

Collision-geometry-based Optimal Guidance for High-speed Target

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Abstract

This paper proposes a new unified form of guidance law based on the collision geometry that can be applied to both head-on (HO) or head-pursuit (HP) engagements for intercepting a target faster than an interceptor. To this end, two possible collision courses for a high-speed target and corresponding nonlinear heading errors are first investigated. The proposed guidance is then determined in a way to specify the desired heading error dynamics that ensures an optimal decreasing pattern. The characteristics of the proposed method are also investigated compared to existing methods. The favorable features are that the engagement geometries between HO or HP can be flexibly selected, and the optimality of the guidance command can be addressed. Moreover, since the proposed guidance law is directly derived from nonlinear collision geometry, the working mechanism is clearly explained, and the nonlinear nature is preserved. Finally, numerical simulations are performed to support our findings.

Keywords: Collision-geometry guidance; High-speed target; Head-pursuit guidance; Optimal error dynamics

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1. Introduction

In the past several decades, many advanced guidance laws have also been devised by modifying classical PN guidance [1, 2, 3, 4] or applying various control theories, such as optimal control [5, 6, 7, 8, 9] and nonlinear control [10, 11, 12, 13, 14], to guidance problems. Most existing guidance laws implicitly presume a specific engagement situation where an interceptor speed is higher than a target speed. Therefore, when applying these guidance laws, they converge to head-on or tail-chase engagement as reported in [15, 16].

For an engagement situation against a tactical ballistic threat, the target speed is typically higher than the interceptor speed. Investigation on the engagement kinematics reveals that two collision courses are possible in that case: the head-on (HO) [17] and the head-pursuit (HP) [18]. The potential issue of the existing guidance laws arises here. Although the two interception strategies, HO or HP, are both available, employing the existing guidance laws for the high-speed target leads to HO engagement because of its underlying assumption, as mentioned in [16]. In that case, a huge acceleration could be demanded because the closing speed between the interceptor and target is typically high under HO case compared to HP case. Thus, the capturability for the high-speed target could be considerably limited [16]. In other words, since the existing guidance laws result in HO engagement even if HP engagement is less tricky than HO engagement, they might have limitations in performance improvement.

Recently, some studies on guidance laws exploit the advantage of HP interception engagement against a high-speed target [15, 16, 18, 19, 20, 21] because of the reason mentioned above. The basic idea of HP interception strategy was first introduced in [18]. It has also been found that HP interception trajectory can be achieved by maintaining a simple geometric condition [19, 20]. Based on the geometric rule, a new guidance law called the head-pursuit guidance was first suggested using the sliding mode control (SMC) approach. The head-pursuit guidance was extended to the three-

dimensional space in [21]. However, since these guidance laws could be applied only to HP engagement, these guidance laws' operation might be complicated. The scalability of these guidance laws might be limited. As a remedy, guidance laws that can be applied to both HO or HP engagements were studied in [15, 16]. The method studied in [15] can select the type of engagement geometries through the choice of the intercept angle with respect to a target's flight direction. In [16], the concept of retro-PN guidance was suggested. Retro-PN guidance means PN guidance with negative navigation constant, and it has been proven that employing retro-PN guidance converges to HP collision geometry. Therefore, in this approach, the interception geometries between HO or HP can be selected by choosing a sign of the navigation constant.

Although the previous studies on the head-pursuit guidance have contributed to extending our knowledge on HP engagement against a high-speed target, there is still some room for improvement and some points to be further studied. First, the optimality of the guidance laws was unable to be addressed in the previous works due to the limitation of design approaches used in their studies. Therefore, the previous guidance laws might not be effective in terms of saving control energy. Second, it has been less understood how HO or HP engagements are exactly accomplished by employing those guidance laws from the collision geometry standpoint and how the heading error behaves under these guidance laws. Third, a rigorous analysis of the magnitude of acceleration demands depending on the interception geometries was absent in the previous works. Accordingly, a guideline to select a better interception strategy between the two possible collision courses (HO or HP) was not provided for a given engagement condition.

Motivated by these observations, this study proposes a new guidance law for HO or HP engagements that handle the above issues. Since the guidance's underlying principle is to achieve the collision course for a given engagement condition, guidance can be considered as a problem nullifying the heading error. Based on this fundamental

55 principle, the collision courses for HO or HP engagements and their corresponding heading error dynamics are first derived. The proposed method is then determined by applying a nonlinear control methodology to the nonlinear heading error dynamics. In this study, we use the feedback linearization control [22] in conjunction with a specific form of the error dynamics [23] guaranteeing an optimal decreasing pattern of the tracking error. The proposed guidance law consists of two terms: an optimal feedback 60 command nullifying the heading error and a bias command compensating for the collision course's deformation. Compared to the previous studies, the proposed method can specify the type of engagement geometries by choosing the heading errors to be nullified.

65 The characteristics of the proposed method are analyzed to provide better insights into the proposed method. In a near-collision course, the proposed guidance law is compared with true proportional navigation (TPN) guidance [24] and pure proportional navigation (PPN) guidance [25]. The magnitudes of acceleration demands between HO and HP collision courses for different initial engagement conditions are also 70 investigated. Furthermore, the optimality of the proposed guidance law and the corresponding cost function are analyzed by utilizing our previous study [23]. Finally, the performance of the proposed guidance law is investigated through numerical simulations.

The contributions of this study are threefold. First, a new guidance law capable of 75 both HO and HP engagements is proposed based on an approach substantially different from the previous works. Since the proposed method is directly derived from the collision course and the nonlinear heading error dynamics, the working principle is clearly explained from the collision course. The heading errors are effectively nullified under the nonlinear engagement scenario, compared to other methods [16, 15]. Second, our 80 study can be considered as the first attempt to investigate the optimality of a guidance law for HP engagement. By examining the dynamics of heading error, the cost function

of the proposed guidance law for HP engagement is determined. Third, our findings can provide better insight into HO or HP interception strategies. The analysis results provide the underlying relationship between HO or HP engagements and PN guidance. 85 The comparison of acceleration demands between HO and HP cases is offered. These analysis results could be utilized to develop a guideline to select a favorable interception strategy between HO or HP as well as an appropriate guidance gain by guidance designers.

This paper is constructed as follows. In Section 2, the problem formulation is 90 stated. In Section 3, the derivation and analysis of the proposed guidance law are provided. In Section 4, numerical simulations are performed to demonstrate the performance of the proposed guidance law. Finally, the concluding remarks are provided in Section 5.

2. Problem Formulation

95 In this section, the engagement kinematics used in this study is first described. For a given initial engagement condition, possible interception strategies are discussed. The guidance problem to be tackled is then stated.

2.1. Derivation of Engagement Kinematics

In this study, an interception engagement between an interceptor and an incoming 100 target with a high-speed (such as a tactical ballistic threat) is considered. To derive the engagement kinematics, the following assumptions are used. First, the bandwidth of the control loop is much higher than one of the guidance loop. The guidance problem can reasonably be regarded as the problem that only pertains to the kinematics. In this context, it is assumed that the interceptor and target can be treated as the point 105 masses. Second, it is assumed that the interceptor and target speeds are constant based on the fact that variations of the interceptor and target speeds during a short homing time can be negligible compared to other state variables. Third, it is assumed that

the target is non-maneuvering, or the target maneuver during the short homing time is insignificant. Fourth, we assume that the target speed is higher than the interceptor speed. Fifth, the roll channel autopilot is designed to achieve a fast response compared to the pitch/yaw channel autopilots. Based on this fact, it is assumed that the interceptor's roll motion can be rapidly stabilized so that the interceptor motion can be decoupled into the pitch channel and the yaw channel. From the engagement kinematics standpoint, the characteristic of the two perpendicular channels is identical. Thus, planar engagement kinematics is considered, as shown in Fig. 1. It is worth noting that the third and fourth assumptions are typically valid for most tactical ballistic targets [15, 16, 19, 20], and other assumptions have been widely adopted for designing guidance laws [15, 16, 19, 20, 21, 23, 26, 27].

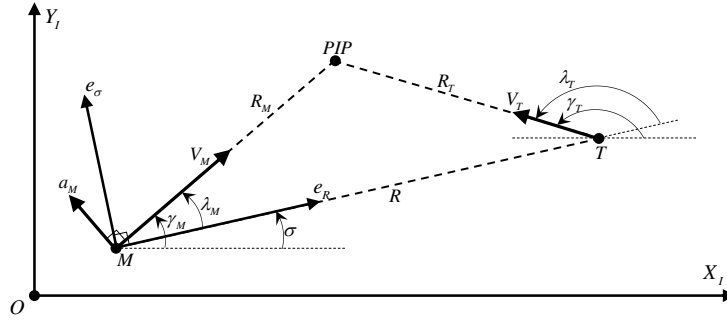


Figure 1: The engagement geometry between the interceptor and target.

In Fig. 1, the inertial reference frame is denoted by $X_I - O - Y_I$. In this frame, the interceptor and target are denoted by M and T , respectively. The line-of-sight (LOS) angle and the relative range are expressed by σ and R , respectively. The variables V_M , γ_M , a_M , and λ_M denote the interceptor speed, flight path angle, normal acceleration, and lead angle, respectively. The normal acceleration a_M acts on the interceptor in a perpendicular to the velocity direction so that it alters γ_M . The governing equation of

this kinematics rule can be expressed as

$$\dot{\gamma}_M = \frac{a_M}{V_M} \quad (1)$$

λ_M is defined to be the angle between the LOS and velocity directions, as shown in Fig. 1. By definition, it can be written as

$$\lambda_M \triangleq \gamma_M - \sigma \quad (2)$$

In a similar way, the variables regarding the target can be defined. We omit the explanation of the target variables because they are self-explanatory.

