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Individual Research Project
Assessment of Multiple Reference Frame (MRF) and Sliding
Mesh methods on a brake system and wheel model

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Computational Fluid Dynamics

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Academic Year: 2024–2025

Supervisor: Dr Tom Teschner
Associate Supervisor: Matthew Sorrell - Head of
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This thesis is submitted in partial fulfilment of the
requirements for the degree of MSc.

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Acknowledgments

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Abstract

This project investigates the application of the Multiple Reference Frame (MRF) and Sliding Mesh (SM) methods for simulating the aerothermal performance of a rotating wheel and brake disc assembly. The study aims to evaluate the accuracy and computational trade-offs between the steady-state MRF approach and the more computationally intensive, transient SM method, and to determine if a correction factor can be derived to enhance MRF results.

The methodology began with the aerodynamic validation of an isolated wheel model against experimental Particle Image Velocimetry (PIV) data, establishing solver settings and mesh independence. Later, a comprehensive model incorporating a 205/55/R16 tyre, rim, and a vented brake disc with pads and caliper was developed. Four steady-state MRF simulations were conducted at varying rotational speeds, and a single transient SM simulation was performed for comparison. Key quantities of interest included the drag coefficient (Cd), heat transfer coefficient (HTC), and temperature distribution.

Results indicated that both MRF and SM methods produced aerodynamically valid results for the isolated wheel, with the SM method capturing wake structures with marginally higher fidelity. For the full brake system, significant differences emerged. The SM method predicted a 14.7% higher HTC and a more uniform temperature distribution across the brake disc compared to the MRF result at the same speed, suggesting it more accurately captures the transient convective cooling. A strong linear relationship was observed between rotational speed and HTC in the MRF data. A correction factor was developed and implemented via a User-Defined Function (UDF) to scale the MRF-predicted HTC. While this adjustment successfully brought the maximum temperature prediction to within 2.2% of the SM value, it did not fully replicate the SM results for other thermal variables.

The study concludes that while the MRF method offers a computationally efficient approximation, the SM method provides a more physically accurate representation of the convective heat transfer process. A simple single-variable correction factor for MRF is insufficient to fully emulate SM results, as the differences are multi-variate and inherent to the fundamental approaches of each method. Future work should focus on longer-duration SM simulations for robust validation and exploring multi-variable correction methodologies.

Keywords:

Computational Fluid Dynamics (CFD), Multiple Reference Frame (MRF), Sliding Mesh, Brake Disc Cooling, Heat Transfer Coefficient (HTC), Aerothermal Analysis, Automotive Aerodynamics.

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1 Nomenclature

AR	Asymptotic range
C_d	Drag coefficient
C_f	Skin-friction coefficient
C_p	Pressure coefficient
C_s	Roughness constant
D	Wheel diameter [m]
d_i	Difference between ranks for Spearman's correlation
F_b	Braking force [N]
F_d	Drag force [N]
F_{normal}	Normal force applied by pads [N]
F_s	Safety factor for GCI
GCI	Grid convergence index
$\mathbf{g}$	Gravitational acceleration vector [m/s^2]
gr	Growth ratio for inflation layers
h	Representative grid size; Heat transfer coefficient (HTC) [m; $\text{W}/(\text{m}^2\text{K})$]
HBP	Hydraulic brake pressure [Pa]
k	Thermal conductivity; Turbulent kinetic energy [$\text{W}/(\text{mK})$; m^2/s^2]
k_L	Laminar kinetic energy [m^2/s^2]
N	Number of inflation layers; Number of data points
N_{pads}	Number of brake pads
N_{pis}	Number of pistons in the caliper
P_k	Production of turbulent kinetic energy [m^2/s^3]
$P_{\text{dyn,ref}}$	Dynamic reference pressure [Pa]
p	Pressure; Order of convergence [Pa; -]
p_∞	Freestream pressure [Pa]
$\dot{Q}$	Total heat generation rate [W]
q''	Heat flux [W/m^2]
q'''	Volumetric heat source [W/m^3]
R_e	Effective radius for brake torque [m]
Re	Reynolds number
$\mathbf{r}$	Position vector [m]
r	Grid refinement ratio
S	Volumetric energy source term (UDF) [W/m^3]
S_ϕ	Volumetric source term for a scalar
T	Temperature; Rotation period [K; s]
T_{avg}	Average temperature [K]
T_{max}	Maximum temperature [K]
T_o	Temperature at standard conditions [K]
T_{ref}	Reference temperature [K]
$T_{\text{tot,ref}}$	Local stagnation temperature [K]
T_{wall}	Wall temperature [K]
t	Time [s]
Δt	Time step size [s]
$\mathbf{u}$	Fluid velocity vector [m/s]

u	Freestream flow velocity [m/s]
$\mathbf{u}_g$	Mesh velocity vector [m/s]
$\mathbf{u}_r$	Velocity relative to a rotating frame [m/s]
u_{ref}	Reference velocity [m/s]
u_τ	Friction velocity [m/s]
V	Volume of a cell or domain [m ³]
V_x	Streamwise (x) velocity component [m/s]
y^+	Dimensionless wall distance
y_1	First layer height [m]
Δy	First layer height [m]
y_t	Total height of inflation layers [m]
α	Thermal diffusivity [m ² /s]
$\beta_0, \beta_1, \beta_2$	Regression coefficients
γ	Rate of specific heats for air
Γ	Diffusion coefficient
ΔP	Pressure drop [Pa]
ε	Relative error
ε_i	Residual error in regression
θ	Angular position on the wheel [°, rad]
μ	Dynamic viscosity [Pa s]
ν	Kinematic viscosity; Eddy viscosity [m ² /s]
ν_T	Turbulent eddy viscosity [m ² /s]
ρ	Fluid density [kg/m ³]
ρ_{ref}	Reference density [kg/m ³]
ρ_∞	Freestream density [kg/m ³]
τ_b	Brake torque [N m]
τ_w	Shear wall stress [Pa]
τ	Viscous stress tensor [Pa]
ϕ	Transported scalar quantity
ϕ_{ext}	Extrapolated value from grid convergence study
ω	Specific dissipation rate; Angular velocity [1/s; rad/s]
Ω	Angular velocity vector [rad/s]

2 Introduction

2.1 Project Background

In the automotive industry and motorsports applications, the wheel and brake system are components that need to be considered in a deep analysis so that their reliability can ensure the vehicle's safety and performance. Brakes can be subjected to thermal fatigue, friction vibration and correlation between them, which can lead to structural damages as presented by Yang (1). Additionally, brakes can present overheating with excessive wear, cracking or fluid vaporisation according to Emery (2). This has led to studies to improve the cooling performance by testing different cooling vanes configurations, adding the use of Computational Fluid Dynamics tools through the design process, like the work pre-

sented by Li (3) or Jafari (4).

In the braking phase, the friction between the brake disc and the pad generates heat, and although this heat is dissipated through the process of convection to the ambient, the adjacent components are affected by the conduction and radiation (Cravero (5)). Therefore, it is necessary to introduce such additional components to the analysis, among them, there are the tyre and wheel. The brake's overheating can influence the tyre and rubber performance by means of temperature variations, which alter the rim and tyre's air and this, its pressure Stefanelli, (6). Also this temperature increase can influence overheating of the internal surfaces of the tyre and contribute to degradation phenomena like blistering, West (7). An additional interest on tyre analysis is that forces like driving, cornering, and braking are created by tyres according to Katz (8), and aerodynamic loads may affect car's performance through the influence of drag and lift on the car's tyres and their interaction with additional vehicle's components such as the vehicle chassis or the front wing. In this scenario, Cravero (5) presented first an isolated wheel analysis with stationary and unsteady flow conditions to later study the interactions of flow structures with a Formula 1 car's front wing.

Additionally, it is important to identify the brake's functionality before applying any computational analysis. A brake system consists of the brake disc, which is assembled to the wheel hub, both rotating with the same rotational speed. When the brake pedal is pressed, the hydraulic pump provides the brake fluid to the caliper. This fluid pressure acts on caliper's pistons to push the pads against the disc brake and thus, it generates the friction between the pads and disc permitting that a vehicle can stop, Li (3). For vehicles with these friction braking systems, the friction between the brake disc or rotor and the pads fosters the rate of heat generation, and heat is dissipated through convection to the surrounding flow and through conduction and radiation to the close components.

Computational Fluid Dynamics has become a useful and regular tool to analyse the aerodynamic influence of vortical structures on the rest of the car's components, but it has also been used to optimise the cooling design of the brake disc and test different internal vane configurations which affect the heat transfer coefficient (HTC). The assessment of a rotating geometry, such as the wheel or a brake disc, can be analysed based on stationary or rotating assumptions, and in most studies, the analysis has been limited to isolated wheels. This approach is helpful to test different turbulent and meshing methods. Axerio-Cilies (9) studied and isolated a 60% scaled Formula 1 wheel with rotating and stationary conditions to assess the flow structures and their accuracy through the use of URANS (Unsteady Reynolds Average Navier-Stokes) by comparing against experimental results.

One of the main objectives of different studies has focused on assessing the tyre rotation method. CFD codes such as Ansys Fluent gives the opportunity to use different numerical methods for rotational geometries, which have been used on different studies. Among them, it can be found the Rotating Wall (RW), Sliding Mesh (SM) and Moving Reference Frame (MRF). Patel (10) used the RW method to investigate the wake behind an isolated wheel and compared the results against PIV (Particle Image Velocimetry) data obtained through a wind tunnel experiment for a 50% scaled cover wheel; the covers

were applied on both sides of the wheel, neglecting the influence of the rim. In the case of MRF, Zhao (11) used MRF and RW methods on different regions of the tyre and brake disc to assess the different topologies of the cooling vanes and their influence on the enhancement of the heat dissipation. And Schnepf (12) used a Sliding Mesh approach to assess the flow around a rotating wheel with a Lattice-Boltzmann solver and full SM applied to the tyre without ground contact, and compared its results to experimental data with similar conditions for the contact patch. Additionally, it is possible to use all three methods on the same geometry to catch the different effects of the different regions of the tyre and wheel on the flow structures and velocity estimations such as Hobeika (13), who uses a hybrid approach by applying MRF, MW and SM on the tyre's grooves, tyre contact surface, and rim, respectively.

2.2 Aim and objectives of the project.

In those studies, the different conditions to model rotating geometries have been used to evaluate the flow structures and aerodynamic load to compare with experimental data in the case of tyre/wheel analysis, or they have been used in the design process of the brake disc and its different vanes configurations. It can also be found that for a tyre-only system that the MRF method can offer a superior computational efficiency, and the SM method presents a higher accuracy according to Hobeika (13). However, there is no evidence of extensive studies that are focused on the difference between MW, MRF and SM methods regarding their accuracy and computational cost applied on both aerodynamic and heat transfer for a wheel and brake system.

Therefore, the present project aims to investigate the differences and similarities among MRF and SM methods on a full tyre, rim and brake system, by comparing quantities of interest such as the drag coefficient, maximum temperature and heat transfer coefficient (HTC) between both methods and experimental data found in the literature review. In addition, it will be determined whether a correction factor or expression could be obtained that could be used for MRF simulations and that could approximate the SM results, reducing computational time.

The project uses first an isolated wheel without a rim and with lateral covers, with a moving wall (MW) approach on the wheel to establish the solver settings and perform a grid convergence study. Then, the MRF and SM methods are used on the same model to validate the pressure coefficients and aerodynamic loads against experimental data. Finally, a complete wheel, rim, and brake system model is used to evaluate the influence of the MRF and SM methods on the temperature distribution and HTC values. With this analysis, an expression is derived for a correction factor, and its use in the MRF method to approximate the SM results is assessed.

3 Literature Review

The literature review will examine the different studies performed on wheel models and brake systems, and the CFD setups used on them. It will also show those studies which have been validated against an experimental approach.

3.1 Rolling wheel aerodynamic study

The performance of a race car relies on the efficiency of its aerodynamics, and the influence of the wheels is one of the main components to be analysed due to they can be responsible of 30% to 40% of the total drag according to Elofsson (14) and Katz (15). CFD tools have been used to perform a simulation and obtain flow features through the comparison of the pressure coefficient C_p . Croner et al (16) introduce an unsteady study to determine the flow's structures by using URANS simulations on an isolated wheel model and perform a direct comparison with experimental data of the same model obtained by PIV and hot wire anemometry measurements in the Aero Concept Engineering (ACE) facility in Magny-Cours, France. Although the study only considers the moving wall condition for the entire wheel and the wheel has covers on both lateral sides, it provides a comparison between numerical and experimental data of the C_p and the velocity in the x component V_x for different planes behind the wheel. Therefore, the geometry model and the experimental data obtained will be useful for the present project, as they provide a good insight into calibrating the wheel model, which is going to be used to analyse the different MRF and SM approaches.

Likewise, Patel (10), uses a similar model and wind tunnel validation through PIV as well, to investigate the wake of an isolated, rotating wheel and the development of different vortex systems generated by the rotation of the wheel. Previously, Diasinos (17) established the different ground vortex that could be formed behind the wheel. Although Patel found differences in the wake for the mean and instantaneous flow-fields, the comparison between CFD and the data from PIV showed good agreement. Therefore, the PIV data obtained from Patel's study is being used in the present project as a qualitative comparison of the different structures obtained through the MW, MRF and SM methods.

3.2 Moving Wall, Multi-Reference-Frame, Sliding Mesh

Moving wall (MW), Multi-Reference-Frame (MRF) and Sliding Mesh (SM) are different techniques that have been used for modelling wheel rotation. The works from Croner (16) and Patel (10), use a wheel as a rotating no-slip condition wall and an angular velocity so that the tangential velocity of the wheel match with the freestream velocity. In the work from Fu (18), a comparison between the three methods is performed only through the values of the drag coefficient C_d obtained on the contact surface area of the tyre, however, a comparison of the C_p between the methods is omitted. A similar comparison of the C_d over the contact surface of the tyre is performed by Mears (19), thus, this analysis is implemented on the present project as an additional tool to assess the CFD methods between

them and against experimental data.

The work of Fu also presents an interesting insight regarding the use of SM. To avoid singularities in the mesh and high skew on its cells, a patch contact is necessary between the tyre and the ground. If a sliding mesh method is used around the entire tyre geometry, the contact patch will also rotate and therefore, a hybrid approach can be required for the model with SM and MW. This hybrid approach is also used for the different tyre regions, such as the tyre, rim and tyre's grooves according to Hobeika (13). Hobeika uses this analysis to evaluate the assignment accuracy of different methods in different regions, and although initially, his work does not consider the influence of ground, it is useful to identify the areas to which it is appropriate to assign MRF, SM or MW in the case of our project to analyse both the wheel and the brake system.

3.3 Brake system and thermal analysis

The brake system is a crucial component in a vehicle's safety. And it is a component able to convert kinetic and potential energy into thermal energy. Through the friction between pads and the brake disc, a deceleration rate is achieved, but also a heat generation which can be dissipated through convection to the surroundings or this heat can be transferred through conduction and radiation to the adjacent components. High temperatures can be the cause of excessive wear and failure of brakes Day, A.J (20). To analyse the conditions of brake systems, CFD tools have been used. Cravero (5) analyses the flow and the heat transfer after a braking event for a wheel of an F1 car and its interaction with a front wing. His study focuses on the heat extraction after a braking event and not on the braking process and the heat transfer from the pads to the brake disc. However, the study obtains a thermal source of 60.2 MW/m³, which is similar to the one calculated for the purpose of the present project and will be discussed in the methodology section.

The work presented by Vdovin (21) presents a coupling between FEA (Finite Element Analysis) and CFD to predict the temperatures of the brake system components during a brake test. Although this coupling methodology is beyond the scope of the current project, Vdovin's work gives experimental data regarding the temperature reached by the braking test and provides information on the braking force which has been used in this test. This information is being used as an entry for the current project to determine the volumetric thermal source that was used for the pads. Thus, it is possible to use the experimental validation from Vdovin to compare against the MRF and SM results for the brake simulations. Additionally, Vdovin's paper gives a list of the different components that can be used in the analysis of the brake system. The exact model used by Vdovin was not used in the current project, but it was considered the components such as the pads, caliper, brake disc, upright-knuckle, hub and the cooling duct, which are common on configurations for race-car. Zhang (22) also uses a CFD and FEA coupling as a method to solve first the aerodynamic loads and compare the C_d with experimental data, and then uses FEA for the thermal analysis. The purpose of the paper is to present a grey relational analysis and entropy weight method to provide suggestions for the design process of wheel and brake cooling vanes, and although the methodology is different concerning the scope and ob-

jective of the present project, data such as material properties of brake pads and disc are valuable as well as the maximum temperature and HTC data observed by Zhang through a steady state and RANS scenario and by using MW condition for the wheel and brake disc.

Dekker (23) provides a thermal analysis focused only on the brake disc. By using MRF and MW on the brake geometry and experimental validation, the paper assessed the importance of different parameters on the cooling performance for a brake disc. The author explores how different factors, such as the turbulence models or the conditions, techniques to treat the rotating wall of the brake disc. Even though here only a brake disc is evaluated, the data of the relation between temperature, HTC and rotational speed is used to compare the results obtained with the MRF and Sliding Mesh approaches for the full wheel and brake system model of the present project.

4 Methodology

This section is going to introduce first the geometry used for the modelling and its domain, also a brief introduction into the theoretical background used for the CFD solver, a description of the brake system is presented as well, and the statistical tools are shown, which are used for an additional quantitative comparison.

4.1 Domain modelling

4.1.1 Wheel only domain modelling

An isolated wheel is used initially as a model to calibrate the solver settings. The wheel corresponds to a 205/55/R16 standard tyre with a width of $L = 0.220$ m and an external diameter of $D = 0.631$ m. This wheel is covered on both lateral with circular smooth covers, and a support is attached to one of the lateral covers as presented in Fig. 1. The wheel-only model used for this project corresponds to the experimental model used by Croner (16), and it has been validated through an experimental setup in the Aero Concept Engineering (ACE) facility in Magny-Cours, France with Particle Image Velocimetry (PIV).

The domain of this model resembles the ACE wind tunnel Aero-CE (24), the width = 2.3m and height = 2.5 m from the ACE wind tunnel are used as presented in Fig. 2. However, for the length of the domain a value of 6.4 m is used following the Axerio-Cilies (9) advice for the upstream and downstream lengths with respect to the centre and diameter of the wheel, according to Tab. 1 for the Moving Wall case (MW) and Fig. 3. An additional body of influence has been added to create a refinement around the wheel and behind from it to capture the wake.

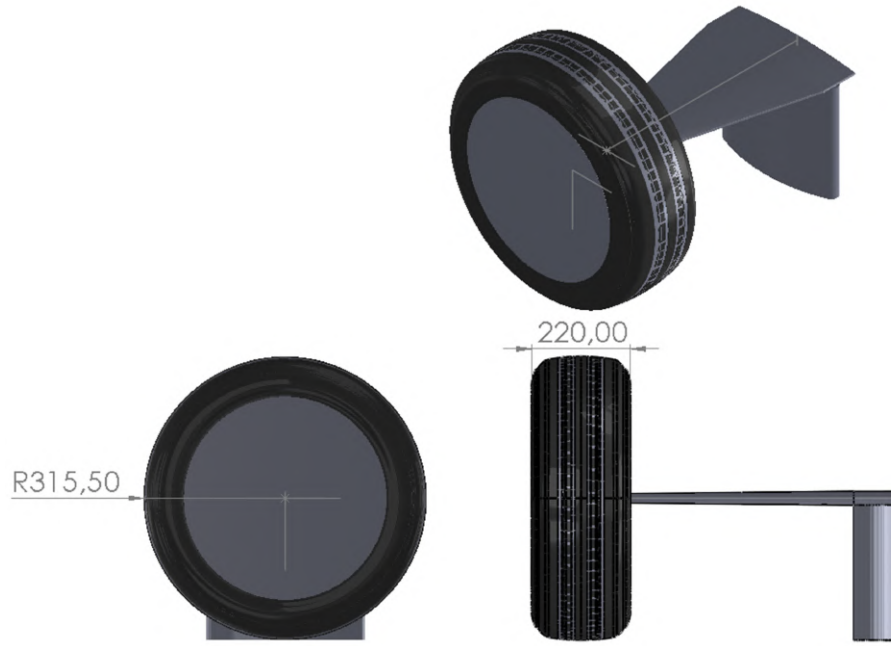


Figure 1: Wheel and support model

Length	Relation Value Case MW	Relation Value Case MRF/SM
Upstream	4.5D	8.1D
Downstream	4.9D	14.8D
Total Height		16.4D
Total Lateral		16.4D

Table 1: Upstream and downstream lengths relations for the domain boundaries. For MW case and MRF/SM case

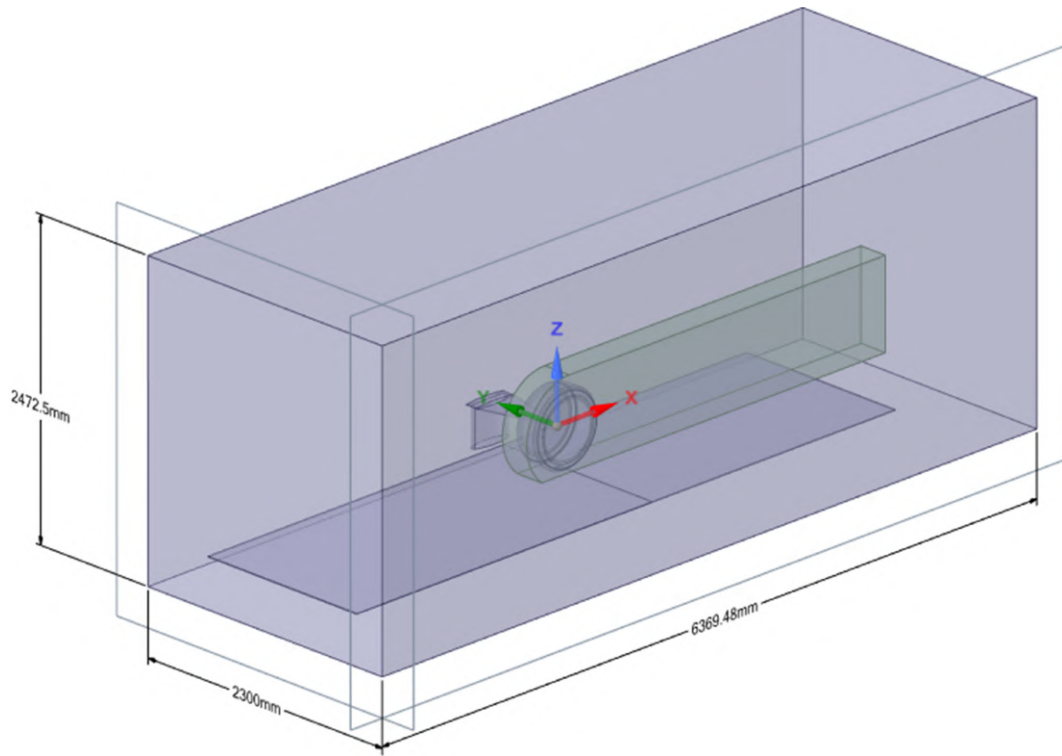


Figure 2: Domain dimensions isolated wheel model

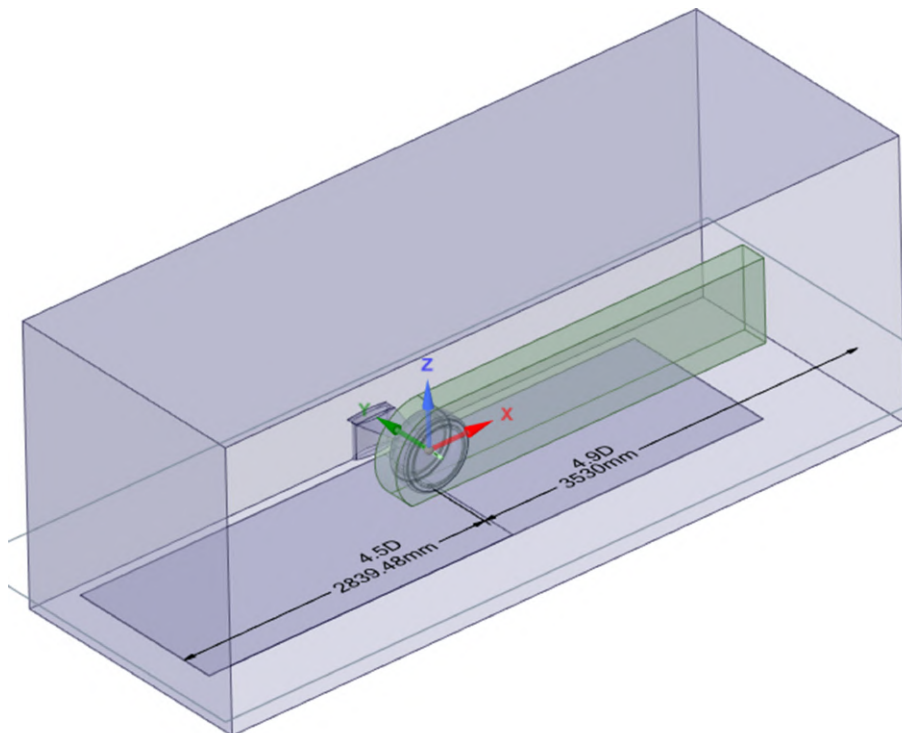


Figure 3: Dimensions of the boundaries in the domain for the isolated wheel model

As can be seen in the Fig. 2 for the domain, a ground has also been added under the wheel. A patch has been added, as Mlinaric, P (25), accentuated the importance of the

contact patch. Thus, following the procedure from Croner (16) to reproduce the ground contact of the wheel, the contact patch was created with a cut at 0.5 mm from the bottom of the wheel as presented in Fig. 4

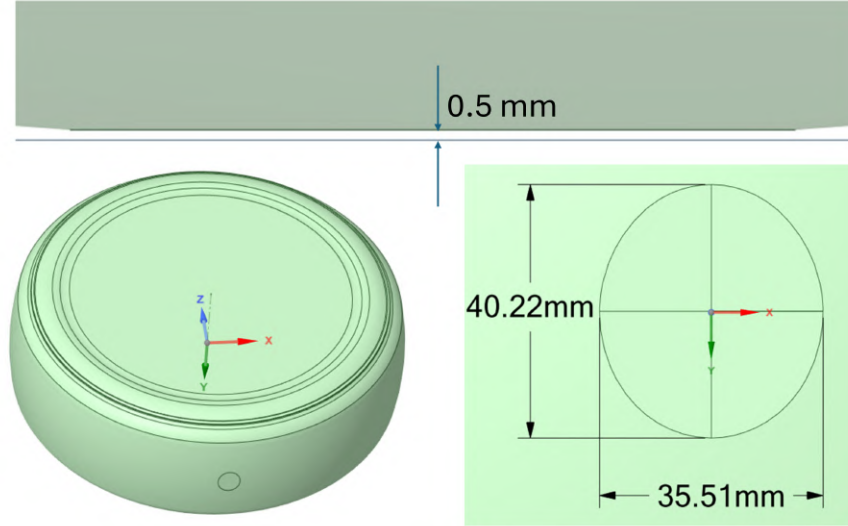


Figure 4: Contact patch treatment

This model is used for the moving wall condition for the rotational modelling of the tyre, and it is used for three different meshes with different refinement levels: Coarse, Medium and Fine meshes.

