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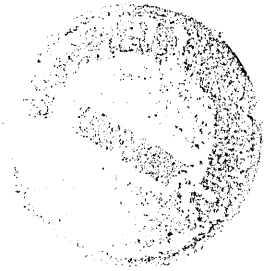
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Uncertainty Analysis in the Management of
Gas Metering Systems.

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September 2000



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*So far as the laws of mathematics refer to reality they are not certain.
And so far as they are certain they do not refer to reality.*

Albert Einstein
Geometry and Experience

Do not confuse the moon with the finger that points at it.

Zen Proverb

Everything is vague to a degree you do not realise till you have tried to make it precise

Bertrand Russell
The Philosophy of Logical Atomism

What we observe is not the nature itself, but nature exposed to our method of questioning

Werner Heisenberg
Physics and Philosophy

God eternally Geometrizes

Plato

All things are a matter of degree. They may be probabilistically deterministic or deterministically probabilistic.

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ABSTRACT

*In the natural gas market, open access along with gas brokering and marketing has resulted in multiple gas contracts through one physical measuring point. Accurate metering of natural gas has become more important than ever as deregulation subjects pipeline companies to competition. A more competitive market is driving the need for real-time accurate electronic flow measurement. Modern electronic natural gas metering systems (ENGMS) introduced additional, though necessary, complexity in the estimation and verification of the reported results. Additionally, it becomes more and more important to be able to verify these results. The application of Monte Carlo simulation as a combined energy flow measurement uncertainty estimation method seems to offer specific advantages over the more complex, traditional uncertainty estimation methods while at the same time fully conforms with the method of the ISO/GUM, (the authoritative document for uncertainty evaluation). Since, Monte Carlo simulation relies on randomness, it seems to capture more naturally and to have a more direct connection with the underlying physics of measurement uncertainty. Typical comparisons of estimated uncertainties using the Monte Carlo method and the conventional method (RSS) have been carried out. In general Monte Carlo gave slightly higher estimated uncertainties. This is due to the fact that the simplified conventional methods inevitably neglect correlations between the variables. A methodology for gas management decision making based on Monte Carlo uncertainty estimation is proposed. According to this, uncertainty analysis could be incorporated in any management decision making process regarding metering systems and importantly through **all** stages, from selection and design of a metering system up to real operation. In addition to the Monte Carlo simulation, work regarding possible improvements to orifice flow measurement quality is considered.*

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NOTATION

B_Y	Systematic uncertainty in the result Y
c	Speed of sound
D, D_{p_a}	Actual pipe diameter
D_{p_c}	Calibrated pipe diameter
do, do_a	Actual orifice diameter
do_c	Calibrated orifice diameter
d_{vc}	Vena Contracta diameter
C and $ C $	Covariance matrix and its determinant
$ C_{ij} $	Cofactor matrix for matrix C_{ij}
C_d	Discharge coefficient
C_c	Contraction Coefficient
C_j	Jet Necking Coefficient
C_v	Velocity Coefficient
E	Expense in monetary units
E	Velocity of approach factor
E_{Flow}	Energy flow rate (Kcal/month)
I_Q	Measurement information Quality
P	Purchased Gas price (\$/Kcal)
k	Isentropic exponent
K_{Po}	Pressure coefficient (based on orifice throat mean velocity)
K_{PD}	Pressure coefficient (based on pipe mean velocity)
K_{Loss}	Pressure Loss Coefficient
K_R	Pressure Recovery Coefficient
K	Flow Coefficient
m	Maximum metrological discrimination capability in bits
K_2	Convective acceleration factor
MW	Molecular Weight
P_{RP}	Pressure recovery parameter
P_{LP}	Pressure loss parameter
P_{LR}	Pressure loss recovery parameter
u_D	Pipe mean velocity
u_{vc}	Vena contracta mean velocity
Q_m	Mass flowrate
Q_v	Volume flowrate
$p(X_i)$	Probability density function
$p(X_i)_{X_j}$	Marginal probability density function
$p(X_i/X_j)$	Conditional probability density function
$P(X_1, X_2)$	Second order or joint Gaussian probability density function
$P(X_i)$	Cumulative Probability distribution function
P	Static pressure
S	Entropy
S_Y	Random uncertainty in the result Y
Z	Compressibility factor
z_i	Standard normally distributed random numbers

T_1, T_2	Upstream and downstream from the orifice gas temperatures respectively
T_{p_c}, T_{o_c}	Calibration temperatures of pipe and orifice respectively
$u_c(X_i), u(X_i)$	Combined standard uncertainties
t_E	Response time in sec
a	Alfa ratio (ratio of vena contracta diameter to pipe diameter)
β	Beta ratio (ratio of orifice diameter to pipe diameter)
β^x	Ratio of pressure recovery to differential pressure
$\Delta P_{1,3}$	Differential Pressure (Upstream and vena contracta)
$\Delta P_{4,3}$	Pressure recovery
$\Delta \varpi$	Permanent Pressure Loss
$\frac{\partial Y}{\partial X_i}$	Sensitivity Coefficient
$\frac{\partial \ln Y}{\partial \ln X_i}$	Influence Coefficient
λ_p, λ_o	Thermal expansion coefficients of pipe and orifice respectively.
ρ_{X1X2}	Correlation coefficient
$\sigma_{(X1X2)}$	Covariance
$\sigma(X), \sigma_c(X)$	Standard deviations or combined uncertainties
$\frac{\sigma(y)}{y}, \frac{u(y)}{y}$	Relative uncertainties
v	Specific volume

ABBREVIATION

ISO/GUM	International Standards Organisation/Guide for the Uncertainty of Measurement.
RSS	Root Sum Squares
AGA	American Gas Association
ADC	Analogue - digital converter
DMA	Direct memory access
ESD	Electronic system design
ENGMS	Electronic Natural Gas Metering System
GCV	Gross Calorific Value (Kcal/Rm ³)
PCA	Principal Components Analysis
FFT	Fast Fourier Transformation
cdf	Cumulative distribution function

CHAPTER 1

1. INTRODUCTION

Gas measurement is said to be the cash register of the gas industry. The efficient and accurate measurement of Natural gas is of major importance in the present-day world of optimum energy management. In every purchase, sales, or transportation and exchange contract, general terms and conditions with reference to measurement are clearly spelled out. These general terms and conditions address four typical areas: a) Definitions of terms, b) Gas quality specifications, c) Method of measurement and measurement uncertainty and d) Measurement equipment. After careful examination it can be seen that directly or indirectly, three out of four of the above terms are related with measurement and uncertainty.

The cost of purchased gas is calculated using a formula as shown below which is an example of the relationship between money exchange and flow accuracy:

$$\text{\$} = E_{Flow} * P$$

where E_{Flow} : is the energy flow in Kcal/Month and P: is the Gas energy price in \$/Kcal.

Usually the payment system is based on monthly payment invoices. Both energy flow and Gas energy price are not fixed quantities. Gas energy price is usually calculated from a basic agreed price that is modified between agreed time intervals by international gas prices and relevant indexes that are published or are available to the parties involved.

Furthermore, energy flow is calculated by:

$$E_{Flow} = Q * GCV$$

where Q is the volumetric flow rate expressed in Rm^3 /Month (where R means reference conditions of pressure and temperature) and GCV is the Gross or Superior Calorific Value of the N. gas in Kcal/ Rm^3 .

The open access concept along with gas brokering and marketing has resulted in a new style of gas purchase contracts that is multiple contracts through one physical measuring point and this has resulted in nomination and allocation challenges. Accurate metering of natural gas is more important than ever as deregulation subjects

pipeline companies to competition. A more competitive market is driving the need for real-time electronic flow measurement (Valenti, M., 1997).

So, all the interested parties require faster and more reliable (accurate) information from the measuring point to make quick and right business decisions and to minimise imbalances. All these aspects are specifically addressed and emphasised in the **Directive 98/30/EC** of the European Parliament and of the Council of the European Union of **22 June 1998** concerning common rules for the internal (EC) market in natural gas. Further details are also included in two subsequent reports on harmonisation requirements. Coherent technical as well as commercial trading rules are considered to be critical to the proper functioning of the internal EC market. A harmonisation of gas metering and accounting methods is demanded.

As electronic gas meters (EGM) replace the more traditional chart recorders as the method of recording and calculating custody transfer volumes in the natural gas industry, it becomes more and more important to be able to audit the volumes produced by these devices. The economic importance of natural gas metering is presented by Cascetta F., et.al., 1999. He mentions that in 1995 the worldwide natural gas production was more than $2000 \times 10^9 \text{ m}^3$ and consequently the world natural gas market was about $\$420 \times 10^9$ (assuming an average unitary price of $\$0.20/\text{m}^3$).

Orifice meters account for more than 80 percent of installed process plant meters. Over the last 20 years or so the renewed interest in improving the uncertainty of N. Gas metering using orifice meters has been accompanied by a growing realisation that it is also necessary to have more reliable methods of estimating the uncertainty in the flow rate measurement. The traditional simple orifice meters used for natural gas flow measurements have now been replaced by modern electronic orifice metering systems. These employ a whole set of advanced electronic equipment ranging from the latest smart pressure and temperature transmitters, on line densitometers, relative density analysers, and flow computers to on line gas chromatographs for the continuous measurement of gas energy content, flow rate and gas quality.

Although these systems have offered, to a certain extent, the necessary fast transmission of information, they have created, due to their complexity, difficulties in the proper estimation of combined measurement uncertainties. The difficulty of reliable measurement uncertainty estimation increases almost exponentially with

complexity of the measurement system. The complexity generated due to interdependencies of direct and model based measurements is usually extremely difficult to be accounted for in a combined measurement uncertainty analysis. An important aspect of the methodology developed in this thesis is that the existing computational operation of such modern electronic Gas metering systems can be considered in detailed.

...A new reference meter would have cost \$400,000. As the meter monitors product worth \$20 million per day, an improvement in uncertainty of 0.01% would have saved up to \$700,000 per year! (Cousins, T., 1997).

...It is therefore most important to distinguish between random uncertainties, which eventually balance out to zero, and systematic errors, which the industry and government agencies take great pains to eliminate. (Gold, R.C., 1986).

If the measurement uncertainty cannot be calculated reliably, then there is no way of predicting whether or not, increased costs due to purchasing more expensive instrumentation, installing longer straight lengths of pipe upstream of the orifice meters, or introducing more time-consuming and costly measuring methods, will provide good value for money in terms of better accuracy (F., Kinghorn, 1986). It has been projected that the impact of the orifice measurement research will be on the order of \$120 million/year in the year 2000 (GRI/MRF, 1996).

During the last few years, there has been substantial developments in the field of uncertainty analysis as well, leading to the publication by International Standards Organisation (ISO) of the Guide to the expression of Uncertainty in Measurement (GUM) where an international widely accepted methodology has been established. GUM has become the authoritative document for all aspects of uncertainty evaluation and terminology. Thus, the ISO/5168, 1978, which covers flow measurement uncertainty estimation issues, is currently under revision to conform to the GUM.

The traditional analytical methods in orifice flow measurement uncertainty estimation, as specified in ISO/5168, 1978, requires the consideration of all the interdependencies (partial derivatives) amongst all the respective variables. This renders the 'complete' estimation of the combined measurement uncertainty a difficult if not an impossible task. Clause 7.1.5 in ISO 5168 (1978) includes numerous examples of partial derivatives that actually cannot be calculated directly and numerical methods are the only alternatives. ISO-5168 does not take into account other interdependencies that

could be generated within a modern electronic orifice metering system due to different measurement models and computational methods employed.

Conventional numerical methods are based on Taylor series expansion usually up to 2nd order. They are not only inefficient in terms of programming effort but also their intrinsic linear interpolation characteristics have limitations. For example, it can be proved that in systems with large uncertainties or those that utilise multiple common measurements, covariance uncertainty terms may be significant.

1.1. Objectives.

The objectives of this thesis can be summarized as follows:

- The development of methodologies for reliable energy flow rate quality (uncertainty) estimation and measurement control, that could be utilised in every stage of a gas measurement process, from planning and design to a continuous performance evaluation of modern natural gas fiscal metering systems. In order to meet this objective it was regarded necessary to analyse and present an integrated picture of the subject of modern electronic natural gas energy flow metering. The literature survey carried out and presented, covered multiple areas relevant directly or indirectly to the uncertainty of Natural Gas energy measurements. The research effort was focused in the modeling and methods' effects on combined measurement uncertainty and on the effects of correlated measurement inputs in combined measurement results.
- The optimal use of available multi-sensor/source data based on uncertainty analysis, for further reduction of energy flow uncertainty (uncertainty management for measurement quality improvement) and further cost savings. The immense importance of a management approach based on the methodologies of Quality Management (QM) had to be emphasised. The optimal use of multi-sensor data, combined with the first objective, could finally lead in improved reliability of a N. gas fiscal metering system. The reliability of such a system can be defined as: "The probability that any element of and subsequently the whole metering system will operate to an agreed level of performance, for a specified period, subject to specified environmental and process operating conditions." For the purpose of this project, the **agreed level of performance** has been specified as the **uncertainty** (in a sense of a tolerance) interval within which a modern gas metering systems must and can operate. Usually, such operational uncertainty intervals are carefully specified within Gas purchase contracts, however they create a lot of controversies among the interested

parties. The reasons for these controversies have to do mainly with the difficulty in estimating the uncertainty of such complex (multi-component) measurement systems by analytical methods. Additionally, there are cases where analytical methods are extremely difficult to apply and unjustifiable assumptions must be adopted that reduce the confidence of the uncertainty estimation results. So, the measurement uncertainty estimation methodologies of these systems was the target of this project that could ultimately also lead to improved overall performance. Improved reliability as defined above, could lead to significant cost savings.

In terms of the developed methodology, additional effort took place in order to be compatible with the recent international recommendations regarding uncertainty estimation which are present in the Guide to the estimation of Uncertainty in Measurement (GUM).

Summarizing in a sentence, the target was the development of a reliable applied methodology for on-line energy flow measurement uncertainty analysis incorporated within the management decision making process regarding modern Natural gas metering. Meeting these two objectives inevitably would lead to an improvement in the reliability of such metering systems and a reduction in their Total Lifetime Operating Cost. The later is possible to be achieved by the reduction of two factors. Firstly, reduction in the cost of maintenance over lifetime of the system (by avoiding unnecessary calibrations) and secondly by the reduction of the cost of measurement error over lifetime, the significance of which in natural gas metering has already been presented.

1.2. Proposed Methodology

Particular elements from various sensor fusion methods available have been properly modified and applied in this project.

An important feature of the methods developed in this project is that they have been considered in detail the existing operational methodology and characteristics of modern electronic Gas metering systems.

In a N. Gas metering system and virtually in any other metering system two main types of sensors are considered.

- a. Direct sensors (or concrete/physical sensors) that 'directly' sense the variable in question.

- b. Model based sensors or otherwise 'software based sensors' (Lu,Y.Z., 1996) or 'virtual sensors' that are actually infer or estimate the variable in question through other measurements and model(s) that relate the variable in question with the other measurements. These models or functional relations are referred to the "Guide" as the measurement equations.

It may be said that virtually all the existed sensors today belong to the second type since even the physical sensors give an output (in voltage, current e.t.a) that is functionally related to the measurand (measured variable).

For the estimation and analysis of combined measurement uncertainty issues, elements from the **Bayesian statistical theory** have been adopted since it forms the mathematical foundation of recent international recommendations included in the Guide for the estimation of uncertainty in measurement. These elements have been used and combined by the author within the compatible computer based **Monte Carlo simulation** method and have been applied to the error propagation research studies. Additionally, covariance and correlation coefficients where extensively used in these studies. Elements from a variety of other available mathematical methods were employed that combined with Monte Carlo uncertainty analysis could allow the on-line performance verification of a N. gas metering system.

For the study and observation of temporal effects on orifice meter, **Gabor transforms** (time-dependent Fourier transforms) were applied to pressure transducer signals collected upstream and downstream of an orifice meter. The respective 3D frequency-time-amplitude plots (**spectrograms**) appeared to give a compact and satisfactory method that can be used for the on line visualization of process variability and offer further aid in on line uncertainty assessments.

Analytical redundancy techniques when combined with on line uncertainty analysis can provide a rigorous and powerful tool in the measurement validation problem.

The combination and synergetic use of the above methods within a general methodology for a continuous and reliable uncertainty analysis, provides a powerful aid in the decision making, from planning and design, to real operation of measurement systems that could lead to significant cost savings. The proposed methodology consists of certain timely separated steps. The whole methodology is presented in chapter 6 after the presentation of the uncertainty analysis results.

1.3. Scope of the thesis.

In **Chapter 2** an overview of the latest developments towards an internationally accepted methodology for measurement uncertainty estimation is presented. A review of calibration and traceability issues is also included as well as the role of uncertainty as the proper index of information quality. Previous works on measurement uncertainty estimation in various fields is also presented, together with recent theoretical works on uncertainty estimation methods and especially the introduction of the Monte Carlo simulation method. Finally, a review of performance monitoring and evaluation of natural gas fiscal metering systems is included with emphasis mainly on the aging effects on measurement uncertainty and the efficient use of available instrumentation for flow measurement uncertainty monitoring. In **Chapter 3**, Monte Carlo simulation is introduced along with a new algorithm developed to generate truly normally distributed random numbers.

Chapter 4 gives the author's integrated theoretical approach on combined measurement uncertainty and its estimation methods in conjunction with the most recently processed theoretical background by others. The association of this approach with older very basic concepts that seem to have been neglected, as well as with the more modern concepts of Measurement Process Quality Management has been also analysed. Original theoretical work carried out by the author in terms of correlated uncertainties has been also incorporated within this integrated philosophy. Monte Carlo simulation results of input measurement correlation/covariance effects in simple measurement systems' results have been analysed and graphically demonstrated.

Chapter 5 describes the Monte Carlo computational results of natural gas energy flow rate uncertainty analysis that have been carried out by the author. Justification of the Monte Carlo simulation method as a proper computer based uncertainty estimation method, as well as its absolute compatibility with the internationally accepted GUM methodology has been clarified. A description of the Monte Carlo uncertainty simulation/estimation software and methodology developed in Simulink/Matlab is presented step by step. Improvements in terms of computational efficiency, its reliability, as well as the potential of the Monte Carlo method for on line uncertainty estimation have been also demonstrated.

Experimental analysis results regarding orifice flow metering and monitoring with the aid of new models and interrelationships between the various orifice coefficients

(Discharge, Contraction, Pressure loss coefficients, etc.) that were developed, is present in **Chapter 6**. A discussion on the findings and the relevance and interconnections of various phenomenological diverse practical and theoretical issues has been also included. In **Chapter 7** the role and incorporation of a continuous uncertainty analysis in the management decision making process has been developed. The general methodology, as outlined in section 3.1, has been further expanded taking into account the results of this project. In **Chapter 8** the conclusions drawn regarding the theoretical and practical results have been described. Finally, areas of further work required have been identified and outlined.

CHAPTER 2

2. LITERATURE REVIEW

2.1. Measurement uncertainty and information quality.

It has been repeatedly mentioned the last two decades that quality control and quality assurance are the technological allies of evolutionary and innovative changes taking place in our social and individual lives (ICQC'87, 1987; Kohoutek, 1988).

Ishikawa and Lu, (1985) have mentioned that in order to assure quality, the old concept of simply using just quality testing of an outgoing product has now to be replaced by the modern philosophy of total quality control (TQC). TQC is applied to all stages of life cycle of a process (like in a Natural Gas fiscal measurement process), with emphasis not only on the operation or the results of the operation but also to the entire quality cycle, from planning and design through operation, and preservation of the process.

Sydenham, (1982) & (1983) and Kemeny & Havrilla, (1986) further mentioned that due to the existence of extensively developed world-wide structures and globally recognised services related to legal and plant metrology, quality parameters must be well defined and analysed by physically measurable quantities or their measurable representatives. *Such a parameter is the measurement uncertainty.*

Hofmann, (1983) & (1986), stated that legal metrology has the task to ensure that there exists a unified system of measures and an adequate level of correctness of measurements made.

Plant metrology has the task of providing the measurements for process control and quality assurance. It must also make available quality management information at the right time and at the right place. The INQUAMESS (intelligent quality measurements) philosophy states:

'Objective quality information acquisition and comparable quality audit can only be accomplished if enough, appropriate measurements are made.' Process automation is the technological continuation of the human senses. It is the necessary tool to assist the brains and the hands of an operator. Process control gives us the means to follow and to efficiently control the process. The only method to get the necessary data about the

process' state is by taking samples of the process (measurements/calibrations), analyse them and re-adjust the control according to this data. Poor process control results in oscillating quality: the process quality is then either unacceptable (rejections) or too high (unnecessarily high quality and poor efficiency). The process control has its rules and limits: *'The control can never become superior to the measurements made on the process'*. The more accurate the data obtained from the process the more accurately the process can be controlled. Thus, an important component of the system must be the **quality of the measurement information itself** as it is the «feedstock» of the information system on which success depends so much.

Uncertainty can be regarded as this parameter/measure that can actually express this quality of measurement information.

The technico-economic relationship for assessment of the quality of the measurement information I_Q is given by Sydenham, P.H., & Thorn, R., (1992) as:

$$I_Q = \frac{\log_2 m}{t_E E} \text{ which is expressed as } \frac{\text{bit}}{s * \$} \quad 2.1$$

where:

m - maximum metrological discrimination capability in bits

t_E - response time in seconds

E - expense in monetary units

bit - binary digit of information transmitted

s - second

\$ - monetary unit

From the above function it is clear that the higher the quality and the amount of measurement information (thus the lower the measurement uncertainty), the higher is the technological value, the commercial competitiveness and subsequently the quality of the whole process that the measurement supports. The same conclusion has also been presented extensively by Napper, J., (1986) and especially for flow measurements by Archibald, G., (1993). The other two equally important parameters included in the above formula are the **timeliness** of the measurement information as well as the **cost**. The first quality pioneer was W. Edwards Deming who took his message to Japan in 1950 and helped to turn a failing industry into an economic world power. (Oberle, J.

1990, Burkhalter, B.B., et al., 1996). A few years later, Joseph Juran arrived in Japan and built a comparable record focused on planning and quality control. In 1979, Philip Crosby emerged as the third leader committed to total quality management with an emphasis on zero defects. Although the focus of these three quality gurus was different there was a common message that laid the foundation for the total quality management movement. The following five assumptions are common to all three gurus:

- Commit to quality improvement throughout the organization
- Attack the process or the system, not the employees
- **Strip down the process to find and eliminate problems that diminish quality**
- Identify customers and satisfy their requirements
- Instill teamwork and create an atmosphere for innovation and quality improvement.

Table 3 shows the differences between these assumptions and of other quality leaders that appeared at a later stage such as Feigenbaum, Taguchi and Ishikawa.

2.2. Measurement Uncertainty estimation

The importance of uncertainty analysis has become widely recognized in the last decades. Its importance has been recognised not only as a method that can give us a measure of the reliability of measurement results, but also as a powerful tool for decision making during all stages (from specifications and design up to the proper interpretation of the results) of an experiment or a measurement process. **Kline S.J., (1985)** has mentioned that although a great deal of work has been done on the "trees" of uncertainty estimation procedures, it would seem that there is no general recognition that a "forest" called uncertainty analysis exists as a valid part of engineering experimental work. He also pointed out that uncertainty analysis is the heart of quality control in experimental work. As **Kottegonda N.T., and Rosso R.,(1998)** have said, in past decades there has been an irreversible change in emphasis from descriptive to inferential statistics. In this respect, since all statistical inferences and the risk and reliability of decision making are taking place under *uncertainty*, the necessity of an optimum practical assessment of what is partially known or uncertain is self evident. During the last decade, there have been major developments in the field of uncertainty analysis that led to the publication by International Standards Organisation (ISO) of the Guide to the expression of Uncertainty in Measurement (GUM) where an international

and widely accepted methodology has been established. However, discussions on the proposed methodology of GUM are still active among various working groups including matters regarding the potential fields of application of uncertainty estimation.

2.2.1. Latest developments towards an International methodology.

Over the years, many different approaches in evaluating and expressing the uncertainty of measurement results have been used. Because of this lack of agreement, back in 1977, the International Committee for Weights and Measures (**CIPM**, Comité International des Poids et Mesures), the world's highest authority in the field of measurement science (i.e., metrology), asked the International Bureau of Weights and Measures (**BIPM**, Bureau International des Poids et Mesures), to address the problem in collaboration with the various national metrology institutes and to propose a specific recommendation for its solution. This led to the development of Recommendation **INC-1 (1980)** by the Working Group on the Statement of Uncertainties convened by the **BIPM**, a recommendation that **CIPM** approved in 1981 and reaffirmed in 1986 via its own Recommendations **1 (CI-1981)** and **1 (CI-1986)**

The above recommendation **INC-1 (1980)** is a brief outline rather than a detailed prescription. Consequently, the **CIPM** asked the International Organisation for Standardisation (**ISO**) to develop a detailed guide based on the recommendation because ISO could more easily reflect the requirements stemming from the broad interests of industry and commerce. The ISO Technical Advisory Group on Metrology (**TAG4**) was given this responsibility. It in turn established Working group 3 and assigned it the following terms of reference:

"To develop a guidance document based upon the recommendation of **BIPM** Working group on the statement of Uncertainties which provides rules on the expression of measurement uncertainty for use within standardisation, calibration, laboratory accreditation, and metrology services". The end result of the work of **ISO/TAG 4/WG 3** is the 100-page *Guide to the Expression of Uncertainty in Measurement* (or **GUM** as it is now often called). It was published in 1993 (corrected and reprinted in 1995) by ISO in the name of the seven international organisations that supported its development in **ISO/TAG 4**. These are **BIPM** (Bureau International de Poids et Mesures), **IEC** (International Electrotechnical Commission), **IFCC** (International Federation of Clinical

Chemistry), **ISO** (International Organisation for Standardisation), **IUPAC** (International Union of Pure and Applied Chemistry), **IUPAP** (International Union of Pure and Applied Physics), **OIML** (International Organisation for Legal Metrology).

The focus of the ISO Guide or GUM is the establishment of general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields, from the shop floor to fundamental research. As a consequence, the principles of the GUM are intended to be applicable to a broad spectrum of measurements, including those required for:

- maintaining quality control and quality assurance in production
- complying with and enforcing laws and regulations
- conducting basic research, applied research and development, in science and engineering
- calibrating standards and instruments and performing tests throughout a national measurement system in order to achieve traceability to national standards
- developing, maintaining and comparing international and national physical reference standards, including reference materials.

The GUM method of evaluating and expressing measurement uncertainty has been adopted widely by EC and U.S. industry. The National Conference of Standards Laboratories (NCSL), which has some 1500 members, has prepared and widely distributed **Recommended Practice RP-12, *Determining and Reporting Measurement Uncertainties***, based on the GUM.

ISO published the French, German and Chinese translations of the GUM in 1995, and an Italian translation was published in 1997. Translations in many other languages have been completed or are underway.

GUM has been adopted by various regional metrology and related organisations including **NORAMET** (North American Collaboration in Measurement Standards), **NAVLAP** (National Voluntary Laboratory Accreditation Program), **A2LA** (American Association for Laboratory Accreditation), **EUROMET** (European Collaboration in Measurement Standards), **EUROLAB** (A focus in analytical chemistry in Europe), **EAL** (European Cooperation for Accreditation of Laboratories). Moreover, the GUM has been adopted by **NIST** (National Institute for Standards and Technology), **NIST/TN-1297**,

and most NIST's sister national metrology institutes throughout the world such as the **NRC** (National Research Council) in Canada, **NPL** (National Physical Laboratory) in U.K and **PTB** (Physikalisch-Technische Bundesanstalt) in Germany. Most recently GUM has been adopted by **ANSI** (American National Standards Institute) as an American National Standard, **ANSI/NCSL, Z540-2-1997**.

Recently, a new international organisation has been established to assume responsibility for the maintenance and revision of the GUM and its companion document, the **VIM** - International Vocabulary of Basic and General terms in Metrology. The name of the new organisation is **JCGM** (Joint Committee for Guides on Metrology) and its members are the seven international organisations listed above that supported the development of the GUM. Furthermore, ISO/TAG 4 has been reconstituted as the **Joint ISO/IEC TAG**, and will focus on metrological issues internal to ISO and IEC as well as represent ISO and IEC on the JCGM. More detailed information on the subject can be found in NIST (National Institute of Technology) web pages at [www@nist.gov](http://www.nist.gov).

2.2.1.1. Calibration, traceability and Standardisation issues.

Today's process personnel all over the world are well aware of both economic and quality constraints. The professionals of process automation know that no process instruments can permanently remain at their optimal level of precision. The drift of the various process instruments requires periodical calibration. In order to maintain the accuracy of the process control or generally of the measurement instruments, these need to be verified against reliable calibration equipment (of higher accuracy classification than the instruments). Process maintenance is the key supplement of good process operation. Applying **ISO 9000** quality assurance system, and systematic calibration, improves the ability to control efficiently the quality and the entire cost effectiveness of a process. Planning the calibration is a part of the quality assurance for the measurements. And at this planning stage **Kline S.J., (1985)** and **Moffat R.J. (1985)**, have strongly emphasised the importance of the uncertainty analysis and the advantages offered by it.

A good implementation of an ISO 9000 quality assurance system requires both traceable calibration and good documentation of all the instruments directly involved with the quality of the process. The **ISO 9001, (1994)**, and **ISO 9002, (1994)**, quality assurance

systems require producers to state what measurements are used during the process, what level of error tolerance is allowed for these measurements and which measuring instruments are used for the measurements. **ISO 10012-1, (1992)** presents the quality assurance requirements for measuring equipment. Furthermore, it is a useful guide regarding specification and requirements for a measurement and calibration system while **ISO-10012, 1997** gives useful guidelines to control the measurement processes.

The adaptation of a **Quality Assurance system** in N. Gas fiscal metering (or MAP - Measurement Assurance Program), for respective inter-laboratory/facilities measurements, as referred by **Croarkin, M.C., (1984)**, is the most economic way to ensure a continuous optimal quality measurement process corresponding to the "sold" instrumentation quality.

Measurement and Calibration is unavoidably accompanied by the concept of traceability. This is based upon the need to check and ensure measurement results. **Spitzer D.W., (1991)** has mentioned that traceability has come to mean many things to many people while **Belanger, B.C., (1979)** has also stated that there are a number of definitions for traceability. A prevalently used definition for traceability is *"to calibrate into hierarchical scheme of measurements that leads, ultimately, to the national references for the respective measurements."*

However, the difficulty in flow measurement of fluids arises from the fact that there is not an '*identity*' standard or artifact such as for mass, or length. For example, the platinum kilogram known as K-20 is the ultimate artifact to provide the fundamental basis for mass measurement. Thus, for flow measurement systems, the metrology is based upon '*derived*' standards. **Mattingly, G.,E., (1981) & (1986)**, and **Mattingly, G.,E. & Spencer, E.,A, (1978)**, have analysed this subject and they have presented some procedures for quantifying the respective random and systematic errors in the various flow calibration facilities.

It can be said that traceability is a declaration stating to which national (and international) standard a certain instrument has been compared. Measurement devices form a kind of hierarchy with international and national standards on the top. There is a network comprising of organisations from the industrialised countries world wide, in

which standards are compared to each other in order to make sure that measurement readings mean exactly the same everywhere.

Traceability could also be expressed as a chain hanging down from the true value. The links represent International Standards, national standards, reference standards, working standards (or secondary standards) and process instruments. The true value is thus transferred with a certain degree of accuracy.

It could be also said that without primary standards, measurement equipment would only show changes and repeat indefinable indications. In case of no traceability to primary standards, measurements would be just some kind of values. This means that without internationally agreed quantities and without primary standards, everyday technical activity would be impossible. The calibration chain begins with the artifacts and primary standards and ends with process instruments. Thus, when investing in calibration equipment for process instruments it is very important that the supplier can guarantee that the calibrators are traceable by providing a calibration certificate that indicates, among others, against which reference standard and national standard the calibrators have been calibrated.

2.2.2. Measurement Uncertainty estimation – Previous work and fields of application.

Uncertainty estimation (or analysis in general) has been used in many scientific and technological fields and for various purposes during the last few years. **Cahill D.M., (1992)** presented an uncertainty estimation methodology implemented during an effort to reevaluate the uncertainty of the instrumentation systems used to obtain wind tunnel data in the test facilities of Arnold Engineering Development Center (AEDC). **Cahil D.M, (1994)** also mentions that the standardisation of a single uncertainty methodology is a tremendous accomplishment but it does not ensure that the methodology will be applied correctly. He analysed the case where estimating uncertainty by employing the root of sum squares (RSS) procedure while considering and using partial derivatives between dependent variables in this estimation, could result in a significant error in the estimated uncertainty. He also stated that the partial derivatives with respect to dependent variables could be used, if the correct correlated biases present in the relationships of dependent variables are accounted for in the uncertainty analysis, a task

next to impossible as he added. **Wright F.L., (1994)** examined the power of least squares curve fit approach in terms of reducing the confidence interval of mean values in calibration data compared with single sample statistical analysis results. **Blumenthal P.Z., (1995)** developed an off line PC program that is based on RSS method, for estimating measurement uncertainty for aeronautics test instrumentation at Lewis Research Center. **Blanton J.N., (1995)** applied the RSS analytical methodology for the uncertainty estimation of test section pressure and velocity in the Large Cavitation Channel (LCC). The LCC is a large recirculating variable pressure water tunnel at the Memphis Detachment of the CARDEROCKDIV, Naval Surface Warfare Center/David Taylor Model Basin and it is also used for measurements of non-cavitating flow fields around fully appended hydrodynamic models. However as he admitted, he has not considered conceptual bias errors which are errors resulting from not actually measuring all the variables whose symbols appear in the data reduction equations. These errors are described by **Moffat R.J., (1988)** and are coming from effects that are not usually considered since they are difficult to be quantified in an uncertainty analysis. For example blockage effects on a fluid velocity measurement by pitot tube. **Greiner B., and Frederick, Jr.R.A., (1995)** mentioned that in order to reduce the cost and complexity of experimental programs quite often experimenters choose to use sub-scale or lab-scale test rigs. From these lab-scale tests, data are collected in order to gain more information about the basic operational phenomena and physics. This methodology has been widely applied in the area of propulsion testing for the past fifty years as in many other fields. However, and until recently, as they said, little more than afterthought has been given to the experimental uncertainty of the data from which any empirical correlations were made. A particular case is with lab-scale testing surrounding hybrid rocket motors. A test program conducted by the University of Alabama in Huntsville Propulsion Research Center into the mechanism of hybrid rocket instability led to a detailed uncertainty analysis of a lab-scale hybrid motor used in the investigation. Their results showed that the typical uncertainties in the measurements of the fuel regression rate ($\pm 8.7\%$), oxidizer flux ($\pm 10.8\%$), motor characteristic velocity ($\pm 10.5\%$) and the oxidiser fuel to mass ratio ($\pm 9.2\%$), were relatively high, thus limiting the use of the lab-scale hybrid motor for quantitative investigations such as the generation of curve fits.

Hedlund E.R., and Kammeyer M.E., (1996) presented an assessment of experimental uncertainty in the Naval Surface Warfare Center's Hypervelocity Wind Tunnel 9 facility as applied to tunnel conditions, static stability and drag, pressure and heat transfer testing. They followed an approach that was in accordance with AIAA Standard S-071. Additionally, they emphasised the high interest in terms of reliable measurement uncertainty estimation and methodology standardisation in wind tunnel testing. This interest is driven, as they said, by the need to quantify the data validity so that design changes to aerospace vehicles are sensibly engineered, computational fluid dynamic models are properly validated and meaningful data comparisons between facilities can be performed.

The development and status of a Data Quality Assurance program initiated at NASA Langley Research Center as part of an effort to re-engineer their wind-tunnel testing process is presented by **Hemsch M.J., (1996)**. The program implementation was divided in the following four major phases:

1. Simplified Uncertainty Prediction and Reporting
2. Detailed Uncertainty Analysis and Statistical Control
3. Certification and National Standards
4. Continuous Improvement.

The importance that NASA has set on Uncertainty estimation through this program, is self-evident. Hemsch has also adopted the point of view that statistical thinking and designing wind tunnels tests, using uncertainty analysis, are fundamental paradigm shifts and a cultural change of huge proportions is involved. He stresses also the fact that changes in the wind-tunnel testing measurement uncertainty levels may need systematic changes in the process itself. However, although Hemsch recognised the value of uncertainty predictions in the design of experiments, he also stated that it was not yet an idea whose time had come. **Belter D.L., (1996)**, presented the application of an uncertainty analysis methodology at the BOEING Aerodynamics laboratory and described how uncertainty analysis can help in decision making. Presenting an example from their testing facility, he described that they were in the process of applying uncertainty analysis in order to uncover the areas where instrumentation is less than adequate to meet routine customer requirements. This would be proved very useful

when upgrading or replacing capital equipment since it is easier to identify and justify the allocation of funds for a direct payoff to the customer. For this purpose, they developed a computer program, often known as a "jitter" program, for the computation of sensitivity coefficients, in which each independent variable is perturbed by the same percent and the resulting change in the parameter of interest is observed. **Springer A.M., (1999)** presented results of an uncertainty analysis performed on the NASA Marshall Space Flight Center 14-inch trisonic wind tunnel in accordance with **AIAA S-071, (1995)**. Their analysis was able to reveal the major contributors in the uncertainty of wind tunnel data that were the wind tunnel calibration and a strain gauge balance.

An area of extremely high importance in uncertainty estimation, from a theoretical and a practical point of view, that seems to have received the necessary attention only during the last few years, is the correlated uncertainties between input variables and their effects in a measurement result.

2.2.3. Uncertainty estimation theory and the role of the Monte Carlo simulation method.

The use of Monte Carlo as an uncertainty estimation method has received proper attention only the last few years. It's potential as a computer based method as well as the fields of application are currently under extensive research.

One of the most recent Monte Carlo works with regard to the work carried out in this thesis, was that of **Mullen M., Pratt E.L., (1999)** in Los Alamos National Laboratory. They used a Monte Carlo method to test the significance of trends observed in radiochemical data sets in order to answer the question:

"When an apparent trend is observed, is it really "significant" or are the reported results so uncertain that the apparent trend could be due to chance alone."

Bekesi G., et.al., (1999) have applied Monte Carlo simulation for modeling and uncertainty estimation of regional groundwater recharge in the Manawatu Region, New Zealand. Because groundwater is inherently susceptible to contamination from landuse activities, they carried out a vulnerability assessment study to examine those factors that predominantly control this vulnerability. **Morgan M.G., et.al., (1990)** have selected Monte Carlo simulation techniques for their respective uncertainty assessments because, as they said, Monte Carlo simulation could provide a truly global measure for

uncertainty. **Basil, M., & Jamieson, A.W., (1998)** have actually introduced and applied Monte Carlo simulation methods to assess the uncertainty of Natural gas density and calorific value in gas metering. These provide elementary measurement uncertainties of total natural gas energy flow uncertainty estimation.

In terms of the theoretical aspects introduced in the Guide and their applications in the uncertainty analysis of elementary uncertainties in natural gas energy flow measurement processes can be found in **Van der Grinten J.G.M., (1998)** of Nmi and **Midttveit O., Nilsson J., (1998)** of Christian Michelsen research AS.

Additionally, **Lira H.I., and Woger W., (1998)** theoretically addressed the problem of evaluating the uncertainty in knowing a directly measured quantity with a measuring instrument. **Lira H.I., and Woger W., (1997)** also presented a treatment of the problem of evaluating the uncertainty that arises from the limited resolution of the indicating devices. A Bayesian theory of measurement uncertainty is presented by **Woger W., and Weise K., (1992)**. Bayesian statistics and the principle of maximum entropy constituted the basis of this theory. **Meyn A.L., (1998)** described how uncertainty analysis has become an important tool in planning and executing tests of high complexity and how beneficial would be if uncertainty analysis could be automated and integrated into the data reduction process. He also presents a process named "automatic differentiation". **Maciejewski K.P.,** presented an approach to the assessment of random uncertainty (1996) and of systematic uncertainty (1998) in transient measurements. He used an inverse model method for the estimation of the unobserved input (measurand) to a linear time invariant measurement system from the continuous observed output from the measured system. Inverse methods have become recently quite popular while it is known from **Bentley, P.J., (1988)** and **Beck V.J., et.al. (1998)**, that inverse equations are useful for error reduction. **Sakariassen R., (1998)** demonstrated a method for on line quality control of ultrasonic gas flow meters which is actually based, although not specifically mentioned, in analytical redundancy. Finally, **Mylvaganam K.S.P., (1998)**, presents the importance of software testability, testing and verification in fiscal oil and gas flow metering. He also referred, how the Norwegian Petroleum Directorate stipulates that the calculation accuracy of the flow computers shall be reviewed

independently at least on an annual basis, with the usage, for example, of a reliable independent PC based program.

Some other important reference works on the subject are included in the presentation of theoretical and experimental work.

2.3. Natural gas Flow Measurement System with orifice meters – Performance monitoring and evaluation.

2.3.1. Factors affecting flow measurement uncertainty and on line performance monitoring using signal analysis methods.

The orifice meter is the most widely used and well-documented flow meter in Natural Gas fiscal metering industry. ISO-5167, (1992), Battye J.S., (1985), Miller R.W., (1983), Spitzer D.W., (1991), Spink, L.K.(9th ed.) et. al provide detailed information on the design, installation and operation of this meter.

The accurate measurement of small differential pressures in the order of 0 - 500mbar, at high line pressure (100 to 200bar), has assumed growing importance in recent years. This is particularly evident in the field of offshore flowrate measurements associated with custody transfer and allocation.

The measurement of gas flow is one of metrology's most misunderstood 'sciences' and carries an elevated degree of uncertainty. Typical errors in measurement include: failing to clearly specify terms used for the measurement conditions (e.g. Normal, reference, STP, Contract etc.) that could create a lot of confusion; or concepts like considering rotameters as 'primary devices!!'. (D. M. Korpi, 1994)

Because gas is a compressible medium, accurate measurement requires the use of all the standard disciplines. In order to measure flow, or perform relevant calibrations, it is required to use three of the primary standards (time, length, mass). As such, the potential for error propagation is much greater. A gas flow rate calibration error analysis typically yields no better than a 0.2% of reading (NIST traceable mass flow Standard). The next level transfer standard offers a 0.8% accuracy. This demonstrates why the only accurate means to calibrate gas flow rate is with a primary standard.

All new orifice metering system must be designed and comply with certain specifications listed in ISO-5167: Section1.1. A vital issue of the proper operation of an orifice meter is the existence and preservation of a fully developed flow velocity profile

upstream of it and a steady flow. The velocity profile effects on orifice meter accuracy have been extensively described in the existing literature. A significant degradation in accuracy may occur however from other causes with the passage of time. (BS 1042: Section1.5, 1996).

This degradation may result from several causes:

- deformation of the orifice plate (elastic bending or buckling (plastic deformation))
- deposition of the upstream face of an orifice plate
- deposition in the meter tube
- orifice plate sharp edge rounding.

Experimental studies on the above as well as on their effects on orifice meter accuracy have been carried out by Jepson, P.& Chipchase R.,1975, Norman, R., et.al., 1983, Norman, R., et.al., 1984, and Hobbs, J.M., 1990

Jepson & Chipchase concluded that plastically buckled orifice plates will in general under-register the flow. Plastic deformation arises when an orifice plate has been subjected to excessive differential pressures such as those that may occur during over-rapid pressurisation or venting of a line containing a compressible fluid, or through abnormal flow conditions. The differential pressure required to yield a simply supported orifice plate may be estimated from:

$$\Delta p_y = \sigma_y \left(\frac{E}{D_2} \right)^2 \left(\frac{1}{0.681 - 0.651\beta} \right)$$

where E: orifice plate thickness, D_2 : Plate support diameter, σ_y : Yield stress of orifice plate material and β : beta ratio.

For a plate simply supported at its rim, a first approximation for the percentage increase in discharge coefficient is given by:

$$\Delta C = \frac{100\Delta P}{Y} \left(\frac{D_2}{E} \right) \left(\frac{a_1 D_2}{E} - a_2 \right)$$

where

$$a_1 = \beta(0.135 - 0.155\beta)$$

$$a_2 = 1.17 - 1.06\beta^{1.3}$$

Furthermore the flow measurement error due to elastic deflection in the flow direction can be related to the geometry and mechanical properties of an orifice plate by (Jepson,P & Chipchase, R.):

$$\frac{\Delta q_m}{q_m} = -\frac{1}{(1-C_c^2 \beta^4)} \left\{ aN + bN^2 - \frac{cNE}{D_2} \right\}$$

where a, b, c: coefficients, C_c : Contraction coefficient and $N = \frac{\Delta P}{Y} \left(\frac{D_2}{E} \right)^3$ where Y is

the elastic modulus of the orifice plate material.

Orifice plates that do not have the specified sharpness of the inlet edge (edge radius $r < 0.0004d$ (in accordance with BS EN ISO 5167-1,1992) will have progressively increasing discharge coefficients as the edge radius increases. Tests have shown that for r/d of 0.001 the effect on the C_d is to increase it by 0.5% and for r/d of 0.01 by about 5%. This is an approximately linear relationship. (BS 1042:Section:1.5,1996, Hobbs, J.M., et. al., 1990).

Duddy, E., 1988, and Hobbs, J.M., 1988, have carried out several tests at NEL with simulated damages on orifice plates and their effects of the discharge coefficient. Duddy studied the effects of local damage to the upstream edge of a 0.75 diameter ratio plate and he noticed that the C_d seems to depend on the ratio of the dimensions of the indentation to that of the bore. Furthermore, Hobbs plotted a clear trend of gradually increasing discharge coefficients for three different beta ratio plates as the radius of their sharp edge increases. Possible causes of sharp edge deterioration are:

- Erosion
- cavitation
- mechanical damage
- careless handling

Excessive thickness of the orifice edge can significantly increase the discharge coefficient (Husain, Z.D., et. al., 1986). Local damage to the upstream face or edge of an orifice plate does not adversely affect the discharge coefficient provided that the damage is far away from the pressure tapping as possible. (Hobbs, J.M., et.al., 1990). The discharge coefficient is much less sensitive to the surface condition of the downstream face of the plate.

The measurement diagnostic capabilities of the modern signal analysis techniques have been also emphasised by many researchers in the field of flow measurement.

Higham, E.H. et al, 1986, and **Ajayi, A.A., 1987** analysed the output signals from a turbine meter using signals analysis techniques and noticed the changes to the 'signatures' (Power Spectrum Densities) under various conditions. A change in the signature was the result of a change either of the status of the meter or its operating conditions. Thus, by monitoring these signatures over a period of time they could initiate diagnosis actions if an alteration was indicated.

Amadi - Echendu, 1988, and **Amadi- Echendu et.al., 1990**, have undertaken much of the research in this area. A review work related to this area could be found in **Papadopoulos, C., 1996** and **Herne-Smith R.S., 1996**. The analysis of the noise spectrum from a resonant wire differential pressure transducer, operated in conjunction with an orifice plate was suggested by **Hurren, P.J. et. al., 1988** and **Zhu, H., et. al., 1992**. Using the FFT (Fast Fourier Transformation) algorithm, he successfully demonstrated the feasibility of using such a procedure to obtain a 'signature' for different operating conditions. **Amadi - Echendu, 1988**, in addition to the power spectrum signatures that represent the signals in the frequency domain, used the coefficient of variance (equivalent to the sample variance/sample mean) as a statistical parameter in order to measure the relative strength of the perturbations with respect to the mean flowrate. Relatively high values were observed when distinct pulsations were present in the flow. That was an expected observation, since the variance and the power spectrum density are actually related.

The effects of acoustic noise on orifice meters have been studied by **Norman, R., et. al., 1989** at the British Gas test facility at Bishop Auckland. A description of the test facility at Bishop Auckland is given by **Norman R., et. al., 1986**. According to Norman, in-pipe acoustic noise could cause errors in orifice plate metering and the international metering community regarded this topic as one of high priority for further research. Acoustic noise tests on a 0.4β (β : beta ratio) orifice plate in a 600mm line at a pressure approximately of 50 bar have produced the following findings:

- A pipe noise level of 170dBA caused a decrease in the C_d by 0,5% for D, D/2 taps and by 0.4% for flange taps.
- The introduction of an appropriately designed silencer reduced the noise levels of 170dBA by 20 - 150dBA. The effect of acoustic noise could be partly explained by an extension of root-mean square error theory to high frequency random noise.
- The differential pressure transmitter was unaffected by the noise levels to which it was exposed.

The last conclusion of Norman et. al. must be carefully examined by considering also the fact to what extend the differential pressure transducer can actually read the true wall differential pressure when it is connected to the impulse lines, where a viscous damping was actually present.

A considerable amount of work, such as the work of **Botros, K.K., et. al., 1986**, as well as of many other workers, has been carried out on sound velocity effects at relatively low frequencies and below the cut-off frequency, where plane waves only propagate along the pipe. The presence of the acoustic field near the orifice has been also reported by **Agarwal, N.K., 1987** and **Bull, M.K., 1983**. This presence was evidenced by the presence, in the wall pressure spectra, of sharp peaks, the frequencies of which can be identified with those corresponding to the onset of propagation of the various orders of acoustic modes that can be generated within a circular pipe. The peaks were not very prominent in the separated flow region, indicating a major contribution to the pressure fluctuations in this region by turbulence or non-propagating acoustic modes or both. Peaks were prominent in spectra both upstream of the orifice and in the reattached and redeveloping flow downstream. Similar results have been obtained by the author as well (**Papadopoulos, C., 1996**).

In the Draft Revision **ISO CD TR 3313, March 1997**, the effects of flow pulsation on flow measuring instruments have been extensively described. A review of experimental data for orifice metering error in pulsating flow conditions is given by **Gajan, P., et.al., 1992**. The effects of the impulse lines to the Differential pressure transmitters response under pulsating flow is described by **Botros, K.K., et. al., 1992**, **Hebrand, P., et. al., 1985** and **Clark, C., 1992**. The later author has also emphasised the dramatic effect of bubbles in liquid-filled impulse lines upon their resonant frequency. **Kothari, K.M.,**

1988 in the 13th report of the 600mm orifice plate project that was carried out at Gasunie's Bernoulli Laboratory in Netherlands, reported experiments regarding the DP impulse lines and showed their significant effects on the DP measurements. These effects are caused by the energy losses of the acoustic waves traveling through the lines. The acoustic waves are generated by the turbulent flow passing the pressure taps. Acoustic resonance (standing waves) in the lines can occur very easily in which cases the effect on the DP measurements is strong. The effect was explained by considering a sudden change in cross section of the measuring line. When the gas is going from the large into the small diameter pipe, a *vena contracta* will be present in the small diameter pipe. When the gas is going into the opposite direction, a significantly different streamline profile is present. So, during one half of the periodic time of the oscillating flow the pressure loss across the diameter change differs from the pressure loss during the second half of the periodic time. The time averaged value of the differential pressure across the diameter change is **non-zero**; the time averaged value of the mass flow, however, is **zero**. In summary it has been stated that if the time varying difference can be faithfully followed by the secondary measurement system and the square root of its signal averaged, then the square root error is eliminated for the indicated time-mean flow rate. For a fast response secondary measurement system, a differential pressure transmitter with very small internal volume and high natural frequency is required. In addition the following rules must be applied:

- The mechanical and electronic frequency limits of the secondary system should be at least ten times greater than the flow pulsation frequency.
- Impulse lines should be as short as possible (if possible the transducer to be flush-mounted) and their length must be less than 10% of the pulsation quarter-wavelength ($l < 0.1 * c / (4 * f_r)$) where c velocity of sound and f_r , resonant frequency.
- The secondary system for liquids must not have any gas bubbles trapped in the impulse lines and the opposite for a gas system.

Recently, **Zhang, J., et.al., 2000**, used wavelet transforms of the DP pressure signals across an orifice meter for detecting the incipience of blockage of the impulse lines and its effect on the DP signal output. They reported that the blockage in the sensing lines influenced the wide-band noise of the differential pressure sensor output. The discharge

coefficient uncertainties and the uncertainties in the differential pressure and line density measurements are the main contributors to gas flow measurement uncertainty.

2.3.2. Multi-measurement data integration, fusion and validation in

Natural Gas flow measurement.

A modern N. gas metering system can be defined as a multi-sensor system and as such can be analysed using respective multisensor data fusion and integration methods. These methods have been successfully applied in other fields such as aerospace, military, or even in finance for risk analysis and decision making. A review on the subject and its application areas could be found in **Luo, R.,C.,et.al.(1989)** and **Bloch, I. (1996)**. In those fields it has been repeatedly reported the ability of uncertainty reduction using sensor fusion and integration techniques. A data fusion approach to improve the performance capability of multiphase flow meters within an expert system has been recently presented by **Annunziato M., Bertini I., Lopez S., et.al. (1999)**. An application for controlling and optimising oil production wells based on a methodology of process behavior trajectory monitoring has been presented by **Zambrano,A., Vazquez, M. et.al., 1999**. They developed a control system which it performs fusion of all the process variables into a single point (the current process state) and process changes are interpreted as a deviation from the expected trajectory. **McCoy, (1992)** described an application of data fusion for fuel management requirements of military aircraft. A probabilistic approach to sensor data validation has been presented by **Bickmore, T.W., 1992**. His method was used for validating rocket engine sensor data in real time using analytical redundancy Bayesian Belief networks, which are actually graphical representations of joint probability distributions of sets of random variables. Finally, **Osder, S., 1999** has presented a practical view of redundancy management application and theory and analysed the advantages and limitations of analytical redundancy as opposed to physical redundancy. He concluded that analytical redundancy does not substitute physical redundancy but it can be useful supplement *if implemented in a manner that properly exploits the physical redundancy*. More technical details are given in the following chapters leading to work done by the author.

CHAPTER 3

3. MONTE CARLO SIMULATION METHOD AND NEW ALGORITHM TO GENERATE TRULY STANDARD NORMALLY DISTRIBUTED RANDOM NUMBERS.

3.1. Monte Carlo Simulation method.

With the availability of digital computers, numerical experiments have become an increasingly popular method to analyse physical engineering systems. In many applications they are also substituted for physical experiments although physical experiments are traditionally the ones that could give confidence to any numerical result. According to **Kottegoda, N.T., and Rosso, R., 1998**, *simulation* is generally defined as the process of replication of the real world based on a set of assumptions and conceived models of reality.

Monte Carlo simulation can be classified as a *computer-intensive method* (**Manly, B.F., 1997**), and particularly as a *computer based random sampling method*. However with today's ever-increasing computer power and considering the benefits of the method in uncertainty analysis compared with analytical methods, the word intensive becomes quite relative. Initially, it was devised as an experimental probabilistic method to solve difficult deterministic problems since computers can easily simulate a large number of experimental trials that have random outcomes.

The term Monte Carlo is said to have been introduced by the physicist John Von Neumann as a code word connected with his secret work on the atomic bomb at Los Alamos during World War II.

When applied to uncertainty estimation, random numbers are used to randomly sample *measurement inputs' parameter uncertainty space* instead of single point selection/calculation carried out by conventional methods. So the Monte Carlo method relies on "*randomness*".

The Monte Carlo Simulation procedure, as it has been applied for combined measurement uncertainty estimation purposes in this project, is actually based, as the various numerical integration techniques (trapezoidal, Simpson's e.t.a), on the basic Reimann definition of the definite integral as a limit described as follows.

The integral of a function $f(x)$ from a to b is approximated by first partitioning the interval a - b in n segments and evaluating the sum as: $I = \sum_{i=1}^n f(x_i) \Delta x_i$, where Δx_i is the width of the i^{th} segment and $f(x_i)$ is a constant value of $f(x)$ for some x within the segment or otherwise a polynomial fit of degree zero. Usually, a partition of equally sized segments is selected so $\Delta x = \frac{(b-a)}{n}$ and as $n \rightarrow \infty$, $\Delta x_i \rightarrow 0$. The integral then corresponds to:

$$\int_a^b f(x) dx = \sum_{i=1}^n f(x_i) \frac{(b-a)}{n} = (b-a) \overline{f(x)} \quad 3.1$$

In the end, this expresses the *mean value theorem* which states that the definite integral of a function from a to b is approximated by the area of the rectangle of height $\overline{f(x)}$ and width $(b-a)$, where $\overline{f(x)}$ is the mean value of the function between the limits. It can be associated with "random" sampling from a uniform distribution, in such a way that equally spaced x values between a and b are selected so $\overline{f(x)} = \frac{1}{(b-a)} \int_a^b f(x) dx$. 5.2

The term $\frac{1}{(b-a)}$ can also represent a uniform probability density function (or replaced by other probability functions, depending on the case being modeled) and if inserted into the integral, corresponds to the fundamental definition (equation) of the mean value.

3.2. Random numbers generation - Sampling the input distributions

The first step in the application of a Monte Carlo uncertainty estimation method is the representation of every measurement model's input with some probability distribution. This has been carried out through the generation of sets of *standard random variables*. It mainly involves the computer generation of sequences, usually of uniform random deviates, on the interval $[0,1]$. All the methods of generating standard uniform deviates are in reality deterministic, in the sense that systematic procedures are used after one or more initial values are randomly selected (Kottegoda, N.T., Rosso, R., 1998).

The most common computer-coded algorithms that are used as random number generators usually apply a recursive formula, where each new digit generated is in some

way based on the digit generated prior to it. Given an initial number (called a *seed*), the algorithm can generate a sequence of digits in apparently random order. Eventually, the sequence of digits starts over, and the list begins to repeat itself or cycle. The length of the cycle or the number of digits that begin to repeat is called the *period* of the generator. Of course the longer the period the better, since as it can be easily understood, in a truly random number generation procedure it is undesirable to have repetition of the digits in the same order over and over again. Today, primarily three types of generators are in use:

- Congruential generators that are based on modular arithmetic (Remainder after division).
- Generators that use the binary (bit) structure of computer-stored information (Shift Register generators or Linear Feedback Shift Registers, LFSR described by Gershenfeld, N., 1999).
- Generators based on number theory.

Linear congruential generators, and more sophisticated variants, are the most common random number generators in most digital computers. The *multiplicative congruential generator*, is the simplest and most common algorithm used, given by: $x_{I+1} = ax_i \pmod{m}$. It is part of the more generalised class of algorithms given by: $x_{I+1} = (ax_I + c) \pmod{m}$ when $c=0$. The later general algorithm is implemented with a recursive calculation of a sequence of integers $x_1, x_2, \dots, x_n$ each between 0 and $m-1$ where m is a large number. The factors a and c are positive integers called the *multiplier* and the *increment* respectively, and the notation $\pmod{m}$ dictates that x_{I+1} is the remainder obtained after dividing $(ax_I + c)$ by m . The modulus m determines the maximum length of the period. In reality, m must be a very large prime number to get a good generator with a long period. Since these numbers are not truly random, because on the one hand they have been generated via a set of deterministic rules and on the other hand they are repeated with a given period, they are generally referred as '*pseudo-random numbers*'.

According to the author's point of view, although the sequence of numbers generated by these generators, which are the most common today, is regarded as a random sequence, it has more the characteristics of a *chaotic* sequence and in the end must be regarded as such. On the other hand, **Bennett, D.J, 1998** referred that even the inventor of the Monte Carlo method, Von Neumann, has stated that anyone who considers arithmetic methods

of view, Neumann's statement has to do exactly with this *deterministic* character of random number generators in general and of Monte Carlo methods in particular, which are based on deterministic rules and on pseudo-random numbers generation.

A new class of generators based on number theory has been developed recently by **Marsaglia, G., & Zaman, A., 1991**. Their technique that was called add-with-carry and subtract-with-borrow, relies on the Fibonacci sequence and the so-called lagged-Fibonacci sequence, both named after the 12th century Italian mathematician Leonardo Fibonacci. The Fibonacci sequence consists of the numbers 0,1,1,2,3,5,8,13,21,34 and so on, where each number except the first two (0 and 1) is obtained by summing the two numbers preceding it. If the sum is 10 or more, for example 14, we use the right-most digit only (e.g. 4) and carry the 1 to be used for the calculation of the next digit. A lagged-Fibonacci sequence begins with two enormously large starting numbers, or seeds, instead of 0 and 1. Marsaglia and Zaman have been demonstrated that their generators have very long periods and they deserve serious consideration for Monte Carlo work. Algorithms that generate pseudo-random numbers, which closely approximate independent random variates, are widely available in any computers statistical software today.

Random variables with Gaussian probability distributions, $N(0,1)$, have been selected for sampling the inputs to the measurement system in this project as it has been justified in section 4.1.2.1. The particular procedure followed is presented next.

Assuming that the two quantities X_1 and X_2 with mean (best) values $x_1=5$ and $x_2=6$ and uncertainties, corresponding to approximately 95% confidence level, $u(x_1)=2\sigma(x_1)=0.1$ and $u(x_2)=2\sigma(x_2)=0.8$ respectively, represent the inputs to a measurement system equation. In order these quantities to be modeled as normal random variables, a set of independent identically distributed standard normal variables, $n_i, \sim N(0,1)$ is generated using a proper random generation algorithm. This can be accomplished with many different ways:

- A self-built generator is employed, as the author did for various testing cases. A linear congruential generator similar to these employed in digital computer generators, gives sequences of numbers as output that follow standard uniform distributions $u_i \sim U[0,1]$. The input measurands are expressed then as functions, generally known as

distributions $u_i \sim U[0,1]$. The input measurands are expressed then as functions, generally known as *random functions*, of the generated uniform distributions. Actually, this is carried out by employing transformation methods for random variables and calculating the respecting *Jacobians* of the transformations as those described by **Walpole, R.E., & Myers, R.H., 1993**. These are generally inverse transform methods, and they can not be applied straightforwardly in cases where probability models cannot be inverted analytically such as when we want to transform a uniform $U(0,1)$ to a normal $N(0,1)$ distribution as in this project. In such cases, other methods can be employed such as the *decomposition* or *rejection* methods, as described by **Kalos, M.H., et. al., 1986** and **Kottegoda, N.T., et.al., 1998**.

The author though has adopted the most straightforward Box-Muller method as described by **Press et.al., 1992**. So, two independent standard normal distributions $n_1 \sim N(0,1)$ and $n_2 \sim N(0,1)$ are obtained by two independently generated uniform $u_1 \sim U[0,1]$ and $u_2 \sim U[0,1]$ distributions from the functions $n_1 = \sqrt{-2 \ln(u_1)} \cos(2\pi u_2)$ and $n_2 = \sqrt{-2 \ln(u_1)} \sin(2\pi u_2)$. These are shown in **figure 3.1.1**.

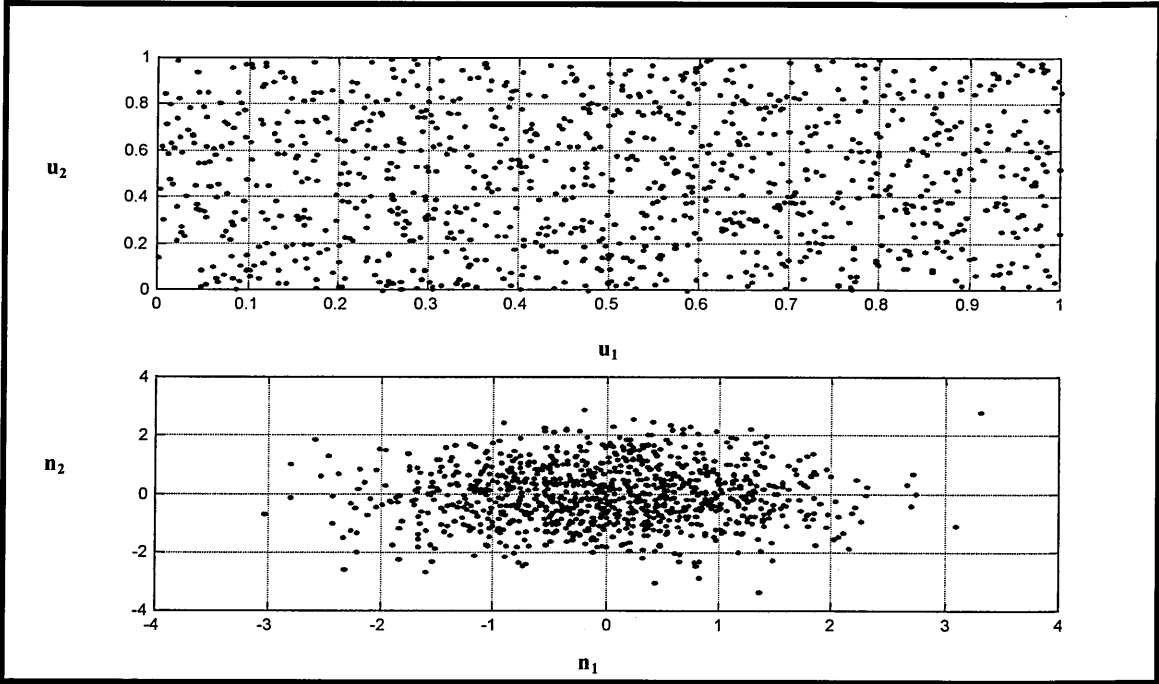


Figure 3.1.1 Box-Muller transformation of two independent generated u_1, u_2 uniform $U[0,1]$ distributions to two independent n_1, n_2 normal $N(0,1)$ distributions.

Once the sets of standard normal distributions $\mathbf{n}_1, \mathbf{n}_2 \sim \mathbf{N}(0,1)$ have been generated in this way, common Monte Carlo methods use them to directly model the normal inputs $\mathbf{X}_i \sim \mathbf{N}(\mu, \sigma)$ as random functions of these standard normal distributions using the transformation $\mathbf{x}_i = \mu + \sigma \mathbf{n}_i$. So, the two input measurements x_1 and x_2 with their uncertainties can be modeled as $\mathbf{x}_1 = 5 + 0.1\mathbf{n}_1$ and $\mathbf{x}_2 = 6 + 0.8\mathbf{n}_2$.

- Another similar procedure can be also followed using the computer random number generator functions and producing either standard uniform distributions ('rand'), and transform them to normal similarly as above, or producing normal distributions in one go using the normal random numbers computer generator function ('randn') which actually performs similar transformations.

3.2.1. A new algorithm to generate truly standard normally distributed random numbers

Up to this point, all the above methods are virtually equivalent. These directly computer produced random sequences have sampling variability that is decreased by increasing the number of sample points or sample sets, converging after many sampling cycles to the known input mean 0 and standard deviation 1 with a certain rate and accuracy.

Since one of the interests of this project was to examine the inputs' covariance errors and their effects in a complex multivariable measurement result, it was important for the means and uncertainties of the simulated inputs to have minimum possible errors when these inputs were modeled as independent normal random variables. However, approximation errors always exist in Monte Carlo mean and standard deviation estimation (generation) and depend on the spread of the values of the generated samples and on the number of samples used. So, a goal would be the sampling means and standard deviations of the generated x_1 and x_2 practically not to differ significantly from their nominal input values.

In a straightforward application of Monte Carlo using the raw generated random distributions, the error term on the estimated mean decreases as $n^{-1/2}$, where n is the number of samples, which is generally a slow convergence rate. Thus, depending on the test case, an increased number n of simulated samples is usually needed, which consequently results in regarding Monte Carlo as an inefficient method in terms of computation time and effort. However, there are various methods proposed to overcome

this problem, aiming in reducing the variance of simulated results without increasing the sample size. *Importance sampling*, *control variates* and *antithetic variates* are the main methods proposed. For example, the antithetic variates method is actually based on the properties of the mean of negatively correlated samples added together which cause variance reduction. These particular properties, which are also significant for other reasons, will be further discussed during the presentation of the test results.

The author invented another more simple method and managed to obtain estimates for the input variables almost down to computer round off accuracy. This was a key point in the uncertainty propagation study as well as in the later analytical evaluation and validation of the numerical results obtained. More importantly, the method can give reliable and reproducible results even for single sample runs, which is a significant improvement and favors the wider application of Monte Carlo for uncertainty estimation.

Unfortunately, the method was discovered at the latest stages of this project and so the main results obtained for energy gas metering uncertainties have been obtained from long sets of simulations cycles. However, as it was said, the method was proved extremely valuable in the analytical verification of these results and in the better comprehension of the uncertainty propagation mechanism.

The procedure is quite simple. After having obtained the first generated standard normal distributions $n_1, n_2 \sim N(0,1)$ as described previously, a second (2nd) transformation is

applied by calculating $z_1 = \frac{(n_{1i} - \bar{n}_1)}{\sigma(n_1)}$ and $z_2 = \frac{(n_{2i} - \bar{n}_2)}{\sigma(n_2)}$ respectively. Since n_1 and n_2

follow normal distributions (standard), z_1 and z_2 will also follow standard normal

distributions given by: $f(z_i) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z_i^2}{2}}$ which actually defines the distribution of the

mean (0 in this case) of a random sample from a normal population. In other words it is obtained the standard normal distribution of the raw generated standard normal distribution. So, the mean over the same sample of the generated standard normal variable, after the 2nd transformation, obtained analytically is:

$$\bar{z} = \sum_{i=1}^k \frac{\left(\frac{(n_i - \bar{n})}{\sigma(n)} \right)}{k} = \sum_{i=1}^k \frac{(n_i - \bar{n})}{k\sigma(n)} = \frac{\sum_{i=1}^k n_i}{k\sigma(n)} - \frac{\sum_{i=1}^k \bar{n}}{k\sigma(n)} = \frac{\bar{n}}{\sigma(n)} - \frac{k\bar{n}}{k\sigma(n)} = \frac{\bar{n}}{\sigma(n)} - \frac{\bar{n}}{\sigma(n)} = 0$$

Similarly, the variance of z_i obtained analytically is:

$$\sigma(z_i)^2 = \frac{\sum_{i=1}^k (z_i - \bar{z})^2}{k-1} = \frac{\sum_{i=1}^k \left(\frac{n_i - \bar{n}}{\sigma(n)} - 0 \right)^2}{k-1} = \frac{1}{k-1} \sum_{i=1}^k \frac{(n_i - \bar{n})^2}{\sigma(n)^2} = \frac{1}{k-1} \sum_{i=1}^k \frac{(k-1)(n_i - \bar{n})^2}{\sum_{i=1}^k (n_i - \bar{n})^2} = 1$$

So the variable z_i obtained after the 2nd transformation is a truly standard normal variable having a mean 0 and a standard deviation 1 down to the computer round off accuracy.

A series of results of this method have follows

Raw Mean	Raw Std.Dev.	2 nd Trans.Mean	2 nd Trans. Std.Dev.
9.98841046014976	2.14239440383019	10.00000000000000	2.00000000000000
10.00487505777973	2.17107575759270	10.00000000000000	2.00000000000000
9.93265892679448	2.06122749978007	10.00000000000000	2.00000000000000
10.18566266489137	2.01042973433044	10.00000000000000	2.00000000000000
9.93286438754131	1.99543404543892	9.99999999999999	2.00000000000000
9.97125485866152	1.82627709201304	10.00000000000000	2.00000000000000
10.06439103082642	2.04660779843629	10.00000000000000	2.00000000000000
10.08195148903928	2.30034692888027	10.00000000000000	2.00000000000000
10.04246249069982	2.02802067131690	10.00000000000000	2.00000000000000
10.10593191140007	1.94058848649218	10.00000000000000	2.00000000000000
9.62400833675698	2.10211067374829	10.00000000000000	2.00000000000000
9.99207166932287	2.08644127478734	10.00000000000000	2.00000000000000
9.99449193513517	2.16103127499955	10.00000000000000	2.00000000000000
9.86370081926398	2.21657232795509	10.00000000000000	2.00000000000000
10.04789035844397	1.92350660588923	10.00000000000000	2.00000000000000
Mean			
9.98884175978045	2.06747097169937	10.00000000000000	2.00000000000000
Mean error %			
-0.11158240219552	3.37354858496832	0	0
Max error %			
5.83597092273143	25.95826443536400	0.00000000000007	0.00000000000007

They are presented the results for 15 generated normal random sequences of 1000 point each, for a specified input variable with mean **10** and standard deviation **2**, using the raw outputs (1st and 2nd columns) and using the 2nd transformation method (3rd and 4th columns). **The improvement is more than obvious.** Further verification of the method could be seen in **figures 3.1.2 and 3.1.3**. They have been generated 3 standard normal distributions (raw outputs of the computer generator) of 1000, 10000, and 100000 points respectively, and compared with the standard normal distribution obtained after applying the 2nd transformation to the first raw sequence of 1000 points. Particularly, in **figure 3.1.3**, it can be easily seen how by increasing the number of sample points the computer

generated raw distributions approach the one obtained from the 2nd transformation of only 1000 points.

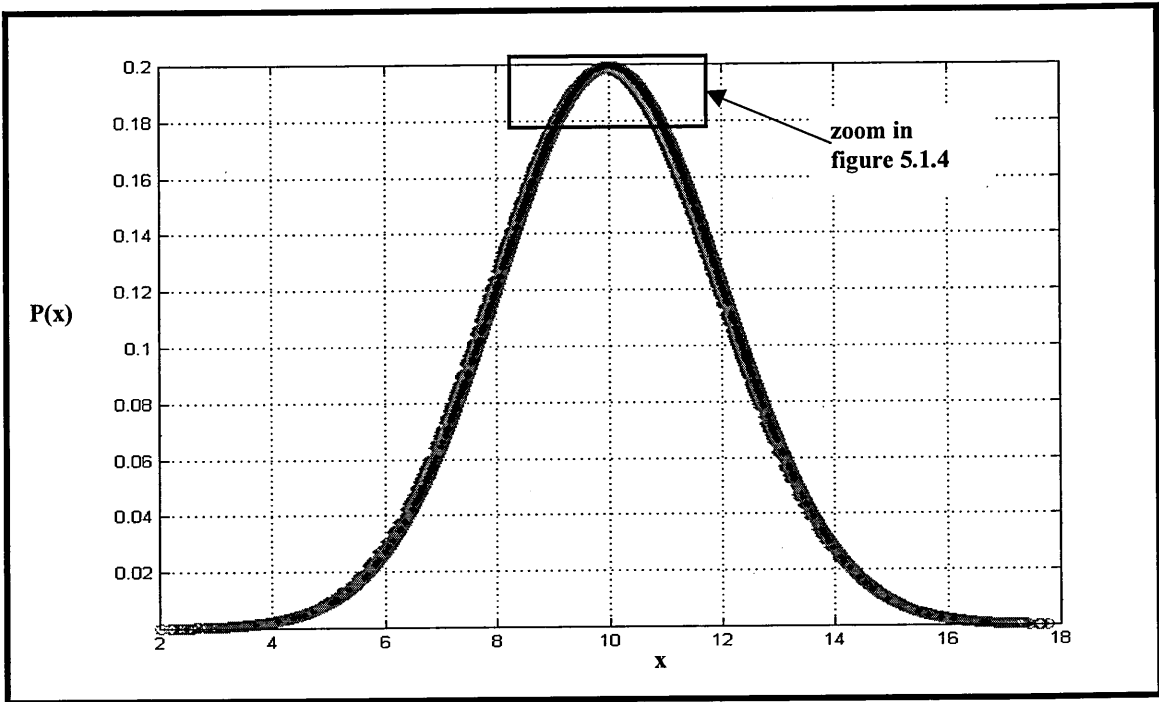


Figure 3.1.2 - Four generated Normal distributions of 1000, 10000, 100000 points and applying a 2nd transformation to the 1000 points.

