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EXPERIMENTAL AND THEORETICAL STUDIES INTO THE
FATIGUE CRACK PROPAGATION BEHAVIOUR OF THE 7075
ALUMINIUM-ZINC ALLOY UNDER CONSTANT AND VARIABLE
AMPLITUDE LOADING

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TO FAMILY AND FRIENDS
WITH APPRECIATION

A B S T R A C T

This thesis describes both theoretical and experimental work performed in order to study the fatigue crack propagation behaviour of the 7075 T7351 Aluminium-Zinc alloy, under constant and variable amplitude loading.

For the theoretical aspects of the work, Static, Elastic and Elastic-Plastic Finite Element methods were utilised for the evaluation of residual stresses in the vicinity of the crack tip caused by overloading. The results have demonstrated that the plasticity caused by the loading part of the load cycle induces compressive residual stresses in the vicinity of the crack upon partial and/or full unloading.

The experimental work has been carried out aiming to ascertain the constant load amplitude fatigue crack growth behaviour. It has been established that the material does not exhibit a sigmoidal crack growth curve but, one of parabolic like shape. An empirical crack growth law has been derived to describe the constant load amplitude behaviour.

It has been established that overloads cause substantial extension of the total life to failure due to delays caused by overloading. The observed delay increases if block overloads are introduced, and crack arrest is possible to occur. Finally, it is concluded that crack closure is responsible for the observed crack growth rate reductions.

A C K N O W L E D G E M E N T

The author would like to take this opportunity to express his deepest gratitude to a number of people without whose efforts this work could not have been accomplished.

Firstly, a special note of appreciation to Dr A. M. El Zafrany, supervisor, who, in addition to genuine friendship, provided support, encouragement and invaluable assistance throughout the preparation of this thesis.

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CHAPTER 1

INTRODUCTION

1.0 Preface

1.2 Justification and Objectives of study

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CHAPTER ONE

INTRODUCTION

1.0 PREFACE

Fatigue has been identified as the largest contributing factor to the failure of engineering components in service. Extensive investigations have been carried out in order to establish the mechanism of the fatigue failure. In the course of detail research carried out by numerous researchers and organisations, it has been diagnosed that fatigue failure can be sub-divided into two distinct regimes. These are referred to, as the initiation dominated and the crack propagation dominated fatigue failures. This realisation has resulted to the establishment of two separate schools of investigation.

The initiation dominated failures are examined from the point of view of stress conditions that contribute to the initiation of cracks under cycling loading. The approach is based upon the concepts of cumulative damage which states that, loading conditions contribute to the initiation of cracks and, their contributions are dependent upon the magnitude and type of time variable loading (bending, torsion, tension), the presence of stress concentrations etc. The methodology developed due to this approach has been proven to be a very effective and invaluable tool in the development and design of machines and structures.

The applicability of this approach is limited to the prediction of crack initiation. The design philosophy developed in parallel to the evolution of the initiation

dominated fatigue calculation procedures defines failure as the initiation of a detectable crack.

Experience, has shown that the presence of flaws is an inevitable reality as crack like defects can be introduced in metallic materials during machining, fabrication or heat treatment. It has therefore become apparent that cracks cannot render a component unstable. The foundations of fracture mechanics introduced by Griffith have provided the required theoretical concepts for the development of models able to cope with the presence of cracks in engineering materials. In the last thirty years the interests of research, have expanded to include the study of crack growth and propagation. This novel approach to the problem of fatigue failure is concerned with the crack propagation dominated fatigue.

A new methodology has evolved, providing new tools for the prediction of service life, based upon an integrated approach to fatigue, combining the traditional stress analysis concepts and the more modern fracture mechanics theories.

The determination of the fatigue crack propagation curve is an essential part of the fracture mechanics design approach. This curve can be considered to be an extension of the (S-N) curve which is a graphic presentation of the total fatigue life of materials under cyclic loading conditions. As the data for the (S-N) curve is determined with the use of smooth specimens, this does not allow the clear distinction between the crack initiation and the crack propagation periods.

Fatigue crack propagation curves on the other hand, are produced by means of testing specimens that contain cracks and notches. Experimentally determined data of load cycling and crack length measurements can be analysed in

an appropriate manner to develop fatigue crack growth curves. A number of such curves can be produced by utilising different loading conditions and standardised test procedures.

Pre-existing flaws or crack like defects within the material reduce or may eliminate the fatigue crack initiation portion of the fatigue life of components thus, permitting the study of the fatigue crack propagation. The fracture mechanics methodology enhances the understanding of the crack initiation and propagation of fatigue cracks and assists in solving the problem of designing to prevent fatigue failures. Empirical crack propagation laws can be employed at the design stage in order to enable the residual strength assessment.

1.2 JUSTIFICATION AND OBJECTIVES OF STUDY

It has been established that fatigue crack growth and propagation is a phenomenon dependent upon a large number of variables and parameters. The material behaviour and mechanical properties are of paramount importance. It has been found that under identical conditions different materials give different results of crack growth rates. Hence, it is essential to study each material separately, in order to obtain detail knowledge of the exhibited behaviour.

Any one material can be used in a variety of applications which implies a near infinite range of possible loading conditions. The fatigue crack propagation behaviour is dependent upon the load history, in other words the sequence of loading. The standard testing methods only cater for the constant amplitude loading case.

In service components are rarely experiencing constant load amplitude cycling which limits the use of standard

test information. In the case of the investigated material (7075-T7351 Aluminium-Zinc alloy), the information available regarding the fatigue crack growth behaviour is very limited. It is a high strength material frequently and extensively utilised by the aerospace industry in aircraft wings, spars and under-carriage systems. Despite of the extensive usage, published information regarding the fatigue crack growth behaviour is only limited to data of constant load amplitude. The particular heat treatment condition has not been extensively investigated and this has been concluded by the lack of published data.

The broad aims of the presented research effort were,

- a. to study the fracture mechanics theory as adopted in fatigue life prediction methodologies and,
- b. investigate the parameters, variables and macroscopic phenomena influencing the fatigue crack growth.

The specific objectives of the undertaken research project can be classified into two main categories, theoretical and experimental.

Theoretical Objectives

1. Utilise finite element analysis methods for the design of appropriate specimens.
2. Investigate the accuracy and reliability of a variety of finite element modelling methods in the determination of fracture parameters.
3. Utilise elastic-plastic finite element methods for the determination of residual stress fields in the vicinity of cracks, due to overloading.

Experimental Objectives

1. Carry out mechanical tests for the determination of the mechanical properties of the 7075-T7351 Al-Zn alloy.

2. Experimentally investigate the constant load amplitude fatigue crack growth behaviour.
3. Examine the influence of the mean load upon the constant load amplitude fatigue crack growth.
4. Determine the effects of load ratio variation.
5. Utilise the experimental data to determine a reliable crack propagation law governing the constant load amplitude behaviour.
6. Investigate the effects of the peak magnitude of single overloads upon the crack growth.
7. Study the influence of block overloading upon the fatigue crack propagation life.

1.3 THESIS LAYOUT

The presented thesis is divided into six main parts and contains nine chapters in total.

The first part provides a brief introduction to, and description of this study, as well as justification of the need for work in this area and is presented in Chapter One.

Part two, incorporates a critical review of current literature with special reference to general topics of crack growth as related to this study. In addition to the literature, a general overview of fatigue and fracture mechanics as related to modern design practices for service life predictions is presented. Chapters Two and Three comprise part two.

Part three, is contained in Chapter Four. This is totally devoted to the stress analysis aspects of fracture mechanics by means of Finite Element Methods. Methods for the determination of fracture parameters as calculated from FE data and utilised in the present study are

extensively discussed. A detailed elastic and elasto-plastic analysis of the CT specimen is also presented.

Part four, concentrates to the experimental aspects of the research. Methods for the detection and sizing of cracks and flaws are presented and their merits and limitations are discussed. Further, the specific equipment utilised in the present study, supplemented by design and calibration methods of specimens and transducers are introduced. The preliminary tests carried out on the material to establish the mechanical properties are also included. Chapter Five constitutes part four.

Part five, comprises of Chapters Six, Seven and Eight. These are devoted to the experimental research and data analysis aspects. The constant load amplitude studies and the derivation of an empirical mathematical model for crack propagation predictions is presented. The effects of variable amplitude loading upon the fatigue crack growth are discussed in Chapter Seven, where experimental evidence of crack delay and arrest are presented.

The sixth and final part of the thesis presents a general discussion of the findings and identifies areas of further research interest related to fatigue crack growth and propagation.

C H A P T E R 2

LITERATURE REVIEW

2.0 Introduction

2.1 Damage Tolerant Design and Retirement for Cause

2.2 Crack Propagation Phenomena Research

2.2.1 Load Interaction Effects

2.2.2 Crack Closure

2.3 Crack Propagation Modelling

References

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

The presence of small flaws and defects in engineering components is an inevitable reality. These flaws, may be formed in the metal lattice during the solidification of the ingot upon casting, or formed subsequently as a result of machining or material processing.

Due to the nature of internal defects, their detection is a very difficult task. Even, with the current advances in non-destructive testing techniques and the introduction of total quality control on highly specified components, some defective parts are still entering service.

Engineers have acknowledged the fact that no material exists which is defect free and they have been seeking ways and means of accounting for the presence of defects and material irregularities in their calculations. Uncertainty factors are used such as the various safety factors employed in all sorts of calculation procedures. The problem of defects becomes of great interest when the presence of a defect is identified during service. The questions raised are whether the presence of the defect means that component failure is imminent or, the component can be returned to service and, when should the next routine inspection take place. These questions call for the understanding of defect behaviour and the mechanics of crack propagation not only in the constant load amplitude regime but also under spectra and random service loads.

A critical review of published technical reports and papers concerned with current design trends, mechanics of crack growth and, the modelling of crack propagation, is presented in this Chapter.

2.1 DAMAGE TOLERANT DESIGN AND RETIREMENT FOR CAUSE

The development of advanced stress analysis techniques, allows the reduction of safety factors and consequently an increase of service stress levels. Thus, the probability of crack development from existing flaws increases. In applications such as aircraft and general aerospace components where, the structural weight is of fundamental importance the safety factors are reduced further and, the occurrence of cracks becomes even more likely.

In fail-safe design the view is that fatigue cracks will not cause failure before they are detected and repaired. This design philosophy is the brain child of the aircraft industry. Fail-safe designs employ multiple load paths and crack arresting features are built into the structure along with rigid regulations and criteria for inspection and detection of defects.

An extension of the fail-safe philosophy is the damage tolerant design approach [2.1]. This approach assumes the presence of fatigue cracks in an engineering structure. The techniques of fracture mechanics are used to determine whether the cracks will grow to dangerous proportions causing failure, before they are detected during a periodic routine inspection. The emphasis in the design is the use of high fracture toughness materials which exhibit slow crack growth rates. The success of this design procedure is very much dependent upon the sensitivity of non-destructive inspection methods for detecting, locating and sizing crack like defects and, the correct specification of inspection intervals.

In certain components such as turbine discs, the prediction of safe operation time is based upon low cycle fatigue crack initiation criteria. It has though been identified that this approach for the determination of a safe period of operation, yields severe underestimates with the consequence of high maintenance costs. As a result components have for many years been retired from service because they have reached an analytically predetermined period of service life. Inspections of such retired components have revealed that large numbers of those discs had long residual lives after retirement, Annis et al [2.2].

In an attempt to reduce maintenance costs through longer service life, a great deal of research has been carried out to optimise the utilisation of discs. The outcome of this work has been the introduction of the Retirement for Cause philosophy or RFC by Pratt and Whitney [2.2]. This philosophy states that a component will be retired from service only when a defect has been identified to exist (cause) and not because a predetermined service life limit has been reached; usually based upon statistical fatigue life predictions. Even components with small defects in non-critical areas can be returned to service for a further period. Clearly, the approach makes use of the fracture mechanics concepts for the determination of return to service intervals and use of state of the art non destructive evaluation methods, (NDE). The confidence on the approach is relying upon the accurate knowledge of the material behaviour, the crack propagation behaviour and the sensitivity of NDE methods.

2.2 CRACK PROPAGATION PHENOMENA RESEARCH

The fundamental requirement of the state of the art design philosophies is the accurate knowledge of the defect behaviour under cyclic loads of regular and mostly irregular patterns. A large number of research has been stimulated by the need to understand the mechanics of crack growth.

A characteristic of the research is the fact that it is directed to specific materials for specific applications. An ample supply of experimental data regarding the constant load amplitude fatigue crack growth can be found in the literature such as the Atlas of Fatigue Curves edited by Boyer [2.3]. For specific materials such as Aluminium alloys for aerospace applications the ESDU [2.4], provides a comprehensive selection of fracture data. The information contained in publications such as [2.3, 2.4], deals with the constant load amplitude behaviour of fatigue crack growth and is of limited use.

Research work by many individuals and organisations has revealed that the fatigue crack growth is very much dependent upon the load history. Schijve [2.5], has given a detail account of many of the studies carried out prior to 1976, related to the non-linear effects on fatigue crack growth due to load sequencing. The discussion presents the fact that an overwhelming variety of types of variable amplitude loading could be examined in order to fully investigate the load interaction effects. Table 2.1 is an extract from the paper showing the multitude of possibilities and permutations of loading types with the main variables involved.

Overloads	<ul style="list-style-type: none"> Single overload Repeated overloads Block overloads Magnitude of overloads R effects Sequence of Overloads
Step Loading	<ul style="list-style-type: none"> Sequence of steps (hi-lo, lo-hi) Magnitude of steps R effects
Programmed Block Loading	<ul style="list-style-type: none"> Sequence of amplitudes Size of blocks Distribution funct. of amplitudes
Random Loading	<ul style="list-style-type: none"> Spectral Density Function Crest factor (clipping ratio) Irregularity Factor
Flight Simulation Loading	<ul style="list-style-type: none"> Distribution function of load cycles Sequence of flights Sequence of loads in flight Maximum load in test

Table 2.1 Types of Variable-Amplitude Loading
With Main Variables [2.5]

The above table from Schijve [2.5], does not include the effects of environment, load cycling frequency and, temperature in the list of variables which have been found to affect the crack growth; Speidel [2.6], Achter [2.7] and, Takezono et al [2.8] are examples of reported work dealing with the effects of such variables.

The identified main goals of the research are to

1. Increase the fundamental understanding of the crack growth mechanism under variable-amplitude loading
2. Generate data from which useful empirical trends might be derived
3. Check the validity of crack growth models

2.2.1 Load Interaction Effects

Aluminium is the most extensively investigated material and in particular it's derivatives, the high strength aerospace alloys. Research has been stimulated by the extensive employment of such alloys by the aerospace industry, due to their advantageous specific strength characteristics and good corrosion resistance. Extensive test programs were conducted predominantly by the aircraft industries in the early and mid-1970's with the purpose of assessing and quantifying the effect of retardation on the crack growth in aircraft structures.

In the early 1960's Hudson and Hardrath [2.9, 2.10], carried out investigations of the crack propagation patterns in aluminium 2024-T4 alloy spar caps, subjected to variable amplitude loading. The block loading spectra investigated did show that the crack propagation at lower stress levels was retarded by the prior application of higher stress cycles. The retardation phenomena were explained by means of the localised plasticity in the vicinity of the crack tip due to the presence of residual compressive stresses. Similar work was also conducted by Schijve et al [2.11] and Probst et al [2.12], which ascertained the fact that, a single overload interrupting an otherwise constant load amplitude cycling, did cause a propagation delay which was found to be proportional to the magnitude of the overload. In addition, the effects of the yield strength upon the delay were examined by Gallagher and Hughes [2.13], where it was established that

reduced yield strength causes higher delay for the same level of overloading.

The effects of overloading have been found to increase if periodic or block overloads are employed. The study of the multiple block overloading by Trebules et al [2.14], revealed that not only the magnitude of the overload can affect crack growth retardation but, also the number of overload cycles. Increasing the number of overloads increases the subsequent delay and in some cases crack arrest may be achieved. In addition, periodic overloading can result to reduction of the crack growth rates as it has been identified by Ohrloff et al [2.15]. As shown in Fig.2.1, the larger the number of constant amplitude cycles between overloads, the greater the reduction of the crack growth rates.

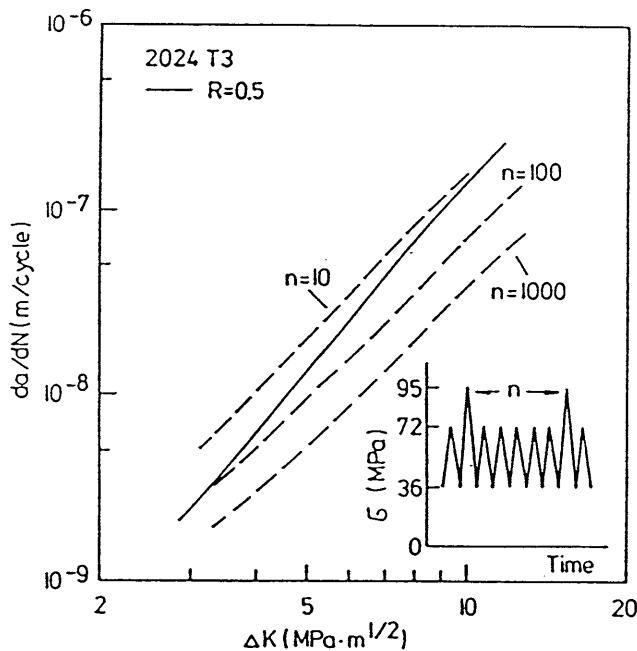


Fig.2.1 Effect of Periodic Overloads on the Crack Propagation Behaviour of 2024-T3 Aluminium [2.15]

One interesting result of the post overload behaviour was found to be the fact that the crack delay did not reach a minimum immediately on application of the overload but, when the crack had grown a small distance into the plastic zone caused by the overload, von Euw et al [2.16]. This phenomenon has been termed the delayed retardation and is clearly depicted in Fig.2.2.

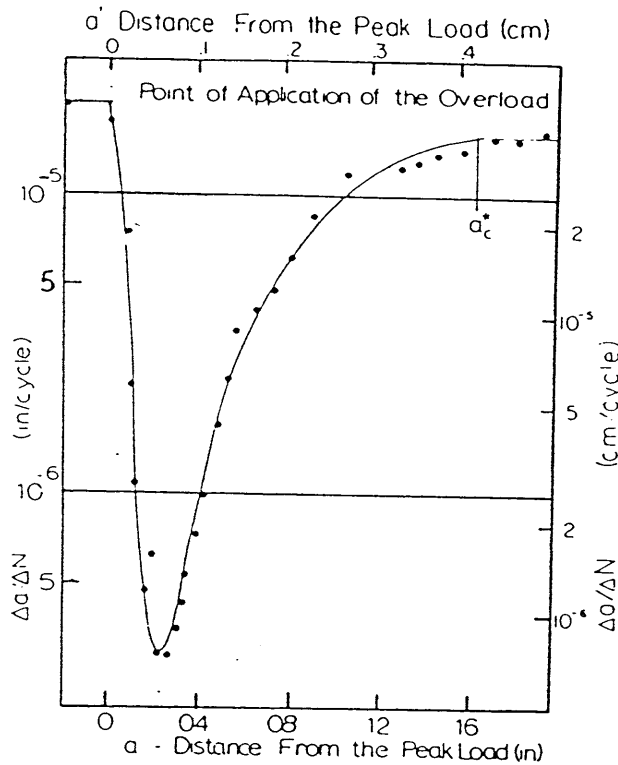


Fig.2.2 Crack Growth Rate Through the Delay Region [2.16]

Where overloads have been found to delay or even arrest crack growth, underloads have demonstrated that they reduce the retardation effect of overloads. In particular, an underload following a tensile overload can cause reduction of the delay period by as much as 40%, Hsu and Lassiter et al [2.17]. Repeated compressive underloads can in some cases completely eliminate the beneficial effects of overloading, Stephens et al [2.18] and Robin et al [2.19]. Figure 2.3, clearly depicts the reduction of post-

overload delay in the presence of compressive underloads on a 316L stainless steel. The observed reduction of the delay in the case of overload/underload sequences has also been noted even when the underloads are positive, Alzos et al [2.20].

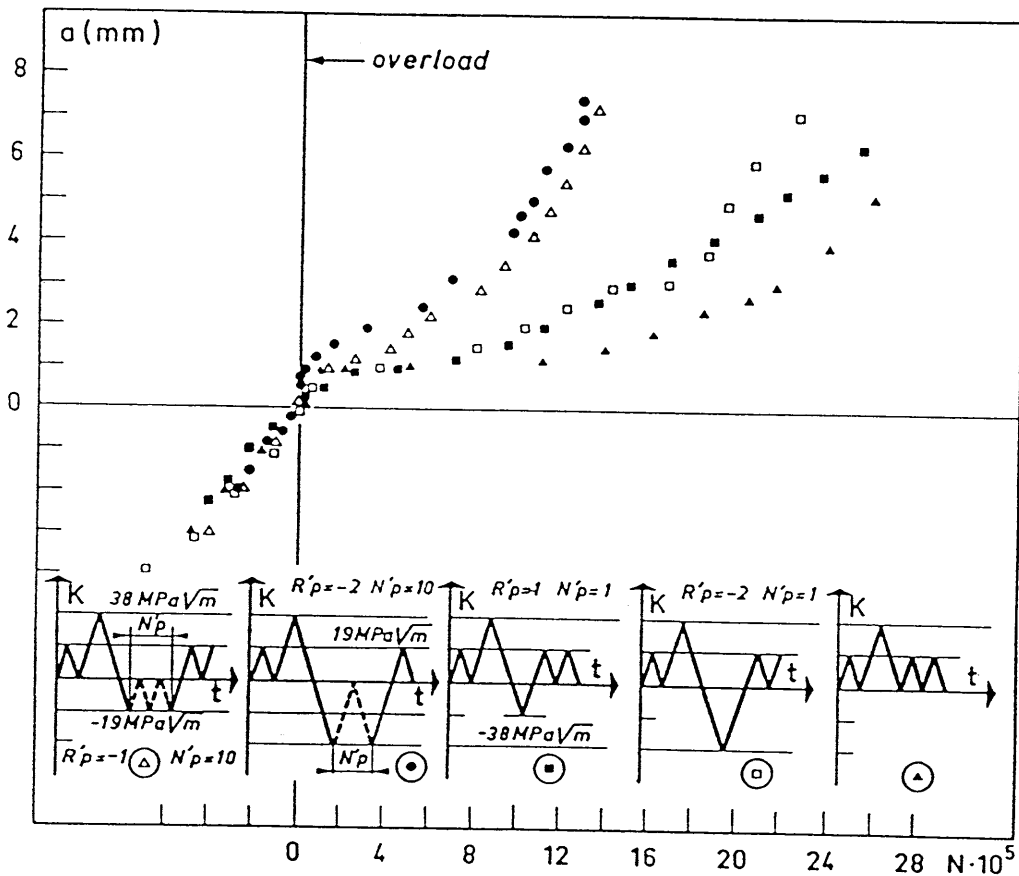


Fig.2.3 Influence of the Number of Compressive Underloads on the Fatigue Crack Growth in 316L [2.19]

The periodic application of overload/underload sequences result to a retardation of growth which is relatively smaller than the delay observed from periodic overloads, Ohrloff et al [2.15], see Fig.2.4.

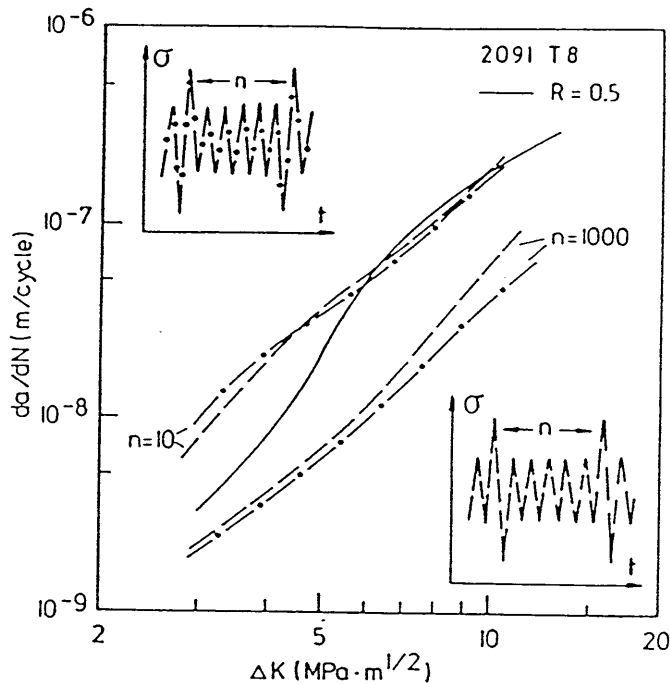


Fig.2.4 Effect of the Periodic Application of Overload/Underload and Underload/Overload Sequences on the Propagation Behaviour of 2019-T8 Aluminium, [2.15]

2.2.2.2 Crack Closure

From the early days of research of the effects of variable amplitude loading upon the fatigue crack growth, plasticity in the vicinity of the crack tip was utilised as a means of interpreting the observed phenomena. The delay observed due to the application of block overloads in the early studies of Hudson and Hardrath [2.10], had been attributed to the presence of residual compressive stresses due to the high amplitude preceding low amplitude stress cycling.

But, this explanation was inadequate as it implies that in the case of a single overload the reduction of the crack growth rate should only occur while the crack tip penetrated through the material affected by the

overloading, (plastic zone). Thus, subsequent growth rates should be identical to the ones observed under constant load amplitude conditions. This explanation fails to offer any reason for the presence of delayed retardation or the observed generalized post overload behaviour.

An adequate explanation of the observed behaviour was only established when the concept of the plasticity induced crack closure, introduced by Elber in the 1970's [2.20, 2.21], became available. Crack closure is related to the residual deformations in the wake of an advancing crack. According to Elber, in the wake of an advancing crack tensile residual deformations are left due, to the plasticity occurring in the vicinity of the crack tip. The plastic zone consists of monotonically stretched material and, as the size of the crack increases the stress intensity factor is higher so the plastic zone size increases. Figure 2.5 depicts the development of a plastic zone envelope around an advancing fatigue crack.

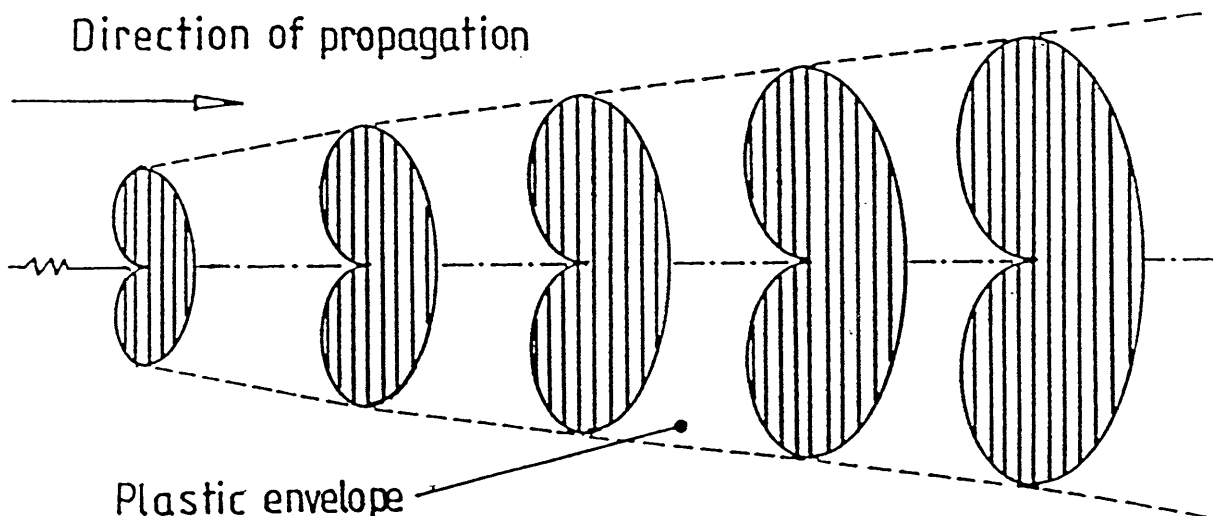


Fig.2.5 Plastic Zone Envelope Around a Fatigue Crack

The residual deformations are the consequence of tensile loading, during the loading half cycle. The monotonically stretched material in the crack flanks is elongated normal to the crack surfaces and has to be accommodated by the surrounding elastically deformed material. There is no problem in containing the plastic deformations as long as the crack is loaded, thus open.

During the unloading half cycle the crack surfaces come into contact before the minimum load has been attained. This is due to the swelling of the crack ligament surfaces caused by the plastic deformation of the crack flank material. As a consequence, the crack closes and it will remain closed for part of the subsequent loading half cycle, see Fig.2.6.

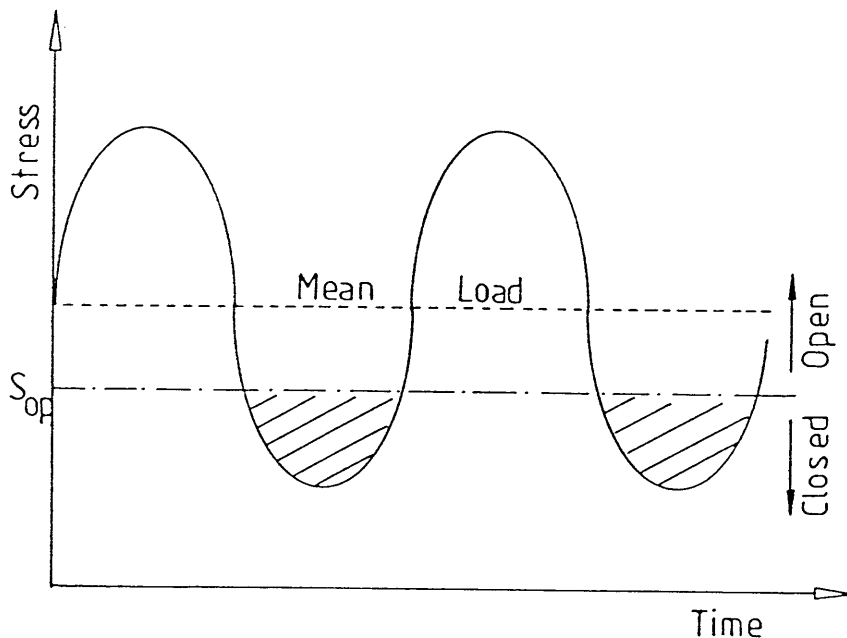


Fig.2.6 Definition of Closure Stress Level

Thus, as the applied stress is reduced the closure of the crack occurs which results into the reduction of the stress intensity range ΔK . The stress level that achieves the opening of the crack is called the opening stress, S_{op} .

Due to the crack closure, an effective stress range S_{eff} may be defined as

$$S_{eff} = S_{max} - S_{op} \quad \dots (2.1)$$

and, an effective stress range ratio is then defined as

$$U = (S_{max} - S_{op}) / (S_{max} - S_{min}) = \Delta S_{eff} / \Delta S \quad \dots (2.2)$$

thus,

$$U = \Delta K_{eff} / \Delta K \quad \dots (2.3)$$

In view of the above results the crack propagation law is modified to the following format

$$da/dN = A (\Delta K_{eff})^n \quad \dots (2.4)$$

Elber established an empirical linear expression for the crack closure parameter U , valid for the 2024-T3 Aluminium alloy and for the range $-0.1 < R < 0.7$,

$$U = 0.5 + 0.4 R \quad \dots (2.5)$$

Other researchers, have carried out studies similar to Elber in order to establish the crack closure parameter U for a variety of materials, Schijve [2.23]. Table 2.2 shows some of the results determined.

2219-T851	$U = 0.68 + 0.91 R$
Ti-6AL-4V	$U = 0.73 + 0.82 R$
7175-T651	$U = 0.40 + 0.40 R$
Mild Steel	$U = 0.69 + 0.45 R$
Mild Steel	$U = 1 / (1.5 - R)$
2024-T3	$U = 0.55 + 0.33 R + 0.12 R^2$
7475-T73	$U = 0.618 + 0.565 R + 0.139 R^2$

Table 2.2 Empirical Crack Closure Parameters [2.23]

In view of the crack closure phenomenon, it becomes possible to explain the delayed retardation phenomenon observed in the post overload fatigue crack growth. The elastic material surrounding the overload induced plastic zone acts like a clamp on this zone, causing the compressive residual stresses. As long as the plastic zone is ahead of the crack tip, this clamping action does not influence the crack opening. But, when the crack penetrates the plastically deformed region the clamping action acts on the new fracture surfaces. This clamping action closes the crack thus, larger external stress is required to open the crack; hence, the crack will propagate at a decreasing rate into this zone, delayed retardation, and under some circumstances it will come to a standstill. In recent publications it has been revealed that crack closure is not only caused by localised plasticity, but a variety of different processes can be involved in the development of closure. McEvily, [2.24], has grouped these processes under six headings as follows:

1. *Plasticity Induced Closure*:- Closure induced due to residual stretch in the wake of the fatigue crack tip.
2. *Roughness Induced Closure*:- This type of closure arises from the mismatch of opposing fracture surface features, asperities.
3. *Crack Filling Closure*:- Due to crack filling with products of corrosion, oxides, fretting debris, liquids and so forth.
4. *Transitional Closure*:- This type of closure occurs on reduction of K or reduction of R and results to crack retardation.
5. *Transformation Induced Closure*:- Due to the presence of phase transformations in the crack path, particularly noticeable in steels.
6. *Grain Boundary Closure*:- Due to the presence of stiffer grain ahead of an advancing crack.

In view of the success of the crack closure phenomena to provide adequate explanations to the questions related to retardation and delay, a large number of research work and effort has been dedicated to the measurement of closure stress levels. Elber's original work did employ the crack mouth opening displacement or CMOD. In recent years other methods have been employed, namely the Back Face Strain (BFS), Ultrasonics, Potential Drop, Interferometry and, Crack Mouth Opening Displacement Gauges, Ray et al [2.25]. Despite, the sophisticated nature of the aforementioned techniques, a variety of results have been obtained which, have been found to depend upon the location of measurement and the data interpretation method, Banerjee [2.26].

The research into the mechanics and effects of crack closure is an on going event as the literature indicates. In recent years, the publication of the proceedings of an international symposium on the mechanics of crack closure, has provided an overview of the developments and the current research interests in the area, [2.27].

2.3 CRACK PROPAGATION MODELLING

The phenomena observed in experimental work can only be used if mathematical models/laws can be derived which can be utilised by the practising engineering designer. As it has already been discussed the problem of fatigue crack growth is of a very complex nature, especially if variable amplitude load histories are encountered.

Predictive methods for variable amplitude fatigue are classified into analytical and empirical models. Many of these models are based upon the Paris-type crack growth rate equation, where others are based upon the concept of crack closure and plasticity in the vicinity of the crack tip.

The Willenborg [2.28] and, the Wheeler [2.29] models, were proposed to explain crack growth delays induced by high loads. Both models employ the plastic zone sizes, however, the concepts are different.

The Wheeler model assumes that reduced crack growth following a tensile overload is expressed by,

$$(da/dN)_{ret} = (c_p) \cdot C \cdot \Delta K^n \quad \dots (2.6)$$

where c_p is retardation parameter bounded by the values of 0.0 (crack arrest) and 1.0 (no retardation). Wheeler, postulated that the retardation parameter (c_p), was a function of the ratio of the current plastic zone to the plastic zone size created by the overload. In contrast to the introduction of the retardation parameter (c_p), Willenborg employed an effective stress to model the crack growth rates through the overload plastic zone.

The reliability of both models has been checked by various authors but, systematic agreement between the models and experimental data was rarely found, Engle et al [2.30]. Both models have utilised a simple assumption about the plastic zone size (Irwin), and they assume that the effects of retardation are restricted into the passage of the crack through the plastic zone and that, maximum retardation occurs immediately after an overload. In real terms this is incorrect as delayed retardation occurs, as it has been discussed earlier. Thus, the Wheeler and Willenborg models are not fully adequate for the prediction of fatigue crack growth, even though the models can deal quite well with some programmed block load sequences, Broek [2.31].

Other more complex models have been used based upon the crack closure concept. The model developed by Newman [2.32], utilises a Dugdale type strip-yielding concept to

include plastic deformation in the wake of an advancing crack. In this model a series of variables have to be experimentally determined, such as the crack opening load and the coefficients of a Paris type crack growth law. These information are utilised in order to determine effective stress ranges and effective stress intensity factors which, in turn are used in a multi variable crack propagation equation. Newman's model has been found to show limited correlation with experimental results but, in simple cases it can provide reasonable predictions, Besuner et al [2.33].

Many of the models that have been developed to date, have been used for the development of fatigue crack growth predictive computer programs. The LOSEQ program developed in West Germany is a good example of a software that combines many of the published theoretical models for prediction of crack growth, Fühling [2.34]. Other programs include the ONERA and PREFFAS developed by Aerospatiale in France for airframe materials, Bleuzen et al [2.35], and, numerous other software packages developed in the United States [2.33]. A good discussion of the advantages and limitations for a number of models and software packages is given by Wanhill et al [2.36], where recommendations are made for further research and development.

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C H A P T E R 3

FATIGUE AND FRACTURE MECHANICS

A GENERAL OVERVIEW

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CHAPTER THREE

FATIGUE AND FRACTURE MECHANICS A GENERAL OVERVIEW

3.0 INTRODUCTION

Fatigue and Fracture are two topics often investigated separately in research, but in reality and in terms of design are linked. Fatigue analysis, treats the problem of crack initiation dominated failures, whereas, Fracture Mechanics is involved with the study of already existing defects. It provides the theoretical background required for analytical and practical tools, in addition to methods for the evaluation of residual strength and life of cracked components.

The overview presented in this chapter discusses briefly the problem of Fatigue with special reference to the fatigue cracking mechanism and life predictions. Fracture is treated independently, with respect to the stress analysis of cracks, elastic and elastic-plastic fracture and fracture criteria.

3.1 FATIGUE FAILURE

The industrial revolution of the 19th century signifies the beginning of a new era for engineering. The introduction of machines changed the working practices and revolutionised all aspects of contemporary life. Demand for the use of metallic materials increased although the understanding of material behaviour under dynamic loading was at a primitive state as compared with current knowledge.

The appearance of the steam engine and the introduction of the railway are significant events of the period. Numerous failures were recorded, which were unexpected and often involved moving parts. Design calculations were believed to have taken into consideration all eventualities. It was though observed that the common denominator in these failures was the fact that all components concerned were subjected to dynamic and/or fluctuating loads. This puzzling failure was attributed to the "tiredness" of the material or "fatigue", whose early dictionary definition [3.1], describes it as : *"The condition of weakness in metal caused by repeated blows or long-continued strain"*.

Surveys concerned with failures of engineering components state that, fatigue accounts for at least 90 percent of all in service failures attributed to mechanical causes, [3.2]. A rigorous definition of fatigue is difficult, but a concise statement has been published by the American Society of Metals (ASM), Boyer [3.3]. Fatigue is defined as, *"The progressive, localised permanent structural change that occurs in materials subjected to fluctuating stresses and strains that may result in cracks or failure after a sufficient number of load fluctuations"*.

3.1.1 Historical Perspective

Early observations of failures in components subjected to cyclic loading at stress levels lower than those necessary to postulate static failure, triggered systematic studies of the *fatigue strength* .

During the 1850's and 1860's Wöhler performed numerous laboratory tests of components subjected to cyclic stresses. The pioneering experimental work of Whöler [3.4], was concerned with railway axles, and the results of his tests were presented in the form of a load versus cycles to failure graph. This type of fatigue data presentation is the well known (S-N) curve, which is still recognised and used today in fatigue design calculations. A typical (S-N) diagram is illustrated in Fig.3.1.

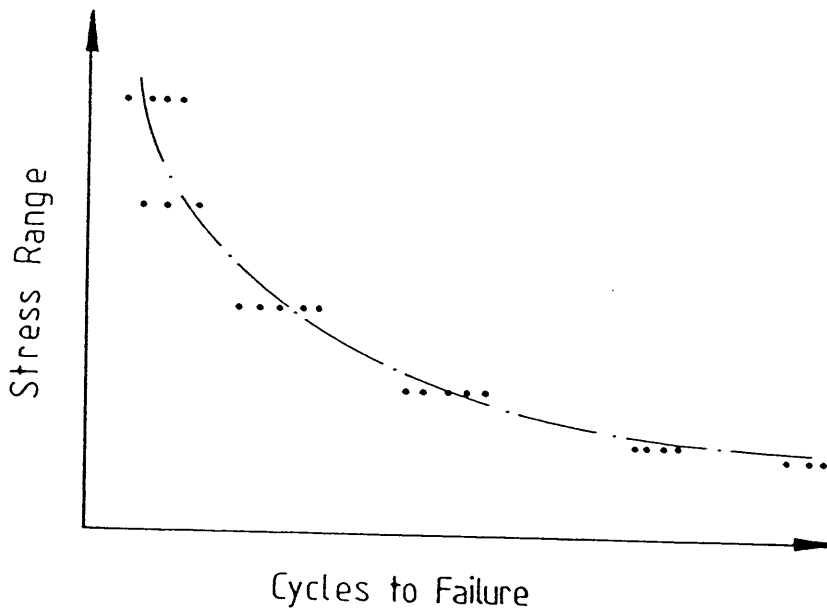


Fig.3.1 A Typical (S-N) or Whöler Diagram.

Whöler's experimental work showed that the fatigue life of a component decreased with increasing loading and further, indicated that below a certain stress amplitude fracture

did not occur thus, infinite life was observed. Hence, the concept of fatigue or endurance limit was introduced to signify the range of finite life.

Early researchers noted that the fatigue life of components was not affected only by the applied stress range but also by the mean cyclic load. The (S-N) curve conveys solely information relating fatigue life to stress amplitude, and the effects of mean load are not accounted for.

Early contributions to this problem were made by Goodman [3.5], who presented diagrams of the relationship between stress amplitude and mean stress that bear his name. A common type of Goodman diagram is shown in Fig.3.2, which is established for a fixed number of cycles or at the fatigue limit, for Ferrous materials.

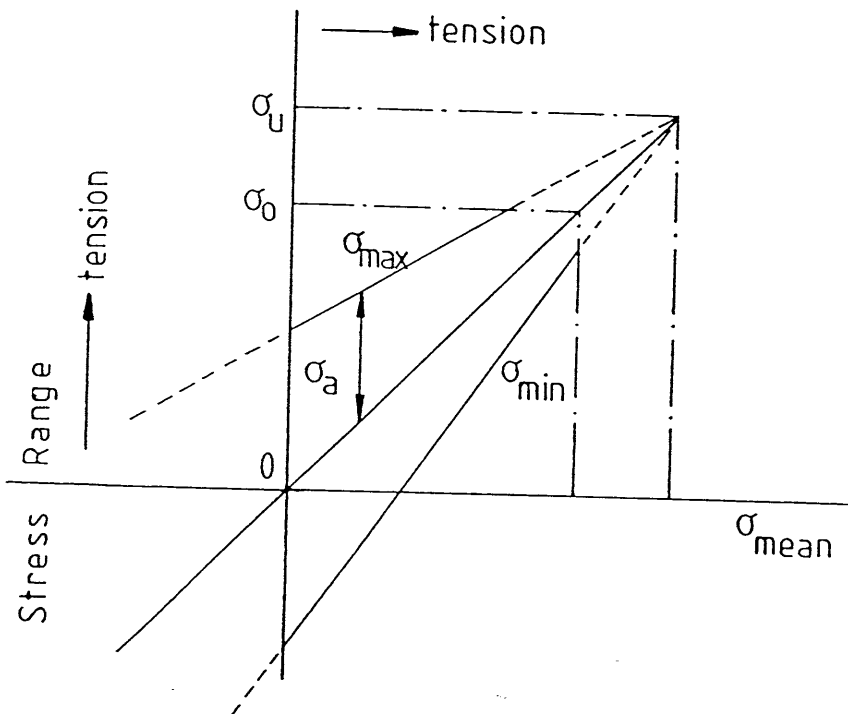


Fig.3.2 Goodman Diagram Relating Stress Amplitude to Mean Stress.

An alternative method of presenting the same information is by means of the Haig-Soderberg diagram [3.6], where the alternating stress is plotted against the mean stress, Fig.3.3

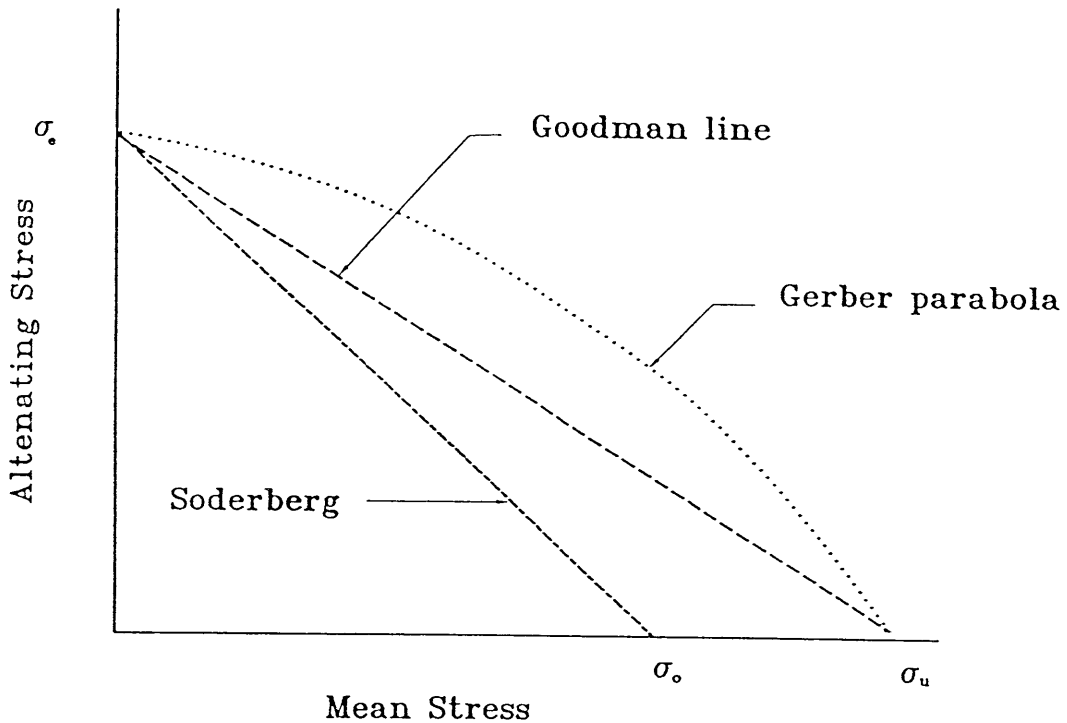


Fig.3.3 Haig-Soderberg Diagram.

A straight line follows the suggestion of Goodman, while a parabolic relationship was proposed by Gerber. Test data generally falls closer to the Goodman line, thus, this formulation is preferred in engineering design.

The relationships proposed by Gerber, Goodman and Soderberg may be expressed by the following mathematical formulations:

$$\text{Gerber} \quad \sigma_a = \sigma_e [1 - (\sigma_m / \sigma_u)^2] \quad \dots (3.1)$$

$$\text{Goodman} \quad \sigma_a = \sigma_e [1 - (\sigma_m / \sigma_u)] \quad \dots (3.2)$$

$$\text{Soderberg} \quad \sigma_a = \sigma_e [1 - (\sigma_m / \sigma_y)] \quad \dots (3.3)$$

Where,

σ_a = Stress amplitude

σ_e = Fatigue limit for fully reversed loading

σ_y = Yield stress

σ_m = Mean stress

σ_u = Ultimate tensile stress

Both Gerber and Goodman base the design upon the ultimate tensile strength σ_u whereas, Soderberg is more conservative and proposes the use of the yield stress σ_y see Fig.3.3.

3.1.2 Fatigue Cracking Mechanism

The mechanism of fatigue failure is a complex process that is influenced by an array of variables. There are factors such as, stress concentrations, residual stresses, corrosion, temperature and the metallurgical structure that may produce beneficial or detrimental effects. Fatigue failure is caused by the simultaneous action of cyclic loading, tensile stress and the presence of plastic strain. The absence of any one of these conditions will neither initiate nor propagate a fatigue crack.

The fatigue process may be subdivided into four distinct stages as proposed by Forsyth [3.7]. These four distinct regimes are illustrated in Fig.3.4.

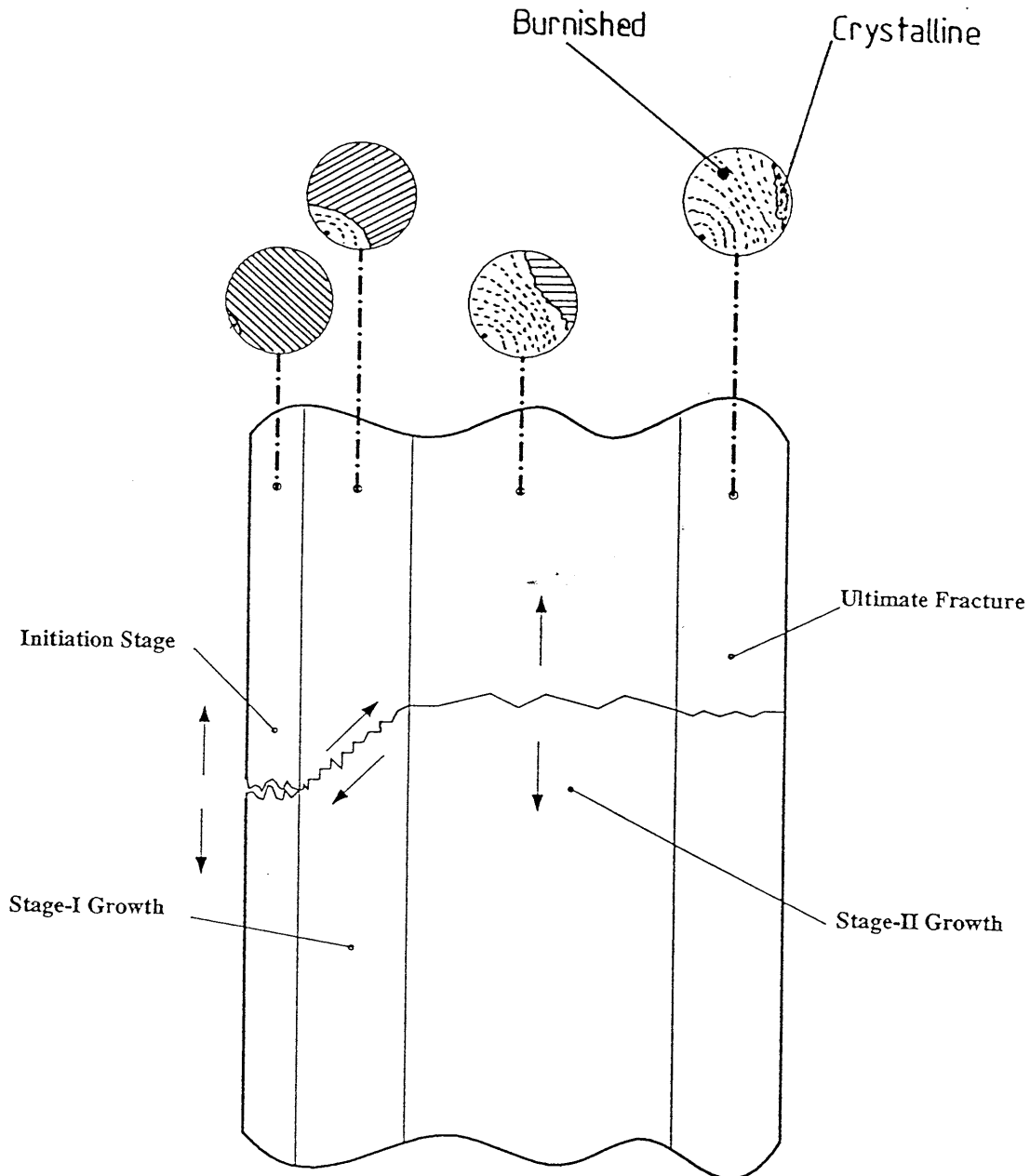


Fig.3.4 Model of Fatigue Failure Process After Forsyth.

Crack initiation occurs at the surface due to the action of cyclic stresses that are well below the static strength of the material. Under certain circumstances cracks may be initiated in the sub-surface, where always an interface is involved, such as the interface of a carburized surface layer with the base metal. Free surfaces are highly suitable crack initiation sites, as surface intersecting grains are less constrained by their surroundings and hence, they exhibit a lower flow stress. The stress levels at free surfaces are higher than in the main bulk of the material. Surface cracking is promoted by the presence of stress concentrations and surface discontinuities due to material inhomogeneity.

An important micro-structural characteristic that appears to be unique to fatigue, is the formation of ridges and grooves, Fig.3.5, on the otherwise polished surface. These formations are called slip-band extrusions and slip-band intrusions, Forsyth et al [3.8]

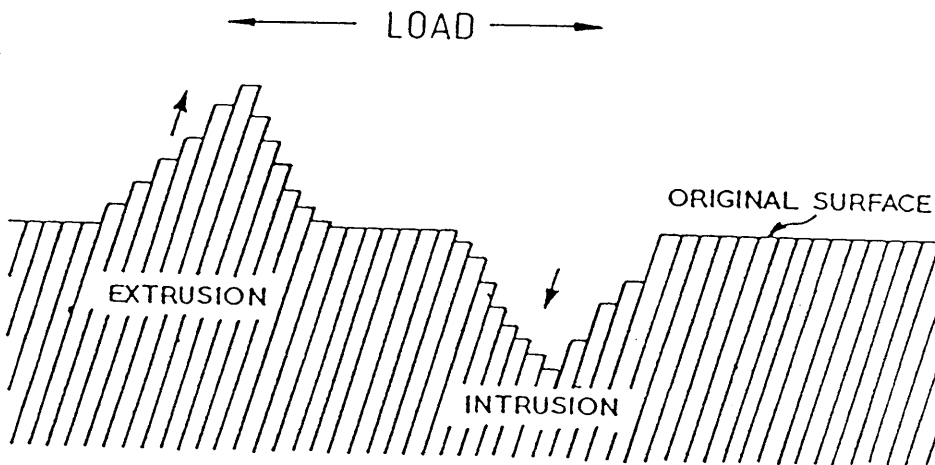


Fig.3.5 Wood's Model for Fatigue Crack Initiation.

The formation of extrusions and intrusions has been interpreted by Wood [3.9], Tetelman et al [3.10], as the systematic buildup of fine slip movements due to cyclic plastic deformation corresponding, slip movements of the order of 10^{-7} cm compared to 10^{-4} - 10^{-5} cm that occur under static loading.

The root of the notch formed by an intrusion is of atomic dimensions and acts as a stress raiser. Such a condition provides the ideal site for the initiation of a fatigue crack.

Once a fatigue crack has been initiated it propagates initially along the slip bands, and this characterises the Stage-I crack growth. The crack propagates along planes of maximum resolved shear stress. These planes are at 45 degrees to the direction of the maximum principal stress. Stage-I crack growth is slow and propagation rates are of the order of Angstroms (10^{-10} m) per cycle. The fracture surface produced at this stage is featureless.

Subsequently the crack changes direction as it enters the Stage-II crack growth phase, after extending for only a few grain diameters under the Stage-I regime. It propagates on planes perpendicular to the direction of the greatest tensile stress. The advancing crack produces a fracture surface characteristic to fatigue, where a pattern of ripples or fatigue fracture striations is present, Fig.3.6

Formation of the characteristic beachmarks on the fracture surface can be successfully explained by the plastic blunting process that takes place during the growth of Stage-II fatigue cracks as described by Laird [3.11] and illustrated in Fig.3.7.

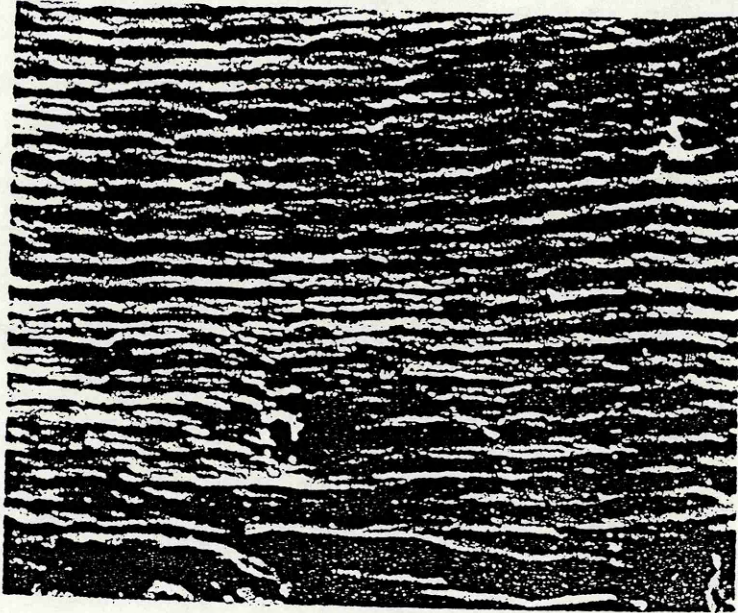


Fig.3.6 Fatigue Striations in β -Annealed Ti-6Al-4V (2000x).

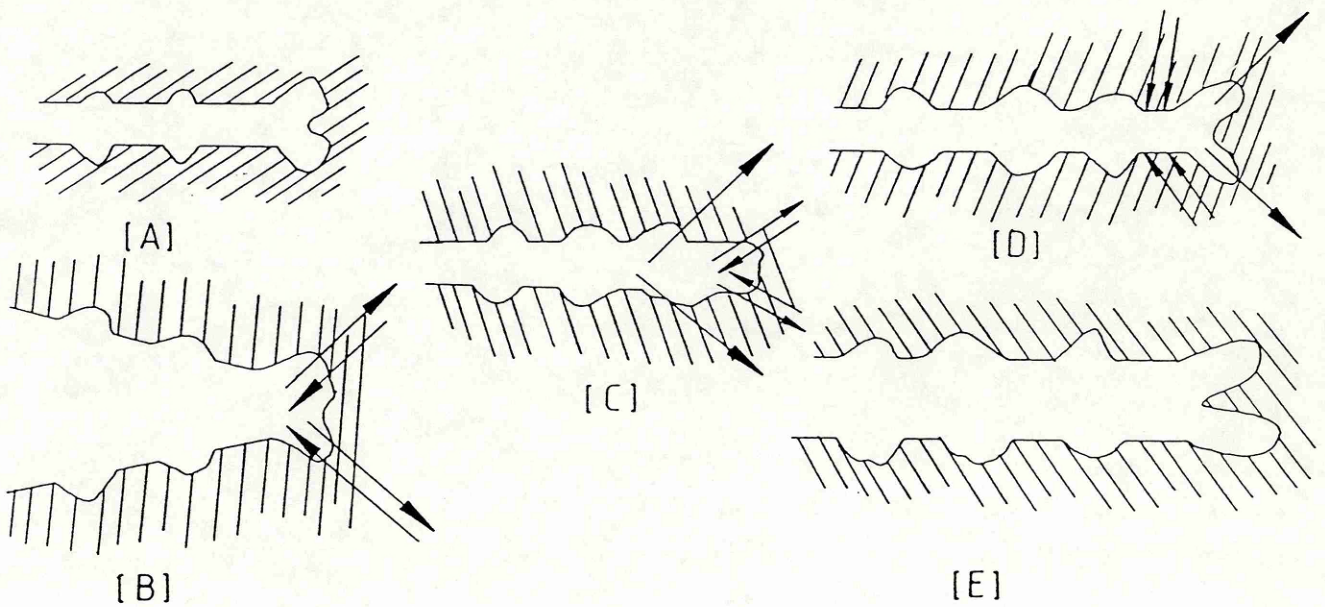


Fig.3.7 Plastic Blunting Process for Growth of Stage-II Fatigue Crack

At the beginning of a loading cycle the crack tip is sharp, Fig.3.7(a). As the load increases the small double notch at the crack tip concentrates the slip along planes at 45 degrees to the plane of the crack, Fig.3.7(b). The crack attains its maximum opening position, when the load is at its maximum Fig.3.7(c), and it grows longer by plastic shearing due to which the crack tip becomes blunt. Upon the unloading part of the cycle the slip direction in the two zones is reversed, Fig.3.7(d). The newly created fracture surfaces by tension, are forced by the surrounding elastic material to fit into the plane of the crack, Fig.3.7(e). This causes folding in the vicinity of the crack tip by means of a buckling collapse and hence, a resharpened crack tip is produced.

Finally, the material experiences catastrophic failure, at the ultimate fracture stage. The final separation of the material occurs during a single tensile loading half-cycle. This fast failure occurs because the cross-sectional area of the remaining un-cracked ligament is unable to sustain the applied load. The fracture surface associated with ultimate fracture is typically crystalline in appearance, whereas the preceding fractured surfaces due to Stage-I and Stage-II growth are burnished by mating surfaces rubbing together under the influence of alternating loads.

3.2 THE CONCEPT OF FATIGUE LIFE

The service life of any component may be defined as the total number of load fluctuations, in kilo-cycles, that the component or structure may sustain prior to either crack initiation or complete separation. It is evident from the discussion concerning the fatigue cracking mechanism, see section 2.3, that a fatigue fracture may be characterized as a crack initiation or a crack propagation dominated failure. Initiation dominated fatigue is treated

by means of local stress/strain history approach and the propagation dominated fatigue is treated with the concepts of Fracture Mechanics.

3.2.1 Initiation Dominated Fatigue

A well defined limit between the crack initiation and crack propagation phases cannot be identified, unless the question of when does a micro-crack become a macro-crack can be answered. An illustration of the relative sizes of micro-structural features and cracks is given in Fig.3.8.

It is often quoted by investigators that, a crack is a defect that can be reliably detected by e.g. a non-destructive testing technique [3.12, 3.13], and it behaves in a predictable manner [3.14]. Hence, defects of greater size are said to be in the propagation phase and those smaller to be in the initiation phase.

Predictions of the crack initiation life are based upon the concepts of notch stress/strain analysis and low cycle fatigue, [3.15-3.17]. One approach is to base life calculations upon the stresses and strains that occur in the vicinity of stress raisers and notches which, are favourable crack initiation sites. Local stresses and strains are very difficult to measure and often it is a formidable task, e.g. measurement of strains in a threaded connection.

However, well validated theoretical models make possible the determination of local elastic-plastic stress/strain histories, utilising numerical, analytic, and experimental techniques such as, Finite and Boundary Element Methods and Photo-Elastic stress analysis.

The assumption upon which this approach is based is that if the stress/strain history at the notch tip and the one applied to a smooth specimen are the same then, the

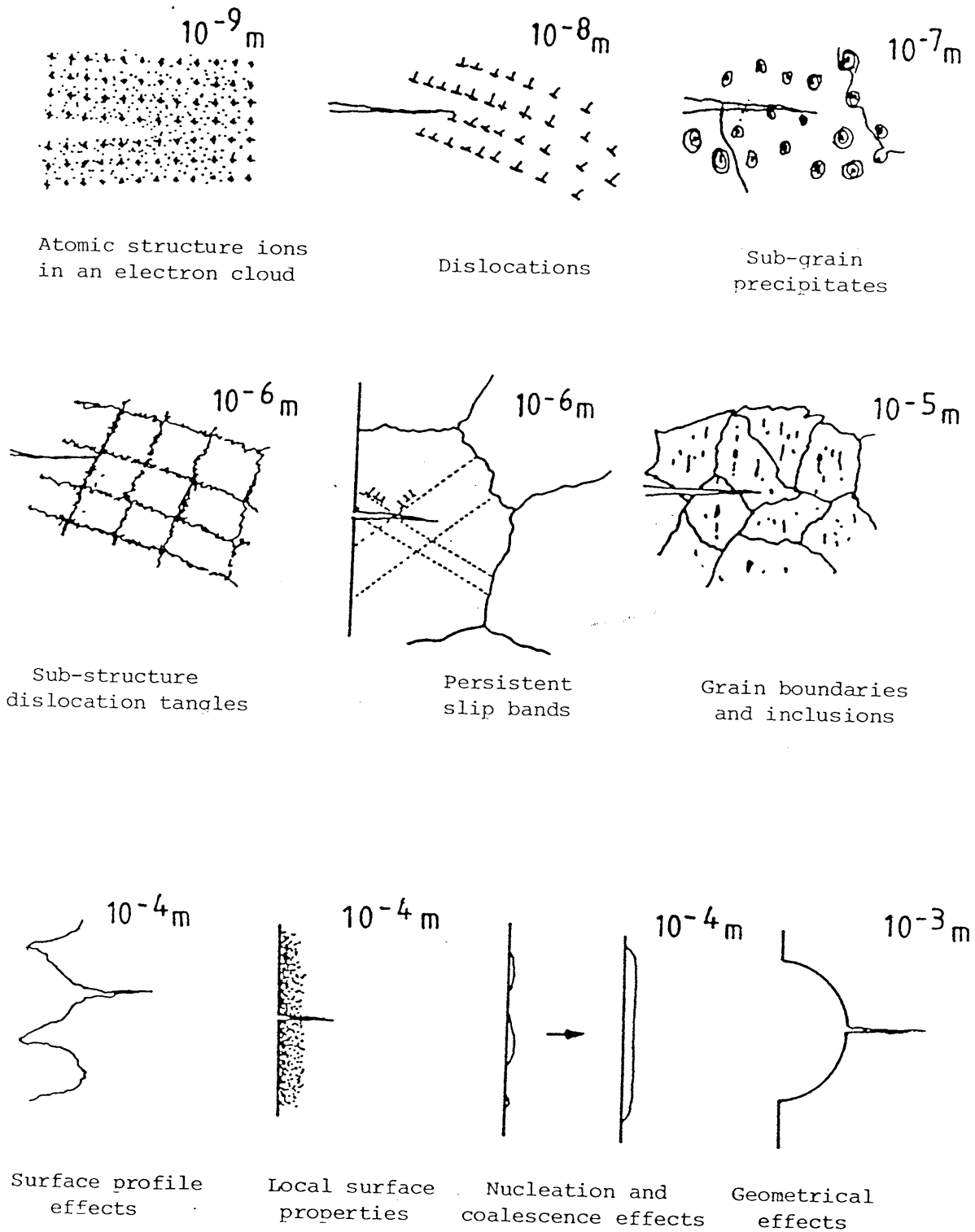


Fig.3.8 Relative scale of cracks and microstructural defects

fatigue life of the specimen will be equal to the fatigue crack initiation life in the notch tip. Specifically, the life of the notched member is expected to be equal to that of the un-notched specimen if failure is defined in both as a common small crack size.

A local strain approach to fatigue life prediction consists of three fundamental steps, Fig.3.9. Firstly, the local notch stress must be determined, Glinka [3.18]. Secondly, a cycle counting procedure has to be employed [3.19, 3.20], and finally the damage must be calculated utilising the concepts of linear cumulative damage approach of Palmgren [3.21] and Miner [3.22]

3.3 LINEAR ELASTIC FRACTURE MECHANICS

3.3.1 Crack Tip Stresses

The damage tolerant design philosophy recognizes the inevitable presence of flaws and crack like defects in engineering materials and structures. This approach to design, Annis et al [3.23], aims at determining safe load and crack length conditions which will preclude catastrophic failure during a conservatively estimated service life. It is a well documented fact that, the presence of crack like defects and other design features such as, keyways, holes etc. causes acute stress gradients and high stress concentrations.

Conventional means of describing the state of stress in the vicinity of a discontinuity is the characterization of the field by a stress concentration factor. When the discontinuity considered is a crack i.e. a slot of infinitesimal width, the stress concentration factor is an inadequate and inappropriate measure of defining the state of stress.

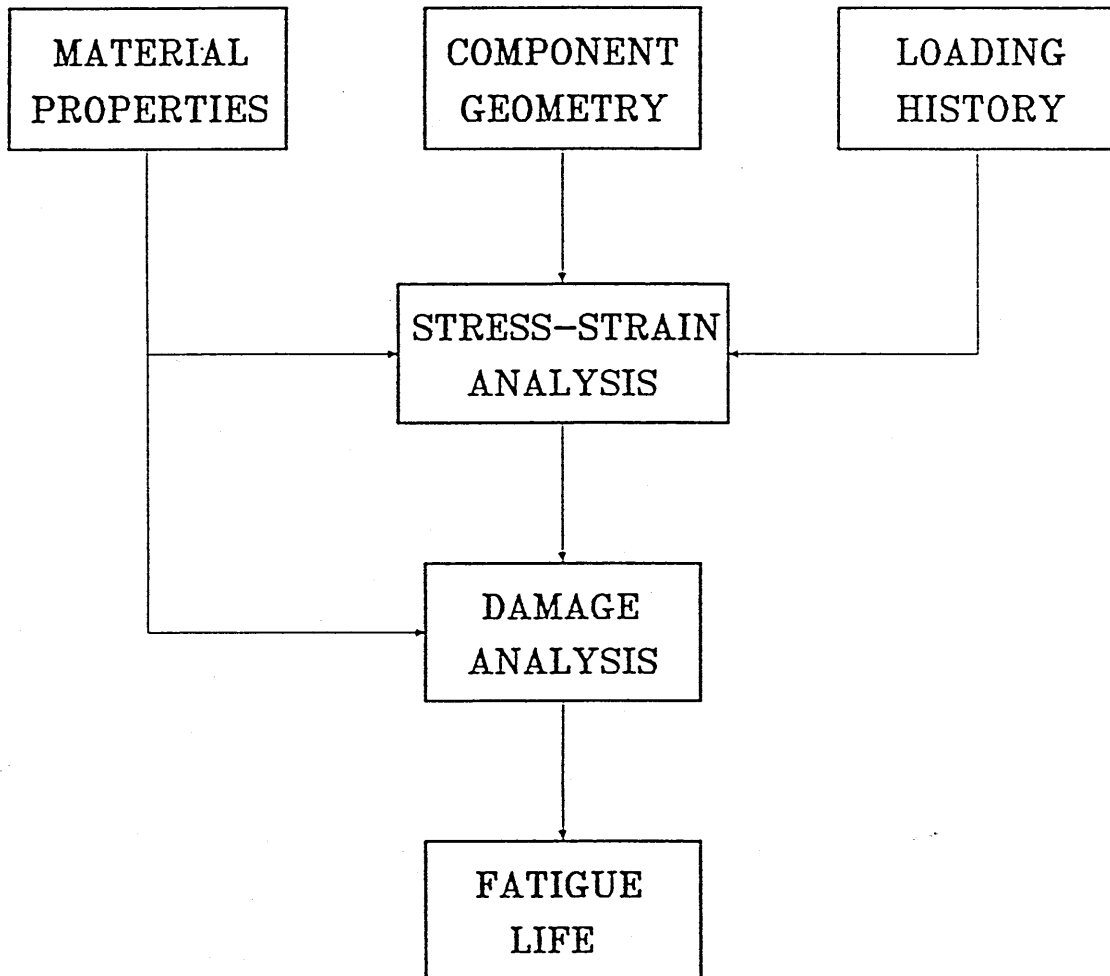


Fig.3.9 Block diagram of Fatigue Life prediction procedures

A proof of the inadequacy of the stress concentration factor as a single parameter for characterization of the stress field in the presence of a crack, is provided by the classic analysis of Inglis presented in 1913 [3.24]. This analysis is based upon the assumption that a crack is equivalent to an elliptical hole with a minor axis approaching the limit of zero.

This modelling assumption leads to the conclusion that the stress concentration at the crack tip is infinity. This paradoxical conclusion implies that a cracked body made out of material with finite mechanical properties (strength), cannot support any load. In reality, cracked bodies have the ability to sustain loads, although their load bearing capacity is reduced. Therefore, the concept of the stress concentration factor collapses in the limiting case of a crack.

The reported findings of Inglis, prompted the detail study of the behaviour of cracked bodies, which has been the impetus for the development of the most modern area of Continuum Mechanics, the Fracture Mechanics. A pioneering exercise was carried out by Griffith [3.25], who studied the mechanical strength of hardened glass [3.26]. The energy balance approach of Griffith postulates that unstable crack propagation takes place when the strain energy released in an incremental crack extension exceeds the energy that is absorbed by creating new crack surfaces. A detailed analysis of the energy balance approach is reviewed and presented in Appendix 1.

Strictly speaking, the work of Griffith may only be applied to materials of brittle nature, where non-linear effects such as plasticity are absent on the continuum scale. This restriction virtually eliminates any consideration of real engineering situations. Practical difficulties in the application of the energy balance

approach in the engineering design environment and limitations presented due to the applicability of the formulation to brittle materials, prompted an early search into an alternative treatment of the cracked body problem. This alternative method of approach was instigated by Irwin in the 1950's, and is known as the stress intensity factor formulation.

Considering the results of theoretical solutions regarding the state of stress in the close proximity of a crack within an elastic continuum, attributed to Westergaard [3.27], and Sneddon [3.28], Irwin [3.29, 3.30] noted that the stresses could be expressed in the following generalized form

$$\sigma_{ij} = (K/\sqrt{2\pi r}) f_{ij}(\theta) \quad \dots (3.4)$$

Where σ_{ij} are the stresses on a material element $dx.dy$ at a distance r from the crack tip and at an angle θ from the plane of the crack, and $f_{ij}(\theta)$ are known functions of θ , refer to Fig.3.10. K represents the stress intensity factor which is a function of the geometry and loading.

The analytic expressions of the stresses in a two dimensional body are represented by the following mathematical statements :

$$\sigma_{xx} = [(\sigma_{\infty}\sqrt{\pi a})/(\sqrt{2\pi r})] \cos(\theta/2) [1 - \sin(\theta/2)\sin(3\theta/2)]$$

$$\sigma_{yy} = [(\sigma_{\infty}\sqrt{\pi a})/(\sqrt{2\pi r})] \cos(\theta/2) [1 + \sin(\theta/2)\sin(3\theta/2)] \dots (3.5)$$

$$\tau_{xy} = [(\sigma_{\infty}\sqrt{\pi a})/(\sqrt{2\pi r})] \cos(\theta/2) [\sin(\theta/2)\cos(3\theta/2)]$$

The above expressions provide the stress field definition in the vicinity of the crack tip. It can be clearly seen that the stresses decay to zero for large values of the distance r , higher order terms have been ignored, and tend to infinity near the crack tip. Figures 3.11, 3.12, 3.13

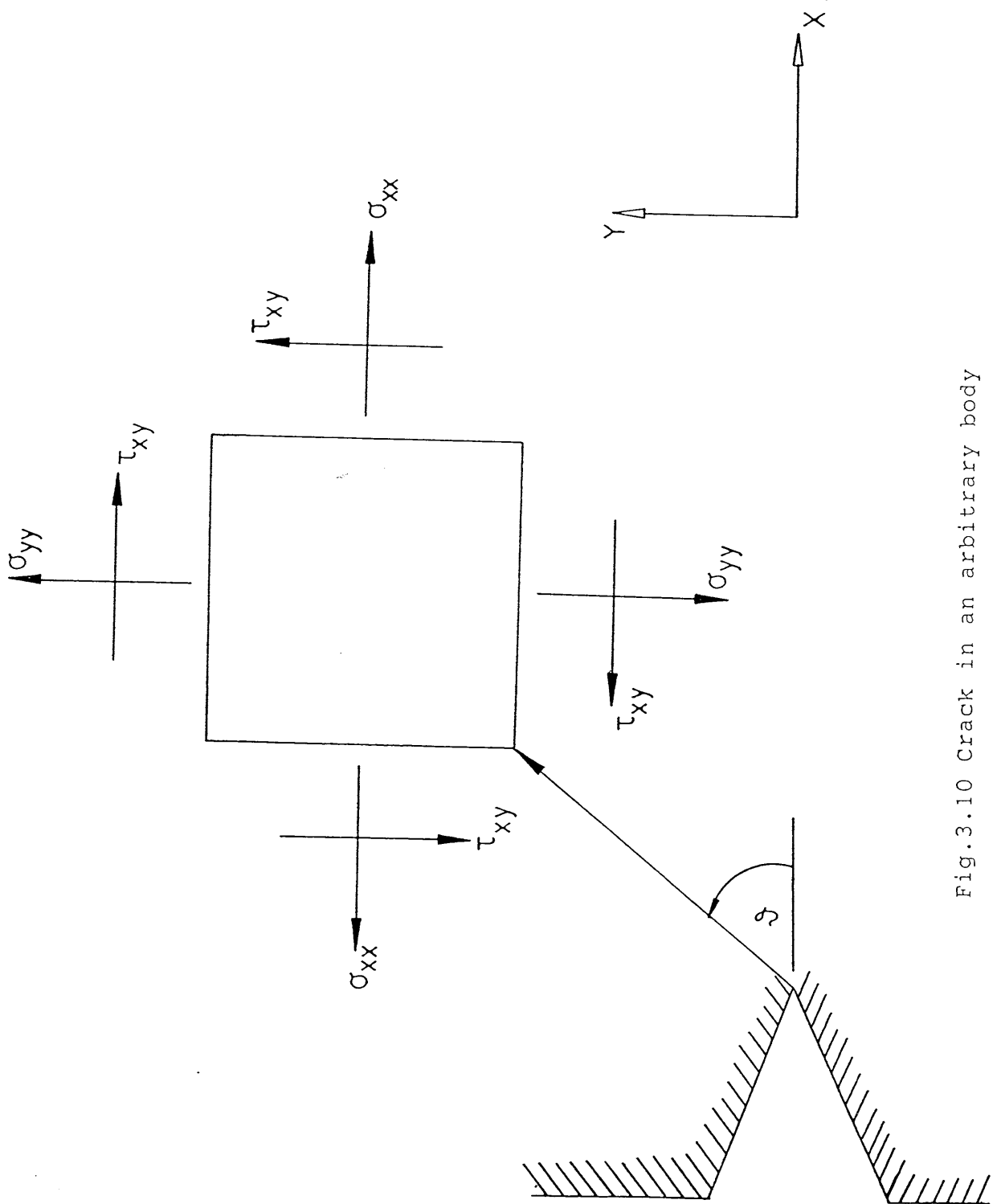


Fig.3.10 Crack in an arbitrary body

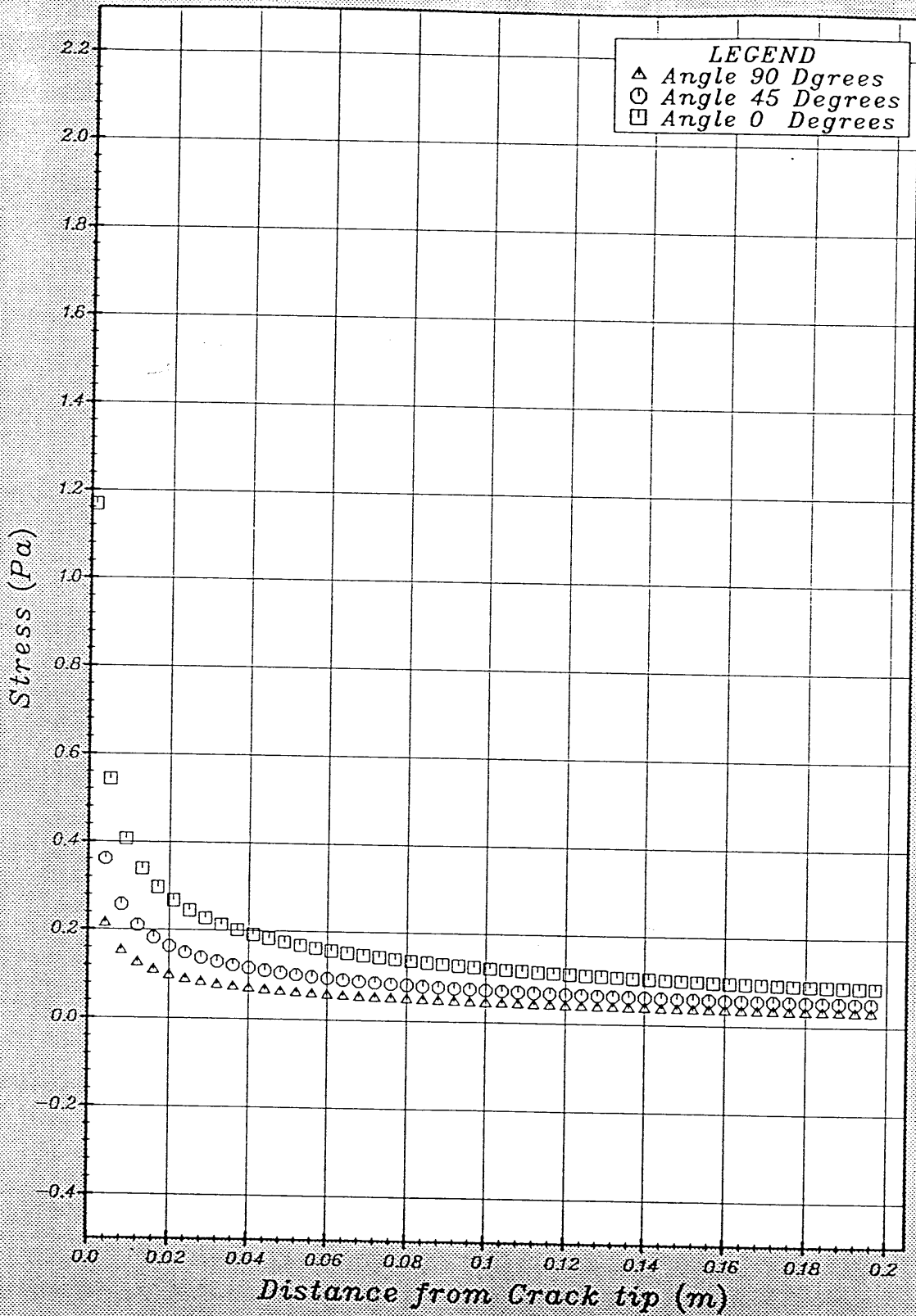


Fig.3.11 σ_{xx} Stress Component
Stress Distribution in the Vicinity
of the Crack Tip, from the Closed Form
Expressions Derived by Irwin [3.29], [3.30]

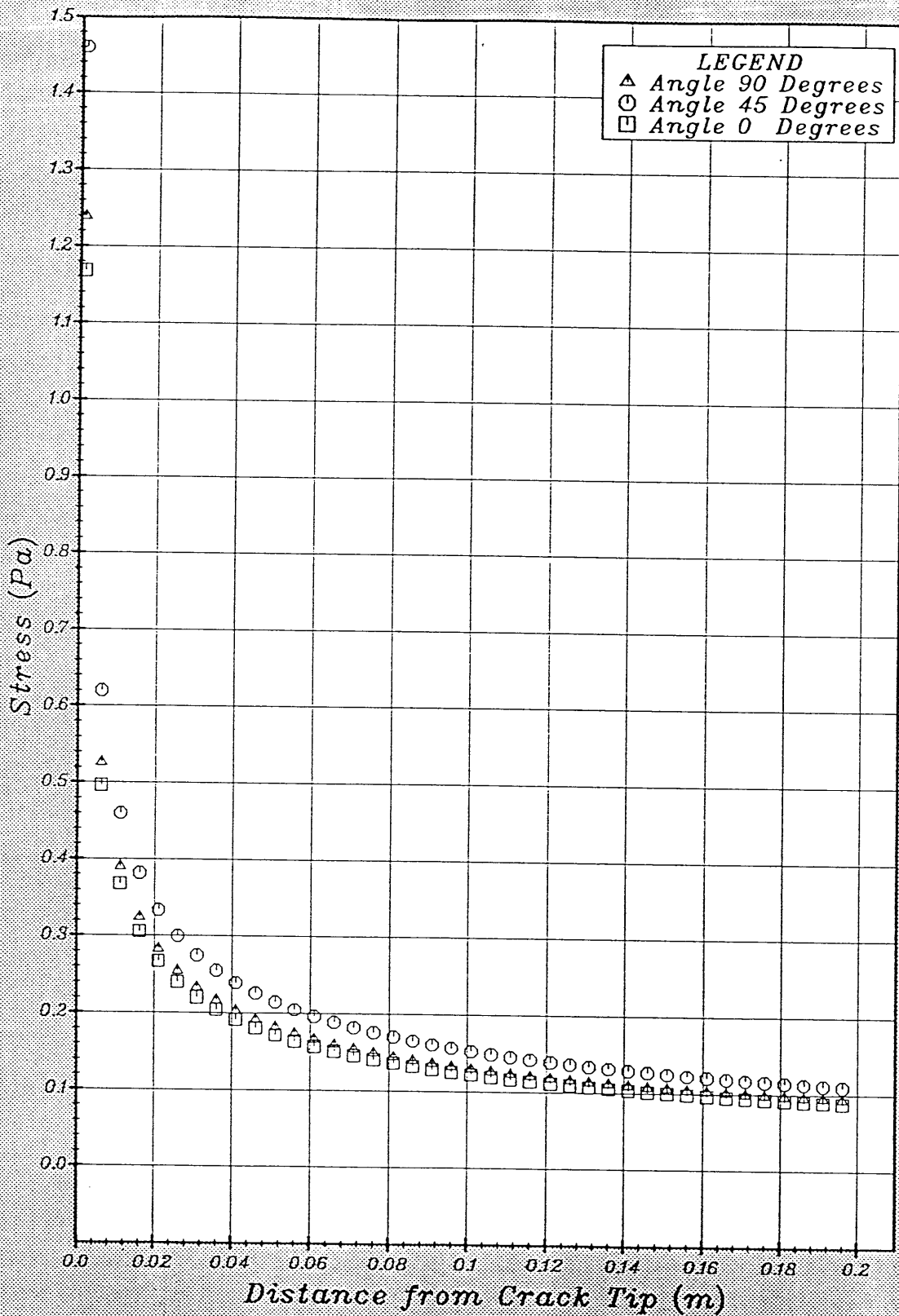


Fig.3.12 σ_{yy} Stress Component
Stress Distribution in the Vicinity
of the Crack Tip, from the Closed Form
Expressions Derived by Irwin [3.29], [3.30]

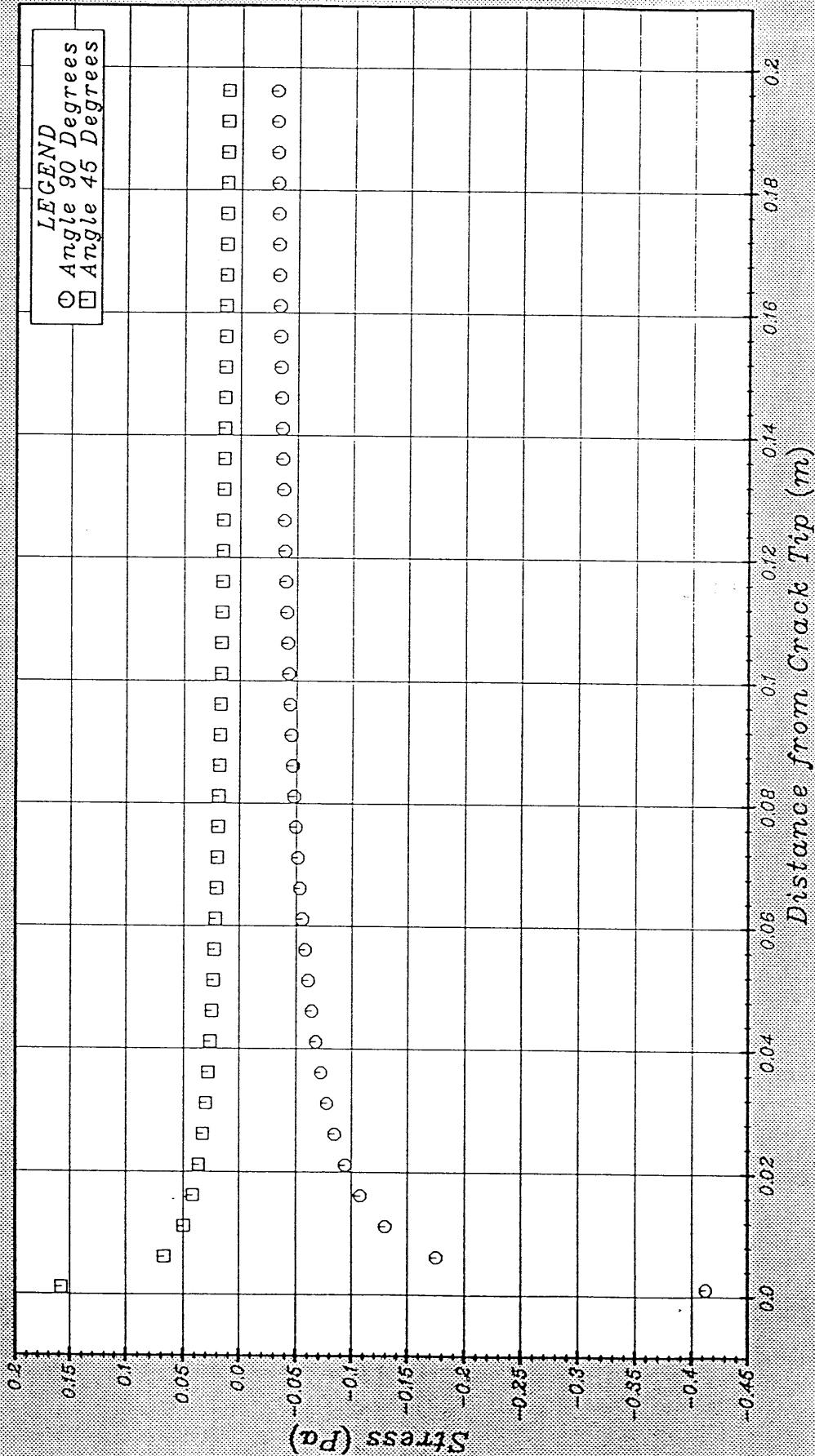


Fig.3.13 τ_{xy} Stress Component
Stress Distribution in the Vicinity
of the Crack Tip, from the Closed Form
Expressions Derived by Irwin [3.29], [3.30]

depict the stress distributions of σ_{xx} , σ_{yy} and τ_{xy} along the plane of the crack ($\vartheta=0^\circ$), and along lines of $\vartheta=45^\circ$ and $\vartheta=90^\circ$ in an anticlockwise direction with respect to the plane of the crack. It is apparent from the illustrations that near the crack tip the stresses tend to infinity, due to the presence of the $(1/\sqrt{r})$ singularity in equations (3.5), which constitutes impossible the precise evaluation of the stresses at the crack tip.

The analysis of the stress field presented above is concerned with a slit-shaped crack with zero crack tip radius. In some applications it is applicable to consider a small radius to be present at the crack tip. A solution may be obtained if the origin of the polar coordinate system is moved an amount equal to the crack tip radius, see Fig.3.14.

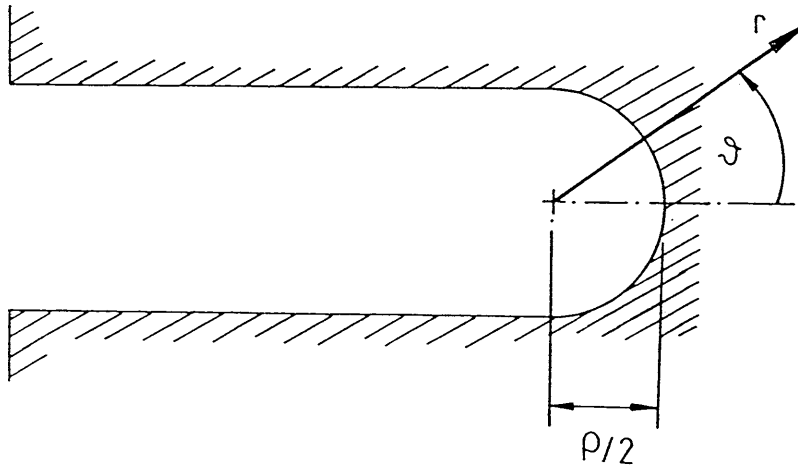


Fig.3.14 Crack With a Finite Tip Radius, $\rho/2$.

The equations that now describe the stresses in the vicinity of the crack in the presence of a finite crack tip radius ($\rho/2$) are :

$$\begin{aligned}\sigma_{xx} &= K' [(1-\sin(\theta/2)\sin(3\theta/2)) - (\rho/2r)\cos(3\theta/2)] \\ \sigma_{yy} &= K' [(1+\sin(\theta/2)\sin(3\theta/2)) - (\rho/2r)\cos(3\theta/2)] \quad \dots(3.6) \\ \tau_{xy} &= K' [(\sin(\theta/2)\sin(3\theta/2)) - (\rho/2r)\cos(3\theta/2)]\end{aligned}$$

$$\text{Where } K' = [K/(\sqrt{2\pi r})]\cos(\theta/2)$$

The above equations eliminate the effects of the $(1/\sqrt{r})$ singularity, it is therefore possible to obtain finite values of the stresses at the crack tip.

3.3.2 The Stress Intensity Factor

Irwin's formulation for the stresses in the vicinity of the crack tip, equations (3.4) & (3.5), contains a constant term,

$$K = \sigma_{\infty} \sqrt{\pi a} \quad \dots(3.7)$$

which defines the magnitude of the elastic stress intensity. This term is called the Stress Intensity Factor (SIF) and is proportional to the applied stress σ_{∞} , and to the square root of the half-crack length a . Thus, the stress intensity factor is dependent upon the geometry of the cracked body and the loading conditions. It can be generally defined as,

$$K = \sigma_{\infty} Q \sqrt{\pi a} \quad \dots(3.8)$$

Where, Q is the configuration factor that accounts for the geometries of the component and crack.

Irwin, has shown, that during an incremental extension of a crack da , the work done by the stress field ahead of the crack when moving through the displacements corresponding

to a crack of length $(a+da)$ is equivalent to the change in the strain energy $G \times da$, where G is the strain energy release rate. This result postulates the equivalence of the energy balance approach with the stress intensity factor formulation. In fact, the Griffith theory requires that a critical value of strain energy release G_c , has to be reached for fracture to take place. Irwin's approach states that a critical stress intensity K_c has to be achieved for fracture.

Thus, it is possible to define the following fracture criterion,

$$\sigma_c Q \sqrt{\pi a} \geq \sigma_c \cdot Q \sqrt{\pi a} = K_c \quad \dots (3.9)$$

Where c denotes critical values.

There exists a relationship between G and K , which in the case of tensile loading takes the form,

$$K = \sqrt{G \cdot E'} \quad \dots (3.10)$$

Where $E' = E$ for plane stress,
 $E' = E / (1 - \nu^2)$ for plane strain.

A crack may be deformed in a variety of ways, but generally three fundamental modes exist, as illustrated in Fig.3.15; Opening Mode (Mode I), Sliding or Shearing Mode (Mode II) and Tearing Mode (Mode III).

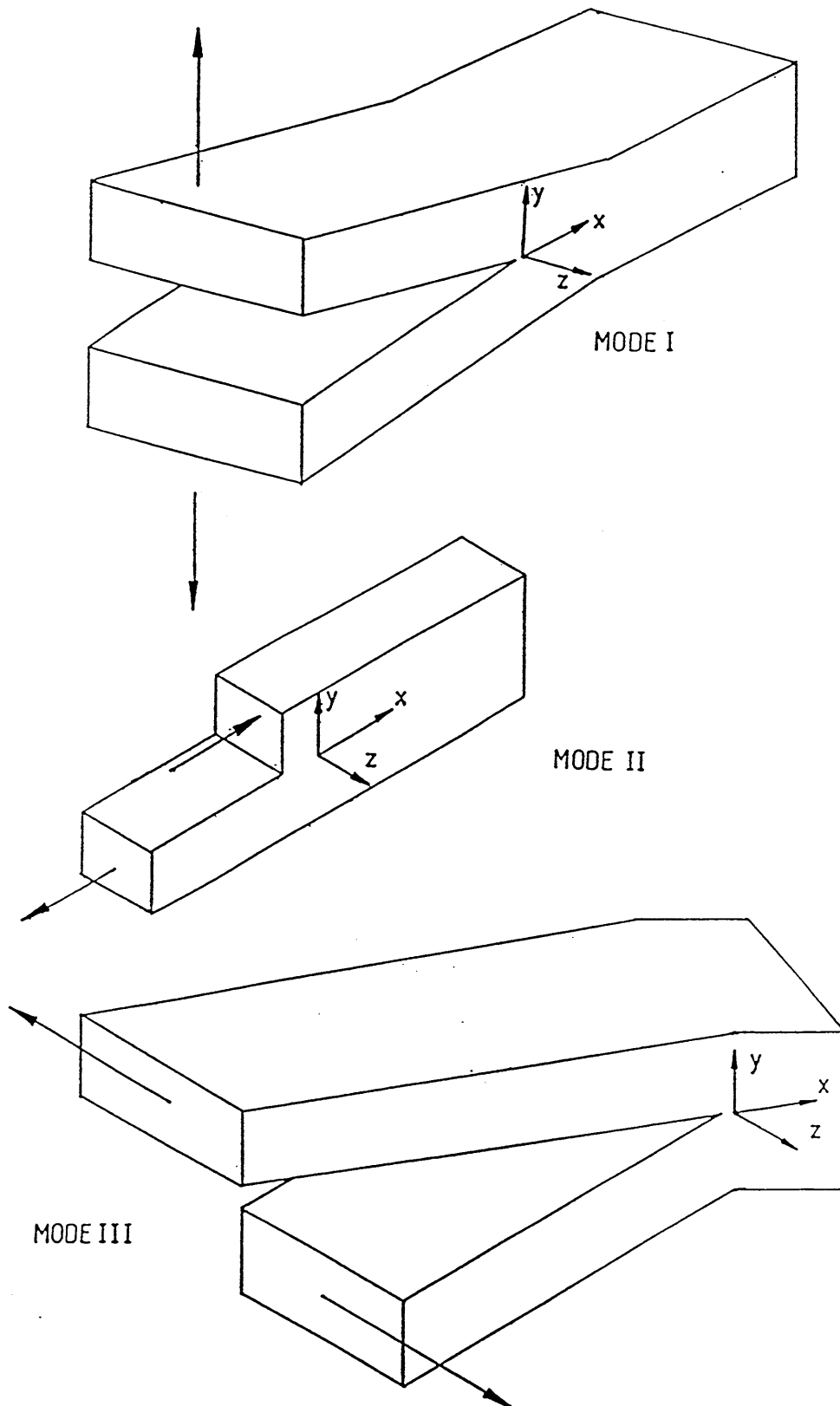


Fig.3.15 Fundamental Modes of Crack Deformation.

The stress intensity factors associated with each of the aforementioned modes, under plane strain conditions are expressed in the following form,

$$\begin{array}{lll} \text{Mode I} & K_I & = \sigma Q\sqrt{\pi a} \\ \text{Mode II} & K_{II} & = \tau_{xy} Q\sqrt{\pi a} \\ \text{Mode III} & K_{III} & = \tau_{xz} Q\sqrt{\pi a} \end{array} \quad \dots (3.12)$$

The state of stress in a cracked body can adequately be described by the stress intensity factor which, provides a powerful tool in the evaluation of strength characteristics of cracked bodies. It allows the determination of critical conditions of load and geometry, that when attained result into catastrophic failure. Geometric configurations bearing crack like defects can be experimentally evaluated so that their load bearing capacity is determined and is taken into account in design procedures and structural integrity assessments.

For each and every material it has been found that a critical characteristic stress intensity factor exists. This characteristic critical stress intensity is regarded as a material property, a constant termed the fracture toughness. This property is a measure of the resistance of the material to brittle fracture. Well defined and justified experimental procedures are utilised for the evaluation of plane strain fracture toughness K_{Ic} and are documented in National and International Standards, [3.31, 3.32].

Some typical values of fracture toughness for some popular materials are given in Table 3.1.

Material	Yield Strength	Fracture Toughness K_{Ic}
Name	MPa	$\text{MPa}\sqrt{\text{m}}$
Maraging Steel	1730	90.00
Ti-6Al-4V	900	57.00
2024-T3 Al alloy	385	26.00
7075-T3 Al alloy	500	24.00

Table 3.1 Typical Values of K_{Ic} [3.33]

Stress intensity factors may also be determined theoretically and appropriate stress intensity factor calibration equations are widely available. Examples of literature containing such mathematical formulations are references [3.34-3.36]

3.4 CRACK TIP PLASTICITY CORRECTIONS

Brittle material behaviour has been the foundation of the Griffith's energy balance approach for the study of fracture phenomena. Further, in the case of the Irwin stress intensity factor formulation the basic assumptions have been the linear elastic and isotropic behaviour of materials. The high stress levels present in the vicinity of a crack like defect arise the question of possible yielding. The linear elasticity approach presents severe limitations on both the accuracy and interpretation of the above concepts when materials capable to undergo plastic deformation are considered. In cases of limited yielding, Linear Elastic Fracture Mechanics may be considered to

adequately treat the problem. The approach adopted in the treatment of small plasticity is that of the use of plasticity corrections, based upon theoretical concepts.

3.4.1 Irwin's Plastic Zone

The elastic stress field solutions have shown that a stress singularity exists at the tip of an elastic crack. In practice such a singularity cannot exist and for metallic materials capable of yielding, this implies that a region of plastic deformation is present around the crack tip.

Irwin [3.37], argued that the presence of plasticity makes a crack behave as if it were longer than its physical size. The displacements are higher and the body stiffness is lower than in the elastic case. He therefore proposed the use of a notional crack length that is effectively the physical length incremented by a finite amount Δa . This incremental extension accounts for the redistribution of stresses due to the presence of limited plasticity.

A first approximation as to the extent of plasticity is obtained by the following expression,

$$\sigma_{yy} = Y = K_I / (\sqrt{2\pi r_y}) \quad \dots (3.13)$$

Where Y = Yield stress

K_I = Opening mode stress intensity factor

r_y = Assumed plastic zone size

Thus, r_y can be shown to be equal to

$$r_y = K_I^2 / (2\pi Y^2) \quad \dots (3.14)$$

If a circular shape for the plastic zone based upon a perfectly plastic material behaviour are assumed then, the stress distribution would be as illustrated in Fig.3.16.

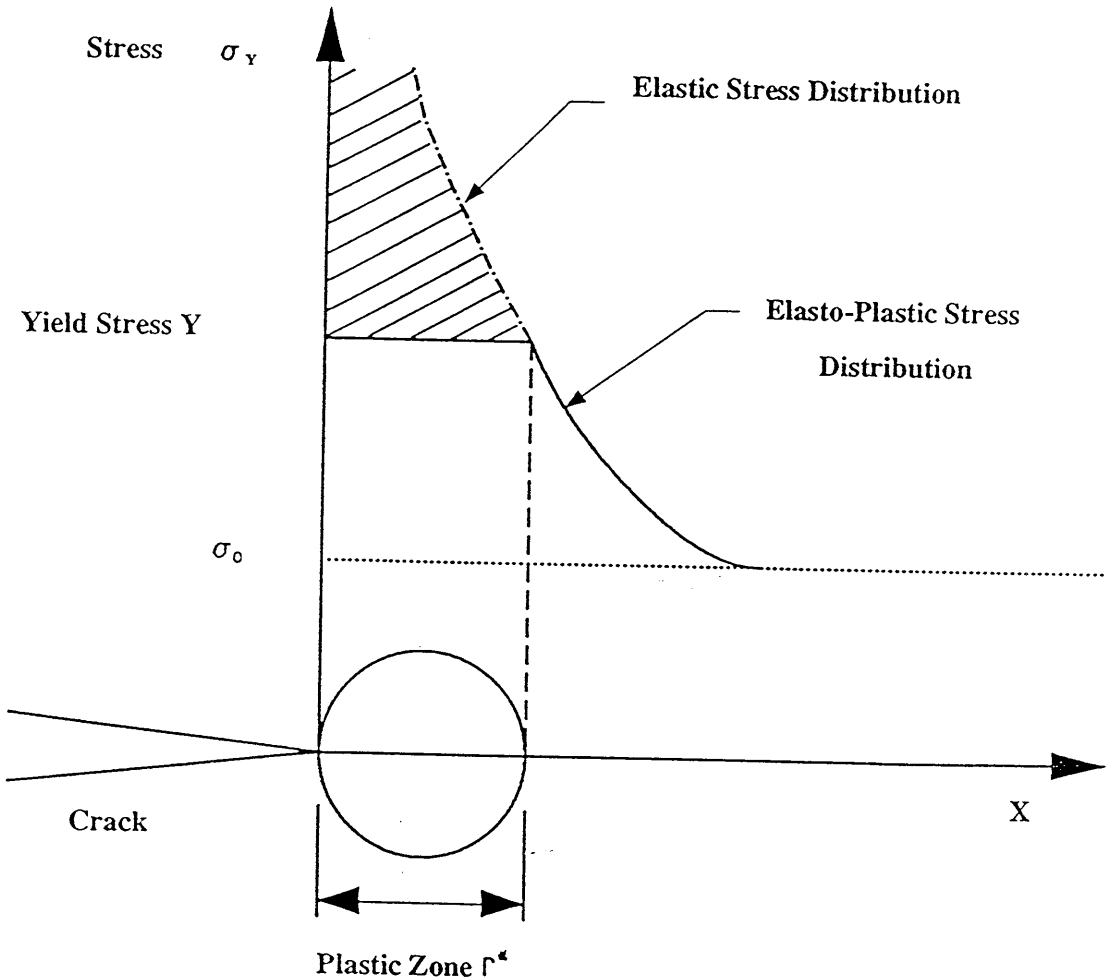


Fig.3.16 First Approximation of Plastic Zone.

The stresses shown in Fig.3.16 (hatched area), are required to be redistributed because their high magnitudes will cause further yielding. In fact, the whole stress curve must be shifted so that equilibrium is maintained. The stress redistribution results in further plastic deformation ahead of the crack tip of a total depth r_p , as illustrated in Fig.3.17.

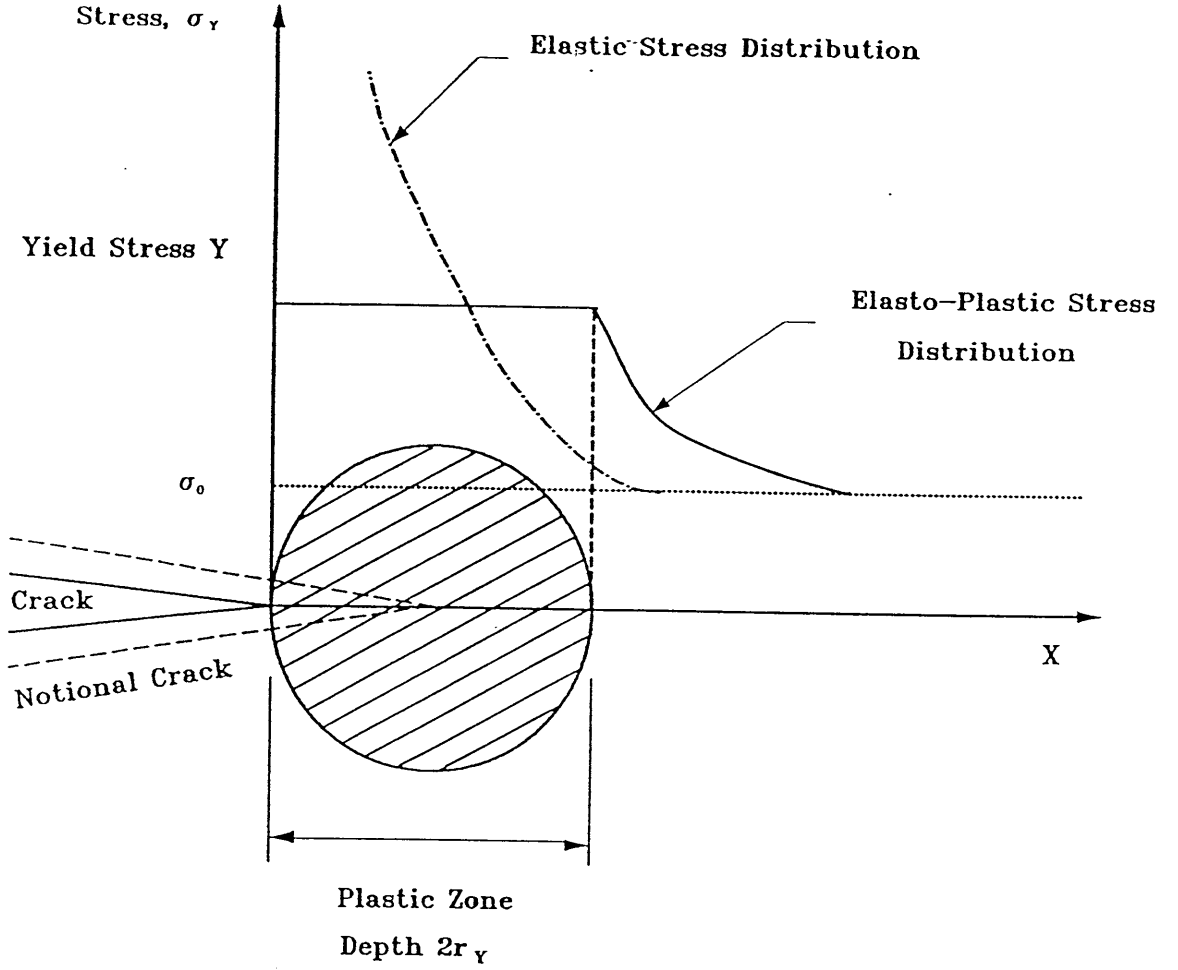


Fig.3.17 Irwin's Notional Crack and Plastic Zone.

Considering the equations (3.5) for the stresses in the vicinity of the crack tip region then for $\vartheta=0^\circ$ i.e. along an axis in the plane of the crack, equilibrium stipulates

$$\int_0^{r_y} \{K_I / (\sqrt{2\pi r})\} dr = r_p Y \quad \dots (3.15)$$

After integration the above expression becomes,

$$(2/\pi)^{1/2} K_I \sqrt{r_y} = r_p Y \quad \dots (3.16)$$

By substitution of equation (3.14) into (3.16) the new estimate of the plastic zone radius is,

$$r_p = K_I^2 / (\pi Y^2) \quad \dots (3.17)$$

which when compared with the first approximation it can be seen that the plastic zone depth in accordance to Irwin is double the first approximation estimate. Assuming the plastic zone shape as being circular then the radius is,

$$\lambda = r_p/2 = K_I^2 / (2\pi Y^2) \quad \dots (3.18)$$

In view of the above result Irwin proposed a plastic zone correction based upon a notional crack which extends to the centre of the plastic zone. Hence, the crack behaves as if it were of length $(a+\lambda)$. The notional crack stress intensity factor is given by:

$$K^* = Q\sigma\sqrt{\pi(a+\lambda)} \quad \dots (3.19)$$

3.4.2 Dugdale's Plastic Zone Model

An alternative approach to that of Irwin was proposed by Dugdale [3.38]. This alternative proposal is based on the assumption that the crack extends right through the plastic zone and the plastic deformation is concentrated in the localised strip in front of the crack, see Fig.3.18.

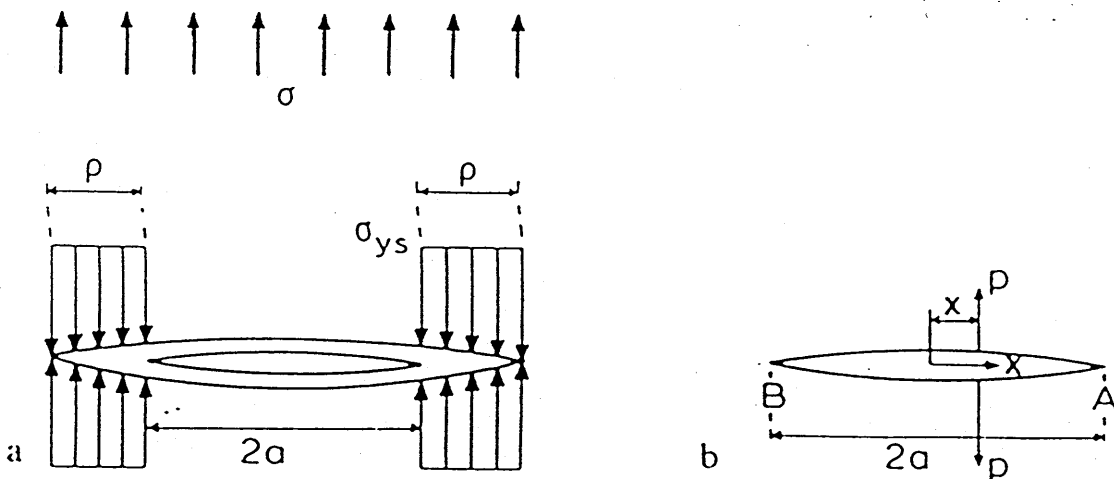


Fig.3.18 Dugdale Approach.

The Dugdale crack is of length $2(a+\rho)$, where ρ is the extent of the plastic zone. The length of the crack within the plastic zone experiences a negative pressure of magnitude equal to the yield strength of the material. The extent of the plastic zone, ρ , is fixed by the requirement that the stress singularity at the crack tip should be eliminated thus, K should be zero. Consequently, the stress intensity factor due to the remotely applied stress, K_σ , has to be compensated by K_p , the stress intensity factor of the wedge forces.

$$K_\sigma = -K_p \quad \dots (3.20)$$

Where the negative sign indicates that the applied forces tend to cause crack closure.

It has been calculated by Dugdale that the K_p is given by,

$$K_p = 2Y [(a+\rho)/\pi] \cdot \arccos(a/a+\rho) \quad \dots (3.21)$$

and K_σ by,

$$K_\sigma = \sigma \sqrt{\pi(a+\rho)} \quad \dots (3.22)$$

Thus, can be determined from

$$\rho = (\pi/8) \cdot (K/Y)^2 \quad \dots (3.23a)$$

$$a/(a+\rho) = \cos(\pi \sigma / 2Y) \quad \dots (3.23b)$$

The values determined by equation (3.23a) are virtually identical to the values obtained by using the Irwin approximation. For high values of the ratio (σ/Y) equation (3.23b) has to be used and the deviations from the Irwin approximation are larger.

Correcting for plasticity is not necessary when dealing with Linear Elastic Fracture Mechanics. In cases of larger plasticity with respect to the crack length the

application of such corrections is uncertain, due to the limited validity of the expressions for K which are based upon elastic solutions.

3.4.3 Plastic Zone Shape

In the previously presented approaches the shape of the plastic zone has been assumed, e.g. circular for the Irwin approximation. However, a more accurate picture of the plastic zone shapes may be obtained by considering proper yield criteria such as Tresca's or Von Mises's.

The crack tip state of stress in terms of principal stresses is given by:.

$$\sigma_1 = [K_I/\sqrt{2\pi a}] \cdot \cos(\vartheta/2) \cdot [1 + \sin(\vartheta/2)]$$

$$\sigma_2 = [K_I/\sqrt{2\pi a}] \cdot \cos(\vartheta/2) \cdot [1 - \sin(\vartheta/2)]$$

...(3.24)

$$\sigma_3 = 2\nu[K_I/\sqrt{2\pi a}] \cdot \cos(\vartheta/2) \quad \text{for plane strain}$$

$$\sigma_3 = 0 \quad \text{for plane stress}$$

In accordance to the Von Mises yield criterion, the plastic zone may be reduced to a function of :

$$r_p(\vartheta) = (K_I/2Y\sqrt{\pi})^2 \cdot [(2/3)\sin^2\vartheta + (1-2\nu)^2 \cdot (1+\cos\vartheta)]$$

for plane strain, and

...(3.25)

$$r_p(\vartheta) = (K_I/2Y\sqrt{\pi})^2 \cdot [1 + (2/3)\sin^2\vartheta + \cos\vartheta]$$

for plane stress.

Adopting Tresca's criterion of yielding,

$$r_p(\vartheta) = (K_1/\sqrt{2}Y\sqrt{\pi})^2 \cdot \{ \cos(\vartheta/2) + [1 + \sin(\vartheta/2)] \}$$

for plane stress and, the larger of

$$r_p(\vartheta) = (K_1/\sqrt{2}Y\sqrt{\pi})^2 \cdot \cos^2(\vartheta/2) \cdot [1 - 2\sqrt{1 + \sin(\vartheta/2)}]^2$$

and

... (3.26)

$$r_p(\vartheta) = (K_1/\sqrt{2}Y\sqrt{\pi})^2 \cdot \cos^2(\vartheta/2)$$

for plane strain.

The extent of the plastic zone shapes according to Tresca are slightly larger than those obtained according to Von Mises, as can be seen in Fig.3.19.