4.1.2 Wheel only domain modelling for MRF and SM

For the MRF and SM cases with only the wheel geometry, the domain dimensions changed with respect to the previous case. This new approach follows the upstream, downstream, total width and total height, which is used by Hobeika (13) and Patel (10) and is presented in Tab. 1 for the MRF/SM case and observed in the Fig. 5, where the length, width and height of the domain are shown as well.

An additional difference with this modelling is the necessity of creating additional enclosures, which later will be used to define a cell-zone for the Frame Motion (MRF) and the Mesh Motion (SM) in Ansys Fluent. The enclosure for the MRF/SM zone is defined surrounding the geometry concerning the rim, where, in this simplified model of the wheel, the rim has been replaced by lateral covers. As can be seen in the Fig. 6 in the cross section area, the lateral covers are contained within the SM/MRF enclosure (grey area), which has a tangential contact with the wheel, but conserving a gap of 1.45mm between the wheel and lateral covers surfaces. This gap is considered to avoid interferences for the moving and stationary zones when the SM case is used.

This type of configuration, where only the internal geometries are subjected to the MRF/SM techniques, exhibits that a hybrid approach is needed. Therefore, a moving wall condition (MW) is used for the wheel geometry, which is the zone in contact with the ground, whereas for the internal components, the MRF and SM methods are applied. This

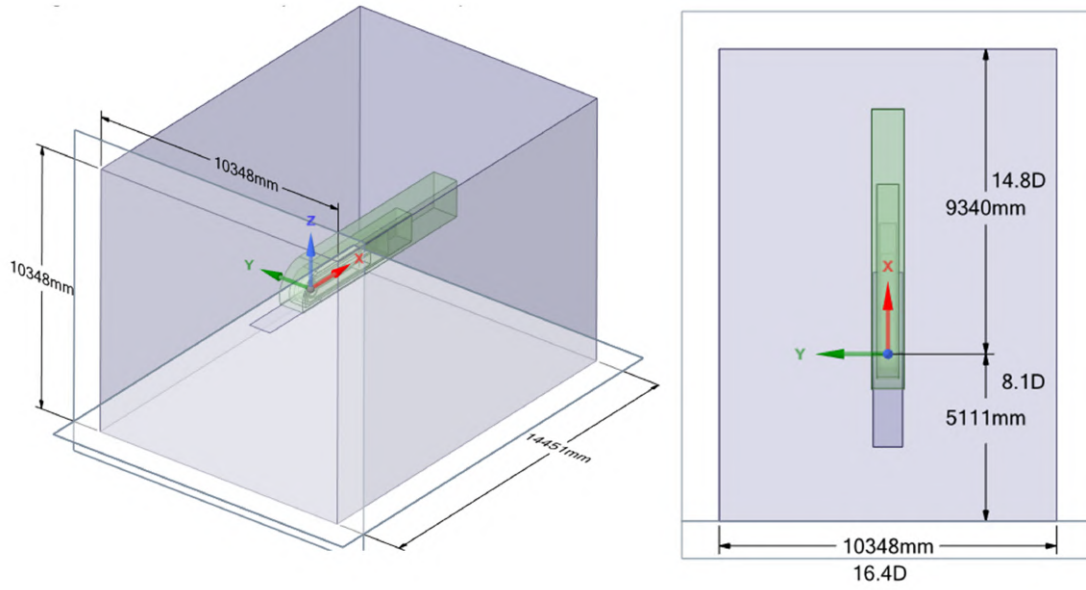


Figure 5: MRF-SM domain of the isolated wheel

approach is similar to the method concluded by Fu (18), and it is explained by the presence of the ground and contact patch, which can physically rotate with the SM method.

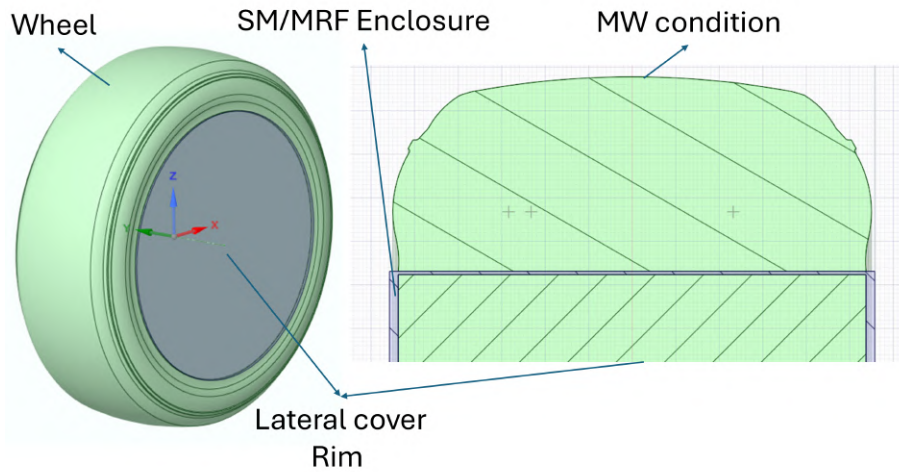


Figure 6: MRF-SM enclosure treatment for the isolated wheel model

4.2 Meshing the isolated wheel model

4.2.1 Boundary Layer and First Layer thickness estimation

As is necessary to ensure a resolution for the flow near the wall, the first layer thickness of three different refinement meshes was calculated based on the dimensionless wall distance (y^+). The calculus follows the turbulence modelling guidelines as established by Cengel (26) or in guides from Fluent (27), where the first cell height is determined

to achieve a specific value of (y^+), allowing the resolution of the viscous sublayer. The Reynolds number was first determined using the characteristic diameter of the wheel as Equation (1), the fluid properties and velocity data were obtained similarly from the work from Croner (16) to emulate their experimental results.

$$Re = \frac{\rho V D}{\mu} \quad (1)$$

The skin-friction coefficient was obtained from empirical correlations, and the wall shear stress was calculated from Equation (2)

$$C_f = \frac{2\tau_w}{\rho V^2} \quad (2)$$

$$\tau_w = \rho u_\tau^2$$

With the skin-friction coefficient and the wall shear stress, the friction velocity was defined as Equation (3)

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \quad (3)$$

By defining the non-dimensional wall coordinate (y^+) value for each of the mesh refinements: 10,4,1 for the coarse, medium and fine levels, it was possible to determine the first cell height for each case with Equation (4)

$$y^+ = \frac{u_\tau y_1}{\nu}$$

$$y_1 = \frac{y^+ \nu}{u_\tau} \quad (4)$$

Additionally, to estimate the total height of the successive inflation layer the Equation (5) was used, where gr is the growth ratio, 1.2 for all the cases, and N is the specified number of layers: 10, 15 and 20 layer for the coarse, medium and fine mesh. These y_i values were important when the boundary layer was applied to zones of MRF/SM enclosures to ensure that the inflationary layers could not create interferences between the moving and stationary cells (SM case).

$$y_i = y_1 \cdot \frac{1 - (gr)^N}{1 - gr} \quad (5)$$

4.2.2 Mesh Convergence Analysis

To ensure that the numerical results obtained in this first section for the isolated wheel are independent of the discretisation, a spatial convergence was assessed. The methodology follows the Richardson extrapolation technique and the Grid Convergence Index (GCI), both specified by Celik (28).

The characteristic mesh size h was defined based on the number of cells N for the computational domain of volume V , thus h is calculated with Equation (6)

$$h = \left(\frac{V}{N} \right)^{1/3} \quad (6)$$

For the three refined mesh levels, the refinement ratio is defined as Equation (7), where h_1 , h_2 , h_3 are the fine, medium and coarse meshes, respectively.

$$r_{21} = \frac{h_2}{h_1}, \quad r_{32} = \frac{h_3}{h_2} \quad (7)$$

The apparent order of convergence p is obtained iteratively using Equation (8), where ϕ_1 , ϕ_2 , ϕ_3 are the numerical solution obtained on each mesh refinement. This value is expected to be near the discretisation order used in the solver.

$$p = \frac{1}{\ln(r_{21})} \left| \ln \left(\frac{\phi_3 - \phi_2}{\phi_2 - \phi_1} \right) \right| \quad (8)$$

Through the Richardson extrapolation in Equation (9), the extrapolated value of the solution as the grid size tends to zero is defined. This estimates a solution towards which the discretisation is expected to converge. For the relative errors between the consecutive mesh levels, the equation Equation (10) is used

$$\phi_{\text{ext}} = \frac{r_{21}^p \phi_1 - \phi_2}{r_{21}^p - 1} \quad (9)$$

$$\varepsilon_{21} = \left| \frac{\phi_1 - \phi_2}{\phi_1} \right|, \quad \varepsilon_{32} = \left| \frac{\phi_2 - \phi_3}{\phi_2} \right| \quad (10)$$

Finally, the GCI is used to quantify the numerical uncertainty related to the mesh discretisation by using the equation Equation (11), where $F_s = 1.25$ is a safety factor recommended for typical CFD applications as Celik (28). And to verify that the solutions are within the asymptotic range of convergence the asymptotic ratio is obtained with eq Equation (12), where an AR nearly equal to 1 is desired.

$$\text{GCI}_{21} = F_s \cdot \frac{\varepsilon_{21}}{r_{21}^p - 1} \cdot 100$$

$$\text{GCI}_{32} = F_s \cdot \frac{\varepsilon_{32}}{r_{32}^p - 1} \cdot 100 \quad (11)$$

$$AR = \frac{r_{21}^p \cdot \text{GCI}_{21}}{\text{GCI}_{32}} \quad (12)$$

4.2.3 Meshing

Ansys Fluent Meshing was used to create the different meshes. For both the isolated wheel model and the model of the isolated wheel used for the MRF and SM methods, different groups were created from Spaceclaim to give them different treatments of localizing. The Tab. 2 offers a visual of the zones and their local sizing technique for the isolated wheel model used with MW and the isolated wheel model for the MRF and SM methods. For the MRF/SM model, a total of four bodies of influence were added, with a

Group (MW-Model)	Local Sizing	Group (MRF/SM-Model)	Local Sizing
Farfield	Face Size	Farfield	Face Size
boi-wake	Body Of Interest	boi-wake-4	Body Of Interest
wheel contact	Curvature	boi-wake-3	Body Of Interest
wheel side	Curvature	boi-wake-2	Body Of Interest
support-pilar	Face Size	boi-wake-1	Body Of Interest
support-arm	Face Size	wheel contact	Curvature
belt surface	Face Size	wheel side	Curvature
belt laterals	Face Size	rim	Curvature
		ground-up-low	Face Size
		ground-up+ref	Face Size
		ground-laterals	Face Size

Table 2: Groups and local sizing techniques used for the wheel isolated models with the MW method and the MRF/SM method

growth ratio between them of 1.3.

To add the boundary layers, the first layer thickness was used according to section 3.2.1 for the following groups: In the case of the isolated wheel for the MW method, the boundary layers were added to the entire wheel geometry, which corresponds to the contact surface of the wheel and the side covers, additionally they were added to the upper ground surface behind the wheel so the vortical structures could be captured. In the MRF/SM wheel isolated model, the boundary layers were added separately for the contact surface and the lateral covers of the wheel, and for the ground surfaces behind the wheel, as well.

For the wheel surfaces, the boundary layers were created with a Uniform method for the offset method type, which depends on the growth rate and the first layer height. Whereas for the ground refinement, a smooth transition is used, which is a robust method and is defined for the number of layers, transition ratio and growth rate. These different values for the smooth transition are obtained from the y^+ and the first layer height for each of the mesh refinement levels and cases.

4.3 Brakes modelling

4.3.1 Brakes geometry and domain for the MRF and SM methods

The brake system constitute one of the most critical components of a vehicle under any work conditions, as stated by Vdovin (21), due to its crucial role on safety scenarios related to its capacity to convert the kinetic energy into heat, and the inherent performance issues related to overheating, which can lead to excessive wear. For the second phase of the present project, the wheel and rim are going to be considered along side with some of the main brake system components, presenting a modified design from the one used by Vdovin (21).

The model for the brake system is composed of the components exhibited in Fig. 7.

Where: 1)Caliper, 2)Two pads, 3)The hub, 4)the wheel nut, 5)Duct scoop which is a usual component that helps to cool down the brake components, 6)Brake bell, 7)Brake Disc, 8)Upright knuckle where the suspension bars are attached to, 9)The 205/55/R16 standard tyre, 10)Rim. The brake disc consists of 40 radial cooling vanes.

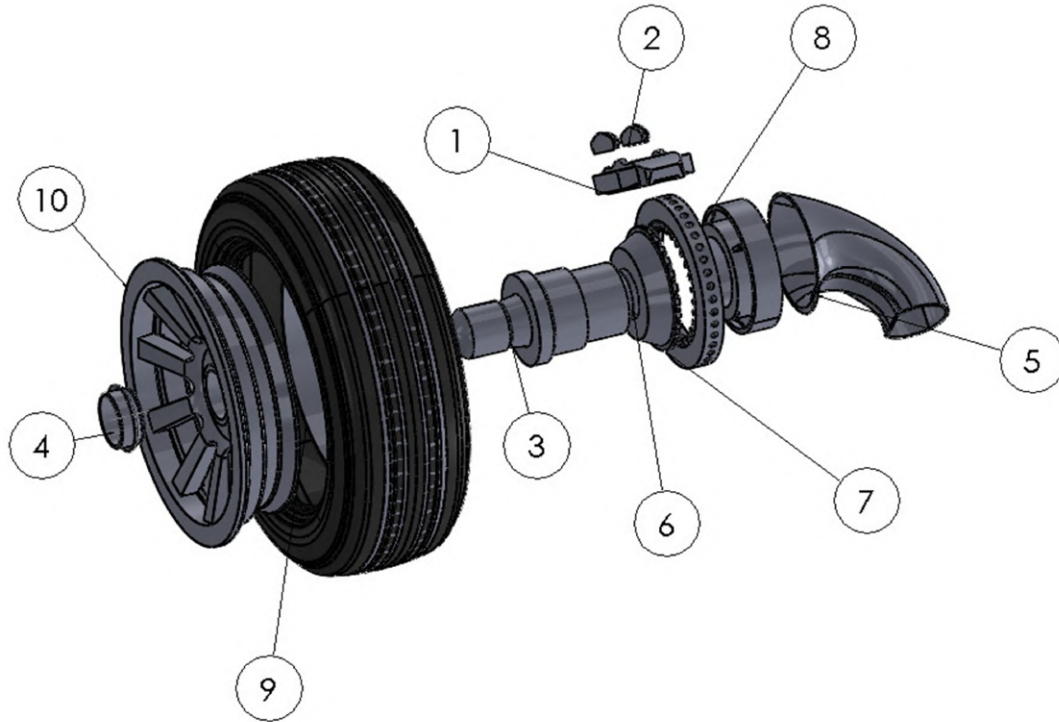


Figure 7: Components used in the brake disc and wheel model

For the domain, the width and length from the domain of the MRF/SM of the isolated wheel model were used. This is Width=10.3m, Length=14.5m, however the height of the domain for the brake system model was modified so that the bottom surface of the enclosure would be close to the lower surface of the ground. This decision was made due to the fact that the vortical structures formed behind the wheel are limited by the ground, which differs from the analysis of Hobeika (13) where no ground is considered initially. The dimensions of the domain/fluid enclosure for the brake and wheel system is shown in Fig. 8

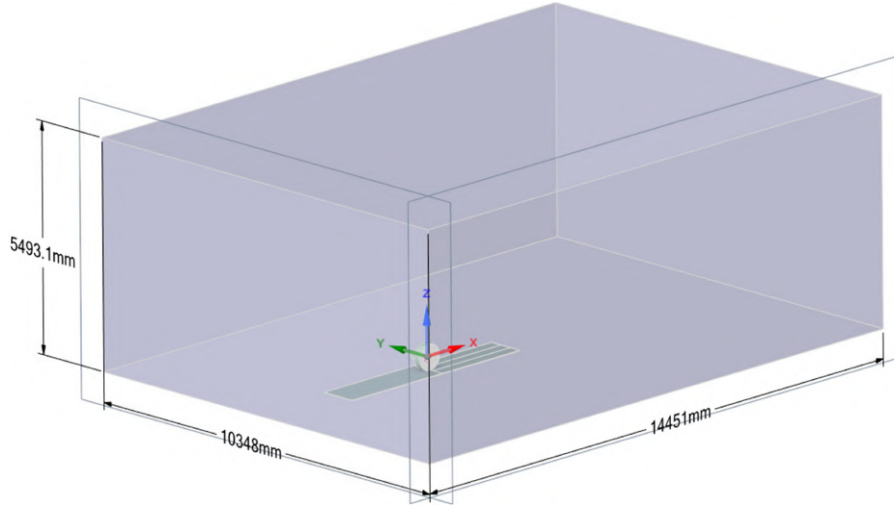


Figure 8: Domain for the brake and wheel model

Two additional enclosures were added for the rim and the brake disc. These enclosures are used for the designation of fluid zones as frame motion (MRF) and mesh motion (SM). For the rim enclosure as shown in Fig. 9, a gap between the tyre and the rim was left and also between the rim and the hub, but the rim-enclosure surface is tangential to the surface of the tyre, additionally, the enclosure here is not a cylinder which covers the rim geometry, but it is an enclosure that follows the rim contour, reducing the amount of cells which are going to be rotating in the case of SM, this procedure has already been used by Haag (29) where concluded that the closer the SM/MRF enclosures are to the rim, the better agreement is achieved between the simulation and the experimental data.

For the brake disc enclosure, the enclosure consists in a geometry similar to a cylinder that follows the brake disc geometry. There was left a gap between the solid components of the brake disc and the pads, and also between the brake disc and the hub and brake bell, as can be seen in the Fig. 10. This means that the solid pads are directly in contact with the surface of the brake disc enclosure, and that the enclosure is tangential to the surfaces of the hub and bell. Additionally, 40 bodies were created to fill the vanes cavities, this has the purpose of avoiding the use of capping in Fluent Meshing and then assigning them as fluid zones to have an accurate air flow through these cooling vanes, also a body for the internal volume of the duct scoop was created with the same purpose.

4.3.2 Meshing

The meshing process for the brake system and wheel model followed a similar process to the one used previously. However more groups were assigned due to the amount of components, four bodies of influence were used as used with the MRF/SM wheel isolated model. One only group was assigned to the rim, but for the brake disc surfaces, it was divided into five zones denoted as Disc-mid and corresponding to the zones of the inlet and outlet surfaces of the vanes, the internal surfaces of the vanes, the laterals faces of the brake disc and the cone of the brake disc which is the internal circumference of the disc that is in contact with the hub. The Tab. 3 exhibits these groups and the groups assigned to the other components of the model and the local sizing technique used for each of them.

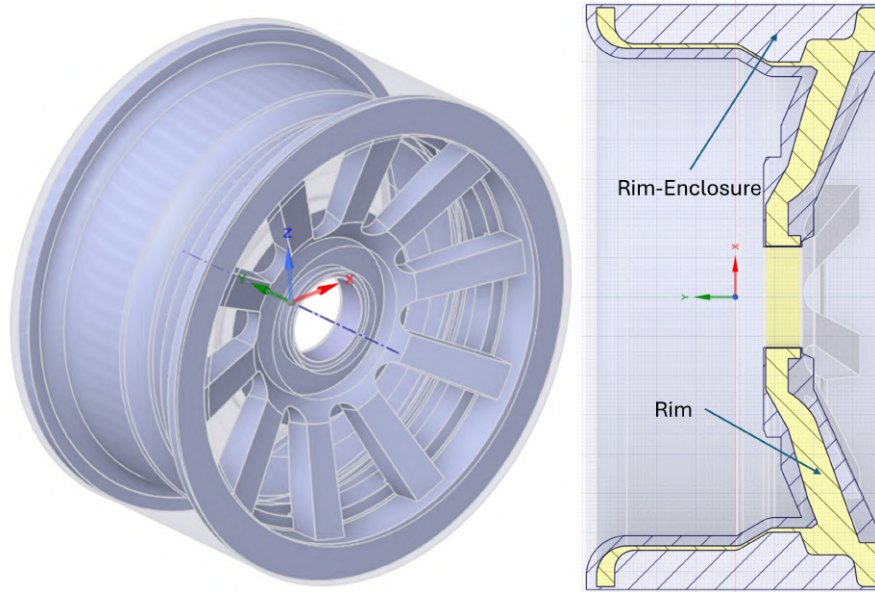


Figure 9: Enclosure MRF/SM for the rim

Group (BrakeDisc/Wheel model)	Local Sizing	Group (BrakeDisc/Wheel model)	Local Sizing
Farfield	Face Size	Hub	Face Size
boi-wake-4	Body Of Interest	Wheel nut	Curvature
boi-wake-3	Body Of Interest	Caliper	Curvature
boi-wake-2	Body Of Interest	Pads	Proximity
boi-wake-1	Body Of Interest	Bell	Curvature
ground-up	Face Size	Disc-lat1	Curvature
ground-laterals	Face Size	Disc-lat2	Curvature
ground-ref-up	Face Size	Disc-mid-lateral	Curvature
tyre	Curvature	Disc-mid-ven-in	Curvature
rim	Curvature	Disc-mid-ven-out	Curvature
Duct-scoop	Curvature	Disc-mid-vanes	Curvature
upright-knuckle	Face Size	Disc-mid-con	Curvature

Table 3: Groups and local sizing techniques used for the brake system and wheel model

Boundary layers were assigned to the different zones of the brake system and the wheel, following the same theory of the boundary layer and first layer thickens used with the previous cases. The Tab. 4 summarises the offset method used for each zone.

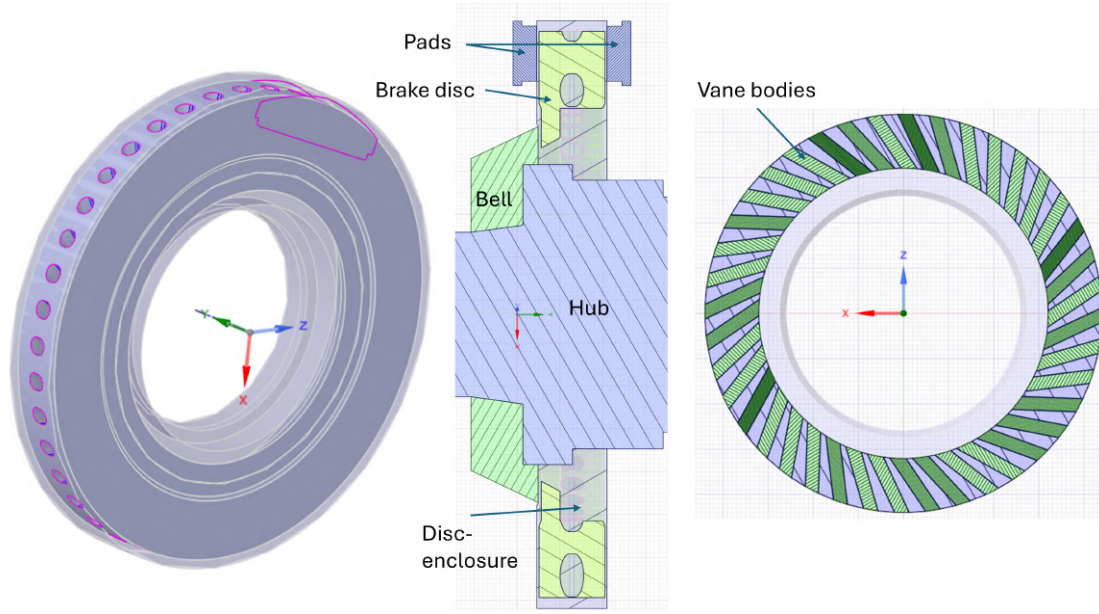


Figure 10: Enclosure MRF/SM for the brake disc and cooling vanes fluid bodies

Zone	Offset Method
ground-ref-up	smooth transition
tyre	Uniform
Rim-total	Uniform
Disc-mid-laterals	Uniform
Disc-mid-in	Uniform
Disc-mid-out	Uniform
Upright	smooth transition
Caliper	smooth transition
Pads	smooth transition
Disc-mid-vanes	Uniform
Duct-scoop	Uniform
Bell	smooth transition
Hub	smooth transition

Table 4: The boundary layer offset methods used for different zones for the brake system and wheel model

In this case, were used not only fluid regions, but also solid regions. The fluid regions were set to the full fluid enclosures and the MRF/SM enclosures of the brake disc and rim, the internal bodies of the vanes and the duct scoop were also designed as fluid. However, as it is desired to capture the physical heat exchange between both pads and the brake disc, these were assigned as solid regions to later give them their material properties. Therefore, the geometry description for the meshing watertight workflow was specified as both fluid and solid regions without the use of capping but it is specified as a non-conformal mesh because this would be necessary for the SM method, where moving cells are rotating relative to a stationary region and the cells will be changing their position.

