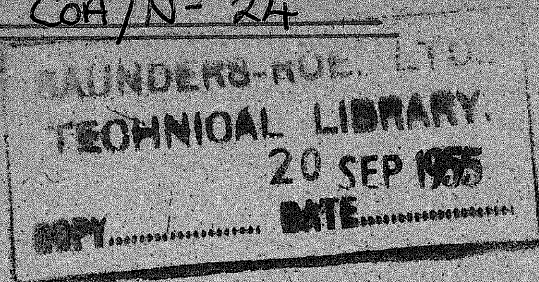
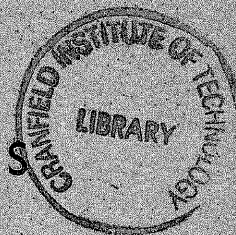


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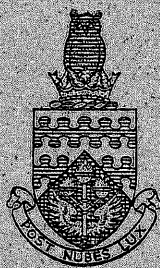
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THE COLLEGE OF AERONAUTICS
CRANFIELD



A PRELIMINARY INVESTIGATION OF THE
FLOW OVER A PARTICULAR WING BODY
COMBINATION AT MACH NUMBER 2

by

J.R.BUSING and G.M.LILLEY



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THE COLLEGE OF AERONAUTICS
CRANFIELD



A Preliminary Investigation of the Flow over a
Particular Wing Body Combination at $M = 2.0$.

-by-

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SUMMARY

Results are presented of visual tests, by surface flow and Schlieren techniques, of the flow over a particular wing body combination at $M = 2.0$, made in the 9in x 9in supersonic wind tunnel at the College of Aeronautics. It should be noted that the photographs in this report represent only a small part of all the data collected.

The effects of providing roughened bands and air jets for transition fixation on the body are discussed. It is shown that although such methods may be satisfactory at zero or low angles of incidence they do not fix transition uniformly over the complete body circumference at higher incidences. For this reason the main part of the tests have been performed with free transition.

The formation of vortex sheets on both wings and body is discussed for a range of incidence and roll angles and some conclusions are drawn as to the shape of the vortex pattern and its induced flow effects.

* Seconded from the Aerodynamics Section of the Navigational Project Division, English Electric, Luton.

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1. Introduction

The present study is part of a programme dealing with wing body interference at supersonic speeds. It was considered essential to understand the nature of the flow in the boundary layer and vortex sheets before attempting to measure the pressure distributions on the wings and body (and each separately). In particular it was found necessary to study the effects of transition producing devices in order to ascertain whether or not they caused marked extraneous effects on the flow pattern over the whole range of incidence and roll. The tests were performed in the new 9in x 9in supersonic wind tunnel and since a detailed report on its design, instrumentation and calibration is in the course of preparation only a brief discussion will be presented here.

The bibliography includes not only the references referred to in this report but also those references studied in connection with the complete investigation.

2. Description of Wind Tunnel

The 9in x 9in supersonic wind tunnel is a continuous return flow type tunnel driven from a ten stage axial flow compressor absorbing 1000 h.p. The pressure ratio across the compressor is approximately 3:1 which enables Mach numbers up to 2.5 to be obtained in the working section. The compressor is driven by two 500 h.p. D.C. motors operating in tandem through a 1:8.16 step up gear box (see figures 1, 2, and 3). The air leaves the compressor section, flows through a specially designed rapid expansion which is fitted with gauze screens to prevent flow breakaway, and enters the indirect contra flow water cooler. The settling chamber is provided with two gauze screens each of 30 mesh. The contraction ratio between the settling chamber and the throat section is about 76/1. The working section is fitted with a single sided curved wooden liner (see Figure 4) fixed to the floor and a flat wooden liner fixed to the roof. Cut outs in the liners are provided for the model support crescent which is operated externally by remotely controlled magslips situated in pressure tight boxes placed above and below the working section. The incidence may be set to within ± 1 minute over the range -5° to 30° from the flat sided liner as datum. The model is rolled by means of a verrier coupling contained in the sting, and roll angles can be set to the same accuracy as incidence. The side walls of the working section consist of full length laminated specially selected plate glass windows of 1.5 inches total thickness. A model catchment unit is placed downstream of the working section.

The tunnel is provided with a vacuum pump of 600 cu.ft./min.

capacity, which enables the stagnation pressure to be varied from 1.5 in. Hg. absolute to 30 in. Hg. absolute. The latter pressure cannot be maintained without overloading the compressor for part of the tunnel Mach number range and when a model is operated at high incidence. The tunnel air is supplied through a cotton wool filter and a silica gel drying bed. At present, the reactivation plant comprises a 10 KW heater and a small centrifugal fan and filter. The gel bed is cooled during operation by passing mains water through a coil immersed in the bed. Because of the large leakage rate at the seals of the compressor it has not been possible to run the tunnel with a moisture content of less than one part in 2000.

The tunnel stagnation pressure is controlled by means of a simple compressed air servo-mechanism which can hold the pressure to within 0.02 in. Hg. The stagnation temperature control is hand operated and is maintained to within 0.5°C in the range 25°C to 40°C .

The tunnel is provided with a 3 bank vertical 48 tube mercury manometer in which the mercury level in the tubes can be raised by applying a pressure to the reservoir. A clamping device enables all levels to be frozen when necessary. Pressure distributions can be measured to an accuracy of ± 0.01 in. Hg.

The optical apparatus consists of a two mirror schlieren system with spark or mercury vapour lamp illumination. The mirrors have a focal length of 10 feet and a diameter of 1 foot. The spark apparatus consists of a 220:1 high voltage transformer the spark being triggered by a conventional current relay device. The primary is connected to a 40V D.C. supply whilst the secondary is connected across a $0.022 \mu\text{F}$ oil filled high voltage condenser. The type of spark gap found to give the longest life and least wander consisted of a forward facing cone with a 0.05 in. diameter hole at the apex and a spherical electrode about $1/8$ in. diameter.

3. Working Section Calibration

The initial calibration was performed in a second working section fitted with continuous steel side walls. Traverses were made at one station 3 feet downstream of the throat with a 17 tube total head comb. The Mach number variation was ± 0.5 per cent on a mean of 1.95 with a maximum variation near the walls, just outside the boundary layer of ± 1 per cent.

The direction of flow was measured with the aid of a 27° total angle cone having four pressure holes symmetrically placed 0.5 in. from the nose. The cone was traversed over the section 3 feet downstream of the throat. The pressure difference between diametrically opposite holes was proportional to the

angles of yaw and pitch of the stream relative to the cone axis, at least for small angles of deviation. It was found that the mean pitch was -0.1° with a maximum deviation of $\pm 0.2^\circ$ and the mean yaw was -0.05° with a maximum deviation of $\pm 0.2^\circ$.

Later tests showed that the pitch and yaw were not constant over a range of stagnation temperatures. Although insufficient time was available to explore this effect thoroughly it was believed to be due to non-uniformity of the temperature distribution after the cooler. It was found, however, that the above figures for pitch and yaw could be maintained at a selected value of the stagnation temperature.

The centre line static pressure distribution in the working section was measured using a 10in. long cylindrical body having a 20° total angle, conical nose. The cylindrical portion was positioned so that it covered the same region which would be occupied by the model. The variation in pressure was found to be ± 1 per cent and the Mach number equalled 2.00 ± 0.01 .

4. Description of the Model

The model (see Figures 5 and 6) is designed to give basic information on wing-body interference using a wing of small aspect ratio mounted on a cylindrical body operating up to high angles of incidence. The cylindrical body is 0.735in. diameter and has a tangent ogive head of length 2.96 calibres and an included nose angle of 40° . The wings are of rectangular plan-form with a gross aspect ratio of 0.664 and a net aspect ratio of 0.419. They have a 6° single wedge section and are symmetrically placed with reference to the body centre line. The leading edge is 7.42 calibres from the nose and has a radius of 0.001in.

The body is made in two halves (see Fig. 6b) to enable pressure tubes from the wing to be connected and led away at the rear of the sting. In order to obtain as detailed a pressure distribution as possible spanwise holes have been drilled in the wing thus allowing a large number of holes to be drilled through the wing surface at selected positions. The base of the model is connected to the main sting through the vernier coupling. The model may also be mounted on a yawed sting at incidences varying by 5° and rotated about an axis parallel to the free stream direction (see Figure 5). Further models have been made to test the body and wing alone as well as the body in the presence of the wing. Of these it was expected that some difficulty would be encountered in mounting the wing alone. For this reason a series of exploratory tests were performed, using a dummy wooden wing, to establish that the model arrangement was satisfactory with regard to strength and model-sting

interference. The wing alone tests, when completed may show some differences in flow since the wing has no trailing edge (see Fig. 18). The complete model was tested at a Reynolds number of 2.5×10^6 per foot.

5. Experimental Details

5.1. Transition Tests

In these tests a sublimation technique was used. The model was sprayed with a mixture of azobenzene and 10 per cent petroleum ether, and the tunnel was run for about 20 minutes at a stagnation temperature of 35°C . The results were always difficult to interpret since sublimation, which is a function of local skin friction, occurred close to the nose of the body and the leading edge of the wings in addition to the true transition region further downstream. A qualitative guide to transition was found from observing the fuzziness which developed along the edges of the turbulent boundary layer as seen in enlarged schlieren photographs. In addition laminar layers could be detected by observation of the wakes behind suitably positioned disturbances.

Artificial transition was provided by brass bands 0.1in wide by 0.05in. thick and No. 80 carborundum powder bands 0.1in. wide fixed in turn to the cylindrical body just behind the shoulder of the ogival head and at 1.5 calibres from the body nose. Further tests were made in which air was blown out of a row of holes and a single slot respectively on the body about two calibres downstream of the shoulder.

5.2. Surface flow visualisation tests*

The characteristics of the flow near the surface of the wings and body were obtained by painting the model with a thin coat of titanium dioxide mixed with 60 per cent raw linseed oil and 40 per cent copal varnish. Sufficient titanium dioxide was used to give the mixture a consistency of thick white paint. It was found that after about 30 minutes running at a stagnation temperature of 35°C and a stagnation pressure of 20in. Hg. absolute the paint had been swept into a well defined pattern of flow lines, and had hardened sufficiently so there was no change when the tunnel was shut down. The model was then removed from the tunnel and photographed.

* Details of the technique used at N.A.E. were kindly supplied by Mr. D. Morris. More recently it has been found that improved results are obtained if a small quantity of oleic acid is added to a mixture of the titanium dioxide with gear oil.

5.3. Optical Tests

A set of schlieren photographs using the microsecond spark apparatus were completed over the range 0° to 30° incidence and 0° to 90° roll. The photographs were taken with the second slit vertical and horizontal and with cut off from both directions. These combined pictures enable such defects as overloading of the schlieren system to be detected. The tests were nearly completed before graded filters of the type used at N.P.L. were obtained. Consequently the tests were finished using standard slit and knife edge arrangements. A few shadowgraph pictures were also taken but they yielded little additional information.

Several tests were made with the yawed sting to help establish the spacing of the vortices shed from the body. The vapour screen technique was also tried for the same purpose. From the schlieren photographs taken with the yawed sting at various angles of rotation it was possible to obtain a detailed picture of the cross flow vortex and shock wave configurations. The vapour screen technique proved to be unsuccessful with this size of tunnel as it was found impossible to reach a high enough humidity in the working section using atmospheric air.

6. Experimental Results

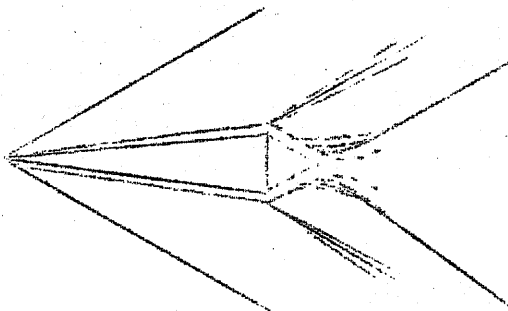
In the following presentation of the main experimental results frequent reference will be made to regions of flow separation on the body and wing. Associated with these regions of separation are vortices or rolled up vortex sheets. These are shown diagrammatically in Figure 19.

The features of the flow around the body and wings which were investigated are as follows

- (i) Movement of the transition front on both body and wings with incidence.
- (ii) Transition fixation by artificial stimulation of the boundary layer.
- (iii) Position of laminar separation on both body and wings, its variation with incidence and the formation of the separation vortex sheets and vortices.
- (iv) The variation with incidence of the angle of the bow shock wave with the direction of the mainstream and the position of the shock waves associated with the separated flow regions.

6.1. Results of transition tests

At zero incidence the azobenzene tests showed that the boundary layer flow was laminar on the body up to the position where the wing leading edge shock waves crossed the body. On the wing the tests showed laminar flow back to the wing trailing edge, although the flow in the root was always turbulent. Since the design of the wing ensured that no shock wave occurred immediately at the trailing edge (see sketch below) it was naturally expected that laminar flow without separation would occur up to the trailing edge. The expansion around the trailing edge caused vortex sheets, the extension of the wing boundary layers to spring from each shoulder and converge towards each other downstream of the trailing edge. The trailing edge shock wave was formed at this location and the wing wake flowed through its centre.



Even when the wing was placed at moderate incidence the vortex sheets, formed behind the trailing edge, gave the wing section the character of a pointed trailing edge. Thus an expansion of the flow around the shoulder was still obtained even when the upper surface was subjected to an expansion around the leading edge.²⁴

The schlieren pictures at zero incidence showed a gradual boundary layer growth from the nose of the body up to the position of the wings. It was found that the boundary layer thickened on crossing the wing shock waves (see Figure 7c) and developed a fuzziness characteristic of a turbulent boundary layer. The wing boundary layers could not be observed in this way since they were obscured by the body.

At incidence transition was maintained far back on the undersurface of the body but large forward transition movements occurred on the upper surface. This forward shift of transition was noticed at quite low incidences. The transition front over the complete body was continuous between the transition points on the upper and lower surfaces of the body. Between incidences of 5° and 10° a separation of the boundary layer in the cross flow plane developed. The separation front commenced on the upper surface, in the plane of symmetry some distance downstream

of the body nose and spread downstream towards the lower surface with increase in distance from the body nose. In cases where separation was present it was not established whether or not transition occurred before separation of the cross flow. On the wing undersurface transition still occurred at the trailing edge, apart from the turbulent flow in the root. On the wing upper surface transition moved quickly forward but again the effect was masked by separation and reattachment of the flow which will be described below. It was found that up to about 15° incidence an expansion followed by a shock wave still occurred around the trailing edge. Above ~~this~~ incidence the flow separated ahead of the trailing edge shock wave and at 30° incidence separation commenced at the leading edge although probably not over the complete span.

It was considered essential to find out if the position of separation of the flow on the body or wings was affected by whether the boundary layer was laminar or turbulent. The tests using brass bands placed 0.4 calibres downstream of the ogival head body junction were disappointing since a fully developed turbulent boundary layer could not be obtained at zero incidence. Bands placed half way along the ogival head were equally unsuccessful in promoting transition. The tests, using a carborundum band, showed more promise since at zero incidence turbulence set in about 0.5 in. downstream of the band. However at high angles of incidence the boundary layer flow on the body undersurface, just downstream of the band, appeared to break up into a number of discrete vortices (see Figure 7a). Little change in the position of the separation front around the body could be observed when it formed either upstream or downstream of the band.

A series of tests were next made using air jets from an annular slot in the body to promote transition¹³ (see Figures 7b and 7c). With this method transition was obtained just downstream of the slot at zero incidence by adjusting the blowing rate. At incidence the separation front commenced ahead of the blowing slot and was therefore upstream of the front for zero blow. The air jets appeared to disturb the separated flow regions at higher angles of incidence (see Figure 8) and blanketed the upper surface from the induced flow of the body vortices in the separated flow region. In the case of the wing a similar method of promoting transition was used by opening the first row of pressure holes nearest the leading edge and blowing air out under pressure. As it was not possible to observe the condition of the boundary layer on the schlieren apparatus, pressure readings were taken for the remaining holes with and without air blowing. No change in the pressures was observed over the whole range of incidence for the two conditions of air on and air off.

The tentative conclusion was reached that although transition was partly promoted by these various methods none appeared to fix transition and/or separation of the boundary

layer flow over the body, at least at high angles of incidence, in a definite manner. The method using air jets, although attractive in many ways, was finally discarded because of the unknown interference effects on the body vortices at high incidence. However, this statement should be treated with some caution since it may not apply when transition is fixed by means of air jets at other body positions, say nearer the nose. In the tests described below transition was left free on both the body and wing since it was felt that the ratio of boundary layer thickness to model size would be more nearly like that at very high Reynolds numbers where the boundary layer would be turbulent from the nose or leading edge. It appeared from the investigation above that the biggest doubt in leaving transition free on the wings was whether, at the higher angles of incidence, a separation of the flow ahead of the trailing edge might be caused by a laminar boundary layer - shock wave interaction. Although it is intended to investigate this further, the present evidence indicates that the commencement of the separated flow region on the wing chord of this wing of small aspect ratio is not greatly affected by the character of the boundary layer ahead of this region.

6.2. Results of the surface flow visualisation

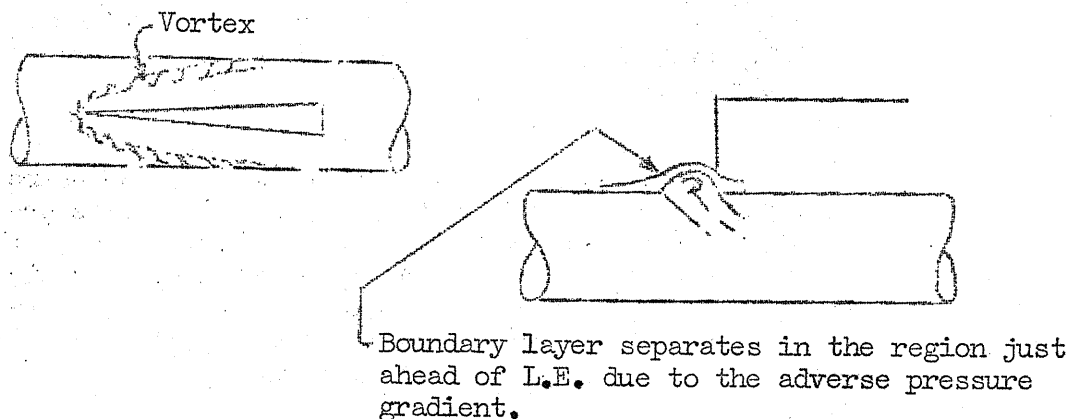
(i) Flow over body and wings at zero incidence

The titanium dioxide pictures taken at 0° incidence (see Figures 9a, 9b and 9c) show how sensitive the boundary layer flow close to the body surface is to small changes in flow angle. Although the pitch and yaw in the tunnel are less than 0.1° , local flow directions on the model surface reach 5° to the body centre line. The wing appears to stabilise the body flow and at the 0.75c position the flow is approximately identical on both upper and lower surfaces. The inward flow from the roots of the wing over the body is probably due to the wing pressure being higher than the body pressures. The flow shows a tendency to separate from the body in the region of the wing root junction. The area of disturbed flow commences slightly ahead of the wing leading edge and increases progressively in the downstream direction. The flow over the body turns back towards the wing wake in the region of the expansion behind the wing trailing edge.

(ii) Flow over body at small and high incidences

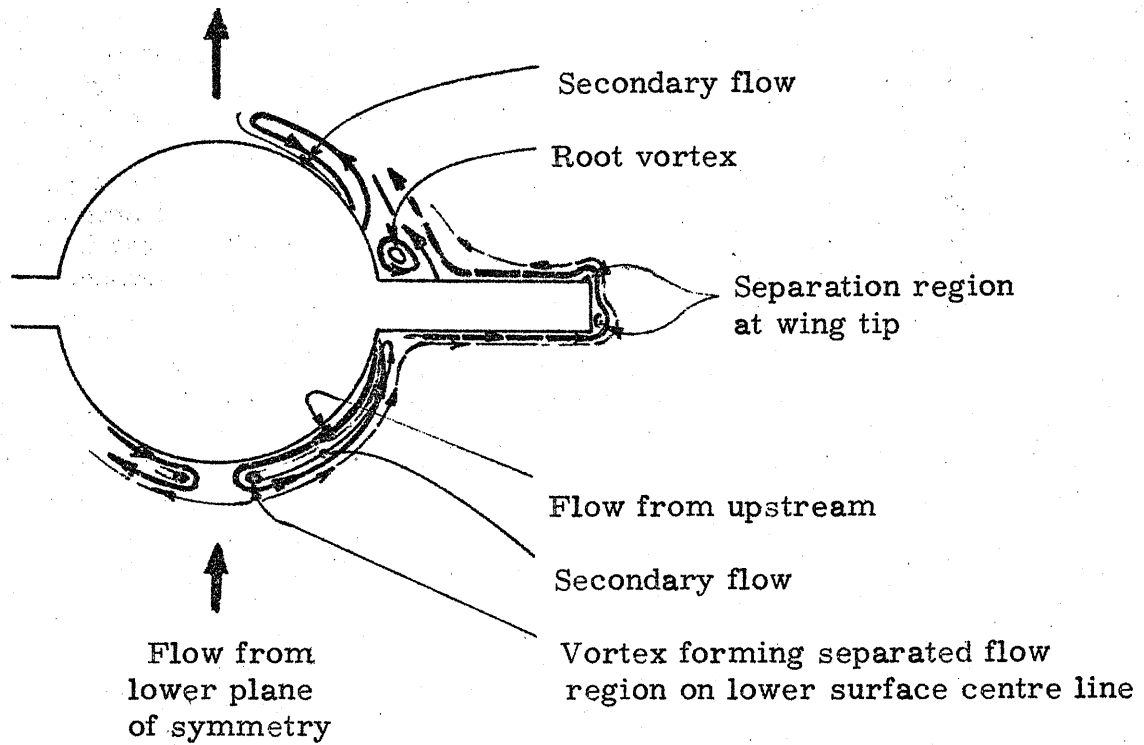
At a small incidence a cross flow around the body develops (see Figure 9b) and local flow angles of 20° in the boundary layer are found at incidences of 5° (see Figure 9c). The upper surface boundary layer on the body separates ahead of the wings at some incidence between 5° and 10° although strong secondary flows in the upper surface boundary layer appear at lower incidences. A short distance downstream of separation a pair of separated vortex sheets are formed above the body in which any

cross section through them appears similar to a symmetric vortex pair (see paragraph (iv) below.) The flow from the main stream is induced to flow between these vortex sheets (see Figure 9a) and reforms the boundary layer on the upper surface of the body. At the wing position the cross flow from the lower to the upper surface of the body is prevented. It can be seen from Figure 9b that the flow on the body undersurface separates in crossing the wing leading edge shock waves as they curve around the body. This separation, which also exists at zero incidence commences on the body slightly upstream of the wing leading edge as a result of the influence of the upstream adverse pressure gradient caused by the wing itself and its bow wave. The separation region has a cross section in the form of a closed bubble or vortex. (Smoke pictures at subsonic speeds, see Figure 16, show the presence of this small vortex on the body slightly ahead of the wing leading edge.)[#] The vortices proceed downstream after curving over the centre line on the body undersurface. On the upper surface of the body the vortices combine with the secondary vortices in the separated flow region above the body.