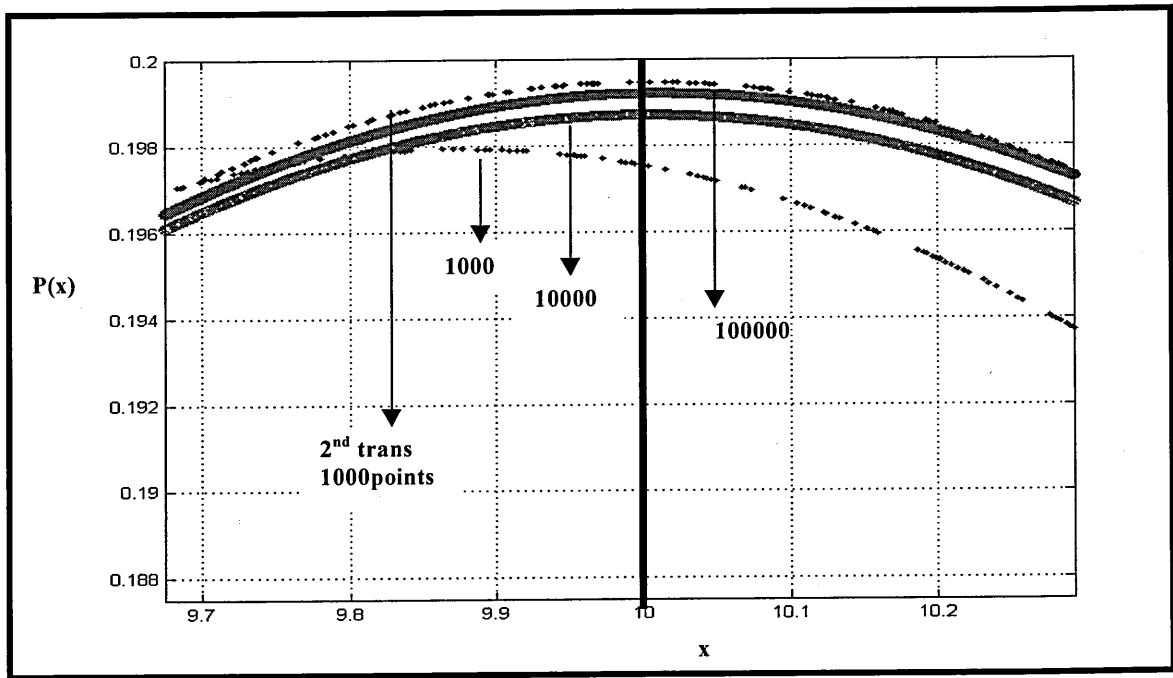


Figure 3.1.3 Zoom view of improvement by applying the 2nd transformation to the raw generated output of 1000 points of the random generator (Bootstrap method).

CHAPTER 4

4. UNCERTAINTY ESTIMATION AND MONTE CARLO SIMULATION

METHOD

4.1. Theory of Measurement Uncertainty Estimation methods.

In this chapter, an integrated approach of the theoretical aspects of combined measurement uncertainty estimation is formulated. The evolution of the various proposed methods in the past, which led to the development of the internationally accepted methodology of GUM, is also described. The compatibility of Monte Carlo simulation method for uncertainty estimation with the GUM methodology is demonstrated. Emphasis has been put not only on the advantages of Monte Carlo as a computer-based method but mainly on its intrinsic theoretical compatibility with GUM and its association to the physical reality of measurements and especially when they are combined in modern complex measurement systems.

4.1.1. Measurement and Uncertainty – A general philosophy.

The author has adopted in this project a philosophy which starts from the point that measurement must be considered as an *Independent Process*. The word independent is used in order to separate the *Measurement Process* and the respective *Measurement System* from the *Actual Process* (and the respective system), the realisation of which is the objective of the measurement. This philosophy has its roots in the fundamental ideas developed in the early 20th century during the formulation of the great physical theories of quantum mechanics and relativity. A similar philosophy has been adopted in **ISO-10012.2 (Draft), (1997)**. It was taught by **Eisenhart, C., (1969)**, it has been adopted by **Shewhart, W.A., (1986)** and his famous successor Deming W.E. who developed it for manufacturing (production) processes and it is also presented by **Scherkenbach, W.W., (1991)**. Generally, measurement is an ordinarily repeatable operation, so that it is appropriate to regard measurement as a production process, the product being the numbers, i.e., the measurements. In this sense, the concepts and techniques of Statistical Process Control (SPC) that have been proved so useful in the quality control of industrial production can be adopted.

The reliable estimation of a measurement process uncertainty is closely associated with all the above. **Measurement Process Quality Management (MPQM)** can be defined as

this complex management structure with the aid of which the measurement process is managed to produce measurement uncertainty levels that are in statistical control within some desired limits. According to the author's point of view, the reliable estimation of measurement uncertainty is necessary for the attainment of measurement process control and can be carried out and actually classified to two timely separated important steps:

1. The *a priori* specification of reasonable desired measurement uncertainty control limits. This can be actually classified as the **uncertainty of the measurement system** (as a characteristic/ability of the system).
2. The *a posteriori* uncertainty estimation of the actually produced uncertainty levels. This can be classified as the **uncertainty of the measurement process** given a specific (selected) measurement system that operates under specific measurement conditions.

In this way, the author attempts to separate the usually confusing notions of uncertainty in the sense of accuracy of a selected metering system, which is usually the one often stated by the manufacturers, and the actually produced uncertainty during the real operation of the same metering system, which is a *conditional* uncertainty, mainly based on metering real operating conditions and metering management procedures.

Both these steps will be further considered in this chapter. Virtually, they compose the basic concepts of Bayesian statistical theory of measurement uncertainty and inevitably, they require good knowledge of the Measurement System, the Measurement Process and by all means, of the Actual Process we try to measure. The measurement system is composed of the instrumentation, the data acquisition and analysis systems, the personnel running the system and performing the procedures and the metrology support system. The later author's ideas appear to integrate the traditional mean of Process Control in Chemical industries within a framework of Total Quality Control. According to them, the control action, depending so much on the measurements, can not be carried out just only by the manipulation of some physical variables simply through feedback or feedforward mathematical algorithms, but now must be also taken into account factors previously totally ignored, such as latest calibration results and personnel running the system!!!. In this way it is allowed the control of the evolution of a continuous measurement process.

The definition of traceability in **VIM, 1993** includes '...an unbroken chain of comparisons all having stated uncertainties'. As **Lira, I.H., and Wolfgang, W., (1998)** said, this requisite acknowledges the fact that the information about the measurand that is provided through a measurement result can rarely be assumed to be complete (in reality is never

complete). Thus, confidence in measurements is possible **only** if a quantitative and reliable expression of their relative quality, the measurement uncertainty, is assessed.

The biggest advantage that any reliable uncertainty estimation offers is that the measurement process acquires some state of **predictability**. Predictability is the key word that lies in the definition of statistical process control (SPC). This is given to be the attainment of a state of predictability (such that) the mean of a large number of measurements will approach a limiting value (limiting mean) and the individual measurements will have a stable distribution, described by their standard deviation.

Hemsch M.J., (1996) has stated that a particular measurement operation cannot be regarded as constituting a measurement process unless statistical stability of the type known as a state of statistical control has been attained.

In ordinary use the word 'uncertainty' does not inspire confidence. However, when used in a technical sense as 'measurement uncertainty' or 'uncertainty of a test result' it carries a specific meaning. Some earliest meanings or definitions to the measurement uncertainty are given below, ending with the latest and most widely accepted:

- The uncertainty of measurement is defined as the **range** on either side of the mean value of a set of measurements between which a given fraction of readings would lie if a very large number of observations were made.
- The uncertainty of measurement is "the **probability** of a reading lying in a given range on either side of the mean value".
- Uncertainty is the **residual error** that may exist in an instrument or workpiece after calibration corrections have been made.

These three definitions of measurement uncertainty as, range, probability and residual error, virtually are conceptually different (**Dietrich C.F., (1991)**).

- Another simple, surprisingly old definition that is still widely accepted is attributed to **Airy, Sir George Biddle, (1879)** who defined the uncertainty as, "the possible value that an error may have".
- Finally, the latest and most widely accepted definition of measurement uncertainty must be considered as: "...a **parameter** associated with the result of a measurement (e.g. a calibration or test) that defines the range (or dispersion, according to ISO/WD, 5168 (1999)) of the values that could reasonably be attributed to the measured quantity (measurand). When uncertainty is evaluated and reported in a specified way, it indicates

the level of confidence that the value actually lies within the range defined by the uncertainty interval".

The underlined phrase must be emphasised since it introduces the concepts of subjectivity (reason) and normal judgement. These concepts are the characteristics of Bayesian statistical thought and compose the basic cause of many high level important arguments between the Classical statistical thought that is based on objectivity or relative frequency of occurrence probabilities and the Bayesian statistical thought that is based on subjective prior probabilities (beliefs). Not enlarging upon the subject, two important common elements shared by both statistical thoughts have to be mentioned. These are that:

- They both conceptualise a problem of decision-making consisting of states of nature and actions that must be taken in the light of sample, or experimental evidence, or observation about these states of nature. So, both consider measurement as a really fundamental process.
- They both use *conditional probabilities*, though in a quite different context, of sample outcomes given states of nature (population parameters) for the decision process.

It is simple to realise the controversy existed over the years even on the definition of uncertainty itself. Furthermore, someone could easily imagine how this controversy, unavoidably, has also been extended to the uncertainty estimation methods. Before proceeding in the development and presentation of a common background theory for measurement uncertainty estimation and its methods, it is necessary to clarify two additional aspects regarding uncertainty. These are the types and the sources of uncertainty. Inevitably, these also carry some controversy. Measurement uncertainty could actually arise from the following 3 major sources.

- **External and Internal measurement system's parameters.**

External system parameters are known sources of uncertainty and include mainly the so-called environmental effects (e.g. temperature, radiation e.t.c.). Since we are talking about measurement (process) uncertainty associated with a specific measurement system, the effects of the actual process variability have been considered and reasonably included by the author **in external measurement system parameters (!!)**. From this, it can be easily realised that uncertainty is a characteristic of the measurement process (or of the system in prior assessments) and not of the **actual process**. According to the author's point of view, it is a common mistake, that the process natural variability is often associated with the measurement uncertainty. Physical process natural variability is actually the unknown that

we try to measure or else reproduce/transform in a desired form. Uncertainty has to do with the measurement process and the measurement system we employ to observe process variability.

Internal system parameters such as materials' and other properties of the various components of a designed measurement system can be sources of uncertainty as well.

External and internal sources of uncertainty can be mutually interacted in the sense that the more somebody tries to control some of the external measurement system's parameters (e.g. environmental temperature, measurands' variability) the more they become part of, or internal to the measurement system's design and finally command its performance.

- Another source of uncertainty is the measurement system's **model**. This uncertainty source was in the focus of this research project and examined from different points of view. The reason was based on the fact that all the latest international recommendations, and other works on measurement uncertainty estimation methods, initiate from the definition of the measurement process. This is actually accomplished through the initial selection or establishment of a functional relation (measurement equation) between the quantity being measured (measurand) and several input variables. The whole matter will be extensively formalised, analysed and presented through this and later chapters.
- A final important source of measurement uncertainty, which has also been examined in this research project, can be assigned to the **methods** that one uses in his attempt to observe the behaviour of the actual system. The selection of a particular method (by the selection of metering principle and of the respective required instrumentation combination) inevitably defines the respective measurement performance as well. Factors such as human perception and sensor error combined with data fusion and integration (physical measurements/model based measurements fusion) are sources of uncertainty that generally could be called as **observational** or **method's** uncertainties.

The next **figure 4.1** summarises the concept that virtually measurement uncertainty could be hierarchically attributed to three major sources, model, parameter and observational uncertainties.

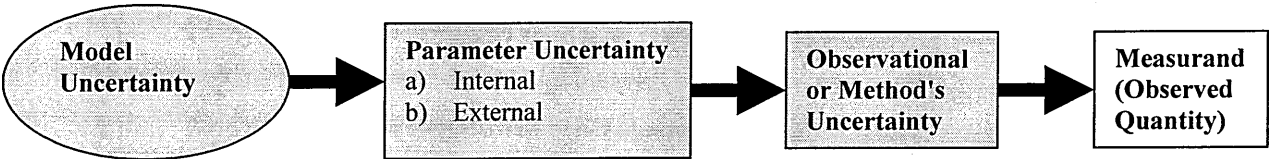


Figure 4.1 – General sources of measurement uncertainty.

Other general or more detailed classifications of measurement uncertainty sources are available in the literature such as the one adopted in the new **ISO/WD 5168.3,1999**. There, uncertainty sources are divided and further analysed into a) Calibration uncertainty, b) data acquisition uncertainty, c) data reduction uncertainty d) uncertainty due to methods e) others. However, it could be an easy exercise to classify and incorporate them into the three major categories presented in the previous section.

In terms of the types of uncertainties, generally they could be either **probabilistic** or **possibilistic**, and so are also the respective available estimation methods. Even aspects that are clearly deterministic, such as the periodic variations of environmental temperature during a 24-hour day or simpler sinusoidal waves can be also characterised using probabilistic methods. Probabilistic uncertainty estimation methods are the more traditional methods and make use of random variables and random functions to describe the various sources of uncertainty. These could be generally referred to as reliability methods, for reasons that will be explained later and the newest standards' recommendations are exclusively referred to them. Possibilistic methods involve the use of **fuzzy set theory** or possibility theory (Dempster-Shafer theory of evidence) to model uncertainty and are useful when dealing with large complex systems or systems that do not lend themselves to statistical treatment. They are particularly applicable when an exact system model may be impractical or even impossible to be obtained.

4.1.2. Combined measurement uncertainty estimation – The Measurement equations.

It was stated previously, that the first stage of an uncertainty evaluation is the definition of the measurement process through the use of those functional relationships that correspond to the selected measurement method, among the various available methods. This is the first and possibly the most vital step. According to the author's point of view, such an approach can be easily justified by the fact that only when the measurement method/principles and the respective functional relationship(s) (models) have been established, can be established the measurement system's detailed design, operation and maintenance. After this step, the other major sources of measurement uncertainty described previously can be then more carefully examined. A widely adopted *general* representation of such a functional relationship (measurement model/equation) between N input variables

$(X_1, X_2, X_3, \dots, X_N)$ and output quantity Y being measured (measurand) in a flow measurement process is:

$$Y = f(X_1, X_2, X_3, \dots, X_N) \quad 4.1$$

X_i includes corrections (or correction factors), as well as quantities that take into account various other sources of uncertainty, such as different observers, instruments, samples, laboratories and times at which observations are made (e.g. different days), e.t.c. Thus, the general functional relationship f in 4.1 should express not simply a physical law but a measurement process, and in particular it should contain all these quantities that can contribute a significant uncertainty to the measurement result. Further information can be found in [www@nist.gov](http://www.nist.gov).

A truly global probabilistic approach regarding the treatment of the general measurement equation will be presented below in such a way that it could consistently cover most of the various uncertainty estimation methods present in the existing literature. Comparisons between the various methods and justification of each one within such a global approach can be proved useful in this effort for the removal of the sometimes unnecessary controversies present on the subject. A sort list of some of the most important identified controversies is as follows:

- a) Examination of the necessity or not of classification of random and systematic measurement uncertainties.
- b) Proper estimation of confidence intervals
- c) Effects of correlated input variables/uncertainties (multicollinearity/non-linearity) on combined measurement uncertainty estimation.
- d) Conceptual (applied methods) errors and Model errors and use of correction factors to resolve them.
- e) Combination of the c) and d).

The author gave a particular emphasis to the last three subjects that involve a relatively new area of research.

Coleman, H.W., et.al., (1997), has been referred to the matter of correlated uncertainty effects in the modelling (case e)) as an unexplored area at this time.

4.1.2.1. Measurement equations and a global probabilistic approach.

Equation 4.1 is a general representation of a measurement process, which gives a quantity Y as a function of a long list of real variables. Some of these variables are under the direct control of the measurement's "operator", some are under indirect control, some are observed but not controlled and some are not even observed. A measurement aiming to obtain the result Y as a function of one or more variables, will probably be designed as a *partial derivative measurement*, where one or more of the controlled variables is controllably varying, hoping that the rest will remain constant throughout the measurement process. The operator's wish is illustrated in the next equation 4.2 where every variable to the left of the semicolon is treated as a controlled, or at least observed (measured) variable and all the other terms as parameters which are not expected to change, or be important during the measurement.

$$Y = f(X_1, X_2, X_3; \quad x_5, x_6, \dots, x_N) \quad 4.2$$

The fact that this is a simplified assumption is brought to light when the controlled variables are held constant and the results display unconsidered scatter. The reason is that by definition the model/equation is only an abstract representation of the real world. Even finite element models with hundreds of thousands of elements may sometimes provide a very simple representation of how the physical (actual) system actually behaves (Moffat, R.J., 1982). While appearing to be deterministic, such modelling uncertainty can also be characterised using probabilistic analysis methods. This global approach could be developed with the use of a simple measurement equation where the measurement result is considered, in the domain of interest, as a continuous function of two other variables X_1 and X_2 :

$$Y = F(X_1, X_2) \quad 4.3$$

Equations 4.2 and 4.3 are equivalent to 4.1 in the sense that each of X_1, X_2 etc. *may actually represent results of partial measurement processes with various sources of elemental uncertainties.*

Estimators are assigned to each of Y, X_1, X_2 whose respective probability density functions are consistent with the available information about the corresponding quantities. The estimators' distributions express the state of incomplete knowledge about the measurands and must not be understood in the sense of the frequency at which values occur. This is a vivid example of the difference between Bayesian and Classical statistical thought.

So, initially a distinction must be made between a real quantity Y and its estimator although they may share the same symbol. The estimator serves the purpose of providing for an estimate of the quantity and for the uncertainty associated with this estimate. Generally, it could be said that the estimator is the decision rule, the estimate is the action, and the possible values that the measurand can assume are the *states of nature*. Note the similarity of the last expression and the latest definition of uncertainty given in section 4.1.1. So, what uncertainty is all about, are these probable states of nature. This issue will be further analysed later in terms of states of thermodynamic systems. This so important distinction between the real quantity and its estimator must receive particular attention. This is a distinction that also lies on ideas flourished during the formulation of the great physical theories of the early 20th century, the quantum mechanics and relativity. **Planck, Max, (1936)**, in *The Philosophy of Physics*, clearly stated that whenever a physical measurement takes place, a distinction must be made between the objective and actual event, which takes place completely independently, and the process of measurement, which is occasioned by the event and renders it perceptible. He also added the fact that physics deals with the actual events and its objective is to discover the laws that these events obey. Actually, the three underlying phrases in the description of Planck form the basis from which we will proceed to the presentation of the measurement uncertainty estimation theory. In this way, the phrase Independent Process employed by the author to characterise the measurement process in the beginning of this chapter is justified.

Some important elements of probability theory and thermodynamics are also employed which are going to be used in the subsequent theoretical developments. Assuming that there is a thermodynamic quantity Y that we want to measure it. The quantity Y , using the appropriate thermodynamic terms, could be any thermophysical property *primitive* or *derived*. Following Planck's ideas it could be said that it exists independently of the measurement process that intends to observe it.

It could also be stated, within the latest mentioned frame of modern physics, that the quantity Y , belonging to the *physical world image*, could have either a "certain" value ($Y=c$, the ideal true value) or it could vary in time (or space) within a range of possible values, depending on its physical context ($Y_1 < Y < Y_2$ or $Y=Y \pm \delta Y$). The mean to acquire knowledge about the quantity Y is by measurement. However, measurement is a process that belongs to the *world of senses*, also in accordance with the view of modern physics. Measurement process uncertainty arises exactly at this point where a measurement is

employed to obtain knowledge about a particular quantity. Uncertainty for a measurand, in general, pre-exists in the form of prior probabilities or beliefs in a Bayesian sense.

An *estimate* of the measurand or output quantity Y , denoted by y , is obtained from equation 4.3 using input estimates x_1, x_2 for the values of input quantities X_1, X_2 . The estimates y, x_1, x_2 are generally the best estimates of the quantities Y, X_1, X_2 , (usually the mean values) and are inferred as the expectation values $E(Y), E(X_1), E(X_2)$ of the corresponding estimators. The standard uncertainties associated with those estimates, denoted as $u(Y), u(X_1), u(X_2)$ are evaluated as the square root of the variances (standard deviations $\sigma(F), \sigma(X_1), \sigma(X_2)$) of the estimators.

Thus the output estimate that is the result of the measurement, is given by:

$$y = f(x_1, x_2) \quad 4.4$$

Equation 4.4 is called in the literature generally as the *data reduction equation* where the function is continuous and has continuous derivatives in the domain of interest. Most measurement uncertainty estimation methods start from 4.4 and various ways to proceed with the calculations have been proposed. Additional features from probability theory are selected in order to proceed in a consistent manner. Particularly, the previously described estimates are referred to random variables that assigned to each of the physical quantities Y, X_1, X_2 showing unpredictable measured values in a repeated measurement. The widths of the corresponding probability densities $p(y), p(X_1), p(X_2)$, given by the respective standard deviations, serve as the *parameters* that qualitatively describe to what extent the unknown values are known, or in other words the width gives the uncertainty. Every quantity of 4.3 is examined separately in order to analyse how uncertainties in the input quantities X_1 and X_2 "propagate" to the measurement result y . Considering the random input variable X_1 , it may represent any of physical gas quantities. Its best value has to be estimated from n statistical independent observations or data points $X_{1,n}$. If X_1 is an ergodic random process then the desired statistical information about the entire random variable X_1 can be obtained from a single sample probability distribution. By all means, ergodicity implies also stationarity. Process stationarity, evaluated through stationarity tests on process samples can significantly simplify further processing (**Bendat, J.S., Piersol, A.G., (1986)**). The subject will be further discussed in the next section where it will be related with another important concept, of conditional probability. However, it could be added that it is also associated with the concept of Statistical Process Control mentioned in the beginning of this chapter. In case that $X_{1,n}$ is obtained from an ergodic random process (and

fortunately, in practice, random data representing stationary physical phenomena are generally ergodic), and as n approaches infinity the limiting distribution of X_1 is a Gaussian (Normal) distribution where it is described by the respective probability density

$$\text{function: } p(X_1) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(X_{1,n} - E(X_1))^2}{2\sigma^2}\right] \quad 4.5$$

where

$$E(X_1) = \frac{1}{n} \sum_{n=1}^n X_{1,n} = \sum_{-\infty}^{\infty} X_1 p(X_1) dX_1 \quad 4.6$$

The last part of the equation 4.6 is really the fundamental definition of mean value as well as the starting point of any Monte Carlo simulation procedure as has been shown in chapter 3. The standard deviation (sample) is given by:

$$\sigma(X_1) = \sqrt{\frac{1}{n-1} \sum_{n=1}^n (X_{1,n} - E(X_1))^2} = \sqrt{\frac{n}{n-1} (E(X_1^2) - (E(X_1))^2)} \quad 4.7$$

where

$$E(X_1^2) = \frac{1}{n} \sum_{n=1}^n X_{1,n}^2 = \sum_{-\infty}^{\infty} X_1^2 p(X_1) dX_1 \quad 4.8$$

and $n \rightarrow \infty$ in all cases.

The closely related probability distribution function, $P(X_1)$, is defined by the equation:

$$P(X_1) = \sum_{-\infty}^{X_1} p(X_1) dX_1 \quad 4.9$$

and gives the probability that a sample value of the random variable is less than X_1 .

The value of $P(X_1)$ lies between zero and unity since,

$$\text{Prob}(-\infty \leq X_1 \leq \infty) = \sum_{-\infty}^{\infty} p(X_1) dX_1 = P(X_1 = \infty) = 1 \quad 4.10$$

Furthermore, as n approaches infinity the former discrete probability distributions approach the respective continuous probability distributions and the summations are changed to integrals. This also applies to the subsequent estimates. It must also be reminded the first-order normal probability density function, $p(X_1)$, is generally defined as (Newland, D.E., 1996):

$$\text{Prob}(X_1 \leq X_{1,n} \leq X_1 + dX_1) = p(X_1) dX_1. \quad 4.11$$

So, as $n \rightarrow \infty$, the data points $X_{1,n}$ become more dense, $dX_1 \rightarrow 0$ and $p(X_1)$ approaches the continuous probability. It follows that the probability density function could be obtained

from the closely related probability distribution function, which is also becoming continuous, as: $p(X_1) = \frac{dP(X_1)}{dX_1}$. This says that the slope of the probability distribution function, (usually referred as the *cumulative probability distribution function*) is equal to the probability density. The same concepts are similarly applied to the other random variable X_2 . The respective functions $p(X_1)$, $p(X_2)$, are *first-order Gaussian probability density functions* that specify the probabilities $p(X_1)dX_1$, $p(X_2)dX_2$ that **each** of the random variables lies in the intervals X_1+dX_1 and X_2+dX_2 respectively. However, the expected value of the function $f(X_1, X_2)$ depends on **both** X_1 , X_2 . The probability that the X_1 random variable lies in the range X_1 to X_1+dX_1 **and** that the X_2 random variable lies in the range X_2 to X_2+dX_2 is given by $p(X_1, X_2)dX_1dX_2$. The $p(X_1, X_2)$ is the *second-order or joint Gaussian probability density function*.

Based on the ideas of **Coleman, H.W., & Steele, W.G., (1995)** it can be further assumed that corrections in the input variables X_1 , X_2 have been made for all of the components of uncertainty arising from known systematic effects (e.g. through calibration). If additionally, it is assumed that the test instrumentation and/or apparatus is changed for each measurement, the remaining uncertainty components from unknown systematic effects are equally as likely to be positive as negative. Therefore, the combined remaining uncertainties from systematic and random effects in X_1 and X_2 , will be random variables following a Gaussian distribution, according to the above as well as similar arguments presented in the ISO Guide. These combined standard uncertainties $u_c(X_1)$, $u_c(X_2)$ as they have named in the Guide, propagate to the measurement result y and are expressed as multiples of standard deviations ($t \cdot \sigma_n(X_i)$) of the respective probability distributions. The numerical factor t used as a multiplier of the combined standard uncertainty is typically in the range 2 to 3 depending on the selected confidence interval (approximately between 95% and 99%) and the effective degrees of freedom. It defines an interval about the X_1 , X_2 that may expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the quantities X_1 , X_2 . The result y that depends on both X_1 and X_2 , is ideally given by the expected value of the function:

$$y = E(f(X_1, X_2)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(X_1, X_2) p(X_1, X_2) dX_1 dX_2 \quad 4.12$$

The last expression suggests that both bands of values extend from $-\infty$ to $+\infty$ so there is 100 per cent certainty that both X_1 and X_2 must lie within these bounds, and thus

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(X_1, X_2) dX_1 dX_2 = 1. \quad 4.13$$

Since, as it was stated previously, a large fraction of the distribution of values of X_1 and X_2 could reasonably be attributed to lie within the finite bands $t^* \sigma_n(X_1)$, and $t^* \sigma_n(X_2)$, the respective joint probability distribution is given by:

$$\begin{aligned} \text{Prob}(X_1 \text{ within } t\sigma(X_1) \cdot \text{and} \cdot X_2 \text{ within } t\sigma(X_2)) &= P(X_1, X_2) = \\ &= \int_{X_1 - t\sigma(X_1)}^{X_1 + t\sigma(X_1)} \int_{X_2 - t\sigma(X_2)}^{X_2 + t\sigma(X_2)} p(X_1, X_2) dX_1 dX_2 \end{aligned} \quad 4.14$$

and the respective joint Gaussian (Normal) bivariate probability density is given by:

$$p(X_1, X_2) = \frac{\partial}{\partial X_1} \left[\frac{\partial P(X_1, X_2)}{\partial X_2} \right] = \left[\frac{\partial^2 P(X_1, X_2)}{\partial X_1 \partial X_2} \right] \quad 4.15$$

This can be expressed, by the author, in the normalised form:

$$\begin{aligned} p(X_1, X_2) &= \frac{1}{2\pi t^2 \sigma(X_1) \sigma(X_2) \sqrt{1 - \rho_{X_1, X_2}^2}} \dots \\ &\dots e^{\left(\frac{-(\sigma(X_2)^2 (X_1 - E(X_1))^2 - 2\rho_{X_1, X_2} \sigma(X_1) \sigma(X_2) (X_1 - E(X_1))(X_2 - E(X_2)) + \sigma(X_1)^2 (X_2 - E(X_2))^2)}{2\sigma(X_1)^2 \sigma(X_2)^2 (1 - \rho_{X_1, X_2}^2)} \right)} \end{aligned} \quad 4.16$$

The term ρ_{X_1, X_2} is the *correlation coefficient* or *normalised covariance* defined as:

$$\rho_{X_1, X_2} = \frac{\sigma(X_1, X_2)}{\sigma(X_1) \sigma(X_2)} \quad 4.17$$

where the covariance $\sigma(X_1, X_2)$ is defined as:

$$\begin{aligned} \sigma(X_1, X_2) &= E[(X_1 - E(X_1))(X_2 - E(X_2))] = \\ &= \int_{X_1 - t\sigma(X_1)}^{X_1 + t\sigma(X_1)} \int_{X_2 - t\sigma(X_2)}^{X_2 + t\sigma(X_2)} (X_1 - E(X_1))(X_2 - E(X_2)) p(X_1, X_2) dX_1 dX_2 \end{aligned} \quad 4.18$$

Note that $\sigma(X_1, X_1) = \sigma(X_1)^2$, the variance of X_1 and similarly for X_2 . A simple though important relation exists between the covariance and the product of variances called the *Cauchy -Schwartz inequality* and is:

$$|\sigma(X_1, X_2)| \leq \sigma(X_1) \sigma(X_2) \text{ or} \quad 4.19a$$

$$\sigma(X_1, X_2)^2 \leq \sigma(X_1)^2 \sigma(X_2)^2 \quad 4.19b$$

From 4.17 and 4.19a we have:

$$-1 \leq \rho_{X_1, X_2} \leq 1.$$

The correlation coefficient is a measure of the linear association between the random variables X_1 and X_2 . An issue that has been further analysed by the author is the nominator of equation 4.16 normalised. The condition:

$$\sigma(X_2)^2 X_1^2 - 2\rho_{X_1, X_2} \sigma(X_1) \sigma(X_2) X_1 X_2 + \sigma(X_1)^2 X_2^2 = k \quad 4.20$$

or the equivalent:

$$\sigma(X_2)^2 X_1^2 - 2\sigma(X_1 X_2) X_1 X_2 + \sigma(X_1)^2 X_2^2 = k \quad (k=\text{constant}) \quad 4.21$$

which actually implies $p(X_1, X_2) = k$, defines the (constant) *probability density isoquants* of the joint distribution.

A simulated joint (bivariate) normal probability density function can be seen in **figure 4.2**.

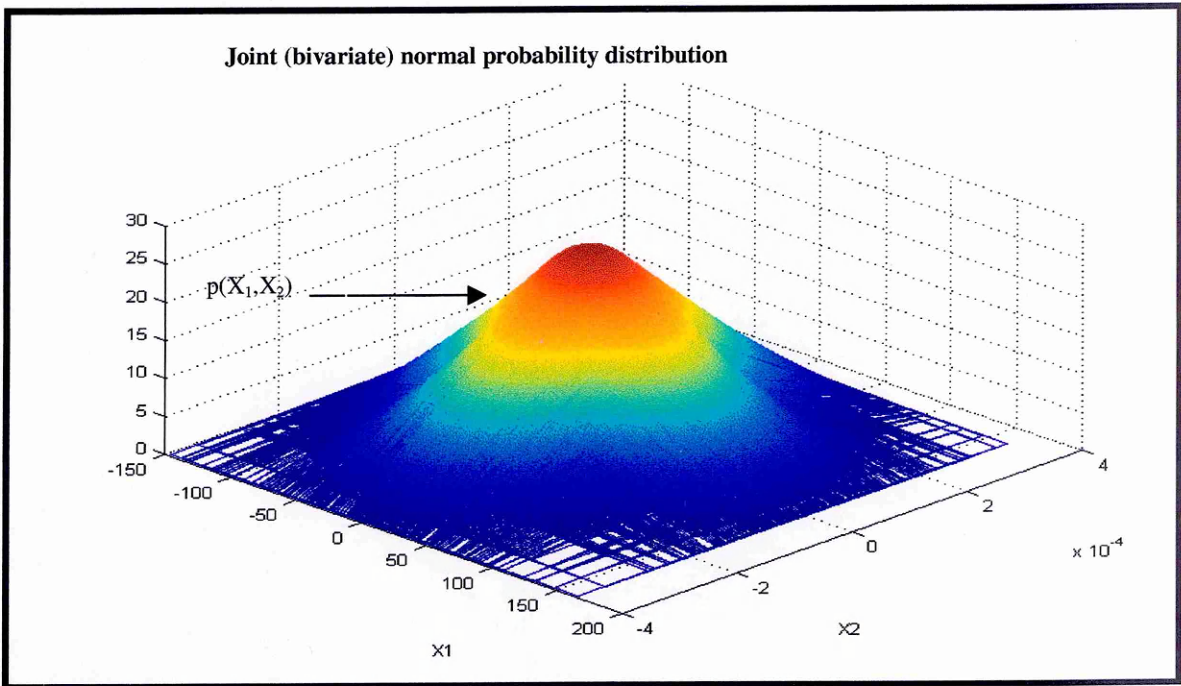


Figure 4.2 A typical joint (bivariate) normal probability density surface.

It is defined by the surface of the single-peaked (unimodal) hill in a three-dimensional space. The probability density isoquants are simply the contour lines of the hill and the enclosed areas define different percentage minimum probabilities regions as shown in **figure 4.3**. The precise form of the contour map (isoquant map) depends crucially, of course, on the form of the joint distribution. The form of the joint distribution in turn depends on the variances and covariance (or correlation) of X_1 and X_2 . From equations 4.12 and 4.16, it can be ascertained that **the result y can assume any value within the**

isoquant area enclosed by $t\sigma(X_1)$, $t\sigma(X_2)$. The later is the evidence of an incomplete knowledge, or a dispersion of the values according to a latest definition of uncertainty given in ISO/WD-5168.5, (1999), of the result y . In this case,"...the parameter that characterises the dispersion of the values that could reasonably attributed to the measurand (**uncertainty**) can be expressed by the **isoquant area** itself.

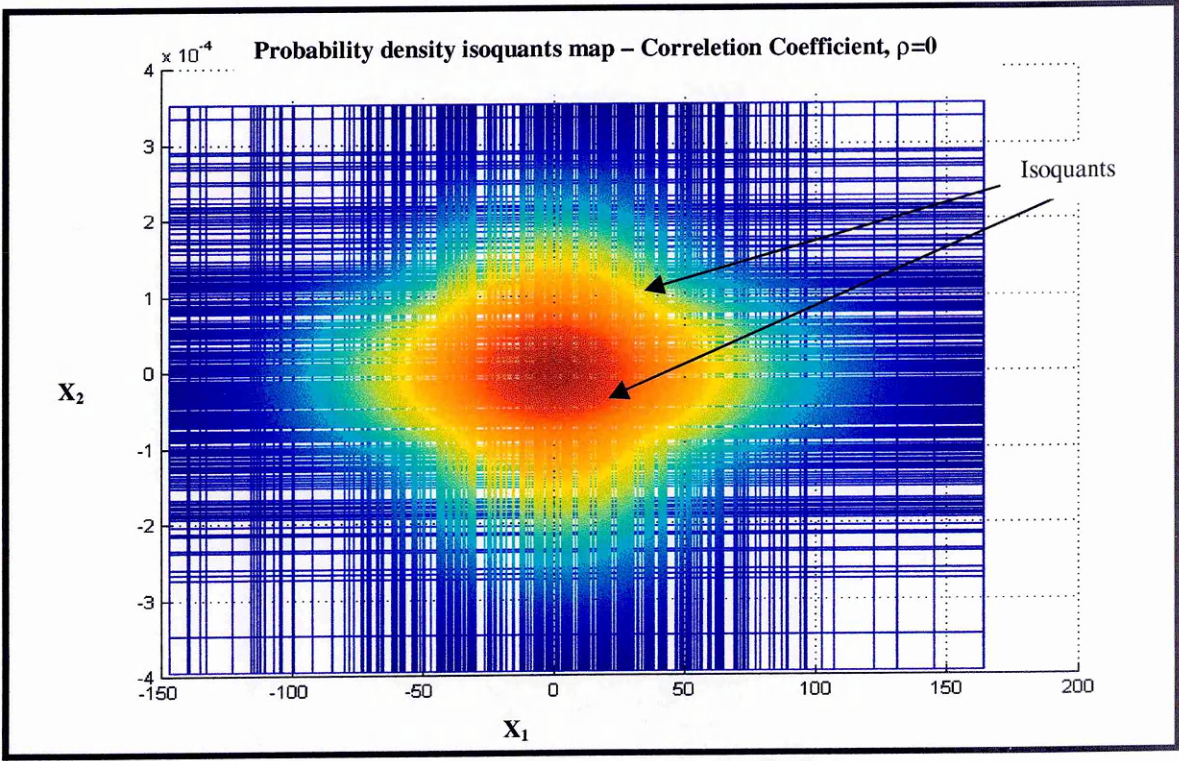


Figure 4.3 Probability density isoquants map – Correlation Coefficient, $\rho=0$).

In full agreement with the recommendations given by **Weise, K., and Woger, W., (1992)**, regarding the evaluation and expression of measurement uncertainty, that are also in line with the GUM, all the above can be generalised for a result obtained from N random variables. They jointly follow an N -dimensional Gaussian (normal) distribution with the associated N -fold probability density function given by:

$$p(X_1, X_2, \dots, X_N) = \frac{1}{(2\pi)^{N/2} |C|^{1/2}} e^{\left[\left(-\frac{1}{2} |C|^{-1} \right) \sum_{i,j=1}^N |C_{ij}| (X_i - E(X_i))(X_j - E(X_j)) \right]} \quad 4.22$$

where C is the *covariance matrix* of the C_{ij} defined below, $|C|$ is the determinant of C , and $|C_{ij}|$ is the cofactor of C_{ij} in determinant $|C|$. Particularly, the cofactor $|C_{ij}|$ of any

element C_{ij} is defined to be the determinant of order $N-1$ formed by omitting the i th row and j th column of C , multiplied by $(-1)^{i+j}$.

The covariance matrix is given by:

$$C = \begin{vmatrix} C_{11} & C_{12} & \dots & C_{1N} \\ C_{21} & C_{22} & \dots & C_{2N} \\ \dots & \dots & \dots & \dots \\ C_{N1} & C_{N2} & \dots & C_{NN} \end{vmatrix} \quad \text{where } C_{ii} = \sigma(X_i)^2 \text{ and} \quad 4.23$$

$C_{ij} = E((X_i - E(X_i))(X_j - E(X_j)))$ are the respective variances and covariances.

So, the joint distribution represents the actual state of knowledge of the multiple measurands. The expectation values of the elementary distributions serve as the measurement results, i.e. as estimates of the elementary measurands. The square roots of the diagonal and the off diagonal elements of the covariance matrix, the standard deviations and covariances, represent the individual and mutual uncertainties of the measurements. In this new approach proposed by the author, **elementary covariance matrices actually may represent group uncertainties**. Additionally, these could be geometrically illustrated by **vectors** with lengths equal to the standard deviations in an Euclidean N -dimensional uncertainty hyperspace. In case of independent Gaussian elementary probability distributions, this could be actually a *N -dimensional hyper-ellipsoid* which is composed of lower dimension ellipsoids corresponding to the group uncertainties. The cosines (or possibly even better, the hyperbolic cosines) of the angles between pairs of the elementary vectors and group vectors equal the corresponding partial and group correlation coefficients respectively.

The author has also noticed that equation 4.21 is a characteristic equation of conic. A further analysis of this equation was regarded necessary since it could offer in the realisation of other important matters regarding measurement uncertainty.

As it is known from analytical geometry the general equation of conic is given by:

$$f(x, y) = ax^2 + 2bxy + cy^2 + 2ex + 2gy + k = 0 \quad 4.24$$

where a, b, c are real and not all zero.

Equation 4.21 is a limiting case of equation 4.24 when $e=g=0$. In the conics' theory, increase interest have the polynomials that are functions of the parameters a, b, c and that **remain invariant** under any reference frame (rotated or translated). Those polynomials are referred as *the invariants of the conic*.

So all the conics of the form 4.22 have geometrical characteristics that are independent of the frames of reference or systems of co-ordinates. These characteristics are analytically expressed with relationships (called invariants) that exist between the parameters of the conic. Such invariants are known to be:

$$D = \begin{vmatrix} a & b & e \\ b & c & g \\ e & g & k \end{vmatrix}, \quad d = \begin{vmatrix} a & b \\ b & c \end{vmatrix}, \quad S = a + c \quad 4.25$$

These do not change value with any change of the reference frames. Furthermore, when:

1. $d > 0$ the conic is an ellipse and if
 - a) $D \neq 0$ and $DS < 0$ is a real ellipse
 - b) $D \neq 0$ and $DS > 0$ is an imaginary ellipse
 - c) $D = 0$ is a pair of imaginary straight lines.
2. $d = 0$ the conic is a parabola and if
 - a) $D \neq 0$ is a real parabola
 - b) $D = 0$ is a pair of straight lines which are
 - b1) Parallel, if $e^2 - ak > 0$
 - b2) Coinciding, if $e^2 - ak = 0$
 - b3) Imaginary, if $e^2 - ak < 0$
3. $d < 0$ the conic is a hyperbola and if
 - a) $D \neq 0$ is a hyperbola
 - b) $D = 0$ is a pair of straight lines that intersect.

Cauchy-Schwartz inequality, as it is expressed in 4.19b, limits 4.21 only to the cases $d > 0$ and $d = 0$. It is useful to develop further the different cases that can be obtained for 4.21 (bivariate case). The invariants for 4.21 are given by:

$$\begin{aligned} D &= k(\sigma(X_1)^2 \sigma(X_2)^2 - \sigma(X_1 X_2)^2) = kd, \\ d &= (\sigma(X_1)^2 \sigma(X_2)^2 - \sigma(X_1 X_2)^2) \geq 0, \\ S &= \sigma(X_1)^2 + \sigma(X_2)^2 > 0 \end{aligned} \quad 4.26$$

1. For $d > 0$ then $|\rho(X_1 X_2)| < 1$ (not perfect (partial) positive or negative correlation) and for $DS < 0$ meaning $D < 0$ (since $S > 0$ always) which is equivalent to $kd < 0$ meaning $k < 0$ (since $d > 0$) then from 4.24 and 4.21:

$$\left| \frac{\sigma(X_2)^2}{\sigma(X_1 X_2)} \frac{X_1}{X_2} + \frac{\sigma(X_1)^2}{\sigma(X_1 X_2)} \frac{X_2}{X_1} \right| = Q > 2 \quad 4.27$$

and the isoquant is a real ellipse.

2. For $d>0$ and for $DS>0$ meaning $D>0$ (since $S>0$ always) which is equivalent to $kd>0$ meaning $k>0$ (since $d>0$) then from 4.24 and 4.21:

$$Q < 2 \quad 4.28$$

and the isoquant is an imaginary ellipse.

3. For $d>0$ and for $D=0$ which is equivalent to $kd=0$ meaning $k=0$ (since $d>0$) then from 4.24 and 4.21:

$$Q = 2 \quad 4.29$$

and a pair of imaginary straight lines is obtained. This case must be very rare for the variances, covariances and generally measurements' variations to be found in such an interrelationship. However, the most important case, quite common and similar with 4.29 is:

4. For $d=0$, then $|\rho(X_1X_2)| = 1$ (perfect positive or negative correlation) we obtain a parabola. Because inevitably from 4.26 also $D=0$ (with $k=0$ or $k \neq 0$), from the case 2b regarding the conics, we obtain actually a pair of straight lines. The term k , as it will be next proved, can be either $k<0$ (case2b1) or $k=0$ (case 2b2), which actually corresponds to equations 4.27 and 4.29 combined. .

$$\left| \frac{\sigma(X_2)^2}{\sigma(X_1X_2)} \frac{X_1}{X_2} + \frac{\sigma(X_1)^2}{\sigma(X_1X_2)} \frac{X_2}{X_1} \right| \geq 2. \quad 4.30$$

However, an important exception must be considered. Since $d=0$ as well, (perfect correlation), 4.30 **could be modified in various ways** one of which is:

$$\left| \frac{\sigma(X_1X_2)}{\sigma(X_1)^2} \frac{X_1}{X_2} + \frac{\sigma(X_1)^2}{\sigma(X_1X_2)} \frac{X_2}{X_1} \right| \geq 2. \quad 4.31$$

Equation 4.31 is the sum of two reciprocal parts having the general form: $\left| X + \frac{1}{X} \right| \geq 2$. This

particular form is valid for any X and can be easily proved.

So, for $X>0$ $X + \frac{1}{X} = 2 \cosh(\ln(X)) \geq 2 \Rightarrow \cosh(\ln(X)) \geq 1$ which is always valid since,

expanded in a Taylor series and for $Y=\ln(X)$,

$$\cosh(Y) = 1 + \frac{Y^2}{2!} + \frac{Y^4}{4!} + \dots + \frac{Y^{2n}}{2n!} \geq 1 \text{ for all } Y.$$

Similarly for $X < 0$ we can obtain $-\left(X + \frac{1}{X}\right) = -2 \cosh(\ln(X)) \leq 2 \Rightarrow \cosh(\ln(X)) \geq 1$ where $X > 0$ again in the latest expression.

Because of the last proofs, which can be justified also by the fact that because k actually defines the joint probability density isoquants, case 4.28 is not a case that exists in reality so all the possible cases are covered by equation 4.30.

The meaning and potential practical applications of the latest considerations will be realised after the presentation of the experimental results.

4.1.3. Conditional probabilities – Conditional and Unconditional (Total) Independence and Causality - Correlation.

In this section two additional relevant theoretical aspects will be introduced. It is known that first order probability densities can be obtained from the corresponding second order probabilities. Equation 4.13 is normalised with respect to each variable so that (Newland, D.E., (1996)):

$$p(X_1)_{X_2} = \int_{-\infty}^{\infty} p(X_1, X_2) dX_2 \text{ and} \quad 4.32$$

$$p(X_2)_{X_1} = \int_{-\infty}^{\infty} p(X_1, X_2) dX_1$$

Integrating a variable out of a joint distribution is called *marginilising* over the variable and the respective $p(X_1)_{X_2}$ and $p(X_2)_{X_1}$ obtained are called the *marginal probability density functions*. The notation $p(X_1)_{X_2}$ has been adopted in order to emphasise an important qualitative difference of marginal probability densities with the respective first order probability densities $p(X_1)$ and $p(X_2)$ described in the previous section.

An important concept concerning random variables is that of *conditional probability distribution* and its respective density function $p(X_i|X_j)$ ("the probability density of $X_i=x_i$ given $X_j=x_j$ ") introduced by Bayes. The importance of its use is better understood when describing how new information is incorporated into probability assessments. This is actually the essence of Bayesian statistics that in turn is the foundation stone of the recent International recommendations for the uncertainty evaluation (GUM). The notion of conditional probability provides the capability of re-evaluating the idea of probability of a random variable in light of additional information, that is, when it is known that another

event has occurred or is occurring. Furthermore, it also enables the better understanding of other important concepts such as **independence**, (Walpole, R.E., & Myers, R.H., (1993)). The symbol "|", read "given" in the conditional probability density function $p(X_1|X_2)$ *conditions*, as Hey, J.D., (1983) very elaborately said, the density of the distribution of values X_1 . In the same sense Gershenfeld, N., 1999, has written that $p(X_1|X_2)$ is the probability of seeing a particular value of X_1 , if we **already** know the value of X_2 . It can be proved more useful, if this is generalised and instead of referring to particular values of X_1 , X_2 , is referred to the probability densities themselves, in the same conditional sense as $p(X_1)p(X_2)$. This actually implies that if it is **already** known the density of the distribution of the values X_2 , the conditional probability density $p(X_1|p(X_2))$ may represent the probability of getting the density of the distribution of values X_1 conditional on the already known density of the values of X_2 . Virtually the word already is used to dictate the prior knowledge in terms of X_2 in a temporal (or even spatial) sense. Apparently similar issues that they present qualitative differences sometimes are not clearly mentioned in the existing literature. These issues will be illustrated below and inevitably they have led to some further important considerations.

The conditional probability density $p(X_1|X_2)$ is defined and obtained by:

$$p(X_1/X_2) = \frac{p(X_1, X_2)}{p(X_2)} \quad 4.33$$

The condition that X_1 and X_2 are *independent* is given by:

$p(X_1/X_2) = p(X_1)$ and $p(X_2/X_1) = p(X_2)$ for all X_1 and X_2 . This is usually the way that independence is expressed. Additionally, from the definition 4.33, another important implication is:

Variables X_1 and X_2 are independent if and only if

$$p(X_1, X_2) = p(X_1)p(X_2). \quad 4.34$$

With reference to Gaussian random variables, the above equations that express independence, could easily lead to the misconception of regarding $p(X_1)$ and $p(X_2)$ as equivalent to the two separate *first order probability densities for X_1 and X_2* as those are expressed by equation 4.5. However, the more correct and general case is to say that in 4.27 the separate probability densities $p(X_1)$ and $p(X_2)$ are actually the *marginal probability densities* $p(X_1)_{X_2}$ and $p(X_2)_{X_1}$ respectively. This sensitive qualitative difference has to be particularly emphasised. The example below will be used as a demonstration for a better understanding and analysis of independence.

Assuming that the two random Gaussian variables X_1 and X_2 with the respective first order probability distributions represent the variations of the share of a commercial company in a stock market and the hourly variations of gas static pressure in a gas pipeline respectively. It is evident that there is not any meaningful *unit of association* or a reasonable link between them and obviously they belong to two completely different systems (in a broad sense) or different *probability spaces* (Kottegoda, N.T., (1998)). Particularly, Hamburg, M., (1987) has employed the term *unit of association* and he used it to analyse cause and effect relationships. Thus, it could be said that in this case there is not any kind of association or any "interaction" between the two random variables and that their first order probability densities exist completely independently the one from the other. Such a complete or *unconditional independence* could be seen in figure 4.4 where two such probability densities, although totally independent, have been generated and plotted together.

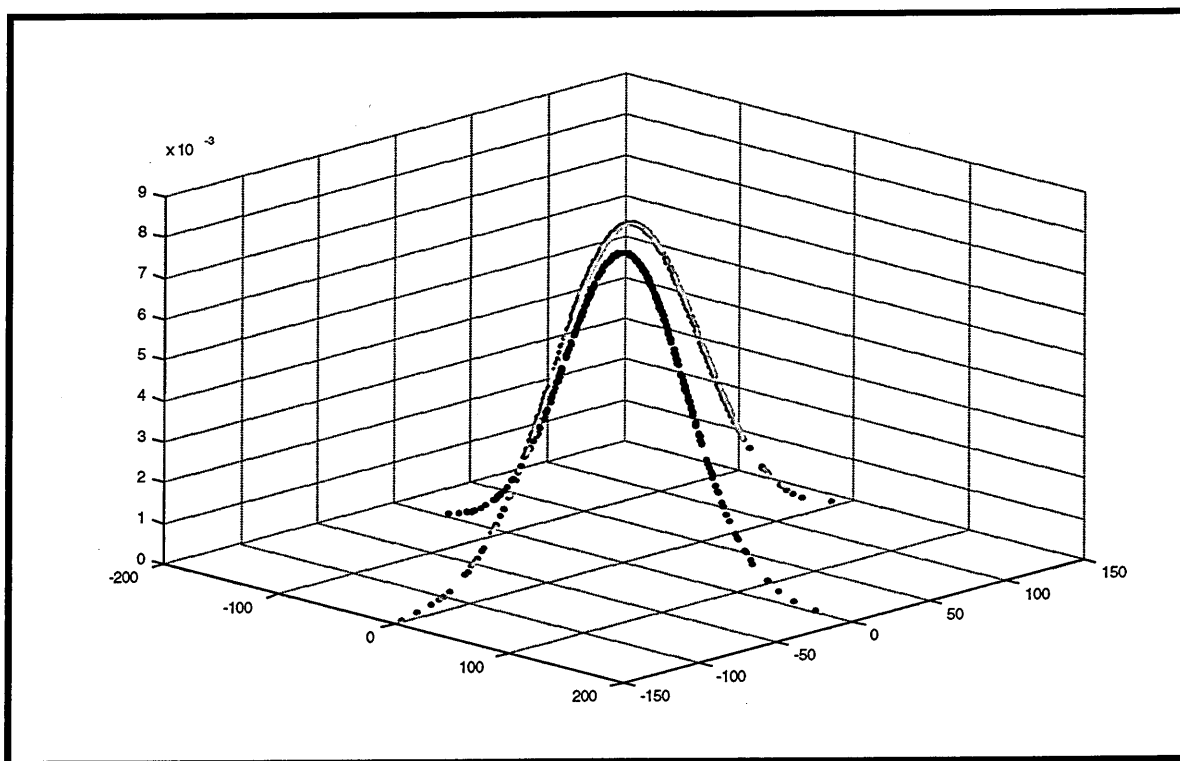


Figure 4.4 Unconditional Independent random variables (1st order probability densities)

The term unconditional independence is introduced in order to describe the fact that neither of the two probability densities can *condition* in any way the density of the other. There is always a probability to observe some time a high correlation between the two variables, however this nonsense correlation is certain to be purely due to *chance*. Additionally,

because rare events must not be excluded in the probability theory, in the extreme case of the existence of some highly improbable link between the two variables, this link have to be something temporarily local. In other words, it would not be able to offer long term explanations on the mutual behaviour of the variables. In summary, unconditional distributions must be associated with a kind of *unconditional or Total Independence*. This will become clearer if it is examined the implied alternative case of *conditional independence* as it is suggested. The starting point of such a concept must be the definition of conditional probability. As it was explained above, the symbol "|" in $p(X_1/X_2)$ conditions the probability of X_1 according to the probability of X_2 . This implies by definition some kind of ability, given *a priori* to the probability density of X_2 , to affect or interact in some way with the probability density of X_1 . In a sense, it does express some dependence, but mainly an acceptance, or a possibility, of the existence of some causal or physical link between the two variables. This could be understood by accepting that the two variables belong to the same or to a close/neighbour system, or probability space, that *moderates* this existence of physical link and the direct communication (interaction) between the two variables.

It has been shown in the previous section how individual and mutual uncertainties are obtained from the covariance matrix of their joint probability density function $p(X_1, X_2)$. Generally, the effect of *conditioning* could be better illustrated in **figure 4.5**.

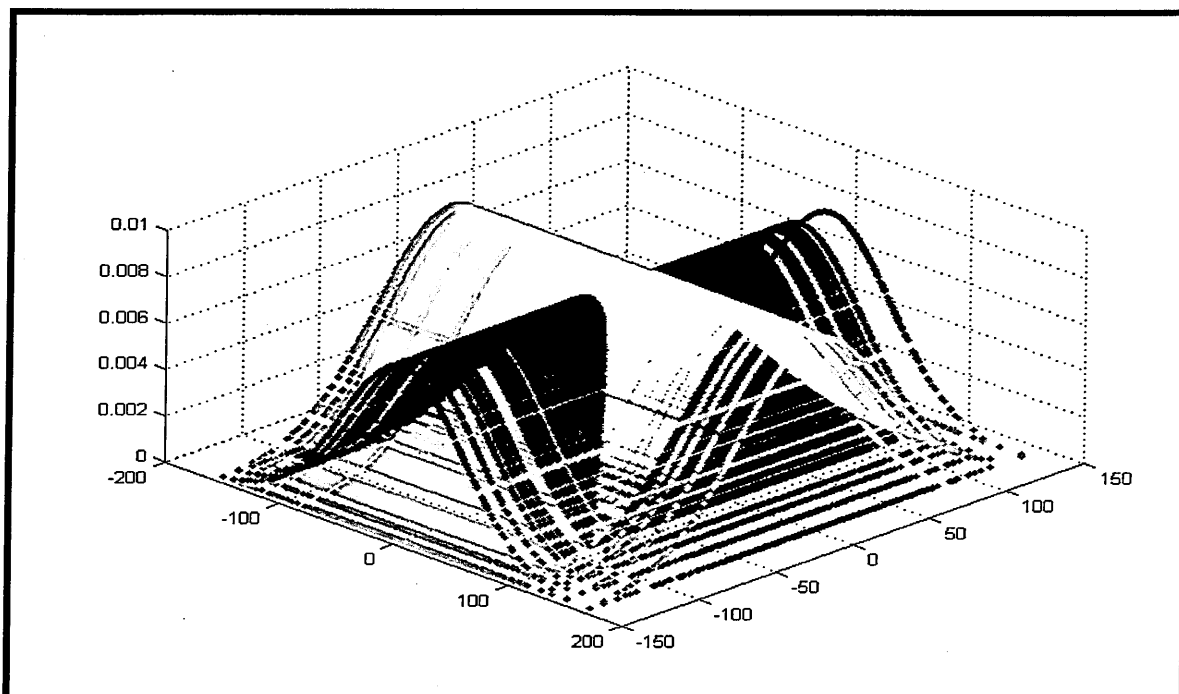


Figure 4.5 Conditional Independent random variables (Marginal probability densities – Joint distribution of probability distributions)

It is displayed the joint distribution of two simulated *conditional independent* variables without considering any other relationship except that they are simply belong to the same system. Additionally, it must be realised that it is no longer simply represents two unconnected distributions but rather conditional (connected) distributions *or better the distributions' co-evolution*. Distribution evolution equations are very fundamental equations originated from quantum mechanics, and offer a natural way to describe stochastic process. This particularly emphasised by Newland, D.E., 1996, where he clearly explained how the term stationary refers to the probability distributions and not to the samples themselves!

Until this moment, time and dynamic measurements have not been entered in any of the previous descriptions but can be naturally appeared at this point by considering the probability to see a sample value X_1 at a time t (written $X_1(t)$) given a history (ensemble) of earlier samples :

$$p(X_1(t)/X_1(t_1), X_1(t_2), \dots)$$

Given prior values for X_1 this determines the probability distribution of the future values. If the distribution depends only on the time differences and not on the absolute time,

$$p(X_1(t)/X_1(t-t_1), X_1(t-t_2), \dots),$$

then the process is said to be stationary. A *strictly stationary* process is one for which all probability distributions of the ensemble are invariant with time. A more modest definition used to describe processes in which the means and covariances are independent of time is that of *weak* or *wide sense* stationary process. If the conditional distribution is limited to a finite history

$$p(X_1(t)/X_1(t-t_1), X_1(t-t_2), \dots, X_1(t-t_N)),$$

this is said to be an *Nth-order Markov process* and if it depends on just the previous sample it is simply called a *Markov process*. Finally, if X_1 and t in a Markov process are discrete variables it becomes a *Markov chain*.

Mutual information in terms of the two variables can now be obtained from the convolution of the two marginal distributions, the joint probability density function $p(X_1, X_2)$. So, in **figure 4.5** it is demonstrated how the probability distribution of the one variable "conditions" the probability distribution of the other when they are conditional independent. This, according to the authors point of view, could be better understood if *the joint distribution is assumed to be the main fundamental (or pre-existing) distribution for*

the two variables that belong to the same system, from which the individual and mutual effects can be realised. So, in the case of conditional independent variables, it can be concluded that the effects of their mutual interaction are limited only to the combined evolution of their density distributions and not to other characteristics of their marginal distributions such as the shape or the orientation. It is necessary to make clear at this moment that the marginal probabilities of figure 4.5 have not simply obtained by evolving the distribution of the one to the values of the other but have been obtained as normalised products of their more fundamental joint distribution. Obviously, the two random variables with their probability densities shown in figure 4.5 are uncorrelated, however, they are not totally independent in the sense that was developed here.

A different procedure from the classical procedures mentioned in chapter 3 in terms of the modeling of the given input variables and their uncertainties was also adopted for test case studies of uncertainty propagation mechanism and visualisation of its effects that were carried out. So, having obtained the improved standard normal distributions z_1 and z_2 as it was described in chapter 3, the random variables $x_1 \sim N(\mu_{x1}, \sigma_{(x1)})$ and $x_2 \sim N(\mu_{x2}, \sigma_{(x2)})$ for example, were not modeled by simply using the random function $x_i = \mu + \sigma z_i$. Instead, they were modeled as bivariate normal distributions, by applying the notions of joint and conditional (dependent) distributions described in this section and particularly using equations 4.16 and 4.33. In practice, the random variable x_1 was first modeled as:

$$x_1 = \mu_{x_1} + \sigma(x_1)z_1$$

Then the x_2 is sampled as a *conditional variate x_2 given x_1* , modelled as:

$$x_2 = \mu_{x_2} + \rho \left(x_1 - \bar{x}_1 \right) \frac{\sigma(x_2)}{\sigma(x_1)} + \sigma(x_2)z_2(1 - \rho^2)^{\frac{1}{2}}$$

where ρ is the correlation coefficient or normalised covariance of x_1 and x_2 . The same procedure can be equally applied, successively, for multivariable cases with mutually dependent components. These are actually most of the cases prevalent in complex measurement systems. The knowledge of all the input covariances in such cases though is really a very difficult task. It can be easily realised that this procedure can covers all the possible cases in terms of the two modelled input variables by simply adjusting the correlation coefficient accordingly. Thus, for example, for a correlation coefficient $\rho=0$, the model for x_2 becomes the same with the one used for the generation of two independent random variables and so this case is part of this later method. A series of generated

covariate samples for x_1 and x_2 , following this procedure, and for different values of correlation coefficient ($\rho=0.9, 0.3, -0.3$, and -0.9) is shown in **figures 4.6 and 4.7**. Finally, another fact that was also realised was that both these procedures could be applied as well to uniform distributions, in cases that a thorough consideration and further modelling of measurements' systematic uncertainty components may wanted to be accounted for separately.

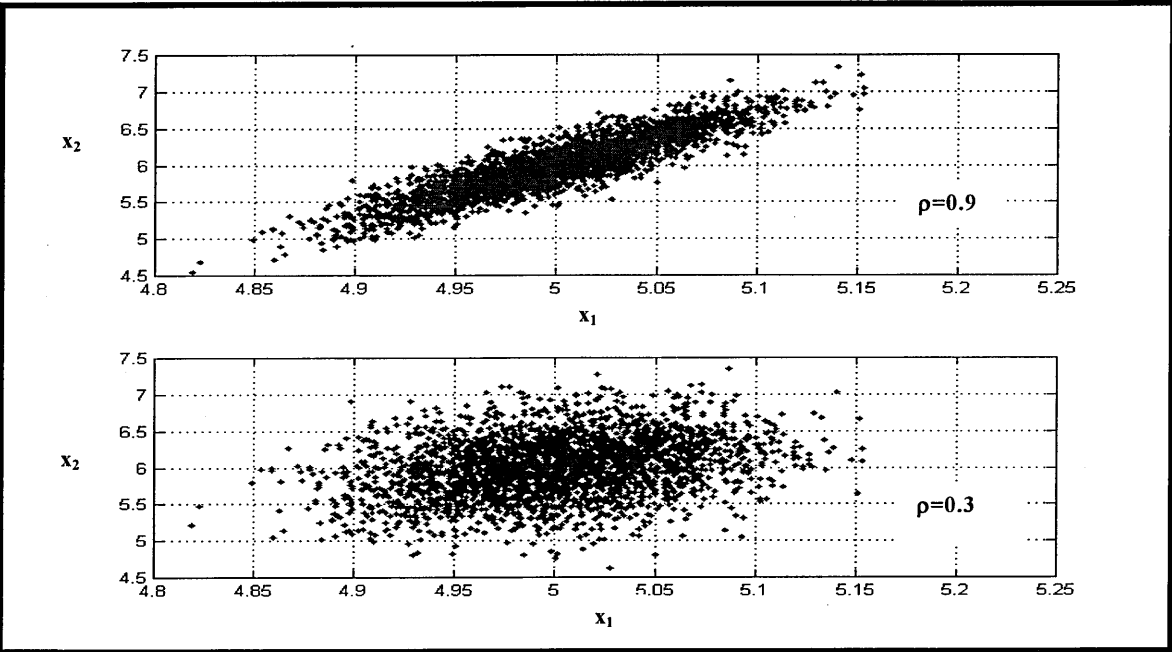


Figure 4.6 Generated covariate samples $x_1 \sim N(5, 0.1)$ and $x_2 \sim N(6, 0.8)$ for $\rho=0.9$ and 0.3 .

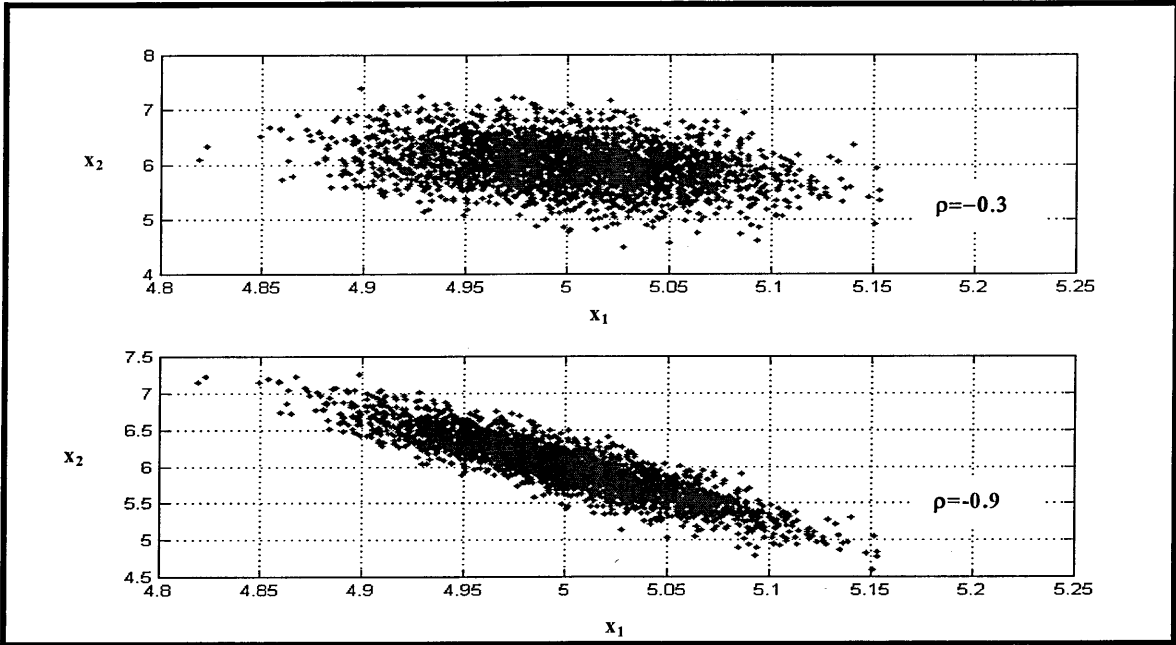


Figure 4.7 Generated covariate samples $x_1 \sim N(5, 0.1)$ and $x_2 \sim N(6, 0.8)$ for $\rho=-0.9$ and -0.3

In **figures 4.8 and 4.9** are shown views of the joint probability density function of two simulated partially positively correlated random variables.

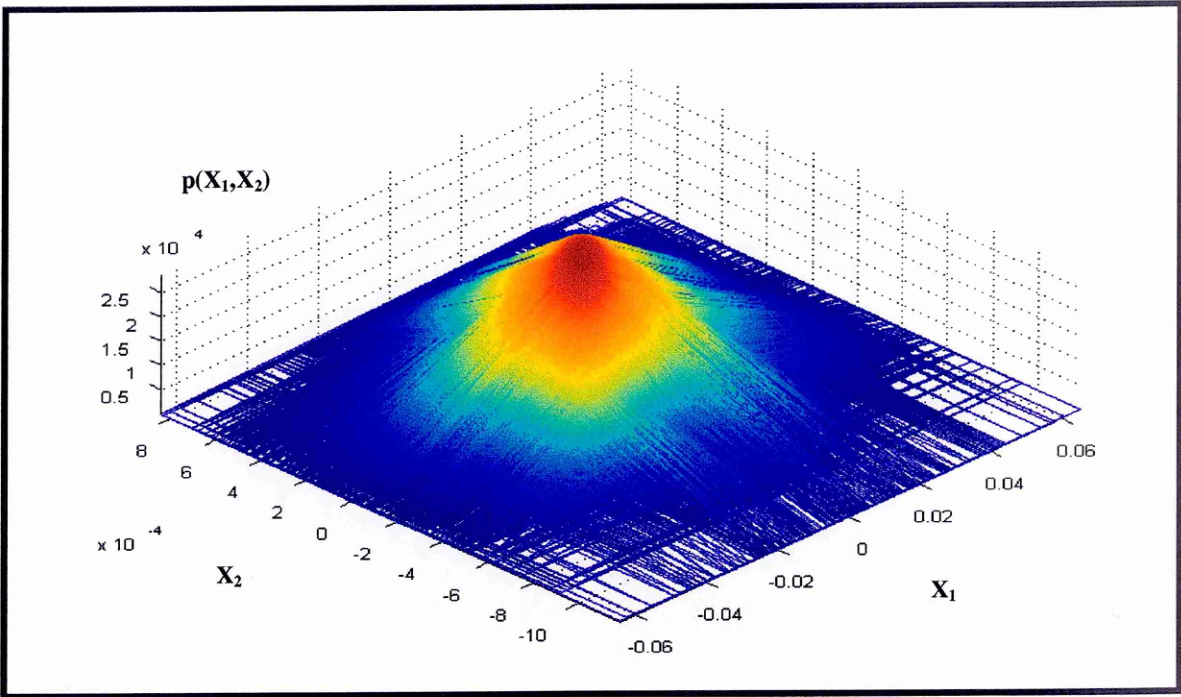


Figure 4.8 Joint probability density of two positively correlated random variables

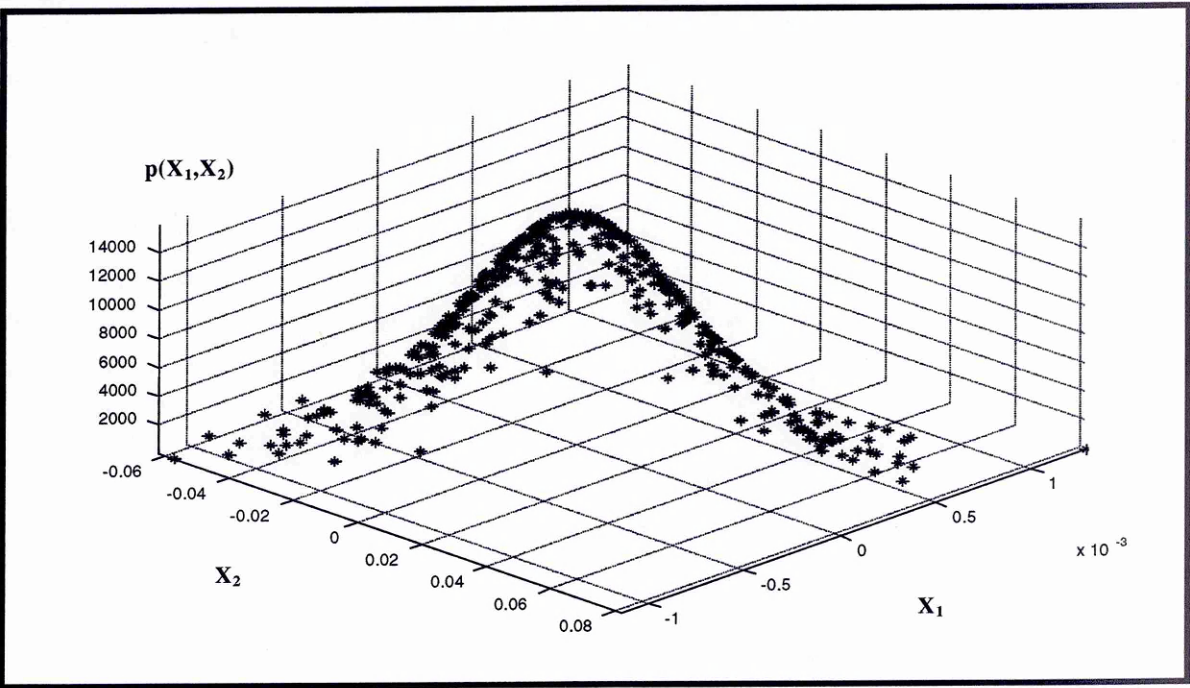


Figure 4.9 Joint probability density of two positively correlated random variables.

The marginal distributions obtained are shown in the **figures 4.10 and 4.11**. The effect of the mutual interaction between X_1 and X_2 is a "rotation" from the assumed independent

distribution lines $X_1=0$ and $X_2=0$, towards the line $X_1=kX_2$ where $k=\tan\phi$, (fig.4.12). The angle ϕ between the two marginal distributions can be regarded as another measure of their mutual information.

As a consequence of all the above it could be ascertained that the general form of equation 4.34 is:

$$p(X_1, X_2) = p(X_1)x_2 p(X_2)x_1 \tag{4.35}$$

This is valid for any conditionally dependent Gaussian variables X_1 and X_2 . In case of total independence, $p(X_1)x_2=p(X_1)$ and $p(X_2)x_1=p(X_2)$. In this way the conditional probability densities $p(X_1/X_2)$ and $p(X_2/X_1)$ can be interchanged with the marginal probability densities.