4.4 Theoretical background and solver settings

4.4.1 Moving wall, Multiple Reference Frame, Sliding Mesh methods

4.4.1.1 -MRF - Multi Reference Frame Model: Allows to divided the computational domain into subregions that are solved in different reference frames. Used in steady state it can be assigned to individual cells different rotational or translational speeds. For rotating subregions, it is treated in a frame that rotates with angular velocity Ω . These rotations introduce fictitious body forces like the Coriolis and Centrifugal. Therefore, the geometry dos not move. In the case of the model, the rim or brake disc remains in the initial position, but the Coriolis and centrifugal acceleration are introduced as volume forces. Thus, it is necessary a velocity transformation according to Equation (13), where $\mathbf{u}$ is the absolute velocity in a inertial frame, $\mathbf{u}_r$ is the velocity relative to the rotating frame and $\mathbf{r}$ is the position vector from the rotation axis according to Fluent (27) and Luo (30).

$$\mathbf{u} = \mathbf{u}_r + \Omega \times \mathbf{r} \quad (13)$$

With Equation (13) the momentum equation in the rotating frame now includes Coriolis and centrifugal terms as Equation (14). For the wheel, both rotating domains for the rim and the brake are small compare with the full fluid domain, therefore an absolute velocity is used, which can reduce the numerical diffusion as presented by Fu (18).

$$\rho \left(\frac{\partial \mathbf{u}_r}{\partial t} + (\mathbf{u}_r \cdot \nabla) \mathbf{u}_r \right) = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g} - 2\rho \Omega \times \mathbf{u}_r - \rho \Omega \times (\Omega \times \mathbf{r}) \quad (14)$$

4.4.1.2 SM - Sliding mesh The sliding mesh is used through a transient simulation. In this case mesh zones are prescribed to move with respect to stationary zones. The coupling between these zones is perform with non-conformal interfaces ant their geometric relationship is updated at each time step to allow the fluxes to pass trough the zones. The motion is rigid, so cell shapes and volume in moving zones remain constant. The conservation equation for a scalar value on a volume V with a sliding boundary is presented as Equation (15). With ρ is density, ϕ is a transported scalar, where for continuity $\phi = 1$, $\mathbf{u}$ is the fluid velocity, $\mathbf{u}_g$ is the mesh (boundary) velocity, Γ is the diffusion coefficient and S_ϕ is the volumetric source.

$$\frac{d}{dt} \int_V \rho \phi dV + \int_{\partial V} \rho \phi (\mathbf{u} - \mathbf{u}_g) \cdot \mathbf{n} dA = \int_{\partial V} \Gamma \nabla \phi \cdot \mathbf{n} dA + \int_V S_\phi dV \quad (15)$$

As the volume stays constant and the mesh motion is rigid, the volume $V^{n+1} = V^n$, and the discrete time derivative for a conserved cell quantity is presented in Equation (16) and equivalently the rate of change of cell volume is zero as Equation (17) and it is simplified in Equation (18) according to Fluent (27), with n_f as the number of faces of the cell, $\vec{u}_{g,j}$ is the mesh speed in the face j and $\vec{A}_j$ is the area vector of the face j and the term $\vec{u}_{g,j} \cdot \vec{A}_j$ represents the volumetric flux related to the mesh motion in the face j

$$\frac{d}{dt} \int_V \rho \phi dV \approx \frac{[(\rho \phi)^{n+1} - (\rho \phi)^n] V}{\Delta t} \quad (16)$$

$$\frac{dV}{dt} = \int_{\partial V} \mathbf{u}_g \cdot \mathbf{n} dA = 0 \quad (17)$$

$$\sum_j^{n_f} \vec{u}_{g,j} \cdot \vec{A}_j = 0 \quad (18)$$

The continuity and momentum for the sliding mesh and control volume form are presented in Equation (19) and Equation (20)

$$\frac{d}{dt} \int_V \rho dV + \int_{\partial V} \rho (\mathbf{u} - \mathbf{u}_g) \cdot \mathbf{n} dA = 0 \quad (19)$$

$$\frac{d}{dt} \int_V \rho \mathbf{u} dV + \int_{\partial V} \rho \mathbf{u} (\mathbf{u} - \mathbf{u}_g) \cdot \mathbf{n} dA = \int_{\partial V} \boldsymbol{\tau} \cdot \mathbf{n} dA - \int_{\partial V} p \mathbf{n} dA + \int_V \rho \mathbf{g} dV \quad (20)$$

4.4.1.3 Moving Wall The moving wall, or in this case the rotating wall, is a Dirichlet boundary condition that define the wall translational or rotational velocity. For a rotational case, the cell velocities on the surface are found with the product of the angular velocity and the distance to the origin axis. The no-slip condition becomes $\mathbf{u}_{wall} = \mathbf{u}_{wall}(t, \mathbf{x})$ and for a rotating cylinder $\mathbf{u}_{wall} = \boldsymbol{\omega} \times \mathbf{r}$. A tangential velocity is applied to the surfaces that rotates but the mesh does not actually moves. The fluid around the object does not has a rotation condition as in the cases of MRF and SM.

Governing equations and turbulence model

For this case, the governing equations for an incompressible flow are resolved through the continuity Equation (21), momentum Equation (22) and energy equations Equation (24), which ensure the conservation of mass, momentum and energy within the domain of interest according to Malalasekera (31)

$$\nabla \cdot \mathbf{u} = 0 \quad (21)$$

For the momentum equation, the first term on the left hand side corresponds to the unsteady acceleration, and the second term is the convective term. Meanwhile, in the right hand side of the equation, the first term is the pressure gradient, the second one is the viscous diffusion term and $\mathbf{f}_b$ is the external body forces

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}_b \quad (22)$$

When heat transfer is relevant, the conservation of energy for incompressible flows is expressed as the temperature equation, where the first two terms in the left hand side of the equation are the unsteady temperature variation and the convective heat transport. And for the right hand side, the first term is the heat conduction with $\alpha = \frac{k}{\rho c_p}$ as the thermal diffusivity and c_p as the specific heat at constant pressure

$$\frac{\partial T}{\partial t} + (\mathbf{U} \cdot \nabla) T = \alpha \nabla^2 T + \frac{q'''}{\rho c_p} \quad (23)$$

4.4.2 Turbulence modeling

For the turbulence model it was chosen the Transitory $k - k_L - \omega$ model. This method is based on the RANS equations and more specifically on the Menster SST $k - \omega$ model. The RANS equations allows the decomposition of the NS equation into mean and fluctuating components and the $k - \omega$ is based on a transformation of the $k - \varepsilon$ model near the wall regions, where the dissipative term is expressed as $\varepsilon = k\omega$. This model has been proved to give a better performance for zero pressure gradient and adverse pressure gradient boundary layers suitable for free shear layers according to Malalaskera (31).

The Transitory $k - k_L - \omega$ model is a three-equation turbulence model from Ansys Fluent which extends the two equation eddy-viscosity approaches to predict laminar to turbulent transition phenomena. The addition of a laminar kinetic energy term k_L allows the model to capture previous transitional effects in the boundary layer, useful for flows where the transitions affect the flow separation. The k is given by Equation (24) with P_k as the production of turbulent kinetic energy and D_k is the destruction due to laminar-turbulent transition effects.

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = P_k - D_k + \frac{\partial}{\partial x_j} \left[(\nu + \sigma_k \nu_T) \frac{\partial k}{\partial x_j} \right] \quad (24)$$

The laminar kinetic energy k_L is presented in Equation (25) with P_{k_L} and D_{k_L} being the generation of laminar kinetic energy and dissipation and transfer into turbulent kinetic energy, respectively.

$$\frac{\partial k_L}{\partial t} + U_j \frac{\partial k_L}{\partial x_j} = P_{k_L} - D_{k_L} + \frac{\partial}{\partial x_j} \left[(\nu + \sigma_{k_L} \nu_T) \frac{\partial k_L}{\partial x_j} \right] \quad (25)$$

For the specific dissipation rate ω , the Equation (26) is presented.

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \alpha \frac{P_k}{\nu_T} - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[(\nu + \sigma_\omega \nu_T) \frac{\partial \omega}{\partial x_j} \right] \quad (26)$$

4.4.3 Heat transfer

4.4.3.1 Volumetric source In this work, the thermal energy is introduced by using volumetric heat sources given to the pads. The value of the source is based on the hydraulic brake pressure from the work of Hu (32) of 0.382 MPa and validated with the constant braking force of 750N used by the experimental setup in the work from Vdovin (21). The value is obtained from the brake equations recommended by the brake manufacturer Twiflex (33). Initially, the internal and external ratios of the contact circumference surface generated by the pads are needed as shown in Fig. 11, where $R - in = 0.115m$ and $R - out = 0.129m$. The normal force applied by the pads is obtained from Equation (27), with $N_{pis} = 4$ is the number of pistons used in the caliper of the current model, $HBP = 0.382MPa$ is the hydraulic brake pressure and $A_{pis} = 4.9e^{-4}m^2$ is the area of the hydraulic cylindrical piston with a diameter of $D = 0.025m$

$$F_{normal} = N_{pis} \cdot HBP \cdot A_{pis} \quad (27)$$

The obtained value was $F_{normal} = 750N$, with Equation (28) and a roughness coefficient value of the disc of $C_s = 0.5$ a value of braking force for a number of pads $N_{pads} = 2$ is obtained $F_b = 750N$

$$F_b = F_{normal} \cdot C_s \cdot N_{pads} \quad (28)$$

The brake torque $\tau_b = 91.75Nm$ is obtained from Equation (29) with $R_e = \frac{R_{in} + R_{out}}{2} = 0.122m$ as the effective ratio.

$$\tau_b = F_b \cdot R_e \quad (29)$$

The total energy added as heat is obtained with Equation (30), obtaining a value of $\dot{Q} = 6395W$ with an angular velocity $\omega = 69.7rad/s$. And finally with Equation (31) and the total volume of the pads $V = 4.7e^{-5}m^3$ the volumetric source per pad $\frac{q'''}{pad} = 6.79e^7W/m^3 = 67.9MW/m^3$ is obtained. This value is in the same order of magnitude to the one obtained with the simulation performed by Cravero (5) of $60.2 MW/m^3$

$$\dot{Q} = \tau_b \cdot \omega \quad (30)$$

$$\frac{q'''}{pad} = \frac{\dot{Q}}{(V_{both,pads} \cdot 2)} \quad (31)$$

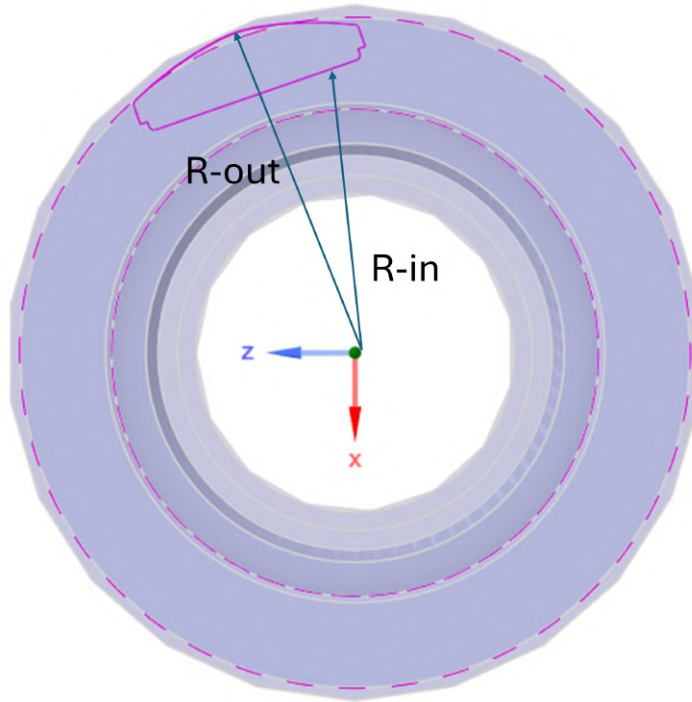


Figure 11: Internal and external ratio of the contact surface between pad and disc

4.4.4 Quantities of interest

4.4.4.1 HTC Heat Transfer Coefficient The heat transfer coefficient is obtained from the simulations with the brake system and wheel mode. This value represents the heat

transfer between a moving fluid and a solid. In Ansys Fluent this value is an output and a boundary condition and when it is solved, it is defined according to Equation (32). In Fluent this value is obtained with an area-weighted average surface report for the solid disc and for the fluid surfaces corresponding to the disc. This value is compared with the values obtained by Dekker (23).

$$h = \frac{q''_{\text{conv}}}{T_{\text{wall}} - T_{\text{ref}}} \quad (32)$$

4.4.4.2 C_d - Drag Coefficient The drag coefficient is a non-dimensional parameter that quantifies the drag or resistance of an object in a fluid environment. According to the Equation (33) and the description from Katz (8), it relates the drag force to the dynamic pressure of the flow and the object's characteristic area. In this project, the characteristic area corresponds to the frontal area of the tyre and the C_d value was obtained with an expression created in Ansys Fluent following Equation (33) where the Drag force was acquire through the force reports obtained only from the contact surface of the tyre, which is a similar procedure used in the experimental set-up from the work of Croner (16).

$$C_d = \frac{F_d}{\frac{1}{2}\rho u^2 A} \quad (33)$$

Where:

C_d is the drag coefficient (dimensionless),

F_d is the drag force (N),

ρ is the fluid density (kg/m³),

u is the freestream flow velocity (m/s),

A is the characteristic reference area (m²).

4.4.4.3 C_p - Pressure coefficient The absolute pressure usually depends on the velocity magnitude and can be difficult to compare the pressure distribution for simulations with different velocities. The C_p is a non-dimensional value which overcomes that issue. This quantity describes the pressure by normalising with the dynamic pressure according to the definition of Katz (8) with the Equation (34). This value is used to compare against the experimental results obtained by Croner (16). The value of C_p is calculated in post-processing from the absolute and static pressures and the velocity components. And is obtained along the wheel contact surface centre-line.

$$C_p = \frac{p - p_{\infty}}{\frac{1}{2}\rho_{\infty}U_{\infty}^2} \quad (34)$$

4.5 Solver Settings

The air is considered the fluid in this case. The brake disc is a grey cast iron HT250 with properties extracted from Zhang (22), and the pads are made from a resin matrix, following the properties presented by Hu (32). The material properties are summarised in the Tab. 5, Tab. 6 and Tab. 7 for the brake disc, pads and air, respectively.

Component	Brake disc	
Material	grey cast iron ht250	
Density	7280	kg/m3
SpecificHeat Cp	480	J/kgK
Thermal Conductivity	45	W/mK
Temperature	293	kg/m3
Heat Flux Area	2.50E+04	q/(W/m2)
Roughness height	0.5	K_s/mm
Roughness Constant Cs	0.5	Cs
Brake disc volume	0.00073334	m3

Table 5: Material properties of the brake disc

Component	Pads	
Material	Resine Matrix	
Density	2250	kg/m3
SpecificHeat Cp	1320	J/kgK
Thermal Conductivity	1.1	W/mK

Table 6: Material properties of the pads

Component	air	
Density	1.225	
SpecificHeat Cp	1006.43	J/kgK
Thermal Conductivity	0.0242	W/mK
Viscosity	1.79E-05	kg/ms

Table 7: Material properties of the fluid air

For all the simulations, the following are the common parameters: the air flow is taken as an incompressible fluid and a pressure-based solution is used. The Transitory $k_L - k - \omega$ model is used for the turbulent region. And for the simulations with the energy model activated, the temperature of the free stream is set as 293 K.

4.5.1 Isolated Wheel model

For the isolated wheel model, where three different mesh refinements were simulated, the Tab. 8 and Tab. 9 present the used numerical settings and boundary conditions.

Solution Term	Solution Method
Turbulence model	Transitory k-kl-w
Pressure Velocity coupling	SIMPLE
Spatial discretisation gradient	Least squares cell based
Pressure	Second Order
Momentum	Second Order Upwind
Turbulent Kinetic Energy	Second Order Upwind
Laminar Kinetic Energy	Second Order Upwind
Specific dissipation rate	Second Order Upwind
Pseudo Time Method	Local Time Step
Pseudo Time Courant Number	5

Table 8: Numerical set-up

Boundary	Zone	Parameters
Velocity inlet	inlet-farfield	u=22 m/s, kl=1e-6m ² /s ² Inlet Turbulent Intensity=0.1 Turbulent Viscosity ratio=0.1
Outlet	outlet-farfield	Pressure outlet, Gauge Pressure=0
Wall	ground	No-slip, Moving Wall Translational, Absolute speed =22m/s, X-direction
Wall	tyre	No-slip, Moving Wall Rotational, Absolute speed =-69.7rad/s, Y-rotation axis
Wall	support	Stationary wall, No-slip

Table 9: Boundary conditions for the isolated wheel model, with a moving wall boundary condition for the rotating model

4.5.2 Isolated Wheel model - MRF - SM

The isolated wheel model with MRF and SM models used similar numerical settings to those used by the previous model, according to Tab. 8. However, for MRF, the cell zone condition for the fluid enclosure of the rim has been designed as Frame Motion with an angular velocity of 69.7 rad/s with a rotation axis direction in the y component and the tyre was set with a Moving Wall boundary condition. The boundary conditions for the MRF method with an isolated wheel are shown in Tab. 10. This approach was used for SM, but with Mesh Motion, and the boundary conditions are similar to MRF in Tab. 10. Additionally, because the SM method needs a transient solver, the time-step-size was calculated according to Equation (35) and the parameters to run the transient simulation are presented in Tab. 11

$$\Delta t \approx \frac{\text{Typical cell size}}{\text{Characteristic flow velocity}} \quad (35)$$

Boundary	Zone	Parameters
Velocity inlet	inlet-farfield	u=22 m/s, $\kappa=1\text{e-}6\text{m}^2/\text{s}^2$ Inlet Turbulent Intensity=0.1 Turbulent Viscosity ratio=0.1
Outlet	outlet-farfield	Pressure outlet, Gauge Pressure=0
Wall	ground	No-slip, Moving Wall Translational, Absolute speed =22m/s, X-direction
Wall	tyre	No-slip, Moving Wall Rotational, Absolute speed =-69.7rad/s, Y-rotation axis
Wall	rim-covers	No-slip, Moving Wall Rotational, Relative speed =0rad/s, Y-rotation axis

Table 10: Boundary conditions for the isolated wheel model for MRF and SM method for the rim and moving wall condition for the tyre

Parameter	Value	
u	22	m/s
time-step size Δt	1.82E-04	s
Minimal Cell Size	0.0004	m
CFL	10	
#Timesteps	550.00	
#Iteration/time-step	30	
Passes/Revolution	495.80	Passes/rev
Physical time	0.10	s
w-rotational speed	69.7	rad/s
T(rotation period)	0.090146131	rad/s
Number of wheel spins	1.11	spins

Table 11: Transient parameters specification for the SM isolated wheel model

4.5.3 Brake system and wheel model - MRF

For the Brake system and wheel model with MRF, four different cases are simulated with a variation of the angular velocity. The angular velocity of 31.7 rad/s which is related to the free stream velocity of 10 m/s from the experimental setup of Vdovin (21) and the case with the angular velocity of 69.7 rad/s is used to assess the drag coefficient, as was made with the isolated wheel model, comparing its value against the results obtained by Croner (16). The Tab. 12 shows the different cases used for the MRF simulations. The values from the column *Flow Speed (m/s)* are used to assign the inlet velocity in the inlet boundary conditions and are also used for the translational velocity assigned to the ground. Meanwhile the column *Angular Speed (rad/s)* is used as the angular velocity designated to the rotational speed for the Frame Motion set up in the cell zone condition for the RIM and Brake Disc enclosures and also for the 40 fluid bodies corresponding to the cooling vanes of the brake disc. The *Angular Speed (rad/s)* values are also used for the Moving Wall boundary conditions used in the tyre.

MRF Case	Flows Speed (m/s)	Speed (km/h)	angular velocity (rad/s)	RPM
MRF-2	10	36	31.7	302.7
MRF-3	13.9	50.04	44.1	420.7
MRF-1	22	79.2	69.7	665.9
MRF-4	30	108	95.1	908.0

Table 12: MRF cases evaluated with the brake system and wheel model.

The numerical settings of the solver are specified in Tab. 13:

Solution Term	Solution Method
Energy	On
Turbulence model	Transitory k-kl-w
Pressure Velocity coupling	SIMPLE
Spatial discretisation gradient	Least squares cell based
Pressure	Second Order
Momentum	Second Order Upwind
Turbulent Kinetic Energy	Second Order Upwind
Laminar Kinetic Energy	Second Order Upwind
Specific dissipation rate	Second Order Upwind
Pseudo Time Method	off

Table 13: Numerical settings for the MRF - Brake system and wheel model

Mesh interfaces were created in order to obtain a heat transference between the solid pads and the fluid disc enclosure designated for the MRF/SM zone, and between the fluid disc enclosure and the solid brake disc. The non-conformal interfaces were created between the solid and fluid domains to allow independent meshing and the rotational models of MRF and specially SM. In Ansys Fluent, the interface between two cell zones is represented by a pair of wall boundary conditions. Therefore a wall is formed for one of the cell zones, and also a shadow wall is formed, which is a topological twin representing the same physical surface, but it belongs to the adjacent cell zone. The non-conformal interfaces are needed when there is no alignment of the mesh nodes on both sides of the interface. The Tab. 14 describes the Ansys Fluent structure of one of the pad interfaces (for the second pad, the setup was the same) and the setup used for the interfaces between the solid brake disc and the MRF/SM enclosure fluid surfaces corresponding to the brake disc.

Interface	Wall ID	Type	Associated Zone	Relevant Active Conditions
4*Pad-Enclosure Disc	1	Original	Pad (solid)	Thermal: Convection (inactive)
	9	Original	Enclosure (fluid)	Momentum: Moving Wall 0 rad/s (inactive)
	6	Interface (Wall)	Pad (solid)	Thermal: Coupled, pad-material
	14	Interface (Shadow)	Enclosure (fluid)	Momentum: Moving Wall, Thermal: Coupled
4*Solid disc-Enclosure disc	15	Original	Enclosure (fluid)	Momentum: Moving Wall 0 rad/s (inactive)
	21	Original	Disc (solid)	Thermal: Convection (inactive)
	32	Interface (Wall)	Enclosure (fluid)	Momentum: Moving Wall, Thermal: Coupled
	37	Interface (Shadow)	Disc (solid)	Thermal: Coupled, cast-iron-disc-material

Table 14: Summary of non-conformal interfaces generated in ANSYS Fluent Meshing

The original type refers to the original geometric faces, where their boundary conditions are suppressed by the new interfaces.

A coupled thermal condition was applied at the interfaces, where the HTC, the heat transfer coefficient, must be solved. With a coupled thermal condition, the continuity of temperature and heat flux across the boundary is enforced to allow energy to be transferred between the solid and fluid zones.

Additional coupled interfaces were created between the solid brake disc and the 40 bodies of fluid that represent the cooling vanes of the brake disc.

4.5.4 Brake system and wheel model - SM

For the Sliding Mesh method applied to the brake system and wheel model the interface procedure was the same as presented for the MRF case. However, for the SM mesh, only

the 69.7 rad/s angular velocity was assessed to compare temperature, HTC and drag coefficient. The numerical setup is specified in the Tab. 15 with volumetric sources applied to the pads. Pseudo time method under relaxation factor were used for this case in order to avoid the divergence of the energy residuals, and they are present in Tab. 16.

Solution Term	Solution Method
Energy	On
Turbulence model	Transitory k-kl-w
Pressure Velocity coupling	SIMPLEC
Spatial discretisation gradient	Least squares cell based
Pressure	Second Order
Momentum	Second Order Upwind
Turbulent Kinetic Energy	Second Order Upwind
Laminar Kinetic Energy	Second Order Upwind
Specific dissipation rate	Second Order Upwind
Pseudo Time Method	Local Time
Pseudo Time Courant Number	1

Table 15: Numerical setup for the SM - Brake system and wheel model

Relaxation Factor	Value
Density	1
Body-force	1
Momentum	0.4
k	0.8
kl	0.8
omega	0.3
Pressure	0.7
Temperature	0.3
turb-viscosity	0.8

Table 16: Pseudo time method - Under Relaxation factor for the SM - Brake system wheel model

For the transient analysis the time step size obtained from the minimal cell size was $2e-4$, but as the full heat transfer was required, this time step size could have caused a total run simulation time of 528 hours or days for one second of simulated physical time. Therefore, a timestep size of 0.01 s was used and the transient setup is exhibited in Tab. 17

Parameter	Value	
u	22	m/s
time-step size Δt	0.0002	s
time-step size Δt	0.01	s
Minimal Cell Size	0.004	m
CFL	55	
#Timesteps	1000	
#Iteration/time-step	30	
Passes/Revolution	90.1461307	Passes/rev
Physical time	1	s
w-rotational speed	69.7	rad/s
T(rotation period)	0.09014613	rad/s
Number of wheel spins	11.0930995	spins

Table 17: Transient parameters setup for the SM brake system and wheel model

4.6 Statistical tools for comparison

Additional statistical analysis was added as a quantitative comparison in order to perform the comparison between the MRF and SM methods beyond the comparison of quantities of interest and the qualitative analysis of temperature maps.

4.6.1 Regression model

Statistical regression models were employed in order to establish the relationship between the angular velocity w and the heat transfer coefficient h using the four data points from the MRF simulation and the SM point. The regression is a method used to quantify how much a dependent variable varies as function of a different variable. Montgomery (34). For this project, the regression was used to provide an analytical description of the MRF simulation results and also evaluation of the SM value to check if it lies within the expected prediction interval of the MRF trend..

A linear regression is first used which assumes a lineal relationship between ω and h according to Equation (36)

$$h_i = \beta_0 + \beta_1, \omega_i + \varepsilon_i \quad (36)$$

Where β_0 is the intercept, β_1 is the slope and ε_i is the residual error assumed to be normally distributed with zero mean and constant variance. However, a quadratic regression model was also tested allowing curvature as presented in Equation (37)

$$h_i = \beta_0 + \beta_1, \omega_i + \beta_2, \omega_i^2 + \varepsilon_i \quad (37)$$

The model was evaluated using the Akaike Information Criteria (AIC) and corrected with AICc, which penalises the model complexity. Equation (38)

$$AIC = 2k - 2\ln(\hat{L}) \quad (38)$$

With k is the number of model parameters and $\hat{L}$ is the maximum likelihood of the fitted model.

This regression were used to generate two types of intervals:

- Confidence bands: Around the regression curve, used to indicate the uncertainty in the estimation of the mean response. Equation (39)

$$\hat{h}(\omega) \pm t_{\alpha/2, df} \cdot SE(\hat{h}(\omega)) \quad (39)$$

- Prediction bands: Are wider and represent the expected variability of future observations of h at a given ω Equation (40)

$$\hat{h}(\omega^*) \pm t_{\alpha/2, df} \cdot SE_{pred} \quad (40)$$

Where SE_{pred} includes both model uncertainty and the residual variance.

4.6.2 Correlation and covariance

A statistical correlation analysis was used to investigate the relationship between the simulation results obtained from the MRF and SM methods. Correlation allows to quantify the strength and direction of association between two numerical variables according to Freund (35) In this case is used the angular velocity ω and max temperature, average temperature, heat flux, drag coefficient and heat transfer coefficient.

- Pearson's correlation coefficient (r): measures the strength of the linear relationship, Equation (41), where x_i and y_i are the compared values and $\bar{x}$ $\bar{y}$ are their means.

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} \quad (41)$$

- Spearman's rank correlation coefficient ρ : Evaluates if a monotonic relationship exists between variable without considering the linearity, Equation (42), where d_i is the difference between the ranks of the variables.

$$\rho = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n(n^2 - 1)} \quad (42)$$

A $p - value < 0.05$ is used to assess the statistical significance of the coefficients, and confidence intervals (CI) at 95% were estimated using Fisher's z transformation for Pearson and a bootstrap resampling technique for Spearman. The CI gives a range in which the true population of the data correlation is expected to lie.

4.6.3 Principal Component Analysis PCA

The PCA is used to transform a set of possibly correlated variables into a new set of variables with no correlation called principal components Jolliffe (36). In the preset work, PCA was investigated to explore the relationships between thermal and aerodynamic variables obtained from MRF and the SM reference case. The analysed data set were the heat

transfer coefficient, maximum and average temperature, total heat transfer and drag coefficient.

The purpose of PCA is to determine whether the MRF simulation at different angular velocity ω cluster around the SM case, also to identify the main variable which later will be used to use a correction factor used in MRF simulations to emulate SM results.

