

Design and operation of single- and multi-deep Shuttle-based Storage and Retrieval Systems (SBS/RS)

Tone Lerher^a, Jakob Marolt^b, Fabio Sgarbossa^c, Banu Y. Ekren^{d,e}, Goran Dukic^f

^aUniversity of Maribor, Faculty of Mechanical Engineering, Smetanova 17, 2000, Maribor, Slovenia

^bUniversity of Maribor, Faculty of Logistics, Mariborska c. 7, 3000, Celje, Slovenia

^cNorwegian University of Science and Technology, Department of Mechanical and Industrial Engineering, Richard Birkelands vei 2B, 7034 Trondheim, Norway

^dCranfield University, School of Management, Logistics & Supply Chain Management, MK43 0AL, Cranfield, United Kingdom

^eYasar University, Department of Industrial Engineering, Izmir, Turkey

^fUniversity of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Ul. Ivana Lučića 5, 10000, Zagreb, Croatia

Abstract

With the rapid growth of e-commerce practices, the efficient management of warehouses and fulfilment centres has become crucial in supply chain operations. One effective strategy for achieving this efficiency is the digital transformation of warehouses through automation. In particular, shuttle-based storage and retrieval systems (SBS/RS) have gained significant attention due to their ultra-high transaction process capability. However, ongoing research focuses on enhancing the operating performance and cost efficiency of these SBS/RS designs. This chapter presents mathematical modelling approaches for estimating the performance of single- and multi-deep SBS/RS designs. These models provide valuable insights for designing warehouses that optimize transaction process rates while ensuring a balanced utilization of elevators and shuttles in the system, tailored to meet the specific requirements of the warehouse. By leveraging these modelling techniques, warehouse managers can make informed decisions regarding the design and configuration of SBS/RSs to improve overall operational efficiency and productivity.

Keywords: automated warehouses, SBS/RS, single- and multi-deep systems, analytical and numerical modeling, performance analysis.

1 Introduction

The field of logistics has undergone significant changes in recent times, driven by the advent of digital transformation and the application of digital innovation. Digital transformation involves the conversion of business processes into a digital format, utilizing digital methods, environments, and technologies. It is important to note that digitalization should not be confused with digitization, which refers to the process of converting analogue information into a digital form. Digital transformation often relies on digitization to achieve its objectives.

During the process of digital transformation, the logistics research community faces numerous challenges, including labour and knowledge shortages, the need for flexible and scalable e-commerce solutions, the implementation of automated and robotized warehouses, the integration of collaborative robots in intralogistics, cost and energy efficiency considerations, the utilization of artificial intelligence (AI) in logistics operations, the development of human-machine teams, the creation of digital platforms and assistance systems, and more.

This chapter specifically focuses on the increasing prevalence of automated warehouses worldwide. Figure 1 illustrates the continuous growth projected in the global warehouse automation market size until 2026. This growth is primarily attributed to the rapid expansion of e-commerce practices. E-commerce warehouses face unique challenges such as handling small order volumes, managing larger product ranges, coping with highly variable order structures, and fulfilling short delivery time requirements due to the growing number of online shoppers. Consequently, there has been a significant surge in the adoption of automation technologies in distribution and fulfilment warehouses to address these challenges.

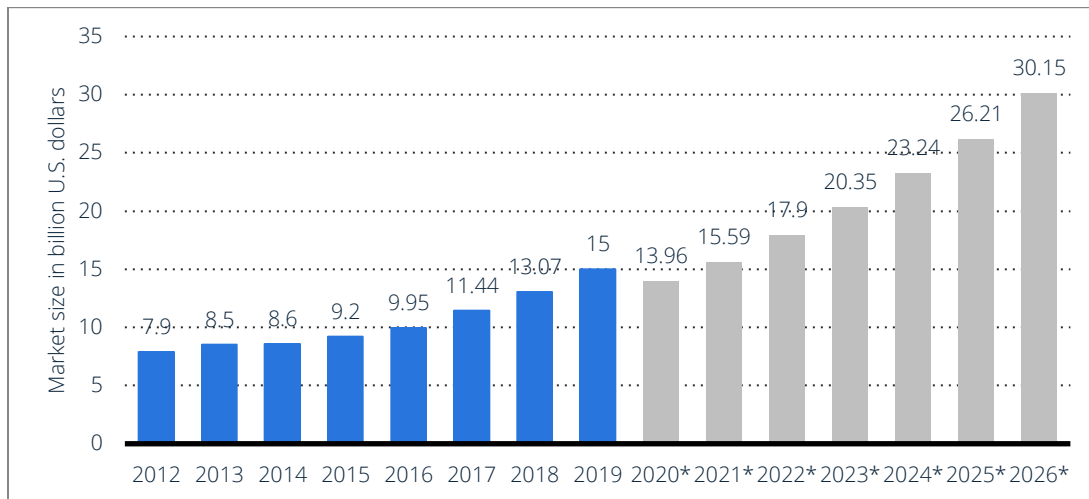


Fig. 1. Global warehouse automation market size 2012-2026

Source: Statista (2020)

Automated high-bay warehouses, also known as crane-based automated storage and retrieval systems (AS/RSs) for pallets, have a longstanding history tracing back to the 1960s. The primary advantage of AS/RSs stems from their ability to optimize space utilization through the use of cranes, particularly storage and retrieval (S/R) machines capable of lifting pallets to heights of 20 meters or higher.

In 1962, DEMAG, a pioneering company in Europe, introduced the first AS/RS, consisting of a rack system with 4,500 storage locations reaching a height of 20 meters. This ground-breaking innovation

marked a new era in the material handling sector, particularly in Europe, and came to be known as the automated high-bay warehouse. These systems are commonly referred to today as unit-load AS/RSs.

Subsequently, mini-load crane-based AS/RSs for totes and boxes entered the market. These systems enabled S/R machines to handle one or even multiple totes simultaneously, employing a multi-shuttle system and operating within single- or double-deep storage racks. Furthermore, these systems offered the capability of order picking for quantities less than a full pallet, working in conjunction with operators situated on the S/R machines, which came to be known as person-on-board AS/RSs.

Around 20 years ago, a new design was introduced to address the limited throughput performance typically observed in mini-load crane-based AS/RSs, which is approximately 120 double cycles per hour (FEM 9.851) (FEM, 2001). This innovative technology is commonly referred to as Shuttle-Based Storage and Retrieval System (SBS/RS) or Autonomous Vehicle-Based Storage and Retrieval System (AVS/RS) in the literature. Within the research community, the term SBS/RS primarily pertains to systems handling smaller loads such as totes, while the abbreviation AVS/RS is used for systems designed to handle larger loads, such as pallets. These distinctions allow for a clear differentiation between the two types of systems based on the load size they are primarily designed to handle.

The SBS/RS incorporates elevators equipped with lifting tables to facilitate the vertical movement of loads, such as totes, to their respective tiers within the racks. Within each tier of the storage rack, a dedicated shuttle vehicle is present, designed to perform storage and/or retrieval operations. The shuttle vehicle operates exclusively within a single tier (i.e., tier-captive), ensuring efficient and precise handling of the loads.

In the SBS/RS, the concept of being tier-captive implies that the shuttle vehicle is restricted to operate within a specific tier of the storage rack and cannot move between different tiers. While elevators facilitate the vertical movement of totes, the shuttles enable their horizontal movement within a single-, double-, or multiple-deep storage rack system. In a single-deep storage rack system, all totes can be retrieved from either side of the storage rack, ensuring instant accessibility to every stored tote. On the other hand, a multiple-deep storage rack system allows for the retrieval of all totes from either side of the storage rack in a high-density storage configuration with multiple depths within a single lane or channel. To retrieve totes located in deeper positions within the storage rack, the use of a satellite vehicle that can travel within the lane or channel is necessary.

In situations where products of the same type are stored within a lane or channel of the storage rack, no relocations are required for accessing specific totes. However, if different products are placed in the same lane or channel, the relocation of blocking totes may be necessary. Comparing both possibilities, storing different products within one lane or channel offers a broader range of product options within the warehouse. The decision to choose one option over the other depends on the product variety and the complexity involved in performing relocations of blocking totes. In many racking designs, the number of shuttles often exceeds the number of elevators, resulting in elevators becoming the bottleneck of the system. Given the widespread utilization of these systems in warehouses, this chapter aims to address the following research questions:

RQ1: How can the average utilization of elevators and shuttles be balanced through alternative SBS/RS designs?

RQ2: Which alternative SBS/RS designs, with balanced elevator and shuttle utilization, yield high performance?

To answer these research questions, we develop analytical and simulation models. Our focus is on both single- and multi-deep SBS/RS racking systems, and we benchmark them using various rack designs that consider the depth (number of lanes), length (number of columns), and height (number of tiers) of the warehouse.

The organization of this chapter is as follows: Section 2 provides a comprehensive literature review on single- and multi-deep SBS/RSs. In Section 3, we introduce the SBS/RS in the context of intralogistics. Sections 4 and 5 present the analytical and simulation models for single- and multi-deep SBS/RSs, respectively. In Section 6, we present the results of the simulation models for alternative SBS/RSs across different storage rack configuration scenarios. Finally, we conclude the chapter by summarizing our work and highlighting the key findings.

2 Literature review on single- and multi-deep SBS/RSs

This section provides an overview of the current state of the art in tier-captive SBS/RS systems, specifically focusing on both single- and multi-deep SBS/RS configurations. The literature on this topic primarily falls into two main categories: analytical modelling and simulation modelling of SBS/RSs. In this section, we present relevant works from these perspectives, highlighting the key findings and contributions in each area. By exploring the existing literature, we aim to provide a comprehensive understanding of the advancements made in analytical and simulation modelling approaches for SBS/RSs.

Single-Deep SBS/RS Literature Works

In the existing literature, a significant portion of the research focuses on the analysis of single-deep tier-captive SBS/RS systems. Carlo et al. (2012) introduce a heuristic modelling approach to estimate the performance of a novel variant of single-deep SBS/RS, featuring two non-passing lifting mechanisms for vertical load travel. They compare the performance of this new variant with the traditional single lifting system design.

Marchet et al. (2012) develop queuing network models to estimate the average waiting and cycle times of transactions in SBS/RS systems. They validate the effectiveness of their models by comparing them with simulation models. In a subsequent study, Marchet et al. (2013) propose a framework for the design of SBS/RS, incorporating cost performance metrics. Their findings indicate that optimal SBS/RS designs tend to favour a large number of aisles.

Lerher et al. (2015a) focus on estimating the average travel time of transactions through analytical modelling of single-deep SBS/RS. Their models consider factors such as acceleration/deceleration, maximum velocity of shuttles and lifting mechanisms, and single and dual command scheduling of transactions. However, this work does not include the estimation of queuing waiting time metrics.

Lerher et al. (2015b) explore the advantages of SBS/RS technology in terms of reduced average cycle time and increased transaction process capacity under different warehouse designs. Their results demonstrate that SBS/RS is a promising technology for enhancing throughput capacity.

Ekren et al. (2015) investigate class-based storage policies in single-deep SBS/RS designs. They find that class-based storage policies perform well, particularly in situations with a relatively high number of warehouse designs.

Wang et al. (2015) analyse a single-deep SBS/RS warehouse by proposing a task scheduling optimization model with three correlated multi-objectives. They employ NSGA-II for solving the multi-objective task scheduling model and observe that the Pareto optimal solution generated by their proposed algorithm yields favourable results.

Tappia et al. (2017) employ queuing network models to predict the performance of both single-tier and multi-tier SBS/RS system designs. Each tier is represented as a multi-class semi-open queuing network, while the vertical transfer is modelled as an open queuing network. The findings suggest that

in order to minimize the expected throughput time in single-tier systems, the depth/width ratio of the warehouse should be around 1.25.

Zou et al. (2016) propose a fork-join queuing network model to estimate the performance of SBS/RS systems, where shuttles and lifts can execute simultaneous movements rather than sequential ones. The results indicate that the fork-join queuing network model provides highly accurate predictions of system performance. Additionally, based on experimental results, it is shown that in system designs with less than 10 tiers, the simultaneous movement policy outperforms the sequential policy by at least 5.51 percent. The advantage of the simultaneous movement policy diminishes with decreasing rack height and aisle length.

Ekren (2017) investigates design trade-offs in single-deep SBS/RS systems through simulation of various designs. The results are presented using a graph-based representation to assist decision-makers, such as warehouse managers, in promptly evaluating predefined system designs and selecting the appropriate design concept based on company requirements.

Ekren et al. (2018) develop a performance calculator tool to estimate travel time, variance, and energy consumption outputs for a predefined SBS/RS design. This tool serves as the foundation for their subsequent queuing network modelling approach, where it is integrated to estimate queuing performance metrics for a predefined system design (Ekren et al., 2021).

Eder (2019) proposes continuous time open queuing network models with limited capacity for SBS/RS. These models directly incorporate the cycle time distribution of shuttles and lifts. The main novelty of this work lies in the utilization of a time continuous queuing model that combines the interactions of lifts and shuttles with a spatial discrete approach. The developed models are validated against simulation models.

Ekren (2020) identifies significant factors that impact the performance of single-deep SBS/RS systems, considering metrics such as average cycle time, energy consumption, and energy regeneration per transaction. Moreover, she determines the optimal levels of these critical design factors for improving each performance measure. To accomplish this, a statistical test, Tukey's test, is employed.

Lerher et al. (2021) present analytical travel time models for estimating average cycle time per transaction and throughput performance in SBS/RS systems with multiple-tier shuttle vehicles. The proposed models consider simultaneous travel of multiple-tier shuttle vehicles in both horizontal and vertical directions. Additionally, the models can estimate performance metrics for single and dual command cycles separately.

Multi-Deep SBS/RS Literature Works

There is a limited number of works available on multi-deep SBS/RS in the literature. We provide a summary of the relevant studies in this section.

Lerher (2016) investigates analytical travel time models for estimating the average cycle time per transaction in double-deep SBS/RS. The advantage of double-deep SBS/RS lies in its reduced footprint compared to single-deep SBS/RS. The proposed models can estimate travel times for single- and dual-command cycles separately.

Manzini et al. (2016) present travel time models for deep-lane AVS/RS and provide guidelines for system layout configuration, including determining the number of depths, optimal shape ratio, and lane length. They validate the proposed models using simulation models.

Wang et al. (2019) examine the task scheduling problem in double-deep SBS/RS. They aim to minimize the total time required for a batch of retrieval tasks using a modified Simulated Annealing Algorithm (SAA). The proposed model and algorithm's efficiency are verified through case studies.