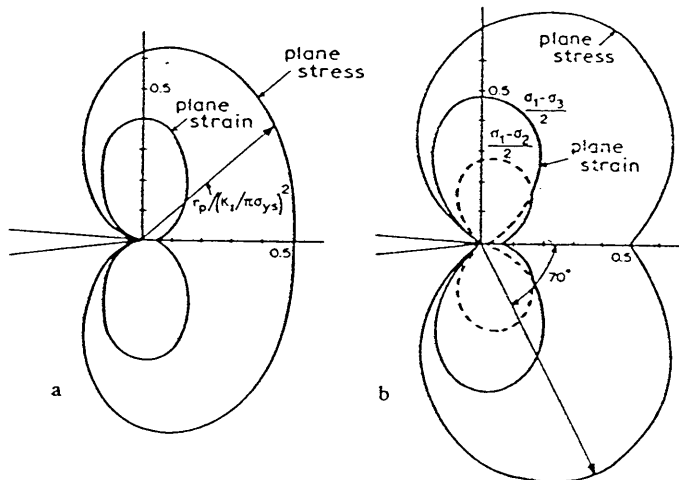


Fig.3.19 Plastic Zone Shapes According to
a. Von Mises; b. Tresca yield criteria.

In real cracked bodies of sufficient thickness predominant plane strain conditions dominate the interior due to substantial constraint, and plane stress conditions prevail at the stress-free surfaces. As a result the shape of the plastic zone changes from that appropriate to plane strain in the main bulk, to that of plane stress at the surface, in the manner shown in Fig.3.20.

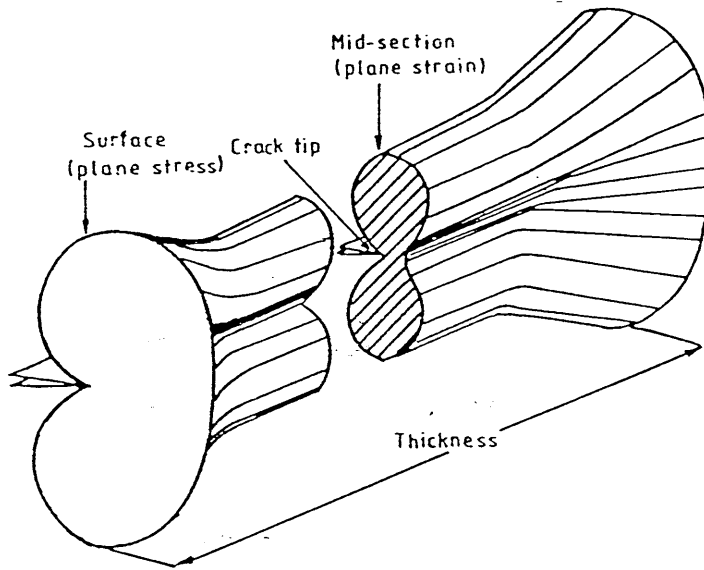


Fig.3.20 Variation of Crack Tip Plastic Zone Through the Thickness of a Plate.

3.5 ELASTIC PLASTIC FRACTURE MECHANICS

In general, the majority of engineering materials are able to undergo significant plastic deformation prior to fracture. Specifically, metallic materials by enlarge exhibit an amount of ductility prior to failure which suggests that, the presence of localised high stresses causes yielding or plastic deformation. The existence of crack like defects within a stressed body constitutes severe stress levels, hence yielding of the material surrounding the crack tip is inevitable.

The presence of plasticity causes redistribution of the stresses as it has already been discussed. Linear Elastic Fracture Mechanics approaches can handle small scale yielding, but the stress intensity factor approach is based upon linear elasticity and it would be incorrect to assume that is applicable to cases of non-linear behaviour.

Gross plasticity associated with fibrous tearing disables LEFM and, the results have no real meaning. In such cases, more general fracture criteria capable to handle plasticity related effects of such predominant nature have to be adopted; this is the concern of Elastic Plastic Fracture Mechanics, (EPFM).

The approach into general yielding fracture mechanics has been to determine criteria that in the presence of plastic deformation are able to predict the conditions which must be achieved in order that fracture takes place. Widely accepted and investigated criteria do exist and are the Crack Opening Displacement, COD, and the J-Integral.

3.5.1 Crack Opening Displacement Fracture Criterion

In the presence of extensive yielding the applied stress cannot be used to calculate either local stresses or strains at the crack tip. Wells [3.39, 3.40], focused his attention to the amount of crack opening prior to crack extension, as a characteristic parameter of the crack tip region. He argued that, at the crack tip the stresses always exceed the yield strength of the material and plastic deformation exists. Hence, extension of the crack will occur if the plastic strains and stresses exceed certain respective limits.

As stresses cannot exceed a critical value in the vicinity of the crack tip, the plastic strains in the region

control the fracture process. It would therefore be feasible to assess the plastic strain levels at the tip by measuring the displacement of the flanks especially, very close to the tip. At the threshold of crack extension it has been experimentally verified that the COD attains a value δ_t that is a constant for a particular material and loading. Thus, a critical value of the COD exists, δ_{crit} , that can be used as a criterion to predict fracture. Burdekin and Stone [3.41], improved the basis for the COD concept.

Using the Dugdale strip yield model they developed a mathematical expression for COD,

$$\delta_t = (8Ya/\pi E) \cdot \ln \sec(\pi\sigma/2Y) \quad \dots(3.27)$$

In the case of LEFM this expression can be simplified and,

$$\delta_t = (K_I^2/YE) = \pi\sigma^2 a/EY \quad \dots(3.28)$$

The Burdekin and Stone formulation has one main disadvantage as equation (3.27) represents the COD for an infinite plate and, it is not possible to derive similar expressions for practical geometries. Thus, the applicability of the equation is very limited. In addition limitations are also due to the fact that the analysis is based upon the Dugdale model that assumes plane stress conditions and an elastic perfectly plastic material behaviour.

3.5.2 The J-Integral

The J-integral is a product of the energy balance approach into the study of cracked bodies. Its introduction is attributed to J.R.Rice [3.42], who showed that a path independent line integral related to the energy in the

vicinity of a crack can be used to analyse two-dimensional crack problems in the presence of plastic deformation.

The original J-integral definition refers to a homogeneous linear or non-linear material free of body forces and subjected to a two-dimensional deformation field (plane strain, generalised plane stress, antiplane strain), so that all stresses σ_{ij} depend only upon a two-dimensional Cartesian coordinate system.

It is defined in two dimensions by the following mathematical expression,

$$J = \oint_{\Gamma} [W \cdot dy - T \cdot (\partial u / \partial x)] dS \quad \dots (3.29)$$

As illustrated in Fig.3.21, Γ is any contour surrounding the crack tip.

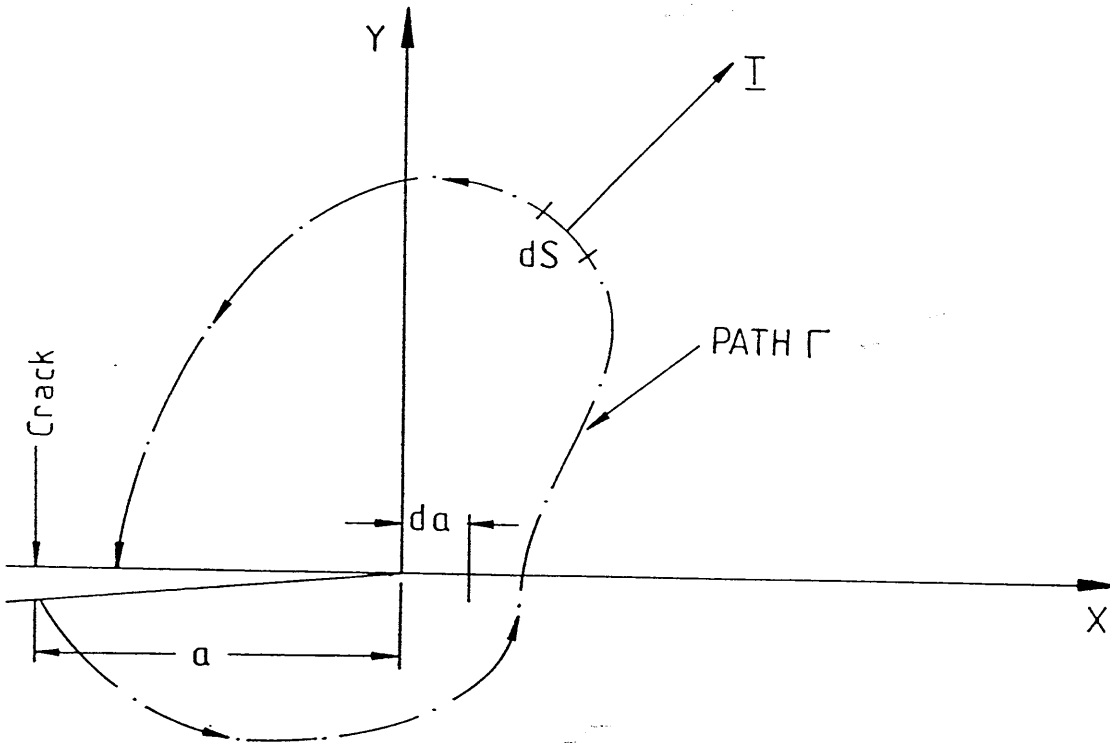


Fig.3.21 Notation and Parameters Used for J-Integral.

W is the strain energy density defined as,

$$W = W(\epsilon) = \int_{\epsilon} \sigma_{ij} \cdot d\epsilon_{ij} \quad \dots (3.30)$$

Where: σ_{ij} , ϵ_{ij} are the stress and strain tensors respectively. T is the traction vector which acts on the contour and is defined in terms of stress components and directional cosines n_{ij} of the outward normal to Γ , $T_{ij} = \sigma_{ij} \cdot n_i$, u is the displacement vector and ds is an element of arc length along Γ .

In the rigorous mathematical analysis presented by Rice [3.42], the path independence of J has been proven. Since, paths can be chosen close to the crack tip, the energy contour integral J, represents some average measure of the deformation field in the crack vicinity which, makes J a valuable analytic tool. In the absence of body forces Rice has shown that J is identical to the energy release rate i.e. the rate of potential energy release with respect to crack length.

$$J = -\partial U_p / \partial a \quad \dots (3.31)$$

Therefore, in the case of linear elastic behaviour J is equal to the LEFM defined energy release rate G and hence, related to the stress intensity factor. Consequently, J can be considered as a measure of the energy available for crack extension in linear elastic cases. Even, in the case of non-linear elastic behaviour J can be regarded as the rate of energy available for crack extension.

Regarding elastic-plastic behaviour, a limitation arises as discussed by Bengley and Landes [3.43]. The derivation of J is based upon a deformation plasticity theory for a non-linear elastic material, Rice [3.42]. This deformation type theory of plasticity is based upon the irreversibility of plastic deformation. When though crack

extension takes place, some unloading occurs in the wake of the crack. In real elastic-plastic materials this unloading is linear elastic. Hence, J can be applied to a stationary crack only in elastic-plastic materials, where its physical significance is that it is a measure of the characteristic crack tip elastic-plastic field and not a measure of the energy available for extension.

The limited application of the J -integral to stationary cracks does not constitute its use possible as a fracture parameter for the interpretation of crack growth. In general, the literature concerned with experimental work illustrates the data with respect to the stress intensity factor range ΔK , i.e. an LEFM parameter even, in cases of elevated temperature testing and in the presence of overloads.

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CHAPTER 3

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CHAPTER FOUR

THEORETICAL INVESTIGATIONS

4.0 INTRODUCTION

The presence of a crack in an elastic stress field causes a serious disturbance in the distribution of stresses. Specifically, the crack causes acute stress concentrations and severe stress gradients. In general, the stress analysis of cracked bodies is a very difficult task even when considering simple geometries and/or somewhat simple boundary conditions. Theoretical, closed form solutions require some very complex mathematical analyses, Muskhelishvily [4.1]. This is the very reason for the limited availability of such solutions regarding cracked body problems.

The immense difficulty encountered when attempting to analyse such cases makes the utilisation of numerical and/or approximate methods a very attractive alternative. A numerical method with wide applicability in stress analysis is the Finite Element Method, F.E.M., and more recently the Boundary Element Method, B.E.M.. Both techniques are characterised by great flexibility and applicability to a large number of boundary and loading conditions as well as many different material behaviours.

The theoretical investigations presented here have all been carried out with the use of the Finite Element Method. The geometry investigated is that of the Compact Tension specimen, under elastic and elasto-plastic conditions. The results of these analyses were utilised to determine the loads that were to be used for the

experimental program. A detail study has been carried out aiming to evaluate the accuracy of a number of analytical techniques utilising the F.E. analysis data for the evaluation of stress intensity factors. The main objective of the theoretical work has been the evaluation of the effect of overloads upon the stress field. Residual stress distributions have been obtained as a result of elasto-plastic analyses as well as diagrams showing the evolution and growth of plasticity in the vicinity of the crack tip.

4.1 THE FINITE ELEMENT METHOD, A GENERAL OVERVIEW

In the Finite Element Method, the continuum is idealised as a structure consisting of a number of individual elements connected only at nodal points. The field variable (displacement), inside the solid represents the unknown parameter, and simple functions are chosen to approximate the variation of displacements over each finite element, Zienkiewicz [4.2]. These functions are known as displacement models or displacement functions, and are defined in terms of the values of the field variables at the nodes. Hence, the final solution yields the approximate displacements at discrete locations in the body, the nodal points. Further, the displacement models can be expressed in terms of simple polynomial or trigonometric functions which are employed in finite element programs and packages for solutions using digital computers.

Equilibrium equations for each element are obtained by adopting a variational principle of mechanics such as, the principle of minimum potential energy. This stipulates that, the potential energy of a loaded body is represented by the sum of the internal energy stored as a result of the deformation of the body due to the action of external

loads and, the work done by the external forces. For the body to attain equilibrium, this energy must be minimum.

For each element a stiffness relationship must be established, which relates the element nodal forces to the corresponding nodal displacements. In order to arrive to this stiffness relationship the following steps should be followed, [4.3, 4.4]: considering a two dimensional example,

Step 1: Definition of nodal parameters. Nodal forces and corresponding deflections expressed in terms of components relative to a global coordinate system of axes, Fig.4.1:

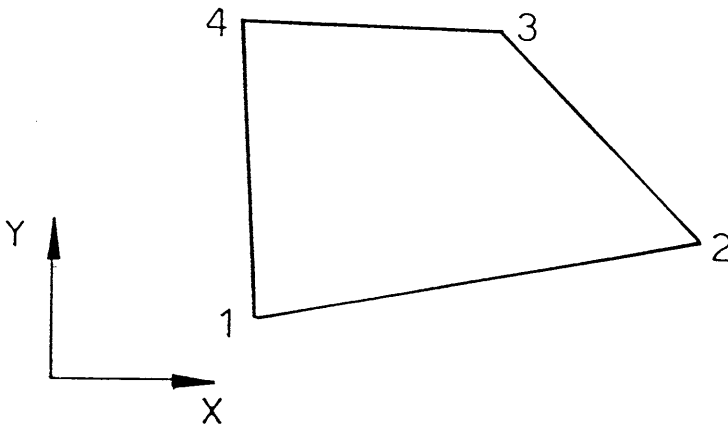


Fig.4.1 Typical Finite Element and Global Axes System

Nodal displacements:

$$\underline{\delta} = \{ U_1 \ V_1 \ U_2 \ V_2 \ \dots \} \quad \dots (4.1)$$

Nodal forces:

$$\underline{F} = \{ F_{x1} \ F_{y1} \ F_{x2} \ F_{y2} \ \dots \} \quad \dots (4.2)$$

Step 2: Displacement components at any point expressed in terms of nodal displacements and shape functions, N_i :

$$U(x,y) = U_1N_1 + U_2N_2 + U_3N_3 + \dots \quad \dots(4.3)$$

$$V(x,y) = V_1N_1 + V_2N_2 + V_3N_3 + \dots \quad \dots(4.4)$$

Step 3: Strain components expressed in terms of nodal displacements and shape functions:

$$\underline{\epsilon} = \underline{B} \cdot \underline{\delta} \quad \dots(4.5)$$

Where $\underline{\epsilon}$ = Strain vector

$\underline{\delta}$ = Displacement matrix

$$\text{and } \underline{B} = \begin{vmatrix} \partial N_1 / \partial x & 0 & \partial N_2 / \partial x & 0 \dots \\ 0 & \partial N_1 / \partial y & 0 & \partial N_2 / \partial y \dots \\ \partial N_1 / \partial y & \partial N_1 / \partial x & \partial N_2 / \partial y & \partial N_2 / \partial x \dots \end{vmatrix} \quad \dots(4.6)$$

Step 4: Express the stress components at any point in terms of nodal displacements and shape functions:

$$\underline{\sigma} = \underline{D} \cdot \underline{\epsilon} = \underline{D} \cdot \underline{B} \cdot \underline{\delta} \quad \dots(4.7)$$

Where $\underline{\sigma}$ = Stress vector

$\underline{D}$ = Material properties matrix

$\underline{\epsilon}$ = Strain matrix

The $\underline{D}$ or material properties matrix can be of two different types, dependent upon the stress condition considered, i.e. plane stress or plane strain.

Plane stress, $\sigma_z = \tau_{xz} = \tau_{yz} = 0$ and $\gamma_{xz} = \gamma_{yz} = 0$.

$$\underline{D} = E/(1-\nu^2) \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{vmatrix} \quad \dots (4.8)$$

Plane strain, $\epsilon_z = \gamma_{xz} = \gamma_{yz} = 0$ and $\tau_{xz} = \tau_{yz} = 0$, $\sigma_z = f(\sigma_x, \sigma_y)$

$$\underline{D} = E/(1+\nu)(1-2\nu) \begin{vmatrix} (1-\nu) & 0 & 0 \\ 0 & (1-\nu) & 0 \\ 0 & 0 & (1-2\nu)/2 \end{vmatrix} \quad \dots (4.9)$$

Step 5: Express the total potential energy of the element in terms of nodal displacements. If U and W represent the strain energy of the structure and the work done by external forces respectively, a variational statement may be written as follows:

$$\chi = U - W \quad \dots (4.10)$$

Where

$$U = (1/2) \iiint \underline{\sigma}^t \cdot \underline{\epsilon} \cdot dx \cdot dy \cdot dz \quad \dots (4.11)$$

and,

$$W = \underline{\delta}^t \cdot \underline{F}$$

$\underline{F}$ = Force matrix

Hence,

$$U = (1/2) \cdot \underline{\delta}^t \cdot (\iiint \underline{B}^t \cdot \underline{D} \cdot \underline{B} \cdot dx \cdot dy \cdot dz) \cdot \underline{\delta} \quad \dots (4.12)$$

and,

$$\chi = (1/2) \cdot \underline{\delta}^t \cdot (\iiint \underline{B}^t \cdot \underline{D} \cdot \underline{B} \cdot dx \cdot dy \cdot dz) \cdot \underline{\delta} - \underline{\delta}^t \cdot \underline{F} \quad \dots (4.13)$$

Step 6: Apply the minimum potential energy theorem

$$\partial X / \partial \underline{\delta} = 0 = (\iiint \underline{B}^t \cdot \underline{D} \cdot \underline{B} \cdot dx \cdot dy \cdot dz) \cdot \underline{\delta} - \underline{F} \quad \dots (4.14)$$

or $\underline{K}_{(e)} \cdot \underline{\delta} = \underline{F}$

If a large number of elements is used the total stiffness matrix $\underline{K}_{(total)}$ is assembled, in which contributions from adjoining elements are summed at their common nodes. Therefore, $\underline{K}_{(total)}$ represents the structural stiffness and it can be used to determine all the nodal displacements $\underline{r}$, due to external forces $\underline{R}$.

$$\underline{R} = \underline{K}_{(total)} \cdot \underline{r} \quad \dots (4.15)$$

Where $\underline{R}$ = Total force matrix, $\underline{r}$ = Nodal displacement matrix.

In order that the above technique is implemented successfully, it is imperative to specify the boundary conditions i.e. specify prescribed displacements (constraints), so that the inversion of the stiffness matrix $\underline{K}_{(total)}$ is possible. If the constraints are not specified then a situation arises in which the whole structure is free to float and, inversion of the singular matrix $\underline{K}_{(total)}$ becomes impossible.

In general, commercial Finite Element packages contain libraries of different elements with defined displacement functions. Such packages allow the transfer of geometry data from modelling programs and the formation of the structure stiffness matrix is carried out automatically while the user specifies the type of elements to be utilised in the discretisation. Boundary, kinematic and loading conditions can also be specified as well as material properties. Thus, the user defines all the required information regarding the particular analysis and subsequently an appropriate solver is selected. Finally, the numerical stress, strain, displacement output is

stored into appropriate formatted data files that can be accessed for further post-processing. Fig.4.2 shows a flow chart of the finite element procedure as implemented by modern engineering software packages.

4.2 NON-LINEAR F.E. ANALYSIS OF ELASTO-PLASTICITY

Equation (4.15) assumes that the stiffness matrix $K_{(total)}$ remains unchanged i.e. constant during the application of the load. This condition can only be true if the structure is loaded so that stress levels keep the material within the elastic region. Solutions of this type are relatively straight forward, as the resulting equations are linear. However, when the applied load causes the material to pass the threshold of elasticity, it is incorrect to adopt a linear analysis approach as the material behaviour is no longer linear.

4.2.1 The Yielding Condition

The material is assumed to exhibit a linear elastic behaviour prior to yield, according to Hooke's law. Loading beyond yield, causes the presence of plastic strains, which upon withdrawal of the load sustain irreversible plastic residual deformations. It is convenient to idealise the stress-strain curve so that it approximates two straight lines of linear elasticity and linear strain hardening, hence, defined by the elastic modulus, the yield stress and plastic stress-strain gradient (tangent modulus). This idealised stress strain behaviour is depicted in Fig.4.3. The well established Von Mises yield criterion is used, which states that when the equivalent stress becomes greater than the uni-axial yield stress, yielding takes place.

where,

$$\sigma_{eq} > \sigma_{yield}$$
$$\sigma_{eq} = (1/\sqrt{2}) \cdot \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} \quad \dots (4.16)$$

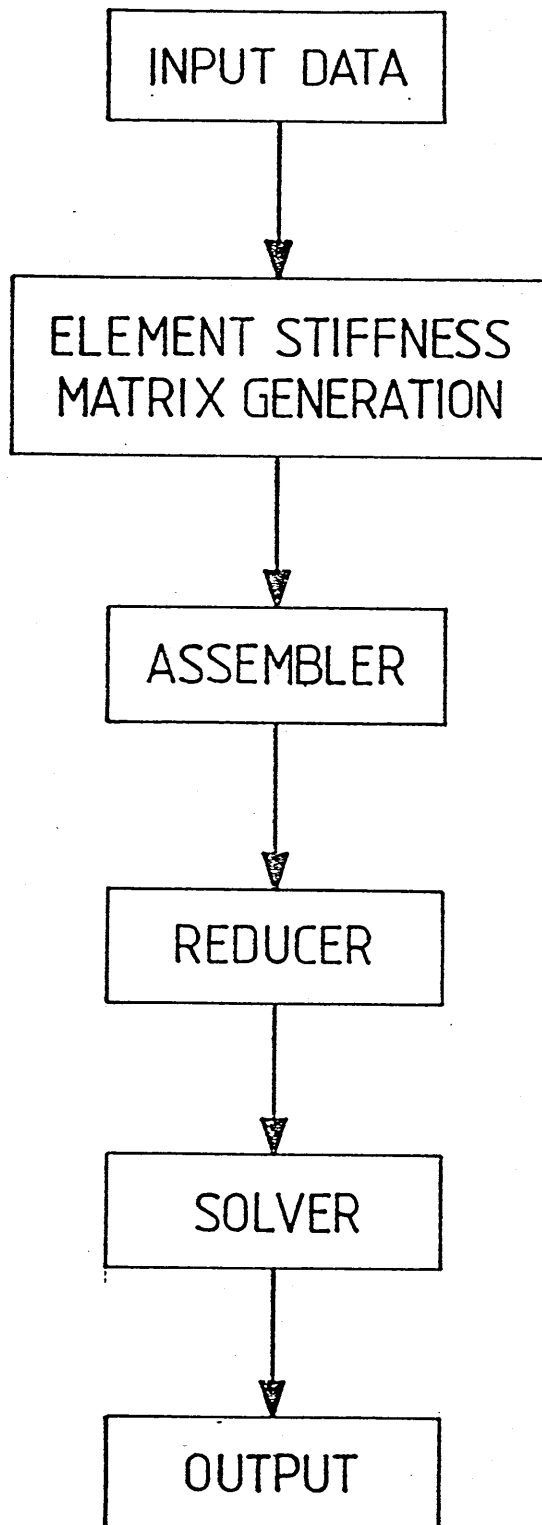


Fig.4.2 Flow chart of Finite Element Methods as implemented by commercial packages

4.2.2 The Initial Strain-Initial Stress

Method of Solution

As the plasticity solution is based upon the incremental Prandtl-Reuss equations [4.5, 4.6], it is necessary to apply the final load on the structure in load increments. These load steps need only to start after the first point in the structure is stressed beyond the point of yielding. The size of subsequent load steps is dependent upon the problem considered; the load stepping continuing until the full load has been applied.

Upon application of the first load increment, the strains and hence the stresses are calculated at the Gauss or integration points for each element. At locations where yielding has taken place the calculated equivalent stress (4.16), gives values which are greater than the yield stress. At the start of each load increment the plastic strains are taken as zero; therefore, the equivalent stress and equivalent strain correspond to a point P on the continuation of the elastic line of the stress-strain curve and not on the strain curve itself, see Fig.4.4.

As the strains are calculated from the applied load via the elastic stiffness [4.7, 4.8], it is necessary to add to the original nodal forces the loads corresponding to the plastic strains to cause these extra required strains.

The plastic strain components are calculated from the plastic strain increment, given by PQ in Fig.4.4. As the nodal forces equivalent to the plastic strains are calculated from the nodes round the element using only the element stiffness, these nodal forces when applied do not necessarily give the expected strains as no account has been made of the effect of the rest of the structure or of the effect of the other elements that have yielded. In general, the calculated new equivalent stress, strain

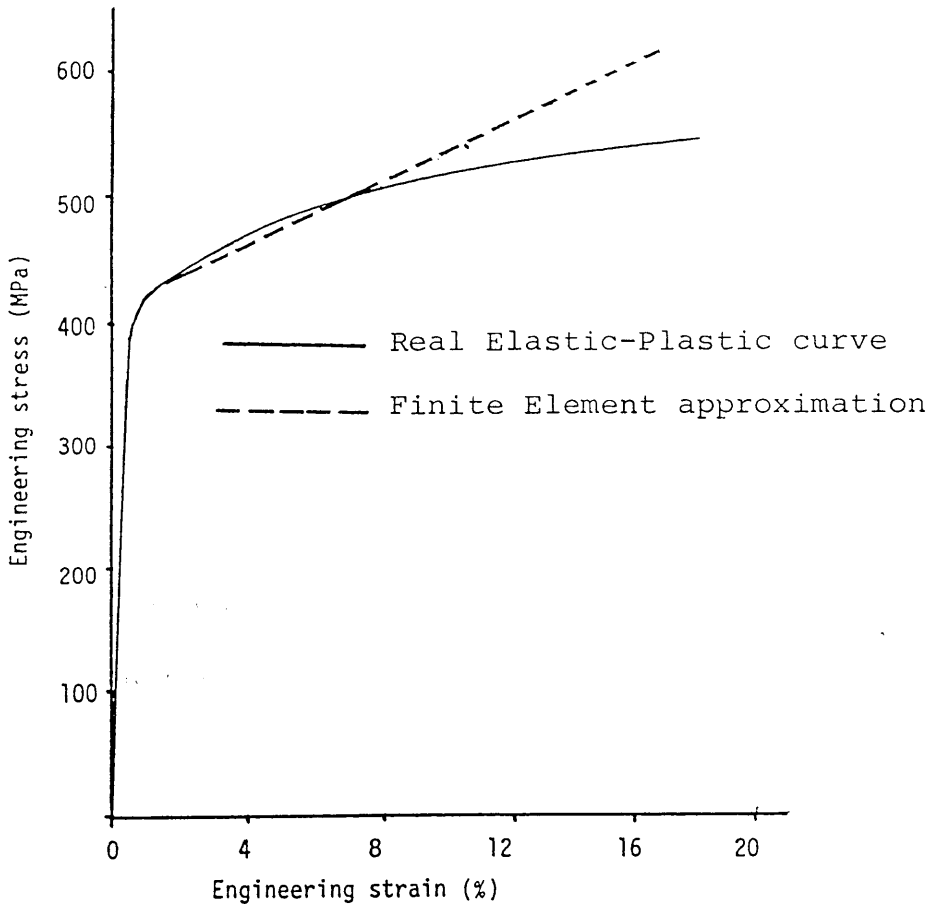


Fig.4.3 Idealised Stress-Strain Curve
of the 7075 Alloy

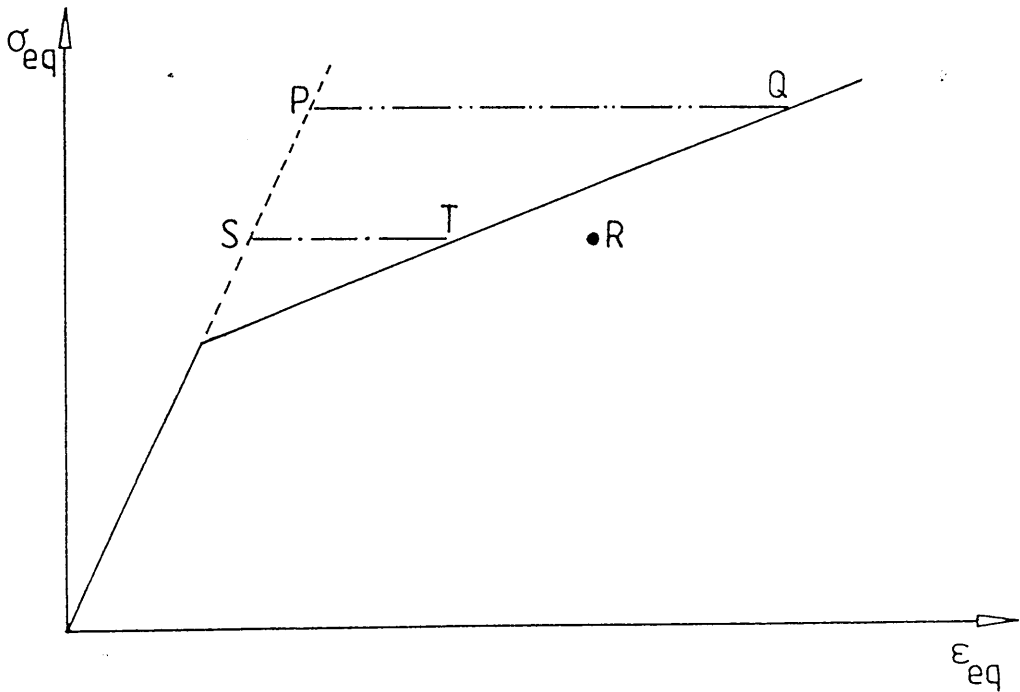


Fig.4.4 Typical iterative solution procedure for Elasto-plasticity

points will not all lie on the curve. If, say for a particular point, the new stress and strain are represented by a point, see Fig.4.4, then a new estimate ST would be made for the equivalent plastic strain increment for the corresponding load step.

The whole process is repeated with this new estimate of equivalent plastic strain. The iterative procedure continues until the equivalent stress and equivalent strain at each point in the structure is close enough to the prescribed material stress-strain curve.

Thus, if values of equivalent stress fall below the set tolerance level the iterations are terminated. The advantages are that, such controls give direct control to the solution accuracy and simultaneously eliminate excessive and costly computations.

4.3 F.E. PRE- AND POST-PROCESSING SOFTWARE

4.3.1 The Pre-Processor

In order to carry out the task of data preparation the SDRC I-DEAS interactive pre-processor was utilised. This menu-driven software allows the development of finite element 2-dimensional and 3-dimensional mesh models which can be transferred to other F.E. Analysis Packages (e.g. PAFEC, ABAQUS, NASTRAN), for solution. The steps followed in order to produce an F.E. model of the compact type tension specimen were,

- I. Geometry Definition
- II. Mapped/Free Mesh Generation
- III. Model Checking
- IV. Mesh Optimisation/Renumbering
- V. Data Transfer

A short description of each step followed in the pre-processing stage is given below.

(I) Geometry Definition

By entering the GEOMETRY DEFINITION task of the pre-processor, the geometry of the model can be defined by means of creating points, lines, arcs and splines. It has to be noted that the creation of these geometric entities are interrelated upon one another and as a consequence they have to be developed with the required mesh pattern in mind. The advantage of using a menu driven interactive package is the fact that the user is able to create and delete entities aiming to achieve a more realistic representation of the structure considered.

(II) Mapped/Free Mesh Generator

The process of representing a continuum by a finite number of sub-domains (elements) which are connected through common points (nodes), is termed discretisation. The interactive pre-processor provides two methods of performing this task MMG (mapped mesh generation) and FMG (free mesh generation).

Using MMG, it is possible to generate nodes and elements on pre-defined mesh areas. The number of elements on each edge of the mesh area and their distribution (bias) is interactively controlled thus, allowing fine or coarse mesh generation. The mesh patterns generated by this method are regular, Fig.4.5. As nodes and elements can be previewed, it is possible at this stage to modify the mesh areas grid definitions and preview many alternatives before finalising the discretisation.

Irregularly shaped areas can cause problems when using MMG. The FMG option allows the generation of meshes within such irregular surfaces or volumes. It provides a relative control upon the distribution of elements by allowing

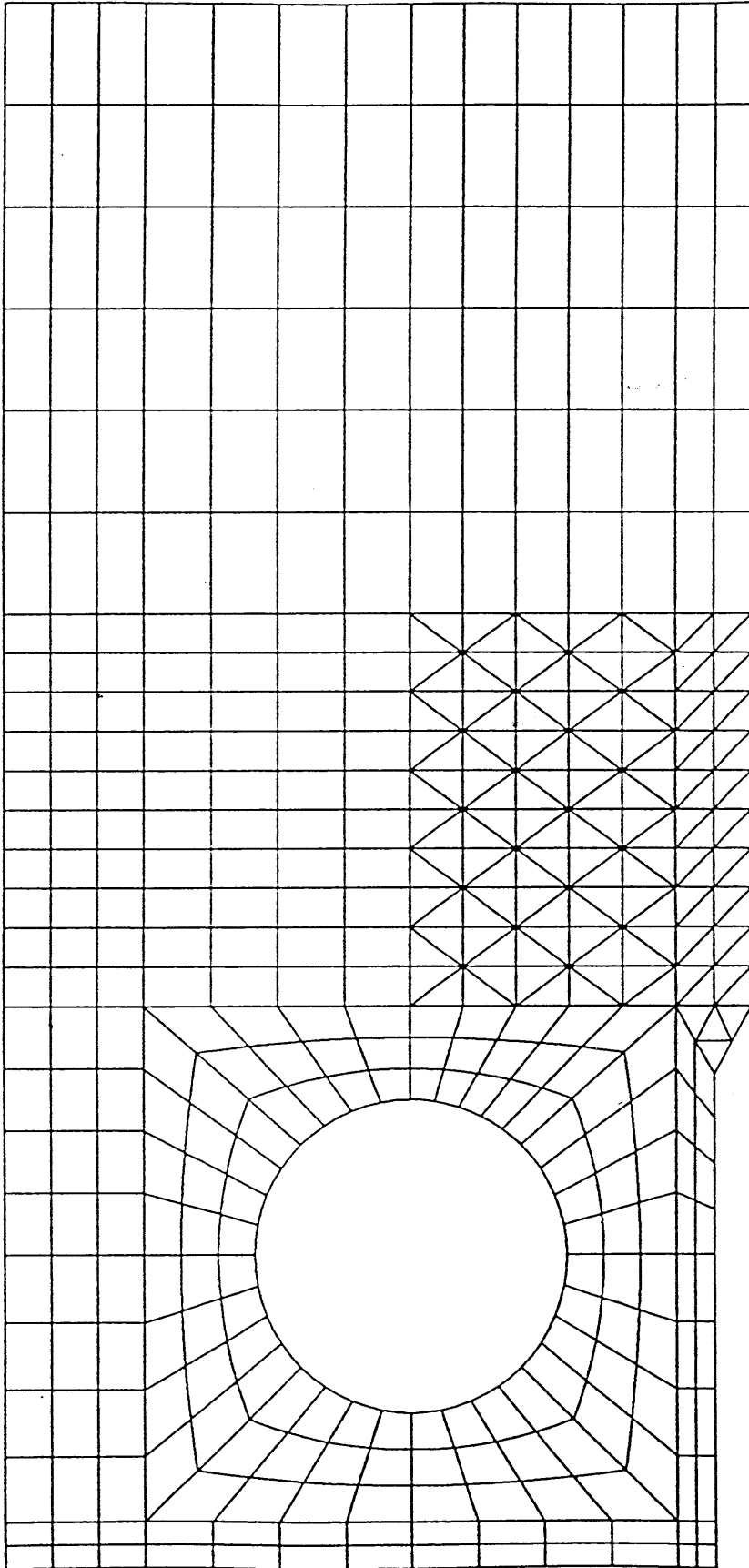


Fig.4.5 Mapped mesh generated F.E. model of CT-specimen

definitions of global and/or local element lengths and number of elements per curve.

The MMG and FMG can be used concurrently and FMG is able to produce automatic transitioning of elements thus, providing relatively regular element distributions throughout the model, Fig.4.6.

(III) Model Checking

Generated meshes have to be checked as to avoid problems and errors during the solution stage. A series of verification checks can be carried out for node coincidence, element warping, element interior angle and distortion, and free edge continuity. The reasons for such checks are to ensure that there are no elements which may have either large aspect ratios or small angles between adjacent sides as their presence diminishes the accuracy of the solution. Free edge continuity ensures that no undesired cracks are present within the model.

(IV) Mesh Optimisation/Renumbering

Due to the mesh generation the numbering of nodes and elements produces matrices with wide bandwidths which in effect require longer processing times. Thus, the wavefront and bandwidth can be checked and a renumbering option allows the adoption of a new numbering order of nodes and elements that can reduce the bandwidth and subsequently the run time by up to 40% in some cases. This is particularly useful when dealing with large models where the CPU-time may add to the cost of the analysis.

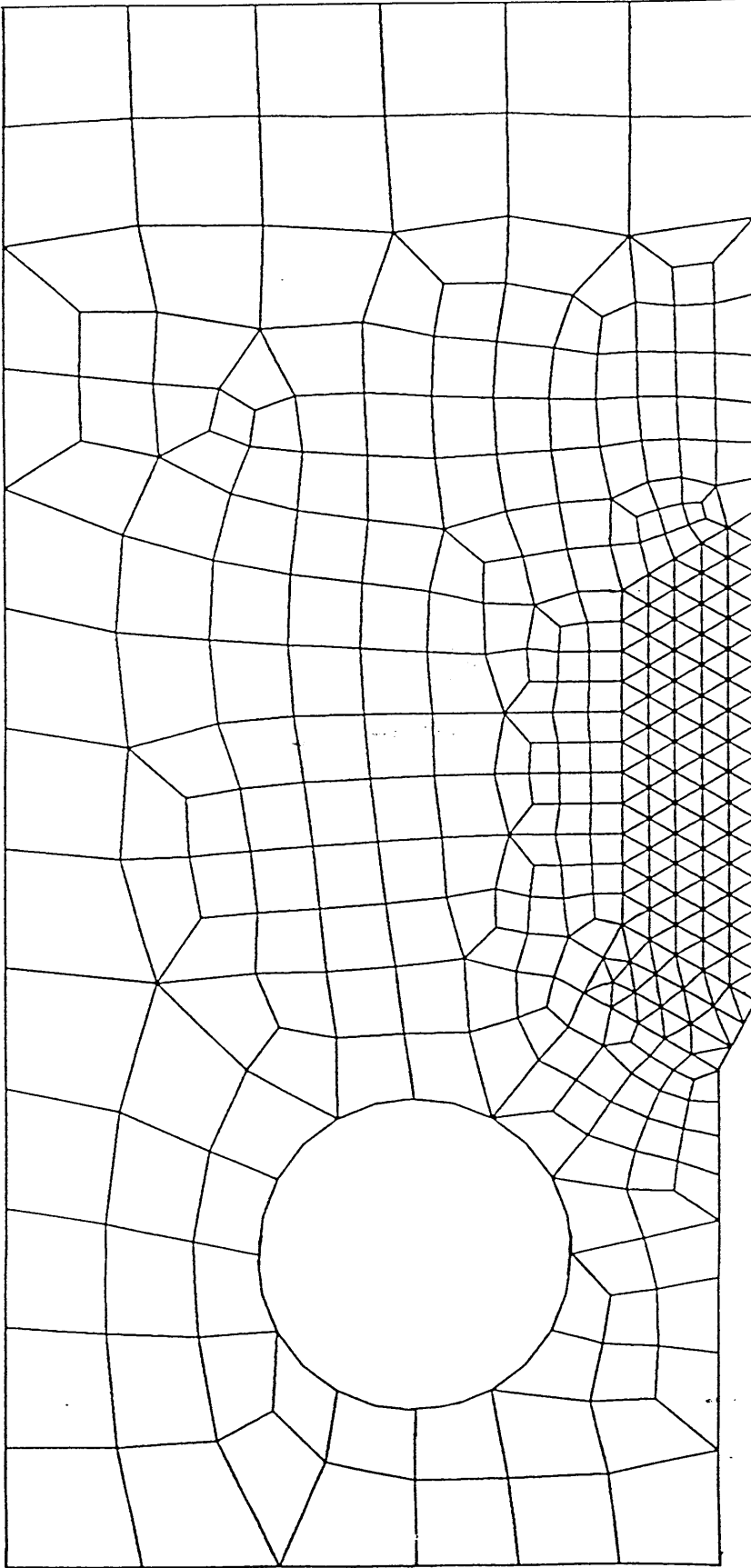


Fig.4.4.6 Combined Mapped-Free meshed model of CT-specimen

(V) Data Transfer

The element topology data and any other information regarding boundary, loading conditions and material properties can be saved into a data file with the required format depending upon the solver (analysis program) that is to be used e.g. PAFEC, ABAQUS, NASTRAN. Such input data files can be edited and additional information may be inserted or changes can be carried out. Special data loaders are available to translate/transfer information between suites of software packages such as IDSPAF, PAFIDS and the PURLE data loader.

4.3.2 The Post-Processor

The finite element analysis results obtained from solutions carried out by the PAFEC and ABAQUS packages, were post-processed utilising the SDRC I-DEAS SUPERTAB post-processor. This versatile software incorporates a data loading module, a utility that allows the import of analysis data from the ABAQUS and PAFEC packages. In effect this utility is a data transfer translator that is capable to read and translate data formatted files and simultaneously import the information into a user defined SUPERTAB database. The post-processor which is interactive menu-driven, allows the user to process and manipulate solution data so as to obtain deformed shapes, stress contours, stress arrow plots, criterion plots and X-Y plots of selected variables and magnitude ranges.

4.4 F.E. MODELLING OF THE COMPACT TYPE TENSION SPECIMEN

The overall shape and dimensions of the compact tension specimen were in accordance to the ASTM E-647/86 standard for constant load amplitude testing. The material considered was Aluminium 7075-T7351 with the following mechanical properties:

Modulus of Elasticity $E=80.0$ GPa

Poisson's Ratio $\nu=0.303$

Ultimate Tensile Strength $\sigma_{UTS}=529$ MPa

0.2% Proof Stress $\sigma_{0.2\%}=465$ MPa

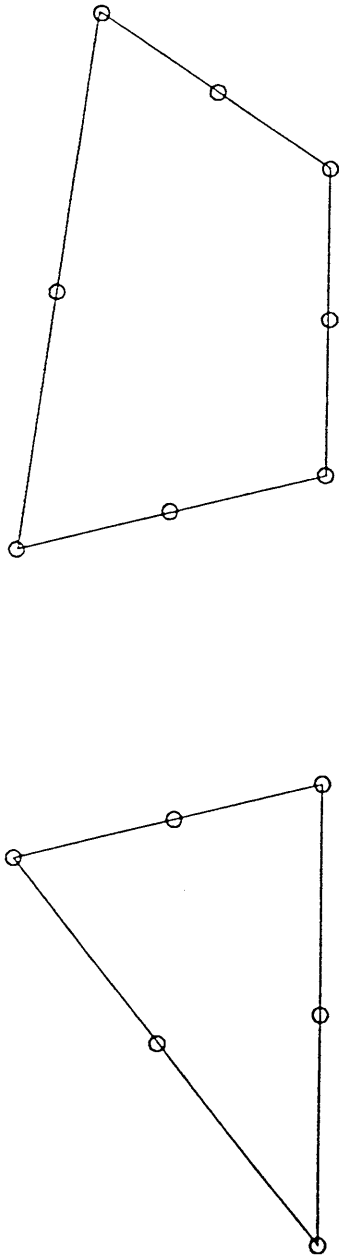
For the development of the F.E. models and the subsequent analysis, the geometric symmetry of the component was taken into consideration. Thus, only one half of the geometry was modelled.

4.4.1 Crack and Crack Tip Modelling

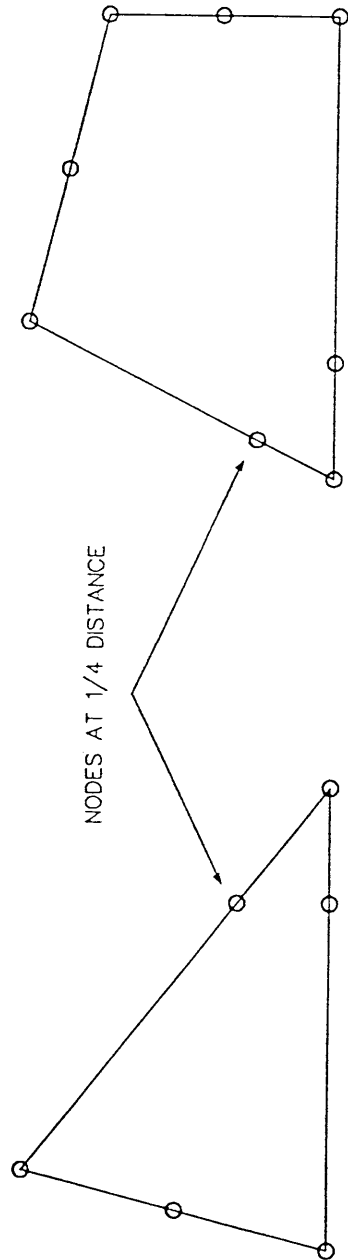
The accuracy of solution when considering cracked body problems is highly dependent upon the correct representation of the crack and sound modelling of the crack tip singularity. The crack tip can be modelled with the use of two different types of elements

- a. six noded triangular, eight noded quadrilateral isoparametric elements,
- b. six noded triangular, eight noded quadrilateral with two mid-side nodes located at the 1/4 distance near the node identifying the crack tip, also known as 'special' crack tip elements.

Figure 4.7, shows the two types of non-singular and singular isoparametric elements frequently utilised. Barsoum [4.10], has shown that by collapsing one side of a quadrilateral to form a triangular element, far better results are obtained, than when using conventional crack



Non-Singular Isoparametric Elements



Singular Isoparametric Elements

Fig.4.7 Isoparametric and Singular crack tip elements

tip elements. Such collapsable elements are used within ABAQUS.

When utilising standard isoparametric elements, reasonable modelling of the crack tip singularity may be achieved if, a large number of elements are concentrated in the vicinity of the tip to form a fine mesh. This large concentration of elements is dictated by the presence of acute stress concentrations and steep stress gradients. The use of specially developed singular crack tip elements eliminates the requirement for high element concentrations. Accurate solutions can be obtained with a relatively coarser mesh which in addition reduces the computer processing time (CPU) and the required scratch memory space thus, reducing the total computational cost.

In the present study both types of elements and modelling have been used and typical meshes of the CT-specimen geometry are shown in Fig.4.8.

Due to the fact that two different analysis packages were utilised namely PAFEC and ABAQUS, different models were produced. PAFEC, has the special capability to produce a crack-tip model with singular isoparametric elements by simply specifying in the input data deck the node that is to be considered as the crack tip, [4.11]. In the ABAQUS package, such a facility is not available. A somewhat more elaborate procedure has to be adopted, where rectangular isoparametric elements with one collapsed side are specified by means of utilising the ABAQUS own pre-processing routine library, [4.12, 4.13], in order to produce a focused mesh. In general, the majority of the modelling was carried out utilising the I-DEAS interactive pre-processor, with the exception of the crack tip region.

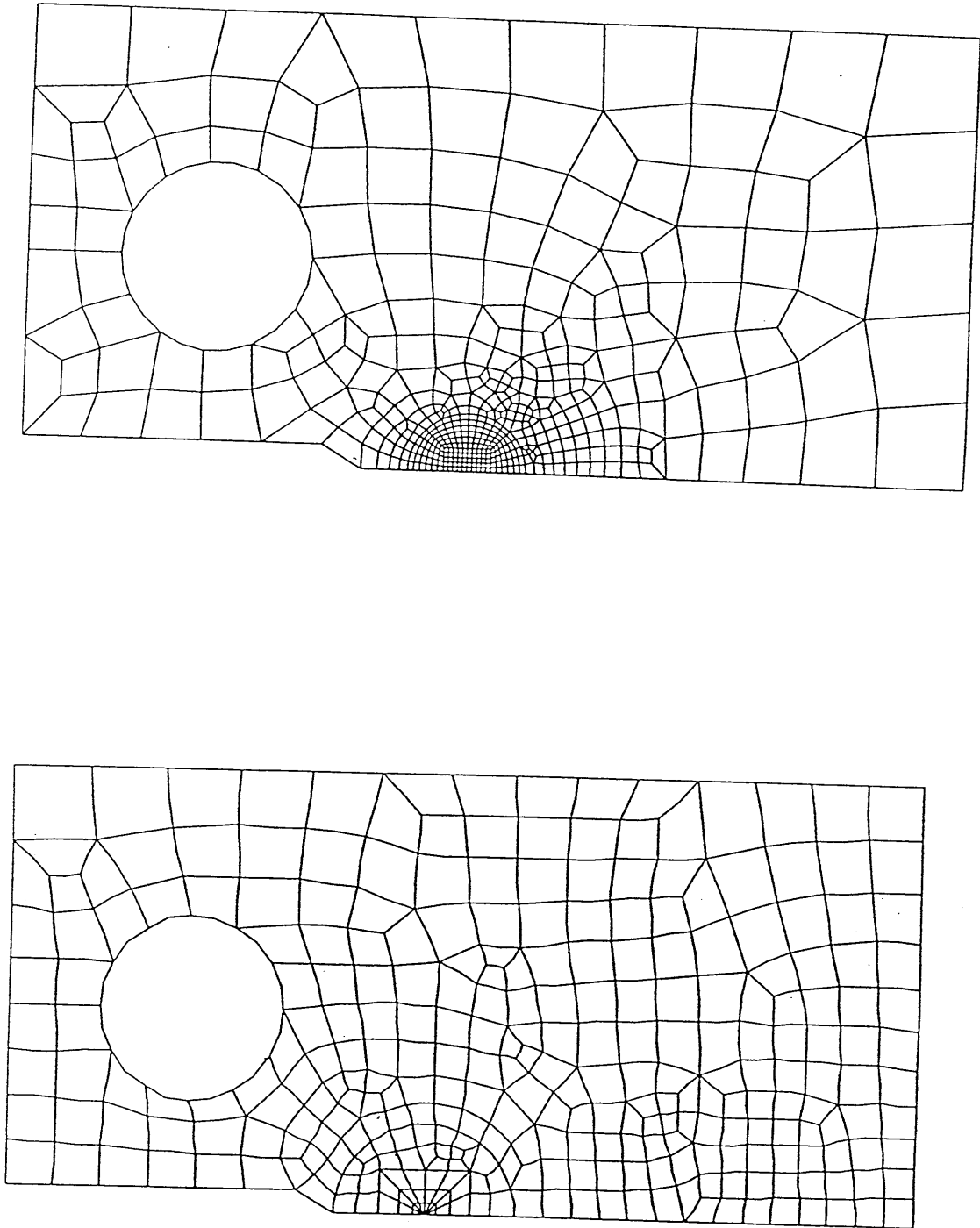


Fig.4.8 Typical F.E. models utilised in the present study

4.4.2 Kinematic Restraints and Loading

The axial symmetry of the component was taken into account so that, only one half of the specimen was modelled. Consequently, appropriate kinematic conditions had to be selected so as to allow the half model to simulate the full component under identical loading conditions.

The crack modelling is achieved by allowing the nodes that represent the crack ligament to move freely on the two dimensional plane. All nodes following the crack tip are allowed to move only along the X-axis; all other freedoms are restrained. An illustration of the kinematic restraints applied to the boundary is shown in Fig.4.9.

The loading applied to the model was attempted to be as realistic as possible. In reality the load is applied by means of the slide fitting pins. Hence, the load is applied to the specimen in the region of the conformal contact of the pin and hole assembly. For the load modelling the Hertzian theory of elastic contact was considered [4.14]. The loading was modelled as nodal point compressive forces Fig.4.10, which were calculated based upon the total load and, the area of contact was accordingly determined from the Hertzian theory, Johnson [4.14], Timoshenko [4.15].

4.5 DETERMINATION OF CRACK PARAMETERS FROM F.E. RESULTS

Numerical solutions of stress analysis problems by means of finite element procedures, provide solutions in the form of displacements, strains and stresses, at nodal points within a discretised domain. In the cases of cracked body problems such information can be utilised to obtain crack parameters such as stress intensity factors and J-integral values.

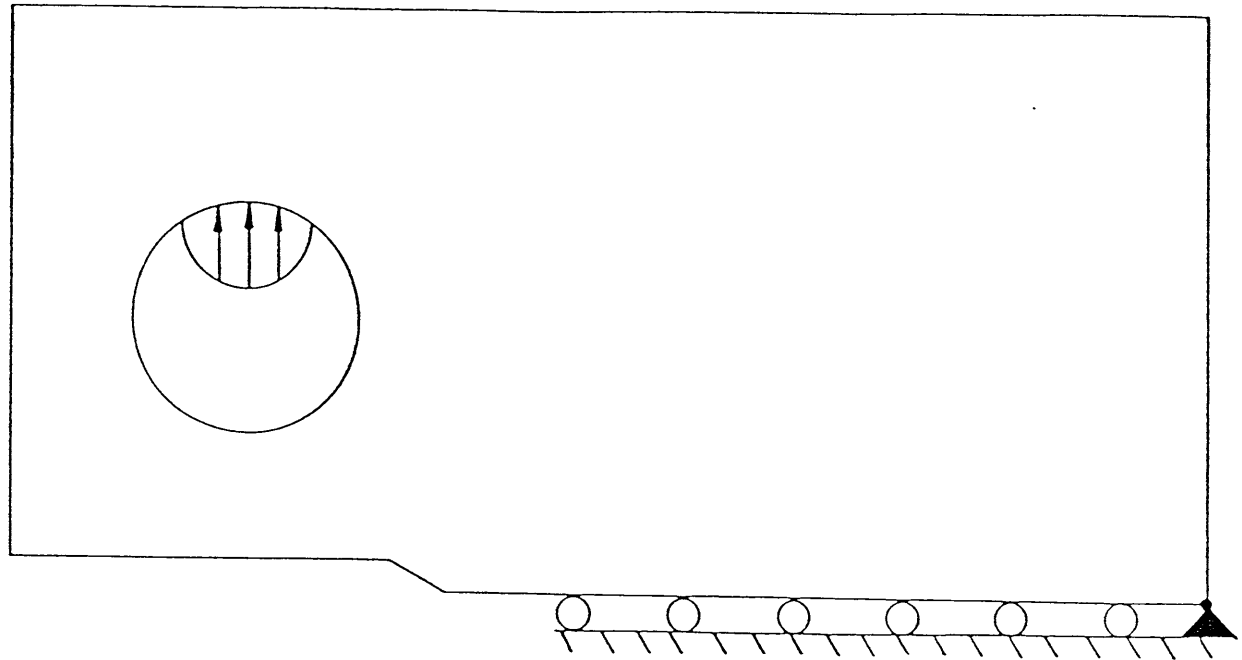


Fig.4.9 F.E. model kinematic restraints and loading conditions

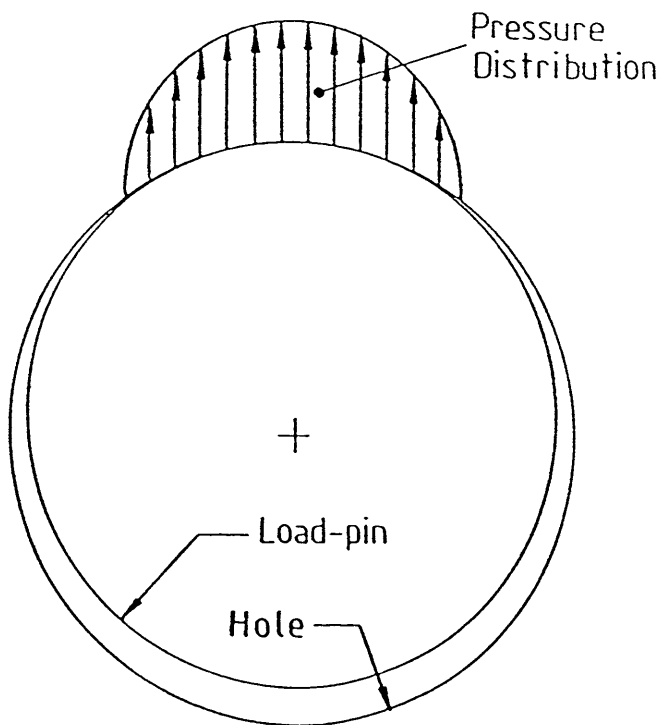


Fig.4.10 Hertzian pressure distribution in the conformal contact of the hole and load pin

Well documented techniques are available that can be used to perform such numerical evaluations using the results of F.E. analyses. Approximations of stress intensity factors may be obtained by means of crack ligament displacements or crack tip vicinity stresses, two approaches that have been presented by Chan et al [4.16]. Further, compliance concepts can be adopted, Sharples [4.17], Dixon and Strannigan [4.18]. The results of F.E. analyses are particularly suited for evaluation of J-integral values along contours encircling a crack, Rice [4.19].

4.5.1 The Displacement Method

The displacement method, involves a correlation of the finite element nodal point displacement with the well known crack tip displacement equations. The plane strain displacement equations can be written as,

$$U_i = (K_I / G) \cdot [r / 2\pi]^{1/2} \cdot f_i(\theta, \nu) \quad \dots (4.17)$$

Where $U_1 = u$ and $U_2 = v$,

$$f_1(\theta, \nu) = \cos(\theta/2) \cdot [1 - 2\nu + \sin^2(\theta/2)],$$

$$f_2(\theta, \nu) = \sin(\theta/2) \cdot [1 - 2\nu - \cos^2(\theta/2)],$$

G = Shear modulus of elasticity,

K_I = Mode-I stress intensity factor.

Substituting a nodal displacement U_i^* at some point (r, θ) near the crack tip into equation (4.17) a quantity K_I^* can be calculated,

$$K_I^* = [2\pi/r]^{1/2} \cdot G \cdot U_i^* / f_i(\theta, \nu) \quad \dots (4.18)$$

Such values can be plotted as a function of the distance from the crack tip and extrapolated to $r=0$. It has been shown by Chan et al [4.16], that the extrapolated value of K_I^* is a good estimate of the stress intensity factor, and the accuracy increases if $\theta = \pi$ and the displacement component used is $U_i^* = U_2 = v$.

4.5.2 The Stress Method

The stress method for determining crack-tip stress intensity factors is similar to the displacement method. In this approach the nodal stresses are correlated with the well known crack tip stress equations. For plane strain conditions, the equations of the stresses near the tip are,

$$\sigma_{ij} = (K_I / \sqrt{2\pi r}) \cdot f_{ij}(\theta) \quad \dots (4.19)$$

Nodal point stresses σ_{ij}^* in the vicinity of the crack tip can be substituted into (4.19) and values of K_I^* can be calculated as,

$$K_I^* = [(2\pi r)^{1/2} / f_{ij}(\theta)] \cdot \sigma_{ij}^*(r, \theta) \quad \dots (4.20)$$

From plots of K_I^* as a function of r , for a fixed angle and a particular stress component, estimates of K_I can be made. The same procedure of linear extrapolation can be adopted as for displacements and K_I^* values thus obtained are good approximations of the stress intensity factor. The accuracy of the approximated values of K_I is highest when obtained from K_I^* curves corresponding to σ_{yy} stresses on the $\theta=0$ plane, i.e. nodal stresses ahead of the crack tip.

4.5.3 J-Integral Evaluation

The J-integral can be evaluated from F.E. stress results. A contour or contours can be selected such as to enclose within the crack. The stresses at nodal points that belong to the selected contour, with information regarding their location, can be supplied to a computer program which carries out the J-integral evaluation. In the course of the present study a FORTRAN computer program has been developed for evaluating J-integrals from 2-dimensional finite element solutions, and for the cases of both plane

stress and plane strain. Information regarding the development of equations and a listing of the short program developed are given in Appendix 2.

4.5.4 Compliance Methods

The compliance method of determining K , involves the determination of the energy release rate G , which is a parameter directly related to the stress intensity factor. Considering an arbitrary body subjected to an arbitrary system of forces, Fig.4.11, the energy available for crack extension is provided by the work done by the applied forces $P_i \cdot d\Delta_i$, and the energy release of strain energy $-dV$ in the total strain energy stored in the body. Thus,

$$G = P_i \cdot (d\Delta_i/da) - dV/da \quad \dots (4.21)$$

Where G = Strain energy release rate per unit thickness

Δ_i = Displacement

P_i = Load

V = Strain energy

a = Crack length

Dixon and Strannigan [4.18], have shown that for constant load the energy release rate is given by the expression,

$$G = (1/2) \cdot (P_i/B) \cdot (d\Delta_i/da) \quad \dots (4.22)$$

Where B = Thickness.

Therefore G may be estimated by determining $d\Delta_i/da$ for different loads P_i , by means of plotting (Δ_i vs a) data and differentiating the curve using interpolation or a least squares method.

This methodology involves displacements at the point of load application. An alternative formulation for specific geometric configurations (CT and WOL specimens), allows calculation of compliance values from displacements at

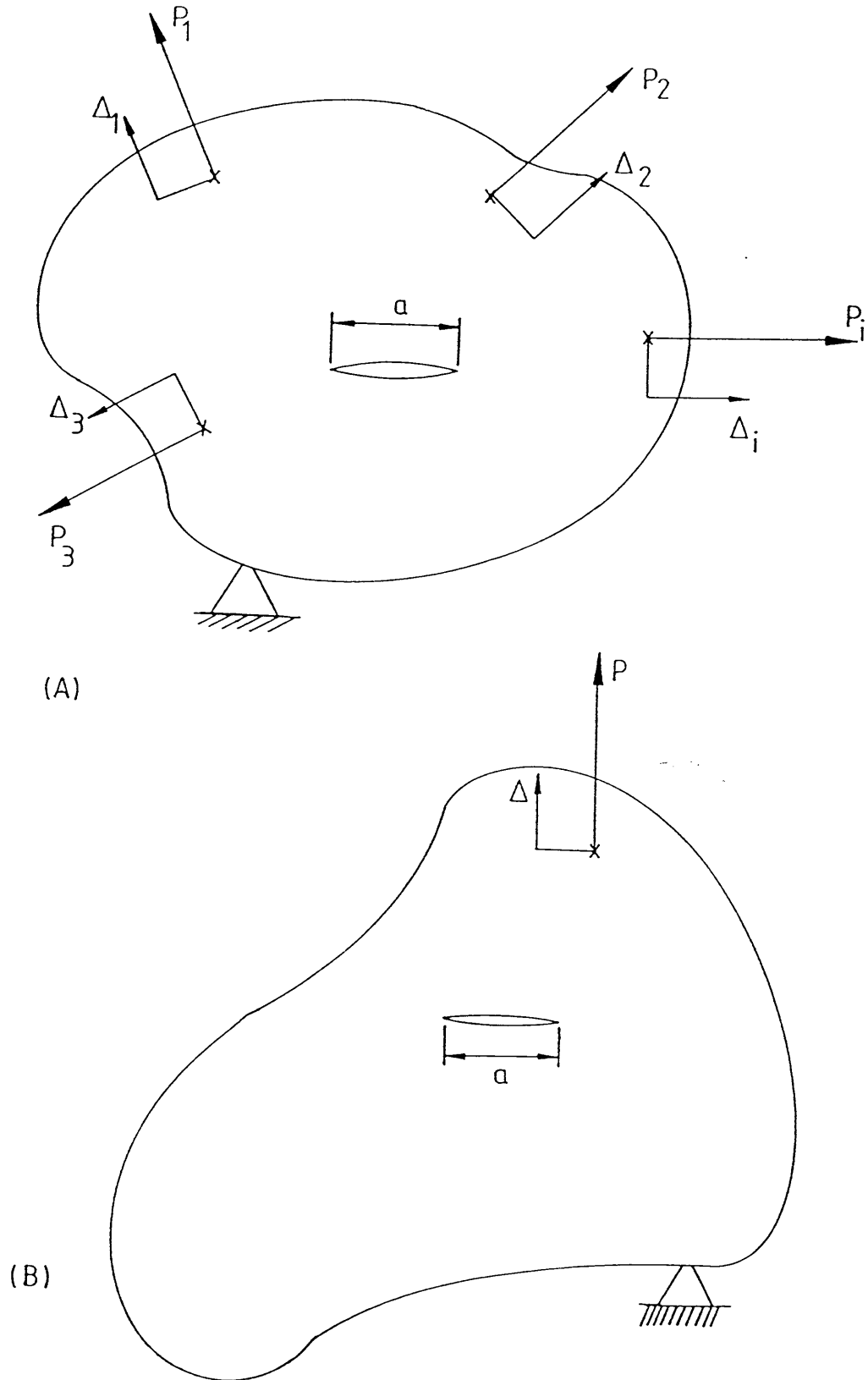


Fig.4.11 Crack in a body of arbitrary shape.

A) Arbitrary forces and displacements

B) Single force and displacement

other locations rather than the loading points. This technique is detailed in a publication by Saxena and Hudak [4.20]. A computer program has been developed based upon the formulation of [4.20] to assist the evaluation of stress intensity factors from F.E. displacement data, see Appendix 3.

4.6 ELASTIC F.E. ANALYSIS OF COMPACT TYPE TENSION SPECIMEN

4.6.1 Displacement Data Post-Processing and Presentation

The finite element analysis has been carried out for three different crack lengths, namely 8.4, 10.8 and 14.4mm. Displacement solutions from the analysis have been used to obtain deformed geometry shapes of the loaded CT specimen. The plots of displaced shapes are shown in Fig.4.12-4.14 and, correspond to a maximum applied load of 600N. It is clearly indicated that the degree of distortion is proportional to the crack length. This distortion is pronoun in the half of the specimen that contains the loading hole. As anticipated, the circular hole with the application of the pressure loading deforms to a near elliptical shape.

Further, the thin section of material at the front face of the specimen bends inwards. The distortion of the hole clearly denotes that the pin will be subjected to compressive loading not only in the contact area at the top part of the hole, but rather to a larger extent. This observation coupled with the fact that testing involves cyclic loading of long durations, indicates that fretting may be a potential problem. This dictates that appropriate measures have to be taken in practice as to ensure minimum wear due to the sliding contact, between the hole and the load pin. Hence, high viscosity lubricants may be used to

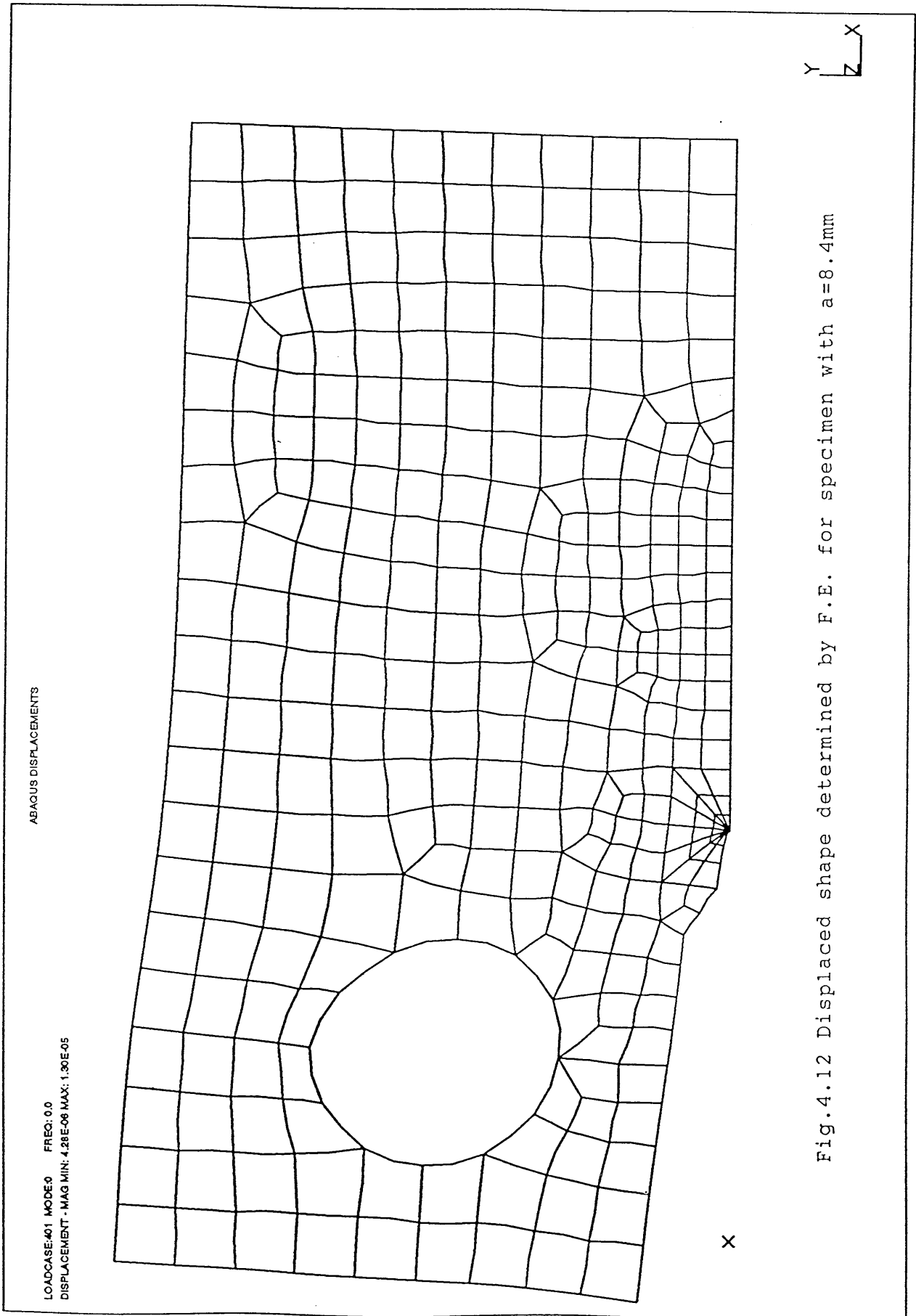


Fig.4.12 Displaced shape determined by F.E. for specimen with $a=8.4\text{mm}$

ABACUS DISPLACEMENTS

LOADCASE:401 MODE:0 FREQ:0.0
DISPLACEMENT - MAG MIN:1.39E-05 MAX:3.15E-05

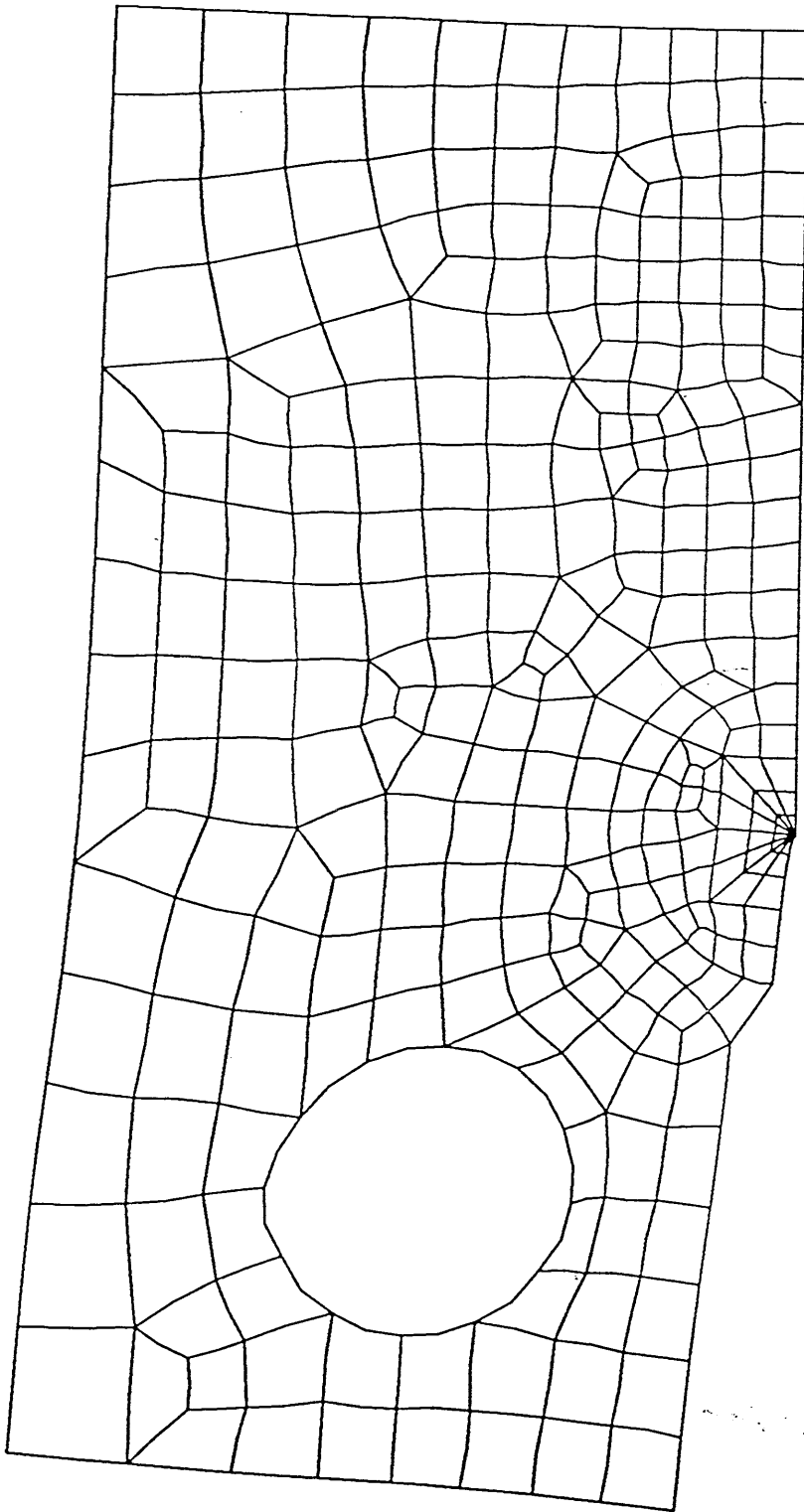
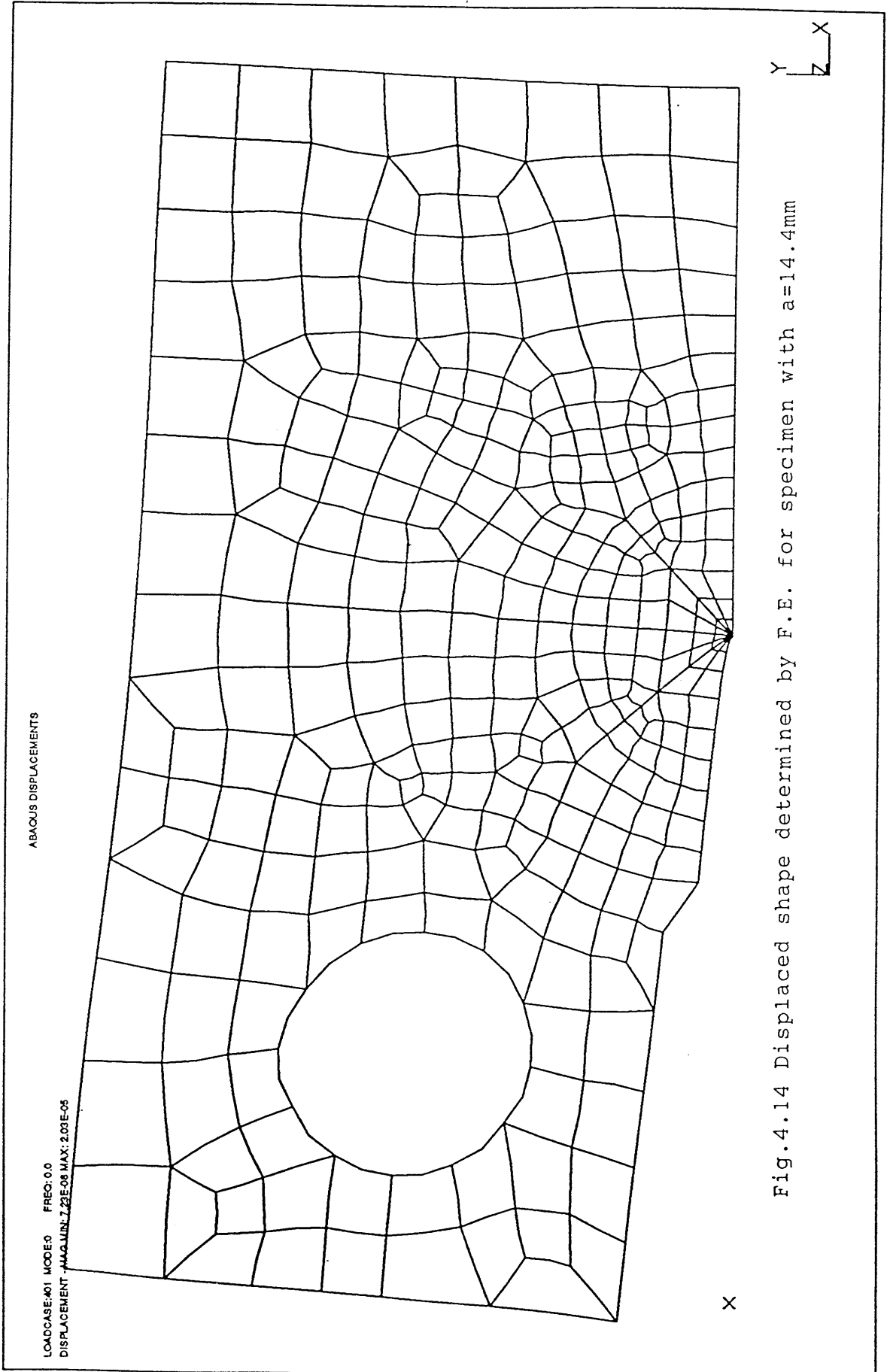


Fig.4.13 Displaced shape determined by F.E. for specimen with $a=10.4\text{mm}$



reduce wear and the possibility of failure in the region, e.g. Molybdenum grease.

Due to the modelling of the crack as a straight slit, the deformation of the crack ligament is of a parabolic like shape, as shown in Fig.4.15. The data for this plot has been extracted from the analysis output data, and displacements represent nodal displacements along the crack ligament.

4.6.2 Stress Data Post-Processing

Stress data obtained from the elastic F.E. analysis of the CT specimen were utilised for the determination of stress contour plots. The purpose of those plots was to provide information of the overall distribution of stresses within the geometry, and especially in the regions of interest.

An arrow plot of the maximum principal stresses is shown in Fig.4.16. This depicts the directions of the principal stresses, and the colour coding of the arrows provides information of the stress magnitudes. It is clear from this plot that the arrows tend to focus to one particular location, the crack tip area. The high concentration of the arrows in the vicinity of the crack tip identifies a region of severe stress concentration. An important observation is the direction of the principal stresses on the loading hole, and in particular at a 45° clockwise location from the top. It is clearly shown that the principal stresses are of a tensile nature, and their magnitude is somewhat higher than any other stresses in the region. The arrow plot illustrates the presence of principal stresses whose direction is perpendicular to the free boundaries on the specimen. In reality this is an impossible situation, and the stresses have to be parallel or tangential to the free surface. This irregularity can

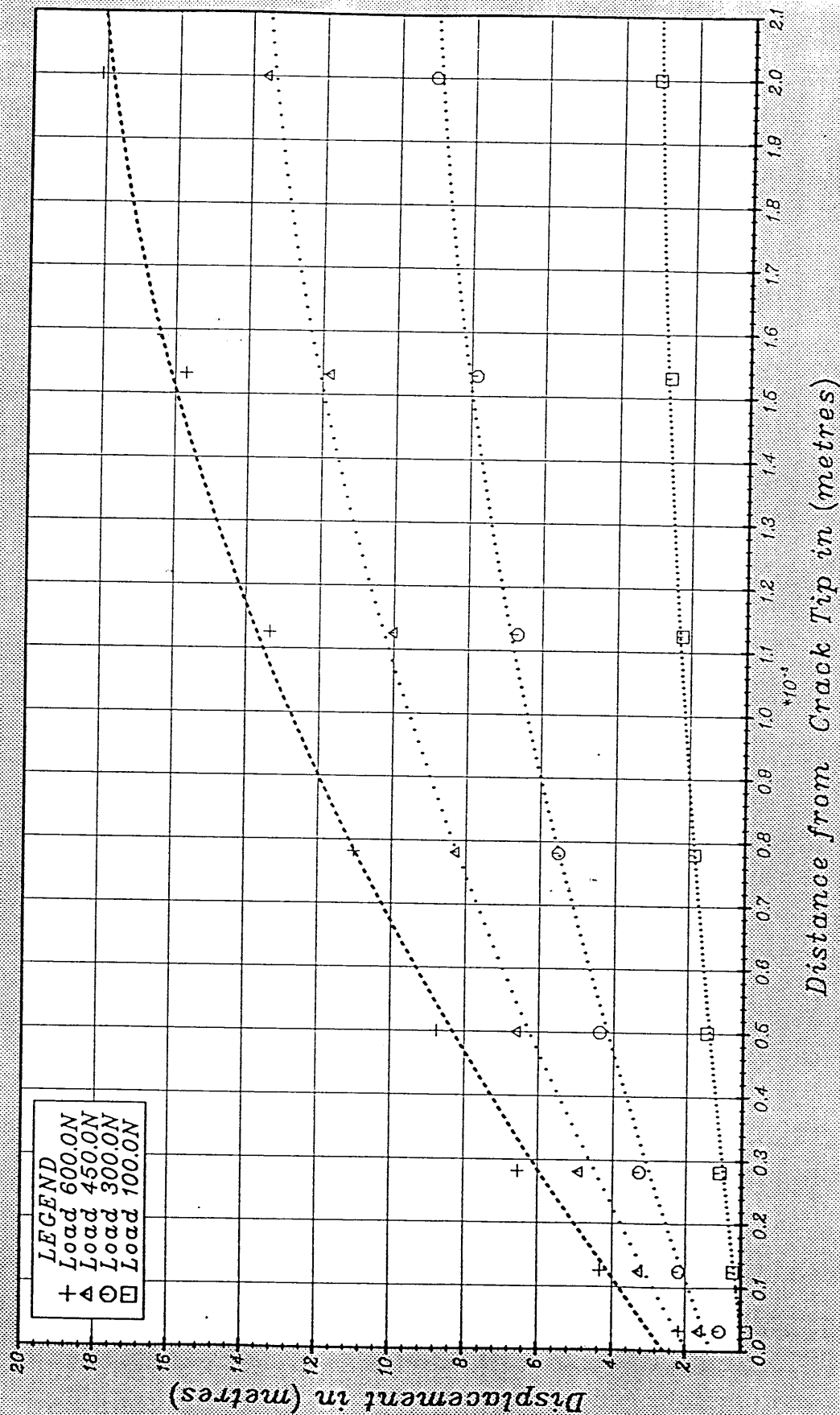


Fig.4.15 Crack ligament Y-direction displacement as a function of distance from the crack tip for a crack $\alpha=2.0\text{mm}$ from notch. Elastic ABAQUS F.E. analysis.

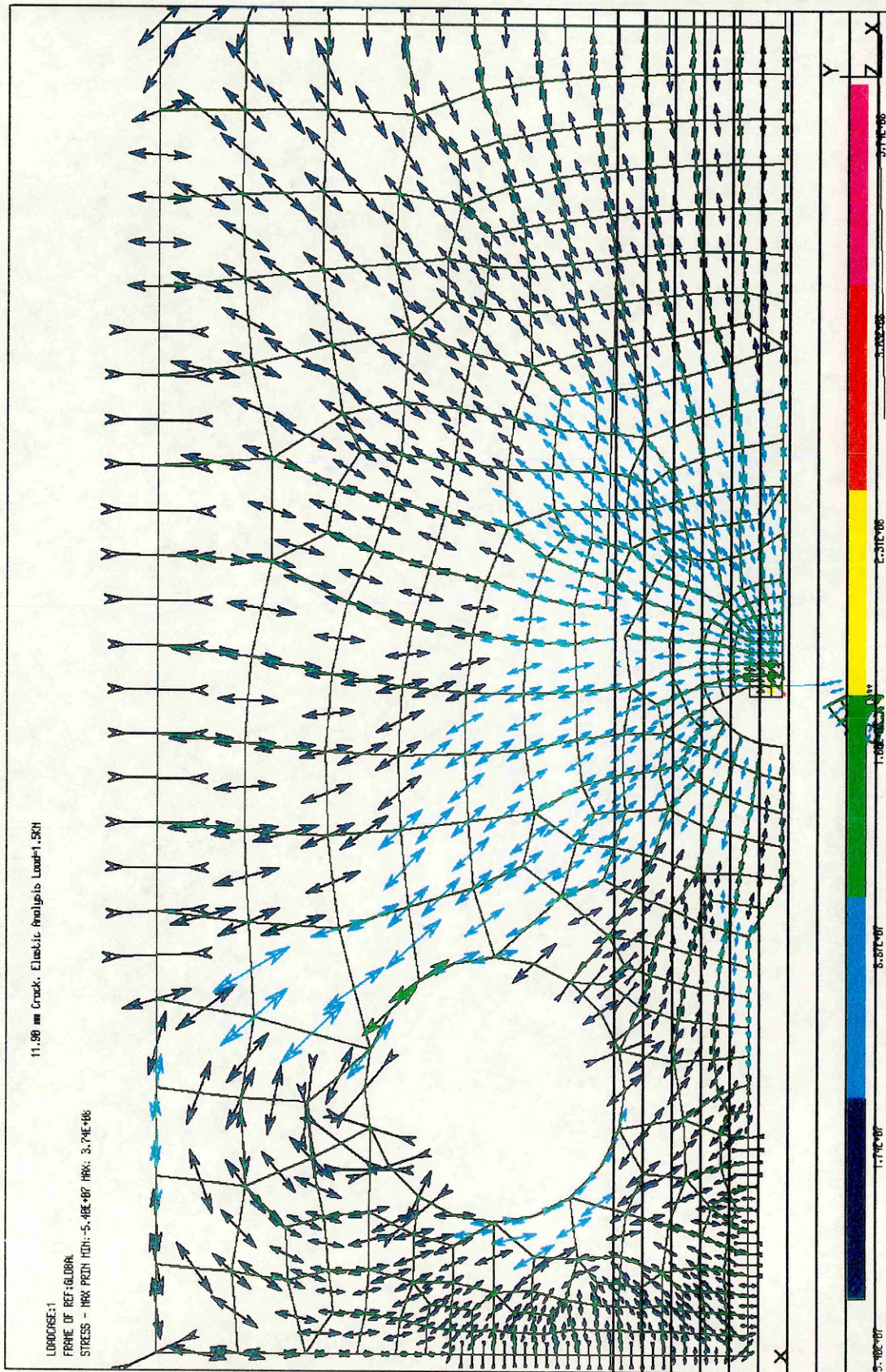


Fig. 4.16 Arrow Plot of Maximum Principal Stresses

only be explained as being the result of the finite element formulation which is based upon the displacements.

In addition to the above observations, Fig.4.17 and Fig.4.18, show the maximum principal stress contours. Each one of these plots deals with a different range of stress values, the objective being to segregate the stresses at various locations. It is clearly indicated in both plots that significant stresses are present in the region of the hole and in the above mentioned location. In fact the direction of these stresses is significant to the effect that, it indicates this as a suitable crack initiation and propagation site. During the experimental program abnormal failures of specimens occurred which after detail examination of the fractured surfaces revealed that the cracks emanated from this region.

A contour plot of the principal stresses in the vicinity of the crack tip superimposed upon the displaced geometry, is illustrated in Fig.4.19. The plot indicates that the crack is enclosed within an intense stress field, and the crack ligament experiences stress levels of very low magnitudes.

Shear stress distributions of maximum shear and Von Mises stresses are presented in Fig.4.20, Fig.4.21 respectively. Both plots show a similar picture of the shear stresses, identifying the crack tip as a stress concentration. The hole region once again is identified to be containing high stress levels. At the apex of the hole where the pressure load is applied, the shear stresses are shown to be higher under the surface which is in agreement to the Hertzian theory of contact. This predicts that shear approaches a maximum beneath the surface.

Finally, the shear stresses in the vicinity of the crack tip are depicted in Fig.4.22 and Fig.4.23, for the Von

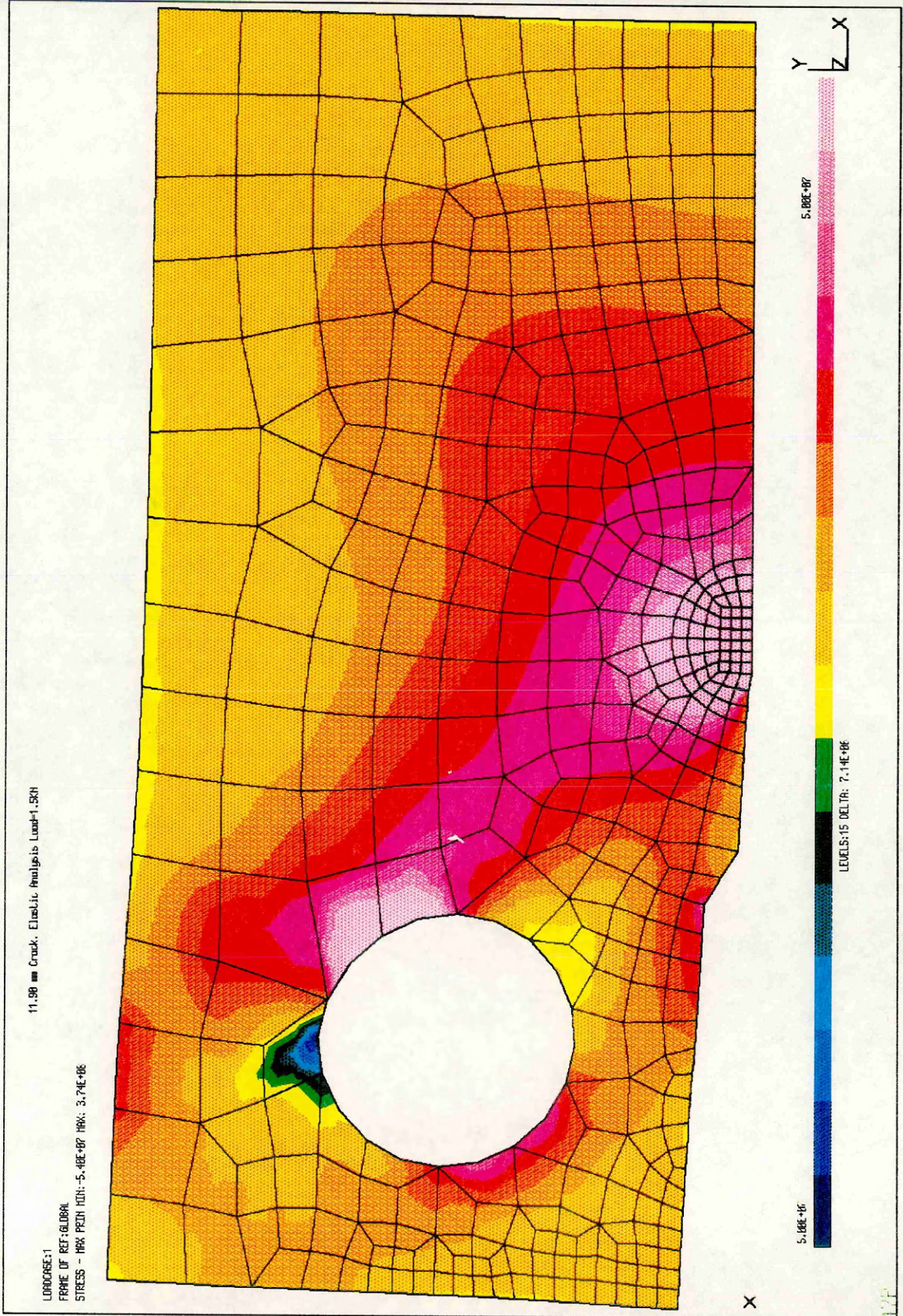


Fig.4.17 Maximum principal stress contours, range 1

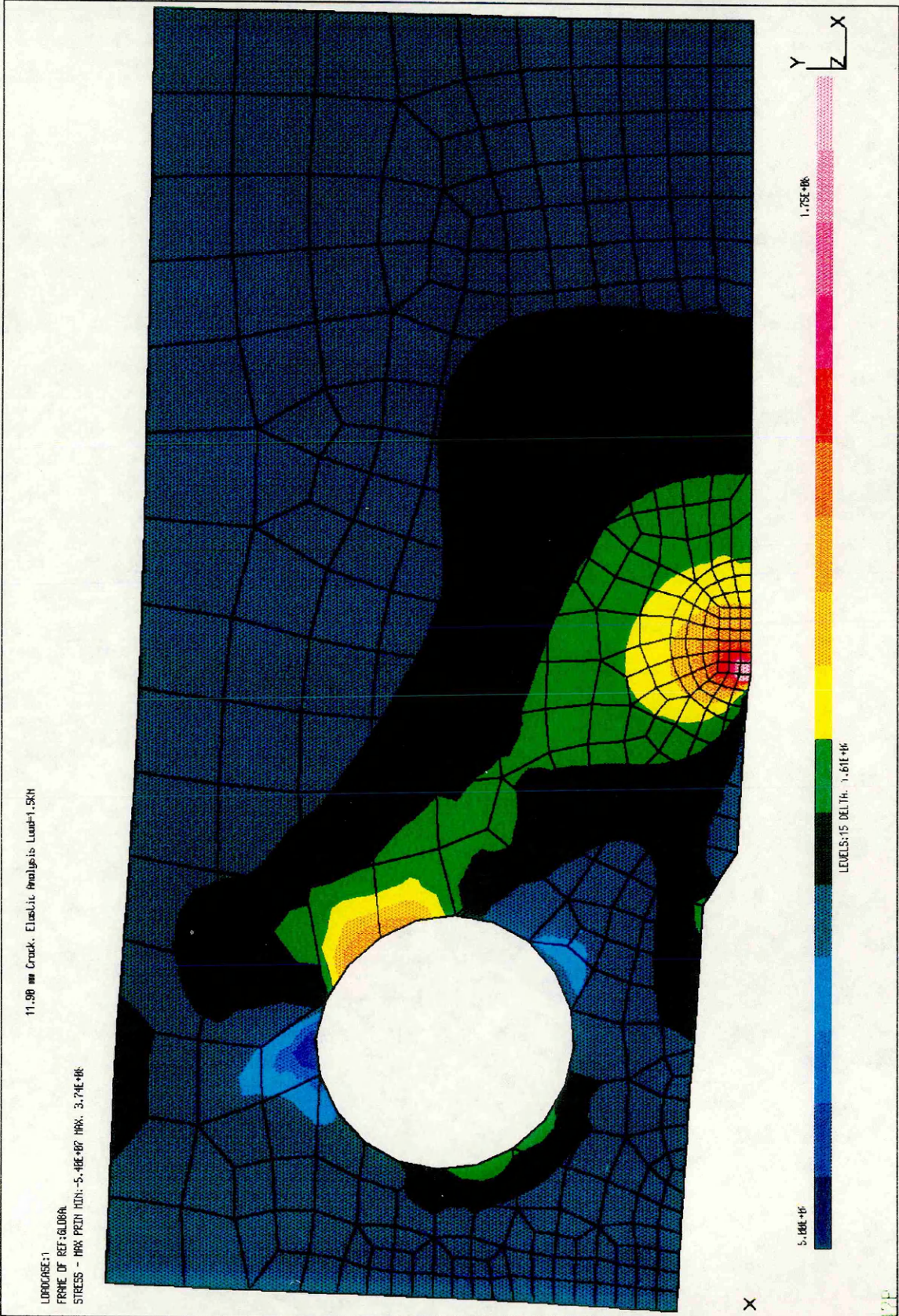
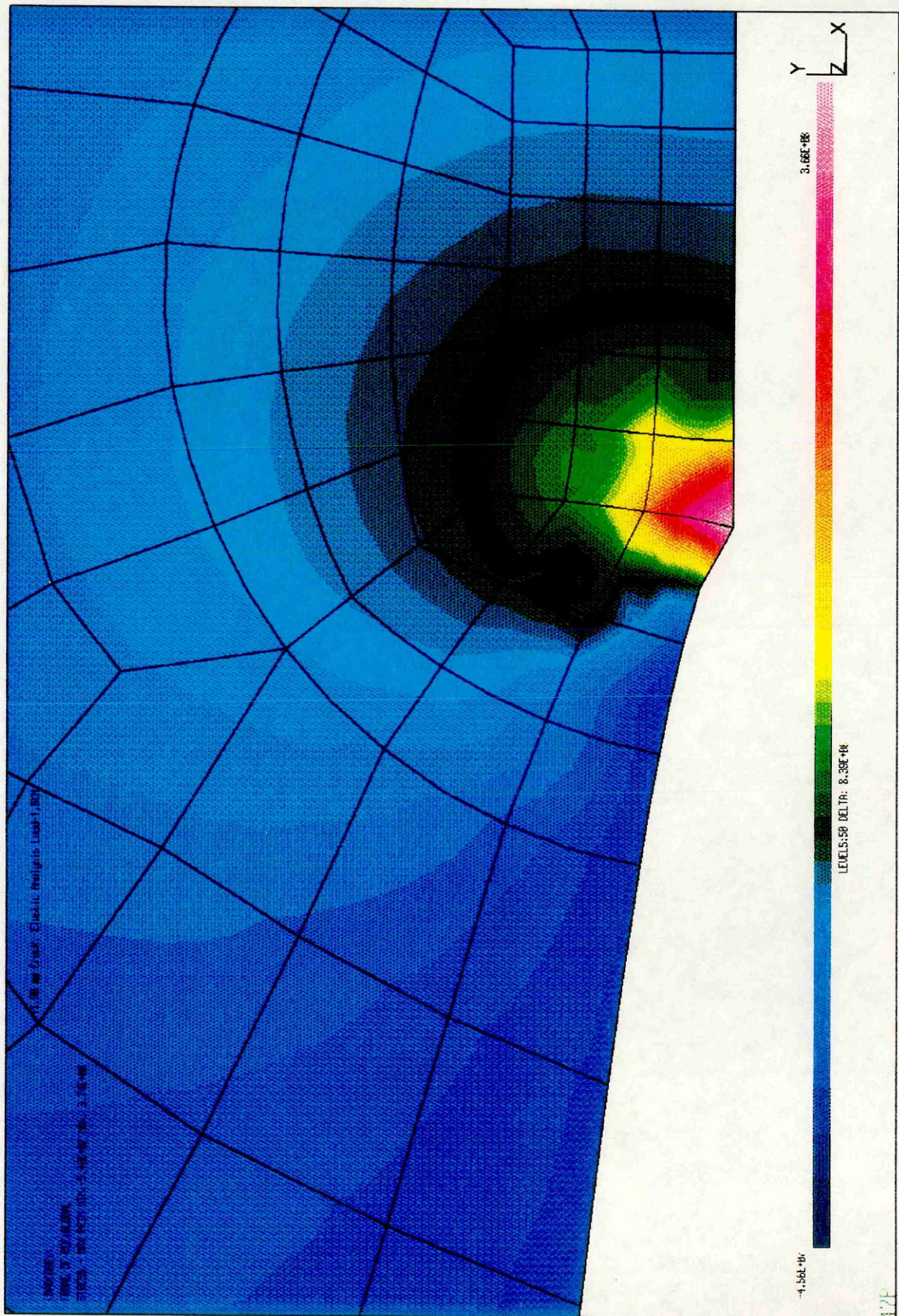


Fig.4.18 Maximum principal stress contours, range 2



g.4.19 Maximum principal stress contours in the vicinity of the crack

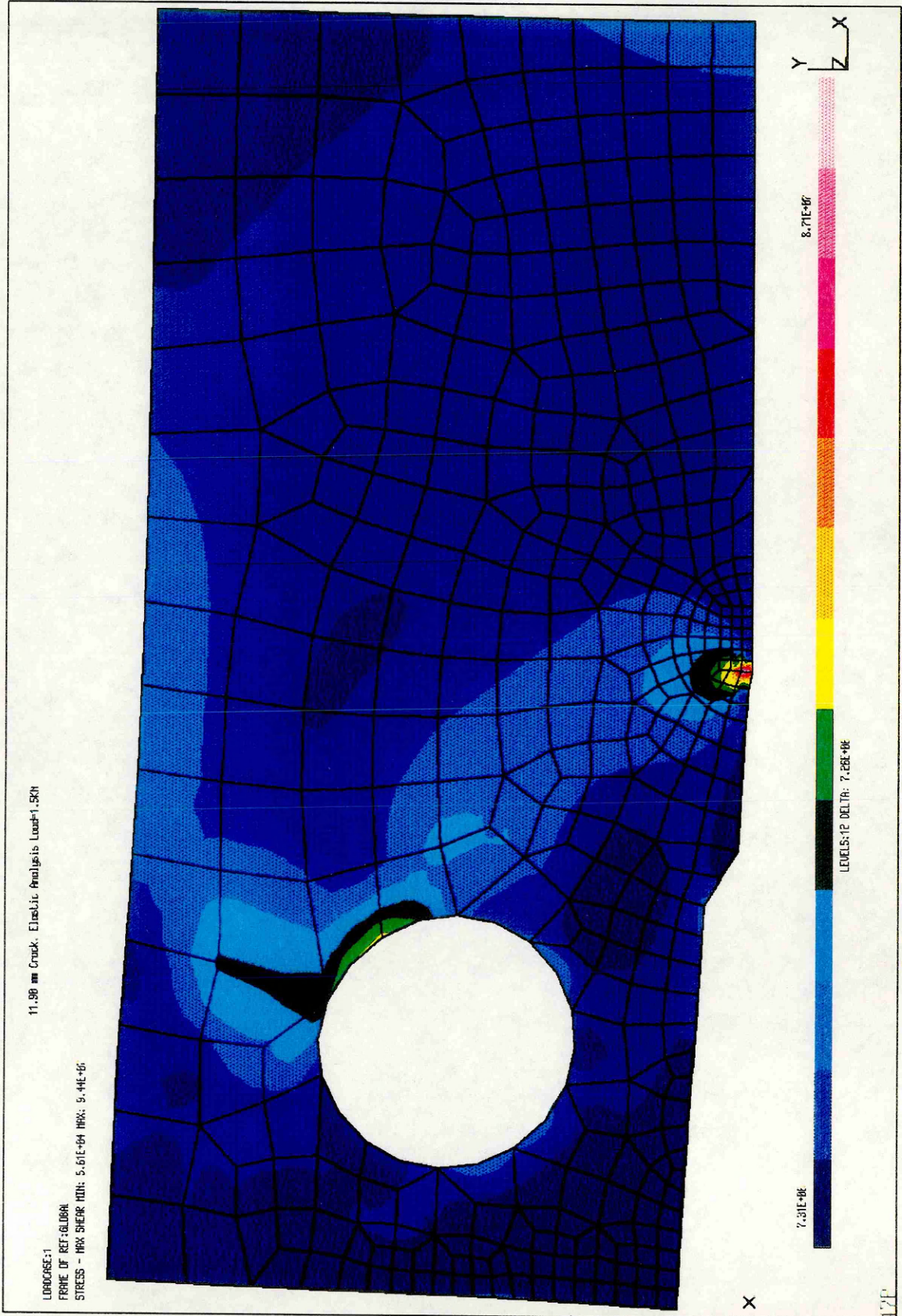


Fig.4.20 Maximum shear stress contours

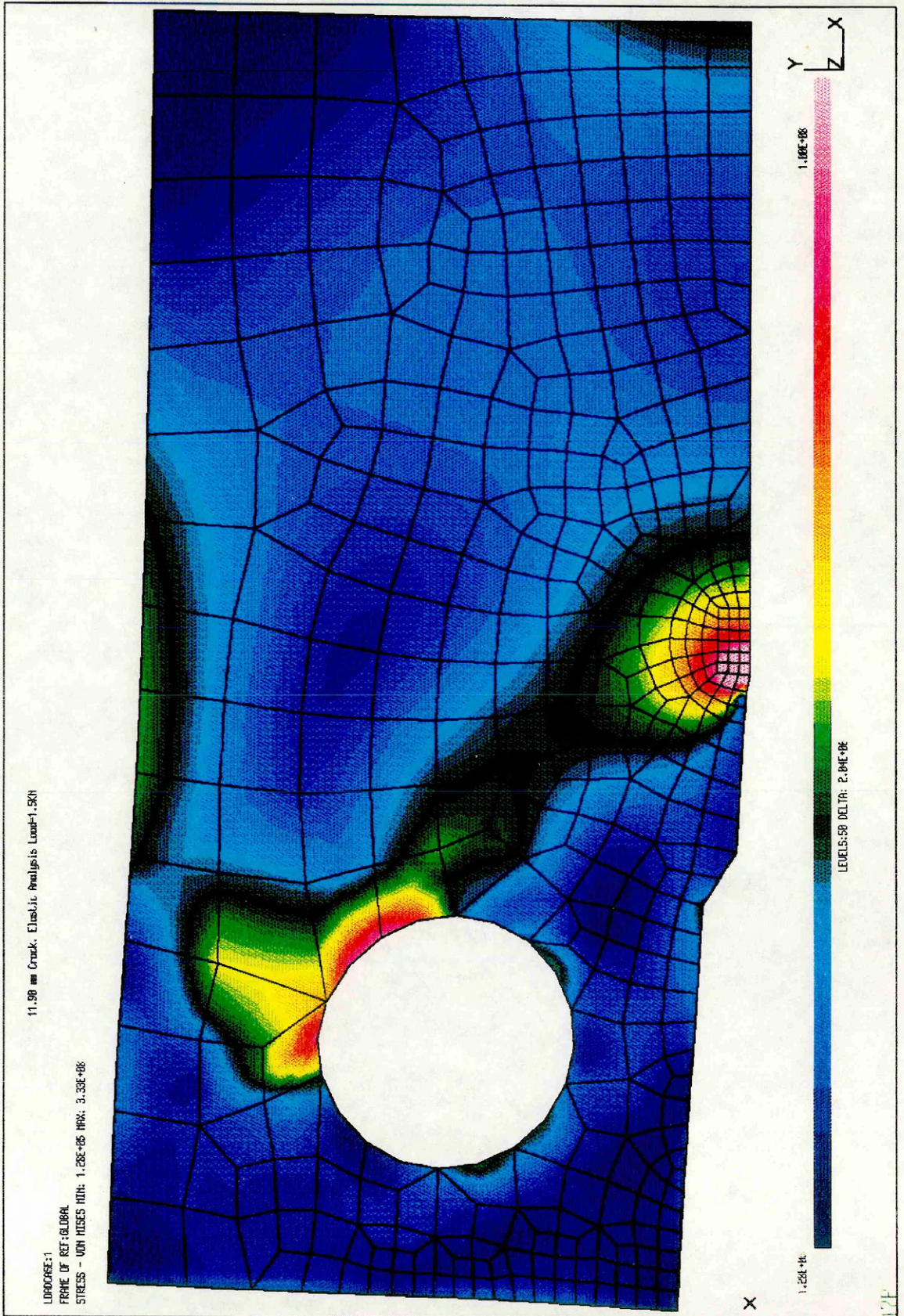


Fig.4.21 Von Mises shear stress contours

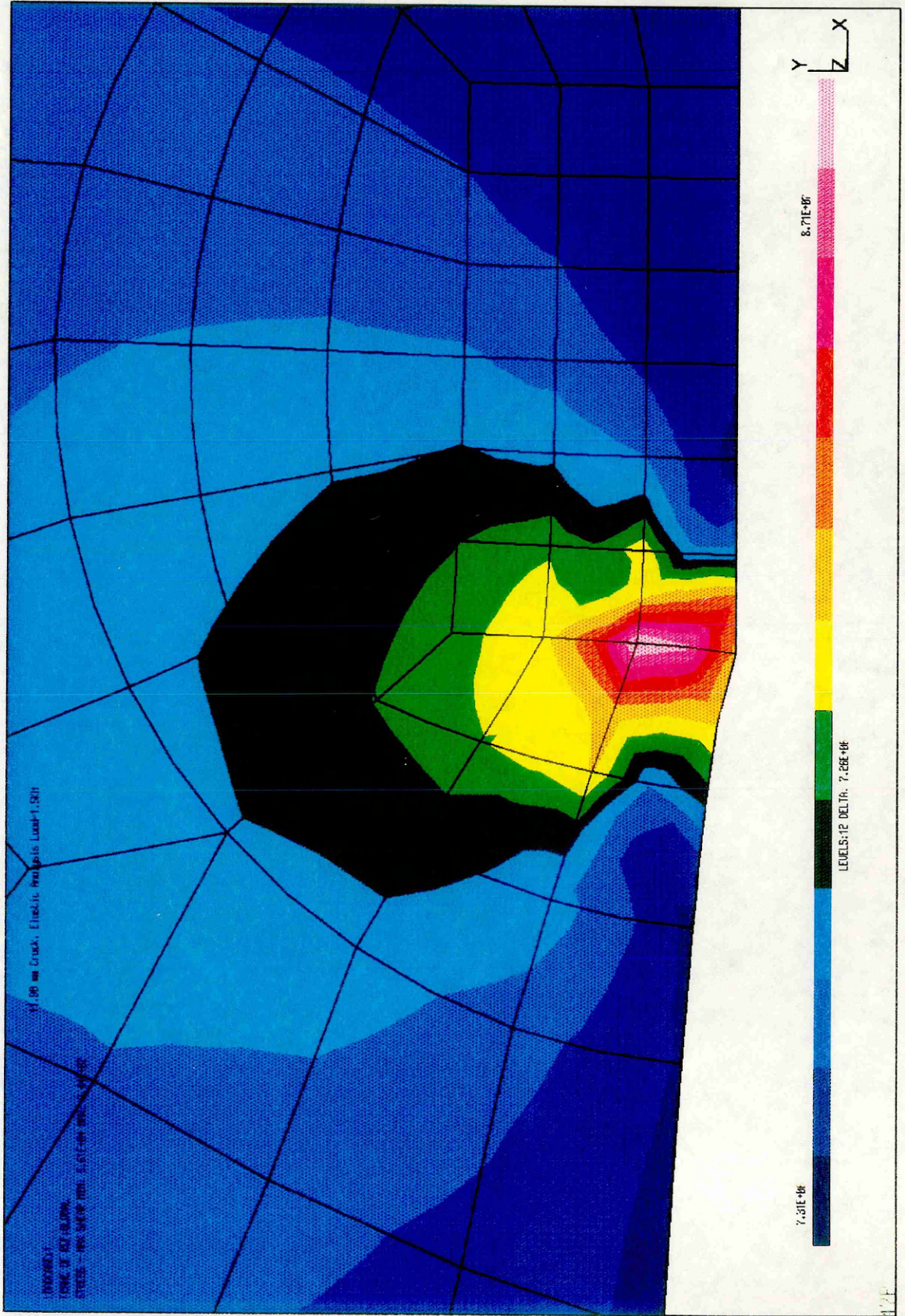


Fig. 4.22 Von Mises Shear Stress Contours
in the Vicinity of the Crack Tip

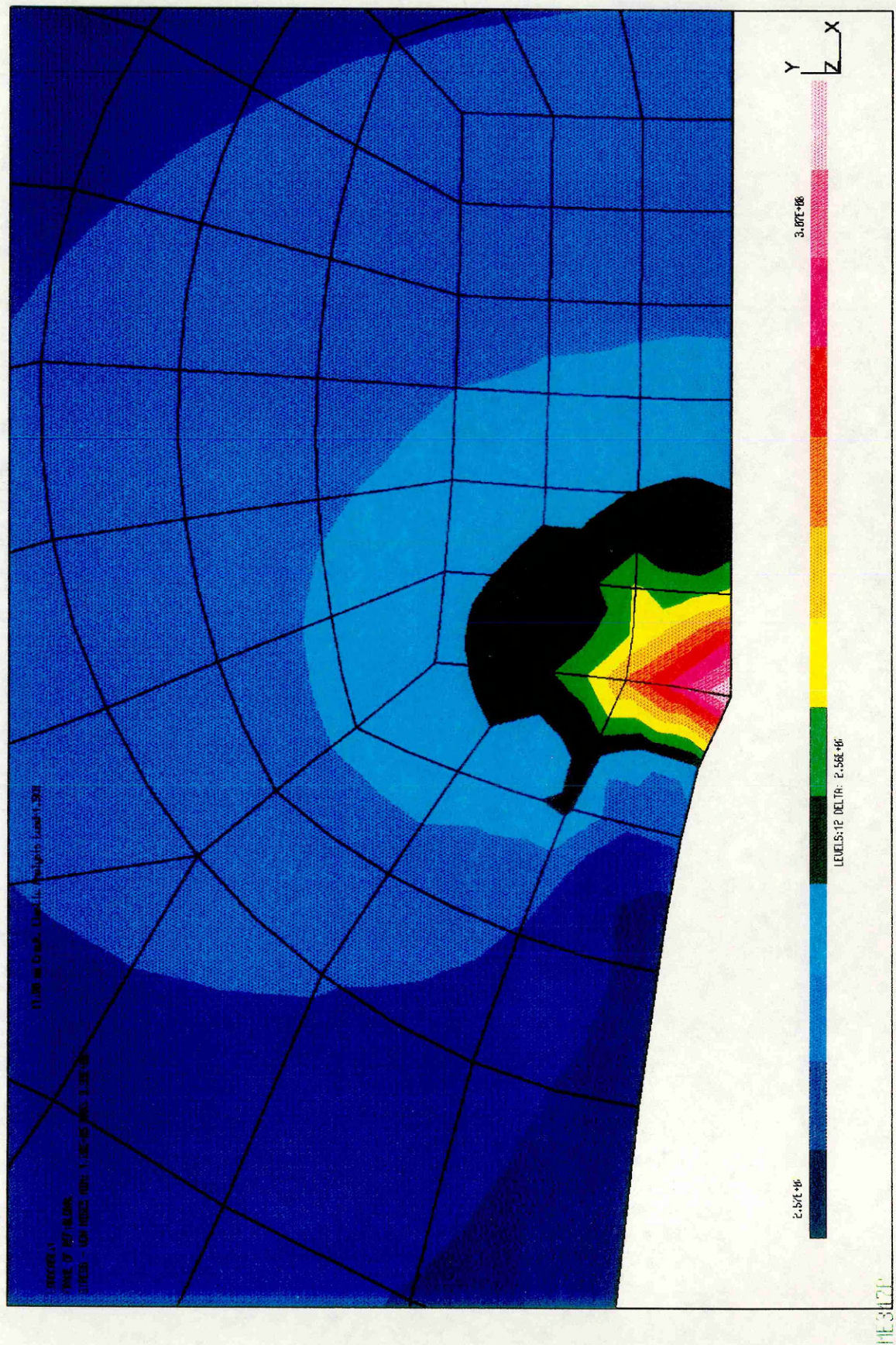


Fig.4.23 Maximum shear stress contours in the vicinity of the crack tip

Mises and Tresca yield criteria respectively. The shape of the shear stress distributions can be related to the shape of the plastic zone since plasticity occurs within this highly stressed region and expands in accordance to the shape of the shear stress distribution.

It is therefore, clear that the shapes of the plastic zone predicted is related to the yield criterion assumed. In the case of the Von Mises yield criterion, Fig.4.22, the shapes of the shear contours surrounding the crack tip are of a bean type, and in the case of the Tresca criterion are of a balloon shape, see Fig.4.23. These configurations compare well with the plastic zone shapes shown in Fig.3.19 determined by a theoretical approach.

It is important to note at this point that the contours presented in the vicinity of the crack tip are rather irregular in shape. This is due to the fact that the plotting program carries out a series of interpolations prior to plotting the contours which near the crack tip are rather inaccurate due to the acute stress gradients present (effect of the stress singularity).

