

CRANFIELD UNIVERSITY

Mohammad Naghavi Zadeh

Static and Dynamic Characterisation of the Zero Poisson's Ratio
Fish Cells Metamaterial

School of Aerospace, Transport, and Manufacturing

PhD Thesis
Academic Year: 2017 - 2020

Supervisor: Dr. Iman Dayyani
Associate Supervisor: Dr. Mehdi Yasaee
August 2020

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ABSTRACT

Morphing structures provide the ultimate aerodynamic efficiency for the aircraft wings during different phases of flight. Achieving a continuously deformed and smooth aerodynamic surface requires a sophisticated morphing skin that provides the necessary strain capacity, prevents stiffness augmentation under actuation, sustains aerodynamic loads, and presents anisotropic behaviour. Cellular metamaterials with zero Poisson's ratio (ZPR) are able to satisfy the morphing requirements when used as a core in the sandwich morphing skin. Therefore, a new zero Poisson's ratio metamaterial called "Fish Cells" with compliance in two planar directions is presented that supports the 2D morphing applications. The ZPR behaviour of the Fish Cells metamaterial is studied and proved analytically, numerically, and experimentally. The stress-strain curves of the Fish Cell metamaterial is studied experimentally and effect of geometrical details on the ZPR performance is investigated. In addition, the in-plane and out of plane homogenized equivalent elastic modulus of the Fish Cells are derived. Effect of Poisson's ratio on dynamic characteristics of cellular structures is studied in two parts. First, hexagonal topology is considered that achieves negative to positive range of Poisson's ratios by tuning the geometry. The modal properties of such metastructures are studied analytically, numerically, and experimentally. As a result, it is shown that modal properties including the damping ratio, natural frequencies and mode shapes can be tuned by tailoring the Poisson's ratio. Second, the dynamic analysis of the Fish Cells metamaterial is considered where effect of gradient properties and pre-stress on the modal parameters are studied. It is found that under morphing strains, the ZPR is a crucial factor to save the stability of the cellular core. In addition, it is shown that Fish Cell geometry enables gradient properties that help in tuning modal properties while the global ZPR behaviour is maintained.

Keywords:

Zero Poisson's ratio, Metamaterial, Cellular material, Morphing skin, Fish Cell, Vibrations, Gradient properties, Damping

ACKNOWLEDGEMENTS

To my family who supported me through this journey...

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LIST OF ABBREVIATIONS

AAW	Active Aeroelastic Wing
EMA	Experimental Modal Analysis
FDM	Fused Deposition Modelling
FE	Finite Element
FishBAC	Fish Bone Active Camber
HECS	Hyper Elliptic Cambered Span
HSDT	Higher Order Shear Deformation Theory
MAW	Mission Adaptive Wing
MCST	Modified Couple Stress Theory
NPR	Negative Poisson's Ratio
PLA	Poly Lactic Acid
PPR	Positive Poisson's Ratio
UAV	Unmanned Aerial Vehicle
ZPR	Zero Poisson's Ratio

1 Introduction

Sustainable and green aviation is the most important aim of the current research and investment in aerospace industry. This is conceived by efforts dedicated in development of technologies that help in increasing the efficiency of flight and reducing the fuel consumption and emissions. Current trending attention and investments in green aviation are revolving about hybrid electric and fully electric propulsion systems that reduce the carbon and NO_x emissions significantly or further optimizing the existing jet engines. However, the propulsion system, although very effective, is not the only source of efficiency improvement and emissions reduction. As traditionally known, the four forces of flight including weight, lift, thrust, and drag have their independent role in adjusting the efficiency of the aircraft flight; therefore, there are three other parameters to adjust for improving the efficiency except thrust.

Weight is a crucial parameter that results in higher efficiency when it is reduced since there is less fuel burnt to carry a unit weight across a unit distance considering a constant unit of thrust. The term weight here refers to the empty weight of the aircraft because the operators cannot make profit from that comparing to the cargo and passenger weight. Most of the studies and research in weight reduction has been dedicated to structural optimization and developing composite structures that have larger specific stiffness and energy absorption than metals and reduce the final structural weight fraction.

Adjusting the lift and drag coefficients of an aircraft depends significantly on the wing planform and aerofoil design. The lift to drag ratio is the main design parameter considered in the wing design and optimization for different phases of flight. Each phase of flight such as take-off, climb, cruise, loiter, descend, and landing have specific requirements in terms of lift and drag that wing design must meet. Therefore, a geometrically variable planform and cross section is required that changes shape in accordance with each phase's requirements. In this regard, moving surfaces including trailing edge flaps, leading edge slats, and ailerons are the most common features for lift and drag control in the commercial aircrafts flying today. In airplanes with more expanded flight

regimes like supersonic fighters, additional shape changes such as variable sweep and variable air intake and exhaust nozzle are also considered. Although the geometrical adjustments applied using these moving surfaces meet the requirements of different airplanes in different phases of flight, they can still become more efficient in terms of lift to drag ratio. Because the wings of current commercial airplanes can obtain a limited number of shapes due to the moving surfaces' limits. Therefore, researchers have pursued concepts with continuously variable geometry and aerodynamically smooth surface for wing design, which is mostly known as "Morphing Structure".

1.1 Morphing structures

There is not a unique and global definition existing for morphing structures, however, there are certain characteristics for a structure mentioned in the literature to be interpreted as a morphing structure. One important feature jointly pointed in the literature is the shape changing capability and adaptation (Sofla et al., 2010; Weisshaar, 2013). The adaptation refers mainly to the reconfiguration of the wing shape and planform to the optimum state in each phase of flight (Ferguson et al., 2007). Therefore, the morphing aircraft can achieve the optimum efficiency in all phases of flight, while current airplanes are optimized for a single phase of flight such as cruise. However, this advantage can be achieved when multidisciplinary system design approach is considered because the overall advantage of morphing must overcome the additional challenges incurred by weight and complexity of the systems (Friswell, 2012). These challenges for morphing aircraft design are further discussed in literature (Jha and Kudva, 2004). As these challenges are extensive for a fully morphing wing design, there are only certain wing planform parameters considered for shape adaptation that have the largest effect on an aircraft's efficiency depending on its mission profile. Therefore, categorization of morphing wings is explicitly based on the geometric parameter that is changed during flight.

A classification of morphing wing categories is presented by Barbarino et al. as shown in Figure 1-1, which considers planar shape, out of plane shape, and cross sectional shape of the wing (Barbarino et al., 2011).

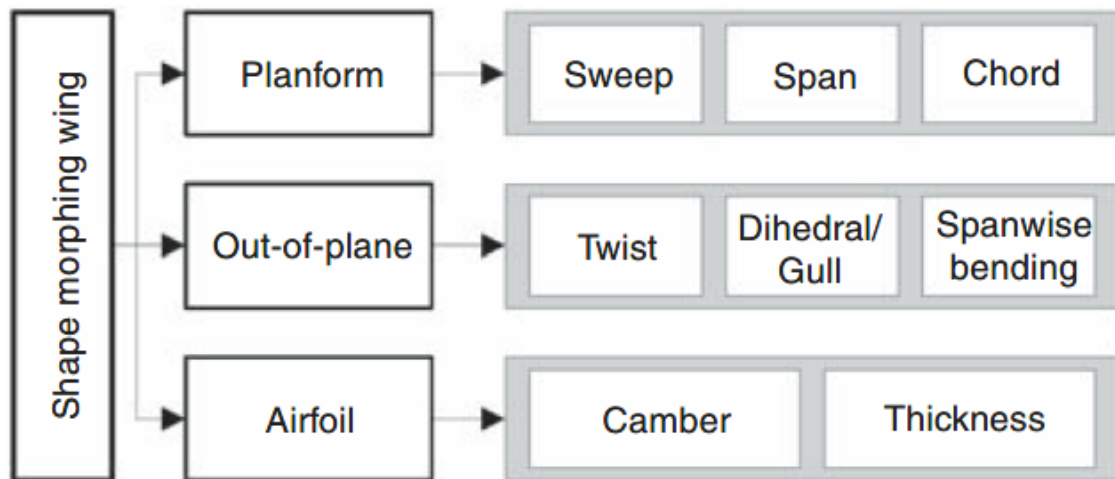
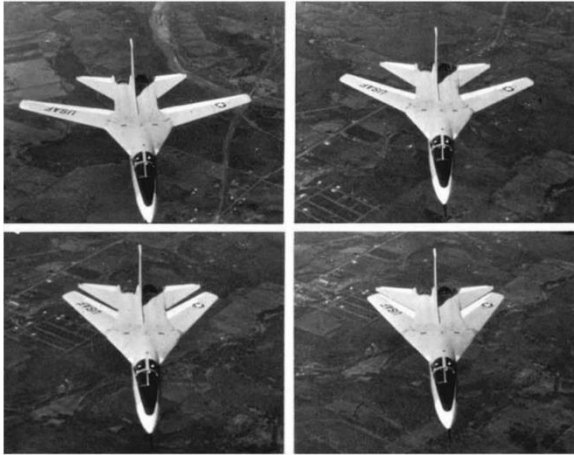


Figure 1-1 The classification of the morphing wing categories presented by (Barbarino et al., 2011)

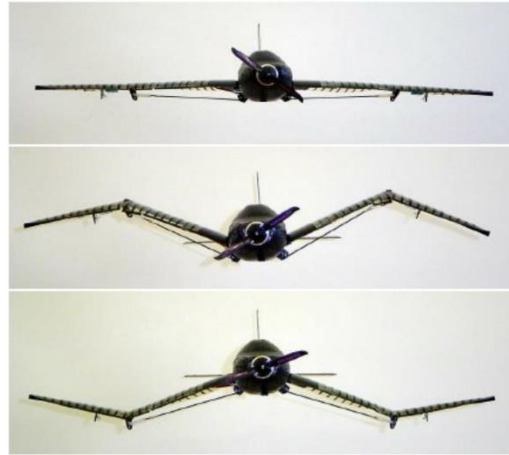
Changing the chord length is achieved in current commercial aircrafts using trailing edge flaps, which also modify the aerofoil camber simultaneously. The sweep morphing has been a common feature among supersonic fighter jets such as General Dynamics F111 as shown in Figure 1-2(a) due to different aerodynamic requirements in subsonic and supersonic flight conditions. This feature is achieved by attaching the wing to the fuselage through a hinge that supports the rotation of the wing and transfers the load to the fuselage. Gull wing has been used in manned airplanes in the stationary state, however, the morphing gull wing is studied with UAVs only as shown in Figure 1-2(b) (Abdulrahim and Lind, 2004) because large and complex actuation system that supports wing bending moment is required. Two types of span morphing has been investigated in experimental aviation as shown in Figure 1-2(c): span extension and span bending. The span morphing concepts such as Hyper-Elliptic Cambered Span (HECS) wings have received attention because they can help in reducing the induced drag (Lazos and Visser, 2006).

The twist morphing has been one of the first methods employed by Wright brothers to control the roll attitude of the aircraft. Although current commercial airplanes use ailerons to control the rolling moment, there are studies ongoing about twist morphing using concepts such as smart structures or cellular lattice structures as shown in Figure 1-2(d) (Jenett et al., 2017).

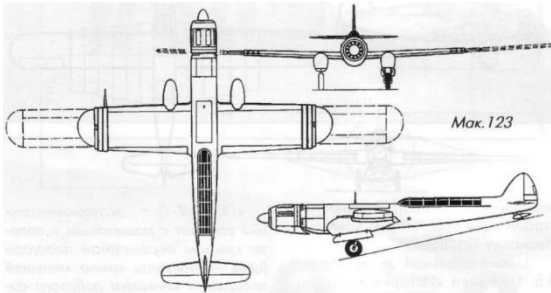
a) Sweep morphing: F111



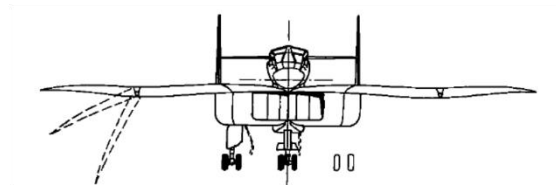
b) Gull wing



c) Span extension: MAK-10



Span bending: XB-70 Valkyrie



d) Twist morphing



e) Camber morphing

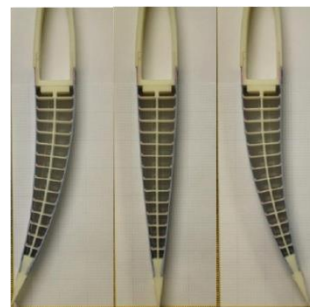


Figure 1-2 Different morphing configurations: (a) variable sweep, (b) variable dihedral (Abdulrahim and Lind, 2004), (c) variable span including extension and bending, (d) variable twist angle (Jenett et al., 2017), and (e) variable aerofoil's camber (Woods and Friswell, 2012).

The simplicity and efficiency of the camber morphing comparing to the other morphing configurations has grown interest among research groups, which has resulted in development of new concepts such as Fish Bone Active Camber morphing (FishBAC) as shown in Figure 1-2(e) (Woods and Friswell, 2012). The camber morphing increases the aircraft performance at aerodynamic design point and beyond it (Renken, 1985). There are several experimental projects developed to study the effect of camber morphing on flight performance as shown in Figure 1-3 including Mission Adaptive Wing (MAW) (Bonnema and Smith, 1988; Gilbert, 1981), Active aeroelastic Wing (AAW) (Pendleton et al., 2007), and FlexFoil equipped wing (Kota, Flick and Collier, 2016).

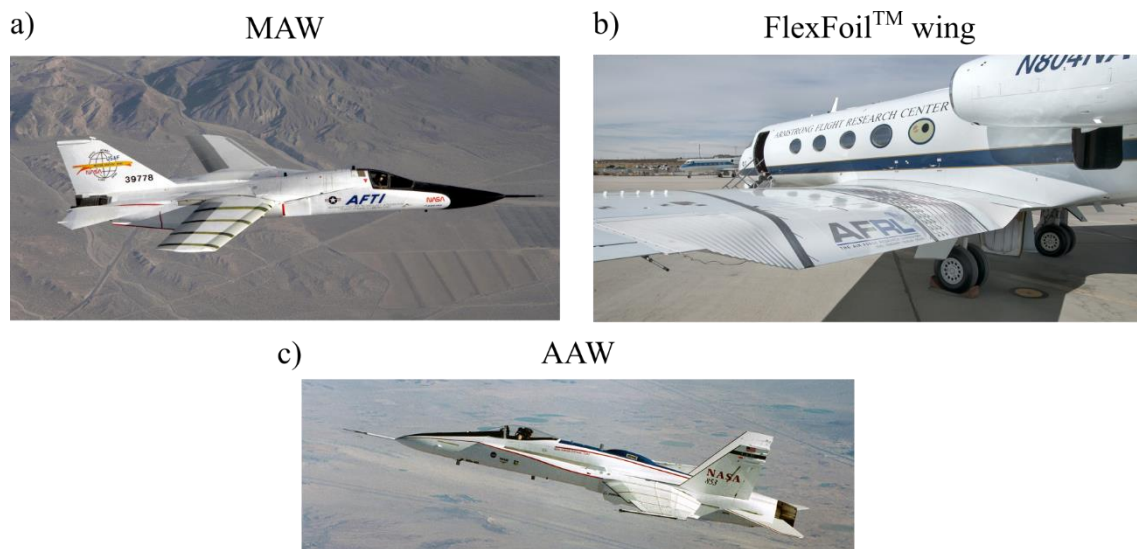


Figure 1-3 Experimental camber-morphing demonstrators: (a) Mission adaptive wing (MAW), (b) FlexFoil equipped wing, (c) Active aeroelastic wing (AAW)

The camber morphing wings are also able to perform twist morphing when modification of the camber along span is possible. This feature is achieved for example by using new manufacturing techniques such as 3D printing that enables producing monolithic parts for the morphing wing (Fasel et al., 2020) or using the cellular lattice structures (Jenett et al., 2017).

Although each of the morphing categories mentioned provide aerodynamic and flight performance improvements, they face many challenges that must be solved such as the integration of compliant and stiff structural components, low actuation force, and compliant and smooth skin with sufficient elastic strain

capacity to support morphing, while stiff enough to withstand aerodynamic loads (Friswell et al., 2006). Among all these challenges, one joint feature in morphing categories like camber, twist, and continuous span bending (such as HECS) is the suitable skin. Since smoothness and continuous deformation of the skin is important in achieving the target configuration in these morphing categories, therefore it should be studied precisely.

1.2 Morphing skin characteristics

An appropriate skin for morphing aerospace structures can be considered from different aspects to study including constructing material, installation, static deformation, and dynamic response. In addition, the morphing mechanism developed in the structure requires the skin to accompany with shape changes; therefore, the skin should have enough strain capacity or employ a deformation mechanism that accommodates and follows the shape changes. A review of the morphing skins by Thill et al. (Thill et al., 2008) discusses different materials and actuation mechanisms. Focusing on the skin from material and structural point of view, there are two main categories discussed and developed in the literature: deployable structures and anisotropic sandwich skins. The concept of deployable structures is similar to what naturally happens in the bird's wings where feathers can move to change the shape of the wing (Matloff et al., 2020). Deployable skins are mostly based on origami and kirigami designs (Ning et al., 2018) with applications in different engineering disciplines including aerospace morphing because, for example in origami structures, the skin is continuous and the stress due to deployment is largely concentrated at crease lines (Zhai, Wang and Jiang, 2018). Therefore, large deployments are achieved with reasonable actuation force; however, the surface is not smooth in all stages of the deployment unless the structure is fully deployed, i.e. during the folding/unfolding process. In order to provide continuous and aerodynamically smooth surface in all deformation ranges, sandwich skins are a promising solution; however, the range of deformation they can achieve might be smaller than deployable skins. Nevertheless, in morphing categories such as camber morphing the elastic strain capacity of about 2% is sufficient and meets the

shape changing requirements (Gandhi and Anusonti-Inthra, 2008). Therefore, sandwich skins are an appropriate candidate for such a morphing category because they provide the aerodynamic smoothness, support continuous shape changes, and exhibit anisotropic behaviour considered for the loading and actuation requirements.

In terms of materials, the sandwich skin consists of a compliant outer layer that provides aerodynamic smoothness and a core that supplies bending stiffness. The materials considered for the outer layer are mostly elastomers that present large strain capacity and a satisfactory surface smoothness, which are studied in detail by Kikuta (Kikuta, 2003). While the outer layer ensures low flow friction, core provides bending stiffness that transfers aerodynamic loads into the wing frame. Although the core must have large transverse stiffness, its in-plane stiffness must be minimized in favour of low actuation load. It should also maintain its stiffness over the morphing range, i.e., stiffness does not augment significantly.

The main solutions offered in the literature for a suitable core in sandwich morphing skin are corrugated core (Xiong et al., 2019) and cellular lattice core (Wadley, 2006) as shown in Figure 1-4. The corrugated cores provide the anisotropic behaviour by reducing the extensional and bending stiffness in the corrugation direction, while increasing them in the transverse direction (Dayyani et al., 2015b). Bidirectional corrugated cores (Ray, 1996) are also developed that provide compliance in two directions for morphing applications in two directions. Corrugated structures can have different cross sectional geometries including trapezoidal, sinusoidal, etc. These geometries can tailor the elastic properties of the corrugated panel such as in-plane stiffness and transverse shear stiffness (Airoidi et al., 2018). The cellular cores provide larger design space as the lattice designs can be either 2D such as honeycombs or 3D such as Kagome and pyramidal lattices. The 2D lattice cores are produced by extruding a cross sectional geometry tailored for a specific behaviour such as hexagonal that results in honeycomb. One advantage of the cellular cores like honeycomb over corrugated cores is preserving the out of plane thickness

under large deformations. Therefore, the moment of area is saved and bending stiffness is less affected. In terms of geometry, the cellular cores with properties that do not exist in natural materials and are suitable for the morphing applications can be designed, which are categorized as extremal mechanical metamaterials that can provide exotic characteristics such as negative or zero Poisson's ratio.

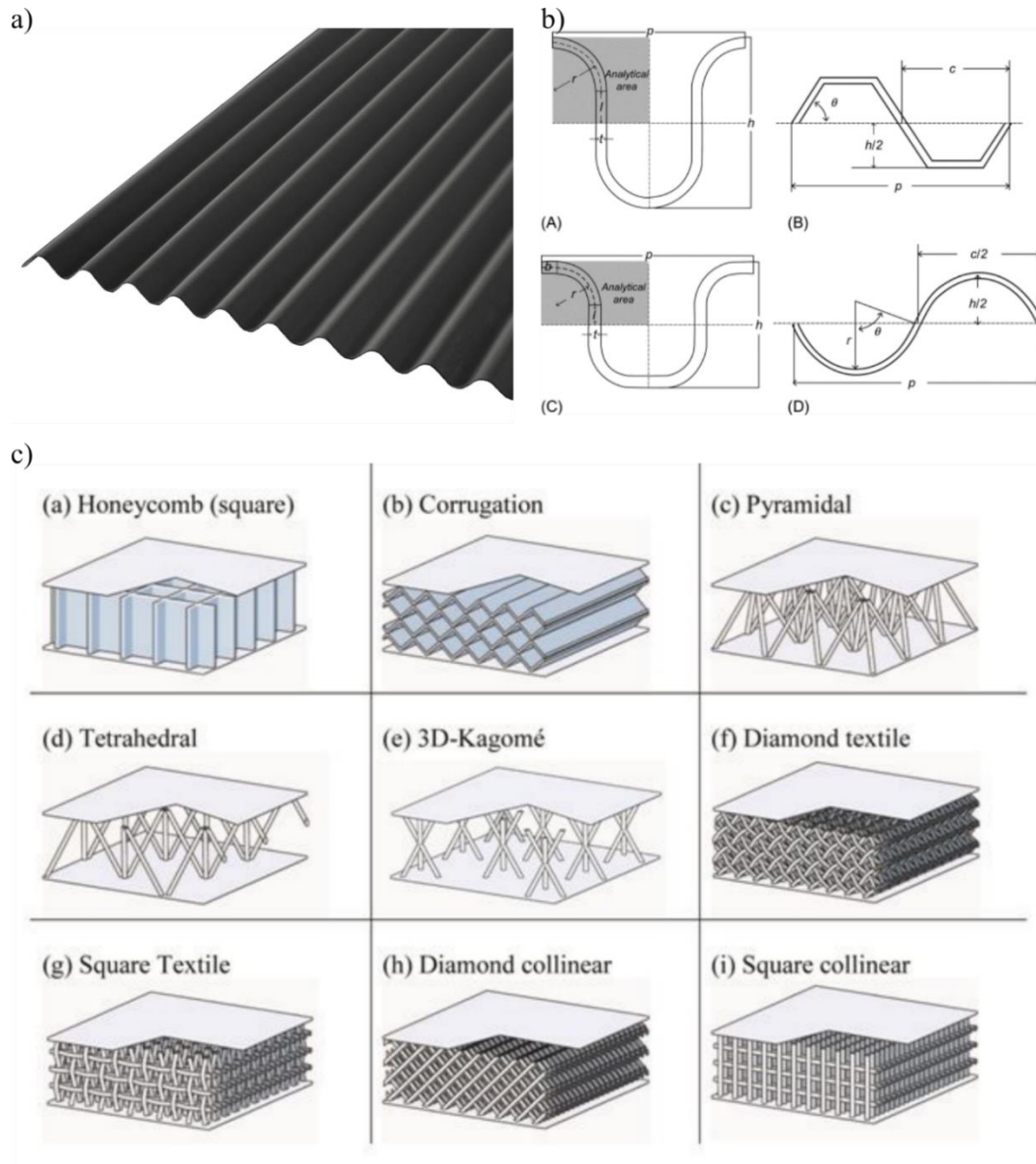


Figure 1-4 (a) A corrugated laminated panel and (b) different corrugation geometries (Airoidi et al., 2018). (c) Cellular core topologies including truss lattice cores and honeycombs (<https://www2.virginia.edu/ms/research/wadley/cellular-materials.html>).

1.3 Metamaterials as cellular cores

Metamaterials are generally synthetic multifunctional materials that can provide tailored exotic properties in different disciplines including mechanics, acoustics, and electrodynamics (Lincoln et al., 2019). The mechanical metamaterials present mechanical properties that cannot be found in the natural materials such as negative Poisson's ratio (NPR or auxetics), zero Poisson's ratio (ZPR), negative or zero thermal expansion, etc., which are mostly achieved by tailoring the geometry of the representative unit cell of the structure (Bertoldi et al., 2017b). For example, honeycomb, which is a well-known cellular core in aerospace industry, can be considered as a metamaterial because it can achieve large Poisson's ratio ranges (larger than one) that natural materials cannot achieve (Greaves et al., 2011). In addition, the hexagonal geometry that honeycomb has, can be tailored by changing its ligaments' angle to achieve negative (re-entrant) or zero Poisson's ratio (Kolken and Zadpoor, 2017a).

The re-entrant design has been studied vastly in the literature because of the negative Poisson's ratio. The interest behind negative Poisson's ratio is its ability to increase the volume while expanded and densify further when squeezed. This high expansion capacity caused by negative Poisson's ratio has raised interest in aerospace applications such as thin and light composite structures that can achieve large areas after deployment (Sakovsky and Ermanni, 2020). Under compressive loads, densification allows for higher energy absorption by auxetic structures; therefore, in components that impact is one of the loading scenarios, internal auxetic cellular design would be an advantage (Qi et al., 2017). However, for morphing skin application, there are two major requirements that must be met: alleviating stiffness augmentation under deformation and providing the highest geometrical conformability. In addition any type of cellular core including conventional honeycombs or tailored metamaterials present a specific value of the Poisson's ratio. Therefore, the role of Poisson's ratio in dealing with these two requirements should be investigated.

Figure 1-5 presents a graphical description of two major types of static loadings applied on a panel where the Poisson's ratio has three values: negative, zero,

or positive. Under planar tension, the negative and positive Poisson's ratio panels experience transverse deformations but in opposite directions. However, the ZPR sample maintains its lateral dimension and no transverse deformation occurs. Similarly, under out of plane bending moment, the negative and positive Poisson's ratio panels expose induced curvatures in the transverse direction but with opposite signs; while, the ZPR panel presents no induced curvature in the transverse direction.

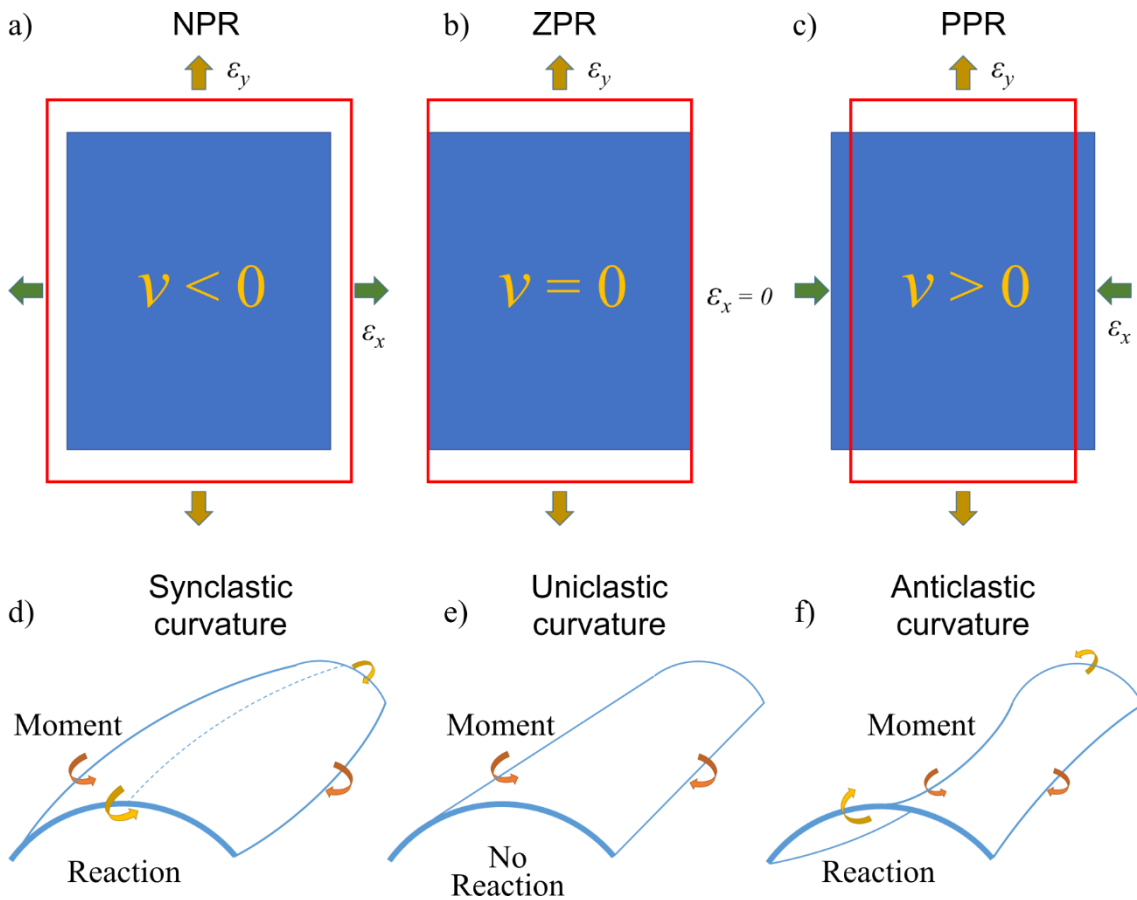


Figure 1-5 Effect of Poisson's ratio on the transverse deformation under tension for (a) auxetic, (b) ZPR, and (c) positive Poisson's ratio structures. The bending deformations of a flat panel with (d) NPR, (e) ZPR, and (f) positive Poisson's ratio

Considering the morphing requirements mentioned above and assembly of the morphing skin with the frame structure, transverse displacements cause reaction loads when lateral edges are constrained. Therefore, the stiffness of positive Poisson's ratio structure under tensile loading increases, while auxetic structure may face buckling instability due to compressive loads induced by the

expansion in the transverse direction. Therefore, the ideal choice for in-plane loading is ZPR because no reaction load on lateral edges (when constraints exist) would be imposed under tensile or compressive loads, hence, neither stiffness augmentation nor instabilities will not occur.

Assuming a set of actuators control the aerodynamic shape of the skin to change the curvature of the aerofoil into the optimum state. In case of negative and positive Poisson's ratio, the induced curvature caused by the Poisson's effect results in lack of full control over the optimal curvature. Thus, larger number of distributed actuators and sensors might be required to control the overall curvature of the skin. This means that negative and positive Poisson's ratio prevent the full geometric conformability of a morphing skin, which is not desired. In contrast, the ZPR panel provides geometric conformability because the curvatures in two orthogonal directions are decoupled and independent of each other.

As a conclusion, zero Poisson's ratio is the desired elastic property that must be pursued for cellular lattice cores of the sandwich morphing skins to ensure both stiffness augmentation alleviation and geometric conformability.

1.4 Zero Poisson's Ratio (ZPR) Metamaterials

Chronologically, Olympio and Gandhi introduced the zero Poisson's ratio hybrid honeycomb and accordion design for the first time as desired core topologies for morphing skin applications (Olympio and Gandhi, 2007, 2010a). The hybrid honeycomb was formed by arranging hexagonal and re-entrant cells side by side, while accordion design was formed by attaching the solid straight strips by bending compliant ligaments. The geometry of solid strips used for designing the accordion core is an effective parameter in determining its bending stiffness and formability (Huang et al., 2016b, 2017a). However, the existence of a continuous solid strip imposes a limit on its strain capacity in the strips direction, which is not a concern in one directional morphing applications. Therefore, other designs based on the accordion topology have been pursued for 1D morphing applications where the main difference has been the design and arrangement of bending ligaments such as chevron-shaped ligaments (Bubert

et al., 2010), cosine shaped optimized ligaments (Dove-Jay, 2015), modified semi re-entrant design (Chen and Fu, 2018), and mixed cruciform geometry (Zou and Zhou, 2017). All these designs share the ZPR property whereas effective elastic properties differ.

The strain capacity limit of accordion design is not observed in hybrid honeycomb because the deformation mechanism of the cell in both planar directions is based on bending of the ligaments. Therefore, the ZPR metamaterials like hybrid honeycomb can meet the strain capacity required for two directional morphing; however, lack of ZPR in both directions as well as compliance is an issue. This condition of 1D-ZPR/2D-compliance is also present in semi re-entrant design (Grima et al., 2010) and Silicomb design (Lira et al., 2009). Therefore, the 2D-ZPR/2D-compliant cellular designs are required, which have been proposed in a few cellular designs (Gong et al., 2015; Wang et al., 2014). Hence, metamaterials with compliance and ZPR in two directions require more attention because they provide more flexibility in optimizing the elastic properties for designers. A detailed review and classification of the ZPR metamaterials is presented in chapter 2.

1.5 Thesis objectives

The advantages of the sandwich morphing skins with ZPR core is recognized. Although ZPR property is introduced as an important feature required for morphing applications, integration of the morphing skins with the frame and effect of local deformations on the integrity efficiency and morphing requirements is not clearly discussed in the literature. In addition, the geometric details of the design such as thickness of the joint might results in deviation from design properties and ZPR itself (Huang, Liu and Tang, 2018). Therefore, the deformations of the ZPR geometry must be studied in detail using full-scale solid models in addition to simplified models like beams.

Considering the morphing application for the sandwich skin, there are considerable loading conditions in aerospace that they have dynamic nature rather than static. Therefore, the performance of a morphing skin should not be based only on static deformations and dynamic characteristics must be also

considered. In this regard, effect of the ZPR design and skin integration on natural frequencies, mode shapes, and damping of the cellular core must be studied systematically.

In order to provide a flexible and novel design for a 2D-ZPR/2D-compliant core, designing and proposing a new metamaterial is considered. The satisfaction of the morphing requirements for this design from both static and dynamic points of view should be studied. Variation of elastic properties with geometrical parameters would be studied through derivation of equivalent elastic properties of the proposed metamaterial. The effect of constraints due to manufacturing limits such as minimum thickness and size effect in static and dynamic analysis also should be considered.

1.6 Summary of chapters

The overall flow of the thesis is about studying the static and dynamic characteristics of the candid core designs for morphing skins. In general, chapters 2 and 3 discuss the static analysis and equivalent model of a new zero Poisson's ratio metamaterial proposed by the author and Chapters 4 and 5 cover the dynamic analysis of the mechanical metamaterials tuning the Poisson's ratio.

In chapter 2, a detail review of the zero Poisson's ratio metamaterials in the literature is presented and a categorization for this class of metamaterials is presented. The importance of developing metamaterials with compliance and zero Poisson's ratio in both planar directions is discussed and a new geometry that meets this requirement is proposed. The new metamaterial called "Fish Cells" is studied using numerical and experimental methods to evaluate its zero Poisson's ratio property and to study the geometrical features that affect deformation mechanism significantly. In addition, the stiffness augmentation as an important criterion in design of the morphing skin is investigated using numerical analysis. The performance of the Fish Cells metamaterial is proved satisfying, where effective stiffness of the structure at strained and free conditions are found equal.

Chapter 3 presents analytical and numerical study of the equivalent elastic properties of the Fish Cells metamaterial. The metamaterial unit cell is modelled as a slender beam structure and appropriate loading and boundary conditions for six equivalent modulus are considered where energy method and Castigliano's theorem are employed to derive the elastic modulus. A detailed parametric study is done to evaluate the effect of geometrical parameters on the elastic properties. In addition, the zero Poisson's ratio of the metamaterial is proved analytically and it is shown that modified geometries that satisfy the required geometric relations of the Fish Cells topology can also achieve zero Poisson's ratio.

In chapter 4, dynamic response of the metamaterials tuning the Poisson's ratio is studied where negative, zero, and positive Poisson's ratio are achieved using hexagonal topology. The scope of the study includes natural frequencies, mode shapes, and damping ratio. The study includes analytical, numerical, and experimental investigation of the 3D printed samples made from poly lactic acid (PLA). This chapter provides an insight on how to tune the damping properties by manipulating the geometrical parameters of the metamaterial where a correlation between the Poisson's ratio and the damping ratio is proposed. In addition, an important parameter less considered in the literature was size effect in cellular metamaterials. Due to the manufacturing limits, the size effect in the cellular structures like honeycombs and metamaterials are inevitable. Therefore, the effect of the length scale on the stiffness and dynamic response of the metamaterials is studied and the trend of the unit cell geometry with length scale parameter is studied based on the modified couple stress theory.

Chapter 5 focuses on the dynamic characteristics of the Fish Cells metamaterial and its geometrical derivatives. Fish Cells has ability to achieve various elastic modulus ranges while unit cell size is constant and zero Poisson's ratio is maintained. This feature, allows for creating zero Poisson's ratio metastructures with different spatially gradient in-plane stiffness such as soft-hard, soft-hard-soft, and hard-soft-hard functions. This is an important feature because variable stiffness materials are a candid design for morphing skins. Therefore, dynamic

properties of the Fish Cells metamaterial and its gradient derivatives are studied numerically and experimentally.

The conclusions obtained from the overall study conducted in this thesis is presented in chapter 6. Zero Poisson's ratio as an important property for candid core structures of sandwich morphing skins is discussed. The performance of the Fish Cells metamaterial under static and dynamic loadings is discussed and suggestions for improving the structure are presented.

1.7 Research Contributions

The research results presented in this thesis have led to preparation of four journal papers where two papers are accepted and published by the time this thesis is submitted. The list of papers are presented in Table 1-1.

Table 1-1 the contributions of this research in terms of peer reviewed journal publications

Paper	Status
Fish Cells, a New Zero Poisson's Ratio Metamaterial. Part I: Design & Experiment	Accepted doi.org/10.1177/1045389X20930079
Fish Cells, a New Zero Poisson's Ratio Metamaterial. Part II: Elastic Properties	Accepted doi.org/10.1177/1045389X20942576
Dynamic Characterization of Mechanical Metamaterials with Tuneable Elastic Properties	Ready to submit
Dynamic Analysis of a Zero Poisson's Ratio Metamaterial: Effect of Pre-stress and Gradient Properties	in preparation

2 “Fish Cell”: A New Zero Poisson’s Ratio Metamaterial

2.1 Introduction

Metamaterials refer to materials designed in micro/nanoscale to expose exotic behaviour in the macroscale. Mechanical metamaterials can tune mechanical properties such as elastic modulus, bulk modulus, Poisson’s ratio, coefficient of thermal expansion, and dynamic response of the structure. The smallest fundamental representative volume element or unit cell of the metamaterial is designed with a specific geometry to be repeated periodically in two or three dimensions and create large-scale structures with the desired tuned property (Bertoldi et al., 2017a). Recent advances in additive manufacturing have enabled the fabrication of structures with arbitrarily complex micro-architecture that has attracted increasing attention to the development of mechanical metamaterials (Chen and Zheng, 2018; Jiang and Wang, 2016; Vyatskikh et al., 2018; Zheng et al., 2014).

Mechanical metamaterials with tuned Poisson’s ratio can possess negative or zero values (Clausen et al., 2015). According to the mathematical definition, ZPR metamaterials expose neither transverse displacements under in-plane longitudinal loadings nor double curvature under out of plane bending moments; because there is no coupling between stresses and strains in two orthogonal directions (Grima et al., 2010). Therefore, ZPR metamaterials present single curvature under bending (unclastic) comparing to the anticlastic curvature of positive Poisson’s ratio and synclastic curvature of the negative Poisson’s ratio (Sanami et al., 2014). These unique characteristics are attractive in many applications: in tissue engineering, Zero Poisson’s Ratio (ZPR) scaffolds have similar function as natural tissue providing better wound healing and tissue regrowth (Soman et al., 2012), in amorphous materials like viscous thin films ZPR materials can be tightly rolled to make soft ultrathin coatings without thickness modification requirements or squeezing force increment (Nguyen, Maheshwari and Saraf, 2012), and in textile industries where weaving patterns resulting in ZPR for fabrics can be developed (Zulifqar, Hua and Hu, 2018). ZPR metamaterials presented in the literature can be classified according to

their compliance in a planar context, i.e. how much flexibility in each direction they demonstrate.

ZPR with compliance in one direction mainly exists in designs where continuous solid strips are connected together by inclined ribs that can bend easily compared to the solid strips. The stiffness in the direction of the solid strips is much larger than normal direction where bending of the inclined ribs increases the compliance. Olympio and Gandhi (Olympio and Gandhi, 2007, 2010b) proposed such a geometry, called accordion design, as a core for morphing skins as shown in Figure 2-1(a). Based on accordion topology, several other designs have been proposed that mostly differ in the inclined rib's geometry such as Figure 2-1(b) and 2-1(c) (Bubert et al., 2010b; Chen and Fu, 2018; Huang et al., 2016a; Liu et al., 2013; Rong and Zhou, 2017; Zou and Zhou, 2017). Chen et al. (Chen, Shen and Li, 2015) investigated in-plane mechanical properties of three different ZPR accordion structures with hexagonal, re-entrant and V-shaped ribs. It was shown that these three geometries with the same parameters have the same in-plane elastic modulus but different shear modulus. Although ZPR metamaterials with compliance in one direction may have large in-plane and bending stiffness along the direction of the solid strips, this can be a disadvantage since the structure will be limited to morphing in only one direction.

ZPR metamaterials proposed with compliance in two in-plane directions provide the opportunity for bidirectional morphing. However, some of the designs do not have ZPR in both directions. For instance, Hybrid honeycomb (Olympio and Gandhi, 2007, 2010b) as shown in Figure 2-1(d), Semi Re-entrant honeycomb (Grima et al., 2010) as shown in Figure 2-1(e), and Silicomb (Lira et al., 2009) as shown in Figure 2-1(f) provide ZPR in one direction, while compliant in both directions. Solving this issue, cellular metamaterials with different geometries are proposed with designed bending deformation mechanism of the elements in the unit cell that neutralizes displacements in the orthogonal direction inside the unit cell or in connection with adjacent cells (Gong et al., 2015a; Liu and Zhang, 2018; Mizzi et al., 2018; Wang et al., 2014).

