

CRANFIELD UNIVERSITY



GUST REJECTION IN MULTIROTOR AIRCRAFT USING SLIDING MODE CONTROL

SCHOOL OF AEROSPACE, TRANSPORT AND MANUFACTURING

Dynamics, Simulation and Control Group
Department of Aerospace Engineering

PhD THESIS

Academic Year: 2022/2023

Supervisor : Dr James F. Whidborne

Submission date: June 18, 2023

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MUKARRAM KHASIMUL SHAIK

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Abstract

Multirotor aircraft are being used in many applications because of their mechanical simplicity and high manoeuvrability. Many potential applications of multirotor aircraft are in gusty and turbulent environments, it is imperative for them to have a stable hovering as well as offer resistance to transient gusts and winds. Many research studies have been carried out with regards to control of the multirotor aircraft but not much work has been undertaken considering the aerodynamic effects impacting the aircraft, in particular the rotor tilt angle. As the Vertical Take Off and Landing (VTOL) aircrafts are light-weight, they are prone to gusty wind conditions and desired landing and hovering is a great challenge during these disturbances.

In this research work, an analysis of the effect of rotor tilt on the stability and gust rejection properties is performed with a conceptual planar birotor initially and then extended to quadrotors. The gust rejection properties of multirotor aircraft are also examined using sliding mode control taking rotor tilt angle into account. Models of the aircraft were developed in MATLAB/Simulink to implement nonlinear dynamic equations of the multirotor vehicles. A range of rotor tilt angles have been considered and investigated for gust rejection through extensive simulation studies using Proportional-Integral-Velocity (PIV) control, Sliding Mode Control (SMC) of altitude, attitude and PIV position control and a complete SMC control of altitude, attitude and position control. The performance of the three controllers were compared through numerical simulations. It was found that the complete sliding mode controller is very robust at rejecting the gust and shows that the rotor tilt angle does not impact on the vehicle stability when used with SMC, even in the presence of parametric uncertainties and external disturbances. Finally, suggestions for further work based on this research are presented on further design and development of sliding mode controllers for different multirotor configurations.

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Acknowledgements

I would like to express my profound gratitude and appreciation to my director of studies and supervisor, Dr James F. Whidborne, for his constant guidance, support, encouragement and spending most of his weekend evenings in supervising and advising me on this research work. I am greatly indebted to the kindness, patience and empathy he has shown during all these years. His valuable suggestions made this work interesting and a great learning experience for me.

I also wish to thank the School of Aerospace, Transport and Manufacturing and I acknowledge the support and facilities provided by Cranfield University all through these years to complete this research work. I am indebted to my co-supervisor, Dr Mudassir Lone for the kind support he offered during my review meetings and his valuable feedback has helped me in finalising this work.

I am thankful to our family friend Mrs Tahirah Yasin for supporting me in looking after my children during university visits and meetings with my supervisor.

I would like to express my gratitude to my friends Sivarama Reddy Gudur, Mohammad Shameem, Mohammad Abdul Mujeeb, Athmeeya and Shaik Mahaboob Basha who cheered me up and supported me all through these years with their love and affection.

I am grateful to my father-in-law, Dr Momin Shaik Ahmed, who always offered me his unquestionable support in all walks of my life and inspired me at every step, including my research.

Finally, I thank my parents, Late Mr Shaik Maqbool Hussain and Mrs Azgari Begum, and my brothers, Siraj, Azmath and Khasim, for their love, sacrifices, prayers and understanding. I am ever so thankful to my wife, Shahina and my kids Faisal, Zuhayr and Fawzan for all the support and sacrifices they made all these years by losing some of their precious weekends and ‘dad time’ so that I could focus on my research!

A promise to my father...fulfilled!

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Nomenclature

α	Angle between the incident flow and the tip-path plane
α_a	Angle of attack of the blade
ω	Vector of angular rates in body frame
$\mathbf{F}_g$	Gravitational force vector
$\mathbf{F}$	Vector of the forces acting on the centre of mass
$\mathbf{I}$	Moments of inertia matrix
$\mathbf{M}$	Vector of the moments acting about the centre of mass
$\mathbf{R}$	Rotation matrix
$\mathbf{V}$	Vector of velocities in body frame
δ	Blade profile drag coefficient
λ	Inflow ratio
$\mathbb{R}$	Real number
μ	Advance ratio
ρ	Air density
σ	Solidity factor
τ_i	Torque from the i^{th} rotor
θ_1	Twist in the blade
$\theta_{rotorblade}$	Rotor blade pitch angle
Υ	Rotor tilt angle
A	Area of the rotor disc
a	Lift slope of the blade
a_1	Backward flapping angle
b_{blade}	Number of blades
c	Constant chord
C_H	Drag coefficient
C_T	Rotor thrust coefficient
g	Acceleration due to gravity
l	Quadrotor arm length
m	Mass of the vehicle
Q	Rotor torque
R_{rotor}	Rotor radius

$S(t)$	Time-varying sliding surface
T_i	Thrust from the i^{th} rotor
v_i	Induced velocity
V_w	Wind velocity

Acronyms

DSF	Direct Switching Function
ITAE	Integral of Time-weighted Absolute Error
LQR	Linear Quadratic Regulator
MPC	Model Predictive Control
MRAC	Model Reference Adaptive Control
PD	Proportional-Derivative
PIV	Proportional-Integral-Velocity
PID	Proportional-Integral-Derivative
PVTOL	Planar Vertical Take Off and Landing
PWM	Pulse Width Modulation
SMC	Sliding Mode Control
UAV	Unmanned Aerial Vehicles
VTOL	Vertical Take Off and Landing

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CHAPTER 1

Introduction

1.1 Context

Multirotors have become the most popular configuration for Unmanned Aerial Vehicles (UAV) as they are less complicated than classical helicopter designs and there have been many of improvements in motor, sensors and battery technology. The control of UAVs is an interesting research topic which has many applications such as traffic surveillance [Coifman et al., 2006], reconnaissance [Girard et al., 2004], search and rescue [Ryan and Hedrick, 2005], fire detection [Casbeer et al., 2006], Human-UAV interaction in payload handover tasks [Peringal et al., 2022], cinematography, and exploration. Researchers of MIT using a draganflyer quadrotor achieved vision-based hovering, takeoff and landing of the quadrotor combined with a computer tracking system [Salih et al., 2010]. Research at Stanford University makes use of an outdoor testbed named STARMAC (Stanford Testbed of Autonomous Rotorcraft for Multi-Agent Control) for testing and validating multi-agent algorithms and control schemes [Hoffmann et al., 2004].

Quadrotor control is quite challenging when the uncertainties in the parameters, external disturbances, motor failures, wind gust, are taken into account. It is difficult to maintain the stability and performance in the presence of all these factors over different operating conditions. Nonlinear control strategies are more suitable for these types of systems considering all the parametric uncertainties and external disturbance, faults, etc., when compared to linear control strategies. In many of the research works undertaken for the control of quadrotors, methods like gain Scheduling, Proportional-Integral-Derivative (PID), Backstepping Control, Model Predictive Control (MPC), SMC and Model Reference Adaptive Control (MRAC) are included [Zhang and Chamseddine, 2012; Sadeghzadeh et al., 2011].

SMC is known for its robustness when dealing with inaccuracies in dynamic modelling, uncertainties in the parameters of the vehicle and external disturbances acting on the system. A lot of interesting research work has been carried out applying SMC for the control of quadrotors in recent years. Sira-Ramirez et al. [1994] used SMC focusing on the altitude control of a miniature radio control helicopter. Xu and Ozguner [2006] proposed a design using SMC for the robust control of the position and the yaw of an autonomous quadrotor helicopter. Runcharoon and Srichatrapimuk [2013] presented a controller based on SMC for a quadrotor splitting the attitude control by SMC and the altitude and position was control by a Proportional-Derivative (PD) controller. In this work, initially a quadrotor was modelled in body frame and simulation studies conducted using with PID and SMC control strategies. A comparative simulation study was carried out using these two control strategies. The modelling was done using the body axes in both conceptual Planar Vertical Take Off and Landing (PVTOL) and a quadrotor. Simulation studies have been carried out using both the PID and SMC controllers. The dynamic modelling of the quadrotors has been carried out by many researchers based on certain assumptions and the dynamic equations of the quadrotors are well established [Bouabdallah et al., 2004a,b; Erginer and Altug, 2007; Zhang et al., 2009; Fang et al., 2010; Sharifi et al., 2010]. But many of the useful applications occur in rugged environments

such as mountain terrains, off-shore structures where in gust windy conditions are encountered. As the multirotor VTOL are very light, their performance is very much prone to wind gusts and thus limiting their operation during hovering and landing. Without accurate hovering and landing, many of the applications of UAV will not function properly.

However, not much research work has been carried out with regards to aerodynamic modelling and gust rejection in VTOLs control and how the rotor tilt angle impacts on gust-rejection properties of the VTOLs. The research work in the field of rotor aerodynamics was taken up by Pounds et al. [2004]; Bangura and Mahony [2012] is based on the established methods used in rotorcraft. In other researches, the gust rejection properties of the controller are evaluated in the controller design by utilising feedback and often by a gust observer [Dean et al., 2017; Yang et al., 2017; Demircioglu and Bastürk, 2017], Kalman-filtering methods [Xing et al., 2019; Belokon' et al., 2013] or even on-board gust measurements [Craig et al., 2020] which raised the computation load and the measurement values are prone to errors due to the unmodelled aerodynamic effects caused by the air surrounding the propellers. Mendez et al. [2022] investigated the use of a ground-based wind measuring LiDAR that captures both the amplitude and the direction of the wind at the landing site of an autonomous quadrotor. LiDAR (Light Detection and Ranging) is a form of laser anemometry which utilises the Doppler-shift in the frequency of backscattered light to measure wind speed. LiDAR's remote sensing capability has been used to detect wind disturbances in the aerospace industry as well. Whidborne et al. [2022] investigated the effect of rotor tilt on the gust rejection properties of multirotor aircraft using standard aerodynamic models of the rotors included in the PVTOL and quadrotor dynamic models. In the first case dealing with PVTOL, pole-placement strategy was used and in the case of quadrotor a Linear Quadratic Regulator (LQR) controller was used. The results showed that the outward rotor tilt significantly improves the gust rejection properties of the multirotor aircraft.

In this work, the research work of [Whidborne et al., 2022] has been extended with the application of PIV and SMC in the control of PVTOL and quadrotor incorporating the standard aerodynamics of the rotors. All the modelling and implementation of the PVTOL and quadrotor has been done in MATLAB/Simulink and simulated with the PIV and SMC controllers. A wind gust step and 1-cosine wind gust have been used to evaluate the gust rejection properties of the SMC for different rotor tilt angles. The results of this work showed that there is no effect of outward rotor tilt angles on the gust rejection properties of the multirotor aircraft when used with SMC. Extensive simulation studies have been carried out with different wind gust scenarios to confirm this behaviour.

The PID control has been used widely in the control of attitude as well as altitude in quadrotors as it has a simple structure and known to provide good performance by tuning the PID gains even if the specified model of the system is not known. When quadrotor measurements can be feedback, a PIV is used where position and velocity errors can be used resulting in less noisy control output with a reduced overshoot as the time derivative of the position error is not considered. PIV is a linear control with good properties and provides a baseline to compare with other control strategies. In this work, for comparison of the simulation studies, it is denoted as Method #1.

Sliding Mode Control is known for its robustness to nonlinearities, parametric uncertainties and external disturbances. Whidborne and Cooke [2017] used a conceptual planar birotor aircraft to investigate the impact of rotor tilt angle on gust rejection using linear control. As a result of their investigation, it was found that an outward rotor tilt angle improves the performance of the multirotor aircraft when considering wind gust into account. Following the work done by Whidborne and Cooke [2017]; Whidborne et al. [2022], the question arises of whether the advantages accrued by outward tilt angle are still present with nonlinear control. SMC is chosen as the method to be investigated because it is known to be robust to parametric variations and external disturbances. Furthermore, a lot of research work has been undertaken using SMC in the control of quadrotors and is a preferred choice due to its robustness properties. Initially, a cascade controller, i.e, in the inner loop SMC is used for attitude and altitude control and PIV position control is used in the outer loop to improve the performance of the control system when compared to PIV alone, especially the model uncertainties, external disturbances and the effect of nonlinearities can be addressed in the inner loop, which is SMC, even before the whole system performance is affected. In this work, for comparison of the simulation studies, it is denoted as Method #2.

Subsequently, a full SMC control both in the inner loop for attitude and altitude control along with the SMC position control in the outer loop was implemented as an extension to Method #2. In this

work, for comparison of the simulation studies, it is denoted as Method #3. With the simulation results presented, it is shown that a full SMC controller (Method #3) performs very well in gust rejection and, unlike the linear controllers, the rotor tilt angles do not have a significant impact on the performance of the multirotor aircraft when using SMC technique.

This chapter describes the context of this research work wherein the effect of rotor tilt angle in a multirotor aircraft has been investigated with different control methodologies. The aims, objectives have been highlighted and the methodology in carrying out this research work has been discussed including the contribution to knowledge. Finally, the organisation of the thesis has been laid out giving a summary of each chapter of thesis.

1.2 Aims and Objectives

The aim of this research work is to provide an understanding of gust rejection properties of multirotor vehicles using SMC. The objectives defined for this work are as follows:

1. To model a quadrotor in body axes and carry out similar simulation studies i.e. PIV and SMC controller using MATLAB/Simulink.
2. To model a conceptual PVTOL model initially without considering aerodynamics and then extending it to include aerodynamics and design a PID controller for it.
3. To determine the effect of rotor tilt angle using the PVTOL model on the gust rejection properties of a PVTOL under sliding mode control.
4. To extend the investigation of the gust rejection properties of the rotor tilt angle to quadrotor VTOL aircraft.

1.3 Methodology

All the work carried out in this thesis is through model-based design and development. Model-based design and development leads to cost-effective development of dynamic systems and their control. The code development time for the product can be very much reduced, eliminating the introduction of manually coded errors, by using the auto code generation tools. This methodology is widely being adopted in industry to reduce the project costs for design, development, verification. With the qualified auto code generators, the software generated based on these model-based design techniques is loaded on many safety-critical systems. Carrying out these modelling and simulation activities iteratively will enhance the design quality as well as time taken in the development of the product. This methodology is also very useful in carrying out testing that is difficult to be reproduced with the hardware equipment alone. The methodology to carry out this work is mainly based on simulation studies using MATLAB/Simulink. Some of the parameters used in simulation were obtained from the wind tunnel experiments carried out by Martinez [2007].

1.4 Contribution to Knowledge

- Analysing the effect of rotor tilt angle on the gust rejection. Whidborne and Cooke [2017] carried out investigations of the effect of rotor tilt angle using classical control techniques. The work done by Whidborne and Cooke [2017]; Whidborne et al. [2022] has been extended in this research work.
- The important contribution of this research work is the analysis of the effect of rotor tilt angle on the multirotor aircraft gust rejection using SMC under the presence of parametric uncertainty and external disturbance. This has been carried out initially on a conceptual PVTOL and then extended to a quadrotor. It was found that the sliding mode controller is very robust at rejecting the gust and shows that the rotor tilt angle does not impact on the vehicle stability when used with SMC, even in the presence of parametric uncertainty and external disturbances.

1.4.1 Publications Resulting from this Work

- Mukarram K Shaik and James F. Whidborne “A Robust Sliding Model Control of the Quadrotor” published as a result of this work in 2016 UKACC 11th International Conference on Control (CONTROL), Belfast, UK, 31st August - 2nd September 2016.
- Mukarram K Shaik and James F. Whidborne “Gust Rejection in a VTOL Aircraft using Sliding Mode Control”, In preparation for submission to an international journal.
- Mukarram K Shaik and James F. Whidborne “Effect of Rotor Tilt Angle on Gust Rejection Properties of a Quadrotor using PIV and Sliding Mode Control”, In preparation for submission to an international journal.

1.5 Potential Impact on Industry

Multirotor aircraft are used in many applications because of their simplicity and high manoeuvrability, these include surveillance, aerial surveying, photography, structure inspection, personal transport aircraft (air-taxis) and low-cost delivery services. For example, HEROTECH8 [2020] have developed solutions for round-the-clock automated drone operation to automatically deploy, recover and recharge drone with the need for an operator to be on-site. Sancho Martínez et al. [2020] have combined the use of UAVs and image processing techniques to identify the issues in wastewater treatment plant inspection using HEROTECH8 autonomous docking station.

Due to the congestion of traffic in urban areas, many companies have started designing and developing VTOL aircraft as potential air taxis. Autonomous operation will be important for their success. Many companies are trying to harness the potential of air-taxis for ferrying passengers across different cities. The landing and take-off of these vehicles in gusty wind conditions will be a big challenge. So, developing controllers taking into account their working environments i.e. gusty wind conditions where accurate hovering and landing is needed, has a great potential impact on the industries dealing with design and development of VTOLs. Robust control strategies for the VTOLs will help the industry in meeting the design standards and being compliant with the challenging regulations that are still being developed for such urban air vehicles.

1.6 Organisation of the Thesis and Contributions

The organisation of this thesis is as follows:

Chapter 2: Literature Review and Background

Chapter 2 deals with the literature review on this subject and work done by different researchers focusing on multirotor VTOL aircrafts and their application, modelling of multirotor VTOL aircraft, study of different control strategies of multirotor VTOL vehicles. Special emphasis is given to sliding mode control of VTOLs. A review of different modelling and control techniques taking into account the aerodynamics of the VTOLs is also considered. This chapter also looks into different researches dealing with gust rejection properties of multirotor VTOLs.

Chapter 3: Dynamic Modelling and Simulation of a Quadrotor

Chapter 3 derives a quadrotor model in the body frame. A sliding mode controller is designed and simulated with the developed dynamic model of the quadrotor. The performance is compared with a proportional plus derivative controller. The robustness of the design is also tested for parametric uncertainty and the effect of disturbances is also investigated. It is found that the sliding mode controller provides good performance, robustness and disturbance rejection when compared to the PD controller.

Chapter 4: Gust Rejection Properties of a Conceptual PVTOL

Chapter 4 considers the modelling and control of a conceptual PVTOL as a toy example in body axes including the aerodynamic effects into the equations of motion associated with the conceptual

vehicle. The PVTOL aircraft offers insight into the investigation of quadrotor dynamics and control as the vehicle can be restricted to a vertical plane as a result it can be analysed using basic control knowledge. PVTOL offers 3 degrees-of-freedom and is constrained to move in vertical x-z plane. This conceptual PVTOL with aerodynamics modelled is then used for simulation with PD and SMC control strategies and its stability is investigated. This chapter deals with the evaluation of gust rejection properties of a conceptual PVTOL as well.

Chapter 5: Effect of Rotor Tilt Angle on Gust Rejection Properties of a Quadrotor using PIV Control

Chapter 5 considers the effect of rotor tilt angle on gust rejection properties of a quadrotor modelled in body axes including the aerodynamic effects into the dynamic equations of the quadrotor. The vast majority of literature on quadrotor control does not explicitly consider the problem of gust rejection except some simulation studies. The effect of the rotor tilt on the stability of the quadrotor aircraft is analysed using PIV control strategy to stabilise the aircraft and the effect of gusts on the lateral position and vertical position is investigated by simulation studies. The idea is that by a small fixed-angle outward tilt of the rotors, the aerodynamics can be tailored so that a horizontal gust on the vehicle causes the aircraft to pitch or roll into the wind thus reducing the horizontal position displacement.

Chapter 6: Gust Rejection Properties of a Quadrotor using Sliding Mode Control

Chapter 6 considers the effect of rotor tilt angle on gust rejection properties of a quadrotor including the aerodynamic effects using sliding mode control. The effect of the rotor tilt on the stability of the quadrotor aircraft is analysed using SMC control strategy to stabilise the aircraft and the effect of gusts on the lateral position and vertical position is investigated by simulation studies. A comparative analysis between the PIV and SMC control is considered and it is demonstrated that the SMC control produces almost identical responses for different rotor tilt angles in the presence of gust. The SMC is very robust at rejecting the gust and shows that the rotor tilt angle does not impact on the vehicle stability even in the presence of parametric uncertainty and external disturbances.

Chapter 7: Conclusions and Recommendations for Further Work

Chapter 7 summarises the tasks achieved, and the conclusions drawn out of this research work including contribution to knowledge. Finally, recommendations for further work have been provided. The appendices sections provide snapshots of all the modelling and development done in MATLAB/Simulink including the MATLAB routines used for simulation.

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CHAPTER 2

Literature Review and Background

In chapter 1, the context of the research, aims and objectives of this work have been established. Chapter 2 details the literature review related to this research and the work done in the field of multirotor aircrafts including aerodynamics modelling and different control strategies applied. This chapter also highlights the research work done on gust rejection properties of multirotors with special emphasis on sliding mode control.

2.1 Introduction

There is lot of interest in multirotor aircraft as they have the ability to hover and vertical take-off and land anywhere, hence there is no requirement for runways. This makes them very much suitable in applications where there is limited space and high manoeuvrability is needed. There are lots of functional areas where the multirotor aircraft can be used both in military and civil applications. Most of the initial applications of multirotor aircraft were in surveillance, reconnaissance [Rodriguez et al., 2006; Ahmadzadeh et al., 2006], target acquisition systems [Quigley et al., 2005]. Gradually the multirotor aircraft found its way into civil applications such as tourism, mapping [Samad et al., 2013], traffic monitoring [Heintz et al., 2007], remote sensing [Nebiker et al., 2008; Jensen et al., 2009], weather studies [Techy et al., 2009], disaster response [DeBusk, 2010; Casbeer et al., 2006; Maza et al., 2011; Ezequiel et al., 2014], agriculture [Göktoğan et al., 2010] etc.

A quadrotor has four inputs and six outputs, and the state variables are highly coupled and is considered as an under-actuated system. It is difficult to control all of the state variables at the same time and to attain a desired position and yaw and maintaining the stability of the vehicle. Many researchers have proposed different modelling approaches for a quadrotor [Goodman et al., 2015; Bouabdallah et al., 2004a; Altug et al., 2002].

There has been a tremendous growth in different control strategies used in the control of multirotor aircraft as well. Research in the field of autonomous multirotors has significantly contributed to the research literature of control strategies used in multirotor VTOLs. Many universities and companies are working together to develop UAVs based on different quadrotor designs and the UAV industry is expected to grow to nearly US\$41.3 billion by 2026 [Schroth and Bodecker, 2021]. Mistler et al. [2001] proposed the exact linearisation method, transforming the nonlinear system into a controllable linear system for a miniature quadrotor. The limitation of this strategy is it requires all the states for feedback to be measured which could be overcome by designing a nonlinear state observer for feedback. This is not a significant limitation nowadays as measuring the states is not hard or expensive. Bouabdallah et al. [2004b] Bouabdallah et al. [2004a] presented linear control techniques such as PID and LQR for the control of an indoor micro quadrotor (OS4). Adir and Stoica [2012] presented an integral LQR control for a star-shaped octocopter.

Alaimo et al. [2013] used a PD controller for a hexacopter, with the rotors located on the corners of a regular hexagon and the propulsion system consisted of three pairs of counter-rotating fixed-pitch propellers, that was modelled by generalising the quadrotor model mathematical equations. The equations of motion of the hexacopter were defined by means of quaternions. Numerical simulations were carried out using MATLAB software. Later on, this work was extended with a PID controller [Alaimo et al., 2014]. Argentin et al. [2013] studied the performance of a Integral of Time-weighted Absolute Error (ITAE) tuned PID, a classic LQR controller and a PID controller tuned with a LQR loop for a quadrotor and simulation results were presented for the vertical attitude. It was found that the LQR controller low steady state error but with big transition delays and when the system needs fast parameters update with no direct access to the states of the system, the LQR controller falls short. The PID controller provides a faster response but not with robust gains as compared to the LQR controller. The classic PID controller when tuned by the LQR results in a robust controller. Cabecinhas et al. [2012] addressed the problem of robust take-off of a quadrotor in the presence of sloped terrains and surrounding obstacles where uncertainties in both the environment and vehicle dynamics are to be considered. The control strategy requires that the current operative mode of the quadrotor is known and for the take-off operation, the information provided by contact or force sensors is used to retrieve the operating mode knowing the velocity and attitude of the system obtained through the inertial navigation unit resulting in a trajectory generation and tracking control system problem. The five operating modes of the quadrotor considered here are Free-Flight mode, Take-off-and-Landing and Landed modes (both sliding and non-sliding situations) in accordance with the nature of the ground contact.

Bouabdallah and Siegwart [2005] presented the application of both back stepping and sliding mode control a micro quadrotor called OS4. The comparative results showed that the back stepping controller performed well in the presence of a relatively high perturbations when compared to the SMC.

Boiko and Fridman [2005] analysed two continuous sliding mode algorithms wherein the shortcomings of the discontinuous classical sliding mode such as chattering and infinite-time convergence are overcome. They are known as the power-fractional sliding mode algorithm and second-order sliding mode algorithm also known as super-twisting. The second-order sliding mode control doesn't offer chattering elimination but it also exhibits chattering phenomenon in the presence of parasitic dynamics [Boiko et al., 2004].

Madani and Benallegue [2006] proposed the controlling of the quadrotor using a back stepping controller. Madani and Benallegue [2006] considered the quadrotor to be three interconnected subsystems relating the pitch and roll angles to the horizontal position and the yaw angle relating to the vertical position and lastly the rotor dynamics. Then a back stepping controller was proposed to control the positions and yaw and the stability properties studied. Belokon' et al. [2013] used the AR. Drone quadrotor developed by Parrot (USA) [Bristeau et al., 2011] to investigate the possibility of estimating the linear velocity components of the quadrotor using a Kalman filter in the presence of measurement noise and time delays in the control channel. Experiments were carried out over various trajectories to study the controller stability and numerical simulation results concurred with the experimental findings. Alexis et al. [2011] proposed a switching model predictive controller for a set induced linearised piecewise affine (PWA) approximations covering a sub-set of the quadrotor's flight envelope in the presence of wind gusts.

Firdaus [2018] investigated the chattering issue by developing sliding mode algorithms such as quasi-sliding mode control, super-twisting SMC algorithm and dynamic sliding mode control driven by sliding mode observers. The super-twisting algorithm of SMC showed better performance when compared to others. Asad et al. [2017] proposed the chattering and stability analysis of the inverse hyperbolic function based reaching law where the calculated gain bound of the control law will result in avoiding the chatter. This strategy was applied to control the speed of a DC motor. The results demonstrate good performance and chatter reduction when compared to the first order SMC.

Zheng et al. [2014] proposed a second order sliding mode controller for a quadrotor in order to perform the position and attitude tracking control by decomposing the system into two subsystems with one as a fully actuated subsystem and another as an underactuated subsystem and defining the sliding manifolds by having a linear combination of position and velocity tracking errors of the state variables and obtaining the switching surface coefficients for the two subsystems. The proposed controller was robust in tracking the reference values and the tracking errors tend to zero with the sliding variable converging to their sliding surfaces.

An excellent survey of a wide variety of control techniques for autonomous quadrotors has been carried out by [Kim et al., 2019]. In this chapter, a brief review of some of the early multirotor control literature

will be carried out. The control of the position and attitude i.e. roll, pitch and yaw angle of the quadrotors is very sensitive to external disturbances such as wind gust but not many research studies have been carried out with regards to gust rejection. Ding and Wang [2018] proposed a linear active disturbance rejection control (LADRC) taking wind disturbance into account and used a linear extended state observer for wind gust rejection. Whidborne and Cooke [2017] used a conceptual planar birotor aircraft to investigate the impact of rotor tilt angle on gust rejection. They carried out a detailed analysis using a planar birotor aircraft including the rotor aerodynamics and studied the stability properties of the birotor during wind gust. As a result of their investigation, it was found that an outward rotor tilt angle improves the performance of the multirotor aircraft when considering wind gust into account.

The focus of this research work is to extend the work done by Whidborne and Cooke [2017]; Whidborne et al. [2022] and analyse the effect of rotor tilt angle on gust rejection using sliding mode control of multirotor aircraft. Extensive simulation studies will be carried out in MATLAB/Simulink to investigate whether the rotor tilt angle has any impact on gust rejection using SMC in a multirotor aircraft.

2.2 Sliding Mode Control

As the name sliding mode control suggests the structure of the control is altered along the switching surfaces in state space by utilising a high-speed control law that drives the nonlinear system's state trajectory onto the sliding surface subsequently so that certain desirable properties are attained [Itkis, 1976]. Once the system state reaches the switching plane, the system response thereafter depends only on the gradients of the switching planes and remains insensitive to disturbances and parametric variations [Dorling and Zinober, 1986].

SMC was proposed by Soviet researchers in the early 1950s but the work in English on the properties of SMC appeared in the late 1970s [Itkis, 1976; Utkin, 1977]. Initially, a linear second-order system, represented in a canonical form, was considered by [Itkis, 1976; Utkin, 1977] and SMC is still being used in the control of different system types that includes MIMO systems, nonlinear systems and stochastic systems etc. A comprehensive survey on SMC is presented by Hung et al. [1993].

To illustrate the basic philosophy of the SMC approach, consider a second-order piece-wise linear system considered by Hung et al. [1993] and as shown in Figure 2.1.

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= 2y - x + u \\ u &= -kx\end{aligned}\tag{2.2.1}$$

and k is defined by:

$$k = \begin{cases} -4, & s(x, y) < 0 \\ 4, & s(x, y) > 0 \end{cases}\tag{2.2.2}$$

Also, the switching function, $s(x, y)$ is taken as:

$$s(x, y) = x\sigma, \sigma = 0.5x + y.\tag{2.2.3}$$

$s(x, y) = 0$ is called the switching surface. As seen in Figure 2.1, the gain k switches in accordance with the sign of $s(x, y)$ and the system performance spans into different regions of the phase plane and is defined by two mathematical models as shown in Figure 2.2;

When $s(x, y) = x\sigma > 0$, i.e. in region I, the system model equations are defined as:

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= 2y - x + -4x = 2y - 5x\end{aligned}$$

When $s(x, y) = x\sigma < 0$, i.e. in region II, the system model equations are defined as:

$$\dot{x} = y$$

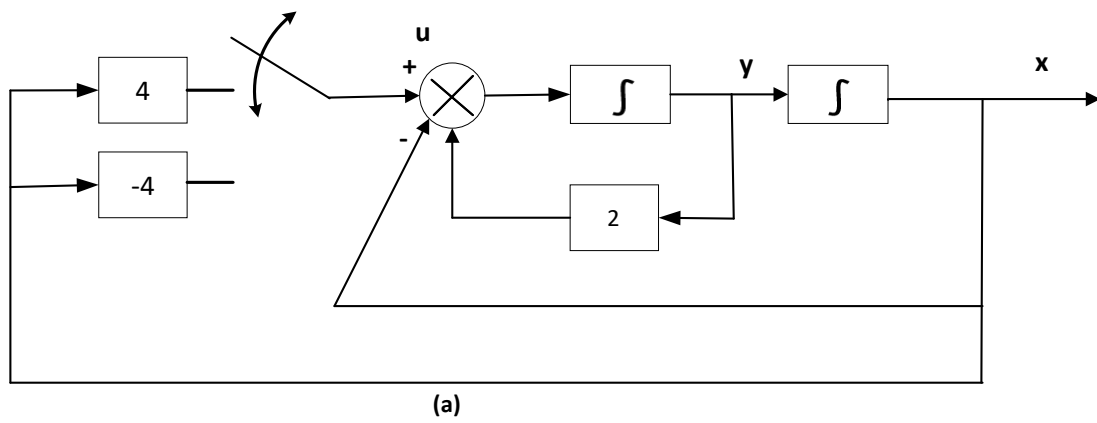
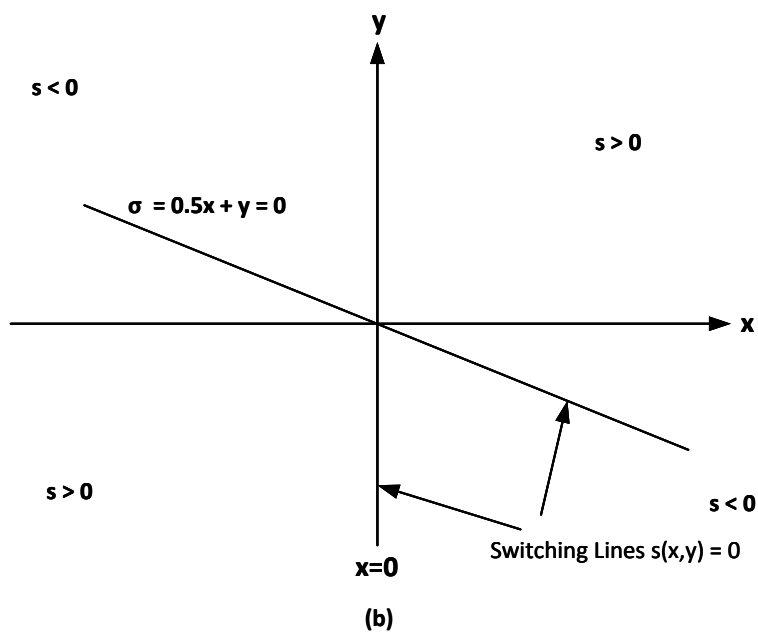


Figure 2.1: An example of SMC. a) Model of the system b) Switching regions

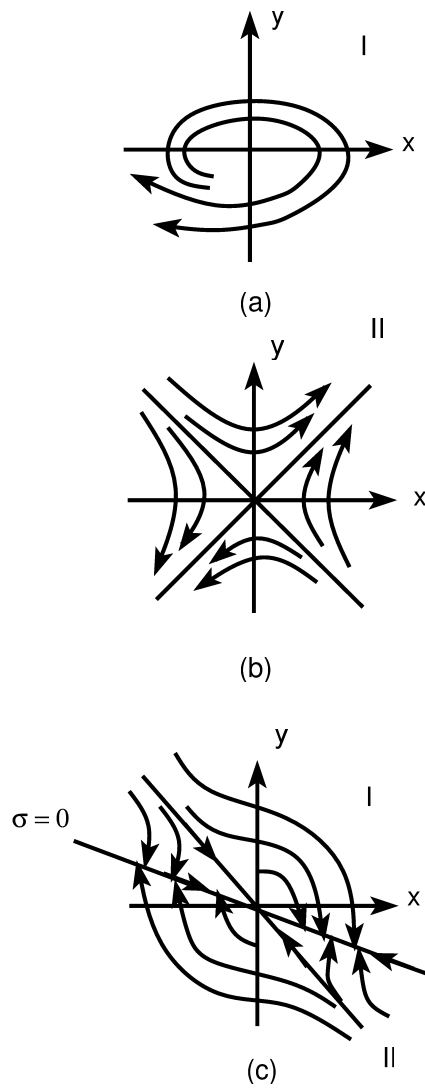


Figure 2.2: (a) System I phase portrait (b) System II phase portrait (c) Phase portrait of the overall system

$$\dot{y} = 2y - x + 4x = 2y + 3x$$

From Figure 2.2, it can be seen from the phase plots of the two system models, neither of them are asymptotically stable but when the two systems are combined, the phase plot of the combined system is asymptotically stable except the discontinuities on the direction of motion of the $x = 0$. This motion on the switching surface is called the sliding mode. The first part of the phase trajectory is known as the reaching mode, wherein the phase trajectory moves from any position on the phase plane towards the switching line and the second part where the trajectory towards to the origin is called the sliding mode. Figure 2.3 illustrates the switching manifold for this example and Figure 2.4 is the associated quiver plot.

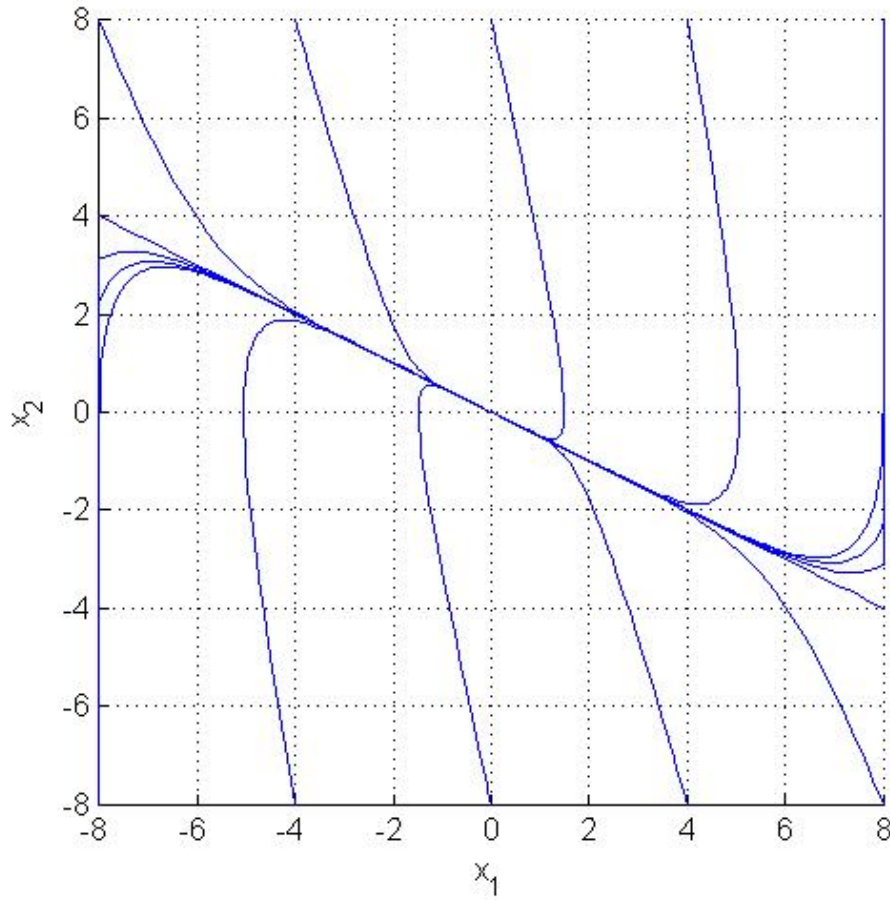


Figure 2.3: Sliding manifold for piece-wise linear example

2.2.1 Design of SMC

The design of SMC systems consists of two independent stages. The first stage is the construction of the switching surface so that the original system restricted to the surface responds in a desired manner. There are various design procedures for constructing these surfaces [Hung et al., 1993].

In the second stage, a SMC system is designed to drive the state of the system from any initial state in state space to reach the switching surface in finite time. This process is called the reaching mode.

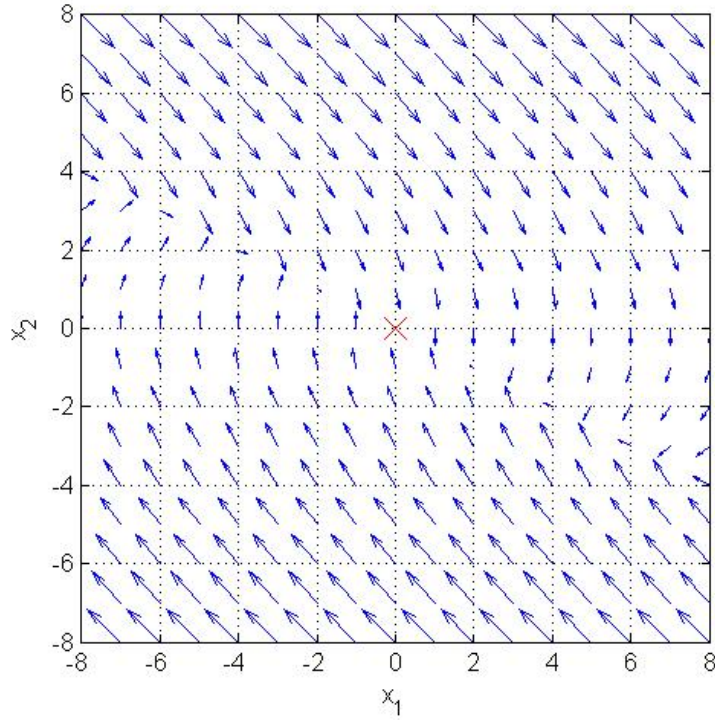


Figure 2.4: Quiver plot for piece-wise linear example

2.2.1.1 Reaching Conditions and the Reaching Mode

Different approaches can be used for specifying the reaching condition, i.e. when the state reaches the sliding surface, in the design of SMC systems. They may be designed by a Direct Switching Function (DSF) approach or by a Lyapunov function approach or by a reaching law approach. The DSF design approach uses the following reaching condition:

$$\lim_{s(x) \rightarrow 0^+} \dot{s}(x) < 0 \quad \text{and} \quad \lim_{s(x) \rightarrow 0^-} \dot{s}(x) > 0$$

The Lyapunov function design approach uses the following reaching condition:

$$V(x, t) = s^T(x)s(x) > 0 \quad \text{where} \quad \dot{V}(x, t) < 0$$

The reaching law approach uses the following law:

$$\dot{s}(x) = -ks(x) - \lambda_1 \operatorname{sgn}(s(x)) \quad (2.2.4)$$

where $\operatorname{sgn}(\cdot)$ is the signum function and is defined by:

$$\operatorname{sgn}(s(x)) = \begin{cases} -1 & \text{if } s(x) < 0, \\ 0 & \text{if } s(x) = 0, \\ 1 & \text{if } s(x) > 0 \end{cases} \quad (2.2.5)$$

The system characteristics during the reaching phase are specified as well when using the reaching law approach in addition to the establishment of the reaching condition.

2.2.1.2 The Control Law

The design of the control law pertaining to SMC is dependent on the choice of the switching surface and whether the control law structure has been already specified or not. The aim is to satisfy a reaching

condition where the control law structure is prespecified or not. For a free control structure, the reaching law approach (2.2.4) can be used where k and λ_1 are positive gains. The term, $-ks(x)$, forces the system state to reach the switching surface faster when $|s(x)|$ is large. The term, $-\lambda_1 \text{sgn}(s(x))$, is used to counteract the parametric variations and disturbances in the system.

2.2.2 Chattering

When using SMC, it is assumed that the controller can switch from one value to another with an infinite speed, but it is not possible because of the finite time delays in the computation effort and the limitations of the actuators. As a result, chattering usually occurs when using SMC and appears as a high-frequency oscillatory signal during steady state but may excite the unmodeled high frequency system dynamics. To reduce the chattering, many methods have been proposed in the research literature such as higher order sliding mode control [Bartolomeu et al., 2008], boundary layer method [Baik et al., 2000], using hyperbolic functions [Asad et al., 2017; Leśniewski and Bartoszewicz, 2015] super-twisting sliding mode control method [Salgado et al., 2011]. However, the chattering aspect in multirotor aircraft is not a big concern as the rotor speed controllers use MOSFET switches along with Pulse Width Modulation (PWM) with frequencies above the bandwidth of the motors and in practice, the chatter will be at the PWM switching frequency.

2.2.3 SMC Features

SMC is a robust controller that can be invariant to system parametric variations and external disturbances as long as they enter the channel of the control input which is defined as a matched uncertainty. In SMC, two or more control structures that may be unstable on their own, can be combined together to provide an asymptotically stable system. In the first phase of the sliding mode design, a switching surface is created and in the second phase, the desired trajectory on the sliding mode is provided by pole placement technique. In sliding mode control, the system in sliding mode is of a lower order than the original system and results in a high-speed response without compromising the stability of the system.

2.3 Example of Sliding Mode Control of Nonlinear Systems

Consider the following system [Slotine and Li, 1991]

$$\ddot{x} = -a(t)\dot{x}^2 \cos(3x) + u \quad (2.3.1)$$

where $a(t)$ is $1 \leq a(t) \leq 2$. A best estimate of the above system, $\hat{f}$, is given by:

$$\hat{f} = -1.5\dot{x}^2 \cos(3x) \quad (2.3.2)$$

Let the switching surface be defined as:

$$s = \dot{x} - \dot{x}_d + \lambda(x - x_d) = \dot{\tilde{x}} + \lambda\tilde{x} \quad (2.3.3)$$

Then,

$$\dot{s} = \ddot{x} - \ddot{x}_d + \lambda\dot{\tilde{x}} = f + u - \ddot{x}_d + \lambda\dot{\tilde{x}} \quad (2.3.4)$$

The control law approximation so that $\dot{s} = 0$, is given by:

$$\hat{u} = -\hat{f} + \ddot{x}_d - \lambda\dot{\tilde{x}} \quad (2.3.5)$$

The sliding condition is given by:

$$s\dot{s} = \frac{1}{2} \frac{d}{dt} s^2 \leq -\eta|s| \quad (2.3.6)$$

A discontinuous term is added to $\hat{u}$ to satisfy the above condition i.e.

$$u = \hat{u} - k \text{sgn}(s) = -\hat{f} + \ddot{x}_d - \lambda\dot{\tilde{x}} - k \text{sgn}(s) \quad (2.3.7)$$

As $s\dot{s} \leq -\eta|s|$, we have

$$(\ddot{x} - \ddot{x}_d + \lambda\dot{\tilde{x}})s \leq -\eta|s| \quad (2.3.8)$$

Substituting $\ddot{x} = f + u$, we get:

$$(f + u - \ddot{x}_d + \lambda\dot{\tilde{x}})s \leq -\eta|s| \quad (2.3.9)$$

Substituting (2.3.7) in (2.3.9):

$$[f - \hat{f} - k \operatorname{sgn}(s)]s \leq -\eta|s| \quad (2.3.10)$$

Assuming η is positive and as $\operatorname{sgn}(s)s = |s|$, for k to be positive, k is represented as:

$$k \geq |f - \hat{f}| + \eta \quad (2.3.11)$$

Choosing the largest bound for f ,

$$k \geq 0.5\dot{x}^2 |\cos(3x)| + \eta \quad (2.3.12)$$

The control law is given by:

$$u = -\hat{f} + \ddot{x}_d - \lambda\dot{\tilde{x}} - k \operatorname{sgn}(s) \quad (2.3.13)$$

Figure 2.5 illustrates the switching manifold for this example and Figure 2.6 is the associated quiver plot.

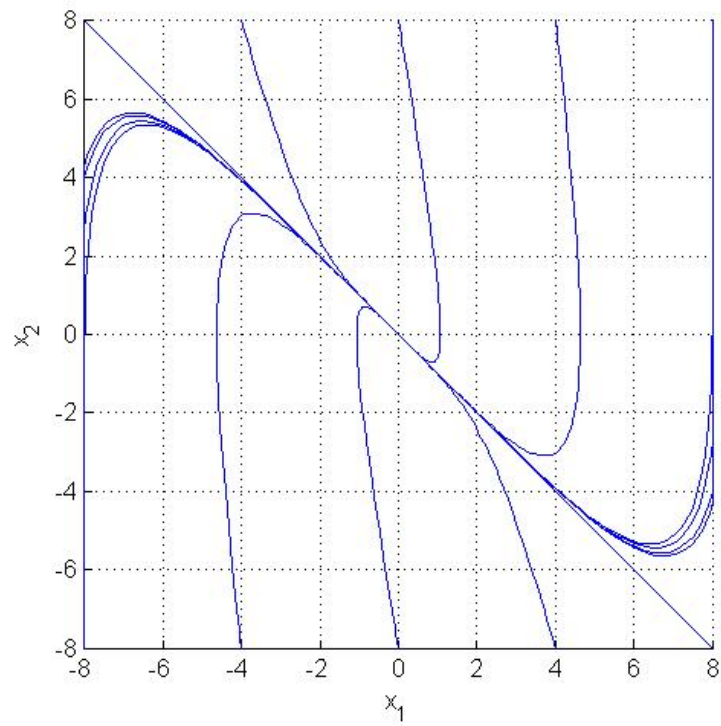


Figure 2.5: Sliding manifold for the nonlinear example

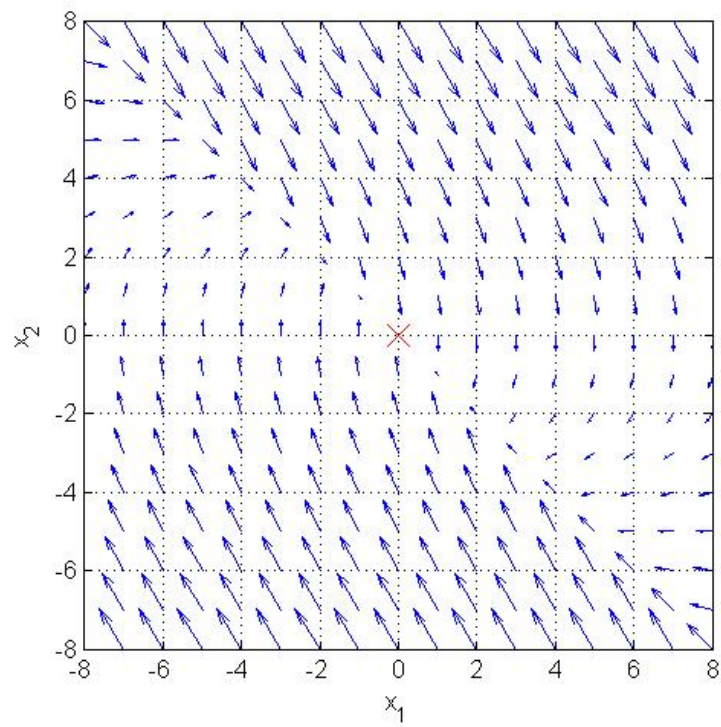


Figure 2.6: Quiver plot for the nonlinear example

2.4 Chapter Summary and Conclusions

In this chapter, the literature review associated with multicopter aircraft, their applications and different control strategies proposed for quadrotors has been described. Also, a brief introduction to the different control strategies used in the control of quadrotor i.e. Linear control, Nonlinear control and Intelligent control, have been explained with special emphasis on sliding mode control with examples. This chapter provides a good background to the design of a PID/PD controller and sliding mode controller for a quadrotor as described in the subsequent chapters.

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CHAPTER 3

Dynamic Modelling and Simulation of a Quadrotor

In the previous chapter, a detailed literature review relating to multirotor aircraft and their applications along with different control strategies proposed by various researchers has been undertaken. This literature review lays a good research background for dynamic modelling of a quadrotor and simulating with different control strategies. In this chapter, a dynamic model of the quadrotor is developed in the body frame and simulated with a PIV controller (Method #1). Subsequently, a sliding mode controller has been designed and simulation work has been carried out with the same quadrotor dynamic model taking into consideration the parametric uncertainties and effect of external disturbances. It is found that the SMC performs well providing robustness to parametric uncertainties and good disturbance rejection, when compared to PIV controller.

3.1 Quadrotor Model in Body Axes

The choice of modelling in inertial axes or body axes depends on the application. If it is an outdoor application using GPS, then body axes rates are measured on-board, so body axes modelling is preferred. But if the application is indoors, for eg., indoor flying lab, the position and attitude rates are measured by ground-fixed cameras, the inertial axes modelling is preferred. And if the application requires high manoeuvrability, then quaternion modelling is suitable. This section covers the modelling of quadrotor in body frame.

A quadrotor is affected by independently varying speed of the four rotors as shown in Figure 3.1. Body-fixed frame has x,y and z axes originating at the center of mass of the vehicle. There are four control inputs i.e. 3 moments and 1 force as shown in Equations (3.1.1), (3.1.2) and (3.1.3) and (3.1.4):

$$L = l(T_4 - T_2), \quad (3.1.1)$$

$$M = l(T_1 - T_3), \quad (3.1.2)$$

$$N = -\tau_1 + \tau_2 - \tau_3 + \tau_4, \quad (3.1.3)$$

$$T = -T_1 - T_2 - T_3 - T_4, \quad (3.1.4)$$

where l is the arm length T_i is the thrust from i^{th} rotor τ_i is the torque from i^{th} rotor. The following assumptions have been made in the modelling of the quadrotor in body axes:

- the structure is rigid and symmetric,
- the propellers are rigid,
- ground effect is neglected,

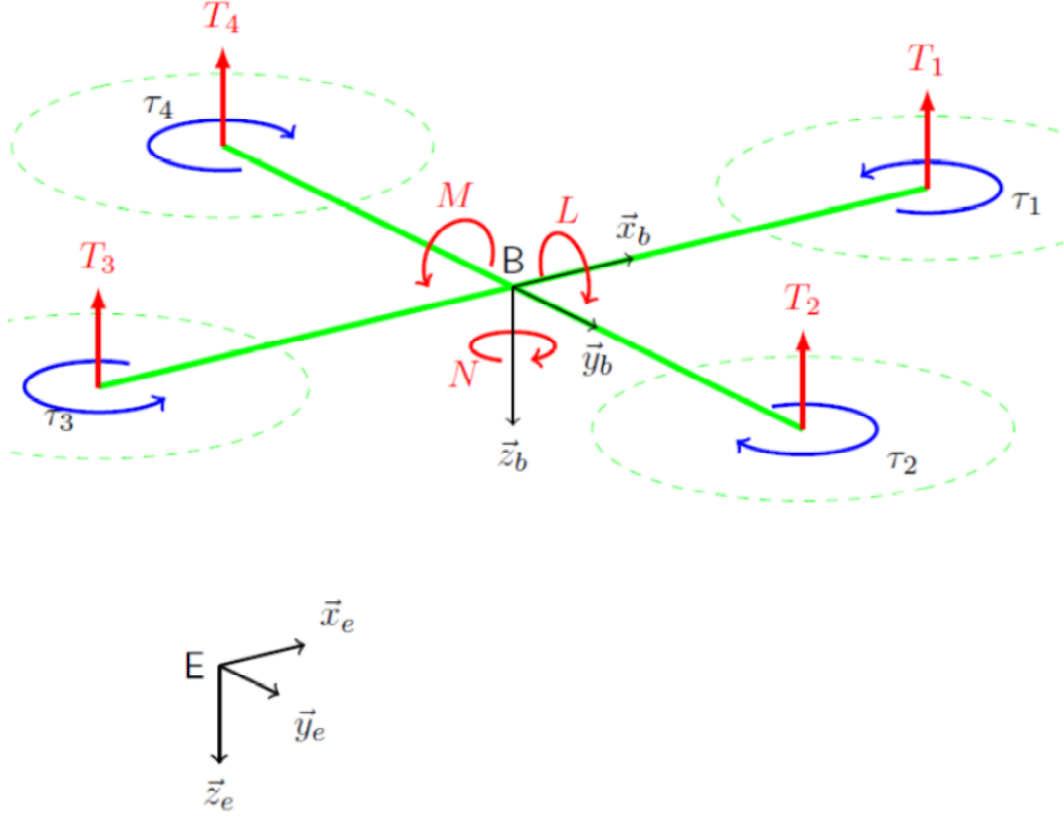


Figure 3.1: Quadrotor vehicle schematic (body axes)

- body aerodynamic forces ignored
- rotor Coriolis forces ignored,
- the wind and drag forces are not included,
- and the motor dynamics are ignored.

The convention in aeronautical engineering is that the axes are represented by considering the aircraft moves in the positive x-direction and the right side pertaining to the positive y-direction and the positive z-direction is the vertical underside. The three angles, i.e. ϕ , θ and ψ are the roll, pitch and yaw respectively. The angles from the body axes to the inertial earth frame can be related by the rotation matrix $\mathbf{R}$, presented in various works [Bouabdallah et al., 2004b,a; Bouabdallah, 2007; Erginer and Altug, 2007; Fang et al., 2010; Zhang et al., 2009; Bouktir et al., 2008; Mian and Daobo, 2008; Sharifi et al., 2010; Li and Li, 2011; Seah et al., 2017; Xu et al., 2016]. The rotation matrix, $\mathbf{R}$, is used to transform the movement of the vehicle from body frame to earth frame. It is given by Cook [2013]:

$$\mathbf{R}(\phi, \theta, \psi) = \begin{bmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \cos \phi \cos \psi + \sin \phi \sin \theta \sin \psi & \cos \theta \sin \phi \\ \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi & \cos \theta \cos \phi \end{bmatrix}$$

The Gravitation Force Vector $\mathbf{F}_g$ in the body axes is given by:

$$\mathbf{F}_g = m\mathbf{R} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad (3.1.6)$$

Simplifying (3.1.6) gives:

$$\mathbf{F}_g = mg \begin{bmatrix} -\sin \theta \\ \cos \theta \sin \phi \\ \cos \theta \cos \phi \end{bmatrix} \quad (3.1.7)$$

where m is the mass of the vehicle and g is the acceleration due to gravity.

The Newton-Euler equations of motion body axes frame are given by [Seah et al., 2017]:

$$\mathbf{F} = m\dot{\mathbf{V}} + \omega \times m\mathbf{V} \quad (3.1.8)$$

$$\mathbf{M} = \mathbf{I}\dot{\omega} + \omega \times \mathbf{I}\omega \quad (3.1.9)$$

where $\mathbf{V} = [u, v, w]^T$ and $\omega = [p, q, r]^T$ are the vectors of velocities and angular rates respectively. $\mathbf{I} = \text{diag}(I_x, I_y, I_z)$ is the moments of inertia matrix, $\mathbf{F} = \mathbf{F}_g + [0, 0, -T]^T$ and $\mathbf{M} = [L, M, N]^T$ are the vector of forces and moments acting on the centre of mass respectively,

Expanding (3.1.8) gives:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} 0 \\ 0 \\ -T \end{bmatrix} + g \begin{bmatrix} -\sin \theta \\ \cos \theta \sin \phi \\ \cos \theta \cos \phi \end{bmatrix} - \begin{bmatrix} qw - rv \\ ru - pw \\ pv - qu \end{bmatrix} \quad (3.1.10)$$

Similarly, expanding (3.1.9) gives:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L/I_x \\ M/I_y \\ N/I_z \end{bmatrix} - \begin{bmatrix} qr(I_z - I_y)/I_x \\ rp(I_x - I_z)/I_y \\ pq(I_y - I_x)/I_z \end{bmatrix} \quad (3.1.11)$$

The position of the vehicle in the inertial earth frame is given by:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \mathbf{R}^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \cos \psi \cos \phi & \cos \psi \sin \theta \sin \phi & \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \\ \sin \psi \cos \theta & \sin \psi \sin \theta \sin \phi & \sin \psi \sin \theta \cos \phi - \sin \phi \cos \psi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (3.1.12)$$

The body angle rates are given by:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \phi & \sin \phi \cos \theta \\ 0 & -\sin \phi & \cos \theta \cos \phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (3.1.13)$$

Equation (3.1.13) can be re-written as:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan \theta \sin \phi & \tan \theta \cos \phi \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (3.1.14)$$

By differentiating (3.1.12) and neglecting $\dot{R}$ terms, it can be shown that:

$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = \begin{bmatrix} \cos \psi \cos \phi & \cos \psi \sin \theta \sin \phi & \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \\ \sin \psi \cos \theta & \sin \psi \sin \theta \sin \phi & \sin \psi \sin \theta \cos \phi - \sin \phi \cos \psi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} \quad (3.1.15)$$

By substituting (3.1.10) in (3.1.15) and based on the assumption that the rotor coriolis forces are negligible, (3.1.15) can be re-written as:

$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = \begin{bmatrix} -T/m(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ -T/m(\cos \phi \sin \theta \sin \psi + \sin \phi \cos \psi) \\ -T/m(\cos \phi \cos \theta) + g \end{bmatrix} \quad (3.1.16)$$

In (3.1.16), X and Y axes are not considered for attitude control but for altitude control, z -axis is needed as the inputs are coupled to obtain the voltage for each rotor.

Table 3.1: Vehicle parameters and variables for a quadrotor

Symbol	Variable Definition	Value	Unit
l	arm length	0.25	m
m	mass	1.56	kg
I_x	inertia on x axis	0.04	kg.m ²
I_y	inertia on y axis	0.04	kg.m ²
I_z	inertia on z axis	0.08	kg.m ²
b	thrust coefficient	20×10^{-6}	N.s ²
d	drag coefficient	0.4×10^{-6}	N.m.s ²
J_r	rotor inertia	90×10^{-6}	kg.m ²
ϕ	roll angle		rad
θ	pitch angle		rad
ψ	yaw angle		rad
ω_i	speed of rotor number i		rad.s ⁻¹

The equations with regards to modelling of the quadrotor in body axes are described by the following equations:

$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta, \quad (3.1.17)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi, \quad (3.1.18)$$

$$\dot{\psi} = q \sin \phi / \cos \theta + r \cos \phi / \cos \theta, \quad (3.1.19)$$

$$\dot{p} = \frac{I_y - I_z}{I_x} qr + \frac{L}{I_x}, \quad (3.1.20)$$

$$\dot{q} = \frac{I_z - I_x}{I_y} rp + \frac{M}{I_y}, \quad (3.1.21)$$

$$\dot{r} = \frac{I_x - I_y}{I_z} pq + \frac{N}{I_z}, \quad (3.1.22)$$

$$\dot{u} = -g \sin \theta - qw + rv, \quad (3.1.23)$$

$$\dot{v} = g \cos \theta \sin \phi - ru + pw, \quad (3.1.24)$$

$$\dot{w} = -\frac{T}{m} + g \cos \theta \cos \phi - pv + qu, \quad (3.1.25)$$

$$\dot{x} = u \cos \theta \cos \psi + v(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) + w(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi), \quad (3.1.26)$$

$$\dot{y} = u \cos \theta \sin \psi + v(\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) + w(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi), \quad (3.1.27)$$

$$\dot{z} = -u \sin \theta + v \sin \phi \cos \theta + w \cos \phi \cos \theta, \quad (3.1.28)$$

All the above equations have been modelled in MATLAB/Simulink and initially a PIV control has been designed and simulated with this quadrotor model. Subsequently, a SMC controller has been designed and simulated with the quadrotor model and a comparison of the two simulation results have been discussed.

The parameters and variables for the quadrotor are as listed in Table 3.1 and are taken from Adir et al. [2011]. There are four control inputs U_1 , U_2 , U_3 and U_4 and six controlled outputs, X , Y , Z , ϕ , θ for this system and ψ . U_1 is used to control the roll angle, U_2 controls the pitch angle, U_3 controls the yaw angle and U_4 determines the altitude of the vehicle. The values of U_i are determined from the rotor speed ω_i with an assumption of known thrust and drag coefficients b and d . The motor dynamics are ignored, and it is assumed that the rotor speeds can be directly manipulated.

3.2 Simulation Results of the PIV Control of Quadrotor

The quadrotor modelled in body axes has been simulated with a PIV Controller. In velocity feedback controller the output rates are available as a measurement and are fed back so that the problem of high-frequency noise amplification that occurs with derivative action is eliminated. For details of the PIV controller implementation in Simulink, refer to Appendix C.1.

The controller gains used for the PIV Controller simulation are give below:

$$K_{px} = -2.3415; K_{py} = 2.3415; K_{pz} = -2.8324;$$

$$K_{ix} = -1; K_{iy} = 1; K_{iz} = -1;$$

$$K_{pu} = -2.2414; K_{pv} = 2.2414; K_{pw} = -3.5114;$$

$$K_{pp} = 2.1780; K_{pq} = 2.1780; K_{pr} = 1.6280;$$

$$K_{p\phi} = 9.3589; K_{p\theta} = 9.3589; K_{p\psi} = 2.0630; K_{i\psi} = 1;$$

Simulations of the dynamic model equations from (3.1.17) to (3.1.28) with both the PIV controller and SMC controller were performed using MATLAB/Simulink. The first set of simulation results show the step responses of the PIV controller for the nominal vehicle system for an initial step perturbation and a demand in the altitude. The second set of simulation tests the performance of the SMC controller in the absence for parametric uncertainty and external disturbance. Then it is extended to study the performance of SMC controller in the presence of parametric uncertainty and external disturbance. When all the actuators are functional, the quadrotor stabilises itself and the desired altitude is reached.

Simulation results demonstrates that the quadrotor is initially configured with a 15° tilt 3-axes skew and it is desired to have an altitude of 20 m with a PIV controller. It is seen that the vehicle is able to stabilise itself, returns to hover condition approximately after 8 seconds and attains the desired altitude 20 m just after 7 seconds as shown in Figure 3.2 and Figure 3.3. The lateral position responses to a 10 m x-position and y-position step demands are as shown in Figure 3.4. It can be seen that the both the x and y positions are attained after approximately 8 seconds.

Figure 3.5 and Figure 3.6 show the control efforts of the PIV controller. For the moment controls, initially the control effort values are negative to correct for the attitude errors, followed by a positive demand to correct the subsequent lateral displacement errors. The initial control effort for the collective thrust, U_1 , is around 27 Nm. This is why the initial altitude response is faster for the PIV controller.

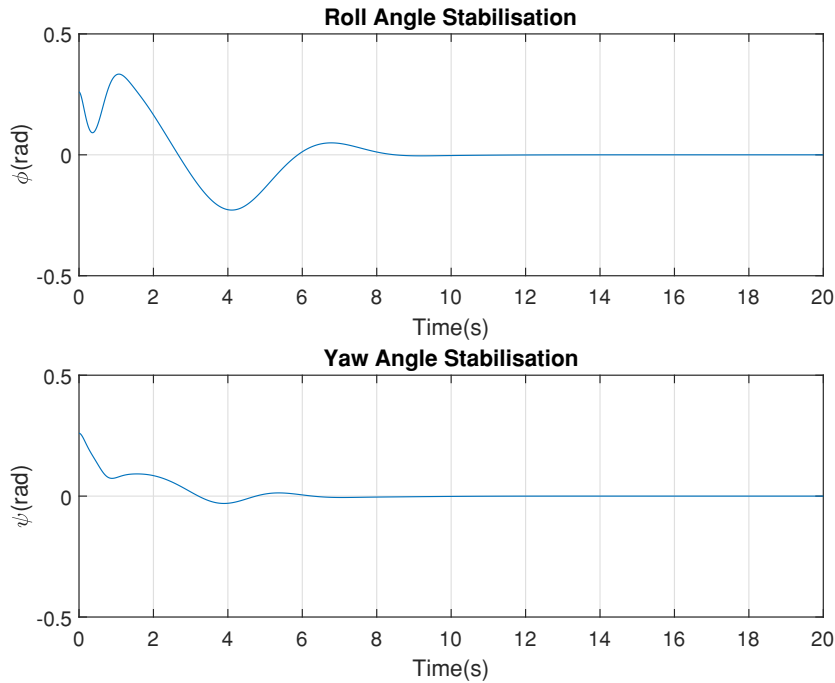


Figure 3.2: Simulation results of quadrotor with PIV control - roll and yaw angle stabilisation

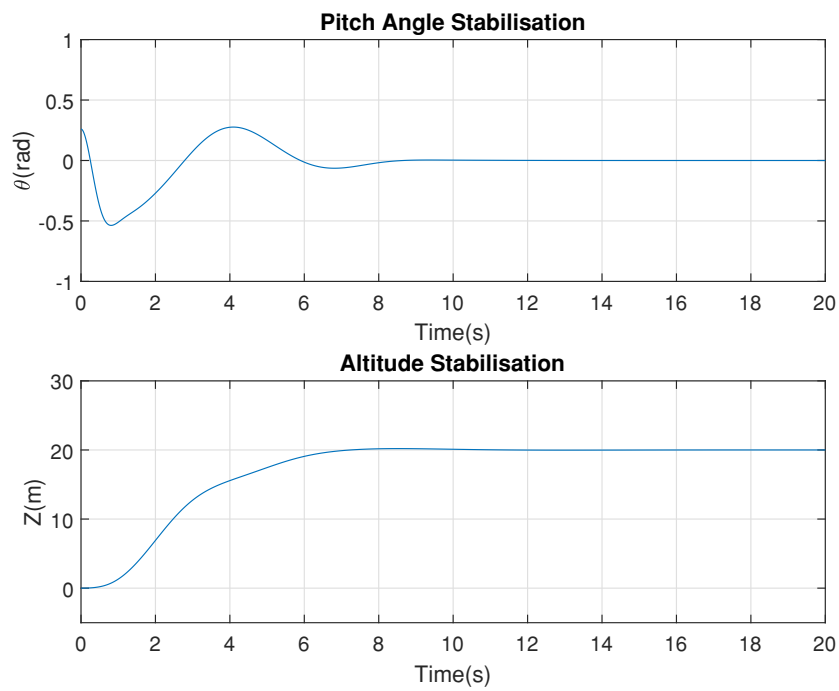


Figure 3.3: Simulation results of quadrotor with PIV control - pitch angle and altitude stabilisation

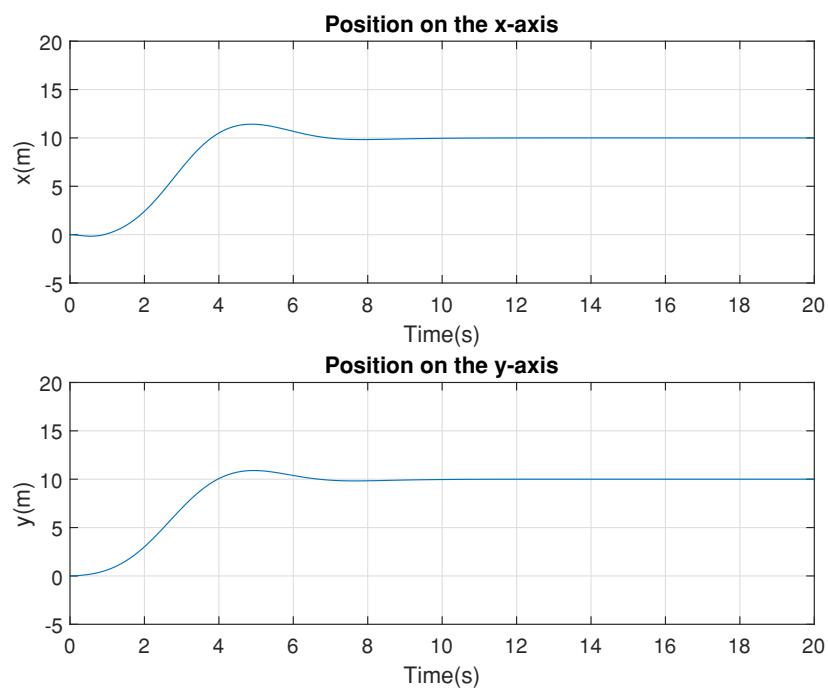


Figure 3.4: Simulation results of quadrotor with PIV control - lateral position response to 10 m demand

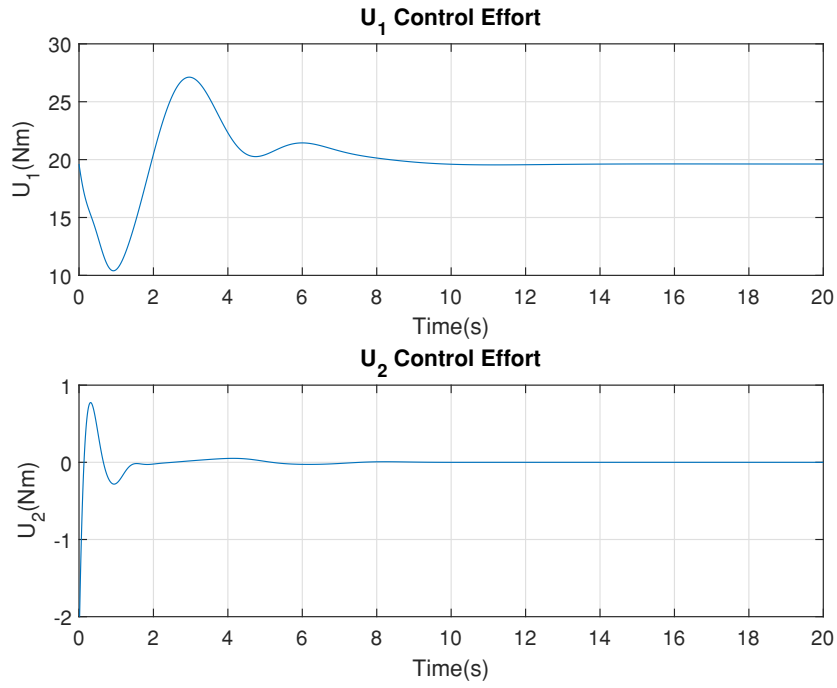


Figure 3.5: Simulation results of quadrotor with PIV control - U_1 and U_2 control efforts

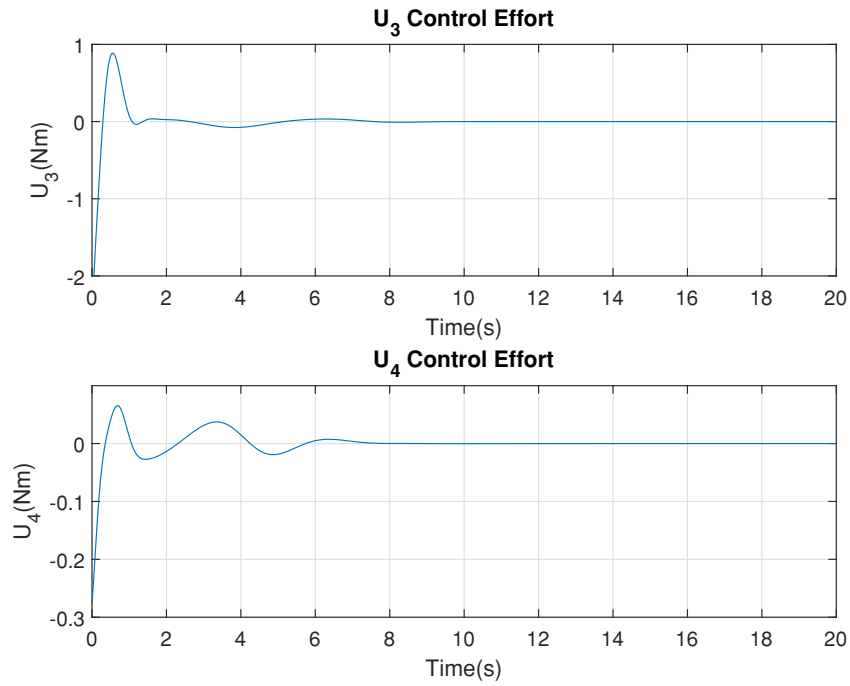


Figure 3.6: Simulation results of quadrotor with PIV control - U_3 and U_4 control efforts

3.3 Simulation Results of the SMC Control of Quadrotor

The quadrotor modelled in body axes has been simulated with a sliding mode controller. There are two steps to be followed in the design of sliding mode control. Firstly, for each output, Z, ϕ, θ, ψ define a time-varying sliding surface, $s(t)$, in the state-space $\mathbb{R}^n$ by the scalar equation $s(\alpha, t) = 0$ where $\alpha = Z, \phi, \theta, \psi$. Secondly, select the feedback control law such that s^2 remains a Lyapunov-like function of the closed-loop system, despite the presence of model imprecision and disturbances and satisfying the necessary sliding condition, $s\dot{s} < 0$ [Slotine and Li, 1991]. The sliding surface equations are

$$s_\alpha = \dot{e}_\alpha + \lambda_\alpha e_\alpha \quad (3.3.1)$$

where $\alpha = \phi, \theta, \psi$ or Z , e_α is the relevant tracking error and λ_α is a constant. The Lyapunov functions can be assumed to be

$$V(s_\alpha) = \frac{1}{2}s_\alpha^2 \quad (3.3.2)$$

By choosing Lyapunov function candidate as $V = \frac{1}{2}s_\alpha^T s_\alpha$, its time derivative is negative definite, that is $\dot{V} = s_\alpha \dot{s}_\alpha < -\eta|s_\alpha|$ and

$$\dot{s}_\alpha = -f - u + \ddot{\alpha}_d + \lambda_\alpha \dot{e}_\alpha \quad (3.3.3)$$

The control inputs for the SMC are given by:

$$U_1 = m[g \cos \theta \cos \phi - pv + qu + \lambda_z \dot{e}_z + k_z \operatorname{sgn}(s_z)] \quad (3.3.4)$$

$$U_2 = I_x[-(\frac{I_y - I_z}{I_x})qr + \lambda_\phi \dot{e}_\phi + k_\phi \operatorname{sgn}(s_\phi)] \quad (3.3.5)$$

$$U_3 = I_y[-(\frac{I_z - I_x}{I_y})pr + \lambda_\theta \dot{e}_\theta + k_\theta \operatorname{sgn}(s_\theta)] \quad (3.3.6)$$

$$U_4 = I_z[-(\frac{I_x - I_y}{I_z})pq + \lambda_\psi \dot{e}_\psi + k_\psi \operatorname{sgn}(s_\psi)] \quad (3.3.7)$$

where λ and k are positive constant values. For details of the SMC controller implementation in Simulink, refer to Appendix C.2

Simulation results demonstrates that the quadrotor is initially configured with a 15° tilt 3-axes skew and it is desired to have an altitude of 20 m with a SMC controller. It is seen that the vehicle is able to stabilise itself, returns to hover condition approximately after 7 seconds and attains the desired altitude 20 m just after 8 seconds as shown in Figure 3.7 and Figure 3.8. The lateral position responses to a 10 m x-position and y-position step demands are as shown in Figure 3.9. It can be seen that the both the x and y positions are attained after approximately 5 seconds.