4.6.3 LEFM Parameter Estimates from Elastic F.E. Results

Elastic quasi-static finite element analyses were performed for a series of crack lengths and loads by utilising two F.E. computer packages, namely ABAQUS and PAFEC. The solution results, of nodal displacements and average nodal stresses were used in order to obtain estimates of stress intensity factors, that were compared with published data [4.16, 4.20] and closed form solution [4.21, 4.22] regarding the compact tension geometric configuration.

The close-form solution of stress intensity factor K_I for the compact tension specimen is given in the form of the K-calibration equation,

$$K = (P/B\sqrt{W}) \cdot (2+\alpha) \cdot (1-\alpha)^{-2/3} \cdot f(\alpha) \quad \dots (4.23)$$

$$f(\alpha) = 0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4$$

Where $\alpha = a/W$

a =Crack length

B =Specimen thickness

W =Specimen length

P =Applied load

The expression being valid for $a/W \geq 0.2$ [4.23,4.24].

The Y-displacements of the cracked ligament with respect to the distance from the crack tip are shown in Fig.4.24. These represent just a sample of the total number of cases investigated. This type of data was processed in the manner described in section (4.5.1) so that estimated values of the K_I^* were obtained. Similarly, Fig.4.25 depicts the σ_{yy} stress distribution in the vicinity of the crack tip. These results were also used in accordance to the method discussed in section (4.5.2).

The estimates of K_I^* were subsequently plotted as a function of distance from the crack tip in order to determine the linear portion of the graphs. As the stress intensity estimates are obtained by extrapolation of the curves to $r=0$ (crack tip), a computer program was utilised to carry out the fit of a straight line by means of a least squares procedure. Fig.4.26, shows a plot of the fitted straight lines for stress and displacement data.

The evaluation of the J-energy integral is particularly suited to finite element calculations as the data at nodal locations on defined contours is used. The J-integral

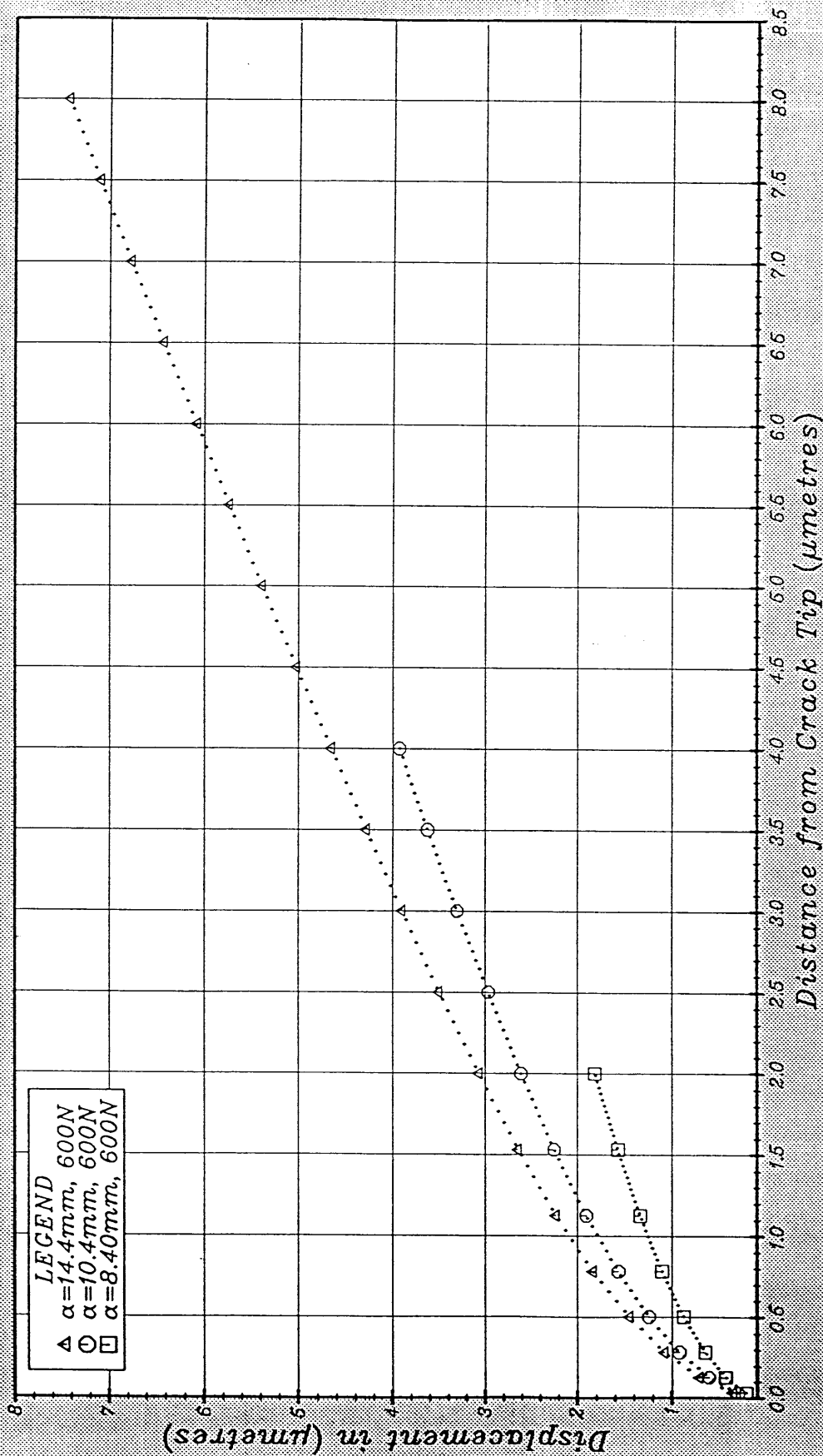


Fig.4.24 Variation of cracked ligament displacement as a function of distance from the crack tip for three different crack lengths. ABAQUS elastic results.

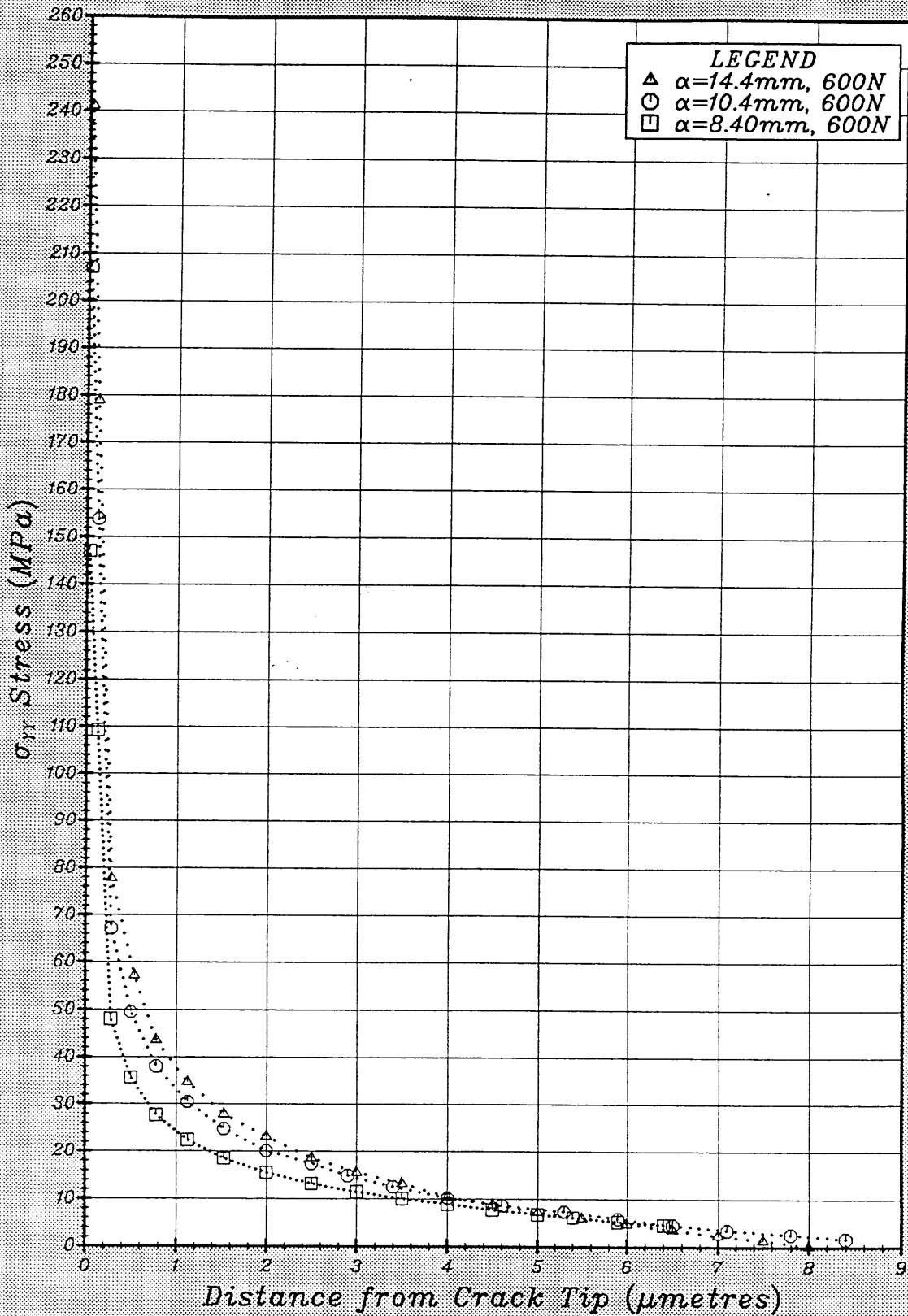


Fig.4.25 σ_{yy} stress as a function of distance form the crack tip for three different crack lengths. Results from elastic ABAQUS solutions.

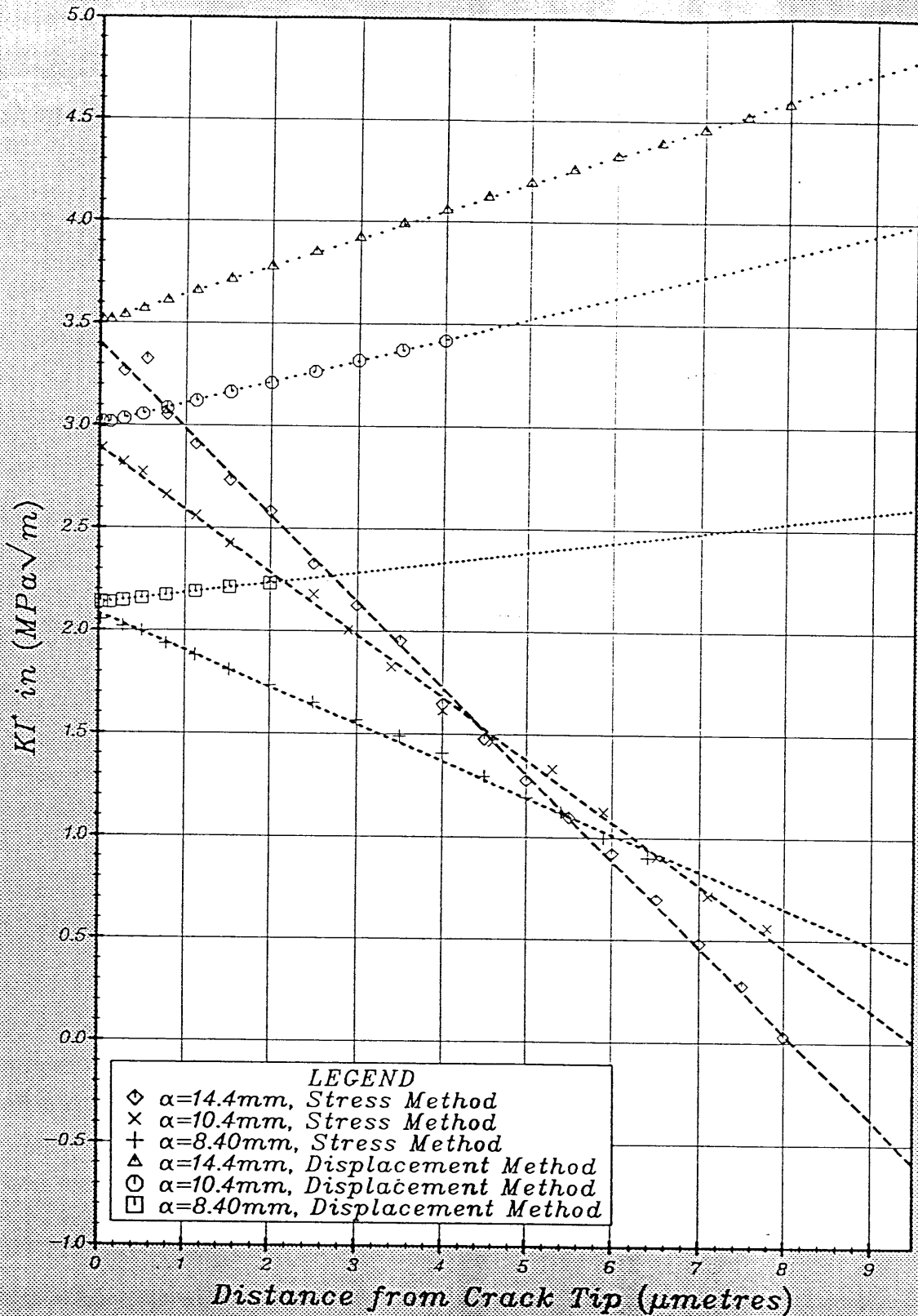


Fig.4.26 K_I variation as a function of distance from the crack tip, estimated by the stress and displacement methods from FE elastic analysis ABAQUS data.

program detailed in Appendix 2, was utilised and thus, it was possible to evaluate J values. These values were converted to stress intensity factors by means of the equation,

$$K=[J.E/(1-\gamma^2)]^{1/2} \quad \dots(4.24)$$

The above equation is limited within the elastic limit and for the plane strain state of stress.

The displacements of the corner nodes at the face of the specimen and adjacent to the slot were utilised for evaluating stress intensity factors by means of the compliance technique. In addition, it is possible to use the displacements in relation to applied load and the geometry to predict the crack length. This is of particular importance when dealing with experimental procedures where a clip-on-gauge may be used for evaluation of the crack length, during fracture toughness or crack propagation testing, [4.25, 4.26]. This has been shown to be a very accurate technique.

The results for stress intensity factor estimates utilising the aforementioned methods are given for the purpose of comparison in Table 4.1.

It can be observed that all techniques compare very well with the K-calibration estimates. The displacement and stress methods give good approximations of stress intensity factors, but it is noticeable that the displacement based formulation leads to more accurate predictions when compared to the stress method. The least time consuming method, is the compliance method based upon a single displacement value. In general, it has been found that the compliance method underestimates the stress intensity factor.

STRESS INTENSITY FACTOR ESTIMATES (MPam^{1/2})

	CASE NUMBER	CRACK LENGTH (mm)	LOAD (N)	K-CALIBRATION ASTM - E647/86	STRESS METHOD	DISPLACEMENT METHOD	COMPLIANCE TECHNIQUE	J-INTEGRAL
ABAQUS NON SINGULAR	1	2.0	100.0	0.356	0.342	0.372	0.332	0.338
	2	4.0	100.0	0.419	0.406	0.423	0.397	0.423
	3	8.0	100.0	0.583	0.561	0.586	0.552	0.583
ABAQUS SINGULAR	4	2.0	300.0	1.068	1.026	1.116	0.997	1.016
	5	4.0	300.0	1.256	1.216	1.269	1.193	1.274
	6	8.0	300.0	1.748	1.682	1.757	1.657	1.754
PAFEC SINGULAR	7	2.0	450.0	1.602	1.538	1.673	1.496	1.597
	8	4.0	450.0	1.884	2.239	2.138	1.789	2.155
	9	8.0	450.0	2.622	2.523	2.636	2.485	2.641
PAFEC NON SINGULAR	10	2.0	600.0	2.136	2.052	2.232	1.995	2.129
	11	4.0	600.0	2.513	2.890	3.017	2.386	3.037
	12	8.0	600.0	3.496	3.365	3.515	3.313	3.522

Table 4.1 Comparison of KI Estimation Methods

The reason for this has been traced to the fact that the procedure is based upon the crack length evaluation with the aid of expressions derived by Saxena and Hudak [4.20]. It is therefore concluded, that the displacements calculated by the finite element method must be the real factor affecting the accuracy, if one accepts that the Saxena and Hudak formulation is accurate.

Finally, the values obtained from the J-integral are satisfactory and generally compare well with all other techniques. Below, the average deviation for each technique estimates from the K-calibration values are shown.

- a. Stress Method 3.7%
- b. Displacement Method 3.0%
- c. Compliance 5.6%
- d. J-Integral 2.9%

All values represent the average deviation from the K-calibration, for the total number of cases examined for each method.

4.7 ELASTO-PLASTIC F.E. ANALYSIS

Localised plasticity in the vicinity of the crack tip, is a direct result of the severe stress concentrations and stress gradients present. The experimental studies included the investigation of the effect of overloads, i.e. loads of higher peak magnitudes than the constant amplitude cyclic loading. The plasticity developed in the vicinity of the crack tip due to the application of overloads is of particular interest. Hence, theoretical analyses utilising elasto-plastic F.E. methods were carried out in order to examine the resulting stress field. The PAFEC software package was utilised for this purpose.

As it has been previously discussed, the finite element analysis of elasto-plasticity is based upon an iterative procedure, and in particular, to the initial strain initial stress method of solution, detailed in Section 4.2.2.

Preliminary detail studies of elastic solutions were carried out, and are detailed above. Specifically, the equivalent or Von Mises stresses were considered and it was possible to relate the equivalent stress levels to the applied loading. By linear scaling, predictions of the loads sufficient to bring the material to the threshold of elasticity in the vicinity of the crack tip were established. Thus, the first load increment was set so as to elevate the localised crack vicinity stresses to the limit of the elastic behaviour. Subsequent load increments, drew the material into the plastic region in small steps of the order of 5% of the total load; each load step incrementally enlarging the plastic deformation. It is advisable that small load increments are adopted in order that the accuracy of solution is maintained.

The application of the total load, was followed by partial elastic unloading to a load level equal to the minimum load encountered in constant load amplitude cycling, e.g. 1.5KN. Finally, the unloading was extended to the zero load level, full unloading. It has therefore, been possible to obtain residual stress distributions in the vicinity of the crack tip.

Full unloading following the overload, yields direct results of residual stresses. In the case of partial unloading, the stresses are the sum of elastic and plastic stress components. Residual stresses, were determined by means of subtracting the average nodal stresses of an elastic solution from the results of the partial unloading

elasto-plastic analysis. Similarly, residual strain components may be determined.

4.7.1 Elasto-Plasticity, Procedures

The elasto-plastic studies carried out utilised the models of the geometry previously developed for elastic analysis. Some of the models were modified in terms of element numbers, whose density was increased in the vicinity of the crack tip as it was intended to study the formation and spreading of plasticity as a function of applied load. In order to account for the post-yielding behaviour of the material, a special module was introduced in the data file to include the plastic properties. In fact, the material was represented by a an elastic linear strain hardening model. The yield was set as the 0.2% proof stress at 460.0MPa, with a strain hardening modulus of 15.0GPa.

The elasto-plastic solutions were obtained using incremental boundary loads and the method of successive elastic solutions. The appropriate solution for the predetermined increment of load was obtained iteratively using a compromise method of solution between initial strain and initial stress approaches. For the solutions reported here, the accuracy criterion adopted was that the discrepancy resulting from successive estimates of the calculated equivalent stress at each Gauss point should not exceed 1.0% of the provided equivalent stress (material property) at a given value of an equivalent plastic strain.

4.7.2 Elasto-Plastic Analysis Results

Two crack lengths were considered, 11.9 and 15.9mm. These crack lengths were selected during the experimental program for the application of overloads, single and block. The cases of loading studied were those of single

overloads interrupting the constant load amplitude cycling with load ratio $R=0.6$ over a mean load $P_{mean}=2.0KN$; the overload ratios being $R_{ovl}=1.6, 1.8, 2.0$, see Fig.4.27 for terminology. Following the incremental loading to the full level of load, elastic unloading was carried out in the first instance partially, to $P_{min}=1.5KN$ representing the minimum of the constant load amplitude cycle, and then to zero load representing the full unloading condition.

Figures 4.28 and 4.29 show sequential illustrations of the development and spread of plasticity ahead of the crack tip for a range of applied loads used in the present study, for crack lengths of 11.9 and 15.9mm respectively. The element sizes in the grids depicted are $330\ \mu m \times 330\ \mu m$. It is clear that as the crack length is increased the amount of plasticity increases as the severity of the stress concentration is amplified. The shapes of the plastic zones presented are by no means indicative of the real shape of the plastic zone. The plasticity evolution sequences have been developed from information regarding the yielding of individual Gauss points within each element.

Figure 4.30, shows the variation of the plastic zone length as a function of the applied load. In addition, the Irwin and Dugdale plastic zone predictions are presented for comparison. It can be seen that there exist significant discrepancies between the finite element results and the two theoretical models. In fact the finite element predictions are lower than the theoretical ones for the whole range. It should be noted that the theoretical solutions of Irwin and Dugdale are based upon an elastic perfectly plastic material whereas, the presented analysis considers an elastic-linear strain hardening material behaviour, and plane strain conditions.

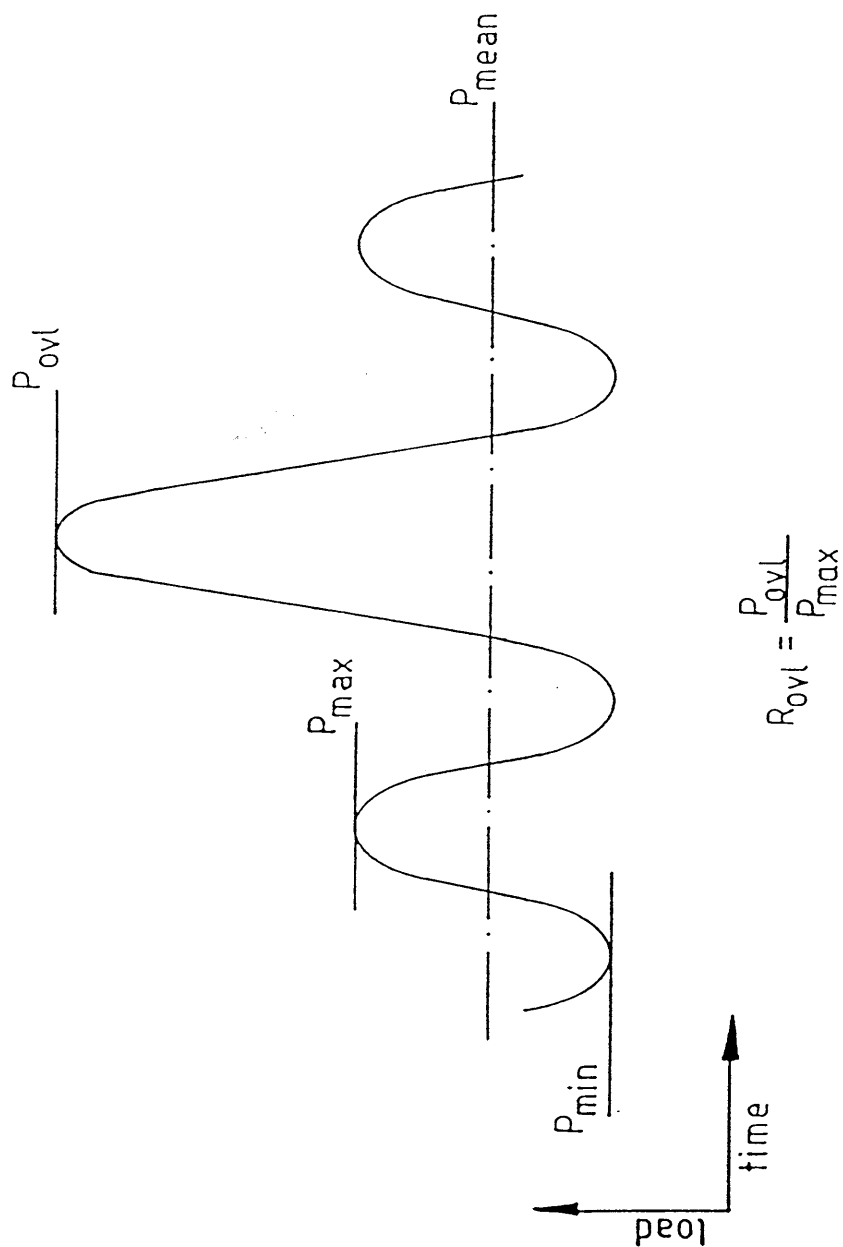


Fig.4.27 Definition of Overload Terminology

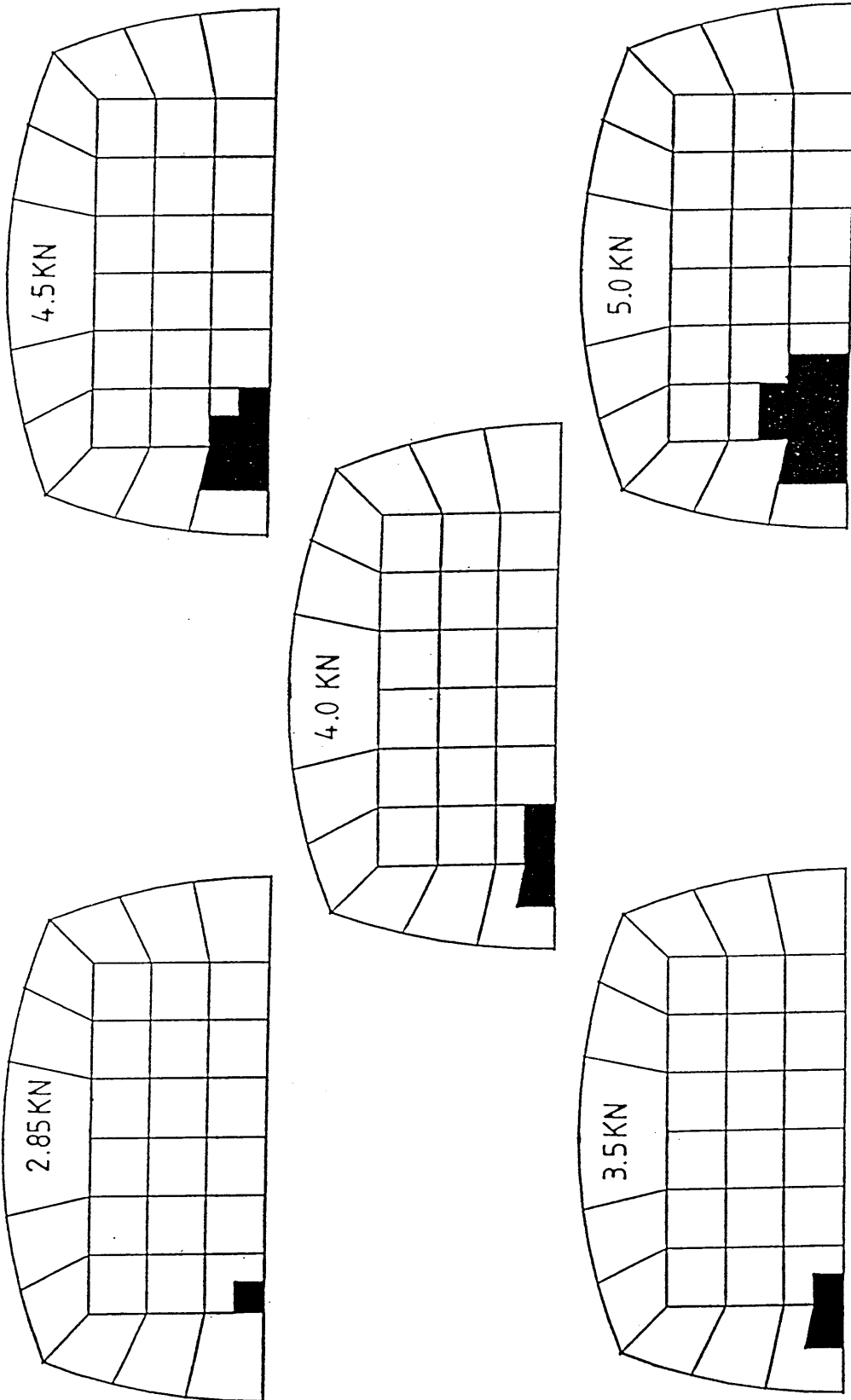


Fig. 4.28 Sequential illustrations of plasticity development as predicted by F.E. analysis for a maximum load of 5.0 kN and crack length $a=11.9\text{mm}$

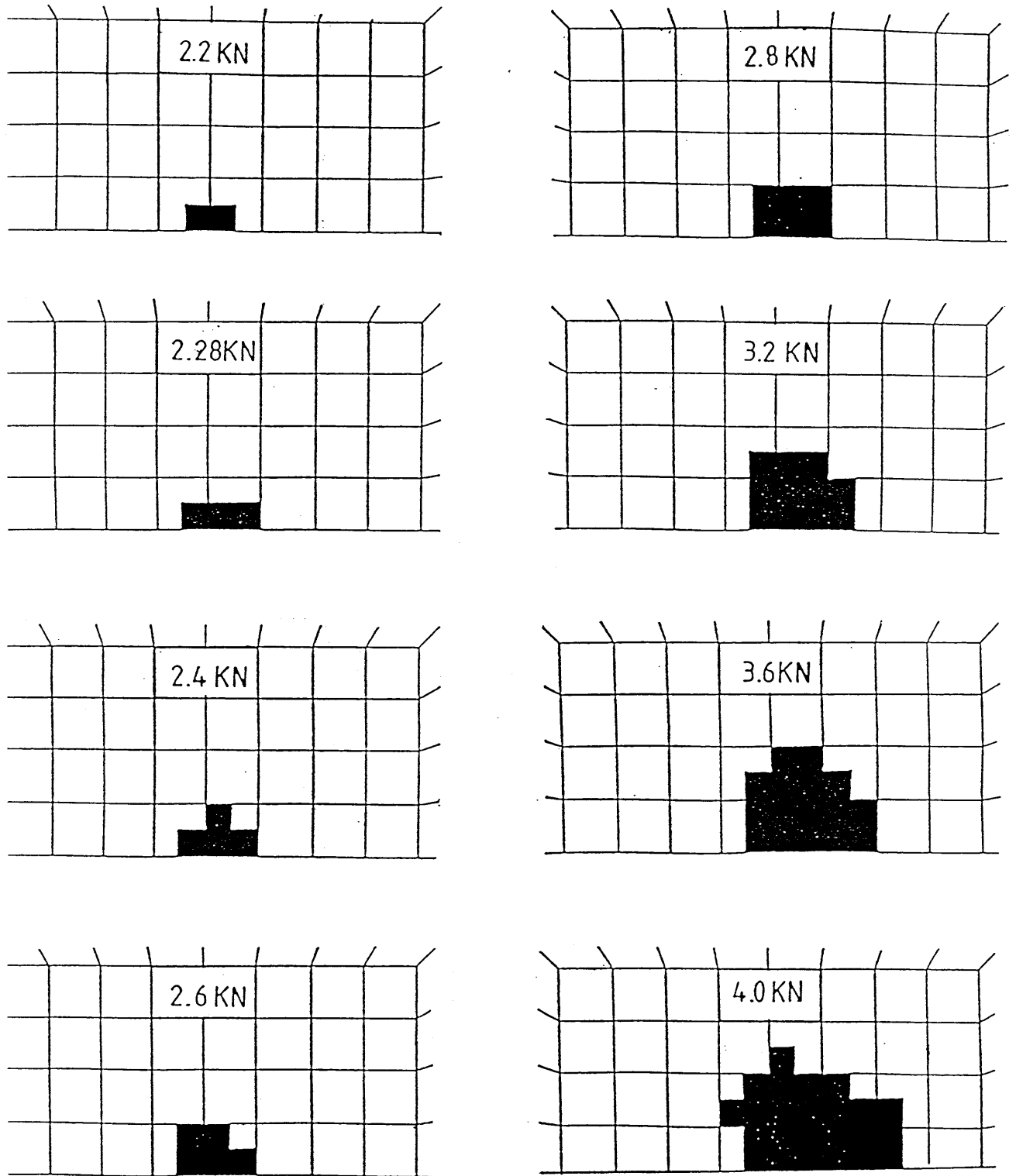


Fig. 4.29 Sequential illustrations of plasticity development as predicted by F.E. analysis for a maximum load of 4.0KN and crack length $a=15.9\text{mm}$

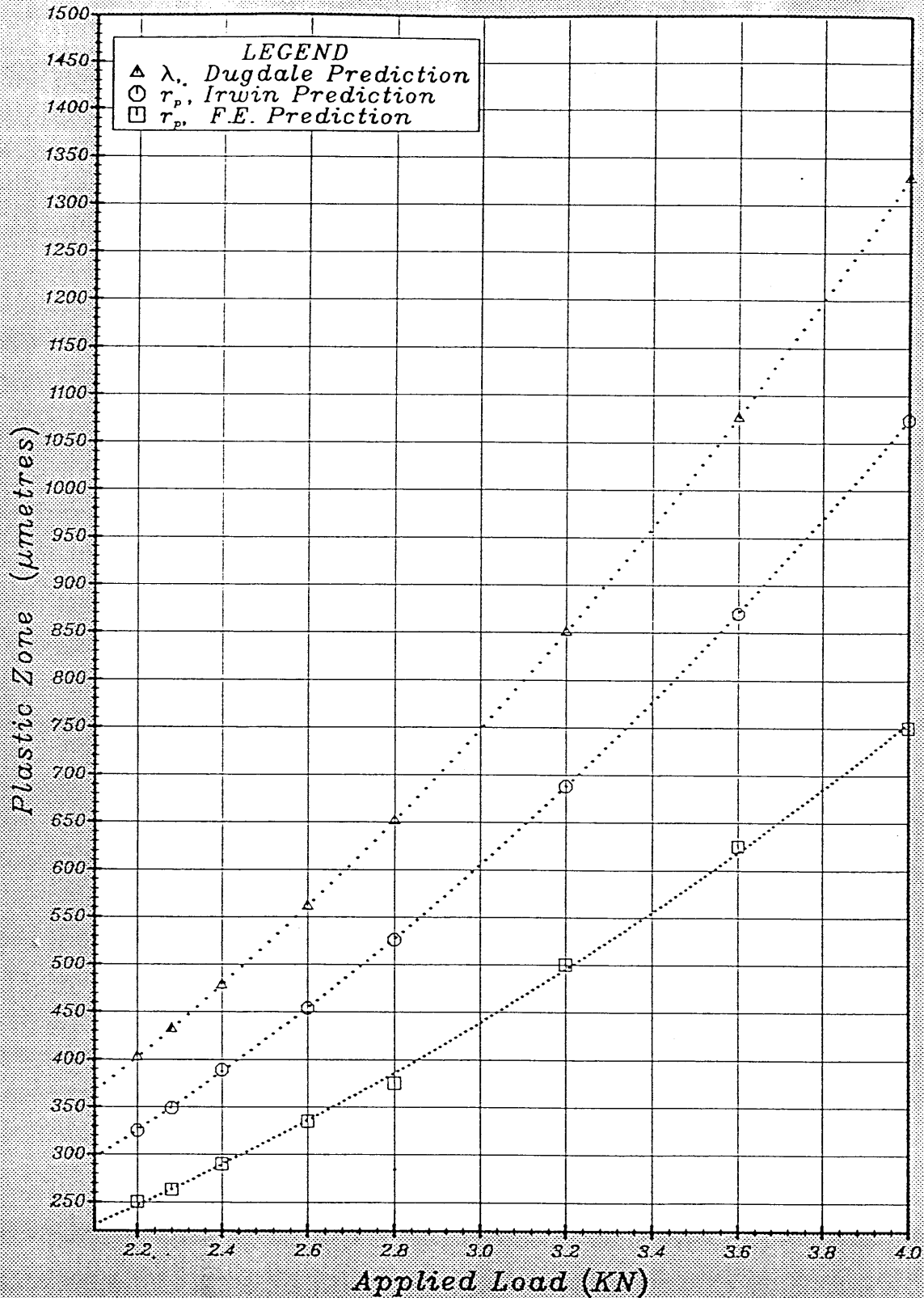


Fig.4.30 Comparison of plastic zone sizes predicted by the Irwin, Dugdale models and plane strain elasto-plastic F.E. analysis, for a crack length $a=15.9$ mm.

Results published by Meguid et al [4.27], show that the extent of the plastic zone predicted by finite elements for a double-edge cracked plate, fall between the Irwin and Dugdale predictions. The analysis though has been carried out utilising a plane stress condition. It has been discussed by Broek [4.21], that the plastic zone in plane strain is smaller than the plane stress one, due to the fact that the effective yield stress in case of plane strain is larger than the uniaxial yield stress. The maximum stress in the plane strain plastic zone can be as high as three times the uniaxial yield stress of the material.

It can therefore be concluded, that the theoretical plastic zone models of Irwin and Dugdale overestimate the size of the plastic zone, and their applicability is restricted to the plane stress state of stress and elastic-perfectly plastic material behaviour.

The stress distribution ahead of the crack after the application of the full load that causes plasticity in the local area is shown in Fig.4.31. It can be observed that the stress distribution of σ_{yy} is very similar to the distribution obtained from an elastic solution. It attains a very high magnitude at the crack tip and decays exponentially as the distance from the crack increases. Similarly the σ_{xx} stress is at its maximum value near the crack tip and it also decays rapidly as the distance from the crack tip increases, which is in agreement with the results obtained from elastic solutions.

Following full elastic unloading to zero load, residual stresses are locked in the material, which are illustrated in Fig.4.32 for the σ_{xx}^r and σ_{yy}^r stress components. The residual stresses in the close proximity of the crack tip are compressive for both components of stress. The σ_{yy}^r stress quickly becomes tensile in nature and decays to

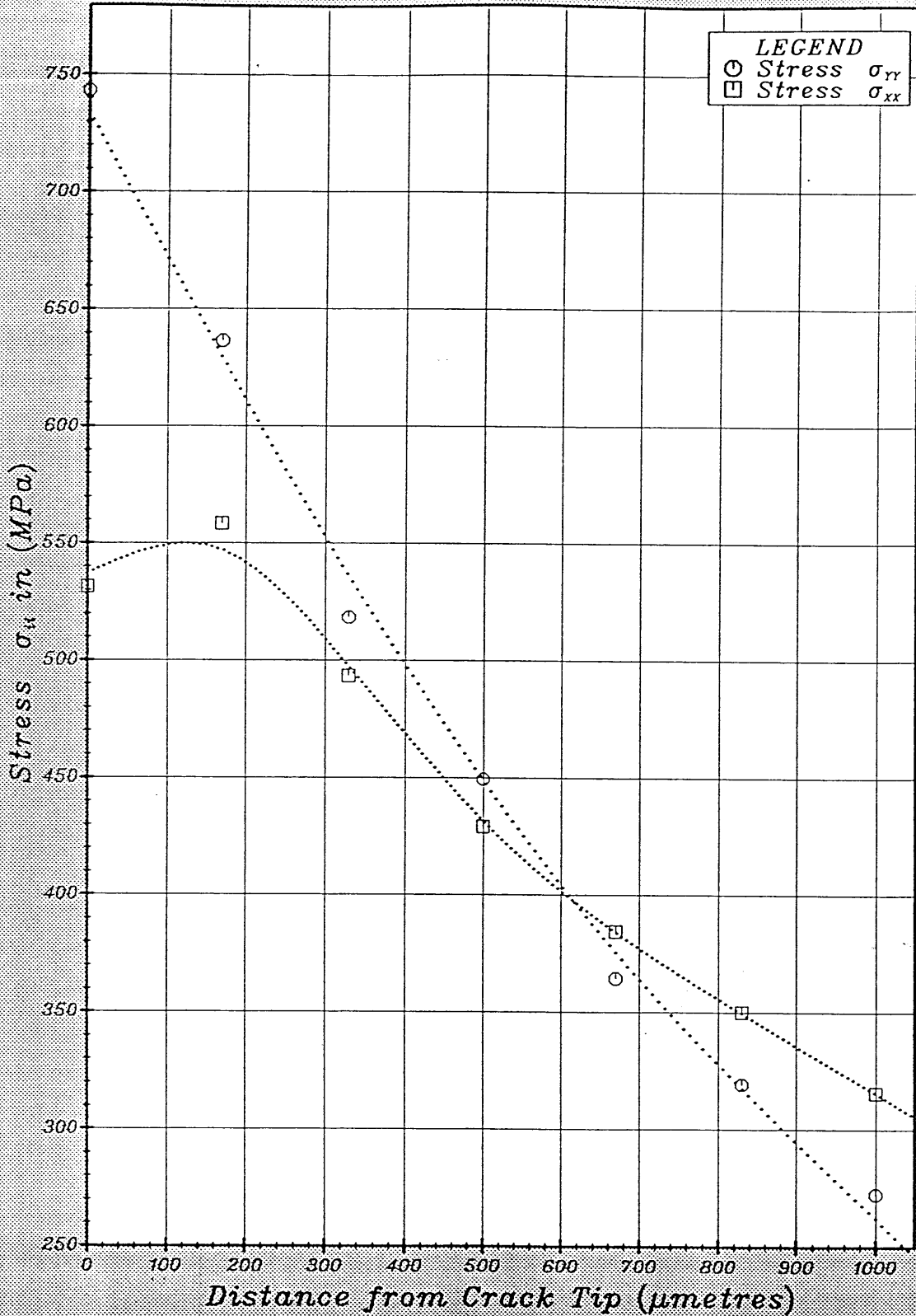


Fig.4.31 Variation of σ_{xx} and σ_{yy} stress components as a function of distance from the crack tip, during the application of an overload of 5.0KN, Crack length $a=11.9\text{mm}$.

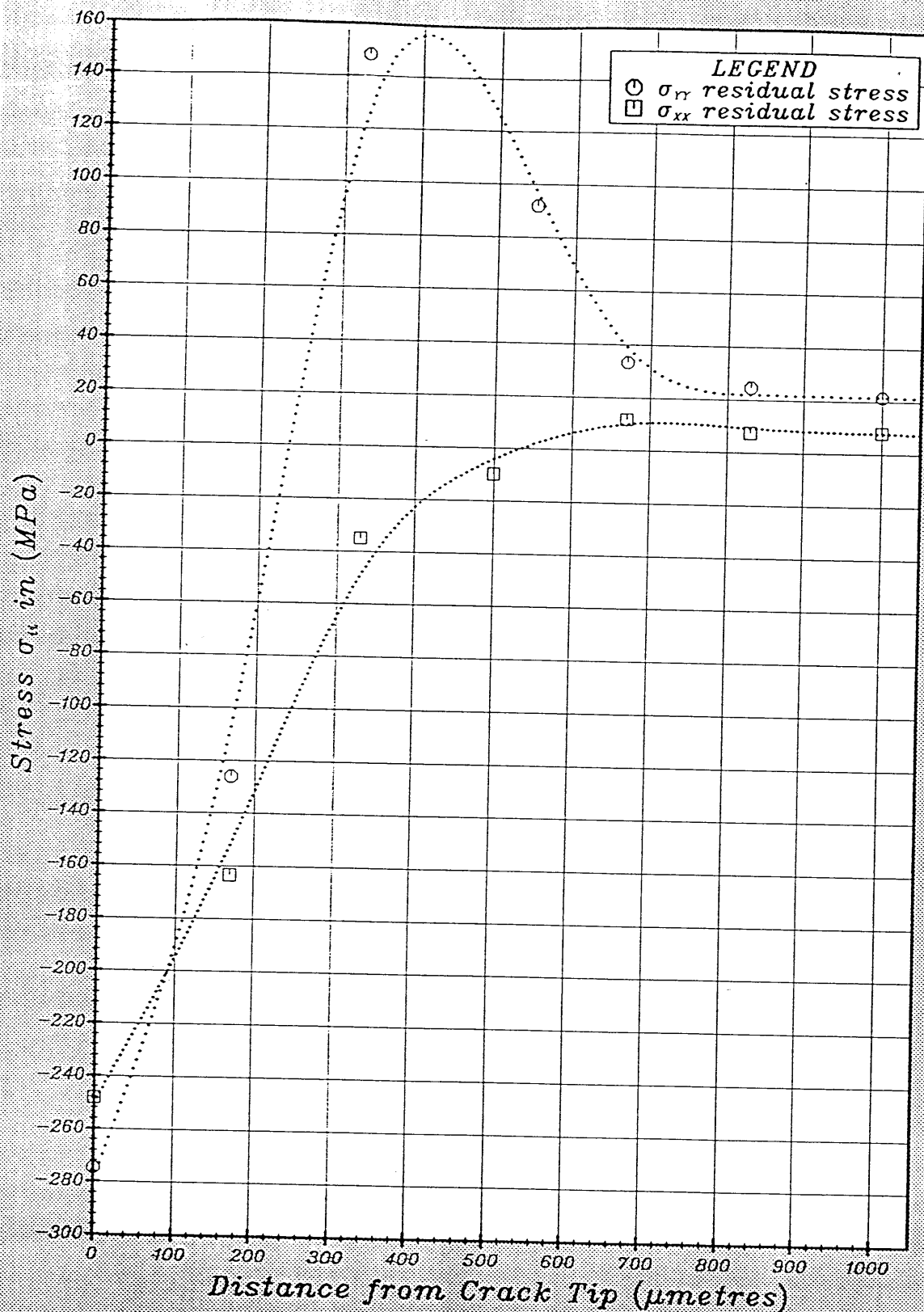


Fig.4.32 Residual stress distributions of σ_{xx} and σ_{yy} , for a crack length $a=11.9\text{mm}$, following full elastic unloading from 5.0 to 0.0 KN.

near zero after its maximum. The residual σ_{xx}^r stresses, though compressive in the proximity of the crack tip, do not become tensile to the degree observed for the σ_{yy}^r component. They tend to degenerate to zero, by acquiring an intermediate very small positive magnitude.

The residual stress distributions due to partial unloading subsequent to the application of an overload are shown in Figures 4.33 and 4.34 for the σ_{xx} and σ_{yy} stress components respectively. The total distribution represents the nodal average stresses following the unloading, whereas, the elastic stresses are obtained by an elastic solution with a load equal to the unloading load level. The residual stresses have been determined by superposition of the total and elastic distributions. It is clearly indicated that the σ_{xx}^r as well as the σ_{yy}^r stresses are compressive in the vicinity of the crack tip; this is in agreement to the results corresponding to total elastic unloading. In addition the residual stress field of σ_{yy}^r is more intense than the σ_{xx}^r .

Experimental investigations regarding the stress field in the vicinity of the crack tip subsequent to the application of an overload are very difficult to trace.

One such publication by Busch and Lebrun [4.28], has used X-ray diffraction as the means of quantifying experimentally the stress field. In the study the case of a single tensile overload was examined. It has been concluded that the residual stress distribution σ_{yy}^r , following the overload is compressive in the proximity of the crack tip and within a small distance from the crack the distribution becomes positive in nature, before finally decaying to near zero. The reloading of the material results to a distribution which is very different to the one obtained near a crack under elastic conditions. In fact the stress at the crack tip is not the maximum,

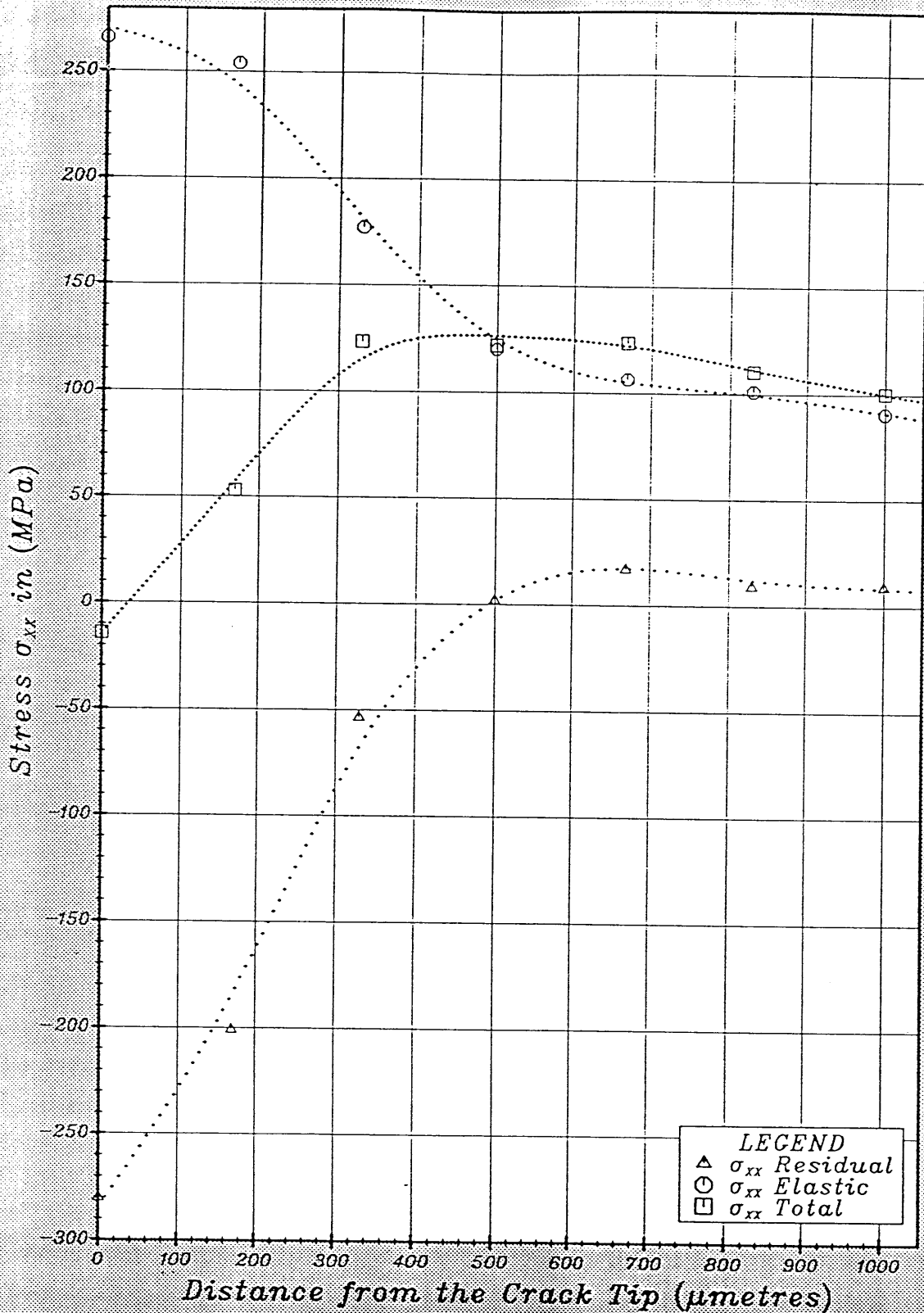


Fig.4.33 Variation of σ_{xx} stress component as a function of distance from the crack tip, for a crack length=11.9mm, following partial unloading from 5.0 to 1.5 KN.

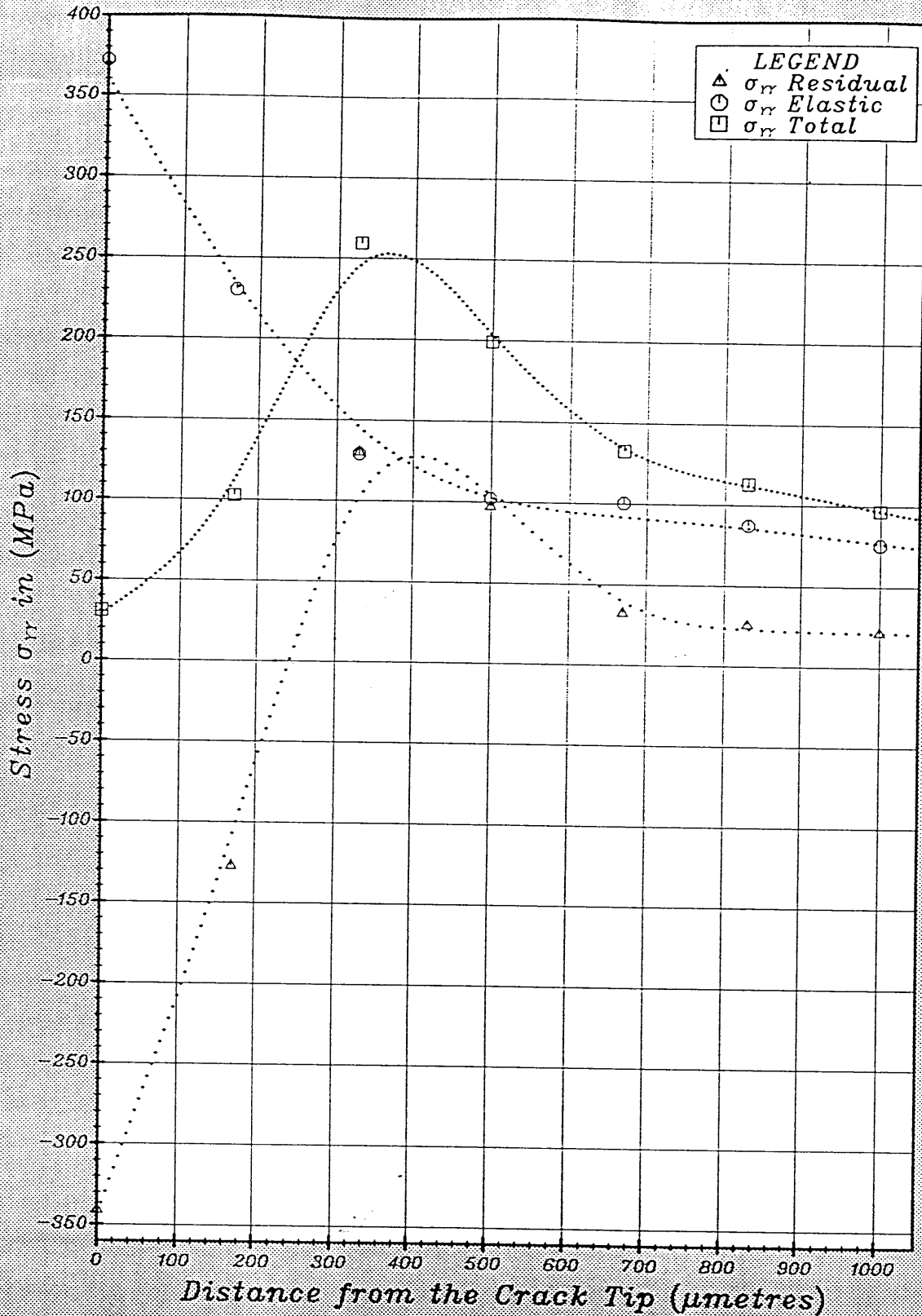


Fig.4.34 Variation of σ_{rr} stress component as a function of distance from the crack tip, for a crack length $a=11.9\text{mm}$, following partial elastic unloading from 5.0 to 1.5KN

but it has been observed that the maximum occurs further away.

In view of the above published results a comparison with the elasto-plastic finite elements has shown that the distributions predicted are behaving in an identical manner as the ones obtained experimentally. Quantitative comparisons though cannot be made as the material used in the experimental study, has different mechanical characteristics and the geometry differs in thickness and length. In addition, in [4.28] no data is presented regarding the σ_{xx}^r stress component. It can though be concluded that the finite element studies carried out as part of the presented work have yielded stress trends that are in good agreement with the experimental investigations of Busch and Lebrun [4.28].

Elasto-plastic finite element solutions for a series of overloads at the same crack length, $a=11.90\text{mm}$, were carried out. Both the cases of partial and total elastic unloading were examined and, the resulting stress distributions are depicted in Figures 4.35 to 4.38 for comparison. Evidently, the increased magnitude of applied load induces plastic deformation which surrounds the crack tip, the size of the plastic zone being proportional to the square of the applied load. The increased size of plasticity results to higher magnitudes of residual stresses upon unloading, partial and/or full.

Considering the σ_{xx}^r stress component distributions depicted in Figures 4.35 and 4.36, the compressive nature of the stresses in the vicinity of the crack is apparent. A consequence of increasing the applied load is the increased magnitude of the compressive residual stress. In addition the extension of the compressive layer increases ahead of the crack. In a similar fashion the magnitude of the σ_{yy}^r in the proximity of the crack becomes more

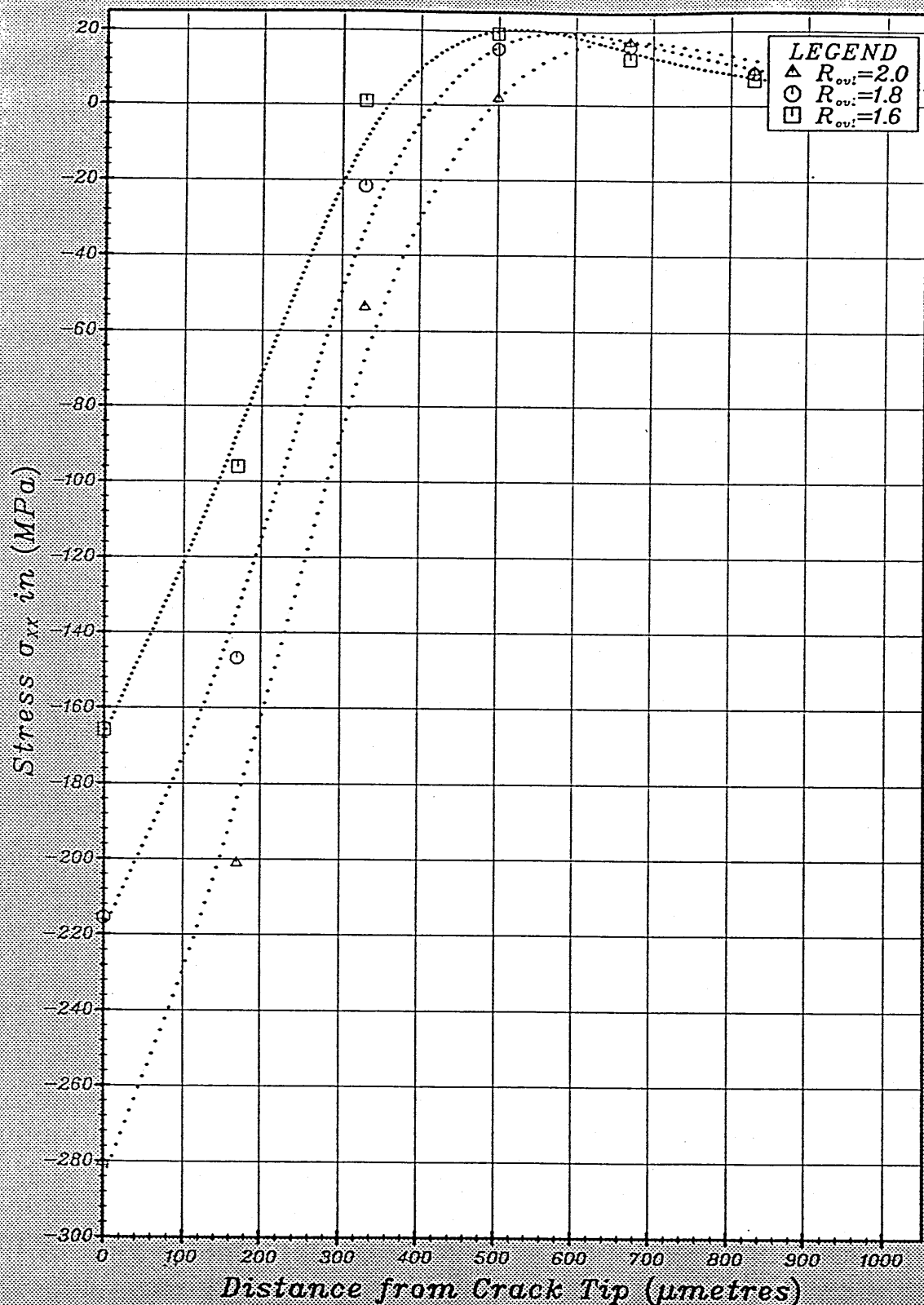


Fig.4.35 Variation of σ_{xx} residual stress as a function of distance from the crack tip. Partial unloading following a single overload for a crack length $a=11.90\text{mm}$ and $R_{ov}=1.6, 1.8, 2.0$.

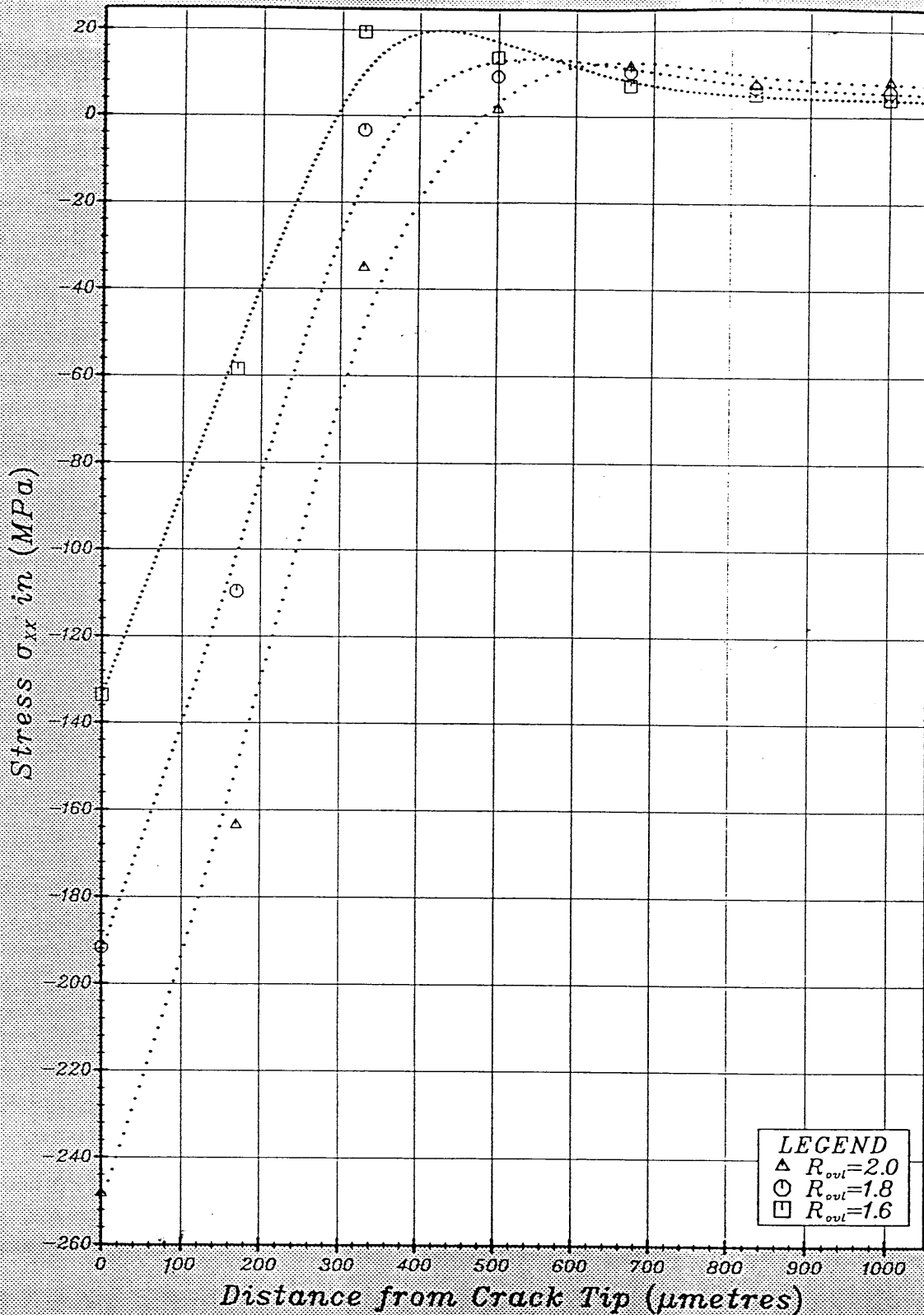


Fig.4.36 Variation of σ_{xx} residual stress component as a function of distance from the crack tip. Full unloading following a single overload for a crack length $a=11.90$ mm and $R_{ovl}=1.6, 1.8, 2.0$.

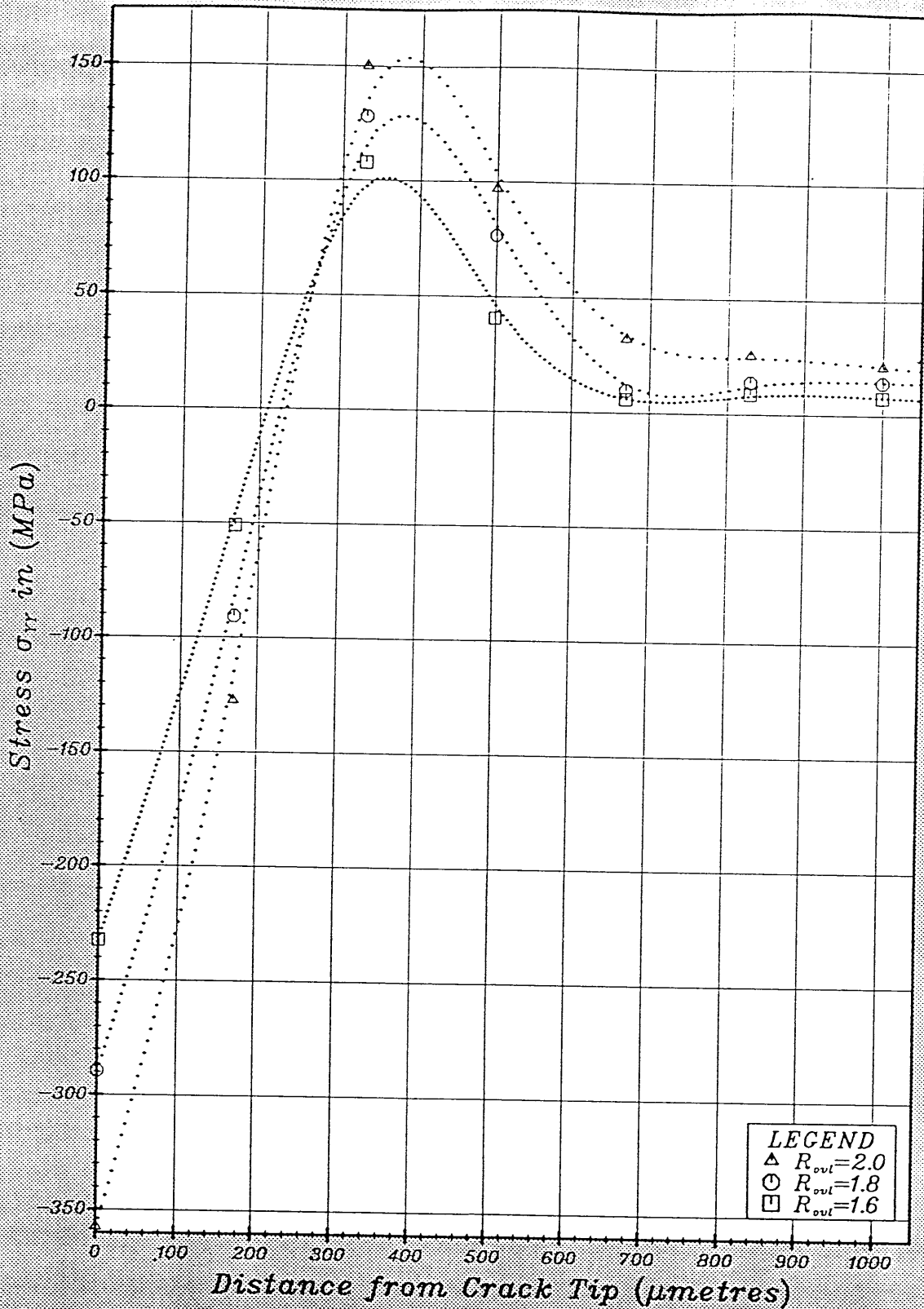


Fig.4.37 Variation of σ_{yy} residual stress component as a function of distance from the crack tip. Partial unloading following a single overload for a crack length $\alpha=11.90$ mm and $R_{ovl}=1.6, 1.8, 2.0$.

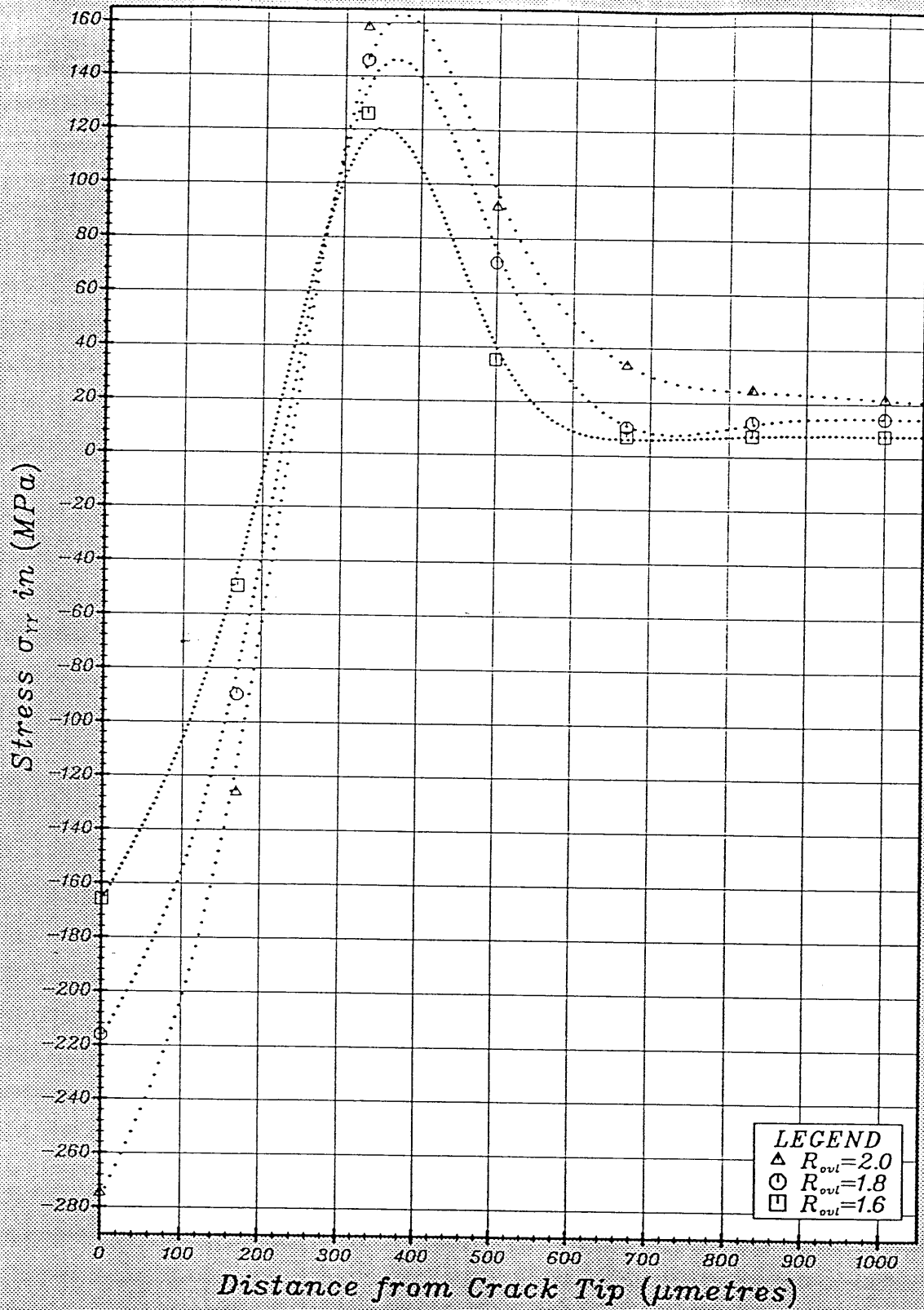


Fig.4.38 Variation of σ_{yy} residual stress as a function of distance from the crack tip. Full unloading following a single overload for a crack length $a=11.9\text{mm}$ and $R_{ovl}=1.6, 1.8, 2.0$.

compressive as a result of increasing the applied load. The magnitude of the tensile residual stress ahead of the crack increases and this is anticipated as equilibrium has to be satisfied.

The presence of the compressive residual stresses in the vicinity of the crack resulting by the application of an overload suggests that, subsequent loading must raise the stress levels to such magnitudes as to overcome and cancel the effect of the compressive residuals, if crack propagation is to take place. By comparison of the stress distributions depicted in Fig.4.34, it is clearly demonstrated that the elastic stress distribution due to a load of 1.5KN is very much more positive than the total stress distribution after the application of the overload. This means that the applied stress intensity factor following an overload is of a magnitude less than the one under the fully elastic condition.

Using the stress method previously discussed it was calculated that the stress intensity factor for a load of 1.50KN prior to an overload was $7.95\text{MPam}^{1/2}$, whereas, after the application of the overload the stress intensity was found to be $6.05\text{MPam}^{1/2}$, a reduction of the order of 23.9%. The calculated reduction of the stress intensity factor is due to the presence of the compressive residual stresses. In practice it has been observed that an overload can significantly reduce the crack propagation rate or even cause crack arrest, von Ew et al [4.29].

The fact that the residual stresses remain compressive within a small distance from the crack tip shows that an affected zone ahead of the crack associated with the compressive residual stress, must be the reason for the experimentally observed reduction of the crack propagation rates following the application of overloads. Effectively, the residual stresses increase the stress intensity

threshold, K_{th} , above which crack propagation is enabled. If the magnitude of the compressive residuals is substantial the post overload reduction of the stresses would be drastic, resulting not only to retardation of the crack growth but also to a temporary or permanent hindrance of the crack.

4.8 CONCLUSIONS

4.8.1 Elastic Finite Element Analysis

The elastic finite element studies of the compact type tension specimen have revealed that,

- a. There exists an interaction between the stress field of the crack and the one associated to the loading holes for small crack lengths.
- b. A possible crack initiation site has been identified on the rim of the loading hole, located at 45° anticlockwise relative to the hole apex.
- c. The crack is enclosed within a very intense stress field, but the cracked ligament is virtually stress free.
- d. Crack tip and ordinary isoparametric elements yield to comparable results, as long as the density of the ordinary isoparametric element mesh is high.
- e. Stress intensity factors can be satisfactorily evaluated, within 5.5%, by a variety of approximation techniques.
- f. The displacement method for evaluating stress intensity factors yields more accurate results than the stress method.

- g. The J-integral evaluations of stress intensity factors are the most accurate as compared to any other method.
- h. The shape of the maximum shear and Von Mises stress contours gives an indication of the plastic zone shape.

4.8.2 Elasto-Plastic Finite Element Analysis

The two dimensional plane strain, elasto-plastic finite element investigations have concluded the following,

- a. The Dugdale and Irwin plastic zones are overestimates of the theoretically determined sizes.
- b. Following the application of an overload a residual stress field is present in the vicinity of the crack tip.
- c. The σ_{yy}^r residual stresses are compressive in the proximity of the crack tip, and attain a high positive magnitude at a short distance from the crack, and compare well with experimental studies.
- d. The σ_{xx}^r residual stresses are compressive near the crack, but they rapidly become positive a short distance from the crack tip.
- e. Increasing the magnitude of the overload increases the magnitude of the residual stresses.
- f. The effective stress field following the application of an overload is significantly lower to the one prior to the overload.

CHAPTER 4

- g. The compressive residual stress field affected zone depth increases with the overload level.
- h. The post-overload stress intensities are considerably lower than the pre-overload ones.
- i. The reduction of the stress intensity due to the residual stress field provides a reasonable explanation of the crack retardation subsequent to overloads.
- j. The partial unloading related residual stresses are lower than the total unloading residuals.

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CHAPTER 5

EXPERIMENTAL FACILITY AND PRELIMINARY TESTING

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References

CHAPTER FIVE

EXPERIMENTAL FACILITY AND PRELIMINARY TESTING

5.0 INTRODUCTION

The main aim of experimental engineering approaches is, the collection, presentation and interpretation of data collected in the course of the investigations. In the presented study a variety of equipment, methods and experimental techniques were utilised.

In the present Chapter, a brief account of the equipment utilised is given, with special emphasis to the operational characteristics of the servo-hydraulic testing facility. For the purpose of data collection a variety of methods are available based upon sound physical principles. The possibilities and limitations of these methods are discussed within the context of the study of fatigue crack growth in metallic materials. In addition, information is given regarding the preliminary testing of the material used and, the design of the utilised specimens. Special transducers designed and built, in the course of the study are also presented, accompanied by detailed information with regard to calibration procedures and ancillary equipment.

5.1 SERVO-HYDRAULIC UNIAXIAL FATIGUE TESTING MACHINE

A block diagram of a typical electro-hydraulic, servo-controlled fatigue testing machine, showing the basic components and layout is depicted in Fig.5.1. In general, machines of this type comprise of a double acting servo-hydraulic actuator mounted in a loading frame. Depending upon the machine load capacity, the required stiffness and other geometric constraints, two or more support columns are present. The hydraulic actuator may be mounted in the base of the machine or alternatively, located in its crosshead. The double acting actuator is driven by a hydraulic power pack (pump).

Due to the mode of operation of such equipment, that involves hydraulically powered moving parts, it is of paramount importance to employ leakage free, low friction pistons. The main source of friction in this system is generally due to the upper and lower piston bearings. In order to minimise the friction, hydrostatic bearings are frequently employed. Furthermore, whilst the stiffness of the frame assembly is important, it is also of paramount interest to maintain coaxility of the loading axis over the full working range of the machine and in addition ensure good dynamic performance. Typically, the maximum error due to lack of coaxility should not exceed $\pm 0.2\text{mm}$ over the full stroke of the actuator for a typical 100KN machine.

In order to achieve machine load capacities of the order of 100KN, hydraulic pressures in the range of 2-40 bar and power packs having a wide range of deliveries (15-120 ltrs/min) are typical of current equipment. Servo-hydraulic equipment are inefficient, and a typical 40 ltrs/min pack may utilise a 20KW electric motor. Most of the power is being dissipated in the form of heat by the cooling system. The viscosity of the hydraulic fluid is

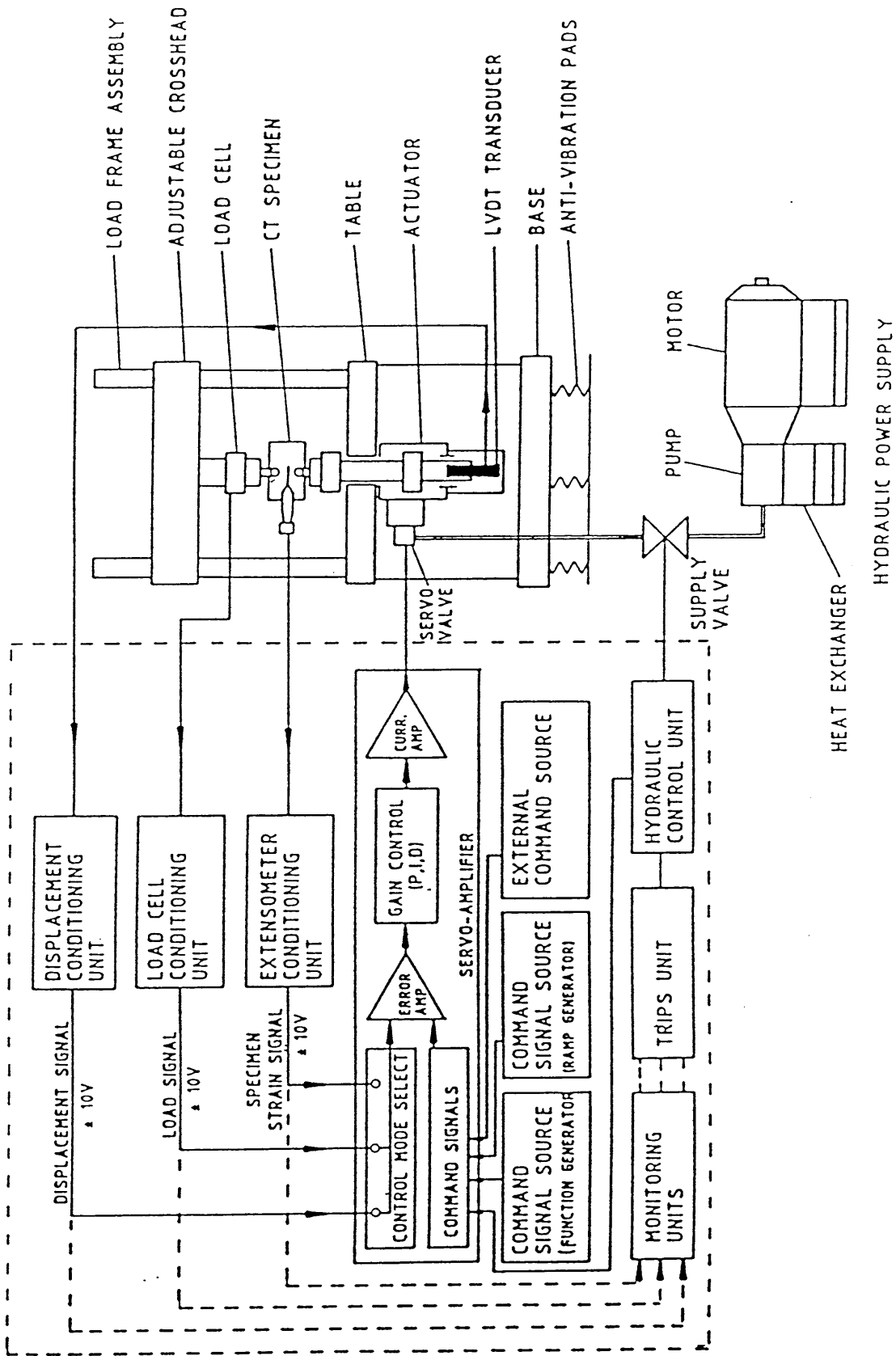


Fig.5.1 Block diagram of the Servo-Hydraulic test machine

dependent upon the operating temperature and is generally maintained below 50°C. The temperature is controlled by means of heat exchangers, with or without forced cooling systems.

In order that such equipment is easily utilised and accurately controlled, a number of sensors, transducers and instruments are either integrally mounted or may be added according to test conditions and degree of monitoring required. A standard feature is an integrally mounted load cell, used to measure the load and an LVDT for the measurement of displacement. There are cases when higher accuracy is required in monitoring and measuring displacement and/or strains. For such cases additional position transducers may be employed such as strain gauges, extensometers or clip-on-gauges. Special amplifier modules are available on the testing equipment and signals from added sensors may be fed, to be either used to enhance machine control or may be directed to recording or display units such as plotters and CRO's. For the purpose of standardisation, it is common practice to process these signals using ± 10.0 Volt voltage range, where the negative sign implies a compressive reading.

Servo-hydraulic machines enable the definition of a required test through the use of an electrical command signal. Control of the system is achieved by closed-loop or negative feedback techniques, utilising an electronic device known as a servo-amplifier. The control function of the servo-amplifier is briefly explained below.

The first operation of the servo-amplifier is to carry out algebraic subtraction of the actual desired value of the specimen load or displacement signal from the generated command signal, thus producing an error signal. Subsequently, this error signal is passed through an amplification unit with variable characteristics, before

being converted to a current drive signal, see Fig.5.1. The current drive signal is applied to the control coil of the servo-valves which control the differential flow of hydraulic oil through the two available ports. In response to this current, the valve preferentially admits oil to the two chambers of the actuator in such a fashion as to reduce the magnitude of the error signal by causing the actuator to move.

In servo-controlled systems, the servo-amplifier represents the heart of the machine. It generally comprises of several modules and performs several functions. Firstly, the feedback or control mode is selected which may be load, displacement or strain. Subsequently, a source has to be selected for the command signal that may either be a function generator, a ramp generator, an external signal or a combination of these. Finally, the command signal is scaled to the required level and any static command signals may be added. Hence, command signals are generated and after comparison with the feedback, an error signal is obtained that controls the servo-valves and consequently drives the hydraulic actuator.

A number of supplementary measuring devices are available, usually digital voltmeters (DVM) aiming to enhance system monitoring. In addition a number of electronic trip circuits are available safeguarding the machine mechanical systems and specimens from various malfunctions.

The specific equipment utilised in the present investigations is shown in Fig.5.2. The system consists of a uniaxial test frame with capacity of 100.0KN and hydraulic crosshead lifts and clamps, a table mounted actuator with total travel of 100.0mm driven by a hydraulic power supply delivering up to 45.0ltrs/min of

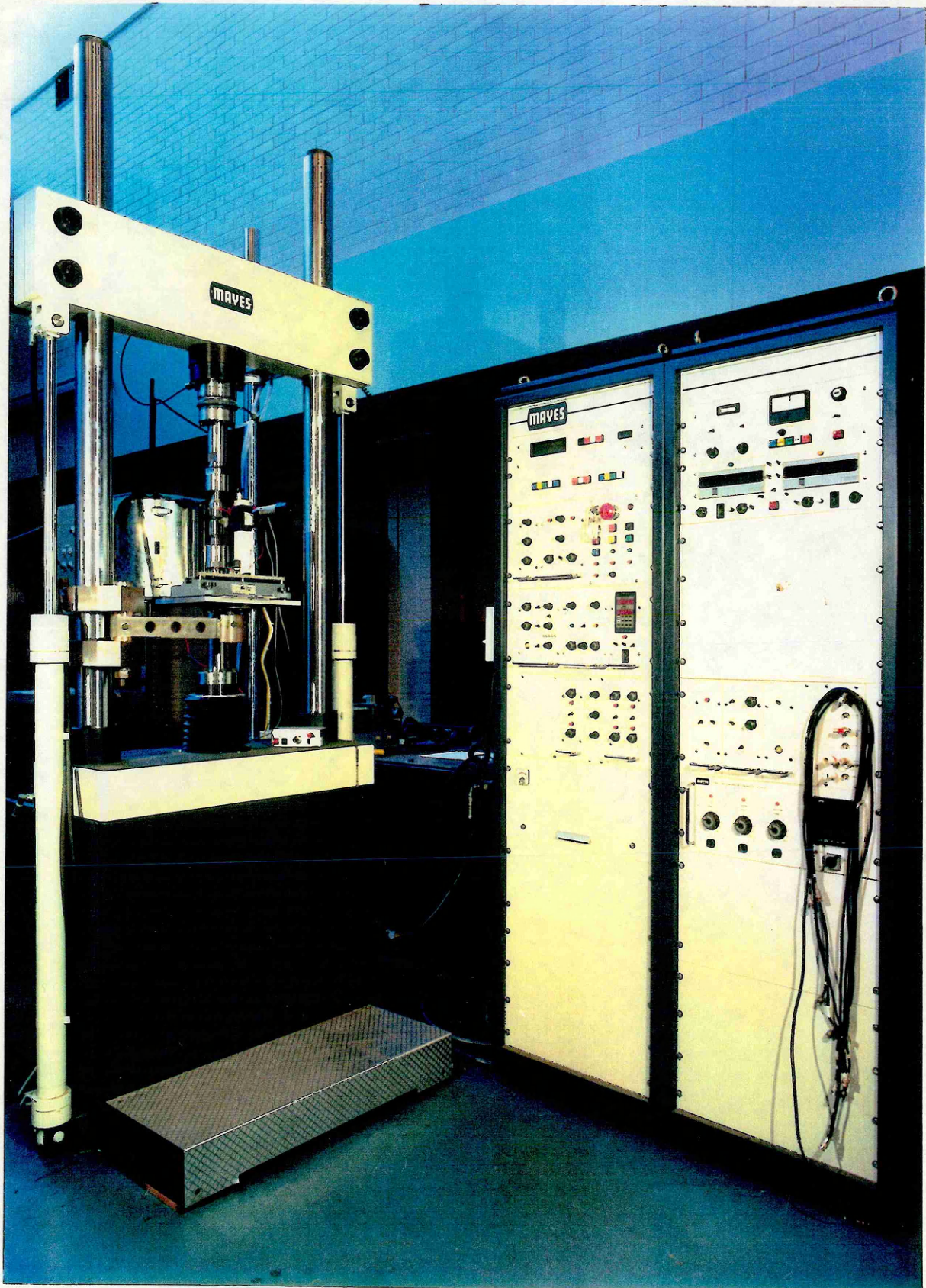


Fig. 5.2 Uniaxial Servo-Hydraulic Fatigue Testing
Machine Utilised in Present Study

oil at a pressure of 20.0bars, and finally, a console accommodating control and readout instrumentation.

The combination of the actuator, servo-valve and the hydraulic pack used, allows actuator displacements of the order of 0.5mm, test frequencies of 50.0Hz for load ranges 20KN depending upon the stiffness of the specimen and any structural resonances that may be set-up.

The load frame incorporates a ± 100.0 KN capacity load cell which in conjunction with its supply and readout instrumentation gives a resolution of 1.0N. The actuator displacement is monitored by means of a linear voltage differential transformer type position transducer with a range of 100.0mm and a resolution better than 5.0microns when used in conjunction with its drive and readout instrumentation.

Two types of signal generators are available aiming to provide versatility to the machine and allow the performance of a wide variety of quasi-static and dynamic tests. The two generators available are a 16 bit resolution ramp and hold function generator and an analogue function generator which may be combined and used to apply complex load, displacement or strain histories to the specimen. The system may be driven from external command signals and, external feedback signals could also be used for control.

For purposes of set-up and continuous monitoring of the equipment a dual beam digital storage oscilloscope with screen dump to an X-Y plotter facility was used. The oscilloscope was employed to visualise signals in cyclic tests and to complement the minimum and maximum readings available from the four-and-a-half significant figure

digital voltmeter display integrally mount on the control console.

Finally, the system includes a flexible system of overtravel, undertravel and cycle counting trips which can be used to interrupt or terminate a test according to variations in any one of the feedback signals independently of the control mode. Machine component protection is achieved by means of a fracture trip, oil level and temperature trips, that interrupt the operation if specimen fracture or overheating or leakage of the hydraulic fluid is experienced.

5.2 CRACK SIZE MEASURING TECHNIQUES

There is a large number of techniques available for the detection and sizing of cracks and other crack like defects and flaws. A number of these techniques have been developed by equipment manufacturers to a high degree of sophistication and performance in order to satisfy the need to identify manufacturing flaws and detect, locate and measure fatigue cracks in highly specified components destined for the nuclear and aerospace industries [5.1]. Other techniques however, have not evolved to such a high degree and are better suited to laboratory applications.

This section outlines some of the more widely used industrial and laboratory techniques in so far as they affect crack growth monitoring and sizing.

5.2.1 Industrial Non-Destructive Techniques

(I) Ultrasonic Inspection [5.2, 5.3]

Ultrasonic techniques use beams of high frequency (0.1-20 MHz) short wavelength (1-10mm) vibration waves, generated by a piezoelectric transducer and transmitted through a component or specimen with velocities of 1000-6000m/s, depending upon the material. The presence of a defect in the path of the travelling waves causes them to reflect back to the source, as the flaw discontinuity presents an impenetrable barrier. The reflected waves are then converted to electrical waves with characteristics proportional to the wave amplitude and elapsed time. A proportion of the waves by-passes the flaw and is reflected back by the opposite side of the specimen. The signature of those waves provides a means of establishing the thickness of the component examined, and a means for calibrating the technique.

An oscilloscope display of probe output as a function of time is shown in Fig.5.3, and is known as the A-scan. In this case, the height of the reflected pulse is proportional to the size of the flaw or defect. This method makes crack sizing difficult due to the relationship of crack size, crack distance and reflectivity.

For the specific case of a CT-specimen the schematic representation of Fig.5.4 could be utilised for crack length measurements. Such a set-up comprises of an oscillator which is used for the excitation of the piezoelectric transducer that transmittes energy pulses through the material. The reflected waves are detected by a pulse receiver and are directed to appropriate recording units, such as an oscilloscope or chart recorder. In this case, a multiple probe arrangement is utilised so that the

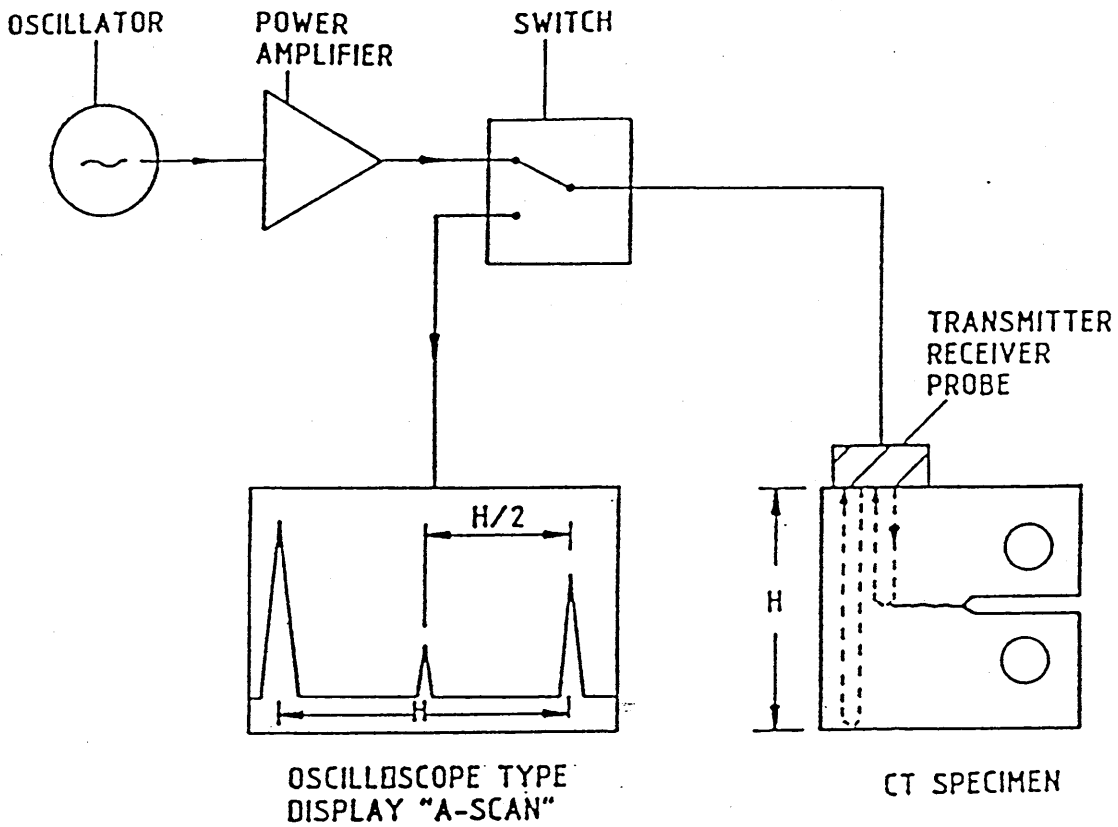


Fig.5.3 Ultrasonic flaw detection, simple setup giving 'A' scan

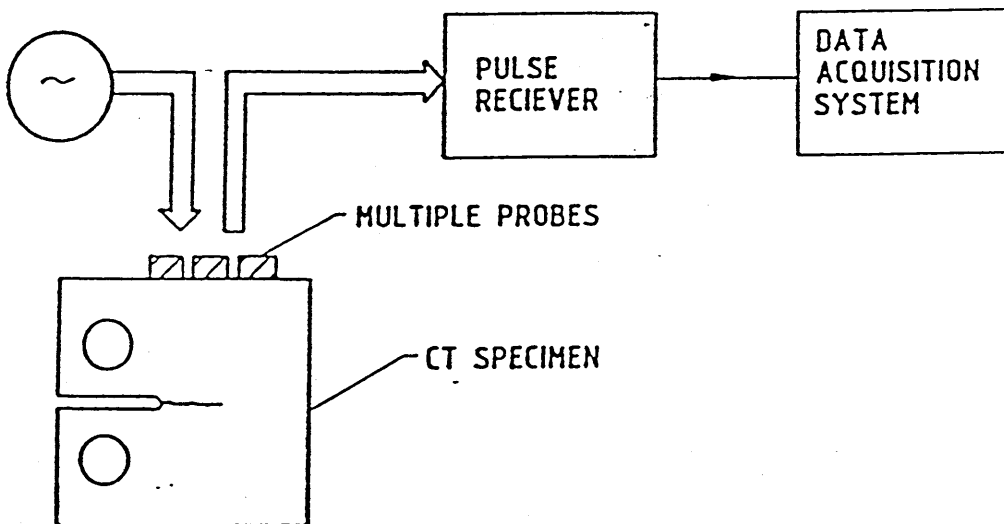


Fig.5.4 Ultrasonic setup utilising multiple probes to size a crack in a CT specimen

location of the crack tip can be established by means of the A-scan technique or alternatively the probe could be traversed over the surface.

Complex multiple systems are also used with mechanical probe movement and digitization of signals, followed by computer storage; methods of computer interpretation are developing rapidly.

The development of ultrasonic techniques has resulted in the use of projection image scanning facilities (P-scan) in off-shore installations, nuclear power plants refineries etc. These highly sophisticated facilities are generally adopted for weld inspection and corrosion mapping by means of three projection images, of the top, side and end views of a component. The thickness scanning technique (T-scan) is a hybrid of (P-scan) and is currently being used by a number of companies to monitor corrosion severity in critical components, [5.5].

The great flexibility exhibited by ultrasonic techniques, in mapping flaws in large and complex structures accounts for its popularity, and not their sensitivity to small defects. For the purpose of fatigue crack growth monitoring a complex facility should be built and adapted to the requirements of the program, which would increase the cost of the research without any significant gains.

(II) Acoustic Emission Methods [5.6]

Solids subjected to sufficiently high stress levels, generate sound that is emitted in discrete pulses. This phenomena is called acoustic emission or stress wave emission. Acoustic emissions of this nature are sound or ultrasound pulses generated by crack initiation and/or crack propagation. They are detectable by special transducers fixed to the surface of components, which in

turn generate electrical pulses. A network of such detectors may be placed on a specimen surface, and knowledge of the time difference for reception of the acoustic signal by the detector in the array can be used for locating the crack or initiating event. Alternatively, if the rate of acoustic emission is monitored on a continuous basis, sudden changes in the rate are indicative of crack propagation or formation of new flaws.

Figure 5.5 shows an arrangement which could be used for monitoring crack propagation in a CT-specimen. In this case, the probe is in direct contact with the surface. In view of the low level of energy generated, some pre-amplification and conditioning of the signal is necessary. The secondary amplifier and event detector are used to drive the event counter, which in turn feeds the data acquisition system. In data handling, a number of options can be used. These include counts distribution, pulse width distribution, energy distribution and linear location.

The inherent advantage of acoustic emission is that it is a non-localised technique; it is not necessary to examine specific regions of a structure and large volumes can be examined at any one time [5.7]. The nature of these techniques makes them suitable for continuous monitoring of flaws.

Limitations of this method arise from the fact that it is susceptible to background noise generated by the loading arrangement, hydraulic power pack noise and other vibration sources. In addition, pulses are distorted during transmission through the material. Different engineering materials exhibit different intensities of emission and it is not infrequent that certain materials

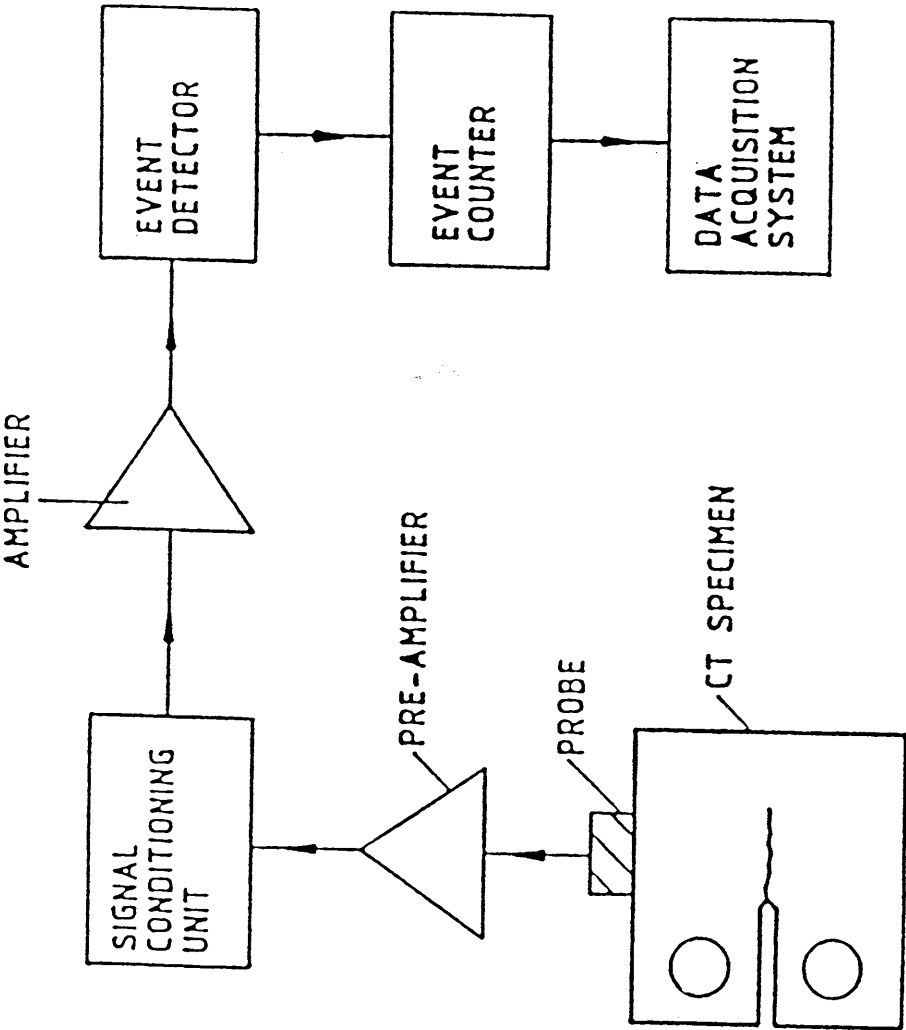


Fig.5.5 Schematic of acoustic emission setup

remain silent or emit very weak pulses until very near fracture.

In the laboratory the technique is interesting in that extremely small defects can generate detectable acoustic emissions. The problem of relating acoustic emission to crack growth rate seems to have no simple solution, [5.8].

(III) Eddy Current Techniques [5.2, 5.4]

The physical principle upon which this technique is based is that, if a metallic material is subjected to a fluctuating magnetic field, eddy currents are induced within the material. If a coil carrying an alternating current is placed in proximity to a conductive material, secondary or eddy currents are induced. These currents produce a magnetic field surrounding the coil. This interaction between fields causes a back e.m.f. in the coil and hence, a change in the coil impedance value. If a material is uniform the impedance value of the coil is the same at all points. In the presence of a discontinuity, the distribution of eddy currents is altered and consequently a reduction in the magnetic field is detected, i.e. a change in the coil impedance.

The main disadvantage of this technique is that it is more qualitative than quantitative with respect to crack size measurements. Thus, its main applicability is in quality control applications and routine component testing. Finally, the technique frequently requires to be supplemented by ultrasonic measurements or radiography.

(IV) Other Techniques [5.2]

A number of other techniques are available for crack and flaw detection. Many of them rely on the crack or flaw being 'open'; to surround an 'empty volume' or void. Radiographic techniques show cracks as changes in shadow

density: Low volume or shallow cracks can be difficult to detect and the direction of illumination is very critical. Liquid penetrant techniques rely on a crack breaking the surface and the coloured die to be drawn by capillary attraction into the defect. It is common practice to use fluorescent liquids which increase the technique's sensitivity when viewed under ultra-violet illumination.

Liquid penetrants can affect crack behaviour by either sealing the crack tip from the atmosphere or acting corrosively in their own right.

Magnetic particle inspection is another alternative but it can only be utilised when dealing with ferro-magnetic materials, as it relies upon magnetisation of the component by means of passing a high magnitude current or using permanent magnets or electromagnets.

5.2.2 Laboratory Crack Sizing Techniques

(I) Electrical Potential Drop Methods

Electrical potential drop techniques are slowly becoming established in industry, but their utilisation in large scale remains limited by comparison to the techniques presented in the previous section.

Potential drop techniques have for a number of years been used for measuring crack lengths in metallic material fatigue and fracture specimens, Vosikovsky [5.9]. The principle of operation is that the electrical resistance of a specimen increases with crack extension. This increase in resistance is reflected as a measurable increase in electrical potential across the specimen, which can with appropriate calibration be directly related to the crack length. An alternative method is through the use of a metallic film deposited onto the specimen (non-conducting materials) or a metal foil bonded onto the

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specimen, or nowadays the use of special crack growth monitoring strain gauges. Both direct current (D.C.P.D.) and alternating current (A.C.P.D.) systems can be used for measuring the changes in potential, utilising standard measurement techniques.

The block diagram of a typical D.C.P.D. setup is illustrated in Fig.5.6. A constant current source supplies a current that passes through the conducting component. A probe may then be used to measure local values of the electrical field in the component at discrete locations. Field values are measured across the two pole pieces of the probe. The presence of a crack dictates an increase in the effective resistance of the local area within the component. The increased resistance gives a higher potential drop across the crack. Continuous monitoring can be carried out by recording the potential drop as a function of time, or by comparing the measurement to a reference uncracked specimen [5.9, 5.10, 5.11].

A similar configuration may be used for alternating current potential drop A.C.P.D., a block diagram of a typical set up is depicted in Fig.5.7. There are two main operation related differences between D.C.P.D and A.C.P.D. techniques. Using A.C. excitation it is possible to condition the output from the probe using a phase sensitive detector. This provides the advantage of almost entirely rejecting signals of electrical noise and thermally induced e.m.f.'s. Secondly, depending upon the permeability of the test piece substratum, the current tends to flow mostly in the vicinity of the surface. This is the so termed skin effect that concentrates the current thus reducing the excitation requirements down to 1-2 ampères. By comparison, D.C.P.D. systems require excitation currents of the order of 10 ampères to achieve

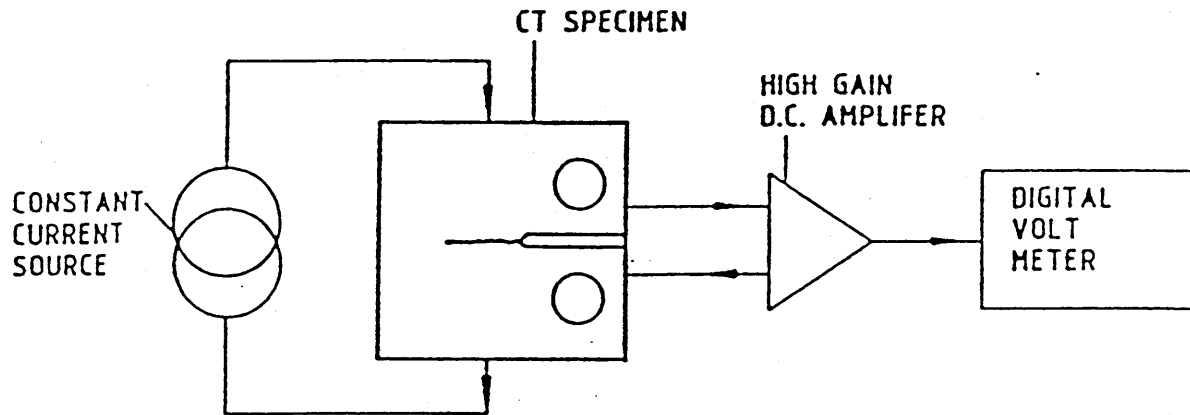


Fig.5.6 D.C. potential drop arrangement

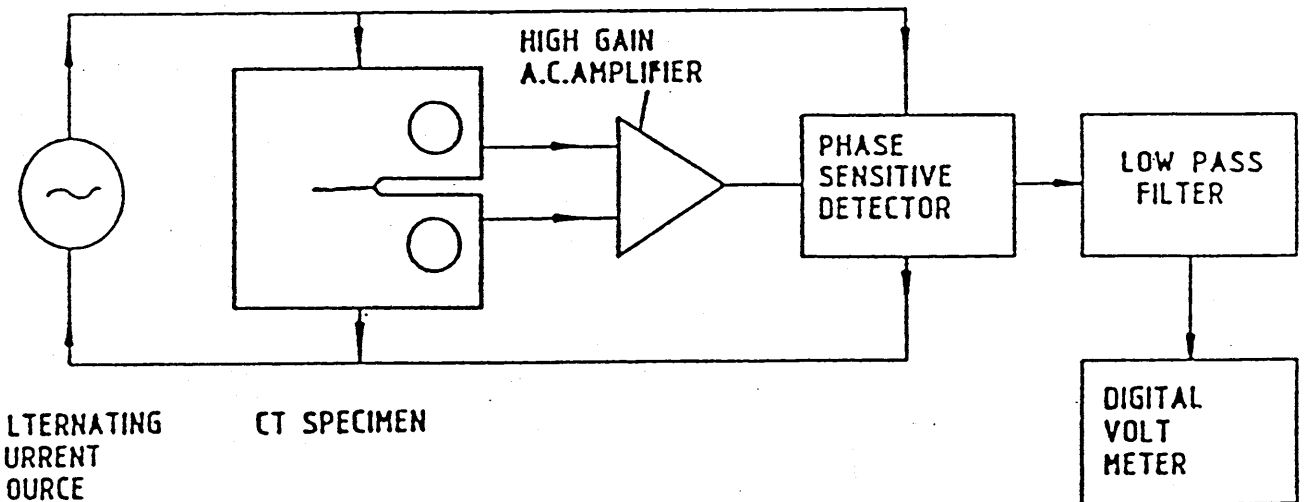


Fig.5.7 A.C. potential drop arrangement

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comparable defect resolution. A comparison of the two system operating characteristics is shown in Table 5.1.

	D.C.P.D.	A.C.P.D.
Base Potential	400.0 μ V	100.0 μ V
Required Current	10.0 A	0.75 A
Noise Level at Output	0.10 μ V	0.02 μ V
Crack Length Resolution	0.013mm	0.010mm
Sensitivity to Therm. e.m.f.	high	none

Table 5.1 Comparison of A.C.P.D. and D.C.P.D.
Operating Characteristics [5.10]

(II) Optical Methods

Optical methods vary from inspection with the naked eye to the use of microscopic examination and frequently the use of contrast enhancing systems.

Conventionally, for the examination of cracks in standard fracture mechanics specimens, a travelling microscope attached to a Vernier scale is employed, focused on the crack tip and follows its progress. The visibility of the crack tip can be inhibited by the following parameters :

- a. The presence of extensive plastic deformation, resulting into blurring of the image not allowing clear definition of the crack tip.
- b. The presence of surface oxidation which constitutes detection of fine cracks against the dull background of the oxidised layer awkward. This is predominant when dealing with elevated temperature fatigue testing.

- c. The intensity of illumination used.
- d. The testing frequency. In cases engaging high frequency fatigue testing, continuous illumination has no effect and is recommended to utilise stroboscopic illumination, to attain good image definition.

Optical methods can only be utilised for monitoring or sizing of surface breaking cracks, and a reliable indication of the average crack length may only be obtained if the crack front is relatively straight through the thickness of the specimen. With the use of a travelling microscope configuration it is possible to detect crack extensions of the order of 0.1mm, but the precise location of the crack tip is probably no better than $\pm 0.3\text{mm}$. An advantage of this technique is that crack extensions are directly obtainable without prior calibration, but however, it has the disadvantage of being manual and operator dependent.

The optical method may be assisted with the use of contrast enhancing dyes such as coloured liquid penetrants. Some experience is needed for the efficient identification and tracking of surface cracks, however, visual tests are inexpensive, simple, fast to perform and very flexible.

In the current research program, a travelling microscope arrangement was utilised for crack tracking and sizing. Due to the relatively high frequency of testing it was essential to use stroboscopic illumination to achieve continuous un-interrupted measuring of crack extensions. In testing of the type undertaken, fatigue crack propagation studies, it is important to avoid test interruptions as the crack propagation can be affected by such events, [5.12, 5.13]. A stroboscopic device was used,

specially designed and built in house as part of another research program [5.14], see Fig.5.8.

The stroboscope circuit is triggered by an electrical signal associated with the movement of the specimen e.g. servo-hydraulic test machine command program. The light source is not triggered simultaneously but after a delay that can be selected by the operator. This gives the capability of obtaining a light flash at different stages of the loading cycle. With respect to an advancing crack, this could be seen either open, close or at an intermediate position. The stroboscopic light source was mounted on the travelling microscope objective lens casing so as to allow the light to be as close to the examined object as possible.

(III) Compliance Techniques

In fracture mechanics testing the potential of the compliance technique to monitor crack growth has been recognised for several years. This technique is relatively simple since it is based upon measurement of two basic quantities in mechanics of materials testing; namely, applied load and specimen deflection.

The compliance which is the inverse of the test specimen stiffness, increases as the crack extends. The crack extension causes increase of the deflection of the cracked ligaments, which can be measured with appropriate instrumentation. The measurements of the crack opening displacement can be directly related to the stress intensity factor, Wells [4.15]. The COD measurement is employed in fracture toughness testing of ductile metals and is covered by materials testing standards as the BS 5762:1979 [4.16].

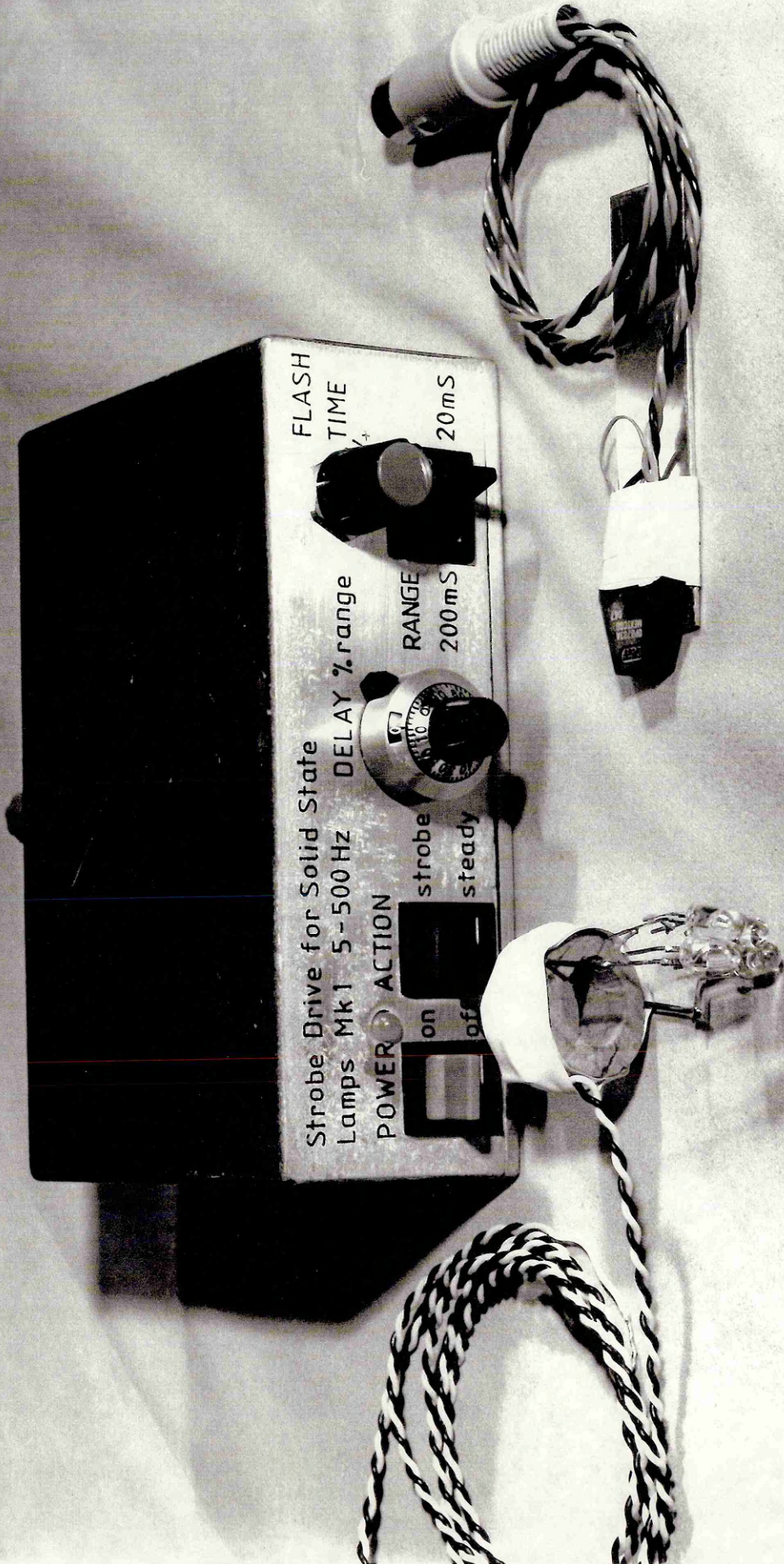


Fig. 5.8 Stroboscopic Illumination Device

In fatigue crack growth testing COD measurements can be used for the evaluation of the compliance from the obtained load, displacement records. This information can be used with appropriate calibration equations, so that the stress intensity factor and in addition the crack length can be determined. The measurement of the displacement has to be carried out at particular locations such as the load line, or in the case of the CT-specimen at the front face of the component. A clip-on-gauge which is a purpose built extensometric transducer has to be used for the measurement of the chosen location displacements. In the course of the present study, such a transducer was designed and built. Full details of the design and calibration of the clip-on-gauge can be found in a subsequent section, 5.5.