Guerrazzi et al. (2019) evaluate energy consumption in a deep-lane AVS/RS with an energy recovery system. Their study demonstrates significant energy savings in deep-lane AVS/RS systems compared to traditional crane-based systems.

D'Antonio et al. (2019) analyze a complex tier-to-tier, multi-shuttle, deep-lane AVS/RS. They present an analytical model for evaluating the expected cycle time and, consequently, assessing the throughput performance.

Eder (2020) introduces an analytical method to estimate performance metrics for SBS/RS with multiple-deep designs. The method is based on a continuous-time open queuing network model with limited capacity. The cycle times of lifts and shuttles are determined using a spatial value approach, allowing for direct application of time distributions in the model. The analytical models are validated using simulation models.

Lin et al. (2021) investigate simulation models to analyse the performance of a double-deep tier-captive SBS/RS under single-elevator and double-elevator patterns. They consider outcomes such as shuttle carrier waiting time, elevator idle time, and total outbound time. An optimization model and an improved NSGA-II algorithm for task scheduling are studied based on these outcomes. Experiments are conducted with various task sizes, load factors, and elevator patterns in the SBS/RS.

Marolt et al. (2022a) focus on a multiple-deep AVS/RS tier simulation study. They analyse nine different combinations of storage and relocation assignment strategies, including random, depth-first, and nearest neighbour strategies. The study only considers Dual Command (DC) cycles. The results show that scenarios with deeper AVS/RS (fivefold and sixfold-deep) not only have better space area utilization but also exhibit higher throughput performance compared to scenarios with fewer depths.

Subsequently, Marolt et al. (2022b) introduce an analytical model for a multiple-deep AVS/RS tier, specifically focusing on "Depth-First storage" and "Depth-First relocation" strategies. Additionally, they provide an empirical model that simplifies the calculation of Single Command (SC) cycles and DC cycles. Both the analytical and empirical models are validated using a simulation model.

Battarra et al. (2022) propose and utilize a novel hybrid analytical-simulative model for the design of a deep-lane and multisatellite AVS-RS. Their model evaluates and controls the system performance of an AVS-RS equipped with multiple free and non-free satellites for each tier of the storage rack.

Table 1 provides an overview of recent publications on single- and multi-deep SBS/RS, summarizing the key information from these studies.

Table 1. Existing literature papers on single and multi-deep SBS/RS

Literature	Rack System	Modelling	Performance Metric
Battarra et al. (2022)	Multi-deep	Analytical and Simulation	System performance
Carlo et al. (2012)	Single-deep	Heuristic approach	Throughput rate
Marchet et al. (2012)	Single-deep	Queuing network	Average waiting and cycle time of a transaction
Marchet et al. (2013)	Single-deep	Simulation	Flow time of a transaction
Lerher et al. (2015b)	Single-deep	Simulation	Average cycle time of a transaction
Lerher et al. (2015a)	Single-deep	Analytical	Throughput rate
Ekren et al. (2015)	Single-deep	Simulation	Utilisation of lifts/shuttles and cycle time of a transaction
Wang et al. (2015)	Single-deep	NSGA-II heuristic approach	Multi-objectives
Tappia et al. (2017)	Single-deep	Queuing network	Throughput rate
Zou et al. (2016)	Single-deep	Queuing network	Multi-objectives
Ekren (2017)	Single-deep	Simulation	Multi-objectives
Borovinšek et al. (2017)	Single-deep	NSGA-II heuristic approach	Multi-objectives (including energy consumption per transaction)

Ekren et al. (2018)	Single-deep	Analytical	Multi-objectives (including energy consumption per transaction)
Ekren et al. (2021)	Single-deep	Queuing network	Multi-objectives (including energy consumption per transaction)
Eder (2019)	Single-deep	Queuing network	Multi-objectives
Ekren (2020)	Single-deep	Simulation	Multi-objectives (including energy consumption per transaction)
Lerher et al. (2021)	Single-deep	Analytical	Average cycle time per transaction and throughput rate
Lerher (2016)	Double-deep	Analytical	Average cycle time of a transaction
Wang et al. (2019)	Double-deep	Simulated Annealing	Average cycle time of a retrieval transaction
Eder (2020)	Multi-deep	Queuing network	Multi-objectives
Lin et al. (2021)	Double-deep	NSGA-II heuristic approach	Multi-objectives
Marolt et al. (2022a)	Multi-deep	Simulation	Throughput rate
Marolt et al. (2022b)	Multi-deep	Analytical, Empirical, Simulation	Throughput rate
Guerrazzi et al. (2019)	Multi-deep	Simulation	Multi-objectives (including energy consumption per transaction)
D'Antonio et al. (2019)	Multi-deep	Analytical, Simulation	Throughput rate
D'Antonio et al. (2018)	Multi-deep	Analytical, Simulation	Throughput rate
Manzini et al. (2016)	Multi-deep	Analytical, Simulation	Throughput rate

In contrast, our study focuses on achieving a balanced average utilization of elevators and shuttles by examining both single and multi-deep SBS/RS systems. We also aim to identify warehouse designs that can enhance the overall throughput performance of the system. To the best of our knowledge, this is the first work that formulates and compares single and multi-deep SBS/RS systems considering various operational aspects. Therefore, the contribution of this chapter can be summarized as follows:

- Evaluation and discussion of SBS/RSs with single and multi-deep storage rack designs, addressing the research gap in the existing scientific literature.
- Development of analytical travel time models for both single and multi-deep SBS/RSs, taking into account the operating characteristics of elevators' lifting tables and shuttle vehicles. We also consider the "depth-first storage policy" for storage and "nearest-neighbour rearrangement policy" for retrieval transaction processing.
- Experimentation with fifteen different configurations, including various velocity profiles for the elevators' lifting tables and shuttle vehicles, as the layout of the SBS/RS significantly impacts system performance.

3 Shuttle-Based Storage and Retrieval Systems in intralogistics

Over the past decade, there has been a notable increase in the implementation of automated warehouse systems in intralogistics, with one such system being the SBS/RS. It is worth mentioning that SBS/RS is sometimes referred to as Automated Vehicle Storage/Retrieval System (AVS/RS) in both literature and practice.

The concept of utilizing a combined system of elevators and automated shuttles can be traced back to the 1970s, as evidenced by technical sketches and patent applications. However, despite the relatively advanced development of this combined system concept, the technology of that time was not yet sufficiently mature to enable practical implementation of SBS/RS.

In terms of professional literature, the German Association of Engineers (Verein Deutscher Ingenieure; VDI) published the technical guidelines "VDI 2692 Blatt 1 - Automated vehicle storage and retrieval systems for small unit loads" in 2015 (VDI, 2015). Additionally, in 2017, the FEM (European

Federation of Materials Handling and Storage Equipment) published the technical guidelines "FEM 9.860 – Cycle time calculation for automated vehicle storage and retrieval systems" (FEM, 2017). These technical guidelines aimed to provide practical users with an understanding of the individual components of SBS/RS and basic analytical models for calculating the system's throughput capacity under various SBS/RS conditions.

3.1 Basic installation of Shuttle-Based Storage and Retrieval Systems

SBS/RSs are specifically designed for the storage and retrieval of small transport unit loads (TUL), including items such as totes and cartons. These systems consist of several key components, including an elevator equipped with an elevator's lifting table, a storage rack (SR) comprising columns and tiers, tier-captive shuttle vehicles, and buffer positions located in front of each tier of the SR. Figure 2 provides a visual representation of these components in an SBS/RS configuration.

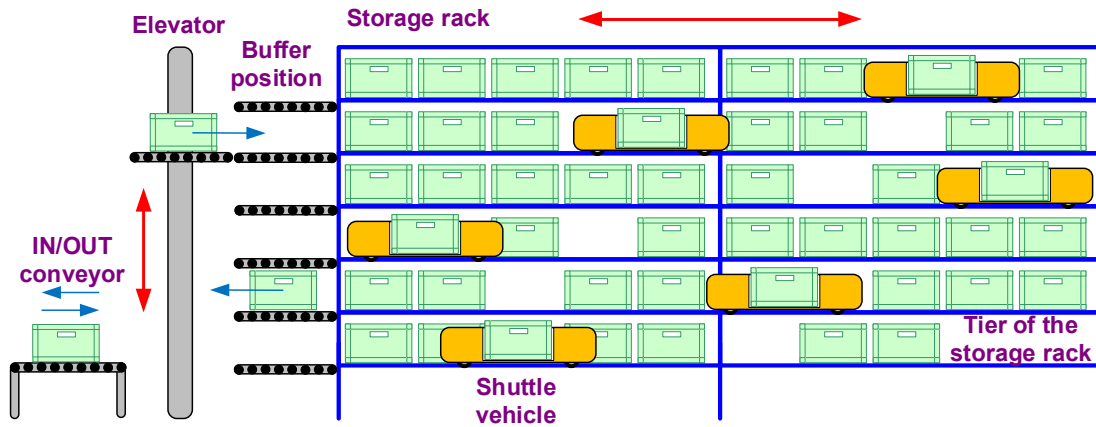


Fig. 2. SBS/RS with automatic “tier-captive” shuttle vehicles in the SR

Source: own

The elevator, equipped with a lifting table, facilitates vertical movement (in the y direction) of the TULs. Buffer positions are utilized to store the TULs in front of each tier of the storage rack (SR). Shuttle vehicles, on the other hand, enable horizontal travel (in the x direction) of the TULs, allowing for efficient storage and retrieval operations. In the case of a "tier captive SBS/RS," each shuttle vehicle operates within a dedicated single tier of the storage rack.

Compared to crane-based mini-load AS/RS, SBS/RS offers several advantages. It can achieve higher throughput performance, providing faster storage and retrieval operations. Additionally, SBS/RS systems offer greater flexibility, allowing for dynamic reconfiguration and adaptation to changing operational requirements. Moreover, SBS/RS systems tend to have lower electricity consumption, making them more energy efficient.

3.1.1 Shuttle vehicles travelling in the horizontal direction

The single-tier shuttle vehicle, depicted in Figure 3a, is responsible for horizontal travel in the x direction. In certain cases, it can also perform simultaneous movement in the vertical y direction, as illustrated in Figure 3b. The maximum weight capacity for the TUL should not exceed 50 kg. The dimensions of the

TUL typically fall within the range of a minimum size of (150 x 200 x 80) mm and a maximum size of (600 x 400 x 250) mm.

The velocity of the shuttle vehicle is determined by the specific design of the SBS/RS and can vary between $v_x = 1.5$ m/s and $v_x = 4$ m/s. Different SBS/RS configurations may have different velocity ranges for the shuttle vehicle, allowing for customization based on operational requirements and system capabilities.

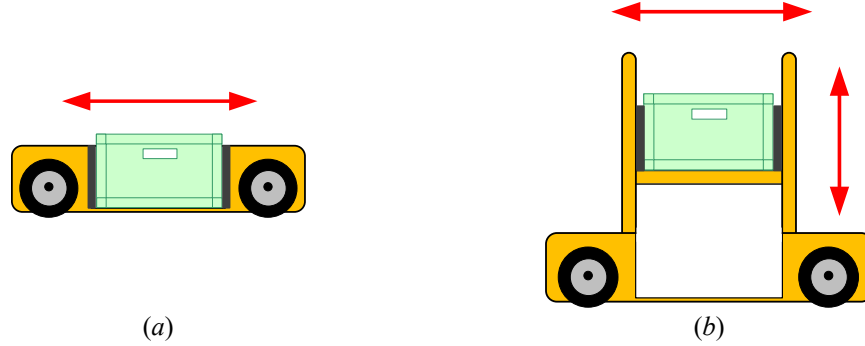


Fig. 3. Designs of the shuttle vehicle for (a) travelling in the horizontal direction (single-tier shuttle vehicle) and (b) travelling in the horizontal direction and moving in the vertical direction (multi-tier shuttle vehicles)

Source: own

In practical implementations, a specialized vehicle called a satellite vehicle is used to travel within the lanes/channels of the multiple-tier storage rack, as depicted in Figure 4. In this configuration, the shuttle vehicle serves as the carrier for the satellite vehicle. The satellite vehicle is responsible for navigating and operating within a specific tier of the storage rack, enabling efficient storage and retrieval operations within the system. This arrangement allows for enhanced flexibility and improved performance in multi-tier SBS/RS setups.

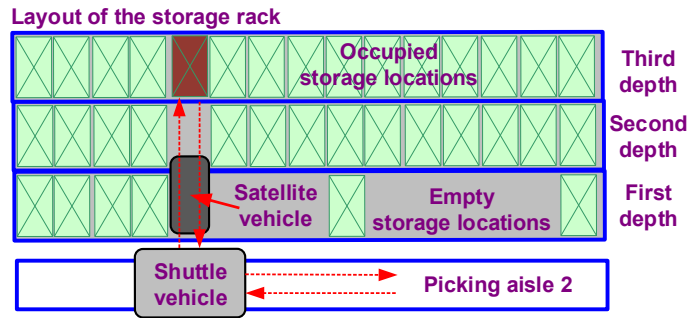


Fig. 4. Satellite vehicle travelling inside the lane/channel of the multiple-tier storage rack.

Source: own

The shuttle vehicle is equipped with various components, including drives, controls, and energy units, all housed within its structure. These components work together to facilitate the operation of the shuttle vehicle. To ensure precise positioning and accurate detection of storage locations, the shuttle vehicle is equipped with multiple sensor systems. These sensors, along with position markers in the rail guides, enable the shuttle vehicle to navigate and position itself accurately within the aisle of the tier.

Typically, the shuttle vehicle is equipped with a minimum of two drive units. These drive units are responsible for propelling the shuttle vehicle horizontally (in the x direction) and for moving the load attachments during the loading and unloading of TULs in the storage rack. In the case of a shuttle vehicle with an integrated hoisted carriage (as shown in Figure 4b), an additional drive unit is required to enable vertical movement of the TULs within the tier (in the y direction). This configuration allows for efficient vertical transfer of TULs within the multi-tier SBS/RS system.

Shuttle vehicles in AS/RS systems can be powered using two main methods. One approach involves using electrical conductors installed in rail guides within the aisle of the S/R system. The shuttle vehicle contacts these conductors to receive power. Alternatively, shuttle vehicles can be equipped with lithium-ion batteries as their power source. This battery-powered configuration offers greater flexibility and eliminates the need for a physical power connection.

Some shuttle vehicles are equipped with energy recovery technology, also known as recuperation technology. This feature allows the shuttle vehicles to capture and store energy generated during deceleration or braking. The stored energy can be later utilized to power the shuttle during acceleration or movement, thereby reducing overall energy consumption. When the shuttle vehicle decelerates or brakes, the regenerative braking system converts the kinetic energy into electrical energy, which is then stored back in the battery or capacitor.

