

ORIGINAL RESEARCH

Statistical analysis of in-flight drone signatures

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Abstract

Drone-monitoring radars typically integrate many pulses in order to improve signal to noise ratio and enable high detection performance. Over the course of this coherent processing interval (CPI), many components of the drone signature change and the signature's amplitude and Doppler modulations may hinder coherent integration performance, even in the absence of range-Doppler cell migrations. A statistical characterisation of these fluctuations aides radar designers in selecting optimal CPI lengths. This paper presents a statistical analysis of experimental data of nine flying drones, collected with a frequency modulated continuous wave Ku-band radar, and examines the statistical features of the amplitude fluctuations of the drone body and blades as well as the signature decorrelation time. The method of moments is used to estimate the probability density function parameters of different drone spectral components with the aim of informing the development of improved theory for predicting drone signatures and ultimately increasing detection performance. Results show that, on average, the Weibull distribution provided the best mean square error fit to the data for most drone spectral components and drone types, with the Rayleigh distribution being the next best match. These results were further corroborated by a study of detection performance for a fluctuating target. Whilst decorrelation times of the various signatures varied significantly, even for the same drone, results show that an approximate inverse relationship between drone spectral component bandwidth and decorrelation time held, with individual spectral lines decorrelating after tens to hundreds of msec.

1 | INTRODUCTION

Drones and drone-monitoring radars have been studied increasingly over the last 10 years. Over time, more drone types and a more diverse set of radars have been developed, with many studies describing radar signatures of various drones. Some of these characterisations are used to inform radar improvements and algorithmic detection and target characterisation logic (including machine learning and artificial intelligence).

The aim of this section is to review previous research which has employed radars at different wavelengths and polarisations, with a number of different drone types, to characterise drone signatures and drone radar cross sections (RCSs) in a variety of ways. [1] provided a useful approach to using micro-Doppler analysis for characterising small drones (a two-bladed helicopter, a quadcopter and a hexacopter). Using a

low-cost X-band radar, it illustrated how to determine the spectral extent of jet engine modulation (JEM) lines, how to estimate the rotation rate using spectrogram analysis and how to calculate blade tip velocities with the aim of identifying the drone type. It also pointed out the frequently changing rotational velocities of the blades and the 'chaotic' behaviour of spectral lines, which complicate signature-based identification techniques.

[2] provided a comprehensive overview of approaches to characterising drone RCSs. It summarised the different methodologies used (including the use of Radio Frequencies from S-band to Ku-band) as well as the overall and relative RCSs of numerous drones. Included in this summary was [3], which performed detailed modelling and analysis of DJI Phantom Vision 2+ RCSs, illustrating the effect of changing aspect angle, carrier frequency and polarisation. In this work, simulated data was corroborated with experimental data. [4] used

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two different radars (operating at 14 and 20 GHz) to collect signatures from two different drones (a DJI Inspire 1 and a HobbyLord F829) using different polarisation combinations. Collecting multi-aspect data in both laboratory and outdoor settings, this work showed different polarisation settings provide stronger target returns. [5] examined the major scattering components on a DJI Phantom 2, a 3DR Solo and a DJI Inspire 1 by creating inverse synthetic aperture radar (ISAR) images of each drone at different frequencies (3–6 GHz and 12–15 GHz) and different polarisations, all within an anechoic chamber. In their work, it was found the non-plastic components of the drone dominate the signature and that rotors are unobservable, even when rotating. Also, [6] studied RCS statistics of a Parrot AR in-flight within a laboratory setting at a number of different angles, using maximum likelihood estimation (MLE) to match amplitude statistics of the drone to a given distribution.

One of the more recent examples of a signature study is [7], which investigated how blade rotation affects drone signatures with a primary goal of demonstrating how blade flashes can be used similarly to micro-Doppler processing for target identification. To this end, it contained multiple examples of DJI Phantom 3 signatures, including amplitude versus range (for a high range resolution radar), amplitude versus time and amplitude versus frequency (looking at S-, X-, and Ku-bands). Supplemented with a high-speed camera, these plots are useful at showing how blade flashes affect the amplitude versus time plots, how much lower in reflectivity the blades are compared to the main body and how the spectrum for various scenarios should appear.

Two examples looking at how signatures change in flight are [8, 9]. The former provided diverse methods to calculate the coherency of drone signatures, including statistical analysis of the amplitude and phase of the return signals as well as the gain from different integration times. It used an X-band FMCW radar and examined three different drones (a Parrot AR, a DJI FJ550 and a DJI Phantom 2), some of which were on different trajectories and in different wind conditions. Results showed that the coherency times varied extensively (over hundreds of milliseconds) for these different scenarios. The latter ([9]) also used an FMCW X-band radar to collect in-flight signatures of a DJI Phantom 4. This analysis included power and RCS versus range (corresponding to hundreds of seconds) as well as a histogram of target amplitudes. It stated that the target is a Swerling 1 because of its Chi Squared density function.

Like [6, 9, 10] attempted to statistically characterise signatures by providing a comprehensive approach to matching empirical data to parameterised statistical distributions with the goal of estimating how RCS varies with frequency, polarisation and azimuth. It examined six different drones (four quadcopters and two hexacopters) on a turntable within an anechoic chamber and for comparison flew one of the quadcopters outdoors to examine RCS variations with elevation angle.

The research listed above, whilst providing a comprehensive set of literature on drone RCSs and the fluctuations

thereof, all work with data after coherent integration. For example, [8, 9] look at tens or hundreds of seconds worth of data, stating or implying that they have already integrated a certain number of pulses. Reference [7] shows time-domain and frequency-domain data after coherent integration but still does not look in depth at amplitude fluctuations over time.

This work analyses unique experimentally collected Ku-band data of flying drones in an uncontrolled environment prior to coherent integration. It examines the statistical characteristics of the individual spectral components that drive the composite signature, with the aim of improving detection efficiency and CPI duration selection. To the best of the authors' knowledge, no previous work has presented Ku-band radar data to look over such a broad set of in-flight drone data, including multiple examples of quadcopters, hexacopters and, particularly, fixed-wing drones. Finally, this analysis compares how drone spectral component statistics (both RCS fluctuations and spectral component decorrelation times) vary with time from drone type to drone type. The analysis of decorrelation times informs on the selection of suitable CPI durations to achieve detection. The longer the decorrelation time the longer is the CPI that provides a useful coherent integration gain. Integrating the target once the return has decorrelated does not offer a significant gain and therefore CPI durations can be optimised to manage radar resources effectively.

Section II reviews drone signature theory. Section III describes the experiments conducted and how the signature spectral components were extracted for further analysis. Section IV provides a detailed methodology for statistical analysis. It examines one specific drone thoroughly and summarises the same statistical metrics across all nine drones in a comprehensive summary table. Section V presents observations with detection performance. Finally, Section VI presents conclusions.

2 | DRONE SIGNATURE THEORY

Let us consider the 2-dimensional (2-D) geometry shown in Figure 1 with an ideal point scatterer (S2) rotating in the XY plane around a stationary centre of rotation (S1) with a radius L_2 ,

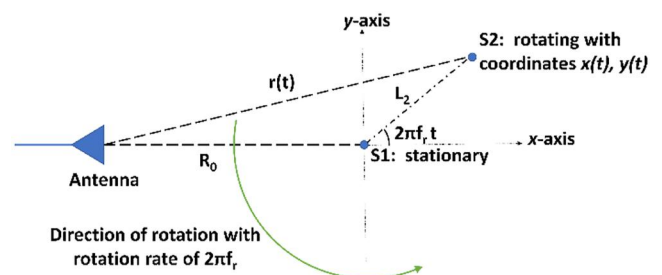


FIGURE 1 Rotating scatterer model

where R_0 is the range from the radar antenna to the centre of rotation, $r(t)$ is the time-varying range between the radar and the scatterer, f_r is the scatterer rotational frequency and $2\pi f_r t$ is the angle at time t that the radius of rotation forms with the x-axis. The coordinates of S2 relative to the origin of the Cartesian plane are $x(t) = L_2 \cos(2\pi f_r t)$ and $y(t) = L_2 \sin(2\pi f_r t)$. Using the Pythagorean theorem and assuming $L_2 \ll R_0$, the range from the antenna to the rotating scatterer is $r(t) = R_0 + L_2 \cos(2\pi f_r t)$ and, as a result, the Doppler induced by the rotating point scatterer is a function of $\sin(2\pi f_r t)$ [11–14]. In [11], the simple case presented in Figure 1 was extended to show that the reflected radar signal from a distributed target with N finite-length rotating propeller blades, in a 3-D Cartesian plane, can be modelled as in Equation (1).

$$x(t) = \sum_{n=0}^{N-1} A_r (L_2 - L_1) \sin\left(\frac{4\pi}{\lambda} \frac{L_2 - L_1}{2} \cos(\alpha) \sin\left(2\pi f_r t + \frac{2\pi n}{N}\right)\right) e^{j\left(2\pi f_c t - \frac{4\pi}{\lambda} \left[R_0 + vt + \frac{L_2 + L_1}{2} \cos(\alpha) \sin\left(2\pi f_r t + \frac{2\pi n}{N}\right)\right]\right)} \quad (1)$$