Calibration equations allowing the determination of compliance for some common fracture mechanics specimens have been developed by Saxena and Hudak [4.17]. The formulation provided allows the determination of compliance for a wide range of crack lengths from measurement of deflections at various locations. The equations presented give the compliance as a function of crack length, and the crack length as a function of the compliance. This latter set of equations can be utilised for the determination of crack lengths during the crack growth process, by continuous monitoring of the applied load and corresponding deflections.

Crack length measurements based upon the compliance, have been found to be very accurate, and in addition continuous monitoring of the crack length may be achieved with the use of a digital data acquisition systems, [5.18, 5.19]. The compliance method was employed in the present study in order to enhance the crack length measurements which were carried out with the use of an optical method. The clip-

on-gauge when attached to the mouth of the specimen, measures the average deflection of the cracked ligament. This implies that the crack lengths calculated are the average values through the specimen thickness.

It was therefore, considered appropriate to develop a computer program for the process of experimentally determined load displacement information and the determination of stress intensity factors and crack lengths. This program is based upon the formulations of Saxena et al [5.17], and a listing can be found in Appendix 3.

5.3 PRELIMINARY TESTING

5.3.1 The Material

The material utilised for the studies of fatigue crack propagation was the high strength Aluminium alloy 7075 T7351 whose equivalent British specification is BS L160: 1976. The chemical composition of the material is shown in Table 5.1

Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	Zr
0.10	0.19	1.28	0.05	2.52	0.20	5.98	0.054	0.05

Table 5.1 Aluminium-Zinc 7075 Alloy Percentage
by Weight Chemical Composition

This solution treated alloy is artificially aged to an overaged condition to obtain maximum strength properties. Finally, it is stress relieved by stretching to a set of 1-3%, and stabilised. The material is utilised in the

aerospace industry as it combines high strength light weight with good stress corrosion cracking resistance [5.20]. The main applications of the alloy are as skin of the lower wing surface in aircraft, and in thicker sections as wing spars, and other forged sections, as well as pressure vessels in landing gear shock absorbers.

5.3.2 Tensile Test

The material was supplied as a drawn bar with diameter of two inches. The material was supplied with a manufactures certificate, where some mechanical properties were quoted. It was therefore considered appropriate that the mechanical properties of the material were verified by means of carrying out mechanical testing.

The first test carried out was a tensile test in order to determine the basic mechanical characteristics of the material; namely, Young's modulus, 0.2% proof stress and ultimate tensile strength. The testing was carried out utilising cylindrical specimens machined in accordance to BS 18: part 1 and 2. The servo-hydraulic uniaxial testing machine was utilised and operated in the stroke control mode. An X-Y plotter was connected to the output ports of the load cell and the LVDT (machine console), thus it was possible to obtain a load displacement record of the test sequence. Finally, this load displacement record was used for the determination of the mechanical properties, a summary of which is shown in Table 5.2. The load displacement record clearly indicated that the material exhibits an elastic nearly linear strain hardening behaviour. A micro-hardness testing machine was utilised for hardness measurement. The Poisson's ratio was also determined by means of a simple uniaxial test.

5.3.3 Fracture Toughness Test

A special test was carried out for the purpose of determining the fracture toughness of the material. This was dictated by the fact that the value quoted by the supplier was for the longitudinal direction, however the test specimens were machined from transverse slices of the bar. Test specimens of the compact tension type were made to comply with the ASTM 399-86 specification for K_{Ic} determination. The specimens were of dimensions $W=32.0\text{mm}$ and $B=10.0\text{mm}$. They were pre-cracked with the use of tensile cyclic loading of frequency 15Hz and amplitude of 1.25KN, $R=0.25$ about a mean of 1.5KN.

Modulus of Elasticity	$E = 80.0 \text{ GPa}$
Tangent Modulus	$E = 15.0 \text{ GPa}$
Poisson's Ratio	$\nu = 0.303$
Ultimate Tensile Strength	$\sigma_{uts} = 529 \text{ MPa}$
0.2% Proof Stress	$\sigma_{0.2\%} = 460 \text{ MPa}$
Fracture Toughness	$K_{Ic} = 26.0 \text{ MPa}\sqrt{\text{m}}$
Hardness	142-146 HB
Elongation to Fracture	10.5%

Table 5.2 Mechanical Properties of 7075 T7351 Aluminium Alloy in Overaged Drawn Form

5.4 COMPACT TYPE TENSION SPECIMEN DESIGN

Investigations in fatigue crack propagation require not only special testing machines, but also purpose made test specimens. The test specimens are geometric configurations that have well established and documented stress intensity factor equations and usually conform with national or international standards. It is possible to use non standard specimens provided that well-established stress intensity calibrations are available and that the

specimens are of sufficient dimensions to remain predominantly elastic during testing.

In general, the test specimens can be of two different types according to the stress condition investigated. For plane stress studies, specimens of thin sections are required. These are made of sheet material of small thickness and are usually of the centre crack type or as they are commonly known CCT (Centre-Cracked-Tension). Plane strain conditions require that the specimens are of substantial thickness to ensure that the state of stress is predominantly plane strain. A popular specimen of this type is the Compact Type specimen (CT), which has been employed in the present investigations and is covered by the ASTM E647/86 [5.21], standard for constant load amplitude fatigue crack growth testing.

The dimensions of the specimen have to comply with set limits, otherwise the crack front may be curved, in which case corrections have to be applied to the measured crack lengths to account for the crack front curvature. The set limits of the thickness with respect to the length of the specimen are as follows, $W/20 \leq B \leq W/4$ where W is the specimen length and B is the thickness. As the specimen is required to behave elastically during testing for all values of applied load the general dimensions have to conform with the following geometric constraint as stated in the specification, $(W-a) \geq 4/\pi (K_{\max}/\sigma_{\text{yield}})^2$, where, a , is the crack length.

Crack initiation features are required, so that a crack is initiated quickly and it propagates with the minimum of deviation along the configuration axis of symmetry. Such starter notch features can be either, a straight or a chevron type notch. This stress concentrator has to be carefully machined as inadequate sharpness may cause prolongation of the crack initiation period. Machining

can either be electrical discharge machining, sawcutting, broaching or milling. For Aluminium it is recommended that milling may be used. The notch feature selected was the straight notch. A special shaped milling cutter, shown in Fig.5.9, was used for the machining of the notch with a 60° inclusive angle. The only modification in the specimen geometry was the addition of two, 2.0mm diameter tapped holes on the front face of the specimen for the fixing of the knife edges supporting the clip-on-gauge.

The specimens were produced from transverse slices cut from a two inch diameter bar. A detail drawing with all the dimensions and machining information of the specimen is shown in Fig.5.10.

5.5 CLIP-ON-GAUGE DESIGN AND CALIBRATION

A double cantilever clip-on-gauge is a special purpose extensometric device, designed to measure the crack opening displacement COD, and more generally used for compliance measurements. This type of transducer is thus designed as to clip-on to the front face of a Compact Type Tension specimen with the aid of purpose-made removable sharp edges. The knife edges are attached onto the specimen with small screws.

The clip-on-gauge consists of two cantilever beams and a spacer block, which are clamped together with a single nut and bolt. The sensing elements used are electrical resistance strain gauges, that are cemented onto the tension and compression surfaces of each cantilever beam.

The size of the cantilever beams is dependent upon the strain gauge limitations, linearity, fatigue life and associated strain ranges. The design of these cantilevers was based upon the principles of the simple theory of

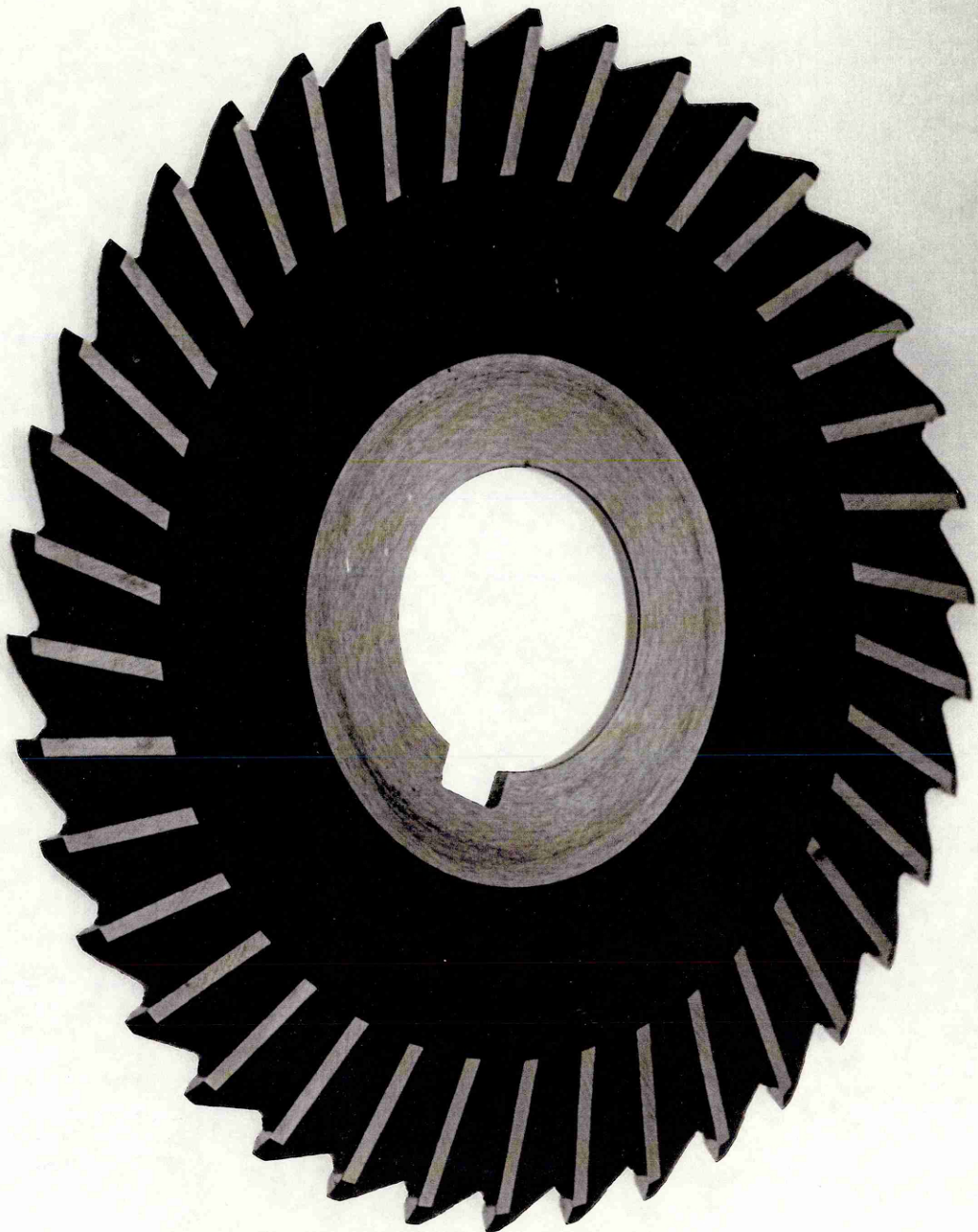


Fig. 5.9 Special Shaped Milling Cutter used for
Machining the Starter Notch

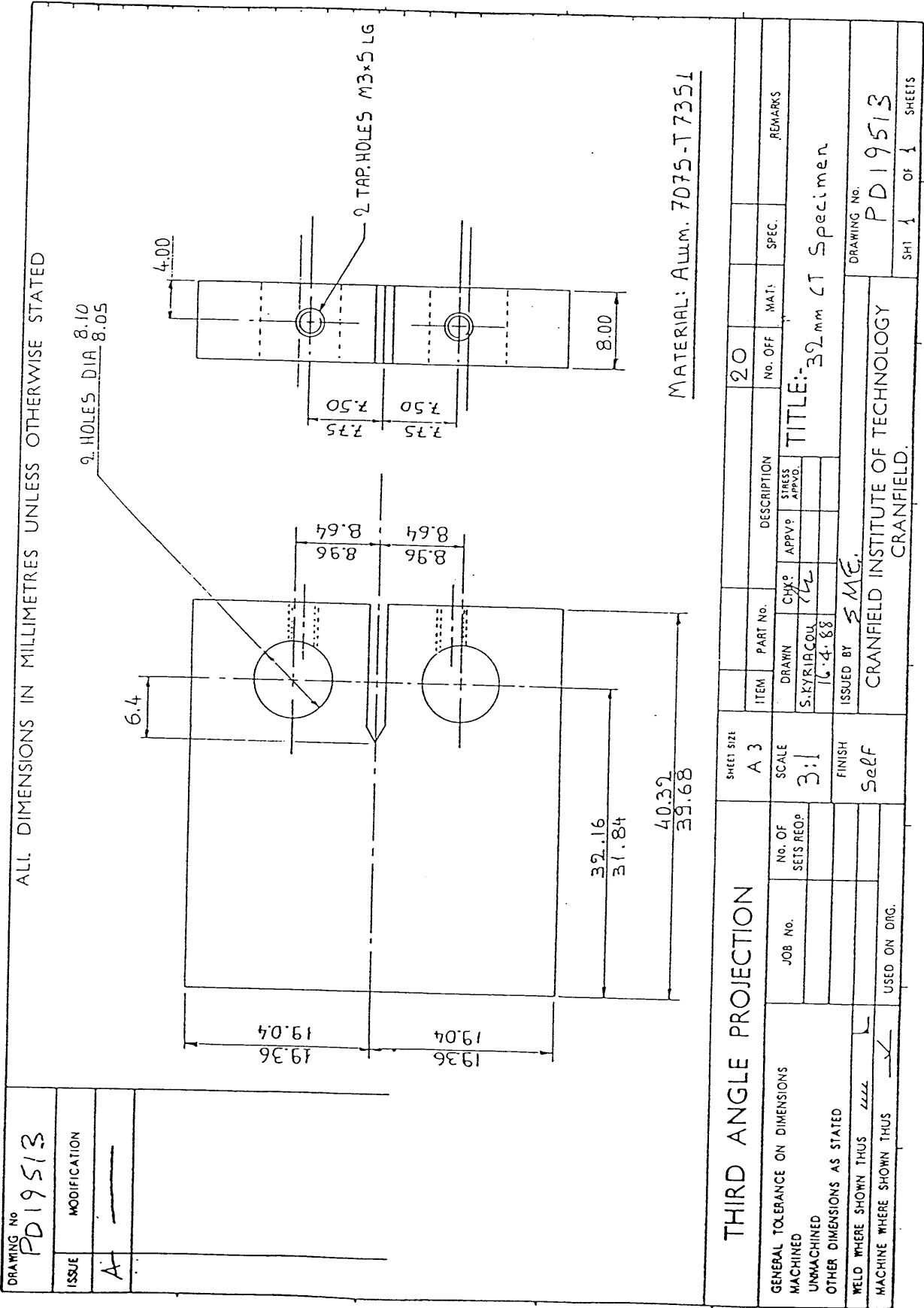


Fig.5.10 Compact Tension specimen, manufacturing drawing

bending with limitations arising from the strain gauge manufacturer's specifications.

5.5.1 Mechanical Design

The deflection U of any point along a cantilever beam with dimensions, length L , width b and thickness t , due to the application of a point load P at the free end, is given by the following equation,

$$U = (Px^2/6EI) \cdot (3L-x) \quad \dots (5.1)$$

where x is the distance of the point measured from the root of the cantilever, see Fig.5.11 for notation.

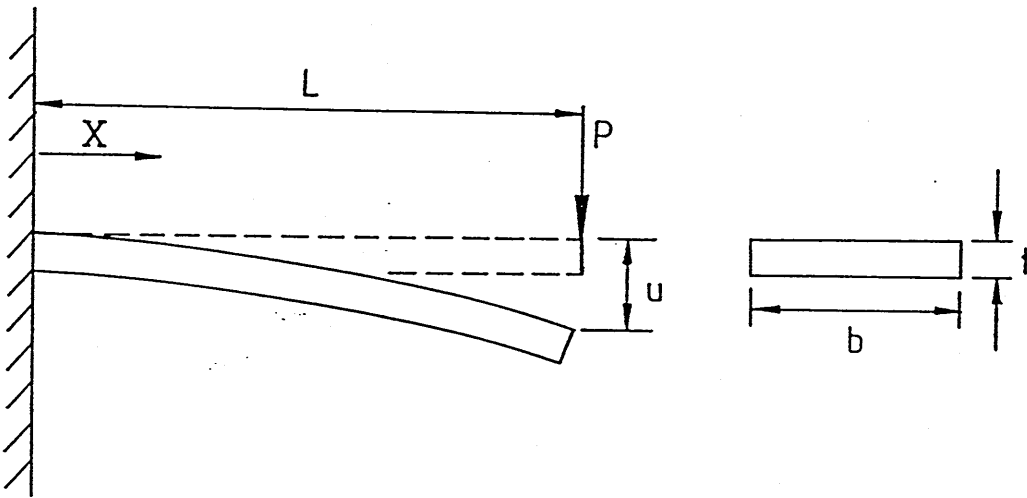


Fig.5.11 Cantilever Beam Dimensions and Notation

The maximum deflection occurs at the point of load application $x=L$ thus,

$$U_{\max} = P \cdot L^3 / 3 \cdot E \cdot I \quad \dots (5.2)$$

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and the maximum bending moment occurs at the root of the cantilever, the magnitude of which is,

$$M_{\max} = P.L \quad \dots (5.3)$$

Thus, the maximum stress occurs at the restrained end of the cantilever and can be calculated by the following expression,

$$\sigma_{\max} = \pm (U_{\max} \cdot 3.E.I/L^3) \cdot (L.y/I) \quad \dots (5.4)$$

Where I = Second moment of area

$$y = t/2$$

The strain at the same location can be shown to be

$$\epsilon_{\max} = \pm (3/2) \cdot (U_{\max} \cdot t/L^2) \quad \dots (5.5)$$

This represents the magnitude of strain for a given deflection U, that is to be measured by the electrical resistance strain gauges.

As the intended use of the clip-on-gauge was to carry out measurements under dynamic loading conditions, the selected ED-series WELWYN gauges are suitable as they are designed to have extended fatigue life. They consist of an isoelastic alloy foil in combination with a tough, flexible polyimide backing. Their gauge factor is of the order of $K=3.6$, and their extended fatigue life makes this type of strain gauges highly suitable for dynamic strain measurements.

For fatigue life of 10^7 cycles, the specified range is restricted to $\pm 2200 \mu\text{strain}$. In view of the above restrictions in the gauge operating strain range, it was decided to design the cantilever to achieve a maximum strain of the order of $\pm 2100 \mu\text{strain}$. This strain level to

be achieved when, the clip-on-gauge is set on the specimen prior to testing.

The final design calculations carried out according to the above methodology and component limitations, have yielded a cantilever with overall dimensions, length 40.0mm, width 9.5mm and thickness 1.0mm, see Fig.5.12.

The material selected for the fabrication of the cantilevers, was a Beryllium-Copper alloy (1.8%Be, 1.4%Co), which was machined in the annealed condition. The cantilevers were subsequently heat-treated, 2.0 hours at 400°C so as to attain optimum strength and toughness. In order to maintain the weight of the assembly to a minimum, the spacer block was machined from an Aluminium alloy bar.

The dimensions and geometry of the spacer block were based upon the recommendations of ASTM E-399/86 standard, for fracture toughness testing.

The completed clip-on-gauge with the knife edge attachments as fitted on the compact type tension specimen is shown in Fig.5.13

5.5.2 Electrical Design

The sensing elements used on the clip-on-gauge transducer were electrical resistance, foil type, strain gauges. The operation of such strain measuring sensors is based upon the fact that the resistance of a conductor changes with the application of load, strain.

Such changes in the electrical conductivity under loading are dependent upon the material used for the fabrication of the gauges. A measure of the strain gauge sensitivity is the gauge factor K , which is defined as the resistance

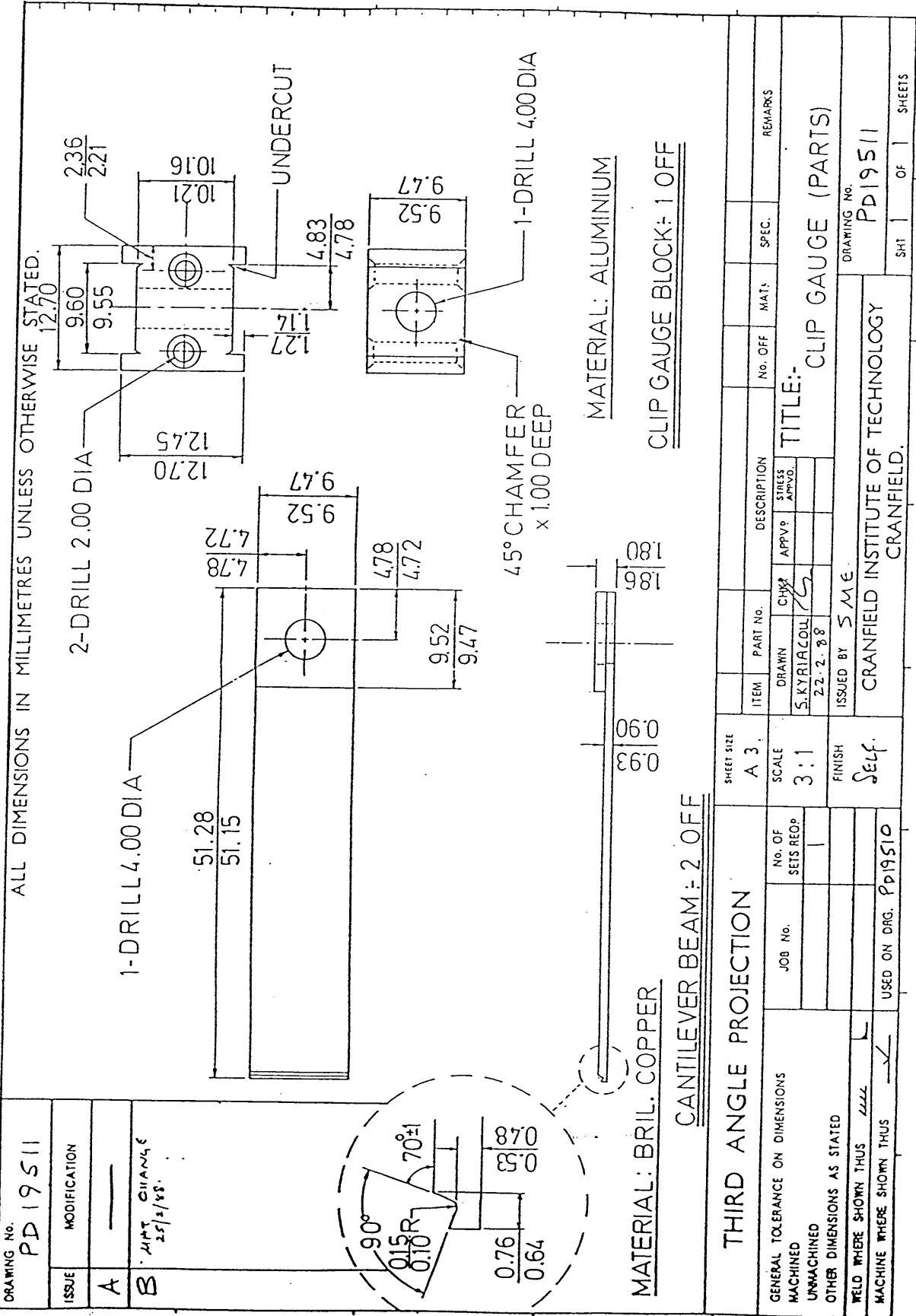


Fig.5.12 Clip-on-Gauge manufacturing drawing

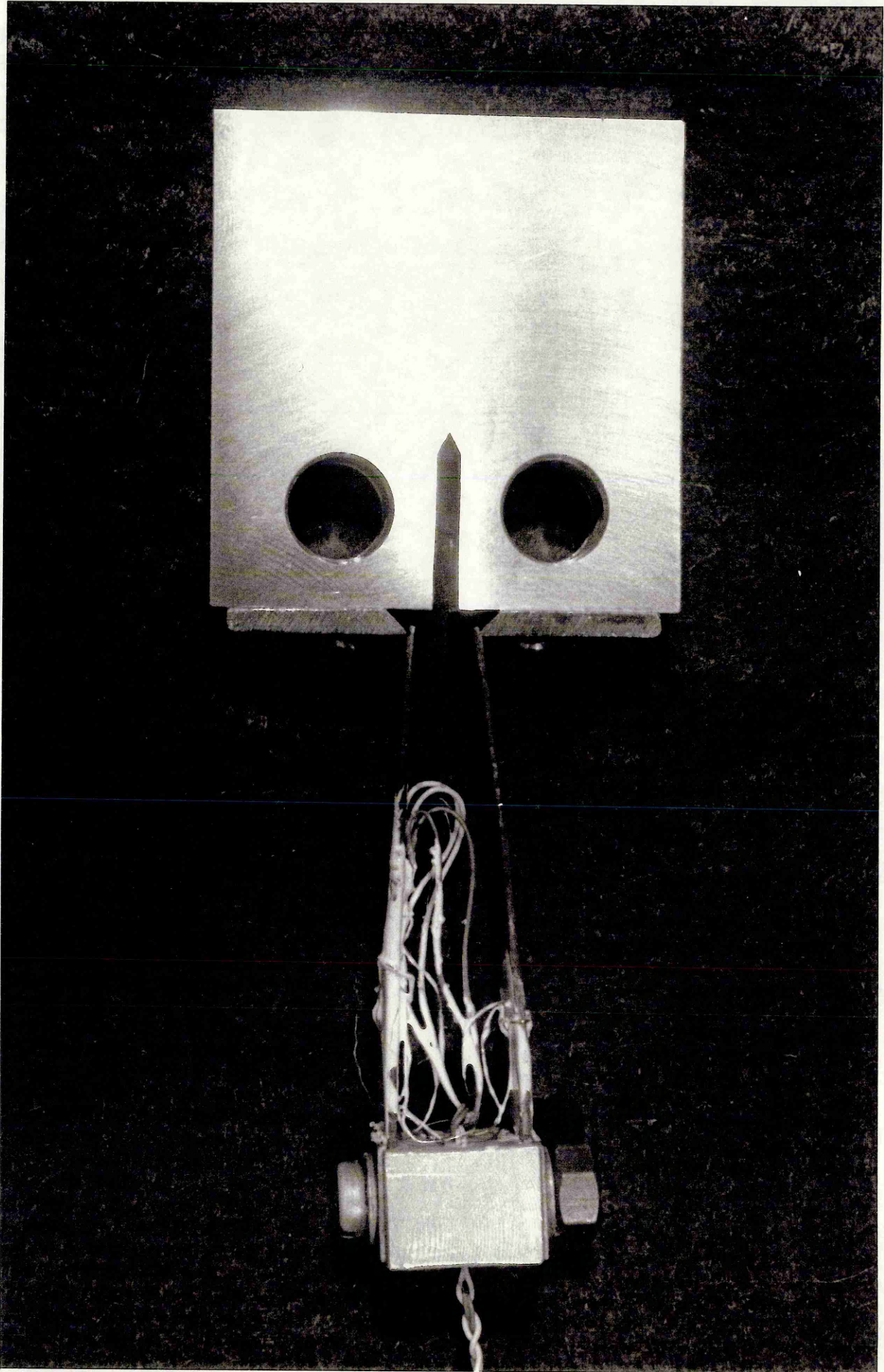


Fig. 5.13 Clip-on-Gauge and Knife Edge Attachments
as Fitted on a CT Specimen

change per unit initial resistance, divided by the applied mechanical and/or thermal strain.

The changes in resistance produced by normal strain levels experienced by engineering components are very small, and sensitive instrumentation is required. All strain gauge instrumentation is based upon the Wheatstone Bridge network, Fig.5.14.

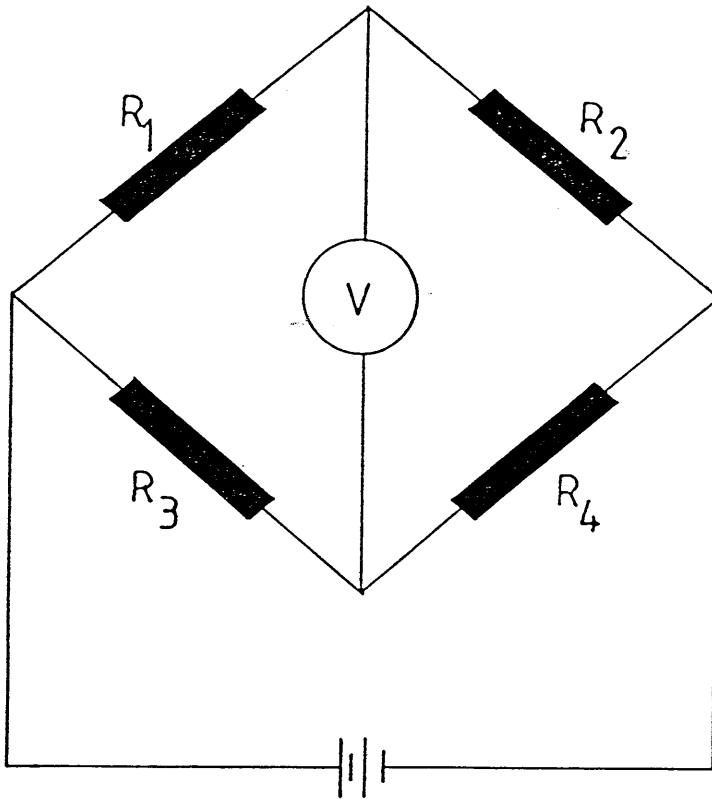


Fig.5.14 Wheatstone Bridge Network.

A variety of bridge configurations may be employed for strain measurement, depending upon the application requirements. Namely, the available configurations are. the quarter, half and full bridge networks with, one, two and four active arms respectively. The full bridge arrangement provides maximum sensitivity and exhibits an inherent temperature compensation capability. The later is

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of primary importance as the resistance of metallic conductors is dependent upon the environmental temperature. Fluctuations in the operating temperature can seriously affect strain measurements, and errors are induced due to thermal loading (strains), on the gauge.

As the clip-on-gauge was to operate under dynamic loading conditions, where the required sensitivity is high, the full bridge configuration was selected for the transducer. The wiring diagram Fig.5.15, depicts the configuration and connection of the strain gauges on the full bridge network to achieve maximum sensitivity.

5.5.3 Transducer Calibration

Calibration of the clip-on-gauge was carried out with the objective of obtaining an analytical expression, relating the clip-on-gauge opening (mm) to the voltage output (Volts) from the bridge.

For the purpose of calibration the following equipment were utilised:

1. A Signal Amplifier
2. A Digital Voltmeter (DVM)
3. A Vernier Calliper

As it is required that accurate measurements of the clip-on-gauge opening are determined, a Vernier Calliper with resolution of 0.02mm was employed. The calibration procedure was as follows:

Initially, the clip-on-gauge was attached to the knife edges of the Vernier. The Opening was adjusted to 5.0mm, as this represents the gauge opening at set up when the transducer is attached to any specimen before testing.

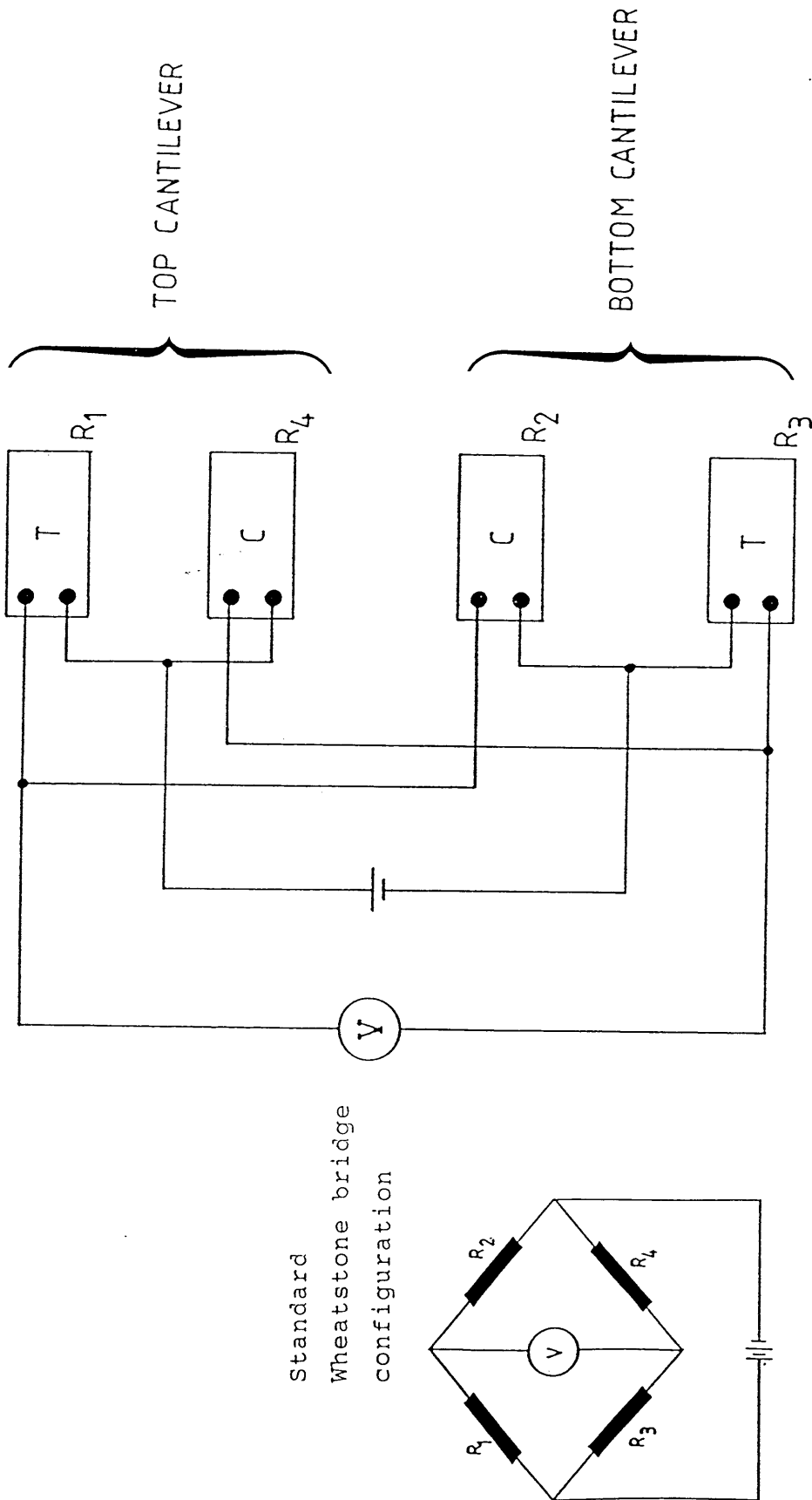


Fig.5.15 Clip-on-Gauge schematic wiring diagram

The bridge excitation voltage was set to 2.0Volts, and the system was zeroed with the use of the adjustable ten-turn potentiometer on the amplifier. Subsequently, the Vernier opening was increased in steps of 0.25mm and the corresponding bridge output voltage were recorded.

Finally, a computer program that carries out least squares polynomial curve fitting was utilised in order to obtain the best fitted first order polynomial through the calibration data. Fig.5.16, shows the calibration line of the clip-on-gauge.

Thus, the following equation was derived for the transducer,

$$GO = 5.182 + 0.425BV \quad \dots(5.6)$$

Where GO =Gauge Opening (mm),
BV =Bridge Voltage Drop (Volts).

5.6 SERVO-HYDRAULIC MACHINE ALIGNMENT

Load eccentricity is the most widely recognised problem in experimental arrangements intended for direct tension testing. When conducting a direct tension test, it is desirable to achieve a uniform tensile stress distribution through the specimen length and cross section. A uniform tensile stress distribution implies that the resultant of the applied forces passes through the elastic centroid of each cross section. If this condition is not satisfied then, bending will be present and as a result an indeterminate stress distribution will be present.

Effectively, this distribution consists of two stress conditions superimposed upon each one another namely, the distribution due to bending and the one due to the direct applied tensile load. The resultant stress distribution caused by an eccentrically applied load is shown in

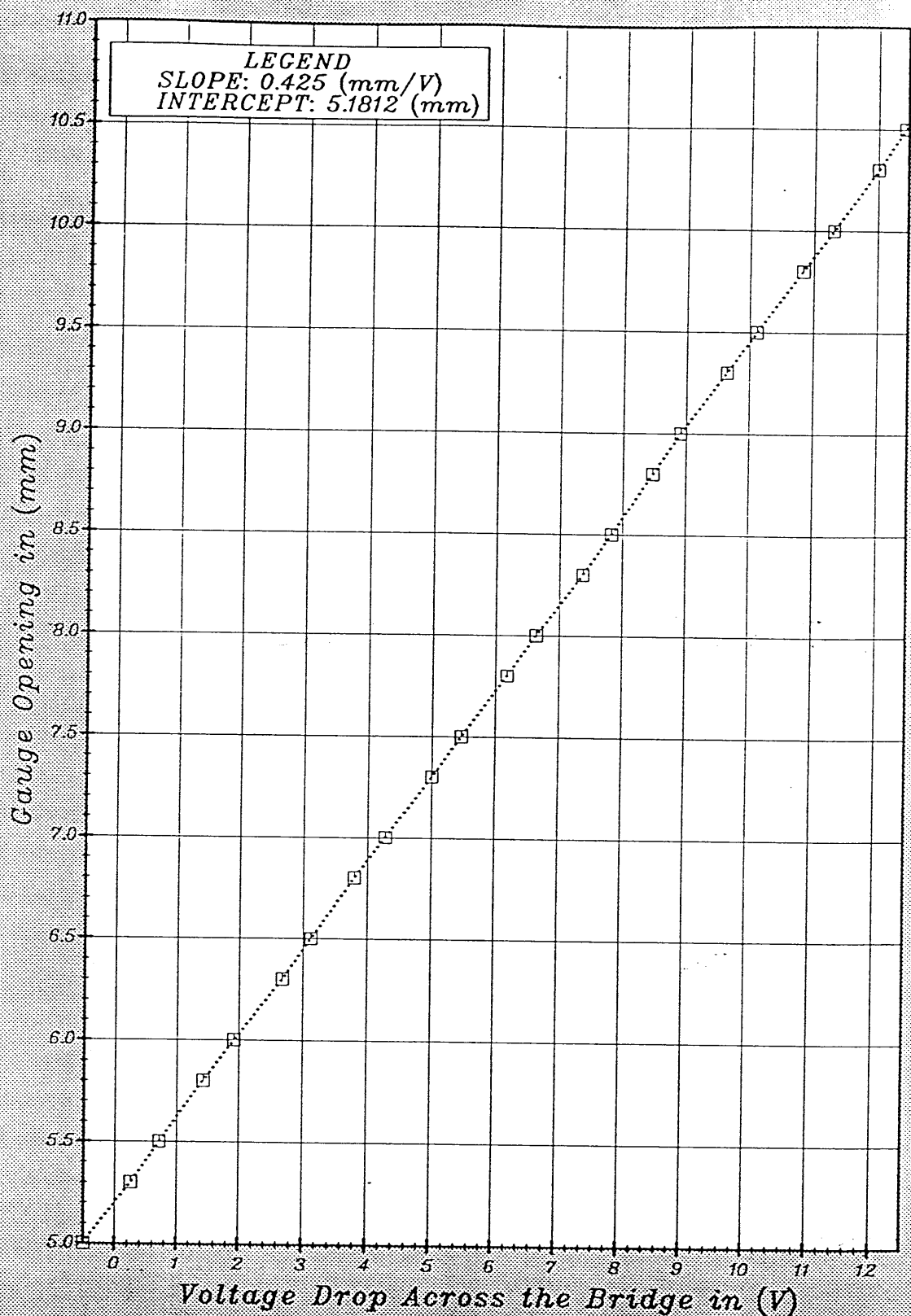


Fig.5.16 Clip-on-gauge calibration curve.
Gauge designed in accordance to ASTM E-399
specification
Calibration supply voltage 2.0V

Fig.5.17. The difference between the expected/desired and the resultant stress distributions (shaded areas) is due to the distribution induced by bending.

Sources of load misrepresentations can be in faults in the loading fixtures due to machining inaccuracies, wear of moving parts, fretting, fatigue, as well as specimen geometric and machining deviations.

Two types of load eccentricity are shown in Fig.5.18. The eccentricity condition depicted in Fig.5.18a, to a first order approximation is constant over the length of the specimen. The other form of eccentricity is more general Fig.5.18b. In this case the load eccentricity varies with the length of the specimen, from e_1 to e_2 at the other. The result of such loading is to cause failure in the vicinity of the highest eccentricity.

With regard to the experimental work undertaken, it was necessary to ensure that the load fixture arrangement would apply a direct tensile load on the specimens. This is of paramount importance as loads applied in any other direction would produce mixed mode opening of the crack which is highly undesirable.

5.6.1 Alignment Test, Equipment and Methodology

In order to carry out the experiments with some degree of certainty regarding the results, it was considered necessary to perform regular checks of the load train alignment. These checks were arranged at set intervals of 100 hours, approximately after 10 experiments.

Such routine checks gave a good picture of the general condition of the loading fixtures. At the initial stages of the work it was observed that the pre-cracking stage preceding each test was becoming considerably longer, while, the original set of clevises made of low grade

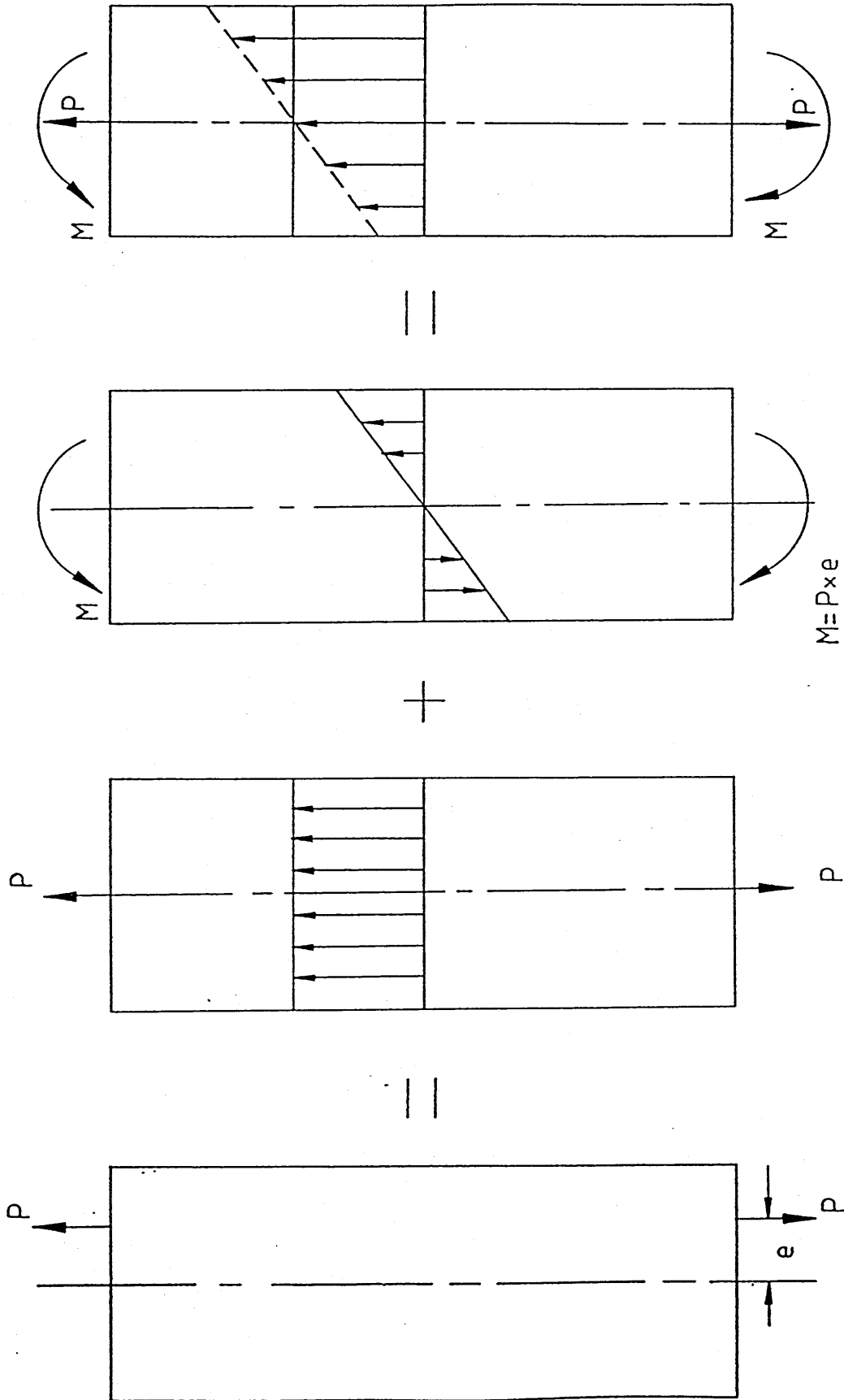


Fig.5.17 Stress distribution caused by load eccentricity

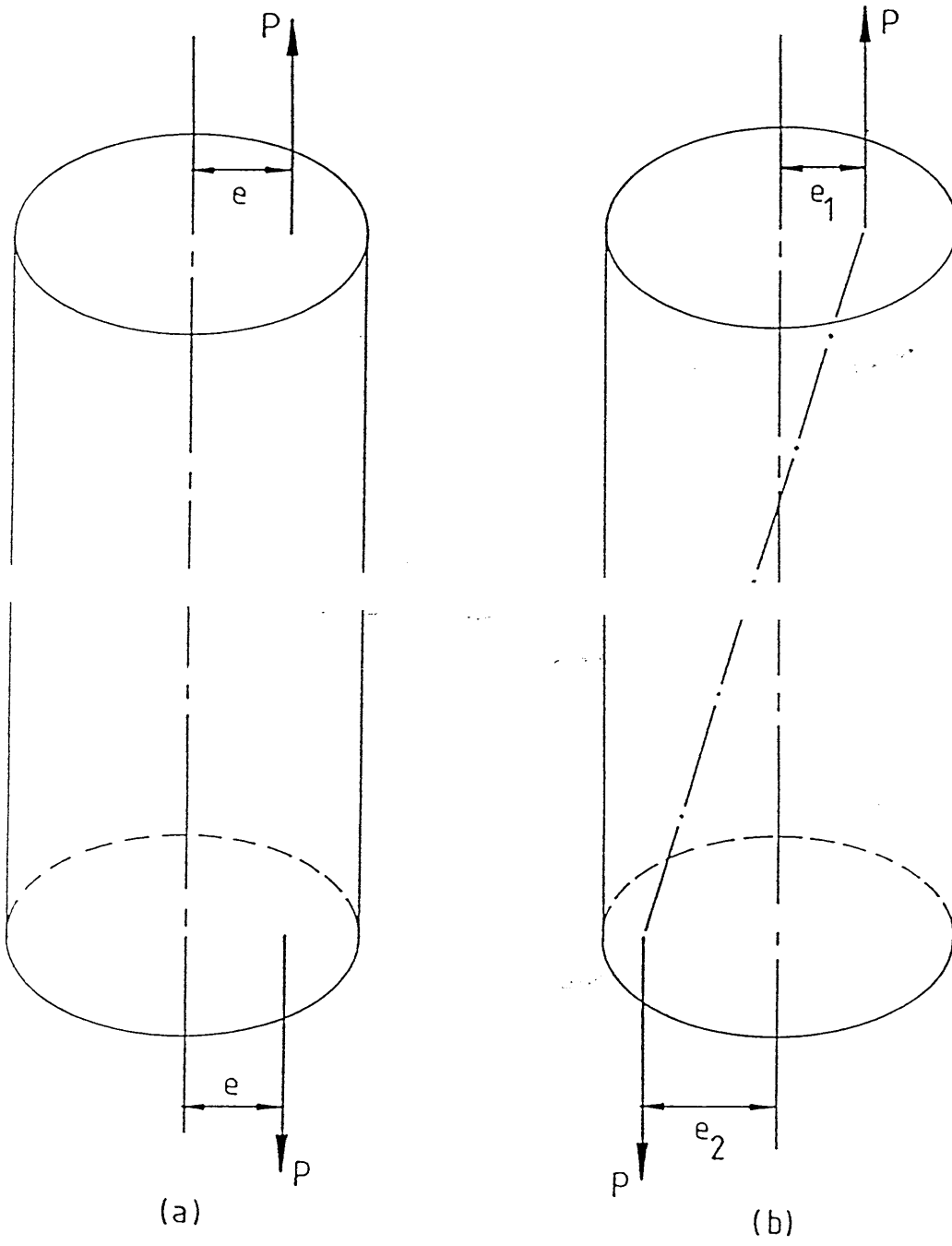


Fig.5.18 Two types of load eccentricity

steel were used. In addition it was observed that the crack lengths on the two faces of the CT-specimens were uneven (slanted crack fronts). A routine alignment test revealed that eccentricity of the order of 5% on the plane at approximately 75° relative to the plane of the specimen was present, see Fig.5.19.

These observations suggested that the test piece was subjected to out of plane bending. Careful examination of the clevises with a magnifying lens (x10), indicated that the screwthreads were damaged and they showed wear due to fretting, caused by the cyclic tensile loading used.

The loading assembly of the machine is shown in Fig.5.20. It can be seen that misalignment may be caused by a fault either on the sub-assembly - clevis location nut, retainer sleeve, actuator arm - or due to a fault to the clevis itself. Therefore, it is essential to check both possibilities, which dictates the use of two different set-ups of measuring equipment.

In order to investigate and quantify the misalignment present in the sub-assembly of actuator arm, clevis location nut and retainer sleeve, a threaded rod specimen was utilised. The specimen was instrumented with three linear strain gauges, bonded with their longitudinal direction parallel to the axis of symmetry of the rod. The strain gauges were located at the mid-length of the specimen at 120° intervals around the circumference.

For the purpose of strain measurements a half bridge arrangement was adopted with a dummy gauge to provide the required temperature compensation. The alignment test was carried out with use of incremental loading and readings of load and bridge output were recorded for each gauge. A plot of the recorded readings is shown in Fig.5.21. The maximum difference between readings at each load level was

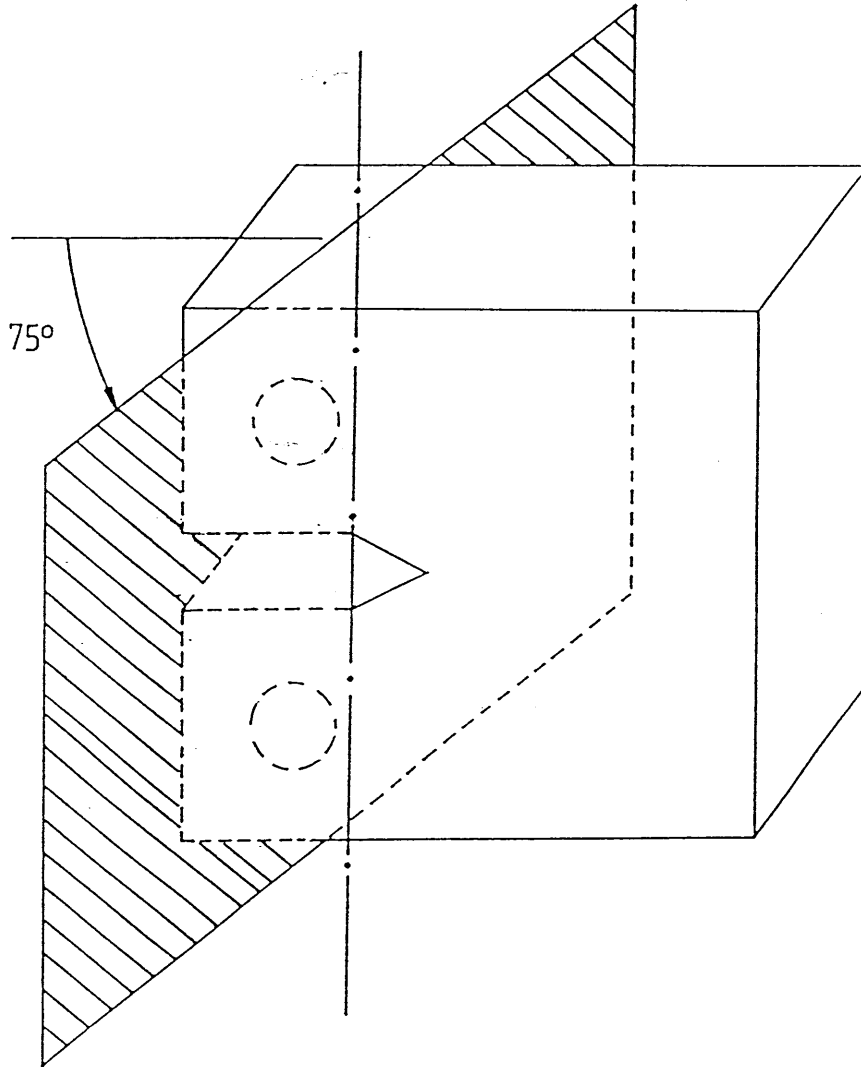


Fig.5.19 Definition of eccentricity plane
for a CT specimen

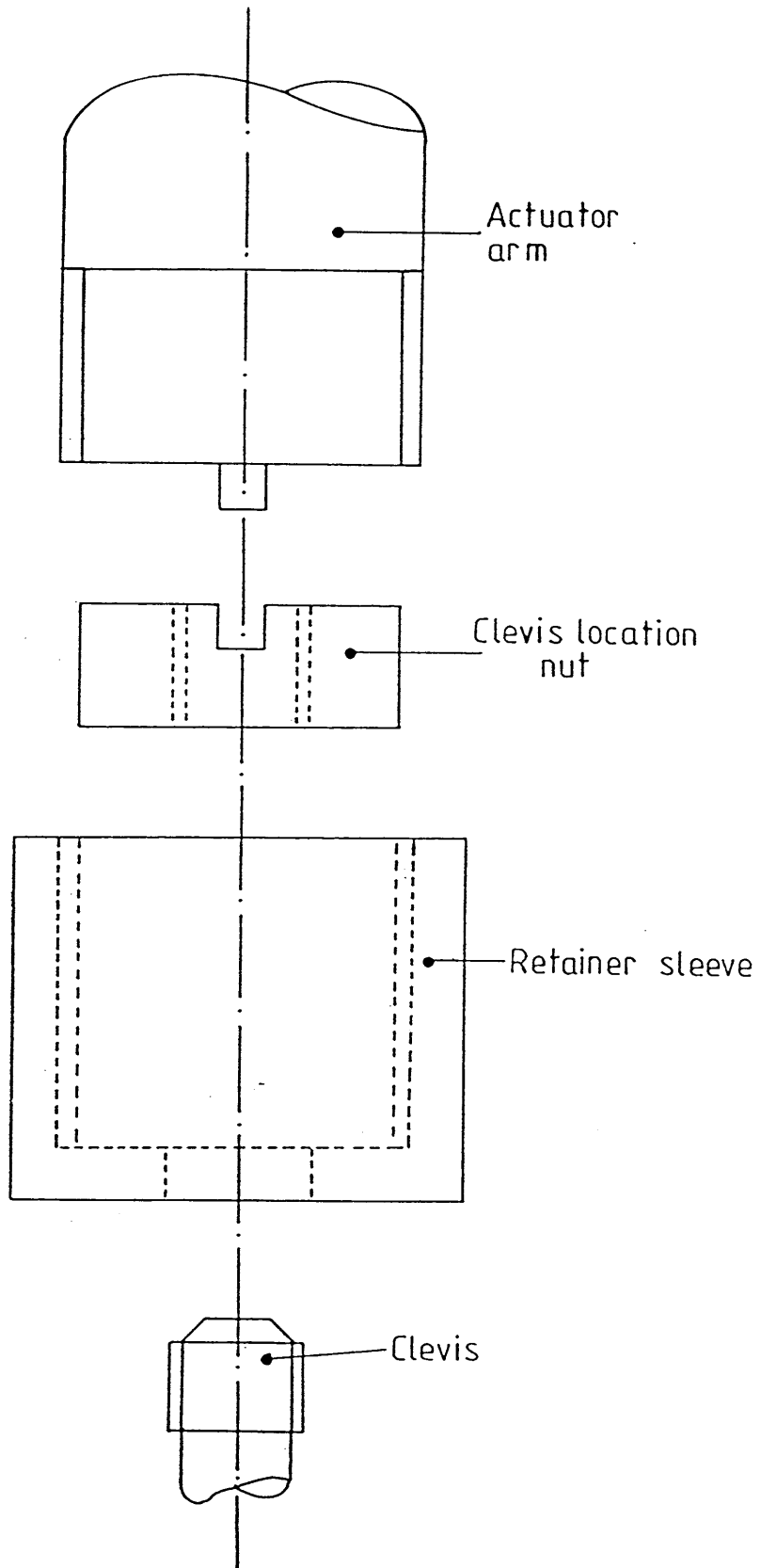


Fig.5.20 Fatigue testing machine
loading assembly

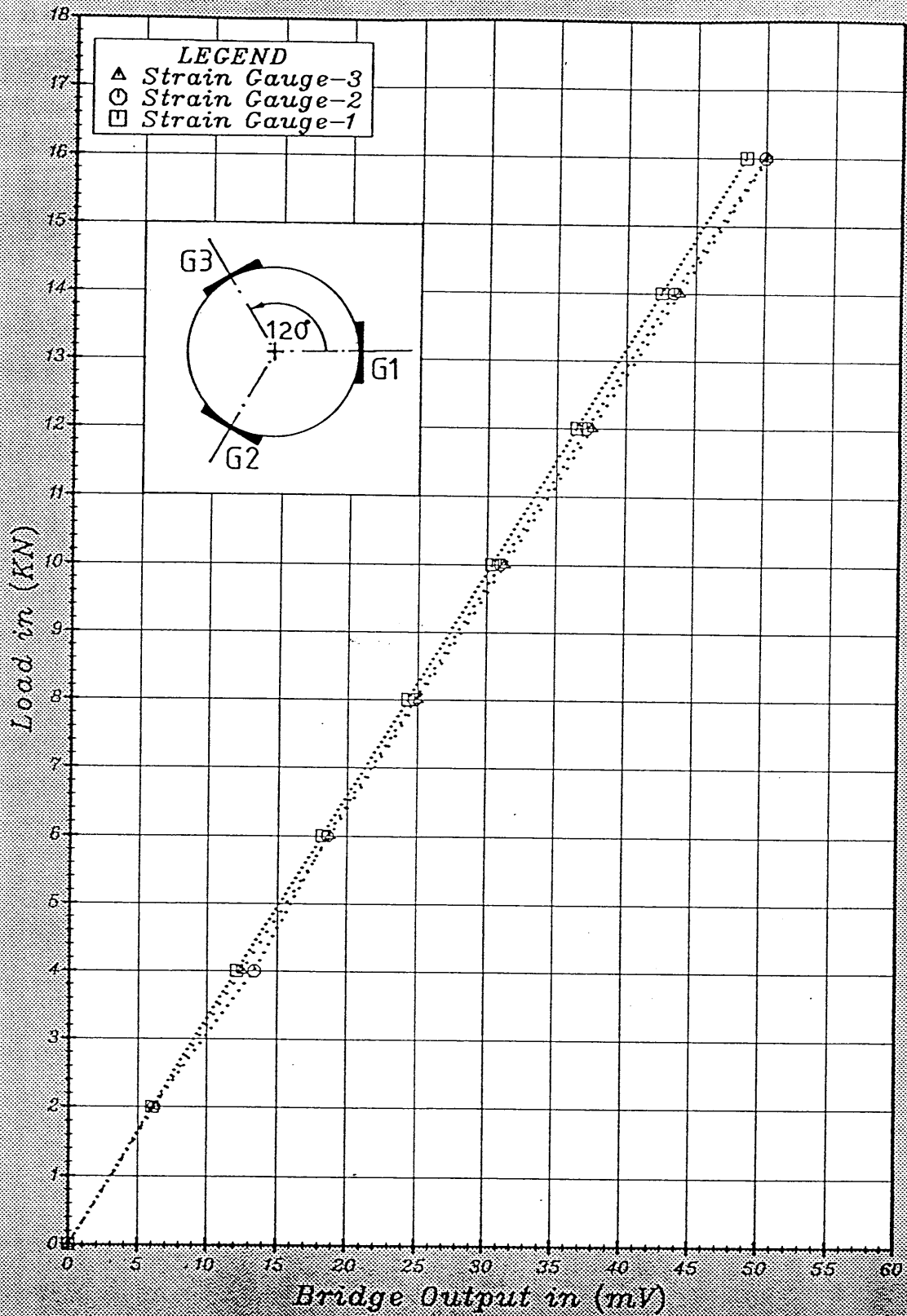


Fig.5.21 Load versus bridge output for cylindrical specimen misalignment tests with three strain gauges at 120 degree intervals

taken as the measure of the misalignment. It was thus found that the maximum difference was of the order of 3% for a maximum load of 15.0KN. The maximum load used during the experimental fatigue crack propagation program was around 5.0KN which from the graph shows an error of less than 1.5%.

In order to check the eccentricity due to the clevises and pins, another type of specimen was designed and used. This strip specimen had thickness of the same nominal value as the compact tension specimens used, and was equipped with two holes, one at either end so as to be held within the clevises by the pins, see Fig.5.22. Two linear strain gauges were bonded on opposite faces of the specimen.

The objective was to isolate and measure the strains due to out of plane bending. This was achieved with the use of the half bridge configuration depicted in Fig.5.23. The output of such a bridge is proportional to twice the actual bending strain. Using the concepts of the engineers theory of bending it is possible to evaluate the load eccentricity by,

$$e = (\epsilon \cdot E \cdot I) / (P \cdot t) \quad \dots (5.7)$$

Where ϵ = Bending strain

E = Modulus of elasticity

I = Second moment of area

P = Applied load

t = Half thickness

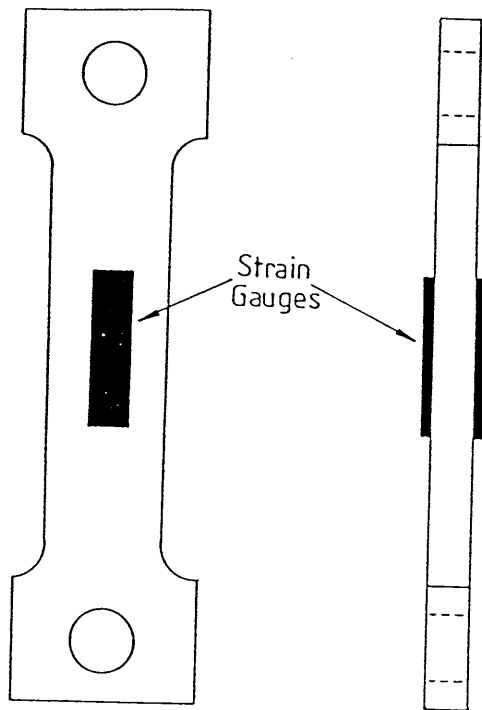


Fig.5.22 Clevice alignment specimen

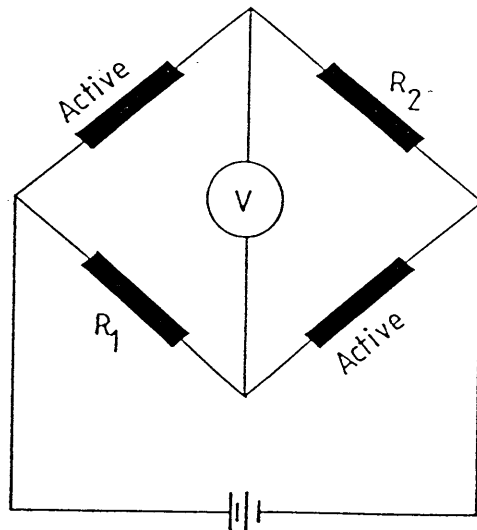


Fig.5.23 Half-bridge configuration for maximum sensitivity

R E F E R E N C E S

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CHAPTER 6

EXPERIMENTAL INVESTIGATIONS (I)

CONSTANT LOAD AMPLITUDE

6.0 Summary

6.1 Experimental Analysis

- 6.1.1 Specimen Selection
- 6.1.2 Specimen and Machine Set-Up
- 6.1.3 Pre-Cracking Procedure
- 6.1.4 Equipment and Procedures
- 6.1.5 Measuring Techniques

6.2 Analysis of Results and Discussions

- 6.2.1 Experimental Programme
- 6.2.2 General Observations
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CHAPTER SIX

EXPERIMENTAL INVESTIGATIONS (I)

CONSTANT LOAD AMPLITUDE

6.0 SUMMARY

The experimental investigations carried out with the purpose of examining the constant load amplitude fatigue crack growth of the 7075 T7351 Aluminum-Zinc alloy are presented within this Chapter. The experimentally determined crack growth trends are discussed with special reference to the effects of the load ratio R , and the mean load P_{mean} .

A special section is included where the derivation of an empirical crack growth law (Second Order Hybrid) is presented in detail. The merits of the derived law are discussed and compared with the Paris-Erdogan linear equation of crack growth and the experimental data.

6.1 EXPERIMENTAL ANALYSIS

6.1.1 Specimen Selection

The tests undertaken were aimed at evaluating the fatigue crack propagation behaviour of the material under investigation. It was therefore considered appropriate to carry out the testing in accordance to well documented procedures using standard specimens, thus allowing the direct comparison of results with published data. The ASTM E647-86 standard for constant load amplitude testing [6.1], was adopted.

The selected specimen geometry is that of the compact tension type, CT as it is commonly known. The nominal dimensions are $W=32.0\text{mm}$ and thickness $B=8.00\text{mm}$. The selection of the dimensions has been based upon the recommendations of the aforementioned standard, in order to ensure that the specimen remains predominantly elastic for the duration of the experiments and, that plane strain conditions prevail.

The specimens were machined from a 2.0 inch diameter bar of an aerospace high strength 7075 T7351 Aluminium alloy. The direction of the material plays a major role on the observed mechanical behaviour, Hertzberg [6.2]. This is due to the direction of the grain structure and other microscopic formations, related to the method of manufacture (forging, rolling e.t.c.), Dieter [6.3] and Smith [6.4]. It was therefore decided to machine all the specimens so that, the direction of the crystal structure was the same. Thus, the specimens machined were of transverse slices of the bar as shown in Fig.6.1.

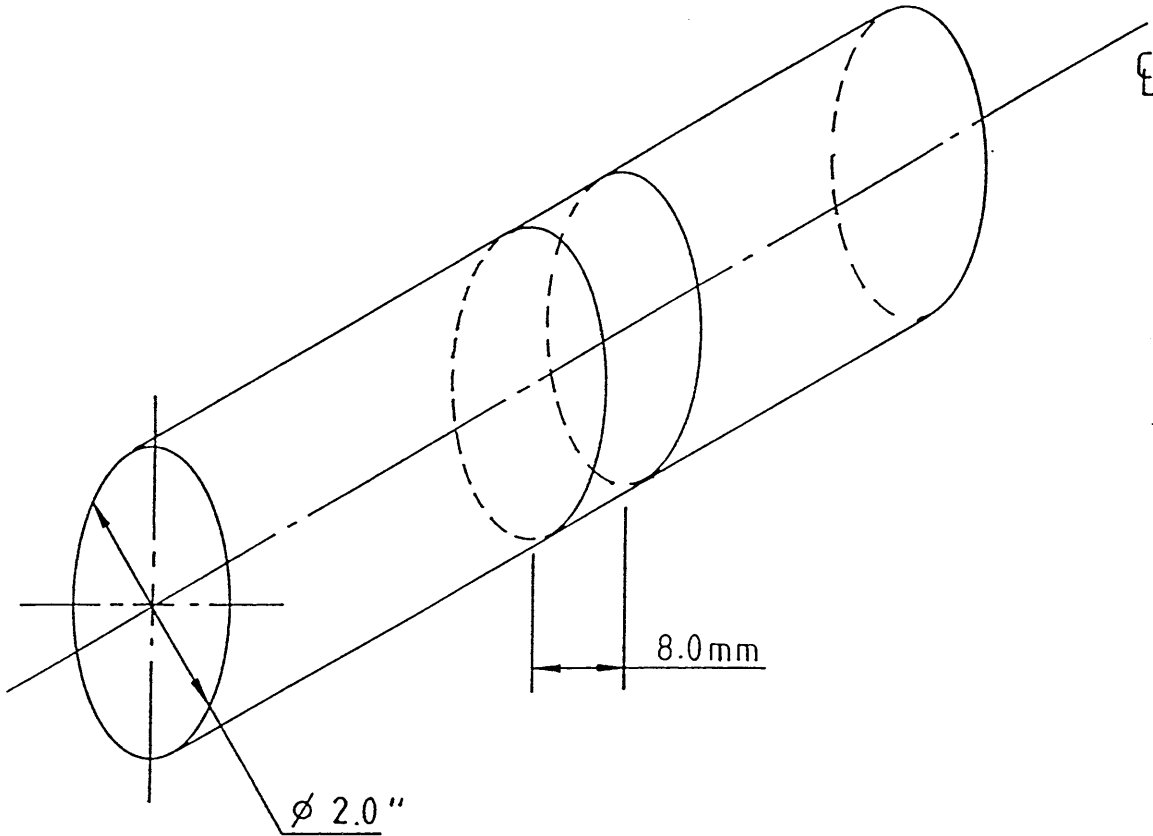


Fig.6.1 Specimen Cutting Direction

The starter notch geometry is important in terms of the pre-cracking time and, it can greatly affect the crack front shape and subsequent growth. The straight through notch geometry was selected. This starter notch configuration is easier to machine with a shaped milling cutter and one final cutting pass is sufficient to produce a reasonably sharp notch. Some attempts were made in machining chevron type notches, which were not successful as the resulting initiated cracks were slanted (uneven crack lengths on either face of the specimen). The reason for the poor performance of the chevron notches was sought and finally attributed to machining inaccuracies.

These inaccuracies originated during the notch preparation stage by the fact that, two cutter settings are required and the machined piece had to be removed, turned over and relocated in the milling machine. Figure 6.2 depicts a specimen with a chevron starter notch, where the eccentric location of the notch tip is clearly identifiable.

In order, to improve the accuracy of determining the crack length by means of the optical method, see Section 5.2.1, the specimens were ground prior to testing in order to improve the imaging of the crack tip. A rotating flat grinder was used with water lubrication and grinding papers of 200 and 500 mesh. The grinding operation was essential to eliminate from the surface marks left by the machining operation that were found to make the identification of the crack tip difficult. Following this operation the specimens were treated with methylated spirit in order to achieve water decontamination. Finally, the methylated spirit was caused to evaporate by means of a warm air jet.

6.1.2 Specimen and Machine Set-Up

The machine control mode was set to displacement control for the purpose of specimen set-up. After locating the specimen in the clevises by means of fine adjustments of the displacement potentiometers, the machine was set to the load control mode of operation. The hydraulic pressure was built in the system by switching to HIGH, and the static load was then applied. This represents the mean load P_{mean} , about which the cycling is to be carried out. Subsequently, the appropriate function generator was selected and the desirable frequency was set. Finally, the cyclic load amplitude is adjusted initially to a very small value which is slowly increased to the desired level.

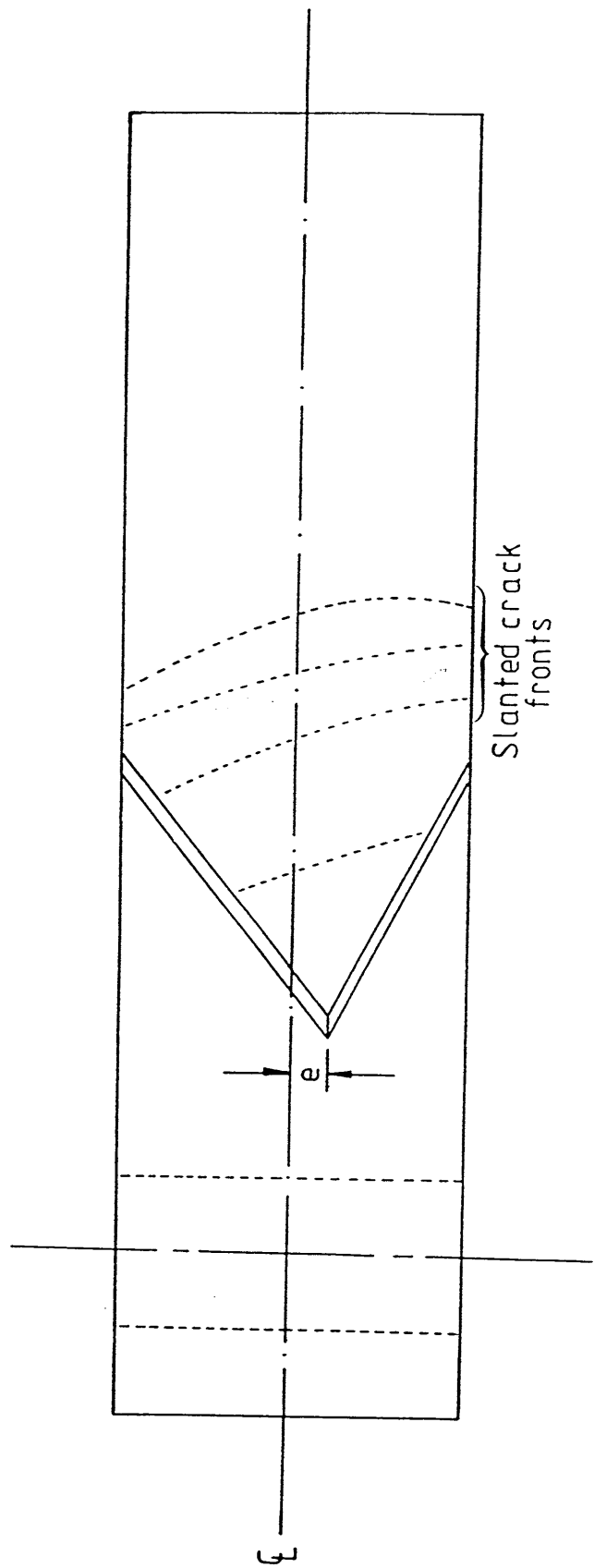


Fig.6.2 Sketch of slanted crack fronts caused by the Chevron notch eccentricity

In addition to the control console instrumentation of the machine, a CRO was connected to the output analogue port of the load cell. This arrangement enables continuous monitoring of the load amplitude and cyclic frequency; the graphical representation of events was found to be easier to follow during testing. Following the correct set-up of the test conditions the fracture trip control was adjusted so that, if the specimen fractured during testing or at the end of the test the machine would be automatically shut down and the pressurized oil would be released to the sump, thus safeguarding the integrity of the equipment.

6.1.3 Pre-Cracking Procedure

In order that the fatigue crack propagation tests commence, it is essential to pre-fatigue the specimens as to produce a small crack. The purpose of pre-cracking is to eliminate from the experimental data information related to the initiation period of a crack at the notch. The crack initiated is allowed to propagate to a small length, thus eliminating the effects of the notch upon the crack tip stress field.

In order to decrease the amount of time needed for initiation of a small crack, it is common practice to carry out the pre-cracking in two stages. Initially, loads higher than the loads intended to be employed in subsequent testing are used. As soon as the crack is visible or otherwise detected, load shedding takes place, i.e. stepwise reduction of the cyclic loads to levels below the ones intended for testing.

The load generally is reduced uniformly to avoid transient effects. Crack growth can be arrested above the threshold stress intensity value due, to the formation of plasticity ahead of the advancing crack tip. Therefore, the step size of the load has to be reduced gradually during pre-

cracking, if the effects of plasticity are to be eliminated. The reduction in the maximum load applied has to be less than 20% of the previous load condition.

In the presented experimental programme the pre-cracking was monitored with an optical arrangement comprising, a travelling microscope and stroboscopic illumination. Readings of the applied loads were taken directly from the digital display of the load cell mounted on the test equipment console.

Ancillary control of the loading was provided by the connected digital storage CRO. The pre-crack was allowed to grow until a crack length of 2.5-3.0mm measured from the notch tip was attained.

6.1.4 Equipment and Procedures

The experimental programme conducted utilised a uniaxial servo-hydraulic testing machine. A set of special clevises were designed and built, Fig.6.3, in order to allow the accurate location of the test specimens within the load train of the machine. The presence of misalignment in the loading arrangement can cause severe deviations of the crack propagation path and in many occasions slanted crack fronts are developed. In addition, bending moments due to eccentric loading can cause abnormal failures of the test specimens. Figure 6.4, depicts some irregular failures caused by the presence of eccentric loading. It is essential to achieve good alignment of the loading arrangement so that spurious load inputs are eliminated. Regular checks of the alignment were carried out utilising purpose built strain gauge instrumented specimens to ensure the good condition of the assembly; Details are presented in Section 5.6. In addition the load cell

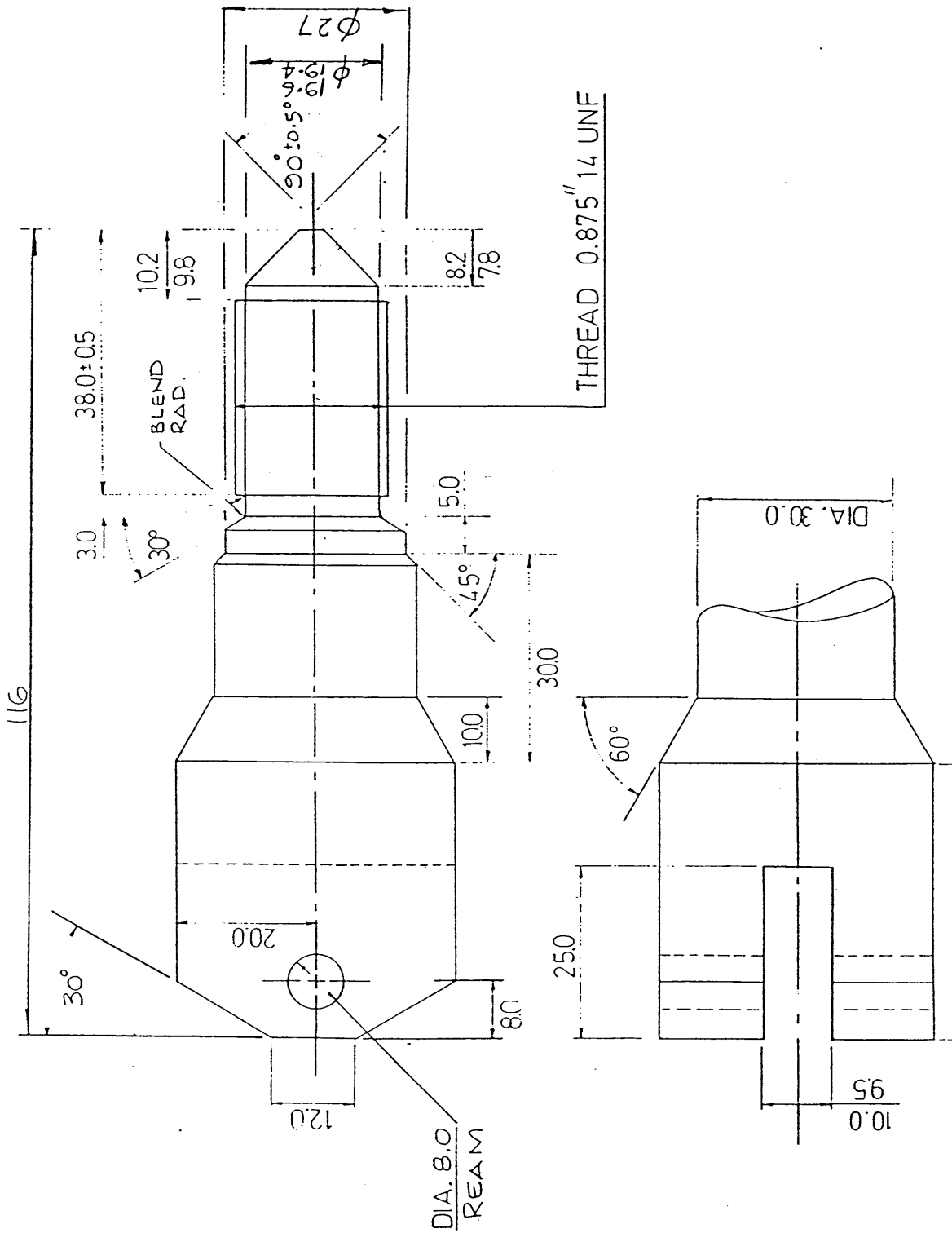


Fig.6.3 Servo-Hydraulic test machine clevis for holding CT specimens

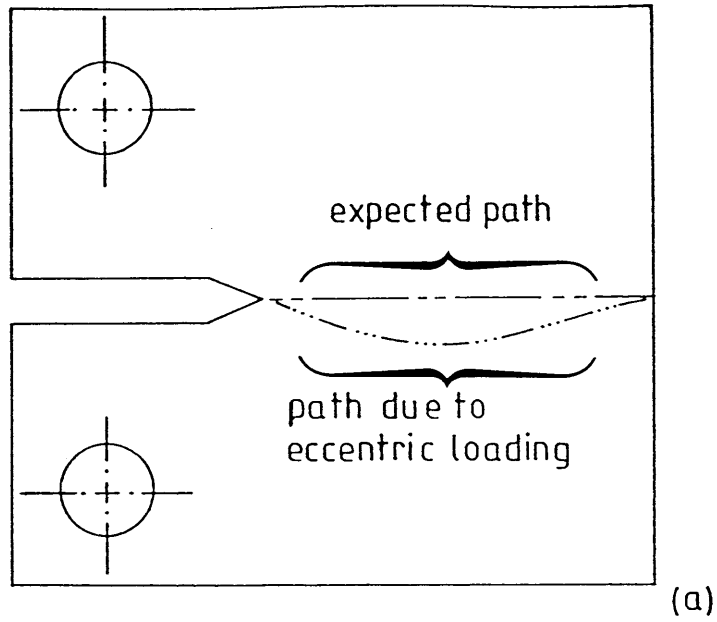
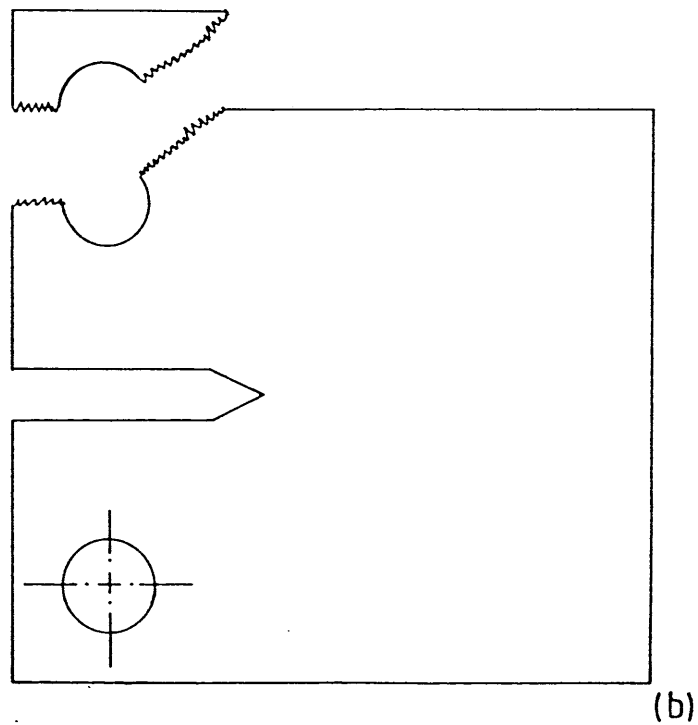


Fig.6.4 Specimen ubnormal failures

a) Slanted crack propagation path

b) Fracture at loading hole

attached CRO was used for checking the load cell digital display for accuracy of displayed readings.

In Fig.6.4 two types of failure are shown, one that relates to the slanted nature of the crack due to load misalignment and the other shows the failure of the specimen at the loading hole. The second failure clearly indicates that cracks have been developed in the loading hole, and only on one side of the specimen. Examination of the loading train in this case did show that the specimen was subjected to a bending load due to incorrect setting of the clevises. The important fact though, is that the Finite Element analysis did predict the location where the cracks appeared as a potential crack initiation site. Figures 4.20-4.22 clearly depict the stress concentration as well as the tensile nature of the stresses in the vicinity of the hole at a location 45° from the hole apex.

6.1.5 Measuring Techniques

The parameters measured during each test were the number of cycles and the crack length. The number of cycles was obtained by means of a digital cycle counter located on the machine console. The crack sizing was carried out by two different methods namely, optical measurements utilising a travelling microscope and the use of a compliance method employing a clip-on-gauge attached onto the mouth of the specimen. An X-Y analogue plotter was connected to the output ports of the clip-on-gauge and load cell, in order to record the necessary load displacement records for further analysis leading to the evaluation of the crack length. Figure 6.5 shows a specimen set in the machine with the crack sizing equipment used i.e. the clip-on-gauge and the travelling microscope arrangement.

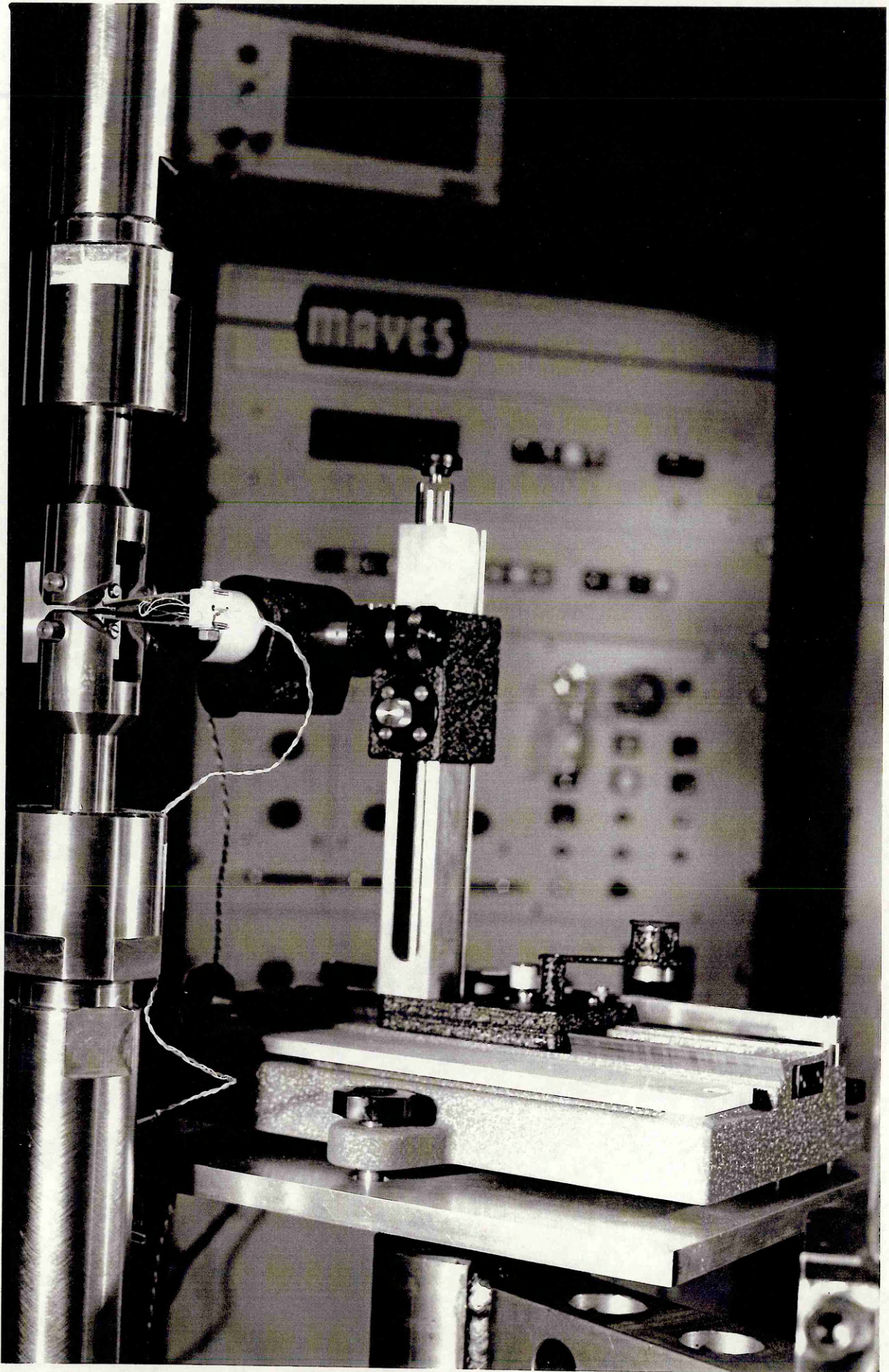


Fig. 6.5 Machine-Specimen Set-up and Crack Sizing Equipment

6.2 ANALYSIS OF RESULTS AND DISCUSSIONS

6.2.1 The Experimental Programme

The experiments conducted were designed so as to reveal the effects of load related parameters, upon the fatigue crack propagation under constant load amplitude conditions. Primarily, the effects of the R load ratio and the mean load P_{mean} , were sought. For this purpose, a matrix of tests was established, where the P_{mean} was varied as well as the R ratio. The matrix of tests conducted is shown in Table 6.1

		R ratio			
P_{mean}	1.0KN	0.1	0.3	0.5	0.6
"	2.0KN	0.1	0.3	0.5	0.6
"	3.0KN	0.1	0.3	0.5	0.6

and a variety of load cycling frequencies

Table 6.1 Constant Load Amplitude Test Matrix

In many published works regarding the fatigue crack growth and propagation in engineering metallic materials, the testing frequencies employed are in the range of 0.10-35 Hz, see references [6.5-6.9]. The majority of testing in the present study has been carried out utilising a load cycling frequency of 3000 r.p.m. or 50 Hz. This is very much higher than commonly employed, but its utilisation has been enabled by the equipment used. The uniaxial testing machine previously described, was found to attain an unstable control state around 57 Hz for the given specimen size and prescribed loading.

By virtue of the machine's sophisticated analogue control system being, of the double feedback closed loop type, at the frequency of 50 Hz the stability of the set waveform was good. It has to be noted that very infrequent

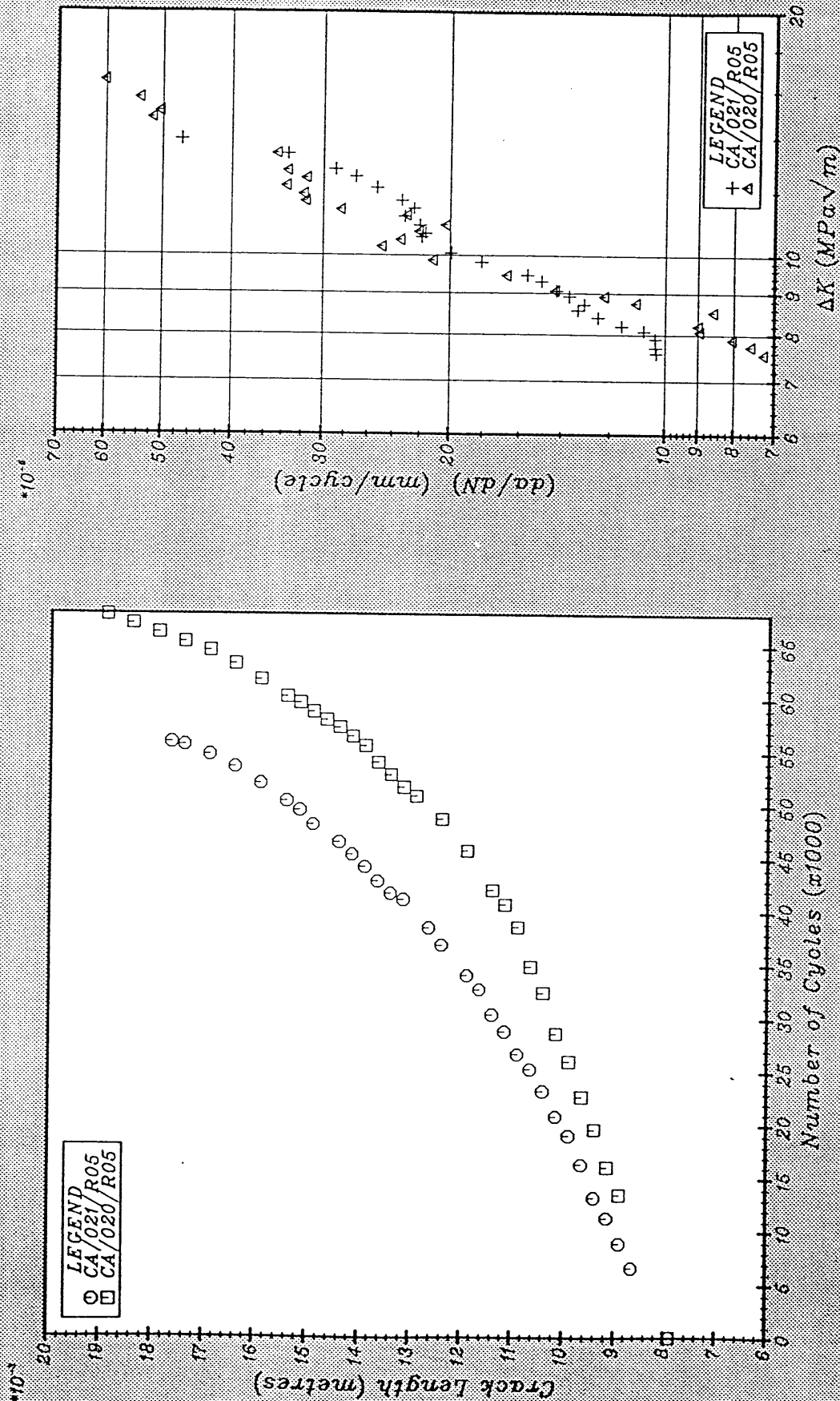
adjustments of the amplitude potentiometer were required, usually at the latter stages of each test. This need arises due to the increase of specimen compliance with increasing crack length. The rapid increase of compliance dictates the need for larger actuator travel in order that the loading is maintained within the preset limits. On the threshold of instability and fast fracture, accurate control of the machine becomes impossible as the inherent system inertia affects the overall response of the loading system to the command signal.

As the experimental programme progressed it was considered essential to expand the research so as to examine the possible influence of the load cycling frequency upon the fatigue crack propagation under constant load amplitude conditions.

6.2.2 General Observations

A large number of experimental data has been generated in the duration of the constant load amplitude work. A sample of experimental data is shown in Figures 6.6-6.9. The combined plots presented, illustrate the experimentally determined crack length (a) versus the number of cycles (N), and the corresponding crack growth curves of (da/dN) versus (ΔK) .

A common characteristic of all the (a versus N) curves, independent of loading condition used, is their shape. In general, this is parabolic and relatively regular. At the initial stages of propagation the curves appear to be shallow i.e. they exhibit small gradients which become gradually more acute especially as the crack length increases. This is an anticipated behaviour, as the increasing crack length implies the presence of higher stress intensities and thus increased availability of strain energy which causes the crack to advance. The



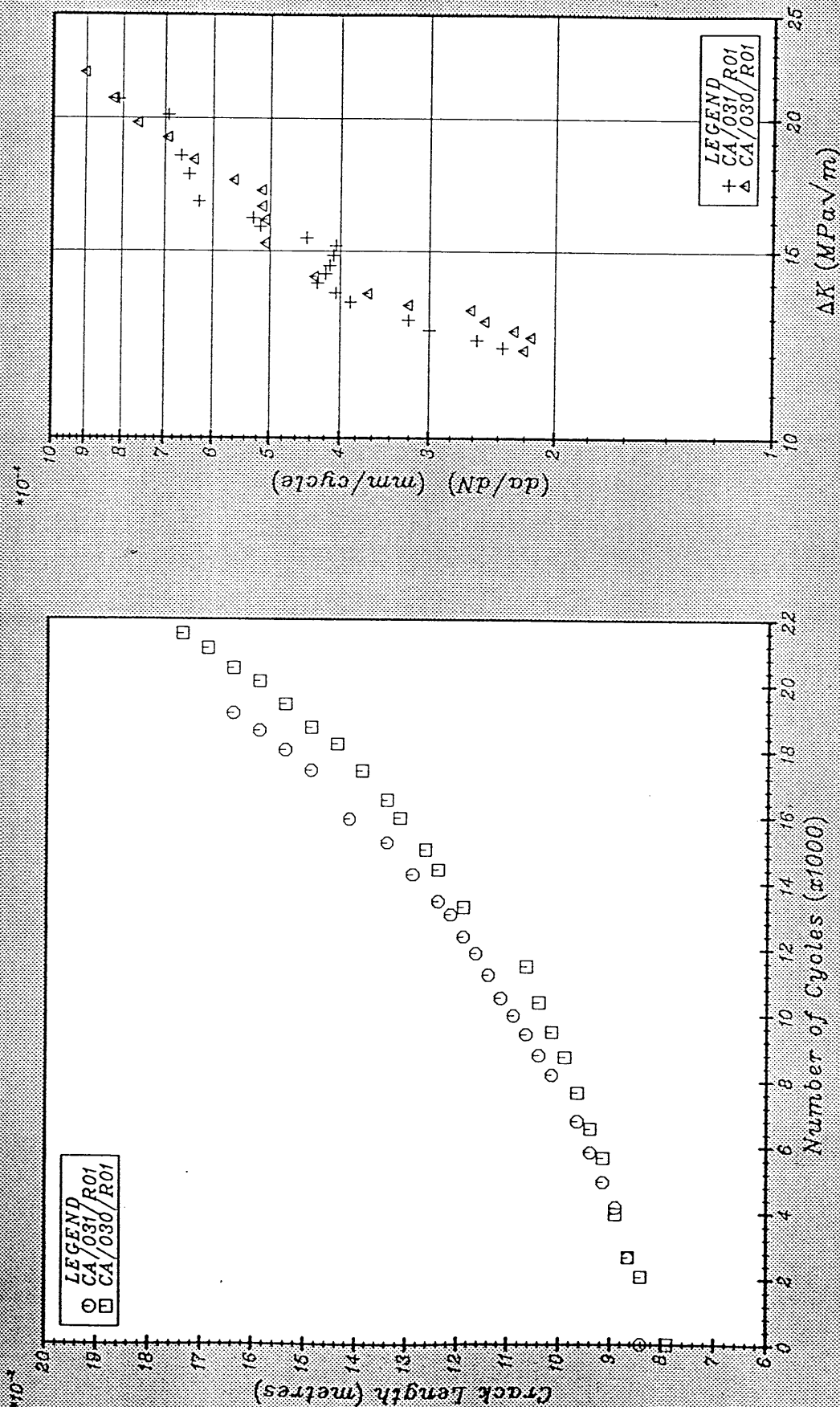


Fig.6.7 Constant load amplitude test material Aluminium-Zinc 7075 T7351 alloy
 Test specification: Load cycling frequency 50 Hz
 $P_{mean}=2.0KN$ $P_{max}=3.636KN$ $P_{min}=0.364KN$ $R=0.1$

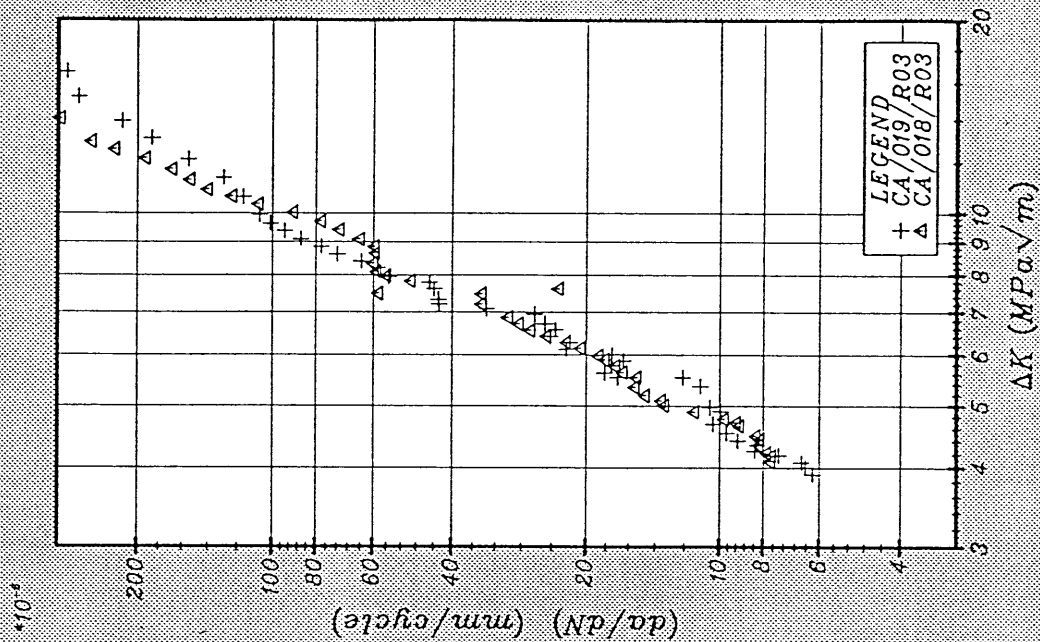
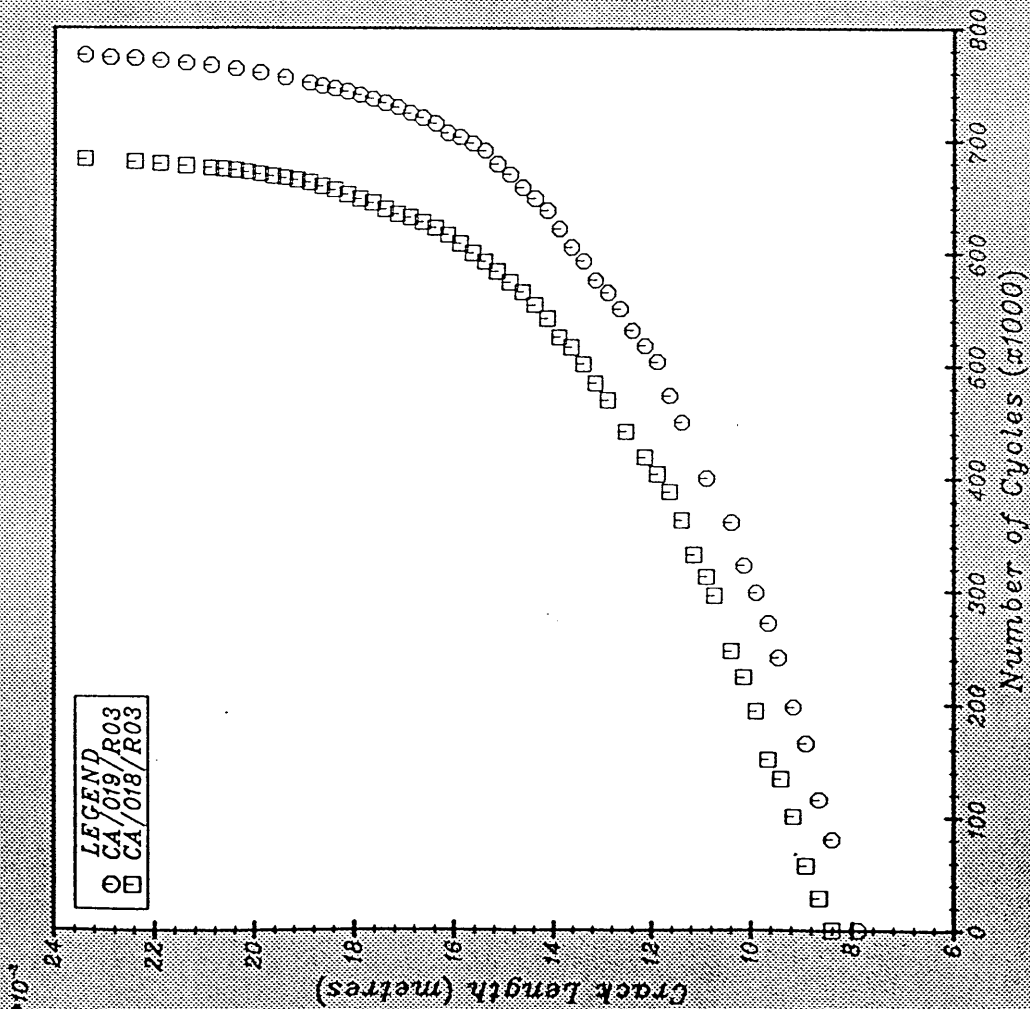
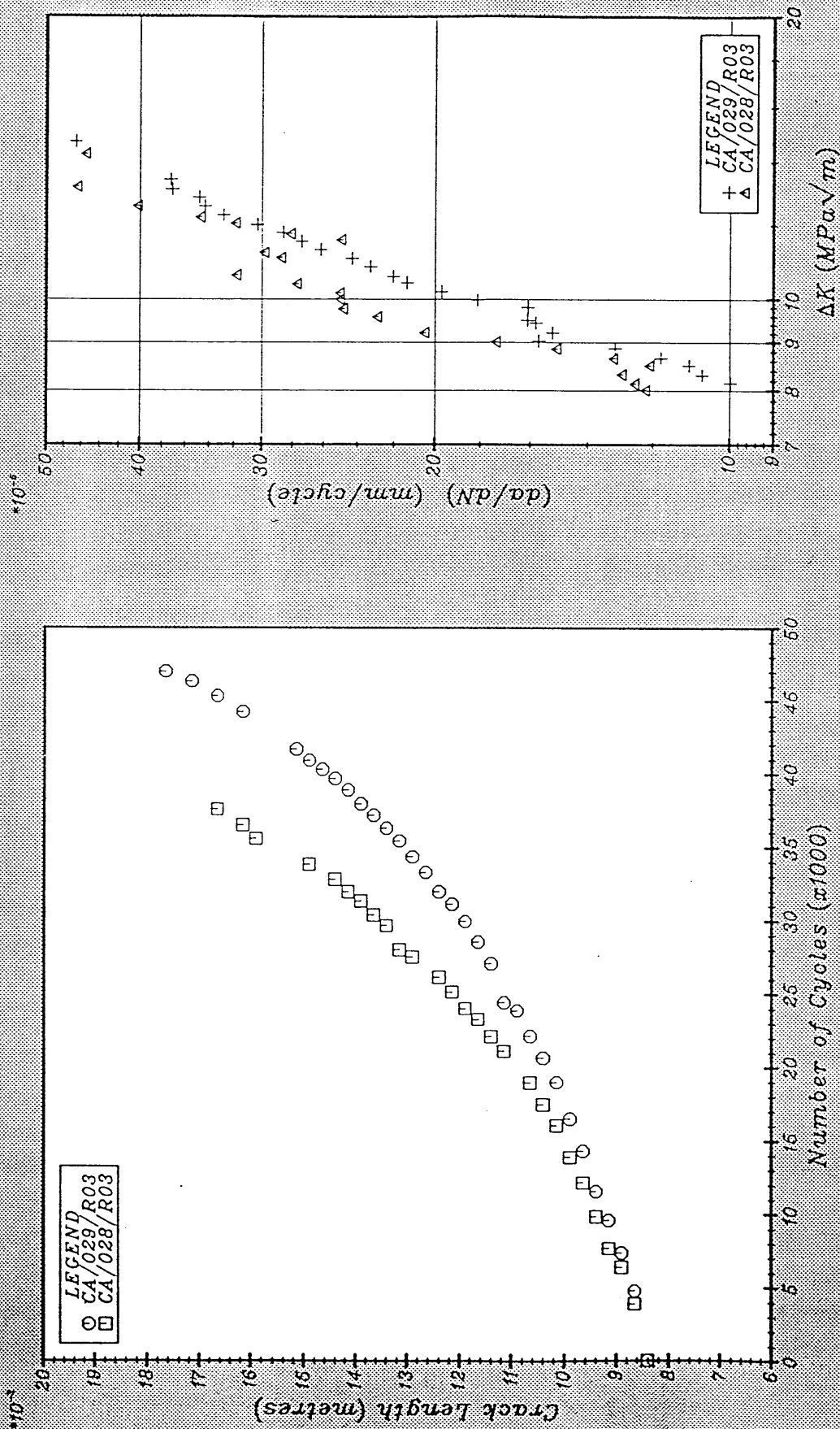


Fig.6.8 Constant load amplitude test material Aluminium-Zinc 7075 T7351 alloy
Test specification: Load cycling frequency 50 Hz
 $P_{mean}=1.0KN$ $P_{max}=1.538KN$ $P_{min}=0.462KN$ $R=0.3$



shapes of these curves are regular and only small *kinks* are present, which are hardly noticeable. The presence of small deviations can be attributed to either micro-structural features and defects or experimental errors in the determination of crack lengths and the inaccurate recording of the corresponding number of cycles. Evidently, micro-constituents are expected to either enhance or hinder the propagation, depending upon whether the constituents are of a brittle or ductile nature. Such events take place in a microscopic scale and the experimental facility used in the current study does not possess either the capability nor the sensitivity to ascertain such occurrences. It is thus considered more probable that the irregularities depicted are primarily due to experimental measurement errors, rather than metallurgical irregularities.

A representation of the crack growth for the 7075 T7351 Aluminium alloy is shown by the logarithmic plots of $(da/dN$ versus $\Delta K)$. It is widely acceptable that the crack growth behaviour of materials is represented by a reflex type sigmoidal shaped curve, [6.10, 6.11, 6.12]. It is true to say that a large number of materials exhibit such a trend, Boyer [6.13]. Examination of the crack growth curves developed in this study, show that the 7075 T7351 alloy does not follow this pattern.

In fact the experimental data suggests that the crack growth curve is parabolic in nature and in some cases it seems to be linear. A series of crack growth curves illustrating clearly the trend of crack propagation in the 7075 T7351 are shown in Figures 6.10-6.15.