The specifics and figures related to energy recuperation can vary depending on the design, technology, and implementation of the shuttle vehicles. Factors such as the shuttle's weight, speed profiles, control algorithms, and the effectiveness of the energy storage system can influence the energy recovery efficiency. Shuttle manufacturers typically provide specifications and performance data regarding energy recovery for their specific models, offering detailed information on energy savings and operational efficiency benefits.

To communicate with the central control system of the AS/RS, shuttle vehicles utilize a wireless WLAN (Wireless Local Area Network) network. This wireless communication enables real-time coordination, control, and monitoring of the shuttle's movements, status, and inventory information, facilitating efficient operation and management of the AS/RS system.

The mean amount of energy regeneration per transaction for a shuttle can be calculated using equations provided in Ekren et al. (2018). Additionally, Ekren et al., (2018) explain the calculations for mean energy consumption of shuttles and the expected value of energy regeneration per transaction, providing further insights into energy-related performance metrics.

3.1.2 Elevator for moving in the vertical direction

The elevator, positioned in front of each SR, plays a crucial role in providing vertical (y) movement for the TULs to and from the buffer locations. It is important to plan the elevators carefully as they often serve as a bottleneck in SBS/RS systems. Depending on the required throughput performance, different elevator designs are used in practice.

For systems with moderate throughput requirements, a single elevator with an elevator's lifting table is commonly employed (Figure 5a). This design allows for vertical transportation of TULs and is widely used in various applications. However, when higher throughput performance is desired, a special design incorporating two independent elevators with their associated lifting tables has been developed (Figure 5b). In this configuration, each elevator can operate individually, effectively doubling the throughput capacity of the system (Borovinšek et al., 2017; Ekren, 2017; Lerher et al., 2015a; Lerher et al., 2015b).

In both designs (Figures 5a and 5b), TULs enter the system via a right roller conveyor and exit the system via a left roller conveyor. The velocity of the elevator's lifting table depends on the specific

SBS/RS design and can range from $v_y = 3$ m/s up to $v_y = 6$ m/s. This velocity parameter is determined based on the system requirements and considerations such as load capacity, operational efficiency, and safety.

Properly configuring the elevator system is essential for achieving optimal throughput performance and efficient operation in SBS/RS systems. The choice between a single elevator or a dual elevator design depends on the desired throughput capacity and the specific operational requirements of the application.

Currently, besides the conventional "tier-captive SBS/RS" configuration, there are several other installations and variations of SBS/RS that have been developed. In this section, we will explore some of these implementations and their unique features.

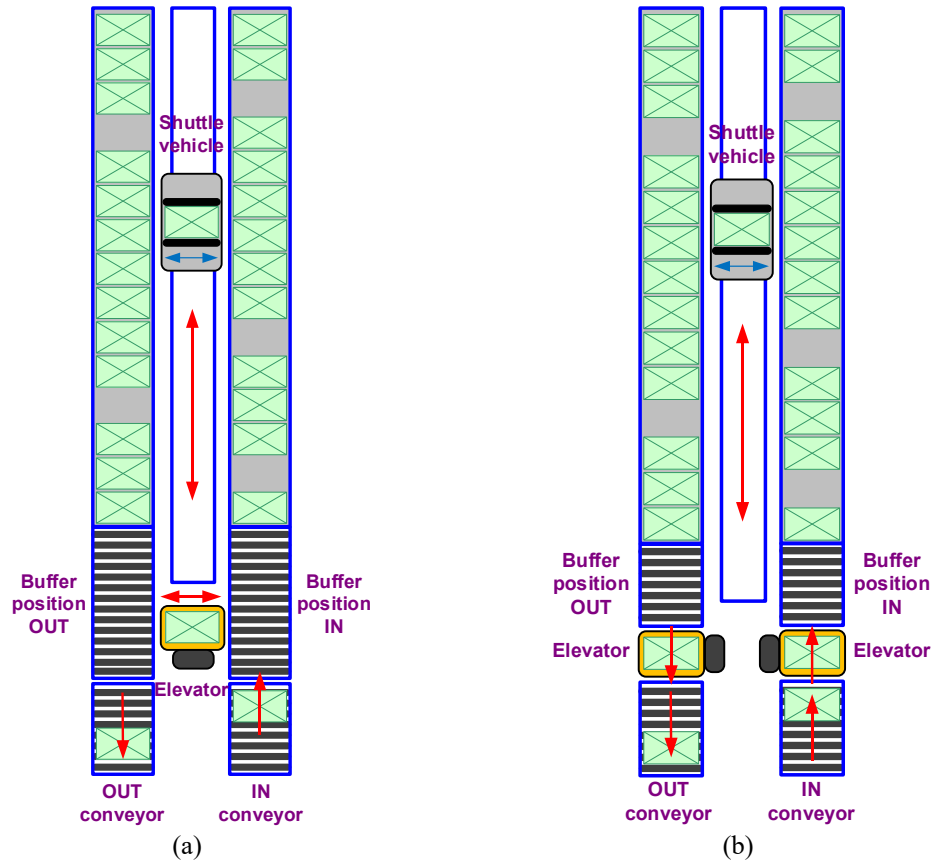


Fig. 5. Design of the (a) single mast and (b) double mast elevator for moving TUL in the vertical direction

Source: own

3.1.3 Special designs of SBS/RS

3.1.3.1 SBS/RS with non-tier captive shuttle vehicles

Figure 6 depicts an alternative configuration of SBS/RS known as "tier-to-tier SBS/RS," where non-tier captive shuttle vehicles are utilized. In this setup, shuttle vehicles can move between tiers through a separate elevator mechanism located at the back of the storage racks. Unlike the tier-captive SBS/RS, there is no dedicated shuttle vehicle assigned to each tier of the storage rack in this configuration. The literature refers to this system as "tier-to-tier SBS/RS" (Arslan et al., 2022; Ekren et al., 2022; Ha et al., 2019; Küçükyavaş et al., 2021; Zhao et al., 2019).

As shown in Figure 6, there are two distinct elevators: one positioned in the front, which moves the TULs to the desired buffer positions at the tiers, and another elevator at the back, responsible for transporting shuttle vehicles between tiers. This configuration offers the advantage of reducing the required number of shuttle vehicles in the system, thanks to the flexible movement capability between tiers. However, it's important to note that implementing the elevator mechanism for shuttle vehicle transportation between tiers introduces an additional cost to the system, which needs to be considered in assessing the overall feasibility and benefits, potentially offsetting the reduced number of shuttle vehicles.

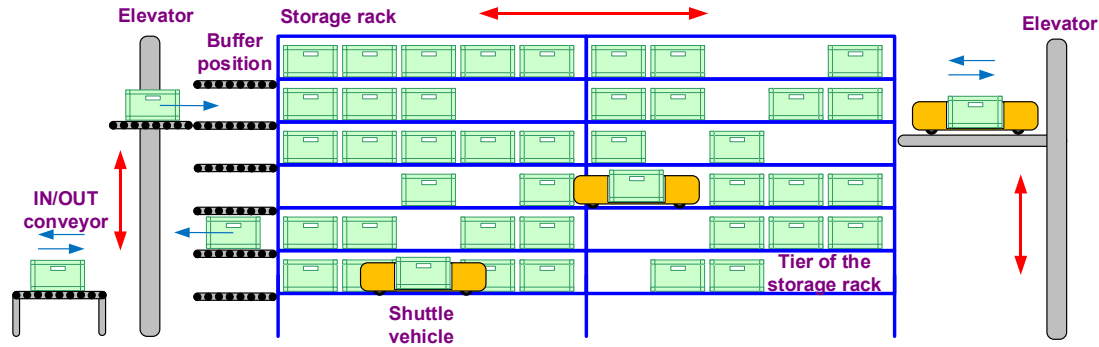


Fig. 6. SBS/RS with non-tier captive shuttle carriers
Source: own

3.1.3.2 SBS/RS with multi-tier captive shuttle vehicles

In the mentioned SBS/RS configuration, shuttle vehicles are capable of operating in both horizontal (x) and vertical (y) directions simultaneously, as depicted in Figure 7. This configuration is commonly known as "multi-tier-captive SBS/RS" (Leher et al., 2021).

In this design, shuttle vehicles are equipped with hoisted carriages that allow them to operate in multiple tiers of the storage rack simultaneously, resulting in improved shuttle vehicle utilization. The elevator positioned in front of the storage rack is responsible for moving the TULs to the buffer positions, while the shuttle vehicles execute storage and retrieval operations across multiple tiers.

By serving multiple tiers of the storage rack, this SBS/RS configuration enables a reduction in the number of shuttle vehicles required in the system, allowing the elevator and shuttle vehicles to operate at maximum performance. However, it's important to note that the special design of the multi-tier captive shuttle carriers introduces an additional cost, which may be offset by the decreased number of shuttle vehicles in the system.

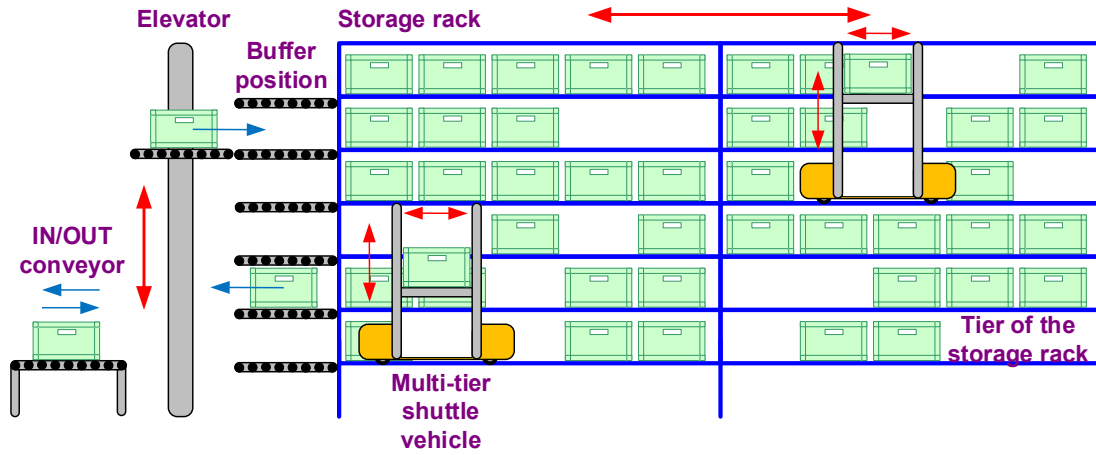


Fig. 7. SBS/RS with multi-tier captive shuttle carriers

Source: own

3.1.3.3 SBS/RS with non-aisle-captive shuttle vehicles

Figure 8 illustrates an SBS/RS configuration featuring non-aisle-captive shuttle vehicles (Turhanlar et al., 2022). In this setup, shuttle vehicles are not dedicated to specific aisles and can move between aisles within the SBS/RS. Big, automated warehouses typically consist of multiple aisles connected by cross-aisles. Shuttle vehicles can traverse these cross-aisles to access different aisles within a tier and perform storage or retrieval operations. This system is commonly referred to as "SBS/RS with storage level-captive shuttle carriers."

The flexibility of this configuration is evident in the number of shuttle vehicles employed in the system, which can be adjusted according to the desired throughput performance. Additional shuttle vehicles can be added or removed as needed. It's worth noting that the special design of shuttle vehicles capable of operating through cross-warehouse aisles within a tier may involve an extra cost. However, this cost can potentially be offset by the reduced number of shuttle vehicles required in the system, resulting in overall efficiency and cost-effectiveness.

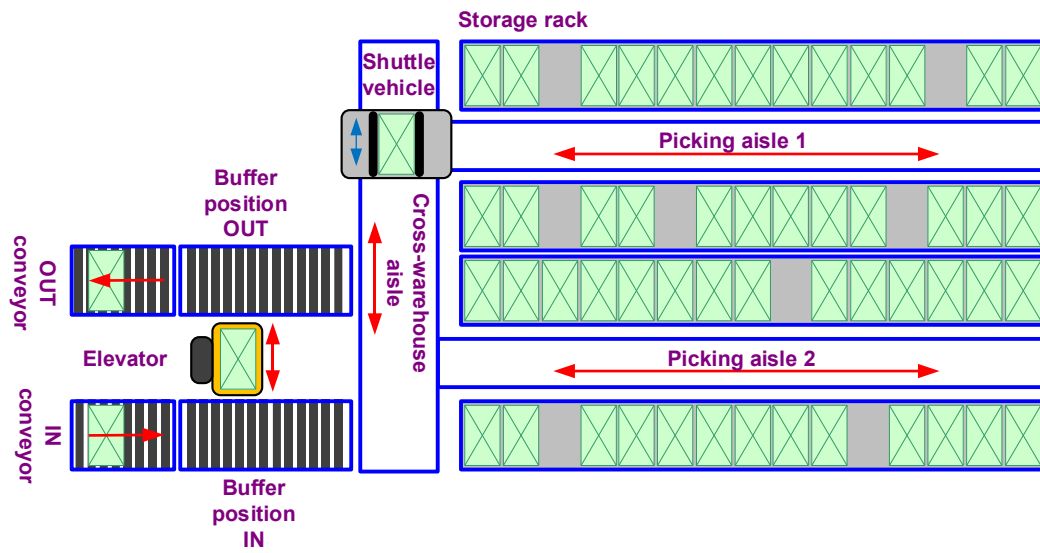


Fig. 8. SBS/RS with storage level-captive shuttle carriers
Source: own

Table 2 provides a concise overview of various SBS/RS configurations, including the basic design and three special designs, enabling readers to gain a rapid and comprehensive understanding of the different solutions offered in the market.

Table 2: Overview of various SBS/RS configurations

	Shuttle		Elevator		Cross-warehouse aisle	Throughput	Flexibility	Complexity
	Single-tier	Multi-tier	For TUL	For shuttle				
Basic SBS/RS	✓		✓			M-H	M	M
SBS/RS with non-tier captive shuttle vehicles	✓		✓	✓		M	H	M-H
SBS/RS with multi-tier captive shuttle vehicles		✓	✓			M	H	M
SBS/RS with non aisle-captive shuttle vehicles	✓	(✓)	✓		✓	M	H	M-H

✓ = the solution has that characteristic; (✓) = the solution can have that characteristics M = medium; H = high

4 Analytical model of single- and multi-deep SBS/RS with tier-captive shuttle vehicles

This section focuses on the development of analytical models for single and multi-deep storage systems for tier-captive SBS/RS configurations. The aim is to estimate the throughput performance, measured in the number of TULs per hour. In the literature, both analytical and numerical (simulation) models have been extensively studied for this purpose. These models typically make two assumptions. First, they consider a uniform velocity-time relationship, where the velocity remains constant throughout the shuttle vehicle's movement. Second, they incorporate the influence of acceleration and deceleration, resulting in a velocity-time relationship that is not constant. Moreover, many models assume a uniform distribution, meaning that each storage location within the system has an equal probability of being selected for either storage or retrieval operations. This assumption simplifies the modelling process and allows for easier calculation of performance metrics.