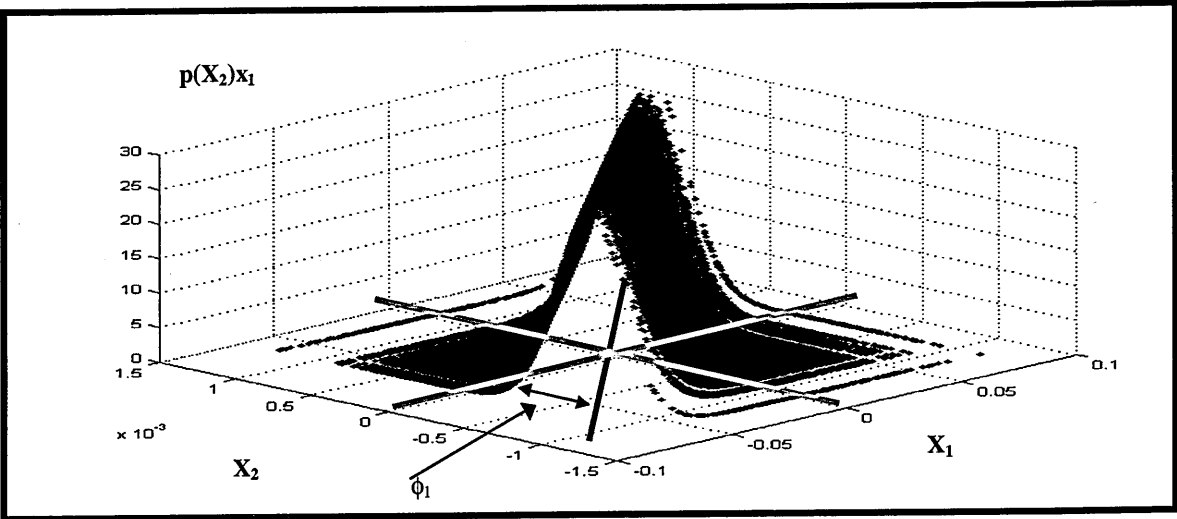


Figure 4.10 Effects of mutual (joint) interaction on the probability density of X_2

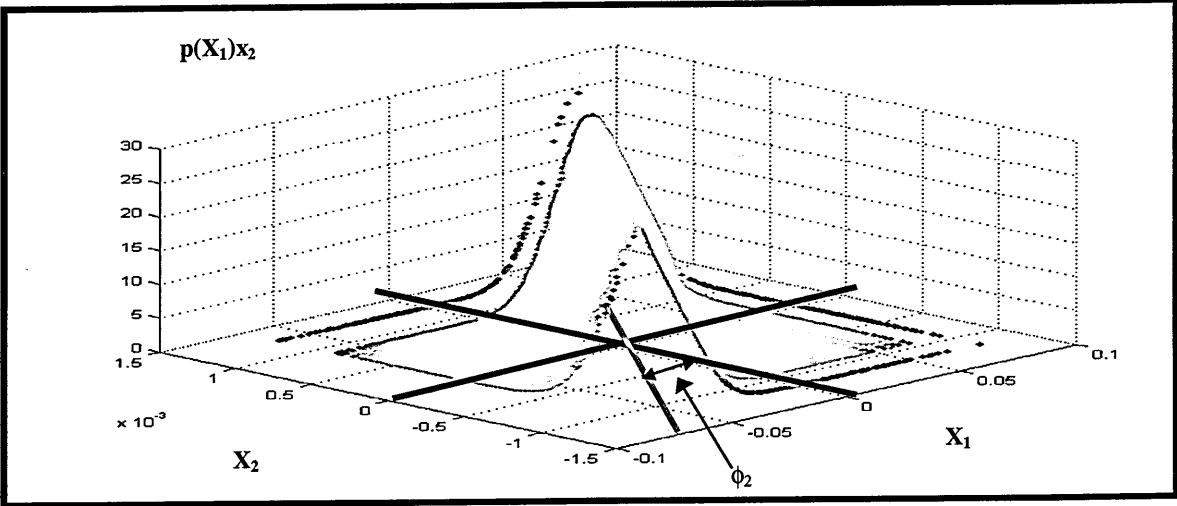


Figure 4.11 Effects of mutual (joint) interaction on the probability density of X_1

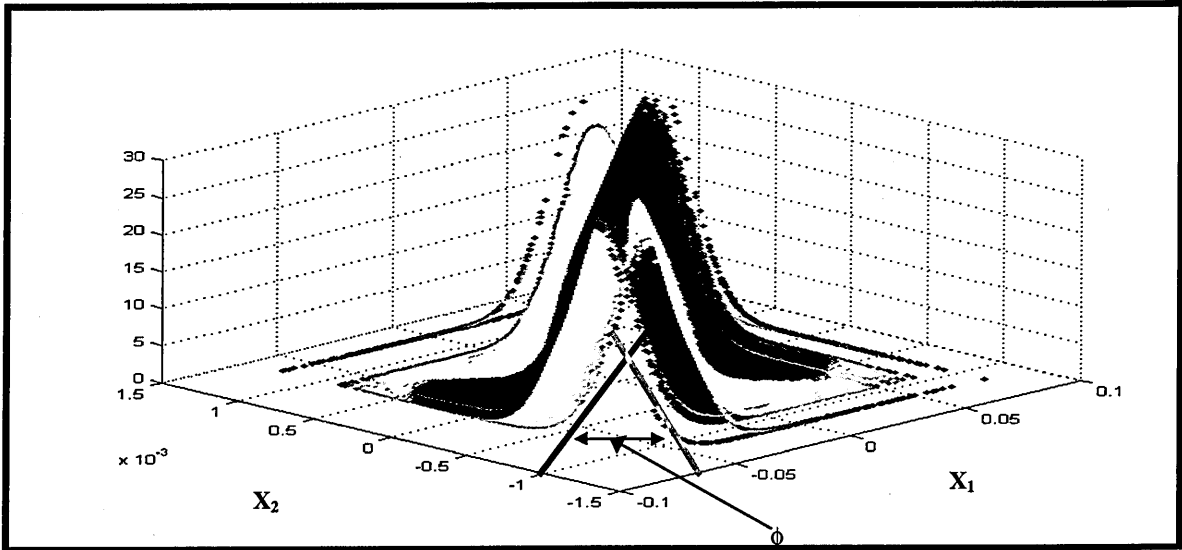


Figure 4.12 Mutual (joint) Information angle ϕ .

4.2. Combined uncertainty estimation methods in practise

4.2.1. Conventional Uncertainty estimation methods and Sensitivity coefficients.

Conventional uncertainty estimation methods start as well from the measurement equation 4.1 or for the case of two random variables, from equation 4.3.

Suppose that the data reduction equation is:

$$Y=f(X_1,X_2)$$

Instead of calculating $E(Y)=E(f(X_1,X_2))$ and $u(Y)=\sigma(Y)$ from the probability density of the estimator Y , from the data reduction equation, Y is now approximated using a polynomial approximation or otherwise a n th order Taylor's series expansion about the means of variables X_1, X_2 . Thus, $Y=f(X_1,X_2)$ that is a function of two variables X_1 and X_2 with means x_1 and x_2 respectively based on multiple measurements x_{1i} and x_{2i} , can be approximated by:

$$f(X_1,X_2)=y_i=f(x_{1i},x_{2i})=f(x_1,x_2)+\frac{\partial f}{\partial X_1}(x_{1i}-x_1)+\frac{\partial f}{\partial X_2}(x_{2i}-x_2)+W \quad 4.36$$

where W is a remainder term given by:

$$W=\frac{1}{2!}\left[\frac{\partial^2 f}{\partial X_1^2}(x_{1i}-x_1)^2+\frac{\partial^2 f}{\partial X_2^2}(x_{2i}-x_2)^2+2\frac{\partial^2 f}{\partial X_1 \partial X_2}(x_{1i}-x_1)(x_{2i}-x_2)\right] \quad 4.37$$

In the last two expressions the first order partial derivatives are computed at $X_1=x_1$ and $X_2=x_2$, therefore are the same for all $i=1,\dots,N$ and all the terms of order higher than 2 in the

expansion are normally excluded. With this approximation, we can obtain the mean as:

$$\begin{aligned} E(Y) = y &= \frac{1}{N} \sum_{i=1}^N y_i = \frac{1}{N} \sum_{i=1}^N f(x_{1i}, x_{2i}) = \\ &= \frac{1}{N} \sum_{i=1}^N \left[f(x_1, x_2) + \frac{\partial f}{\partial X_1} (x_{1i} - x_1) + \frac{\partial f}{\partial X_2} (x_{2i} - x_2) \right] \end{aligned} \quad 4.38$$

This equation gives the mean as the sum of three terms. The first term is just $f(x_1, x_2)$ and the next two are exactly zero since it follows from the definition of mean value that

$$\left[\sum_{i=1}^N (x_{1i} - x_1) = \sum_{i=1}^N (x_{2i} - x_2) = 0 \right]. \quad 4.39$$

Thus it is simply obtained the result $E(Y) = y = f(x_1, x_2)$. So the expectation value of the results y , is obtained by calculating the function $f(X_1, X_2)$ only at the points $E(X_1)=x_1$ and $E(X_2)=x_2$. The residual term W can only be excluded, provided that the uncertainties in X_1 and X_2 are small and hence that all the numbers x_{1i} and x_{2i} are close to x_1 and x_2 respectively. Under this condition, W that contains the sum of the squares of the differences $x_i - x$ will approach zero more quickly than the first order terms. Additionally, the important fact is that *if $f(X_1, X_2)$ is a linear function, then the second order partial derivatives in 4.37 are identically zero and so $W=0$* . Although, this simplifies further the respective calculations, unfortunately, *it is also the most controversial point of reliable uncertainty estimation*, especially regarding fluid flow measurements, driven from the assumption of small independent uncertainties in X_1, X_2 .

The standard deviation in the N values (population) of $y_i=f(x_{1i}, x_{2i})$ that is actually the

$$\text{uncertainty in } Y \text{ is given by: } u(Y) = \sigma(Y) = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - y)^2} \quad 4.40$$

Substituting 4.36 and 4.39, while assuming small uncertainties ($W=0$), in 4.40 we can find that:

$$\begin{aligned} \sigma(Y) &= \sqrt{\frac{1}{N} \sum_{i=1}^N \left[\frac{\partial f}{\partial X_1} (x_{1i} - x_1) + \frac{\partial f}{\partial X_2} (x_{2i} - x_2) \right]^2} = \\ &= \sqrt{\left(\frac{\partial f}{\partial X_1} \right)^2 \frac{1}{N} \sum_{i=1}^N (x_{1i} - x_1)^2 + \left(\frac{\partial f}{\partial X_2} \right)^2 \frac{1}{N} \sum_{i=1}^N (x_{2i} - x_2)^2 + 2 \frac{\partial f}{\partial X_1} \frac{\partial f}{\partial X_2} \frac{1}{N} \sum_{i=1}^N (x_{1i} - x_1)(x_{2i} - x_2)} \end{aligned} \quad 4.41$$

The terms other than the partial derivatives appearing in 4.41 can be easily recognised as the variances and covariance respectively of X_1 , X_2 . So, finally we obtain for the combined uncertainty of the measurement result Y :

$$u(Y) = \sigma(Y) = \sqrt{\left(\frac{\partial f}{\partial X_1}\right)^2 \sigma(X_1)^2 + \left(\frac{\partial f}{\partial X_2}\right)^2 \sigma(X_2)^2 + 2 \frac{\partial f}{\partial X_1} \frac{\partial f}{\partial X_2} \sigma(X_1 X_2)} \quad 4.42$$

or written using the correlation coefficient:

$$u(Y) = \sigma(Y) = \sqrt{\left(\frac{\partial f}{\partial X_1}\right)^2 \sigma(X_1)^2 + \left(\frac{\partial f}{\partial X_2}\right)^2 \sigma(X_2)^2 + 2 \frac{\partial f}{\partial X_1} \frac{\partial f}{\partial X_2} \rho_{X_1 X_2} \sigma(X_1) \sigma(X_2)}$$

This is the basic general equation for the uncertainty of a measurement result Y and is conveniently referred as the **law of propagation of uncertainty**. This equation gives the uncertainty as a standard deviation *whether or not the measurements of X_1 and X_2 are independent or normally distributed*. It is a first order Taylor series approximation of Y while the standard deviations $\sigma(X_1)$ and $\sigma(X_2)$ are referred in the GUM as the standard uncertainties associated with the input estimates x_1 and x_2 .

The partial derivatives of f with respect to the X_i are referred as the *sensitivity coefficients* and they express the effects of each input quantity on the final result or otherwise the sensitivity of the output quantity to each input quantity. The calculation of sensitivity coefficient of each input quantity can be carried out with the aid of a sensitivity analysis, which is an integral part of the conventional combined uncertainty estimation methods, using either analytical or numerical methods. The role of sensitivity analysis will be explained in the next section.

Different approaches in terms of the conventional analytical combined uncertainty estimation methods had been adopted throughout the years.

- An approach that has been adopted from various standard organisations in 1970s and 1980s was the root of sum squares method (RSS) and the associate method of simple addition of partial uncertainty terms (ADD) method (Coleman, H.W., Steele, W.G., 1995). This was also known as the ASME measurement uncertainty methodology and is described by Abernethy, R.B., Benedict, R.P. et.al., 1985. So, according to this method, the uncertainty of a result Y was obtained by:

$$u(Y) = \sqrt{B_Y^2 + (tS_Y)^2} \text{ for a 95\% confidence estimate and} \quad 4.43a$$

$$u(Y) = B_Y + (tS_Y) \text{ for a 99\% confidence interval, where} \quad 4.43b$$

$$B_Y = \sqrt{\left(\frac{\partial f}{\partial X_1} B_{X1}\right)^2 + \left(\frac{\partial f}{\partial X_2} B_{X2}\right)^2} \quad 4.44$$

$$S_Y = \sqrt{\left(\frac{\partial f}{\partial X_1} S_{X1}\right)^2 + \left(\frac{\partial f}{\partial X_2} S_{X2}\right)^2} \quad 4.45$$

where B_Y , B_{X1} , B_{X2} are bias estimates (systematic uncertainties) S_Y , S_{X1} , S_{X2} are random (precision) estimates (random uncertainties) and t is the respective % confidence t value from the t distribution for v degrees of freedom given (approximately) by the so called Welch-Satterthwaite formula as:

$$v = \frac{\left(\left(\frac{\partial f}{\partial X_1} S_{X1}\right)^2 + \left(\frac{\partial f}{\partial X_2} S_{X2}\right)^2\right)^2}{\frac{\left(\frac{\partial f}{\partial X_1} S_{X1}\right)^4}{v_1} + \frac{\left(\frac{\partial f}{\partial X_2} S_{X2}\right)^4}{v_2}} \quad 4.46$$

If S_{X1} and/or S_{X2} has been determined from n readings taken over an appropriate interval, then the numbers of degrees of freedom v_1 and/or v_2 are given by $v_{1/2}=n-1$. The RSS method, as it can be easily seen, ignores correlation effects of the input variables. On the contrary, the ADD method gives the most pessimistic uncertainty bounds, and as it will be shown in the next section, the correlation effects are also accounted for, although there are some additional hidden elements.

- Another approach has been initially developed by **Kline, S.J., and McClintock, F.A., 1953** who did some very significant work in sixties which later was expanded by Coleman, H.W., and Steele, W.G., 1995. A 95% confidence estimate of the uncertainty in the result Y was then proposed as that given by equation 4.43a. In their assessment though, the systematic uncertainty in the result Y is given by:

$$B_Y = \sqrt{\left(\frac{\partial f}{\partial X_1} B_{X1}\right)^2 + \left(\frac{\partial f}{\partial X_2} B_{X2}\right)^2 + 2 \frac{\partial f}{\partial X_1} \frac{\partial f}{\partial X_2} \rho_{BX1X2} B_{X1} B_{X2}} \quad 4.47$$

and the random uncertainty is given by:

$$S_Y = \sqrt{\left(\frac{\partial f}{\partial X_1} S_{X1}\right)^2 + \left(\frac{\partial f}{\partial X_2} S_{X2}\right)^2 + 2 \frac{\partial f}{\partial X_1} \frac{\partial f}{\partial X_2} \rho_{SX1X2} S_{X1} S_{X2}} \quad 4.48$$

where $\rho_{BX_1X_2}$, $\rho_{SX_1X_2}$ the correlation coefficients of the systematic and random uncertainties B_{X_1}, B_{X_2} and S_{X_1}, S_{X_2} respectively and $\rho_{BX_1X_2} B_{X_1} B_{X_2} = B_{X_1X_2}$, $\rho_{SX_1X_2} S_{X_1} S_{X_2} = S_{X_1X_2}$ the respective covariances. S_{X_1} and S_{X_2} are specified as tS intervals where t is determined similarly as above for $v_{1/2}=n-1$ degrees of freedom.

- The *ISO Guide* method accommodates basic aspects of the previous methods though with the addition of some fundamental differences and it expresses the current international view of how to estimate measurement uncertainty. In general, the result of a measurement is regarded as only an approximation or estimate of the value of the specific quantity subjected to measurement (measurand). The uncertainty of the result of a measurement can be obtained by two methods.

Type A evaluations of uncertainty may be based on any valid *statistical* method for treating data. For example, calculation of the standard deviation of the mean, use of least squares method to curve fitting in order to estimate the parameters of the curve and their standard deviations, analysis of variance (ANOVA) for the identification and quantification of random effects in a measurement, are all valid methods.

Type B evaluation of uncertainty may be based on *scientific judgement* and can be carried out by means other than the statistical analysis of a series of observations (data) using all the relevant information available, which may include:

- previous measurement data
- manufacturer's specifications
- uncertainties assigned to reference data taken from handbooks
- data provided in calibration and other reports
- experience with, or general knowledge of, behaviour and properties of relevant materials and instruments e.t.c.

Obviously type A evaluation of uncertainty represents the classical statistical methodology while the introduction of type B evaluation of uncertainty introduces this Bayesian sense already mentioned. Thus the ISO GUM seems to offer this compromise between objective and subjective probabilities and the two major statistical thoughts are combined for improved uncertainty estimation. In the uncertainty calculations method as proposed by the ISO GUM, random and systematic uncertainties are identically treated, justified on the basis that there is not any real need for such a classification by the calculation method. The terms random and systematic uncertainty can actually be misleading when applied in a

sense that does not take into account the use made of the corresponding quantity. Particularly, depending on how that quantity appears in the measurement equation could condition the type of the uncertainty and components of uncertainty arising from random effects could affect in a systematic way the measurement results. Specific examples of such cases can be found in the new ISO/CD-5167, 1999. The *expanded uncertainty* is the term used to express the % confidence interval about the measurement result within which the true value of the measurand is believed to lie. This is given by:

$u(Y) = \sqrt{(tB_Y)^2 + (tS_Y)^2} = t\sqrt{B_Y^2 + S_Y^2} = tu_c(Y)$ where $u_c(Y)$ is called the *combined standard uncertainty* and B_Y , S_Y are given by 4.47 and 4.48 respectively. As it could be noticed, a confidence level t value has been also assigned to the systematic uncertainty term. The next form of Welch-Satterthwaite formula gives (approximately) the effective number of degrees of freedom ν for determining the t value and the respective confidence

$$\text{interval: } \nu = \frac{\left(\left(\frac{\partial f}{\partial X_1} B_{x1} \right)^2 + \left(\frac{\partial f}{\partial X_2} B_{x2} \right)^2 + \left(\frac{\partial f}{\partial X_1} S_{x1} \right)^2 + \left(\frac{\partial f}{\partial X_2} S_{x2} \right)^2 \right)}{\frac{\left(\frac{\partial f}{\partial X_1} B_{x1} \right)^4}{\nu_{B1}} + \frac{\left(\frac{\partial f}{\partial X_2} B_{x2} \right)^4}{\nu_{B2}} + \frac{\left(\frac{\partial f}{\partial X_1} S_{x1} \right)^4}{\nu_{S1}} + \frac{\left(\frac{\partial f}{\partial X_2} S_{x2} \right)^4}{\nu_{S2}}} \quad 4.49$$

$\nu_{B1,2}$ and $\nu_{S1,2}$ are the numbers of degrees of freedom associated with the systematic and random uncertainties respectively. Their calculation method depends on the respective uncertainties evaluation methods used (type A or B). So, if the uncertainties have been obtained as statistical estimates (Type A) from n readings, the numbers of degrees of freedom are given by $\nu_i = n - 1$. In case that the uncertainties have been obtained as non-statistical estimates (Type B), it is suggested in the ISO/GUM that it might be used the

approximation $\nu_i \approx \frac{1}{2} \left(\frac{\Delta u_i}{u_i} \right)^{-2}$. The term $\left(\frac{\Delta u_i}{u_i} \right)$ is the relative uncertainty of the respective

uncertainty (either due to random or systematic effects) and its value is estimated based on the information available by scientific judgement. Additionally, if in Type B evaluations the probability of the quantity lying outside upper and lower limits is practically negligible, the degrees of freedom are infinite ($\nu_i \rightarrow \infty$) and the t value approaches 2 (approximately) considering a respective 95% confidence interval. So, when the available information for a quantity is adequate, based either on a large number of readings obtained or on a rigorous scientific judgement (using prior knowledge), the use of Welch-Satterthwaite formula is

not actually necessary. A t value of 2 then can be used in all cases for a level of confidence of approximately 95%. Coleman, H.W., and Steele, W.G., 1995, present a detailed analysis of the subject, while they applied Monte Carlo simulation to check the percentage coverage in terms of the mean for various number of readings.

4.2.1.1.Sensitivity Coefficients – Calculation methods

In the previous section it was shown that by using the conventional methods for combined uncertainty estimation, it is also necessary to estimate the separate effect of each input quantity on the final result. So, this estimation comes in addition to the estimation of the magnitudes of the component uncertainties. These partial effects are expressed with the partial derivatives of the output with each of the inputs and are called *the sensitivity coefficients*. The reliable estimation of sensitivity coefficients has been the most difficult part of an uncertainty analysis. The knowledge of sensitivity coefficients is useful for understanding the performance characteristics of a measurement system. Particularly, in determining those parameters that are important to pay more attention, and in examining the interactions involved in the uncertainty propagation which are not always so obvious. According to the author's point of view, it is at this point where uncertainty analysis becomes similar to reliability analysis where respective *reliability importance/criticality measures* (improvement potential, Birnbaum's measure - see Aven, T., 1992) are employed for assessment of system's reliability. Reliability and uncertainty analysis complement one another since reliability analysis examines the performance of the "hard" component of a process system while uncertainty analysis examines the "soft" performance of the system. In other words:

We need a measurement system that operates without failures (high "hard" reliability) and consistently produces accurate measurements within reasonable desired control limits (high "soft" reliability or consistently low, known uncertainty). So uncertainty analysis gives the evidences for a reliability analysis of "soft" failures.

More about this matter have been included in chapter 6 within the general framework of quality (uncertainty) management of natural gas fiscal metering systems.

Starting from equation 4.42, the law of uncertainty propagation after substituting the equivalent $\partial f = \partial Y$ is:

$$u(Y) = \sigma(Y) = \sqrt{\left(\frac{\partial Y}{\partial X_1}\right)^2 \sigma(X_1)^2 + \left(\frac{\partial Y}{\partial X_2}\right)^2 \sigma(X_2)^2 + 2 \frac{\partial Y}{\partial X_1} \frac{\partial Y}{\partial X_2} \sigma(X_1 X_2)}. \quad 4.50$$

Another useful non-dimensional form of equation 4.50 can be obtained in the form of a relative uncertainty by dividing 4.50 by the result Y .

$$\frac{u(Y)}{Y} = \frac{\sigma(Y)}{Y} = \sqrt{\left(\frac{\partial Y}{\partial X_1} \frac{\sigma(X_1)}{Y}\right)^2 + \left(\frac{\partial Y}{\partial X_2} \frac{\sigma(X_2)}{Y}\right)^2 + 2 \frac{\partial Y}{\partial X_1} \frac{\partial Y}{\partial X_2} \frac{\sigma(X_1 X_2)}{Y^2}} \quad 4.51$$

The left part of this equation defines the dimensionless *relative uncertainty* of the result as a function of the dimensional uncertainties of the inputs and dimensional sensitivity coefficients. The procedure was applied by Kline, S.J., and McClintock, 1953, in cases of independent variables (without considering the covariance term(s)).

The author has also considered the covariance term(s) and developed the following equations that they can significantly contribute to the best realisation of the uncertainty propagation mechanism. The right part can also become non-dimensional by multiplying the top and bottom of each consecutive term under the square root with the square of the respective variable in question (multiplication by $\frac{X_i^2}{X_i^2} = 1$), except that the covariance term

can be multiplied by $\frac{X_1^2}{X_1^2} \frac{X_2^2}{X_2^2} = 1$.

So, it can be obtained the relative uncertainty of the result as a function of relative uncertainties of the inputs and of the dimensionless sensitivity coefficients that are called in this case the *influence coefficients*:

$$\begin{aligned} \frac{u(Y)}{Y} = \frac{\sigma(Y)}{Y} &= \sqrt{\left(\frac{X_1}{Y} \frac{\partial Y}{\partial X_1} \frac{\sigma(X_1)}{X_1}\right)^2 + \left(\frac{X_2}{Y} \frac{\partial Y}{\partial X_2} \frac{\sigma(X_2)}{X_2}\right)^2 + 2 \frac{\partial Y}{\partial X_1} \frac{\partial Y}{\partial X_2} \frac{\sigma(X_1 X_2)}{Y^2}} = \\ &= \sqrt{\left(\frac{\partial \ln Y}{\partial \ln X_1} \frac{\sigma(X_1)}{X_1}\right)^2 + \left(\frac{\partial \ln Y}{\partial \ln X_2} \frac{\sigma(X_2)}{X_2}\right)^2 + 2 \frac{\partial \ln Y}{\partial \ln X_1} \frac{\partial \ln Y}{\partial \ln X_2} \frac{\sigma(X_1 X_2)}{X_1 X_2}} \end{aligned} \quad 4.52$$

In equations 4.51 and 4.52 various sensitivity measures have appeared. The most common sensitivity coefficient $\frac{\partial Y}{\partial X_i}$, which is actually an approximation of the local (at the

estimated mean) gradient, expresses the local unit change in the output caused by a local unit change of the input. The other sensitivity measure, the influence coefficient, $\frac{X_i}{Y} \frac{\partial Y}{\partial X_i} = \frac{\partial \ln Y}{\partial \ln X_i}$ is in the form of a normalised gradient and expresses the unit percentage

change in the output caused by a unit percentage change in the input.

It must be reminded and strongly emphasised that $\sigma(X_1), \sigma(X_2)$ may be the combined uncertainties of elementary measurement processes and not simply the uncertainty components of single measured variables.

The previous alternative forms of law of uncertainty propagation have been used in the analysis of simulation results of simple models that will be presented later in section 4.2.2. These are useful for the best understanding of the results of this project. The usefulness of these equations and generally the role of the sensitivity analysis can become evident after realising that *uncertainty propagation is not an obvious operation* in the sense that one cannot assume that low uncertainties in the measured inputs will always give low uncertainty in the result or **more interestingly the opposite!!!**

As it was said, one of the most difficult parts of uncertainty analysis is the determination of the sensitivity coefficients shown in the last equations, especially in complex multivariable measurement processes.

The estimation of sensitivity coefficients can be carried out either *analytically* or *numerically*.

It was pointed out in chapter 1 that the traditional analytical methods in orifice flow measurement uncertainty estimation, as specified in ISO/5168, 1978, requires the calculation of a long list of partial derivatives amongst multiple respective physically or model based measured variables.

The normal numerical methods that are used for the calculation of sensitivity coefficients are also based on the definition of partial derivative operation:

$$\frac{\partial Y(x_1, x_2)}{\partial x_1} = \frac{(Y_{x_1+dx_1, x_2} - Y_{x_1, x_2})}{dx_1} \text{ where } x_1, x_2 \text{ are the estimated means of } X_1, X_2. \text{ So, first } Y$$

is calculated using x_1, x_2 and then recalculated using x_1+dx_1, x_2 . The result of the recalculation is expressed as $y+dy_{x_1}$ where dy_{x_1} is the increment caused by dx_1 expressed

$$\text{as: } dy_{x_1} = \frac{\partial Y(x_1, x_2)}{\partial x_1} dx_1 = (Y_{x_1+dx_1, x_2} - Y_{x_1, x_2}). \text{ When, the increment } dx_1 \text{ is taken as large}$$

$$\text{as the uncertainty of } x_1 \text{ (but still regarded as relatively small), then } dy_{x_1} = \frac{\partial Y(x_1, x_2)}{\partial x_1} \sigma(x_1)$$

expresses the partial uncertainty of y due to x_1 .

During the last decades, there have been lots of efforts for the production of reliable computerised uncertainty/sensitivity analysis codes. On line uncertainty analysis/estimation, which is really a difficult task, could be set as the ultimate target of

these efforts. In the meantime, others have proposed various computer-based methods for uncertainty estimation. Moffat, R.J., 1988, has proposed the so-called sequential perturbation method for calculating uncertainty intervals from the data reduction program. It is based on the previously described partial derivative operation, which is basically a finite difference analysis method, and the partial uncertainty terms are calculated by running the data-reduction program $n+1$ times (n , the number of variables) in forward or backward difference calculation, or $2m$ times, if the most accurate central difference method is employed. Applying forward difference, the result y is calculated as usual from the recorded values x_1, x_2 and then once more for each variable, with the value of that variable increased by its uncertainty interval while the other x_i are returned to their recorded baseline value. The method, as Moffat has developed it for the estimation of the uncertainty in the computed heat transfer coefficient from its basic measurements, is illustrated in **Figure 4.13**.

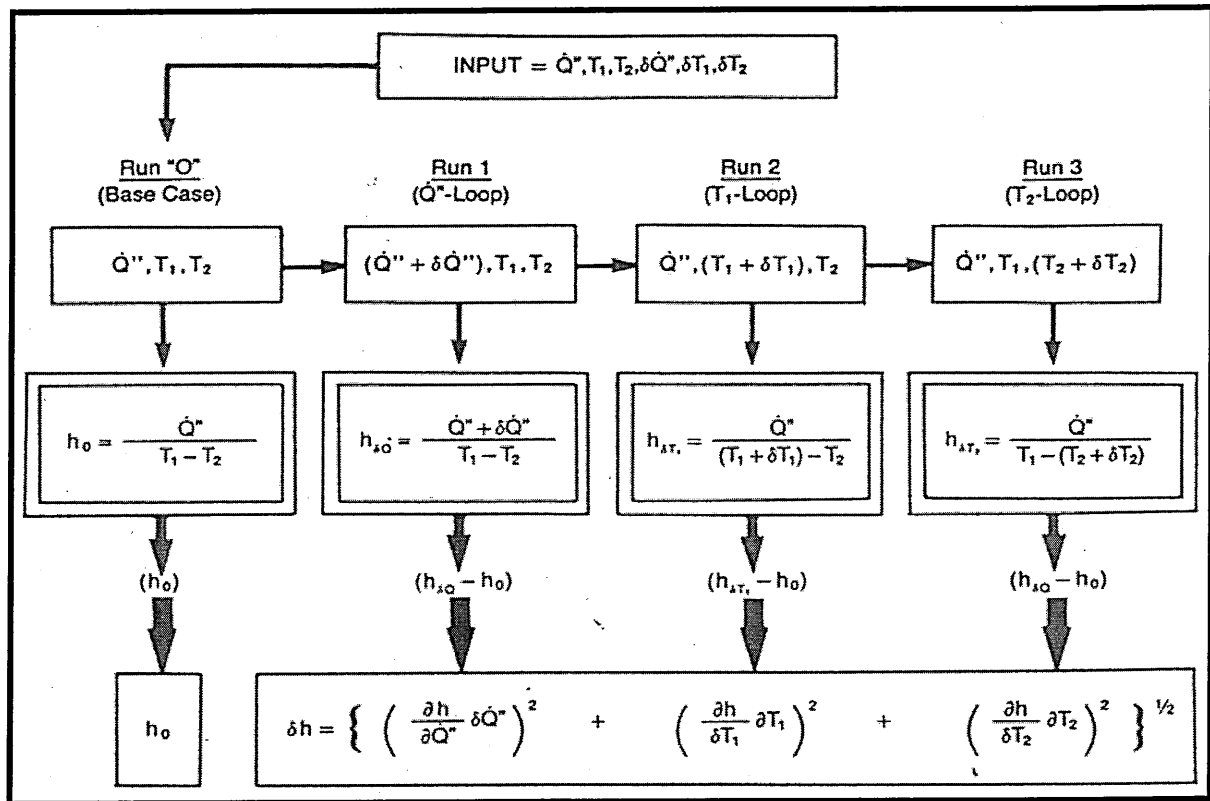


Figure 4.13 The sequential perturbation method for uncertainty estimation of heat transfer coefficient from its basic measurements using the data-reduction program as has been presented by Moffat, R.J., 1988.

Although the fact that by using the data reduction program itself to compute the uncertainties sounds very promising in terms of computer code efficiency, according to the author's point of view, there are significant disadvantages in that application of this method.

- Actually, it is a "computer based *analytical* method" since it requires the sequential estimation of all the partial derivatives, a heavy task as it is described for example in ISO-5168. Thus, its computational efficiency is not really so high.
- It does not take into account covariance error terms.
- In non-linear data-reduction equations and large input uncertainties, the omitted higher order derivatives (terms) may become significant
- It does not offer any statement about confidence intervals.

The most popular computer based method for sensitivity and uncertainty estimation, which also involves the "direct" manipulation of the main data reduction program, although only for the production of a separate partial derivative code, is this of *Automatic Differentiation*. It is based on the basic rules of differentiation (the partial derivative with respect to x, of a constant is zero, of x is one etc.) that they consequently applied and thus are referred to as *chain rules*. So, the computer has to apply these rules with the proper sequence. Meyn, L.A., 1998 has presented a practical example of the method. Although the method calculates precisely the partial derivatives, covariance error terms cannot be accounted for.

4.2.2. Monte Carlo uncertainty estimation method.

As it was said in the previous section in the analysis of the sensitivity coefficients, the relation between the uncertainty in the input measurands and in the results is not obvious a priori. There is a lot of discussion during the last few years about the implications of correlated inputs' uncertainties and their effects in measurement results. The reported results of the respective studies that have been carried out on this subject are usually contradictory and it seems that there is still some confusion on the matter. Mainly, the interest has been limited and focused on the uncertainty propagation in cases of comparative tests (or measurements) where the result is the difference or the ratio of two elemental measurements, Coleman, H.W., et.al., 1995. However, a systematic study and complete treatment of the subject has not been presented yet. Questions such as,

particularly when uncertainties are reduced, or are increased or are possibly cancelled in combined measurements, or when it is viable to use the root sum square method (RSS method) or simply the addition of partial (elementary) uncertainty terms (ADD method) have not received yet clear answers. More generally the question how in particular elementary uncertainties interact in combined measurements, still remains vague and unanswered.

The author has carried out such a systematic study with the aid of Monte Carlo simulations, the results of which cannot only give definite answers to the above questions but also can reveal the inherent mechanisms of measurement and uncertainty propagation. Additionally, it can offer aid in many other important theoretical and practical aspects of measurement and uncertainty in general. The most important thing revealed by these studies was the unlocking and understanding not only of the error propagation mechanism in complex measurements but also of other practical and theoretical matters.

The results of these simulation studies are presented in this chapter. Monte Carlo simulations have been carried out in two steps. A systematic series of simple test cases, which actually were the most revealing, will be presented. The results of these cases will help then in understanding of the most complex results of natural gas energy flow uncertainty estimation.

4.2.2.1. Results in simple test cases and comparisons with analytical estimation methods - Discussion.

For the analytical evaluation of the simulation results, it would be useful to consider the law of propagation of uncertainty as expressed with equations 4.50 - 4.52 developed in section 4.2.1.1. In general, the following characteristics of error propagation mechanism, as those were identified, have to be kept in mind:

- There is not any guarantee that carrying out elementary measurements with low uncertainties will give a low uncertainty in the result or that high inputs' uncertainties will give high uncertainty in the output result.
- From the law of uncertainty propagation as presented in section 4.2.1.1 they could be identified some of those factors that obviously the uncertainty propagation in the result depends on:
 1. The magnitude of input uncertainties
 2. The sensitivity or better the influence coefficients. The preference of the author on the influence coefficients instead of the sensitivity coefficients is justified by the

fact that they involve also the magnitude of the measurands itself which is really a separate factor affecting the propagation of uncertainty in the result as it will be shown.

3. The covariance or correlated uncertainties in the inputs.
4. Last but not least the measurement model's structure (data reduction equation) which is another important factor. Facts such as a linear or non-linear measurement model, and the *sequence* and *type* of the mathematical operations involved (addition, multiplication, etc.) all are playing some role in the uncertainty propagation.

Although these factors are examined and referred to here separately, actually *they are somehow all interrelated*. These interrelationships along with answers to questions made in the beginning of this chapter are those matters that have been revealed with the next simulated simple test cases.

Test Case 1 – Simulation results when the measurand is obtained by the sum or difference of two elementary measurements ($y = x_1 + x_2$ or $y = x_1 - x_2$).

Assuming that we have a case where the measurement result y is coming from the addition of two elementary measurements x_1 and x_2 , so $y = x_1 + x_2$. For example, assuming a case that is used, a model based measurement of stagnation pressure P_{stag} from separate elementary measurements of static and dynamic pressure of a flowing gas in a pipe: $P_{stag} = P_s + \frac{\rho u^2}{2}$.

It can be seen from this (measurement) equation that a measurement of dynamic pressure additionally needs two separate measurements of density and velocity and each must be regarded as an elementary measurement process in the sense developed in chapter 4. Additionally, if for example the density is an equation of state model based measurement that utilises the same static pressure measurement P_s , then the static pressure measurement P_s will be present in both measurements (static and dynamic pressures) and the two terms will share common or correlated uncertainty sources. *These correlated uncertainty sources will range from the calibrations of the respective common pressure sensor up to the various effects on its performance taking place during the actual measurement process.*

Two normal random variables representing the elementary measurement processes and having the same mean, $x_1 = x_2 = 30$, and uncertainties, $u(x_1) = 2\sigma(x_1) = 0.4$ and $u(x_2) = 2\sigma(x_2) = 0.9$ have been generated according to the procedures described in section 5.1.1.1. So, the relative uncertainties of the input simulated measurements are 1.33...% and

3% respectively. The measurement result y , is given by the measurement equation $y=x_1+x_2$. Three different cases have been examined which are

- a) the two elementary measurements are independent thus not sharing common uncertainty sources ($\rho=0$),
- b) the two measurements are subject to positively partial correlated uncertainties and
- c) they are subject to negatively partial correlated uncertainties.

For b) and c) we examine the cases of $\rho=0.7$ and $\rho=-0.7$ respectively. In terms of the case a) it is generally known and **Taylor, J.R., 1997** has also shown analytically, that the probability distribution of the result y of the summation of two normal independent input variates will be also normal and will be centred at $y = E(y_i) = x_1 + x_2$. This is the expectation value of the result y as well as its best estimate that is given by the elementary measurements' best estimates x_1 and x_2 . It actually represents the *true value* of the measurand or better the *conventional true value*.

It is useful to examine analytically the derivation of uncertainty using the various forms of uncertainty propagation law mentioned above. If equation 4.50 is employed, it can be easily shown that the uncertainty of the result y will be given by the root sum squares (RSS) of the elementary uncertainties. Since the result y is estimated by $y = x_1 + x_2$ the sensitivity coefficients calculated in the means will be, $\left(\frac{\partial y}{\partial x_1}\right)_{x_2} = \left(\frac{\partial y}{\partial x_2}\right)_{x_1} = 1$.

So, in general, when the measurement result is obtained only from additions (or subtractions) of elementary measurements, all the respective sensitivity coefficients will be equal to 1. The point is again that x_1 or x_2 express elementary measurement process and may have any functional form that may be based on other measurements.

Since the correlation coefficient $\rho=0$, then the covariance term in 4.50 will be also zero and the RSS of elemental uncertainties finally remains. If equation 4.51 is employed, what remains is similarly that the relative uncertainty of the measurement result y is given as the RSS of the relative input uncertainties in respect to the best estimated **magnitude of the measurement result y** (estimated at the means of x_1, x_2). Similarly, from 4.52, the relative uncertainty of the result y is given by the RSS of the products of relative uncertainties of the inputs with the respective ratios of the input magnitudes to the best estimated magnitude of the result.

From equation 4.50, we obtain $u(y) = 2\sigma(y) = \sqrt{0.4^2 + 0.9^2} \approx 0.9849$ or 1.64%. The reported Monte Carlo simulation results obtained using 15 single samples of 6000 sample points and based on the methodology already described are:

```
ans =
  AVERAGES      2*STD.DEVNS      Uncertainties %, 95.45%cf
INPUTSx1x2 =
  30.0000000000000  0.4000000000000  1.3333333333333
  30.0000000000008  0.9000000000000  2.9999999999999
OUTPUT SUMMATION =
  60.0000000000023  0.98456218359665  1.64093697266108
```

The Monte Carlo uncertainty obtained differed by -0.03% from the analytically estimated value. From each single 6000 points sample the Monte Carlo estimated uncertainty was not differed by more than 0.8% from the analytical result in almost 95% of the cases. Even for much less sample points (around 1000) the results were still satisfactory, fully justifying the application of Monte Carlo method as it has been applied.

The results have been graphically presented with the aid of the various equations developed in section 4.1.3. After the simulated result y has been obtained from the simulated inputs x_1 and x_2 , the joint probability distributions of the result with each of the inputs $p(y, x_1)$ and $p(y, x_2)$ are obtained. In this way, all the necessary information is available in just one 3-D graph and the effects of input correlated uncertainties can be better analysed.

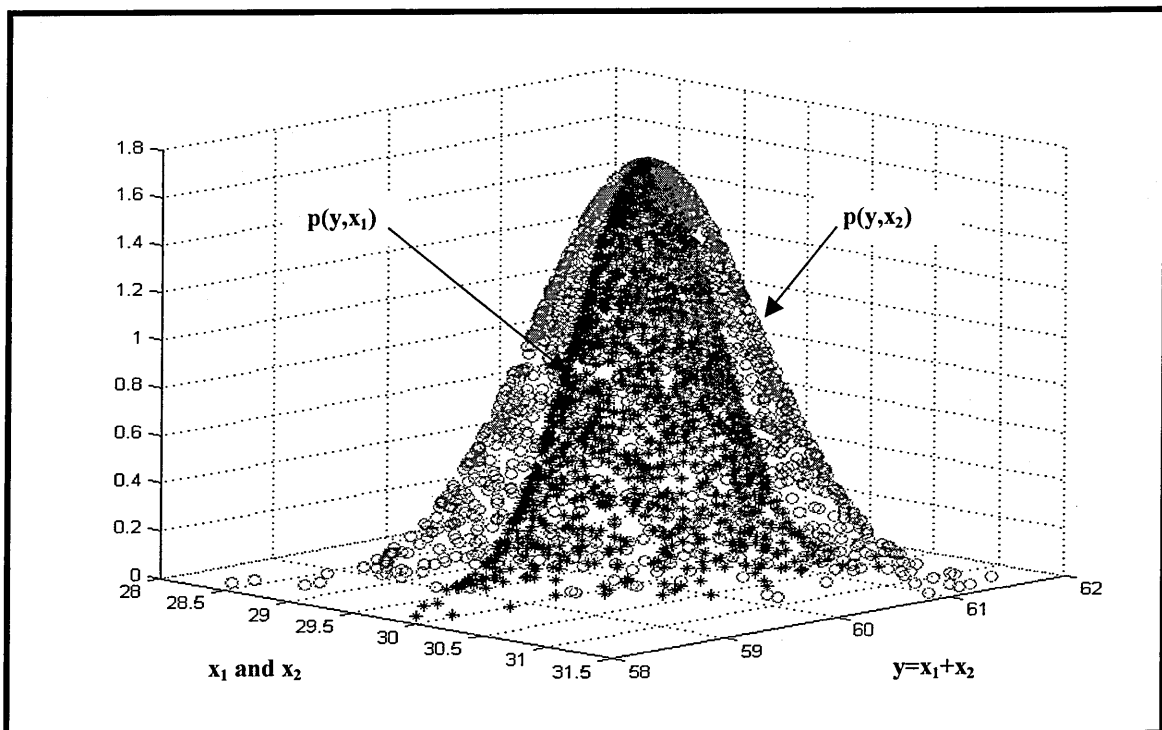


Figure 4.14 Joint distributions $p(y, x_1)$ and $p(y, x_2)$ of result $y = x_1 + x_2$ with each input x_1, x_2 when $\rho_{x_1 x_2} = 0$.

So, in **figure 4.14** have been plotted the joint distributions $p(y,x_1)$ and $p(y,x_2)$ for the case where the two input measurement processes x_1, x_2 do not share any common uncertainty sources ($\rho_{x_1x_2}=0$). In **figure 4.15** are plotted the same joint distributions as viewed from the perspective of the input distributions.

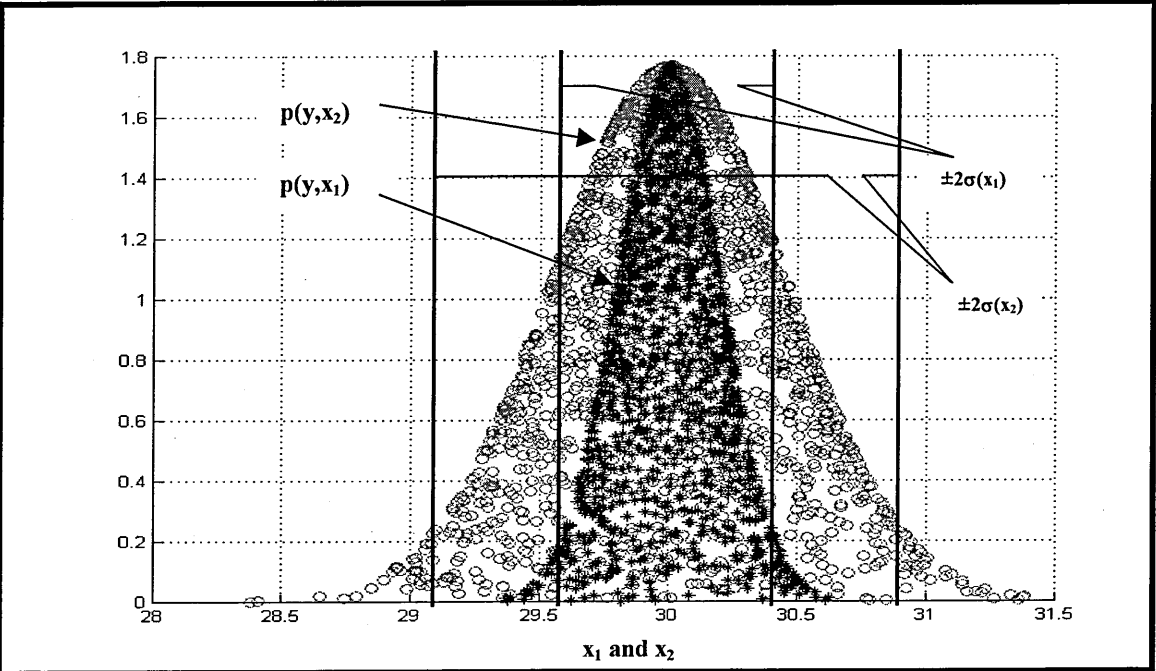


Figure 4.15 Joint distributions $p(y,x_1)$ and $p(y,x_2)$ of result $y=x_1+x_2$ with each input x_1, x_2 when $\rho_{x_1x_2}=0$ (Viewed from the perspective of input distributions x_1, x_2).

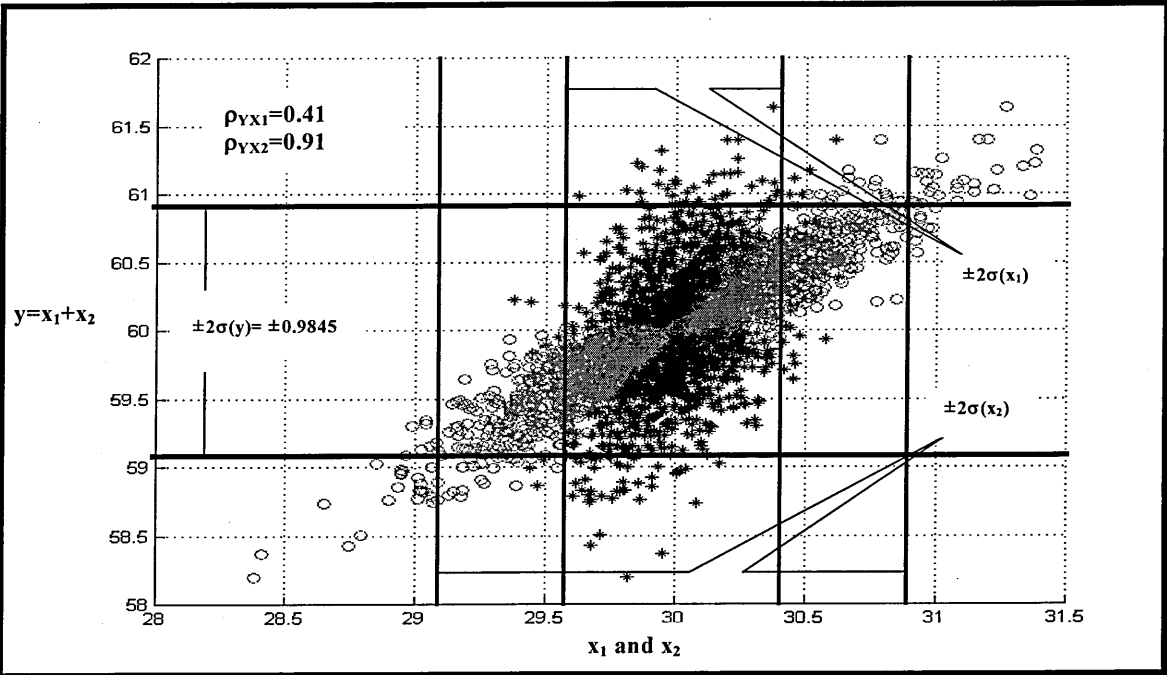


Figure 4.16 Uncertainty of the result $y=x_1+x_2$ when there are not common input uncertainty sources ($\rho_{x_1x_2}=0$).

Another interesting view is that of the plane x_1 , x_2 and y as shown in **figure 4.16** where the inputs' and result's uncertainties are all shown.

Another fact shown in figure 4.16 is the positive correlation, in this particular case, of the result y with both inputs, which is higher with the input x_2 ($\rho_{y x_2}=0.91 > \rho_{y x_1}=0.41$), having the largest uncertainty.

In **figure 4.17** is shown another view of the two joint distributions from the perspective of the result y along with its uncertainty. It can be easily realised the completeness of the information offered by these joint distributions as they have been estimated and plotted by the author.

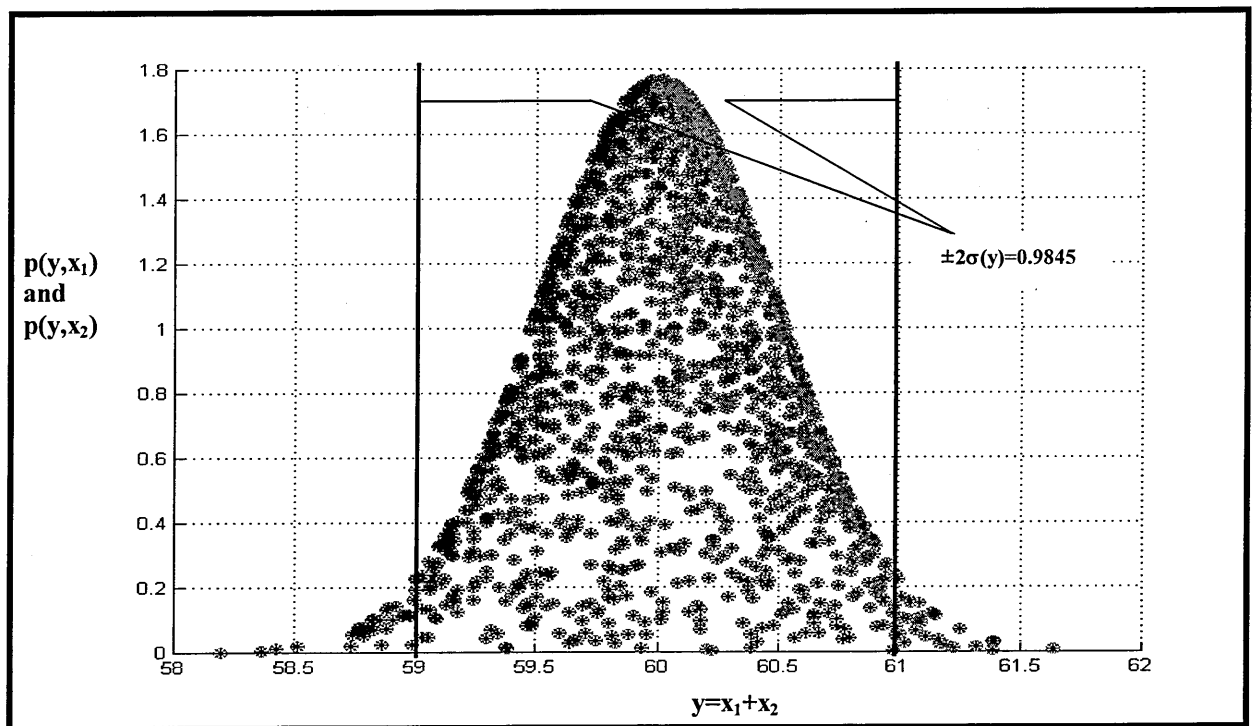


Figure 4.17 Joint distributions $p(y, x_1)$ and $p(y, x_2)$ and uncertainty of the result $y = x_1 + x_2$ when $\rho_{x_1 x_2} = 0$.

In order to study the effects of partial inputs' uncertainty correlation, two other measurements $x_1 = 6 \pm 0.3$ and $x_2 = 5 \pm 0.1$, with relative uncertainties 5% and 2% respectively, have been simulated. The results shown in **figures 4.18** and **4.19** have been obtained for correlation coefficients $\rho_{x_1 x_2} = 0.7$ and $\rho_{x_1 x_2} = -0.7$.

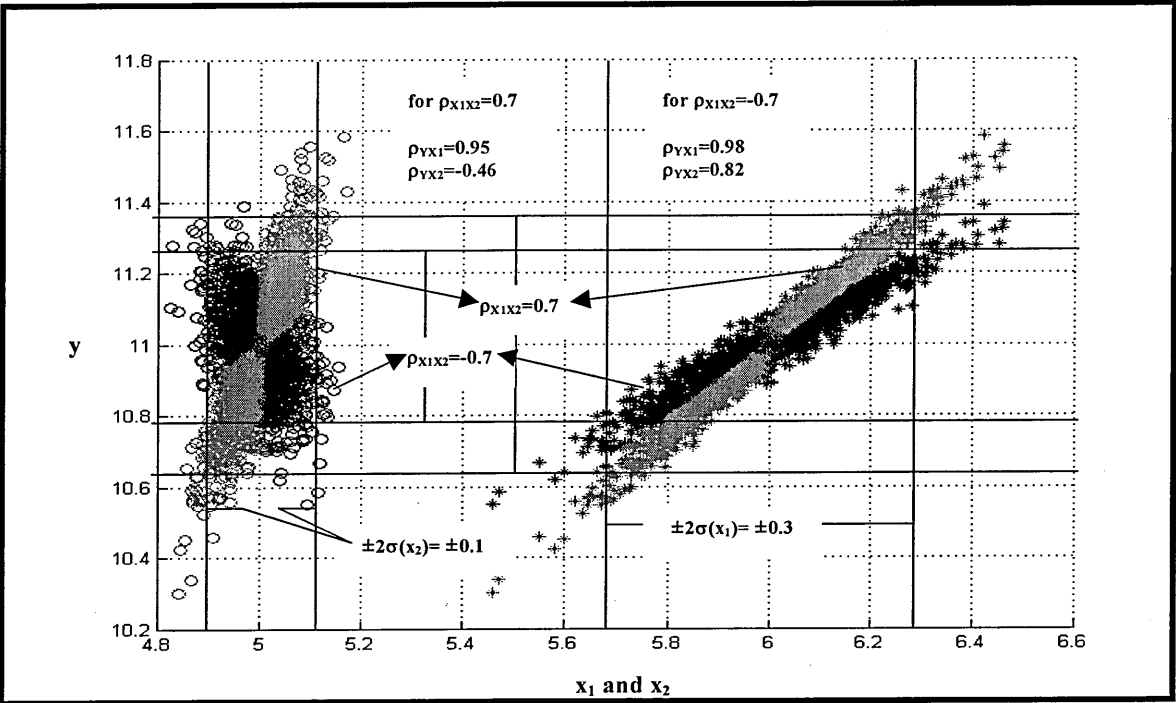


Figure 4.18 Uncertainties of the result $y=x_1+x_2$ when there are partial positive ($\rho_{x_1x_2}=0.7$) and negative ($\rho_{x_1x_2}=-0.7$) correlated input uncertainties.

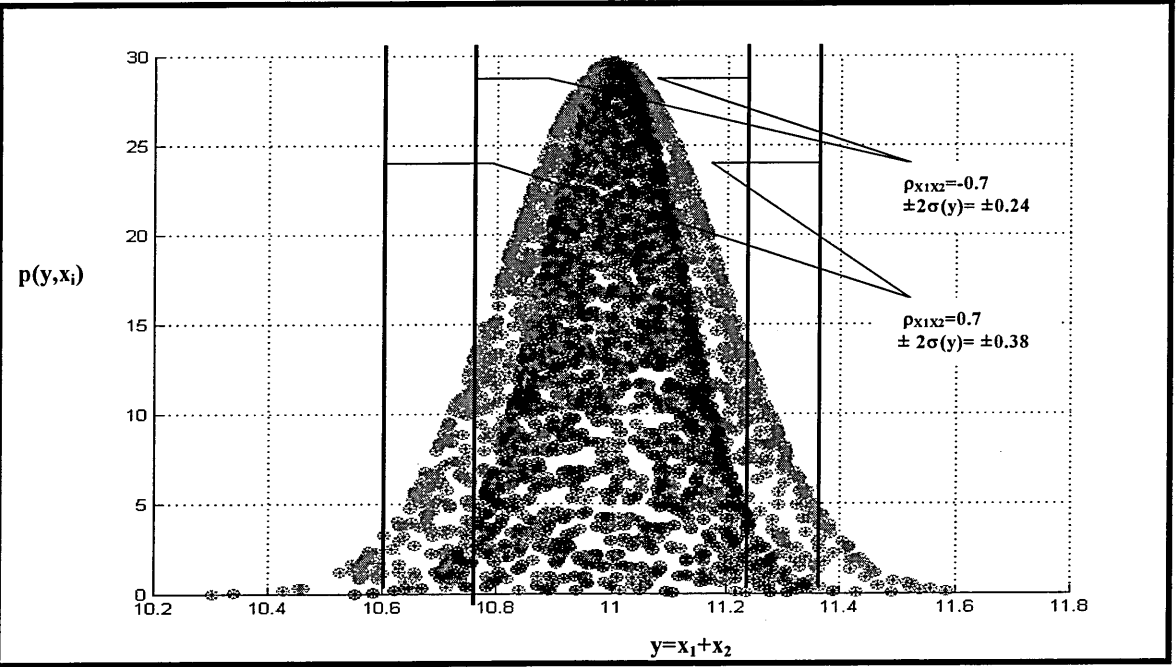


Figure 4.19 Joint distributions $p(y, x_i)$ and uncertainties of the result $y=x_1+x_2$ when there are partial positive ($\rho_{x_1x_2}=0.7$) and negative ($\rho_{x_1x_2}=-0.7$) correlated input uncertainties.

The reported simulation results for the last cases are shown in the following printouts:

ans =

AVERAGES 2*STD.DEVNS Uncertainties %, 95.45%cf

INPUTSx1x2 =

6.000000000000000 0.300000000000000 5.000000000000000

5.000000000000001 0.10002239715478 2.00044794309560

OUTPUT SUMMATION =

11.000000000000001 0.37686029194342 3.42600265403112 (for $\rho_{X1X2} = 0.7$)

11.000000000000001 0.24086244991386 2.18965863558052 (for $\rho_{X1X2} = -0.7$)

An analytical derivation in these two cases can easily validate again the satisfactory accuracy of the computed results.

It can be seen that negative correlated input uncertainties have reduced the uncertainty of the measured result $y=x_1+x_2$. However, exactly the opposite results have been obtained for $y=x_1-x_2$. This will be shown in the next simulation case. In the meantime, the following general conclusions have been drawn and they can be regarded as the uncertainty propagation rules in such cases:

1. The result y is, in any case, a linear function in terms of the inputs x_1, x_2 . Since the first order partial derivatives are all equal to 1, the second order partial derivatives

$\frac{\partial^2 y}{\partial x_1^2} = \frac{\partial^2 y}{\partial x_2^2} = 0$ will all be zero, and so the uncertainty propagation law as expressed by

equations 4.50 to 4.52, is no longer an approximation law. It is valid in any case and free from any assumption (e.g. small uncertainties). This further means that such a measurement system where the measured output is obtained by the addition or subtraction of elementary measurement processes is **always additive** and/or *homogeneous*. In other words it is a *linear system* and so it cannot suffer from (or produce) inherent non-linearities. This is quite important since in such a measurement process the principle of superposition always holds and it does not matter, for example, if elementary measurements are processed before averaging or averaged before processing. This particular characteristic could be very useful in practice if it is considered in the decentralization of the elementary measurements' electronic processing wherever such measurement cases appear. Such a case is for example the one presented in the beginning of this section for the stagnation pressure measurement, P_{stag} .

As it was said, the sensitivity coefficients are $\left(\frac{\partial y}{\partial x_1}\right)_{x_2} = \left(\frac{\partial y}{\partial x_2}\right)_{x_1} = \pm 1$ in all cases where

the measurement equation consists of only summations and subtractions of elementary

measurements. Because of this and from equation 4.50, it could be said that the uncertainty of the measurement result y depends, in all these cases, solely on the uncertainties of the inputs and on their combined uncertainty (covariance). In other words, *it seems to be independent on the magnitudes of the inputs*. This is somehow misleading, since to the contrary, **the magnitude of the measured output y** as will be next proved, may be a significant factor. The uncertainty $u(y)$ of the output as an absolute value does not actually give any feeling of its significance if it is not compared with the magnitude of the output result y . In that respect, the relative uncertainty $\frac{u(y)}{y}$ is the most useful quality index of a

measurement system. *The hidden and most important factor here is the magnitude of the output compared (in relation) with the inputs' magnitudes*. This can be realised from equation 4.52 and the next test case gives a more complete picture of the situation. The simulated result y has been obtained from $y=x_1-x_2$ for x_1, x_2 simulated inputs as before, being now perfectly negative correlated ($\rho_{x_1x_2}=-1$).

```
ans =
  AVERAGES      2*STD.DEVIATIONS  Uncertainties %,95%cf
INPUTSx1x2 =
  6.00000000000000  0.300000000000000  5.00000000000000
  5.00000000000000  0.100000000000000  2.00000000000000
OUTPUT SUBTRACTION =
  1.00000000000000  0.400000000000000  40.0000000000000 (For  $\rho_{x_1x_2}=-1$ )
```

The uncertainty of the result y obtained is 0.4 for input uncertainties 0.3 and 0.1 respectively which sounds as a reasonable value. However, the relative uncertainty, which always gives a better feeling of the uncertainty magnitude, it is now **40%!!** This is a peculiar situation indeed and it is due to the fact that the magnitude of the result y is 6 and 5 times smaller than the input quantities x_1 and x_2 respectively. Because of this, the assumed small input uncertainties acquire a comparable magnitude with the output, which is significant. Thus, it is justified the preference of the author to the influence coefficients and the form of equation 4.52 which includes the relative magnitudes of inputs with the output in the influence coefficient terms. Additionally, it has been shown in this last case that **negative correlation does not reduce uncertainty in all cases**.

Another interesting case tested is presented next in **figures 4.20 and 4.21** for input measurements $x_1=6\pm0.2$ and $x_2=5\pm0.2$ (equal inputs' uncertainties) and $y=x_1+x_2$ for perfect negative correlation ($\rho_{x_1x_2}=-1$) of the inputs' uncertainties.

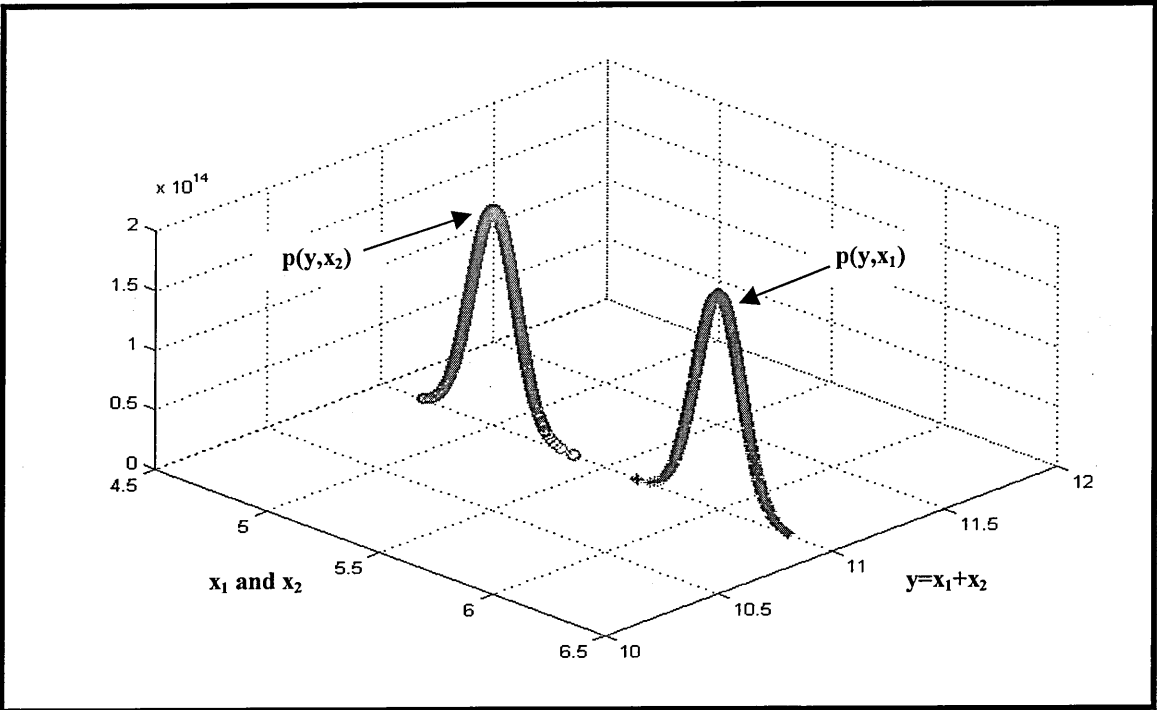


Figure 4.20 Joint distributions $p(y, x_i)$ and uncertainty of the result $y = x_1 + x_2$ for perfect negative correlation of input uncertainties ($\rho_{x_1 x_2} = -1$).

As it could be seen, the inputs' uncertainties are cancelled. If the same distributions are seen from the perspective of the result y , then two coinciding delta functions are obtained, as it is shown in the next figure 4.21.

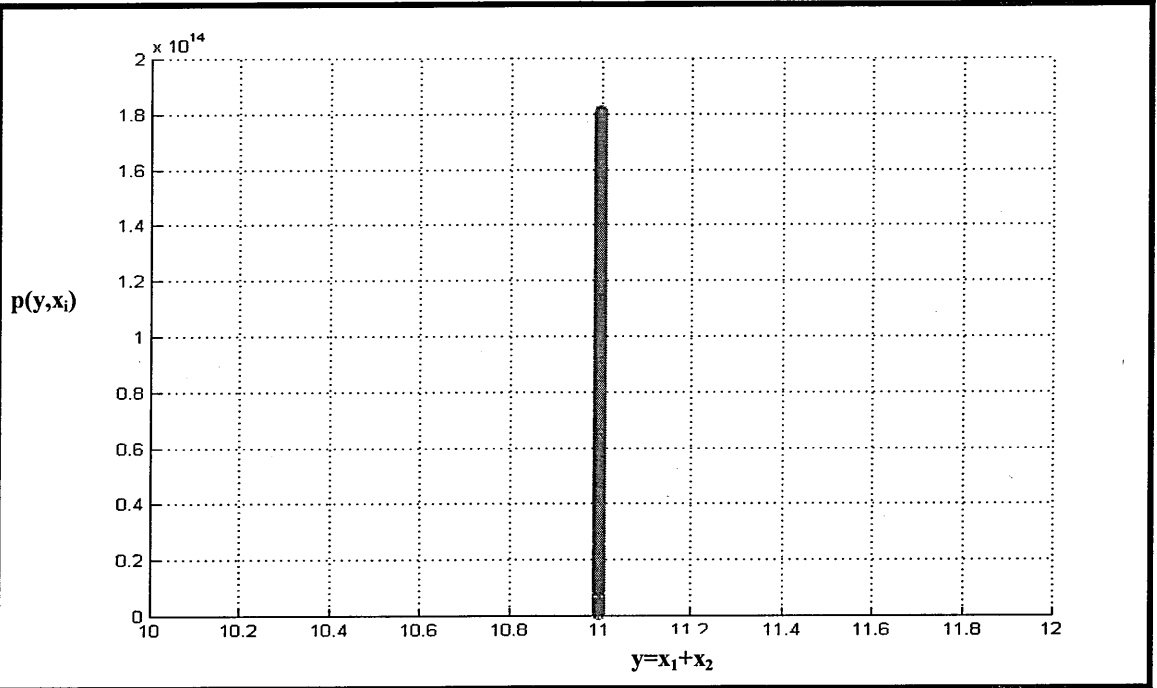


Figure 4.21 Joint distributions $p(y, x_i)$ of the result $y = x_1 + x_2$ for perfect negative correlation of input uncertainties ($\rho_{x_1 x_2} = -1$) (Coinciding Delta functions).

There are now clear answers to the questions set in the beginning of section 4.2.2 in terms of the inputs' correlation effects to output's uncertainty and the proper application of the various existing uncertainty estimation methodologies in each case.

Summarising, when the result y is obtained from the summation or subtraction of input elementary measurements x_1 and x_2 , its relative uncertainty, in all cases, is given by:

$$\frac{u(y)}{y} = \frac{\sigma(y)}{y} = \left\{ \begin{array}{ll} \frac{1}{y} \sqrt{\sigma(x_1)^2 + \sigma(x_2)^2 + 2\rho_{x_1x_2} \sigma(x_1)\sigma(x_2)} \rightarrow \text{for } -1 < \rho_{x_1x_2} < 1 \\ \frac{1}{y} \sqrt{\sigma(x_1)^2 + \sigma(x_2)^2} \rightarrow \text{for } \rho_{x_1x_2} = 0 & \text{(RSS)} \\ \frac{1}{y} |\sigma(x_1) \pm \sigma(x_2)| \text{ (for } y = x_1 + x_2) \rightarrow \text{for } \rho_{x_1x_2} = \pm 1 & \text{(ADD)} \\ \frac{1}{y} |\sigma(x_1) \mp \sigma(x_2)| \text{ (for } y = x_1 - x_2) \rightarrow \text{for } \rho_{x_1x_2} = \pm 1 & \text{(ADD)} \end{array} \right.$$

These, of course, can be extended to more than two elementary measurements and can be regarded as the general laws of uncertainty propagation in these cases and free from any further assumptions (e.g. small input uncertainties). Next, are presented the results of uncertainty propagation simulations of the more interesting test cases of multiplication and division of input measurements, which are more associated with the results of energy uncertainty estimation in gas metering systems.

Test Case 2 – Simulation results when the measurand is obtained by the product or the ratio of two elementary measurements ($y = x_1 x_2$ or $y = x_1/x_2$).

These cases are actually the most common met in measurement systems. Considering for example the volume flow rate measurement equation of orifice meters in which a long sequence of products and ratios among various elementary measurements is taking place. Then the energy flow rate is given again by the multiplication of the flow rate and energy content measurements. All these elementary measurement processes share common (maximum positive correlated) uncertainty sources because in practice they utilise measurements (and model based measurements) from common instruments.

It was regarded useful to examine again analytically, using the uncertainty propagation laws as expressed by equations 4.50 to 4.52, the estimation of results' (combined) uncertainty in the cases of multiplication and ratios of elementary measurements. It must be reminded that these uncertainty propagation laws have been derived from a first order

Taylor series approximation, and as it was proved from the previous Monte Carlo simulations and verified analytically, they give the "true" combined measurement uncertainty in all cases of addition or subtraction of elementary measurements.

Keeping here the assumption of small uncertainties, and when the result is given by $y=x_1x_2$, the sensitivity coefficients obtained analytically at the mean values x_1 and x_2 will

$$\text{be approximately: } \left(\frac{\partial y}{\partial x_1} \right)_{x_2} = x_2 = \frac{y}{x_1} \text{ and } \left(\frac{\partial y}{\partial x_2} \right)_{x_1} = x_1 = \frac{y}{x_2}.$$

These are constants (see also **Taylor, J.R., 1997**) and it is generally believed that are reasonable valid as long as the input uncertainties are really so "small" as to ignore changes and second order effects. From these sensitivity coefficients, equations 4.50 to 4.52 can all take the next general forms:

$$\frac{u(y)}{y} = \frac{\sigma(y)}{y} = \sqrt{\left(\frac{\sigma(x_1)}{x_1} \right)^2 + \left(\frac{\sigma(x_2)}{x_2} \right)^2 + 2 \frac{\sigma(x_1x_2)}{x_1x_2}}$$

or the most useful

4.53

$$\frac{u(y)}{y} = \frac{\sigma(y)}{y} = \sqrt{\left(\frac{\sigma(x_1)}{x_1} \right)^2 + \left(\frac{\sigma(x_2)}{x_2} \right)^2 + 2 \frac{\rho_{x_1x_2} \sigma(x_1)\sigma(x_2)}{x_1x_2}}$$

From 4.52 it can be seen that in this case it is the influence coefficients (or relative sensitivity coefficients as called otherwise) instead of sensitivity coefficients that become 1, which actually corresponds to the dimensionality of the input measurements (exponents of) x_1 and x_2 .