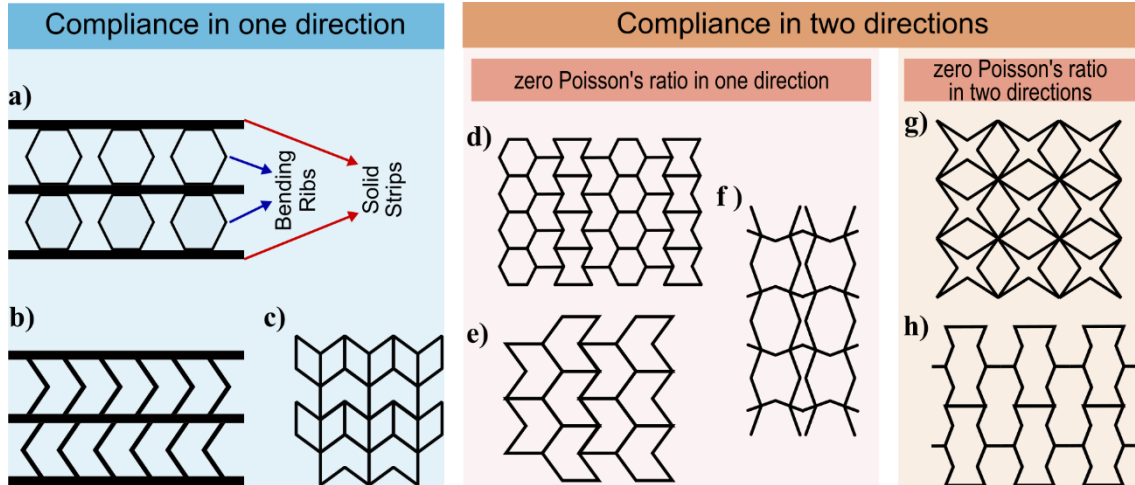


Figure 2-1 Geometrical illustration of different ZPR structures: (a) Accordion design with solid strips connected by bending hexagonal ligaments (Olympio and Gandhi, 2007, 2010; Liu et al., 2013), (b) accordion topology with chevron bending ligaments (Bubert et al., 2010), (c) modified semi re-entrant design (Chen and Fu, 2018), (d) Hybrid honeycomb (Olympio and Gandhi, 2007, 2010), (e) semi re-entrant design (Grima et al., 2010), (f) Silicomb (Lira et al., 2009), (g) a star shape design by Gong et al. (2015), and (h) reverse semi re-entrant design (Wang et al., 2014).

ZPR behaviour can be very valuable for a wide range of engineering applications such as morphing aircraft because of preventing stiffness augmentation and undesired induced curvatures. Although designs for ZPR metamaterials with compliance in two orthogonal in-plane directions are proposed with application in morphing structures (Gong et al., 2015a), integration performance of the metamaterials with the morphing structure and under constraints needs to be studied further.

In this paper, a new metamaterial called Fish Cells is proposed with ZPR and compliance in two directions. Experimental and numerical analysis are used to demonstrate the ZPR behaviour of a homogenized tessellation. Morphing technology is considered as one application of Fish Cells ZPR metamaterial to improve the aerodynamic efficiency and performance of the aircraft (Barbarino et al., 2011). The details of integration study for morphing skin application with local constraints on lateral edges of a homogenized tessellation are conducted in section 2.5. Integration efficiency of the Fish Cells metamaterial is

investigated by stiffness augmentation under tension. Finally, local geometrical enhancements for improved integrity of the Fish Cells metamaterial are presented that result in substantially zero stiffness augmentation, which is required for morphing skins applications.

2.2 Fish Cell Metamaterial

Fish Cells as a novel architecture for metamaterial is shown in Figure 2-2(a) where the unit cell is formed by attachment of two rotated Fish Cells that is similar to the rectangular pattern in 2D Bravais lattices. Six parameters including five parameters in the x-y plane as shown in Figure 2-2(b) and the out of plane thickness, W , are required to define the geometry. The five in-plane parameters are base member length B , inclined rib's horizontal length b , inclined rib's vertical length h , unit cell's height H , and the connector's length C . The uniform thickness of t is assigned to each member, however, the thickness of base members can be different comparing to the bending ribs. The out of plane thickness of the Fish Cells called W , defines the extrusion depth. The values considered for the parameters to study the Fish Cells structure are presented in Table 2-1.

Table 2-1 Values for geometrical parameters of the Fish Cells considered in numerical and experimental studies

Parameter	Value [mm]
B	5
b	3
C	2
H	12
h	3
t	1
W	3

Fish Cells geometry may be considered as a modified assembly of re-entrant and hexagonal cells with negative and positive Poisson's ratio, respectively. Therefore, by using a proper and consistent geometric ratio between re-entrant and hexagonal shapes, the Fish Cells geometry will have ZPR under loading in

the x -direction, i.e., $v_{xy} \cong 0$ as shown in Figure 2-2(c). Fish Cells in a unit cell are positioned next to each other in opposite directions so each hexagonal cell will be connected to the re-entrant cell. As a result, the twin fish in opposite directions cancel each other's displacement in the orthogonal direction (along x -direction) when subjected to tension or compression along the y -direction, i.e., $v_{yx} \cong 0$ as shown in Figure 2-2(d). The tessellation is presented as $m \times n$ where m is the number of unit cells in the x -direction and n is the number of unit cells in the y -direction as shown in Figure 2-2(c) and 2-2(d). Since the unit cell is not symmetric along the y -axis, the tessellations will be also asymmetric in this direction. In order to capture the effects of monoclinic behaviour, i.e., single axis of symmetry, one Fish Cell is added to make the whole tessellation symmetric with respect to the y -axis as denoted by index s in the terminology as shown in Figure 2-2(c). In this paper, the symmetric tessellations are called $(m \times n)_s$ that is discussed in detail in Appendix A. For example, tessellation (10×1) corresponds to 10 and 1 unit cells in the x and y -directions respectively, while tessellation $(10 \times 1)_s$ corresponds to 10 unit cells plus one Fish Cell in the x -direction and one unit cell in the y -direction, respectively. The comparison of the symmetric and asymmetric Fish Cells metamaterials will be discussed in detail in section 2.5.

Moreover, Figure 2-2 highlights Edge_{+y} and Edge_{-y} with normal vectors in $+y$ and $-y$ directions as well as Edge_{+x} and Edge_{-x} with normal vectors in $+x$ and $-x$ directions. This nomination is used for studying local deformations on edges detailed in section 2.5.

The effective elastic modulus is calculated according to Equation 2-1 and Equation 2-2 where P indicates the total reaction force along loading direction, and l_x and l_y are the length of panel in x and y-direction, respectively.

$$E_x = \frac{\sigma_x}{\varepsilon_x} = \frac{P_x}{W \times l_y} / \varepsilon_x \quad (2-1)$$

$$E_y = \frac{\sigma_y}{\varepsilon_y} = \frac{P_y}{W \times l_x} / \varepsilon_y \quad (2-2)$$

To assess the validity of ZPR design, the deformations of the metamaterial in two orthogonal directions is studied using FE analysis.

2.3.2 FEM Analysis

Investigating the response of Fish Cells metamaterial, the structure with geometric parameters presented in section 2-1 is modeled in ABAQUS where elastic constants of Nylon PA2200, as described in section 2-4, with $E = 1130$ MPa and Poisson's ratio $\nu = 0.4$ are used for concept analysis and integration studies. Regarding the boundary conditions, lateral edges are free, all degrees of freedom on the fixed edge are constrained, and displacement in tensile direction is allowed for the moving edge only as shown in Figure 2-2 for each loading direction. Nonlinear effects and strain dependency are neglected and specific displacement is imposed on moving edge according to the tessellation number to achieve a certain strain of $\varepsilon = 0.01$. The metamaterial structure is modeled as a 2D planar part with shear deformable quadratic beam elements B22 and a rectangular cross section with dimensions W , t . Mesh convergence studies are performed, and fine mesh discretization is applied for great accuracy (i.e. less than 0.1% error margin) where the global size of 0.5 mm for each element is chosen.

2.3.3 Homogenization Study

Considering the concept of periodicity and Saint Venant's principle (Mises, 1945) in the theory of elasticity, the behaviour of homogenized tessellation corresponds to sufficiently large tessellations in which the effects of boundary conditions such as stress concentration are minimum (Dayyani et al., 2013).

Therefore, it is important to achieve a tessellation with the right number of cells to represent a homogenized behavior. For this purpose, four structures with clamp boundary conditions at edges including the 5×5, 10×10, 15×15, and 20×20 tessellations are investigated. The previous FEM analysis section describes the simulation parameters of the models.

Figure 2-3(a) demonstrates the homogenization results for ν_{xy} and ν_{yx} calculated based on ABCD rectangle. Considering computation time, the results show that 20×20 tessellation is a good representative of a homogeneous metamaterial and substantially ZPR behavior starts from 5×5 tessellation. Figure 2-3(b) shows the convergence of results for the trend of effective elastic modulus in two directions, where the effective modulus E is normalized with respect to the original constructing material elastic modulus, E_s . **The equivalent elastic modulus of the metastructure is three orders of magnitude smaller than elastic modulus of the constitutive material.** This low stiffness ratio is useful for biomedical applications such as implants where a highly compliant structure is required in interaction with soft tissues (Engelmayr et al., 2008).

The value of Poisson's ratios obtained for 20×20 tessellation are $\nu_{xy} = -7.71 \times 10^{-7}$ and $\nu_{yx} = 9.37 \times 10^{-7}$, which are small enough to be neglected. Hence, the proposed Fish Cells metamaterial has ZPR behavior in two orthogonal directions.

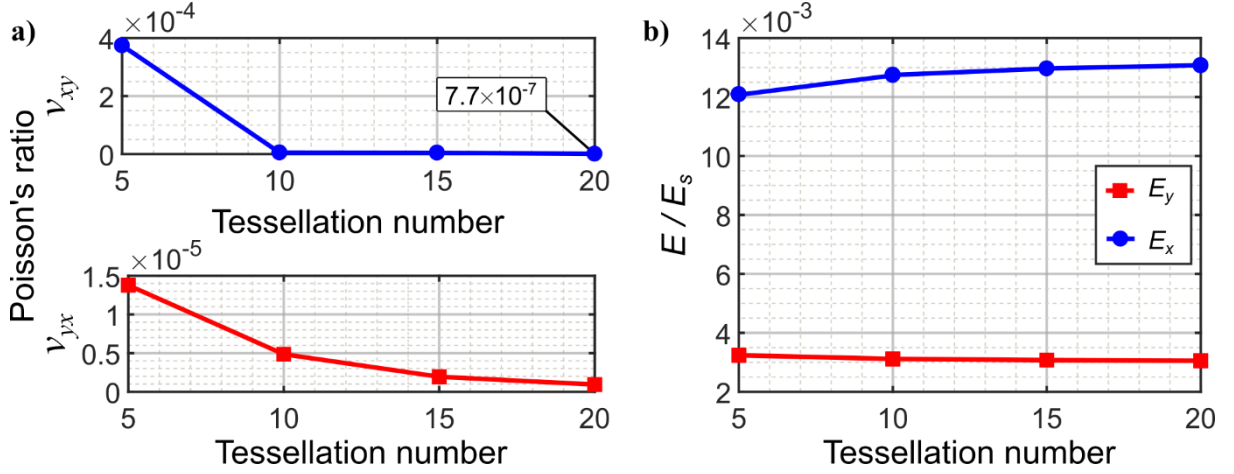


Figure 2-3 Homogenization analysis for 5×5, 10×10, and 20×20 tessellations (a) Poisson's ratio in two directions, and (b) effective elastic modulus in two directions

2.4 Experimental Analysis

2.4.1 Manufacturing

Obtaining homogenous response with minimum boundary condition effects makes manufacturing of test samples a great challenge when the size of Fish Cells should be very small comparing to the sample size. Selective Laser Sintering (SLS) is used for 3D printing of the metamaterials with fine tolerance (EOS P100, 3dprintuk.co.uk). The main limit in the manufacturing of Fish Cells metamaterial is the resolution of the laser for printing minimum thickness of the ribs. The minimum achievable thickness is also dependent on powder grain size for proper bonding and material homogeneity, which in this case was 56 μm on average (Nylon PA2200, EOS GmbH). Hence, the minimum thickness of 1 mm with a manufacturing tolerance of ± 0.1 mm was used.

Considering the manufacturing limitations including minimum thickness and printer desk size, Fish Cells metamaterial samples with $(5 \times 11)_s$ tessellations were manufactured as shown in Figure 2-4(a) and 2-4(b) for v_{xy} and v_{yx} tests, respectively. The reason for selecting a symmetric tessellation number is maintaining the symmetry along y-direction, which is discussed more in section 2.5 of this chapter.

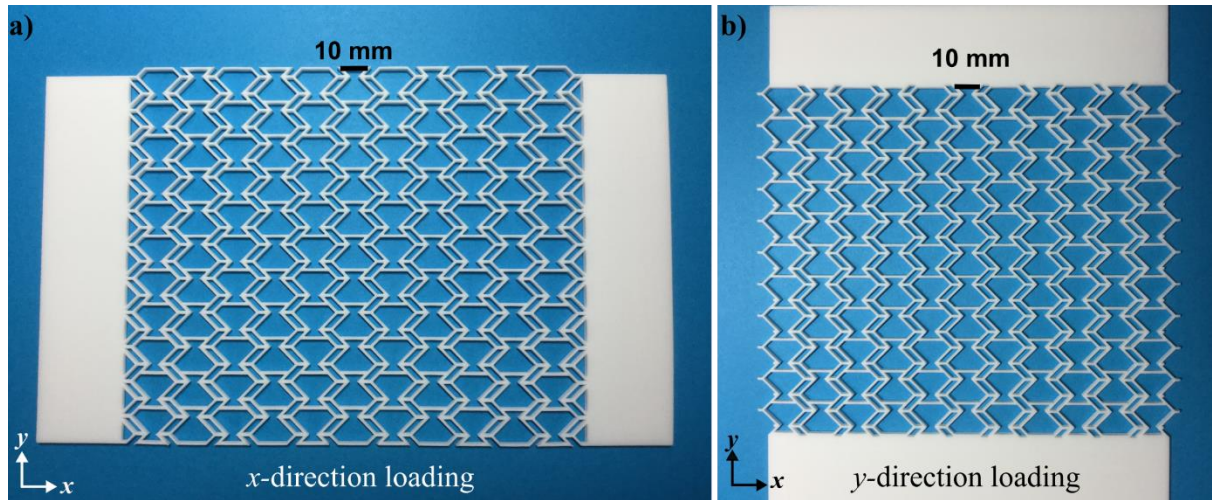


Figure 2-4 3D printed Nylon PA2200 samples for tensile tests in two orthogonal directions for the study of Poisson's ratio (a) ν_{xy} and (b) ν_{yx}

2.4.2 Material Characteristics

In addition to the Poisson's ratio, the force-displacement response of the Fish Cells structure is also considered for investigation. Therefore, the material properties of the Nylon PA2200 as constructing material used in SLS manufacturing are investigated to supply the input data for FEM validation of tests. According to manufacturer datasheet, sintered parts retain the isotropic behavior. To evaluate the mechanical properties of Nylon PA2200, tensile tests are performed on five specimens using Instron 5965 machine with 5kN load sensor. The length, width, and thickness of the specimens were 75 mm, 14 mm and 3 mm, respectively. Figure 2-5 presents experimental stress-strain curves for five test specimens in tension. The average maximum stress reached was about 51.6 MPa, where the sample had a ductile failure at an average strain of 0.27. Poisson's ratio is calculated from the transverse strain to axial strain ratio measured by digital image correlation technique. The calculated average properties from tensile tests are $E = 1130$ MPa and $\nu = 0.4$.

Although slope of the strain-stress curves changes significantly above 40 MPa, the slope is nonlinear even below 20 MPa as shown in Figure 2-5(a). Therefore, cyclic quasi-static tests are conducted on three samples with force limits of 515 N, 695 N and 1000 N. Based on the dimensions of the samples and these

forces, the corresponding stress will be calculated as 10, 14, and 20 MPa. The force-displacements diagrams of three samples at 10, 14, and 20 MPa are shown in Figure 2-5(b). The tenth cycle of case 14 MPa shows the hysteresis initiation, therefore, this stress level is considered for the zero plastic strain (Dayyani, Ziaei-Rad and Friswell, 2014). The average plastic stress-strain values are shown in Figure 2-5(c), where the plastic region is discretized into smaller linear intervals.

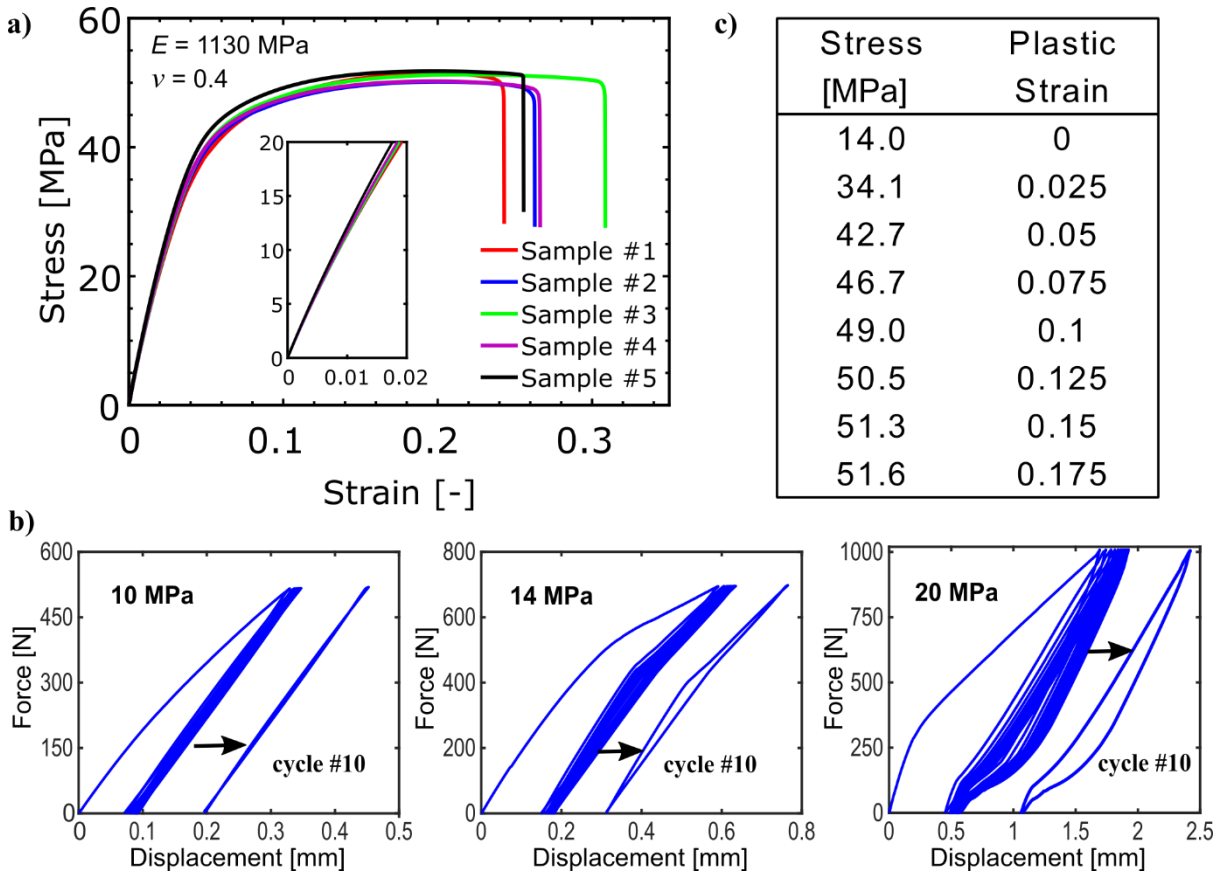


Figure 2-5 (a) Stress-Strain curves of tensile test specimens made from Nylon PA2200, (b) Hysteresis analysis of Nylon PA2200 under quasi-static loading for 10 cycles at different stress amplitudes (c) Nylon PA2200 plastic properties

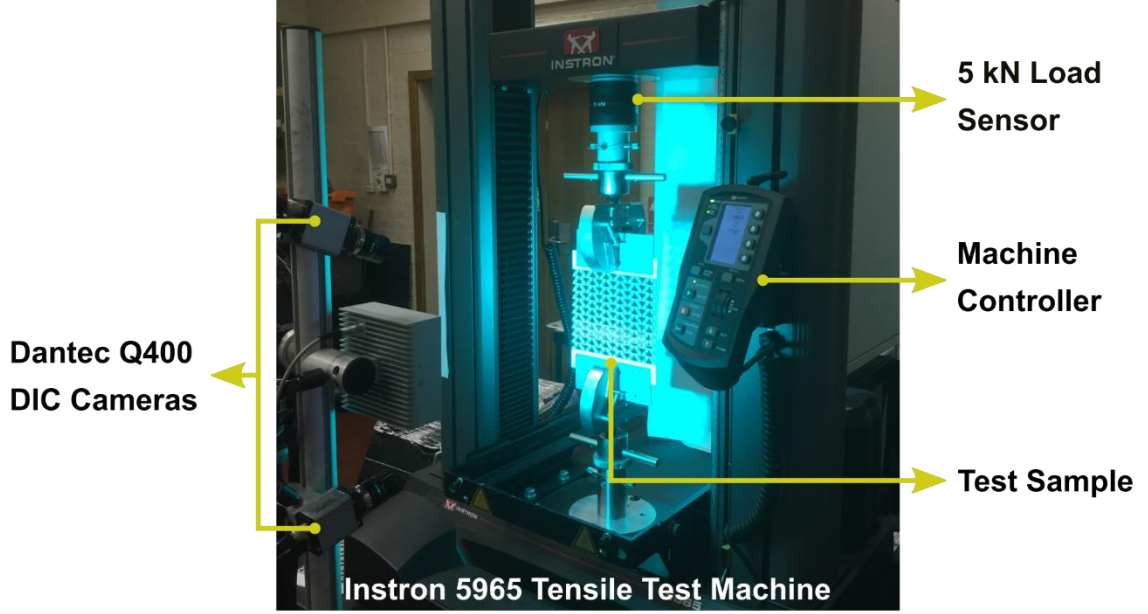
2.4.3 Structural Tests

Measuring displacements and strains for calculation of Poisson's ratio, Digital Image Correlation (DIC) technique is employed by using Dantec Q400 setup. Figure 2-6(a) shows the experimental setup for testing the metamaterial samples in two directions. In each direction, Poisson's ratio is measured using

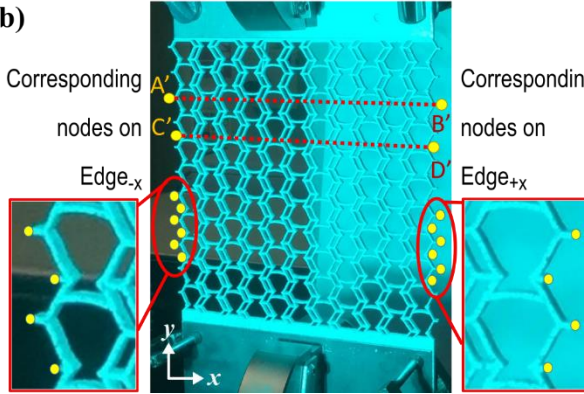
DIC data while force-displacement is directly obtained from the tensile test machine. Location of corresponding nodes on the edges of samples are also highlighted in Figure 2-6(b) and 2-6(c) for integration studies in section 2.5.

Figure 2-7 shows the force-displacement diagrams of metamaterial samples in x - and y -direction. The failure load of experimental metamaterial samples in x and y -direction are 659.3 N and 628.9 N as well as failure displacement of 23.0 mm and 41.7 mm in x and y -direction, respectively. The ultimate displacement capacity of the metamaterial in the y -direction is approximately 1.8 times larger than x -direction. This can be justified by considering the deformation mechanism of the Fish Cell under loading in the x and y -direction as shown in the Figure 2-7(b) and 2-7(c), respectively. Three ribs of a Fish Cell annotated as R_1 , R_2 , and R_3 and the contours of the rotation are plotted in the figures. It is only under loading in y -direction that the rib R_2 rotates and contributes to the total strain. While, under loading in the x -direction, the rib R_2 does not rotate and only equal rotations of ribs R_1 and R_3 contribute to the strain. Therefore, the strain in the y -direction is larger than x -direction for the same stress level. Hence, for applications where large deformations are required, installation of metamaterial with loading along y -direction shall be considered. Moreover, the variation of structural stiffness in the y -direction is smaller than x -direction and maintains stiffness value up to failure, while metamaterial shows softening behavior along the x -direction.

a)



b)



c)

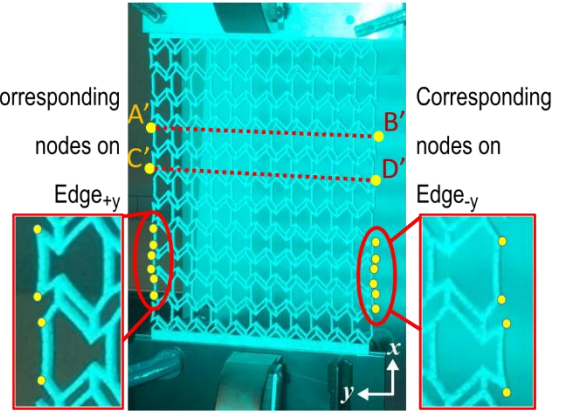


Figure 2-6 (a) Tensile test setup, (b) deformed Fish Cells metamaterial under loading in the y-direction that illustrates the position of corresponding nodes on $Edge_{\pm x}$, (c) deformed Fish Cells metamaterial under loading in the x-direction that illustrates the position of corresponding nodes on $Edge_{\pm y}$.

2.4.4 Results and Discussion

2.4.4.1 Validation

In this section, results of the experiments and numerical analysis performed on Fish Cells metamaterial are compared to investigate the validity of numerical models. Two models of manufactured samples i.e. beam structure and continuum solid structure, with dimensions presented in section 2.2 and material properties presented in section 2.4.2 are simulated in ABAQUS. Beam elements, as described in section 2.3.2, and 2D continuum quadratic plane-

stress elements, known as CPS8, are used for numerical analysis. Mesh convergence studies are performed and fine mesh discretization with a global size of 0.125 mm is applied for plane-stress elements. Figure 2-7(a) shows force-displacement curves of the two models in x and y -directions. FEM model using beam elements has lower stiffness compared to the experiments, while the model with 2D plane-stress elements fits better to the experimental curves in both x - and y -directions. The maximum error between predicted stiffness using 2D plane-stress model and experiment are less than 10% and 5% in the x and y -direction, respectively.

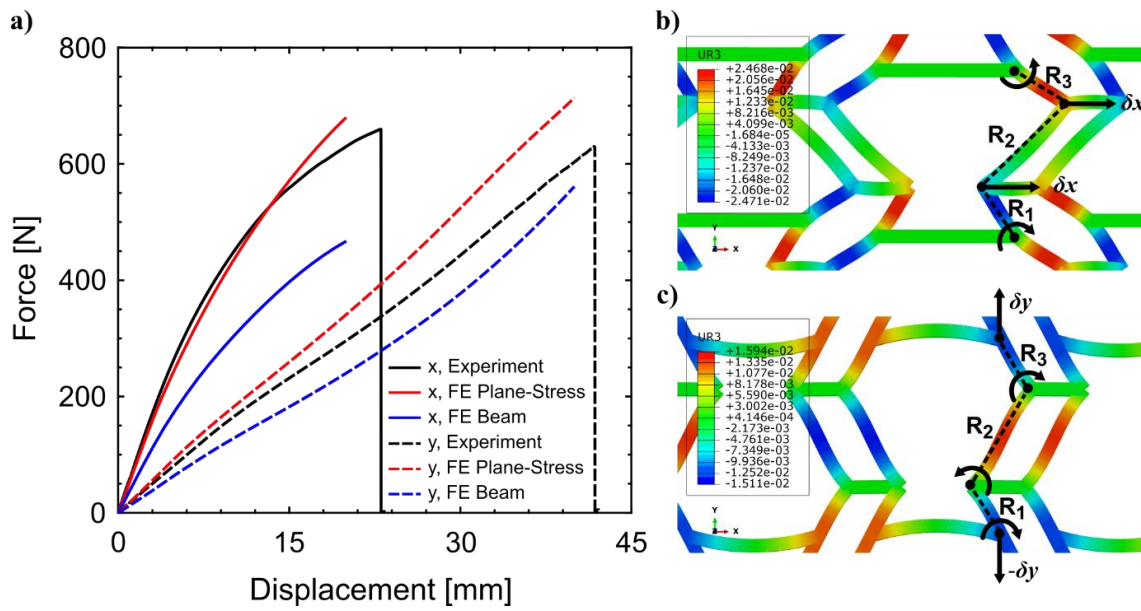


Figure 2-7 (a) Force-displacement diagrams of metamaterial samples in x and y -direction obtained from experiment and FE simulations, (b)schematic illustration of the ribs bending under loading in the x -direction and (c) y -direction, that are plotted over the planar rotation contours obtained from FE simulation

Poisson's ratio in x and y -directions are calculated by measuring displacements of the ABCD rectangle vertices on numerical and experimental models, where DIC system is used to obtain displacement field and strain contours of the metamaterial samples. Poisson's ratio ν_{yx} calculated from simulation with beam elements is $\nu_{yx} = 0.00001$, while $\nu_{yx} = 0.0488$ and $\nu_{yx} = 0.0463$ from simulation with CPS8 elements and experiment, respectively. In the other direction, Poisson's ratio ν_{xy} calculated from a simulation with beam elements is $\nu_{xy} =$

0.0003, while $\nu_{xy} = 0.2087$ and $\nu_{yx} = 0.2265$ from a simulation with CPS8 elements and experiment, respectively. The Poisson's ratios measured from the experiment and numerical model with 2D plane-stress elements are close with less than 8% relative error. The reason for the difference between numerical results with beam and continuum solid models is discussed in the next section.

2.4.4.2 Modelling Approach, Element Discussion

As explained, the desired values for Poisson's ratio are those obtained using beam elements by the numerical model; however, the details of the manufactured samples resulted in Poisson's ratio larger than those predicted by beam model. Chen and Fu (Chen and Fu, 2018) have also reported different results for cellular structure modeled with beam and solid elements. The difference between the two modeling approaches is due to the complex deformations and stress concentration at joints, which is not explicitly captured using the beam element. When tuning the Poisson's ratio for a specific value, the joints should be thin comparing to the ribs to be ideal (Bauer et al., 2017; Bückmann et al., 2014) as considered in the design and mathematical modelling of extremal materials as well (Milton, 1992; Milton and Cherkaev, 1995). If joints are not ideal and beam model is used, the joints' rigidity will result in different Poisson's ratio (Brighenti et al., 2016). Using the beam model that represents an ideal joint, the Fish Cells geometry reaches perfect ZPR behavior, where the interface of ribs and connectors for transferring the load is only a single node. However, considering uniform thickness of the members in manufactured samples, the material builds up at the joints and the thickness at joints has becomes larger than ribs as shown in Figure 2-8(b). Therefore, load transfers along the entire area of the joint's cross-section rather than a single node. In this regard, continuum solid models that are closer to the experiment should be used to justify the ZPR behavior of the Fish Cells metamaterial. In addition, the stress concentration at joints is predicted in the plane stress model that results in more accurate prediction of global structural stiffness and local deformations comparing to beam model. Moreover, aiming to reach the substantially ZPR behavior, a modified Fish Cell with reduced thickness at joints is presented to investigate the ideal joint using solid model as shown in Figure

2-8(c). The thickness at the joint is 0.1 times the thickness of the rib and a chamfer angle of 20° with respect to the rib's midline is applied in the joints. Identical dimensions and material properties as other models are used and tessellation number of experiments is considered for the joint thickness effect study, i.e. $(5 \times 5)_s$. Since the solid FE model with CPS8 elements in ABAQUS is verified with experimental results, it is used similarly to evaluate the thickness reduction effect. Hence, the discretization is locally refined at the joints where the element size reduces from 0.125 mm to 0.01 mm for great accuracy.

In order to understand the effect of modelling approach on the results, beam and solid models are compared. Under tension in the x-direction in the beam model, the re-entrant part of the cell expands as much as hexagonal section contracts, resulting in zero displacements in the orthogonal direction of the Fish Cells as shown in Figure 2-8(d). In the plane-stress model as annotated in Figure 2-8(e), the maximum stress is observed at the connection of the rib with the base member where the bending moment is maximum. Von Mises stress at this point when 1 mm displacement is applied, are 13.6 MPa and 7.1 MPa for plane-stress and beam models, respectively. This large difference is contributed to the stress concentration that can be captured using plane-stress model. The high stress concentration at this point leads to larger rotation of the bending rib in solid model than beam model, i.e., larger contraction in the orthogonal direction; therefore, the metamaterial experiences positive Poisson's ratio. In addition, base members in the beam model do not experience any moments, while in plane-stress model these members do not have uniform stress distribution through thickness, which results in bending moment and curved profile.

Under loading in the y-direction in the plane-stress model, the maximum stress is observed at the joint shared between the connector and the ribs, as annotated in Figure 2-8(h). Von Mises stress at this point are 13.80 MPa and 6.54 MPa for plane-stress and beam models when 2 mm displacement is applied. Comparison of Figures 2-8(g) and 2-8(h) reveals the rotation of connectors in the plane stress model, while there is no rotation in the beam

model. This rotation of connectors in the plane stress model causes contraction in the x -direction and positive Poisson's ratio. The difference of the stress contour in joints due to the stress concentration is highlighted in Figures 2-8(d) to 2-8(i).

Justifying the zero Poisson's ratio behavior of the Fish Cells metamaterial using solid model, the tessellation with reduced thickness at joints is simulated under global strain of 1% in each direction. Figure 2-8(f) and 2-8(i) represent the stress contour of the deformed tessellations in the x and y -direction for the reduced thickness model, respectively. The contour of the stress is better visible at joints in the magnified views because the stress concentrates at the reduced joints and results in the hinging mechanism. The Poisson's ratio of this model are $\nu_{xy} = -0.0617$ and $\nu_{yx} = -0.0071$ that show significant improvement and substantially ZPR behavior comparing to the uniform thickness model.

As a conclusion, thickness reduction at joints for further improvement of the Poisson's ratio value is justified. Therefore, the substantially ZPR behavior of the Fish Cells metamaterial is proved, which was also demonstrated using beam model. In this regard, it is suggested to consider reducing the thickness at joints when a metamaterial design for a specific Poisson's ratio value such as zero is tuned. Figure 2-8 also highlights the approximate number of elements used in the simulation of each FE model. Considering the computation time and complexity of the geometry and fine mesh generation of the solid models with reduced thickness, the beam model can be an effective tool for design purposes; therefore, beam model is employed for integration studies.

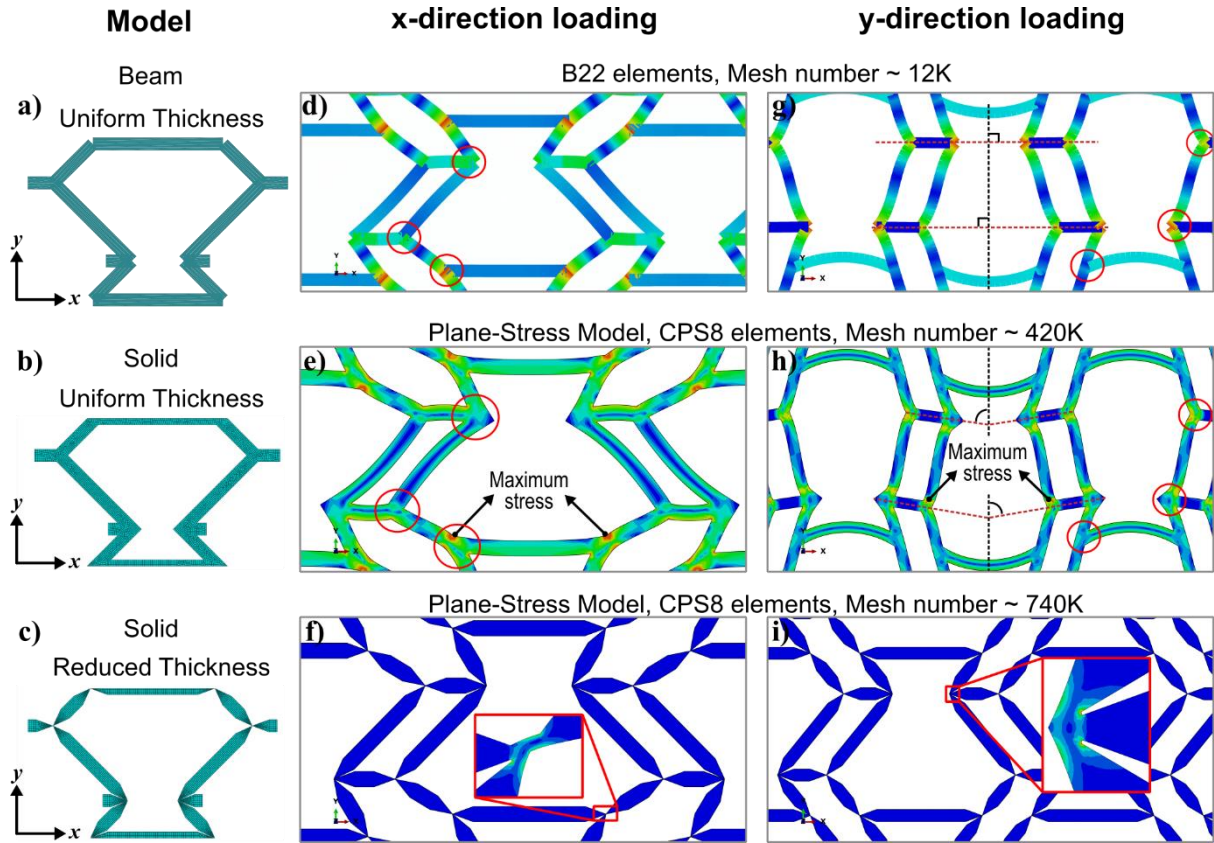


Figure 2-8 Fish Cell: (a) beam model with uniform thickness, (b) solid plane-stress model with uniform thickness, and (c) solid plane-stress model with reduced thickness at joints. Deformed shape and stress contour of the Fish Cells tessellation under tension in the x-direction with (d) beam model, (e) solid model with uniform thickness, and (f) solid model with reduced thickness at joints. Deformed shape and stress contour of the Fish Cells tessellation under tension in the y-direction with (g) beam model, (h) solid model with uniform thickness, and (i) solid model with reduced thickness at joints.

2.5 Structural Integrity for Morphing Application

Morphing technology has diverse applications in many sectors including aerospace, automotive, biomechanics, and renewable energy systems. Focusing on aerospace vehicles, morphing may refer to tools and methods used to improve the aerodynamic efficiency and increase performance by setting up the optimum geometry and configuration of the structure. In this case, the main challenge for the development of morphing structures is a suitable morphing skin for providing a smooth aerodynamic surface, transferring the

distributed aerodynamic loads to the main structural elements and enabling large shape changes through large strain capacity in actuation direction. Hence, low in-plane stiffness and high out-of-plane stiffness are needed in the design of morphing skins to enable large shape changes with low actuation power and to transfer aerodynamic loads efficiently.

2.5.1 Morphing Skin and Fish Cells Metamaterial

The proposed skins for morphing aircraft are usually made of elastomeric covers supported by a core with different geometries and deformation mechanisms, for example, a flexible skin made of a corrugated core and elastomer coatings (Dayyani et al., 2013, 2015a; Dayyani, Ziaei-Rad and Friswell, 2014; Dayyani and Friswell, 2017; Ermakova and Dayyani, 2017). The main advantage of corrugated cores is the anisotropic behavior, i.e., stiffness along the corrugation direction while flexibility in the transverse direction. Corrugated cores have also other remarkable characteristics such as high strength to density ratio, good energy absorption and easy fabrication (Dayyani et al., 2015b). However, the intrinsic weakness of a corrugated core is thickness reduction under deformation, resulting in a significant decrease in moment of inertia and bending stiffness. A solution may be using cellular structures like honeycombs for the core of morphing skins. Although honeycombs satisfy thickness requirements, they have Positive Poisson's Ratio (PPR) that leads to the dependency of longitudinal and transverse stresses. In other words, actuation in morphing direction causes stress in the orthogonal direction. Applying constraints on side edges of morphing skin without causing stiffness augmentation in the actuation directions is a key requirement for skin integration in morphing applications.

Metamaterials with ZPR behavior could be very beneficial for morphing application because of preventing stiffness augmentation and unclastic curvature, which provides better geometric conformability. Substituting ZPR metamaterials with conventional honeycombs can solve the PPR issues. Using Fish Cells metamaterial in morphing skin, stiffness augmentation issues should be solved as the stress coupling will no longer exist. This section demonstrates

the capability of Fish Cells metamaterial in removing stiffness augmentation under deformation with constraints on side edges, representing an assembly of the skin to the main structure.

2.5.2 Edge Smoothness and Boundary Conditions

Although in theory, a homogenized Fish Cells metamaterial exhibits ZPR behavior in two orthogonal directions, in practice perfect ZPR behavior might not be achieved due to size ratio of unit cell to tessellations limited with the current manufacturing technology or applying constraints which may cause local deformations at the edges imposed by boundary conditions, as discussed in section 2.3.3. Therefore, it is important to have a criterion for evaluating integration performance such as edge smoothness and debonding stress where the local deformations at the boundaries are investigated.

In this regard, Poisson's ratio definition is expanded by introducing Edge Smoothness denoted by S to study the edge deformations and the local effects of boundary conditions. While Poisson's ratio of the metamaterial is obtained by measuring strains on ABCD rectangle (shown in Figure 2-2), edge smoothness is defined between two corresponding nodes on two opposite edges connected by a straight virtual line as shown in Figure 2-6(b) and 2-6(c). The corresponding nodes on Edge_{+x} and Edge_{-x}, with normal vectors in +x and -x directions, are located on the free ends of connectors as shown in Figure 2-6(b). Similarly, the corresponding nodes on Edge_{+y} and Edge_{-y}, with normal vectors in +y and -y directions, are located on the intersection of the base and bending rib (corners of the cells), as depicted in Figure 2-6(c). The edge smoothness S that measures the change of the distance between two opposite edges is defined as:

$$S_{xy} = - \frac{\Delta U_y / l_y}{\delta_x / l_x} \quad (2-3)$$

$$S_{yx} = - \frac{\Delta U_x / l_x}{\delta_y / l_y} \quad (2-4)$$

In Equation 2-3, ΔU_y is the difference between displacements of corresponding nodes on Edge_{+y} and Edge_{-y} ($U_y^{B'} - U_y^{A'}$), which means length change of virtual line between them. Parameter l_y is the length of the structure along the y -direction, δ_x is displacement along x -direction imposed on Edge_{+x}, and l_x is the length of the structure along x -direction as shown in Figure 2-2(c). In Equation 2-4, ΔU_x is the difference between the displacements of corresponding nodes on Edge_{+x} and Edge_{-x} ($U_x^{B'} - U_x^{A'}$) and δ_y is the displacement along y -direction imposed on Edge_{+y} as shown in Figure 2-2(d). It should be mentioned that in the presence of homogeneity conditions, edge smoothness of a ZPR metamaterial would be substantially zero. Boundary conditions used for morphing skin with Fish Cells core are presented in Table 2-2 where lateral edges of the metamaterial samples are assessed in both free and constrained conditions. Edge_{+y} and Edge_{-y} are considered as side edges in x -direction loading and Edge_{+x} and Edge_{-x} are side edges in y -direction loading as shown in Figure 2-2(c) and 2-2(d), respectively.