Figure 3.10 and Figure 3.11 show the control efforts of the SMC controller. For the moment controls, initially the control effort values are negative to correct for the attitude errors, followed by a positive demand to correct the subsequent lateral displacement errors. The initial control effort for the collective thrust, U_1 , is around 25 Nm and later settles down to 22 Nm. It is to be noted that both the attitude and altitude responses using an SMC controller are reasonable when compared to PIV controller but in fact bit quicker with regards to attitude and lateral position response timings.

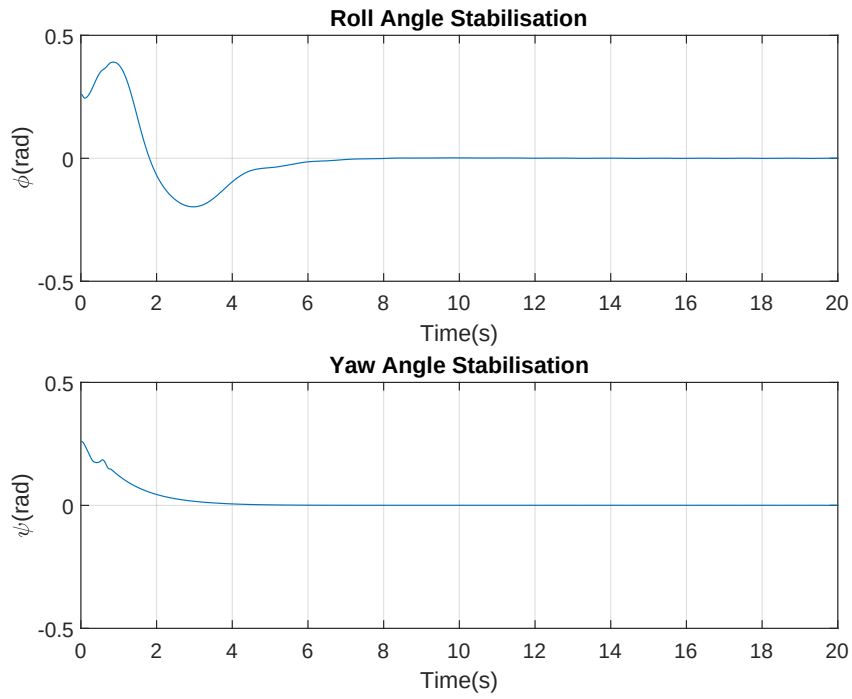


Figure 3.7: Simulation results of quadrotor with SMC control - roll and yaw angle stabilisation

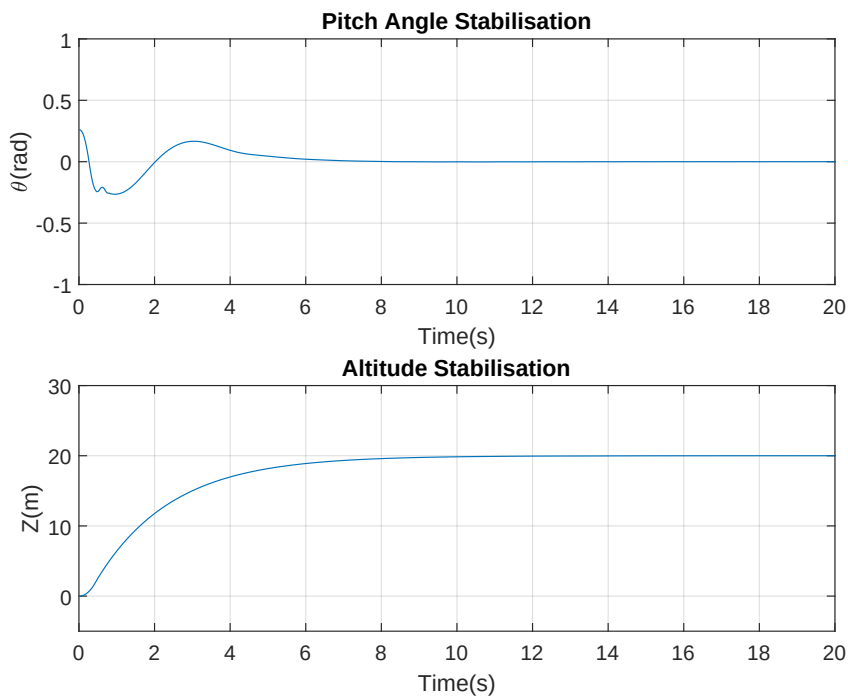


Figure 3.8: Simulation results of quadrotor with SMC control - pitch angle and altitude stabilisation

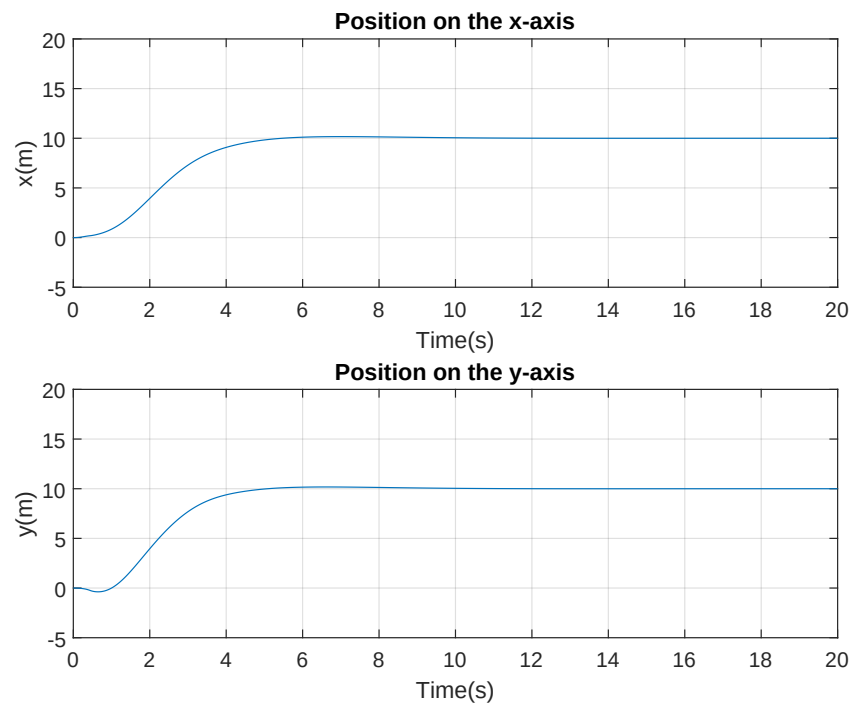


Figure 3.9: Simulation results of quadrotor with SMC control - lateral position response to 20 m demand

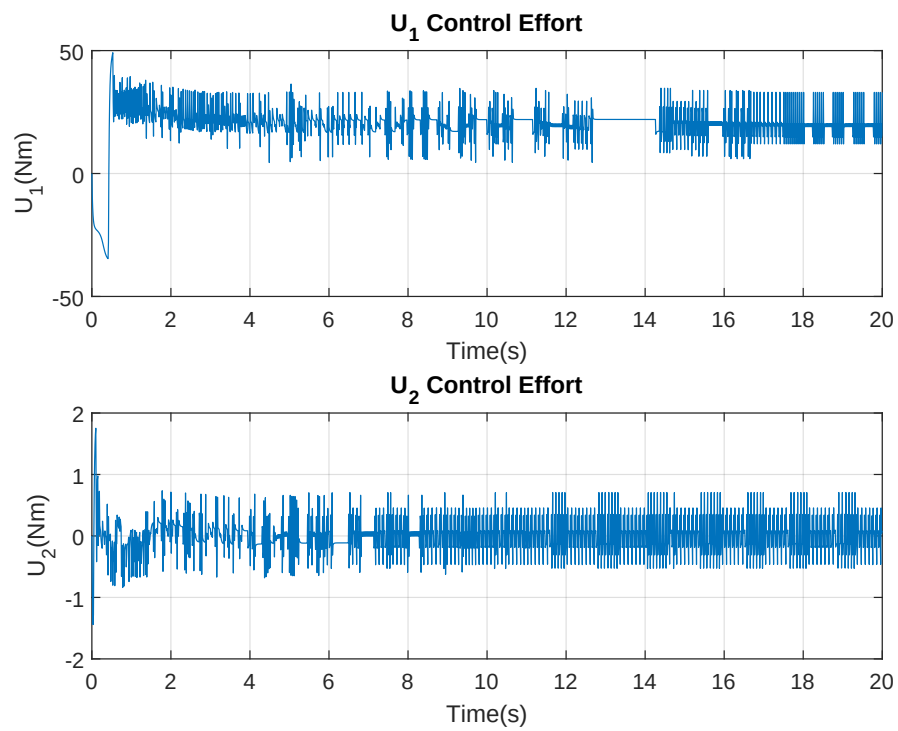


Figure 3.10: Simulation Results of quadrotor with SMC control - U_1 and U_2 control efforts

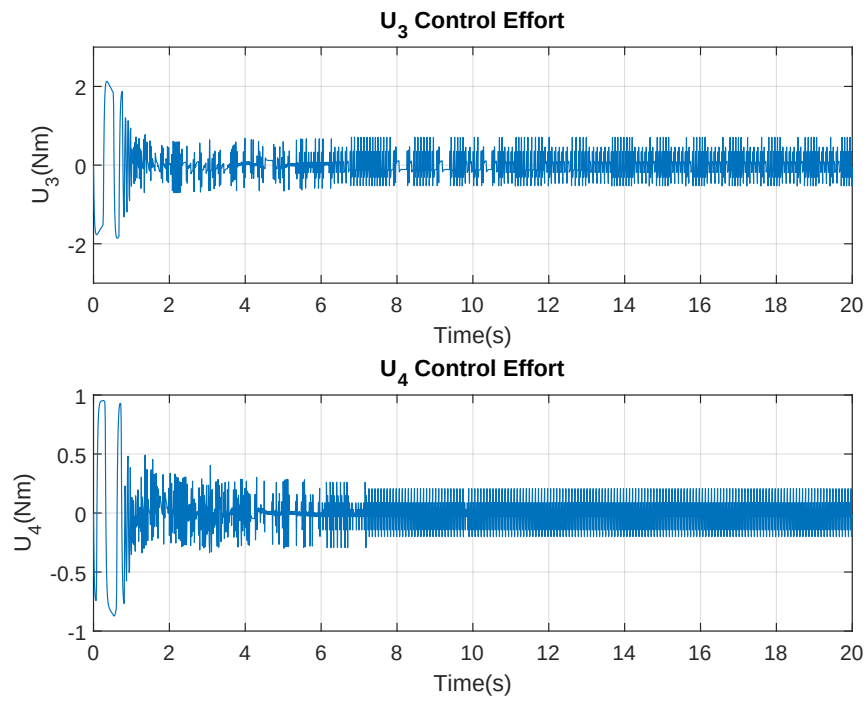


Figure 3.11: Simulation results of quadrotor with SMC control - U_3 and U_4 control efforts

3.4 Simulation Results of the SMC Control of Quadrotor with Parametric Uncertainty and Noise

The quadrotor modelled in body axes has been simulated with a Sliding Mode Controller in the presence of parametric uncertainty and noise. For this simulation test, the model included parametric uncertainties and an external disturbance.

$$\ddot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) + \mathbf{g}(\mathbf{y})\mathbf{u} + \mathbf{d}(t) \quad (3.4.1)$$

where $\mathbf{d}(t) \in \mathbb{R}^n$ is an unknown external disturbance vector. For this simulation the external disturbance, $\mathbf{d}(t)$, is taken as normal gaussian noise:

$$\mathbf{d}(t) = [\mathcal{N}(0, 0.5) \quad \mathcal{N}(0, 0.5) \quad \mathcal{N}(0, 0.5) \quad \mathcal{N}(0, 0.5)]^T \quad (3.4.2)$$

The uncertain constant matrix, $\mathbf{g}(\mathbf{y})$ can be defined as:

$$\mathbf{g}(\mathbf{y}) = \begin{bmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 \\ 0 & 0 & p_3 & 0 \\ 0 & 0 & 0 & p_4 \end{bmatrix} \quad (3.4.3)$$

For the simulations, the uncertain elements of $\mathbf{g}(\mathbf{y})$ are selected as $p_1 = 0.5$, $p_2 = 0.7$, $p_3 = 0.6$ and $p_4 = 0.8$ which correspond to 20 – 50% parametric uncertainty in the inertia, arm length and mass parameters. The initial conditions and altitude demands are the same as defined in section 3.3.

With the uncertainty and external disturbance, the SMC is still able to control the quadrotor for all three axes and quickly stabilize the attitude of the vehicle in 6 seconds, reach the target altitude of 10 metres in 8 seconds and reach the target x and y positions in 5 seconds. Note the greater amount of control effort required, this is because the lower values of the $l/I_{(\cdot)}$ uncertainty terms mean that greater moment forces are required to rotate the vehicle. And a great control effort in U_1 is required for the vehicle to achieve and hold the desired altitude in the presence of parametric uncertainties and noise.

Figure 3.12, Figure 3.13 and Figure 3.14 show the performance of the quadrotor with the SMC controller with parametric uncertainties in inertias and mass and external disturbance in the form of normal Gaussian noise. However, in Figure 3.12 for Roll Angle stabilisation and Figure 3.13 for Pitch stabilisation, ‘hunting’ phenomenon is seen wherein the roll angle and pitch angle don’t stabilise but fluctuates around the final value oscillating between $+/- 0.025$ rads and $+/- 0.05$ rads respectively. This may be due to unmodelled dynamics. Figure 3.15 and Figure 3.16 show the control efforts needed. The noise disturbance is very low in the outputs as seen in the simulation results. However, the controls are sensitive to the noise as is evident in the larger amount of chatter that can be observed. This is of little concern as discussed in section 2.2.2.

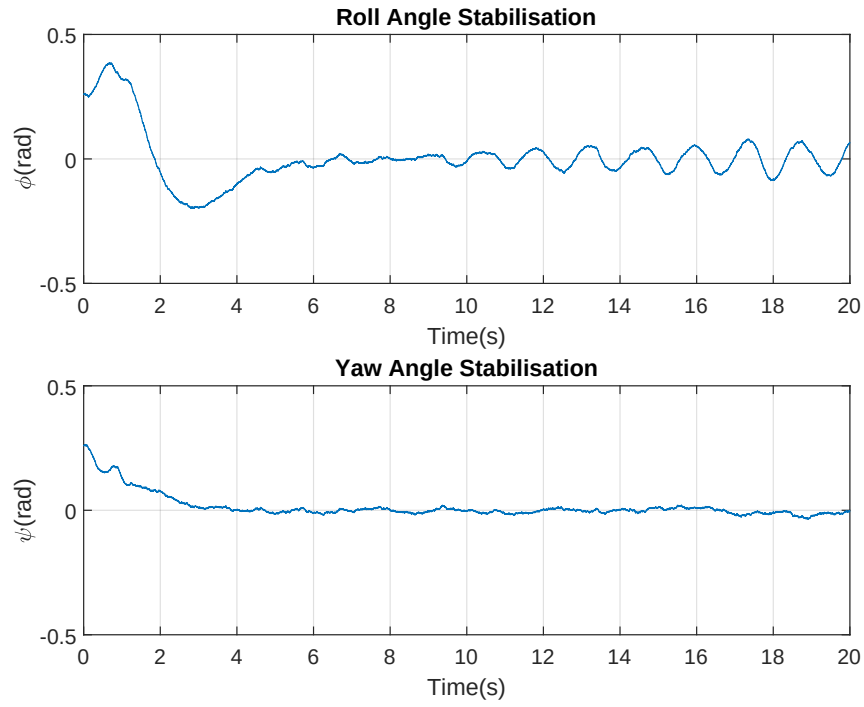


Figure 3.12: Simulation results of quadrotor using SMC control with parametric uncertainty and noise - roll and yaw angle stabilisation

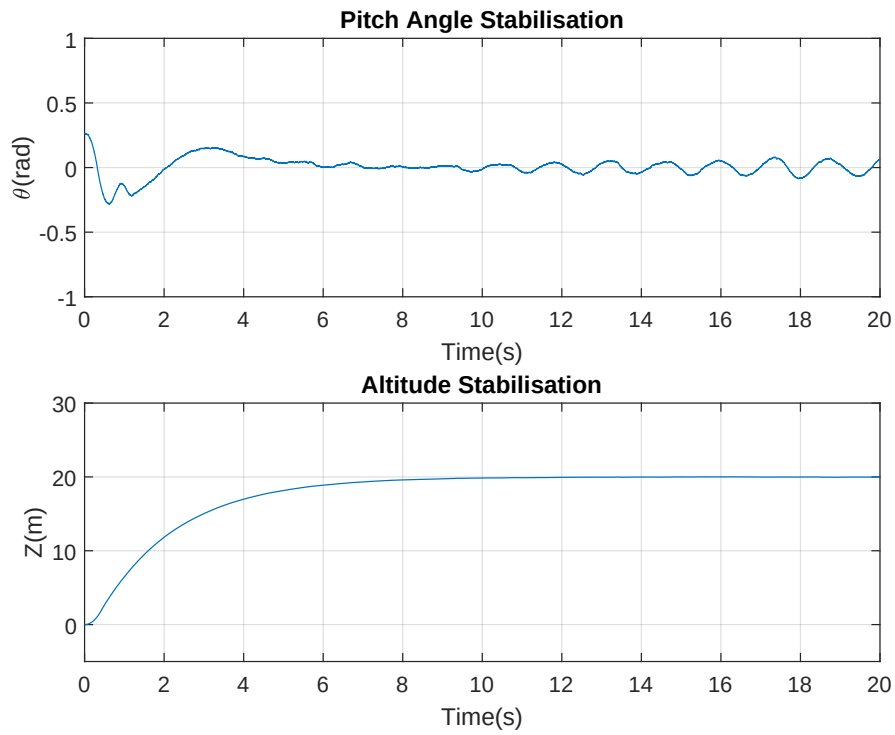


Figure 3.13: Simulation results of quadrotor using SMC control with parametric uncertainty and noise - pitch angle and altitude stabilisation

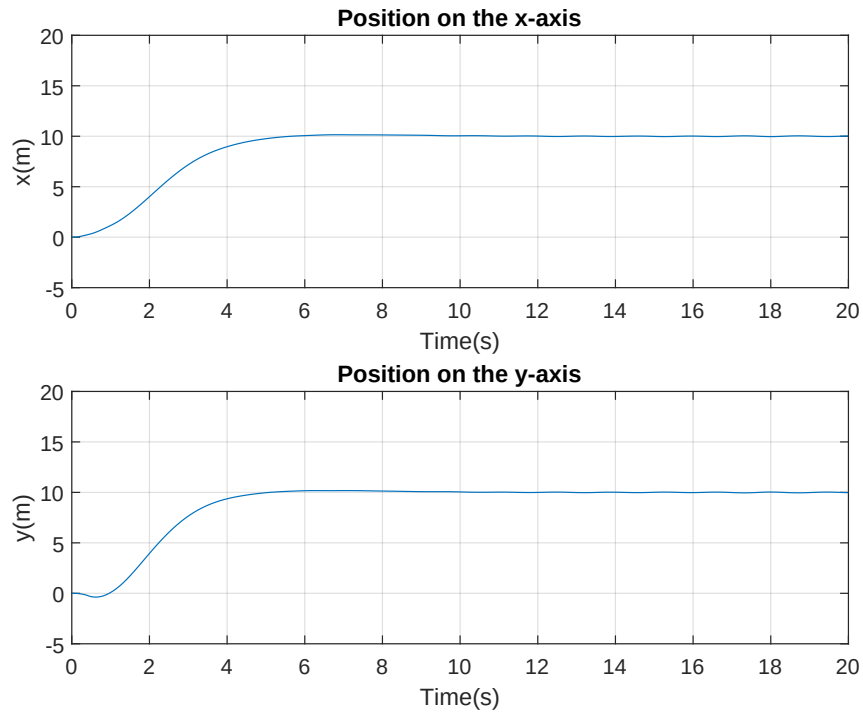


Figure 3.14: Simulation results of quadrotor using SMC control with parametric uncertainty and noise - lateral position response to 20 m demand

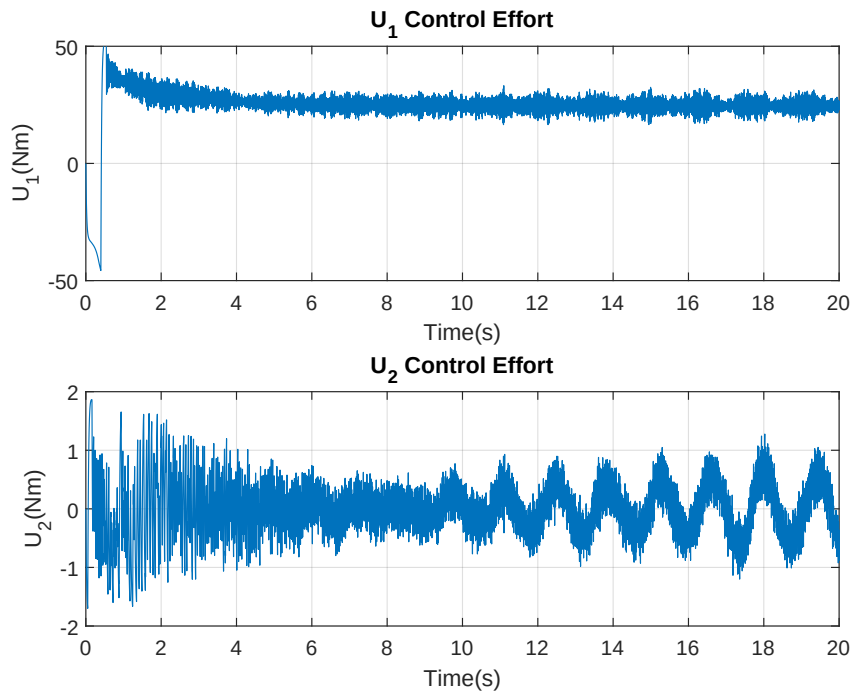


Figure 3.15: Simulation results of quadrotor using SMC control with parametric uncertainty and noise - U_1 and U_2 control efforts

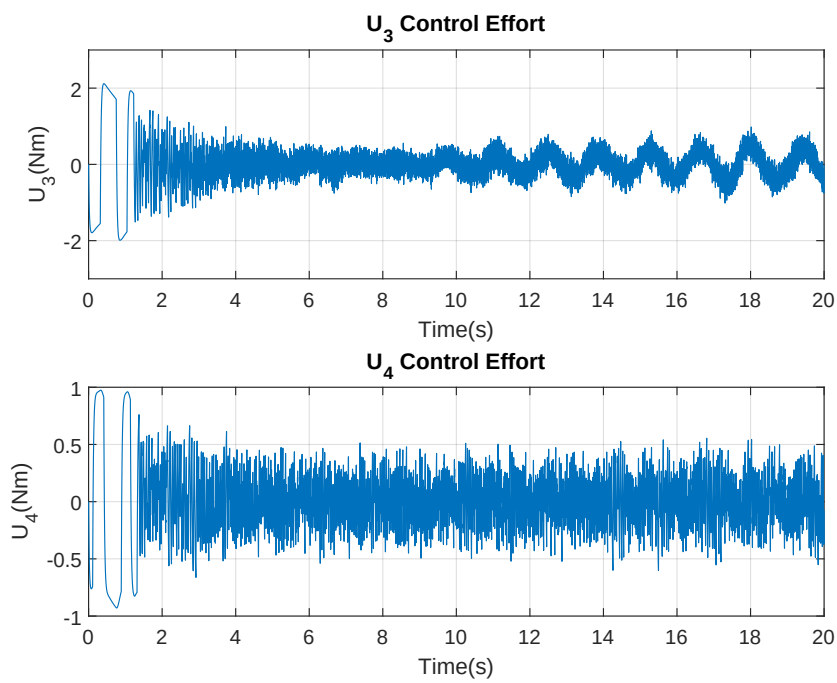


Figure 3.16: Simulation results of quadrotor using SMC control with parametric uncertainty and noise - U_3 and U_4 control efforts

3.5 Chapter Summary and Conclusions

This chapter deals with the modelling and simulation of the quadrotor in body axes. Detailed simulation studies have been carried out for both the PIV and the sliding mode control of quadrotor. Simulation studies with SMC have been carried out taking into consideration parametric uncertainties and external disturbances. The simulation results of the SMC controller demonstrate that the quadrotor is able to stabilise itself at the desired position and altitude. The overall performance of the controllers is seen to be satisfactory with the vehicle returning to a hover condition in a reasonable amount of time. Also, the SMC controller performs well in the presence of parametric uncertainties and external disturbance. This fulfils the objective 1 of this work as mentioned in Chapter 1.

CHAPTER 4

Gust Rejection Properties of a Conceptual Birotor

In chapter 3, a quadrotor was modelled in body axes without considering any aerodynamic effects and simulation studies have been carried out using PIV and SMC control. This lays a good foundation in modelling a quadrotor taking aerodynamics into account and simulating with SMC. In this chapter, a conceptual birotor has been modelled initially without considering the aerodynamics and subsequently developing the conceptual PVTOL model that includes aerodynamic effects into the equations of motion. Stability Analysis for SMC altitude control, pitch control and position control has been carried out. Detailed simulation work has been carried out for the PVTOL without aerodynamics and including aerodynamics. This chapter deals with the evaluation of gust rejection properties of a conceptual PVTOL taking into account the effect of different rotor tilt angles using SMC. It is found that the SMC is almost invariant to different rotor tilt angles and an analysis has been undertaken to investigate the reasons for this behaviour when SMC is used.

4.1 Conceptual PVTOL Model without Aerodynamics Effects

Consider a conceptual birotor VTOL aircraft operating only on the vertical (x-y) plane without considering any associated aerodynamic parameters with it initially. The planar birotor vehicle schematic is as shown in Figure 4.1. The birotor dynamic model, without considering the aerodynamics, is represented by:

$$I\ddot{\phi} = (T_1 - T_2)l \quad (4.1.1)$$

$$m\ddot{x} = (T_1 + T_2)(-\sin\phi) \quad (4.1.2)$$

$$m\ddot{y} = (T_1 + T_2)(\cos\phi) - mg \quad (4.1.3)$$

Let

$$U_1 = (T_1 + T_2) \quad (4.1.4)$$

and

$$U_2 = (T_1 - T_2)l \quad (4.1.5)$$

Then

$$\ddot{\phi} = \frac{U_2}{I} \quad (4.1.6)$$

$$\ddot{x} = -\sin\phi \frac{U_1}{m} \quad (4.1.7)$$

$$\ddot{y} = \cos\phi \frac{U_1}{m} - g \quad (4.1.8)$$

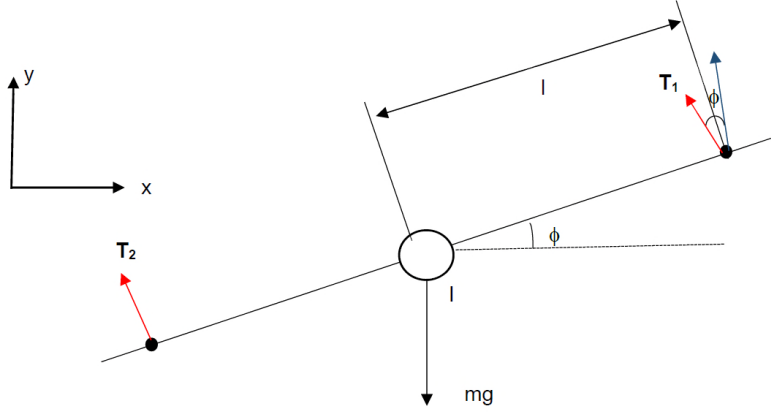


Figure 4.1: Planar birotor vehicle schematic

It is to be noted that:

$$T_1 = b\omega_1^2 \quad (4.1.9)$$

$$T_2 = b\omega_2^2 \quad (4.1.10)$$

where b is the thrust coefficient. When linearised at hover i.e. $U_2 = 0, y = y_0, x = x_0, U_1 = mg, \phi = 0$

$$\ddot{\phi} = \frac{U_2}{I} \quad (4.1.11)$$

$$\ddot{y} \approx \frac{U_1}{m} - g \quad (4.1.12)$$

$$\ddot{x} \approx g\phi \quad (4.1.13)$$

With appropriate PD control on y and x i.e. cascaded control (U_1 and U_2), simulation can be carried out with a linearised model of the birotor. But a PD control with a non-linearised version of the birotor has been analysed. Ignoring the drag coefficients at low speeds, the system inputs U_1 and U_2 are defined as:

$$U_1 = b(\omega_1^2 + \omega_2^2) = (T_1 + T_2) \quad (4.1.14)$$

$$U_2 = bl(\omega_1^2 - \omega_2^2) = (T_1 - T_2)l \quad (4.1.15)$$

From equations (4.1.4) and (4.1.5), it can be shown that:

$$\omega_1^2 = \frac{U_1}{2b} + \frac{U_2}{2bl} \quad (4.1.16)$$

$$\omega_2^2 = \frac{U_1}{2b} - \frac{U_2}{2bl} \quad (4.1.17)$$

The PVTOL parameters are as listed in Table 4.1. In the PVTOL considered, there are two control inputs, U_1 and U_2 and three outputs, x , y and ϕ . U_1 controls the position y and U_2 controls position x via the roll angle, ϕ . Assuming the thrust coefficient b is known, the rotor speeds can be manipulated and ignoring the motor dynamics, the values of the control inputs U_i can be determined from the rotor speeds ω_i . The dynamic equations of the birotor were modelled in MATLAB/Simulink and some numerical simulations were conducted using a PD controller.

Table 4.1: PVTOL parameters

Symbol	Variable definition	Value	Unit
l	arm length	0.45	m
m	mass	2.36/2	kg
I	Moment of inertia on x axis	0.04	kg.m ²
b	thrust coefficient	20×10^{-6}	N.s ²
ϕ	roll angle		radians
ω_i	speed of rotor number i		rad.s ⁻¹

The linearised model of the birotor was simulated with a PD controller. When all the actuators are functional, the birotor stabilises itself and the desired positions are reached. Simulation results demonstrate that the linearised PVTOL stabilises at the desired x-position, y-position and roll angle with a PD controller. From Figures 4.2, 4.3 and 4.4, it is seen that the performance of the PD controller is seen to be satisfactory. The controller gains are given by $k_{py} = 25$, $k_{iy} = 25$, $k_{dy} = 15$, $k_{px} = 5$, $k_{ix} = 0$, and $k_{dx} = 3$, $k_{p\phi} = 11.5758$, $k_{i\phi} = 0$ and $k_{d\phi} = 5$. The initial conditions are taken as:

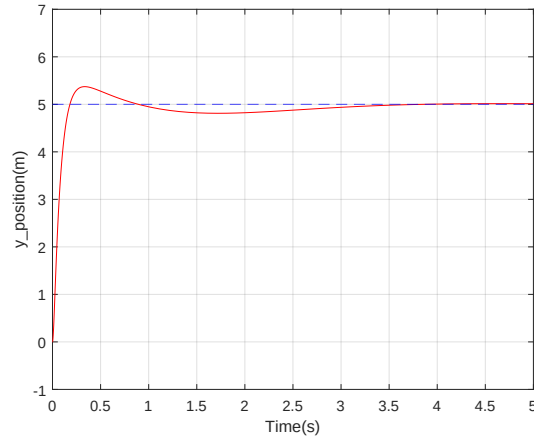


Figure 4.2: y-position control of a linearised birotor with PD control

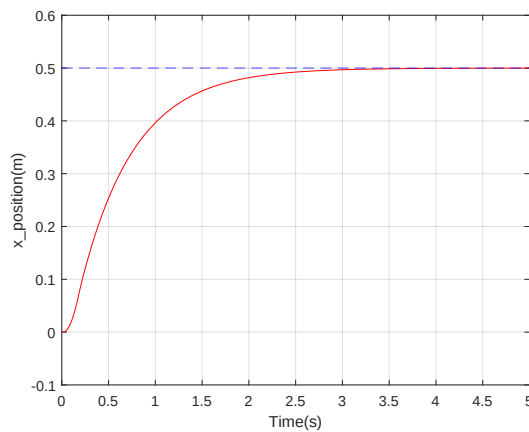


Figure 4.3: x-position control of a linearised birotor with PD Control

$$\mathbf{y}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \quad (4.1.18)$$

The desired output vector is:

$$\mathbf{y}_d = \begin{bmatrix} 5 & 0.5 & 0 \end{bmatrix}^T \quad (4.1.19)$$

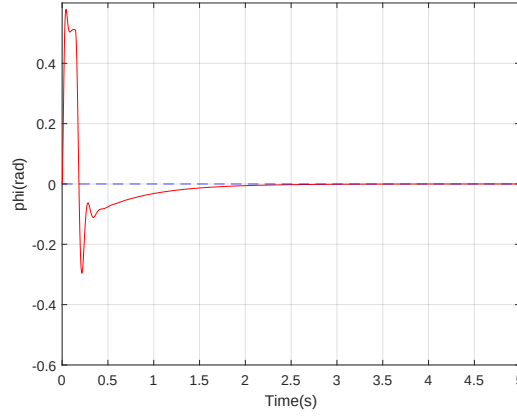


Figure 4.4: Roll angle stabilisation of a linearised birotor with PD Control

The PVTOL starts with zero roll angle and position displacements of 0.5 m as lateral displacement and 5 m as vertical displacement respectively at hover. The main focus of the simulation studies is developing a PD controller for the birotor without linearising the model dynamics. So, the modelling work has been extended and simulation of the model with a PD controller and sliding model controller as well. A comparison with the PD and SMC controllers has been performed and subsequently, the birotor with aerodynamics for gust rejection properties, has been simulated with SMC and extended to a quadrotor.

4.2 Conceptual PVTOL Model including Aerodynamics Effects

A lot of research work on the aerodynamic modelling of the rotor dynamics has been conducted by different researchers [Pounds et al., 2004; Martinez, 2007; Bristeau et al., 2004; Waslander and Wang, 2009; Whidborne and Cooke, 2017; Whidborne et al., 2022] including well-known books in helicopter dynamics written by Bramwell [1976], Seddon [1990].

Whidborne and Cooke [2017], Whidborne et al. [2022] analyse how outward rotor tilt angle affects the stability and investigated the gust rejection properties of a birotor with an outward tilt angle. The aerodynamic modelling of the birotor being considered is mostly based on their work.

Consider a conceptual PVTOL aircraft operating only on the vertical (x-z) plane where the rotors have a tilt angle, Υ . The planar birotor vehicle schematic in Figure 4.5 can be extended to include the geometry of the rotors with tilt angle, Υ , and the vehicle schematic can be represented as shown in Figure 4.5. $W = mg$ is the weight of the vehicle. V_w is the wind velocity and θ is the roll/pitch angle. The wind velocity V_w is assumed to be acting horizontally and $T_i(\Omega_i, \alpha_i, V_i)$ and $H_i(\Omega_i, \alpha_i, V_i)$ are the thrusts and drag factors of the i^{th} rotor given by:

$$T = \rho A (\Omega R_{rotor})^2 C_T \quad (4.2.1)$$

$$H = \rho A (\Omega R_{rotor})^2 C_H \quad (4.2.2)$$

where ρ is the air density, A is the rotor disc area, Ω is the rotor angular velocity, R_{rotor} is the rotor radius, C_T is the rotor thrust coefficient and C_H is the drag coefficient. The rotor torque, Q , is given by

$$Q = \rho A \Omega^2 R_{rotor}^3 C_Q \quad (4.2.3)$$

where C_Q is the rotor torque coefficient. The thrust coefficient, C_T , ignoring the flapping term, is given by:

$$C_T = \frac{1}{2} \sigma a \left[\frac{1}{3} \theta_{rotorblade} \left(1 + \frac{3}{2} \mu^2 \right) - \frac{1}{2} \lambda \right] \quad (4.2.4)$$

where σ is the solidity factor, a is the lift slope of the blade, $\theta_{rotorblade}$ is the rotor blade pitch angle, μ is the advance ratio and λ is the inflow ratio [Seddon, 1990; Bramwell, 1976].

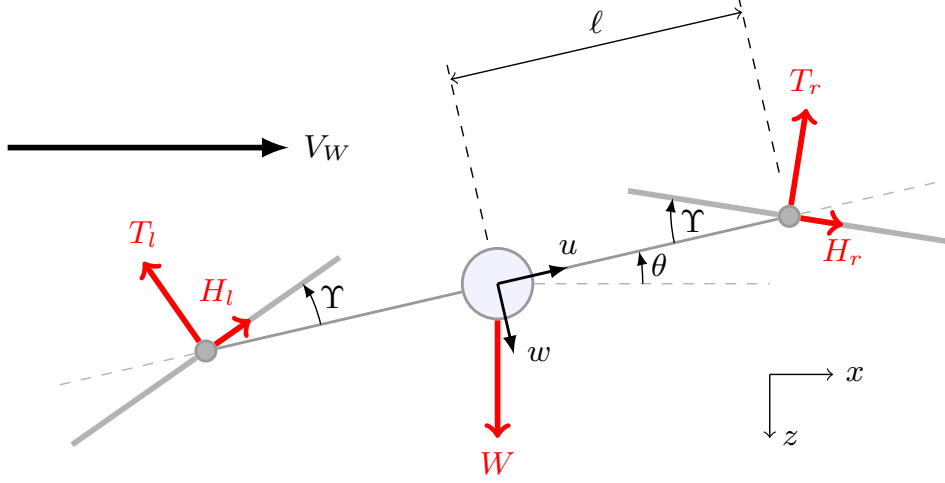


Figure 4.5: Planar birotor vehicle schematic with rotor tilt [Whidborne et al., 2022]

The term σ can be defined as in [Bramwell, 1976] and is given by:

$$\sigma = \frac{b_{blade}c}{\pi R_{rotor}} \quad (4.2.5)$$

where R_{rotor} is the rotor radius and b_{blade} is the number of blades and c is the constant chord.

For a tapering blade, $\theta_{rotorblade} = \theta_0 + 3/4\theta_1$ where θ_0 is the blade pitch at the roots and θ_1 is the twist [Bramwell, 1976].

The parameter μ is as defined in Seddon [1990] and is given by:

$$\mu = \frac{V \cos \alpha}{\Omega R_{rotor}} \quad (4.2.6)$$

where V is the incident flow speed, α is the angle between the incident flow and the tip-path plane as shown in Figure 4.6. Parameter λ , the inflow ratio Seddon [1990], is given by :

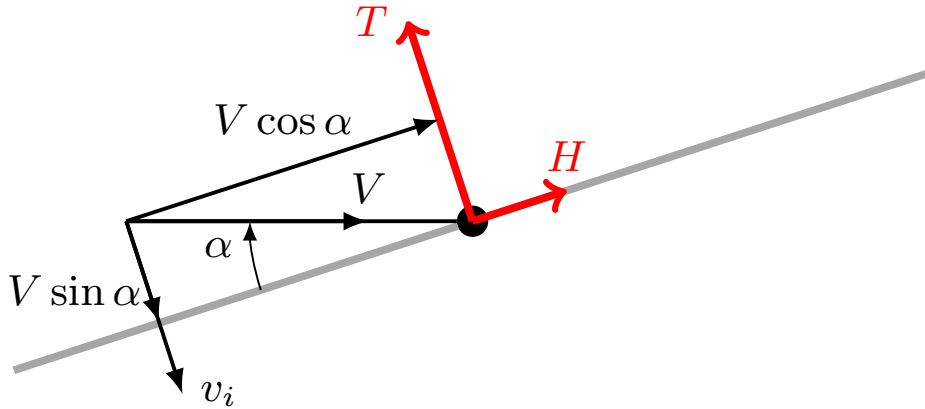


Figure 4.6: Incidence and component velocities on the disc in an airflow [Seddon, 1990]

$$\lambda = \frac{V \sin \alpha + v_i}{\Omega R} \quad (4.2.7)$$

where v_i is the induced velocity and calculated as in Seddon [1990]:

$$\left(\frac{v_i}{v_h}\right)^4 + \left(\frac{V}{v_h}\right)^2 \left(\frac{v_i}{v_h}\right)^2 = 1 \quad (4.2.8)$$

and

$$v_h^2 = \frac{T}{2\rho A} \quad (4.2.9)$$

The coefficient C_H , is represented as Bramwell [1976]:

$$C_H = \frac{a\sigma}{2} \left[\frac{\mu c_{d0}}{2a} + \frac{1}{3}a_1\theta - \frac{3}{4}\lambda a_1 + \frac{1}{2}\mu\theta\lambda + \frac{1}{4}\mu a_1^2 \right] \quad (4.2.10)$$

where a_1 is the backward flapping angle and δ is the blade profile drag coefficient.

The variable, a_1 , can be calculated as done by Seddon [1990]:

$$a_1 = \frac{\mu(8\theta/3 - 2\lambda)}{1 - \mu^2/2} \quad (4.2.11)$$

4.2.1 Blade Torque Coefficient

If the blade profile drag is represented as the polar

$$c_d = c_{d0} + c_{d1}\alpha_a + c_{d2}\alpha_a^2 \quad (4.2.12)$$

where α_a , is the angle of attack of the blade and the torque coefficient, if c_{d2} is insignificant, it can be simplified [Whidborne et al., 2022] to:

$$C_Q = \frac{\sigma}{4} \left[\frac{c_{d0}}{2} (1 + \mu^2) + \frac{c_{d1}}{1 + 3/2\mu^2} \left(\theta_0 (1/2 - 19/36\mu^2 + 3/4\mu^4) + \theta_1 (2/5 - 2/5\mu^2 + \mu^4/2) + \lambda(2 - \mu^2)/3 \right) \right]. \quad (4.2.13)$$

The equations of motion for a conceptual birotor as shown in Figure 4.5 can be represented as:

$$m(\dot{u} + qw) = -W \sin \theta - (T_l - T_r) \sin \Upsilon + (H_l + H_r) \cos \Upsilon \quad (4.2.14)$$

$$m(\dot{w} - qu) = W \cos \theta - (T_l + T_r) \cos \Upsilon - (H_l - H_r) \sin \Upsilon \quad (4.2.15)$$

$$I\dot{q} = -(T_l - T_r)\ell \cos \Upsilon - (H_l + H_r)\ell \sin \Upsilon \quad (4.2.16)$$

$$\alpha_l = \arctan \left(\frac{V_W \sin(\theta + \Upsilon) - u \sin \Upsilon - (w + \ell q) \cos \Upsilon}{V_W \cos(\theta + \Upsilon) - u \cos \Upsilon + (w + \ell q) \sin \Upsilon} \right) \quad (4.2.17)$$

$$\alpha_r = \arctan \left(\frac{V_W \sin(\theta - \Upsilon) + u \sin \Upsilon - (w - \ell q) \cos \Upsilon}{V_W \cos(\theta - \Upsilon) - u \cos \Upsilon - (w - \ell q) \sin \Upsilon} \right) \quad (4.2.18)$$

$$V_l = \sqrt{(V_W \sin(\theta + \Upsilon) - u \sin \Upsilon - (w + \ell q) \cos \Upsilon)^2 + (V_W \cos(\theta + \Upsilon) - u \cos \Upsilon + (w + \ell q) \sin \Upsilon)^2} \quad (4.2.19)$$

$$V_r = \sqrt{(V_W \sin(\theta - \Upsilon) + u \sin \Upsilon - (w - \ell q) \cos \Upsilon)^2 + (V_W \cos(\theta - \Upsilon) - u \cos \Upsilon - (w - \ell q) \sin \Upsilon)^2} \quad (4.2.20)$$

where m is mass, ℓ is the arm-length, I is the moment of inertia and the weight is given by $W = mg$ where g is the gravitational constant.

Re-arranging the above equations and taking the navigation equations into account, the final set of dynamic equations of motion are represented by:

$$\dot{u} = -g \sin \theta - \left(\frac{1}{m}\right)(T_l - T_r) \sin \Upsilon + \left(\frac{1}{m}\right)(H_l + H_r) \cos \Upsilon - qw, \quad (4.2.21)$$

$$\dot{x} = u \cos \theta + w \sin \theta, \quad (4.2.22)$$

$$\dot{w} = g \cos \theta - \left(\frac{1}{m}\right)(T_l + T_r) \cos \Upsilon - \left(\frac{1}{m}\right)(H_l - H_r) \sin \Upsilon + qu, \quad (4.2.23)$$

$$\dot{z} = w \cos \theta - u \sin \theta, \quad (4.2.24)$$

$$\dot{q} = -\left(\frac{\ell}{I}\right)(T_l - T_r) \cos \Upsilon - \left(\frac{\ell}{I}\right)(H_l + H_r) \sin \Upsilon, \quad (4.2.25)$$

$$\dot{\theta} = q \quad (4.2.26)$$

The above equations can be simplified as:

$$\dot{u} = -g \sin \theta - qw - \left(\frac{1}{m}\right)(T_l - T_r) \sin \Upsilon + \left(\frac{1}{m}\right)(H_l + H_r) \cos \Upsilon, \quad (4.2.27)$$

$$\dot{x} = u \cos \theta + w \sin \theta, \quad (4.2.28)$$

$$\dot{w} = g \cos \theta + qu - \left(\frac{1}{m}\right)U_1, \quad (4.2.29)$$

$$\dot{z} = -u \sin \theta + w \cos \theta, \quad (4.2.30)$$

$$\dot{q} = -\left(\frac{l}{I}\right)U_2, \quad (4.2.31)$$

$$\dot{\theta} = q \quad (4.2.32)$$

where

$$U_1 = (T_l + T_r) \cos \Upsilon + (H_l - H_r) \sin \Upsilon, \quad (4.2.33)$$

$$U_2 = (T_l - T_r) \cos \Upsilon + (H_l + H_r) \sin \Upsilon \quad (4.2.34)$$

The equations describing the mapping $(\Omega, \alpha, V) \mapsto (T, H, Q)$ are given by (4.2.1) – (4.2.13). The equations from (4.2.1) to (4.2.13) provide the mapping between $(\Omega, \alpha, V) \mapsto (T, H, Q)$. It is to be noted that in equation (4.2.9), as there is a T term, an explicit solution cannot be obtained as the equation are implicit in nature. Hence the MATLAB routine, **fzero** is used for the calculation of the mapping. Firstly, a PID controller is implemented in controlling this conceptual birotor and then a comparative simulation study is done with sliding mode control.

4.3 PID Control of the PVTOL with Aerodynamics

The PVTOL model was implemented in MATLAB/Simulink as illustrated in Appendix B and was simulated using a PID Controller. All the MATLAB code and parameters used are listed in the .m files used for the simulation as detailed in Appendix D. The controller gains are given by $k_{hp} = 10$, $k_{hi} = 1$, $k_w = -2$, $k_{xp} = -0.05$, $k_{xi} = -0.0115$, and $k_u = -0.10$, $k_{tp} = 17$, $k_q = -8$.

As can be seen from the simulation results, the lateral position response and the vertical position response are as shown in Figures 4.7 and 4.8 for a lateral position step demand of 10 m and different rotor tilt angles. The pitch angle response to 10 m horizontal position step demand is as shown in Figure 4.9. In this case, the wind is not taken into account. The responses are nearly identical and with a maximum pitch angle of 12° , the system remains stable and reaching the step demand of 10 m in 16 seconds. The vertical response for the 10 m horizontal position response as shown in Figure 4.8 has only some small differences for different rotor tilt angles.

Figure 4.10 shows different rotor rotation rates. The rotors have to work hardest for the rotor tilt angle $\Upsilon = 10^\circ$.

Figures 4.11, 4.12, 4.13 and 4.14 show the simulation results in the presence of a lateral wind velocity step of 5 ms^{-1} . As can be seen from the simulation results, for the rotor tilt angle $\Upsilon = 10^\circ$, the position response results in a minimum overshoot as the vehicle actually pitches into the wind when compared to the $\Upsilon = -5^\circ$ where the vehicle pitches away from the wind. With the rotor tilt angle, $\Upsilon = -5^\circ$, the lateral position response is worse due to the wind gust. The gust rejection properties with $\Upsilon > 0$ are as expected with regards to the positive rotor tilt angles (outward tilt) when compared to $\Upsilon < 0$ (inward tilt). The desired lateral position step of 10 m is reached within 16 seconds for $\Upsilon = 10^\circ$. As expected, the rotors have to work hardest to make the vehicle stable.

4.4 Simulation Results of PVTOL with Aerodynamics using PID Control

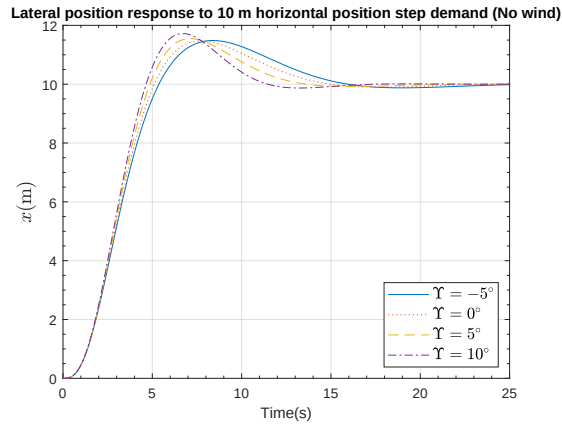


Figure 4.7: Lateral position response to 10 m horizontal position step demand (no wind) with PID Control

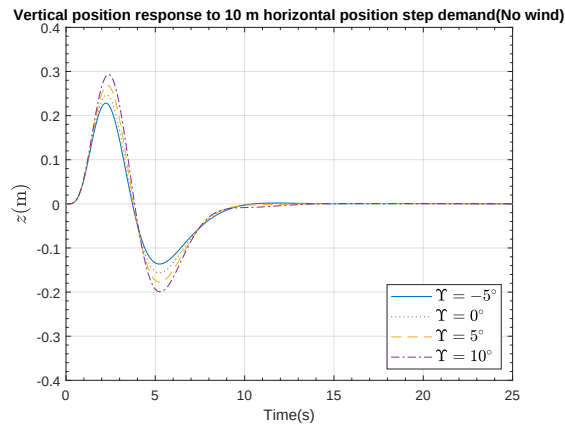


Figure 4.8: Vertical position response to 10 m horizontal position step demand (no wind)

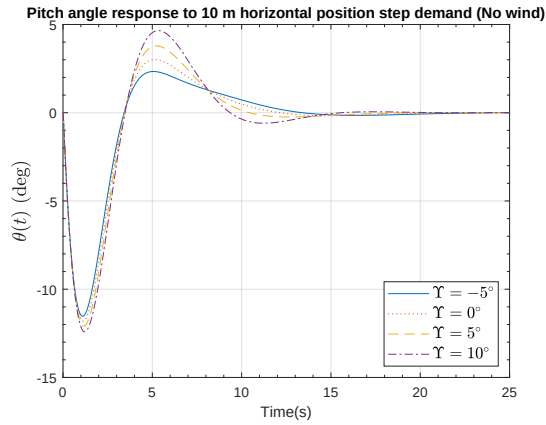


Figure 4.9: Pitch angle response to 10 m horizontal position step demand (no wind)

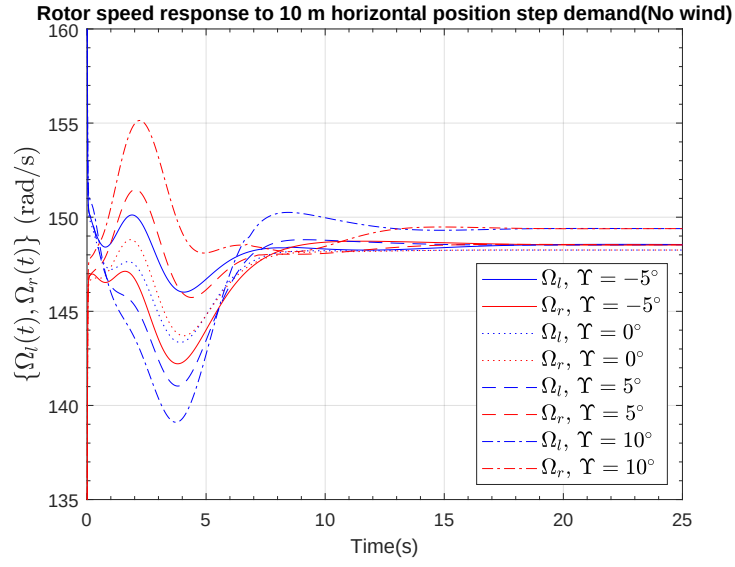


Figure 4.10: Rotor speed response to 10 m horizontal position step demand (no wind)

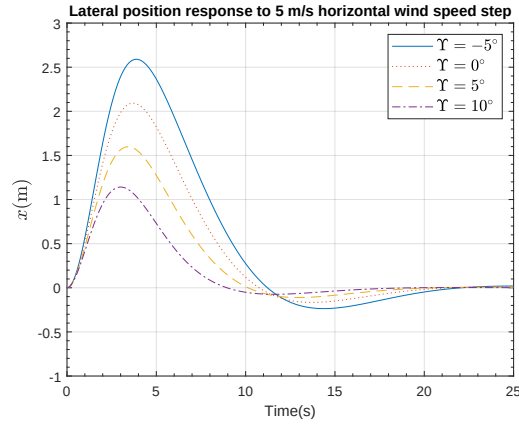


Figure 4.11: Lateral position response to 5 ms^{-1} horizontal wind speed step using PID control

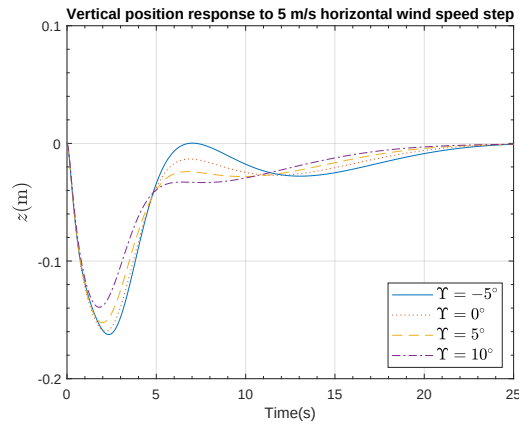


Figure 4.12: Vertical position response to 5 ms^{-1} horizontal wind speed step using PID control

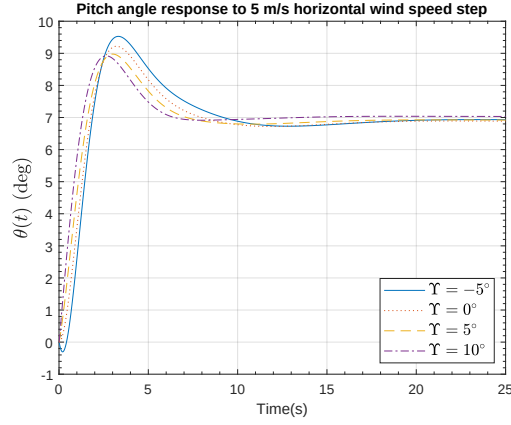


Figure 4.13: Pitch angle response to 5 ms^{-1} horizontal wind speed step using PID control

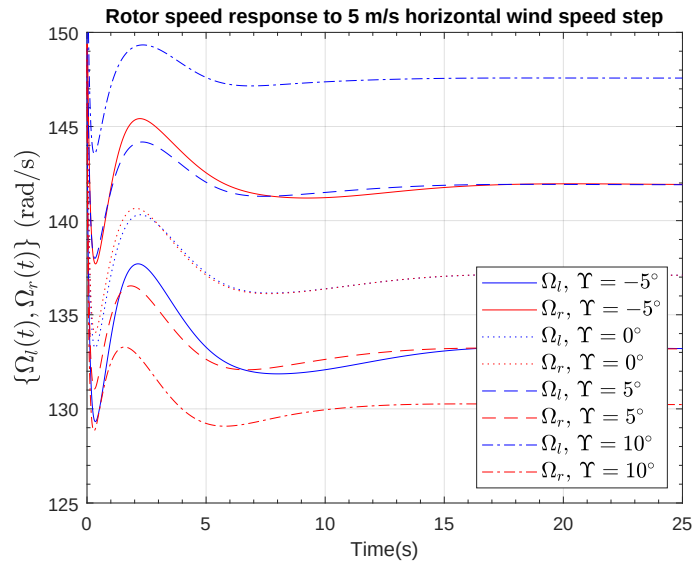


Figure 4.14: Rotor speed response to 5 ms^{-1} horizontal wind speed step using PID control

4.5 SMC Control of the PVTOL with Aerodynamics

Considering the equations of motion of a PVTOL represented by:

$$\dot{u} = -g \sin \theta - \left(\frac{1}{m}\right)(T_l - T_r) \sin \Upsilon + \left(\frac{1}{m}\right)(H_l + H_r) \cos \Upsilon - qw, \quad (4.5.1)$$

$$\dot{x} = u \cos \theta + w \sin \theta, \quad (4.5.2)$$

$$\dot{w} = g \cos \theta - \left(\frac{1}{m}\right)(T_l + T_r) \cos \Upsilon - \left(\frac{1}{m}\right)(H_l - H_r) \sin \Upsilon + qu, \quad (4.5.3)$$

$$\dot{z} = w \cos \theta - u \sin \theta, \quad (4.5.4)$$

$$\dot{q} = -\left(\frac{1}{I}\right)(T_l - T_r)\ell \cos \Upsilon - \left(\frac{1}{I}\right)(H_l + H_r)\ell \sin \Upsilon, \quad (4.5.5)$$

$$\dot{\theta} = q \quad (4.5.6)$$

The above equations can be re-arranged as:

$$\dot{u} = -g \sin \theta - qw - \left(\frac{1}{m}\right)(T_l - T_r) \sin \Upsilon - (H_l + H_r) \cos \Upsilon, \quad (4.5.7)$$

$$\dot{x} = u \cos \theta + w \sin \theta, \quad (4.5.8)$$

$$\dot{w} = g \cos \theta + qu - \left(\frac{1}{m}\right)U_1, \quad (4.5.9)$$

$$\dot{z} = -u \sin \theta + w \cos \theta, \quad (4.5.10)$$

$$\dot{q} = -\left(\frac{\ell}{I}\right)U_2, \quad (4.5.11)$$

$$\dot{\theta} = q \quad (4.5.12)$$

where

$$U_1 = (T_l + T_r) \cos \Upsilon + (H_l - H_r) \sin \Upsilon, \quad (4.5.13)$$

$$U_2 = (T_l - T_r) \cos \Upsilon + (H_l + H_r) \sin \Upsilon \quad (4.5.14)$$

$$(4.5.15)$$

The control inputs for the SMC of PVTOL can be defined as:

$$U_1 = m [g \cos \theta + qu + \ddot{z}_d + \lambda_z \dot{e}_z + k_z \operatorname{sgn}(s_z)] \quad (4.5.16)$$

$$U_2 = -\frac{I}{\ell} [\ddot{\theta}_d + \lambda_\theta \dot{e}_\theta + k_\theta \operatorname{sgn}(s_\theta)] \quad (4.5.17)$$

where U_1 , U_2 are the control inputs and z , θ and x are the outputs of interest and the desired pitch i.e. θ_d with x error is given by:

$$\theta_d = -[\ddot{x}_d + \lambda_x \dot{e}_x + k_x \operatorname{sgn}(s_x)] \quad (4.5.18)$$

and λ and k are constant values and signum function is defined by (2.2.5). The tracking error for position is given by:

$$x_e = x - x_d \quad (4.5.19)$$

The integral sliding function to reduce the steady state error in the position is given by:

$$s_x = \dot{x}_e + \lambda_x x_e + \int_0^\infty \lambda_i x_e \quad (4.5.20)$$

$$(4.5.21)$$

All the parameters that were suitable for the simulation studies are listed in the MATLAB files listed in Appendix D. The MATLAB/Simulink implementation of the desired pitch that is fed into the attitude control block is as shown in Appendix B.

4.6 Stability Analysis

4.6.1 Stability Analysis of PVTOL with Aerodynamics considering only SMC Altitude Control

Consider the equations of motion in a PVTOL relating to Altitude:

$$\dot{z} = w \cos \theta - u \sin \theta, \quad (4.6.1)$$

$$\dot{w} = g \cos \theta + qu - \frac{1}{m}(T_l + T_r) \cos \Upsilon - \frac{1}{m}(H_l - H_r) \sin \Upsilon, \quad (4.6.2)$$

If we consider only altitude control, then we can exclude the pitch i.e. θ is very small and $u \sin \theta$ becomes a disturbance. Then

$$\dot{z} = w + d_1, \quad (4.6.3)$$

$$\dot{w} = g - \frac{1}{m}(T_l + T_r) \cos \Upsilon + qu - \frac{1}{m}(H_l - H_r) \sin \Upsilon, \quad (4.6.4)$$

In the equation (4.6.4), the term $(T_l + T_r) \cos \Upsilon - mg$ can be considered as the control term, u_z and $qu + (\frac{1}{m})(H_l - H_r) \sin \Upsilon$ can be considered as the disturbance term, d_2 . Let

$$u_z = (T_l + T_r) \cos \Upsilon - mg, \quad (4.6.5)$$

then ignoring d_1 and d_2 , we have:

$$\dot{z} = w \quad (4.6.6)$$

$$\dot{w} = -\frac{1}{m}u_z \quad (4.6.7)$$

Let $S = z + w$, hence $\dot{S} = \dot{z} + \dot{w}$. If we define a Lyapunov function as:

$$V = \frac{1}{2}S^2 \quad (4.6.8)$$

Then,

$$\dot{V} = S\dot{S} \quad (4.6.9)$$

$$= S(w - \frac{1}{m}u_z) \quad (4.6.10)$$

Defining the control as $u_z = \kappa_z \text{sgn}(S)$ gives

$$\dot{V} = S(w - \frac{\kappa_z}{m} \text{sgn}(S)) \quad (4.6.11)$$

Assume $|w|m < \zeta_1$, then $|S||w|m < \zeta_1|S|$ and

$$\dot{V} = Sw - \frac{\kappa_z}{m}|S| \quad (4.6.12)$$

$$< |S||w| \quad (4.6.13)$$

$$< |S| \frac{1}{m}(\zeta_1 - \kappa_z) \quad (4.6.14)$$

which is negative definite if $\kappa_z > \zeta_1$. i.e. For positive definite V , we get $\dot{V}$ negative definite, hence stable. The S will converge to 0 and states will converge to 0. It takes infinite time to reach equilibrium point. However, we can show that the trajectories reach S in finite time. In (4.6.14), if we define $\kappa_z = \zeta_1 + \kappa_0$, where $\kappa_0 > 0$ then from equations, (4.6.14) and (4.6.8),

$$\dot{V} < -\frac{\kappa_0}{m}\sqrt{2V} \quad (4.6.15)$$

$$\frac{dV}{dt} < -\frac{\kappa_0}{m}\sqrt{2V} \quad (4.6.16)$$

$$\frac{1}{\sqrt{2V}}dV < -\frac{\kappa_0}{m} \quad (4.6.17)$$

Integrating (4.6.17),

$$\sqrt{V} < -\frac{\kappa_0}{m\sqrt{2}}t \quad (4.6.18)$$

But

$$\sqrt{V} = \frac{1}{\sqrt{2}}S \quad (4.6.19)$$

Therefore,

$$|S| < S(0) - \frac{1}{m}\kappa_0 t \quad (4.6.20)$$

which means, as $S \equiv 0$,

$$S(0) - \frac{1}{m}\kappa_0 t > 0 \quad (4.6.21)$$

$$t < \frac{mS(0)}{\kappa_0} \quad (4.6.22)$$

which implies a finite time in which $S(t)$ reaches zero.

