

The idea under a station-specific weighting scheme is that the equipment, the technical precision, the methodology change station by station and must be considered when assessing the quality of any current or future observations. However, it has soon become clear that, within the same station, other factors exist that can vary the quality of the observations, and that should be accounted in the computation of the weights in order to achieve better results in the OD process. The effects of the epoch of observation, the light curve of the object, the motion rate have been statistically analysed [5] and some of these introduced in the current weighting scheme. Again, the idea is that, speaking of the epoch, over the time, some improvements both technical and to star catalogues, while the physical parameters of the asteroids are expected to have an influence on the observation performance.

In this framework, the aim of this paper is to introduce the study of a dynamic classification of asteroids as a possible parameter to be considered to further enhance the observations weighting scheme.

2.1.1 Influence of asteroids dynamic classification on observation quality

The Solar System minor planets are divided into three main dynamical macro-classes, i.e., Near-Earth Asteroids (NEAs), Main Belt Asteroids (MBAs), Trans-Neptunian Objects (TNOs), and it is possible to state that objects belonging to each one of these families have different characteristics when observed from planet Earth. The magnitude, the number of oppositions, the motion rate are in general different class by class and it is already known that these parameters can influence the observation quality. Also the observing techniques and procedures sometimes change, within the same stations, when dealing with asteroids of one or another class, as in the case of the observatories particularly focused on NEAs, like the *NEO follow-ups*. It is not rare that they manually remeasure with particular care a NEA when it is detected by the automated pipeline, with the aim of providing the astrometric data with the highest accuracy for this class of objects.

In light of these facts, studying the statistics on residuals divided on the basis of the asteroid dynamic classification, is not the same as analysing the impact of factors as magnitude or rate of motion individually, as done in previous works. Considering the residuals for the distinct classes of NEAs or MBAs allows us to consider also other influencing factors of different nature. Then, in case an influence on the residuals is actually found, it will be possible to develop a weighting system that can mitigate this bias and that is only

based on the dynamic class. This direct dependency would make the weights relatively easy to be applied to the observations once the orbit of an asteroid is determined, rather than, for instance, considering for any observation the rate of motion or the magnitude, factors that are not even included in the current weighting scheme.

To analyse this influence, the asteroids and their observations are clustered following these dynamic classes and the observation residuals are statistically studied in order to understand if an effect exists. This study will consider the observation performance of the most productive asteroid surveys towards the already mentioned dynamic classes, following the baseline presented in a previous work [5], including also data from four stations that were not considered there. The effects of observation epoch, magnitude and catalogue used for the astrometric reduction on the obtained results will be analysed too.

The structure of this paper can be summarised as follows: in Section 2.2 a description of how the data are selected and gathered is provided; the results of the RMS analysis is presented in Section 2.3; Section 2.4 introduces the influence of other factors such as epoch of observation, magnitude and catalogue.

2.2 Data gathering

The Minor Planet Center orbit (MPCORB) database¹ included more than 1.2 million asteroids as of January 2023, 619999 of which numbered. The observations of most of these bodies can be collected from the ESA's Near-Earth Objects Coordination Centre (NEOCC)² database, where the .rwo files containing the observations per each object are available. These files list, in each line, the provided observations and it is possible to extrapolate data on the time when it has been taken, the two angular positions RA and Dec and the MPC code of the station. These alphanumeric strings uniquely identify each observatory, and in the following sections they will be used to address the stations for the sake of brevity³.

The lines contain also the (O-C) residuals in RA and Dec, where the *computed* trajectory is provided by the ESA's NEOCC OD pipeline [6] that applies the most suitable dynamical model and an outlier rejection algorithm. These values are used to perform the analyses on RMS values.

¹<https://minorplanetcenter.net/iau/MPCORB.html>

²<https://neo.ssa.esa.int/>

³<https://minorplanetcenter.net/iau/lists/ObsCodesF.html>

Table 2.1: *RMS of the residuals in RA and Dec for 17 of the most productive surveys subdivided among the different asteroids macro-families. The number of available observations for each station for each class of asteroid is shown.*

Obs code	NEAs			MBAs			TNOs		
	RMS_{RA} (")	RMS_{Dec} (")	N. obs	RMS_{RA} (")	RMS_{Dec} (")	N. obs	RMS_{RA} (")	RMS_{Dec} (")	N. obs
704	0.62	0.61	98059	0.65	0.65	30972048	0.22	0.31	8
G96	0.29	0.27	124120	0.29	0.27	56769122	0.32	0.32	72
F51	0.12	0.12	110009	0.10	0.10	55106503	0.13	0.13	84201
F52	0.11	0.11	36608	0.10	0.10	17862574	0.11	0.11	5988
703	0.59	0.55	141209	0.63	0.59	32115514	0.82	0.62	11
691	0.44	0.40	30272	0.35	0.28	13366691	0.56	0.54	315
G45	0.33	0.33	30167	0.34	0.35	16318465	0.35	0.45	176
699	0.60	0.55	10613	0.61	0.56	5082880	N/A	N/A	0
644	0.34	0.40	9985	0.28	0.35	3786679	0.35	0.36	816
D29	0.40	0.38	18819	0.41	0.40	10885377	0.46	0.44	1311
C51	0.53	0.57	40524	0.60	0.68	3403771	N/A	N/A	0
E12	0.38	0.39	14564	0.47	0.50	2085989	N/A	N/A	0
608	0.59	0.65	5385	0.59	0.72	1147931	0.38	0.39	85
J75	0.41	0.38	2051	0.42	0.40	1112119	N/A	N/A	0
I41	0.31	0.27	34232	0.21	0.15	13707787	0.13	0.11	739
T05	0.31	0.31	83962	0.34	0.34	20556412	0.27	0.26	2141
T08	0.32	0.33	90321	0.35	0.36	22261667	0.30	0.32	2780

As already said, the asteroids are divided in dynamical families for the statistical studies. The classification in NEAs, MBAs and TNOs is made comparing the perihelion q of the computed orbit. Although fixed boundaries establishing if an asteroid belongs to a class or another have never been objectively defined, we divide the minor planets upon the following conditions: NEA if $q < 1.3$ au, MBA if q is between 1.78 and 5 au, TNO if $q > 30$ au.

Furthermore, the asteroids have also been filtered on the basis of the number of oppositions for which they have been observed. The interest in having a reasonably high number of oppositions is that in such a way the astrometric points are more spaced in time and the OD is more likely to provide reliable results, making the RMS statistics more significant. That is why only asteroids with strictly more than 2 oppositions have been considered, allowing us to study the residuals for both numbered and many non-numbered minor planets.

2.2.1 RMS computation

The 17 analysed stations are listed in the first column of Table 2.1 and are responsible, overall, for the production of more than 80% of the whole amount of observations. For each of these surveys the residuals are collected from the NEOCC database as of January 2023 and the RMS of residuals are computed dividing the data not only by station but also by class of asteroid. The results are shown in Table 2.1 together with the total number of observations N used to compute the statistics.

The total number of available astrometric data provides information on the statistical degree of confidence of the analysis and on the data distribution in the three asteroid classes. From it, MBAs turn out to have a higher availability of data, due to the fact that MBAs are more present in the Solar System, with almost 92% of asteroids belonging to this class⁴, and that they can benefit of more oppositions from Earth.

The opposite should be said about TNOs for which a very low number of astrometric data is available. There are even five stations (699, C51, E12, J75), mostly characterised by small-aperture telescopes, that have never reported observations of a TNO, while the most productive survey in this sense is F51 with more than 80 000 observations provided. For it, and given a general low availability of data on TNOs, it has been decided not to further analyse the statistics for these objects.

⁴<https://minorplanetcenter.net/mpc/summary>

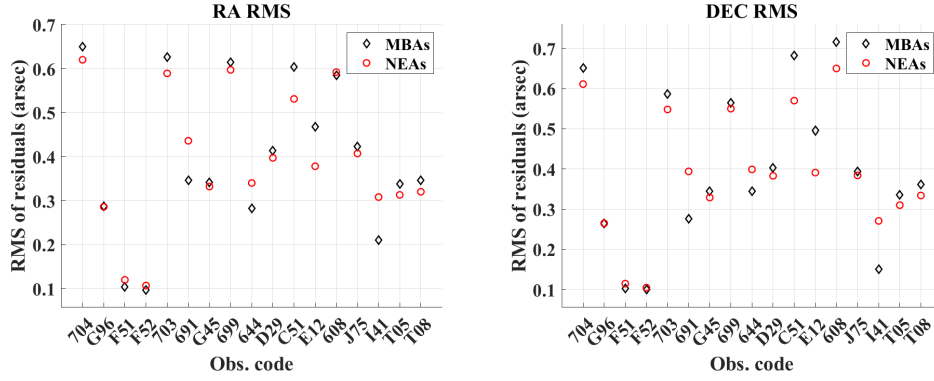


Figure 2.1: RMS of residuals in RA (left panel) and Dec (right panel) for all the stations, considering the values for NEAs and MBAs

2.3 Statistical analysis of RMS results

From the RMS values reported in Table 2.1 and represented also in Figure 2.1 one can see the global measure accuracy of the 17 stations towards the two considered asteroid classes: MBAs and NEAs.

First of all it is possible to compare the observation quality of the stations, from which it stands out that those with the lower RMS level are F51 and F52, the two Pan-STARRS stations on Haleakala. The similarity in the observation quality between these two surveys is not surprising as they share the same location, instruments, equipment, searching algorithms. However, F52 has a slightly lower RMS value, due to the subsequent installation with respect to F51, so that its findings have led to improvements in F52 equipment [7].

If we then focus our attention on the differences standing between NEAs and MBAs observations, it is possible to see that they are in general small enough to be considered within the signal noise. The stations that in this sense show the smallest gap, lower than $0.08''$, are G96 (Mt. Lemmon Survey) and F52 (Pan-STARRS 2, Haleakala). However, there are also four observatories that have high differences in the RMS of residuals in both RA and Dec.

E12 (Siding Spring Survey) and C51 (WISE) have low RMS on NEAs measures while I41 (Palomar Mountain) has a better accuracy for MBAs. This could be explained by the scopes and working procedures of these three stations. The operators of E12, which is currently no longer in operation, were used to manually reprocess and remeasure NEAs detection, fact that could have caused the higher precision on NEAs data. In a similar way, the space-based telescope C51 - WISE (Wide-field Infrared Survey Explorer) has, since

Table 2.2: Average ($\overline{res}$) and standard deviation (σ) of the astrometric data available for station 691 in RA and Dec, divided between MBAs and NEAs.

Ast. Class	$\overline{res}_{RA}$ (")	σ_{RA} (")	$\overline{res}_{Dec}$ (")	σ_{Dec} (")
MBA	-0.15	0.31	-0.026	0.27
NEA	-0.085	0.43	-0.069	0.39

2013, the target of providing NEAs observations, following the introduction of the NASA mission Near-Earth Object Wide-field Infrared Survey Explorer (NEOWISE) missions. This effort, again, could explain the better accuracy for this particular class of objects. I41, instead, has a lower RMS on MBAs data. Since I41 is not a station purely dedicated to asteroids observations [8] it might be reasonable to think that the detection of a relatively faster moving object like a NEA could lead to a worse accuracy.

The same better quality on MBAs data has been found also for 691 (Steward Observatory, Kitt Peak-Spacewatch), which is a productive survey still operating since 1984. For this station a deeper statistical analysis on the residuals is presented below to further investigate the reasons for the discrepancy on observation quality.

2.3.1 Statistics on astrometric data from 691 (Steward Observatory, Kitt Peak-Spacewatch)

In this section a deeper statistical analysis on the residuals provided by station 691 is presented in order to interpret the differences in the RMS levels between NEAs and MBAs.

In Figure 2.2 the histograms of the error distributions in RA and Dec are shown, superimposing the bar charts for MBAs and NEAs for each of the two angular quantities. The curves representing the normal Gaussian distributions, plotted on the basis of the mean and standard deviation values listed in Table 2.2, are represented for comparison. Both the histograms and the Gaussian curves have been normalised according to the definition of probability density function (pdf).

From Table 2.2 is easy to see that the average of the residuals is different from zero, as it would be expected in case of an ideal Gaussian distribution dominated just by random errors. It has already been discussed how also other external and perhaps systematic

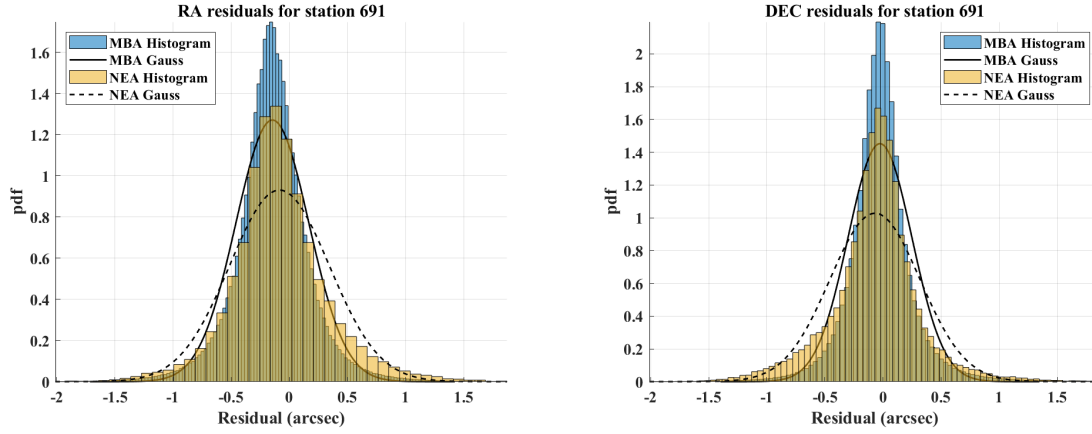


Figure 2.2: Probability density function (pdf) of the distribution of the residuals for station 691 in Right Ascension (left panel) and Declination (right panel). The histogram and the Gaussian Normal Distribution are represented superimposed for the residuals of MBAs and NEAs observations distinctly. The asymmetric tails suggest the presence of systematic biases in the observations, likely due to the working procedure or technical equipment employed for the different asteroid classes.

factors can affect the observations, contributing to a non-zero average of errors, which can be seen as the global bias of the station. This bias seems to be overall smaller for the Dec measures if compared to those in RA. However, the standard deviations of the four cases are similar to the RMS values for this station (Table 2.1), which is typical of data sets with zero or, as in this case, almost-zero averages.

The non compliance to the Gaussian behaviour of the errors can also be deduced by comparing the histograms to the relative ideal Gaussian curves. The concentration of values in the central bins of the plot and the asymmetric tails suggest once again the presence of systematic biases in the observations.

The bias of the station can be numerically evaluated through the $(\overline{res}/RMS)$ ratio, that can be computed for both the asteroid classes and the two angular quantities. For NEAs it is -0.20 in RA and -0.18 in Dec while for MBAs -0.42 in RA and -0.25 in Dec.

These results lead to the conclusion that 691 achieves, statistically speaking, in general, a better accuracy for NEAs observations and a higher precision (smaller standard deviation) for MBAs data. These behaviour might be explained by the working procedures or the technical equipment that may have difficulties in providing the same precision for both the dynamical classes.

2.4 Influence of other factors

Observation accuracy has been proved to be affected by many factors [5], such as the epoch of the measure, the magnitude, and the catalogue used. In this section, in addition to confirming the outcomes already presented in previous works as a secondary scope, adding information that became available recently especially with reference to observation years and adopted catalogues, a correlation is searched between these factors and the asteroids classification.

2.4.1 Dependency on the observation epoch

In Figures 2.3 and 2.4 the RMS of residuals over the years up to 2022 are plotted, highlighting the values for RA and Dec for MBAs and NEAs. In the following graphs, also the error bars for RMS values [9] are represented, in order to indicate the statistical degree of accuracy of the values. It is especially important to understand whether, when a RMS is anomalously high or low, it is due to actually data accuracy or low data availability in that particular bin.

The behaviours highlighted in Fig. 2.1 are here generally confirmed: the stations that had an overall better accuracy for one of the two classes of asteroids show the same trend in these plots for almost all the period of observation. These are the cases of the already analysed E12, C51, I41, 691.

C51 (Fig. 2.4b) has always provided an overall better accuracy for NEO in both the measured quantities, though it experienced a worsening of RMS between 2020 and 2021 for this class of asteroids, fact that might be connected with the approaching end of life of the spacecraft.

E12 case (Fig. 2.4c) is similar, but it is evident from the graph that it is no more in operation since the last available data are from 2013. However, it also shows that from 2005 on the RMS values for MBAs and NEAs started showing very different trends, with clear better performance on NEAs observations.

Also I41 (Fig. 2.4f) shows a similar behaviour. It started producing a high number of NEAs observations, more than 1000 per year, since 2018, approximately when the survey definitely changed denomination and equipment from Palomar Transient Factory (PTF) to Zwicky Transient Facility (ZTF) [8, 10], although the RMS values shown on the

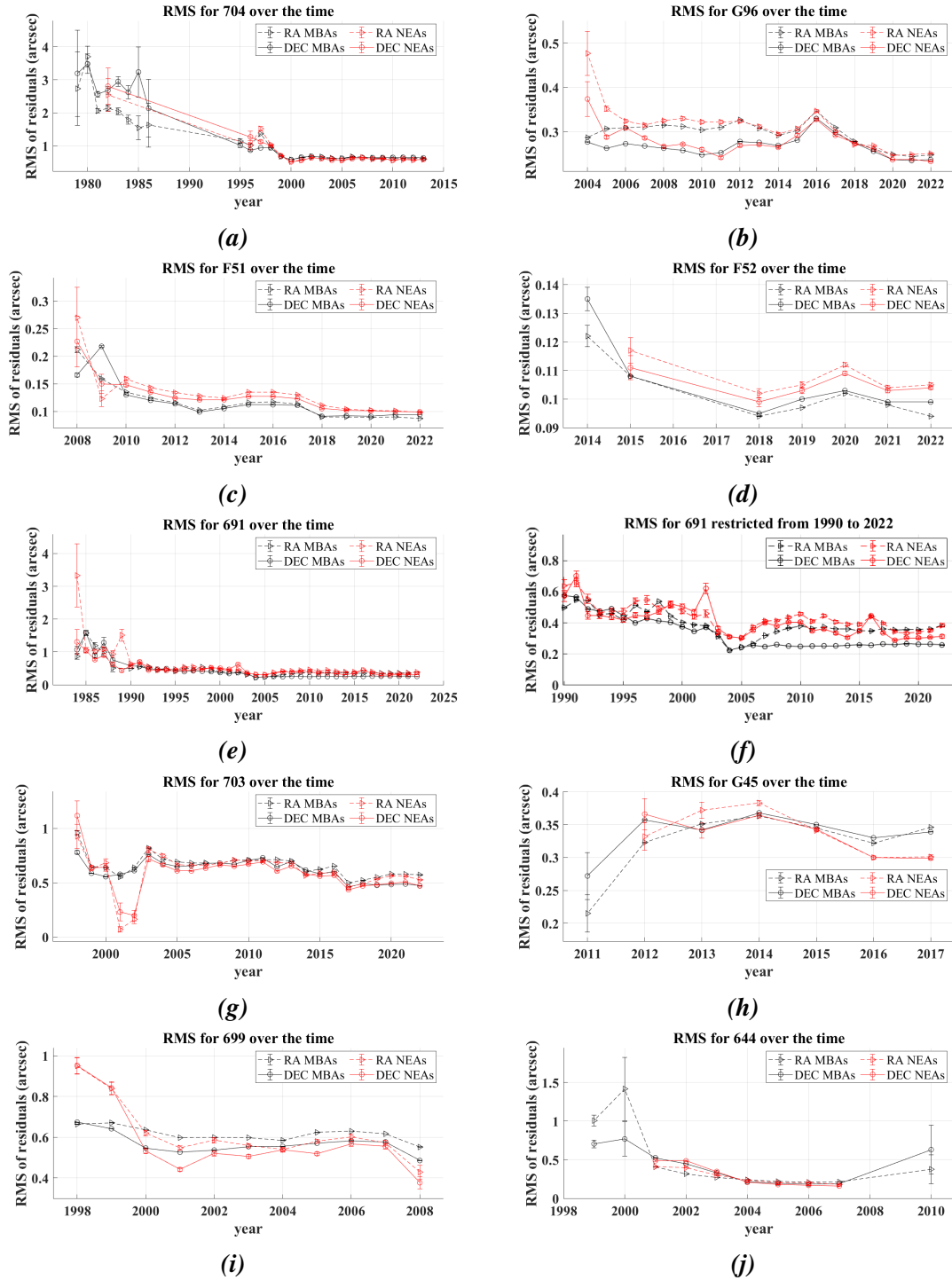


Figure 2.3: RMS of residuals over the years of observation for the considered stations. The plots for Main Belters and Near-Earth asteroids are plotted differentiating between the residuals for Right Ascension and Declination.

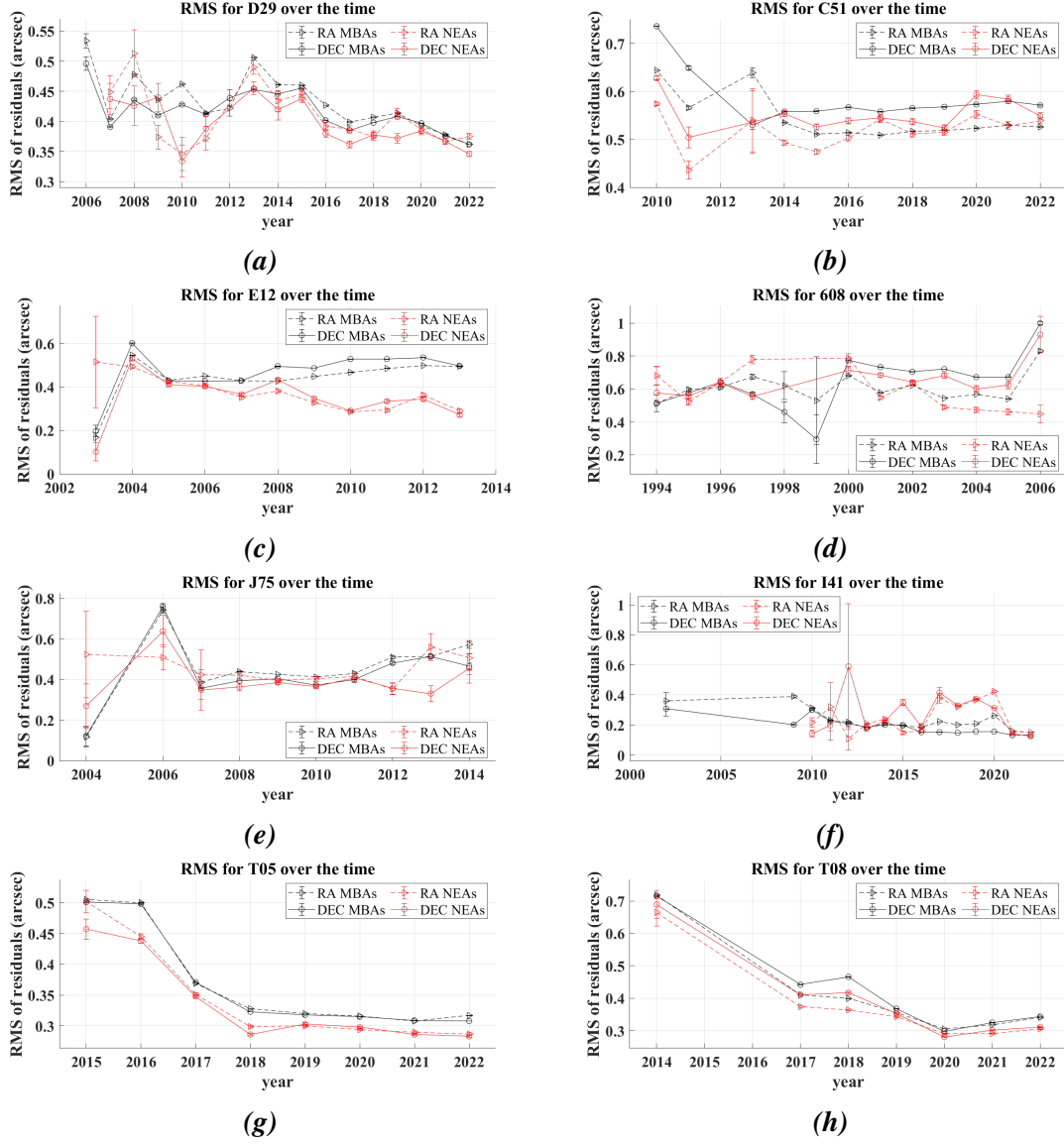


Figure 2.4: RMS of residuals over the years of observation for the considered stations. The plots for Main Belters and Near-Earth asteroids are plotted differentiating between the residuals for Right Ascension and Declination.

graph have an oscillating behaviour. The accuracy for these objects has been relatively low if compared to that for MBAs, but it is evident how the RMS values for NEAs started lowering in 2021, rapidly reaching the same accuracy as Main Belters, probably also thanks to new automated pipelines introduced for the observation of NEAs [11, 12].

691 (Fig. 2.3e) ensures a better accuracy for MBAs during almost all the period of observation. Also in this case the oscillations from 1984 to 1990 could be affected by the low number of astrometric measures produced during that period of time, when also no observations for MBAs have been registered for year 1990. Moving to more recent epochs, more clearly shown in the detailed graph 2.3f, although one can see that this station has overall better performances for MBAs, some improvements are highlighted in the observations of NEAs starting from 2010. This may lead to the same RMS value for the two classes of asteroids in future, since it is also evident that from 2018 the RMS_{RA} for NEAs are constantly lower than those for MBAs.

All the stations but the already discussed E12, show the same trends over the years for both MBAs and NEAs, meaning that, also for those stations that have an accuracy gap based on the asteroids classes, the eventual improvements in the observation pipeline are implemented regardless of the measured object. These changes may reasonably be due to technological improvements or the adoption of new catalogues and in general it is possible to assess a better data quality as time passes.

2.4.2 Dependency on the magnitude

Figures 2.5 and 2.6 illustrate the trend of the RMS of residuals over the magnitude of the observed objects, always subdivided between NEAs and MBAs. To compute the graphs the collected astrometric data have been gathered in bins wide 0.4 units of magnitude from a minimum of 10 to a maximum of 24. All the values have been corrected in order to indicate a magnitude in band *V*. To try to reduce the visualization of outliers, bins containing less than 40 observations have not been plotted.

In general, for all the stations and for both the asteroid classes the curves follow the same trend according to which the fainter objects, characterised by an higher magnitude value, have a worse accuracy. However, also low magnitudes in most of the cases lead to greater RMS values due to the fact that too bright objects tend to saturate the detector and so worsen the quality of the astrometric data [5].

With respect to the differences between the two asteroid classes, it appears that NEAs

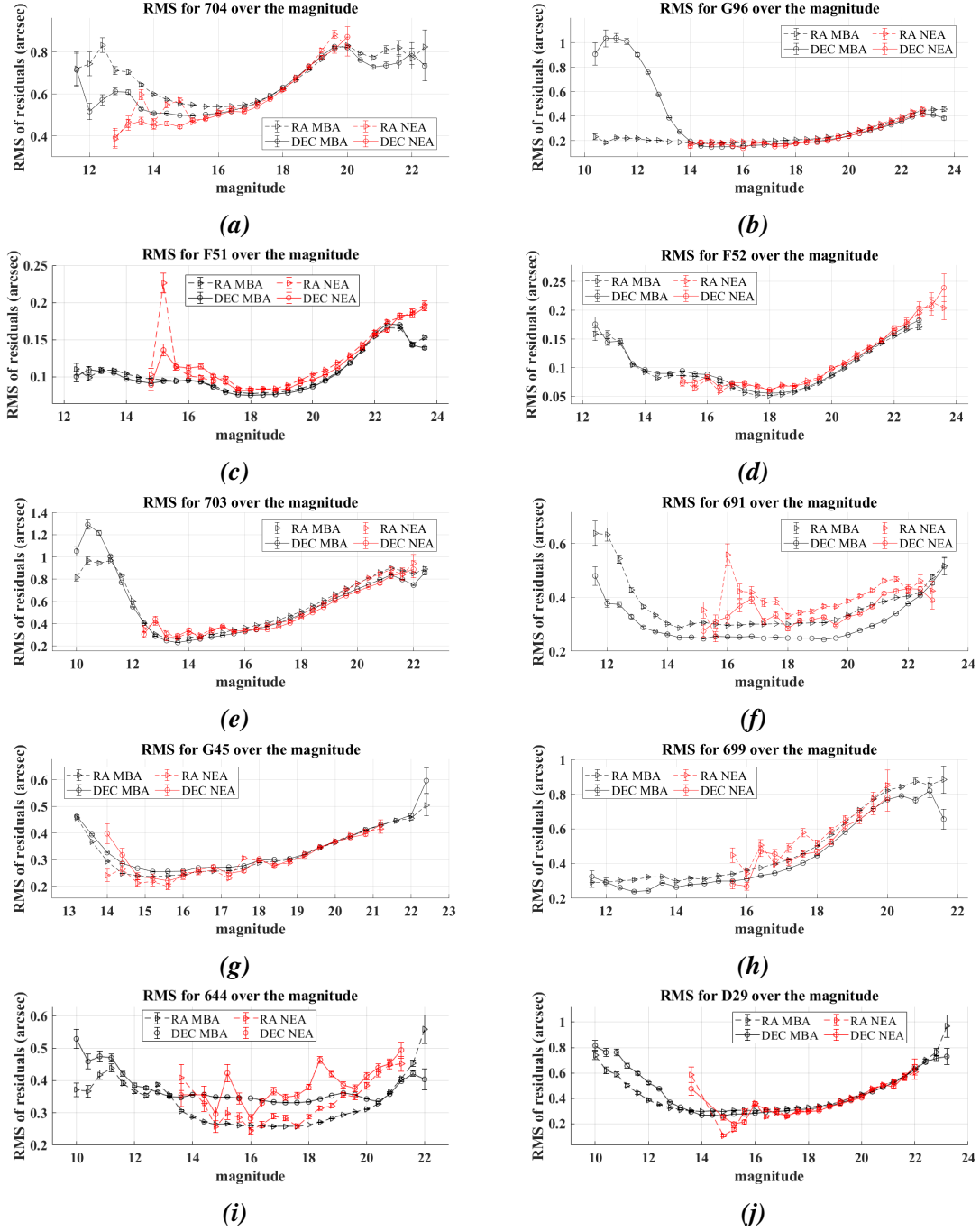


Figure 2.5: RMS of residuals over the magnitudes of observation. The plots for Main Belters and Near-Earth asteroids are plotted differentiating between the residuals for Right Ascension and Declination.