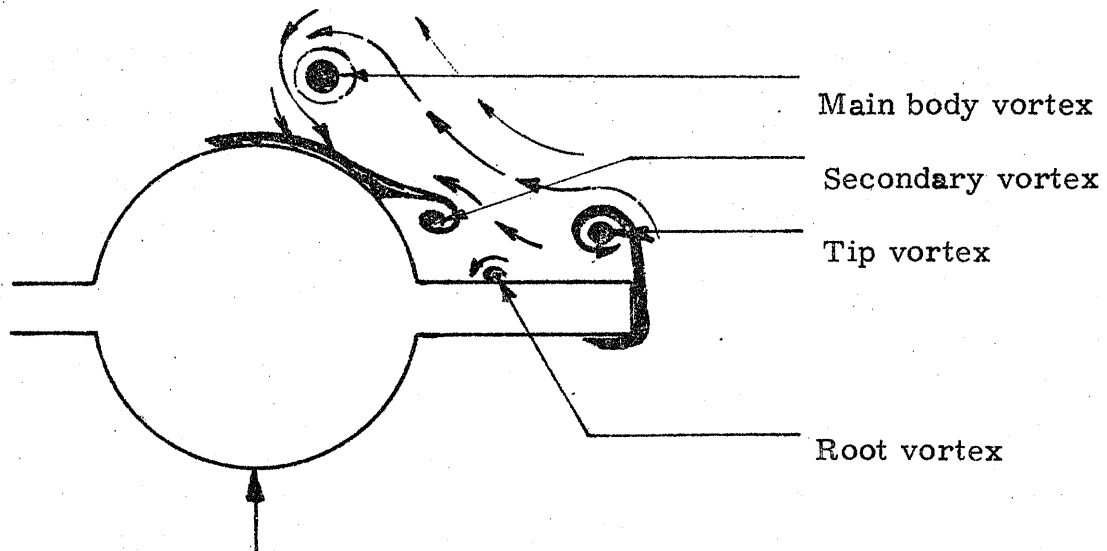


The body undersurface flow reattaches downstream of the wing leading edge shock wave but, owing to the presence of the wing, the surface flow is turned back towards the body centre line in the lower plane of symmetry. Since the flow outside the body boundary layer must still be upwards, toward the wings, it is seen that a strong secondary flow develops.

* These vortices have been independently observed by Stanbrook and Sutton (N.A.E.) who made known their results to the authors.



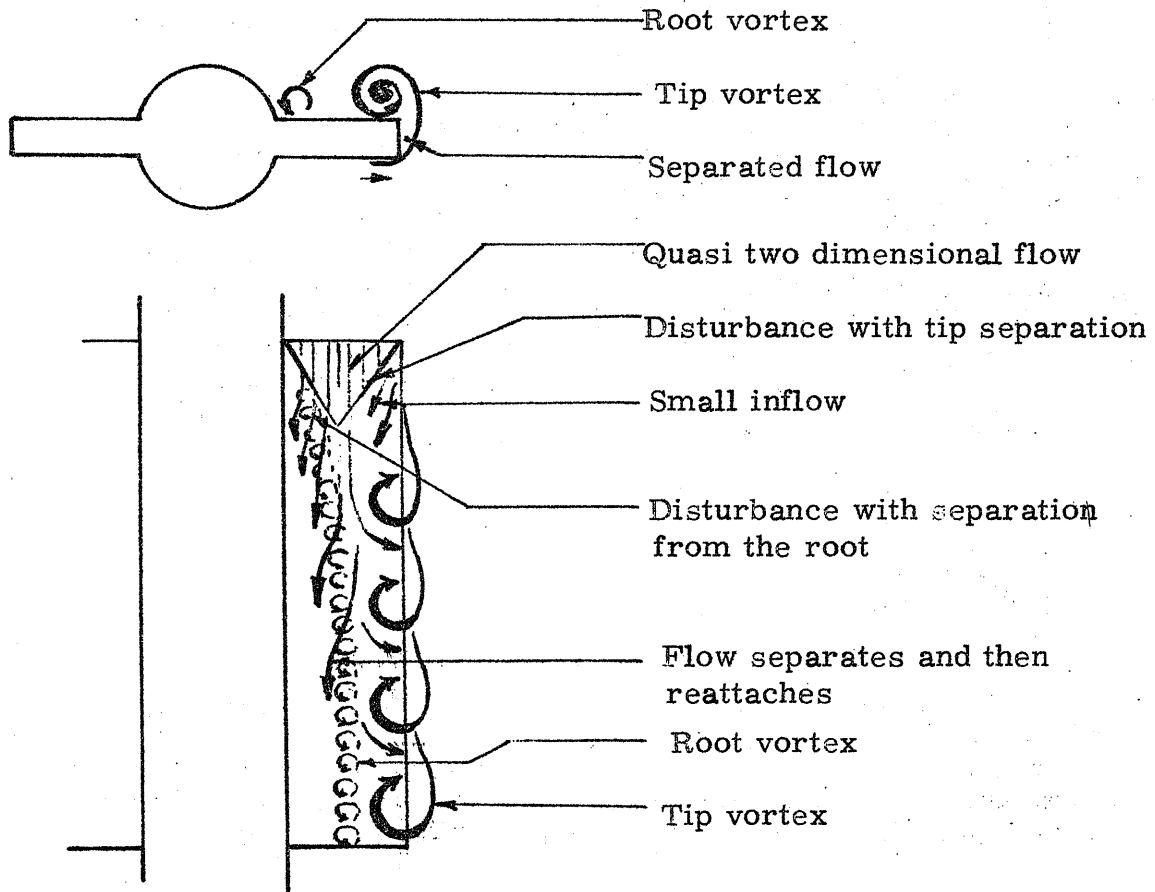
On the upper surface of the body downstream of the wing leading edge, the flow from the undersurface is prevented, by the wing, from feeding into the side vortex sheets of the main body vortices. Thus the anchor of the main body vortices is virtually cut, and their strength no longer increases in an axial direction. The induced flow between the vortices still takes place as is evident from a scrutiny of Figure 9a.



(iii) Flow over the wing at small and high incidences

At incidence the flow over the wing lower surface develops a drift towards the wing tips in and downstream of the quasi-two dimensional flow region near the wing leading edge. This outboard drift of the flow from the lower surface commences at a very small incidence, and as in subsonic flow feeds into the tip vortex formed above the upper wing surface. The flow in passing around the thick tip sections separates from the lower half of the tip but reattaches on the upper half (see Figure 9c). It is noted that on the wing lower surfaces there are no visible effects of disturbances from tips or root which is in complete contrast to the markedly disturbed flow patterns on the upper surface (see Figures 9a and 9b).

On the wing upper surface (see Figure 9a) at a small angle of incidence the flow near the tips drifts inboard and meets the vortex from the separated flow region on the body ahead of the wing leading edge. This vortex passes downstream to the wing wake sandwiched between the root and tip flows. When the incidence has increased to 10° the outward flow from the under-surface separates completely from the tip section in moving towards the upper surface except close to the leading edge. The tip vortex soon develops downstream of the leading edge and induces an outward flow on the upper surface (see Figure 9a).



At 15° the tip vortex commences at the leading edge and the root vortex is swept out towards the tip by the induced outward flow, leaving the tip at about $0.75c$. It is not immediately obvious from these surface streak pictures as to the position above the surface of the wing, of these vortices associated with the regions of separated flow. A close scrutiny of the schlieren photographs suggests that the tip vortex, at these incidences, extends well above the wing whereas the root vortex lies close to the surface (see Figures 8, 10 and 18).

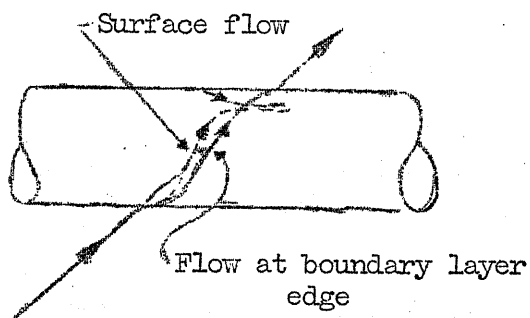
As the incidence is further increased the extent of the separation behind the root vortex increases, the induced outflow by the tip vortex is increased and the root vortex leaves the tip section closer to the leading edge.

(iv) Detailed qualitative description of the body boundary layer flow

As discussed briefly above the vortex flow over the upper surface of the body makes its appearance between 5° and 10° incidence. The surface flow pictures show very clearly the commencement and development of these vortex sheets. It should be noted that in Figures 9a, b and c, the regions of dense white indicate either a very thick boundary layer or a separated flow region. Thin boundary layers are shown either as dense black regions or regions having a series of black streaks. The detailed boundary layer flow picture can be described as follows.

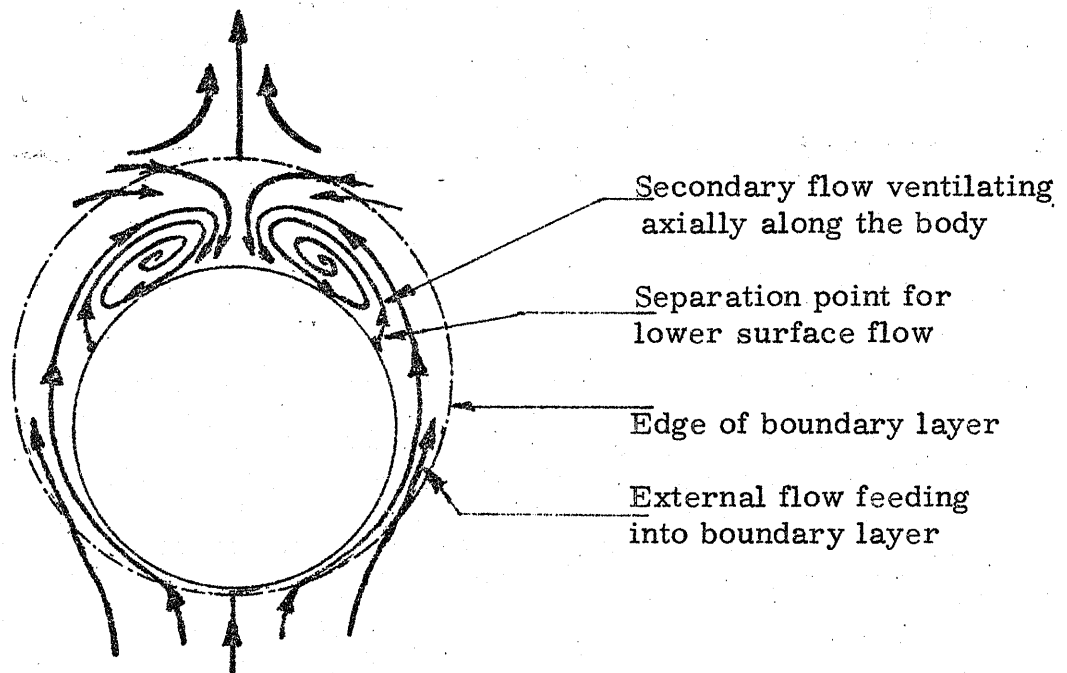
At a small body incidence, say less than 5° , the flow from the lower plane of symmetry sweeps upwards over the body (note the fir-tree like flow pattern near the body centre line in Figure 9c). Since the pressure near the sides of the body is lower than that on the lower surface, the flow near the surface in the boundary layer spirals around the body surface at a greater angle of incidence than the flow near the outer edge of the boundary layer. This explains why the observed surface

flow angles are greater than would have been expected from potential flow theory.



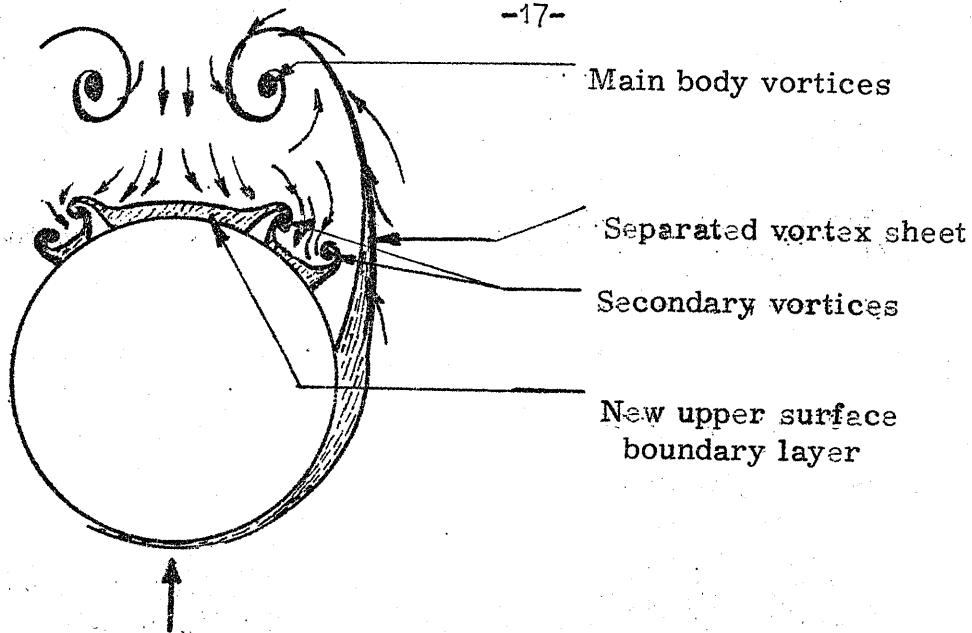
As the flow progresses from the sides to the upper surface of the body the pressure rises, the boundary layer thickens, and the surface flow is turned into the direction of the body axis. This process is accelerated by the upper surface boundary layer flowing down the pressure gradient and thus

forming a strong secondary flow on the upper part of the body, as illustrated in the sketch below.

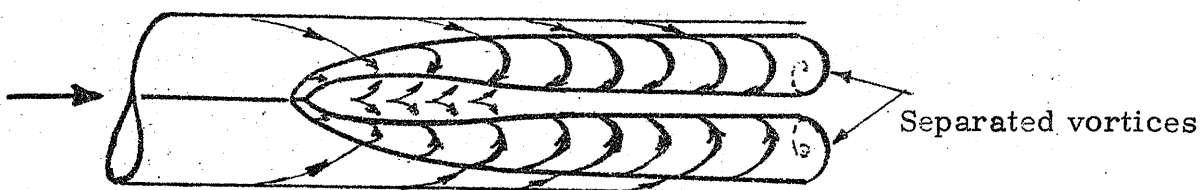


As the incidence increases above 5° , say, the separation of the lower surface flow becomes more marked and the strength of the secondary flow vortex, which is being fed from the lower surface flow, becomes so great that the vortex separates from the body surface. The separation commences at the upper surface

-17-



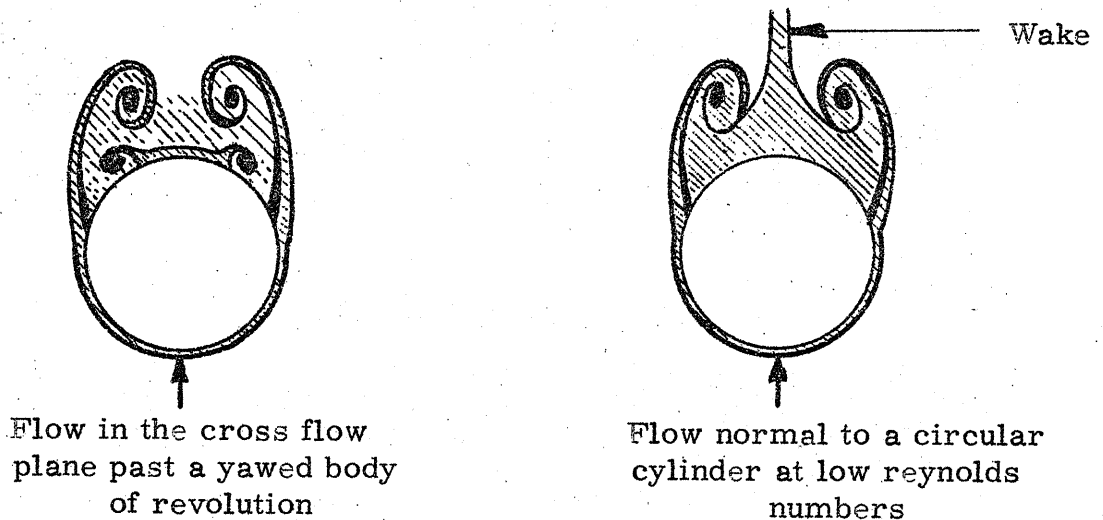
centre line and the vortex formed behind the separation front forms the core of the rolled up separated vortex. In the present tests the separation front on the upper surface occurred roughly at the body nose when the nose tangent was in the stream direction i.e. at an incidence of 20° . Throughout its length the separated vortex sheet is anchored to the side boundary layers of the body, which help to feed the vortex from the lower surface flow.



Downstream of the separation front on the upper surface the external flow from the lower plane of symmetry spirals in between the separated vortices and reforms the boundary layer on the upper surface (see Figure 9a incidences 10° and above). This flow separates at a short distance away from the upper surface centre line and the vortex sheets defining the separated flow

/region ...

region roll up and form secondary vortices (see sketch above). This flow configuration is very similar to the vortices formed on a delta wing of large sweepback angle as reported by Ornberg.⁴⁸ It should be noted that, as in the case of the delta wing, the region of non-zero vorticity is confined to the boundary layers and the separated vortices. No vorticity is shed into the external flow above the vortices near the upper plane of symmetry. It is thus seen that the viscous flow in the cross flow plane past a yawed body of revolution is different from the corresponding two-dimensional steady flow past a circular cylinder.



Shaded area denotes region of non zero vorticity

Since, in the steady flow past a two-dimensional cylinder, it is the instability of the wake which causes the symmetric vortex pair to become unstable at quite low Reynolds numbers it appears reasonable to expect that the symmetric vortex pair, above a yawed body of revolution, will be stable up to much higher values of Reynolds number (based on local diameter and the cross flow velocity) due to the absence of the wake in the cross flow plane. In the present tests the symmetric vortex pair was observed throughout the complete range of incidence for which the maximum cross flow Reynolds number was about 0.7×10^5 . It has been assumed by Allen⁴ that the vortices develop on the leeward side of a yawed body of revolution in a similar way to those produced in the impulsive motion of a two-dimensional cylinder normal to itself from rest. In the analogy, distances from the body nose represent different times in the development of the two-dimensional flow. From what has been said above it is clear that this analogy is not exact. However, it is to be expected that the pressure distribution around a yawed cylinder

will be qualitatively similar to that in the impulsive motion of a two-dimensional cylinder from rest provided that the geometry of the separated vortices is similar in the two cases.

Naturally the type of flow separation on a yawed body of revolution must depend on whether the lower surface boundary layer is laminar or turbulent. The flow must therefore be a function of both the Reynolds numbers of the cross flow and that based on the product of distance from the nose and the axial component of the mainstream velocity. The results of the present tests show, however, that the flow over the lower surface of the body is so stable that except at very high Reynolds numbers a laminar type flow separation will occur. At incidences above about 20° it has been observed by Allen³ that the symmetric vortex pair above the body is no longer stable. The pattern becomes asymmetric and one or more additional vortices are formed. Further examples of this type of flow are shown in section 6.3. below. It is outside the range of the present equipment to show how dependent these vortex flows are on Reynolds number.

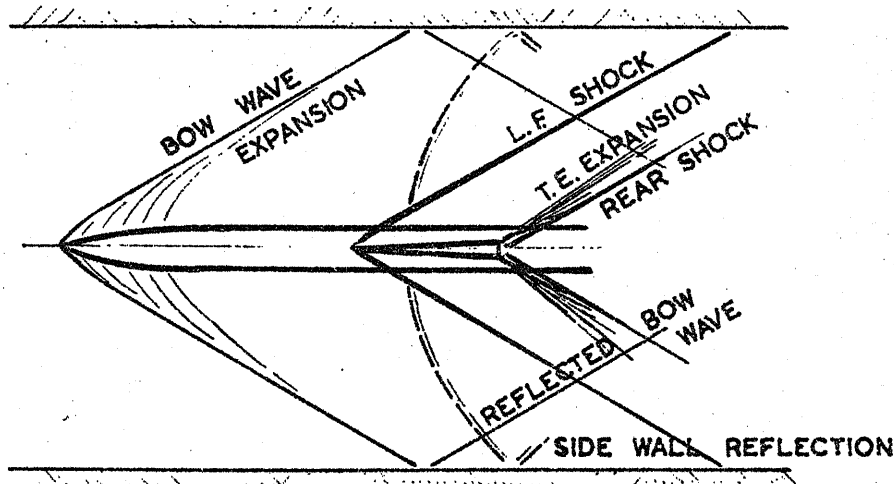
6.3. A Study of the Schlieren Pictures

From the tests on the surface flow visualisation a picture has been obtained of the boundary layer flow, its separation, and its interaction with shock waves. In the present section some further light is thrown on the character of the boundary layer, mainly on the body, but more important is the attempt made to explain the formation of the shock waves, expansion waves and vortex sheets springing from the body and wings. The mode of interpretation of shock waves and expansion regions in schlieren photographs is well known but that relating to vortices and vortex sheets has as yet received little attention. For this reason a short discussion is included of the grading of light intensity in schlieren pictures of these phenomena.