4.6.2 Stability Analysis of PVTOL with Aerodynamics considering only SMC Pitch Control

Consider the equations of motion in a PVTOL relating to Pitch:

$$\dot{\theta} = q \quad (4.6.23)$$

$$\dot{q} = -\frac{1}{I}(T_l - T_r)\ell \cos \Upsilon - \frac{1}{I}(H_l + H_r)\ell \sin \Upsilon \quad (4.6.24)$$

and re-organising (4.6.24) as:

$$\dot{\theta} = q \quad (4.6.25)$$

$$\dot{q} = -\frac{\ell}{I}[(T_l - T_r) \cos \Upsilon] - \frac{\ell}{I}[(H_l + H_r) \sin \Upsilon] \quad (4.6.26)$$

In the equation (4.6.26), the term $[(T_l - T_r) \cos \Upsilon]$ can be considered as the control term, u_θ and $-\frac{\ell}{I}[(H_l + H_r) \sin \Upsilon]$ can be considered as the disturbance term, d_3 . At hover, $u = mg$. Let $\Delta u = u - mg$. Then (4.6.26) can be represented as:

$$\dot{\theta} = q \quad (4.6.27)$$

$$\dot{q} = -\frac{\ell}{I}u_\theta + d_3 \quad (4.6.28)$$

Ignoring d_3 , (4.6.28) can be reduced to:

$$\dot{\theta} = q \quad (4.6.29)$$

$$\dot{q} = -\frac{\ell}{I}u_\theta \quad (4.6.30)$$

Let $S = \theta + q$, hence $\dot{S} = \dot{\theta} + \dot{q}$. If we define a Lyapunov function as:

$$V = \frac{1}{2}S^2 \quad (4.6.31)$$

Then,

$$\dot{V} = S\dot{S} \quad (4.6.32)$$

$$= S(q - \frac{\ell}{I}u_\theta) \quad (4.6.33)$$

Defining the control as $u_\theta = \kappa_\theta \operatorname{sgn}(S)$ gives

$$\dot{V} = S(q - \frac{\kappa_\theta}{\frac{I}{\ell}} \operatorname{sgn}(S)) \quad (4.6.34)$$

Assume $|q| \frac{I}{\ell} < \zeta_2$, then $|S| |q| \frac{I}{\ell} < \zeta_2 |S|$ and

$$\dot{V} = Sq - \frac{\kappa_\theta \ell}{I} |S| \quad (4.6.35)$$

$$< |S| |q| - \frac{\kappa_\theta \ell}{I} |S| \quad (4.6.36)$$

$$< |S| \frac{\ell}{I} (\zeta_2 - \kappa_\theta) \quad (4.6.37)$$

which is negative definite if $\kappa_\theta > \zeta_2$. i.e. For positive definite V , we get $\dot{V}$ negative definite, hence stable. The S will converge to 0 and states will converge to 0. It takes infinite time to reach equilibrium point. However, we can show that the trajectories reach S in finite time. In (4.6.37), if we define $\kappa_\theta = \zeta_2 + \kappa_0$, where $\kappa_0 > 0$ then from equations, (4.6.37) and (4.6.31),

$$\dot{V} < -\frac{\kappa_0 \ell}{I} \sqrt{2V} \quad (4.6.38)$$

$$\frac{dV}{dt} < -\frac{\kappa_0 \ell}{I} \sqrt{2V} \quad (4.6.39)$$

$$\frac{1}{\sqrt{2V}} dV < -\frac{\kappa_0 \ell}{I} \quad (4.6.40)$$

Integrating (4.6.40),

$$\sqrt{V} < -\frac{\kappa_0 \ell}{I\sqrt{2}} t \quad (4.6.41)$$

But

$$\sqrt{V} = \frac{1}{\sqrt{2}} S \quad (4.6.42)$$

Therefore,

$$|S| < S(0) - \frac{\ell}{I} \kappa_0 t \quad (4.6.43)$$

which means, as $S \equiv 0$,

$$S(0) - \frac{\ell}{I} \kappa_0 t > 0 \quad (4.6.44)$$

$$t < \frac{IS(0)}{\ell \kappa_0} \quad (4.6.45)$$

which implies a finite time in which $S(t)$ reaches zero.

4.6.3 Stability Analysis of PVTOL with Aerodynamics considering only SMC Position Control

Consider the equations of motion in a PVTOL relating to Position:

$$\dot{x} = u \cos \theta + w \sin \theta, \quad (4.6.46)$$

$$\dot{u} = -g \sin \theta - qw - \frac{1}{m}(T_l - T_r) \sin \Upsilon + \frac{1}{m}(H_l + H_r) \cos \Upsilon, \quad (4.6.47)$$

If we consider only position control, then we can exclude the pitch i.e. θ is very small and $w \sin \theta$ becomes a disturbance. Then

$$\dot{x} = u + d_5, \quad (4.6.48)$$

$$\dot{u} = -\frac{1}{m}(T_l - T_r) \sin \Upsilon - qw + \frac{1}{m}(H_l + H_r) \cos \Upsilon, \quad (4.6.49)$$

In the equation (4.6.49), the term $(T_l - T_r) \sin \Upsilon$ can be considered as the control term, u_x and $-qw + (\frac{1}{m})(H_l + H_r) \cos \Upsilon$ can be considered as the disturbance term, d_6 . Let

$$u_x = (T_l - T_r) \sin \Upsilon \quad (4.6.50)$$

then ignoring d_5 and d_6 , we have:

$$\dot{x} = u \quad (4.6.51)$$

$$\dot{u} = -\frac{1}{m}u_x \quad (4.6.52)$$

Let $S = x + u$, hence $\dot{S} = \dot{x} + \dot{u}$. If we define a Lyapunov function as:

$$V = \frac{1}{2}S^2 \quad (4.6.53)$$

Then,

$$\dot{V} = S\dot{S} \quad (4.6.54)$$

$$= S(u - \frac{1}{m}u_x) \quad (4.6.55)$$

Defining the control as $u_x = \kappa_x \text{sgn}(S)$ gives

$$\dot{V} = S(u - \frac{\kappa_x}{m} \text{sgn}(S)) \quad (4.6.56)$$

Assume $|u| m < \zeta_3$, then $|S| |u| m < \zeta_3 |S|$ and

$$\dot{V} = Su - \frac{\kappa_x}{m} |S| \quad (4.6.57)$$

$$< |S| |u| \quad (4.6.58)$$

$$< |S| \frac{1}{m}(\zeta_1 - \kappa_x) \quad (4.6.59)$$

which is negative definite if $\kappa_x > \zeta_3$. i.e. For positive definite V , we get $\dot{V}$ negative definite, hence stable. The S will converge to 0 and states will converge to 0. It takes infinite time to reach equilibrium point. However, we can show that the trajectories reach S in finite time. In (4.6.14), if we define $\kappa_x = \zeta_3 + \kappa_0$, where $\kappa_0 > 0$ then from equations, (4.6.59) and (4.6.53),

$$\dot{V} < -\frac{\kappa_0}{m} \sqrt{2V} \quad (4.6.60)$$

$$\frac{dV}{dt} < -\frac{\kappa_0}{m} \sqrt{2V} \quad (4.6.61)$$

$$\frac{1}{\sqrt{2V}}dV < -\frac{\kappa_0}{m} \quad (4.6.62)$$

Integrating (4.6.17),

$$\sqrt{V} < -\frac{\kappa_0}{m\sqrt{2}}t \quad (4.6.63)$$

But

$$\sqrt{V} = \frac{1}{\sqrt{2}}S \quad (4.6.64)$$

Therefore,

$$|S| < S(0) - \frac{1}{m}\kappa_0 t \quad (4.6.65)$$

which means, as $S \equiv 0$,

$$S(0) - \frac{1}{m}\kappa_0 t > 0 \quad (4.6.66)$$

$$t < \frac{mS(0)}{\kappa_0} \quad (4.6.67)$$

which implies a finite time in which $S(t)$ reaches zero.

Similar stability analysis can be undertaken for quadrotor as just it is an extension of the PVTOL stability analysis and quadrotor being a modular extension of a PVTOL.

4.7 Simulation Results of PVTOL with Aerodynamics using SMC for Attitude and Altitude Control and PIV for Position Control

The MATLAB/Simulink model of the SMC control of the PVTOL is described in Appendix B. As can

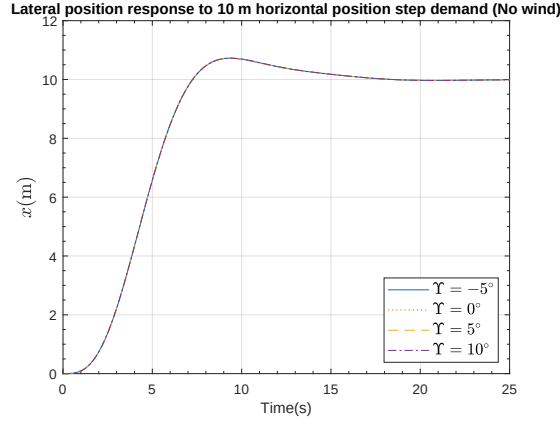


Figure 4.15: Lateral position response to 10 m horizontal position step demand (no wind gust)

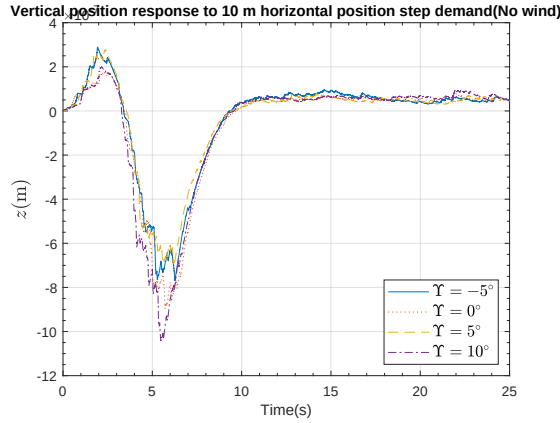


Figure 4.16: Vertical position response to 10 m horizontal position step demand (no wind gust)

be seen from the simulation results, the lateral position response and the vertical position response using a sliding mode controller are as shown in Figures 4.15 and 4.16 for a lateral position step demand of 10 m and different rotor tilt angles. The pitch angle response to 10 m horizontal position step demand is as shown in Figure 4.17. In this case, still air is considered. The responses are nearly identical and with a maximum pitch angle of 6.5° , the system remains stable and reaching the step demand of 10 m in 17 seconds. The vertical response is small for the 10 m horizontal position response as shown in Figure 4.16 and has some small differences for different rotor tilt angles. Figure 4.18 shows different rotor rotation rates. The rotors have to work hardest for the rotor tilt angle $\Upsilon = 10^\circ$ and least for $\Upsilon = 0^\circ$. Figure 4.19, Figure 4.20, Figure 4.21 and Figure 4.22 show the simulation results in the presence of a lateral wind velocity step of 5 ms^{-1} . As can be seen from the simulation results, using SMC control the responses are nearly identical with a maximum pitch angle of 11° for all the rotor tilt angles and remains steady after 19 seconds and as expected the rotors have to work hardest for $\Upsilon = 10^\circ$ to make the vehicle stable. The rotor title angles do not impact on gust rejection properties when the PVTOL is controlled with SMC. The sliding mode control performs very well in rejecting the gust.

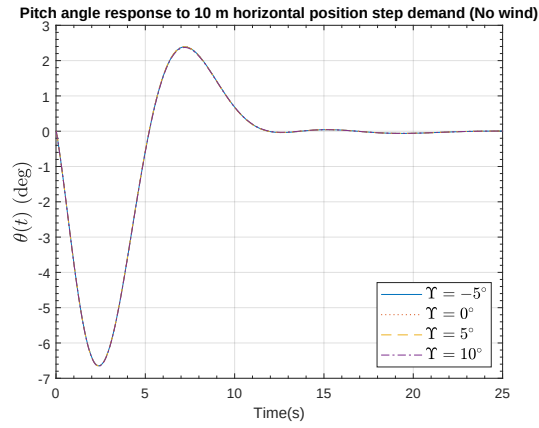


Figure 4.17: Pitch angle response to 10 m horizontal position step demand (no wind gust)

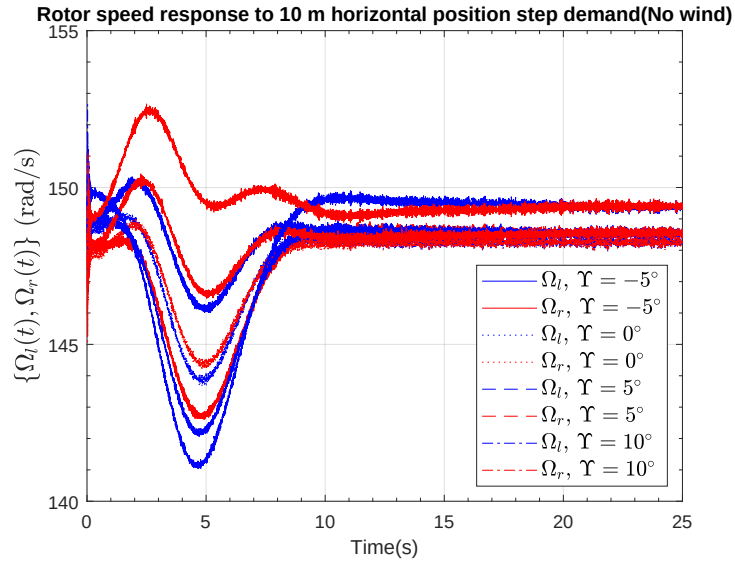
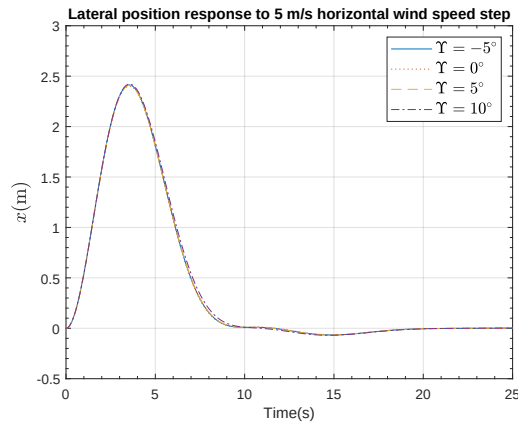


Figure 4.18: Rotor speed response to 10 m horizontal position step demand (no wind gust)

Figure 4.19: Lateral position response to 5 ms⁻¹ horizontal wind speed step

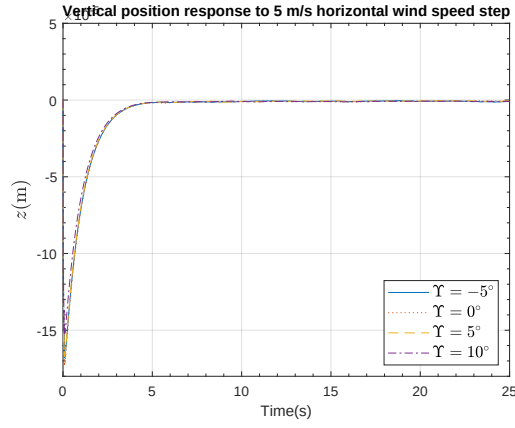


Figure 4.20: Vertical position response to 5 ms^{-1} horizontal wind speed step

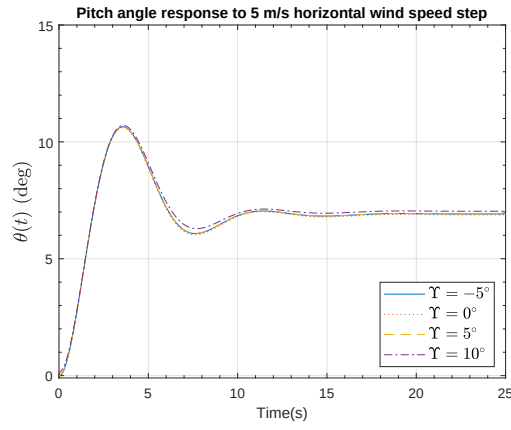


Figure 4.21: Pitch angle response to 5 ms^{-1} horizontal wind speed step

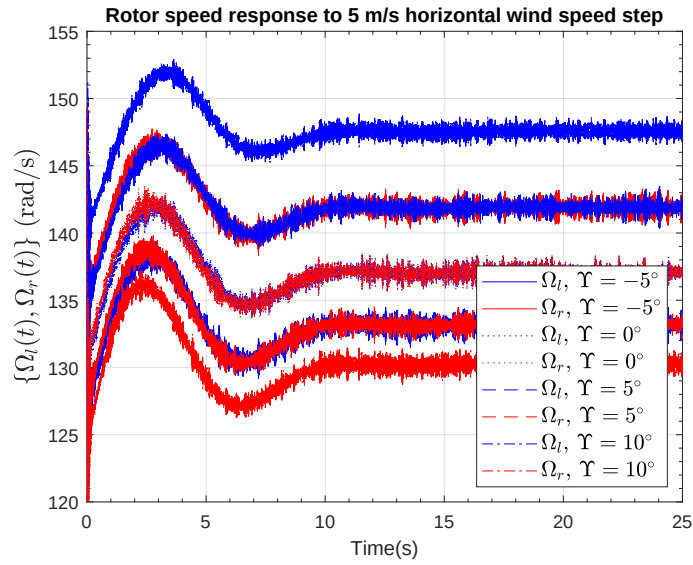


Figure 4.22: Rotor speed response to 5 ms^{-1} horizontal wind speed step

4.8 Simulation Results of PVTOL with Aerodynamics using SMC for Attitude, Altitude Control and Position Control

Figure 4.23, Figure 4.24, Figure 4.25 and Figure 4.26 show the simulation results of PVTOL with sliding mode control of Altitude, Attitude and Position. As can be seen from the simulation results, using SMC control the responses are nearly identical with a maximum pitch angle of 14° for all the rotor tilt angles and remains steady after 14 seconds and as expected the rotors have to work hardest for $\Upsilon = 10^\circ$ to make the vehicle stable. The rotor tilt angles do not impact on gust rejection properties when the PVTOL is controlled with SMC. The sliding mode control performs very well in rejecting the gust.

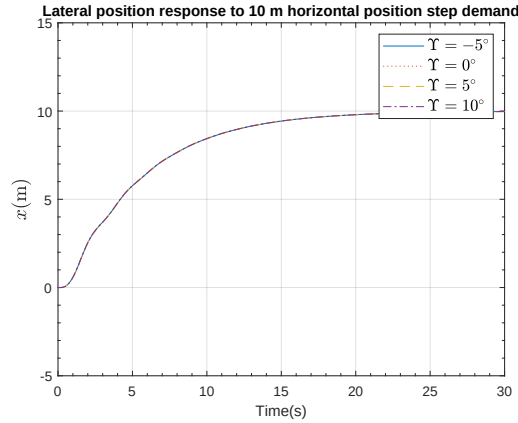


Figure 4.23: Lateral position response to 10 m horizontal position step demand (no wind) using SMC control

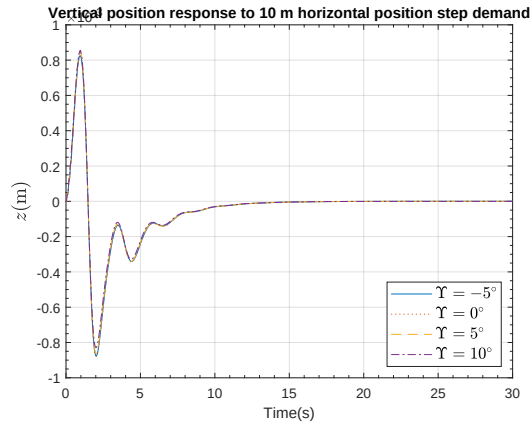


Figure 4.24: Vertical position response to 10 m horizontal position step demand (no wind) using SMC control

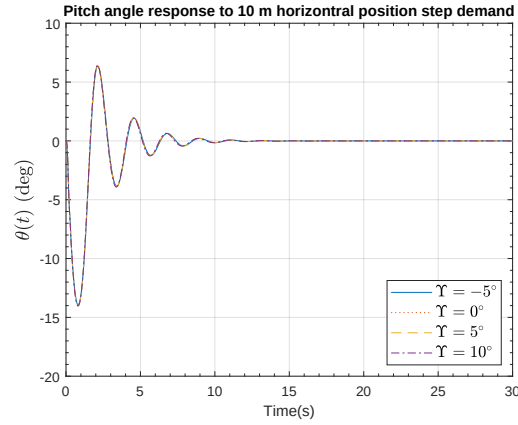


Figure 4.25: Pitch angle response to 10 m horizontal position step demand (no wind) using SMC control

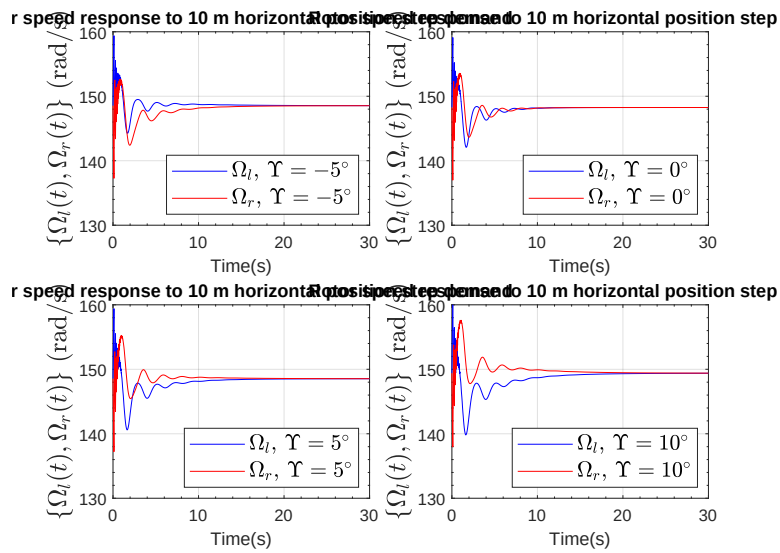


Figure 4.26: Rotor speed response to 10 m horizontal position step demand (no wind) using SMC control

4.8.1 Simulation Results of PVTOL with Aerodynamics using SMC for Altitude, Attitude and Position Control with wind gust step at Hover

Figure 4.27, Figure 4.28, Figure 4.29 and Figure 4.30 show the simulation results of PVTOL with sliding mode control of Altitude, Attitude and Position at hover with wind gust step. As can be seen from the simulation results, using SMC control the responses are nearly identical with a maximum pitch angle of 11° for most of the rotor tilt angles and remains steady after 14 seconds and as expected the rotors have to work hardest for $\Upsilon = 10^\circ$ to make the vehicle stable. The rotor tilt angles do not impact on gust rejection properties when the PVTOL is controlled with SMC. The sliding mode control performs very well in rejecting the gust.

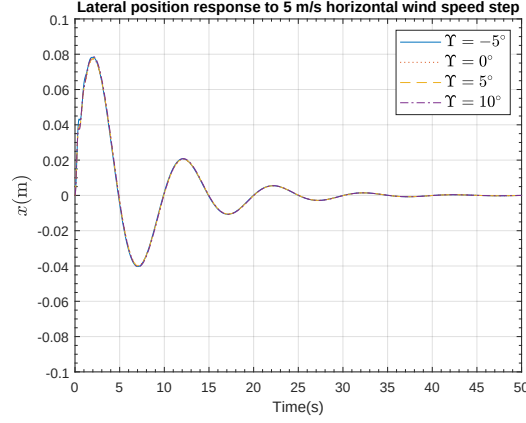


Figure 4.27: Lateral position response to 5 ms^{-1} horizontal wind speed step using SMC control

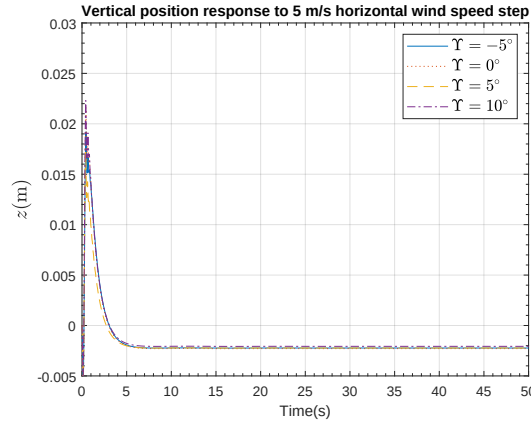


Figure 4.28: Vertical position response to 5 ms^{-1} horizontal wind speed step using SMC control

Figure 4.31 illustrates the phase portrait of altitude of the PVTOL with 10° rotor tilt angle considering a 5 ms^{-1} horizontal wind gust step. The system reaches the sliding phase and stays on that surface until it meets the equilibrium point where the error and derivative or error approaches zero. It can also be seen that no chattering occurs on the sliding surface and it's control effort during the altitude tracking.

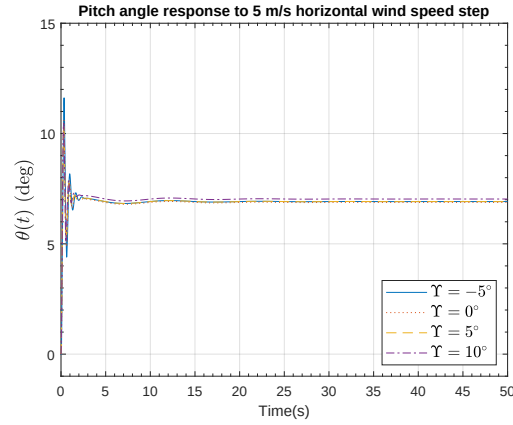


Figure 4.29: Pitch angle response to 5 ms^{-1} horizontal wind speed step using SMC control

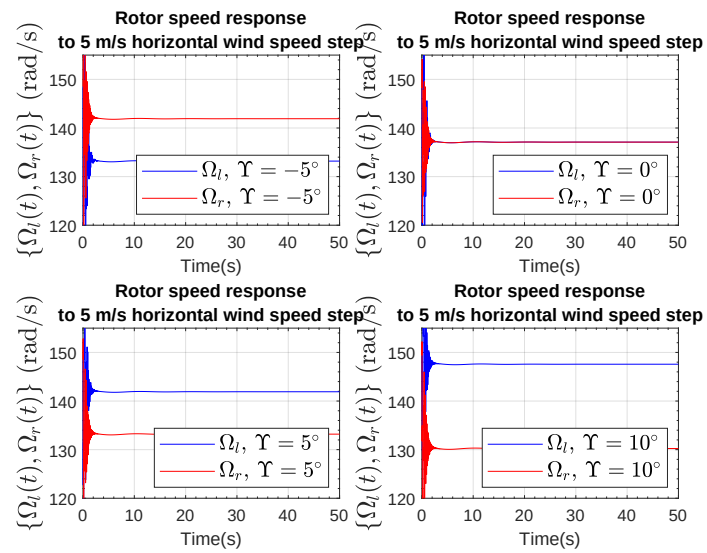


Figure 4.30: Rotor speed response to 5 ms^{-1} horizontal wind speed step using SMC control

Phase Portrait of Altitude with 5 m/s wind and 10 deg rotor tilt angle

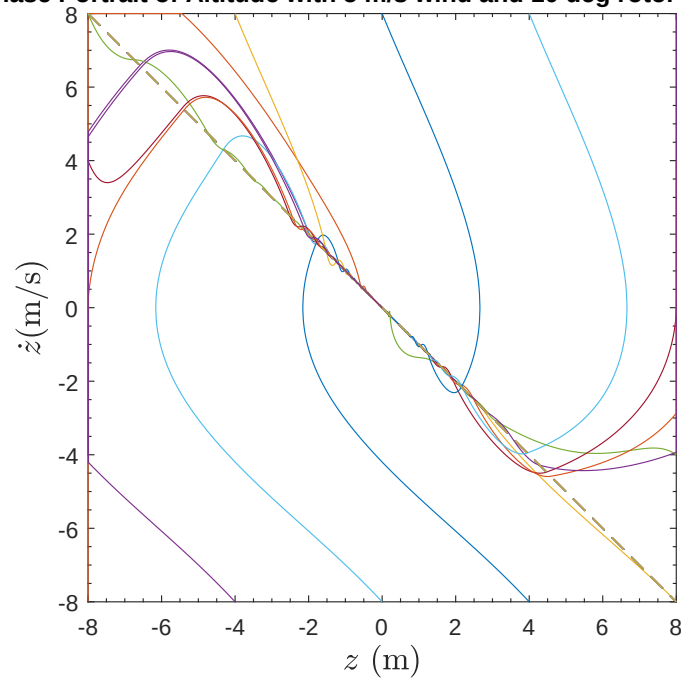


Figure 4.31: Phase portrait of altitude with 5 ms^{-1} horizontal wind speed step and 10 deg rotor tilt angle using SMC control

4.8.2 Simulation Results of PVTOL with Aerodynamics using SMC for Attitude, Altitude Control and Position Control with demands considering wind gust step

Figure 4.32, Figure 4.33, Figure 4.34 and Figure 4.35 show the simulation results of PVTOL with sliding mode control of Altitude, Attitude and Position to a 10 m horizontal position demand considering a 5 ms^{-1} horizontal wind gust step. As can be seen from the simulation results, using SMC control the responses are nearly identical with a maximum pitch angle of 50° for most of the rotor tilt angles and remains steady after 40 seconds and as expected the rotors have to work hardest for $\Upsilon = 10^\circ$ to make the vehicle stable. The rotor tilt angles do not impact on gust rejection properties when the PVTOL is controlled with SMC. The sliding mode control performs very well in rejecting the gust.

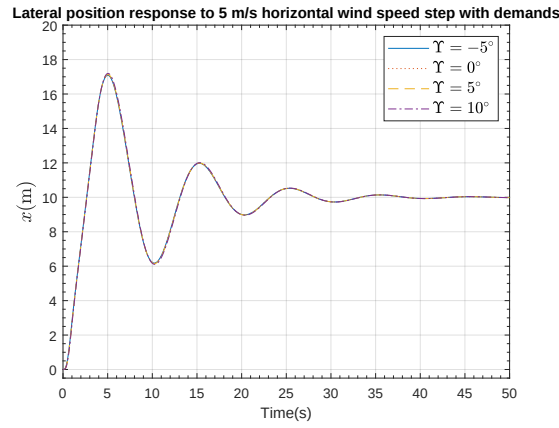


Figure 4.32: Lateral position response to 5 ms^{-1} horizontal wind gust step using SMC control

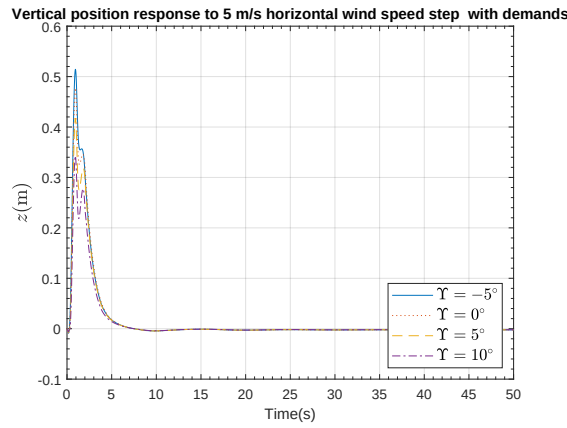


Figure 4.33: Vertical position response to 5 ms^{-1} horizontal wind gust step using SMC control

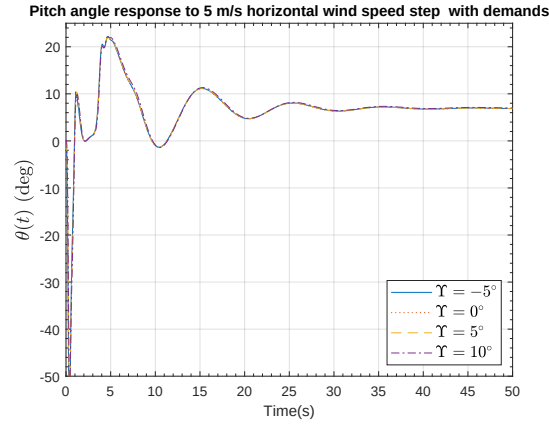


Figure 4.34: Pitch angle response to 5 ms^{-1} horizontal wind gust step using SMC control

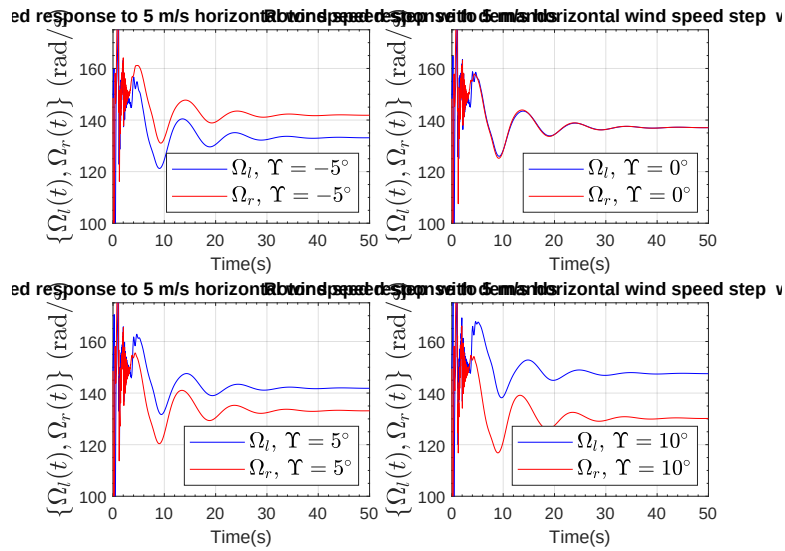


Figure 4.35: Rotor speed response to 5 ms^{-1} horizontal wind gust step using SMC control

4.9 Comparison of the Simulation Results

Defining the PID Control as Method #1, SMC control of Altitude & Attitude Control and PIV Position Control as Method #2 and SMC Control of Altitude, Attitude and Position as Method #3, the Table 4.2 illustrates a comparison of the lateral position responses to $10m$ horizontal position step demand in a PVTOL for different rotor tilt angles and without considering wind gust step.

Lateral position response in a PVTOL (without wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	−5°	0°	5°	10°	−5°	0°	5°	10°	−5°
Method #1	11.5	11.5	11.75	11.5	5.25	5.15	5	5.5	23	20	18	24
Method #2	10.6	10.6	10.6	10.6	7	7	7	7	22	22	22	22
Method #3	10	10	10	10	13	13	13	13	25	25	25	25

Table 4.2: Comparison of lateral position responses to 10m horizontal position step demand in a PVTOL without wind gust step for different rotor tilt angles

Vertical position response in a PVTOL (without wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	−5°	0°	5°	10°	−5°	0°	5°	10°	−5°
Method #1	0.24	0.26	0.29	0.25	1.8	2.2	2	2.2	13	14	14	14
Method #2	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0
Method #3	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0

Table 4.3: Comparison of vertical position responses to 10m horizontal position step demand in a PVTOL without wind gust step for different rotor tilt angles

Table 4.3 illustrates a comparison of the vertical position responses to 10m horizontal position step demand in a PVTOL for different rotor tilt angles and without considering wind gust step.

Table 4.4 illustrates a comparison of the pitch angle responses to 10m horizontal position step demand in a PVTOL for different rotor tilt angles and without considering wind gust step.

Table 4.5 illustrates a comparison of the lateral position responses to 10m horizontal position step demand in a PVTOL for different rotor tilt angles and considering $5ms^{-1}$ horizontal wind gust step.

Table 4.6 illustrates a comparison of the vertical position responses to 10m horizontal position step demand in a PVTOL for different rotor tilt angles and considering $5ms^{-1}$ horizontal wind gust step.

Table 4.7 illustrates a comparison of the pitch angle responses to 10m horizontal position step demand in a PVTOL for different rotor tilt angles and considering $5ms^{-1}$ horizontal wind gust step. It is evident from the comparison of the simulation results that the SMC control of Altitude, Attitude and Position Control of the PVTOL i.e. Method #3 performs better than the PID control in terms of Overshoot, Rise Time and Settling Time even in the presence of wind gust step.

Pitch angle response in a PVTOL (without wind gust step)	Max Overshoot (degrees)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	−5°	0°	5°	10°	−5°	0°	5°	10°	−5°
Method #1	-11.75	-12	-12.5	-11.5	0.9	0.9	0.8	0.5	22	22	20	22
Method #2	-6.6	-6.6	-6.6	-6.6	1.8	1.8	1.8	1.8	22	22	22	22
Method #3	-14	-14	-14	-14	0.6	0.6	0.6	0.6	14	14	14	14

Table 4.4: Comparison of pitch angle responses to 10m horizontal position step demand in a PVTOL 5ms horizontal considering wind gust step for different rotor tilt angles

Lateral position response in a PVTOL (with wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	−5°	0°	5°	10°	−5°	0°	5°	10°	−5°
Method #1	2.1	1.6	1.1	2.6	3	2.5	2	3	21	20	20	25
Method #2	2.4	2.4	2.4	2.4	2.5	2.5	2.5	2.5	20	20	20	20
Method #3	0.078	0.078	0.078	0.078	1.5	1.5	1.5	1.5	40	40	40	40

Table 4.5: Comparison of lateral position responses to 10m horizontal position step demand in a PVTOL with $5ms^{-1}$ horizontal wind gust step for different rotor tilt angles

Vertical position response in a PVTOL (with wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	−5°	0°	5°	10°	−5°	0°	5°	10°	−5°
Method #1	-0.16	-0.15	-0.14	-0.16	1.5	1.25	1.25	1.5	23	23	24	24
Method #2	-19	-18	-15	-19	0.5	0.5	0.5	0.5	10	10	10	10
Method #3	0.022	0.02	0.02	0.02	0.5	0.5	0.5	0.5	7	7	7	7

Table 4.6: Comparison of vertical position responses to 10m horizontal position step demand in a PVTOL with $5ms^{-1}$ horizontal wind gust step for different rotor tilt angles

Pitch angle response in a PVTOL (with wind gust step)	Max Overshoot (degrees)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	−5°	0°	5°	10°	−5°	0°	5°	10°	−5°
Method #1	9.2	9	8.9	9.5	2.25	2	1.5	2.5	25	25	15	25
Method #2	10.5	10.5	10.5	10.5	2.5	2.5	2.5	2.5	17	17	17	17
Method #3	10.5	10.5	10.5	11.5	0.5	0.5	0.5	0.5	15	15	15	15

Table 4.7: Comparison of pitch angle responses to 10m horizontal position step demand in a PVTOL with $5ms^{-1}$ horizontal wind gust step for different rotor tilt angles

4.10 Analysis of the SMC control of PVTOL with Aerodynamics

From the simulation results as shown in Figure 4.19, Figure 4.20, Figure 4.21 and Figure 4.22, it is observed that the responses are nearly identical for all the rotor title angles considered. The SMC performance is nearly invariant to the change in the rotor tilt angle, Υ . Assuming a linearised PVTOL model of the dynamics in still air i.e. $V_w = 0$, the effect of the rotor tilt angle Υ can be analysed. The equations of motion of a PVTOL represented by (4.10.3) - (4.5.11), can be represented as

$$\dot{\bar{\mathbf{x}}} = \mathbf{f}(\bar{\mathbf{x}}, \bar{\mathbf{u}}) \quad (4.10.1)$$

where the state vector is given by $\bar{\mathbf{x}} = [z, w, x, u, \theta, q]^T$; output vector is given by $\bar{\mathbf{y}} = [z, x, \theta]^T$ and the control vector is given by $\bar{\mathbf{u}} = [T_l + T_r, T_l - T_r]$.

The general form of the linearised model can be represented by:

$$\Delta \dot{\mathbf{x}} = \left[\frac{\partial f_i}{\partial x_j} \right]_{\mathbf{x}=\mathbf{x}_{trim}} \Delta \mathbf{x} + \left[\frac{\partial f_i}{\partial u_j} \right]_{\mathbf{u}=\mathbf{u}_{trim}} \Delta \mathbf{u} \quad (4.10.2)$$

Assuming, (4.10.3) as f_1 and (4.5.9) as f_2 and so on, the linear model can be obtained as follows:

$$f_1 = -u \sin \theta + w \cos \theta, \quad (4.10.3)$$

At trim, $\theta_{trim} = 0$.

$$\frac{\partial f_1}{\partial z} = 0; \quad (4.10.4)$$

$$\frac{\partial f_1}{\partial w} = \cos \theta_{\theta=\theta_{trim}} = \cos 0 = 1 \quad (4.10.5)$$

$$\frac{\partial f_1}{\partial x} = 0; \quad (4.10.6)$$

$$\frac{\partial f_1}{\partial u} = -\sin \theta_{\theta=0} \quad (4.10.7)$$

$$\frac{\partial f_1}{\partial \theta} = -u \cos \theta - w \sin \theta_{\theta=0; u=0} = 0; \quad (4.10.8)$$

$$\frac{\partial f_1}{\partial q} = 0; \quad (4.10.9)$$

Assuming small perturbations about hover trim and repeating for f_2 , f_3 , f_4 , f_5 and f_6 , the small perturbation linear model is given by $\dot{\mathbf{x}} = \mathbf{A}_T \mathbf{x} + \mathbf{B}_T \mathbf{u}$ where

$$\mathbf{A}_T = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_6} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_6}{\partial x_1} & \cdots & \frac{\partial f_6}{\partial x_6} \end{bmatrix} \quad (4.10.10)$$

Performing the partial derivatives will result in $\mathbf{A}_T$ being represented as:

$$\mathbf{A}_T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.10.11)$$

$$\mathbf{B}_T = \begin{bmatrix} 0 & 0 \\ -\frac{\cos \Upsilon}{m} & 0 \\ 0 & 0 \\ 0 & -\frac{\sin \Upsilon}{m} \\ 0 & 0 \\ 0 & -\frac{l \cos \Upsilon}{I} \end{bmatrix} \quad (4.10.12)$$

Similarly, $\mathbf{B}_T$ is given by: And the system transfer function matrix, $\mathbf{G}_T$ is given by

$$\mathbf{G}_T = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \quad (4.10.13)$$

Using Gauss-Jordan method and by simplifying, $\mathbf{G}_T$ is given by:

$$\mathbf{G}_T = \begin{bmatrix} -\frac{\cos \Upsilon}{ms^2} & 0 \\ 0 & \frac{(mgl \cos \Upsilon - I \sin \Upsilon s^2)}{mIs^4} \\ 0 & -\frac{l \cos \Upsilon}{Is^2} \end{bmatrix} \quad (4.10.14)$$

Now, considering the perturbation, $\dot{\bar{\mathbf{x}}} = \mathbf{A}_T \bar{\mathbf{x}} + \mathbf{B}_T \bar{\mathbf{u}} + \mathbf{f}_{\text{pert}}(\bar{\mathbf{x}})$, the perturbation matrix can be found as below:

$$\dot{\bar{\mathbf{x}}}_1 = f_1(\bar{\mathbf{x}}) \quad (4.10.15)$$

$$f_{\text{pert}1} = f_1(\bar{\mathbf{x}}) - \mathbf{A}_T(1, :) \bar{x} - \mathbf{B}_T(1, :) \bar{u} \quad (4.10.16)$$

where $\mathbf{A}_T$ is given by (4.10.11), $\mathbf{B}_T$ is given by (4.10.12), $\bar{\mathbf{x}}$ is given by $\bar{\mathbf{x}} = [z, w, x, u, \theta, q]^T$ and $\bar{\mathbf{u}} = [T_l + T_r, T_l - T_r]$. So,

$$\begin{aligned} f_{\text{pert}1} &= f_1(\bar{\mathbf{x}}) - w \\ &= -u \sin \theta + w \cos \theta - w \\ \implies f_{\text{pert}1} &= -u \sin \theta - w(1 - \cos \theta) \end{aligned} \quad (4.10.17)$$

Similarly .

$$\begin{aligned} f_{\text{pert}2} &= f_2(\bar{\mathbf{x}}) - \mathbf{A}_T(2, :) \bar{\mathbf{x}} - \mathbf{B}_T(2, :) \bar{\mathbf{u}} \\ &= g \cos \theta + qu - \frac{1}{m}[(T_l + T_r) \cos \Upsilon \\ &\quad - (H_l - H_r) \sin \Upsilon - 0 \\ &\quad - [-\frac{\cos \Upsilon}{m}(T_l + T_r)] \\ \implies f_{\text{pert}2} &= g \cos \theta + qu + \frac{1}{m}(H_l - H_r) \sin \Upsilon \end{aligned} \quad (4.10.18)$$

Proceeding in a similar way, $f_{\text{pert}3}$, $f_{\text{pert}4}$, $f_{\text{pert}5}$ and $f_{\text{pert}6}$ can be calculated. Finally, the perturbation matrix $\mathbf{f}_{\text{pert}}$ is given by:

$$\mathbf{f}_{\text{pert}} = \begin{bmatrix} -u \sin \theta - w(1 - \cos \theta) \\ g \cos \theta + qu + \frac{1}{m}(H_l - H_r) \sin \Upsilon \\ -u(1 - \cos \theta) + w \sin \theta \\ -g \sin \theta - qw + g\theta + \frac{1}{m}(H_l + H_r) \cos \Upsilon \\ 0 \\ \frac{l}{I}(H_l + H_r) \sin \Upsilon \end{bmatrix} \quad (4.10.19)$$

If we consider Υ , i.e. the rotor tilt angle as a perturbation, the uncertainty i.e. 2^{nd} , 4^{th} and the 6^{th} elements of the $\mathbf{f}_{\text{pert}}$ matrix is acting on the input channels and this a a matched uncertainty as shown in (4.10.12). The uncertainty acting in the input channels is referred to as matched uncertainty and the uncertainty which does not act in the input channels is referred to as unmatched uncertainty [Edwards and Spurgeon, 1998, p.17 and p.37]. The unmatched uncertainty is as shown in (4.10.19). $(H_l - H_r)$ and $(H_l + H_r)$ will be very small and $\sin \Upsilon$ can be approximated to Υ and $\cos \Upsilon$ can be approximated to 1, for small Υ values and can be neglected. The SMC performance is nearly invariant to the change in the rotor tilt angle, Υ . The reason for this is a change in the tilt angle can be modelled as a change in the dynamics of the aircraft i.e. plant perturbation. The uncertainty in the tilt angle represented by (4.5.9), (4.5.7) and (4.5.11) act in the channel of the input. The SMC completely rejects the uncertainty acting in the input channels i.e. matched uncertainty [Edwards and Spurgeon, 1998, p.17 and p.37]. The rotor tilt angle, Υ , terms in (4.5.1) can be treated as a disturbance and is small because of the $\sin \Upsilon$ term multiplied by the difference in the thrust coefficients and the small values of the drag coefficients, that can be neglected. As the uncertainties are acting in the channel of the input, the SMC completely rejects them, and nearly identical results are obtained for different rotor tilt angles. This explains why the outward tilt angle has an impact on the gust rejection properties of the PVTOL using PID control but does not have any significant impact when used with SMC. Similar reasoning applies to the case of a quadrotor as well.

4.11 Chapter Summary and Conclusions

In this chapter, a conceptual birotor has been modelled in body axes without considering the aerodynamics and then the linearised model of the birotor was simulated with a PD controller. Subsequently, the PVTOL has been modelled taking aerodynamics into account and simulated with PID and SMC controllers. Stability Analysis for SMC altitude Control, pitch control and position control has been carried out. The effect of different rotor tilt angles has been studied. Although the outward tilt angle has an impact on the gust rejection properties using PID control, the SMC control outweighs the performance of the PID control and is almost invariant to different rotor tilt angles. An analysis has been undertaken to investigate the reasons for this behaviour when the PVTOL is controlled by SMC. This chapter lays a foundation to extend the PVTOL modelling and simulation studies to a quadrotor taking aerodynamics into account. This fulfils the objectives 2 and 3 of this work as mentioned in Chapter 1.

CHAPTER 5

Effect of Rotor Tilt Angle on Gust Rejection Properties of a Quadrotor using PIV Control

In chapter 4, an evaluation of gust rejection properties of a conceptual PVTOL taking into account the effect of different rotor tilt angles using SMC has been undertaken. Additionally, stability analysis for SMC has been undertaken which can be extended to quadrotors. This offers a good insight into investigating the effect of rotor tilt angle in evaluating the gust rejection properties of a quadrotor taking into account the aerodynamic effects. In this chapter, the effect of the rotor tilt on the stability of the quadrotor aircraft is analysed using PIV control strategy initially considering the idea that by a small fixed-angle outward tilt of the rotors, the aerodynamics can be tailored so that a horizontal gust on the vehicle causes the aircraft to pitch or roll into the wind thus reducing the horizontal position displacement.

5.1 Introduction

In this chapter, a quadrotor model in body axes that includes aerodynamics has been developed. The gust rejection aspects of the quadrotor have been explored using PIV control. The vehicle schematic of the quadrotor is as shown in Figure 5.1. All the four rotors have an outward tilt of Υ . The inertial earth-reference axes frame is $\mathbf{E}$, and the body axes frame is $\mathbf{B}$ and the rotor frames are denoted by R_1 to R_4 . The dynamic model uses the following assumptions:

- the structure is rigid and symmetric,
- the propellers are rigid,
- ground effect is neglected,
- the inertia matrix is diagonal,
- the centre of mass coincides with the origin of the body axes,
- the rotor corialis forces are insignificant,
- and the motor dynamics are ignored.

The Newton-Euler equations of motion body axes frame are given by (3.1.8) and (3.1.9). The body frame translational rates are related to the earth frame $\mathbf{E}$, rates by $\dot{\mathbf{p}} = \mathbf{R}(\phi, \theta, \psi)\mathbf{v}_\mathbf{B}$ where $\mathbf{p} = [x, y, z]^T$ is the position of the centre of gravity in the earth frame, (ϕ, θ, ψ) is the Euler angle triple. The vehicle

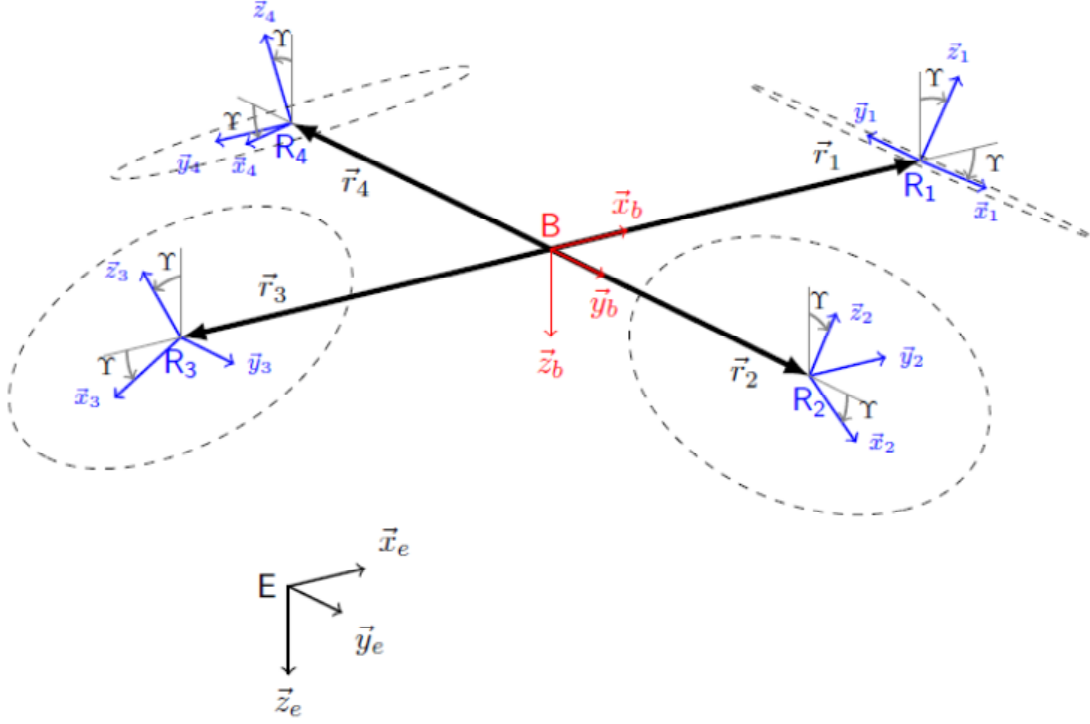


Figure 5.1: Quadrotor vehicle schematic [Whidborne et al., 2022]

rotates through three angles (ϕ, θ, ψ) to move from earth axes to body axes. These are the Euler angles and the rotation matrix, $\mathbf{R}$, is given by equation (3.1.5). The thrust, horizontal disc force and torque triple, (T_i, H_i, Q_i) , for the i^{th} rotor depend on air flow speed and incident angle on the rotor tip plane. Denoting the i^{th} rotor axis unit vector by $\mathbf{z}_i$ as shown in Figure 5.1 and the corresponding air flow vector by $\mathbf{v}_{a_i}$, the incident angle on the i^{th} rotor tip plane, α_i , is given by:

$$\alpha_i = -\arcsin\left(\frac{\mathbf{z}_i \cdot \mathbf{v}_{a_i}}{\|\mathbf{v}_{a_i}\|}\right) \quad (5.1.1)$$

where: $\mathbf{v}_{a_i}$ is the air flow vector given by

$$\mathbf{v}_{a_i} = \boldsymbol{\omega} - \mathbf{b} - \boldsymbol{\omega} \times \mathbf{r}_i \quad (5.1.2)$$

where $\mathbf{v}_\omega$ is the wind vector and $\mathbf{r}_i$ is the i^{th} vehicle arm vector, that is the vector from $\mathbf{B}$ to R_i as shown in Figure 5.1. The resulting force and moment vectors are given by:

$$\mathbf{f}_i = \mathbf{T}_i \mathbf{z}_i + \mathbf{H}_i (\mathbf{v}_{a_i} / \|\mathbf{v}_{a_i}\| + \mathbf{z}_i \sin \alpha_i) \sec \alpha_i \quad (5.1.3)$$

$$\mathbf{q}_i = (-1)^i \mathbf{Q}_i \mathbf{z}_i \quad (5.1.4)$$

The resulting total force, $\mathbf{F}$, is given by:

$$\mathbf{F} = \sum_{i=1}^4 \mathbf{f}_i + mg \mathbf{z}_e \quad (5.1.5)$$

where g is the gravitational constant and the total moment vector, $\mathbf{M}$ is given by:

$$\mathbf{M} = \sum_{i=1}^4 (\mathbf{r}_i \times \mathbf{f}_i + \mathbf{q}_i) \quad (5.1.6)$$

For the quadrotor as shown in Figure 5.1, the equations of motion can be written as shown below:

$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \quad (5.1.7)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (5.1.8)$$

$$\dot{\psi} = q \sin \phi / \cos \theta + r \cos \phi / \cos \theta \quad (5.1.9)$$

$$\dot{p} = \left(\frac{I_y - I_z}{I_x} \right) qr + \frac{L}{I_x}, \quad (5.1.10)$$

$$\dot{q} = \left(\frac{I_z - I_x}{I_y} \right) rp + \frac{M}{I_y}, \quad (5.1.11)$$

$$\dot{r} = \left(\frac{I_x - I_y}{I_z} \right) qr + \frac{N}{I_z}, \quad (5.1.12)$$

$$\dot{u} = F_x/m - qw + rv \quad (5.1.13)$$

$$\dot{v} = F_y/m - ru + pw \quad (5.1.14)$$

$$\dot{w} = F_z/m - pv + qu \quad (5.1.15)$$

$$\dot{x} = u \cos \theta \cos \psi + v(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) + w(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \quad (5.1.16)$$

$$\dot{y} = u \cos \theta \sin \psi + v(\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) + w(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \quad (5.1.17)$$

$$\dot{z} = -u \sin \theta + v \sin \phi \cos \theta + w \cos \phi \cos \theta \quad (5.1.18)$$

where $\mathbf{F}$ and $\mathbf{M}$ are the vectors of the forces and moments acting on the centre of mass as defined in Equations (3.1.1), (3.1.2) and (3.1.3) and (3.1.4) of Chapter 3.

A quadrotor with aerodynamics has been simulated with PIV control using different rotor tilt angles. All the parameters that were suitable for the simulation studies are listed in the MATLAB files listed in Appendix D. The MATLAB/Simulink implementation of the desired pitch and roll angles that are fed into the attitude control block are as shown in Appendix C1.

The simulation results are as illustrated in the subsequent sections. Simulation studies have been conducted firstly without considering any wind and then a horizontal step wind of 10 ms⁻¹ in the x-direction has been considered.

5.2 Simulation Results of PIV Control of a Quadrotor at Hover with 10 ms⁻¹ Wind Gust Step

5.2.1 PIV Control with 10 ms⁻¹ Wind Gust Step for Different Rotor Tilt Angles at Hover

Initially, the position response at hover with a 10 ms⁻¹ wind gust step has been considered. Figures 5.2, 5.3, 5.4 and 5.5 show the simulation results for the position response at hover with a 10 ms⁻¹ wind gust step in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

Figures 5.6, 5.7, 5.8 and 5.9 show the simulation results for the attitude response at hover for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

Figures 5.14 to 5.19 show the comparison of the position responses in both x and y direction and attitude responses at hover for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

For rotor tilt angle $\Upsilon = 0^\circ$ i.e. without any rotor tilt, the PIV controller responds by pitching the aircraft into the wind until the x-position is recovered and settled with a trimmed pitch of 13.5 degrees. The maximum displacement is about 2 m and the maximum pitch is 22 degrees occurring at around 1.8 s. For rotor tilt angle 10 degrees, the PIV controller responds by pitching into the wind after a short downwind displacement, causes an upwind displacement. The controller responds by reducing the pitch so that the x-position is recovered, settling with a trimmed pitch of 14 degrees. The initial maximum downwind displacement is 1.6 m. The maximum pitch is around 23 degrees occurring at around 1.7 s. This demonstrates that the outward tilted configuration has the ability to reject the wind gust.

The altitude responses result in an initial altitude gain due to the additional lift from the lateral wind in all the cases.

Figures 5.10, 5.11, 5.12 and 5.13 show the simulation results for the rotor speed response at hover for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively. It can be seen from the rotor rotation rates, the rotors have to work hardest for $\Upsilon = 10^\circ$ and least for $\Upsilon = 0^\circ$ as expected.

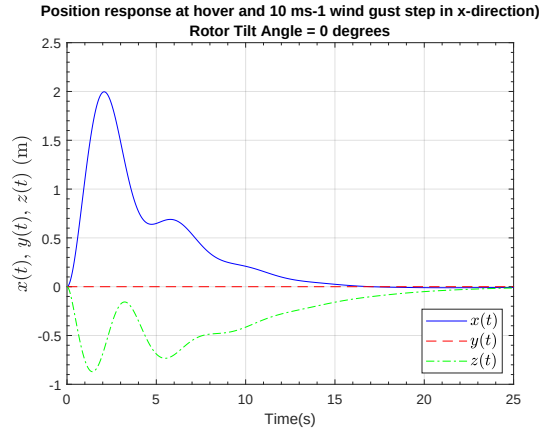


Figure 5.2: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

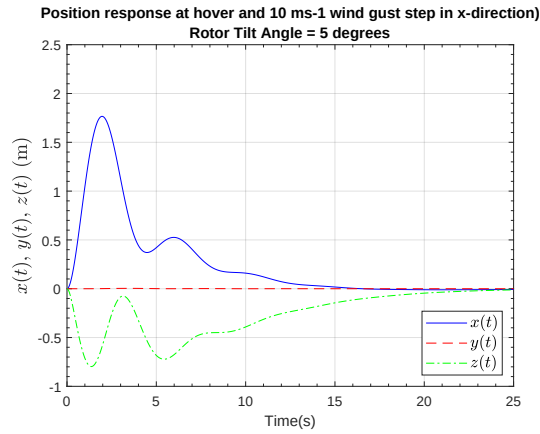


Figure 5.3: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

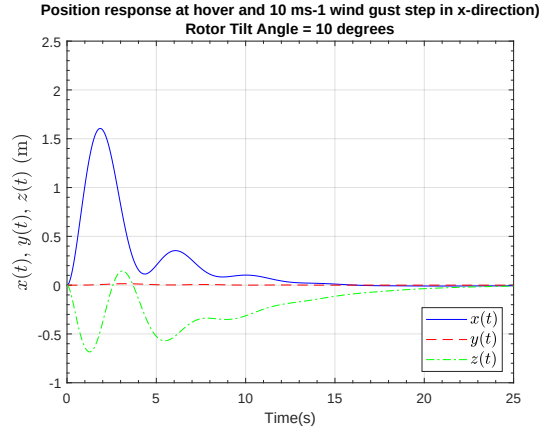


Figure 5.4: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

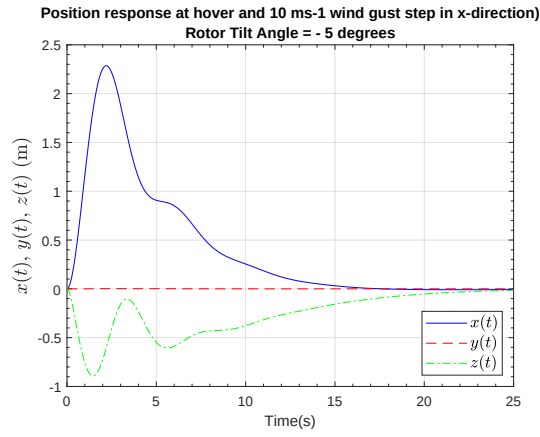


Figure 5.5: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

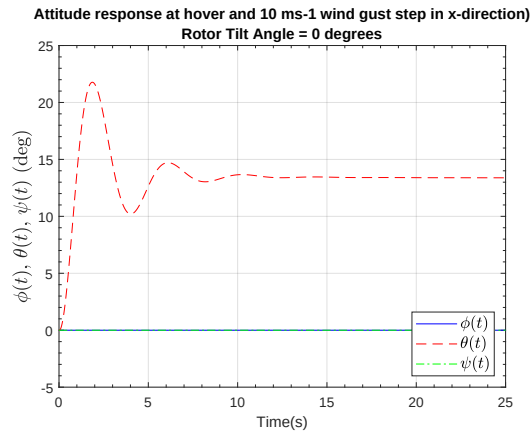


Figure 5.6: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

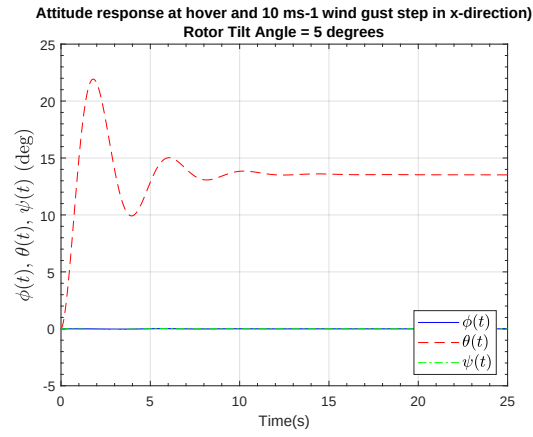


Figure 5.7: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

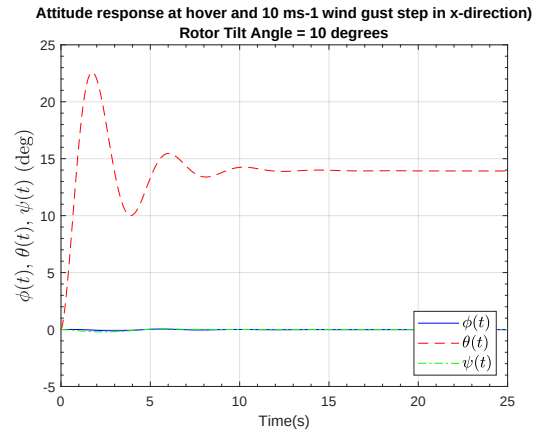


Figure 5.8: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

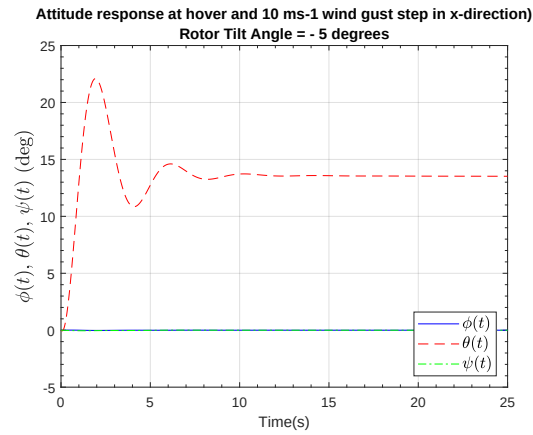


Figure 5.9: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

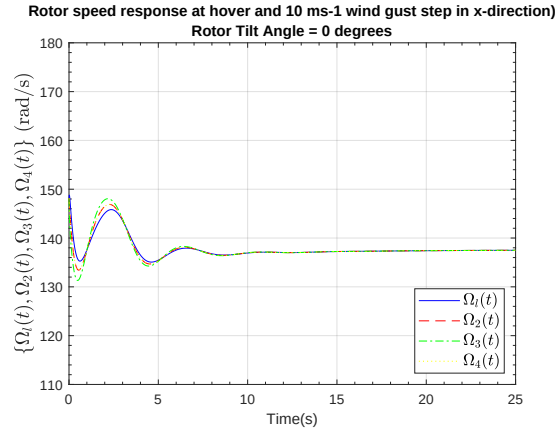


Figure 5.10: Rotor speed response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

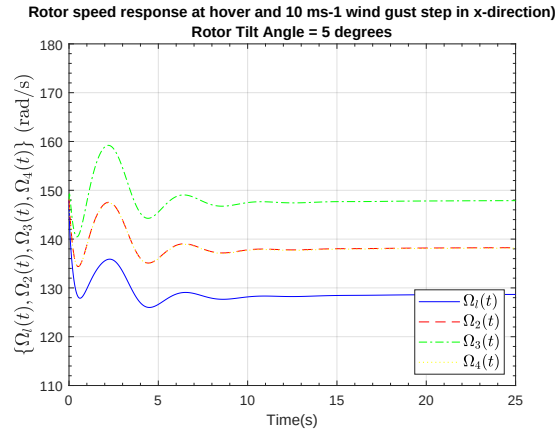


Figure 5.11: Rotor speed response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

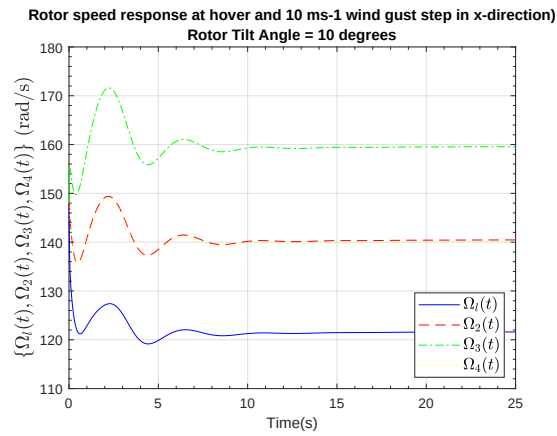


Figure 5.12: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

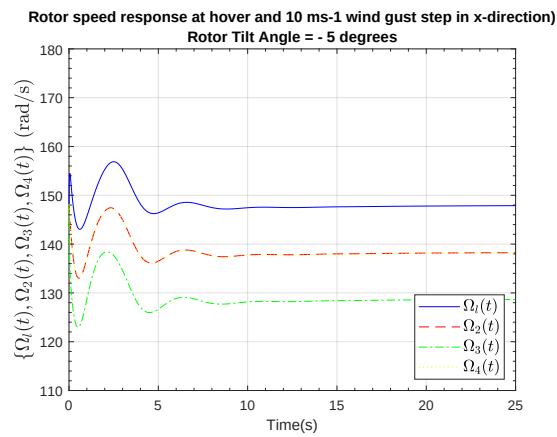


Figure 5.13: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

5.2.2 Comparison of PIV Control with 10 ms⁻¹ Wind Gust for Different Rotor Tilt Angles at Hover

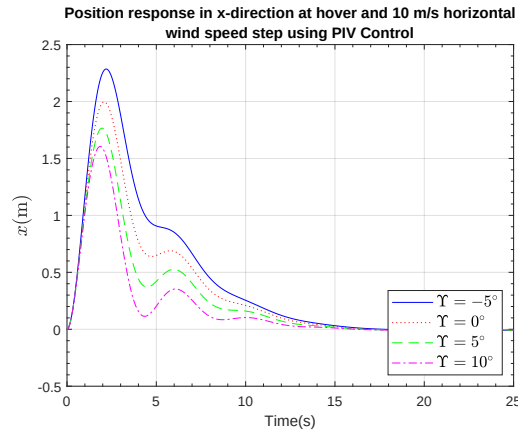


Figure 5.14: Position response to 10 m position step demand in x-direction (with 10 ms⁻¹ horizontal wind gust step)

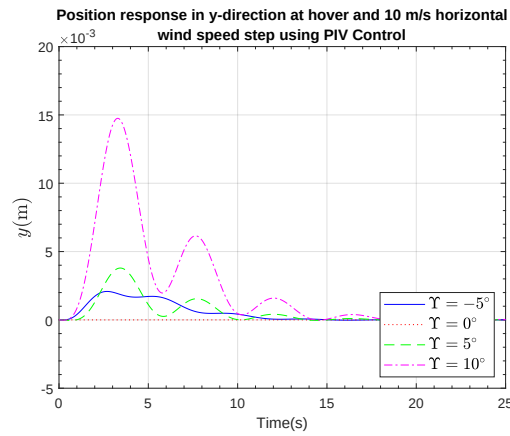


Figure 5.15: Position response to 10 m position step demand in y-direction (with 10 ms⁻¹ horizontal wind gust step)

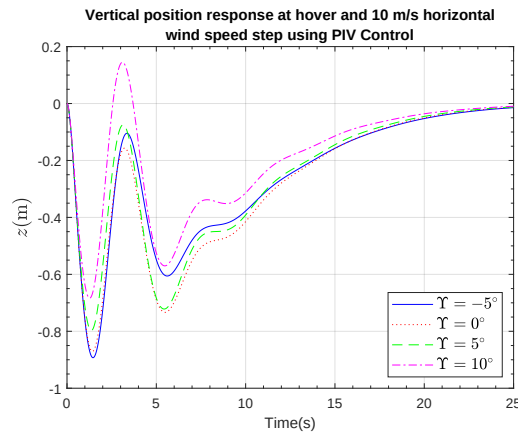


Figure 5.16: Vertical response to 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step)

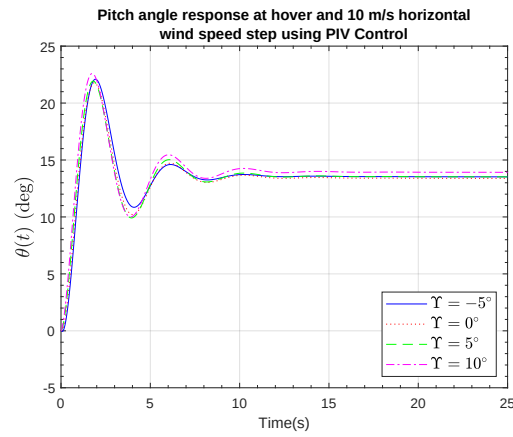


Figure 5.17: Pitch angle response (with 10 ms^{-1} horizontal wind gust step)

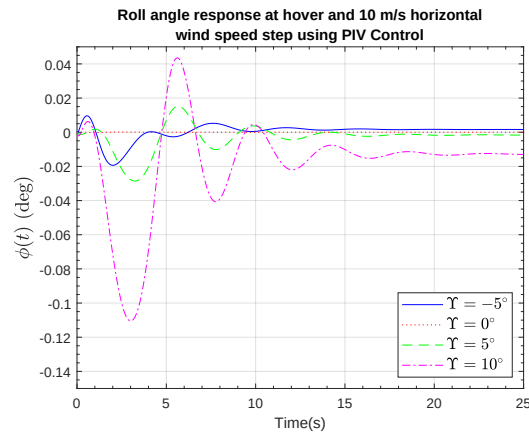


Figure 5.18: Roll angle response (with 10 ms^{-1} horizontal wind gust step)

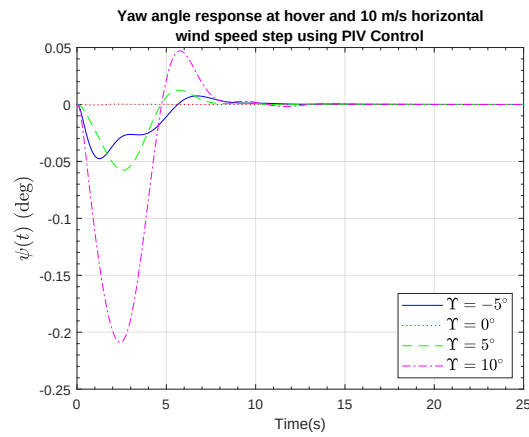


Figure 5.19: Yaw angle response (with 10 ms^{-1} horizontal wind gust step)

5.3 Simulation Results of PIV Control of a Quadrotor for Position and Altitude Step Demands

5.3.1 PIV Control of a Quadrotor for Position and Altitude Step Demands without Wind Gust

Initially, the PIV control of a quadrotor for a position step demand of 10 m in x-direction, 10 m in y-direction and 20 m altitude step demand without wind gust using different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 0^\circ$ and $\Upsilon = -5^\circ$ are considered.