The latest uncertainty propagation equations, as obtained here, are those that can be practically applied in all such cases as implied in the latest uncertainty estimation international proposals.

The questions here are of course how small they have to be these input uncertainties, where do these second order effects come from and subsequently if and when we can ignore them. The next simulation results aimed in answering these questions.

Simulating again the input measurements $x_1=6\pm0.3$ and $x_2=5\pm0.1$ and the result $y=x_1x_2$, for the case of uncorrelated inputs' uncertainties ($\rho_{x_1x_2}=0$), the computed results from one of many single samples of 10000 points, were:

```
ans =  
  AVERAGES      2*STD.DEVIATIONS  Uncertainties %, 95%cf  
INPUTSx1x2 =  
  x1: 6.00000000000000 0.30000000000000 5.00000000000000  
  x2: 5.00000000000000 0.10000000000000 2.00000000000000  
OUTPUT PRODUCT =  
  y: 30.00000335838123 1.61483667662676 5.38278831950736
```

The Monte Carlo output uncertainty average result of 10 samples was in more than 95% of the cases within 0.1% from the analytical calculation result using the uncertainty propagation law as expressed by equation 4.53 (or 4.50 to 4.52). It must be noticed as well that there is a small **bias** in the estimated result y , in this case, which is anyway insignificant and has actually random nature. It corresponds to some residual covariance left due to the single sample used and it is averaged down to zero with a rate $n^{1/2}$ where n is the number of samples. The results of the estimated joint distributions $p(y,x_1)$ and $p(y,x_2)$ are shown in figures 4.22 to 4.25.

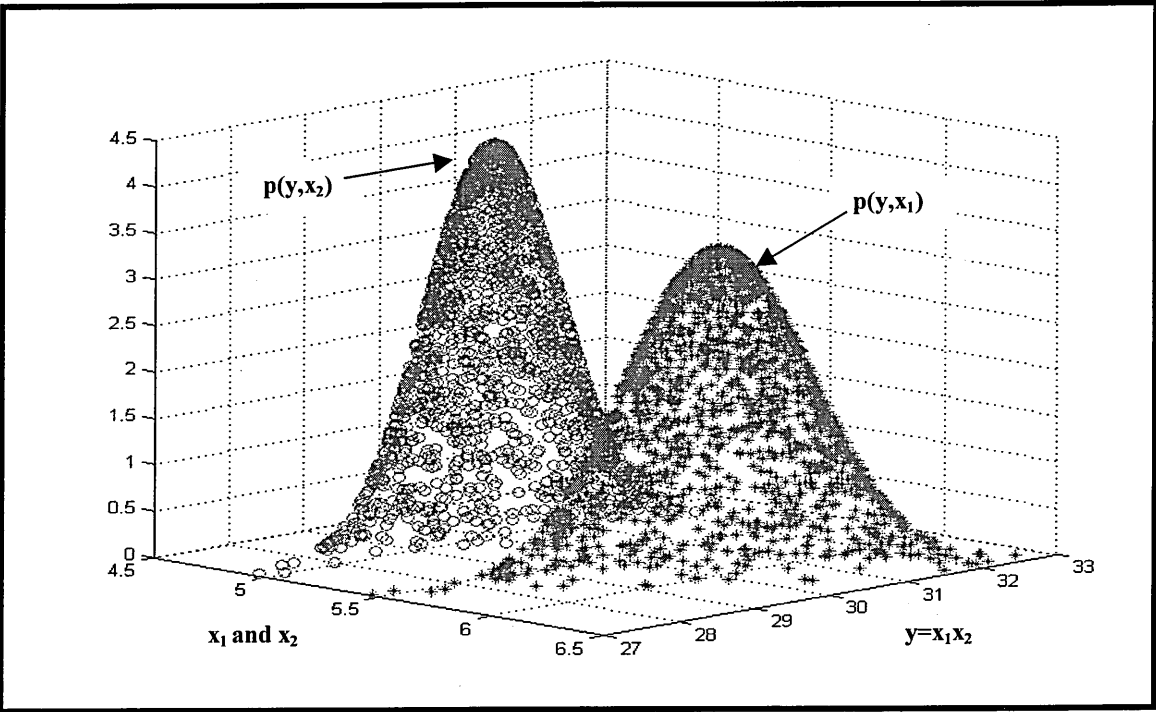


Figure 4.22 Joint distributions $p(y,x_1)$ and $p(y,x_2)$ of the result $y=x_1x_2$ with each input x_1, x_2 when $\rho_{x_1x_2}=0$.

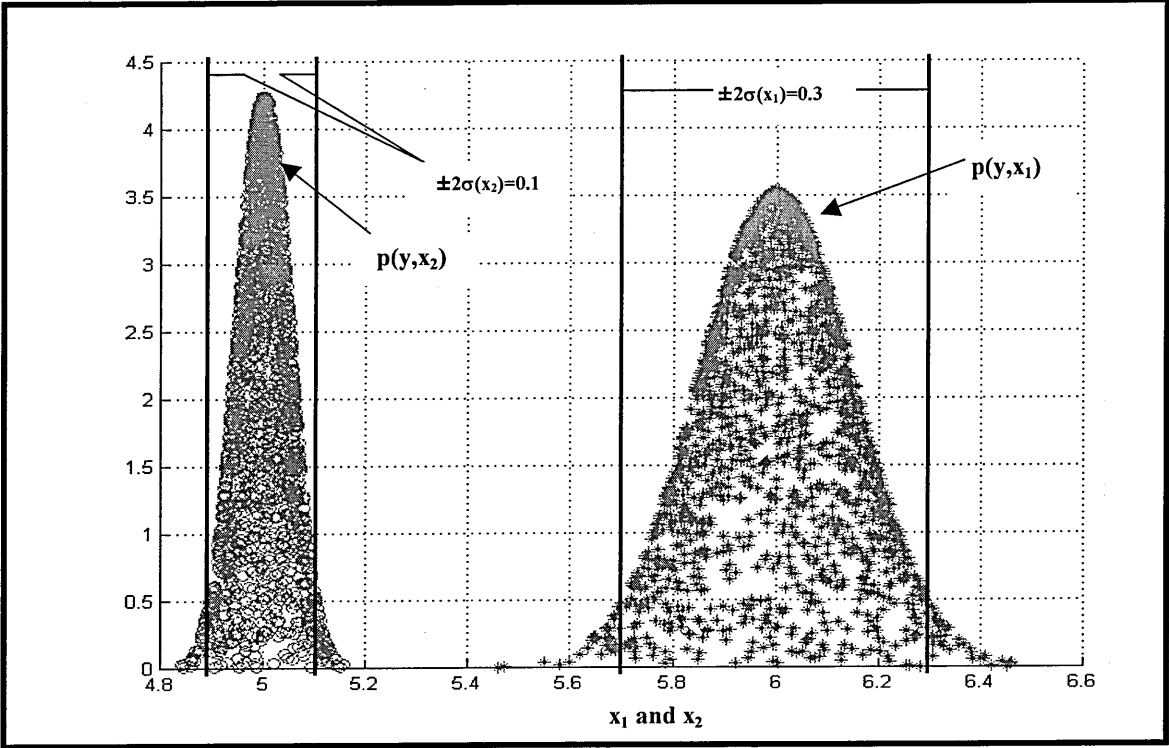


Figure 4.23 Joint distributions $p(y, x_1)$ and $p(y, x_2)$ of the result $y=x_1x_2$ with each input x_1, x_2 when $\rho_{x_1x_2}=0$ (Viewed from the perspective of input distributions x_1, x_2).

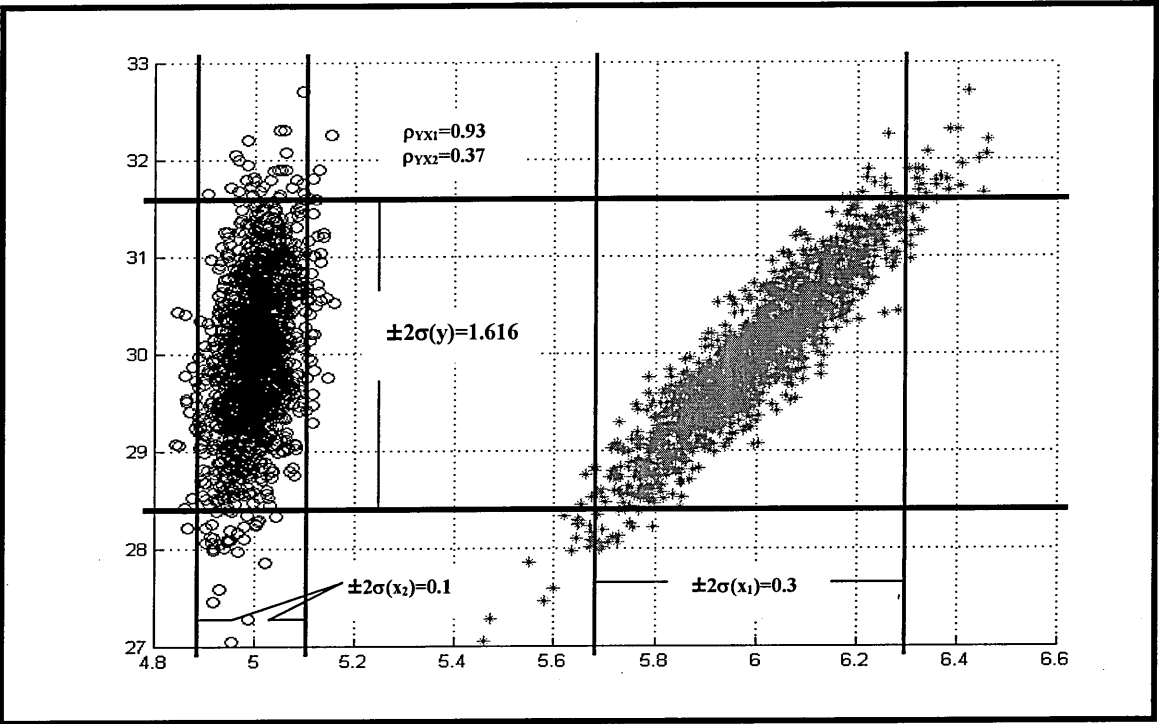


Figure 4.24 Uncertainty of the result $y=x_1x_2$ when there are not common input uncertainty sources ($\rho_{x_1x_2}=0$).

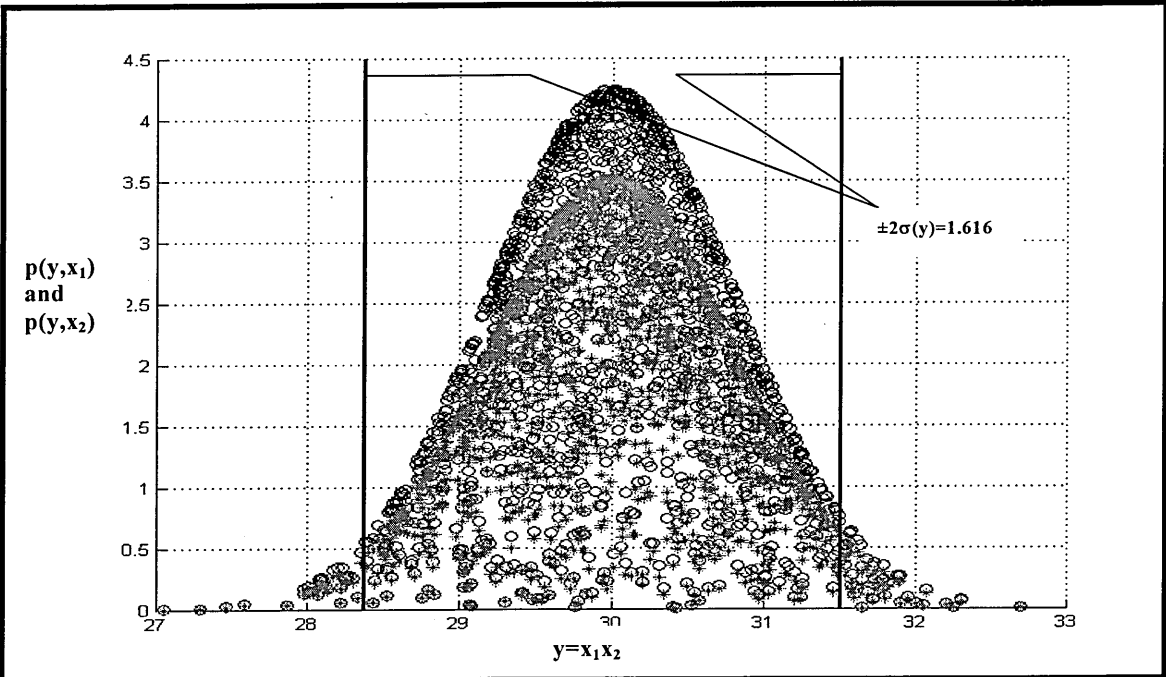


Figure 4.25 Joint distributions $p(y, x_1)$ and $p(y, x_2)$ and uncertainty of the result $y = x_1 x_2$ when $\rho_{x_1 x_2} = 0$.

A general discussion on all the results will follow after considering the cases of correlated input uncertainties, and the respective results for ratios of input measurement processes, which are the most interesting. Using the same measurement inputs as above and for $\rho_{x_1 x_2} = 1$, 10 simulated samples of 10000 points each, they have given results for $y = x_1 x_2$ that are shown in the next reported table:

Input x_1	2std(x_1)	Input x_2	2std(x_2)
6.000000000000000	0.300000000000000	4.999999999999998	0.100000000000000
6.000000000000001	0.300000000000000	4.999999999999999	0.100000000000000
5.999999999999999	0.300000000000000	4.999999999999999	0.100000000000000
6.000000000000003	0.300000000000000	5.000000000000001	0.100000000000000
5.999999999999999	0.300000000000000	5.000000000000001	0.100000000000000
5.999999999999998	0.300000000000000	5.000000000000001	0.100000000000000
6.000000000000002	0.300000000000000	5.000000000000001	0.100000000000000
5.999999999999998	0.300000000000000	4.999999999999997	0.100000000000000
6.000000000000002	0.300000000000000	5.000000000000000	0.100000000000000
6.000000000000002	0.300000000000000	5.000000000000003	0.100000000000000
Output y	2std(y)	%Error for each single sample from Analytical Uncer.	
30.029997000000001	2.09926244893371	0.03512147934737	
30.029997000000005	2.10029182669058	-0.01389650907533	
30.029997000000002	2.09977699444010	0.01061931237605	
30.029997000000003	2.09982874999672	0.00815476206098	
30.029996999999995	2.09981954830847	0.00859293769194	
30.029996999999993	2.09945932879322	0.02574624794194	
30.029997000000013	2.10181404118616	-0.08638291362676	
30.029997000000000	2.10038846883580	-0.01849851599035	
30.029997000000004	2.10015571051177	-0.00741478627484	
30.029996999999997	2.09941145950807	0.02802573771118	

Uncertainty Propagation Law (Analytical) =2.10000000000000
 Monte Carlo Uncertainty=2.10002085772046 (10 samples)
 Error = -9.932247837730029e-004
 Relative Uncertainty of result y=7%

Table 4.1 Monte Carlo computed results for $y=x_1x_2$ and $\rho_{x_1x_2}=1$.

From these results, the following conclusions can be drawn:

- Perfect positive correlation increased the uncertainty of the result (from 5.4% for the uncorrelated case to 7% for perfect positive correlation). The uncertainty $u(y)=2.1$, is the maximum uncertainty in this case. However, there is another more important effect is in this case.
- The inputs' positive correlated uncertainties have resulted in an additional **positive systematic error** in the estimated result y . This is not of random nature as it was the case of uncorrelated input multiplicative measurements. It is an **additional** systematic error that can not be averaged down to zero and actually represents a **mean** residual term (non-linear terms) omitted in equation 4.38, in the theoretical derivations in section 4.2.1 that led to the formulation of the general uncertainty propagation law. This additional error corresponds to the inputs covariance term (it is actually the term $\sigma(x_1, x_2)$).

This observed systematic error becomes smaller for partial input correlated uncertainties while it changes sign for negative correlations. This is an important fact, lying in the center of controversy about the inputs' correlated uncertainty effects. Another important fact it is also the ability given by the previously used joint distributions $p(y, x_1)$ and $p(y, x_2)$ to visualise the nature and the mechanism of this additional systematic error.

In order to realise what actually happens in the vicinity of mean value and how the generated non-linearity **due** to the input covariances leads to this additional systematic error in the estimated mean, instead of simulating measurement inputs with "small" uncertainties, two input measurements with much larger uncertainties were simulated. This can be regarded as a zooming in the area around the mean value, which is the area of interest. So, two input measurements $x_1=30\pm 9$ and $x_2=30\pm 4$ have been used for the simulations of measurements $y=x_1x_2$ and $y=x_1/x_2$ or $y=x_2/x_1$ the results of which are presented for various cases of input correlated uncertainties in **figures 4.26 to 4.31**.

In **figures 4.26 to 4.28** are shown with the aid of the joint distributions $p(y, x_1)$ and $p(y, x_2)$ the effects on the result $y=x_1x_2$ of perfect positive and negative correlations in the

measurement inputs uncertainties ($\rho_{x_1x_2}=1$ and $\rho_{x_1x_2}=-1$). By projecting the different perspectives of these joint probabilities, they can be clearly seen these effects and described by a **Skewing** of the distribution (positive or negative), a **Flattening** (high kurtosis) with respect to Gaussian distribution and a **Bending** that is not clearly shown in these figures.

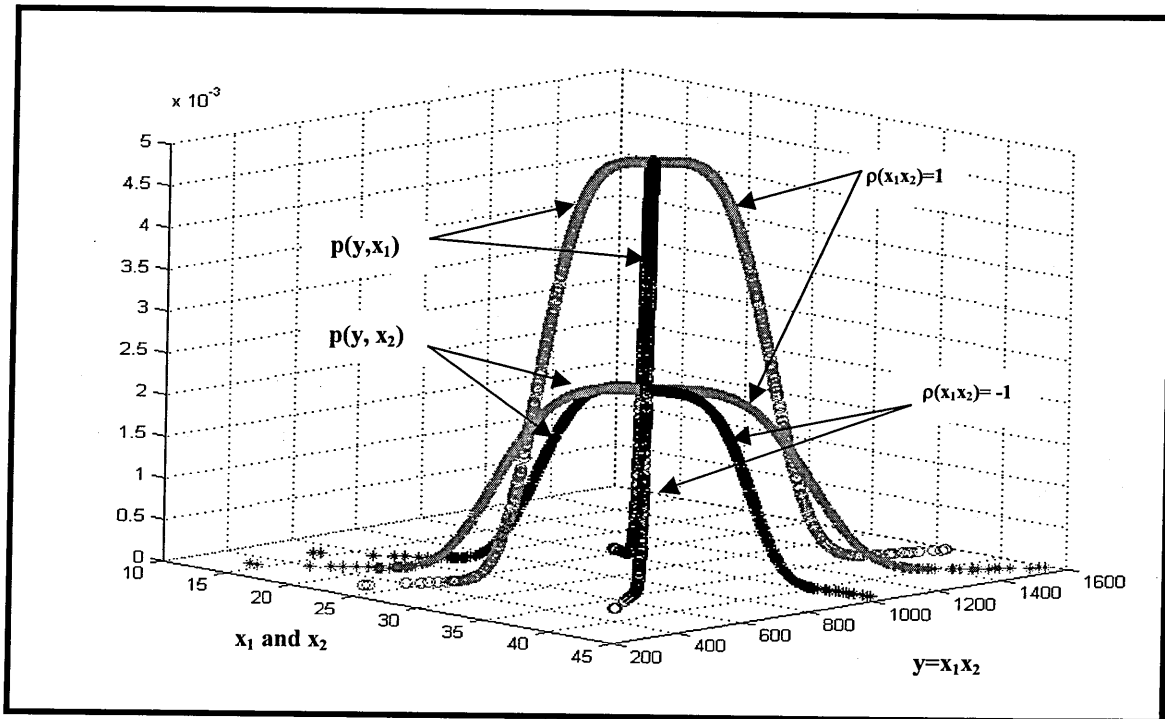


Figure 4.26 Joint distributions $p(y, x_1)$ and $p(y, x_2)$ of the result $y=x_1x_2$ if $\rho_{x_1x_2}=1$ and $\rho_{x_1x_2}=-1$.

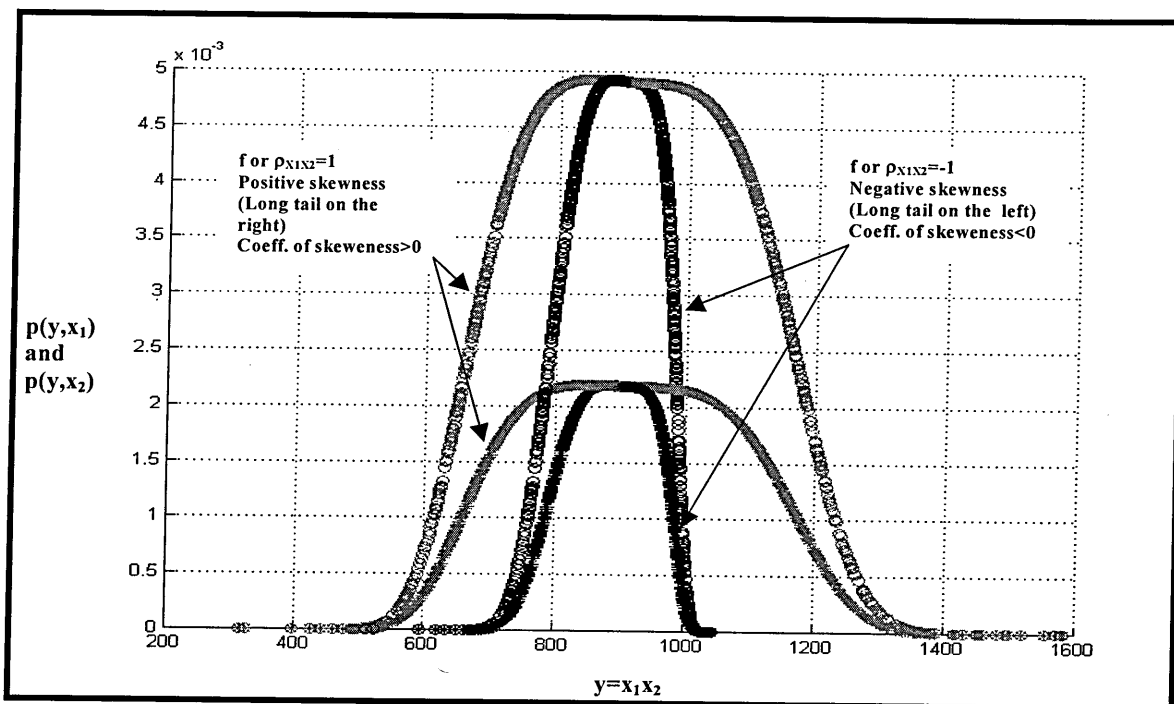


Figure 4.27 The skewed and flattened joint distributions $p(y, x_1)$ and $p(y, x_2)$ of the result $y=x_1x_2$ if $\rho_{x_1x_2}=1$ and $\rho_{x_1x_2}=-1$.

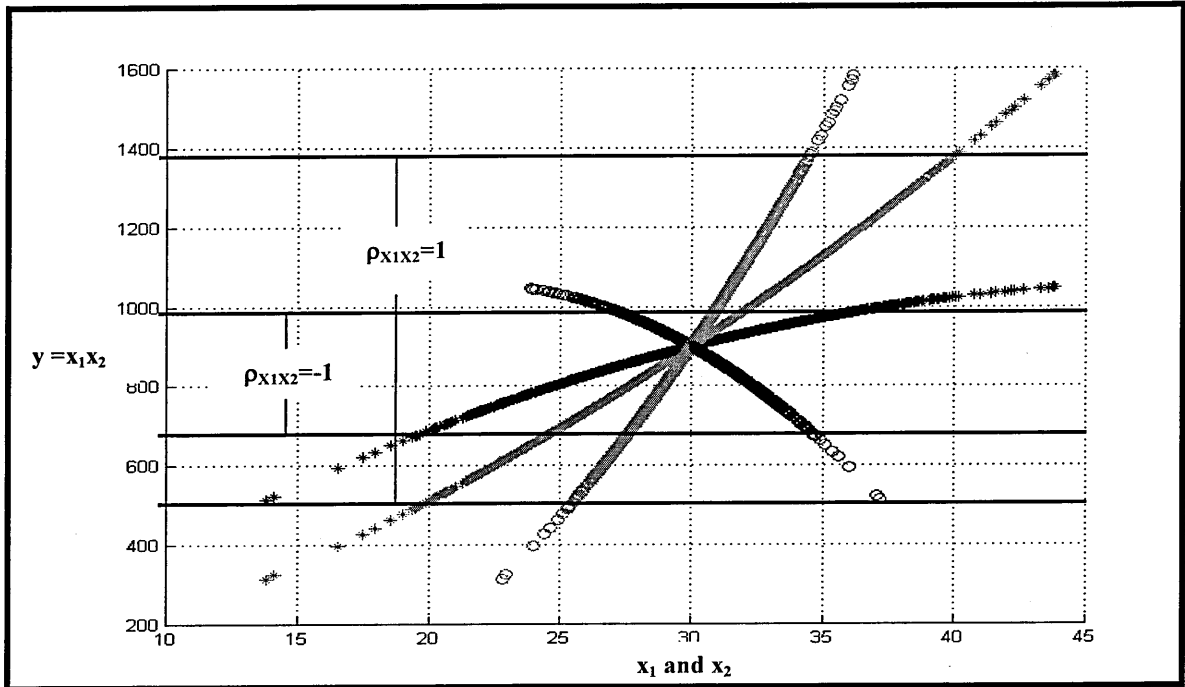


Figure 4.28 Uncertainties of the result $y=x_1x_2$ when there are perfect positive ($\rho_{x_1x_2}=1$) and negative ($\rho_{x_1x_2}=-1$) correlated input uncertainties.

In figures 4.29 to 4.31 are shown results for the respective cases where a measurement result is obtained by the division of the simulated input measurements ($y=x_1/x_2$ and $y=x_2/x_1$).

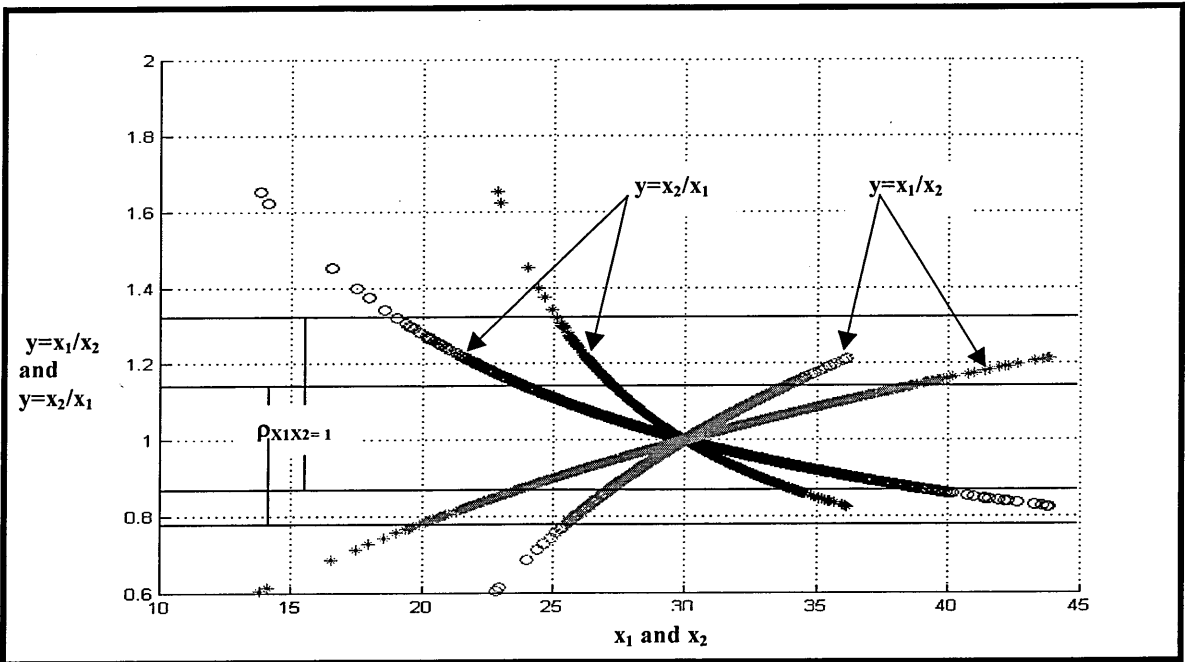


Figure 4.29 Uncertainties of the results $y=x_1/x_2$ and $y=x_2/x_1$ when there are perfect positive correlated input uncertainties ($\rho_{x_1x_2}=1$) and $u(x_1)/x_1 > u(x_2)/x_2$.

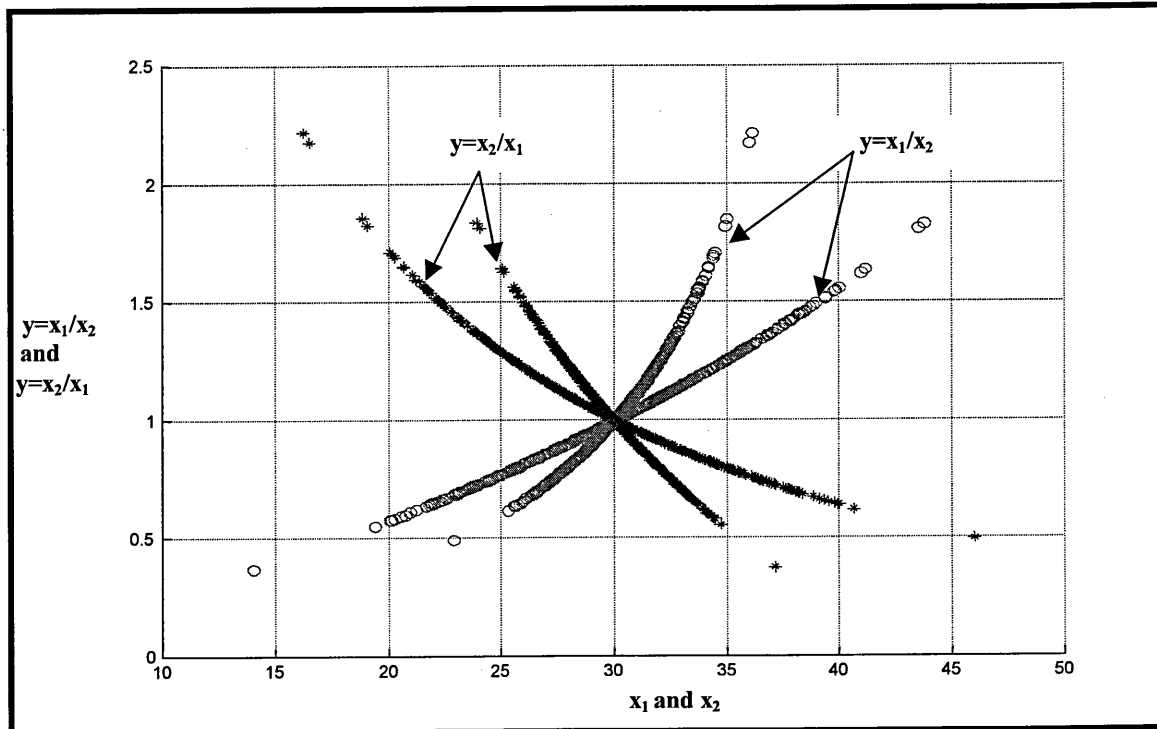


Figure 4.30 Uncertainties of the results $y=x_1/x_2$ and $y=x_2/x_1$ for perfect negative correlated input uncertainties ($\rho_{x_1x_2}=-1$).

Especially, in **figure 4.31** are clearly demonstrated all the effects of measurement inputs' covariance and subsequent generated non-linearity, on the result's distribution (**Skewing**, **Flattening** with respect to Gaussian and **Bending**). This, to the author's knowledge, has never been demonstrated before and it is a significant contribution of this project along with the fact that can explain the results obtained in more complex cases.

These effects of course, are present to a lesser extend in partial input correlated uncertainties as it is next shown in **figure 4.32** for the result $y=x_2/x_1$ and a negative input correlation coefficient ($\rho_{x_1x_2} = -0.8$) for the same inputs as above. As it can be seen, the isoquant area is no longer elliptic but performs a **convexity**, or else the result y is convex with respect to x_1 and x_2 .

Thus for a multi-dimensional case, the geometrical representation of combined uncertainties does not longer correspond to multi-dimensional hyper-ellipsoids *but rather to multi-dimensional crescents*.

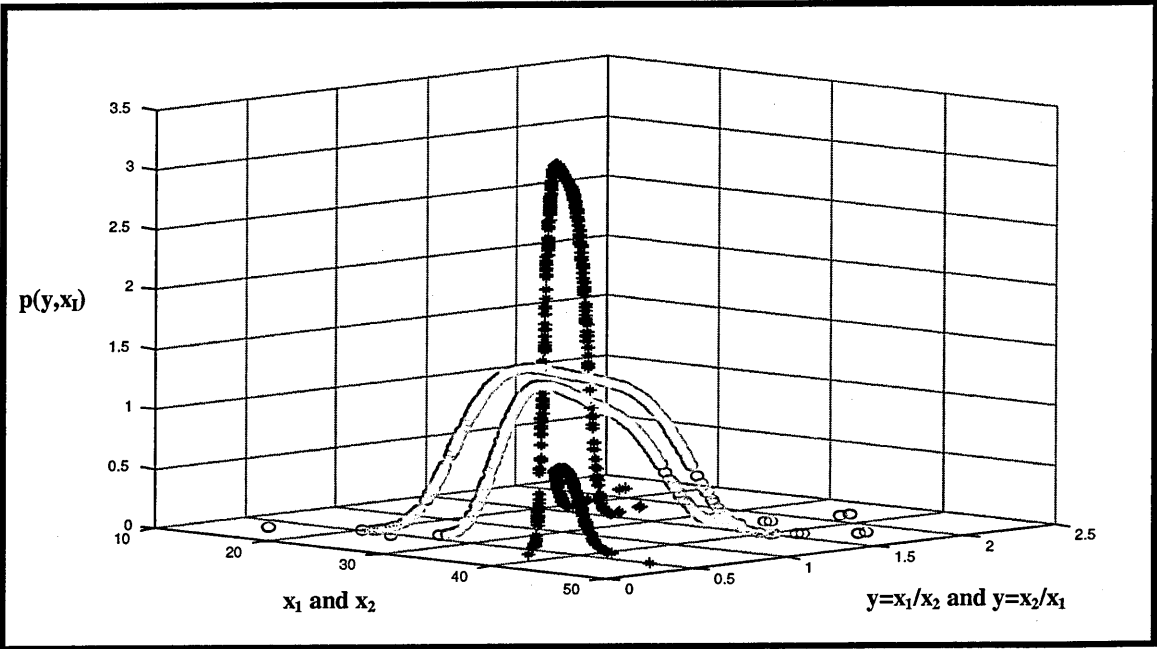


Figure 4.31 The skewed, flattened and bended (effects of generated non-linearity) joint distributions $p(y, x_1)$ for result $y = x_1/x_2$, and $p(y, x_2)$ for result $y = x_2/x_1$, and perfect negative correlated uncertainties ($\rho_{x_1x_2} = -1$).

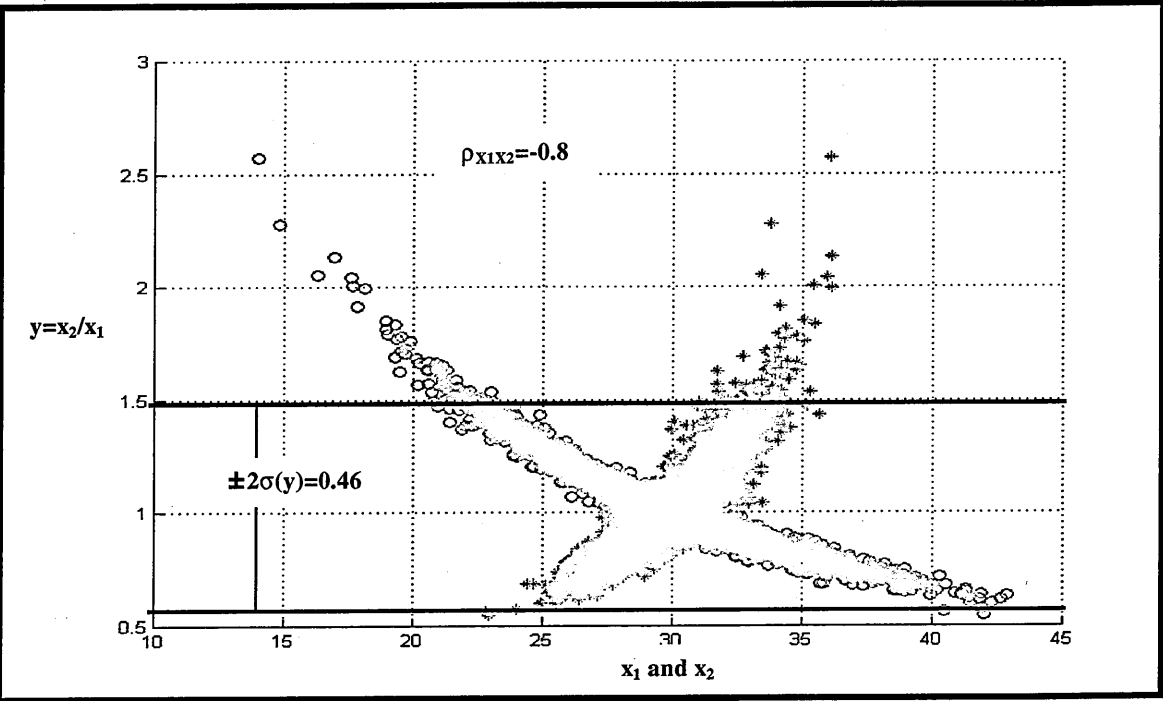


Figure 4.32 The joint probability $p(y, x_1)$ and $p(y, x_2)$ isoquants (crescents) and uncertainty of the result $y = x_2/x_1$ for partial positive inputs' uncertainties correlation $\rho_{x_1x_2} = -0.8$.

Some other interesting perspectives of the inputs' uncertainty covariance effects on the output are also shown in figures 4.33 and 4.34.

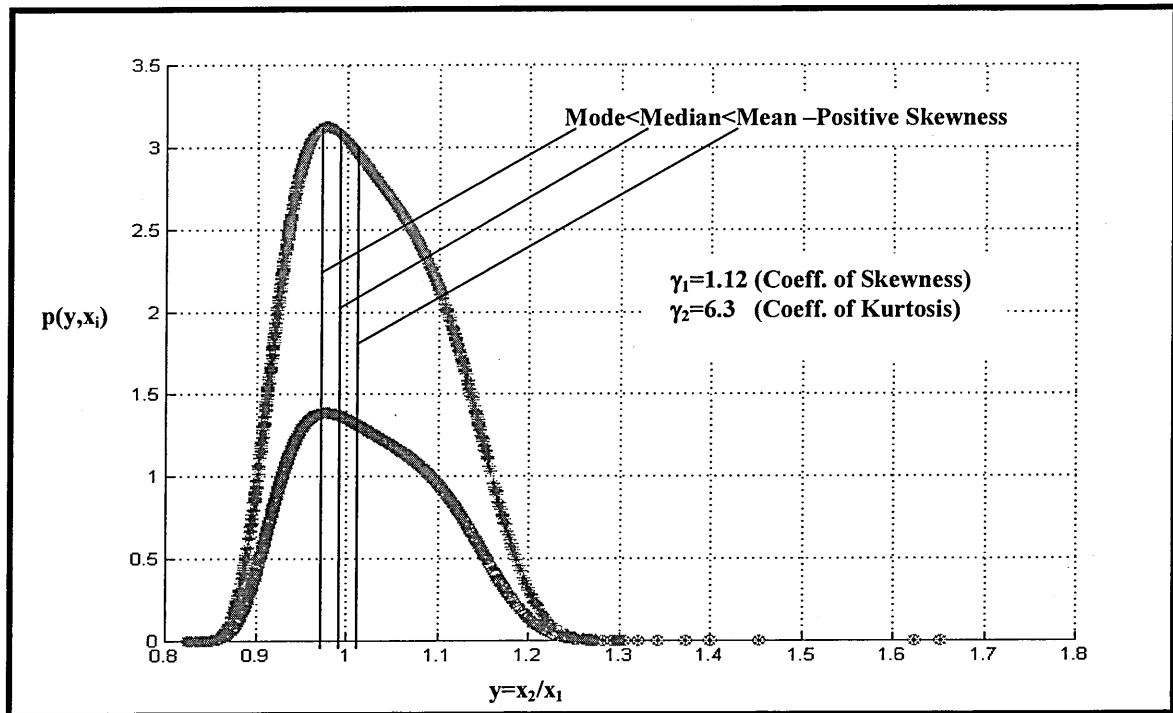


Figure 4.33 The skewed and flattened (also bended) joint probabilities $p(y, x_1)$ and $p(y, x_2)$ for a measurement result $y = x_2/x_1$ and for perfect positive input correlated uncertainties ($\rho_{x_1 x_2} = 1$).

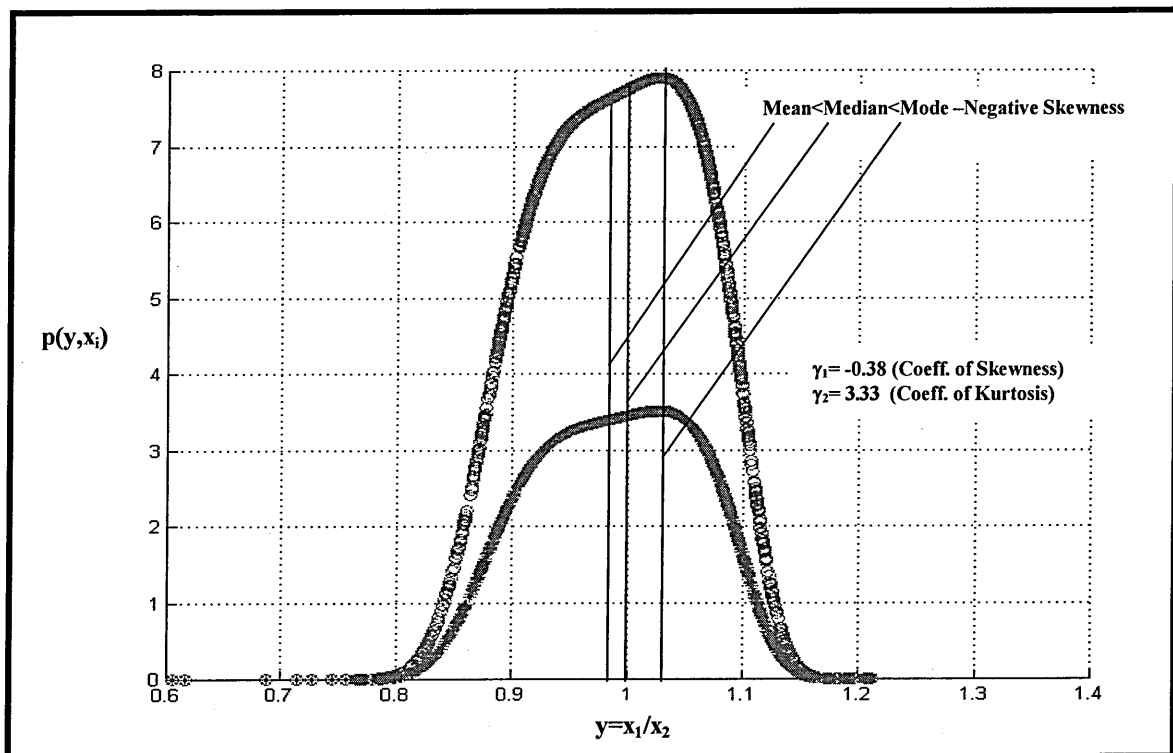


Figure 4.34 The negatively skewed and flattened (also bended) joint probabilities $p(y, x_1)$ and $p(y, x_2)$ for a measurement result $y = x_1/x_2$ and for perfect positive input correlated uncertainties ($\rho_{x_1 x_2} = 1$).

In **figure 4.33**, the joint distributions $p(y, x_1)$ and $p(y, x_2)$ of the result $y = x_2/x_1$, for the same inputs as above and perfect positive correlated uncertainties ($\rho_{x_1 x_2} = 1$), are positively skewed with a long tail on the right. Additionally, y , although not shown here, is again **convex** with respect of x_1 and x_2 . The *coefficient of skewness*, $\gamma_{1y} = 1.12 > 0$, indicating this positive skewness has been obtained from:

$$\gamma_{1y} = \frac{1}{N\sigma(y)^3} \sum_{i=1}^N (y_i - y)^3 \quad 4.54$$

In **figure 4.34**, again the two respective joint distributions of the result $y = x_1/x_2$ for the same inputs and perfect positive correlated uncertainties ($\rho_{x_1 x_2} = 1$) are negatively skewed with a long tail on the left, with an estimated coefficient of skewness $\gamma_{1y} = -0.38 < 0$. Additionally, y , although not shown, is **concave** with respect of x_1 and x_2 .

For the case shown in figure 4.33, if the normal procedure of obtaining the univariate probability density of the result y had been selected, we would obtain the distribution shown next in **figure 4.35**.

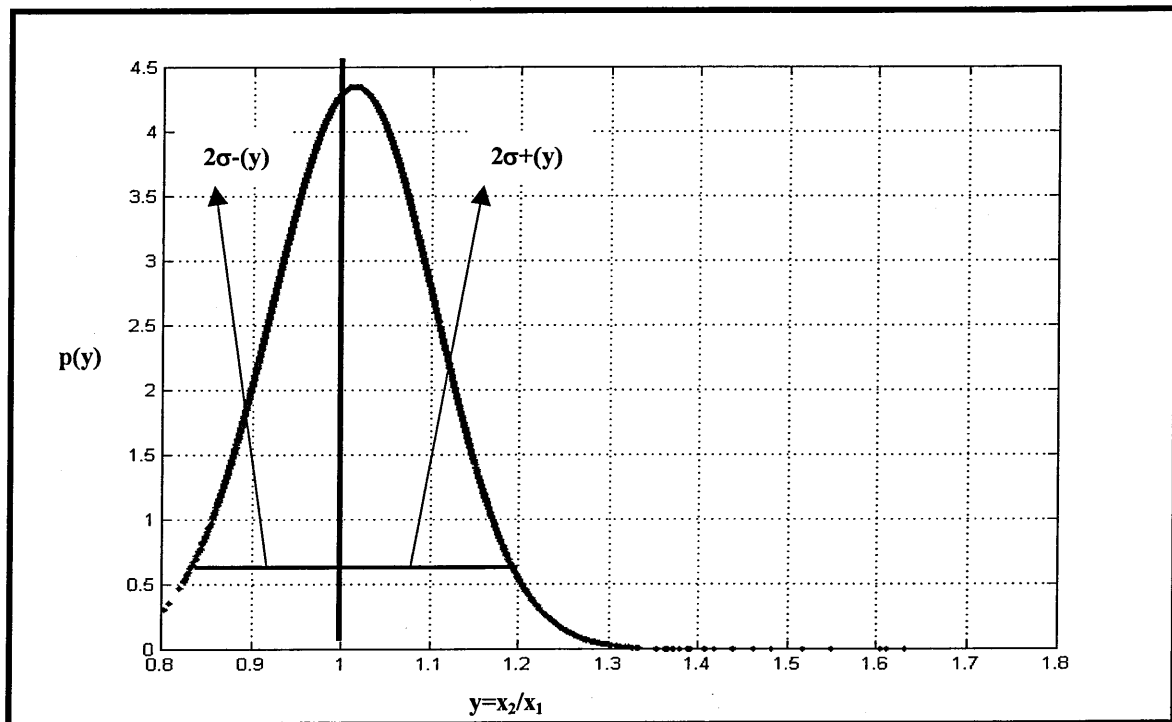


Figure 4.35 Univariate probability density $p(y)$ of the result $y = x_2/x_1$ (see figure 4.33) with its Total (asymmetric) uncertainties $2\sigma-(y)$ and $2\sigma+(y)$.

Although it is clearly shown the skewness of the distribution along with the actual asymmetric uncertainty intervals of the result y , this does not reveal anything about the

flattening (compared to the Gaussian) and the *bending* (non-linearity) of the distribution. In fact, a coefficient of kurtosis, $\gamma_{2y}=6.3 \gg 3$ has been obtained for this case from:

$$\gamma_{1y} = \frac{1}{N\sigma(y)^4} \sum_{i=1}^N (y_i - \bar{y})^4 \quad 4.55$$

The respective estimated coefficient of kurtosis for the distribution of figure 4.34 was $\gamma_{2y}=3.33 > 3$, indicative of its flattening also in this case. All the previous results that have been presented revealed the complexity and multiplicity of the cases that can appear. However, things have been actually simplified and a respective uncertainty propagation law that covers all of these cases has been developed.

Considering equations 4.36 to 4.39 from section 4.2 and including now also the residual W term, we can obtain for the result of multiplication of two input measurements with correlated uncertainties:

$$\begin{aligned} E(Y) = y = E(x_1 x_2) &= \frac{1}{N} \sum_{i=1}^N \left[(x_1)(x_2) + \frac{\partial f}{\partial X_1}(x_{1i} - x_1) + \frac{\partial f}{\partial X_2}(x_{2i} - x_2) + W \right] \Rightarrow \\ E(Y) = y &= (x_1)(x_2) + \frac{1}{N} \sum_{i=1}^N W \end{aligned} \quad 4.56$$

The mean residual term can be easily associated with the inputs covariance term $\sigma(x_1, x_2)$ corresponding to the bias (systematic uncertainty) caused by the input measurements correlated uncertainty sources. In order to get the form of an asymmetric uncertainty propagation law for the presented multiplication case, combining equations 4.36, 4.40 and 4.56 we can get:

$$u(y) = \sigma(y) = \sqrt{\frac{1}{N} \sum_{i=1}^N \left[\frac{\partial f}{\partial X_1}(x_{1i} - x_1) + \frac{\partial f}{\partial X_2}(x_{2i} - x_2) + W - \bar{W} \right]^2} \quad 4.57$$

The last two residual terms are zeroed out leaving the random uncertainty component the same (as in uncertainty propagation law) as it is alternatively expressed by equation 4.42. Thus the **asymmetric total uncertainty** propagation law that gives the uncertainty intervals similar to these shown in **figure 4.35**, is now taking the next general form:

$$\frac{u(y)}{y} = \left\{ \begin{array}{l} \frac{\sigma+(y)}{y} = \sqrt{\left(\frac{\sigma(x_1)}{x_1}\right)^2 + \left(\frac{\sigma(x_2)}{x_2}\right)^2 + \frac{2\rho_{x_1x_2}\sigma(x_1)\sigma(x_2)}{x_1x_2} + \frac{\rho_{x_1x_2}\sigma(x_1)\sigma(x_2)}{x_1x_2}} \rightarrow \text{for } -1 < \rho_{x_1x_2} < 1 \\ \frac{\sigma-(y)}{y} = \sqrt{\left(\frac{\sigma(x_1)}{x_1}\right)^2 + \left(\frac{\sigma(x_2)}{x_2}\right)^2 + \frac{2\rho_{x_1x_2}\sigma(x_1)\sigma(x_2)}{x_1x_2} - \frac{\rho_{x_1x_2}\sigma(x_1)\sigma(x_2)}{x_1x_2}} \rightarrow \text{for } -1 < \rho_{x_1x_2} < 1 \\ \frac{\sigma(y)}{y} = \sqrt{\left(\frac{\sigma(x_1)}{x_1}\right)^2 + \left(\frac{\sigma(x_2)}{x_2}\right)^2} \rightarrow \text{for } \rho_{x_1x_2} = 0 \quad (\text{RSS}) \\ \frac{\sigma+(y)}{y} = \left| \left(\frac{\sigma(x_1)}{x_1}\right) \pm \left(\frac{\sigma(x_2)}{x_2}\right) \right| + \frac{\rho_{x_1x_2}\sigma(x_1)\sigma(x_2)}{x_1x_2} \rightarrow \text{for } \rho_{x_1x_2} = \pm 1 \quad (\text{ADD}) \\ \frac{\sigma-(y)}{y} = \left| \left(\frac{\sigma(x_1)}{x_1}\right) \pm \left(\frac{\sigma(x_2)}{x_2}\right) \right| - \frac{\rho_{x_1x_2}\sigma(x_1)\sigma(x_2)}{x_1x_2} \rightarrow \text{for } \rho_{x_1x_2} = \pm 1 \quad (\text{ADD}) \end{array} \right.$$

Jensen's Uncertainty Inequality can give general explanations in all these cases. According to the author's point of view, Jensen's Uncertainty inequality is the most important uncertainty inequality among the various relevant uncertainty inequalities present in the existing literature. A general discussion for all of the previous and next findings will follow next after the presentation of the results of Monte Carlo simulation of natural gas energy measurement systems.

CHAPTER 5

5. MONTE CARLO ESTIMATION OF NATURAL GAS ENERGY FLOW RATE UNCERTAINTY – FLOW MONITORING.

5.1. Monte Carlo simulation method application to Natural Gas energy flow measurement uncertainty estimation – Justification.

There are properties in thermodynamics that appear to behave randomly even in principle, and inevitably in these cases, as already described in the previous section, probability statements and statistical averages are employed rather than explicit mathematical relationships. This can be easily realised by considering the definition of a primitive thermodynamic property, such as pressure, as it is given by **Shavit, A., and Gutfinger, C., 1995**. According to this definition, a primitive thermodynamic property is "a characteristic quantity of a thermodynamic system, which can be determined by a test, i.e. a measurement. The outcome of the measurement is the value of the property".

Roley, R.L., (1994), gives a good insight to these aspects and emphasises our inability to devise even in principle (due to the Heisenberg's Uncertainty principle as he refers) the sensor that could detect the relationship between molecular motion and a macroscopic property like pressure. The best we can obtain is a sensor that would measure a spatial average of molecular states at an instant of time or a temporal average at a particular position. Actually, all real sensors do neither. Because of their size and response times they actually sense an average over both time and space of what is actually happening on a molecular level.

So, macroscopic primitive thermodynamic properties, such as pressure or temperature, are obtained by definition from measurements and are really averages of molecular properties with respect to time or position, almost always both.

As it was already pointed out, Monte Carlo uncertainty analysis is a probabilistic method that relies on "*randomness*". Such a kind of analysis, according to the authors point of view, seems to be closer to the underlying physics of actual measurement processes that are also probabilistic in nature. **Nicolis, G. (1995)** has also pointed out that in nature the process of measurement, by which the observer communicates with a physical system, is limited by a finite precision. As a result, a 'state' of a system must in reality be understood

and subsequently represented, not as a point in a multidimensional phase space but rather as a small region whose size reflects the finite precision of the measuring apparatus. So, on the probabilistic view we look at our system through a 'window' (phase space cell), whereas in the deterministic (analytical) view it is understood that we are exactly running on a *phase space trajectory*, which is clearly an unrealistic assumption in view of the presence of the errors.

According to the Ergodic Hypothesis, time averages are equivalent to ensemble averages. Monte Carlo uncertainty simulations deal specifically with the probability of generating ensembles of micro or even as in our case macro states and use ensemble averages to obtain or simulate macroscopic properties.

Thus, the application of Monte Carlo simulation in the measurement uncertainty estimation of different measured states of a system seems to offer a theoretically more realistic and closer approach to the physical reality, even at the macroscopic level. In this project, the additional practical benefits of the method have been also revealed.

Basil, M., and Jamieson, A.W., (1998) have described some advantages of Monte Carlo simulation compared to analytical methods in terms of its application in measurement uncertainty estimation and they presented its underlying simplicity and reliability. Its advantages, among others, are that it can handle both small and large uncertainties in the input quantities and in addition complex partial differentiations are no longer necessary.

One of the most important advantages of Monte Carlo uncertainty estimation method is that it could also be used, if there are known joint distributions, for input bias or systematic error covariances in the measured variables of a particular measurement system. (**Gilks, W.R., et.al., 1996**). These correlated uncertainties for example, could be due to calibrations of two pressure transmitters using the same dead weight tester.

Monte Carlo is a numerical method that extensively uses random numbers. Its advantage as an uncertainty estimation method can be realised when used for the uncertainty analysis of complex systems and especially in cases where there is known to be covariance but we want a tighter uncertainty bound than that given by the ordinary sum of partial uncertainties. As **Taylor, J.R., 1997** has proved, this is actually the maximum and most pessimistic uncertainty bound. Monte Carlo uncertainty procedure mainly involves random sampling from the distribution of inputs (if there are known joint

distributions we can sample from them, thereby accounting for input covariances) and compute successive model runs until a significant distribution of outputs is obtained. In this sense, it can be used to solve problems with physical probabilistic structure such as uncertainty propagation in models, or to solve non-probabilistic problems such as finding the area under a curve (Monte Carlo integration). The author has mainly distinguished two main problems associated with the practical implementation of Monte Carlo uncertainty simulation method:

1. Selection of the probability distribution function for the input variables
2. Generation (sampling) of random numbers from that distribution

In terms of the selection of the probability distribution function for the input variables, this can be accomplished by using either statistical estimation techniques or expert judgement. Statistical estimation techniques, as they have already been described in the presentation of the new widely accepted GUM method, involve estimating probability distributions from available data or by collection of a large number of representative samples (Type A evaluation of uncertainty). In cases where limited data are available, expert judgement could provide the information about the input probability distribution (Type B evaluation). For example, a uniform probability distribution can be selected in cases where only a range of possible values of the input are available but no information about which values are more likely to occur. Maximum instrument drift between calibrations or errors due to limited resolution of an instrument's display can be represented by uniform distributions. Similarly, a normal distribution can be typically used to describe unbiased measurement errors such as those produced as a result of instruments' calibration and after calibration corrections (adjustments). A list of the common probability distributions with their respective standard uncertainties in terms of input variables can be found in the new ISO/WD 5168. Although they are presented for use in analytical estimation methods, the author suggests that they could be equally applied in Monte Carlo estimation.

The later descriptions regarding selection of input probability distributions and subsequent assignments of uncertainties as standard deviations of those distributions, has been seen by the author as clear evidence of the absolute compatibility of the Monte Carlo uncertainty methodology with the GUM methodology.

Generally, a suggested complete Monte Carlo uncertainty analysis could be carried out in four stages

- Selection/Estimation of input uncertainties
- Uncertainty propagation and estimation of output uncertainties by statistical methods
- Estimation of uncertainty due to the selected methods (different available elementary measurement models fusion/integration).
- Estimation of uncertainty due to selected models (comparison with other possibly available models).

5.1.1. Software Development - Results and comparisons with analytical estimation methods.

As it was said in chapter 1, in every gas purchase, sale, transportation and exchange contract, general terms and conditions with reference to measurement are clearly spelled out. These terms and conditions address four typical areas, a) Definitions of terms, b) Gas quality specifications, c) Method of measurement and d) Measurement equipment. All directly or indirectly point to the importance of measurement and its associated uncertainty.

5.1.1.1. A typical modern Electronic Natural Gas Metering System (ENGMS).

From the results of a typical modern electronic natural gas metering system, the cost of purchased gas is calculated by:

$$\text{\$} = E_{Flow} * P \quad 5.1.1$$

E_{Flow} : is the energy flow in Kcal/Month and

P : is an agreed Gas energy price in \\$/Kcal.

It is of interest to note that the energy flow rate measurement is associated with an uncertainty but of course there cannot be any kind of uncertainty in terms of money exchange!!. As it was said in section 4.1.1, the reliable estimation of measurement uncertainty becomes necessary for the selection of the appropriate metering system, its design and operation optimisation and subsequently for the continuous evaluation of its performance through:

- The a priori specification of reasonable desired measurement control limits;
- The a posteriori uncertainty estimation of what is actually produced.

As long as what is actually produced is within the desired control limits then these limits define the *tolerance interval* of an indisputable energy purchase transaction between the parties involved, with the elimination of any further uncertainty in the money exchange.

The energy flow rate is calculated by:

$$E_{Flow} = Q * GCV \quad 5.1.2$$

where Q is the volumetric flow rate in $Rm^3/$ Month and GCV is the Gross or Superior Calorific Value in $Kcal/Rm^3$. It is important to note that both Q and GCV must be referred to the same reference conditions.

The flow rate measurement can be characterised as a *quantitative* measurement while the measurement of the GCV is a *qualitative* measurement, based on the determination of the gas composition by mole fraction that virtually describes its quality in terms of its useful energy content.

The measured instantaneous dry natural gas mass flowrate (in Kg/sec) through a single meter run is given by:

$$q_m = C_d E \varepsilon \frac{\pi d_o^2}{4} \sqrt{2 \Delta P \rho_i} \quad 5.1.3$$

The respective volumetric flow rate at reference conditions (e.g. 15 °C and 1.01325 bar)

$$\text{is: } Q = \frac{q_m}{\rho_{ref}} = \frac{C_d E \varepsilon \frac{\pi d_o^2}{4} \sqrt{2 \Delta P \rho_i}}{\rho_{ref}} \text{ in ref. m}^3/\text{sec} \quad 5.1.4$$

where ρ_{ref} is the gas density at reference conditions.

Modern large Natural gas metering facilities operate at relatively high line pressures (up to 250 bar) and consist of several parallel meter runs each one equipped with an instrumentation array that is necessary for the measurement of flow rates.

A typical P&ID of a single meter run with an orifice meter is shown in **figure 5.1.1**. It can be seen that an instrumentation array is needed for the determination of mass and energy flow rate.

Daily and monthly totalisation of the volume flowrate gives the accounted gas volume. It is a useful procedure to examine how every term is obtained and integrated in the last equation, which is the main flow equation. This is shown in **figure 5.1.2** in a form of a **Flow Cognitive Network**.

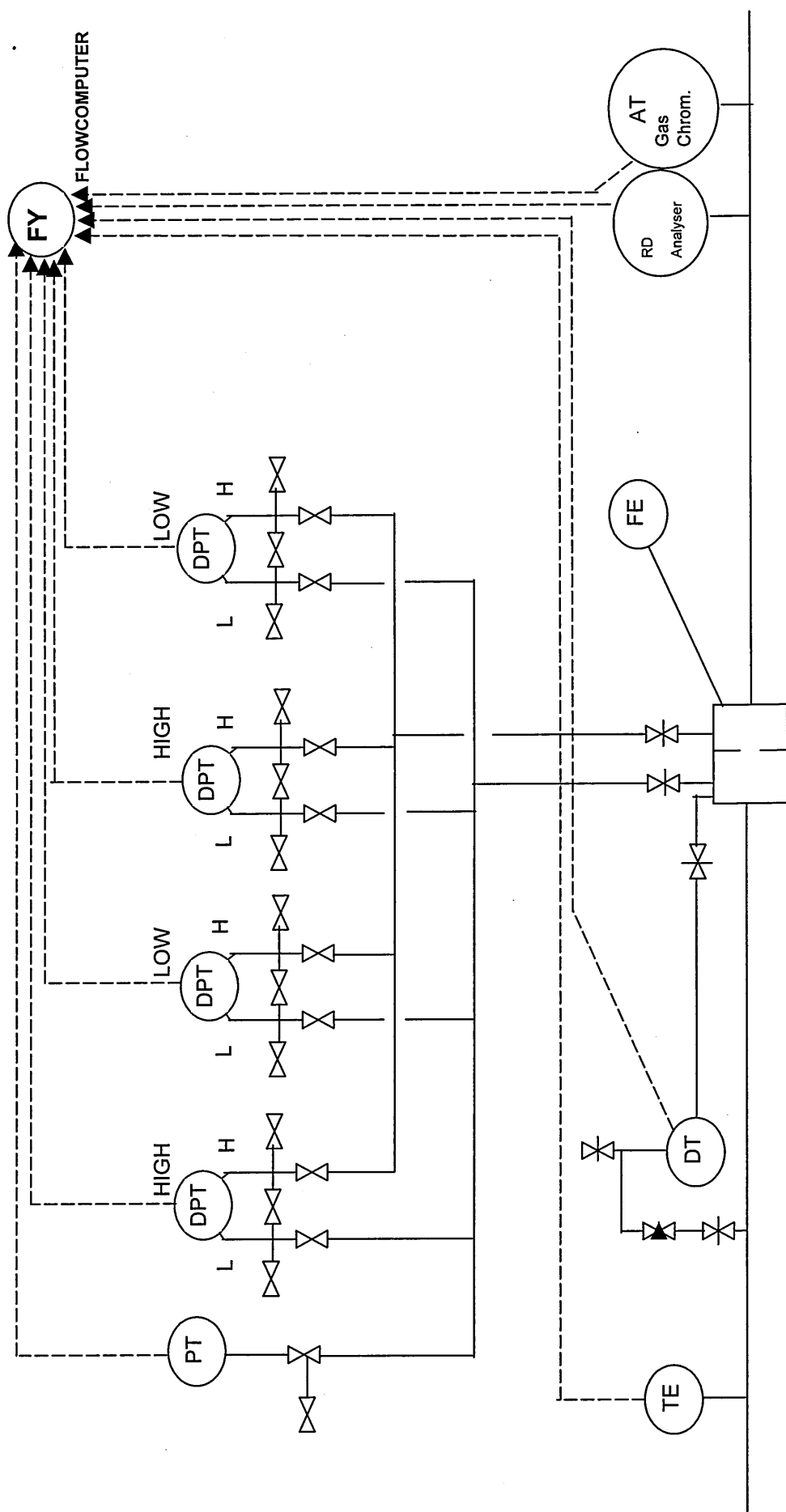


Figure 5.1.1 A typical Process and Instrumentation diagram of an orifice meter run and additional systems

5.1.1.2. Description of the Monte Carlo simulation software developed

A PC based model of a modern Electronic Natural Gas Metering Station (ENGMS) has been developed using MATLAB/SIMULINK. The model and its computational methodology are based on a typical design as shown in **figures 5.1.1 and 5.1.2**.

Energy and mass flow rate uncertainties are estimated using Monte Carlo simulation methods. The model also allows the study of error propagation through the system and the visualisation of not so obvious (obscured) correlations between various parameters in a complex system.

The PC-model developed reports the energy flow rate (MJ/s), mass flow rate (kg/s) and the reference volume flow rate (Rm^3/s). The flow rate is calculated at the upstream conditions (P1, T1) in accordance with ISO-5167. It consists of two main computational systems, the flow (quantity) system and the energy (quality) system. In **Appendix A** some of the main subsystems of the gas energy metering system simulation model are shown. The main subsystems are:

- A field input subsystem that can accept real or simulated input signals;
- An initialisation subsystem that is necessary for the initialisation of the program,
- A corrections subsystem that deals with the downstream to upstream corrections (e.g. the conversion of downstream temperature and line density to the upstream values)
- A discharge coefficient subsystem, for the discharge coefficient (C_d) simulations,
- A line compressibility subsystem based on AGANX-19 method.

The energy system includes all the computational modules of a gas chromatograph for the calculation of energy content (Gross Calorific Value) and other necessary parameters such as reference density and reference compressibility according to the relevant **ISO-6976 (1996)**. The final reporting subsystem reports the flow quantity results and secondary model based measurements such as sound velocity as well as various statistical parameters (e.g. cumulative means and variances). Variables' values can be extracted at any points of the propagation links allowing the study of uncertainty propagation through the system. Prior to the main Monte Carlo simulation a similar version was run for a few cycles and for constant measurement inputs corresponding to the best input estimates and a satisfactory Reynolds' number and Discharge coefficient. Convergence is usually

reached after about 3 cycles. The mean values of these initially estimated constant parameters are used to obtain the methods' uncertainties estimates for further use in the main program, for example the 0.6% uncertainty for the discharge coefficient or the 0.2% uncertainty of the AGANX19 line compressibility estimation method. So, these initially estimated methods' uncertainties were then incorporated in the modeling of the inputs in the main simulation program. Actually, both programs have been incorporated in a Matlab-M program, and they run sequentially. Of course the computational time of the first constant inputs program is insignificant.

In figure 5.1.1 are shown the *direct* (instrumentation mainly based) measurements that are usually taking place in a gas metering system. There are also other direct measurements required which are not actually shown. For example, pipe and orifice diameters are *calibration-based measurements* because they are occasionally measured and recorded according to a specific calibration program. These occasional calibration measurements are entered then as constants in the model based measurements of the 'actual' orifice and pipe diameters, which they are corrected for the real time gas temperature variations (deviations) through the next equation:

$$do_a = do_c(1 + \lambda_o(T_1 - T_{o_c})),$$

$$Dp_a = Dp_c(1 + \lambda_p(T_1 - T_{p_c})) \quad \mathbf{5.1.5}$$

where, do_c , Dp_c are the direct (calibration) measurements of orifice and pipe diameters respectively, T_{o_c} and T_{p_c} are the respective calibration temperatures, λ_o and λ_p are the respective materials' thermal expansion coefficients and T_1 is the gas temperature under real operating conditions upstream to the orifice plate.

The temperature T_1 is also a model based measurement obtained from the directly measured downstream temperature T_2 and considering an isenthalpic expansion across the orifice (Joule-Thomson expansion).

The discharge coefficient, given by Stoltz or Reader-Harris/Gallagher equations, is a function of beta ratio, pipe Reynolds number, upstream fluid density (calculated from a state equation also functionally related to the temperature) and gas pipe velocity. Thus, it is not difficult to visualise that the empirical discharge coefficient also acquires a complex, although weak, dependency on temperature. In fact, all the measurements involved in equation 5.1.1 acquire similar interdependencies through model-based

measurements used in the flow rate computation. The upstream temperature T_1 is given by T_2 by the following Joule-Thompson equation: $T_1 = T_2 \left(\frac{P_3}{P_1} \right)^{k/4}$ where $k = \frac{1-k}{k}$ where k is the isentropic exponent. For real gas compressibility effects, k must be also considered and is defined by:

$$k = \left(\frac{\partial P}{\partial \ln(v)} \right)_S \quad 5.1.6$$

where v is the specific volume and S is the entropy. An analysis of the flow across the orifice along with further theoretical work is present in **Appendix B**.

In order to estimate the overall measurement uncertainty, all the 'direct' measurements (directly observed and calibrated) were modeled as random variables and used as inputs to the simulation program. The procedures followed in the modeling of the input measurements were similar to those described in chapters 3 and 4. Their respective uncertainties corresponded to those uncertainties that they should have been obtained from a large number of repeated measurements of the same quantities during respective calibrations. The inputs were modeled as normal distributions since in reality, the results of such calibrations would follow normal distributions. The possible correlations between the input measurements have not been considered since the respective instruments could have been calibrated using different standard equipment and under different conditions. However, it would be possible to examine various scenarios for input correlated uncertainties with the aid of the developed model, which is the main advantage of the Monte Carlo simulation approach.

Another important matter about the operation of the simulation model is that all the parameters, after the injection of the random field inputs, are allowed to be perturbed according to their model equations. So, a truly realistic picture of the operation of such a complex system can be obtained. As it may be seen in the simulation program's printouts of Appendix A, there are numerous interactions between the various measurement systems and subsystems. For example the energy main system feeds the flow system with those data such as MW, reference density, normalised N_2 , CO_2 contents (necessary for AGANX-19 compressibility calculations) etc., which are also necessary for the flow rate calculations.

It is this fact that allows us to draw this realistic picture of the operation of such a complex measurement system and to study the various possible correlations generated between elementary measurement processes as a result of the models interactions.

5.1.1.3.Results of Monte Carlo natural gas energy flow uncertainty estimation and comparison with analytical estimation methods (RSS).

A summary of the main characteristic series of results obtained for a specific metering system operating within a range of different flow rates as well as the respective simulated input uncertainties are shown in **Table 4**. These have also been presented in FLOMEKO 2000 conference, (Yeung, H., Papadopoulos,E.C., 2000). The results for mass flow, reference volume flow and energy flow rate uncertainties are plotted in **figures 5.1.3 to 5.1.6** along with the respective results obtained using the conventional root of sum squares (RSS) method.

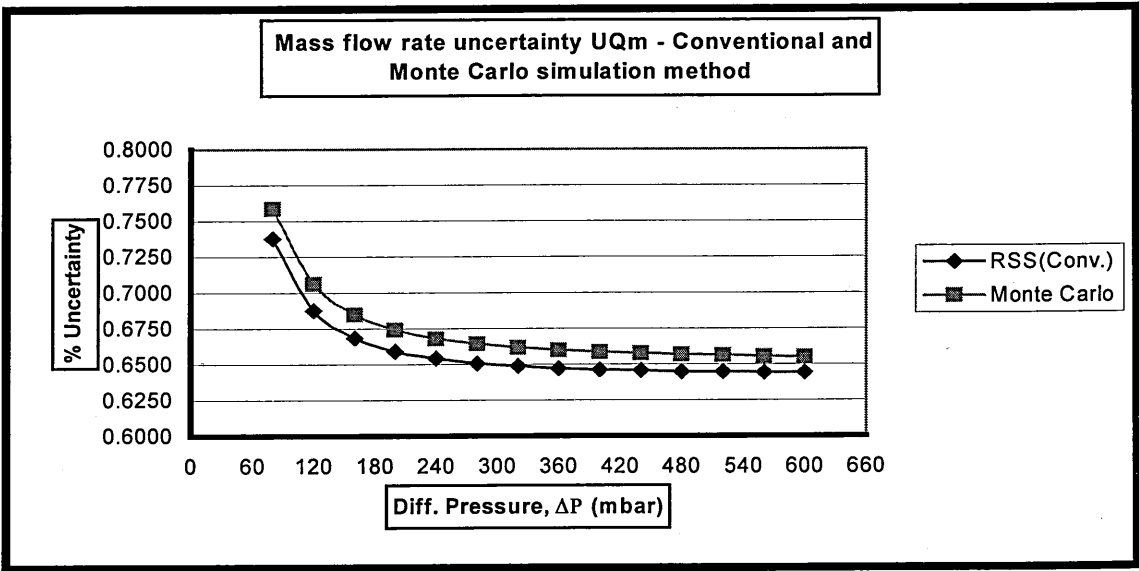


Figure 5.1.3 Comparative mass flow rate uncertainty estimation results of Monte Carlo simulation and RSS methods.