If the polar coordinate system $e_R - M - e_\sigma$ is introduced, as shown in Fig. 1, the relative engagement kinematics can be written, as follows.

$$\begin{aligned} \dot{R} &= V_T \cos \lambda_T - V_M \cos \lambda_M \\ R\dot{\sigma} &= V_T \sin \lambda_T - V_M \sin \lambda_M \end{aligned} \quad (3)$$

The overall goal of the guidance is to drive an interceptor towards a target. Thus, the interception condition can be determined based on the fact that an interceptor should be on a collision course to intercept a target from the collision geometry standpoint. To be more specific, for a given engagement condition shown in Fig. 1, the triangle connecting $M - PIP - T$ represents the collision triangle. In Fig. 1, the point PIP represents the predicted-intercept-point, and the parameters R_M and R_T denote the distances to be traveled by the interceptor and target until reaching PIP. By applying the law of sine to the collision triangle $M - PIP - T$, the following condition can be obtained.

$$\frac{\sin \lambda_T}{R_M} = \frac{\sin \lambda_M}{R_T} = \frac{\sin \lambda_C}{R} \quad (4)$$

where $\lambda_C = \lambda_T - \lambda_M$. From Eq. (4), the interception condition can be determined as

$$R_M \sin \lambda_M = R_T \sin \lambda_T \quad (5)$$

The physical meaning of this condition is that the distances to be traveled by the interceptor and target, perpendicular to the LOS direction, should be the same. If the interceptor and target speeds are assumed to be constant, the distances to be traveled by the interceptor and target can be approximated as $R_M \approx V_M t_{go}$ and $R_T \approx V_T t_{go}$, where t_{go} represents the remaining time of interception called the time-to-go. By substituting these approximations into Eq. (5), the interception condition can further be simplified, as follows

$$V_M \sin \lambda_M = V_T \sin \lambda_T \quad (6)$$

For convenience, let us define a dimensionless parameter called the target-to-interceptor speed ratio as

$$\rho \triangleq \frac{V_T}{V_M} \quad (7)$$

The fourth assumption implies $\rho > 1$. By using this parameter, the above interception condition can be rewritten, as follows

$$\sin \lambda_M = \rho \sin \lambda_T \quad (8)$$

In this study, the relative engagement kinematics, as given in Eqs. (3), and the corresponding interception condition, as shown in Eq. (8), will be used to analyze the interception geometries as well as to develop the proposed guidance law in the following sections.

125 2.2. Possible Types of Interception Geometries

In this section, we discuss possible interception geometries for a given engagement condition. Since an incoming target is mainly tackled in this study, the condition $-\frac{\pi}{2} \leq$

$\lambda_T \leq \frac{\pi}{2}$ is not considered in this analysis. Accordingly, it is assumed that λ_T lies in the range of values as

$$\frac{1}{2}\pi < \lambda_T < \frac{3}{2}\pi \quad (9)$$

Additionally, according to Lemma 1, the collision course against a high-speed target exists only for a certain range of λ_T . It is worth noting that this condition, as given in Eq. (9), can be readily met at the beginning of the homing guidance phase in practice because the homing guidance typically follows an accurate handover from proper mid-course guidance.

Lemma 1. *For a high-speed incoming target ($\rho > 1$ and Eq. (9)), the collision course can exist only when the lead angle of the target λ_T satisfies the following condition.*

$$\pi + \sin^{-1}\left(-\frac{1}{\rho}\right) \leq \lambda_T \leq \pi + \sin^{-1}\left(\frac{1}{\rho}\right) \quad (10)$$

Proof. The collision course only exists when the interception condition, as given in Eq. (8), is feasible for a given λ_T . Based on the fact that the magnitude of the sine function cannot exceed unity, the feasible region of λ_T can be determined, as follows.

$$|\sin \lambda_M| = |\rho \sin \lambda_T| \leq 1 \quad (11)$$

Since $\rho > 1$, we have

$$-\frac{1}{\rho} \leq \sin \lambda_T \leq \frac{1}{\rho} \quad (12)$$

Additionally, since an incoming target (i.e., $\frac{1}{2}\pi < \lambda_T < \frac{3}{2}\pi$) is mainly considered in this study, Fig. 2 shows that the feasible region of λ_T can be determined as Eq. (10), which completes the proof. $\square$

In the feasible region of λ_T for $\rho > 1$, as given in Eq. (10), there are two possible

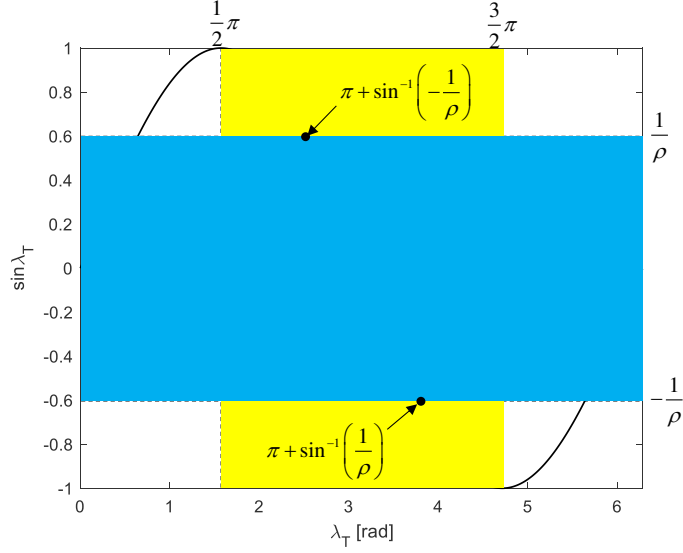


Figure 2: The illustration of the feasible region of λ_T .

solutions that satisfy the interception condition, namely

$$\lambda_{M,1}^* = \sin^{-1}(\rho \sin \lambda_T), \quad \lambda_{M,2}^* = \pi - \sin^{-1}(\rho \sin \lambda_T) \quad (13)$$

In the case of $|\rho \sin \lambda_T| = 1$, the above two solutions are the same as $\lambda_{M,1}^* = \lambda_{M,2}^* = \frac{1}{2}\pi$, otherwise $\lambda_{M,1}^* \neq \lambda_{M,2}^*$. This fact implies that there exist two possible collision courses when $|\rho \sin \lambda_T| < 1$. In this study, we mainly focus on the case of $|\rho \sin \lambda_T| < 1$. From Eq. (2), the interceptor's desired flight path angles to form the collision courses can be determined, as follows

$$\gamma_{M,1}^* = \sigma + \sin^{-1}(\rho \sin \lambda_T), \quad \gamma_{M,2}^* = \sigma + \pi - \sin^{-1}(\rho \sin \lambda_T) \quad (14)$$

Fig. 3 shows two possible collision courses in the feasible region of λ_T for $\rho > 1$. According to the definition, as provided in [15, 16], the first collision course $M - PIP_1 - T$ corresponds to HO engagement. In Fig. 3, the parameter $\gamma_{M,1}^*$ represents the desired

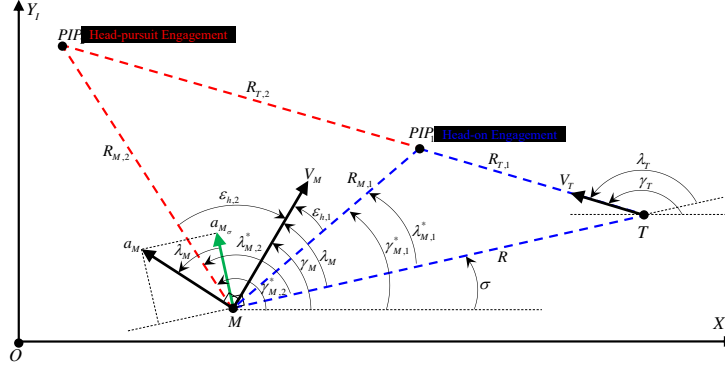


Figure 3: The two possible interception geometries.

flight path angle for achieving the first collision course, and the parameter $\varepsilon_{h,1}$ denotes the corresponding heading error. The second collision course $M - PIP_2 - T$ denotes HP engagement, as provided in [15, 16]. Similarly, the parameters $\gamma_{M,2}^*$ and $\varepsilon_{h,2}$ are the desired flight path angle and heading error to accomplish this interception geometry. From Fig. 3, the heading errors concerning HO and HP interception geometries are defined as

$$\varepsilon_{h,1} = \gamma_M - \gamma_{M,1}^*, \quad \varepsilon_{h,2} = \gamma_M - \gamma_{M,2}^* \quad (15)$$

Hereafter, let us discuss the characteristics of these collision courses. As shown in Fig. 3, it can be observed that $R_{T,2}$ is typically longer than $R_{T,1}$. For a constant speed target, this fact implies that HP case requires more homing engagement time compared to HO case. Additionally, other properties on these collision courses can be summarized as Properties 1, 2, and 3.

Property 1. For the case where γ_M lies in the range of value between $\gamma_{M,1}^* \leq \gamma_M \leq \gamma_{M,2}^*$, as shown in Fig. 3, HO engagement can be achieved when the velocity direction is rotated in the clockwise direction. On the other hand, HP collision course can be accomplished when the velocity direction is rotated in the counter-clockwise direction. In this region, the condition of γ_M that leads to the same magnitude of two heading

errors (i.e., $|\varepsilon_{h,1}| = |\varepsilon_{h,2}|$) can be written as

$$\gamma_M = \sigma + \frac{\pi}{2} \quad \text{or} \quad \lambda_M = \frac{\pi}{2} \quad (16)$$

Accordingly, when $\gamma_M < \sigma + \frac{\pi}{2}$ (or $\lambda_M < \frac{\pi}{2}$), $|\varepsilon_{h,1}|$ is smaller than $|\varepsilon_{h,2}|$.

140 **Property 2.** If $\gamma_M > \gamma_{M,2}^*$, HO and HP collision courses can be achieved when the velocity direction is rotated in the clockwise direction. In this region, $|\varepsilon_{h,2}|$ is smaller than $|\varepsilon_{h,1}|$.