Figures 5.20, 5.21, 5.22 and 5.23 show the simulation results for the position response for 10 m position step demand both in x and y directions and with a 20 m altitude step demand and different rotor angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 0^\circ$ and $\Upsilon = -5^\circ$, respectively. It can be seen in the case of $\Upsilon = 0^\circ$, the position response is almost identical to the case of $\Upsilon = 10^\circ$ which would be a fair means of evaluating the impact of rotor tilt angle on position response. The altitude response $\Upsilon = 0^\circ$ takes a bit longer to reach the steady state when compared to $\Upsilon = 10^\circ$ but not by a significant value as such.

Figures 5.24, 5.25, 5.26 and 5.27 show the simulation results for the attitude response for 10 m position step demand both in x and y directions and with a 20 m altitude step demand and different rotor angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 0^\circ$ and $\Upsilon = -5^\circ$, respectively. It can be seen in the case of $\Upsilon = 0^\circ$, the pitch response is greater than that of $\Upsilon = 10^\circ$. This is because when the aircraft pitches to move forward, the front rotor (R_1) has an additional 10° tilt and because of this an additional force in the x-direction is given by the controller but requiring less pitch.

Figures 5.28, 5.29, 5.30 and 5.31 show the simulation results for the rotor speed response for 10 m position step demand both in x and y directions and with a 20 m altitude step demand and different rotor angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 0^\circ$ and $\Upsilon = -5^\circ$, respectively. It can be seen from the rotor rotation rates, the rotors have to work hardest for $\Upsilon = 10^\circ$ and least for $\Upsilon = 0^\circ$ as expected.

Figures 5.50 to 5.55 shows the comparison of the position responses in both x and y direction and attitude responses for position and altitude step demands for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

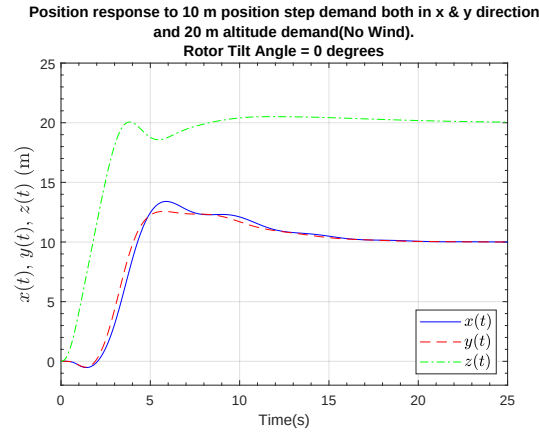


Figure 5.20: Position response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = 0^\circ$

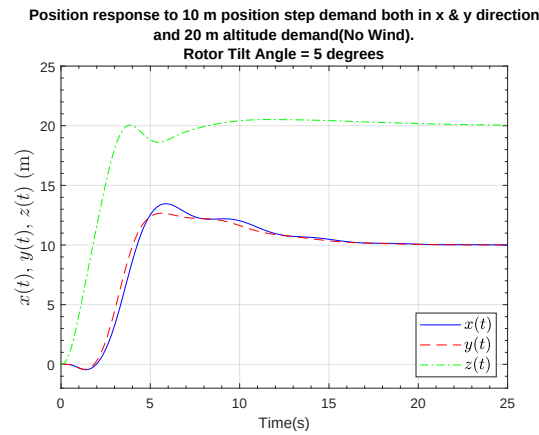


Figure 5.21: Position response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = 5^\circ$

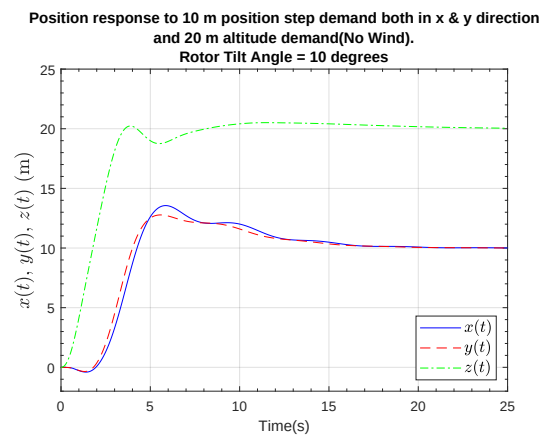


Figure 5.22: Position response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = 10^\circ$

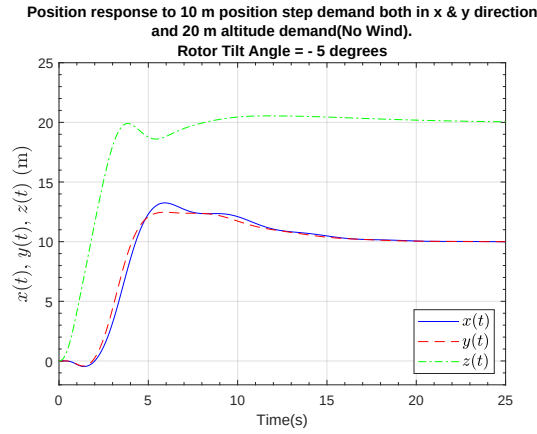


Figure 5.23: Position response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = -5^\circ$

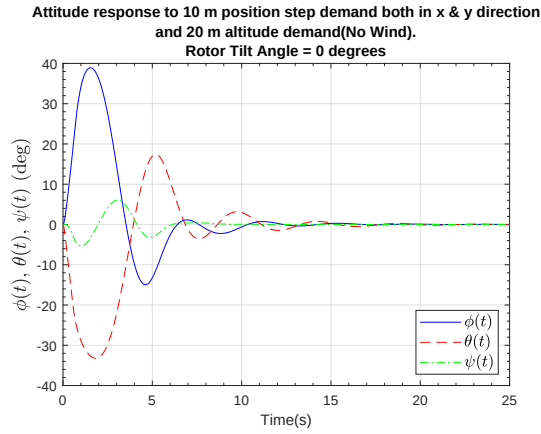


Figure 5.24: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = 0^\circ$

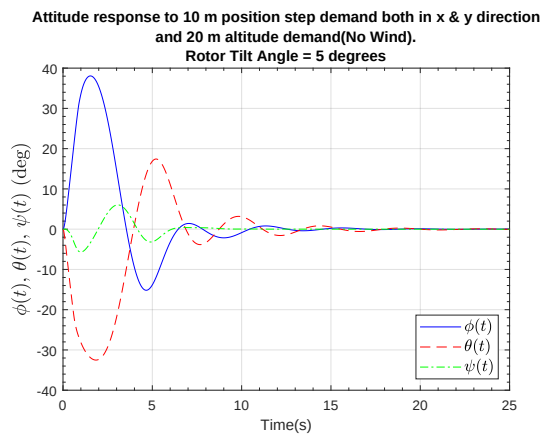


Figure 5.25: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = 5^\circ$

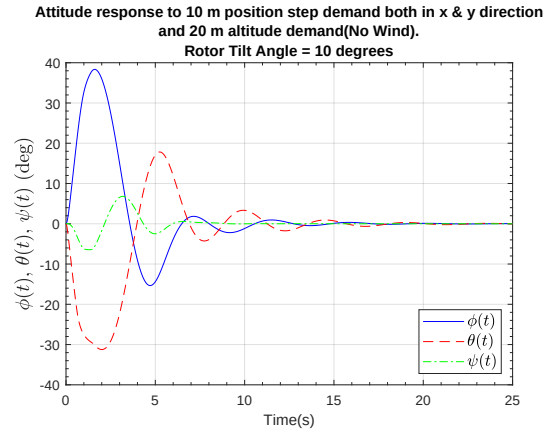


Figure 5.26: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = 10^\circ$

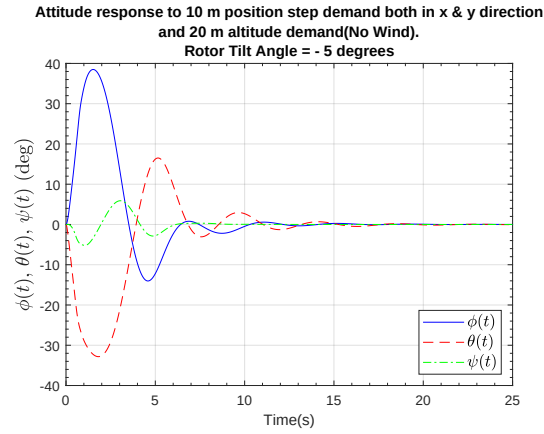


Figure 5.27: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = -5^\circ$

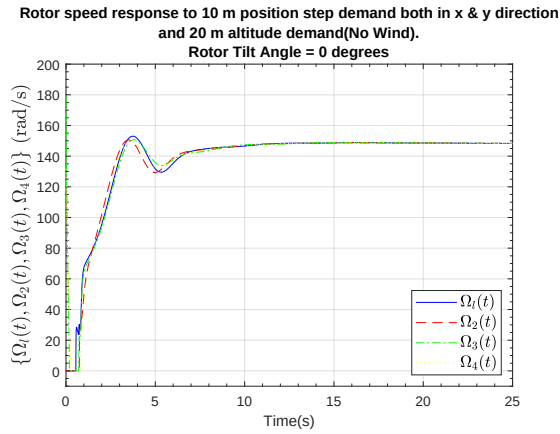


Figure 5.28: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = 0^\circ$

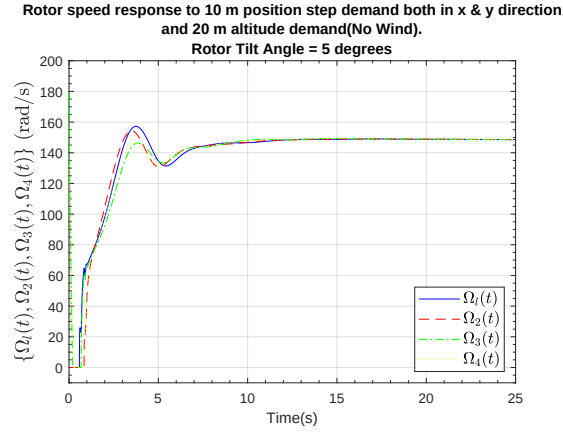


Figure 5.29: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = 5^\circ$

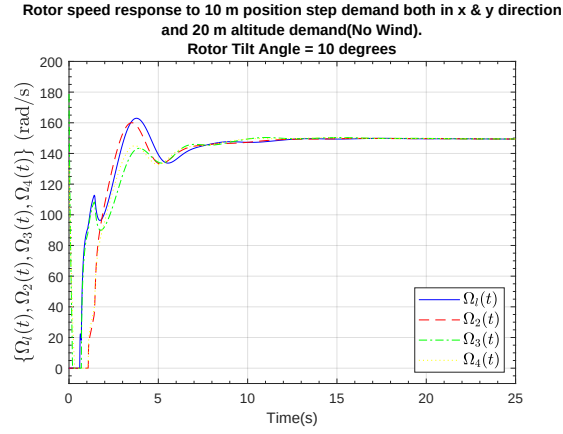


Figure 5.30: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = 10^\circ$

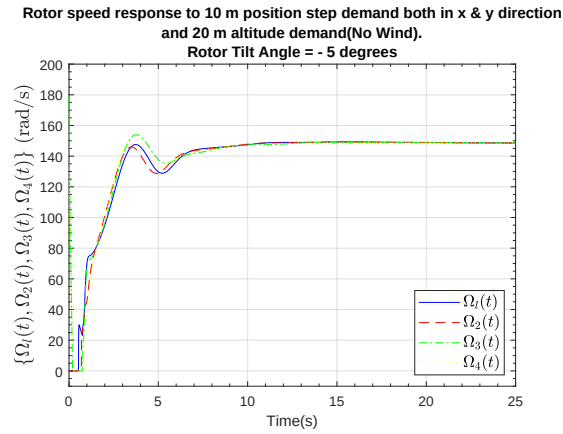


Figure 5.31: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (no wind). $\Upsilon = -5^\circ$

5.3.2 Comparison of PIV Control for Position and Altitude Step Demands without Wind Gust for Different Rotor Tilt Angles

Figure 5.32 shows the comparative position response in x-direction, Figure 5.33 shows the comparative position response in y-direction, Figure 5.34 shows the comparative vertical response and Figures 5.35 to 5.37 shows the comparative attitude responses without wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

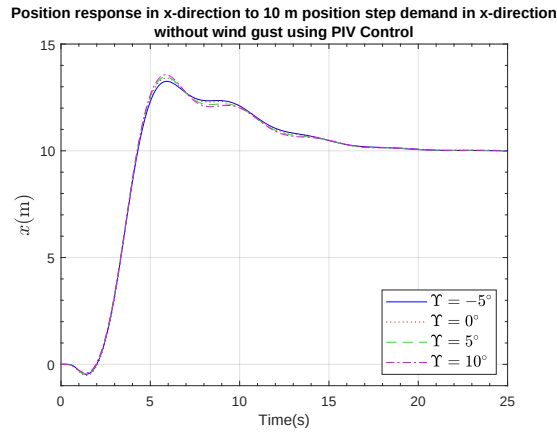


Figure 5.32: Position response to 10 m position step demand in x-direction (without wind gust)

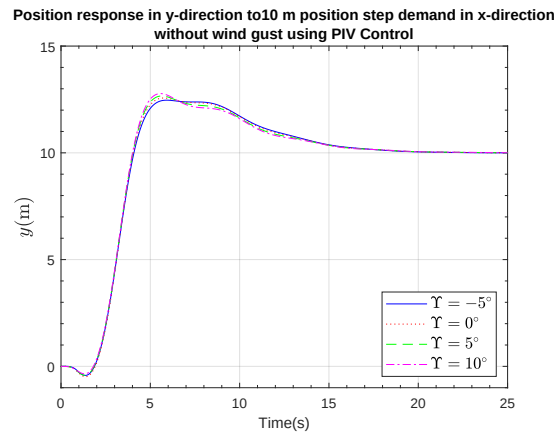


Figure 5.33: Position response to 10 m position step demand in y-direction (without wind gust)

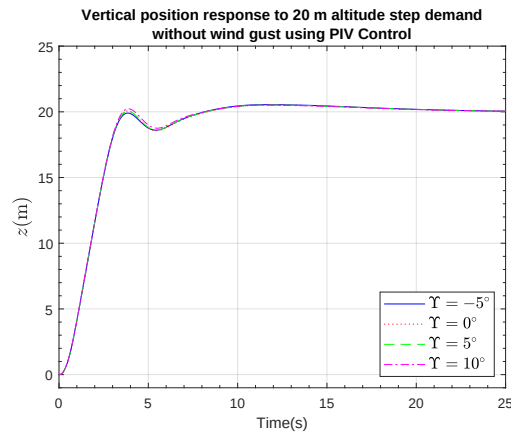


Figure 5.34: Vertical response to 20 m altitude step demand (without wind gust)

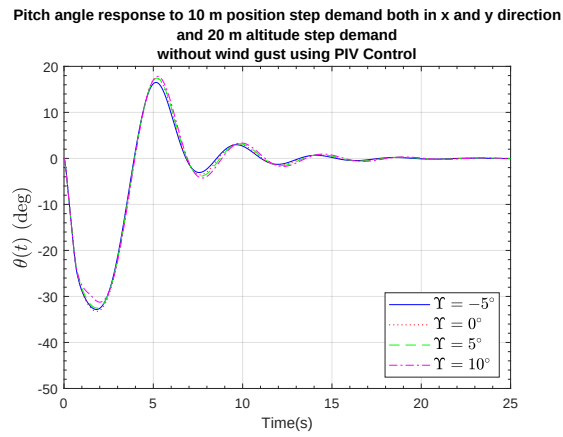


Figure 5.35: Pitch angle response (without wind gust)

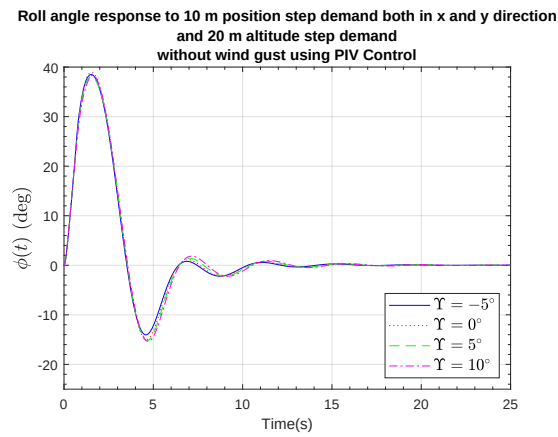


Figure 5.36: Roll angle response (without wind gust)

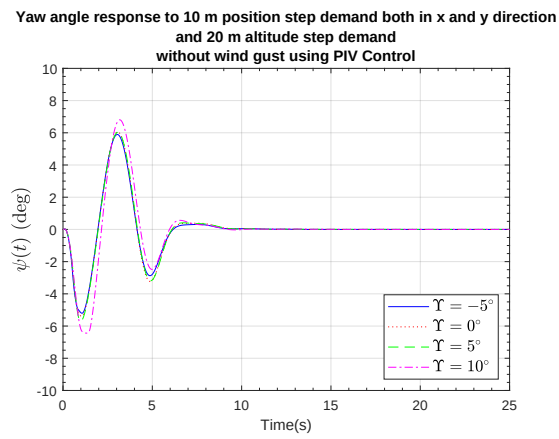


Figure 5.37: Yaw angle response (without wind gust)

5.3.3 PIV Control of a Quadrotor for Position and Altitude Step Demands with 10 ms^{-1} horizontal Wind Gust Step

In this section, the PIV control of a quadrotor for a position step demands of 10 m in x and y-directions respectively and 20 m altitude step demand with a 10 ms^{-1} wind gust in x-direction using different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ are considered.

Figures 5.38, 5.39, 5.40 and 5.41 show the simulation results for the position response for 10 m position step demand both in x and y directions and with a 20 m altitude step demand and different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively. As can be seen from the simulation results, the position response in x-direction for the case of $\Upsilon = 10^\circ$ reaches the steady state quicker when compared to $\Upsilon = 0^\circ$ case and has a lesser overshoot. The position response in y-direction is identical but with a small initial undershoot resulting from the non-minimum phase in the step response of all the cases. The altitude response for a 20 m altitude step demand remains identical for all the 4 rotor tilt angle cases except for a lower settling time in the case of $\Upsilon = 10^\circ$ but with a bit more overshoot when compared to other cases.

Figures 5.42, 5.43, 5.44 and 5.45 show the simulation results for the attitude response for 10 m position step demand both in x and y directions and with a 20 m altitude step demand and different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively. The pitch response for a 20 m altitude step demand remains identical for all the 4 rotor tilt angle cases reaching a maximum pitch of 20° . The maximum pitch is about 2° greater than the untilted case and occurs just after 0.5s.

Figures 5.46, 5.47, 5.48 and 5.49 show the simulation results for the rotor speed response for 10 m position step demand both in x and y directions and with a 20 m altitude step demand and different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively. It can be seen from the rotor rotation rates, the rotors have to work hardest for $\Upsilon = 10^\circ$ and least for $\Upsilon = 0^\circ$ as expected.

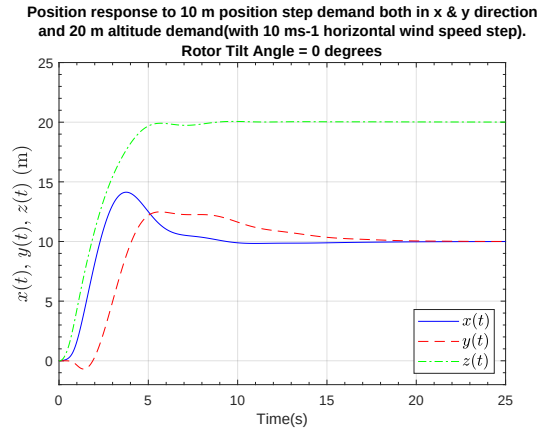


Figure 5.38: Position response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms^{-1} horizontal wind gust step). $\Upsilon = 0^\circ$

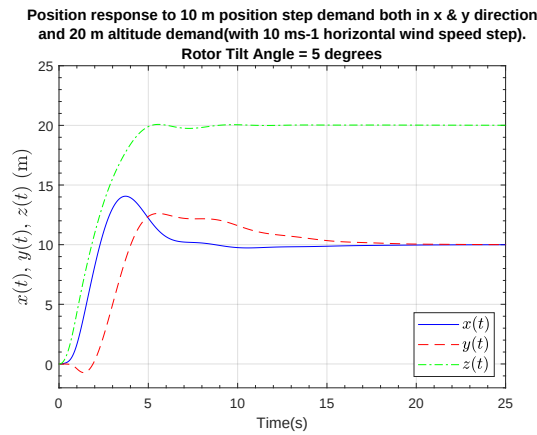


Figure 5.39: Position response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

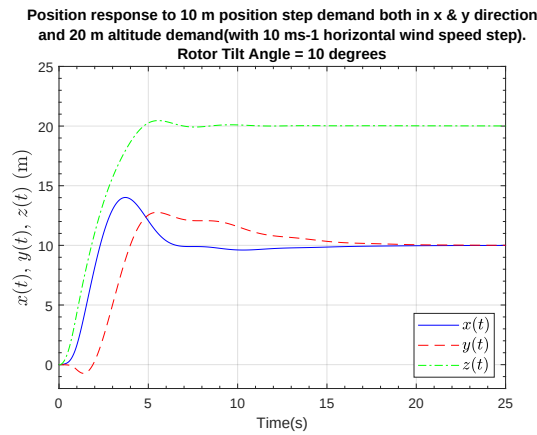


Figure 5.40: Position response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

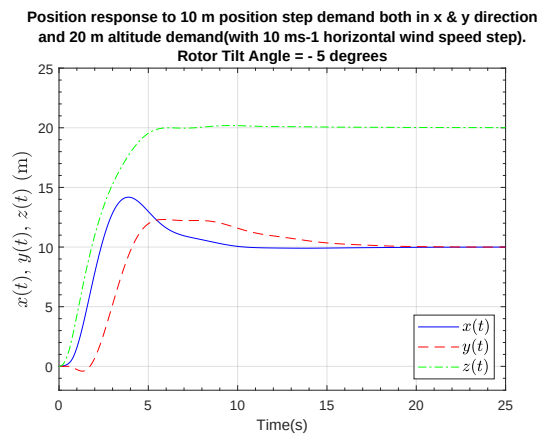


Figure 5.41: Position response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

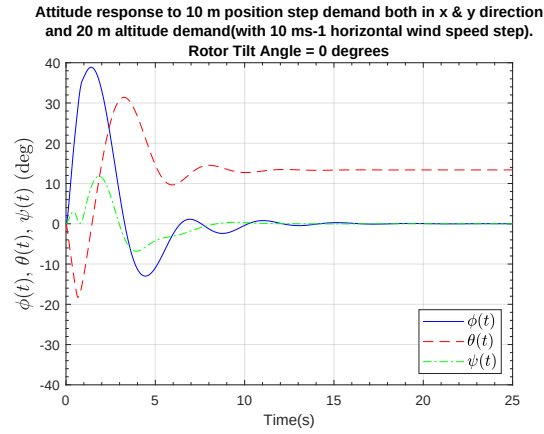


Figure 5.42: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

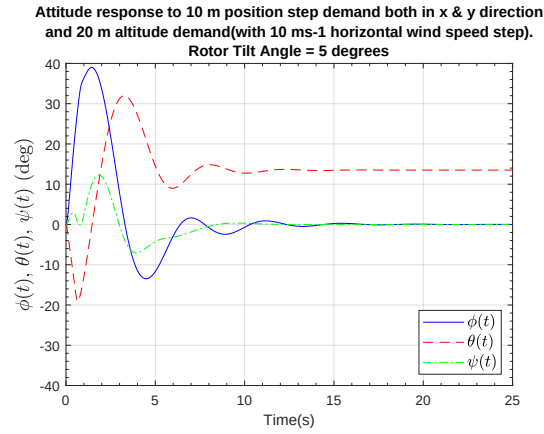


Figure 5.43: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

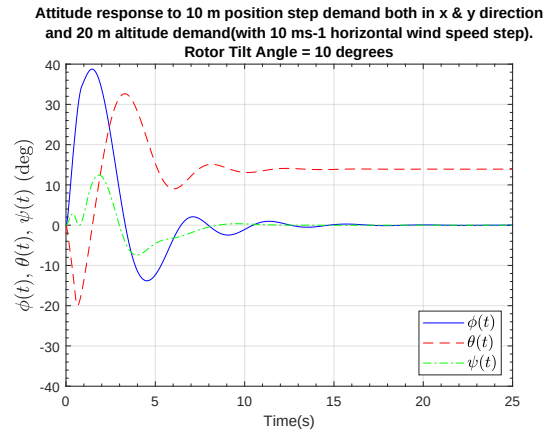


Figure 5.44: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

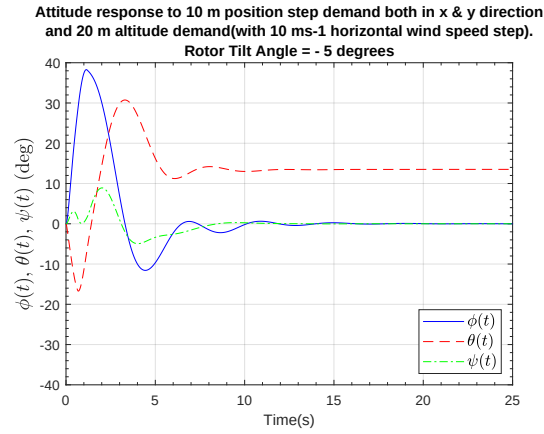


Figure 5.45: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

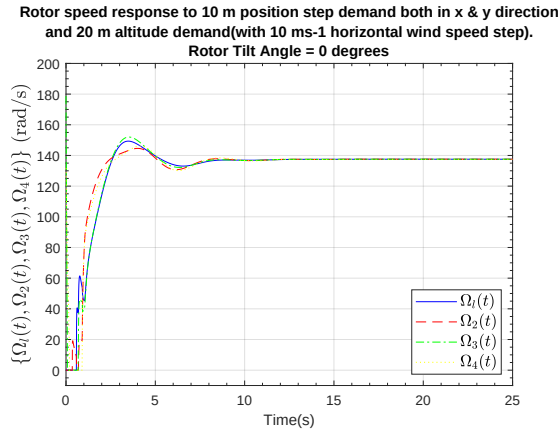


Figure 5.46: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

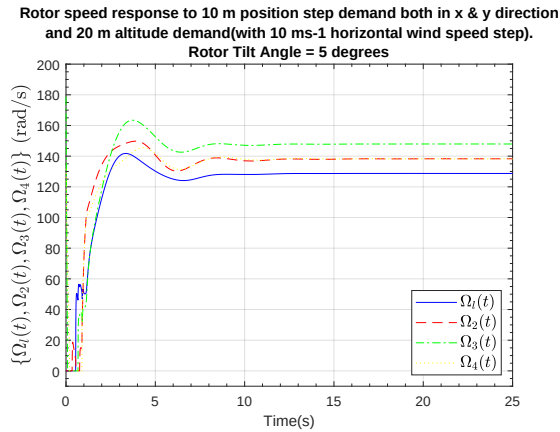


Figure 5.47: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

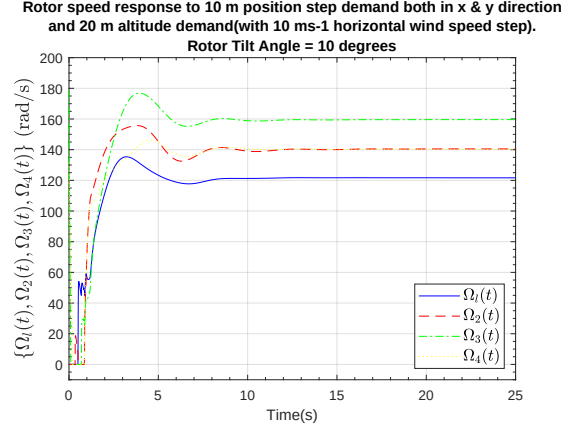


Figure 5.48: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

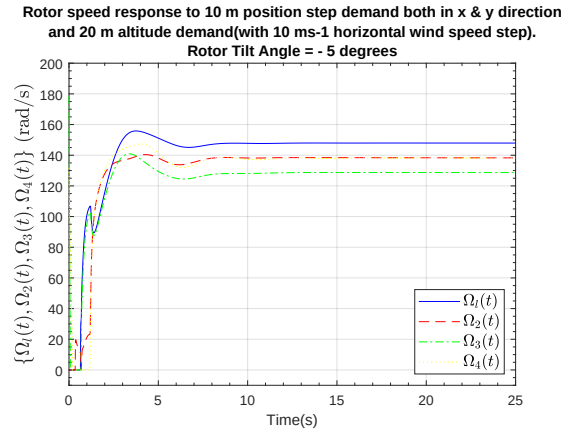


Figure 5.49: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude step demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

5.3.4 Comparison of PIV Control for Position and Altitude Step Demands with 10 ms^{-1} Horizontal Wind Gust for Different Rotor Tilt Angles

Figure 5.50 shows the comparative position response in x-direction, Figure 5.51 shows the comparative position response in y-direction, Figure 5.52 shows the comparative vertical response and Figures 5.53 to 5.55 shows the comparative attitude responses with 10 ms^{-1} wind gust step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

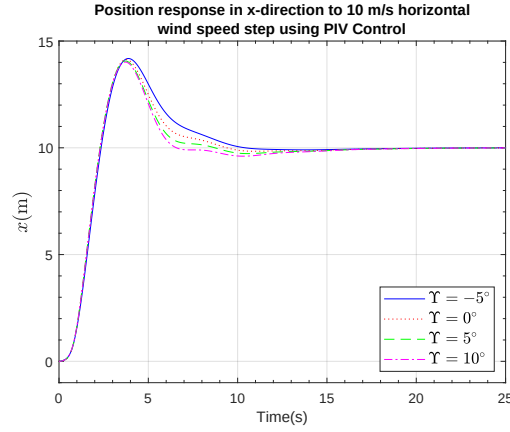


Figure 5.50: Position response to 10 m position step demand in x-direction (with 10 ms^{-1} horizontal wind gust step)

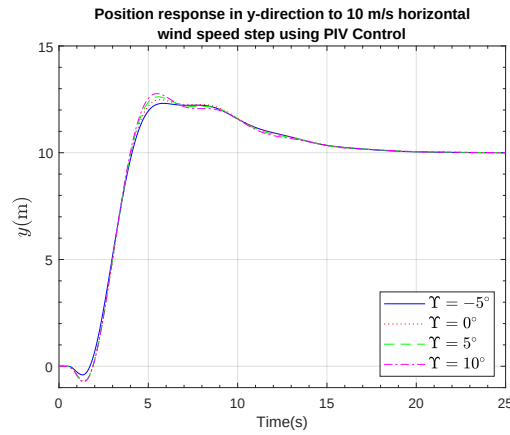


Figure 5.51: Position response to 10 m position step demand in y-direction (with 10 ms^{-1} horizontal wind gust step)

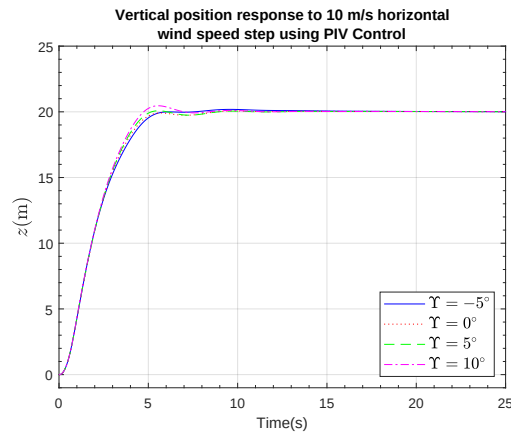


Figure 5.52: Vertical response to 20 m altitude step demand (with 10 ms^{-1} horizontal wind gust step)

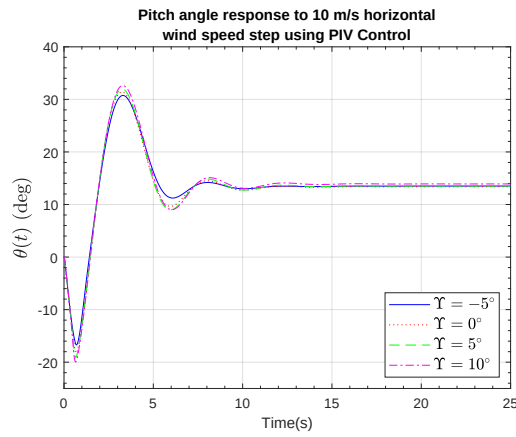


Figure 5.53: Pitch angle response (with 10 ms^{-1} horizontal wind gust step)

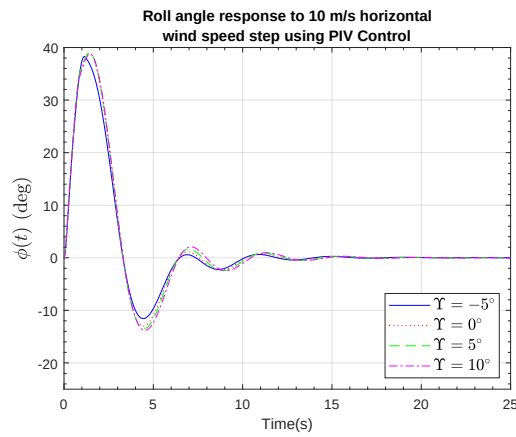


Figure 5.54: Roll angle response (with 10 ms^{-1} horizontal wind gust step)

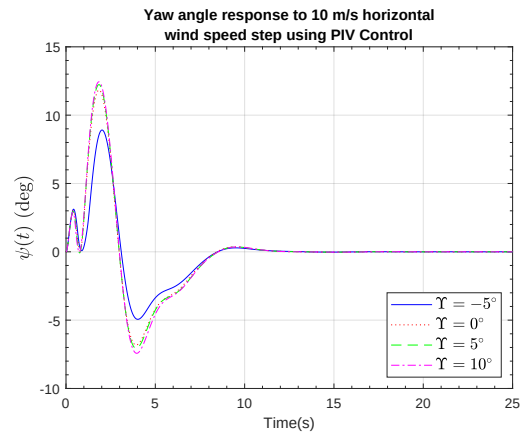


Figure 5.55: Yaw angle response (with 10 ms^{-1} horizontal wind gust step)

5.4 Chapter Summary and Conclusions

In this chapter, effect of rotor tilt angle on gust rejection has been studied using PIV control. The quadrotor in body axes has been modelled taking into account the aerodynamics as well and the effect of outward rotor tilt has been considered and simulation studies carried out. Simulation studies have been carried out at hover without the effect of wind and also with wind for different rotor tilt angles. Subsequently, simulation studies have been carried out for position and altitude step demands firstly without the effect of wind and then with a steady horizontal wind gust step of 10 m/s have been considered. The simulation also illustrated the attitude and rotor speed responses as well. With the simulation results presented, it is shown that an outwards rotor tilt, which introduces anhedral into the multirotor aircraft, can reduce the sensitivity of the vehicle position to gusts using PIV control. The results obtained in this research work are not as significant with regards to the impact of rotor tilt angle on gust rejection properties as those obtained by Whidborne et al. [2022] where LQR controller has been used. This may be due to the optimal tuning of the PIV controller gains. This chapter opens a window of opportunity to explore more towards investigating the effect of rotor tilt angle on gust rejection properties of a quadrotor with a sliding mode controller which is discussed in chapter 6. This fulfils the objective 4 of this work as mentioned in Chapter 1 w.r.t PIV control of the quadrotor.

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CHAPTER 6

Gust Rejection Properties of a Quadrotor using Sliding Mode Control

In chapter 5, the effect of the rotor tilt on the stability of the quadrotor aircraft is analysed using PIV control strategy and offers the inspiration to explore the wind gust rejection properties of quadrotors using SMC and evaluating the effect of rotor tilt angle. In this chapter, a comparative analysis between the PIV and SMC control is considered and it is demonstrated that the sliding mode control produces almost identical responses for different rotor tilt angles in the presence of gusts and is very robust at rejecting the gust and shows that the rotor tilt angle does not impact on the vehicle stability even in the presence of parametric uncertainty and external disturbances.

6.1 Introduction

In this chapter, the same dynamic model as designed in chapter 5, is used for investigating the gust rejection aspects of the quadrotor using SMC. The dynamic equations of the quadrotor used are as follows:

$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \quad (6.1.1)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (6.1.2)$$

$$\dot{\psi} = q \sin \phi / \cos \theta + r \cos \phi / \cos \theta \quad (6.1.3)$$

$$\dot{p} = \left(\frac{I_y - I_z}{I_x} \right) qr + \frac{L}{I_x}, \quad (6.1.4)$$

$$\dot{q} = \left(\frac{I_z - I_x}{I_y} \right) rp + \frac{M}{I_y}, \quad (6.1.5)$$

$$\dot{r} = \left(\frac{I_x - I_y}{I_z} \right) qr + \frac{N}{I_z}, \quad (6.1.6)$$

$$\dot{u} = F_x/m - qw + rv \quad (6.1.7)$$

$$\dot{v} = F_y/m - ru + pw \quad (6.1.8)$$

$$\dot{w} = F_z/m - pv + qu \quad (6.1.9)$$

$$\dot{x} = u \cos \theta \cos \psi + v(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) + w(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \quad (6.1.10)$$

$$\dot{y} = u \cos \theta \sin \psi + v(\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) + w(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \quad (6.1.11)$$

$$\dot{z} = -u \sin \theta + v \sin \phi \cos \theta + w \cos \phi \cos \theta \quad (6.1.12)$$

The quadrotor modelled in body axes has been simulated with a sliding mode controller. The control inputs for the SMC are given by:

$$U_1 = m [g \cos \theta \cos \phi - pv + qu + \ddot{z}_d + \lambda_z \dot{e}_z + k_z \operatorname{sgn}(s_z)] \quad (6.1.13)$$

$$U_2 = I_x [-(\frac{I_y - I_z}{I_x})qr + \ddot{\phi}_d + \lambda_\phi \dot{e}_\phi + k_\phi \operatorname{sgn}(s_\phi)] \quad (6.1.14)$$

$$U_3 = I_y [-(\frac{I_z - I_x}{I_y})pr + \ddot{\theta}_d + \lambda_\theta \dot{e}_\theta + k_\theta \operatorname{sgn}(s_\theta)] \quad (6.1.15)$$

$$U_4 = I_z [-(\frac{I_x - I_y}{I_z})pq + \ddot{\psi}_d + \lambda_\psi \dot{e}_\psi + k_\psi \operatorname{sgn}(s_\psi)] \quad (6.1.16)$$

where λ and k are positive constant values and signum function is defined by (2.2.5). The desired pitch, θ_d , and the desired roll angle, ϕ_d , are given as:

$$\theta_d = [u_x \cos \psi - u_y \sin \psi] \quad (6.1.17)$$

$$\phi_d = [u_x \sin \psi - u_y \cos \psi] \quad (6.1.18)$$

where

$$u_x = [\ddot{x} + \lambda_x \dot{e}_x + k_x \operatorname{sgn}(s_x)] \quad (6.1.19)$$

$$u_y = [\ddot{y} + \lambda_y \dot{e}_y + k_y \operatorname{sgn}(s_y)] \quad (6.1.20)$$

In order to address the slow simulation time and chattering issue, the $\operatorname{sgn}(s)$ has been replaced by the hyperbolic tangent function[Leśniewski and Bartoszewicz, 2015]. All the parameters that were suitable for the simulation studies are listed in the MATLAB files listed in Appendix D. The MATLAB/Simulink implementation of the desired pitch and roll angles that are fed into the attitude control block are as shown in Appendix C5. For details of the SMC controller implementation in Simulink, refer to Appendix C.4 and C.5.

6.2 Simulation Results of Quadrotor with Aerodynamics using SMC for Altitude and Attitude Control and PIV for Position Control

The simulation results of quadrotor with aerodynamics at hover and with altitude and position demands in the absence of any parametric variations and external disturbance are included in Appendix E.1. The simulation results of quadrotor with aerodynamics at hover without and with wind gust and in the presence of any parametric variations and external disturbance are included in Appendix F. In this section, the simulation results of quadrotor for position and altitude demands, in the presence of parametric variations and external disturbance, without and with a 10 ms^{-1} wind gust are considered. In the cascade controller, i.e. SMC control for altitude, attitude and PIV for position control, in the inner loop SMC is used for attitude and altitude control and PIV control is used in the outer loop to improve the performance of the control system when compared to PIV alone, especially in the case of disturbances. The model uncertainties, the external disturbances and reduce the effect of nonlinearities can be addressed in the inner loop which is SMC even before the whole system performance is affected.

6.2.1 SMC Control of Altitude and Attitude and PIV Position Control for Position and Altitude Demands without Wind Gust Step

Simulation results have also been obtained for the case of position and altitude demands initially in the absence of wind gust and thereafter by considering wind gust as well. Simulation results have been obtained for the case of a 10 m position step demand both in x and y direction and 20 m altitude demand using SMC in the presence of parametric uncertainty and external disturbance.

Figures 6.1, 6.2, 6.3 and 6.4 show the simulation results for the position response for 10 m position step demand both in x and y direction and 20 m altitude demand without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

Figures 6.5, 6.6, 6.7 and 6.8 show the simulation results for the attitude response for 10 m position step demand both in x and y direction and 20 m altitude demand without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

As can be seen from the simulation results, there is no significant difference in the position response except for a maximum displacement of around 14 m for x when $\Upsilon = 10^\circ$ whereas other tilt angles have an initial maximum displacement of about 12 m.

With regards to attitude responses, the SMC controller responds by pitching the aircraft into the wind until the x-position is recovered and stabilises. Except for the case of $\Upsilon = 10^\circ$, where after pitching into the wind, it has a maximum pitch of around 20° more when compared to other cases.

Figures 6.9, 6.10, 6.11 and 6.12 show the simulation results for the rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance. It can be seen from the rotor rotation rates, the rotors work with the same rates for different rotor tilt angles with a mean value around 150 rad/s. But for the case of $\Upsilon = 10^\circ$, there is lot of variation in the rotor speeds until 5 seconds but eventually settles down at 150 rad/s.

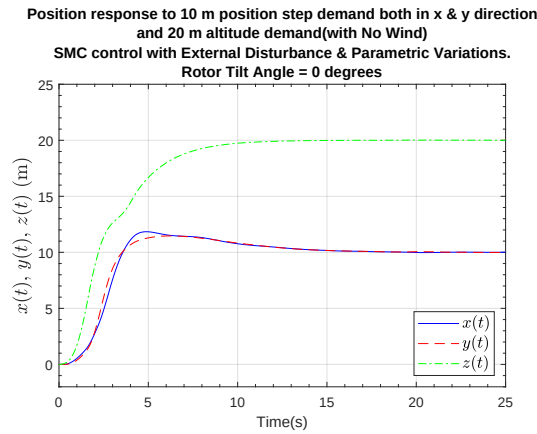


Figure 6.1: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = 0^\circ$

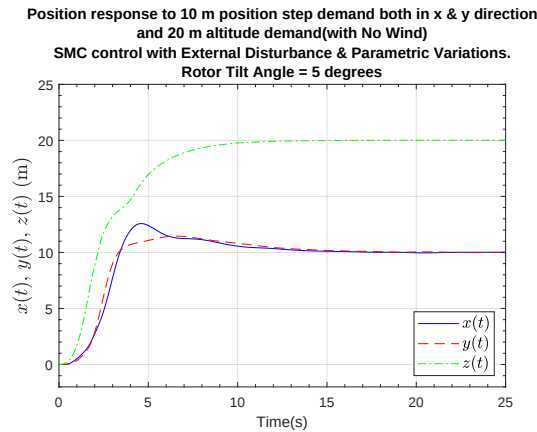


Figure 6.2: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = 5^\circ$

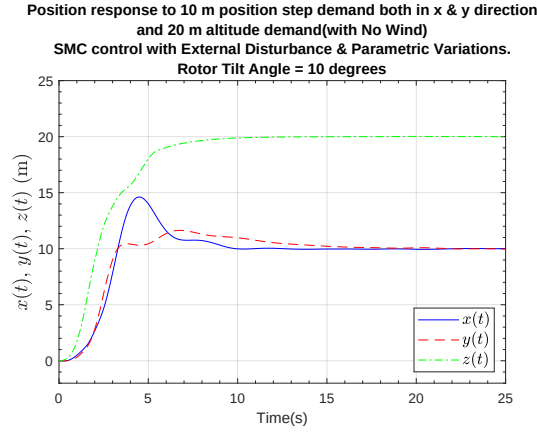


Figure 6.3: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = 10^\circ$

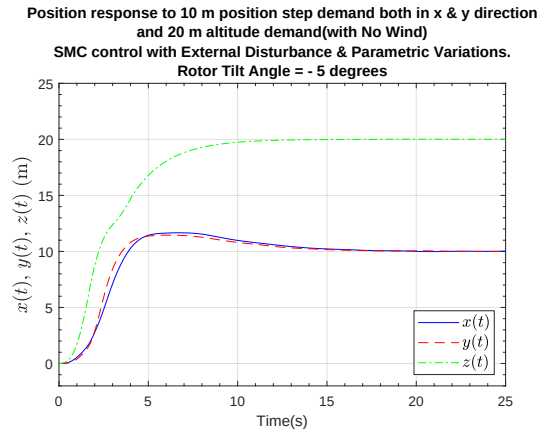


Figure 6.4: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = -5^\circ$

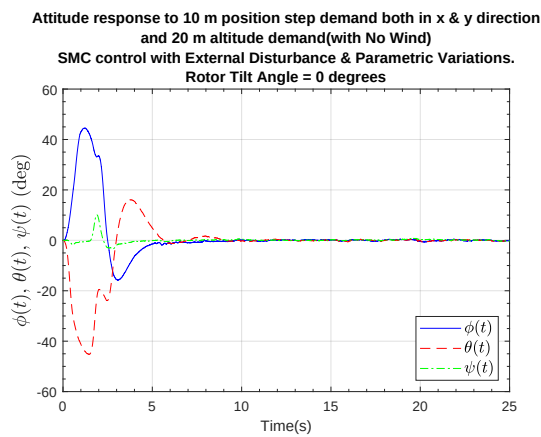


Figure 6.5: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = 0^\circ$

6.2.2 Comparison of SMC Control of Altitude and Attitude and PIV Position Control of Quadrotor for Position and Altitude Demands and without any Wind Gust for Different Rotor Tilt Angles

Figure 6.13 shows the comparative position response in x-direction, Figure 6.14 shows the comparative position response in y-direction, Figure 6.15 shows the comparative vertical response and Figures 6.16

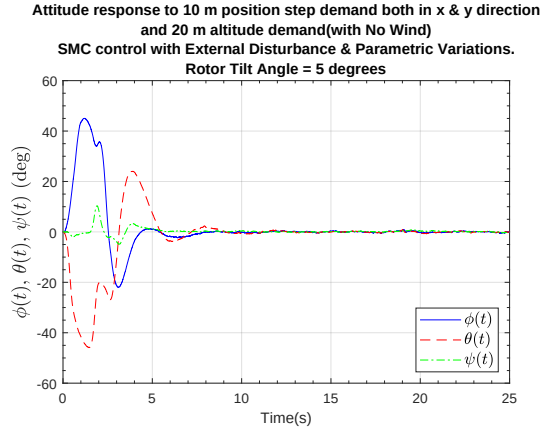


Figure 6.6: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = 5^\circ$

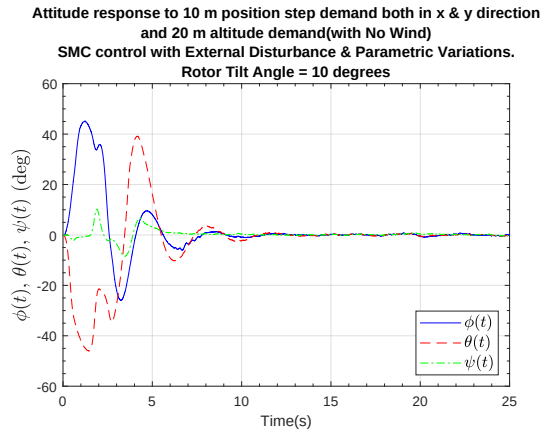


Figure 6.7: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = 10^\circ$

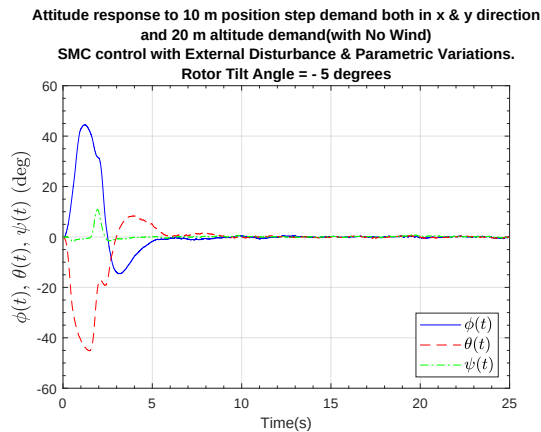


Figure 6.8: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = -5^\circ$

to 6.18 shows the comparative attitude responses without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

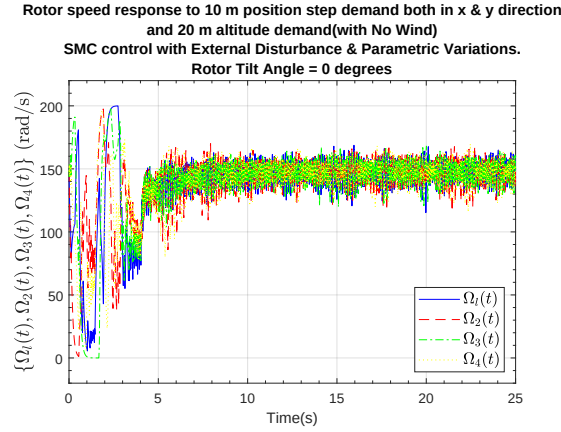


Figure 6.9: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = 0^\circ$

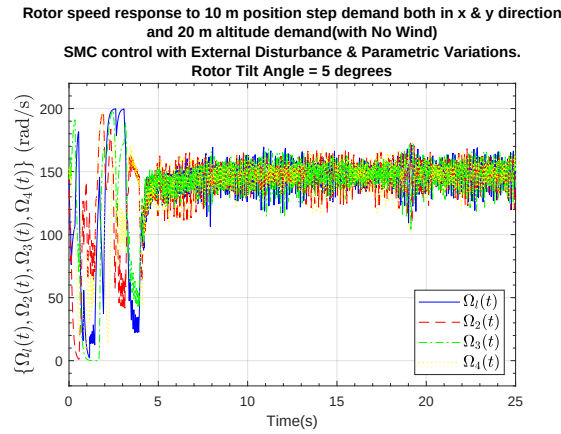


Figure 6.10: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = 5^\circ$

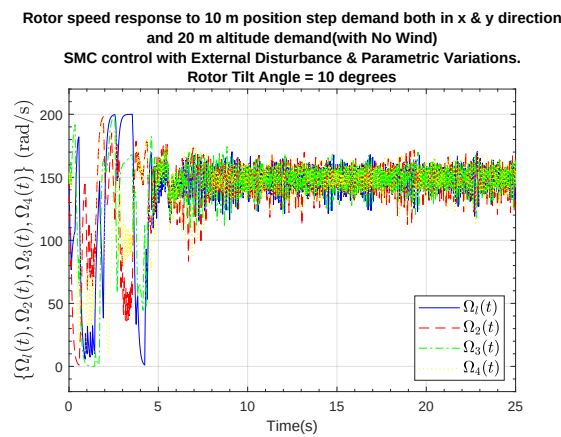


Figure 6.11: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = 10^\circ$

6.2.3 SMC Control of Altitude and Attitude and PIV Position Control for Position and Altitude Demands with 10 ms^{-1} Wind Gust Step

Simulation results have been obtained for the case of position and altitude demands by considering wind gust as well. Simulation results have been obtained for the case of a 10 m position step demand both in

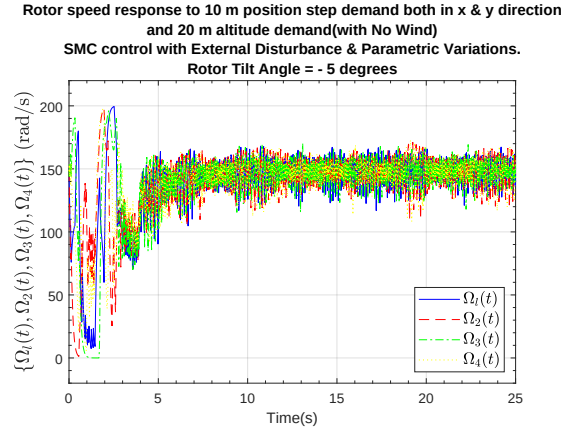


Figure 6.12: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind). $\Upsilon = -5^\circ$

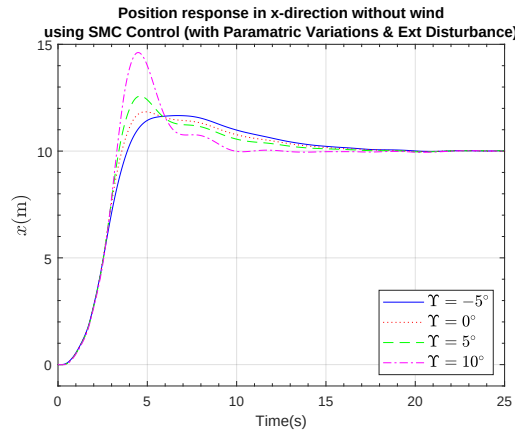


Figure 6.13: Position response in x-direction at hover without wind gust

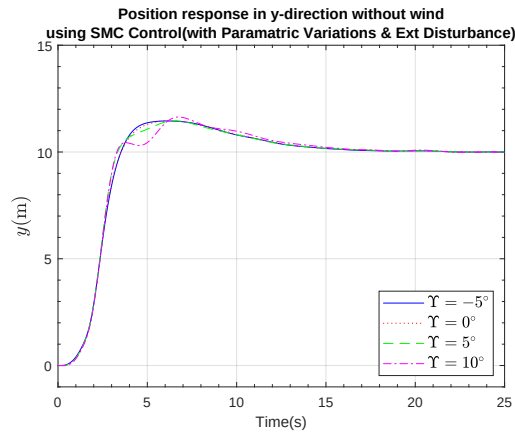


Figure 6.14: Position response in y-direction at hover without wind gust

x and y direction and 20 m altitude demand using SMC in the presence of parametric uncertainties and external disturbance as defined in Chapter 5.

Figures 6.19, 6.20, 6.21 and 6.22 show the simulation results for the position response for 10 m position step demand both in x and y direction and 20 m altitude demand with 10 ms^{-1} wind gust step for

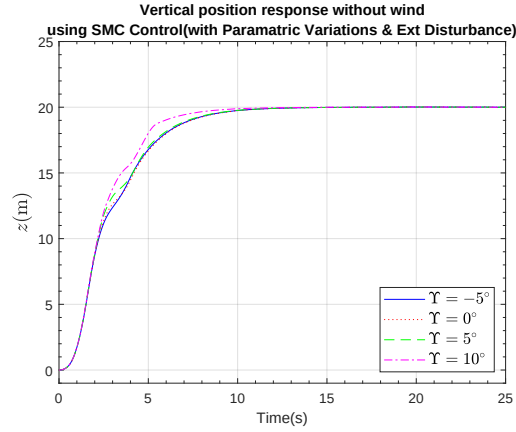


Figure 6.15: Vertical response at hover without wind gust

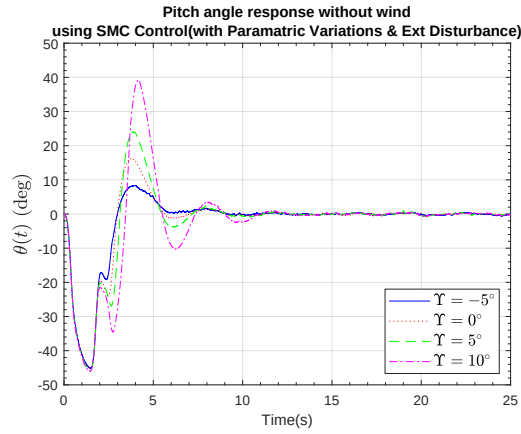


Figure 6.16: Pitch angle response at hover without wind gust

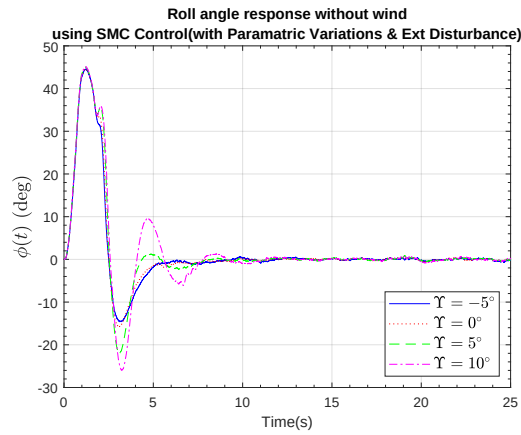


Figure 6.17: Roll angle response at hover without wind gust

different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

Figures 6.23, 6.24, 6.25 and 6.26 show the simulation results for the attitude response for 10 m position step demand both in x and y direction and 20 m altitude demand with 10 ms^{-1} wind gust step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

As can be seen from the simulation results, there is no significant difference in the position response in

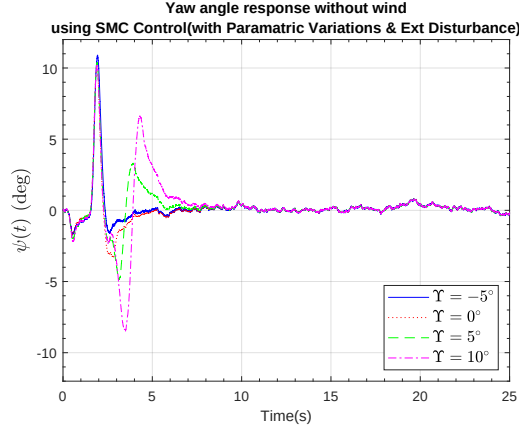


Figure 6.18: Yaw angle response at hover without wind gust

x-direction except for the case of $\Upsilon = 10^\circ$ where the maximum displacement is slightly lesser than the other cases and settles 4 seconds earlier than other cases. The position response for the case of $\Upsilon = 10^\circ$ has slightly increased initial maximum displacement when compared to other cases but again there is not much significant differences between different rotor tilt angle cases. The settling time for all the cases is nearly 12 seconds.

With regards to attitude responses, the SMC controller responds by pitching the aircraft into the wind until the x-position is recovered and stabilises. All of the rotor tilt angle cases have similar pitch angle responses and settling with a trimmed tilt of just under 12° .

Figures 6.27, 6.28, 6.29 and 6.30 show the simulation results for the rotor speed response with 10 ms^{-1} Wind Gust Step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance. It can be seen from the rotor rotation rates, the rotors work with the same rates for different rotor tilt angles with a mean value around 140 rad/s. But in all the rotor tilt cases, there is lot of variation in the rotor speeds until 5 seconds but eventually settle down at 140 rad/s.

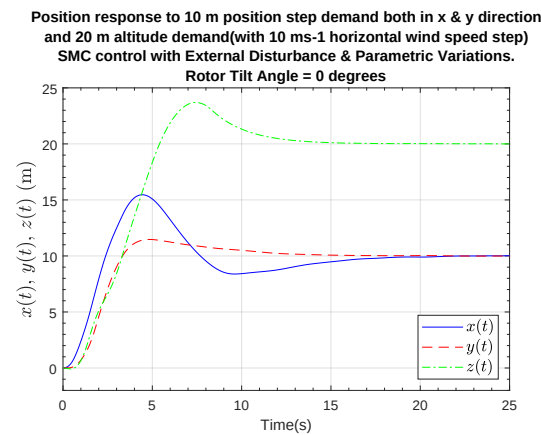


Figure 6.19: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms^{-1} horizontal wind gust step). $\Upsilon = 0^\circ$

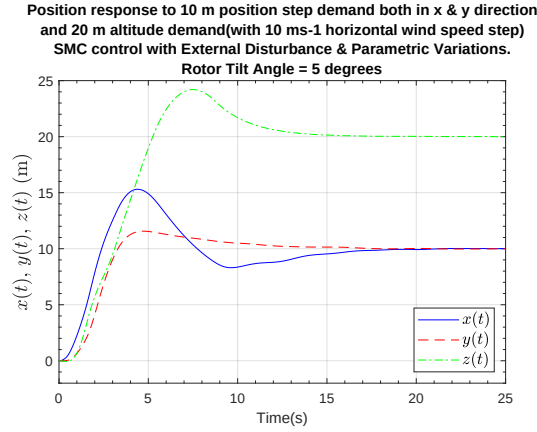


Figure 6.20: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

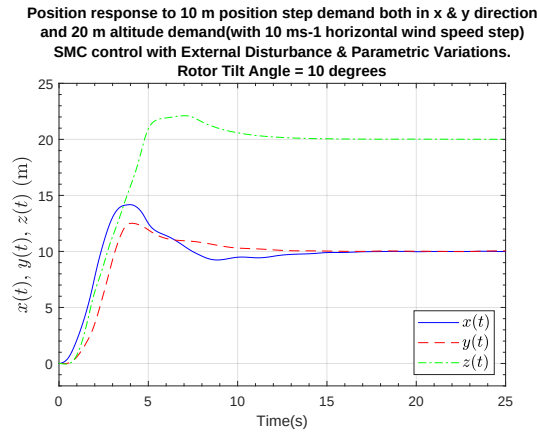


Figure 6.21: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

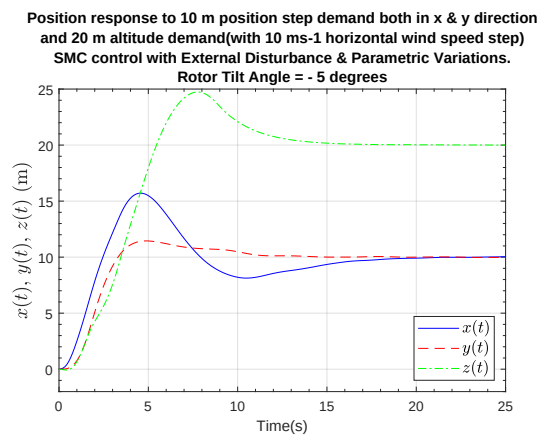


Figure 6.22: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

6.2.4 Comparison of SMC Control of Altitude and Attitude and PIV Position Control for Position and Altitude Demands with 10 ms⁻¹ Wind Gust Step

Figure 6.31 shows the comparative position response in x-direction, Figure 6.32 shows the comparative position response in y-direction, Figure 6.33 shows the comparative vertical response and Figures 6.35 to

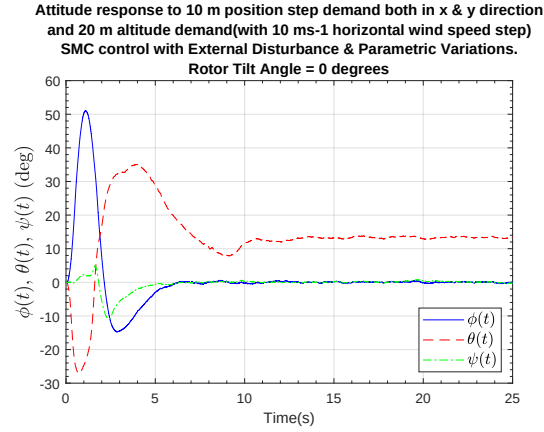


Figure 6.23: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

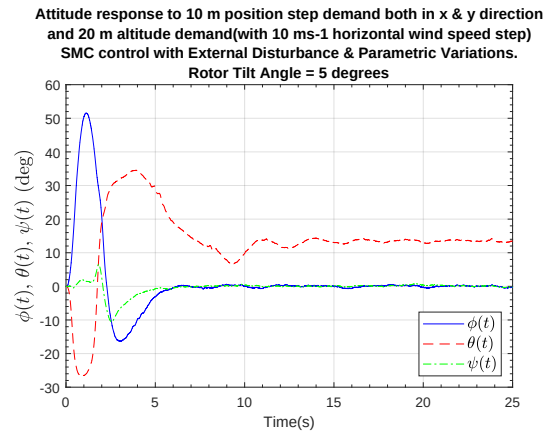


Figure 6.24: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

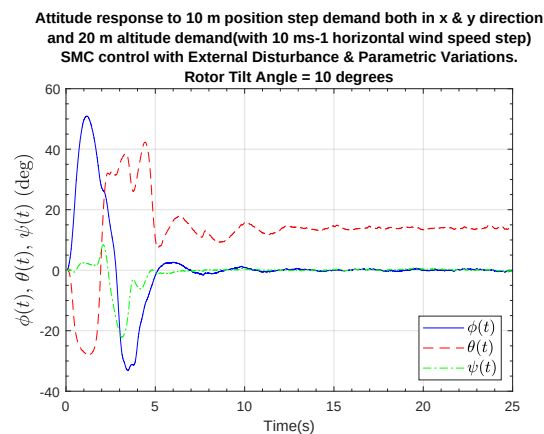


Figure 6.25: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

6.36 shows the comparative attitude responses with 10 ms⁻¹ wind gust step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

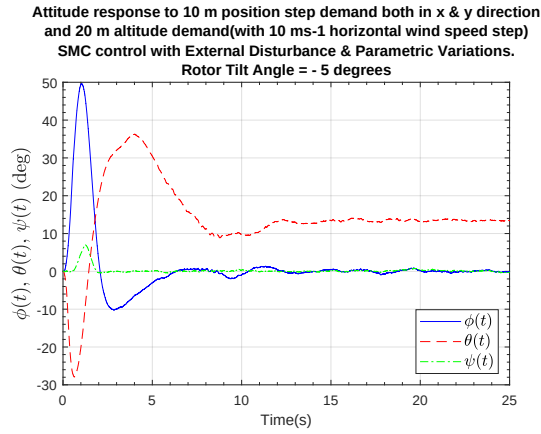


Figure 6.26: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

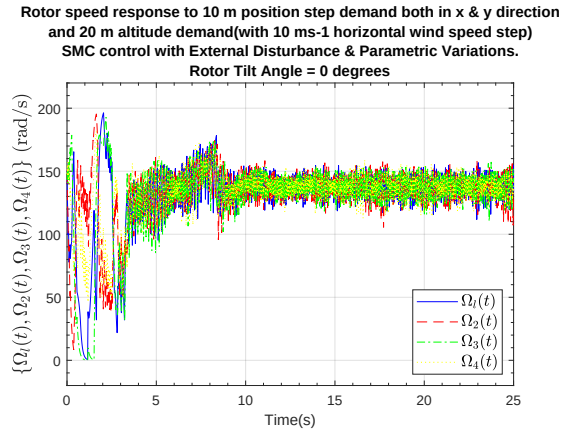


Figure 6.27: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

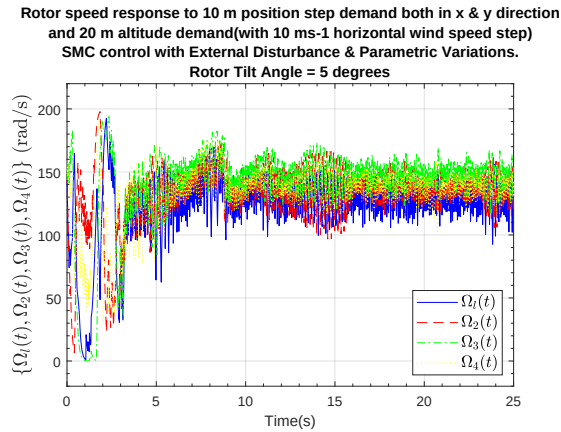


Figure 6.28: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

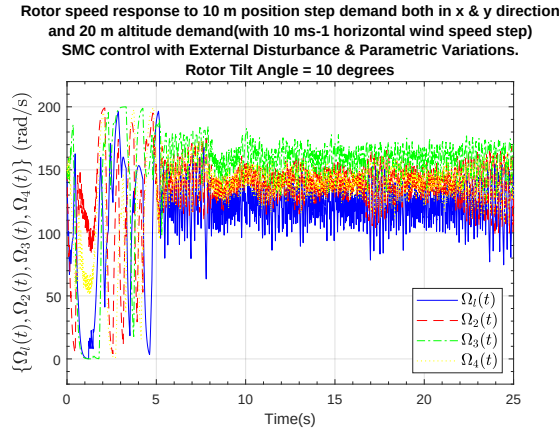


Figure 6.29: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

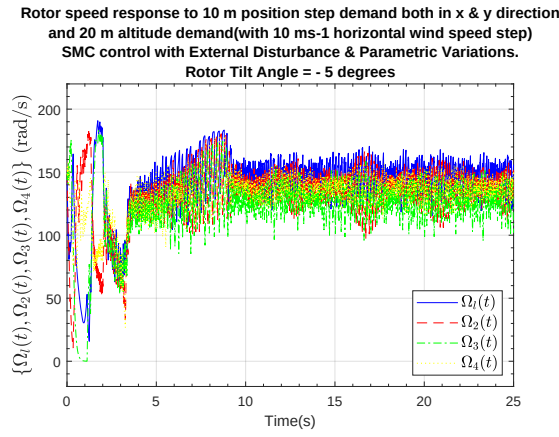


Figure 6.30: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

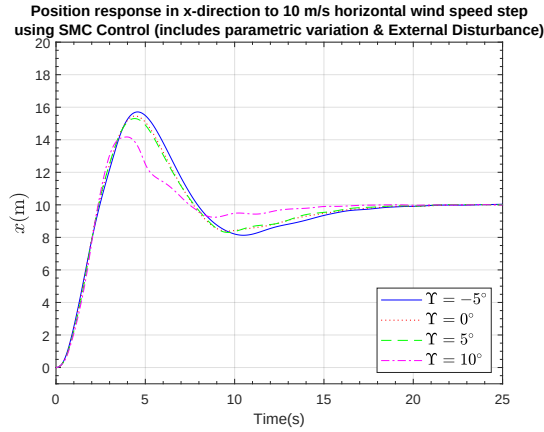
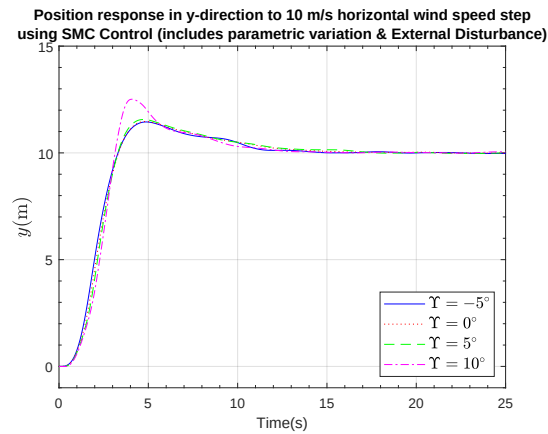
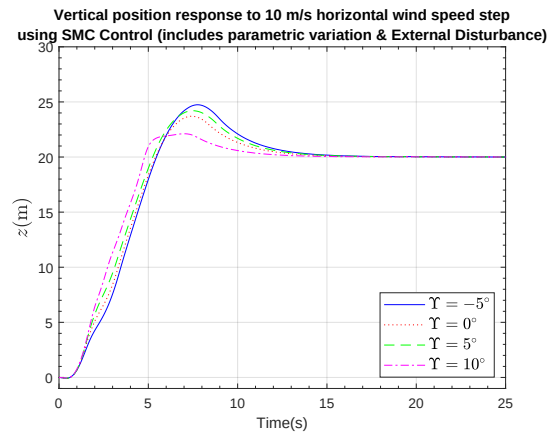
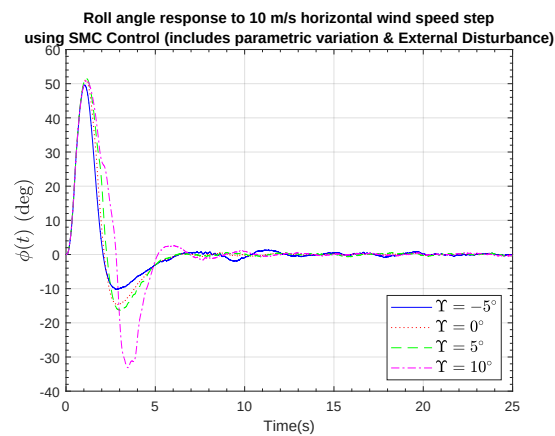


Figure 6.31: Lateral position response in x-direction to 10 ms⁻¹ horizontal wind gust step

Figure 6.32: Lateral Position response in y-direction to 10 ms^{-1} horizontal wind gust stepFigure 6.33: Vertical Position response in z-direction to 10 ms^{-1} horizontal wind gust stepFigure 6.34: Roll angle response to 10 ms^{-1} horizontal wind gust step

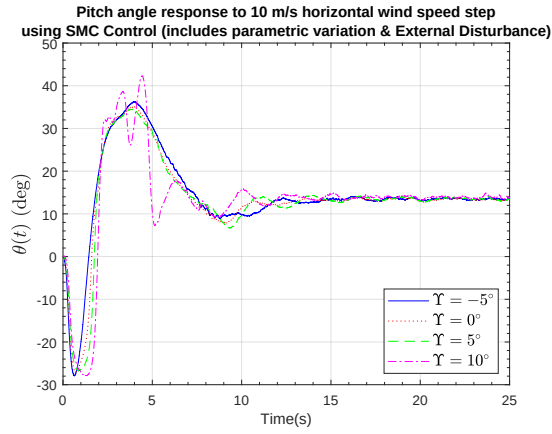


Figure 6.35: Pitch angle response to 10 ms^{-1} horizontal wind gust step

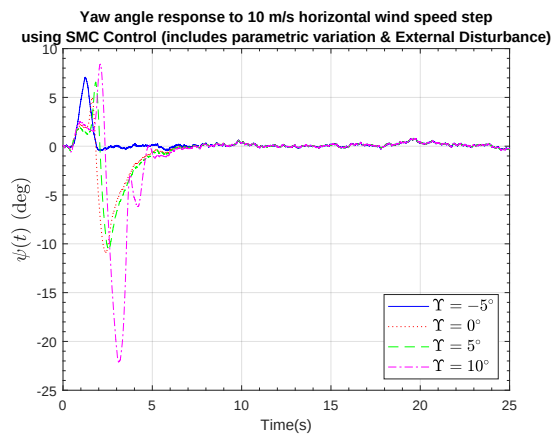


Figure 6.36: Yaw angle response to 10 ms^{-1} horizontal wind gust step

6.3 Simulation Results of Quadrotor with Aerodynamics using SMC for Altitude, Attitude and Position Control

The simulation results of quadrotor with aerodynamics for position and altitude demands in the absence of any parametric variations and external disturbance are included in Appendix E.2. The simulation results SMC control of altitude, attitude and position control of quadrotor at hover in the presence of any parametric variations and external disturbance are included in Appendix G.