Table 2-2 Boundary conditions imposed on edges under loading in x - and y -directions

	x-direction loading		y-direction loading	
	Free side edges	Constrained side edges	Free side edges	Constrained side edges
Edge _{+x}	$U_x = 0.01 \times l_x$ $U_y = 0, U_R = 0$	$U_x = 0.01 \times l_x$ $U_y = 0, U_R = 0$	free	$U_x = 0, U_R = 0$
Edge _{-x}	$U_x = 0$ $U_y = 0, U_R = 0$	$U_x = 0$ $U_y = 0, U_R = 0$	free	$U_x = 0, U_R = 0$
Edge _{+y}	free	$U_y = 0, U_R = 0$	$U_y = 0.01 \times l_y$ $U_x = 0, U_R = 0$	$U_y = 0.01 \times l_y$ $U_x = 0, U_R = 0$
Edge _{-y}	free	$U_y = 0, U_R = 0$	$U_x = 0, U_y = 0$ $U_R = 0$	$U_x = 0, U_y = 0$ $U_R = 0$

2.5.3 Edge Smoothness Homogenization

It is important to perform the edge smoothness study on a tessellation with a homogenized behavior. Hence, a numerical study is done using a beam model as described in section 2.3.2 with different tessellation numbers. Figure 2-9 shows edge smoothness S_{xy} and S_{yx} versus normalized length of the metamaterial for 5×5 , 10×10 , and 20×20 tessellation numbers. For each tessellation, the strain imposed on the moving edge is equal to 1%. This strain range is consistent with maximum strain requirements for a flexible skin in camber morphing (Gandhi and Anusonti-Inthra, 2008).

The local deformations near the boundaries cause S_{xy} to have absolute values larger than zero, which is due to clamp boundary conditions that happens in reality when a skin is attached to the structure. Value of S_{xy} is shown in Figure 2-9(a), where 20×20 tessellation has the least affected edge smoothness by local deformations due to the boundary conditions, while S_{xy} is substantially zero far from boundaries in 5×5 and 10×10 tessellations. For 20×20 tessellation, magnitudes of the edge smoothness S_{xy} is affected by boundary conditions in less than 10% of the domain length from each side. In this regard, the magnified diagram in Figure 2-9(a) illustrates the edge smoothness S_{xy} over the $x/l_x = (0.1, 0.9)$ range where the magnitudes are from the order of 10^{-4} . The pattern of the edge smoothness S_{xy} is symmetric due to symmetric boundary conditions. This justifies considering ABCD rectangle in the center of the metamaterial to measure Poisson's ratio. As the number of tessellation increases, boundary condition effect decreases and edge smoothness tends to zero for a larger range of tessellation length.

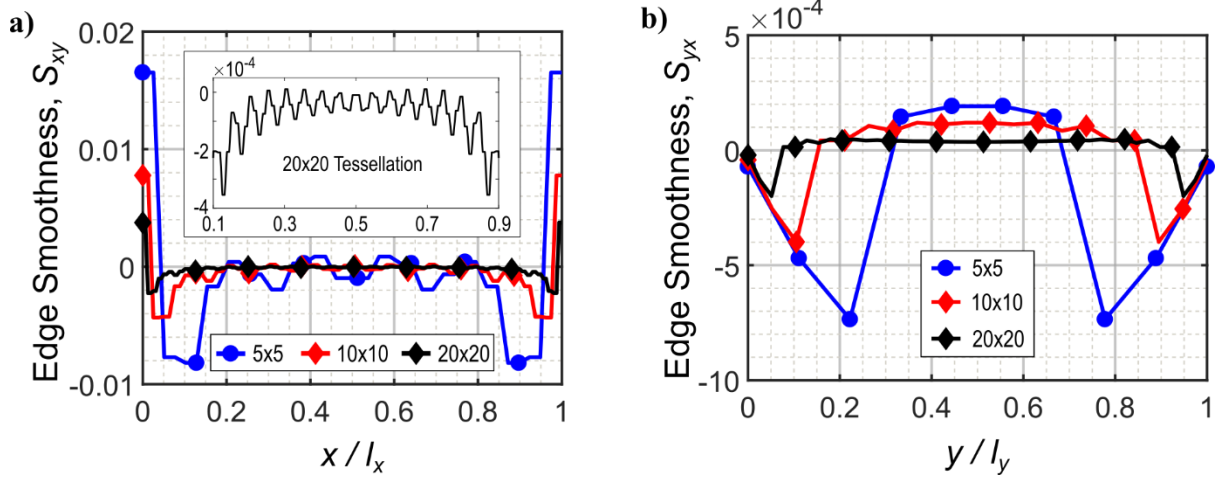


Figure 2-9 Edge Smoothness homogenization analysis in two orthogonal directions for 5×5, 10×10, and 20×20 tessellations: (a) S_{xy} where load is applied on Edge_{+x} in the x-direction and (b) S_{yx} where load is applied on Edge_{+y} in the y-direction.

2.5.4 Edge smoothness S_{xy} and stiffness analysis for loading in the x-direction

This section discusses the effect of symmetry along y-direction on the edge smoothness S_{xy} . In this regard, two cases of 20×20 and (20×20)_s tessellations are compared in Figure 2-10(a). Although both cases demonstrate a similar trend, the 20×20 tessellation shows small repeating fluctuations which are due to shear-extension coupling found in Fish Cells metamaterial, as depicted in Figure 2-10(b). However, for a very large tessellation, the effect of symmetry is negligible as it is in the same order as deformations. The edge smoothness S_{xy} is symmetric in both cases along the length of metamaterial as shown in Figure 2-10(a), which is because of the symmetric boundary conditions of the edges.

Figure 2-10(b) and 2-10(c) show the ratio of nodal displacements for both cases of 20×20 and (20×20)_s along Edge_{+y} and Edge_{-y} as demonstrated in Figure 2-10(d). The displacements of nodes located on hexagonal cells of Edge_{+y} and corresponding nodes located on the re-entrant cell of Edge_{-y} are $-U_y^1$ and $-U_y^2$, respectively. Similarly, nodal displacements for the re-entrant cell on Edge_{+y} and hexagonal cell on Edge_{-y} are also $+U_y^3$ and $+U_y^4$. These displacements are in the same direction but with different values near the boundaries resulting in large values for S_{xy} . However, the displacements difference is very small for corresponding nodes in the mid-length of metamaterial, resulting in S_{xy} close to zero as shown in Figure 2-10(a). The difference in directions of

U_y^1 and U_y^2 with U_y^3 and U_y^4 is due to the opposite arrangement of Fish Cells. Therefore, columns of Fish Cells experience a shift in the y -direction with a maintained width l_y , when stretched in the x -direction. Hence, corresponding nodes on Edge_{+y} and Edge_{-y} between two columns of the tessellation, move up and down as demonstrated in Figure 2-10(d) resulting in very small shear deformations.

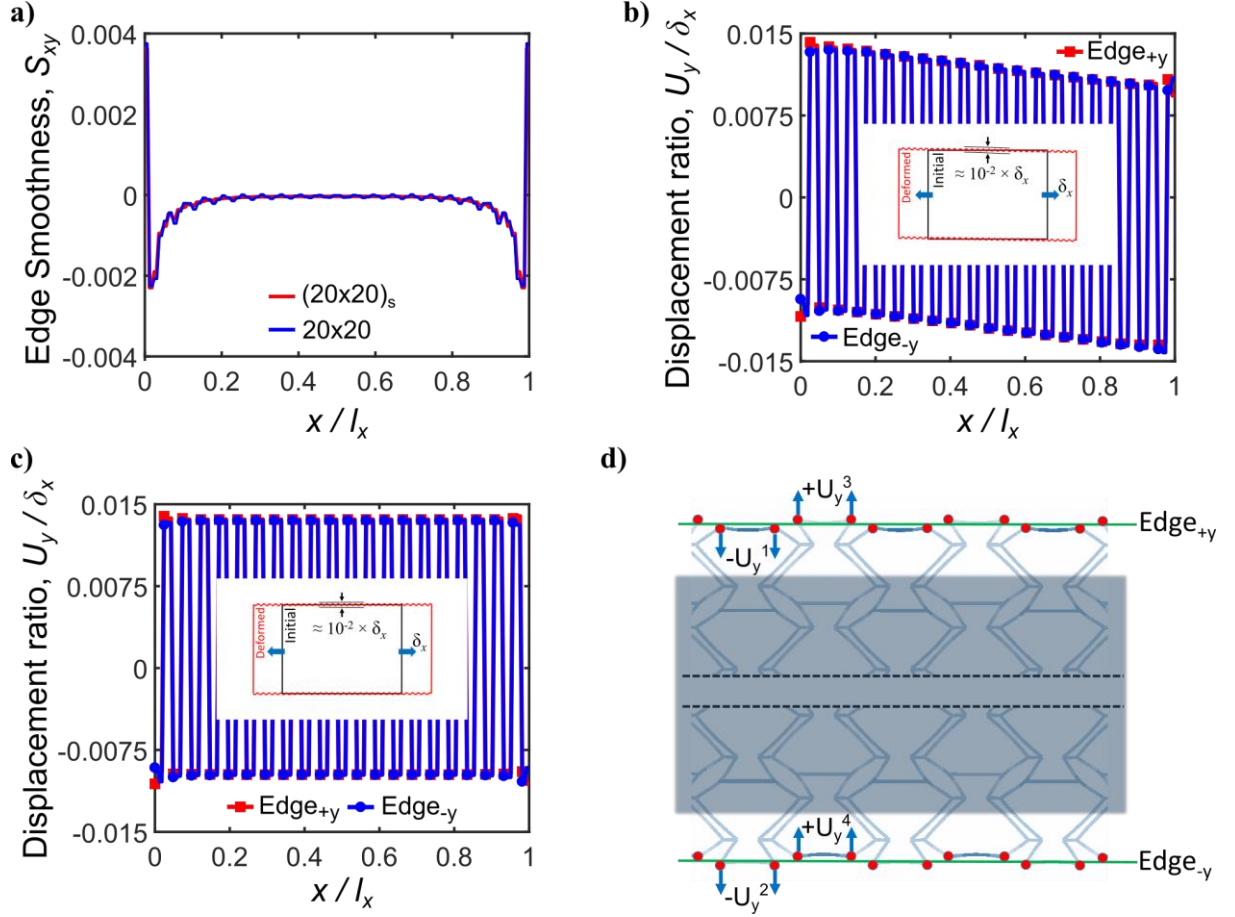


Figure 2-10 Edge Smoothness S_{xy} analysis for loading in the x -direction; (a) for 20×20 and $(20 \times 20)_s$ tessellation numbers, (b) displacement ratio of nodes located on Edge_{+y} and Edge_{-y} for 20×20 tessellation, (c) displacement ratio of nodes located on Edge_{+y} and Edge_{-y} for $(20 \times 20)_s$ tessellation, (d) schematic of displacements of the corresponding nodes on Edge_{+y} and Edge_{-y}.

Table 2-3 presents the stiffness values of 20×20 Fish Cells tessellation in the x -direction for free and constrained side edges expressed earlier in Table 2-2. The results show that the stiffness of Fish Cells metamaterial with constrained side edge is 10% larger than the stiffness of metamaterial with free side edges. Hence, stiffness

augmentation may be defined as this structural stiffness ratio, i.e., $K_{x_constrained} / K_{x_free} = 1.10$.

Table 2-3 Stiffness values of 20×20 Fish Cells tessellation in x-direction loading for free and con-strained side edges

	Free side edges	Constrained side edges
K_x (N/m)	19.49×10^3	21.48×10^3

2.5.5 Edge smoothness S_{yx} and stiffness analysis for loading in the y-direction

Figure 2-11(a) illustrates the edge smoothness S_{yx} for 20×20 and (20×20)_s tessellations where the effect of the symmetry on the edge smoothness is shown. Figure 2-11(b) and 2-11(c) show the ratio of nodal displacements for 20×20 and (20×20)_s tessellations along Edge_{+x} and Edge_{-x} as demonstrated schematically in Figure 2-11(d) and 2-11(e), respectively. The symmetric (20×20)_s tessellation has a zigzag trend of edge smoothness S_{yx} , while 20×20 tessellation has a smooth and substantially zero S_{yx} due to antisymmetric positions of re-entrant (r) and hexagonal (h) cells as shown in Figure 2-11(d) and 2-11(e). For asymmetric tessellations, two corresponding nodes belong to different r and h cells, which move with the same magnitude in the same direction as shown in Figure 2-11(d), therefore $\Delta U_x = 0$ and a smooth trend is achieved. However, for symmetric tessellations two corresponding nodes belong to the same cells, i.e., r-r or h-h, which move with the same magnitude in the opposite directions as shown in Figure 2-11(e). Therefore, $\Delta U_x \neq 0$ and the virtual line between them will either stretch or compress; i.e., negative S_{yx} due to expansion or positive S_{yx} due to contraction.

Table 2-4 presents the stiffness values of Fish Cells metamaterial with 20×20 tessellation in the y-direction for free and constrained side edges expressed earlier in Table 2-2. The results show that the stiffness value of Fish Cells metamaterial with constrained side edge is about 27% larger than the stiffness value of metamaterial with free side edges, i.e. $K_{y_constrained} / K_{y_free} = 1.27$. Although not very large, this stiffness augmentation in y-direction loading is due to the locations of corresponding nodes on connectors' free end of Edge_{-x} and Edge_{+x}, as shown in Figure 2-6(b). These nodes have large displacements under loading due to bending of ribs, hence large reaction forces are expected when they are constrained.

Table 2-4 Stiffness values of Fish Cells metamaterial with 20×20 tessellation in the y-direction loading for free and constrained side edges

	Free side edges	Constrained side edges
K_y (N/m)	24.81×10^3	31.44×10^3

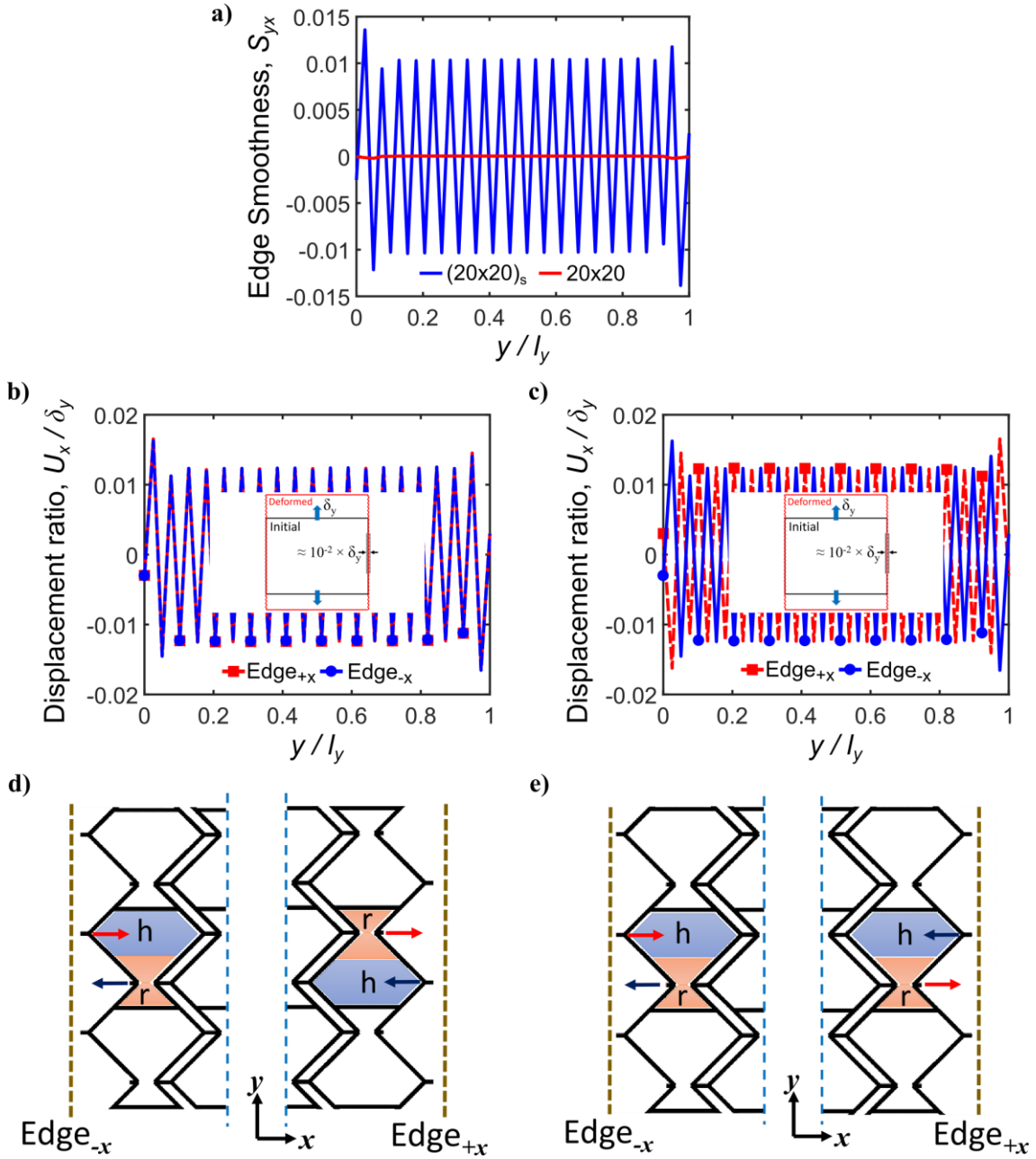


Figure 2-11 Edge Smoothness S_{yx} analysis for loading in the y-direction; (a) for 20×20 and $(20 \times 20)_s$ tessellation numbers, (b) displacement ratio of nodes located

on Edge_{+x} and Edge_{-x} for 20×20 tessellation number, (c) displacement ratio of nodes located on Edge_{+x} and Edge_{-x} for $(20 \times 20)_s$ tessellation number, (d) antisymmetric positions of re-entrant (r) and hexagonal (h) parts of the cell for asymmetric 20×20 tessellation and the corresponding nodes' displacement vector, (e) symmetric positions of re-entrant (r) and hexagonal (h) parts of the cell for symmetric tessellations and the corresponding nodes' displacement vector.

2.5.6 Design Perspective

The obtained results show the effect of tessellation's symmetry along y -direction as follows:

- For asymmetric tessellations, S_{yx} trend is smooth and substantially zero but S_{xy} will have small fluctuations due to shear–extension coupling.
- For symmetric tessellations, S_{yx} has zigzag trend and non-zero values but S_{xy} will have smooth trend because shear–extension coupling does not exist and values are close to zero.

Therefore, the primary direction for morphing integration can be determined. Shear-extension coupling might be a positive aspect of the design in some applications where desirable shear displacement is resultant of tension or similarly, desirable torsion is resultant of axial tension in cylindrical tessellations.

2.5.7 Edge Modification

The stiffness augmentation of the Fish Cells metamaterial is shown to be small under loading with clamp side edges. However, further topology enhancements on the edges are proposed in this section to reduce the stiffness augmentation further. The proposed enhancement for the geometries of $\text{Edges}_{\pm y}$ is shown in Figure 2-12(a) where Fish Cells are removed alternately. In addition, the proposed enhancement for the geometries of $\text{Edges}_{\pm x}$ includes an additional half of Fish Cells for each edge, as highlighted in Figure 2-12(b). FE analysis on structures with modified edges is performed using dimensions presented in section 2.2 and parameters discussed in section 2.3.2. Boundary conditions and loading are also according to Table 2-2; therefore, results of edge enhancement can be compared to conventional edges.

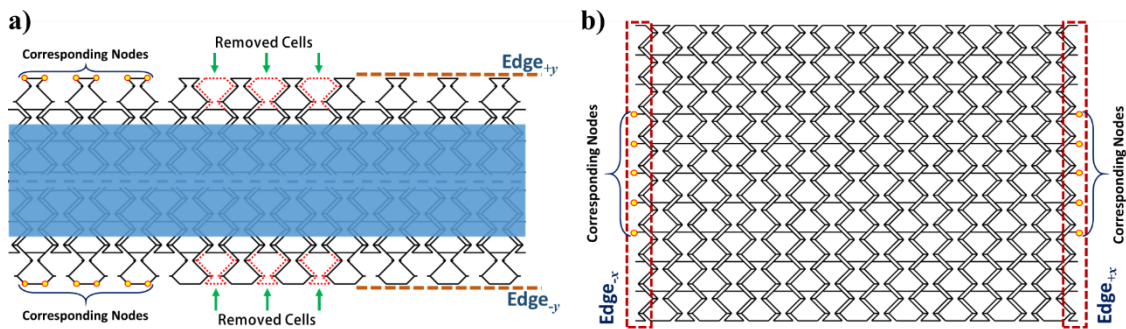


Figure 2-12 (a) Enhanced geometries of $\text{Edges}_{\pm y}$ with alternately removed Fish Cells, (b) enhanced geometries of $\text{Edges}_{\pm x}$ with an additional half of Fish Cells.

2.5.7.1 Discussion of enhanced S_{xy}

Figure 2-13 highlights the positive effect of $\text{Edge}_{\pm y}$ enhancements on edge smoothness S_{xy} . The geometry of the enhanced and conventional Edge_{+y} are compared in Figure 2-13(a) where uniform displacement contour is observed for enhanced geometry. The displacement of the corresponding nodes on the $\text{Edge}_{\pm y}$ in the y -direction is very small. This is because the cells in the enhanced design are not loaded through connectors due to the removal of one Fish Cell of each unit cell. Therefore, the reaction forces reduce significantly, when $\text{Edge}_{\pm y}$ are constrained resulting in stiffness augmentation alleviation.

S_{xy} has maintained the substantially zero values far from the boundaries, as shown in Figure 2-13(b). Figure 2-13(c) shows the displacement ratio of corresponding nodes on Edge_{+y} and Edge_{-y} for the $(20 \times 20)^{\text{enhanced}}$ tessellation, where shear-extension coupling occurs due to tessellation asymmetry along the y -axis. Moreover, the fluctuation of displacements is removed and values are positive compared to Figure 2-10(b).

Figure 2-13(d) shows the displacement ratio of corresponding nodes on Edge_{+y} and Edge_{-y} for the enhanced $(20 \times 20)^{\text{s-enhanced}}$ tessellation where the fluctuating trend has become very smooth compared to Figure 2-10(c). Far from boundaries, Edge_{+y} and Edge_{-y} have equal displacements in the same direction under loading in the x -direction. The deformed and initial schematic of a Fish Cells metamaterial under loading in the x -direction is also shown in Figure 2-13(c) and 2-13(d).

The most important achievement due to this edge enhancement is improving the stiffness augmentation ratio, as presented in Table 2-5 where stiffness remains approximately constant for both cases of free and constrained side edges. This means that Fish Cells metamaterial has excellent integration capabilities in addition to its ZPR performance in the x -direction.

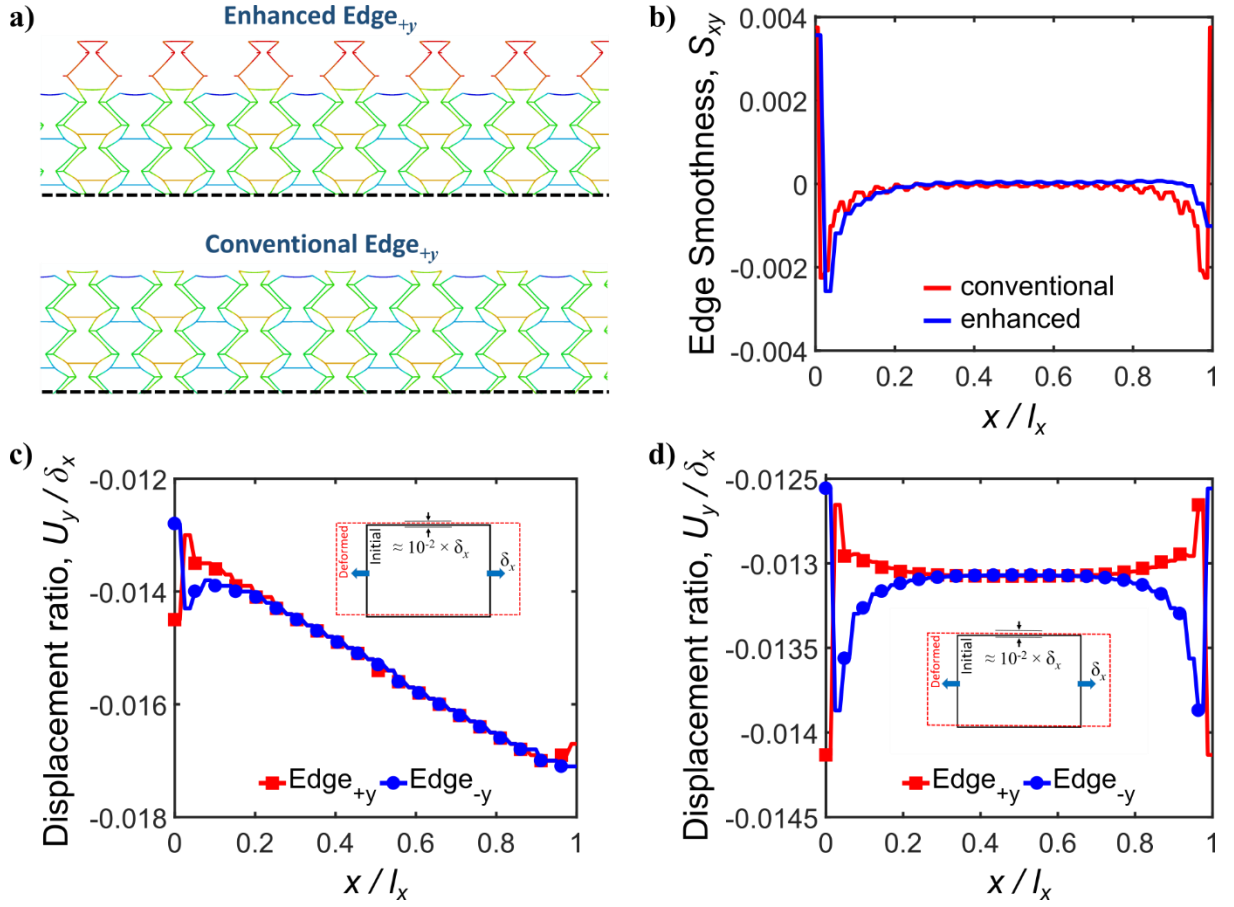


Figure 2-13 Edge enhancement effect on displacement ratio and edge smoothness S_{xy} , (a) enhanced and conventional edge deformations under tension in the x-direction, (b) comparison of Edge Smoothness S_{xy} for 20×20 and $(20 \times 20)^{enhanced}$ tessellations, (c) displacement ratio of corresponding nodes located on Edges _{$\pm y$} for $(20 \times 20)^{enhanced}$ tessellation, (d) displacement ratio of corresponding nodes located on Edges _{$\pm y$} for $(20 \times 20)_s^{enhanced}$ tessellation.

Table 2-5 Stiffness ratios of free and constrained panel with an enhanced edge in the x-direction

Enhanced Edges _{$\pm y$}	Free Edges	Constrained edges	$K_{constrained} / K_{free}$	
K_x (N/m)	19.55×10^3	19.56×10^3	Enhanced	Conventional
			1.0005	1.10

2.5.7.2 Discussion of enhanced S_{yx}

The deformed geometries of conventional and enhanced Edge_{-x} are compared in Figure 2-14(a). Under loading in the y -direction, the base of the Fish Cell bends without displacement in the x -direction, while the connectors have large displacements in the x -direction. Therefore, the proposed enhancement results in the integration of the Edges _{$\pm x$} of metamaterial via base members, i.e. the constraints are applied at the center of the base members. Hence, a uniform displacement contour is achieved by the enhanced geometry that results in edge smoothness improvement and alleviates stiffness augmentation further.

The positive effect of Edges _{$\pm x$} enhancements on edge smoothness S_{yx} is shown in Figure 2-14(b) where the enhanced edges present one order of magnitude smaller values than conventional edges. Figure 2-14(c) depicts the nodal displacement ratio of enhanced Edge_{+x} and Edge_{-x} for $(20 \times 20)^{enhanced}$ tessellation, which exhibits an antisymmetric trend. In contrast to the conventional edge, the edge smoothness S_{yx} for enhanced geometry is not dependent on tessellation symmetry along y -direction, because metamaterial skin is connected to the external frame via Fish Cells base instead of connectors. In this regard, nodal displacements of $(20 \times 20)^{enhanced}$ and $(20 \times 20)_s^{enhanced}$ tessellations with enhanced edge geometries are illustrated in Figure 2-14(c) and 2-14(d) where similar range and trend of values are observed in both tessellations. Moreover, the deformed and initial schematics of the rectangular structure under loading in the y -direction are also shown in Figure 2-14(c) and 2-14(d).

The most important achievement by this edge enhancement is improving the stiffness augmentation ratio as presented in Table 2-6, where stiffness remains substantially constant for both cases of free and constrained side edges. This means that the Fish Cells metamaterial has excellent integration capabilities in addition to its ZPR performance in the y -direction.

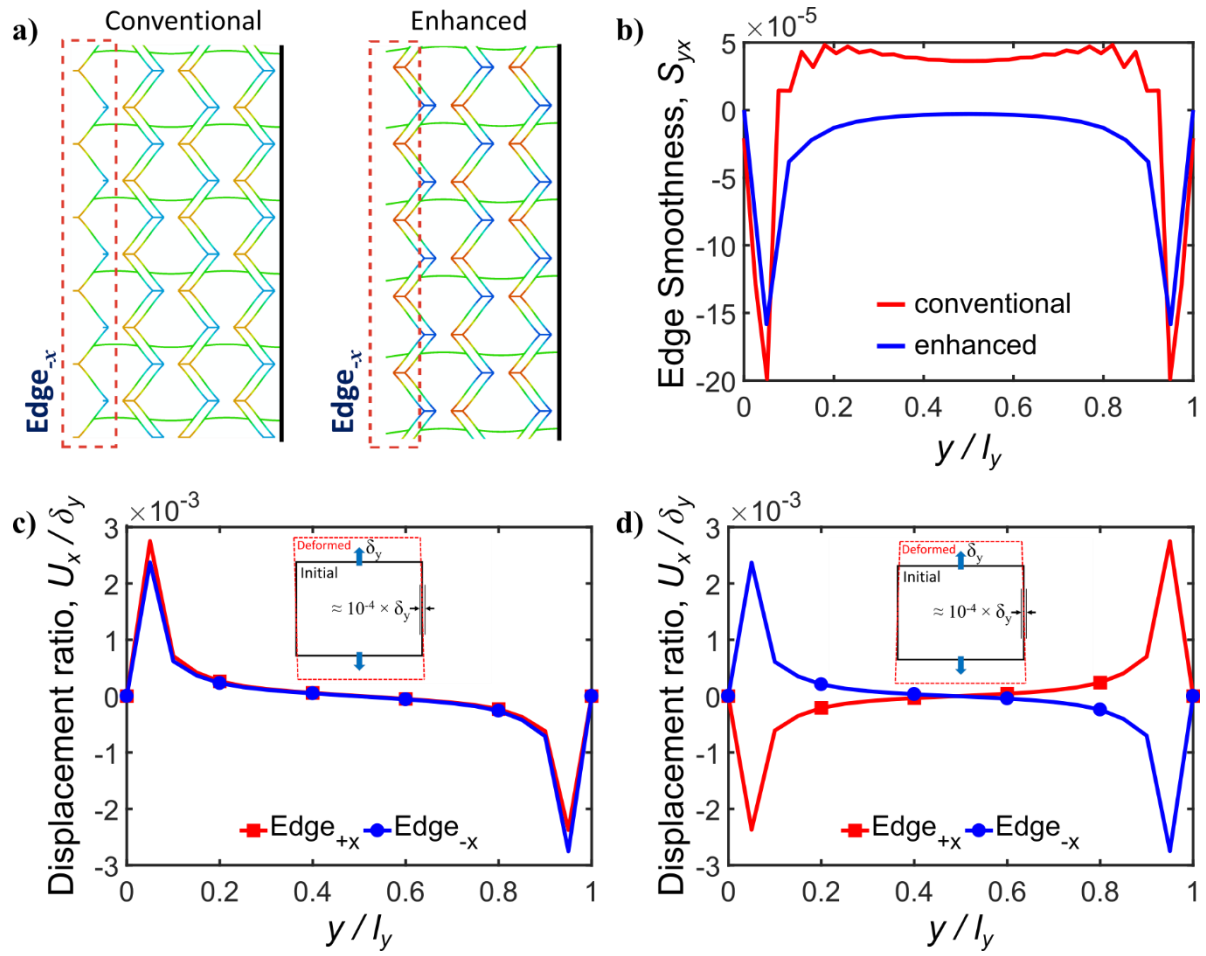


Figure 2-14 Edge enhancement effect on displacement ratio and edge smoothness S_{yx} , (a) enhanced and conventional edge deformations under tension in the y -direction, (b) comparison of Edge Smoothness S_{yx} for $(20 \times 20)^{enhanced}$ and 20×20 tessellations, (c) displacement ratio of corresponding nodes located on Edges $_{\pm x}$ for $(20 \times 20)^{enhanced}$ tessellation, (d) displacement ratio of corresponding nodes located on Edges $_{\pm x}$ for $(20 \times 20)_s^{enhanced}$ tessellation.

Table 2-6 Stiffness ratios for free and constrained panel with an enhanced edge in the y -direction.

Enhanced Edges $_{\pm x}$	Free Edges	Constrained edges	$K_{constrained} / K_{free}$	
K_y (N/m)	25.99×10^3	26.07×10^3	Enhanced	Conventional
			1.0031	1.27

2.6 Conclusion

Fish Cells as a new mechanical metamaterial is proposed with zero Poisson's ratio in two planar directions. Homogenization study is performed on Poisson's ratio and effective elastic modulus in two directions to obtain the right scale of the metamaterial unit cell. Numerical analysis corroborates the ZPR behavior of Fish Cells design in two directions for a homogenized tessellation. The experimental study validated the Poisson's ratio measurement and elastic-plastic behavior of the samples in both directions. A detailed discussion on the effect of the joints is presented by the analysis of the metamaterial using beam and plane-stress models. In addition, the reduced thickness solid model is presented justifying ZPR behavior of the Fish Cells metamaterial architecture.

The application of the Fish Cells metamaterial as the core of morphing skin is considered and integration of metamaterial with morphing structures is critically discussed, where requirements such as relieving stiffness augmentation and fine edge geometry are considered. Edge smoothness is defined as a criterion to assess edge integration of the metamaterial in both orthogonal directions. Concepts for geometric edge enhancements are proposed to further improve stiffness augmentation as the main issue in the development of morphing structures. The numerical studies on homogenized tessellations with enhanced edges show that Fish Cells metamaterial has perfect integration capabilities and achieves substantially zero stiffness augmentation. In addition, Fish Cells metamaterial offers the flexibility for the designer to create symmetric and asymmetric tessellations that can have different edgewise deformations. In case of asymmetry, Fish Cells metamaterial presents extension-shear coupling, which is a very important advantage for applications where bending-twist in out of plane direction is required such as twisting in large span wings under bending. The future study may include topology and shape optimization for maintaining ZPR over large strain range.

3 Equivalent Elastic Properties of the Fish Cell Metamaterial

3.1 Introduction

Cellular materials have received extensive attention in the aerospace engineering applications due to their high performance and low density. Considering improved functionality of the aerospace structures, mechanical metamaterials and their exotic properties have attracted interest in their design and development. The aim of metamaterial design is tailoring specific properties such as bulk modulus, elastic modulus, Poisson's ratio, and coefficient of thermal expansion. Many of these mechanical metamaterials possess cellular architecture while the geometry of the unit cell is tailored to result in a desired deformed shape under loading which allows for exotic behaviour in macroscale. Among mechanical metamaterials, those focused on Poisson's ratio are Zero Poisson's Ratio (ZPR) metamaterials and auxetic metamaterials with Negative Poisson's Ratio (NPR). ZPR is of particular interest given their useful applications in morphing skin's core. The optimal design of these cores requires simple models of the metamaterial that may be incorporated into multidisciplinary system models. Therefore, equivalent structural models are required that retain the dependence on the geometric parameters of the metamaterial.

Due to the high demand for cellular structures, previous studies have led to many analytical and numerical models. The focus of these studies has been to develop analytical formulations or a numerical homogenization framework for equivalent elastic properties of cellular structures based on their geometrical parameters. The main approach for modelling the elastic properties of cellular materials has been energy method where strain energy is developed for each member of the unit cell under loading in each direction (Abd El-Sayed F.K., Jones R. and Burgess I.W., 1979). Having the displacements of the cell measured using the energy method under a specific load, the equivalent elastic modulus in that direction can be obtained. The formulation of the modulus obtained using this method is dependent on the deformation mechanism

considered, e.g. the inclined members of the honeycomb unit cell can be considered as a cantilever beam with bending as the primary mechanism (Gibson et al., 1982) or also account for extension and hinging (Masters and Evans, 1997). Beam model can simplify the equivalent modulus derivation. However, it is accurate enough when cellular structure's relative density is low and the effect of joints at the intersection of inclined members of a cellular structure such as honeycomb is not significant (Malek and Gibson, 2015). Although the beam model is acceptable for low relative density cellular structures, modification of the member's effective length can help to improve the results (Grima et al., 2011). Similar to beam, if thickness to length ratio of members is not small enough, shell model also results in different in-plane elastic properties comparing to 3D or plane stress models due to the negligence of the out of plane stress in shell walls (Catapano and Montemurro, 2014). In this regard, slender members should be considered to keep the density low and be able to neglect the effect of joints. Therefore, the beam model can be used to derive the equivalent elastic modulus of the metamaterials that bending is their dominant deformation mechanism.

Shear modulus derivation is more complicated than elastic modulus in two orthogonal directions of the metamaterial. The first parameter to consider for the correct derivation of the shear modulus is the type of shear loading which can be simple or pure shear. Applying simple shear in two orthogonal directions results in two different equivalent shear modulus when the material lacks symmetry because of different deformations in the unit cell. The discussion of the simple and pure shear for different categories of materials is complex and less investigated in the literature. It is shown that simple and pure shear cannot be considered as equivalent (Destrade, Murphy and Saccomandi, 2012). To obtain the shear modulus of the hexagonal honeycomb, Gibson (Gibson, 1989; Gibson et al., 1982) has considered the pure shear loading where bending was considered as the main deformation mechanism for each member of the cell. If the simple shear loading on the honeycomb is considered, then loading in two orthogonal directions should be considered (Chen, Das and Battley, 2016), which technically means $G_{xy} \neq G_{yx}$.

For cellular metamaterials also similar approach like honeycombs has been followed for shear analysis. Ju et al. (Ju et al., 2011) studied the re-entrant cellular material under simple shear loading with several geometrical parameters in order to find the best yield strength for a predefined equivalent shear modulus. Their results show that shear resistance was independent of the Poisson's ratio value but dependent on the cell topology. In another attempt, Huang et al. (Huang et al., 2017b) considered re-entrant cell with a thin plate attached and calculated the shear modulus by application of pure shear loading where bending of the members is considered using Timoshenko beam theory to derive the closed form equation for shear modulus. Similarly, for ZPR cellular metamaterials with orthotropic cell topology, pure shear loading is applied and bending as the main deformation of the members was considered to derive the equivalent shear modulus formulae (Gong et al., 2015a; Liu et al., 2019).

In this chapter, the Fish Cell metamaterial with ZPR in two planar directions is introduced and its equivalent elastic properties are investigated. The importance of a homogenized equivalent model for elastic properties is in reducing the time to model and analyse the structure significantly while maintaining the accuracy level (Steenackers et al., 2016). The in-plane and out of plane equivalent elastic modulus and in-plane shear modulus of the Fish Cell are obtained by considering small deformations of a 2D unit cell, in which the members will be modelled as thin beams. The importance of RVE selection and periodic boundary conditions in the shear analysis is discussed and pure shear analysis on the material is performed. Due to the complexity of the structure and boundary conditions in shear loading, the FE method is employed to obtain the equivalent in-plane shear modulus. Finally, the effect of geometrical parameters of the Fish Cell on equivalent elastic properties is studied. The importance of this work is to provide an equivalent model, which uses the geometric parameters of the Fish Cell metamaterial as variables and obtains equivalent elastic properties that can be applied for further optimization studies.

3.2 Equivalent Elastic Modulus

The unit cell of the Fish Cell metamaterial is shown in Figure 3-1(a), which satisfies the periodicity conditions to create a tessellation. The unit cell is formed by two Fish Cells that are attached together followed by a rotation. Fish Cell, as shown in Figure 3-1(b), has been introduced by one axis of symmetry along which the cell can be assembled periodically but in the normal direction, the cell should rotate or shift to assemble. Although Fish Cell is the basis of the metamaterial design, the unit cell is selected in a way to satisfy periodicity in both in-plane orthogonal directions and appropriate boundary conditions can be applied considering symmetries (Li, 2008).