This rather unconventional crack growth behaviour of the 7075 Aluminium has been previously observed and documented by Hudson [6.7] and Walker [6.14]. In these publications, it has been shown that similar behaviour is exhibited by

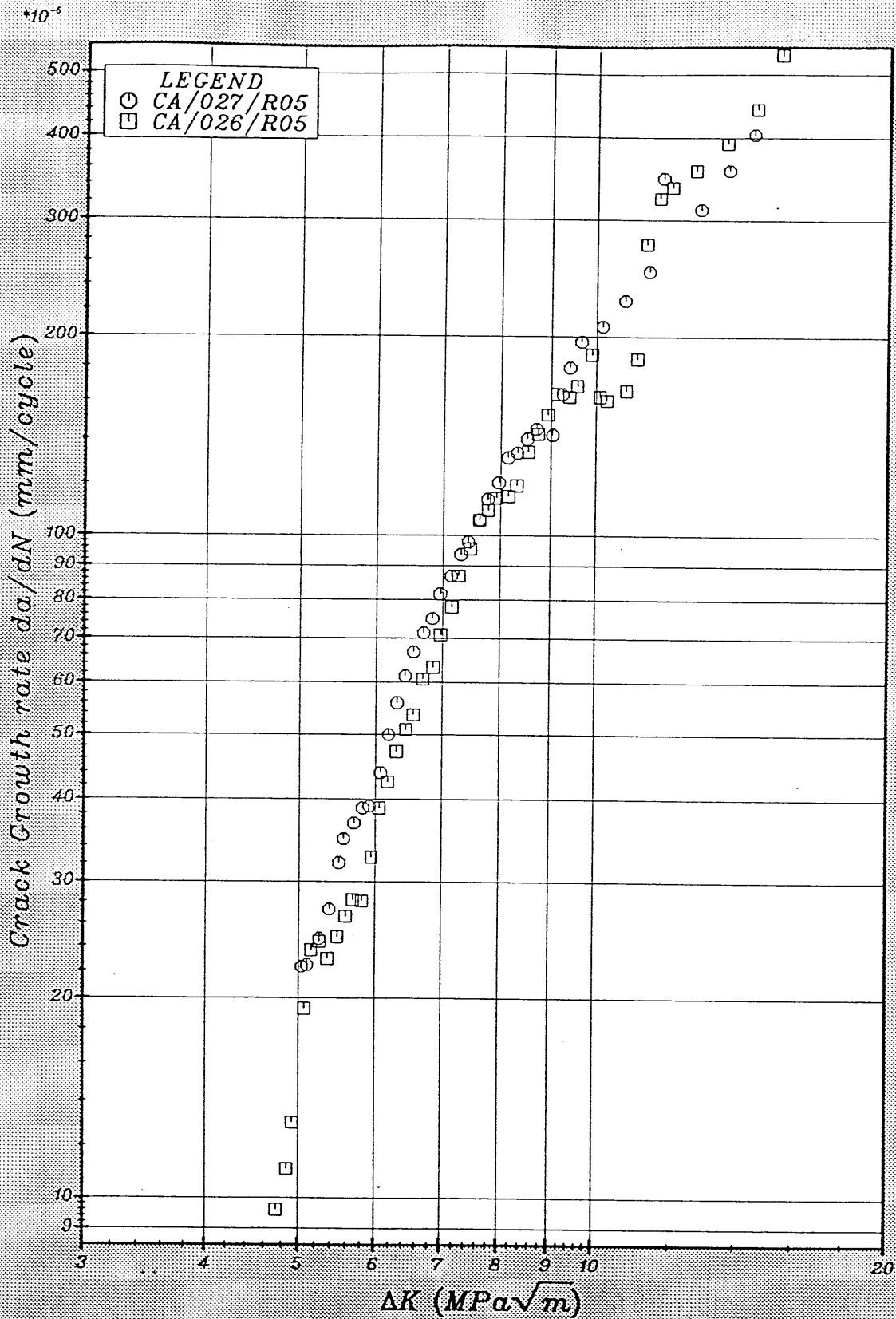


Fig.6.10 Experimental crack growth data
Aluminium-Zinc alloy 7075 T7351
 $P_{mean}=2.0KN$ $R=0.5$ Frequency 50.0 Hz

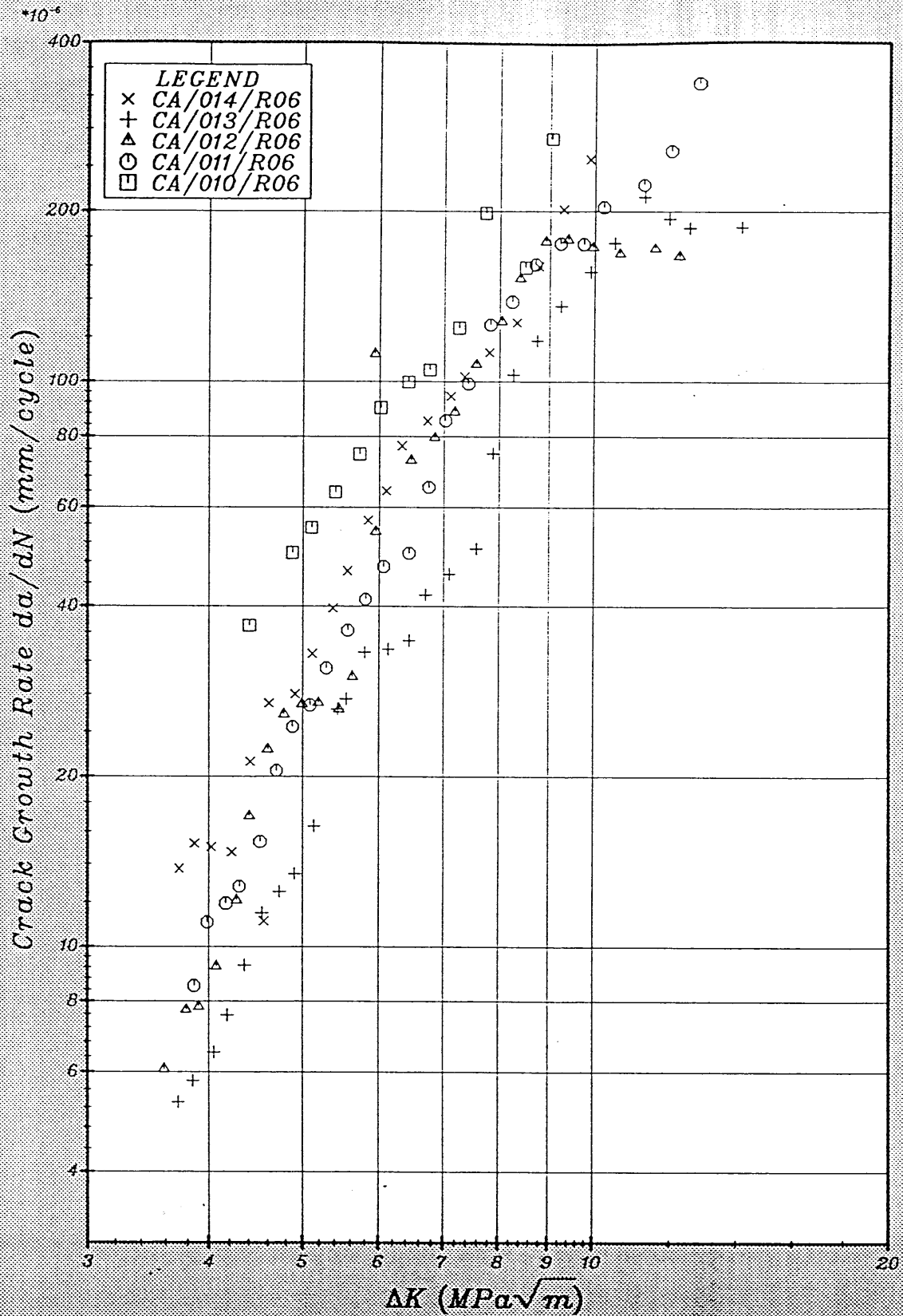


Fig.6.11 Experimental crack growth data
 Aluminium-Zinc alloy 7075 T7351
 $P_{mean}=2.0KN$ $R=0.6$ Frequency 50.0 Hz

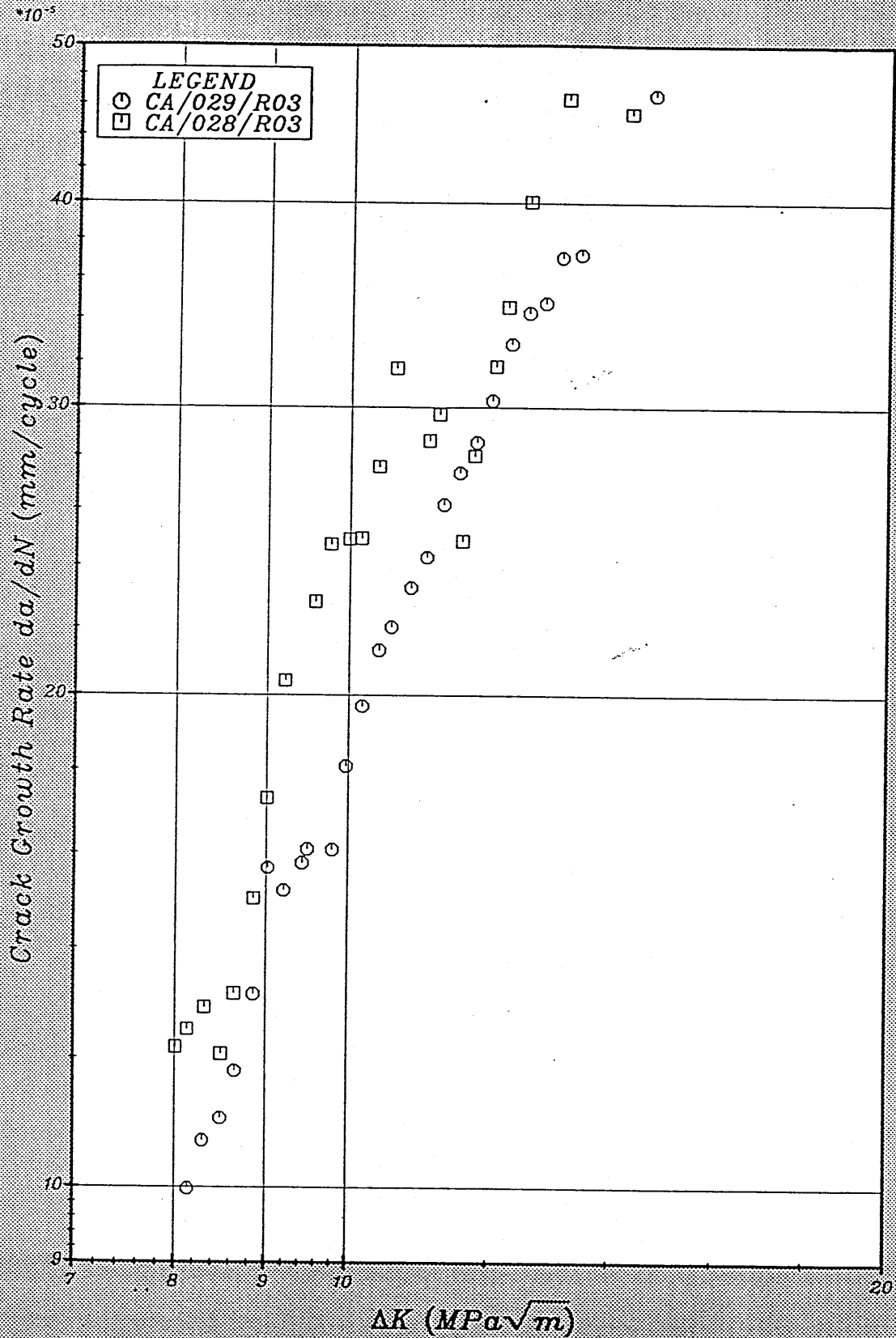


Fig.6.12 Experimental crack growth data
 Aluminium-Zinc alloy 7075 T7351
 $P_{mean}=2.0\text{KN}$ $R=0.3$ Frequency 50.0 Hz

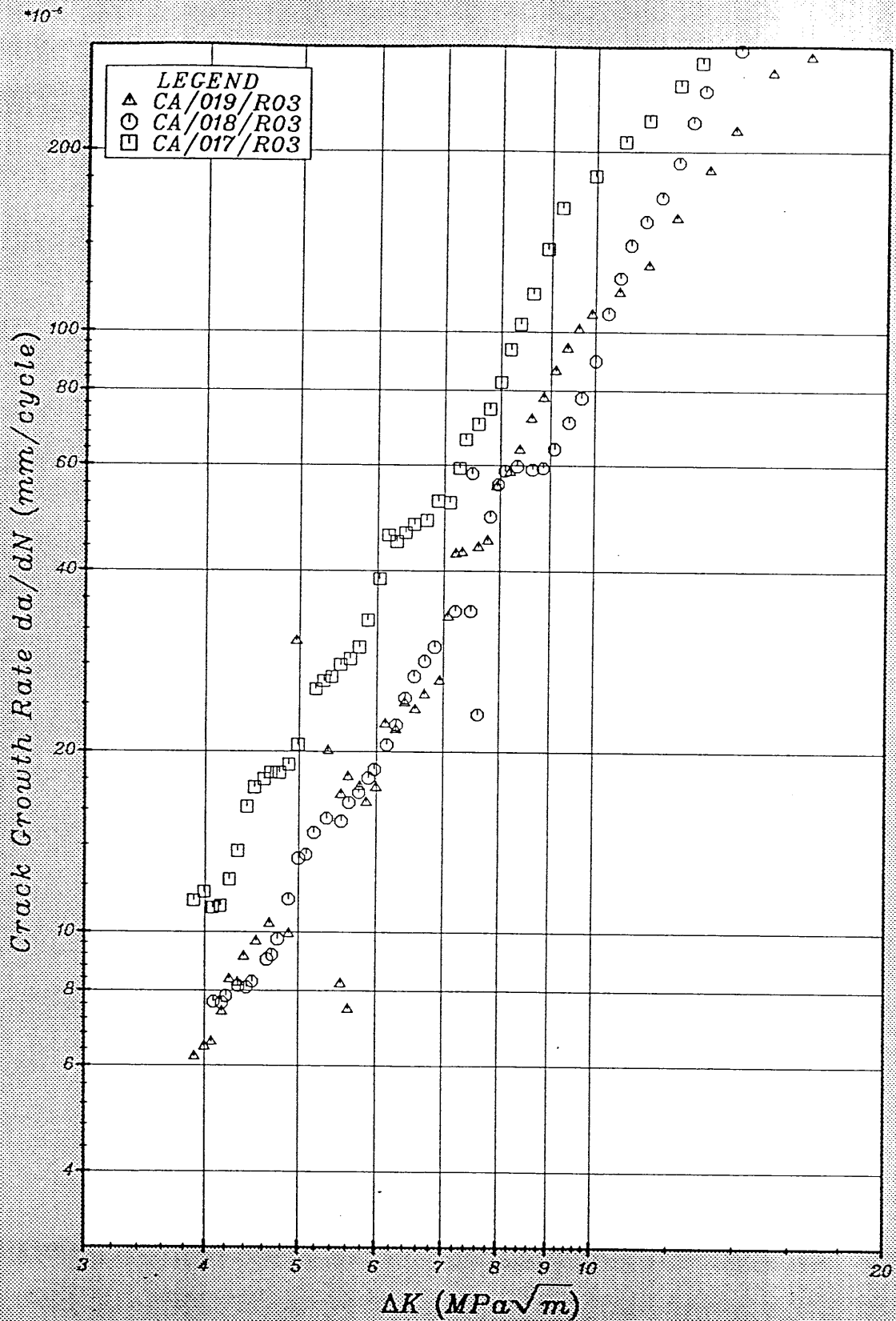


Fig.6.13 Experimental crack growth data
 Aluminium-Zinc alloy 7075 T7351
 $P_{mean}=1.0KN$ $R=0.3$ Frequency 50.0 Hz

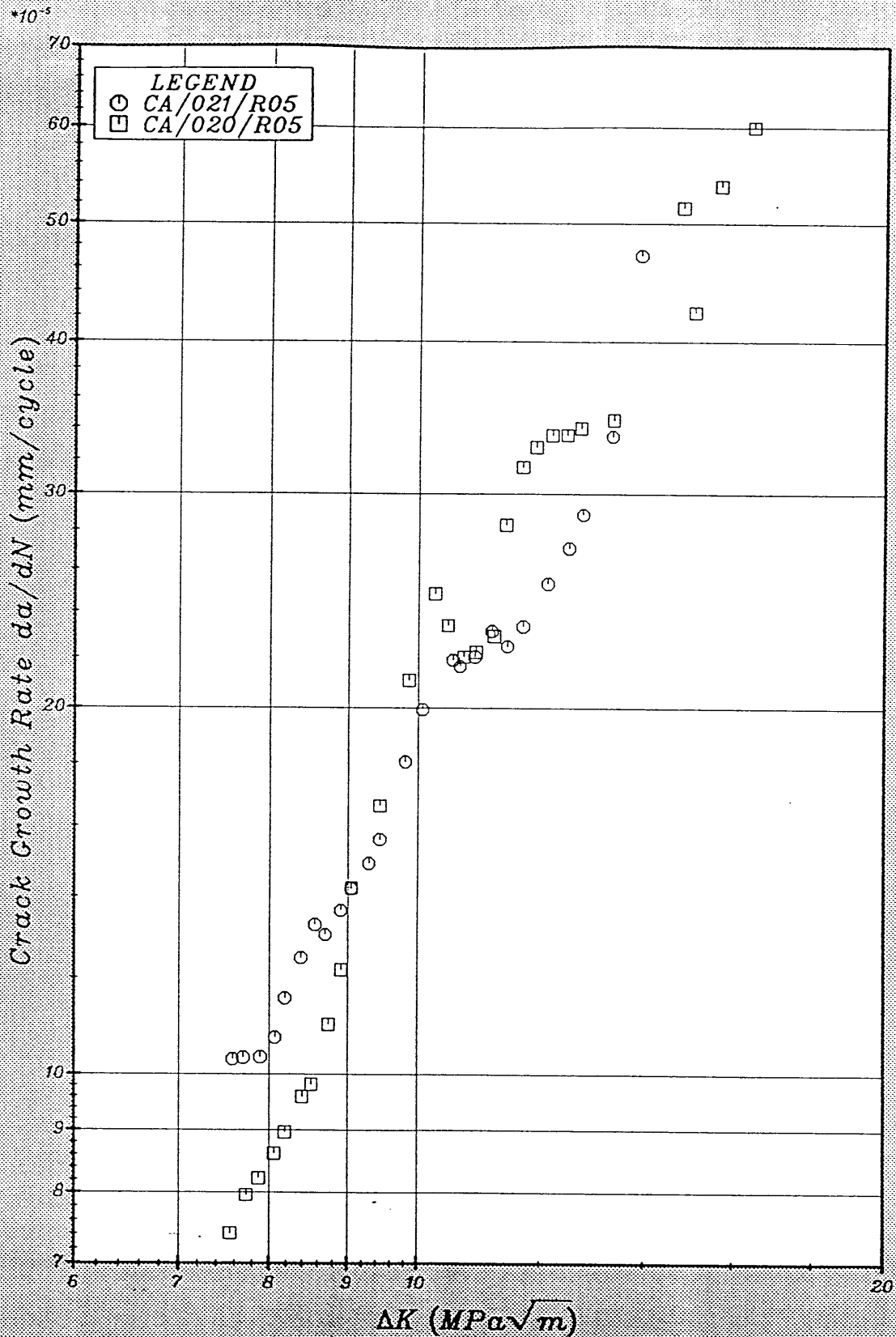


Fig.6.14 Experimental crack growth data
Aluminium-Zinc alloy 7075 T7351
 $P_{mean}=3.0KN$ $R=0.5$ Frequency 50.0 Hz

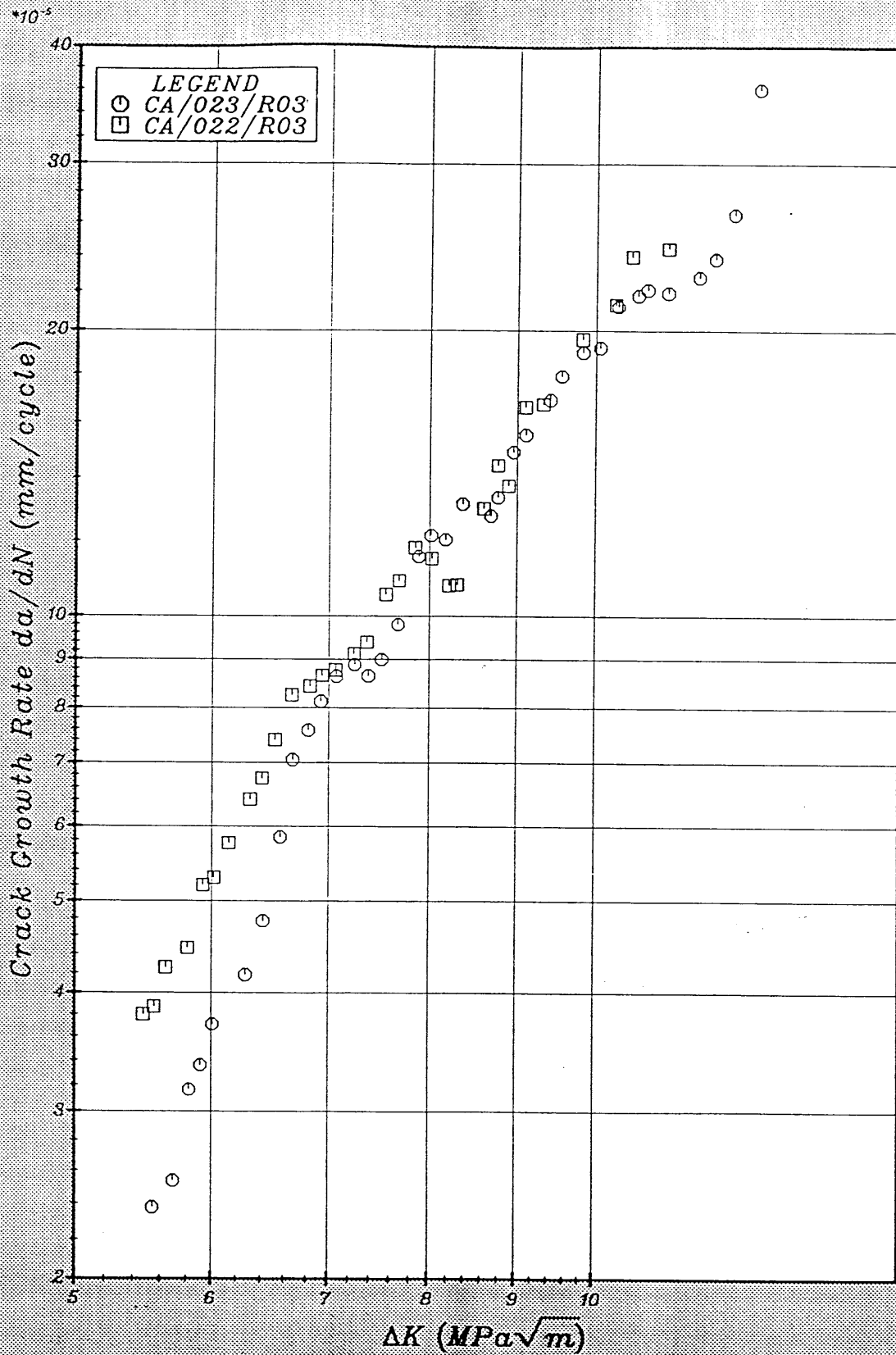


Fig.6.15 Experimental crack growth data
Aluminium-Zinc alloy 7075 T7351
 $P_{mean}=3.0\text{KN}$ $R=0.6$ Frequency 50.0 Hz

the 2024 T3 Aluminium alloy. Figures 6.16 and 6.17 show the crack growth curves for 7075 T6 and 2024 T3 respectively, as presented by Hudson [6.7].

6.2.3 Mean Load Variation

Experimentally determined crack growth curves for the material, tested under identical R ratios but different mean loads, are shown in Figures 6.18-6.21. The mean load variation has been found to influence the crack growth. In fact the crack growth rates for higher mean loads have been found to be higher for all R ratios.

Clearly, the higher mean load causes proportional increase in the stress intensity factors which are higher for identical crack lengths. This more intense state of stress in the vicinity of the crack tip creates a more energetic state where the strain energy has increased. The metal lattice in the vicinity of the crack front experiences larger deformations and higher stresses that cause localised failure which is reflected by the crack growth.

The larger deformations experienced at higher mean loads were reflected on the surface of the specimens in the form of larger localised plasticity in the vicinity of the advancing crack tip. The surface, plane stress plasticity was found to be larger the smaller the R ratio.

As it can be seen in Fig.6.18-6.21, the data clearly indicates that as the P_{mean} increases the range of the stress intensity ΔK is increased. This is understandable, since for the same crack length the corresponding stress intensity factors are higher. The number of data points is also smaller as the rate of growth is higher and consequently the experiments are shorter. An indication of the effect of increased mean load upon the duration of the

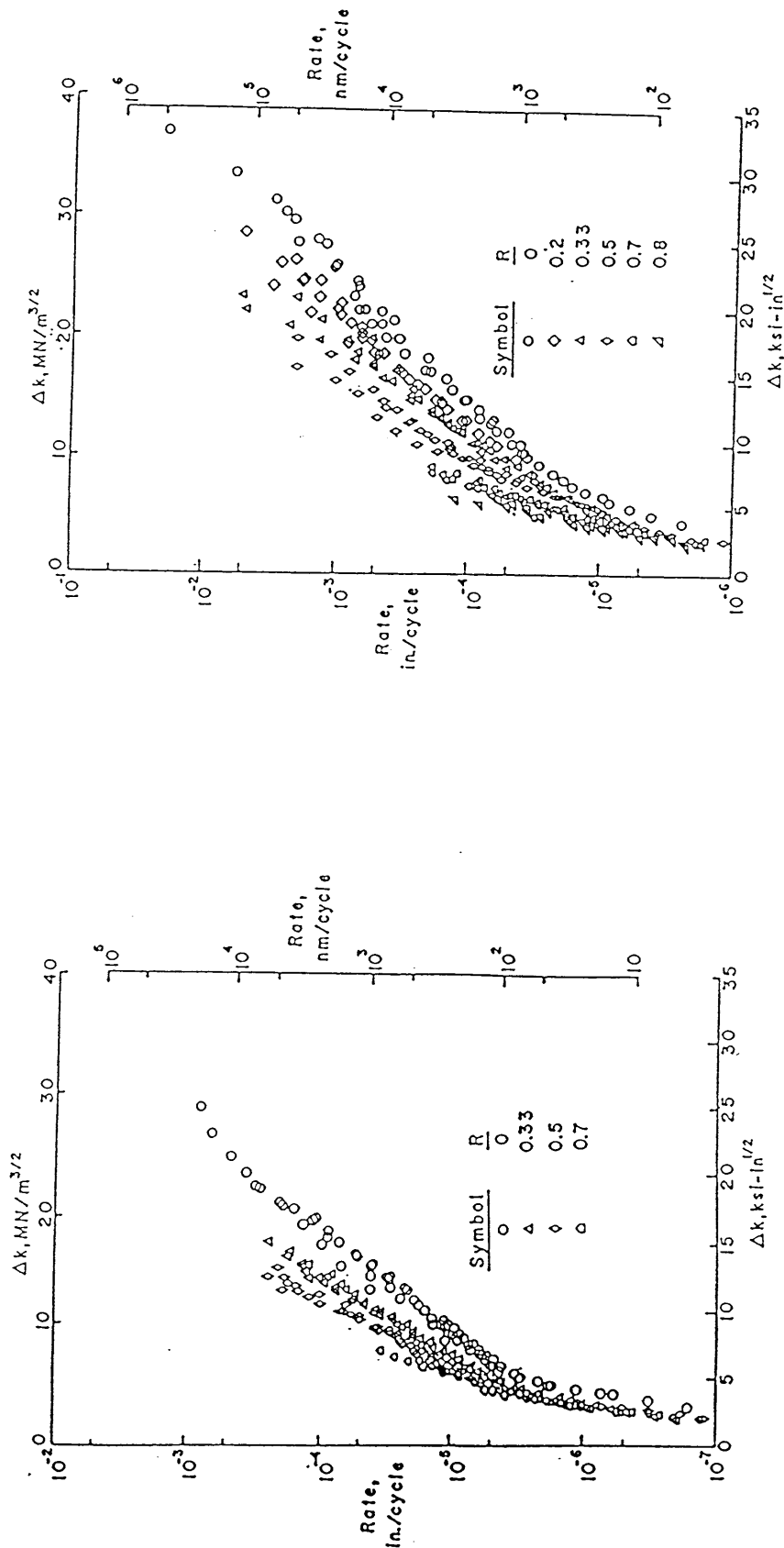


Fig.6.17 Effect of R load ratio upon the constant load amplitude for the 7075 T6 AL-Zn alloy [6.7]

Fig.6.16 Effect of R load Ratio upon the constant load amplitude for the 2024 T3 Aluminium alloy [6.7]

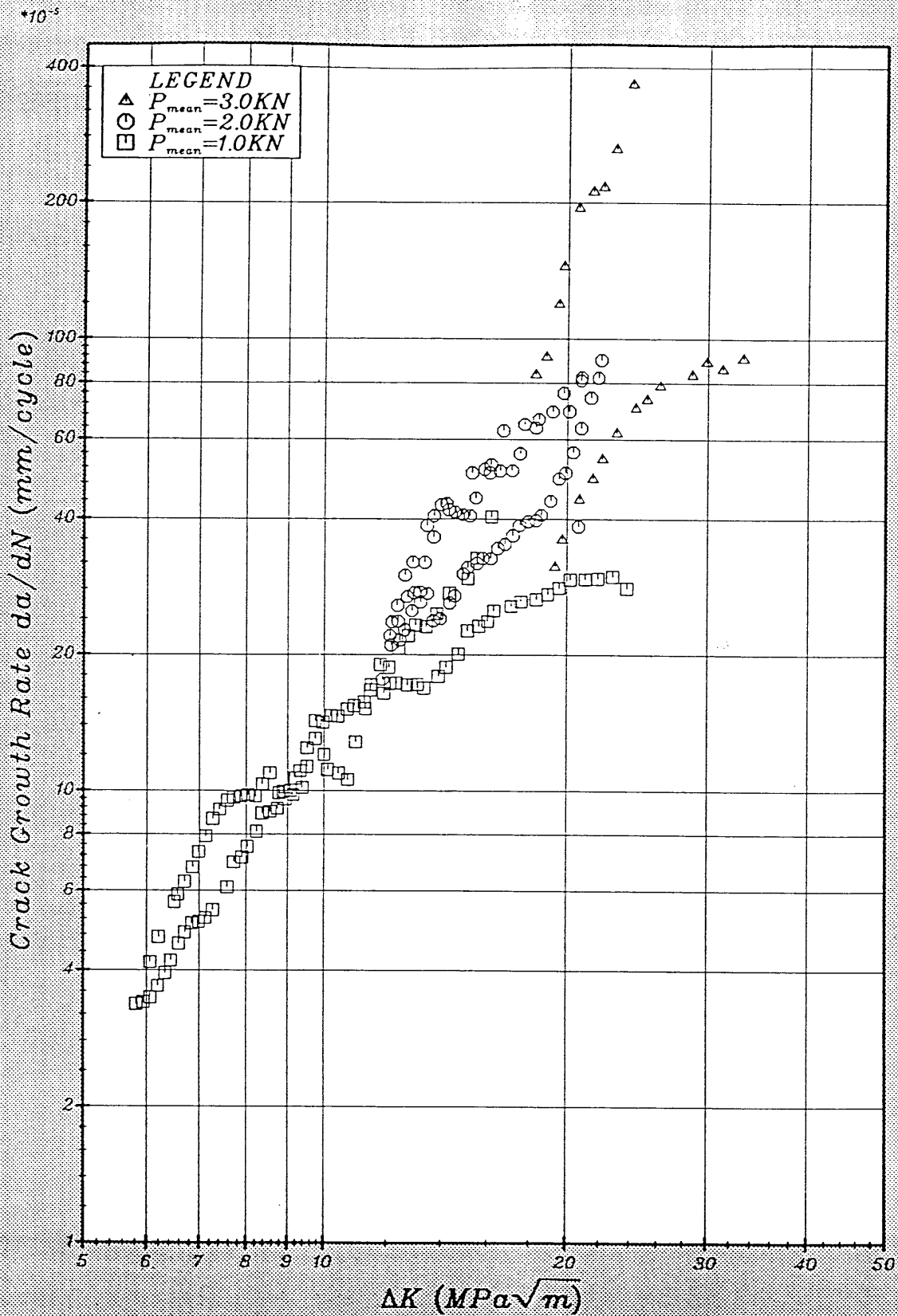


Fig.6.18 Experimental constant load amplitude crack growth data showing the effects of mean load variation for $R=0.1$

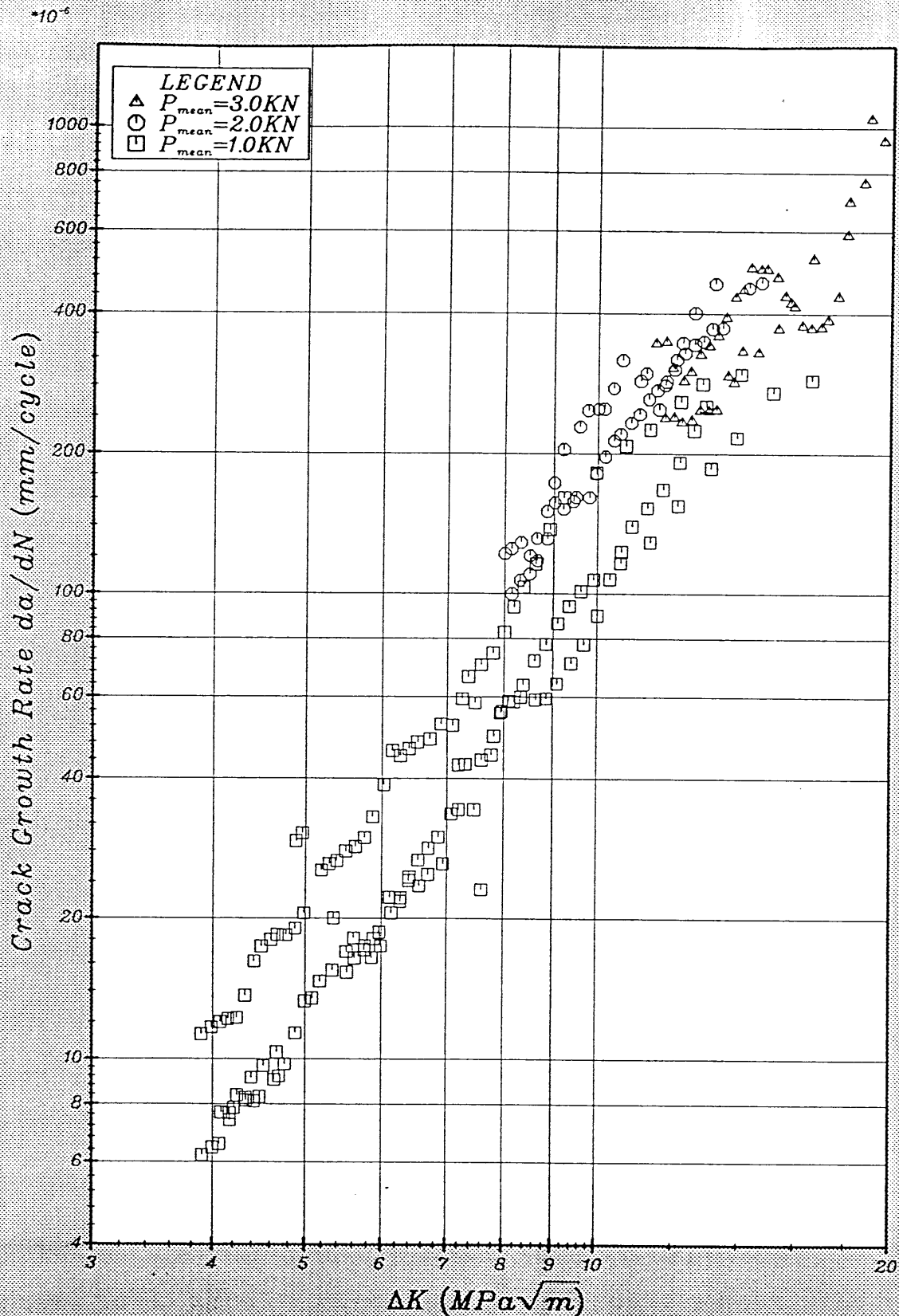


Fig.6.19 Experimental constant load amplitude crack growth data showing the effects of mean load variation for $R=0.3$

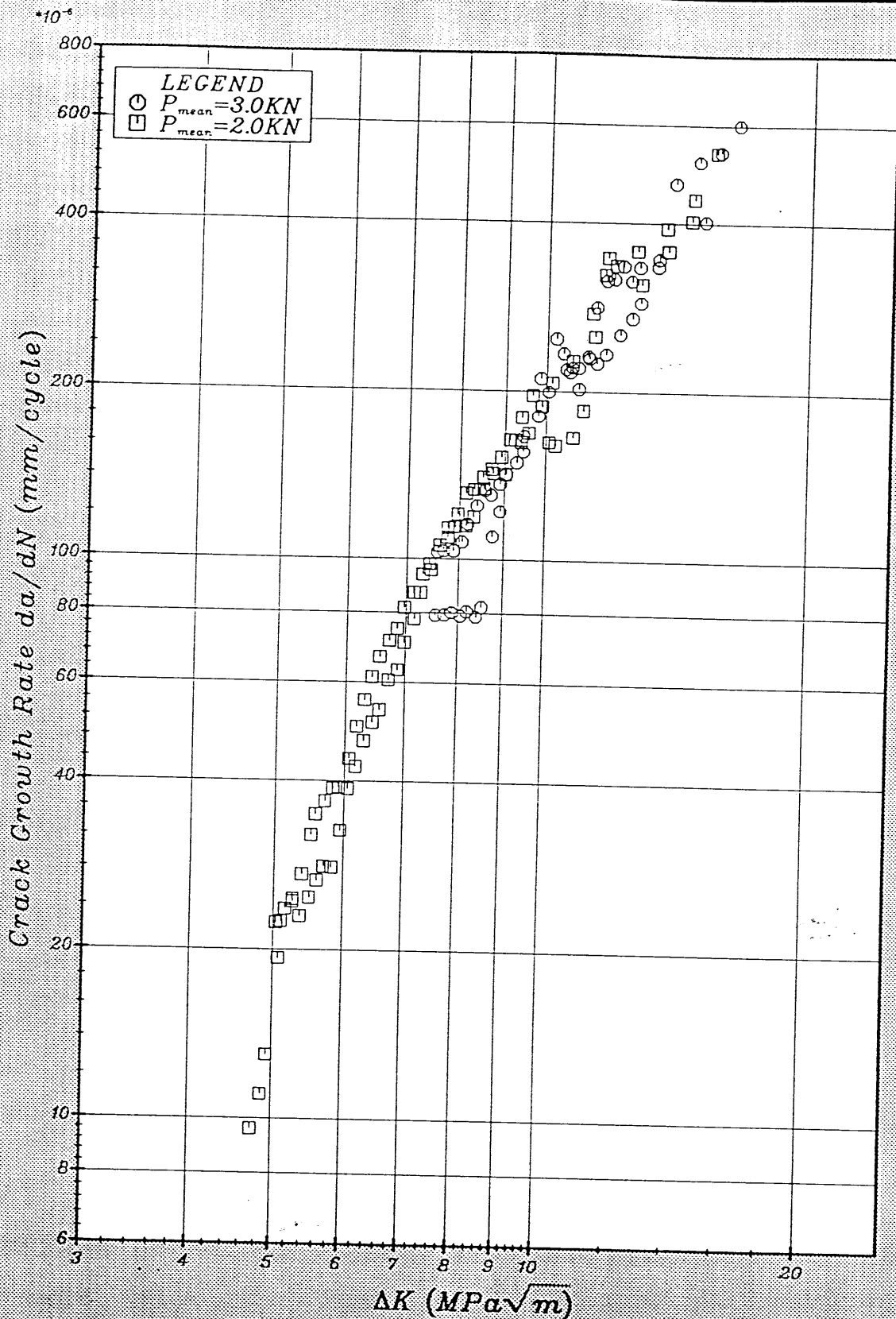


Fig.6.20 Experimental constant load amplitude crack growth data showing the effects of mean load variation for $R=0.5$

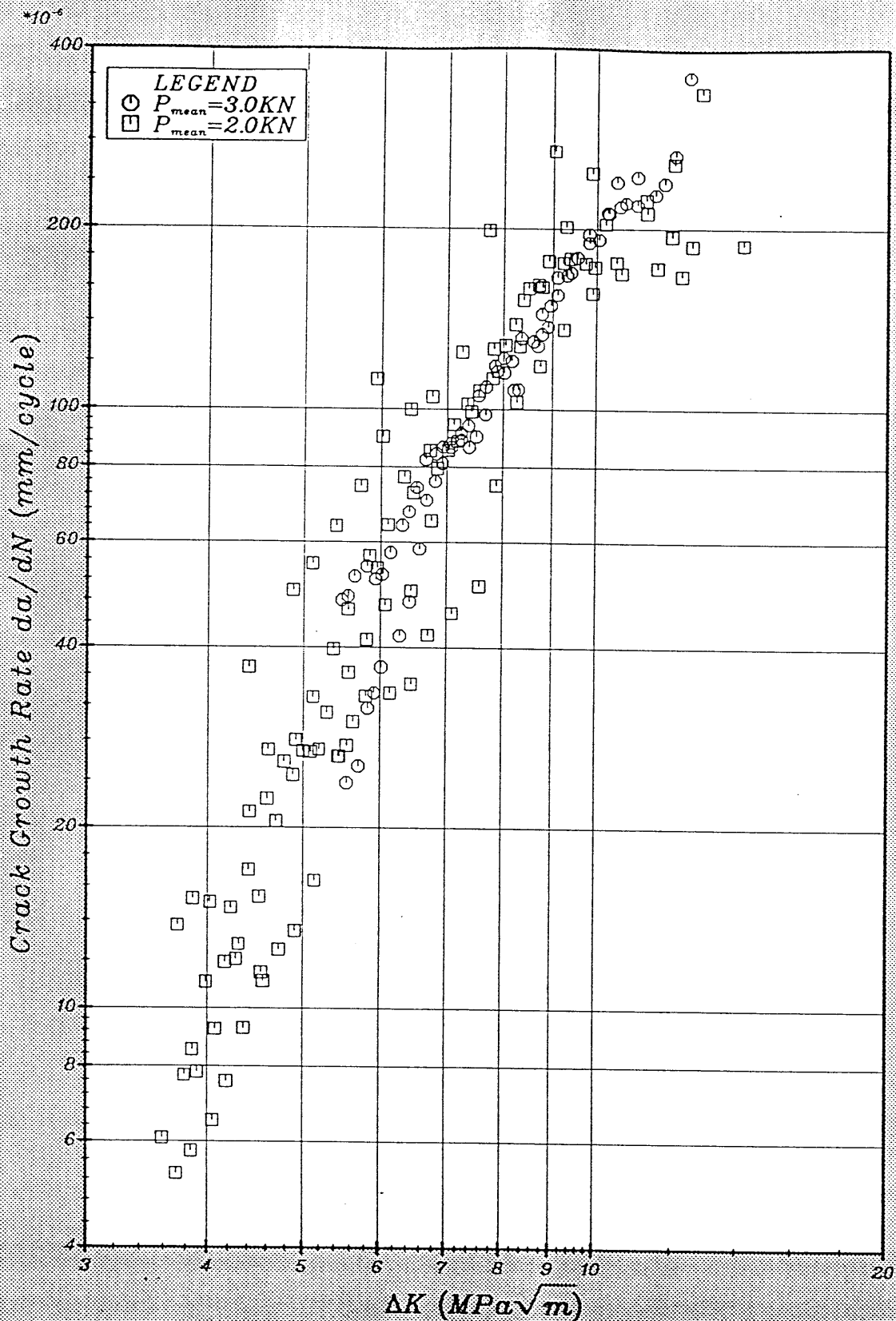


Fig.6.21 Experimental constant load amplitude crack growth data showing the effects of mean load variation for $R=0.6$

fatigue crack propagation life is illustrated below in Table 6.2.

	P_{mean}	N_{total}
R=0.1	1.0KN	210,000
	2.0KN	48,000
	3.0KN	16,000
R=0.3	1.0KN	630,000
	2.0KN	57,000
	3.0KN	21,000
R=0.5	1.0KN	---
	2.0KN	270,000
	3.0KN	60,000
R=0.6	1.0KN	---
	2.0KN	350,000
	3.0KN	154,000

Table 6.2 Average Duration of Fatigue Crack Growth
as Affected by the Mean Load and R Ratio

The information given in Table 6.2 represent the algebraic average of crack propagation lives to failure determined for the minimum of five experiments per reported case.

6.2.4 R Load Ratio Variation

Fatigue crack propagation can be affected by any load variation which causes changes to the stress intensity at the crack tip. In the present investigation the effects of the R loading ratio upon the crack growth rates with respect to load cycling upon the same mean load has been examined.

A series of experiments were conducted in order to ascertain the effects of the load amplitude upon the fatigue crack growth. It has been found that the lower the value of the R ratio, i.e. the higher the load amplitude, the lower the crack growth rate for the same ΔK . These observations are in agreement with the findings of Hudson [6.7] and Walker [6.14]. Figures 6.22-6.24 depict the crack growth curves for experiments carried out where the same mean load was employed but the R ratio was varied. It is shown that the reduced load amplitude causes small increase in the crack growth rates. Examination of the total fatigue crack propagation lives indicates that tests where small amplitudes are involved are significantly longer in duration, see Table 6.2, and a larger number of data points is experimentally obtained.

It was observed during the experiments that the growth of cracks under large load amplitudes was accompanied by visible surface plasticity at the crack tip, appearing as a shaded dimple under stroboscopic illumination.

The reduction of the crack growth rates with increasing load amplitudes can be attributed to this localised crack tip plasticity. At the tip of a growing crack each loading half cycle generates a monotonic plastic zone. Thus the crack has to penetrate this plastically deformed region. As a result of the compressive residual stresses that are generated by the plastic deformation the effective stress intensity factor is smaller than anticipated, reduced by an amount equal to the stress intensity factor corresponding to the residual stress field. Part of the work done by the test machine is expended for plastic deformation which in turn reduces the energy available for crack extension. In fact higher R values result to less crack tip plasticity and effectively the crack growth rates are higher.

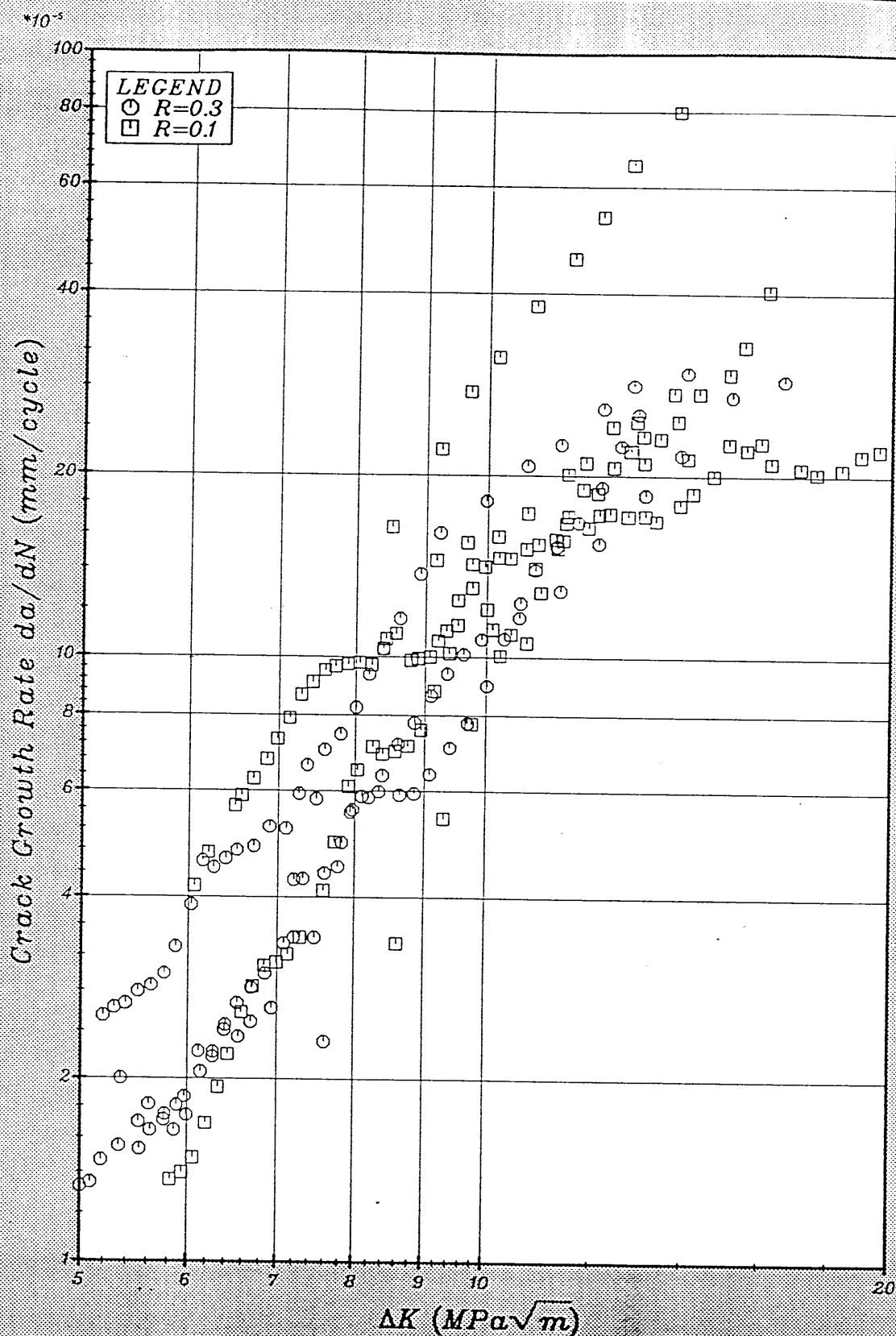


Fig.6.22 Experimental constant load amplitude crack growth data showing the effects of R load ratio variation for $P_{mean}=1.0KN$

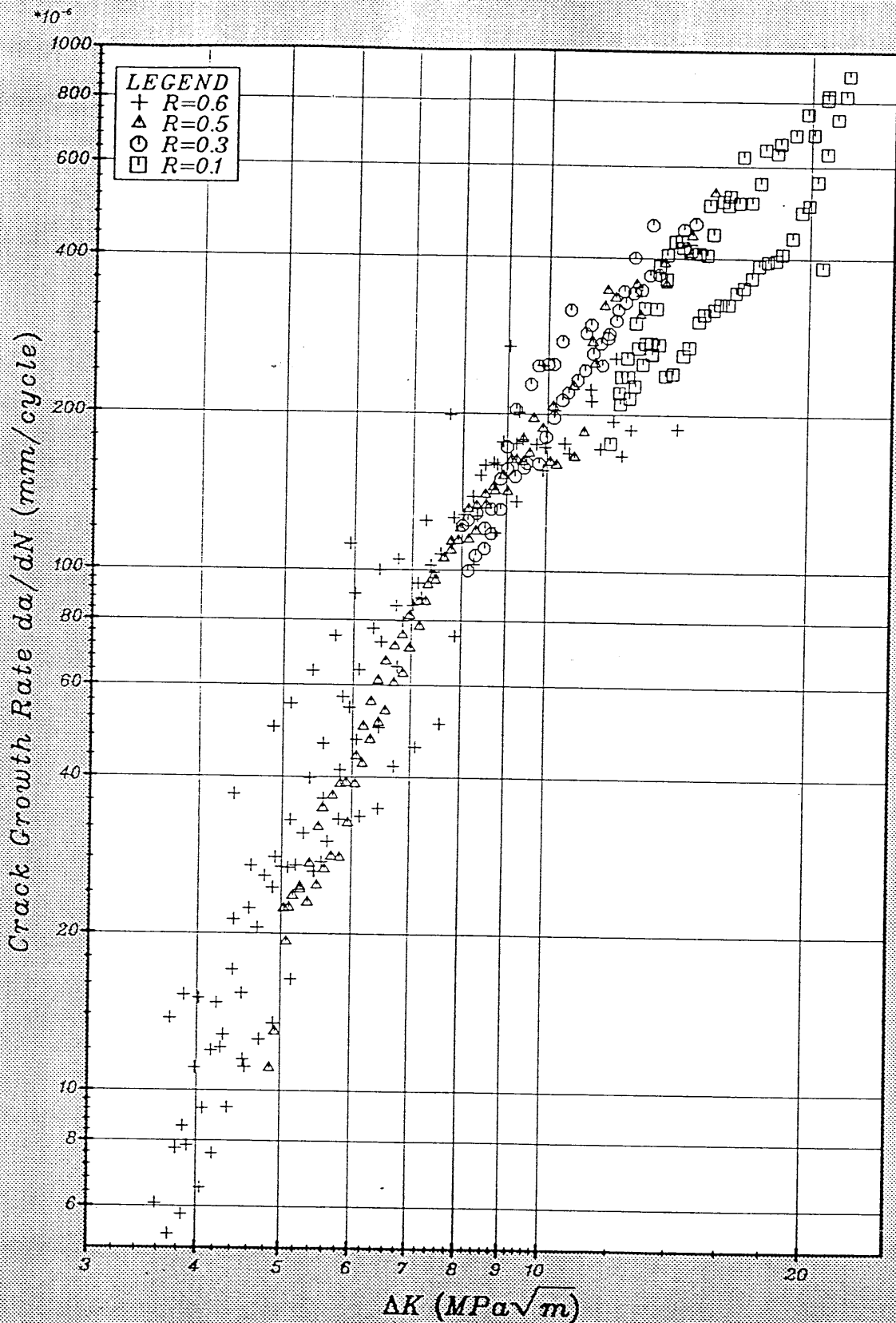


Fig.6.23 Experimental constant load amplitude crack growth data showing the effects of R load ratio variation for $P_{mean}=2.0\text{KN}$

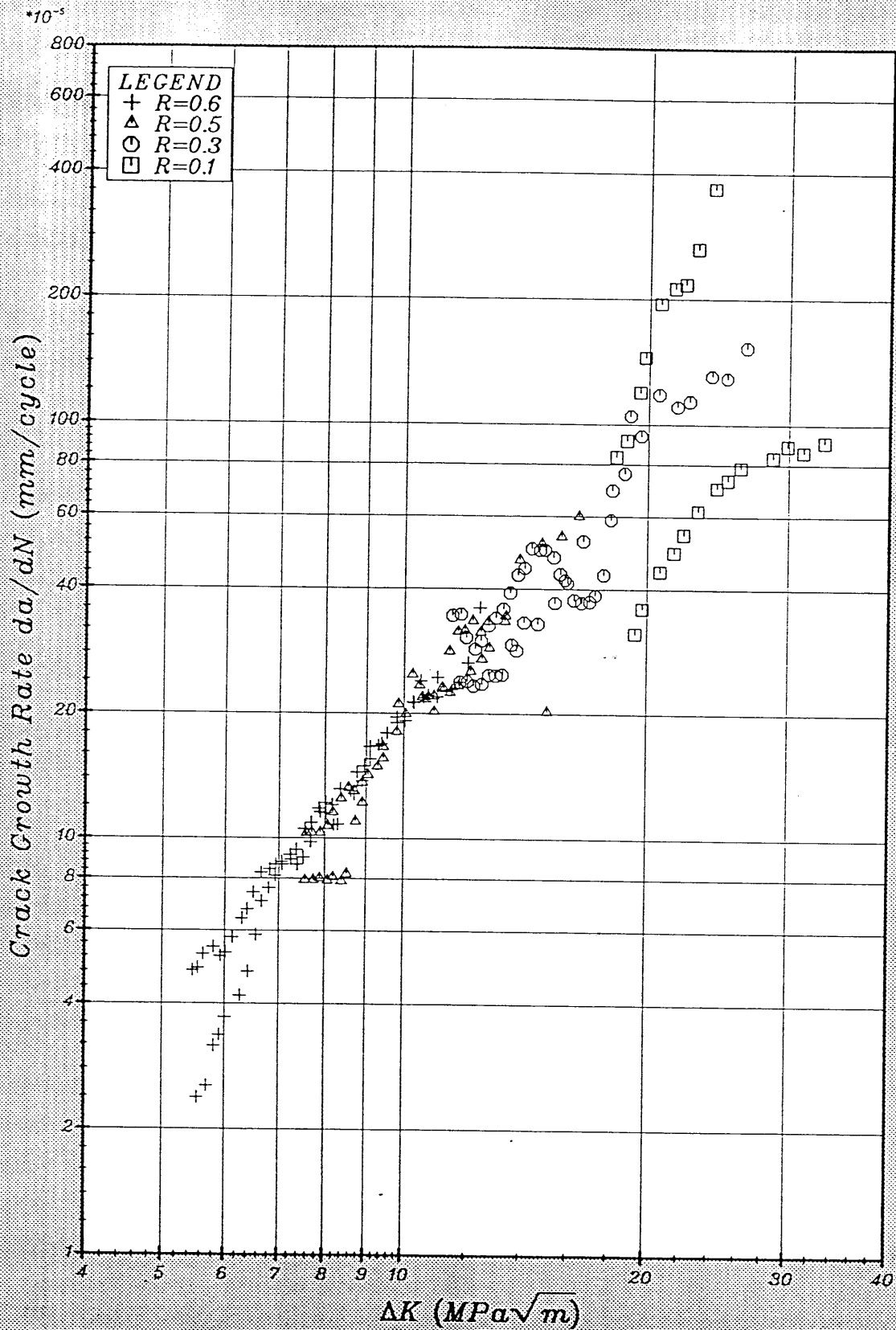


Fig.6.24 Experimental constant load amplitude crack growth data showing the effects of R load ratio variation for $P_{mean}=3.0KN$

6.2.5 Some Observations Related to Cyclic Frequency Variations

The majority of tests were carried out at the same load cycling frequency of 50Hz. It was though observed that frequency changes influenced the duration of pre-cracking. It was found that frequency changes resulted to either faster or shorter crack initiation periods. In fact the lower frequencies (10-25 Hz), resulted to faster crack initiations than high frequencies (25-50 Hz).

These observations triggered a short investigation into the effects of frequency upon the fatigue crack propagation. A small number of experiments were conducted in order to clarify the effects of frequency. A case study which clearly indicates the effects of frequency is shown in Fig.6.25. The (a versus N) curve clearly shows that the lower the frequency the shorter the fatigue life i.e. faster the crack growth. In fact the difference in the total number of cycles to failure is as much as 650.0×10^3 cycles.

The differences in the rate of growth due to changes in frequency, are also reflected in the crack growth curves of (da/dN versus ΔK); Where for the same ΔK the lower frequencies illustrate higher crack growth rates.

Despite of the observed variation in the total number of cycles to failure the crack growth curves of (da/dN versus ΔK) for different frequencies form a narrow scatter band. Even though, the frequency was found to influence the crack initiation or growth there was no noticeable consequence related to any changes between pre-cracking and testing frequencies upon the fatigue crack growth or to the total number of cycles to failure.

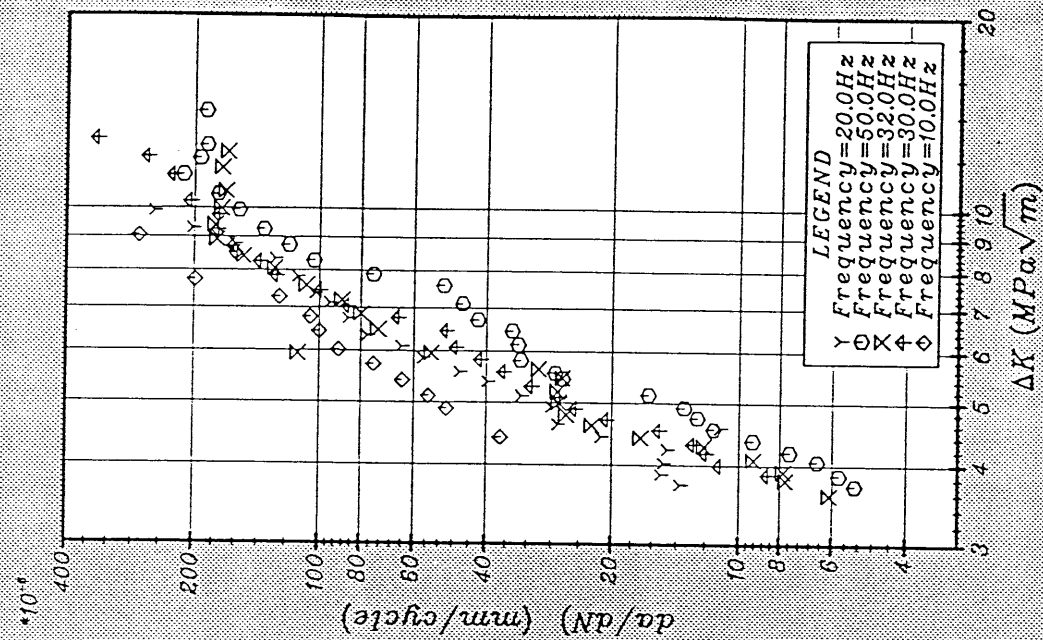
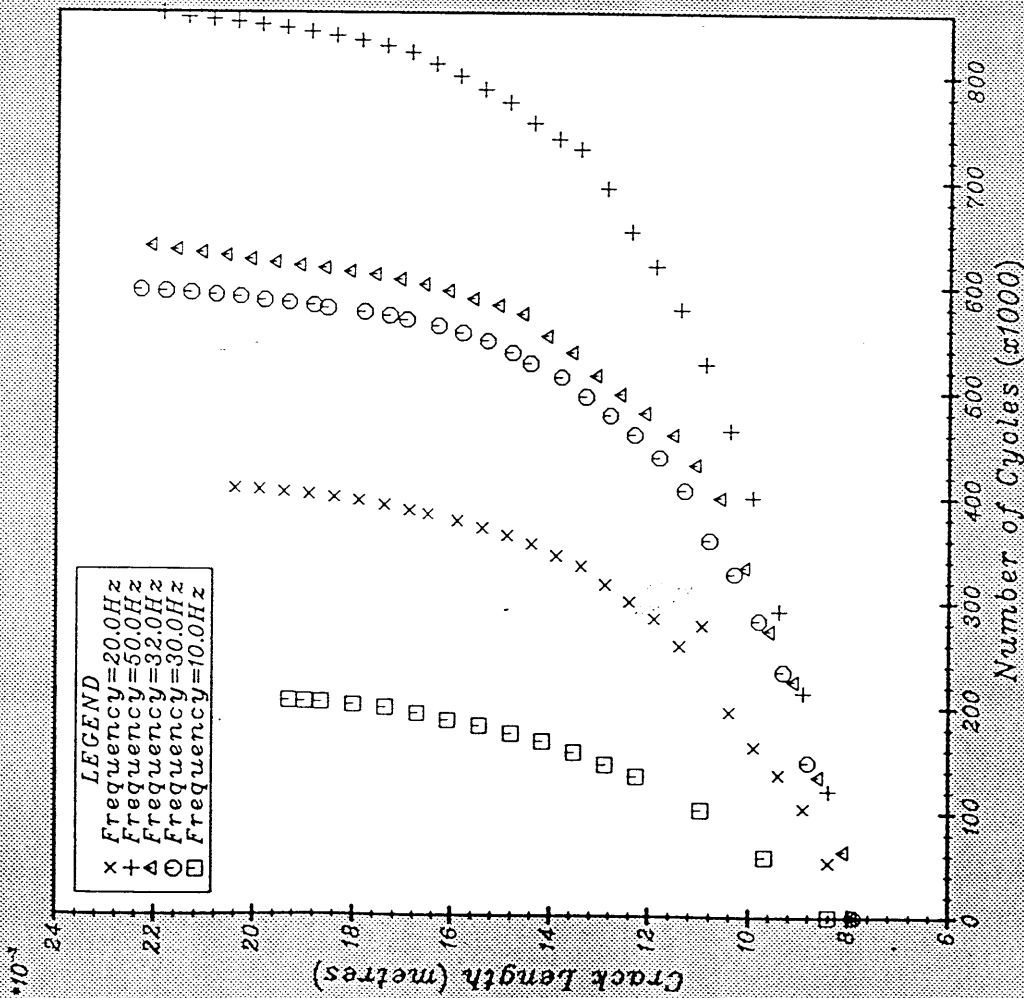


Fig.6.25 Constant load amplitude tests material Aluminium-Zinc 7075 T7351 alloy Test specification: $P_{mean}=2.0KN$ $R=0.72$ and different load cycling frequencies.

In the literature studied no conclusive discussion about the effects of frequency upon the fatigue crack growth in atmospheric conditions has been found. On the other hand studies regarding crack growth in aggressive environments have shown that lower frequencies enhance the fatigue crack growth, Jewett and Walter [6.15]. In the case of aggressive environments it has been concluded that corrosion and related effects are responsible for crack growth enhancement at lower frequencies. Only Hudson [6.7], in his study of 7075 T6, has given a brief indication that in air lower frequencies resulted to higher crack growth rates, without discussing the possible reasons. In general, the crack growth laws that are widely used in design do not take into account the effects of load cycling frequency, Erdogan [6.16].

6.2.6 Correlation of Data With Fatigue Crack Growth Equations

(I) The Paris-Erdogan Crack Growth Law

From the data presented above it has become clear that the data obtained during the experimental programme, exhibit a certain trend of crack growth. In the literature many laws for fatigue crack propagation prediction have been proposed as shown by Erdogan [6.16]. It is widely acceptable, that different materials comply to different crack growth mathematical models. It is important to note that all so called laws are empirical and despite the vast amount of research in the area of fatigue crack growth modelling, no law can be substantially supported with theoretical arguments, Kanninen and Popelar [6.17].

As it can be seen in the previous illustrations of crack growth, the propagation of the 7075 T7351 exhibits an unconventional shape and not the conventional sigmoidal reflex trend expected. In many cases the data exhibits

considerable scatter and from simple observations it is difficult to establish an accurate picture.

One of the mathematical models, is the well known law attributed to Paris and Erdogan [6.18]. This is a power law that describes the crack growth as a function of the applied stress intensity factor range,

$$(da/dN)=f(\Delta K)=C.\Delta K^n \quad \dots (6.1)$$

This equation is represented by a straight line when plotted on logarithmic scales.

With the utilisation of computational methods based upon a least squares technique for curve fitting, the factor C and exponent n of the above equation were determined. The analysis employed the logarithmic values of crack growth rate and stress intensity factor range. As the objectives of the work included the investigation of the effects of mean load and load ratio, separate analyses for the two cases were conducted.

The crack growth data from tests with identical mean loads were fitted with the Paris linear law equation. The assessment of the goodness of fit was based upon statistical procedures, and more precisely the percentage of data uncertainty explained by the fitted line. The statistical measure of this uncertainty is given by the coefficient of determination, Charpa et al [6.19].

The summary of the values of coefficient C and exponent n with the corresponding coefficients of determination and other statistical information regarding the fitted lines are given in Table 6.3. It can be clearly observed from the statistical information given that, the linear crack growth law fitted in the data, gives a good approximation of the behaviour.

$P_{mean} = 1.0KN$	$C = 2.923 \times 10^{-11}$	$n = 2.62$
$C.d = 84.09\%$	$S.e.e = 0.198$	$C.C = 0.917$
$P_{mean} = 2.0KN$	$C = 7.987 \times 10^{-11}$	$n = 2.52$
$C.d = 91.70\%$	$S.e.e = 0.147$	$C.C = 0.956$
$P_{mean} = 3.0KN$	$C = 7.053 \times 10^{-11}$	$n = 2.15$
$C.d = 89.63\%$	$S.e.e = 0.134$	$C.C = 0.947$

$C.d$ Coefficient of Determination

$S.e.e$ Standard Error of Estimate

$C.C$ Correlation Coefficient

Table 6.3 Linear Crack Growth Law Coefficients
Accounting for R Load Ratio Variations

A plot representing the linear equations fitted through data from tests with the same mean load is shown in Fig.6.26. The trend indicated by the crack growth linear equations is that for the same stress intensity range, higher rates are determined for higher mean loads. This is more predominant for low ΔK values but as stress intensity range increases the lines converge.

A similar analysis has been carried out with data obtained from tests with identical load ratios R. The Paris-Erdogan equations fitted through the experimental data are shown in Fig.6.27, and statistical information as well as the coefficients determined are shown in Table 6.4. In this case as the R ratio decreases the crack growth rates for the same ΔK decrease.

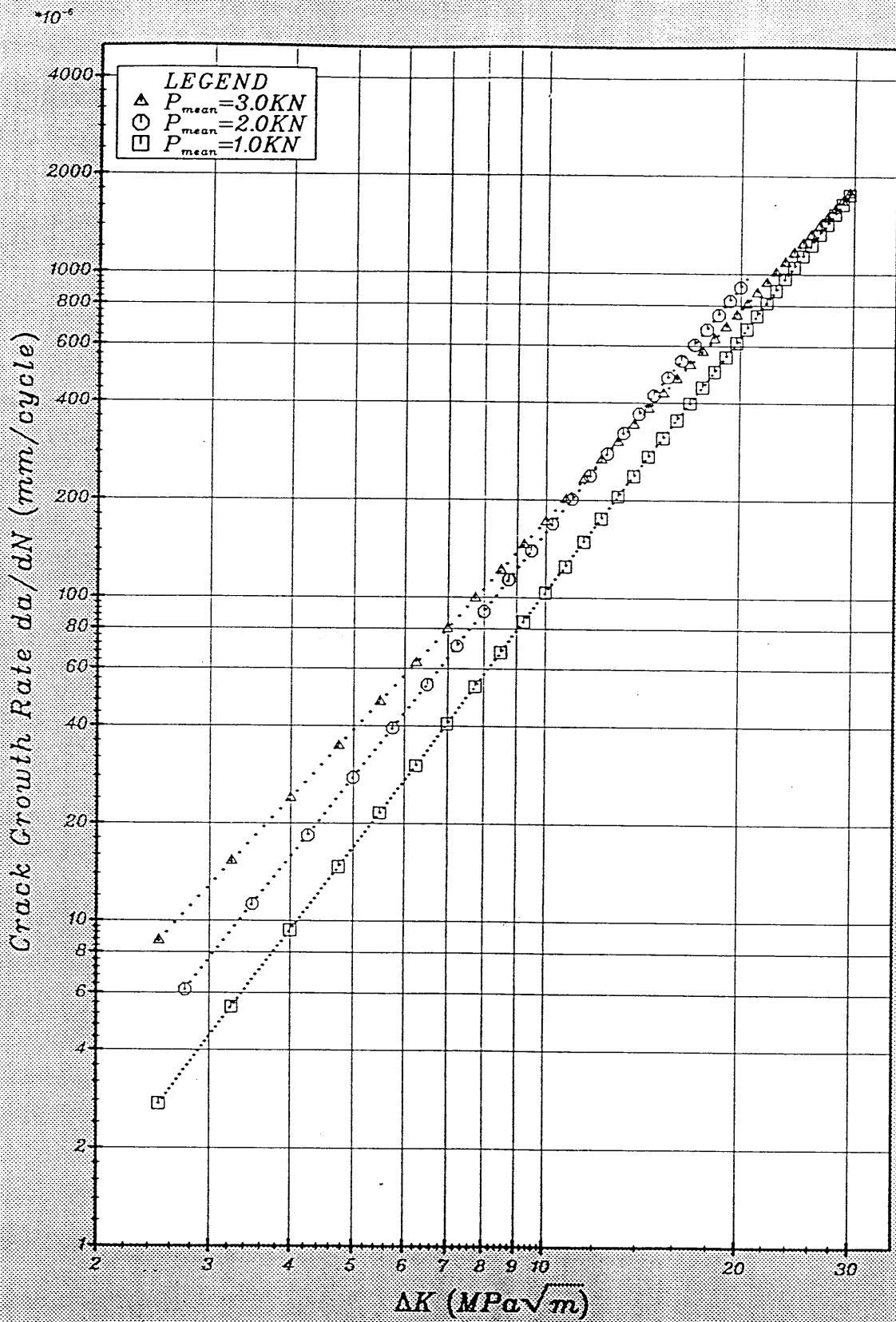


Fig.6.26 Paris-Erdogan linear crack growth laws accounting for mean load variation

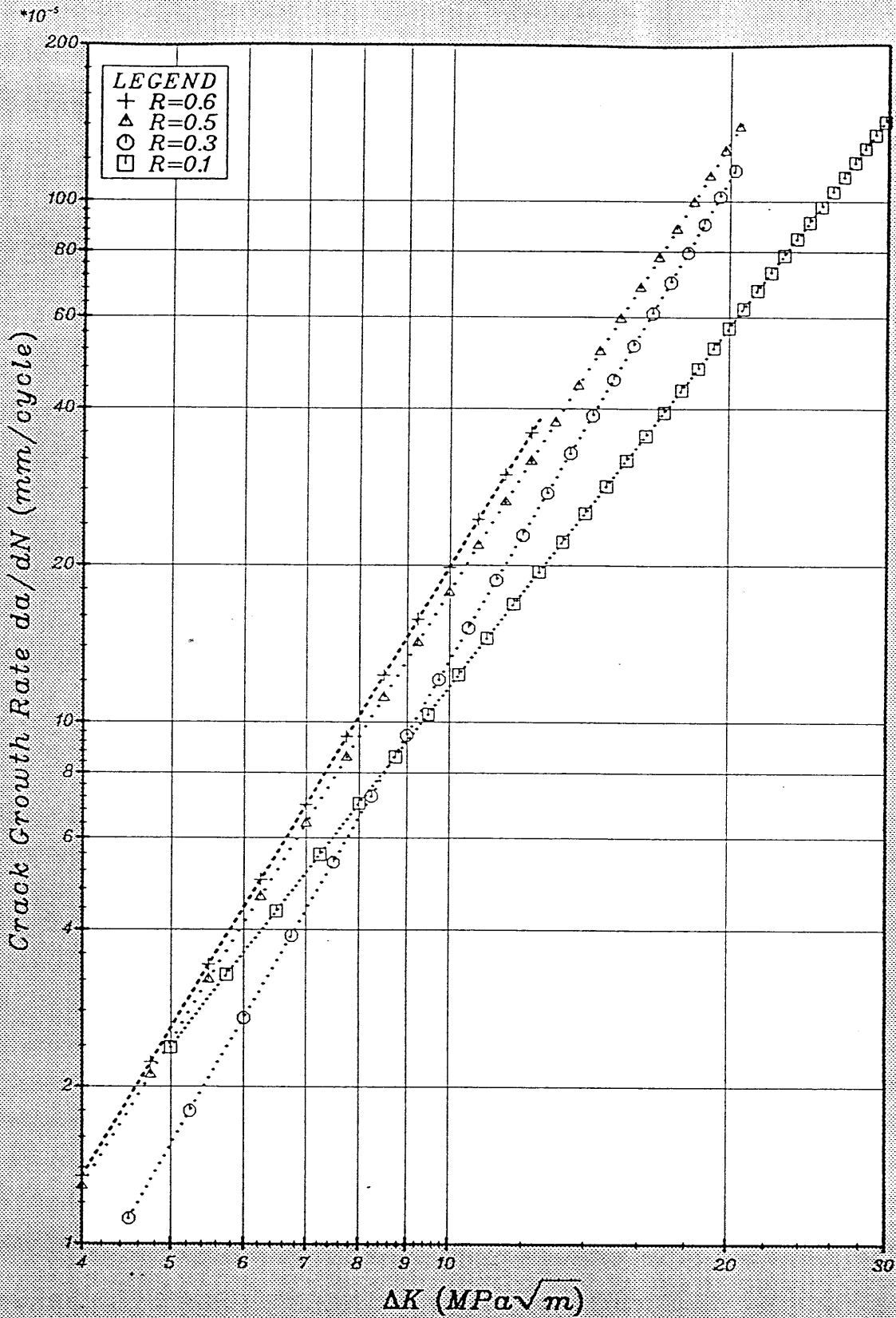


Fig.6.27 Paris-Erdogan linear crack growth laws accounting for R load ratio variations

R=0.1	C=2.153x10 ⁻¹⁰	n=2.23
C.d =82.30%	S.e.e =0.186	C.C =0.907
R=0.3	C=2.640x10 ⁻¹²	n=3.07
C.d =92.66%	S.e.e =0.166	C.C =0.926
R=0.5	C=1.231x10 ⁻¹¹	n=2.86
C.d =94.40%	S.e.e =0.093	C.C =0.967
R=0.6	C=9.026x10 ⁻¹²	n=2.94
C.d =93.69%	S.e.e =0.146	C.C =0.942

C.d Coefficient of Determination

S.e.e Standard Error of Estimate

C.C Correlation Coefficient

Table 6.4 Linear Crack Growth Law Coefficients
Accounting for Mean Load Variations

By combining the equations representing the lines fitted through different families of experimental data (i.e. same R and same P_{mean}) a generalized equation was determined. This generalized law, describing the crack propagation behaviour of the 7075 T7351 has been obtained by fitting a single first order polynomial through the total number of the experimental data obtained in the course of the study. This mathematical model describing the crack propagation of the tested Aluminium alloy is shown in Fig.6.28 supplemented by the individual equations obtained from the sub-studies of R and P_{mean} effects. This generalized law takes the following form,

$$(da/dN)=7.031 \times 10^{-11} \cdot (\Delta K)^{2.512} \quad \dots (6.2)$$

The above equation is the result of considering a total of 17129 experimental points with a coefficient of determination C.d=86.15%. The coefficient of determination

is high thus indicating that the experimental data uncertainty is significantly explained by the presented linear crack growth law. All individual equations form a narrow band around this final law equation as depicted in Fig.6.28.

(II) The Hybrid Second Order Law

This is a concept developed as a result of the analysis of data and the findings within the present study. It has already been discussed that the material crack propagation is rather unconventional. It has also been mentioned that the trend of growth indicates the possibility of the crack growth curve being of a parabolic like shape. This last idea has been examined in detail with the utilisation of the experimental data from this study.

If the Paris law is written in logarithmic form, is described by the following expression,

$$\log(da/dN) = \log(C) + n \cdot \log(\Delta K) \quad \dots (6.3)$$

This may be modified by adding an additional term $S = m \cdot (\log(\Delta K))^2$, which now gives the law as a second order polynomial function of ΔK ,

$$\log(da/dN) = \log(G) + q \cdot \log(\Delta K) + m \cdot (\log(\Delta K))^2 \quad \dots (6.4)$$

The second order polynomial function of ΔK contains three unknown factors namely G , q and m . The evaluation of these terms was enabled by means of utilising a computational least squares method of curve fitting through the experimental data. By considering the inverse logarithms of the above equation the following expression is obtained,

$$(da/dN) = G \cdot \Delta K^q \cdot 10^S \quad \dots (6.4)$$

where $S = m \cdot (\log(\Delta K))^2$

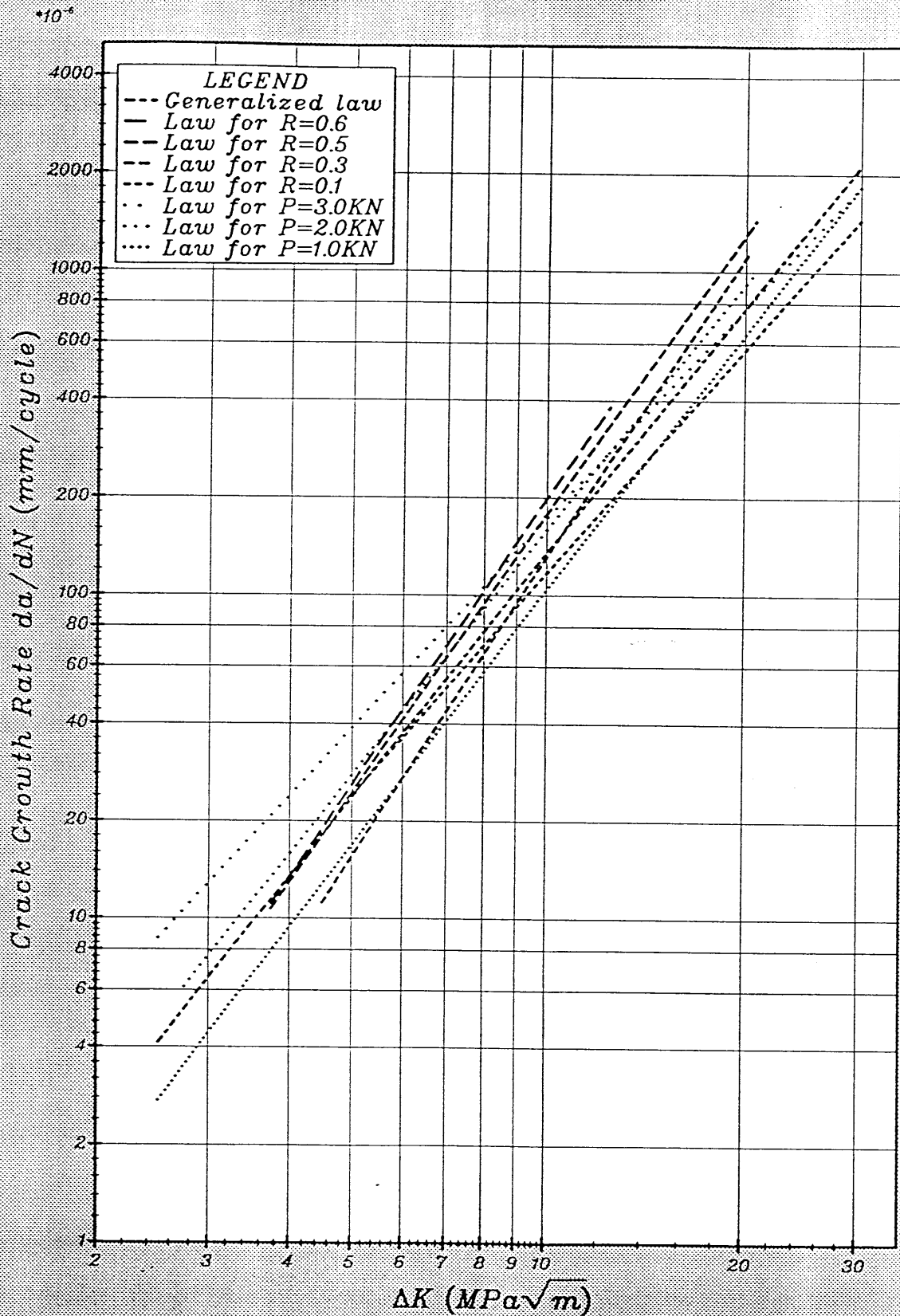


Fig.6.28 The generalized Paris-Erdogan crack growth law and expressions derived from the separate analysis of R and P_{mean}

CHAPTER 6

This modified expression of crack propagation was fitted through experimental data by considering two cases, the constant P_{mean} and constant R ratio. The numerical solutions for the evaluation of the coefficients of equation (6.4) and statistical information regarding the goodness of fit are given in Tables 6.5 and 6.6 below.

$P_{mean} = 1.0 \text{ KN}$	$G = 3.853 \times 10^{-26}$	$q = 14.853$	$m = -2.533$
$C.d = 87.67\%$	$S.e.e = 0.175$	$C.C = 0.946$	
$P_{mean} = 2.0 \text{ KN}$	$G = 7.720 \times 10^{-25}$	$q = 14.049$	$m = -2.357$
$C.d = 95.08\%$	$S.e.e = 0.113$	$C.C = 0.975$	
$P_{mean} = 3.0 \text{ KN}$	$G = 1.074 \times 10^{-18}$	$q = 8.998$	$m = -1.321$
$C.d = 90.94\%$	$S.e.e = 0.126$	$C.C = 0.954$	

Table 6.5 Hybrid Crack Growth Law Coefficients
Accounting for R Load Ratio Variations

$R = 0.1$	$G = 8.192 \times 10^{-24}$	$q = 12.657$	$m = -1.992$
$C.d = 84.47\%$	$S.e.e = 0.174$	$C.C = 0.919$	
$R = 0.3$	$G = 6.093 \times 10^{-19}$	$q = 8.555$	$m = -1.123$
$C.d = 93.07\%$	$S.e.e = 0.161$	$C.C = 0.964$	
$R = 0.5$	$G = 3.772 \times 10^{-30}$	$q = 18.132$	$m = -3.147$
$C.d = 96.73\%$	$S.e.e = 0.070$	$C.C = 0.984$	
$R = 0.6$	$G = 1.800 \times 10^{-32}$	$q = 20.743$	$m = -3.821$
$C.d = 94.73\%$	$S.e.e = 0.121$	$C.C = 0.960$	

Table 6.6 Hybrid Crack Growth Law Coefficients
Accounting for Mean Load Variations

Comparison of the statistical information regarding the goodness of fit for the linear and hybrid laws shown in Tables 6.3-6.6 indicates that, the hybrid expression is marginally better. The improvement in the interpretation of the experimental data uncertainty is of the order of 2.0% at the best.

A generalized hybrid expression has been obtained by utilising the total of the available data points. This yields the following equation,

$$(da/dN)=G.\Delta K^q.10^S \quad \dots(6.5)$$

where $G=5.725 \times 10^{-22}$

$q=11.500$

$S=-1.810 \times (\log(\Delta K))^2$

The coefficient of determination for this equation is of the order of 90.05%. This represents an improvement of approximately 4.0%. Figure 6.29 shows the generalized second order hybrid law supplemented by the expressions from the sub-studies of R and mean load.

A graphic representation of the generalized first order and the 2nd order hybrid expressions is depicted in Fig.6.30, where the good agreement of the two laws in the range of $\Delta K=4.0-20.0(\text{MPam}^{1/2})$. A set of plots representing the linear and hybrid law fit through individual sets of data is given in Appendix 4.

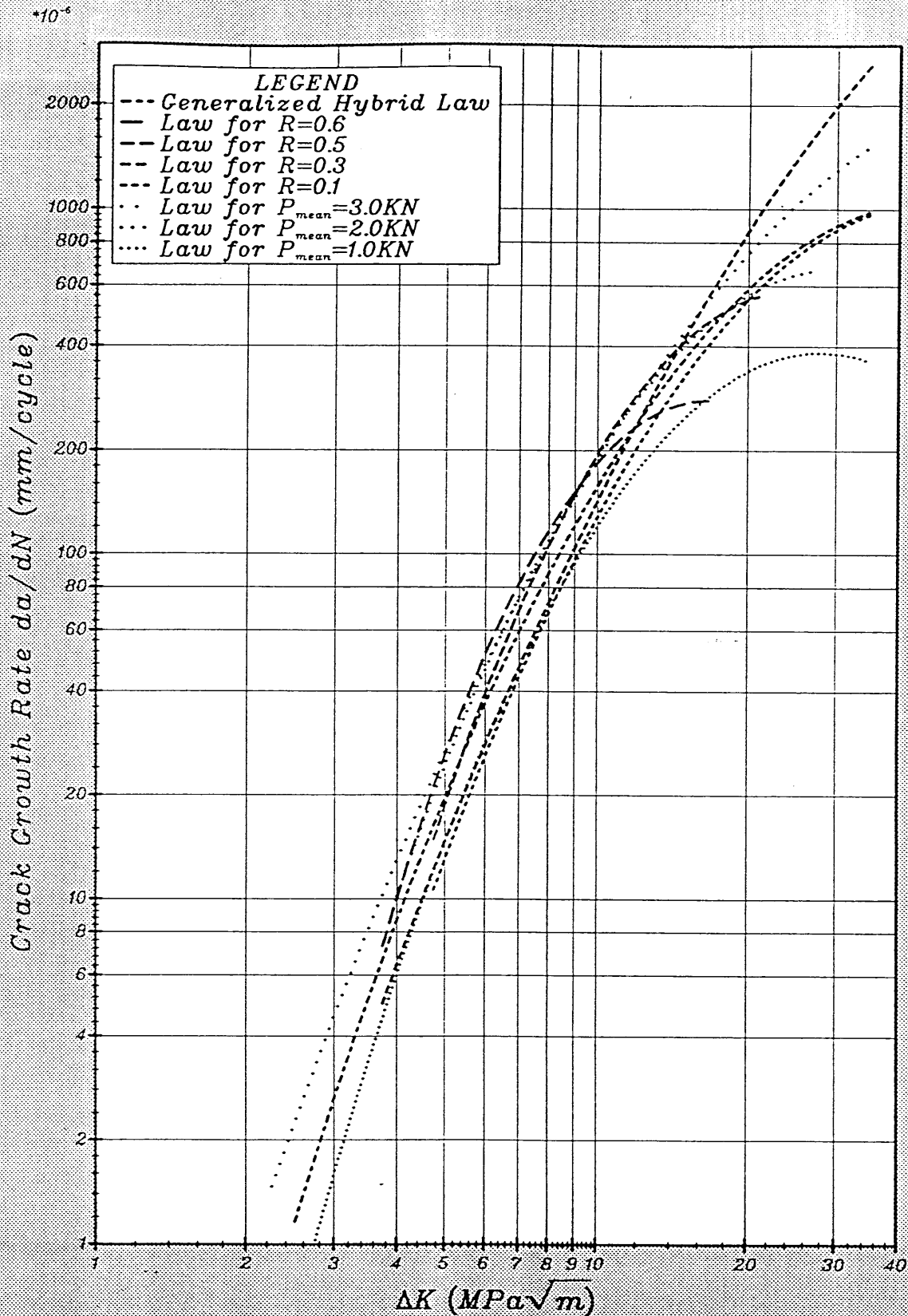


Fig.6.29 The generalized 2nd Order Hybrid crack growth law and expressions derived from the separate analysis of the R load ratio and P_{mean}

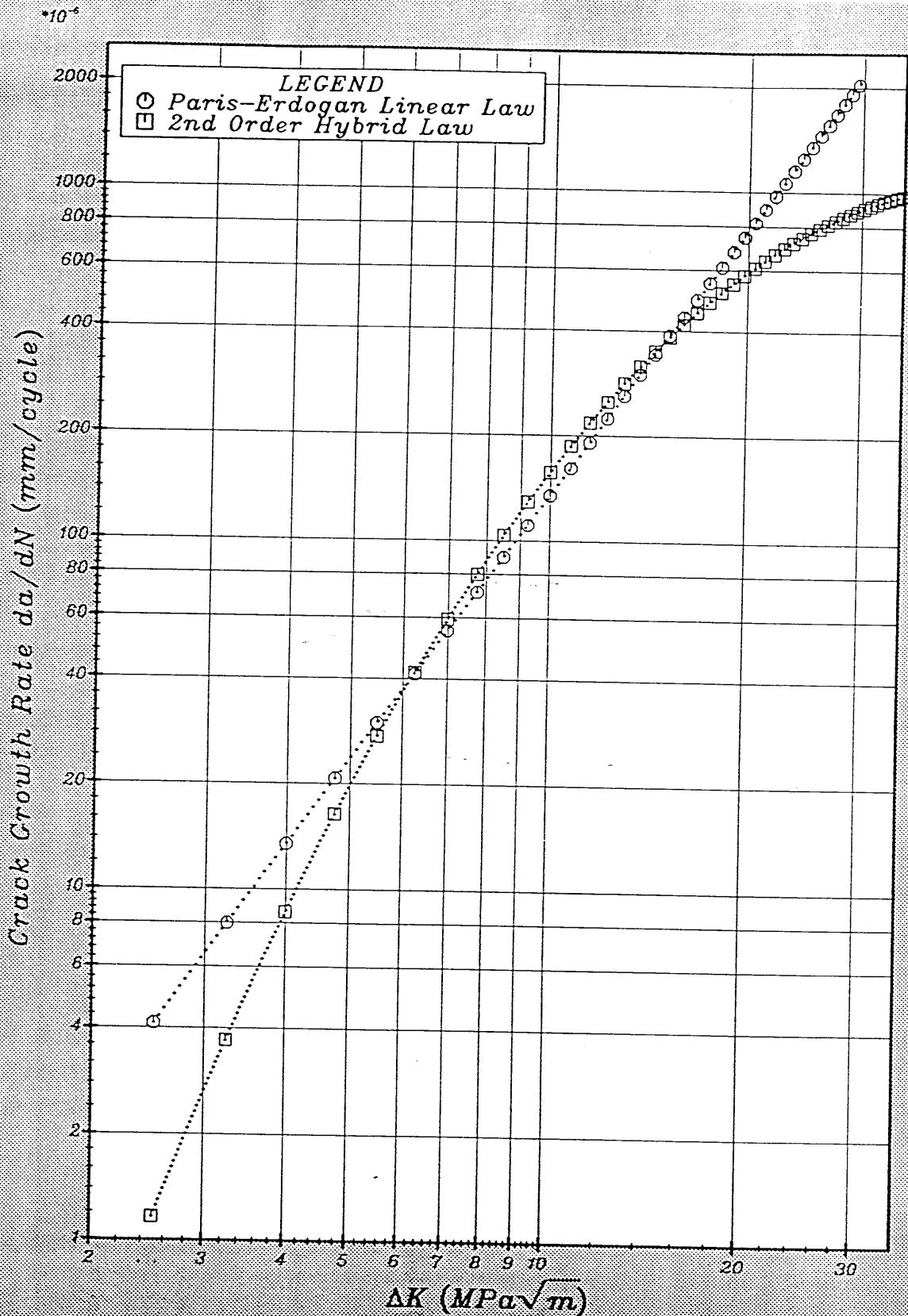


Fig.6.30 Pictorial comparison between the generilized Paris-Erdogan and the 2nd order Hybrid crack growth laws.

6.3 CONCLUSIONS

The investigations in the fatigue crack propagation behaviour of the 7075 T7351 Aluminium alloy under constant load amplitude conditions, have illustrated that,

- a. The straight starter notch performs better than the chevron equivalent. It has more consistently produced straight crack fronts.
- b. The 7075 T7351 Aluminium alloy fatigue crack propagation curve does not exhibit a sigmoidal reflex shape, but one that resembles a parabola.
- c. The experimental facility does not possess the sensitivity to ascertain microscopic scale fracture related events.
- d. An increase of the mean load for the same R load ratio results into an increase of the crack growth rates, for the same ΔK .
- e. Increasing the applied load amplitude, i.e. reducing the R ratio results into an acceleration of the crack growth.
- f. Decreasing the R ratio over the same mean load, results to slower crack propagation rates accompanied by increased surface crack tip plasticity for the same ΔK .
- g. The combined effect of maximum mean load and minimum R ratio results to shorter fatigue crack propagation lives.
- h. For the same mean load a decrease in the load amplitude prolongs the fatigue life.

- i. The load cycling frequency affects the crack growth. Low frequencies in the range of (10-25Hz) cause faster crack growth rates as compared to frequencies in the range (25-50Hz).
- j. The use of different frequencies in pre-cracking and subsequent testing does not influence the crack growth.
- k. The Paris-Erdogan crack growth law fits the experimental data reasonably.
- l. The derived second order hybrid mathematical model provides a marginally better correlation with the experimental data.
- m. The elastic Finite Element analysis of the CT specimen has provided an accurate prediction of a crack initiation site in the vicinity of the loading hole.

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CHAPTER 7

EXPERIMENTAL INVESTIGATIONS (II)

SINGLE AND BLOCK OVERLOADS

7.0 Introduction

7.1 Definitions

7.2 Experimental Analysis

7.3 Single Peak Overload Studies

7.3.1 The Experimental Programme

7.3.2 Analysis of Results and Discussions

7.4 Block Overloads

7.4.1 The Experimental Programme

7.4.2 Analysis of Results and Discussions

7.5 Fractographic Examinations

7.6 Conclusions

7.6.1 Single Overload Studies

7.6.2 Block Overload Studies

References

CHAPTER SEVEN

EXPERIMENTAL INVESTIGATIONS (II)

SINGLE AND BLOCK OVERLOADS

7.0 INTRODUCTION

It has been previously discussed and experimentally demonstrated that the crack propagation is dependent upon changes in the load variable characteristics, Chapter 6. The vast majority of engineering components subjected to dynamic loads, employed in the aerospace, nuclear and offshore industries are seldomly experiencing constant load amplitude type loading. In reality, the loads are of variable amplitude and the associated load histories are very complex. In many applications, the time function of load amplitude variation can only be described as a random process, Schijve [7.1].

The study of complex load histories with the purpose of arriving in predictions of the total number of cycles to failure, is an exercise that requires the study and understanding of the mechanics of more fundamental matters, such as simple load sequences. These sequences comprise of fundamental damage events whose effects cannot be accounted for, by considerations based upon the linear damage accumulation theories alone, Larsen and Annis [7.2]. Such fundamental damage events are, segregated from complex load sequences and studied separately in order to establish the exact nature of their effects upon the fatigue crack propagation, Pook [7.3]. Single peak overloads and block overloads interrupting an otherwise constant amplitude loading history, are examples of such fundamental damage events. An important consequence of variable amplitude loading is that ΔK need not increase

7.2 EXPERIMENTAL ANALYSIS

As in the case of the constant load amplitude studies, the material used was the high strength 7075 T7351 Aluminium Zinc alloy. Specimens of the compact tension type CT, detailed in Section 5.4 were utilised for the experiments with, a straight through starter notch geometry. The uniaxial fatigue testing machine previously described in Section 5.1, was employed.

In this series of experiments the analogue signal and the 16 bit resolution ramp and hold generators, available on the test machine, were employed. In particular, the analogue signal generator was utilised for the purpose of pre-cracking and constant load amplitude. The 16 bit ramp and hold generator was specifically employed for the generation of the required single peak and multiple/block overloads.

The pre-cracking procedure was carried out in the same manner detailed and discussed in Section 6.1.3. The initiated crack was allowed to reach a length of 3.0mm, measured from the machined notch. At this stage of the work, in order to accelerate the crack initiation, the cyclic frequency of 25 Hz was adopted and, cycling about a mean load $P_{\text{mean}} = 1.5\text{KN}$ and $R = 0.4$ were utilised. The lower frequency was employed in view of the constant amplitude observations which revealed that the precracking frequency does not affect any subsequent crack growth but seems to accelerate crack initiation. It was thus possible to achieve the desired pre-cracking crack length within 60,000-80,000 cycles, the number of cycles depending upon the sharpness of the machined starter notch.

7.3 SINGLE PEAK OVERLOAD STUDIES

7.3.1 The Experimental Programme

The main objective of this part of the research has been to ascertain the effects of the introduction of single peak overloads in an otherwise constant load amplitude history. The magnitude of the overload was varied so as to establish the relationship of the overload ratio R_{ovl} , upon the subsequent fatigue crack growth. The pre- and post-overload constant load amplitude sequences were kept the same in all experiments, and only in the cases of crack arrest the post-overload sequence was varied. The employed constant load amplitude cycling frequency, was maintained to the level of 50 Hz throughout the experimental programme.

The experimental investigations of the constant load amplitude fatigue crack growth have shown that, the load sequence with load ratio $R=0.6$ was appropriate for this type of work. The total fatigue lives of specimens under constant amplitude conditions were 350,000 cycles and 154,000 cycles for mean loads of 2.0KN and 3.0KN respectively. This implies that, a large number of experimental data can be gathered in the duration of the experiments. As the crack growth rates involved are not very high, (6.0×10^{-6} - 4.0×10^{-4} mm/cycle), more accurate measurements of the crack length could be made with the utilisation of the travelling microscope and stroboscopic illumination facility. In this type of experiments the crack length was measured in increments of 0.25mm using the optical arrangement. Another reason for the selection of the aforementioned constant load amplitude sequence is that, the associated crack tip plasticity is very small. This is of great importance as the existence of large

plasticity due to the constant load amplitude cycling could obscure the real effects of overloads.

The tests conducted under the single peak overload regime are shown in Table 7.1 below.

Overload Ratio R_{ovl}						
P_{mean}	2.0KN	1.2	1.4	1.6	1.8	2.0
P_{mean}	3.0KN	1.2	1.4	1.6	1.8	2.0

Pre-, Post-Overload Constant Amplitude Frequency 50 Hz

Table 7.1 Single Peak Overload Test Matrix

7.3.2 Analysis of Results and Discussions

A number of experiments were conducted with the aim of investigating the effect of the overload magnitude upon the subsequent crack growth under the constant load amplitude regime. The single overloads were applied in all cases at the same crack length of 11.9mm and 15.9mm. It is essential to maintain the crack length at which the overload is applied the same, because if comparison of results from different crack lengths is attempted then different retardation effects can be obtained, Broek [7.4].

As shown in Table 7.1 the smaller overload ratio investigated was that of $R_{ovl}=1.2$. When the overload was introduced in a load history defined by $P_{mean}=2.0KN$, $R=0.6$ and load frequency 50 Hz, the effect of the overload was minimum. A typical (a versus N) curve of such experiments is shown in Fig.7.2. It can be clearly seen that the crack growth in the post overload region is undisturbed, no crack growth delay is evident. A small concentration of data points following the overload application can be

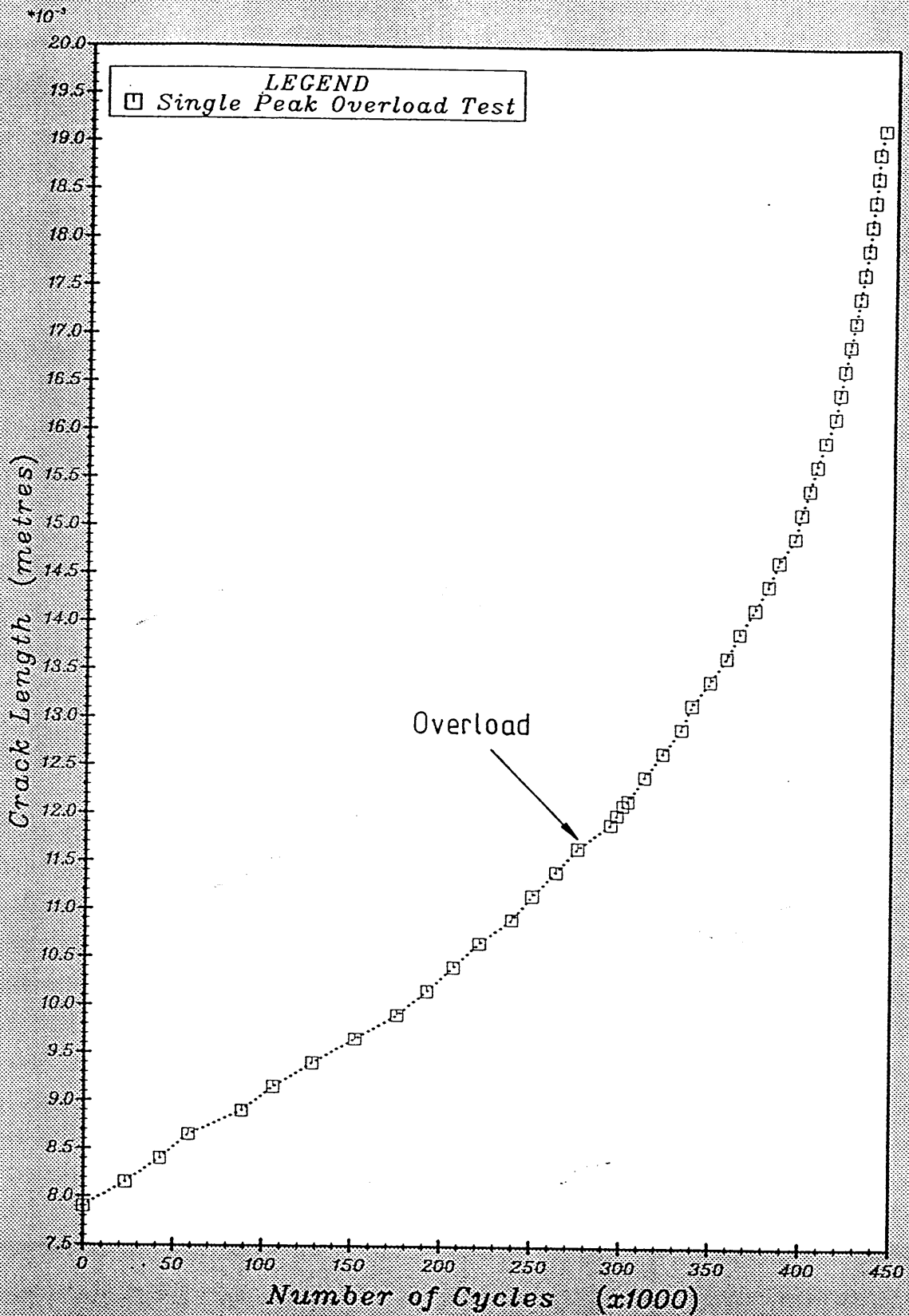


Fig.7.2 Experimental results from a single peak overload test.

Constant Amplitude $P_{mean}=2.0\text{KN}$ $R=0.6$

Overload Ratio $R_{ovl}=1.2$

observed in the (a versus N) curve. This in fact indicates that the crack accelerates in order to attain the crack growth rate corresponding to the constant load amplitude conditions. The apparent localised acceleration is of short duration and is also depicted in the (da/dN versus a) curve shown in Fig.7.3.

Further evidence of the non-influence of the overload under these loading conditions is given by the plot of (da/dN versus ΔK), shown in Fig.7.4. The experimental data is accompanied by the crack growth laws for comparison purposes; These laws, were determined during the constant load amplitude investigations, see Chapter 6. It is evident in Fig.7.4, that the crack growth has hardly been disturbed by the application of the overload and, by comparison with the constant load amplitude corresponding laws the overall agreement is very good.

The state of events has been observed to change when the constant load cycling is over a $P_{\text{mean}} = 3.0\text{KN}$ and all other load variables are kept the same as above. In this case, the crack growth following the overload indicates that a short period of retardation is present, see Fig.7.5. In fact, examination of the crack growth curve (da/dN versus ΔK) shows a clear disturbance of the growth by the overload. Following the first overload the crack growth rate indicates a high growth rate at the initial stages which is subsequently followed by a clearly marked retardation. This retardation reaches a minimum after the crack has advanced by a small amount. By examining the results of (da/dN versus a), Fig.7.6, the crack has been found to advance approximately by 1.26mm before the retardation effect is fully active (i.e. maximum retardation), and approximately 7,520 cycles have elapsed, following the overload application.

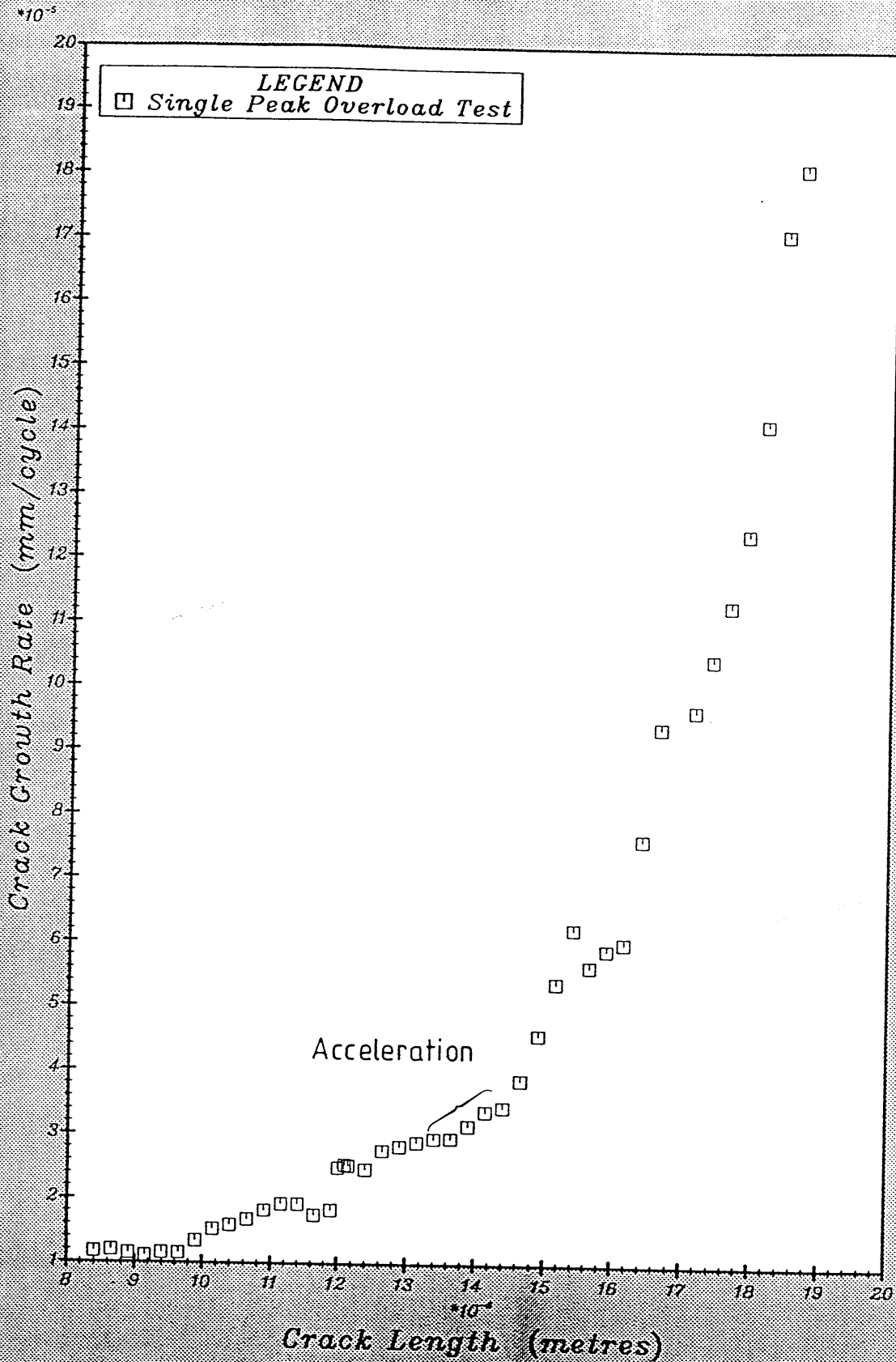


Fig.7.3 Variation of the crack growth rate as a function of crack length, for a single peak overload ($R_{ovl}=1.2$) applied in a constant amplitude sequence $P_{mean}=2.0KN$ $R=0.6$

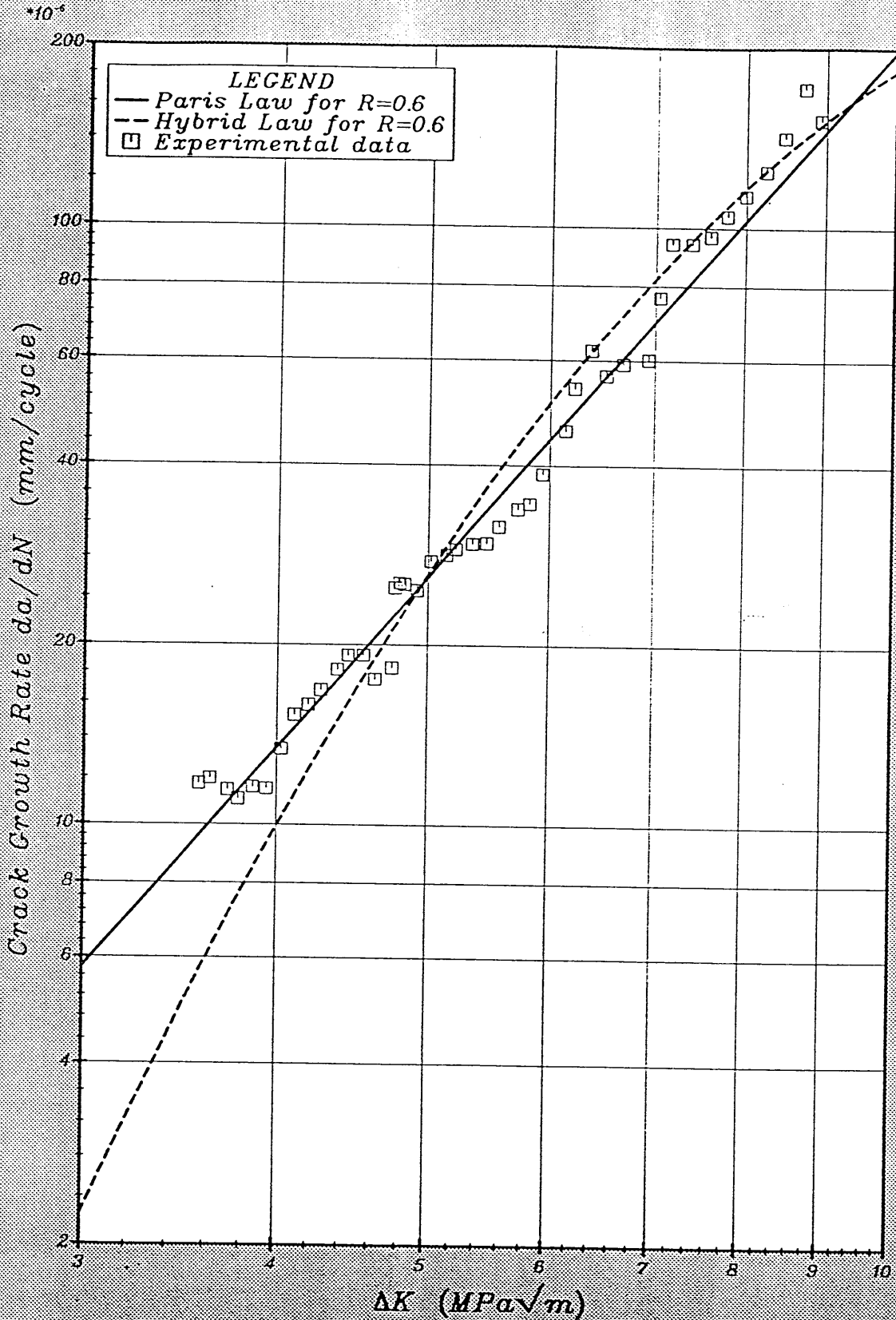


Fig.7.4 Comparison of experimentally determined crack growth data with constant load amplitude mathematical models.

$P_{mean}=2.0KN$ $R=0.6$ $R_{ovl}=1.2$

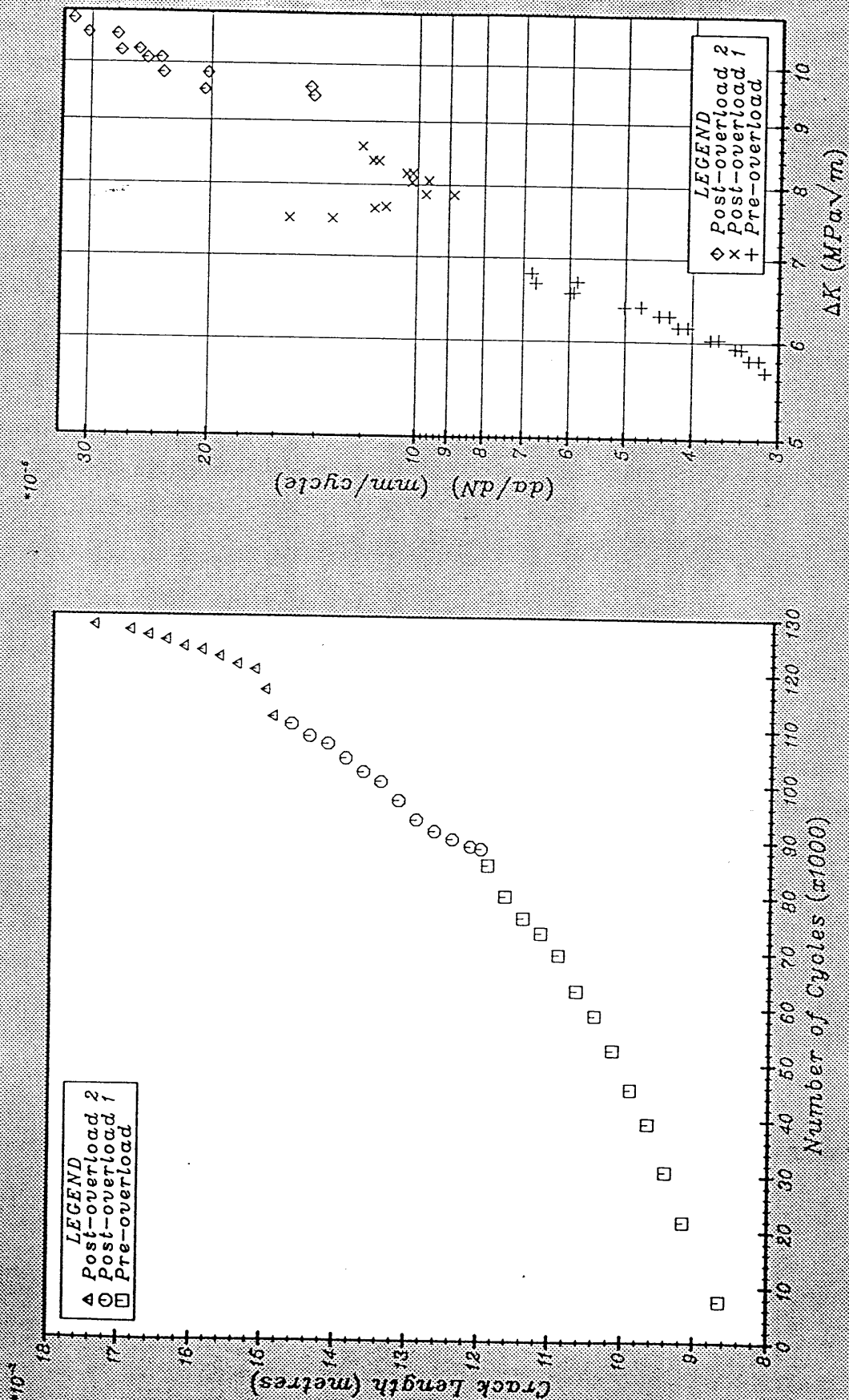


Fig.7.5 Experimental results and calculated crack growth data for a test with two single peak overloads. Constant Amplitude : $P_{mean}=3.0KN$ $R=0.6$ Overload Ratio $R_{ovl}=1.2$

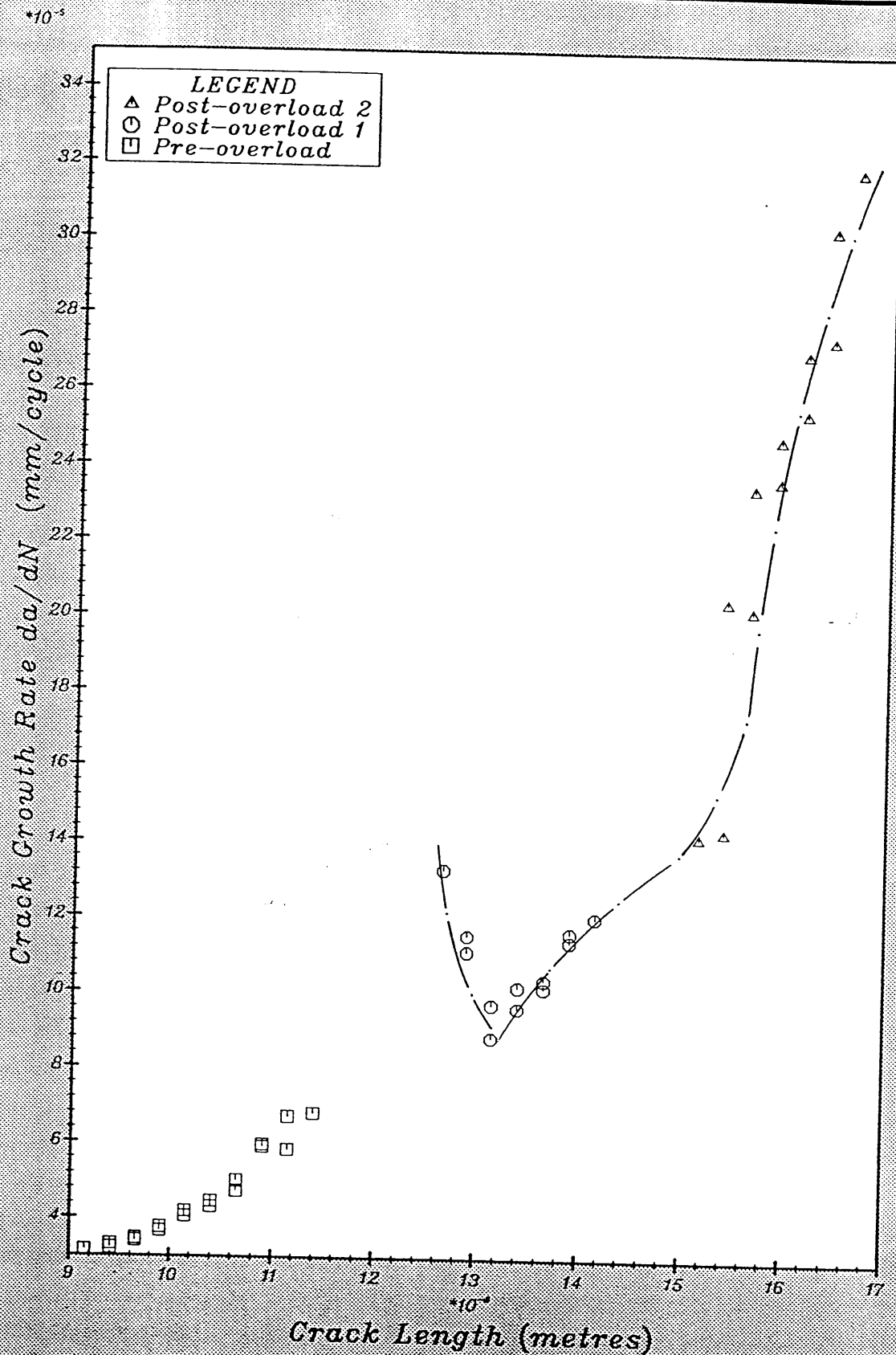


Fig.7.6 Variation of the crack growth rate as a function of crack length, in the presence of two peak overloads ($R_{ovl}=1.2$) in a constant amplitude sequence $P_{mean}=3.0\text{KN}$ $R=0.6$

This effect has been observed by other investigators, Kanninen et al [7.5] and von Euw et al [7.6], who have termed the phenomenon, *delayed retardation*. The second overload does not show a clearly marked retardation, but is believed that this is due to the lack of experimental data points. An investigation in the possible advancement of the crack during the application of the overload has shown that no measurable extension occurred.

The results depicted in Figures 7.4 and 7.5, illustrate that the growth of the crack, after it has passed through the overload affected zone, follows a parabolic like trend similar to the one identified previously for the purely constant load amplitude loading, see Chapter 6.