Property 3. If $\gamma_M < \gamma_{M,1}^*$, HO and HP engagements can be accomplished when the velocity direction is rotated in the counter-clockwise direction. In this region, $|\varepsilon_{h,1}|$ is
145 smaller than $|\varepsilon_{h,2}|$.

2.3. Problem Statement

As observed in Section 2.2, the two interception strategies, HO or HP, are possible when intercepting a high-speed target. This fact implies the possibility of a guidance law that can exploit advantages of both HO and HP collision courses depending on the engagement conditions. However, even though there are two possible collision courses
150 for a high-speed target, most guidance laws have been developed to stick only to HO engagement. Although some studies on guidance laws for HP engagement have been recently reported [16, 15, 18, 19, 20, 21], there is still some room for improvement and some points to be further studied. Therefore, this paper aims to devise a new
155 guidance law that can be applied to both HO and HP interception strategies based on direct regulation of heading error, the approach which has not been tried in the previous studies. In this way, the proposed method could not only improve guidance performance but also provide new insights into HP engagement, compared to the earlier works.

160 3. Proposed Guidance Law

This section devotes to describe the proposed guidance law. The guidance concept used in this study is first introduced. Based on this guidance concept, the proposed guidance law is derived using the feedback linearization control methodology [22] in conjunction with a specific form of desired error dynamics. Then, the characteristics
165 of the proposed guidance law are analyzed. Finally, the contributions of the proposed method are discussed compared to the previous related works [15, 16, 19, 20].

3.1. Guidance Concept

The ultimate goal of guidance is to accomplish the interception by satisfying the collision condition. From the collision geometry, it can be observed that nullifying
170 the heading error is identical to achieving the interception condition. Based on this observation, the heading error is adopted as the variable representing the guidance error in the proposed method. Accordingly, the guidance problem can be considered as the problem to find an appropriate acceleration command in order to nullify the heading error in the remaining time of interception.

Since two possible collision courses exist for a high-speed target, as discussed in Section 2.2, there are two distinct definitions of the heading errors, as shown in Eq. (15). The unified form of the heading errors can be written as

$$\varepsilon_{h,i} = \gamma_M - \gamma_{M,i}^*, \quad \text{where } i \in \{1, 2\} \quad (17)$$

175 The next step is to determine a guidance command that can nullify $\varepsilon_{h,i}$. This problem can be thought of as a finite-time tracking problem with respect to $\varepsilon_{h,i}$, and it can be solved by applying various control methodologies such as the feedback linearization control and sliding mode control (SMC) [22]. Among these possible approaches, the feedback linearization method is adopted in this study because of its favorable charac-
180 teristics, as follows.

1. We can shape the decreasing pattern of the tracking error as desired through appropriate choices of the desired error dynamics.
2. A guidance law of simple form can be obtained. Thus, it is easy to investigate the physical meaning (or working principle) of the obtained guidance law.

In this approach, the desired error dynamics is first chosen. A guidance command is then determined by making the system equation follow the chosen error dynamics. The desired error dynamics adopted in this study is represented as

$$\dot{\varepsilon}_{h,i} + \frac{k}{t_{go}} \varepsilon_{h,i} = 0, \quad \text{where } i \in \{1, 2\} \quad (18)$$

185 where the design parameter k is a positive constant value. One of the merits of this error dynamics is that the control energy can be minimized if the tracking error follows the decreasing pattern of this error dynamics [23]. Therefore, when designing guidance laws using the feedback linearization control, the optimality of the control energy could be incorporated into the design. Additionally, this error dynamics can ensure the finite-
190 time convergence of the tracking error, as shown in Lemma 2.

Lemma 2. *The desired error dynamics as given in Eq. (18) with $k > 0$ guarantees the finite-time convergence of the tracking error; i.e., $\varepsilon_{h,i} \rightarrow 0$ as $t_{go} \rightarrow 0$.*

Proof. Let us define the Lyapunov candidate function, as follows

$$V_i = \frac{1}{2} \varepsilon_{h,i}^2, \quad \text{where } i \in \{1, 2\} \quad (19)$$

By taking the time-derivative of V_i and by substituting Eq. (18) into $\dot{V}_i$, we have

$$\dot{V}_i = \varepsilon_{h,i} \dot{\varepsilon}_{h,i} = -\frac{k}{t_{go}} \varepsilon_{h,i}^2 < 0 \quad (20)$$

Since $\dot{V}_i < 0$, the error dynamics as shown in Eq. (18) is asymptotically stable according to the well-known Lyapunov stability theory [22]. Thus, the tracking error

decreases monotonically with respect to time. Hereafter, let us show that the tracking error converges to zero in a finite-time. From Eqs. (19) and (20), we have

$$\dot{V}_i = -\frac{2k}{t_{go}} V_i \quad (21)$$

It is worth noting that the above equation is a Cauchy-Euler equation. It means that the above equation can be converted into an ordinary differential equation by substituting $t_{go} = e^x$.

$$\frac{dV_i}{dx} - 2kV_i = 0 \quad (22)$$

The solution is then determined as $V_i = V_{0,i} t_{go}^{2k}$, where $V_{0,i}$ represents the initial value of the Lyapunov candidate function. From this result, it can be observed that V_i converges to zero as $t_{go} \rightarrow 0$. Since $V_i = \frac{1}{2} \varepsilon_{h,i}^2$, the tracking error $\varepsilon_{h,i}$ will converge to zero as $t_{go} \rightarrow 0$, which completes proof. $\square$

3.2. Derivation of Proposed Guidance Law

The proposed guidance law is derived based on the guidance concept as mentioned above. First, from Eq. (17), the heading error dynamics can be obtained by taking the time-derivative of $\varepsilon_{h,i}$, as follows

$$\dot{\varepsilon}_{h,i} = \dot{\gamma}_M - \dot{\gamma}_{M,i}^*, \quad \text{where } i \in \{1, 2\} \quad (23)$$

Since $\dot{\gamma}_M = \frac{a_M}{V_M}$ and $\gamma_{M,i}^* = \sigma + \lambda_{M,i}^*$ from Eqs. (13) and (14), the above equation can be rewritten as

$$\dot{\varepsilon}_{h,i} = \frac{a_M}{V_M} - \dot{\sigma} - \dot{\lambda}_{M,i}^* \quad (24)$$

From Eq. (8), the time-derivative of $\lambda_{M,1}^*$ can be determined as

$$\dot{\lambda}_{M,1}^* \cos \lambda_{M,1}^* = \rho \dot{\lambda}_T \cos \lambda_T \quad (25)$$

For a non-maneuvering target (i.e., $\dot{\gamma}_T = 0$), the time-derivative of λ_T is approximated as $\dot{\lambda}_T = -\dot{\sigma}$. Thus, by rearranging Eq. (25) with $\dot{\lambda}_T = -\dot{\sigma}$, we have

$$\dot{\lambda}_{M,1}^* = -\frac{\rho \cos \lambda_T}{\cos \lambda_{M,1}^*} \dot{\sigma} \quad (26)$$

In a similar way, we can determine the time-derivative of $\lambda_{M,2}^*$ from Eq. (8). The unified form of $\dot{\lambda}_{M,i}^*$ is written as

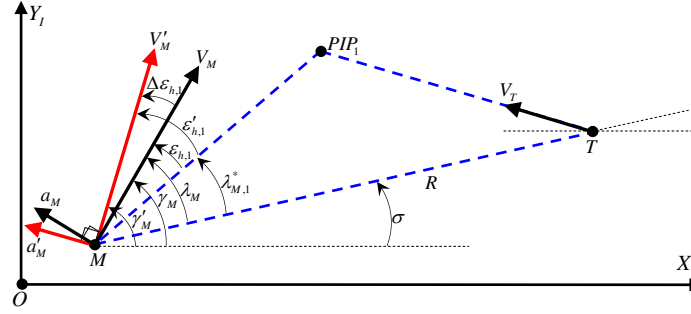
$$\dot{\lambda}_{M,i}^* = -\frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*} \dot{\sigma}, \quad \text{where } i \in \{1, 2\} \quad (27)$$

In this derivation, the fact that $\cos \lambda_{M,2}^* = \cos(\pi - \lambda_{M,1}^*) = -\cos \lambda_{M,1}^*$ is used.

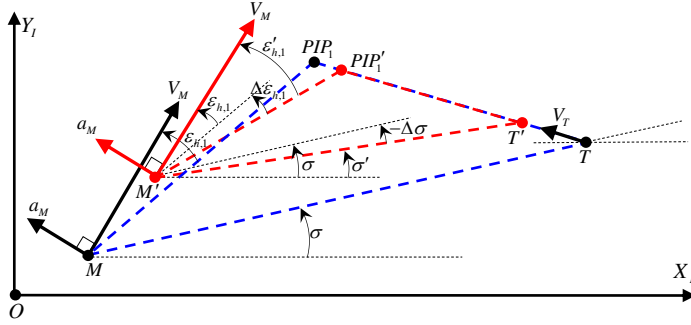
Finally, the heading error dynamics can then be determined by substituting Eq. (27) into Eq. (24), as follows.

$$\dot{\varepsilon}_{h,i} = \frac{a_M}{V_M} - \left(1 - \frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*} \right) \dot{\sigma} \quad i \in \{1, 2\} \quad (28)$$

From Eq. (28), it can be observed that there are two distinct terms leading to $\dot{\varepsilon}_{h,i}$. The first term represents $\dot{\varepsilon}_{h,i}$ by altering γ_M using a_M , as shown in Fig. 4 (a). The second term means $\dot{\varepsilon}_{h,i}$ due to the shift in the interceptor and target positions. Simply put, it can be regarded as $\dot{\varepsilon}_{h,i}$ due to the deformation of the collision course as the interceptor approaches the target, as shown in Fig. 4 (b). In the above equation, the second term is an indirect term for altering $\varepsilon_{h,i}$, and it cannot be arbitrarily controlled. Accordingly, in the heading error dynamics, a_M can be considered as the control input. Once the heading error dynamics is obtained, the proposed guidance command can be determined based on the feedback linearization methodology. To be more specific, the proposed guidance command can be determined to specify the error dynamics by its



(a) Effect of change in flight path angle



(b) Effect of deformation of collision course

Figure 4: The graphical interpretations of the heading error dynamics under HO engagement.

desired form provided in Eq. (18) as

$$a_{M,i} = -\frac{kV_M\varepsilon_{h,i}}{t_{go}} + \left(1 - \frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*}\right) V_M \dot{\sigma} \quad (29)$$

The relative range can be approximated as $R \approx V_c t_{go}$ by the distance formula, where $V_c \triangleq -\dot{R}$ represents the closing speed. Based on this approximation, the above expression can be rewritten as

$$a_{M,i} = a_{M,i}^B + a_{M,i}^F \quad (30)$$

where

$$a_{M,i}^B = -\frac{kV_M V_c \varepsilon_{h,i}}{R} \quad (31)$$

$$a_{M,i}^F = \left(1 - \frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*}\right) V_M \dot{\sigma} \quad (32)$$

In the above equation, the first term $a_{M,i}^B$ represents a feedback guidance command for reducing $\varepsilon_{h,i}$ to achieve the interception engagement. The second term $a_{M,i}^F$ is an additional guidance command that compensates for $\dot{\varepsilon}_{h,i}$ due to the deformation of the collision course according to the shift in the interceptor and target positions.