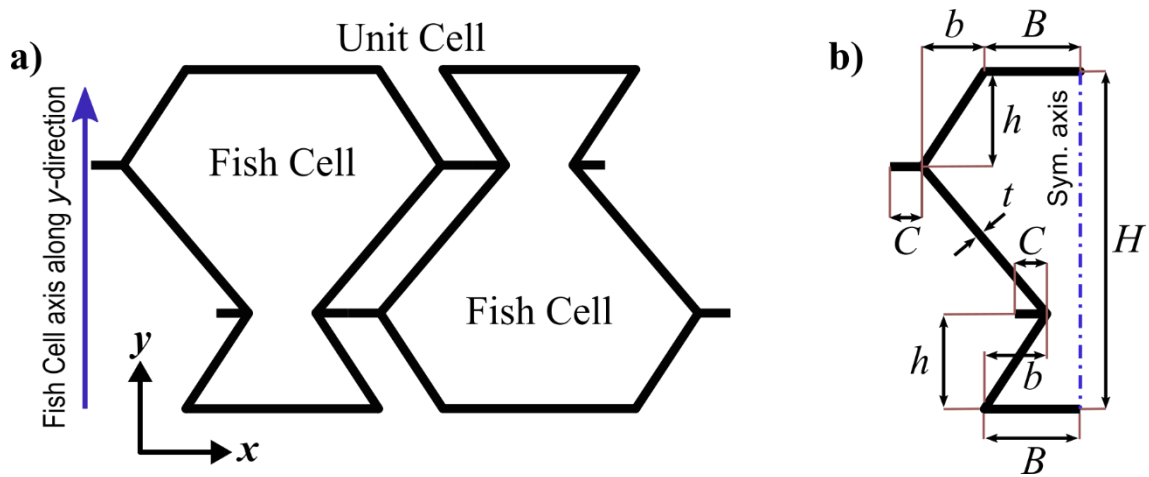


Figure 3-1 (a) unit cell of the Fish Cells metamaterial, (b) Fish Cell parameters' definition

Fish Cell configuration is defined by independent parameters B , H , C , h , and b where B is the half base length, H is the height of the cell, and C is connector's length. Parameters b and h are horizontal and vertical components of the inclined rib's length, respectively. It should be noted that half thickness is allocated to the base of the cell that creates a full thickness member in tessellation. The feasible range for b and h are described in Equation 3-1 and 3-2. Uniform rectangular cross section is assigned to all members with thickness t and extruded out of plane thickness W .

$$0 < b/B < 1 \quad (3-1)$$

$$0 < h/H < 0.5$$

(3-2)

3.2.1 Loading in the x-direction

As detailed in Chapter 2, the main deformation in the unit cell under tension in the x-direction occurs in the inclined members where the convex section of the Fish Cell contracts and the concave section expands in the y-direction. The deformed shape of the cell in the middle of a tessellation is shown in Figure 3-2(a) where FE analysis using beam elements with parameters discussed in Chapter 2 were conducted. The base member of the Fish Cell experiences no bending as moments exerted by inclined members on the base member neutralize each other. This simplifies the model required for bending analysis by neglecting the base member. Considering the symmetry of Fish Cell as shown in Figure 3-2(a), half of one Fish Cell can be modelled as a representative for the unit cell under loading in the x-direction.

A loading diagram of this model is presented in Figure 3-2(b) where loads and reactions are depicted based on the deformation of a cell in tessellation. The accurate boundary condition type for periodic cellular structures is Periodic Boundary Conditions (PBC) (Xia et al., 2006; Xia, Zhang and Ellyin, 2003). However, for normal in-plane loadings, using homogenized boundary conditions that set the value of a degree of freedom equal to zero does not affect the deformed shape adversely and results verify with those obtained from tessellation. Moreover, Castigliano's theorem can be used to derive a closed form solution for the equivalent elastic modulus. The boundary conditions and loading for the half Fish Cell shown in Figure 3-2(b) are defined in Table 3-1. Displacements in the x-direction at nodes "F" and "A" are constrained because of the symmetry and a reaction load is considered at node "A" due to the support. Displacement in the y-direction at node "F" is also constrained to prevent rigid body motion, while node "A" is free to move in the y-direction. Since base members of the cell (AB & EF) experience no bending moment, rotation of the node "E" is constrained and a reaction moment is applied at node "B". Rotations of the nodes "G" and "H" are also zero where reaction moments

are imposed to enable derivation of the total strain energy of the half-cell. In order to calculate the lateral displacement in y -direction using Castigliano's theorem, which helps to derive the Poisson's ratio, P_1 and P_4 are applied as virtual loads. To verify the boundary conditions used in the analytical model, FE analysis is performed by ABAQUS on the half Fish Cell where fine mesh using quadratic shear deformable beam elements B22 was employed. The deformed shape of the half Fish Cell is in agreement with the deformed shape of a half Fish Cell in tessellation as depicted in Figure 3-2(c) where boundary conditions of Table 3-1 were imposed.

Table 3-1 Boundary conditions applied on half Fish Cell in FE analysis under loading in the x -direction

Node	Degree of Freedom			Load	
	u	v	$rotation$	x -direction	y -direction
A	0	<i>free</i>	<i>free</i>	0	0
B	<i>free</i>	<i>free</i>	0	0	$P_1 = 0$ (Virtual)
G	<i>free</i>	<i>free</i>	0	$P_2 = P_x$	0
H	<i>free</i>	<i>free</i>	0	$P_3 = P_x$	0
E	<i>free</i>	<i>free</i>	0	0	$P_4 = 0$ (Virtual)
F	0	0	<i>free</i>	0	0

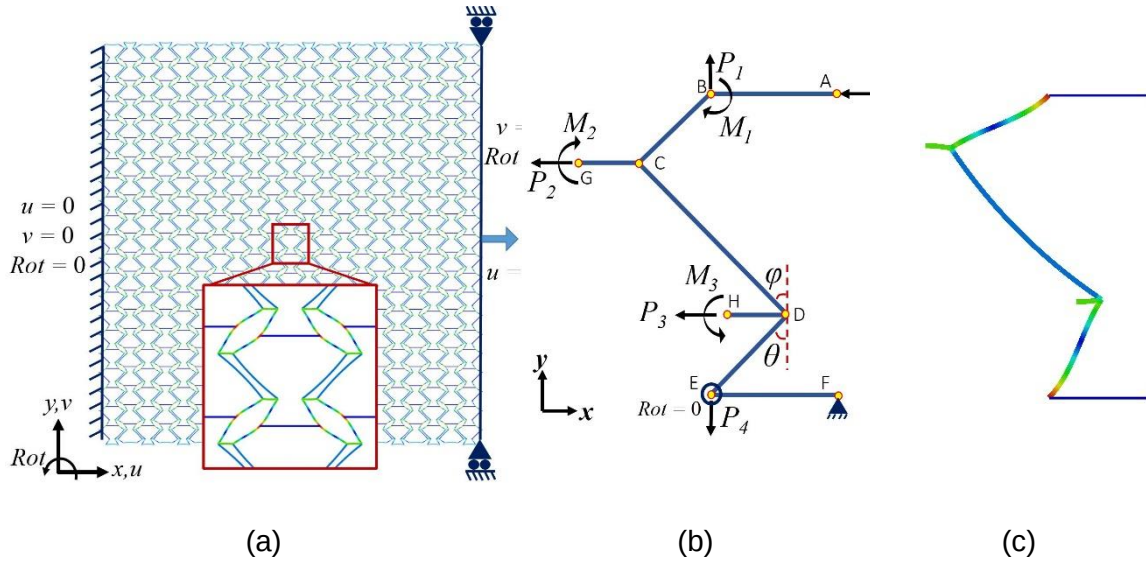


Figure 3-2 (a) deformed shape of the Fish Cell in a tessellation under loading along the x-direction, (b) Loading diagram of half Fish Cell, (c) deformed shape of the half Fish Cell obtained from FE analysis.

Since equilibrium equations should be written for each member, local parameters are defined for inclined members to simplify the equations. Parameter θ is defined as the angle between inclined ribs and y-direction as shown in Figure 3-2(b). Length of the two bending ribs connected to the base members (AB & EF) is assumed L and for the middle rib “CD”, the length is D . The angle between the middle rib and y-direction is called φ . However, length D is a dependent parameter with respect to L , θ , and φ as $D=2L\sin(\theta)/\sin(\varphi)$. These local parameters are correlated to the main Fish Cell parameters by Equation 3-3.

$$\theta = \tan^{-1} \left(\frac{b}{h} \right) \quad (3-3a)$$

$$L = \sqrt{b^2 + h^2} \quad (3-3b)$$

$$\varphi = \tan^{-1} \left(\frac{2b}{H - 2h} \right) \quad (3-3c)$$

Starting from node “B”, the bending moment of each member can be written as

$$M_{BC} = M_1 - P_1 x \sin \theta - R x \cos \theta \quad (3-4a)$$

$$M_{CG} = -M_2 \quad (3-4b)$$

$$M_{CD} = M_1 + P_1 x \sin \varphi - (P_2 + R)x \cos \varphi - P_1 L \sin \theta - RL \cos \theta - M_2 \quad (3-4c)$$

$$M_{DH} = M_3 \quad (3-4d)$$

$$M_{DE} = M_1 + M_3 - M_2 - P_1 x \sin \theta - (P_2 + P_3 + R)x \cos \theta - P_1 L \sin \theta - RL \cos \theta + P_1 D \sin \varphi - (P_2 + R)D \cos \varphi \quad (3-4e)$$

while there is no moment on members “AB” and “EF”. Similarly, the axial force at each member can be derived as

$$F_{AB} = R \quad (3-5a)$$

$$F_{BC} = P_1 \cos \theta - R \sin \theta \quad (3-5b)$$

$$F_{CG} = P_2 \quad (3-5c)$$

$$F_{CD} = (P_2 + R) \sin \varphi + P_1 \cos \varphi \quad (3-5d)$$

$$F_{DH} = P_3 \quad (3-5e)$$

$$F_{DE} = -(P_2 + P_3 + R) \sin \theta + P_1 \cos \theta \quad (3-5f)$$

$$F_{EF} = P_2 + P_3 + R \quad (3-5g)$$

Considering the bending and stretching of the members, the total strain energy is obtained.

$$U_{tot} = U_{bending} + U_{axial} \quad (3-6a)$$

$$U_{bending} = \frac{1}{2E_s I} \left[\int_0^L (M_{BC}^2 + M_{DE}^2) dx + \int_0^D M_{CD}^2 dx + \int_0^C (M_{CG}^2 + M_{DH}^2) dx \right] \quad (3-6b)$$

$$U_{axial} = \int_0^B \frac{1}{2E_s A_{t/2}} (F_{AB}^2 + F_{EF}^2) dx + \int_0^L \frac{1}{2E_s A_t} (F_{BC}^2 + F_{DE}^2) dx + \int_0^D \frac{1}{2E_s A_t} F_{CD}^2 dx + \int_0^C \frac{1}{2E_s A_t} (F_{CG}^2 + F_{DH}^2) dx \quad (3-6c)$$

Bending is the main deformation mechanism that contributes the most to the total energy comparing to the axial deformation. Since very thin members are considered, the effect of shear deformation is neglected. The contribution of axial loads and bending moments to the strain energy are written in Equation (3-6) where E_s is the constructing material modulus and moment of inertia $I = Wt^3/12$. Uniform rectangular cross-section with area A_t is assigned to each inclined member and connectors where $A_t = W \times t$. Since base members of the Fish Cell are shared between two cells, half of the thickness is allocated for the base members in each cell, which affects the axial term of the base members' strain energy denoted by subscript $t/2$ where $A_{t/2} = W \times t/2$ as shown in Equation (3-6c).

The deformation corresponding to each load can be obtained using Castigliano's theorem by derivation of total energy function with respect to that load. Since node "A" has no displacement in x-direction due to symmetry condition, the total energy derivation with respect to load R is zero. In addition, rotations at nodes "B", "G", and "H" are also zero, therefore

$$\frac{\partial U_{tot}}{\partial R} = 0 \quad (3-7a)$$

$$\frac{\partial U_{tot}}{\partial M_1} = \frac{\partial U_{tot}}{\partial M_2} = \frac{\partial U_{tot}}{\partial M_3} = 0 \quad (3-7b)$$

Loads P_2 and P_3 are equal due to periodicity and are assigned with a constant value P_x . Expanding the Equation (3-7a) and substituting P_2 and P_3 with P_x results in $R = -P_x$. Substituting these values in Equation (3-7b), the unknown reaction moments are obtained as

$$M_2 = M_3 = P_x L \cos \theta \left[D + \frac{L}{2} - \frac{(L+D)^2}{2L+D} \right] / \left[C + D - \frac{D(L+D)}{2L+D} \right] \quad (3-8a)$$

$$M_1 = P_x L \cos \theta \left[\frac{(D+L)^2 + DL - 2(D+L)[C(2L+D) + DL]}{C(2L+D) + DL} \right] / [2(2L+D)] \quad (3-8b)$$

3.2.1.1 Equivalent Elastic Modulus

Using Hooke's law, the equivalent elastic modulus of the homogenized Fish Cell metamaterial in the x-direction is obtained by the ratio of stress to strain as:

$$E_x = \sigma_x / \varepsilon_x \quad (3-9)$$

The stress σ_x is equal to the total force in x-direction divided by the effective area $A_x = W \times H$.

$$\sigma_x = 2P_x / A_x \quad (3-10)$$

The strain ε_x is equal to the displacement in x-direction divided by the effective length $l_x = B + C$.

$$\varepsilon_x = \delta_x / l_x \quad (3-11)$$

Displacement δ_x is obtained by calculating strain energy and applying Castigliano's theorem to the load P_2 . The displacements due to P_2 and P_3 have the same value in the x-direction, therefore, derivation with respect to P_2 will be:

$$\delta_x = \frac{\partial U_{tot}}{\partial P_2} = \frac{P_x}{E_s} \left\{ \frac{L^3 \cos^2 \theta (4C \times D + 2C \times L + D \times L)}{12I(C \times D + 2C \times L + D \times L)} + \frac{B + C + b}{W \times t} \right\} \quad (3-12)$$

Combining Equation 3-9 to 3-12, leads to the closed form solution for the equivalent elastic modulus in the x-direction. It is evident that equivalent modulus relation is independent of the load P_x because the load term in σ_x is omitted by the load term in δ_x .

$$E_x = \frac{2E_s(B + C)}{W \times H} \left/ \left\{ \frac{L^3 \cos^2 \theta (4C \times D + 2C \times L + D \times L)}{12 \times I \times (C \times D + 2C \times L + D \times L)} + \frac{B + C + b}{W \times t} \right\} \right. \quad (3-13)$$

Elastic modulus obtained by analytical model from Equation 3-13 for a Fish Cell with $B = 0.8$ cm, $H = 2.4$ cm, $h = 0.6$ cm, $b = 0.6$ cm, $C = 0.2$ cm, $W = 0.1$ cm, and $t = 0.01$ cm is compared with FE analysis results in Table 3-2 for both unit cell and tessellation. The relative error relation is $(E_x^{Tes.} - E_x^{cell}) / E_x^{Tes.}$ where the equivalent modulus of the unit cell obtained from the FE analysis and analytical method are compared to the elastic modulus of tessellation. The constitutive

material considered in FE analysis is steel with $E_s = 200\text{GPa}$ and $\nu = 0.3$, however, the normalized parameter E_x / E_s is considered in parametric studies to bypass material dependency.

Table 3-2 Elastic modulus of the Fish Cell metamaterial normalized by constitutive material modulus E_s , obtained using FE and analytical methods – Relative errors are calculated with respect to the tessellation.

Analysis Method	Normalized Young's Modulus E_x/E_s	Relative Error = $100 \times (E_x^{Tes.} - E_x^{cell}) /E_x^{Tes.}$
FEM (Tessellation)	1.8508e-6	
FEM (unit cell)	1.8416e-6	0.5 %
Analytical	1.8416e-6	0.5 %

Using Equation 3-13, the effect of geometrical parameters on elastic modulus is studied where aspect ratio (H/B) of the Fish Cell and three normalized parameters h/H , b/B , and C/B are considered. Results are shown in Figure 3-3 where elastic modulus E_x is normalized with respect to the constructing material modulus E_s where FE analysis is also used to validate the trend obtained. Two parameters, which cause large changes in modulus from orders of magnitude, are aspect ratio and the vertical component of rib's length as shown in Figure 3-3(a). This large effect of the vertical component of rib's length is because it acts as the moment arm of the load P_x . Equivalent elastic modulus in x -direction reduces by increasing both the vertical component of rib's length and aspect ratio because of the moment arm increase. Variation of the modulus with the horizontal component of the rib's length and connector's length is from the same order of magnitude as shown in Figure 3-3(b). The equivalent modulus reduces by increasing the horizontal component of rib's length since the beam stiffness reduces by increasing the length. The trend for connector's length is not linear where a minimum equivalent modulus for a specific C/B ratio exists.

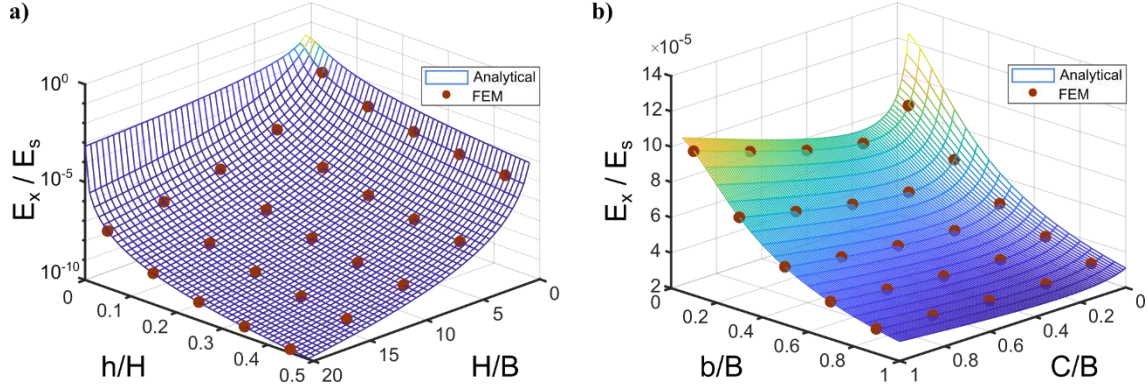


Figure 3-3 Variation of effective elasticity modulus in x-direction for a Fish Cell metamaterial with $t = 0.01B$ and $W = B$ with respect to (a) aspect ratio of the cell and vertical component of inclined rib's length for $C = 0.1B$ & $b = 0.5B$, (b) connector's length and horizontal component of rib's length for $H = B$ & $h = 0.25H$.

3.2.1.2 Poisson's Ratio ν_{xy}

Poisson's ratio is derived using Equation 3-14 where displacement in the x-direction, δ_x , is offered by Equation 3-12. Length of the cell in the y-direction, l_y , is equal to the height H . Displacement in the y-direction, δ_y , at point "B" is obtained by derivation of total energy with respect to P_1 as $\delta_y = \delta_y^{bending} + \delta_y^{axial}$ where contributions of the bending and axial strain energy to δ_y are presented in Equation 3-15 and 3-16, respectively.

$$\nu_{xy} = -\varepsilon_y/\varepsilon_x = -(\delta_y/l_y)/(\delta_x/l_x) \quad (3-14)$$

$$\begin{aligned} \delta_y^{bending} = \frac{1}{2E_s I} [& D(M_1 - M_2 + PL \cos \theta)(D \sin \varphi - 2L \sin \theta) \\ & - 2L^2(M_1 \sin \theta + PL \sin \theta \cos \theta) \\ & + 2L^2 \sin \theta (M_1 + PL \cos \theta)] \end{aligned} \quad (3-15)$$

$$\begin{aligned} \delta_y^{axial} = \frac{1}{2E_s W t} \{ & -L[2R \sin \theta \cos \theta + 2(P_3 + P_2 + R) \sin \theta \cos \theta] \\ & + 2D \cos \varphi \sin \varphi (P_2 + R) \} \end{aligned} \quad (3-16)$$

Substituting $D = 2L \sin \theta / \sin \varphi$ and $P_2 = P_3 = -R = P_x$ in Equation 3-15 and 3-16 and simplification results in $\delta_y^{bending} = \delta_y^{axial} = 0$, which means there is no

lateral displacement in the Fish Cell under loading in the x -direction. This implies that $\epsilon_y = 0$ and as a result, Poisson's ratio $\nu_{xy} = 0$ for any arbitrary values of Fish Cell parameters.

3.2.2 Loading in the y -direction

Similar to loading in the x -direction and considering the deformation mechanism of the cell in a tessellation, loading in the y -direction is studied. The deformed shape of the Fish Cell tessellation is shown in Figure 3-4(a). The main deformation in the unit cell under loading in the y -direction is bending of the inclined members as understood from the tessellation. Extension in y -direction causes the convex section of the cell to contract, while the concave section expands with the same value along the x -direction. The magnified view of the Fish Cell in Figure 3-4(a) indicates the symmetry of the deformed shape about the cell axis. Moreover, deformations in the Fish Cell and the adjacent rotated cell are the same. As a result, half Fish Cell as shown in Figure 3-4(b) represents the unit cell deformation to derive the equivalent elastic modulus in the y -direction, E_y . Applying the symmetry conditions, displacement in x -direction and rotation are constrained at node "F" and the corresponding reaction load R and moment M_1 are considered at point "A". Displacement in the y -direction at point "F" is also constrained to prevent rigid body motion, which results in the clamp boundary condition. Since load transfer from one cell to another happens through inclined members, loads P_1 and P_4 that are equal to P_y are applied with opposite directions at nodes "B" and "E", respectively. Loads P_2 and P_3 are virtual loads since nodes "G" and "H" have the same but opposite displacements in the x -direction. When two Fish Cells are attached by rotation to form the unit cell, node "G" of one cell will be connected to the node "H" of the other via connectors. Because of the same displacement and no rotation at nodes "G" and "H", connectors will experience neither tension/compression nor moment. Similar to section 3.2.1, the FE analysis is performed on a half-cell to confirm the validity of the boundary conditions employed. The boundary conditions in FE analysis for the half Fish Cell shown in Figure 3-4(b) are presented in Table 3-3. The deformed shape of the half Fish Cell under

described loading is shown in Figure 3-4(c) that is compliant with the deformed shape of the cell in tessellation.

Table 3-3 Boundary conditions applied on half Fish Cell in FE analysis under loading in the y-direction

Node	Degree of Freedom			Load	
	u	v	$rotation$	x -direction	y -direction
A	0	free	0	0	0
B	free	free	free	0	$P_1 = P_y$
G	free	free	free	$P_2 = 0$ (Virtual)	0
H	free	free	free	$P_3 = 0$ (Virtual)	0
E	free	free	free	0	$P_4 = -P_y$
F	0	0	0	0	0

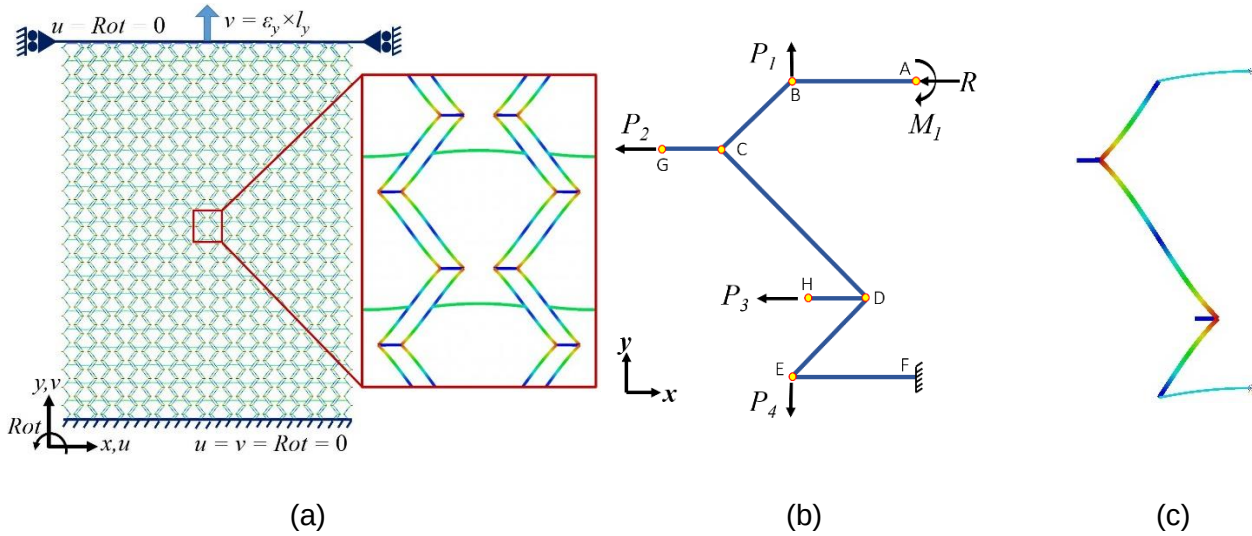


Figure 3-4 (a) deformed shape of the Fish Cell in a tessellation under loading along the y-direction, (b) Loading diagram of half Fish Cell, (c) deformed shape of the half Fish Cell obtained from FE analysis.

Starting from node "A", the bending moment of each member is:

$$M_{AB} = M_1$$

(3-17a)

$$M_{BC} = M_1 - P_1 x \sin \theta - Rx \cos \theta \quad (3-17b)$$

$$M_{CD} = M_1 + P_1 x \sin \varphi + (P_2 - R)x \cos \varphi - P_1 L \sin \theta - RL \cos \theta \quad (3-17c)$$

$$M_{DE} = M_1 - P_1 x \sin \theta + (P_2 - R)x \cos \theta - P_1 L \sin \theta - RL \cos \theta \\ + (P_2 - R)D \cos \varphi + P_1 D \sin \varphi \quad (3-17d)$$

$$M_{EF} = M_1 - 2P_1 L \sin \theta - RL \cos \theta + (P_2 - R)D \cos \varphi + P_1 D \sin \varphi \\ + (P_2 - R)L \cos \theta \quad (3-17e)$$

Similarly, the axial force at each member is derived as:

$$F_{AB} = R \quad (3-18a)$$

$$F_{BC} = P_1 \cos \theta - R \sin \theta \quad (3-18b)$$

$$F_{CG} = P_2 \quad (3-18c)$$

$$F_{CD} = (P_2 + R) \sin \varphi + P_1 \cos \varphi \quad (3-18d)$$

$$F_{DH} = P_3 \quad (3-18e)$$

$$F_{DE} = -(P_2 + P_3 + R) \sin \theta + P_1 \cos \theta \quad (3-18f)$$

$$F_{EF} = P_2 + P_3 + R \quad (3-18g)$$

Using forces and moments at each member, the total strain energy is obtained as:

$$U_{tot} = U_{bending} + U_{axial} \quad (3-19a)$$

$$U_{bending} = \frac{1}{2EI/2} \int_0^B (M_{AB}^2 + M_{EF}^2) dx \\ + \frac{1}{2EI} \left[\int_0^D M_{CD}^2 dx + \int_0^L (M_{BC}^2 + M_{DE}^2) dx \right] \quad (3-19b)$$

$$U_{axial} = \int_0^B \frac{1}{2E_s A_{t/2}} (F_{AB}^2 + F_{EF}^2) dx + \int_0^L \frac{1}{2E_s A_t} (F_{BC}^2 + F_{DE}^2) dx + \int_0^D \frac{1}{2E_s A_t} F_{CD}^2 dx + \int_0^C \frac{1}{2E_s A_t} (F_{CG}^2 + F_{DH}^2) dx \quad (3-19c)$$

where bending and stretching of the members are considered. Loading and boundary conditions in Table 3-3 are considered to model the half Fish Cell deformation. Since there is no bending moment applied on nodes “G” and “H”, members “CG” and “DH” are neglected in bending strain energy derivation. Displacement in the x-direction and rotation of the point “A” are zero due to symmetry along the Fish Cell axis. This means that total strain energy derivative with respect to the corresponding reaction load and moment at point “A” is zero, i.e.

$$\frac{\partial U_{tot}}{\partial R} = 0 \quad (3-20a)$$

$$\frac{\partial U_{tot}}{\partial M_1} = 0 \quad (3-20b)$$

Using Equation (3-20a), reaction load R is derived as:

$$R = P_1 \left[\frac{\eta}{2E_s I} - \frac{2D \sin^2 \varphi - 2L \sin 2\theta}{2E_s W t} \right] / \left[\frac{\xi}{2E_s I} + \frac{4B + 2D \sin^2 \varphi + 4L \sin^2 \theta}{2E_s W t} \right] \quad (3-21)$$

where

$$\begin{aligned} \eta = & a_1 \sin \varphi - a_2 L \sin \theta \\ & + a_3 (-8BD \sin \varphi + 16BL \sin \theta - 2LD \sin \varphi + 2L^2 \sin \theta \\ & - D^2 \sin \varphi + 2L(L + D) \sin \theta) - a_4 \sin \theta \\ & + a_5 (-L \sin \theta + D \sin \varphi) - a_6 \sin \theta \\ & + 2B(D \cos \varphi + 2L \cos \theta)(-2L \sin \theta + D \sin \varphi) \end{aligned} \quad (3-22a)$$

$$\begin{aligned}\xi = & a_1 \cos \varphi + a_2 L \cos \theta \\ & - a_3 (8BD \cos \varphi + 16BL \cos \theta + 2LD \cos \varphi + 2L^2 \cos \theta \\ & + D^2 \cos \varphi + 2L(L + D) \cos \theta) + a_4 \cos \theta \\ & + a_5 (D \cos \varphi + L \cos \theta) + a_6 \cos \theta + 2B(D \cos \varphi + 2L \cos \theta)^2\end{aligned}\quad (3-22b)$$

$$a_1 = \frac{2}{3} D^3 \cos \varphi + D^2 L \cos \theta \quad (3-22c)$$

$$a_2 = D^2 \cos \varphi + 2DL \cos \theta \quad (3-22d)$$

$$\begin{aligned}a_3 = & [D^2 \cos \varphi + 2DL \cos \theta + 2L^2 \cos \theta + 2L(D \cos \varphi + L \cos \theta) \\ & + 2B(D \cos \varphi + 2L \cos \theta)] / (2D + 16B + 4L)\end{aligned}\quad (3-22e)$$

$$a_4 = L^2 (D \cos \varphi + L \cos \theta) \quad (3-22f)$$

$$a_5 = 2L(D \cos \varphi + L \cos \theta) + L^2 \cos \theta \quad (3-22g)$$

$$a_6 = \frac{4}{3} L^3 \cos \theta \quad (3-22h)$$

and moment M_1 is derived using Equation (3-20b) as:

$$\begin{aligned}M_1 = & P_1 \{ [2L(D + L) \cos \theta + D^2 \cos \varphi + 2L(D \cos \varphi + L \cos \theta) \\ & + 8B(D \cos \varphi + 2L \cos \theta)] R \\ & + [2L(D + L) \sin \theta - D^2 \sin \varphi + 2L(L \sin \theta - D \sin \varphi) \\ & + 8B(2L \sin \theta - D \sin \varphi)] \} / (4L + 2D + 16B)\end{aligned}\quad (3-23)$$

3.2.2.1 Equivalent Elastic Modulus

Using Hooke's law, the equivalent elastic modulus of the homogenized Fish Cell metamaterial in the y -direction is obtained by the ratio of stress to strain as:

$$E_y = \sigma_y / \varepsilon_y \quad (3-24)$$

The stress σ_y is equal to the total force in y -direction divided by the effective area $A_y = W(B+C)$.

$$\sigma_y = P_y/A_y \quad (3-25)$$

The strain ε_y is equal to the displacement in y-direction divided by the effective length $l_y = H$.

$$\varepsilon_y = \delta_y/l_y \quad (3-26)$$

Displacement δ_y is obtained by calculating strain energy and applying Castigliano's theorem to the load P_1 .

$$\delta_y = \delta_{bending} + \delta_{axial} \quad (3-27a)$$

$$\delta_{bending} = \frac{-1}{2E_s I} (g_1 + g_2 + g_3 + g_4 + g_5) \quad (3-27b)$$

$$\delta_{axial} = \frac{1}{2E_s W_t} [2D(P_1 + R) \sin^2 \varphi + 4L \cos \theta (P_1 \cos \theta - R \sin \theta)] \quad (3-27c)$$

where reaction load R and moment M_1 are described in Equation 3-21 and 3-23. The coefficients g_i ($i=1:5$) in Equation 3-27 are

$$g_1 = -\frac{4}{3} L^3 b_1 \sin \theta \quad (3-28a)$$

$$g_2 = b_2 D^2 \left(\frac{2}{3} D \sin \varphi - L \sin \theta \right) \quad (3-28b)$$

$$g_3 = 2Lb_3b_4 + L^2(b_1b_4 + M_1 \sin \theta - b_3 \sin \theta) \quad (3-28c)$$

$$g_4 = 8B(RD \cos \varphi - P_1 D \sin \varphi - M_1 + 2RL \cos \theta + 2P_1 L \sin \theta)(D \sin \varphi - 2L \sin \theta) \quad (3-28d)$$

$$g_5 = (P_1 L \sin \theta + RL \cos \theta - M_1)(D^2 \sin \varphi - 2DL \sin \theta) \quad (3-28e)$$

where

$$b_1 = R \cos \theta + P_1 \sin \theta \quad (3-29a)$$

$$b_2 = R \cos \varphi - P_1 \sin \varphi \quad (3-29b)$$

$$b_3 = RD \cos \varphi - P_1 D \sin \varphi + P_1 L \sin \theta + RL \cos \theta - M_1 \quad (3-29c)$$

$$b_4 = D \sin \varphi - L \sin \theta \quad (3-29d)$$

Combining Equation 3-24 to 3-27 leads to the closed form solution for equivalent elastic modulus in the y -direction.

$$E_y = \frac{2E_s H}{W(B + C)} / \delta_y \quad (3-30)$$

It is evident that equivalent modulus relation is independent of load P_y because the load term in σ_y is omitted by the load term in δ_y . The elastic modulus of the Fish Cell with material and geometrical parameters mentioned in section 3.2.1 is obtained using FE analysis and Equation 3-30. The results from the analytical method and FEM are compared in Table 3-4 where FE results are developed for both unit cell and tessellation. Because the base members are shared between two cells, half of bending stiffness is considered for each base member. As a result, elastic modulus from the analytical method is verified with FE results for the tessellation with 1.7 % relative error.

Table 3-4 Elastic modulus of the Fish Cell metamaterial in y -direction normalized by constitutive material modulus E_s , obtained using FE and analytical methods

Analysis Method	Normalized Modulus E_y/E_s	Relative Error = $100 \times (E_y^{Tes.} - E_y^{cell}) /E_y^{Tes.}$
FEM (Tessellation)	5.6565e-7	
FEM (unit cell)	5.7552e-7	1.7 %
Analytical	5.7559e-7	1.7 %

Effect of Fish Cell geometrical parameters on the elastic modulus in the y -direction is shown in Figure 3-5 where further validation of the trend using FE analysis is also provided. The diagrams show that elastic modulus is highly sensitive to variations of b where values decline up to three orders of magnitude by increasing b , while the other three parameters vary in the same order of

magnitude. This is due to the moment arm of the loading in the y -direction which is length b . Increasing the connector's length also results in elastic modulus reduction as shown in Figure 3-5(b). Variation of elastic modulus with vertical component of rib's length is negligible for small aspect ratios, while a declining trend is seen at large aspect ratios as shown in Figure 3-5(a). Elastic modulus increases by increasing the aspect ratio for constant b/B and C/B ratios.

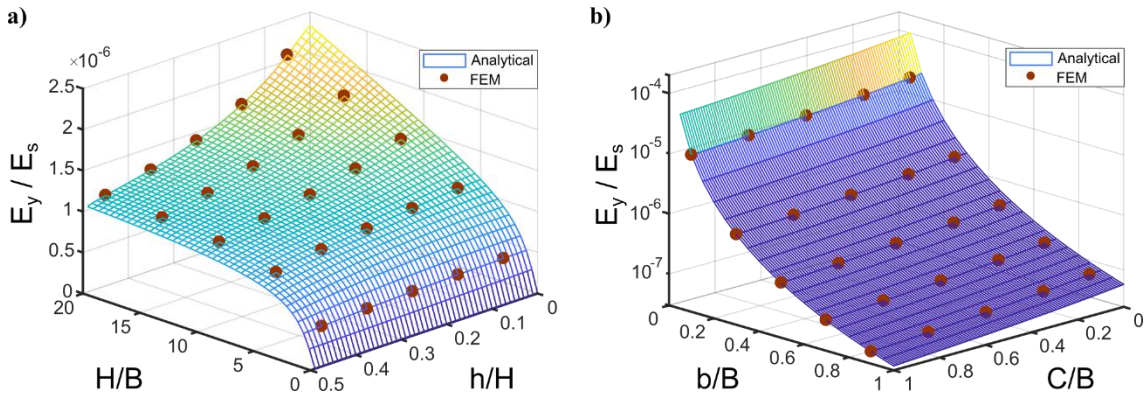


Figure 3-5 Variation of effective elasticity modulus in y -direction for a Fish Cell metamaterial with $t = 0.01B$ and $W = B$ with respect to (a) aspect ratio of the cell and vertical component of inclined rib's length for $C = 0.1B$ & $b = 0.5B$, (b) connector's length and horizontal component of rib's length for $H = B$ & $h = 0.25H$.

3.2.2.2 Poisson's Ratio ν_{yx}

Poisson's ratio ν_{yx} can be proved mathematically equal to zero; however, for the sake of brevity and length of the chapter, the formulation is not presented. The detailed justification of zero ν_{yx} is discussed in chapter 2. Deformations at nodes "C" and "D" are equal but in the opposite directions, which are followed by nodes "G" and "H". When two cells are attached by rotation, nodes "G" and "H" meet each other. Because of the same displacement, no load will be exerted on the connectors, which means no force in the x -direction is transferred from one cell to another. Since bending deformation of the inclined ribs occurs inside the cell domain and no lateral loads exist to cause lateral deformation, Poisson's ratio ν_{yx} is zero.

3.2.3 Loading in the z-direction

Considering the Hooke's law, the ratio of the stiffness of cellular material to the stiffness of bulk solid material can be obtained for the same cross-section area. For a solid material with a cross-section area of A_s , stiffness can be obtained as $k_s = E_s A_s / l$ where l is the length of the sample in the loading direction. The stiffness ratio only depends on the effective area ratio of the cellular structure and the bulk material as:

$$\frac{k_z}{k_s} = \frac{A}{A_s} \quad (3-31)$$

A is the load bearing area of the unit cell and is equal to the total length of members times thickness. The total area of the unit cell domain that is the same as bulk material is $A_s = (B+C)H$. The stiffness ratio in the z-direction for Fish Cell is obtained as:

$$\frac{k_z}{k_s} = \frac{B + 2C + 2L + D}{(B + C)H} t \quad (3-32)$$

The validity of the Equation 3-32 is investigated using FE analysis where the tensile load has been applied to the unit cell as shown in Figure 3-6 and dimensions of the cell as mentioned in section 3.2.1 are considered. To conduct FE analysis, displacement in the z-direction on the bottom surface is constrained and on the top surface, uniform displacement in the z-direction is applied while other degrees of freedom are free. Only at one point at the bottom surface, all degrees of freedom are closed to prevent the rigid body motion. FE analysis is performed using two models: shell with S8R elements and solid with C3D20 elements where fine mesh in both cases is applied for great accuracy. The results of the two FE models are compliant with each other and are also verified by the value obtained for tessellation as shown in Table 3-5. It should be noted that under real tensile or compression test, all degrees of freedom on the top and bottom surface will be constrained. However, this causes a large error with respect to results from Equation 3-32 because the effect of boundary conditions and lateral displacement of the members due to the Poisson's ratio

are not considered in Equation 3-32. The relative error of the analytical model in Table 3-5 is calculated with respect to the tessellation value.

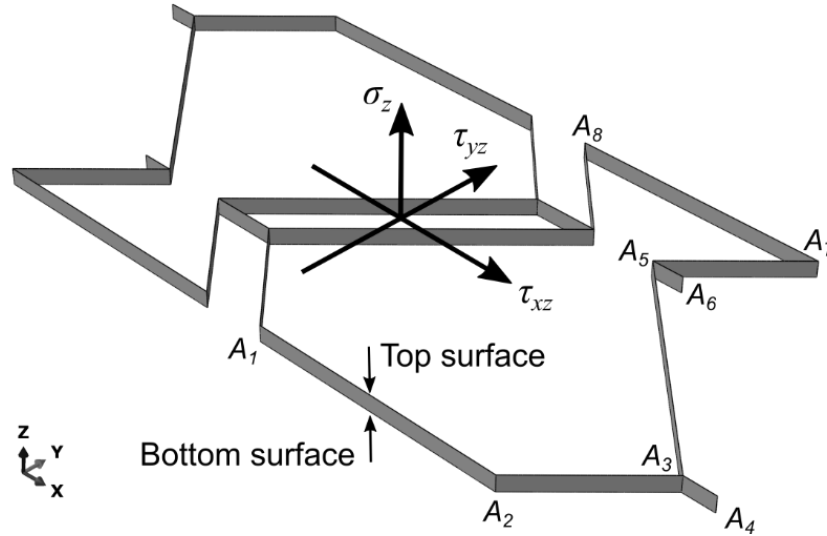


Figure 3-6 Loading and boundary conditions imposed on the unit cell to measure stiffness in the z-direction

Table 3-5 Elastic modulus of the Fish Cell metamaterial in z-direction normalized by constitutive material modulus E_s , obtained using FE and analytical methods.

Analysis Method	Normalized Stiffness k_z/k_s	Relative Error = $100 \times (k_z^{Tes.} - k_z^{cell}) /k_z^{Tes.}$
FEM-S8R (tessellation)	1.9120e-2	0.1 %
FEM-S8R (unit cell)	1.9142e-2	
FEM-C3D20 (unit cell)	1.9083e-2	
Analytical	1.9142e-2	

A parametric study is conducted for out of plane stiffness k_z as shown in Figure 3-7. The trend of stiffness variation with geometric parameters is also verified using FE analysis to further validate the Equation 3-32. It is found from Figure

3-7 that aspect ratio H/B and the horizontal component of the rib's length b , have a larger effect on stiffness in the z -direction. Variation of stiffness with respect to the vertical component of rib's length h is negligible. At small b/B values, connector's length variation does not affect the stiffness value, while at b/B close to unity; stiffness reduces when the connector's length increases.

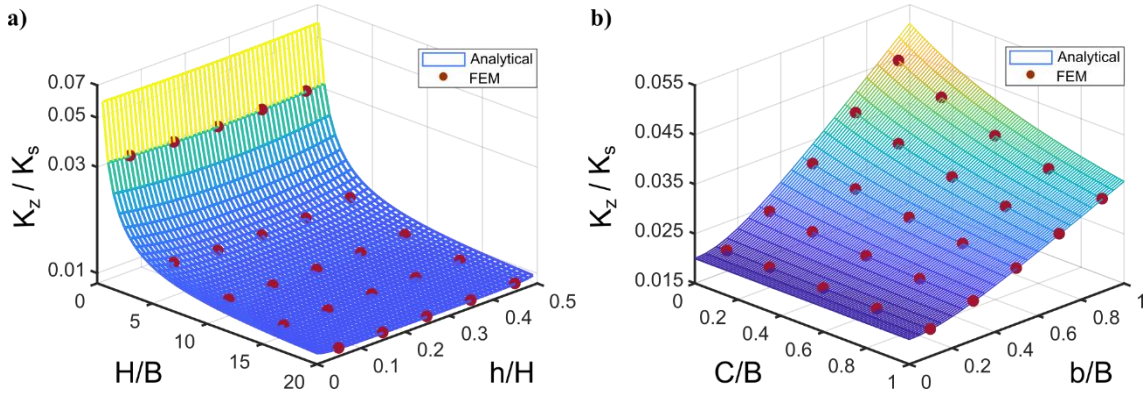


Figure 3-7 Variation of effective elastic modulus in z -direction for a Fish Cell metamaterial with $t = 0.01B$ and $W = B$ with respect to (a) aspect ratio of the cell and vertical component of inclined rib's length for $C = 0.1B$ & $b = 0.5B$, (b) connector's length and horizontal component of rib's length for $H = B$ & $h = 0.25H$. The red circles indicate the values obtained from FE analysis using ABAQUS.