In Equation (1), A_r is an amplitude scale factor, v is the bulk translational velocity of the target, L_1 is the distance from the centre of rotation to the blade root, L_2 is the length of the blade, λ is the radar's wavelength at the carrier frequency f_c and α is the angle between the radar's line of sight and the rotor's plane of rotation. Equation (1) captures the critical variables shaping the Doppler signature of distributed rotating blades. Its Fourier Transform is [11]

$$X(f) = \sum_{n=-N_1}^{N_1} c(n) \delta(f - f_c - f_d - N n f_r) \quad (2)$$

where n is the spectral line index, N_1 refers to the highest significant sideband number, f_d is the Doppler frequency and

$$c(n) = \sum_{l=0}^{\infty} 2 \frac{(-1)^{N n u(n)} A_r N}{\frac{4\pi}{\lambda} \cos(\alpha)} \left[J_{|N n|+2l+1}\left(\frac{4\pi}{\lambda} L_2 \cos(\alpha)\right) - J_{|N n|+2l+1}\left(\frac{4\pi}{\lambda} L_1 \cos(\alpha)\right) \right] e^{-j \frac{4\pi R}{\lambda}} \quad (3)$$

In (3), $u(n)$ is the unit step function which is equal to 1 for $n \geq 0$ and 0 elsewhere, and $J_k(\cdot)$ is a Bessel function of the first kind of the k th order. Equations (2) and (3) provide an analytic expression for the spectrum of a distributed rotating scatterer and assume only some spectral lines will be observable. They indicate that the spectrum will be the superposition of a delta function corresponding to the target stationary components and additional delta functions at multiples of the rotational rate, all Bessel-weighted by the rotor length to wavelength ratio and the line of sight (LOS) angle.

When α approaches 90° (e.g. for a typical fixed wing drone flying radially towards the radar), no micro-Doppler signature is produced. However, in practice, Doppler sidebands are caused by complex modulations induced by the blade RCS as the rotor comes in and out of alignment with the radar's polarisation direction. This amplitude modulation generates periodical or quasi-periodical returns and flashes that create sidebands in the spectrum which the mathematical model used in [13, 14] does not describe. An interested reader can refer to [15] for more details on this topic.

In real-world scenarios, many of the above variables will change value randomly throughout a CPI and the signal will be corrupted by additive white Gaussian noise. For these reasons, a statistical analysis of experimental data represents a very important step to allow the validation of mathematical and simulation models and to provide a full characterisation of real flying drone signatures.

3 | SIGNATURE EXTRACTION AND EXPERIMENTS

This section explains the theory and experimental background to the statistical analysis. Various spectral components of the drone signature are extracted and their SNR estimated. Detailed results are provided for one specific test.

3.1 | Signature extraction

Let us assume a ground-based FMCW radar that provides a target measurement return for every transmitted frequency sweep. The beat signal of interest is

$$z(n) = s(n) + w(n), \quad (4)$$

where $s(n)$ refers to the radar beat signal reflected from the drone for one particular range cell, n indicates the sweep number of the FMCW radar and $w(n)$ is additive white Gaussian noise. Next, the discrete Fourier transform is taken by

$$Z(k) = \sum_{n=1}^N z(n) e^{-j 2\pi n k / K} \quad (5)$$

where N is the length of $z(n)$, K is the even number of points of the Fourier transform and k refers to the frequency bin axis, spanning $-K/2$ to $K/2 - 1$. Typical radar data is such that the spectrum $Z(k)$ contains large ground clutter returns that must be removed. Assuming the target is moving with a sufficient radial velocity, a high pass filter is applied by zeroing out bins near 0 Hz, producing a resulting spectrum $Z_0(k)$. Next, the decluttered signal is converted back into the time-domain via the inverse Fourier transform as

$$z(n)_o = \sum_{k=1}^K Z_o(k) e^{j2\pi kn/N} \quad (6)$$

In Equation (6), $z(n)_o$ is the time domain signal with stationary clutter removed. Since the Doppler bandwidth of $z(n)_o$ spans the sweep repetition frequency and the target contribution only covers a limited portion of this bandwidth, the SNR can be improved by band pass filtering to reject the thermal noise outside the target bandwidth (i.e. because the power spectral density of the noise is uniform within the Doppler bandwidth). Various band pass filters have been applied to the data, to isolate the signature spectral components of interest, improve the SNR and obtain the resulting signal

$$z(n)_f = z(n)_o * h(n)_f \quad (7)$$

to be analysed. In (7), $h(n)_f$ is the time response of the flat bandpass filter tuned to match the bandwidth of a specific drone spectral component of interest. This was manually selected to pass and analyse specific spectral component(s) of interest. Finally, a check of the SNR of $z(n)_f$ can be performed to ensure that the statistics developed reflect the behaviour of the spectral components of interest and not the noise.

3.2 | Statistical and correlation analysis

Once the individual drone spectral components have been separated, it is possible to study their specific statistical properties. When generating signal statistics, the measured amplitude $|z(n)_f|$ is seen as the realisation of a random variable X with probability density function (PDF) $f_X(x)$. The correlation analysis is carried out on the complex signal $z(n)_f$, under the assumption that the samples of the individual drone spectral components are the realisation of a stationary random process, to characterise the first and second order statistics. Decorrelation times are also estimated to examine the duration over which the signal is coherent to inform the development of detection algorithms.

For characterising the drone spectral component statistics, attempts are made to fit the data to four candidate PDFs: the Rayleigh, Gamma, Weibull and Log-Normal PDFs. Each candidate PDF is defined by one or two parameters which are a function of the first order moment $E\{X\}$ and second order moment $E\{X^2\}$ of X . After estimating the data distribution from the experimental data with a histogram, the mean squared error (MSE) between the histogram and each candidate theoretical PDF was calculated and compared to find the 'best fit'. The expressions of the theoretical PDFs follow:

(1) Rayleigh Density Function

The Rayleigh PDF is defined as

$$f_X(x) = \frac{x}{\gamma^2} e^{-x^2/2\gamma^2} \quad (8)$$

for $x \geq 0$ and is fully characterised by the parameter $r = \mu/\sqrt{\pi/2}$, where $E\{X\} = \mu$ is the mean value of X .

(2) Gamma Density Function

The Gamma PDF is defined as

$$f_X(x) = \frac{1}{\theta^g \Gamma(g)} x^{g-1} e^{-x/\theta} \quad (9)$$

for $x \geq 0$. It is fully characterised by the parameters g , referred to as the shape parameter, and θ , referred to as the scale parameter. These two parameters are determined using the mean and the variance of X and the relationships $E\{X\} = \mu = g\theta$ and $E\{X^2\} - E\{X\}^2 = \sigma^2 = g\theta^2$ (where σ^2 is the variance of X).

(3) Weibull Density Function

The Weibull PDF is defined as

$$f_X(x) = \frac{c}{d^c} x^{c-1} e^{-(x/d)^c} \quad (10)$$

for $x \geq 0$, also with c referred to as the shape parameter and d as the scale parameter. They are determined by solving $c = \left(\frac{\mu}{\sigma}\right)^{1.086}$ and $d = \frac{\mu}{\Gamma(1+1/c)}$ with μ and σ defined above [16].

(4) Log-Normal Density Function

The Log-Normal PDF is defined as

$$f_X(x) = \frac{1}{xv\sqrt{2\pi}} \left(-\frac{1}{2} \left(\frac{\ln x - m}{v} \right)^2 \right) \quad (11)$$

for $x \geq 0$, where m and v are the distribution parameters and are determined using equations $m = \ln\left(\frac{\mu^2}{\sqrt{\sigma^2 + \mu^2}}\right)$ and $v = \sqrt{\ln\left(\frac{\sigma^2}{\mu^2 + 1}\right)}$ with μ and σ^2 defined above [17].

The distribution parameters were estimated with the method of moments as in [18] by calculating the moments from the data as

$$E\{X\} = \frac{1}{N} \sum_{n=1}^N |z(n)_f| \quad (12)$$

and

$$E\{X^2\} = \frac{1}{N} \sum_{n=1}^N |z(n)_f|^2 \quad (13)$$

After fitting the data to candidate histograms, the decorrelation time of each signal was estimated. The biased

autocorrelation function (ACFs) $R(m)$ was estimated using the complex data as [19]

$$R(m) = \frac{1}{N} \sum_{n=0}^N z(n)_f z^*(n+m)_f. \quad (14)$$

and normalised for ease of comparison. The decorrelation time was defined as the time at which the value of the ACF decreased below 10% of the peak value [20].

3.3 | Data collection

A monostatic FMCW Ku-band radar operating at 16.6 GHz was used to collect signatures of nine separate flying drones, including three fixed-wing drones, each flying random trajectories in uncontrolled outdoor conditions. The drones used, together with their type and dimensions, are listed in Table 1 and represent multiple categories of airframes with different body shapes, numbers of propellers and flight dynamics. Over two hundred data files from the drone flights were provided.