In this section, we present and discuss expressions for the SC and DC cycles of both shuttle vehicles and the elevator's lifting table. These cycles represent the time required to complete a single storage or retrieval command. The analysis of these cycles is crucial in understanding and optimizing the overall throughput performance of the SBS/RS system.

The model proposed in this study focuses on analysing the SC and DC operations of lifts and shuttles separately. This approach involves evaluating the throughput of lifts and shuttles individually, considering their respective capacities and performance characteristics. The overall system throughput is then determined by the bottleneck component within the system.

It is worth mentioning that this approach differs from the one presented in Liu et al. (2018), where the integrated movements of transport TULs by lifts and shuttles were considered. In contrast, our model specifically examines and illustrates the individual operations of lifts and shuttles, allowing for a more detailed analysis of their throughput performance and system efficiency. The proposed models for single- and multi-deep SBS/RS are based on the following set of assumptions:

- The SBS/RS is divided into storage racks with single- and multiple-depth on both sides (left and right), allowing totes to be stored on either side in each tier of the SR.
- The input/output (I/O) location of the SBS/RS is positioned at the first tier of the SR.
- The idle position of the tier-captive shuttle carrier in each tier of the SR is at the buffer location.
- The SR is divided into columns and tiers.
- Each tier of the storage rack has two buffer locations (left and right) and a single tier-captive shuttle carrier.
- The elevator with its lifting table feeds the SR with totes following the SC sequence. Each aisle of the SR has one inbound and one outbound elevator.
- The shuttle vehicles operate in both SC and DC modes.
- The elevator's lifting table, shuttle vehicles, and satellite vehicles follow predefined sequences of acceleration, constant velocity, and deceleration for their movements.
- The tier-captive shuttle vehicle travels horizontally in sync with the elevator's movements.
- The satellite vehicle travels in the cross-warehouse direction inside the lane simultaneously.
- The drive characteristics of the elevator's lifting table, tier-captive shuttle vehicle, and satellite vehicle, as well as the dimensions of the SR and lane, are known in advance.
- The height of the SR allows the elevator's lifting table to reach its maximum vertical velocity, $v_{\max}$.
- The length of the SR enables the tier-captive shuttle carrier to reach its maximum horizontal velocity, $v_{\max}$.

- The length of the lane allows the satellite vehicle to reach its maximum cross-warehouse velocity, $v_{\max}$.
- The inbound elevator handles storage requests, while the outbound elevator handles retrieval requests, both following a "First-In-First-Out (FIFO) policy."
- For storage requests in the i^{th} tier of the SR, a "depth-first policy" is applied, prioritizing the deepest empty location in that tier.
- Retrieval requests follow a "random policy," where each TUL has an equal probability of being selected for retrieval.
- In the case of a blocking TUL, a "nearest-neighbor policy" is applied, selecting the closest empty location in the deepest lane nearest to the elevator.
- The study assumes a diverse SKU environment, where each TUL represents a unique item. Therefore, a single SKU policy within an individual lane of the SR is not applicable.

These assumptions provide the basis for the analytical models and allow for the estimation of throughput performance in the SBS/RS system.

Abbreviation

AS/RS	Automated storage and retrieval systems.
DC	Dual command.
COM	Compartment
FIFO	First-in-first-out
I/O	Input/output location.
LIFT	Elevator (elevator's lifting table).
S/R	Storage and retrieval.
SBS/RS	Shuttle-based storage and retrieval systems.
SC	Single command.
SHUT	Shuttle vehicle.
SR	Storage rack
SATEL	Satellite vehicle
TUL	Transport unit load

Notations

T_{SC}^{LIFT}	Elevator's lifting table SC cycle.
$t_{P\&D}^{\text{LIFT}}$	Pickup & Drop off times of the elevator.
$t_{I/O, m_i}^{\text{LIFT}}$	Movement of the elevator's lifting table from I/O location to a randomly selected tier of the SR.
$t_{m_i, I/O}^{\text{LIFT}}$	Movement of the elevator's lifting table from destination position to I/O location.
$\lambda_{SC}^{\text{LIFT}}$	Elevator's lifting table throughput performance in case of SC cycle.
k	Decision variable; if $k = 1 \rightarrow$ SC cycle otherwise if $k = 2 \rightarrow$ DC cycle.
w_y	Number of elevators in one aisle.
T_{DC}^{LIFT}	Elevator's lifting table DC cycle.
$t_{m_i, n_i}^{\text{LIFT}}$	Movement of the elevator's lifting table from destination position 1 to destination position 2.

$t_{n_i, I/O}^{LIFT}$	Movement of the elevator's lifting table from destination position 2 to I/O location.
λ_{DC}^{LIFT}	Elevator's lifting table throughput performance in case of DC cycle.
T_{SC}^{SHUT}	Shuttle vehicle's SC cycle.
$t_{P\&D}^{SHUT}$	Pickup & Drop off times of the shuttle vehicle.
t_{BL_i, m_i}^{SHUT}	Movement of the shuttle vehicle from the buffer position to a randomly selected open storage location.
t_{m_i, BL_i}^{SHUT}	Movement of the shuttle vehicle from the destination position to the buffer position.
λ_{SC}^{SHUT}	Shuttle vehicle's throughput performance in case of a SC cycle.
w_x	Number of shuttle vehicles in one aisle (equals number of tiers).
T_{DC}^{SHUT}	Shuttle vehicle's DC cycle.
t_{m_i, n_i}^{SHUT}	Movement of the shuttle vehicle from destination position 1 to destination position 2.
t_{n_i, BL_i}^{SHUT}	Movement of the shuttle vehicle from destination position 2 to the buffer position.
λ_{DC}^{SHUT}	Shuttle vehicle's throughput performance in case of a DC cycle.
t_{BL_i, m_i}^{SHUT}	Movement of the shuttle vehicle from the buffer position to the destination position of the storage lane.
$t_{m_i, lane_i}^{SATEL}$	Movement of the satellite vehicle from the shuttle vehicle towards the storage location inside the lane.
t_{lane_i, m_i}^{SATEL}	Movement of the satellite vehicle from the storage location towards the shuttle vehicle inside the lane.
t_{m_i, BL_i}^{SHUT}	Movement of the shuttle vehicle from the destination position of the storage lane to the buffer position.
t_{m_i, n_i}^{SHUT}	Movement of the shuttle vehicle from the destination position of the storage lane to the destination position of the retrieval lane.
$t_{n_i, lane_j}^{SATEL}$	Movement of the satellite vehicle from the shuttle vehicle towards the retrieval location inside the lane.
t_{lane_j, n_i}^{SATEL}	Movement of the satellite vehicle from the retrieval location towards the shuttle vehicle inside the lane.
t_{n_i, BL_i}^{SHUT}	Movement of the shuttle vehicle from the destination position of the retrieval lane to the buffer position.
ω	Blocking probability in case of the retrieval cycle.
$t_{n_i, lane_i}^{SATEL}$	Movement of the satellite vehicle from the shuttle vehicle towards the blocking location inside the lane.
t_{lane_i, n_i}^{SATEL}	Movement of the satellite vehicle from the blocking location towards the shuttle vehicle inside the lane.
t_{n_i, r_i}^{SHUT}	Movement of the shuttle vehicle from the destination position of the retrieval lane to the nearest-neighbour location.
$t_{r_i, lane_i}^{SATEL}$	Movement of the satellite vehicle from the shuttle vehicle towards the storage location inside the lane.
t_{lane_i, r_i}^{SATEL}	Movement of the satellite vehicle from the storage location towards the shuttle vehicle inside the lane.
t_{r_i, n_i}^{SHUT}	Movement of the shuttle vehicle from the nearest-neighbour location to the destination position of the retrieval lane.

W	Selected i^{th} tier of the SR configuration.
c_{shuttle}	Shuttle's vehicle current position.
s_{SKU}	blocking TULs position
d	Decision variable.
l_{tote}	Tote's length.
w_{tote}	Tote's width.
h_{tote}	Tote's height.
l_{COM}	Length (depth) of the column.
w_{COM}	Width of the column.
h_{COM}	Height of the column (tier).
N_x	Number of columns in the horizontal direction.
N_y	Number of columns in the vertical direction.
A	Fillgrade factor.
D	Depth level.
Q	Warehouse volume.
$N_{y\text{Min}}$	Decision variable.
$N_{y\text{Max}}$	Decision variable.
$N_{x\text{Min}}$	Decision variable.
$N_{x\text{Max}}$	Decision variable.
ΔQ	Tolerance.
vp	Velocity profile.
L	Length of the SR.
H	Height of the SR.
v_x	Maximum velocity of the shuttle vehicle in the horizontal (x) direction.
a_x^+	Acceleration of the shuttle vehicle in the horizontal (x) direction.
a_x^-	Deceleration of the shuttle vehicle in the horizontal (x) direction.
v_y	Maximum velocity of the elevators's lifting table in the vertical (y) direction.
a_y^+	Acceleration of the elevators's lifting table in the vertical (y) direction.
a_y^-	Deceleration of the elevators's lifting table in the vertical (y) direction.
$t_{P/S}$	Pick-up and set-down times.
t_{SAT}	Times associated with moving of the satellite vehicle.
v_{SAT}	Velocity of the satellite vehicle.
a_{SAT}	Acceleration/deceleration of the satellite vehicle.
η_{SHUT}	Utilization of the shuttle vehicle.
$\eta_{\text{LIFT(I)}}$	Utilization of the inbound elevator.
$\eta_{\text{LIFT(O)}}$	Utilization of the outbound elevator.
$\lambda_{\text{SC}}^{\text{LIFT(I)}}$	Throughput performance of the inbound elevator.
$\lambda_{\text{SC}}^{\text{LIFT(O)}}$	Throughput performance of the outbound elevator.
$\lambda_{\text{DC}}^{\text{SHUT(*)}}$	Throughput performance of all shuttle vehicles.
ψ	Decision variable.
$\lambda_{\text{SBS/RS}}$	Throughput performance of the SBS/RS as a whole.

4.1 Models for elevator's lifting table

4.1.1 Single command cycle of the elevator's lifting table

Figure 9 illustrates and explains the operation of the SC cycle for the elevator's lifting table. The SC cycle involves the movement of the elevator's lifting table with a TUL from the SR's I/O location to a randomly selected buffer position in the i^{th} tier of the SR for the storage process. During this process, the TUL is set down at the designated buffer position. Once the storage process is completed, the elevator's lifting table moves back to the I/O location of the SR.

Alternatively, the SC cycle can also involve the empty movement of the elevator's lifting table from the SR's I/O location to a randomly selected buffer position in the i^{th} tier of the SR to retrieve a TUL and deliver it to the I/O location of the SR for the retrieval process.

The SC cycle of the elevator's lifting table represents a single storage or retrieval request within one cycle. It consists of variable times associated with the movement of the elevator's lifting table and constant times related to the pick-up and set down of the TUL.

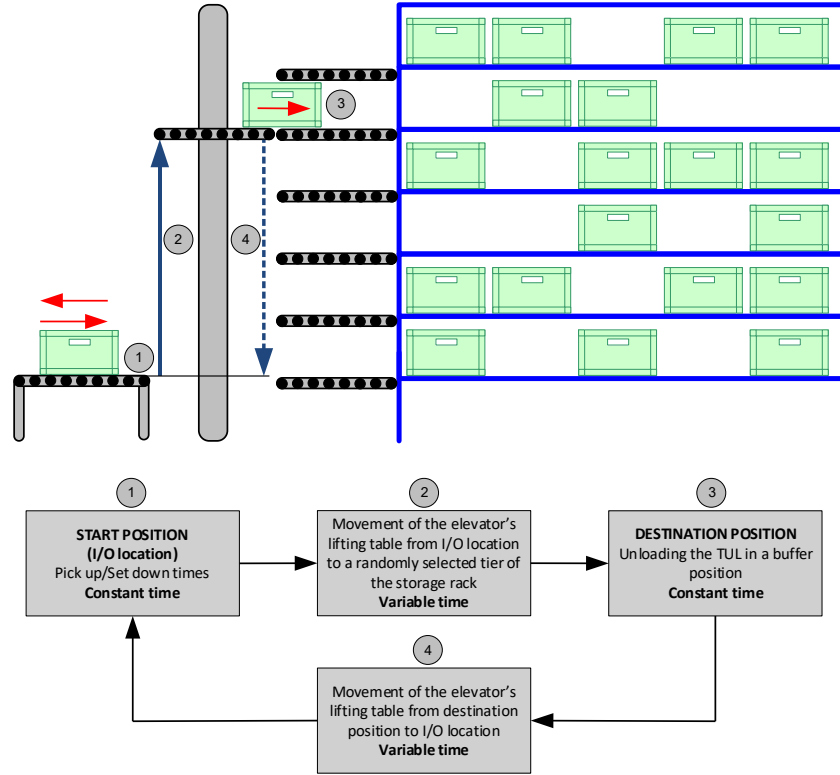


Fig. 9. Elevator's lifting table single command cycle

Source: own

Elevator's lifting table SC cycle T_{SC}^{LIFT} equals (1):

$$T_{SC}^{\text{LIFT}} = 2 \cdot t_{\text{P\&D}}^{\text{LIFT}} + t_{\text{I/O},m_i}^{\text{LIFT}} + t_{m_i,\text{I/O}}^{\text{LIFT}} \quad (1)$$

Elevator's lifting table throughput performance for one aisle in case of a SC cycle $\lambda_{SC}^{\text{LIFT}}$ equals (2):

$$\lambda_{SC}^{LIFT} = \frac{3600}{T_{SC}^{LIFT}} \cdot k \cdot w_y \quad (2)$$

where $k = 1$, due to the SC cycle, and w_y is a number of elevators in one aisle.

4.1.2 Double command cycle of the elevator's lifting table

Figure 11 illustrates the operation of the DC cycle for the elevator's lifting table. The DC cycle involves the movement of the elevator's lifting table with a TUL from the SR's I/O location to a randomly selected buffer position in the i^{th} tier of the SR for the storage process. During this process, the TUL is set down at the designated buffer position.