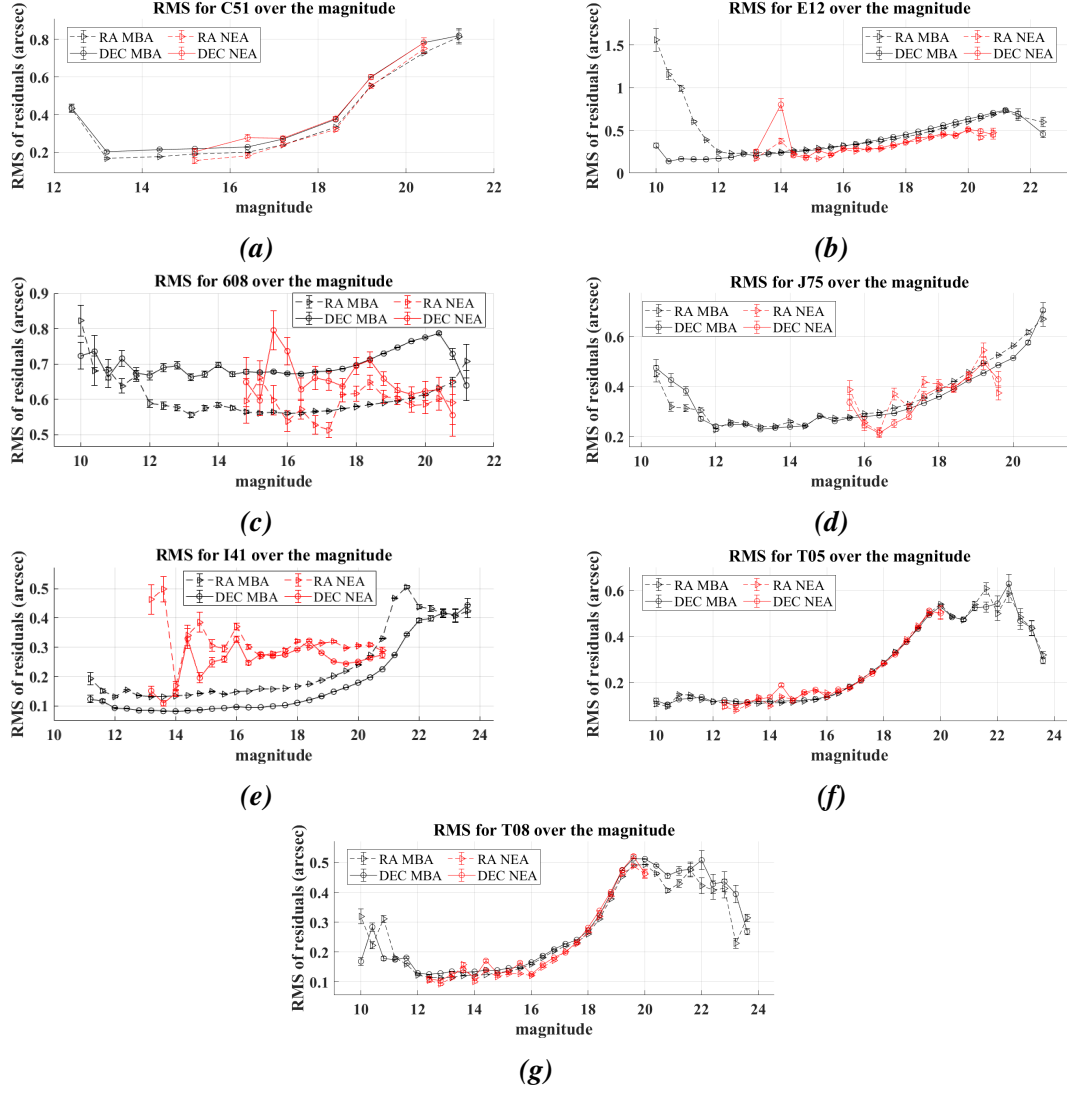


Figure 2.6: RMS of residuals over the magnitudes of observation. The plots for Main Belters and Near-Earth asteroids are plotted differentiating between the residuals for Right Ascension and Declination.

are observed in a narrower magnitude range with respect to MBAs. It is indeed rare for a station to observe a Near-Earth Object brighter than $\text{mag} = 14$ and fainter than $\text{mag} = 22$, for reasons related to the limits imposed by NEAs physical characteristics. Observations of bright NEAs are rarer than MBAs, and the higher motion rate makes it difficult to detect faint NEAs for most of the stations.

In general, the measure of magnitude seems to be less conditioned by the type of observed asteroid. Also for C51, MBAs and NEAs have the same behaviour (Fig. 2.6a), even if for this survey some considerations on the magnitude must be done.

WISE is an orbiting telescope that provides thermal infrared and not optical observations, which leads to a difference in the definition of the magnitude perceived by this station. In fact, it is not comparable to the *optical* one derived just from the brightness of the object, but also from other parameters such as heat and reflectance. Moreover, WISE's way of reporting magnitude data for their astrometric points has been object of dispute [13, 14], also due to the presence of bias and systematic errors. It is also interesting to notice that just 6% of NEAs observations and 3.5% of MBAs observations have a magnitude value reported, and always characterised by an integer value. These reasons in particular make the graph relative to WISE magnitude performance not comparable with the other results, but has been reported for the sake of completeness.

For the other three surveys that had a high sensibility on the asteroid class, the situation is instead that for 691 the accuracy, with the same magnitude, is better for Main Belters on the whole range, as expected. This is verified also for I41, which, however, shows the same RMS level at higher magnitudes. E12 behaves better with NEAs over the entire range.

2.4.3 Dependency on the catalogues

The two angular quantities that describe the position of celestial objects are measured with respect to a set of reference stars, which positions are known thanks to proper star catalogues. Relying on accurate catalogues to perform the astrometric data reduction is crucial as they may introduce biases that can affect the goodness of the measure. Previous studies have dealt with the analysis and correction of systematic errors introduced by these catalogues, and the aim of this section is to verify if a residual bias has remained following the corrections applied in the considered data. Furthermore, the whole analysis can help to assess whether a correlation between catalogues and asteroid class exists and if

Table 2.3: *Most used catalogues for astrometry reduction of MBAs and NEAs observations.*

Cat. name	Cat. code	N. obs (MBAs)	N. obs (NEAs)
Gaia-DR2	<i>V</i>	72 509 341	276 363
Gaia-DR1	<i>U</i>	67 990 790	162 350
2MASS	<i>L</i>	37 770 003	109 592
USNO-A2.0	<i>c</i>	37 210 202	108 127
UCAC-2	<i>r</i>	29 581 421	81 311
UCAC-4	<i>q</i>	26 943 310	59 780
USNO-B1.0	<i>o</i>	13 582 457	32 127
SST-RC4	<i>R</i>	13 449 616	26 602
USNO-A1.0	<i>a</i>	2 028 455	7 864

the accuracy of the catalogues are independent or not from the performance of the station.

Many factors can determine the accuracy of a catalogue. In general, those that are space based, i.e. not affected by atmospheric refraction and with a higher star density, are likely to guarantee a better accuracy. Also more recent catalogues tend to ensure a better quality than the old ones, as it happened with the second release of Gaia catalogue (Gaia-DR2) [15], which since its introduction in 2018 increased the accuracy of astrometric observations. This catalogue's major improvement is the addition of proper motions for more than 1 billion stars, which lowered catalogue errors by allowing observers to use stellar positions for the epochs of their observations. Proper motions will of course be present in all subsequent versions of Gaia, starting from the recently released Gaia-DR3 [16].

From the analysis of the astrometric data, it is possible to know the nine most used catalogues for the reduction of both MBAs and NEAs, listed in Table 2.3 in descending order. The name of the catalogue, the identification code and the number of observations related to the given catalogue are also shown.

These catalogues have been used to reduce, according to the data, more than 98.8% of the total observations, leaving the remaining to other less represented ones.

The most used is Gaia-DR2, followed by its first release, for both MBAs and NEAs. Then, 2MASS and USNO-A 2.0 mark a big gap with the remaining catalogues in the list.

Similarly to the previous sections, Figures 2.7 and 2.8 show the RMS of residuals

for the given stations again divided in MBAs and NEAs. In the graphs only those catalogues that, for each station, provide more than 1‰ of the observations provided by the catalogue which provides the highest number of observations are represented to avoid outliers.

The plots describe pretty well the level of accuracy of the catalogues. *U* and *V* (Gaia-DR1 and DR2) seem in general to have the lowest RMS levels, even if, as expected, the actual performance depend on the stations and the number of observations available for the two catalogues. As an example, F51 and F52 have a higher accuracy with *U* (Gaia-DR1) while G96 with *V* (Gaia-DR2), but this happens just because the first ones have much more observations reduced with Gaia-DR1, and probably kept, historically, this catalogue for their reduction and improved their performance with it.

One can also see that, while most of the catalogues have the same accuracy regardless of the station, a few of them show a dependency on it, such as *L* (2MASS), that has low RMS when used by F51 and F52, with a considerable number of observations, but behaves worse with T05, T08, C51 just to name a few. This implies that a statistics on the performance and accuracy of the catalogues, and studies on the biases and systematic errors derived by them, must not neglect the dependency on the station, on how the astrometric data are collected and treated.

It is also true, in fact, that these graphs can not highlight the performance of the catalogue alone, but of the station as a whole. The observations are, in general, characterised by an error which is the sum of several sources of error of different magnitude. One of these could be, if not corrected at the source, the intrinsic bias of the catalogue, that, however, is expected to have the same precision regardless of the station, while others, such as the already mentioned technical equipment, depend strongly on the observatory. This is why, when an observation is performed with high accuracy, the catalogue's precision becomes perceptible, but when the measure is of bad quality itself, the relatively small error induced by the catalogue is overwhelmed by the other sources. It would be interesting, for this purpose, when available, to check how these results would change applying any correction introduced by the newest Gaia release.

Moreover, in the charts, there are no differences in the catalogues accuracies between MBAs and NEAs. It is in fact expected that a catalogue does not behave differently with respect to the observed body, as it is just used to determine the position of the object. However, it may be interesting to investigate the usual four stations that showed a gap between NEAs and MBAs to see if a certain catalogue influence can be found.

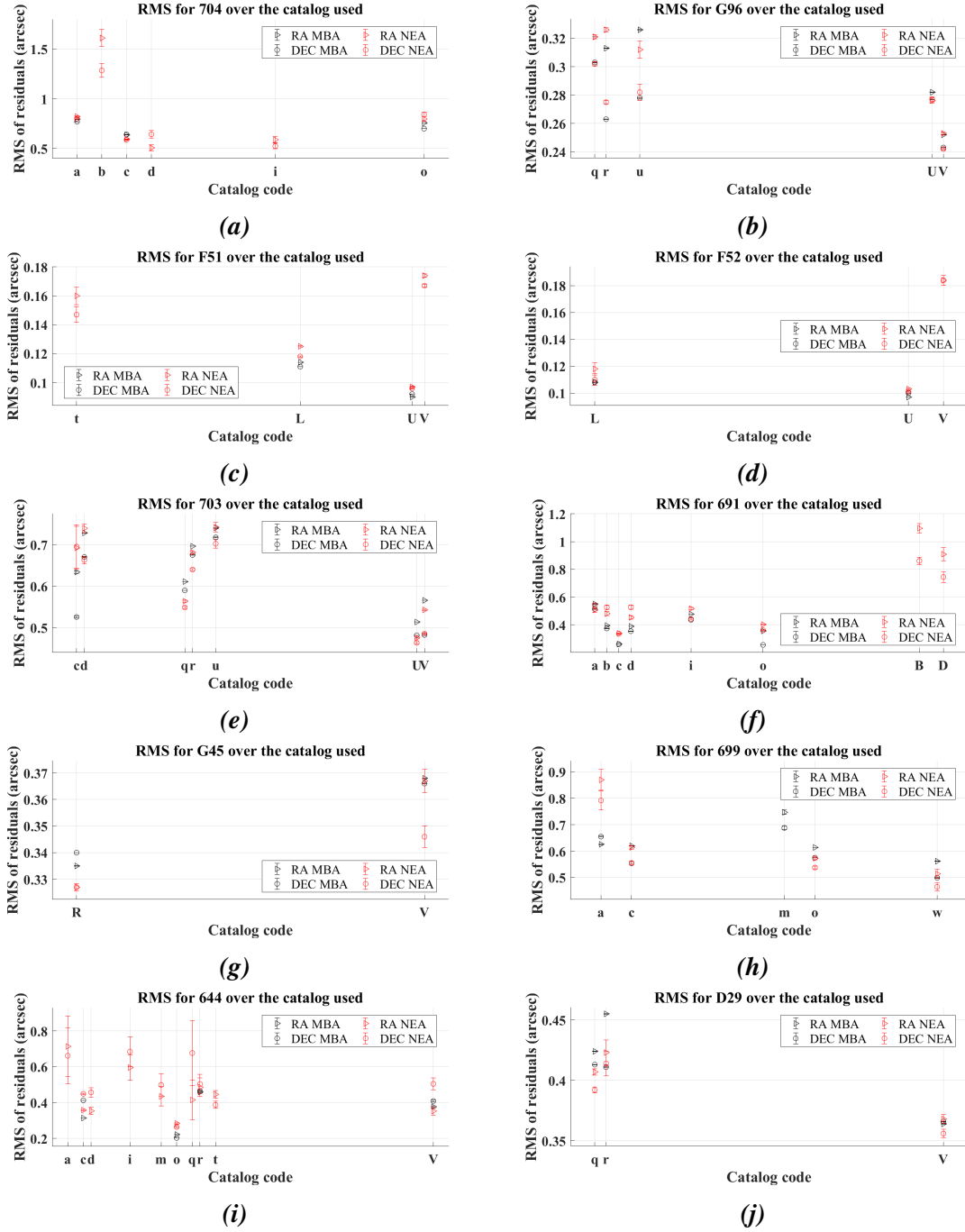


Figure 2.7: RMS of residuals over the catalogues of observation. The plots for Main Belters and Near-Earth asteroids are plotted differentiating between the residuals for Right Ascension and Declination.

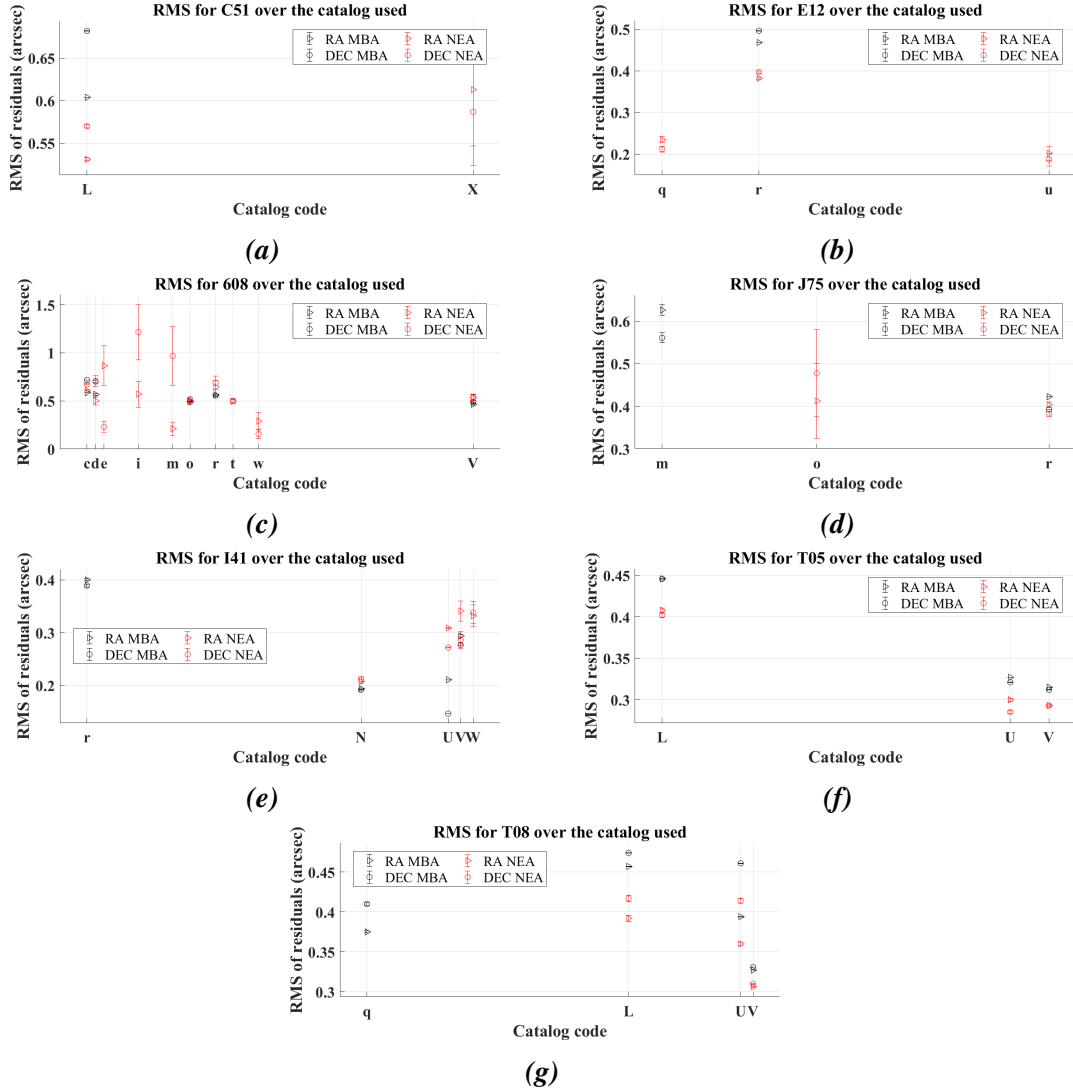


Figure 2.8: RMS of residuals over the catalogues of observation. The plots for Main Belters and Near-Earth asteroids are plotted differentiating between the residuals for Right Ascension and Declination.

691 uses or used a (USNO-A1.0), b (USNO-SA1.0), c (USNO-A2.0), d (USNO-SA2.0), i (GSC-1.1), o (USNO-B1.0) for both the classes with the same RMS level, but it relied on B (SAO 1984) and D (AGK 3) for a small amount of NEAs observations (respectively 552 and 185). These catalogues are only used by this station with its earliest data and are old and characterised by a low accuracy.

For E12 it is unlikely that the difference between MBAs and NEAs is given by the catalogues as it exploited only r (AGK 3) for the first ones and q (UCAC-4), achieving the highest accuracy registered for this catalogue, r (AGK 3), u (UCAC-3) for the others with globally higher RMS levels for MBAs.

The vast majority of observations for MBAs in I41 are reduced with U (Gaia-DR1), characterised by a much higher accuracy than r (AGK 3), which, however, represents a minority of the observations. Anyway, also in this case, as it is in most of the stations, the differences in RMS are definitely low.

C51 uses only L (2MASS), which is an expected outcome since WISE observes in the infrared and near-infrared band as well as 2MASS is a catalogue in the infrared band. The results in Figure 2.8a are therefore the same highlighted in Fig. 2.1.

2.5 Conclusions

A statistical analysis on the astrometric residuals of asteroids observations from among the 17 most productive surveys has been performed focussing our attention on the influence of the dynamic classification of minor planets in the results. The RMS of residuals in Right Ascension (RA) and Declination (Dec) have been computed for the stations and for Main Belt Asteroids (MBAs) and Near-Earth Asteroids (NEAs) and it turned out that for most of the stations the type of asteroid does not influence the quality of the measure.

However, four stations have shown to have different accuracies when observing either MBAs or NEAs.

An analysis of the mutual influence of this asteroids dynamic classification and other factors such as year of observation, magnitude, reference catalogue for astrometric reduction is presented. In general, the stations that have differences in measure quality according to the asteroid type keep these over the years and magnitude, but no influence of the catalogue has been registered. The absence of the discrepancy in the catalogue can be explained by the fact that the observations have been properly de-biased with respect

to the catalogue reduction. The result on the magnitude instead underlines that for those stations an influence on the object class is present. Indeed, otherwise, one would expect, for the same magnitude, the same result in RMS for MBAs and NEAs, which is not the case especially in I41, 691, E12.

Future developments of this work will lead to the definition of a weighting scheme for minor planets observations based on the outcomes of this statistical analysis, that takes into account this dynamic classification. For those stations that show different accuracies when observing different types of asteroids, having observation weights that change with the dynamic class may improve the whole OD pipeline. This could be the case of the so-called *NEO Follow-up stations*, that may have better accuracies when observing NEAs as it is shown in the previous paragraphs for E12.

Data availability

The data underlying this article were accessed from ESA NEOCC asteroids observations database (<https://neo.ssa.esa.int/>). The derived data generated in this research will be shared on reasonable request to the corresponding author.

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Chapter 3

Review of flyby methods and application to close encounters among asteroids

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Foreword

This Chapter serves as a study of flybys among asteroids, explored in this thesis as a possible dynamical cause of the broken orbits. Data on actual past close encounters are compiled and analysed to characterise their geometric and dynamical properties. This dataset also serves as a foundation for the simulations presented in Chapter 5. The chapter demonstrates how unmodelled flybys can affect RMS residuals and evaluates the applicability of two simplified flyby theories—patched conics and the Keplerian map—to the asteroid–asteroid encounters under study.

Abstract

Orbital flybys have been extensively studied for spacecraft missions, resulting in effective mathematical and physical models. However, these models' applicability to natural encounters involving asteroids has not been explored. This paper examines the applicability of two such theories, patched conics (PC) and the Keplerian map (KM), to asteroid encounters. A review of the two methods will be provided, highlighting their assumptions and range of applicability. Simulations of asteroid–asteroid encounters will then be performed to evaluate their effectiveness in these scenarios. The simulation parameters are set by collecting data on actual asteroid–asteroid encounters, hereby presented, generally characterised by high close approach distances and small masses of the perturbing bodies, if compared to those used to build the flyby theories. Results show that the PC theory's effectiveness diminishes with increasing approach distances, aligning with its assumptions. Moreover, the prediction of the model is better in the geometric configurations where the flyby has major effects on the orbital energy change. The KM theory has shown good effectiveness for encounters occurring outside the sphere of influence of the perturbing body, even for very high distances. This research investigates flyby models' strengths and weaknesses in asteroid encounters, offering practical insights and future directions.

*Keywords:*asteroids; review; orbital mechanics; flyby; patched-conics; circular restricted three-body problem; statistics

3.1 Introduction

As of July 2024, more than 1.3 million asteroids have been discovered in the Solar System (www.minorplanetcenter.net/mpc/summary), with approximately 95% in the main belt. The presence of these asteroids is not only known to influence the motion of other planets and asteroids [1, 2, 3] but it also justifies the possibility of flybys among asteroids. Flybys are also confirmed by the presence of asteroid families generated by impacts among minor planets, and by the theories that prove these to be responsible for the dynamical evolution of the asteroids in the solar system [4, 5, 6]. A flyby occurs when the trajectories of two bodies, orbiting around a common centre of mass, come close enough that the Keplerian trajectory of at least one of the two bodies is modified by the gravitational perturbation of the other.

Close approaches (CA) between solar system objects occur regularly, such as the near misses of near earth objects (NEOs) with Earth. CAs between minor bodies and planets are also recognised as significant contributors to the mechanisms of solar system formation [7, 8] and already in the 19th century, Laplace and Tisserand began observing and studying encounters between comets and Jupiter [9]. The 1976 work by Öpik [10] was aimed at studying collisions and close encounters among planetesimals in the solar system, highlighting that the rare events of collisions and spontaneous near misses provide a unique opportunity to investigate the mechanisms of solar system formation.

The outcome of flybys can be studied with numerical simulations of n-body gravitational systems, a method that tends to be complex and computationally heavy, since it involves direct numerical propagations.

Over the course of space exploration, orbital flybys have emerged as a technique for spacecraft trajectory design, enabling missions to navigate through the solar system, reaching paths otherwise unfeasible with the current engine allowances. These manoeuvres, also known as gravity assists, leverage the gravitational pull of celestial bodies to alter a spacecraft's trajectory and achieve desired destinations. Mathematical models have been developed to describe and optimise flybys applied to space mission design, with proper assumptions and simplifications, to reduce the number of parameters involved and chase analytic and more immediate solutions to the flyby problems.

The first foundations of a gravity assist manoeuvre for interplanetary travel were laid back in 1918 by the Soviet author Yuri Vasilievich Kondratyuk [11]. However, in this document, only the suggestion of a close approach as a potential manoeuvre to gain or lose velocity was proposed without providing any mathematical or physical interpretation of the problem. Nevertheless, this can be considered the first work that recognised the significance of a third body in orbital motion, following the pioneering works on the restricted three-body problem by Poincaré in the late 19th century [12].

The first mathematical models can be dated to 1924–1925 when another soviet engineer, Friedrich A. Tsander [13], presented equations to evaluate the energy and velocity change following a planar flyby. These formulas used parameters still employed today: velocity of approach of the small body, heliocentric velocity of the perturbing body, and deflection angle of the small body's planetocentric trajectory resulting from the encounter. Furthermore, he was among the first to assume the deflection to occur in a single point [13], a concept still used in the patched conics (PC) theory. A more comprehensive historical account on the first studies on the gravity assist performed before the advent of the actual interplanetary missions can be found in a previous review [14].

Eventually, the PC theory was presented in 1961 by Micheal Minovitch [15], with the purpose of solving the issues encountered by Battin [16] in describing the three-dimensional hyperbolic trajectory for flybys around Mars. The set of parameters employed for trajectory determination, namely the six orbital elements, was too intricate to include the gravitational fields of both the Sun and the target planet. Minovitch used instead the dynamical quantities, position and velocity vectors, as parameters, which proved to be more useful in a three-dimensional frame and allowed him to have equations for analytical solution of the three-body problem [17].

The key idea was to break the trajectory into three legs, each dominated by a different centre of gravity and characterised by a conic section, that can be jointed together to obtain the overall trajectory. Upon this concept, this approach took the name of patched conics. In this way, Minovitch provided a description of the physical mechanism of energy exchange during a close encounter, and was also able to present a formula for the computation of the energy change.

The theory has been expanded over the years, mainly to account for the complex encounter geometries [18] and to provide a more rigorous geometrical description of the flyby trajectory and its outcomes [10]. Graphical and analytical techniques that simplify the search for feasible and effective transfer trajectories, such as the Tisserand Map [19], have also been successfully proposed. However, as the understanding of celestial mechanics deepened, researchers recognised the limitations of the two-body approximation and sought more accurate mathematical models.

The limitations of the PC theory primarily stem from its fundamental assumptions. The flyby is assumed to follow a hyperbolic trajectory, but in many cases, such as in distant and slow flybys, the assumptions break down [20] and the combined influence of both the Sun and the secondary body must be taken into account.

In this context, the restricted three-body problem (RTBP) represented a step forward in the description of the physics involved in the scenario where a small body is subject to the gravitational action of two massive bodies [21]. A popular application is the circular restricted three-body problem (CRTBP) [22], where the two primaries are in a circular orbit around their centre of mass. In the context of the elliptical restricted three-body problem (ERTBP) [22], an additional level of complexity is introduced by considering the primaries in an elliptical orbit.

These methods require numerical integration of a simplified set of equations of motion for the general solution. However, efforts have been dedicated to achieving even greater

levels of simplification, striving to shift from numerical to more analytically driven approaches. Among these, methods that integrate the PC theory in the CRTBP to refine the problem's parameters have been proposed [23, 24, 25]. Others have expanded the Tisserand maps to the patched-CRTBP, resulting in what is known as the Tisserand–Poincaré (T–P) Map [23]. In 2007, Ross and Scheers proposed the Keplerian map (KM) method [26, 27, 28], a semi-analytical tool that describes encounter outcomes occurring far from the secondary body. Another semi-analytical tool is the Flyby map [29], derived from the KM and the T–P map.

The developed models have proven to work well in spacecraft mission design, or for describing the outcomes of encounters involving solar system planets or the Moon, but their range of applicability in close encounters involving asteroids has not been investigated in depth. These are characterised by masses smaller than the planets, and orbits with inclinations and eccentricities different from the typical planetary ones.

In this paper, the PC and KM theories will be reviewed and tested on flybys among asteroids, to evaluate their range of applicability and outcomes. The selection of these two theories is motivated by the fact that they allow for the solution of a flyby, without recurring to the explicit propagation of the equations of motion. PC and KM also allow for analytical and semi-analytical solutions of the direct flyby problem. Another advantage of the revised methods is that they reduce the parameters used for the description of CEs and explain the mechanism of energy exchange with the geometrical and physical features involved.

Their performance in the test cases will be assessed in two ways: considering an inverse flyby solution scenario, where the PC theory is used to obtain the perturbing mass over approach distance ratio given the pre and post flyby orbital states, and the relative error in the prediction of the orbital energy change following a CE for both PC and KM.

In Section 3.2, data concerning several asteroid encounters will be collected, presented, and analysed. Section 3.3 will review the flyby methods, while in Section 5.6 these will be tested against the conditions of a typical asteroid–asteroid encounter.

3.2 Dynamical Characterisation of Asteroid Encounters

Dynamical data of asteroid encounters serve as a basis for understanding their physical and geometrical characteristics, and for realistic simulation setup. The scope of this

section is to provide an overview of the physical conditions in which asteroid flybys typically occur. Statistical analyses on these data are hereby presented, to link the dynamic outcomes of flybys with their geometrical features. Ultimately the collected data will be used as a baseline for the simulations presented in Section 5.6.

The selected database consists of CAs identified on the basis of the difference in the residuals $\Delta(\text{RMS})$ of the orbit determination (OD) performed, with the software *Find.Orb* (www.projectpluto.com/find_orb.htm (accessed on 1 July 2024)), with and without considering these 300 asteroids, as described in a previous work [30]. The observations for the OD are taken from the ESA Neo Coordination Centre (NEOCC) database (<https://neo.ssa.esa.int/> (accessed on 1 July 2024)) for all the asteroids, both numbered and unnumbered. This enables the identification of asteroids that have experienced a flyby with one of the 300 most massive asteroids, and the signature used to assess the strength of the CA is the $\Delta(\text{RMS})$ between the perturbed and unperturbed solutions of the OD. This methodology allows the identification of the flyby epoch, the perturbing body and, through the Horizon-JPL ephemerides, the state vectors of the bodies at encounter.

In this section, the encounter parameters will be collected and analysed in terms of relative distances and velocities and approach geometries. In total, over 11,500 close encounters have been found.