In schlieren photographs the light intensity is proportioned to the gradient of the density. Thus the side view of a vortex, in which the pressure, density and temperature fall towards the centre, will appear as a bright band adjacent to a dark band (see free vortices in Figure 12). A vortex sheet on the other hand will appear as a bright or dark narrow band*, depending on the setting of the second knife edge, since the temperature and density will vary across the layer although the pressure will be approximately constant.

* It should be noted that the very bright and dark bands adjacent to the tunnel roof and floor do show to some extent the boundary layers on these surfaces. However, part of this light contrast is due to turned down edges on the working section windows and reflections from the surface of the liners.

At zero incidence the expansion over the ogival nose interacts on the bow shock wave. The reflections of this shock wave from the roof and floor boundaries are seen together with the reflections from the side walls (see Figure 10).



On the wing the expansion around the trailing edge is visible together with the rear shock from the wake. At zero incidence the boundary layers on the top and bottom centre lines of the body increase in thickness rapidly behind the wing leading edge shocks. The titanium dioxide pictures indicate that this is associated with the outflow from the wing-body root junction.

At 5° incidence the upper and lower parts of the body bow wave are respectively reduced and increased in strength. On the wing the expansion around the leading edge follows a shock wave. The presence of this shock wave is partly due to the finite nose thickness and partly due to the formation of vortices at the root and tip of the wings. The body lower surface boundary layer is scarcely detectable ahead of the wing leading edge shock wave but thickens rapidly beyond it (see Figure 8). With increase in incidence up to 15° very little change in thickness of the whole body lower surface boundary layer occurs. At 5° incidence on the top surface of the body the boundary layer thickens slowly up to the wing leading edge shock wave. It probably increases beyond the leading edge shock, but the schlieren pictures do not show this clearly since it is masked by the wing flow.

At 10° incidence the upper surface boundary layer thickens downstream of the point indicated by the titanium

dioxide pictures as the start of the formation of the body vortices. It is supposed that the body vortices are not observed in this position since they are relatively small in dimensions and are hidden by the body. A similar effect occurs at higher incidences. The true boundary layer on the upper surface of the body, formed by the induced flow between the body vortices, is not clearly visible since it is masked by the body vortices. However, photographs taken at 90° roll and all incidences (see figure 10) show that this boundary layer is very thin. Between 10° and 15° the reformed body upper surface boundary layer separates as indicated by a shock wave at approximately 60 per cent of the wing chord behind the leading edge (see Figure 8) and agrees qualitatively with the separation indicated by the titanium dioxide pictures. At small incidences this shock wave is very weak and behind it reattachment of the boundary layer occurs. With increase in incidence the shock wave becomes stronger.

If we assume a distribution of density in the body vortices as discussed above and then superpose the fall in density consequent on the acceleration of the induced flow we find that the density gradient in the upper half of the vortex is increased but that in the lower half is reduced and may be zero or even negative. Thus the schlieren pictures show a region of increasing blackness reaching a maximum close to the upper part of the vortex. This is followed by a region of reduced blackness or neutral shade, indicating the lower half of the vortex or vortices. The bright line near the surface, see Figure 10, must, with this interpretation, indicate a shock wave formed in the induced flow above the body, and can only occur when the induced flow is supersonic. The presence of this shock has also been observed in many of the vapour screen photographs taken in America.*

When the model is rolled double shocks appear at the wing leading edge. These are part of the shock waves from each wing tip and each wing root (see Figure 10). The double shock waves a short distance downstream of the bow wave are presumably caused by the separation flow region from each wing root. At 90° roll shock waves can be seen arising from the body just ahead of the leading edge root. These are probably caused by the local boundary layer separation just ahead of the leading edge. The boundary layer development on the wing tip is masked by the body vortices except at very low incidences. It appears that a vortex formation develops at very low incidence which is probably similar in form to that found at low speeds for the flow at incidence over a delta wing of small nose angle.⁴⁸

* The presence of this shock wave has been confirmed by Treadgold (R.A.E.) from pressure measurements.

The development of the vortex sheets behind the wing trailing edge is clearly visible together with the resulting fans of expansion waves around the tip trailing edges. It appears that the whole trailing edge is effectively pointed by these vortex sheets.

7. Bodies of Revolution at Incidence

(i) Preliminary Discussion

Following on the treatment by Munk, R.T. Jones^{25,26} and others, of the flow over slender bodies and wings, Allen² has shown that the lift, drag and pitching moment on slender pointed bodies of revolution can be derived from the forces experienced by the body in an inviscid flow plus the viscous forces (i.e. drag) arising out of the eddying wake as seen in the cross flow (considered in a plane normal to the body axis). The upstream flow in the cross flow plane is taken as $V_N = V_0 \sin \alpha$ where V_0 is the free stream velocity ahead of the bow shock wave and α the body incidence. The presence of the shock wave is not allowed for. In this connection it should be noted that except when the body is set at a large incidence or when the mainstream Mach number is large the component V_N will correspond to a subsonic Mach number.

In applying the theory the cross flow drag is taken to be equal to that present on a two dimensional cylinder having the same free stream Mach number M_N (corresponding to V_N). These values of drag are taken at the same M_N and Reynolds number of the cross flow even when M_N exceeds the critical Mach number or unity. In many cases qualitative agreement is obtained with experimental results, but some differences have been noted, especially at the higher incidences, and these differences are such as to emphasise the limitations of Allen's method and the need to consider the nature of the approximations involved for the cases with which we are here concerned. The method is in the first instance strictly applicable to incompressible flows far downstream of the nose of a cylindrical body for small values of the body incidence. In such cases R.T. Jones has shown that the flow in the laminar boundary layer in the cross flow is independent of the flow parallel to the body axis. However the range of applicability is capable of some extension insofar as it is found that under some conditions of Mach number, incidence and distance from the nose the pressure distribution in the cross flow is like that for a cylinder in two dimensional flow.

(ii) Flow in the cross flow plane

However, consider the cross flow that occurs in

practice for supersonic flow over a body of revolution at incidence in a plane not far from the junction of the nose with the cylindrical after body. On a meridian line at azimuth 0° , i.e. in the lower plane of symmetry, we see that although the velocity is $V_N = V_0 \sin \alpha$ ahead of the shock the speed in the cross flow plane will be reduced by the shock and may be sufficiently so to be everywhere very small or even zero approaching the cylinder. With increase of azimuth angle up to 90° this effect will steadily decrease but may nevertheless be appreciable over a large range of azimuth angle. At angles greater than 90° the flow separates and the start, at least, of the vortex motion will be reproduced in the cross flow plane. Similarly with increase of azimuth angle from 90° to 180° the speed in the cross flow plane will steadily decrease and will only return to $V_N = V_0 \sin \alpha$ on the far side of the shock wave or expansion from the body nose. Thus we see that the longitudinal body flow near the nose of the body essentially modifies the cross flow pattern from that obtained in two dimensional flow. This does not imply that the methods using cross flow entirely lose their validity but rather that they should be confined only to the flow far downstream of the nose. In such cases the bow wave from the nose will be relatively far distant from the body in the cross flow plane and will be sufficiently weak for the flow to be consistent with the assumption that $V_N = V_0 \sin \alpha$ is the uniform cross flow over a sufficiently wide region downstream of the shock.

Further it must be emphasised that the formation of body vortices and wing tip vortices, on wings of low aspect ratio, are essentially : three dimensional problems. The vorticity present in the rolled up vortex sheet at a particular station springs originally from the separated boundary layer on the body some distance upstream of this station. The vortices are thus helical in nature as can be seen from the photographs in the smoke tunnel (see Figure 16). This aspect of the flow is of course ignored in the simplified approach of the cross flow method.

In the real flow we have in fact, rolled up vortex sheets (regions of diffused vorticity) formed by a cross flow separation, whose axes lie midway approximately between the main stream direction and the body axis. These vortices induce a downwash between them. Since the induced flow will have a direction somewhat inclined towards the body a shock wave must exist, if the induced flow velocity in the constriction between the vortex pair has a supersonic cross flow component, to turn the flow back parallel to the body surface. The trace of this wave must be present in the cross flow plane. It is therefore apparent that the real cross flow is somewhat different from the assumed cross flow representation by Allen and others. It appears that this is one of the factors which helps to explain why the lift on bodies of revolution cannot always be calculated

in terms of the simple cross flow viscous contribution. It is worth noting, in this connection, that many of the flow details, peculiar to yawed bodies of revolution, can be visualised in a plane parallel to the mainstream flow and at right angles to the plane of incidence.

(iii) Tests using the yawed sting (supersonic speeds)

By using the yawed sting model support²³ it has been possible to plot the positions of the vortices and associated shock waves in the cross flow plane. This has been done for a parallel circular body fitted with a tangent ogive nose of 40° total nose angle the length from nose to sting support being 12.5 diameters (see Figure 11). Within this length and at 30° incidence a symmetrical pair of vortices has been observed and the cross flow pattern is shown in Figure 13. It should be pointed out that the positions of the vortices and shocks are not well defined and mean positions have been sketched in. This is probably due to the fact that for each roll the model occupies a different position in the tunnel and hence a slight unsteadiness in the wake would cause the shock wave pattern to change. It is difficult to decide where the vortex core lies but since the free discharged vortices which are shed from the 12° cone (see Figure 12) show black and white regions as for an ordinary wake it is assumed that the cores of the vortex pair lie on the demarcation line between black and white nearest the body. The shock wave which runs parallel to the body in between the separated sheets (see Figure 13a) is of small width in the cross flow plane and dies out as the vortices get further away from the body surface. The shock waves which are formed external to the separated vortex sheets (see Figures 11 and 13a) are, when plotted in a cross flow plane, not unlike the shock wave pattern associated with the two dimensional flow past a circular cylinder at a high subsonic Mach number and a very low Reynolds number (see para. (v) below). In the supersonic flow past a yawed body of revolution these shock waves are the envelope of compression waves generated from the separated vortex sheets and assist in turning back the flow over them into the mainstream direction. The shock pattern around the body and vortices observed in these tests is similar to the pattern observed on the vapour screen by many American investigators.

A 12° total nose angle circular cone was also mounted on the yawed sting. It was found that at 30° incidence it was possible to obtain the steady asymmetric pattern with vortices discharged in the free stream direction (see Figure 12). The corresponding cross flow pattern for this condition has been plotted in Figure 13b. In this case it was noted that no shock waves running approximately parallel to the body axis either inside or outside the separated vortex sheets were visible. It can therefore be tentatively assumed that these

shock waves at these Mach numbers and incidences are associated with the symmetric vortex pair only.

The angles between the shock wave from the nose of the cone and the free stream direction were measured at several angles of rotation for 30° incidence. They are compared below with the values given by the M.I.T. cone tables for cones in inviscid flow at large angles of yaw.

Angle of Rotation	Shock wave angle	
	M.I.T. Tables	Measured
0	31.7°	43.5°
30	31.5	41.3
60	30.7	36.5
90	30.5	32.6

The cone tables for a 6° semi nose angle cone give a conical shock wave of circular cross section and 30.5° semi nose angle yawed at 1.2° . The measured results on the other hand indicate that the lower part of the shock wave is conical and almost circular in cross section (within 0.5°) with a semi nose angle of 32.5° and yawed at 10.9° .* It is apparent that at these very large angles of yaw the effect of the vortex separation in modifying the shock pattern is very large. A discussion on the inviscid flow past yawed cones in supersonic flow is given in Appendix II.

(iv) Low speed tests

Figure 15 shows the formation of symmetrical body vortices on a cylindrical body in a small smoke tunnel at low Reynolds number. At higher incidences it was possible to obtain the steady asymmetrical pattern with free vortices. At even higher incidences an unsteady pattern developed. In the latter case a symmetric vortex pair still developed at the nose. These changed into the steady asymmetric pattern further along the body and this in turn developed into the unsteady state at greater distances from the nose. For further details see Appendix I.

In order to get some idea of the surface flow patterns, the tests were repeated, using lamp black and kerosene, on a

* It was not possible to observe the upper part of the bow wave since this was too weak to be seen with the existing schlieren system.

large body, of 4 in. diameter fitted with an ogival nose, mounted in an open jet wind tunnel with a wind speed of about 100 f.p.s. Three typical sets of surface flow patterns are shown in Figure 14. At 15° incidence the pattern is typical of a symmetrical pair of vortices and closely resembles the patterns obtained in the supersonic tunnel. Tests on a smaller but similar body, $3/4$ in. in diameter showed that at this incidence and the same tunnel conditions the symmetric pair of vortices existed up to 40 body diameters from the nose. At about 25° incidences asymmetric vortex patterns began to develop. The upper surface photograph still showed, however, the 'fir tree' pattern associated with the reattached boundary layer on the body upper surface. It was also possible at this incidence to obtain the symmetrical vortex pattern; the different patterns at this one incidence being undoubtedly due to minor surface irregularities and to a lesser extent on the instability of the symmetric vortex pattern. Above 40° incidence it was clear that the symmetrical vortex pair became detached at the shoulder of the ogive and asymmetric trailing vortices developed from the body surface. These are shown in Figure 14 for an incidence of 60° .

(v) Tests using the hydraulic analogy

In order to get a qualitative picture of the shock wave formation due to the normal flow past a circular cylinder tests were made in a hydraulic analogy tank over a range of equivalent Mach numbers. The values of Reynolds number varied from 3000 to 5000. These are shown in Figure 17 and it is interesting to note the marked contraction in the wake with the formation of the rear shock waves (hydraulic jumps). It should be noted that in the photographs the light is coming from the right and hence the actual hydraulic 'jumps' occur on the right of the dark lines which are the shadows cast by the 'jumps'.

Although no definite deductions can be drawn from these photographs it is clear that important changes in the cross flow must be expected as the cross flow Mach number approaches and exceeds unity.

8. Wing Alone Test

Several schlieren pictures of the wing alone are included in this report (see Figure 18). Since the wing was a wooden 'mock up' the tunnel stagnation pressure was kept below about 6 in. Hg. absolute to avoid excessive loads on the model when it was at high incidence. Consequently the Reynolds number of the test was only 0.8×10^6 per foot. Although the model has no trailing edge the formation of the tip vortices should not be unduly different from that on the wing of the wing-body combination. The inclusion of these pictures will therefore

help with the interpretation of the schlieren pictures for the complete model. The relevant information obtained from these photographs has been used in the discussion above.

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The tests were made under a Ministry of Supply Contract. The model was manufactured and supplied by English Electric Co.Ltd.

10. Conclusions

A complete range of visual tests by surface flow using oil bound titanium dioxide, transition tests using azobenzene and schlieren techniques were made on a single wedge section, rectangular planform wing in combination with an ogival-head body at a Mach number of 2.0.

The methods for producing artificial transition by means of smooth brass bands, carborundum bands and continuous air jets were considered satisfactory at zero incidence. At high incidence, however, uniform transition over the complete body circumference was not obtained by any of these methods as a result of the inherently stable boundary layer flow on the lower surface of the body and the early boundary layer separation on the body upper surface. In addition, in certain cases, the transition stimulators caused interference effects on the vortex patterns associated with the separated boundary layer flows at high angles of incidence.

The results of the flow visualisation tests showed that on the body, separation of the upper surface boundary layer commenced at about 5° incidence. The separated boundary layers, on each side of the body, rolled up and formed a symmetric pair of vortices with a strong induced flow between them. This induced flow was responsible for the reattachment of the upper surface boundary layer. At incidences above 15° the induced flow velocity was extremely high and as a result a shock wave, nearly parallel with and close to the body upper surface, was formed.

The changes in the body vortex pattern associated with distance from the body nose, the interference with the wing, and development with incidence for a tangent ogive and conical nose

are all discussed qualitatively. These latter results were supplemented by tests made in a low speed tunnel and certain similarities between the supersonic and subsonic flows were noted.

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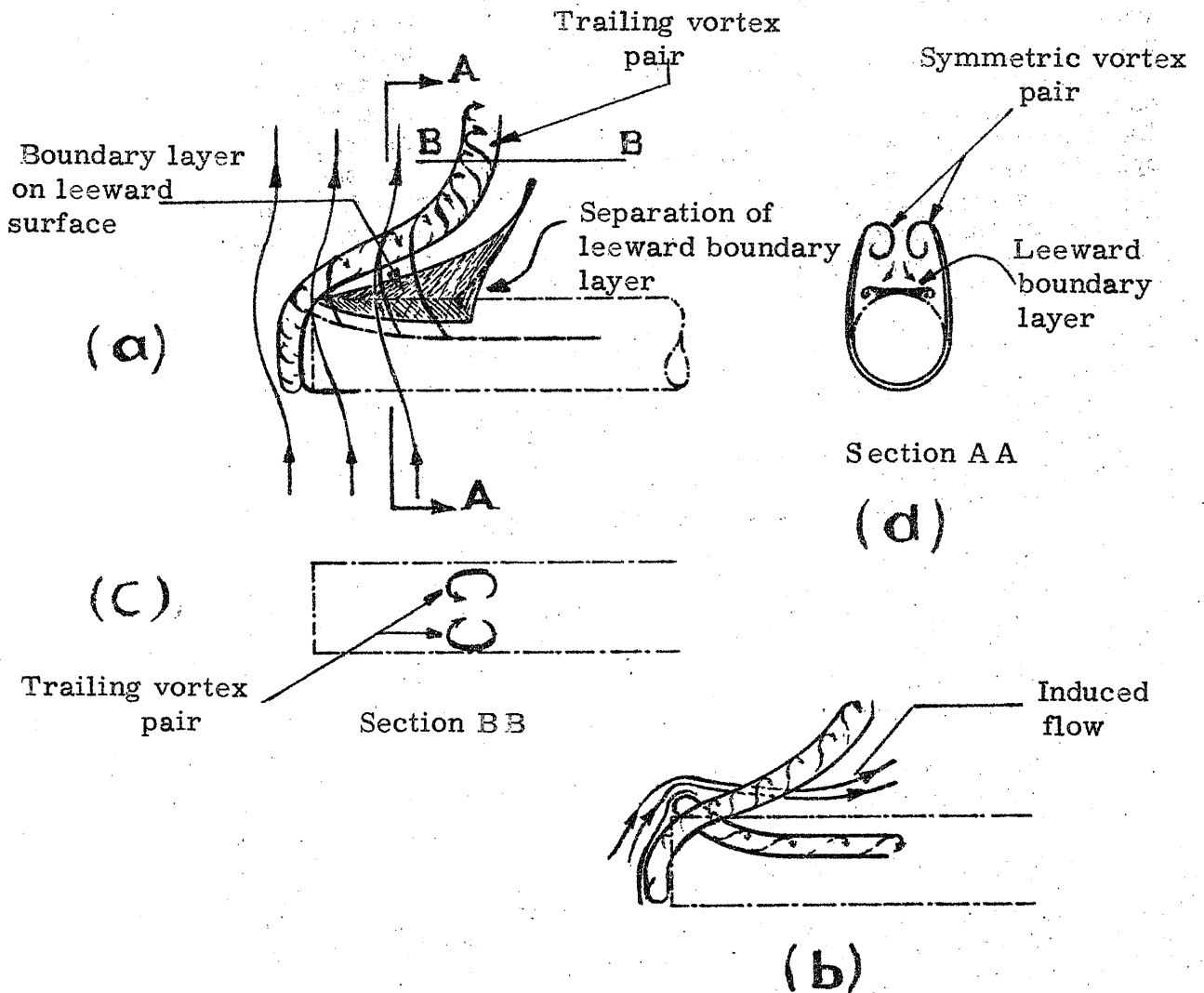
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APPENDIX I

Some further observations on the subsonic flow around bodies of revolution at incidence

When a body of revolution is set at a high angle of incidence the flow on the leeward side is known to be very similar to the two dimensional flow past a circular cylinder at very low Reynolds numbers. In particular at incidences greater than about 8° the leeward flow pattern consists of a symmetric pair of vortices. These vortices arise from the separation of the upper surface flow near the nose of the body and it has been suggested by Allen⁴ that the flow is similar to that produced in the impulsive motion of a cylinder from rest, where, in the analogy, distances from the body nose represent different times in the development of the two-dimensional flow. Observations made by Allen and others have shown that qualitatively this 'flow model' is correct.

However, there are reasons why this simple analogy does not represent all the features of the essentially 3-dimensional flow past yawed bodies of revolution. For instance in the limiting case of a cylinder of finite length yawed at 90° no flow along the length of the cylinder near the free ends would develop according to the analogy. In practice, however, as visual flow observations demonstrate, the flow around the free ends is very similar to that around the nose of the cylinder under conditions of smaller incidence. The essential details of the flow around the free ends of a circular cylinder set at 90° yaw are shown below.