6.3.1 SMC Control of Altitude and Attitude and Position Control for Position and Altitude Demands without Wind Gust

Simulation results have also been obtained for the case of position and altitude demands initially in the absence of wind gust and thereafter by considering wind gust as well. Simulation results have been obtained for the case of a 10 m position step demand both in x and y direction and 20 m altitude demand using SMC in the presence of parametric uncertainty and external disturbance.

Figures 6.37, 6.38, 6.39 and 6.40 show the simulation results for the position response for 10 m position step demand both in x and y direction and 20 m altitude demand without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

Figures 6.41, 6.42, 6.43 and 6.44 show the simulation results for the attitude response for 10 m position step demand both in x and y direction and 20 m altitude demand without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

As can be seen from the simulation results, there is no significant difference in the position response except for a maximum displacement of around 14 m for x when $\Upsilon = 10^\circ$ whereas other tilt angles have an initial maximum displacement of about 12 m.

With regards to attitude responses, the SMC controller responds by pitching the aircraft into the wind until the x-position is recovered and stabilises. Except for the case of $\Upsilon = 10^\circ$, where after pitching into the wind, it has a maximum pitch of around 20° more when compared to other cases.

Figures 6.45, 6.46, 6.47 and 6.48 show the simulation results for the rotor speed response at hover without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance. It can be seen from the rotor rotation rates, the rotors work with the same rates for different rotor tilt angles with a mean value around 150 rad/s. But for the case of $\Upsilon = 10^\circ$, there is lot of variation in the rotor speeds until 5 seconds but eventually settles down at 150 rad/s. The rotor speed responses are much better when compared to the responses obtained from the SMC-PIV controller w.r.t the variation in the rotor speeds.

6.3.2 Comparison of SMC Control of Altitude and Attitude and PIV Position Control of Quadrotor for Position and Altitude Demands and without any Wind Gust for Different Rotor Tilt Angles

Figure 6.49 shows the comparative position response in x-direction, Figure 6.50 shows the comparative position response in y-direction, Figure 6.51 shows the comparative vertical response and Figures 6.52 to 6.54 shows the comparative attitude responses without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

6.3.3 SMC Control of Altitude and Attitude and Position Control for Position and Altitude Demands with 10 ms^{-1} Wind Gust Step

Simulation results have been obtained for the case of position and altitude demands by considering wind gust as well. Simulation results have been obtained for the case of a 10 m position step demand both in

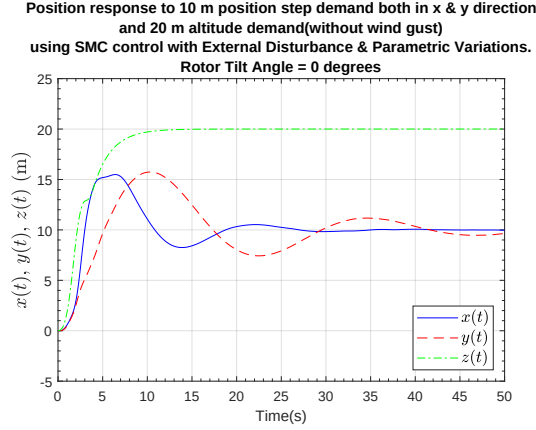


Figure 6.37: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = 0^\circ$

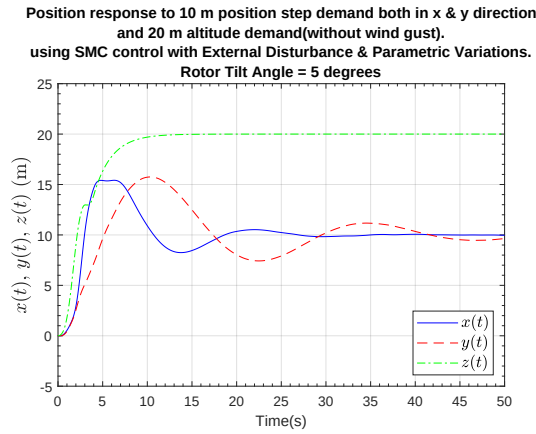


Figure 6.38: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = 5^\circ$

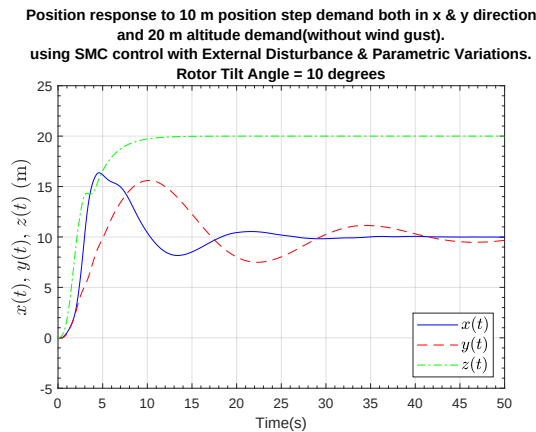


Figure 6.39: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = 10^\circ$

x and y direction and 20 m altitude demand using SMC in the presence of parametric uncertainties and external disturbance as defined in Chapter 5.

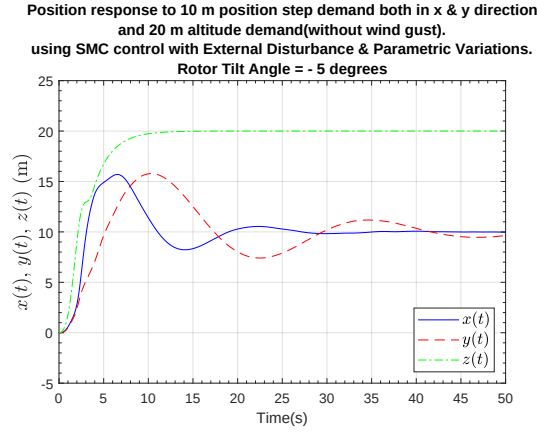


Figure 6.40: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = -5^\circ$

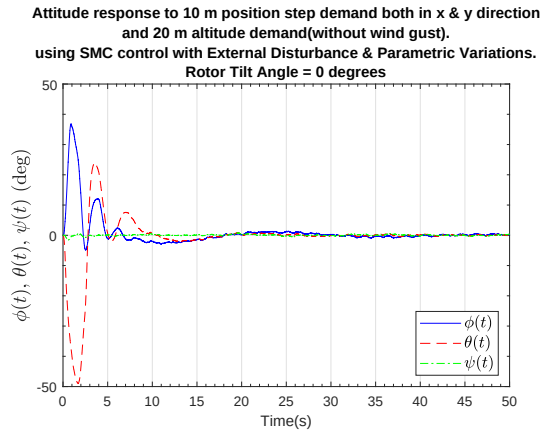


Figure 6.41: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = 0^\circ$

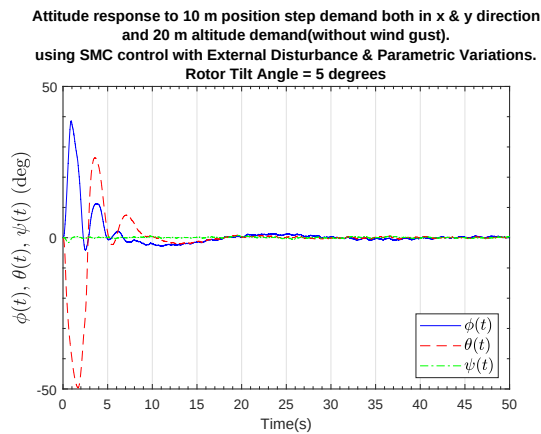


Figure 6.42: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = 5^\circ$

Figures 6.55, 6.56, 6.57 and 6.58 show the simulation results for the position response for 10 m position step demand both in x and y direction and 20 m altitude demand with 10 ms^{-1} wind gust step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

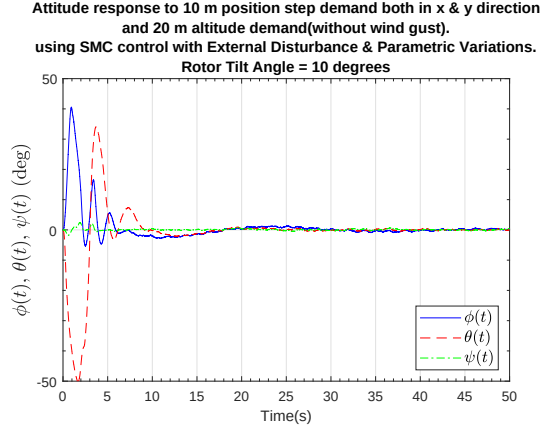


Figure 6.43: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = 10^\circ$

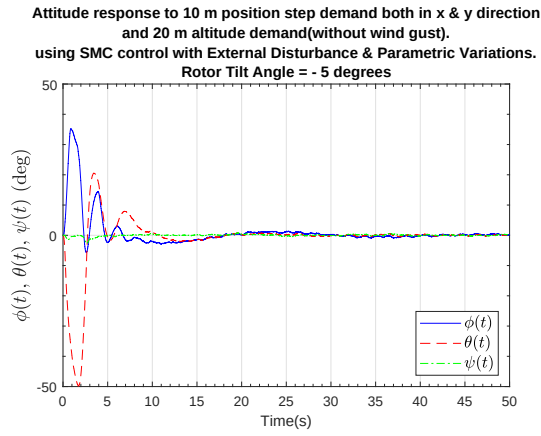


Figure 6.44: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = -5^\circ$

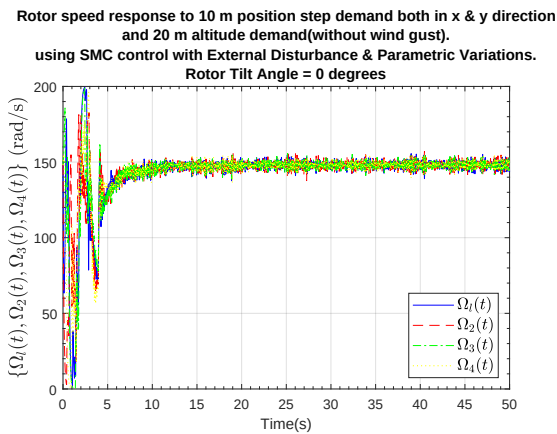


Figure 6.45: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = 0^\circ$

Figures 6.59, 6.60, 6.61 and 6.62 show the simulation results for the attitude response for 10 m position step demand both in x and y direction and 20 m altitude demand with 10 ms^{-1} wind gust step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

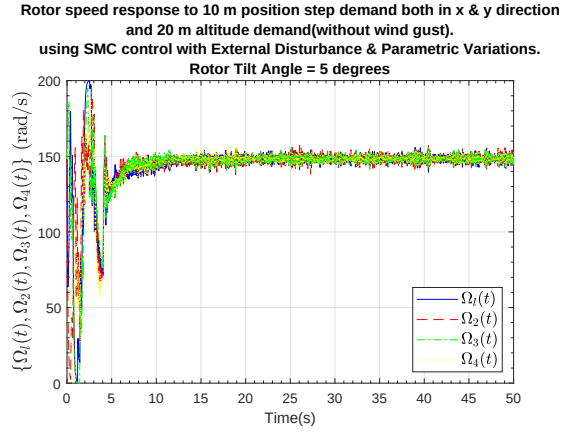


Figure 6.46: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = 5^\circ$

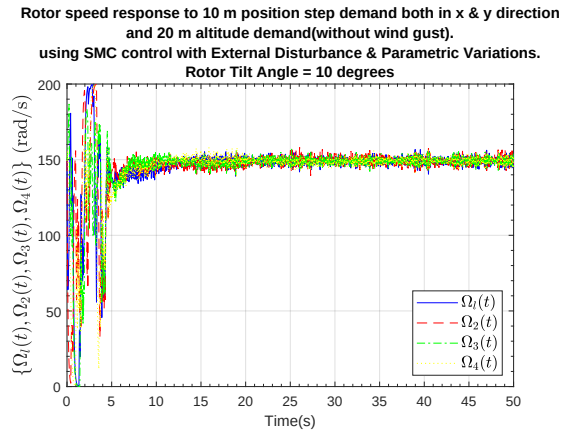


Figure 6.47: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = 10^\circ$

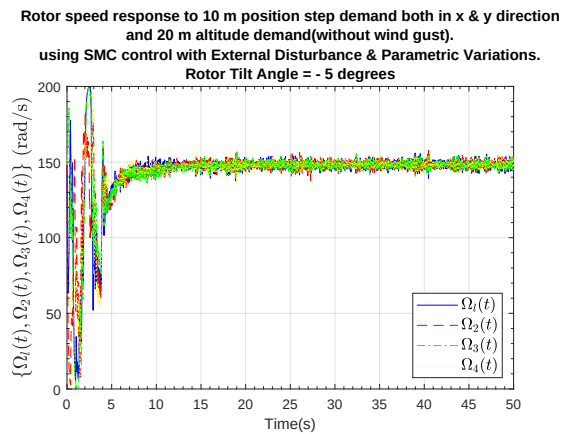


Figure 6.48: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust) using SMC control. $\Upsilon = -5^\circ$

As can be seen from the simulation results, there is no significant difference in the position response in x-direction except for the case of $\Upsilon = 10^\circ$ where the maximum displacement is slightly lesser than the other cases and settles 4 seconds earlier than other cases. The position response for the case of $\Upsilon = 10^\circ$

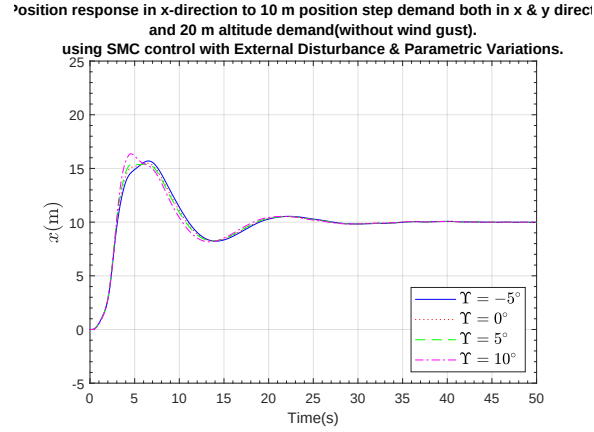


Figure 6.49: Position response in x-direction at hover without wind gust)

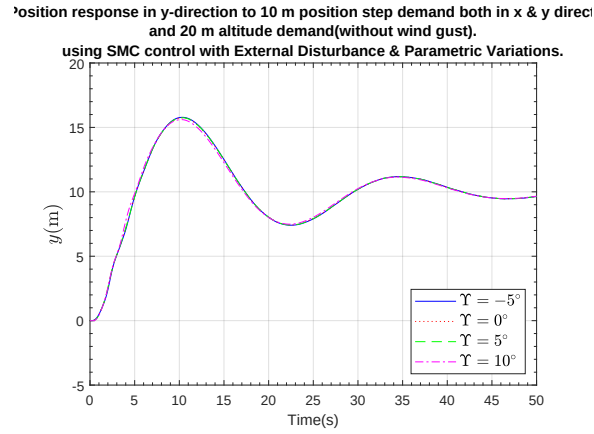


Figure 6.50: Position response in y-direction at hover without wind gust

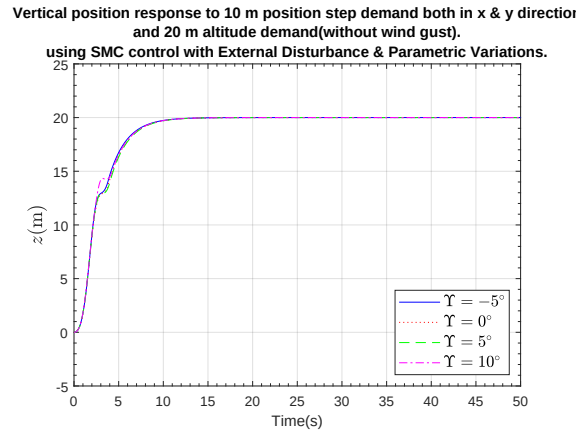


Figure 6.51: Vertical response at hover without wind gust

has slightly increased initial maximum displacement when compared to other cases but again there is not much significant differences between different rotor tilt angle cases. The settling time for all the cases is nearly 12 seconds.

With regards to attitude responses, the SMC controller responds by pitching the aircraft into the wind until the x-position is recovered and stabilises. All of the rotor tilt angle cases have similar pitch angle

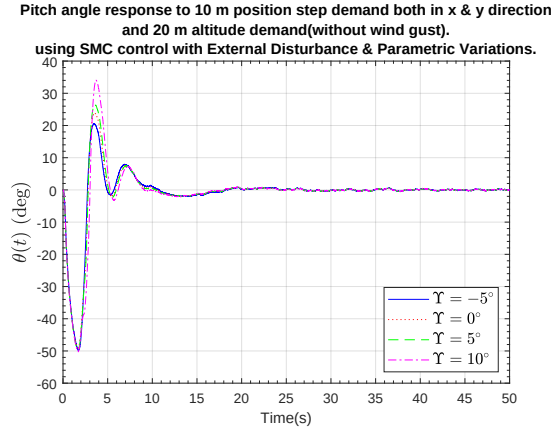


Figure 6.52: Pitch angle response at hover without wind gust

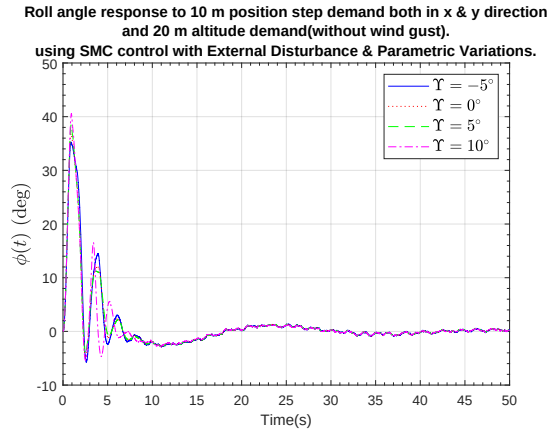


Figure 6.53: Roll angle response at hover without wind gust

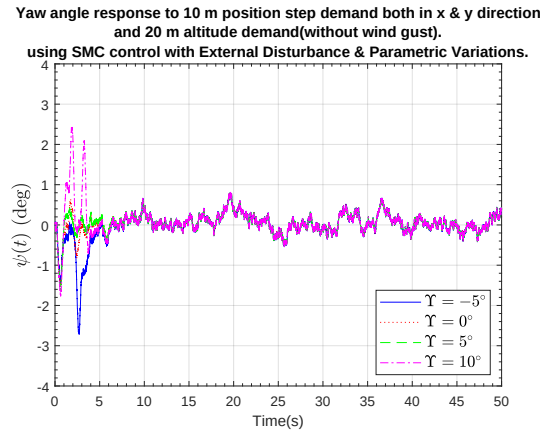


Figure 6.54: Yaw angle response at hover without wind gust

responses and settling with a trimmed tilt of just under 12° .

Figures 6.63, 6.64, 6.65 and 6.66 show the simulation results for the rotor speed response with 10 ms^{-1} Wind Gust Step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance. It can be seen from the rotor rotation rates, the rotors work with the same rates for different rotor tilt angles with a mean value around 140

rad/s. But in all the rotor tilt cases, there is lot of variation in the rotor speeds until 5 seconds but eventually settle down at 140 rad/s. This variation is not due to the chattering in the SMC but is due to the Gaussian Noise that has been added with the Attitude. This variation doesn't exist in simulation studies with no parametric variation or noise and when only parametric uncertainty is taken into account as depicted in Appendix H. Note that in the presence of the noise in the pitch, roll and yaw angles, the SMC performs well as expected.

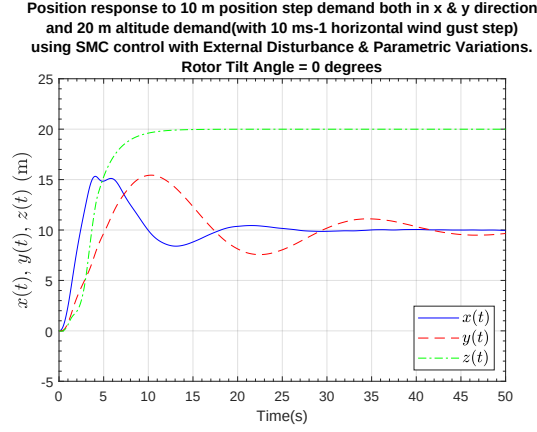


Figure 6.55: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = 0^\circ$

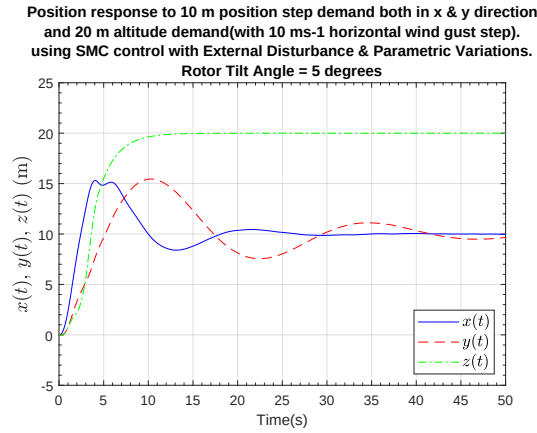


Figure 6.56: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = 5^\circ$

6.3.4 Comparison of SMC Control of Altitude and Attitude and Position Control for Position and Altitude Demands with 10 ms⁻¹ Wind Gust Step

Figure 6.67 shows the comparative position response in x-direction, Figure 6.68 shows the comparative position response in y-direction, Figure 6.69 shows the comparative vertical response and Figures 6.71 to 6.72 shows the comparative attitude responses with 10 ms⁻¹ wind gust step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

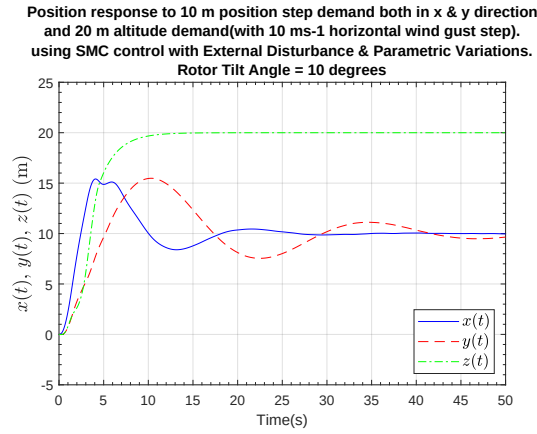


Figure 6.57: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = 10^\circ$

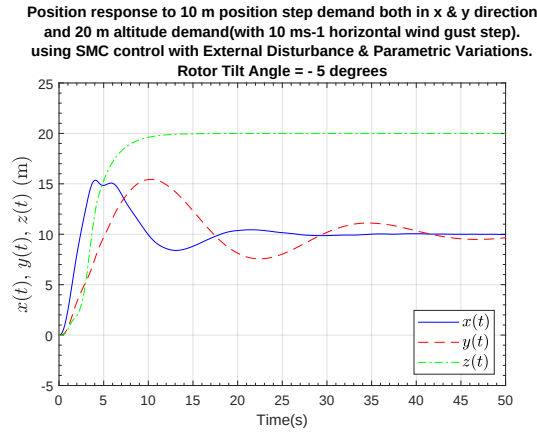


Figure 6.58: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = -5^\circ$

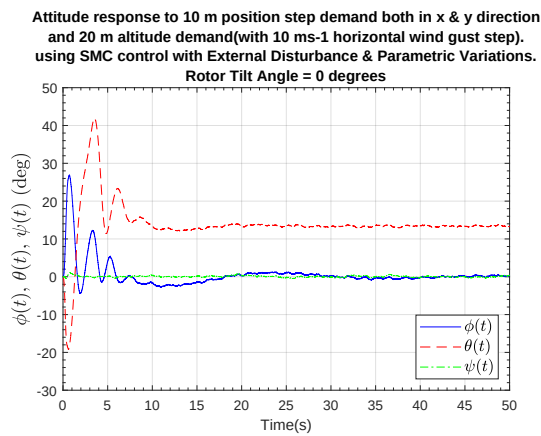


Figure 6.59: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = 0^\circ$

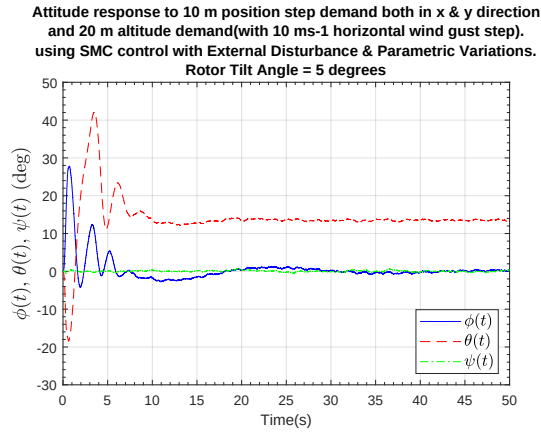


Figure 6.60: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = 5^\circ$

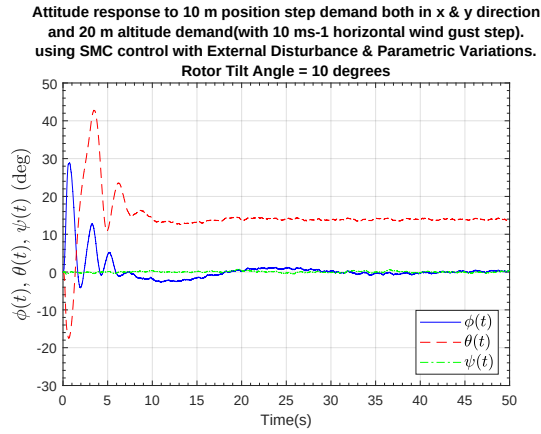


Figure 6.61: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = 10^\circ$

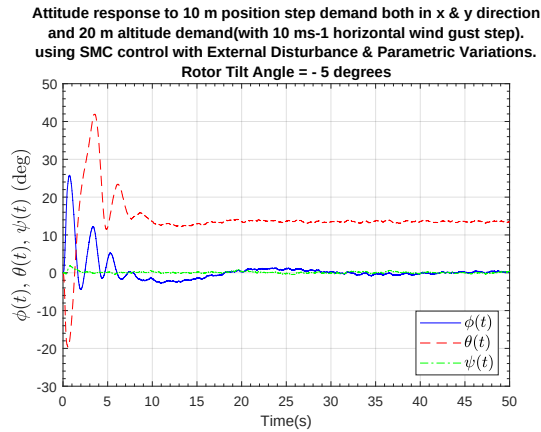


Figure 6.62: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = -5^\circ$

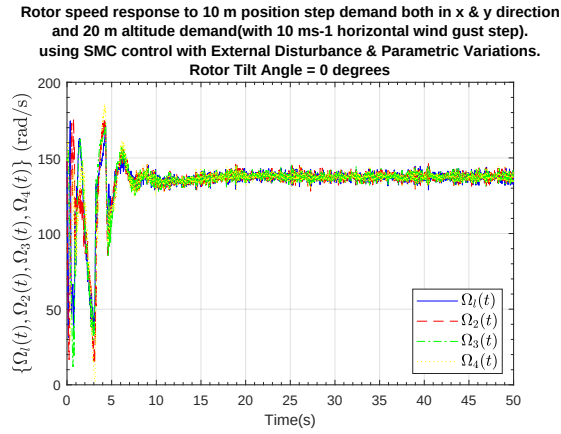


Figure 6.63: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = 0^\circ$

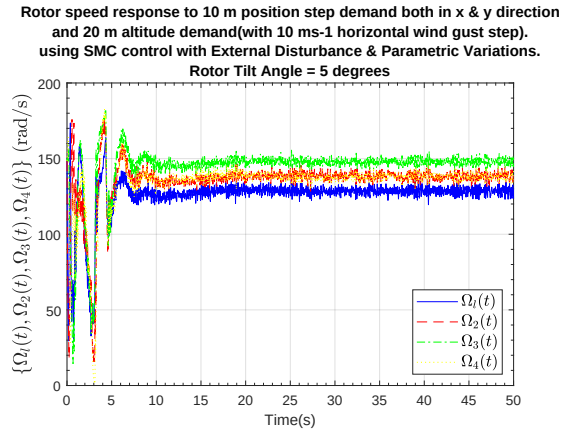


Figure 6.64: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = 5^\circ$

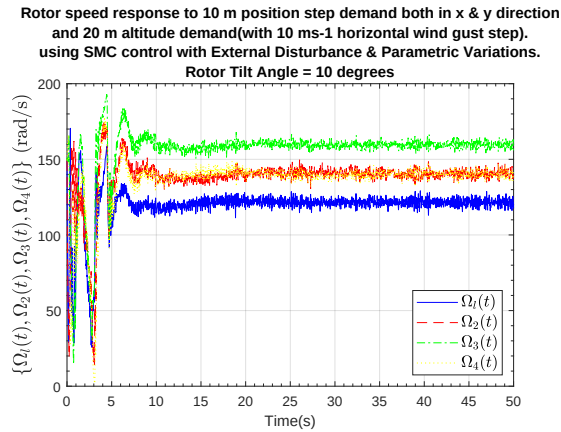


Figure 6.65: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = 10^\circ$

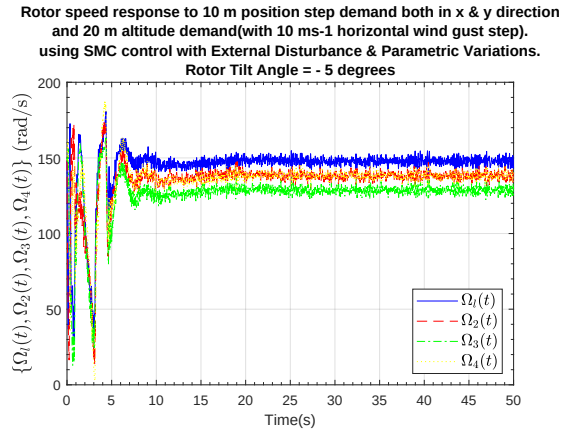


Figure 6.66: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step) using SMC control. $\Upsilon = -5^\circ$

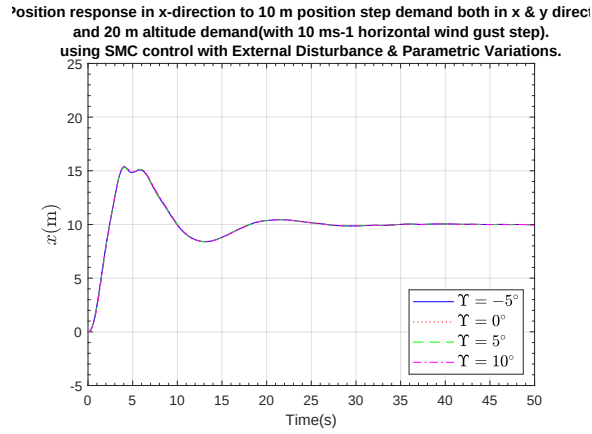


Figure 6.67: Lateral position response in x-direction to 10 ms⁻¹ horizontal wind gust step using SMC control

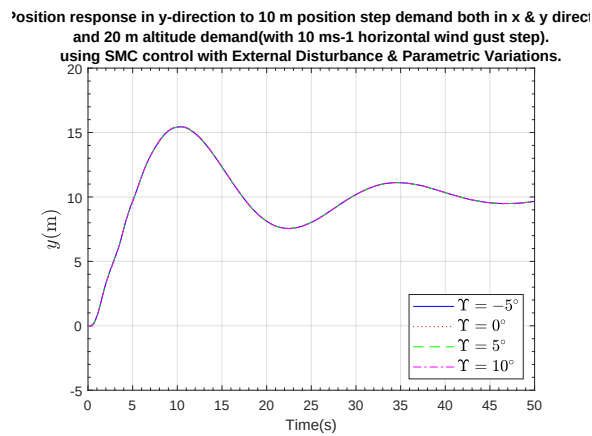


Figure 6.68: Lateral Position response in y-direction to 10 ms⁻¹ horizontal wind gust step using SMC control

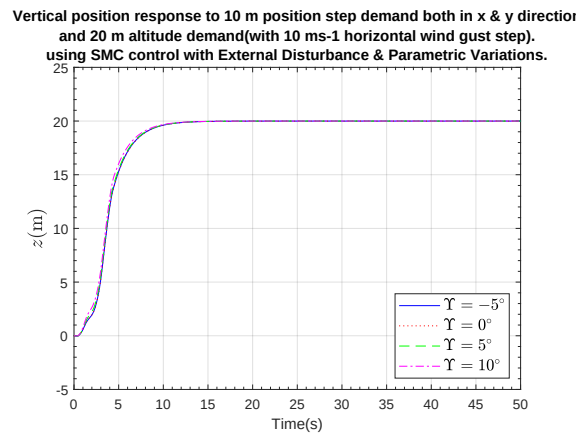


Figure 6.69: Vertical Position response in z-direction to 10 ms⁻¹ horizontal wind gust step using SMC control

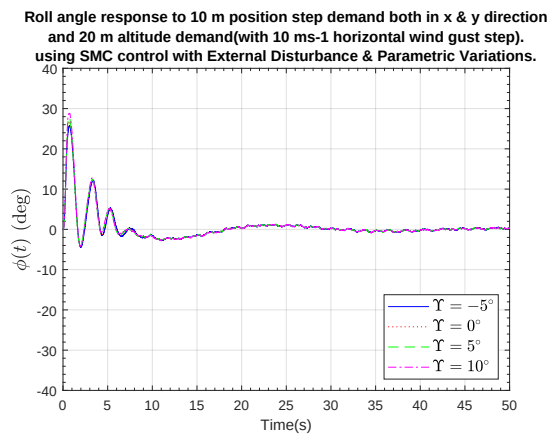


Figure 6.70: Roll angle response to 10 ms⁻¹ horizontal wind gust step using SMC control

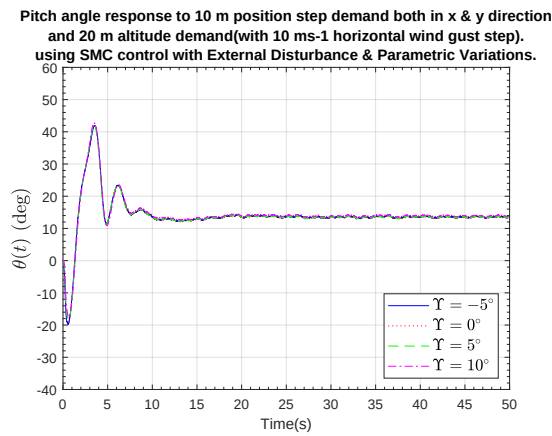


Figure 6.71: Pitch angle response to 10 ms⁻¹ horizontal wind gust step using SMC control

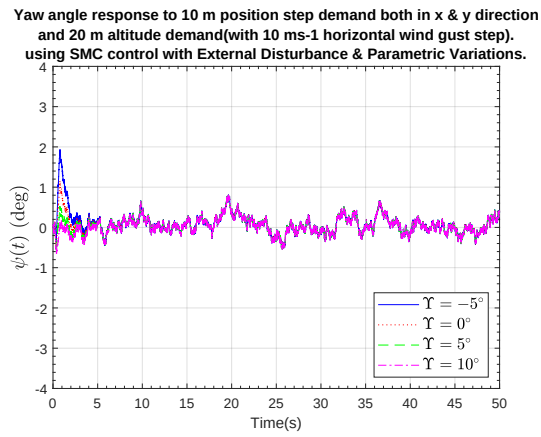


Figure 6.72: Yaw angle response to 10 ms⁻¹ horizontal wind gust step using SMC control

6.4 Comparison of the Quadrotor Simulation Results

Defining the PIV control as Method #1, SMC control of Altitude & Attitude Control and PIV Position Control as Method #2 and SMC Control of Altitude, Attitude and Position as Method #3, the Table 6.1 illustrates a comparison of the position response in x-direction for a position step demand in a Quadrotor for different rotor tilt angles and without considering wind gust step.

Lateral position response in x-direction in a Quadrotor (without wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	-5°	0°	5°	10°	-5°	0°	5°	10°	-5°
Method #1	13.4	13.5	13.6	13.3	4.75	4.75	4.7	4.8	21	21	21	21
Method #2	11.75	12.5	14.75	11.75	4	3.75	3.75	4	18	18	15	20
Method #3	15.25	15.5	16.5	15.75	3.75	3.75	3.75	4.25	32	32	32	32

Table 6.1: Comparison of lateral position responses in x-direction to 10m horizontal position step demand in a Quadrotor without wind gust step for different rotor tilt angles

Lateral position response in y-direction in a Quadrotor (without wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	-5°	0°	5°	10°	-5°	0°	5°	10°	-5°
Method #1	12.6	12.7	12.8	12.5	4.3	4.3	4.25	4.5	20.5	20.5	20.5	20.5
Method #2	11.5	11.5	11.75	11.5	3.75	3.5	5.25	3.75	19	19	20	19
Method #3	15.5	15.5	15.5	15.75	7.5	7.5	7.5	7.5	50	50	50	50

Table 6.2: Comparison of lateral position responses in y-direction to 10m horizontal position step demand in a Quadrotor without wind gust step for different rotor tilt angles

Table 6.2 illustrates a comparison of the position response in y-direction for a position step demand in a Quadrotor for different rotor tilt angles and without considering wind gust step. Table 6.3 illustrates a comparison of the vertical position responses to 20m altitude step demand in a Quadrotor for different rotor tilt angles and without considering wind gust step. Table 6.4 illustrates a comparison of the pitch angle responses to 10m horizontal position step demand and 20m altitude step demand in a Quadrotor for different rotor tilt angles and without considering wind gust step. Table 6.5 illustrates a comparison of the lateral position responses to 10m horizontal position step demand in x-direction in a Quadrotor for different rotor tilt angles and considering $10ms^{-1}$ horizontal wind gust step. Table 6.6 illustrates a comparison of the lateral position responses to 10m horizontal position step demand in y-direction in a Quadrotor for different rotor tilt angles and considering $10ms^{-1}$ horizontal wind gust step. Table 6.7 illustrates a comparison of the vertical position responses to 20m altitude step demand in a Quadrotor for different rotor tilt angles and considering $10ms^{-1}$ horizontal wind gust step. Table 6.8 illustrates a comparison of the pitch angle responses to 10m horizontal position step demand and 20m altitude step demand in a Quadrotor for different rotor tilt angles and considering $10ms^{-1}$ horizontal wind gust step.

From the table 6.1 and table 6.5 w.r.t Method #1 i.e. PIV control of the quadrotor, it can be seen that, an outwards rotor tilt, which introduces anhedron into the quadrotor, can reduce the sensitivity of the vehicle position to gusts using PIV control. The rise time and the settling time is reduced although there is a minor change in the overshoot when the simulation results without wind gust and wind gust step are considered. From the table 6.2 and table 6.6 w.r.t Method #1, the position response in y-direction has a small initial undershoot resulting from the non-minimum phase in the step response of all the cases. But the demanded position in the y-direction is achieved in the presence of wind gust step as well. From the table 6.3 and table 6.7 w.r.t Method #1, the demanded altitude of 20m is achieved with a reduced

Vertical position response in a Quadrotor (without wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	-5°	0°	5°	10°	-5°	0°	5°	10°	-5°
Method #1	20.5	20.5	20.5	20.5	2.5	2.5	2.5	2.5	25	25	25	25
Method #2	20	20	20	20	6.25	7.75	5	6	13	13	13	13
Method #3	20	20	20	20	6	6	6	6	13	13	13	13

Table 6.3: Comparison of vertical position responses to 20m altitude step demand in a Quadrotor without wind gust step for different rotor tilt angles

Pitch angle response in a Quadrotor (without wind gust step)	Max Overshoot (degrees)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	-5°	0°	5°	10°	-5°	0°	5°	10°	-5°
Method #1	-34	-33	-31	-33	1.25	1.25	1.5	1.25	23	23	23	23
Method #2	-45	-46	-46	-45	1	1	1	1	15	15	15	15
Method #3	-50	-50	-50	-50	1.25	1.25	1.25	1.25	20	20	20	20

Table 6.4: Comparison of pitch angle responses to 10m horizontal position step demand in x-direction, 10m horizontal position step demand in y-direction and 20m altitude step demand in a Quadrotor without wind gust step for different rotor tilt angles

Lateral position response in x-direction in a Quadrotor (with wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	-5°	0°	5°	10°	-5°	0°	5°	10°	-5°
Method #1	15.2	15.1	15	15.3	3.25	3.25	3.25	3.25	18	18	18	18
Method #2	15.5	15.25	14.25	15.75	3.5	3.5	3	3.75	21	20	17	21
Method #3	15.5	15.5	15.5	15.5	3.5	3.5	3.5	3.5	34	34	34	34

Table 6.5: Comparison of lateral position responses to 10m horizontal position step demand in x-direction in a Quadrotor with $10ms^{-1}$ horizontal wind gust step for different rotor tilt angles

Lateral position response in y-direction in a Quadrotor (with wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	-5°	0°	5°	10°	-5°	0°	5°	10°	-5°
Method #1	12.5	12.6	12.75	12.25	4.4	4.3	4.25	4.5	20	20	20	20
Method #2	11.5	11.5	12.5	11.5	3.5	3.5	3.25	3.5	16	17	14	16
Method #3	15.5	15.5	15.5	15.5	7.5	7.5	7.5	7.5	50	50	50	50

Table 6.6: Comparison of lateral position responses to 10m horizontal position step demand in y-direction in a Quadrotor with $10ms^{-1}$ horizontal wind gust step for different rotor tilt angles

Vertical position response in a Quadrotor (with wind gust step)	Max Overshoot (m)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	-5°	0°	5°	10°	-5°	0°	5°	10°	-5°
Method #1	20	20	20.5	20.1	3.85	3.8	3.75	4	11	11	12	14
Method #2	23.75	24	22	24.75	6	6	5	6.25	16	16	14	17
Method #3	20	20	20	20	7	6.75	6.75	7	15	15	15	15

Table 6.7: Comparison of vertical position responses to 20m altitude step demand in a Quadrotor $10ms^{-1}$ horizontal wind gust step for different rotor tilt angles

Pitch angle response in a Quadrotor (with wind gust step)	Max Overshoot (degrees)				Rise Time (sec)				Settling Time (sec)			
Rotor Tilt Angle	0°	5°	10°	-5°	0°	5°	10°	-5°	0°	5°	10°	-5°
Method #1	-18	-19	-20	-17	0.5	0.5	0.5	0.6	15	15	15	15
Method #2	-27	-27	-28	-28	0.5	0.5	0.5	0.5	17	17	18	18
Method #3	-20	-18	-17.5	-20	0.25	0.25	0.25	0.25	20	20	20	20

Table 6.8: Comparison of pitch angle responses to 10m horizontal position step demand in x-direction, 10m horizontal position step demand in y-direction and 20m altitude step demand in a Quadrotor with $10ms^{-1}$ horizontal wind gust step for different rotor tilt angles

overshoot and settling time in the presence of wind gust step. From the table 6.4 and table 6.8 w.r.t Method #1, it is seen that the pitch response for a 20 m altitude step demand remains almost identical for all the 4 rotor tilt angle cases reaching a maximum pitch of 20° in the presence of wind gust step. The maximum pitch is about 2° greater than the untilted case and occurs just after 0.5 s and settling with a trimmed tilt of just under 12° . The results obtained in this research work are not as significant with regards to the impact of rotor tilt angle on gust rejection properties as those obtained by Whidborne et al. [2022] where LQR controller has been used. This may be due to the optimal tuning of the PIV controller gains.

From the table 6.1 and table 6.5 w.r.t Method #2 i.e. SMC control of altitude and attitude and PIV position control in the presence of parametric uncertainties and external disturbances (Gaussian Noise), the demanded position of 10m in the x-direction is achieved with some minor differences in the performance relating to different rotor tilt angles. It can be noted that the 10° has a lower overshoot, rise time and setting time when compared to other tilt angles but not too significant. From the table 6.2 and table 6.6 w.r.t Method #2, the demanded position of 10m in the y-direction is achieved in the presence of wind gust step with almost similar behaviour for all rotor tilt angles although there is an added 0.75m overshoot for 10° when compared to other tilt angles. From the table 6.3 and table 6.7 w.r.t Method #2, the demanded altitude of 20m is achieved with a better performance for 10° case with a reduced overshoot and settling times in the presence of wind gust step. From the table 6.4 and table 6.8 w.r.t Method #2, it is seen that the pitch response for a 20 m altitude step demand remains almost identical for all the 4 rotor tilt angle cases reaching a maximum pitch of 28° in the presence of wind gust step. The maximum pitch is about 2° greater than the untilted case and occurs just after 1 s and settling with a trimmed tilt of just under 12° .

From the table 6.1 and table 6.5 w.r.t Method #3 i.e. SMC control of altitude and attitude and position control in the presence of parametric uncertainties and external disturbances (Gaussian Noise), the demanded position of 10m in the x-direction is achieved with identical responses for different rotor tilt angles. Although, there is an increase in the settling time because of the oscillatory response, this can be fixed by adding a proportional action while tracking the error [Visioli, 2006; Firdaus, 2018]. From the table 6.2 and table 6.6 w.r.t Method #2, the demanded position of 10m in the y-direction is achieved in the presence of wind gust step with almost similar behaviour for all rotor tilt angles. From the table 6.3 and table 6.7 w.r.t Method #3, the demanded altitude of 20m is achieved with identical behaviour for all rotor tilt angles. From the table 6.4 and table 6.8 w.r.t Method #3, it is seen that the pitch response for a 20 m altitude step demand remains identical for all the 4 rotor tilt angle cases reaching a maximum pitch of 20° in the presence of wind gust step and settling with a trimmed tilt of just under 12° .

With the simulation results presented, it is shown that SMC controller performs very well in gust rejection and the rotor tilt angles do not have a significant impact on the performance of the multirotor aircraft.

6.5 SMC Control of Altitude and Attitude and Position Control at Hover with 1-Cosine Wind Gust

In addition to considering wind gust step, simulation studies have been carried out using 1-Cosine wind gust with maximum value of 10 ms^{-1} that starts from 5 secs and lasts until 20 secs.

Figures 6.74, 6.75, 6.76 and 6.77 show the simulation results for the position response at hover with a 1-cosine wind gust in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance as considered in Chapter 5.

Figures 6.78, 6.79, 6.80 and 6.81 show the simulation results for the attitude response at hover with a 1-cosine wind gust in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance.

Figures 6.82, 6.83, 6.84 and 6.85 show the simulation results for the rotor speed response at hover with a 1-cosine wind gust in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance.

The 1-Cosine wind gust is represented as in Figures 6.73

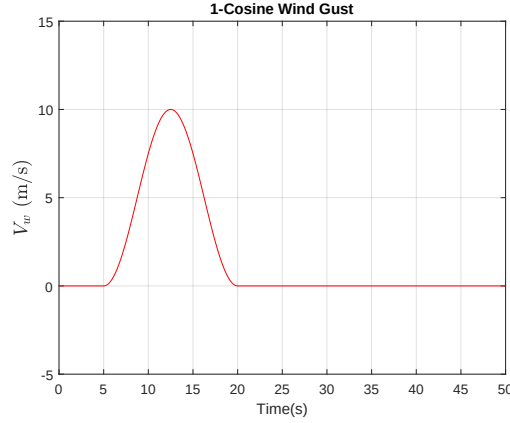


Figure 6.73: 1-cosine wind gust with maximum value of 10 ms^{-1} that starts from 5 secs and lasts until 20 secs

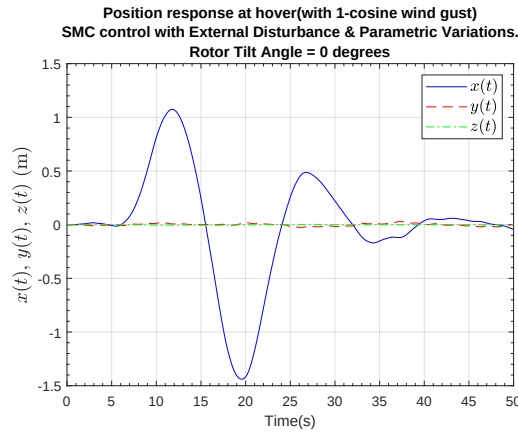


Figure 6.74: Position response at hover (with 1-cosine wind gust). $\Upsilon = 0^\circ$

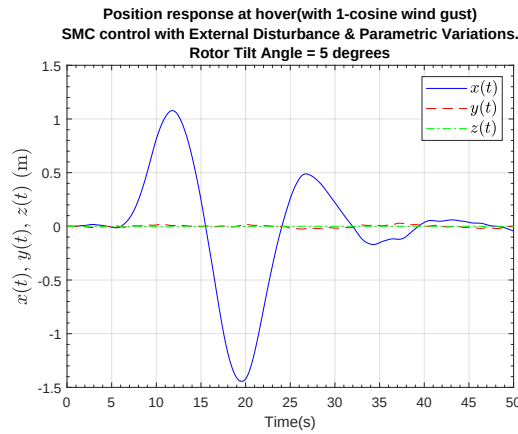


Figure 6.75: Position response at hover (with 1-cosine wind gust). $\Upsilon = 5^\circ$

6.5.1 Comparison of SMC Control of Altitude and Attitude and Position Control of Quadrotor at Hover with 1-Cosine Wind Gust

Figure 6.86 shows the comparative position response in x-direction, Figure 6.87 shows the comparative position response in y-direction, Figure 6.88 shows the comparative vertical response and Figures 6.89

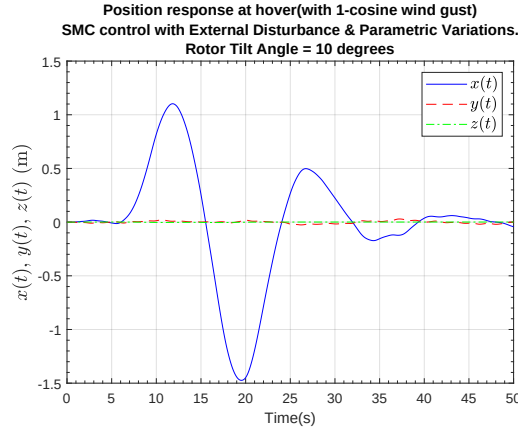


Figure 6.76: Position response at hover (with 1-cosine wind gust). $\Upsilon = 10^\circ$

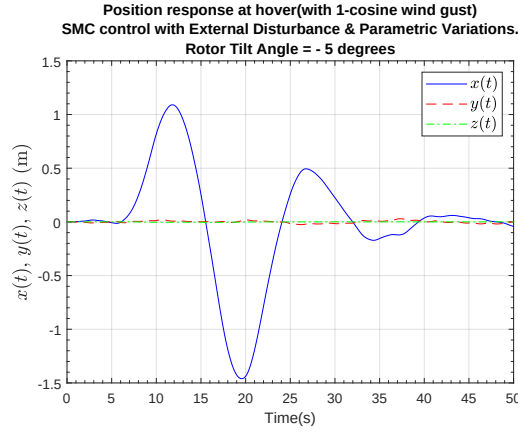


Figure 6.77: Position response at hover (with 1-cosine wind gust). $\Upsilon = -5^\circ$

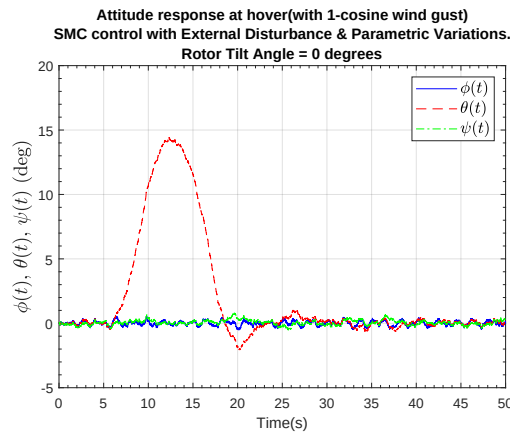
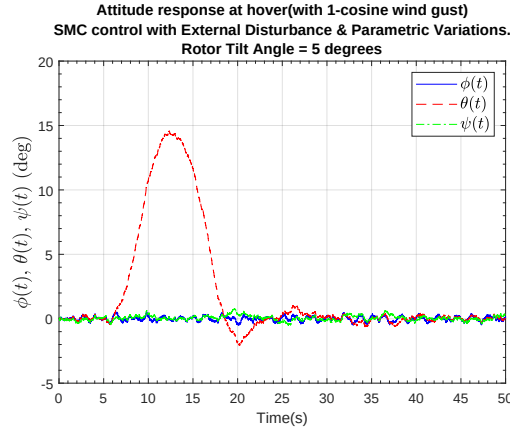
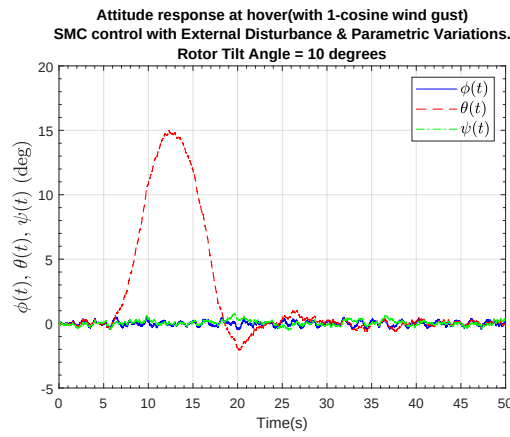
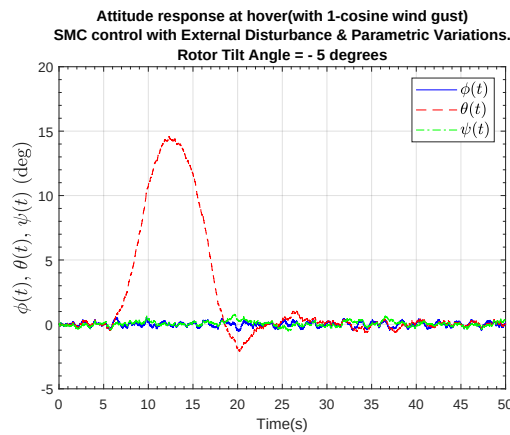


Figure 6.78: Attitude response at hover (with 1-cosine wind gust). $\Upsilon = 0^\circ$

to 6.91 shows the comparative attitude responses with 1-cosine wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively. As can be seen from the simulation results, the Altitude, Attitude and Position responses are identical in the presence of parametric uncertainties and external disturbances.

Figure 6.79: Attitude response at hover (with 1-cosine wind gust). $\Upsilon = 5^\circ$ Figure 6.80: Attitude response at hover (with 1-cosine wind gust). $\Upsilon = 10^\circ$ Figure 6.81: Attitude response at hover (with 1-cosine wind gust). $\Upsilon = -5^\circ$

6.6 SMC Control of Altitude and Attitude and Position Control for Position and Altitude Demands with 1-Cosine Wind Gust

Simulation results have been obtained for the case of position and altitude demands by considering wind gust as well. Simulation results have been obtained for the case of a 10 m position step demand both in

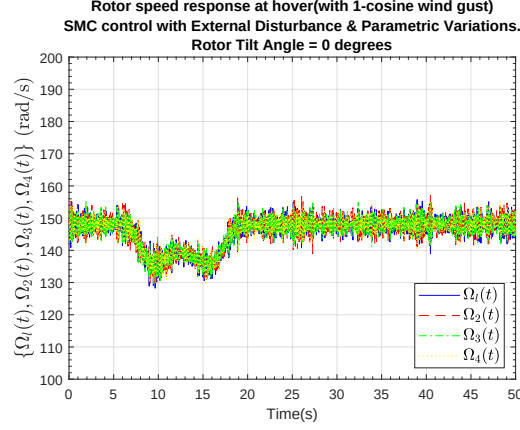


Figure 6.82: Rotor speed response at hover (with 1-cosine wind gust). $\Upsilon = 0^\circ$

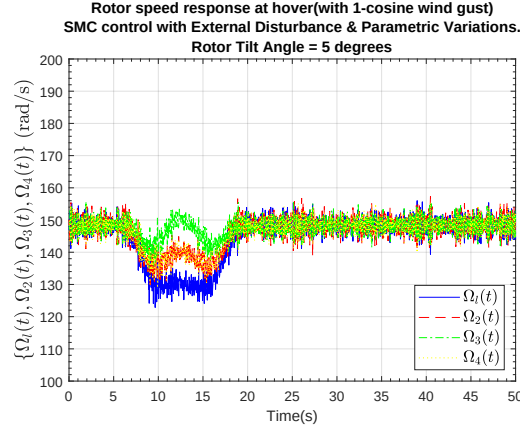


Figure 6.83: Rotor speed response at hover (with 1-cosine wind gust). $\Upsilon = 5^\circ$

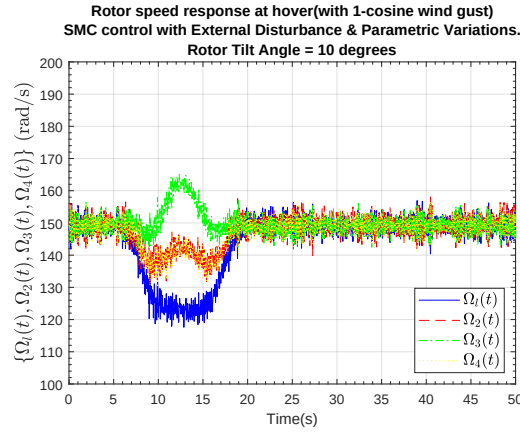


Figure 6.84: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 1-cosine wind gust). $\Upsilon = 10^\circ$

x and y direction and 20 m altitude demand using SMC in the presence of parametric uncertainties and external disturbance as defined in Chapter 5.

Figures 6.92, 6.93, 6.94 and 6.95 show the simulation results for the position response for 10 m position step demand both in x and y direction and 20 m altitude demand with 1-cosine wind gust for different

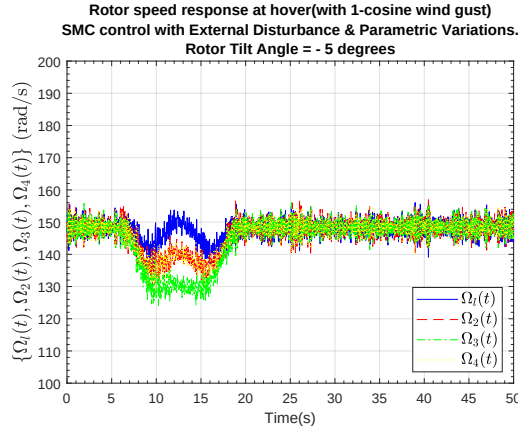


Figure 6.85: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 1-cosine wind gust). $\Upsilon = -5^\circ$

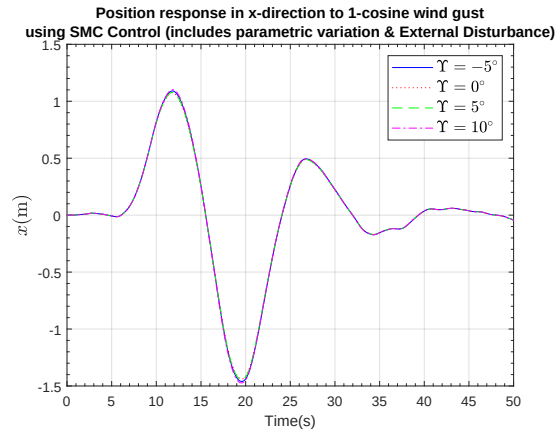


Figure 6.86: Position response in x-direction at hover with 1-cosine wind gust

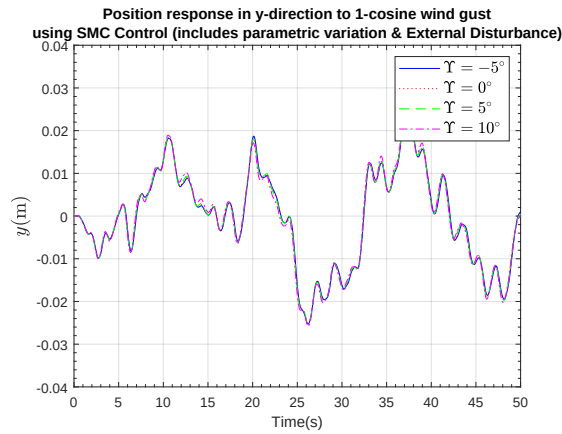


Figure 6.87: Position response in y-direction at hover with 1-cosine wind gust

rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

Figures 6.96, 6.97, 6.98 and 6.99 show the simulation results for the attitude response for 10 m position step demand both in x and y direction and 20 m altitude demand with 1-cosine wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively.

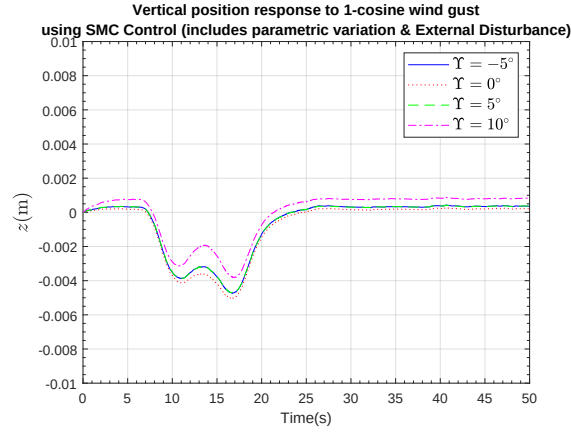


Figure 6.88: Vertical response at hover with 10 ms^{-1} horizontal wind gust

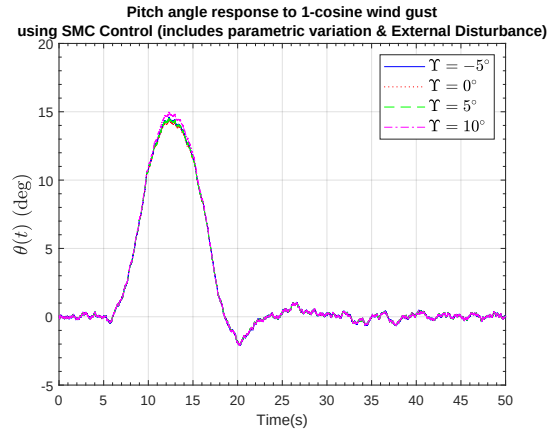


Figure 6.89: Pitch angle response at hover with 1-cosine wind gust

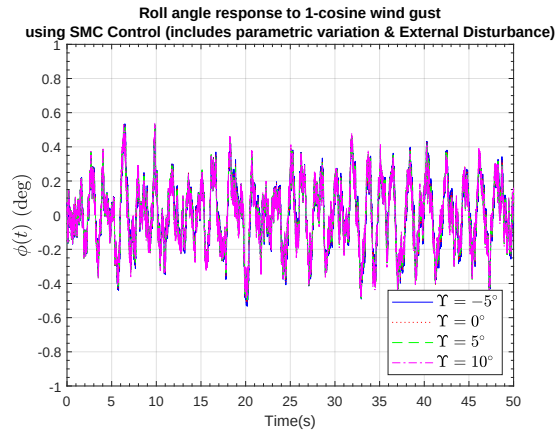


Figure 6.90: Roll angle response at hover with 1-cosine wind gust

As can be seen from the simulation results, there is no significant difference in the position response in x-direction except for the case of $\Upsilon = 10^\circ$ where the overshoot is slightly higher than the other cases but settles in the way as other cases but again there is not much significant differences between different rotor tilt angle cases. The settling time for all the cases is nearly 35 seconds.

With regards to attitude responses, the SMC controller responds by pitching the aircraft into the wind

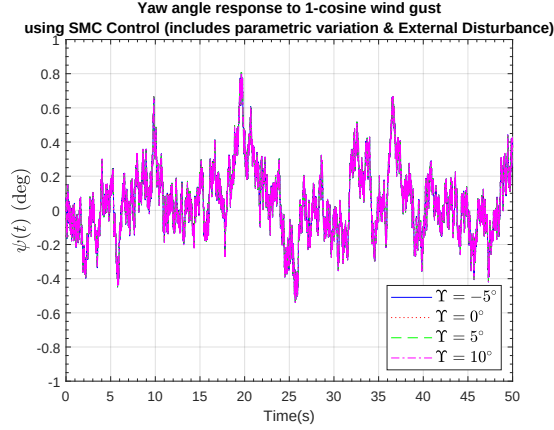
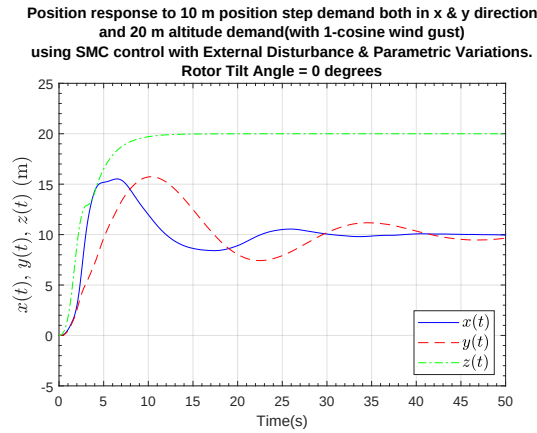


Figure 6.91: Yaw angle response with 1-cosine wind gust

until the x-position is recovered and stabilises. All of the rotor tilt angle cases have similar pitch angle responses and settling with a trimmed tilt of just under 12° .

Figures 6.100, 6.101, 6.102 and 6.103 show the simulation results for the rotor speed response with 1-cosine wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance. It can be seen from the rotor rotation rates, the rotors work with the same rates for different rotor tilt angles with a mean value around 140 rad/s. But in all the rotor tilt cases, there is lot of variation in the rotor speeds until 5 seconds but eventually settle down at 140 rad/s.