The combined uncertainties shown, correspond to approximately 95.5% confidence intervals, (coverage factor $t=2$). The uncertainties of AGANX-19 and discharge coefficient empirical methods were considered to be 0.2%, and 0.6% respectively. These empirical methods' uncertainties were also sampled and incorporated within the software using separate normal distributions. Various tests were carried out to estimate the number

of trials that were necessary in order to obtain an approximately 95.5% coverage for the results (or alternatively 95% for $t=1.96$).

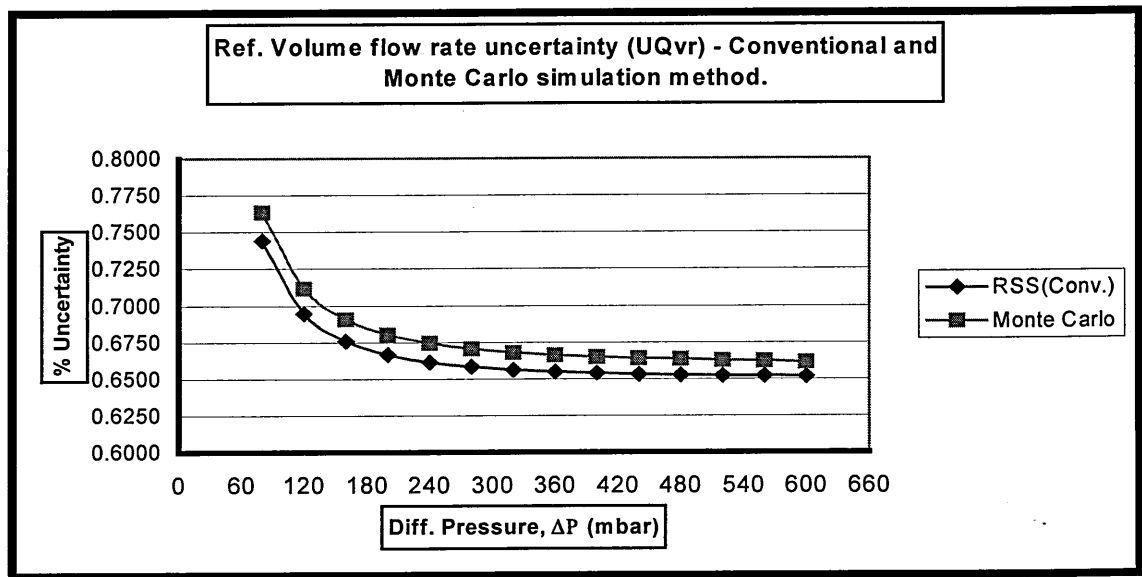


Figure 5.1.4 Comparative ref. volume flow rate uncertainty estimation results of Monte Carlo Simulation and RSS methods.

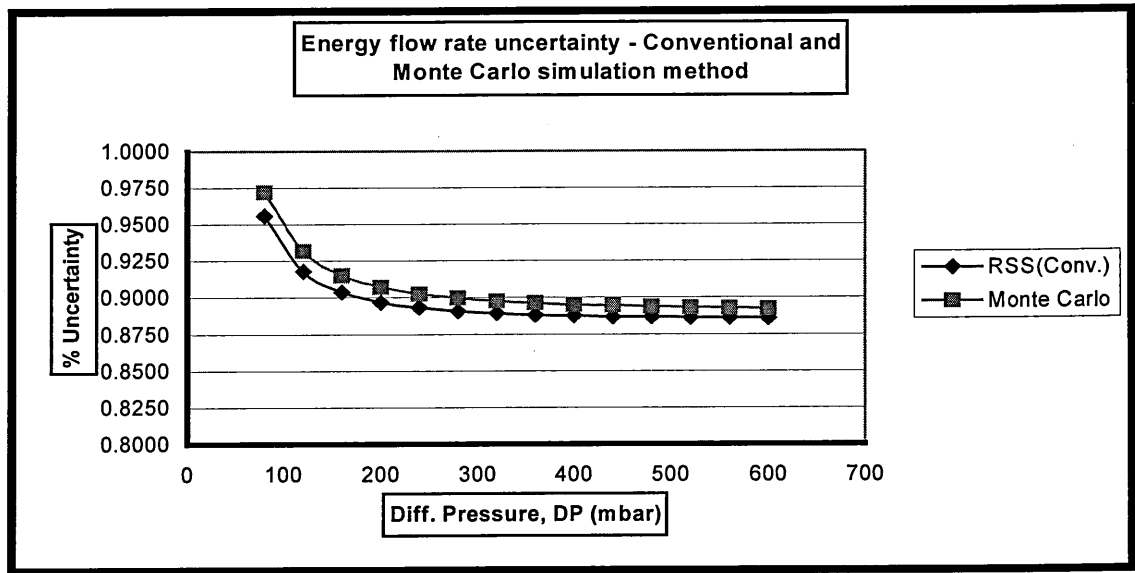


Figure 5.1.5 Comparative energy flow rate uncertainty estimation results of Monte Carlo Simulation and RSS methods.

All the results presented in this project correspond to those obtained from 1000 simulation cycles (with multiple seeds) of 100.000 points since as it was said in section 3.2.1 the improved method was discovered at the latest stages of this project and it was

used mainly for verification purposes. The sampling means and standard deviations of the input variables estimated from single 100.000 points cycles overlapped with those used as inputs to the simulation.

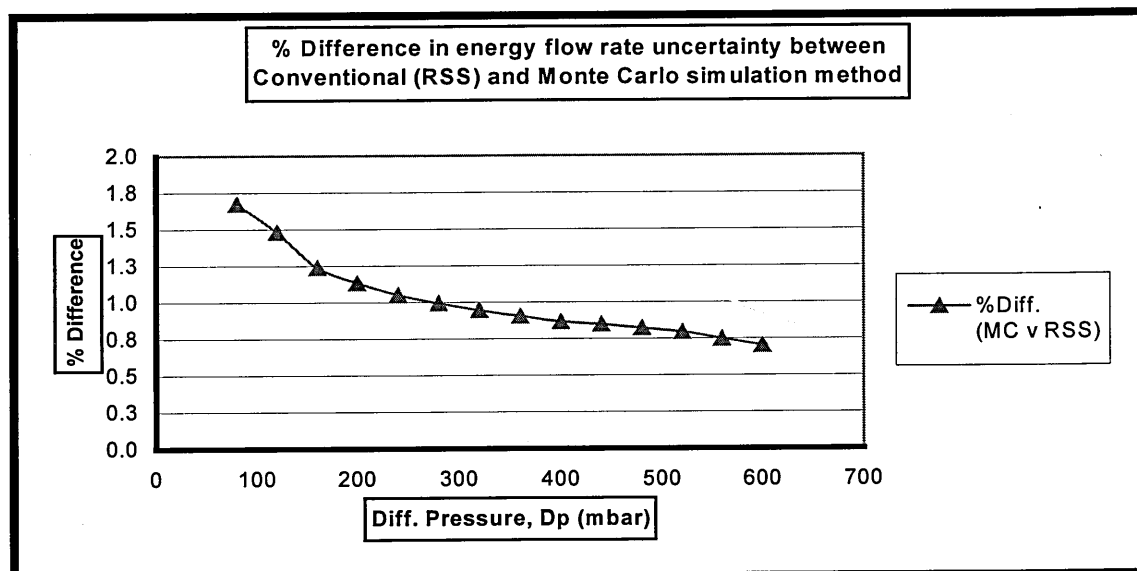


Figure 5.1.6 %Difference in energy flow rate uncertainty estimation between Monte Carlo simulation and RSS methods.

It can be seen that the estimated uncertainties from Monte Carlo method are between **0.8** to **1.8%** higher than the respective estimated uncertainties using the RSS method. This can be explained by the fact of multiple generated correlations during the estimation, which predominately must be positive.

Another probably more important fact studied at the latest stages of this project was that an additional systematic uncertainty component ranging from around **0.1%** at the maximum differential pressure (and respective flow rate) 600mbar to around **0.17%** for the very low differential pressure at 120mbar. The existence and the subsequent search for this possible systematic error initiated after the results obtained for the simple test cases that gave a clearer insight into the uncertainty propagation mechanism and the correlated input uncertainty effects in complex measurement results.

This additional systematic error was estimated at the latest stages of this project and only for two flow rates using the following methodology.

Since the program is initially run with the input measurements having constant values corresponding to the inputs' best estimates, mean flowrates were estimated and then compared with those estimated from the Monte Carlo series of experiments. In the first case for example, at 600mbar differential pressure the constant inputs estimated mean mass flowrate was **26.451 Kg/sec**, while the respective one obtained from the second latest Monte Carlo experimental series was **26.477 Kg/sec** corresponding to an about **0.1%** estimated positive systematic error. The respective estimated flow rates at a lower differential pressure of 120mbar were **11.882 Kg/sec** and **11.902 Kg/sec** respectively, corresponding to an estimated positive systematic error of around **0.17%**. The cause of these observed systematic errors must have again as a source the various generated correlations among the elementary measurements processes as well as the pre-existed (various exponents in the models) and further generated non-linearities as the results obtained and presented in section 4.2.2.1 have revealed. A small difference though exists between the two cases, which in general does not invalidate their intrinsic similarity and this will be discussed in the next section.

Generated obscure correlations can be readily visualised using the Monte Carlo approach developed here where some of them may further imply obscure physical relationships.

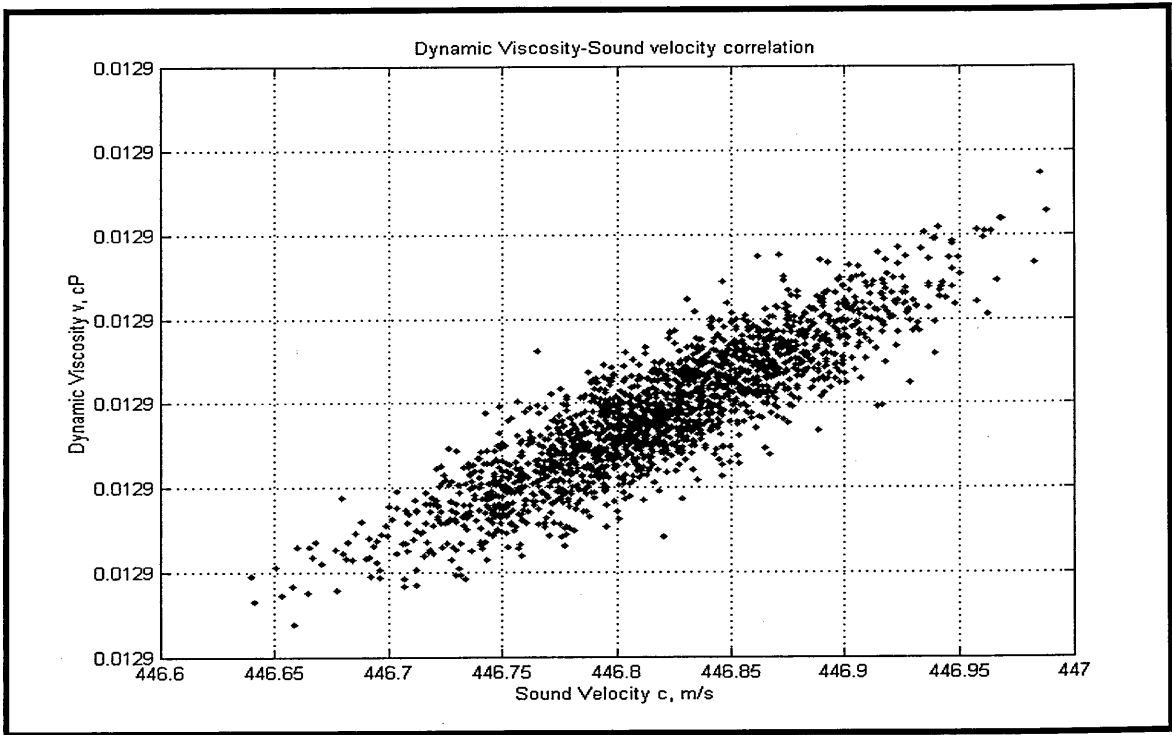


Figure 5.1.7 Dynamic viscosity – Sound velocity correlation obtained.

For example, in **figures 5.1.7 to 5.1.10** are shown the generated correlations between viscosity, compressibility factor, upstream pressure and temperature with the speed of sound. In the simulation program, sound velocity was calculated using $c = \sqrt{\frac{kP_1}{\rho}}$.

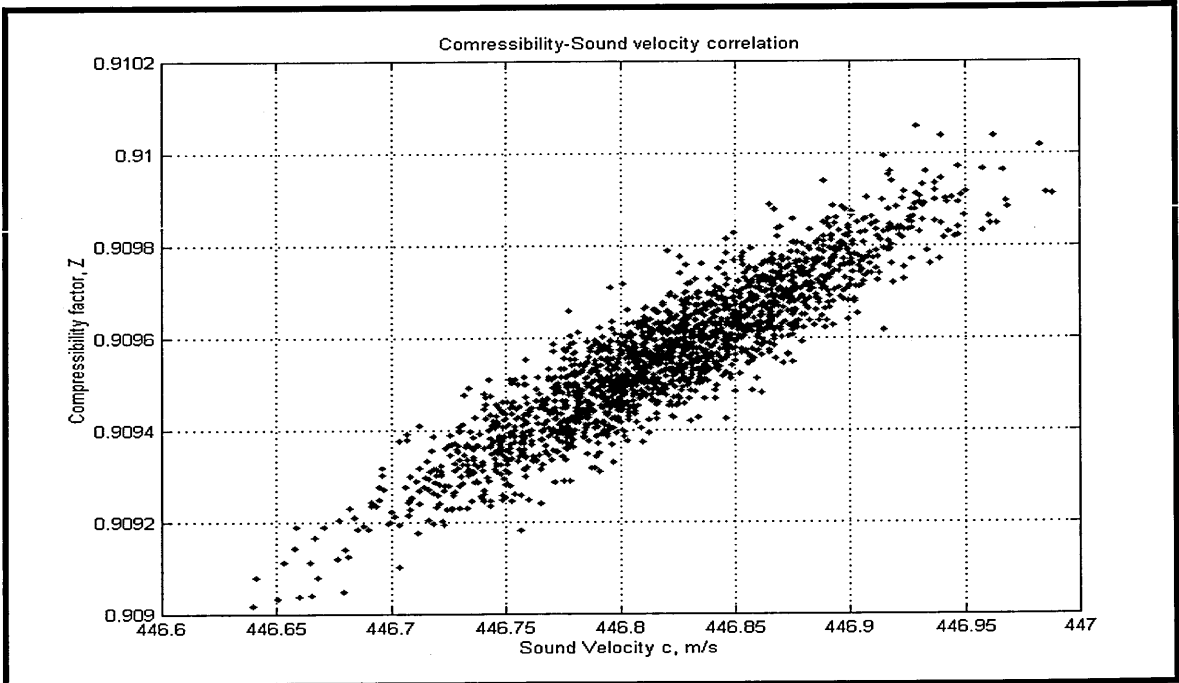


Figure 5.1.8 Line compressibility factor – Sound velocity correlation obtained.

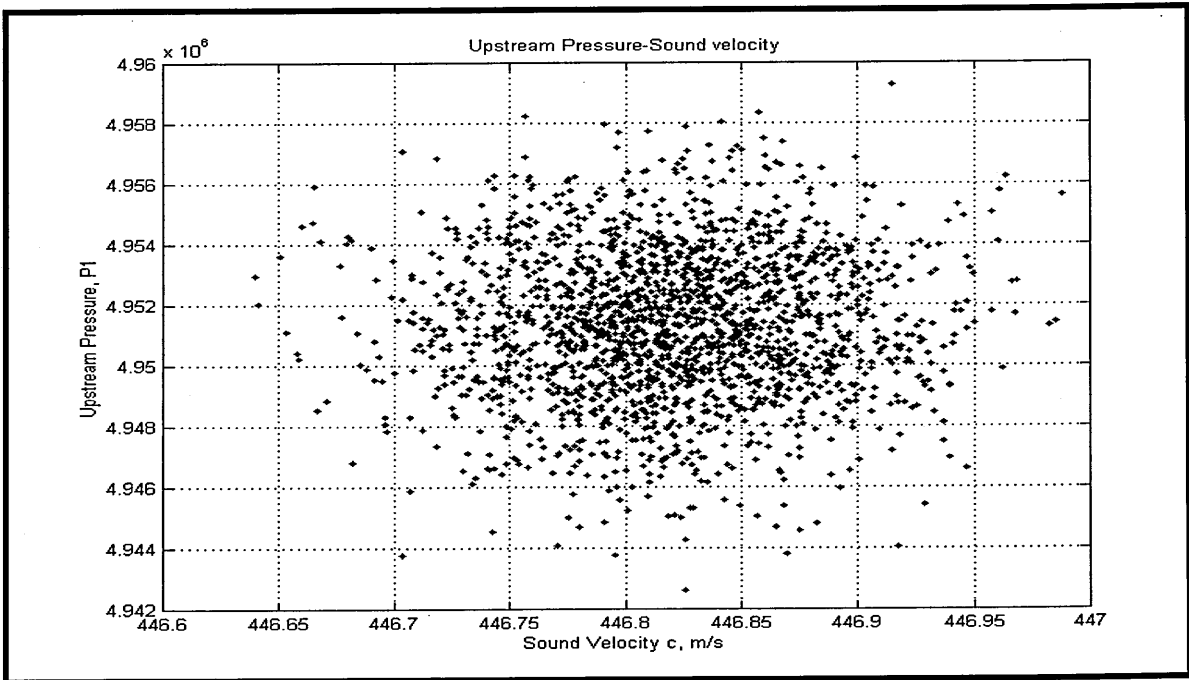


Figure 5.1.9 Upstream static pressure – Sound velocity correlation obtained.

Particularly, it is interesting to see in **figure 5.1.7** that a small **convexity** can be distinguished in a possible obscured relationship between viscosity and sound velocity.

In **figure 5.1.9** it can be seen that sound velocity did not show any correlation with pressure while it acquired a high correlation with temperature. The respective density and isentropic exponent variations that actually de-couple the sound velocity pressure dependence can explain this fact.

Density can be obtained from various model-based measurements. Either from state equations' combination under different conditions, $\rho_1 = \rho_{Ref} \frac{Z_{Ref} T_{Ref} P_1}{Z_1 T_1 P_{Ref}}$ or simply from a

state equation, $\rho_1 = \frac{P_1 MW}{Z_1 R T_1}$. Both models are included in the program giving different

estimated uncertainties. So the program can be also used for testing of alternative models.

Whatever is the case when density enters the previous velocity of sound equation, its pressure dependency "weakens" and c is actually given by: $c = \sqrt{\frac{k Z_1 R T_1}{MW}}$.

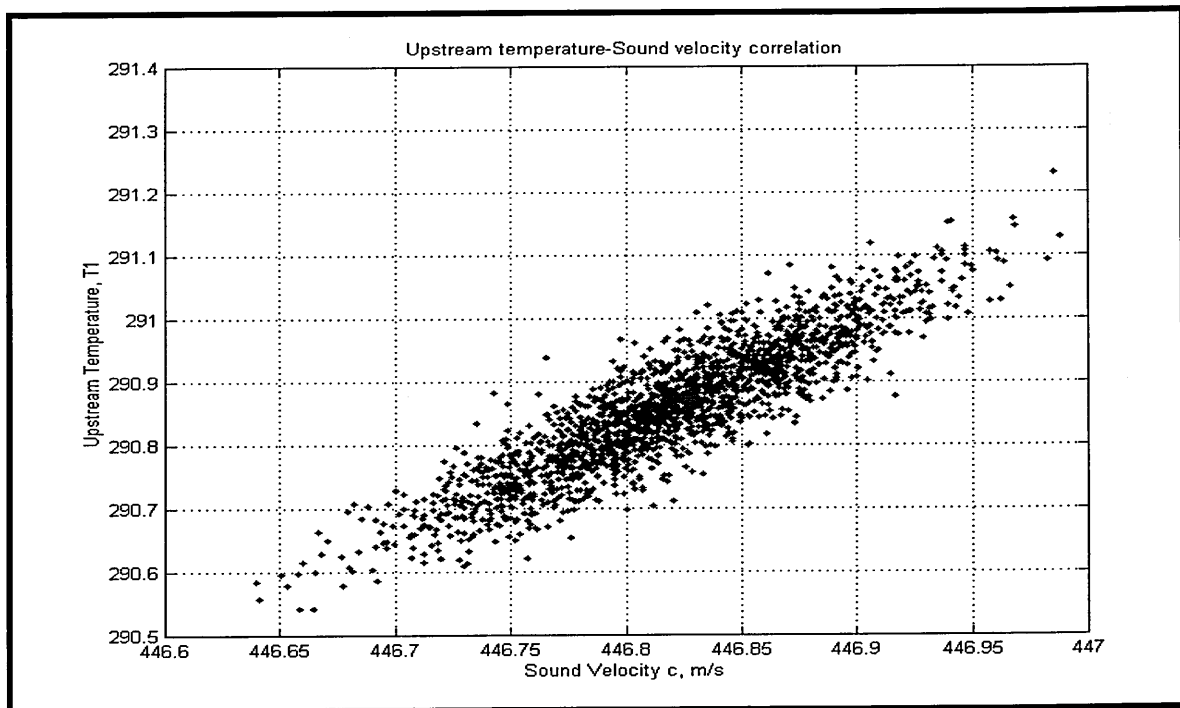


Figure 5.1.10 Upstream Temperature – Sound velocity correlation obtained.

The word 'weakens' was used because although k and Z_1 are also model-based measured from pressure and temperature, generally the temperature dependence of sound velocity predominated. The sound velocity temperature correlation obtained is shown in **figure 5.1.10**. The RSS method mass flow rate uncertainty results have been obtained using the next practical formula present in ISO-5167:

$$\frac{u(Q_m)}{Q_m} = \left[\left(\frac{u(C_d)}{C_d} \right)^2 + \left(\frac{u(\varepsilon)}{\varepsilon} \right)^2 + \left(\frac{2\beta^4}{1-\beta^4} \right)^2 \left(\frac{u(D)}{D} \right)^2 + \left(\frac{2}{1-\beta^4} \right)^2 \left(\frac{u(d)}{d} \right)^2 + \frac{1}{4} \left(\frac{u(\Delta P)}{\Delta P} \right)^2 + \frac{1}{4} \left(\frac{u(\rho)}{\rho} \right)^2 \right]^{1/2} \quad \dots 5.1.7$$

This formula of course considers all the elementary measurements as independent and does not take into account input uncertainty correlations.

5.1.1.4. Monte Carlo simulation results obtained for an interesting elementary test case of uncertainty estimation.

Before presenting one of the most interesting cases studied with the aid of the Monte Carlo software developed, it is necessary to explain the comments made in the end of section 4.2.2.1 about the role of Jensen Uncertainty inequality.

Jensen's Uncertainty inequality can be applied here in terms of a general measurement equation expressing a measurement process, as described in chapter 4:

$$E(f(X_i)) \neq f(E(X_i)) \quad 5.1.8$$

In words, it express that *the end result of a measurement process that combines elementary measurement processes is different from the result of a measurement process of combined elementary measurement processes results*. Simply, as it is employed at this moment, it specifies the difference of pre-processing elementary measurements before we obtain a combined measurement result or post-processing the result after the combination of elementary measurements. Equality holds when the measurement system is **linear**, which practically means that the ultimate result of the combined measurement of two elementary measurements is simply the superposition of each elementary measurement result obtained individually. As it was shown and proved in section 4.2.2.1, such a linear system is the one where the measurement result is obtained from the addition or

subtraction of two elementary measurement processes. To the contrary, a measurement process where the result is obtained from the multiplication or division of elementary measurement processes may or may not be linear, due to the possible correlated uncertainties shared by the elementary measurement processes, as it was shown in section 4.2.2.1.

The case of the effects of normalisation process of the gas composition, as it has been reported by **Basil, M., and Jamieson, A.W., 1998**, falls into the second category. As they stated, when one gas constituent is dominant, such as methane (which is actually a common case for the known natural gases distributed in Europe e.g. Northsea, Norwegian, and Russian natural gases), normalisation leads to a reduction in the spread of uncertainty results and an offset in the final density from the un-normalised result.

Normalisation can actually be seen as an elementary process within the gas energy metering system. The following gas composition in mole % was used to analyse these effects: **C1: 95.5%, C2: 2.8%, C3: 0.5%, NC4: 0.05%, IC4: 0.05%, NC5: 0.03, IC5: 0.02, NC6: 0.005%, CO2: 0.054, N2: 0.6%** with respective components uncertainties, **UC1: 0.1%, and UC2,UC3...UnC6,UN2,UCO2: 0.02%**. The results obtained in this project actually come to confirm the reported results of the authors above, and to offer certain explanations to the effects they reported by employing also the results obtained in section 4.2.2.1.

Two different cases can actually appear in practice. One case is when the un-normalised sum of mole % of gas components is less than 100%. In the above gas composition, the sum of the concentrations of all the components add to **99.6%**. Normalisation to 100% is

usually carried out by means of the next equation:
$$X_n = \frac{X_u}{\sum_{i=1}^k X_u} 100 \quad 5.1.9$$

where X_u is the un-normalised mole % of substance X, $\sum_{i=1}^k X_u$ is the summation of the un-normalised mole % of all the gas components, k is the total number of gas components and X_n is the mole % of substance X after normalisation, which is used in further calculations (MW etc.). This equation is applied for each component separately. The following Monte Carlo simulation results of normalisation for the above gas composition have been obtained and compared with the un-normalised results.

Un-normalised Sum of gas components mole % less than 100% - $\Sigma X_u=99.6\%$, $U\Sigma X_u=0.095\%$										
Mol%	C1	C2	C3	NC4	IC4	NC5	IC5	NC6	CO2	N2
Unor.	95.5	2.8	0.5	0.05	0.05	0.03	0.02	0.005	0.054	0.6
Norm.	95.875	2.8109	0.502	0.0502	0.0502	0.0301	0.0201	0.00502	0.05421	0.6024
%Unc.	C1	C2	C3	NC4	IC4	NC5	IC5	NC6	CO2	N2
Unor.	0.1	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Norm.	0.0041	0.096	0.097	0.097	0.097	0.097	0.097	0.097	0.097	0.097

Table 1. Monte Carlo simulation results for the study of normalisation for $\Sigma X_u=99.6\%<100\%$.

In figure 5.1.11 is shown the correlation of the un-normalised mole % of C1 (dominant component) with the summation of the un-normalised mole % of all the gas components (nominator versus denominator of equation 5.16). Both C1 (modeled input with a normal distribution) and ΣX_u (summation of uncorrelated normal distributions) are normal random variables which as it can be seen, are nearly perfectly positive correlated ($\rho_{C1,\Sigma X_u}=0.999981$).

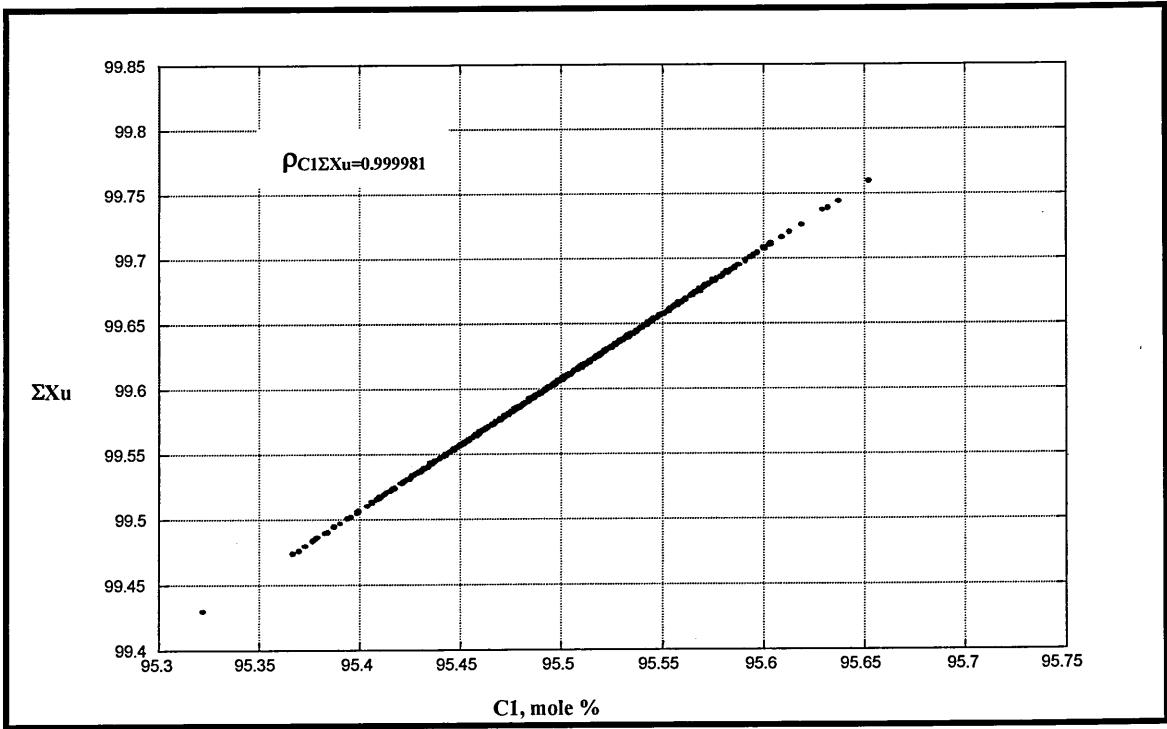


Figure 5.1.11 Un-normalised mole % of C1 versus summation of the un-normalised mole % of all the gas components.

Additionally, it can be seen in **table 3**, that the relative uncertainty of the un-normalised C1 obtained, $UC1/C1 = 0.1/95.5 = 0.001047$ was higher than the respective relative uncertainty of the summation of the un-normalised gas components, $U\Sigma X_u/\Sigma X_u = 0.095/99.6 = 0.000954$. These results, classify this particular case as similar to the one shown in figure 4.29. There, it was obtained the uncertainty of the result $y = x_1/x_2$ (division of two positively correlated Gaussians) when there are nearly perfect positive correlated input uncertainties and $u(x_1)/x_1 > u(x_2)/x_2$. This led to a **concave** dependency of the result y with both the inputs and consequently in an uncertainty reduction. In the end, what is actually happening is an *almost canceling* of the two nearly similar in magnitude input uncertainties $UC1 = 0.1\%$ and $U\Sigma X_u = 0.095$ (or better nearly similar in magnitude relative uncertainties) down to an uncertainty of $0.0041\%!!$ This canceling of uncertainties actually, seems to correspond to the case reported by Basil, M and Jamieson, A.W., 1998, in which they had properly stated that it can become apparent only when applying Monte Carlo simulation and cannot be seen by any other method.

Additionally, also in table 3, it can be seen that the uncertainty of all the other components has significantly increased, becoming nearly the same for all of the components and around **0.097%**. This is happening because all the rest of the components become **uncorrelated** with the un-normalised sum ΣX_u and the resulting uncertainty of the division with it becomes as high as the uncertainty of the summation $U\Sigma X_u = 0.095\%$ which is the predominant uncertainty.

In terms of the shift of the gas density that also reported by the above authors, this is actually due to a respective shift in the normalised molecular weight. Consequently, this

is due to the multiplication factor $\frac{100}{\sum_{i=1}^k X_u}$ of the un-normalised gas components, which is

higher than 1, if the summation is less than 100%, as was in the case developed here. This ultimately led to a positive shift in all the components concentrations as shown in table 3.

In case that the sum of the un-normalised gas components is more than 100%, normalised concentrations become **smaller** than the respective un-normalised ones leading to a negative shift in the molecular weight and a respective negative shift to an equation of state based density measurement. However, all the other effects are similar in both cases.

5.2. Modeling and Methods effects on measurement uncertainty estimation -

Discussion .

As it was developed in chapter 4, general equations 4.1 or 4.2 usually offer a simplified (abstract) representation of the physical (actual) reality in terms of a quantity Y that belongs to the physical world image. The general representation of a measurement process through equations 4.1 and 4.2 implies the possible existence of various *specific* formulations/models for the same measurand Y. Generally, this kind of uncertainty can be appointed to two main sources.

- Uncertainty in structural models' formulation/evaluation domain.
- Uncertainty in models' application/evaluation domain.

Careful considerations of uncertainties associated with the formulation of specific mathematical models' (data reduction equations) are required for a particular measurement process. For the same measured quantity several different *specific* measurement model/equation structures may be available, each corresponding to different physical/mathematical concepts, assumptions, e.t.c, and in the end, their structure generally depends on the level of detail that has been considered during their formulation. Often models are simplified for computability purposes. Such simplifications, for example, include assumptions for the conversion of complex non-linear models to simpler linear models in the parameter space of interest. In such cases, the uncertainty in the results of the simplified models can be tested for example by comparison to the results of more detailed models. Different available model based measurements generate different interdependencies and inevitably different uncertainties. Alternative available model formulations that give different conclusions inevitably suggest for further model evaluation. Uncertainties associated with measurement data used for model evaluation are an additional source of model uncertainty.

Moffat, R.J., 1988, has already said that *an uncertainty estimate is only as good as the equation(s) it is based on*. If those equations are incomplete and do not acknowledge all the significant factors that affect the result or exaggerate some of them, the analysis will give false results.

In order to estimate uncertainties associated with model formulation, several different models available or future developed, each corresponding to a different formulation of

the mathematical problem corresponding to the original physical system, could be tested. This is the best option available at this moment. Obtaining results corresponding to several alternative possible models/methods could give a raw estimate of the associated model uncertainty.

Model uncertainties can also be classified as *subjective* and *objective*. *Subjective* uncertainties are those uncertainties that are generally due to lack of knowledge by the experimenter and can be reduced by improved measurement methods, improved instrumentation and improved model formulations. *Objective* model uncertainties generally reflect the current knowledge and thus cannot be reduced for the time being. This could possibly change if improved theories and further knowledge describing the phenomena under consideration become available in the future.

Generally, in complex measurement systems both physical and model based measurements are usually combined in order to obtain the measurement result. In most of these cases, model based measurements share common physical measured variables and measurements respectively. This situation appears more than often in modern measurement processes, which definitely have become more complex. One of the main contributions of this project comes after the recognition that although physical measurements may be really independent, thus not sharing common sources of systematic or random uncertainties (zero uncertainty covariances), it does not happen the same with model based measurements sharing common physical measurements. When such measurements combined in a complex measurement process, correlated uncertainties can lead to generated non-linearities additional to those already present in the measurement models' structures in the form of elementary measurements' exponents. All the effects of such correlated uncertainties have been graphically presented in a way that has never been demonstrated before, with the aid of Monte Carlo simulations and the use of joint probability densities of the various results with their inputs. It has been also shown that the general form of the law of uncertainty propagation, in the case of multiplication (or division) of two input measurements, must be modified to take into account the asymmetry of the output distribution. Division has been left out from these considerations because it can be actually treated similarly as the multiplication case although it performs particular interest.

The methodology for uncertainty analysis developed here is not exhaustive when practical applications are considered. In practise, residual systematic errors are always left after calibrations. However, they are relatively small if proper traceable calibrations are carried out. Additionally, if these are modelled as uniform distributions, they can be treated by Monte Carlo uncertainty estimation method in a way similar to this already presented for normal input distributions and they do not affect the general concepts of the developed methodology.

There is a long list of relevant important methods and concepts that can be associated with input measurement uncertainty correlations and their physical and practical interpretation. It can be generally said that in a measurement system it is always desirable to obtain some economy of the physical redundancy (instrumentation used) though without loosing any necessary information. It is actually this balance between physical and analytical redundancy that calls for exploitation based on a total quality management approach. These issues will be developed in chapter 7. At this moment, they could be mentioned some of those methods available that mainly exploit analytical redundancies, targeting in generating a subset of the original variables that carries the same amount of information as the original variables. The *Orthogonal transformations* (as they generally called) like Fast Fourier transforms (FFT), Wavelets and Gabor transforms (Short time Fourier transforms) are among the most popular. These are applied to measurements that have a temporal or spatial order. Additionally, methods that employ the measurements' covariance matrix and aim to find a transformation to new variables that are uncorrelated are the *Principal Component Analysis (PCA)* and *Factor Analysis*, with the later directly seeking a smaller set of variables that can explain the covariance structure of the measurements. In factor analysis the variables are transformed using a specific algorithm, and this is equivalent to a rotation of axes of reference. As it was already developed in chapter 4, elementary and group measurement uncertainties can be also represented with vectors and uncertainties analysis can be carried out by employing vector calculus and respective orthogonal transformations.

It could be easily realised and stated that all these considerations actually call for a new way of thinking about uncertainty estimation, not only as a tool for measurement systems

performance evaluation in general, but also in particular on its potential use as measurement's process and systems optimisation tool.

Thus, the reliable estimation of measurement uncertainty becomes necessary for the selection of the most appropriate metering system(s), its design optimisation and subsequently for the continuous (online/real time) **monitoring, evaluation and optimisation** of its performance.

It is well known that uncertainty has been also associated with the other important concept of entropy (thermodynamic and information). In general, entropy is a measure of some property of a system or process that has inherent uncertainty. Entropy and uncertainty can both be seen from a similar perspective, that of information content or knowledge. Because in the end, *what measurements are all about is this knowledge acquisition about a physical process.*

Another also relevant concept from thermodynamics that is necessary to be mentioned for a more complete picture of all the results, including those that will be presented in the next chapter, is that of *State postulate*. The state postulate actually fixes the number of independent properties of a system in a stable state. It says that the stable equilibrium state of a system is uniquely determined by all its quasistatic work parameters and its energy. So, considering the state postulate, it can be said that **it sets a target of a successful factor analysis**. It is remarkable as well to note that the state postulate was formulated as late as 1956 in a paper by **Kline, S.J.**, and **Koenig, F.O.**, with the first author being also a pioneer in combined measurement uncertainty analysis(!), with the publication of some classical papers on the subject as early as in 1953.

6. ORIFICE PLATE FLOW MEASUREMENT AND MONITORING

6.1. New equations for Pressure loss and Discharge coefficient of orifice meter –Interrelationships between the various coefficients and variables

6.1.1. Interrelationships in orifice flow metering.

A research program initiated in Cranfield University since 1995 into possible methods for on line monitoring of the performance of flow meters. Under this PhD project, multiple new models have been developed through theoretical and experimental work that associated the discharge coefficient of orifice meters with the contraction coefficient and the pressure loss coefficients since all the coefficients are related. A summary table of the equations developed in this project and of previously existed equations is present in **Appendix B** along with a theoretical derivation of them.

We could generally say that in high Reynolds number flow through the orifice, inertial forces predominate while viscous forces have only historical effects and are quite negligible. So the theory has been developed on the relatively accurate assumption of inviscid flow. Another important feature of this theory is that of irrotational flow and of the use of Impulse theorem as it is described by **Lighthill, J., 1986**.

For better understanding of the equations developed, some additional remarks are required about the coefficients. **Zanker, K.J., 1961** since the early sixties had pointed out the main objections regarding discharge coefficient that arise from its lack of physical significance. He had also strongly emphasised that the contraction coefficient is the most important factor in orifice flows and its observed variations could be used to explain not only the C_d values but to describe also the flow conditions across the orifice. As he also mentioned, the idea that the contraction coefficient is related to the momentum deflected by the orifice plate was at that time to a certain extent justified both physically and numerically. However, it was not giving a complete picture. So, the inclusion of the effects of contraction and of all the other effects into a single correction coefficient (C_d) was the general practice that was followed. As a result of the above, the contraction coefficient has been neglected for a long time in most publications and the most practical discharge coefficient was adopted in all cases. Actually, Zanker had obtained theoretical formulas for all the coefficients (Loss, Discharge, and Contraction) that agree with **all** the rest of the formulas developed by

the present author. However, it seems that they did not received the proper attention the following years.

The extensive use of C_d as the correction coefficient of every single effect inevitably created some fundamental questions. For example, Zanker also mentioned that there was no reason for assuming that C_d remains the same in both compressible and incompressible flow. Furthermore, the followed practice of multiplying any additional correction factor for such effects as tap position, pipe roughness e.t.c by C_d is accompanied by the *unjustifiable assumption that the relationship between C_d and the variables entering these correction factors is one of independent variables.*

From Monte Carlo simulation results presented in the previous sections, it was made clear that input correlated uncertainties might have significant effects in flow measurement results. In fact, as **Nicolis, G, 1995** has analysed, all the equations constituting the closed set of fluid dynamic equations, feature a universal non-linearity of the form $u\nabla x$, x being ρ (mass density), u (velocity) or T (temperature). This actually reflects the fact that *the properties of a fluid are transported by the fluid velocity – itself a fluid property*. This mechanism is known as the "*hydrodynamic feedback*". The presence of this mechanism can be realised in an orifice metering system where the discharge coefficient, is a function of pipe Reynolds number that is a function of pipe velocity, forming an iterative calculation procedure.

Engel F.V. et al., 1964, has also adopted similar ideas to Zanker and he also mentioned a heuristic principle introduced by A. Einstein namely, that "constants or coefficients in mathematical equations related to physical phenomena should be of the order of unity". This statement has been repeatedly quoted in connection with dimensional analysis. This virtually means that when we assume that a relationship exists between certain quantities, then by dimensional analysis we can find what the form of the relation may be. Substituting then into the relation the numerical values of the physical quantities we get the numerical value of any unknown residual coefficient. If this coefficient is of the order of unity (meaning actually not extremely big) the suspected relation is quite probable and so we continue to discover what the precise relation between the physical quantities may be. To the contrary, a very large or very small coefficient suggests a quite improbable relation in the assumed form between the physical quantities in question.

Experimental measurements that were obtained from a water flow loop in Cranfield are presented next. These measurements were also used by the author on a research project of the flow monitoring capabilities of fast response pressure transducers. They have been further analysed from a different point of view mainly to examine and to discuss some of the theoretical models developed in Appendix B as well as additional aspects originating from them. Further research is necessary for their complete validation and potential applications.

6.1.2. Description of the flow rig and data acquisition.

Figure 6.1 shows the layout of the main components of the rig. The water was circulated via a tank contained approximately 2.5m^3 of water using a pump rated at 2900 rpm that produced a maximum head of 350Kpa. The tank was occasionally drained, cleaned and refilled in order to ensure clean water supply for the measurements. Water temperature at the tank for all measurements was $20 \pm 2^\circ\text{C}$. About 26D downstream the pump outlet, a 3" turbine reference meter calibrated in Cranfield was used and its calibration curve is shown in **figure 6.2**. This had an estimated uncertainty of less than $\pm 1\%$ of reading excluding installation effects since it was calibrated against a 4" turbine meter which had a certified uncertainty of around 0.5% of reading and was calibrated in another laboratory.

The orifice plate test section was 4" ABS pipe and was started with a single 90° bend and three pairs of flanges attached 6D, 20D and 40D downstream the bend. A sharp edged orifice plate was used having a beta ratio of 0.68 with D, D/2 pressure tapplings that virtually coincided with the Vena Contracta tapplings as shown in **figure 6.3**. Thus we could say that the measured differential pressure in all cases was approximately the maximum and according to the arguments expressed regarding the numerical results for the loss coefficient must be quite realistic. Four of the nine tests that were actually carried out have been further analysed in this project for the preliminary study of the resulted equations. In tests 1 and 2 the orifice plate was positioned 40D downstream the single 90° bend while in test 1 a Mitsubishi flow conditioner was positioned 6D downstream the bend as well. In tests 3 and 4 the orifice plate was positioned 20D downstream the bend and unlike in test 3 a Mitsubishi flow conditioner was installed 6D downstream the bend as well. Absolute

static pressure transducers of strain gauge type, with an effective frequency response up to 3KHz were used. Their I/O ranges were 0-10bar/0-10Volts respectively. They provided separate pressure signals at the upstream and downstream tapping of the orifice plate and upstream to the Mitsubishi conditioner. For the collection of the data a PC with a 25MHz-processor clock speed was used and a 12bit analogue to digital converter (ADC) while the sample operation was performed by a Sample and Hold amplifier inserted prior to ADC with a typical track time 2 μ Secs. In order to obtain the widest possible frequency response from the unit, no low pass filter was installed in the signal conditioning circuit and a sample rate 8KHz was used.

Various time records from the pressure transducers were obtained within a mass flow range of 19.7 to 40.6 Kg/sec.

During the measurements, the data collection was starting when flow meter readings had been stabilised, although some transient signals were also recorded due to some unexpected "kicks" in the flow control valve during some measurements. These signals have been discarded from the calculation of the ensemble average values of static and differential pressures however some non-stationarity was present in most of the signals, as it was proved by further analysis. This was mainly due to the valve's performance.

6.1.3. Presentation and analysis of the results – Discussion

In **tables 6.1a,b** are shown the orifice plate theoretical calculations of the various factors for the reference (turbine meter) flowrates and so they are common for all the tests. In **tables 6.2a,b** are shown the orifice plate experimental results for **Test 1** if we had assumed that standard conditions (fully developed velocity profile, e.t.a., according to ISO5167) were prevailing upstream the orifice. In other words, the flow rate was measured using the Reader/Gallacher equation for the calculation of Cd and the measured differential pressures across the orifice. This would have been actually the case of the use of an uncalibrated orifice meter and presented only as a sample for comparison purposes with the actual case described next. Finally, in **tables 6.3a,b** to **6.6a,b** are shown the actual results (calibrated Cd) for the **Tests 1** to **4**, for the reference flow rates, as well as the respective calculations of all the other factors according to the model equations developed in this project. In **table 6.1a**, column 1 is

the turbine meter frequency and in columns 2 and 3 the respective reference volume and mass flow rates of the tests. Reynolds number (Re_D) in column 4 has been calculated by iteration using:

$$Re_D = \frac{4q_m 10^6}{\pi D \mu}$$

where q_m in Kg/s, D pipe diameter in mm, and μ dynamic viscosity in centipoise.

The differential pressure (column 7) was calculated also by iteration according to ISO5167. More importantly, discharge coefficient, in column 6, was calculated using the new Reader-Harris/Gallagher (1998) equation as has been adopted in the new ISO/CD-5167, 1999 and is:

$$C_d(\infty) = 0.5961 + 0.0261\beta^2 - 0.216\beta^8$$

$$C_d(Re) = 0.00521 \left(\frac{10^6 \beta}{Re_D} \right)^{0.7} + (0.0188 + 0.0063A)\beta^{3.5} \left(\frac{10^6}{Re_D} \right)^{0.3}$$

$$C_d(\text{taps}) = (0.043 + 0.08e^{-10L_1} - 0.123e^{-7L_1})(1 - 0.11A) \frac{\beta^4}{1 - \beta^4} - 0.031(M'_2 - 0.8M_2^{1.1})\beta^{1.3}$$

where $A = \left(\frac{19000\beta}{Re_D} \right)^{0.8}$, $M'_2 = \frac{2L'_2}{1 - \beta}$ while for D, D/2 tappings $L_1=1$ and $L'_2=0.47$.

So, finally,

$$C_d = C_d(\infty) + C_d(Re) + C_d(\text{taps}).$$

In figures 6.4 and 6.5 are plotted the theoretical discharge coefficients calculated for the reference Reynolds numbers from both the Stolz and Reader-Harris/Gallagher equations and their percentage difference. This is actually the percentage difference in the flow calculation from using one or the other. It is clear that a significant correction has been introduced with the new equation. In figure 6.6 it is shown the variation of the theoretical pressure loss calculated using the old equation (9d, in equations table/ Appendix B) and one of the new equations developed in this project. It must be said that a particular form of them (9b) is included in the new ISO/CD-5167, 1999. In figure 6.7 it has been drawn, using the old and new equations, the variation of the orifice pressure loss coefficient with the reference Reynolds number. We can notice in this case, that 1.97 would be a more representative value for the exponent x if the approximation formula (9h) is to be used. The theoretical estimated loss coefficient

was around 5.8 while the theoretical vena contracta was estimated around 0.055m. So the reduction of the flow area compared with the orifice throat was estimated around 34.6%. The estimation of probably one of the most important parameters, the pressure loss parameter in column 17, revealed that a pressure loss 54.6% of the maximum differential pressure is expected under standard upstream conditions in this case. In **table 6.1b** all the calculated parameters and coefficients are non-dimensional and are virtually interrelated. Their formulation and respective interrelations are shown in the equations' summary table in the end. Subsequently, a more complete picture has been obtained. Zanker had very properly pointed out that “ *If C_d could be brought to a more rational basis, our general understanding of flow through orifices would be increased, prediction of C_d values under non standard conditions or cases outside our field of practical experience would become possible and general tolerances on C_d could be reduced.* ”. He added also that a much better understanding of C_d is necessary to examine the possible inter-dependence of variables and C_d .

In the results for test 1 shown in tables 5.2a,b, we have assumed standard conditions upstream the orifice. The laboratory flow rig used, despite all the efforts to meet ISO-5167 standard specifications and recommendations, could not be characterised as a truly “Standard installation” because of the various limitations to design a flow metering system in a laboratory “scale” following such strict specifications. However, we can put the necessary arguments in order to get a more complete view to the insight of the existing problems that have been extensively described in chapter 1. These could actually arise after a long term use of an uncalibrated orifice meter. The fact that the orifice was installed 40D downstream the 90° bend and the flow conditioner (Mitsubishi) was positioned 20D downstream the bend could have ensured good flow conditions upstream the orifice according to ISO-5167 current specifications regarding upstream straight lengths. So, to a certain extent, the use of the uncalibrated orifice meter would be reasonable in this case. However, as we can observe, a flow calculation even in this “good” case (that virtually represents sometimes the normal practice followed by the industry in standard installations) would lead to a significant systematic underestimation (around 4%) of the flowrate as shown in **figures 6.8 & 6.9**. The intention in this moment though, is not to analyse the causes of the above systematic error itself but mainly to use it as an argument for comparing with a similar case that could gradually take place in any standard industrial flow metering installation. Such a case could be any of those that have

already been described in section 2.3.1 and have to do mainly with the effects of deterioration of the metering installation (ageing). Those effects that are usually reflected to a distorted velocity profile or other systematic deviations would lead eventually to an undetectable systematic error under the current applied practice.

In other words, as it become evident from the above arguments, the apparent and everlasting trust of a correct flow computation based only on the assumptions of standard installations can not be fully justified in practice. There is no need to mention of course the case of non-standard installations. The important conclusions of all these arguments are that changes in the process in most cases can not be detected by the metering systems under the current operation practice that rely on the use of uncalibrated orifice meters. More importantly, and as could be seen if compare the results of tables 6.2a and 6.3a, we could not acquire this *capability of detection* (ISO 11843-1,1997, ISO/DIS 11843-21) of any systematic error even with the use of the various models developed in this project. The fact is that these effects will be similarly “masked” in these factors as well.

Two “axioms” have been emphatically expressed by Keyser, D.R., 1998. The first is that no flowmeter can be more accurate that the laboratory which calibrated it and the second more relevant is that “*..no uncalibrated flowmeter can be more accurate than a like one that is calibrated in a recognised standards laboratory*”. He also pointed out the fact that mass flow rate, unlike many other physical quantities, is not a single measurand which can be traced to an international standard (in the sense of an artefact) since flow calibration laboratories are systems of instruments and apparatus. He finally added that systematic uncertainties that are not traceable could result from the fact that *apparatus, installation and plant are unique to each laboratory*.

Despite all the above, all these models developed can be proved extremely important and practically useful for the on line monitoring and control of the performance of a calibrated metering system that employs orifice meters, leading possibly to a further reduction of the combined measurement uncertainty. Additionally, important diagnostic information about the flow through the orifice could be obtained. These later are based on the fact of the early detection of metering systems drifts and on the given permission for built-in self-correction that could be possible with the proper use of the various models. Some of the latest capabilities can be revealed now after comparing the results for test 1 with the results of the other tests. So, test 1 could be

regarded as the calibration signature of this particular meter and the respective tables 6.2a,b as its reference tables.

The removal of the flow conditioner in Test 2, once nothing else has been changed compared with test 1, could be seen as the removal of a resistance from the system. Something similar possibly would be the full opening of a partially closed valve 20D upstream the orifice meter. The effects of this action can be further analysed.

We should notice that the recorded differential pressure increased, while discharge coefficient decreased (**figure 6.10**), more prominently in the lower flowrates, suggesting a quite improved velocity profile and a lower acceleration than in test 1 (C_d is still higher than the theoretical value). Normally, and after the last considerations, we would expect a higher contraction (a smaller vena contracta diameter) something that was actually predicted by the models developed and can be shown in **figure 6.11** or comparing the columns 15 of tables 6.3b and 6.4b. Additionally, and because of the smaller vena contracta we would expect a higher pressure loss coefficient and this also has been predicted by the new models. Similar conclusions, that are in full agreement with our current knowledge regarding orifice meter performance, can be drawn also for all the other models and parameters in all the tests, and they really give a strong support to the validity of the models developed. The benefits and the improved capabilities that could arise from a combine use of these models with a calibrated meter or even better with a permanently installed check meter (watch-dog) could be significant. A similar practice has been adopted recently by Gasunie under a big renovation of their export stations (**Van der Kam, P.M.A., & Dam. A.M., 1993**) while Nilsson U.R., et al., (1997) presented a *fingerprint* method for the in situ detection of inaccurate domestic gas flow meters.

6.2. An interesting mathematical function and additional forms of measurement equations - Discussion.

Lord NAPIER in 1614 introduced first the logarithms with base e and so these logarithms were named after him. The same logarithms are called as well Hyperbolic logarithms or otherwise Natural logarithms. The reason is that the natural logarithmic functions are closely related with the hyperbolic functions. The decimal logarithms introduced quite later from another mathematician named Briggs.

By definition the hyperbolic sine (sinh) is: $\sinh x = \frac{e^x - e^{-x}}{2}$, while the hyperbolic

cosine (cosh) is: $\cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{2n!}$ after expanded in a Taylor series.

Now if the variable x is actually a hyperbolic function itself (natural logarithmic function) we obtain for $x > 0$ the next very usefull and important functions according to the authors point of view:

$$x - \frac{1}{x} = 2 \sinh(\ln(x)) \Rightarrow \frac{1}{x} = x - 2 \sinh(\ln(x)) \text{ and}$$

$$x + \frac{1}{x} = 2 \cosh(\ln(x)) \Rightarrow \frac{1}{x} = 2 \cosh(\ln(x)) - x$$

There are many reasons why the author regards the last two equations very important. Some of them will be now presented. First of all, the fact that this function of a function (compound) relates a number with its reciprocal sounds very interesting in itself. And becomes more interesting by the fact that reciprocity as is a concept that is met very often in physics (Space/time, e.t.a.) and lies to the very heart of the physical sciences. We will present next for the reader some interesting properties and expressions that have been obtained for this peculiar function:

We could very easily obtain

$$a) \quad x^2 = 2x \cosh(\ln(x)) - 1 = \left(2 \cosh(\ln(x)) - \frac{1}{x} \right) x$$

and see how the square of a variable can be obtained by a linear function of the form $y = ax$ where the parameter a (slope) is obtained by the subtraction of two non-linear functions of x . For higher powers of x similar more complex forms can be obtained for example $x^3 = (4 \cosh^2(\ln(x)) - 1)x - 2 \cosh(\ln(x))$ of the form $y = ax + b$ where both a and b respectively are non-linear functions of x .

$$b) \quad \cosh(\ln(x)) + \sinh(\ln(x)) = x$$

$$c) \quad \cosh(\ln(x)) - \sinh(\ln(x)) = \frac{1}{x}$$

$$d) \quad \lim_{x \rightarrow \infty} \frac{\cosh(\ln(x))}{x} = \frac{1}{2}$$

However, among the main reasons that it was put an emphasis to the presentation of this peculiar functions are the next:

- In the equations developed and presented in **Appendix B**, functions of reciprocal expressions such as the expression $\frac{1}{\sqrt{P_{LP}}} - \sqrt{P_{LP}}$ appeared in the theoretical derivations.
- More interestingly, it must be noted equation 10f) in which the above expression is present. Squaring both sides of this mass flow rate equation, the previous expression will take the form $X + \frac{1}{X} - 2$.

It is worth mentioning a few important aspects about the last expression and its relevance with the various interrelated concepts and ideas developed in this project. This expression was actually used in the end of section 4.12.1 in uncertainty inequalities expressions developed there. As it was later discovered, it is remarkable that is also associated with stability criteria of finite difference schemes (based on Taylor series expansions) to the solutions of 1D wave (hyperbolic) equations, such as Courant stability criterion (!). Remarkably also, the inequality $X + \frac{1}{X} \geq 2$ is associated with the difference of *geometric* and *arithmetic mean* of two variables. The hidden aspect that makes the whole analysis so important can be revealed after considering that geometric mean is actually used in averaging values which represent a rate of change, such as for example mass or energy flow rates. More importantly, and by examining the definition of geometric mean for a sample $x_1, x_2, \dots, x_n$, given as the positive n th root of the product of the n values, it can be seen that this is the same as the antilog of the mean of the logarithms.

$$\overline{X_{gm}} = (x_1 x_2 \dots x_n)^{\frac{1}{n}} = e^{\left(\frac{1}{n} \sum_{i=1}^n \log(x_i) \right)} = \prod_{i=1}^n x_i^{\frac{1}{n}}.$$

Completing the whole picture, entropy can enter in the whole analysis and discussion, naturally appearing in the exponent of the geometric mean if we replace sample values x_i , with probability values since entropy (and subsequently uncertainty) can be actually defined as a measure of the average amount of information. So, **entropy** can now be defined as **the logarithm of the geometric mean of probability distributions evolved in time**, as it has been developed in section 4.1.3.

So,

$$H(P_n) = \log(\overline{X_{gm}}) = E\left(\log\left(\frac{1}{P_n}\right)\right) = \frac{1}{n} \sum_{i=1}^n \log\left(\frac{1}{P_n}\right) = \sum_{i=1}^n P_n \log\left(\frac{1}{P_n}\right).$$

The last expectation operator of the mean logarithm of probabilities formally introduces and reveals the role and importance of Jensen Uncertainty inequality as it has been already stated. Employing Jensen inequality it can be easily shown that the mean of logarithm of probabilities is always smaller than the logarithm of the mean probabilities or that the geometric mean is always smaller than the arithmetic mean due to the concavity of the log function. Considering an ensemble of probability distributions and starting from an initial probability (a prior), the general tendency of entropy to increase (or to remain constant), could be explained by the fact that as **consequent different** probability values enter the geometric mean formula, they generate a multiplicity of possible states (degenerate states), further increasing the distance of geometric and arithmetic mean values, and subsequently increasing the entropy. Probability distributions of the ensemble that remain invariant with time (definition of stationary process in section 4.1.3) can be defined as **consequent equiprobable events** that asymptotically drive the probability as well as the geometric mean, and subsequently the entropy, to a single constant value (to an isentropic condition). This corresponds to the arithmetic mean of multiple measurements, as the most probable and unbiased estimator. In this case for

$$x_1 = x_2 = \dots = x_n = \frac{1}{P_n} = \frac{1}{k} \text{ we have:}$$

$$\overline{X_{gm}} = (x_1 x_2 \dots x_n)^{\frac{1}{n}} = \frac{1}{n} (x_1 + x_2 \dots + x_n) = \overline{X_{am}}$$

Additionally, Jensen Uncertainty inequality becomes equality. The importance of all these matters is self-evident.

- It can be introduced and analysed under the same sense the appearance of Mach number as another important non-dimensional parameter in orifice flows (compressible and incompressible) since it appears as a consequence of Lorentz transformations, which are generally associated with a hyperbolic rotation in the

algebra through an equation of the form $\tanh \varphi = \frac{u}{c}$

where u is the relative velocity of two reference systems and c in our case is the velocity of sound. Thus, an equation obtained that relates C_d with **Mach** number and the volumetric strain and is:

$$C_d = \sqrt{\left(\frac{D^4}{d_o^4} - 1\right) \left(\frac{1}{\sqrt{1 - M_1^2}} - 1\right) \frac{\gamma P_1}{\Delta P}} .$$

The term $\frac{\gamma P_1}{\Delta P}$ virtually represents the volumetric strain or otherwise the fractional change of volume due to the applied differential pressure.

- It can be examined and analysed the temporal inertial effects that can now be assigned as acoustic effects. This has been carried out with the aid of analysis methods of non-stationary processes (spectograms, non-stationary statistical methods) in order to reveal the relationship of the non-stationarity in the pressure signals upstream and downstream the orifice with the random uncertainties of orifice discharge coefficients.

Using these functions additional forms of the equations have been obtained (see **Appendix B**)

Thus, as it has been revealed in this project, generally the combined uncertainty of Discharge coefficient and subsequently of the orifice flow rate measurement can be viewed as consisting of:

1. A systematic error component due to convective acceleration effects (velocity profile effects) as well as of other systematic effects (tappings position, edge sharpness e.t.a.) and
2. A random error component due to temporal acceleration effects or random temperature variations, etc. The former, although can be removed only with calibration as explained previously, can be monitored on line in a calibrated metering system with the aid of the models developed while the later can be monitored on line as well with the aid of the methods for non-stationary process. These, as a dynamic uncertainty estimation and visualisation method will be presented next.

A formula for the velocity at the vena contracta readily obtained from Bernoulli and

continuity equation is: $u_{vc} = \sqrt{\frac{2 \left(\Delta P_{1,3} - \rho \int_1^3 \frac{\partial u}{\partial t} ds \right)}{\left(1 - \frac{d_{vc}^2}{D^2} \right) \rho}}$.

6.2.1. Random Uncertainty and temporal inertia effects

The temporal inertial term, that virtually expresses the inertial resistance of the fluid to changes in its motion, enters the latest equation as an excess differential pressure term. From this equation we could say that if the fluid is decelerating, $\left(\frac{\partial u}{\partial t} < 0 \right)$, unsteadiness increases the velocity at the vena contracta. The inertial of the fluid resists this deceleration, resulting in a higher velocity estimated by the quasi-steady theory, which neglects the inertia of the fluid. Assuming a constant K_2 parameter as it is given by equation 8a) and subsequently constants C_d , d_{vc} , etc., the results shown in the next table have been obtained for test 1. They were examined, within this sense of excess pressure, if the causes could be detected from the pressure signals upstream and downstream of the orifice. For this reason, a very useful formula that gives the cumulative mean of a signal as a function of two separate terms has been developed and used.

So,

$$\frac{d \left(\frac{\sum_{i=1}^n X_i}{n} \right)}{dn} = \frac{d \left(\frac{\sum_{i=1}^n X_i^2}{\sum_{i=1}^n X_i} \right)}{dn} - \frac{d \left(\frac{(n-1)\sigma X_i^2}{\sum_{i=1}^n X_i} \right)}{dn}$$

The first term, on the right hand side, is the cumulative square over the cumulative sum of the signal. It varies, as will be shown, in identical manner as the cumulative mean. The second term, in every instant, is quite similar to the known coefficient of variance, where σX_i^2 is the variance of the signal. It can be so expressed as a cumulative coefficient of variance. For a stationary random process (or a signal from such a process) with a Gaussian distribution, this cumulative coefficient of variance

must vanish to zero as the measurement sample increases. This is due to the equal probabilities of positive and negative deviations from the estimated mean of a Gaussian process as well as to the $n(n-1)$ term in the denominator in the formula for the variance estimate.

Contraction Coefficient, C_c			0.67635	Constants
Vena Contracta diameter, d_{vc}			0.055924	
Discharge Coefficient, C_d			0.634519	
$\Delta P_{1,3}$ Pa	u_{qs} m/sec	u_{ref} m/sec	exc. ΔP Pa	%Diff. Pipe vel.
29086	2.427	2.441	-342	0.58
34449	2.641	2.639	54	-0.08
39812	2.840	2.837	65	-0.08
43154	2.956	2.969	-376	0.43
48827	3.145	3.167	-701	0.71
53683	3.297	3.299	-58	0.05
59355	3.467	3.497	-1029	0.86
65726	3.649	3.629	699	-0.54
72110	3.822	3.827	-204	0.14
78383	3.984	3.959	996	-0.64
84976	4.149	4.157	-343	0.20
91249	4.299	4.289	426	-0.23
93583	4.354	4.355	-56	0.03
98852	4.475	4.487	-548	0.28
106035	4.634	4.619	702	-0.33
112011	4.763	4.751	573	-0.26
115651	4.840	4.817	1097	-0.48
119298	4.916	4.883	1583	-0.67
124058	5.013	5.015	-106	0.04
u_{ref} : Pipe velocity calculated from the reference meter or allowing changes on K_2 parameter, C_d e.t.a.				
u_{qs} : Quasi-steady pipe velocity assuming constant K_2 parameter and neglecting temporal acceleration.				
exc. ΔP : Temporal inertia term expressed as excess differential pressure.				
$\Delta P_{1,3}$: Measured differential pressure				

Table 2. Excess differential pressure term.

In figures 6.12 and 6.13 are shown the signals obtained from the upstream and downstream pressure transducers of the orifice meter, as well as their respective amplitude spectra, in the case before the last in the previous table where a positive excess pressure of 1583 Pa has been estimated. Except the very high frequency fluctuations observed in the upstream pressure (around 1.5KHz), but at lower

amplitude than the downstream, nothing can be clearly seen about the stationarity of the signals.

It is a fact, that the Central Limit theorem as it is called in short the theorem describing the fact of this vanishing cumulative coefficient of variance mentioned previously, tells us the shape of approached distribution, not the shape of the "approaching" distribution nor the rapidity of the approach. This is illustrated comparing **figure 6.14** of a cumulative mean of a signal obtained for the upstream pressure and **figure 6.15** of a cumulative mean of a simulated random signal with a Gaussian distribution and the same expected mean and variance as the previous one. It can be easily seen the rapidity of the approach in the second signal (after about 600 samples has been 'relaxed' in its expected value) compared with the significant variations of the measured signal. In **figures 6.16** and **6.17** are shown the respective cumulative variances that perform in a similar way. The latest is evidences of the Central Limit effect, that has to do with the degree of approximation to normality and the rapidity of this approach at particular time instants and must be distinguished from the notion of the Central Limit theorem that does not guarantee anything about these concepts.

In **figure 6.18** is shown the cumulative mean as has been obtained for the whole upstream pressure signal of figure 6.14. It is clearly observed this positive tendency in the upstream pressure cumulative mean while the downstream cumulative mean shown on **figure 6.19** seems to have been relaxed in a more steady value.

In **figures 6.20** and **6.21** are shown plotted *spectrograms* obtained for the previous upstream and downstream pressure signals after applying short-time Fourier transform (STFT), also known as the time-dependent Fourier transform (or Gabor transform). The STFT of a sequence $X[n]$ is defined by:

$$X_{STFT}(e^{j\omega}, n) = \sum_{k=-\infty}^{\infty} x[n-k]w(k)e^{-j\omega k}$$

where $w(k)$ is a properly selected window function. The display of the magnitude of the STFT in a time/frequency plot is referred usually as the *spectrogram*, Since it is virtually a 3D plot the magnitude is represented by the darkness of the plot. It could be easily seen their potential as a very good uncertainty visualization method and its evolution with time. The later is better shown in **figures 6.22** to **6.25** where using a *moving time window* it can be clearly seen the evolution of process variability (and

subsequently of the uncertainty) and particularly the passage from a low uncertainty to a higher uncertainty state.

In the STFT analysis of non-stationary signals, the length and the shape of window play an important role. The plots that are presented have been obtained using properly adapted Hamming windows given by:

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi k}{n-1}\right).$$

Other window functions have been tested as well as various techniques for the best possible improvement of the spectrogram resolution.

THE WATER FLOW LOOP (CRANFIELD)

(All the valves are gate)

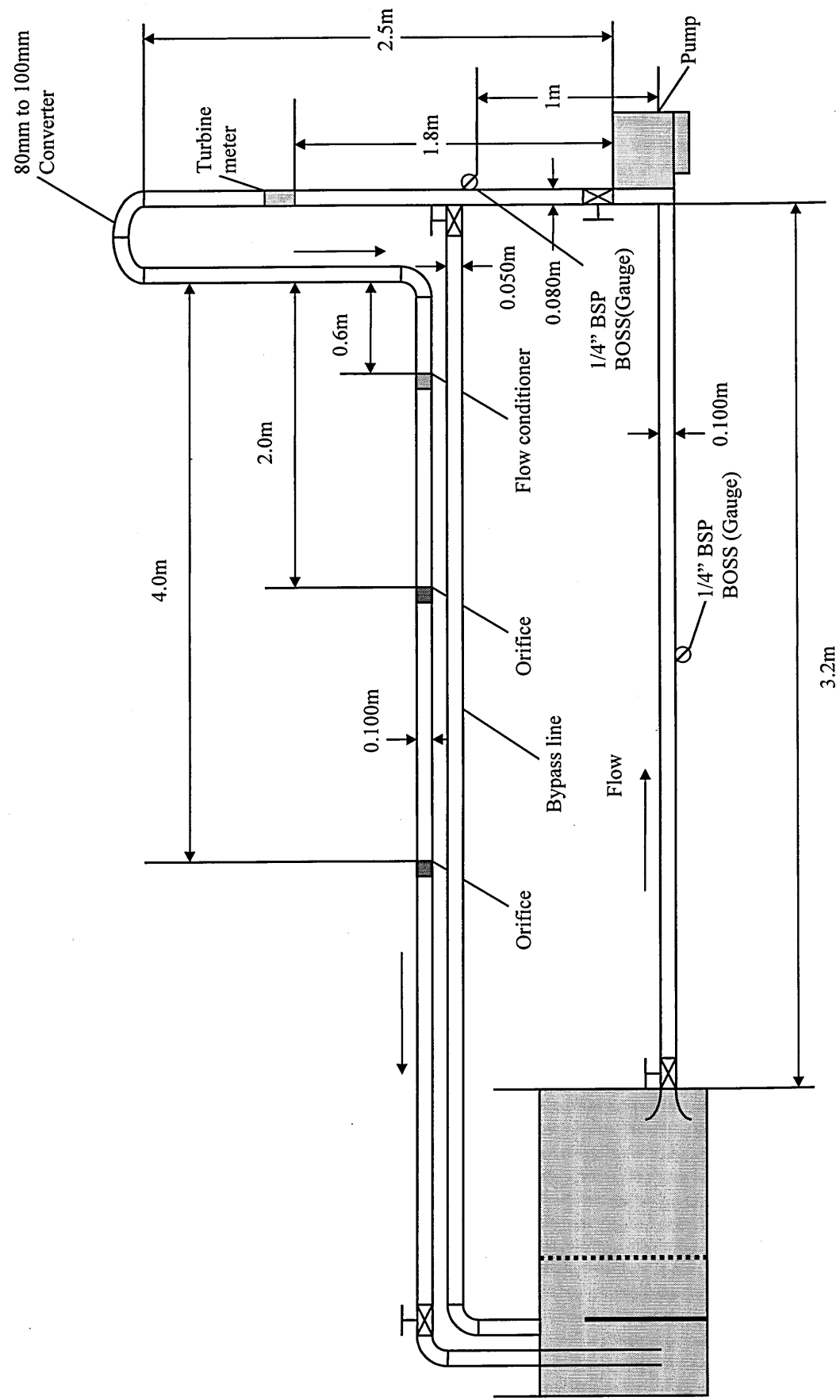


Figure 5.3.1 The water flow loop.