Remark 1. It is worth noting that the proposed guidance law is given by a unified form that can be applied to both HO or HP engagements. In the proposed guidance law, HO or HP engagement geometries can be determined by selecting $\lambda_{M,i}^*$.

Remark 2. For exploiting the HP engagement geometry, a specific gimbaled seeker with a hemispheric field-of-view is required so that the seeker can be pointed along the relative LOS direction with respect to the missile body axis even in the presence of a huge lead angle [16].

3.3. Relations to PN and Retro-PN Guidance Laws in Close Proximity to Collision Courses

Hereafter, the characteristics of the proposed guidance law are analyzed. First, the physical meaning of the proposed guidance law is discussed. Lemma 3 provides the information on the alternative form of the proposed guidance law in a near-collision course.

Lemma 3. *If $\varepsilon_{h,i}$ are small, the proposed guidance law will behave like TPN guidance under both HO and HP engagements as*

$$a_{M_{\sigma},i} \triangleq a_{M,i} \cos \lambda_M = N' V_c \dot{\sigma} \quad (33)$$

where N' represents the effective navigation constant of TPN guidance. In the pro-

posed guidance law, it has a specific value as

$$N' \triangleq k + 1 \quad (34)$$

Proof. In the proposed guidance law, $a_{M,i}^B$ can be rewritten in the term of the LOS rate. To be more specific, the following condition should be met from the engagement geometries and Eq. (6).

$$V_M \sin \lambda_{M,i}^* = V_T \sin \lambda_T \quad (35)$$

In the engagement geometries as shown in Fig. 3, we have the following relationship.

$$\lambda_{M,i}^* = \lambda_M - \varepsilon_{h,i} \quad (36)$$

Substituting Eq. (36) into Eq. (35) gives

$$V_M \sin (\lambda_M - \varepsilon_{h,i}) = V_T \sin \lambda_T \quad (37)$$

From Trigonometric formula, the left-hand side of Eq. (37) can be expanded as

$$V_M \sin (\lambda_M - \varepsilon_{h,i}) = V_M \sin \lambda_M \cos \varepsilon_{h,i} - V_M \cos \lambda_M \sin \varepsilon_{h,i} \quad (38)$$

Under the small-angle approximation of $\varepsilon_{h,i}$ (i.e., $\sin \varepsilon_{h,i} \approx \varepsilon_{h,i}$ and $\cos \varepsilon_{h,i} \approx 1$), the above equation can be approximated as

$$V_M \sin (\lambda_M - \varepsilon_{h,i}) \approx V_M \sin \lambda_M - V_M \cos \lambda_M \varepsilon_{h,i} \quad (39)$$

Substituting Eq. (39) into Eq. (37) yields

$$V_T \sin \lambda_T - V_M \sin \lambda_M \approx -V_M \cos \lambda_M \varepsilon_{h,i} \quad (40)$$

By combining Eqs. (3) and (40), the relationship between $\varepsilon_{h,i}$ and $\dot{\sigma}$ is determined as

$$\varepsilon_{h,i} \approx -\frac{R\dot{\sigma}}{V_M \cos \lambda_M} \quad (41)$$

The alternative form of $a_{M,i}^B$ can then be determined by applying Eq. (41) to Eq. (31), as follows

$$a_{M,i}^B = \frac{kV_c}{\cos \lambda_M} \dot{\sigma} \quad (42)$$

Next, the second term, as provided in Eq. (32), can be rewritten as

$$\left(1 - \frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*}\right) V_M \dot{\sigma} = \left[V_M \cos \lambda_M - V_T \cos \lambda_T \frac{\cos \lambda_M}{\cos \lambda_{M,i}^*} \right] \frac{\dot{\sigma}}{\cos \lambda_M} \quad (43)$$

Under the small-angle approximation of $\varepsilon_{h,i}$, the term $\cos \lambda_M$ can be approximated as

$$\cos \lambda_M \approx \cos \lambda_{M,i}^* \quad (44)$$

By substituting Eq. (44) into Eq. (43), we have

$$\left(1 - \frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*}\right) V_M \dot{\sigma} \approx (V_M \cos \lambda_M - V_T \cos \lambda_T) \frac{\dot{\sigma}}{\cos \lambda_M} \quad (45)$$

In the right-hand side of Eq. (45), the term $(V_M \cos \lambda_M - V_T \cos \lambda_T)$ is identical to $V_c = -\dot{R}$. Thus, we have

$$\left(1 - \frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*}\right) V_M \dot{\sigma} \approx \frac{V_c \dot{\sigma}}{\cos \lambda_M} \quad (46)$$

Therefore, by combining Eqs. (42) and (46), the alternative form of the proposed guidance law in a near-collision course can be expressed as

$$a_{M,i} = \frac{kV_c \dot{\sigma}}{\cos \lambda_M} + \frac{V_c \dot{\sigma}}{\cos \lambda_M} = \frac{(k+1) V_c \dot{\sigma}}{\cos \lambda_M} \quad (47)$$

From the engagement geometry as shown in Fig. 3, the proposed guidance command perpendicular to the LOS vector $a_{M\sigma,i}$ can be determined as

$$a_{M\sigma,i} = a_{M,i} \cos \lambda_M = (k+1) V_c \dot{\sigma} \quad (48)$$

Which completes the proof. $\square$

From Lemma 3, it can be predicted that the proposed guidance law would provide a similar performance of TPN guidance in a near-collision course. Additionally, the result obtained implies that TPN guidance could terminate both HO or HP collision courses. Under TPN, the intercept engagement is decided by the initial engagement condition rather than the design parameters. To be more specific, TPN guidance under the condition $\lambda_{M,0} > \frac{\pi}{2}$ terminates in HP engagement. In the case of $\lambda_{M,0} < \frac{\pi}{2}$, TPN guidance results in HO engagement. It implies that it is less flexible to select HO or HP engagements under TPN, unlike the other guidance laws [15, 16] including the proposed method, and it might be a drawback for guidance operations. Another interesting point is that the same form of TPN guidance command, as shown in Eq. (33), is used for both HO or HP engagements, and the only different thing is a way of projecting TPN guidance command into the normal acceleration. Namely, a guidance command that leads to HP collision course is not shown as retro-PN guidance from TPN guidance standpoint.

Additionally, the proposed guidance law in a near-collision course can be written in the form of PPN guidance by leveraging the relationship between TPN guidance and PPN guidance.

$$a_{M,i} = N V_M \dot{\sigma} \quad (49)$$

with

$$N = \frac{(k+1) V_c}{V_M \cos \lambda_M} \quad (50)$$

In this equation, it can be observed that the navigation constant for HO engagement is positive as $N > 0$ because of $\cos \lambda_M > 0$, and it has a negative value for HP case due to $\cos \lambda_M < 0$. Therefore, it can be understood that the proposed guidance law in a near-collision course can be considered as PPN guidance with time-varying navigation constant under HO engagement. It can also be observed that the proposed guidance law in a near-collision course can be thought of as retro-PPN guidance with time-varying navigation constant, under HP engagement. Therefore, our findings could be utilized for explaining why retro-PN guidance, as studied in [16], terminates in HP collision course. Compared to the proposed guidance law for HP engagement, a guidance law as suggested in [16] is given by retro-PPN with a constant N . This characteristic might be a drawback for effectively correcting ε_h . To be more specific, by substituting Eqs. (41), (46), and (49) into Eq. (28), the heading error dynamics under retro-PN guidance [16] with $N = k + 1$ (which is the same value of the proposed method) can be determined as

$$\dot{\varepsilon}_{h,i} + \frac{k'}{t_{go}} \varepsilon_{h,i} = 0 \quad (51)$$

with

$$k' \triangleq \frac{(k + 1) V_M \cos \lambda_M}{V_c} - 1 \quad (52)$$

where k' can be considered as the effective guidance gain, similar to k in Eq (18). From Eq. (52), we can observe that the value of k' is not constant. Thus, this value might become zero depending on the engagement conditions, unlike the value of k in Eq. (18). This fact implies that there might be some loss of control effectiveness under retro PN guidance [16], depending on the engagement conditions when nullifying ε_h . On the other hand, it is anticipated that ε_h can be effectively nullified under the proposed method because the value of k is always fixed.