Figure 6.92: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = 0^\circ$

6.6.1 Comparison of SMC Control of Altitude and Attitude and Position Control for Position and Altitude Demands with 1-Cosine Wind Gust

Figure 6.104 shows the comparative position response in x-direction, Figure 6.105 shows the comparative position response in y-direction, Figure 6.106 shows the comparative vertical response and Figures 6.108 to 6.109 shows the comparative attitude responses with 1-cosine wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

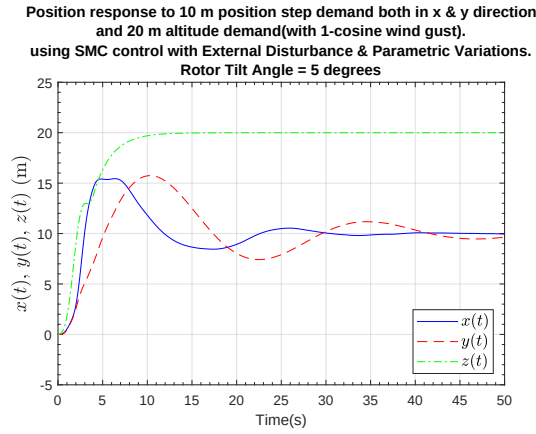


Figure 6.93: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = 5^\circ$

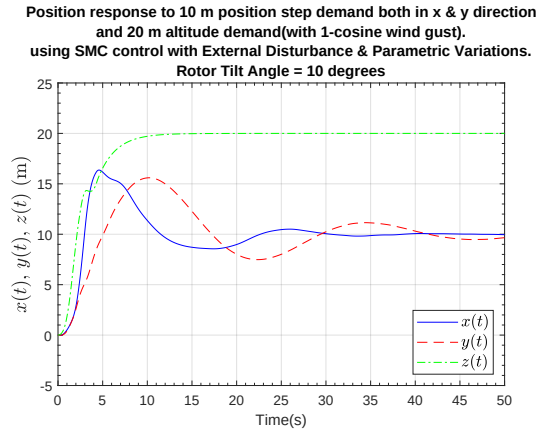


Figure 6.94: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = 10^\circ$

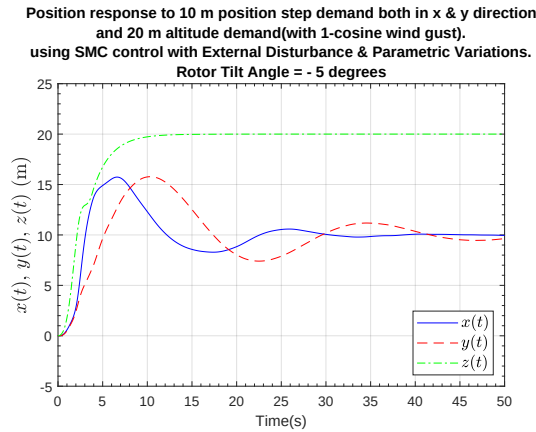


Figure 6.95: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = -5^\circ$

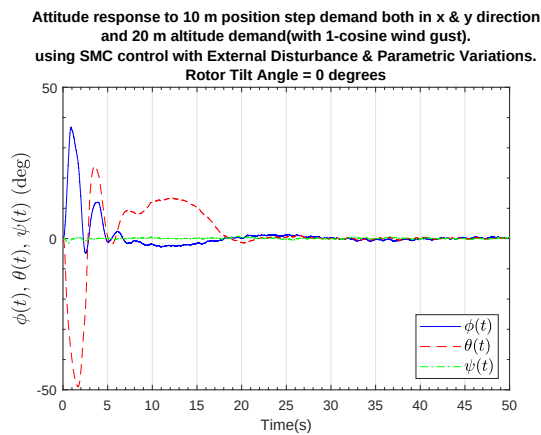


Figure 6.96: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = 0^\circ$

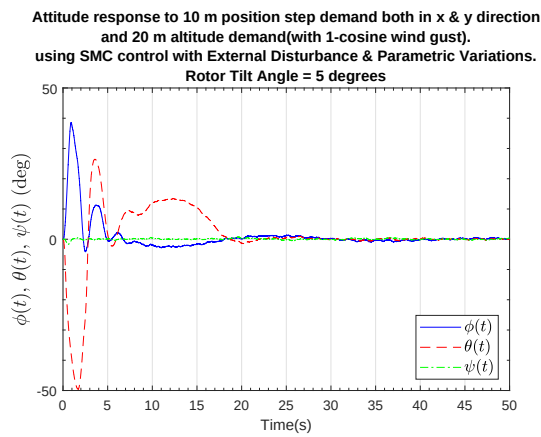


Figure 6.97: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = 5^\circ$

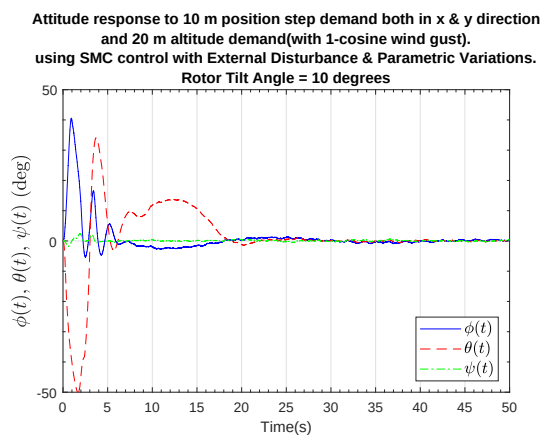


Figure 6.98: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = 10^\circ$

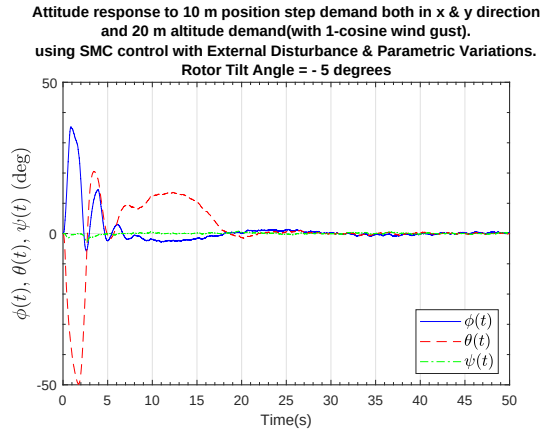


Figure 6.99: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = -5^\circ$

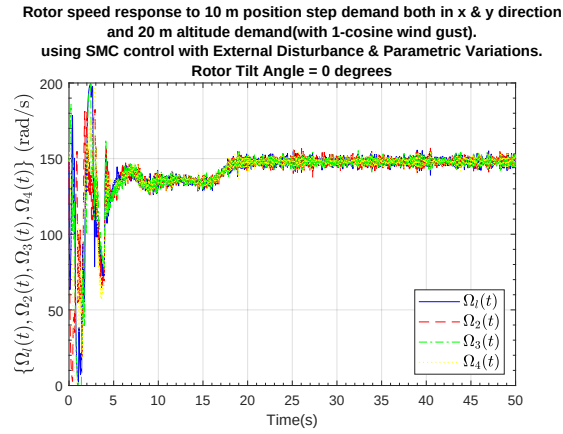


Figure 6.100: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = 0^\circ$

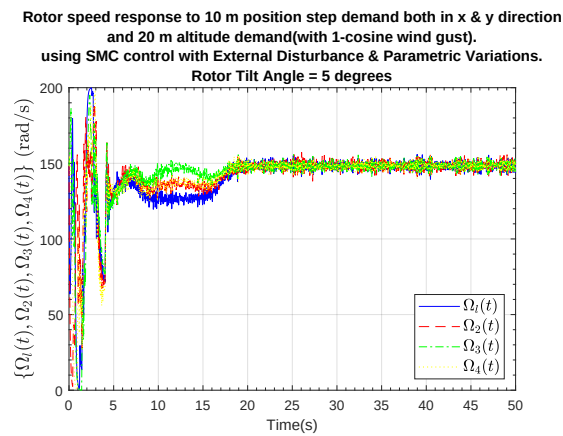


Figure 6.101: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = 5^\circ$

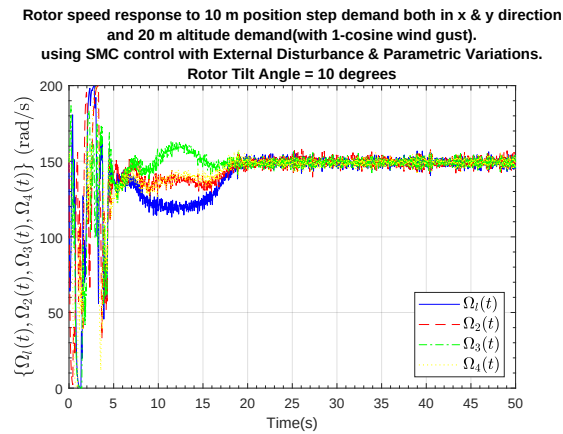


Figure 6.102: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = 10^\circ$

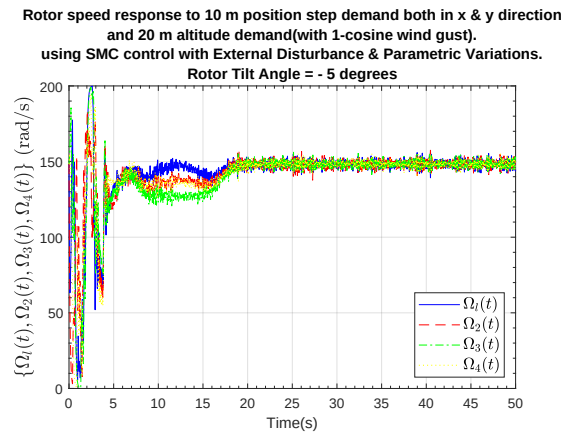


Figure 6.103: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (1-cosine wind gust) using SMC control. $\Upsilon = -5^\circ$

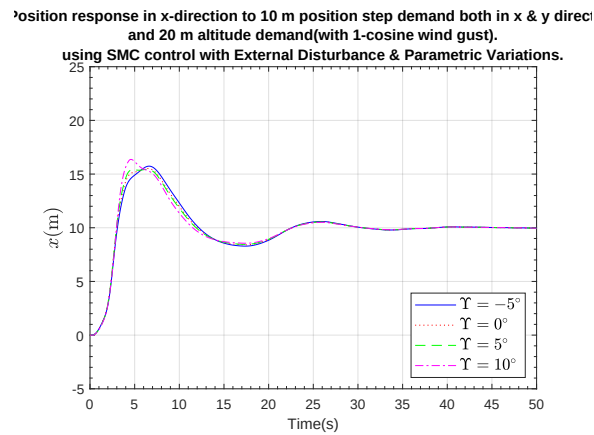


Figure 6.104: Lateral position response in x-direction to 1-cosine wind gust using SMC control

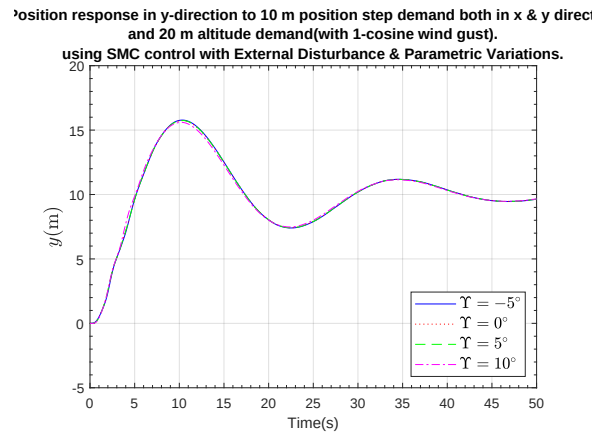


Figure 6.105: Lateral Position response in y-direction to 1-cosine wind gust using SMC control

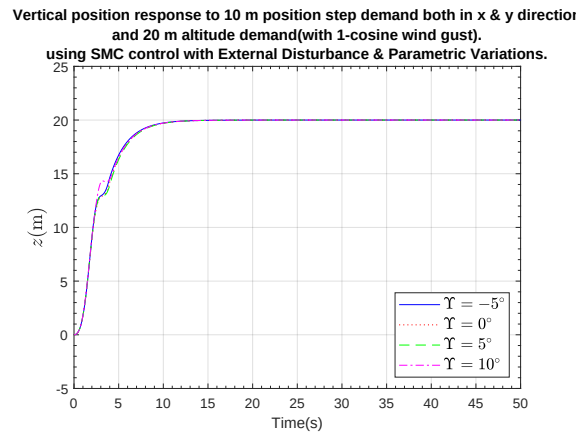


Figure 6.106: Vertical Position response in z-direction to 1-cosine wind gust using SMC control

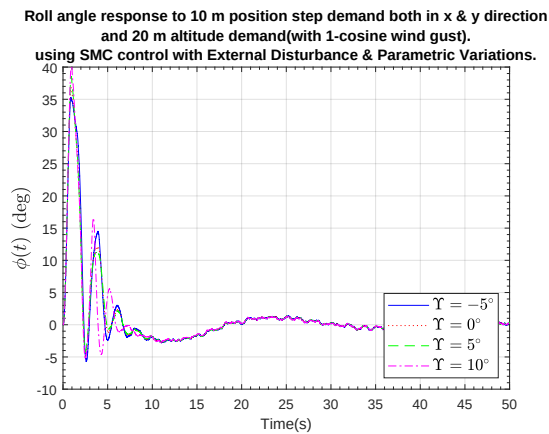


Figure 6.107: Roll angle response to 1-cosine wind gust using SMC control

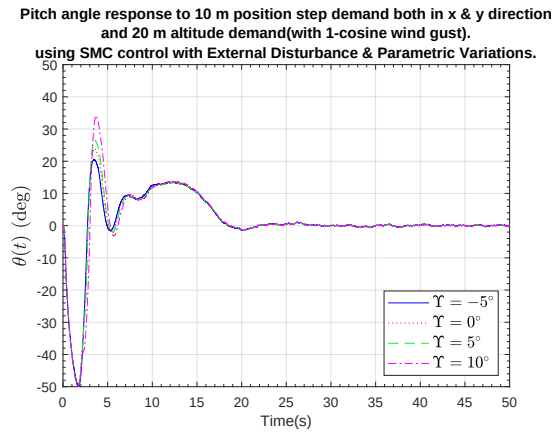


Figure 6.108: Pitch angle response to 1-cosine wind gust using SMC control

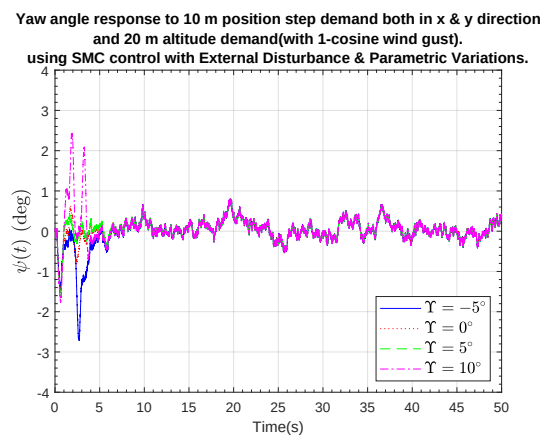


Figure 6.109: Yaw angle response to 1-cosine wind gust using SMC control

6.7 Chapter Summary and Conclusions

In this chapter, effect of rotor tilt angle on gust rejection has been studied using SMC control. The quadrotor in body axes has been modelled taking into account the aerodynamics as well and the effect of outward rotor tilt has been considered and simulation studies carried out.

Initially, simulation studies with SMC control for Altitude, Attitude and PIV Control for Position have been conducted and then the SMC control was extended to Altitude, Attitude and Position Control as well. Simulation studies have been carried out at hover without the effect of wind and also with wind for different rotor tilt angles. Subsequently, simulation studies have been carried out for position and altitude step demands firstly without the effect of wind and then with a steady horizontal wind gust step of 10 m/s have been considered. The simulation also illustrated the attitude and rotor speed responses as well. With the simulation results presented, it is shown that SMC controller performs very well in gust rejection and the rotor tilt angles do not have a significant impact on the performance of the multirotor aircraft when using SMC technique.

A separate set of simulation results have also been presented where the wind gust was taken as a 1-cosine wind gust with maximum value of 10 m/s and acting for 15 secs and the simulation studies have been carried out for the case of hover and position and altitude demands. In this case as well, it was noted that SMC controller performs very well in gust rejection and the rotor tilt angles do not have a significant impact on the performance of the multirotor aircraft.

The results obtained in this research work show that sliding mode controller is very robust at rejecting the gust and shows that the rotor tilt angle does not impact on the vehicle stability when used with SMC which is not in-line with the findings of Whidborne et al. [2022] where LQR controller has been used. The main aim of the work i.e. investigating effect of rotor tilt angle on gust rejection properties in a multirotor aircraft using SMC has been achieved and a comparison of similar investigation based on linear control strategies as undertaken by Whidborne et al. [2022] was done. This chapter opens a window of opportunity to explore more towards investigating the SMC controller for different multirotor aircraft configurations and study the gust rejection properties using different nonlinear control strategies such as backstepping, MPC etc., This fulfils the objective 4 of this work as mentioned in Chapter 1 w.r.t SMC control of the quadrotor.

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7

CHAPTER

Conclusions and Recommendations for Further Work

The aim of this research was to provide an understanding of gust rejection properties of multirotor vehicles using SMC. This chapter summarises how this aim was achieved, highlights the contributions to knowledge and recommends areas for future work.

7.1 Research Summary

As part of this research work, in chapter 2, a detailed review of the work done in the field of multirotor aircraft and the different control strategies used in the control of multirotor aircraft has been presented along with examples with a special emphasis on SMC. A review of different modelling and control techniques taking into account the aerodynamics of the VTOLs is also considered. This chapter also looks into different researches dealing with gust rejection properties of multirotor VTOLs. The initial phase of the research work was to study in detail different configurations of multirotor aircraft and the control strategies associated with them.

The dynamic modelling of a quadrotor in body frame has been performed taking into account the aspects which were previously neglected in many researches. A sliding mode controller has been designed and simulated with the developed quadrotor model and the stability and performance is compared with a PIV controller. The robustness of the SMC controller is tested for parametric uncertainty and external disturbance. It was found that the SMC controller provides good performance and is robust when compared to a PIV controller.

Initially, a conceptual PVTOL model considering aerodynamics was developed and then extended to a quadrotor. This PVTOL model was simulated using PID, SMC control of altitude and attitude and PIV position control and it was extended to SMC control of altitude, attitude and position control and stability investigated. Additionally, the gust rejection properties of a PVTOL were studied including the stability analysis of the PVTOL with aerodynamics was conducted as part of chapter 4.

The effect on gust rejection properties of a quadrotor were investigated initially with PIV control and then by SMC altitude, attitude control and PIV position control. This was extended to a complete SMC control of altitude, attitude and position control as well. The effect of gusts on the lateral and vertical position was investigated. Initially with a small fixed-angle outward tilt of the rotors, it was investigated whether it would result in rejecting the gust or not. A PIV control was simulated with the developed quadrotor dynamic model and it was found that it indeed rejects the wind gust but not as significantly as proposed by Whidborne et al. [2022].

Additionally, the effect of the rotor tilt angle on the stability of the quadrotor is investigated using SMC and a comparative simulation study was done on the three strategies. It was demonstrated that SMC

control produces almost identical responses for different rotor tilt angles in the presence of wind gust and is very robust at rejecting the gust and shows that the rotor tilt angle does not impact on the vehicle stability even in the presence of parametric uncertainties and external disturbances.

The snapshots of all the modelling and development done in MATLAB/Simulink including the MATLAB routines used for simulation have been listed in the appendices.

7.2 Contributions to Knowledge

The contributions to knowledge that have been made as part of this research work are as follows:

- Analysed the effect of rotor tilt angle on the gust rejection. Whidborne and Cooke [2017] carried out investigations of the effect of rotor tilt angle using classical control techniques. The work done by Whidborne and Cooke [2017]; Whidborne et al. [2022] has been extended in this research work.
- The important contribution of this research work is the analysis of the effect of rotor tilt angle on the multirotor aircraft gust rejection using SMC under the presence of parametric uncertainty and external disturbance. This has been carried out initially on a conceptual PVTOL and then extended to a quadrotor. It was found that the sliding mode controller is very robust at rejecting the gust and shows that the rotor tilt angle does not impact on the vehicle stability when used with SMC, even in the presence of parametric uncertainty and external disturbances.

7.3 Suggestions for Further Work

There can be number of possibilities to extend the research presented in this thesis. Firstly, although the main emphasis was placed on quadrotor with SMC control, but this can be extended to other multirotor aircraft configurations using reasonable assumptions made in this work. Secondly, as the MATLAB/Simulink work was done using a safe set of blocks, this model-based design could be directly converted to code using real time workshop in MATLAB/Simulink and use it in validating this controller on different multirotor aircraft.

Finally, other nonlinear control strategies such as backstepping, model predictive control etc., can be explored to investigate whether similar results as SMC will be obtained while studying the effect of rotor tilt angle on gust rejection in multirotor aircraft.

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APPENDIX A

System Modelling of a Birotor without Aerodynamics in MATLAB/Simulink

The dynamic modelling of the birotor without aerodynamics has been done in MATLAB/Simulink. Figure A.1 and Figure A.2 represent the birotor simulink model. The simulation is with a sample time of 0.1 ms using ode4 (Runge-Kutta) solver for PD Control. Figure A.3 is snapshot of the control allocation of a birotor. Figure A.4 and A.5 show snapshots of the model dealing with PD control of a birotor.

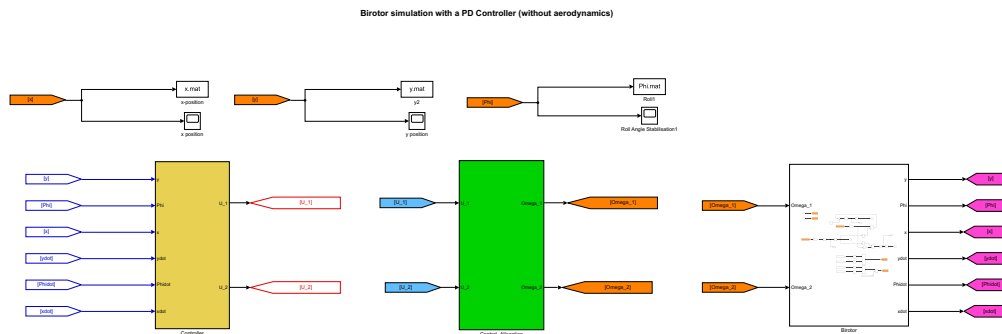


Figure A.1: PD control of a birotor



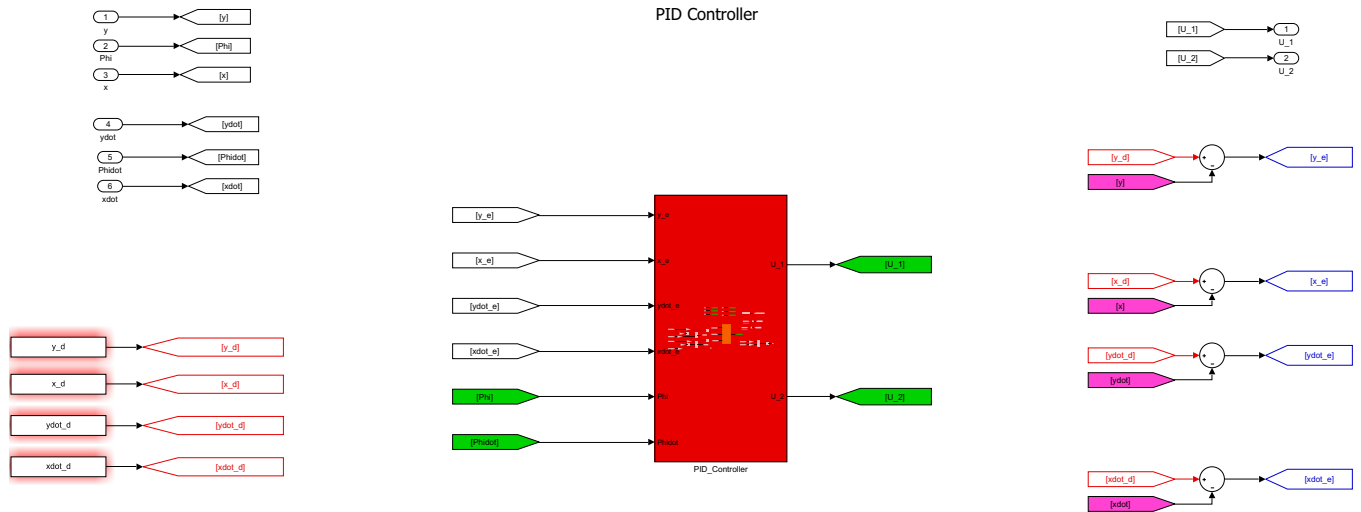


Figure A.4: Birotor PD controller

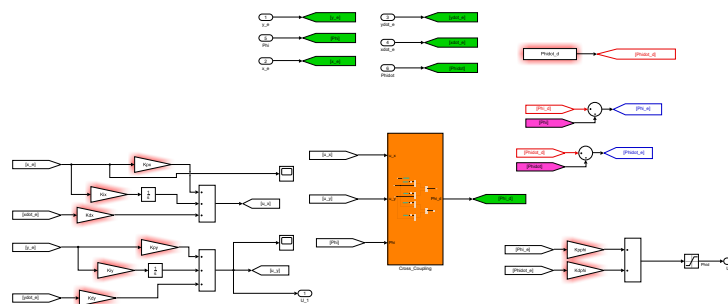


Figure A.5: PD controller design

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APPENDIX B

System Modelling of a PVTOL with Aerodynamics in MATLAB/Simulink

The dynamic modelling of the PVTOL with aerodynamics in body axes has been done using MATLAB/Simulink. Figures B.1, B.2, B.3 and B.4 represent the PVTOL with aerodynamics simulink model. The simulation is run with a sample time of 0.01 s using ode4(Runge-Kutta) solver for PID Control.

Figure B.5 is snapshot of the control allocation of a conceptual PVTOL.

Figure B.6 shows the snapshot of the model dealing with PID control of a conceptual PVTOL.

Figures B.7, B.8, B.9, B.10 and B.11 show the snapshots of the model dealing with SMC Altitude and Attitude and PIV position control of a conceptual PVTOL. The SMC position control is as depicted in Figure B.12.

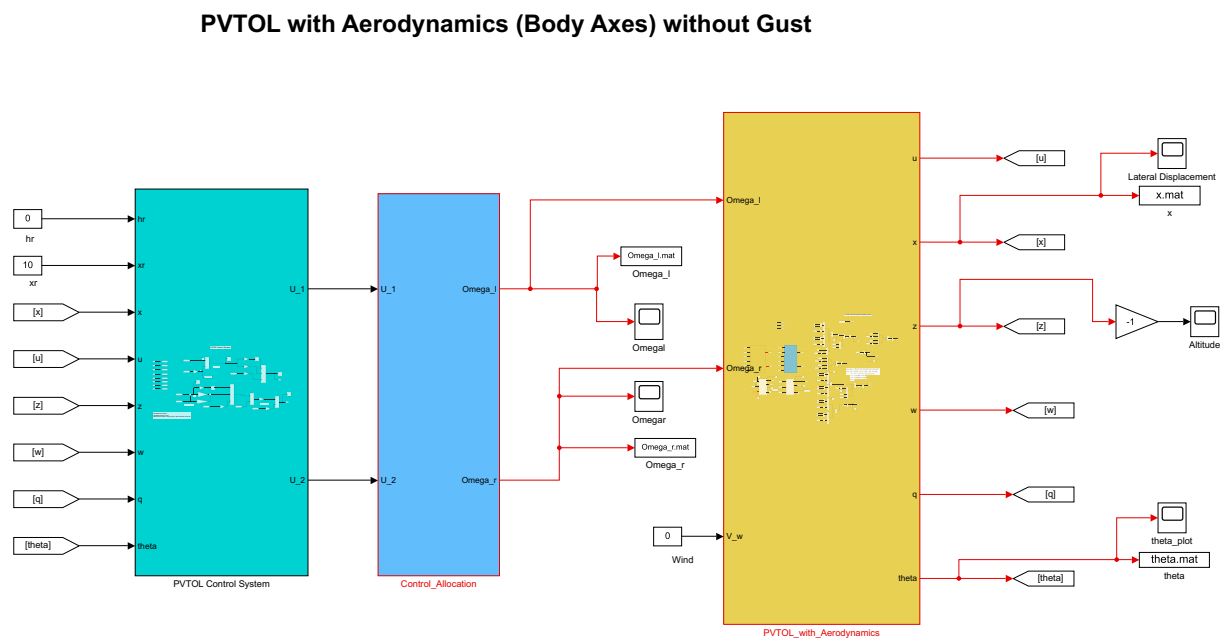


Figure B.1: PID control of a PVTOL

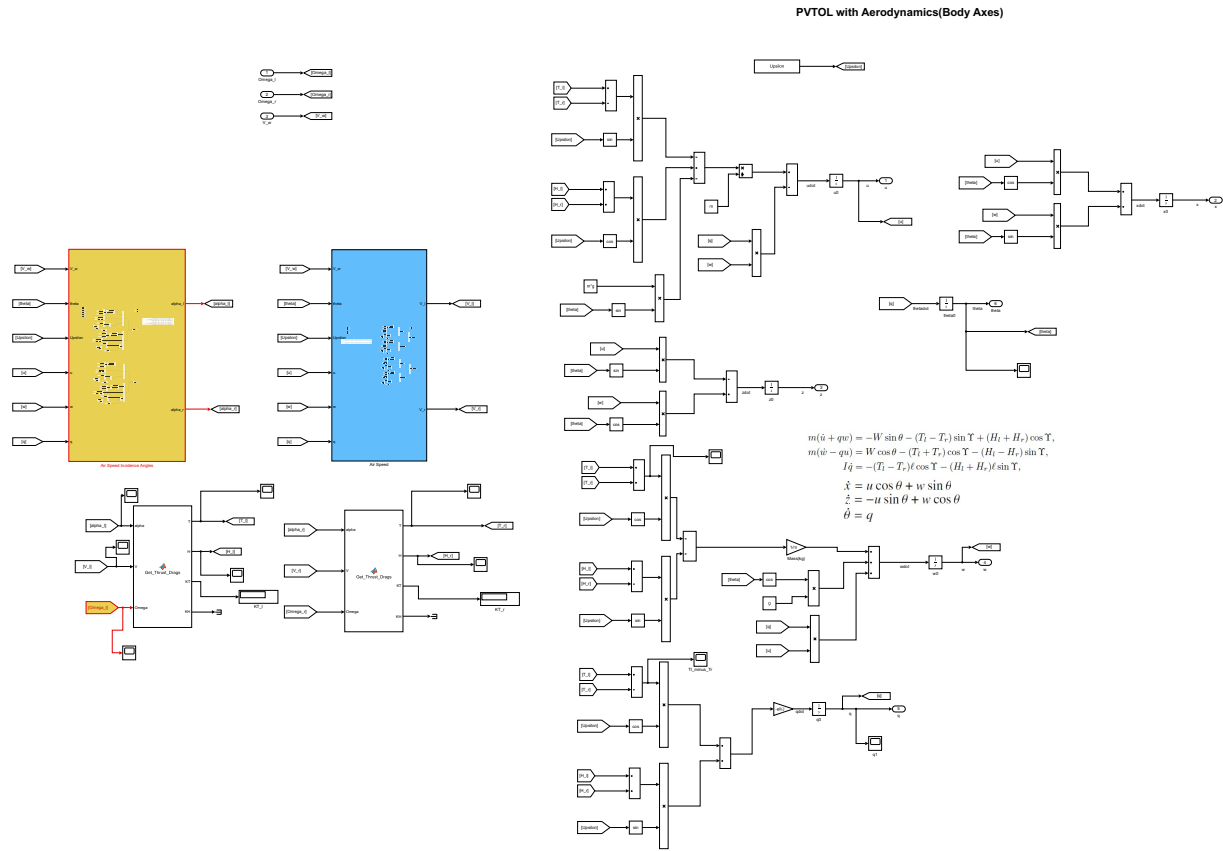


Figure B.2: PVTOL model with aerodynamics

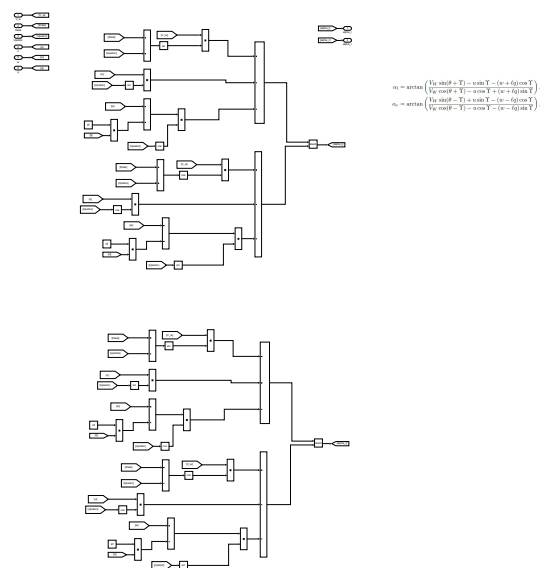


Figure B.3: PVTOL model with aerodynamics - air speed incidence angles

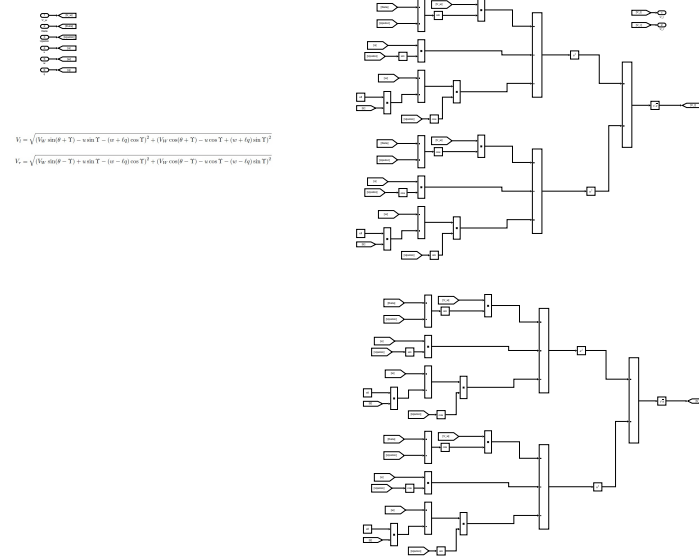


Figure B.4: PVTOL model with Aerodynamics - air speed model block

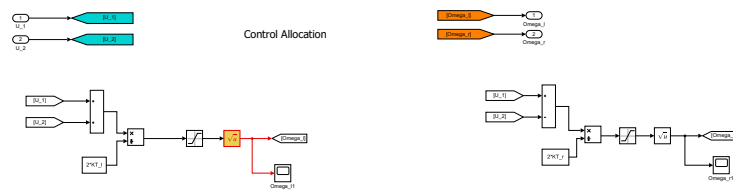


Figure B.5: PVTOL with aerodynamics - control allocation

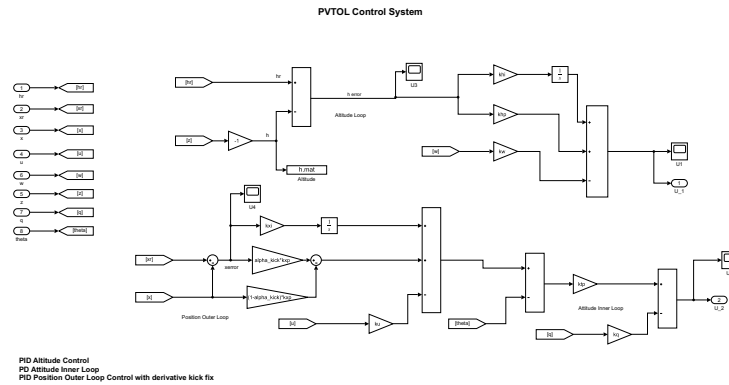


Figure B.6: PVTOL with aerodynamics PID control system

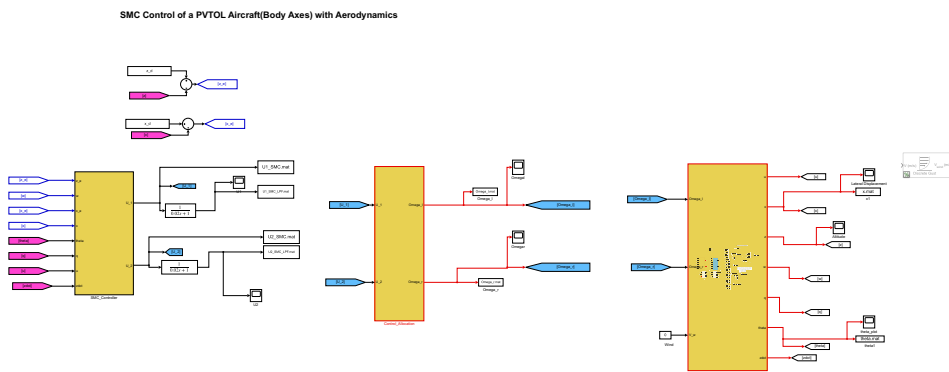


Figure B.7: SMC control of a PVTOL

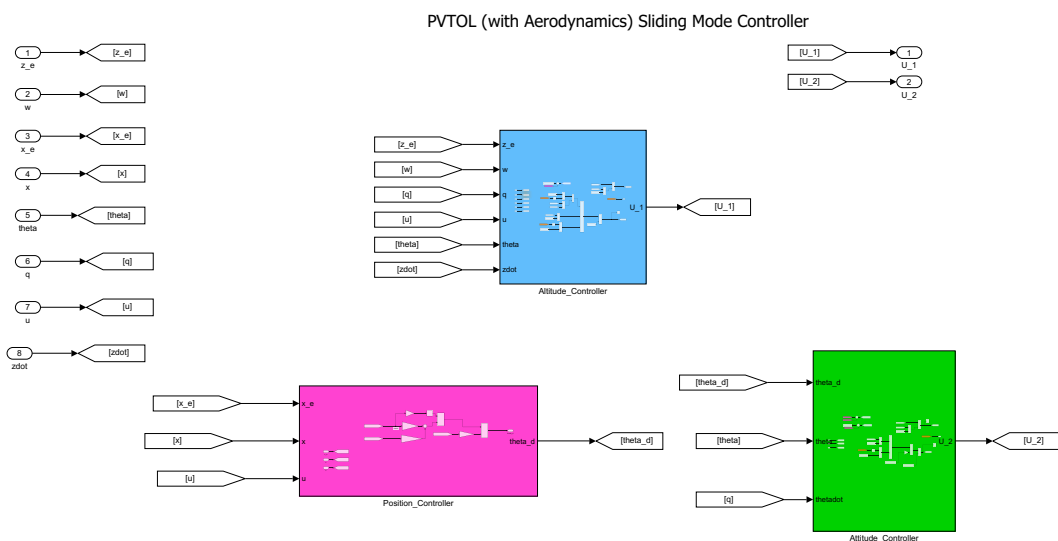


Figure B.8: PVTOL with aerodynamics SMC control system

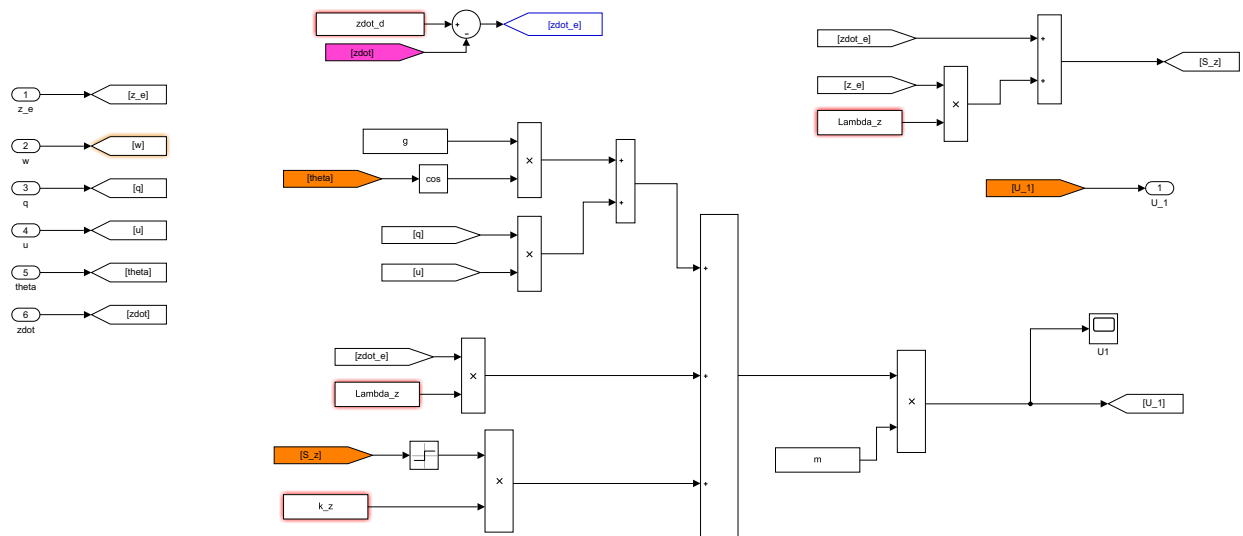


Figure B.9: PVTOL with aerodynamics SMC control system - altitude control

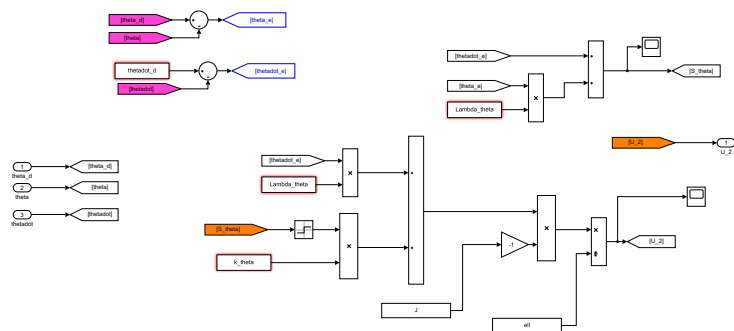


Figure B.10: PVTOL with aerodynamics SMC control system - attitude control

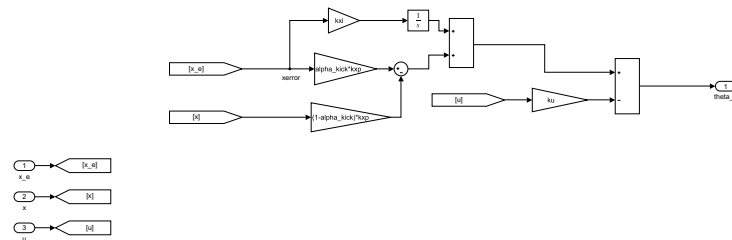


Figure B.11: PVTOL with aerodynamics control system - position control

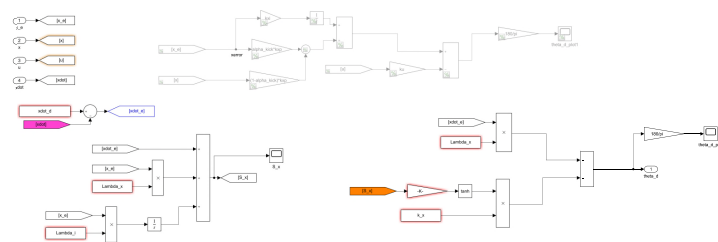


Figure B.12: PVTOL with aerodynamics SMC control system - position control

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APPENDIX C

System Modelling of a Quadrotor with Aerodynamics in MATLAB/Simulink

C.1 System Modelling of a Quadrotor with PIV Control

The dynamic modelling of the quadrotor in body axes has been done in MATLAB/Simulink. Figure C.2 represents the quadrotor simulink Model in body axes. The simulation is run with a sample time of 0.01 s using ode4(Runge-Kutta) solver for PIV control (Figure C.1).

Figures C.3, C.4, C.5 and C.6 show the snapshots of the subsystems developed for PIV control of a quadrotor in body axes.

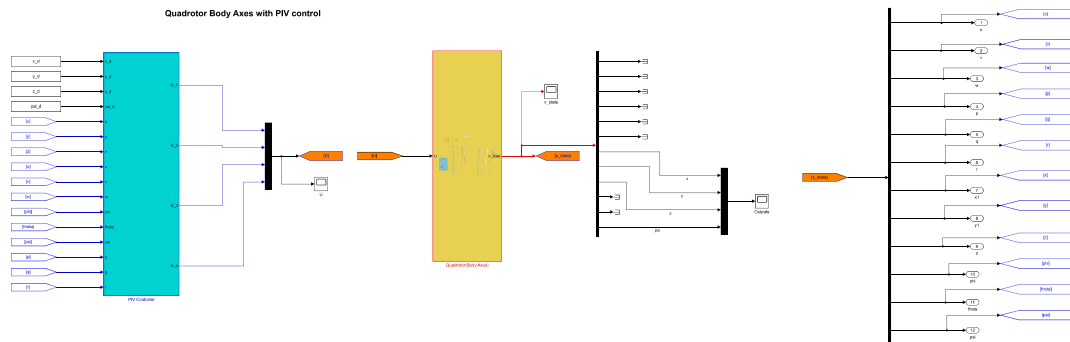


Figure C.1: Simulink model of the quadrotor with PIV Control

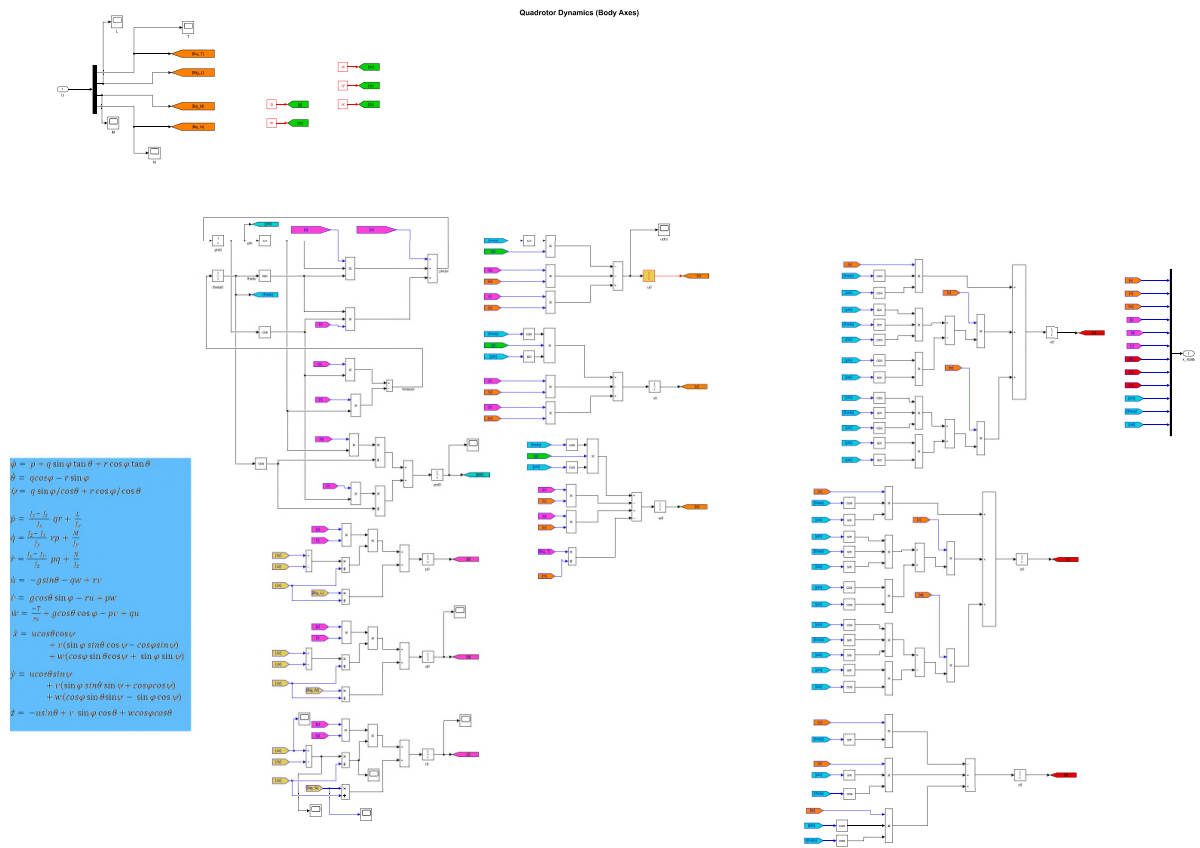


Figure C.2: Simulink model of the quadrotor

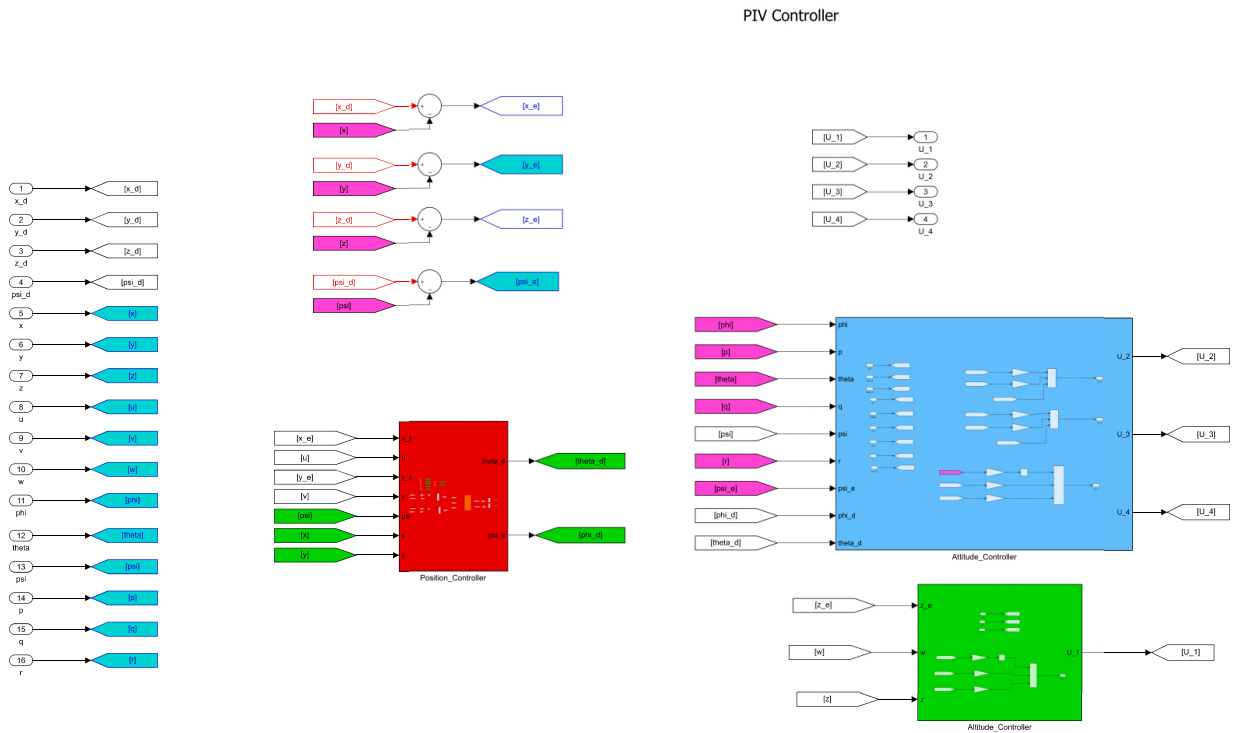


Figure C.3: Quadrotor with PIV control

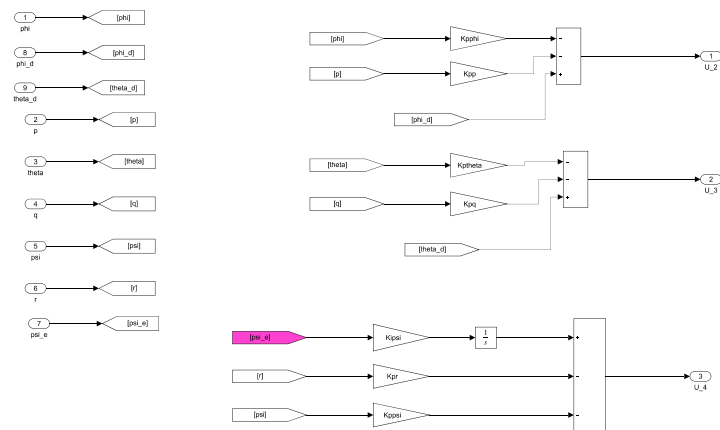


Figure C.4: Altitude controller

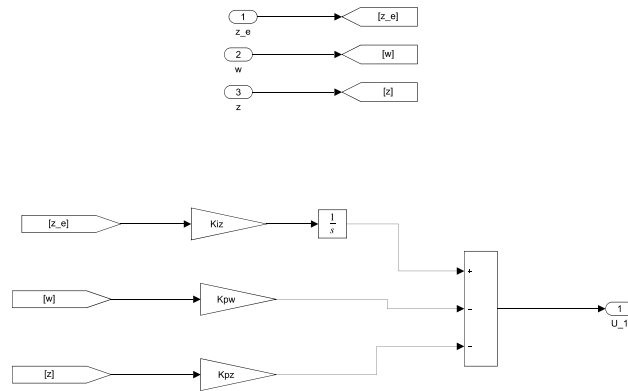


Figure C.5: Attitude controller

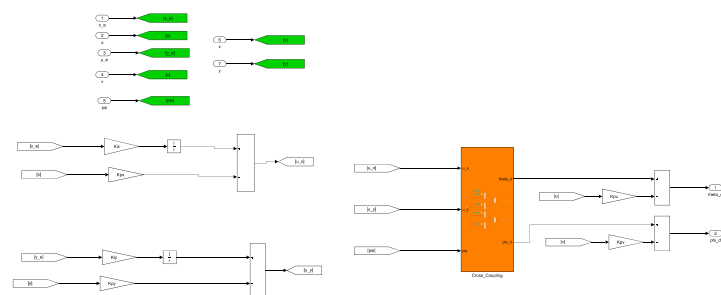


Figure C.6: Position controller

C.2 System Modelling of a Quadrotor with SMC Control

The dynamic modelling of the quadrotor has been done in MATLAB/Simulink. Figure C.7 represents the quadrotor simulink model with SMC controller. The simulation is run with a sample time of 0.1 ms using ode4(Runge-Kutta) solver for sliding mode control.

Figures C.8, C.9, C.10 and C.11 show the snapshots of the subsystems developed for SMC control of a quadrotor.

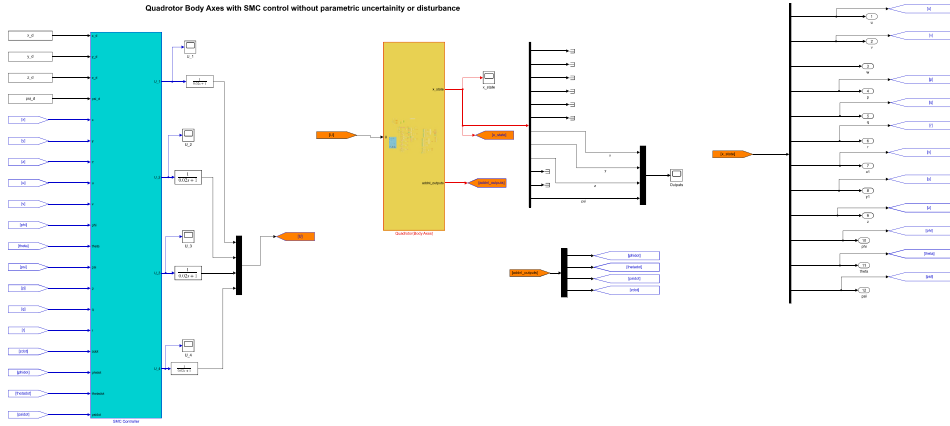


Figure C.7: Sliding mode controller for the quadrotor

C.3 Quadrotor with Aerodynamics with PIV Control MATLAB/Simulink Modelling and Design

The dynamic modelling of the quadrotor with aerodynamics in body axes has been done using MATLAB/Simulink. Figures C.12, C.13, C.14, C.15, C.16 and C.17 represent the quadrotor with aerodynamics simulink model. The simulation is run with a sample time of 0.01 s using ode4(Runge-Kutta) solver for PIV Control.

Figure C.18 is snapshot of the Control Allocation of the quadrotor.

Figure C.19 shows the snapshot of the model dealing with PID control of the quadrotor.

Figures C.20 and C.21 show the snapshots of the model dealing with position control of the quadrotor.

Figure C.22 shows the snapshot of the model dealing with attitude control of the quadrotor

Figure C.23 shows the snapshot of the model dealing with altitude control of the quadrotor

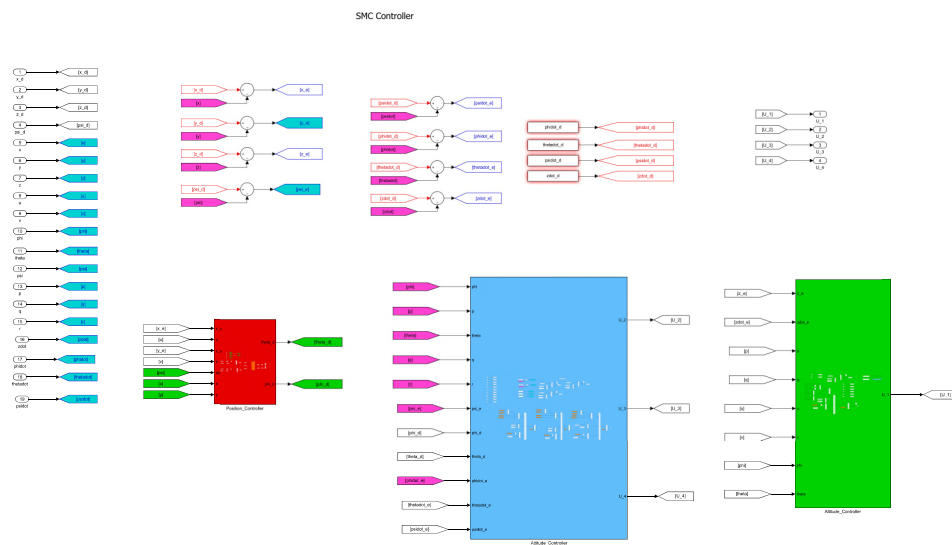


Figure C.8: Sliding mode controller

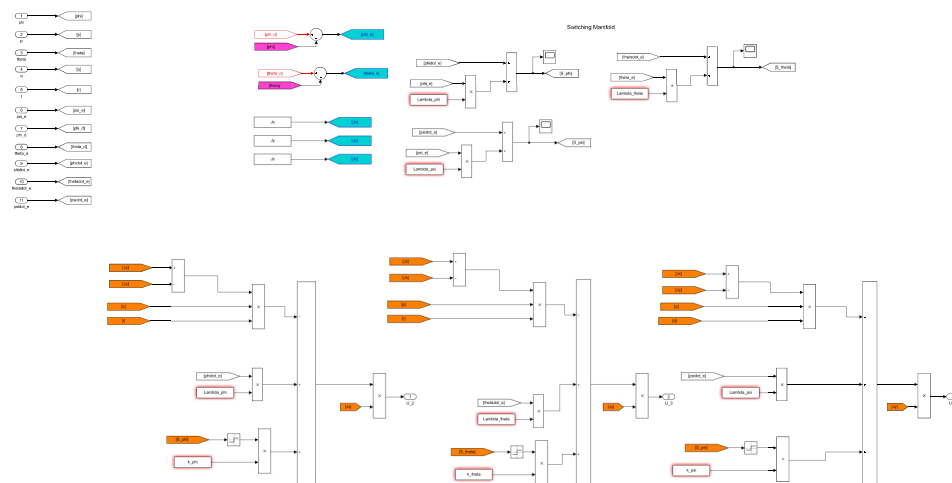


Figure C.9: Quadrotor - sliding mode attitude controller

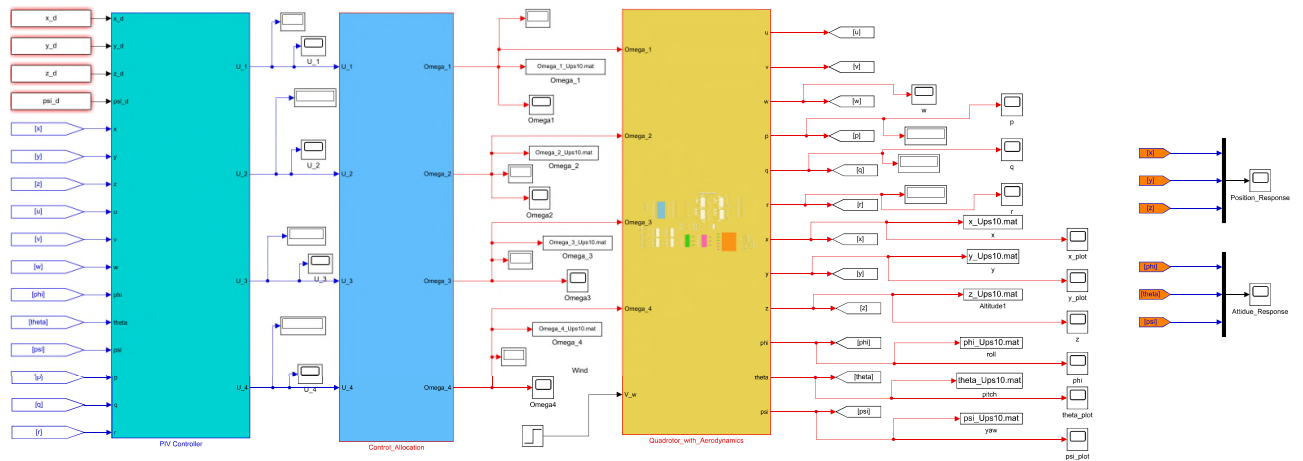


Figure C.12: PID control of a quadrotor

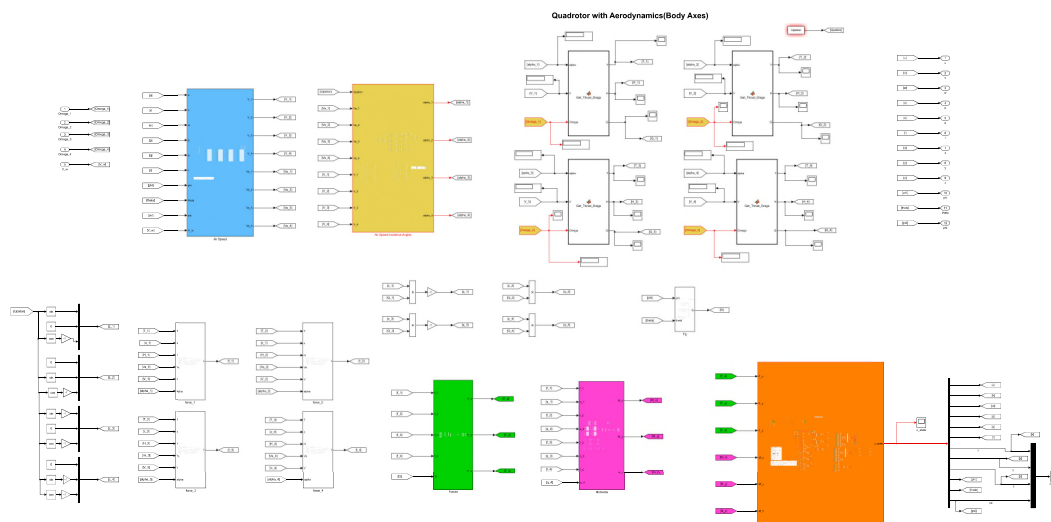


Figure C.13: Quadrotor model with aerodynamics

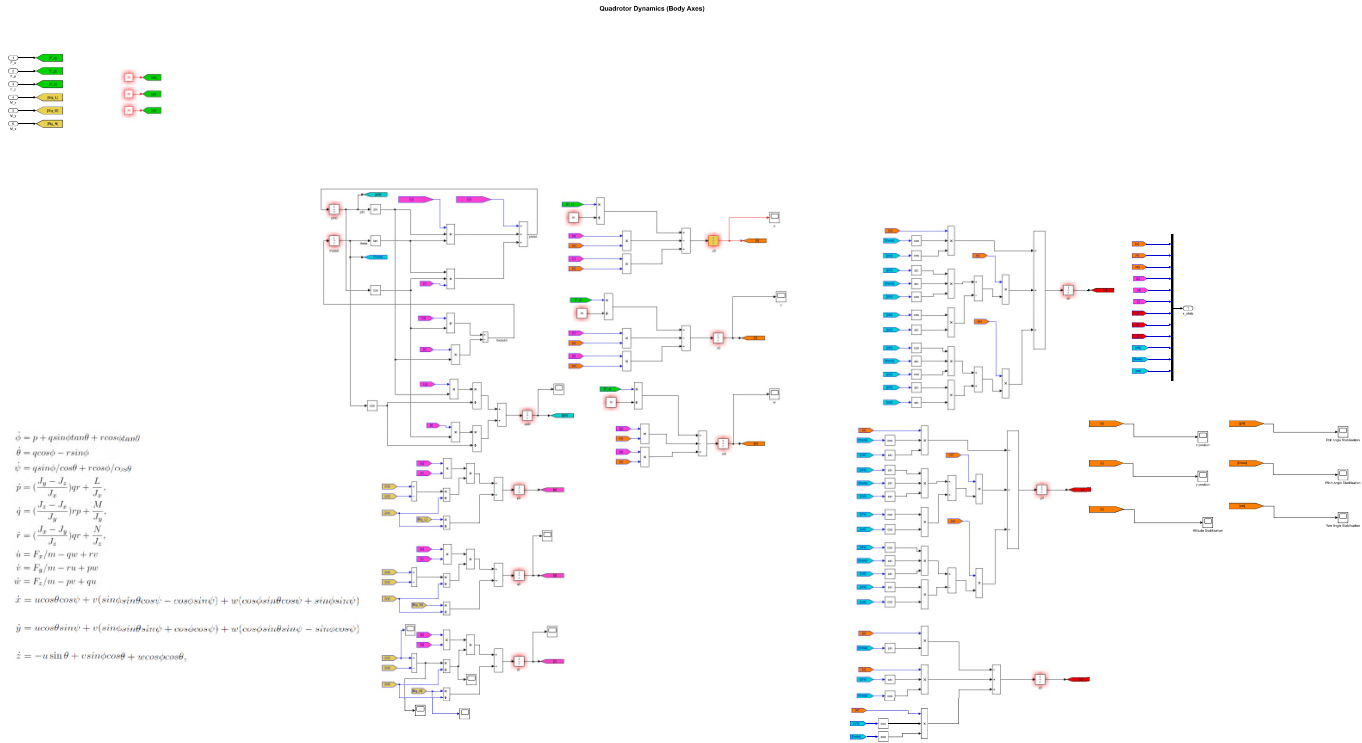


Figure C.14: Quadrotor model with aerodynamics - detailed

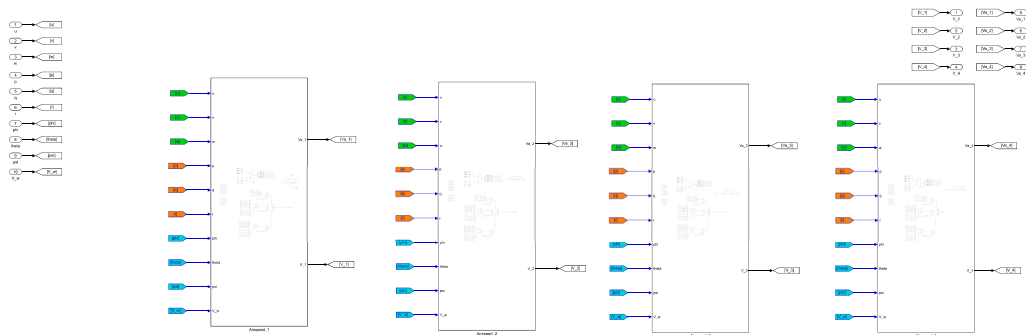
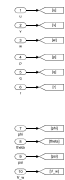
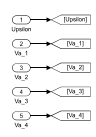


Figure C.15: Quadrotor model with aerodynamics - air speed model block



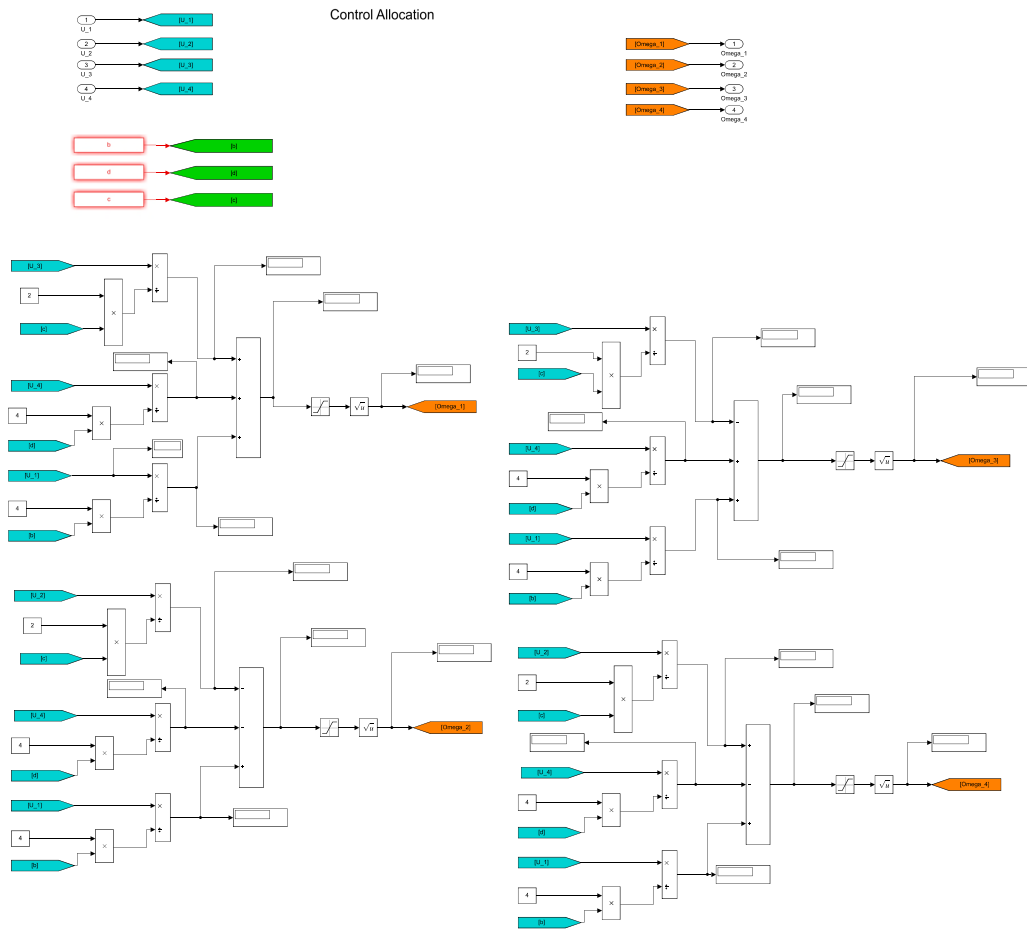


Figure C.18: Quadrotor model with aerodynamics - control allocation

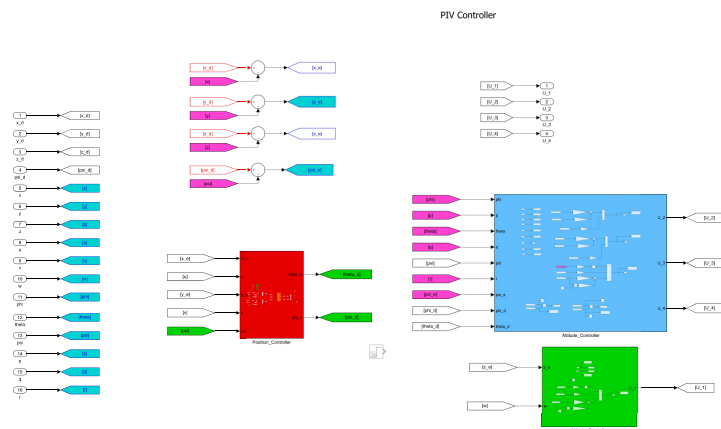


Figure C.19: Quadrotor with aerodynamics - PIV controller

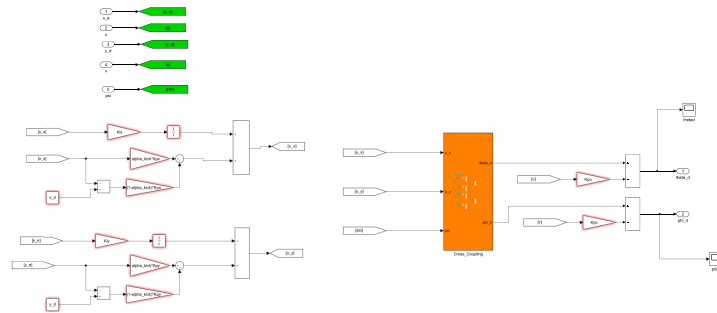


Figure C.20: Quadrotor with aerodynamics - position controller

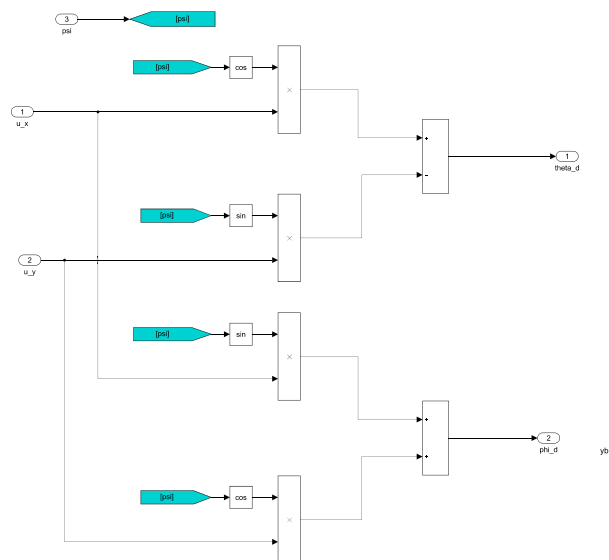


Figure C.21: Quadrotor with aerodynamics - cross coupling in position controller

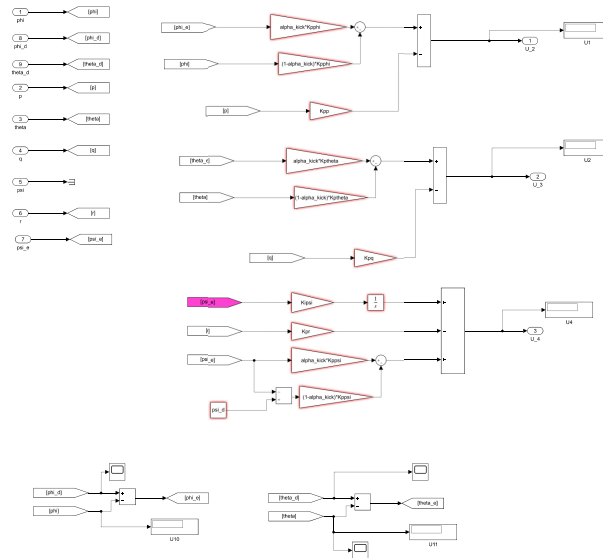


Figure C.22: Quadrotor with aerodynamics - attitude controller

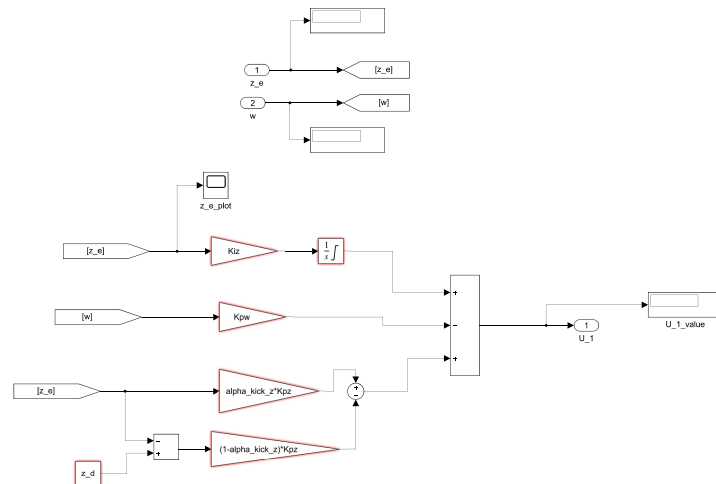


Figure C.23: Quadrotor with aerodynamics - altitude controller

C.4 Quadrotor with Aerodynamics with SMC Altitude and Attitude control and PIV Position Control MATLAB/Simulink Modelling and Design

Figures C.24, C.25, C.26, C.27 and C.28 show the snapshots of the model dealing with SMC control of a quadrotor.