In order to investigate the statistics and carry out an analysis of correlation aimed at informing integration in the absence of range-Doppler migrations, the overall data set was surveyed and specific files that contained high SNR and minimally manoeuvring targets were selected. Three data files per drone type were identified for further analysis. With a 20 MHz waveform, the range resolution of the radar used was 7.5 m, which ensured all spectral components fit into one range cell for each drone. The radar was HH polarised. With a PRF of 6617 Hz, all close-in spectral lines were measured unambiguously within the available Doppler bandwidth (± 3308 Hz corresponding to ± 30 m/s) and no appreciable aliasing was noted for any of the drones. For each drone, data was arranged in frames of one-second-long sequences providing 6617 data samples per range cell. Four range cells on either side of the target's range cell were also provided to ensure possible range migrations could be captured and to extract close-range environmental (clutter/interference/noise)

information. The data clearly identified which range cell contained the target and other range cells could be used for noise estimation.

The analytic techniques described in Section III.A and III. B were applied to the flight data files for all nine drone types. Due to space limitations, in this article, plots for only one file and trajectory of one specific drone type, a Yuneec H920 hexacopter (Figure 2), are reported. The file corresponds to a trajectory on a nearly constant velocity flight path to illustrate the key analytic concepts in the absence of range-Doppler migrations.

This sizeable drone can fly up to 21 m/s, providing a large RCS to the radar. The remainder of Section III as well as Section IV illustrate the analysis of this data file. Results from the statistical analysis of the other eight drones are summarised in Table 5.

3.4 | Data analysis

The time domain signal for the Yuneec H920 that has been analysed in detail shows a high average SNR of over 20 dB. The short time Fourier transform (STFT) of the data is shown in Figure 3 and was created using 8192 FFT points and a time window length of 150 sweeps (corresponding to a dwell time of 22.7 msec and to a Doppler resolution of about 44 Hz). This signal corresponds to $z(n)_0$ from Equation (6), indicating



FIGURE 2 Image of yuneec 920 [21]

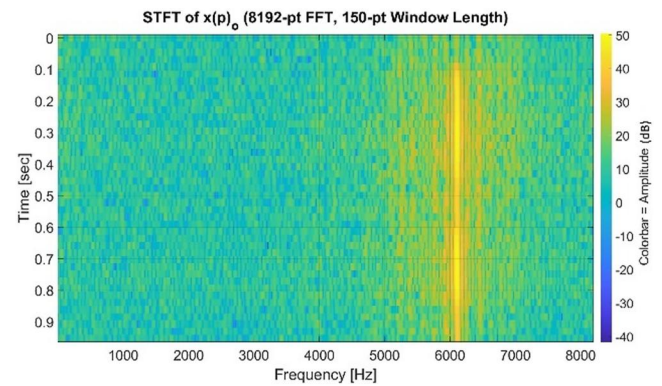


FIGURE 3 Short time Fourier transform (STFT) of yuneec H920 with nearly constant velocity

TABLE 1 Drone types

Name	Type	Dimensions (mm)
DJI M210	Quad	790 × 390 × 290 [22]
DJI Mavic air 2	Quad	183 × 253 × 77 [23]
DJI phantom 3	Quad	590 (diagonal) [24]
DJI mavic 2 pro	Quad	322 × 242 × 84 [25]
Yuneec H920	Hex	797 × 920 × 461 [26]
Yuneec H520	Hex	520 × 457 × 310 [27]
FMS east trainer	Fixed-wing pusher	1280 × 910 [28]
Ares gamma pro V2	Fixed-wing puller	940 × 775 [29]
Dynam A10 warthog	Fixed-wing electric ducted fan (EDF)	1080 × 995 [30]

that the STFT was calculated after the stationary clutter was removed, albeit prior to any further filtering.

The signal clearly demonstrates two large amplitude plateaus (between 200 and 400 msec and between 650 and 800 msec), likely corresponding to drone aspect changes. Notably, results show how the drone amplitude can significantly increase in value in a few hundred milliseconds, or even less. This plot however does not show details of how the amplitude of individual drone spectral components change with time. To isolate the individual spectral components, spectral analysis is needed. Figure 4 shows the spectrum of the drone $Z_0(k)$, which spans the Doppler bandwidth ± 3308 Hz (corresponding to ± 30 m/s) together with the spectrum of the receiver background noise.

The blue line in Figure 4 was obtained with an 8192-point FFT of the range cell containing the target return and the red line from a range cell containing only noise. The high-pass filter used to remove the stationary clutter retained energy above ± 80 Hz, which for this file included the body and all significant drone spectral lines.

For spectral component-specific analysis, this paper analyses selected sections of this spectrum: 1) the entire drone signature, 2) the drone's body, 3) all the spectral lines (without the body) and 4) two individual spectral lines. For each drone file studied, the authors examined the spectrum and defined the drone spectral components as portions of the spectrum

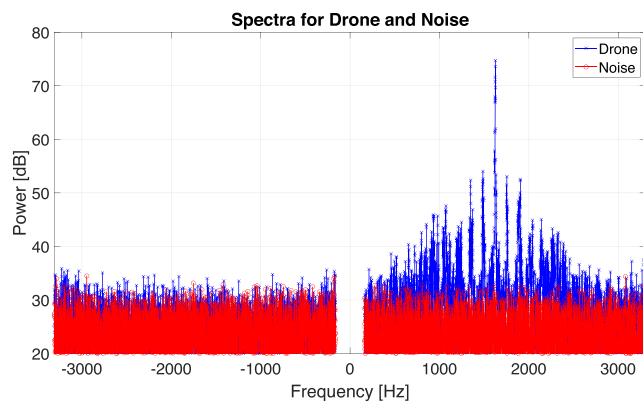


FIGURE 4 Spectra of drone signal and noise signal

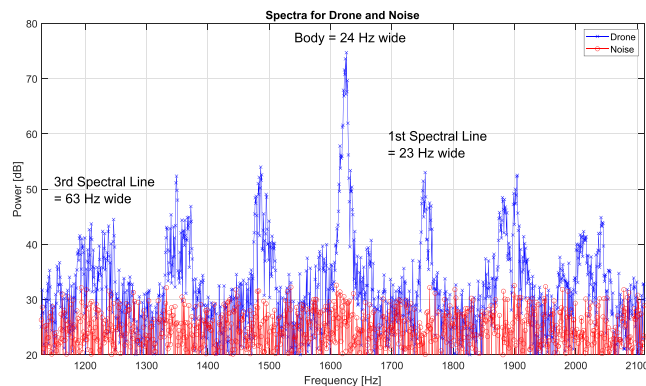


FIGURE 5 Zoomed spectra

where the drone returns were above the noise level and remained above the noise for more than four adjacent bins. The boundaries of a specific spectral component were defined as the bins at which the frequency content was no longer discernible, having decreased to the noise level. Due to spectral line asymmetry and the desire to broaden the analysis, the first spectral line on the positive (right) side of the body was selected and third spectral line on the negative (left) side of the body was selected.¹ Figure 5 shows a zoomed in version of Figure 4, focussing on the body and the first 3 spectral lines.

Results show that the body's return is prominent around 1626 Hz and three spectral lines are shown on either side of it. The spectral lines widen as they move outward from the body's return, decreasing their overall amplitude. This spectral spreading is possibly due to slightly varying rotational velocities and higher ordered mixing products.

Next, the SNRs were estimated. First, the entire drone spectrum was filtered to isolate the body and all appreciable spectral lines with a 2040 Hz bandpass filter, centred at 1470 Hz.² The time-domain signal (on a linear scale) of the entire drone is shown, along with the estimated SNR (on a logarithmic scale), in Figure 6.

The two large amplitude plateaus are still visible after bandpass filtering, as well as some minor fluctuations from the blades. In the bottom plot, the SNR is shown to be around 10 dB to nearly 30 dB at times. To create the SNR estimate, the noise from a range cell with no discernible signal was filtered using the same $b(n)_f$ used to isolate the drone in the range cell with the target. Then the mean noise power was calculated as the mean value of $|w(n)_f|^2$ and the instantaneous power of the filtered target signal was divided by the noise mean power estimate, providing an SNR-versus-time estimate.

The drone body was studied by isolating it with a 24 Hz bandpass filter, centred at 1626 Hz, and the noise signal was isolated using the same filter on the reference range cell. The noise power was estimated by the process described above. The body signal amplitude is shown in Figure 7 with the estimated SNR. Due to the much narrower bandpass filter, the body's SNR was well above 20 dB for most of the one-second acquisition time.

To isolate the spectral lines a bandpass filter covering all spectral line frequencies yet excluding the body frequencies was applied. The amplitude and SNR of the spectral lines are shown in Figure 8.

It is clear that the large amplitude plateaus from Figures 3, 6 and 7 are no longer present, confirming that those notable fluctuations were related to the body instead of the blades. The SNR of the spectral lines fluctuates mainly between 5 and 20 dB for the entire acquisition time, except for very brief SNR drops.

¹Whilst in most data files the body and the first three spectral lines were outside the clutter notch, some SNRs were so significant and spectral extents so broad that a few spectral lines fell in the clutter notch and were not analysed.

²In many cases, the filter was not centred on the body due to the asymmetry of the spectral line pattern. For example, many cases were studied with more spectral lines on one side of the body than the other.