3.4. Comparison between Magnitude of Commands for Two Collision Courses

Next, we analyze the magnitude of the acceleration demands for HO or HP collision courses, depending on engagement conditions. As shown in Fig. 5, the engagement regions can be divided into three areas, as follows

1. Area 1: $0 \leq \lambda_M < \lambda_{M,1}^*$
2. Area 2: $\lambda_{M,1}^* \leq \lambda_M < \lambda_{M,2}^*$
3. Area 3: $\lambda_{M,2}^* \leq \lambda_M \leq \pi$

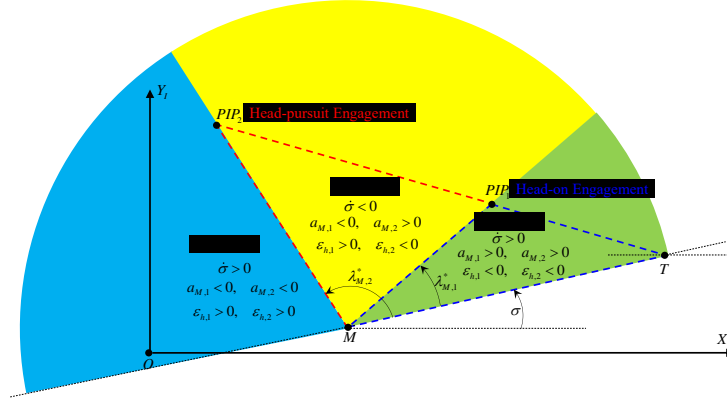


Figure 5: The segmentation of engagement regions.

Lemma 4, 5, and 6 compare the guidance command between HO and HP collision courses for different engagement regions. Lemma 4 implies that $\dot{\epsilon}_{h,i}$ due to the non-zero LOS rate under HP engagement is smaller than under HO engagement for $\rho > 1$. This property can be understood by examining the heading error dynamics, as given in Eq. (28). Since the two terms in parentheses have different signs in the case of HP engagement, these two terms are acting to reduce $\dot{\epsilon}_{h,i}$ due to the non-zero LOS rate. However, for HO engagement, these two terms introduce $\dot{\epsilon}_{h,i}$ due to the non-zero LOS rate in the same direction. This discrepancy brings the gap of $a_{M,i}^F$ between HO and HP engagements.

Additionally, Lemma 5 implies that using HO interception strategy is more beneficial compared to HP interception strategy in terms of reducing the magnitude of the initial guidance command when the initial λ_M lies in Area 1. As mentioned before, the guidance can be thought of as nullifying ε_h in a given remaining time of interception, and the magnitude of initial guidance command is typically proportional to the magnitude of the initial ε_h . Since the initial ε_h in Area 1 is much smaller under HO engagement than under HP engagement, as in Property 3, HO engagement is advantageous in Area 1 in terms of reducing the magnitude of the initial guidance command. Likewise, if the initial λ_M lies in Area 3, employing HP interception strategy is more beneficial compared to HO interception. This is because the initial ε_h in Area 3 is much smaller under HP engagement than under HO engagement, as in Property 2.

In Area 2, there might be ambiguity when selecting a more beneficial interception strategy between HO or HP. This is because as shown in Property 1 there might exist a case where the magnitude of initial ε_h with respect to HO or HP interception geometries might be similar if the initial λ_M lies in Area 2. In that case, Lemma 6 could be a guideline to select a more advantageous interception geometry between HO or HP in terms of reducing the magnitude of the initial guidance command.

Lemma 4. *For λ_T lying in the region specified by Eq. (10), the following condition is met.*

$$\left| \frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*} \right| > 1 \quad (53)$$

Additionally, the magnitude of $a_{M,2}^F$ is smaller than $a_{M,1}^F$.

Proof. From Eq. (8), we have $|\sin \lambda_T| < |\sin \lambda_{M,i}^*|$ because of $\rho > 1$. From the characteristic of sine and cosine functions, the above condition results in $|\cos \lambda_T| > |\cos \lambda_{M,i}^*|$. Accordingly, we have the result as shown in Eq. (53).

Additionally, since $\cos \lambda_{M,2}^* = -\cos \lambda_{M,1}^*$, $a_{M,1}^F$ and $a_{M,2}^F$ can be expressed as

$$a_{M,1}^F = \left(1 - \frac{\rho \cos \lambda_T}{\cos \lambda_{M,1}^*}\right) V_M \dot{\sigma}, \quad a_{M,2}^F = \left(1 + \frac{\rho \cos \lambda_T}{\cos \lambda_{M,1}^*}\right) V_M \dot{\sigma} \quad (54)$$

275 In Eq. (54), since the term $\cos \lambda_T$ is negative for a given value of λ_T , the magnitude of $a_{M,2}^F$ is smaller than the magnitude of $a_{M,1}^F$, which completes the proof. $\square$

Lemma 5. *If the initial collision triangle is given to satisfy Eq. (55), the magnitude of $a_{M,1}$ is smaller than the magnitude of $a_{M,2}$, at the beginning of the homing phase, when the initial λ_M lies in Area 1. Likewise, the magnitude of $a_{M,2}$ is smaller than the magnitude of $a_{M,1}$, at the beginning of the homing phase, if the initial λ_M lies in Area 3.*

$$\lambda_{M,1}^* < \frac{\pi}{2} + \frac{R\dot{\sigma}\rho \cos \lambda_T}{kV_c \cos \lambda_{M,1}^*} \quad (55)$$

Proof. From Fig. 5, it can be observed that in Area 1 we expect $a_{M,1} > 0$, $a_{M,2} > 0$, $\varepsilon_{h,1} < 0$, $\varepsilon_{h,2} < 0$, and $\dot{\sigma} > 0$. Accordingly, the condition that the magnitude of $a_{M,1}$ is smaller than the magnitude of $a_{M,2}$ can be written as $a_{M,2} > a_{M,1}$, because those are positive values. From Eq. (30), we have

$$a_{M,2} - a_{M,1} = -\frac{kV_M V_c}{R} (\varepsilon_{h,2} - \varepsilon_{h,1}) - V_M \dot{\sigma} \rho \cos \lambda_T \left(\frac{1}{\cos \lambda_{M,2}^*} - \frac{1}{\cos \lambda_{M,1}^*} \right) > 0 \quad (56)$$

By using Eqs. (13) to (15) and the fact that $\cos \lambda_{M,2}^* = -\cos \lambda_{M,1}^*$, the above equation can be rearranged as

$$\pi - 2\lambda_{M,1}^* + \frac{2R\dot{\sigma}\rho \cos \lambda_T}{kV_c \cos \lambda_{M,1}^*} > 0 \quad (57)$$

By rearranging the above expression, we can obtain the condition as provided in Eq. (55).

In Area 3, we have $a_{M,1} < 0$, $a_{M,2} < 0$, $\varepsilon_{h,1} > 0$, $\varepsilon_{h,2} > 0$, and $\dot{\sigma} > 0$. The condition that the magnitude of $a_{M,2}$ is smaller than the magnitude of $a_{M,1}$ can be written as $-a_{M,1} > -a_{M,2}$. Thus, similarly, we can obtain the same result by applying
280 Eq. (30) into the condition $-a_{M,1} > -a_{M,2}$. $\square$

Lemma 6. *If the initial λ_M lies in Area 2 and satisfies the following condition given in Eq. (58), the magnitude of $a_{M,2}$ is smaller than the magnitude of $a_{M,1}$, at the beginning of the homing phase.*

$$\lambda_M > \frac{\pi}{2} + \frac{R\dot{\sigma}}{kV_c} \quad (58)$$

Proof. From Fig. 5, it can be found that in Area 2 we expect $a_{M,1} < 0$, $a_{M,2} > 0$, $\varepsilon_{h,1} > 0$, $\varepsilon_{h,2} < 0$, and $\dot{\sigma} < 0$. Therefore, the condition that the magnitude of $a_{M,2}$ is smaller than the magnitude of $a_{M,1}$ can be written as $-a_{M,1} > a_{M,2}$. From Eq. (??), we have

$$a_{M,1} + a_{M,2} = -\frac{kV_M V_c}{R} (\varepsilon_{h,1} + \varepsilon_{h,2}) + 2V_M \dot{\sigma} - V_M \dot{\sigma} \rho \cos \lambda_T \left(\frac{1}{\cos \lambda_{M,2}^*} + \frac{1}{\cos \lambda_{M,1}^*} \right) < 0 \quad (59)$$

From Eqs. (13) to (15) and the fact that $\cos \lambda_{M,2}^* = -\cos \lambda_{M,1}^*$, the above equation can be rearranged as

$$-2\lambda_M + \pi + 2V_M \dot{\sigma} < 0 \quad (60)$$

By rearranging the above expression, we can obtain the condition as provided in Eq. (58), which completes the proof. $\square$

3.5. Optimality of Proposed Guidance Law

285 Next, the optimality of the proposed guidance law is investigated. Lemma 7 provides the information on the cost function of the proposed guidance law from the optimal control standpoint.

Lemma 7. *If the heading error decreases under the specific error dynamics as shown in Eq. (18), the corresponding cost function is given by*

$$\min J_i = \frac{1}{2} \int_t^{t_f} \frac{1}{(t_f - \tau)^{k-1}} [a_{M,i}^B(\tau)]^2 d\tau \quad (61)$$

subject to $\dot{\varepsilon}_{h,i} = \frac{1}{V_M} a_{M,i}^B$.