5 Results & Discussions

5.1 Meshing isolated wheel model

For the coarse, medium and fine meshes, the boundary layer was defined for each of the mesh to establish a refinement. Different values were obtained for the first layer height following different y^+ values according to Tab. 18. A uniform boundary layer method was used in the tyre and a smooth transition for the ground behind the wheel as can be seen in Fig. 12

Uniform Parameter	COARSE		MEDIUM		FINE	
y+	10		4		1	
#Layers	10		15		20	
y1	2.85E-04	m	1.14E-04	m	2.85E-05	m
y1 mm	0.285	mm	0.114	mm	0.028	mm
gr	1.2		1.2		1.2	
yt	0.0074	m	0.0082	m	0.0053	m
ytotall mm	7.39	mm	8.21	mm	5.32	mm
Ground Smooth Transition						
y1	2.85E-04	m	1.14E-04	m	2.85E-05	m
gr=r	1.2		1.2		1.2	
# Layers	5		10		10	
Total Height	2.12E-03	m	2.96E-03	m	2.05E-03	m
Dx	0.008	m	0.008	m	0.008	m
TransitionRatio	0.265		0.370		0.256	

Table 18: Boundary layer parameters for coarse, medium and fine mesh

The quality obtained for the three mesh refinements is presented in Tab. 19 and in Tab. 20 is presented additional mesh info where h is the characteristic mesh size calculated for a total volume mesh of 36.11 m^3 and r is the grid refinement factor which is recommended to be higher than 1.5 according to Celik (28)

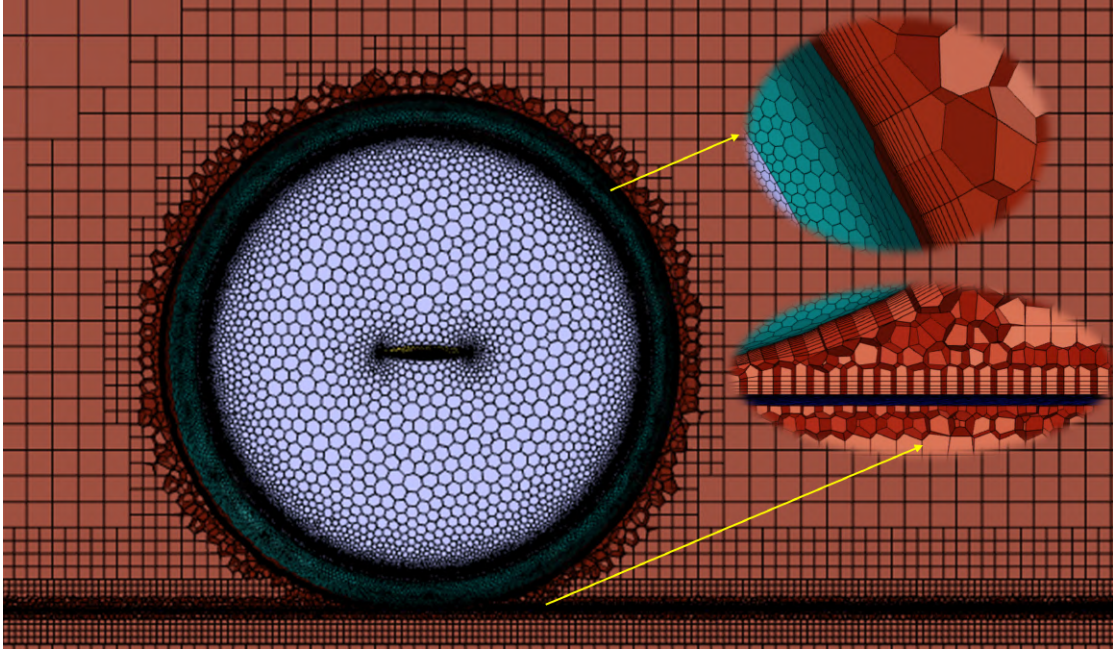


Figure 12: Uniform boundary layers for the tyre and smooth transition boundary layers for the ground

Mesh	min Orthogonal	max Skewness	Size Change	Aspect ratio
fine	0.15	0.75	1.24	1.37
medium	0.1	0.79	42.5	1.36
coarse	0.15	0.85	3.12	3.15

Table 19: Mesh quality with three refinement levels

Mesh	#Cells	h	r
fine	85423959	0.0075	1.76
medium	15558127	0.0132	1.60
coarse	3818019	0.0211	

Table 20: Number of cells and characteristic mesh size

5.2 GCI study

After performing the simulation for each of the mesh refinements, the drag coefficient was obtained to evaluate a GCI study. The results obtained for each mesh and with the extrapolated value are summarised in Tab. 21. The subscripts 1,2,3 correspond to the fine, medium and coarse mesh respectively. The values of C_d were 0.3770 (coarse), 0.4020 (medium), and 0.4120 (fine), while the Richardson extrapolated value was estimated as 0.4161 as presented in Fig. 13. The refinement ratios between the meshes were $r_{21} = 1.76$ $r_{32} = 1.5973$ r_{21} , and the order of accuracy $p = 2,18$ which is close to the theoretical second-order behaviour expected for the discretisation scheme.

The relative errors with respect to the extrapolated value were $\varepsilon_{21} = 0.98\%$ (fine extrapolated) and $\varepsilon_{32} = 2.43\%$ (medium to fine). The corresponding GCIs were 0.50% (fine/medium) and 1.71% (medium/coarse). The asymptotic ratio (AR), defined as the ratio of the two GCIs scaled by the refinement factor, was found to be 1.01, which is very close to the theoretical ideal value of unity. This indicates that the solutions obtained are in the asymptotic range of convergence.

Overall, the GCI analysis shows that the computed drag coefficient is grid independent with an uncertainty of less than 1% for the fine mesh. However, the medium mesh was chosen for the MRF/SM analysis as its error is considered acceptable with 2.4% and a GCI of 1.71%

Mesh Type	Mesh Size (cells)	Cd	Richardson Extrapolated Cd	Refinement Ratio r	Error (%)	GCI (%)
Coarse	3,818,019	0.377	—	—	—	—
Medium	15,558,127	0.402	—	r32=1.5973	2.43	1.7099
Fine	85,423,959	0.412	—	r21=1.7642	0.98	0.5024
Extrapolated		—	0.4161	—	—	—

Table 21: GCI values

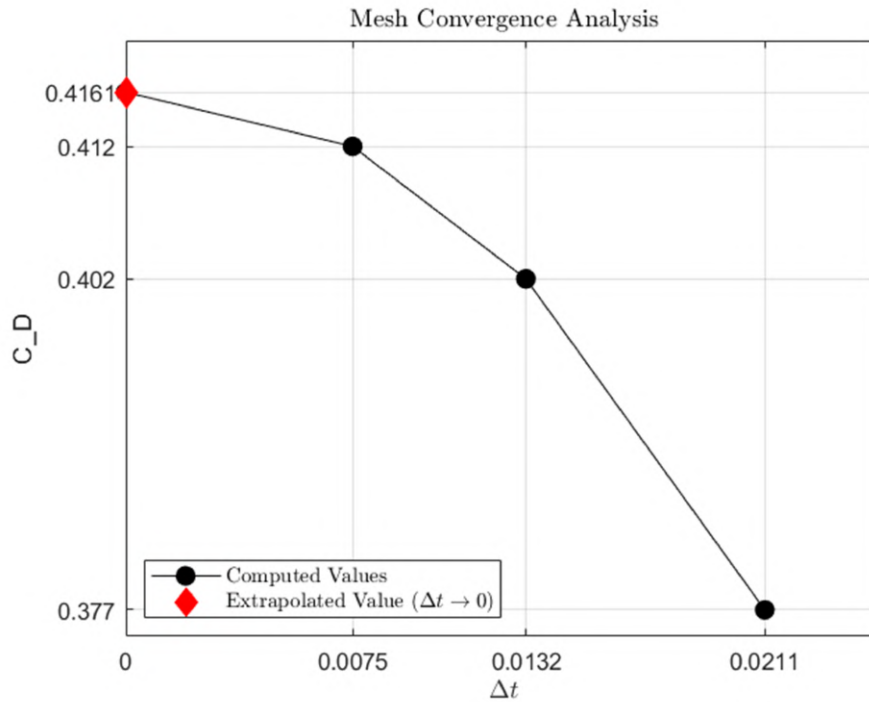


Figure 13: Grid convergency study

5.3 Meshing MRF/SM isolated wheel model

For the mesh of the isolated wheel model with the MRF and SM enclosures, the boundary layer parameters was based on a value $y^+ = 1$ and they are presented in Tab. 22. The resulting polyhexacore mesh, presented in Fig. 14, has 24,923,430 cells, which is higher than the number of cells from the medium mesh used for the GCI study and the quality of this case is presented in Tab. 23

Uniform Parameter	MRF/SM Values	
y+	1	
#Layers	10	
y1	2.85E-05	m
y1mm	0.0285	mm
gr	1.2	
y total	0.00074	m
y total mm	0.739	mm
Ground Smooth transition		
y1	2.85E-05	m
gr=r	1.2	
#Layers	5	
T	2.12E-04	m
Dx	0.008	m
TransitionRatio	0.0265	

Table 22: Boundary layer parameters for the MRF/SM isolated wheel model

Mesh	min Orthogonal	max Skewness	Size Change	Aspec ratio
MRF/SM	0.42	0.58	1.26	1.66

Table 23: Quality mesh for the MRF/SM isolated wheel model

In Fig. 15 it can be seen the refinement zones defined for the wheel and lateral covers, additionally, the purple zone corresponds to the MRF/SM enclosures of the lateral covers.

5.4 Simulation Results isolated Wheel model

Qualitative and quantitative comparison between the results of the simulations against the experimental data was done. Initially, the experimental data from the PIV values from Patel (10) was used. This data was acquired in a planes behind the wheel at a distance of $5D/8$ from the centre of the wheel, where D is the wheel diameter and also in a plane showing the lateral wake formation. This experimental data is compared against the results obtained for the medium mesh with full Moving Wall (MW) boundary condition, the hybrid MRF case (MRF in lateral covers and MW in tyre) and the hybrid SM (SM in

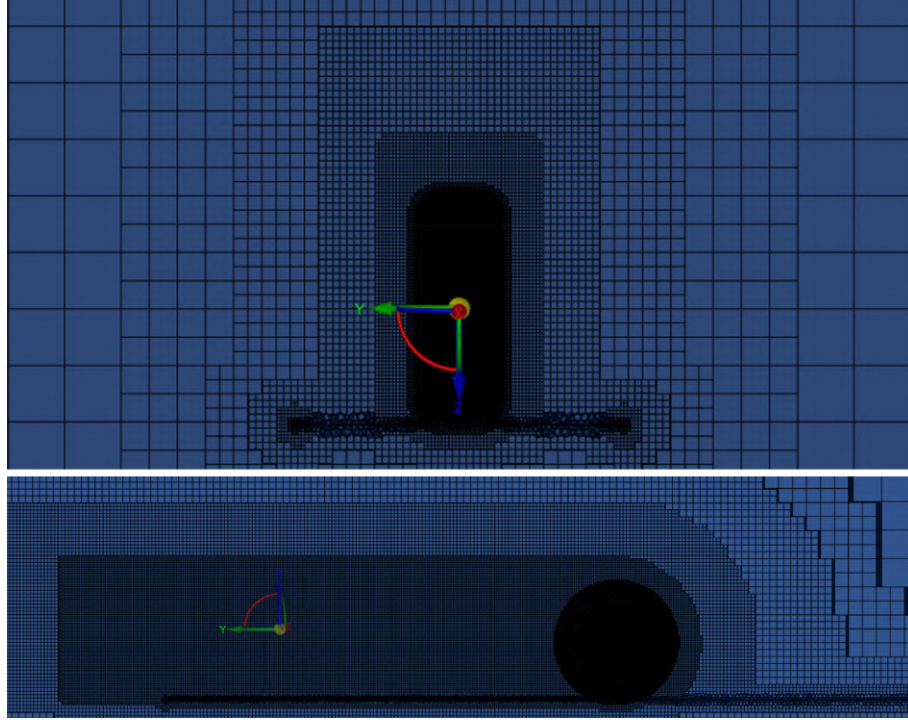


Figure 14: Polyhexacore mesh of the isolated wheel model

lateral covers and MW in tyre).

For the medium mesh with MW, MRF and SM cases, the plane YZ behind the wheel at a distance $5D/8$ is presented in Fig. 16, Fig. 17 and Fig. 18 respectively

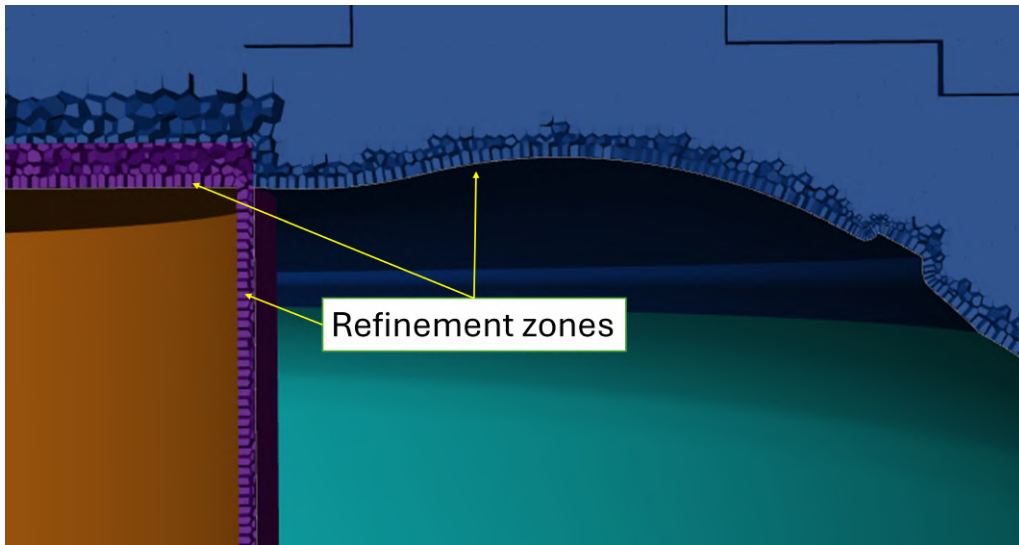


Figure 15: Refinement zones applied to the tyre and lateral covers

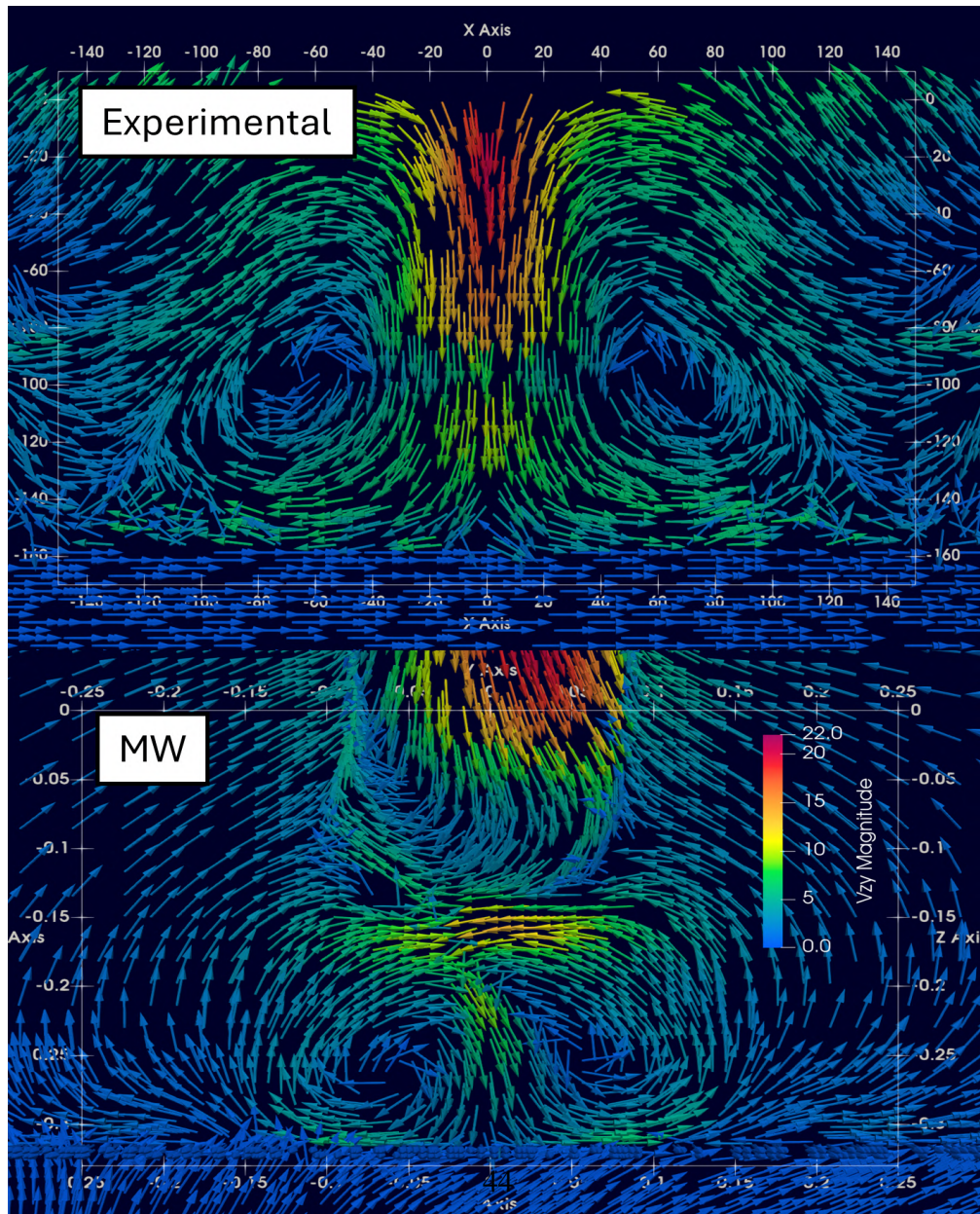


Figure 16: Experimental and MW comparison for the velocity magnitude in the YZ plane at $X=5D/8$

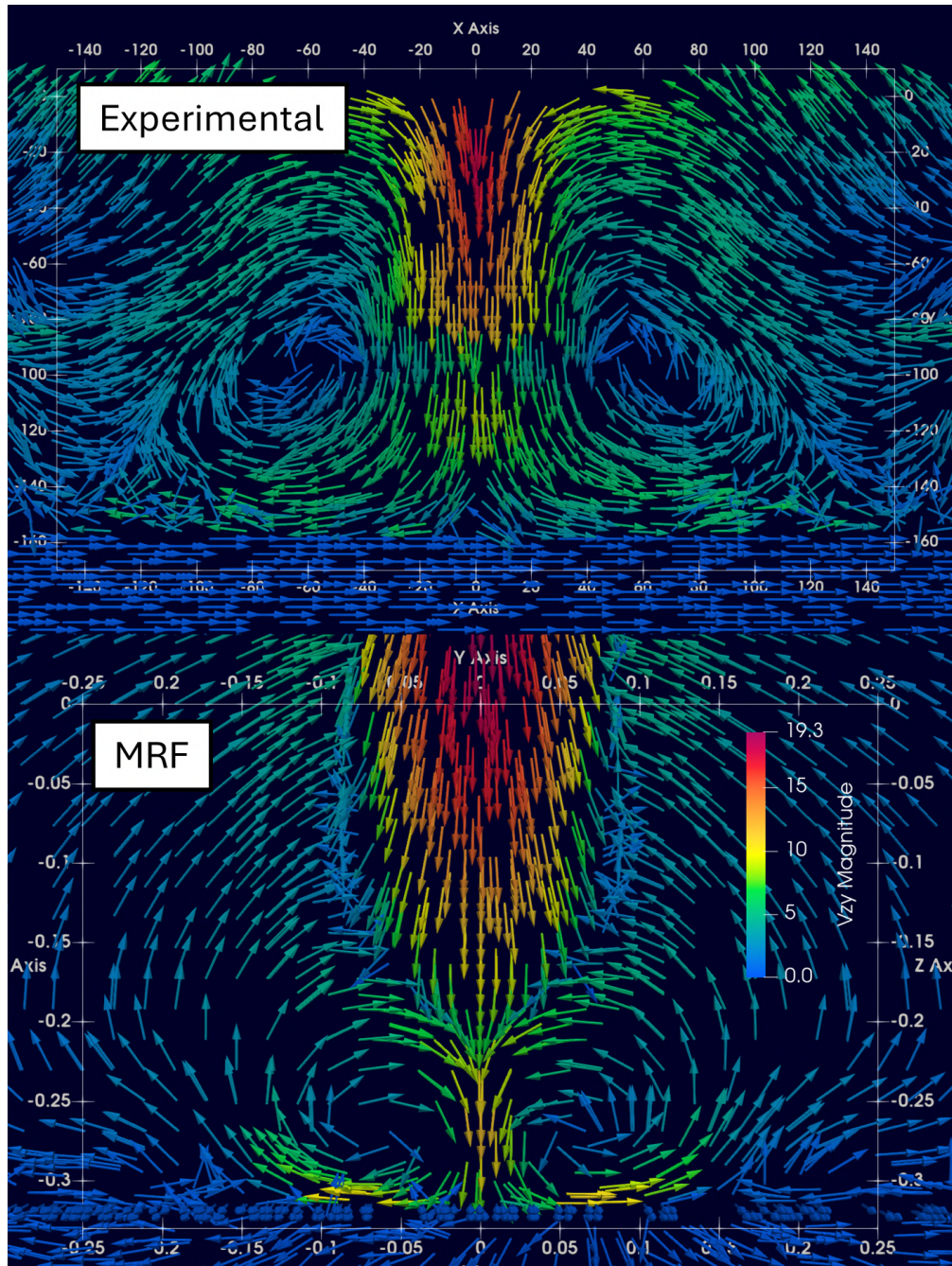


Figure 17: Experimental and MRF comparison for the velocity magnitude in the YZ plane at $X=5D/8$

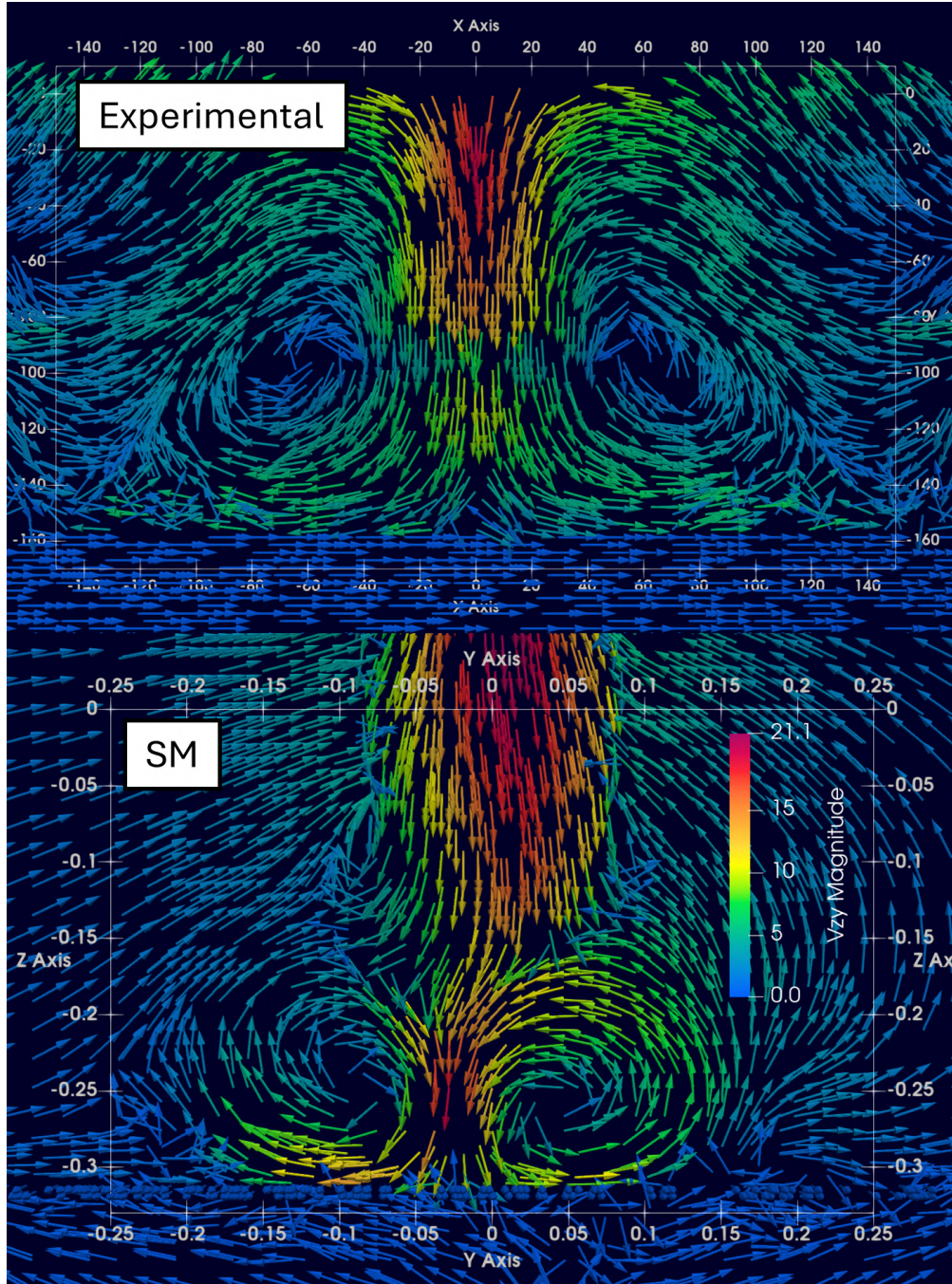


Figure 18: Experimental and SM comparison for the velocity magnitude in the YZ plane at $X=5D/8$

As can be seen in the images for the vectors of Velocity YZ, behind the wheel and close to the ground, two vortical structures are formed similar to the experimental data, with anti-clock wise and clock wise directions for the left and right vortex. This trend is similar to the one observed by Patel (10) of an inverted “T” topology for the vortex behind the wheel. The red color shows the highest velocity achieved in that section, where for the MRF and SM cases the value is closer to the free stream velocity of 22 m/s given as inlet velocity.

A plane XZ showing the lateral distribution of the vectors of velocity XZ is also used to compared the experimental PIV data and the three simulated results (MW,MRF,SM). The results are presented in Fig. 19, Fig. 20 and Fig. 21. It is possible to initially analyse, that the separation point of the three simulations is allocated at the top of the wheel at 0.1 m behind the centre line of the wheel, similar to the experimental figure.

However, with respect to the experimental figure, by observing the recirculation region, it is possible to conclude that the MW, and MRF methods tend to underestimate this region but the SM method have a better fit. Additionally, for the three simulated results, there is visible the formation of a laminar bubble (highlighted with a yellow circle) which is more evident than for the experimental figure. According to Katz (8), the increased thickness of the boundary layer results in a transition to a turbulent boundary layer, which is less sensitive to stall, thus, the flow reattaches allowing the creation of the bubble enclosed in a recirculating flow.