An increase of the overload ratio R_{ovl} from 1.2 to 1.4, has shown that the overload causes a more significant effect. In Fig.7.7, the results of a typical experiment carried out with $P_{mean}=2.0KN$ are shown, where two overloads of $R_{ovl}=1.4$ have been applied, the first at a crack length $a=11.9mm$ and the second at $a=15.9mm$. The crack growth is affected as shown by the small shift of the (a versus N) curve to the right following the overload application. Once again the small concentration of data points is observed following the overload. Examining the (da/dN versus ΔK) curve, Fig.7.8, in the case of the first overload the subsequent growth does not reach an obvious minimum. The crack growth rates change slowly, and after the crack has propagated by 2.0mm following the overload, a sudden increase in the growth is apparent. By comparison of the experimental data, related to the first overload, with the constant load amplitude laws, Fig.7.9, it can be seen that the affect of the overload has been relatively insignificant, as good agreement between the data is illustrated.

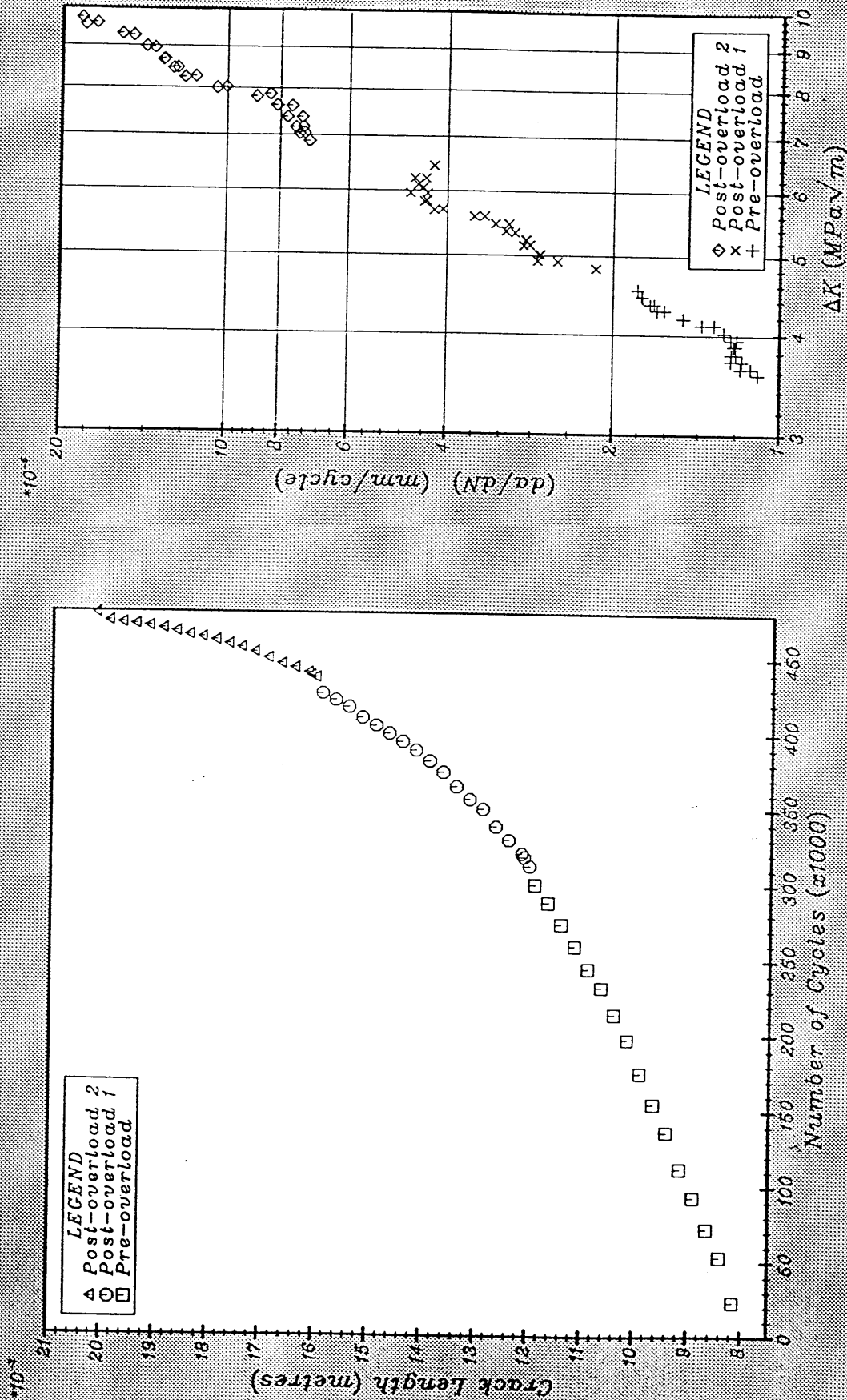


Fig.7.7 Experimental results and calculated crack growth data for a test with two single peak overloads
Constant Amplitude: $P_{mean}=2.0KN$ $R=0.6$
Overload Ratio $R_{ov}=1.4$

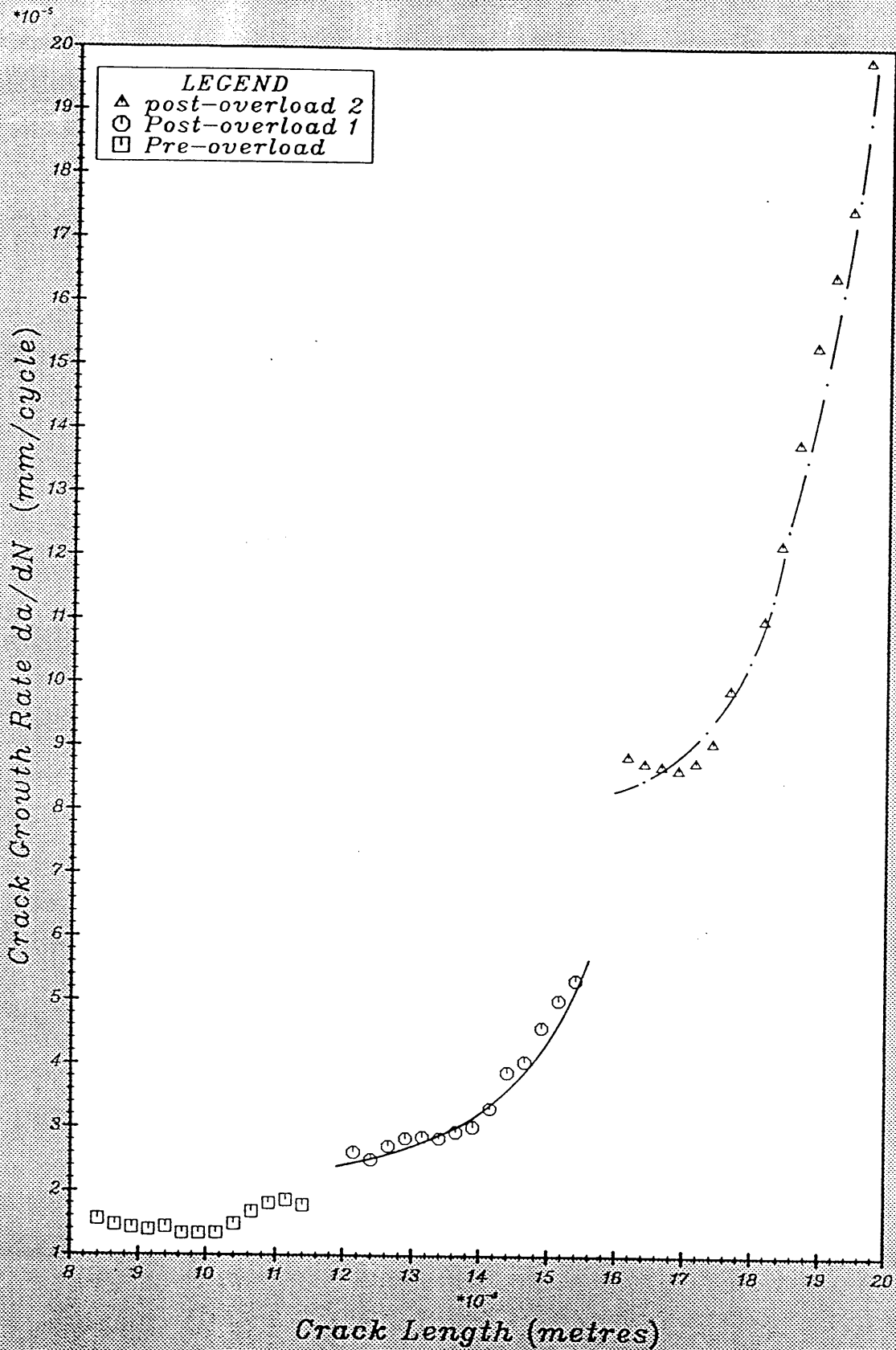


Fig.7.8 Variation of the crack growth rate as a function of crack length in the presence of two peak overloads ($R_{ovl}=1.4$) in a constant amplitude sequence $P_{mean}=2.0KN$ $R=0.6$

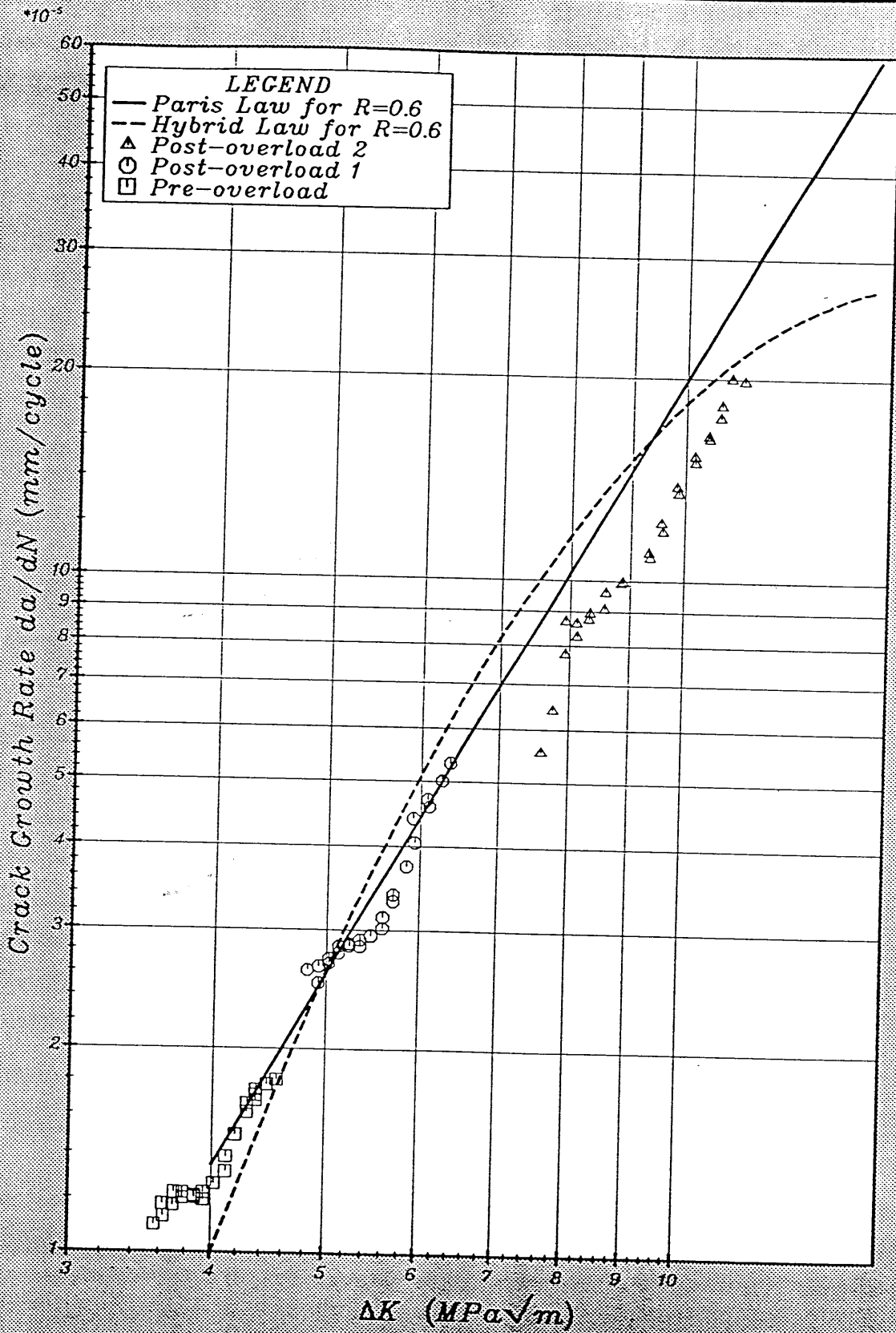


Fig.7.9 Comparison of experimentally determined crack growth data with constant load amplitude mathematical models.

$P_{mean}=2.0\text{KN}$ $R=0.6$ $R_{ov1}=1.4$

The second overload results demonstrate a reduction in the post-overload growth, Fig.7.9. By comparison of the second post-overload crack growth data with data obtained from constant load amplitude studies it was revealed that the post overload crack growth rates were reduced. Table 7.2 provides some indicative results of the effect of the second overload upon the fatigue crack growth as compared with the observed constant load amplitude behaviour.

ΔK (MPam ^{1/2})	Constant Load da/dN (mm/cycle)	Post-Overload
5.0	28x10 ⁻⁶	23.5x10 ⁻⁶
7.0	69x10 ⁻⁶	54.2x10 ⁻⁶
8.0	101x10 ⁻⁶	83.1x10 ⁻⁶
9.0	154x10 ⁻⁶	132.0x10 ⁻⁶
10.0	195x10 ⁻⁶	162.7x10 ⁻⁶

Table 7.2 Crack Growth Rates from Constant Load Amplitude and Second Post-Overload Growth, for $R_{ovl}=1.4$

The data shown in Table 7.2 above, clearly indicate that the second overload has caused reduction of the subsequent crack growth rates by 15-20%. Again in this case it was not possible to determine whether the overload caused any extension of the crack front.

The effects of overloading became more significant when the overload ratio employed was $R_{ovl}=1.6$. Typical results of studies conducted with $R=0.6$ and mean loads of 2.0KN and 3.0KN are illustrated in Figures 7.10 and 7.11 respectively. In both cases it is clearly shown that following the overload the crack length remains unchanged, for a substantial number of cycles following the application of the overload. It is clear that the overload

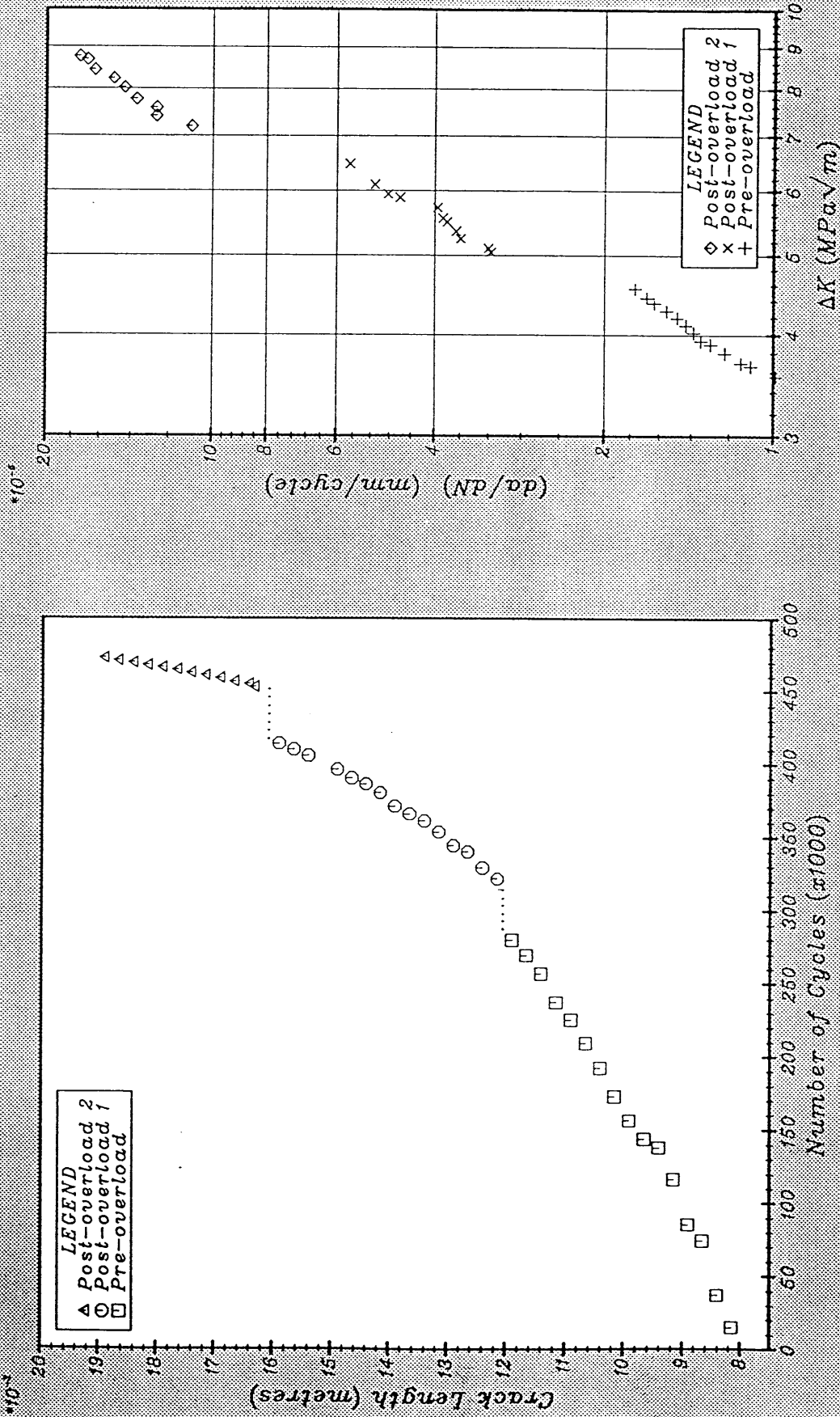


Fig.7.10 Experimental results and calculated crack growth data for a test with two peak overloads.
Constant Amplitude : $P_{mean}=2.0KN$ $R=0.6$
Overload Ratio : $R_{ov}=1.6$

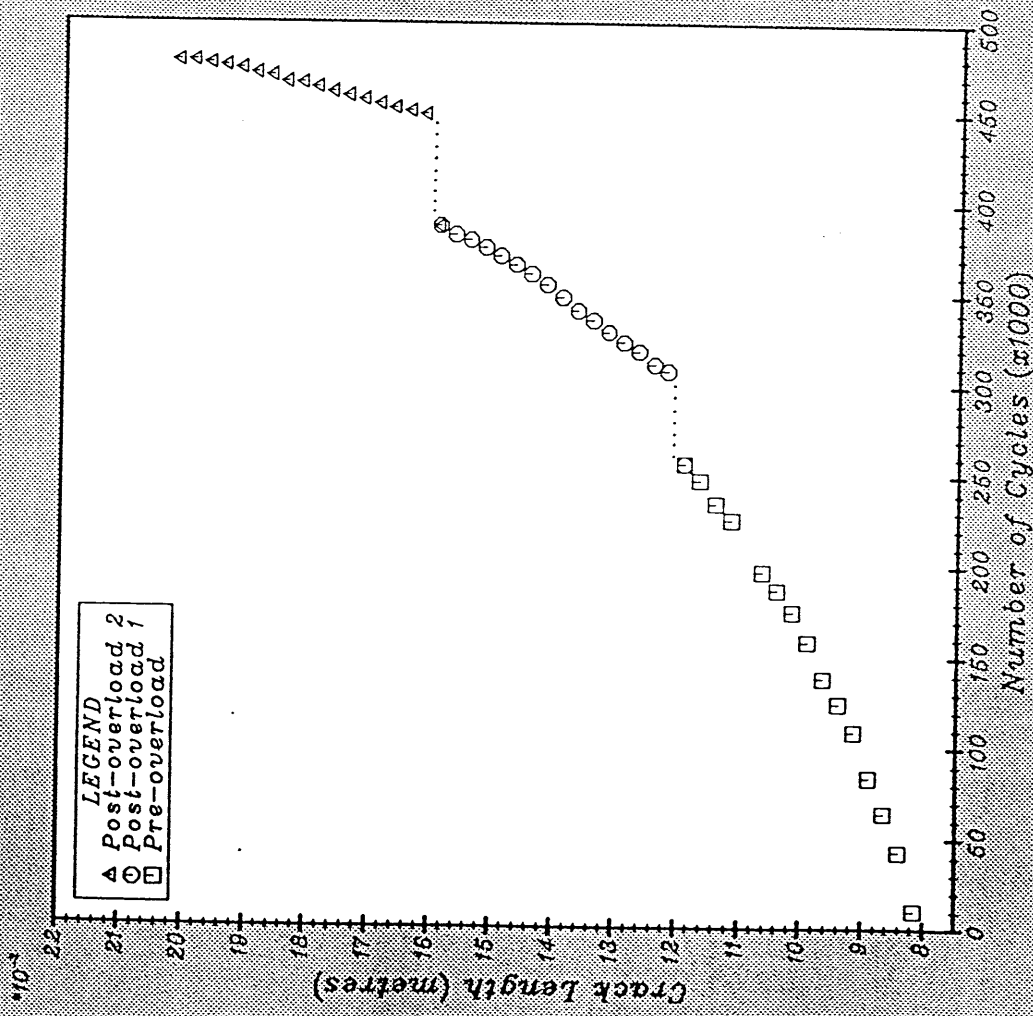
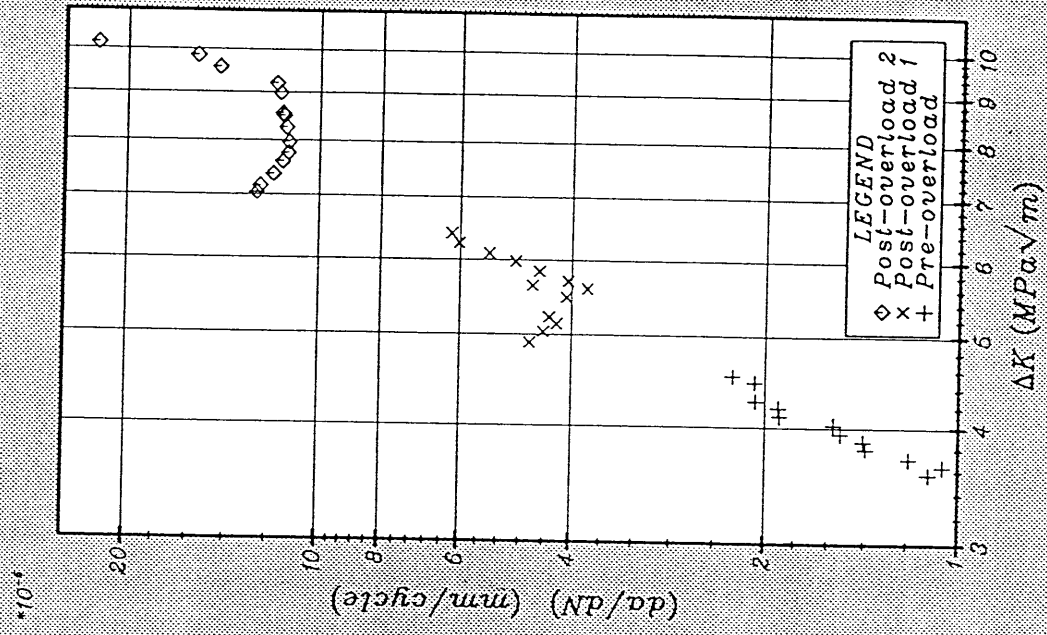


Fig.7.11 Experimental results and calculated crack growth data for a test with two peak overloads. Constant Amplitude : $P_{mean}=3.0KN$ $R=0.6$ Overload Ratio : $R_{ovl}=1.6$

causes temporary arrest of the advancing crack, depicted by the plateau in the (a versus N) curves.

This standstill period is observed to be longer in the case of the second overload. In detail, for $P_{\text{mean}}=2.0\text{KN}$, these periods are 39,050 and 43,120 cycles for the first and second overloads respectively. The respective standstill periods observed in the case of $P_{\text{mean}}=3.0\text{KN}$ were 49,810 and 61,000 cycles. These values represent the average of four duplicate tests.

The post-overload crack growth corresponding to the 3.0KN mean load clearly demonstrates the presence of delayed retardation. Once again, the crack growth rates calculated for the post overload propagation are consistently lower than the crack growth rates determined in the absence of an overload, Fig.7.12.

Further increase of the overload ratio to a value of $R_{\text{ovl}}=1.8$ has shown that substantial crack growth disturbance is caused, demonstrated by the temporary crack arrest periods observed. In Fig.7.13, representative results of a single overload of $R_{\text{ovl}}=1.8$ introduced in a load sequence with $R=0.6$ and $P_{\text{mean}}=2.0\text{KN}$ are depicted. It is evident in this instance, that the plateau following the application of the overload represents a substantial period of the total fatigue life, about 40%. The crack propagation has been found to be significantly affected by the overload as the period over which the crack remains dormant, extends to an average of 233,000 cycles. This delay is substantial if one considers that the total number of cycles to fracture was about 549,000 cycles. By examination of the plot of (da/dN versus a), Fig.7.14, the presence of delayed retardation following the overload application is clearly manifested. This occurred about

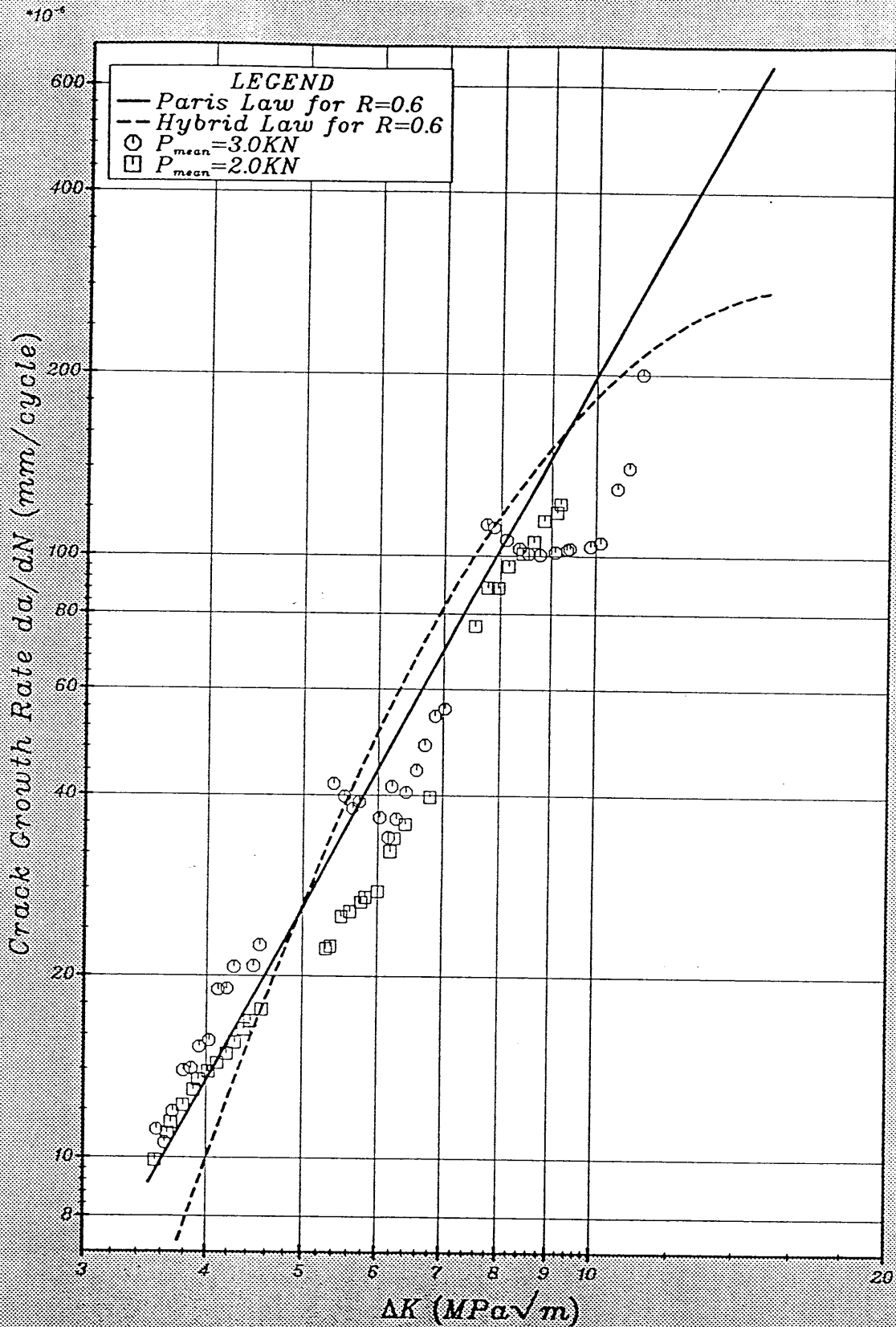


Fig.7.12 Comparison of experimentally determined crack growth data with constant load amplitude mathematical models.

$P_{mean}=2.0\text{KN}$ $R=0.6$. $P_{mean}=3.0\text{KN}$ $R=0.6$ and $R_{ovl}=1.6$

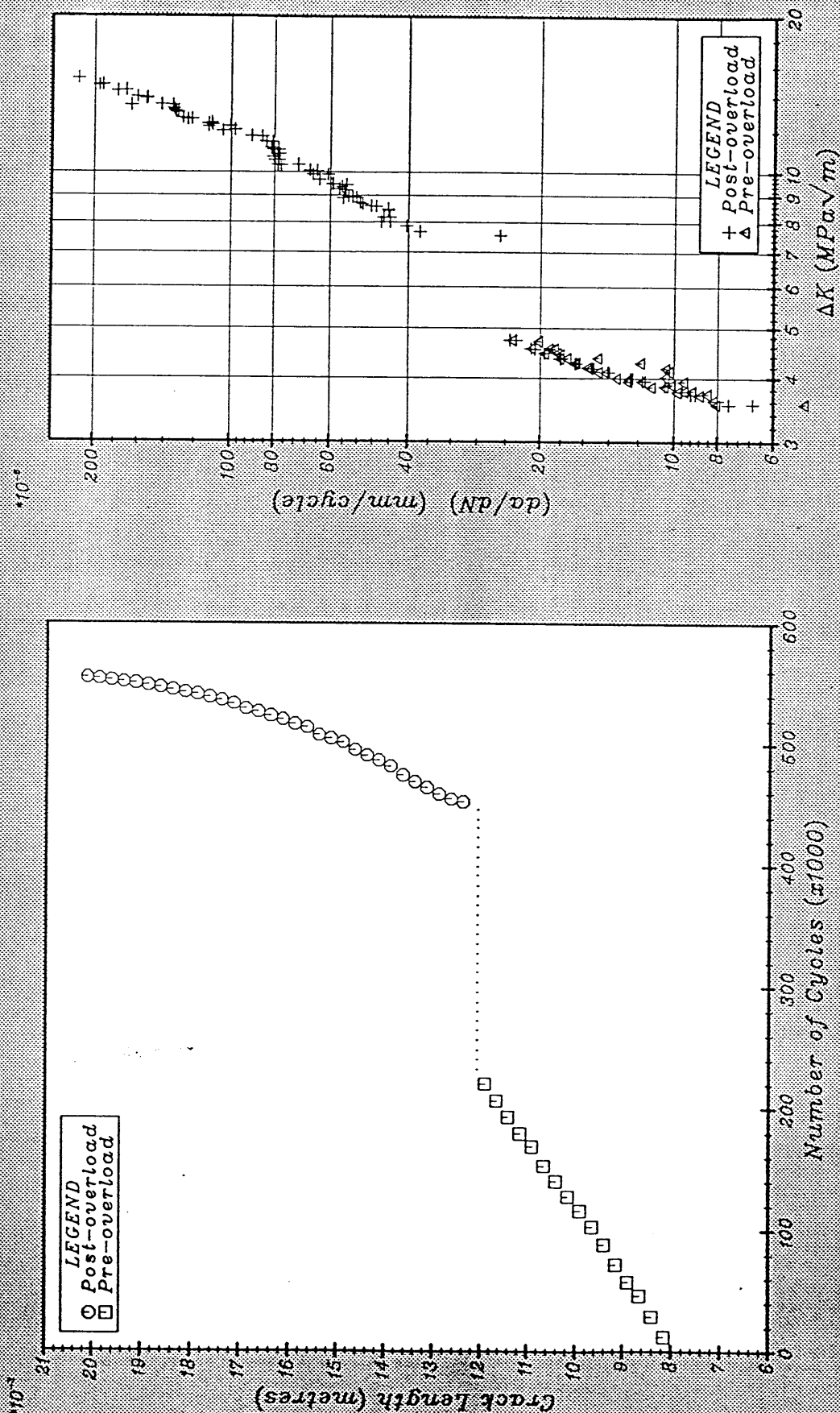


Fig.7.13 Experimental results and calculated crack growth data for a single peak overload test.
Constant Amplitude : $P_{mean}=2.0KN$ $R=0.6$
Overload Ratio : $R_{ovl}=1.8$

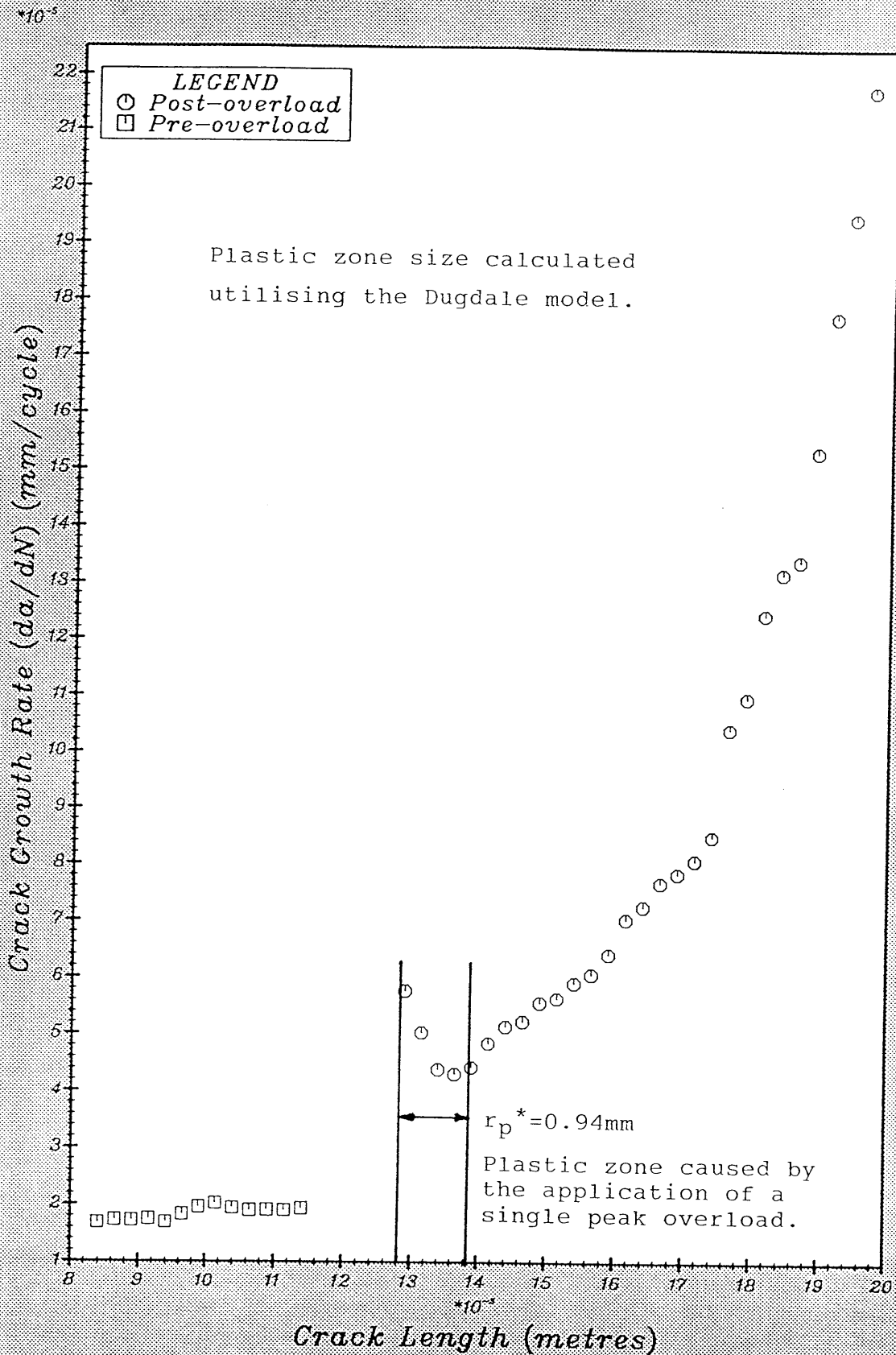


Fig.7.14 Variation of the crack growth rate as a function of the crack length, in the presence of a single peak overload ($R_{ovl}=1.8$) in a constant amplitude sequence $P_{mean}=2.0\text{KN}$ $R=0.6$

30,000 cycles after the constant amplitude loading was resumed and the crack had advanced approximately 1.75mm .

By comparison of the experimentally determined crack growth data with the Paris and Second Order Hybrid laws, Fig.7.15, the effect of the overload is detectable. Table 7.3, provides a quantitative comparison of the crack growth rates observed under constant load amplitude and in the presence of an overload.

ΔK (MPam ^{1/2})	Constant Load da/dN (mm/cycle)	Post-Overload da/dN (mm/cycle)
7.0	69×10^{-6}	33.0×10^{-6}
8.0	101×10^{-6}	42.0×10^{-6}
9.0	154×10^{-6}	51.0×10^{-6}
10.0	195×10^{-6}	74.0×10^{-6}

Table 7.3 Crack Growth Rates from Constant Load Amplitude and Post-Overload Growth, for $R_{ovl}=1.8$

The post-overload crack growth rates are incontrovertibly lower than those predicted by the two laws. The crack growth rate reductions due to overloading have been calculated to be of the order of 50-65%.

A single experiment was carried out where short blocks of four overloads were applied at each of the crack lengths of 11.9mm and 15.9mm. The aim of this work was to investigate whether the use of more than one overloads would enhance further the observed temporary arrest periods and cause any other significant effects.

The results of this experiment are shown in Fig.7.16. It is clearly illustrated that the (a versus N). curve comprises of three discrete segments. The first is the

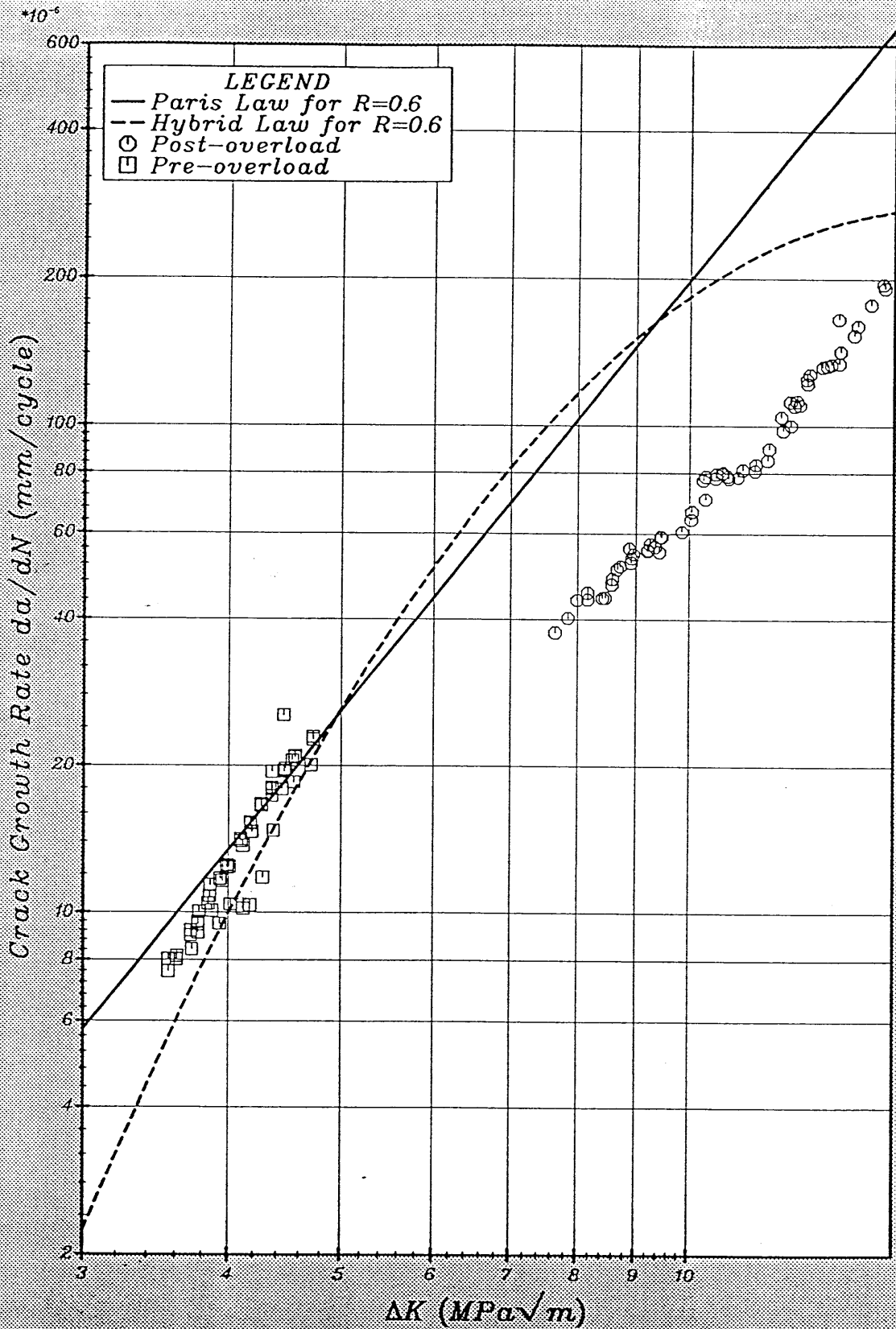


Fig.7.15 Comparison of experimentally determined crack growth data with constant load amplitude mathematical models.

$P_{max}=2.0\text{KN}$ $R=0.6$ $R_{ov}=1.8$

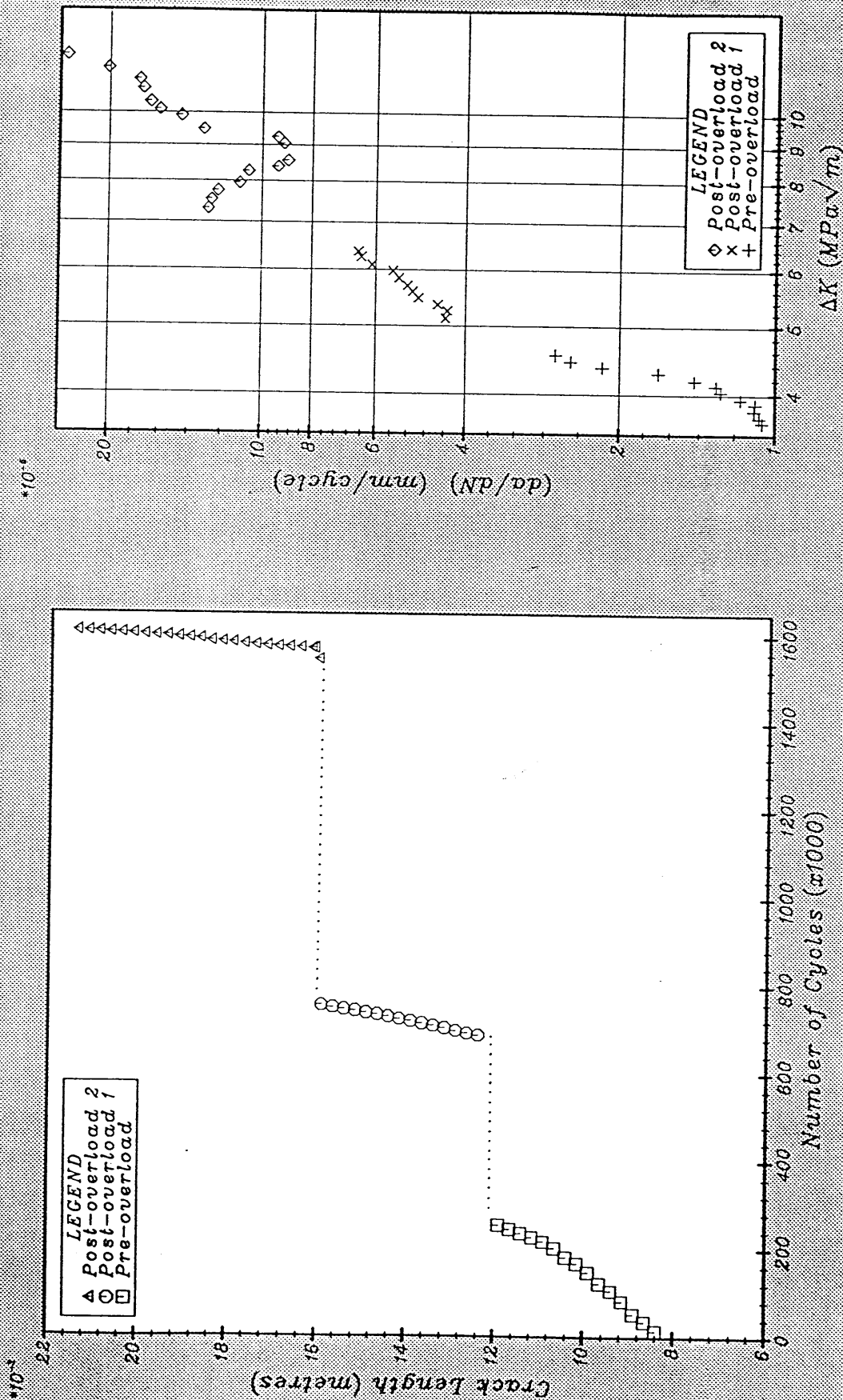


Fig.7.16 Experimental results and calculated crack growth data for a test with two short block overloads
Constant Amplitude : $P_{mean}=2.0KN$ $R=0.6$
Overload Ratio : $R_{ovl}=1.8$

part of the curve obtained by the initial growth of the fatigue crack under constant load amplitude conditions. The second and third sections are situated further to the right and represent the crack growth after the application of the first and second four overload blocks of $R_{ovl}=1.8$. Thus, the curve illustrates the presence of two plateaux which identify the standstill periods caused by the overloads. The first standstill period is 405,000 cycles whereas the second is 720,000 cycles. By comparison the standstill period associated with the first block is almost double the equivalent event due to a single overload. Consequently, the recorded total number of cycles to failure is increased by approximately 1×10^6 cycles. This result has demonstrated that substantial increase in the delay of growth can be achieved by the introduction of short block loads. The second overload block is accompanied by a clearly defined delayed retardation taking place after the crack has advanced 2.46mm after the application of the short block overloads Fig.7.17.

Raising the mean load to 3.0KN and maintaining the overload ratio to $R_{ovl}=1.8$ a very interesting phenomenon was observed. The crack after the application of the overload did not show any indication of advancing. In fact, the crack was subjected to a further 1.25×10^6 cycles following the application of the overload. At this point the crack was considered to have been arrested and the test was discontinued. Duplicate tests were carried out, yielding the same conclusion. With the use of the optical arrangement it was possible to observe the surface of the specimen during the application of the overload. The presence of substantial plasticity in the vicinity of the crack tip was identified. The plasticity could be

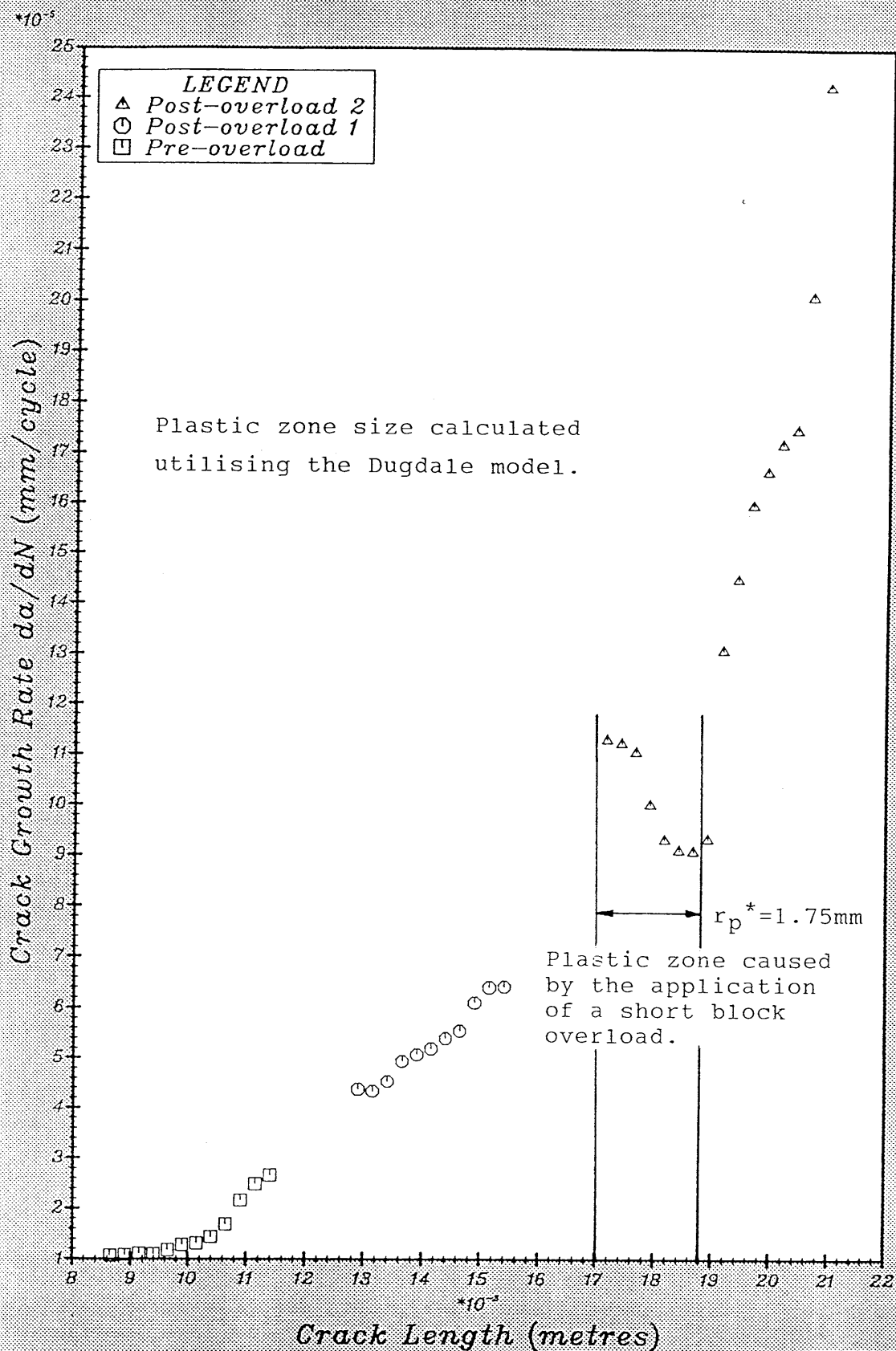


Fig7.17 Variation of the crack growth rate as a function of crack length, in the presence of two short block overloads ($R_{ov}=1.8$) in a constant amplitude sequence $P_{max}=2.0\text{KN}$ $R=0.6$

distinguished since it appeared as a dimple on the surface of the specimen with a rather contrasting dull appearance.

In view of the arresting result of the overload, it was considered essential to investigate whether the crack would propagate by changing the post-overload constant amplitude loading. Thus, after the application of the overload the loading was maintained to $R=0.6$ for a further 470,000 cycles. As anticipated the crack remained dormant. At this point the loading levels were changed to $R=0.3$, by maintaining the same level of mean load i.e. 2.0KN. Continuous monitoring of the crack length revealed that the crack began to advance almost immediately after changing the R-ratio. The results from this experiment are shown in Fig.7.18. As in all previous cases the post-overload crack growth is disturbed. Comparing the resumed growth caused by the reduced R-ratio, with the crack growth law for $R=0.3$, Fig.7.19, it is clear that the rates are significantly lower, about 40-50%. In addition the experimental results indicate a change in the pattern of growth. This change is more easily identified by the graphical presentation of $(da/dN \text{ versus } N)$, Fig.7.20. It is apparent that delayed retardation takes place which is followed by a period of growth over which the crack propagation rates level off and subsequently accelerate until failure.

The highest overload ratio examined was that of $R_{ovl}=2.0$. As it has already been discussed the overload of $R_{ovl}=1.8$ did result to significant temporary arrest, for $P_{mean}=2.0\text{KN}$, and total arrest in the case of a 3.0KN mean load. In view of the results related to 3.0KN, no further tests were carried out.

Applying an overload of $R_{ovl}=2.0$ did in fact result to the arrest of the crack, ($P_{mean}=2.0\text{KN}$, $R=0.6$). A series of experiments did in fact cause the same effect every time.

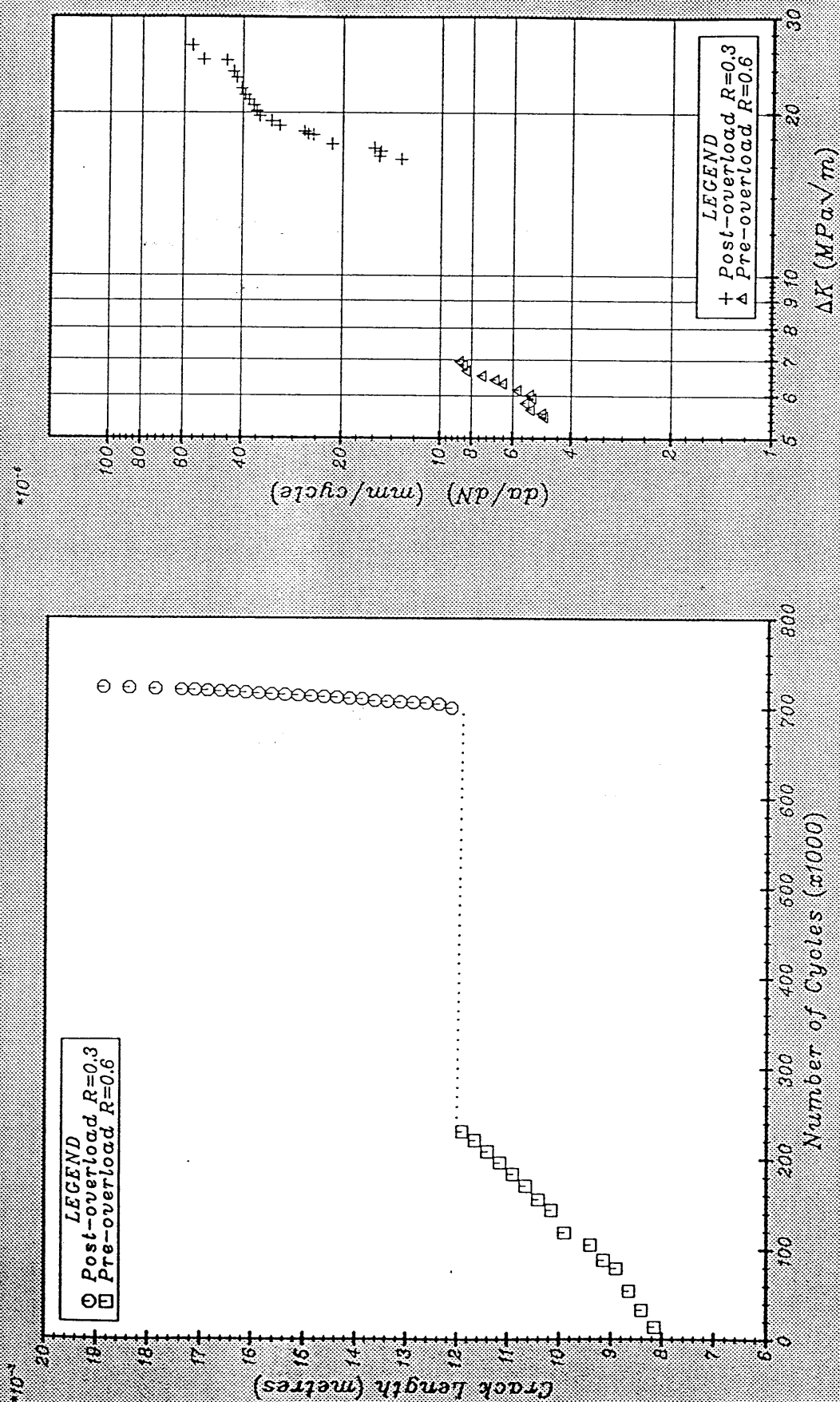


Fig.7.18 Experimental results and calculated crack growth data for a single peak overload test.
Constant Amplitude : $P_{mean} = 3.0 \text{ KN}$ $R=0.6$
Overload Ratio : $R_{ovl} = 1.8$

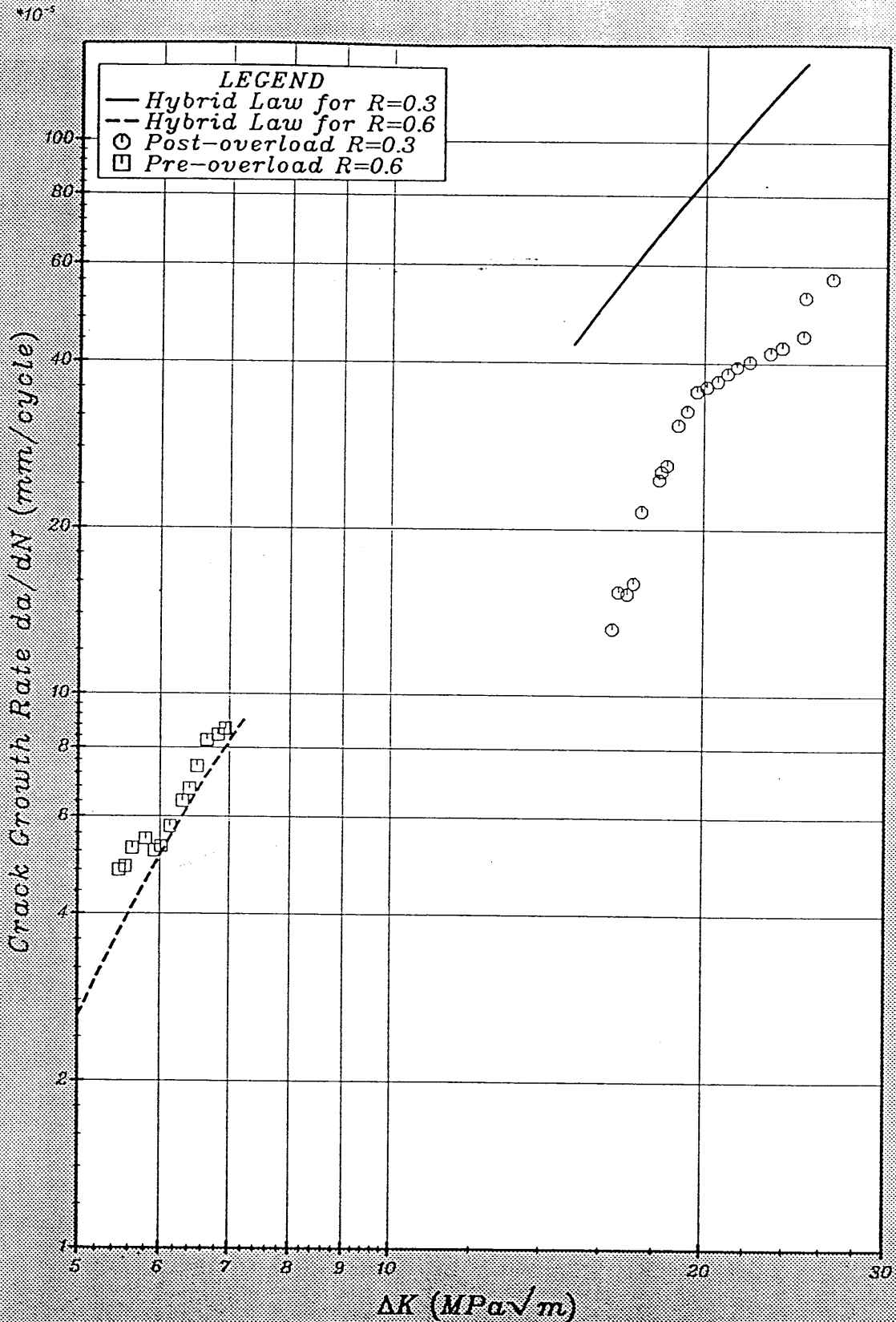


Fig.7.19 Comparison of experimentally determined crack growth data with constant load amplitude mathematical models.

$P_{mean}=3.0KN$ $R=0.6$ $R_{ovl}=1.8$

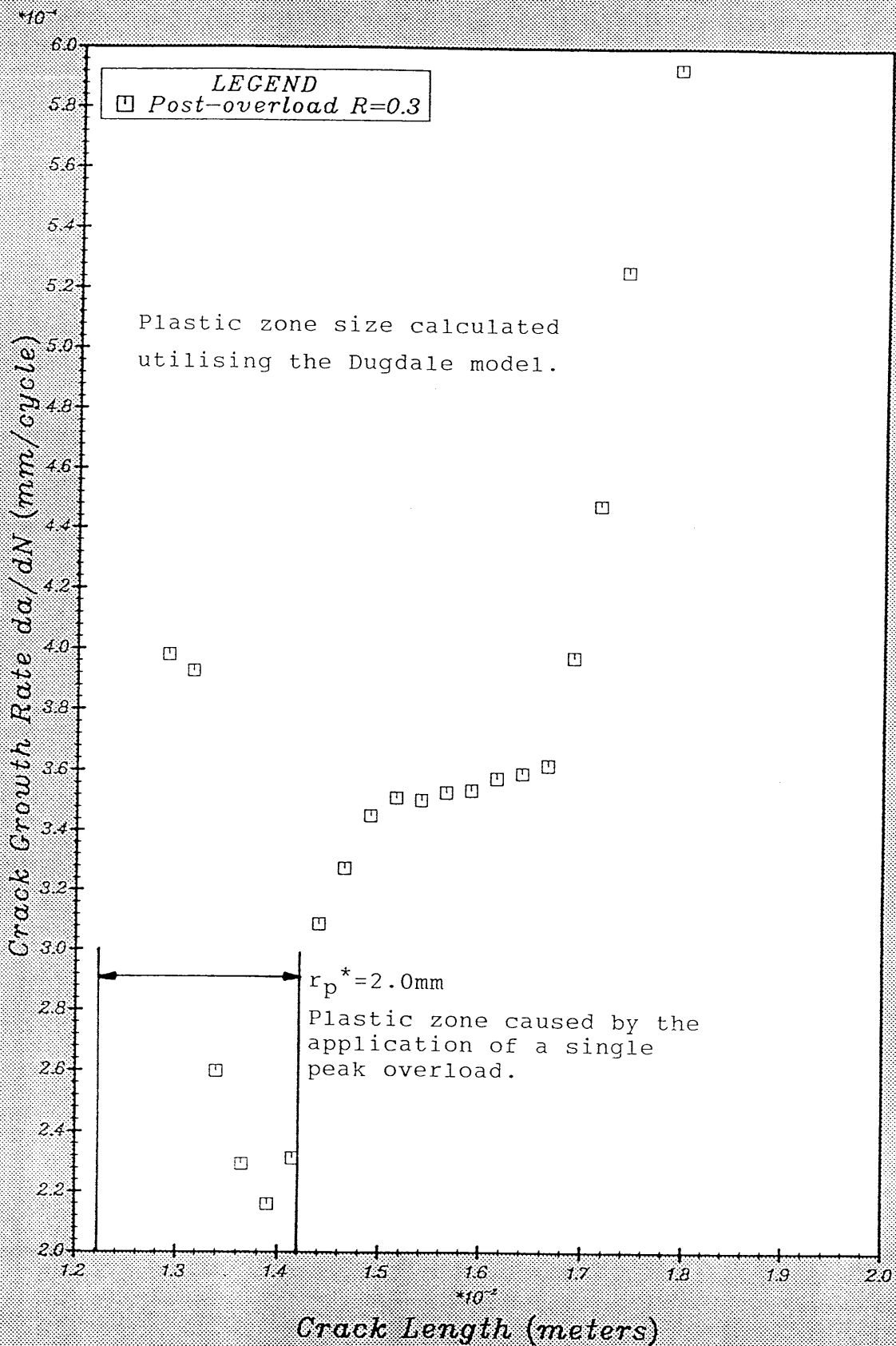
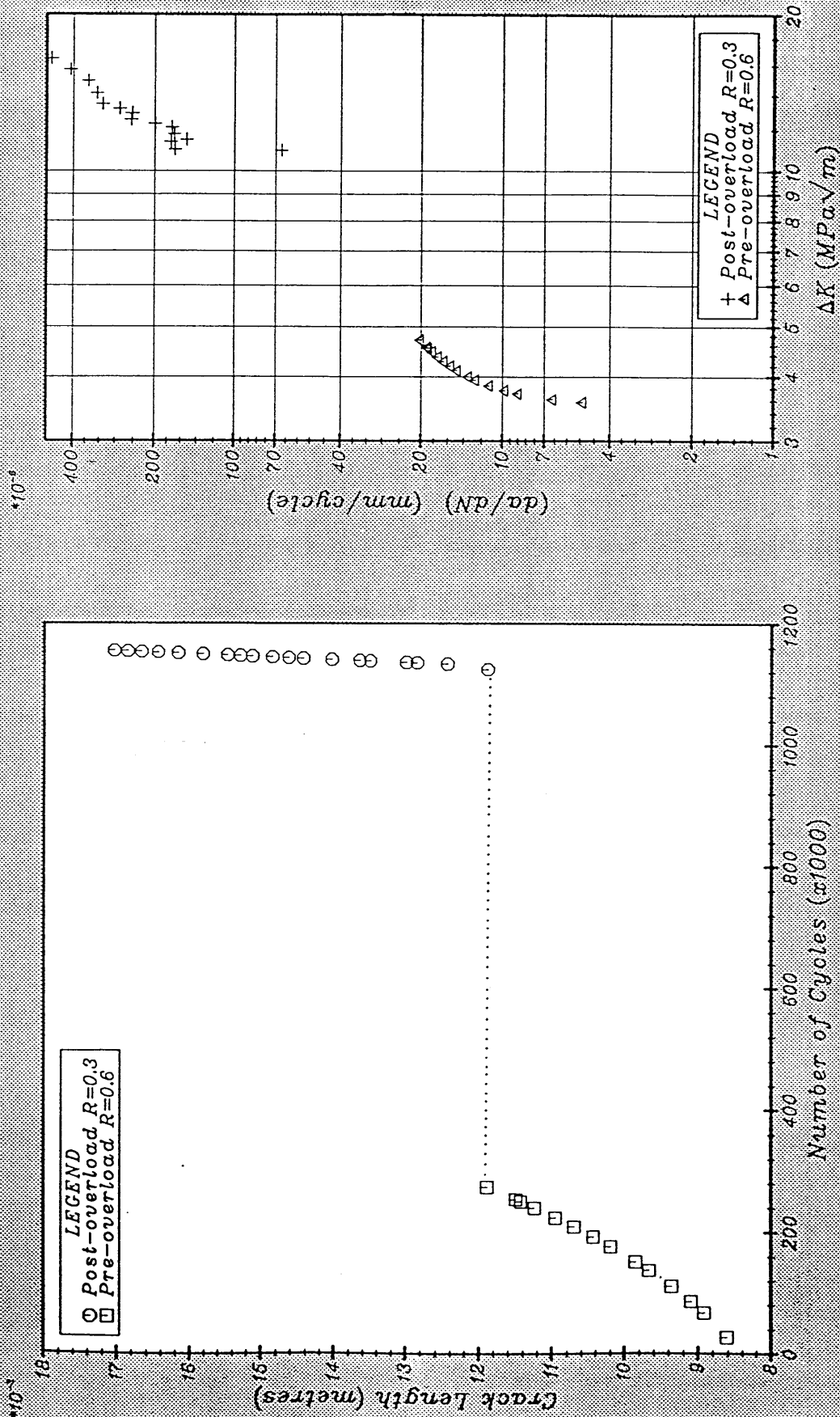


Fig.7.20 Variation of the crack growth rate as a function of crack length, for a single peak overload ($R_{ovl}=1.8$) applied in a constant amplitude sequence $P_{mean}=3.0\text{KN}$ $R=0.6$



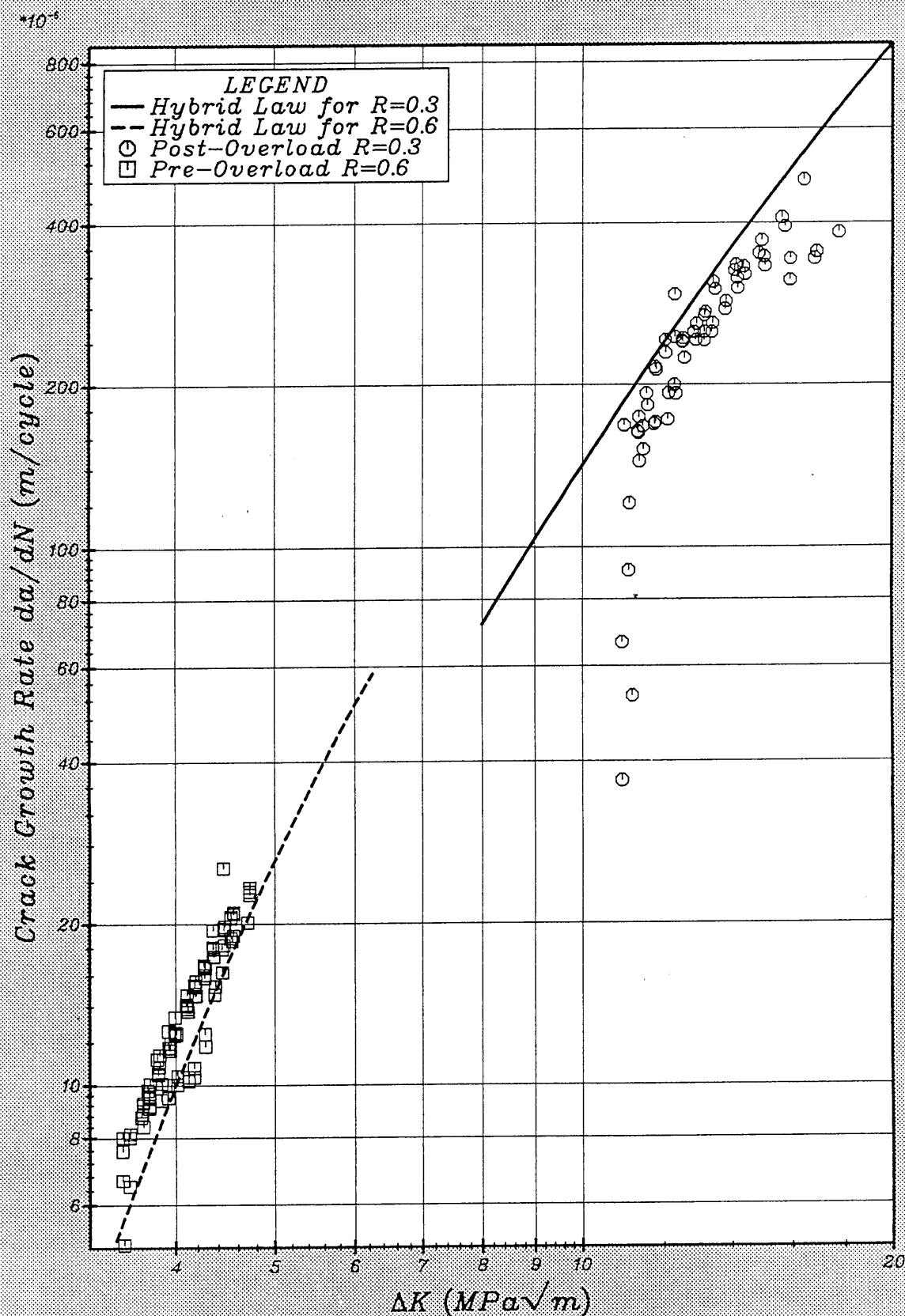


Fig.7.22 Comparison of experimentally determined crack growth data with constant load amplitude mathematical models for two identical tests.
 $P_{mean}=2.0\text{KN}$ $R=0.6$ $R_{ovl}=2.0$

Thus, two tests were performed where after the overload the pre-overload cycling was resumed for a further one million cycles without this causing any propagation. Subsequently, the load amplitude was increased ($R=0.3$), this effectively promoting propagation. The results of a typical test are illustrated in Fig.7.21. The crack growth data from both the aforementioned tests are depicted on Fig.7.22, accompanied by the equivalent crack growth laws. It is apparent that the resumed crack growth is initially lower than the one predicted by the mathematical model but very quickly the propagation reaches the same levels, thus, overcoming the effects of overloading.

7.4 BLOCK OVERLOAD STUDIES

7.4.1 The Experimental Programme

This part of the experimental work is an extension of the single overload studies. The main objective of the investigations has been to ascertain the effect of the presence of a block overload upon the total fatigue crack propagation life.

For the purposes of the investigation the previously designed compact tension specimens were utilised. These, were pre-cracked in accordance to the procedure detailed in Sections 6.1.3 and 7.2. The loading history of each of the experiments can be divided into three main portions. The first part is the constant load amplitude loading applied following the pre-cracking operation. This was interrupted as soon as the crack had propagated by 5.5mm. At this point, the loading was modified and a block overload was introduced. The cyclic duration of the block loading was controlled by means of utilising the programmable cyclic counter trip of the testing machine. Finally, upon interruption of the overload sequence, the constant load amplitude cycling was resumed. The constant

amplitude loads prior and successive to the block overload were identical.

All experimental testing reported within, has utilised a constant amplitude sequence which is defined by, $P_{mean}=2.0KN$, $R=0.6$ and frequency of 50 Hz. This is the same loading utilised in the single overload investigations. The overload ratio, R_{ovl} , was in addition varied during testing. Two ratios were included in the study that of 1.4 and 1.6. The duration of the blocks was an other variable investigated and this was varied between 1,000 and 10,000 cycles. Table 7.4 shows the matrix of tests carried out.

R_{ovl}	Block Overload Cyclic Duration					
1.4	1000	2000	4000	6000	8000	
1.6	1000	2000	4000	6000	8000	10000
1.8	1000	2000				

Table 7.4 Block Overload Test Matrix

It has to be pointed out that in the presented work the block overload cyclic frequency utilised was of the order of 10Hz.

7.4.2 Analysis of Results and Discussions

The experimental information acquired in the duration of each test was of the form of data pairs of crack length and corresponding numbers of elapsed cycles. Examination of the collected data has revealed that the (a versus N) curve for block overloads, cannot be satisfactorily described by a single mathematical function which, could be subsequently differentiated in order to evaluate the crack growth rates of (da/dN) . A graphical illustration of

a typical set of block overload experimental data is shown in Fig.7.23. The nature of the data distribution and the knowledge of the loading history, show that the data can be separated into individual sets, each representing a particular section of the loading history applied, either constant load amplitude or block overloading.

The primary use of the experimentally acquired information is for the determination of crack growth curves (da/dN versus ΔK). Clearly, the discontinuous nature of the experimental data calls for caution in the utilisation of data reduction methods. The implementation of the incremental polynomial methods recommended by the ASTM E647-86 specification, considering all the points of a typical data set, would yield erroneous estimates of the crack growth rates, particularly in the vicinity of discontinuities. This shortcoming of the methods does not by any means preclude their employment. These methods as well as the piece-wise discrete polynomial methodology suggested by McCartney and Cooper [7.7], can still be used. It is though essential to apply these methods of data reduction into artificially discretised data sets. Each such data set should comprise only of experimental data from particular regimes of loading i.e. constant load amplitude or block overloading. This latter artificial data separation procedure has been utilised in the analysis of block overload test data. The methods of experimental data reduction employed in the presented research project are detailed in Chapter 8.

Real time observations of the events in the vicinity of the crack tip during the application of block overloading were enabled with the aid of the travelling (15x) microscope and, the utilised stroboscopic illumination equipment. At the first stages of the block loading and in particular during the first 10-20 cycles, it was observed

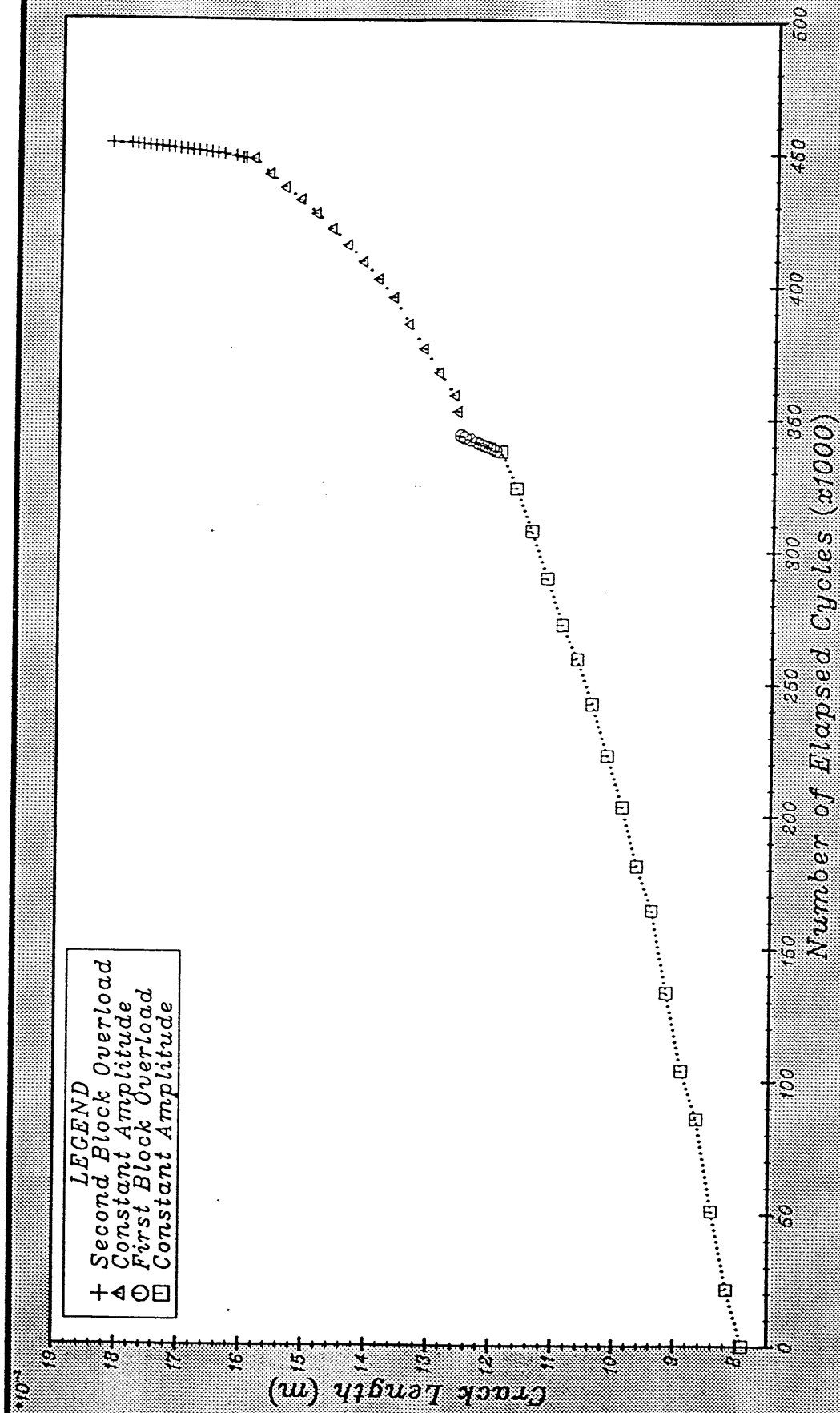


Fig.7.23 Typical experimental data from a block overload test. Two overload blocks of 6000 cycles, $R_{ovl}=1.4$

that the material in the vicinity of the crack tip suffered plastic deformation, the extent of deformation ahead of the crack was proportional to the load. This was depicted as a small dimple in the vicinity of the crack tip, accompanied by a localised change in the brightness of the image. During a period of 250-400 cycles the cracks generally appeared to remain dormant. The number of cycles over which the crack did remain dormant was dependent upon the value of the overload employed. The higher the value of R_{ovl} the longer the stationary period recorded.

Subsequently, the crack appeared to propagate under the influence of the block loading. The observed propagation occurred during the loading half cycle, always surrounded by a small plastic deformation. Another observation is the fact that the crack under the influence of the block loading and the presence of plasticity did show some branching while propagating, an observation also mentioned by Venkateswara et al [7.8]. This branching has been explained as a deflection of the crack path caused by the presence of shear bands on the specimen surface due to substantial localised plastic deformation. The crack is deflected along the shear bands thus localised branching is observed. This explanation is in line with the model of Wood, see Fig.3.7.

As shown in Table 7.4, three overload ratios were utilised for the study. Not all overload ratios caused the same extension of the crack during the application of a block. In the case of $R_{ovl}=1.4$, the recorded extensions of the crack were small and in the duration of the experiments these were only sensed by the clip-on-gauge. In the case of $R_{ovl}=1.6$ the crack extensions were also observed and measured through the travelling microscope as the crack opening was much larger and this facilitated the better definition of the crack tip.

The crack extensions recorded at the end of each block overload application as a function of the block length are depicted in Fig.7.24. The values of extension presented are the average values from five duplicate tests. It is clearly demonstrated that the crack extensions due to $R_{ovl}=1.6$ are very much larger than the corresponding extensions due to $R_{ovl}=1.4$. Examination of the above mentioned crack extensions at the end of overload blocks shows that the increased number of cycles in each block results to an increased extension.

It is therefore apparent that the application of the block overloads results to finite damage i.e. extension of the crack. As the crack extension during the application of the block overload is incremental, it has been possible to record the values of (a,N) and subsequently determine the crack growth curves. The interesting result of this analysis is the fact that the crack growth curves can be clearly considered as three separate curves. Each curve represents a section of the load history. Typical data obtained during the experimental programme are presented in Appendix 5.

It is evident that the crack propagation is disturbed by the introduction of block overloads. The curves are clearly discontinuous due to the presence of the growth attributed to the block load. As the block load stress intensity factor range (ΔK) is higher than the pre-overload constant load amplitude, the corresponding block growth data is shifted to the right. This results to the observed discontinuity in the growth.

When utilising block overloads of $R_{ovl}=1.6$, it was observed that substantial delays in the crack growth were induced upon resumption of constant load amplitude loading. It is interesting to note that the growth delays did not increase proportionally to the length of the block. In

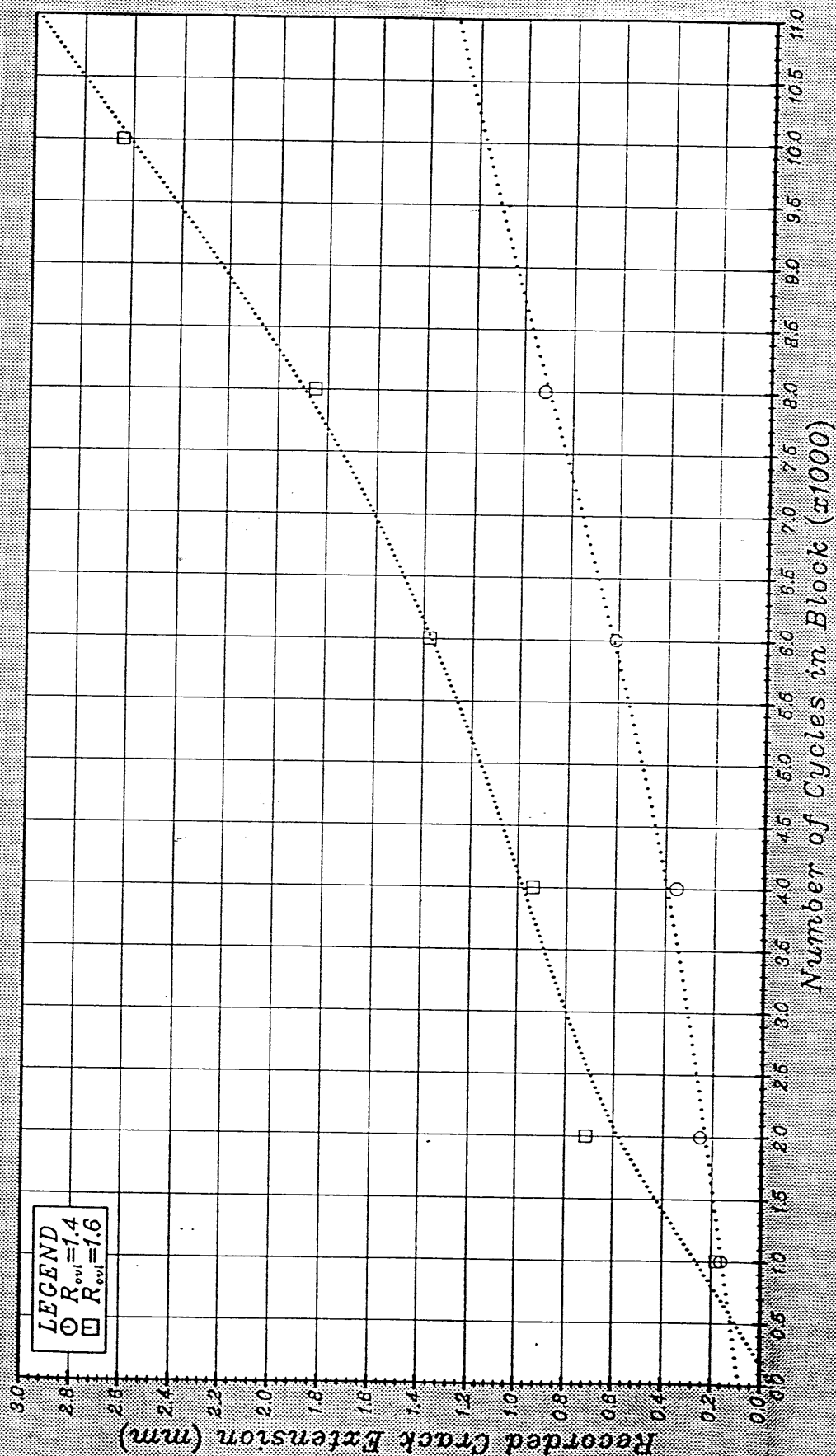


Fig.7.24 Crack extension as a function of the block overload length, for two overload ratios $R_{ovl}=1.4$ and $R_{ovl}=1.6$

fact the delay periods increase initially as the number of overloads increase and a maximum delay is attained around 2,000 cycles of block length. Subsequently, the delay periods are systematically reduced as the number of overloads increases. These observations regarding the effect of the number of cycles in a block and the corresponding post block-overload delay are illustrated in Fig.7.25. In the same manner the results of post-overload delays for the $R_{ovl}=1.4$ show a peak delay when the block length is 1,000 cycles. The delays due to an overload ratio of 1.4 are much lower than the values recorded for $R_{ovl}=1.6$. This is though expected, as the studies of single peak overloads did show that the overload ratio of 1.4 did cause a reduced effect in the growth.

It can be clearly concluded from the above observations that block overloading does not always produce beneficial effects. The presented results suggest that an optimum number of overload cycles exists that produces the maximum post-overload crack growth delay. The delays produced by multiple overloads are clearly higher than the delays produced by a single peak overload of the same magnitude.

As in the case of a single overload the ratio of 1.8 was tested and the results obtained were very similar. By applying blocks of 1,000 and 2,000 cycles crack growth arrest was obtained. Attempts were made to increase the number of cycles in the block but the result was to arrest the growth but, when the block exceeded 5,750 cycles catastrophic failure of the specimens occurred.

Examination of the total fatigue crack propagation life for each of the load cases gives a rather confusing picture of the effect of block loading. The total average fatigue crack growth life for the constant load amplitude loading of $R=0.6$ and $P_{mean}=2.0\text{KN}$ has been found to be 350,000 cycles. In general the total life has been

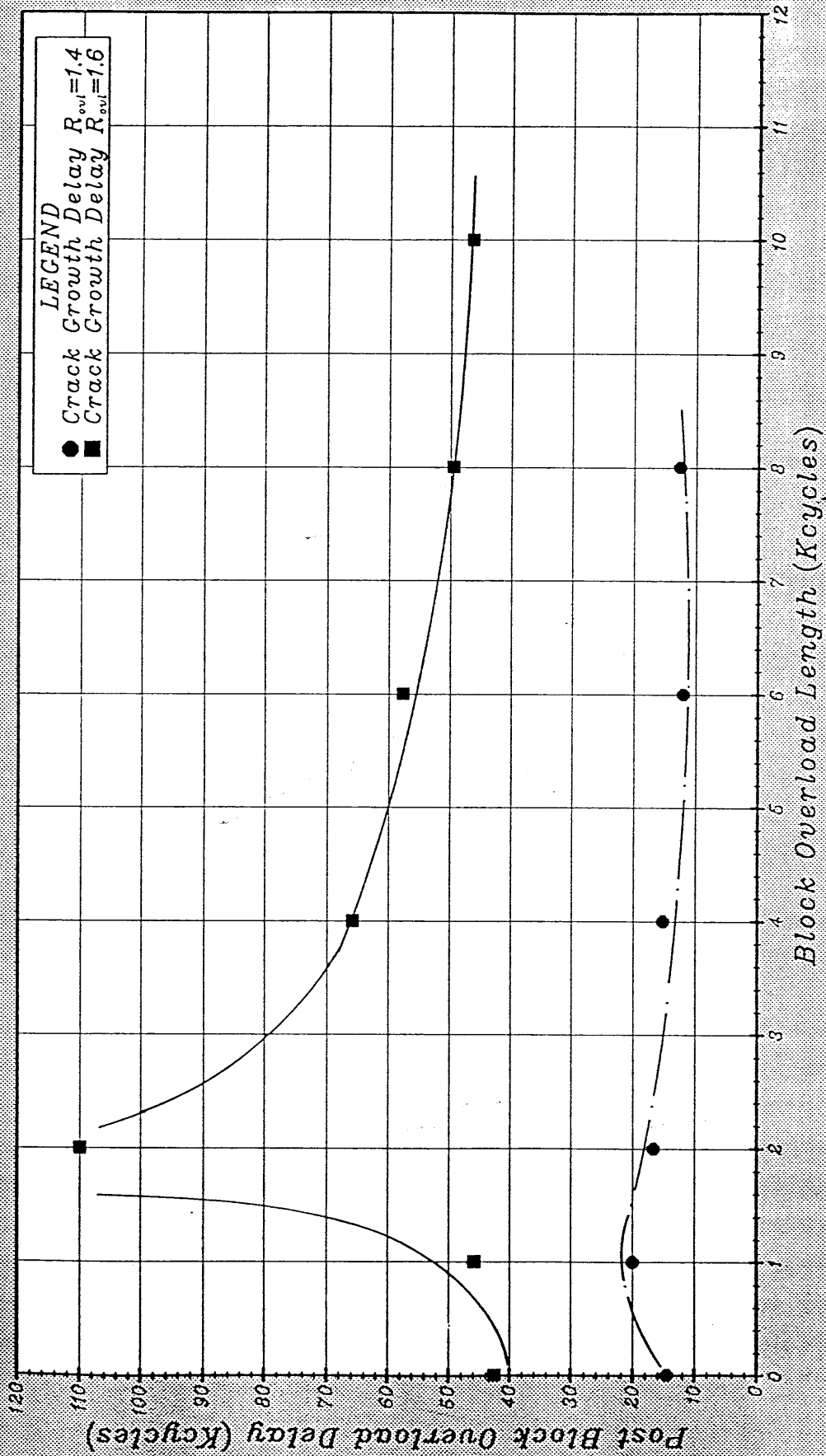


Fig.7.25 Effect of block overload cyclic length upon the post-block overload delay. Results of experiments with $R_{ovl}=1.6$ and $R_{ovl}=1.4$

CHAPTER 7

extended by as much as 30% but in some cases it has been shortened. Table 7.5 shows the total lives recorded for the cases examined, as percentage values of the average constant load amplitude life. Examining Table 7.5 there is evidence to suggest that reduction in the total fatigue life is associated with large extensions of the crack during the application of the block overloading. It has not been possible to establish a clear pattern of the block effects upon the total life to failure.

R_{ovl}	Block Length	Total Life
1.4	1000	120.0%
1.4	2000	128.0%
1.4	4000	133.4%
1.4	6000	117.1%
1.4	8000	99.60%
1.6	1000	111.5%
1.6	2000	113.0%
1.6	4000	106.0%
1.6	6000	120.0%
1.6	8000	98.00%
1.6	10000	96.00%

Table 7.5 Effect of block length and magnitude
upon the total life to failure

In agreement to the findings of the single overload studies the delay caused by overloading is related to the magnitude of the applied overload. It has been established that the higher the magnitude of the overload ratio the higher the delay for the same block length. It has to be noted that the increased magnitude of the overload ratio R_{ovl} , the more substantial the damage caused (crack extension). Thus it is not possible to firmly conclude if block overloading is beneficial.

The observed variation in the post-block overload recorded delays shown in Fig.7.25 may be explained by considering the crack extensions caused by the block.

In order for a crack to propagate the stress intensity must be higher than the threshold stress intensity factor value which is raised if plasticity is present in the vicinity of the crack. As longer cracks experience higher stress intensities they are more able to overcome the effects of crack closure than smaller length cracks under identical loading conditions.

Examining the crack extensions due to block overloading, is apparent that blocks over 3000 cycles long result into rapidly increasing extensions, Fig.7.24. Therefore, these cracks by being longer overcome more quickly the closure effects caused by the prior higher stress levels, so the temporary arrest periods associated with these cracks and block lengths are effectively shorter.

7.5 FRACTOGRAPHIC EXAMINATIONS

Fractography can be defined as the microscopic examination of fractured surfaces, aiming to ascertain the microscopic events taking place during fracture. Such studies can be carried out utilising either traditional reflection microscopes or modern state of the art scanning electron microscope facilities.

The fractured surfaces are three dimensional and this can present real problems when utilising optical equipment, as focus is limited to a single depth of optical field, the remainder of the image appears blurred. Scanning Electron Microscopes on the other hand, do not have such limitations and magnifications of ($\times 10^6$) can be easily achieved. The main

advantages of SEM are the superior magnification capability and, good definition and sharpness of the image.

In the current study a Scanning Electron Microscope was utilised for the examination of a single specimen subjected to a constant load amplitude interrupted by two 6000 cycle block overloads of $R_{ovl}=1.6$.

The low magnification image shown in Fig.7.26, gives a general view of the fracture surface appearance. Two relatively bright bands are visible which indicate the crack growth under the influence of block overloading. Prior to the first overload and between the two overloads some straight bands are apparent across the width of the specimen. These bands at higher magnifications are shown in Fig.7.27-7.31. The fracture appears to take place across crystals (trans-crystalline fracture). These bands represent the locations of the crack tip at various moments during constant amplitude crack propagation. Figure 7.30 shows an enlargement of such a band where a rippled pattern is revealed. These ripples or striations represent incremental extensions of the crack at individual load cycles. The fracture of an individual crystal is shown enlarged in Fig.7.31. The trans-crystalline failure is clearly visible.

Further, the region within the first block overload is shown in Fig.7.32. The fracture surface appears to be rather different than the ones corresponding to constant load amplitude. Any clearly marked bands across the specimen width are apparent. In the zoom detail included, a striation pattern is visible, a characteristic of fatigue fracture. Finally, the fractographs obtained from within the second block overload region are depicted in Fig.7.33-7.34. The fibrous appearance of the surface is clearly visible. This fibrous fracture is a characteristic of the presence of plastic deformation during crack propagation.

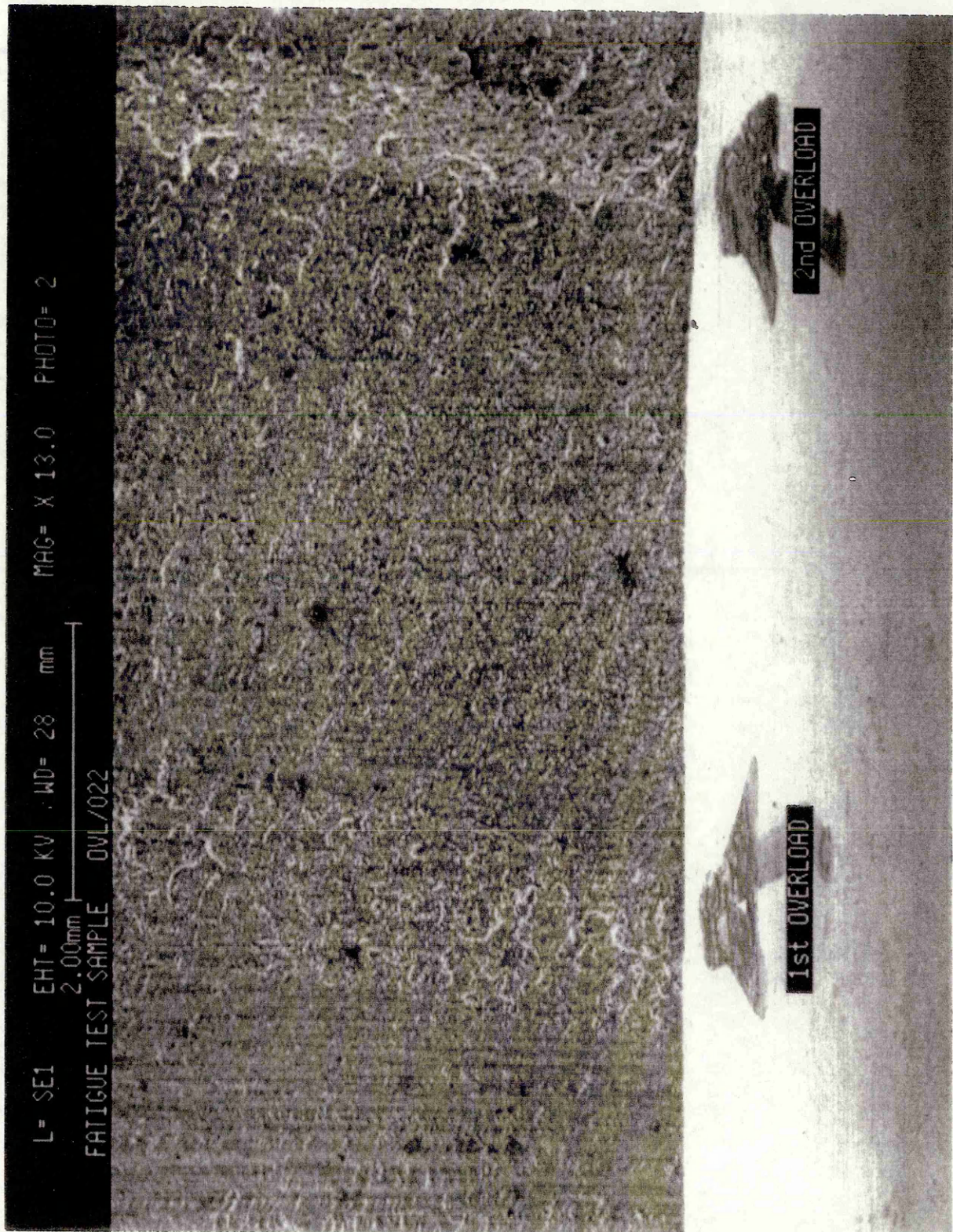


Fig.7.26 General overview of fractured specimen

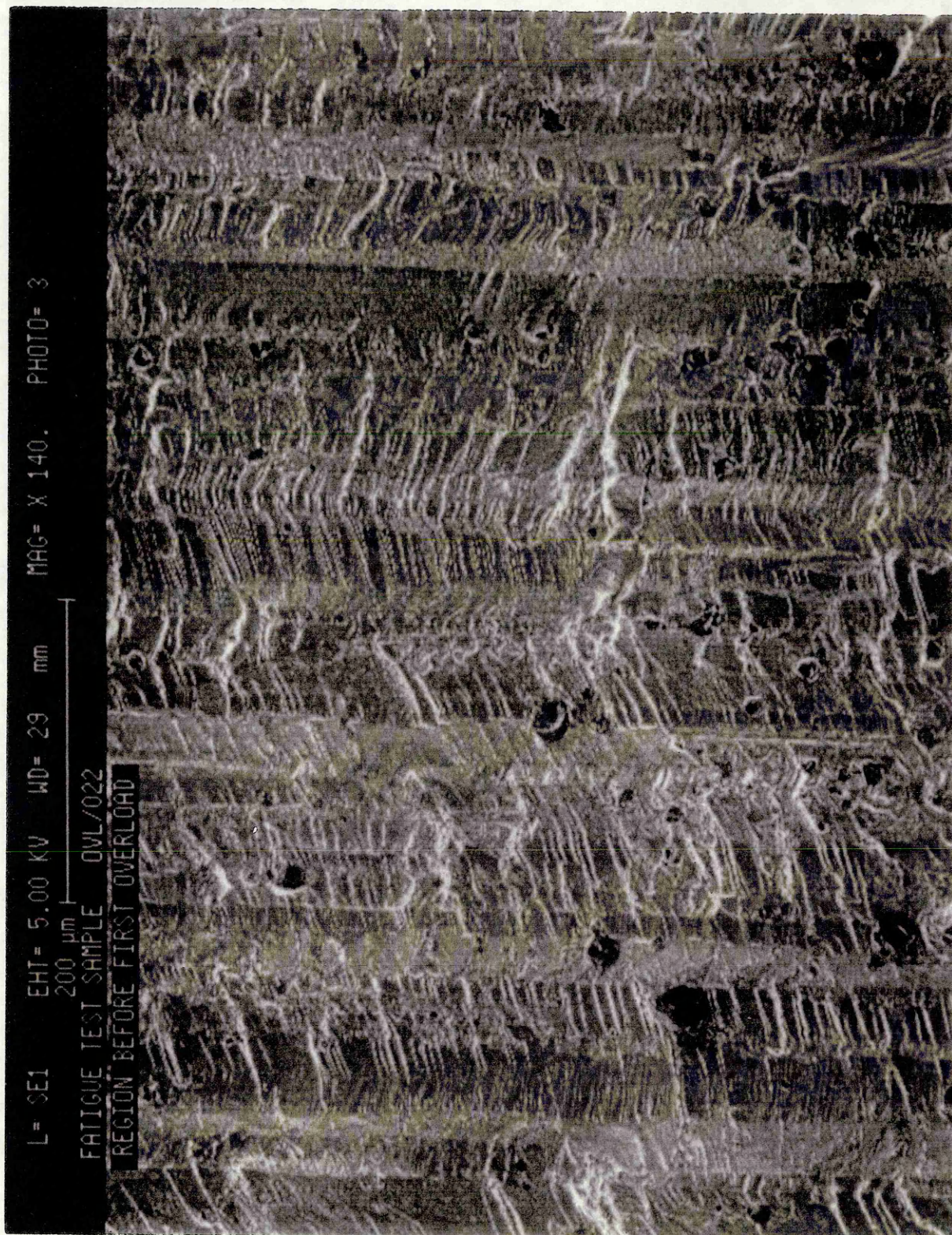


Fig.7.27 Region before first block overload
(x252) magnification

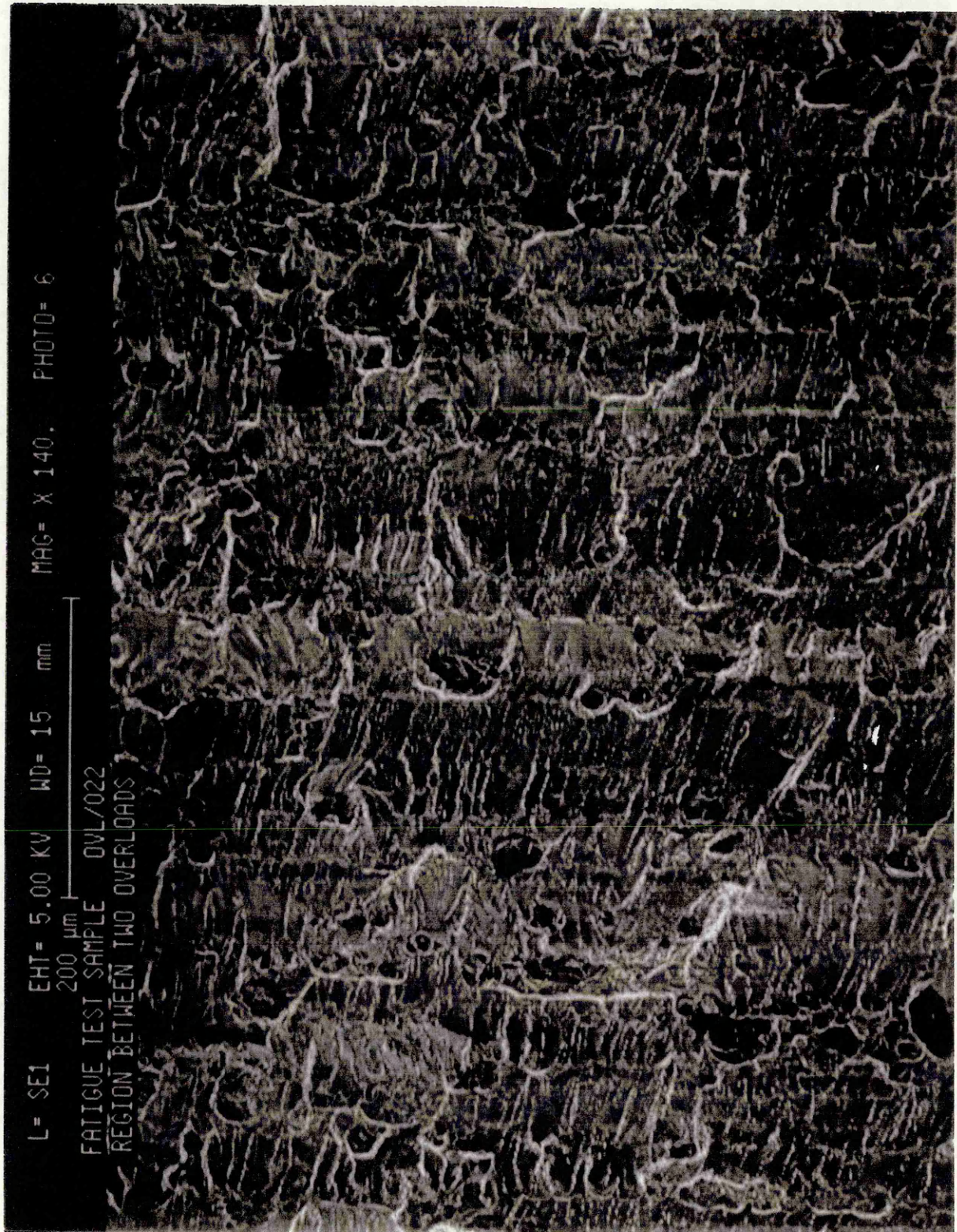


Fig.7.28 Region between the two block overloads
(x252) magnification

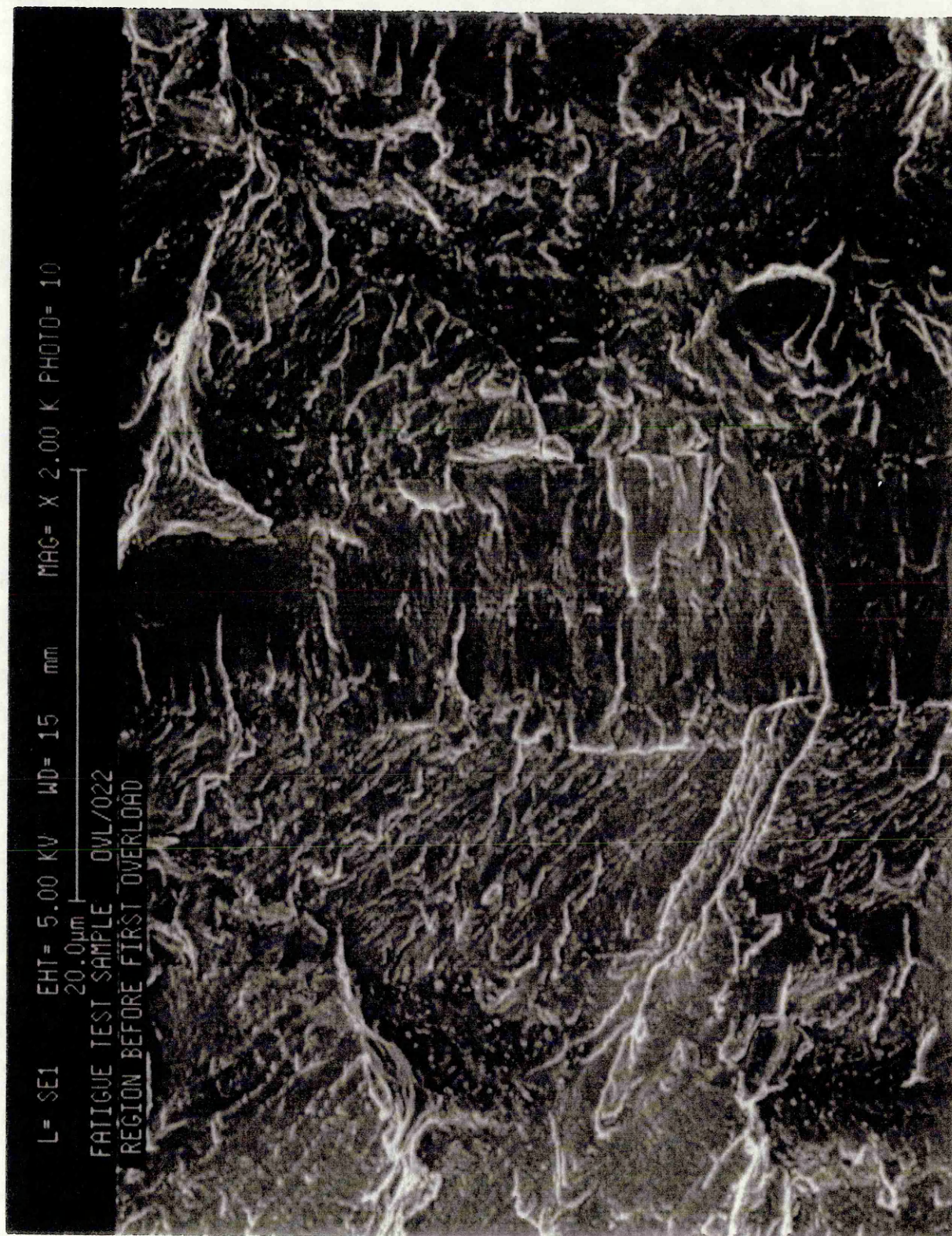


Fig.7.29 Region before first block overload
(x3600) magnification

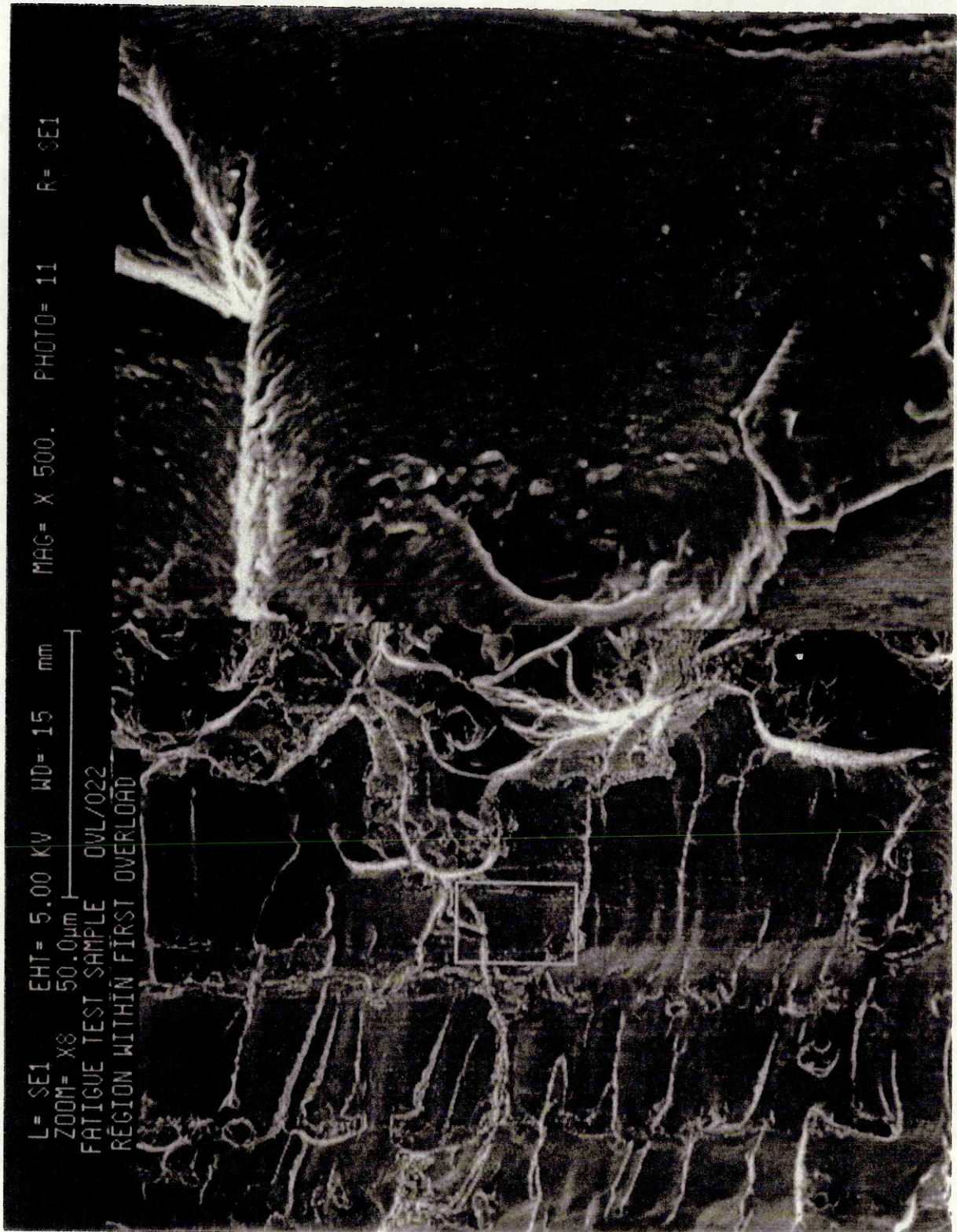


Fig.7.30 Region within first block overload showing the incremental extension striations (x900) magnification

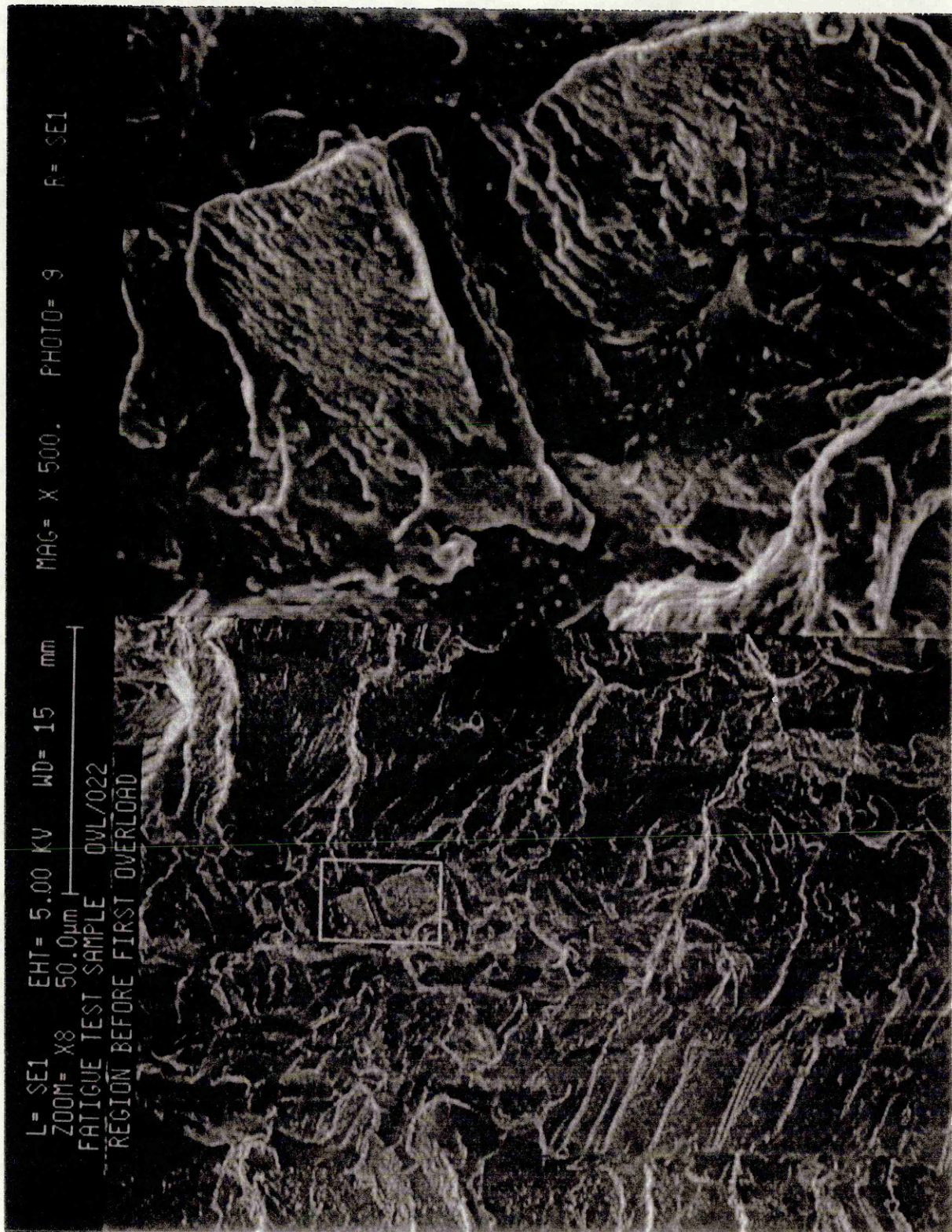


Fig.7.31 Trans-crystalline failure
(x7200) magnification

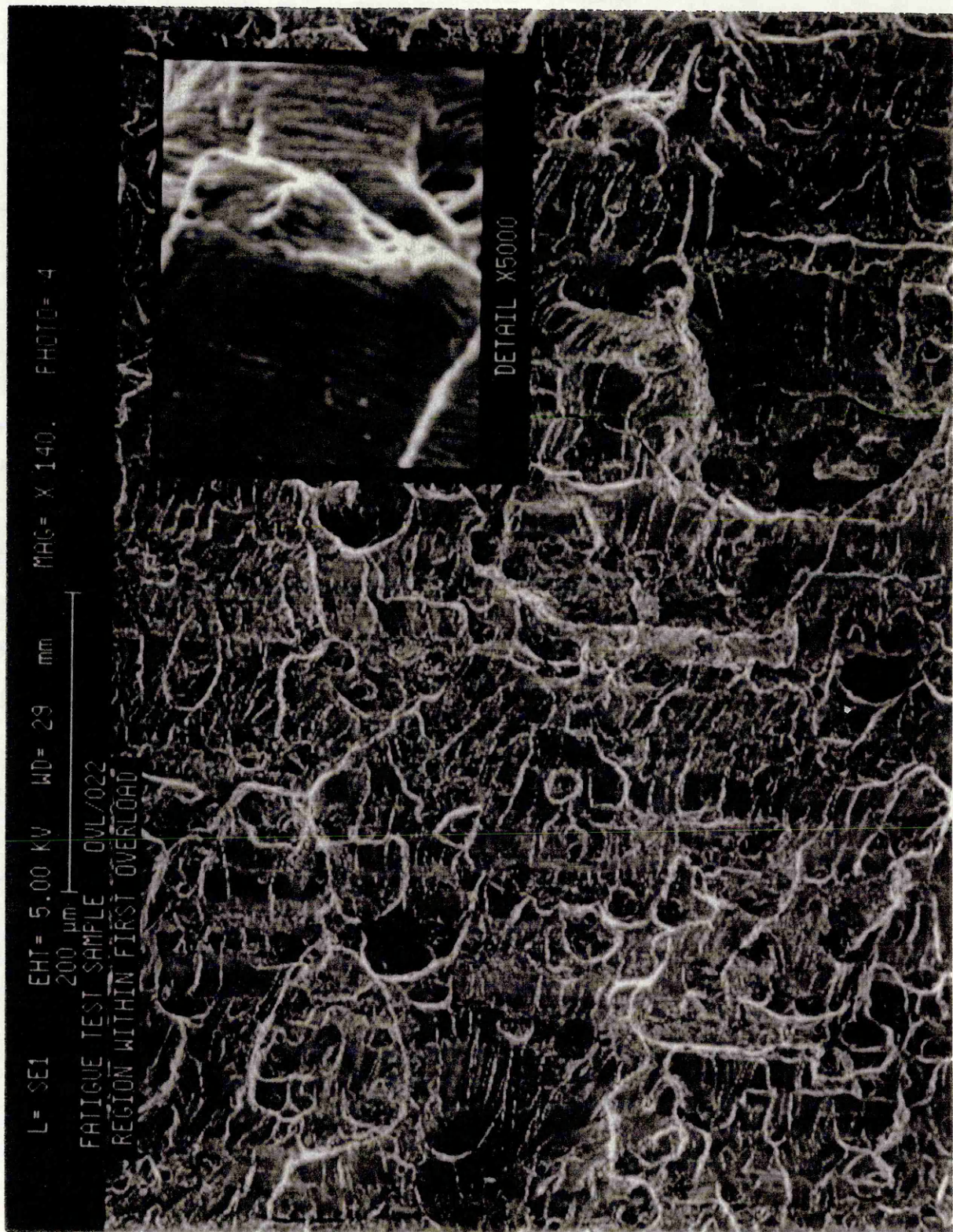


Fig.7.32 Region within the first block overload
(x252) magnification

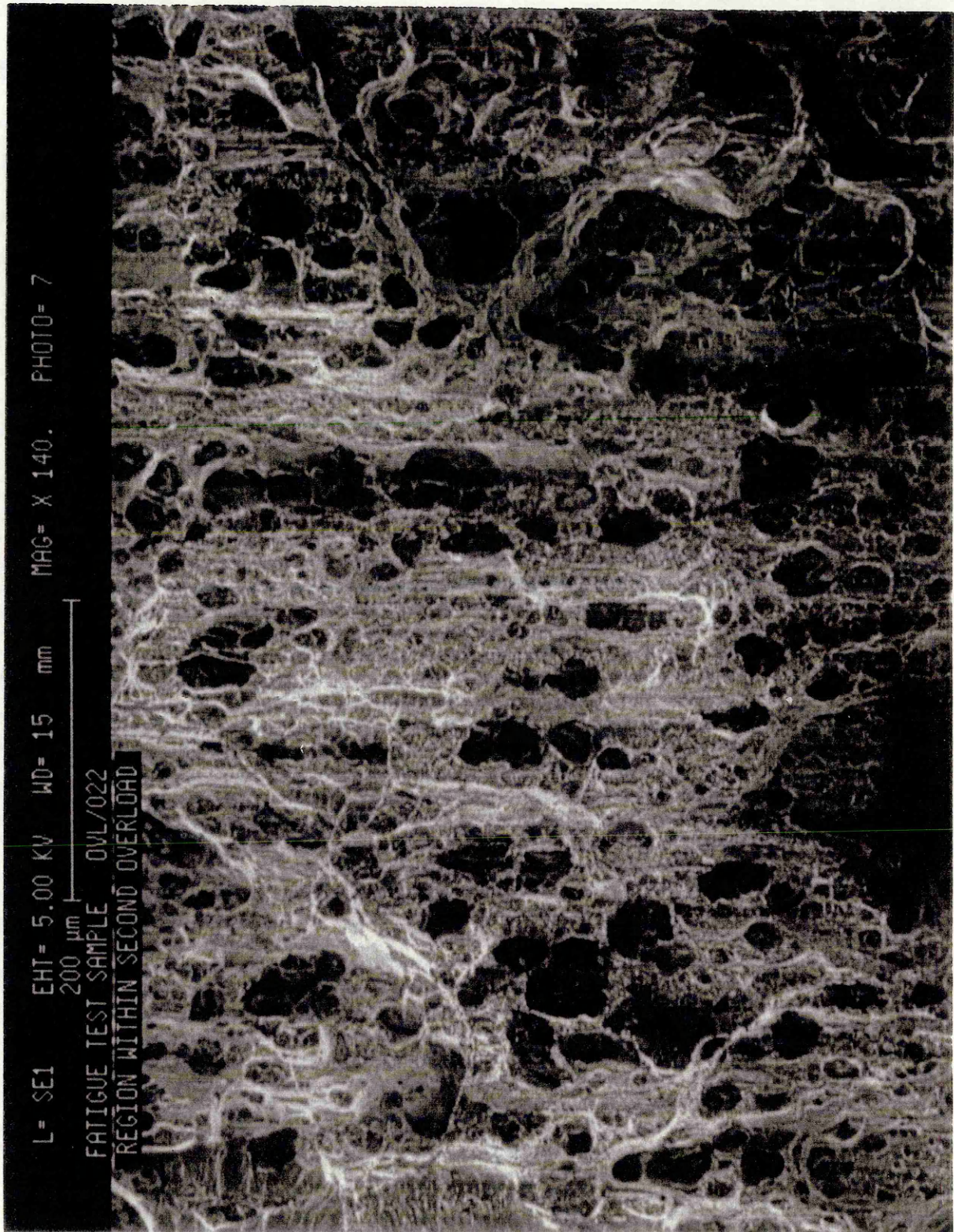


Fig.7.33 Region within the second block overload
(x252) magnification

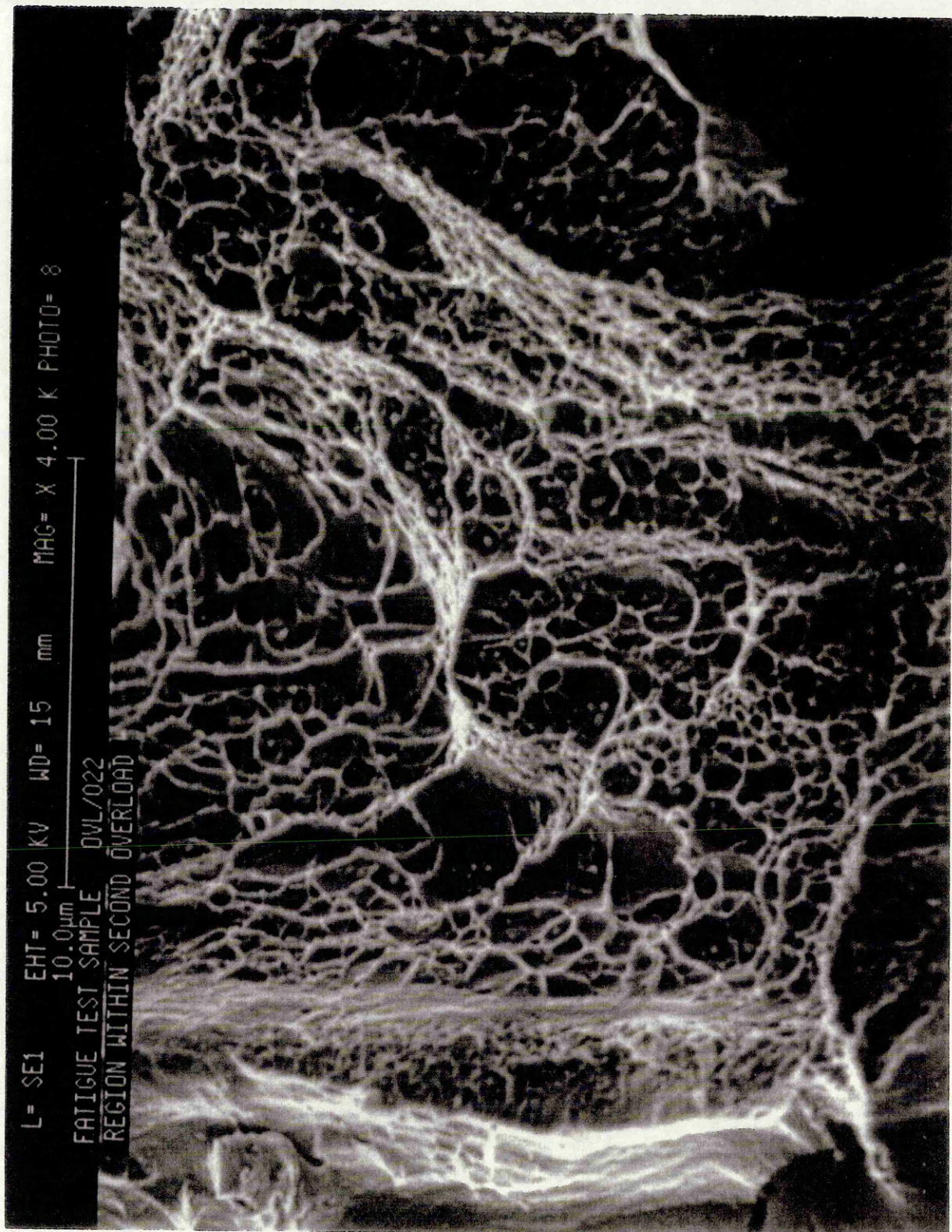


Fig.7.34 Fibrous appearance of fracture surface
within the second overload region
(x7200) magnification

7.6 CONCLUSIONS

7.6.1 Single Overload Studies

The investigations in the effects of a single peak overload upon the constant load amplitude fatigue crack propagation behaviour, have established that,

- a. The introduction of a single overload causes plastic deformations in the vicinity of the crack tip which, are visible under small magnifications.
- b. The post-overload crack growth experiences a stand still period which is proportional to the magnitude of the overload ratio R_{ovl} .
- c. The post-overload crack growth rates are significantly reduced, the greater the overload ratio R_{ovl} the larger the growth rate reduction.
- d. Following the application of the single overload and after a number of cycles under constant amplitude have elapsed the crack growth attains a minimum crack growth rate, *delayed retardation*.
- e. Following the *delayed retardation* the crack tends to accelerate.
- f. Increasing the mean load and maintaining the same overload ratio the stand still periods are extended.
- g. Crack arrest can be achieved if the value of the overload ratio is higher than $R_{ovl}=2.0$.
- h. Reducing the post-overload R load ratio the arrested crack resumes propagation.