Proof. The general form of the tracking problem can be written as

$$\dot{\varepsilon} = f(t) + g(t)u \quad (62)$$

where the parameter ε represents a tracking error such as the heading error. According to our previous work [23], it has been shown that if the tracking error is nullifying under the specific error dynamics as provided in Eq. (18), the corresponding cost function is given by

$$\min J = \frac{1}{2} \int_t^{t_f} \frac{g^2(\tau)}{(t_f - \tau)^{k-1}} \left[u(\tau) + \frac{f(\tau)}{g(\tau)} \right]^2 d\tau \quad (63)$$

By comparing Eq. (61) and Eq. (63), it can be observed that the heading error dynamics as given in Eq. (28) is a specific form of the general tracking problem as shown in Eq. (62), as follows

$$f(t) = - \left(1 - \frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*} \right) \dot{\sigma}, \quad g(t) = \frac{1}{V_M}, \quad u = a_M \quad (64)$$

By substituting the expressions as provided in Eq. (64) into Eq. (63), we have

$$\min J_i = \frac{1}{2} \int_t^{t_f} \frac{1}{V_M^2 (t_f - \tau)^{k-1}} \left[a_{M,i}(\tau) - \left(1 - \frac{\rho \cos \lambda_T}{\cos \lambda_{M,i}^*} \right) V_M \dot{\sigma}(\tau) \right]^2 d\tau \quad (65)$$

From Eqs. (32) and (65), it can be observed that the second term in brackets is identical to $a_{M,i}^F$. Therefore, by using Eq. (30), we have

$$\min J_i = \frac{1}{2} \int_t^{t_f} \frac{1}{V_M^2 (t_f - \tau)^{k-1}} [a_{M,i}^B(\tau)]^2 d\tau \quad (66)$$

From Eq. (66), since a constant value in the cost function does not affect the optimal solution in the above equation, the effective cost function can be written as Eq. (61), which completes the proof. $\square$

The result obtained is the cost function for the feedback guidance command $a_{M,i}^B$.

Thus, it implies that the proposed guidance law is a suboptimal guidance law, as it is not a minimizer of the acceleration command but is merely a minimizer of the pseudo-control. From Eq. (61), it can be readily observed that the weighting function of the cost function is given by $1/t_{go}^{k-1}$. In that case, since the weighting value under $k > 1$ increases as $t_{go} \rightarrow 0$, the cost function becomes more expensive as $t_{go} \rightarrow 0$. Accordingly, it implies that the feedback guidance command $a_{M,i}^B$ gradually decreases as the interceptor approaches the target (i.e., $t_{go} \rightarrow 0$) for $k > 1$. To ensure a finite command of $a_{M,i}^B$, $k > 1$ is recommended. Additionally, if the design parameter k is selected as the unity (i.e., $k = 1$), the cost function is identical to the pure control energy.

4. Simulation Study

In this section, the performance and feasibility of the proposed guidance law are demonstrated through three different nonlinear simulations. The first simulation investigates the performance of the proposed guidance law for different initial heading angles. In the second simulation, we analyze the characteristics of the proposed method according to changes in the design parameter. The third simulation compares the proposed guidance law with other guidance laws applicable to both HO and HP engagements. The proposed guidance command as shown in Eq. (30) is used for these simulation studies. The design parameter is chosen as $k = 2$ for the default value because this value is equivalent to $N' = 3$ in TPN guidance, as shown in Eq. (34). In these simulations, a sample engagement scenario described by the parameters in Table 1 is considered. In this case, HO and HP interception geometries can be determined as shown in Fig. 6. The speed ratio between the interceptor and target is $\rho = 1.25$, and the desired flight path angles for HO and HP collision courses are determined as $\gamma_{M,1}^* = 107.1^\circ$ and $\gamma_{M,2}^* = 162.9^\circ$, respectively. The desired lead angles for HO and HP collision courses are also computed as $\lambda_{M,1}^* = 62.1^\circ$ and $\lambda_{M,2}^* = 117.9^\circ$, respectively.

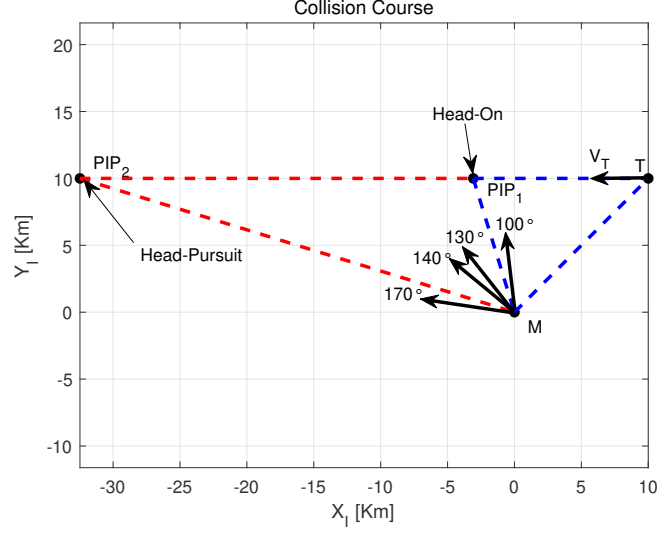


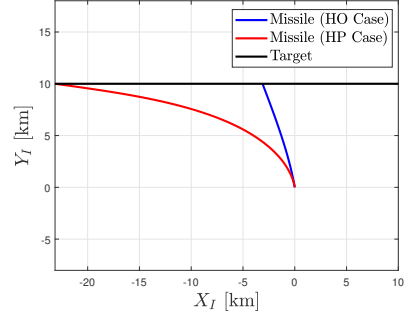
Figure 6: The collision courses for the given engagement conditions.

Table 1: The initial engagement conditions.

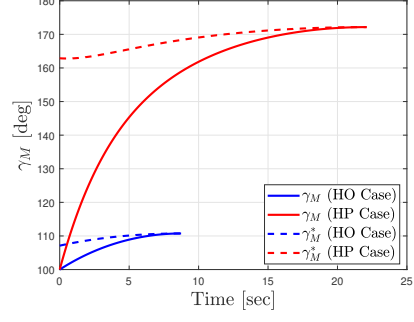
Parameter	Values
Initial Interceptor Position, M_0	(0, 0) km
Initial Target Position, T_0	(10, 10) km
Interceptor Speed, V_M	1200m/s
Target Speed, V_T	1500m/s
Interceptor's Initial Flight Path Angle, $\gamma_{M,0}$	$100^\circ, 130^\circ, 140^\circ, 170^\circ$
Target's Flight Path Angle, γ_T	180°
Design Parameter, k	2, 4, 6

4.1. Performance Analysis for Different Initial Heading Angles

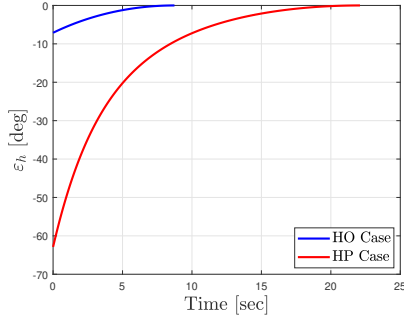
Figures 7, 8, and 9 show the nonlinear simulation results of the proposed guidance law with $\gamma_{M,0} = 100^\circ, 170^\circ$, and 130° , respectively, in order to examine the performance of the proposed method according to changes of initial engagement conditions. In the cases of $\gamma_{M,0} = 100^\circ$ and $\gamma_{M,0} = 170^\circ$, the interception engagements start from Area 1 and Area 3, respectively, as shown in Fig. 5. In both cases, the terms in the right-hand side of Eq. (55) are determined as 87.6° and 78.7° , respectively. It means that the condition shown in Eq. (55) is satisfied in both cases so that the statement in Lemma 5 holds. When $\gamma_{M,0} = 130^\circ$, the interception engagement occurs at



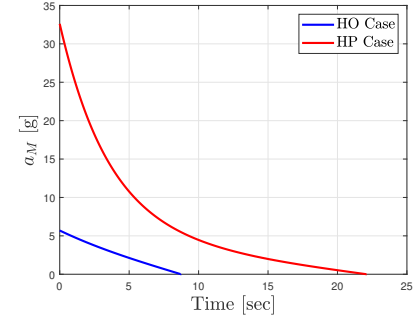
(a) Interception Trajectory



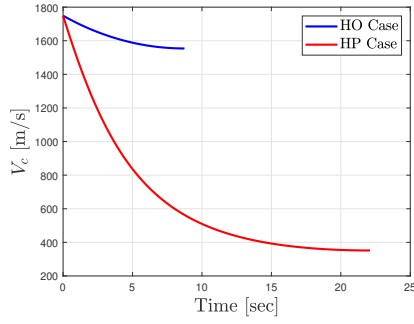
(b) Flight Path Angle



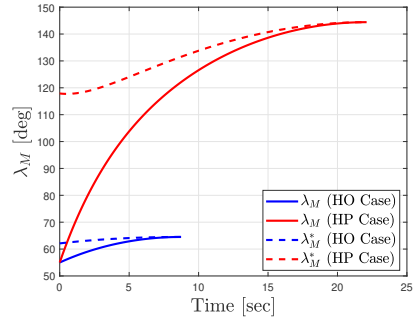
(c) Heading Error



(d) Guidance Command

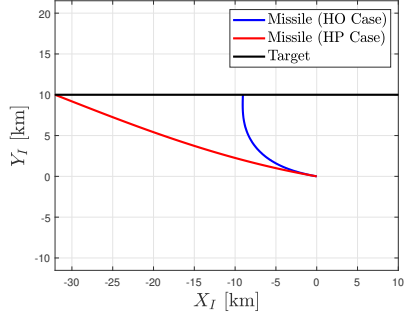


(e) Closing Speed

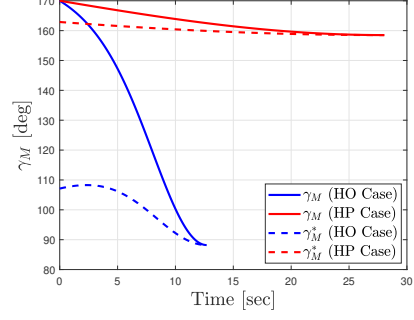


(f) Lead Angle

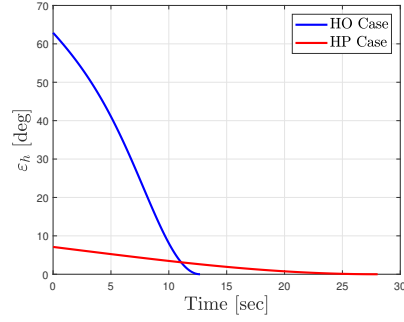
Figure 7: The simulation results of the proposed method for $\gamma_{M,0} = 100^\circ$.