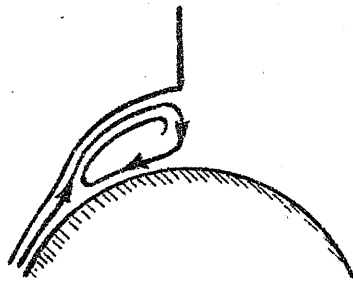


Since the pressure to leeward is slightly higher than at the sides of the cylinder a secondary flow develops close to the surface which results in a separation of the windward flow. This separation, which is a continuation of the separation around the free end of the cylinder forms a symmetric pair of vortices, with axes approximately parallel to that of the cylinder, which develop from the free end along the length of the cylinder. The low pressure region formed between the vortex pair induces a strong axial flow, commencing at the free end, and thus forms a boundary layer on the leeward side, which separates at a short distance from the rear centre line of the body (see (d)). In addition the windward flow moving past the vortices and their side vortex sheets, is induced to flow around them thereby increasing their

size and strength. A short distance from the free end the vortex sheets connecting the vortices to the cylinder become unstable and as a result the vortices break away and trail in the stream direction (see (a)). New vortices reform at the edges of the vortex sheets and periodically separate from them. The shedding of vorticity from the boundary layers of the cylinder into the wake occurs inside the region bounded by the trailing vortex pair. This type of vorticity shedding is different from that occurring in the impulsive motion of a two-dimensional cylinder from rest, where vorticity is shed in the streamwise direction in the plane of symmetry and is bounded by a vortex pair at rest in the fluid. At very low Reynolds numbers in the range, where, in steady flow, a symmetric vortex pair exists downstream of the two-dimensional cylinder, a similar type of flow exists along the entire length of a cylinder of finite length. In this case there is no trailing vortex pair. Vorticity is shed into the wake inboard of the separation of the boundary layer on the leeward side near the free ends.

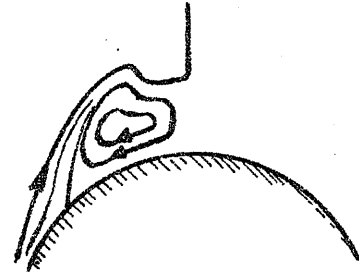
The flow around the nose of a cylinder at smaller angles of yaw is similar except that the symmetric vortex pair form the trailing vortex pair at greater distances from the nose of the cylinder. In addition the boundary layer on the leeward surface, formed by the induced flow between the vortices, only separates along the axis at large distances from the nose.

The flow over bodies, having streamlined ends or noses, is very similar to that around the bluff ended cylinder described above. The formation of the symmetric vortex pair around such a body at incidence can be described as follows. Near the nose a cross flow develops from the lower to the upper surface. On the upper surface fluid near the surface moves down the pressure gradient and so forms a secondary flow which opposes the motion of the cross flow close to the wall of the cylinder.

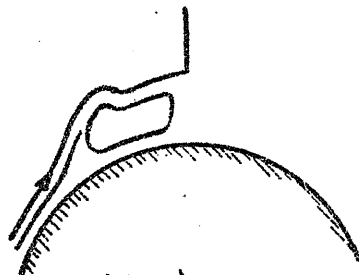


(a)

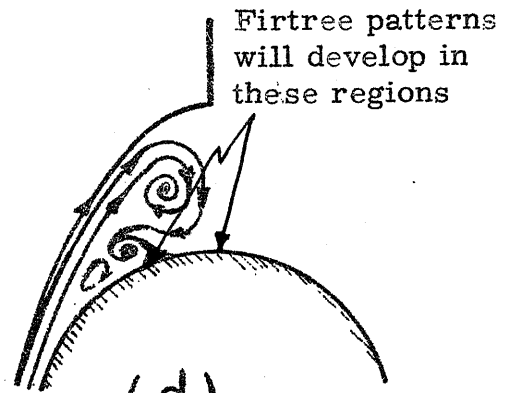
Initial Stage



(c)



(b)

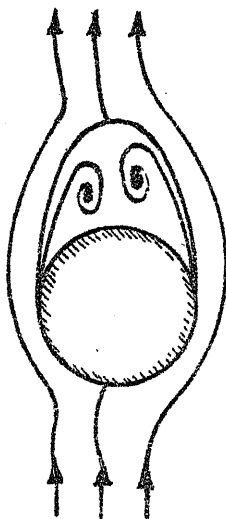


(d)

Final Vortex Configuration

A separation of the cross flow in the boundary layer develops but, owing to the axial motion, the boundary layer as a whole does not separate. Downstream of the nose the secondary flow develops into an elongated vortex which is fed from the cross flow in the upper plane of symmetry. However, at a certain distance from the nose the secondary flow vortices breakaway and form the symmetric vortex pair. The flow outside the boundary layer in the cross flow curls around the vortices and either feeds the upper surface boundary layer or leaves approximately in the stream direction between the vortices. It is remarked that all the boundary layers ventilate into the space occupied by the vortices on the upper surface.

Farther downstream of the nose the side vortex sheets and vortices, whose size and strength are being increased continuously by feeding on the cross flow, become unstable and an asymmetric vortex pattern develops in which the free vortex trails in the stream direction.



Development of side force
on a body of revolution due
to assymmetric growth of
vortex pair.

APPENDIX II

A short discussion on the inviscid supersonic flow past cones at incidence

At supersonic speeds the flow around a body of revolution at incidence is similar to that at subsonic speeds, although the actual pattern is modified by the bow shock wave. One important effect of the bow shock wave is that for bodies of revolution at incidence the strength of the shock wave varies continuously from a maximum in the lower plane of symmetry to a minimum in the upper plane of symmetry. Thus in meridian planes, passing through the body axis, the flows on crossing the shock wave will have different values of entropy and stagnation pressure. In discussing the flow over bodies of revolution at supersonic speeds it is therefore important to consider how the flow is affected by this variation in entropy.

Consider the case of a cone set at incidence in an inviscid uniform flow at a supersonic speed, such that the bow wave is attached to the nose. At zero incidence the flow is along the generators of the cone surface from the nose, and the entire surface is 'wetted' by the upstream 'streamline' coincident with the body axis. The entire surface is one of constant entropy and in this case the entire region between the shock wave and the cone is at constant entropy also. At incidences smaller than the semi-nose angle, the flow in the lower plane of symmetry,

which has a lower stagnation pressure than the flow in the upper plane of symmetry downstream of the bow shock wave, 'wets' the lower part of the cone. The flow pattern near the lower centre line is fir-tree like (see sketch below), and the flow no longer sweeps down the cone, as at zero incidence. In fact, over the lower part of the cone, the flow is more nearly represented as being parallel with the mainstream. Since the lower part of the cone is 'wetted' by flow from the lower plane of symmetry this part of the cone surface is at constant entropy. The question now arises as to whether or not the entire surface is one of constant entropy.

If the entire surface of the cone is to be one of constant entropy, equal to its value in the lower plane of symmetry, the flow in this plane must 'wet' the entire surface of the cone, and in particular that part near and in the upper plane of symmetry. Although this is possible when the incidence is greater than the semi-nose angle, at smaller incidences, it would imply that the upper surface flow was displaced upwards to accommodate the flow from the lower plane of symmetry, or, expressed in another way, it would imply a relatively large circumferential displacement of the lines of flow, from their zero incidence configuration, for an infinitesimal incidence. Neither the first nor the second implication appear to have any foundation from visual flow observations. In this incidence range the upper part of the cone appears to be 'wetted' by the nose streamline which in consequence must split up into a surface of variable entropy. Thus the cone surface is divided up into a region of constant entropy and variable entropy in the lower and upper parts respectively. A discontinuity in the entropy must occur along the lines dividing one region from the other since the lines of flow over the upper part of the cone come from only a restricted section of the total possible meridional planes. As the angle of incidence increases so the region of variable entropy over the upper part of the cone must get less and less until for angles, greater than the semi-nose angle, the entire surface must be one of constant entropy i.e. it is 'wetted' by the flow in the lower plane of symmetry. At some intermediate angle of incidence the variable entropy region will collapse onto the upper centre line and a discontinuity in entropy must exist there. As the incidence is increased so the line discontinuity in entropy must exist in the upper plane of symmetry above the cone.

As yet we have not discussed the form the streamlines take on in the variable entropy region. Ferri¹⁷ and Holt²⁵ have shown, from the conical flow equations behind a conical shock wave, that the conditions for the flow normal to the cone surface to be zero are

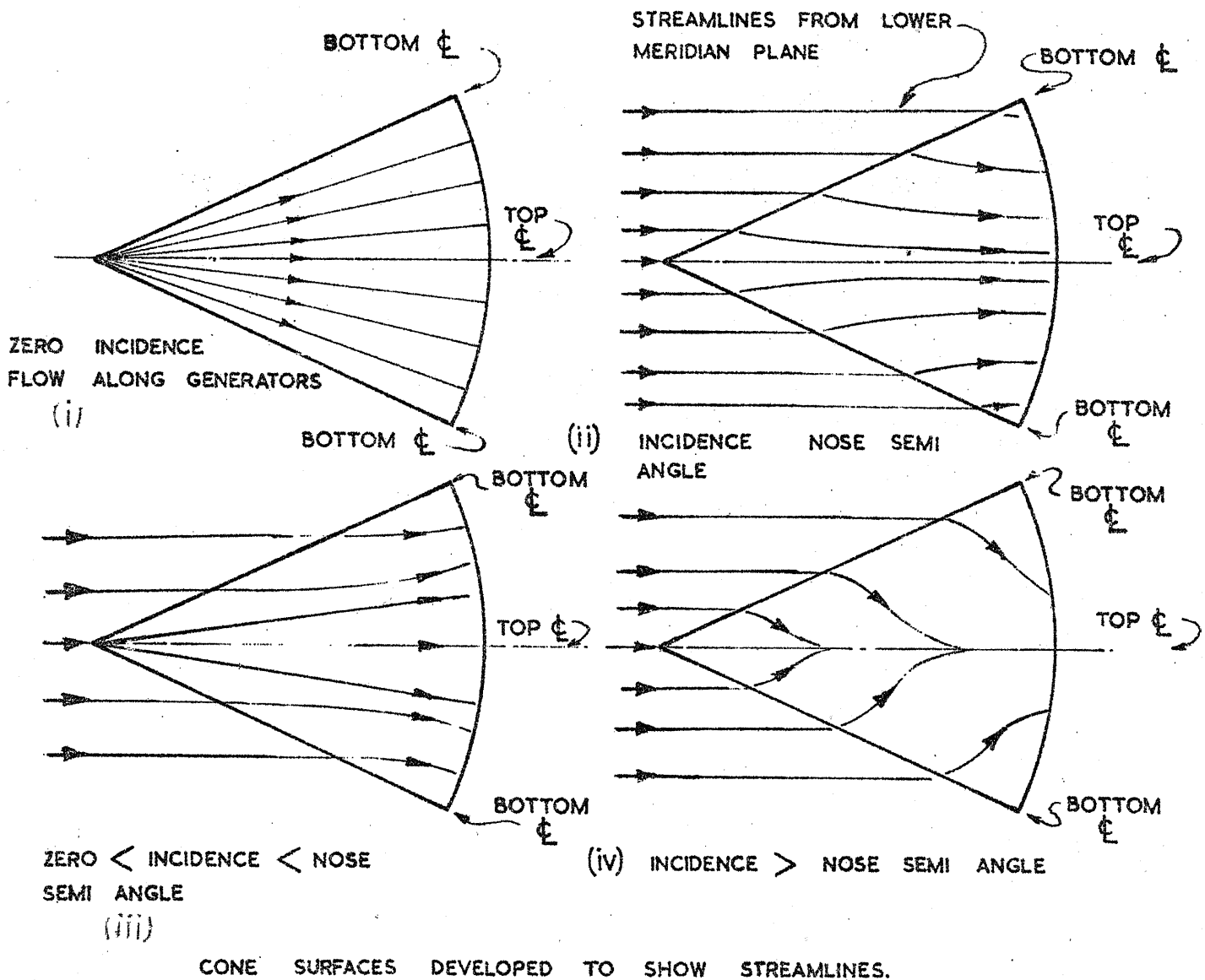
- or (a) the circumferential velocity is zero
- (b) the gradient of entropy in a circumferential direction is zero,
- or (c) a discontinuity in entropy occurs in which case either (a) or (b) must be satisfied on each side

or (d) of the discontinuity,
the gradient of entropy normal to the surface
is infinite.

It can be seen from conditions of flow symmetry that condition (a) is always satisfied in the upper and lower planes of symmetry. Also, (b) is satisfied whenever the surface is one of constant entropy. In addition (a) is satisfied when the flow is along the generators of the cone surface. It is further assumed that (d) does not represent a physically possible flow.

It follows that in the case discussed above, when the angle of incidence was less than the nose semi-angle, condition (b) is satisfied over the lower part of the cone. It can be seen that since condition (b) is not satisfied over the upper part of the cone then the flow over this part of the cone surface can only exist if it is along the generators of the cone. It follows that the discontinuity in entropy (c) must lie along one of the generators of the cone.

In the range of incidence, where the line discontinuity in entropy exists above the upper surface of the cone, we have already discussed that viscous effects considerably modify the inviscid flow pattern, and in particular these viscous effects must modify the position of the discontinuity as calculated from inviscid flow theory as well as its diffusion.



It has been shown above that the inviscid flow past yawed cones is dependent on the values of incidence and semi-nose angle. The range of incidence can be divided up as follows:

- (i) zero incidence -flow along generators of cone (surface at constant entropy).
- (ii) Between zero incidence and an incidence approximately equal to the nose semi-angle - transition region, lower part of cone surface at constant entropy and separated by a discontinuity in entropy from variable entropy over the upper part of cone surface.
- (iii) Incidence approximately equal to the nose semi-angle - entire surface at constant entropy, but a discontinuity in entropy exists on upper meridian line.
- (iv) Incidence greater than nose semi-angle - surface at constant entropy, but a discontinuity exists in upper meridian plane.

Similar results must exist for pointed nosed bodies of revolution of arbitrary shape but since the bow wave will be curved the entropy at the surface must vary both circumferentially and axially.

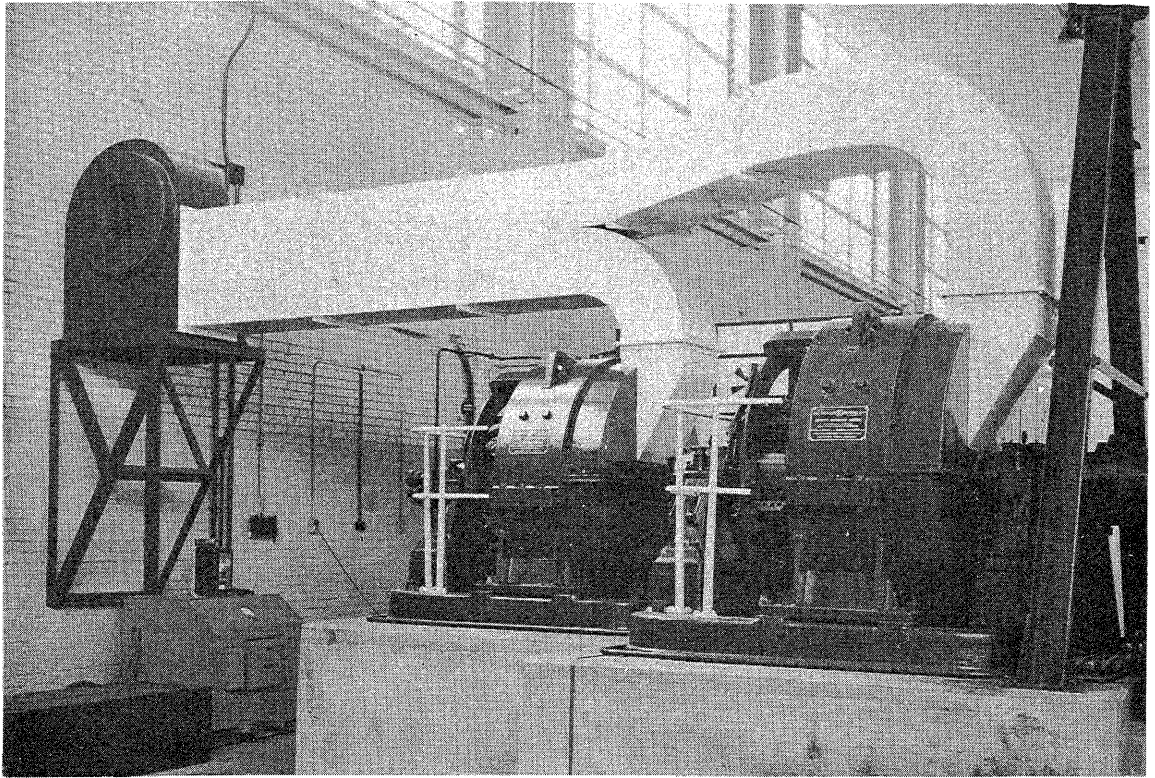


FIG. 2. LAYOUT OF TUNNEL MOTORS AND COOLING FAN .

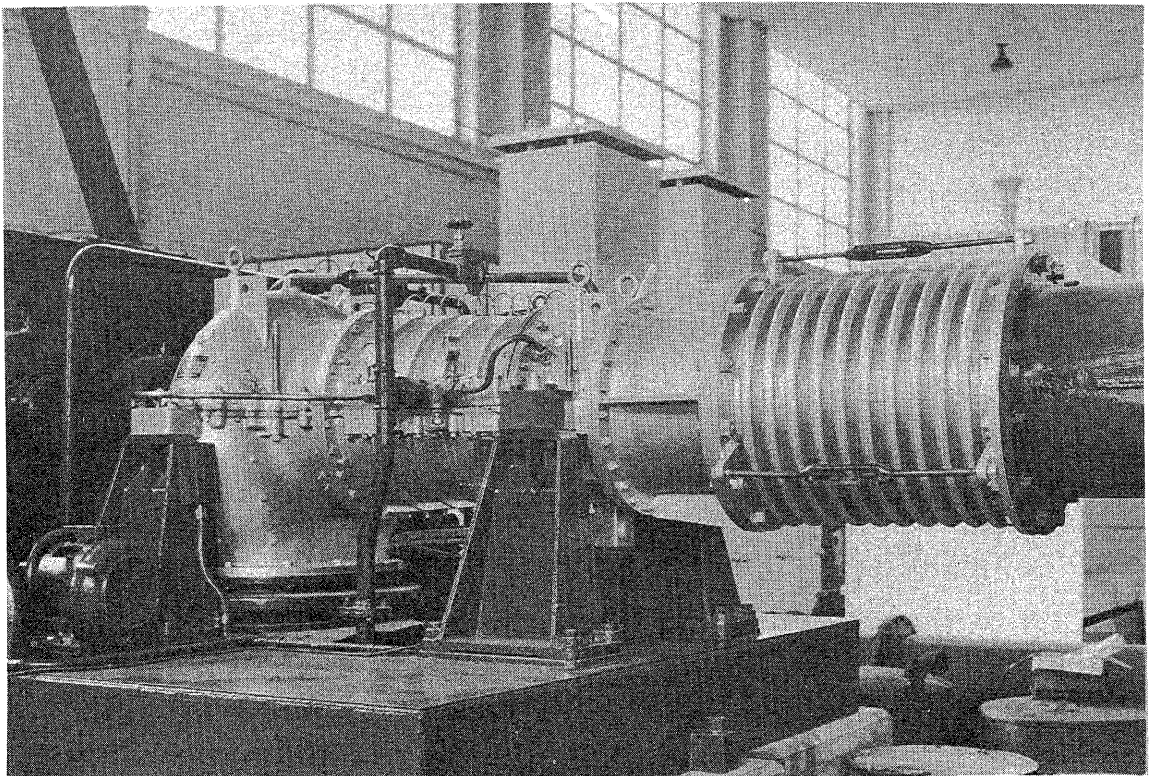


FIG. 3. LAYOUT OF TUNNEL GEAR BOX AND COMPRESSOR.

Z.9213.R.

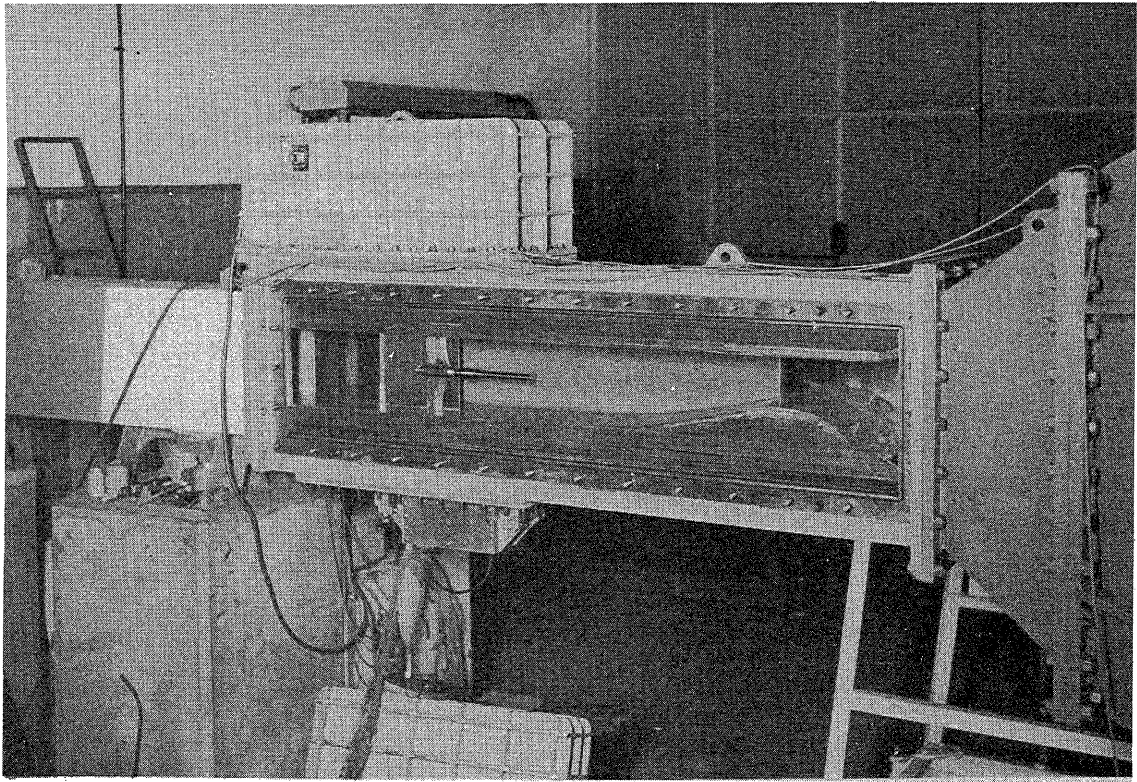


FIG.4. WORKING SECTION WITH ONE WINDOW REMOVED.