3.2.1 Role of the Encounter Distance

As a metric to compare the encounter distances, we use the Hill's radius, R_H , as a representation of the size of a body's sphere of influence (SOI), i.e., the region where the perturbing asteroid's gravitational influence is assumed to be stronger than that of the Sun. R_H can be computed as [31]:

$$R_H = \sqrt[3]{\frac{m_P}{3M}} \cdot r_s \quad (3.1)$$

where m_P is the mass of the perturbing asteroid, M the mass of the Sun, and r_s is the distance between the perturbing asteroid and the Sun at the moment of encounter.

The absolute frequency distribution of the recorded CA distances, relative to R_H , is shown in Figure 3.1. The graph represents only the encounters within a distance of 200 Hill's radii. These are approximately 5500 out of the total events recorded. The plot

shows that encounters take place at every distance, with most of them being within the range between 10 and 40 R_H . The long distances observed from these data allow us to exclude the effect of the asphericity of the perturbing asteroid on the overall perturbation of the flyby. This effect diminishes as $1/r^n$, with $n > 2$ [32], and the distances we deal with in these cases are long enough to neglect this effect. Moreover, we are considering dimensions much greater than the radii of the asteroids, which is the typical distance at which asphericity becomes prominent over other major perturbations [33].

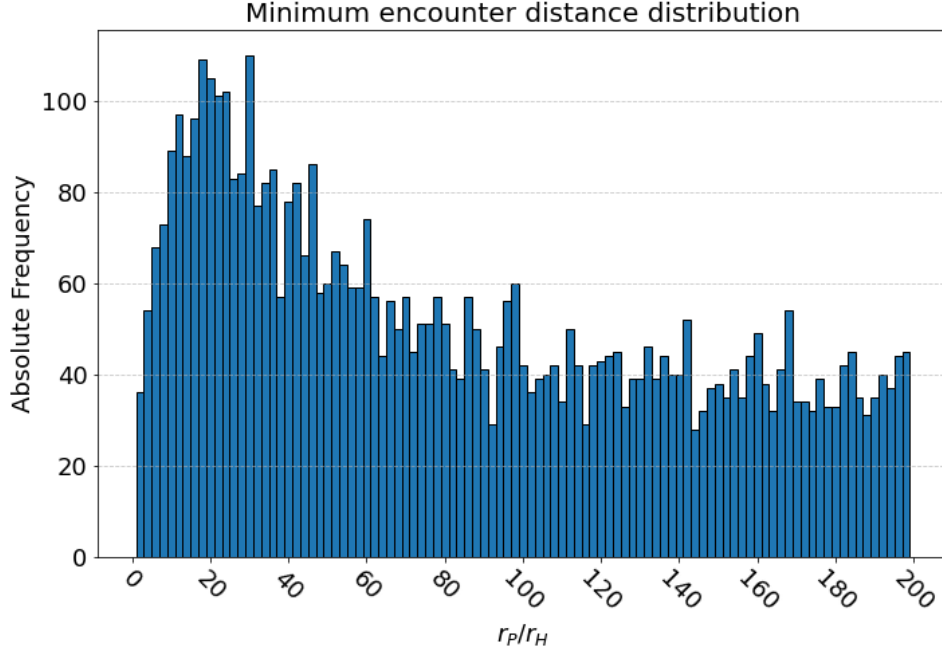


Figure 3.1: Bar chart of the absolute frequency of minimum encounter distances (r_p) with respect to the perturbing body's Hill's radius (R_H). The plot includes only the encounters between 1 and 200 R_H . The bins for the bar chart have a uniform span of $2 r_p/R_H$.

Only 19 encounters were located inside the SOI of the perturbing asteroid, suggesting that very close encounters in a natural and random scenario are rather rare.

Data show that, even for high encounter distances, the flyby has a measurable effect on the RMS residuals, even if, as per Figure 3.2, distance has an effect on this parameter. The scatter plot shows that the $\Delta(\text{RMS})$ is higher for encounters closer to the perturbing body. Indeed, the encounter distance, as will be shown later, is an important parameter for assessing the effect of a flyby.

The plot in Figure 3.2 contemplates only single encounters, i.e., cases where an asteroid has one and only one close approach with one of the 300 perturbing asteroids in the model. Only 1 700 resulted to be single encounters. In these cases, the effect of the

flyby on the residuals is direct while, on the contrary, when multiple encounters occur, the impact of each flyby is challenging to disentangle.

For a similar reason, in cases where the $\Delta(\text{RMS})$ is very high, exceeding $1 \div 2$, it is possible that the cause of the residual change is not solely due to the encounter, but rather a continuous gravitational influence from one of the more massive asteroids such as Ceres or Vesta.

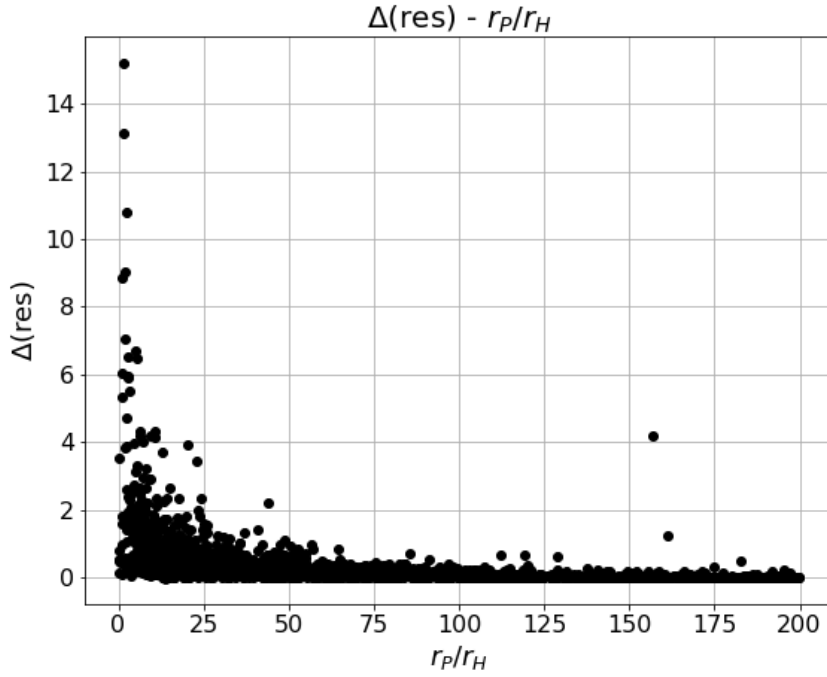


Figure 3.2: Relationship between the $\Delta(\text{RMS})$ of the detected single encounters and the encounter distance, relative to the Hill's radius, R_H .

3.2.2 Role of the Relative Inclination

The relative inclination, in the context of a flyby, is the angle between the perifocal planes of the perturbed and perturbing bodies, as shown in Figure 3.3. This transformation gains importance in asteroids, as they in general do have an inclination with respect to the ecliptic. Especially for the 300 most massive asteroids, the average inclination is 10.9 deg with a standard deviation $\sigma = 6.4$ deg, so the inclination of the perturbing body and the orientation of its perifocal plane have to be accounted for when evaluating the relative inclination.

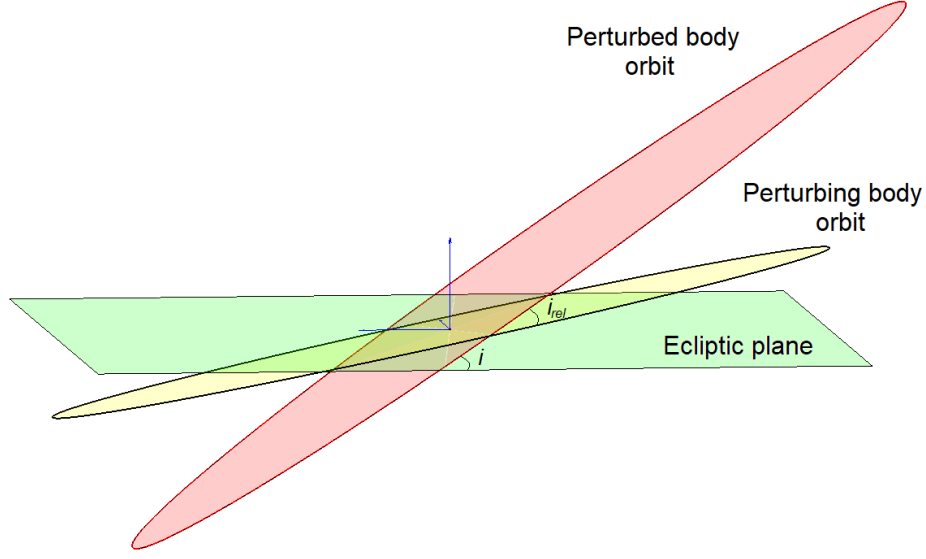


Figure 3.3: Representation of the relative inclination between two different orbits. i_{rel} is the inclination that one orbit has with respect to the perifocal plane of the other object, so this was taken as the new reference frame instead of the ecliptic.

This can be calculated analytically from the osculating Keplerian orbital elements of the two bodies using geometric relations, transformation between the two orbits and the law of cosines as:

$$i_{rel} = \cos^{-1}[\sin i_1 \sin i_2 \cos(\Omega_1 - \Omega_2) + \cos i_1 \cos i_2] \quad (3.2)$$

where i and Ω are, respectively, the inclination and the right ascension of the ascending node (RAAN), and the subscripts 1 and 2 indicate the two different bodies.

A geometrical formula derived by Öpik [10] relates i_{rel} with the relative velocities at encounter, in the context of two intersecting orbits, for analysing flybys or collisions. This relationship is valid for the special case where the perturbing body is circular with a unit semimajor axis, i.e., in the Jacobi normalised units. Under these assumptions, according to Öpik's geometrical formulation, the relative velocity, U , normalised with respect to the perturbing body's circular velocity, reads:

$$U = \sqrt{3 - T} = \sqrt{3 - 1/a^* - 2\sqrt{a^*(1 - e^2)} \cos i_{rel}} \quad (3.3)$$

where T is the Tisserand parameter [34] and $a^* = a/r_{ref}$ is the normalised semimajor axis of the perturbed body. The reference normalising distance is $r_{ref} = r_S$. U is the normalised relative velocity at encounter $U = v_P/v_{ref}$, with $v_{ref} = \sqrt{\mu_{SUN}/r_{ref}}$ is the pseudo-circular velocity at encounter. Also, μ_{SUN} is the Sun's gravitational parameter.

In Figure 3.4, the values for relative inclination and U for the detected cases are shown. It should be noted that a normalisation has to be performed for the values collected in the real-case scenario involved in Equation (3.3). Semimajor axes are reported as a^* .

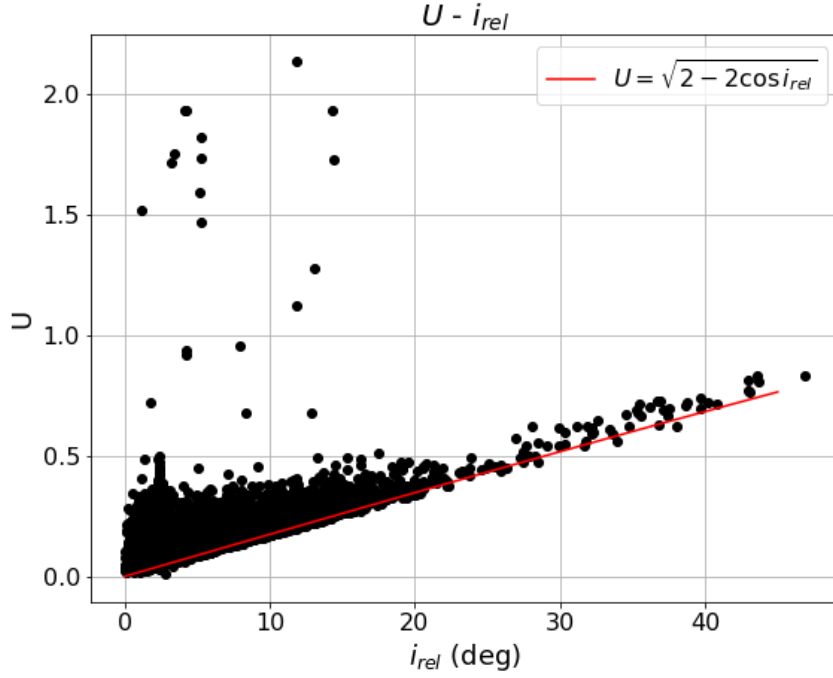


Figure 3.4: Relationship between the relative inclination, i_{rel} , and the normalised relative velocity, U .

The region for high inclinations and low U is void, as per the natural domain of Equation (3.3). An example of limiting curve has been superimposed to the plot, representing the $(i_{rel} - U)$ relationship for the encounter with a circular orbit with $a = 1$. Although this only represents a very particular case, the majority of the points are above the red line. For those below the line, the reason has to be found in values of eccentricities of both the perturbing and perturbed body different from zero.

This trend can be explained by considering the velocity vectors of the two bodies. When the magnitudes are equal, a greater inclination usually implies a larger angle between the two vectors, resulting in a greater difference vector. Similarly, eccentricity also contributes to an increase in relative velocity.

3.2.3 Relative Velocity

Another factor known to play a role in the outcome of a flyby is the relative velocity at encounter between the perturbed and the perturbing bodies. In general, this has an effect on the time spent close to the SOI of the perturbing body. A slower flyby allows for more time for the gravitational perturbation to act on the perturbed body and get a major deflection. A faster flyby, on the other hand, will lower the time spent close to the perturbing body, reducing the effect of the flyby itself.

The relative velocity has an upper limit, due to the relative inclinations and differences in eccentricity. From the perspective of a simulation, the relative velocity should be limited in order to avoid, especially at low relative inclinations, the generation of unfeasible orbits, either too eccentric or even with hyperbolic heliocentric trajectories.

A lower limit should stand in order to ensure the escape from the perturbing body. The escape velocity from a body depends on its mass, m_P , and the distance from its centre of mass, and can be expressed as [31]:

$$v_E = \sqrt{2 \frac{m_P G}{r_P}} \quad (3.4)$$

The relative velocity, at the moment of the minimum approach distance, should be greater than v_E . This is expected from the analysis of real cases.

Considering the big encounter distances and the low masses involved in the cases of interest, Equation (3.4) will likely provide a very low threshold, easy to satisfy. This is shown in Figure 3.5, where the relative velocity and the approach distances are plotted compared to the curve of the escape velocity as a function of the distance. In order to represent in the same graph encounters with objects with different masses and semimajor axes, the quantities have been normalised accordingly, having on the x -axis r_P/r_{ref} and on the y -axis $U/\sqrt{\mu}$, with $\mu = m_P/m_{SUN}$ being the mass parameter of the perturbing body.

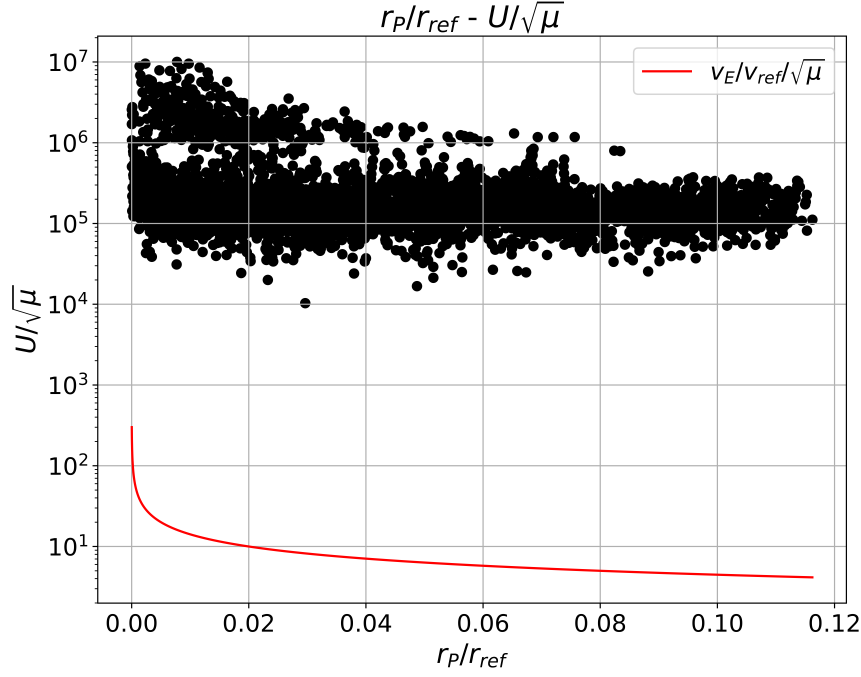


Figure 3.5: Relationship between the relative velocity and the encounter distance, compared with the normalised escape velocity, $v_E/v_{ref}/\sqrt{\mu}$ (red line). The y-axis is in logarithmic scale.

The escape condition is always satisfied, as expected. Moreover, in this normalisation the velocity has a downwards trend with the encounter distance.

3.2.4 Geometrical Parameters in the Planetocentric Frame

In this section, the geometrical encounter parameters are collected and investigated, to see if a correlation between the intensity of the flyby and the directions of approach can be found. These values can also be used as a dataset for simulation setups. The reference frame used is a planetocentric frame (XYZ), centred in the perturbing body, the X -axis in the Sun-body direction, with the Sun on the negative axis, the Y -axis in the direction of motion that the perturbing body would have if it moved on a circular trajectory, and the Z -axis parallel to the momentum vector, in order to complete the right-handed frame. The radius of the minimum approach point, r_P , and the relative velocity vector in that point of the trajectory, v_P , can be referred to this reference frame through proper geometric transformations. Once the quantities in the heliocentric-ecliptic (IJK) frame and the orbital elements of the perturbing body, indicated with the subscript P , are known, one can

apply the proper rotation matrices to find the vectors in the XYZ frame:

$$\begin{aligned}\mathbf{r}_{P,XYZ} &= L_3(\omega_P + \nu_P) L_1(i_P) L_3(\Omega_P) \mathbf{r}_{P,IJK} \\ \mathbf{v}_{P,XYZ} &= L_3(\omega_P + \nu_P) L_1(i_P) L_3(\Omega_P) \mathbf{v}_{P,IJK}\end{aligned}$$

where $L_3(\bullet)$ and $L_1(\bullet)$ are rotation matrices about the local z and x axes, respectively. Ω_P and ω_P are the right ascension of the ascending node and the argument of periapsis of the perturbing body's orbit, and ν_P is its true anomaly at the moment of encounter.

These vectors can be represented in a tridimensional set of polar coordinates [18, 35], as shown in Figure 5.1. The three angles used in the representation are defined as:

$$\beta = \sin^{-1} \left(\frac{r_{P,Z}}{\|\mathbf{r}_P\|} \right) \quad (3.5)$$

$$\alpha = \text{atan2}(r_{P,Y}, r_{P,X}) \quad (3.6)$$

$$\sin \gamma = \frac{v_{P,Z}}{\|\mathbf{v}_P\| \cos \beta} \quad (3.7)$$

$$\cos \gamma = - \frac{v_{P,X} + v_{P,Z} \tan \beta \cos \alpha}{\|\mathbf{v}_P\| \sin \alpha} \quad (3.8)$$

$$\gamma = \text{atan2}(\sin \gamma, \cos \gamma) \quad (3.9)$$

where $r_{P,\bullet}$ and $v_{P,\bullet}$ represent the direction of the respective vectors in the indicated (X, Y, Z) directions.

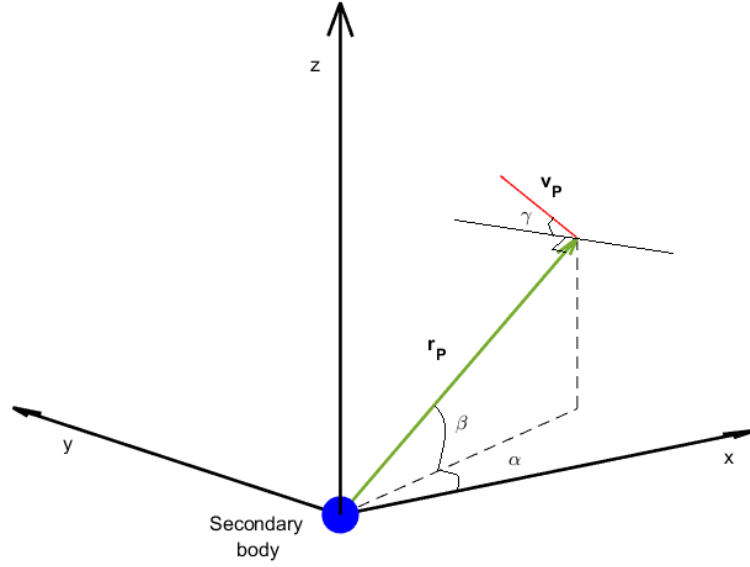


Figure 3.6: Geometric representation of the planetocentric frame.

The angles used for representing $\mathbf{r}_P$, α , β , and γ , derived from the detected cases, and their relationships are illustrated in Figures 3.7 and 3.8. Upon analysing the scatter plot in Figure 3.7, one can observe fewer occurrences where $\beta = \pm\pi/2$, indicating encounters happening directly above or below the perturbing body. Similarly, the same pattern is observed when $\alpha = 0$ or $\alpha = \pm\pi$, signifying cases of encounters parallel to the perturbing body's direction of motion.

These plots (Fig. 3.7, 3.8) encompass all the encounters identified through the $\Delta(\text{RMS})$ method. Considering the stochastic nature of the analyses, one would anticipate a random distribution of encounters across various parameter ranges. However, the observed distribution deviates from this expectation, revealing regions with notably fewer occurrences. This anomaly suggests that encounters characterized by these specific geometrical parameters may indeed exist in our real-case dataset. The rarity of recorded instances in these regions implies that encounters with such parameters tend to be too faint to be reliably detected with the $\Delta(\text{RMS})$ signature method.

The angle γ is the parameter that defines how much the relative velocity is tilted with respect to the plane of the perturber. This links γ to the relative inclination. The latter is confirmed by Figure 3.9, where the values of i_{rel} are plotted against the respective values of γ angle. Their relationship is easily readable, where at vertical γ , close to $\pm\pi/2$, correspond higher values in i_{rel} . Hence, eventually in the context of simulations, relative inclinations and γ angles have to be tuned accordingly.

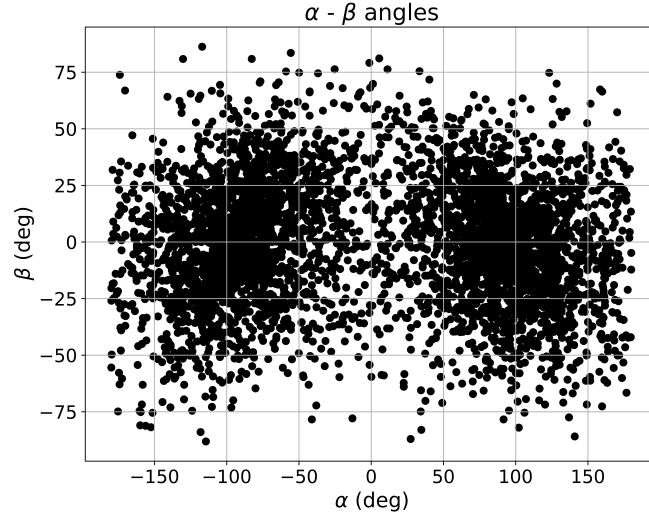


Figure 3.7: Distribution of α and β angles for the analysed encounters

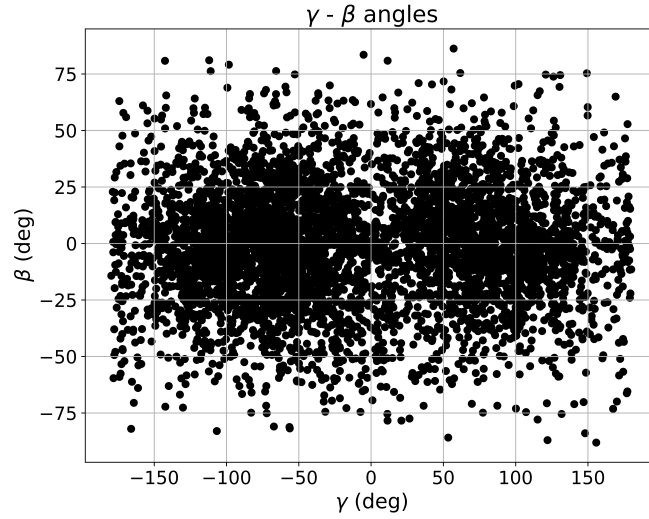


Figure 3.8: Distribution of γ and β angles for the analysed encounters.

However, from the analysis of Figure 3.4 it was said that the relative inclination should be related to the normalised relative velocity, U , so that one would expect the same relationship between γ and i_{rel} to be valid also for γ and U . The latter is shown in Figure 3.10.

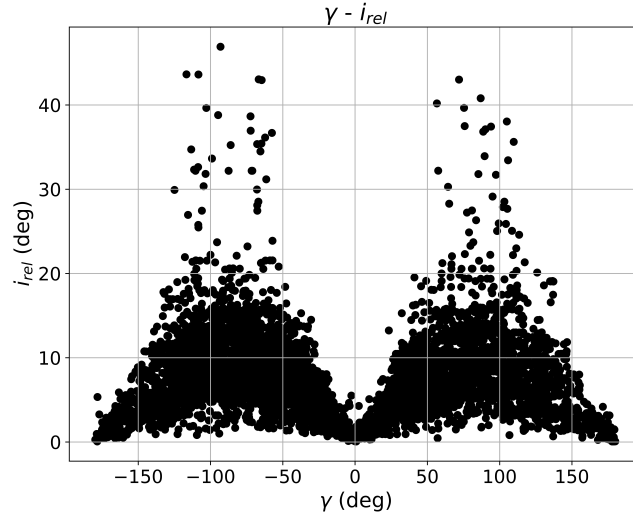


Figure 3.9: Relationship between γ and i_{rel} .

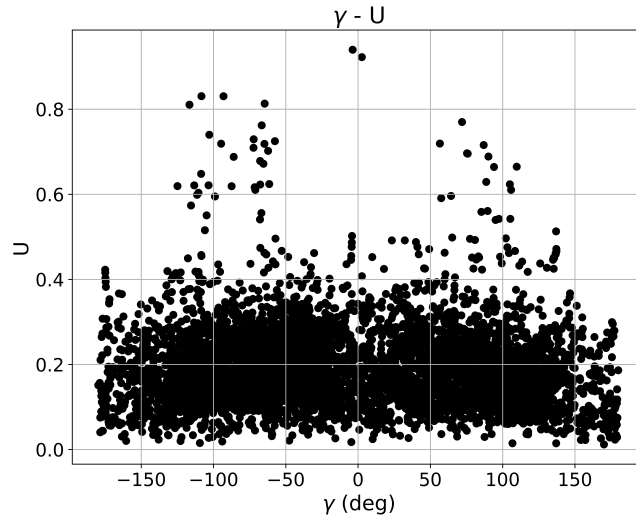


Figure 3.10: Relationship between γ and U .

From this plot, one can say that higher values of U can take place only in the presence of γ values close to $\pm\pi/2$. The points are more uniformly distributed than in the previous plot, especially in the regions around $\gamma = 0$ and $\pm\pi$. Indeed, γ and U are not physically related, while the drivers for i_{rel} are both γ and U . For γ values close to a horizontal direction, no matter how much the velocity is, the relative inclination will always be low, while, on the contrary, it is possible to lower down U to get low i_{rel} for seemingly vertical γ s.

Furthermore, from the values in Figure 3.10 we can assume U ranging typically between 0.05 and 0.38.

3.3 Flyby Theories

In the following paragraphs, the technical contents of PC and KM will be reviewed, together with their assumptions, limitations, mathematical foundations, and relevant reference systems.

3.3.1 Patched Conics

The PC theory, also known as the linked conics approach [31], is an analytical method used to model flyby trajectories. It simplifies the three-body problem, breaking it down into a series of two body problems modelled by conics of simpler mathematical interpretation. It involves three bodies: a small massless particle, that experiences the flyby, a perturbing or secondary body, and a primary; the first two bodies orbit around the primary. A planetary flyby can be decomposed into three distinct trajectories: two elliptic arcs about the primary body, namely the Sun, before and after the encounter, and a two-body Rutherford-like scattering hyperbola about the secondary body, which connects the two heliocentric arcs and represents the flyby itself. The fundamental idea at the basis of this method is that every arc is influenced only by the gravitational attraction of one body, and the presence of the two massive bodies is never considered simultaneously. The elliptic arcs are influenced by the Sun and the hyperbola only by the planet, and the concept of SOI [36] is used to determine when to switch from one body to another. Within the SOI of the secondary, its gravitational effect dominates that of the Sun, allowing the PC theory to neglect the influence of the Sun. Outside the SOI, instead, the primary has the strongest influence and is considered as the only centre of attraction. A flyby, in this context, takes place when the small body enters the SOI of the secondary.

The SOI can be mathematically represented by the Hill's sphere, which is the sphere around the secondary where its gravitational influence dominates over the larger primary body. Its physical interpretation comes from the solution of the Lagrangian three-body problem, since its dimension is almost the distance between the first and second Lagrangian points. A possible formula to compute the radius of the Hill's sphere is shown in equation 5.1, where, in the more general context of the PC formulation, m_P is the mass

of the perturbing (or secondary) body, M the mass of the primary, and r_s is the distance between the secondary and the primary at the moment of encounter.

Inside the SOI, the massless particle moves in a hyperbolic trajectory around the secondary, as in Figure 3.11. This trajectory is relative to the secondary, which is also in motion around the Sun. At the edges of the sphere of influence, the massless particle has a relative velocity, which takes the name of hyperbolic excess velocity, of $\mathbf{v}_\infty = \mathbf{V}_1 - \mathbf{V}_P$, where $\mathbf{V}_1$ is the heliocentric velocity of the massless particle before the encounter and $\mathbf{V}_P$ is the heliocentric velocity of the perturbing body. Throughout the following pages, heliocentric velocities will be denoted in capital letters, while relative velocities with respect to the perturbing body will be represented in lowercase letters.

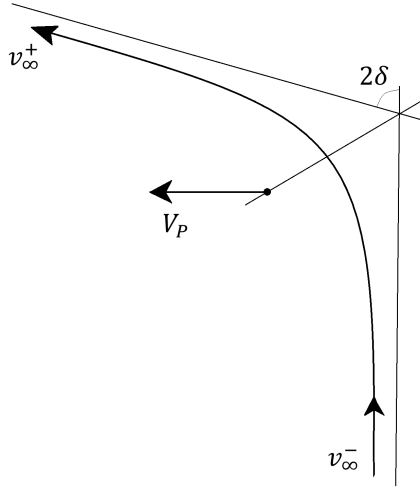


Figure 3.11: Hyperbolic trajectory of the massless particle around the secondary during a flyby. $\mathbf{V}_P$ represents the velocity vector of the secondary, $\mathbf{v}_\infty^+$ and $\mathbf{v}_\infty^-$ are, respectively, the velocities at the entry and exit points of the hyperbolic trajectory, and 2δ is the deflection angle between the two asymptotes of the hyperbola, caused by the flyby.