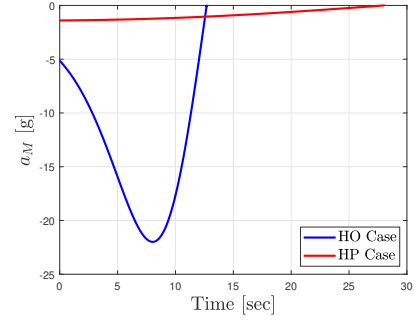
(a) Interception Trajectory



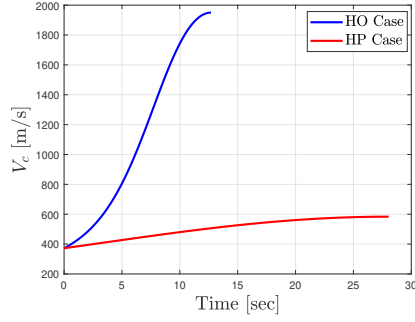
(b) Flight Path Angle



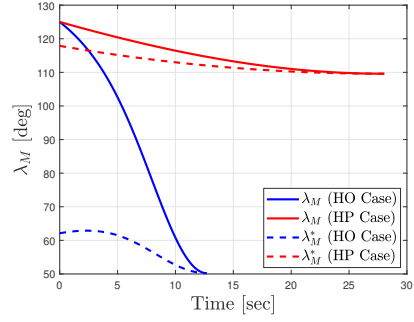
(c) Heading Error



(d) Guidance Command

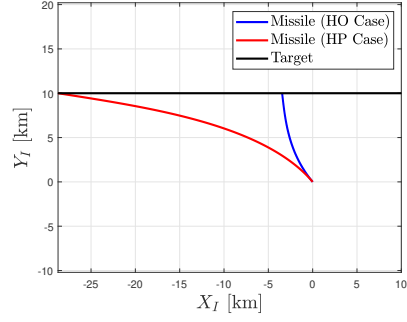


(e) Closing Speed

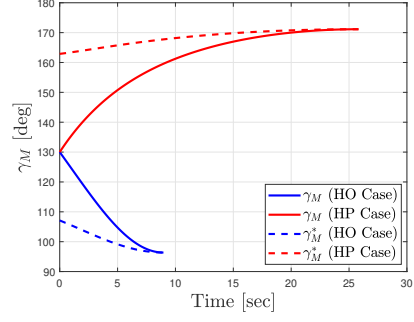


(f) Lead Angle

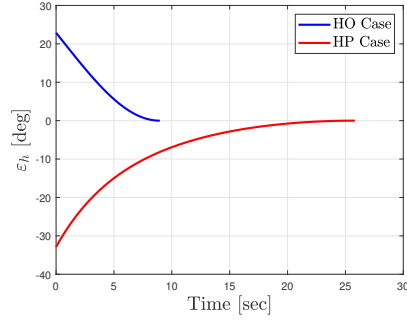
Figure 8: The simulation results of the proposed method for $\gamma_{M,0} = 170^\circ$.



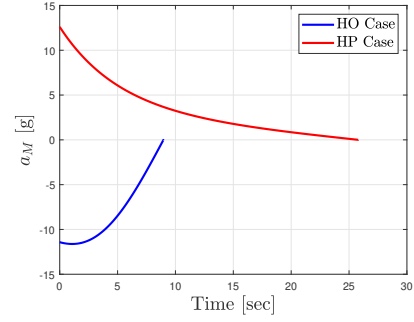
(a) Interception Trajectory



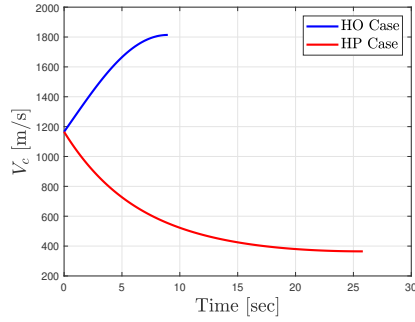
(b) Flight Path Angle



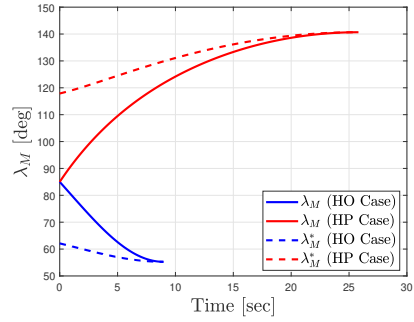
(c) Heading Error



(d) Guidance Command



(e) Closing Speed



(f) Lead Angle

Figure 9: The simulation results of the proposed method for $\gamma_{M,0} = 130^\circ$.

Area 2. Since the term in the right-hand side of Eq. (58) is computed as 86.0° in that case, it can be expected that the magnitude of $a_{M,1}$ is smaller than the magnitude of $a_{M,2}$ at the beginning of the homing phase, according to Lemma 6.

As shown in Figs. 7, 8, and 9, the heading errors converge to zero for all the simulation cases. It indicates that the interceptor can successfully intercept the target by employing the proposed guidance law. An interesting property of the proposed guidance law is that the guidance command converges to zero as the interceptor approaches the target. This property is beneficial to ensure operational margin to cope with unexpected situations during the terminal homing phase. As shown in Fig. 7 (d), more initial acceleration is demanded under HP case in Area 1, as expected. It implies that HO engagement is more beneficial than HP engagement in this area. Conversely, in Area 3, HO engagement requires more initial acceleration demand, as shown in Fig. 8 (d). In Area 2, the magnitude of the initial guidance command depends on the magnitude of the initial heading error and the remaining time of interception. Since the closing speed for HP engagement typically is lower than one for HO engagement as shown in Figs. 7 (e), 8 (e), and 9 (e), it could bring the effect of increasing the remaining time of interception in HP case. Therefore, even if the magnitude of the initial heading error for HP engagement is greater than the magnitude of the heading error for HO engagement, the magnitude of the acceleration demand could be similar in Area 2. In this case, the result of Lemma 6 can be utilized for a guideline to choose a beneficial interception strategy between HO and HP in terms of reducing the magnitude of the initial guidance command. As expected, it can be observed that the magnitude of $a_{M,1}$ is smaller than the magnitude of $a_{M,2}$ at the initial time, as shown in Fig. 9 (d).

4.2. Performance Analysis for Different Design Parameters

Figures. 10 and 11 show the simulation results of the proposed guidance law with $k = 2, 4$, and 6 under HO and HP engagements, respectively, in order to investigate the characteristics of the proposed method according to changes of the design parameter.

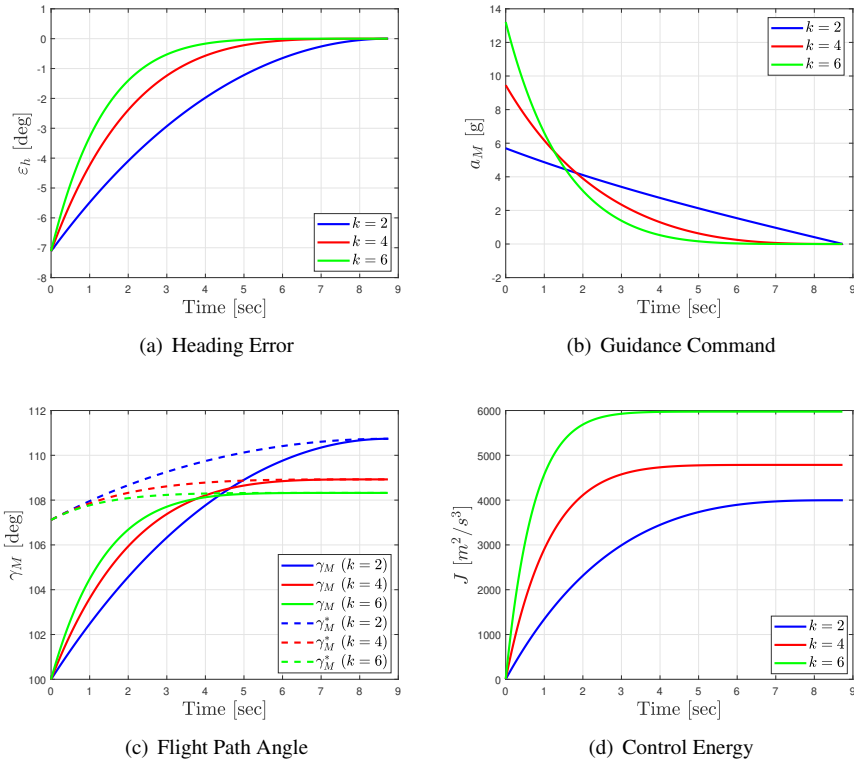


Figure 10: The simulation results of the proposed method for various k under HO engagement.