In the subsequent step of the DC cycle, the empty elevator's lifting table is moved to a randomly selected occupied buffer position in the j^{th} tier of the storage rack for the retrieval process. It picks up the TUL from the occupied buffer position and delivers it to the I/O location of the SR.

The DC cycle of the elevator's lifting table combines a storage process and a retrieval process within one cycle. It includes the movement of the elevator's lifting table with a TUL for storage and the subsequent movement to retrieve another TUL from a different tier.

In summary, the DC cycle of the elevator's lifting table combines the storage and retrieval processes in one cycle, with variable times for movement and constant times for pick-up and set-down operations.

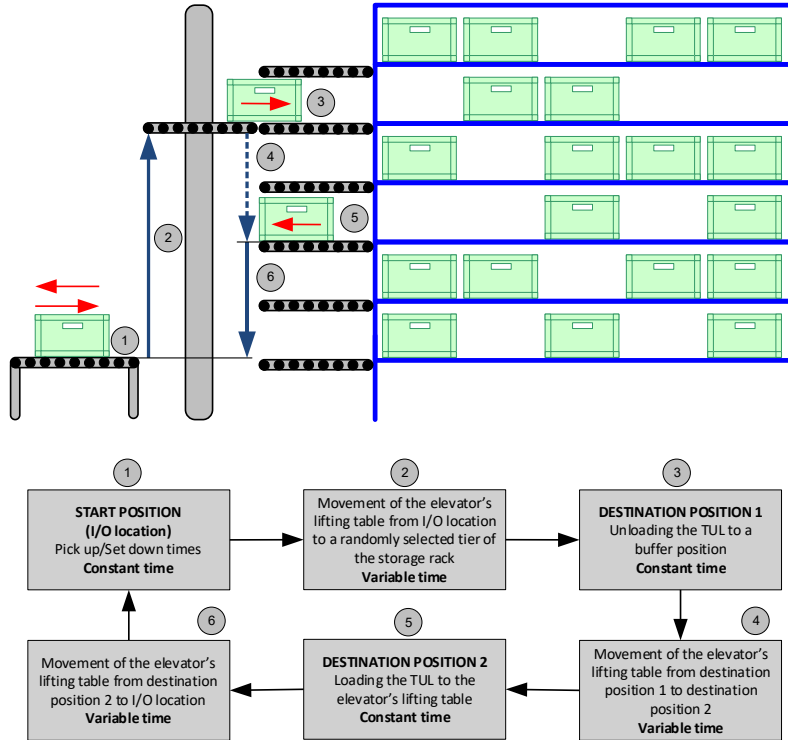


Fig. 10. Elevator's lifting table double command cycle

Source: own

Elevator's lifting table DC cycle T_{DC}^{LIFT} equals (3):

$$T_{DC}^{LIFT} = 4 \cdot t_{P\&D}^{LIFT} + t_{I/O, m_i}^{LIFT} + t_{m_i, n_i}^{LIFT} + t_{n_i, I/O}^{LIFT} \quad (3)$$

Elevator's lifting table throughput performance for one aisle in case of a DC cycle λ_{DC}^{LIFT} equals (4):

$$\lambda_{DC}^{LIFT} = \frac{3600}{T_{DC}^{LIFT}} \cdot k \cdot w_y \quad (4)$$

where $k = 2$, due to the DC cycle, and w_y is a number of elevators in one aisle.

4.2 Shuttle vehicle for single-deep SBS/RS

4.2.1 Single command cycle of the shuttle vehicle

The SC cycle of the shuttle vehicle is illustrated in Figure 11. This cycle involves the movement of the shuttle vehicle with a TUL from the buffer position in the i^{th} tier of the SR to a randomly selected free storage location within the same tier. Once the storage request is fulfilled, the shuttle vehicle returns to the buffer position in the i^{th} tier of the SR.

Conversely, the shuttle vehicle may also encounter the opposite scenario, where it moves empty from the buffer position in the i^{th} tier of the SR to a randomly selected occupied retrieval location. In this case, the shuttle vehicle picks up the TUL and transports it to the buffer position in the i^{th} tier of the SR.

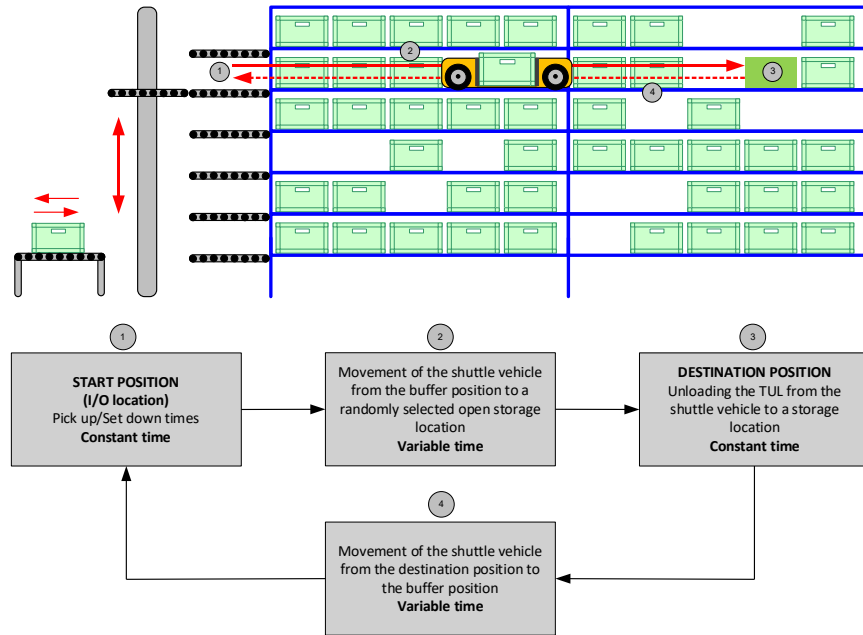


Fig. 11. Shuttle vehicle's single command cycle for single-deep SBS/RS

Source: own

In summary, the SC cycle of the shuttle vehicle corresponds to either a storage or retrieval request within a single cycle. It consists of two types of times: variable times, which account for the movement of the shuttle vehicle, and constant times, which encompass the pick-up and set-down durations of the TUL.

Shuttle vehicle's SC cycle T_{SC}^{SHUT} equals (5):

$$T_{SC}^{SHUT} = 2 \cdot t_{P\&D}^{SHUT} + t_{BL_i, m_i}^{SHUT} + t_{m_i, BL_i}^{SHUT} \quad (5)$$

Shuttle vehicle's throughput performance in case of a SC cycle λ_{SC}^{SHUT} equals (6):

$$\lambda_{SC}^{SHUT} = \frac{3600}{T_{SC}^{SHUT}} \cdot k \cdot w_x \quad (6)$$

where $k = 1$, due to the SC cycle, and w_x is number of shuttle vehicles in one aisle; equals number of tiers.

4.2.2 Double command cycle of the shuttle vehicle

The DC cycle of the shuttle vehicle, as depicted in Figure 12, involves the movement of a shuttle vehicle with a TUL from a buffer position in the i^{th} tier of the storage rack to a randomly selected (available) storage location, where the TUL is placed during a storage request. Subsequently, the empty shuttle vehicle is directed to a randomly selected (occupied) retrieval location to retrieve the TUL and transport it back to the buffer position in the i^{th} tier of the storage rack, fulfilling a retrieval request.

The DC cycle combines both storage and retrieval operations within a single cycle and comprises variable times (for the shuttle vehicle's movement) as well as constant times (for the pick-up and set-down actions of the TUL).

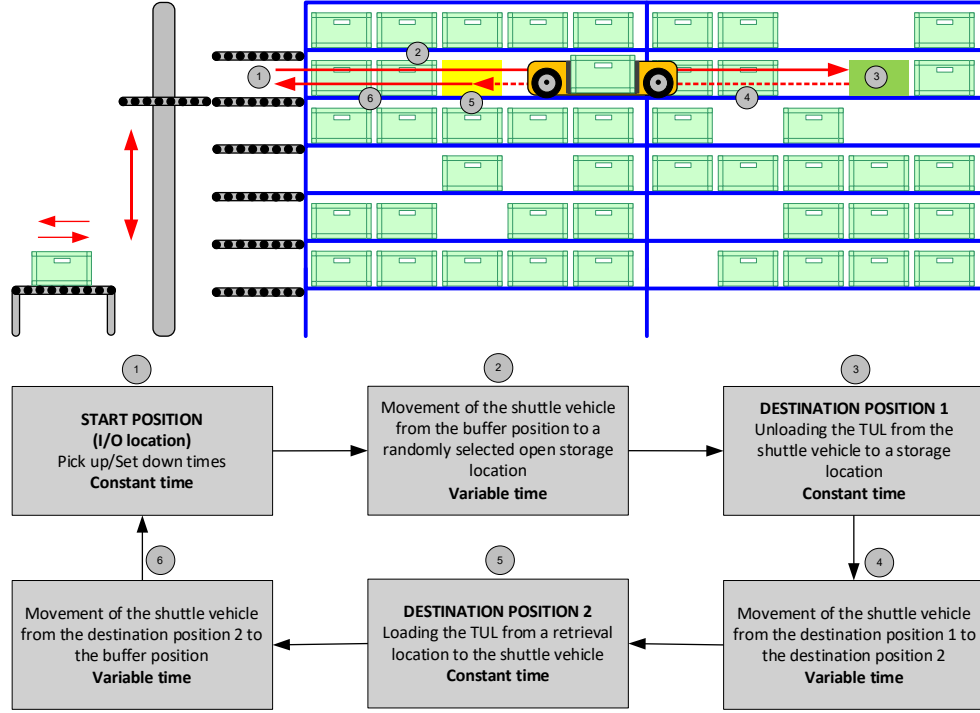


Fig. 12. Shuttle vehicle's double command cycle for single-deep SBS/RS

Source: own

Shuttle vehicle's DC cycle T_{DC}^{SHUT} equals (7):

$$T_{DC}^{SHUT} = 4 \cdot t_{P\&D}^{SHUT} + t_{BL_i, m_i}^{SHUT} + t_{m_i, n_i}^{SHUT} + t_{n_i, BL_i}^{SHUT} \quad (7)$$

Shuttle vehicle's throughput performance in case of a DC cycle λ_{DC}^{SHUT} equals (8):

$$\lambda_{DC}^{SHUT} = \frac{3600}{T_{DC}^{SHUT}} \cdot k \cdot w_x \quad (8)$$

where $k = 2$, due to the DC cycle, and w_x is number of shuttle vehicles in one aisle; equals number of tiers.

4.3 Models for shuttle vehicle for multi-deep SBS/RS

4.3.1 Single and double command cycle of the shuttle vehicle

In the case of a multi-deep SBS/RS (as illustrated in Figure 13), the average travel time for the DC cycle, denoted as T_{DC}^{SHUT} , is increased due to the potential rearrangement of a blocking TUL to the nearest available storage location. This rearrangement follows the "nearest-neighbour rearrangement policy" within the storage rack (as depicted in Figure 13).

In our model, which incorporates the "depth first" storage policy, rearrangements cannot take place during the storage request itself. However, the rearrangement process occurs in the following sequence

(as shown in Figure 14): the blocking TUL is loaded onto a satellite vehicle, the shuttle vehicle is repositioned to the storage location using the "nearest-neighbour rearrangement policy", the formerly blocking TUL is unloaded by the satellite vehicle, the shuttle vehicle is repositioned back to its previous retrieval location, and finally, the TUL is retrieved by the satellite vehicle.

It's important to note that during this process, the shuttle vehicle carries a satellite vehicle, which in turn holds the attached TUL.

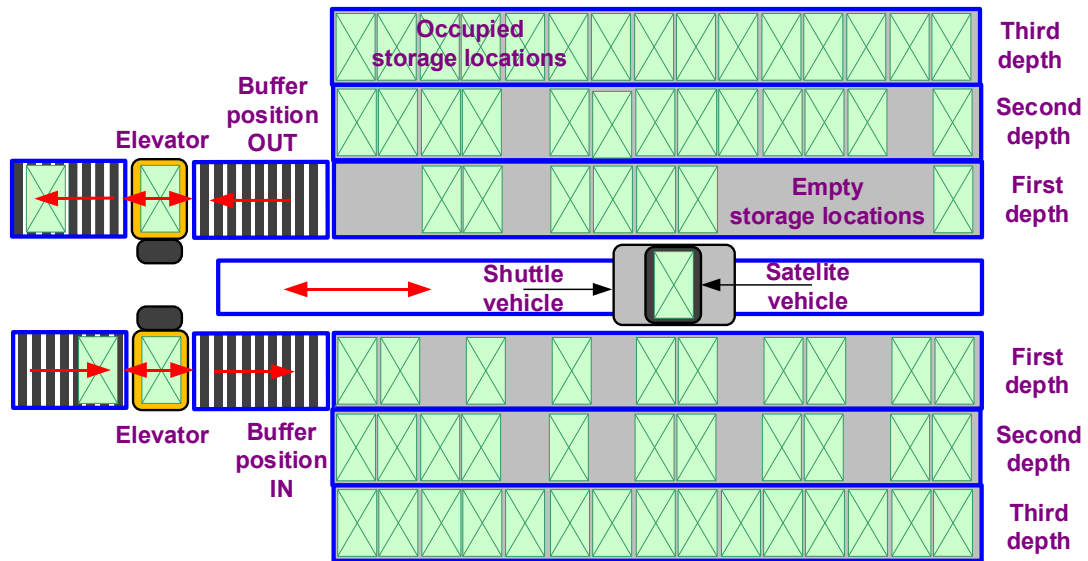


Fig. 13. Multi-deep SBS/RS

Source: own

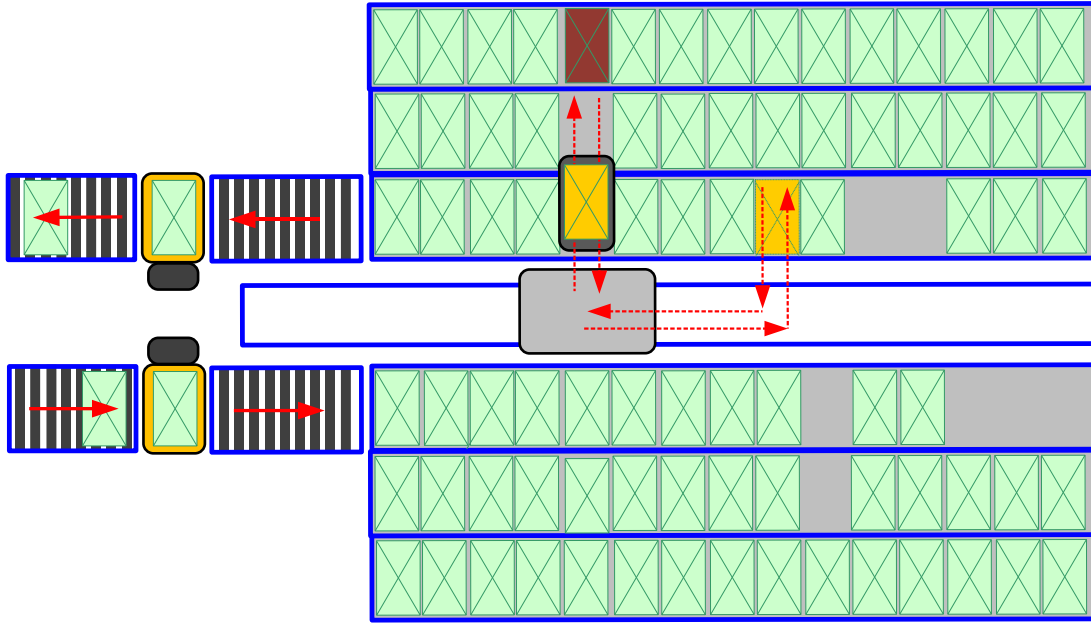


Fig. 14. Nearest-neighbour rearrangement policy
Source: own

The algorithmic flow for storage and retrieval processes in the case of the DC cycle, with and without the rearrangement of the blocking TUL, is illustrated in Figure 15.