The direction of $\mathbf{v}_\infty$ also indicates the direction of the incoming asymptote of the hyperbola. Due to the gravitational interaction between the two bodies, the asymptote is rotated by an angle, 2δ , that depends on the mass of the secondary, the hyperbolic excess velocity, and the minimum distance of the approach trajectory [35]:

$$\sin \delta = \frac{1}{1 + r_p v_\infty^2 / \mu_P} \quad (3.10)$$

where δ is half of the rotation of the asymptotes, r_p is the minimum distance between the

two bodies during the encounter, and μ_P is the gravitational parameter of the secondary. This rotation represents the deflection of the flyby itself.

After the encounter, the relative velocity, at the exit of the SOI, is rotated with respect to $\mathbf{v}_\infty$ by an angle, 2δ , and has the same magnitude as $\mathbf{v}_\infty$. The post-encounter heliocentric velocity is obtained by the vector sum $\mathbf{V}_2 = \mathbf{v}_\infty^+ + \mathbf{V}_P$.

In this model, the rotation of the velocity, which represents the effect of the encounter in the PC approach, is assumed to occur at a single point: the point of minimum distance between the unperturbed heliocentric trajectory and the perturbing body.

In this way, the rotation of the hyperbola and the Δv after the encounter can be calculated [31]:

$$\Delta v = 2v_\infty \sin \delta = \frac{\mu_P}{r_p v_p} \quad (3.11)$$

where v_p is the magnitude of the velocity at the minimum distance. From the previous assumptions, the Δv related to the planetocentric trajectory is equal to the ΔV related to the heliocentric trajectory.

As showed by Minovitch [15], the use of the vectors allows for the evaluation of the energy change due to the flyby, which can be computed as:

$$\Delta E = \mathbf{V}_P \cdot (\mathbf{V}_2 - \mathbf{V}_1) = \frac{V_2^2 - V_1^2}{2} \quad (3.12)$$

The formula of ΔE is also important to understand the mechanism of energy exchange during a flyby. In this process, the smaller body undergoes a change in momentum relative to the secondary body. The rotation of the relative velocity, followed by its vector summation with the perturbing body's velocity, not only alters the magnitude of the velocity but also modifies the flight path angle. In non-coplanar flybys, the inclination can undergo changes as well. It is important to note that this manoeuvre affects the secondary body, subjecting it to mutual gravitational perturbations from the smaller body. However, due to the substantial difference in masses, with the secondary having significantly greater mass, the impact on the secondary from the flyby becomes practically negligible.

The ΔE can also be evaluated analytically once the geometric encounter parameters are defined. Broucke [18] proposed a set of parameters that enabled him to derive an analytical formula for the energy change in the co-planar flyby involving just three parameters. This set was later expanded by Prado [35] to account for the three-dimensionality

of a generic flyby, and this approach will be considered in the following pages, to better describe the real cases encountered. The three-dimensional set of parameters is shown in Figure 5.1, and the ΔE can be computed [35]:

$$\Delta E = -2v_{\infty}V_P \cos \beta \sin \alpha \sin \delta \quad (3.13)$$

Öpik's Theory

Öpik's theory of close approaches is a formalism based on a two-body scenario to study and predict encounters [10, 7]. Öpik's theory models the flyby, similarly to the PC theory, as a two-body problem and can describe, through the use of the Öpik variables, the pre- and post-encounter parameters of a flyby and systematically study its outcome. Originally it was thought and widely used as a tool to analyse planetary encounters, especially in the context of comet capture and planetary formation, collisions of planetesimals, and resonant encounters. However, with the advent of numerical computation, it has progressively transformed from a predictive tool to a tool for understanding the dynamic and geometric features of close approaches. The formalism was developed for intersecting orbits, and although it was originally valid only at collision, in 2003 [37] an extension of the theory was proposed to account for non-collision cases, using a first-order approximation of the particles hyperbolic motion.

In the Öpik's formulation, the massless particle and the secondary body move in their heliocentric trajectories around the Sun. They are placed in a Jacobi frame. In this non-dimensional frame, the components of the relative velocity, $\mathbf{U}$, at encounter can be derived geometrically from the orbital elements of the massless particle [10]:

$$U_X = \pm \sqrt{2 - 1/a - a(1 - e^2)} \quad (3.14)$$

$$U_Y = \sqrt{a(1 - e^2)} \sin i - 1 \quad (3.15)$$

$$U_Z = \pm \sqrt{a(1 - e^2)} \sin i \quad (3.16)$$

where a, e, i are the heliocentric orbital elements of the perturbed body. The $\pm$ symbol depends on if the encounter takes place at the ascending or descending node defined upon the perifocal plane of the perturbing body. U is the direction of the incoming asymptote of the pre-encounter unperturbed trajectory, and the magnitude of $\mathbf{U}$ is as in Equation (3.3).

This vector can also be expressed in the XYZ local planetocentric reference frame, making use of angular coordinates in the form of $\mathbf{U} = \mathbf{f}(U, \phi, \vartheta)$, as shown in Figure 3.12.

With such a definition, the vector $\mathbf{U}$ can be rewritten:

$$U_X = U \sin \vartheta \sin \phi$$

$$U_Y = U \cos \vartheta$$

$$U_Z = U \sin \vartheta \cos \phi$$

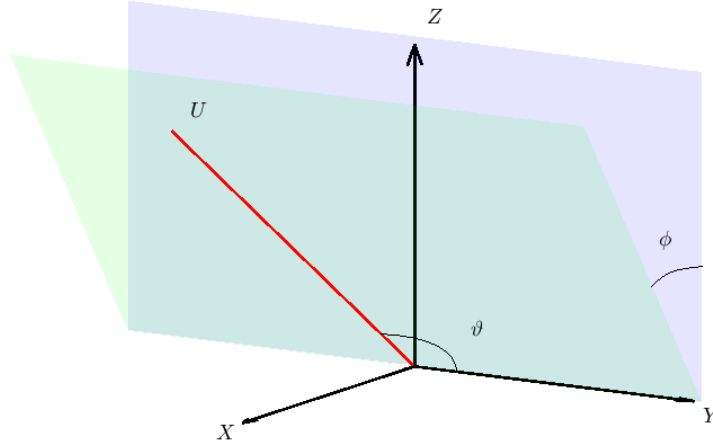


Figure 3.12: Representation of the relative velocity vector, $\mathbf{U}$, in the XYZ planetocentric frame. $\vartheta \in [0, 2\pi)$ is the angle between the Y -axis and $\mathbf{U}$, while $\phi \in [0, \pi]$ is the angle between the YZ plane and the YU plane.

The orientation of $\mathbf{U}$ determines the *b-plane* or *impact plane* [38], a plane centred at the planet and orthogonal to $\mathbf{U}$. This plane encompasses vector $\mathbf{b}$, extending from the planet to the point of intersection with the incoming asymptote. The magnitude of $\mathbf{b}$ corresponds to the impact parameter b .

As a consequence of the encounter, the relative velocity is rotated according to the two-body scattering scenario with the angle 2δ , such that:

$$\tan \frac{2\delta}{2} = \frac{\mu}{bU^2} \quad (3.17)$$

As this is the rotation of the asymptotes of the flyby hyperbola, Öpik's theory enables the computation of post-encounter Öpik parameters $\mathbf{U}'$, ϕ' , ϑ' , determining the direction of the outgoing asymptote and, consequently, the post-encounter heliocentric trajectory.

Öpik's theory has been developed over the years and used to analytically study the energy distribution in CAs [39], to analytically solve the direct flyby problem, allowing one to find the pre-encounter trajectory given the desired post-encounter condition and

the perturbing body [40].

Other studies have highlighted the constraints of this approach, typically sharing the limitations inherent in the PC theory. Challenges become prominent, especially in cases where encounters unfold slowly and cannot be accurately approximated as occurring at a single point. Additionally, complications arise when the secondary body influences the particle, even outside the context of an ongoing flyby [20]. Encounters between co-planar bodies are critical for Öpik’s theory [41], where the heliocentric velocities approach tangency, and the Tisserand parameter tends towards 3 [39].

3.3.2 Keplerian Map

The KM serves as a semi-analytical tool for assessing the consequences of a flyby within the framework of the CRTBP. Specifically, it enables the computation of changes in the orbital elements of a small body influenced by the third-body effect over one period of revolution. The KM has also demonstrated effectiveness beyond the SOI of the perturbing body [27, 28], and this hypothesis will be discussed in the results section. In the upcoming paragraph, the focus will be on examining the change in the semimajor axis, denoted as Δa , crucial for the energy variation [27].

The KM method facilitates the mapping of the six orbital elements of the small body, illustrating their alterations after one revolution while considering the perturbation from the secondary. The chosen reference system is the synodic frame, shown in Figure 3.13, characterised by an x -axis fixed along the line connecting the primary and secondary, and rotating in accordance with the two bodies with the angular velocity $\omega_R = \sqrt{GM/R^3}$, where R is the distance between the primaries. The y -axis is orthogonal to the x -axis, in the direction of the velocity of the secondary. Finally, the z -axis completes the right-handed frame. Additionally, the same Jacobi units described in the previous paragraph are used to simplify the equations and non-dimensionalise the problem. This reference frame, used in the mathematical definition of the CRTBP and in the context of the KM, allows for the immediate definition of the positions of the two primaries, with the small body’s position determined from its orbital elements.

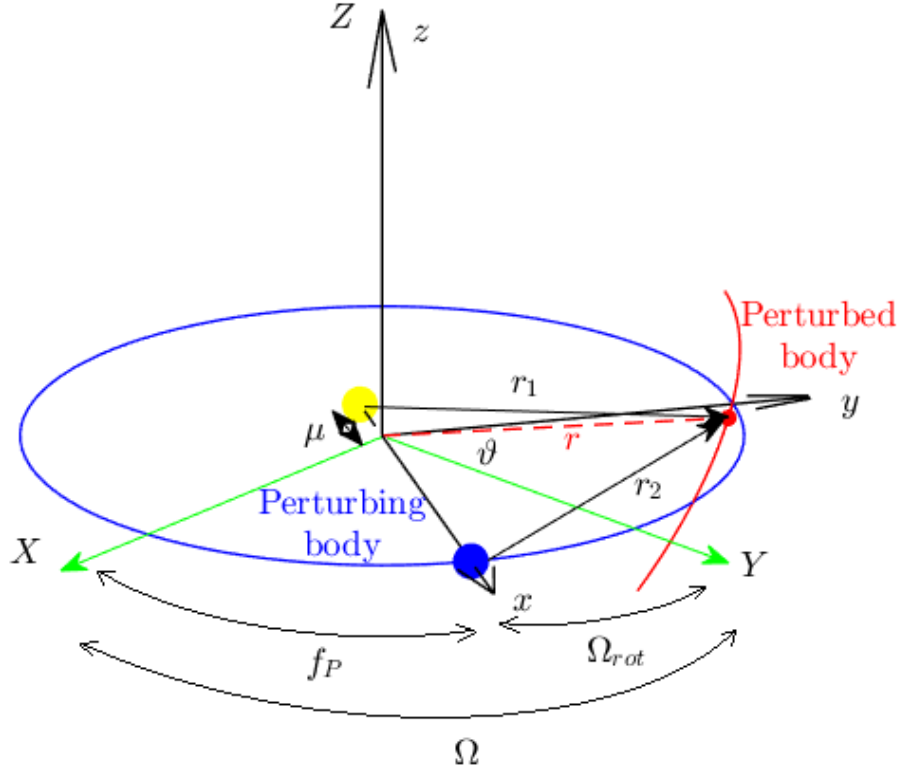


Figure 3.13: Representation of the synodic reference frame.

A first Picard iteration of the Lagrange planetary equations (LPE), numerically integrated over one period of revolution of the small body, provides the estimate of the change of the orbital parameters. Assuming $\mu \ll M$ and $\mu \ll r$, with $\mathbf{r}$ the radius connecting the small body to the centre of the synodic frame, the Hamiltonian for the motion of the massless particle can be written as per [27]:

$$\mathcal{H} = \mathcal{K} + \mathcal{U} + \mathcal{O}(\mu^2) \quad (3.18)$$

where $\mathcal{K}$ is the Keplerian term of the nominal trajectory and $\mathcal{U}$ is the disturbing potential. The variation of the orbital elements can be retrieved with the LPE [32] applied to the disturbing function $\mathcal{R}$ [28]:

$$\mathcal{R} = -\mathcal{U} \approx -\mu \left(\frac{1}{r} \frac{\cos \vartheta}{r^2} - \frac{1}{\sqrt{1 + r^2 - 2r \cos \vartheta}} \right) \quad (3.19)$$

Here, r is, under the approximations, comparable to r_1 , distance between the small body and the primary. ϑ is the angle between the lines connecting the two primaries and

the centre of the reference frame to the small body. Hence, the distance between the small and the perturbing body is $r_2 \approx \sqrt{1 + r^2 - 2r \cos \vartheta}$.

The variation of the semimajor axis can be obtained by applying a first Picard iteration on the LPE over a period of time [28]:

$$\Delta a = \int_{t_1}^{t_2} \frac{da}{dt} dt = \int_{t_1}^{t_2} \frac{2}{na} \frac{\partial \mathcal{R}}{\partial \mathcal{M}} dt \quad (3.20)$$

in which $n = \sqrt{M/a_0^3}$ is the mean motion of the small body and $\mathcal{M}$ is its mean anomaly. t_1 and t_2 are two arbitrary values of time.

However, from the analytical point of view, it is more convenient to express the integrals with respect to the true anomaly, f , taking as a reference point the orbital elements at the periapsis of the Keplerian orbit of the small body. The derivatives can then be evaluated with respect to the true anomaly and the integration can be done over one period of revolution from $-\pi$ to $+\pi$, allowing the method to inevitably consider the effect of the close encounter. The formula for the semimajor axis change over an orbital period then becomes [28]:

$$\Delta a = \frac{2}{n_0^2 a_0} \int_{-\pi}^{+\pi} \frac{\partial \mathcal{R}}{\partial f} df \quad (3.21)$$

The values with subscript "0" are intended to be those evaluated at the periapsis of the orbit, as it has been taken as the centre of integration. The integral has to be evaluated numerically, although an analytical expression for the derivative of the disturbing function $\mathcal{R}$ in Equation (3.19) can be provided [28]:

$$\begin{aligned} \frac{\partial \mathcal{R}}{\partial f} = & -\mu \left(\frac{-e \sin f}{r(1 + e \cos f)} - \frac{1}{r^2} (\cos(\Omega_{\text{rot}}) \sin(\omega + f) \right. \\ & + \sin(\Omega_{\text{rot}}) \cos(\omega + f) \cos i) - \frac{2e \cos \theta \sin f}{r^2(1 + e \cos f)} \Big) \\ & - \frac{\mu}{(1 + r^2 - 2r \cos \theta)^{3/2}} \\ & \left(r^2 \frac{e \sin f}{1 + e \cos f} + r(\cos(\Omega_{\text{rot}}) \sin(\omega + f) \right. \\ & + \sin(\Omega_{\text{rot}}) \cos(\omega + f) \cos i) \Big) \\ & + \frac{\mu r \cos \theta}{(1 + r^2 - 2r \cos \theta)^{3/2}} \frac{e \sin f}{1 + e \cos f} \end{aligned} \quad (3.22)$$

All the quantities in this equation can be evaluated from the orbital elements of the small body, given the geometry of the CRTBP. The quantity Ω_{rot} is defined as the angle between the RAAN Ω of the small body and the true anomaly of the secondary in its circular motion: $\Omega_{\text{rot}} = \Omega - f_P$. Since the secondary is in circular motion in the synodic normalised frame, where its period is exactly 2π , and considering dimensionless variables, it can be written as a function of the orbital elements of the small body, in particular its true anomaly, f , to account for its variation in time, as in [27]:

$$f_P = \sqrt{\frac{a^3}{1-\mu}} \left[2 \tan^{-1} \left(\sqrt{\frac{1-e}{1+e}} \tan(f/2) \right) - e \frac{\sqrt{1-e^2} \sin f}{1+e \cos f} \right] + t_0 \quad (3.23)$$

Since f_P can be directly connected to the time of integration, t_0 is the time at which the small body passes through its periapsis, so that $f_P(f=0) = 0$ and $f_P = 0$ at encounter. For a successful numerical integration, it is crucial that the value of f_P remains within the bounds of $-\pi$ and π . Furthermore, with these definitions, the angle between the secondary and the small body ϑ can also be analytically defined [27]:

$$\cos \vartheta = \cos \Omega_{\text{rot}} \cos(\omega + f) - \sin \Omega_{\text{rot}} \sin(\omega + f) \cos i \quad (3.24)$$

Finally, the numerical evaluation of the integral in Equation (3.21) can be calculated. The method has proven to be reliable for encounters outside the SOI and is faster than the numerical integration of the CRTBP equations of motion [27, 28].

3.4 Applicability of the Theories on Asteroid Encounters

In Section 3.2, it was shown that the majority of asteroid encounters take place far from the SOI of the perturbing body, a condition that goes against the assumption for the PC theory and that instead is beneficial for the KM theory. However, previous studies have shown that Öpik's theory, and hence the two-body approach, can work well, in certain cases, even where the encounters are slow and distant [20], even those several times the Hill's radius of the perturbing body [41].

The applicability of the PC theory is hereafter verified through numerical simulations of close encounters among asteroids. The flybys are modelled in the planetocentric frame at the moment of closest approach using the three-dimensional Broucke parametrisation illustrated in Figure 5.1 and normalising the physical quantities according to the

Jacobi units.

The physical parameters at encounter used for the simulations have been retrieved from the data presented in Section 3.2 and properly set within the following ranges:

- The angles α , β , γ are set in order to obtain all the possible combinations of geometric positions for the minimum distance radius and velocity; the ranges are: $\alpha \in [0, 2\pi]$, $\beta \in [-\pi/2, \pi/2]$, $\gamma \in [-\pi, \pi]$;
- The mass parameters μ of the perturbing body span from the mass of the fourth most massive asteroid, Hygiea ($\mu = 4.42 \times 10^{-11}$), to the 299th in the list (Ganymed with $\mu = 3.362 \times 10^{-14}$);
- The velocity at the moment of the closest encounter, v_P , ranges between 0.08 and 0.25;
- The minimum approach distance, r_P , spans from 2 to 100 times the R_H of the perturbing body. To analyse the differences with a flyby inside the SOI, the values $0.5 \cdot R_H$ and $0.8 \cdot R_H$ are also included in the simulation.

These quantities define the encounter configuration, and for getting the pre- and post-encounter conditions, the trajectory of the massless particle is propagated backward and forward, respectively, for half a period of the perturbing body, i.e., $\pm\pi$, considering the use of the normalised frame. This is implemented to ensure that the massless particle does not experience the gravitational influence of the perturbing body at the initial and final moments of the propagation, and to avoid multiple encounters. The simulation setup comprises only the Sun with unit mass, the perturbing body, and the massless particle, allowing the neglect of the disturbing effects of other bodies. The propagation is performed with the integration of the Gauss variational equations (GVE) [32, 42]. The disturbing, or perturbing, acceleration is represented by the gravitational acceleration of the secondary body [43], and the integration is performed in Python using the `solve_ivp` function with the DOP853 routine¹, which implements an explicit Runge–Kutta method of order 8 with time step control.

¹https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.solve_ivp.html

3.4.1 Inverse Flyby Solution

Let us consider a scenario where the epoch of the flyby, along with the Runge–Kutta and post-encounter trajectories, is known, but no information is available regarding the mass and orbit of the perturbing body. The PC theory was initially developed to address the direct flyby problem, where all characteristics of the perturbing body are known, and the design focuses on achieving a specific geometry for the encounter based on the desired pre or post trajectory. We can reverse Equation (3.10) to extract information about the perturbing body when given the pre- and post-encounter arcs. In fact, encounter distance-to-perturbing mass ratio is:

$$\left(\frac{r_P}{m_P}\right)_{PC} = \frac{1 - \sin \delta}{\sin \delta v_P^2 / G} \quad (3.25)$$

Its validity extends to all cases where the PC theory is applicable. The deflection angle, δ , and the value of the relative velocity at minimum approach, v_P , can be evaluated, even without information on the perturbing body. Its orbit can be assumed as circular, with a radius equal to the radius at which the encounter, i.e., the deflection, takes place. If we assume the secondary is moving in a circular orbit, the problem is simplified (with the assumption of a fixed eccentricity, $e_P = 0$) and its velocity at encounter, v_P , corresponds to the circular velocity for the given radius. The angle δ is instead obtained as the angle between the pre and post unperturbed relative velocities, obtained through a two-body heliocentric propagation of the pre- and post-encounter trajectories.

The results of this inverse PC can give only information on the ratio (r_P/m_P) , and these two quantities cannot be decoupled through only using the two-body theory. However, this tool is powerful also because it does not require any assumption on the approach direction. In fact, if one also knew the approach distance and direction, the mass of the perturbing body could be retrieved with a reverse KM approach.

The applicability of this reversed formula in the context of asteroid encounters can be verified both by considering the real data presented in Section 3.2 and numerical simulations, comparing the actual value of the distance over mass ratio and the result of Equation (3.25). For the real cases, the formula can predict the ratio with a relative error smaller than 5% for 33% of the cases. However, it is with the results of the numerical simulations that one can obtain a better idea of how this formula, hence the PC approach, is working for asteroid flybys, given a higher level control on the encounter parameters.

The findings emphasize that the accuracy of predictions, as per the assumptions of the PC theory, is influenced by the control parameters. An increase in r_P leads to a corre-

sponding increase in error, whereas an increase in the mass of the perturbing body or the relative velocity at encounter results in a reduction of the relative error. Despite aligning with the limitations of the PC theory, the predicted values remain accurate, even for encounter distances significantly beyond the Hill's radius.

3.4.2 Errors in the Energy Variations Estimations

From Equation (3.13), one can calculate the energy change resulting from the flyby using the PC theory, while for KM:

$$\Delta E_{KM} = \frac{GM}{2a_0} - \frac{GM}{2(a_0 + \Delta a_{KM})} \quad (3.26)$$

where a_0 is the semimajor axis of the pre-encounter trajectory and Δa_{KM} is given by Equation (3.21).

Multiple encounters are simulated to compare the variation in energy differences (ΔE) described by both the PC and KM theories, with the actual values between the pre- and post-encounter arcs.

A previous study [44] conducted a similar analysis to evaluate the applicability scope of the PC theory when varying parameters in the 3D Broucke parametrisation. That publication analysed the absolute difference in ΔE between the PC theory and CRTBP propagation, concluding that larger masses of the perturbing body, lower r_P , and lower v_P were leading to higher absolute errors. These findings were justified by asserting that, with that combination of parameters, the duration of the small particle's influence by the secondary is extended, and it is inappropriate to consider the flyby as instantaneous. While these results may appear to contradict the assumptions of the PC theory, especially concerning the approach distance and the mass of the perturbing body, they are accurate because the absolute error served as the control parameter. This absolute error is computed as the difference between ΔE_{PC} and ΔE_{CRTBP} . Indeed, with an increase in distance or a decrease in the mass parameter, the overall ΔE becomes smaller, resulting in smaller differences between the values obtained through the two different methods.

A more accurate way to assess the performance of methods like PC or KM in predicting the energy change is using the relative error, calculated as:

$$\text{Err rel} = \left\| \frac{\Delta E_{PC/KM} - \Delta E_{GVE}}{\Delta E_{GVE}} \right\| \quad (3.27)$$

In the following paragraphs, the relative errors for the two revised theories will be analysed and compared. The results for the PC theory will be used to see if the conclusions of the previous work [44] can stand also in the relative error, even if, in this case, the simulation parameters will be based on the asteroid encounters data, so larger distances and smaller mass parameters than in the benchmark paper.

Relative Errors for the PC Theory

The plots in Figure 3.14 show the relative error in ΔE for an asteroid flyby at different distances from the perturbing body. To reduce the complexity of the problem and simplify the readability of the results, some of the parameters have been fixed, and only r_P , α and γ vary over the respective ranges. The other variables are $v_P = 0.04$, $\mu = 9.823 \times 10^{-13}$, and $\beta = 77$ deg.

The sequence of plots show a degradation of the performance of the PC theory to predict the actual flyby outcome as the distance increases. The trend is also sensitive to the approach geometry, with higher values around the encounters taking place at $\alpha = \pi$, which is incidentally also where $\Delta E_{PC} = 0$, according to Equation (3.13), and in general where ΔE is lower, as also shown later. Getting much further from the SOI, the relative errors become too high to consider the PC prediction as a reliable one, with errors greater than 60% that have been blanked out from the plots for an easier visualisation.

In general, what is represented in the graph can be explained by noting that the lower relative errors take place where the ΔE is higher, and vice-versa. The reason can be twofold. Specifically, when ΔE_{GVE} is low but not zero, i.e., when $\alpha = 0$, Equation (3.13) yields zero, failing to capture even a minor ΔE that might exist in reality. This deficiency results in a substantial relative error, potentially reaching 100%. This occurs because the PC model, when confronted with minimal ΔE , inaccurately approximates the encounter outcome as having zero energy change. The second aspect contributing to this phenomenon is the presence of a low denominator value in Equation (3.27), amplifying the relative error. This dual effect is more pronounced for scenarios with lower ΔE values. In contrast, higher ΔE values tend to alleviate this issue, although a discernible increase in absolute error may persist, as indicated in [44].

The same assumption on high ΔE –low error can be verified when looking at the planar case, as per in Figure 3.15. When $\beta = 0$, Equation (3.13) is maximised in β and indeed the results show a lower error if compared to the previous one, with $\beta = 77$ deg.

In this scenario, the errors are more steadily below the 60% threshold, even for larger distances. This confirms a dependency of the relative error in β that goes accordingly to that in α , following the trend of the respective sinusoidal functions present in Equation (3.13).

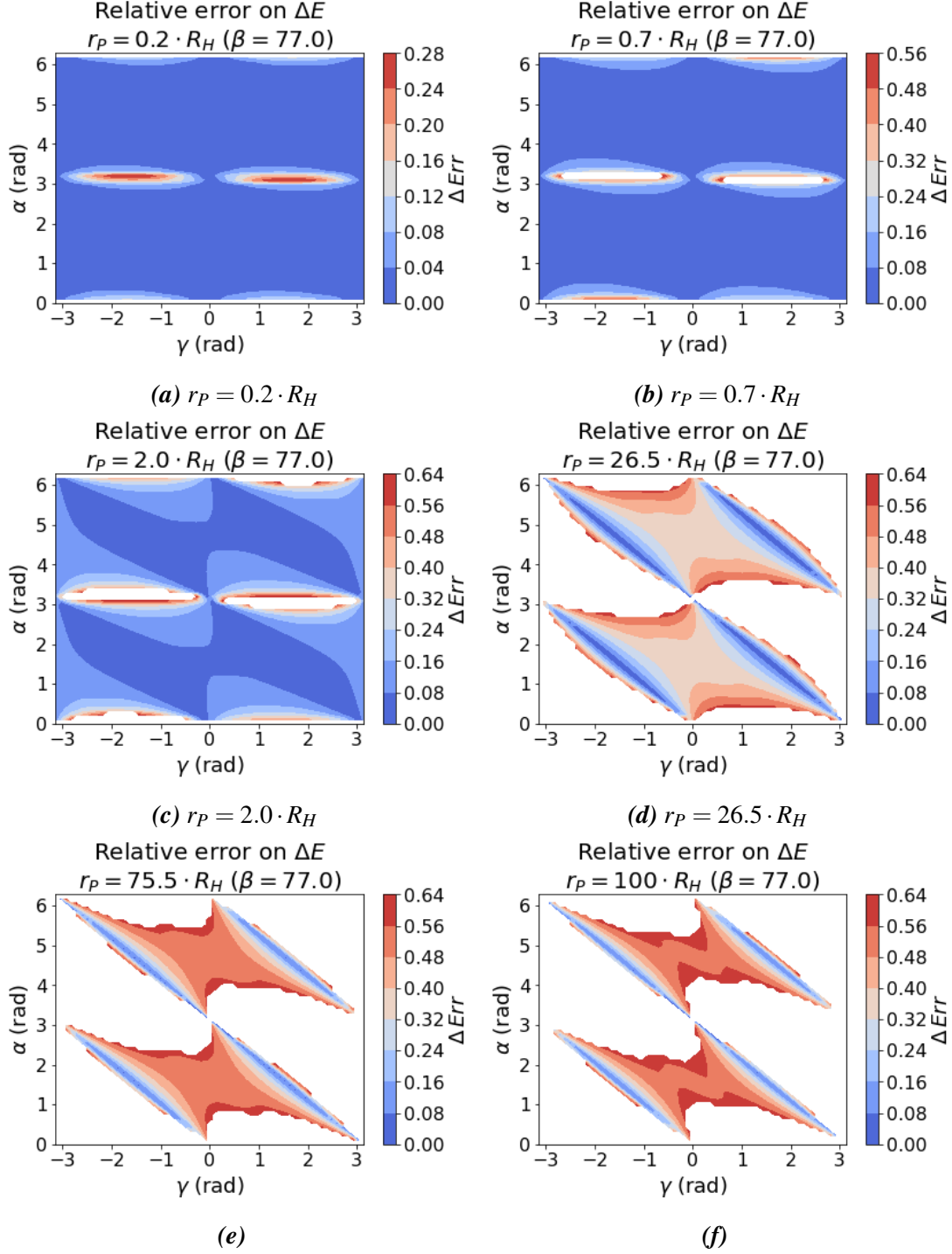


Figure 3.14: Relative error of the ΔE_{PC} with respect to ΔE_{GVE} at different r_P/R_H and $\beta = 77$ deg. The values above 0.6 are blanked out in the heatmap for a better visualisation.