3.3 Shear Loading

3.3.1 In-plane Shear G_{xy}

Pure shear loading is considered for shear analysis where the loading and boundary conditions on the tessellation are shown in Figure 3-8(a). The magnified view of the unit cell in tessellation shows that main deformation under pure shear loading is bending of the base members. For pure shear loading, same shear stress $\tau_{xy} = \tau_{yx} = \tau$ has been applied to the edges of the tessellation by exerting concentrated forces on the corresponding nodes of each edge. The sum of these concentrated forces divided by the area of the edge is equal to the shear stress. Displacements of one node at the corner of tessellation are constrained in x and y -direction to prevent the rigid body motion.

For the unit cell, one full thickness base member is used as shown in Figure 3-8(b). Similar to the tessellation, stress on the edges of the unit cell has been applied as concentrated loads P_x and P_y on the corresponding nodes of each edge where P_x and P_y are defined in Equation 3-33. Loads and boundary conditions imposed on the corresponding nodes of the unit cell are defined in Table 3-6. The deformed shape of the unit cell under described loading is shown in Figure 3-8(c) where dotted rectangle shows the unit cell domain before shear deformation and dashed parallelogram shows the domain after pure shear loading.

$$P_x = \tau \times (B + C)W \quad (3-33a)$$

$$P_y = \tau \times HW/2 \quad (3-33b)$$

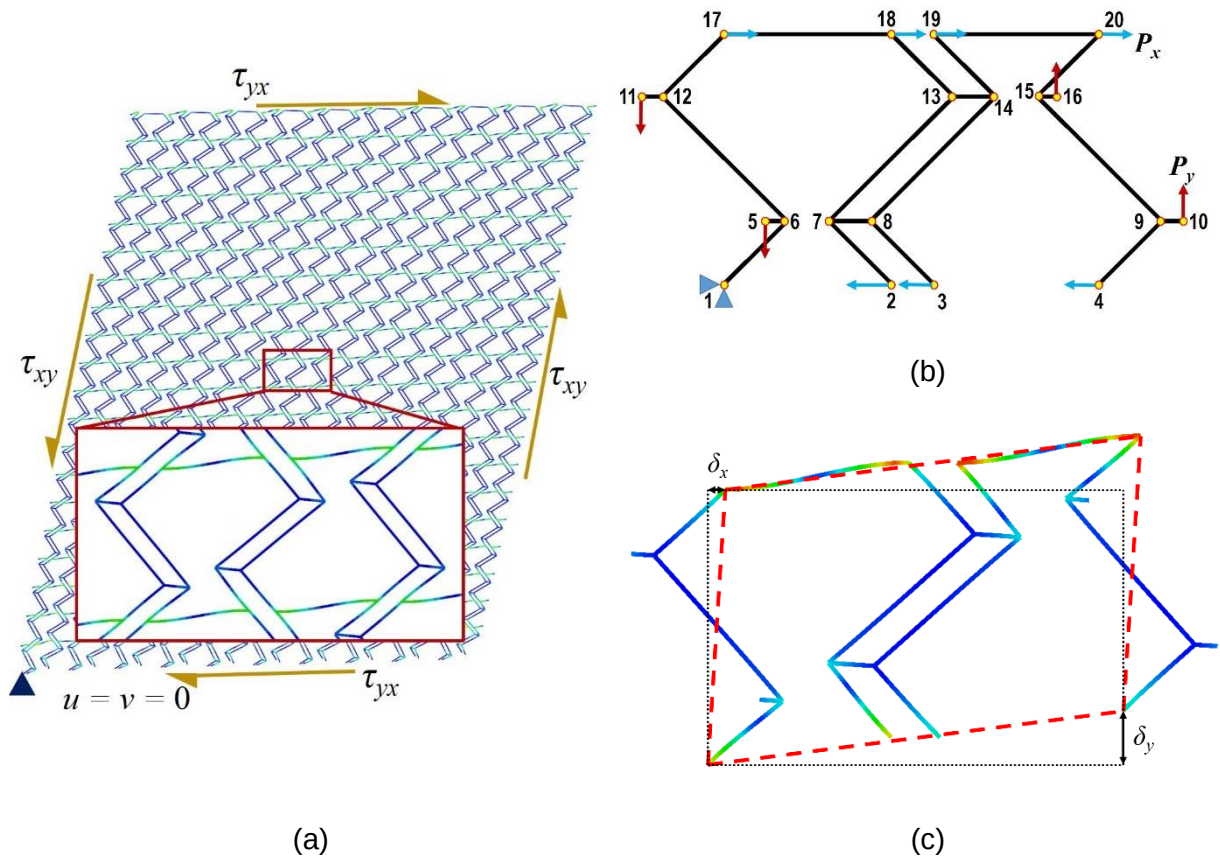


Figure 3-8 (a) Deformation of the tessellation under pure shear loading with a magnified view of the unit cell, (b) schematic of the unit cell with loads and boundary conditions for pure shear analysis, (c) deformed shape of the unit cell under pure shear.

The most important parameter affecting the shear response and deformation is the boundary conditions. In order to obtain deformation for a unit cell, similar to that shown in the magnified view in Figure 3-8(a), periodic boundary conditions must be applied. The only reason homogenous boundary conditions on node 1 was applied, is to prevent rigid body motion. Xia et al. (Xia et al., 2006; Xia, Zhang and Ellyin, 2003) presented an explicit unified form of boundary conditions for a periodic RVE that satisfies periodicity conditions and satisfies any combination of multiaxial loads. They also showed that homogenous boundary conditions on a periodic unit cell can be valid only when normal tractions are applied on boundaries. Therefore, homogenous boundary conditions used for deriving E_x and E_y in this chapter are valid but cannot be expanded to the shear loading. Because the assumption that planes remain plane after deformation is not valid anymore and also violates stress and strain periodicity conditions, homogenous boundary conditions cannot be used in the shear analysis. Hence, periodic boundary conditions are considered for shear loading to meet periodicity requirements, which means the boundaries of the unit cell after deformation must exactly meet each other without overlap or separation. In this regard, periodic boundary conditions for the unit cell are presented in Table 3-6. These conditions engage the degrees of freedom of the corresponding nodes on two opposite edges with each other.

Table 3-6 Boundary conditions and concentrated loads imposed on the unit cell for pure shear analysis.

Node	Boundary Condition			Load	
	u	v	$rotation$	x -direction	y -direction
1	0	0	<i>free</i>	0	0
2	<i>free</i>	<i>free</i>	<i>free</i>	$-2P_x$	0
3	<i>free</i>	<i>free</i>	<i>free</i>	$-P_x$	0
4	<i>free</i>	<i>free</i>	<i>free</i>	$-P_x$	0
17	<i>free</i>	$v_{17} = v_1$	$rot_{17} = rot_1$	P_x	0
18	<i>free</i>	$v_{18} = v_2$	$rot_{18} = rot_2$	P_x	0
19	<i>free</i>	$v_{19} = v_3$	$rot_{19} = rot_3$	P_x	0
20	<i>free</i>	$v_{20} = v_4$	$rot_{20} = rot_4$	P_x	0
5	<i>free</i>	<i>free</i>	<i>free</i>	0	$-P_y$
11	<i>free</i>	<i>free</i>	<i>free</i>	0	$-P_y$
10	$u_{10} = u_5$	<i>free</i>	$rot_{10} = rot_5$	0	P_y
16	$u_{16} = u_{12}$	<i>free</i>	$rot_{16} = rot_{12}$	0	P_y

In order to perform parametric study and obtain equivalent shear modulus for a range of geometrical parameters, the unit cell FE analysis code was written in MATLAB. Bending and axial deformations were considered for each member and cubic shape function was used where each member was assigned one element. In order to solve the multi-constraint problem in MATLAB, Master-Slave Method (MSM) was used (Felippa, 2004) which allows solving models with multi-point constraints and PBC. In order to derive the equivalent shear modulus of the unit cell, the displacements δ_x and δ_y as shown in Figure 3-8(c) were measured to calculate the shear strain as in Equation 3-34. Substituting

shear strain in the Hooke's law as in Equation 3-35, the effective shear modulus was obtained.

$$\gamma_{xy} = \frac{\delta_y}{l_x} + \frac{\delta_x}{l_y} \quad (3-34)$$

$$G_{xy} = \tau_{xy} / \gamma_{xy} \quad (3-35)$$

Periodic boundary conditions can be applied in ABAQUS between two nodes as MPCs by writing a subroutine that allows defining a relation between desired degrees of freedom as in Table 3-6. The shear modulus obtained from two FE models with material and dimensions mentioned in section 2.1 is presented in Table 3-7 for the unit cell and tessellation. The relative errors of the unit cell shear modulus are calculated with respect to the tessellation.

Table 3-7 Comparing the in-plane equivalent shear modulus of the Fish Cell metamaterial normalized by constitutive material modulus E_s using two FE models.

Analysis Method	Normalized Shear Modulus G_{xy} / E_s	Relative Error = $100 \times (G_{xy}^{Tes.} - G_{xy}^{cell}) / G_{xy}^{Tes.}$
FEM (tessellation)	1.3061e-7	
FEM-ABAQUS (unit cell)	1.3246e-7	1.4 %
FEM-MSM (unit cell)	1.3061e-7	0.0 %

Parametric study on the shear modulus is conducted using Equation 3-35 and the results are presented in Figure 3-9. It is evident that effective shear modulus of the Fish Cell metamaterial is highly dependent on the aspect ratio of the unit cell. Shear modulus reduces by several orders of magnitude when aspect ratio increases. For a constant height H , the shear modulus reduces by reducing the base members' length. By increasing the aspect ratio, variations of shear modulus with the vertical component of rib's length also increase as shown in Figure 3-9(a). Variations of shear modulus with connector's length and the horizontal component of rib's length are from the same order of magnitude as

shown in Figure 3-9(b). Shear modulus increases generally by increasing the connector's length and has a maximum before C/B reaches unity. Generally, for most of the b/B domain, shear modulus reduces by increasing the horizontal component of the rib's length.

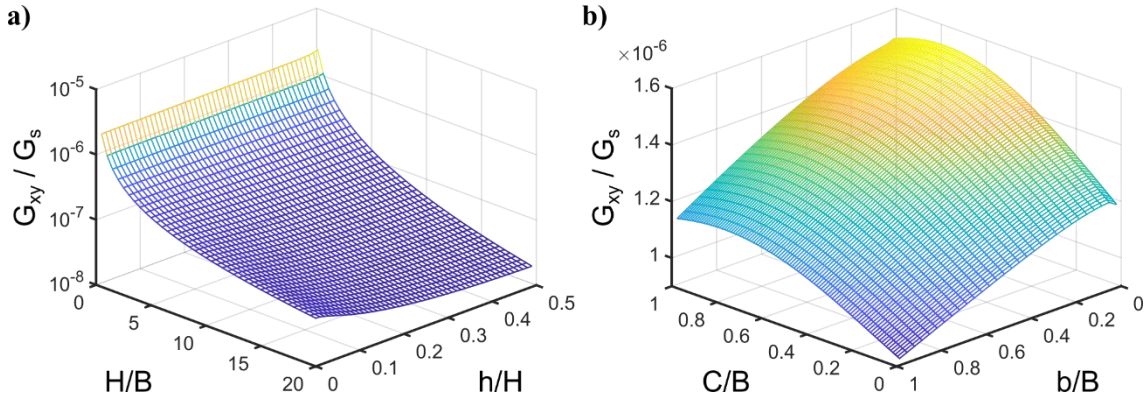


Figure 3-9 Variation of the in-plane shear modulus with respect to (a) aspect ratio of the cell and vertical component of inclined rib's length for $C = 0.1B$ & $b = 0.5B$, (b) connector's length and horizontal component of rib's length for $H = B$ & $h = 0.25H$.

3.3.2 Transverse Shear Loading

The Fish Cell transverse shear stiffness is important to study when the structure sustains shear loads such as torsion on the wing skin. The equivalent transverse shear modulus of the honeycomb structures is studied using unit load and displacement method (Kelsey, Gellatly and Clark, 1958) that leads to an upper and lower boundary for the effective transverse shear modulus. These lower(Reuss) and upper(Voigt) bounds are used to evaluate the transverse shear modulus of several 2D cellular structures (Hohe and Becker, 2002) because the accurate values can be obtained only using numerical models due to complex deformations and stress distribution (Gibson and Ashby, 1999; Malek and Gibson, 2015). One reason that upper and lower bounds are presented is that the assumption of uniform stress distribution in the cell walls, which was considered in derivation of the other equivalent elastic properties, is not valid for transverse shear loading anymore. The general stress field in the cell walls is complex and depends on the boundary conditions as well. Therefore, two assumptions are considered to enable an estimation of the

equivalent transverse shear modulus in two directions: a statically compatible uniform stress field and a kinematically compatible uniform strain field. The first assumption using the minimum complementary energy provides the lower bound, while the latter provides the upper bound using the minimum potential energy theorem. Hence, equivalent transverse shear modulus of the Fish Cell metamaterial in two directions are presented by the lower and the upper bounds. The unit cell in 3D format and coordinate system are shown in Figure 3-6 where bottom and top surfaces are also annotated. Considering the upper bound for shear modulus G_{xz} , the uniform strain γ_{xz} is applied on the unit cell. The equilibrium states that the strain energy of a structure with equivalent shear modulus is equal or smaller than the sum of the strain energy of each member as defined by Equation 3-36, where G_s is the solid material's shear modulus.

$$\frac{1}{2} G_{xz} \gamma_{xz}^2 V \leq \frac{1}{2} \sum_i G_s \gamma_i^2 V_i \quad (3-36)$$

The contribution of each member to the strain energy is calculated using the shear strain and the volume of each member presented in Equations 3-37 and 3-38, respectively. The notation of the members are presented in Figure 3-6.

$$\begin{cases} \gamma_{A_1 A_2} = \gamma_{A_7 A_8} = \gamma_{xz} \\ \gamma_{A_2 A_3} = \gamma_{A_5 A_7} = \gamma_{xz} \cos \theta \\ \gamma_{A_3 A_4} = \gamma_{A_5 A_6} = \gamma_{xz} \\ \gamma_{A_3 A_5} = \gamma_{xz} \cos \varphi \end{cases} \quad (3-37)$$

$$\begin{cases} V_{A_1 A_2} = V_{A_7 A_8} = BW \frac{t}{2} \\ V_{A_2 A_3} = V_{A_5 A_7} = Wt \sqrt{b^2 + h^2} \\ V_{A_3 A_4} = V_{A_5 A_6} = W C t \\ V_{A_3 A_5} = Wt \sqrt{4b^2 + (H - 2h)^2} \end{cases} \quad (3-38)$$

Substituting Equations 3-37 and 3-38 in 3-36 and using the definition of the L and D presented in section 3.2.1 and Equation 3-3, the closed form relation for the upper bound of the G_{xz} is obtained as Equation 3-39.

$$\frac{G_{xz}}{G_s} \leq \frac{t}{(B + C)H} \left[B + 2C + 2b^2 \left(\frac{1}{L} + \frac{2}{D} \right) \right] \quad (3-39)$$

Similarly, by considering the shear strains of the members presented in Equation 3-40 for the orthogonal direction, the upper bound of the equivalent shear modulus G_{yz} is obtained as Equation 3-41. It must be noted that under transverse shear loading in the y -direction, the base and connector members (A_1A_2 , A_3A_4 , A_5A_6 , and A_7A_8) do not sustain any shear load because they are normal to the shear load and only bending happens which has negligible contribution to the overall shear stiffness; therefore, their shear strain is neglected.

$$\begin{cases} \gamma_{A_2A_3} = \gamma_{A_5A_7} = \gamma_{yz} \sin \theta \\ \gamma_{A_3A_5} = \gamma_{yz} \sin \varphi \end{cases} \quad (3-40)$$

$$\frac{G_{yz}}{G_s} \leq \frac{2t}{(B+C)H} \left[\frac{2h^2}{L} + \frac{(H-2h)^2}{2D} \right] \quad (3-41)$$

The lower bound of the equivalent transverse shear modulus is obtained from the complementary energy as

$$\frac{1}{2} \frac{\tau_{xz}^2}{G_{xz}} V \leq \frac{1}{2} \sum_i \frac{\tau_i^2}{G_s} V_i \quad (3-42)$$

The base and connectors of the Fish Cell have the same value of stress because they are parallel to the loading direction. According to the equilibrium in the z -direction at the joints, the stresses in the members are defined as

$$\begin{cases} \tau_{A_2A_3} + \tau_{A_3A_5} = \tau_{A_3A_4} \\ \tau_{A_1A_2} = \tau_{A_2A_3} + \tau_{A_5A_7} \\ \tau_{A_2A_3} = \tau_{A_5A_7} \\ \tau_{A_1A_2} = \tau_{A_7A_8} = \tau_{A_3A_4} = \tau_{A_5A_6} \end{cases} \quad (3-43)$$

The stress values are derived by writing the equilibrium of loads in the x -direction.

$$\tau_{xz} H(B+C) = \tau_{A_1A_2} t \left(B + 2C + L \cos \theta + \frac{1}{2} D \cos \varphi \right) \quad (3-44)$$

Substituting the stresses and volumes of the members in Equation 3-42, the lower bound is derived as:

$$\frac{G_{xz}}{G_s} \geq \frac{t(2B + 4C + 2L\cos\theta + D\cos\varphi)^2}{H(B + C)(4B + 8C + 2L + D)} \quad (3-45)$$

Under shear stress in the y -direction, the base and connectors do not sustain **transverse shear loading** because they are normal to the loading direction. According to the equilibrium of the **shear flow** at joints, the shear stress in inclined ribs are equal; *i.e.*, $\tau_{A_2A_3} = \tau_{A_3A_5}$. Considering the equilibrium of the loads in the y -direction, the shear stress in the inclined ribs is derived.

$$\tau_{yz}H(B + C) = \tau_{A_2A_3}t(2L\sin\theta + D\sin\varphi) \quad (3-46)$$

Similarly, the lower bound of the transverse shear modulus G_{yz} is obtained from the equilibrium of the complementary energy.

$$\frac{G_{yz}}{G_s} \geq \frac{t(2L\sin\theta + D\sin\varphi)^2}{H(B + C)(2L + D)} \quad (3-47)$$

Due to the complexity of the stress distribution in transverse shear loading, the FE models are also developed using ABAQUS with quadratic shell elements S8R and fine mesh size to study the equivalent transverse shear modulus. The results of FE analysis are dependent on the boundary conditions imposed on the unit cell. The translational boundary conditions considered for Fish Cell metamaterial are $u_x = u_y = u_z = 0$ at the bottom surface. At the top surface (Lira, Innocenti and Scarpa, 2009), $u_y = u_z = 0$ and $u_x = \gamma_{xz} \times W$ for shear loading in the x -direction, while $u_x = u_z = 0$ and $u_y = \gamma_{yz} \times W$ for shear loading in the y -direction, where $\gamma_{xz} = \gamma_{yz} = 0.01$. The rotational degrees of freedom are $rot_x = rot_z = 0$ for shear loading in the x -direction and $rot_y = rot_z = 0$ for shear loading in the y -direction on both surfaces (Lira and Scarpa, 2010). The effective stress is then calculated by dividing the sum of reaction forces on a face to the effective area of the unit cell. The normalized transverse shear modulus obtained from FE analysis are compared to the upper and lower bounds in both directions in Table 3-8. The difference of the shear modulus obtained from a unit cell and a

tessellation is negligible as shown in Table 3-8; therefore, the parametric studies are verified with FE models of a unit cell.

Table 3-8 Transverse shear modulus G_{xz} and G_{yz} obtained for a unit cell with $B = 8\text{mm}$, $W = 1\text{mm}$, $t = 0.1\text{mm}$, $b = h = 6\text{mm}$, and $H = 24\text{mm}$. $G_s = E_s / [2(1+\nu)]$ is the shear modulus of the solid material.

Shear Modulus	Analytical		FEM	
	Lower bound	Upper bound	Unit Cell	5×5 tessellation
G_{xz} / G_s	1.17e-2	1.21e-2	1.19e-2	1.20e-2
G_{yz} / G_s	7.07e-3	10.61e-3	7.16e-3	7.20e-3

The deformed Fish Cell under shear loading in the x and y -direction are presented in Figure 3-10(a) and 3-10(b), respectively. Figure 3-10(b) verifies the assumption of negligible contribution of the base and connectors to the shear modulus G_{yz} as the stress levels are close to zero comparing to the inclined ribs. The effect of changing aspect ratio H/B on the shear modulus G_{xz} is strong as shown in Figure 3-10(c) where the normalized modulus reduces orders of magnitude by increasing the aspect ratio. In the other direction, G_{yz} changes at small aspect ratios, while the vertical component of rib's length, h , has no significant effect on neither G_{xz} nor G_{yz} . Figures 3-10(d) and 3-10(f) show that shear modulus G_{yz} obtained from FE is closer to the lower bound.

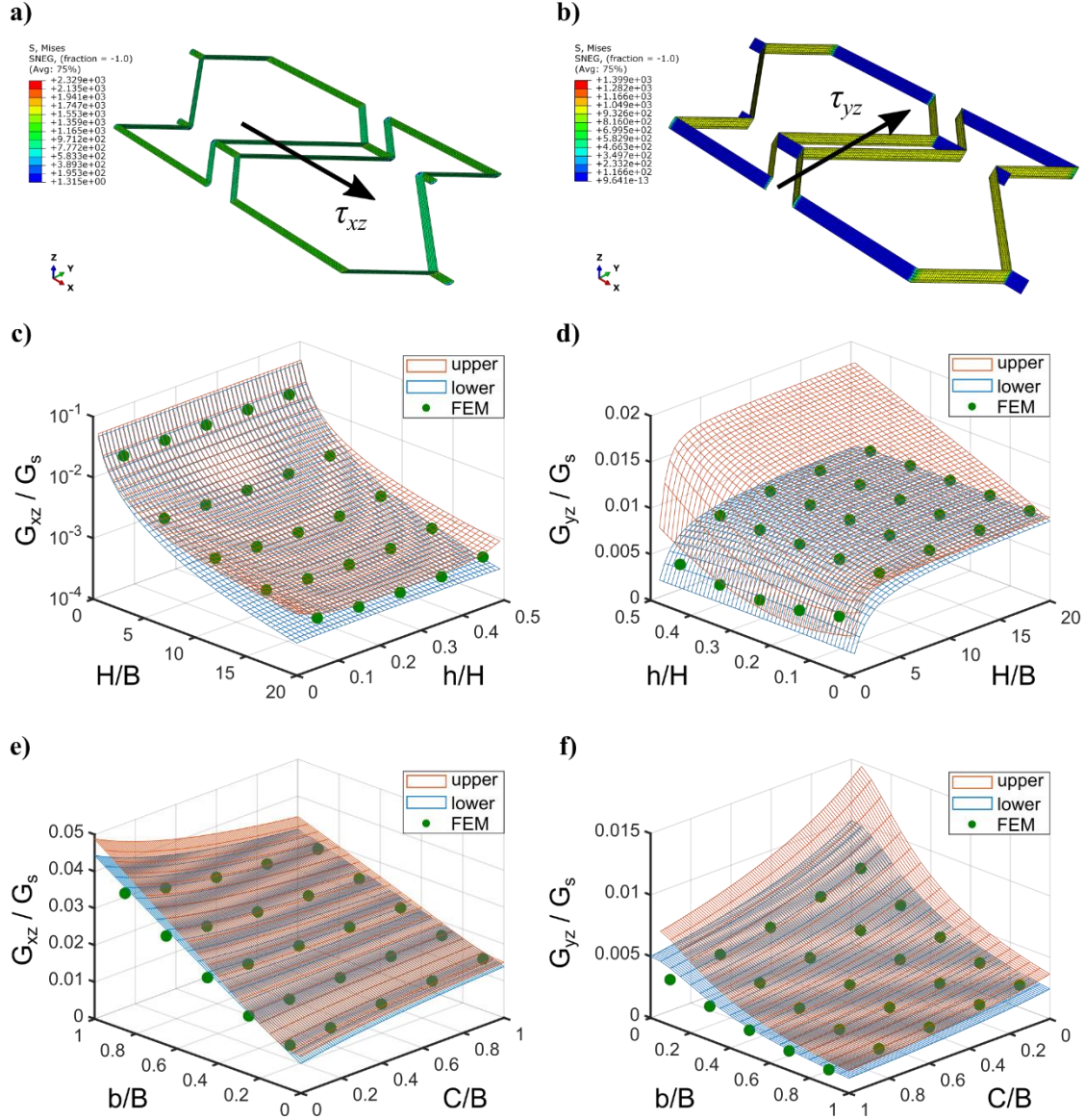
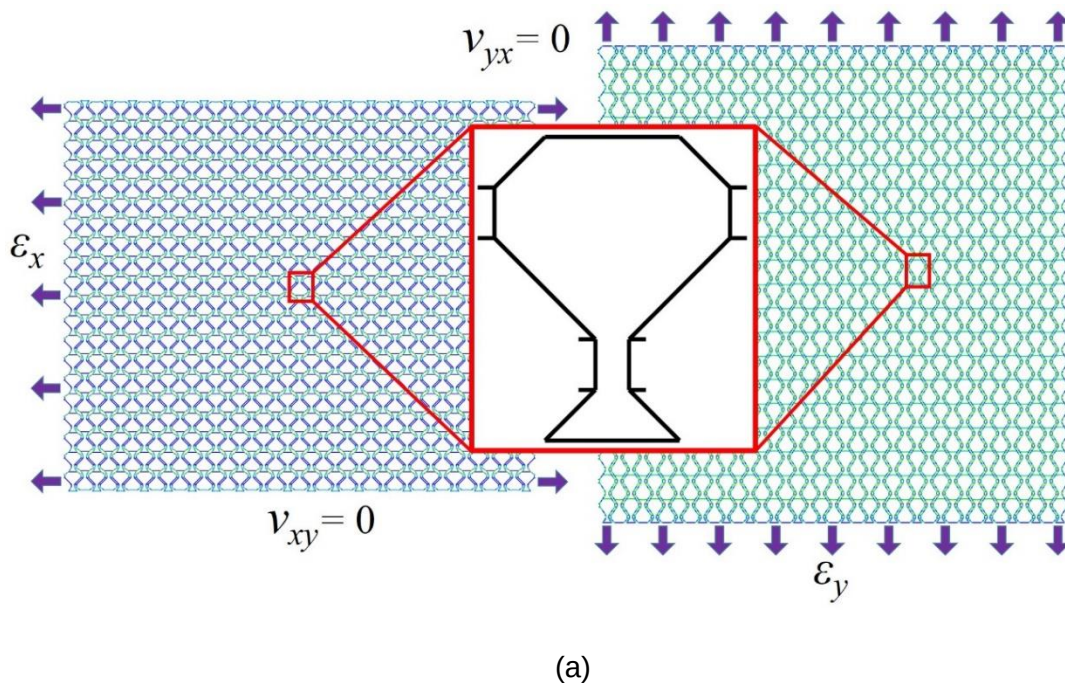


Figure 3-10 Deformed shape and stress distribution in Fish Cell metamaterial under shear loading (a) τ_{xz} and (b) τ_{yz} . Effect of the aspect ratio H/B and the vertical component of the rib's length, h , on the shear modulus (c) G_{xz} and (d) G_{yz} where $C = 0.1B$ & $b = 0.5B$. Effect of the connector's length and horizontal components of the rib's length, b , on the shear modulus (e) G_{xz} and (f) G_{yz} where $H = B$ & $h = 0.25H$. For all cases $B = 10\text{mm}$, $W = B$, and $t = 0.01B$.

3.4 Discussion

In section 3.2, it was shown that Fish Cell's ZPR is independent of the parameters' value and originates from the cell topology. Therefore, an optimized range of parameters based on desired equivalent elastic properties can be

obtained for Fish Cell. In addition to optimization of the parameters' values, the topology can also be subject of optimization by considering geometrical requirements. Members BC, CD, and DE have **point symmetry** with respect to the midpoint of member CD as shown in Figure 3-2(b). This geometrical relation is the basis of the Fish Cell topology that results in ZPR. Saving the pointwise symmetry of the inclined members, other geometries can be also applied for the inclined section to optimize the elastic properties of the Fish Cell. The Fish Cell analyzed in sections 3.2 and 3.3, was based on the combination of two convex and concave hexagons by the omission of the mutual base as discussed in **chapter 3**. In addition to the hexagon, higher order regular polygons with even number such as octagon can be also used as shown in Figure 3-11(a). The FE analysis of the octagonal Fish Cell tessellation is performed using ABAQUS in two orthogonal directions with beam elements B22 and fine mesh where ZPR in both directions is proved. Analytical curves such as spline that provide better control on the inclined member's profile design can be also used and result in ZPR as shown in Figure 3-11(b). This flexibility of the Fish Cell topology provides opportunities for future optimization studies.



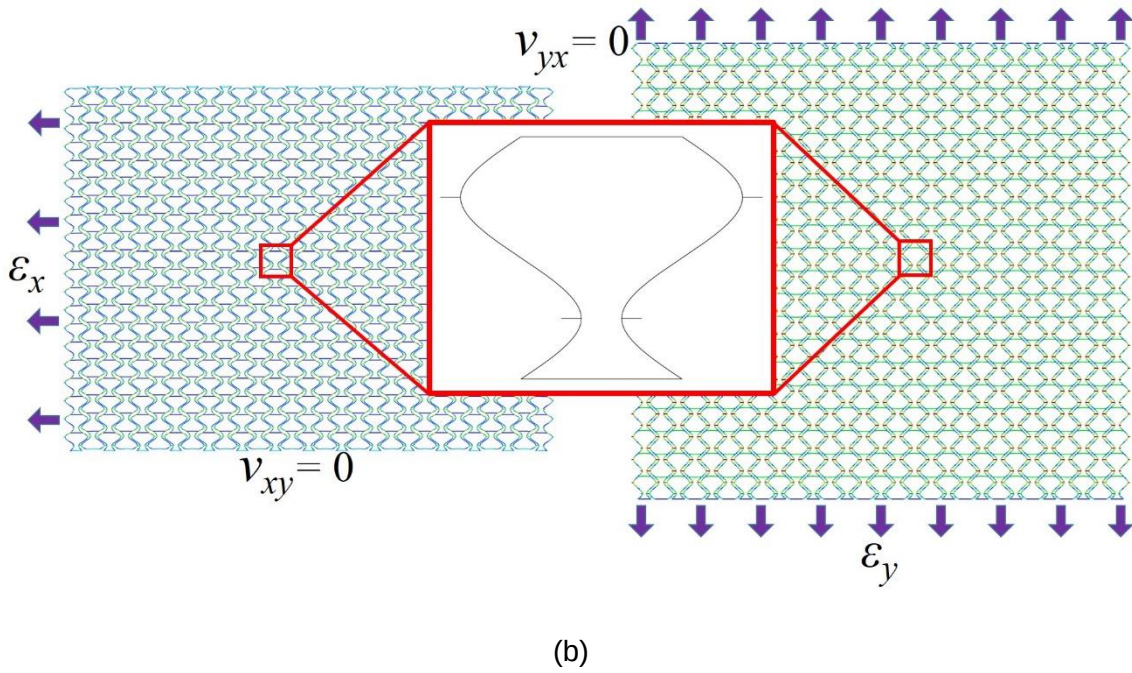


Figure 3-11 Different topologies for Fish Cell inclined section with ZPR: (a) Octagonal polygon, and (b) Spline curve.

3.5 Conclusion

Fish Cell as a new ZPR metamaterial is presented and its elastic properties are investigated. The equivalent elastic properties of the Fish Cell are obtained and verified using analytical and numerical models. The study shows that Fish Cell geometry achieves zero Poisson's ratio in both in-plane directions for any arbitrary value of its geometrical parameters. Equivalent elastic modulus in planar directions and equivalent stiffness in the transverse direction are derived. Importance of periodic boundary conditions (PBC) in the shear analysis is shown and equivalent shear modulus is obtained by considering proper PBC in the pure shear loading to obtain the same deformation in the unit cell as in the tessellation. The transverse shear modulus are studied based on Reuss and Voigt bounds and verified with FE analysis where results are closer to the lower bound. The elastic properties analysis is followed by parametric studies where the effect of aspect ratio, horizontal and vertical components of the inclined rib's length, and connector's length are studied. The results show that aspect ratio of the Fish Cell, H/B , has a strong effect on the transverse elastic properties.

Results suggest that based on design requirements, an optimum range of geometrical parameters can be obtained using optimization methods. Based on the relations obtained in this paper, optimization of the Fish Cell metamaterial for morphing skin's core or other applications can be conducted in future studies.

4 Dynamic Characteristics of Lattice Metamaterials Tuning the Poisson's Ratio

4.1 Introduction

Mechanical metamaterials (Zadpoor, 2016) belong to the class of designer materials whose unprecedented tailor-made properties originate from their designs at a smaller scale. One of these unusual properties is to tune the values of Poisson's ratio through the rational design of their microstructures, *i.e.*, negative Poisson's ratio or so-called auxetics (Kolken and Zadpoor, 2017b) and zero Poisson's ratio (ZPR). The values of the Poisson's ratio of a meta-structure can be either locally (Hedayati et al., 2018) or globally tuned to achieve unusual properties such as shape-shifting or shape-matching (Mirzaali et al., 2018b).

The microstructures of the mechanical metamaterials can be composed of several unit cells repeated in the space while comforting the integrity of the whole lattice structure. Thereby, by adjustment of the geometry of each unit cell, specific values of the Poisson's ratio can be achieved. The hexagonal (with positive or negative angle) (Kolken & Zadpoor, 2017) and chiral unit cells (Wu et al., 2019) were some of the examples of particular unit cell designs used in the past for the design of the mechanical metamaterials with tunable values of Poisson's ratio. This suggests that only by rational design of the unit cell's geometry one can reach a broader range of elastic properties (*i.e.*, elastic stiffness and Poisson's ratio) (Gibson et al., 1982; Malek and Gibson, 2015; Masters and Evans, 1996; Reis and Ganghoffer, 2012). Moreover, even the range of elastic properties can be extended by random integration of unit cells in the design space of the lattice structures (Mirzaali et al., 2017, 2018a; Mirzaali, Pahlavani and Zadpoor, 2019), which allows designing for targeted properties and (multi-)functionalities.

As far as ZPR or auxetic properties are concerned, they have several applications in the field of aeronautical (Gong et al., 2015b; Olympio and Gandhi, 2010b) and biomedical engineering (Douglas, Phani and Gagnon,

2014; Engelmayer et al., 2008; Soman et al., 2012). These classes of the mechanical metamaterials have been successfully designed and tested under specific quasi-static conditions and have already shown properties that cannot be achieved using traditionally engineered materials. However, it is still unclear how these unusual properties (*i.e.*, auxetic and ZPR) can influence the dynamic responses of, for example, auxetic structures as compared to non-auxetic ones. Additionally, it is not clear how freedom in devising the geometry of unit cells in mechanical metamaterials can help to tailor their dynamic characteristics. Therefore, understanding the evolution of dynamic characteristics (*e.g.*, damping properties, natural frequencies, and mode shapes) of mechanical metamaterials and the capabilities to tune those dynamic properties independently is of high importance as it can open up new opportunities in finding advanced materials with multi-stable/multifunctional properties.

The advent of additive manufacturing techniques (also known as 3D printing), has provided prospects and freedom to precisely design, control, and fabricate mechanical metamaterials with complex geometrical features; however, the scale of the samples manufactured with respect to the representative unit cell is one of their limits. This is because the maximum build volume of the commercial 3D printers and their minimum manufacturing size and tolerance are limited. Due to this limitation, the specimen size can be maximum 2 to 3 orders of magnitude larger than unit cell size. This limit raises concerns about the size effects in the analysis of the mechanical metamaterials (Ameen et al., 2018) and investigating the non-classical elasticity theories due to the length scale of the metamaterial where rotations of the microstructure come into account (Kadic, Frenzel and Wegener, 2018). In non-classical elasticity, the length scale can be used as a tool to alter the properties of the metamaterial, especially when rotation is the main deformation mechanism of the unit cell (Coulais, Kettenis and van Hecke, 2018; Frenzel, Kadic and Wegener, 2017). Besides, the fine length scale can emerge as an important parameter that helps in achieving extraordinary properties such as extremely high specific strength in metamaterials (Li and Gao, 2016; Montemayor and Greer, 2015). This size effect is also observed in elastic properties of cellular structures with negative

Poisson's ratio, such as auxetic foams (Lakes, 1991) and large positive Poisson's ratio structures like honeycombs (Wang and Stronge, 1999). It is also shown experimentally that length scale parameter and size effects are larger for lattices that have unit cells with bending dominated deformation mechanism than lattices where axial deformation mechanism is dominant (Rueger and Lakes, 2018). Based on this discussion, the length scale can be an effective parameter in altering the properties of the metamaterials tuning the Poisson's ratio because the bending of members and rotations are main deformation mechanisms. Therefore, a systematic study is required to investigate the length scale of the lattice metamaterials tuning the Poisson's ratio and its possible correlation with geometry and Poisson's ratio.

Towards this aim, here, we used a combination of analytical approach, numerical simulation, and dynamic testing to investigate the effect of Poisson's ratio and length scale in mechanical metamaterials by designing, additively manufacturing, and mechanically testing a set of 2D metamaterial lattices. The analytical model of an equivalent plate of the specimens was developed using higher order shear deformation theory and the modified couple-stress theory that can study the effect of length scale parameter on the structural response. Using the experimental results and utilizing the analytical model, a trend between the length scale and the Poisson's ratio was recognized and presented. Also, the effects of Poisson's ratio on the dynamic characteristics of the metamaterials, including natural frequency, mode shapes, and damping ratios, were investigated.

4.2 Experimental Samples and Tests

4.2.1 Samples' Design

The geometry of the unit cell and its parameters (*i.e.*, ligaments thickness, $t = 0.6$ mm, and length, $l = 5$ mm) are shown in Figure 4-1(a) where the out of plane thickness $h = 5$ mm is considered. While the ligaments' length remain constant, the values of the Poisson's ratio in two directions can be tuned by changing the angle θ in each unit cell (Figure 4-1b). The range of negative, zero, and positive Poisson's ratio was achieved by choosing five values for

angle θ as -20, -10, 0, 14, and 20 degrees. The relative density of the structure (ρ), elastic modulus, and Poisson's ratio of the cellular material were calculated based on the formulas in the literature (Malek & Gibson, 2015) (see the Appendix B). In addition, the finite element simulation was used to evaluate the effective mechanical properties considering the thickness at joints that result in higher stiffness. The effective Poisson's ratio and elastic modulus of the lattice in the x and y-directions are presented in Figure 4-1(b-d).

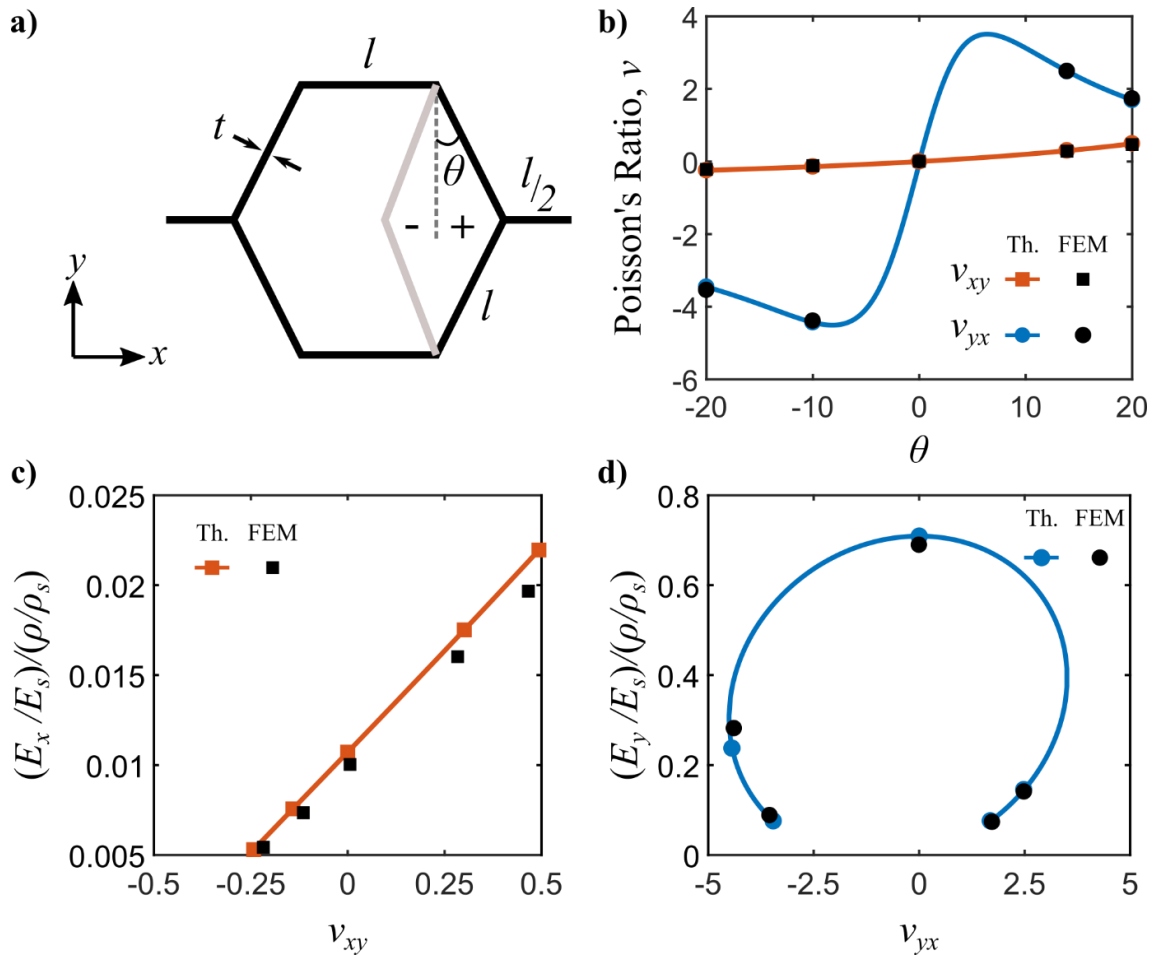


Figure 4-1 (a) Unit cell design of the hexagonal topology used for designing the samples where angle θ can possess different values, while other parameters are constant. (b) Variation of the Poisson's ratio in two planar directions are plotted versus angle θ which shows that large Poisson's ratios are achieved for two samples and FE results are also in good agreement with theory. The variation of the non-dimensional equivalent elastic modulus versus its corresponding Poisson's ratio is plotted in (c) x-direction and (d) y-direction

4.2.2 Samples' Manufacturing

In order to study the dynamic characteristics, five specimens with different values of Poisson's ratio were additively manufactured (Ultimaker 2+, The Netherlands) using commercial Polylactic Acid (PLA) filaments. The maximum planar build volume of the printer (200mm×200mm) limited the dimension of the samples. Three negative, zero, and positive Poisson's ratio samples manufactured using this method are shown in Figure 4-2.