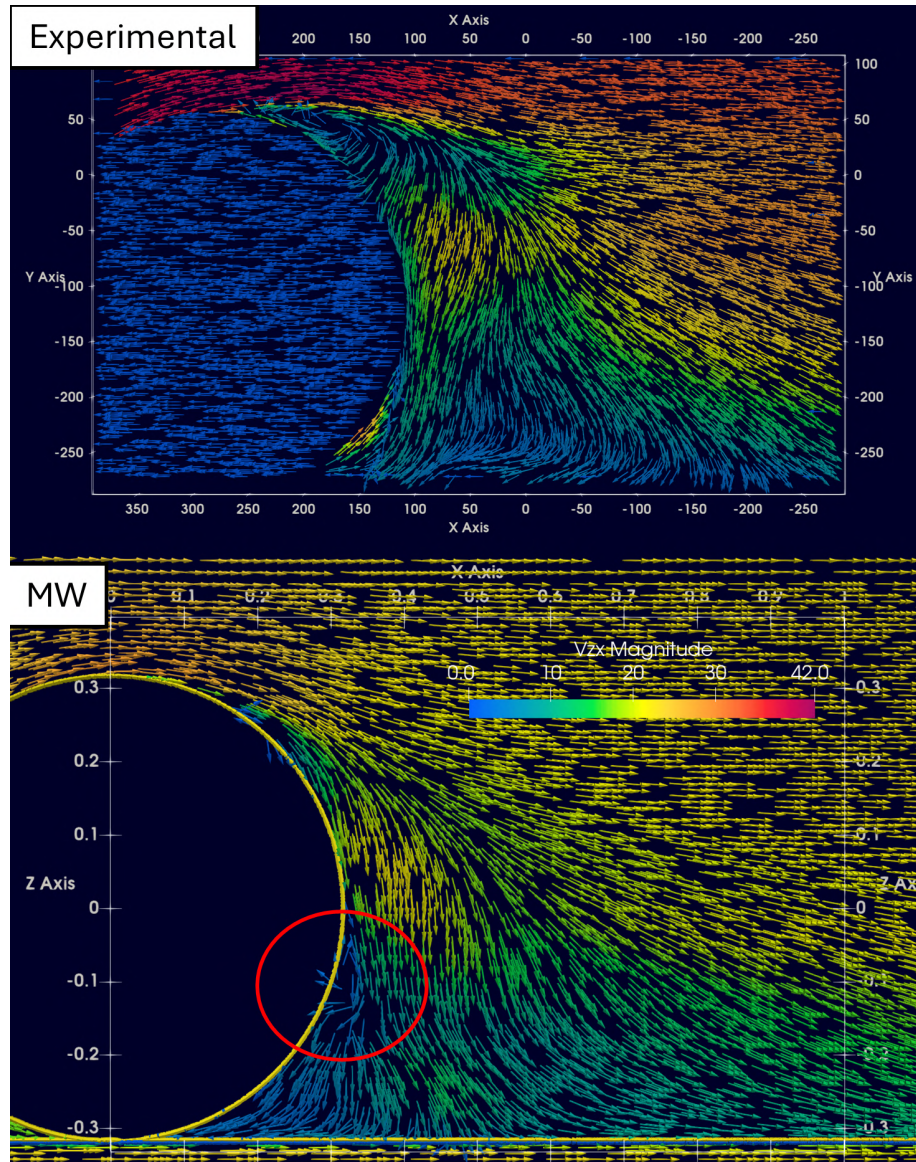


Figure 19: Experimental and MW comparison for the velocity magnitude in the XZ plane at Y=0

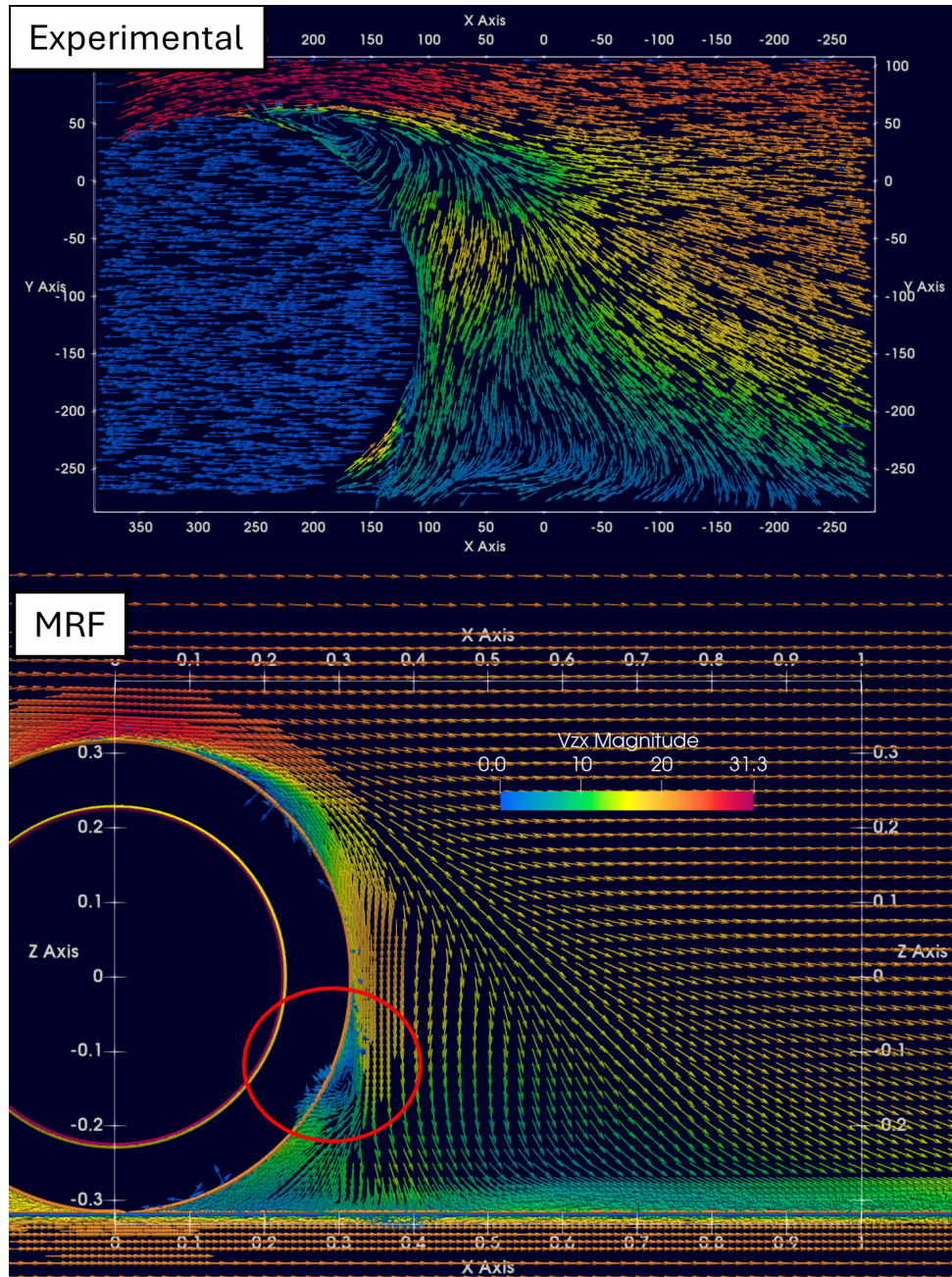


Figure 20: Experimental and MRF comparison for the velocity magnitude in the XZ plane at Y=0

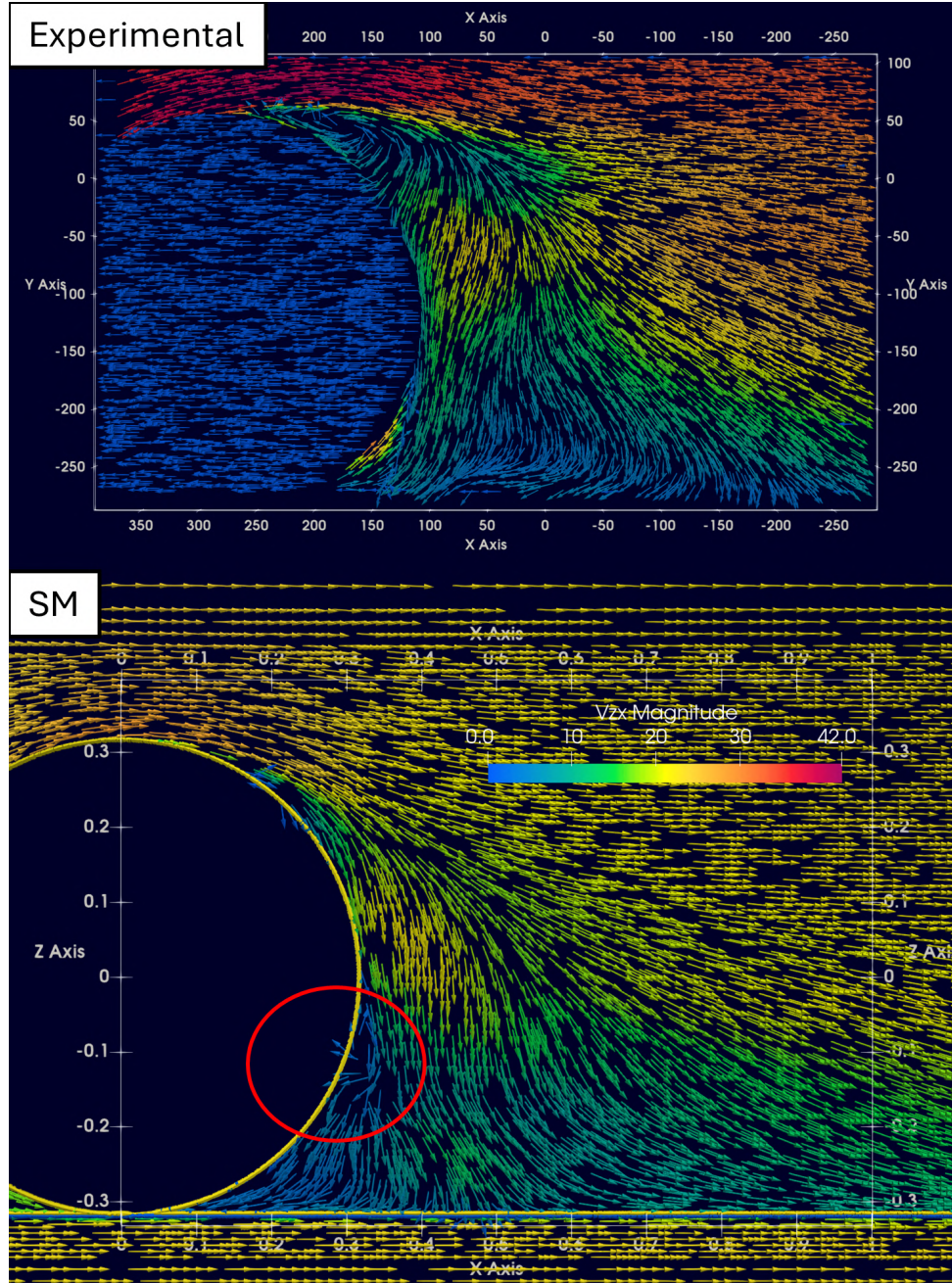


Figure 21: Experimental and SM comparison for the velocity magnitude in the XZ plane at $Y=0$

Additionally, Fig. 22 and Fig. 23 for the planes XZ and YZ respectively are presented to show a contours and vector fields through the Line Integral Convolution Method (LIC). In Fig. 22 the inverted "T" topology of both vortex is observed, but with symmetrical shapes for the MRF and SM cases respect to the MW method.

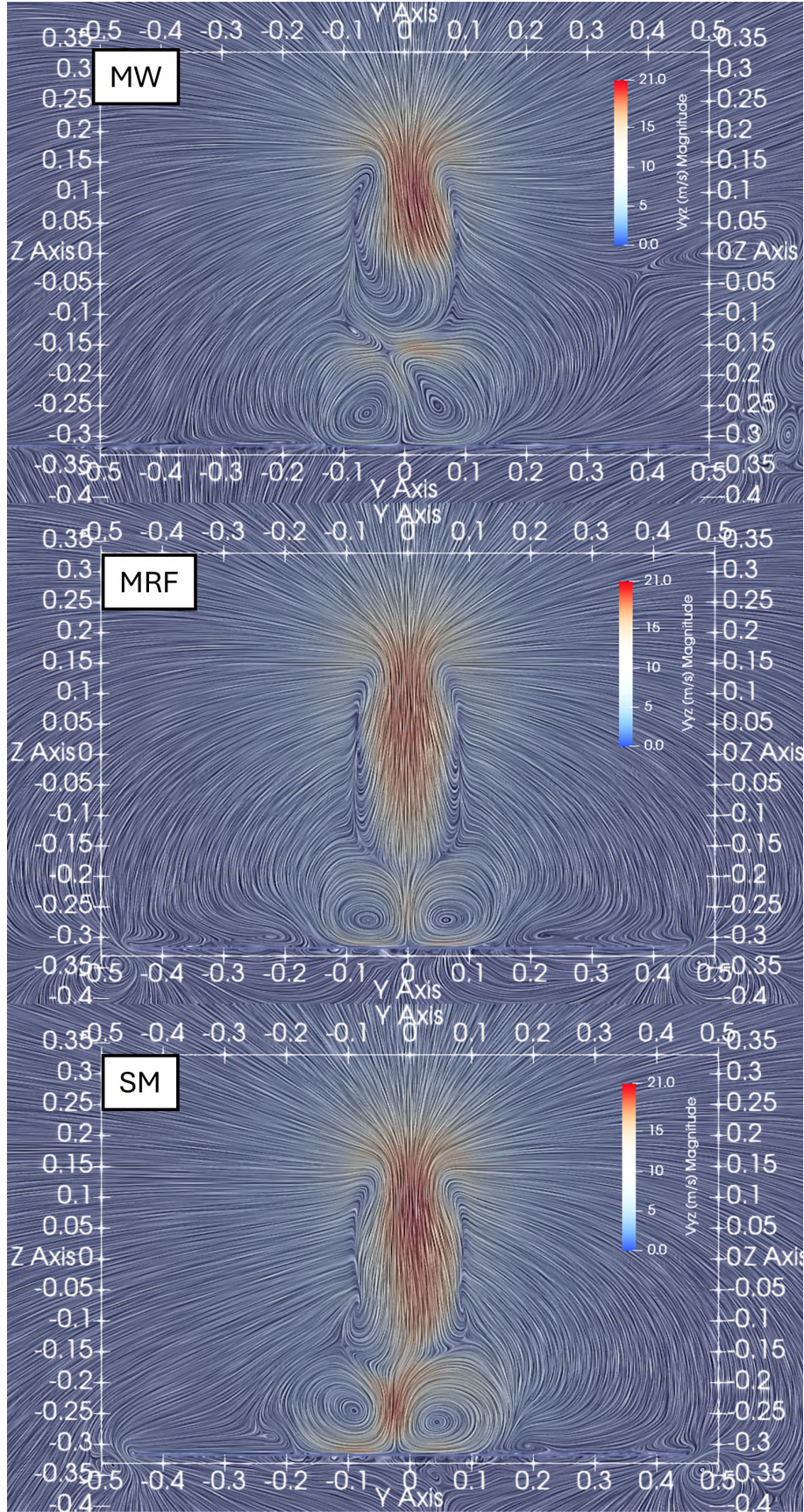


Figure 22: Comparative of LIC surfaces at a plane YZ at $X=5D/8$

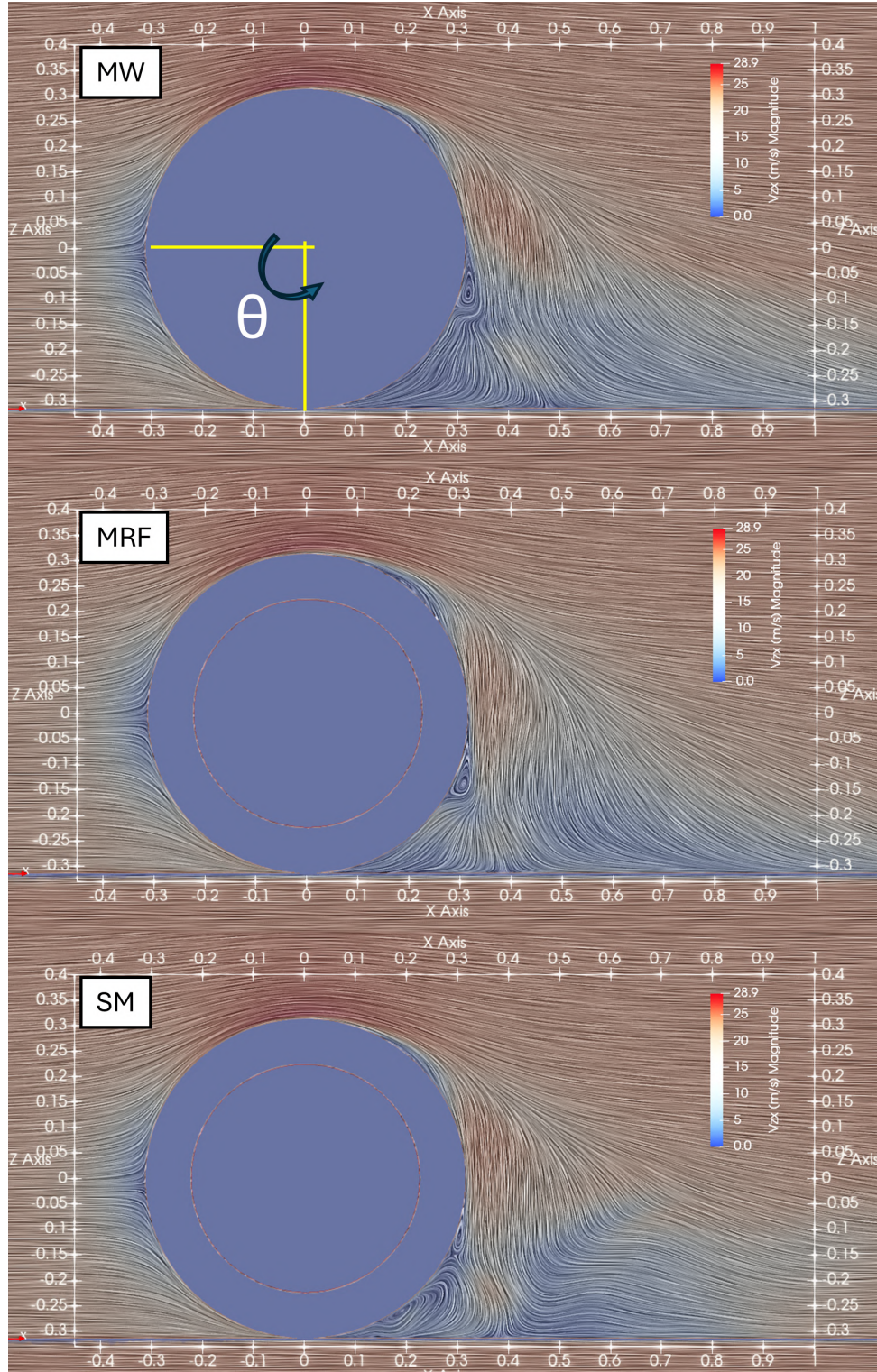


Figure 23: Comparative of LIC surfaces at a plane XZ at Y=0

To assess the accuracy of the simulated results beyond a qualitative comparison of the flow, three quantitative methods have been adopted to perform a comparison against experimental results:

- C_p comparison: The pressure coefficient distribution was obtained along the wheel

centre-line. The plot shows the C_p value vs θ , the angular position which starts at the frontal stagnation point and goes through the wheel centre-line in anti clock wise direction as shown in the MW case in Fig. 23. Experimental data obtained from Croner (16) and Mears (19).

- V_x profile comparison: The comparison of streamwise x-velocity profiles (V_x) in the midplane $Y = 0$ at three different X locations behind the wheel: $5D/8$, $6D/8$ and $7D/8$, with D as the wheel's diameter. Experimental data obtained from Croner (16).
- C_d comparison: The drag coefficient is obtained in the contact surface of the tyre in the experimental and simulated results. Experimental data obtained from Croner (16). This comparison method will be used later in the brake system and wheel model.

The Fig. 24 represents the C_p comparison between the experimental data and the results from the three different mesh refinements for the isolated wheel model with the Moving Wall boundary condition. The Fig. 25 shows the comparison for the MW-medium mesh and the hybrid methods for MRF and SM. For both images, there is a good agreement for the $C_p = 1$ allocated at $\theta = 0$ and $\theta = 360$ with respect to the experimental data. These points represent the frontal stagnation point and its corresponding point behind the steering wheel, where the local velocity is zero.

The three different mesh refinements and the MRF and SM cases follow the same trend of the experimental data. However, an special attention must be given at the position $\theta = 90$ where the contact patch is allocated and there is a discontinuity where an almost instantaneous jump from negative to positive can be seen in the C_p value. Behind this point $\theta < 90$, the C_p is negative due to the air flow is accelerated by the curvature of the wheel, thus the local velocity is higher than the free stream velocity. For $\theta > 90$ the flow has passed through the physical obstruction of the contact patch and the local velocity is lower than the free stream velocity. Therefore C_p is positive, this generates recirculation zones and flow detachment. This behavior can be confirmed with Fig. 23 where the velocity at the front of the tire contact patch is greater than at the rear according to the color map.

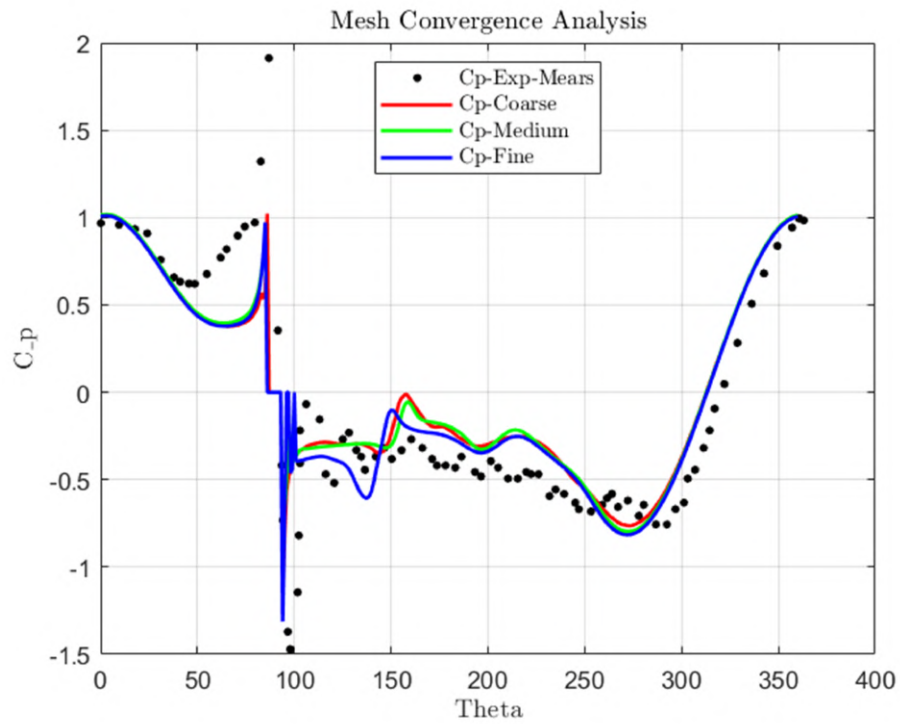


Figure 24: Pressure coefficient C_p comparison for the mesh convergency analysis

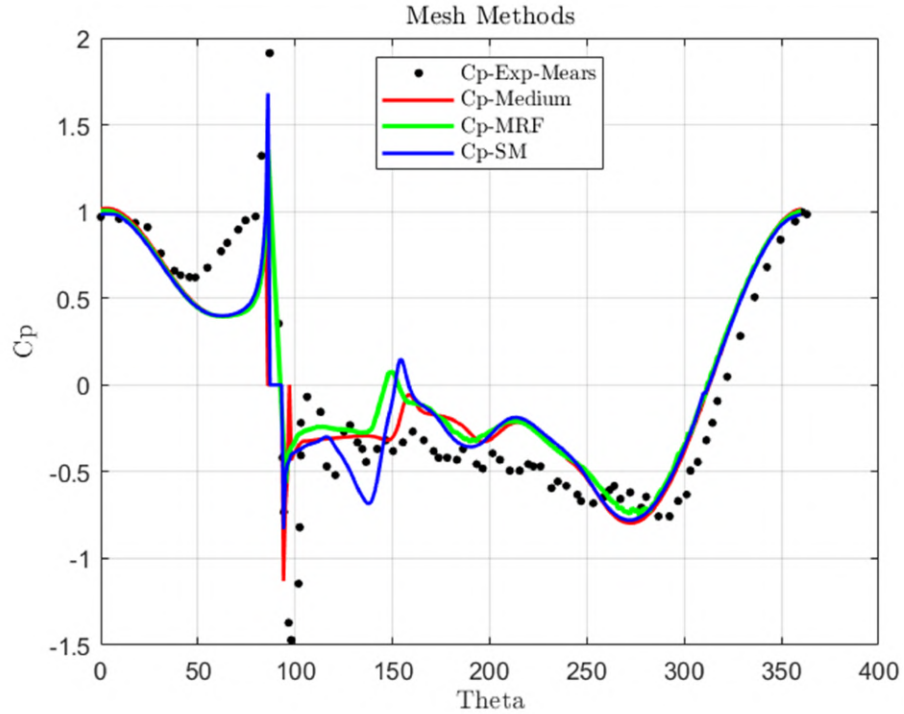


Figure 25: Pressure coefficient C_p comparison for MW,MRF and SM methods with the isolated wheel model

For the fine and medium mesh it can be seen a closer agreement for the negative C_p

but an underestimation for the positive C_p behind the wheel. In the Fig. 25, the MRF and SM methods have a better agreement for the negative C_p but specially for the positive C_p compared to the medium mesh with full moving wall boundary condition. This demonstrates that accurate results can be obtained with a mesh size larger than the medium mesh of 15,558,127 cells. Furthermore, the hybrid MRF and SM methods show a closer trend to experimental data.

The Fig. 26 and Fig. 27 show the results for the V_x profile in the midplane $Y = 0$. In the Fig. 26, all three mesh refinements show similar trend for V_x , however the medium mesh has a better fit compared to the coarse and fine meshes. This differences are more evident below $Z = 0$ which is the zone below the midpoint of the wheel, closer to the ground.