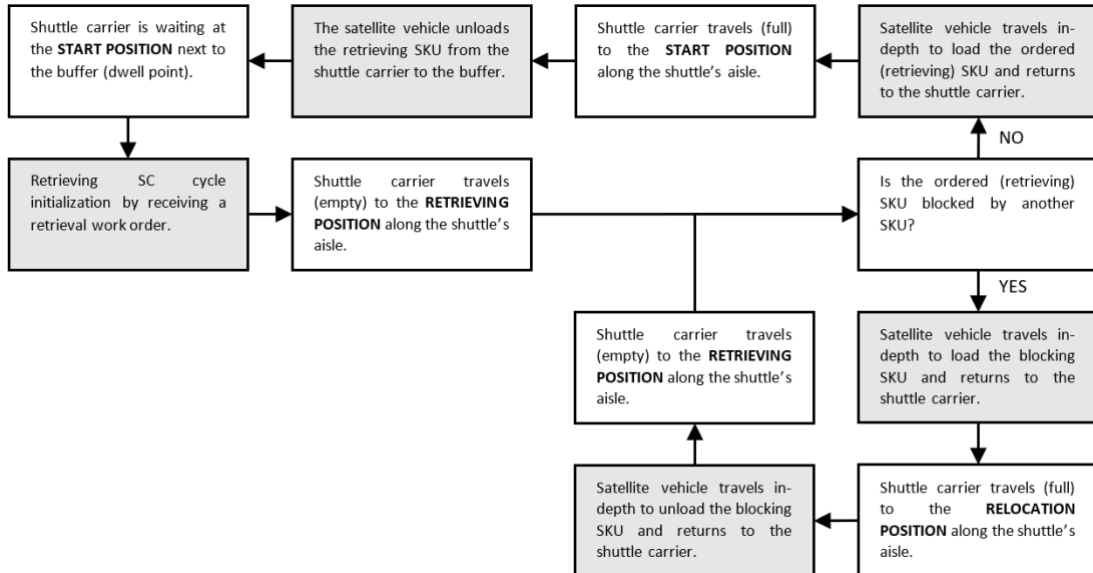


Fig. 15. The algorithm for the retrieval sequence of TULs in multi-deep SBS/RS
Source: own

Shuttle vehicle's SC cycle T_{SC}^{SHUT} for the storage sequence equals (9):

$$T_{SC}^{SHUT} = 2 \cdot t_{P\&D}^{SHUT} + t_{BL_i, m_i}^{SHUT} + t_{m_i, lane_i}^{SATEL} + t_{lane_i, m_i}^{SATEL} + t_{m_i, BL_i}^{SHUT} \quad (9)$$

Shuttle vehicle's throughput performance in case of a SC cycle λ_{SC}^{SHUT} for the storage sequence equals (10):

$$\lambda_{SC}^{SHUT} = \frac{3600}{T_{SC}^{SHUT}} \cdot k \cdot w_x \quad (10)$$

where $k = 1$, due to the SC cycle, and w_x is number of shuttle vehicles in one aisle; equals number of tiers.

Shuttle vehicle's DC cycle T_{DC}^{SHUT} with no rearrangements equals (11):

$$T_{DC}^{SHUT} = 2 \cdot t_{P\&D}^{SHUT} + t_{BL_i, m_i}^{SHUT} + t_{m_i, lane_i}^{SATEL} + t_{lane_i, m_i}^{SATEL} + t_{m_i, n_i}^{SHUT} + t_{n_i, lane_j}^{SATEL} + t_{lane_j, n_i}^{SATEL} + t_{n_i, BL_i}^{SHUT} \quad (11)$$

Shuttle vehicle's DC cycle T_{DC}^{SHUT} with rearrangements equals (12):

$$T_{DC}^{SHUT} = 2 \cdot t_{P\&D}^{SHUT} + t_{BL_i, m_i}^{SHUT} + t_{m_i, lane_i}^{SATEL} + t_{lane_i, m_i}^{SATEL} + t_{m_i, n_i}^{SHUT} + \omega(t_{n_i, lane_i}^{SATEL} + t_{lane_i, n_i}^{SATEL} + t_{n_i, r_i}^{SHUT} + t_{r_i, lane_i}^{SATEL} + t_{lane_i, r_i}^{SATEL} + t_{r_i, n_i}^{SHUT}) + t_{n_i, lane_j}^{SATEL} + t_{lane_j, n_i}^{SATEL} + t_{n_i, BL_i}^{SHUT} \quad (12)$$

Shuttle vehicle's throughput performance in case of a DC cycle λ_{DC}^{SHUT} equals (13):

$$\lambda_{DC}^{SHUT} = \frac{3600}{T_{DC}^{SHUT}} \cdot k \cdot w_x \quad (13)$$

where $k = 2$, due to the DC cycle, and w_x is number of shuttle vehicles in one aisle; equals number of tiers.

5 Simulation model of single- and multi-deep SBS/RS

A simulation model is employed to assess the performance of single- and multi-deep SBS/RS configurations. The simulation experiments are carefully designed to ensure a comprehensive and reliable analysis. In total, 30 simulation runs are conducted for each SBS/RS configuration, with each run representing a different random sequence of S/R requests. Performance measures are collected for a simulation period of 100,000 seconds (27.8 hours) for each run, excluding a warm-up period of 10,000 seconds (2.78 hours) to mitigate the influence of initial conditions. The simulation procedure for single-

and multi-deep SBS/RS, implemented in MATLAB Simulink, is outlined in the algorithm depicted in Figure 16.

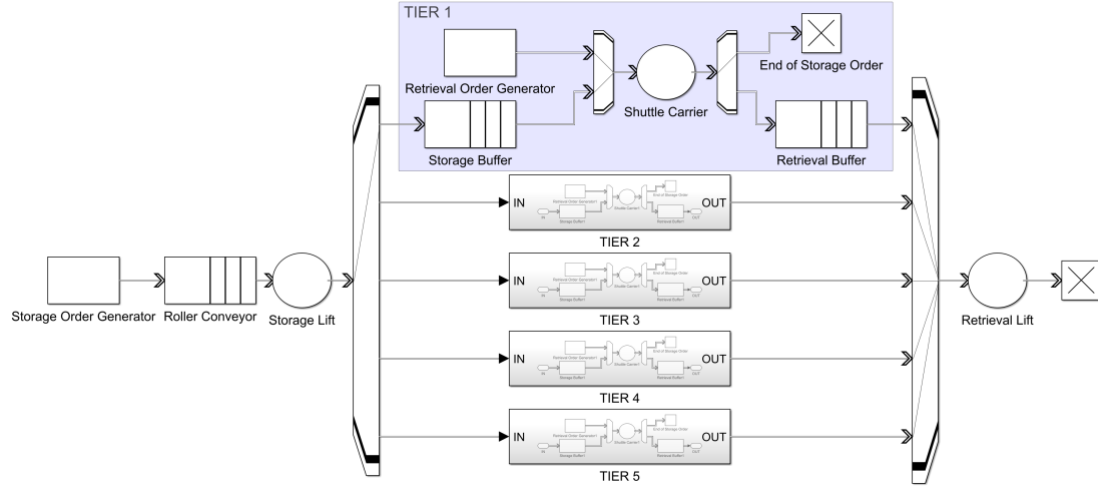


Fig. 16. Simulation model of single- and multi-deep SBS/RS in MATLAB Simulink

Source: own

Figure 16 illustrates the simulation model's workflow for the generation and processing of TULs in the SBS/RS. The process begins with the "Storage Order Generator" block, which generates TULs and directs them to the "Roller Conveyor" buffer. From there, the TULs are served by the "Storage Lift" server, where their processing time is determined by the kinematic model explained in Section 4. Once the service time is completed, the TULs are transferred to the "Storage Buffer" in the i^{th} tier of the S/R system.

The TULs remain in the "Storage Buffer" until the "Shuttle Carrier" server becomes available. Similar to the elevator, the shuttle vehicle serves as a server and holds the TULs for a designated service time based on the kinematic model described in Section 4. Finally, the TULs exit the system through the "End of Storage Order" entity termination block.

On the other hand, the initiation of retrieval orders takes place in the i^{th} tier of the S/R system, with the TULs being directed to the aforementioned "Shuttle Carrier" server. After the completion of the retrieval service time, the TULs are moved to the "Retrieval Buffer" and await processing by the "Retrieval Lift" server. Similar to the storage process, the service time is determined by the kinematic model (Section 4), and the TULs are terminated from the model through an entity terminator.

It is important to note that the "Shuttle Carrier" server can accommodate only one TUL at a time. Hence, only a storage or retrieval TUL can be present on the "Shuttle Carrier" server at any given moment. To maintain clarity in the image, a specific case of the dual command cycle is not depicted in Figure 16. Additionally, the elevator's lifting table exclusively performs storage and retrieval cycles due to the specific design of the SBS/RS with separate elevators (refer to Figure 5b). One elevator handles storage requests while the other processes retrieval requests, following the First-In-First-Out (FIFO) policy.

The SBS/RS system utilizes tier-captive shuttle vehicles to handle SC and DC requests. When it comes to the storage request of a TUL in the i^{th} tier of the SR, a "depth-first policy" is employed. In contrast, for retrieval requests, a "random policy" is applied, ensuring that each TUL has an equal chance

of being selected. The "depth-first storage policy" gives priority to the deepest empty location in the i^{th} tier of the storage rack. In cases where multiple storage locations are available in the "order list," our policy selects the location that is closest to the current position of the shuttle vehicle. The pseudo-algorithm for the "depth-first storage policy" is outlined in Figure 17. It considers the configuration of the selected i^{th} tier of the SR (W), the current position of the shuttle vehicle (c_{shuttle}), and the position of any blocking TULs (s_{SKU}) to determine the appropriate storage location.

```

1. Input:  $W, c_{\text{shuttle}}, s_{\text{SKU}}$ 
2. for  $d = D$  to 1 (step = -1) do
3.    $s = \frac{2 \cdot s_{\text{SKU}}}{s_{\text{SKU}}}$ 
4.   if  $W(d, c_{\text{shuttle}}, s) = 0$  then
5.     output =  $[d, c_{\text{shuttle}} - c, s]$ 
6.     exit
7.   else
8.     for  $c = 1$  to ( $C - 1$ )
9.       for  $s = 1$  to 2
10.        if  $c_{\text{shuttle}} + c \leq C$  then
11.          if  $W(d, c_{\text{shuttle}} + c, s) = 0$  then
12.            output =  $[d, c_{\text{shuttle}} + c, s]$ 
13.            exit
14.          if  $c_{\text{shuttle}} - c \geq 1$  then
15.            if  $W(d, c_{\text{shuttle}} - c, s) = 0$  then
16.              output =  $[d, c_{\text{shuttle}} - c, s]$ 
17.              exit

```

Fig. 17. "Depth-first storage policy" pseudo algorithm
Source: Marolt et al. (2022a).

The utilization of multiple depths in the SBS/RS system allows for potential rearrangements of blocking TULs. The "depth-first storage policy" guarantees that the TULs are always stored in the deepest lane of the SR during the storage assignment. If all storage locations in the deepest lane are occupied, storage in the next deepest lane commences. As a result, the opportunity for rearranging blocking TULs during the storage assignment is eliminated.

On the other hand, in the retrieval request, access to a TUL stored in the j^{th} lane of the SR may be obstructed by a blocking TUL in the i^{th} lane. To address this issue, a "nearest-neighbour rearrangement policy" is applied, which selects an alternative empty storage location for rearranging the blocking TUL. The policy gives priority to the storage location that is closest to the current position of the shuttle vehicle. Ideally, the storage location on the opposite side of the aisle is preferred as it minimizes the shuttle vehicle's movement. If there are multiple storage locations available in the "order list," the "nearest-neighbour policy" primarily selects the closest empty location in the deepest lane, which is nearest to the elevator. The pseudo-algorithm for the "nearest-neighbour policy" is presented in Figure 18. It considers the selected configuration of the i^{th} tier of the SR (W), the current position of the shuttle vehicle (c_{shuttle}), and the position of blocking TULs (s_{SKU}) to determine the suitable rearrangement location.

```

1. Input:  $W, c_{shuttle}, s_{SKU}$ 
2.  $s = \frac{2 \cdot s_{SKU}}{s_{SKU}}$ 
3. for  $d = D$  to 1 do
4.   if  $W(d, c_{shuttle}, s) = 0$  then
5.     output =  $[d, c_{shuttle}, s]$ 
6.     exit
7.   for  $c = 1$  to  $(C - 1)$ 
8.     for  $d = D$  to 1 (step = -1)
9.       for  $s = 1$  to 2
10.        if  $c_{shuttle} + c \leq C$  then
11.          if  $W(d, c_{shuttle} + c, s) = 0$  then
12.            output =  $[d, c_{shuttle} + c, s]$ 
13.            exit
14.          if  $c_{shuttle} - c \geq 1$  then
15.            if  $W(d, c_{shuttle} - c, s) = 0$  then
16.              output =  $[d, c_{shuttle} - c, s]$ 
17.              exit

```

Fig. 18. “Nearest-neighbour rearrangement policy” pseudo algorithm
Source: Marolt et al. (2022a).