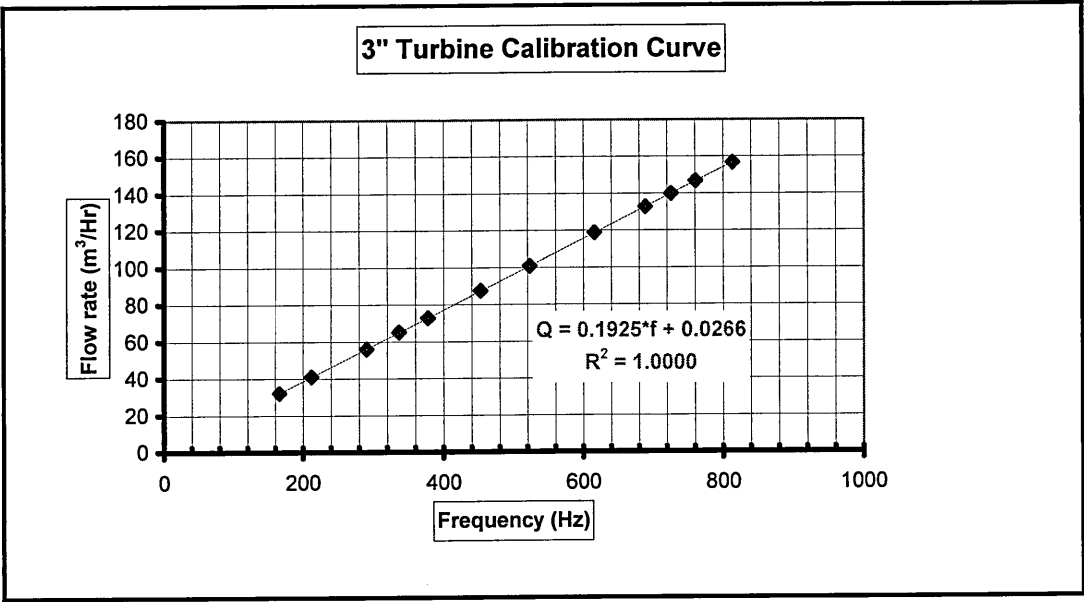


Figure 6.2: Reference Turbine Meter Calibration Curve

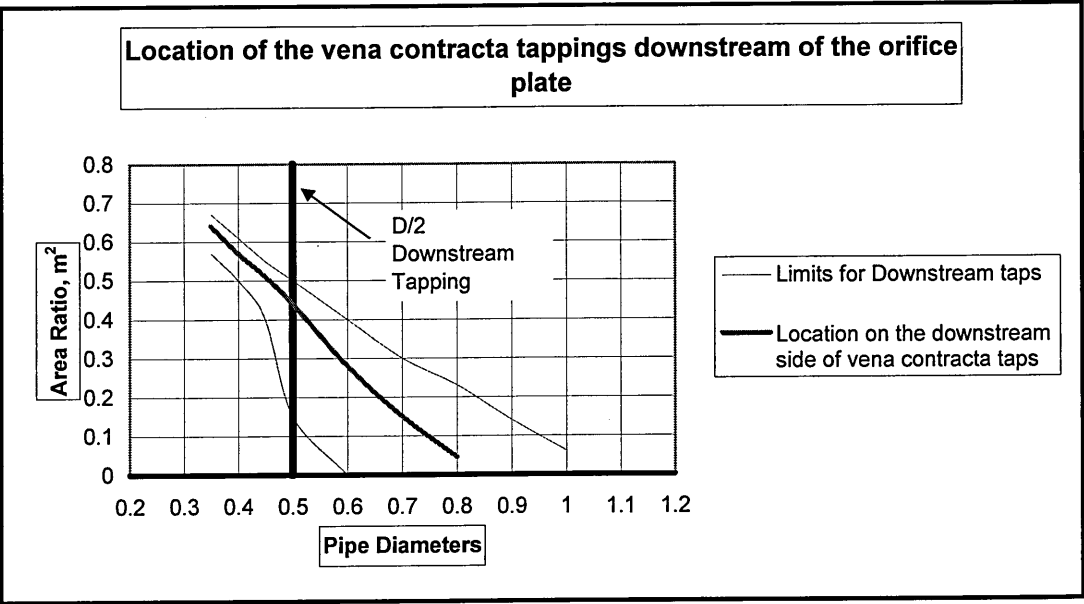


Figure 6.3 - Location of the vena contracta tappings downstream of the orifice plate expressed in pipe diameters

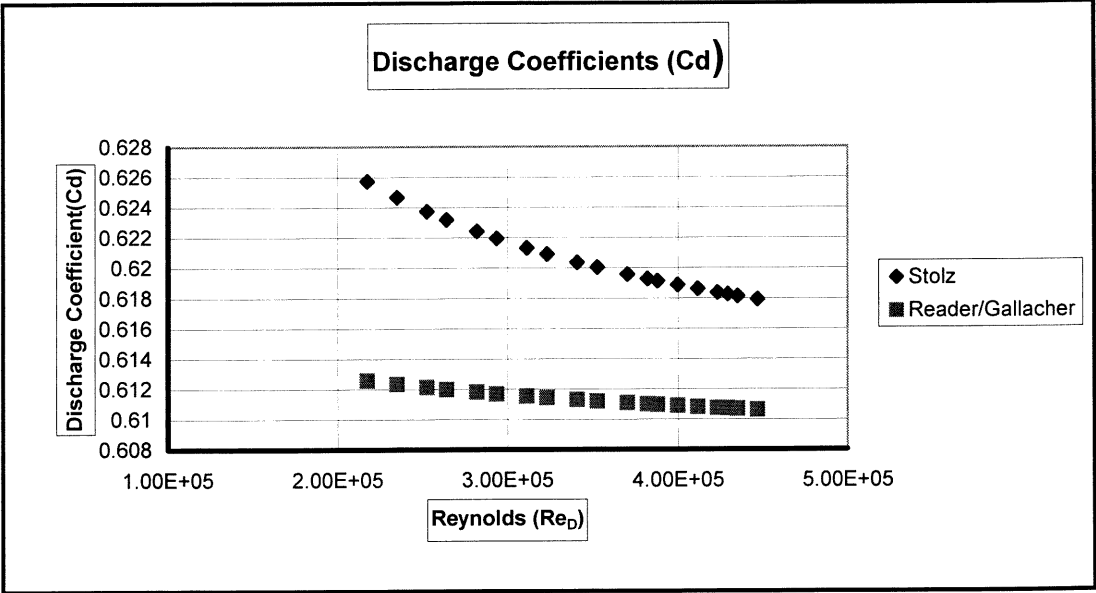


Figure 6.4 - Stolz & Reader/Gallagher Discharge Coefficients for the reference flowrates

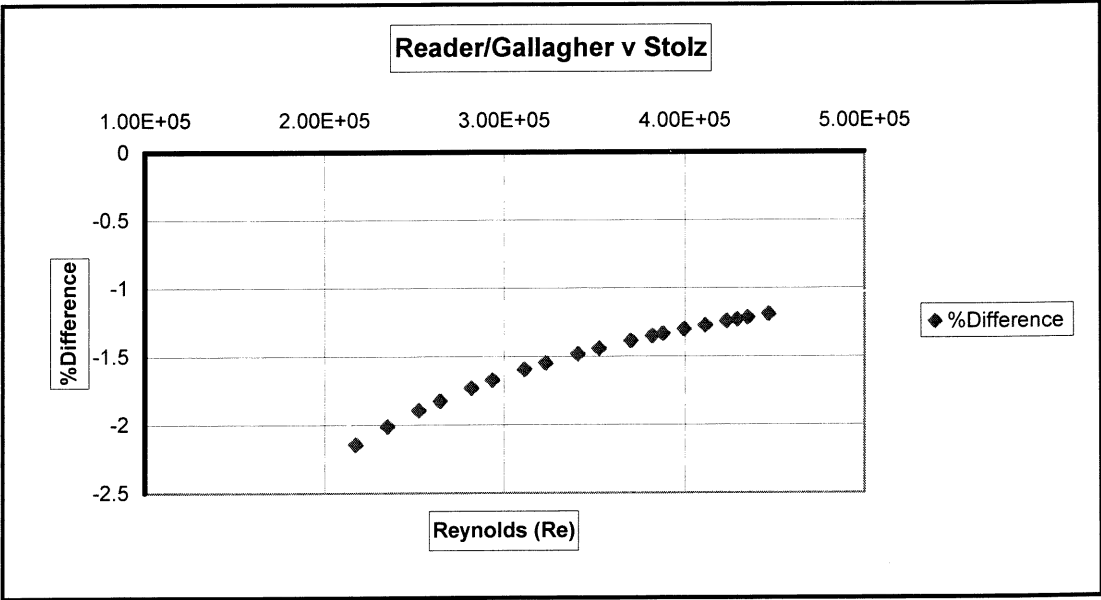


Figure 6.5 - %Difference Reader/Gallagher v Stolz equations

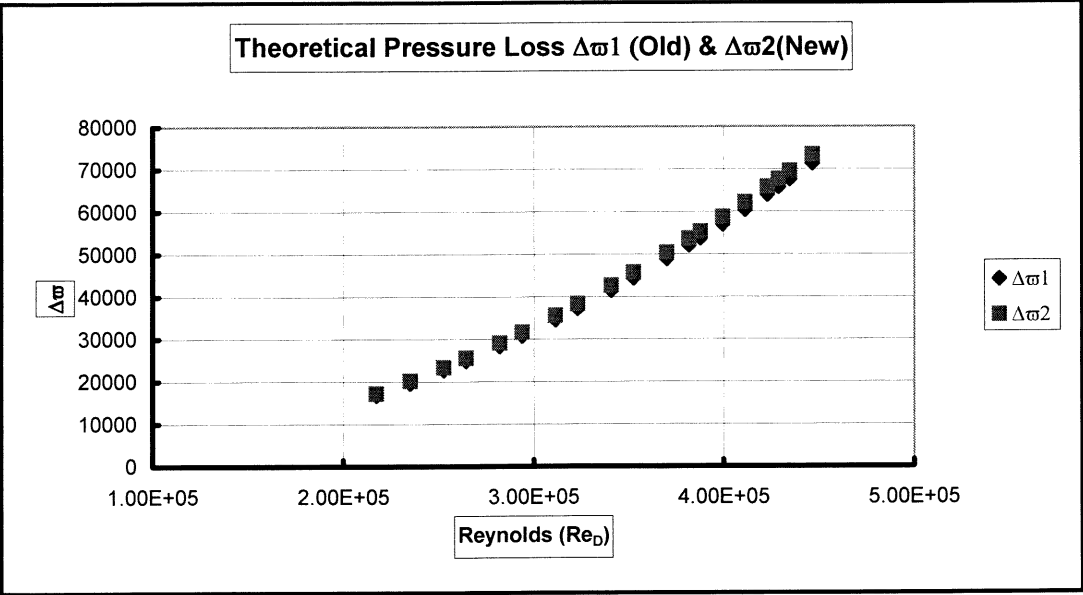


Figure 6.6 - Old and new equation for the pressure Loss

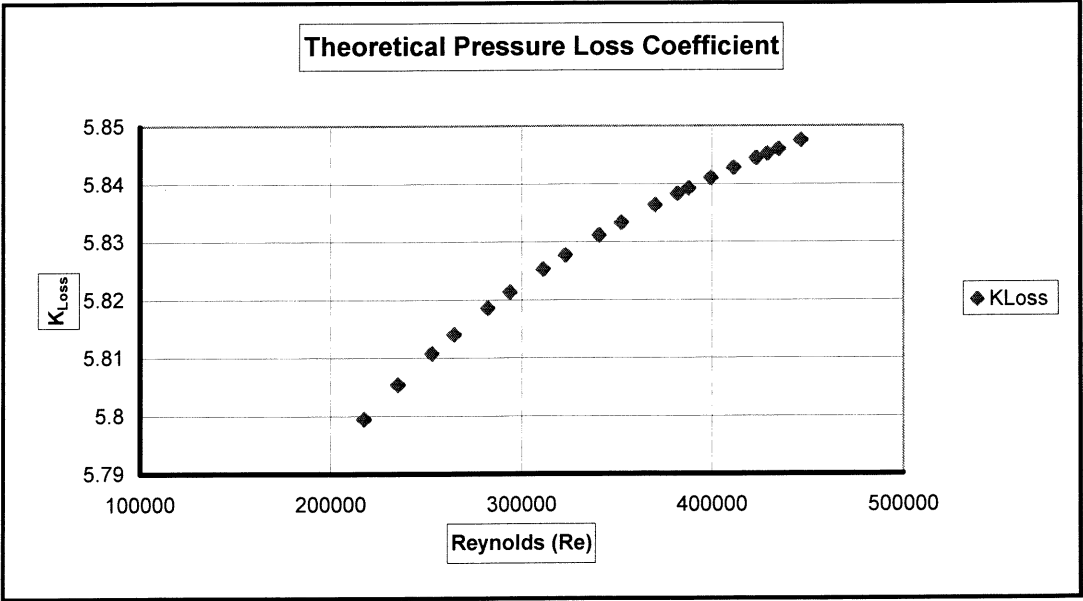


Figure 6.7 - Calculated Theoretical pressure Loss coefficient using the new equations

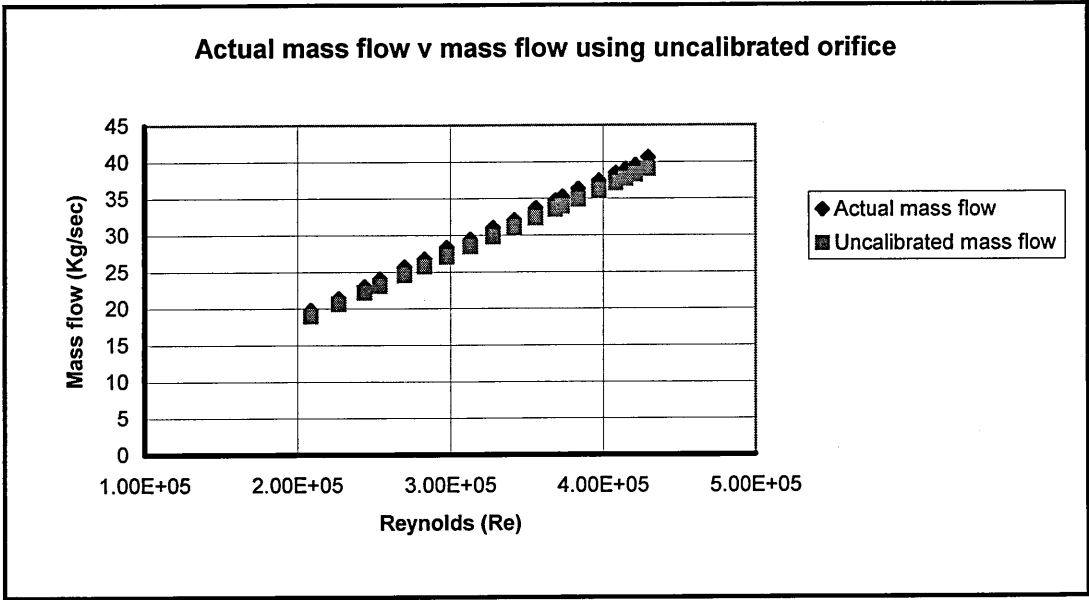


Figure 6.8 - Actual mass flow rate and mass flow rate obtained from uncalibrated orifice.

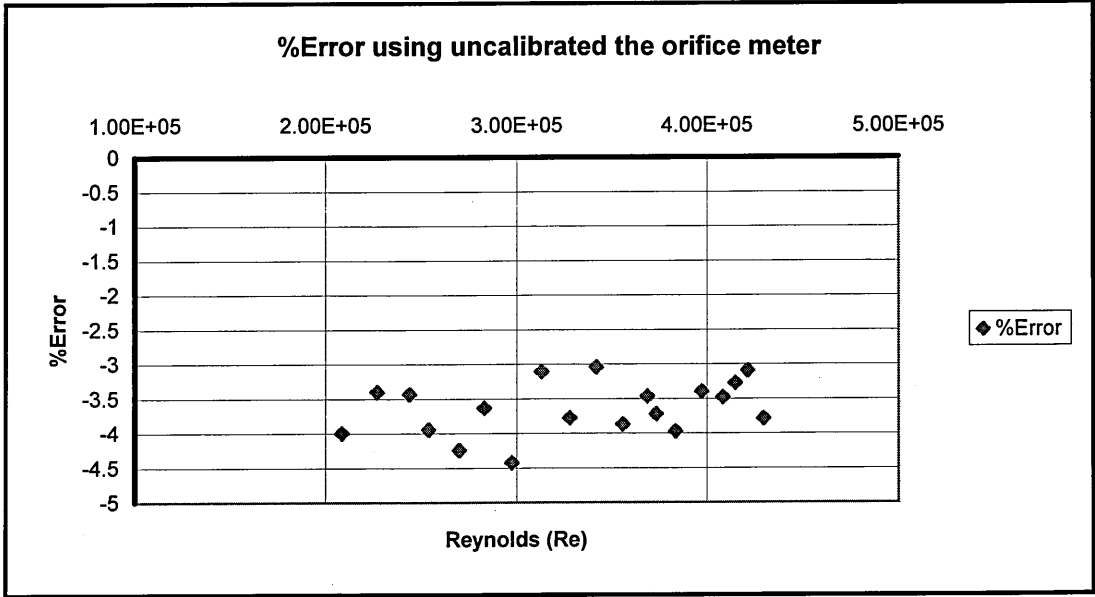


Figure 6.9 - % Error in the calculated flow rate from the uncalibrated orifice.

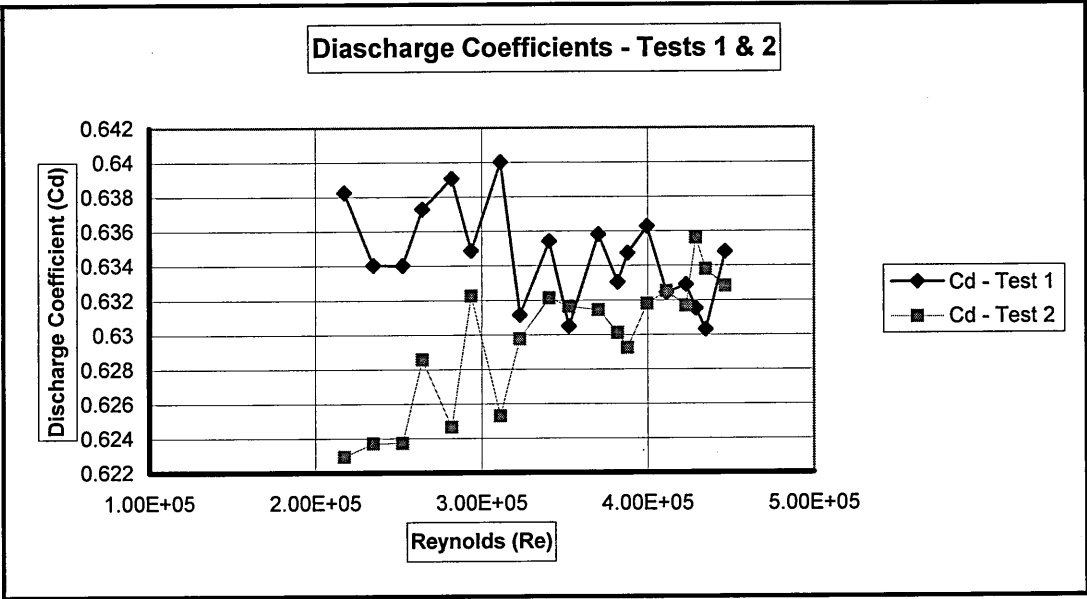


Figure 6.10 - Discharge coefficient C_d in Tests 1 & 2.

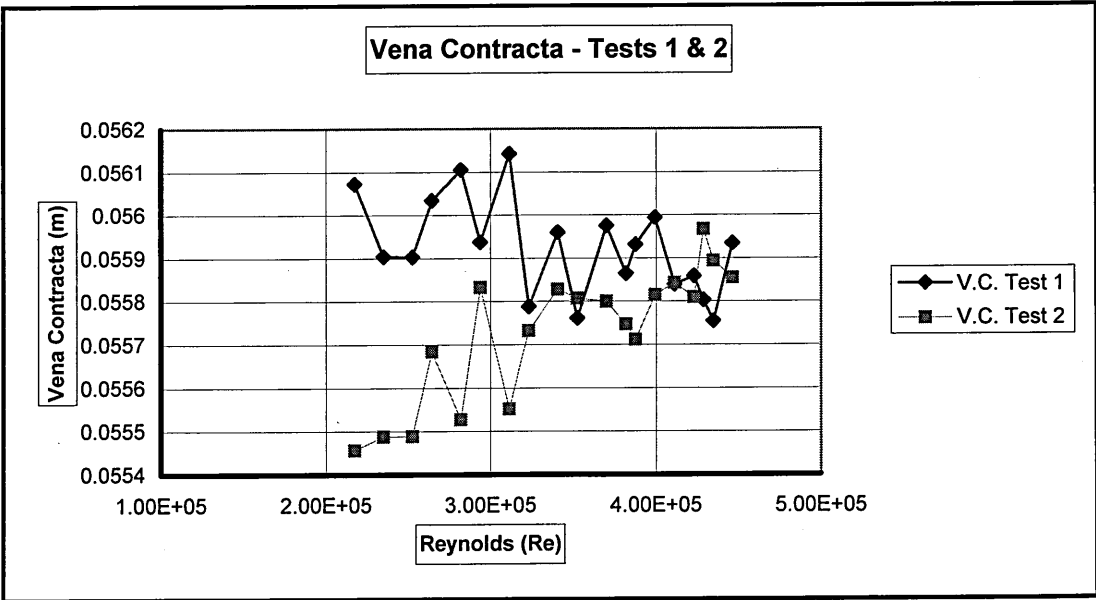


Figure 6.11 - Vena contracta calculated for Tests 1 & 2

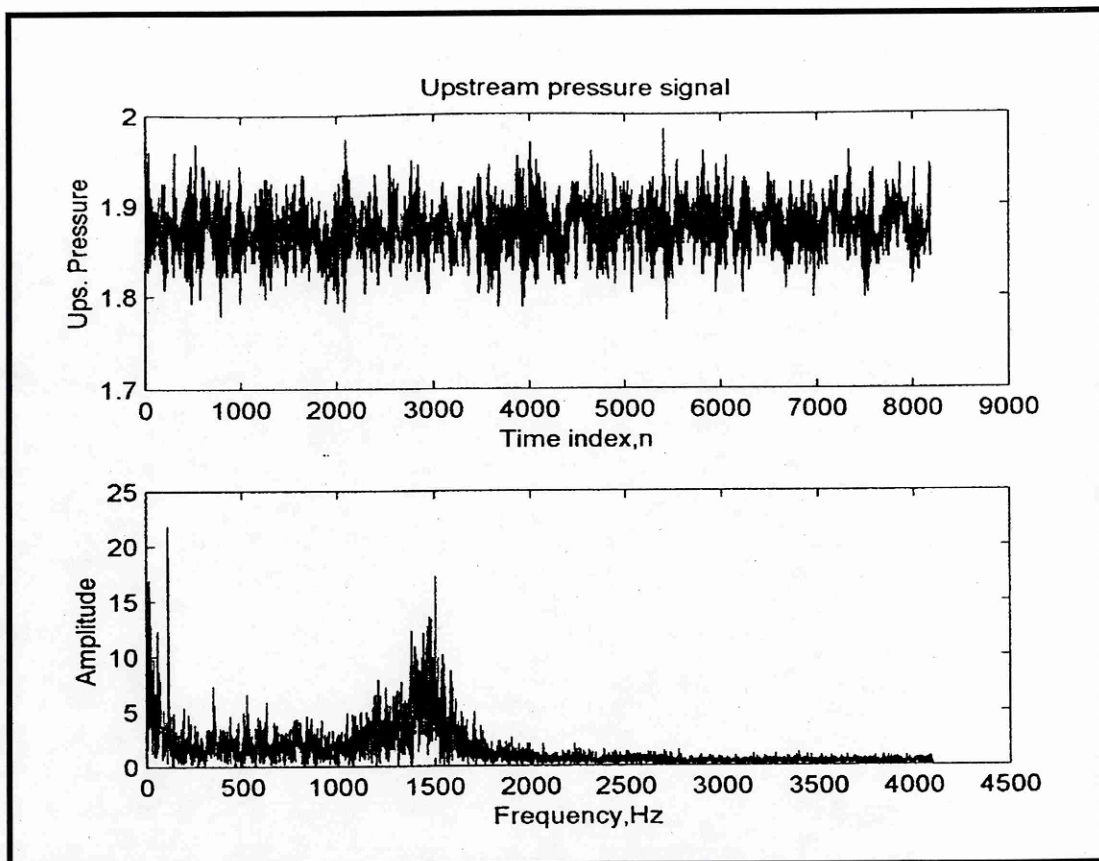


Figure 6.12 Upstream pressure signal and amplitude spectrum

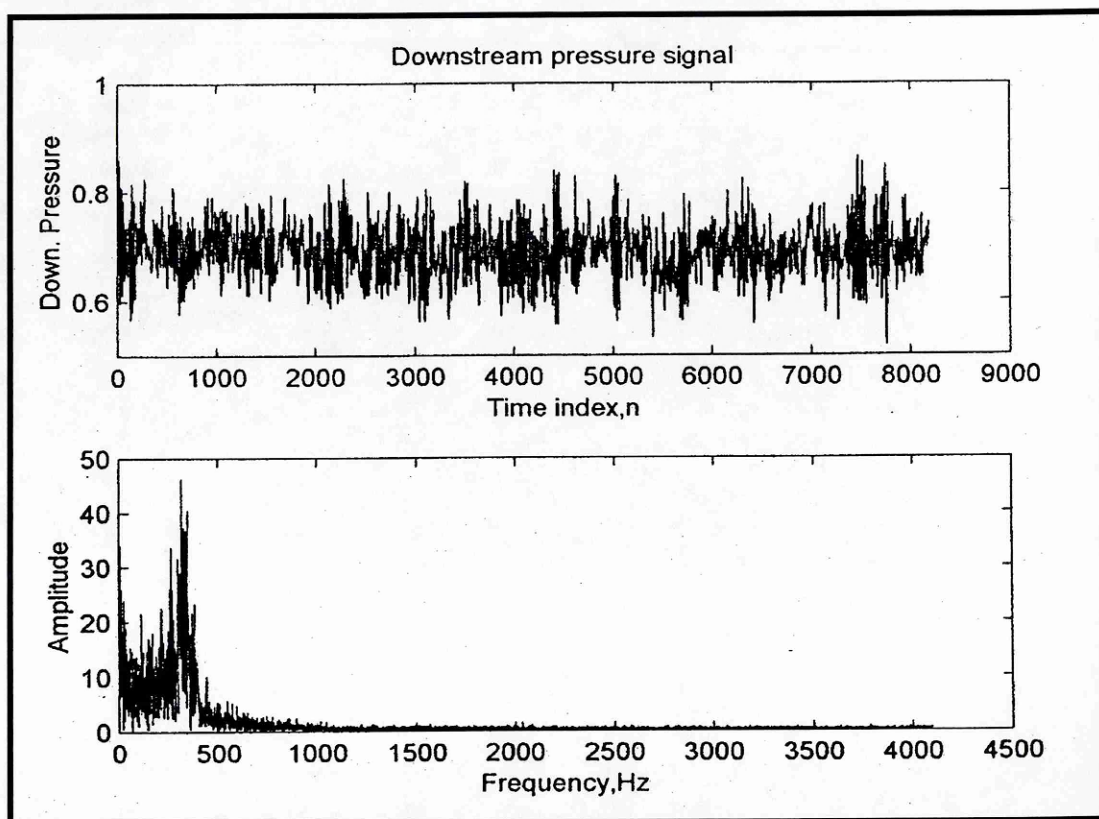


Figure 6.13 Downstream pressure signal and amplitude spectrum

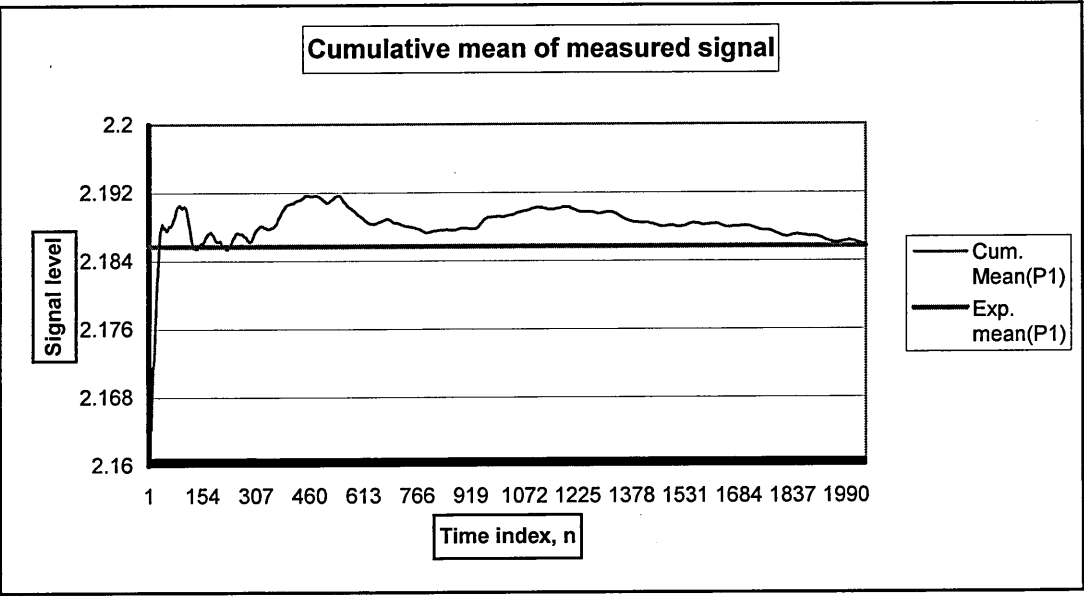


Figure 6.14 Cumulative mean of a pressure signal obtained upstream the orifice

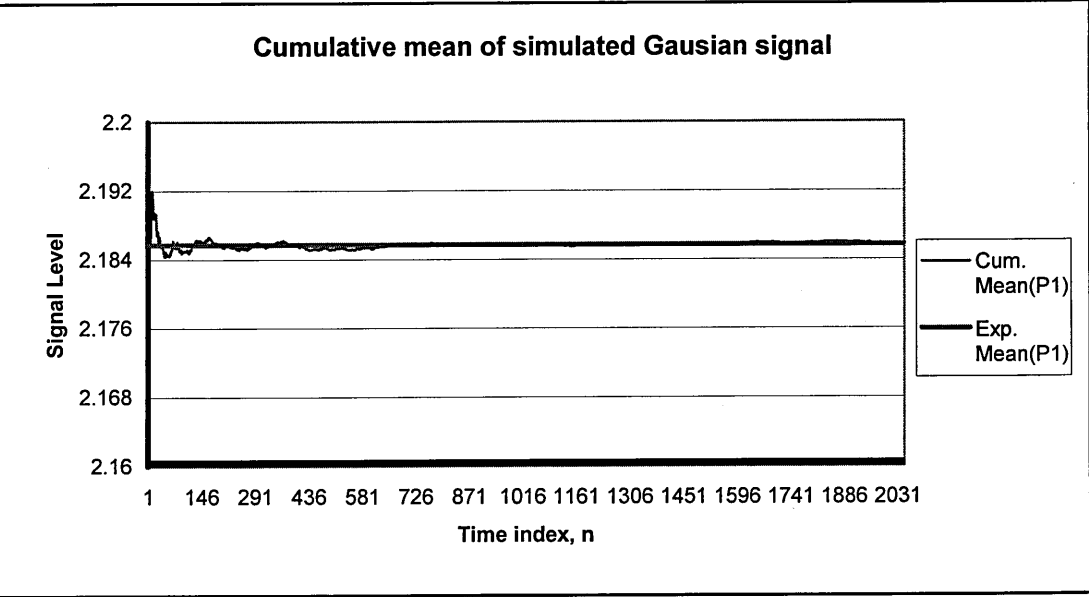


Figure 6.15 Cumulative mean of a simulated Gaussian pressure signal with the same expected value and variance as that of figure 6.14

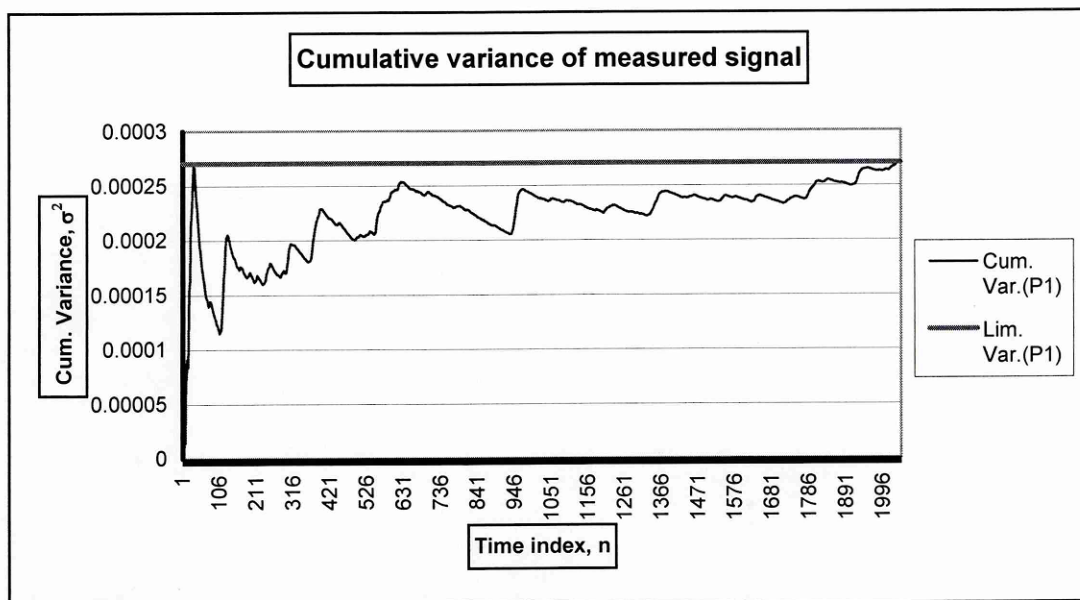


Figure 6.16 Cumulative variance of the pressure signal of figure 6.14 obtained upstream the orifice

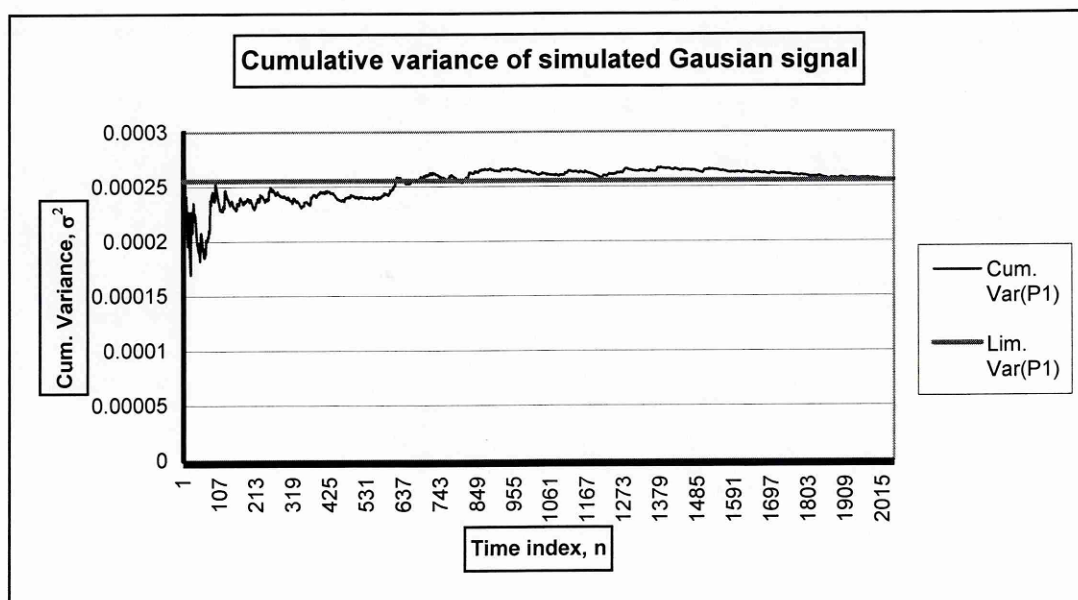


Figure 6.17 Cumulative variance of the simulated Gaussian pressure signal of figure 6.15

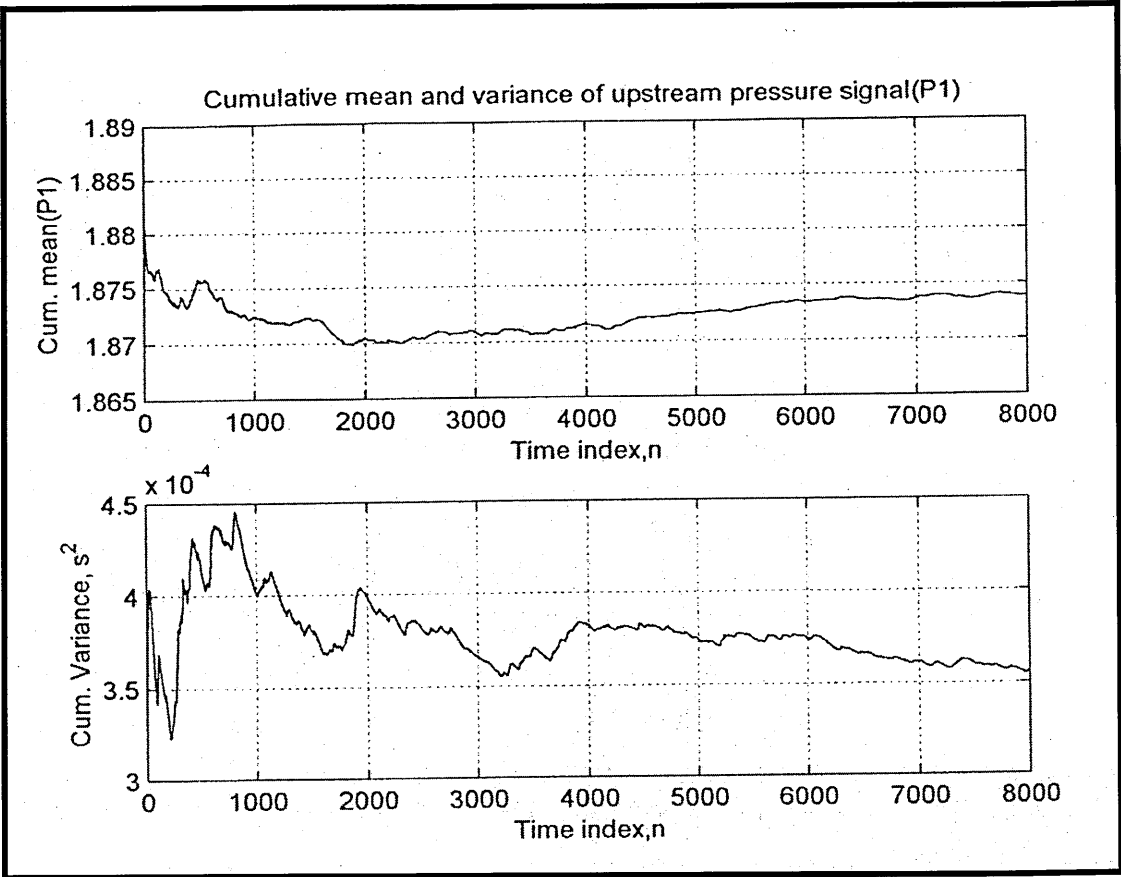


Figure 6.18 Cumulative mean and variance of upstream pressure signal (P1)

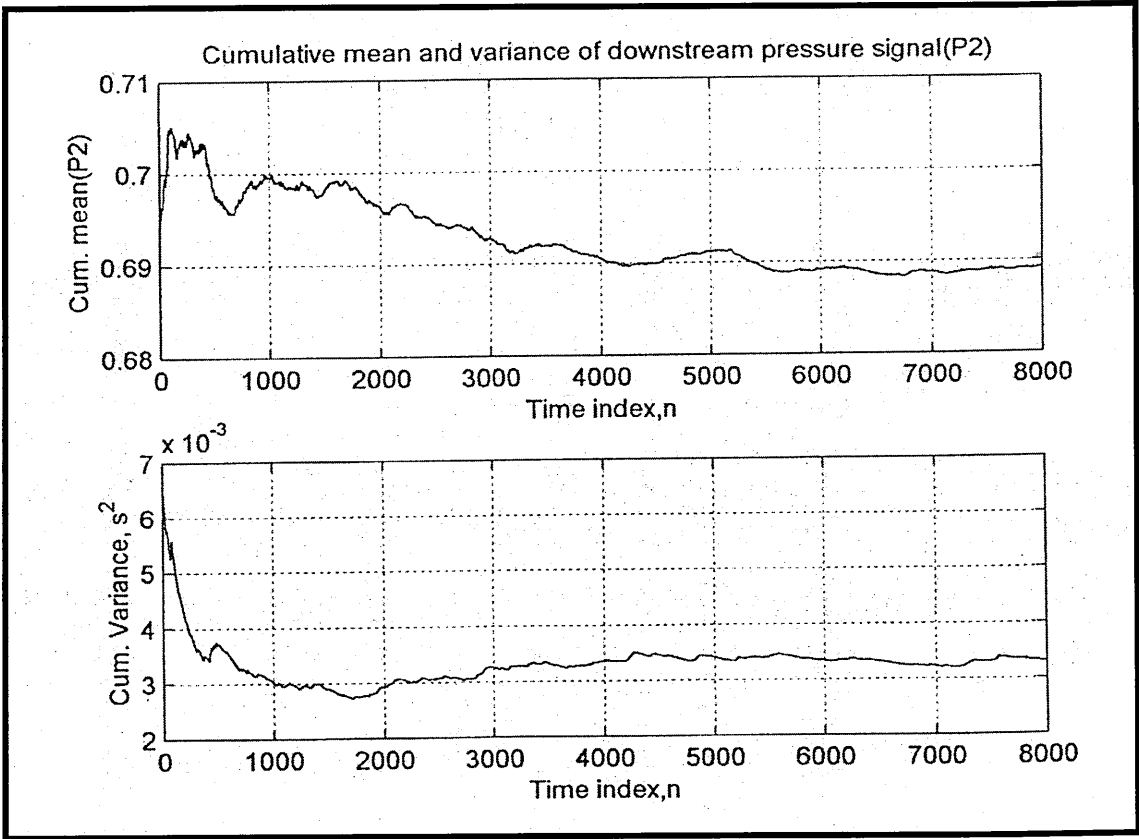


Figure 6.19 Cumulative mean and variance of downstream pressure signal (P2)

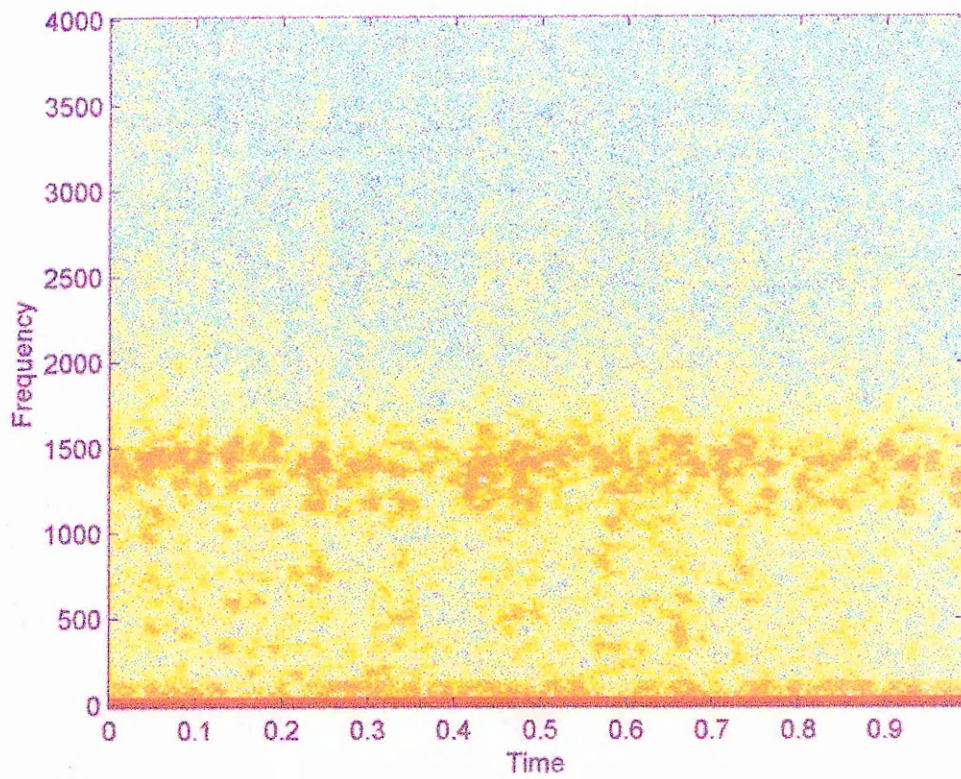


Figure 6.20 Spectrogram of the upstream pressure signal P1

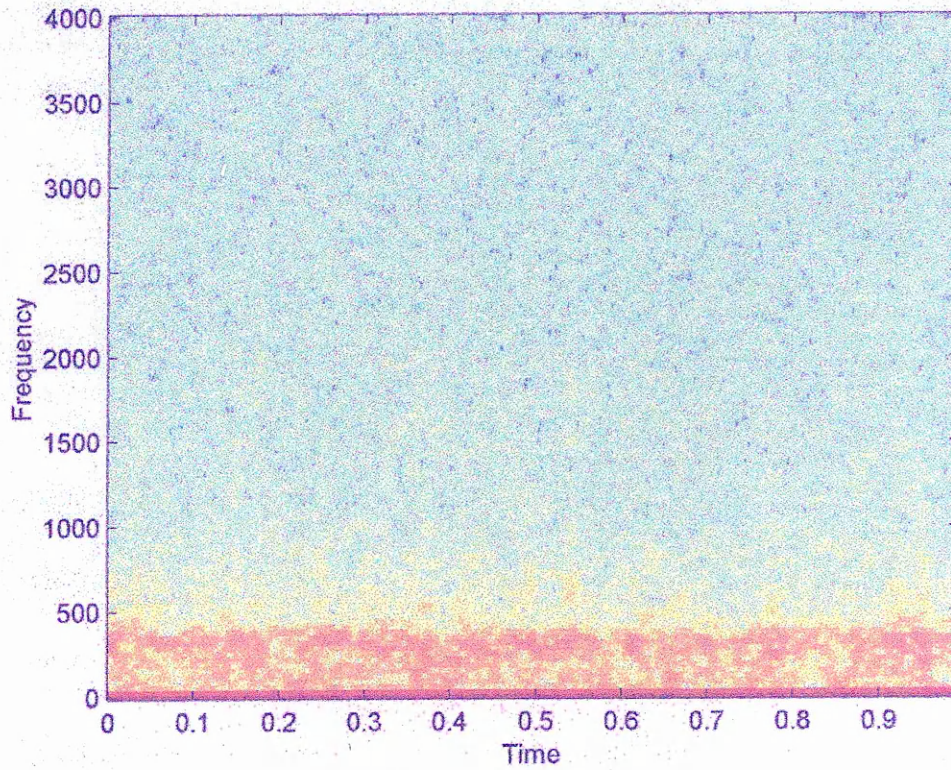


Figure 6.21 Spectrogram of the downstream pressure signal P2

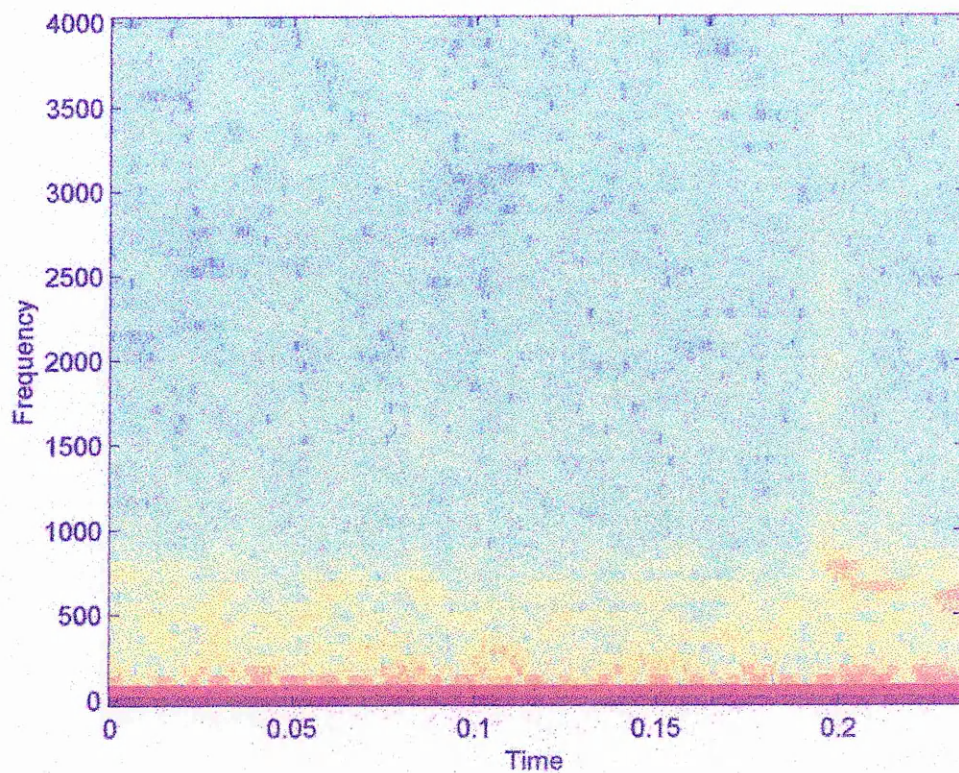


Figure 6.22 Visualisation of uncertainty in time of a pressure signal

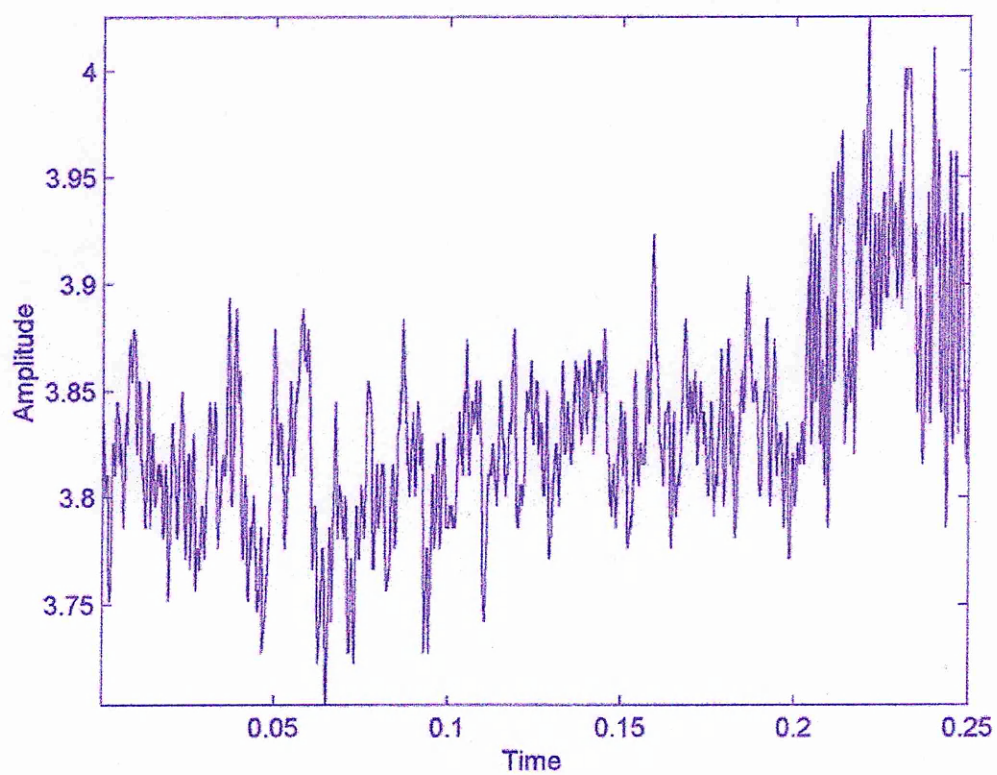


Figure 6.23 A time window of a pressure signal

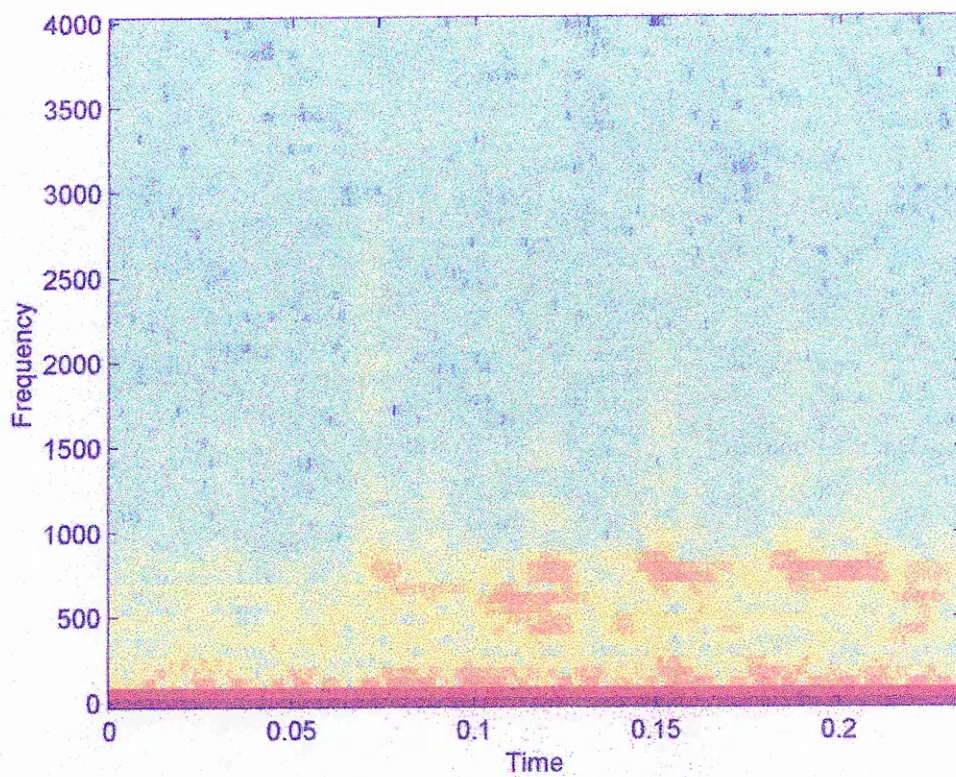


Figure 6.24 Visualisation of uncertainty in time of a pressure signal

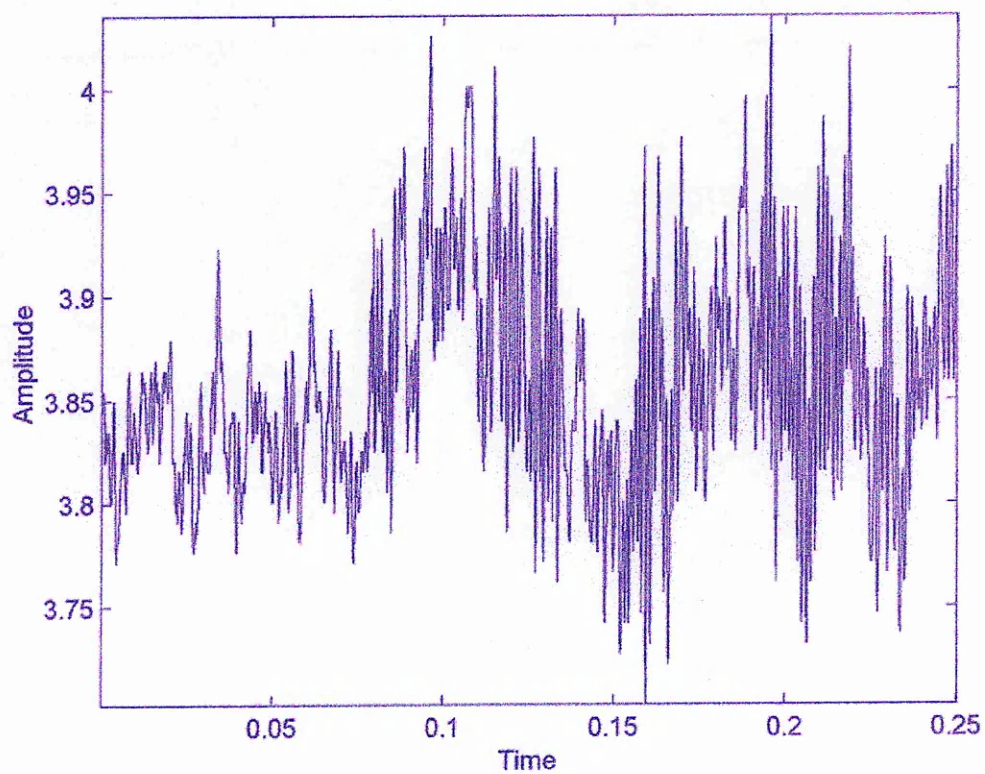


Figure 6.25 A time window of the pressure signal of figure 6.23

CHAPTER 7

7. MANAGEMENT OF NATURAL GAS METERING SYSTEMS. A MULTI-MEASUREMENT INTEGRATION AND FUSION METHODOLOGY FOR ON LINE MEASUREMENT VALIDATION AND DECISION-MAKING.

7.1. Managing gas flow measurement uncertainty for on line measurement validation and further cost savings.

In this chapter, the benefits of a continuous and reliable uncertainty analysis incorporated in the whole measurement process management is developed and justified.

Natural gas fiscal measurement is based on a combination of the laws of physics, chemistry, engineering and accounting and is not an exact science. However it is an applied science and with changing technology and business environment, measurement technology is changing as well (Barua L.D., 1992). This multidisciplinary nature of gas fiscal metering demands a special and sometimes unavoidably complex management of the produced information and results (**Data management**) since information must be distributed to various departments and people within or outside the Gas Company. Thus, usually the presentation form of these results must be adapted in every case to the users needs.

Virtually when we refer to the term fiscal metering in N. Gas industry we mean all these measurements that need to be of the highest possible quality since they usually have big financial interest. Measurements in science and industry are carried out using identical or disparate multiple sensors that generate large amounts of data of similar or different nature which need to be processed in a meaningful way. The increasing demand for more accurate information dictates the development of procedures for efficient data management that are more practical, robust and of a lesser cost.

Upp, E.L., 1993, described how measurement system's uncertainty commercially is used as a sales' tool, by both suppliers and users of metering equipment. The supplier with the «best number» wins the bid. On the other hand, the user sometimes requires measurement uncertainties beyond the capabilities of any meter available. As Upp, E.L., has also added, in either case, the definition of uncertainty may serve a purpose for the user or supplier but has little relevance.

It has been already described in the previous chapters, that the measurement system output is the conventional true value (estimator) for the measured quantity. This is a value approximating to the true value of a quantity such that, for the purpose for which that value is used, the difference between the two can be regarded negligible. (ISO/DTR 14253-2, 1997).

Another important issue that is usually ignored by people involved in gas fiscal metering is that *"accounting of gas can be no better than measurement of gas."*

Accounting, as it applies to the gas industry is based on conditions in contracts between the producer, or seller and the buyer (or shipper and transporter) and on regulatory standards. In these contracts as it was already said, are specified to the minutest detail all the factors that are necessary for the accurate and reliable measurement of the delivered material.

For most of the flow measurements (such as control applications), a repeatable measurement may be more important than an accurate measurement but this is not always true. Accurate flow measurement is required in custody transfer measurements since the uncertainty of the measurement may have major financial implications. In addition to revenue considerations, contractual obligations can have far-reaching impact resulting from improper measurement of the Gas or accounting thereafter, thereby resulting in possible litigation. The adoption of International practice that can be established through the adoption of International Standards is the only safe way to avoid such disagreements and to ensure reliable and correct generation of information and results. This can be implemented safely through a Total Measurement Quality Management scheme.

In section 2.1, a technico-economic formula for the assessment of measurement information, as it has been given by Sydenham, P.H., & Thorn, R., (1992), was presented. In this formula the term in the nominator was associated with the uncertainty, of which fast and reliable analysis methods were the main objectives of this project. In the denominator two other important terms are also included. The importance of these two additional terms, which are more associated with the management and economics of a measurement process and further characterise its quality, will be expanded in this chapter.

One of the terms is associated with the **cost** of generated measurement information. This may be the Total lifetime operating cost of a measurement system as given by **Bentley, J.P, (1988)**:

**T.L.O.C.= Initial cost of system (purchase, delivery, installation and commissioning)
+ cost of failures and maintenance over lifetime of the system
+ cost of measurement error over lifetime of the system. 7.1**

Obviously the best measurement system for a given application is the one with the minimum T.L.O.C. We can expand the relevant terms in the last formula as:

$$T.L.O.C. = C_I + [C_R + (C_L + C_P)T_R]\lambda T + (C_M + C_M T_M)mT + T \int_{-\infty}^{+\infty} C_E p(E)dE \quad 7.2$$

where C_I is the initial cost of the system, C_R is the average cost of materials per breakdown maintenance repair, C_L is the repair labour cost per hour, C_P is the process cost per hour, C_M the average cost of materials per preventative maintenance service, C_E is the cost penalty associated with a measurement system error in monetary units/year, T_R is the average repair time in hours, T_M is the average maintenance time in hours, T is the system lifetime in years, λ is the average failure rate (faults/year), m is the number of times that preventative maintenance is carried on the measurement system per year (eg. calibrations). The last integral term of equation 6.2 that represents the total lifetime cost of measurement error, is the one profound term that actually dictates a continuous and reliable uncertainty analysis of gas metering process where $p(E)dE$ is a probability of measurement error being between $E+dE$. It could be generally said that in gas metering systems, because of the cost and today's high flow rates of the purchase product that are due to the fast growth and expansion of gas industry, the last term becomes extremely significant. The fact is that if it is not properly accounted for, it is possible to outweigh in very short time operation intervals the other terms, something that has been realised and emphasised during the last few years within the gas metering community.

In Chapter 3 it was presented a summary of the general methodology proposed in this project for uncertainty analysis management aiming to measurement quality improvement and subsequent cost reductions. This methodology, which consists of a series of distinct steps, virtually targets to the reduction of two cost factors included in the total lifetime operating cost of a gas measurement system. The first is obviously the cost

of total lifetime error, the last term in equation 6.2. The second factor is the cost of preventative maintenance, which for gas metering systems is mainly due to the usually scheduled calibrations, carried out in every piece of equipment of the measurement system. Calibrations are carried out according to a calibration plan and at fixed time intervals, which could range from daily (gas chromatographs) and monthly/quarterly (usually all other metering instrumentation) to six monthly/yearly (pipe inspections etc.). It would be extremely beneficial if these calibration intervals, can be optimised by on line monitoring the calibration status of the instruments. Unnecessary scheduled calibrations not only increase the preventative maintenance costs but some times could have counter effects since they may reduce the expected lifetime of the respective instrumentation by speeding up aging effects. On the other hand, drifts or malfunctions in measurement process and instrumentation taking place between large calibration intervals may pass unnoticed and subsequently increase the cost of measurement error or put additional labor cost in correcting an identified error with delay.

The on line monitoring of the calibration status and more general the on line measurement process validation as well as the attainment of measurement process control, are the implicit aims of the proposed methodology that is based on a reliable and continuous uncertainty analysis. This introduces also the last term of the technico-economic formula for the assessment of measurement information quality, which is the **timeliness** of the measurement information, critical in the management's decision making process within the newly competitive natural gas market. A methodology based on the results of this project has been formulated and presented in the next section.

7.2. A multi-measurement integration/fusion methodology based on Monte Carlo uncertainty analysis and decision making under uncertainty.

The first step of the proposed methodology, is the:

1. Assignment of target (specification limits) measurement tolerance intervals for the combined uncertainty of the overall metering system. These are actually the limits as have been specified in section 5.1.2.1 of an indisputable energy purchase transaction between the parties involved. If this is applied before the selection of a particular metering system, uncertainty analysis comes as an aid in decision making or as an

optimisation tool during the selection of an appropriate metering method and system as Moffat, R.J., 1985 has analysed. This uncertainty analysis during the qualification process of various metering methods and systems would be based on any available information, so it would be actually a Type B uncertainty analysis if applying conventional methodologies. Usually the best available information is manufacturer's data and recommendations about the capabilities of the metering instrumentation.

An alternative improved method proposed here for this stage, could be an of-line Monte Carlo uncertainty analysis, that can be characterised as a mixed type (A and B). It is characterised as a mixed type because it can utilise all the available information, transforming them into statistical parameters (probability distributions). This is actually preferable than a Type B uncertainty analysis that would otherwise be carried out. With Monte Carlo uncertainty analysis, particular characteristics other than just the capabilities of the measurement instrumentation (such as common uncertainty sources by use of common instruments, and sensors/models interactions) can be accounted for, even at this early metering system's selection stage, offering additional aid in decision making.

In terms of an existing (already selected) measurement system, the same procedure can be applied using the additional information gained from the following second and third steps of the methodology which are the:

2. Calibration of all the physical sensors and evaluation of their uncertainty which can be expressed as a multiple of standard deviation (usually 2σ for an approximately 95.4% confidence interval).
3. Calibration of the model based sensors using the *a priori* information from the results of step 2. Utilising the information gained from calibration results and the specific calibration conditions and modeling them in a Monte Carlo uncertainty analysis, a more realistic (updated) picture in terms of the capabilities of the specific metering system can be gained. Additionally, the correlated uncertainties due to calibration of instruments using for example the same calibrator would be then also taken into account. At this stage also, analytical redundancy formulas can be Monte Carlo calibrated for further use in fault detection and isolation subroutines.

The next most important and difficult step is the:

4. On line uncertainty analysis for on line measurement performance monitoring and evaluation. This mainly includes the following:

- *Test of consistency of every single measurement.* If at least three replicated physical or model based sensors, corresponding wherever possible to independent measurements, are available for every single measurement, this could also aid further in fault detection and isolation. The test must be applied in pairs for the three sensors of every single measurement. In this case, most of the available conformity criteria based on Bayesian statistical theory of measurement uncertainty turn out to be essentially equivalent to one another and yield the quite reasonable condition for a pair of measurement results y_1 and y_2 of the same measurand:

$$|y_1 - y_2| \leq \beta s \text{ but with different suitable numbers of } \beta \text{ typically in the range 1 to 3.}$$

s is a characteristic uncertainty corresponding to the difference $y_1 - y_2$. Initially it could be obtained using again the *a priori* information from calibrations of steps 2 & 3. Then can be updated using the *a posteriori* information from the actual measurements.

4a) If the test of consistency is successful then we proceed in sensor fusion using one of the available methods. A method with certain preference since it has a sound theoretical statistical basis (obtained by the principle of maximum likelihood) is the use of weighted average (or center of gravity) formula and the fused best estimate is given by:

$$Y_{best} = \frac{\sum w_i x_i}{\sum w_i}$$

where the weight w_i is the reciprocal square of the corresponding uncertainty of each sensor measurement which is approximately 2 standard deviations of the posterior distribution of the measurement mean. It is this fusion process that could also lead to a reduced uncertainty.

4b) If the test of consistency fails then fault detection and diagnosis and finally re-calibration of the fault sensor(s).

The uncertainty analysis methodology proposed here up to the step 4 could be readily applied and is generic. The rest of the steps of the presented methodology are coming mainly as proposals for future directions of an on-line uncertainty analysis rather than as a readily applied method and are non-generic.

CHAPTER 8

8. CONCLUSIONS AND PROPOSALS FOR FURTHER WORK.

It was generally shown that there is an increasing need within N. gas industry and not only, for an uncertainty analysis methodology that would be reliable and could take into account most of the factors contributing in a combined measurement uncertainty result. Additionally, such a method would be more beneficial if it would be possible to be implemented in a wide range of conditions and at various stages of a measurement process. Finally, uncertainty analysis on a real time basis is the outmost target. It has been concluded that:

- Monte Carlo uncertainty simulation methods and particularly those that also utilise Bayesian statistics based on conditional distributions, seems to offer most of the above wanted features and additionally they have the potential for on line implementation.
- The rigorous theoretical basis and the practical advantages of the method have been demonstrated. Importantly, the method is fully compatible with the International proposed methodology as expressed in the GUM.
- Theoretically, it seems to be closer to the underlying physics of complex measurement process that are actually affected by a chaotic net of interactions (correlations) between physical and model based measurements, calibrations etc. Monte Carlo method seems to be the only method that could capture the effects of these correlations and provide illustrative visual presentations.
- Simulation of simple and complex test cases revealed most of the characteristics of uncertainty propagation mechanism, which are not so obvious a priori. Partial and perfect correlated uncertainties in measurement inputs affect differently measurement results coming from the addition/subtraction or the product/ratio of input measurements. In the first case and for input measurements modeled as conditional Gaussian distributions (correlated), the measurement output is an unbiased Gaussian. Inputs' uncertainty correlations have only random effects, affecting only the width of the result's distribution. To the contrary, in the most common cases of measurements results obtained as products/ratios of input measurements with correlated

uncertainties, the effects are not only limited on the width of the resulting distribution but they also cause a systematic shift. This shift leads to asymmetric uncertainty intervals that in order to be accounted for, a slight modification of the current uncertainty propagation law seems to be necessary.

- These effects have been also graphically demonstrated with the aid of joint and conditional distributions and present particular interest. The inputs' uncertainty correlations effects on the measurement result distribution obtained were **Skewing** of the distribution, **Flattening** with respect to Gaussian and a **Bending**. All these perform increased interest since they seem to correspond and further introduce **non-linear type** distributions.
- Monte Carlo energy flow rate uncertainty analysis in gas metering systems was carried out with a Monte Carlo software program developed in Matlab/Simulink and it was compared with those from an analytical RSS uncertainty estimation method. It was revealed that interactions of physical/model and model/model based measurements (correlations) generally have a positive impact on uncertainty intervals (estimated around 0.8% to 1.8%) as well as a positive shift (estimated around 0.1%).
- The effects of gas composition normalisation process when a gas component (methane in natural gas) is dominant have been studied using Monte Carlo and they confirmed the results obtained by other authors. Additionally, certain explanations were given to the reported shift in density estimations and the uncertainty reduction of normalised results (canceling of input uncertainties).
- The whole research study led also to the development of a long list of theoretically derived interrelationships between the various orifice meter coefficients and variables (Discharge, Contraction, pressure loss etc.). The interrelationships developed in this project fully agree with some of those presented by Zanker, K.J., who has questioned the possible interdependence between orifice meter variables, as early as in 1961. However, it seems they have been neglected for long time now and especially the importance of contraction coefficient as the main coefficient in orifice flow. Several other conclusions mainly regarding various important theoretical matters associated with uncertainty analysis have been drawn which generally call for further research.

Further work is necessary that must be concentrated on the more detailed modeling of real operating instrumentation systems and the proper selection/combination of distributions corresponding to their measurements under real calibration/operation conditions. This alone is a heavy task. However, it is a necessary step in order to get into a reliable real time Monte Carlo uncertainty estimation.

Further research is necessary to resolve practical problems such as stability considerations of the actual process as well as different time responses and reporting time delays of instrumentation and the appropriate selection of uncertainty analysis time intervals based on this information. For example, gas chromatographs usually provide updated information in terms of gas composition every 10 to 15 mins. On line or real time are generally *relative* terms depending on the application and the specific needs for updated information of measurement uncertainty type. So, 15 mins or even longer time intervals may be appropriate for updates of gas flow measurement uncertainty analysis results, which could possibly allow an online Monte Carlo software to operate in parallel with the main flow measurement computation program.

It is also necessary the further exploitation of the various available transforms within a sense of measurement systems and processes optimisation as it was described in section 5.2, as well as the possible development of other transforms and amongst other things methods to speed up simulations in complex systems. This process may lead also in a better comprehension of the actual processes under measurement.

An integration of modeling and simulation that could start from instruments level up to complex metering or other process systems in networks could be possible and may need serious consideration.

Finally, Monte Carlo simulation needs to be more widely published as a really promising and proper method for on line combined uncertainty estimation, as well as it needs to be adopted in the various Standards regarding uncertainty estimation. ISO 5168 seems to have promptly responded in a way to such calls.