Finally, two specific spectral lines were isolated to compare the properties of different spectral components. The first and third spectral lines were selected as they were frequently observable in the spectral analysis of the dataset (i.e. their SNRs are usually higher than spectral lines further away from the body). In analysing the broader set of drones, every attempt was made to select spectral lines from opposite sides of the body. However, in some cases a first and third spectral line on opposite sides of the body were not observable, so an

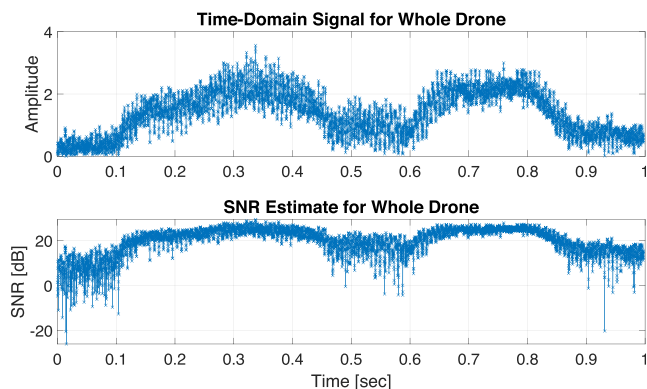


FIGURE 6 Signal and estimated signal to noise ratio (SNR): whole drone

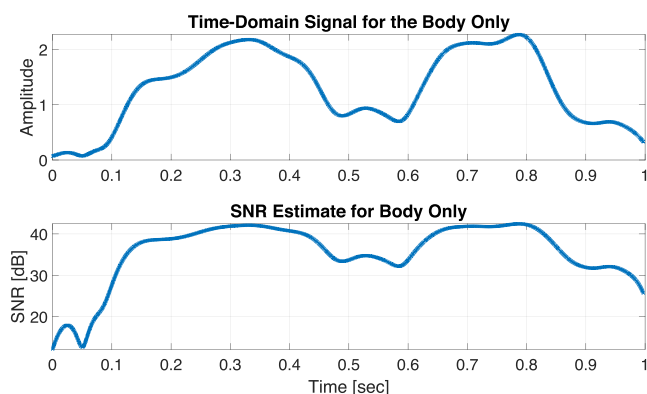


FIGURE 7 Signal and estimated signal to noise ratio (SNR): body only

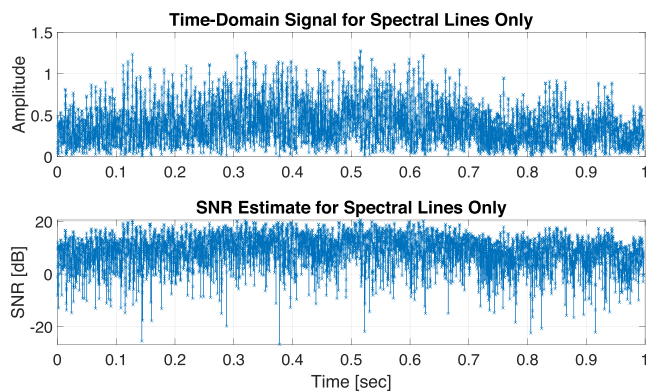


FIGURE 8 Signal and estimated signal to noise ratio (SNR): all spectral lines

adjustment was made to take them from the same side. Both spectral line time-domain plots in Figures 9 and 10 show, to varying degrees, periodic amplitude fluctuation.

In order to isolate the first spectral line, a 23 Hz band pass filter, centred at 1760 Hz, was used. As can be seen in the bottom plot of Figure 9, the SNR fluctuated extensively with peaks around 20–25 dB.

In order to isolate the third spectral line, a 63 Hz bandpass filter, centred at 1217 Hz, was used. As can be seen in the bottom plot of Figure 10, the SNR also fluctuated with peaks between 10 and 20 dB. Table 2 summarises the key parameters of each drone spectral component (for the specific data file in this section). The mean target signal power was calculated as the mean of $|z(n)_f|^2$ minus the mean noise power.

4 | STATISTICAL ANALYSIS

This section presents a detailed statistical analysis of the data described above. It applies the theory presented in Section II and discusses the results. Finally, it presents a summary table of the results of the application of the same analysis to the eight other drones.

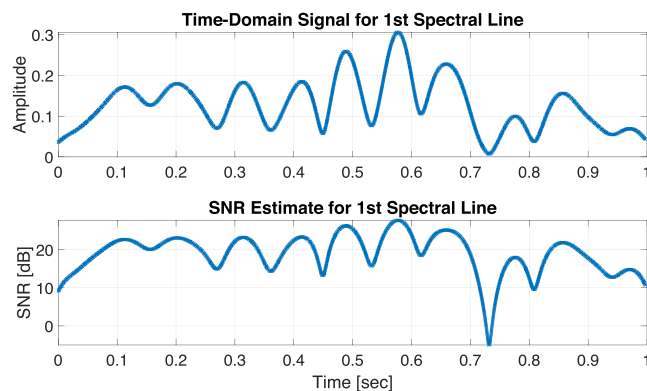


FIGURE 9 Signal and estimated signal to noise ratio (SNR): first spectral line

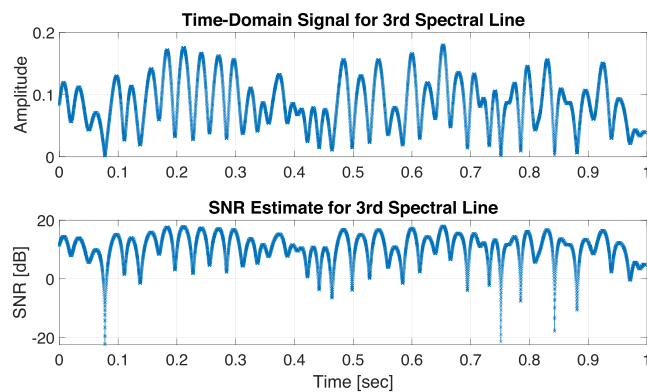


FIGURE 10 Signal and estimated signal to noise ratio (SNR): third spectral line

First a histogram was developed of the amplitudes reflected from the entire drone for 6600 sweeps (approximately one second). To create this histogram (as well as that of the body-only), 82 amplitude bins were used, corresponding to the

TABLE 2 Bandwidth, signal and noise parameters

Drone spectral component	Bandwidth (Hz)	Mean signal Power (dB)	Mean NoisePower (dB)	Mean SNR (dB)
Whole drone	2039.5	2.8	-18.3	21.1
Body	24.2	2.4	-35.3	37.7
All SLs*	2013.7	-7.9	-18.4	10.5
1 st SL	23.4	-17.8	-38.0	20.2
3 rd SL	63.0	-21.3	-32.8	11.6

Abbreviations: SL, Spectral Line; SNR, signal to noise ratio.

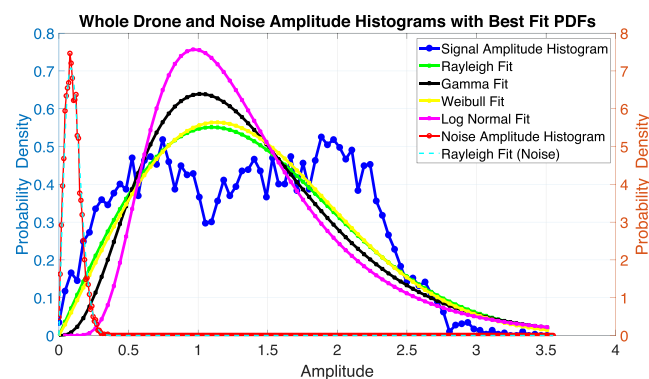


FIGURE 11 Magnitude and histogram compared: whole drone

TABLE 3 Distribution parameters. They grey bold boxes indicate the distributions that provide the best MSE fit

Drone Spectral Component	Calculated Parameters		Fitted Parameters			
	μ	σ^2	Rayleigh (R)	Gamma (g, θ)	Weibull (c, d)	Log Normal (m, v)
Whole Drone	1.379	0.497	1.101	3.825, 0.361	2.072, 1.557	0.205, 0.482
Body Only	1.315	0.456	1.050	3.792, 0.347	2.062, 1.485	0.157, 0.482
All Spectral Lines	0.401	0.050	0.320	3.254, 0.123	1.900, 0.452	-1.047, 0.517
1 st Spectral Line	0.129	0.004	0.103	4.318, 0.030	2.212, 0.145	-2.154, 0.456
3 rd Spectral Line	0.087	0.002	0.069	4.813, 0.018	2.347, 0.0977	-2.541, 0.434

Abbreviation: MSE, mean square error.

square root of the number of samples of the time-domain signal plus an additional amplitude bin. This number was selected through trial and error with the aim of providing fine amplitude resolution but also allowing a useful number of occurrences per amplitude bin. To create the noise histograms, five times more amplitude bins were used to ensure sufficient occurrences (i.e. most noise values fell far to the left on the x-axis due to the high SNR). Finally, to ensure the area under the histogram equals unity, the amplitude bin counts (y-axis) were scaled by the bin width, which was on the order of 10^{-3} , thus giving probability density values much larger than one in some cases.