- i. Overloads applied at different crack lengths cause different crack retardation effects.
- j. The presence of single peak overloads extends significantly the total fatigue life to failure.

7.6.2 Block Overload Studies

Studies of the effects of introducing block overloading in the otherwise constant load amplitude load history, have illustrated that,

- a. The crack advances under the influence of the block load thus, causing a finite. damage.
- b. The crack extension is proportional to the cyclic length of the block.
- c. The higher the magnitude of the block overload ratio R_{ovl} , the larger the crack extensions at the end of block loading.
- d. Visible plastic deformation always accompany the advancing crack and some branching is apparent.
- e. Following the application of a block overload the crack remains dormant for a period of time.
- f. The stand still periods following a block overload are higher than the periods observed after the application of an equivalent single peak overload.
- g. It has been found that the temporary crack arrest period following the application of a block overload is not proportional to the block cyclic length but, an optimum number of cycles exists that causes the longest stand still period. This optimum number of cycles is 2,000 for $R_{ovl}=1.6$.

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CHAPTER 8

EXPERIMENTAL DATA ANALYSIS

TECHNIQUES

8.0 Introduction

8.1 The Secant Method

8.2 Incremental Polynomial Methods

8.3 Total Fitted Polynomials

8.4 The Piece-Wise Continuous Polynomial Technique

8.4.1 N.P.L. Analysis Technique, Theory

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CHAPTER EIGHT

EXPERIMENTAL DATA ANALYSIS

TECHNIQUES

8.0 INTRODUCTION

During fatigue crack propagation experiments the data obtained is that related to the crack length, the number of elapsed cycles and information regarding the load history. In order to determine the crack propagation curves, it is essential to combine the above information in an appropriate manner so as to obtain crack growth rates (da/dN) and, the stress intensity ranges ΔK . As far as the calculation of ΔK is concerned, the use of the standard CT specimen geometry provides with a polynomial expression in the form of the stress intensity factor calibration equation.

The main difficulty lies in the determination of crack growth rates. In the literature, a variety of techniques have been proposed by a number of researchers for the evaluation of the (da/dN) information, see references [8.1-8.3]. The common idea in all techniques, is a procedure of curve fitting through the a versus N data so as to allow the numerical evaluation of the crack growth rates, by formal differentiation of the fitted functions. The American ASTM E-647/86 specification [8.4], which has been followed throughout the experimental programme, recommends the use of the secant method and/or incremental polynomial techniques. In the course of this research the recommended techniques and other methods of analysis were attempted. The methods used and evaluated are detailed in this chapter.

8.1 THE SECANT METHOD

The idea behind the secant method for the evaluation of crack growth rates (da/dN) , is based upon the fact that, for a continuous function the slope at a particular location can be approximated as the gradient of the straight line connecting two adjacent data points of the curve. In the case of discrete data points obtained by means of an experimental procedure, this technique proves to be quite suitable for the evaluation of the required gradients.

In fact, the recommendation of the E-647/86 standard is that the slope is calculated by means of utilising two adjacent discrete data points on the a versus N curve. This corresponds to the average gradient in the interval between the two points. Consequently, the slope is assumed to correspond to the average crack length calculated by the two interval defining data points considered. In mathematical terms, the technique can be formally expressed as follows:

$$(da/dN)_{\bar{a}} = (a_{i+1} - a_i) / (N_{i+1} - N_i) \quad \dots (8.1)$$

Where, (a_i, N_i) and (a_{i+1}, N_{i+1}) are the coordinates of two adjacent data points on the a versus N experimental curve

Since the computed (da/dN) is an average rate of crack growth over the interval defined by $(a_j - a_i)$, the average crack length, $\bar{a} = \frac{1}{2}(a_{i+1} + a_i)$, is normally used for the evaluation of ΔK .

It is clearly shown in Fig.8.1, that the accuracy of the slope is very much dependent upon the distance of the two points on the considered interval. The inaccuracies stem from a variety of factors such as the distance of the two points and the localised curvature of the curve.

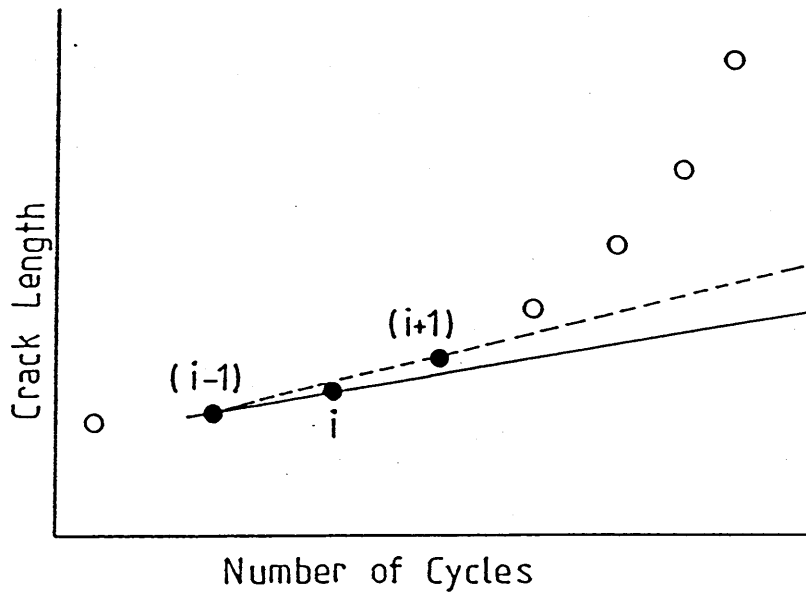


Fig.8.1 The Secant Method
for Crack Growth Rate Evaluation

8.2 INCREMENTAL POLYNOMIAL METHODS

This approach of computing (da/dN) crack growth rates, involves the fitting of a second order polynomial (parabola), to sets of $(2n+1)$ successive data points, where n is usually 1, 2, 3 or 4. The selection of the second order polynomial as a suitable function for the incremental curve fitting is due to the fact that, the experimentally determined $(a$ versus $N)$ curves have a parabolic shape, as it has been discussed in Section 6.2.2.

The E-647/86 specification, recommends the use of two types of procedures based upon the 5-point and 7-point incremental polynomials, i.e. successive intervals on the

(a versus N) curve consisting of 5 or 7 points respectively. To illustrate these methods the 5-point technique is taken as an example. Five data points are selected through which a second order polynomial is fitted. This polynomial is then formally differentiated, and the gradient corresponding to the mid-point of the interval, (3rd point), is evaluated. This procedure is repeated by incrementing the imaginary five point template one position along the data curve, such that, an updated interval is formed by four data points from the previous interval plus one new point, as shown in Fig.8.2.

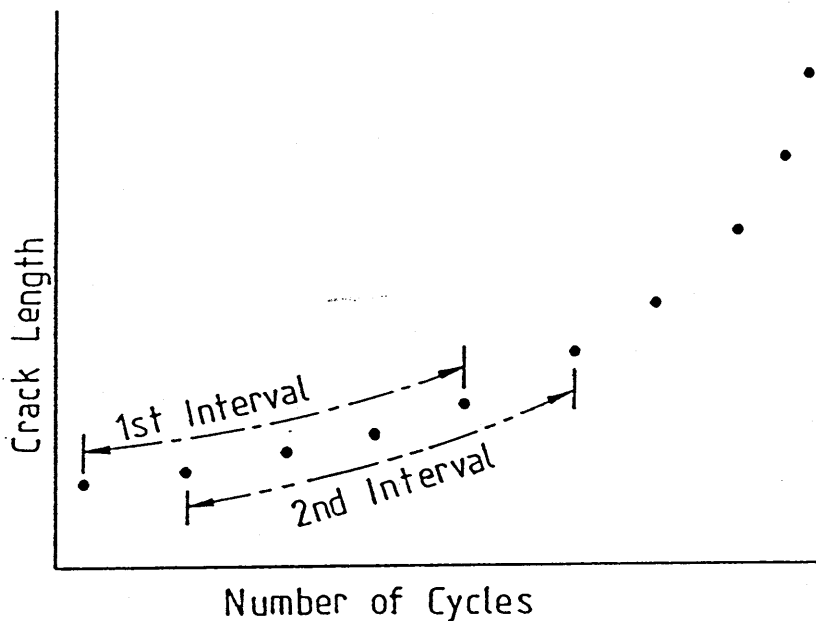


Fig.8.2 Incremental Polynomial Techniques

This incremental curve fitting process is repeated until all the data points have been considered. The seven point polynomial technique is exactly the same, but seven points are considered, and consequently the crack growth rate is evaluated at the 4th point of each interval.

Figure 8.3 illustrates the results of the application of the 5 and 7 incremental polynomial methods in set of constant load amplitude results. It is clear that the data predicted by these methods describes the original data very well. However, a disadvantage of this approach is the fact that either four or six data points, depending upon whether the five or seven point method is used, fail to provide any crack growth information. In the presence of data scatter, which is frequently encountered in fatigue crack growth testing, the fitting of the parabola is inevitably affected by the scatter. Thus the presence of one point that does not follow the regular pattern of the curve, is liable to affect the calculations in five or seven successive intervals, depending upon the interval size employed. It is thus clear that, the 5-point incremental polynomial technique can lead to more reliable experimental data reduction as compared with the seven point method.

8.3 TOTAL FITTED POLYNOMIALS

This type of analysis considers the total number of data comprising the data set. The idea is, to fit through the experimental data a single polynomial function. It is reasonable to assume that a single polynomial may adequately fit the experimental data as the (a versus N) curves show that they follow a near second order polynomial (parabolic) shape, starting with a small gradient which gradually becomes steeper.

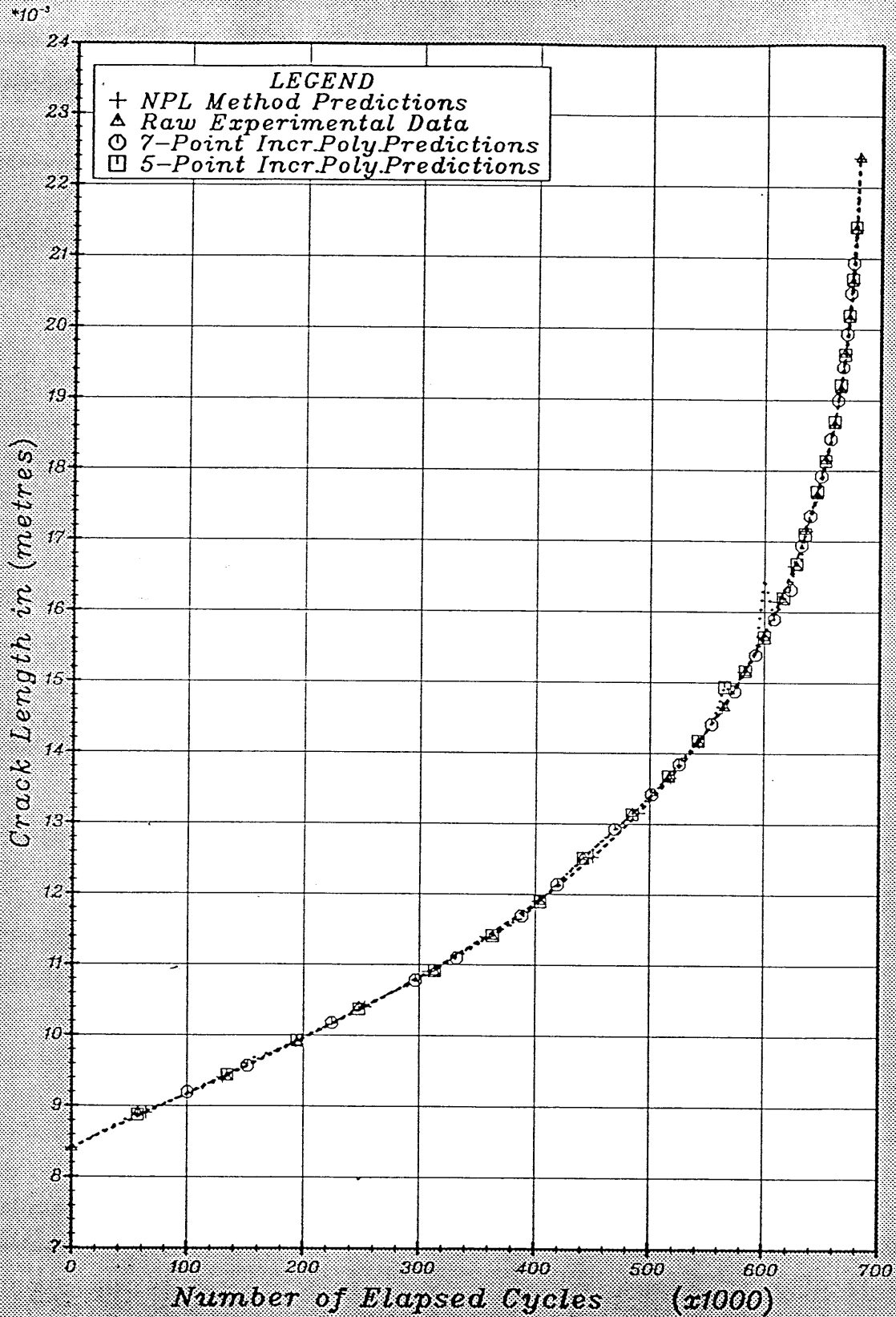


Fig.8.3 Experimental data and predictions obtained by the piece-wise and incremental polynomial methods.

This approach was attempted in the present study, but the results were rather poor. In fact a series of polynomials with orders 2, 3, 4, and 5 were utilised in an attempt to improve the fit of a single polynomial by means of increasing the number of correlation parameters. The curve fitting was based upon a least squares procedure.

If the goodness of fit is to be judged by the coefficient of determination alone, the fit is greatly improved the higher the degree of polynomial used. Table 8.1 shows the correlation coefficients obtained by series of polynomials for a particular set of constant load amplitude experimental data.

Polynomial Order	Coefficient of Determination
2nd	93.64%
3rd	97.08%
4th	98.40%
5th	99.08%

Table 8.1 Improvement of Fit Achieved by the use
of Higher Order Polynomials

These improvements of the fit are artificial and only better in statistical terms. In reality, the curves fitted do not represent the original data. In fact, the fitted polynomial functions are poor in their representation of events. This is made clear by the plot of predicted data in relation to the original experimental data, see Fig.8.3. The total polynomial curves calculated by the least squares procedures show, a type of oscillation about the original data, which is dependent upon the degree of the polynomial. This is rather an incorrect representation of events as the oscillation implies that, the crack grows with a variable rate; sometimes it accelerates and other times it slows down, which is unrealistic.

In general terms, the idea of fitting a single polynomial through the experimental data can lead to incorrect evaluations of the crack growth rates due to the oscillation of the fitted polynomials. The agreement of these methods with experimental data is only reasonable at the latter stages of experiments where they follow the experimental data fairly well, see Fig.8.4.

8.4 THE PIECE-WISE CONTINUOUS POLYNOMIAL METHOD

This method of experimental data reduction with the purpose of determining the crack growth rates (da/dN), is also based upon the principle of curve fitting as the incremental polynomial techniques previously described. The main difference is the use of spline functions rather than polynomials. These functions allow the discretisation of the data set by a number of small intervals that are connected with common data points or knots. The main property of the spline function is that the continuity of slope is preserved at the knots, thus resulting to the fitting of a smooth curve through the original data points.

This approach has been pioneered by the scientists of the National Physical Laboratory, (NPL), in the U.K., [8.5]. The technique has been based upon a damage accumulation law formulated by McCartney and Gale [8.6, 8.7]. The basic equation of the damage accumulation law is augmented with a spline function of degree, 1, which enhances the smoothness of the derivative and improves the fit by introducing additional correlation parameters.

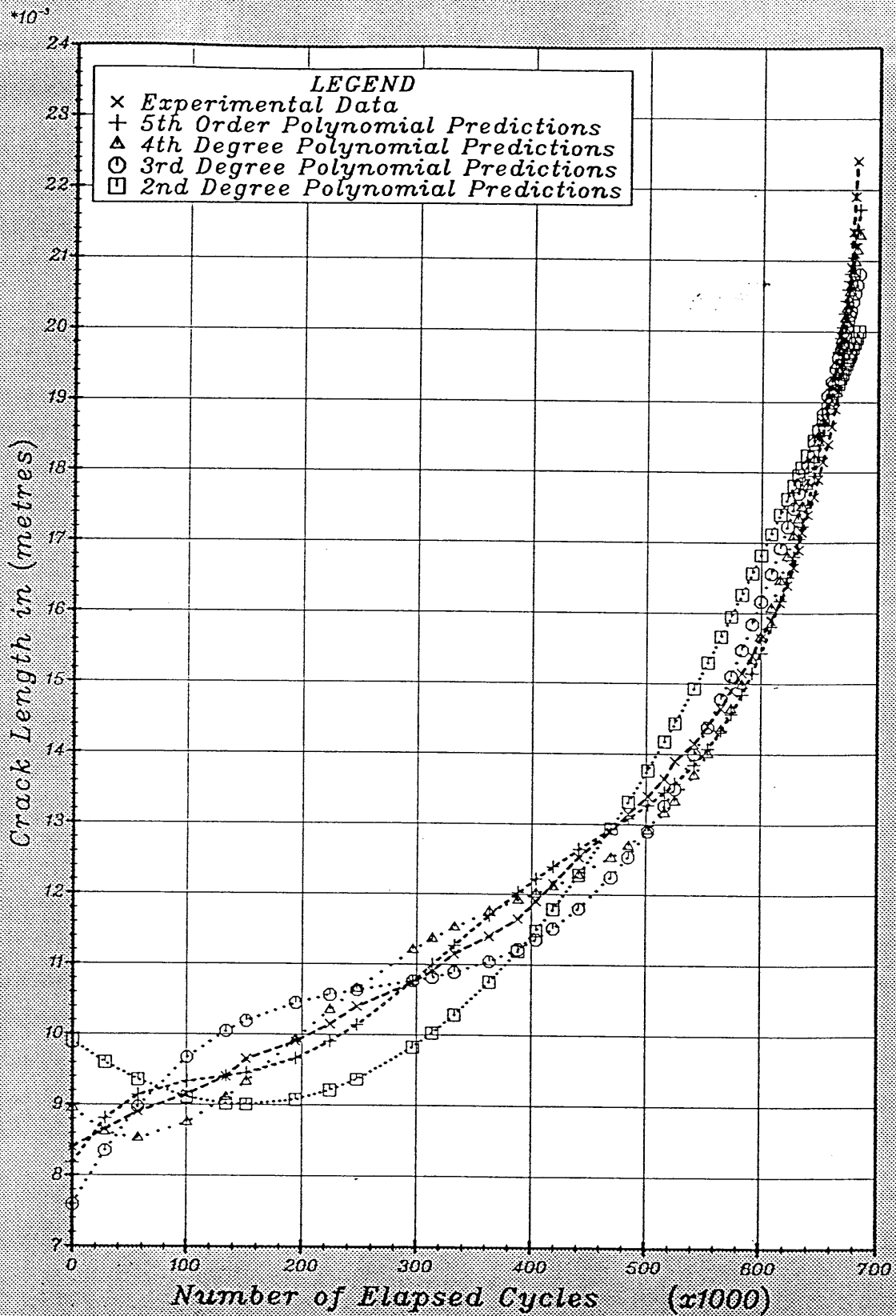


Fig.8.4 Experimental fatigue crack growth data and predictions resulting by the curve fitting of simple polynomials.

8.4.1 N.P.L. Analysis Technique, Theory

From fatigue crack propagation experiments, data is obtained of the form (a_j, N_j) where $j=1,2,3,\dots,m$ and m is the total number of data points. These can be conveniently transformed to (x_j, Y_j) data by mean of

$$x_j = (a_j/a_1) \quad Y_j = (N_j - N_1)/1000 \quad \dots(8.2)$$

The damage accumulation law derived by McCartney et al [8.8, 8.9], is described by the following mathematical expression,

$$Y(x) = A_1 + (A_2/x) + A_3 \ln(x) \quad \dots(8.3)$$

Where A_1, A_2, A_3 , are unknown correlation parameters. In order to generalize the above expression and improve the curve fitting a spline function is introduced to equation (8.3). A special spline function is selected for this purpose which, is of the form

$$G(x) = \sum_{p=i}^{n+1} \lambda_p \cdot (k_p - x)^l \quad \dots(8.4)$$

Where $k_{i-1} < x \leq k_i$, $i=1,2,\dots,n+1$ and is defined within the interval $k_0 < x \leq k_{n+1}$. This is a piece-wise analytic polynomial function of degree l , which contains n points of discontinuity (knots), $x=k_p$, $p=1,2,\dots,n$. As a consequence, the $G(x)$ function can be differentiated $l-1$ times within the $k_0 < x \leq k_{n+1}$ interval. Clearly, the larger the value of the exponent l , the smoother the resulting function.

CHAPTER 8

Augmenting expressions (8.3) and (8.4), gives the following equation,

$$y(x) = A_1 + (A_2/x) + A_3 \ln(x) + G(x)$$

or,

$$y(x) = A_1 + (A_2/x) + A_3 \ln(x) + \sum_{p=1}^{n+1} \lambda_p (k_p - x)^l \quad \dots (8.5)$$

The end points of the spline function are chosen such that

$$k_0 = x_1 = 1 \quad k_{n+1} = x_m \quad \dots (8.6)$$

The knots define the limits of each interval, and are so selected as to contain an equal number of data points

$$k_{i-1} < x \leq k_i \quad i=1, 2, \dots, n+1$$

It is required to partition the data transformed by equation (8.2), into $n+1$ intervals defined by n knots, so as to create appropriate arrays defined as

$$(x_{ij}, y_{ij}) \quad \begin{matrix} i=1, 2, \dots, n+1 \\ j=1, 2, \dots, s_i \end{matrix} \quad \dots (8.7)$$

where, s_i is the number of data points per defined interval and, i is the interval number.

From the fatigue crack growth theory of McCartney and Gale [8.8], it has been found that the crack growth at the latter stages can be adequately described by the derived law, equation (8.3). Thus, the function $G(x)$ in the last interval, $(n+1)$, is set to zero by choosing $\lambda_{n+1} = 0$. Therefore the curve to be fitted is defined as follows

$$\dots y(x) = A_1 + (A_2/x) + A_3 \ln(x) + \sum_{p=1}^n \lambda_p (k_p - x)^l$$

$$\text{for } k_{i-1} < x \leq k_i \quad i=1 \dots n \quad \dots (8.8.a)$$

and,
$$y(x) = A_1 + (A_2/x) + A_3 \cdot \ln(x) \quad \dots (8.8.b)$$
 in last interval $k_n < x \leq k_{n+1}$

The sum of squares of the residuals of equation (8.8) is defined as

$$F(\{A_u\}, \{\lambda_q\}) = \sum_{i=1}^{n+1} \sum_{j=1}^{s_i} r_{ij}^2$$

where, ... (8.9)

$$r_{ij} = A_1 + (A_2/x_{ij}) + A_3 \cdot \ln(x_{ij}) + \sum_{p=i}^{n+1} \lambda_p \cdot (k_p - x_{ij})^l - y_{ij}$$

representing the distance of the point from the curve measured in a direction parallel to the y-axis. The correlation parameters A_u , $u=1,2,3$ and λ_q , $q=1,2,\dots,n$ have to be evaluated thus, the function $F(\{A_u\}, \{\lambda_q\})$, must be minimised so that the best fit of (8.8) is determined, by means of a least squares procedure. The minimisation process is defined by,

$$\begin{aligned} \partial F / \partial A_1 &= 0 \\ \partial F / \partial A_2 &= 0 \\ \partial F / \partial A_3 &= 0 \\ \partial F / \partial \lambda_q &= 0 \end{aligned} \quad \dots (8.10)$$

which can be rewritten in the form

$$\sum_{r=1}^{n+3} K_{sr} A_r = f_s \quad s=1,2,3,\dots,n+3 \quad \dots (8.11)$$

By considering the transformed data and equations (8.10), it is possible to form a set of linear equations, (8.11), that contain only the unknown parameters of A_u and λ_q and, form the matrix K_{sr} with dimensions $[(n+3) \times (n+3)]$. The vector A_r , containing the correlation parameters can be evaluated by solving the system of simultaneous linear equations described by (8.11), by matrix inversion or Gaussian elimination procedures.

The method uses a series of correlation parameters in order to achieve an improvement of fit and continuity of the slopes. As it has been discussed by McCartney and Cooper, [8.6], the method is very much dependent upon the number of data intervals and the degree l , of the spline used.

A FORTRAN computer program was prepared for the analysis of data based on the above theory. Special subroutines were written in order to select the optimum values of the variables for the curve fitting. The user provides the program with an initial minimum number of intervals and a minimum degree of the spline, l . The program utilises those values for an initial solution. Then it varies the number of intervals and for each selection uses a series of spline orders up to a maximum of 6th order. For each pair parameters it calculates the standard error of estimates and the coefficients of determination for each interval. Subsequently, the program arrives to a recommendation of two solutions that are selected automatically, one based upon the minimum average standard error of estimates and another based upon the highest average coefficient of determination. The user can use these proposed solutions to plot the data and exercise his judgement upon the best one.

8.5 DISCUSSION AND CONCLUSIONS

An important part of the study of the fatigue crack propagation is the analysis of the experimental data. For this purpose the researchers have attempted to produce a unified method for the analysis of the (a, N) data. This is reflected by the recommendations of the ASTM E-647/86 standard. It is important to note that the techniques recommended are based upon extensive scientific research and long term evaluations.

The secant method of evaluation of da/dN from experimental data can suffer from inaccuracies if the data collected is rather poor. The interval between data points is very important as the low density of data points fails to adequately describe the trend of crack growth, especially when severe changes in the curvature are present.

The total fitted polynomial procedure is an inefficient type of analysis. In statistical terms the fit is excellent, but in real terms it is totally wrong. This is due to the oscillations of the fitted polynomials through the original data sets. In fact, the inaccuracies that can appear in the data due to the oscillations are clearly shown in Fig.8.5, where the crack propagation curves are shown. It is obvious that the total fitted polynomials have provided curves that are smooth but the rates of crack growth calculated are inaccurate.

The methods based upon incremental polynomials are better as compared with the total fitted polynomial methods. Their advantage is the fact that curves fitted between limited sub-sets of data can lead to good correlations between the calculated curve and the original data. On the other hand the existence of scatter in the data, i.e. points that do not follow the general data trend, can cause localised inaccuracies, as such points are considered in more than one sub-sets, see Fig.8.5. The resulting crack growth rates are affected by the scatter and the resulting crack growth curves depict this by the lack of smoothness and the kinks that are present

Finally, the piece-wise method proposed by the National Physical Laboratory, is superior to all other methods. The continuity of calculated gradients is very much better than any other technique. It has been found that using five or less intervals and $l=4$ the smoothness of the resulting crack growth curve is very good, and the results

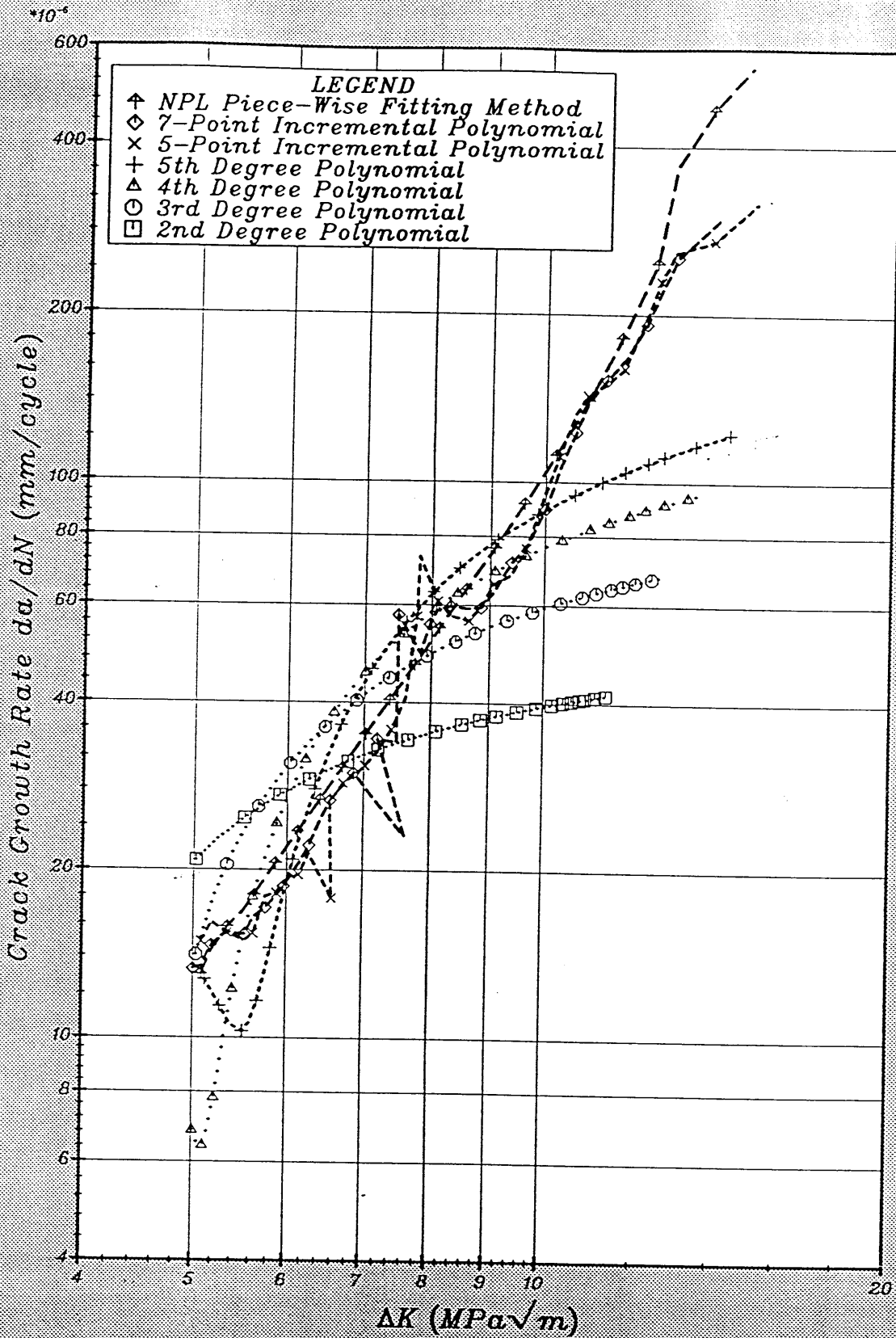


Fig.8.5 Comparison of the crack growth curves, determined by the use of seven different methods of fatigue crack growth data reduction.

CHAPTER 8

are comparable to the incremental polynomial methods, see Fig.8.5.

The computer programs prepared in the course of the present study, each supplemented by a sample run output are shown in Appendix 6. The information presented in Fig.8.3-8.5, is the output data shown in the appended sample listings.

R E F E R E N C E S

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CHAPTER NINE

CONCLUSIONS

The objectives of the presented research have been previously outlined in Chapter 1. The theoretical background, the methods of approaching each objective, the results obtained and their analysis have been presented in Chapters 2-8. All the objectives of the work have been satisfactorily achieved and, a summary of the conclusions drawn is given below.

Elastic finite element analysis results of displacements and stresses have been found to allow the accurate evaluation of stress intensity factors with the aid of approximation techniques. Potential crack initiation sites were also identified by the finite element methods and were experimentally verified..

The theoretical studies of the effects of overloading by means of finite element methods have shown that plastic deformations are developed in the vicinity of the crack tip. The extent of plasticity ahead of the crack is dependent upon the value of the maximum load applied. Upon unloading, compressive residual stresses are developed. These residual stresses are compressive in the proximity of the crack tip but, attain a positive magnitude a short distance from the crack. The effect of the presence of the compressive residual stresses in the vicinity of the crack tip is to reduce the intensity of the stress field. This reduction is evident by means of a reduction of the stress intensity factor after the application of an overload. The extent of the plastic zone as determined by finite element elastic-plastic analysis is lower than the ones predicted by the Irwin and Dugdale models.

The experimental investigations of the constant load amplitude fatigue crack propagation behaviour of the 7075 T7351 Al-Zn alloy, have demonstrated that the material does not exhibit the general reflex-type behaviour. There is evidence to suggest that the frequency of the applied constant amplitude loading affects the growth, low frequencies causing faster failures.

It has experimentally been established that, the introduction of single peak overloads in an otherwise constant amplitude load history, cause substantial reductions in the post-overload growth. The crack growth rate reductions are preceded by a temporary arrest period and delayed retardation. Increased magnitudes of overloading result to larger disturbances in the subsequent growth. The presence of the delay retardation demonstrates that the cause for the reductions in growth is due to crack closure caused by the overload induced plastic deformations. Significant magnitude overloading can result to the total arrest of the fatigue crack.

Introducing block overloading in the constant amplitude load history causes severe changes in the crack propagation but, there is no evidence to firmly establish that block overloads are beneficial. The post-overload stand still periods are not proportional to the cyclic length of the overload. Significant damage is caused by the block loading as the cracks advance substantially under their influence. The finite extensions of the cracks are proportional to the block overload cyclic durations. In some cases the total life to failure is observed to be extended but in other occasions there is no real change and some reductions in life have been recorded.

The phenomena of post-overload temporary arrest, crack growth retardation and delayed retardation observed in the experimental work are all related to the presence of a single overload or block overloading. It has been stated that in all cases the introduction of high loads i.e. higher stress intensities are always accompanied by plastic deformations which were clearly marked on the specimen surface. The theoretical investigations have shown that the application of a single overload causes plastic deformations which upon unloading produce considerable levels of compressive residual stresses in the proximity of the crack tip. The compressive residual stresses cause reduction of the stress intensity factors by approximately 20%. Effectively, the residual stresses raise the stress intensity threshold, K_{th} , above which crack propagation is enabled. Thus, crack closure is responsible for the observed phenomena. This conclusion is further supported by the experimental observations regarding crack arrest, where it was established that reduction of R i.e. increase of the load amplitude causes resumption of growth. The increase of the load ratio R causes stress intensity levels higher than the threshold value thus overcoming the effects of crack closure.

The presented research has by no means fully studied the crack growth behaviour of the 7075 T7351 Al-Zn alloy. Some factors that influence the constant load amplitude propagation have been identified. There must be some further work carried out to establish the effects of load frequency upon the crack growth. If possible a crack growth model for constant load amplitude crack growth able to account for frequency effects should be developed as it is believed it will greatly enhance the predictions of the crack propagation life.

Some effects of overloading upon the crack growth behaviour

have been studied but the list of possible loading combinations is endless. It is suggested that effects of periodic overloading are investigated as well as the combinations of overload-underload, underload-overload. It is essential to continue the research in order to establish the crack closure characteristics of the alloy, so as to enable the development of a mathematical model for the prediction of crack growth in the presence of variable amplitude loading.

There is Nothing Like

CHALLENGE

To Bring the Best Out In Man

A P P E N D I X 1

G r i f f i t h E n e r g y B a l a n c e

A p p r o a c h

Griffith Energy Balance Approach

A1 MATHEMATICAL FORMULATION

Consider an infinite plate of unit thickness that contains a through thickness crack of length $2a$, and is subjected to uniform tensile load, applied at infinity, see Fig.A1.1.

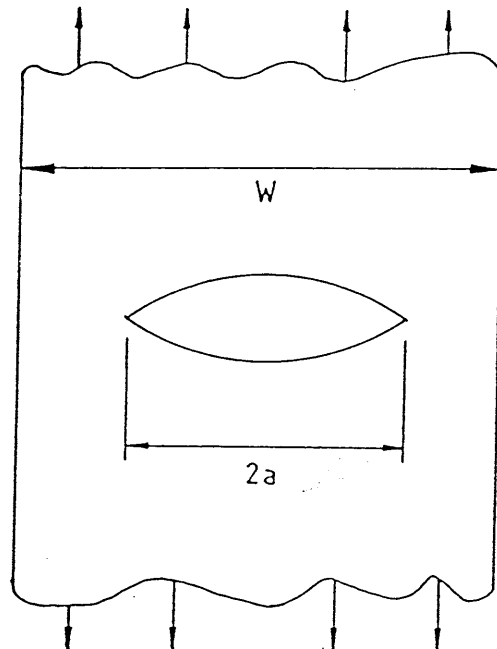


Fig.A1.1 Through Cracked Plate of Infinite Dimensions

The total energy U of the cracked plate described above may be written in the following mathematical form,

$$U = U_0 + U_s + U_\gamma - F \quad \dots (A1.1)$$

APPENDIX 1

From the above equation it is clearly indicated that the total energy of a cracked body can be divided into four components.

- a. The uncracked plate elastic strain energy U_o ,

$$U_o = \int_V (\sigma^2 / 2E) \cdot dV \quad \dots (A1.2)$$

Where V is the volume.

This strain energy term is a constant and represents the elastic strain energy stored in the uncracked body for a given loading condition.

- b. Elastic strain energy release U_a ,

$$U_a = \pm (\pi \cdot \sigma^2 \cdot a^2) / E \quad \dots (A1.3)$$

As calculated by Griffith [3.25].

The energy release is due to the introduction of the crack of length $2a$, whose formation causes localised relaxation of the material in the vicinity of the cracked ligaments

- c. Change in the elastic surface energy U_f ,

$$U_f = 4 \cdot a \cdot \gamma_e \quad \dots (A1.4)$$

Where γ_e is the elastic surface energy of the material, the change is caused by the formation of the new crack surfaces.

- d. Work done by the externally applied forces F,

$$F = \int \delta P \cdot dy \quad \dots (A1.5)$$

Where P = load and δ = displacement

APPENDIX 1

Hence, the total energy of the cracked body is represented by the following expression,

$$U = \int_V (\sigma^2/2E) .dV \pm (\pi . \sigma^2 . a^2)/E + 4 . a . \gamma_e - \int P . dy \dots (A1.6)$$

Generally two loading conditions may arise,

- I. Fixed grip or constant displacement
- II. Constant load

I Fixed Grip Analysis

The fixed grip loading condition implies that the externally applied forces do not produce any work i.e. $F=0$, in equation (A1.1). Since, crack extension is associated with a reduction in the component stiffness or increase in compliance, the associated applied load decreases. The reduction in the applied load results to a reduction of the elastic strain energy term U_a ,

$$-(\pi . \sigma^2 . a^2)/E \dots (A1.7)$$

In order to attain equilibrium following an infinitesimal crack extension, the change in the total energy due to this extension must be zero.

$$\partial U / \partial a = \partial (U_o - U_a + U_g) / \partial a = 0 \dots (A1.8)$$

As U_o is constant, the above expression may be rewritten as,

$$\partial (U_g - U_a) / \partial a = 0 \dots (A1.9)$$

or

$$\begin{aligned} \partial (4 . a . \gamma_e - (\pi . \sigma^2 . a^2)/E) / \partial a &= 4 . \gamma_e - 2 . (\pi . \sigma^2 . a)/E = 0 \\ \therefore \sigma . a^{1/2} &= (2 . \gamma_e . E)^{1/2} \end{aligned}$$

Instability will occur if,

$$\partial U / \partial a < 0 \quad \partial U / \partial a < \partial U_a / \partial a \dots (A1.10)$$

The physical explanation of the above conditions is that:

If the elastic energy release due to a potential increment of crack growth exceeds the demand for the same crack growth, then the introduction of a crack will lead to unstable propagation.

II Constant Load Analysis

In the case of constant load, an infinitesimal crack extension results to a change in displacement. Hence, the externally, applied loads produce work F , as defined in equation (A1.5). In order that the balance of energy is maintained, the strain energy release U_a is increased and accordingly,

$$\pm(\pi.\sigma^2.a^2)/E \quad \dots (A1.11)$$

Once again, to attain equilibrium following an infinitesimal crack extension, the change in the total energy due to this extension must be zero.

$$\partial U/\partial a = \partial (U_0 + U_a + U_y - F)/\partial a = 0 \quad \dots (A1.12)$$

If instability is to occur then,

$$\partial U/\partial a \leq 0 \quad \partial (U_a + U_y - F)/\partial a \leq 0 \quad \dots (A1.13)$$

as U_0 is constant.

Thus, the instability condition becomes,

$$\partial (F - U_a)/\partial a \geq \partial U_y/\partial a \quad \dots (A1.14)$$

In the left hand side of the above equation the term $\partial F/\partial a$ represents the energy provided by the external work F for an infinitesimal crack extension and $\partial U_a/\partial a$ is the increase of elastic energy owing to the external work $\partial F/\partial a$. Thus, $\partial (F - U_a)/\partial a$ represents the amount of energy that is available for crack extension. Hence, the work input by the external forces provides both the increase of

APPENDIX 1

elastic energy in the cracked body and the energy required for crack extension.

A2 THE GRIFFITH POSTULATE

Unstable crack propagation takes place when the strain energy released in an incremental crack extension exceeds the energy that is absorbed by creating new crack surfaces.

The energy available to cause increase to the crack length is the so termed *Elastic Strain Energy Release Rate*, per unit crack tip, G .

$$\begin{aligned} G &= (\pi \cdot \sigma^2 \cdot a) / E && \text{Plane stress} \\ &&& \dots (A1.15) \\ G &= (1 - \nu^2) \cdot (\pi \cdot \sigma^2 \cdot a) / E && \text{Plane strain} \end{aligned}$$

A P P E N D I X 2

G e n e r a l i s e d F o r m u l a t i o n o f
J - I n t e g r a l i n T w o D i m e n s i o n s

Generalised Formulation of J-Integral
in Two Dimensions

Consider an inclined crack, lying along the ξ -axis as shown in Fig.A2.1. The equation of the J-integral in terms of the local coordinates (ξ, η) is given by,

$$\begin{aligned} J &= \int_{\Gamma} (W \cdot d\eta - T_i \cdot (\partial U_i / \partial \xi) \cdot ds) \\ \text{or} \quad J &= \int_{\Gamma} (W \cdot d\eta - (\sigma_n, \tau_n) \begin{pmatrix} \partial U_i / \partial \xi \\ \partial V_i / \partial \xi \end{pmatrix} ds) \quad \dots (A2.1) \end{aligned}$$

Where :

$$W = (1/2E) \cdot (\sigma_x + \sigma_y + \sigma_z)^2 + [(1+2\nu)/E] \cdot (\tau_{xy}^2 - \sigma_x \sigma_y - \sigma_x \sigma_z - \sigma_y \sigma_z)$$

$$\sigma_n = \sigma_x \cos^2 \alpha + \sigma_y \sin^2 \alpha + \tau_{xy} \cdot \sin \alpha \cdot \cos \alpha$$

$$\tau_n = (\sigma_y - \sigma_x) \cdot \sin \alpha \cdot \cos \alpha + \tau_{xy} \cdot (\cos^2 \alpha \cdot \sin^2 \alpha)$$

$$U_n = u \cdot \cos \alpha + v \cdot \sin \alpha$$

$$V_n = -u \cdot \sin \alpha + v \cdot \cos \alpha$$

For plane strain $\sigma_z = \nu (\sigma_x + \sigma_y)$

and,

for plane stress $\sigma_z = 0$

After manipulation and changing to the global coordinate system, equation (A2.1) becomes :

$$\begin{aligned} J = \int W \cdot (-\sin \phi \cdot dx + \cos \phi \cdot dy) - \int (\sigma_x \cdot \epsilon_x \cdot \cos \phi \cdot dy + \tau_{xy} \cdot \epsilon_y \cdot \sin \phi \cdot dy \\ - \tau_{xy} \cdot \epsilon_x \cdot \sin \phi \cdot dx - \sigma_y \cdot \epsilon_y \cdot \sin \phi \cdot dx + \tau_{xy} \cdot \gamma_{xy} \cdot \cos \phi \cdot dy \\ - \tau_{xy} \cdot \gamma_{xy} \cdot \sin \phi \cdot dx + A) \quad (A2.2) \end{aligned}$$

Where $A = (\sigma_x \cdot \sin \phi - \tau_{xy} \cdot \cos \phi) \cdot du - (\sigma_y \cdot \cos \phi - \tau_{xy} \cdot \sin \phi) \cdot dv$

APPENDIX 2

Numerical evaluation of equation (A2.2) can be achieved by summing the values of each portion of the contour dS between adjacent nodes of the contour. Hence, for linear integration between the i th and $(i+1)$ th node, equation (A2.2) may be rewritten in the following form:

$$\begin{aligned}
 J = & \sum_{i=1}^n (1/2) \cdot (W_i + W_{i+1}) \cdot (-\sin\phi \cdot \Delta x + \cos\phi \cdot \Delta y) \\
 & - (\sigma_{xi} \cdot \epsilon_{xi} + \sigma_{x(i+1)} \cdot \epsilon_{x(i+1)}) \cdot \cos\phi \cdot \Delta y / 2 \\
 & - (\tau_{xyi} \cdot \epsilon_{yi} + \tau_{xy(i+1)} \cdot \epsilon_{y(i+1)}) \cdot \sin\phi \cdot \Delta y / 2 \\
 & + (\tau_{xyi} \cdot \epsilon_{xi} + \tau_{xy(i+1)} \cdot \epsilon_{x(i+1)}) \cdot \cos\phi \cdot \Delta x / 2 \quad \dots (A2.3) \\
 & + (\sigma_{yi} \cdot \epsilon_{yi} + \sigma_{y(i+1)} \cdot \epsilon_{y(i+1)}) \cdot \sin\phi \cdot \Delta x / 2 \\
 & - (\tau_{xyi} \cdot \gamma_{xyi} + \tau_{xy(i+1)} \cdot \gamma_{xy(i+1)}) \cdot \cos\phi \cdot \Delta y / 2 \\
 & - (\tau_{xyi} \cdot \gamma_{xyi} + \tau_{xy(i+1)} \cdot \gamma_{xy(i+1)}) \cdot \sin\phi \cdot \Delta x / 2 - A
 \end{aligned}$$

A further simplification may be achieved by considering the crack to lie along the global x-axis, i.e. $\phi = 0$.

$$\begin{aligned}
 J = & \sum_{i=1}^n (\Delta y / 2) \cdot (W_i + W_{i+1}) - (\sigma_{xi} \cdot \epsilon_{xi} + \sigma_{x(i+1)} \cdot \epsilon_{x(i+1)}) \cdot \Delta y / 2 \\
 & + (\tau_{xyi} \cdot \epsilon_{xi} + \tau_{xy(i+1)} \cdot \epsilon_{x(i+1)}) \cdot \Delta x / 2 \quad \dots (A2.4) \\
 & - (\tau_{xyi} \cdot \gamma_{xyi} + \tau_{xy(i+1)} \cdot \gamma_{xy(i+1)}) \cdot \Delta y / 2 - A
 \end{aligned}$$

Where $A = (\tau_{xyi} + \tau_{xy(i+1)}) \cdot (\Delta u / 2) - (\sigma_{yi} + \sigma_{y(i+1)}) \cdot \Delta v / 2$

APPENDIX 2

Equations (A2.4) can be written in terms of stresses so that the stress output from a Finite Element Analysis can be utilised,

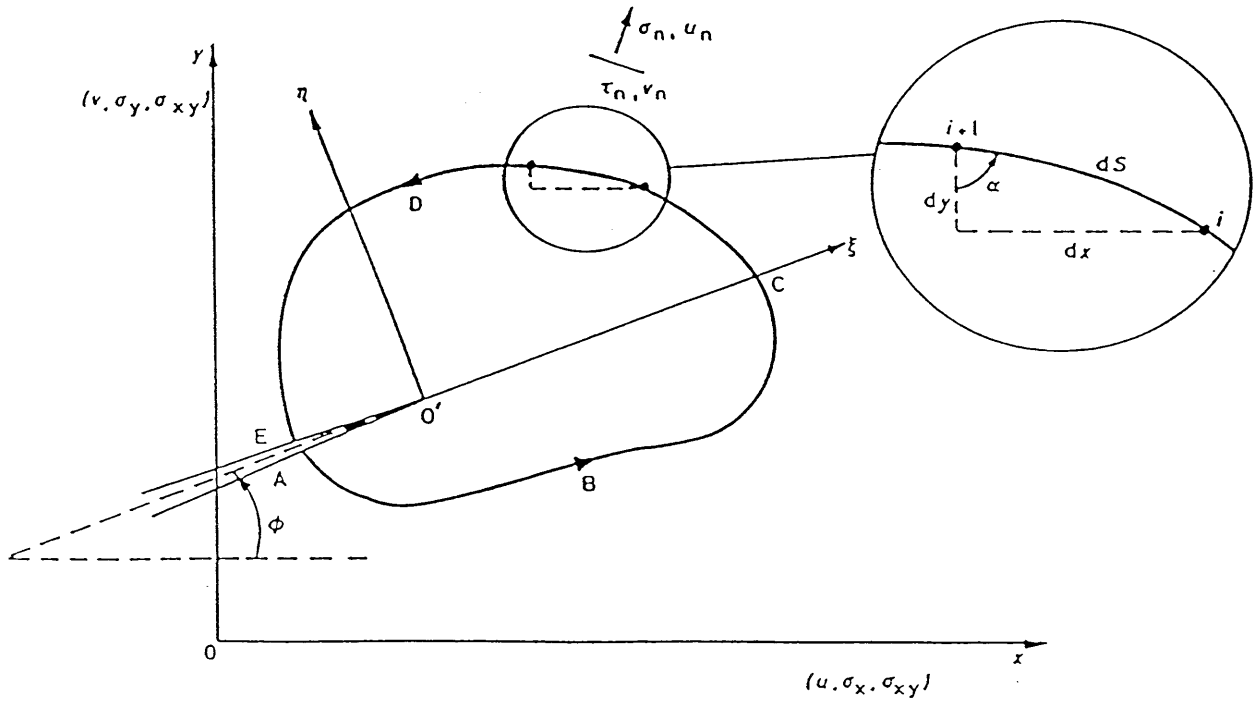
$$\begin{aligned}\epsilon_x &= (1/E) \cdot (\sigma_x - \nu \cdot \sigma_y) \\ \epsilon_y &= (1/E) \cdot (\sigma_y - \nu \cdot \sigma_x) \\ \gamma_{xy} &= \tau_{xy}/G\end{aligned}\quad \dots (A2.5)$$

Where $G=E/2 \cdot (1+\nu)$, the shear modulus of elasticity

Thus, the J-integral equation in terms of stresses is expressed by the following mathematical expression,

$$\begin{aligned}J &= \sum_{i=1}^n (\Delta y/2) \cdot (W_i + W_{(i+1)}) \\ &\quad + (\tau_{xyi} \cdot \sigma_{xi} + \tau_{xy(i+1)} \cdot \sigma_{x(i+1)}) \cdot \Delta x/2E \quad \dots (A2.6) \\ &\quad - (\sigma_{xi}^2 + \sigma_{x(i+1)}^2) \cdot \Delta y/2E - (\tau_{xyi}^2 + \tau_{xy(i+1)}^2) \cdot \Delta y/2G - A\end{aligned}$$

The above equation (A2.6) was utilised for the development of of a FORTRAN computer program, able to calculate the J-integral from stress results obtained from Finite Element results. A listing of the program can be found in the following pages.



O' POSITION OF THE CRACK TIP

ϕ INCLINATION OF THE CRACK WITH RESPECT TO THE X-AXIS

α INCLINATION OF THE CONTOUR WITH RESPECT TO THE Y-AXIS

dS DISTANCE BETWEEN NODES i AND $i+1$ ON THE CONTOUR

xOy GLOBAL CARTESIAN CO-ORDINATES

$\xi O\eta$ LOCAL CARTESIAN CO-ORDINATES

Fig.A2.1 Description of Arbitrarily Inclined Crack
Surrounded by an Arbitrary Contour

```

C.....
C      This program uses the data obtained from the F.E. analysis
C      and calculates the J-integral as defined by J.Rice for the
C      plane stress, plane strain loading conditions.
C      The definition of each contour is inserted in the program
C      by means of a data file that contains the nodal stresses,
C      displacements and coordinates of each node.

```

NOTATION

```

SIGX : DIRECT STRESS X-DIRECTION
SIGY : DIRECT STRESS Y-DIRECTION
SIGZ : DIRECT STRESS Z-DIRECTION
SIGXY : DIRECT STRESS 2-DIRECTION
SIGXZ : DIRECT STRESS 2-DIRECTION
SIGXYZ : SHEAR STRESS IN X-Y PLANE
X      : NODAL X-COORDINATE
Y      : NODAL Y-COORDINATE
U      : X-DISPLACEMENT
V      : Y-DISPLACEMENT

```

Stellos Kyriacou
PhD Applied Mechanics
Cranfield Inst. of Tech.
June 1988.

PROGRAM JINTEGRAL

```

DIMENSION SIGX(50), SIGY(50), SIGZ(50),
      X(50), Y(50), W(50), A(50), B(50),
      F(50), C(50), D(50), Z(50), U(50), T(50)
CHARACTER FNAME*12

```

Interactive Input

```
WRITE(5,*) ' What is the name of the data file with FE results ?'
READ(6,*) FNAME
OPEN(1,NAME=FNAME, STATUS='OLD', ACCESS='SEQUENTIAL')

WRITE(5,*) ' Is the problem one of plane stress or plane strain ?'
WRITE(5,*) ' Please insert 1= Plane Stress'
WRITE(5,*) ' OR'
WRITE(5,*) ' 2= Plane Strain'
READ(6,*) KS
```

```
F (KS.NE.1,OR.KS.NE.2) THEN  
    . . . E R R O R . .  
    . . . UNACCEPTABLE INPUT . .  
    . . . E R R O R . .  
    . . .  
    . . . PHOTO 1  
    . . . AND IF
```

WRITE(S,*) ' What is the modulus of elasticity of the material ?'
 WRITE(S,*) ' Input the value in N/mq.m'
 READ (*,*) E
 ! value of Young's modulus
 WRITE(S,*) ' What is the value of the Poisson Ratio ?'

```

READ(1,*) NP      ! Number of points lying on chosen contour
DO 10 N=1, NP
C..... read stresses and displacements
      READ(1,*) SIGX(N), SIGY(N), SIGXY(N), U(N), T(N)
10  CONTINUE

```

```

DO 20 N=1, NP
C..... read nodal coordinates
READ(1,*) X(N), Y(N)
CONTINUE

G=E/(2*(1+V)) ! Shear modulus

DO 50 N=1, NP
IF (KSE.EQ.1) THEN
SIGZ(N)=0 ! Plane Stress
ELSE
SIGZ(N)=V*(SIGX(N)+SIGY(N)) ! Plane Strain
END IF

W(N)=(((SIGX(N)+SIGY(N)+SIG(N))*2)/(2.*E))+((SIGXY(N)**2)-
(SIGX(N)*SIGY(N)-SIGX(N)*SIGZ(N)-SIGY(N)*SIGZ(N))*(1+(2.*V))/2
CONTINUE
SUM=0.0

DO 70 I=1, NP
A(I-1)=((-SIGXY(I-1)-SIGXY(I))*U(I)-U(I-1))/2
!
B(I-1)=(SIGXY(I-1)**2+SIGXY(I)**2)*(Y(I)-Y(I-1))/(2.*G)
F(I-1)=(SIGX(I-1)**2+SIGX(I)**2)*(Y(I)-Y(I-1))/(2.*E)
C(I-1)=((SIGXY(I-1)*SIGX(I-1)+SIGXY(I)*SIGX(I))*X(I)-X(I-1))/(2.*E)
D(I-1)=(W(I-1)+W(I))*(Y(I)-Y(I-1))/2
Z(I-1)=D(I-1)+C(I-1)-F(I-1)-B(I-1)-A(I-1)
SUM=SUM+Z(I-1)
SIF=SORT(SUM**E*2.0)
CONTINUE

WRITE(6,100)
WRITE(6,200)
WRITE(6,300)
WRITE(6,400)
WRITE(6,500)
WRITE(6,600)
WRITE(6,700)
WRITE(6,800) SUM, SIF

C C C
C FORMATS FORMATS FORMATS FORMATS FORMATS FORMATS
C *****
FORMAT(15X,'SIGX','9X','SIGY','7X','SIGXY','9X','U','15X','T',/)
FORMAT(F11.5,F13.5,F15.5,2F16.12)
FORMAT(////,5X,'X','7X','Y',/)
FORMAT(2F8.3)
FORMAT(////,7X,'A','15X','B','15X','F','15X','C','15X','D','15X','2',/)
FORMAT(16F16.10)
FORMAT(////,6X,'SUM','12X','SIF',/,6X,'(J)','12X','(K)',/)
FORMAT(F15.0,F15.4)

STOP

```

stop

APPENDIX 3

A P P E N D I X 3

P r o g r a m f o r t h e E v a l u a t i o n o f
C r a c k L e n g t h s b y C o m p l i a n c e
M e t h o d s


```

AR= 0.0
SIF= 0.0
C***** IF (ICODE.EQ.1) THEN
      CALL TABLE1
C-----
C.....NORMALISED - COMPLIANCE.....
C.....
      F1= A/N
      F2=1-(X/F1)
      F3=(1+F1)/(1-F1)
      F4=F3**2
      G=B1+ B2*F1+ B3*(F1**2)+ B4*(F1**3)+ B5*(F1**4)+ B6*(F1**5.)
C-----
C.....NORMALISED COMPLIANCE
C.....
      AR= AXROT1(F1)*W
C.....
C.....K-CALIBRATION.....
C.....
      SIF= XCALL1(F1)
C.....
C.....END OF CALCULATION.....
C*****SCREEN PRESENTATION
C-----
      WRITE (5,10)
      WRITE (5,55) B, W
      WRITE (5,15) LOCATION
      WRITE (5,50) P
      WRITE (5,20) A
      WRITE (5,25) COMPL
      WRITE (5,30) AR
      WRITE (5,35) SIF
C*****
C.....RUN FILE PRESENTATION.....
C.....
      IF (ABC.EQ.1.0) THEN
        WRITE (10,10)
        WRITE (10,55) B, W
        WRITE (10,15) LOCATION
        WRITE (10,50) P
        WRITE (10,20) A
        WRITE (10,25) COMPL
        WRITE (10,30) AR
        WRITE (10,35) SIF
        END IF
C-----
C-----CONTROL SCREEN SCROLL-----
C-----
      WRITE (5,111)
      WRITE (5,100)
      READ (5,'(A5)') SCROLL
C-----
      ELSE
C-----
C.....NORMALISED - CRACK LENGTH.....
C.....
      CALL TABLE2
C-----

```

```

C-----
      R=SQRT((B*E*V*1.E-06/P)+1)
      U=1/R
      ! 1E-06 as units of B,V in (mm)
C-----
      AW=C1+ C2*U+ C3*(U**2)+ C4*(U**3)+ C5*(U**4)+ C6*(U**5.)
      ! AW = normalised crack length
C-----
C.....AXIS OF ROTATION.....
C-----
      AR=AXROT1(AW)*W
C.....
C.....K-CALIBRATION.....
C.....
      SIF=XKCALL(AW)
C.....
C.....END OF CALCULATION.....
C*****SCREEN PRESENTATION
C-----
      WRITE (5,40)
      WRITE (5,55) B, W
      WRITE (5,15) LOCATION
      WRITE (5,65) E
      WRITE (5,50) P
      WRITE (5,45) V
      WRITE (5,60) AW
      WRITE (5,30) AR
      WRITE (5,35) SIF
C*****
C.....RUN FILE PRESENTATION.....
C.....
      IF (ABC.EQ.1.0) THEN
        WRITE (10,40)
        WRITE (10,55) B, W
        WRITE (10,15) LOCATION
        WRITE (10,65) E
        WRITE (10,50) P
        WRITE (10,45) V
        WRITE (10,60) AW
        WRITE (10,30) AR
        WRITE (10,35) SIF
        END IF
C-----
C-----CONTROL SCREEN SCROLL-----
C-----
      WRITE (5,100)
      READ (5,'(A5)') SCROLL
C-----
      END IF
C-----
C-----
C-----
      10 +
      15
      20
      FORMAT(5X,'LOCATION:',2X,A10,/)
      FORMAT(5X,'CRACK LENGTH=',F7.3,1X,'(mm)',/)

```



```

C-----
      WRITE(2,200)
      WRITE(2,110) B, W
      WRITE(2,120) LOCATION
C*****
      DO 60 J=1,100
        R(J)=SORT(B+E*V(J)*(1.E-06)/P(J))+1      ! 1E-06 B,V are in (mm)
        U(J)=1/R(J)
      AM(J)=C1+ C2*U(J)+ C3*(U(J)**2.)+ C4*(U(J)**3.)+ C5*(U(J)**4.)+
      + C6*(U(J)**5.)      ! normalised crack length
C-----
C.....AXIS OF ROTATION.....
C
      AR = 0.0
      AR = AXROT2(AW(J))*W      ! in (mm)
C-----
C.....K-CALIBRATION.....
C
      SIF = 0.0
      SIF = XKCAL2(AW(J))      ! dimensionless
C-----
C.....END OF CALCULATION.....
C-----
      WRITE(2,220) AW(J), P(J), V(J), AR, SIF
C-----
60 CONTINUE
   END IF
C-----
C-----
C-----
C-----
100 FORMAT(5X,'CALCULATION OF NORMALISED COMPLIANCE',/,/,
          5X,/,/)
110 FORMAT(5X,'CT SPECIMEN',/,/,
          5X,'B=',F6.3,'(mm)',5X,'W=',F6.3,'(mm)',/,/)
120 FORMAT(5X,'LOCATION ',A10,/)
130 FORMAT(5X,'CRACK',5X,'NORMALISED',5X,'AXIS OF',5X,'NORMALISED',/,/,
          5X,'LENGTH',5X,'COMPLIANCE',5X,'ROTATION',8X,'SIF',/,/,
          6X,'(mm)',23X,'(mm)',/,/)
140 FORMAT(4X,F6.3,7X,F7.3,7X,F7.4,5X,F9.4)
200 FORMAT(5X,'CALCULATION OF NORMALISED CRACK LENGTH',/,/,
          5X,/,/)
210 FORMAT(5X,'NORMALISED',5X,'LOAD',5X,'DISPLACEMENT',5X,'AXIS OF',
          5X,'NORMALISED',/,/,
          8X,'CRACK',8X,'(N)',9X,'ROTATION',8X,'SIF',/,/,
          7X,'LENGTH',21X,'(mm)',/,/)
220 FORMAT(6X,F7.4,6X,F7.3,8X,F8.5,8X,F7.4,8X,F8.5)
      RETURN
      END
C-----
C.....FUNCTION DEFINITIONS.....
C-----

```

```

ELSE
B4=-91.534
B5=107.40
B6=-40.792

X= 0.0
B1=2.1630
B2=12.219
B3=-20.065
B4=-0.9925
B5=20.609
B6=-9.9314

```

END IF
RETURN
END

```

*****
SUBROUTINE TABL22 ! contains the values
of constants and selects the correct set
for the location selected, for the
calculation of normalised crack length.
*****

```

SUBROUTINE TABLE2

```
COMMON/ DATA2/ C1, C2, C3, C4, C5, C6
COMMON/ LOC/ IXTM
```

*****INITIALISE VARIABLES

1-0.0
2-0.0
3-0.0
4-0.0
5-0.0
6-0.0

```
*****
IF (XN.EQ.1) THEN
  C1= 1.0012
  C2= -4.916
  C3= 23.057
  C4= -323.9
  C5= 1798.30
  C6= -3513.2

```

ELSE IF (IXW.EQ.2) THEN

(IXM.EQ.2) TH
C1= 1.001000
C2= -4.66950
C3= 18.46000
C4= -236.820
C5= 1214.900
C6= -2143.60

```
ELSE IF (IXW.EQ.3) THEN
  C1= 1.000800
  C2= -4.44730
  C3= 15.40000
  C4= -180.550
  C5= 870.9200
  C6= -1411.30
```

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C1	1.00020
C2	-4.0632
C3	11.2420
C4	-106.04
C5	464.330
C6	-650.68

END IF

RETURN
END

```

*****
SUBROUTINE INTER: Prompts the user to input
                  the name of an output file that stores the
                  details of the present interactive run.
*****

```

SUBROUTINE INTERF

.....
COMMON/WR T/ABC

.....

CHARACTER*10 ANS, ENAME

```
ABC=0.0
! Prevents write into file.
```

WRITE (6,10)

IF (ANS.EQ.'Y'.OR.ANS.EQ.'V') THEN

```

BC=1.0
RITE(6,20)
! prompts write statements in call subroutine.

```

READ (5, '(A10)') FNAME

```
PEN(10,FILENAME,      ! run file open on Unit...10
      STATUS='NEW',
      ACCESS='SEQUENTIAL',
      FORM='FORMATTED')
```

EST

GO TO 99

44

```
FORMAT(//,5X,'Do you wish to store the present run details',  
1X,'into a file?',//
```

```
FORMAT(//,5X,'-----',//,
5X,'What is the name of the file?',//,
5X,'10 Char.max.',//,
5X,'-----',//)
```

RETURN
ID

**PAGE
MISSING**

APPENDIX 4

APPENDIX 4

Fitting of Experimental Data
With Empirical Crack Growth
Law Equations

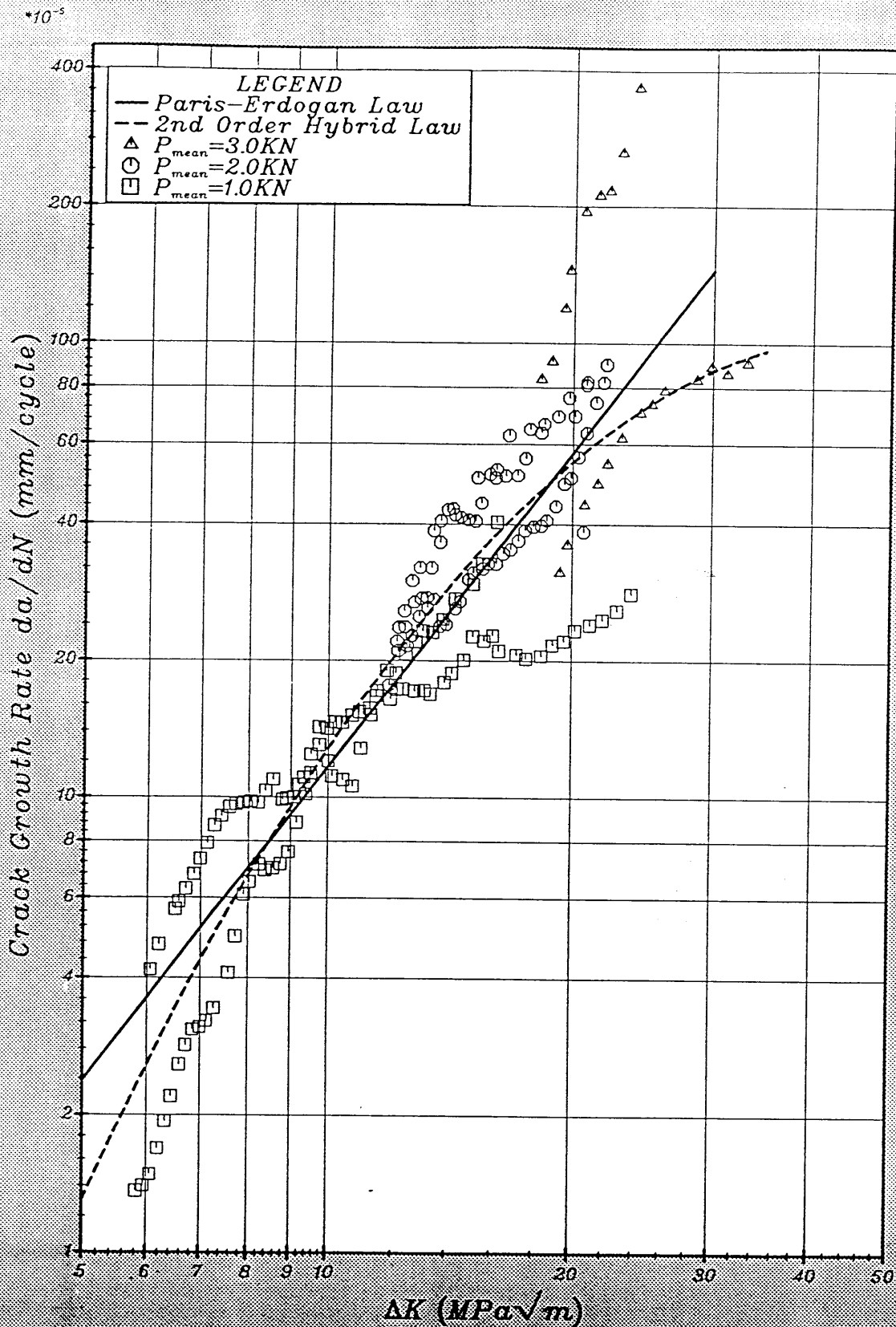


Fig.A4.1 Constant load amplitude fatigue crack growth data for different mean load levels and, correlation with empirically derived equations for $R=0.1$

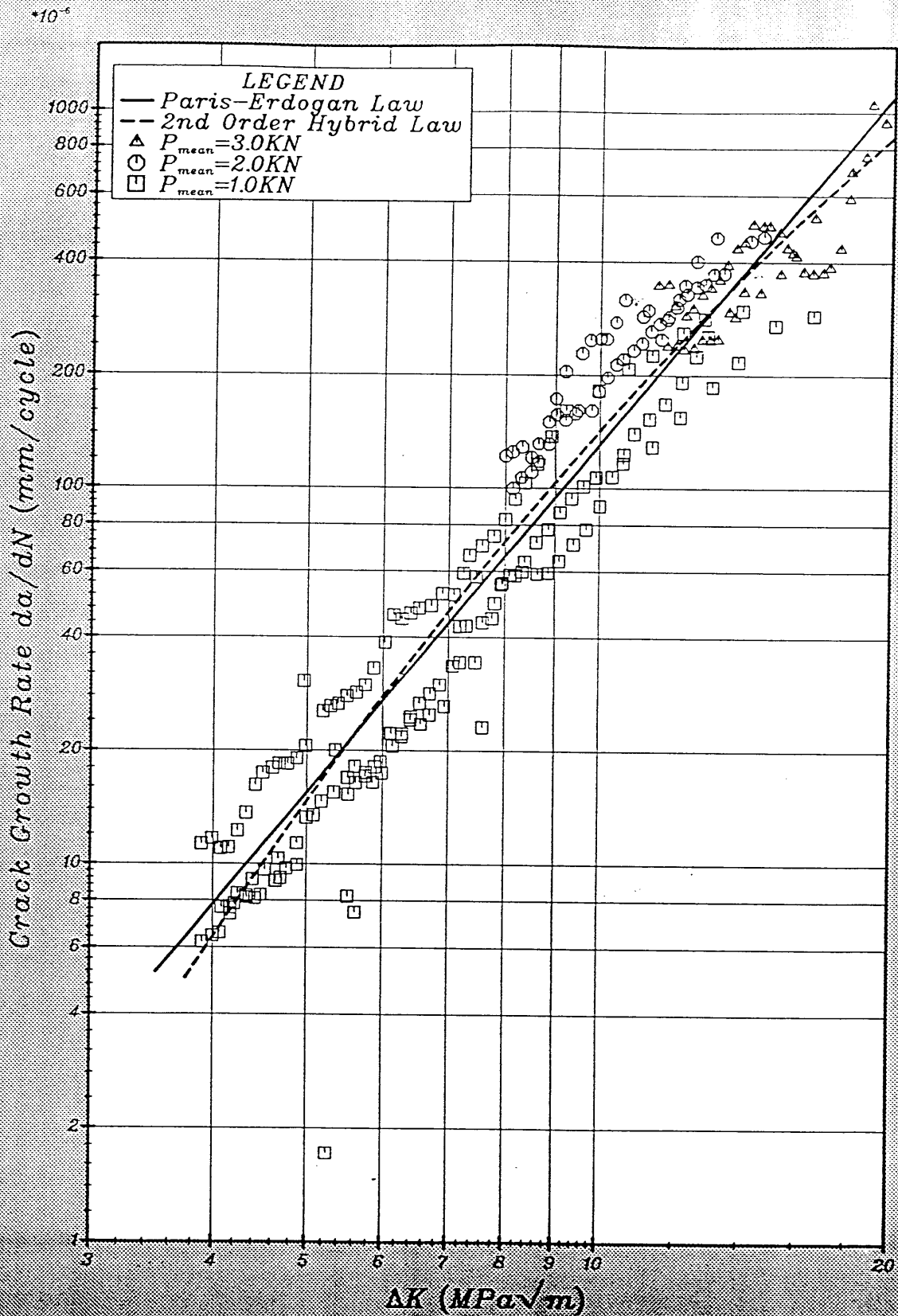


Fig.A4.2 Constant load amplitude fatigue crack growth data for different mean load levels and, correlation with empirically derived equations for $R=0.3$

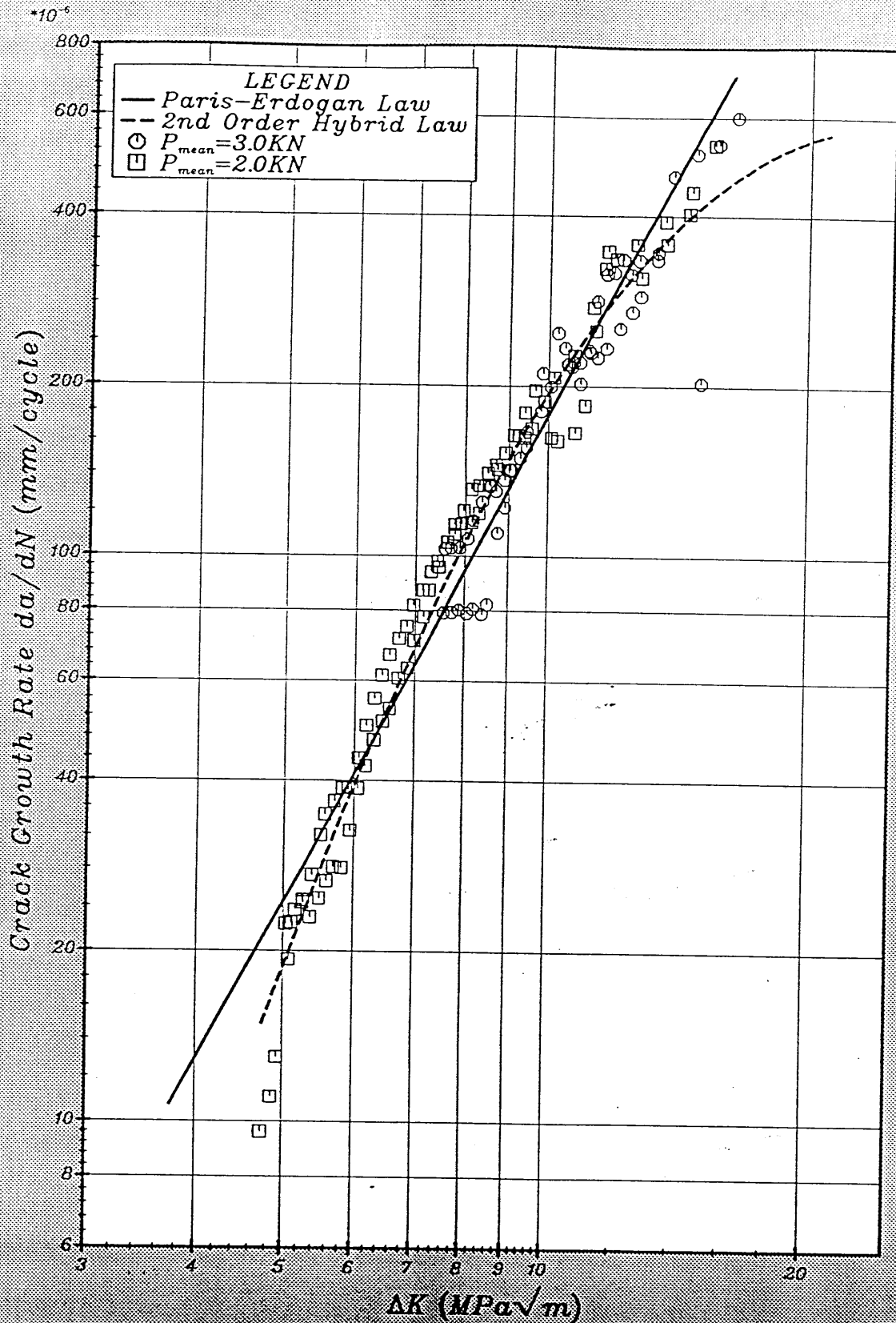


Fig.A4.3 Constant load amplitude fatigue crack growth data for different mean load levels and, correlation with empirically derived equations for $R=0.5$

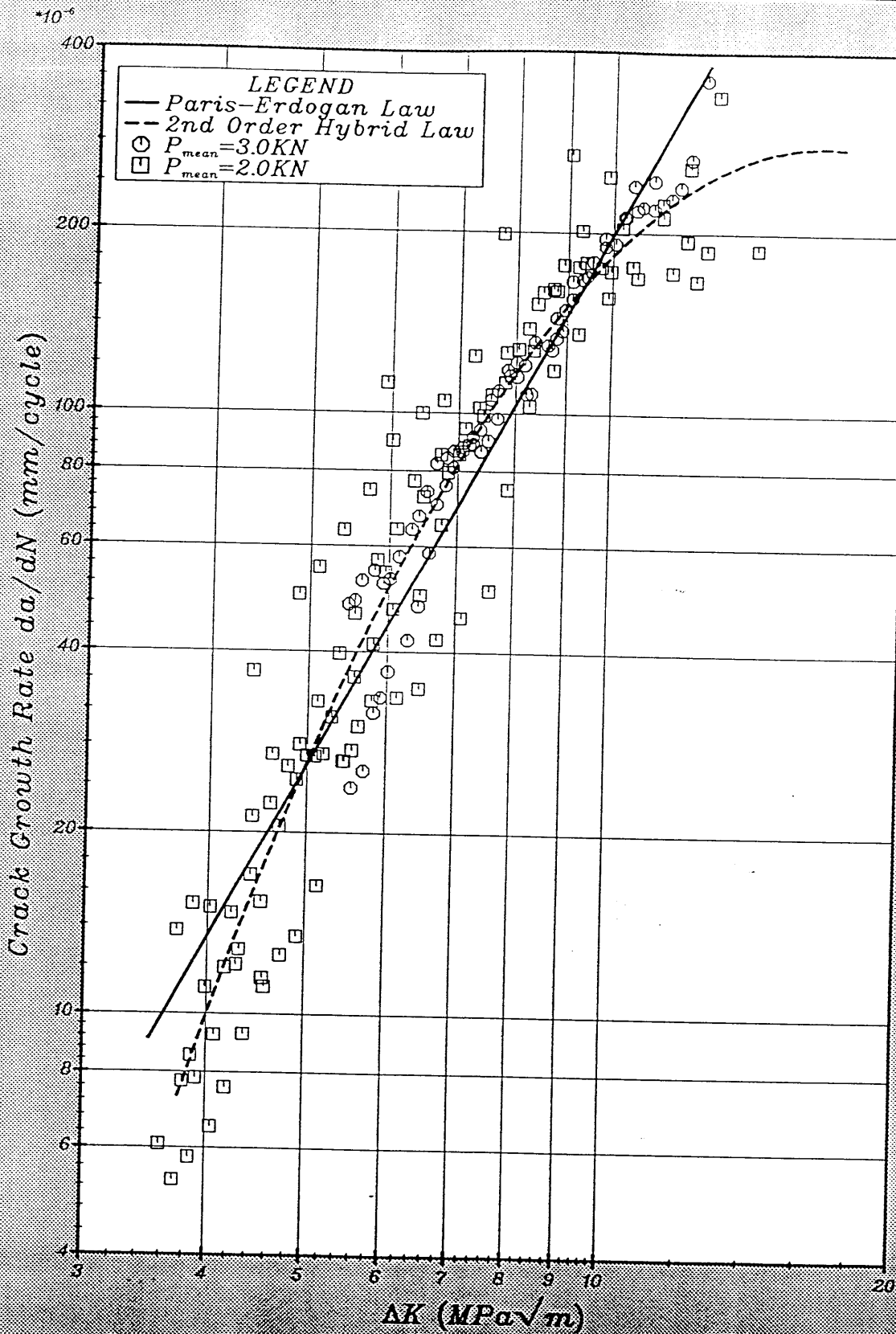


Fig.A4.4 Constant load amplitude fatigue crack growth data for different mean load levels and correlation with empirically derived equations for $R=0.6$

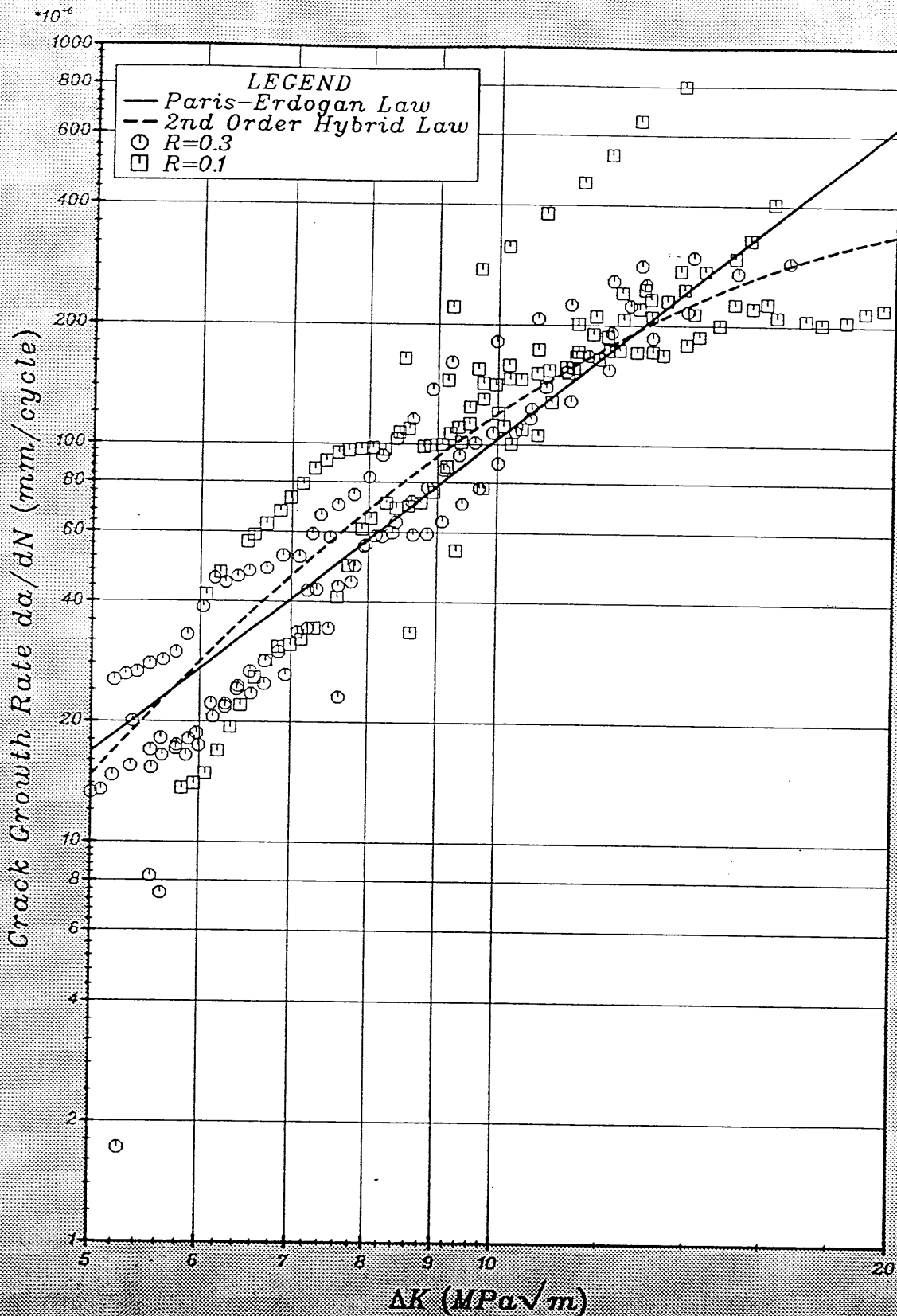


Fig.A4.5 Constane load amplitude fatigue crack growth data for different load ratios and, correlation with empirically derived equations for $P_{mean}=1.0KN$

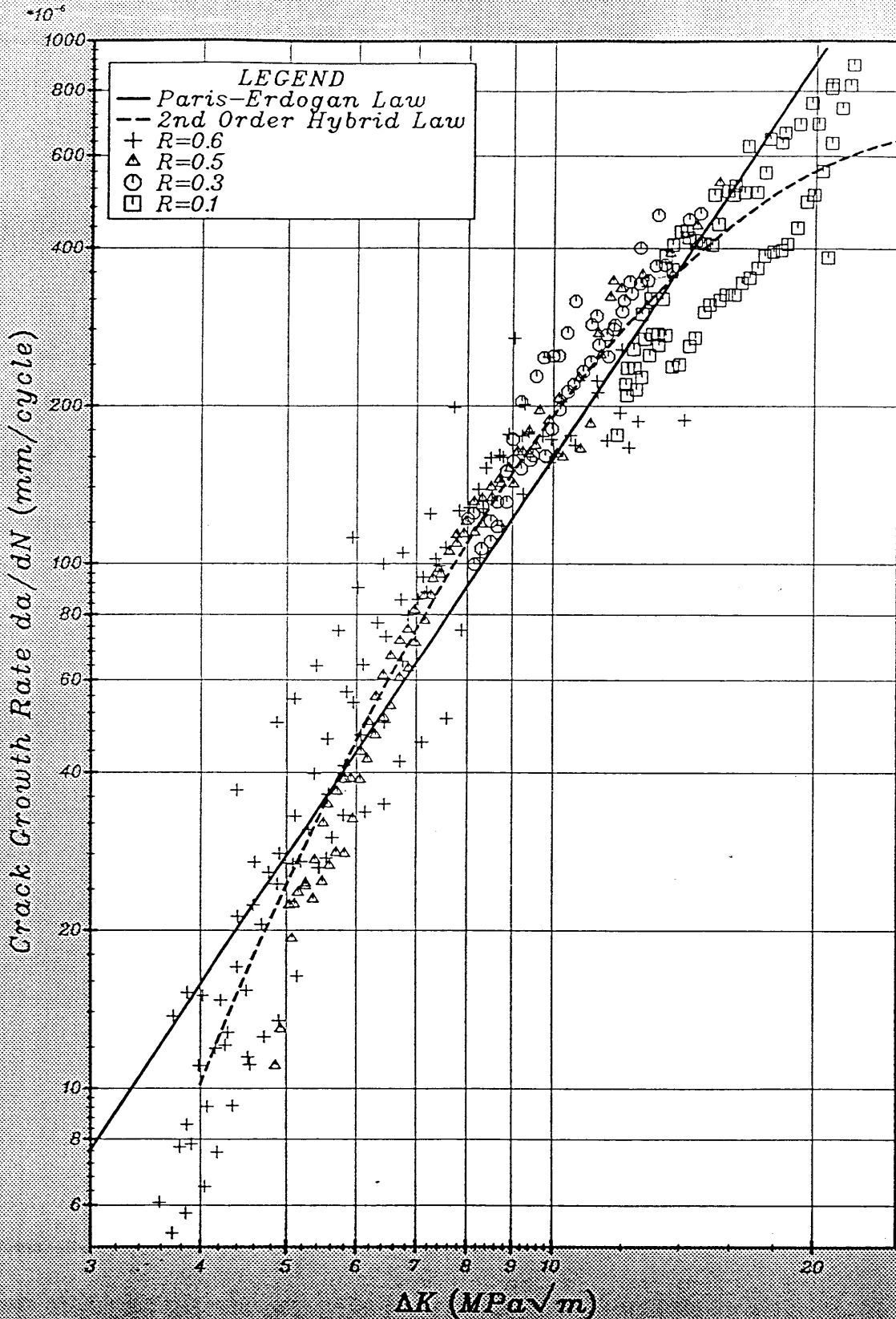


Fig.A4.6 Constant load amplitude fatigue crack growth data for different load ratios and, correlation with empirically derived equations for $P_{mean}=2.0KN$

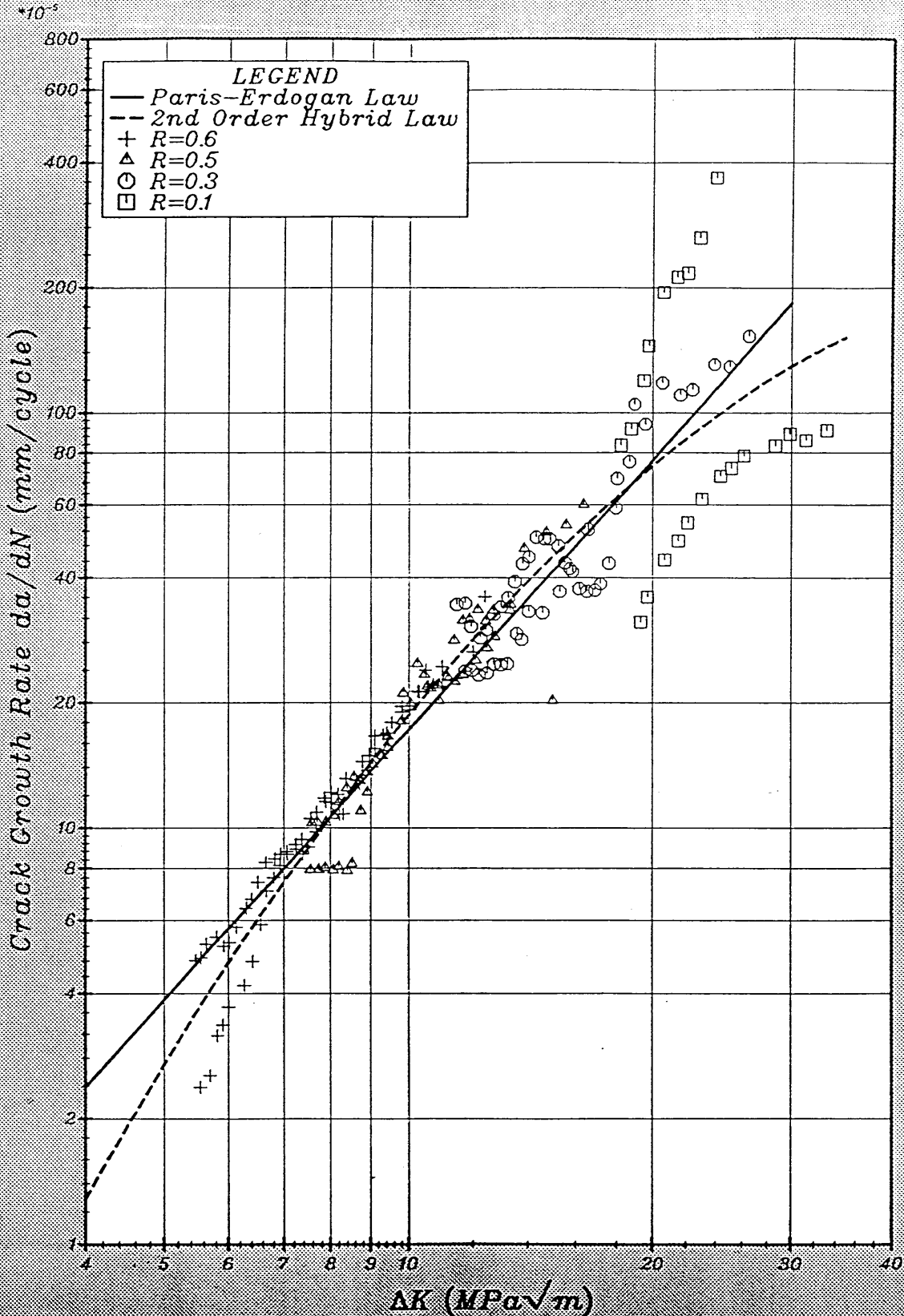


Fig.A4.7 Constant load amplitude fatigue crack growth data for different load ratios and, correlation with empirically derived equations for $P_{mean}=3.0KN$

A P P E N D I X 5

B l o c k O v e r l o a d R e s u l t s

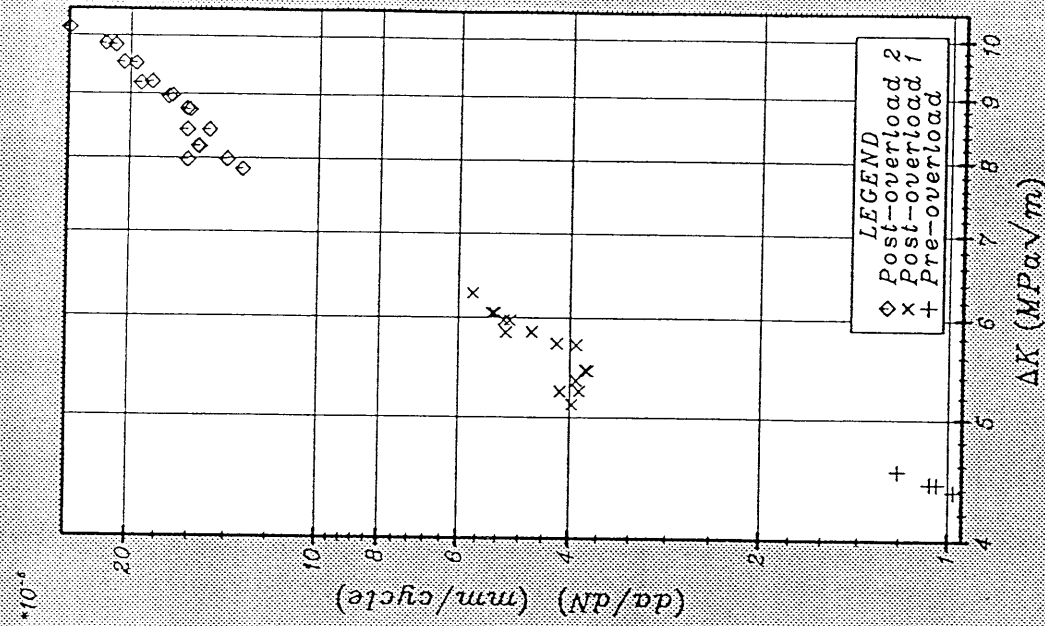
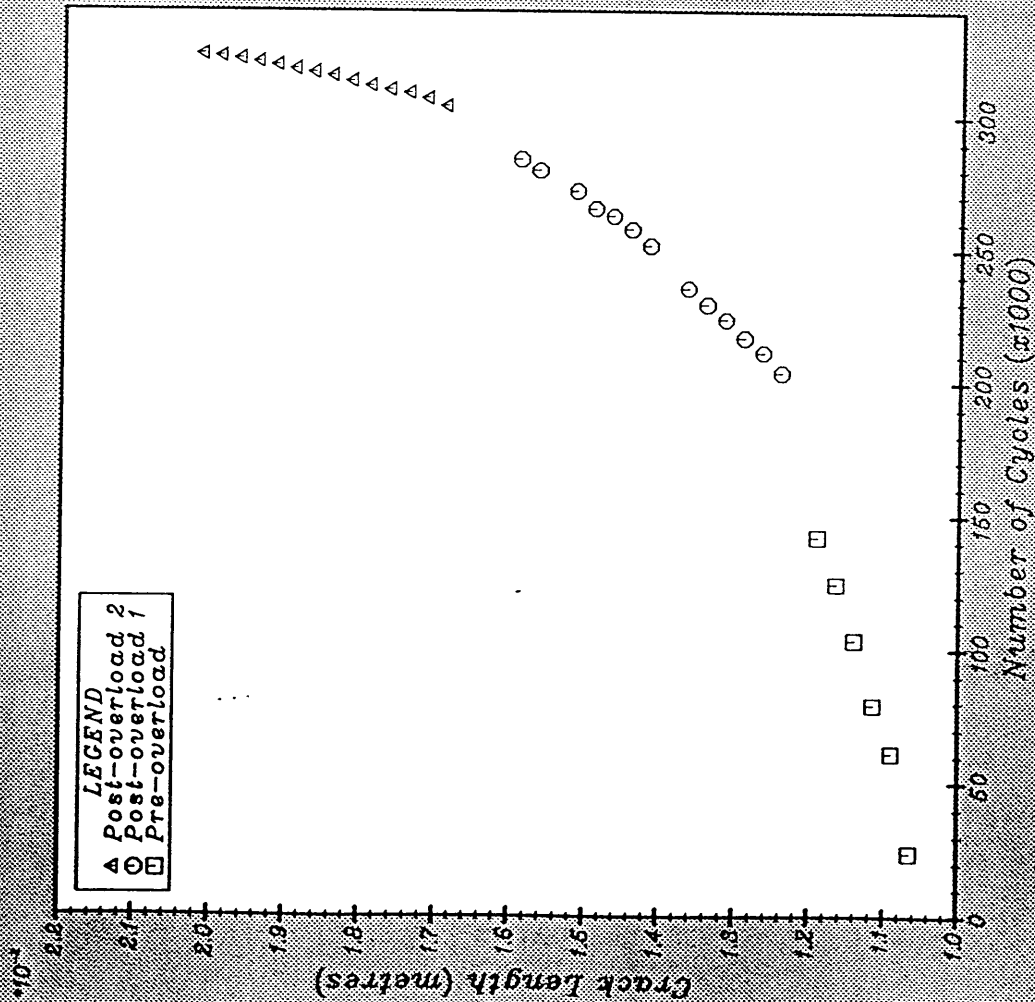


Fig.A5.1 Experimental fatigue crack growth data from a block overload test.
Block duration 1000 cycles
Overload Ratio $R_{ov}=1.4$

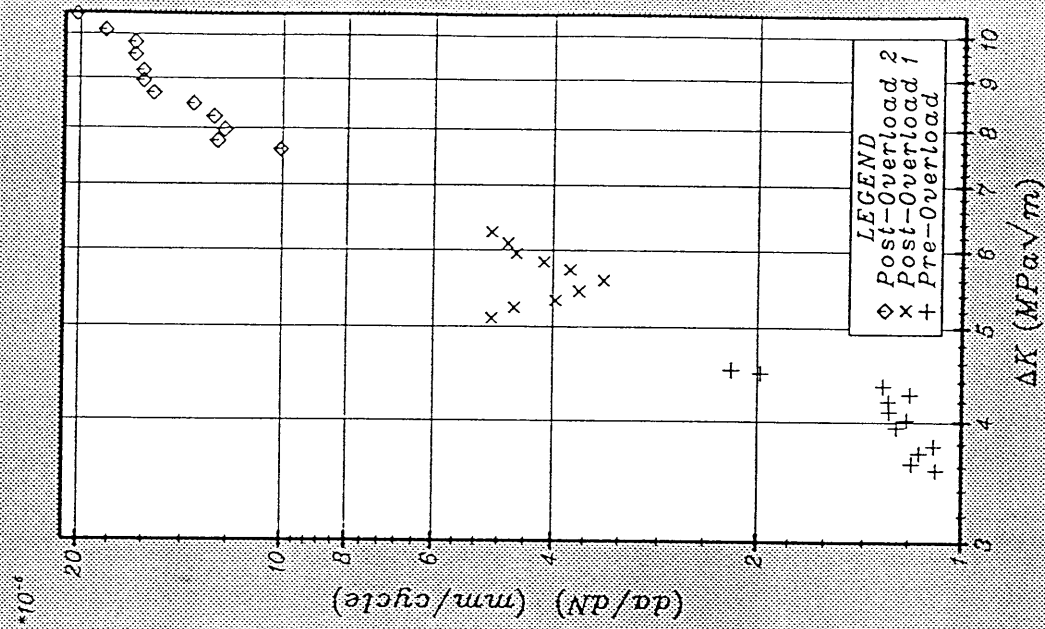
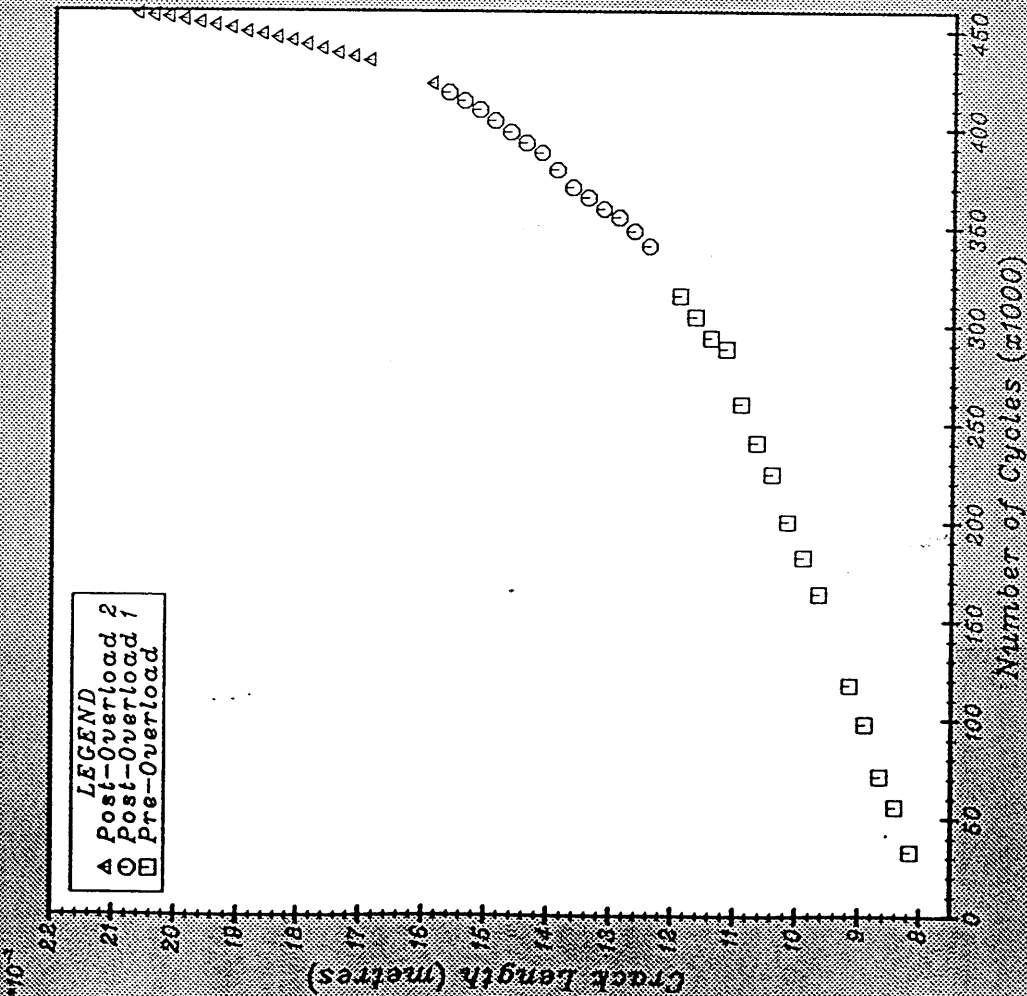


Fig.A5.2 Experimental fatigue crack growth data from a block overload test. Block duration 2000 cycles Overload Ratio $R_{ov}=1.4$