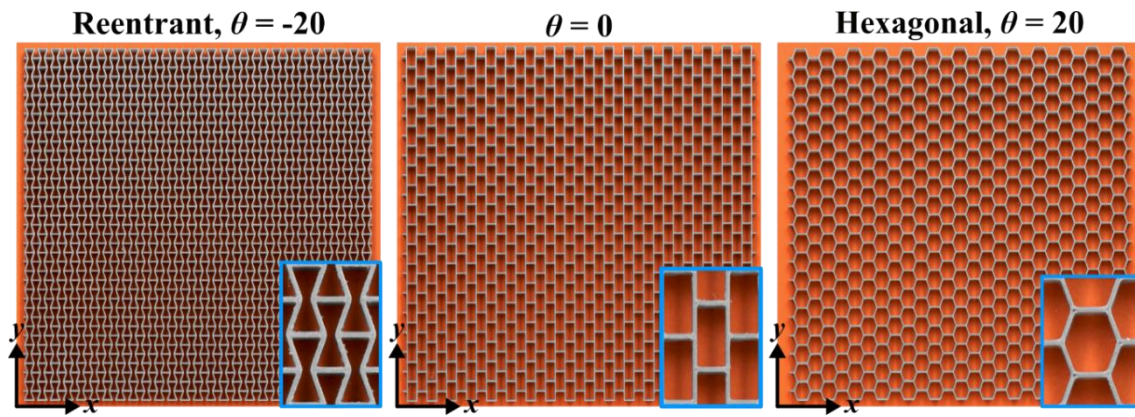


Figure 4-2 additively manufactured samples using Ultimaker FDM 3D printer and PLA material. Three samples with negative, zero, and positive Poisson's ratio are shown where the unit cell is illustrated in the magnified view. The thickness of each ligament is $t = 0.6$ mm

4.2.3 Modal Test Setup

We used experimental modal analysis (EMA) to measure the natural frequencies, damping ratio, and to visualize the corresponding mode shapes of the metamaterial samples. The test setup as shown in Figure 4-3 includes a signal generator (OFV-5000, Polytec) that controls the excitation parameters transferred to the shaker (Type 4809, Bruel&Kaer). The specimen was excited by the shaker via stinger that had an approximately pointwise contact with the specimen's surface. The velocity response of the specimen under the excitation was measured using Laser Doppler Vibrometer (LDV) (PSV-400, Polytec). Since the optical method was employed for the velocity measurement, targets for laser beam reflection were installed on the surface of each test sample using reflective tape (3M 3430). About 100 – 120 targets were installed on each lattice

structure depending on its size and geometry. In each test run, the specimen was excited using a periodic chirp function over 200 Hz frequency band where three complex averages using 6400 FFT lines were considered to measure the frequency response. The Polytec built-in software used the circle fit method to measure the resonance frequency and damping ratio at each peak in the frequency response.

The boundary conditions considered in this experiment were free conditions at all four edges of the specimens. Two fish wires that were connected to the rubbers installed on a frame, suspended the lattice structure loosely while it was free to move, which has been proved efficient in achieving free boundary conditions (Alijani et al., 2013). The stinger of the shaker was connected to the specimen in a point with about 20-30 mm offset from the center of the lattice structure. This excitation point was found to provide the best modal response after several trials because it was not on the node of the first five natural modes.

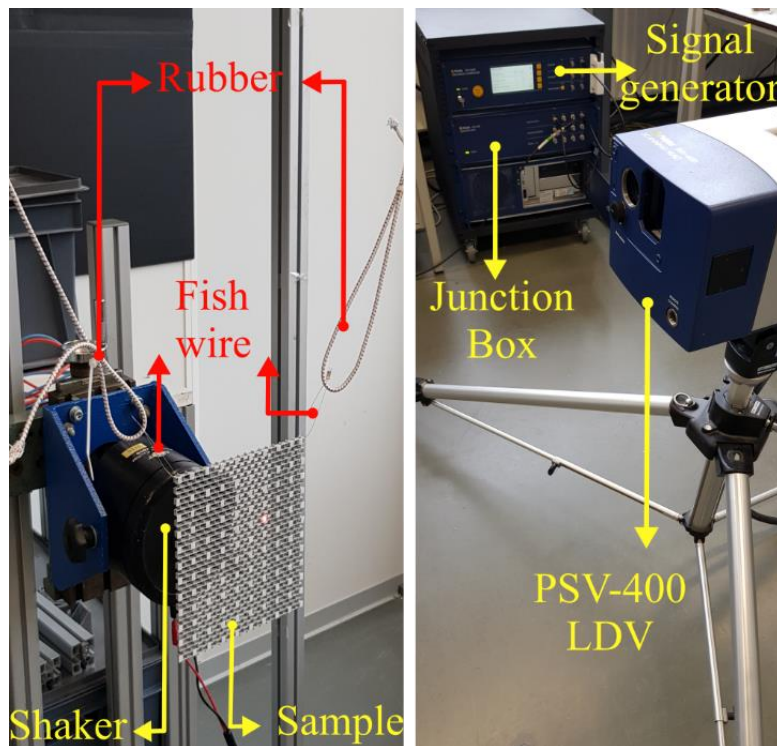


Figure 4-3 Modal test setup consisting of the shaker, test specimen and its installation, Polytec LDV sensor, and signal generator. The rubber and fish wire meet the free boundary condition requirements and the shaker has a point contact via stinger

4.2.4 Finite Element Analysis

The FE analysis of the samples was conducted by ABAQUS 2018 commercial package using fully integrated solid elements C3D20 and approximate global size of 0.3 mm. The Lanczos solver was used to solve the Eigen-value problem numerically and results were obtained for the 0-200 Hz frequency band. Six zero frequency modes were obtained due to rigid body motions caused by free boundary conditions of the samples and the natural frequencies of first five deflecting modes are reported in section 4.4.

4.3 Analytical Model and Formulation

In order to derive analytical model for this study, the samples are considered as moderately thick plates with equivalent homogenous properties obtained from FE analysis and constitutive models of the literature presented in the supplementary document. The equivalent plate with length a , width b , and thickness h is shown in Figure 4-4 where the Cartesian coordinate origin is on the mid-surface of the plate.

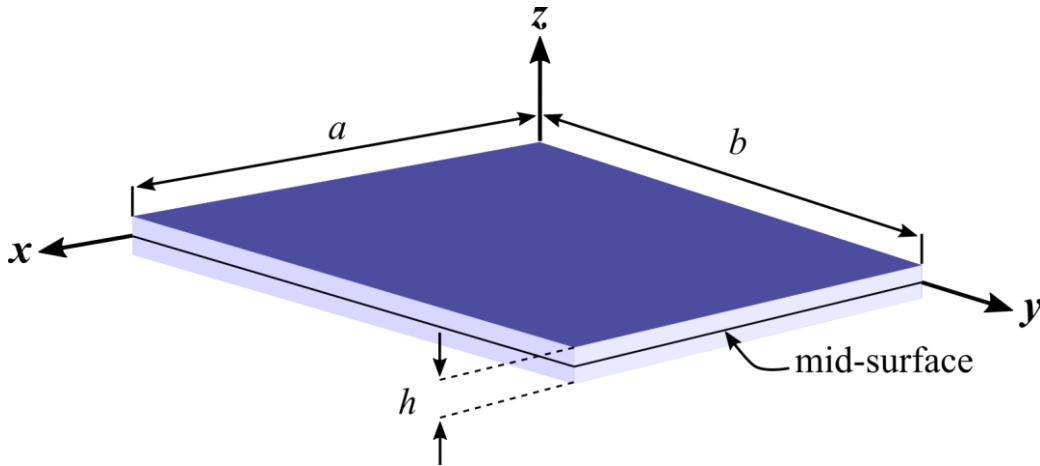


Figure 4-4 Geometrical parameters and coordinate system of an equivalent plate representing the metamaterial samples. The x-y axes of the plate are aligned with those of the unit cell

4.3.1 Reddy's Higher Order Plate Theory

The plate is moderately thick ($a/h \approx 36$) and shear deformations and rotary inertia have accountable effect on the deformations of the plate; therefore, the displacements of an arbitrary point on the plate are defined using three components based on the Reddy's higher-order shear deformation theory (HSDT) (Reddy, 2003).

$$u(x, y, z) = u_0 + z\phi_x - \frac{4}{3h^2}z^3\left(\phi_x + \frac{\partial w}{\partial x}\right) \quad (4-1a)$$

$$v(x, y, z) = v_0 + z\phi_y - \frac{4}{3h^2}z^3\left(\phi_y + \frac{\partial w}{\partial y}\right) \quad (4-1b)$$

$$w(x, y, z) = w_0 \quad (4-1c)$$

Displacements u_0 , v_0 , and w_0 in Equation (4-1) represent the displacements of a point on the middle-surface in the x , y , and z -directions, respectively and ϕ_x and ϕ_y represent the rotations of the cross section at that point. Considering the plane-stress condition, the strains are derived as Equation (4-2).

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^{(0)} \\ \varepsilon_y^{(0)} \\ \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x^{(0)} \\ \kappa_y^{(0)} \\ 0 \\ 0 \\ \kappa_{xy}^{(0)} \end{Bmatrix} + z^2 \begin{Bmatrix} 0 \\ 0 \\ \kappa_{yz}^{(1)} \\ \kappa_{xz}^{(1)} \\ 0 \end{Bmatrix} + z^3 \begin{Bmatrix} \kappa_x^{(2)} \\ \kappa_y^{(2)} \\ 0 \\ 0 \\ \kappa_{xy}^{(2)} \end{Bmatrix} \quad (4-2)$$

The relations between strains at the mid-surface and curvatures with displacements are defined in Equation (4-3).

$$\varepsilon_x^{(0)} = \frac{\partial u}{\partial x} \quad (4-3a)$$

$$\varepsilon_y^{(0)} = \frac{\partial v}{\partial y} \quad (4-3b)$$

$$\gamma_{xy}^{(0)} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad (4-3c)$$

$$\gamma_{xz}^{(0)} = \phi_x + \frac{\partial w}{\partial x} \quad (4-3d)$$

$$\gamma_{yz}^{(0)} = \phi_y + \frac{\partial w}{\partial y} \quad (4-3e)$$

$$\kappa_x^{(0)} = \frac{\partial \phi_x}{\partial x} \quad (4-3f)$$

$$\kappa_y^{(0)} = \frac{\partial \phi_y}{\partial y} \quad (4-3g)$$

$$\kappa_{xy}^{(0)} = \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \quad (4-3h)$$

$$\kappa_{xz}^{(1)} = -\frac{4}{h^2} \left(\phi_x + \frac{\partial w}{\partial x} \right) \quad (4-3i)$$

$$\kappa_{yz}^{(1)} = -\frac{4}{h^2} \left(\phi_y + \frac{\partial w}{\partial y} \right) \quad (4-3j)$$

$$\kappa_x^{(2)} = -\frac{4}{3h^2} \left(\frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) \quad (4-3k)$$

$$\kappa_y^{(2)} = -\frac{4}{3h^2} \left(\frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2} \right) \quad (4-3l)$$

$$\kappa_{xy}^{(2)} = -\frac{4}{3h^2} \left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} + 2 \frac{\partial^2 w}{\partial x \partial y} \right) \quad (4-3m)$$

Using the strains obtained in Equation (4-2), the stresses at each point of the plate were obtained from Hooke's law by developing the stiffness matrix presented in Equation (4-4).

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \quad (4-4)$$

The elements of stiffness matrix are derived using cellular material theory from literature and FE analysis of the samples, which are presented in the supplementary document.

4.3.2 Refined Modified Couple Stress Theory (MCST)

In the modified couple-stress theory (Yang et al., 2002), the couple-stress moments and curvatures are correlated according to Equation (4-5).

$$m_{ij} = \mu \chi_{ij} \quad (4-5)$$

where μ is the couple-stress tensor, χ_{ij} ($i, j = x, y, z$) is the vector of curvatures, and m_{ij} is the vector of couple-stress moments, which are defined in Equation (4-6) with all components where the coordinate system presented in Fig. 4-4 is used.

$$\chi_{ij} = \{\chi_x, \chi_y, \chi_z, \chi_{xy}, \chi_{yx}, \chi_{yz}, \chi_{zy}, \chi_{xz}, \chi_{zx}\}^T \quad (4-6a)$$

$$m_{ij} = \{m_x, m_y, m_z, m_{xy}, m_{yx}, m_{yz}, m_{zy}, m_{xz}, m_{zx}\}^T \quad (4-6b)$$

The curvature components χ_{ij} are derived from rotations ω_{ij} of an element of the plate using Equation (4-7), which are correlated to the displacement field $\tilde{u}(u, v, w)$ according to Equation (4-8).

$$\chi_{ij} = \frac{1}{2}(\nabla \omega + (\nabla \omega)^T) \quad (4-7)$$

$$\omega_{ij} = \frac{1}{2} \text{Curl}(\tilde{u}) \quad (4-8)$$

The important parameter in the modified couple-stress theory is the couple-stress tensor that is discussed extensively in the literature. The modified couple-stress theory of Yang et al. (Yang et al., 2002) presented a symmetric couple-stress tensor with one length scale parameter, which is appropriate only for isotropic materials. For orthotropic materials such as composite laminates, Chen et al. (Chen, Xu and Li, 2012) presented a new couple-stress tensor that considered the directional properties of the composite material using two length scale parameters. They also assumed that rotation about z-axis of the material is zero ($\omega_z = 0$), which simplified the theory by reducing the number of curvature components considered. Mohammadabadi and Daneshmehr developed a model by considering additional rotations and three length scale parameters (Mohammad-Abadi and Daneshmehr, 2015), while rotation about z-axis was

neglected. A more complete model considering all nine rotation components was presented by Thanh et al. (Thanh et al., 2019) that uses three length scale parameters in three directions, which is adopted in this paper. In order to evaluate the effect of simplifications applied by Chen et al. (Chen, Xu and Li, 2012) in their couple-stress tensor, a correction factor CF that has two values of 0 or 1 is used in the couple-stress tensor of the current study as presented in Equation (4-9).

$$\mu = \begin{bmatrix} 2G_{xz}l_x^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2G_{yz}l_y^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & CF \times 2G_{xy}l_z^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{xz}l_x^2 & G_{yz}l_y^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{xz}l_x^2 & G_{yz}l_y^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & CF \times G_{yz}l_y^2 & CF \times G_{xy}l_z^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & CF \times G_{yz}l_y^2 & CF \times G_{xy}l_z^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & CF \times G_{xz}l_x^2 & CF \times G_{xy}l_z^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & CF \times G_{xz}l_x^2 & CF \times G_{xy}l_z^2 \end{bmatrix} \quad (4-9)$$

where l_x , l_y , and l_z are length scale parameters of the material in three directions. The important point about the values of the length scale parameters in modified couple-stress theory is that they are not constant and depend on the unit cell to the structure size ratio (Khorshidi, 2018). In this regard, a standard method for determining the length scale parameters is the experimental tests such as vibration analysis and then fitting the experimental data using the developed analytical model to extract the length scale parameters (Li et al., 2018), which is the method pursued in this paper.

4.3.3 Displacement Functions and Boundary Conditions

The trigonometric displacement functions are very efficient for simply supported boundary condition; however, they require higher degrees of freedom for other boundary conditions. Therefore, alternative solutions like polynomial expansions such as Chebyshev polynomials that require similar degrees of freedom for any boundary conditions are more efficient (Kurylov and Amabili, 2010). The recursive formula for Chebyshev polynomials of the first kind is presented in Equation (4-10) where n is the order of the polynomial (Alijani and Amabili, 2013).

$$T_0(x) = 1 \quad (4-10a)$$

$$T_1(x) = x \quad (4-10b)$$

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \quad (4-10c)$$

The natural modes of vibration considering synchronous motion with a harmonic time function $g(t) = \exp(i\omega t)$ are presented in Equation (4-11).

$$u_0(x, y, t) = U(x, y)g(t) \quad (4-11a)$$

$$v_0(x, y, t) = V(x, y)g(t) \quad (4-11b)$$

$$w_0(x, y, t) = W(x, y)g(t) \quad (4-11c)$$

$$\phi_x(x, y, t) = \Phi_x(x, y)g(t) \quad (4-11d)$$

$$\phi_y(x, y, t) = \Phi_y(x, y)g(t) \quad (4-11e)$$

Considering Chebyshev polynomials in both x and y -directions, the mode shapes U , V , W , Φ_x , and Φ_y are presented in Eq. (4-12). It should be noted that domain of the variable in Chebyshev polynomials is from -1 to 1, therefore, transformations $2\xi-1$ and $2\eta-1$ are applied to normalize the plate's coordinates where $\xi = x/a$ and $\eta = y/b$. The same orders M and N of the Chebyshev polynomials for all mode shapes is assumed in the x and y -direction, respectively.

$$U(\xi, \eta) = \sum_{m=0}^M \sum_{n=0}^N U_{m,n} T_m(2\xi - 1) T_n(2\eta - 1) \quad (4-12a)$$

$$V(\xi, \eta) = \sum_{m=0}^M \sum_{n=0}^N V_{m,n} T_m(2\xi - 1) T_n(2\eta - 1) \quad (4-12b)$$

$$W(\xi, \eta) = \sum_{m=0}^M \sum_{n=0}^N W_{m,n} T_m(2\xi - 1) T_n(2\eta - 1) \quad (4-12c)$$

$$\Phi_x(\xi, \eta) = \sum_{m=0}^M \sum_{n=0}^N \Phi_{x,m,n} T_m(2\xi - 1) T_n(2\eta - 1) \quad (4-12d)$$

$$\Phi_y(\xi, \eta) = \sum_{m=0}^M \sum_{n=0}^N \Phi_{y_{m,n}} T_m(2\xi - 1) T_n(2\eta - 1) \quad (4-12e)$$

Since the boundary conditions of the test samples are free on all edges, there is no geometric constraints applied on the shape functions of Equation (4-12).

4.3.4 Equations of Motion

The potential energy U_p of the system consists of the strains and rotations of the equivalent plate as in Equation (4-13a). The kinetic energy of the equivalent plate is also presented as in Equation (4-13b).

$$U_p = \frac{1}{2} \int_0^a \int_0^b \int_{-\frac{h}{2}}^{+\frac{h}{2}} (\sigma^T \varepsilon + m^T \chi) dz dy dx \quad (4-13a)$$

$$T_p = \frac{1}{2} \int_0^a \int_0^b \int_{-\frac{h}{2}}^{+\frac{h}{2}} \rho (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dz dy dx \quad (4-13b)$$

Using the Rayleigh-Ritz method, the equations of motion were obtained, which form the Eigen value problem presented in Equation (4-14) when free vibration is considered.

$$(K_p - \Omega^2 M_p) P = 0 \quad (4-14)$$

Where K_p is the plate stiffness matrix, M_p is the inertia matrix, and Ω is the circular frequency. The Eigen-vector P is the vector of amplitudes of each degree of freedom of the mode shapes as described in Equation (4-15). The size of the vector P depends on the Chebyshev polynomials' order, which is equal to $5 \times (M+1) \times (N+1)$.

$$P = (U_{1,1}, \dots, U_{M,N}, V_{1,1}, \dots, V_{M,N}, W_{1,1}, \dots, W_{M,N}, \Phi_{x_{1,1}}, \dots, \Phi_{x_{M,N}}, \Phi_{y_{1,1}}, \dots, \Phi_{y_{M,N}}) \quad (4-15)$$

In order to have a non-trivial solution, the determinant of the coefficients of Equation (4-14), for each metamaterial sample with its equivalent properties, must be zero. Solving this problem, the natural frequencies and mode shapes can be obtained.

4.4 Results and Discussion

In order to study the analytical results, the components of the stiffness and inertia matrices are obtained using MATLAB software where the Eigen-value problem of the Equation (4-14) is solved for every sample using its equivalent properties discussed in the supplementary document. Chebyshev polynomials of order 7 are considered for all mode shapes that results in 320×320 stiffness and inertia matrices. The validation of the code and results are presented in the Appendix B.

4.4.1 Natural Frequencies

The natural frequencies of the first five modes of each sample are presented in Table 4-1 where experimental (average value), numerical, and analytical values using both classic and couple-stress theories are presented. The average experimental natural frequencies are higher than full scale FE analysis results, which can be due to manufacturing of samples that results in thicker joints and higher stiffness as a result; however, the trend of the results are consistent. Based on the first mode, the natural frequencies increase with the Poisson's ratio of the sample. The natural frequencies obtained by solving Equation (4-14) with length scales equal to zero, i.e. only classical HSDT considered, are much lower than experimental and FE values. The relative error of the natural frequencies obtained from HSDT with respect to experiment are presented as Error-1 where 30-50% error exists for different mode. This large difference in the natural frequency prediction using HSDT stems from the size effect in the samples, which can be modified by considering length scale using couple-stress theory.

In order to use the couple-stress theory, the length scales of the material are required to form the couple-stress tensor of Equation (4-9). In this case, the equal length scale values $l_x = l_y = l_z$ are assumed and the contribution of the transverse rotations on the overall stiffness of the structure is evaluated by setting CF as 0 or 1 in the analysis. Using the experimental results, the length scale of each sample is estimated by minimizing the average error of the five modes. Therefore, Error-2_{ave} is defined as the minimization function where

Error-2 is the relative error between natural frequencies obtained from experiment and refined MCST.

$$\text{Error-2}_{ave} = \frac{1}{5} \sum_{i=1}^5 \sqrt{\frac{(\Omega_{experiment_i} - \Omega_{MCST_i})^2}{\Omega_{experiment_i}^2}} \quad (4-16)$$

Table 4-1 Natural Frequencies of the first five modes of each samples obtained from Experimental Modal Analysis (EMA), FE simulation, and analytical model. The analytical model results include classical elasticity with HSDT and non-classical refined Modified Couple-Stress Theory (MCST)

θ	mode	Natural Frequency, Ω (Hz)							
		Experiment	FEM	HSDT	Error-1 %	Modified Couple Stress Theory			
						length scale (mm) $l_x = l_y = l_z$	CF		Error-2 %
							0	1	
-20	1	26.16	21.77	13.45	48.58	0.2851	26.16	26.16	0.00
	2	29.87	32.10	20.61	31.00		28.15	28.15	5.77
	3	58.42	52.86	33.29	43.02		58.79	58.79	0.64
	4	76.12	85.64	55.59	26.97		78.07	78.07	2.57
	5	113.45	105.13	67.17	40.79		109.48	109.48	3.49
-10	1	28.92	25.27	16.64	42.46	0.2950	29.55	29.55	2.17
	2	36.08	39.05	24.76	31.37		33.70	33.70	6.58
	3	68.47	62.94	41.43	39.49		68.47	68.47	0.00
	4	86.01	107.31	68.80	20.01		95.63	95.63	11.1
	5	134.30	126.89	82.83	38.32		128.63	128.63	4.21
0	1	30.59	27.24	18.44	39.72	0.3044	30.50	30.50	0.28
	2	36.06	42.36	26.66	26.07		36.06	36.06	0.00
	3	72.36	69.16	46.24	36.10		72.99	72.99	0.87
	4	93.08	116.37	73.35	21.20		101.33	101.33	8.86
	5	145.89	140.46	92.52	36.58		139.74	139.74	4.21
14	1	37.18	30.95	22.82	38.62	0.3474	37.18	37.18	0.00
	2	50.90	52.93	36.26	28.76		52.69	52.69	3.51
	3	93.77	83.14	60.54	35.44		96.12	96.12	2.51
	4	137.63	126.11	92.27	32.96		135.29	135.29	1.69
	5	176.62	149.31	123.22	30.23		166.36	166.36	5.81
20	1	39.04	35.69	24.46	37.35	0.3558	39.03	39.03	0.009

	2	59.29	61.43	39.29	33.73		58.68	58.68	1.02
	3	99.02	97.90	66.43	32.91		104.54	104.54	5.57
	4	124.91	125.24	85.63	31.45		129.37	129.37	3.57
	5	173.93	143.83	106.70	38.65		148.95	148.95	14.36

The estimated length scale parameter for each sample by minimizing the Equation (4-16) is presented in Table 4-1. The couple-stress moments and rotations contribute to the increment in strain energy of the structure; therefore, the natural frequencies increase. The corresponding natural frequencies of the refined MCST using the estimated length scale values are presented for both $CF = 0$ and $CF = 1$ in Table 4-1. The natural frequencies obtained from MCST present significant improvement comparing to the HSDT, which is conceived by comparing the values of Error-1 and Error-2 for each mode. Therefore, the size effects in analytical models for dynamic analysis of the mechanical metamaterial structures, which are an order of magnitude larger than unit cell size, must be considered.

In addition, the effect of transverse rotations is negligible as the natural frequencies obtained with $CF = 0$ and $CF = 1$ are equal. Therefore, the 9×9 couple-stress tensor of Equation (4-9) can be reduced to a 4×4 tensor for further analysis to reduce the number of equations. Moreover, it should be noted that samples in this study were 2D lattices and if the structure is a 3D lattice where the number of cells in the z-direction of the plate varies, then possibility of using these simplifications must be further studied.

4.4.2 Length scale variation with angle θ in hexagonal metamaterials

Considering the estimated length scale parameters in Table 4-1 for the five samples, it is found that the length scale parameter increases by the angle θ in the samples. Although this trend is obtained by considering the equal length scales, it is consistent for the geometrical locus plot of the length scale parameters in two directions for the first natural mode as shown in Figure 4-5. In contrast to the Table 4-1 that presented the length scale by considering all five natural modes, the locus plots of the length scale in Figure 4-5 are obtained for the natural frequency of the first mode of each sample. Figure 4-5(a) presents

the locus plot of the length scales by considering the experimental natural frequencies, while Figure 4-5(b) considers FE simulation natural frequencies to derive locus plots. Both figures indicate a consistent trend that the length scale increases with angle θ and Poisson's ratio as a result. Each locus plot was obtained from the coincidence of the admissible surface of natural frequency versus length scale with corresponding natural frequency of the experiment or FEM. The admissible surface locus of the first three natural modes of the zero Poisson's ratio sample ($\theta = 0$) are presented in Figure 4-5(c) where incremental trend of the natural frequency with length scale parameters is shown.

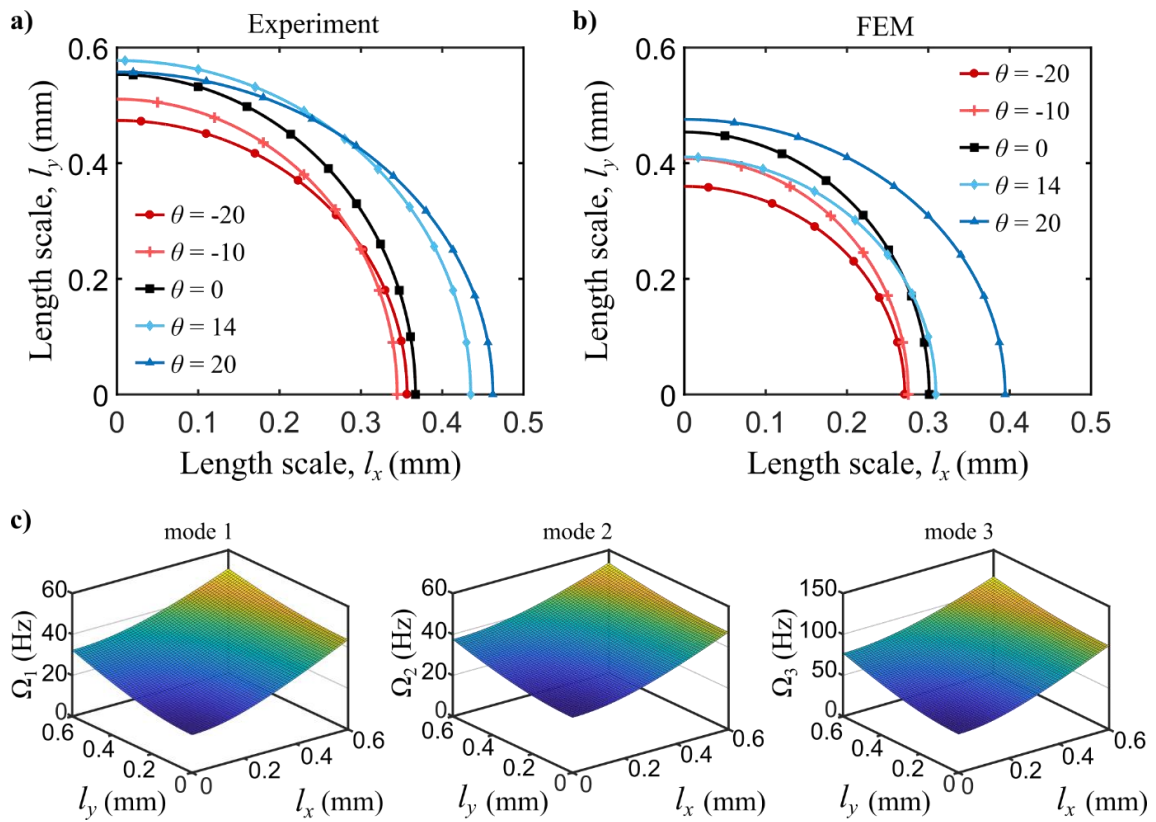


Figure 4-5 Locus plot of the length scale parameters for each test specimen obtained using the natural frequencies of (a) experiment and (b) FE analysis. (c) 3D locus surface of the natural frequencies and the length scale parameters for the first three modes of the zero Poisson's ratio sample

4.4.3 Effect of Poisson's ratio on the mode shapes

The Poisson's ratio causes a coupling between bending of the plate in two directions, which means applying a moment in one direction induces bending in

the normal direction, too. The curvature and induced curvatures of the samples with positive Poisson's ratio are in opposite directions (anticlastic) because the stiffness matrix elements Q_{11} and Q_{12} have opposite signs. In contrast, negative Poisson's ratio metamaterials expose curvature and normal induced curvatures of the same directions (synclastic). While, zero Poisson's ratio metamaterials do not present any induced curvature because $Q_{12} = Q_{21} = 0$. This effect of the Poisson's ratio is observed in the natural mode shapes of the test specimens obtained from FE simulations, EMA, and analytical model, which are consistent for all modes. The two positive Poisson's ratio and the two negative Poisson's ratio samples presented the same five natural mode shapes; therefore, only one set for each is presented in Figure 4-6. The mode shapes are categorized according to the symmetry (S) or asymmetry (A) of the deformation with respect to the x and y -directions (Alijani and Amabili, 2013). The first four natural modes of all samples share the same shapes; however, the deformation shape is different because the Poisson's ratio affects the induced curvature. The first mode shape is asymmetric about both directions (AA), which is dominated by the shear stiffness of the sample; therefore, the deformations are identical in three categories. The second, third, and fourth mode shapes include symmetric deformations, which are strongly dependent on normal planar stiffness of the samples and their coupling. The second mode shape is the first symmetric bending mode of the samples where the effect of Poisson's ratio is observed. The negative and positive Poisson's ratio are strongly affecting the deformation shape of the samples in this mode by causing synclastic and anticlastic deformations, respectively.

These strong induced curvatures due to Poisson's ratio are also visible in the fourth mode where symmetric deformation is formed about x -direction. The third mode shape also has a symmetric deformation about y -direction; however, the Poisson's ratio ν_{xy} is about one tenth of the ν_{yx} , therefore, the synclastic and anticlastic curvatures in this mode are small to observe. In contrast, zero Poisson's ratio results in a clean unicastic bending deformation with a single curvature, which is observed in the second and fourth mode shapes, clearly.

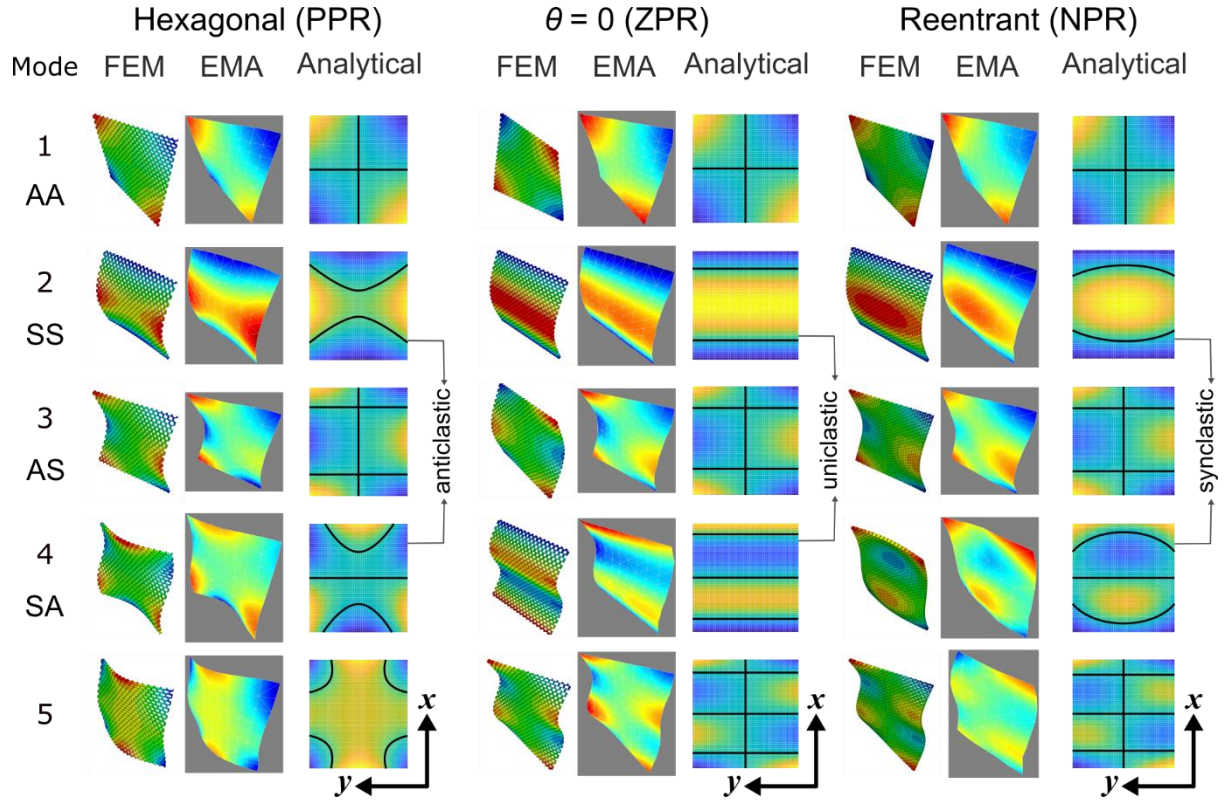


Figure 4-6 Natural mode shapes of first five modes of the test samples with negative, zero, and positive Poisson's ratio. The black lines in analytical figures represent the zero displacement paths of each mode shape, approximately. The effect of Poisson's ratio on curvature coupling of the symmetric deformation shapes is highlighted. In the mode shape nomination, the first and second letter stand for the symmetry (S) or asymmetry (A) of the deformation with respect to the central axis along the x and y-direction, respectively.

4.4.4 Parametric study

The analytical model developed in this study was verified by the experimental results using the refined modified couple-stress theory. In metamaterial samples considered in the experiments, the hexagonal topology with angle θ as the only variable was considered, which results in different Poisson's ratio, density, and equivalent elastic properties for each sample. In this section, effect of different geometrical and elastic parameters are studied using analytical model while only one parameter changes and the others are constant. The elastic constants were $E = 3500$ MPa, $\rho = 1250$ kg/m³, $\nu = 0.3$, and $G = E / [2(1+\nu)]$ and in all cases the reciprocal relation was used as $\nu_{yx}/\nu_{xy} = E_y/E_x$.

a) Poisson's ratio has a significant effect on both the value of natural frequency and the mode shape. Figure 4-7(a) presents the dimensionless frequency, $\tilde{\Omega} = \Omega a^2 \sqrt{\rho/(E_x h^2)}$, versus Poisson's ratio for three length scale values $l/h = 0$, $l/h = 1$, and $l/h = 10$. The elastic and geometric parameters considered were $a = b = 100h$, $h = 1$ mm, $E_x = 2E_y = E$, $\nu_{xy} = \nu$, $G_{xy} = G_{xz} = G_{yz} = G$, and $I_x = I_y = I$. The frequency plots of the first three natural modes show that variation in Poisson's ratio causes different trends for three modes. While natural frequency of the first mode increases with Poisson's ratio, the second and third modes have declining trends, which results in mode transition at a certain point. This means different first mode shape and different rate of variation in frequency with Poisson's ratio increment, which has resulted in maximum natural frequency of the first mode in transition point. Based on the trend of natural frequencies for different length scales, it is concluded that mode transition occurs at higher Poisson's ratio by increasing the length scale parameter. In addition, the Poisson's ratio variation results in significant changes in the value of natural frequency especially for second and third modes where bending has dominant effect.

b) Another important parameter affecting the dynamic properties of the mechanical metamaterials is size effect. In this regard, the variation of the dimensionless natural frequency is studied for different values of the length scale parameter ranging from 0 to 10. The elastic and geometric parameters considered were $a = b$, $h = 1$ mm, $E_x = E_y = E$, $\nu_{xy} = \nu$, $G_{xy} = G_{xz} = G_{yz} = G$, and $I_x = I_y = I$. Three values for length to thickness ratio ($a/h = 20, 50, 100$) are also considered to study the effect of plate thickness on the size effect. The dimensionless natural frequencies of the first three modes are illustrated in Figure 4-7(b). Natural frequencies increase with length scale parameter as expected; however, the rate of increase in each mode is different with the others; therefore, mode transition occurs at certain length scale values. For example, for a plate with $a/h = 20$, the mode transition of the first mode shape (labelled by arrow in the figure) happens at about $l/h = 2.7$, while this value reduces to $l/h = 1.1$ and $l/h = 0.6$ for $a/h = 50$ and $a/h = 100$, respectively. This

trend shows that the higher the length to thickness ratio, the lower length scale of the mode transition is.

c) In addition to elastic parameters, the geometric parameters can also affect the dynamic properties, however, the values and trend are dependent on the Poisson's ratio value. Therefore, three Poisson's ratio values of $\nu = -0.4, 0$, and 0.4 are considered to study the aspect ratio of the plate when $b = 100h$, $h = 1$ mm, $E_x = E_y = E$, $\nu_{xy} = \nu$, $G_{xy} = G_{xz} = G_{yz} = G$, and $I_x = I_y = I$. Increasing the aspect ratio of the plate (a/b) results in the dimensionless natural frequency increase. In this case, the mode transition occurs first for positive Poisson's ratio and then zero Poisson's ratio model, while first mode shape of the negative Poisson's ratio model does not experience transition and its rate of increase slows at aspect ratios larger than one. In addition, the positive Poisson's ratio case experiences larger number of the mode transitions by changing the aspect ratio. The results show that for aspect ratios larger than two the variation of dimensionless natural frequency of the first mode is negligible for all three values of the Poisson's ratio.

d) The Poisson's ratio and elastic modulus of two planar directions are correlated by $\nu_{yx}/\nu_{xy} = E_y/E_x$. Considering $E_x = E$, $\nu_{xy} = \nu$, $a = b = 100h$, $h = 1$ mm, and $I = 0$, the variation of modulus ratio in two directions is studied in Figure 4-7(d) where three values of $\nu = -0.4, 0$, and 0.4 are assumed. It is evident that natural frequency increases by increasing the stiffness of the structure; however, the rate of this increase varies due to the mode transition. The mode transition of the first mode shape for $\nu = 0.4, 0$, and -0.4 happens at $E_y/E_x = 0.34, 0.47$, and 1.96 , respectively, which means that reducing the Poisson's ratio results in mode transition at higher modulus ratios. Due to the mode transition, the rate of dimensionless natural frequency increments slows down significantly at high modulus ratios.