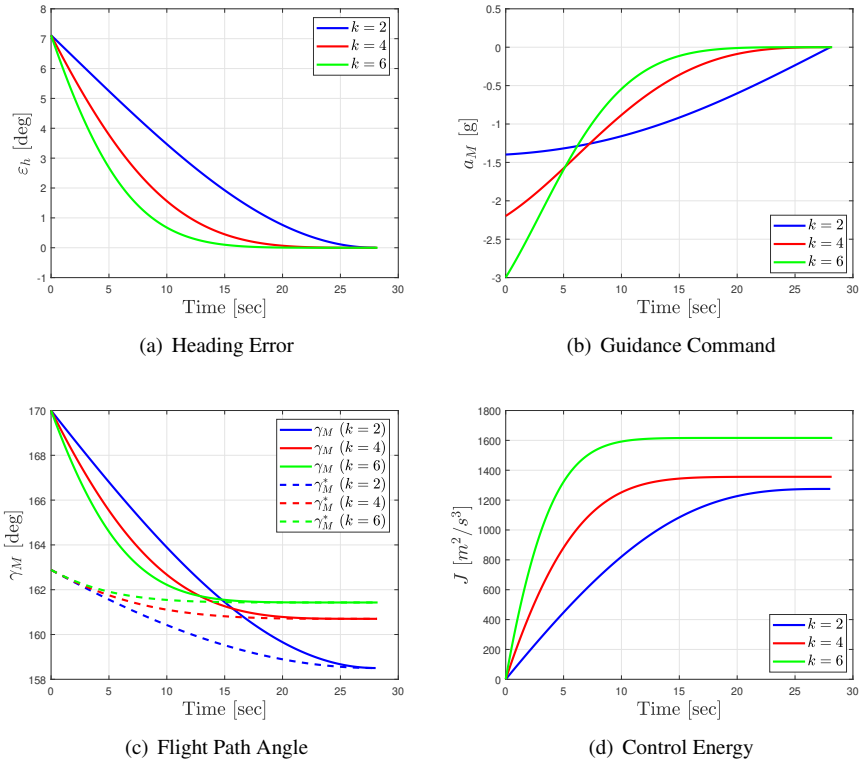


Figure 11: The simulation results of the proposed method for various k under HP engagement.

355 In these simulations, the initial flight path angles are chosen as $\gamma_M = 100^\circ$ for HO interception strategy and as $\gamma_M = 170^\circ$ for HP interception strategy. As shown in Figs. 10 (a) and 11 (a), the heading errors converge to zero more rapidly as the value of k increases. Additionally, it can be observed that as k increases, the initial acceleration demands increase, and the magnitude of acceleration demands rapidly converge to zero, 360 as shown in Figs. 10 (b) and 11 (b). Accordingly, more control energies are required as k increases, as shown in Figs. 10 (d) and 11 (d). This tendency matches well with the desired error dynamics given by Eq. (18) and the cost function given by Eq. (61).

4.3. Comparison with Other Guidance Laws

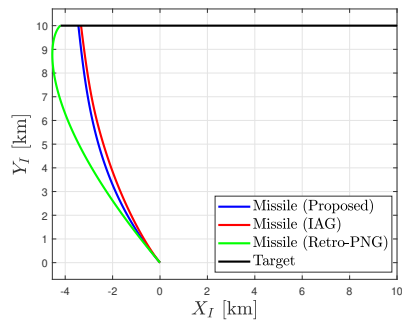
In this subsection, the performance of the proposed guidance law is compared with other guidance laws called the intercept-angle guidance (IAG) [15] and the retro-PN guidance [16] that consider HO and HP engagements. Since the original IAG has been developed for a first-order lag system, the following modified version of IAG for a lag-free system is used in this study.

$$a_M = \frac{1}{\cos \lambda_M} \left(2V_c \dot{\sigma} + \frac{R\dot{\sigma}}{\tau} - \frac{R}{\tau} K \text{sat}(s, s_{bl}) \right) \quad (67)$$

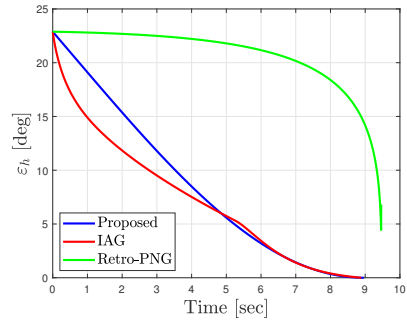
In the above equation, $\text{sat}(s, s_{bl})$ represents the standard saturation function. The variables s and s_{bl} are the sliding surface and the boundary layer, respectively. τ and K represent the design parameters that determine the convergence rate of the reaching phase. The sliding surface s is defined as

$$s = -\tau \dot{\sigma} + (\lambda_T - \lambda_{T,d}) \quad (68)$$

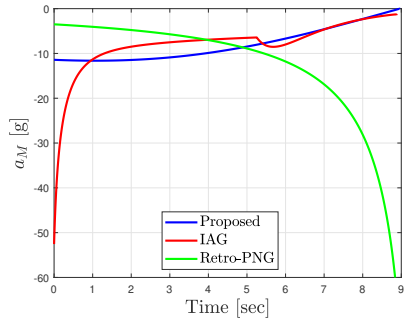
where $\lambda_{T,d}$ denotes the desired lead angle of the target, which is also the design parameter. In these simulations, the design parameters of IAG are chosen as $\tau = 0.5$, $K = 0.5 \frac{\pi}{180^\circ}$, $s_{bl} = 1.0^\circ$. Additionally, $\lambda_{T,d}$ is selected as the terminal value of the target's lead angle, which are recorded in the simulation results of the proposed method, for 365



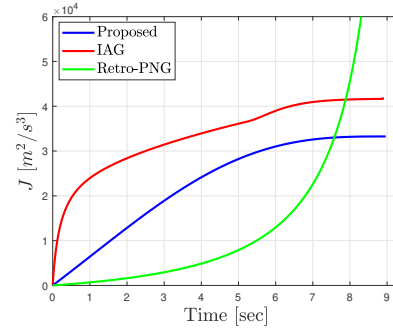
(a) Interception Trajectory



(b) Heading Error

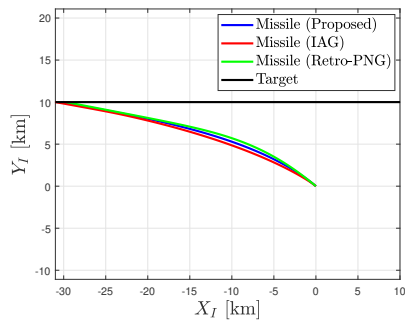


(c) Guidance Command

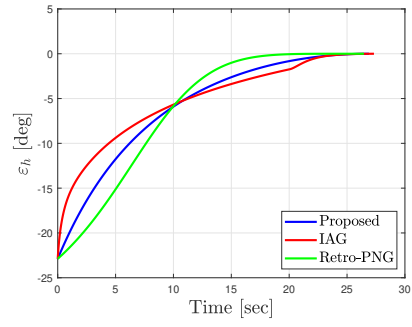


(d) Control Energy

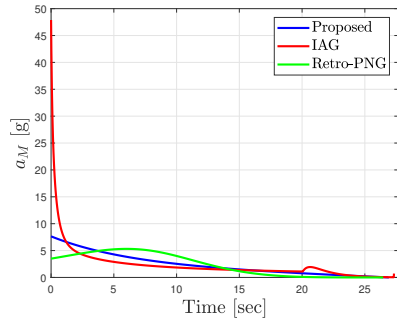
Figure 12: The comparison with other guidance laws under HO engagement.



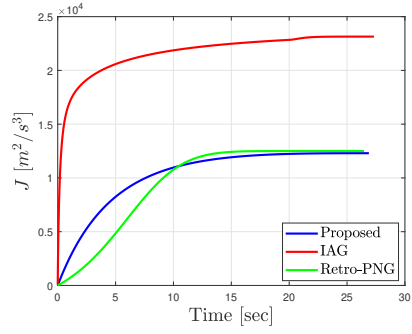
(a) Interception Trajectory



(b) Heading Error



(c) Guidance Command



(d) Control Energy

Figure 13: The comparison with other guidance laws under HP engagement.

a fair comparison. In retro-PN guidance, we use the guidance command, as shown in Eq. (49), with $N = 3$ for HO case and $N = -3$ for HP case. The initial flight path angles are chosen as $\gamma_{M,0} = 130^\circ$ for HO case and as $\gamma_{M,0} = 140^\circ$ for HP case.

The simulation results for HO engagement under the three guidance laws are provided in Fig. 12. As shown in Fig. 12 (d), IAG requires a huge acceleration demand at the initial time under HO engagement. In the case of retro-PN guidance, the acceleration demand blows up as the missile approaches the target, under HO engagement. As a result, the heading error does not converge to zero under retro-PN guidance, as shown in Fig. 12 (b). Additionally, as shown in Fig. 12 (d), the proposed method requires less control energy defined as $J = \frac{1}{2} \int_{t_0}^{t_f} a_M^2 dt$ than IAG and retro-PN guidance. Figure 13 shows the comparison of the three guidance laws under HP engagement. In this case, IAG also produces a huge guidance command at the beginning of the terminal homing phase, as shown in Fig. 13 (c). Additionally, Figs. 13 (b), (c), and (d) indicate that retro-PN guidance provides a similar guidance performance compared to the proposed method in this case, however it requires a slightly more control energy. Therefore, we can know that the proposed guidance law can provide satisfactory performance in terms of control energy, interception performance, and magnitude of guidance command under both HO and HP engagements, unlike the other guidance laws.

5. Conclusions

In this paper, a unified form of guidance laws for HO or HP collision courses is devised using nullification of the heading error. The proposed guidance law has a simple form, and it is composed of two terms: the feedback command for reducing the heading error and the bias command for rejecting the heading error rate due to the deformation of the collision course. The optimality analysis reveals that the first term in the proposed guidance law is the optimal feedback command. In this study, the relationship between the proposed guidance law and PN guidance laws is also investigated. Our

findings show that the proposed guidance law will provide a similar guidance performance of TPN guidance in a near-collision course under HO and HP engagements. It is found that TPN guidance can achieve both HO or HP interception geometries by choosing the interceptor's initial lead angle. Besides, it is shown that the proposed guidance law can be considered PPN guidance with time-varying navigation constant under HO engagement, or retro-PPN guidance with time-varying navigation constant under HP engagement. The magnitude of the guidance command depending on the choice of the desired collision course between HO and HP is investigated. The analysis results could be used for evaluating the reliability and the performance of the proposed guidance law. Moreover, the analysis results could be utilized as a guideline for choosing a more beneficial interception strategy between HO or HP as well as an appropriate guidance gain by guidance designers. The performance and feasibility of the proposed guidance law are examined through nonlinear simulations. The simulation results indicate that the proposed guidance law can provide satisfactory performance for intercepting a high-speed target.

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