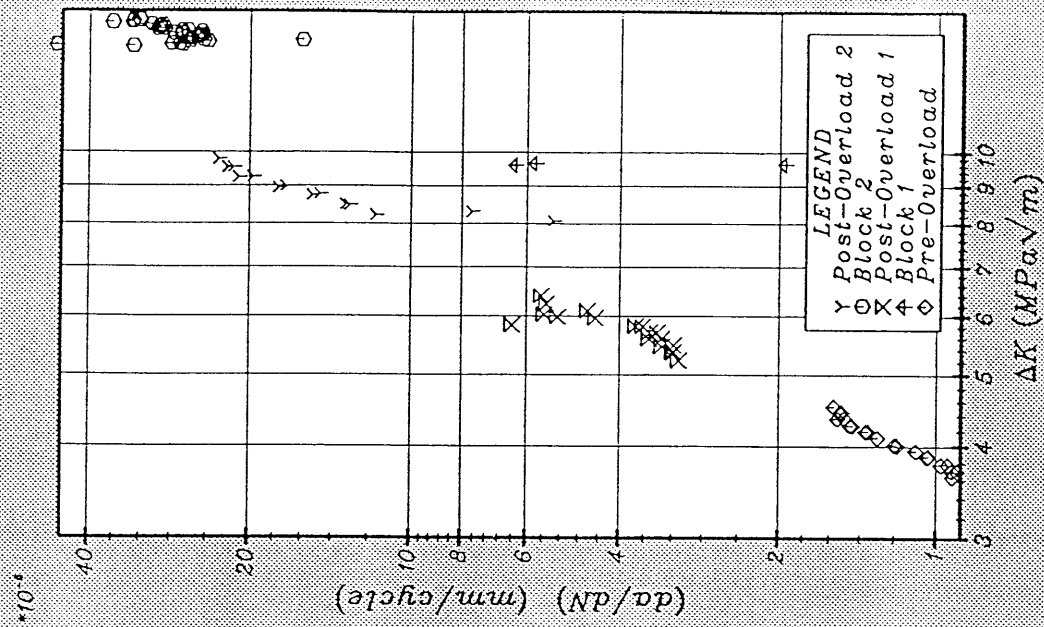
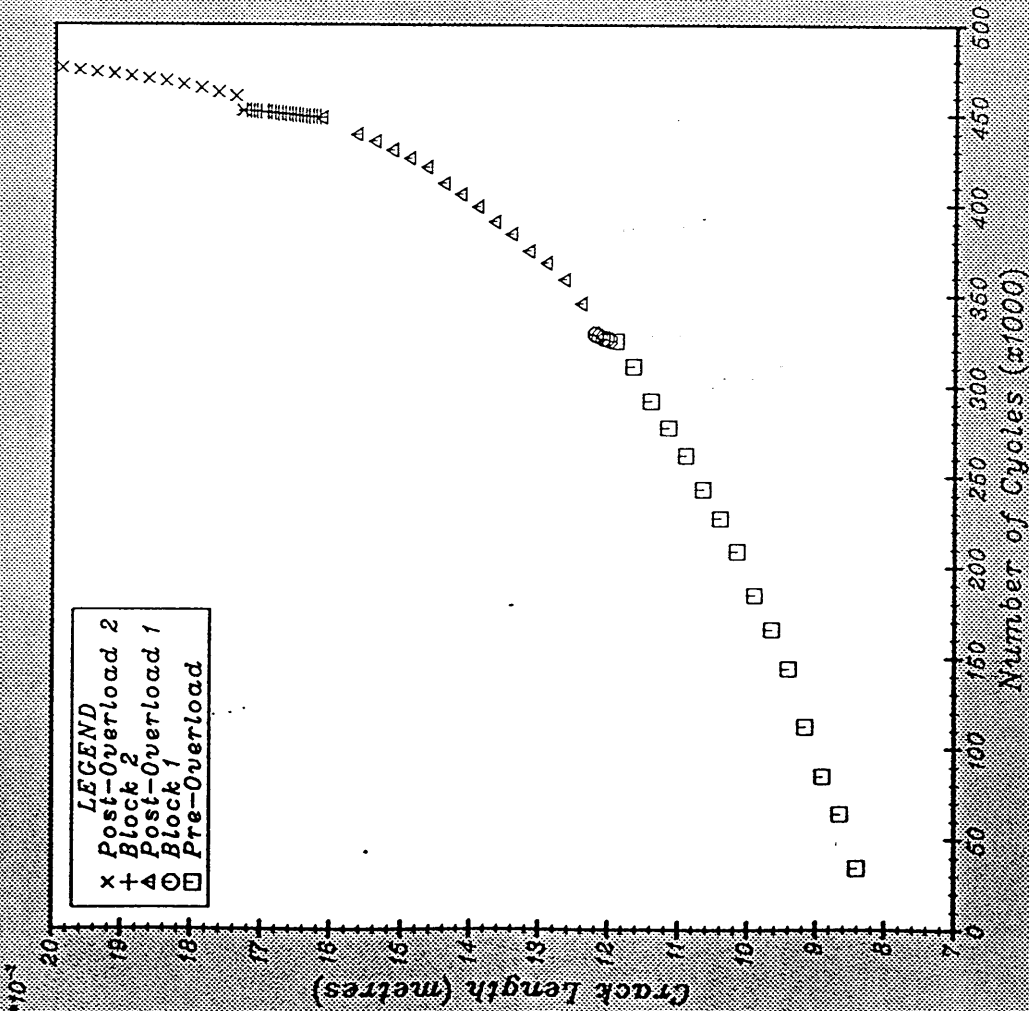
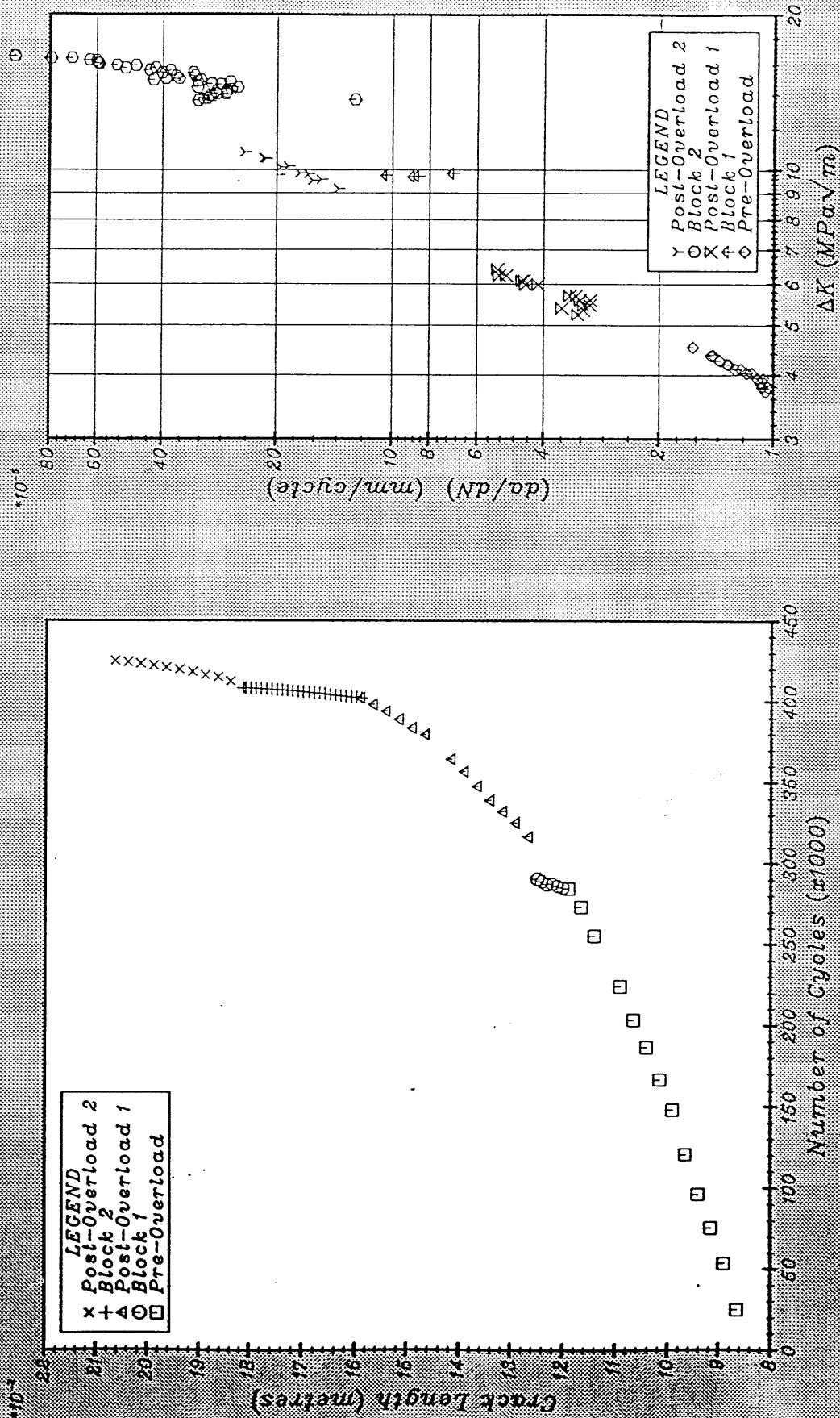
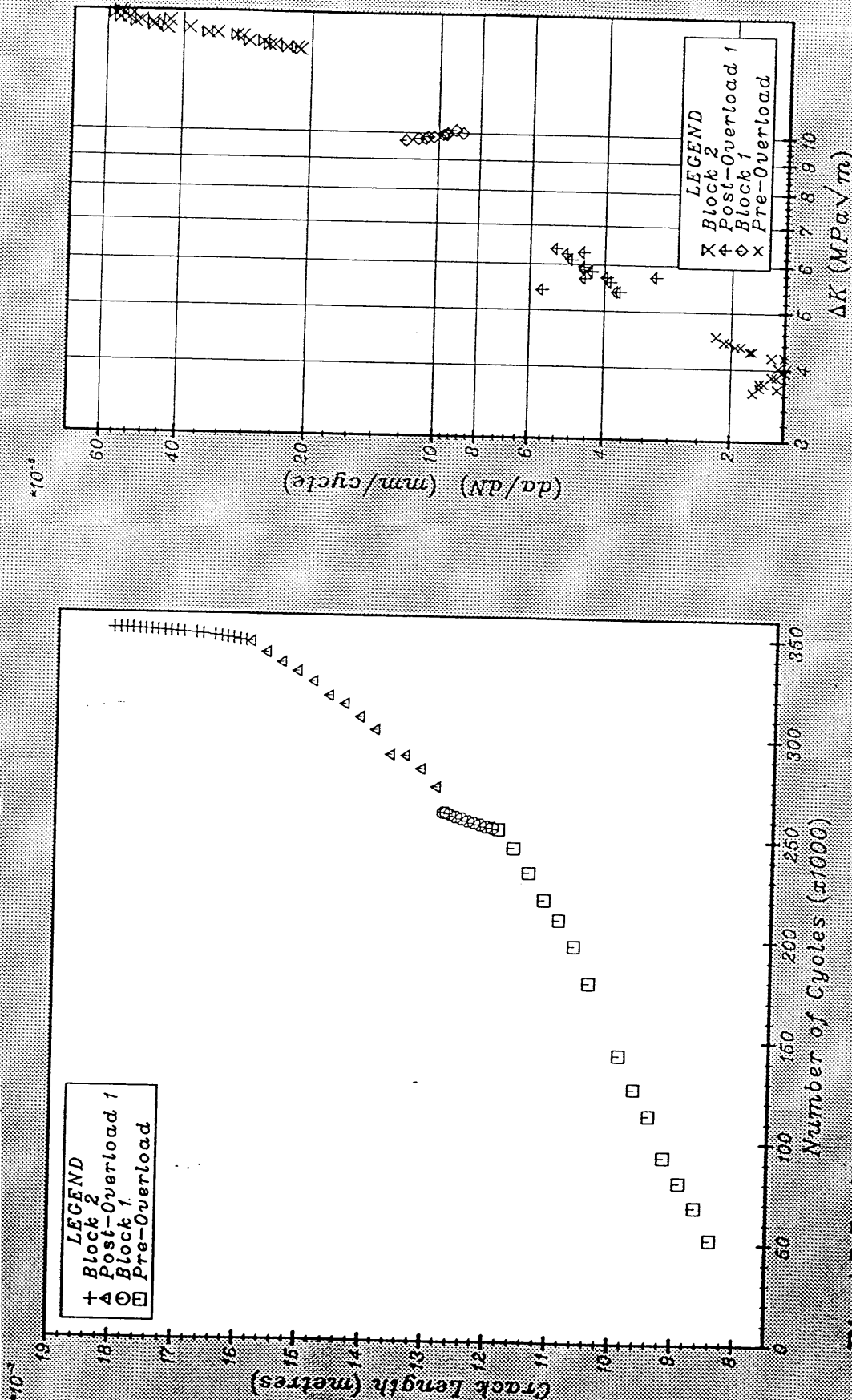


Fig.A5.3 Experimental fatigue crack growth data from a block overload test.
Block duration 4000 cycles
Overload Ratio $R_{ov}=1.4$



FigA5.4 Experimental fatigue crack growth data from a block overload test.
Block duration 6000 cycles
Overload Ratio $R_{ov}=1.4$



FigA5.5 Experimental fatigue crack growth data from a block overload test.
Block duration 8000 cycles
Overload Ratio $R_{ovl}=1.4$

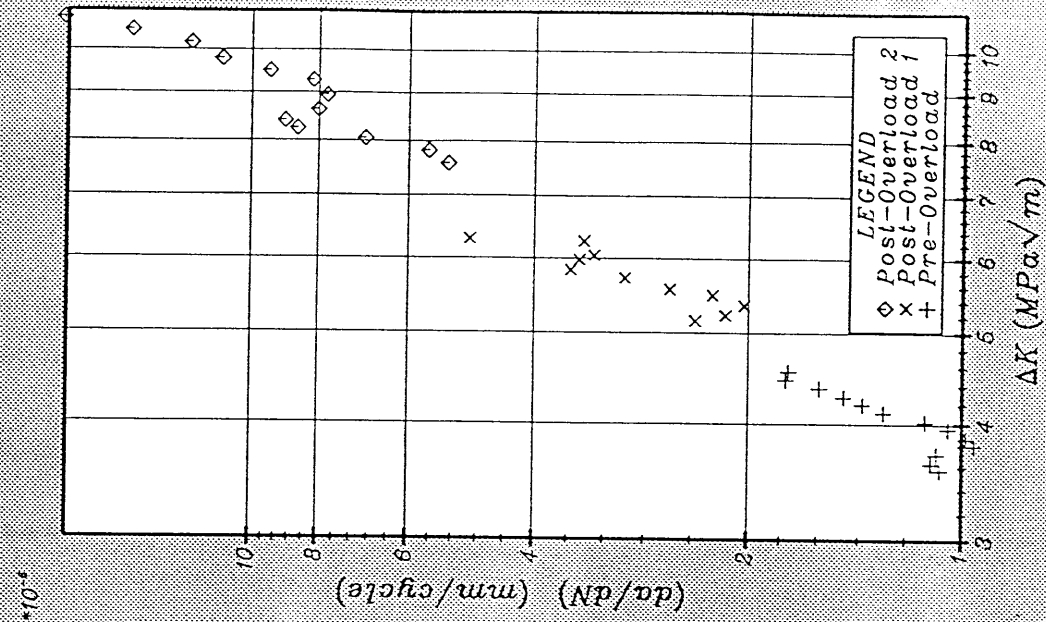
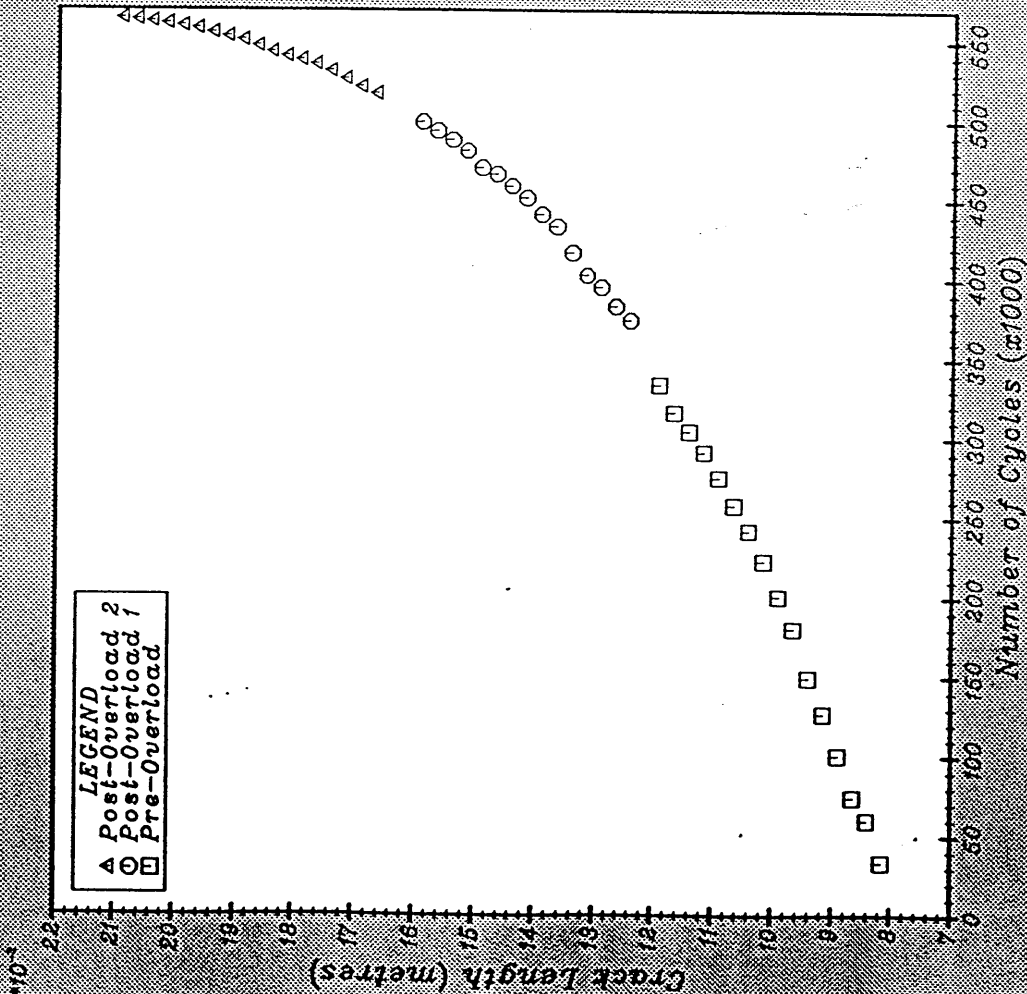
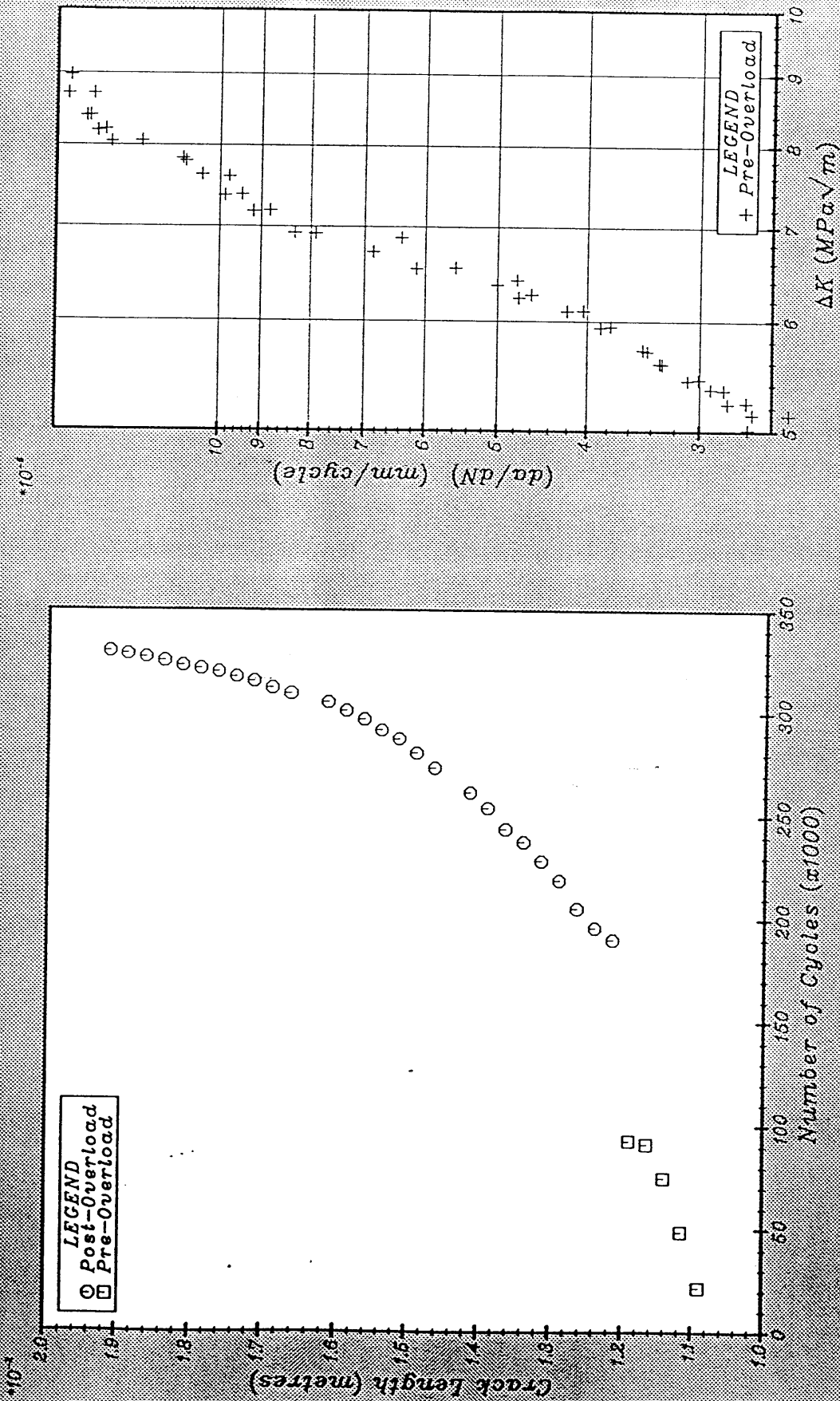


Fig.A5.6 Experimental fatigue crack growth data from a block overload test.
Block duration 1000 cycles
Overload Ratio $R_{ov}=1.6$



FigA5.7 Experimental fatigue crack growth data from a block overload test.
Block duration 2000 cycles
Overload Ratio $R_{ov}=1.6$

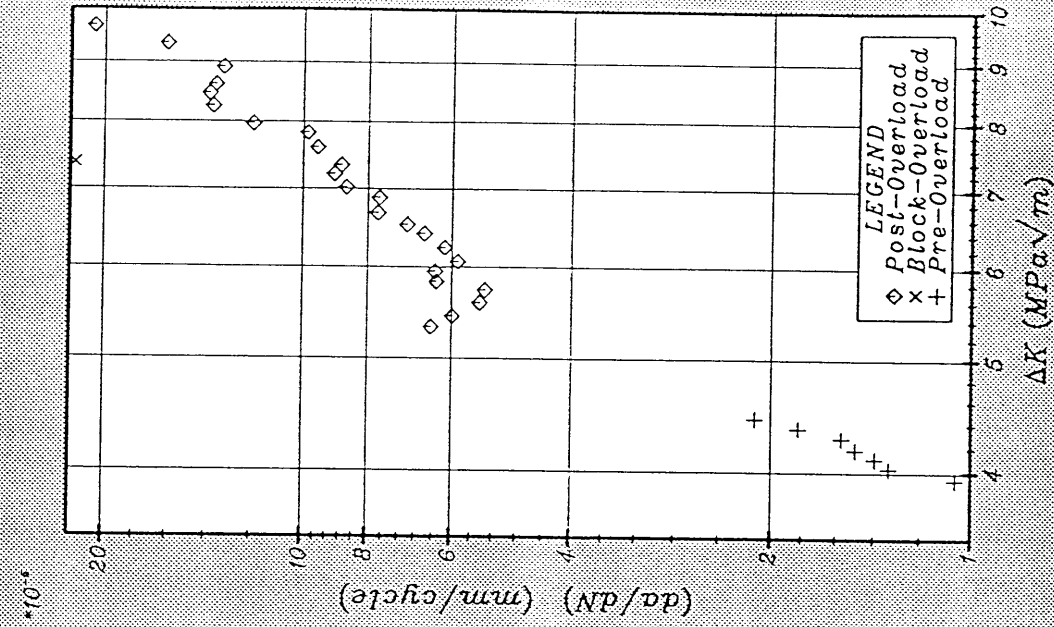
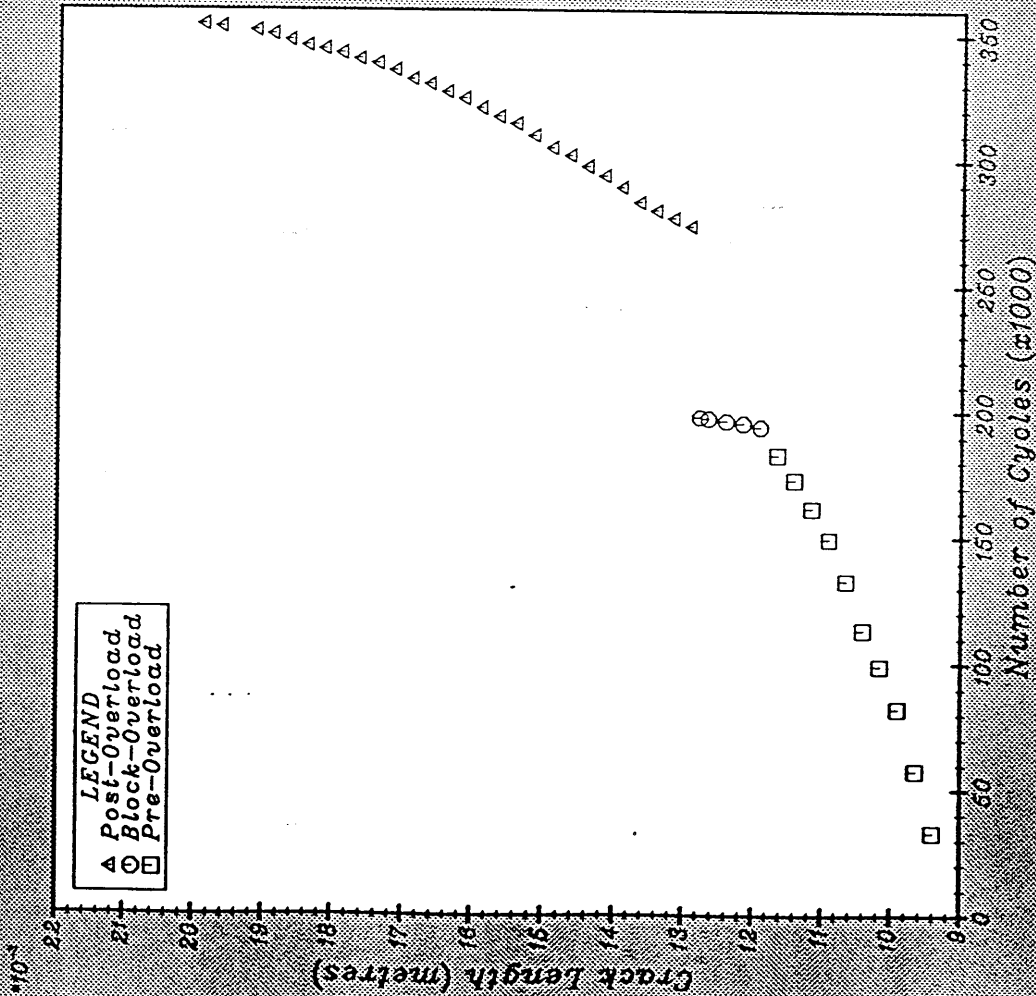


Fig.A5.8 Experimental fatigue crack growth data from a block overload test.
Block duration 4000 cycles
Overload Ratio $R_{ov}=1.6$

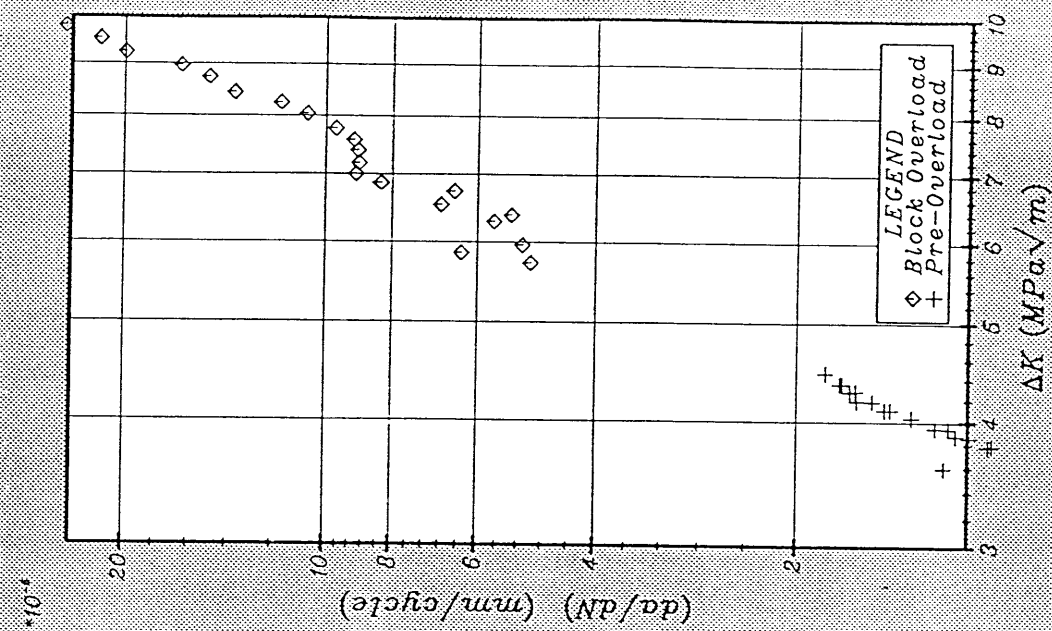
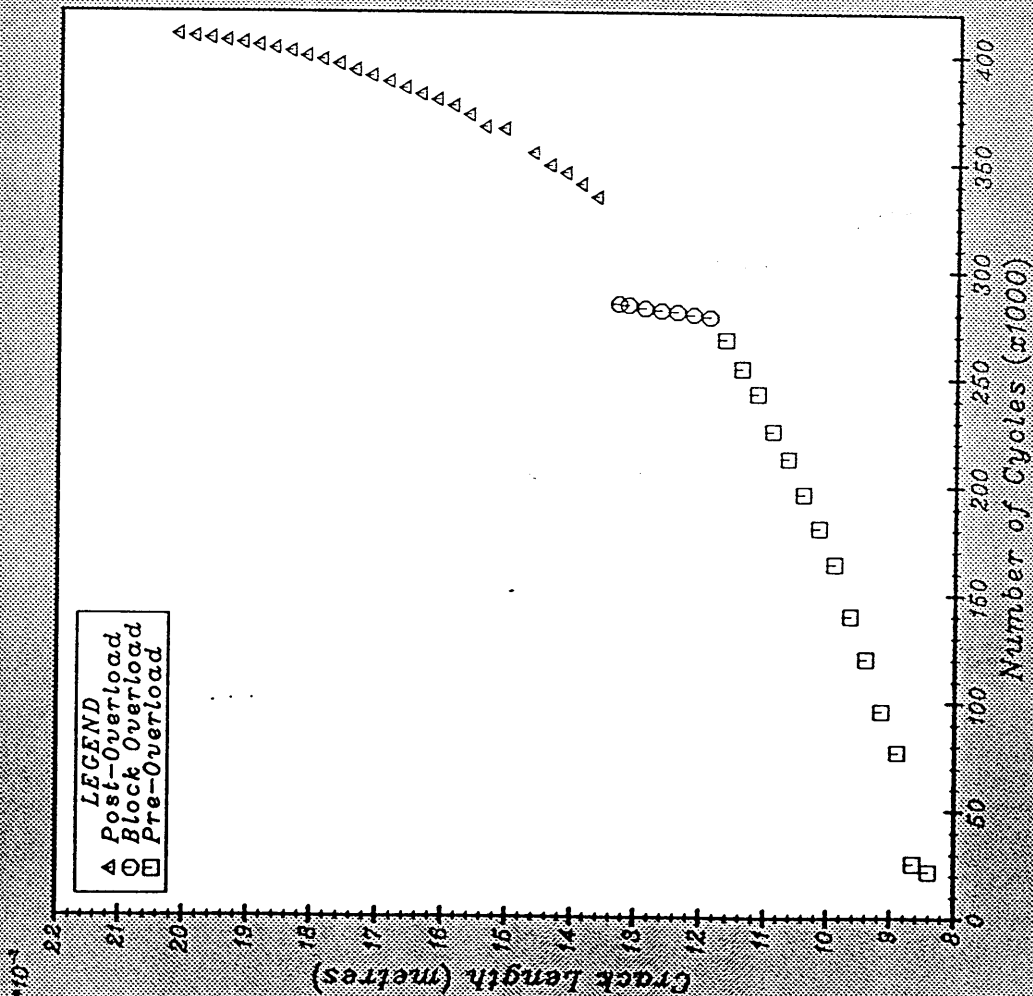
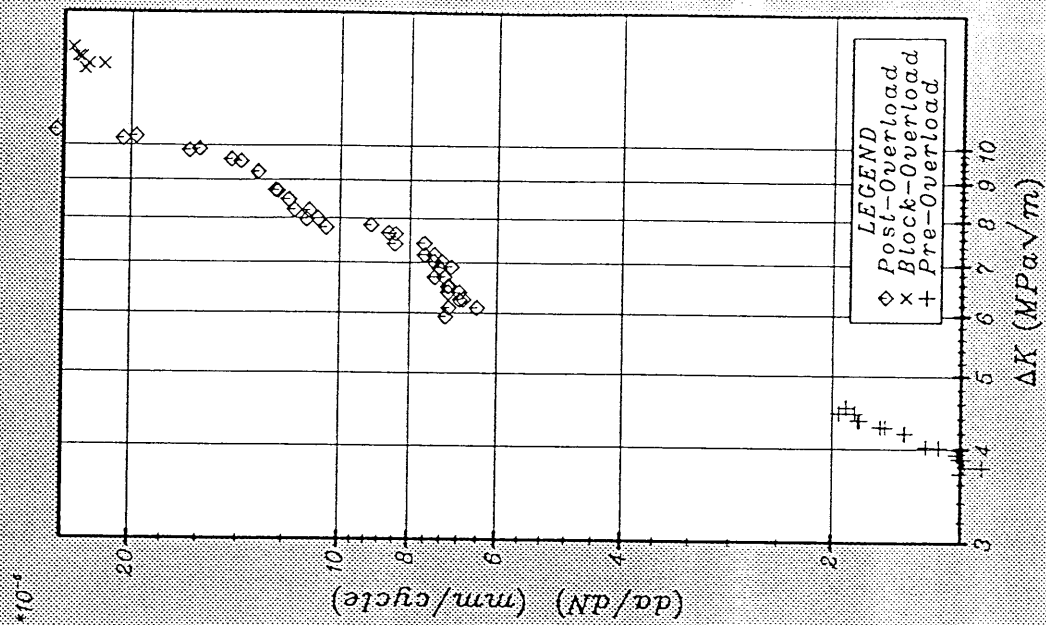


Fig.A5.9 Experimental fatigue crack growth data from a block overload test.
Block duration 6000 cycles
Overload Ratio $R_{o,1}=1.6$



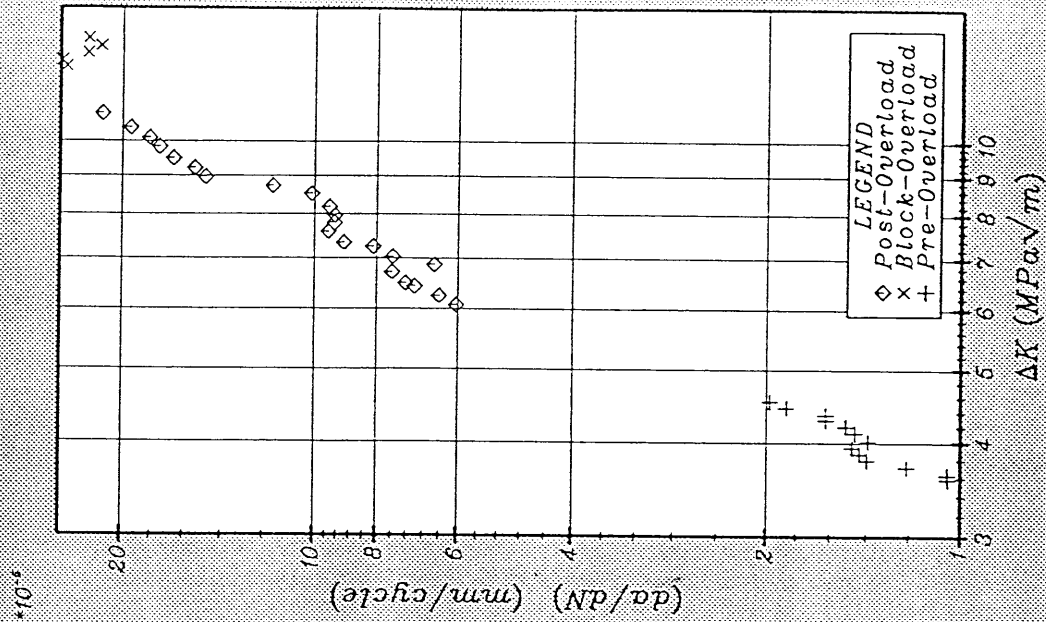
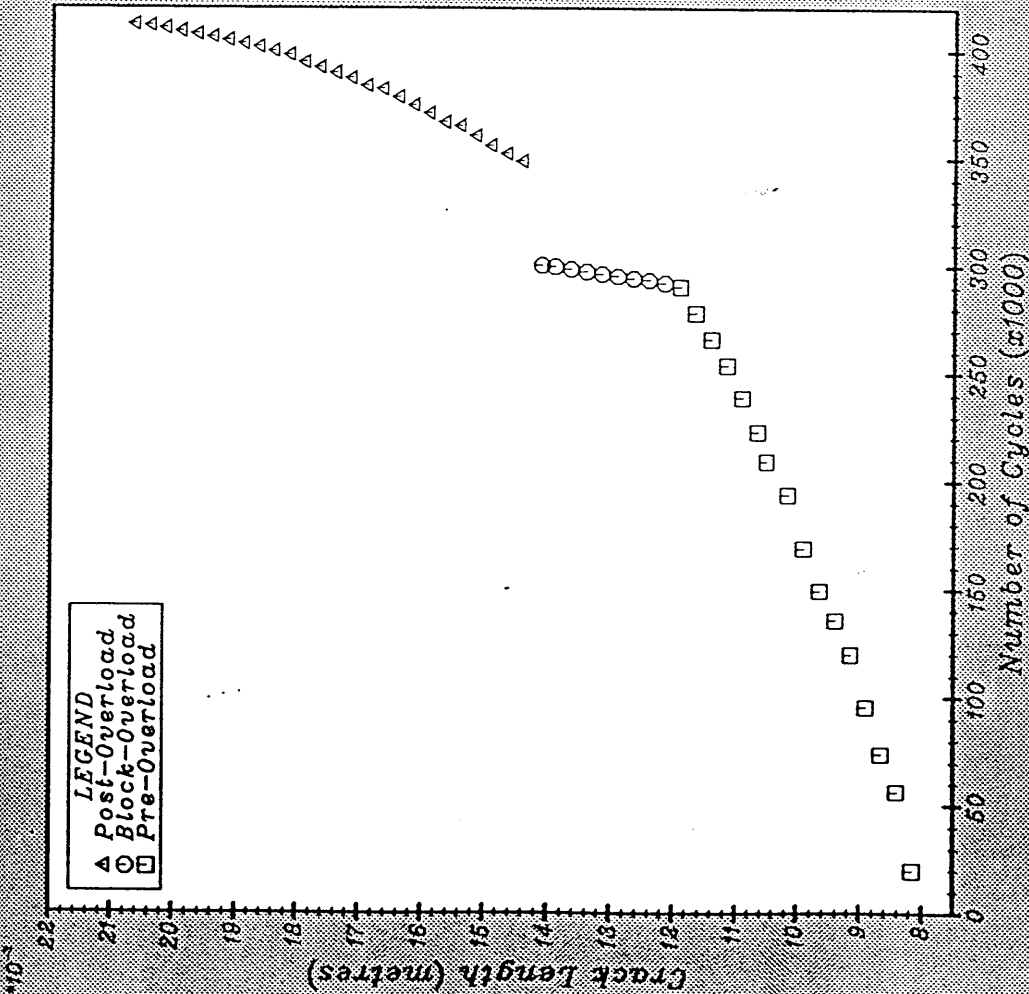


Fig.A5.11 Experimental fatigue crack growth data from a block overload test.
Block duration 10000 cycles
Overload Ratio $R_{o,n}=1.6$

A P P E N D I X 6

P r o g r a m s f o r R e d u c t i o n
o f E x p e r i m e n t a l D a t a f o r
D e t e r m i n a t i o n o f C r a c k G r o w t h
R a t e s

```

OPEN(1,ERR=20, FILE=INFILE,
+ STATUS='OLD',
+ ACCESS='SEQUENTIAL')
I open file.

ZZZ=1.0
I Just to skip the IF and continue to main
IF (ZZZ.NE.1.0) THEN I program procedures when no error.

20 CALL ERRORS (G,INFILE)

IF (G.EQ.1.0) THEN
GO TO 999
ELSE IF (G.EQ.2) THEN
GO TO 74
END IF
END IF

C .....
C INTERACTIVE INPUT OF TEST DETAILS
C .....

WRITE(6,221)
WRITE(6,105)
READ(5,(A15)) STN I test code number
WRITE(6,221)
WRITE(6,110)
READ(5,(A3)) TYPE

IF (TYPE.EQ.'CT'.OR.TYPE.EQ.'CCT') THEN
GO TO 51
ELSE
WRITE(6,*)
WRITE(6,*)
GO TO 50
END IF

51 WRITE(6,221)
WRITE(6,115)
READ(5,(F6.3)) B1
WRITE(6,221)
WRITE(6,120)
READ(5,(F6.3)) W1
WRITE(6,221)
WRITE(6,125)
READ(5,(F6.3)) An1
WRITE(6,221)
WRITE(6,130)
READ(5,(A50)) MAT
WRITE(6,221)
WRITE(6,135)
READ(5,(F7.3)) YS1
WRITE(6,221)
WRITE(6,140)
READ(5,(F6.3)) Pmin1
WRITE(6,221)
WRITE(6,145)
READ(5,(F6.3)) Pmax1
WRITE(6,221)
WRITE(6,150)
READ(5,(F6.2)) TEMP
WRITE(6,221)
WRITE(6,155)
READ(5,(A15)) ENV
WRITE(6,221)
WRITE(6,160)
READ(5,(F5.3)) FREQ
WRITE(6,221)

```

PROGRAM WRITTEN BY : STELIOS KYRIACOU
PHD APPLIED MECHANICS
CRANFIELD INST. OF TECH.

-1- THIS PROGRAM CARRIES OUT THE 5 & 7 POINT INCREMENTAL POLYNOMIAL REGRESSION REDUCTION OF EXPERIMENTAL DATA FROM FATIGUE TESTS. THE DATA FOR INPUT ARE THE GEOMETRY, LOADING CONDITION AND MATERIAL INFORMATION.

-2- A DATA FILE WITH THE TEST DATA OF CRACK LENGTH FROM THE NOTCH TIP(MM) AND THE NUMBER OF CYCLES ARE ONLY REQUIRED. THE PROGRAM CALCULATES THE DADN AND DELTA-K FOR EACH CRACK LENGTH AND GIVES THE PREDICTED CRACK LENGTH OF THE POLYNOMIAL FIT.

-3- THE DATA IS WRITTEN IN A DATA FILE WHOSE NAME IS CHOSEN BY THE USER, THIS INCLUDES A FULL REPORT OF THE TEST.

-4- IN ADDITION TO THE DATA REDUCTION BY THE TWO TECHNIQUES RECOMMENDED BY THE STANDARD, 2ND AND THIRD ORDER POLYNOMIALS ARE FITTED THROUGH THE EXPERIMENTAL (A-N) DATA. STATISTICAL ANALYSIS OF THE CURVE FITTING IS CARRIED OUT. IN ADDITION TO THIS DATA FILE 2 OTHER DATA FILES ARE CREATED WHERE DATA IS WRITTEN IN THE FORMAT REQUIRED BY THE PLOTTING ROUTINES.

PROGRAM ASTM I..Incremental polynomial data reduction...

CHARACTER*50 INFILE*20, STN*15, TYPE*3, ENV*15, MAT

DIMENSION DELK5(100), DELK7(100), I temporary arrays.

DADN5(100), DADN7(100),

AX5(100), AX7(100),

CYC(100), CRK(100),

CRACK(100), CYCLE(100),

F(15,16), R(3),

ARS(100), CYCLE3(100), AR7(100), CYCLE4(100),

APR(100), DADNPR(100), DELKPR(100)

DO 333 K=1,5

WRITE(6,*)

CONTINUE

333 WRITE(6,*)FIVE & SEVEN POINT INCREMENTAL

+ POLYNOMIAL DATA REDUCTION....

WRITE(6,*) Stellos Kyriacou

999 WRITE(6,221)

WRITE(6,*)

WRITE(6,1)

WRITE(6,2)

WRITE(6,1)

WRITE(6,3)

READ(5,(A20)) INFILE I read file name.

```
WRITE(6,221)
CALL DATAR (CYC, CRK, NTRUE, AN1) I read data from Input file
.....
END OF INTERACTIVE INPUT OF TEST DETAILS
.....
conversion to SI system of UNITS and definition of constants.
IORD=2 : I polynomial order.
N5=5
N7=7
PI=4.0*ATAN(1.0)
B=B1*1E-03 I conv. to metres
W=W1*1E-03 I conv. to metres
Y=YS*1E+06 I conv. to N/sqr.m
PMIN=PMINI*1E+03 I conv. to N
PMA=PMAX*1E+03 I conv. to N
PP=PMAX-PMIN
RR=PMINI/PMAX1
.....
***** END OF UNITS CONVERSION *****
.....
MAIN PROGRAM PROCEDURES
.....
CALL OUTPUT I define and open output file
CALL INTER (FOUR) I define and open plot files unit=8,9,10
WRITE(6,299)
.....
OUTPUT INTO THE DATA FILE
.....
WRITE(2,165)
WRITE(2,170) STN, NTRUE
WRITE(2,175) TYPE, B1, W1, AN1
WRITE(2,180) MAT
WRITE(2,185) YS1
WRITE(2,190) ENV, TEMP
WRITE(2,191) ENV, TEMP
WRITE(2,192) FREQ
WRITE(2,195) PMIN1, PMA=1, RR
WRITE(2,200)
WRITE(2,201)
.....
END OF OUTPUT INTO FILE
.....
.....
FIVE POINT INCREMENTAL POLYNOMIAL
DATA REDUCTION
.....
WRITE(2,250) I identifying header
WRITE(2,200)
```

```
WRITE(2,201)
DO 1005 I=1,2 I write into file first 2 data pairs.
WRITE(2,220) I,CYC(I),CRK(I)
1005 CONTINUE
NPSS=NTRUE-4
K=0
DO 5000 I=1,NPSS I=number of passes through the data.
I2=I+2 I Counter used for printing of data only.
CALL ZEROS (F,CYCLE,CRACK,R)
L=0
K=K+1
KP4=K+4
DO 1006 J=K, KP4
L=L+1
CRACK(L)=CRK(J) I conversion
CYCLE(L)=CYC(J) I of array names.
1006 CONTINUE
CALL POLREQ (CYCLE,CRACK,IORD,N5,F)
CALL GAUSS (F,IORD,R)
CALL STATS (CYCLE,CRACK,N5,IORD,R,SRT,SEE,COFDET,CORCOF,SDEV)
C
DADN5(I)=R(2)+2.0*R(3)*CYCLE(3) I calculate dA/dN at 3rd data point
CYCLES(I)=CYCLE(3) I array to write UNIT3 file.
AR=R(1)+R(2)*CYCLE(3)+R(3)*CYCLE(3)**2. I crack length predicted by reg.
AR5(I)=AR I array to write UNIT3 file.
S=1E+10 I this value is used in the case of CCT specimen for size req.
IF (TYPE.EQ.'CT') THEN
T=AR*W
FT=(2+T)/(0.886+4.64*T-13.32*T**2+14.72*T**3-5.6*T**4)/(1-T)**1.5
S=YS*SQRT((PI*W*(1-T))/2)
ELSE
T=2*AR*W
SEC=1.0/(COS(PI*T/2))
FT=SQRT((PI*T*SEC)/2)
SNET=PMAX1/(B*W*(1-T))
END IF
DELK5(I)=(1E-06)*(FT*PP)/(B*SQRT(W)) I converted to MPa.m(1/2)
AX5(I)=DELK5(I)/(1-RR)
IF (AX5(I).GE.S) GO TO 301 I* print
IF (SNET.GE.YS) GO TO 301 I* print
WRITE(2,205) I2, CYCLE(3), CRACK(3), AR, COFDET, DELK5(I), DADN5(I)
GO TO 5000
301 WRITE(2,210) I2, CYCLE(3), CRACK(3), AR, COFDET, DELK5(I), DADN5(I),
+ '...'
5000 CONTINUE
```

```

J=NPSS5+3
K=NPSS5+4

DO 1007 I=J,K      I write into file last 3 data pairs.
WRITE(2,220) I,CYC(I),CRK(I)

1007 CONTINUE
WRITE(2,215).
WRITE(2,'(,"")') I write onto next page.

C SEVEN POINT INCREMENTAL POLYNOMIAL
C DATA REDUCTION
C .....
WRITE(2,255)      I identifying header
WRITE(2,200)
WRITE(2,201)

DO 15 I=1,3 I write into file first 3 data pairs.
WRITE(2,220) I,CYC(I),CRK(I)

15 CONTINUE
NPSS7=NPSS7-6
K=0

DO 10000 I=1, NPSS7 I=number of passes through the data.
I=counter used for printing of data only.
CALL ZEROS (F,CYC,CRACK,R)
L=0
K=K+1
KP6=K+6
DO 500 J=K, KP6
L=L+1
CRACK(L)=CRK(J) I conversion
CYCLE(L)=CYC(J) I of array names.
500 CONTINUE

CALL POLREG (CYCLE,CRACK,IORD,N7,F)
CALL GAUSS (F,IORD,R)
CALL STATS (CYCLE,CRACK,N7,IORD,R,SRT,SEE,COFDET,CORCOR,SDEV)

C DADN7(I)=R(2)*2.0/R(3)*CYCLE(4) I calculate dA/dN at 4th data point.
CYCLE4(I)=CYCLE(4) I array to write UNIT3 file.
AR=R(1)*R(2)*CYCLE(4)*R(3)*CYCLE(4)**2. I crack length predicted by reg.
AR7(I)=AR I array to write UNIT3 file.
S=1E+10 I this value is used in the case of CCT specimen for size reg.
IF (TYPE.EQ.'CT') THEN
T=ARW
FT=(2+T)/(0.886-4.64*T-13.32*T**2+14.72*T**3-5.6*T**4)/(1-T)**1.5
S=Y5*SQRT(PI*W*(1-T))/2
ELSE

```

```

T=2*ARW
SEC=1.0/(COS(PI*T/2))
FT=SQRT(PI*T*SEC/2)
SNET=PMAX1/(B*W*(1-T))
END IF

DELK7(I)=(1E-06)*(FT**PP)/(B*SQRT(W)) I converted to MPa.m(1/2)
AX7(I)=DELK7(I)/(1-RR)

IF (AX7(I).GE.S) GO TO 30 I* print
IF (SNET.GE.Y5) GO TO 30 I* print

WRITE(2,205) I3, CYCLE(4), CRACK(4), AR, COFDET, DELK7(I), DADN7(I)

GO TO 10000
30 WRITE(2,210) I3, CYCLE(4), CRACK(4), AR, COFDET, DELK7(I), DADN7(I),
+ '...'

10000 CONTINUE

J=NPSS7+4
K=NPSS7+6

DO 17 I=J,K I write into file last 3 data pairs.
WRITE(2,220) I,CYC(I),CRK(I)

17 CONTINUE
WRITE(2,215)
WRITE(2,'(,"")') I write onto next page.

C .....
C END OF DATA REDUCTION
C .....

CALL CALCS (CYC, CRK, NTRUE, W, B, PP, APP, DADNPR, DELKPR)
CLOSE(2,STATUS='KEEP') I close and save the report file UNIT 2

C .....
C UNIT 3 PLOT FILE a vs N
C .....
C DO 35 I=1, NTRUE I experimental data.
C WRITE(3,'') CYC(I), CRK(I)
C35 CONTINUE
C WRITE(3,'') 999, 999
C
C DO 37 K=1, NPSS5 I 5 point regression.
WRITE(3,'') CYCLE3(K), AR5(K)
CONTINUE
37 WRITE(3,'') 999, 999
C
C DO 38 L=1, NPSS7 I 7 point regression.
WRITE(3,'') CYCLE4(L), AR7(L)
CONTINUE
38 WRITE(3,'') 999, 999
C
C .....
C UNIT 4 PLOT FILE dA/dN vs Delta K
C .....
C
DO 39 I=1, NPSS5 I 5 point regression.
WRITE(4,'') DELK5(I), DADN5(I)

```



```

C C INFORMATION INTO TWO ARRAYS N(I) & A(I)
C N: number of cycles
C A: crack length measured from notch tip
C N: total number of data points read
C .....
C C
C C SUBROUTINE DATAR (CYCLES,CRACK,NTRUE,AN1)
C DIMENSION CYCLES(100), CRACK(100)
C
C L=0
C DO 1 I=1,150
C READ(1,FMT=*,END=2) CYCLES(I), CRACK(I)
C L=L+1
C CYCLES(I)=CYCLES(I)/1000 ! Scale cycles: division by 1000
C CRACK(I)=(CRACK(I)+AN1)/1000
C CONTINUE
C NTRUE=L ! The counter is used as the true number of points
C ! in the data set read.
C CLOSE(1,STATUS='KEEP') ! Close and save the input file
C RETURN
C END
C .....
C C SUBROUTINE CALCS : USES THE EXPERIMENTAL DATA
C C TO FIT A SECOND ORDER POLYNOMIAL AND CARRIES
C C OUT CALCULATIONS TO PREDICT THE CRACK LENGTHS,
C C DADN AND DELTA-K VALUES FROM THE REGRESSION
C C FIT. A STATISTICAL ANALYSIS IS ALSO PROVIDED.
C C .....
C C SUBROUTINE CALCS (CYC, CRK, NTRUE, W, B, PP, APR, DADN, DELK)
C DIMENSION CYC(NTRUE), CRK(NTRUE), R(10), A(15,16), APR(100),
C + DADN(100), T(100), FT(100), DELK(100)
C DO 999 IORD=2,5 ! Carry out the data reduction by fitting polynomials
C ! of orders of 2 to 5.....
C CALL POLREG (CYC, CRK, IORD, NTRUE, A)
C CALL GAUSS (A, IORD, R)
C CALL STATS (CYC, CRK, NTRUE, IORD, R, SRT, SEE, COFDET, CORCOF, SDEV)
C DO 10 I=1, NTRUE
C IF (IORD.EQ.1) THEN
C APR(I)=R(1)+R(2)*CYC(I)
C DADN(I)=R(1)
C ELSE IF (IORD.EQ.2) THEN
C APR(I)=R(1)+R(2)*CYC(I)+R(3)*(CYC(I)**2)
C DADN(I)=R(2)+2.0*CYC(I)*R(3)
C ELSE IF (IORD.EQ.3) THEN
C APR(I)=R(1)+R(2)*CYC(I)+R(3)*(CYC(I)**2)+R(4)*(CYC(I)**3)
C DADN(I)=R(2)+2.*R(3)*CYC(I)+3.*R(4)*(CYC(I)**2)

```

```

WRITE(6,540)
READ(5,(A10)) OUT3 ! Input name of the out3 file.
OPEN(7,FILE=OUT3,
+ STATUS='NEW',
+ ACCESS='SEQUENTIAL') ! Open out3 file.
C .....
C C FORMATS FORMATS FORMATS FORMATS FORMATS
C .....
C C
C 499 FORMAT(//,5X,"" No Graphical out-put has been requested ""//,
+ 5X,"" TYPE <CR> In following questions ""//)
C 500 FORMAT(5X,"Do you wish to obtain graphical output ?"/,
+ 5X,""....."/,5X,"Please type Y or N"/,
+ 5X,"".....")
C 520 FORMAT(//,5X,"What is the name of the output file"/,
+ 5X,"for (a vs N) data?"/,,"" up to 10 Characters"/,
+ 5X,"Type 'filename.type' up to 10 Characters"/)
C 530 FORMAT(//,5X,"What is the name of the output file"/,
+ 5X,"for (dA/dN vs delta-K) data?"/,
+ 5X,"Type 'filename.type' up to 10 Characters"/)
C 540 FORMAT(//,5X,"What is the name of the output file"/,
+ 5X,"for (dA/dN vs A) data?"/,
+ 5X,"Type 'filename.type' up to 10 Characters"/)
C RETURN
C END
C .....
C C SUBROUTINE ZEROS, TO RE-INITIALISE THE ARRAYS
C C TO THE VALUE OF ZERO SO THAT THE SUBSEQUENT
C C INCREMENTED DATA SETS ARE STORED.
C C .....
C C SUBROUTINE ZEROS (F,CYCLE,CRACK,R)
C DIMENSION F(15,16), CYCLE(100), CRACK(100), R(3)
C DO 10 I=1,100
C CRACK(I)=0.0
C CYCLE(I)=0.0
C CONTINUE
C 10
C DO 12 I=1,15
C DO 11 J=1,16
C F(I,J)=0.0
C CONTINUE
C 12
C DO 14 I=1,3
C R(I)=0.0
C CONTINUE
C 14
C RETURN
C END
C .....
C C SUBROUTINE DATAR, READS THE DATA FROM THE DATA FILE AND STORES THE

```

```

ELSE IF (IORD.EQ.4) THEN
  APR(I)=R(1)*R(2)*CYC(I)*R(3)*(CYC(I)**2)+R(4)*(CYC(I)**3.)
  + R(5)*(CYC(I)**4.0)
  + DADN(I)=R(2)*2.*R(3)*CYC(I)+3.*R(4)*(CYC(I)**2.)
  + 4.*R(5)*(CYC(I)**3.)
  + 5.*R(6)*(CYC(I)**4.)
ELSE IF (IORD.EQ.5) THEN
  C- new section.....
  APR(I)=R(1)+R(2)*CYC(I)+R(3)*(CYC(I)**2)+R(4)*(CYC(I)**3)
  + R(5)*(CYC(I)**4)+R(6)*(CYC(I)**5)
  + DADN(I)=R(2)+
  + 2.*R(3)*CYC(I)+
  + 3.*R(4)*(CYC(I)**2)+
  + 4.*R(5)*(CYC(I)**3)+
  + 5.*R(6)*(CYC(I)**4)
END IF
T(I)=APR(I)/W
FT(I)=(2+T(I))*(0.882+4.64*T(I)-13.32*(T(I)**2)+14.72*(T(I)**3.)
-5.6*(T(I)**4.))/((1-T(I))**1.5)
DELK(I)=(1.E-06)*(FT(I)*PP)/(B*SQRT(W))
10 CONTINUE
WRITE(2,51) IORD
WRITE(2,54)
DO 79 I=1,IORD+1
  WRITE(2,55) I-1, R(I)
  CONTINUE
79 CONTINUE
WRITE(2,56)
WRITE(2,57) SDEV I STANDARD DEVIATION
WRITE(2,58) SEE I STANDARD ERROR OF ESTIMATE
WRITE(2,59) COFDET I COEFFICIENT OF DETERMINATION
WRITE(2,60) CORCOF I CORRELATION COEFFICIENT
WRITE(2,70)
WRITE(2,80)
DO 15 J=1,NTRUE
  WRITE(2,90) J, CYC(J), CRK(J), APR(J), DELK(J), DADN(J)
  WRITE (3,1) CYC(J), APR(J)
  WRITE (4,1) DELK(J), DADN(J)
  WRITE (7,1) APR(J), DADN(J)
15 CONTINUE
WRITE (3,1) 999, 999
WRITE (4,1) 999, 999
WRITE (7,1) 999, 999
DO 3 JJ=1,100
  APR(JJ)=0.0
  DELK(JJ)=0.0
  DADN(JJ)=0.0
  CONTINUE
3 IF (IORD.LT.5) THEN
  WRITE(2,('1,1')) I write onto next page.
END IF
999 CONTINUE

```

```

C ===== FORMATS ===== FORMATS
C ===== FORMATS ===== FORMATS
51 FORMAT(/5X,'"',1,' ORDER REGRESSION POLYNOMIAL PREDICTIONS "')
54 FORMAT(/5X,'Coefficients of the polynomial in ascending order',
/5X,'-----')
55 FORMAT(/20X,'A(',12,')=' ,E20.14)
56 FORMAT(/5X,'DATA STATISTICS:',/5X,'=====')
57 FORMAT(/5X,'STANDARD DEVIATION =',F10.6)
58 FORMAT(5X,'STANDARD ERROR OF ESTIMATE =',F10.6)
59 FORMAT(5X,'COEFFICIENT OF DETERMINATION =',F10.6)
60 FORMAT(5X,'CORRELATION COEFFICIENT =',F10.6)
'0 FORMAT(/2X,'OBS.No',5X,'CYCLES',5X,'A(Meas)',5X,'A(Reg)',
+ 5X,'Delta K',5X,'dA/dN')
80 FORMAT(12X,'(x1000)',7X,'(m)',10X,'(m)',6X,'(MPam*.5)',
+ 3X,'mm/Cycle',/)
90 FORMAT(3X,13,5X,F8.3,4X,F8.5,4X,F8.5,5X,F7.3,4X,E10.3)
RETURN
END
*****
C SUBROUTINE POLREG, TO READ A SET OF NUMBERS FROM A DATA FILE
C AND FORM THE COEFFICIENT AUGMENTED MATRIX OF THE NORMAL EQUATIONS
C REQUIRED TO SOLVE THE POLYNOMIAL REGRESSION EQUATIONS THROUGH A SET
C OF DATA POINTS.
C X : X coordinate
C Y : Y coordinate
C A : Augmented matrix of coefficients
C N : Number of data points
C IORD : Order of polynomial to be used for curve fitting
*****
SUBROUTINE POLREG (X,Y,IORD,N,A)
DIMENSION X(100),Y(100),A(15,16)
-----
DO 2100 sets the matrix row by row
DO 2060 sets the columns
DO 2050 calculates the coefficient of the equations
DO 2090 calculates the right hand side coefficients
-----
DO 1 I=1,15
DO 2 J=1,16
  A(I,J)=0.0 I Initialise A(I,J) matrix
  CONTINUE
2 1 CONTINUE
IP=IORD+1
DO 2100 I=1,IP
  DO 2060 J=1,JP

```

```

14      CONTINUE
15      CONTINUE
20      CONTINUE

C      BACK SUBSTITUTION STAGE
C      -----
C      R(N)=A(N,NP1)
C      DO 30 K=2,N
C      I=NP1-K
C      R(I)=A(I,NP1)
C      IP1=I+1
C      DO 30 J=IP1,N
C      R(I)=R(I)-A(I,J)*R(J)
30      CONTINUE

RETURN
END

C      SUBROUTINE STATS, TO CARRY OUT STATISTICAL ANALYSIS OF THE
C      REGRESSION POLYNOMIALS.
C      IT CALCULATES THE SUM OF THE SQUARES OF THE RESIDUALS ABOUT THE
C      MEAN AND THE SUM OF THE SQUARE OF THE RESIDUALS ABOUT THE REGRESSION
C      LINE (CURVE).
C      STATS (N,Y,A,IORD,SRT,SEE,CORCOF,CORCOF)
C      N : Number of data points
C      Y : Array of dependent variable
C      A : Coefficients of the polynomial
C      IORD : Order of the polynomial
C      SRT : Sum of sq. of residuals about the regression curve
C      SEE : Standard error of estimate
C      CORCOF : Coefficient of determination
C      CORCOF : Correlation coefficient
C      -----

SUBROUTINE STATS (X,Y,N,IORD,A,SRT,SEE,CORCOF,CORCOF,SDEV)
  DIMENSION SR(150), B(150),
  + X(100), Y(100), A(10)

  SUM1=0.0
  DO 50 I=1,N
  SUM1=SUM1+Y(I)
  CONTINUE
  AVER=SUM1/N      I MEAN OF DEPENDENT VARIABLE

  SUM2=0.0
  DO 60 I=1,N
  ST=(Y(I)-AVER)**2
  SUM2=SUM2+ST
  CONTINUE

  SRES=SUM2
  ZZ=N-1
  I SUM OF SQ. OF RESIDUALS ABOUT THE MEAN
  I change variable from integer to real
  VAR=SRES/ZZ      I VARIANCE
  SDEV=SQRT(VAR)   I STANDARD DEVIATION

  C      CALCULATION OF SUM OF SQUARES OF RESIDUALS ABOUT THE REGRESSION
  C      LINE (CURVE).
  C      -----
  SUM3=0.0
  DO 70 I=1,N

```

```

K=I+J-2
DO 2050 L=1,N
IF (K.EQ.0.0) THEN
  A(I,J)=A(I,J)+1
ELSE
  A(I,J)=A(I,J)+X(L)**K
END IF
CONTINUE
CONTINUE

2050
2060
C
DO 2090 L=1,N
IR=IP1
IF ((I-1).EQ.0.0) THEN
  A(I,IR)=A(I,IR)+Y(L)
ELSE
  A(I,IR)=A(I,IR)+Y(L)*(X(L)**(I-1))
END IF
CONTINUE
CONTINUE

2090
2100
RETURN
END

C      SUBROUTINE GAUSS, TO CARRY OUT THE GAUSSIAN ELIMINATION
C      FOR THE SOLUTION OF THE SYSTEM OF LINEAR SIMULTANEOUS
C      EQUATIONS.
C      GAUSS (A,IORD,R)
C      A : Augmented matrix of coefficients
C      IORD : Order of polynomial fitted
C      R : Roots of normal equations, coefficients of polynomial
C      -----

SUBROUTINE GAUSS (A,IORD,R)
  DIMENSION A(15,16), R(10)

  DIMENSION B(15,16)
  N=IORD+1
  NP1=N+1

  C      this DO Loop saves the A(I,J) into B(I,J) so that
  C      it may be used to check the solution, it can be omitted.
  C      -----
  DO 1000 I=1,N
  DO 1000 J=1,NP1
  B(I,J)=A(I,J)
  CONTINUE

  C      ELIMINATION STAGE
  C      -----
  DO 20 K=1,N
  KP1=K+1
  DO 10 J=KP1,NP1
  A(K,J)=A(K,J)/A(K,K)
  CONTINUE
  IF (K.EQ.N) GO TO 20
  DO 15 I=KP1,N
  DO 14 J=KP1,NP1
  A(I,J)=A(I,J)-A(I,K)*A(K,J)

```

```

IP=IORD+1
SUM4=0.0
DO 65 J=1,IP
  K=J-1
  IF (J.EQ.1.0) THEN
    B(J)=A(J)
  ELSE
    B(J)=A(J)*X(I)**K
  END IF
  SUM4=SUM4+B(J)
65  CONTINUE
  SR(I)=(Y(I)+SUM4)**2
  SUM3=SUM3+SR(I)
70  CONTINUE
  SRT=SUM3
  ZZ=N-(IORD+1)
  SEE=SQRT(SRT/ZZ)
  COFDET=(SRES-SRT)/SRES
  CORCOF=SQRT(ABS(COFDET))
  RETURN
END
      ***** END OF PROGRAM *****
000

```

FIVE & SEVEN POINT INCREMENTAL POLYNOMIAL METHOD FOR
DETERMINING dA/dN AND Delta-K IN ACCORDANCE TO ASTM E-647

Spec.No. CA/018/R03 No. Points= 54

CT Specimen B= 8.000 mm W= 32.000 mm An= 6.400 mm

Material : ALUMINIUM ALLOY

Material Yield Stress = 469.000MPa

= T E S T - R E P O R T =

Environment :Air Test Temperature : 10.60 deg.C

Test Frequency = 50.000Hz

Pmin= 0.462KN Pmax= 1.538KN R= 0.300

*** 5-POINT INCREMENTAL POLYNOMIAL METHOD ***

OBS.No	CYCLES (x1000)	A(Meas.) (m)	A(Reg.) (m)	M.C.C.	Delta K (MPa ^m .5)	dA/dN (mm/Cycle)
1	0.000	0.00840				
2	28.250	0.00865			3.904	0.754E-05
3	56.890	0.00890		.99693	4.079	0.705E-05
4	100.750	0.00915		.90430	4.171	0.766E-05
5	134.280	0.00940		.90900	4.223	0.827E-05
6	151.540	0.00965		.90566	4.337	0.825E-05
7	194.530	0.00990		.90325	4.423	0.773E-05
8	224.850	0.01015		.99706	4.497	0.844E-05
9	247.940	0.01040		.99226	5.009	0.102E-04
21	516.250	0.01365		.99200	5.900	0.191E-04
22	525.210	0.01390		.99332	5.800	0.196E-04
23	541.980	0.01415		.99096	6.148	0.214E-04
24	553.640	0.01440		.95110	6.270	0.178E-04
25	555.340	0.01465		.99733	6.539	0.265E-04
26	574.080	0.01490		.99849	6.715	0.286E-04
27	583.540	0.01515		.99936	6.868	0.296E-04
28	592.380	0.01540		.99758	7.019	0.308E-04
29	599.810	0.01565		.99583	7.402	0.358E-04
30	608.420	0.01590		.99383	7.608	0.409E-04
31	616.060	0.01615		.99236	7.766	0.572E-04
32	622.280	0.01640		.99022	7.804	0.733E-04
33	627.380	0.01665		.98862	8.107	0.609E-04
34	631.410	0.01690		.98622	8.340	0.574E-04
35	634.220	0.01715		.99292	8.642	0.562E-04
36	638.840	0.01740		.99426	8.835	0.594E-04
37	644.410	0.01765		.99840	9.064	0.655E-04
38	647.950	0.01790		.99575	9.379	0.680E-04
39	651.750	0.01815		.99879	9.652	0.760E-04
40	656.100	0.01840		.90196	10.272	0.930E-04
41	659.380	0.01865		.99428	10.272	0.111E-03
42	662.450	0.01890		.99898	10.567	0.132E-03
43	664.840	0.01915		.99842	10.839	0.143E-03
44	666.760	0.01940		.99184	11.236	0.149E-03
45	668.280	0.01965		.99184	11.646	0.160E-03
46	670.180	0.01990		.98463	12.079	0.190E-03
47	671.850	0.02015		.99223	12.494	0.229E-03
48	673.330	0.02040		.99223	12.850	0.250E-03
49	674.480	0.02065		.99908	13.904	0.271E-03
50	675.350	0.02090				
51	677.400	0.02140				

*** 7-POINT INCREMENTAL POLYNOMIAL METHOD ***

OBS.No	CYCLES (x1000)	A(Meas.) (m)	A(Reg.) (m)	M.C.C.	Delta K (MPa ^m .5)	dA/dN (mm/Cycle)
1	0.000	0.00840				
2	28.250	0.00865				
3	56.890	0.00890				
4	100.750	0.00915		.99342	4.089	0.766E-05
5	134.280	0.00940	0.00919	.99408	4.173	0.703E-05
6	151.540	0.00965	0.00957	.99382	4.215	0.763E-05
7	194.530	0.00990	0.00993	.99494	4.342	0.814E-05
8	224.850	0.01015	0.01018	.99473	4.427	0.809E-05
9	247.940	0.01040	0.01034	.99606	4.486	0.827E-05
21	516.250	0.01365	0.01368	.99727	5.893	0.187E-04
22	525.210	0.01390	0.01384	.99725	5.974	0.181E-04
23	541.980	0.01415	0.01417	.99635	6.145	0.205E-04
24	553.640	0.01440	0.01441	.99640	6.277	0.222E-04
25	555.340	0.01465	0.01465	.99911	6.410	0.246E-04
26	574.080	0.01490	0.01488	.99946	6.543	0.267E-04
27	583.540	0.01515	0.01514	.99944	6.703	0.283E-04
28	592.380	0.01540	0.01540	.99891	6.860	0.299E-04
29	599.810	0.01565	0.01565	.99346	7.609	0.231E-04
30	608.420	0.01590	0.01590	.99843	7.196	0.345E-04
31	616.060	0.01615	0.01630	.94596	7.475	0.339E-04
32	622.280	0.01640	0.01631	.96157	7.488	0.575E-04
33	627.380	0.01665	0.01675	.97006	7.822	0.491E-04
34	631.410	0.01690	0.01695	.99005	7.977	0.554E-04
35	634.220	0.01715	0.01709	.99402	8.096	0.587E-04
36	638.840	0.01740	0.01736	.99404	8.332	0.590E-04
37	644.410	0.01765	0.01770	.99500	8.635	0.591E-04
38	647.950	0.01790	0.01792	.99527	8.051	0.594E-04
39	651.750	0.01815	0.01816	.99399	9.009	0.639E-04
40	656.100	0.01840	0.01845	.99540	9.391	0.717E-04
41	659.380	0.01865	0.01872	.90420	9.690	0.776E-04
42	662.450	0.01890	0.01899	.97340	10.013	0.895E-04
43	664.840	0.01915	0.01924	.98112	10.313	0.107E-03
44	666.760	0.01940	0.01946	.98720	10.603	0.123E-03
45	668.280	0.01965	0.01965	.99577	10.065	0.139E-03
46	670.180	0.01990	0.01993	.99586	11.263	0.153E-03
47	671.850	0.02015	0.02021	.98782	11.691	0.167E-03
48	673.330	0.02040	0.02051	.97446	12.170	0.192E-03
49	674.480	0.02065	0.02074	.98674	12.568	0.224E-03
50	675.350	0.02090	0.02093	.99461	12.915	0.253E-03
51	677.400	0.02140	0.02149	.98643	14.033	0.296E-03
52	679.150	0.02190				
53	680.900	0.02240				
54	683.130	0.02340				

*2 ORDER REGRESSION POLYNOMIAL PREDICTIONS *

Coefficients of the polynomial in ascending order

A(0)=-0.98998732864857E-02
A(1)=-.11817310223705E-04
A(2)=-0.38934896195997E-07

DATA STATISTICS:

STANDARD DEVIATION = 0.004121
STANDARD ERROR OF ESTIMATE = 0.001059
COEFFICIENT OF DETERMINATION = 0.936419
CORRELATION COEFFICIENT = 0.967687

OBS.No	CYCLES (x1000)	A(Meas.) (m)	A(Reg.) (m)	Delta K (MPam*.5)	dA/dN mm/Cycle
1	0.000	0.00840	0.00990	4.318	-0.118E-04
2	28.250	0.00865	0.00960	4.213	-0.962E-05
3	56.890	0.00890	0.00935	4.131	-0.735E-05
4	100.750	0.00915	0.00910	4.048	-0.397E-05
5	134.280	0.00940	0.00902	4.018	-0.136E-05
6	151.540	0.00965	0.00900	4.014	-0.169E-07
7	194.530	0.00990	0.00907	4.038	0.333E-05
8	224.850	0.01015	0.00921	4.083	0.569E-05
9	247.940	0.01040	0.00936	4.134	0.749E-05
21	516.250	0.01365	0.01418	6.134	0.284E-04
22	525.210	0.01390	0.01443	6.271	0.291E-04
23	541.980	0.01415	0.01493	6.556	0.304E-04
24	553.840	0.01440	0.01530	6.778	0.313E-04
25	565.340	0.01465	0.01566	7.013	0.322E-04
26	574.080	0.01490	0.01595	7.205	0.329E-04
27	583.540	0.01515	0.01626	7.428	0.336E-04
28	592.380	0.01540	0.01656	7.651	0.343E-04
29	599.810	0.01565	0.01682	7.852	0.349E-04
30	608.420	0.01590	0.01712	8.100	0.356E-04
31	616.060	0.01615	0.01740	8.336	0.362E-04
32	622.280	0.01640	0.01762	8.540	0.366E-04
33	627.380	0.01665	0.01781	8.716	0.370E-04
34	631.410	0.01690	0.01796	8.861	0.374E-04
35	634.220	0.01715	0.01807	8.965	0.376E-04
36	638.840	0.01740	0.01824	9.143	0.379E-04
37	644.410	0.01765	0.01845	9.367	0.384E-04
38	647.950	0.01790	0.01859	9.516	0.386E-04
39	651.750	0.01815	0.01874	9.681	0.389E-04
40	656.100	0.01840	0.01891	9.879	0.393E-04
41	659.380	0.01865	0.01904	10.033	0.395E-04
42	662.450	0.01890	0.01916	10.183	0.398E-04
43	664.840	0.01915	0.01925	10.302	0.400E-04
44	666.760	0.01940	0.01933	10.400	0.401E-04
45	668.280	0.01965	0.01939	10.479	0.402E-04
46	670.180	0.01990	0.01947	10.580	0.404E-04
47	671.850	0.02015	0.01953	10.670	0.405E-04
48	673.330	0.02040	0.01959	10.752	0.406E-04
49	674.480	0.02065	0.01964	10.816	0.407E-04
50	675.350	0.02090	0.01968	10.865	0.408E-04
51	677.400	0.02190	0.01976	10.982	0.409E-04
52	679.150	0.02190	0.01983	11.084	0.411E-04
53	680.900	0.02240	0.01990	11.188	0.412E-04

*3 ORDER REGRESSION POLYNOMIAL PREDICTIONS *

Coefficients of the polynomial in ascending order

A(0)=-0.75912289321423E-02
A(1)=-0.29544236895163E-04
A(2)=-.10093284430468E-06
A(3)=-0.12589267128771E-09

DATA STATISTICS:

STANDARD DEVIATION = 0.004121
STANDARD ERROR OF ESTIMATE = 0.000724
COEFFICIENT OF DETERMINATION = 0.970897
CORRELATION COEFFICIENT = 0.985341

OBS.No	CYCLES (x1000)	A(Meas.) (m)	A(Reg.) (m)	Delta K (MPam*.5)	dA/dN mm/Cycle
1	0.000	0.00840	0.00759	3.565	0.295E-04
2	28.250	0.00865	0.00835	3.802	0.241E-04
3	56.890	0.00890	0.00897	4.003	0.193E-04
4	100.750	0.00915	0.00967	4.239	0.130E-04
5	134.280	0.00940	0.01004	4.368	0.925E-05
6	151.540	0.00965	0.01019	4.419	0.763E-05
7	194.530	0.00990	0.01045	4.511	0.452E-05
8	224.850	0.01015	0.01056	4.554	0.325E-05
9	247.940	0.01040	0.01063	4.579	0.271E-05
21	516.250	0.01365	0.01326	5.672	0.260E-04
22	525.210	0.01390	0.01351	5.788	0.277E-04
23	541.980	0.01415	0.01400	6.037	0.311E-04
24	553.840	0.01440	0.01438	6.242	0.336E-04
25	565.340	0.01465	0.01478	6.468	0.361E-04
26	574.080	0.01490	0.01511	6.660	0.381E-04
27	583.540	0.01515	0.01548	6.892	0.404E-04
28	592.380	0.01540	0.01584	7.134	0.425E-04
29	599.810	0.01565	0.01617	7.359	0.443E-04
30	608.420	0.01590	0.01656	7.647	0.465E-04
31	616.060	0.01615	0.01692	7.933	0.485E-04
32	622.280	0.01640	0.01723	8.189	0.502E-04
33	627.380	0.01665	0.01749	8.417	0.516E-04
34	631.410	0.01690	0.01770	8.609	0.527E-04
35	634.220	0.01715	0.01785	8.750	0.534E-04
36	638.840	0.01740	0.01810	8.995	0.547E-04
37	644.410	0.01765	0.01840	9.315	0.563E-04
38	647.950	0.01790	0.01861	9.534	0.573E-04
39	651.750	0.01815	0.01883	9.784	0.584E-04
40	656.100	0.01840	0.01908	10.090	0.597E-04
41	659.380	0.01865	0.01928	10.337	0.606E-04
42	662.450	0.01890	0.01947	10.580	0.616E-04
43	664.840	0.01915	0.01962	10.780	0.623E-04
44	666.760	0.01940	0.01974	10.947	0.629E-04
45	668.280	0.01965	0.01983	11.083	0.633E-04
46	670.180	0.01990	0.01995	11.259	0.639E-04
47	671.850	0.02015	0.02006	11.418	0.644E-04
48	673.330	0.02040	0.02016	11.564	0.649E-04
49	674.480	0.02065	0.02023	11.681	0.652E-04
50	675.350	0.02090	0.02029	11.770	0.655E-04
51	677.400	0.02140	0.02042	11.988	0.661E-04

*4 ORDER REGRESSION POLYNOMIAL PREDICTIONS *

Coefficients of the polynomial in ascending order

A(0)=-0.89480541646481E-02
A(1)=-.17186539480463E-04
A(2)=0.19838580556097E-06
A(3)=-.51970083791986E-09
A(4)=0.44650998935573E-12

DATA STATISTICS:

STANDARD DEVIATION = 0.004121
STANDARD ERROR OF ESTIMATE = 0.000541
COEFFICIENT OF DETERMINATION = 0.984058
CORRELATION COEFFICIENT = 0.991997

OBS.No	CYCLES (x1000)	A(Meas.) (m)	A(Reg.) (m)	Delta K (MPam*.5)	dA/dN mm/Cycle
1	0.000	0.00840	0.00895	3.996	-0.172E-04
2	28.250	0.00865	0.00861	3.886	-0.718E-05
3	56.890	0.00890	0.00852	3.858	0.669E-06
4	100.750	0.00915	0.00874	3.930	0.879E-05
5	134.280	0.00940	0.00910	4.048	0.123E-04
6	151.540	0.00965	0.00933	4.122	0.134E-04
7	194.530	0.00990	0.00993	4.327	0.141E-04
8	224.850	0.01015	0.01035	4.476	0.135E-04
9	247.940	0.01040	0.01065	4.585	0.128E-04
21	516.250	0.01365	0.01316	5.622	0.179E-04
22	525.210	0.01390	0.01333	5.703	0.199E-04
23	541.980	0.01415	0.01370	5.883	0.242E-04
24	553.840	0.01440	0.01400	6.041	0.277E-04
25	565.340	0.01465	0.01435	6.222	0.315E-04
26	574.080	0.01490	0.01463	6.383	0.347E-04
27	583.540	0.01515	0.01498	6.584	0.383E-04
28	592.380	0.01540	0.01533	6.801	0.420E-04
29	599.810	0.01565	0.01566	7.010	0.453E-04
30	608.420	0.01590	0.01607	7.287	0.493E-04
31	616.060	0.01615	0.01646	7.572	0.531E-04
32	622.280	0.01640	0.01680	7.834	0.564E-04
33	627.380	0.01665	0.01709	8.074	0.591E-04
34	631.410	0.01690	0.01733	8.282	0.614E-04
35	634.220	0.01715	0.01751	8.437	0.630E-04
36	638.840	0.01740	0.01781	8.712	0.650E-04
37	644.410	0.01765	0.01818	9.082	0.669E-04
38	647.950	0.01790	0.01843	9.342	0.712E-04
39	651.750	0.01815	0.01870	9.645	0.736E-04
40	656.100	0.01840	0.01903	10.027	0.764E-04
41	659.380	0.01865	0.01929	10.343	0.786E-04
42	662.450	0.01890	0.01953	10.663	0.807E-04
43	664.840	0.01915	0.01972	10.931	0.823E-04
44	666.760	0.01940	0.01990	11.150	0.837E-04
45	668.280	0.01965	0.02001	11.346	0.847E-04
46	670.180	0.01990	0.02017	11.593	0.861E-04
47	671.850	0.02015	0.02032	11.821	0.873E-04
48	673.330	0.02040	0.02045	12.032	0.883E-04
49	674.480	0.02065	0.02055	12.202	0.892E-04

*5 ORDER REGRESSION POLYNOMIAL PREDICTIONS *

Coefficients of the polynomial in ascending order

A(0)=-0.81753283739090E-02
A(1)=-0.29594928491861E-04
A(2)=-.29067018658679E-06
A(3)=-0.13338661286610E-08
A(4)=-.24801276483910E-11
A(5)=-0.16391434311757E-14

DATA STATISTICS:

STANDARD DEVIATION = 0.004121
STANDARD ERROR OF ESTIMATE = 0.000413
COEFFICIENT OF DETERMINATION = 0.990887
CORRELATION COEFFICIENT = 0.995433

OBS.No	CYCLES (x1000)	A(Meas.) (m)	A(Reg.) (m)	Delta K (MPam*.5)	dA/dN mm/Cycle
1	0.000	0.00840	0.00818	3.748	0.296E-04
2	28.250	0.00865	0.00881	3.951	0.161E-04
3	56.890	0.00890	0.00914	4.059	0.773E-05
4	100.750	0.00915	0.00933	4.124	0.234E-05
5	134.280	0.00940	0.00940	4.148	0.233E-05
6	151.540	0.00965	0.00945	4.164	0.319E-05
7	194.530	0.00990	0.00966	4.234	0.664E-05
8	224.850	0.01015	0.00990	4.318	0.937E-05
9	247.940	0.01040	0.01014	4.401	0.112E-04
26	574.080	0.01490	0.01456	6.343	0.279E-04
27	583.540	0.01515	0.01485	6.505	0.320E-04
28	592.380	0.01540	0.01540	6.685	0.364E-04
29	599.810	0.01565	0.01543	6.864	0.406E-04
30	608.420	0.01590	0.01581	7.108	0.459E-04
31	616.060	0.01615	0.01618	7.366	0.512E-04
32	622.280	0.01640	0.01651	7.610	0.558E-04
33	627.380	0.01665	0.01680	7.839	0.599E-04
34	631.410	0.01690	0.01705	8.041	0.633E-04
35	634.220	0.01715	0.01723	8.194	0.657E-04
36	638.840	0.01740	0.01755	8.470	0.699E-04
37	644.410	0.01765	0.01795	8.851	0.753E-04
38	647.950	0.01790	0.01822	9.125	0.788E-04
39	651.750	0.01815	0.01853	9.451	0.828E-04
40	656.100	0.01840	0.01890	9.871	0.876E-04
41	659.380	0.01865	0.01919	10.228	0.913E-04
42	662.450	0.01890	0.01948	10.597	0.949E-04
43	664.840	0.01915	0.01971	10.911	0.978E-04
44	666.760	0.01940	0.01990	11.182	0.100E-03
45	668.280	0.01965	0.02005	11.409	0.102E-03
46	670.180	0.01990	0.02025	11.712	0.104E-03
47	671.850	0.02015	0.02043	11.995	0.107E-03
48	673.330	0.02040	0.02059	12.261	0.109E-03
49	674.480	0.02065	0.02071	12.478	0.110E-03
50	675.350	0.02090	0.02081	12.649	0.111E-03
51	677.400	0.02140	0.02104	13.075	0.114E-03
52	679.150	0.02190	0.02124	13.466	0.116E-03


```

*****
THIS PROGRAM CARRIES OUT THE FATIGUE CRACK GROWTH DATA
REDUCTION IN ACCORDANCE TO THE 'computerised processing of
fatigue crack propagation data' REPORT PUBLISHED BY
L.N.McCARTNEY AND P.M.COOPER OF THE NATIONAL PHYSICAL LABORATORY
UNITED KINGDOM.
IN ACCORDANCE TO THIS PROCEDURE A FUNCTIONAL EXPRESSION IS FITTED
THROUGH THE DATA, IN TERMS OF LEAST SQUARES. FURTHER A SPLINE
FUNCTION IS SIMULTANEOUSLY FITTED, WHICH PROVIDES DATA SMOOTHING.
THE ORDER OF THE SPLINE FUNCTION IS A VARIABLE THAT CAN BE
INTRODUCED IN THE PROGRAM BY THE USER.
THUS THE SLOPES da/dN CAN BE CALCULATED FROM RAW EXPERIMENTAL DATA,
OF A VS N DATA RECORDS.
THE DATA IS FURTHER PROCESSED TO PROVIDE A COMPLETE REPORT OF
DATA ANALYSIS IN TERMS OF da/dN vs Delta-K
*****

```

```

PROGRAM NPLNEW
CHARACTER*20 INFILE, OUTFILE, TYPE*2, STN*15, MAT*50
DIMENSION XRAW(100),YRAW(100),XIJ(15,20),YIJ(15,20)
+ AUGMAT(30,30),APPI(15),KNOT(0:15),V(KNOT(0:15)),
+ F(20),R(30),ESTDY(100),DELK(100),CYCLES(100)
REAL MSR

```

```

900 WRITE(6,10)
WRITE(6,221)
WRITE(6,1)
READ(5, '(A20)') INFILE
+ I read file name.
WRITE(6,221)
OPEN(1,ERR=20,FILE=INFILE,
+ STATUS='OLD',
+ ACCESS='SEQUENTIAL')
+ I open file.
WRITE(6,2)
READ(6,*) NINT I read number of intervals
WRITE(6,221)
WRITE(6,3)
READ(6,*) L I read order of spline
WRITE(6,221)
WRITE(6,4)
READ(6, '(A20)') OUTFILE
WRITE(6,221)
OPEN(2,FILE=OUTFILE, I open output report file
+ STATUS='NEW',
+ ACCESS='SEQUENTIAL',
+ FORM='FORMATTED')
WRITE(6,221)

```

```

ZZZ=1.0 I just to skip the IF and continue to main
IF (ZZZ.NE.1.0) THEN I program procedures when no error.
20 CALL ERRORS (Q,INFILE)
IF (Q.EQ.1.0) THEN
GO TO 909
ELSE IF (Q.EQ.2) THEN
GO TO 74
END IF
END IF

```

```

*****
INTERACTIVE INPUT OF TEST DETAILS
*****

```

```

WRITE(6,105)
READ(5, '(A15)') STN I test code number
WRITE(6,221)
TYPE='CT'
C
WRITE(6,115)
READ(5, '(F6.3)') B1
WRITE(6,221)
WRITE(6,120)
READ(5, '(F6.3)') W1
WRITE(6,221)
WRITE(6,125)
READ(5, '(F6.3)') AN1
WRITE(6,221)
WRITE(6,130)
READ(5, '(A50)') MAT
WRITE(6,221)
WRITE(6,135)
READ(5, '(F7.3)') YS1
WRITE(6,221)
WRITE(6,140)
READ(5, '(F6.3)') Pmin1
WRITE(6,221)
WRITE(6,145)
READ(5, '(F6.3)') Pmax1
WRITE(6,221)
WRITE(6,150)
READ(5, '(F6.2)') TEMP
WRITE(6,221)
WRITE(6,155)
READ(5, '(A15)') ENV
WRITE(6,221)
WRITE(6,160)
READ(5, '(F5.3)') FREQ
WRITE(6,221)
WRITE(6,221)

```

```

*****
END OF INTERACTIVE INPUT OF TEST DETAILS
*****
conversion to SI system of UNITS and definition of constants.
PI=4.0*ATAN(1.0)
B=B1*1E-03 I conv. to metres
W=W1*1E-03 I conv. to metres
YS=YS1*1E+06 I conv. to N/sq.m
PMIN=PMIN1*1E+03 I conv. to N
PMAX=PMAX1*1E+03 I conv. to N
DeltaP=PMAX-PMIN
RatioP=PMIN1/PMAX1
*****
END OF UNITS CONVERSION *****

```

```

C
C
C
C

```

```

C CALL DATAR (YRAW,XRAW,NTRUE,ANI,FACTOR,CYCLES)
C CALL DCONVR (XRAW,YRAW,XIJ,YIJ,NTRUE,NINT,NM1,NPPI,KNOT,VKNOT)
C CALL MATRICES (XIJ,YIJ,KNOT,VKNOT,NPPI,NM1,NINT,AUGMAT,L)
C CALL GAUSS (AUGMAT,NM1,R)
C CALL ESTIMATE (NTRUE,L,VKNOT,XRAW,NM1,R,FACTOR,ESTY,ESTDY)
C CALL STATS (ESTY,YRAW,NTRUE,MSR,COFDET)
C CALL SIFS (XRAW,NTRUE,B,W,DeltaP,FACTOR,DELK)
C CALL INTER (FLAG)
C ----- output data into the report data file -----
C
WRITE(2,165) STN,NTRUE
WRITE(2,170) TYPE,B1,W1,ANI
WRITE(2,175) MAT
WRITE(2,180) YS1
WRITE(2,185) YS1
WRITE(2,190) ENV,TEMP
WRITE(2,191) FREQ
WRITE(2,192) PMIN1, PMA1, RATIO
WRITE(2,195) PMIN1, PMA1, RATIO
WRITE(2,200) I headers
WRITE(2,201) I headers
DO 169 I=1,NTRUE-1
CL=XRAW(I)*FACTOR/1000.0
CLS=CYCLES(I)/1000.0
WRITE(2,205) I,CLS,CL,DELK(I),ESTDY(I)
WRITE(3,*) CLS,CL
WRITE(4,*) DELK(I),ESTDY(I)
WRITE(7,*) CL,ESTDY(I)
169 CONTINUE
WRITE(3,*) 999,999
DO 1700 I=1,NTRUE-1
CL=XRAW(I)*FACTOR/1000.0
WRITE(3,*) ESTY(I-1),CL
1700 CONTINUE
WRITE(2,210) NINT,L,MSR,COFDET
WRITE(3,*) 999,999
WRITE(4,*) 999,999
WRITE(7,*) 999,999
C
WRITE(6,*) 'MEAN SQR.RESIDUAL =',MSR
WRITE(6,*)
WRITE(6,*) 'COFDET=',COFDET
C
IF (FLAG.EQ.1.0) THEN
CLOSE(3,STATUS='KEEP') I (A,N) DATA FILE
CLOSE(4,STATUS='KEEP') I (A,N) DELTA-K DATA FILE
CLOSE(7,STATUS='KEEP') I (A,N) DATA FILE
ELSE
CLOSE(3,STATUS='DELETE') I delete data files
CLOSE(4,STATUS='DELETE') I no graphical output required
CLOSE(7,STATUS='DELETE')
END IF
C
74 WRITE(6,300) I end of run message
C
C

```

```

C FORMATS for interactive input
C
1 FORMAT(8X, '8X, What is the name of the input file ?', /,
+ 8X, 'TYPE THE "filename.type" max 10 characters', /,
+ 8X, /)
2 FORMAT(8X, '8X, HOW MANY INTERVALS ARE TO BE CONSIDERED ?', /,
+ 8X, /)
3 FORMAT(8X, '8X, PLEASE INSERT THE ORDER OF THE SPLINE', /,
+ 8X, /)
4 FORMAT(8X, '8X, WHAT IS THE NAME OF THE REPORT FILE ?', /,
+ 8X, /)
10 FORMAT(/////15X, 'FATIGUE CRACK PROPAGATION', /,
+ 15X, 'DATA REDUCTION PROGRAM', /,
+ 15X, 'Stelios Kyriacou PhD Applied Mechanics', /)
105 FORMAT(5X, '5X, INPUT THE SPECIMEN or TEST IDENTIFICATION NUMBER', /,
+ 5X, /)
115 FORMAT(5X, '5X, WHAT IS THE THICKNESS OF THE SPECIMEN (B)? (in mm)', /,
+ 5X, /)
120 FORMAT(5X, '5X, WHAT IS THE SPECIMEN LENGTH (W)? (in mm)', /,
+ 5X, /)
125 FORMAT(5X, '5X, WHAT IS THE MACHINED NOTCH LENGTH (An)? (in mm)', /,
+ 5X, /)
130 FORMAT(5X, '5X, WHAT IS THE SPECIMEN MATERIAL ?', /,
+ 5X, /)
135 FORMAT(5X, '5X, YIELD STRESS or 0.2% PROOF STRESS ? (in MPa)', /,
+ 5X, /)
140 FORMAT(5X, '5X, Pmin (in KN)', /,
+ 5X, /)
145 FORMAT(5X, '5X, Pmax (in KN)', /,
+ 5X, /)
150 FORMAT(5X, '5X, TEST TEMPERATURE (in deg.C)', /,
+ 5X, /)
155 FORMAT(5X, '5X, TEST ENVIRONMENT ?', /,
+ 5X, /)
160 FORMAT(5X, /)

```



```

        DIMENSION KNOT(0:15), XRAW(100), YRAW(100), X(15,20), Y(15,20),
        + NPPI(15), VKNOT(0:15)

        NKNT=NINT+1
        NM1=NINT-1
        A=NP/NM1
        NCP=INT(A)-2
        LI=NP-(NM1*NCP+NM1+1)-1

C----- positioning of knots -----
C
        KP=0
        KNOT(0)=1
        VKNOT(0)=XRAW(1)

        DO 10 I=1,NINT
            ID=NINT-I
            IF (ID.NE.0) THEN
                KNOT(ID)=KNOT(I-1)+NCP+1
                KP=KNOT(I)
                VKNOT(I)=XRAW(KP)
            ELSE
                KNOT(I)=KNOT(I-1)+LI+1
                KP=KNOT(I)
                VKNOT(I)=XRAW(KP)
            END IF
        END DO

10 CONTINUE

C----- data array conversion -----
C
        IC=0
        DO 15 I=1,NINT
            LIM1=KNOT(I-1)
            LIM2=KNOT(I)
            LL=LIM2-LIM1+1
            NPPI(I)=LL
            IC=LM1
            DO 15 J=1,LL
                X(I,J)=XRAW(IC)
                Y(I,J)=YRAW(IC)
                IC=IC+1
            END DO
        END DO

15 CONTINUE

        RETURN
        END

C----- SUBROUTINE MATRICES, USES THE DATA GENERATED BY SUBROUTINE
C----- DCONVR, AND FORMS THE MATRIX OF COEFFICIENTS THAT ARE REQUIRED
C----- FOR THE DATA FITTING EQUATIONS.
C
C----- NOTATION:-
C
C----- NINT : TOTAL NUMBER OF INTERVALS
C----- NM1 : NUMBER OF INTERNAL KNOTS
C----- NPPI : NUMBER OF POINTS PER INTERVAL
C----- X : X-COORDINATES OF DATA PAIRS
C----- Y : Y-COORDINATES OF DATA PAIRS
C----- KNOT : ARRAY HOLDING THE LOCATION OF THE KNOTS

```

```

        READ(5,'(A20)') OUT2
        OPEN(4,FILE=OUT2,
        + STATUS='NEW',
        + ACCESS='SEQUENTIAL')
        WRITE(6,540)
        READ(5,'(A20)') OUT3
        OPEN(7,NAME=OUT3,
        + STATUS='NEW',
        + ACCESS='SEQUENTIAL')

C----- FORMATS FORMATS FORMATS FORMATS FORMATS
C----- FORMATS FORMATS FORMATS FORMATS FORMATS
499 FORMAT(//5X,'** No Graphical out-put has been requested **',
        + 5X,'** TYPE <CR> in following questions **',//)
500 FORMAT(5X,'Do you wish to obtain graphical output ?',
        + 5X,'-----',//5X,'Please type Y or N',//)
510 FORMAT(5X,'-----')
520 FORMAT(//5X,'What is the name of the output file ',
        + 5X,'for (a vs N) data?',
        + 5X,'Type "filename.type" up to 10 Characters',//)
530 FORMAT(//5X,'What is the name of the output file ',
        + 5X,'for (dA/dN vs delta-K) data?',
        + 5X,'Type "filename.type" up to 10 Characters',//)
540 FORMAT(//5X,'What is the name of the output file ',
        + 5X,'for (dA/dN vs A) data?',
        + 5X,'Type "filename.type" up to 10 Characters',//)

        RETURN
        END

C----- SUBROUTINE DCONVR, THIS SUBROUTINE USES THE RAW DATA FROM THE DATA
C----- READING ROUTINE AND CONVERTS THE READ ONE DIMENSIONAL ARRAYS INTO
C----- TWO TWO-DIMENSIONAL ARRAYS DENOTING THE INTERVAL NUMBER AND THE
C----- LOCATION OF THE DATA PAIR WITHIN THE INTERVAL.
C
C----- NOTATION:-
C----- NP : NUMBER OF DATA POINTS
C----- NINT: TOTAL NUMBER OF INTERVALS
C----- NKNT: TOTAL NUMBER OF KNOTS
C----- NM1 : TOTAL NUMBER OF INTERVALS MINUS 1, NUMBER OF INTERNAL KNOTS
C----- NCP : NUMBER OF CONTAINED POINTS PER NM1 INTERVAL
C----- LI : NUMBER OF CONTAINED POINTS IN LAST INTERVAL
C----- KNOT(I): ARRAY HOLDING THE LOCATION OF THE KNOTS
C----- NPPI: NUMBER OF POINTS PER INTERVAL
C----- VKNOT: X-COORDINATE VALUE OF KNOT
C----- XRAW: DATA INPUT
C----- YRAW: DATA INPUT
C----- X : DATA OUTPUT
C----- Y : DATA OUTPUT
C-----
C----- SUBROUTINE DCONVR (XRAW,YRAW,X,Y,NP,NINT,NM1,NPPI,KNOT,VKNOT)

```

```

C VKNOT: X-COORDINATE VALUE OF KNOT
C A(I,J): MATRIX OF COEFFICIENTS
C F(I): VECTOR OF R.H.S. COEFFICIENTS
C L: SPLINE FUNCTION EXPONENT user defined in main prog.
C .....
SUBROUTINE MATRICES (X,Y,KNOT,VKNOT,NPPI,NM1,NINT,AUGMAT,L)
  DIMENSION A(30,30), NPPI(15), X(15,20), Y(15,20), KNOT(0:15),
  * VKNOT(0:15), AUGMAT(30,30), F(20)
  INTEGER P,R
  C WRITE(6,*) ' Matrix Formation...'
  C----- Initialisation of matrix arrays -----
  DO 10 I=1,30
    DO 10 J=1,30
      A(I,J)=0.0
    AUGMAT(I,J)=0.0
  10 CONTINUE
  C----- formation of first 3x3 matrix terms -----
  DO 100 I=1, NINT
    DO 100 J=1, NPPI(I)
      A(1,1)=A(1,1)+1.0
      A(1,2)=A(1,2)+1.0/X(I,J)**L
      A(1,3)=A(1,3)+LOG(X(I,J))
      A(2,2)=A(2,2)+1.0/X(I,J)**(2.)
      A(2,3)=A(2,3)+LOG(X(I,J))*X(I,J)
      A(3,3)=A(3,3)+LOG(X(I,J))**2.
    100 CONTINUE
  C----- formation of remaining symmetric matrix terms -----
  DO 200 R=4, (NM1+3)
    DO 200 I=1, (R-3)
      DO 200 J=1, NPPI(I)
        A(1,R)=A(1,R)+(VKNOT(R-3)-X(I,J))**L
        A(2,R)=A(2,R)+(VKNOT(R-3)-X(I,J))**L/X(I,J)
        A(3,R)=A(3,R)+(VKNOT(R-3)-X(I,J))**L*LOG(X(I,J))
      200 CONTINUE
  C----- formation of diagonal matrix terms -----
  DO 300 R=4, (NM1+3)
    DO 300 P=R, (NM1+3)
      DO 300 I=1, (R-3)
        SUM=0.0
        DO 270 J=1, NPPI(I)
          SUM=SUM+(VKNOT(P-3)-X(I,J))**L*((VKNOT(R-3)-X(I,J))**L)
        270 CONTINUE
      AUGMAT(I,I)=SUM
    300 CONTINUE
  END

```

```

270 CONTINUE
  A(R,P)=A(R,P)+SUM
300 CONTINUE
  DO 305 I=1, (NM1+3) ! formation of complete symmetric matrix
  DO 305 J=1, (NM1+3) ! from diagonal form.
    A(I,J)=A(I,J)
305 CONTINUE
  C----- formation of right hand side vector -----
  C
  DO 400 I=1, (NM1+3) ! Initialisation of array
    F(I)=0.0
400 CONTINUE
  DO 500 I=1, (NM1+1)
    DO 500 J=1, NPPI(I)
      F(1)=F(1)+Y(I,J)
      F(2)=F(2)+Y(I,J)/X(I,J)
      F(3)=F(3)+Y(I,J)*LOG(X(I,J))
500 CONTINUE
  DO 480 R=4, (NM1+3)
    DO 480 I=1, (R-3)
      DO 480 J=1, NPPI(I)
        F(R)=F(R)+Y(I,J)*(VKNOT(R-3)-X(I,J))**L
480 CONTINUE
  C----- formation of augmented matrix of coefficients -----
  C----- to be used by the Gauss solver -----
  C
  DO 600 I=1, (NM1+3)
    DO 600 J=1, (NM1+3)
      AUGMAT(I,J)=A(I,J)
600 CONTINUE
  DO 700 I=1, (NM1+3)
    AUGMAT(I, (NM1+4))=F(I)
700 CONTINUE
  RETURN
  END
C----- SUBROUTINE GAUSS, TO CARRY OUT THE GAUSSIAN ELIMINATION
C----- FOR THE SOLUTION OF THE SYSTEM OF LINEAR SIMULTANEOUS
C----- EQUATIONS.
C----- GAUSS (AJORD,R)
C----- A: Augmented matrix of coefficients
C----- IORD: Number of equations to be solved
C----- R: Roots of normal equations, coefficients of polynomial
C-----
SUBROUTINE GAUSS (A,NM1,R)

```

```

SUBROUTINE ESTIMATE (NTRUE,L,VKNOT,XRAW,NM1,R,FACTOR,ESTY,ESTDY)
DIMENSION VKNOT(0:15),XRAW(100),R(30),ESTY(100),ESTDY(100)
DO 100 I=1,(NTRUE-1)
  X=XRAW(I+1)  ! avoids 1st data point
  Y=R(1)+R(2)/X+R(3)*LOG(X)
  DY=(-R(2)/(X**2))+R(3)/X
  IF (X.GT.VKNOT(NM1)) THEN  ! If the point is located in the
    GOTO 1111  ! last interval no more calcs.
  END IF
  DO 50 J=1,NM1  ! for all intervals but last
    IF (X.GT.VKNOT(J-1).AND.X.LE.VKNOT(J)) THEN
      DO 40 M=J,NM1
        Y=Y+R(M+3)*(VKNOT(M)-X)**L
        DY=DY-L*R(M+3)*(VKNOT(M)-X)**(L-1)
      40 CONTINUE
    END IF
  50 CONTINUE
  1111 ESTDY(I)=1.0/DY*(FACTOR/1000.)  ! In mm/cycle
  100 CONTINUE
RETURN
END
.....
SUBROUTINE SIFS, USES THE RAW EXPERIMENTAL DATA OF CRACK LENGTHS
IN ORDER TO CALCULATE THE STRESS INTENSITY (DELTA-K) FROM THE
THEORETICAL EQUATION GIVEN IN THE ASTM-E647 STANDARD.
NOTATION:
NTRUE : NUMBER OF DATA POINTS
XRAW : EXPERIMENTAL CRACK LENGTHS (m)
B : SPECIMEN THICKNESS (m)
W : SPECIMEN WIDTH (m)
AOW : A OVER W RATIO
FAOW : FUNCTION OF A OVER W
DELTA P : LOAD AMPLITUDE
.....
SUBROUTINE SIFS (XRAW,NTRUE,B,W,DELTA P,FACTOR,DELK)
DIMENSION XRAW(100), DELK(100)
DO 10 I=1, NTRUE
  AOW=XRAW(I)*FACTOR*(1.0E-3)/W
  FAOW1=(2+AOW)/(1-AOW)**(1.5)
  FAOW2=(0.886+4.64*AOW-13.32*AOW**2+14.72*AOW**3-5.6*AOW**4)
  FAOW= FAOW1*FAOW2
  DELK(I)=(1.0E-06)*(FAOW*Delta P)/(B*SQRT(W))
10 CONTINUE

```

```

DIMENSION A(30,30), B(30,30), R(30)
WRITE(6,'') Gauss Solver...
N=NM1+3
NP1=N+1
.....
this DO Loop saves the A(i,j) into B(i,j) so that
it may be used to check the solution, it can be omitted.
.....
DO 1000 I=1,N
  DO 1000 J=1,NP1
    B(I,J)=A(I,J)
  CONTINUE
1000
.....
ELIMINATION STAGE
.....
DO 20 K=1,N
  KP1=K+1
  DO 10 J=KP1,NP1
    A(K,J)=A(K,J)/A(K,K)
  CONTINUE
  IF (K.EQ.N) GO TO 20
  DO 15 I=KP1,N
    DO 14 J=KP1,NP1
      A(I,J)=A(I,J)-A(I,K)*A(K,J)
    CONTINUE
  15 CONTINUE
20
.....
BACK SUBSTITUTION STAGE
.....
R(N)=A(N,NP1)
DO 30 K=2,N
  I=NP1-K
  R(I)=A(I,NP1)
  IP1=I+1
  DO 30 J=IP1,N
    R(I)=R(I)-A(I,J)*R(J)
  CONTINUE
30
RETURN
END
.....
SUBROUTINE ESTIMATE, USES THE RAW DATA AND THE SOLUTIONS
OF THE SPLINE-POLYNOMIAL EXPRESSION IN ORDER TO ESTIMATE
THE VALUES OF N (CYCLES), WITH RESPECT TO THE EXPERIMENTAL
VALUES OF CRACK LENGTH.
NOTATION:
NTRUE : NUMBER OF DATA POINTS
L : EXPONENT OF SPLINE FUNCTION
VKNOT : VALUE OF X AT THE KNOT
XRAW : EXPERIMENTAL VALUE OF DATA POINT
NM1 : NUMBER OF INTERNAL KNOTS
R : ROOTS OF SOLUTION
ESTY : ESTIMATED VALUE OF Y (CYCLES)
ESTDY : ESTIMATED VALUE OF GRADIENT (DA/DN) In mm/cycle
.....

```

RETURN
END

CCCCCCCCCCCCCCCCCCCC
SUBROUTINE STATS, TO CARRY OUT STATISTICAL ANALYSIS OF THE
REGRESSION POLYNOMIALS.
IT CALCULATES THE SUM OF THE SQUARES OF THE RESIDUALS ABOUT THE
MEAN AND THE SUM OF THE SQUARE OF THE RESIDUALS ABOUT THE REGRESSION
LINE (CURVE).

NOTATION:

NTRUE: Number of data points
Y: Array of dependent variable
MSR: Sum of sq. of residuals about the regression curve
COFDET: Coefficient of determination

C SUBROUTINE STATS (ESTY,YRAW,NTRUE,MSR,SDEV,SRES,COFDET)
SUBROUTINE STATS (ESTY,YRAW,NTRUE,MSR,COFDET)
DIMENSION ESTY(100),YRAW(100)
REAL MSR

60 SUM1=0.0
DO 60 I=1,NTRUE
SUM1=SUM1+YRAW(I)
CONTINUE
AVER=SUM1/NTRUE I MEAN OF DEPENDENT VARIABLE
WRITE(6,*) 'AVERG.', AVER

C ST=0.0
SUM2=0.0
DO 60 I=1,NTRUE
ST=(YRAW(I)-AVER)**2
SUM2=SUM2+ST
CONTINUE

60 SRES=SUM2 I SUM OF SQ. OF RESIDUALS ABOUT THE MEAN
ZZ=NTRUE-1 I change variable from integer to real
VAR=SRES/ZZ I VARIANCE
SDEV=SQRT(VAR) I STANDARD DEVIATION

C SR=0.0
DO 70 I=2,NTRUE
SR=SR+(ESTY(I-1)-YRAW(I))**2.0
CONTINUE
MSR=SR/(NTRUE-1)

70 COFDET=(SRES-MSR)/SRES

RETURN
END

CCCCCCCCCCCCCCCCCCCC
SUBROUTINE ERRORS TO INFORM THE USER OF ERRORS ENCOUNTERED
DURING OPENING THE SPECIFIED INPUT FILE.
CCCCCCCCCCCCCCCCCCCC

SUBROUTINE ERRORS (G,INFILE)

==== TROUBLE-SHOOTING MESSAGES =====

C CHARACTER*20 INFILE

C WRITE(6,*)
WRITE(6,*)
WRITE(6,21)
WRITE(6,22) INFILE
WRITE(6,21)
WRITE(6,23)

READ(5, '(A1)') ANS
G=2.0
IF (ANS.EQ.'Y'.OR.ANS.EQ.'Y') THEN
G=1.0
END IF

21 FORMAT(2X, '*** ERROR ***')
22 FORMAT(2X, '*** ERROR ***', 5X, A20,
+ '*** FILE NOT FOUND ***')
23 FORMAT(//2X, 'Do you wish to continue?')
RETURN
END

DATA REDUCTION TECHNIQUE FOR DETERMINING

CAV/DN AND Delta-K
IN ACCORDANCE TO NPL-REPORT MAT-APP.23 (1972)

Spec.No.CA/V018/R03 No.Points= 54
CT-Specimen B= 8.000 mm W= 32.000 mm An= 6.400 mm
Material : Aluminium-Zinc Alloy, 7075 T7351
Material Yield Stress = 469.000MPa

= T E S T - R E P O R T =

Environment : Air Test Temperature : 10.60 deg.C

Test frequency = 50.000Hz

Fmin= 0.462kN Fmax= 1.538kN R= 0.300

OBS.No	CYCLES (x1000)	A(Meas.) (m)	Delta K (MPa ^{1/2} .S)	da/dN (mm/cycle)
1	0.000	0.00840	3.830	0.719E-05
2	28.250	0.00865	3.910	0.737E-05
3	56.890	0.00890	3.992	0.755E-05
4	100.750	0.00915	4.074	0.773E-05
5	134.280	0.00940	4.158	0.793E-05
6	151.540	0.00965	4.243	0.815E-05
7	194.530	0.00990	4.330	0.840E-05
8	224.850	0.01015	4.418	0.870E-05
9	247.940	0.01040	4.507	0.919E-05
10	296.780	0.01074	4.632	0.947E-05
11	313.550	0.01090	4.692	0.998E-05
12	332.900	0.01115	4.787	0.106E-04
13	363.500	0.01140	4.884	0.114E-04
14	388.460	0.01165	4.983	0.121E-04
15	404.000	0.01190	5.085	0.130E-04
16	418.980	0.01215	5.189	0.143E-04
17	441.670	0.01253	5.352	0.158E-04
18	469.710	0.01290	5.518	0.169E-04
19	484.600	0.01315	5.634	0.180E-04
20	501.500	0.01340	5.754	0.193E-04
21	516.250	0.01365	5.877	0.206E-04
22	525.210	0.01390	6.004	0.220E-04
23	541.980	0.01415	6.135	0.235E-04
24	553.840	0.01440	6.270	0.251E-04
25	565.340	0.01465	6.411	0.268E-04
26	574.080	0.01490	6.556	0.286E-04
27	583.540	0.01515	6.706	0.306E-04
28	592.380	0.01540	6.862	0.328E-04
29	599.810	0.01565	7.025	0.351E-04
30	608.420	0.01590	7.193	0.377E-04
31	616.060	0.01615	7.368	0.404E-04
32	622.280	0.01640	7.551	0.435E-04
33	627.380	0.01665	7.741	0.468E-04
34	631.410	0.01690	7.940	0.505E-04
35	634.220	0.01715	8.147	0.546E-04
36	638.840	0.01740	8.364	0.591E-04
37	644.410	0.01765	8.591	0.641E-04
38	647.950	0.01790	8.828	0.698E-04
39	651.750	0.01815	9.077	0.761E-04
40	656.100	0.01840	9.338	0.833E-04
41	659.380	0.01865	9.612	0.916E-04

42	662.450	0.01890	9.900	0.101E-03
43	664.840	0.01915	10.204	0.112E-03
44	666.760	0.01940	10.523	0.125E-03
45	668.280	0.01965	10.860	0.140E-03
46	670.180	0.01990	11.216	0.159E-03
47	671.850	0.02015	11.591	0.182E-03
48	673.330	0.02040	11.989	0.210E-03
49	674.480	0.02065	12.410	0.247E-03
50	675.350	0.02090	12.056	0.360E-03
51	677.400	0.02140	13.834	0.666E-03
52	679.150	0.02190	14.942	0.255E-02
53	600.900	0.02240	16.204	-0.644E-03

NUMBER OF INTERVALS= 5
SPLINE FUNCTION ORDER= 3
SUM OF SQ. OF RESIDUALS = 11.80663490
COEFF. OF DETERMINATION = 0.99999446