6 Analysis of alternative Single- and multi-deep SBS/RS designs

6.1 Main input data for the analysis

This section presents the key input data used in the analysis. The TUL being considered is a tote with the following dimensions: length (l_{tote}) = 0.6 m, width (w_{tote}) = 0.4 m, and height (h_{tote}) = 0.24 m. Regarding the storage location for the TUL, the column has the following dimensions: length (depth) of the column (l_{COM}) = 0.6 m, width of the column (w_{COM}) = 0.5 m, and height of the column (tier) (h_{COM}) = 0.4 m. The distance from the buffer position to the first column was set to 0.8 m.

The dimensions of the Single-Deep Block SBS/RS (L and H) depend on the number of columns (N_x) in the horizontal direction and the number of tiers (N_y) in the vertical direction, respectively. The fill grade factor (alpha, α) was set to 80%, indicating the occupancy level of the storage locations.

To facilitate the selection of SBS/RS configurations, a selection manager tool has been developed in MATLAB. This tool allows users to choose configurations based on their specific requirements. By adjusting the number of tiers (N_y), the number of columns (N_x), and depths (D) of the SR, the selection manager can generate SBS/RS configurations with approximately equal warehouse volumes (Q).

Here's how the selection manager tool works:

- The user provides the desired warehouse volume (Q), the minimum and maximum range of N_y (N_{yMin} , N_{yMax}), the minimum and maximum range of N_x (N_{xMin} , N_{xMax}), and the maximum allowed tolerance (ΔQ).
- The selection manager tool generates various alternative SBS/RS configurations based on the input parameters.

- The tool identifies the SBS/RS configurations whose warehouse volumes (Q) fall within the specified tolerance ΔQ . These configurations are considered as potential candidates.
- All generated SBS/RS configurations, along with their respective warehouse volumes, are presented in a table. The configurations that have similar warehouse volumes to the desired volume are highlighted.
- The table also includes the difference between the calculated warehouse volume and the desired warehouse volume, providing users with information on how closely each configuration matches their requirements.

By utilizing the selection manager tool, users can easily explore and compare different SBS/RS configurations, ultimately selecting the one that best suits their needs based on warehouse volume and the specified tolerance. The resulting configurations, along with their corresponding details, are presented in Table 3.

Table 3: Single- and multi-deep SBS/RS configuration

SBS/RS configuration	Number of tiers $N_y (l)$	Number of depths $D (l)$	Number of columns $N_x (l)$	Length of the SR $L (m)$	Height of the SR $H (m)$	Warehouse volume $Q (l)$
1	15	5	80	40	6	12000
2	15	4	100	50	6	12000
3	15	3	132	66	6	11880
4	20	2	150	75	8	12000
5	20	4	75	37.5	8	12000
6	20	5	60	30	8	12000
7	20	3	100	50	8	12000
8	20	1	300	150	8	12000
9	15	2	200	100	6	12000
10	15	1	400	200	6	12000
11	10	5	120	60	4	12000
12	10	4	150	75	4	12000
13	10	3	200	100	4	12000
14	10	2	300	150	4	12000
15	10	1	600	300	4	12000

The elevator's lifting table and the shuttle vehicle velocity profiles are presented in Table 4.

Table 4: Velocity scenarios of the shuttle vehicle and the elevator's lifting table

Velocity profile	Shuttle vehicle travelling in the horizontal direction			Elevator's lifting table movement in the vertical direction		
	$v_x (m/s)$	$a_x^+ (m/s^2)$	$a_x^- (m/s^2)$	$v_y (m/s)$	$a_y^+ (m/s^2)$	$a_y^- (m/s^2)$
(vp)	3.0	2.0	2.0	5.0	6.0	6.0

Table 5 provides additional information regarding the time durations associated with various operations involving the lift, shuttle vehicle, and satellite vehicle. These durations are crucial factors in the overall performance analysis of the system.

Table 5: Other time-dependent data used in the analysis

Variable	Unit of measure	Data
$t_{P/S}$ (lift)	s	1.50
$t_{P/S}$ (shuttle vehicle)	s	3.40

t_{SAT} (satellite vehicle 2 nd depth)	s	8.94
t_{SAT} (satellite vehicle 3 rd depth)	s	10.94
t_{SAT} (satellite vehicle 4 th depth)	s	12.94
t_{SAT} (satellite vehicle 5 th depth)	s	14.94

Note: t_{SAT} Please take note that in the context of this system t_{SAT} refers to the time taken by the satellite vehicle to travel to the i^{th} depth within the lane of the SR, carry out the storage or retrieval of the TUL, and return to the shuttle vehicle. The velocity of the satellite vehicle was predetermined as $v_{SAT} = 0.6$ m/s, while the acceleration and deceleration values were set at $a_{SAT} = 0.4$ m/s².

6.2 Analysis and evaluation of the results

This section provides an analysis of the utilization of the elevator's lifting table and shuttle vehicles, as well as the throughput performance for the chosen single- and multi-deep SBS/RS configurations. Tables 6-9 present the results obtained from these analyses, considering the selected configurations from Table 3 and the velocity profile specified in Table 4 for both the shuttle vehicle and the elevator's lifting table.

6.2.1 Utilization and throughput performance of the shuttle vehicle

The utilization values and throughput performances of the shuttle vehicle for the selected single- and multi-deep SBS/RS configurations have been determined based on the simulation analyses. The analyses are conducted considering the configurations from Table 3, where the possibility of rearranging the blocking TULs by a satellite vehicle during retrieval assignments exists in multi-deep SBS/RS. The selected configurations correspond to a fill grade factor ($\alpha = 0.80$), the velocity profile of the shuttle vehicle as defined in Table 4, and the velocity profile of the satellite vehicle as defined in Table 5.

The distribution of utilization levels (η_{SHUT}) reveals that the length of the storage rack (L) and the number of depths (D) have a significant impact on utilization and, consequently, on the throughput performance of the shuttle vehicle. In the case of a single-deep SBS/RS, the shuttle vehicle operates at a relatively high utilization level ($\eta_{SHUT} = 88\%$) for long storage racks (configuration no. 8). On the other hand, in the case of a multi-deep SBS/RS, the shuttle vehicle operates at an extremely high utilization level ($\eta_{SHUT} = 96\%$) for short storage racks (configuration no. 1). Conversely, a multi-deep SBS/RS with relatively short storage racks results in a low utilization of the shuttle vehicle ($\eta_{SHUT} = 58\%$) (configuration no. 7).

The SBS/RS configurations 1, 2, and 3 demonstrate the best results in terms of throughput performance, with a value of $\lambda_{DC}^{SHUT} = 65$ TUL/hour. These configurations have shown superior performance compared to others in the analysis.

Table 6: Utilization and throughput performance of the shuttle vehicle

Velocity profile (vp) and the fill grade factor (α)	SBS/RS configuration	Shuttle vehicle utilization	Throughput performance of the shuttle vehicle
		η_{SHUT} (/)	λ_{DC}^{SHUT} (TUL/hour)
$v_x = 3$ m/s $a_x^+ = 2$ m/s ² $a_x^- = 2$ m/s ² $\alpha = 0.80$	1	0.96	64.91
	2	0.95	65.34
	3	0.97	65.12
	4	0.63	43.20
	5	0.57	43.20
	6	0.60	43.16
	7	0.58	43.24
	8	0.88	43.16
	9	1.00	55.87

	10	1.00	38.88
	11	1.00	56.45
	12	1.00	53.68
	13	1.00	51.12
	14	1.00	40.25
	15	1.00	26.03

Note: The operation of the shuttle vehicles is primarily driven by the DC requests. Table 5 provides the velocity profile of the satellite vehicle, which includes the corresponding times for its movement within the i^{th} depth of the lane in the SR. These values represent the average performance of an individual shuttle vehicle and encompass the overall performance of all the shuttle vehicles operating within the system. The presented values refer to one individual shuttle vehicle and are an average that encapsulates the overall performance of all the shuttle vehicles operating within the system.

6.2.2 Utilization and throughput performance of the inbound and the outbound elevator

The utilization values and throughput performances of the inbound and outbound elevators in the selected single- and multi-deep SBS/RS configurations have been determined through simulation analyses. These analyses consider the SBS/RS configurations listed in Table 3, where the inbound elevator delivers TUL to buffer positions in the i^{th} tier of the SR based on SC requests, and the outbound elevator retrieves TUL from buffer positions in the j^{th} tier of the SR also based on SC requests. The velocity profile of the elevators, as presented in Table 4, corresponds to the selected inbound and outbound elevator configurations.

The utilization levels of the inbound elevator ($\eta_{\text{LIFT(I)}}$) and outbound elevator ($\eta_{\text{LIFT(O)}}$) are influenced by the height of the storage rack (H) and have a significant impact on the throughput performance of both elevators. Notably, the best results are achieved in SBS/RS configurations 1, 2, and 3, with a throughput performance of $\lambda_{\text{SC}}^{\text{LIFT(I)}} = 488$ TUL/hour for the inbound elevator (Table 7) and $\lambda_{\text{SC}}^{\text{LIFT(O)}} = 491$ TUL/hour for the outbound elevator (Table 8).

However, for SBS/RS configurations beyond configuration 10, both elevators operate with relatively low utilization levels. Since the shuttle vehicles operate at high utilization levels, the inbound and outbound elevators spend most of their time idle, waiting for storage and retrieval orders to be executed.

Table 7: Utilization and throughput performance of the inbound elevator

Velocity profile (vp)	SBS/RS configuration	Inbound elevator utilization	Throughput performance of the inbound elevator
		$\eta_{\text{LIFT(I)}} (/)$	$\lambda_{\text{SC}}^{\text{LIFT(I)}} \text{ (TUL/hour)}$
$v_y = 5 \text{ m/s}$ $a_y^+ = 6 \text{ m/s}^2$ $a_y^- = 6 \text{ m/s}^2$	1	0.98	480.96
	2	0.99	488.41
	3	0.99	488.23
	4	1.00	431.10
	5	1.00	431.10
	6	1.00	431.10
	7	1.00	431.10
	8	1.00	429.91
	9	0.81	400.28
	10	0.56	274.72
	11	0.48	277.42
	12	0.46	267.70
	13	0.44	252.50
	14	0.34	194.36

	15	0.22	126.94
--	----	------	--------

Note: Operation of the inbound elevator is based on the SC sequences.

Table 8: Utilization and throughput performance of the outbound elevator

Velocity profile (vp)	SBS/RS configuration	Outbound elevator utilization	Throughput performance of the outbound elevator
		$\eta_{\text{LIFT(O)}} (/)$	$\lambda_{\text{SC}}^{\text{LIFT(O)}} (\text{TUL/hour})$
$v_y = 5 \text{ m/s}$ $a_y^+ = 6 \text{ m/s}^2$ $a_y^- = 6 \text{ m/s}^2$	1	1.00	492.05
	2	1.00	491.90
	3	1.00	491.69
	4	1.00	430.92
	5	1.00	430.85
	6	1.00	430.81
	7	1.00	430.78
	8	1.00	430.63
	9	0.87	430.45
	10	0.60	296.46
	11	0.50	288.68
	12	0.49	280.73
	13	0.46	263.34
	14	0.36	206.17
	15	0.23	135.25

Note: Operation of the outbound elevator is based on the SC sequences.

6.2.3 Throughput performance of single- and multi-deep SBS/RS

Table 9 provides a summary of various performance metrics for the single- and multi-deep SBS/RS system. It includes the throughput performance of the shuttle vehicle ($\lambda_{\text{DC}}^{\text{SHUT}(*)}$), the inbound elevator ($\lambda_{\text{SC}}^{\text{LIFT(I)}}$), and the outbound elevator ($\lambda_{\text{SC}}^{\text{LIFT(O)}}$). The utilization levels of the shuttle vehicle (η_{SHUT}), the inbound elevator ($\eta_{\text{LIFT(I)}}$), and the outbound elevator ($\eta_{\text{LIFT(O)}}$) are also presented. Additionally, the bottleneck of the system (Ψ) and the overall throughput performance of the SBS/RS ($\lambda_{\text{SBS/RS}}$) are included.

To calculate the overall throughput performance of the SBS/RS, it is essential to identify the bottleneck within the system, which can either be the inbound and outbound elevators or the shuttle vehicles, depending on the configuration. Generally, in SBS/RS systems, the inbound and outbound elevators serve as the bottleneck. However, in cases where there is many columns and depths (multi-deep SBS/RS), the shuttle vehicles become the bottleneck.

The best results are typically achieved in SBS/RS configurations 1, 2, and 3, where the overall throughput performance ($\lambda_{\text{SBS/RS}}$) ranges from 973 to 980 TUL/hour. These configurations demonstrate high utilization levels for both the inbound and outbound elevators, as well as the shuttle vehicles, resulting in optimal system performance.

Table 9: Throughput performance of single- and multi-deep SBS/RS

No.	$\lambda_{DC}^{SHUT(*)}$ (totes/h)	$\lambda_{SC}^{LIFT(I)}$ (totes/h)	$\lambda_{SC}^{LIFT(O)}$ (totes/h)	η_{SHUT} (/)	$\eta_{LIFT(I)}$ (/)	$\eta_{LIFT(O)}$ (/)	Ψ (/)	$\lambda_{SBS/RS}$ (TUL/h)
1	973.62	480.96	492.05	0.96	0.98	1.00	LIFT	973.01
2	980.10	488.41	491.90	0.95	0.99	1.00	LIFT	980.32
3	976.86	488.23	491.69	0.97	0.99	1.00	LIFT	979.92
4	864.00	431.10	430.92	0.63	1.00	1.00	LIFT	862.02
5	864.00	431.10	430.85	0.57	1.00	1.00	LIFT	861.95
6	863.28	431.10	430.81	0.60	1.00	1.00	LIFT	861.91
7	864.72	431.10	430.78	0.58	1.00	1.00	LIFT	861.88
8	863.28	429.91	430.63	0.88	1.00	1.00	LIFT	860.54
9	838.08	400.28	430.45	1.00	0.81	0.87	SHUT	838.08
10	583.20	274.72	296.46	1.00	0.56	0.60	SHUT	583.20
11	564.48	277.42	288.68	1.00	0.48	0.50	SHUT	564.48
12	536.76	267.70	280.73	1.00	0.46	0.49	SHUT	536.76
13	511.20	252.50	263.34	1.00	0.44	0.46	SHUT	511.20
14	402.48	194.36	206.17	1.00	0.34	0.36	SHUT	402.48
15	260.28	126.94	135.25	1.00	0.22	0.23	SHUT	260.28

Note: $\lambda_{DC}^{SHUT(*)} = \lambda_{DC}^{SHUT} \cdot N_y$, where N_y is the number of tiers.