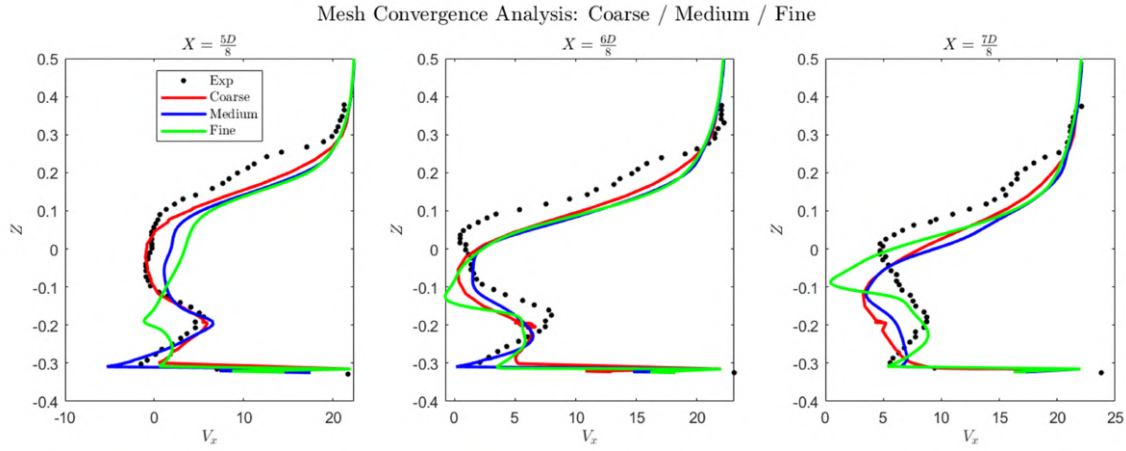


Figure 26: X-Velocity V_x comparison at different X distances behind the wheel for the the mesh convergency analysis

The Fig. 27 shows a good fit at the position $X = 5D/8$ for all the cases, but the MRF and specially the SM method have a better fit in the first two X positions. Although, in $X = 7D/8$ the results try to follow the experimental trend, it is clear the differences near the ground. This is not only the case for the MW, MRF and SM methods, but also is visible for the three refinement meshes. The higher differences near the ground can be explained due to that vortical structures are formed near the wall, as was possible to see in Fig. 16 to Fig. 18, the fine mesh has the highest differences because the smallest cells can catch more variations in the velocity profile. The position $X = 7D/8$ is a further point respect to the wheel, therefore the wake can include stronger turbulent structures which can influence the differences for all the models and refinements.

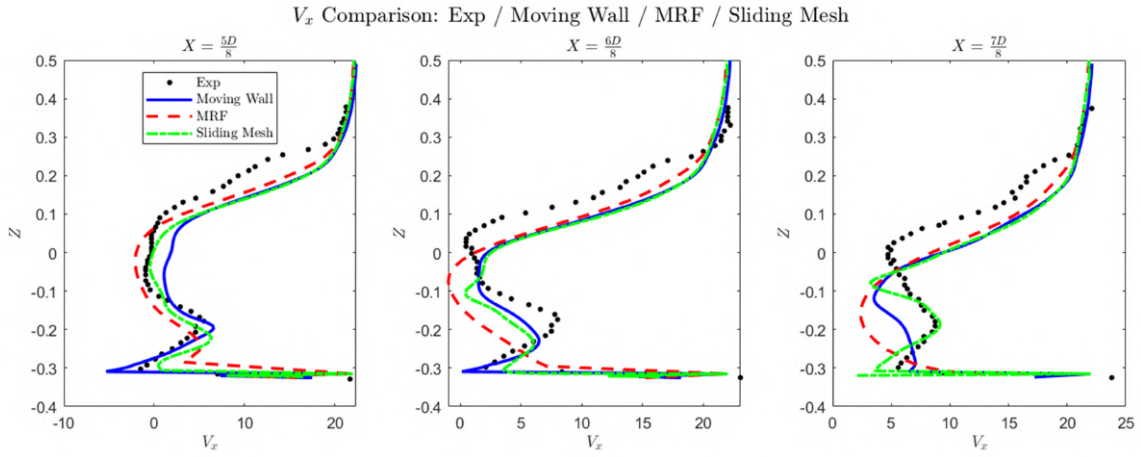


Figure 27: X-Velocity V_x comparison at different X distances behind the wheel for MW, MRF and SM methods with the isolated wheel model

5.5 Meshing MRF/SM Brake system and wheel model

5.5.1 Meshing

For the brake system and wheel model, a polyhexacore mesh was created with 5.784.101 cells as presented in Fig. 28. For this case, the number of cells is reduced compared to the previous cases, due to the face size of the cells from the farfield were considered with a size of 40% the characteristic length of the domain, which corresponds to the wheel diameter. However, four bodies of interest were used covering the wheel, brakes and the wake, as used for the isolated wheel model with MRF/SM, where the target size for the closest boi to the geometry was 30 mm, with a uniform rate of growth of 1.5 for the rest of the bodies of influence. The purpose of this coarse mesh is to reduce the computational load due to the use of the SM method along side with the energy solver. For this reason, the C_d value obtained with this simulation for the angular velocity $\omega = 69.7 \text{ rad/s}$ is going to be compared with the experimental results, as well.

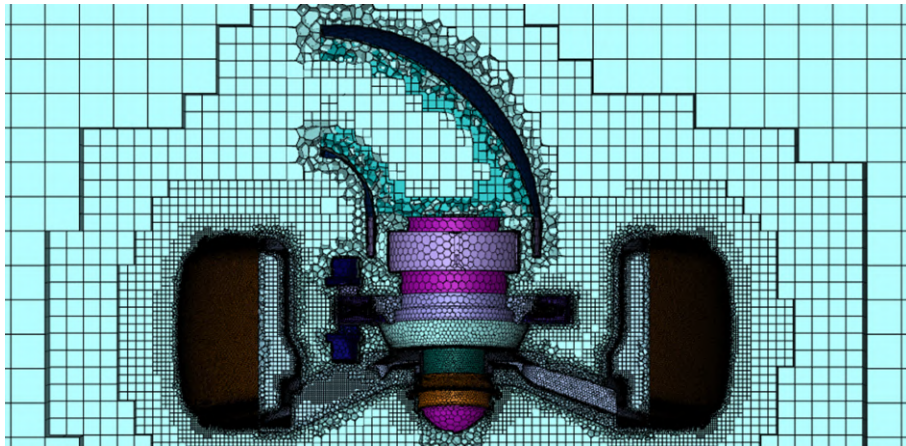


Figure 28: Polyhexacore mesh for the brake and wheel model

Zone	Offset Method
ground-ref-up	smooth transition
Tyre/Wheel	Uniform
Rim	Uniform
Disc-mid-laterals	Uniform
Disc-in	Uniform
Disc-out	Uniform
Upright	smooth transition
Caliper	smooth transition
Pads	smooth transition
Disc-mid-vanes	Uniform
Duct-scoop	Uniform
bell	smooth transition
Hub	smooth transition

Table 24: Boundary condition parameter for the Brake system and wheel model

The boundary layer and first height were used with a $y^+ = 4$ for all the surfaces of the components of the model, and the zones with their corresponding methods are shown in Tab. 24. Additionally, the number of layers for each surface was established as five, as can be seen in the Fig. 29 for the boundary layers of the brake disc and the internal boundary layers for the cooling vanes.

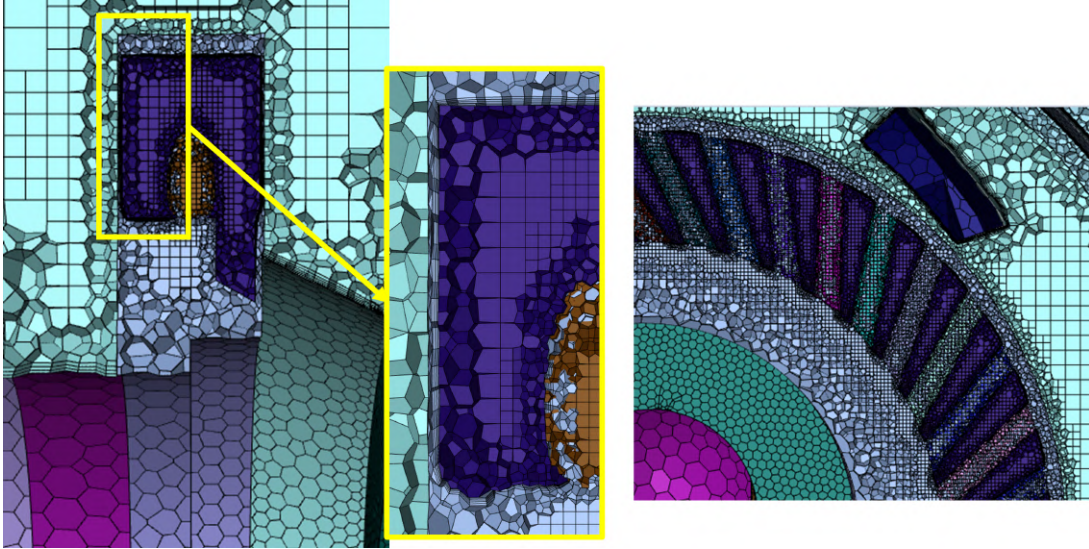


Figure 29: Boundary layers of the brake model

5.5.2 MRF & SM results

The MRF results, the V_{zx} velocity magnitude is first observed for the four different angular velocities in Fig. 30 for the cases MRF2 ($\omega = 31.7rad/s$), MRF1 ($\omega = 44.1rad/s$) and in Fig. 31 for the for the cases MRF3 ($\omega = 69.7rad/s$), MRF1 ($\omega = 95.1rad/s$).

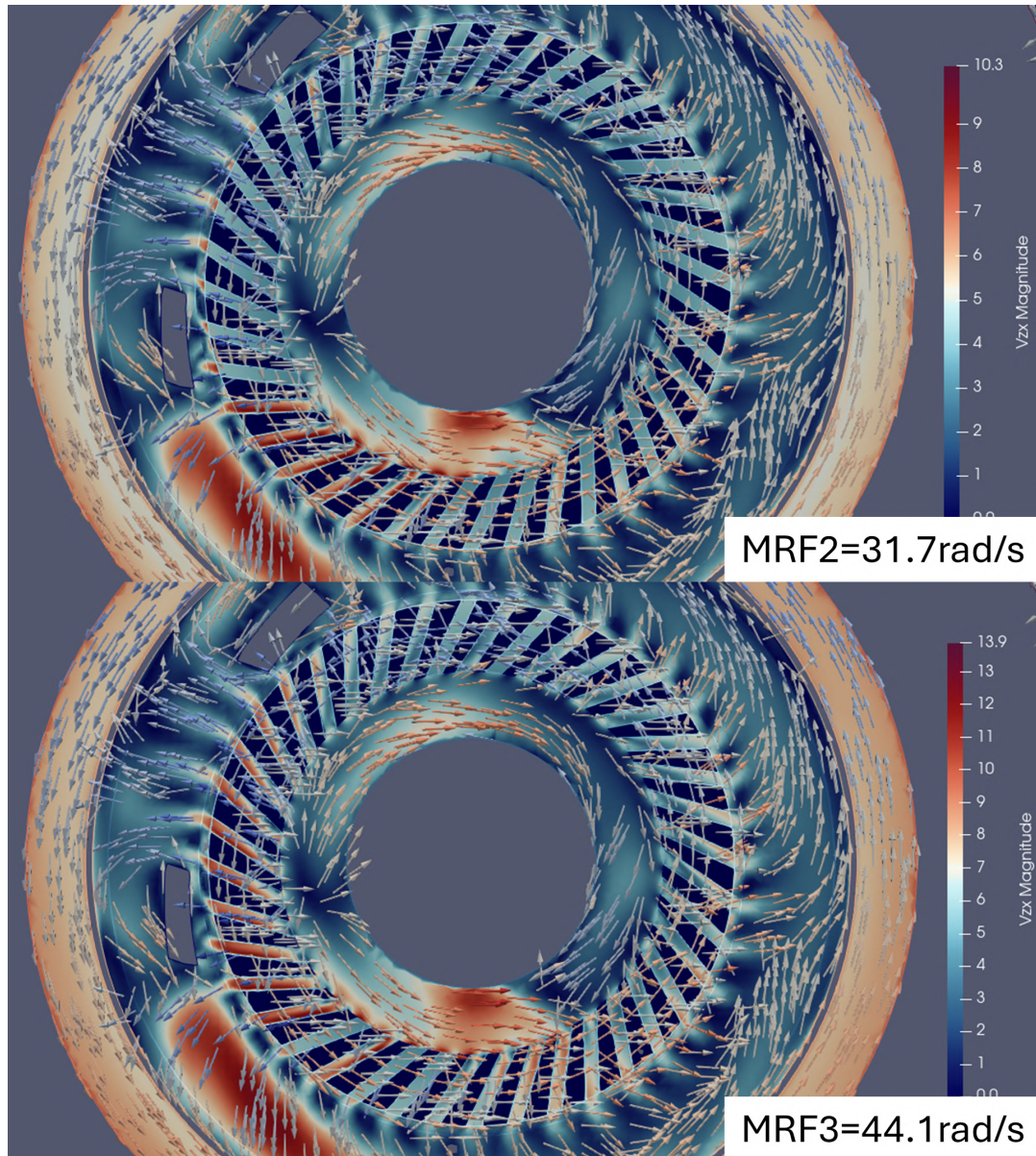


Figure 30: Velocity V_{zx} for the cases MRF-2 and MRF-3 in an internal plane of the brake

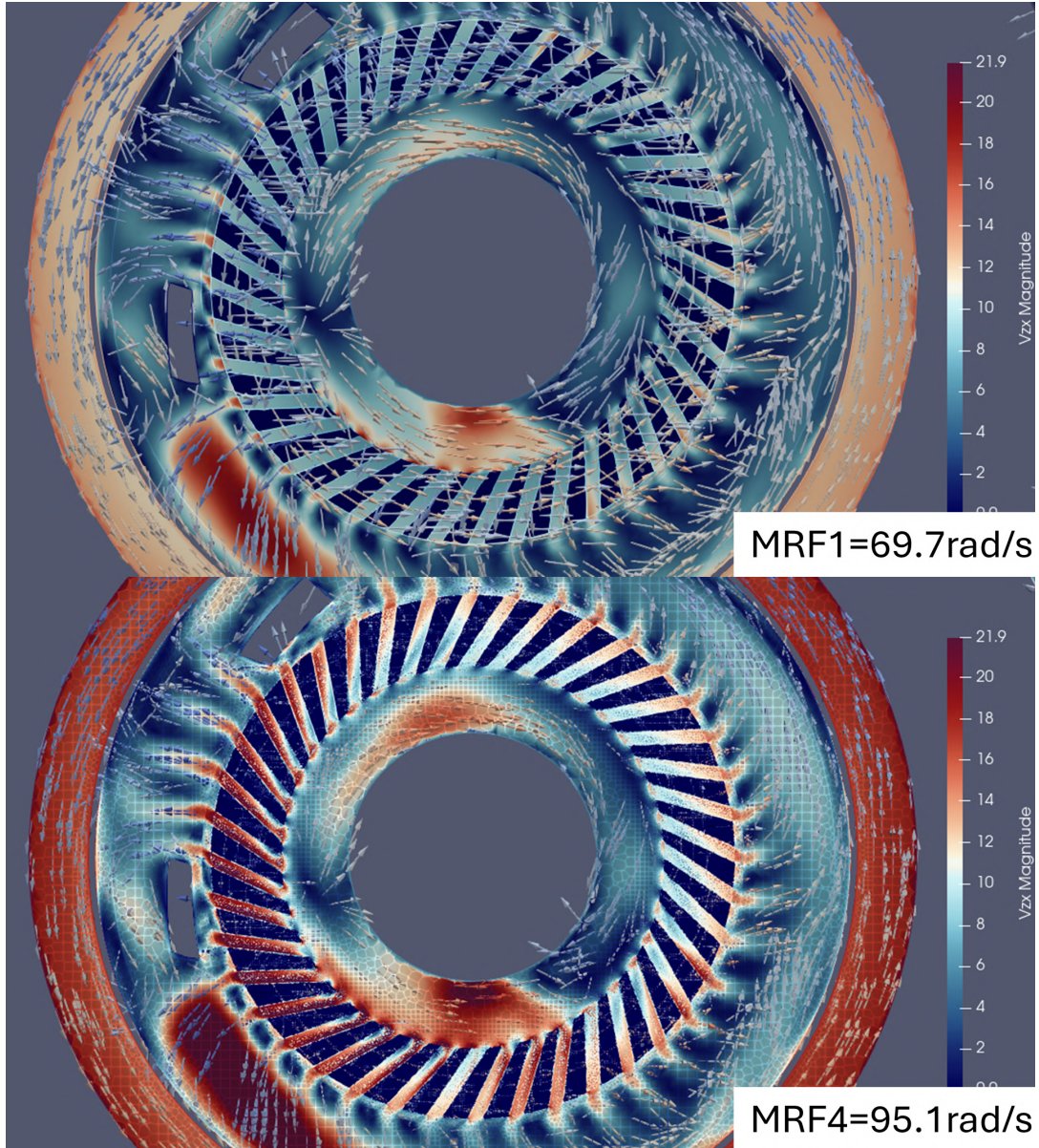


Figure 31: Velocity V_{zx} for the cases MRF-1 and MRF-4 in an internal plane of the brake

In these images, the flow through the vanes of the brake disc is noticed from the internal to the external surfaces of the brake vanes. This flow is generated for the free stream and the flow that enters through the duct-scoop, contributing to the cooling of the brake disc. Additionally, the V_{zx} magnitude increases as the rotational speed increases, which is consistent with the expected physical behaviour. The velocity through the vanes was also assessed through an area-weighted average surface report obtained from the fluid bodies of the vanes. The Tab. 25 summarises the vanes' velocities and experimental data obtained from THURESSON (37), where the velocity is known as pump flow, obtained through an experimental set up for the freestream velocities of $V = 30\text{km/h}$ and $V = 100\text{km/h}$, which are compared with the simulated results obtained for the angular velocities of $\omega = 31.7\text{rad/s}$ ($Speed_{flow} = 36\text{km/h}$) and $\omega = 95.1\text{rad/s}$ ($Speed_{flow} = 108\text{km/h}$). As can be seen in Tab. 25, in both cases, the pump flows data between simulated and exper-

imental results are close, and the difference can be explained due to the higher velocity used for the simulated cases.

Case	Rot-speed (rad/s)	Speed (km/h)	V_vanes (m/s)	Exp V_vanes (m/s)
MRF-2	31.70	36.00	3.60	2.03
MRF-3	44.06	50.04	5.01	-
MRF-1	69.73	79.20	7.92	-
MRF-4	95.09	108.00	10.81	9.05
SM	69.73	79.20	7.92	-

Table 25: Pump velocities through vanes

In the case of SM, which is compared against the MRF case with $\omega = 69.7 \text{ rad/s}$ and visible in the Fig. 32, there is a more uniform distribution of the velocity through the hole brake disk and cooling vanes, the direction of the vectors is similar in both cases in the region of the outlet of the cooling ducts. However, the average value of the pump velocity in the SM case is the same as obtained in the MRF result with the same angular velocity.

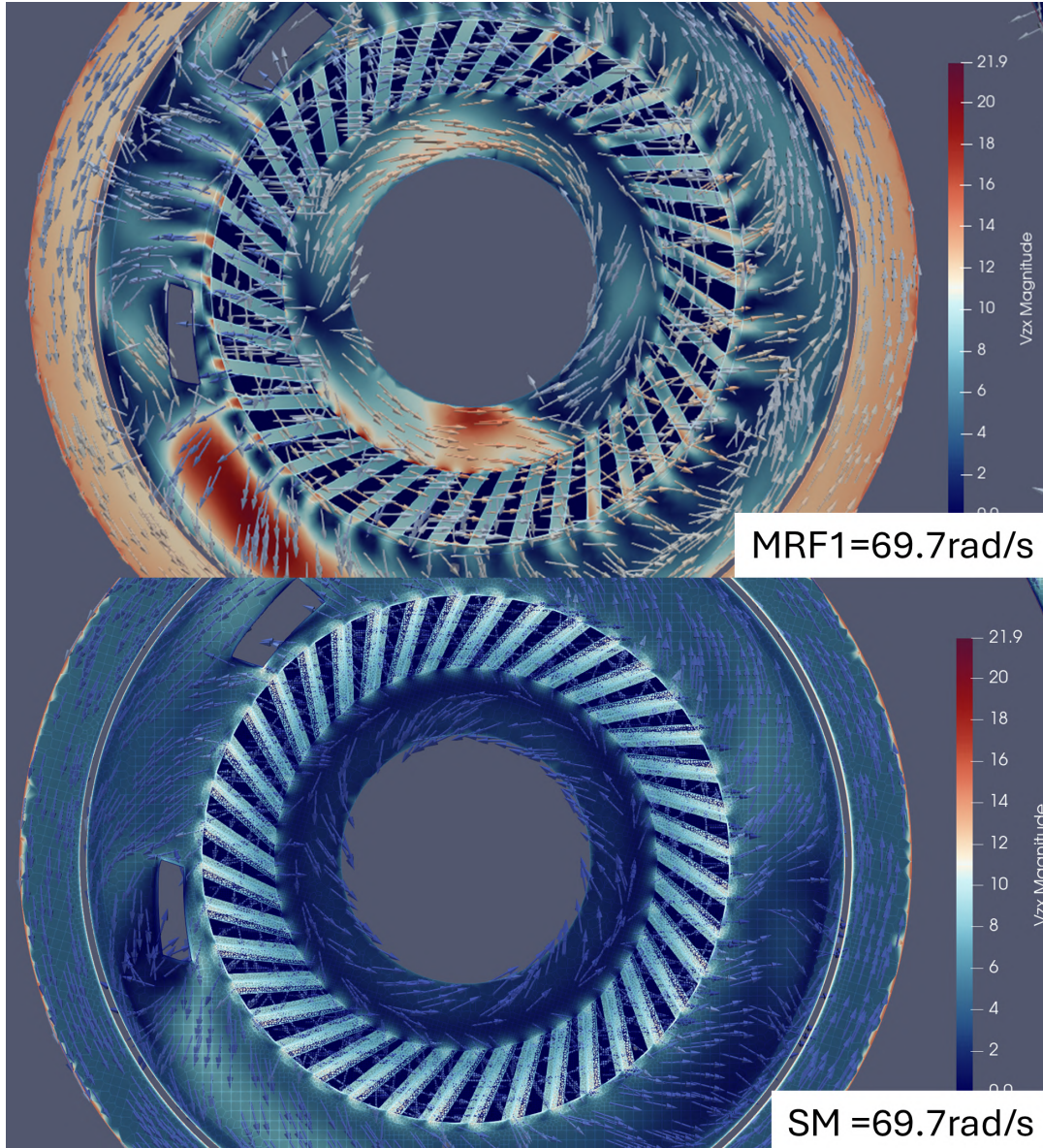


Figure 32: Velocity V_{zx} for the cases MRF-1 and SM in an internal plane of the brake

Following the thermal analysis, and qualitative comparison is presented for the variables HTC (Heat Transfer Coefficient) and the temperature through contour maps. The Fig. 33 gives a visual of the temperature distribution for the MRF cases for the different angular velocities. The temperature tends to decrease with the increase in the angular velocity, but it is also seen that for all the MRF cases, the maximum temperature is always observed in the area where the pads are located and from where the heat transfer is received from the thermal volumetric source.

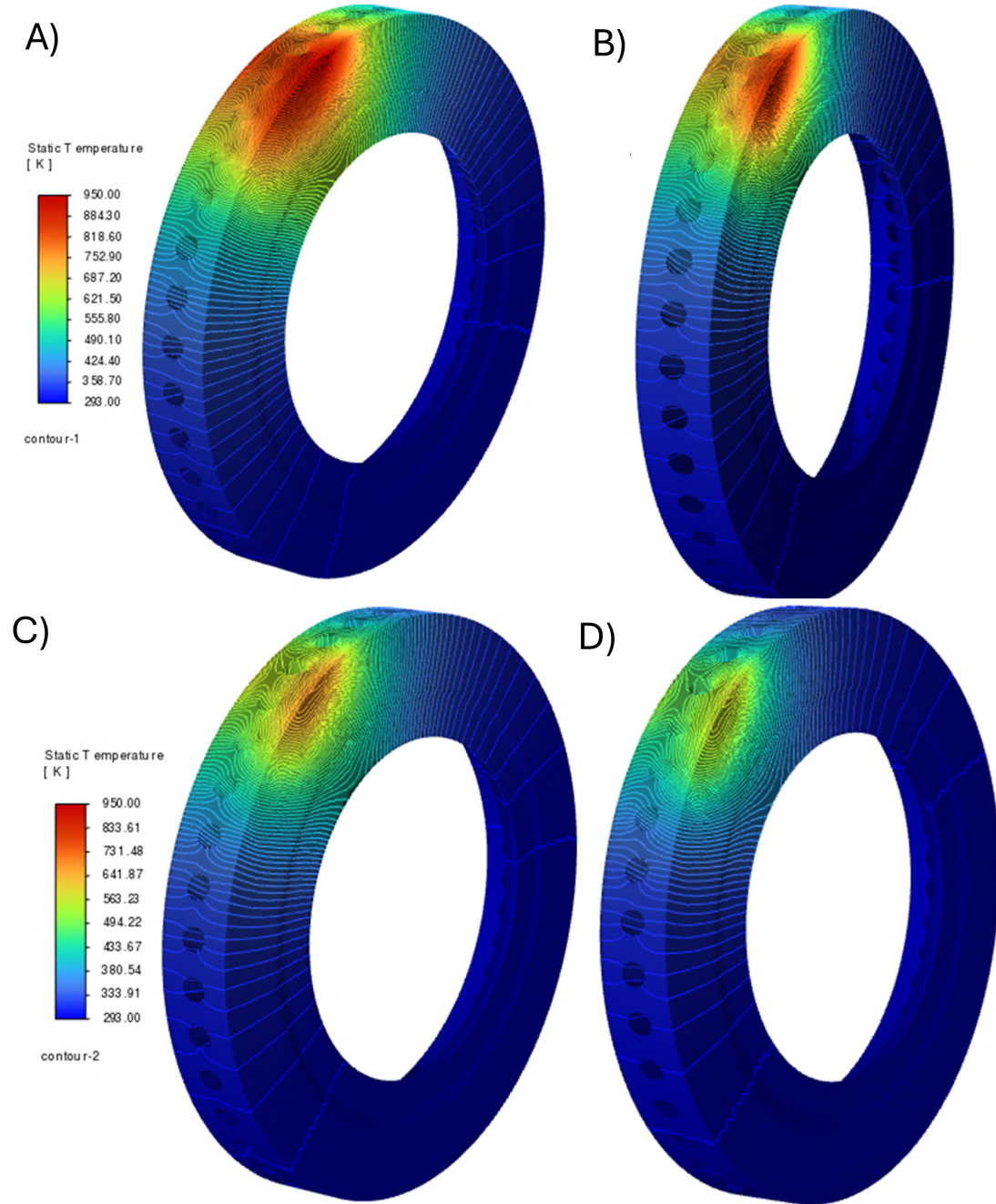


Figure 33: Average temperature distribution for the MRF cases: A)31.7 rad/s, B)44.1 rad/s, C)69.7 rad/s, D)95.1 rad/s

A comparison between the MRF and the SM case shows a marked difference in the temperature distribution map, in Fig. 34. While in the case of MRF, the maximum temperature is found in the area adjacent to the pads, and the temperature change contours are located around this area, in the case of transient-state SM, a uniform mean temperature distribution can be seen across the entire outer surface of the disc. However, a lower temperature is observed in the vane area, corresponding to the effect of the flow in the cooling veins.