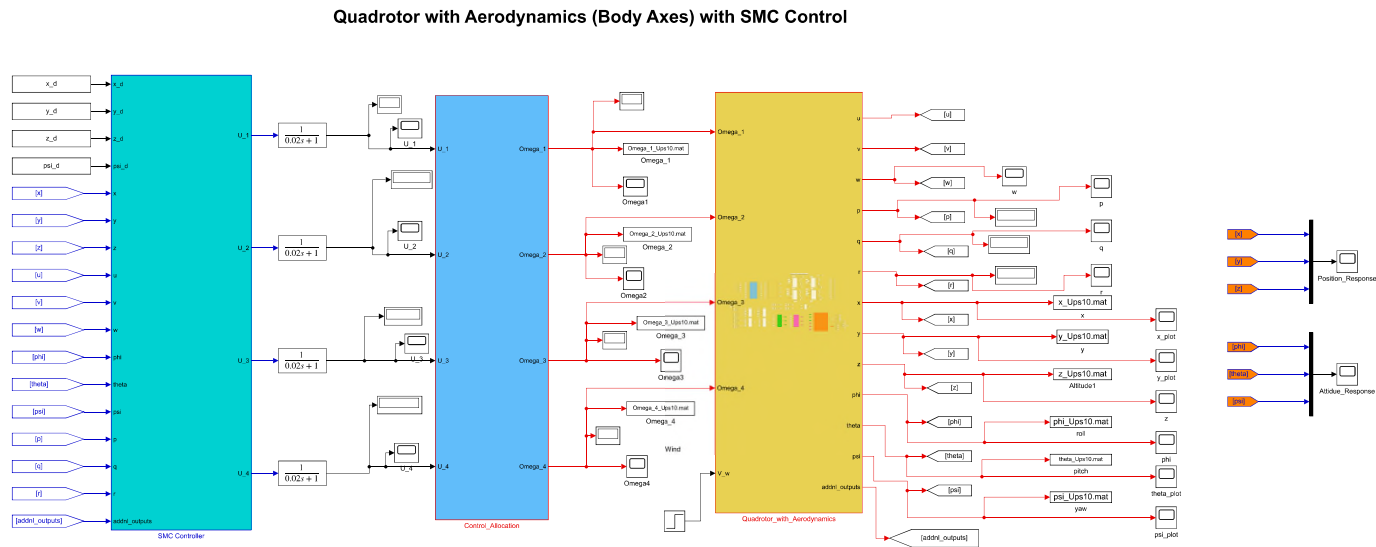


Figure C.24: SMC control of a quadrotor

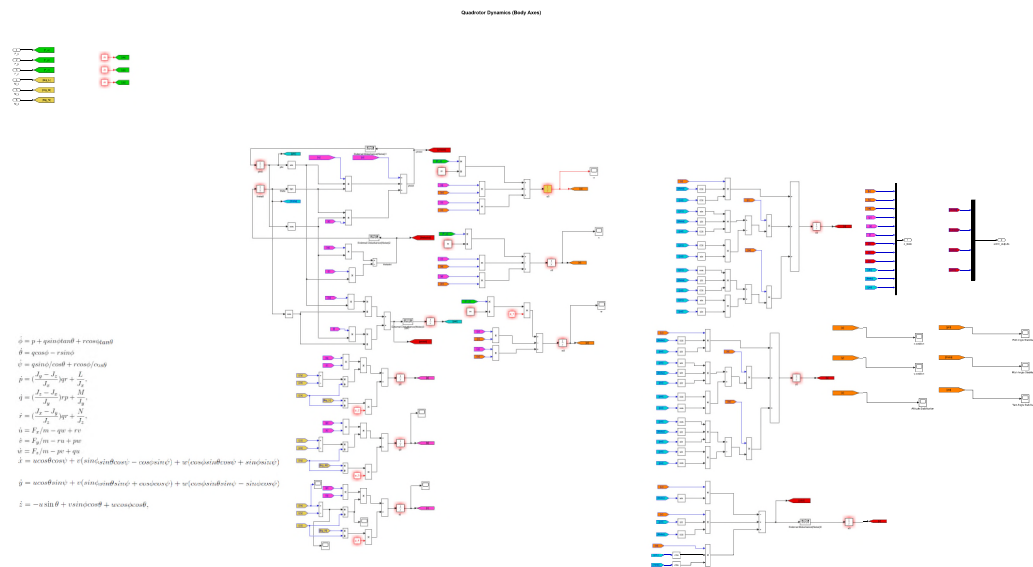


Figure C.25: Quadrotor with aerodynamics - detailed

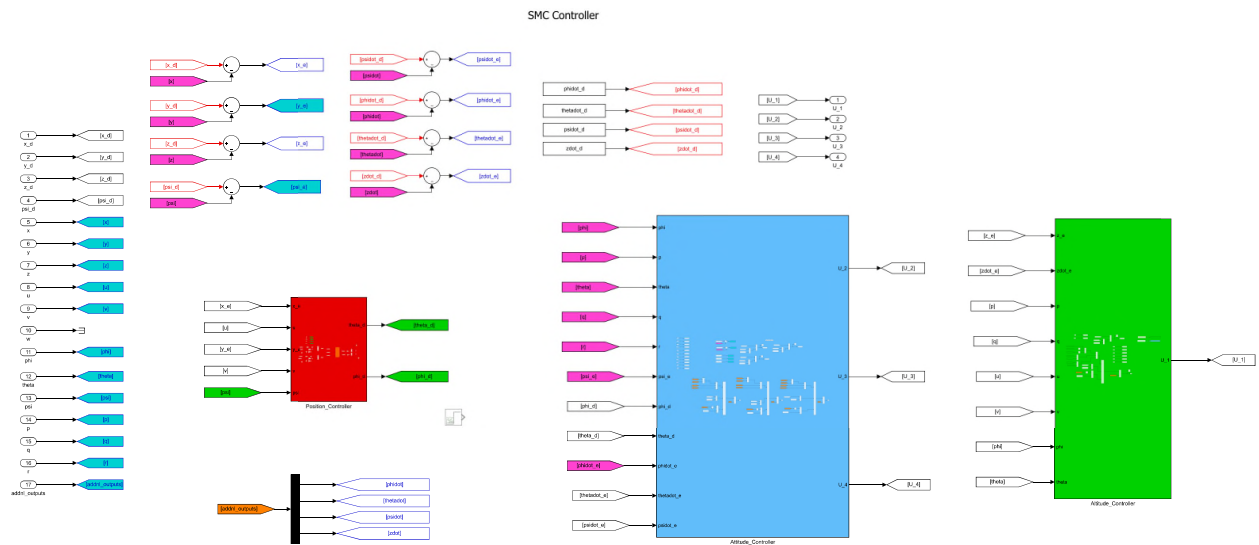


Figure C.26: Quadrotor with aerodynamics SMC controller

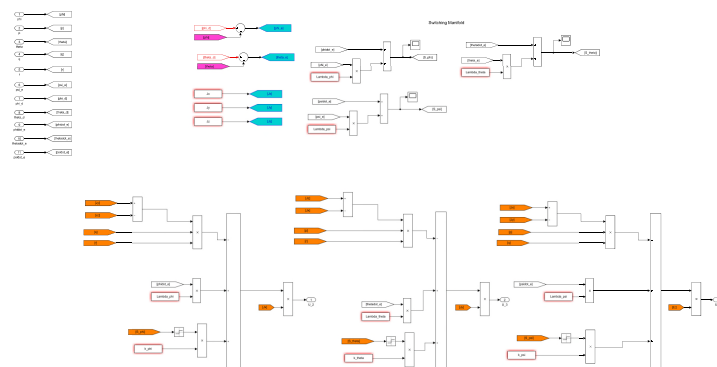


Figure C.27: Quadrotor with aerodynamics SMC attitude controller

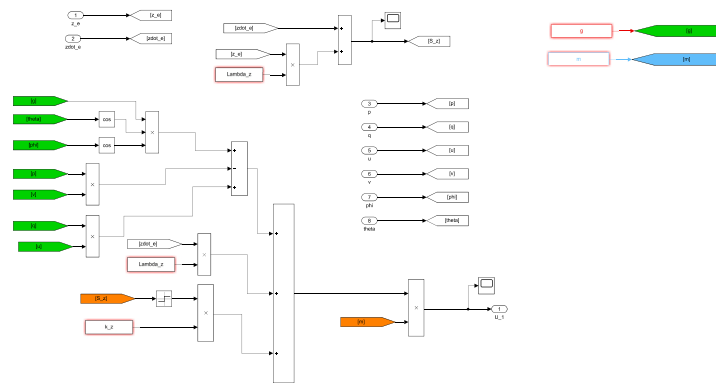


Figure C.28: Quadrotor with aerodynamics SMC altitude controller

C.5 Quadrotor with Aerodynamics with SMC Altitude and Attitude control and Position Control MATLAB/Simulink Modelling and Design

Figures C.29 and C.30 show the snapshots of the model dealing with SMC Altitude, Attitude and Position control of a quadrotor.

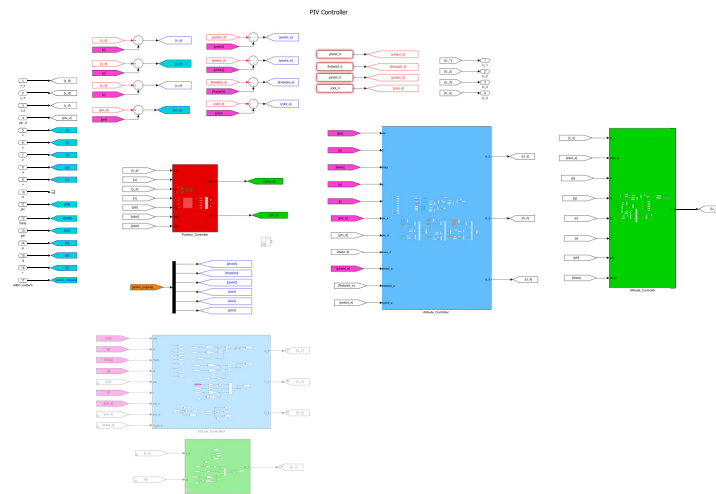


Figure C.29: Quadrotor with aerodynamics with SMC Altitude and Attitude control and Position Control controller

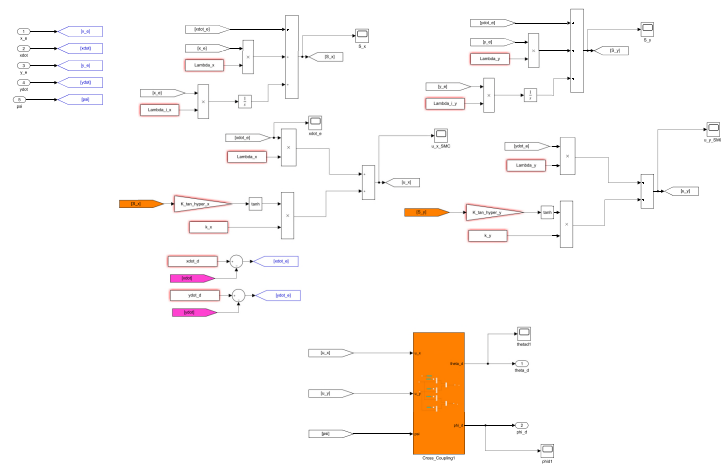


Figure C.30: Quadrotor with aerodynamics SMC Position controller

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APPENDIX D

MATLAB Code

D.1 MATLAB Code for PVTOL with Aerodynamics PID and SMC Control

Some of the m-files used for PVTOL and quadrotor modelling with aerodynamics have been adapted from Whidborne [2020].

D.1.1 Initialisation for PID Control

```
1 %Initialisation file for the PVTOL(Body Axes) with
2 %Aerodynamics Simulation using PID controller
3
4 [~, pvtol, rotor, rho] = Coefficients; % PVTOL coefficients
5
6 m=1.18;      % mass (kg)
7 J=0.0625;    % moment of inertia (kgm^2).
22 % It is same as I=J=0.0625. I and J are used interchangeably in init files
24
8 ell = 0.45;  % armlength (m)
9 g = 9.81;    % gravitational constant
10 Upsilon = 0; %ranging from -6 to +7
11
12 khp=10;     % proportional
13 khi=1; % integral
14 kw = -2.0;
15 kxp = -0.050; % proportional
16 kxi = -0.0115; % integral
17 ku = -0.10;
18 %% classical attitude controller
19 ktp=-17.0;  % proportional
20 kq=-8.0;
21
22 % KT_l = rho*A*(rotor.R)^2*CT_l; % T/Omega^2: ~6/(150^2);
23 % KT_r = rho*A*(rotor.R)^2*CT_r; %T/Omega^2: ~6/(150^2);
24
25 KT_l = 0.00026459;
```

```
26 KT_r = 0.00026459;
27
28 % initial state
29 u0 = 0;
30 x0=0;
31 w0 = 0;
32 z0=0;
33 q0 = 0;
34 theta0=0;
35
36 % linear
37 alpha_kick=1; % derivative kick fix
```

D.1.2 Initialisation for PIV control of Quadrotor

```
1
2 %init
3 % mod jfw 14 Dec 2020
4 Upsilon = 10*pi/180; %ranging -5 to +10 degrees.
5 [quad, ~, rotor, rho] = Coefficients(Upsilon); % Draganfly XPro quadrotor parameters
6
7 m=2.356; % mass (kg)
8 %m = 2; % mass (kg)
9 Jx = 0.1676; % inertia of x axis [kg.m^2]
10 Jy = 0.1676; % inertia of y axis [kg.m^2]
11 Jz = 0.29743; % inertia of z axis [kg.m^2] (should be at least 2I_x)
12
13 %Jx = 0.2; % inertia of x axis [kg.m^2]
14 %Jy = 0.2; % inertia of y axis [kg.m^2]
15 %Jz = 0.4; % inertia of z axis [kg.m^2] (should be at least 2I_x)
16
17 %J = diag([Jx;Jy;Jz]); % moment of inertia matrix
18 %ell = 0.4534; % armlength (m) (Martinez p 273)
19 ell = 0.25; %l (m) Arm Length
20 g = 9.81; % gravitational constant
21
22 % parameters of control allocation
23
24 b = 2.6333*10^-4; %thrust coefficient (N.s^2) - Gains of the Control Allocation - b = F0
25 c = 6.5833*10^-5; % Gains of the Control Allocation - c = b*l; M = c*OmegaTrim^2;
26 %d = tau/148.127^2;
27 d = 9.5843e-06; % Gains of the Control Allocation - tau = d*OmegaTrim^2; Use Q_Trim rout
28
29
30 %-----
31 %With Upsilon = 0; wiht V_Wind = 0; %Finalised
32
33 %x & y gains should be same as the aircraft is symmetrical
34
35 Kpx = -0.195; Kix = -0.0375; Kpu = -0.3;
36 Kpy = -Kpx; Kiy = -Kix; Kpv = -Kpu;
37 Kpz = -3.1; Kiz = -0.45; Kpw = -3.75;
38 Kpphi = 4.75; Kptheta = 6.5; Kppsi = 1.6;
39 Kpp = 4; Kpq = 6; Kpr = 1.5;
40 Kipsi=1;
41
```



```

42
43 Omega_max = 200;
44 Omega_min = 1; %To avoid divide by zero.
45
46
47 %initial condition
48 %U0 = transpose([m*g 0 0 0]);
49
50 p_d = 0;
51 q_d = 0;
52 r_d = 0;
53
54 u_d = 0;
55 v_d = 0;
56 w_d = 0;
57
58 x_d = 10;
59 y_d = 10;
60 z_d = 20;
61 psi_d = 0;
62
63
64 % initial state
65 u0 = 0;
66 v0 = 0;
67 w0 = 0;
68 x0 = 0;
69 y0 = 0;
70 z0 = 0;
71 p0 = 0;
72 q0 = 0;
73 r0 = 0;
74 phi0 = 0;
75 theta0 = 0;
76 psi0 = 0;
77
78 % linear
79 alpha_kick=0.75; % derivative kick fix
80 alpha_kick_z=0.75; % derivative kick fix
81
82
83 Vw = 0; % constant horizontal wind airspeed (m/s)
84 wind=[0;0;0]; % wind disturbance
85
86 % initial trim guess values
87 %Omega_1 = 150; Omega_2 = 150; Omega_3 = 150; Omega_4 = 150;
88
89
90 %% trim & lin
91 % options for trim residual minimization
92 trimoptions = optimset('TolFun',1e-7,'TolX',1e-7,...
93     'MaxFunEvals',20000,'MaxIter',20000,...
94     'Display','on');
95
96 % set initial trim points guess
97 xtrim0=zeros(12,1); utrim0=148.1287*ones(4,1);
98
99

```

```

100 %trim & linearize
101 [A,Bomega,xtrim,Omegatrim] = QuadrotorTrimLin( quad,wind,xtrim0,utrim0,trimoptions);
102 B = Bomega*quad.CA;
103
104 Bu=B(:,1:2); % extract control matrix
105 Bw=B(:,3); % extract disturbance matrix
106 C=[1 0 0 0 0 0; % output matrix
107     0 0 1 0 0 0;
108     0 0 0 0 1 0];
109
110 thetatrim=xtrim(5);
111
112 Omegaltrim = Omegatrim(1);
113 Omega2trim = Omegatrim(2);
114 Omega3trim = Omegatrim(3);
115 Omega4trim = Omegatrim(4);
116
117
118 %KT_1 = (quad.m*g/4)/Omegaltrim^2;
119 %KT_2 = (quad.m*g/4)/Omega2trim^2;
120 %KT_3 = (quad.m*g/4)/Omega3trim^2;
121 %KT_4 = (quad.m*g/4)/Omega4trim^2;
122

```

D.1.3 Initialisation for SMC control of Attitude and PIV Control of Position of Quadrotor

```

1
2 %Initialisation File for the SMC control of PVTOL with Aerodynmics
3 [~, pvtol, rotor, rho] = Coefficients; % PVTOL coefficients
4
5 m=1.18; % mass (kg)
6 J=0.0625; % moment of inertia (kgm^2)
21
22 % It is same as I=J=0.0625. I and J are used interchangeably in init files
24
7 ell = 0.45; % armlength (m)
8 g = 9.81; % gravitational constant
9 Upsilon = 0; %ranging from -6 to +7
10
11 %SMC Controller parameters
12 Lambda_theta = 1;
13 k_theta = 10;
14
15 Lambda_z = 1;
16 k_z = -3;
17
18 z_d = 0;
19 x_d = 10;
20
21 kxp = -0.050; % proportional
22 kxi = -0.0115; % integral
23 ku = -0.10;
24 %% classical attitude controller
25 ktp=-17.0; % proportional
26 kq=-8.0;

```

```

27
28 KT_l = 0.00026459;
29 % KT_l = rho*A*(rotor.R)^2*CT_l; % T/Omega^2: ~6/(150^2);
30 KT_r = 0.00026459;
31 % KT_r = rho*A*(rotor.R)^2*CT_r; %T/Omega^2: ~6/(150^2);
32
33 % initial state
34 u0 = 0;
35 x0=0;
36 w0 = 0;
37 z0=0;
38 q0 = 0;
39 theta0=0;
40 thetadot_d = 0;
41 zdot_d=0;
42
43 % linear
44 alpha_kick=1; % derivative kick fix

```

D.1.4 Initialisation for SMC control of Attitude and Position Control of Quadrotor

```

1
2 %Initialisation File for the SMC control of PVTOL with Aerodyanmics
3 [~, pvtol, rotor, rho] = Coefficients; % PVTOL coefficients
4
5 m=1.18; % mass (kg)
6 J=0.0625; % moment of inertia (kgm^2)
7 ell = 0.45; % armlength (m)
8 g = 9.81; % gravitational constant
9 Upsilon = 10*pi/180; %degrees
10
11 %SMC Controller parameters
12 %Lambda_theta = 1;%Good
13 %k_theta = 30; %Good
14 Lambda_theta = 5;%5
15 k_theta = 80;%50
16
17 Lambda_z = 1;
18 %k_z = -5;
19 k_z = -6; %Good
20 %k_z = -50;
21
22 %Lateral Position Control
23
24 Lambda_x = 0.35;%Good
25 k_x = 1.5;%Good
26 Lambda_i = 0.5;%Good
27
28 z_d = 0;
29 x_d = 10;
30
31
32 kxp = -0.055; % proportional
33 kxi = -0.012; % integral
34 ku = -0.1;

```

```
35
36 %Tf = 50; %Actuator Filter Time Constant. ORIGINAL
37 Tf = 5;%Good
38
39
40 K_tan_hyper = 100;%Good
41 K_tan_hyper_x = 1;%Good
42
43 %K_tan_hyper = 50;
44 %K_tan_hyper_x = 1;
45
46 % initial state
47 u0 = 0;
48 x0 = 0;
49 w0 = 0;
50 z0 = 0;
51 q0 = 0;
52
53 thetadot_d = 0;
54 zdot_d=0;
55 xdot_d=0;
56
57 % initial trim guess values
58 Omegal0 = 150; Omegar0 = 150; theta0 = 0;
59 Vw = 0;          % constant horizontal wind airspeed (m/s)
60
61 % options for trim residual minimization
62 trimoptions = optimset('TolFun',1e-7,'TolX',1e-7);
63
64 xtrim0=[0;0;0;0;0;theta0;0];
65 utrim0=[Omegal0+Omegar0;Omegal0-Omegar0;Vw];% set initial trim points guess
66
67 %trim & linearize
68 [ A,B,xtrim,utrim ] = PvtolTrimLin(pvtol,Vw,Upsilon,xtrim0,utrim0,trimoptions);
69
70 Bu=B(:,1:2);      % extract control matrix
71 Bw=B(:,3);        % extract disturbance matrix
72 C=[1 0 0 0 0 0; % output matrix
73     0 0 1 0 0 0;
74     0 0 0 0 1 0];
75
76 thetatrim=xtrim(5);
77
78 Omegaltrim = (utrim(1)+utrim(2))/2;
79 Omegartrim = (utrim(1)-utrim(2))/2; % record trim
80
81 KT_l = (m*g/2)/Omegaltrim^2;
82 KT_r = (m*g/2)/Omegartrim^2;
83
84
85
86 % linear
87 alpha_kick=0.1; % derivative kick fix
88
```

D.2 MATLAB Code Associated with Parameters/Coefficients of the PVTOL

```

1 function [ quad, pvtol, rotor, rho ] = Coefficients(Upsilon)
2 %   [ quadrotor, vtol, rotor ] = Coefficients(Upsilon)
3 %   Defines the rotor and airframe coefficients for the
4 %   Draganflyer XPro quadrotor and a conceptual PVTOL aircraft
5 %   based on measurements from Martinez (2007).
6 %
7 %   Inputs:
8 %       Upsilon = outwards rotortilt angle
9 %
10 %   Outputs:
11 %   quadrotor = Draganflyer XPro quadrotor coefficients data structure
12 %   pvtol = conceptual PVTOL aircraft coefficients data structure
13 %   rotor = Draganflyer XPro quadrotor rotor coefficients data structure
14 %   rho = atmosphere air density (kg/m3) at sea level
15 %
16 %   BiBTeX References:
17 %   @MastersThesis{MartinezMSC2007,
18 %   Title      = {Modelling of the Flight Dynamics of a Quadrotor Helicopter}
19 %   Author     = {Mart\'\{i\}nez, V. M.},
20 %   School     = {Cranfield University},
21 %   Year       = {2007},
22 %   Address    = {Bedfordshire, U.K.},
23 %   File       = {http://public.cranfield.ac.uk/eh3081/reports/
24 %                 VicenteMartinezMSc2007.pdf}
25 %   }
26 %   @Book{SeddonBook1990,
27 %   Title      = {Basic Helicopter Aerodynamics},
28 %   Author     = {Seddon, J.},
29 %   Publisher  = {BSP Professional},
30 %   Year       = {1990},
31 %   Address    = {Oxford, U.K.}
32 %   }
33 %   @Book{BramwellBook2001,
34 %   Title      = {Bramwell's Helicopter Dynamics},
35 %   Author     = {Bramwell, A. R. S. and Done, G. and Balmford, D.},
36 %   Publisher  = {Butterworth-Heinemann},
37 %   Year       = {2001},
38 %   Address    = {Oxford, U.K.},
39 %   Edition    = {2nd}
40 %   }
41
42 %   (c) James Whidborne
43 %   Cranfield University, 15 August 2020
44
45 if nargin==0, Upsilon=1; end
46
47 %% atmosphere
48 rho= 1.225; % air density (kg/m3)
49
50
51 %% rotor parameters (Martinez p 133-143 & 274)
52
53 rotor.a = 5.5;           % Aerofoil lift slope
54 rotor.R = 0.258;         % Rotor disc radius (including the hinge offset)

```

```

55 c = 0.04; % Mean chord (0.032 on p 135)
56 b = 2; % Number of blades
57 rotor.sigma = b*c/(pi*rotor.R); % Rotor solidity
58 rotor.theta0 = 0.37; % Blade pitch at the root
59 rotor.theta1 = -0.09;
60 % Twist (Seddon p 27  $\theta = \theta_0 + r \theta_{tw}$ ,  $r \in [0,1]$ )
61 rotor.theta = rotor.theta0 + rotor.theta1*3/4; % twist at 3/4 radius
62 % drag coefficient is constant in Seddon, and is quadratic in
63 % Bramwell (ed2) p 135. Here we take the linear expression obtained
64 % by Martinez  $C_d = C_{d0} + C_{d1} \alpha$ 
65 rotor.Cd0=0.05; %  $C_d = 0.05 + 0.7 \alpha$  (Martinez p 143)
66 rotor.Cd1=0.7; %
67
68 %% quadrotor vehicle parameters (Martinez p 273-274)
69
70 % mass and inertias
71 quad.m = 2.356; % total mass[kg]
72 Ix = 0.1676; % inertia of x axis [kg.m^2]
73 Iy = 0.1676; % inertia of y axis [kg.m^2]
74 Iz = 0.29743; % inertia of z axis [kg.m^2]
75 % (should be at least 2I_x)
76 quad.J = diag([Ix;Iy;Iz]); % moment of inertia matrix
77 %quad.rBC = [0;0;0]; % vector from body frame to center of mass
78
79 % arm vector/rotor geometry
80 quad.l = 0.4534; % armlength (Martinez p 273)
81 % arm vectors from B to Ri
82 quad.rBi = quad.l*[1 0 -1 0;
83 0 1 0 -1;
84 0 0 0 0];
85 % rotor rotation matrices from Body frame to ith rotor frame
86 RBi = zeros(3,3,4); % 3 rows, 3 cols, 4 pages
87 RBi(:,:,1) = DCM(Upsilon*[0;1;0]);
88 RBi(:,:,2) = DCM(Upsilon*[-1;0;0]);
89 RBi(:,:,3) = DCM(Upsilon*[0;-1;0]);
90 RBi(:,:,4) = DCM(Upsilon*[1;0;0]);
91 % rotor direction vectors for 4 rotors
92 quad.zi = squeeze(-RBi(:,3,:));
93 % extract third column (i.e. negative z-direction)
94
95 % rotor inclination angle
96 %quad.Upsilon = Upsilon; % not used - just to keep track of it
97
98 % rotor parameters
99 quad.rotor = rotor; % rotor parameters
100
101 % Control Allocation matrix  $\Omega = CA u$ 
102 % Maps the controls to the channels:
103 %  $u_1 \rightarrow c(\Omega_1 - \Omega_3)$  : x-channel
104 %  $u_2 \rightarrow c(-\Omega_2 + \Omega_4)$  : y-channel
105 %  $u_3 \rightarrow b(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4)$  : z-channel
106 %  $u_4 \rightarrow d(\Omega_1 - \Omega_2 + \Omega_3 - \Omega_4)$  : psi-channel
107 %  $CA = \text{inv}(\begin{bmatrix} c & 0 & -c & 0; \\ 0 & -c & 0 & c; \\ b & b & b & b; \\ -d & d & -d & d \end{bmatrix})$ 
108 %
109 %
110 %
111 %
112 b = 0.033; c = 0.2; d = 0.01; % parameters of control allocation

```

```

113 quad.CA = [-1/(2*c),      0,      1/(4*b), - 1/(4*d);
114             0,      -1/(2*c),      1/(4*b),      1/(4*d);
115             1/(2*c),      0,      1/(4*b), - 1/(4*d);
116             0,      1/(2*c),      1/(4*b),      1/(4*d)];
117
118 % environment parameters
119 quad.rho = rho; % air density (kg/m3)
120
121 %% PVTOL vehicle airframe parameters
122 % conceptual vehicle parameters - approx based on Draganflyer XPro
123 pvtol.m = 1.18; % mass (kg) - half quad mass because only 2 rotors
124 pvtol.I = 0.0625; % mass is reduced, so inertia is arbitarily set
125 pvtol.l = 0.45; % armlength (m)
126 % rotor parameters
127 pvtol.rotor = rotor; % rotor parameters
128 % environment parameters
129 pvtol.rho = rho; % air density (kg/m3)
130
131 end
132
133 %% DCM (Rotation matrix)
134 function T=DCM(rpy)
135 % Direction Cosine Matrix
136 phi=rpy(1,:);theta=rpy(2,:);psi=rpy(3,:); %roll, pitch, yaw
137 % Rotation matrix
138 T=[cos(theta).*cos(psi),      cos(theta).*sin(psi), -sin(theta);
139     sin(phi).*sin(theta).*cos(psi) - cos(phi).*sin(psi),...
140     sin(phi).*sin(theta).*sin(psi) + cos(phi).*cos(psi),...
141     sin(phi).*cos(theta);
142     cos(phi).*sin(theta).*cos(psi) + sin(phi).*sin(psi),...
143     cos(phi).*sin(theta).*sin(psi) - sin(phi).*cos(psi),...
144     cos(phi).*cos(theta)];
145 end

```

D.2.1 Rotor Thrust Coefficient

```

1 function [CT]=RotorCT(T,alpha,V,Omega,rho,rotor)
2 %ROTORCT
3 % [CT]=RotorCT(T,alpha,V,Omega,rho,rotor)
4 % Returns rotor thrust coefficient CT(T)
5 % the subsequent thrust is calculated from
6 % T = rho*A*(Omega*rotor.R)^2*CT(T,alpha,V,Omega,rho,rotor)
7 % NOTE that this is an implicit equation and must be solved iteratively
8 % using fzero.m or similar
9 %
10 % CT: rotor thrust coefficient
11 %
12 % T: rotor thrust
13 % alpha : angle of attack/wind incidence angle (rad)
14 % V : incident windspeed (m/s)
15 % Omega : rotor speed (rad/sec)
16 % rho: air density (kg/m^3)
17 % rotor: vehicle rotor coefficients structure (see Coefficients.m)
18
19 % (c) James Whidborne
20 % Cranfield University, 19 August 2020

```

```
21
22 [CT, ~]=RotorC(T, alpha, V, Omega, rho, rotor);
23 % Returns rotor thrust coefficient
24
25 end
26
27
```

D.2.2 Rotor Drag Coefficient

```
1 function [CH, vi, lambda]=RotorCH(T, alpha, V, Omega, rho, rotor)
2 %ROTORCH
3 % [CH]=RotorCH(T, alpha, V, Omega, rho, rotor)
4 % Returns rotor drag coefficient CH(T)
5 % the subsequent drag is calculated from
6 %  $H = \rho \cdot A \cdot (\Omega \cdot \text{rotor}.R)^2 \cdot CH(T, \alpha, V, \Omega, \rho, \text{rotor})$ 
7 %
8 % CH: rotor drag coefficient
9 %
10 % T: rotor thrust
11 % alpha : angle of attack/wind incidence angle (rad)
12 % V : incident windspeed (m/s)
13 % Omega : rotor speed (rad/sec)
14 % rho: air density (kg/m^3)
15 % rotor: vehicle rotor coefficients structure (see Coefficients.m)
16
17 % (c) James Whidborne
18 % Cranfield University, 19 August 2020
19
20 [~, CH, ~]=RotorC(T, alpha, V, Omega, rho, rotor);
21 % Returns rotor thrust coefficient
22
23 end
24
25
```

D.2.3 Rotor Blade Torque Coefficient

```
1 function [CQ]=RotorCQ(T, alpha, V, Omega, rho, rotor)
2 %ROTORCQ
3 % [CQ]=RotorCQ(T, alpha, V, Omega, rho, rotor)
4 % Returns rotor blade torque coefficient CQ(T)
5 % the subsequent blade torque is calculated from
6 %  $Q = \frac{1}{2} \rho A \Omega^2 R^3 CQ(T, \alpha, V, \Omega, \rho, \text{rotor})$ 
7 %
8 % CQ: rotor drag coefficient
9 %
10 % T: rotor thrust (N)
11 % alpha : angle of attack/wind incidence angle (rad)
12 % V : incident windspeed (m/s)
13 % Omega : rotor speed (rad/sec)
14 % rho: air density (kg/m^3)
15 % rotor: vehicle rotor coefficients structure (see Coefficients.m)
16
```



```

17 % (c) James Whidborne
18 % Cranfield University, 19 August 2020
19
20 [~,~,CQ]=RotorC(T,alpha,V,Omega,rho,rotor);
21 % Returns rotor blade torque coefficient
22
23 end
24
25

```

D.2.4 Function for C_T , C_H and C_Q

```

1 function [CT,CH,CQ,lambda]=RotorC(T,alpha,V,Omega,rho,rotor)
2 %ROTORC
3 % [CT,CH,CQ]=RotorC(T,alpha,V,Omega,rho,rotor)
4 % Returns rotor thrust and drag coefficients CT(T) and CH(T) and the
5 % blade torque coefficient CQ(T)
6 % the subsequent thrust and drag are calculated from
7 % T = rho*A*(Omega*rotor.R)^2*CT(T,alpha,V,Omega,rho,rotor)
8 % H = rho*A*(Omega*rotor.R)^2*CH(T,alpha,V,Omega,rho,rotor)
9 % Q = rho*A*Omega^2*rotor.R^3*CQ(T,alpha,V,Omega,rho,rotor)
10 % NOTE that this is an implicit equation and must be solved
11 %iteratively using fzero.m or similar
12 %
13 % CT: rotor thrust coefficient
14 % CH: rotor drag coefficient
15 % CQ: blade torque coefficient
16 %
17 % T: rotor thrust
18 % alpha : angle of attack/wind incidence angle (rad)
19 % V : incident windspeed (m/s)
20 % Omega : rotor speed (rad/sec)
21 % rho: air density (kg/m^3)
22 % rotor: vehicle rotor coefficients structure (see Coefficients.m)
23 % References:
24 % Bramwell, "Bramwell's Helicopter Dynamics". Butterworth, 2nd ed.,
25 % 2001 Seddon, "Basic Helicopter Aerodynamics". BSP Professional, 1990.
26
27
28 % (c) James Whidborne
29 % Cranfield University, 1 March 2020
30
31 A= pi*rotor.R^2; % disc area
32 mu=V*cos(alpha)./(rotor.R*Omega); % advance ratio
33 if mu>0.5, mu=0.5; end % max mu = 0.5 (Bramwell, p. 96)
34
35
36 % calculate vh (Seddon[p52]: eq (5.4) Bramwell[p78]: eq (3.2) )
37 vhsquared= (T/(2*rho*A));
38 visquared = (sqrt( V.^4+4*vhsquared.^2) - V.^2)/2;
39 vi=sqrt(visquared); % induced downwash
40 lambda = (V.*sin(alpha) + vi)./(Omega*rotor.R);% rotor inflow ratio
41 %- note different sign in Bramwell - this is the Seddon convention
42 % Seddon[p59]: eq(2.27) , Bramwell[p98]: eq (3.33)
43 CT = rotor.sigma/2 * rotor.a *(rotor.theta/3 * (1+3*mu.^2/2) - lambda/2);
44

```

```

45 % flapping coefficient Seddon: eq 5.50, , Bramwell[p107]: eq (3.56)
46 %a1=mu.*(8*rotor.theta0/3 - 2*lambda)./(1+3*mu.^2/2);% Seddon: eq 5.53
47 a1=mu.*(8*rotor.theta0/3 - 2*lambda)./(1-mu.^2/2);% Seddon: eq 5.50
48
49 % Bramwell[p 100]: eq (3.39)
50 %CH= rotor.a*rotor.sigma/2*(mu*delta/(2*rotor.a) + a1*rotor.theta/3 ...
51 %      + 3*lambda.*a1/4 - mu.*rotor.theta.*lambda/2 + mu.*a1.^2/4);
52 CH = rotor.a*rotor.sigma/2*(mu*rotor.Cd0/(2*rotor.a) + a1*rotor.theta/3 ...
53      - 3*lambda.*a1/4 + mu.*rotor.theta.*lambda/2 + mu.*a1.^2/4);
54 % corrected sign on lambda
55 % CH = rotor.sigma*delta* mu/4; % alternative expression
56 % Seddon p61 eq(5.32)
57
58 % Seddon p 62 eq (5.37)
59 %CQ = rotor.sigma*delta*(1+3*mu^2)/8+lambda*CT-mu*CH;
60 % this equation gives very poor results
61 % Prouty (1990)
62 CQ = rotor.sigma/4*(rotor.Cd0*(1+mu^2)/2 ...
63      + rotor.Cd1/(1+3/2*mu^2)*(rotor.theta0*(1/2-19/36*mu^2+3/4*mu^4) ...
64      + rotor.theta1*(2/5-2/5*mu^2+mu^4/2) - (2-mu^2)/3*lambda));
65 end
66
67

```

D.3 MATLAB Code Associated with Calculation of Thrusts and Drags of a PVTOL

```

1 function [T, H, KT, KH] = Get_Thrust_Drags(alpha, V,
2      Omega, pvtol, rotor, rho, g)
3 %
4 % INPUT VARIABLES
5 % alpha - angle of attack (rad)
6 % V      - airspeed (m/s)
7 % Omega - rotor speed
8 %
9 % OUTPUTS
10 % T      - vertical thrust (N)
11 % H      - horizontal thrust (N)
12 %
13 % CONSTANT PARAMETERS (from MATLAB base workspace)
14 % pvtol - structure with pvtol.m - vehicle mass (kg)
15 % rotor - structure with rotor.R - rotor radius (m)
16 % rho   - air density (kg/m^3)
17 % g     - standard gravitational acceleration constant (m/s^2)
18 %
19 %
20 % NOTE on constant parameters
21 % see https://uk.mathworks.com/help/simulink/ug/parameter-arguments-in-
22 % matlab-function-block-functions.html
23 % -----
24 % Parameter arguments for MATLAB Function blocks should not take
25 % their values from signals in the Simulink model. Instead, Simulink
26 % searches the workspace. model workspace and then the MATLAB®
27 % base workspace.
28

```

```

29 % In the MATLAB Function Block Editor, add an argument to the function
30 % header of the MATLAB Function block. The name of the argument must
31 %match the name of the MATLAB variable that you want to pass to the
32 %MATLAB Function block.
33 %
34 % The new argument appears as an input port on the MATLAB Function
35 %block in the model.
36 %
37 % In the MATLAB Function Block Editor, click Edit Data.
38 %
39 % Select the new argument.
40 %
41 % Set Scope to Parameter and click Apply.
42 %
43 % The input port for the parameter argument no longer appears in the
44 % MATLAB Function block.
45 %
46
47 A = pi*(rotor.R^2); % rotor disc area
48
49 % initial trim guess values
50 %Omega0 = 150;
51 T0 = pvtol.m*g/2; %initial Thrust.
52
53 %T=zeros(length(V),Omega,length(alpha)); H=T; % pre-allocate arrays
54 % get thrusts and drags - solve using fzero
55 T = fzero(@(x)rho*A*(Omega*rotor.R)^2*RotorCT(x,alpha,V,Omega,...
56     rho,rotor)-x,T0);
57 H = rho*A*(Omega*rotor.R)^2*RotorCH(T,alpha,V,Omega,rho,rotor);
58 KT = rho*A*(rotor.R)^2*RotorCT(T,alpha,V,Omega,rho,rotor);
59 KH = rho*A*(rotor.R)^2*RotorCH(T,alpha,V,Omega,rho,rotor);
60
61 end

```

D.3.1 Initialisation for SMC Altitude, Attitude and PIV Position Control of Quadrotor with Aerodynamics

```

1
2 %init
3 % mod jfw 14 Dec 2020
4 Upsilon = 10*pi/180; %ranging -5 to +10 degrees.
5 [quad, ~, rotor, rho] = Coefficients(Upsilon); % Draganfly XPro quadrotor parameters
6
7 m=2.356; % mass (kg)
8 %m = 2; % mass (kg)
9 Jx = 0.1676; % inertia of x axis [kg.m^2]
10 Jy = 0.1676; % inertia of y axis [kg.m^2]
11 Jz = 0.29743; % inertia of z axis [kg.m^2] (should be at least 2I_x)
12
13 % It is same as Ix=Jx, Iy=Jy; Iz=Jz. I and J are used interchangeably in init files
14
15 %Jx = 0.2; % inertia of x axis [kg.m^2]
16 %Jy = 0.2; % inertia of y axis [kg.m^2]
17 %Jz = 0.4; % inertia of z axis [kg.m^2] (should be at least 2I_x)
18

```

```
17 %J = diag([Jx;Jy;Jz]); % moment of inertia matrix
18 %ell = 0.4534; % armlength (m) (Martinez p 273)
19 ell = 0.25; %l (m) Arm Length
20 g = 9.81; % gravitational constant
21
22 Tf = 0.1; %Actuator Filter Time Constant.
23 p = 1/Tf;
24
25 % parameters of control allocation
26
27 b = 2.6333*10^-4;
28
29 %thrust coefficient (N.s^2) - Gains of the Control Allocation
30
31 %b = F0/OmegaTrim^2; = (mg/4)/OmegaTrim^2;
32
33 c= 6.5833*10^-5;
34
35 % Gains of the Control Allocation - c = b*l; M = c*OmegaTrim^2;
36
37 %d = tau/148.127^2;
38 d = 9.5843e-06;
39
40 % Gains of the Control Allocation
41
42 %tau = d*OmegaTrim^2; Use Q_Trim routine. d= 0.2103/148.1287^2;
43
44 %-----
45 %With Upsilon = 0; wiht V_Wind = 0; %Finalised
46
47 %x & y gains should be same as the aircraft is symmetrical
48
49 Lambda_phi = 1.25;
50 Lambda_theta = 1.25;
51 Lambda_psi = 1;
52 Lambda_z = 0.5;
53
54 k_z = -5;
55
56 %k_phi = 12; k_theta = 12; k_psi = 2.0630;%DONE- ORIGINAL
57 k_phi = 12; k_theta = 12; k_psi = 2.0620;%DONE
58 %Kpx = -0.27; %DONE
59 %Kix = -0.05; %DONE
60 %Kpu = -0.45; %DONE
61
62
63 Kpx = -0.195; Kix = -0.0375; Kpu = -0.3;
64
65 %Kpx=-0.255; Kix = -0.02;Kpu = -0.45;
66
67 Kpy = -Kpx; %DONE
68 Kiy = -Kix; %DONE
69 Kpv = -Kpu; %DONE
70
71 Omega_max = 200;
72 Omega_min = 1; %To avoid divide by zero.
73
74 %Uncertainty in parameters (20% - 50%) variation in mass & inertia
```

```

64 %parameters
65
66 p_1 = 0.8;
67 p_2 = 0.5;
68 p_3 = 0.7;
69 p_4 = 0.6;
70
71 %No parametric Uncertainty
72 %p_1 = 1;
73 %p_2 = 1;
74 %p_3 = 1;
75 %p_4 = 1;
76
77 %initial condition
78 %U0 = transpose([m*g 0 0 0]);
79
80 p_d = 0;
81 q_d = 0;
82 r_d = 0;
83
84 u_d = 0;
85 v_d = 0;
86 w_d = 0;
87
88 x_d = 10;
89 y_d = 10;
90 z_d = 20;
91 psi_d = 0;
92
93 phidot_d = 0;
94 thetadot_d = 0;
95 psidot_d = 0;
96 zdot_d = 0;
97
98 % initial state
99 u0 = 0;
100 v0 = 0;
101 w0 = 0;
102 x0 = 0;
103 y0 = 0;
104 z0 = 0;
105 p0 = 0;
106 q0 = 0;
107 r0 = 0;
108 phi0 = 0;
109 theta0 = 0;
110 psi0 = 0;
111
112 % linear
113 alpha_kick=0.75; % derivative kick fix
114 %alpha_kick_z=0.75; % derivative kick fix
115
116
117 Vw = 0; % constant horizontal wind airspeed (m/s)
118 wind=[0;0;0]; % wind disturbance
119
120 % initial trim guess values
121 %Omega_1 = 150; Omega_2 = 150; Omega_3 = 150; Omega_4 = 150;

```

```

122
123
124 %% trim & lin
125 % options for trim residual minimization
126 trimoptions = optimset('TolFun',1e-7,'TolX',1e-7 ,...
127     'MaxFunEvals',20000,'MaxIter',20000,...
128     'Display','on');
129
130 % set initial trim points guess
131 xtrim0=zeros(12,1); utrim0=148.1287*ones(4,1);
132
133
134 %trim & linearize
135 [A,Bomega,xtrim,Omegatrim] = QuadrotorTrimLin( quad,wind,xtrim0,utrim0,trimoptions);
136 B = Bomega*quad.CA;
137
138 Bu=B(:,1:2); % extract control matrix
139 Bw=B(:,3); % extract disturbance matrix
140 C=[1 0 0 0 0 0; % output matrix
141     0 0 1 0 0 0;
142     0 0 0 0 1 0];
143
144 thetatrim=xtrim(5);
145
146 Omegaltrim = Omegatrim(1);
147 Omega2trim = Omegatrim(2);
148 Omega3trim = Omegatrim(3);
149 Omega4trim = Omegatrim(4);
150
151
152 %KT_1 = (quad.m*g/4)/Omegaltrim^2;
153 %KT_2 = (quad.m*g/4)/Omega2trim^2;
154 %KT_3 = (quad.m*g/4)/Omega3trim^2;
155 %KT_4 = (quad.m*g/4)/Omega4trim^2;
156
157

```

D.3.2 Initialisation for SMC Altitude, Attitude and Position Control of Quadrotor with Aerodynamics

```

1
2 %init
3 clear;
4 Upsilon = 10*pi/180; %ranging -5 to +10 degrees.
5 [quad, ~, rotor, rho] = Coefficients(Upsilon); % Draganfly XPro quadrotor parameters
6
7 m=2.356; % mass (kg)
8 %m = 2; % mass (kg)
9 Jx = 0.1676; % inertia of x axis [kg.m^2]
10 Jy = 0.1676; % inertia of y axis [kg.m^2]
11 Jz = 0.29743; % inertia of z axis [kg.m^2] (should be at least 2I_x)
12
13 %Jx = 0.2; % inertia of x axis [kg.m^2]
14 %Jy = 0.2; % inertia of y axis [kg.m^2]
15 %Jz = 0.4; % inertia of z axis [kg.m^2] (should be at least 2I_x)
16

```

```

17 %J = diag([Jx;Jy;Jz]); % moment of inertia matrix
18 %ell = 0.4534; % armlength (m) (Martinez p 273)
19 ell = 0.25; %l (m) Arm Length
20 g = 9.81; % gravitational constant
21
22
23 K_tan_hyper = 100;%Good
24
25
26 Tf = 0.1; %Actuator Filter Time Constant.
27 %Tf=5;
28 p = 1/Tf;
29
30 % parameters of control allocation
31
32 b = 2.6333*10^-4; %thrust coefficient (N.s^2) - Gains of the Control Allocation - b = F0
33 c= 6.5833*10^-5; % Gains of the Control Allocation - c = b*l; M = c*OmegaTrim^2;
34 %d = tau/148.127^2;
35 d = 9.5843e-06; % Gains of the Control Allocation - tau = d*OmegaTrim^2; Use Q_Trim routine
36
37
38 %-----
39 %With Upsilon = 0;wiht V_Wind = 0; %Finalised
40
41 %x & y gains should be same as the aircraft is symmetrical
42
43 Lambda_phi = 1.25;
44 Lambda_theta = 1.25;
45 Lambda_psi = 1;
46 Lambda_z = 0.5;
47 k_z = -5;
48
49 %SMC Position Control Parameters
50
51 Lambda_x = 0.2; %PVTOL 0.3 %Good 0.1
52 Lambda_y = 0.2;
53
54 k_x = 40; %PVTOL 0.625 %Good 25
55 k_y = 40; %Good 25;
56
57 K_tan_hyper_x = -0.0125;%PVTOL 1 %Good 1/100
58 K_tan_hyper_y = 0.0125;
59
60 Lambda_i_x = 0.1; %PVTOL 0.5 %Good 0.00025
61 Lambda_i_y = 0.1;
62
63
64 k_phi = 12; k_theta = 12; k_psi = 2.0620;%DONE
65
66 Kpx = -0.195; Kix = -0.0375; Kpu = -0.3;
67
68 Kpy = -Kpx; %DONE
69 Kiy = -Kix; %DONE
70 Kpv = -Kpu; %DONE
71
72 Omega_max = 200;
73 Omega_min = 1; %To avoid divide by zero.
74

```

```
75 %Uncertainty in parameters (20% - 50%) variation in mass & inertia
76 %parameters
77
78 p_1 = 0.8;
79 p_2 = 0.5;
80 p_3 = 0.7;
81 p_4 = 0.6;
82
83 %No parametric Uncertainty
84 %p_1 = 1;
85 %p_2 = 1;
86 %p_3 = 1;
87 %p_4 = 1;
88
89 %initial condition
90 %U0 = transpose([m*g 0 0 0]);
91
92 p_d = 0;
93 q_d = 0;
94 r_d = 0;
95
96 u_d = 0;
97 v_d = 0;
98 w_d = 0;
99
100 x_d = 10;
101 y_d = 10;
102 z_d = 20;
103 psi_d = 0;
104
105 phidot_d = 0;
106 thetadot_d = 0;
107 psidot_d = 0;
108 zdot_d = 0;
109 xdot_d=0;
110 ydot_d=0;
111
112 % initial state
113 u0 = 0;
114 v0 = 0;
115 w0 = 0;
116 x0 = 0;
117 y0 = 0;
118 z0 = 0;
119 p0 = 0;
120 q0 = 0;
121 r0 = 0;
122 phi0 =0;
123 theta0 = 0;
124 psi0 =0;
125
126 % linear
127 alpha_kick=0.75; % derivative kick fix
128 %alpha_kick_z=0.75; % derivative kick fix
129
130
131 Vw = 0; % constant horizontal wind airspeed (m/s)
132 wind=[0;0;0]; % wind disturbance
```


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170
171

```
% initial trim guess values
%Omega_1 = 150; Omega_2 = 150; Omega_3 = 150; Omega_4 = 150;

%% trim & lin
% options for trim residual minimization
trimoptions = optimset('TolFun',1e-7,'TolX',1e-7 ,...
    'MaxFunEvals',20000,'MaxIter',20000,...
    'Display','on');

% set initial trim points guess
xtrim0=zeros(12,1); utrim0=148.1287*ones(4,1);

%trim & linearize
[A,Bomega,xtrim,Omegatrim] = QuadrotorTrimLin( quad,wind,xtrim0,utrim0,trimoptions);
B = Bomega*quad.CA;

Bu=B(:,1:2); % extract control matrix
Bw=B(:,3); % extract disturbance matrix
C=[1 0 0 0 0 0; % output matrix
    0 0 1 0 0 0;
    0 0 0 0 1 0];

thetatrim=xtrim(5);

Omegaltrim = Omegatrim(1);
Omega2trim = Omegatrim(2);
Omega3trim = Omegatrim(3);
Omega4trim = Omegatrim(4);

%KT_1 = (quad.m*g/4)/Omegaltrim^2;
%KT_2 = (quad.m*g/4)/Omega2trim^2;
%KT_3 = (quad.m*g/4)/Omega3trim^2;
%KT_4 = (quad.m*g/4)/Omega4trim^2;
```

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APPENDIX E

Additional Simulation Results - Part 1

E.1 Simulation Results of Quadrotor with Aerodynamics at Hover without Parametric Variation and External Disturbance using SMC for Altitude and Attitude Control and PIV for Position Control

E.1.1 SMC Altitude and Attitude Control and PIV Position Control at Hover without Wind Gust

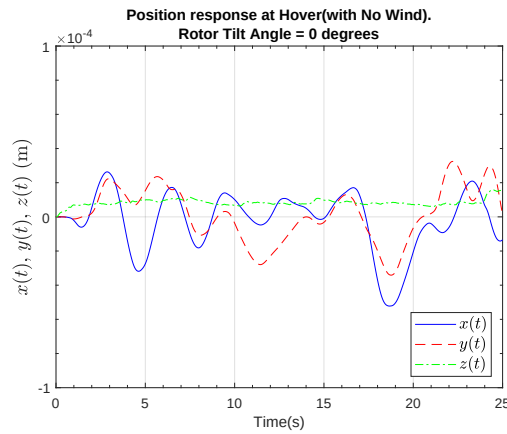
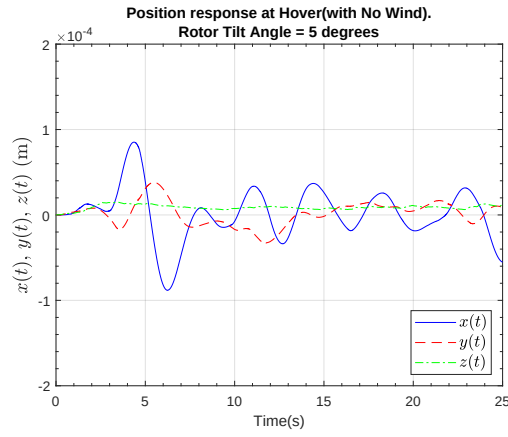
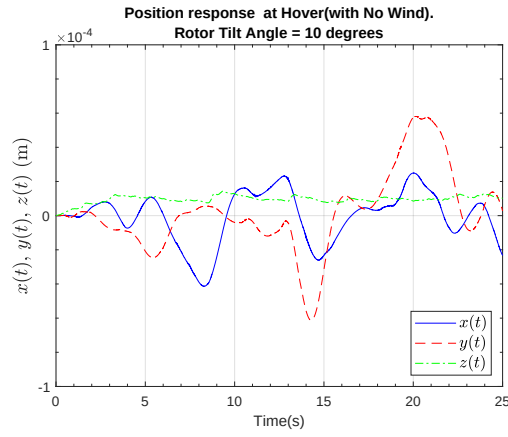
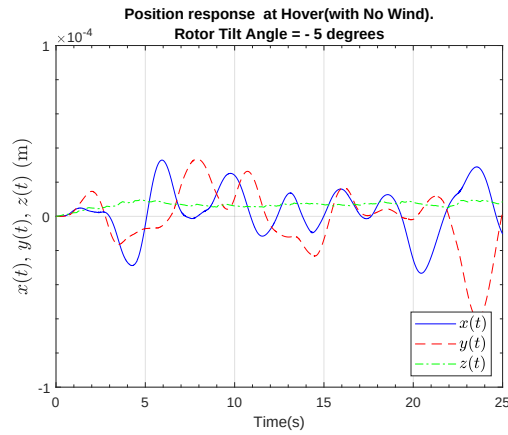


Figure E.1: Position response at hover (no wind gust). $\Upsilon = 0^\circ$

Figure E.2: Position response at hover (no wind gust). $\Upsilon = 5^\circ$ Figure E.3: Position response at hover (no wind gust). $\Upsilon = 10^\circ$ Figure E.4: Position response at hover (no wind gust). $\Upsilon = -5^\circ$

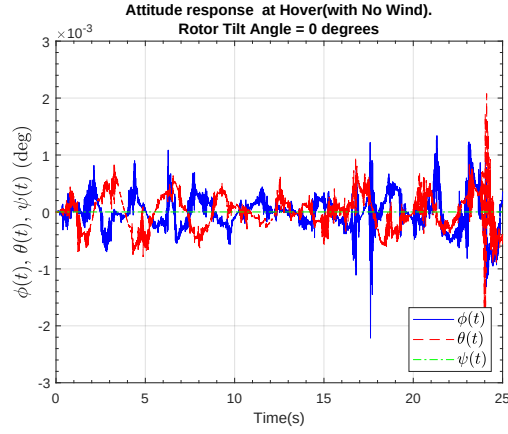


Figure E.5: Attitude response at hover (no wind gust). $\Upsilon = 0^\circ$

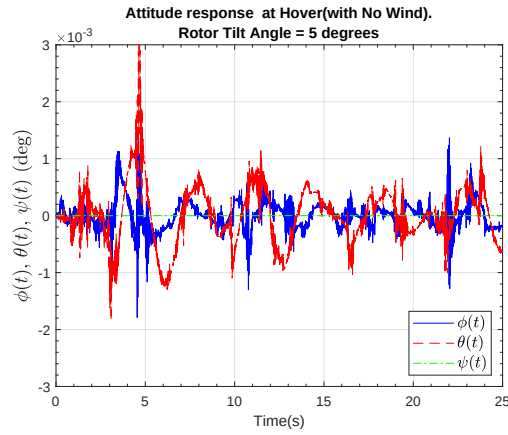


Figure E.6: Attitude response at hover (no wind gust). $\Upsilon = 5^\circ$

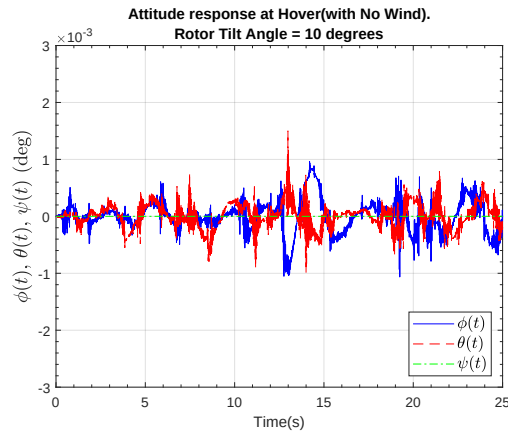
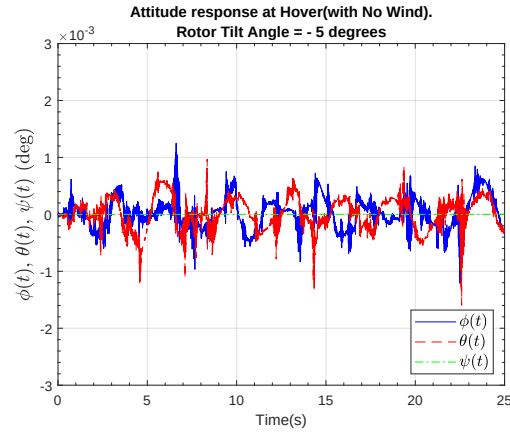
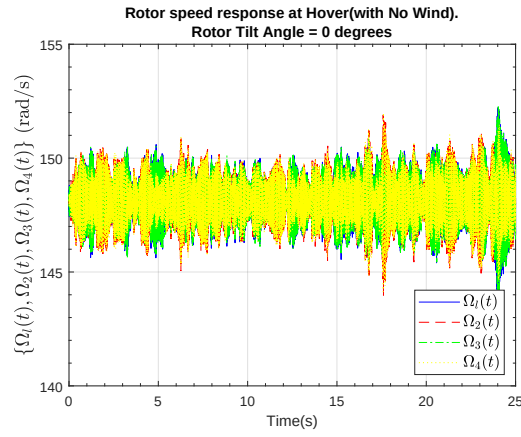
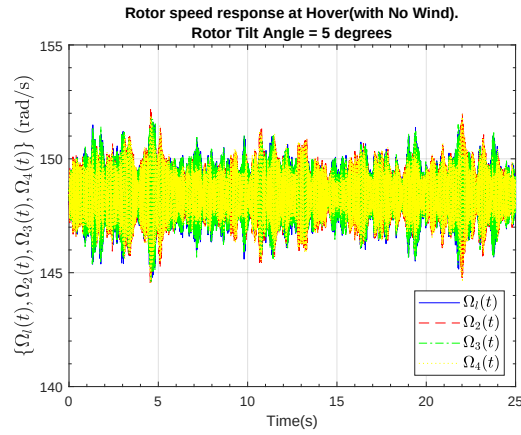


Figure E.7: Attitude response at hover (no wind gust). $\Upsilon = 10^\circ$

Figure E.8: Attitude response at hover (no wind gust). $\Upsilon = -5^\circ$ Figure E.9: Rotor speed response at hover (no wind gust). $\Upsilon = 0^\circ$ Figure E.10: Rotor speed response at hover (no wind gust). $\Upsilon = 5^\circ$

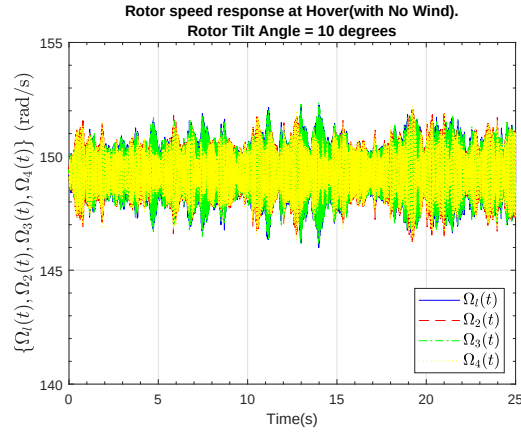


Figure E.11: Rotor speed response at hover (no wind gust). $\Upsilon = 10^\circ$

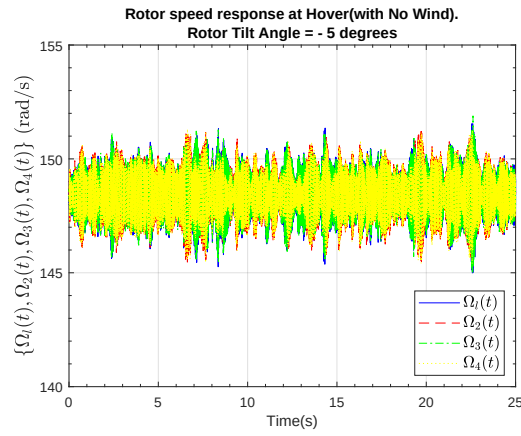


Figure E.12: Rotor speed response at hover (no wind gust). $\Upsilon = -5^\circ$

E.1.2 SMC Altitude and Attitude Control and PIV Position Control at Hover with 10 ms^{-1} Wind Gust Step

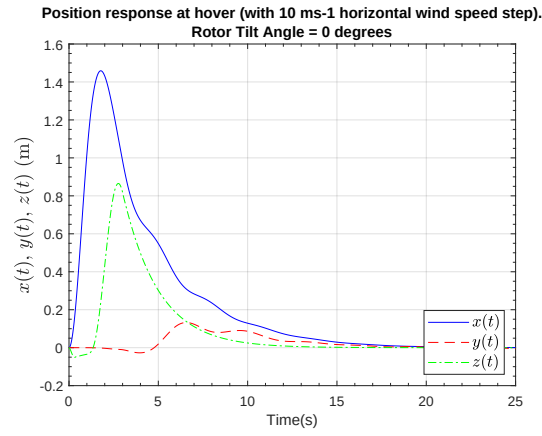


Figure E.13: Position response at hover (with 10 ms^{-1} horizontal wind gust step). $\Upsilon = 0^\circ$

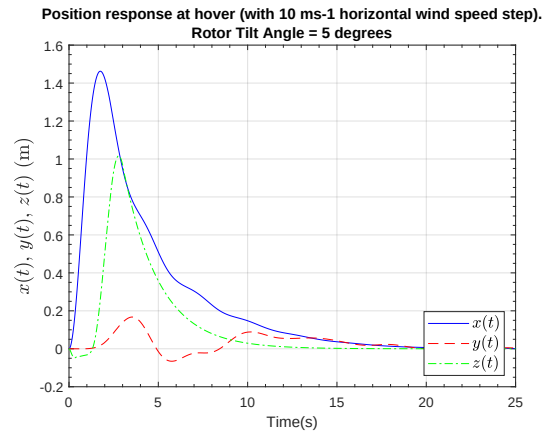


Figure E.14: Position response at hover (with 10 ms^{-1} horizontal wind gust step). $\Upsilon = 5^\circ$

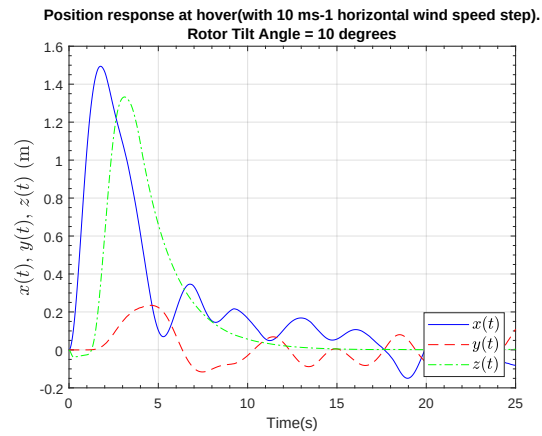


Figure E.15: Position response at hover (with 10 ms^{-1} horizontal wind gust step). $\Upsilon = 10^\circ$