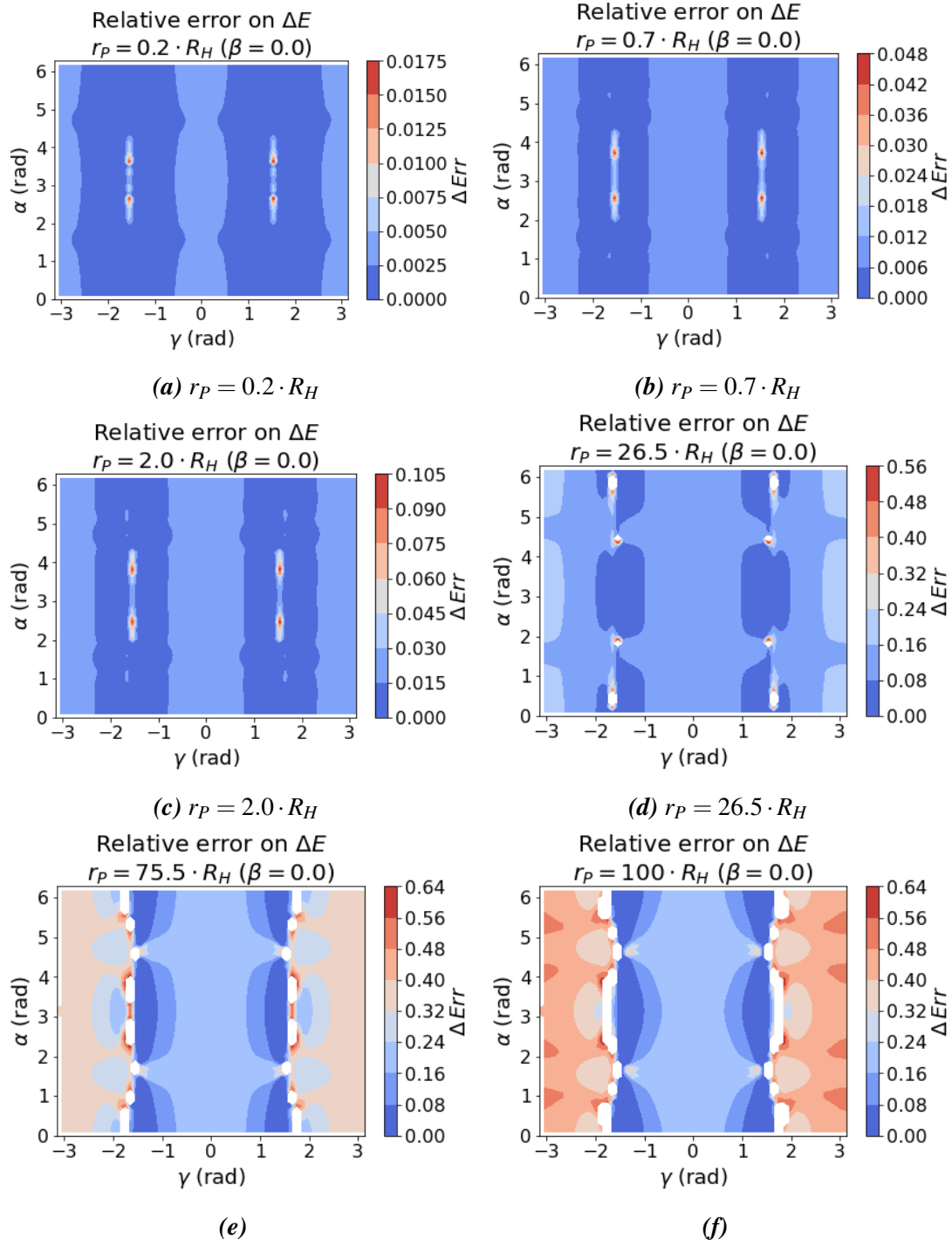


Figure 3.15: Relative error of the ΔE_{PC} with respect to ΔE_{GVE} at different r_p/R_H and $\beta = 0$ deg. The values above 0.6 are blanked out in the heatmap for a better visualisation.

However, the impact of the parameter δ on the results is different. This parameter is not incorporated into Equation (3.13) and does not play any role in ΔE_{PC} . This absence of γ influence is also evident in Figure 3.16, where ΔE_{PC} is plotted for two values of r_p/R_H , and no effect of γ is observable. Nonetheless, in Figures 3.14 and 3.15, the relative errors

are impacted by γ , indicating that its effect is significant in real cases. This suggests that the real case scenario, unlike the PC formulation, is influenced by γ and that its absence in the PC equation may lead to discrepancies in relative errors.

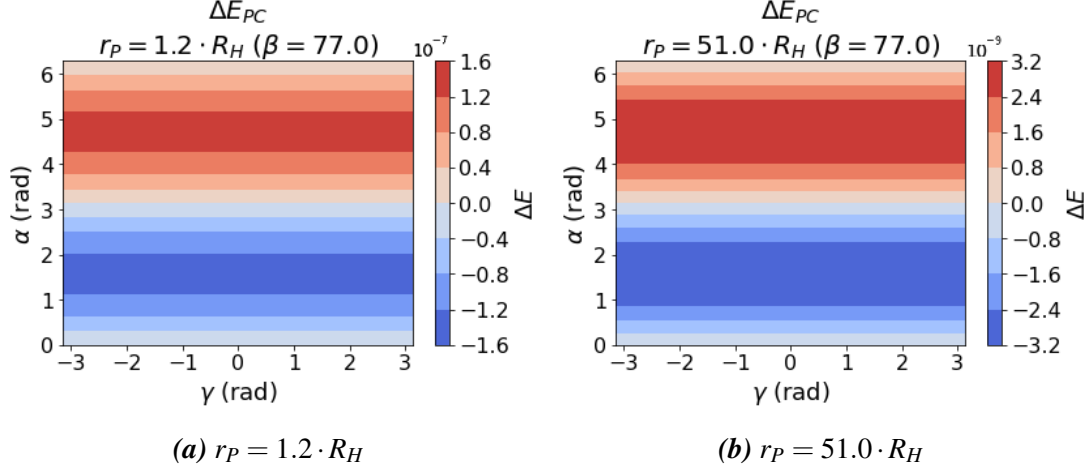


Figure 3.16: ΔE_{PC} for $\beta = 77$ deg. According to Equation (3.13), there is no effect of parameter γ .

The latter is confirmed by Figure 3.17, where the ΔE_{GVE} is plotted for the same distance ratios, and here the role of γ becomes more clear, especially when the distance from the perturbing body increases. This suggests that, once again, equations related to the PC theory tend to be a valid representation of the reality only when close to the SOI of the perturbing bodies.

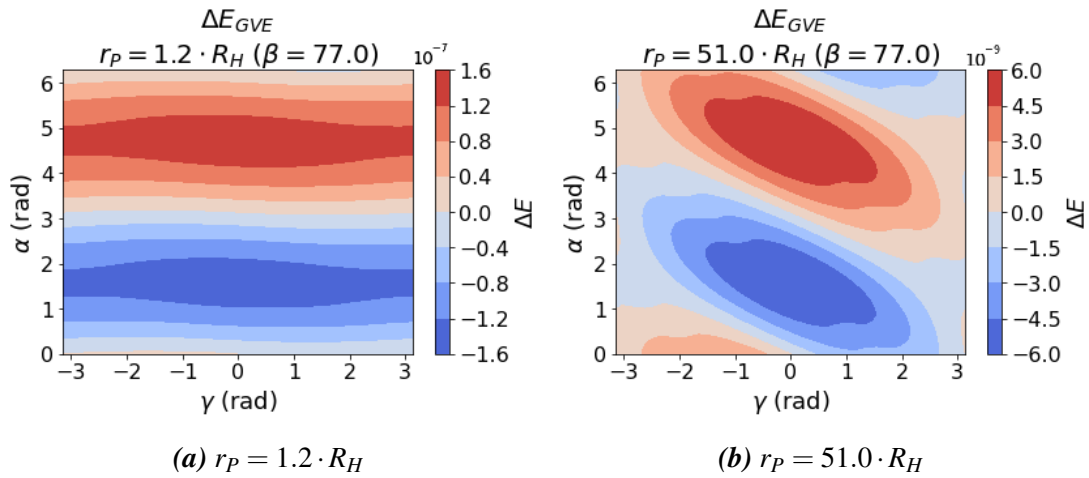


Figure 3.17: ΔE_{GVE} for $\beta = 77$ deg.

Relative Errors for the KM Theory

The same plots, at $\beta = 77^\circ$ and 0° , respectively, are reported for the KM in Figures 3.18 and 3.19.

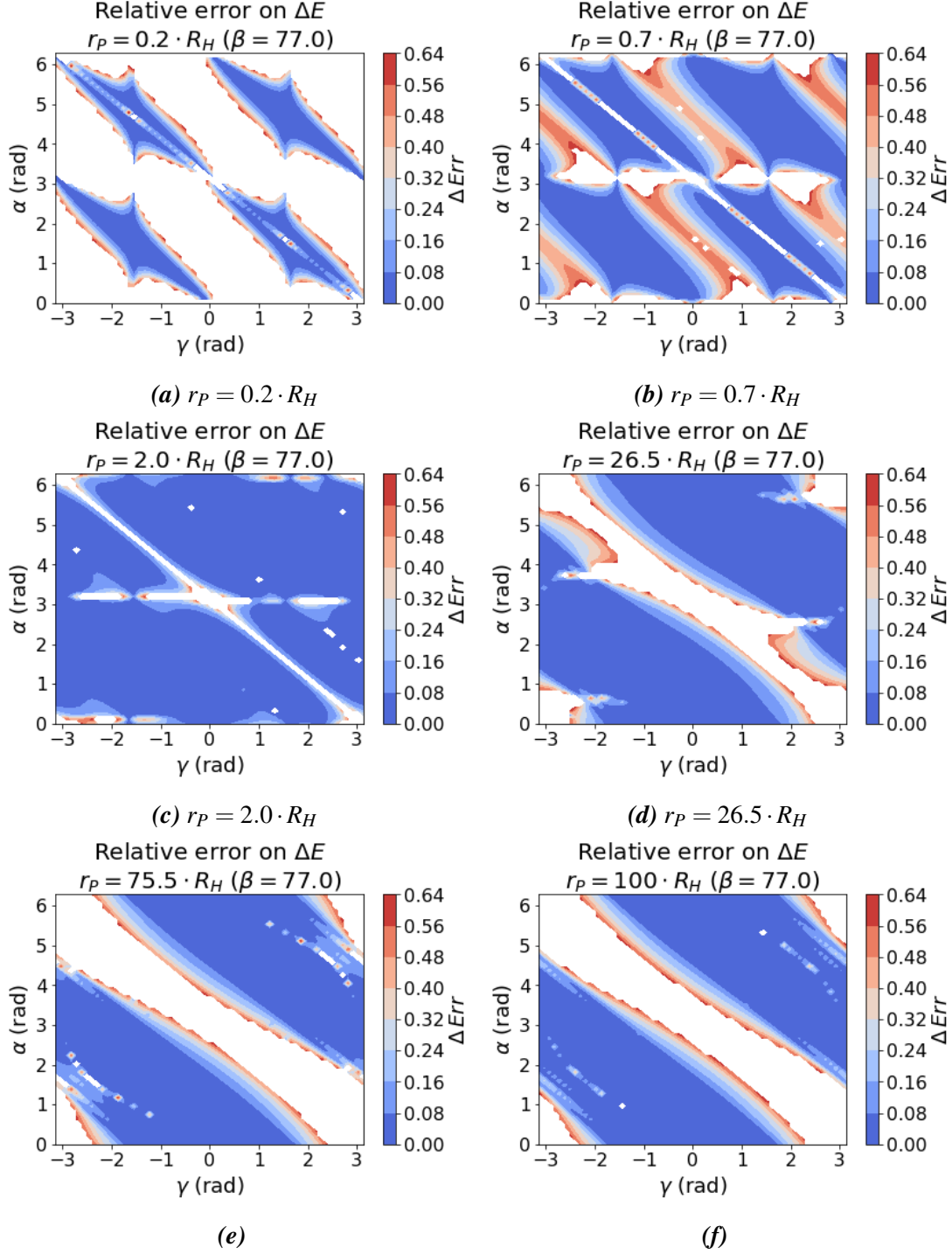


Figure 3.18: Relative error of the ΔE_{KM} with respect to ΔE_{GVE} at different r_P/R_H and $\beta = 77^\circ$. The values above 0.6 are blanked out in the heatmap for a better visualisation.

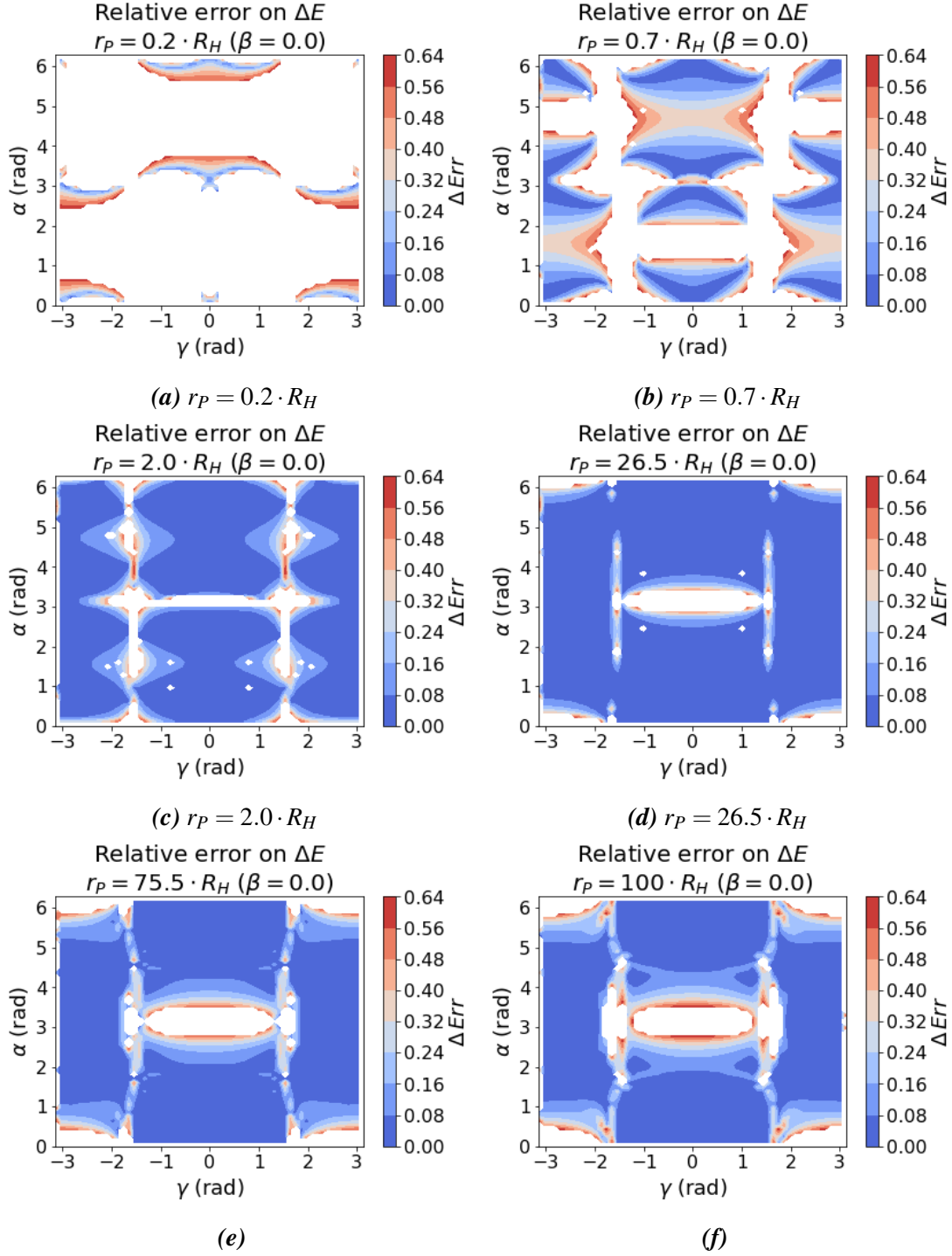


Figure 3.19: Relative error of the ΔE_{KM} with respect to ΔE_{GVE} at different r_p/R_H and $\beta = 0$ deg. The values above 0.6 are blanked out in the heatmap for a better visualisation.

In the first case, the outcome is much more positive than the PC theory for the areas far from the SOI. The differences in the performance of the method between 75.5 and 100 times the Hill's radius is almost inappreciable. The behaviour inside the SOI is in-

stead much worse than the PC results, confirming what was stated in Section 3.3.2 on the effectiveness of KM outside the SOI.

The plots at $\beta = 0$ deg highlight a similar response, but with a wide area around $\alpha = \pi$, where the method is not able to give a reliable prediction at any distance.

3.5 Conclusions

A total of 11 500 close encounters among asteroids have been identified from the OD of all the objects present in the ESA’s NEOCC database. The data highlight the distant nature of typical asteroid encounters, with only 5 500 events happening within a distance from the perturbing body of 200 Hill’s radius and only 19 inside the SOI of the perturbing body. This is attributed to the chaotic distribution of asteroids in the solar system, which is mirrored in their encounter patterns. Additionally, the small masses of asteroids result in smaller SOI, further reducing the geometric probability of encounters occurring within the SOI. The collected data can also help in statistically evaluating the geometric parameters that characterise a typical asteroid–asteroid encounter. Furthermore, the survey can assist in setting up numerical simulations of asteroid flybys. Particularly, the parameters of the 3D-Broucke parametrisation of the actual encounters have been shown, emphasising the typical range of normalised relative velocity, which spans between 0.05 and 0.38.

Theories for describing flyby outcomes have been developed and are widely available in the literature. A focus has been placed on methods capable of analytically or semi-analytically solving the flyby problem using a limited set of parameters, which are constrained either to encounter geometry or the orbital elements of the involved bodies. The PC and Öpik’s theory have been outlined, along with their respective assumptions and limitations, emphasising that these theories are not designed for describing encounters that are too distant from the perturbing body. Particularly significant in the context of asteroid dynamics is Öpik’s theory, explicitly developed for planetary flybys and collisions, providing geometric descriptions of encounter parameters. Another theory tailored for planetary encounters is the KM theory, which can semi-analytically solve flybys and predict changes in post-encounter orbital elements. The fact that these theories are implemented for planetary encounters means that the eccentricity and inclination of the perturbing bodies are considered as zero. This assumption, though acceptable for planets, becomes no more valid for generic asteroid encounters, characterised by higher eccentricities and inclinations.

The applicability of these theories to asteroid encounters challenges their fundamental assumptions. By testing the analytical formulations of the PC theory against data on asteroid encounters through simulations, this aspect has been investigated. The results indicate a degradation in the ability of the PC theory to predict the actual outcome of a flyby as the distance from the central body increases, as expected. However, it is also demonstrated that the PC theory exhibits the capability to describe encounters outside the SOI, contrary to the assumptions of the PC theory.

A dependency on the geometry approach is evident. Indeed, the relative errors are smaller when Equation (3.13) tends to higher values, i.e., when the flyby has a stronger effect. Also, a dependency on the geometric parameter γ in the energy change, ΔE , has been observed in the numerical simulations. This dependency is absent in the PC formulations. However, the results indicate that this dependency becomes significant when approaching the edge of the SOI and is important when very far from the perturbing body. This aligns with the limits of validity of the PC theory, suggesting a co-influence of the distance and γ .

The KM theory has similarly shown a dependency on the geometry but has proven to work better than the PC theory for the cases outside the SOI, even at very high distances. This is due to the fact that it is a semi-analytical method that takes into account the equations of motion. However, the performances inside the SOI have demonstrated to be non sufficient for flyby analyses.

Future works should focus on better understanding the mechanism governing very distant encounters. The physical explanation for why γ has a significant effect only outside the SOI remains unclear. One potential explanation could be that γ contributes to the out-of-plane component of velocity, leading to a higher relative inclination (Figure 3.9). However, investigations in this direction have not yielded conclusive results.

Furthermore, studying distant encounters could now lead to a new, general definition of the boundaries of application for the PC-two-body theory, following recent works on enhanced dynamical definitions of planetary SOIs [45]. As demonstrated, the boundary of the SOI can be surpassed.

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Chapter 4

Overview on astrometric mass determination for asteroids and significance of the dynamic model

Foreword

The flybys identified in Ch. 3 are here used to perform mass determination from astrometry for the perturbing asteroids included in the dynamical model. This is done to contribute to the mass determination of asteroids, which has its scientific and technical relevance discussed in the Chapter, and to provide a first dynamical improvement to asteroid OD. The Chapter focuses on finding masses for asteroids with low to no existing mass determinations using the deflection method, and manages to find the new mass of two asteroids, and refine the mass of another four.

Abstract

Astrometric mass determination is a direct way to estimate the mass of an asteroid from the analysis of the gravitational effect it exerts on other smaller bodies, when they experience a close encounter. The accuracy of the results is connected to the number of

considered encounters per target asteroid and to the dynamical strength of these encounters. This paper presents an algorithm capable of systematically scanning current asteroid observation databases, and automatically selecting existing close approaches between asteroids, based on their observed-computed residuals, to perform their astrometric mass determination. The algorithm, scanning a sample of more than 1 million minor planets, autonomously found and analysed more than 5000 strong dynamic interactions, leading to the first mass determinations with the astrometric method for (32) Pomona and (91) Aegina, and the best available determinations, in terms of accuracy, for (94) Aurora, (110) Lydia, (111) Ate and (747) Winchester.

4.1 Introduction

Accurate knowledge of asteroid masses is essential for determining the trajectories of objects within the solar system. Asteroids, through their gravitational fields, can significantly affect the motion of other asteroids and planets [1, 2, 3], especially Mars [4]. Therefore, accurate mass determinations improve the reliability of planetary and small bodies ephemerides. Furthermore, bulk densities and porosities can be accurately determined when masses and volumes are well known, enhancing our knowledge of the solar system bodies in terms of composition and physical characteristics [5]. Also, improving our understanding of asteroid masses and compositions contribute to our ability to study their orbits [6] and long-term evolution, which is especially important when considering Near-Earth Asteroids (NEA) [7]. Ultimately, in the context of planetary defense missions, a good preliminary knowledge of NEAs masses can help tune better missions for active asteroid deflection and assess the effects of an asteroid impact on Earth [8, 9].

The determination of asteroid masses can be done through several ways, which are mainly divided into indirect and direct methods, each characterised by a different degree of uncertainty [10]. Primary estimates are done by means of indirect methods, which derive an asteroid mass by its volume and density. Volumes can be retrieved by absolute magnitudes and albedos [11, 12], while information on densities can be obtained by the taxonomic class [13]. Other techniques to evaluate their sizes include radar observations, occultation or thermal infrared observations, or the detection of the Yarkovsky effect [14]. However, indirect methods fall off being with rather high uncertainties [15] if compared to other techniques, mainly due to the uncertainties on the observable characteristics of the body and assumptions like spherical shapes and average densities equal to surface density may not be accurate [16]. The grain density of minor planets are ultimately derived by

direct methods, when available, or by analyses of bulk density and porosity of meteorites fallen on Earth [17].

Direct methods generally allow for more precise mass determinations (MDs). These methods use the gravitational effect that a *target* asteroid has on other *test* objects [18]. Depending on the test bodies used, direct methods can be categorised into various sub-groups, which we describe below.

The ephemeris method involves analysing the positions and velocities of numerous celestial bodies by applying the laws of celestial mechanics. The masses of asteroids (targets) are inferred based on the perturbations they cause in the orbits of planets or other massive bodies (treated as test particles) [19].

The test bodies can be also other asteroids: the deflection method is based on observations of close encounters (CEs) among asteroids. From the deflection that the test asteroid(s) has after a CE with the more massive target one, the mass of the latter can be inferred by using methods like modified orbit determination (OD) (e.g., as in [5]) or the Bielicki method [20, 21]. In this context, the availability of additional and more accurate observations, along with improved astrometric data, enables higher observation accuracy. This is further supported by the increased astrometric accuracy provided by missions such as Gaia, especially with Gaia Focused Product Release (FPR) [22], and improved weighting systems for observations [23, 24]. These advancements contribute to improved orbit and mass determinations [25, 26, 27, 28]. Anyway, uncertainties remain too large for less massive asteroids, still close to 100% [18].

A satellite of the target asteroid can be used as well as a test object, when available. From optical or radar images of the system of bodies one can understand the dynamics of the involved objects. In particular, the period of the satellite's orbit can be determined by its lightcurve, and the mass of the main asteroid can be retrieved by applying Kepler's third law [29, 30]. This method allows to gain uncertainties around 15%, but it is subject to the condition of dealing with a well-characterized binary system.

Much rarer, but way more precise, with uncertainties around 1%, is the MD method that relies on a spacecraft encountering a target body. Doppler shifts in radio signals transmitted from a spacecraft during a flyby can provide an estimate of an asteroid's mass by analysing the spacecraft's velocity changes due to the asteroid's gravitational influence. However, this method has limitations in precision. In contrast, a rendezvous mission, where the spacecraft remains in proximity to the asteroid for an extended period, allows for more precise MD through prolonged tracking and detailed gravitational studies.

The first mission to accomplish this result was the NEAR-Shoemaker, which allowed for the determination of the masses of (253) Mathilde [31] in 1997 and (433) Eros [32] in 1999. Later, other missions have contributed to the direct observation of asteroids, like Dawn for (1) Ceres and (4) Vesta [33, 34], Hayabusa for (25143) Itokawa [35], Rosetta for (21) Lutetia [36], OSIRIS-REx for (101955) Bennu [37], Hayabusa2 for (162173) Ryugu [38]. This kind of missions can also provide on-site images of the asteroid, providing also knowledge on the shape, volume and distribution of the gravity field of the bodies.

The amount of known masses has increased during the past years, going from roughly 24 in 2002 [11] to the 132 masses derived with the astrometric method in 2014 by [39]. This is one of the latest most comprehensive astrometric MD surveys available, while more updated information can be found through online resources [40]¹. In the near future, we expect an increase in both the number and quality of available estimated masses, driven by, as mentioned before, the availability of more accurate astrometric data and the implementation of improved observation campaigns and surveys. This could lead to the discovery of new asteroids, providing more opportunities to detect close encounters and increasing the number of available test subjects.

The number of asteroids with known mass and the precision of the already available values can be improved for many asteroids. For some of them zero or only a few CEs have been observed, even due to their orbital properties, such as high inclinations, rendering the astrometric method either inapplicable or subject to significant uncertainties. In order to use the astrometric method, and extend the available masses determined dynamically, it is necessary to develop reliable strategies for identifying suitable encounters to be used for the astrometric method, also involving smaller asteroids. As it will be shown later, this is particularly challenging, as not all the CEs can lead to the convergence of the method and to reliable mass values.

The aim of this chapter is to present a novel algorithm for autonomously finding and selecting CEs suitable for asteroid MD with an astrometric method, already in the OD phase. The impact of this chapter lays in the enhancement of our current understanding of the Solar System and its dynamics, our capability of planning space resource exploration and utilisation, or, in case of potential impacts, predicting severity and prioritising mitigation efforts. Section 4.2 describes the CE selection and MD methods, while section 4.3 introduces the results obtained, which enhance literature. Finally, in section 4.4 the physical validity of the results is discussed.

¹<https://astro.kretlow.de/simda/>

4.2 Methodology

4.2.1 Astrometric method

The conventional OD is performed through an algorithm based on the least-squares method, that aims at minimising the cost function [41]:

$$Q(x) = b^T \Lambda^{-1} b, \quad (4.1)$$

where x is the state vector to be estimated, i.e., the 6 orbital parameters, $b(x)$ is the vector containing the observed-computed (O-C) residuals for each observation and Λ is the covariance matrix, made diagonal thanks to the assumption of uncorrelated observations. With suitable initial condition for the state vector, the differential corrections algorithm [41] iteratively refines the solution until it finds the parameters that minimise the cost function.

The astrometric MD method is a modification of the OD for the test body, with the mass of the target asteroid as a 7th parameter of the state vector [5, 42, 43, 25]. This is possible when the target body gravitationally influences the test body's trajectory through a flyby or resonant effects. In this way, by considering accurate observations of the test asteroid both before and after the CE, the influence of the target may be visible on the O-C residuals of the observations. The least-squares method can thus provide the mass of the perturbing body that minimises equation (4.1), together with the test body's orbital parameters.

The mass of the perturbing body is contemplated by the force model, which includes all the perturbations that can affect the test body's orbit evolution within the observation timespan. To get reliable results for both OD and MD, it is important that the perturbation models are as accurate as possible, including all the relevant forces, i.e., presence of third bodies, other than those involved in the flyby, solar radiation pressure, relativity or Yarkovsky effect, as all of these may have a significant impact on the O-C residuals and hence on the solution.

In the present study, for the force model, the BC-405 ephemerides model from [5] for the positions and masses of the planets and the most massive main belt asteroids (MBAs) has been employed, while other perturbations are considered just when relevant to the

single analysed test asteroid.

4.2.2 CE detection and selection

The conventional technique for finding CEs to be used in the MD method requires for the propagation of the motion of the test and target asteroids to see if they get close enough to experience a flyby [44, 45, 25, 46, 47].

This technique requires propagating a large number of asteroid orbits over extended periods of time. However, not all of the identified flybys are suitable for MD, so a selection of the most appropriate ones must be made. This selection can be based on factors such as the minimum approach distance [45], the deflection angle caused by the flyby [44], the predicted change in mean motion [25, 46]. Other authors have also analysed the difference in residuals on right ascension (RA) between the orbit perturbed and unperturbed by the target [47, 42].

This strategy offers limitations in the computational time needed, the number of included asteroids, and therefore analysed encounters. The CEs are usually filtered upon two parameters, one of which is the minimum distance of the encounter following the N-body propagation. The threshold distance for which a CE can lead to a significant deflection and being valuable for MD can however be unpredictable [48, 49], and can result in the creation of false positives and the inclusion of false negatives. The encounters are then usually further filtered upon another parameter, thus increasing the method's uncertainty. Furthermore, the OD has to be computed, to see if an effect on the O-C residuals from the target body is actually significant, impacting on the global computational time.

The methodology proposed in this work aims to serve as an alternative to the standard brute-force propagation method while also unifying the phases of OD of the test asteroids and the selection of suitable cases for MD. Moreover, its purpose is to avoid the preliminary propagation of the asteroid trajectories, and perform the detection and MD on top of the OD. It is based, similarly to what suggested by [42], on an analysis of the O-C root mean square (RMS) residuals, normalised in σ , of the OD.

The dynamical model applied to the numerical method depends on the force model used and has an effect on the OD results that can be reflected on the RMS residuals. In the force model, the gravitational influence of the perturbing asteroids in BC-405 can be included or ignored. If an asteroid undergoes a flyby that causes a significant deflection with one of the modelled bodies, the inclusion or exclusion of the perturbing body's ef-

fect will impact the RMS value. In this chapter, we assume that the masses included in the dynamic model are close to their true values. Under this assumption, the RMS of the perturbed solution, $(RMS)_{pert}$, will be lower than that of the unperturbed solution, $(RMS)_{unpert}$. However, this introduces a limitation: if the assumed mass of the perturbing body is significantly larger than its actual value, even up to twice the true value, $(RMS)_{pert}$ could end up higher than $(RMS)_{unpert}$, as the solution would deviate further from reality. This assumption is used to focus on cases where excluding the perturbing body from the OD solution leads to a significant increase in residuals. We define the difference as: $(RMS)_{unpert} - (RMS)_{pert} = \Delta(RMS)$.