Figure 11 shows the results for the whole drone and indicates that, within the acquisition time, the amplitude fluctuates widely. This histogram does not neatly fit into known PDFs, as it is somewhat flat across the top (i.e. from bins 0.5 to 2) but shows somewhat gradual tails. Despite the mismatch, for this data file, the Rayleigh PDF was the best MSE fit (as it resulted in the lowest MSE).

This histogram does not neatly fit into known PDFs, as it is somewhat flat across the top (i.e. from bins 0.5 to 2) but shows somewhat gradual tails. Despite the mismatch, for this data file, the Rayleigh PDF was the best MSE fit (as it resulted in the lowest MSE). The estimated parameters for each PDF are shown in Table 3 for the Yuneec H920 case. The MSEs between each theoretical PDF curve and the data histograms are shown in Table 5. A histogram of the drone body alone was created and results are shown in Figure 12. Results show the body reflected a wide variation in signal amplitude across the one-second interval with 'spikes' corresponding to the amplitude values on which the drone dwelled for 10 or more msec, as seen by comparing the x-axis from Figure 12 with the y-axis from the top plot in Figure 7. Furthermore, the body

histogram rapidly drops off between amplitudes of 2 and 2.5. This is largely consistent with Figure 11, which sees a more gradual drop off due to the presence of the blades producing brief amplitudes above 2.5. For this data file, despite poor fitting performance, the Log-Normal PDF demonstrated the lowest MSE.

Despite the initial implicit stationarity assumption, results suggest the body's statistics are non-stationary and further analysis was performed on the data to corroborate this. A study of the mean and variance of a subsection of the time-domain signal indicate significant variations as a function of the subsection of sweeps analysed. For example, the first 200 msec of data (1323 sweeps) will produce a different mean and variance than data collected in the interval 200–400 msec, and so on. To illustrate this point, a sliding window probability density function of the body was estimated. A window of 25 msec, which corresponds to 165 sweeps, was selected through trial and error to capture how the data distribution changes with time. A histogram was obtained using the data in a window and this process repeated for sequential sections of the time domain signal, moving 165 sweeps at a time. The results obtained are shown on the left of Figure 13, illustrating that the fluctuation of the body's amplitude can vary significantly between different 25-msec subsections. On the right of Figure 13, the same plot is coloured by the mean value, illustrating the difference in the mean on a window-to-window basis. The purpose of these plots is to demonstrate the non-

stationarity of the body's amplitudes. Fitting the histogram of each window to the distributions presented in Section IV resulted in much better fitting performance than those obtained in Figures 11 and 12.

The histogram of all the spectral lines is shown in Figure 14. For the spectral lines (and associated noise), the amplitudes did not vary as much as the whole drone and body-only signals. Therefore, 25 amplitude bins were used that were equally spaced between 0 and the maximum amplitude. The Weibull PDF best fits this data (although the MSEs of the Rayleigh and Gamma PDFs were very close to the Weibull's).

Figures 15 and 16 show the histograms of the first and third spectral lines, respectively. The results illustrate a few key

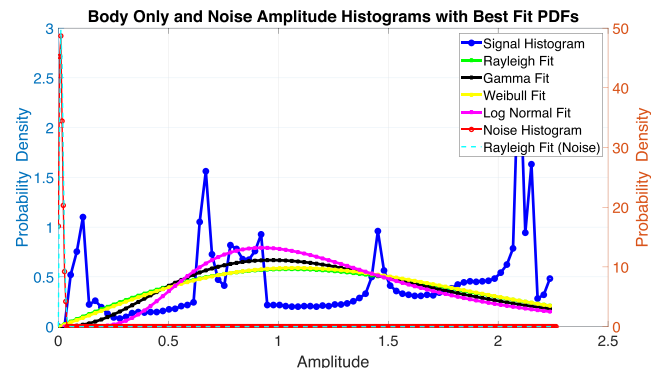


FIGURE 12 Magnitude and histogram compared: body only

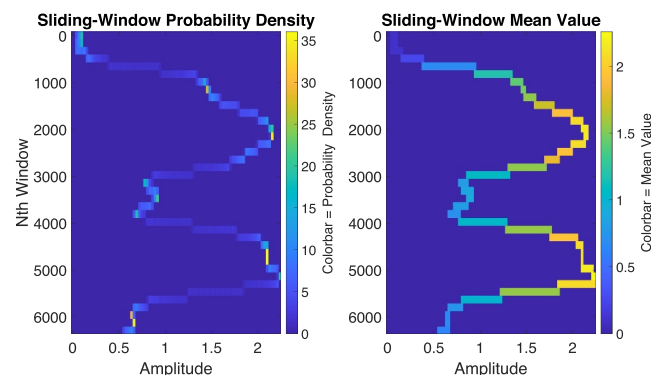


FIGURE 13 Sliding-window density and mean values: body only

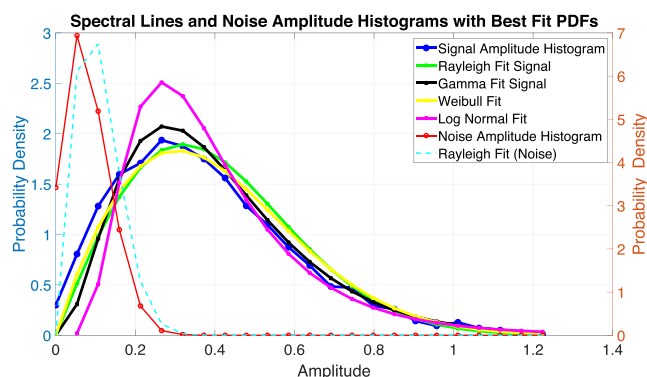


FIGURE 14 Magnitude and histogram compared: all spectral lines

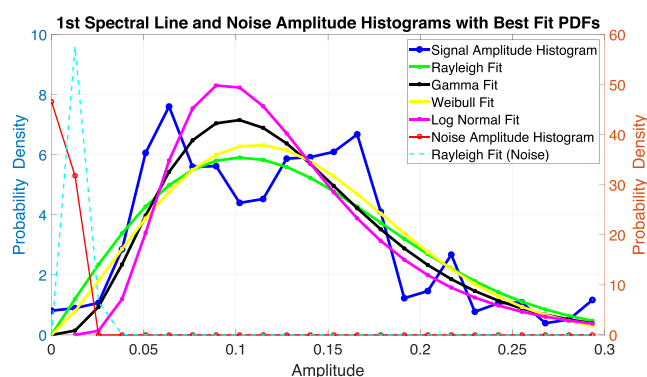


FIGURE 15 Magnitude and histogram compared: first spectral line

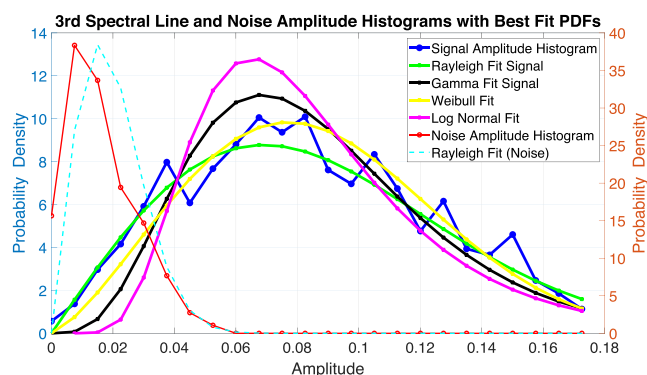


FIGURE 16 Magnitude and histogram compared: third spectral line

points. First, the amplitude span of all the spectral lines (x-axis of Figure 14, above) is much wider than that of the individual spectral lines (x-axes of Figures 15 and 16). Similarly, the noise level is much higher for all the spectral lines than in the individual spectral lines due to the much wider extent of the Doppler of the latter. Of all the spectral lines, the separation between the signal and the noise is greatest for the first spectral line. This arises because the spectral spreading of the first spectral line is less than other spectral lines and this narrower bandwidth passes less noise, improving the SNR. The probability density curve of the first spectral line (Figure 15) shows two minor peaks which produce two ridges (these will be discussed in more detail below). However, the fitted PDFs are still able to approach the slopes on the left and right side of the data distribution. The Rayleigh PDF was the best fit to this data file.

The histogram of the third spectral line (Figure 16) is similar to that in Figure 14, but the SNR is lower and therefore the most likely values of the signal and noise are closer. Since the third spectral line contains more minor spikes compared to the two more prominent spikes in the first spectral line, the data distribution is smoother and better matches the reference PDFs. The Rayleigh PDF was also the best fit to this data file and its MSE was slightly more than half of the MSE of the first spectral line's histogram with the Rayleigh PDF.

The histograms clearly show that the SNR was sufficient to place the signal plus noise higher in amplitude than the noise only. For a radar using a detector in which the threshold is some level higher than the estimated noise power, the three different SNRs would present three very different probabilities of detection. Therefore, the spectral component-specific analysis shown above helps develop more tailored detection logic.