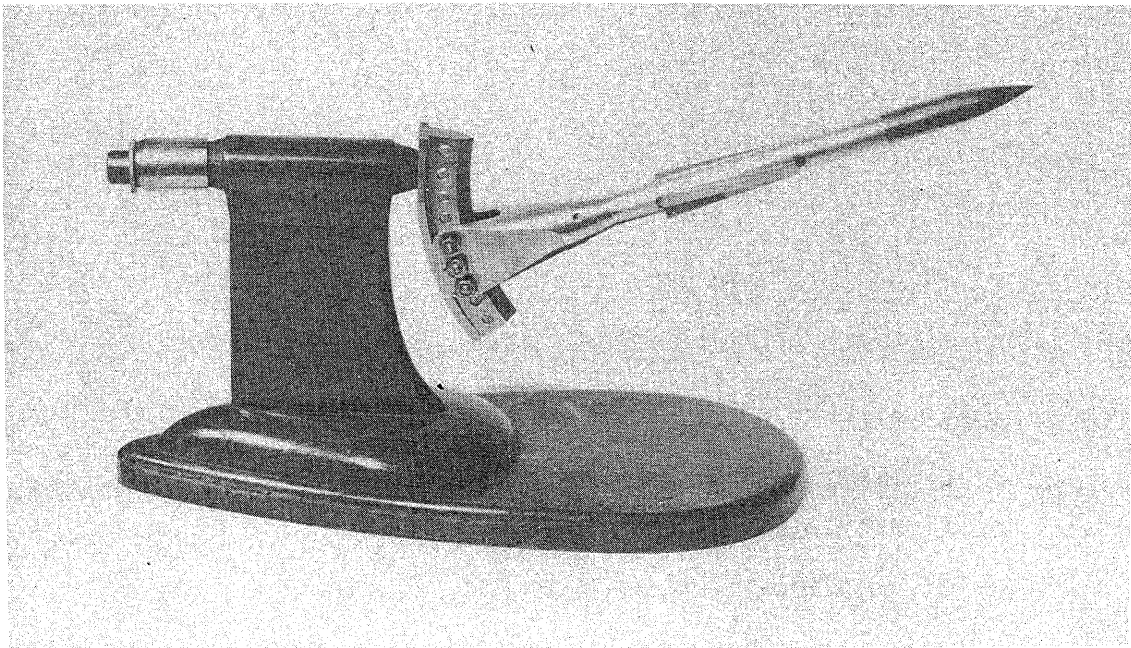


FIG.5. MODEL MOUNTED ON YAWED STING.

Z.9213.R.

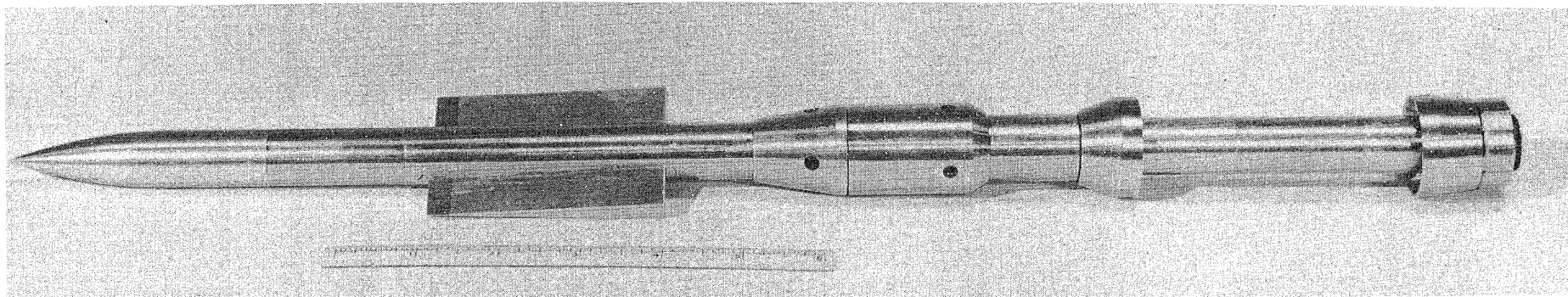


FIG. 6a.

LAYOUT OF MODEL AND STING.

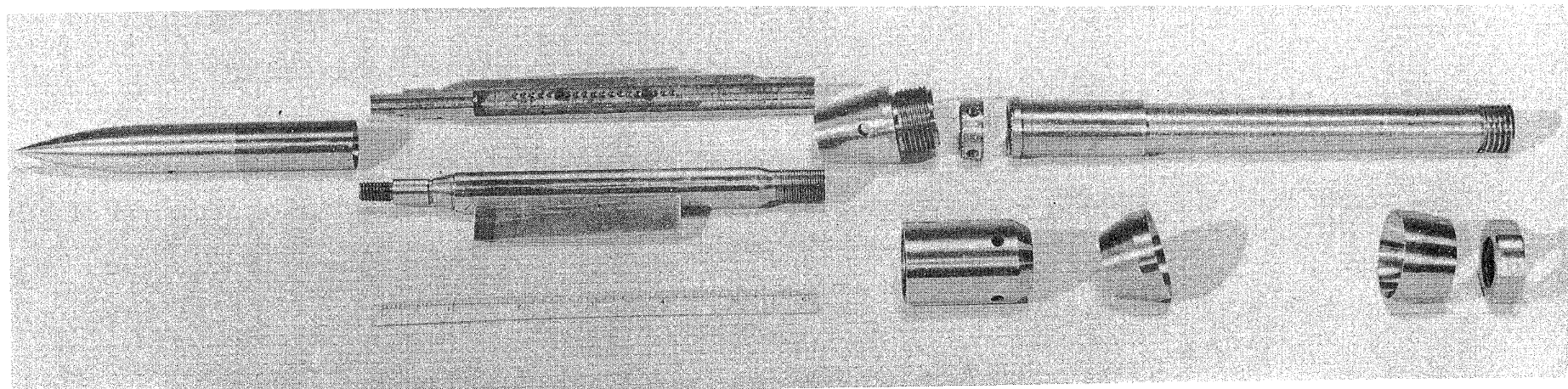
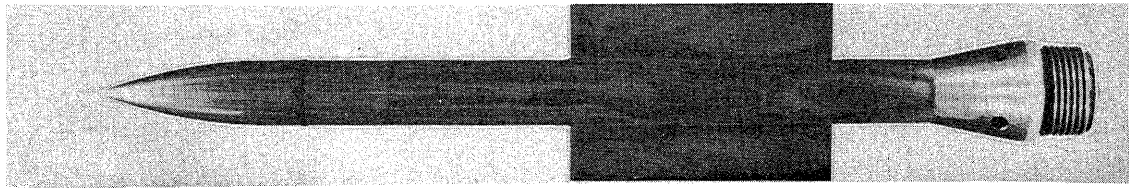


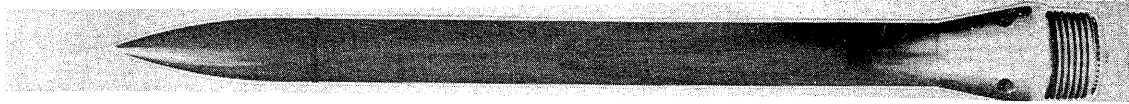
FIG. 6b.

LAYOUT OF MODEL SECTIONED

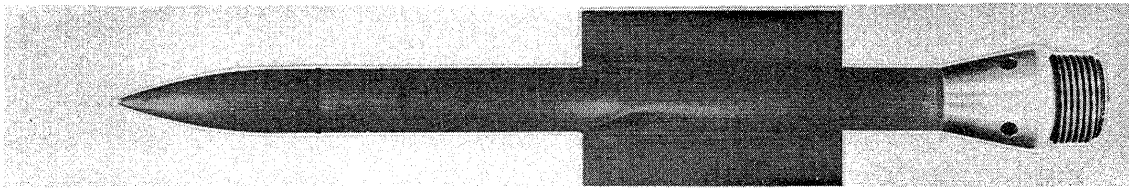
Z.9213.R.



UPPER SURFACE

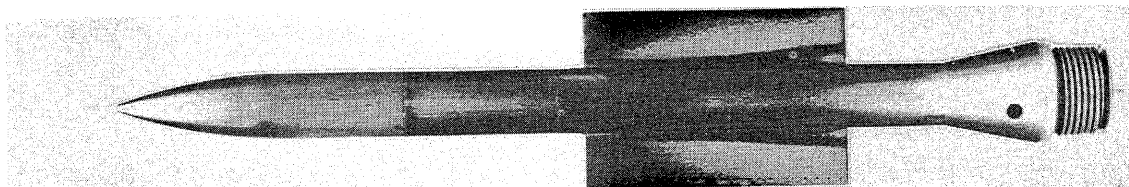


STARBOARD SIDE

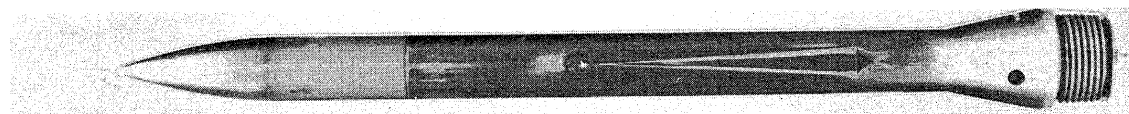


LOWER SURFACE

FIG. 7a. CARBORUNDUM BAND AT SHOULDER.
25° INCIDENCE.



LOWER SURFACE

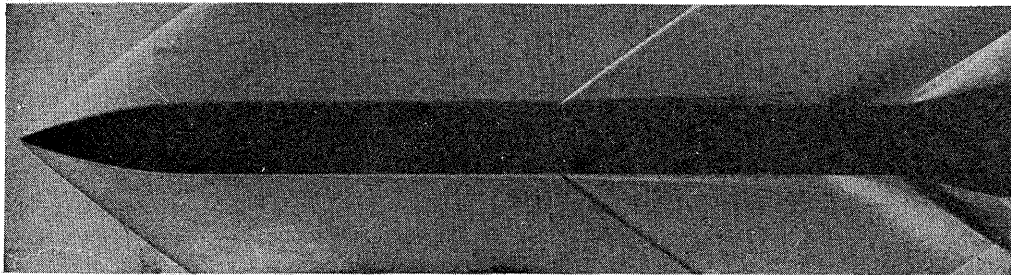


PORT SIDE

FIG. 7b. AIR JET ISSUING FROM ANNULAR ORIFICE.
0° INCIDENCE.

AZO-BENZENE SURFACE PATTERNS.

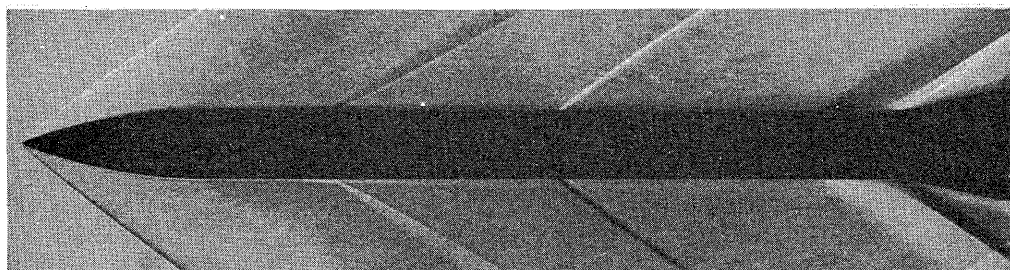
Z.9213.R.



NO AIR



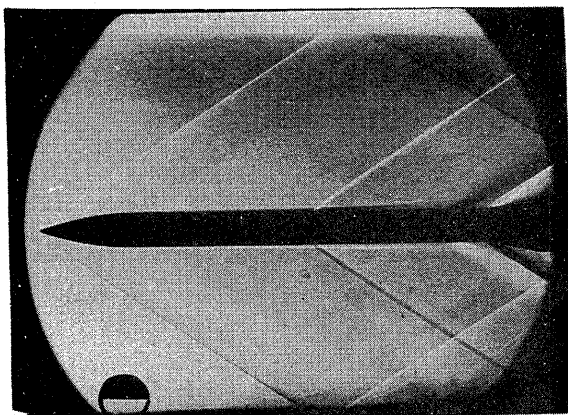
PRESSURE RATIO ACROSS ANNULUS $\sim 6:1$



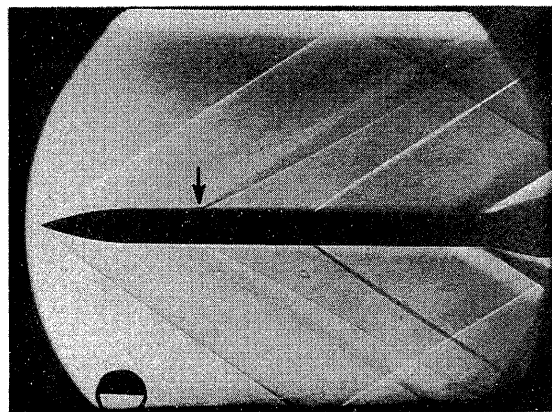
PRESSURE RATIO ACROSS ANNULUS $\sim 12:1$

FIG. 7c. SCHLIEREN PHOTOGRAPHS SHOWING THE FORMATION OF A TURBULENT BOUNDARY LAYER USING AN ANNULAR AIR JET.

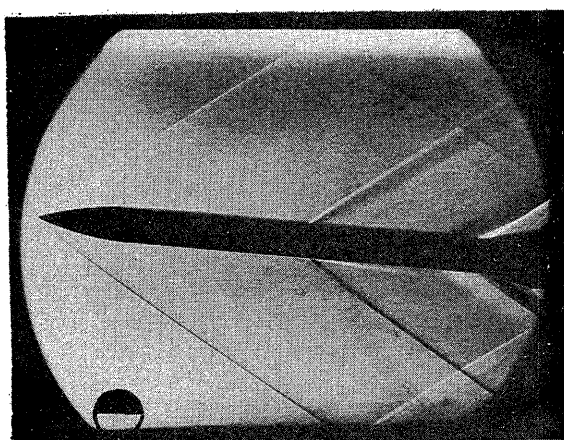
Z.9213.R.



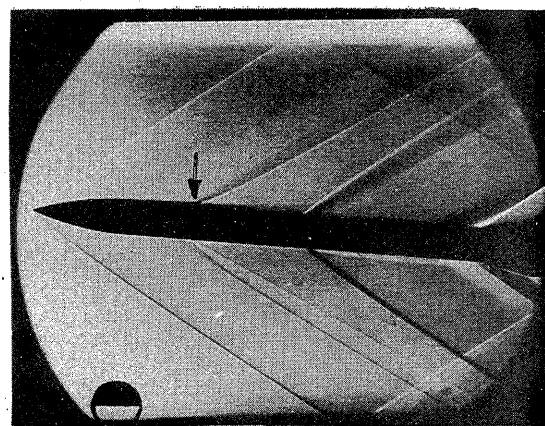
0°



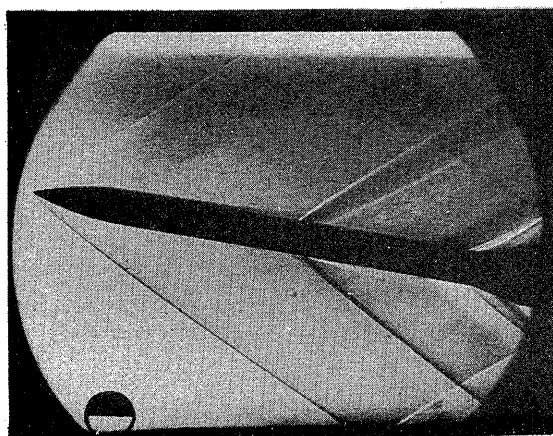
0°



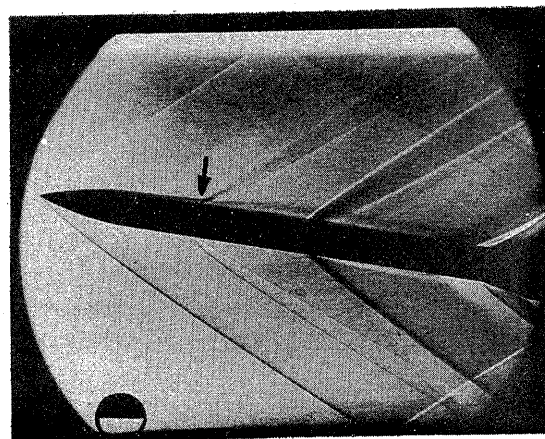
5°



5°



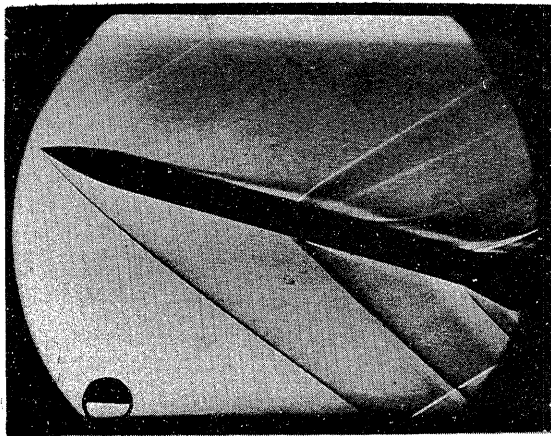
10°



10°

FIG. 8A. SCHLIEREN PHOTOGRAPHS SHOWING THE EFFECT OF AIR JETS, USED TO PROMOTE A TURBULENT BOUNDARY LAYER, ON THE GROWTH OF VORTEX SHEETS. MODEL AT ZERO ROLL.

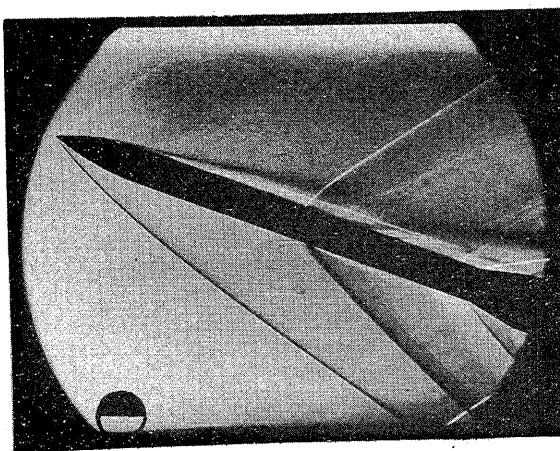
↓ Denotes position of air jets.