The simulation study conduct a comprehensive comparison between single-deep and multi-deep SBS/RS configurations, analysing various performance and utilization parameters. The results revealed distinct differences between the two configurations.

Multi-depth configurations, such as configurations 1, 2, and 3, demonstrated superior performance and utilization. These configurations exhibited high shuttle utilization and maximized system throughput by fully utilizing the elevators. The synchronized operation of shuttles and elevators resulted in efficient material handling and increased overall system productivity.

In contrast, the single-depth configurations, such as configuration 8 for 20 tiers and configuration 10 for 15 tiers, exhibited a noticeable decline in system throughput performance, despite having high shuttle utilization. The main contributing factor to this decrease is the underutilization of elevators, which introduced a potential bottleneck in the system. Consequently, the system's overall performance is hindered.

Furthermore, the study investigated configurations with fewer tiers but increased depth, such as configurations 11 through 15. In these cases, the system throughput continues to decrease. Although shuttle utilization remained high, there is a significant decrease in elevator utilization. This finding emphasizes the trade-off between storage depth, shuttle utilization, and elevator utilization. It indicates that while deeper storage may enhance shuttle utilization, it can lead to a decline in overall system performance if the elevators are not adequately utilized.

These findings contribute to a deeper understanding of the comparative performance between single- and multi-depth SBS/RS configurations. They provide valuable insights for warehouse design and optimization, enabling decision-makers to make informed choices based on their specific requirements and priorities.

7 Conclusions

This chapter focuses on addressing the design and operation issues of SBS/RSs, aiming to achieve a balanced design that maximizes productivity. Through an extensive literature review, various models, variants, operation policies, and approaches of SBS/RSs have been analysed. While different designs can achieve the desired throughput, it is often observed that systems comprising elevators and shuttle

vehicles exhibit an imbalance, with either elevators or shuttles being highly underutilized. Typically, elevators become the bottleneck in the system. Therefore, this chapter develops a procedure for designing SBS/RSs, considering both tier-captive and aisle-captive systems with single-deep and multi-deep racking structures.

The design procedures are based on existing models for calculating the throughputs of elevators and shuttle vehicles in single command cycles and dual command cycles or utilizing simulation models of the system. The analysis of proposed alternative designs considers different single- and multi-deep configurations, as well as the length (number of columns) and height (number of tiers) of the storage rack, while maintaining the same warehouse volume (storage capacity).

For the case study, a storage capacity of 12000 TULs is selected, considering practical velocity profiles of elevators and shuttle vehicles, as well as the dimensions of TULs and storage locations. A MATLAB tool is developed to generate possible alternatives, resulting in 15 feasible options with single and multi-deep storage racks. Numerical simulations are conducted to calculate the throughputs of the alternative systems and determine the utilization of elevators and lifts.

The results indicate that different alternatives are feasible, but they can lead to significantly different system throughputs depending on the storage depth and rack shape. Throughputs range from 260 to 980 TULs/hour. Furthermore, the study demonstrates that it is possible to design alternatives with high throughputs while maintaining balanced utilization of elevators and vehicles.

When transitioning from single-depth to multi-depth Shuttle-Based Storage and Retrieval Systems (SBS/RS), there is a distinct shift in performance and utilization dynamics. In multi-depth configurations (such as configurations 1, 2, and 3), shuttle utilization remains high, leading to maximum system throughput as elevators are fully utilized. However, in single-depth configurations like configuration 8 (20 tiers) and configuration 10 (15 tiers), we observe a significant drop in system throughput despite high shuttle utilization. This drop is primarily caused by long shuttle vehicle aisles, resulting in underutilization of elevators and creating a bottleneck in the system.

Furthermore, as we consider configurations with fewer tiers (configurations 11 to 15), the system throughput continues to decrease due to the sustained high shuttle utilization but a notable decrease in elevator utilization. Thus, achieving a balance between storage depth, shuttle utilization, and elevator utilization is crucial when transitioning from single depth to multi-depth configurations.

Designs with underutilized elevators should be avoided as they result in lower throughputs. The key to achieving systems with high throughputs lies in finding a balance between shuttle vehicle utilization and fully utilizing elevators. This study highlights that the best results are obtained when the number of tiers, rack dimensions, and depth of the storage lines enable a balanced utilization of both shuttle vehicles and elevators.

Acknowledgement

This research work was supported by the Slovenian Research Agency (ARRS) in the framework of the Applied research project entitled: “Warehousing 4.0 – Integration model of robotics and warehouse order picking systems”; grant number: L5-2626.

References

- Arslan, B., & Ekren, B. Y. (2022). Transaction selection policy in tier-to-tier SBSRS by using Deep Q-Learning. *International Journal of Production Research*, 1-14. <https://doi.org/10.1080/00207543.2022.2148767>
- Battarra, I., Accorsi, R., Manzini, Rubin, S. (2022). Hybrid model for the design of a deep-lane multisatellite AVS/RS. *Int J Adv Manuf Technol* 121, 1191–1217 <https://doi.org/10.1007/s00170-022-09375-x>

- Borovinšek, M., Ekren, B. Y., Burinskienė, A., & Lerher, T. (2017). Multi-objective optimisation model of shuttle-based storage and retrieval system [Article]. *Transport*, 32(2), 120-137. <https://doi.org/10.3846/16484142.2016.1186732>
- Carlo, H. J., & Vis, I. F. A. (2012). Sequencing dynamic storage systems with multiple lifts and shuttles. *International Journal of Production Economics*,
- D'Antonio, G., & Chiabert, P. (2019). Analytical models for cycle time and throughput evaluation of multi-shuttle deep-lane AVS/RS [Article]. *International Journal of Advanced Manufacturing Technology*, 104(5-8), 1919-1936. <https://doi.org/10.1007/s00170-019-03985-8>
- D'Antonio, G., Maddis, M. D., Bedolla, J. S., Chiabert, P., & Lombardi, F. (2018). Analytical models for the evaluation of deep-lane autonomous vehicle storage and retrieval system performance [Article]. *International Journal of Advanced Manufacturing Technology*, 94(5-8), 1811-1824. <https://doi.org/10.1007/s00170-017-0313-2>
- Eder, M. (2019). An analytical approach for a performance calculation of shuttle-based storage and retrieval systems [Article]. *Production and Manufacturing Research*, 7(1), 255-270. <https://doi.org/10.1080/21693277.2019.1619102>
- Eder, M. (2020). An approach for a performance calculation of shuttle-based storage and retrieval systems with multiple-deep storage [Article]. *International Journal of Advanced Manufacturing Technology*, 107(1-2), 859-873. <https://doi.org/10.1007/s00170-019-04831-7>
- Ekren, B. Y. (2017). Graph-based solution for performance evaluation of shuttle-based storage and retrieval system [Article]. *International Journal of Production Research*, 55(21), 6516-6526. <https://doi.org/10.1080/00207543.2016.1203076>
- Ekren, B. Y. (2020). A simulation-based experimental design for SBS/RS warehouse design by considering energy related performance metrics [Article]. *Simulation Modelling Practice and Theory*, 98, Article 101991. <https://doi.org/10.1016/j.simpat.2019.101991>
- Ekren, B. Y., & Akpunar, A. (2021). An open queueing network-based tool for performance estimations in a shuttle-based storage and retrieval system [Article]. *Applied Mathematical Modelling*, 89, 1678-1695. <https://doi.org/10.1016/j.apm.2020.07.055>
- Ekren, B. Y., Akpunar, A., Sari, Z., & Lerher, T. (2018). A tool for time, variance and energy related performance estimations in a shuttle-based storage and retrieval system [Article]. *Applied Mathematical Modelling*, 63, 109-127. <https://doi.org/10.1016/j.apm.2018.06.037>
- Ekren, B. Y., & Arslan, B. (2022). A reinforcement learning approach for transaction scheduling in a shuttle-based storage and retrieval system. *International Transactions in Operational Research*, n/a(n/a). <https://doi.org/10.1111/itor.13135>
- Ekren, B. Y., Sari, Z., & Lerher, T. (2015). Warehouse design under class-based storage policy of shuttle-based storage and retrieval system. IFAC-PapersOnLine,
- FEM. (2001). Blatt 9.851. In *Leistungsnachweise für Regalbediengeräte, Spielzeiten*.
- FEM. (2017). 9.860. *Cycle time calculation for automated vehicle storage and retrieval systems*.
- Guerrazzi, E., Mininno, V., Aloini, D., Dulmin, R., Scarpelli, C., & Sabatini, M. (2019). Energy evaluation of deep-lane autonomous vehicle storage and retrieval system [Article]. *Sustainability (Switzerland)*, 11(14), Article 3817. <https://doi.org/10.3390/su11143817>
- Ha, Y., & Chae, J. (2019). A decision model to determine the number of shuttles in a tier-to-tier SBS/RS. *International Journal of Production Research*, 57(4), 963-984. <https://doi.org/10.1080/00207543.2018.1476787>
- Küçükyaşar, M., Y. Ekren, B., & Lerher, T. (2021). Cost and performance comparison for tier-captive and tier-to-tier SBS/RS warehouse configurations. *International Transactions in Operational Research*, 28(4), 1847-1863. <https://doi.org/https://doi.org/10.1111/itor.12864>
- Lerher, T. (2016). Travel time model for double-deep shuttle-based storage and retrieval systems [Article]. *International Journal of Production Research*, 54(9), 2519-2540. <https://doi.org/10.1080/00207543.2015.1061717>
- Lerher, T., Ekren, B. Y., Dukic, G., & Rosi, B. (2015a). Travel time model for shuttle-based storage and retrieval systems [Article]. *International Journal of Advanced Manufacturing Technology*, 78(9-12), 1705-1725. <https://doi.org/10.1007/s00170-014-6726-2>

- Lerher, T., Ekren, Y. B., Sari, Z., & Rosi, B. (2015b). Simulation analysis of shuttle based storage and retrieval systems [Article]. *International Journal of Simulation Modelling*, 14(1), 48-59. [https://doi.org/10.2507/IJSIMM14\(1\)5.281](https://doi.org/10.2507/IJSIMM14(1)5.281)
- Lerher, T., Ficko, M., & Palčič, I. (2021). Throughput performance analysis of Automated Vehicle Storage and Retrieval Systems with multiple-tier shuttle vehicles [Article]. *Applied Mathematical Modelling*, 91, 1004-1022. <https://doi.org/10.1016/j.apm.2020.10.032>
- Lin, Y., Wang, Y., Zhu, J., & Wang, L. (2021). A Model and a Task Scheduling Method for Double-Deep Tier-Captive SBS/RS with Alternative Elevator-Patterns [Article]. *IEEE Access*, 9, 146378-146391. <https://doi.org/10.1109/ACCESS.2021.3120418>
- Liu, T., Gong, Y., & De Koster, R.B.M. (2018). Travel time models for split-platform automated storage and retrieval systems [Article]. *International Journal of Production Economics*, 197, 197-214. <https://doi.org/10.1016/j.ijpe.2017.12.021>
- Manzini, R., Accorsi, R., Baruffaldi, G., Cennerazzo, T., & Gamberi, M. (2016). Travel time models for deep-lane unit-load autonomous vehicle storage and retrieval system (AVS/RS) [Article]. *International Journal of Production Research*, 54(14), 4286-4304. <https://doi.org/10.1080/00207543.2016.1144241>
- Marchet, G., Melacini, M., Perotti, S., & Tappia, E. (2012). Analytical model to estimate performances of autonomous vehicle storage and retrieval systems for product totes [Article]. *International Journal of Production Research*, 50(24), 7134-7148. <https://doi.org/10.1080/00207543.2011.639815>
- Marchet, G., Melacini, M., Perotti, S., & Tappia, E. (2013). Development of a framework for the design of autonomous vehicle storage and retrieval systems [Article]. *International Journal of Production Research*, 51(14), 4365-4387. <https://doi.org/10.1080/00207543.2013.778430>
- Marolt, J., Kosanić, N., & Lerher, T. (2022a). Relocation and storage assignment strategy evaluation in a multiple-deep tier captive automated vehicle storage and retrieval system with undetermined retrieval sequence [Article]. *International Journal of Advanced Manufacturing Technology*, 118(9-10), 3403-3420. <https://doi.org/10.1007/s00170-021-08169-x>
- Marolt, J., Šinko, S., & Lerher, T. (2022b). Model of a multiple-deep automated vehicles storage and retrieval system following the combination of Depth-First storage and Depth-First relocation strategies [Article]. *International Journal of Production Research*. <https://doi.org/10.1080/00207543.2022.2087568>
- Statista. (2020). *Size of the warehouse automation market worldwide from 2012 to 2026 (in billion U.S. dollars)*. <https://www.statista.com/statistics/1094202/global-warehouse-automation-market-size/>
- Tappia, E., Roy, D., De Koster, R., & Melacini, M. (2017). Modeling, analysis, and design insights for shuttle-based compact storage systems [Article]. *Transportation Science*, 51(1), 269-295. <https://doi.org/10.1287/trsc.2016.0699>
- Turhanlar, E. E., Ekren, B. Y., & Lerher, T. (2022). Autonomous mobile robot travel under deadlock and collision prevention algorithms by agent-based modelling in warehouses. *International Journal of Logistics Research and Applications*, 1-20. <https://doi.org/10.1080/13675567.2022.2138290>
- Van Wijngaarden, B., & Ketelaars, T. (2014). Shuttles in Warehouse Logistics. Logistik Werkstatt Graz. VDI. (2015). 2692 Blatt 1. In *Automated vehicle storage and retrieval systems for small unit loads*.
- Wang, Y., Mou, S., & Wu, Y. (2015). Task scheduling for multi-tier shuttle warehousing systems. *International Journal of Production Research*, 53(19), 5884-5895. <https://doi.org/10.1080/00207543.2015.1012604>
- Wang, Y., Zhang, R., Liu, H., Zhang, X., & Liu, Z. (2019). Task scheduling model of double-deep multi-tier shuttle system [Article]. *Processes*, 7(9), Article 604. <https://doi.org/10.3390/pr7090604>
- Zhao, X., Wang, Y., Wang, Y., & Huang, K. (2019). Integer Programming Scheduling Model for Tier-to-Tier Shuttle-Based Storage and Retrieval Systems. *Processes*, 7(4), 223. <https://www.mdpi.com/2227-9717/7/4/223>
- Zou, B., Xu, X., Gong, Y., & De Koster, R. (2016). Modeling parallel movement of lifts and vehicles in tier-captive vehicle-based warehousing systems [Article]. *European Journal of Operational Research*, 254(1), 51-67. <https://doi.org/10.1016/j.ejor.2016.03.039>