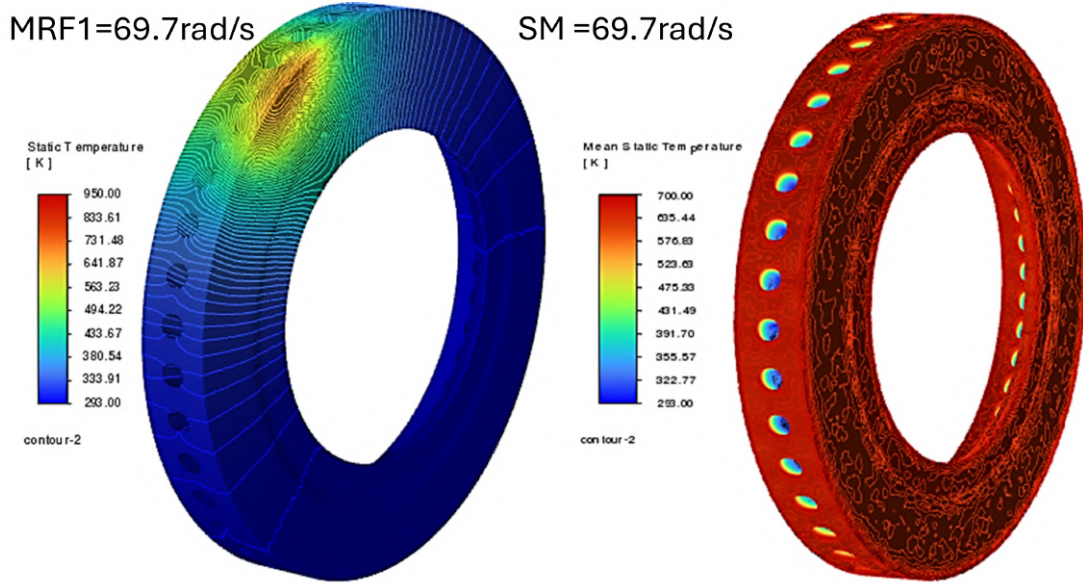


Figure 34: Temperature distribution for the MRF-1 and SM cases both with ω 69.7 rad/s

An increasing value of the angular velocity leads to an increase in the HTC value, according to the Fig. 35 for the MRF cases. However, for the brake disc in the SM case, the HTC distribution is more uniform in Fig. 36, as was the case with the temperature. Another difference is the HTC sign, a positive HTC, the heat flux and thermal gradient are both in the same direction, which is when the disc has a higher temperature than the surrounding flow, when the heat flux is directed to the flow. This positive HTC is more evident in the MRF cases. But in a negative HTC, as in the SM case, the heat flux and thermal gradient are in opposite directions, and the air has a higher temperature than the disc surface, which means that the heat goes towards the solid brake disc. This behaviour can be explained due to the pads being in direct contact with the fluid enclosure of the brake disc, which is the enclosure where the MRF and SM are assigned, but for the SM case, the physical rotation of the enclosure allows a more uniform heat transfer, which then is transferred to the solid disc visible in the Fig. 36

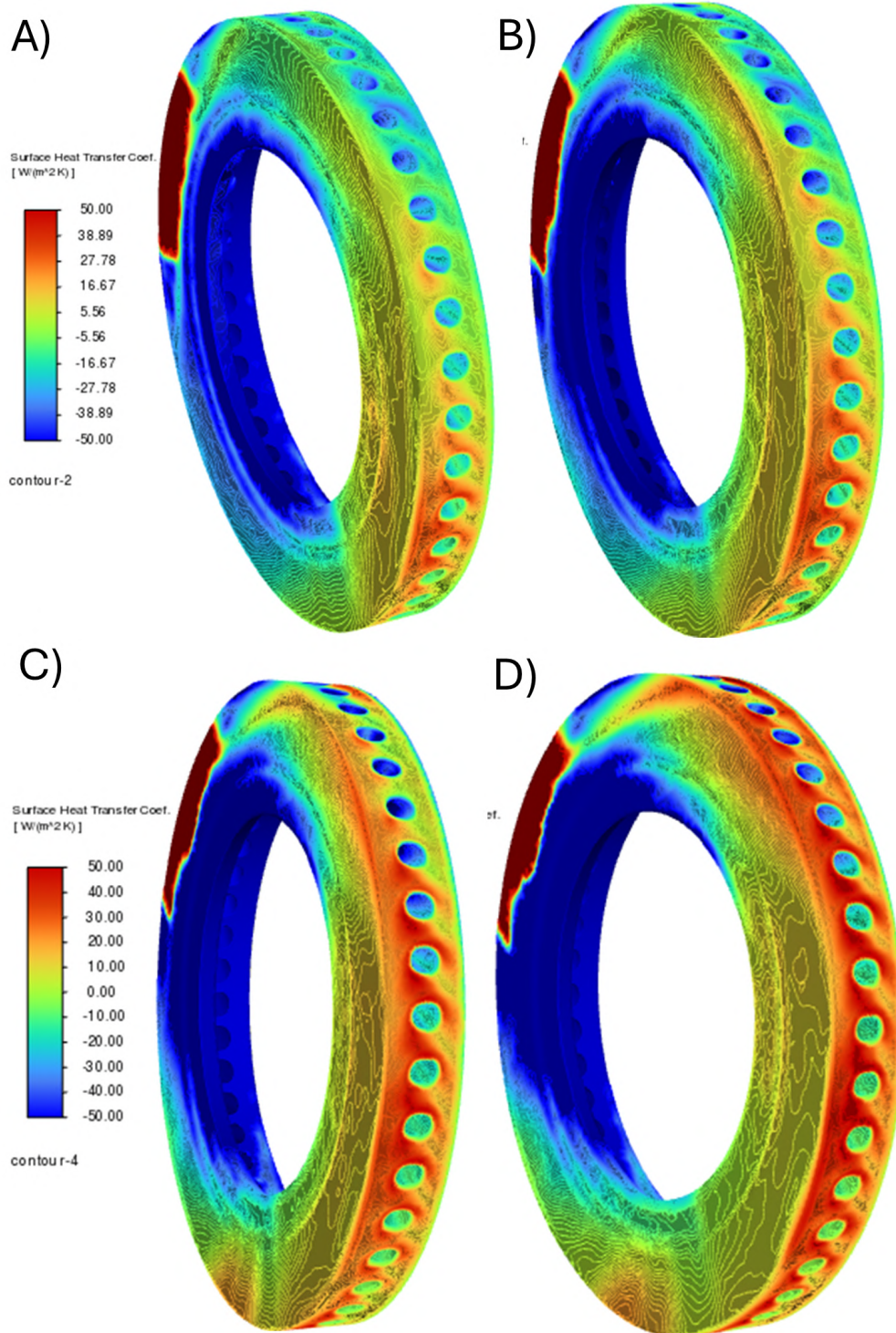


Figure 35: Heat transfer coefficient distribution for the MRF cases: A)31.7 rad/s, B)44.1 rad/s, C)69.7 rad/s, D)95.1 rad/s

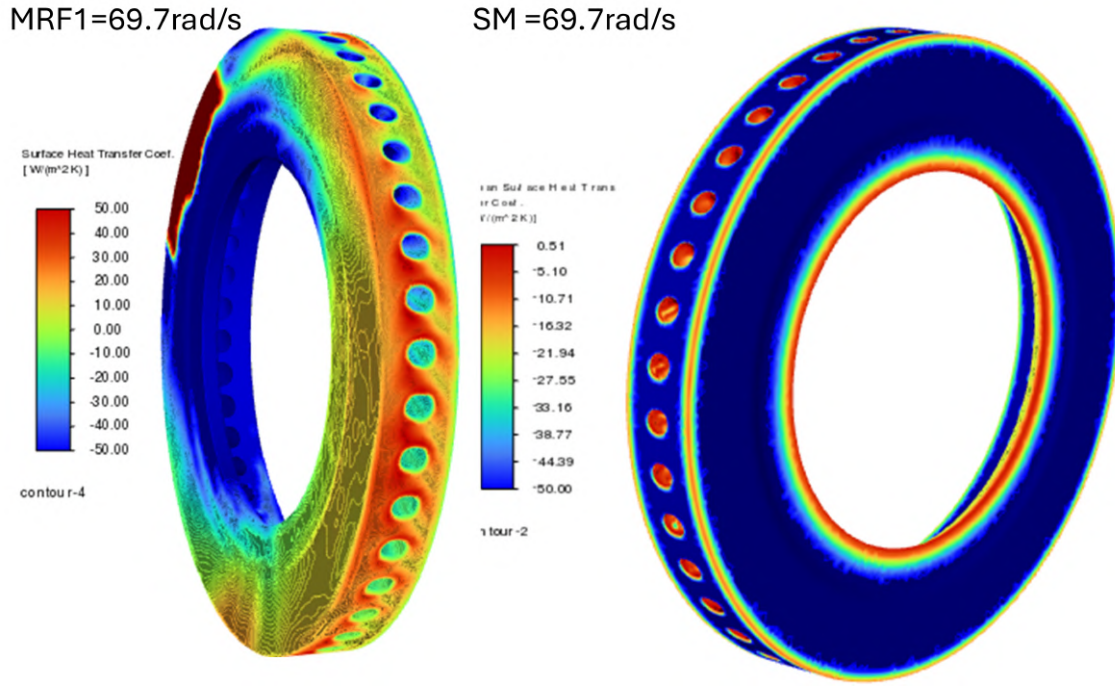


Figure 36: Heat Transfer Coefficient distribution for the MRF-1 and SM cases both with ω 69.7 rad/s

As established previously, the drag coefficient C_d is one of the methods used to validate the different simulated results, the comparison for all the cases was performed with a similar angular velocity of $\omega = 69.7 \text{ rad/s}$. This method was used initially to assess the different drag coefficients obtained for the three mesh refinements and the MRF and SM. The Fig. 37 presents the comparison of the drag coefficients where the experimental results come from Croner (16). The fine mesh with the full MW method achieves the closest value with a relative error of 6.4%, followed by the medium and coarse mesh with 13.2% and 14.1% of relative error. The SM and MRF hybrid method with the mesh of 24,923,430 cells achieved a close value, but with a higher relative error, where the SM case conserves a lower relative error of 14.8% with respect to the MRF with the same mesh.

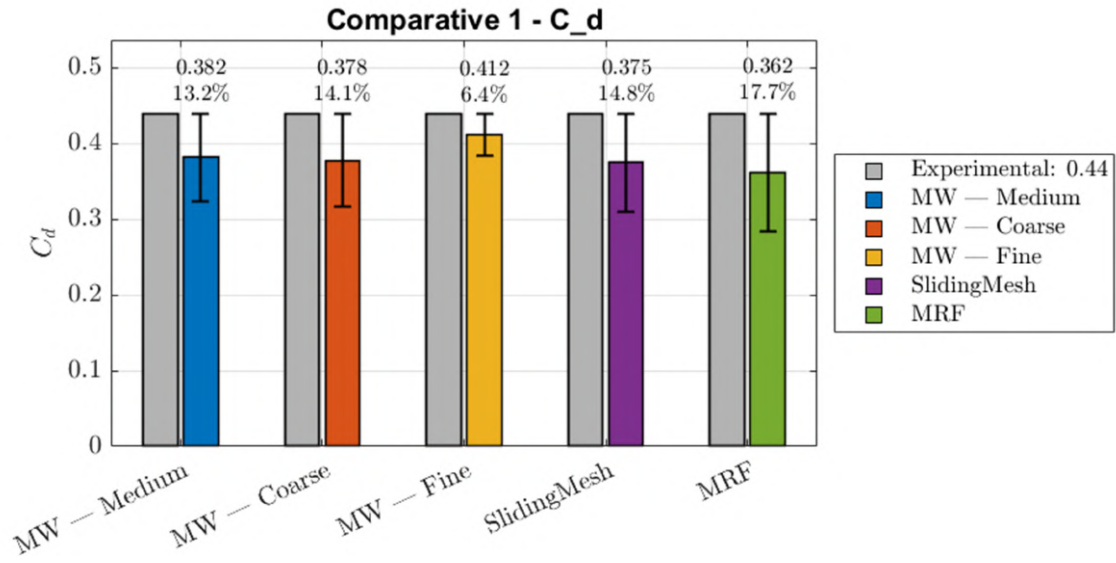


Figure 37: Drag coefficients comparison for the three refinement meshes and for the MRF and SM methods in the isolated wheel model

The Fig. 38 presents a similar comparison for the drag coefficient, but focused on the MRF and SM cases with the brake system and wheel model; in this case, the mesh for both cases has 5,784,101 cells. The relative errors are lower with respect to the MRF and SM results of the isolated wheel. But this could be explained due to the use of a higher number of iterations for the case of the MRF with brakes and a longer time for the SM-brakes case. The isolated wheel was simulated for a physical time of 0.1 s with a time-step size of 1.82×10^{-4} s, and the SM-brake case was solved for a physical time of 1 s with a bigger time-step size of 0.01 s. In all the simulated cases, an underestimation is noted, and an interesting approach to improve this for the SM case could be to add a better refinement for the surfaces of the brake and wheel model and simulate for a longer physical time, thus taking the insights from the fine mesh and the SM-brake model results for the drag coefficient.

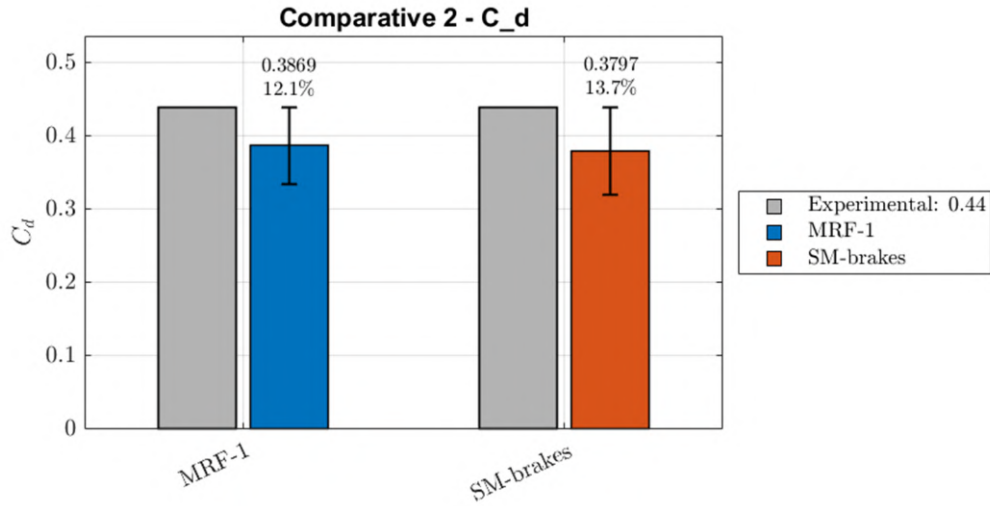


Figure 38: Drag coefficients comparison between the MRF and SM methods for the brake and wheel model

The previous validation was supported on an aerodynamic basis. Nevertheless, a thermal validation is also needed for the brake-wheel model. First, the Fig. 39 shows the comparison of the average temperature and the average HTC from the experimental data obtained in Vdovin (21) against the simulated cases MRF-2 corresponding to the brake-wheel model with an angular speed of $\omega = 31.7 \text{ rad/s}$. For both variables, the simulated results underestimate the experimental data, with temperature showing the greatest relative error, 21.4%, while the HTC is closer to the experimental data, with an 8.1% relative error. The large difference in temperature may be due to the experimental method used to obtain it, using four probes on the tire surface, while the temperature for the simulation was obtained using a weighted average surface report.

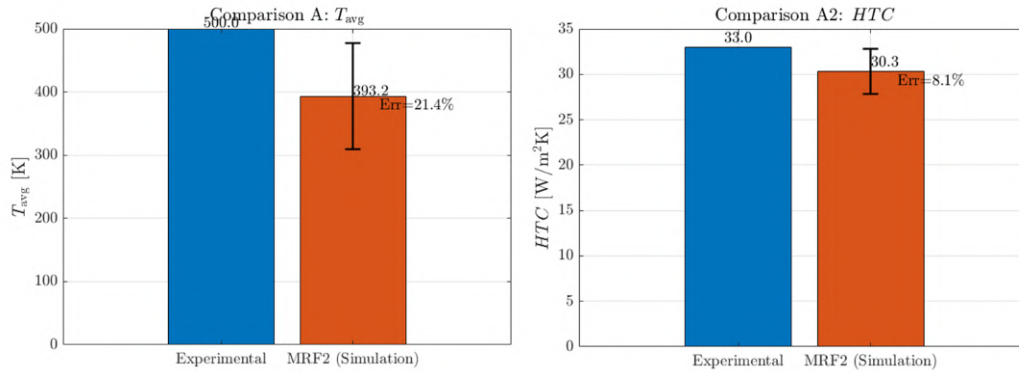


Figure 39: Comparison A: Average temperature and Comparison A2: Heat transfer coefficient for the experimental and the MRF-2 case with $\omega = 31.7 \text{ rad/s}$

Additionally, the experimental results obtained by Dekker (23) are used to compare the behaviour of the HTC, Temperature and the angular speed. The model used by Dekker consists only of a brake disc without a wheel, in addition, the brake pressure is different, and therefore, the amount of heat transferred is also different with respect to the model

used for the simulations of the present work. However, these experimental data are useful to validate the behaviour of the MRF and SM results.

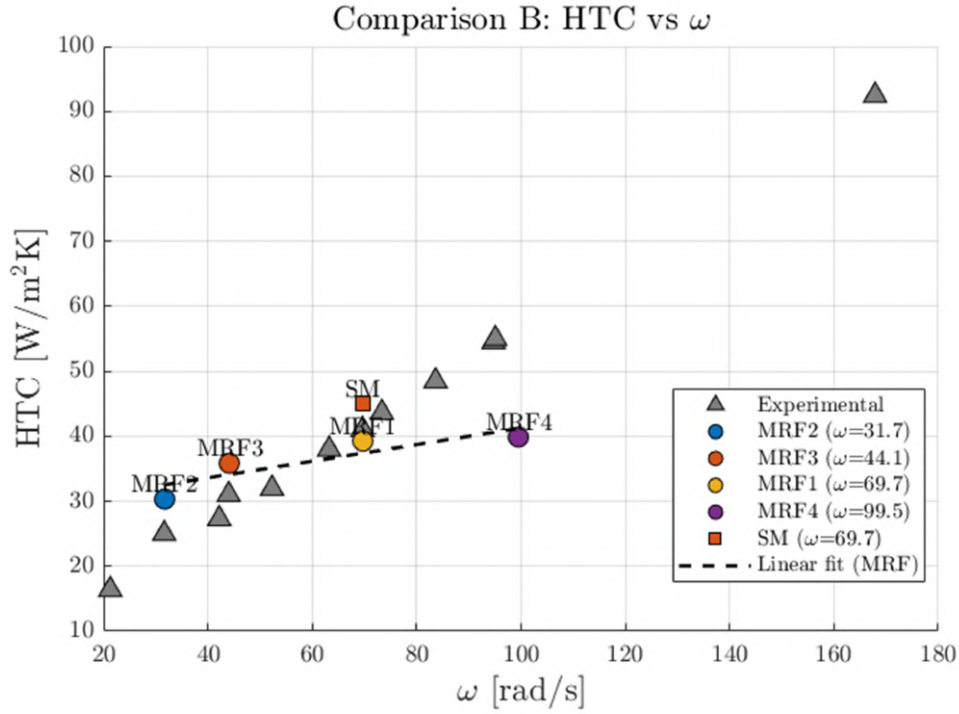


Figure 40: Relation of HTC vs the angular speed of the experimental, MRF and SM data

The Fig. 40 shows initially the relation of the HTC with the angular speed. The experimental data exhibit a linear trend, which is emulated by the MRF results. Additionally, the value of the SM method of the absolute average HTC is also plotted, showing an over-estimation with respect to the MRF result at the same angular speed $\omega = 69.7$, the HTC value for the MRF-1 case was $39.5 W/m^2K$ and for the SM cases was $abs(45.02 W/m^2K)$, which corresponds to the values observed in the Fig. 36.

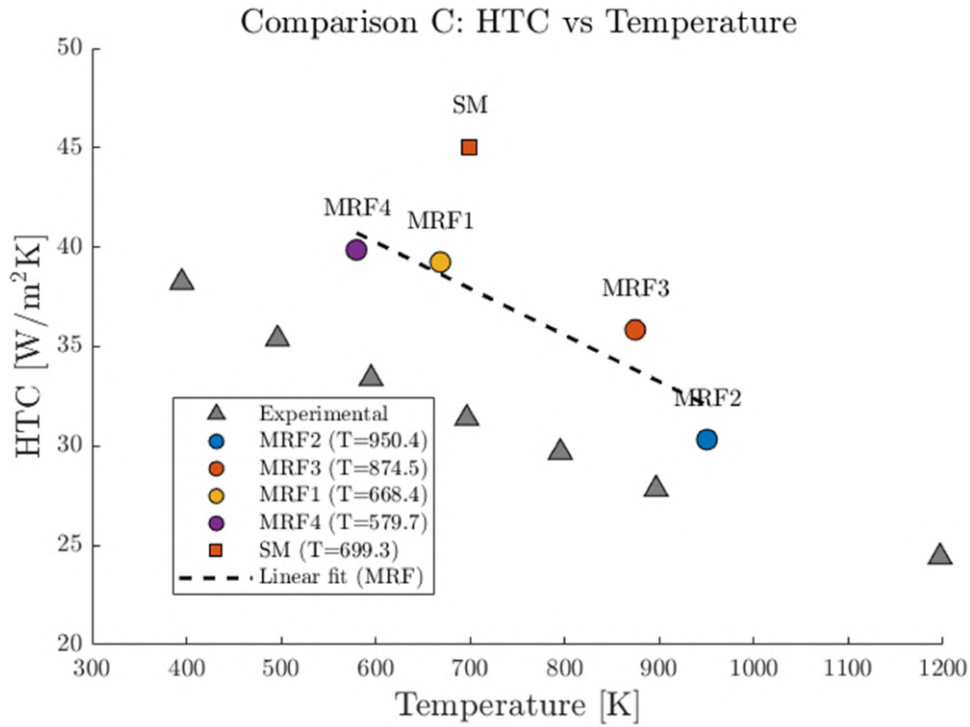


Figure 41: Relation of HTC vs the maximum temperature of the experimental, MRF and SM data

Furthermore, the Fig. 41 shows the relation between the HTC and the temperature, where both the experimental and simulated results of the MRF case follow a decreasing linear trend, while the value for the SM case presents again an overestimation with respect to the MRF1 for both variables, the HTC and maximum temperature. For SM, $HTC = 45.02 \text{ W/m}^2\text{K}$ and $T_{max} = 699 \text{ K}$, while MRF-1 has the values of $HTC = 39.5 \text{ W/m}^2\text{K}$ and $T_{max} = 668 \text{ K}$.

The linear relationship exhibited for HTC vs ω and for HTC vs T_{max} led to the analysis of whether this linear relationship existed for the other variables. Therefore, it was evaluated through the use of Pearson and Spearman correlations. First, the relation between the variables and the angular speed is evaluated through the Spearman Method and it is shown in Fig. 42, where it is shown that the quantities HTC and drag coefficient have an increasing linear trend and the T_{max} and the total heat have a decreasing trend, which is an expected behaviour for the physics of the model. It must be noticed that for both the Pearson and Spearman correlations a value of p , is present to show the statistical significance of those values. As there are only four values simulated for the MRF case, this statistical analysis must be considered as an exploration of the behaviour of the data and not as a conclusive mark.

In addition, the heat map from Pearson correlation is presented in Fig. 43 where the direction and magnitude of the relations between all the analysed variables is presented

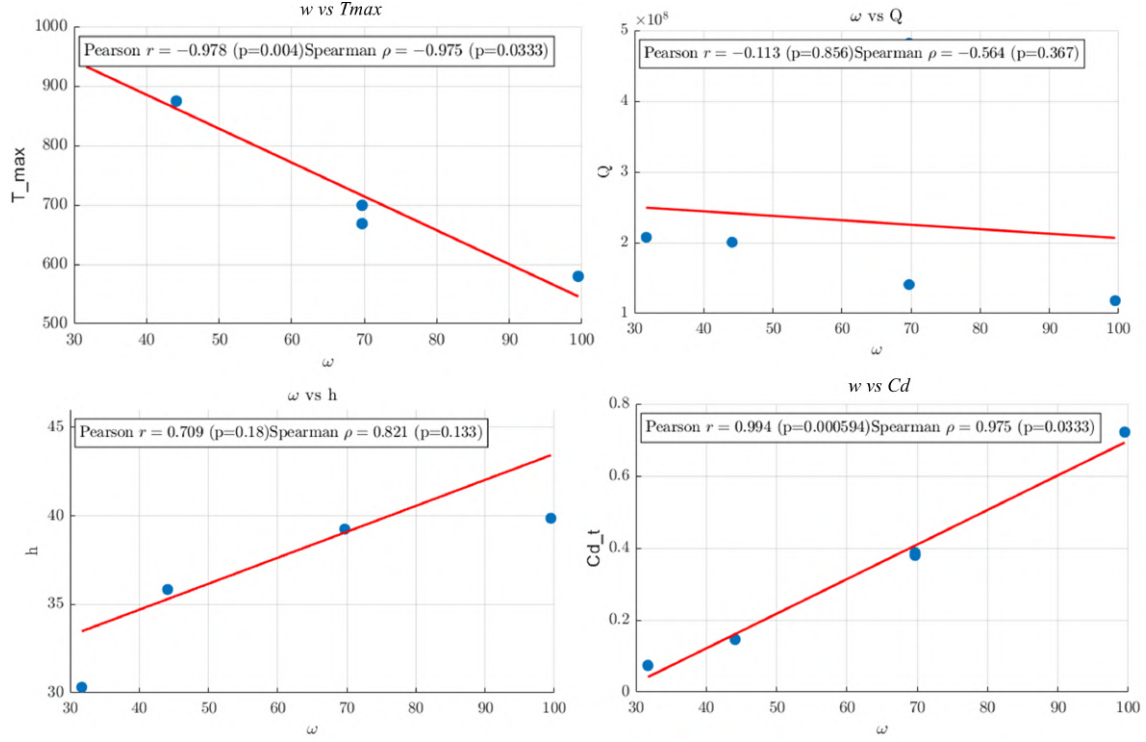


Figure 42: Pearson and Spearman correlations for the different quantities respect the angular velocity ω

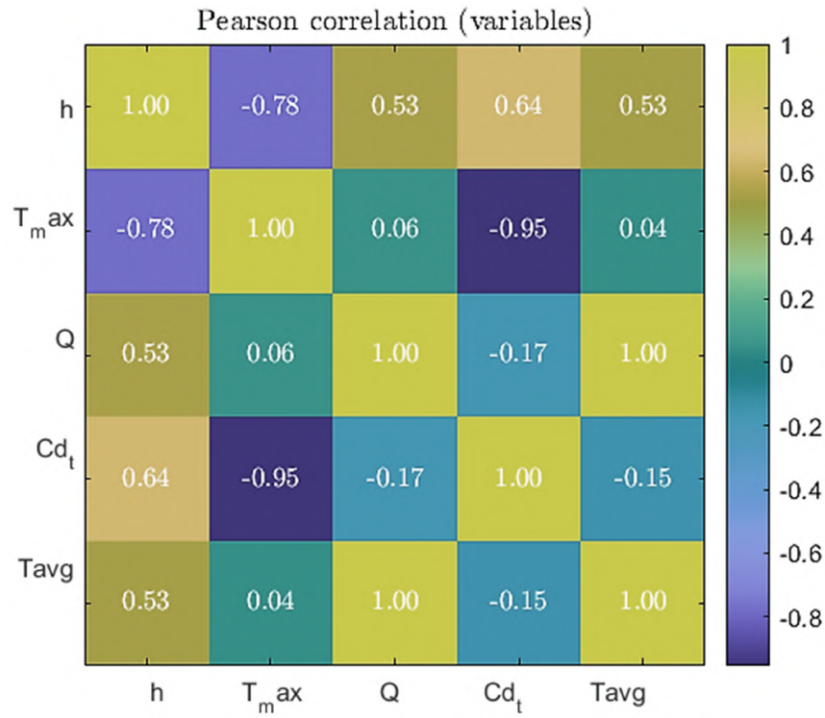


Figure 43: Heat map of the Pearson correlation values between the different quantities of interest

A regression analysis was also used as a tool to identify the results behaviour. The simulated outputs of ω and the heat transfer coefficient were used for a linear and a quadratic fitting models.