It is important to pause briefly to consider the minor bifurcation in the histogram of the first spectral line. Reverting to (2), in an ideal scenario, the spectral lines are generated by a function of the term $e^{j\sin(\omega t)}$. The distribution of a sinewave is an arcsine function and the distribution of the real and imaginary components of $e^{j\sin(\omega t)}$ are simply arcsine functions scaled in amplitude [31]. The authors theorise the reasons the pure arcsine function is not observed in the histograms above is the rotor frequencies are constantly changing (f_r is a random variable itself) and the amplitudes of the returns are also changing due to drone aspect changes. Minor changes to a sinewave behaviour are likely as well as the presence of additive Gaussian noise. The result of these changing variables is the peaks at the edges of an arcsine distribution change into Gaussian-like distributions.³ Since mixing products cause greater spectral spreading in the third spectral line, it is probable the arcsine distribution becomes more like the fitted PDFs.

Additionally, it is well known from statistics theory that the probability distribution of the sum of two independent random variables is the convolution of the individual distributions. Based on the observed data, it is reasonable to treat the distribution of the body as independent from the blades. Whilst a

drone manoeuvre imparts a Doppler shift on both the body and the blades, the amplitudes still fluctuate mostly independently from each other. Therefore, an estimate of a whole-drone distribution can be developed. Treating the spectral lines with a given distribution (i.e. Rayleigh), they can be convolved with the body's distribution. The difficulty in this approach is that the body is non-stationary, and its distribution cannot reasonably be predicted. However, if simple assumptions of the drone's body can be made or parameterised, then through convolution, the composite distribution can be created. Depending on the needs of the radar designer, this approach can be adapted extensively.

Finally, the autocorrelation functions (ACFs) of the data were calculated, starting with the entire drone. The analysis of the ACFs is key as it helps identify the coherency of the drone signature versus time and therefore informs suitable coherent integration times for radar detection.

Figure 17 shows the ACF of the whole drone dropped below 10% at an offset of ± 162.5 msec. Furthermore, the ACF contains minor amplitude spikes at intervals of approximately 7.5 msec, which corresponds to the spectral line spacings of approximately 130 Hz (i.e. the reciprocal) in Figures 4 and 5. These amplitude spikes do not occur on the body-only ACF, as the bandpass filter that isolates the body excludes the higher frequency spectral line activity. The body-only ACF is shown in Figure 18.

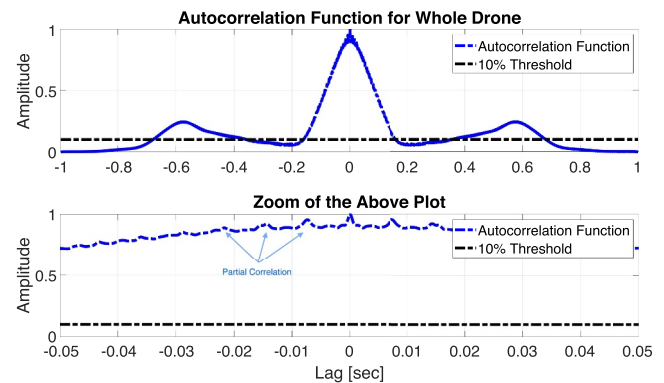


FIGURE 17 Autocorrelation function: Whole drone

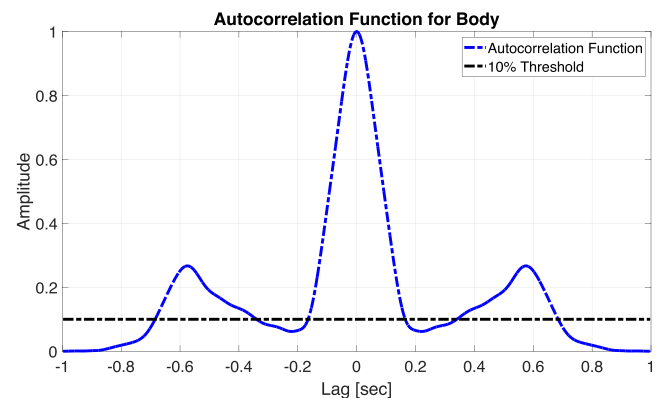


FIGURE 18 Autocorrelation function: Body only

³The authors conducted simulations in MATLAB of Equation (1) using different SNRs as well as with varying rotational velocities. The results corroborated the hypothesis discussed above.

The decorrelation time of the body is ± 168.4 msec, quite similar to that of the entire drone. In this particular ACF, partial correlations occur around ± 600 msec, which correspond with the similarly shaped amplitude ridges occurring on the top plot of Figure 7. Overall, the analysis of the ACFs of the spectral lines shows significant variations in shape and demonstrates much shorter decorrelation times. An ACF of all the spectral lines is shown in Figure 19 along with a zoomed-in version. Results show a decorrelation time of just over 1 millisecond.

Moving to the individual spectra lines, the ACF of the first spectral line decorrelates after ± 349.6 msec, based on the 10% definition described above (see Figure 20). However, a local minimum – which approaches but does not drop below 10% – occurs at ± 122.7 msec.

Finally, the ACF of the third spectral line is shown in Figure 21 and shows the 10% value is reached around ± 13.3 msec offset.

Table 4 summarises the decorrelation times for each spectral component. Results indicate the body's decorrelation time drives the whole drone's decorrelation time. Beyond this specific correlation, there is an approximately inverse relationship between bandwidth and decorrelation time (if the first minimum of the first spectral line's ACF is taken as the point of decorrelation), a point elaborated on further in Section V.

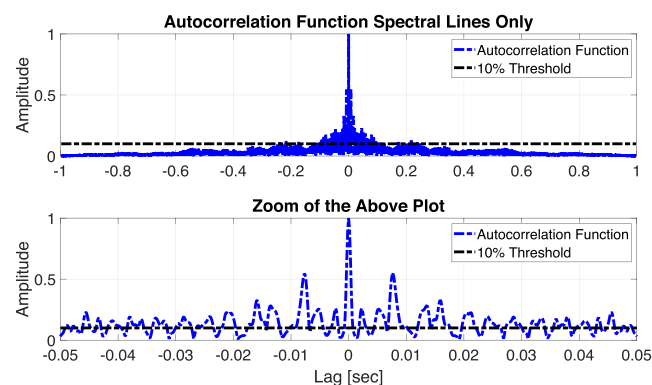


FIGURE 19 Autocorrelation function: All spectral lines

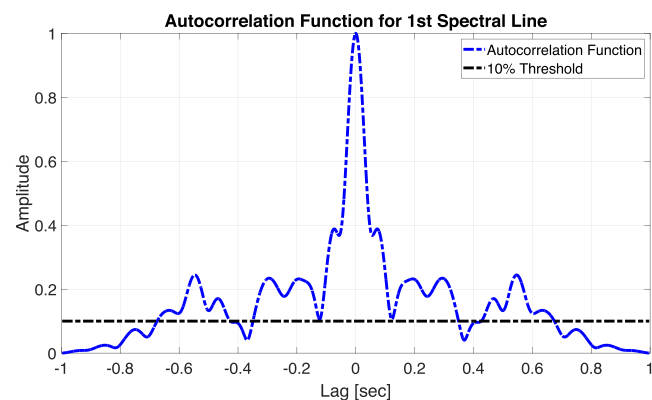


FIGURE 20 Autocorrelation function: first spectral line

5 | OBSERVATIONS

The methods developed in Sections III and IV were applied to the wider set of drones. For each drone type, three files were analysed. The results were grouped by drone type and placed in Table 5 for comparison. The grey highlighted cells in Table 5 correspond to the lowest MSE measurements and the file names correspond to the specific data files analysed. Similar to the case shown above, the drone spectral components were manually identified in the spectrum. Each file contained a unique set of SNRs and sometimes, in cases with lower SNRs, the outer spectral lines were more difficult to identify. In those cases, the overall bandwidth of the drone (including all identifiable spectral lines) may be narrower than other, higher SNR cases. Also, in some cases, the drone was manoeuvring significantly throughout the acquisition time which led to cell-migration, wider bandwidth and lower SNR spectral components. Manoeuvres also meant aspect changes within the acquisition time, leading to further SNR changes. Of the many dozens of data files available, the higher SNR, less manoeuvring drones were used in order to provide the most consistent statistics and very few migration effects.

A few findings are notable. Regardless of drone type, the body is nearly always non-stationary, with histograms like that in Figure 12. Even seemingly minor manoeuvres or drone aspect changes can create large amplitude fluctuations in the body returns. Furthermore, the similarities between whole drone and body decorrelation times held across the wider

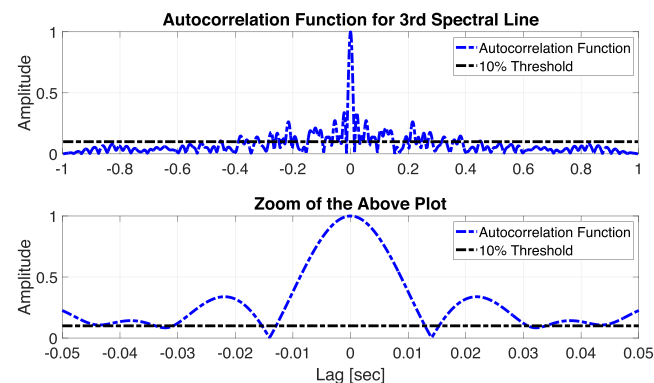


FIGURE 21 Autocorrelation function: third spectral line

TABLE 4 Decorrelation times

Drone spectral component	Decorrelation time (msec)
Whole drone	± 162.3
Body	± 165.5
All spectral line	± 1.1
1 st SL	$\pm 349.6^*$
3 rd SL	± 13.3

* As seen in Figure 20, the main lobe's nulls reach minima at ± 122.7 msec, but the value is still slightly above 10%.