The OD is performed with the software *Find_Orb*² on all the asteroids included in the ESA NEO coordination centre (NEOCC)³ database, both numbered and not numbered. The perturbation model applied for the perturbed solution is the 7007fe model, which includes the gravitational effects of the nine planets, the Moon, and the 300 most massive asteroids of the solar system, in accordance to the BC-405 ephemerides and mass model. We weight the observations according to the VFCC17 scheme [23] and the astrometric positions de-biased following [50].

The algorithm for the detection and selection of the effective CE is as follows, and is also summarised in Figure 4.1. The orbit of each asteroid in the NEOCC database is solved in the perturbed case, as described above, to get the related value of residuals $(RMS)_{pert}$. The same orbit is solved in the unperturbed case, which is without the 300 perturbing asteroids of the BC-405 model, to get the $(RMS)_{unpert}$. None of the available observations for each asteroid has been rejected. This may seem like a controversial choice, but there is no outlier rejection in the CE detection phase because there are no differences in the $\Delta(RMS)$, as we aim to observe the effect on $\Delta(RMS)$ solely due to the dynamics. This is crucial as it cannot be known a - priori whether the single observation could be rejected due to astrometric issues or for force model deficiencies. Instead of an outlier rejection on the single observations, a simple rejection criterion has been applied on the obtained residual values to neglect the asteroids with poor observation accuracy. Operatively, if the normalised residuals $(RMS)_{pert} > 5$ or $(RMS)_{unpert} > 100$ the solution was considered to be affected by outliers and rejected for the CE determination. We select these values according to the fact that a proper OD without considering outliers, should provide a residual value smaller or closer to 1 in the fully perturbed scenario. However, the threshold values should undergo a refinement process in future developments of the method. In the case the orbits are accepted, if the difference in the two val-

²www.projectpluto.com/find_orb.htm

³<https://neo.ssa.esa.int/>

ues $\Delta(RMS) = (RMS)_{unpert} - (RMS)_{pert}$ is greater than a certain threshold an encounter among asteroids has been found. The threshold has been set at 0.07 as, according to empirical investigations, it has proven to be a good compromise between computational time and exclusion of false negatives.

The $\Delta(RMS)$ due to a single flyby can be treated as a signature of the strength of the same flyby [48]. It is also known that high accuracies in MD, and also the effectiveness of the determination itself, are better achieved with strong flybys in terms of greater deflection angles [26].

To identify the asteroid responsible for the perturbation, an internal loop is performed over the 300 perturbing asteroids in the model, evaluating the gravitational effect of each asteroid individually to determine its influence on reducing the residuals. Firstly, the asteroids are filtered upon the minimum orbital intersection distance (MOID) with respect to the test object, to evaluate whether a CE is feasible or not. If the MOID of a given perturbing asteroid is less than 0.02 au, then the orbit of the test body is solved using just that particular perturbing asteroid in the perturbation model for the asteroids. We address this solution as the *single asteroid-perturbed solution*, which is associated to the residual value $(RMS)_{ast}$. In case $(RMS)_{ast} < (RMS)_{unpert}$, a CE between the test and the target asteroids has been found.

4.2.3 Mass determination method

After the detection of a CE, it may be possible to determine the mass of the target, although it is important to remember, as discussed later, that not all the CEs lead to a precise MD. Figure 4.2 illustrate the algorithm presented for MD. The method proposed in this study seeks to determine the mass value that minimises the residuals of the solution perturbed by a single asteroid, and is not currently able to resolve simultaneous determination as in [25, 27], following a similar approach to that suggested in [42]. The target mass value is found by iteratively adjusting the initial mass value in the BC-405 model, denoted as m_0 . An interval of $(\pm 3.5 \cdot m_0)$ around m_0 is discretised into 8 points, and the RMS is computed for each of these test masses. This new RMS is the value of residuals of the perturbed solution with the test mass value as the mass of the perturbing body. A parabolic fit is then applied to these mass - RMS points. If the fit exhibits a parabolic trend, verified using the R-squared test, and has a minimum not at the extremes of the interval, the algorithm is considered converged, and the mass value that minimises the parabolic spline interpolation is selected as the target mass value.

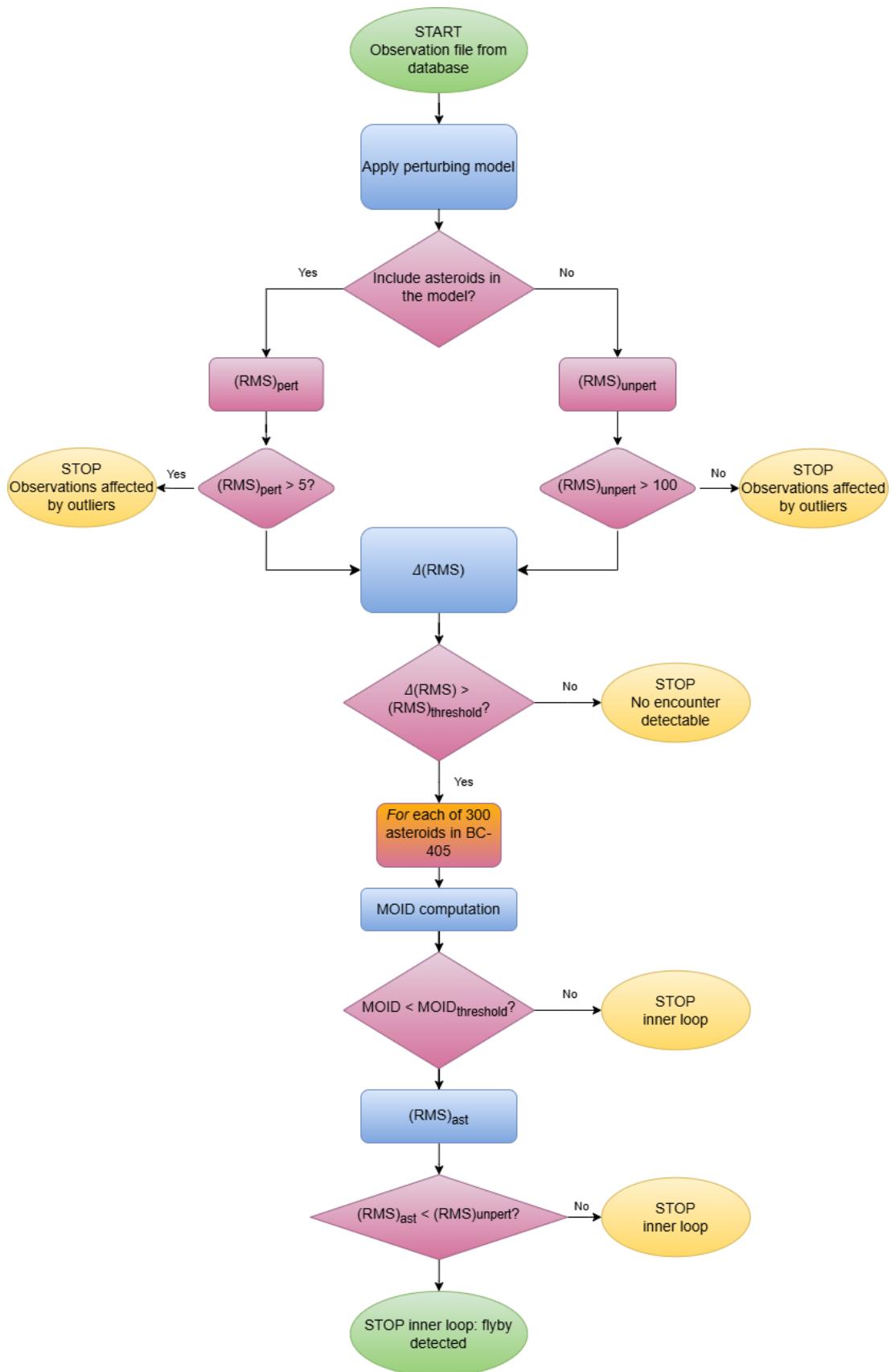


Figure 4.1: Schematic description of the algorithm for CE selection.

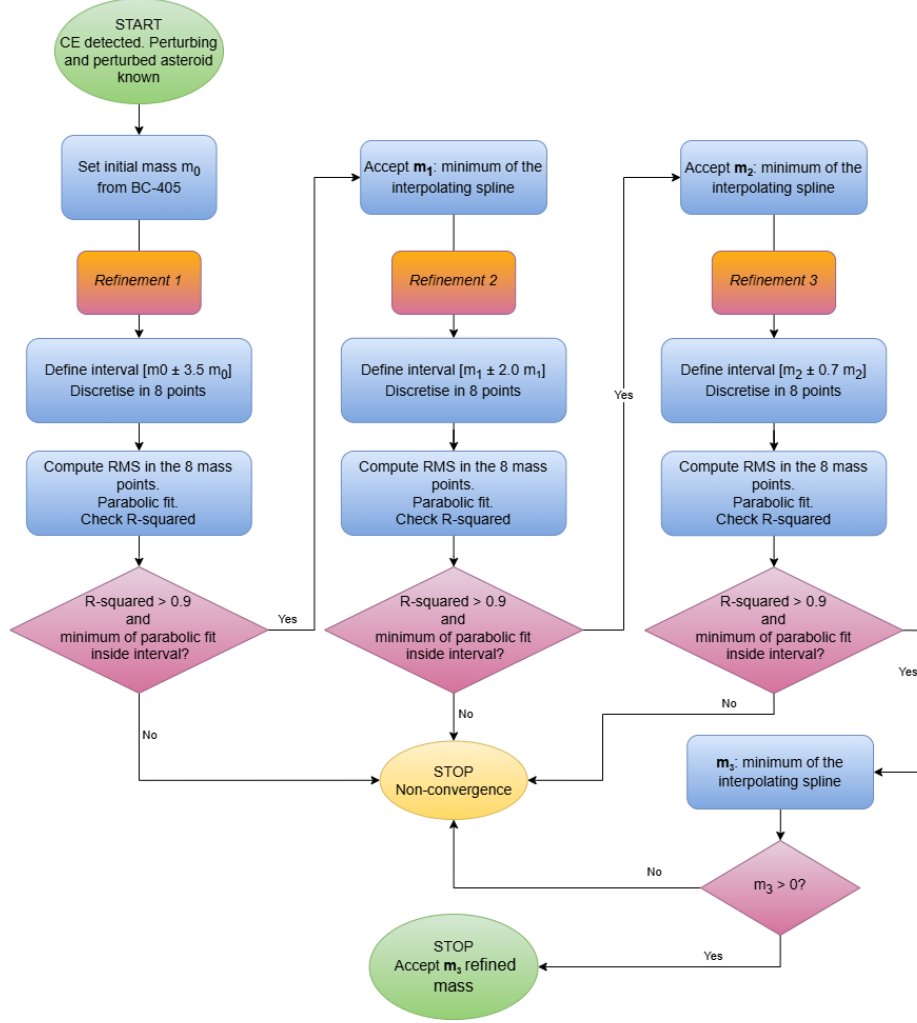


Figure 4.2: Schematic description of the algorithm for CE selection.

If the method converges, then the mass value is refined through 3 refinement cycles, which repeats the parabolic fit test and the minimisation spline to find the updated refined mass value:

1. m_1 found from the initial mass value m_0 in the interval $(m_0 \pm 3.5 \cdot m_0)$;
2. m_2 found from m_1 in the interval $(m_1 \pm 2 \cdot m_1)$;
3. m_3 found from m_2 in the interval $(m_2 \pm 0.7 \cdot m_2)$. This is considered to be the mass resulting from the method.

The convergence of this algorithm is however not guaranteed, as per the astrometric method, for all the CEs. The non-convergence can be determined by many factors, that range from the dynamical characteristics of the CE to the quantity, quality, temporal distribution of the observations, along with the numerical strategy employed for OD [51].

While these masses can also be determined using the conventional differential correction algorithm, applying it within an automated pipeline is challenging, as convergence information may not always be readily available from *Find_Orb*. Since non-convergence in *Find_Orb* also implies non-convergence of the method introduced in this section, *Find_Orb* is used to obtain results and estimate uncertainties, while the minimisation method serves as an operational routine to verify the convergence of MD and determine whether an MD solution is available. It is worth noting that, at its current stage of development, the minimisation method does not yet support uncertainty calculations.

The solutions are obtained without accounting for the Yarkovsky effect, as the following analyses focus on large MBAs, for which this dynamical effect is weak due to their low surface-to-mass ratio and the time frames considered [52]. Moreover, this assumption has been verified by analysing the impact of the Yarkovsky effect on the presented results, which was found to be negligible.

4.3 Results

We analysed the astrometry of 1 264 404 asteroids, all those present in the NEOCC database at 1 February 2023, both numbered and non numbered, with the method presented in section 4.2.2, resulting in the detection of more than 5000 encounters, occurring at different distances and with different dynamical outcomes.

Most of the detected gravitational effects involve massive asteroids with well-established masses, such as (1) Ceres, (2) Pallas, (3) Juno, (4) Vesta, (10) Hygiea, (16) Psyche, among others. It is important to note that, particularly for these massive bodies, the observed effects might arise from either CEs or other gravitational interactions, such as resonances due to similar orbital periods between the test and target asteroids. These phenomena can still be visible in the astrometric residuals. Nevertheless, such events can still be utilized for MD purposes [15].

The main objective of this chapter is to estimate and present masses just for those asteroids among the 300 listed in the BC-405 model with four or fewer MD with the deflection method, as indicated by the SsODNet database⁴ [53]. Just these were considered as target asteroids, to obtain results from previously unobserved CEs and contribute to improving the uncertainty of poorly known mass values or providing initial estimates for asteroids that do not have a dynamical MD published yet. Therefore, the well-known

⁴<https://github.com/maxmahlke/rocks>

masses of larger asteroids have been partly employed to verify and fine-tune the algorithm. Specifically, the consistency between the results obtained from the fully converged minimization method described in Section 4.2.3 and *Find_Orb*'s MD was verified, as well as the reasonable agreement between the derived mass values and those already available in the literature. Moreover, we notice that when both the methods converge, they do it to the same value, thus verifying the numerical approach. Some of the results used for fine-tuning and verifying the method are listed in Table 4.1, using massive asteroids with well established masses.

Table 4.1: *Some of the mass values used for verification of the algorithm. The test and target asteroids are listed, together with the masses obtained by the RMS minimisation m_M , from Find_Orb m_{FO} and its uncertainty, from the BC-405 ephemerides m_0 , derived from Goffin m_G . Masses are reported as a ratio of the Sun mass.*

Target	Test	m_M	m_{FO}	δm_{FO}	m_0	m_G
(3) Juno	(431840)	1.23×10^{-11}	1.23×10^{-11}	1.55×10^{-12}	1.50×10^{-11}	1.26×10^{-11}
(6) Hebe	(568133)	6.36×10^{-12}	6.36×10^{-12}	3.99×10^{-13}	6.30×10^{-12}	4.50×10^{-12}
(13) Egeria	(582282)	2.38×10^{-11}	2.35×10^{-11}	2.18×10^{-11}	8.00×10^{-12}	4.70×10^{-12}
(16) Psyche	2014 HS116	1.12×10^{-11}	1.12×10^{-11}	1.16×10^{-12}	1.14×10^{-11}	1.17×10^{-11}
(19) Fortuna	2016 RM33	4.39×10^{-12}	4.37×10^{-12}	1.21×10^{-13}	4.19×10^{-12}	4.50×10^{-12}
(19) Fortuna	(125910)	4.38×10^{-12}	4.39×10^{-12}	1.62×10^{-13}	4.19×10^{-12}	4.50×10^{-12}
(451) Patientia	(449924)	6.93×10^{-12}	6.82×10^{-12}	2.63×10^{-12}	4.02×10^{-12}	6.80×10^{-12}

The Table shows the target and test bodies for each encounter, and the masses obtained with both the minimisation algorithm m_M and with *Find_Orb* m_{FO} , together with the uncertainty associated to *Find_Orb* determination δm_{FO} . The table also shows the difference of the derived masses with respect to the original masses m_0 stored in the BC-405 model used by *Find_Orb* for the ephemerides of asteroids and those derived by [39] with the astrometric method (m_G). These two models frequently exhibit substantial discrepancies, primarily due to the fact that the former masses were established in 2013, predating Goffin's work. The mass values are presented as ratios relative to the Sun mass.

4.3.1 Encounters that enhance previous results

Some of the detected encounters are previously unseen, and they can help contribute to the improvement of the uncertainty of already known asteroid masses. These encounters may be novel for various reasons, such as occurring after the latest MD studies or being excluded by the selection methods used in those studies. Additionally, they may not have been considered as studies often focused solely on numbered asteroids.

The identified results, able to refine or enhance current literature, focussing on as-

teroids with very limited masses published with the deflection method, are presented in Table 4.2, which displays the target and test bodies along with the values of initial mass in the BC-405 model m_0 , the mass obtained with the minimisation method m_M and the mass given by *Find_Orb* with the classic least-squares method m_{FO} . The $1 - \sigma$ uncertainty of the measure given by *Find_Orb* δm_{FO} and the relative error $\delta m_{FO}/m_{FO}$ are also reported.

Table 4.2: *Masses with enhanced uncertainty. The test and target asteroids are listed, together with the initial masses m_0 , the masses obtained from the RMS minimisation m_M , the masses from Find_Orb m_{FO} , the uncertainties derived from Find_Orb and the relative uncertainties. Masses are reported as a ratio of the Sun mass.*

Target	Test	m_0	m_M	m_{FO}	δm_{FO}	σ_{rel}
(94) Aurora	(427341)	3.03×10^{-12}	3.28×10^{-12}	3.28×10^{-12}	4.08×10^{-13}	12.4 %
(110) Lydia	(66778)	8.90×10^{-13}	7.34×10^{-13}	7.38×10^{-13}	1.68×10^{-14}	2.3 %
(111) Ate	(232126)	3.06×10^{-12}	6.43×10^{-12}	9.96×10^{-12}	8.35×10^{-12}	83.8 %
(747) Winchester	(209638)	1.79×10^{-12}	2.68×10^{-12}	2.78×10^{-12}	6.91×10^{-13}	24.9 %

Asteroid (110) Lydia has few available MDs, with only one obtained through the astrometric method by Goffin in 2014, who presented a mass value of $(6.988 \pm 0.503) \times 10^{-13}$. The value presented in table 4.2 confirms Goffin’s determination, with a slightly higher value but a lower uncertainty, marking the lowest uncertainty ever reported for this asteroid. The favourable conditions of the analysed encounter with asteroid (66778) 1999 TL221 played a significant role in our determination, as it occurred at a distance of 14810 km, just 0.58 times the target’s Hill’s radius. The $\Delta(\text{RMS})_{(110)}$ due to (110) Lydia is also particularly relevant, with a value of 0.43. It should be noted that this encounter occurred in 2010, i.e., before Goffin’s work and was indeed included in his investigation. The lower uncertainty in our result may be attributed to a larger number of available observations, spanning almost ten years more, which likely contributed to refining the orbit of the test asteroid. Other possible explanations involve the adoption of more accurate debiasing catalogues for astrometric observations, as already said, such as Gaia DR2. Figure 4.3 illustrates the computation and refinement process for Lydia’s mass through a sequence of iterations.

The MD proposed for asteroid (747) Winchester comes from a CE with (209638) 2005 BT10 which took place in November 2014, at a distance of 109004 km ≈ 3.42 Hill’s radii of (747). The encounter is close enough to generate a strong effect on the residuals with a $\Delta(\text{RMS})_{(747)} = 0.038$ and to impact positively on the relative uncertainty. This asteroid has numerous determinations using the ephemerides method with low uncertainty, but the result with lower uncertainty using the dynamical method is reported in 2011, before the encounter analysed here, by [15], with a relative uncertainty of 72%.

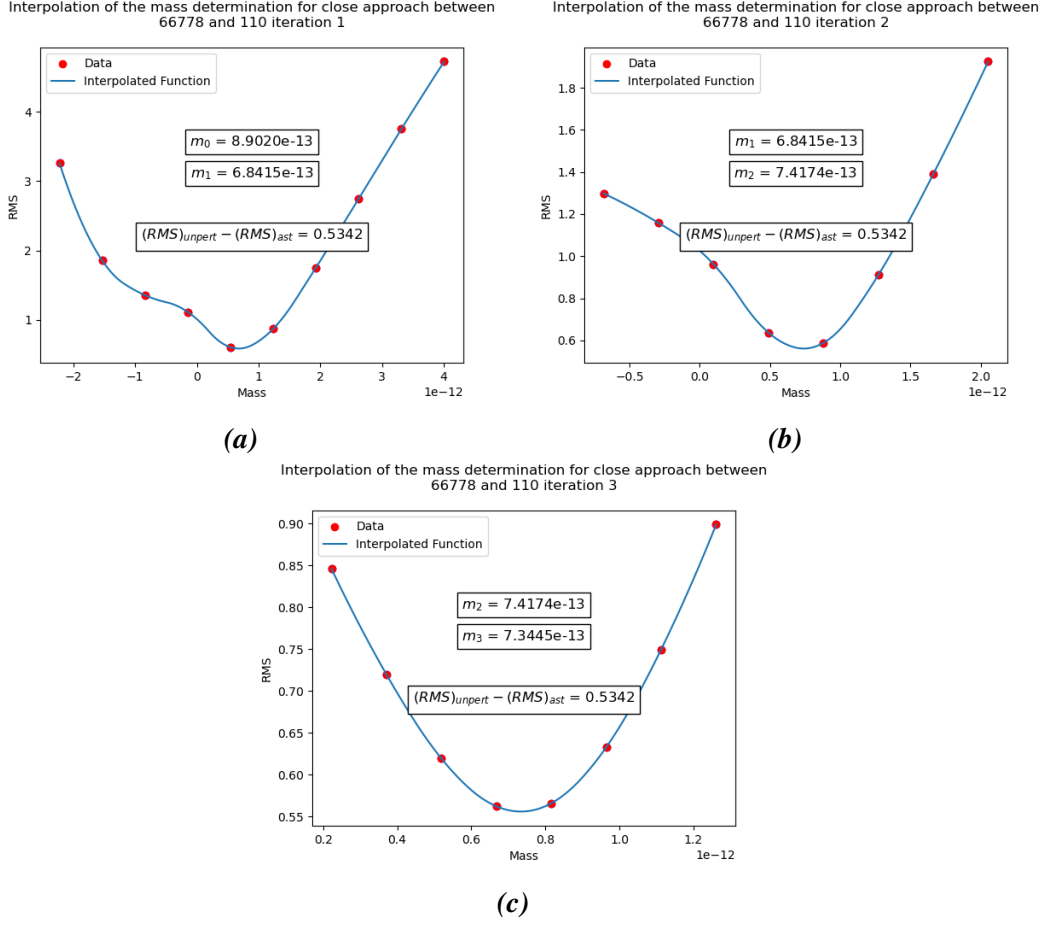


Figure 4.3: Iterative approach to converge to the mass of (110) Lydia following the encounter with (66778) 1999 TL221

The determination for asteroid (94) Aurora is the one that deviates the least from m_0 , although it remains significantly distant from the value reported by Goffin in 2014, which, at $(8.999 \pm 3.001) \times 10^{-13}$, represented the highest precision to date, with a relative uncertainty of 33%. It should be noticed, on the opposite side, the much higher uncertainties presented by [15], proving the difficulties in finding reliable encounters for this asteroid. The event we are considering here, indeed, is quite recent if compared to the previous publications, taking place in July 2019 with asteroid (427341) 2014 WN348 at a distance of 111 753 km, 2.36 times (94) Aurora's Hill's radius.

The value for (111) Ate carries the highest uncertainty, at 83.8%. This highlights the difficulties in achieving an efficient mass determination. Additionally, as shown in Table 4.2, there are significant differences between the initial mass m_0 , m_M and also m_{FO} , further reflecting these challenges. The previous best MD presented a mass value of $(1.362 \pm 1.251) \times 10^{-12}$, lower than that presented in this chapter, and with a relative uncertainty of 92%. It must be noticed that, for this MD, the encounter with asteroid

(232126) 2002 AE114 is very far, 2227512.29 km, more than 57 times the perturbing body's Hill's radius, with a $\Delta(\text{RMS})_{(111)}$ of 0.0036. This CE occurred in August 2006, which may correspond to the event analysed by [15]. Our improved precision is likely due to the availability of more observations or the implementation of an enhanced debiasing scheme.

The most accurate result for (111) Ate presented in [40] actually comes from a 2008 study [54]. However, this study reports the excessively high value of $(8.39 \pm 1.90) \times 10^{-11}$, and it is based on a distant encounter from 2003, where all observations were affected by low accuracy. Recalculating with updated observations yields $(1.927 \pm 2.81) \times 10^{-14}$. And, using only observations up to 2003 and applying current weights, the astrometric method converges to a negative value in *Find_Orb*.

Finally, the discrepancy between m_M and m_{FO} arises as, unlike in the other cases, the minimization method did not converge beyond the second iteration of refinement. Moreover, our result seems very sensitive to the application of outlier rejection on observations.

4.3.2 Asteroids with unpublished masses

Table 4.3 lists two cases for which a first dynamic MD is presented in literature to date. It has to be noted that for both of them the $\Delta(\text{RMS})_{ast}$ is pretty small, lower than 0.006, due to the distant nature of the CEs.

Table 4.3: *Masses of asteroids (91) Aegina and (32) Pomona determined for the first time through the deflection method. The initial masses m_0 , the masses obtained from the RMS minimisation m_M , the masses from Find_Orb m_{FO} , the uncertainties derived from Find_Orb and the relative uncertainties are shown. Masses are reported as a ratio of the Sun mass.*

Target	Test	m_0	m_M	m_{FO}	δm_{FO}	σ_{rel}
(32) Pomona	(113675)	3.74×10^{-13}	4.22×10^{-13}	5.06×10^{-13}	4.92×10^{-13}	97.2 %
(32) Pomona	(90813)	3.74×10^{-13}	-	4.12×10^{-12}	1.26×10^{-12}	30.6 %
(91) Aegina	(89100)	4.67×10^{-13}	1.36×10^{-12}	1.57×10^{-12}	1.48×10^{-12}	94.2 %

For (91) Aegina, only ephemerides-based mass determinations are available, with the value of $(3.404 \pm 1.654) \times 10^{-13}$ [19]. The m_0 value aligns closely at approximately 4.67×10^{-13} . The analysed encounter with asteroid (89100) 2001 TN194 occurred in 2014, at a minimum distance of 660618 km, equivalent to 31 times Aegina's Hill's radius, as calculated with m_0 . This encounter had a minor impact on the residuals, moving them by only 0.0047. Consequently, the mass determination comes with a significant un-

certainty, δm_{FO} , nearly equal to m_{FO} . Moreover, the mass derived in [39] falls within 1σ in our result.

The resulting mass from our calculations (visible in figure 4.4) and the one obtained in *Find_Orb* are not perfectly aligned, and are considerably higher than that obtained using the ephemerides method. However, this disparity is not necessarily unfavourable. Still, the uncertainty is too large to accept this as a definitive result. Nevertheless, this analysis demonstrates our capability to identify distant encounters that may prove valuable for MD purposes.

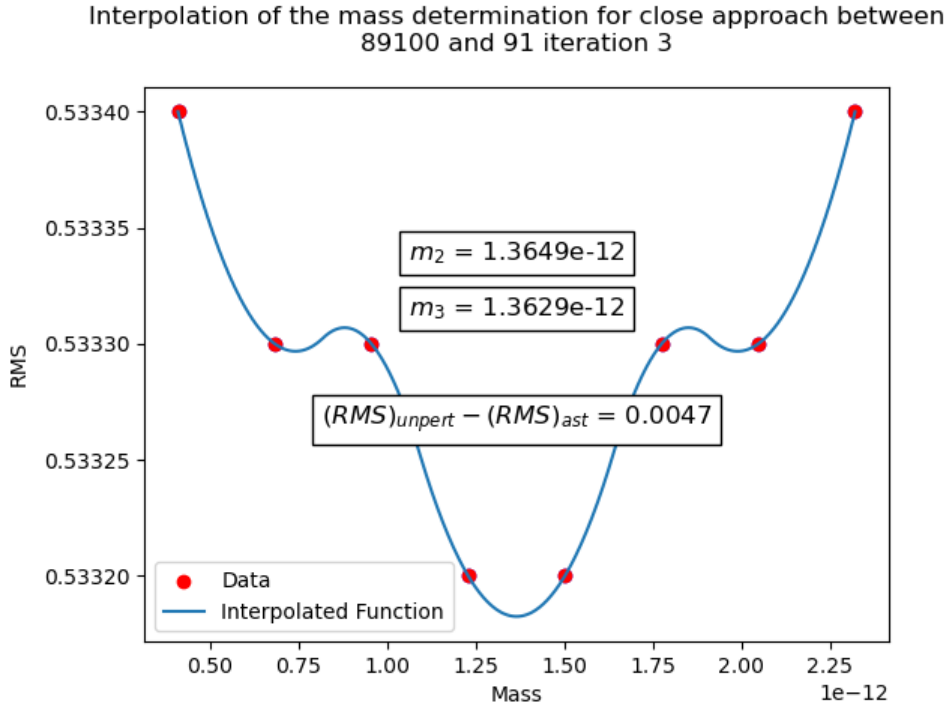


Figure 4.4: MD of (91) Aegina

In the case of asteroid (32) Pomona, we have found two CEs able to give results for MD purposes. For this asteroid, one determination using the ephemerides method is available in [19], marking a value of $(3.585 \pm 1.674) \times 10^{-13}$. This value is again close to m_0 . The first analysed CE is with asteroid (113675) 2002 TN102, and took place on December 26th 2015 at a distance of 430096 km, corresponding to 22.23 times (32) Hill's radii, and caused a $\Delta(\text{RMS})_{(32)} = 0.0058$. We get a value of $m_M = 4.220 \times 10^{-13}$, although the algorithm doesn't converge after the second refinement iteration. The value is however reasonably close to the previously available MDs, and also close to the value of m_{FO} . However, it must be noticed that this mass value shows a sensitivity to the outlier rejection. Indeed, no convergence of m_{FO} is achieved considering all the observations – probably the same happening to m_M – while this value can be obtained, with a 97%

uncertainty, applying a 3σ rejection on the total number of observations available.