The linear fit returns an intercept of 28.48 and a slope of 0.1279, with RMSE = 2.56 and a $R^2=0.771$, and the quadratic fit improves the linear fit with a RMSE =1.23 and $R^2=0.974$ and with coefficients $\beta_1 = 0.59516$ and $\beta_2 = -0.0035391$, the negative quadratic term implies a concave response, so the HTC value increases with ω but only until a peak and then, the values decreases.

The linear model fitted with the four MRF points is $h = 28.482 + 0.12793 \omega$, at $\omega = 69.7 \text{ rad/s}$ the linear prediction gives $\text{HTC}=37.40 \text{ W/m}^2\text{K}$. The obtained SM value of $45.020 \text{ W/m}^2\text{K}$ exceeds the linear prediction with 16.9 % of relative error. In this regression, the slope is positive following the previous results where HTC increases with ω . The quadratic model presents a better prediction with $\text{HTC} = 39.8$ and with a 95% confidence interval for the mean of (25.987, 53.644) which make the actual SM value fit in this interval.

The Fig. 44 shows the fitted linear and quadratic models for the MRF data and the confidence intervals for the fitted lines themselves. The bands represent the uncertainty in estimating the true average value of h for a given ω , and the Fig. 45 is the prediction bands for which shows the same fitted models but with prediction intervals. These bands represent the uncertainty for predicting a single new observation. And according to those figure, the simulated SM value falls inside the quadratic mean 95% CI (Confidence intervals) consistent with the quadratic mean. The simulated value of SM is above the linear mean by $7.62 \text{ W/m}^2\text{K}$ but it falls within the prediction bands of the linear regression.

Although the quadratic method has a better fit, the linear regression is considered for this project due to the previous results observed with the experimental data and also with the Pearson and Spearman correlations. Additionally, the linear regression shouldn't be despised due to the low amount of MRF data used to build the regression, a conclusion remarked by the value $p = 0.122$ which is higher to the 0.05 value needed to be considered statistical significant at the standard 5% level.

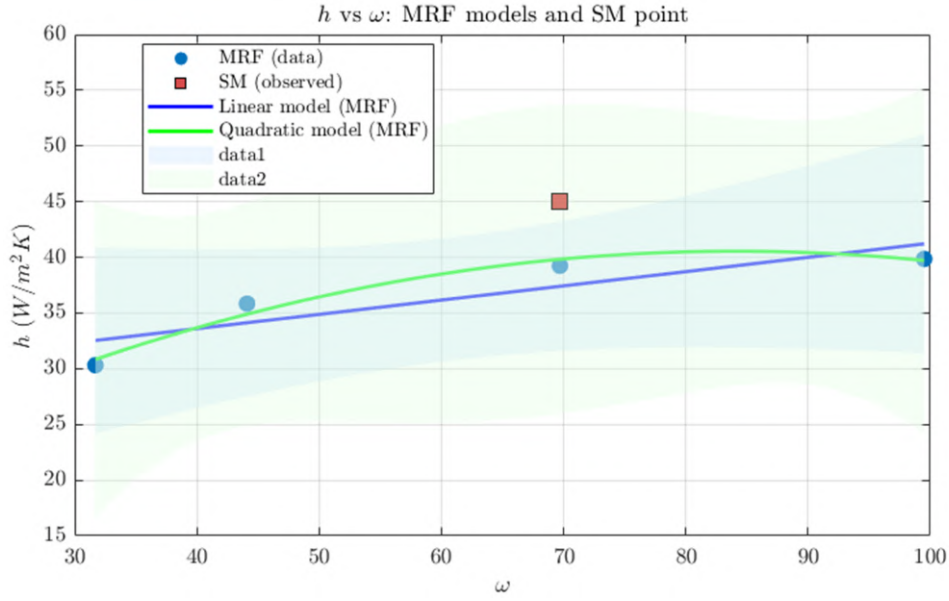


Figure 44: Confidence Bands for the Regression Mean

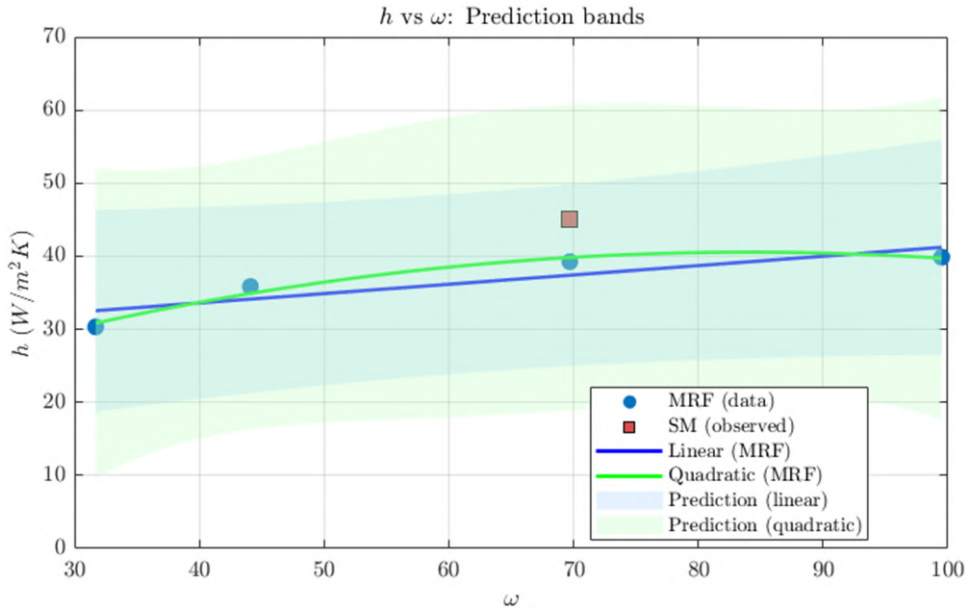


Figure 45: Prediction bands for the regression models

Finally, to assess which one of the variables must be chosen to modify the MRF results through the implementation of a UDF, a PCA (Principal Component Analysis) is performed with variables of HTC, maximum temperature, heat transfer Q and the average temperature. According to the Fig. 46, the PC1 captures the maximum variability in the data with 61.8%, whereas the PC2 captures the 37.1%, which means the PC1 and PC2 explain approximately the 99% of the variability, this allows to reduce 4 variables into 2 axes.

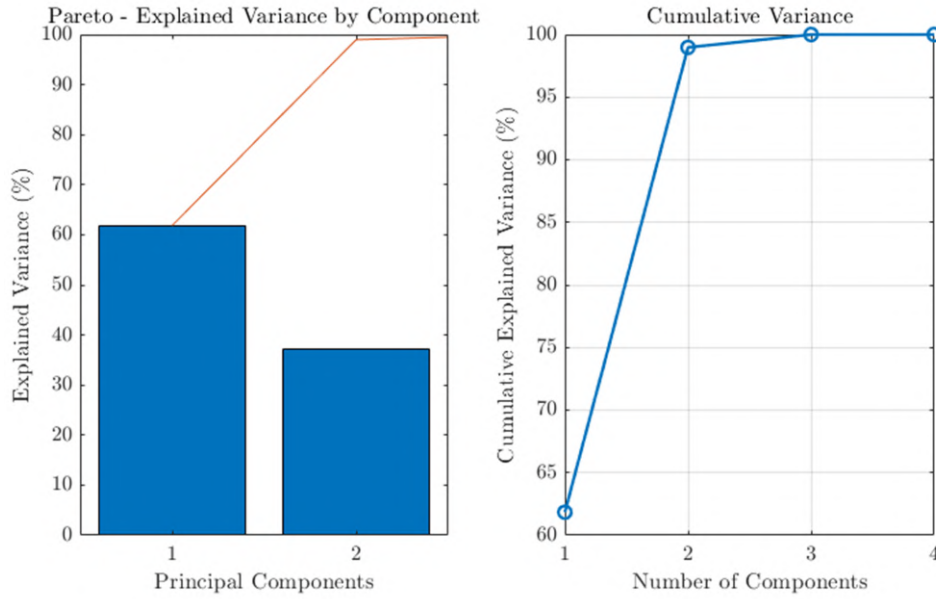


Figure 46: Pareto and cumulative variance for the Principal Component Analysis

The Fig. 47 presents the loadings for the PC1 and PC2, the black arrows shows how each original variable contributes to the principal components. The variables Q and T_{avg} point almost in the same direction which indicates a strong correlation between them, additionally they both dominate for positive PC1, the heat transfer coefficient also is positive for PC1, so that this component reflects the average thermodynamic behaviour. The maximum temperature T_{max} contributes mainly to PC2, but also does the heat transfer coefficient h with its negative value, therefore this second principal component reflects the peak/maximum behavior of temperatures.

In the Fig. 47, there are also the coordinates of the MRF and SM data. The values of MRF-2 (31.7rad/s) and MRF-3 (44.1 rad/s), which are the cases with the lowest angular speed are in the positive PC2 which is distinguished by the high maximum temperatures. And the The values of MRF-1 (69.7 rad/s) and MRF-4 (99.5 rad/s) are allocated in the negative PC2, influenced by low heat transfer coefficient. For the SM case, it is allocated in the positive PC1, with a high influence from the variables Q (heat transfer) and T_{avg} (average temperature). The variables h , Q and T_{avg} have a strong influence in how the model can be reduce to the PC1, with loadings of 0.53, 0.57 and 0.57 respectively. Therefore, any of these variables can be used as the basis for a correction factor to fit the MRF data through a UDF.

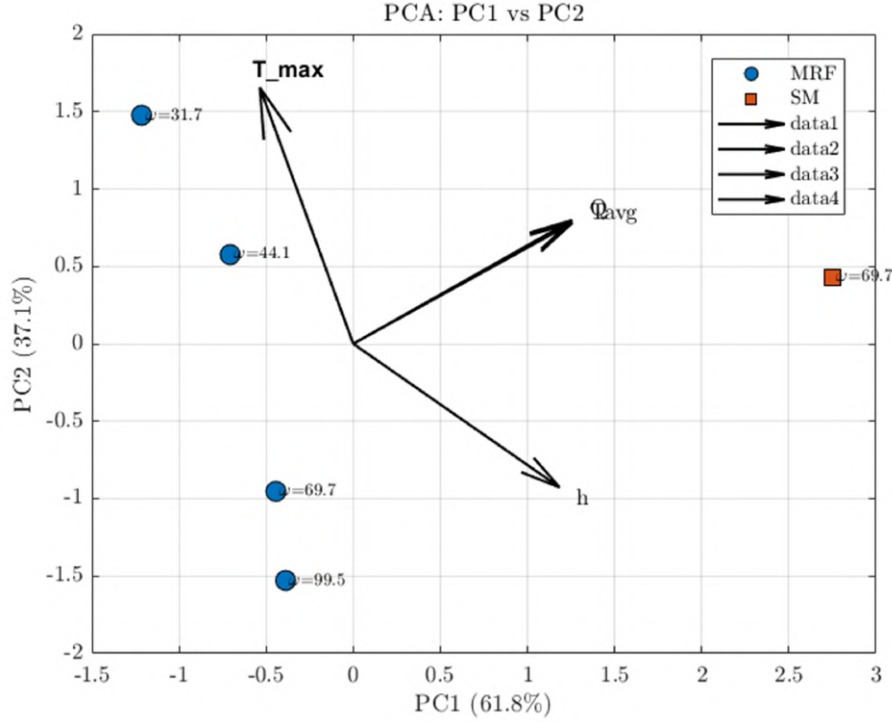


Figure 47: Graph of the loadings of the variables in PC1 and PC2

5.5.3 UDF procedure

To solve the differences between both methods MRF and SM a correction methodology was implemented in terms of the heat transfer coefficient. First, a MATLAB script that calculates a correlation factor from simulation data is implemented and then, it is applied a User-Defined Function (UDF) in ANSYS Fluent that uses this factor during the energy balance in the solid regions of the brake disc.

The script reads the averaged results from the set of MRF simulations and the reference SM case. A factor f_{ref} is computed to scale the MRF-predicted heat transfer coefficient so that it matches the SM prediction at the same angular velocity. With the values $h_{mrf}(\omega_{mrf})$ and $h_{sm}(\omega_{sm})$ at the same angular speed $\omega_{mrf} = \omega_{sm}$, the factor is defined as Equation (43)

$$f_{ref} = \frac{h_{SM}(\omega_{SM})}{h_{MRF}(\omega_{SM})} \quad (43)$$

If the the value $h_{sm}(\omega_{sm})$ is obtained with a different angular velocity from those of the MRF dataset the value is estimated via linear regression or interpolated from the MRF data set. The script then rescales the entire set of MRF results as shown in Equation (44), which ensures the consistency with the SM prediction:

$$h_{MRF,adj}(\omega) = f_{ref} \cdot h_{MRF}(\omega) \quad (44)$$

The Matlab code also generates a summary table and plot as shown in the Fig. 48to compare the original and adjusted heat transfer coefficients. Additionally, the scalar factor

f_{ref} is exported into a text file, which will be later used through the UDF.

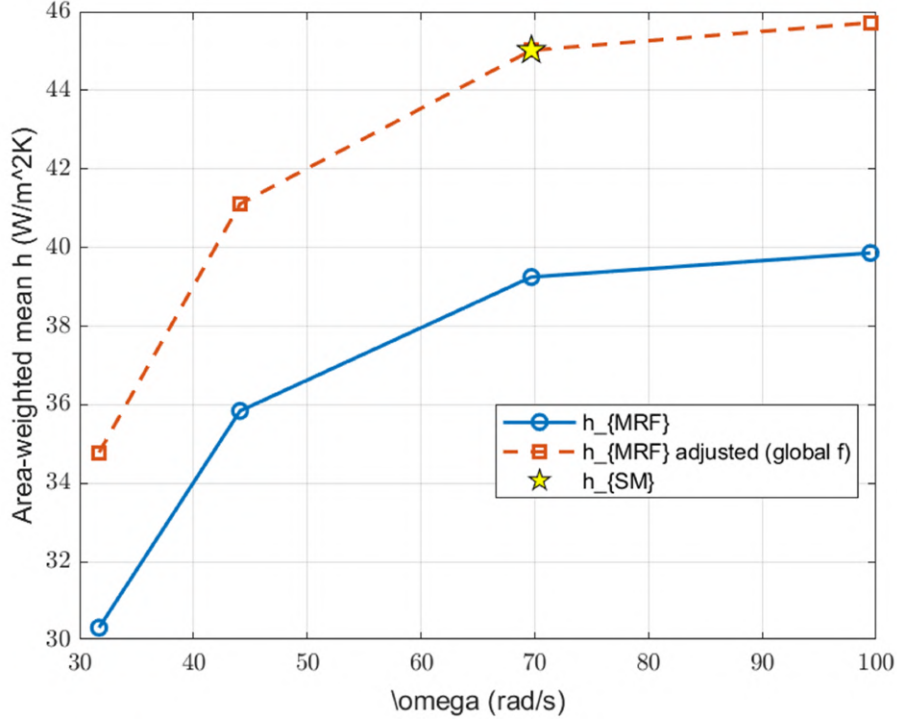


Figure 48: Adjusted heat transfer coefficient through the correction factor f_{ref}

The UDF introduces a volumetric energy source in the brake disc solid region to emulate the adjusted convective heat transfer predicted by the correction factor. The initialisation of the UDF reads the file generated by the Matlab script with the factor f_{ref} . This initialisation also contains a fluid reference temperature $T_{\infty} = 293K$. Later, for each solid cell of volume V_{cell} and temperature T_{cell} the UDF evaluates a characteristic length, Equation (45), and computes an effective surface coefficient in Fig. 46

$$\delta \approx \sqrt[3]{V_{cell}} \quad (45)$$

$$k_{surf} = (f_{ref} - 1) h_{base} \quad (46)$$

This coefficient is converted into a volumetric form with Equation (47), which leads to the additional volumetric energy source term applied in the energy equation of the solid region, Equation (48)

$$k_{vol} = \frac{k_{surf}}{\delta} \quad (47)$$

$$S = k_{vol} (T_{cell} - T_{\infty}) \quad (48)$$

The source term is positive when $f_{ref} > 1$ and $T_{cell} > T_{\infty}$, where additional heat is extracted from the solid to mimic the stronger convective cooling predicted by the SM method. And if $f_{ref} < 1$, the UDF reduces the effective heat loss.

Based on the results obtained from the MRF-1 and SM simulations with the brake and wheel model for the angular velocity $\omega = 69.7 \text{ rad/s}$. The UDF procedure was applied in the MRF-1 setting to asses if an improvement or closer results could be achieved respect to the SM results. The Tab. 26 shows the results for the average values obtained from the MRF1-UDF simulation, and according to this, the maximum temperature value increased compared to the MRF1 data and now the relative error with respect to the SM case is 2.16%. An increase is also seen in the heat flux Q and in the average temperature, but a loss in the value of the heat transfer coefficient is evident, which is even lower than the MRF1 value.

Method	Rot-speed (rad/s)	h	T_max	Q	Cd	Tavg	V_vanes
MRF1	69.7	39.25	668.38	1.41E+08	0.39	330.65	7.92
SM	69.7	45.02	699.28	4.82E+08	0.38	608.75	7.92
MRF1-UDF	69.7	31.69	714.13	1.84E+08	0.39	347.49	7.92

Table 26: Comparison of the MRF and SM methods of the brake and wheel model against the MRF1 fixed through UDF factor

The qualitative comparison for the temperature in Fig. 49 allows to conclude that although the maximum temperature is higher in the modified MRF case, the distribution keeps far from the SM method. For the heat transfer coefficient the results are more critical where not only the distribution of the modified MRF is different from the SM method, but also the HTC is lower than the initial MRF and the SM methods, as is observed in Fig. 50

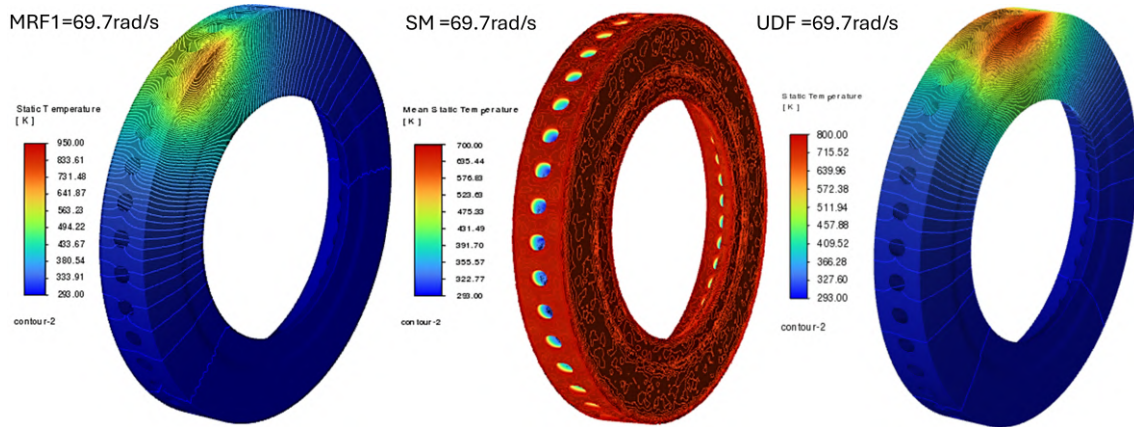


Figure 49: Temperature distribution comparison for the MRF-1, SM and the Modified MRF through the factor f_{ref} and the UDF

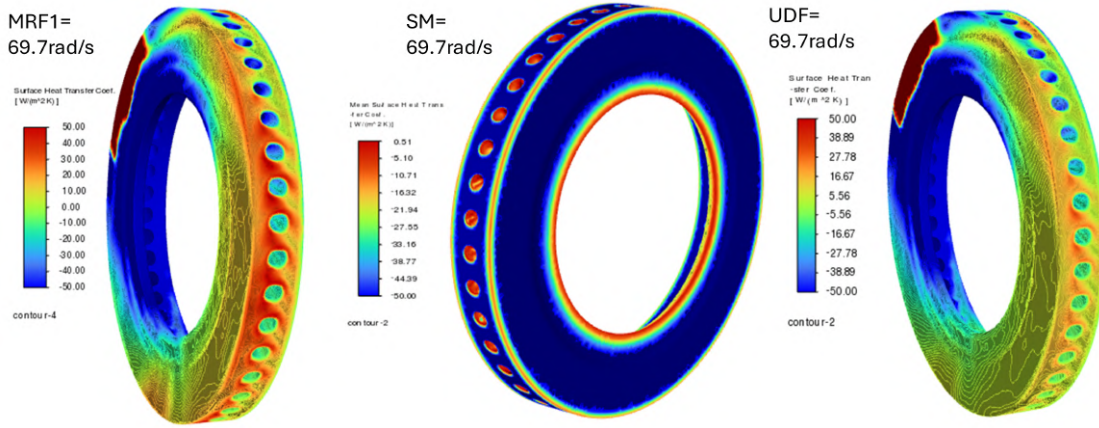


Figure 50: Heat transfer coefficient comparison for the MRF-1, SM and the Modified MRF through the factor f_{ref} and the UDF

With these results it can be said that exists a relation between the different thermal quantities that allow the development of a correction factor for the MRF simulations. However, it is not possible to conclude that a factor based only in the heat transfer coefficient is enough. Although, the maximum temperature of the SM was reached with the factor, the additional quantities such as the heat transfer coefficient and average temperature from the SM method, could not be reached with the modified MRF case. For a better fit, more variable could be implemented such as the values of the quantities at different angular positions.

The Matlab script and the UDF scripts are available in the repository: <https://github.com/rb18arteaga/MRF-SM-Project.git>

6 Conclusion & Feature Work

- This project successfully developed and executed a comprehensive methodology to assess the Multi-Reference Frame (MRF) and Sliding Mesh (SM) methods for simulating the aerodynamics and thermal behaviour of a rotating wheel and brake system. The investigation progressed from validating an isolated wheel model against experimental data to a full thermal analysis of a brake assembly, culminating in a statistical evaluation and a proposed correction methodology.
- A validations of aerodynamic conditions was performed through the drag coefficient and pressure coefficient. The initial validation with an isolated wheel model confirmed that both the MRF and SM methods can produce results that can align with the experimental data for C_d and C_p . But also for the streamwise velocity V_x . The SM method showed a marginally superior ability to capture the complex wake structures and recirculation zones behind the wheel, as observed in the Particle Image Velocimetry (PIV) data.
- For the coupled brake and wheel model, a significant discrepancy was identified between the steady-state MRF and the transient approach with SM. The SM method resulted in a more uniform distribution of temperature and heat transfer coefficient

(HTC) across the surface of the disc. The SM simulation predicted a higher convective cooling effect, shown by a 14.7 % higher HTC and a 4.6% higher maximum temperature compared to the MRF result at the same angular velocity (69.7 rad/s). Therefore, this suggest that the transient SM method can capture more accurately the physical heat transfer process.

- The MRF results across four different angular velocities captured correctly the expected physical behavior of quantities like the HTC and drag coefficient increasing with the rotational speed, while the maximum and average temperature decreased. Statistical analysis performed must be observed like exploratory tools to assess the trend of the results due to the low quantity of MRF data available. The statistical analysis with Pearson/Spearman correlation and regression confirmed strong linear relationships between these variables and the angular velocity. The SM result for the HTC at 69.7 rads fell within the prediction interval of the linear and quadratic regression models fitted to the MRF data, and although the predicted value for the HTC with the linear regression presented higher error, the linear regression has been chosen as the representative model due to the comparison with the experimental data and the results of the previous statistical analysis which support the linear behaviour.
- Correction Factor Potential: A correction methodology was proposed and implemented via a User-Defined Function (UDF) to scale the MRF-predicted HTC based on a factor derived from the SM result. While the application of this factor successfully adjusted the maximum temperature closer to the SM value (reducing the error to 2.16%), it did not fully replicate the SM results for other key variables like average HTC and temperature distribution. This indicates that a simple scalar correction based on a single variable is insufficient; the difference between MRF and SM is multi-variate and influenced by the methods' fundamental treatment of flow rotation and heat transfer.

Future work can be performed with extended transient simulations. In this project the SM simulation was limited by computational resources. Running the SM case for a longer physical time and with a smaller time-step would provide robust reference result for validating the steady-stat MRF approach and any correction factors. A GCI should be employed with thermal quantities to asses the brake and wheel model, to ensure that the observed differences between the MRF and SM are not influenced by discretisations error and it would define an appropriate mesh resolution for future thermal studies. Additionally, through this work it was observed that the settings for the boundary conditions at the walls or interfaces which interact with the heat transfer process can influence the results of the SM method

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