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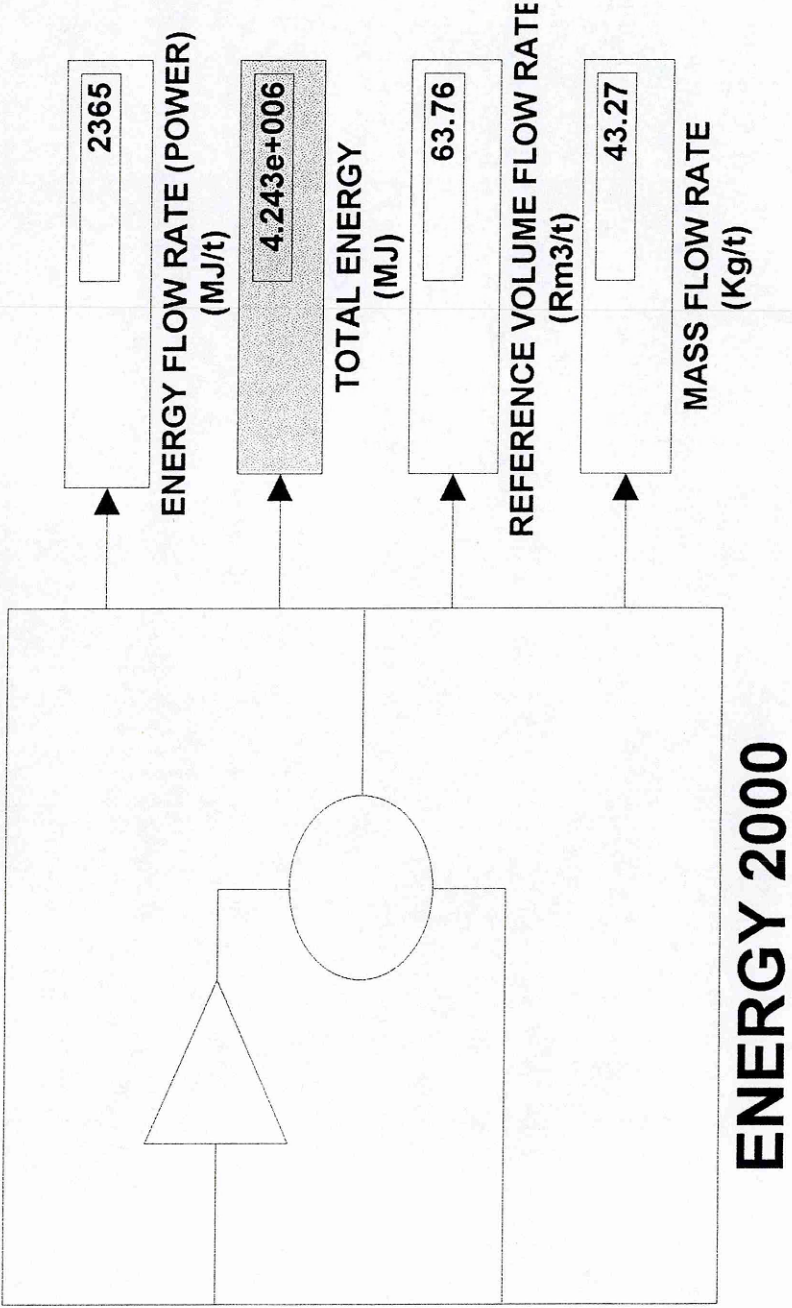
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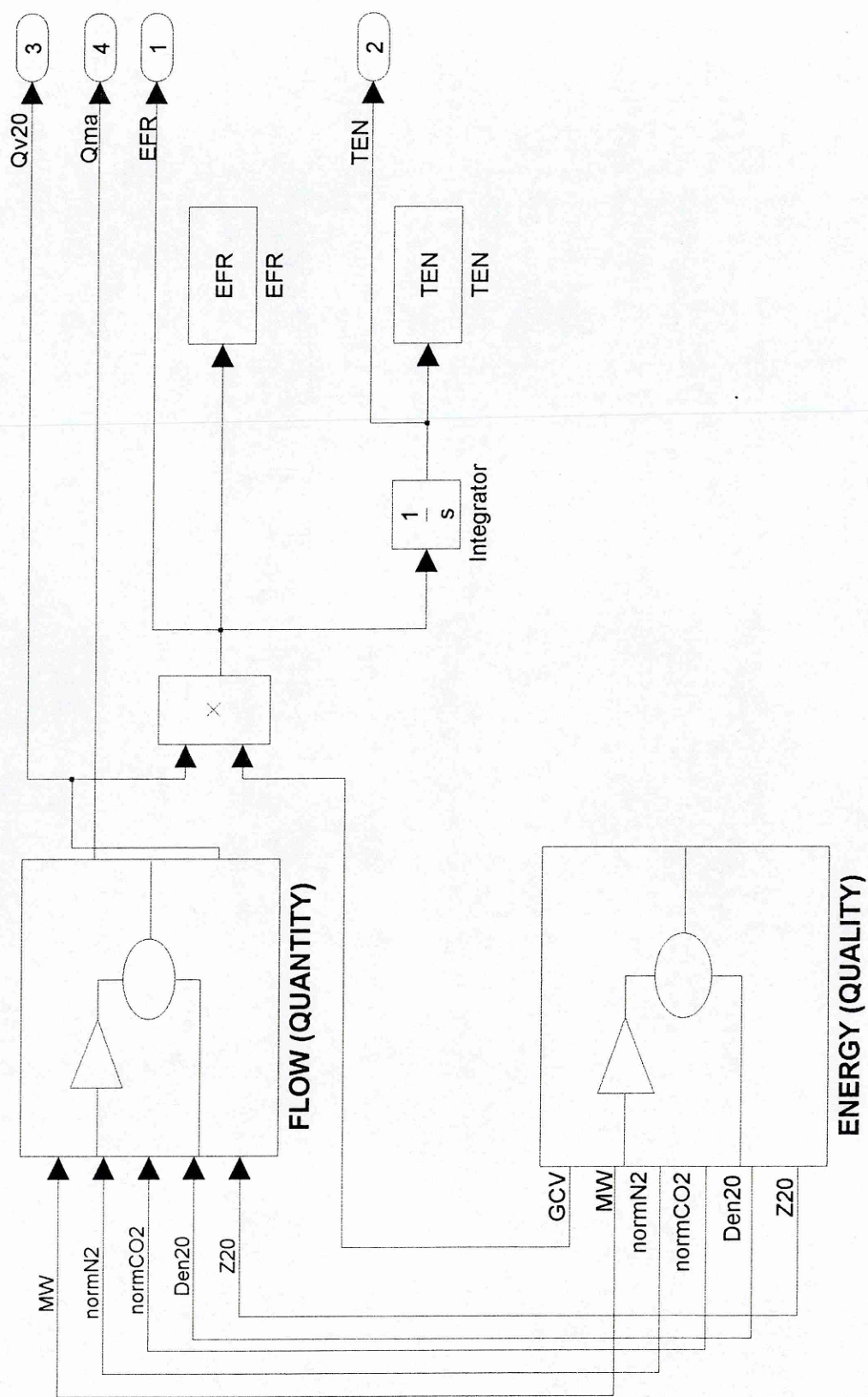
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APPENDIX A

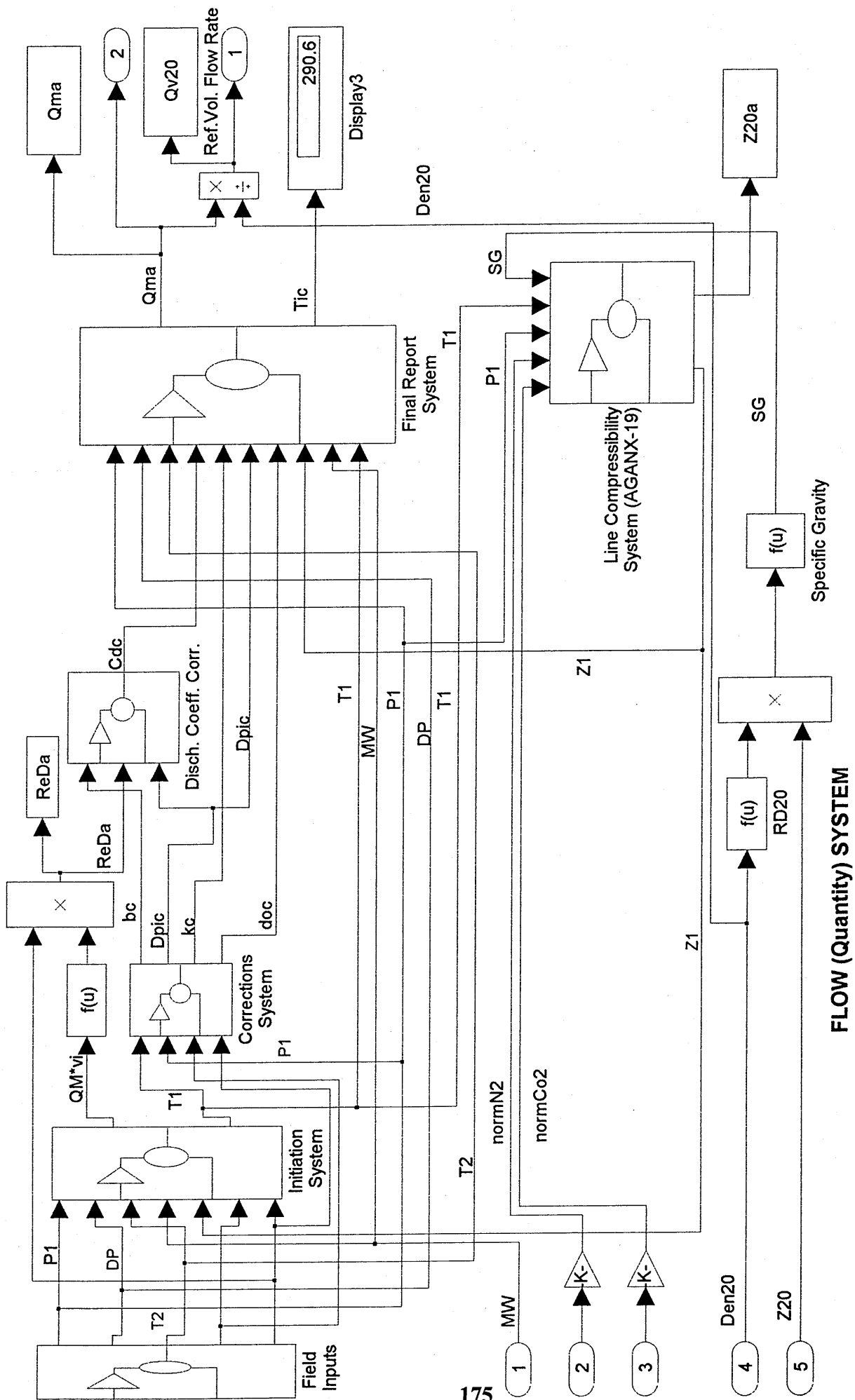
- **Sample of the Monte Carlo Uncertainty simulation program and its subsystems (Simulink)**

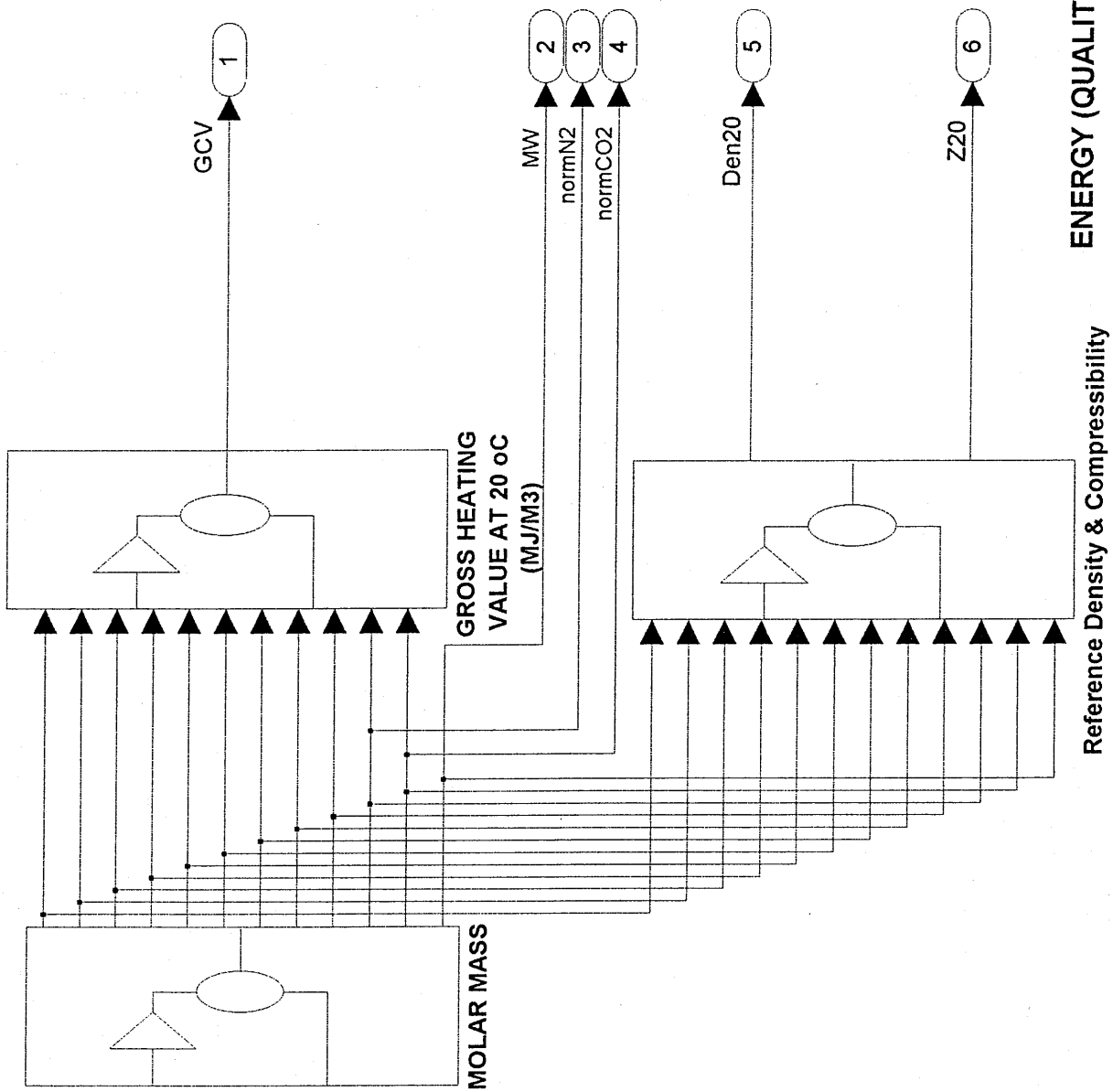


MAIN SYSTEM (OUTPUTS)

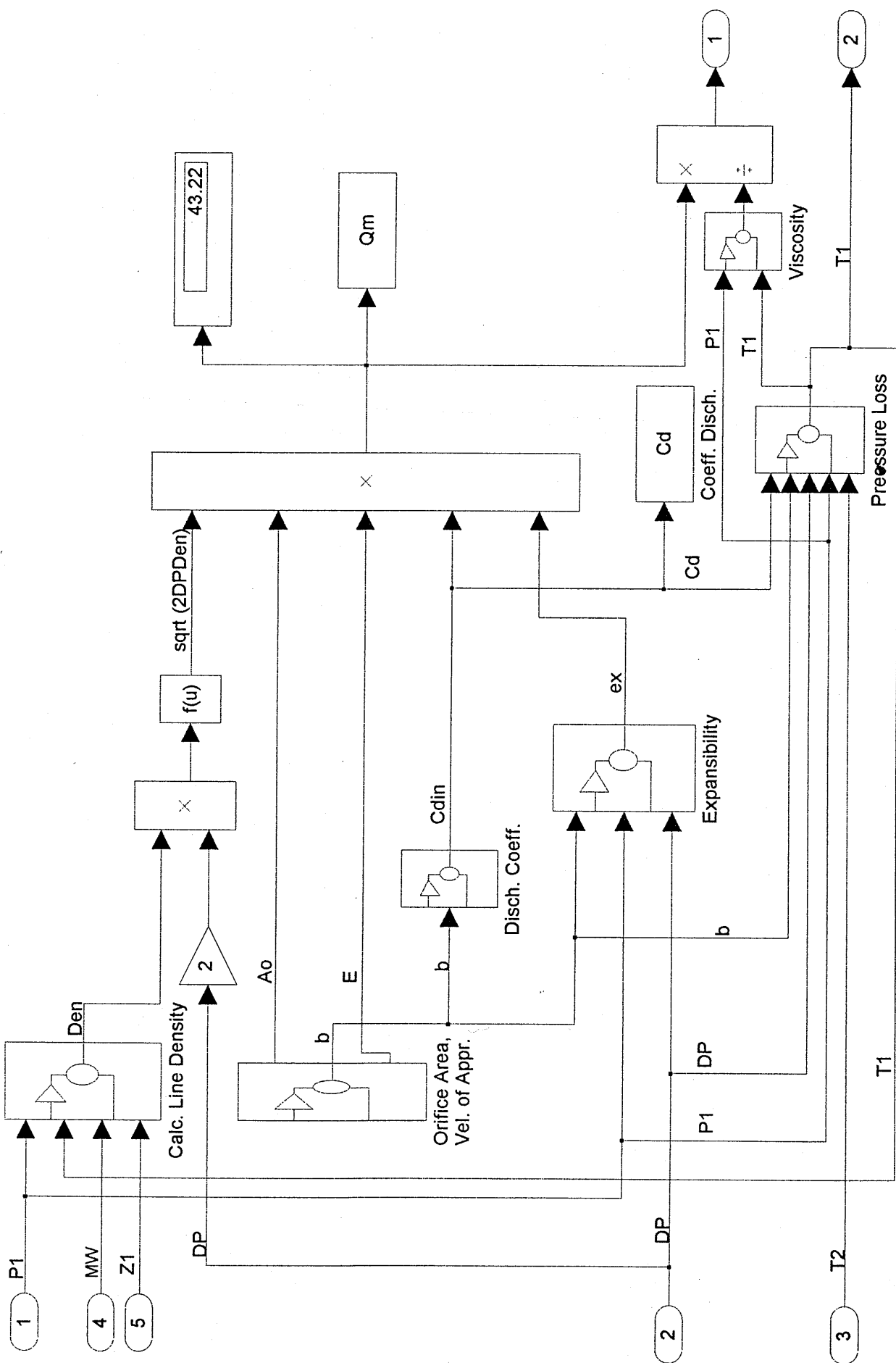


QUANTITY & QUALITY SUBSYSTEMS

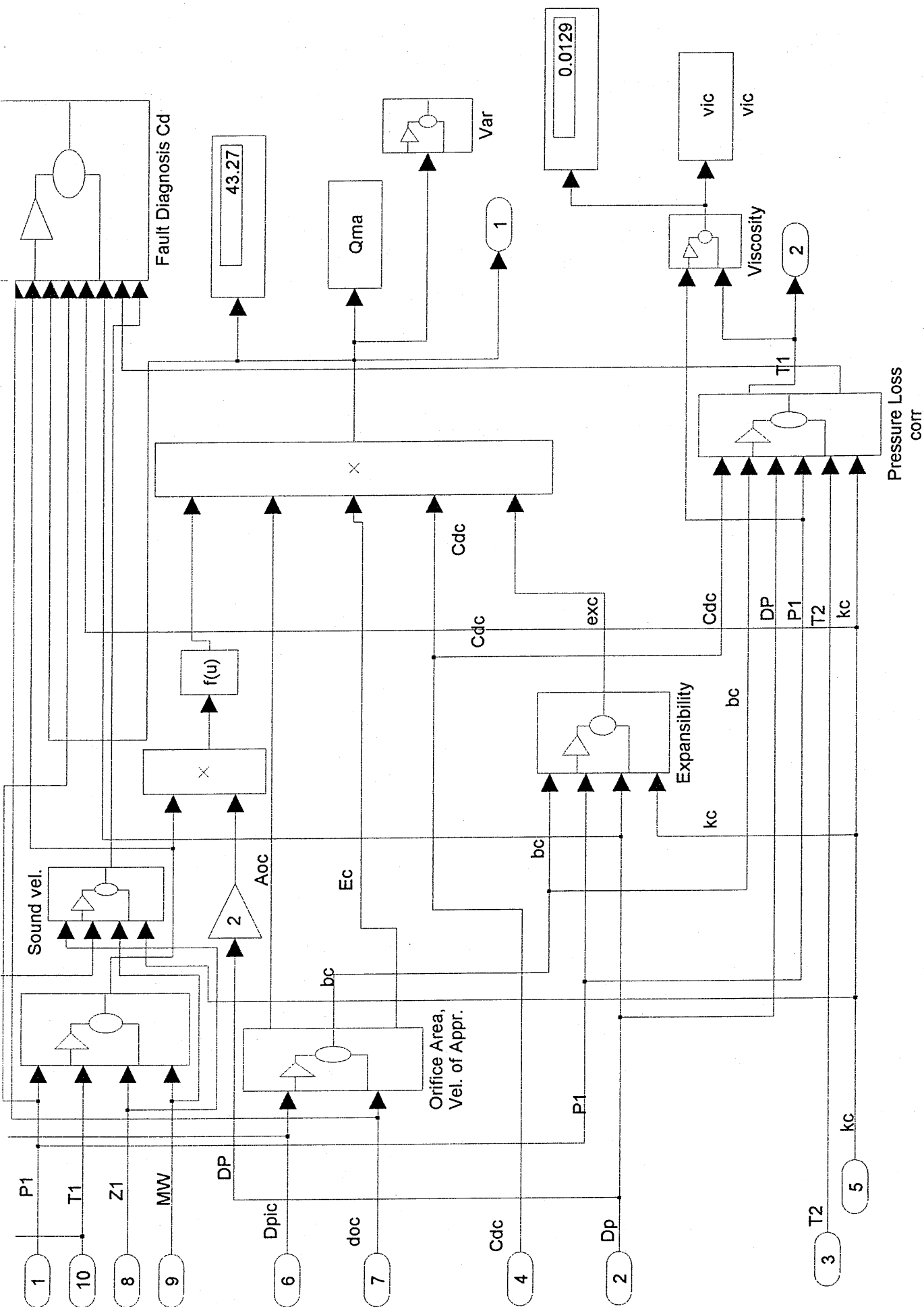


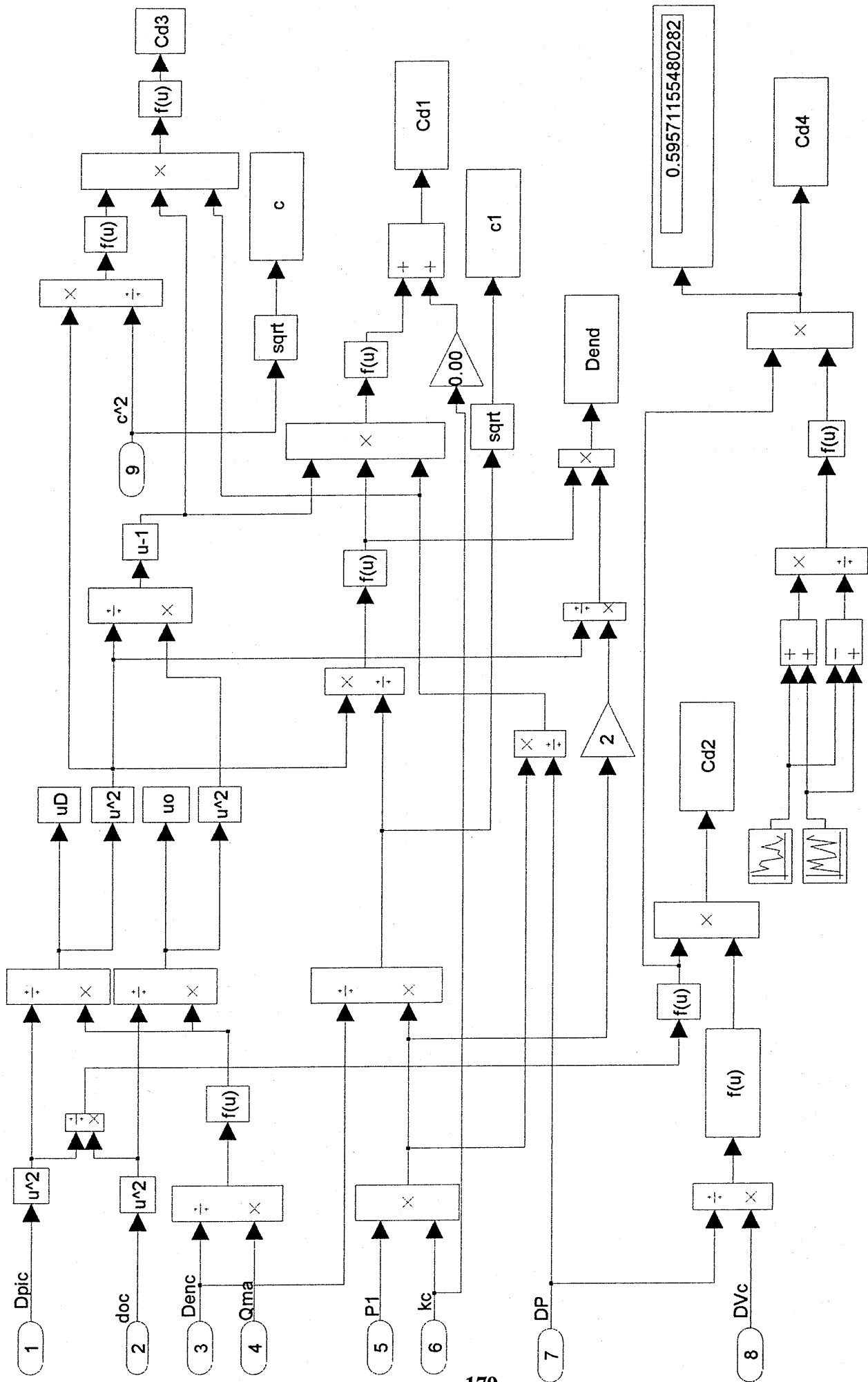


ENERGY (QUALITY) MAIN SUBSYSTEMS

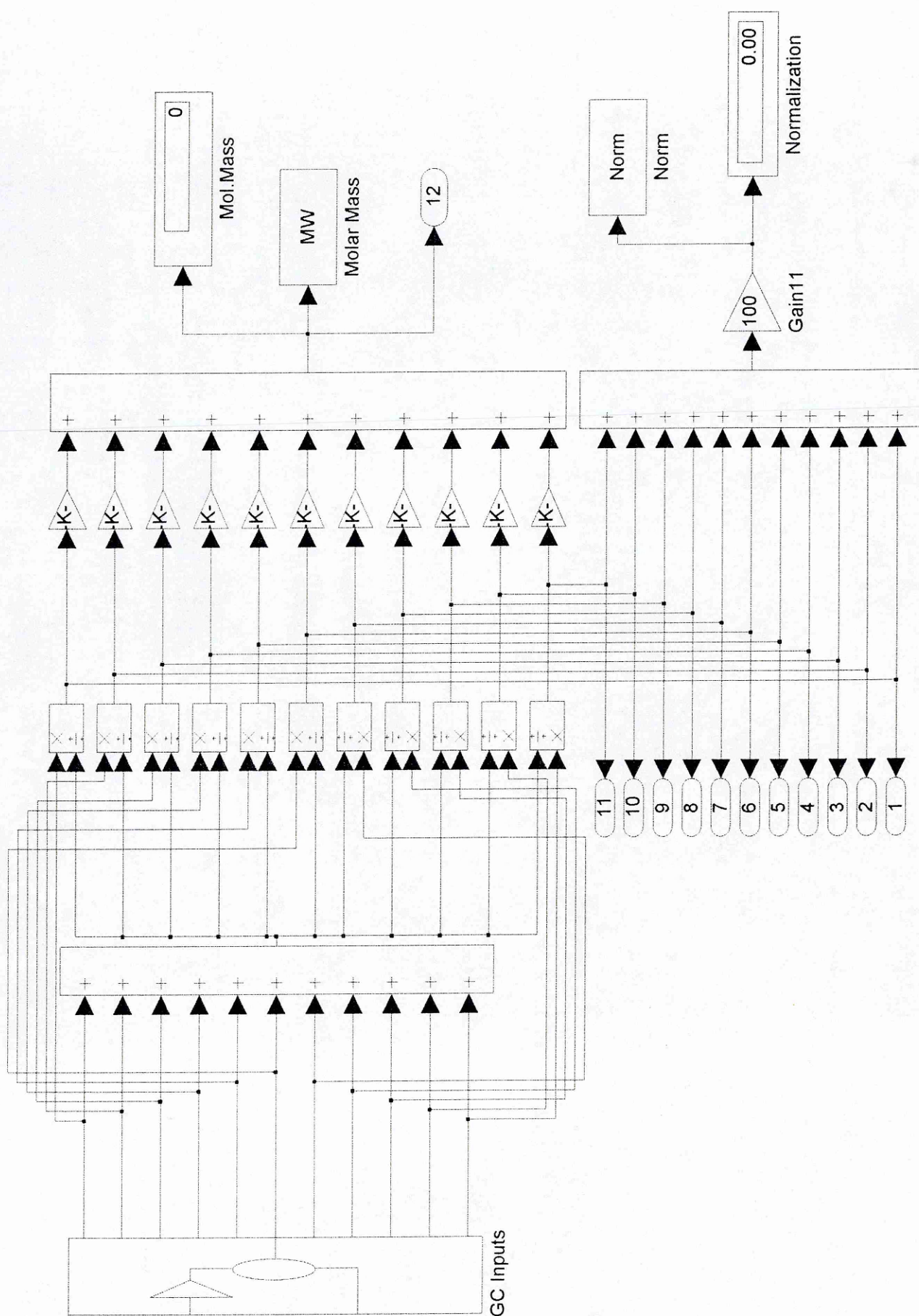


Initiation Subsystem

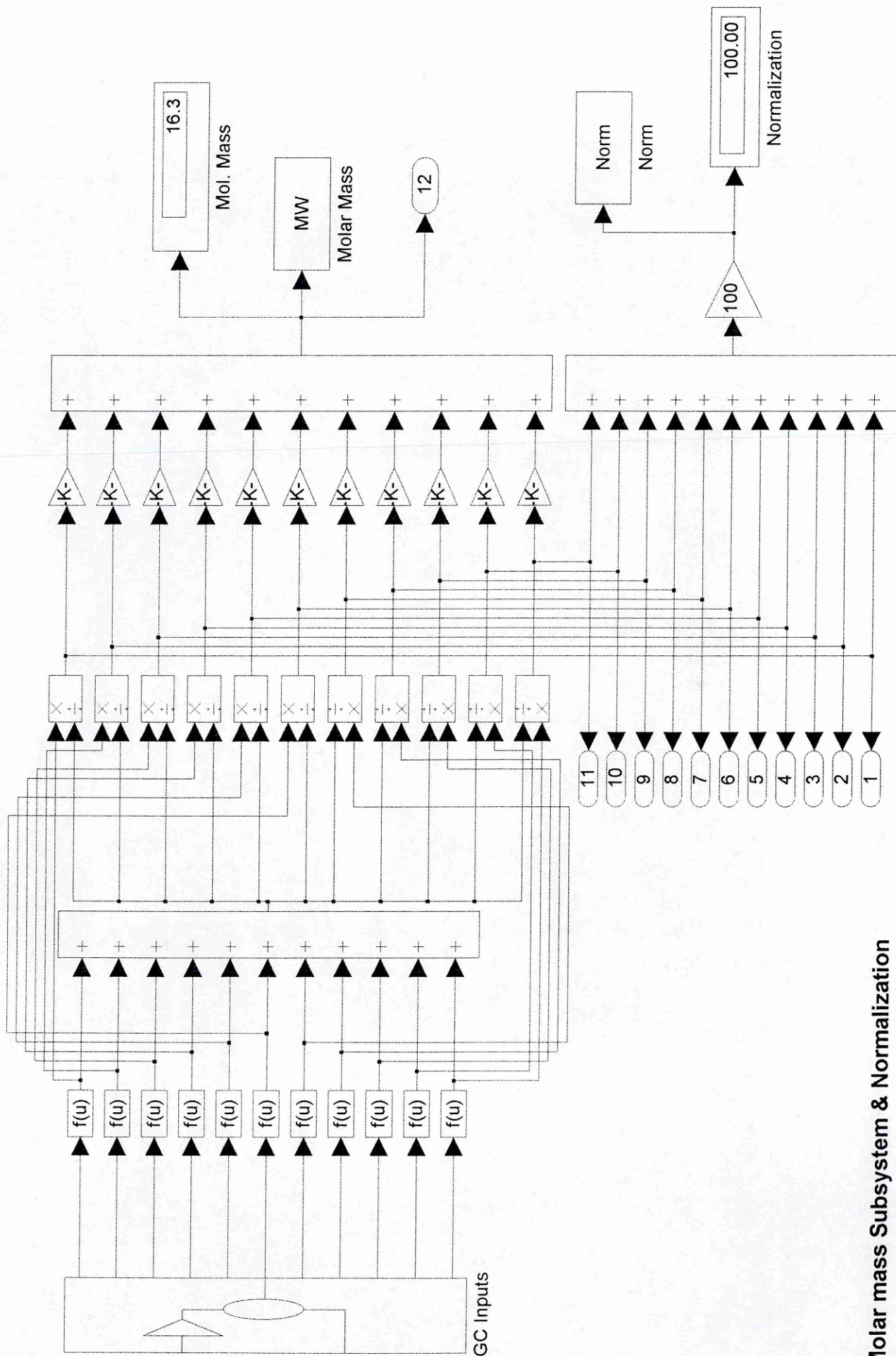




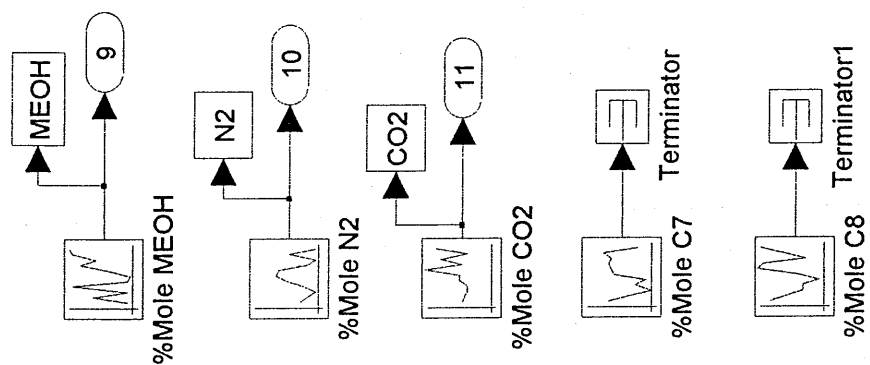
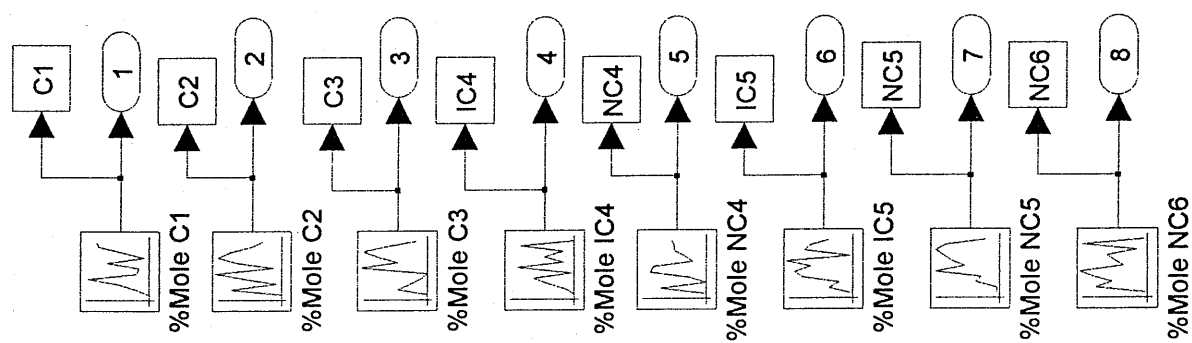
FAULT DIAGNOSIS (Cd) SUBSYSTEM



GAS NORMALISATION MODULE



Molar mass Subsystem & Normalization



Gas Chromatographic Inputs

APPENDIX B

- **Development of new Orifice flow theoretical equations – Summary Table of developed interrelationships**

B.1 Discharge Coefficient – The unverifiable factor with the highest uncertainty

Over the years a large body of experimental information has accumulated on the pressure drop characteristics of the orifice meter. Orifice meter is simply a member of a single family of flow elements named as "constrictions". Orifices, perforated plates, nozzles e.t.a. are all members of this family because of their geometrical similarities. The performance of these elements has been studied and analysed by their pressure drop characteristics. Furthermore, this pressure drop characteristics have been expressed in some cases in terms of the pressure loss coefficient (Kloss), while in other cases in terms of the discharge coefficient (Cd), or more generally in terms of some kind of pressure coefficient (Cp). The information about their performance is simply presented as a plot of pressure drop against flow rate. These pressure drop characteristics have been used, depending on the particular device, in a number of different contexts. For example, perforated plates have been used to reduce non-uniformities in ducts' flows, for boundary layer control applications e.t.a. while it is well known that the pressure drop characteristics of the orifice are extensively used for flow metering purposes.

The effort to describe the performance of the square edge orifice plate in various flow conditions has a very long history. The factor that in conclusion has been used to describe this performance is the coefficient of discharge (Cd). This is actually the quotient:

$$C_d = \frac{Q_{\text{actual}}}{Q_{\text{theoretical}}} \text{ where } Q \text{ is the volumetric flow rate.}$$

Analysing how the Cd appears in the description of flow through the orifice and is used in the calculation of flow rate, it is necessary to describe the derivation of the relationship between flow rate and differential pressure across the orifice. We must say that in high Reynolds number flow through the orifice inertial forces predominate while viscous forces have only historical effects and are quite negligible. So the theory has been developed on the relatively accurate assumption of inviscid flow.

B.1.1 New equations for Pressure loss and discharge coefficient of orifice meter.

A research program initiated in Cranfield University since 1995 into possible methods for on line monitoring of the performance of flow meters. Recently, and under a Ph.D. project, multiple theoretical models have been developed that associated the discharge coefficient of orifice meters with the contraction coefficient and the pressure loss coefficients since all

the coefficients are related. These were successfully supported by respective experimental work. A systematic derivation of the equations is presented. It is regarded necessary to present these equations in the very many possible forms in order to be quite practical for someone to select the most appropriate for his purpose. Some forms of the equations are presented for first time while others are quite old that only recently have been accepted by ISO and are present in the new **ISO 5167-2, 1999**. **Gerd Urner, 1997**, a member of the VDI/VDE-GMA-Committee for Measurement of Fluid Flow by means of differential pressure methods, published some of the equations regarding the above mentioned coefficients and virtually supports a suggestion of Prof. F. Brandt to correct the pressure loss equation given in **ISO 5167, 1991**. The permanent pressure loss in ISO 5167 is given as a function of measured differential pressure, discharge coefficient and beta ratio. In addition, it is given a simple empirical equation for the permanent pressure loss as a function only of beta ratio. It was also proposed, and has been accepted, the inclusion of an equation for the pressure loss coefficient that can be obtained as a function of discharge coefficient and beta ratio. The research program conducted by the author in the subject comes to support the acceptance of the new equations by ISO, as well as to propose the inclusion of another more simple empirical equation for the pressure loss coefficient. The later coefficient can also be obtained as a function only of beta ratio, similarly with the permanent pressure loss. In addition, it is proposed the inclusion of a table that is present here and includes a large list of the interrelationships between the various factors and coefficients regarding orifice metering.

B.1.1.1 Recent developments

A literature survey carried out by the author revealed that the equations presented by Gerd Urner (1997), regarding the permanent pressure loss and the pressure loss coefficient were actually known for many years. However, they had not been adopted by ISO until recently. The new ISO-5167-2, which was circulated as a Committee draft, (ISO/CD 5167-2, March 1999) includes these 'new' equations for permanent pressure loss and the loss coefficient in clause 4.4. Particularly, the old equation for the permanent pressure loss as a function of measured pressure drop has been modified and additionally, an equation for the pressure loss coefficient, since none was present in the previous ISO-5167 standard, has been adopted.

B.1.1.2 Derivation of the flow equation through the orifice and discharge coefficient.

In order to express the Newton's law of motion for a fluid particle in the form mass $\times$ acceleration, we need first an expression for the acceleration of a fluid particle. This can be expressed using the material time derivative $\frac{DU}{Dt}$ of the equation:

$$\frac{DU}{Dt} = \frac{\partial U}{\partial t} + (U \nabla)U \quad (B.1)$$

because we need the time rate of change following the fluid particle. When expressed in cylindrical coordinates the acceleration becomes:

$$\begin{aligned} \frac{DU}{Dt} = \frac{\partial U}{\partial t} + (U \nabla)U = & \\ & \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial z} \right) i_x + \\ & \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial z} \right) i_y + \\ & \left(\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} + \omega \frac{\partial \omega}{\partial z} \right) i_z \end{aligned} \quad (B.2)$$

Each of the three components of the acceleration vector requires one time derivative and three spatial derivatives of a velocity component in its expression or a total of twelve derivatives are needed for terming the accelerations of a fluid particle. There are two additional non-derivative terms when expressed in cylindrical coordinates, the centrifugal acceleration $-\frac{U_{\theta}^2}{r}$, in the radial direction and the Coriolis acceleration, $\frac{U_r U_{\theta}}{r}$, in the tangential direction.

For a selected volume element δV of a fluid having mass $\rho \delta V$, Newton's law of motion for a fluid particle can be obtained by equating the product of mass times acceleration of the fluid particle to the sum of the pressure force and the gravity force acting on the element and take the form:

$$\begin{aligned} (\rho \delta V) \left(\frac{\partial U}{\partial t} + (U \nabla)U \right) = & \\ = (-\nabla P) \delta V + (\rho \delta V)g & \end{aligned} \quad (B.3)$$

where dividing by $\rho \delta V$ we obtain Euler's equation of motion:

$$\left(\frac{\partial \mathbf{U}}{\partial t} \right) + (\mathbf{U} \nabla) \mathbf{U} = \frac{(-\nabla P)}{\rho} + \mathbf{g} \quad (\text{B.4})$$

Using the above form of Euler's equation after we replace the term $(\mathbf{U} \nabla) \mathbf{U}$ by using the vector identity of the equation

$$(\mathbf{U} \nabla) \mathbf{U} = \nabla \left(\frac{U^2}{2} \right) - \mathbf{U} \times (\nabla \times \mathbf{U}) \quad (\text{B.5})$$

and rearranging the terms we obtain the next form of Euler's equation:

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \left(\frac{U^2}{2} \right) + \frac{1}{\rho} \nabla P - \mathbf{g} = \mathbf{U} \times (\nabla \times \mathbf{U})$$

We are able to derive Bernoulli's equation by integrating the previous form of Euler's equation along a streamline. We can integrate between two points 1 and 2 and form the scalar product of Euler's equation and ds , where ds is the element of length and is parallel

to $\mathbf{U}$ at each point along the line. Thus the integral $\int_1^2 \mathbf{U} \times (\nabla \times \mathbf{U}) ds$ is zero because its

integrand is perpendicular to $\mathbf{U}$ and therefore the scalar product of the integrand with ds is identically zero. The condition ds is parallel to $\mathbf{U}$ can be expressed as $ds \times \mathbf{U} = 0$. Assuming incompressible flow, and eliminating explicitly the gravity term, two of the remaining integrals can be evaluated directly as follows:

$$\left. \begin{array}{l} \nabla U = 0 \\ \text{or} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \end{array} \right| \quad (\text{Incompressible flow-from continuity equation})$$

$$\begin{aligned} \int_1^2 \nabla \left(\frac{U^2}{2} \right) ds &= \\ &= \frac{1}{2} \int_1^2 \left(\frac{\partial U^2}{\partial x} dx + \frac{\partial U^2}{\partial y} dy + \frac{\partial U^2}{\partial z} dz \right) \\ &= \frac{1}{2} \int_1^2 d(U^2) = \frac{U_2^2 - U_1^2}{2} \end{aligned} \quad (\text{B.6})$$

and

$$\int_1^2 \frac{1}{\rho} \nabla P ds = \frac{1}{\rho} (P_2 - P_1) \quad (\text{B.7})$$

At this moment we can derive Bernoulli's equation - in its one-dimensional form - as it applies between two planes upstream and downstream the orifice, in order to derive the relationship between flowrate and differential pressure:

$$\int_1^2 \frac{\partial u}{\partial t} dx + \frac{u_2^2 - u_1^2}{2} - \frac{P_1 - P_2}{\rho} = 0 \quad (\text{B.8})$$

The integrand of the first term in the previous form of Bernoulli's equation is the unsteady term and expresses the *local or temporal acceleration* of the fluid due to change of velocity in the longitudinal direction. This must be known at all points along the instantaneous streamline for the integral to be evaluated.

The author would like to express at this point a very important fact that sometimes may be disregarded. Since we have assumed incompressible flow for the above derivation of Bernoulli's equation along a streamline, we must say that it is not sufficient that the flow be incompressible for the density to be constant along the streamline ($\rho = c$), unless the flow

is also steady ($\frac{\partial U}{\partial t} = 0$). On the other hand, it is sufficient but not necessary for the density to be constant everywhere in the flow field in order to be constant along a streamline.

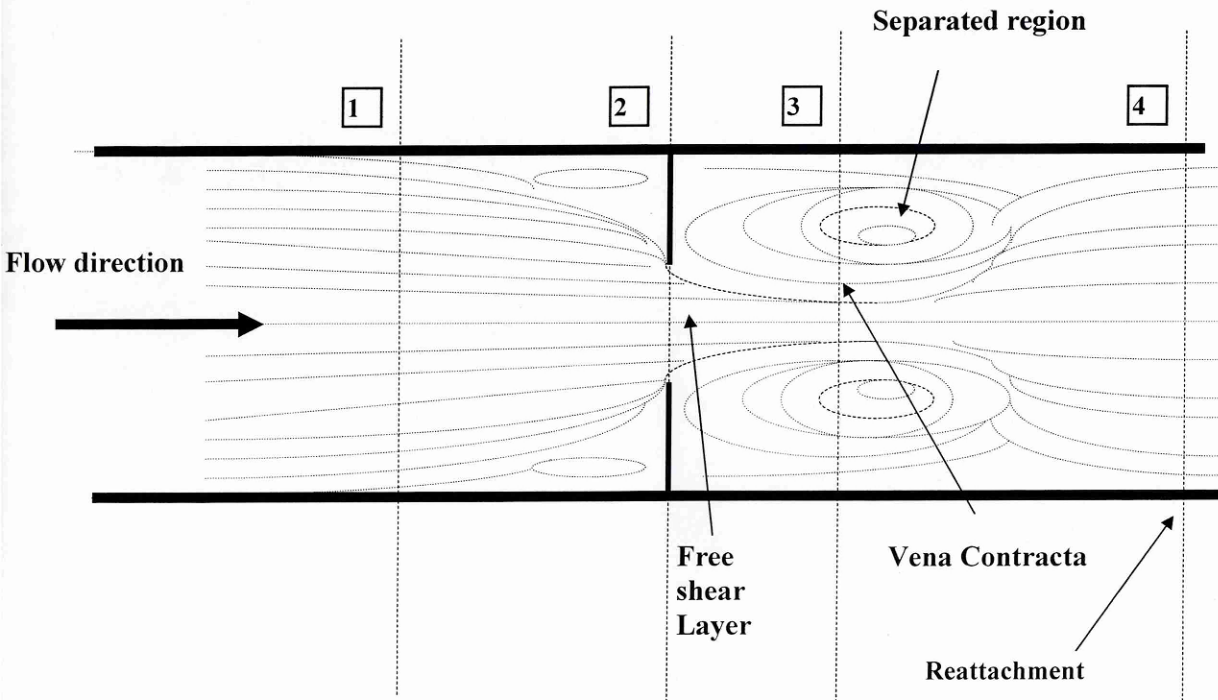


Figure B.1. Flow through the orifice plate

From this point and using the last very convenient form of Bernoulli's equation many different interpretations can be given and different theories have been proposed.

Particularly, in order to analyse the appearance of discharge coefficient (C_d) in the flow rate-differential pressure relationship we must first describe the characteristics of the flow through the orifice, as those have been extensively presented after many years measurements and observation. In the **figure B.1** is shown the flow through the orifice plate. It is assumed that between planes 1 and 2 of integration there is a flow without friction so $P_{1Tot}=P_{2Tot}$.

In **figure B.2**, in the next page, is shown the static pressure distribution along the pipeline containing an orifice plate. Note that $P_{1...2} - P_{2...3}$ (the subscript 1...2 means that theoretically we could use an upstream pressure tapping anywhere between the planes 1 and 2 and 2...3 means unlikely the same for the downstream pressure tapping) is the pressure difference (ΔP) that is created by the flow through the orifice and is used for flow metering purposes. Obviously, it depends on the pressure tappings position where P_1-P_4 is the permanent pressure loss ($\Delta \varpi$) caused by the presence of the orifice in the pipe. The two differential pressures are not independent. In ISO-5167 are specified the internationally agreed positions of the pressure tappings (flange, corner, D-D/2 tappings) on which various empirical equations for C_d have been developed (Stoltz, Reader-Harris/Gallagher).

In the two figures, Plane 1 is the plane where separation starts. Plane 2 is the plane of orifice throat. Plane 3 is the *vena contracta* plane and plane 4 is the pressure recovery plane where reattachment of the flow occurs.

Describing the flow through the orifice we can say that the presence of the orifice meter cause an acceleration to the fluid stream by imposing a contraction on the flow area. The flow separates away from the surface of the orifice to form a discrete jet, which contracts to a minimum cross-sectional area at the vena contracta, together with a separated flow region, as shown in figure B.1. The separating streamline is well defined between the orifice entry and the vena contracta, in which region the flow has the characteristics of a laminar (inviscid) free shear layer as it is subjected to a favourable pressure gradient. Downstream from the vena contracta, transition takes place and the distinct identities of the jet and separated flow are rapidly lost due to turbulent mixing in a region dominated by an adverse pressure gradient until reattachment at the pipe wall occurs about 6D downstream from the orifice plate. Thus the flow decelerates, "expanding" back to the original pipe diameter. As the fluid accelerates, its static pressure is reduced. As the fluid decelerates,

the pressure rises again, but not without losses. Losses are related to the irreversibility in the flow field, eddy structures and turbulence. Mechanical energy is dissipated (or better is transformed) by these mechanisms, while viscosity has no immediate effect as has been already stated. This mechanical energy (mainly kinetic energy for the orifice jet) that is transformed into internal or thermal energy must be provided by flow work done by the main stream. The result of this energy transfer is a loss of pressure and an increase of the entropy of the fluid in contrast to the flow up to the vena contracta that is in fact *isentropic*, something that is readily justified by the favorable pressure gradient in this region. So, the overall flow process through the orifice from plane 1 to 4 is an irreversible adiabatic expansion, thermodynamically equivalent to a Joule-Thomson expansion. The new standard ISO-5167 takes into account all these facts and has been modified accordingly.

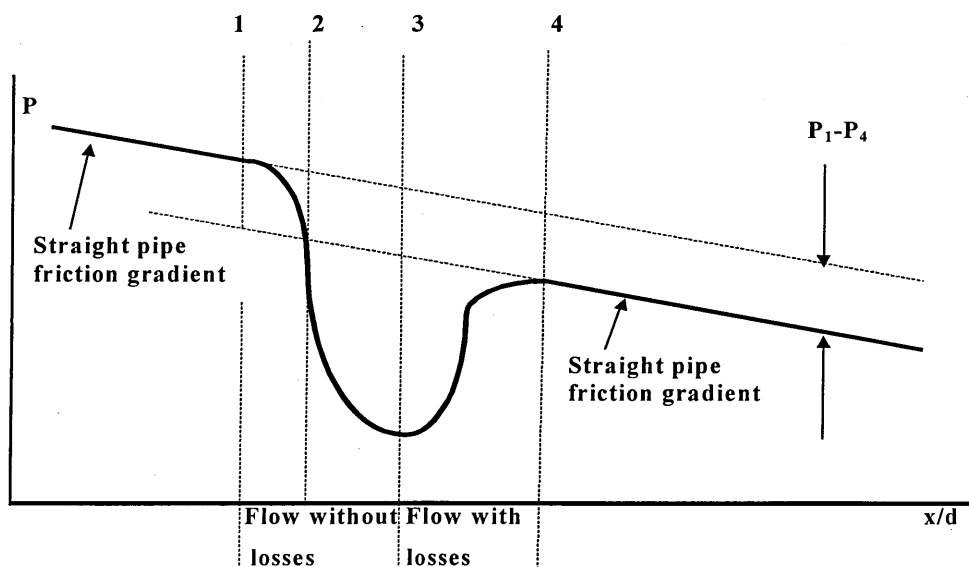


Figure B.2 Static pressure distribution along a pipeline with an orifice plate installed

The ratio of the vena contracta area to the cross sectional area of the orifice is called the coefficient of contraction (C_c). The Contraction coefficient is the principal flow parameter influencing the pressure drop characteristics of the orifice as it will become evident in the following analysis and hence it will form the focal point of ensuing discussions.

Considering all the assumptions stated above and using proper pressure tapings upstream and downstream of the orifice meter, we measure the differential pressure across the orifice

$P_{1...2} - P_{2...3}$. In addition, assuming that the *temporal acceleration* of the fluid $\left(\frac{\partial u}{\partial t}\right)$

between the two tappings is negligible compared with the *convective acceleration* $u \frac{\partial u}{\partial x}$, the last form of Bernoulli's equation applied at plane 1 upstream and to the throat, plane 2, takes the forms:

$$\left(\frac{\rho u_o^2}{2(P_1 - P_2)} - \frac{\rho u_D^2}{2(P_1 - P_2)} \right) = 1 \quad (\text{B.9})$$

or

$$u_D^2 = u_o^2 - \frac{2(P_1 - P_2)}{\rho} \quad (\text{B.10})$$

Equation **B.10** is the one that is used in conjunction with *mass continuity* equation ($q_m = \rho_D u_D A_D = \rho_o u_o A_o$) for flow metering purposes. Since we have assumed incompressible flow ($\rho_D = \rho_o$), the pipe velocity and the volumetric flow rate, after we replace

$$u_o^2 = u_D^2 \frac{A_D^2}{A_o^2} = u_D^2 \frac{D^4}{d_o^4}$$

are given by:

$$u_D = \frac{1}{\sqrt{\frac{D^4}{d_o^4} - 1}} \sqrt{\frac{2\Delta P_{1,2}}{\rho}} = \frac{\beta^2}{\sqrt{1 - \beta^4}} \sqrt{\frac{2\Delta P_{1,2}}{\rho}} \quad (\text{B.11a})$$

$$Q = \frac{\beta^2}{\sqrt{1 - \beta^4}} \frac{\pi D^2}{4} \sqrt{\frac{2\Delta P_{1,2}}{\rho}} = \frac{1}{\sqrt{1 - \frac{d_o^4}{D^4}}} \frac{\pi d_o^2}{4} \sqrt{\frac{2\Delta P_{1,2}}{\rho}} \quad (\text{B.11b})$$

where the last equations are referred to **mean** quantities and Q is the mean volumetric flow in m^3/sec , D , d_o is the pipe and the orifice throat diameter respectively, $\beta = \frac{d_o}{D}$ is the so called beta ratio, A_D , A_o the respective areas and u_D , u_o the respective mean velocities. Although everything seems clear until this point since D and d_o are known (after precision measurements) and the density can be measured or calculated from a state equation for gases, the differential pressure used in the last equation is referred to planes 1 and 2. This means that the downstream pressure tapping must be exactly in the orifice throat for applying equations **B.9** to **B.11**. However, in practice the downstream pressure is measured in the region between planes 2 and 3 in the respective tappings' positions that are referred

in ISO-5167. Subsequently, the measured differential pressure ($\Delta P_{1,2...3}$) may be significantly higher than the one used in equations B.9 to B.11.

Although the previous considerations may be well known to people relevant with the subject, it is regarded necessary to present in detail the subject in order to become more clear how the discharge coefficient appears in the flow equation and to proceed to more significant considerations. Thus, the discharge coefficient (C_d) appears as an empirical correction factor in the flow equation. It accounts for this discrepancy between the theoretical integration interval of ideal flow calculation between planes 1 and 2 (including the subsequent interval of application of mass continuity equation) and the practically measured differential pressure that is between plane 1 and a plane between the orifice throat and the vena contracta. It must be emphasised at this moment, that C_d has virtually the characteristics of a pressure coefficient (C_p) with all the consequences that this can have in its uncertainty in various orifice installations and flow conditions. The need for standardisation of the orifice meter as a flow metering device, that was driven by its wide use in various industries, brought the necessity for standardisation of all the parameters that could possibly affect this flow rate-differential pressure relationship. Some of the most important are the *upstream fully developed velocity profile*, the tapping position, the orifice edge sharpness e.t.a. However, there are some other parameters that cause extensive difficulties in this standardisation effort since unfortunately they are extremely difficult to control. They introduce significant effects on the above relationship and subsequently on the target for improved metering capabilities with the orifice meter. These are pipe wall roughness or else pipe friction effects and interference effects from other pipe components. Equation B.10 (combined with the mass continuity equation) and equation B.11 are modified with the inclusion of discharge (correction) coefficient and can take the forms:

$$u_D = C_d \frac{\beta^2}{\sqrt{1-\beta^4}} \sqrt{\frac{2\Delta P_{1,2...3}}{\rho}} \quad (\text{B.12})$$

and

$$Q = C_d \frac{1}{\sqrt{1-\frac{d_o^4}{D^4}}} \frac{\pi d_o^4}{4} \sqrt{\frac{2\Delta P_{1,2...3}}{\rho}} \quad (\text{B.13})$$

Using the form of equation B.9, from B.12 we can simply obtain:

$$C_d = \sqrt{\frac{\rho u_o^2}{2(P_1 - P_{2...3})} - \frac{\rho u_D^2}{2(P_1 - P_{2...3})}} \Rightarrow \quad (B.14)$$

$$C_d = \sqrt{\frac{1}{C_{Po}} - \frac{1}{C_{PD}}} < 1$$

where C_{Po} , C_{PD} are pressure coefficients based on the mean velocity at the orifice throat and at the pipe respectively. The value of C_d that is obtained from **B.14** is the value that is used for flow measurement purposes and is around 0,6.

B.1.3 Development of the new forms of the various equations

Within the context of the previous approach, equation **B.14** reveals those characteristics of the discharge coefficient as a pressure coefficient that have been mentioned previously. If the planes of application of Bernoulli's equation are the same with the planes that the differential pressure is measured then C_d has its limiting value that is 1. Thus, the left form of **B.14** can be presented in the next more general form that it can be applied between planes 1 (just at the point that separation starts) and any plane between 2 and 3 where a favourable pressure gradient exists and the flow is still regarded inviscid:

$$C_d = \sqrt{\frac{\rho u_{2...3}^2}{2(P_1 - P_{2...3})} - \frac{\rho u_D^2}{2(P_1 - P_{2...3})}} = 1 \quad (B.15)$$

In order to introduce the contraction coefficient $\left(C_c = \frac{d_{vc}^2}{d_o^2}\right)$ (where d_{vc} is the vena contracta diameter) as well as to explore its significance and its relation with C_d , we assume that the planes of application of Bernoulli's equation are 1 and 3 (vena contracta plane). We introduce also another coefficient $\left(C_j = \frac{d_{vc}^2}{D^2}\right)$ that could be called *Jet necking*

Coefficient and the respective ratio $\frac{d_{vc}}{D}$ called an *Alfa (a)* ratio in order to be used later for simplicity purposes.

Then, considering all the above, for planes 1 and 3 where are the planes that differential pressure is measured as well, we have from **B.11a**:

$$u_D = \frac{1}{\sqrt{\frac{D^4}{d_{vc}^4} - 1}} \sqrt{\frac{2\Delta P_{1,3}}{\rho}} \quad (B.16)$$

and from **B.12**:

$$u_D = \frac{C_d \frac{d_o^2}{D^2}}{\sqrt{1 - \frac{d_o^4}{D^4}}} \sqrt{\frac{2\Delta P_{1,3}}{\rho}} \quad (\text{B.17})$$

Equating the last two equations after squaring we have:

$$\begin{aligned} C_d^2 &= \frac{d_{vc}^4}{d_o^4} \frac{(D^4 - d_o^4)}{(D^4 - d_{vc}^4)} \Rightarrow \\ C_d &= C_c \sqrt{\frac{1 - \beta^4}{1 - a^4}} \end{aligned} \quad (\text{B.18})$$

After proper manipulation of the last equation, a long series of some new very useful equations can be obtained. They will be presented next, in order to explore all the interrelationships between the variables to be easier for someone to collect those that are the most proper for his purposes. In addition, they can lead us to reconsider some of the established equations that currently are present in ISO-5167.

It is evident that the next series of equations could be viewed in a more general sense and could be applied to any member of the constriction device family as long as all the previous assumptions are considered. They can be applied to the whole region of inviscid flow and particularly between plane 1 and any plane between 2 and 3. In this occasion, d_{vc} can be replaced by a diameter $d_{2...3}$ that represents the diameter of the jet at any downstream pressure measurement point until the vena contracta.

We must also emphasise that under this broad sense, all the relevant coefficients ($C_{d1,2...3}$, $C_{c1,2...3}$, $C_{j1,2...3}$ e.t.a.) are based on the same conditions and are tappings position dependent. So, from **B.18** we have:

$$\begin{aligned} C_d &= C_c \sqrt{\frac{1 - \beta^4}{1 - \frac{d_{vc}^4}{D^4}}} \Rightarrow \\ C_d &= C_c \sqrt{\frac{1 - \beta^4}{1 - \frac{d_{vc}^4}{D^4} \frac{d_o^4}{d_o^4}}} \Rightarrow \\ C_d &= C_c \sqrt{\frac{(1 - \beta^4)}{(1 - C_c^2 \beta^4)}} \end{aligned} \quad (\text{B.19})$$

$$C_c = \frac{C_d}{\sqrt{1-\beta^4(1-C_d^2)}} \quad (\text{B.20})$$

In the last equations is shown the relationship between the C_d and C_c . We can easily see that if $C_c=1$ ($d_{vc}=d_o$) then also $C_d=1$, and vice versa. However, the most important functions that can be derived from equation B.20 have to do with the pressure loss $\Delta\varpi$ and the pressure loss coefficient K_{loss} and will be developed after the presentation of some other series of equations.

Since $E = \frac{1}{\sqrt{1-\beta^4}}$ (Velocity of approach factor) then from B.19:

$$K = C_d E = \frac{C_c}{\sqrt{1-C_c^2\beta^4}}$$

(Where K is the so called Flow coefficient)

The term $C_c\beta^2$ represents the jet necking coefficient C_j and is a very important term. We can also obtain:

$$C_j^2 - \frac{C_j^2}{C_d^2} - \frac{C_c^2}{C_d^2} = 1 \quad (\text{B.21})$$

$$\frac{C_d^2 - C_c^2}{C_c^2(C_d^2 - 1)} = \beta^4 \quad (\text{B.22})$$

from which we can obtain:

$$C_c = \frac{1}{\beta^2} \frac{1}{\sqrt{\frac{2\Delta P_{1,3}}{\rho u_D^2} + 1}} \quad (\text{B.23})$$

$$C_{pD} = \frac{\Delta P_{1,3}}{\frac{\rho u_D^2}{2}} = \frac{1}{C_c^2 \beta^4} - 1 \quad (\text{B.24})$$

and respectively

$$C_{po} = \frac{\Delta P_{1,3}}{\frac{\rho u_o^2}{2}} = \frac{1}{C_c^2} - \beta^4 \quad (\text{B.25})$$

and

$$C_{pvc} = \frac{\Delta P_{1,3}}{\frac{\rho u_{vc}^2}{2}} = 1 - C_c^2 \beta^4 \quad (\text{B.26})$$

Certainly, from the last three equations various interrelationships could be created.

We can continue presenting a few other equations mainly because it is necessary to show that this nexus of interrelationships is almost unlimited. An effort has been made to present them with a proper order. Thus, continuing this process we can obtain the usually unknown vena contracta diameter d_{vc} from:

$$d_{vc} = \frac{d_o \sqrt{C_d}}{\sqrt[4]{1 - \beta^4 (1 - C_d^2)}} \text{ or } d_{vc} = \sqrt[4]{\frac{1}{\frac{1}{D^4} + \frac{1}{C_d^2} \left(\frac{1}{d_o^4} - \frac{1}{D^4} \right)}} \quad (\text{B.27})$$

(d_{vc} as a function of C_d , d_o , D)

$$d_{vc} = \frac{D}{\sqrt[4]{\frac{\Delta P_{1,3}}{\rho u_D^2} + 1}} = \frac{D}{\sqrt[4]{C_{pD} + 1}} \quad (\text{B.28})$$

(as a function of a pressure coefficient and D)

$$\begin{aligned} d_{vc} &= \frac{D}{\sqrt[4]{\frac{\pi^2 \Delta P_{1,3} D^4}{8 \rho Q^2} + 1}} = \\ &= D \sqrt[4]{Q \sqrt{\frac{8 \rho}{\pi^2 \Delta P_{1,3} D^4 + 8 \rho Q^2}}} = \\ &= D \sqrt[4]{q_m \sqrt{\frac{8}{\rho \pi^2 \Delta P_{1,3} D^4 + 8 q_m^2}}} \end{aligned} \quad (\text{B.29})$$

(as function of Q or q_m where all the terms in the last three equalities that are after D it can be easily seen that are equal with the Jet necking Coefficient C_j).

Inevitably, someone could say that this interdependency among the variables is not really giving a clear and simple picture and possibly adds much complexity if the intention is to use them for practical flow measurement purposes. However, all these equations can provide us the tools to study in a more detail the flow rate - differential pressure relationship. They could be very useful in cases of in situ-calibration and for monitoring purposes of the flow measurement process. Since, it is mainly desirable to study the flow measurement - differential pressure relationship, they will be presented next the series of equations that could best describe this relationship, based on all the above considerations. For example, the volumetric flow rate could be obtained by:

$$\begin{aligned}
 Q &= \frac{1}{\sqrt{\frac{D^4}{d_{vc}^4} - 1}} \frac{\pi D^2}{4} \sqrt{\frac{2 \Delta P_{1,3}}{\rho}} = \\
 &= \pi \frac{1}{\sqrt{\frac{1}{d_{vc}^4} - \frac{1}{D^4}}} \sqrt{\frac{\Delta P}{8\rho}}
 \end{aligned} \tag{B.30}$$

B.1.4 Quasi steady theory and quasi steady theory with inertial term.

In the last equation, d_{vc} is practically unknown when the orifice meter is used in industrial applications. That is why we use d_o which can be measured and the C_d as the correction factor. However, it is necessary to consider the fact that d_o , under normal operating conditions, has an almost constant value (is modified slightly by the temperature changes). On the other hand, d_{vc} is an unsteady parameter, a fact that has been verified by various research observations of the flow through the orifice (**Hebrand, P., 1985**). Thus, we could say that all the factors such as C_d , C_c , C_j vary in such a manner, in order to take into account all the effects that can modify the differential pressure-flow rate square (more correctly) relationship. This particular fact was and continues to be, under the scope of current research by many other researchers in the field. **ISO CD/TR 3313,1997** describes the effects of flow pulsation on orifice plates and other flow meters.

Particularly, the problem is focused in the unsteady flow disturbances (turbulence, pulsation, acoustic disturbances, flow steps e.t.a.) and mainly in the inertia of the fluid

between the two pressure taps. Since, if the temporal inertial term $\int_1^3 \frac{\partial u}{\partial t} ds$ is also

considered, an expression that can be obtained for the flow rate - differential pressure relationship is:

$$K_1 \frac{dQ_{(t)}}{dt} + K_2 Q_{(t)}^2 = \frac{\Delta P_{1,3}}{\rho} \tag{B.31}$$

In this equation, the first term on the left-hand side corresponds to the inertia of the fluid between the two pressure taps. The K_2 parameter can be linked to discharge coefficient C_d . From this equation different theories have been proposed. (**Mottram, R.C., Gajan, P. et.al., 1992**).

In **Quasi-steady theory** it is regarded that the normal steady flow rate relationship between flowrate and differential pressure holds "*instantaneously*" during the period of unsteadiness. The author would like to express his considerations regarding the term *instantaneously* since it is as **fuzzy** as the difficulties encountered in the description of the whole subject and have lead to the development of the different theories. In this theory, the influence of the inertia of the fluid is regarded negligible ($K_1=0$) and the K_2 parameter retains the steady flow value which is directly linked with the discharge coefficient. Thus, during these conditions of unsteadiness, flowrate is deduced from the corresponding differential pressure:

$$Q_{(t)} = \sqrt{\frac{\Delta P_{1,3}}{\rho K_2}}$$

It is possible, according to this theory, to determine the mean flowrate $\bar{Q}$ by measuring the time-mean square root of the differential pressure $\left(\sqrt{\Delta P_{1,3}}\right)$ when the differential pressure is measured with fast response sensors. However, industrial DP sensors have generally a low frequency response and often have additional built-in electronic damping to reduce fluctuations in the output signal. This "averaging" procedure has as a consequence the overestimation of the flowrate because of the "well-known" square root effect $\sqrt{\overline{\Delta P_{1,3}}} > \overline{\sqrt{\Delta P_{1,3}}}$ or otherwise Jensen's Uncertainty Inequality. The coefficient K_2 focuses researcher's attention for many years (**Sparks, C.R., 1966, Mosely, M.S., 1966,)** and more recently of **McCloy, D., 1996** and particularly **Mottram R.C., Gajan, P., et. al., 1992**, because it is the one that is connected with all the coefficients presented in this chapter and tend to describe the flow rate-differential pressure relationship. Thus, considering all the equations above, we obtain the following expressions for K_2 :

$$\begin{aligned}
 K_2 &= \frac{8(1-\beta^4 C_c^2)}{\pi^2 \beta^4 C_c^2 D^4} \text{ or} \\
 &= \frac{8(1-C_j^2)}{\pi^2 d_{vc}^4} \text{ or} \\
 &= \frac{8(D^4 - d_{vc}^4)}{\pi^2 d_{vc}^4 D^4} \text{ or} \\
 &= \frac{8(1-\beta^4)}{\pi^2 C_d^2 d_o^2} \text{ or} \\
 &= \frac{8}{\pi^2} \left(\frac{1}{d_{vc}^4} - \frac{1}{D^4} \right)
 \end{aligned} \tag{B.32}$$

The combinations could be many however the last expression is the simplest. The first of the last expressions for K_2 presented in this project is the same with the expression obtained from the air-flow study of **Wilson, M.P., & Teyssandier, R.G., 1969**, and has been used recently by **Gajan, P., Mottram, R.C., et. al., 1992**, in order to model the relationship between instantaneous flow rate and differential pressure giving very satisfactory results. In **quasi-steady theory with inertial term**, the temporal acceleration term is not neglected ($K_1 \neq 0$). Mottram have linked this coefficient to an inertia length l_e defined as: $K_1 = \frac{4l_e}{\pi d_o^2 C_c}$. In acoustic terminology the quantity $\rho \frac{4l_e}{\pi d_o^2}$ is the inertance of the restriction. It is also emphasised that in this theory it is regarded that the unsteadiness does not affect the value of the coefficient K_2 during its cycle.

B.1.5 Permanent pressure loss and loss coefficient new forms, (K_{loss}).

As it has been explained previously and looking in **figure B.2**, in the flow through orifice mechanical losses take place between planes 3 and 4 in a region of adverse pressure gradient. The flow is no longer isentropic, but is an irreversible adiabatic process for which only enthalpy and mass are conserved. So, the overall flow process through the orifice from plane 1 to 4 is an irreversible adiabatic expansion, thermodynamically equivalent to a Joule-Thomson expansion. The new standard ISO-5167 takes into account all these facts and has been modified accordingly. A new equation has been adopted as well for the expansibility factor based on the concept of isenthalpic expansion between planes 1 and 4 as shown in figure B.1. The mechanical losses that appear in the form of *permanent pressure losses* $\Delta \varpi = P_1 - P_4$ can be calculated as a function of the **maximum** pressure

drop $\Delta P_{1,3}$. The pressure recovery between planes 3 and 4 can be calculated using the

impulse theorem: $P_4 - P_3 = \rho u_D u_{vc} - \rho u_D^2$ (B.33)

So, we could take:

$$\begin{aligned} \frac{\Delta \varpi}{\Delta P_{1,3}} &= \frac{\Delta P_{1,3} - \Delta P_{4,3}}{\Delta P_{1,3}} = \\ &= \frac{\frac{\rho}{2} u_{vc}^2 - \frac{\rho}{2} u_D^2 - \rho u_D u_{vc} + \rho u_D^2}{\frac{\rho}{2} u_{vc}^2 - \frac{\rho}{2} u_D^2} = \\ &= \frac{u_D^2 - 2 u_D u_{vc} + u_{vc}^2}{u_{vc}^2 - u_D^2} = \\ &= \frac{(u_D - u_{vc})^2}{(u_D - u_{vc})(u_D + u_{vc})} = \frac{1 - \frac{u_D}{u_{vc}}}{1 + \frac{u_D}{u_{vc}}} \end{aligned} \quad (B.34)$$

However, from continuity we have:

$$\frac{u_D}{u_{vc}} = \frac{d_{vc}^2}{D^2} = \frac{d_{vc}^2}{D^2} \frac{d_o^2}{d_o^2} = C_e \beta^2 = C_j \quad (B.35)$$

where C_j is the jet necking coefficient.

Thus, the pressure loss can be obtained now as:

$$\frac{\Delta \varpi}{\Delta P_{1,3}} = \frac{1 - C_e \beta^2}{1 + C_e \beta^2} = \frac{1 - C_j}{1 + C_j} \quad (B.36)$$

If we combine equations B.20 and B.36 we obtain:

$$\frac{\Delta \varpi}{\Delta P_{1,3}} = \frac{\sqrt{1 - \beta^4 (1 - C_d^2)} - C_d \beta^2}{\sqrt{1 - \beta^4 (1 - C_d^2)} + C_d \beta^2} \quad (B.37a)$$

Two alternative quite similar but simpler forms obtained by the author are:

$$\frac{\Delta \varpi}{\Delta P_{1,3}} = \frac{\left(\sqrt{1 - \beta^4 (1 - C_d^2)} - C_d \beta^2 \right)^2}{1 - \beta^4} \quad (B.37b)$$

$$\frac{\Delta \varpi}{\Delta P_{1,3}} = \frac{(1 - \beta^4)}{\left(\sqrt{1 - \beta^4 (1 - C_d^2)} + C_d \beta^2 \right)^2} \quad (B.37c)$$

Equations **B.37a,b,c** differ from the respective equation that is currently present in ISO-5167 and is extensively used by the industry worldwide. The *permanent pressure loss* is calculated at this moment, according to ISO-5167, from:

$$\Delta \varpi = \frac{\sqrt{1 - \beta^4 - C_d \beta^2}}{\sqrt{1 - \beta^4 + C_d \beta^2}} \Delta P_{1,2,3} \quad (\text{B.38})$$

The author would like to make two comments on equations **B.37a,b,c** and **B.38**. Equation **B.37**, which is currently used by industry, practically underestimates the permanent pressure loss for two apparent reasons:

1. The differential pressure practically is measured between planes 1 and a plane between 2 and 3 using various tappings configurations (flange, corner, D D/2). However the permanent pressure loss, as has been calculated previously and after reminding that represents purely separation losses, depends on the maximum differential pressure, which is $\Delta P_{1,3}$. So, from both equations **B.37** and **B.38** we could calculate only approximately the permanent pressure loss in the sense that has been developed until this moment. In order to calculate it more accurately, we must be able to measure $\Delta P_{1,3}$ which practically means that:

- a) the upstream pressure tapping must not be within the upstream separation flow region caused by the presence itself of the orifice and not too far away upstream from the separation point, in order to avoid taking into account pipe friction losses and
- b) the downstream pressure loss tapping must be just after the reattachment point (approximately after 6D from the upstream face of the orifice plate) for the same reason as above.

The later is mainly important in flow measurement of liquids were the pipe friction losses and consequently the viscous effects are predominant. In gas flows, there could be a higher tolerance regarding the position of the downstream pressure loss tapping and even a reasonably higher distance than 6D could not add any significant uncertainty.

2. The remaining term of equation **B.38** is smaller than the respective term of **B.37**.

So, generally we could say that:

$$\Delta \varpi(\text{from B.37}) > \Delta \varpi(\text{from B.38}).$$

The practical consequences of this discrepancy have to do mainly with the calculation of upstream to downstream corrections of density and mainly in flow metering of

compressible fluids such as in natural gas. From the above equations the pressure loss coefficient could be calculated as well and is:

$$K_{\text{loss}} = \frac{\Delta \varpi}{\frac{\rho u_D^2}{2}} = \left(\frac{1}{C_c \beta^2} - 1 \right)^2 = \left(\frac{\sqrt{1 - \beta^4 (1 - C_d^2)}}{C_d \beta^2} - 1 \right)^2 \quad (\text{B.39})$$

Equations **B.37** and **B.39** are these equations that recently have been adopted in the new ISO/CD 5167-2,1999. It is interesting to mention that actually both equations have been published over 20 years ago! **Benedict R.P, 1977**, obtained an equation for the loss coefficient as a function of contraction coefficient and discharge coefficient which was based on the mean velocity at the orifice throat and has the form:

$$K_{\text{Loss } 2} = \left(\frac{1 - \beta^4}{C_d^2} \right) - 2\beta^2 \left(\frac{1}{C_c} - \beta^4 \right) \quad (\text{B.40})$$

where $K_{\text{Loss}} = \beta^4 K_{\text{Loss } 2}$

He also related the dimensionless meter loss parameter, P_{ASME} , given by:

$$P_{\text{ASME}} = \frac{P_{t1} - P_{t4}}{P_1 - P_3} \quad (\text{B.41})$$

(which expresses the loss in total pressure in terms of the maximum static pressure drop across the fluid meter) with the loss and discharge coefficients.

$$P_{\text{ASME}} = \frac{K_{\text{Loss}} C_d^2}{1 - \beta^4} \quad (\text{B.42})$$

The last three equations are giving the same results and are equivalent with the respective equations that have been developed in this project with the following remark:

$$P_{t1} - P_{t4} = P_1 - P_4 = \Delta \varpi \quad (\text{B.43})$$

The last expression implies that since we have accepted an adiabatic/isenthalpic process between sections 1 and 4, the losses are completely transformed to internal energy.