Abbreviation: SL, Spectral Line.

TABLE 5 Results Chart. The grey highlighted bold cells correspond to the lowest Mean Square Error (MSE) measurements and the file names correspond to the specific data files analysed (SL = spectral line, LN = Log-Normal and EIDF = electric ducted fan)

Parameter	Distribution		DJI M210		DJI Mavic Air 2		DJI Phantom 3		DJI Mavic Pro 2		Yuneec H920		Yuneec H520		FMS Easy Trainer (Pusher)		Ares Gamma Pro V2 (Puller)		Dynam A10 Warthog (EDF)											
	8.8	8.7	15.7	126.9	126.20	115.13	78.11	78.14	72.8	50.5	50.6	52.7	40.58	40.60	27.8	95.5	95.8	96.32	137.6	137.26	135.15	149.5	149.6	150.16	156.22	156.44	156.33			
MSE	Whole Drone	Rayleigh	0.0156	0.0208	0.0008	0.2013	0.0746	0.0772	0.0091	0.0987	0.022	0.0407	0.055	0.021	0.0074	0.009	0.01	0.0007	0.0241	0.005	0.0039	0.0249	0.0039	0.0092	0.0059	0.0349	0.0427	0.0836	0.0316	0.011
		Gamma	0.0099	0.0085	0.0011	0.0618	0.0571	0.0229	0.0166	0.009	0.024	0.0091	0.011	0.008	0.0034	0.007	0.013	0.02	0.0007	0.0254	0.002	0.0349	0.0078	0.0097	0.0091	0.008	0.0289	0.0307	0.0638	0.047
		Weibull	0.0098	0.0073	0.0009	0.0535	0.0162	0.0219	0.0098	0.0308	0.008	0.0039	0.006	0.008	0.0083	0.006	0.011	0.055	0.0076	0.0036	0.003	0.009	0.0041	0.005	0.0064	0.0085	0.0217	0.0077	0.0165	0.012
	Body Only	LN	0.1675	0.1214	0.0107	0.5579	0.4694	0.2155	0.0144	0.0034	0.024	0.0308	0.024	0.047	0.0705	0.101	0.055	0.0076	0.0036	0.003	0.1529	0.0802	0.1255	0.0748	0.0631	0.132	0.06	0.1971	0.1687	0.087
		Rayleigh	1.7961	1.0588	0.0268	1.7207	10.005	2.2088	0.1223	0.8602	0.572	0.9753	1.789	0.227	0.1424	0.207	0.198	0.0139	0.2245	0.016	1.4346	0.0587	0.1168	0.3631	0.2033	0.2214	0.3744	0.9142	4.007	4.807
		Gamma	1.7528	1.1397	0.0254	1.3009	4.6869	1.4025	0.1416	0.5708	0.566	0.7331	1.399	0.193	0.193	0.1588	0.2	0.213	0.0137	0.2262	0.012	1.0719	0.071	0.1246	0.3639	0.134	0.1957	0.258	0.9685	1.988
	All SLs	Weibull	1.7554	1.0918	0.0279	1.2728	5.4697	1.3964	0.128	0.7078	0.472	0.6787	1.269	0.2	0.1465	0.181	0.2	0.0135	0.2313	0.013	1.2491	0.061	0.1092	0.3667	0.1385	0.1836	0.1789	0.911	1.639	0.911
		LN	1.5329	0.6394	0.0195	3.5584	0.0306	2.5324	0.0453	0	0.05	0.0004	0.005	0.304	0.2778	0.294	0.122	0.0315	0.0071	0.026	0.1609	0.0895	0.4491	0.4563	0.3392	0.3174	0.0967	0.1862	5.452	
		Rayleigh	0.1516	0.1367	0.0237	0.0952	0.1228	0.0534	0.0143	0.0894	0.047	0.0467	0.074	0.032	0.0476	0.021	0.024	0.0068	0.0251	0.03	0.0809	0.1315	0.0883	0.1552	0.1412	0.1412	0.0781	0.0716	0.071	0.116
	1st SL	Gamma	0.0486	0.0434	0.0074	0.0706	0.1835	0.0478	0.0271	0.0599	0.038	0.0388	0.054	0.037	0.0533	0.049	0.024	0.0056	0.0426	0.025	0.0705	0.0488	0.0839	0.051	0.0323	0.0663	0.1304	0.1811	0.112	0.112
		Weibull	0.0513	0.0442	0.0077	0.0503	0.0973	0.0353	0.0155	0.0721	0.02	0.0313	0.044	0.024	0.0357	0.02	0.015	0.0046	0.0207	0.014	0.056	0.0536	0.0545	0.0458	0.0379	0.0535	0.0745	0.0818	0.114	0.114
		LN	0.1843	0.1043	0.0235	0.2537	0.6023	0.1874	0.0787	0.0388	0.14	0.1261	0.166	0.115	0.194	0.154	0.098	0.02	0.1434	0.09	0.2319	0.1876	0.3007	0.1725	0.1091	0.2331	0.3873	0.6162	0.195	0.195
De-correlation time (msec)	Whole Drone	Rayleigh	64.545	44.284	0.3592	7.0047	1.8982	2.0824	0.829	14.922	1.306	55.912	0.747	6.157	2.8285	1.97	1.35	0.7638	3.0763	0.177	220.817	14.884	11.596	26.623	26.379	39.676	0.8479	65.574	0.851	0.851
		Gamma	97.139	67.007	0.5126	5.4146	2.84	0.5048	1.8808	1.2149	1.064	26.01	1.153	4.734	2.0717	2.741	1.664	0.3203	5.8668	0.351	256.744	23.502	5.4726	25.932	28.514	33.392	3.846	18.4677	1.851	1.851
		Weibull	70.6248	46.763	0.4334	3.8168	2.0787	0.5617	0.8651	2.309	0.911	25.263	0.867	5.104	2.1622	1.707	1.414	0.3447	3.2034	0.204	219.265	15.104	0.983	19.16	23.062	26.125	29.96	29.96	0.964	0.964
	3rd SL	LN	150.955	100.42	0.958	11.459	5.0845	0.9191	3.4084	1.5885	2.459	43.729	2.68	5.979	2.6653	5.458	2.845	0.7961	12.555	0.656	307.914	37.089	7.1224	32.293	44.297	62.639	10.163	19.7166	5.104	5.104
		Rayleigh	0.4975	4.0777	0.1719	0.6551	7.7903	10.156	17.623	8.8403	1.707	3.5671	0.305	0.294	5.8973	3.028	0.728	0.4335	10.128	1.371	71.8068	31.272	14.136	69.172	7.4075	63.448	0.986	130.367	0.562	0.562
		Gamma	1.2453	3.5594	0.1408	2.0189	6.5015	9.3629	22.168	15.051	3.241	1.8614	0.604	0.844	6.552	4.951	2.134	0.5555	13.471	0.678	33.0172	31.612	7.9305	66.794	8.8887	78.824	33.39	143.369	1.366	1.366
	All SLs	Weibull	0.6022	2.6301	0.1903	0.7884	7.0193	7.6314	16.499	10.336	1.767	1.2129	0.312	0.33	5.5897	2.709	1.024	0.2965	10.715	0.685	26.6065	26.537	7.1772	47.921	33.554	60.34	1.0397	127.3	0.728	0.728
		LN	2.8836	7.2901	0.1974	4.7303	13.717	17.777	28.696	24.966	6.38	3.5973	1.845	2.178	10.3	3.888	4.965	0.8854	18.054	1.471	37.4806	54.567	69.593	84.063	101.358	119.55	108.08	185.915	42.3	42.3
		Whole Drone	81.2	510.7	24	26.1	148	103.4	199.5	561.9	13.5	35.8	167.3	279.4	66.2	124.5	162.3	125.9	30.5	293.3	151.4	25.4	23.6	49.3	380.5	79	65.9	6	34.8	0.2
	Bandwidth (Hz)	Body	230.9	563.2	24.9	27.1	363	132.7	202.4	563.1	13.8	36.9	365.3	300	193.6	126.6	165.5	128.8	537.1	293.2	433.9	54.9	24	56.2	388.4	83.1	66.2	124.8	23.7	23.7
		All SLs	1.7	1.8	2.3	1.2	11.3	2.3	0.8	2.3	15.9	1.1	1.1	16.3	2	1.8	1.1	2	1.5	76.8	2.9	0.6	0.8	1.5	1.4	0.9	1.4	0.2	0.2	0.2
		1st SL	178.3	450.2	50	14.5	160.9	48.2	82.5	622	10.1	143.4	34	68	139	92.8	349.6	160.2	143.1	114.6	644.1	127.6	59.1	486.6	51.1	35.1	7.7	34.2	5	5
Bandwidth (Hz)	3rd SL	291.7	329.9	97.2	46.8	20.7	200.2	97.6	141.9	22.4	81.3	5.7	27.1	129.8	186.5	13.3	35.5	322	9.5	0.309	101.0	102.9	200.7	331.9	78.4	4.7	417.4	15.9	15.9	
	Whole Drone	261.3	256.7	298.7	3103	2271	2367	3416	2328	2565	3043	3642	1146	2065	3287	2040	1890	1890	1890	5099	6159	5653	2719	1664	4443	4663	4663	4663	4663	
	Body	27	20	110	105	43	71	40	23	141	81	48	119	36	40	24	28	53	89	24	194	91	24	40	117	125	40	131	131	
Bandwidth (Hz)	All SLs	2586	2547	2877	2998	2229	2296	3376	2305	2424	2962	3594	3027	2029	3246	2014	3409	1452	1801	5015	5965	5561	2695	6576	4326	4538	1753	5973	5973	
	1st SL	24	20	79	109	70	114	48	39	162	18	73	47	23	53	23	65	46	75	8	79	41	19	23	28	156	36	201	201	201
	3rd SL	82	40	125	263	74	49	27	34	88	40	323	161	40	34	63	77	18	116	116	28	69	33	20	24	222	15	293	293	293