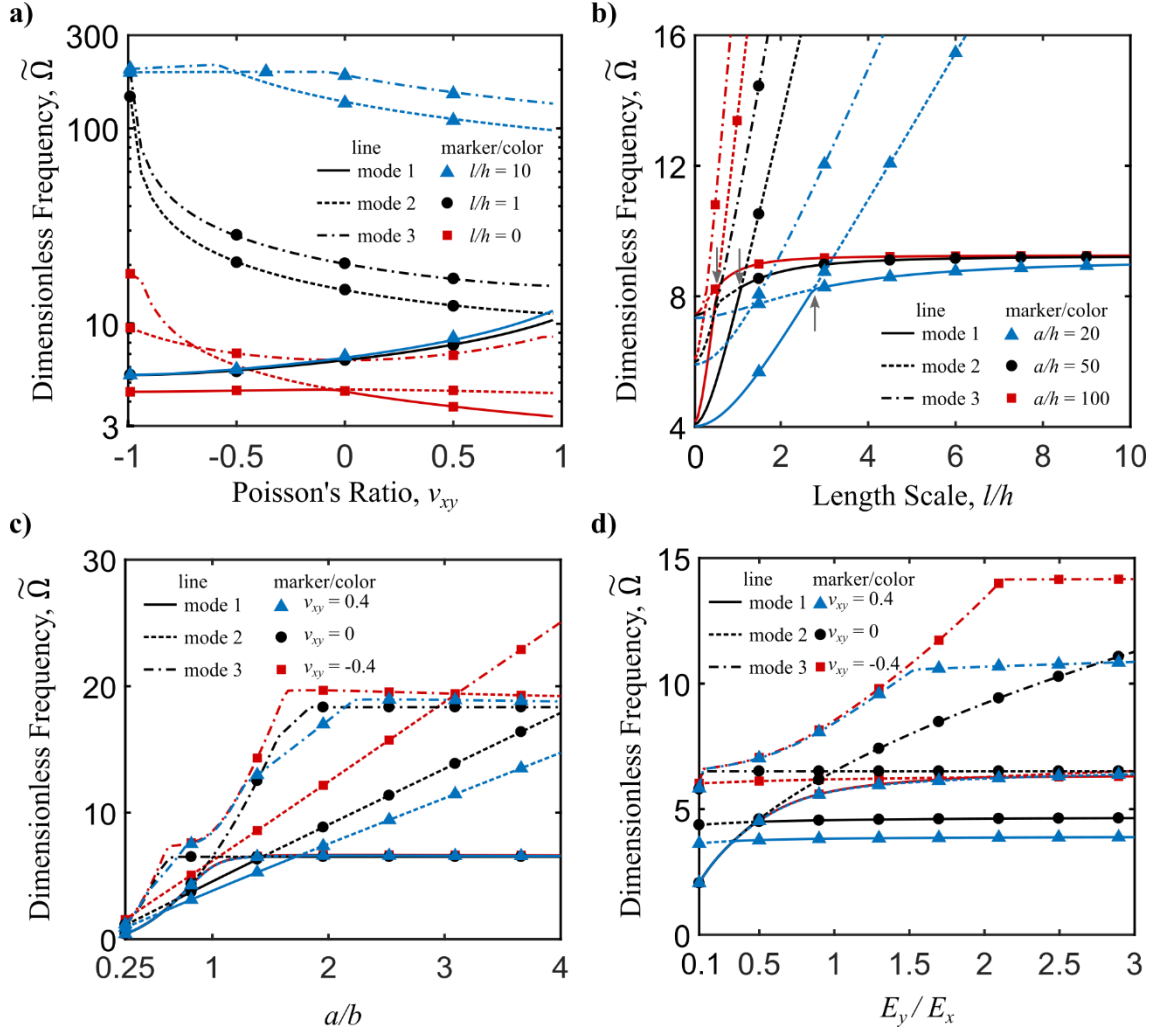


Figure 4-7 Effect of geometrical and elastic parameters on the dimensionless natural frequency of first three modes of a free plate, $\tilde{\Omega} = \Omega a^2 \sqrt{\rho/(E_x h^2)}$. (a) Variation of natural frequencies with Poisson's ratio for three length scale parameters, (b) effect of length scale parameter on the natural frequency and the mode shape transition for three thickness ratios, (c) effect of aspect ratio for negative, zero, and positive Poisson's ratios, (d) effect of stiffness ratio in two planar directions.

4.4.5 Damping ration and dissipation analysis

4.4.5.1 Damping coefficient analysis using EMA

The damping coefficient, ζ , of the first three modes obtained from EMA are depicted in the Figure 4-8(a). Based on the first three mode shapes, the damping coefficient of the SS mode is higher in zero and positive Poisson's

ratio than negative Poisson's ratio samples. The average damping coefficients of the zero angle structure for the first three modes are 0.026, 0.043, and 0.034, respectively, which are larger than corresponding values of positive and negative Poisson's ratio structures according to Figure 4-8(a).

The trend of damping coefficient variation versus angle θ in hexagonal designs suggests that damping coefficient has an extremum for each natural mode at zero angle among samples with $-20^\circ < \theta < 20^\circ$. For the first mode, ζ reduces for $\theta = 14^\circ$ and $\theta = -10^\circ$ samples but starts increasing when absolute angle value increases to 20 degrees where similar trend is also observed in second and third mode. Considering this trend for damping ratio comparing to the Figure 4-1(b), it is concluded that absolute magnitude of the Poisson's ratio has an inverse correlation with damping ratio. This means that minimum damping ratio has occurred at angles close to the extremum of the absolute Poisson's ratio. This can be potentially an important result that in metamaterials with hexagonal geometry, the damping coefficient can be tuned with the angle θ or Poisson's ratio consequently. However, the trend discussed for the damping coefficients must be validated using a second method to ensure the independency of the trend from probable errors in EMA.

The damping measurement in EMA is based on the vibration at resonance frequency where strains are infinitesimal. The major sources of damping in EMA are (Salari-sharif, Schaedler and Valdevit, 2014): 1) intrinsic damping of the material, 2) microscale internal friction, and 3) viscous air damping. The intrinsic damping differs based on the class of material: metal alloys and polymers. Polymers have much larger damping capacity than metals, however, they have poor stiffness, strength, and creep characteristics comparing to metals and suffer from strongly temperature-dependent properties (Graesser and Wong, 1992). In terms of internal friction, sliding of long polymer chains and micro-cracks that stem from the layered FDM manufacturing process can be a source of internal friction. The viscous air damping correlates to the effective area of the structure in the plane of vibration and it is important at high frequencies. Since large number of reflective targets were installed in support of LDV scanning, the effective area normal to the oscillation direction increases that

can increase the viscous damping. However, the effect of viscous damping on lattice structures is about 15% between ambient and vacuum conditions (Salari-sharif, Schaedler and Valdevit, 2014). This means that damping and energy loss in the structure are mainly a function of the intrinsic properties and microscale friction; therefore, the viscous damping should not affect the trend of damping coefficient significantly when damping in metamaterials is studied in ambient air. In addition, viscous damping effects are negligible when quasi-static loading is employed for dissipation analysis. Therefore, hysteresis analysis of the metamaterials under quasi-static loading is an efficient second method for validation of the damping trend.

4.4.5.2 Loss factor analysis using quasi-static cyclic tests

Along with the damping coefficient, other mostly used parameters in the literature is loss coefficient $\eta = \Delta U_p / (2\pi U_p)$, that is the energy loss per radian of a vibratory system and depends on the amplitude and frequency of the vibration (Ashby et al., 2000). Additional parameters are logarithmic decrement δ , tangent of the phase lag $\tan(\phi)$, the quality factor Q , and the ratio of loss to storage modulus E''/E' (Graesser & Wong, 1992). There is also loss factor Ψ that similar to the loss coefficient is considered as a measure for global damping of the metamaterial structure rather than hysteretic property of the constitutive material and can take nonlinearities of the system into account (Bert, 1973; Carfagni, Lenzi and Pierini, 1998).

The loss factor $\Psi = \Delta U_p / U_p$ is the ratio of the dissipated energy in a hysteresis cycle, ΔU_p , to the total strain energy, U_p , in a loading and is considered in this paper as the second verification parameter for the damping trend. This parameter is used in the literature in-line with the specific damping capacity to derive the analytical model for energy loss in plates, laminates, and sandwich panels where the damping coefficients form a diagonal matrix (Bert, 1973; Berthelot, 2006; Berthelot and Sefrani, 2004; Maheri, Adams and Hugon, 2008; Maheri and Adams, 1994). Therefore, the trend of damping ratio and loss factor obtained here can be used for further analysis of forced vibration and dynamic response of metamaterials tuning the Poisson's ratio. In order to measure the loss factor, quasi-static tests using Instron tensile test machine and loading rate

of 1 mm/min up to 1% strain in the y -direction are conducted as shown in Figure 4-8(b). Loading in the y -direction is considered because the effective modulus E_y is an order of magnitude larger than x -direction and has larger contribution to the total strain energy of the structure. The hysteresis curves of the samples are obtained for five cycles where the loss factors are plotted versus angle θ and cycle number in Figure 4-8(c). The loss factor for the first cycle is much larger than next ones because the main irreversible fracture mechanisms in the structure happen during the first loading cycle (Salari-sharif, Schaedler and Valdevit, 2014). After second cycle, the slope of the loss factor curve becomes smooth and maintains approximately a constant value. This shows the amount of dissipation in each cycle that causes damping in the structure; hence, the trend of the loss factor can be associated with the damping coefficient.

4.4.5.3 Damping ratio and loss factor comparison

Considering the fifth cycle, the loss factor values for samples are compared with the corresponding damping ratio of the first vibration mode in Table 4-2. The trend of the loss factor and damping coefficient are fully compatible according to the Table 4-2. The loss factor and damping coefficient have a maximum at zero degree and by reducing or increasing the angle θ , the loss factor and damping coefficient first decrease and then increase. Therefore, it is concluded that the trend obtained from EMA for damping coefficients is valid. As a conclusion, an inverse correlation between the loss factor/damping ratio of the hexagonal structures and absolute value of the Poisson's ratio ν_{yx} is obtained. The experimental results suggest that by increasing the absolute Poisson's ratio ν_{yx} , the loss factor/damping ratio of the structure reduces and vice versa.

Table 4-2 Loss factor values from the fifth hysteresis cycle for hexagonal structures versus the damping coefficient of the first natural mode of vibration

θ (degree)	ψ	ζ
-20	0.052	0.027
-10	0.044	0.021
0	0.050	0.026
14	0.029	0.013
20	0.039	0.016

4.4.5.4 Damping Merit Index (DMI) analysis

In order to evaluate the damping merit of different designs, a figure of merit is used (Chen, Wang and Ma, 2020; Salari-Sharif, Schaedler and Valdevit, 2018), which is proportional to the ratio of cubic root of elastic modulus to the density of the structure, $E^{1/3}/\rho$. The damping merit can be used also as a measure in optimization frameworks (Haghpanah et al., 2017; Izard et al., 2017). The Damping Merit Index (DMI) is studied using the relation $\left(\frac{E_y}{E_s}\right)^{1/3} \psi / \left(\frac{\rho}{\rho_s}\right)$ as shown in Figure 4-8(d) where modulus and density obtained from FE simulations are used. Since DMI is correlated to the ratio of effective modulus to the density, its trend versus Poisson's ratio is similar to Figure 4-1(d). The results show that the DMI in a metamaterial is strongly dependent on its dominant effective modulus, in this case E_y . Therefore, if damping merit of the cellular structure is a concern, the zero Poisson's ratio metamaterial provides better performance than negative and positive Poisson's ratio structures. The constant loss factor plots in Figure 4-8(d) also indicate that cellular structures with Poisson's ratio close to zero present a higher damping merit.

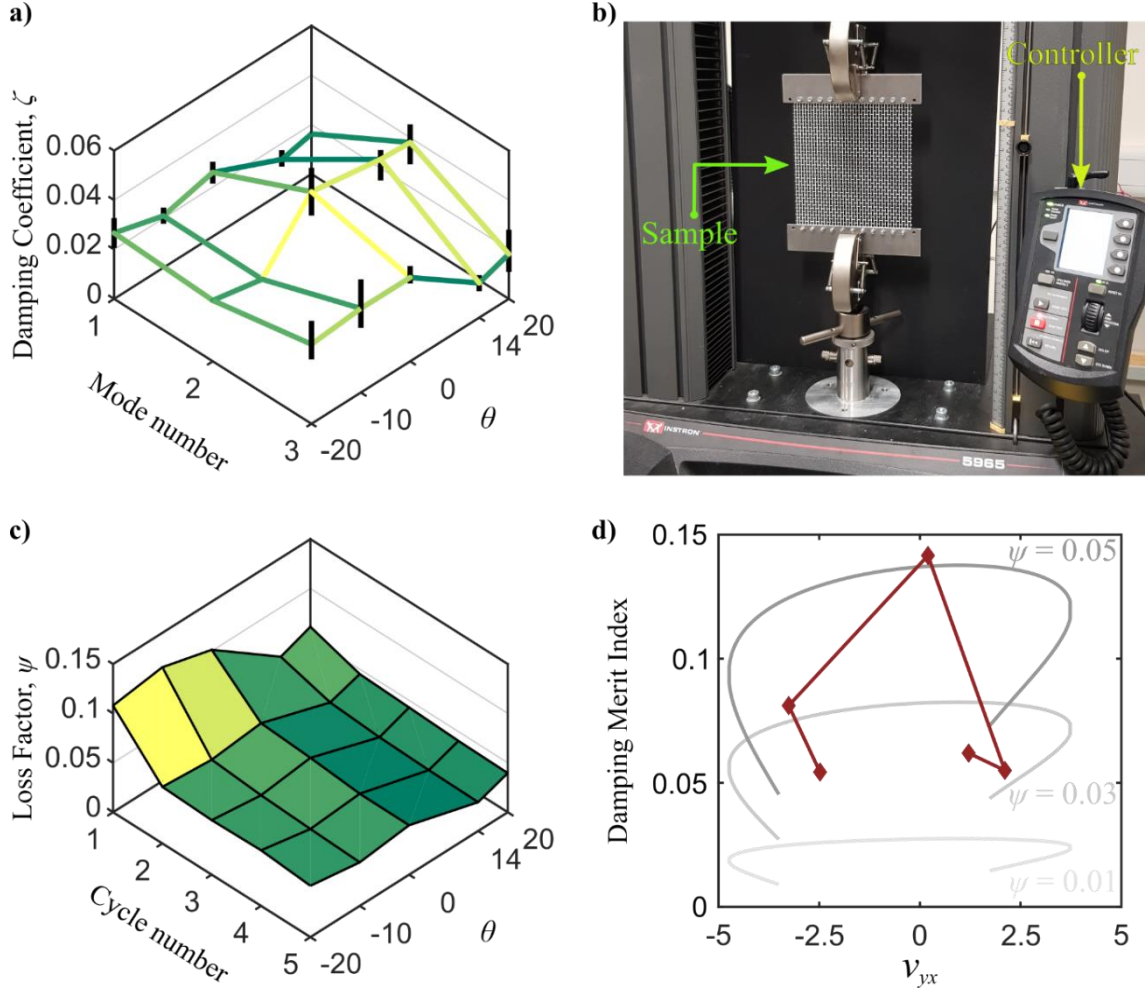


Figure 4-8 (a) Average damping coefficients of the first three natural modes of the test samples obtained from EMA where the black lines represent the error bar of each mode. (b) Tensile test setup for cyclic quasi-static analysis of the metamaterials hysteresis. (c) Variation of the loss factor in each cycle for the five test samples. (d) Damping merit index of the test samples (red line) versus the Poisson's ratio and geometric locus of the constant loss factor DMI curves for three different loss factors.

4.5 Conclusion

The dynamic characteristics of mechanical metamaterials tuning the Poisson's ratio with hexagonal topology were investigated using experimental, numerical, and analytical methods. The experimental samples were manufactured using 3D printing where the unit cell size was one order of magnitude smaller than sample size. The experimental modal analysis (EMA) was employed to study

the natural frequencies, mode shapes, and damping coefficients of the samples with free boundary conditions. An analytical model that enables the study of size effects was developed using refined modified couple-stress theory for orthotropic material based on displacement functions of the Reddy's higher order shear deformation theory. The equations of motion were derived using Chebyshev polynomials as the solution where free conditions were considered. The results revealed the significance of size effect consideration in dynamic analysis of mechanical metamaterials where the average relative errors in natural frequencies prediction was reduced considerably by using modified couple-stress theory. Therefore, it is recommended to consider the size effects in designing the cellular metamaterials when the unit cell size is not much smaller than sample size.

The effect of Poisson's ratio on the length scale parameter was studied for metamaterials with hexagonal topology. It was shown that the length scale parameter increases by increasing the Poisson's ratio when the ligaments' angle is the only parameter in tuning the Poisson's ratio. In addition, the effect of the Poisson's ratio on the deformations of mode shapes was studied where Poisson's ratio sign and value significantly affected bending dominated deformations. Using the analytical model, effect of Poisson's ratio, thickness, stiffness ratio, and aspect ratio of a homogenous sample were studied. The first mode of the structure as the fundamental mode changed caused by mode transition, which resulted in different rate of change in natural frequency.

The damping and dissipation properties of the metamaterials tuning the Poisson's ratio were systematically studied **in this chapter**. Damping coefficients extracted from the EMA presented a maximum value for the zero Poisson's ratio sample. In addition, an inverse trend with absolute value of the dominant Poisson's ratio was found; *i.e.*, the damping coefficient was smaller in samples with larger Poisson's ratio. The trend was also admitted by hysteresis tensile cyclic quasi-static tests where loss factor was determined as a measure of structural dissipation. The damping merit index was also found to be the highest for the zero Poisson's ratio sample as a measure for energy dissipation.

Therefore, the Poisson's ratio can be also considered as a parameter in tuning and optimizing the damping properties of a cellular structure.

5 Dynamic Characteristics of Zero Poisson's Ratio Metamaterials

5.1 Introduction

The importance of zero Poisson's ratio for a cellular core in the morphing skin was proved in chapter two. In chapter four, the dynamic characteristics of the hexagonal shaped metamaterials tuning the Poisson's ratio were discussed in detail and it was shown that ZPR results in higher damping ratio and presents uniaxial bending deformation. However, the hexagonal geometry that results in ZPR (ligament's angle $\theta = 0$) has the accordion design and presents large stiffness in the solid strips direction, which is not desired for two directional morphing. Therefore, the Fish Cell metamaterial as a potential 2D compliant ZPR geometry is a candid substitute. In this regard, the dynamic characteristics of Fish Cell metamaterial should be also studied and design flexibilities that it provides in terms of tuning the modal behaviour should be explored.

One significant advantage of the Fish Cell design is the wide range of elastic properties that it can possess while the ZPR behaviour and size of the unit cell are maintained. The parametric studies performed in chapter 3 indicate that the equivalent elastic modulus in two planar directions are strongly dependent on the vertical and horizontal components of the bending rib's length. Therefore, the unit cell elastic properties can be tuned by changing the bending rib's length and angle to tune the stiffness, density, and modal properties as a result. In addition, this tuning of the geometry can be applied locally, *i.e.*, the tessellation of the Fish Cell metamaterial can possess locally tailored elastic properties. This advantage of the Fish Cell design allows for creating variable stiffness core geometries that help in tuning the static deformation and dynamic properties such as mode shapes and natural frequencies. This stiffness variation can follow a distribution function and enable the design of functionally graded ZPR metamaterials, which has not been studied specifically.

In terms of elastic properties, the cellular Voronoi structures have shown improved modulus and yield strength when density was functionally graded

(FG) (Ajdari et al., 2009). In addition to randomly distributed structures, deterministic designs based on void geometry with gradient functions in two planar directions were studied which presented higher bending stiffness with FG structure than uniform design with the same mass (Niknam et al., 2018). Gradient functions applied on the geometry are used to tune the effective elastic modulus, whereas the target parameter can be the effective Poisson's ratio such as tailoring the gradient auxeticity through the thickness of a beam structure (Li, Shen and Wang, 2019).

The gradient design has significant potential in improving the response of a structure under dynamic loading such as tuning the modal properties or increasing the absorbed impact energy (Wu, Su and Shi, 2020). Applying gradient into the Poisson's ratio, Lira et al. studied the dynamic characteristics of the negative to positive Poisson's ratio cellular structures based on hexagonal topology where reduction in the displacements and natural frequencies were observed (Lira, Scarpa and Rajasekaran, 2011). In another study, optimization of the cell angle and thickness was applied on the hexagonal re-entrant geometry with gradient properties where significant reduction in the radiated sound pressure level was observed, while natural frequencies had increased (Mazloomi et al., 2018; Ranjbar et al., 2016). When Poisson's ratio in one direction has a negative to positive gradient law, the tailored dynamic response is between the bounds of the fully re-entrant and hexagonal structures (Boldrin et al., 2016). In curved structures such as a cylinder, the direction of the gradient from high to low Poisson's ratio magnitude presented different dynamic response where improved dynamic response in terms of radiated sound pressure level was achieved by gradient design comparing to the regular design (Li and Yang, 2020). In addition, combination of the positive and negative Poisson's ratio hexagonal cells was also shown as an effective tool in increasing the gap width in certain frequency bands (Mukherjee, Scarpa and Gopalakrishnan, 2016). Therefore, cellular structures with a functionally graded Poisson's ratio have shown a potential breakthrough in designing optimum cores; however, it is ZPR that is desired for morphing skins.

In this chapter, the functionally graded elastic properties are investigated for a homogenized plate made of ZPR Fish Cell metamaterial. The unique property of gradient Fish Cell design is that elastic modulus and density are tailored spatially, while Poisson's ratio remains zero. This property enables the gradient Fish cell metastructure to satisfy the morphing requirements especially the stiffness augmentation alleviation. To understand the effect of stiffness augmentation on dynamic properties, this chapter also provides a detailed numerical study on three structures with positive, negative, and zero Poisson's ratio. The boundary conditions and constraints of an integrated skin on all edges are applied and pre-strain that represents the morphing actuation is applied on the cellular core.

5.2 Materials and Methods

5.2.1 Experimental Modal Analysis (EMA)

Two samples with tessellation number of 11×18 with uniform and gradient Fish Cell geometry were manufactured by 3D printing method (Ultimaker 2+, The Netherlands) using commercial Polylactic Acid (PLA) filaments as shown in Figure 5-1(a) and 5-1(b), respectively. The length and width of both samples are $L_x = 176$ mm and $L_y = 180$ mm, respectively. The dimensions of the Fish Cell are $B = 2.5$ mm, $b = 1.73$ mm, $H = 10$ mm, $h = 3$ mm, $C = 1.5$ mm, and $t = 0.6$ mm, where the out of plane thickness of the sample is $W = 5$ mm. Similar dimensions are used for gradient design except horizontal length of the bending rib, b , which follows a gradient function in the y -direction. The length b decreases as much as 0.192 mm in each row up to the middle of the sample in the y -direction and then returns to the original value with the same rate. Therefore, the sample has larger equivalent modulus in the middle of the tessellation than two ends along the y -direction.

Modal test setup was used to measure the natural frequencies, damping ratio, and to visualize the corresponding mode shapes of the uniform and gradient Fish Cell metastructures. The test setup included a signal generator (OFV-5000, Polytec), shaker (Type 4809, Bruel & Kaer), and Laser Doppler Vibrometer (LDV) (PSV-400, Polytec), as shown in Figure 5-1(c). For the sake

of optical measurement, targets for laser beam reflection were installed on the surface of each test sample using reflective tape (3M 3430). In each test run, the sample was excited using a periodic chirp function over 200 Hz frequency band with three complex averages and 6400 FFT lines to measure the frequency response. All edges free was the boundary conditions considered in this experiment. The details of the samples installation and data acquisition were explained in chapter 4. In addition, quasi-static cyclic hysteresis data with loading rate of 1 mm/min up to 1% strain in the y-direction of the samples was also studied to evaluate the effect of gradient design on the loss factor.

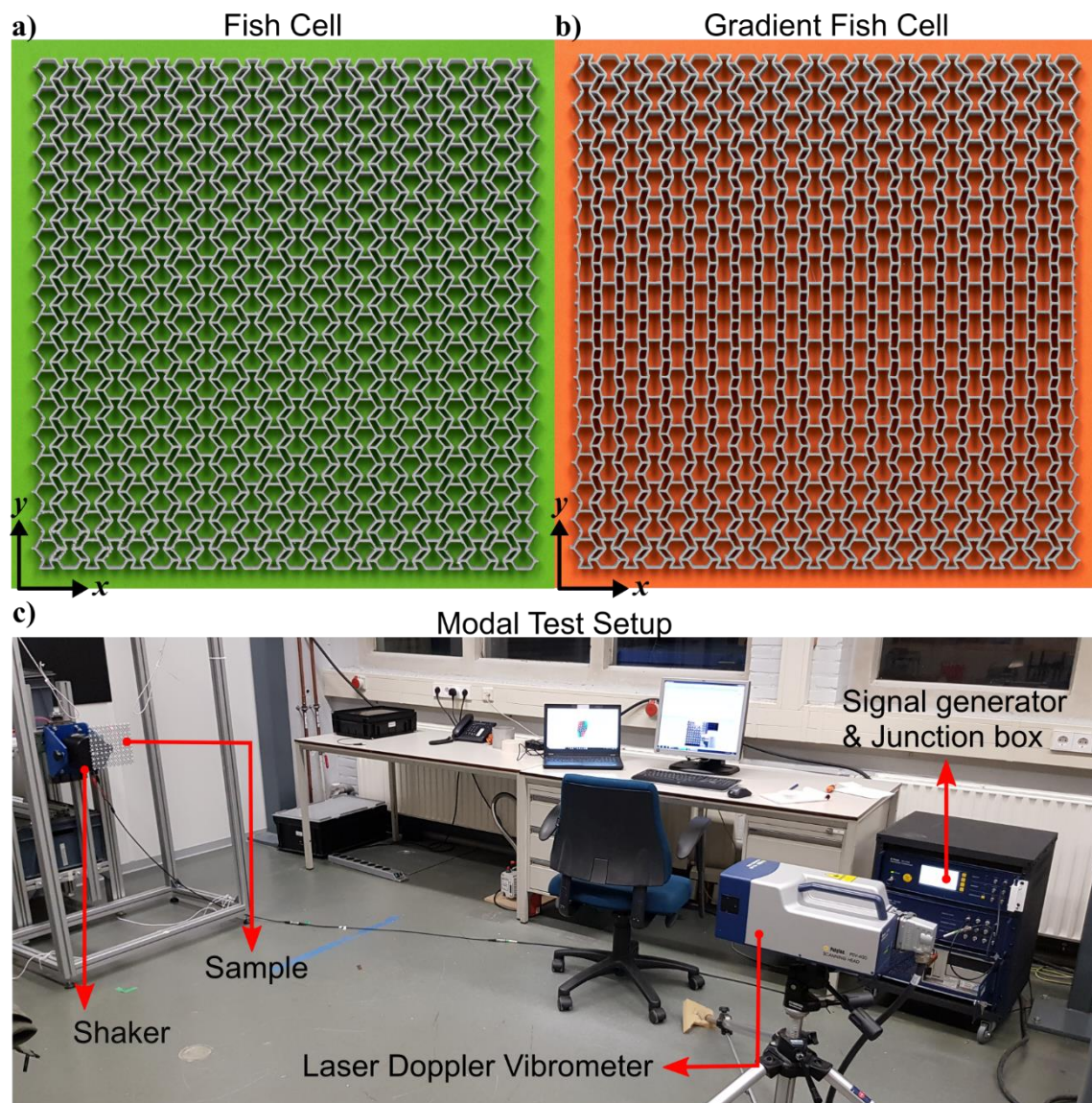


Figure 5-1 the 3D printed samples made of PLA: (a) uniform Fish Cell and (b) gradient Fish cell design. (c) the modal test setup.

5.2.2 FE Analysis

The finite element analysis was conducted using ABAQUS commercial software. The solid model that provides detailed representation of the cellular structure with free boundary conditions and PLA elastic properties ($E = 3500$ MPa, $\nu = 0.36$, and $\rho = 1250$ kg/m³) is generated using C3D20 elements to verify the experimental results. Due to large size of the tessellation and computational resource limits, approximate global size of 0.3 mm was used to discretize the model.

5.3 Results and Discussion

5.3.1 Natural Frequencies, Mode Shapes, and Damping Coefficients

The frequencies of the natural modes in the 200 Hz band are presented for uniform and gradient Fish Cell samples in Figure 5-2(a) and 5-2(b), respectively. The natural frequencies obtained from solid FE analysis are close to the experimental results of uniform Fish Cell design for first two modes, while a larger gap is observed between FE and experimental results for the next three modes. For gradient Fish Cell, the full scale solid FE model verifies the natural frequencies obtained by the modal analysis.

The results show that increasing of the stiffness in the middle of the tessellation using gradient function has resulted in lower modal density in the frequency band as expected where there are six natural modes observed for gradient Fish Cell versus the seven modes of uniform design. Considering the mode shapes presented in Figure 5-3(c), the gradient design shares the same first three natural mode shapes, however, the mode shapes change from the fourth mode. Therefore, the gradient function can be used as a tool to suppress specific mode shapes in a certain frequency band of desire. The effect of the zero Poisson's ratio is also observed through the uniaxial curvature of the symmetric bending modes

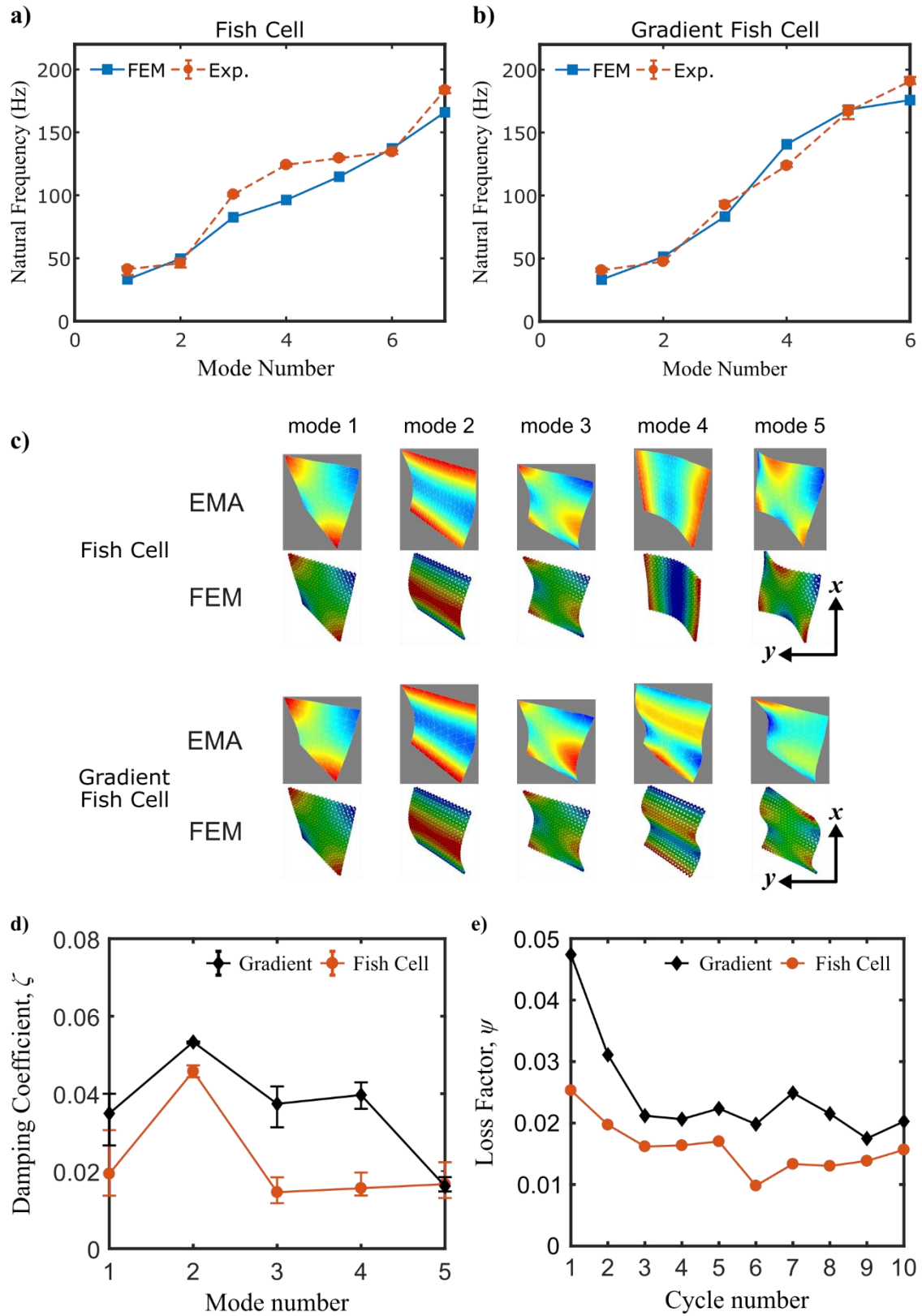


Figure 5-2 Natural frequencies of (a) Fish Cell and (b) Gradient Fish Cell. (c) First five mode shapes of two samples, (d) Damping coefficients of the first five modes, (e) Loss factor of the two samples.

In terms of damping coefficient, the gradient design presents higher damping coefficient as shown in Figure 5-2(d). The average damping coefficients of the first three natural modes of gradient design, which have the same mode shape in both samples, are larger than uniform Fish Cell design. Therefore, for applications that higher damping ratio is desired, gradient design with increased stiffness can be used to increase the damping of the metastructure. This dissipation trend of the Fish Cell design is also verified by quasi-static hysteresis tests where average loss factor of the gradient design in the first ten cycles is larger than uniform Fish Cell design as shown in Figure 5-2(e).

5.3.2 Effect of Gradient Function

The spatially gradient functions can be applied on the geometrical parameters of the Fish Cell metamaterial to tune the elastic and modal properties. In this chapter, gradient function along the y -direction is considered because it was shown in chapter 2 that Fish Cell metamaterial presents larger strain capacity in the y -direction than x -direction. Three types of gradient are applied as Soft-Hard (SH), Soft-Hard-Soft (SHS), and Hard-Soft-Hard (HSH), which are categorized based on the stiffness of the sample along y -direction. Therefore, horizontal length of the bending rib, b , which has the dominant effect on the modulus in the y -direction (as discussed in chapter 3), is considered as the geometric parameter to apply the gradient function. Twelve gradient functions of three categories are considered as shown in Table 5-1 where values of the b cover the maximum admissible range, which means $0 < b < B$. The spatial variable $\eta = y/L_y$ transforms the interval $0 < y < L_y$ into $-1 < \eta < 1$. In addition, the effect of changing the maximum stiffness position along y -direction in SHS and HSH samples from $\eta = 0$ to $\eta = 0.25, 0.50$, and 0.75 is also studied.

The homogenized planar shell model was used based on equivalent elastic properties of the Fish Cell metamaterial derived in chapter 3. In this case a $200 \times 200 \text{ mm}^2$ plate with out of plane thickness $W = 5 \text{ mm}$ was considered. The plate is divided into 40 equal partitions along the y -direction where each partition was assigned with a specific equivalent properties based on the gradient function. The quadratic S8R shell element with approximate global size

of 1 mm was considered for analysis. The base of the Fish Cell dimensions for gradient study was $t = 0.1$ mm, $H = 5$ mm, $B = H/2$, $C = B/2$, and $h = H/4$ where the value of $b = B/2$ was considered for the uniform design. The lower thickness was assigned to ensure the values of the equivalent modulus are accurate as their formulas are derived based on slender beams theory in chapter 3. The boundary conditions in all simulations are all edges free.

Table 5-1 Bounds of values for length b in different linear gradient functions and locations along y -direction. The normalized position along y -direction is used as $\eta = y/L_y$.

	b		
Name	$\eta = -1$	$\eta = 0$	$\eta = 1$
Uniform	$B/2$	$B/2$	$B/2$
HSH-1	0	B	0
HSH-2	$B/2$	$3B/2$	$B/2$
SHS-1	B	0	B
SHS-2	$3B/2$	$B/2$	$3B/2$
SH-1	0	$B/2$	B
SH-2	$B/4$	B	$3B/4$
	$\eta = -1$	$\eta = 0.25$	$\eta = 1$
HSH-3	0	B	0
SHS-3	B	0	B
	$\eta = -1$	$\eta = 0.5$	$\eta = 1$
HSH-4	0	B	0
SHS-4	B	0	B
	$\eta = -1$	$\eta = 0.75$	$\eta = 1$
HSH-5	0	B	0
SHS-5	B	0	B

The effect of symmetric softening or hardening gradients along y -direction on natural frequencies and mode shapes are demonstrated in Figure 5-3. The natural frequency of each mode is normalized with respect to the similar mode of the uniform Fish Cell design. The HSH gradient samples present lower

natural frequencies (except first symmetric bending mode, which is slightly higher than uniform) comparing to the uniform design as shown in Figure 5-3(a).

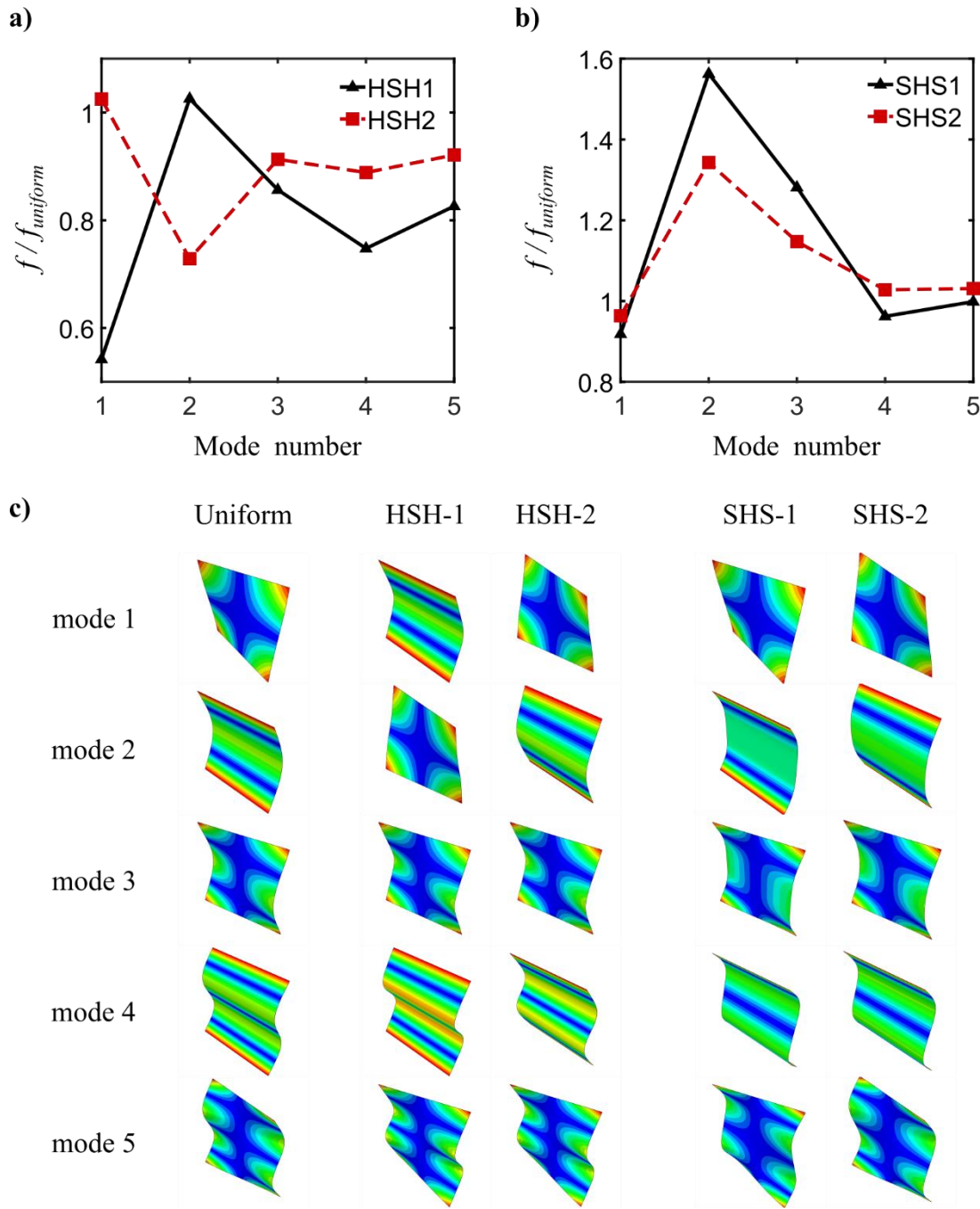


Figure 5-3 The natural frequencies ratio of (a) HSH-1&2 and (b) SHS-1&2 gradient samples with respect to the same modes of the uniform Fish Cell, (c) mode shapes of the gradient samples comparing to the uniform Fish Cell.

Mode transition has happened for HSH-1 sample between first two modes, which is caused by reduced stiffness in the middle of the tessellation. The SHS gradient samples present significantly higher natural frequencies for the second and third modes, while for the other modes, values are close to the uniform design as shown in Figure 5-3(b). The HSH-1 with SHS-1 samples and HSH-2 with SHS-2 samples are peer designs as they have equal overall planar stiffness in the y-direction and total mass. The important result is that by adjusting the distribution of the Fish Cell geometric parameters, the natural frequencies can significantly change. For example, for the first symmetric bending mode of HSH-1 and SHS-1 the normalized frequencies are 0.54 and 1.56, respectively, which offers a large domain for tuning. Therefore, gradient design capability of the Fish Cell allows tuning the natural frequencies while overall zero Poisson's ratio behaviour is maintained. In terms of mode shapes, the stiffness distribution by gradient laws affects the deformation shape as well. This difference is observed by comparing for example the first symmetric mode of SHS and HSH samples where the curvature of the bending differs significantly.

The asymmetric gradient functions, *i.e.* soft to hard, result in the reduced natural frequencies in all modes as shown in Figure 5-4(a).

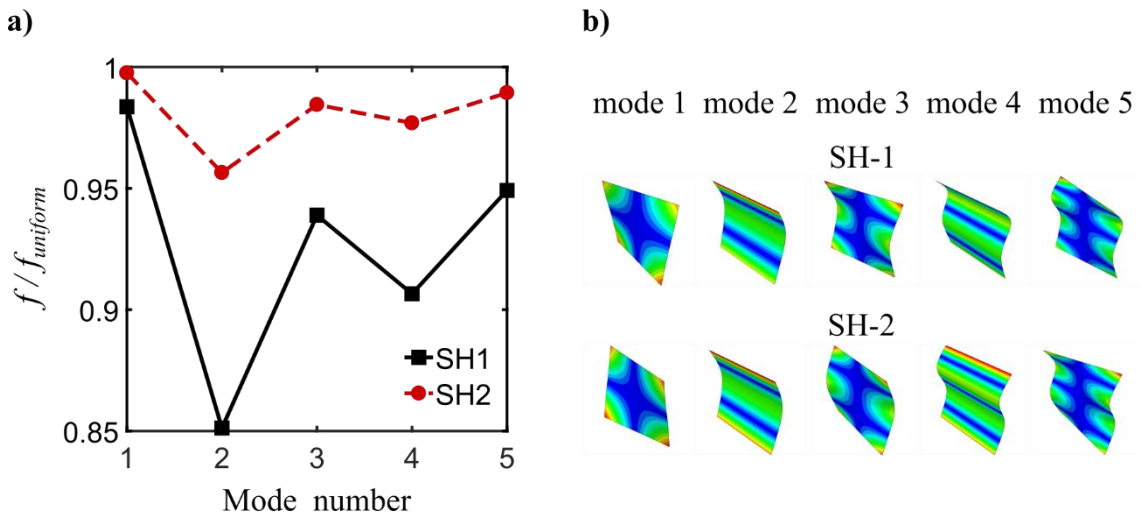


Figure 5-4 (a) The frequency ratio of the SH samples with respect to the same modes of the uniform Fish Cell, (b) mode shapes of the SH-1 and SH-2 samples.