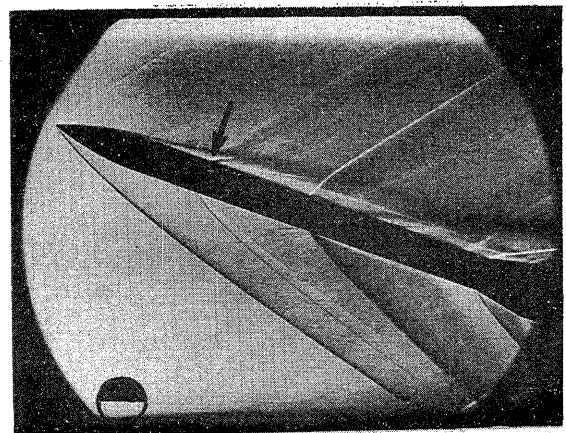
15°



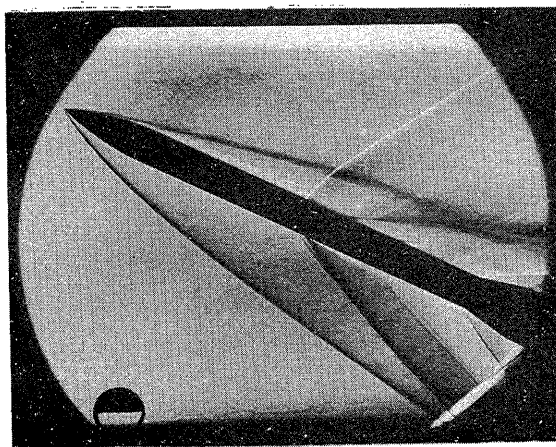
15°



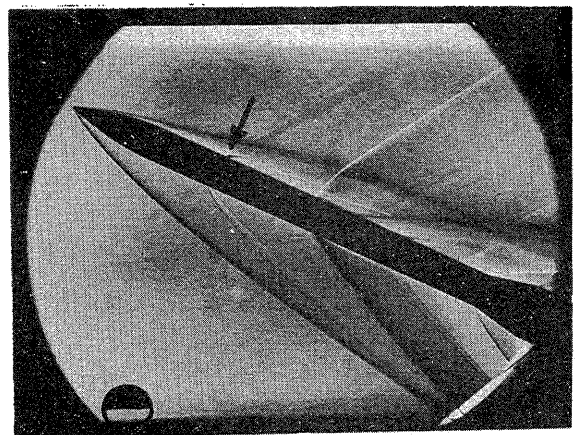
20°



20°



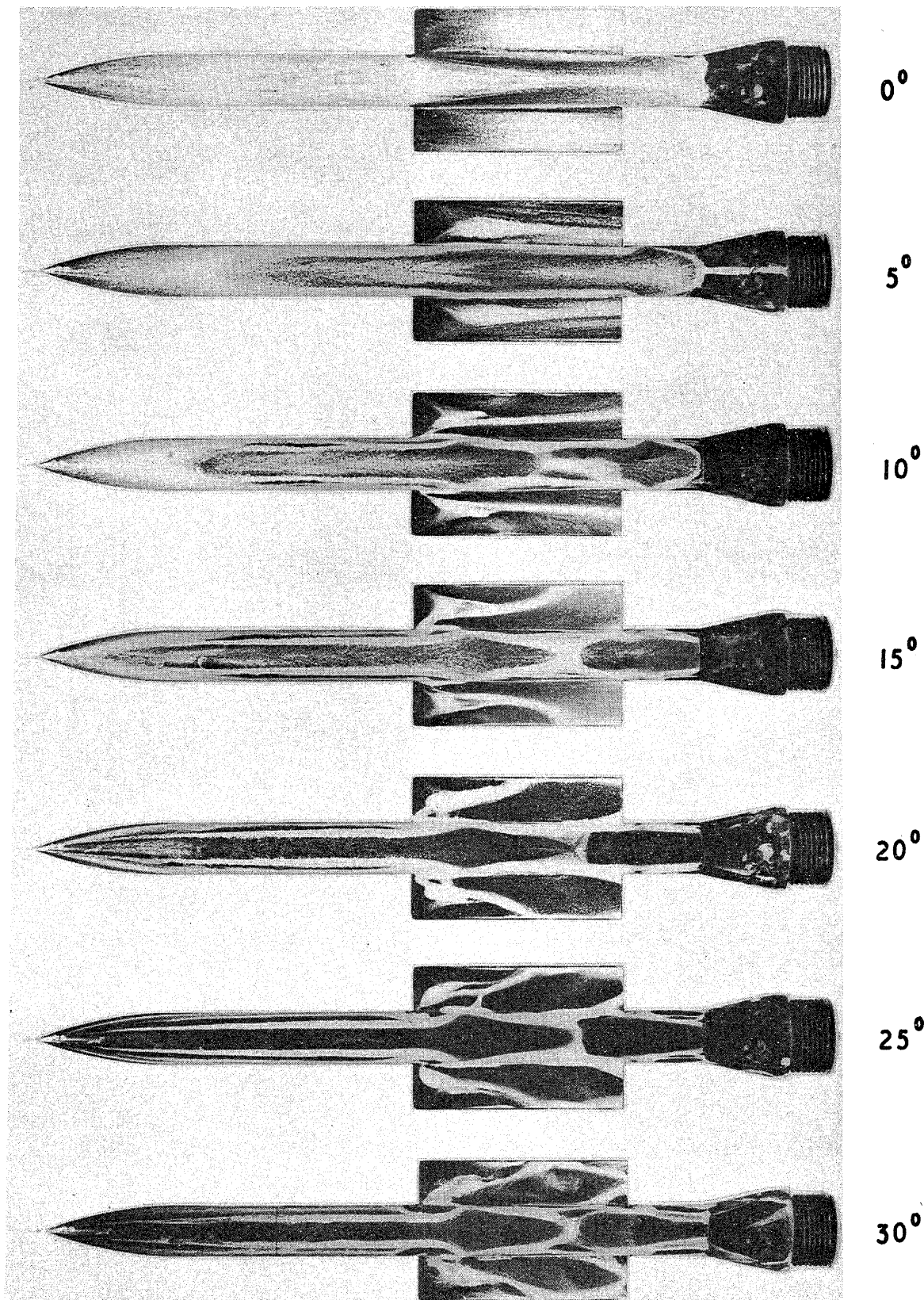
25°



25°

FIG. 8B. SCHLIEREN PHOTOGRAPHS SHOWING THE EFFECT OF AIR JETS, USED TO PROMOTE A TURBULENT BOUNDARY LAYER, ON THE GROWTH OF VORTEX SHEETS. MODEL AT ZERO ROLL.

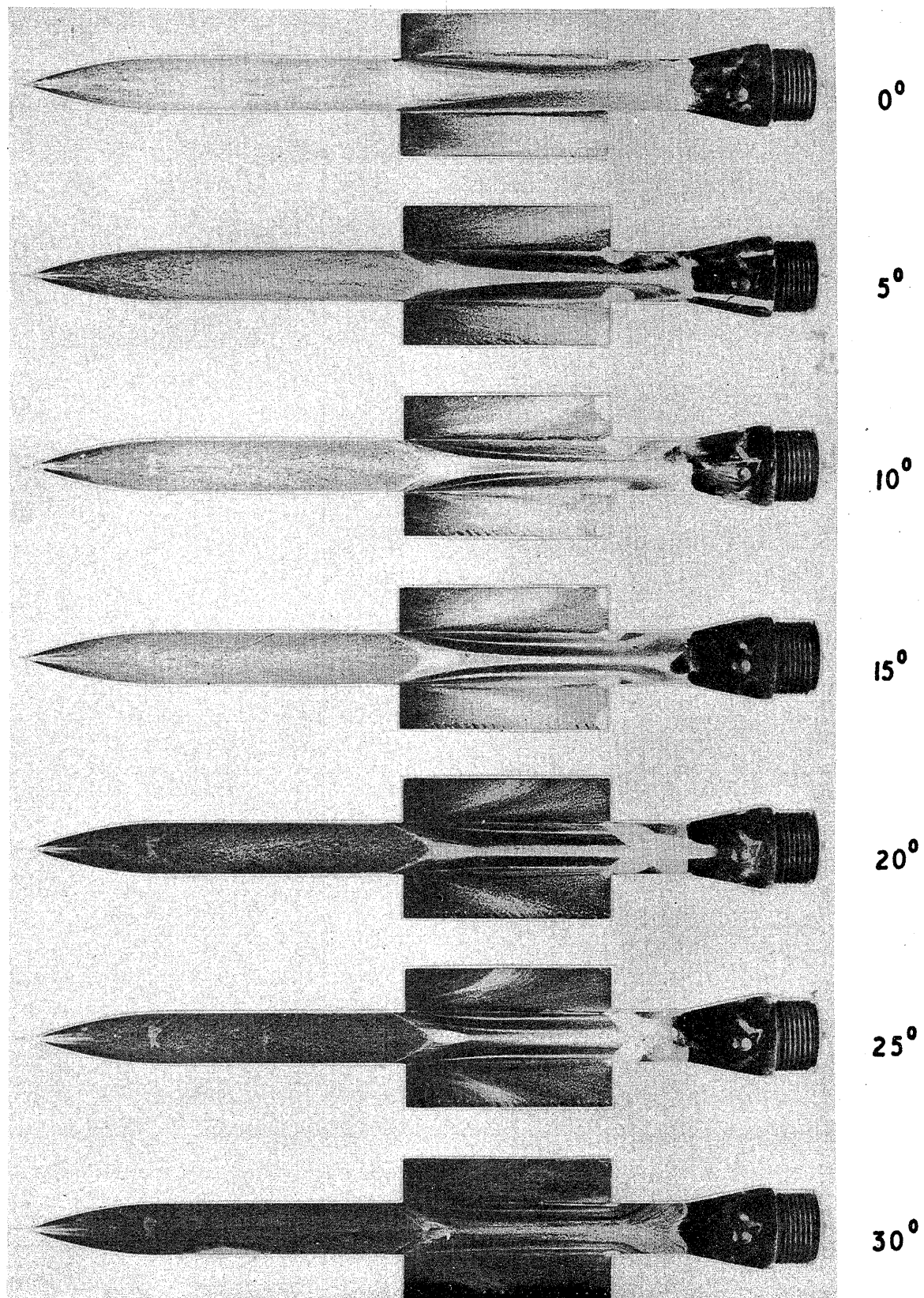
↓ Denotes position of air jets.



UPPER SURFACE ZERO ROLL

FIG.9a. SURFACE FLOW PATTERNS OBTAINED WITH
TITANIUM DIOXIDE MIXED IN OIL.

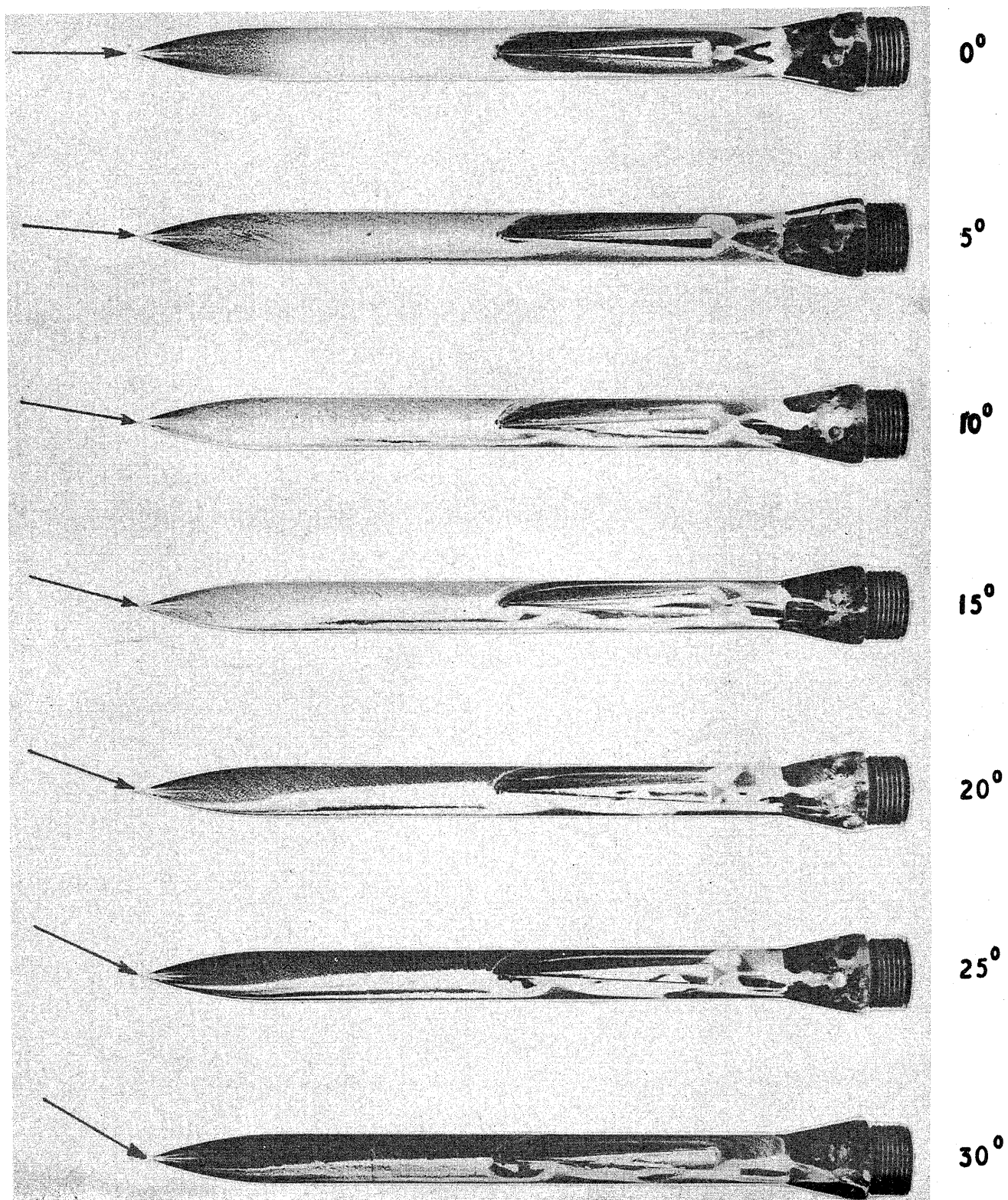
Z.9213.R.



LOWER SURFACE ZERO ROLL

FIG.9b. SURFACE FLOW PATTERNS OBTAINED WITH
TITANIUM DIOXIDE MIXED IN OIL.

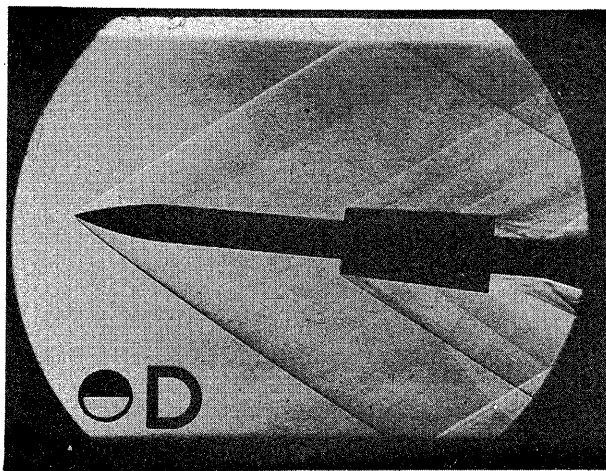
Z.9213.R.



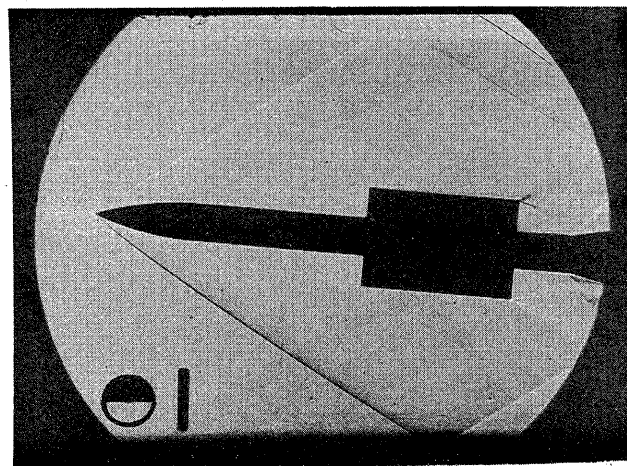
PORT SIDE ZERO ROLL

**FIG.9c. SURFACE FLOW PATTERNS OBTAINED WITH
TITANIUM DIOXIDE MIXED IN OIL.**

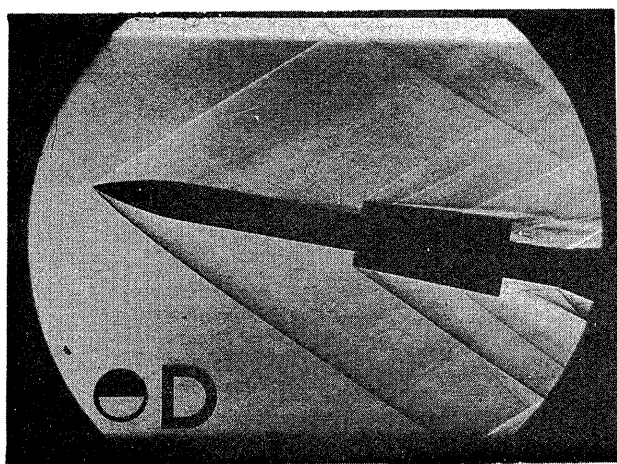
Z.9213.R.



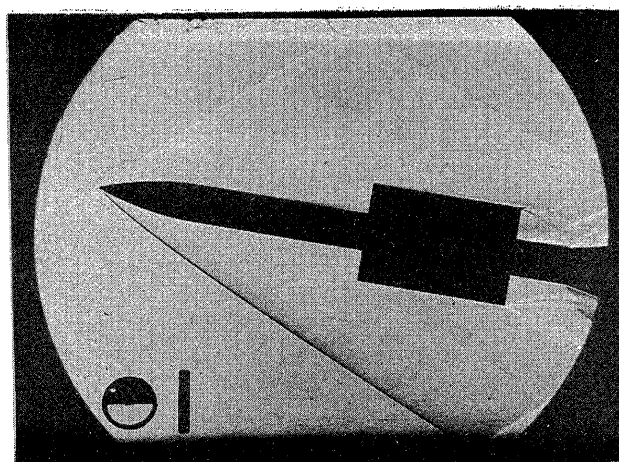
5° INC. 40° ROLL.



5° INC. 90° ROLL.



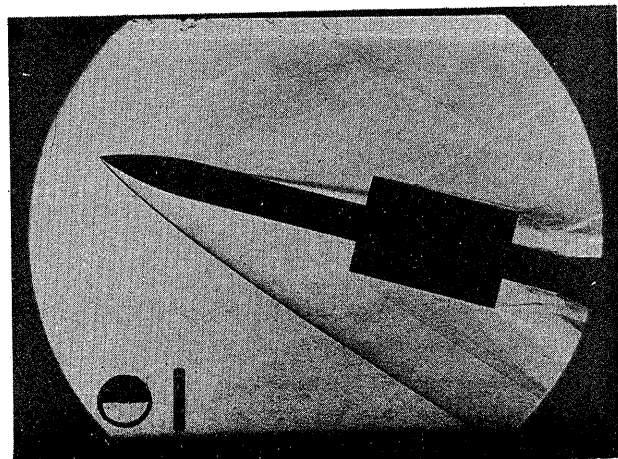
10° INC. 40° ROLL.



10° INC. 90° ROLL.

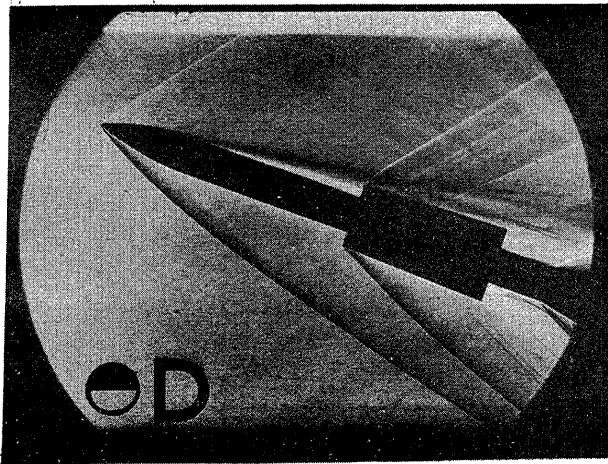


15° INC. 40° ROLL.

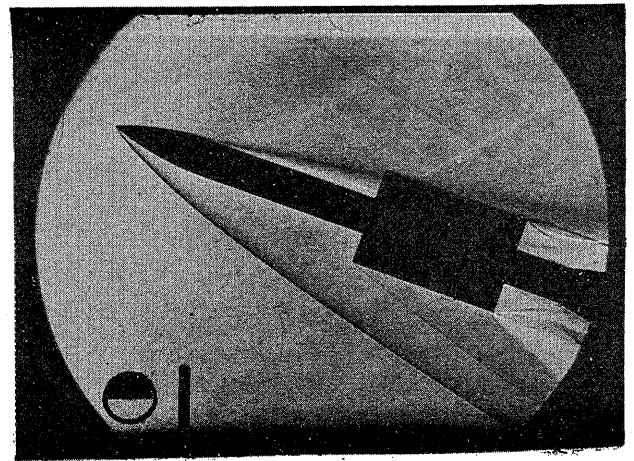


15° INC. 90° ROLL.

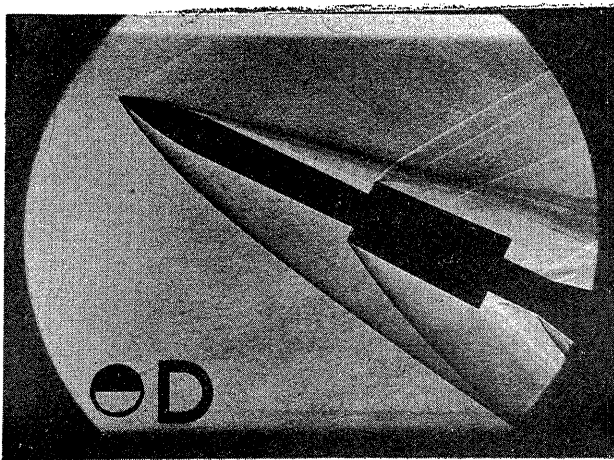
FIG. 10A. TYPICAL SCHLIEREN PHOTOGRAPHS OF THE MODEL
AT VARYING INCIDENCE AND AT 40° AND 90° ROLL.



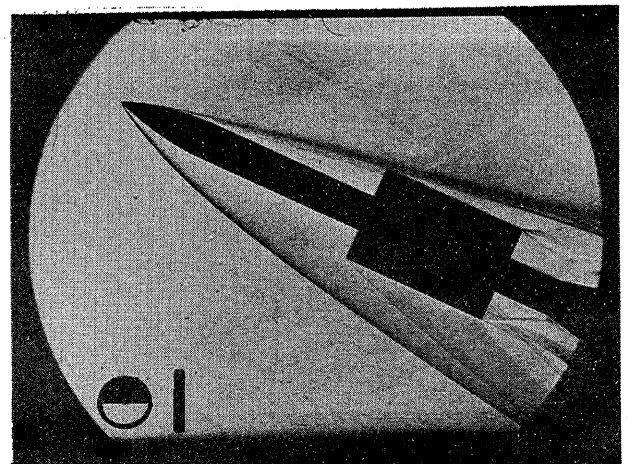
20° INC. 40° ROLL.



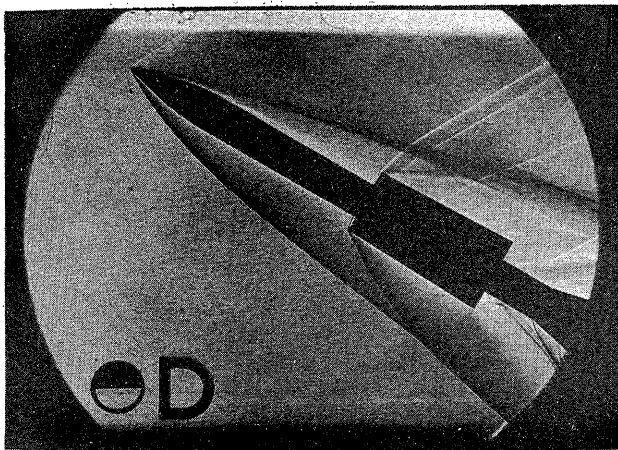
20° INC. 90° ROLL.