Alvi, S.H. et.al.,1978, presented an equivalent form of the loss parameter $\frac{\Delta \varpi}{\Delta P}$ as:

$$G = 1 - \frac{2\beta^2 C_d^2}{1 - \beta^4} \left(\frac{1}{C_c} - \beta^2 \right) \quad (\text{B.44})$$

where he has not clearly specified if G is referred to total pressure loss or static pressure loss. However, equation **B.44** is present as well in **Miller, R.W., 1989** and in the latest revision (**1996**) and is equivalent to equation **B.37** which have been adopted only recently by ISO.

Thus, this project supports as well these equations, considering though the concepts that have already been put into the processing of these equations and on the other hand presents all the other completely new forms based on the same concepts.

From equation **B.36** we could obtain:

$$\begin{aligned}\frac{\Delta P_{1,3}}{\Delta \varpi} &= \frac{1 + C_c \beta^2}{1 - C_c \beta^2} \Rightarrow \\ \Delta P_{1,3} + \Delta \varpi &= \frac{1 + C_c \beta^2 + 1 - C_c \beta^2}{1 - C_c \beta^2} \Delta \varpi \Rightarrow \\ \Delta P_{1,3} + \Delta \varpi &= \frac{2}{1 - C_c \beta^2} \Delta \varpi\end{aligned}\tag{B.45}$$

Similarly,

$$\Delta P_{1,3} - \Delta \varpi = \frac{2 C_c \beta^2}{1 - C_c \beta^2} \Delta \varpi\tag{B.46}$$

Dividing the last two equations we obtain a very interesting formula:

$$\frac{\Delta P_{1,3} + \Delta \varpi}{\Delta P_{1,3} - \Delta \varpi} = \frac{1}{C_c \beta^2} = \frac{1}{C_j}\tag{B.47}$$

From **B.47** and **B.39** we can obtain some new very useful formulas for the loss coefficient, K_{loss} :

$$K_{\text{loss}} = \left(\frac{\Delta P_{1,3} + \Delta \varpi}{\Delta P_{1,3} - \Delta \varpi} - 1 \right)^2 = \left(\frac{2}{\frac{\Delta P_{1,3}}{\Delta \varpi} - 1} \right)^2\tag{B.48}$$

or alternatively and using **B.34**

$$K_{\text{loss}} = \left(\frac{u_{vc}}{u_D} - 1 \right)^2\tag{B.49}$$

Finally, and after proper manipulation of the above equations we obtain:

$$\begin{aligned}
 u_D &= (\Delta P_{1,3} - \Delta \varpi) \frac{1}{\sqrt{2 \Delta \varpi \rho}} = \\
 &= \frac{\Delta P_{1,3}}{\sqrt{2 \Delta \varpi \rho}} - \sqrt{\frac{\Delta \varpi}{2 \rho}} = \frac{\Delta P_{4,3}}{\sqrt{2 \Delta \varpi \rho}}
 \end{aligned} \tag{B.50}$$

where $\Delta P_{4,3}$ is the pressure recovery. Subsequently, the volume and mass flow rate can be obtained from:

$$\begin{aligned}
 Q &= (\Delta P_{1,3} - \Delta \varpi) \frac{1}{\sqrt{2 \Delta \varpi \rho}} \frac{\pi D^2}{4} \\
 &= \frac{\Delta P_{4,3}}{\sqrt{2 \Delta \varpi \rho}} \frac{\pi D^2}{4}
 \end{aligned} \tag{B.51}$$

(Volume flow rate)

and

$$\begin{aligned}
 q_m &= (\Delta P_{1,3} - \Delta \varpi) \sqrt{\frac{\rho}{2 \Delta \varpi}} \frac{\pi D^2}{4} \Rightarrow \\
 q_m &= \frac{\Delta P_{4,3} \sqrt{\rho}}{\sqrt{2 \Delta \varpi}} \frac{\pi D^2}{4}
 \end{aligned} \tag{B.52}$$

(Mass flow rate)

Since $\Delta P_{4,3} = \rho u_D u_{vc} - \rho u_D^2$ (Impulse theorem) and using B.50 we have:

$$\begin{aligned}
 u_D &= \frac{\rho u_D u_{vc} - \rho u_D^2}{\sqrt{2 \Delta \varpi \rho}} \Rightarrow \\
 \Delta \varpi &= \frac{\rho}{2} (u_{vc} - u_D)^2
 \end{aligned} \tag{B.53}$$

additionally we can obtain:

$$\Delta P = \frac{\rho}{2} (u_{vc}^2 - u_D^2) \tag{B.54a}$$

and

$$u_{vc} = \frac{(\Delta P + \Delta \varpi)}{\sqrt{2 \Delta \varpi \rho}} \tag{B.54b}$$

B.1.6 Flow rate and discharge coefficient monitoring

The last equations can be very useful for flow measurement monitoring purposes. It is almost clear that they provide us with a lot of information for the insights of the flow through the orifice plate while the methodology could be possibly applied to all the other

constriction devices as well. However, the more important fact is that they can also give us the ability to improve the performance of the orifice meter and subsequently the flow measurement accuracy. By using another differential pressure transmitter to measure the pressure loss $\Delta\varpi$ we could monitor continuously the flow through the orifice or alternatively the discharge coefficient, C_d .

From **B.39** someone can obtain:

$$C_d = \frac{1}{\beta^2} \sqrt{\frac{1 - \beta^4}{K_{\text{loss}} + 2\sqrt{K_{\text{loss}}}}} \Rightarrow \quad (\text{B.55a})$$

$$C_d E = K_{\text{flow}} = \frac{1}{\beta^2 \sqrt{K_{\text{loss}} + 2\sqrt{K_{\text{loss}}}}}$$

and also:

$$C_c = \frac{1}{\beta^2 \sqrt{K_{\text{loss}} + 2K_{\text{loss}} + 1}} \quad (\text{B.55b})$$

and using **B.48** we have:

$$C_d = \frac{\sqrt{1 - \beta^4}}{\beta^2} \frac{1}{\sqrt{\left(\frac{\Delta P_{1,3} + \Delta\varpi}{\Delta P_{1,3} - \Delta\varpi}\right)^2 - 1}} \quad (\text{B.55c})$$

where $\Delta P_{1,3}$ and $\Delta\varpi$ could be continuously measured by two separate differential pressure transmitters. The importance of the meter pressure loss parameter $P_{\text{LP}} = \frac{\Delta\varpi}{\Delta P_{1,3}}$ must be

strongly emphasised since it is the factor that associates the pressure loss coefficient and the discharge coefficient. This can become evident by the presentation of the next series of equations. It can be easily shown

$$\begin{aligned} \left(\frac{\Delta P_{1,3} + \Delta\varpi}{\Delta P_{1,3} - \Delta\varpi}\right)^2 - 1 &= \left(\frac{\Delta P_{1,3} + \Delta\varpi}{\Delta P_{1,3} - \Delta\varpi} - 1\right)^2 \frac{\Delta P_{1,3}}{\Delta\varpi} = \\ &= K_{\text{Loss}} \frac{\Delta P_{1,3}}{\Delta\varpi} \end{aligned} \quad (\text{B.56})$$

So from **B.55c** we can obtain the simpler equation

$$C_d = \frac{\sqrt{1 - \beta^4}}{\beta^2} \frac{1}{\sqrt{K_{\text{Loss}} \frac{\Delta P_{1,3}}{\Delta\varpi}}} \quad (\text{B.57})$$

from which we obtain the next simple but *very important equation*:

$$\frac{\Delta \varpi}{\Delta P_{1,3}} = K_{\text{Loss}} C_d^2 E^2 \beta^4 \quad (\text{B.58})$$

Equation **B.58** is probably one of the most important equations presented in this thesis. It reveals the simplicity of the interdependence of almost all the significant parameters that contribute in the orifice meter performance. Using equation **B.58** it can be revealed as well another series of useful equations and also be explained the results and the appearance itself of one of those formulas in ISO 5167.

In ISO-5167 there is an approximate formula for $\Delta \varpi$ that can be written as:

$$\Delta \varpi = (1 - \beta^{1,9}) \Delta P_{1,3} \quad (\text{B.59})$$

From this formula and from **B.51** and **B.52** we could obtain the next useful approximate equations for the flow rate:

$$Q = \frac{\beta^{1,9}}{\sqrt{1 - \beta^{1,9}}} \sqrt{\frac{\Delta P_{1,3}}{2\rho}} \frac{\pi D^2}{4} \quad (\text{B.60})$$

and

$$q_m = \frac{\beta^{1,9}}{\sqrt{1 - \beta^{1,9}}} \sqrt{\frac{\Delta P_{1,3} \rho}{2}} \frac{\pi D^2}{4} \quad (\text{B.61})$$

The last two equations can give very good results under normal operating conditions and furthermore could be useful when a "raw" or fast estimation of the flow rate is needed, something that is very common in industrial applications. Additionally, someone could take a more exact solution if the constant 1,9 will be replaced by a variable x . Then, we could take:

$$\frac{C_d \beta^2}{\sqrt{1 - \beta^4}} \sqrt{\frac{2 \Delta P_{1,3}}{\rho}} = \frac{\beta^x}{\sqrt{1 - \beta^x}} \sqrt{\frac{\Delta P_{1,3}}{2\rho}} \quad (\text{B.62})$$

where after manipulation of the variables we obtain the next quadratic function of β^x :

$$(1 - \beta^4)(\beta^x)^2 + 4 C_d^2 \beta^4 \beta^x - 4 C_d^2 \beta^4 = 0 \quad (\text{B.63})$$

from which we obtain only one solution for x (the other solution lead to the logarithm of a

negative value and so is not defined) and is: $x = \frac{\log\left(\frac{2}{\sqrt{K_{\text{loss}}} + 2}\right)}{\log(\beta)}$ (B.64)

More importantly it can easily obtained:

$$\beta^x = 1 - \frac{\Delta \varpi}{\Delta P_{1,3}}$$

which is another quite important formula. Thus, it is evident that the dimensionless pressure loss parameter and subsequently the product of the other important dimensionless parameters on the right hand side of equation **B.58**, collapses in a single parameter x , an exponent of beta ratio.

Lastly, after proper manipulation of the above equations we can obtain:

$$\beta^x = 1 - K_{\text{Loss}} C_d^2 E^2 \beta^4 \quad (\text{B.65})$$

from which

$$x = \frac{\log(1 - K_{\text{Loss}} C_d^2 E^2 \beta^4)}{\log(\beta)} \quad (\text{B.66})$$

Furthermore, and more importantly, the pressure loss coefficient can be obtained by

$$K_{\text{Loss}} = 4 \left(\frac{1}{\beta^x} - 1 \right)^2 \quad (\text{B.67})$$

Since in ISO-5167-2 has been adopted equation **B.59** as an approximate formula for the permanent pressure loss, under the same sense, it is proposed the inclusion of the next approximate formula for the pressure loss coefficient which is actually based on the same concepts like formula **B.59**.

$$K_{\text{Loss}} = 4 \left(\frac{1}{\beta^{1.9}} - 1 \right)^2 \quad (\text{B.68})$$

In addition, the next equation for discharge coefficient can be obtained

$$\begin{aligned} C_d &= \frac{1}{2} \frac{\beta^x}{\beta^2} \frac{\sqrt{(1 - \beta^4)}}{\sqrt{(1 - \beta^x)}} = \\ &= 0.5 \beta^{(x-2)} (1 - \beta^4)^{0.5} (1 - \beta^x)^{-0.5} \end{aligned} \quad (\text{B.69})$$

or as function of pressure loss parameter $P_{\text{LP}} = \frac{\Delta \varpi}{\Delta P_{1,3}}$,

$$C_d = \frac{1}{2 \beta^2 E} \left(\sqrt{\frac{1}{P_{\text{LP}}}} - \sqrt{P_{\text{LP}}} \right) \quad (\text{B.70})$$

Other various forms can be also obtained and are summarised in the next table

Summary table of the Old and New equations – Identified Sources

Parameter	Function	Source
1. Discharge Coefficient, (Cd)	a) $C_d = \sqrt{\frac{\rho u_{2,3}^2}{2(P_1 - P_{2,3})} - \frac{\rho u_D^2}{2(P_1 - P_{2,3})}} = 1$	PhD project
	b) $C_d = \sqrt{\frac{1}{C_{Po}} - \frac{1}{C_{PD}}} < 1$	PhD project
	c) $C_d = \frac{d_{vc}^2}{d_o^2} \sqrt{\frac{(D^4 - d_o^4)}{(D^4 - d_{vc}^4)}} = \sqrt{\frac{(u_o^2 - u_D^2)}{(u_{vc}^2 - u_D^2)}}$	PhD project
	d) $C_d = C_c C_v = C_c \sqrt{\frac{1 - \beta^4}{1 - a^4}}$	Zanker, 1961
	e) $C_d = \frac{1}{\beta^2} \sqrt{\frac{1 - \beta^4}{K_{Loss} + 2\sqrt{K_{Loss}}}}$	PhD project
	f) $C_d = \frac{\sqrt{1 - \beta^4}}{\beta^2} \frac{1}{\sqrt{\left(\frac{\Delta P_{l,3} + \Delta \varpi}{\Delta P_{l,3} - \Delta \varpi}\right)^2 - 1}}$	PhD project
	g) $C_d = \frac{\sqrt{1 - \beta^4}}{\beta^2} \frac{1}{\sqrt{K_{Loss} \frac{\Delta P_{l,3}}{\Delta \varpi}}}$	PhD project
	h) $C_d = \frac{1}{2} \frac{\beta^x}{\beta^2} \frac{\sqrt{(1 - \beta^4)}}{\sqrt{(1 - \beta^x)}} =$ $= 0.5 \beta^{(x-2)} (1 - \beta^4)^{0.5} (1 - \beta^x)^{-0.5}$	PhD project
	i) $C_d = \frac{1}{2\beta^2 E} \left(\frac{1}{\sqrt{P_{LP}}} - \sqrt{P_{LP}} \right)$	PhD project
	j) $C_d = \frac{1}{2} \frac{\Delta P_{l,3}}{\sqrt{\Delta P_{l,3} \Delta \varpi}} \sqrt{\frac{1}{\beta^4} - 1}$	PhD project
	k) $C_d = \frac{1}{2} \sqrt{\frac{1 - \beta^4}{\beta^{4-x} (\beta^{-x} - 1)}}$	PhD project
	l) $C_d = \sinh \ln \left(\frac{1}{\sqrt{P_{LP}}} \right) \sqrt{\frac{1}{\beta^4} - 1}$	PhD project
	m) $C_d = \sqrt{\frac{1}{\beta^4} - 1} \left(\frac{\sinh \ln(\sqrt{\Delta \varpi}) \cosh \ln(\sqrt{\Delta P_{l,3}}) -}{\cosh \ln(\sqrt{\Delta \varpi}) \sinh \ln(\sqrt{\Delta P_{l,3}})} \right)$	PhD project
2. Loss Coefficient, (K _{Loss})	a) $K_{Loss} = \frac{\Delta \varpi}{\frac{\rho u_D^2}{2}} = \left(\frac{1}{C_c \beta^2} - 1 \right)^2$	Zanker, 1961
	b) $K_{Loss} = \left(\frac{\sqrt{1 - \beta^4 (1 - C_d^2)}}{C_d \beta^2} - 1 \right)^2$	a) ISO/CD-5167-1999, New model adopted

Summary table of the Old and New equations – Identified Sources

2. Loss Coefficient (K_{Loss})	c) $K_{Loss2} = \left(\frac{1 - \beta^4}{C_d^2} \right) - 2\beta^2 \left(\frac{1}{C_c} - \beta^2 \right)$ where $K_{Loss} = \beta^4 K_{Loss2}$	a) Benedict R.P., 1977, (Based on orifice throat mean velocity)
	d) $K_{Loss} = \left(\frac{\Delta P_{1,3} + \Delta \varpi}{\Delta P_{1,3} - \Delta \varpi} - 1 \right)^2$	PhD project
	e) $K_{Loss} = 4 \left(\frac{1}{\frac{\Delta P_{1,3}}{\Delta \varpi} - 1} \right)^2 = 4 \left(\frac{\Delta \varpi}{\Delta P_{4,3}} \right)^2$	PhD project
	f) $K_{Loss} = \left(\frac{u_{vc}}{u_D} - 1 \right)^2 = \left(\frac{D^2}{d_{vc}^2} - 1 \right)^2 = \left(\frac{1}{\alpha^2} - 1 \right)^2$	PhD project
	g) $K_{Loss} = \frac{P_{LP}}{C_d^2 E^2 \beta^4}$	PhD project
	h) $K_{Loss} = 4 \left(\frac{1}{\beta^x} - 1 \right)^2$	PhD project
	i) $K_{Loss} = 4 \left(\frac{1}{\beta^{1,9}} - 1 \right)^2$ (approximately)	PhD project
	j) $K_{Loss} = 8\beta^{-x} (\cosh \ln \beta^x - 1)$	PhD project
	k) $K_{Loss} = \frac{8[\cosh \ln(1 - P_{LP}) - 1]}{1 - P_{LP}}$	PhD project
	l) $K_{Loss} = \frac{P_{LP}}{\sinh^2 \ln(\sqrt{P_{LP}})}$	PhD project
3. Flow Coefficient (K)	a) $K = C_d E = \frac{C_c}{\sqrt{1 - C_c^2 \beta^4}} = \frac{C_c}{\sqrt{1 - \alpha^4}}$	PhD project
	b) $K = \frac{1}{2\beta^2} \frac{(\Delta P_{4,3})}{\sqrt{\Delta P \Delta \varpi}} = \frac{1}{2\beta^2} \left(\frac{1}{\sqrt{P_{LP}}} - \sqrt{P_{LP}} \right)$	PhD project
	$K = \frac{l}{\beta^2} \sqrt{\frac{\Delta \varpi}{K_{Loss} \Delta P}} = \frac{l}{\beta^2 \sqrt{K_{Loss} + 2\sqrt{K_{Loss}}}} =$	PhD project
	c) $\frac{\sinh \ln \left(\frac{l}{\sqrt{P_{LP}}} \right)}{\beta^2}$	PhD project
	d) $K = \frac{1}{\beta^2} \sqrt{\frac{1 - \beta^x}{K_{Loss}}}$	PhD project
4. Vena contract diameter, (d_{vc})	a) $d_{vc} = \frac{d_o \sqrt{C_d}}{\sqrt[4]{1 - \beta^4 (1 - C_d^2)}}$	PhD project

Summary table of the Old and New equations – Identified Sources

4. Vena contract diameter, (d_{vc})	b) $d_{vc} = \sqrt[4]{\frac{1}{\frac{1}{D^4} + \frac{1}{C_d^2} \left(\frac{1}{d_o^4} - \frac{1}{D^4} \right)}}$	PhD project
	c) $d_{vc} = D \sqrt{\frac{\Delta P_{4,3}}{\Delta P_{1,3} + \Delta \varpi}} = D \left(\tanh \ln \left(\frac{1}{\sqrt{P_{LP}}} \right) \right)^{\frac{1}{2}}$	PhD project
5. Alfa ratio, (a) $a = \frac{d_{vc}}{D} = \sqrt{C_j}$	a) $a = \frac{1}{\sqrt[4]{\frac{\Delta P_{1,3}}{\rho u_D^2} + 1}} = \frac{1}{\sqrt[4]{C_{plD} + 1}}$ $= \sqrt{\frac{u_D}{u_{vc}}}$	PhD project
	b) $a = \frac{1}{\sqrt[4]{\frac{\pi^2 \Delta P_{1,3} D^4}{8 \rho Q^2} + 1}}$	PhD project
	c) $a = \sqrt{Q \sqrt{\frac{8 \rho}{\pi^2 \Delta P_{1,3} D^4 + 8 \rho Q^2}}}$	PhD project
	d) $a = \sqrt{q_m \sqrt{\frac{8}{\rho \pi^2 \Delta P_{1,3} D^4 + 8 q_m^2}}}$	PhD project
	e) $a = \frac{1}{\sqrt[4]{K_{Loss} \frac{\Delta P}{\Delta \varpi} + 1}} = \sqrt{\frac{1 - P_{LP}}{1 + P_{LP}}} = \sqrt{\frac{\Delta P_{4,3}}{\Delta P_{1,3} + \Delta \varpi}}$	PhD project
	f) $a = \frac{1}{\sqrt[4]{\frac{4(1 - \beta^x)}{\beta^{2x}} + 1}} = \sqrt{\frac{\beta^x}{2 - \beta^x}} =$ $= \frac{1}{\sqrt[4]{\frac{1}{\sinh^2 \ln(\sqrt{P_{LP}})} + 1}}$	PhD project
6. Contraction Coefficient, (C_c) $C_c = \frac{d_{vc}^2}{d_o^2} = \frac{a^2}{\beta^2}$ (Can be obtained similarly from all the previous equations for (a) ratio.)	a) $C_c = \frac{C_d}{\sqrt{1 - \beta^4 (1 - C_d^2)}}$	a) Alvi S.H., et.al., 1978, b) Miller R.W., 1989
	b) $C_c = \frac{1}{\beta^2 \sqrt{\frac{K_{Loss}}{P_{LP}} + 1}}$	PhD project
	c) $C_c = \frac{1}{\beta^2 \sqrt{K_{Loss} + 2 \sqrt{K_{Loss}} + 1}} = \frac{1}{\beta^2 (\sqrt{K_{Loss}} + 1)}$	The right part - Zanker, 1961
	d) $C_c = \frac{\Delta P_{4,3}}{\beta^2 (\Delta P_{1,3} + \Delta \varpi)} = \frac{\Delta P_{4,3}}{\beta^2 (2 \Delta \varpi + \Delta P_{4,3})}$	PhD project

Summary table of the Old and New equations – Identified Sources

7. Other relationships (C_d , C_c , C_j)	a) $\frac{(C_j^2 - 1)(C_d^2 - 1)}{1 - C_c^2} = 1$	PhD project
	b) $\frac{C_d^2 - C_c^2}{C_c^2(C_d^2 - 1)} = \beta^4$	PhD project
8. Convective acceleration factor K_2	a) $K_2 = \frac{8(1 - \beta^4 C_c^2)}{\pi^2 \beta^4 C_c^2 D^4}$	a) Mottram, P, Gajan, P., et.al., 1992
	b) $K_2 = \frac{8(1 - C_j^2)}{\pi^2 d_{vc}^4}$	PhD project
	c) $K_2 = \frac{8(1 - \beta^4)}{\pi^2 C_d^2 d_o^4}$	PhD project
	d) $K_2 = \frac{8}{\pi^2} \left(\frac{1}{d_{vc}^4} - \frac{1}{D^4} \right)$	PhD project
	e) $K_2 = \frac{16\beta^{-x} \sqrt{K_{LOSS}}}{\pi^2 D^4} = \frac{32\beta^{-x} (\beta^{-x} - 1)}{\pi^2 D^4}$ $= \frac{8(K_{LOSS} + 2\sqrt{K_{LOSS}})}{\pi^2 D^4}$	PhD project
	f) $K_2 = \frac{32\Delta P_{1,3} \Delta \varpi}{\pi^2 D^4 \Delta P_{4,3}^2}$	PhD project
	g) $K_2 = \frac{8}{\pi^2 D^4 \sinh^2 \ln(\sqrt{P_{LP}})} = \frac{8 \coth^2 \ln(\sqrt{P_{LP}}) - 1}{\pi D^4}$	PhD project
9. Pressure Loss Parameter, $P_{LP} = \frac{\Delta \varpi}{\Delta P_{1,3}}$	a) $\frac{\Delta \varpi}{\Delta P_{1,3}} = \frac{1 - C_c \beta^2}{1 + C_c \beta^2} = \frac{1 - C_j}{1 + C_j} =$ $= \frac{1 - a^2}{1 + a^2} = \frac{u_{vc} - u_D}{u_{vc} + u_D} = \frac{C_v - C_d \beta^2}{C_v + C_d \beta^2}$	PhD project
	b) $\frac{\Delta \varpi}{\Delta P_{1,3}} = \frac{\sqrt{1 - \beta^4(1 - C_d^2)} - C_d \beta^2}{\sqrt{1 - \beta^4(1 - C_d^2)} + C_d \beta^2}$	a) ISO/CD 5167- 2,1999 b) Umer, G., 1997
	c) $\frac{\Delta \varpi}{\Delta P_{1,3}} = \frac{(\sqrt{1 - \beta^4(1 - C_d^2)} - C_d \beta^2)^2}{1 - \beta^4}$	PhD project
	d) $\frac{\Delta \varpi}{\Delta P_{1,3}} = \frac{1 - \beta^4}{(\sqrt{1 - \beta^4(1 - C_d^2)} + C_d \beta^2)^2}$	PhD project
	d) $\frac{\Delta \varpi}{\Delta P_{1,3}} = \frac{\sqrt{1 - \beta^4} - C_d \beta^2}{\sqrt{1 - \beta^4} + C_d \beta^2}$	a) ISO 5167,1991 Old equation
	e) $\frac{\Delta \varpi}{\Delta P_{1,3}} = 1 - \frac{2\beta^2 C_d^2}{1 - \beta^4} \left(\frac{1}{C_c} - \beta^2 \right)$	a) Alvi, S.H., et.al.,1978 b) Miller, R. W.,19 89
	f) $\frac{\Delta \varpi}{\Delta P_{1,3}} = K_{LOSS} C_d^2 E^2 \beta^4$	PhD project

Summary table of the Old and New equations – Identified Sources

9. Pressure Loss Parameter, $P_{LP} = \frac{\Delta \varpi}{\Delta P}$	g) $\frac{\Delta \varpi}{\Delta P_{1,3}} = 1 - \beta^x = 1 - \frac{\alpha}{\cosh \ln(\alpha)}$	PhD project
	h) $\frac{\Delta \varpi}{\Delta P_{1,3}} = 1 - \beta^{1.9}$ (approximately)	a)ISO-5167,1991
10. Mass flowrate, (q_m)	$q_m = \frac{1}{\sqrt{1 - \beta^4}} \frac{\pi d_o^2}{4} \sqrt{2\Delta P_{1,2} \rho} =$ a) $= \frac{1}{\sqrt{1 - \frac{d_o^4}{D^4}}} \frac{\pi d_o^2}{4} \sqrt{2\Delta P_{1,2} \rho}$	From Bernoulli and mass continuity equations
	b) $q_m = C d_{1,2,3} \frac{1}{\sqrt{1 - \frac{d_o^4}{D^4}}} \frac{\pi d_o^2}{4} \sqrt{2\Delta P_{1,2,3} \rho}$	a)ISO-5167
	$q_m = \frac{1}{\sqrt{\frac{D^4}{d_{vc}^4} - 1}} \frac{\pi D^2}{4} \sqrt{2\Delta P_{1,3} \rho} =$ c) $= \frac{a^2}{\sqrt{1 - a^4}} \frac{\pi D^2}{4} \sqrt{2\Delta P_{1,3} \rho}$	PhD project from Bernoulli and mass continuity
	d) $q_m = \pi \frac{1}{\sqrt{\frac{1}{d_{vc}^4} - \frac{1}{D^4}}} \sqrt{\frac{\Delta P_{1,3} \rho}{8}}$	PhD project
	$q_m = (\Delta P_{1,3} - \Delta \varpi) \sqrt{\frac{\rho}{2\Delta \varpi}} \frac{\pi D^2}{4} \Rightarrow$ e) $q_m = \frac{\Delta P_{1,3} \sqrt{\rho}}{\beta^2 \sqrt{2\Delta \varpi}} \frac{\pi d_o^2}{4}$	PhD project
	f) $q_m = \frac{1 - P_{LP}}{\beta^2 \sqrt{P_{LP}}} \sqrt{\frac{\Delta P_{1,3} \rho}{2}} \frac{\pi d_o^2}{4}$	PhD project
	g) $q_m = \frac{\beta^{x-2}}{\sqrt{1 - \beta^x}} \sqrt{\frac{\Delta P_{1,3} \rho}{2}} \frac{\pi d_o^2}{4}$	PhD project
	h) $q_m = \sqrt{\frac{2\Delta P \rho}{\beta^4 (K_{LOSS} + 2\sqrt{K_{LOSS}})}} \frac{\pi d_o^2}{4}$	PhD project
	i) $q_m = \sqrt{\frac{\Delta P_{1,3} \rho}{2\beta^{-x}(\beta^{-x} - 1)}} \frac{\pi D^2}{4}$	PhD project
	j) $q_m = \sinh \ln \left(\frac{1}{\sqrt{P_{LP}}} \right) \sqrt{\frac{2\Delta P \rho}{\beta^4}} \frac{\pi d_o^2}{4}$	PhD project
11. Velocity Coefficient, (C_v)	a) $C_v = \sqrt{\frac{1 - \beta^4}{1 - \alpha^4}}$	PhD project
	b) $C_v = \sqrt{1 - \beta^4 (1 - C_d^2)}$	PhD project

Summary table of the Old and New equations – Identified Sources

12. Beta parameter, β^x or Pressure Recover. parameter, P_{PR}	a) $\beta^x = P_{RP} = \frac{\Delta P_{4,3}}{\Delta P_{1,3}} = 1 - \frac{\Delta \varpi}{\Delta P_{1,3}}$	PhD project
	b) $\beta^x = 1 - K_{LOSS} C_d^2 E^2 \beta^4$	PhD project
	c) $\beta^x = \frac{2}{\sqrt{K_{LOSS}} + 2}$	PhD project
	d) $\beta^x = \frac{2}{1 + \frac{1}{\alpha^2}} = \frac{2\alpha^2}{\alpha^2 + 1} = \frac{\alpha}{\cosh \ln(\alpha)}$	PhD project
13. Loss Recovery Parameter (P_{LR})	$P_{LR} = \frac{\Delta P_{4,3}}{\Delta \varpi}$	PhD project
14. Pressure Recover. Coefficient (K_R)	$K_R = \frac{\Delta P_{4,3}}{\frac{\rho u_D^2}{2}}$	
15. Pressure Coefficient (K_p)	$K_p = \frac{\Delta P_{1,3}}{\frac{\rho u_D^2}{2}}$	
16. Other supportive equations	a) $x = \frac{\ln\left(\frac{2}{\sqrt{K_{LOSS}} + 2}\right)}{\ln(\beta)}$	PhD project
	b) $x = \frac{\ln(1 - K_{LOSS} C_d^2 E^2 \beta^4)}{\ln(\beta)}$	PhD project
	c) $x = \frac{\ln(\Delta P_{4,3}) - \ln(\Delta P_{1,3})}{\ln(\beta)}$	PhD project
	c) $\Delta \varpi = \frac{\rho}{2} (u_{vc} - u_D)^2$	PhD project
	d) $\Delta P = \frac{\rho}{2} (u_{vc}^2 - u_D^2)$	
	e) $u_{vc} = \frac{(\Delta P_{1,3} + \Delta \varpi)}{\sqrt{2\Delta \varpi \rho}}$	PhD project
	f) $u_D = \frac{(\Delta P_{4,3})}{\sqrt{2\Delta \varpi \rho}}$	PhD project
	g) $\frac{\Delta P_{1,3} + \Delta \varpi}{\Delta P_{1,3} - \Delta \varpi} = \frac{1}{C_c \beta^2} = \frac{1}{C_j}$	PhD project
	h) $C_v \alpha^2 = C_d \beta^2$	PhD project
	i) $\left(\frac{\Delta P_{1,3} + \Delta \varpi}{\Delta P_{1,3} - \Delta \varpi}\right)^2 - 1 = \left(\frac{\Delta P_{1,3} + \Delta \varpi}{\Delta P_{1,3} - \Delta \varpi} - 1\right)^2 \frac{\Delta P_{1,3}}{\Delta \varpi} =$ $= K_{LOSS} \frac{\Delta P_{1,3}}{\Delta \varpi}$	PhD project

Summary table of the Old and New equations – Identified Sources

17. Other supportive equations	j) $\frac{1}{\sqrt{P_{LP}}} - \sqrt{P_{LP}} = \frac{\Delta P_{4,3}}{\sqrt{\Delta P_{1,3} \Delta \varpi}} = 2\beta^2 C_d E$	PhD project
18. Special Formulas (Constraints)	a) $P_{dyn} = \frac{1}{2} \rho u_D^2 = \Delta P_{1,3} \sinh^2 \ln(\sqrt{P_{LP}}) = \frac{\Delta P_{4,3}^2}{4\Delta \varpi}$	PhD project
	b) $K_P = K_{Loss} + K_R$	PhD project
	c) $\frac{K_R^2}{K_{Loss}} = 4$	PhD project
	d) $P_{LP} + P_{RP} = 1$	PhD project
	e) $P_{LR} K_R = 4$	PhD project
	f) $K_2 = \frac{8K_P}{\pi^2 D^4}$	PhD project

Summary table of the Old and New equations – Identified Sources

Notation

D	Pipe diameter
d_o	Orifice diameter
d_{vc}	Vena Contracta diameter
C_d	Discharge coefficient
C_c	Contraction Coefficient
C_j	Jet Necking Coefficient
C_v	Velocity Coefficient
K_{Po}	Pressure coefficient (based on orifice throat mean velocity)
K_{PD}	Pressure coefficient (based on pipe mean velocity)
α	Alfa ratio (ratio of vena contracta diameter to pipe diameter)
β	Beta ratio (ratio of orifice diameter to pipe diameter)
K_{Loss}	Pressure Loss Coefficient
K_R	Pressure Recovery Coefficient
K	Flow Coefficient
K_2	Convective acceleration factor
$\Delta P_{1,3}$	Differential Pressure (Upstream and vena contracta)
$\Delta P_{4,3}$	Pressure recovery
$\Delta \varpi$	Permanent Pressure Loss
P_{RP}	Pressure recovery parameter
P_{LP}	Pressure loss parameter
P_{LR}	Pressure loss recovery parameter
u_D	Pipe mean velocity
u_{vc}	Vena contracta mean velocity
E	Velocity of approach factor
β^x	Ratio of pressure recovery to differential pressure
q_m	Mass flowrate
Q	Volume flowrate

TABLES

Characteristics	W. Edwards Deming	Joseph B. Croby	Armand Feigenbaum	Genichi Taguchi	Kaoru Ishikawa
A. General 1. Definition of quality 2. Main emphasis 3. Dominant factor 4. Scope of application 5. Applicability—Sector	A. General 1. Customer-led 2. Process 3. Control of variation 4. Holistic 5. Manufacturing emphasis	A. General 1. Customer-led 2. Conformance to requirements/performance 3. Zero defects 4. Implicitly functional 5. Manufacturing emphasis	A. General 1. Customer-led 2. Process 3. Principles of total quality control—methodology 4. Holistic and integrative 5. Manufacturing emphasis, but also applies to services	A. General 1. Supply-led—value to society 2. Reducing variation of product process around target values 3. Control quality to minimize total cost to society 4. Off-line in design and on-line in production 5. Manufacturing emphasis	A. General 1. Value-led 2. Individual quality 3. Perceived consumer value 4. Holistic 5. Manufacturing emphasis, but also applies to services
B. External 1. Customer focus 2. Market focus (environment) 3. Vendor/buyer relationship	B. External 1. Surprise, delight customers; anticipate customer needs 2. Quality needed to stay in business 3. Considered integral; quality requirement of vendors	B. External 1. Implicit in customer requirements 2. Not addressed 3. Not specified	B. External 1. Customer ultimately defines quality 2. Quality as competitive advantage 3. Close relationship for quality assurance	B. External 1. Implicit in customer requirements and society as a whole 2. Reducing society's loss 3. [no information]	B. External 1. Integral to quality definition and management 2. Quality—an important source of competitive advantage 3. [no information]
C. Organizational Context 1. Primary change agent 2. Top management role 3. Management style 4. Organizational culture	C. Organizational Context 1. Top management 2. To promote quality: to work on the system 3. Participative 4. Quality as an organizational culture, integrated—need to break down barriers	C. Organizational Context 1. Top management 2. To understand costs of quality, to promote zero defects 3. Instructional, promotional 4. Quality as an organizational culture	C. Organizational Context 1. Top management 2. To promote total quality control; as designer of Q program 3. Participative 4. Spirit of 'quality mindedness'	C. Organizational Context 1. Management 2. Creating a continuous improvement ethics 3. Participative 4. Continuous improvement	C. Organizational Context 1. Top management 2. To promote quality on a company-wide basis and across all functions 3. Participative/open communications 4. Continuous improvement—learning organization
D. Organizational Requirements 1. Top management commitment 2. Employee involvement/participation 3. Education and training 4. Investment in non-human resources	D. Organizational Requirements 1. Essential starting point 2. Essential 3. Essential—a vigorous program 4. Essential—to reduce total costs	D. Organizational Requirements 1. Essential 2. Essential 3. Essential 4. As part of improvement programs	D. Organizational Requirements 1. Essential 2. Involvement and support essential/integral 3. Essential—to promote company-wide 'quality mindedness' 4. An integral component of total quality control	D. Organizational Requirements 1. Essential 2. Essential 3. Essential for all employees 4. As a part of overall quality drive	D. Organizational Requirements 1. Essential 2. Essential 3. Essential for all employees 4. As a part of overall quality drive
E. Means/Techniques 1. Costs of quality (appreciated) 2. Training programs 3. Participation/feedback (e.g. communication, quality circles) 4. Statistical methods 5. Recognition/rewards systems 6. Inspection procedures	E. Means/Techniques 1. Reduce total costs 2. Essential—especially training on the job for all to retrain 3. Achieve in both positive way (e.g. teamwork) and negative way (e.g. drive out fear, eliminate exhortations) 4. Statistical process control 5. Increased job satisfaction through quality and pride in workmanship. Performance appraisal irrelevant 6. End mass inspection, which accepts errors/defects. Obtain statistical evidence of built-in quality	E. Means/Techniques 1. Important, to motivate management 2. Training and motivational program, including 'zero-defects' days 3. Teamwork on zero defects; consultation on problems 4. Not specific, but measuring results is stressed 5. Recognize and reward those who achieve quality (zero-defects) goals 6. Not stressed	E. Means/Techniques 1. Integral to business strategy, planning, investment, programming 2. Customized to fit circumstances, quality problem centered 3. Essential, e.g. via quality circles and quality of working life programs 4. As appropriate, useful, but only part of TOC pattern 5. Recognition via participation of employees for commitment to quality 6. Improved by total quality programs	E. Means/Techniques 1. Important to define and minimize—explicitly identified 2. Enhance knowledge and understanding of employees. Train in statistical methods 3. Quality circles, teamwork 4. Taguchi methods of process control 5. The need for recognition recognized 6. Continuous measurement—operatives responsibility	E. Means/Techniques 1. Improvement in quality reduces other costs 2. Essential. All employees should be competent in statistical process control and problem-solving 3. Advocate of quality circles as a means for participation and problem-solving 4. Statistical process control 5. Quality behavior and performance to be rewarded 6. Inspection part of operatives role, essential in manufacturing
F. Change 1. Pace of change 2. Nature of change	F. Change 1. Implicitly gradual—in implementing the 14 points 2. Continuous improvement philosophy	F. Change 1. Gradual 2. Sequential (though ordering of steps may vary)	F. Change 1. Gradual 2. Step by step, starting with selected areas	F. Change 1. Gradual 2. Continuous improvement in acceptable levels of variation	F. Change 1. Gradual 2. Continuous improvement

Table 3. The different views of Quality Gurus.

Pipe diameter, D		300	mm	Gas Composition - Mole%						U _{C1}		0.1%	U _D	0.4%
Orifice diameter, d _o		160	mm	C1	98.7	iC5	0.057	MEOH	0.0017	U _{C2,...nC6}	U _{do}	0.02%	U _{do}	0.07%
Beta ratio, b		0.53		C2	0.383	nC5	0.0048			U _{GCV}		0.6%		
Upstream Pres. P1		49.5	Bara	C3	0.206	nC6	0.003	MW	16.295	U _{line dens}	reading	0.2%	reading	
Upstream Tem. T1		17.7	°C	iC4	0.02	N2	0.64	Sound sp.	446.6 m/s	U _{ref. dens}	reading	0.1%	reading	
GCV		37.1	Mj/Rm ³	nC4	0.027	C02	0.015			U _{Cd}	reading	0.6%	reading	
Range		Est. Uncertainties		Conventional method				Monte Carlo Simulation				% Difference in estimated Uncertainty (MCS v Conv.)		
DP mbars	U _{DP} %	U _{Ec} %	UQ _m %	UQ _v %	Uengy %	UQ _m %	UQ _v %	Uengy %	%Diff. UQ _m	%Diff. UQ _v	%Diff. Uengy			
600	0.1676	0.0485	0.6441	0.6518	0.8859	0.6548	0.6614	0.8922	1.6	1.4	0.7			
560	0.1733	0.0452	0.6443	0.6520	0.8860	0.6554	0.6622	0.8927	1.7	1.5	0.7			
520	0.1799	0.0420	0.6445	0.6522	0.8862	0.6562	0.6627	0.8933	1.8	1.6	0.8			
480	0.1877	0.0388	0.6448	0.6525	0.8865	0.6568	0.6633	0.8938	1.8	1.6	0.8			
440	0.1970	0.0355	0.6453	0.6530	0.8868	0.6575	0.6641	0.8944	1.8	1.7	0.8			
400	0.2083	0.0323	0.6461	0.6538	0.8874	0.6584	0.6649	0.8951	1.9	1.7	0.9			
360	0.2222	0.0291	0.6471	0.6548	0.8881	0.6598	0.6662	0.8962	1.9	1.7	0.9			
320	0.2399	0.0259	0.6485	0.6562	0.8891	0.6616	0.6680	0.8976	2.0	1.8	0.9			
280	0.2629	0.0226	0.6506	0.6583	0.8907	0.6642	0.6705	0.8996	2.0	1.8	1.0			
240	0.2940	0.0194	0.6538	0.6614	0.8930	0.6680	0.6743	0.9025	2.1	1.9	1.1			
200	0.3380	0.0162	0.6590	0.6666	0.8968	0.6741	0.6802	0.9071	2.2	2.0	1.1			
160	0.4047	0.0129	0.6683	0.6757	0.9037	0.6846	0.6905	0.9150	2.4	2.1	1.2			
120	0.5169	0.0097	0.6873	0.6945	0.9178	0.7062	0.7118	0.9316	2.7	2.4	1.5			
80	0.7432	0.0065	0.7373	0.7441	0.9558	0.7585	0.7634	0.9721	2.8	2.5	1.7			

Table 4. Results of Comparison between conventional (RSS) method with Monte Carlo simulation method for energy flow rate uncertainty estimations

	Tappings	D, D/2 ($\equiv$ Vena Contracta)									
Pipe diameter, D Orifice diameter, d Diameter ratio, B Vel of approach, E Pipe area, A Orifice area, a Density, rho Kinematic visc,nu	1	2	3	4	5	6	7	8	9	10	11
Reference Turbine Frequency	Reference Volume Flowrate	Reference Mass Flowrate	Reference Reynolds Number	Reference Pipe Velocity	Calculated Discharge Coefficient	Calculated Differential Pressure	Calculated Pressure Loss (Equation 9d)	Calculated Pressure Loss (Equation 9b)	Calculated Pressure Loss (Equation 9c)	Calculated Pressure Recovery	Calculated Convective acc. Term (Equation 8)
Symbol	f Hz	q _v m³/sec	q _m Kg/sec	Re	u _b m/sec	C _d	ΔP _{1,3} Pa	Δσ Pa	Δσ Pa	ΔP _{4,3} Pa	K ₂
	370	0.01979	19.75	2.17E+05	2.441	0.6126	31575	16745	17249	14326	80755
	400	0.02140	21.36	2.35E+05	2.639	0.6123	36932	19591	20181	16751	80819
	430	0.02300	22.96	2.53E+05	2.837	0.6121	42710	22661	23343	19367	80876
	450	0.02407	24.02	2.64E+05	2.969	0.6120	46796	24833	25579	21217	80912
	480	0.02568	25.63	2.82E+05	3.167	0.6118	53275	28277	29126	24149	80960
	500	0.02675	26.69	2.94E+05	3.299	0.6117	57829	30698	31619	26210	80991
	530	0.02835	28.30	3.11E+05	3.497	0.6115	65010	34516	35551	29459	81033
	550	0.02942	29.36	3.23E+05	3.629	0.6114	70032	37187	38301	31731	81059
	580	0.03103	30.97	3.41E+05	3.827	0.6113	77916	41379	42618	35298	81096
	600	0.03210	32.03	3.53E+05	3.959	0.6112	83406	44299	45625	37781	81119
	630	0.03370	33.63	3.70E+05	4.157	0.6111	91992	48866	50328	41665	81152
	650	0.03477	34.70	3.82E+05	4.289	0.6110	97951	52036	53591	44359	81173
	660	0.03531	35.24	3.88E+05	4.355	0.6110	101000	53658	55262	45738	81183
	680	0.03638	36.30	4.00E+05	4.487	0.6109	107240	56977	58680	48560	81202
	700	0.03745	37.37	4.11E+05	4.619	0.6108	113667	60397	62201	51466	81221
	720	0.03852	38.44	4.23E+05	4.751	0.6107	120281	63916	65825	54457	81239
	730	0.03905	38.97	4.29E+05	4.817	0.6107	123659	65713	67675	55984	81247
	740	0.03959	39.51	4.35E+05	4.883	0.6107	127083	67535	69551	57532	81256
	760	0.04066	40.58	4.47E+05	5.015	0.6106	134073	71255	73381	60692	81272

Table 6.1a - Orifice meter theoretical calculations based on the reference flow rates

ORIFICE PLATE THEORETICAL CALCULATIONS FOR THE REFERENCE FLOWRATES

Pipe diameter, D		Tappings											D, D/2 ($\equiv$ Vena Contracta)										
Orifice diameter, d																							
Diameter ratio, B																							
Vel of approach, E																							
Pipe area, A _p																							
Orifice area, A _o																							
Density																							
Kinematic visc, ν																							
12	13	14	15	16	17	18	19	20	21	22	23												
Calculated Flow Coefficient	Calculated Contraction Coefficient	Calculated Velocity Coefficient	Calculated V.Contracta diameter	Calculated Alfa ratio	Calculated Beta or P _{RP} Parameter	Calculated Exponent	Calculated Pressure Loss Parameter	Calculated Loss Recovery Parameter	Calculated Pressure Loss Coefficient	Calculated Pressure Recovery Coefficient	Calculated Pressure Coefficient												
K	C _c	C _v	d _c	a	b ^x or P _{RP} Parameter	x	P _{LP}	P _{LR}	K _{Loss}	K _R	K _P												
0.6852	0.6550	0.9352	0.055034	0.5417	0.4537	1.9682	0.5463	0.8305	5.799	4.816	10.616												
0.6849	0.6548	0.9352	0.055024	0.5416	0.4536	1.9689	0.5464	0.8301	5.805	4.819	10.624												
0.6846	0.6546	0.9352	0.055015	0.5415	0.4535	1.9696	0.5465	0.8297	5.811	4.821	10.632												
0.6845	0.6544	0.9351	0.055010	0.5414	0.4534	1.9699	0.5466	0.8295	5.814	4.822	10.636												
0.6843	0.6542	0.9351	0.055002	0.5414	0.4533	1.9705	0.5467	0.8291	5.819	4.824	10.643												
0.6842	0.6541	0.9351	0.054997	0.5413	0.4532	1.9708	0.5468	0.8289	5.821	4.825	10.647												
0.6840	0.6540	0.9351	0.054991	0.5412	0.4531	1.9713	0.5469	0.8287	5.825	4.827	10.652												
0.6839	0.6539	0.9351	0.054987	0.5412	0.4531	1.9715	0.5469	0.8285	5.828	4.828	10.656												
0.6837	0.6537	0.9351	0.054981	0.5412	0.4530	1.9720	0.5470	0.8282	5.831	4.830	10.661												
0.6836	0.6537	0.9350	0.054977	0.5411	0.4530	1.9722	0.5470	0.8281	5.833	4.830	10.664												
0.6835	0.6535	0.9350	0.054972	0.5411	0.4529	1.9726	0.5471	0.8279	5.836	4.832	10.668												
0.6834	0.6535	0.9350	0.054969	0.5410	0.4529	1.9728	0.5471	0.8277	5.838	4.832	10.671												
0.6834	0.6534	0.9350	0.054968	0.5410	0.4529	1.9729	0.5471	0.8277	5.839	4.833	10.672												
0.6833	0.6534	0.9350	0.054965	0.5410	0.4528	1.9731	0.5472	0.8275	5.841	4.834	10.675												
0.6832	0.6533	0.9350	0.054962	0.5410	0.4528	1.9733	0.5472	0.8274	5.843	4.834	10.677												
0.6831	0.6532	0.9350	0.054959	0.5409	0.4527	1.9735	0.5473	0.8273	5.844	4.835	10.679												
0.6831	0.6532	0.9350	0.054958	0.5409	0.4527	1.9736	0.5473	0.8272	5.845	4.835	10.681												
0.6830	0.6532	0.9350	0.054956	0.5409	0.4527	1.9737	0.5473	0.8272	5.846	4.836	10.682												

Table 6.1b - Orifice meter theoretical calculations based on the reference flow rates

Tappings											D, D/2 (≡Vena Contracta)										
Pipe diameter, D											0.1016 m										
Orifice diameter, d											0.068 m										
Diameter ratio, B											0.67										
Vel of approach, E											1.118										
Pipe area, A											0.0081 m2										
Orifice area, a											0.0036 m2										
Density, rho											998 kg/m3										
Kinematic visc,nu											1.141E-06 m2/s										
1											2										
Reference											3										
Mass											4										
Flowrate											5										
2											6										
3											7										
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Table 6.2a - Orifice meter calculations for Test 1 assuming upstream standard conditions (uncalibrated results)

	Tappings						D, D/2 ($\cong$ Vena Contracta)					
Pipe diameter, D	0.1016 m											
Orifice diameter, d	0.068 m											
Diameter ratio, B	0.67											
Vel of approach, E	1.118											
Pipe area, A _p	0.0081 m ²											
Orifice area, A _o	0.0036 m ²											
Density	998 kg/m ³											
Kinematic visc, v	1.14E-06 m ² /s											
	12	13	14	15	16	17	18	19	20	21	22	23
Calculated Flow Coefficient	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated
Contraction Coefficient	Contraction	Contraction	Velocity	Calculated V.Contracta	Alfa ratio	Beta or P _{RP}	Exponent	Pressure Loss Parameter	Loss Recovery Parameter	Pressure Loss Coefficient	Pressure Recovery Coefficient	Pressure Coefficient
C _c	C _c	C _c	C _v	d _c	a	b ^x or P _{RP}	x	P _{LP}	P _{LR}	K _{Loss}	K _R	K _P
0.6853	0.6551	0.9352	0.055039	0.5417		0.4538	1.9679	0.5462	0.8307	5.796	4.815	10.611
0.6850	0.6549	0.9352	0.055028	0.5416		0.4536	1.9686	0.5464	0.8303	5.803	4.818	10.621
0.6848	0.6547	0.9352	0.055019	0.5415		0.4535	1.9693	0.5465	0.8299	5.808	4.820	10.628
0.6846	0.6545	0.9352	0.055014	0.5415		0.4534	1.9696	0.5466	0.8297	5.811	4.821	10.632
0.6844	0.6544	0.9351	0.055007	0.5414		0.4534	1.9701	0.5466	0.8293	5.816	4.823	10.639
0.6843	0.6542	0.9351	0.055002	0.5414		0.4533	1.9705	0.5467	0.8291	5.819	4.824	10.643
0.6841	0.6541	0.9351	0.054996	0.5413		0.4532	1.9709	0.5468	0.8289	5.822	4.826	10.648
0.6840	0.6540	0.9351	0.054990	0.5412		0.4531	1.9713	0.5469	0.8286	5.826	4.827	10.653
0.6838	0.6538	0.9351	0.054985	0.5412		0.4531	1.9717	0.5469	0.8284	5.829	4.829	10.657
0.6837	0.6537	0.9350	0.054981	0.5411		0.4530	1.9720	0.5470	0.8282	5.831	4.830	10.661
0.6836	0.6536	0.9350	0.054976	0.5411		0.4530	1.9723	0.5470	0.8280	5.834	4.831	10.665
0.6835	0.6535	0.9350	0.054973	0.5411		0.4529	1.9725	0.5471	0.8279	5.836	4.832	10.668
0.6835	0.6535	0.9350	0.054971	0.5411		0.4529	1.9726	0.5471	0.8278	5.837	4.832	10.669
0.6834	0.6534	0.9350	0.054969	0.5410		0.4529	1.9728	0.5471	0.8277	5.839	4.833	10.671
0.6833	0.6534	0.9350	0.054965	0.5410		0.4528	1.9731	0.5472	0.8276	5.841	4.834	10.674
0.6832	0.6533	0.9350	0.054962	0.5410		0.4528	1.9733	0.5472	0.8274	5.842	4.834	10.677
0.6832	0.6533	0.9350	0.054961	0.5410		0.4528	1.9734	0.5472	0.8274	5.843	4.835	10.678
0.6831	0.6532	0.9350	0.054959	0.5409		0.4527	1.9735	0.5473	0.8273	5.844	4.835	10.679
0.6831	0.6532	0.9350	0.054957	0.5409		0.4527	1.9736	0.5473	0.827285	5.845	4.835	10.681

Table 6.2b - Orifice meter calculations for Test 1 assuming upstream standard conditions (uncalibrated results)

TEST 1 - ACTUAL ORIFICE PLATE MEASUREMENTS & CALCULATIONS FOR THE SPECIFIED FLOWRATES

Tappings		D, D/2 (≡Vena Contracta)										
Pipe diameter, D	0.1016 m											
Orifice diameter, d	0.068 m											
Diameter ratio, B	0.67											
Vel of approach, E	1.118											
Pipe area, A	0.0081 m2											
Orifice area, a	0.0036 m2											
Density, rho	998 kg/m3											
Kinematic visc,nu	1.141E-06 m2/s											
		1	2	3	4	5	6	7	8	9	10	11
Reference Turbine Frequency	Reference Volume Flowrate	Reference Mass Flowrate	Reference Reynolds Number	Reference Pipe Mean Velocity	Calibrated Discharge Coefficient	Measured Differential Pressure	Calculated Pressure Loss (Equation 9d)	Calculated Pressure Loss (Equation 9b)	Calculated Pressure Recovery	Calculated Convective acc. Term (Equation 8)		
Symbol	f	q _v	q _m	Re	u _b	C _d	ΔP _{1,3}	Δσ	Δσ	Δσ	ΔP _{4,3}	K ₂
Unit	Hz	m ³ /sec	Kg/sec		m/sec		Pa	Pa	Pa	Pa	Pa	
	370	0.01979	19.75	2.17E+05	2.441	0.6382	29086	14991	15505	13582	74390	
	400	0.02140	21.36	2.35E+05	2.639	0.6340	34449	17839	18437	16011	75384	
	430	0.02300	22.96	2.53E+05	2.837	0.6340	39812	20616	21308	18504	75388	
	450	0.02407	24.02	2.64E+05	2.969	0.6373	43154	22266	23025	20130	74616	
	480	0.02568	25.63	2.82E+05	3.167	0.6391	48827	25143	26007	22820	74201	
	500	0.02675	26.69	2.94E+05	3.299	0.6349	53683	27773	28708	24975	75184	
	530	0.02835	28.30	3.11E+05	3.497	0.6400	59355	30532	31587	27768	73984	
	550	0.02942	29.36	3.23E+05	3.629	0.6311	65726	34145	35274	30452	76075	
	580	0.03103	30.97	3.41E+05	3.827	0.6354	72110	37283	38543	33568	75053	
	600	0.03210	32.03	3.53E+05	3.959	0.6305	78383	40751	42094	36290	76235	
	630	0.03370	33.63	3.70E+05	4.157	0.6358	84976	43917	45403	39573	74963	
	650	0.03477	34.70	3.82E+05	4.289	0.6330	91249	47304	48883	42366	75619	
	660	0.03531	35.24	3.88E+05	4.355	0.6347	93583	48423	50053	43529	75221	
	680	0.03638	36.30	4.00E+05	4.487	0.6363	98852	51061	52793	46059	74851	
	700	0.03745	37.37	4.11E+05	4.619	0.6324	106035	55007	56838	49197	75767	
	720	0.03852	38.44	4.23E+05	4.751	0.6329	112011	58076	60014	51997	75653	
	730	0.03905	38.97	4.29E+05	4.817	0.6315	115651	60057	62047	53605	75986	
	740	0.03959	39.51	4.35E+05	4.883	0.6303	119298	62034	64077	55221	76278	
	760	0.04066	40.58	4.47E+05	5.015	0.6348	124058	64187	66348	57710	75202	

[illegible]

Table 6.3b - Orifice meter actual measurements calculations for Test 1

TEST 2 - ACTUAL ORIFICE PLATE MEASUREMENTS & CALCULATIONS FOR THE REFERENCE FLOWRATES

Pipe diameter, D	Tappings										
Orifice diameter, d	D, D/2 ($\equiv$ Vena Contracta)										
Diameter ratio, B	0.1016 m										
Vel of approach, E	0.068 m										
Pipe area, A	0.67										
Orifice area, a	1.118										
Density, rho	0.0081 m2										
Kinematic visc, nu	0.0036 m2										
	998 kg/m3										
	1.141E-06 m2/s										
Quantity	1	2	3	4	5	6	7	8	9	10	11
Reference	Reference	Reference	Reference	Reference	Reference	Calibrated	Measured	Calculated	Calculated	Calculated	Calculated
Turbine	Volume	Mass	Flowrate	Reynolds	Pipe Mean	Discharge	Differential	Pressure	Pressure	Pressure	Conductive
Frequency	Flowrate	Flowrate	Flowrate	Number	Velocity	Coefficient	Pressure	Loss	Loss	Recovery	acc. Term
Symbol	f	q _v	q _m	Re	u _b	C _d	$\Delta P_{1,3}$	$\Delta \sigma$	$\Delta \sigma$	$\Delta P_{4,3}$	K ₂
Unit	Hz	m ³ /sec	Kg/sec		m/sec		Pa	Pa	Pa	Pa	
	370	0.01979	19.75	2.17E+05	2.441	0.6229	30534	16008	16516	14018	78093
	400	0.02140	21.36	2.35E+05	2.639	0.6237	35681	18690	19286	16395	77901
	430	0.02300	22.96	2.53E+05	2.837	0.6237	41231	21597	22286	18945	77896
	450	0.02407	24.02	2.64E+05	2.969	0.6286	44463	23165	23921	20541	76701
	480	0.02568	25.63	2.82E+05	3.167	0.6247	51221	26802	27660	23561	77660
	500	0.02675	26.69	2.94E+05	3.299	0.6322	54253	28150	29086	25167	75807
	530	0.02835	28.30	3.11E+05	3.497	0.6253	62324	32589	33636	28688	77506
	550	0.02942	29.36	3.23E+05	3.629	0.6298	66166	34427	35557	30609	76409
	580	0.03103	30.97	3.41E+05	3.827	0.6321	73031	37898	39157	33873	75837
	600	0.03210	32.03	3.53E+05	3.959	0.6316	78280	40645	41993	36288	75959
	630	0.03370	33.63	3.70E+05	4.157	0.6314	86356	44848	46333	40023	76005
	650	0.03477	34.70	3.82E+05	4.289	0.6301	92310	48010	49590	42720	76322
	660	0.03531	35.24	3.88E+05	4.355	0.6292	95439	49686	51314	44125	76536
	680	0.03638	36.30	4.00E+05	4.487	0.6318	100490	52167	53898	46593	75916
	700	0.03745	37.37	4.11E+05	4.619	0.6325	106248	55112	56947	49301	75745
	720	0.03852	38.44	4.23E+05	4.751	0.6317	112706	58517	60457	52249	75947
	730	0.03905	38.97	4.29E+05	4.817	0.6356	114427	59150	61150	53277	75009
	740	0.03959	39.51	4.35E+05	4.883	0.6338	118261	61257	63309	54952	75441
	760	0.04066	40.58	4.47E+05	5.015	0.6328	125125	64882	67045	58079	75674

Table 6.4a - Orifice meter actual measurements calculations for Test 2

[illegible]

Table 6.4b - Orifice meter actual measurements calculations for Test 2

Pipe diameter, D	0.1016 m
Orifice diameter, d	0.068 m
Diameter ratio, B	0.67
Vel of approach, E	1.118
Pipe area, A	0.0081 m ²
Orifice area, a	0.0036 m ²
Density, rho	998 kg/m ³
Kinematic visc, nu	1.141E-06 m ² /s

Quantity	1	2	3	4	5	6	7	8	9	10	11
Reference Turbine Frequency	Reference Volume Flowrate	Reference Mass Flowrate	Reference Reynolds Number	Reference Pipe Mean Velocity	Calibrated Discharge Coefficient	Measured Differential Pressure	Calculated Pressure Loss (Equation 9d)	Calculated Pressure Loss (Equation 9b)	Calculated Pressure Recovery	Calculated Convective acc. Term (Equation 8)	
Symbol	f	q_v	q_m	Re	u_b	C_d	$\Delta P_{1,3}$	$\Delta\varpi$	$\Delta\varpi$	$\Delta P_{4,3}$	K_2
Unit	Hz	m^3/sec	Kg/sec		m/sec		Pa	Pa	Pa	Pa	
	370	0.01979	19.75	2.17E+05	2.441	0.6309	29827	15500	16012	13815	76141
	400	0.02140	21.36	2.35E+05	2.639	0.6354	34369	17771	18371	15998	75068
	430	0.02300	22.96	2.53E+05	2.837	0.6345	39819	20608	21301	18518	75260
	450	0.02407	24.02	2.64E+05	2.969	0.6320	43958	22813	23571	20386	75860
	480	0.02568	25.63	2.82E+05	3.167	0.6308	50214	26097	26959	23256	76164
	500	0.02675	26.69	2.94E+05	3.299	0.6269	55158	28790	29722	25436	77103
	530	0.02835	28.30	3.11E+05	3.497	0.6252	62324	32593	33640	28684	77537
	550	0.02942	29.36	3.23E+05	3.629	0.6268	66770	34856	35984	30786	77136
	580	0.03103	30.97	3.41E+05	3.827	0.6206	75747	39816	41065	34682	78689
	600	0.03210	32.03	3.53E+05	3.959	0.6259	79689	41641	42983	36706	77357
	630	0.03370	33.63	3.70E+05	4.157	0.6212	89173	46837	48312	40861	78515
	650	0.03477	34.70	3.82E+05	4.289	0.6249	93818	49078	50652	43166	77600
	660	0.03531	35.24	3.88E+05	4.355	0.6248	96744	50612	52234	44510	77614
	680	0.03638	36.30	4.00E+05	4.487	0.6307	100790	52385	54114	46676	76174
	700	0.03745	37.37	4.11E+05	4.619	0.6321	106342	55186	57020	49323	75843
	720	0.03852	38.44	4.23E+05	4.751	0.6287	113708	59229	61165	52543	76653
	730	0.03905	38.97	4.29E+05	4.817	0.6265	117741	61485	63472	54269	77212
	740	0.03959	39.51	4.35E+05	4.883	0.6302	119563	62179	64226	55338	76303
	755	0.04039	40.31	4.44E+05	4.982	0.6285	125112	65184	67312	57800	76703

Tappings														D, D/2 (≡Vena Contracta)													
Pipe diameter, D																											
Orifice diameter, d																											
Diameter ratio, B																											
Vel of approach, E																											
Pipe area, A _p																											
Orifice area, A _o																											
Density																											
Kinematic visc, v																											
12	13	14	15	16	17	18	19	20	21	22	23	12	13	14	15	16	17	18	19	20	21	22	23				
Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated	Calculated				
Flow	Contraction	Velocity	V.Contracta	Alfa	Beta or	Exponent	Pressure	Loss	Recovery	Pressure	Pressure	Flow	Contraction	Velocity	V.Contracta	Alfa	Beta or	Exponent	Pressure	Loss	Recovery	Pressure	Pressure				
Coefficient	Coefficient	Coefficient	diameter	ratio	P _{PR}		Parameter	Parameter	Parameter	Parameter	Parameter	Coefficient	Coefficient	Coefficient	diameter	ratio	P _{PR}		Parameter	Parameter	Parameter	Coefficient	Coefficient				
K	C _c	C _v	d _c	a	b ^x or P _{PR}	x	P _{LP}	P _{LR}	K _{Loss}	K _R	K _P	K	C _c	C _v	d _c	a	b ^x or P _{PR}	x	P _{LP}	P _{LR}	K _{Loss}	K _R	K _P				
0.7164	0.6821	0.9377	0.055777	0.5490	0.4632	1.9168	0.5368	0.8628	5.373	4.636	10.009	0.7164	0.6821	0.9377	0.055777	0.5490	0.4632	1.9168	0.5368	0.8628	5.373	4.636	10.009				
0.7087	0.6755	0.9383	0.055957	0.5508	0.4655	1.9044	0.5345	0.8708	5.275	4.593	9.868	0.7087	0.6755	0.9383	0.055957	0.5508	0.4655	1.9044	0.5345	0.8708	5.275	4.593	9.868				
0.7089	0.6757	0.9382	0.055925	0.5504	0.4651	1.9067	0.5349	0.8694	5.292	4.601	9.893	0.7089	0.6757	0.9382	0.055925	0.5504	0.4651	1.9067	0.5349	0.8694	5.292	4.601	9.893				
0.7146	0.6806	0.9378	0.055824	0.5494	0.4638	1.9136	0.5362	0.8649	5.347	4.625	9.972	0.7146	0.6806	0.9378	0.055824	0.5494	0.4638	1.9136	0.5362	0.8649	5.347	4.625	9.972				
0.7176	0.6832	0.9376	0.055773	0.5489	0.4631	1.9170	0.5369	0.8626	5.375	4.637	10.012	0.7176	0.6832	0.9376	0.055773	0.5489	0.4631	1.9170	0.5369	0.8626	5.375	4.637	10.012				
0.7128	0.6790	0.9371	0.055618	0.5474	0.4611	1.9277	0.5389	0.8558	5.462	4.674	10.136	0.7128	0.6790	0.9371	0.055618	0.5474	0.4611	1.9277	0.5389	0.8558	5.462	4.674	10.136				
0.7209	0.6860	0.9369	0.055547	0.5467	0.4602	1.9326	0.5398	0.8527	5.502	4.691	10.193	0.7209	0.6860	0.9369	0.055547	0.5467	0.4602	1.9326	0.5398	0.8527	5.502	4.691	10.193				
0.7074	0.6743	0.9371	0.055612	0.5474	0.4611	1.9281	0.5389	0.8555	5.465	4.675	10.140	0.7074	0.6743	0.9371	0.055612	0.5474	0.4611	1.9281	0.5389	0.8555	5.465	4.675	10.140				
0.7158	0.6816	0.9363	0.055360	0.5449	0.4579	1.9455	0.5421	0.8446	5.608	4.736	10.344	0.7158	0.6816	0.9363	0.055360	0.5449	0.4579	1.9455	0.5421	0.8446	5.608	4.736	10.344				
0.7067	0.6738	0.9370	0.055576	0.5470	0.4606	1.9306	0.5394	0.8540	5.485	4.684	10.169	0.7067	0.6738	0.9370	0.055576	0.5470	0.4606	1.9306	0.5394	0.8540	5.485	4.684	10.169				
0.7161	0.6819	0.9364	0.055388	0.5452	0.4582	1.9436	0.5418	0.8458	5.592	4.729	10.321	0.7161	0.6819	0.9364	0.055388	0.5452	0.4582	1.9436	0.5418	0.8458	5.592	4.729	10.321				

Table 6.5b - Orifice meter actual measurements calculations for Test 3

Tappings											D, D/2 (≡Vena Contracta)																																																																																																																																														
Pipe diameter, D											0.1016 m																																																																																																																																														
Orifice diameter, d											0.068 m																																																																																																																																														
Diameter ratio, B											0.67																																																																																																																																														
Vel of approach, E											1.118																																																																																																																																														
Pipe area, A											0.0081 m ²																																																																																																																																														
Orifice area, a											0.0036 m ²																																																																																																																																														
Density, rho											998 kg/m ³																																																																																																																																														
Kinematic visc,nu											1.141E-06 m ² /s																																																																																																																																														
1											2											3											4											5											6											7											8											9											10											11																																											
Reference Turbine Frequency											Reference Volume Flowrate											Reference Mass Flowrate											Reference Reynolds Number											Reference Pipe Mean Velocity											Calibrated Discharge Coefficient											Measured Differential Pressure											Calculated Pressure Loss (Equation 9d)											Calculated Pressure Loss (Equation 9b)											Calculated Pressure Recovery											Calculated Convective acc. Term (Equation 8)																																											
Symbol											f											q _v											q _m											Re											u _b											C _d											ΔP _{1,3}											Δσ											Δσ											Δσ											ΔP _{4,3}											K ₂																					
Unit											Hz											m ³ /sec											Kg/sec																						m/sec																						Pa											Pa											Pa											Pa											Pa											Pa																					
											370											19.75											2.17E+05											2.441											0.6220											30686											16104											16613											14072											78332																																											
											400											21.36											2.35E+05											2.639											0.6221											35846											18810											19404											16442											78294																																											
											430											22.96											2.53E+05											2.837											0.6230											41308											21655											22342											18966											78073																																											
											450											24.02											2.64E+05											2.969											0.6195											45760											24083											24835											20926											78971																																											
											480											25.63											2.82E+05											3.167											0.6251											51127											26740											27598											23530											77549																																											
											500											26.69											2.94E+05											3.299											0.6240											55681											29158											30089											25592											77834																																											
											530											28.30											3.11E+05											3.497											0.6220											62957											33040											34084											28873											78324																																											
											550											29.36											3.23E+05											3.629											0.6233											67528											35390											36515											31012											78012																																											
											580											30.97											3.41E+05											3.827											0.6286											73812											38452											39708											34104											76679																																											
											600											32.03											3.53E+05											3.959											0.6267											79478											41493											42835											36643											77152																																											
											630											33.63											3.70E+05											4.157											0.6287											87074											45359											46841											40233											76668																																											
											650											34.70											3.82E+05											4.289											0.6275											93046											48534											50111											42935											76962																																											
											660											35.24											3.88E+05											4.355											0.6260											96383											50357											51981											44402											77324																																											
											680											36.30											4.00E+05											4.487											0.6280											101653											52992											54717											46936											76826																																											
											700											37.37											4.11E+05											4.619											0.6257											108536											56728											58554											49982											77407																																											
											720											38.44											4.23E+05											4.751											0.6285											113807											59298											61234											52573											76720																																											
											730											38.97											4.29																																																																																																																								

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TEST 4 - ACTUAL ORIFICE PLATE MEASUREMENTS & CALCULATIONS FOR THE REFERENCE FLOWRATES

Pipe diameter, D	0.1016 m	Tappings										D, D/2 ($\equiv$ Vena Contracta)									
Orifice diameter, d	0.068 m																				
Diameter ratio, B	0.67																				
Vel of approach, E	1.118																				
Pipe area, A_p	0.0081 m ²																				
Orifice area, A_o	0.0036 m ²																				
Density	998 kg/m ³																				
Kinematic visc, ν	1.14E-06 m ² /s																				
12	13	14	15	16	17	18	19	20	21	22	23										
Calculated Flow Coefficient	Calculated Contraction Coefficient	Calculated Velocity Coefficient	Calculated V.Contracta diameter	Calculated Alfa ratio	Calculated Beta or P_{PR} Parameter	Calculated Exponent	Calculated Pressure Loss Parameter	Calculated Loss Recovery Parameter	Calculated Pressure Loss Coefficient	Calculated Pressure Recovery Coefficient	Calculated Pressure Coefficient										
K	C_c	C_v	d_c	a	b^* or P_{PR}	x	P_{LP}	P_{LR}	K_{Loss}	K_R	K_p										
0.7194	0.6848	0.9365	0.055418	0.5455	0.4586	1.9415	0.5414	0.8470	5.575	4.722	10.297										
0.7132	0.6794	0.9365	0.055424	0.5455	0.4587	1.9411	0.5413	0.8473	5.571	4.721	10.292										
0.7129	0.6791	0.9366	0.055460	0.5459	0.4591	1.9386	0.5409	0.8489	5.551	4.712	10.263										
0.7189	0.6843	0.9361	0.055315	0.5444	0.4573	1.9486	0.5427	0.8426	5.634	4.747	10.381										
0.7196	0.6849	0.9369	0.055545	0.5467	0.4602	1.9327	0.5398	0.8526	5.503	4.692	10.194										
0.7138	0.6799	0.9367	0.055498	0.5462	0.4596	1.9359	0.5404	0.8506	5.529	4.703	10.232										
0.7220	0.6870	0.9365	0.055419	0.5455	0.4586	1.9414	0.5414	0.8471	5.574	4.722	10.296										
0.7086	0.6754	0.9366	0.055470	0.5460	0.4593	1.9379	0.5407	0.8493	5.545	4.710	10.255										
0.7130	0.6792	0.9374	0.055688	0.5481	0.4620	1.9229	0.5380	0.8589	5.423	4.657	10.080										
0.7065	0.6735	0.9371	0.055610	0.5473	0.4610	1.9283	0.5390	0.8554	5.466	4.676	10.142										
0.7136	0.6797	0.9374	0.055689	0.5481	0.4621	1.9228	0.5379	0.8589	5.422	4.657	10.079										
0.7099	0.6765	0.9372	0.055641	0.5476	0.4614	1.9261	0.5386	0.8568	5.449	4.668	10.117										
0.7129	0.6791	0.9370	0.055582	0.5471	0.4607	1.9302	0.5393	0.8542	5.482	4.683	10.165										
0.7145	0.6805	0.9373	0.055663	0.5479	0.4617	1.9246	0.5383	0.8578	5.436	4.663	10.099										
0.7096	0.6763	0.9370	0.055568	0.5469	0.4605	1.9311	0.5395	0.8536	5.490	4.686	10.176										
0.7094	0.6761	0.9373	0.055681	0.5480	0.4619	1.9234	0.5381	0.8586	5.426	4.659	10.085										
0.7071	0.6741	0.9374	0.055708	0.5483	0.4623	1.9215	0.5377	0.8597	5.412	4.653	10.064										
0.7056	0.6728	0.9373	0.055678	0.5480	0.4619	1.9236	0.5381	0.8584	5.428	4.660	10.088										
0.7122	0.6785	0.9373	0.055673	0.5480	0.4618	1.9239	0.5382	0.8582	5.431	4.661	10.092										

Table 6.6b - Orifice meter actual measurements calculations for Test 4