The SH-1 sample presents the largest variation of E_y possible between two ends of the sample as horizontal length b , changes from zero to its maximum possible value, B . In this case, the largest reduction of the natural frequency with respect to the uniform design, belongs to the second mode (first symmetric bending mode) with 0.85 ratio. In terms of mode shapes, the order of the modes is similar to the uniform design; however, the bending curvature is not symmetric. Considering modes 2 to 5, it is observed that the maximum displacement point of each mode shape has shifted towards the hard end of the sample as shown in Figure 5-4(b).

The position of minimum or maximum stiffness along y -direction in the HSH and SHS samples is also an important parameter to tune the modal response. The HSH samples have the same equivalent elastic modulus at both ends similar to the SHS. However, the location of the minimum or maximum stiffness can shift from the middle of the tessellation towards one of the ends. In this case, the normalized natural frequencies of HSH-3, 4, and 5 samples, which share the same bounds of length b with HSH-1, are presented in Figure 5-5(a). The results show that the first symmetric bending mode has been affected more than other modes where mode transition has happened in HSH-5 sample comparing to the HSH-1. In SHS samples, the variation of the response to the change in maximum stiffness location is more significant. Considering mode 2, the frequency ratio for SHS-3 and SHS-5 samples are 1.36 and 0.95, respectively, while for SHS-1 this ratio was 1.56. Similarly, mode 3 also experiences a decrease in frequency ratio by shifting the maximum stiffness location towards ends. In contrast, the frequency ratio of the modes 1, 4, and 5 increased by shifting the maximum stiffness location. Therefore, shifting the maximum stiffness position from center towards end in the y -direction, resulted in a different response for different modes. In this regard, the spatial distribution of the linear gradient functions can be tailored in favour of the tuning of a specific natural mode.

The effect of shifting the maximum stiffness position is also observed on the deformation of the mode shapes where maximum displacement position shifts

towards the ends as shown in Figure 5-5(c). This means that density of the modal nodes can be tuned also spatially, which can be useful in high frequency modes.

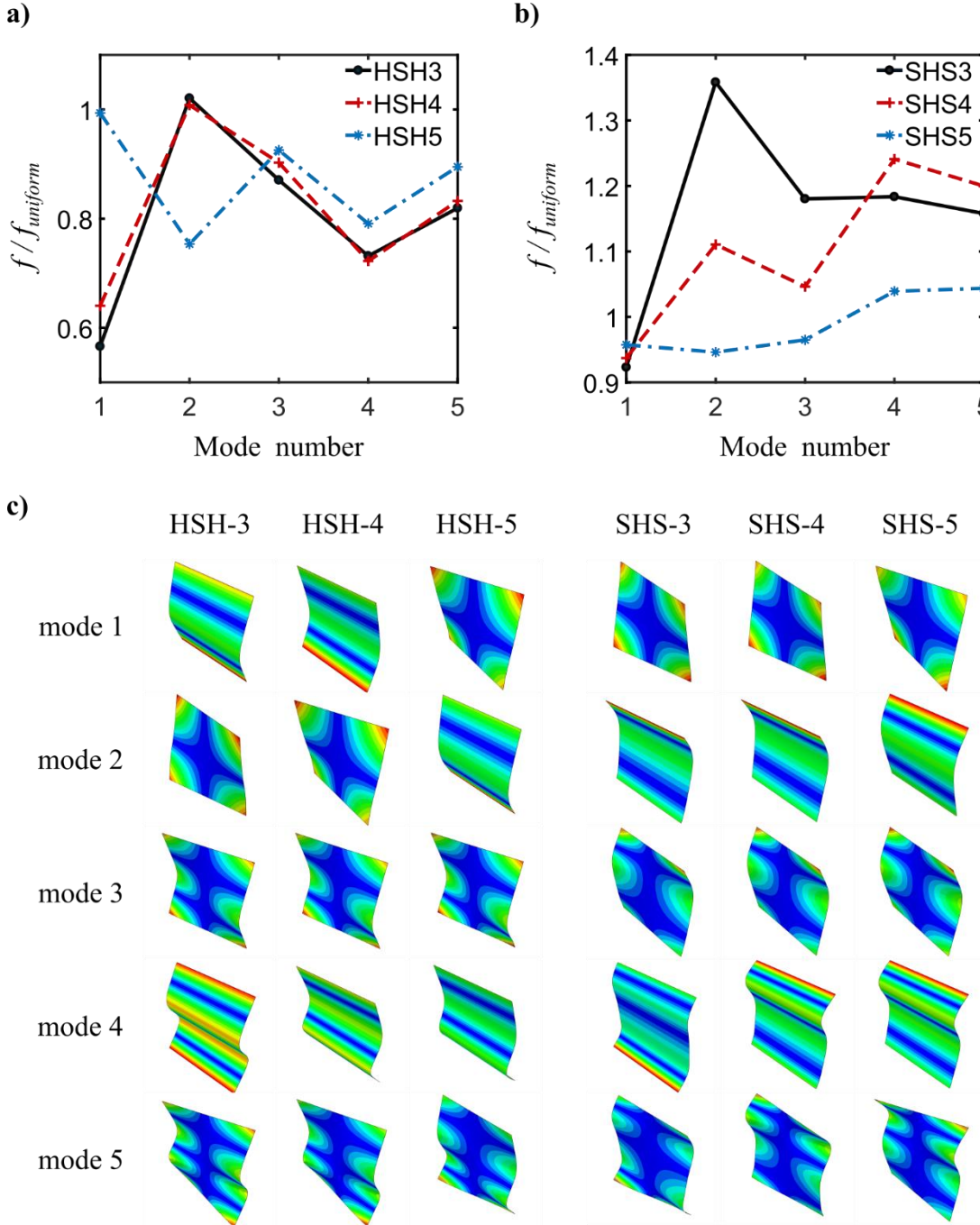


Figure 5-5 The natural frequencies ratio of the gradient samples (a) HSH-3,4,5, and (b) SHS-3,4,5 with respect to the same modes of the uniform Fish cell design. (c) Mode shapes of the gradient samples.

5.3.3 Combined Effect of Integration and Poisson's Ratio

An integrated morphing sandwich skin will be attached to the frame structure using fasteners and it is usually allowed for displacements only in designated direction by prismatic joints. Therefore, the boundary conditions are clamped on all edges, while only displacement in the morphing direction is allowed. Under such a condition, the effect of the zero Poisson's ratio comparing to negative and positive Poisson's ratio should be studied. In this regard, three geometries including Fish Cell, hexagonal honeycomb, and re-entrant honeycomb are considered. The dimensions of the hexagonal and re-entrant unit cells are designed for achieving equal elastic modulus in the y -direction because this direction is considered for morphing. The dimensions of the Fish Cell and tessellation number are mentioned in section 5.2.1. According to the cellular material theory (Malek and Gibson, 2015), the dimensions of the re-entrant and hexagonal unit cell are $h = l = 7$ mm and $t = 0.6$ mm where the ligament angle for hexagonal and re-entrant cell are $\theta = 17.66$ and $\theta = -28.2$ degrees, respectively. The hexagonal honeycomb has 10×14 tessellation number and re-entrant honeycomb has 24×15 tessellation number to represent the similar length and width of tessellations close to the Fish Cell. The greater Poisson's ratio for the re-entrant and hexagonal samples are $\nu_{yx} = -3.0$ and $\nu_{yx} = 2.1$, respectively.

The numerical simulations are performed in ABAQUS using shell models with quadratic S8R elements and approximate global size of 0.5 mm where material properties of the PLA are considered. All degrees of freedom of edges are constrained; however, the two edges parallel to the y -direction and one of the transverse edges where pre-strain is applied are free to have translational displacement in the y -direction. The problem is solved in two steps including first a static step with geometric nonlinearities allowed to obtain the stiffness matrix due to pre-strain and the second step that solves the Eigen-value problem of free vibration. The maximum tensile strain considered in the y -direction is $\varepsilon_y = 0.01$ because this range of strain is sufficient for morphing applications such as camber morphing (Gandhi and Anusonti-Inthra, 2008). The strain is applied in $\varepsilon_y = 0.001$ steps and the natural frequencies of the first three

modes are normalized with respect to the frequency of the structure at initial load-free state, f_0 . The variation of the natural frequencies of the structure with increasing the pre-strain magnitude is presented in Figure 5-6.

The effect of the Poisson's ratio and stiffness augmentation on natural frequencies is significant when pre-strain is applied. Comparing Figure 5-6(a) and 5-6(b), it is evident that positive Poisson's ratio hexagonal sample experiences much larger variation in the value of natural frequencies with respect to the zero Poisson's ratio Fish Cell. The natural frequency of the first mode of hexagonal sample almost doubles at $\epsilon_y = 0.01$, while for Fish Cell this ratio is 1.2. In addition, the higher modes of Fish Cell sample experience even smaller increase in the natural frequency when strain is increased; however, a mode transition is observed at strain values above 0.003 that results in a change in the trend of the second mode shape. Therefore, the positive Poisson's ratio and stiffness augmentation had significant effect on the value of the natural frequencies.

On the other side, the negative Poisson's ratio sample presents a very different behaviour under tensile load imposed. In this case, the frequency ratio reduces by increasing the strain and sample faces buckling instability due to the compressive transverse loads at certain strain in the y -direction. Before buckling also, the order and deformation shape of the modes changes rapidly. Therefore, the negative Poisson's ratio is not a desired property under tensile strain when integration of the skin with frame structure is considered in analysis of the morphing systems. Similarly, under compressive load, the positive Poisson's ratio sample is prone to buckling instability due to the lateral loads under strain. This case can happen in camber morphing when the skin of the upper surface of the wing is under tension, while the lower surface is under compression. Hence, the only satisfying option for morphing requirements from dynamic point of view is zero Poisson's ratio where Fish Cell has presented a promising performance in retaining the dynamic characteristics under pre-strains.

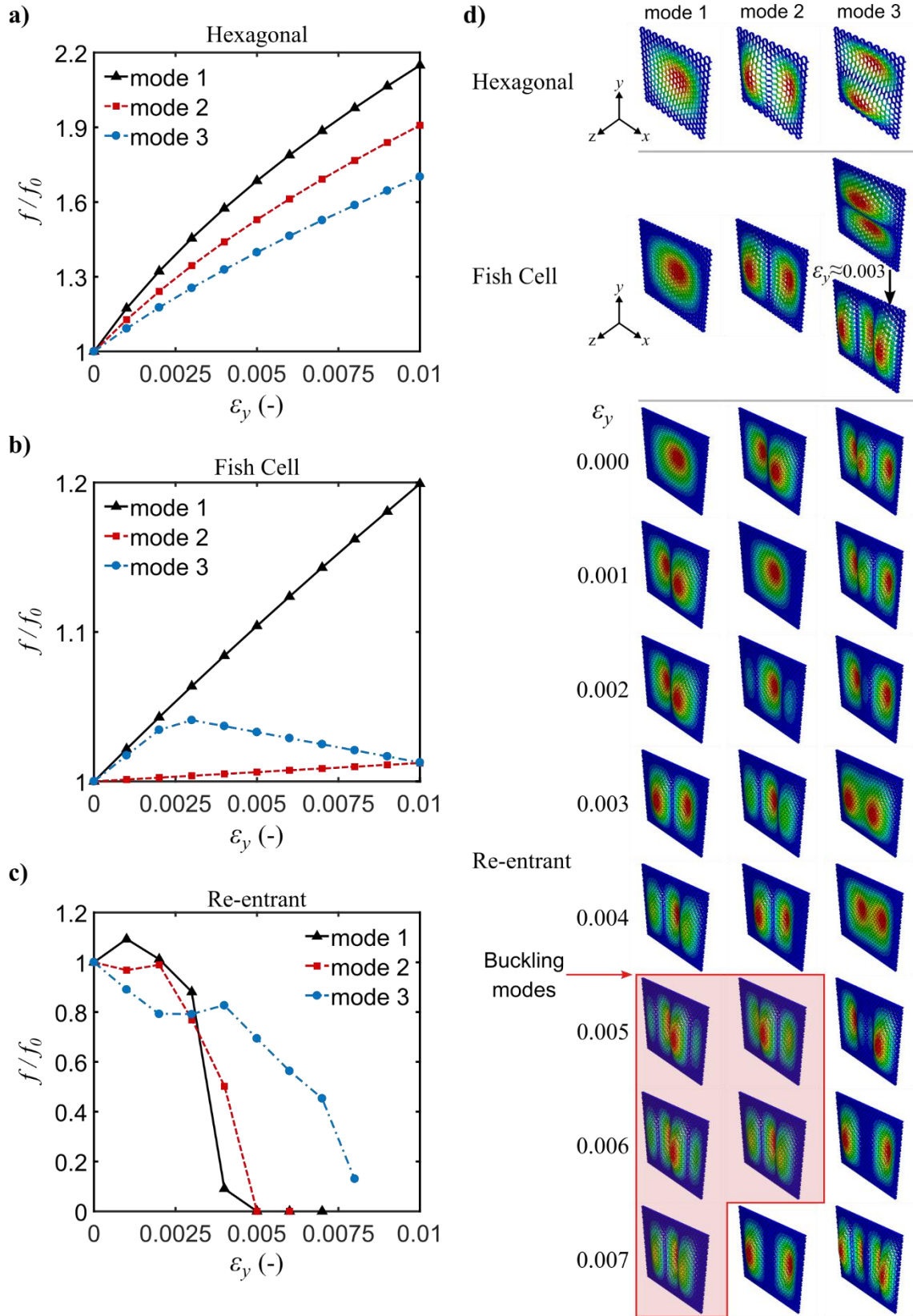


Figure 5-6 Variation of normalized frequencies of (a) hexagonal, (b) Fish cell, and (c) re-entrant samples with strain; (c) mode shapes of three samples.

5.4 Conclusions and Remarks

Dynamic characteristics of the Fish Cell metamaterial as a **candidate** ZPR core for morphing sandwich skins was studied using numerical and experimental methods. Effect of gradient properties on the modal parameters was investigated and it was shown that increasing the stiffness by a symmetric gradient function leads to the increase in the natural frequencies and damping coefficients of the samples. The effect of three different gradient functions including soft-hard, soft-hard-soft, and hard-soft-hard, which determine the stiffness of the sample along y-direction was investigated numerically. The important results obtained was that for the same overall planar stiffness and total mass, the dynamic properties including mode shapes order and natural frequencies can change by tuning the gradient of the stiffness in the sample.

The integration of the morphing skin as the main application for the ZPR sandwich core was considered where positive and negative Poisson's ratio cellular cores with the same equivalent modulus were considered as the margin of comparison for Fish Cell metamaterial. The numerical analysis of the three samples considering clamp boundary conditions and application of pre-strain was performed. The results indicate clearly that Poisson's ratio is a challenging parameter on the stability of the cellular core under strains even smaller than 0.01. Therefore, the ZPR property of the Fish Cell was proved as an important achievement to save the cellular core from going into instable zone under strain and even maintain the dynamic characteristics in an acceptable range over the strain domain.

6 Conclusions and Future Work

The core of a sandwich morphing skin is recognized as a critical component in efficiency and performance of an aerospace morphing structure. Therefore, Fish Cell metamaterial with zero Poisson's ratio behaviour is proposed as a potential solution for an efficient core in this thesis. The elastic properties of the Fish Cell metamaterial are studied using analytical, numerical, and experimental methods where following results are concluded:

- The Fish Cell metamaterial presents zero Poisson's ratio behaviour in both planar directions. This behaviour is independent of the constitutive material and values assigned to the geometrical parameters where ZPR exists for all admissible values of the Fish Cell's parameters.
- Fish Cell metamaterial is a promising solution for a morphing skin's core because the stiffness augmentation under loading in both planar directions is negligible; hence, the actuation load required for morphing will not increase.
- The Fish Cell design allows for tailoring the equivalent elastic properties over a broad range and achieve anisotropic behaviour while zero Poisson's ratio is maintained.
- The edge deformations for integration of the core with a structure must be considered in morphing applications, which are negligible for Fish Cell metamaterial and local enhancements improve the edgewise integrity of the Fish Cell with structure further.
- The geometrical details such as thickness at the joints affect the elastic properties significantly. For the best results, thin bending ribs must be considered, which can satisfy the range of strain capacity required for morphing applications such as camber morphing.
- Since main limiting factor in the strain capacity is the initiation of the plastic deformation in joints, optimizing the design at joints and considering the nonlinear geometric deformations can help to improve the joint design. Hence, as a part of future studies, joint optimization can be an important topic for designing cellular structures with applications

that require large strain capacity. In addition, the choice of the constitutive material can be studied further such as using superelastic materials like NiTiNOL to improve the elastic deformation range of the Fish Cell metamaterial.

- The relations for equivalent elastic properties of the Fish Cell metamaterial are provided in this thesis; therefore, the sizing optimization can be used to tailor the values of the geometrical parameters for a desired range of elastic properties.
- The derivatives of the Fish Cell geometry with non-straight bending ribs such as polygonal or B-spline curved ribs can be developed that help in improving the Fish Cell properties. For example, a shape optimization as a part of future studies can focus on reducing the stress concentration at joints and achieving larger range of elastic deformations using non-straight bending ribs.

In addition to the static analysis, the dynamic characterization of the mechanical metamaterials is conducted to understand first how the Poisson's ratio affect modal properties of a structure and then how the Fish Cell metamaterial can tune the dynamic characteristics as a core. In order to answer the first question and for a systematic study on the effect of Poisson's ratio on the dynamic characteristics, a consistent geometry with minimum change in parameters was required that enables tuning the elastic properties. Therefore, the hexagonal geometry is considered, which can achieve a large range of negative, zero, and positive Poisson's ratio by tuning its ligament's angle. The dynamic analysis of the 3D printed structures with negative, zero, and positive Poisson's ratio obtained by hexagonal geometry is done using analytical methods, numerical simulation, and experimental modal tests to answer the first question where the concluding remarks are:

- The damping coefficient of the hexagonal mechanical metamaterials has an inverse correlation with the absolute value of the Poisson's ratio. This means that the larger the absolute value of the Poisson's ratio the smaller is the damping coefficient.

- The natural frequencies of the positive Poisson's ratio structures are larger than negative ones. Because the natural frequencies increase with increasing the angle of hexagonal geometry from negative to positive values.
- Due to manufacturing limits, the size effects must be considered in analytical modelling of the 3D printed mechanical metamaterials. This conclusion is verified by validation of results using modified couple stress theory comparing to the classical elasticity theory.
- The value of the length scale parameter associated with modified couple stress theory increases for structures with hexagonal unit cells by increasing the angle of the ligament from negative to positive.
- The deformations of each mode shape of the structure is strongly dependent on the Poisson's ratio where synclastic, uniclasic, and anticlastic curvatures are caused by negative, zero, and positive Poisson's ratio, respectively.

Regarding the dynamic analysis of the Fish Cell metamaterial, two types of geometries are considered to study the dynamic characteristics: the uniform Fish Cell geometry and gradient Fish Cell geometry where the latter offers flexibility to tune the elastic properties along the length of the structure. This gradient feature for Fish Cell metamaterial is a great advantage because the designer can tune the elastic properties without changing the unit cell size while ZPR behaviour in two planar directions is maintained. In this case, the experimental modal analysis and numerical simulations are conducted on the all edges free 3D printed samples where the conclusions are:

- Gradient design creates variable stiffness Fish Cell metastructure that can achieve tailored modal properties including the natural frequency, mode shape, and damping ratio. The natural frequencies can be higher or lower than uniform Fish Cell metastructure depending on the gradient function.

- For two gradient Fish Cell metastructures with the same total mass and the same equivalent stiffness but different gradient functions, the natural frequencies can change significantly.
- Fish Cell metastructure mode shapes are uniaxial because Poisson's ratio is zero. However, the symmetry of the mode shape in gradient Fish Cell metastructure depends on the symmetry of the gradient function. This means that position of the node (zero displacement point) of the symmetric mode shapes can change in the gradient Fish Cell metastructure by tuning the gradient function.

Another aspect of the Poisson's ratio effect on the dynamic characteristics is understood when boundary conditions of a morphing structure are applied on the edges of the structure. This case is studied by comparing three structures with the same equivalent elastic modulus in the morphing direction while edges parallel to morphing direction were constrained. The conclusions obtained from numerical simulations are:

- Under tensile loading and by increasing the strain, the natural frequencies of the positive Poisson's ratio sample increase significantly, mainly due to the stiffness augmentation caused by constrained edges. On the other side, the negative Poisson's ratio structure becomes unstable and buckles due to the compressive reaction forces on the constrained edges. Therefore, positive and negative Poisson's ratio structures cannot maintain modal properties under morphing loads.
- The natural frequencies of the zero Poisson's ratio Fish Cell metastructure experience small changes because the stiffness augmentation is alleviated. Therefore, Fish Cell metastructure is also an ideal solution for morphing applications from dynamic point of view.

Apart from the design, manufacturing of the cellular cores for morphing application is also an important factor in achieving the right performance. Additive manufacturing used in this thesis for producing the samples is an efficient method in terms of satisfying complex geometries' fabrication; however, the ability to manufacture in large scales and sizes are the limits that current 3D

printers are bounded with. Therefore, other methods that help in scalable manufacturing can be investigated in future studies. Considering the geometrical complexities of the Fish Cell design that includes concave (negative angle) features, using traditional sheet metal or composite lamination techniques used in fabricating hexagonal honeycombs requires further modifications. Therefore, more innovative techniques such as laser cut of sheets with kirigami and origami fabrication can be studied in the future as a potential manufacturing method.

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APPENDICES

Appendix A Supplements of Chapter 2

A.1 Terminology of the tessellation numbers

This section introduces a convention for tessellation numbering used in this paper. Figure A-1(a) illustrates the numbering method for a 5×5 tessellation by assembling $m = 5$ unit cells in the x -direction and $n = 5$ unit cells in the y -direction.

Figure A-1(b) illustrates the terminology for a symmetric tessellation along y -direction. In the terminology shown as $(m \times n)_s$, the subscript s mean that there are m unit cells plus one Fish Cell assembled in the x -direction. Therefore, the tessellation will have an axis of symmetry in the y -direction. In this figure, tessellation $(5 \times 5)_s$ is presented.

Figure A-1(c) presents the enhancements proposed for $\text{Edge}_{\pm x}$ and $\text{Edge}_{\pm y}$. On $\text{Edge}_{\pm x}$, half of a Fish Cell is added at each edge and as a result, the midpoint in base members become the interface of the connection rather than connectors. On $\text{Edge}_{\pm y}$, a Fish Cell is removed from each unit cell and as a result, a single Fish Cell remains. In this terminology presented as $(m \times n)^{\text{enhanced}}$, m is the number of unit cells assembled in the x -direction and superscript “*enhanced*” indicates the edges are modified by the procedure explained. This enhancement terminology can be also applied to the symmetric tessellations as $(m \times n)_s^{\text{enhanced}}$.

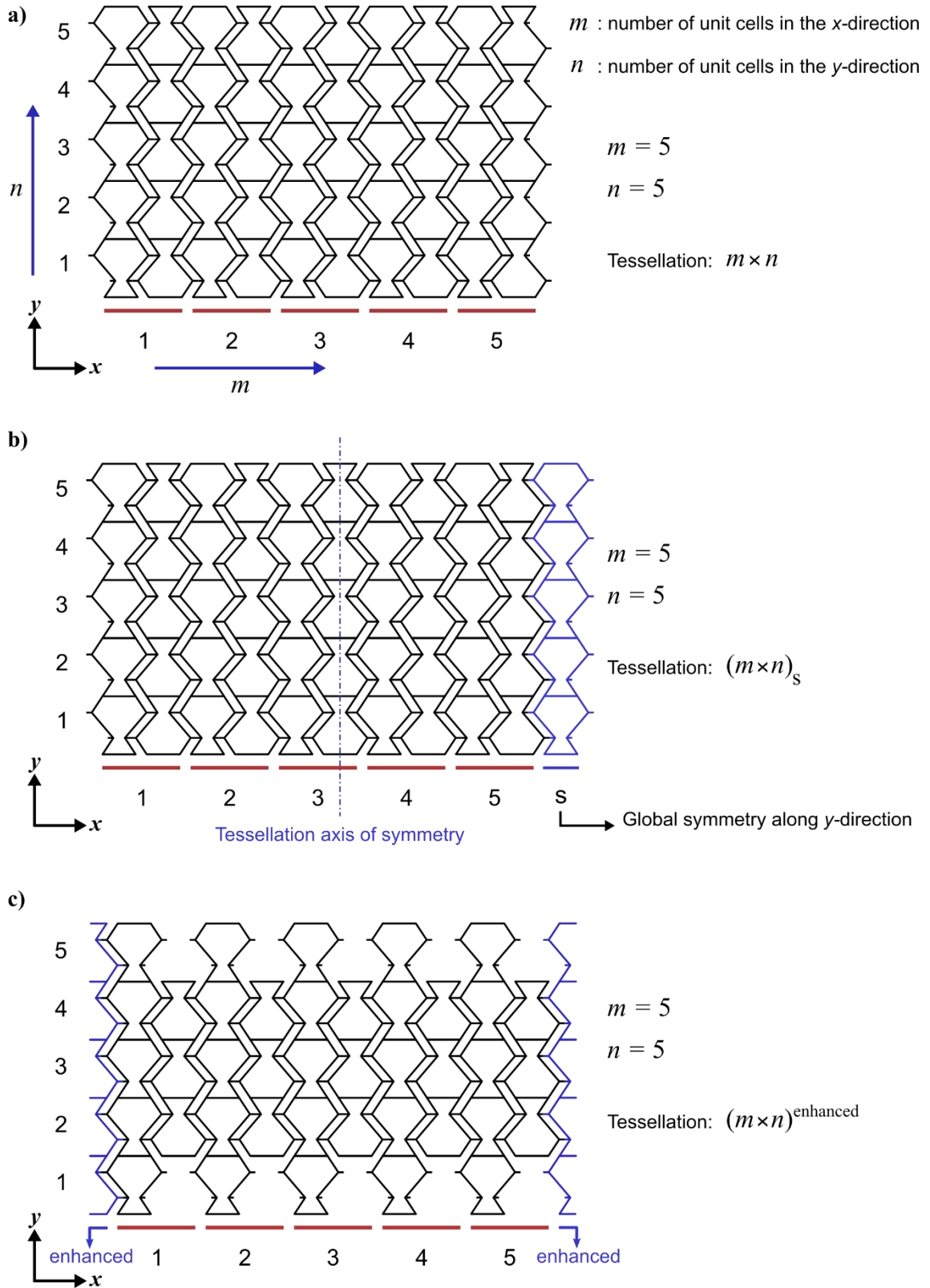


Figure A-1 a) Tessellation numbering terminology illustration for the conventional tessellation, b) symmetric tessellation along y-direction, and c) tessellation with enhanced Edges_{±x} and Edges_{±y}

A.2 DIC Measurements

In this section, the data captured using Dantec Q400 DIC system and its processing in the Istra 4D V4.3.1 is discussed. The DIC setup is shown in Figure 2-6(a). In order to recognize the displacements under loading, the samples were painted white with fine speckle patterned black dots. The data obtained for tests in two directions were analyzed in the Istra 4D software as shown in Figure A-2. The dataset in each direction consists of several steps divided by specific time increments (0.5 Hz in this case) and the load and displacement of the Instron tensile test machine are also recorded for each step. By defining the gauge points A, B, C, and D in the first step as shown in Figure 2-2(c) and 2-2(d), *i.e.*, initial undeformed state, the software will compute the displacements of those points in the proceeding steps. Therefore, the displacements of the gauge points in two directions and strains are computed consequently. The Poisson's ratio will be also calculated based on the average strain of the gauge points at each step.

Figure A-2(a) and A-2(b) illustrate the sample under loading in the *x*-direction in initial and deformed states, respectively. The four red dots in the figures also represent the gauge points. The colors represent the contour of displacement in the perpendicular direction, *i.e.* *y*-direction. Figure A-2(c) and A-2(d) illustrate the sample under loading in the *y*-direction in initial and deformed states, respectively. The colors represent the contour of displacement in the perpendicular direction, *i.e.* *x*-direction.

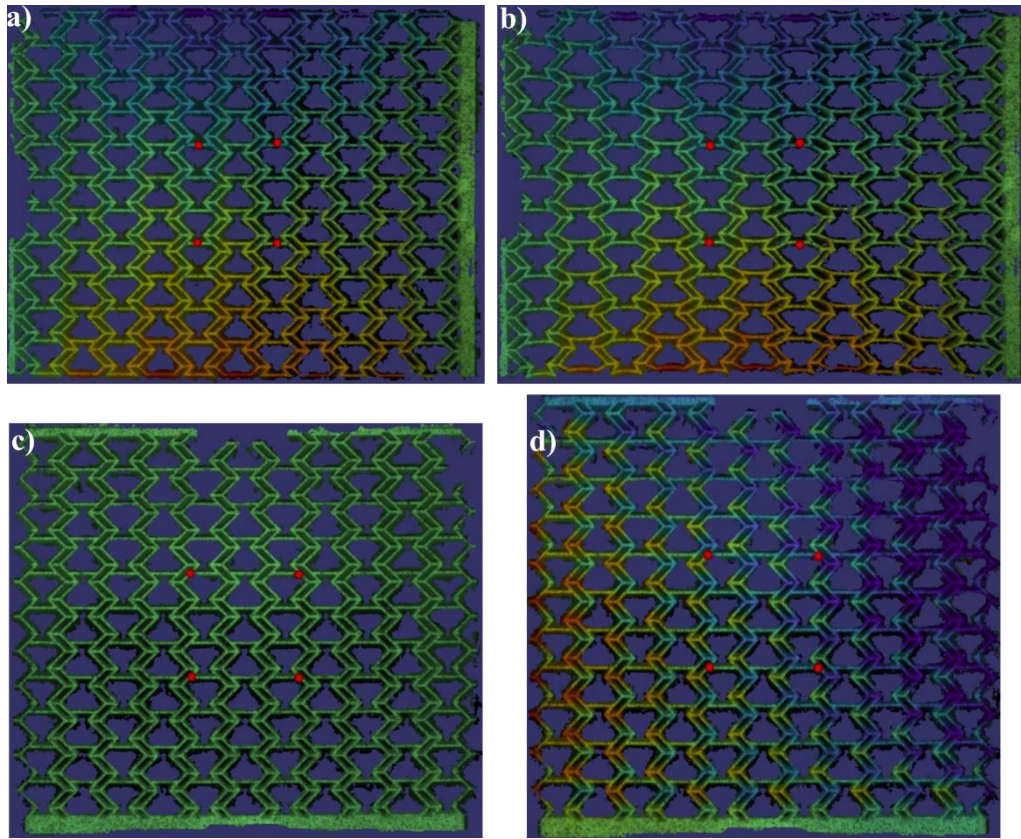


Figure A-2 Post processing of the data recorded from experimental test in the x-direction in (a) initial state and (b) deformed state. (c) initial state and (d) deformed state under loading in the y-direction.

Appendix B Supplements of Chapter 4

B.1 Test Sample Properties

There are five samples designed and manufactured for the Experimental Modal Analysis (EMA). All samples are manufactured using PLA material (Ultimaker, NL) with elastic modulus $E_s = 3500$ MPa, Poisson's ratio $\nu = 0.36$, and density $\rho_s = 1250$ kg/m³. The manufacturing method was Fused Deposition Modelling (FDM) where Ultimaker 2+ machine was employed. The limits imposed by manufacturing method are the maximum size of the samples and minimum thickness of the ligaments. The extruding nozzle diameter was 0.4 mm and two adjacent lines of material deposition was considered to print the sample walls. In order to have a consistent study, regular hexagonal cells were considered with length of each ligament $l = 5$ mm and only the angle θ changed in each sample. Five samples were designed to study the effect of Poisson's ratio on dynamic properties including positive, negative, and zero Poisson's ratio. Two negative angles $\theta = -10^\circ$, -20° were considered for negative Poisson's ratio samples, $\theta = 0$ degree that results in zero Poisson's ratio, and $\theta = 14^\circ$ and 20° that result in positive Poisson's ratio were manufactured. The number of unit cells in the tessellation in the x and y -direction are presented by N_x and N_y for each sample, respectively. The total length of the samples in the x and y -direction are also presented by a and b , respectively. Considering the size limits of the 3D printer and maintaining the total size of the samples in an acceptable range, the tessellations of different geometries were created to the closest values of a 180×180 mm rectangular planform. The number of cells for tessellations in x and y -direction and the final size of the samples are presented in Table B-1.

Table B-1 the dimensions and number of unit cells for the experimental samples

Sample		N_x	N_y	a (mm)	b (mm)
Hexagonal	$\theta = 20$	13	19	174.46	179.14
	$\theta = 14$	14	18.5	173.58	179.60
$\theta = 0$		18	18	180	180
Reentrant	$\theta = -10$	21	18	173.53	177.87
	$\theta = -20$	27	19	177.65	178.54

In order to calculate the values of equivalent elastic properties, formulas presented by Malek and Gibson (Malek & Gibson, 2015) are considered. These formulas were modified for the coordinate system and equal length of ligaments as presented in Eq. B.1-1.

$$\frac{\rho}{\rho_s} = 1 - \frac{l_b(h_b + l_b \sin \theta)}{l^2(1 + \sin \theta)} \quad (\text{B.1-1a})$$

$$E_x = E_s \left(\frac{t}{l_b} \right)^3 \frac{1 + \sin \theta}{\cos^3 \theta} \left[\frac{1}{1 + \left(2.4 + 1.5\nu + \tan^2 \theta + \frac{2(h_b/l_b)}{\cos^2 \theta} \right) \left(\frac{t}{l_b} \right)^2} \right] \quad (\text{B.1-1b})$$

$$E_y = E_s \left(\frac{t}{l_b} \right)^3 \frac{\cos \theta}{(1 + \sin \theta) \sin^2 \theta} \left[\frac{1}{1 + (2.4 + 1.5\nu + \cot^2 \theta)(t/l_b)^2} \right] \quad (\text{B.1-1c})$$

$$\nu_{yx} = \frac{\sin \theta (1 + \sin \theta)}{\cos^2 \theta} \left[\frac{1 + (1.4 + 1.5\nu)(t/l_b)^2}{1 + \left(2.4 + 1.5\nu + \tan^2 \theta + \frac{2(h_b/l_b)}{\cos^2 \theta} \right) \left(\frac{t}{l_b} \right)^2} \right] \quad (\text{B.1-1d})$$

$$\nu_{yx} = \frac{\cos^2 \theta}{\sin \theta (1 + \sin \theta)} \left[\frac{1 + (1.4 + 1.5\nu)(t/l_b)^2}{1 + (2.4 + 1.5\nu + \cot^2 \theta)(t/l_b)^2} \right] \quad (\text{B.1-1e})$$

$$G_{xy} = E_s \left(\frac{t}{l_b} \right)^3 \frac{1 + \sin \theta}{(h_b/l_b)^2 \cos \theta} \left(\frac{1}{C} \right) \quad (\text{B.1-1f})$$

$$G_{xz} = G_s \frac{t/l}{(1 + \sin \theta) \cos \theta} \left[\sin^2 \theta \left(\frac{l_b}{l} \right) + \frac{h_b}{2l} + \frac{3}{4} \left(\frac{t}{l} \right) \tan \theta - \frac{\sin^2 \theta}{2 \cos \theta} \left(\frac{t}{l} \right) (2 \sin \theta - 1) \right] \quad (\text{B.1-1g})$$

$$G_{yz} = G_s \frac{t/l}{(1 + \sin \theta) \cos \theta} \left[\cos^2 \theta \left(\frac{l_b}{l} \right) + \frac{3}{4} \left(\frac{t}{l} \right) \tan \theta - \frac{\cos \theta}{2} \left(\frac{t}{l} \right) (2 \sin \theta - 1) \right] \quad (\text{B.1-1h})$$

where additional parameters are defined as

$$C = 1 + 2 \left(\frac{h_b}{l_b} \right) + \left(\frac{t}{l_b} \right)^2 \left[\frac{2.4 + 1.5\nu}{h_b/l_b} (3 + \sin \theta) + \frac{1 + \sin \theta}{(h_b/l_b)^2} ((1 + \sin \theta) \tan^2 \theta + \sin \theta) \right] \quad (\text{B.1-2a})$$

$$G_s = \frac{E_s}{2(1 + \nu)} \quad (\text{B.1-2b})$$

$$h_b = l - t(1 - \sin \theta) / \cos \theta \quad (\text{B.1-2c})$$

$$l_b = l - t / (2 \cos \theta) \quad (\text{B.1-2d})$$

The strength of the theory presented by Malek and Gibson is consideration of the joint's thickness in the derivation of equivalent elastic properties; therefore, it can present higher accuracy than existing theories in the literature. However, there is a singularity in determination of the Poisson's ratio at $\theta = 0$, which is not a concern because the Poisson's ratio values in the neighborhood are close to zero. In addition, FE analysis was conducted using ABAQUS 2018 software to evaluate the accuracy of the analytical model employed for estimation of the elastic properties. The samples were modeled as 2D plane-stress solids and were discretized with fine mesh for great accuracy as shown in Figure B-1. The continuum shear deformable quadratic CPS8 elements were used to improve the accuracy with lower mesh elements number. The aim of using solid model was to consider the effect of solid joints on the deformations, which cannot be explicitly captured using beam element and beam theories.

The results obtained for equivalent properties of the samples are presented and compared in Table B-2 for both Malek and Gibson's theory and FE

analysis. The results show that the theory predicts higher stiffness values than FEM for positive Poisson's ratio and zero Poisson's ratio, while it presents lower stiffness values for negative Poisson's ratio. Therefore, there is a small difference between the Poisson's ratio values of theory and simulation; however, the trend is consistent for both cases.

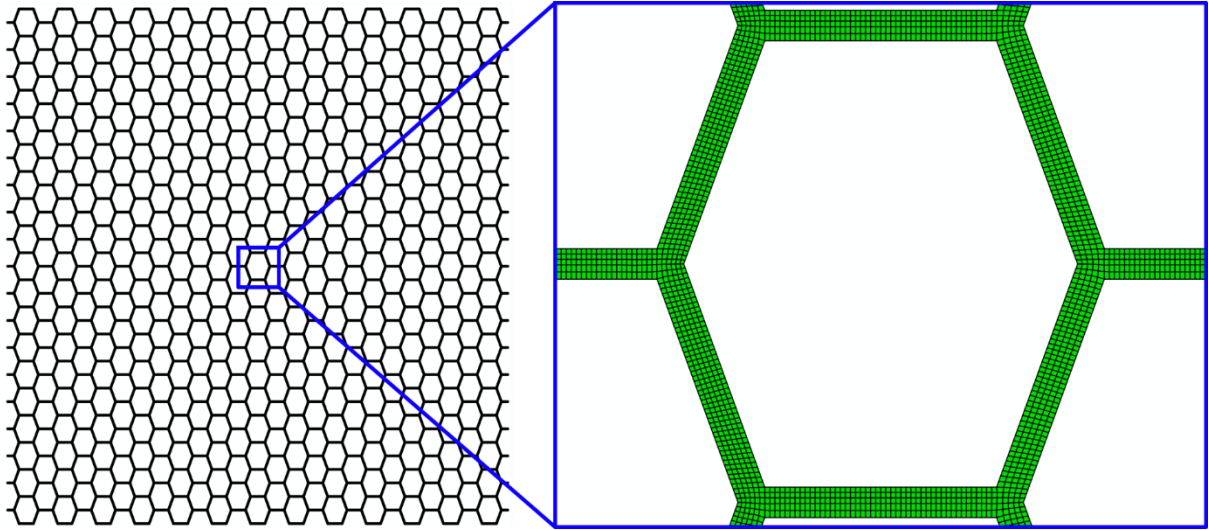


Figure B-1 the plane-stress FE analysis of the hexagonal lattice with angle $\theta = 20$ modelled in ABAQUS where the magnified view presents the mesh quality

Table B-2 Comparison of the elastic properties of test samples obtained from Malek and Gibson theory versus FE analysis

θ	Malek and Gibson's Theory				FEM				error %
	E_x	E_y	ν_{xy}	ν_{yx}	E_x	E_y	ν_{xy}	ν_{yx}	
20	10.969	37.639	0.493	1.693	9.55	35.52	0.466	1.733	-5.96
14	9.167	75.736	0.301	2.484	8.12	71.24	0.283	2.488	-6.31
0	6.752	446.808	0	-	6.12	420.45	0	0	-6.27
-10	5.855	183.472	-0.141	-4.432	5.48	210.01	-0.114	-4.387	12.64
-20	5.395	76.770	-0.243	-3.454	5.289	86.03	-0.218	-3.542	10.76

Having the equivalent elastic properties, the compliance matrix of the material was obtained as Eq. B.1-3a. The elastic properties E_x , E_y , ν_{xy} , and ν_{yx} were substituted by values obtained from FE analysis and equivalent shear

properties were derived from Malek and Gibson's relations. The stiffness matrix was finally derived by inversion of the compliance matrix as in Eq. B.1-3b.

$$S = \begin{bmatrix} 1/E_x & -\nu_{yx}/E_y & 0 & 0 & 0 \\ -\nu_{xy}/E_x & 1/E_y & 0 & 0 & 0 \\ 0 & 0 & 1/G_{yz} & 0 & 0 \\ 0 & 0 & 0 & 1/G_{xz} & 0 \\ 0 & 0 & 0 & 0 & 1/G_{xy} \end{bmatrix} \quad (\text{B.1-3a})$$

$$Q = S^{-1} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \quad (\text{B.1-3b})$$

B.2 Validation of the code for free vibration analysis of the free plate

The code developed for the calculation of the natural frequencies of a plate with free boundary conditions was validated by the reference results (Messina and Soldatos, 1999). The dimensions of the plate are $h = 0.15$ m, $a = 1.5$ m, and $b = 1$ m. The plate is orthotropic and effective elastic properties relations are $E_x/E_y = 20$, $G_{xy}/E_y = G_{xz}/E_y = 0.5$, $G_{yz}/E_y = 1/3$, and $\rho = 1600$ kg/m³. The values of the non-dimensional frequency $\tilde{\Omega} = \Omega a^2 \sqrt{\rho/(E_x h^2)}$ of the first five modes were calculated by present method using Chebyshev polynomials of order 7 and compared to the results in the literature in Table B-3 where the accuracy of the code is verified.

Table B-3 the non-dimensional frequencies obtained from the code developed in this paper comparing to the reference values

Mode	Non-dimensional Frequency, $\tilde{\Omega}$		Error %
	Ref. (Messina & Soldatos, 1999)	Present	
1	1.4512	1.4511	0.007
2	3.0048	3.0048	0.0
3	3.9772	3.9772	0.0
4	5.2502	5.2502	0.0
5	5.7032	5.7032	0.0