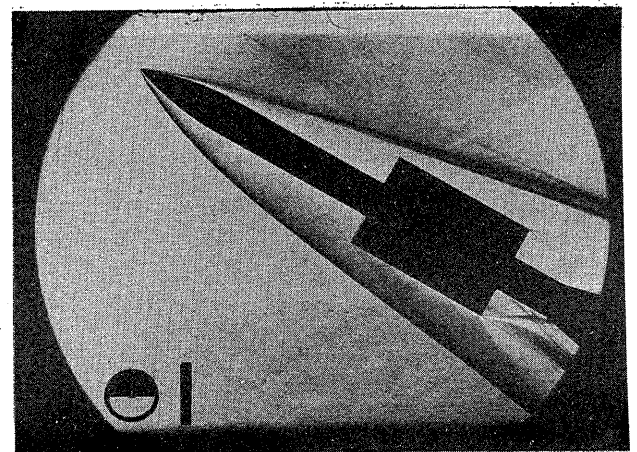
25° INC. 40° ROLL.



25° INC. 90° ROLL.

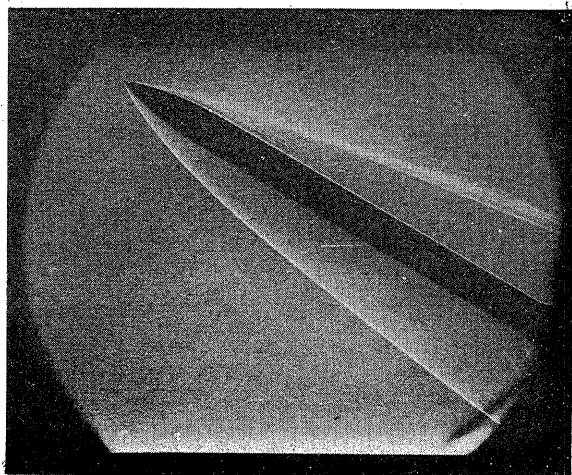


30° INC. 40° ROLL.

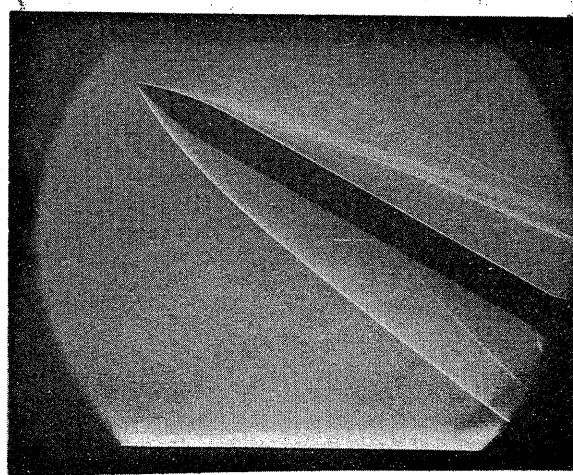


30° INC. 90° ROLL.

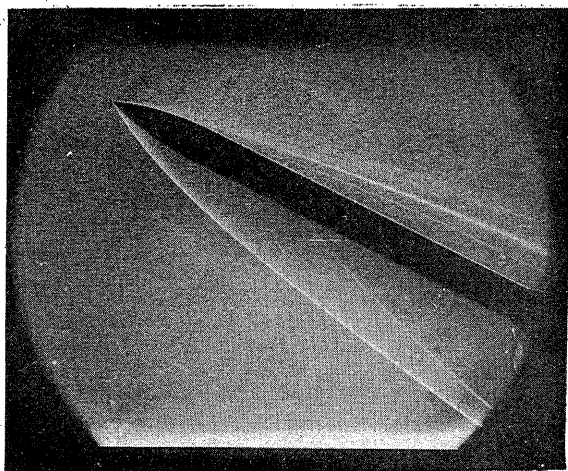
FIG. 10B. TYPICAL SCHLIEREN PHOTOGRAPHS OF THE MODEL
AT VARYING INCIDENCE AND AT 40° AND 90° ROLL.



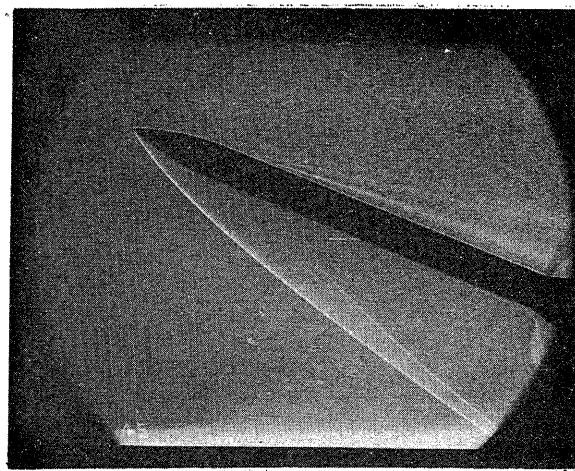
0°



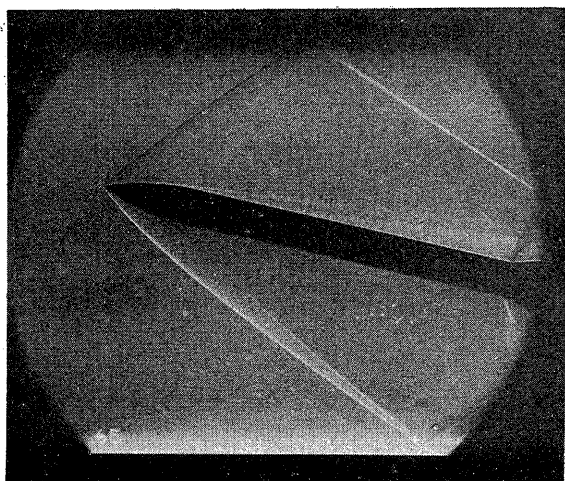
15°



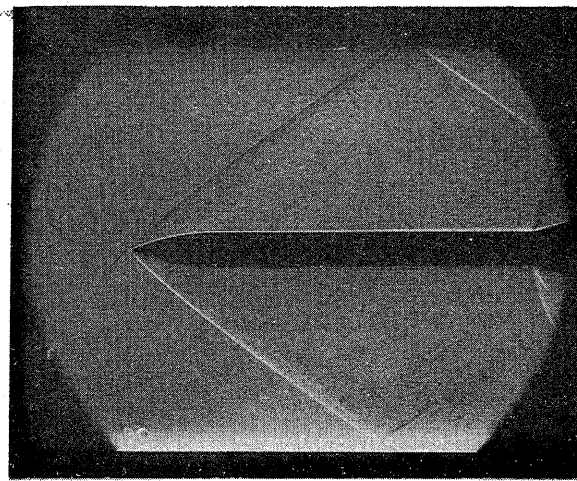
30°



45°

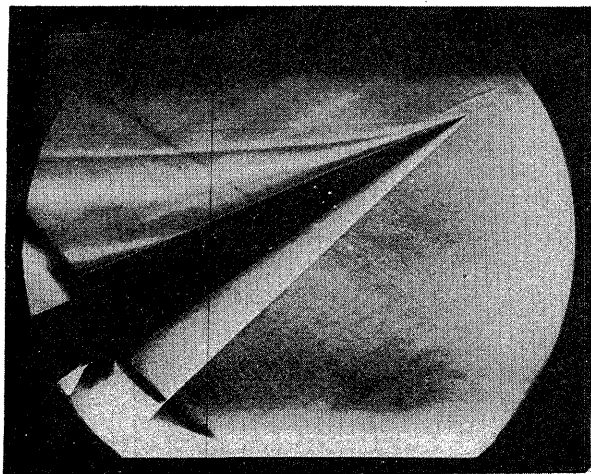


65°

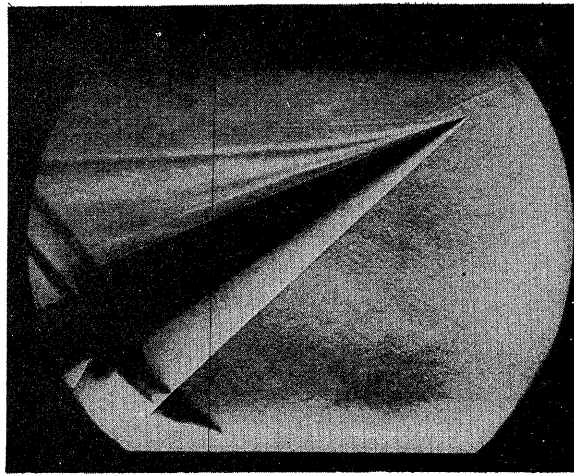


90°

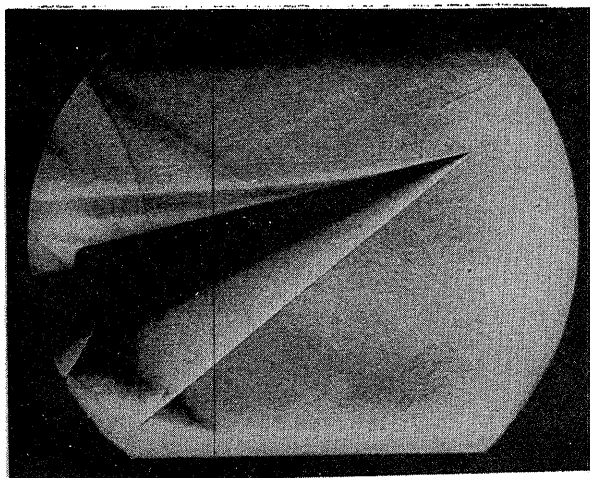
FIG. II. SCHLIEREN PHOTOGRAPHS OF BODY ALONE MOUNTED ON YAWED STING AT 30° INCIDENCE AND ROTATED TO VARIOUS ANGLES BETWEEN 0° AND 90°. GRADED FILTER CUT OFF.



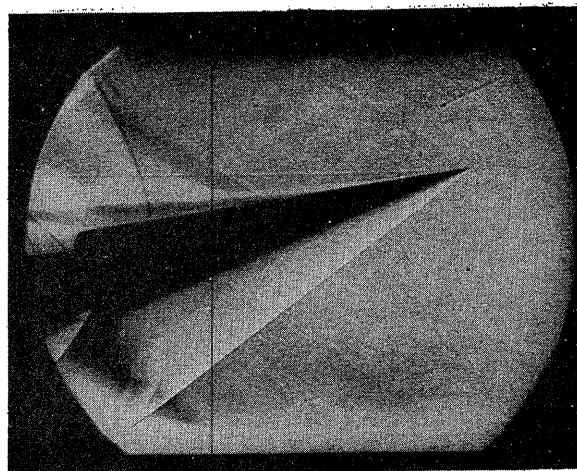
0°



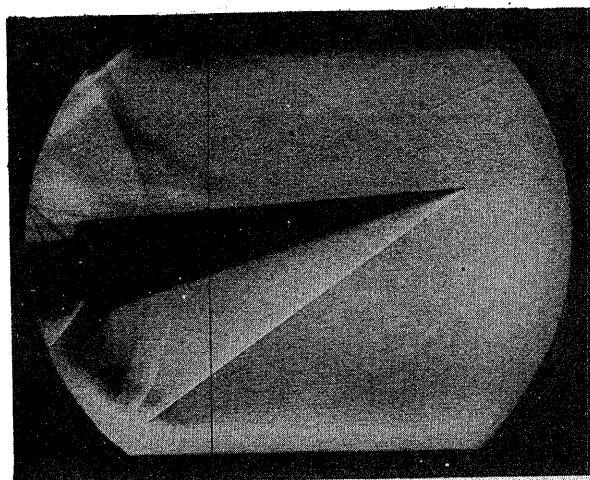
10°



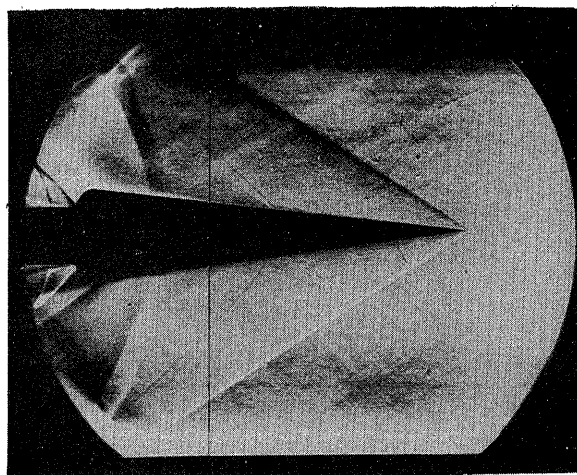
50°



60°



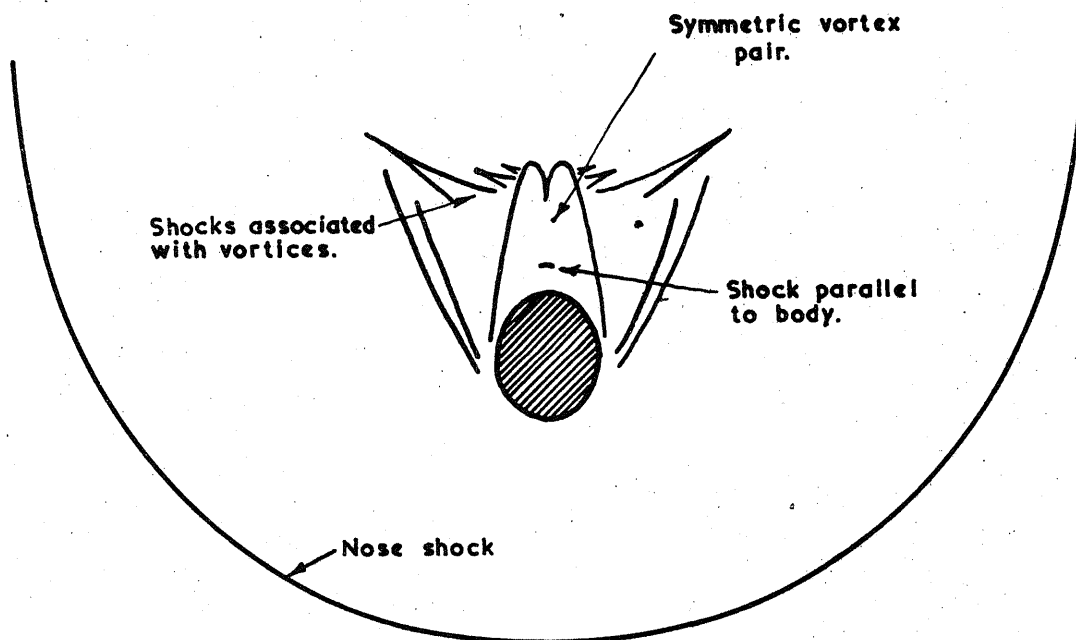
70°



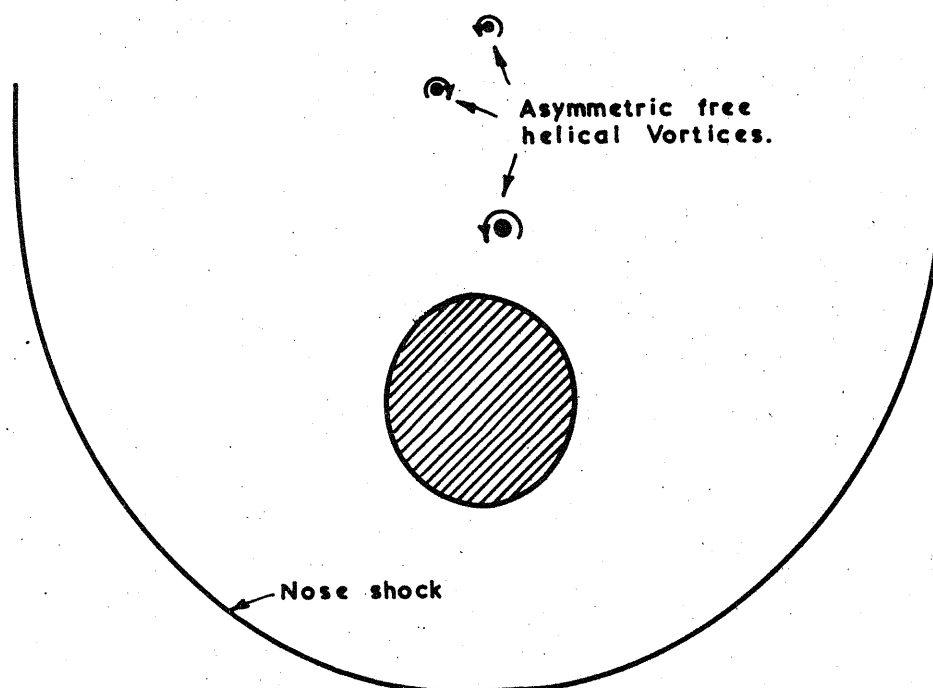
90°

FIG. 12. SCHLIEREN PHOTOGRAPHS OF CONE MOUNTED ON YAWED STING AT 30° INCIDENCE AND ROTATED TO VARIOUS ANGLES BETWEEN 0° AND 90°.

Z.9213.R.



a. BODY ALONE. CROSS SECTION 10 BODY DIAS. FROM NOSE. SEE FIG. 11.



b. 13° CONE. CROSS SECTION 6.5 INCHES FROM NOSE. SEE FIG. 12.

FIG. 13 CROSS FLOW SHOCK WAVE AND VORTEX SHEET CONFIGURATIONS AS OBTAINED FROM SCHLIEREN PHOTOGRAPHS OF BODIES MOUNTED ON A YAWED STING.

Z.9213.R.

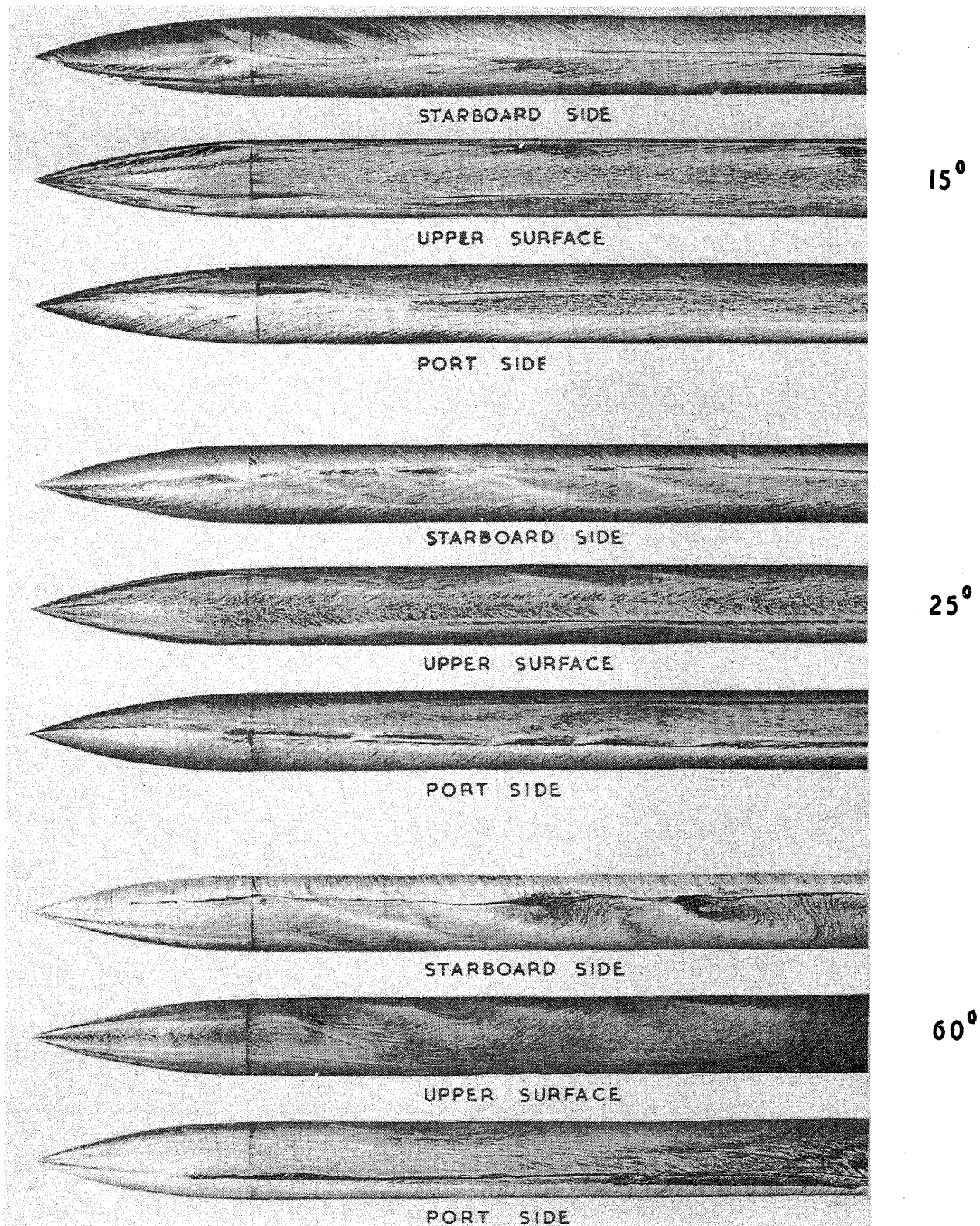
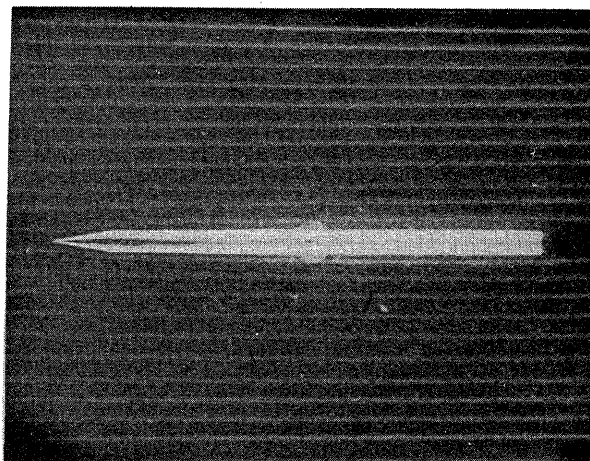


FIG. 14. LOW SUBSONIC SURFACE FLOW PATTERNS USING LAMP BLACK AND KEROSENE ON A CIRCULAR CYLINDER WITH OGIVAL HEAD AT VARIOUS INCIDENCES.