The other analysed encounter, with asteroid (90813) 1995 ES7 occurred in June 2011 at a distance of 675 501 km, corresponding to 34.92 times the Hill's radius of the perturbing body. This encounter isn't good enough to get to the convergence of the minimisation method, but it seems strong enough, with its $\Delta(\text{RMS})_{(32)} = 0.0036$, to get to the convergence of m_{FO} . The overall uncertainty is acceptable, although too high with respect to the previous determinations and the expected results, and hence should be considered as an outlier and not accepted as a valid result for (32) Pomona MD.

4.4 Discussion of results

The dynamical properties of the studied CE, represented on the plot as (target, test), are reported in Figure 4.5. On the x axis, the normalised relative velocity perturbed-perturbing at encounter $U = v_P/V_C$ is represented, where v_P is the relative velocity at encounter and the normalisation term V_C is the circular velocity of the target body at encounter. On the y axis is reported the normalised minimum approach distance r_P/R_H , where r_P is the minimum distance target-test at encounter and R_H the Hill's radius of the perturbing body.

The plot shows how the CEs with higher $\Delta(\text{RMS})$, and also able to provide a better MD, are those with stronger dynamical characteristics [48]. In particular, very close encounters have high probability of impacting significantly on the residuals.

A way to interpret these results and understand how reliable the obtained values are, is to compute the bulk density and porosity from the mass values [5]. The bulk density is the density derived by the ratio between mass and volume. It differs from the grain density, which refers to the density of the material composing the asteroid. This difference arises because the material making up the asteroid is not entirely compact. Additionally, variations in asteroid's core and surface composition [55] contribute to voids, leading to porosity [5, 18]. Defined the porosity as:

$$\mathcal{P}_{\%} = 1 - \frac{\rho_B}{\rho_G}, \quad (4.2)$$

where ρ_B is the bulk density and ρ_G is the grain density. Only a positive porosity can be physically acceptable, which means that one should expect $\rho_B < \rho_G$, such that the voids present in the real structure make the asteroid less dense than it would be if compact.

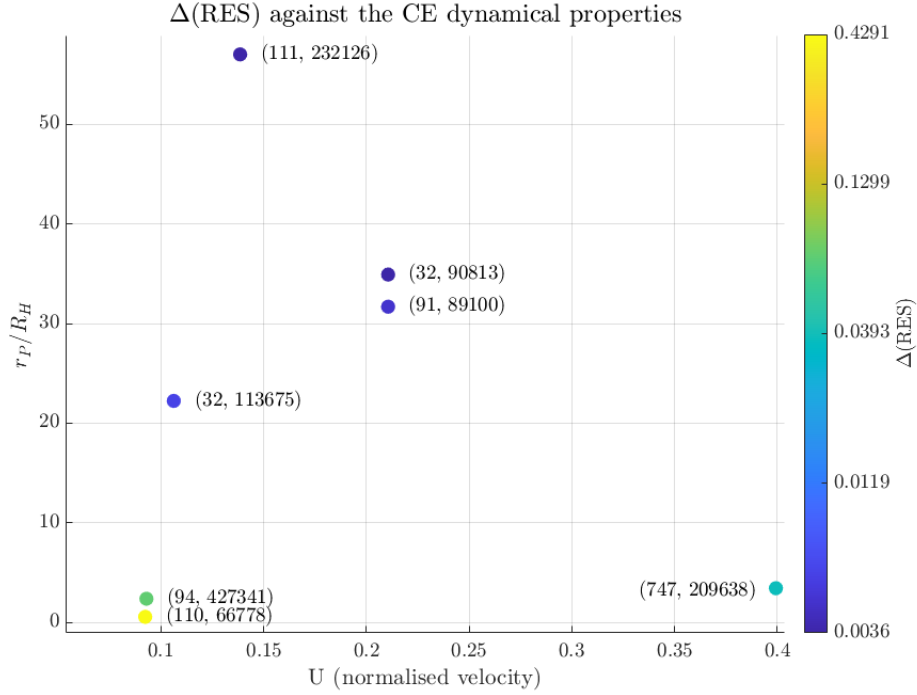


Figure 4.5: Influence of the dynamical properties of the CE, normalised relative velocity U and normalised encounter distance r_P/R_H on the $\Delta(\text{RMS})$. The pairs of (perturbing, perturbed) asteroids associated with the analysed encounters are indicated in parentheses.

This is generally true even if the void is in fact filled by other less heavy materials such as ice [18]. The data on volumes, bulk densities, assumed grain densities and derived porosities, with related uncertainties, is presented in Table 4.4.

The negative porosity values for asteroid (91) Aegina and (32) Pomona mean that we are computing much higher bulk densities with respect to the hypothesised ρ_G which suggest that the obtained masses are inconsistent with the taxonomic classification. Nevertheless, the uncertainty on $\mathcal{P}$ is also big enough to contemplate for positive porosities in the range $\mathcal{P} \pm \delta \mathcal{P}$, with the uncertainties coming from those in the mass, the diameter and the grain densities and taxonomic classes which are assumed as a pure value from previous publications [56, 5, 18, 58, 17]. This suggests that the value of $m \pm \delta m$ can be considered an acceptable guess for the state-of-the-art determination with the dynamic method, also in light of the high uncertainty. This is however typical in cases of first determinations.

Similarly, the MD for (111) Ate provides a bulk density much greater than the supposed grain one and also physically meaningless, suggesting a higher mass, but justified by also the high δm .

Table 4.4: Masses m , diameters D , bulk and grain densities, taxonomic classes and porosities of the investigated asteroids. The uncertainties of the physical values are stated as well. The densities have been computed assuming that the mass of the sun is 1.989×10^{30} kg. The references from where the data are collected are indicated.

Asteroid	m	δm	D (km)	δD (km)	ρ_B (g/cm ³)	$\delta \rho_B$ (g/cm ³)	Tax. Class ⁽¹⁾	$\rho_G^{(2)}$ (g/cm ³)	$\mathcal{P}_\%$	$\delta \mathcal{P}_\%$
(32) Pomona	5.06×10^{-13}	4.92×10^{-13}	80.0 ⁽³⁾	3.8	3.76	3.69	S	3.56	-5.50	103.63
(91) Aegina	1.57×10^{-12}	1.48×10^{-12}	106.9 ⁽³⁾	5.5	4.89	4.66	Ch	2.46	-98.71	189.58
(94) Aurora	3.28×10^{-12}	4.08×10^{-13}	186.35 ⁽⁴⁾	8.8	1.93	0.364	P	2.90	33.57	12.55
(110) Lydia	7.38×10^{-13}	1.68×10^{-14}	88.5 ⁽³⁾	4.6	4.05	0.383	M	5.50	26.45	6.97
(111) Ate	9.96×10^{-12}	8.35×10^{-12}	142.85 ⁽⁴⁾	5.94	13.0	11.0	Ch	2.46	-275.28	317.95
(747) Winchester	2.78×10^{-12}	6.91×10^{-13}	170.07 ⁽⁴⁾	6.7	2.15	0.591	C	2.46	12.73	24.02

References: (1) [56]; (2) [5]; (3) [18]; (4) [57].

(747) Winchester shows a positive porosity, but with a $\delta \mathcal{P} / \mathcal{P} > 100\%$. The computed porosity is much lower than that in [5], which however reported it too with a relative error greater than 100%. If our value will be confirmed by future determinations, it will suggest a monolithic compact structure for asteroid (747), however expected for even bigger bodies [5].

Better results, in terms of central value of porosity and relative uncertainty, are presented for (94) Aurora and (110) Lydia. The density for (94) Aurora is consistent with what presented by [18], and the porosity is not far from what presented in [5]. However, our value is higher, suggesting a more fractured structure, more likely to be the actual case, and a better precision.

4.5 Conclusions

In this chapter, an alternative algorithm for performing asteroid MD with the dynamic method has been presented, marking the contribution to knowledge introduced with this publication. The method is based on the use of OD residuals for filtering the CEs to be used for MD. The residuals can be seen as a signature of the outcome of the flyby, in terms of deflection angle, and can already provide an indication of the effectiveness of the encounter on MD. Moreover, with this strategy, the propagation of the orbit of all the necessary asteroids in an N-body simulation environment can be avoided, and the flyby detection and MD can be performed on top of the OD.

As per the novelty introduced in this chapter, we present the mass values of 6 asteroids found with our method. These results aim at improving the uncertainty of the MD for 4 asteroids, (94) Aurora, (110) Lydia, (111) Ate and (747) Winchester, and presenting the first MD with the astrometric method for (32) Pomona and (91) Aegina. The obtained mass values have been verified through the analysis of the derived bulk density and porosity, resulting in:

- The mass value presented for (111) Ate is big with respect to its taxonomic classification, yet obtained with a large uncertainty, 84%, which is however the best to date;
- The masses for (32) Pomona and (91) Aegina are presented for the first time with an uncertainty of 97% and 92% respectively;

- The masses for (94) Aurora, (110) Lydia and (747) Winchester have a lower uncertainty than previously published (12%, 22%, 25%), thanks also to the dynamically strong encounters detected. The mass values are also consistent with the expected porosities and can be considered as the best MD available to date.

However, the method still needs to be further refined to establish a precise definition of the $\Delta(\text{RMS})$ threshold for encounter detection. Currently set at 0.001, this threshold is intended to encompass as many dynamical events as possible. However, it has been observed that cases with a RMS difference of 0.003 do not guarantee convergence. Therefore, it is essential to conduct a statistical analysis to assess the significance of the $\Delta(\text{RMS})$ threshold.

In general, all the parameters used in the algorithm may require refinement, from the $\Delta(\text{RMS})$ thresholds to the criteria for rejecting high-residual orbits. These parameters can be fine-tuned in successive runs based on a statistical analysis of the results obtained in the initial iteration. Additionally, an outlier rejection approach could be considered to exclude poor astrometric measurements rather than discarding orbits with high residuals solely due to low-accuracy astrometry. However, since the same dataset of observations should be used for both the perturbed and unperturbed orbit determinations, outlier rejection should be applied to the perturbed solution first, with the same dataset then used for the unperturbed case. While this increases the algorithm's complexity, it enhances its analytical capabilities.

The algorithm could also be improved by relaxing the assumption that the masses in the BC-405 model are close to their actual values and exploring cases where $\Delta(\text{RMS}) < 0$, potentially leading to the inclusion of new scenarios and results. Finally, the current limitation of the method in performing simultaneous mass determinations could be addressed by modifying the algorithm or implementing an iterative refinement process for MD and OD when an asteroid has multiple detected encounters.

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Chapter 5

Effect of asteroid flybys on astrometric residuals, under strong dynamic conditions

Foreword

This Chapter investigates the effect of expanding the dynamical model through the inclusion of additional asteroids. This represents the second dynamical improvement proposed in the thesis to enhance asteroid OD. Numerical simulations of asteroid–asteroid flybys are carried out, with the definition of initial and boundary conditions supported by the outcomes of Ch.3 as well as by other supporting analyses presented in this chapter. Synthetic observations generated from these simulations are then processed through the OD software to evaluate the resulting $\Delta(\text{RMS})$ for a typical asteroid–asteroid flyby. These values can then be compared with the statistical analysis in Sec.1.4 to assess whether an extended dynamical model is capable of correcting the broken orbits.

Abstract

This paper examines asteroid-asteroid flybys and their impact on residuals of orbit determination (OD), which is the process of fitting asteroid's astrometric observations to get its orbital trajectory. When an asteroid undergoes a close encounter with another, their mutual gravitational interaction causes a deflection in their trajectories. In traditional OD, a dynamical model is applied based on a pre-determined set of dynamical perturbations, which should include the most significant disturbing forces acting on the asteroid during the observational period. If considered relevant, flybys among asteroids, along with planetary encounters and other dynamical perturbations, are modelled and accounted for in the dynamical model. In this work, the effect of an unmodeled perturbing asteroid is estimated by comparing OD solutions with and without an additional hypothetical perturbing body. The flyby is modelled and simulated to maximise its impact on residuals: the encounter geometry is optimised for maximum deflection, as based on previous findings, and an analysis is conducted to determine the optimal distribution of observations. The trajectories of the two asteroids are numerically propagated, and synthetic observations are generated. These observations are processed using the OD software Find_Orb to compute root mean square (RMS) residuals of the solutions. Results show that, given current astrometric accuracy, a flyby at the edge of the perturbing body's sphere of influence can produce only a very small change in RMS residuals, on the order of 0.01.

5.1 Introduction

Asteroid orbit determination (OD) is the process of fitting the astrometric observations of an asteroid, which provide the object's position at a certain epoch, to obtain the asteroid's osculating orbital elements, through a least-squares method [2]. OD accounts for asteroid flybys, and other relevant dynamical perturbations affecting the asteroid's motion during the observational period, by applying an appropriate dynamic model. By excluding part of the dynamic model in the fitting, one may see a difference in the observed - computed (O-C) root mean square (RMS) residuals of the solution, which quantifies the discrepancy between the predicted and observed positions of the asteroid. If the effects are accounted for correctly in the dynamical model (e.g., the masses of the perturbing bodies are correct), then the change in residuals between the unperturbed and the perturbed solutions

(unperturbed - perturbed) is $\Delta(\text{RMS}) > 0$ [3]. Thus, in case of a flyby, by excluding the perturbing asteroid from the dynamic model, one can see the impact of the flyby itself on the $\Delta(\text{RMS})$.

The objective of this paper is to estimate, in terms of maximum $\Delta(\text{RMS})$, the perturbation that the smallest asteroid in the BC-405 dynamic model [4] has on the OD of a test asteroid. The BC-405 dynamic model comprises the 300 asteroids with greater gravitational influence in the solar system, and is the model implemented by the software here used for OD, *Find_Orb*¹. Other models exist, including more asteroids, such as the DE-421 model [5] comprising 343 asteroids, or less such as the sixteen or just the three more massive. The choice of model depends on its expected impact on the results and the available computational resources for computing the asteroids' ephemerides.

To fulfil the objective of the paper, numerical simulations of asteroid - asteroid close encounters (CE) are performed and synthetic observations are generated, as outlined in Section 5.2.

To evaluate the maximum $\Delta(\text{RMS})$, the flybys to be simulated have to represent a *worst - case* scenario, a condition in which the flyby produces the maximum observable effect on the residuals. This is defined by dynamic and geometric conditions that maximise deflection while remaining representative of a typical asteroid-asteroid flyby. These initial conditions are outlined in Section 5.3 based on results from [1].

Also the distribution of observations have a role in maximising the $\Delta(\text{RMS})$ output, and their optimal distribution is discussed in Section 5.4. Other boundary conditions useful for the generation of synthetic observations, such as typical number of observations or observation time span, are outlined in Section 5.5.

The results and the influence of the boundary conditions are finally discussed in Sections 5.6 and 5.7 respectively, by analysing the obtained $\Delta(\text{RMS})$ from *Find_Orb* and the relative error on the mass determination (MD) made possible by the flyby occurrence. In fact, as described in Ch. 4, when a flyby occurs, it is possible to estimate the mass of the perturbing body upon the deflection experienced by another asteroid.

By analysing these results and how they relate to the boundary and initial conditions, this paper can set the basis for the development of a flyby - $\Delta(\text{RMS})$ map, which could be used to infer dynamical events from OD residual.

¹https://www.projectpluto.com/find_orb.htm

5.2 Numerical propagation and synthetic observations

A numerical simulation is used to study and evaluate the outcome of CEs among asteroids. The trajectories of an asteroid of mass M and another of mass $m \ll M$ are propagated, and a flyby among the two is set to take place at the half of the propagation arc. The trajectory of the asteroid of mass m is then sampled in n points to get the N synthetic observations that will feed the OD software.

- *Propagation model:* the orbits of the two asteroids are propagated in the barycentric ecliptic reference frame of the Solar System using the full set of equations of motion, integrated with an explicit Runge-Kutta method order 8 with adaptive time-step control solver. The perturbing effects of third bodies (namely, the planets and the three most massive asteroids, Ceres, Pallas and Vesta) can be included numerically. However, in most simulations, these effects have been neglected to reduce computational time, as their inclusion has been found to have no significant impact on the investigated results.
- *CE and orbit definition:* the position R_P and velocity V_P of the perturbing body are specified in the barycentric ecliptic frame at the epoch of the CE, thereby defining the perturbing body's trajectory. The flyby is then described using the three-dimensional Broucke reference frame [1, 6, 7] (Figure 5.1), which provides the angles α , β , γ , along with the relative position and velocity at encounter, r_P and v_P . This fully defines the state vectors of the two bodies at encounter, allowing for the description of the flyby and the propagation of the two asteroids orbits. These are propagated both forward and backward in time for a predefined duration from the moment of the encounter.
- *Synthetic observations generation:* the trajectory of the perturbed body is then sampled at N points, representing the synthetic observations to be generated. These observations are produced from any desired ground-based station, identified by its Minor Planet Center (MPC) code², while accounting for a first-order light-time correction. Additionally, artificial noise can be introduced to simulate a more realistic observational scenario, representing the intrinsic astrometric error in the measurements [8].

The synthetic observations are formatted into an Astrometry Data Exchange Standard (ADES) observation file [9], which includes the observer's station code and

²<https://minorplanetcenter.net/iau/lists/ObsCodesF.html>

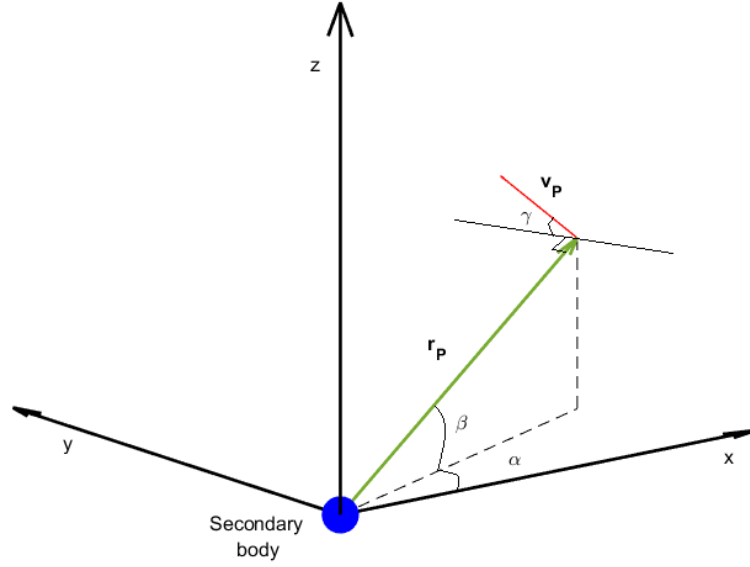


Figure 5.1: Geometric representation of the planetocentric frame for CE representation.

the observables (RA and Dec) with a precision of up to the seventh decimal place. A function is implemented to compute the apparent magnitude of the observed asteroid from its relative position, following the H-G magnitude system [10]. The calculated magnitude is also included in the file.

5.2.1 Orbit determination

OD fits the asteroid's observations to provide the osculating orbital elements through a least-squares method relying on a differential corrections algorithm [2, 11]. A dynamic model, which accounts for the relevant gravitational and non-gravitational perturbations, is applied to the fit, and observations are associated with a weight, to account for their expected uncertainties [2, 3, 12, 13]. The weights are assigned to the observations based on their expected uncertainty. In the case of this synthetic data generation, all the observations are produced with the same noise, hence a uniform weight of 0.36 is applied to the observations in the OD phase. The nominal solution in terms of orbital parameters or determined mass, and the (O-C) residuals of each observations, do not change depending on the weight value applied. However, the covariance, and the associated uncertainties and the weighted RMS residuals, depend on a multiplication factor equal to the assigned weight [2].

After the dynamic simulation and the synthetic data production, the actual OD to

Table 5.1: Typical ranges for parameters characterising a flyby among asteroids, as derived from [1]. r_P and v_P are the relative position and velocity at minimum distance respectively. They are indicated with respect to the Hill's radius of the perturbing body (R_H) and the circular velocity at the perturbing body's distance from the Sun at the moment of encounter ($V_{P,circ}$).

α (deg)	$[0, 360]$
β (deg)	$[-90, 90]$
γ (deg)	$[0, 360]$
$r_P (\times R_H)$	$\approx > 1.2$
$v_P (\times V_{P,circ})$	$[0.05 - 0.25]$

generate the results that will be analysed is performed with the software *Find_Orb*: the solution in terms of orbital elements, MD and RMS residuals is thus obtained. By toggling on and off the perturbation of the synthetically generated perturbing asteroid in the solution³, it is possible to assess the flyby's influence on the OD residuals, allowing for a direct quantification of the simulated flyby's effect on the OD and MD.

5.3 Strongest dynamic flyby conditions

The conditions that lead to the strongest dynamical deflection can be inferred from a previous study on asteroid flybys [1], which explored these events in detail and suggested a dataset for setting up simulations. A summary of the dataset is provided in Table 5.1. Building on that work, this paper focuses on how such close encounters affect RMS residuals and, in particular, which configurations produce the largest deviations. The flybys are described through a set of geometric and dynamical parameters, including the relative velocity at approach, the minimum distance, and the angles that define the bodies' positions at closest encounter.

In terms of geometric angles defined in Figure 5.1, it is set:

- $\alpha = \pi/2$
- $\beta = 0$

³The influence of the artificially created perturbing asteroid is incorporated by modifying the BC-405 asteroid ephemerides files used by *Find_Orb*, by replacing the ephemerides of asteroid (433) Eros. This has been chosen because, as a near-Earth asteroid (NEA), it has no significant impact on the orbits of the synthetic Main Belt Asteroids (MBAs) considered in this study.

- $\gamma = 0$

This configuration results in the perturbed body's trajectory being nearly parallel to that of the perturbing body, with the consequence of having two coplanar orbits (perturbed - perturbing asteroids). Such an arrangement maximises the dynamical effects of the flyby in terms of changes in the semi-major axis (a) [1, 7].

The velocity at closest encounter is set to $v_P = 0.05 \times V_{P,circ}$, where $V_{P,circ} = \sqrt{\mu_{SUN}/R_P}$ is the circular velocity at the perturbing body's distance from the Sun. The value corresponds to the lowest velocity found in the statistical dataset [1] but remains well above the capture velocity for the cases of interest. Lowest velocities guarantee more deflections, and, in first approximation [1] this can also be seen by the analytical expressions for the flyby Δv according to the patched conics (PC) theory, where $\Delta v \propto 1/v_P$ [14].

The closest approach distance is set to $r_P = 1.2 \times R_{SOI}$, where R_{SOI} is the radius of the sphere of influence (SOI) of the perturbing asteroid, defined as its Hill radius [14]:

$$R_H = \sqrt[3]{\frac{\mu_P}{3}} \cdot R_P \quad (5.1)$$

where μ_P is the ratio between the mass of the perturbing asteroid M and the mass of the Sun.

Again, following the PC approximation, the Δv and the deflection during a flyby are $\propto 1/r_P$, which means that higher deflections are obtained with closer approach distances. The choice of $1.2 \times R_{SOI}$ has been made as it is close enough to the SOI of the perturbing body, but still within the range of typical asteroid-asteroid encounters. As shown in [1], encounters within the SOI itself are rare, whereas those within $1 - 2 \times R_{SOI}$, while still uncommon, occur more frequently. In Section 5.7, this distance will be varied over a broader range to better assess its influence on $\Delta(\text{RMS})$.

5.4 Best conditions for observation

In this section, several observational factors influencing the $\Delta(\text{RMS})$ will be analysed. These have been identified by analysing typical asteroid observation datasets:

1. Total number of observations.

2. Number of observations before and after the CE.
3. Observing time before/after the CE.
4. Distribution of observations around the CE.

The influence of these observing conditions on finding an optimal observation distribution is tested by generating synthetic observation files describing the same flyby while varying the observation distribution. To assess the impact and determine the best strategy, the relative precision of the MD obtained for the perturbing body from the deflection after the flyby (Ch. 4) is used as a control parameter: the optimal distribution will be assumed to be the one maximising the MD precision. This approach is preferred over comparing $\Delta(\text{RMS})$, as $\Delta(\text{RMS})$ is usually close to zero in the cases of interest or shows only minor variations. Moreover, the link between MD from astrometry and $\Delta(\text{RMS})$ has been established in previous works [15] and in Ch. 4.

By evaluating the precision of the MD, one can also use the results of this section to develop a strategy for future observation campaigns aimed at observing a flyby to achieve the best results in terms of MD from astrometry. Although some of the results presented in this section may not be feasible due to real-life constraints, they can still be assessed numerically.

The parameters for these simulations are based on those introduced in Section 5.3. The mass of the perturbing body, m_P , expressed in solar masses, and the CE distance are specified in the legends of the reference figures. Unless stated otherwise, the number of observations is set to 800, with a propagation time span of 10 years. The orbital elements of the perturbing body are listed in Table 5.2, along with its state vectors in terms of position and velocity at the time of encounter, which is set to occur on 15 September 2024 at 00:00 UTC. From these values, the relative velocity at encounter is determined to be 1.04 km s^{-1} . The geometric angles are defined as $\alpha = \pi/2$, $\beta = 0$, $\gamma = 0$.

Table 5.2: *Orbital parameters of the perturbing body for the simulations for assessing the best observation scenario. Semimajor axis a_P , eccentricity e_P , inclination i_P are reported together with R_P and V_P .*

a_P (au)	e_P	i_P (deg)	R_P (au)	V_P (km/s)
1.89	0.073	17.9	$[-1.53, 1.30, 0.03]$	$[-11.8, -15.3, -6.2]$

5.4.1 Total number of observations

OD improves as more observations constrain the trajectory, and short-arc OD remains highly uncertain due to limited data. A possible concern is whether too many observations could lead to overfitting [2, 16].

The analysis presented here considers two different cases, with varying the mass of the perturbing body and the minimum approach distance. As shown in Figure 5.2, increasing the number of observations, up to a capped maximum of 190 per year, improves the MD precision and is preferable for accurately capturing flyby effects. The relative error, in these cases, reaches a plateau beyond approximately 60 observations per year, indicating that additional data points provide no further benefit. This finding is particularly important in real-world scenarios, where technical constraints often limit the number of possible observations.

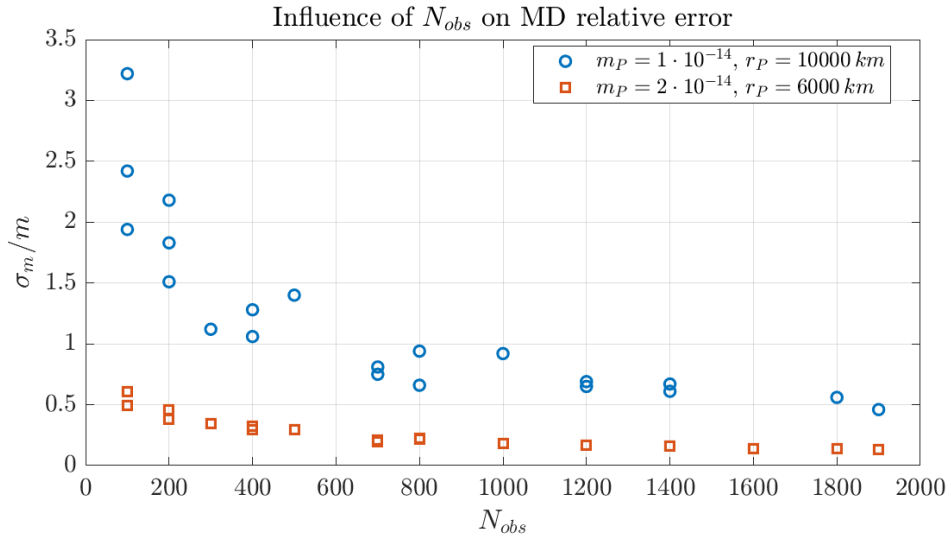


Figure 5.2: Influence of the total number of observations on the relative uncertainty of the MD σ_m/m .

5.4.2 Number of observations before and after the CE

This study evaluates how the distribution of number of observations before and after a CE affects MD accuracy. The propagation time before and after the CE is kept equal while systematically varying the observation distribution. Figure 5.3 shows the MD relative error as a function of the ratio of post-CE to pre-CE observations.

Four different cases are considered, varying the mass of the perturbing body and the

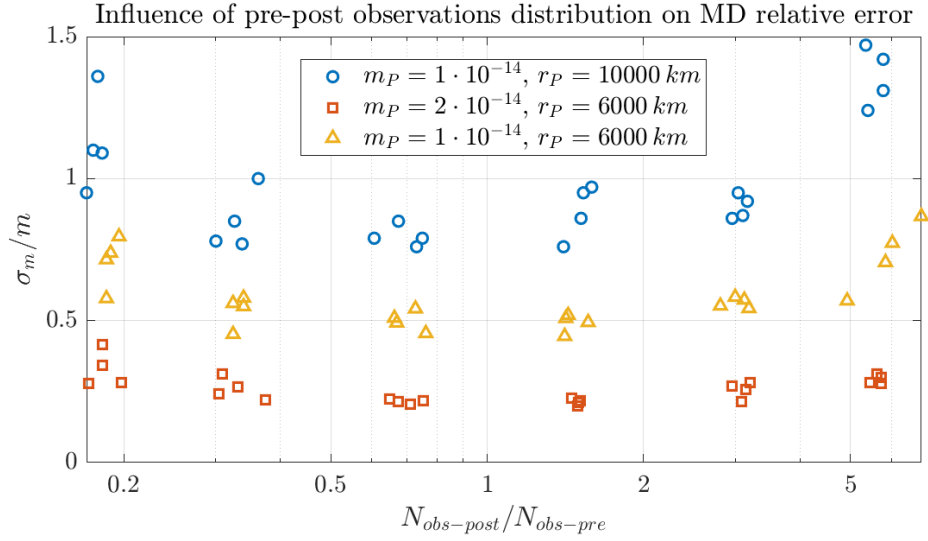


Figure 5.3: Influence of the distribution of observations before or after the CE on the relative uncertainty of the MD σ_m/m .

CE distance. In general, the scatter plot suggests that a symmetric distribution of observations is the most effective. However, for flybys characterised by higher perturbing masses and shorter encounter distances the differences become less significant, reducing the necessity for optimising the observation distribution.

5.4.3 Observing time before/after the CE

Keeping the optimal symmetric distribution of observations before and after the CE, this section investigates the effect of the duration, in time, of the observation arc before and after the flyby. The study wants to understand the best placement of the flyby within the whole observation arc.

Figure 5.4 shows the MD relative error as a function of the ratio of years of observations after CE to years of observations before. The total observation period is 10 years; the number of observations is varied (three cases use 194 observations and a fourth case using 270), to assess the role of the density of observations. Also, the mass of the perturbing body and the minimum approach distance are changed to explore different flyby conditions.