dataset: the difference in decorrelation times between the whole drone and body was less than 10 msec in well over half the cases (17 out of 27).

The Weibull PDF most often produced the lowest MSEs. For the whole drone and all the spectral lines, the Weibull PDF was most often the best match. For the individual spectral lines, the Rayleigh PDF was most often the best match, but the Weibull was a close second. Of the 135 MSEs calculated, the Weibull was the best match in 60 cases, with Rayleigh in second place with 34 lowest MSEs. For the body, the Log-Normal was the most often the best match.

Another observation has to do with the number of blades: generally, the more blades on the drone, the better the individual spectral line histograms fit the theoretical PDFs. This point is quantitatively illustrated by averaging all the MSEs for each drone spectral component for three data files per drone category. The average MSEs for the fixed-wing drones was 18.2, for the quadcopters was 5.2 and for the hexacopters was 1.5. The dominant contributions to these averages are the MSEs for the spectral lines. The average MSEs for the spectral lines (that is, all the spectral lines, the first individually and the third individually), are 30.1 for the fixed-wing drones, 8.3 for the quadcopters and 2.4 for the hexacopters. Much less MSE variation is observed for the body histograms: 0.70 for the fixed-wing drones, 1.218 for the quadcopters and 0.132 for the hexacopters.

The drones with multiple blades all have slightly different rotational rates, meaning that the first and third spectral lines for the multi-bladed drones are wider than those with fewer blades. The wider spectral lines produce histograms that match better to the reference PDFs. The average of the first and the third spectral line bandwidths of the pusher and puller drones was 33 Hz, while it was 87.87 Hz for the quadcopters and 52.75 Hz for the hexacopters.

The decorrelation time of the body is highly affected by the consistency (or lack thereof) of the body's SNR throughout the CPI. Drone trajectories that had a comparatively low SNR or produced large fluctuations in the SNR of the body often had shorter decorrelation times. The decorrelation times of all the spectral lines are categorically short – on the order of a few milliseconds. The individual spectral lines can have very different decorrelation times with the fixed-wing drones usually having the longest decorrelation times. The average first and the third spectral line decorrelation times was ± 211.5 msec for the fixed-wing drones, ± 134.6 msec for the quadcopters and ± 141.3 for the hexacopters. As discussed above, the fixed-wing drones probably use aerodynamic stabilisation techniques meaning their propeller varies its rotational velocities less, whereas multi-blade copters must use individual rotors to achieve stability. In general, these extended correlation times (and narrower bandwidth spectral lines) result in a lower relative amplitude among the single-blade drones compared to the multi-blade drones (i.e. the difference in SNR between the body and the first spectral line and the body and the third spectral line is less among the fixed-wing drones). This trend is further seen in the difference in first and third spectral line bandwidths between the single- and multi-blade drones.

Across all the drones summarised in Table 5, the whole drone bandwidth varies extensively (from just under 2 kHz to over 6 kHz), as does the whole drone decorrelation time (from a few 10 s of msec to well over 600 msec). However, the body bandwidths do not vary nearly as much as the whole drones, yet the body-only decorrelation times still vary similarly to the whole drones. For the spectral lines, the bandwidth of all the spectral lines varies extensively (like the whole drone), but the decorrelation times do not vary much, usually only a few msec. The bandwidths of the first and third spectral lines do not vary much – they are almost all on the order of a few tens of Hz – but their decorrelation times do vary extensively. If any decorrelation time trend is evident across the wider dataset, it is that there is an approximate inverse relationship between bandwidth and decorrelation time up to approximately 200 Hz. From there, the decorrelation times remain on the order of a few msec.

The results of the statistical analysis were further corroborated with a study of detection performance. The experimental probability of detection (PD) performance was examined and compared to performance obtained with simulated data with amplitude fluctuations that followed each of the listed distributions used for fitting the data. A Neyman-Pearson test for the detection of a fluctuating target with non-Gaussian distributions was conducted. For the case of a constant amplitude target (Swearing 0), the probability of detection is determined by solving the Marcum Q function as [32].

$$P_D = Q_M \left(\sqrt{\frac{2A^2}{\sigma_n^2}}, \sqrt{-2 \ln(PFA)} \right) \quad (15)$$

In (15), A is the target amplitude, σ_n^2 is the noise mean power and PFA is the desired probability of false alarm. When the target amplitude fluctuates stochastically, P_D is obtained as an average across the range of target amplitude values as

$$P_D = \frac{1}{N} \sum_{i=1}^N Q_M \left(\sqrt{\frac{2A_i^2}{\sigma_n^2}}, \sqrt{-2 \ln(PFA)} \right) \quad (16)$$

where A_i represents the different simulated or experimental random amplitude values. In the dataset used for this paper, the amplitude data were taken from the same 6600 samples used to generate the histograms. In order to plot a PD curve as a function of SNR, the mean SNR ($\overline{SNR}$) is defined to express Equation (16) as

$$P_D = \frac{1}{N} \sum_{i=1}^N Q_M \left(\sqrt{\frac{2A_i^2 \overline{SNR}}{\frac{1}{N} \sum_{k=1}^N A_k^2}}, \sqrt{-2 \ln(PFA)} \right) \quad (17)$$

Detection performance was analysed for the whole Yuneec H920 drone for a probability of false alarm of 10^{-7} . A plot was generated for $\overline{SNR}$ ranging from 0 to 40 dB. The experimental detection curve is shown in Figure 22, along with the simulated curves associated with the four other distributions used and

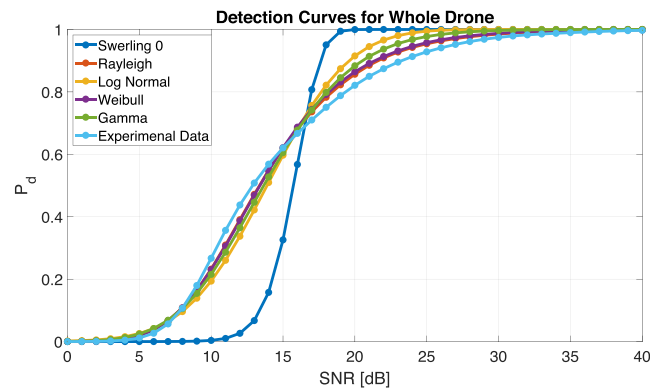


FIGURE 22 P_d curves versus mean signal to noise ratio (SNR) for the experimental data and fitting distributions: whole drone

the performance for a Swerling 0 target. The theoretical curves were created by simulating random data for each distribution using the distribution parameters obtained in the statistical analysis with the method of moments.

Results show that the Rayleigh and Weibull assumptions provide a closer detection curve to the experimental probability of detection than the Swerling 0 assumption. Given that the Rayleigh was the best fit for the whole drone, if a Rayleigh distribution was assumed, at 20 dB, the detection prediction would have been approximately 3.45% less. If a Weibull was assumed (a close second place to the Rayleigh distribution), the detection prediction would have been 4.18% less.

6 | CONCLUSIONS

This research provided comprehensive statistical analysis of individual drone spectral components of nine different drone types. It also estimated decorrelation times of the individual drone spectral components and highlighted effects due to non-stationary properties of the signature. It compared and contrasted the statistics of the same drone spectral components from different drones and, ultimately, it provided useful information for more intelligent selection of CPI durations and useful guidelines for refining detector logic for drones. The results of the statistical analysis were also further corroborated by a study of detection performance for the whole drone.

Subsequent work could perform similar analysis at different radio frequencies or at different polarisations. Additionally, further exploration of high-SNR spectral lines could be useful to better understand the transition from noisy arcsine distributions into the single-peak distributions studied here.

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CONFLICT OF INTEREST

This is to confirm there is no conflict of interest associated with this research and this publication.

DATA AVAILABILITY STATEMENT

Research data cannot be shared with anyone (there are caveats that need to be dealt with on a case by case). We have added a statement in the acknowledgements regarding this.

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