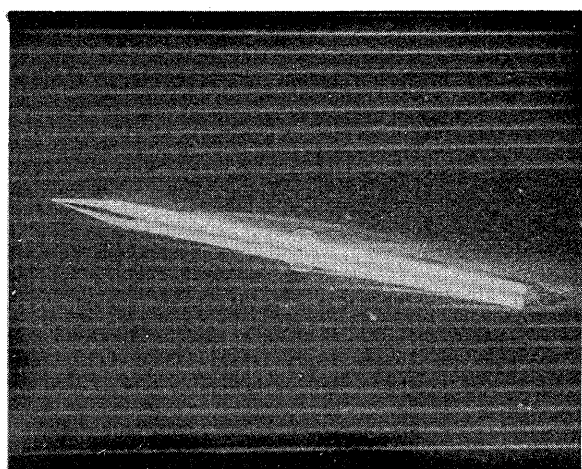
Z.9213.R.



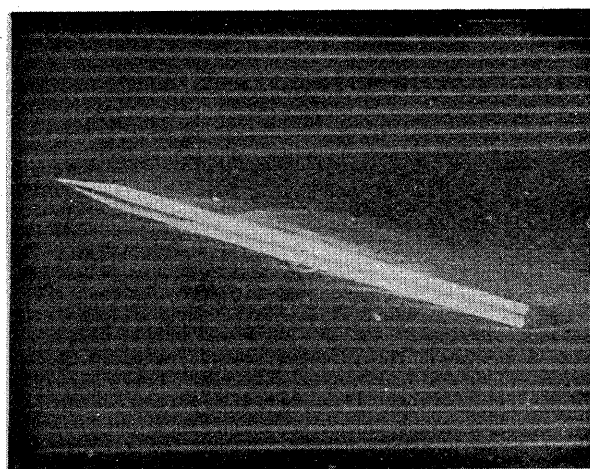
0°



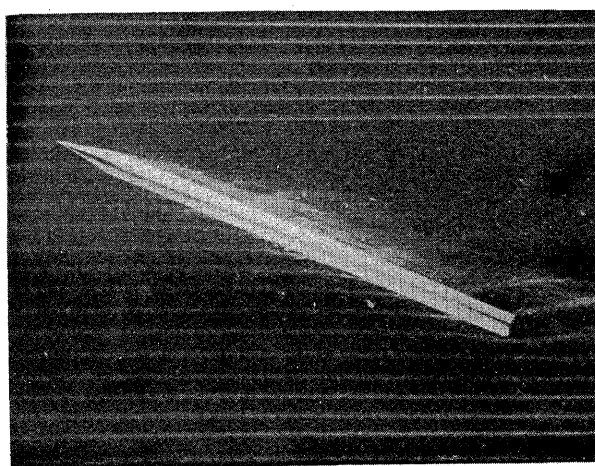
8°



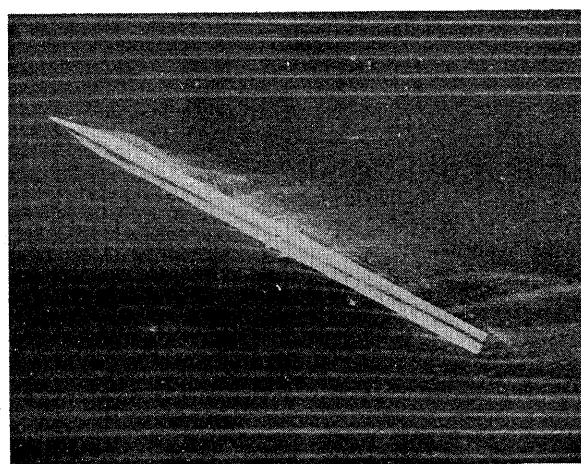
12°



17°



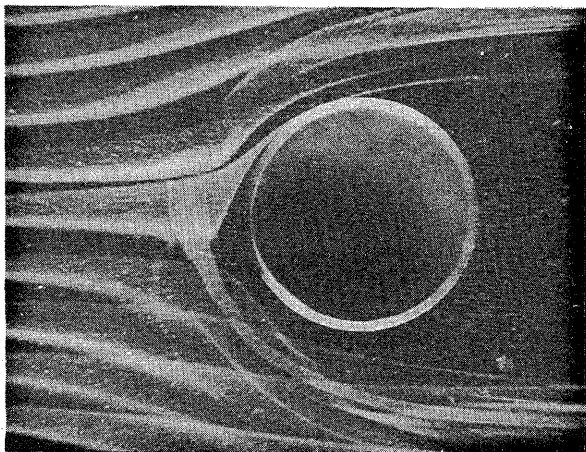
23°



28°

FIG. 15. SMOKE TUNNEL PHOTOGRAPHS SHOWING THE DEVELOPMENT OF BODY VORTICES WITH INCREASING INCIDENCE.

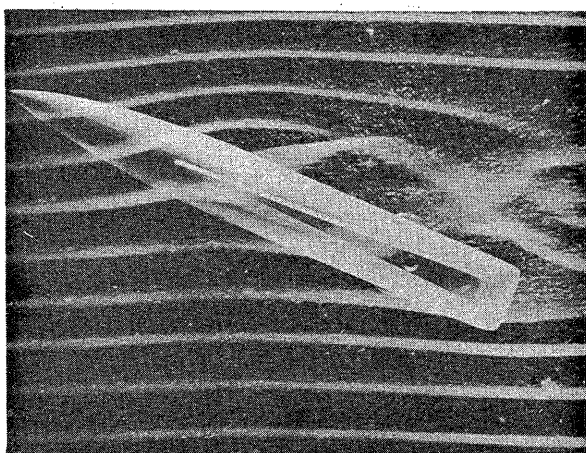
Z.9213.R.



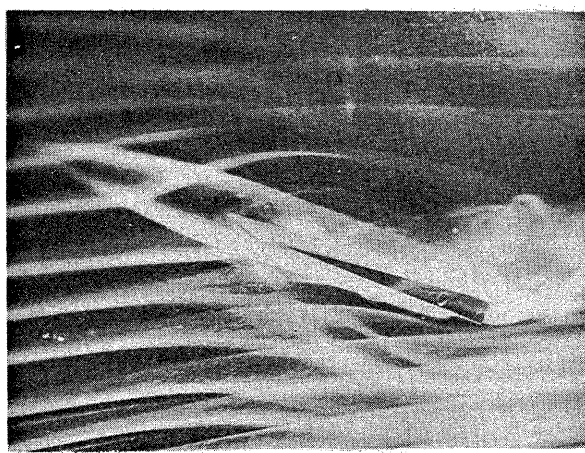
Cylinder mounted on wall showing vortex on wall ahead of cylinder.



Sharp nose aerofoil at incidence on wall. Incidence $\sim 15^\circ$

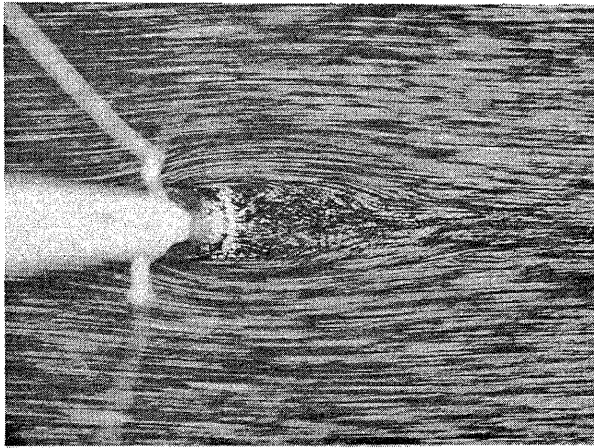


Half model of wing mounted on cylindrical body showing wing tip vortex. Incidence $\sim 20^\circ$

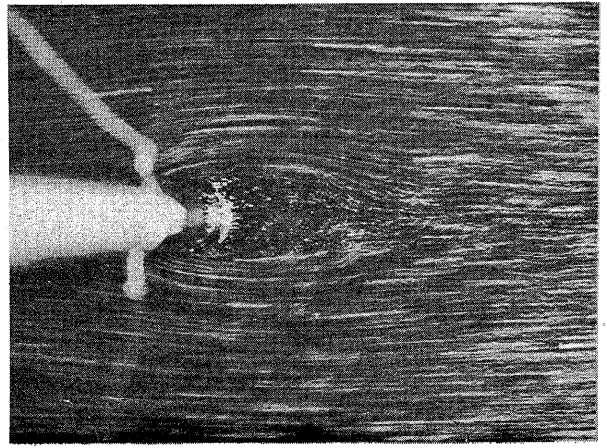


Half model of wing mounted on cylindrical body showing root vortex. Incidence $\sim 20^\circ$

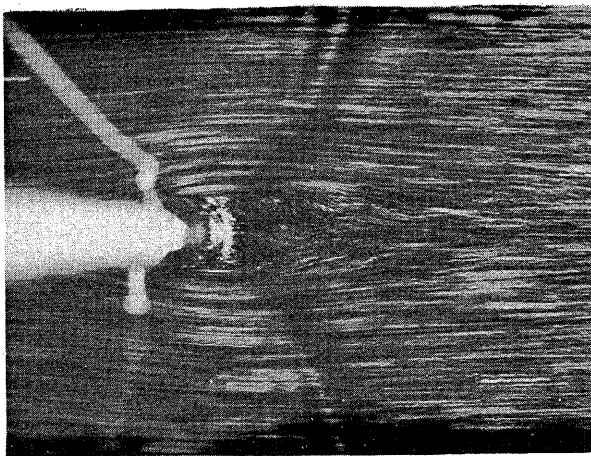
FIG. 16. SMOKE TUNNEL PHOTOGRAPHS SHOWING THE PRESENCE OF WING — BODY JUNCTION VORTICES.



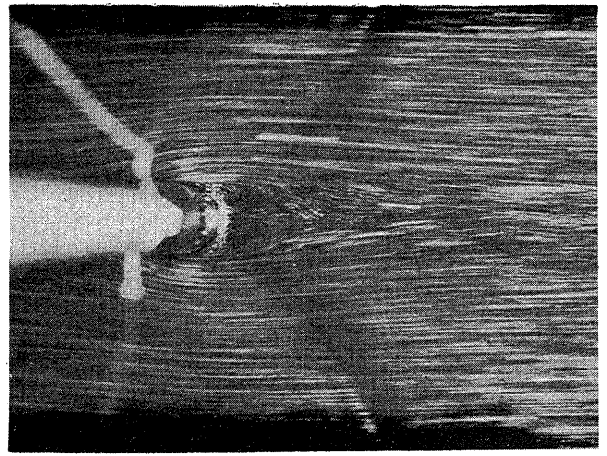
$M = 0.585$



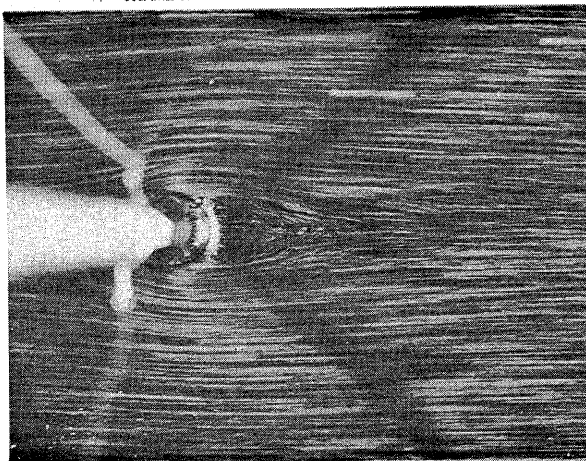
$M = 0.785$



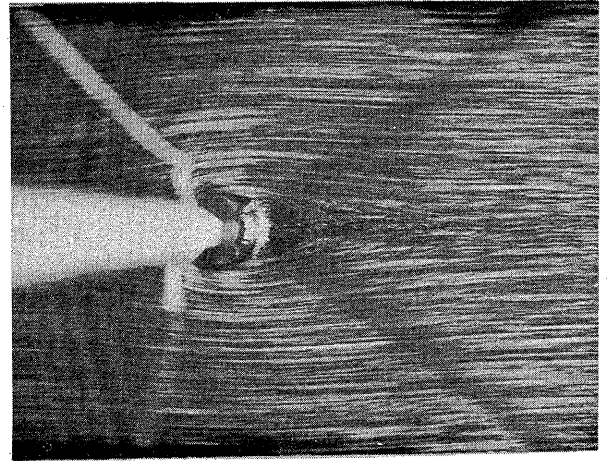
$M = 0.840$



$M = 0.895$

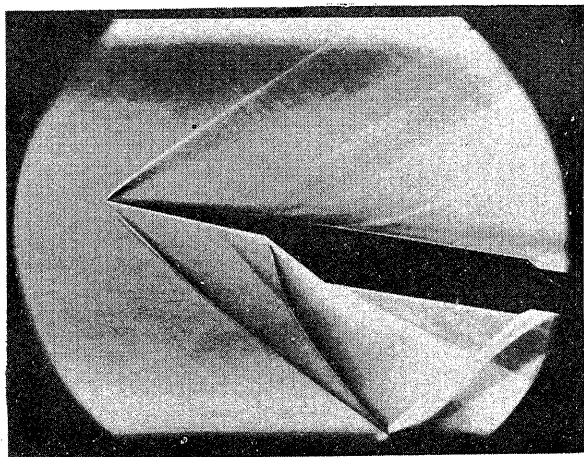


$M = 0.975$

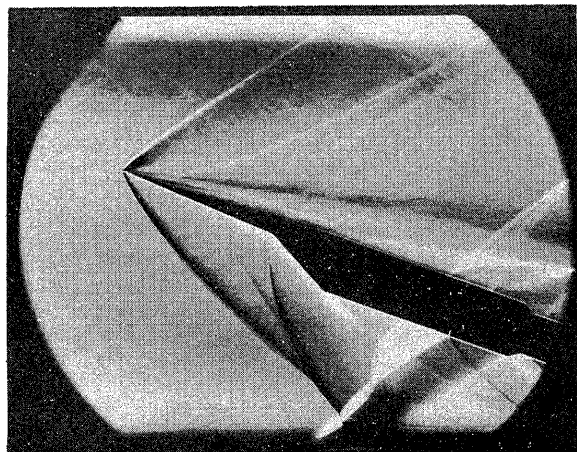


$M = 1.120$

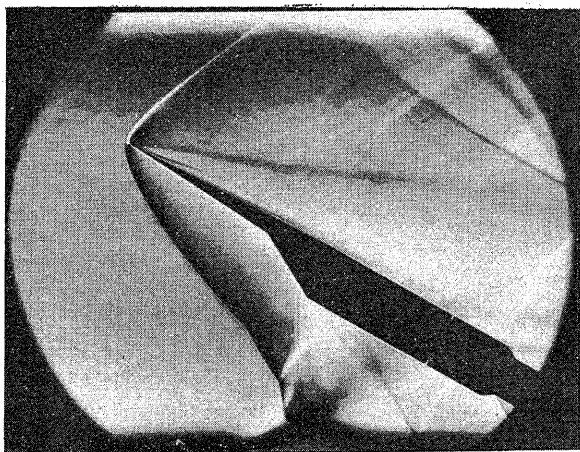
FIG. 17. CROSS FLOW PAST A CIRCULAR CYLINDER IN THE HYDRAULIC ANALOGY TANK AT VARIOUS EQUIVALENT MACH NOS.



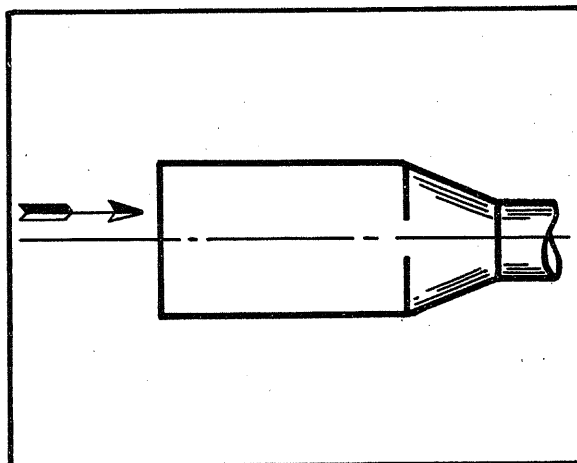
10°



20°



30°



PLAN OF WING

FIG. 18. SCHLIEREN PHOTOGRAPHS OF WOODEN MODEL OF WING ALONE AT ZERO ROLL AND VARIOUS ANGLES OF INCIDENCE.

Z.9213.R.

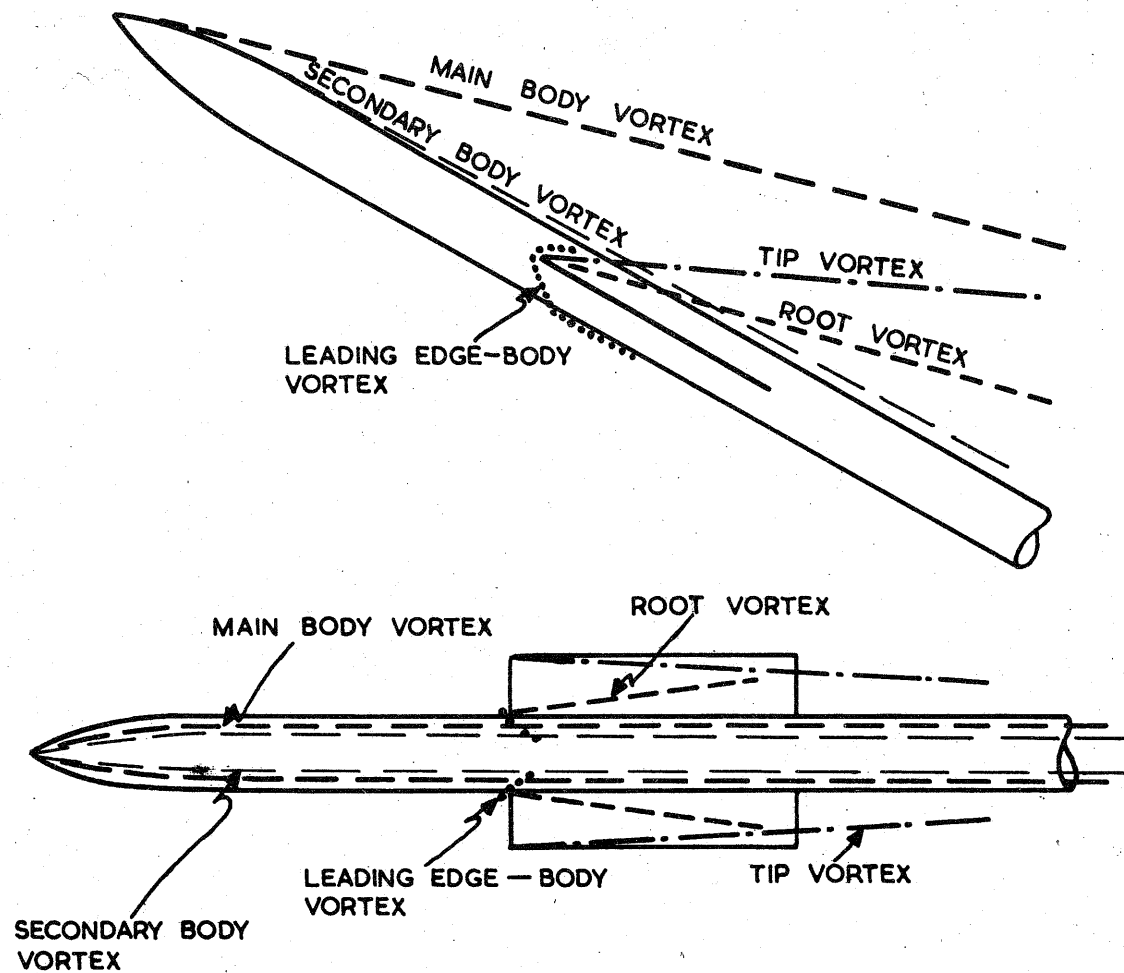
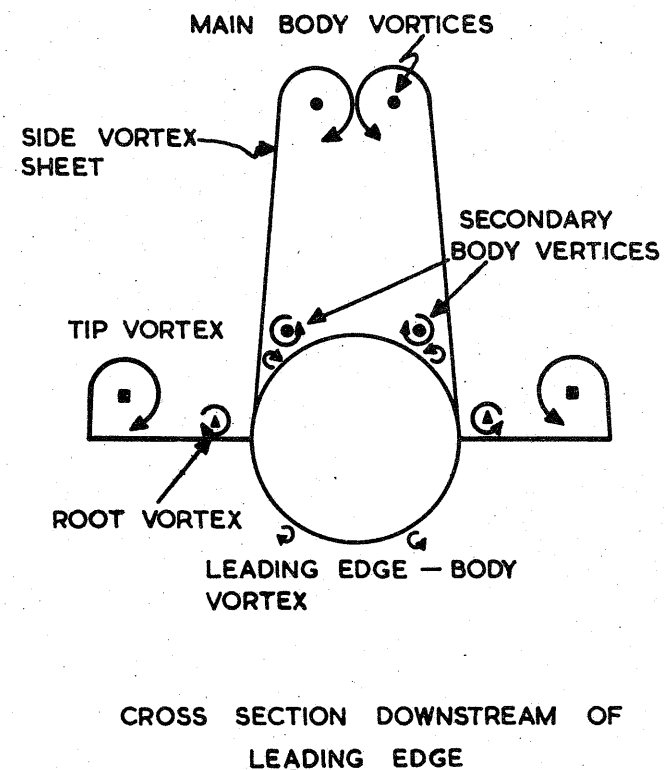


FIG. 19. APPROXIMATE CONFIGURATION OF VORTICES ASSOCIATED WITH A WING BODY COMBINATION AT INCIDENCE.