A symmetric time distribution seems preferable, with a similar mitigation effect on flybys involving higher perturbing masses and shorter distances, as observed previously.

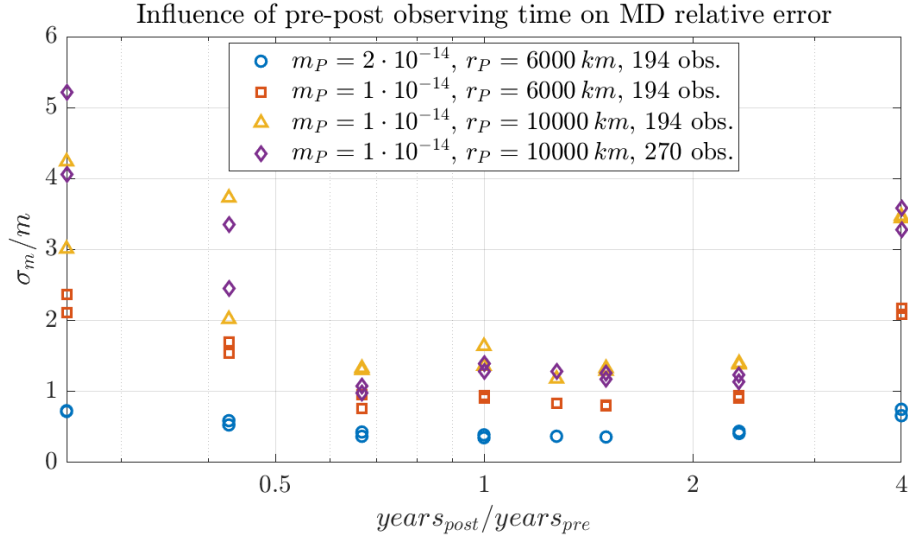


Figure 5.4: Influence of the amount of observing time before or after the CE on the relative uncertainty of the MD σ_m/m .

5.4.4 Distribution of observations around the CE

Visibility constraints often prevent observations from being taken exactly at the moment of the CE. The first available observation, before or after the encounter, may be delayed by up to half a synodic period with Earth. To assess this effect, a time window around the CE is defined where no observations are allowed, and its impact is analysed. Figure 5.5 shows the relative error on MD as a function of the time span (in days) between the CE and the first available observation.

In this study, observations are artificially enforced even when they would not be physically feasible. Since *Find_Orb* integrates them numerically without considering their observational feasibility, their impact can be examined in an idealised setting.

The plot shows that having observations closely spaced around the CE significantly improves the characterisation of the flyby, reducing the error in MD. As the gap between the CE and the first available observation increases beyond a certain threshold, the MD accuracy degrades. Moreover, different flyby characteristics influence both the overall relative error and the threshold beyond which accuracy declines. These results suggest that, if future campaigns for MD are planned, a well-designed strategy should aim to specifically target close-encounter events with well-timed observations.

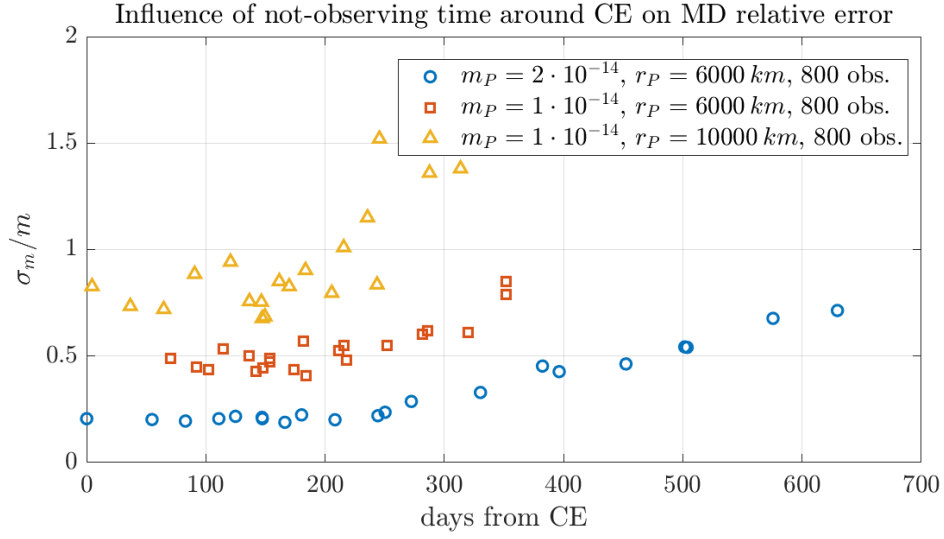


Figure 5.5: Influence of the observing time gap before and after the CE on the relative uncertainty of the MD σ_m/m .

5.5 Setting boundary conditions for simulation

The boundary conditions for these simulations include parameters affecting astrometric observation quality and OD results, excluding the dynamical conditions discussed in Section 5.3 and the observation distribution in the ADES file (Section 5.4). These parameters include the observation timespan, the total number of observations (already analysed in Section 5.4, where the conclusion was that more observations and a longer timespan are generally beneficial, though a practical limit must be considered), and the intrinsic ground residual $(RMS)_0$. The latter represents the fundamental observational uncertainty that cannot be eliminated unless significant improvements in measurement precision are achieved. In the previous Section 5.4, the assumption was of a zero ground residual, an unrealistic condition since real-world observations always involve a degree of uncertainty. For a more realistic scenario, reasonable values for these parameters must be established to define a typical case, with a broader range explored later (Section 5.7) to assess their influence.

For the observation timespan, the chosen baseline is 20 years. This choice reflects the fact that, after two decades of observations—typically covering about 20 oppositions for a main-belt asteroid—an orbit can be considered well determined. This is twice the 10-year threshold required for an asteroid to be numbered and is short enough to be relevant for real-data verification and for planning future observation campaigns for mass determination.

The number of observations within this timespan must also be constrained to a feasible range. A practical way to estimate this is by analysing observational statistics from the NEOCC database. Examining all available data files, we find an average observation rate of 13.96 per year, with a median of 8.37. However, this dataset includes non-numbered asteroids, lost objects, and bodies with poorly determined orbits. Restricting the analysis to numbered asteroids only, the average rises to 19.35, with a median of 13.48. Based on this, we approximate the number of observations as $19.35 \times \text{years of observations}$ for a best-case scenario.

For the ground residual, a similar statistical approach is followed, considering the average and median values for numbered asteroids. These are 0.564 and 0.561, respectively, which we aim to use as a reference, although numerical oscillations may introduce minor variations in the second decimal place. This measure provides insight into the overall precision of OD, which, in the case of simulated or synthetic data, is entirely dependent on the artificial noise added to observations. Consequently, it directly impacts OD [3, 13] and MD precision. Lower residual values improve the description of the flyby effect. A typical ground residual of approximately 0.55 is adopted for these investigations, however, also a lowest practical value of $(RMS)_0$ is to be set. This value is set as the lowest RMS residual shown by a numbered MBA in the NEOCC database, which is $(RMS)_0 \approx 0.2$, for asteroids (729) and (643304), describing an *ideal best-case scenario*.

As a mass for these simulations, $2.255 \times 10^{-14} \times M_S$ is selected, which represents, in Sun masses M_S , the mass of (1036) Ganymed, the smallest mass presented in the BC-405 model, if NEA (433) Eros is excluded. This will provide the simulation with the worst case possible for a mass excluded by the BC-405 model.

5.6 Results

The simulations in terms of orbit propagation and synthetic observations generation can be done on top of the previously set parameters to present the resulting $\Delta(RMS)$. To ensure consistency with Section 5.4, the adopted time distribution is symmetric, with an equal number of observations before and after the CE. These observations are taken from various topocentric observatories worldwide to maximise coverage. In accordance with Section 5.4.4, the flyby is set to occur at opposition, with a maximum phase angle with Earth of at most 10 degrees, ensuring that the event is well observed. Additionally, to guarantee that the observations could realistically be obtained, it is imposed an elongation constraint above 160 degrees and is required that the asteroid remains above the horizon.

The results for the typical best-case scenario, with $(RMS)_0 \approx 0.55$, are presented in Table 5.3, along with the initial and boundary conditions used for the propagation and data generation. The resulting $\Delta(RMS) = 0.01$ indicates that a flyby involving an asteroid with a mass smaller than those of the 300 most massive asteroids does not significantly alter the OD residuals, even under the optimal dynamical conditions and observations distribution.

Consequently, such flyby effects remain undetectable in the observational data. As an outcome, this result implies that extending the dynamical model employed by *Find_Orb* beyond the 300 most massive asteroids to include lower-mass bodies would not yield significant improvements in OD, given the observational accuracy considered in this case. The MD obtained in this case is $(2.191 \pm 0.445) \times 10^{-14}$, yielding a relative uncertainty of 20.3%, which still represents a reasonable level of precision.

To assess the impact of improved astrometric precision, the OD analysis is repeated using $(RMS)_0 \approx 0.20$. The results for this case are presented in Table 5.4.

The MD in this scenario is $(2.159 \pm 0.384) \times 10^{-14}$, with a relative uncertainty of 17.7%. The change in $\Delta(RMS) \approx 0.05$ is a more noticeable value that, albeit minimal, could be beneficial for improving OD and might be detectable by an algorithm similar to that proposed in Ch. 4. However, achieving this level of astrometric precision for all asteroid observations remains challenging at the current state of the art. Nonetheless, with continued advancements and improvements [17], such precision could become observable, making the extension of the dynamical model to include additional asteroids, such as those in the DE-421 model, a possible approach.

5.7 Effect of the boundary conditions

To further discuss the obtained results and analyse the variation of $\Delta(RMS)$ with the influencing factors identified so far, the response of $\Delta(RMS)$ and MD to factors such as the

Table 5.3: Parameters for the simulation of the typical best case scenario and result of the OD in terms of $\Delta(RMS)$.

Flyby Scenario					Boundary Conditions			Pert. params		Result $\Delta(RMS)$
α (deg)	β (deg)	γ (deg)	r_P (km)	v_P (km/s)	$(RMS)_0$	Years	Obs.	m_P M_{SUN}	a_P (au)	
90	0	0	9721.3	0.89	0.53	20	386	2.255×10^{-14}	2.82	0.01

Table 5.4: Parameters for the simulation of the "best-best" case scenario and result of the OD in terms of $\Delta(RMS)$.

Flyby Scenario					Boundary Conditions			Pert. params		Result $\Delta(RMS)$
α (deg)	β (deg)	γ (deg)	r_P (km)	v_P (km/s)	$(RMS)_0$	Years	Obs.	m_P M_{SUN}	a_P (au)	
90	0	0	9721.3	0.89	0.20	20	386	2.255×10^{-14}	2.82	0.05

following is studied:

- $(RMS)_0$.
- Duration of observation period.
- r_P (relative position at encounter).
- a_P (semimajor axis of the perturbing body).

The study of the variations of $\Delta(RMS)$ and MD relative error against these parameters is reported in Figure 5.6.

In terms of sensitivity to the boundary conditions, lower $(RMS)_0$ values and longer observation periods lead to improved results, specifically higher $\Delta(RMS)$ and lower relative uncertainties in MD. Both effects indicate that a higher-quality fit lead to a more accurate description of the entire trajectory, including the flyby, as anticipated and previously discussed.

The effect of $(RMS)_0$ (Figure 5.6a), as previously discussed in Section 5.5 and evident in the results presented in Tables 5.3 and 5.4, shows that observations with lower astrometric uncertainty produce better results in capturing the effect of a flyby, under identical boundary and dynamical conditions—for these results, changing only the noise introduced in the synthetic observations. This indicates that improving the quality of observations—an ongoing process (see Chapter 2 [3])—enhances the description of the flyby and its outcome on the asteroid MD, even in cases characterised by weaker dynamical interactions.

For the duration of the observation arc, with $(RMS)_0 \approx 0.52$ (Figure 5.6c), an observation time-span of 40 years leads to a $\Delta(RMS)$ comparable to the value of 0.05 reported in Table 5.4. This refers to a flyby that results in a high deflection, which enables good precision in MD, with relative errors ranging between 60% and 10%. Overall, the results related to the number of years of observations, shown in both plots 5.6b and 5.6c, reflect



Figure 5.6: Effect of variation of boundary conditions on $\Delta(RMS)$ and on the relative uncertainty of the MD.

a similar outcome to that observed in Figure 5.2. A longer time-span allows for extended monitoring of the asteroid’s trajectory, thereby increasing the length of the observational arc (despite maintaining, in these cases, the same number of observations), which in turn has a positive effect on the quality of the fit. However, while longer observation periods are clearly beneficial, this parameter is ultimately constrained by the physical availability of time.

Regarding the dynamical parameters of the flyby, the effect of r_P (plots 5.6d and 5.6e) aligns with expectations: smaller distances result in increased $\Delta(\text{RMS})$. As noted in Section 5.3, shorter distances lead to flybys with greater deflections, which, unsurprisingly, affect both $\Delta(\text{RMS})$ and the precision of MD. Similarly, although not explicitly shown here, a comparable trend is expected for variations in v_P and m_P , given their well-established influence on flyby dynamics [1]. These results reinforce the conclusion that, in general, flybys occurring under dynamical conditions favourable to high deflection, whether due to mass or geometry, are more likely to provide accurate outcomes in MD.

Finally, the impact of the orbital elements of the perturbing body and, indirectly, of the perturbed body, is examined. More specifically, this analysis assesses how the distance of these bodies from Earth, where observations are conducted, affects the results. Figure 5.6f illustrates the influence of a_P , semimajor axis of the perturbing body. The analysis is constrained within the typical orbital range of a MBA and extended to include values representing trans-Jupiter objects. Jupiter’s orbit, and consequently its Trojan asteroids, have been excluded from the analysis, as the propagation is performed in an unperturbed framework, making such cases less representative of reality. The results indicate that as the perturbing body’s distance from Earth increases, $\Delta(\text{RMS})$ decreases, for the same $(\text{RMS})_0$. This trend is likely driven by the relative velocity between the observed asteroid and Earth: closer bodies have shorter synodic periods, increasing the probability of observation. Additionally, faster-moving objects (i.e., those at smaller heliocentric distances) exhibit greater positional variation within a given observational window, making their motion more pronounced. This, in turn, enhances the information available for least-squares integration in OD.

5.8 Conclusions

This paper examined the effect of a flyby, geometrically and dynamically characterised to produce a high deflection, on the residual change $\Delta(\text{RMS})$ in asteroid OD. The influence of initial conditions, observation parameters, and boundary conditions on $\Delta(\text{RMS})$

has been explored. Using a numerical approach to propagate asteroid orbits and simulate asteroid-asteroid CEs, the conditions that best characterise a CE for MD purposes have been identified. These include a relatively high number of observations, a balanced distribution before and after the CE, and observations taken as close as possible to the CE to improve its characterisation.

After establishing the optimal conditions for maximising the dynamical outcome of a flyby [1] and ensuring that observations effectively capture the CE, OD outcomes have been evaluated in terms of $\Delta(\text{RMS})$ using numerical propagation, a synthetic observation generator, and the *Find_Orb* software for processing the generated observations.

For a typical scenario, where the initial residual level is $(\text{RMS})_0 \approx 0.53$, the resulting $\Delta(\text{RMS})$ due to the flyby, and, more generally, the gravitational influence, does not exceed 0.01. This value is nearly imperceptible within the noise expected in real observational scenarios.

A more favourable outcome is observed when $(\text{RMS})_0$ is reduced. Lowering this value enhances the overall accuracy of the observations and the astrometric data upon which OD relies. The lowest feasible limit is the accuracy achieved by the MBA with lowest RMS residuals in the NEOCC database: $(\text{RMS})_0 \approx 0.2$. In this case, the resulting $\Delta(\text{RMS}) = 0.05$. This suggests that significant improvements in observational precision are necessary to achieve better results in this context.

Despite the limitations in $\Delta(\text{RMS})$ detection, the MD remains reliable in both cases. This is because the CE under consideration causes a dynamical deflection high enough to provide a robust estimate of the astrometric MD.

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Chapter 6

Conclusions and discussion

The need for further developments in OD of asteroids is made evident by the statistical analysis of the residuals, where a relevant number of poorly fitted orbits with high residuals can be observed. Patterns such as *broken orbits* justify efforts to find a dynamical explanation for these phenomena. This thesis examines ways to improve the fitting of asteroid OD, both through non-dynamical (Ch. 2) and dynamical analyses (Ch. 3, 4, 5), with a particular focus on the latter. Specifically, contributions are made to the study of the effect of asteroid flybys on OD residuals.

Figure 6.1 illustrates the overall research approach adopted in this thesis to address the aim and individual objectives introduced in Sec. 1.2 (pg. 8). A summary of the research flow and its outcomes is provided below.

The dataset considered is ESA’s NEOCC asteroid orbital database. The orbits and associated residuals in the dataset have been analysed in Section 1.4, providing a first systematic characterisation of this database, in terms of residual range and statistical characterisation. The average residual has been computed, and a threshold for acceptable residuals has been set, responding to objective (1A).

Orbits with residuals above the threshold require further analyses. In Section 1.4.1, these orbits are pruned to exclude those affected by biases in observation quality or strategy, as they are unlikely to benefit from a dynamical refinement. The applied filters identified 2626 asteroids exceeding the quality threshold, where the cause cannot be directly attributed to observation quality, aligning with objective (1B).

This analysis in Section 1.4 provides a first systematic characterisation of the NEOCC

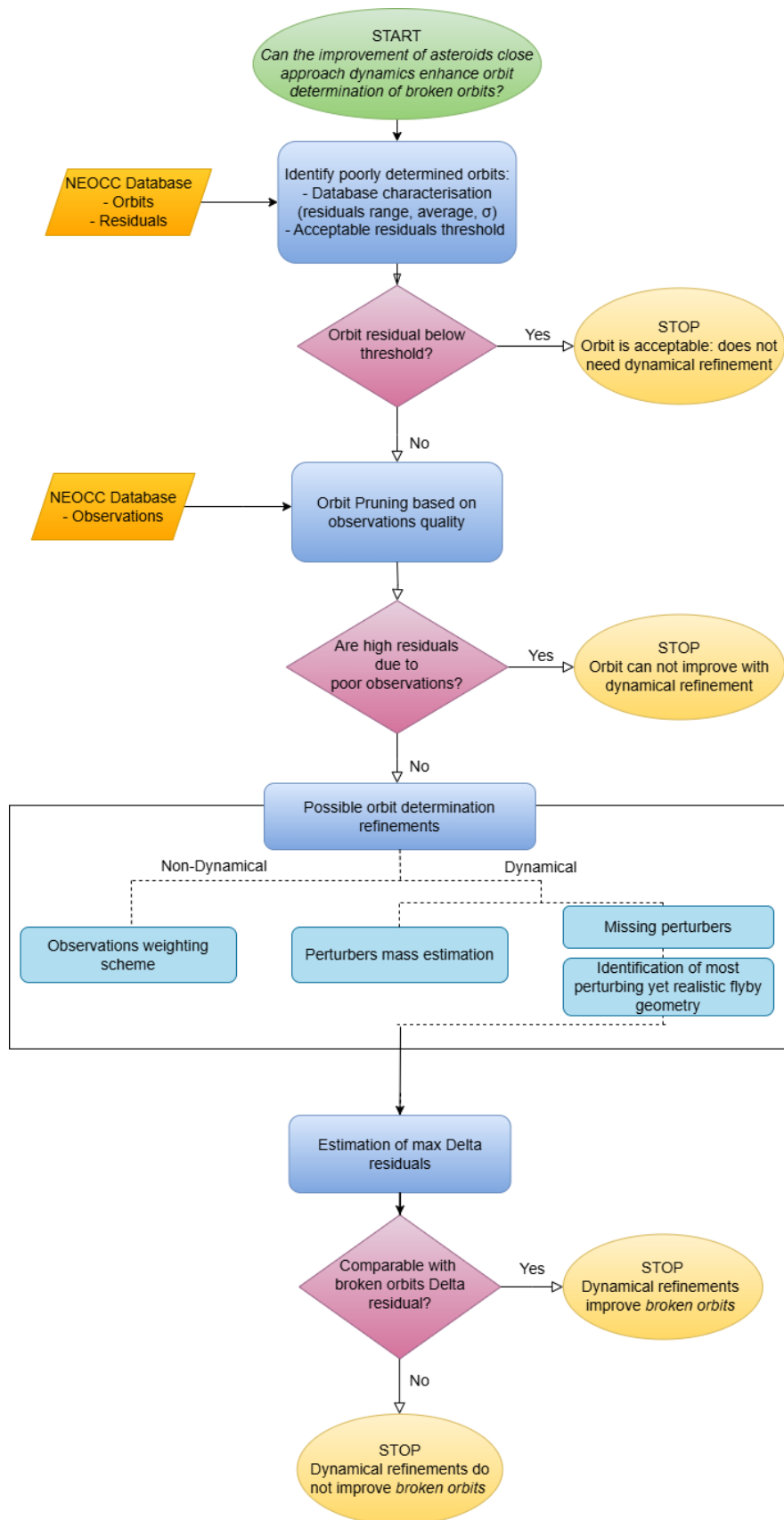


Figure 6.1: Overall flow of the research approach.

orbital database in terms of residuals. It offers insights into the quality of the dataset and potential corrections. The scripts used for the statistical analysis of the database are available online¹, together with the outcome of the pruning in terms of asteroid orbit characterisation, as presented in Section 1.4.

After analysing the current state of the dataset, potential strategies for improving asteroid OD have been identified and divided into two categories: non-dynamical and dynamical:

- A non-dynamical approach to improving OD is addressed in Ch. 2, which examines the role of weighting schemes applied to astrometric observations. This chapter presents an extensive statistical analysis of the performance of major asteroid surveys to assess the current state of astrometric precision. The varying accuracy of certain stations, relative to the dynamical classification of asteroids, has led to the proposal of a new weighting scheme for asteroid observations that incorporates this classification into the weights. This chapter addressed objective (1C). The scripts used for the study are published online², along with both the raw data—organised by station and sourced from the NEOCC database—and the resulting output³.
- Possible improvements in the asteroid-related section of the dynamical model are derived from the preliminary study of asteroid flybys presented in Ch. 3. This chapter compiled and analysed data on asteroid flybys, quantifying residual changes between perturbed and unperturbed orbital solutions. It provided the first systematic characterisation of dynamical and geometric parameters associated with these encounters, forming a dataset to support future numerical simulations. The study also highlighted that, while some flybys could be approximated using simplified models, such as patched conics or Keplerian maps, a comprehensive analysis required full numerical integration of the dynamical equations, which was implemented in Ch. 5. This work addressed objectives (2.IIA) and (2.IIB).

The data gathered on asteroid flybys enabled the investigation of two strategies for dynamical improvement:

- To further improve the dynamical model, the masses of the asteroids already included can be refined for greater accuracy. Chapter 4 examines mass determination from astrometry as a method to enhance the existing mass estimates.

¹https://github.com/StroNico/thesis-code/tree/main/orbit_pruning

²https://github.com/StroNico/thesis-code/tree/main/observation_statistics

³<https://doi.org/10.5281/zenodo.16739406>

Methodological details, along with the full set of scripts and libraries for performing automatic MD via *Find_Orb* as the OD software, are openly available on GitHub⁴. The focus is on asteroids with limited previous determinations using the deflection method. This study provides the first mass estimates for (32) Pomona and (91) Aegina using this approach and achieves the most precise determinations to date for (94) Aurora, (110) Lydia, and (747) Winchester. This addressed objective (2.I).

- Expanding the dynamical model to include more asteroids, specifically beyond the 300 considered in the BC-405 model, would account for additional perturbing bodies and introduce new potential flybys. Chapter 5 examined the impact of an expanded model on orbit determination and residuals. This chapter also investigated the optimal distribution of observations to maximize the effect of a flyby on residuals within the observational window, a strategy that could be used in future asteroid observation campaigns for MD. The study concluded that for the smallest asteroids in BC-405, the maximum deviation in RMS residuals under typical conditions is approximately 0.01, starting from a baseline of $(RMS)_0 \approx 0.53$. This addresses objective (2.IIC). Work in this chapter relied on a dedicated set of Python libraries developed for asteroid orbit propagation, flyby simulation, and synthetic observation generation. These tools are openly accessible online⁵.

This result can be compared with the findings of Section 1.4. The simulated change in residuals, representing the maximum variation expected from a typical close flyby with an asteroid just outside the BC-405 dynamical model, is insufficient to significantly alter the OD residuals and explain the presence of *broken orbits* in the database. Given an initial ground residual comparable to the average RMS, the required residual increase should exceed the threshold of 0.838. However, the observed change of just 0.01 is too small to account for the anomalously high residuals of these orbits. Even in the most favourable case of the lowest achievable ground residual, the resulting orbits would not exhibit behaviour matching the identified outliers. This analysis leads to the conclusion that flybys with objects beyond the BC-405 model cannot explain the broken orbits, thus addressing objective (3).

However, the NEOCC employs a more compact dynamical model, consisting of only the *big-16* (plus Pluto) for NEAs and the Ceres-Pallas-Vesta (CPV) model for MBAs,

⁴<https://github.com/StroNico/mass-determination>

⁵<https://github.com/StroNico/ACCESSO>

incorporating the 16 and 3 most massive asteroids, respectively⁶. This approach is motivated by computational efficiency, given the vast amount of data to manage, and by their focus on NEAs and planetary defense, where massive MBAs exert minimal gravitational influence and have almost no chance of undergoing a significant flyby. Consequently, some of the *broken orbits* may be resolved by adopting a more extensive model, such as BC-405, as appointed by Alessandro Odasso in February 2022 for the case of the flyby between (310585) 2001 UU20 and (324) Bamberga⁷. However, further expansion beyond this would offer little benefit for addressing these issues while significantly increasing computational costs. Alternative solutions for the *broken orbits* are discussed in Section 6.2.

6.1 Other contributions

Beyond the primary objectives of this work, the project has led to additional contributions as by-products of the activities. One such contribution is the improvement of the open-source software *Find_Orb*, used for OD. Since the focus of this study was not on developing new OD software, modifying existing algorithms, or proposing alternative computational approaches, but rather on evaluating the interaction between dynamics and OD, a reliable and open-source software was chosen as a tool. However, certain features necessary for this research were not available in the original version at the start of the project. To address this, specific modifications were requested from the developer, such as the ability to adjust the threshold for rejecting outlier observations. These changes were essential for the analyses carried out in Chapters 3 and 4. Furthermore, while working extensively with *Find_Orb*, a thorough examination of its code led to the identification of bugs, whose fixes were proposed and subsequently implemented by the developer. These adjustments not only met the needs of this research but also enhanced the software for the broader community, with the changes now publicly available for all users.

Another side contribution comes from the pruning presented in Section 1.4.1. The orbits with high residuals, identified as resulting from incorrect OD procedures or misidentifications, such as the cases of 2020 CD3 and 2015 HX69, can be reported to the MPC, which maintains and assigns observations, to help correct the global asteroid database.

⁶Marco Fenucci et al. *The Aegis Orbit Determination and Impact Monitoring System and services of the ESA NEOCC web portal*. 2024. arXiv: 2411.03763 [astro-ph.EP]. URL: <https://arxiv.org/abs/2411.03763>.

⁷<https://groups.io/g/mpml/message/37369?p=,,20,0,0,0::created,,2001+uu20+bamberg,20,2,0,89302907>

6.2 Future Developments

The future directions of this research project fall into two categories: those aimed at overcoming the limitations of the methodology, imposed by assumptions, technical constraints, or time restrictions, and those that build upon the ideas and outcomes of this thesis.

- *Overcoming limitations:*

1. The focus so far has been on the stakeholder's objective to investigate the occurrence and possible causes of orbits with high weighted RMS residuals. However, orbits with very low residuals, as discussed in Section 1.4.1 and Ch. 5, may also be affected by biases related to weighting, limited observations, OD, or incorrect outlier rejection. This issue has been observed in the NEOCC database, as in the case of (310585) 2001 UU20, where pre-close-encounter observations with (324) Bamberga were just rejected instead of including the perturber in the dynamical model. Also, investigating non-weighted residuals in arcseconds could provide additional insights.
2. Related to the previous point, the focus has been on strategies to reduce residuals. However, as noted in Chapter 4, if the mass in the dynamical model is significantly inaccurate, a negative $\Delta(\text{RMS})$ between perturbed and unperturbed cases may occur. This possibility has not been explored, and doing so could extend the analyses in Chapters 3 and 4 by identifying additional flybys and MDs.
3. Section 1.1 introduced the concept of *broken orbits* and demonstrated empirically how breaking an orbit can reduce residuals. Developing an automated and rigorous method for identifying and processing *broken orbits* would be beneficial. This could be linked to various factors, such as observation timing, weighting schemes, observational baselines, and unfiltered outliers. A systematic statistical study of broken orbits is necessary to establish reliable criteria for this process.

- *Building on the outcomes:*

1. Expanding on the previous point, machine learning (ML) techniques could be used to identify optimal *breakpoints* in orbits. Additionally, ML could help address the *inverse flyby problem*, where the goal is to determine the orbit of a

perturbing body based solely on the deflection it causes on the perturbed body. Largely, during this project, there have been attempts to solve this problem analytically or semi-analytically using results from Chapter 3, but without success. ML and time series forecasting techniques could offer a promising alternative.

2. The weighting scheme proposed in Chapter 2 could be formally implemented.
3. As suggested in the conclusions of Chapter 3, a new definition of the SOI, more consistent with the dynamics of distant flybys, could be developed.
4. The root cause of *broken orbits* remains unresolved. If they are not the result of a flyby, other dynamical events could be considered. SRP and Yarkovsky effect are unlikely to be significant for MBAs due to their distance from the Sun. Another possibility is asteroid collisions, which are impulsive events that can occur, but are rare to track. However, past observations of asteroid impacts indicate that their effects on trajectories are minimal, with residual changes smaller than those required to explain the broken orbits⁸.
5. Alternative explanations for *broken orbits* could be non-dynamical, related instead to observational quality and improperly applied weighting schemes. Weights assigned to older observations are often higher than those for recent ones, which could contribute to differences in residuals between pre- and post-encounter orbits. However, if this were the primary cause, orbits based solely on older observations should consistently show high residuals, which is not always observed. Further investigation is needed to assess the impact of weighting and observational inconsistencies in the cases of interest.

⁸Yu Chernetenko. “Collisions of Asteroids: Asteroid (596) Sheila”. In: *Astrophysical Bulletin* 74 (2019-04), pp. 203–207. DOI: 10.1134/S1990341319020093.