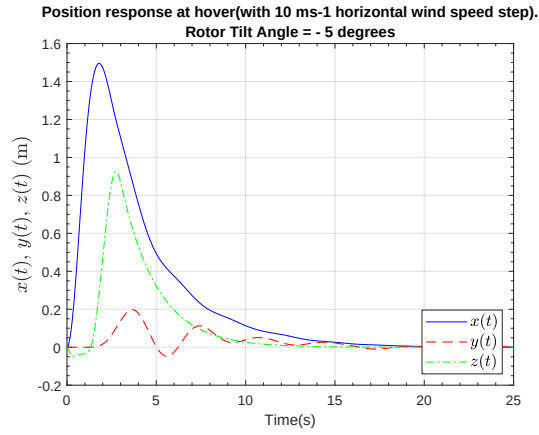


Figure E.16: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

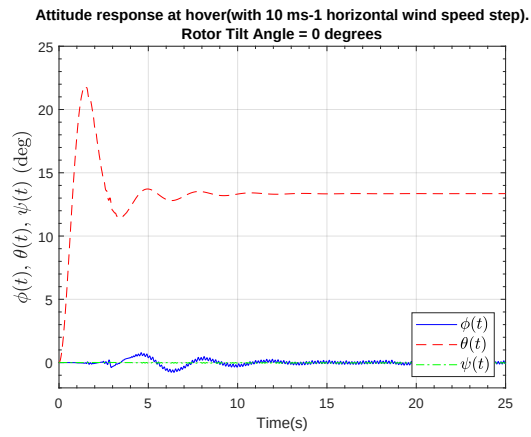


Figure E.17: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

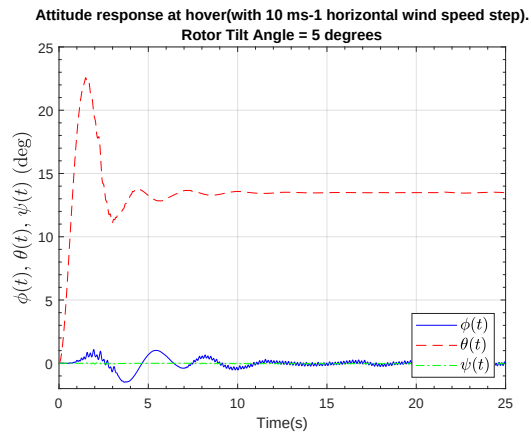
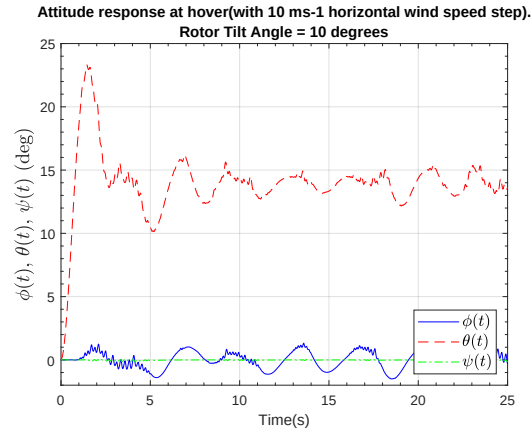
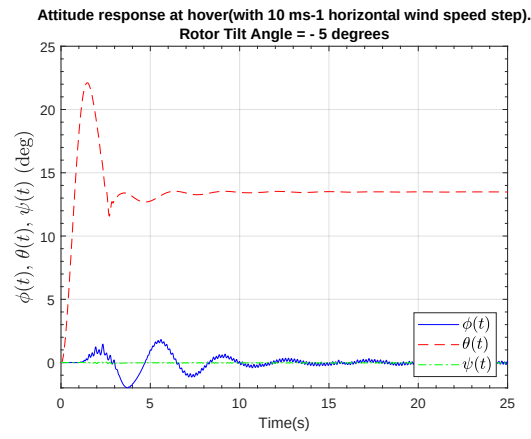
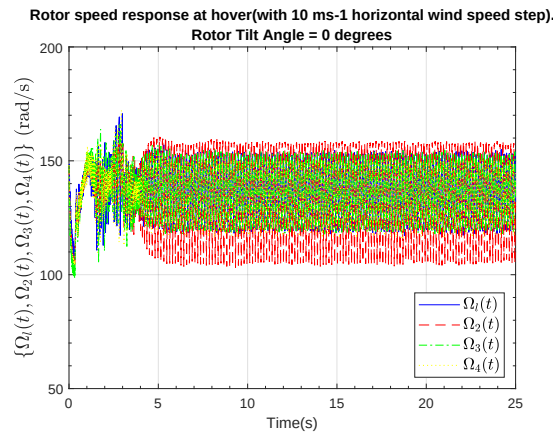


Figure E.18: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

Figure E.19: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$ Figure E.20: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$ Figure E.21: Rotor speed response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

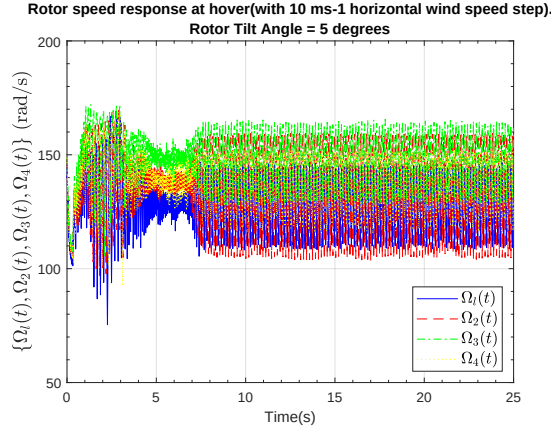


Figure E.22: Rotor speed response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

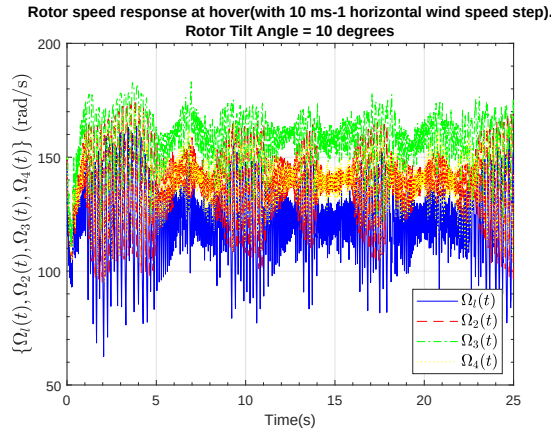


Figure E.23: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

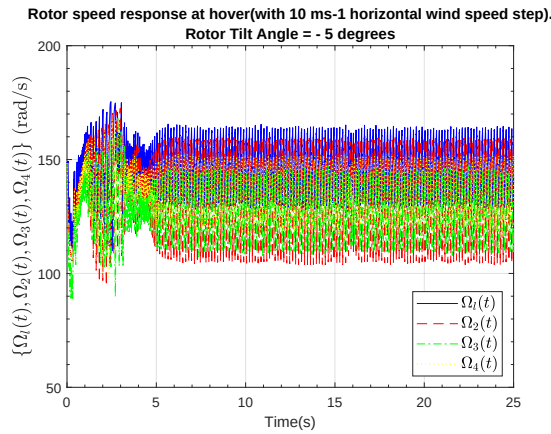


Figure E.24: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

E.1.3 Comparison of SMC Altitude and Attitude Control and PIV Position Control with 10 ms^{-1} Wind Gust Step for Different Rotor Tilt Angles at Hover

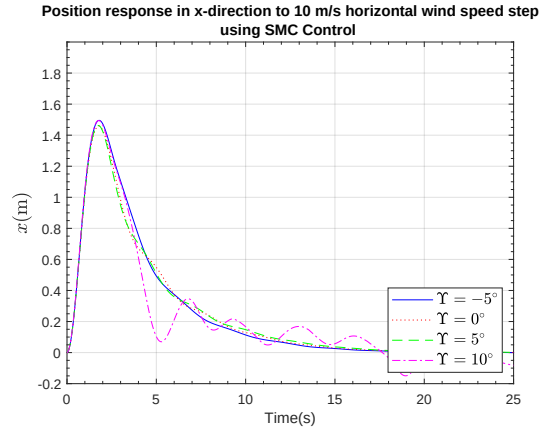


Figure E.25: Position response to 10 m position step demand both in x-direction (with 10 ms^{-1} horizontal wind gust step)

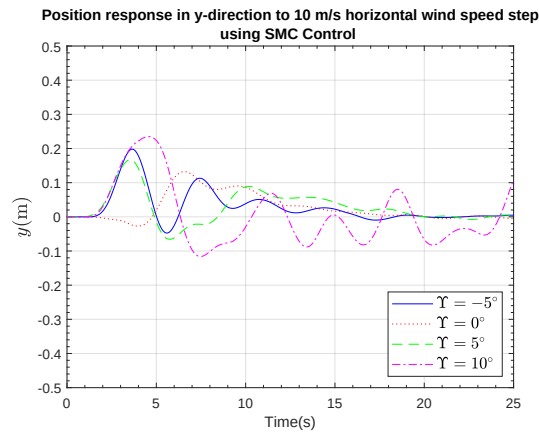


Figure E.26: Position response to 10 m position step demand both in y-direction (with 10 ms^{-1} horizontal wind gust step)

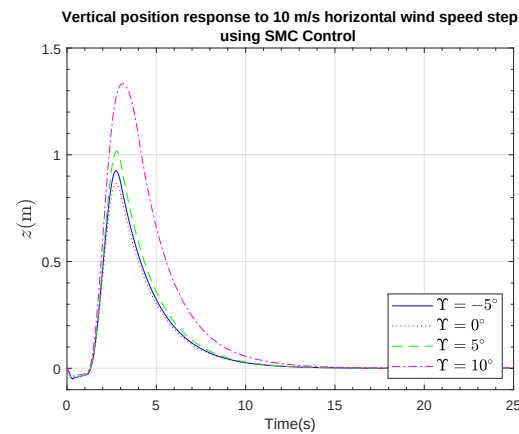


Figure E.27: Vertical response to 20 m altitude step demand (with 10 ms^{-1} horizontal wind gust step)

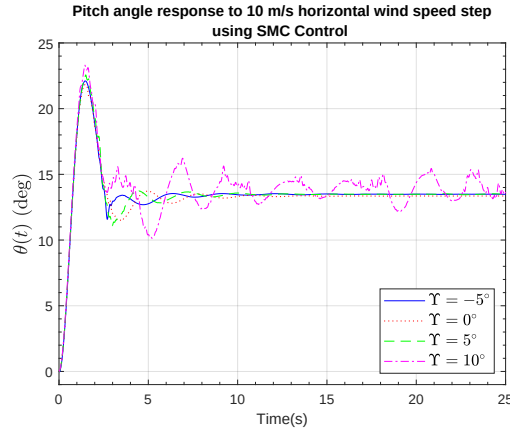


Figure E.28: Pitch angle response (with 10 ms^{-1} horizontal wind gust step)

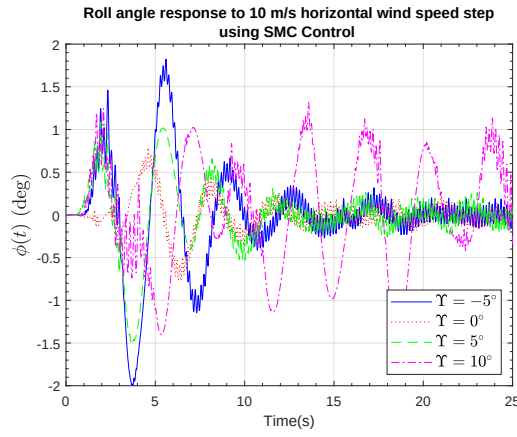


Figure E.29: Roll angle response (with 10 ms^{-1} horizontal wind gust step)

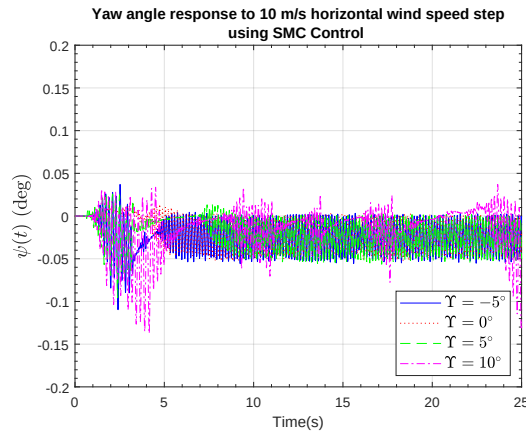


Figure E.30: Yaw angle response (with 10 ms^{-1} horizontal wind gust step)

E.1.4 SMC Altitude and Attitude Control and PIV Position Control for Position and Altitude Demands without Wind Gust

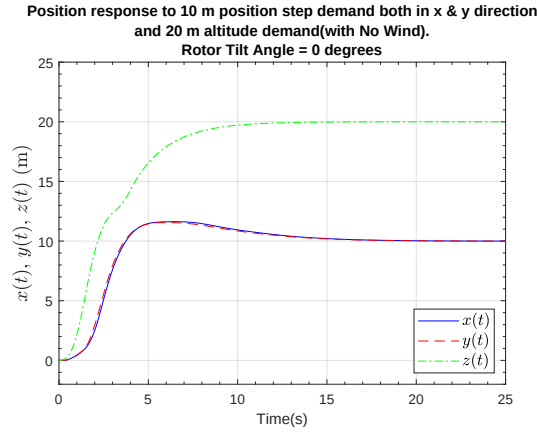


Figure E.31: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = 0^\circ$

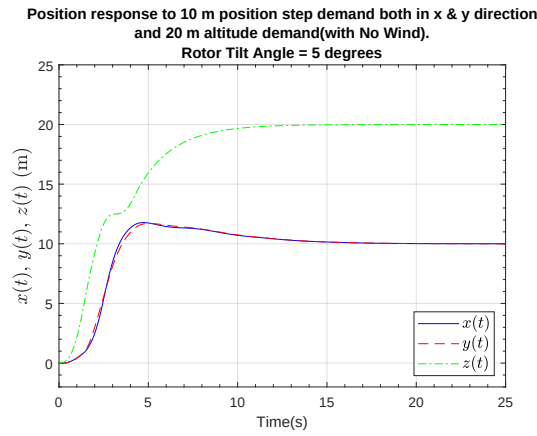


Figure E.32: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = 5^\circ$

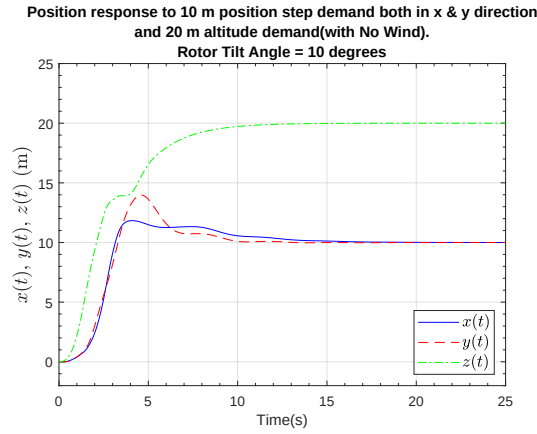


Figure E.33: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = 10^\circ$

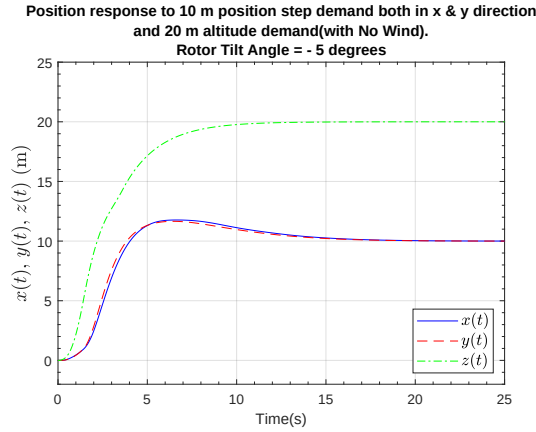


Figure E.34: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = -5^\circ$

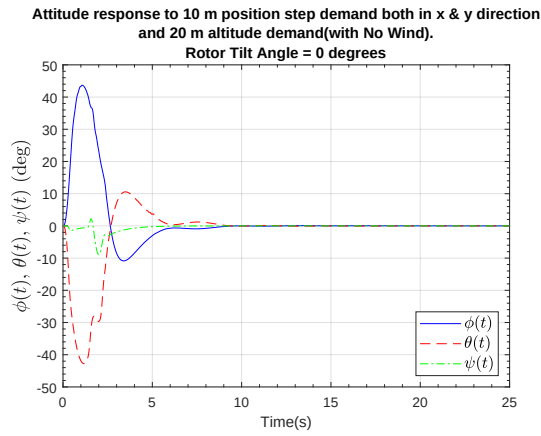


Figure E.35: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = 0^\circ$

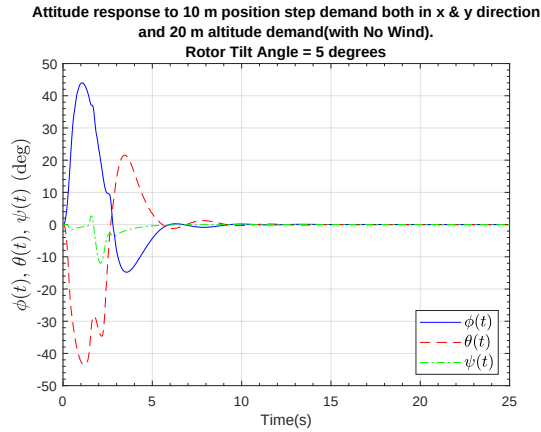


Figure E.36: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = 5^\circ$

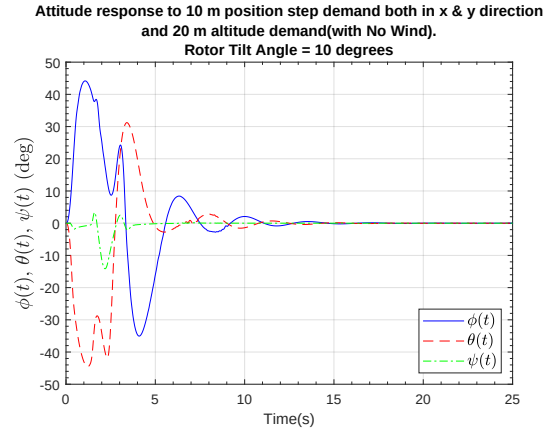


Figure E.37: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = 10^\circ$

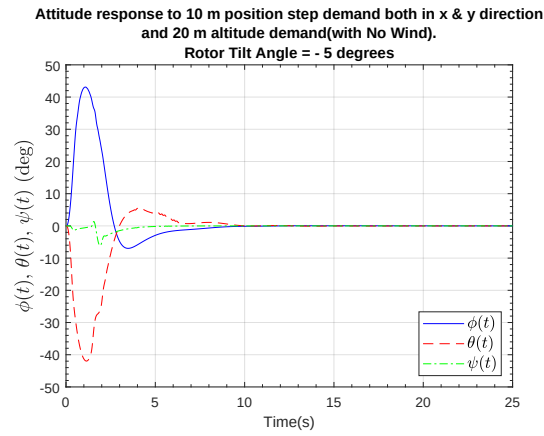


Figure E.38: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = -5^\circ$

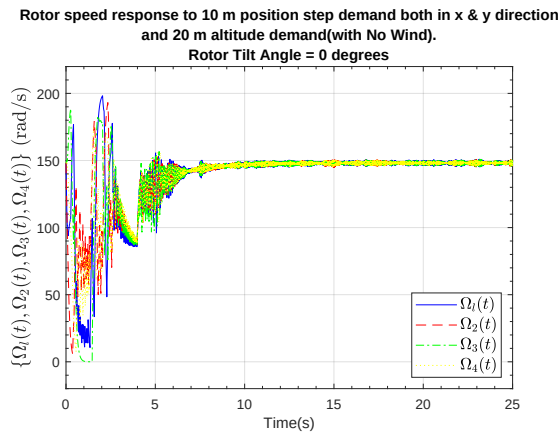


Figure E.39: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = 0^\circ$

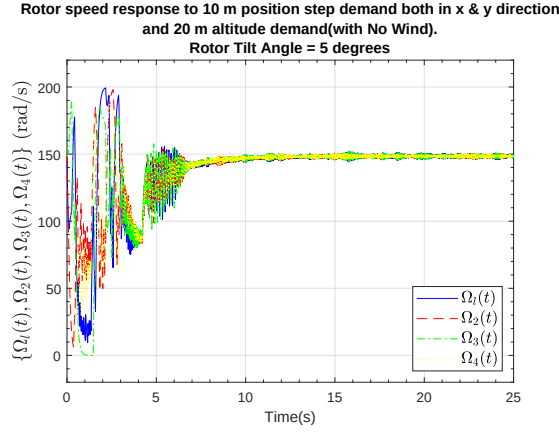


Figure E.40: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = 5^\circ$

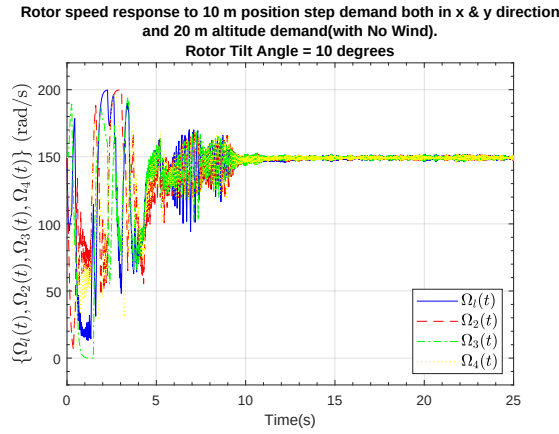


Figure E.41: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = 10^\circ$

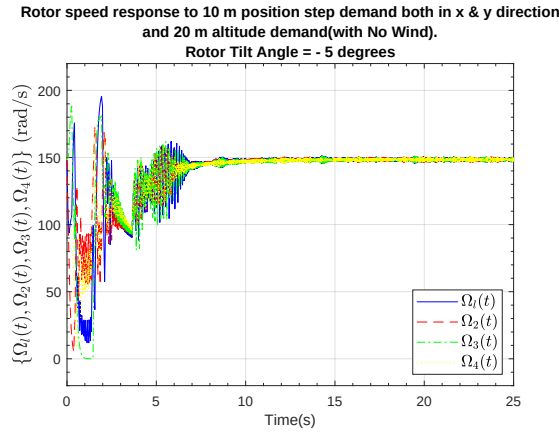


Figure E.42: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (no wind gust). $\Upsilon = -5^\circ$

E.1.5 SMC Altitude and Attitude Control and PIV Position Control for Position and Altitude Demands with 10 ms^{-1} Wind Gust Step

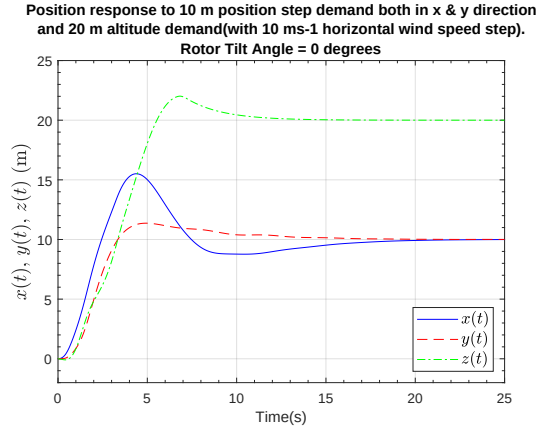


Figure E.43: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms^{-1} horizontal wind gust step). $\Upsilon = 0^\circ$

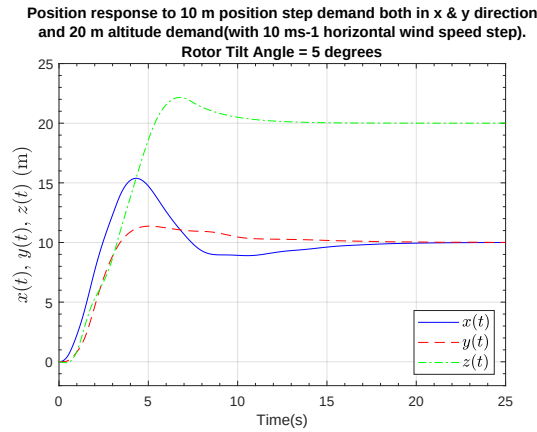


Figure E.44: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms^{-1} horizontal wind gust step). $\Upsilon = 5^\circ$

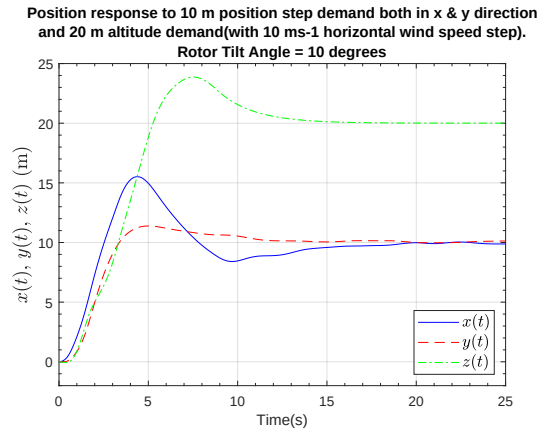


Figure E.45: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms^{-1} horizontal wind gust step). $\Upsilon = 10^\circ$

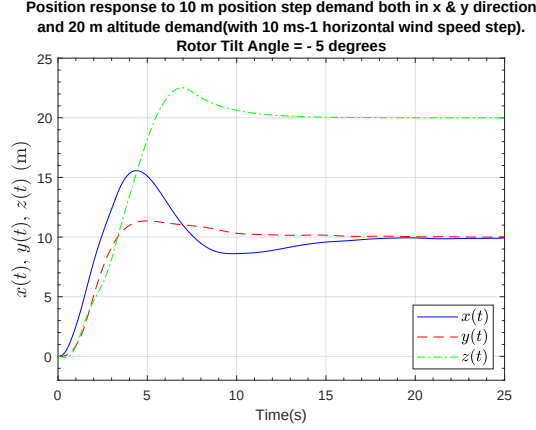


Figure E.46: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

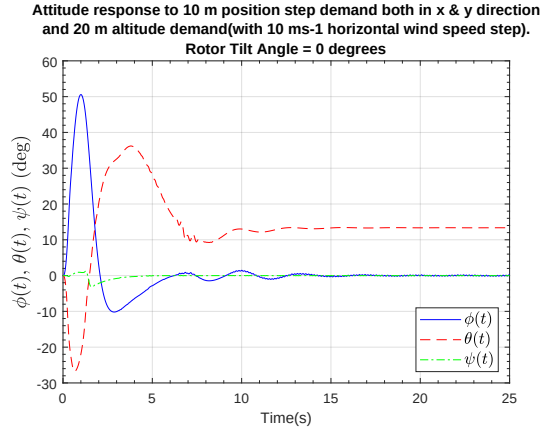


Figure E.47: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

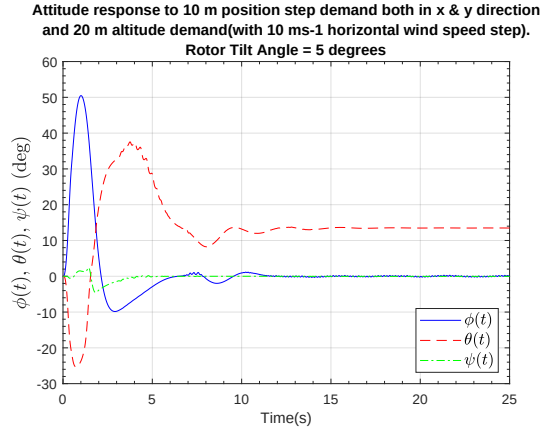


Figure E.48: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

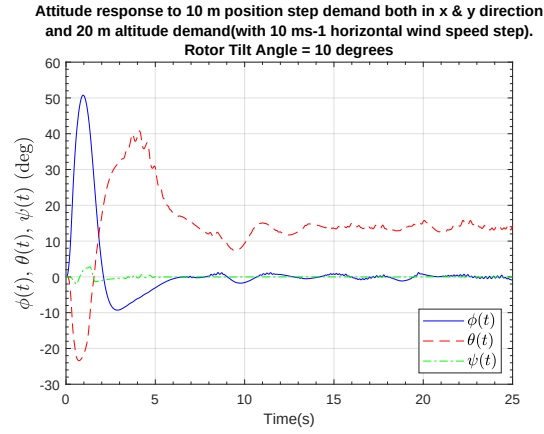


Figure E.49: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

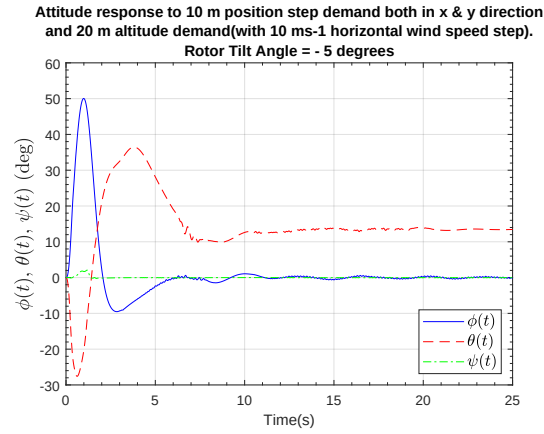


Figure E.50: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

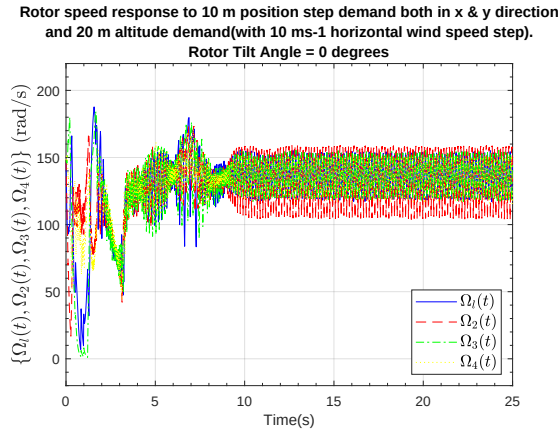


Figure E.51: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

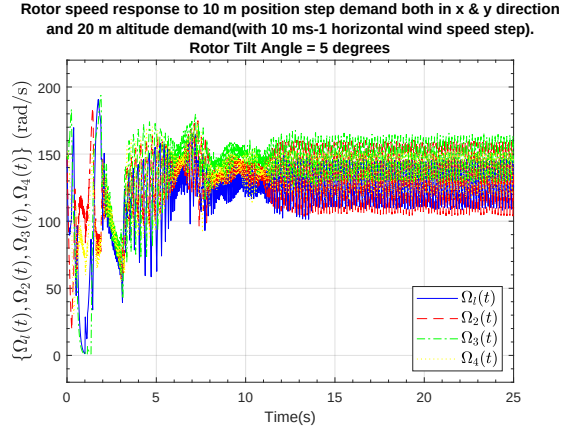


Figure E.52: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

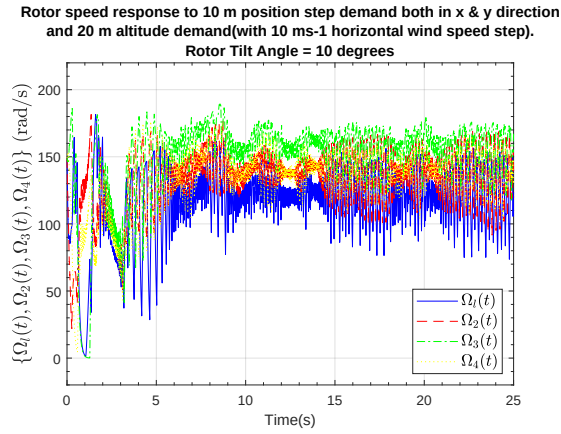


Figure E.53: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

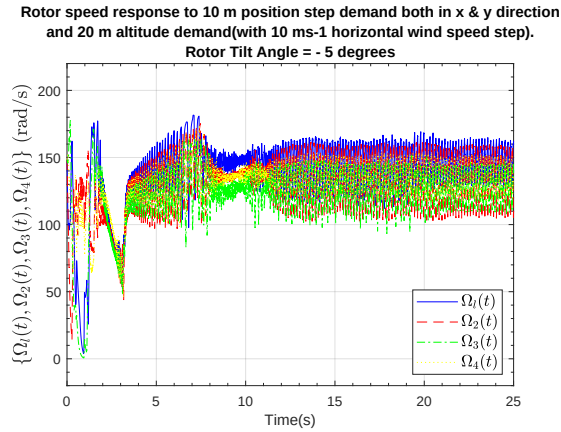


Figure E.54: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

E.1.6 Comparison of SMC Altitude and Attitude Control and PIV Position Control of a Quadrotor for Position and Altitude Demands with 10 ms^{-1} Wind Gust Step

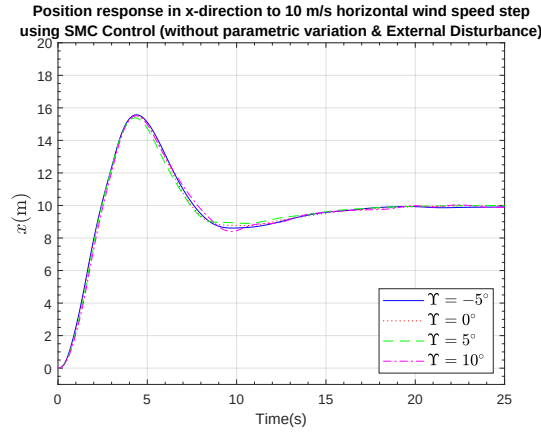


Figure E.55: Lateral position response in x-direction to 10 ms^{-1} horizontal wind gust step

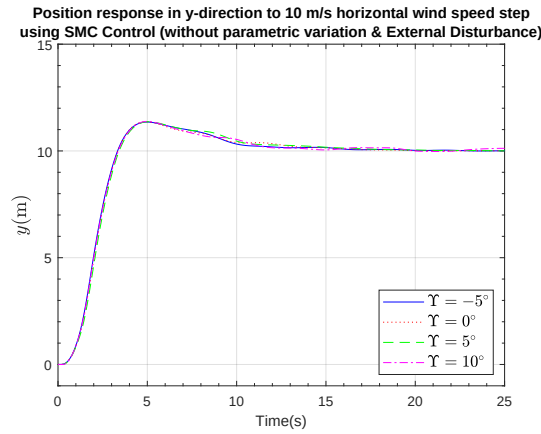


Figure E.56: Lateral position response in y-direction to 10 ms^{-1} horizontal wind gust step

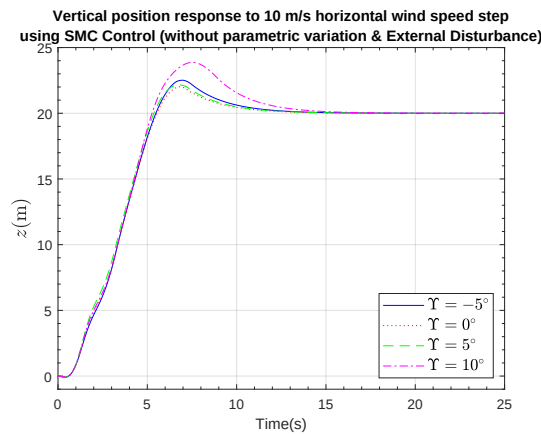


Figure E.57: Vertical position response in z-direction to 10 ms^{-1} horizontal wind gust step

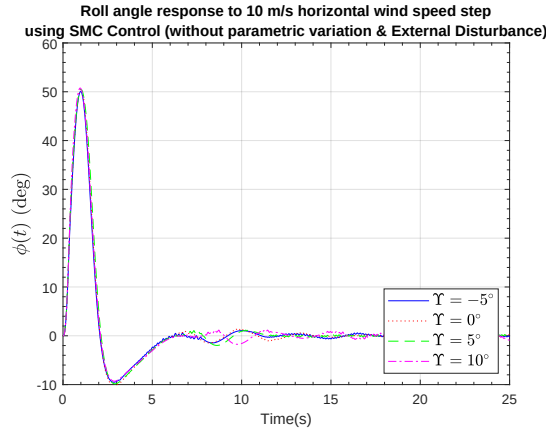


Figure E.58: Roll angle response to 10 ms^{-1} horizontal wind gust step

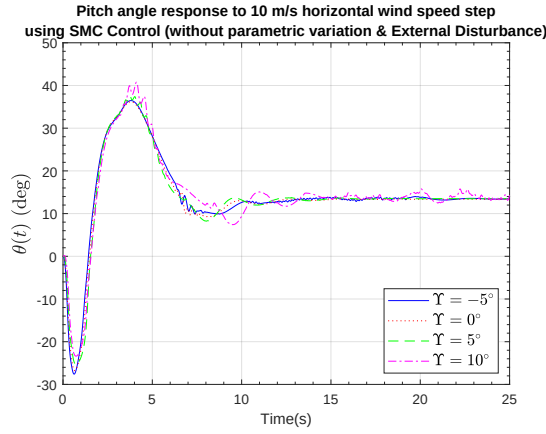


Figure E.59: Pitch angle response to 10 ms^{-1} horizontal wind gust step

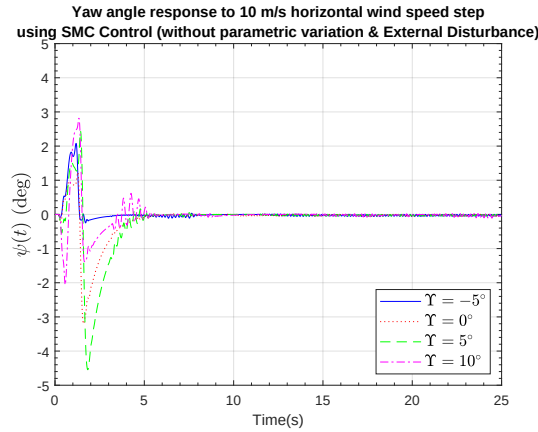


Figure E.60: Yaw angle response to 10 ms^{-1} horizontal wind gust step

E.2 Simulation Results of Quadrotor with Aerodynamics for Position and Altitude Demands without Parametric Variation and External Disturbance using SMC for Altitude and Attitude Control and Position Control

E.2.1 SMC Altitude and Attitude Control and Position Control for Position and Altitude Demands with 10 ms^{-1} Wind Gust Step

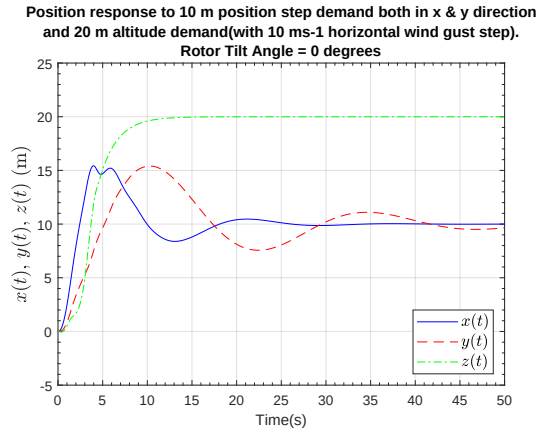


Figure E.61: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

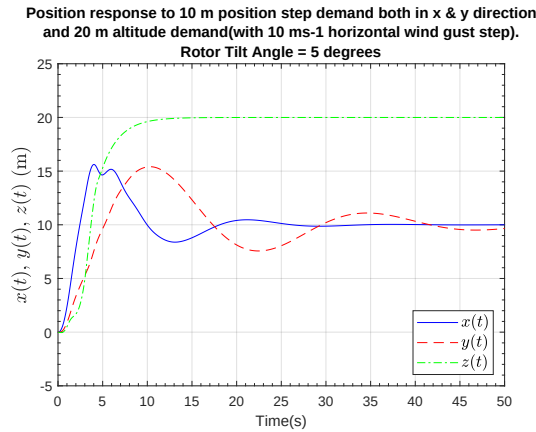


Figure E.62: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

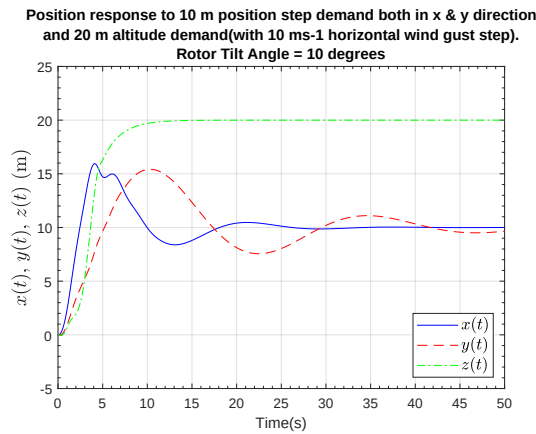


Figure E.63: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

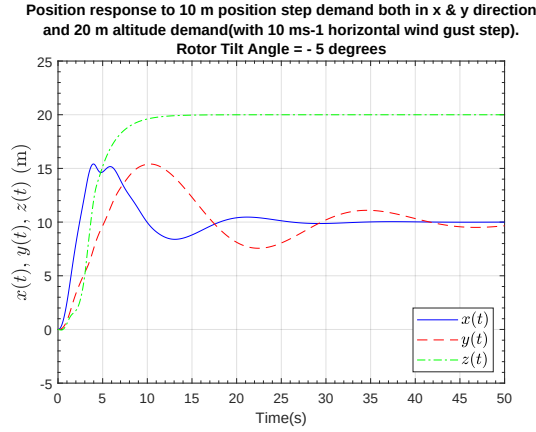


Figure E.64: Position response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

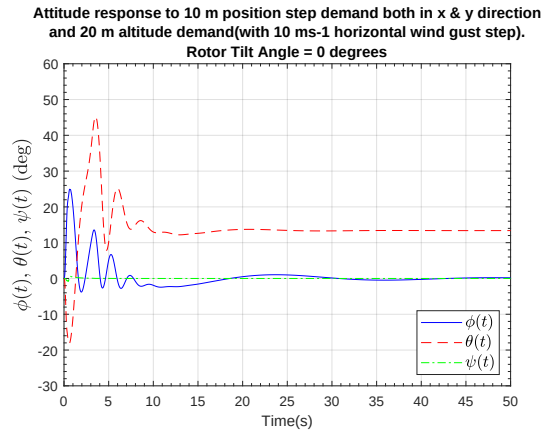


Figure E.65: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

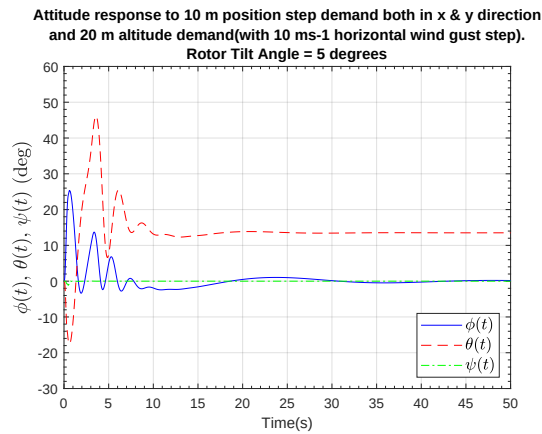


Figure E.66: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

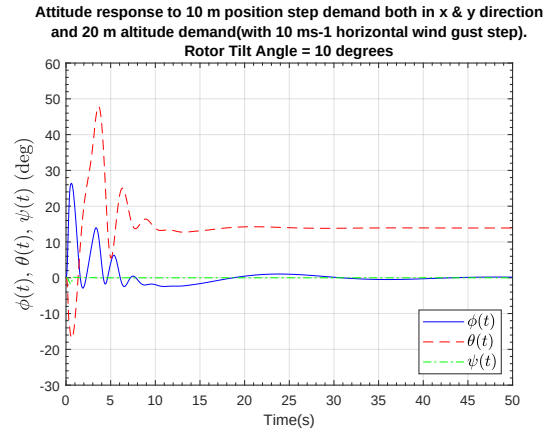


Figure E.67: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

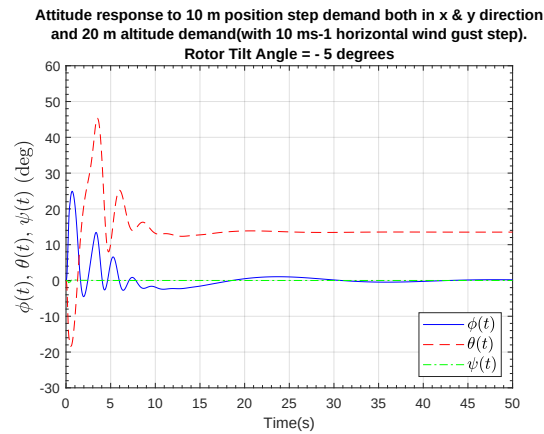


Figure E.68: Attitude response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

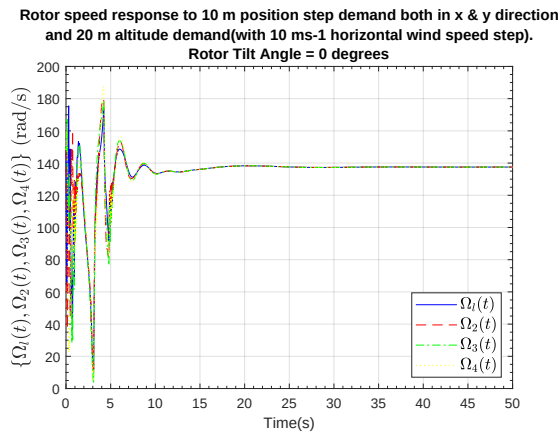


Figure E.69: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

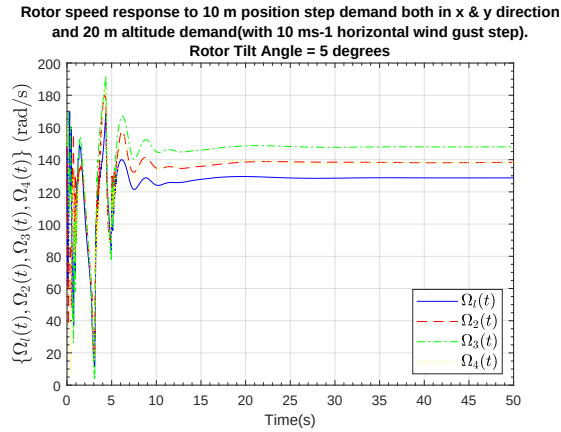


Figure E.70: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

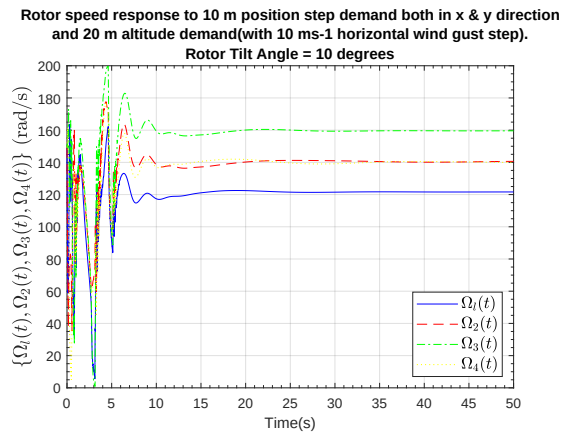


Figure E.71: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

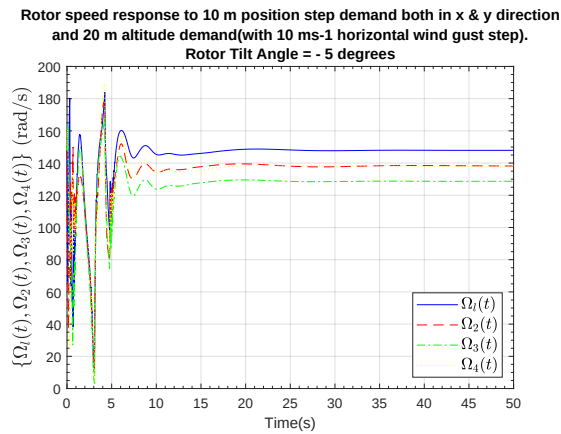


Figure E.72: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

E.2.2 Comparison of SMC Altitude and Attitude Control and PIV Position Control of a Quadrotor for Position and Altitude Demands with 10 ms^{-1} Wind Gust Step

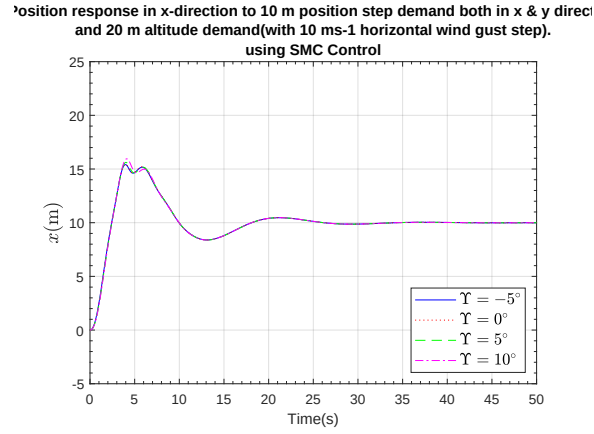


Figure E.73: Lateral position response in x-direction to 10 ms^{-1} horizontal wind gust step

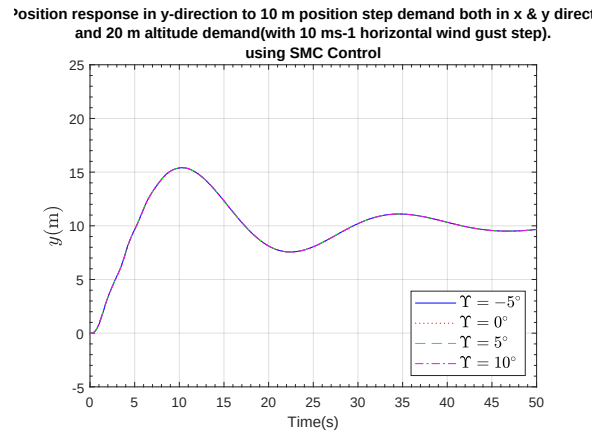


Figure E.74: Lateral position response in y-direction to 10 ms^{-1} horizontal wind gust step

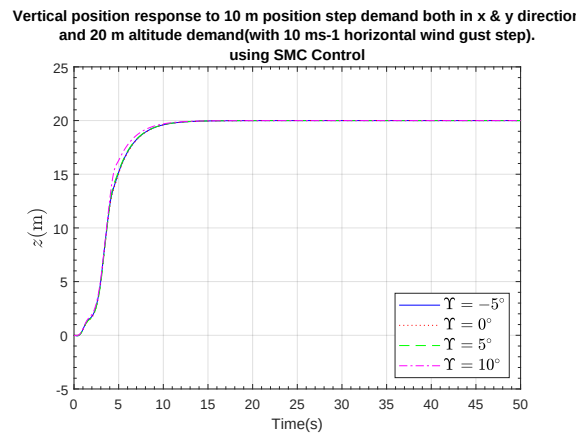


Figure E.75: Vertical position response in z-direction to 10 ms^{-1} horizontal wind gust step

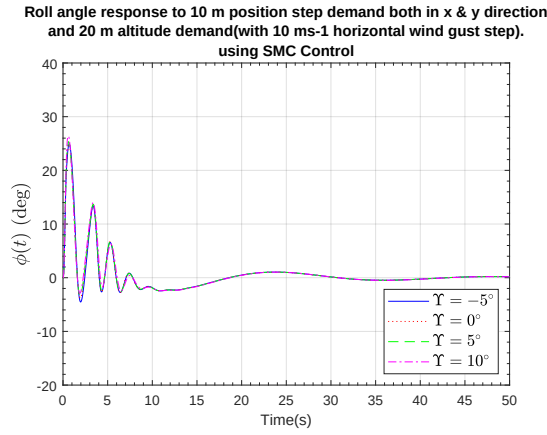


Figure E.76: Roll angle response to 10 ms⁻¹ horizontal wind gust step

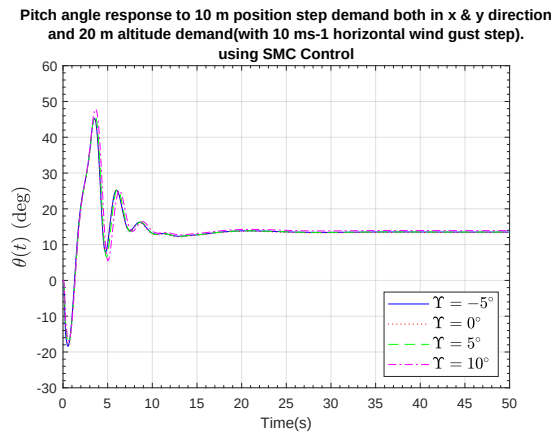


Figure E.77: Pitch angle response to 10 ms⁻¹ horizontal wind gust step

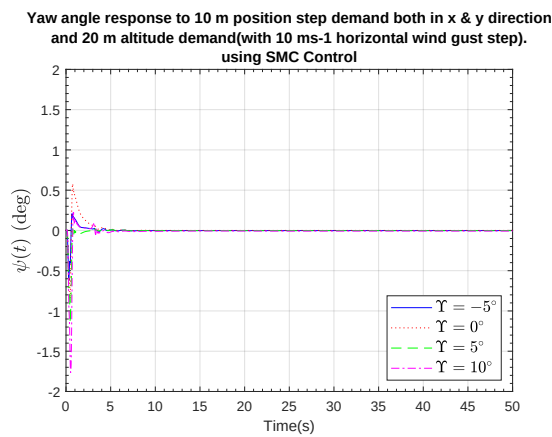


Figure E.78: Yaw angle response to 10 ms⁻¹ horizontal wind gust step

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APPENDIX F

Additional Simulation Results - Part 2

F.0.1 SMC Control of Altitude and Attitude and PIV Position Control of Quadrotor at Hover without Wind Gust

Initially, the simulation results for SMC control of quadrotor at hover without any wind gust in the presence of parametric variations and external disturbance is considered. The position response, the attitude response and the rotors speed response are studied.

Figures F.1, F.2, F.3 and F.4 show the simulation results for the position response at hover without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance as considered in Chapter 5.

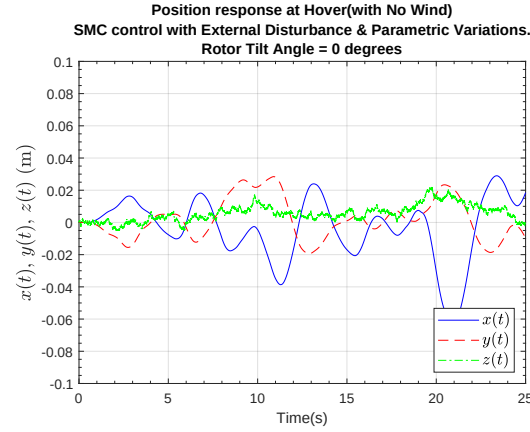
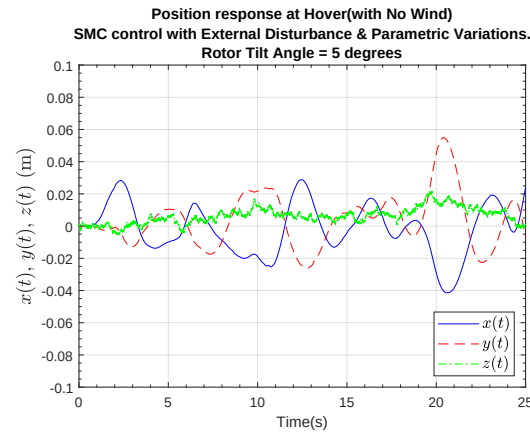
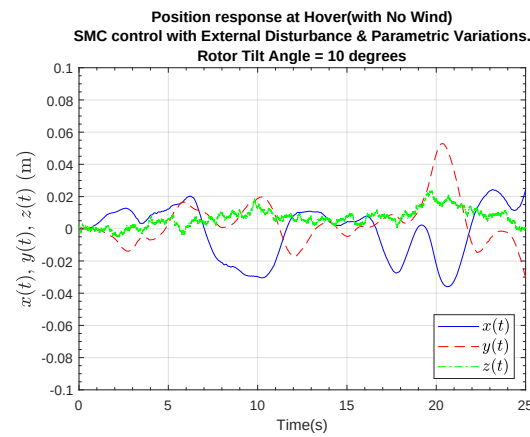
Figures F.5, F.6, F.7 and F.8 show the simulation results for the attitude response at hover without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance.

As can be seen from the simulation results, there is no significant differences in the position and attitude responses which gives an earlier indication that the SMC controller behaves in the same way for different rotor tilt angles which needs to be further explored in the case of taking wind gust into account.

Figures F.9, F.10, F.11 and F.12 show the simulation results for the rotor speed response at hover without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance. It can be seen from the rotor rotation rates, the rotors work with the same rates for different rotor tilt angles with a mean value around 150 rad/s.

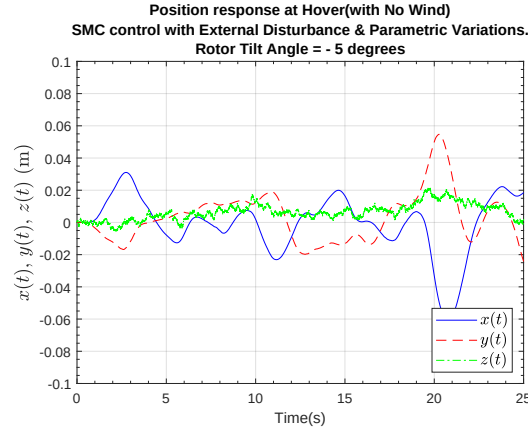
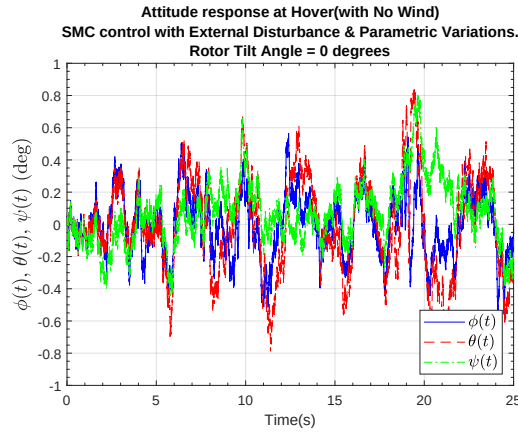
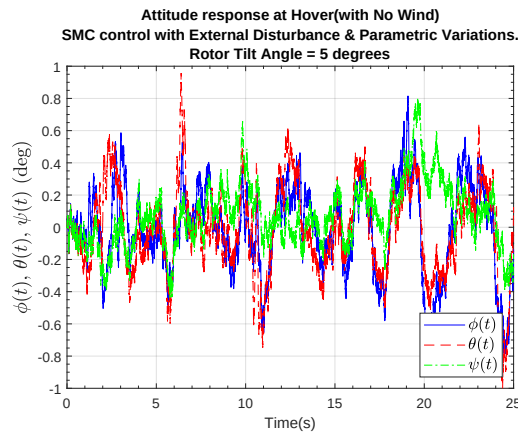
F.0.2 Comparison of SMC Control of Altitude and Attitude and PIV Position Control of Quadrotor at Hover without any Wind Gust

Figure F.13 shows the comparative position response in x-direction, Figure F.14 shows the comparative position response in y-direction, Figure F.15 shows the comparative vertical response and Figures F.16 to F.18 shows the comparative attitude responses without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

Figure F.1: Position response at hover (no wind). $\Upsilon = 0^\circ$ Figure F.2: Position response at hover (no wind). $\Upsilon = 5^\circ$ Figure F.3: Position response at hover (no wind). $\Upsilon = 10^\circ$

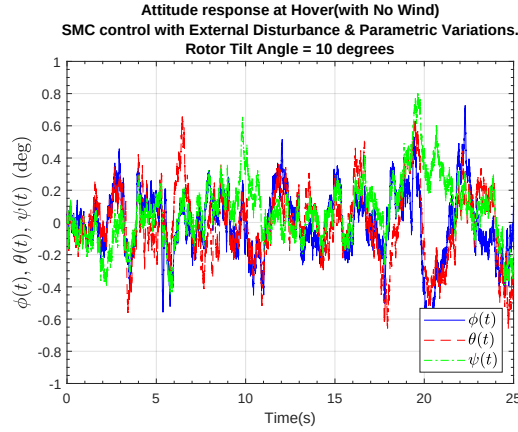
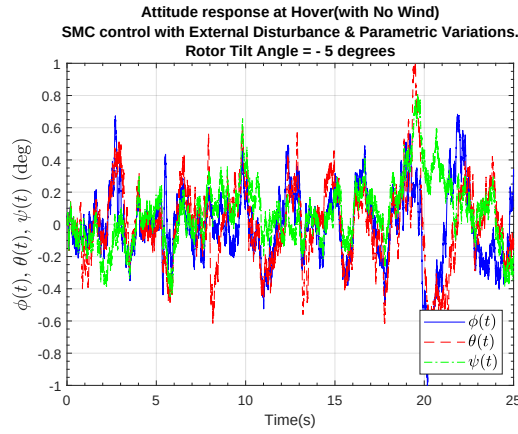
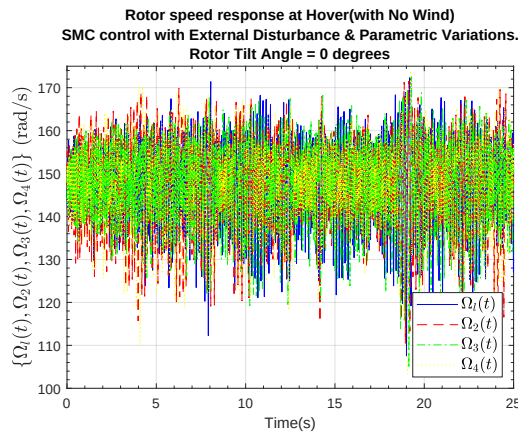
F.0.3 SMC Control of Altitude and Attitude and PIV Position Control of Quadrotor at Hover with 10 ms^{-1} Wind Gust Step

Figures F.19, F.20, F.21 and F.22 show the simulation results for the position response at hover with a 10 ms^{-1} wind gust step in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and

Figure F.4: Position response at hover (no wind). $\Upsilon = -5^\circ$ Figure F.5: Attitude response at hover (no wind). $\Upsilon = 0^\circ$ Figure F.6: Attitude response at hover (no wind). $\Upsilon = 5^\circ$

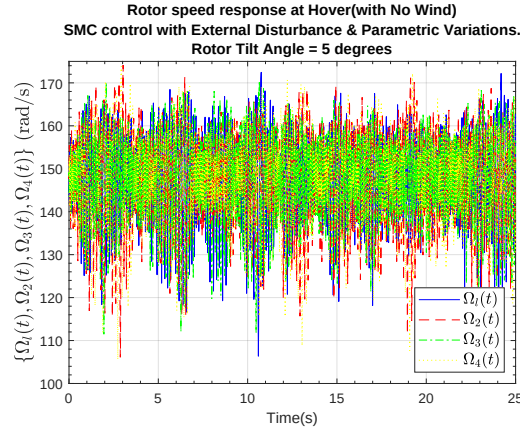
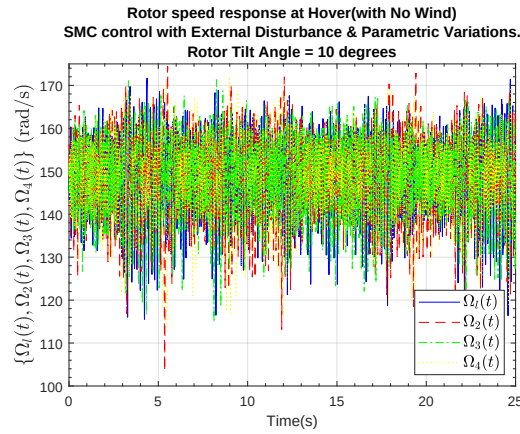
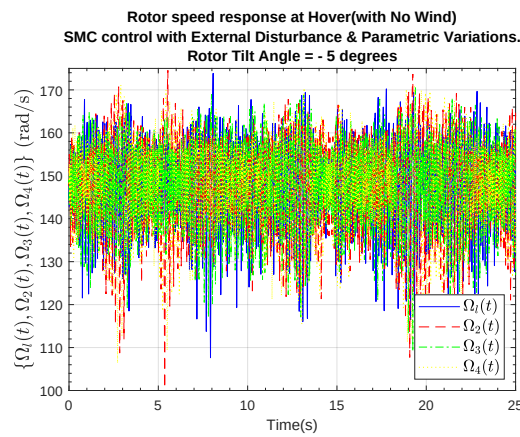
$\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance as considered in Chapter 5.

Figures F.23, F.24, F.25 and F.26 show the simulation results for the attitude response at hover with a 10 ms^{-1} wind gust step in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance.

Figure F.7: Attitude response at hover (no wind). $\Upsilon = 10^\circ$ Figure F.8: Attitude response at hover (no wind). $\Upsilon = -5^\circ$ Figure F.9: Rotor speed response at hover (no wind). $\Upsilon = 0^\circ$

Figures F.27, F.28, F.29 and F.30 show the simulation results for the rotor speed response at hover with a 10 ms^{-1} wind gust step in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance.

As can be seen from the simulation results, there is no significant differences in the position and attitude responses which indicates that the SMC controller behaves in the same way for different rotor tilt angles

Figure F.10: Rotor speed response at hover (no wind). $\Upsilon = 5^\circ$ Figure F.11: Rotor speed response at hover (no wind). $\Upsilon = 10^\circ$ Figure F.12: Rotor speed response at hover (no wind). $\Upsilon = -5^\circ$

even in the presence 10 ms^{-1} wind gust step, parametric uncertainty and external disturbances. From the position response for different rotor tilt angles, the maximum displacement for x is around 1.5 m and stabilises after 17 secs, the position response in y -direction remains almost similar to different rotor tilt angles but is sensitive to noise. The maximum displacement for $\Upsilon = 10^\circ$ is greater when compared to other tilt angles but they have the nearly the settling times of around 15 seconds. The attitude responses for different rotor tilt angles are almost the same.

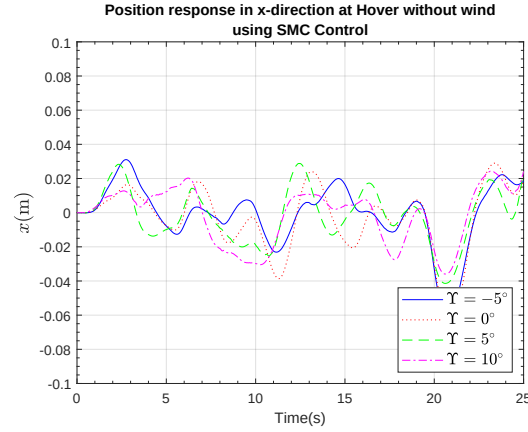


Figure F.13: Position response in x-direction at hover without wind gust step

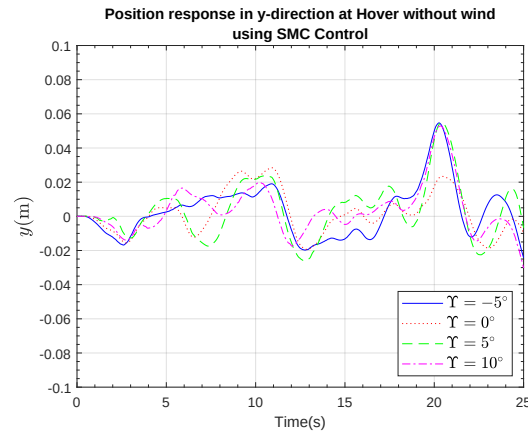


Figure F.14: Position response in y-direction at hover without wind gust step

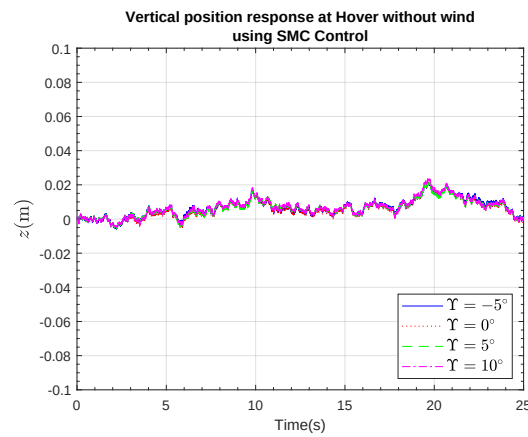


Figure F.15: Vertical response at hover without wind gust step

F.0.4 Comparison of SMC Control of Altitude and Attitude and PIV Position Control of Quadrotor at Hover with 10 ms^{-1} Wind Gust Step

Figure F.31 shows the comparative position response in x-direction, Figure F.32 shows the comparative position response in y-direction, Figure F.33 shows the comparative vertical response and Figures F.34

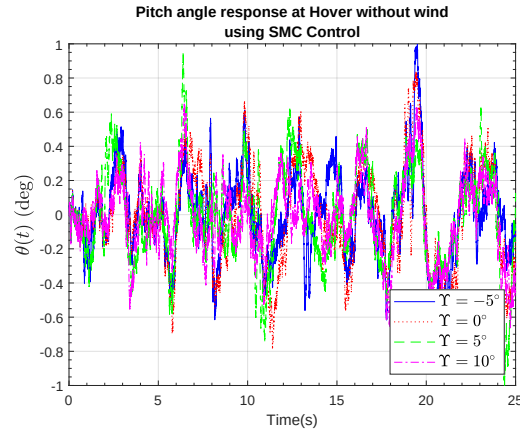


Figure F.16: Pitch angle response at hover without wind gust step

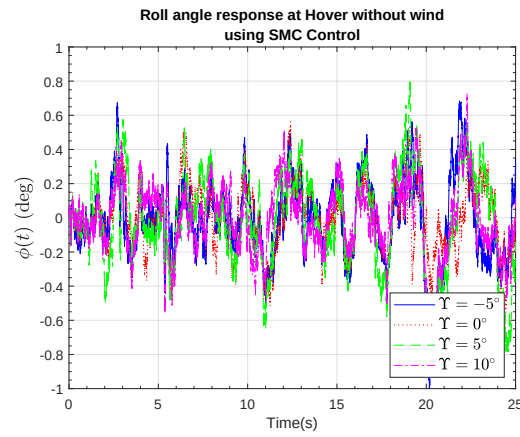


Figure F.17: Roll angle response at hover without wind gust step

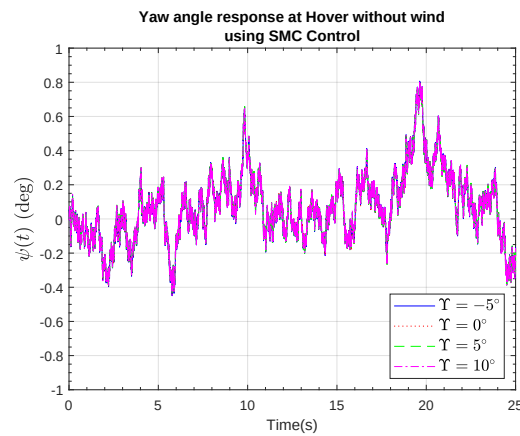
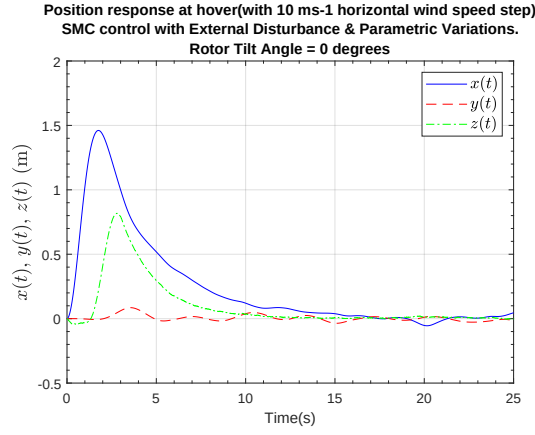
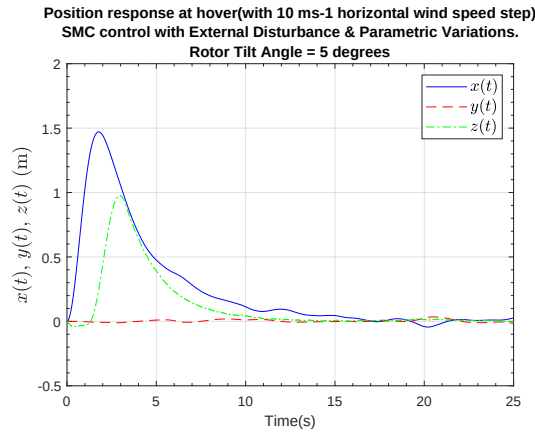
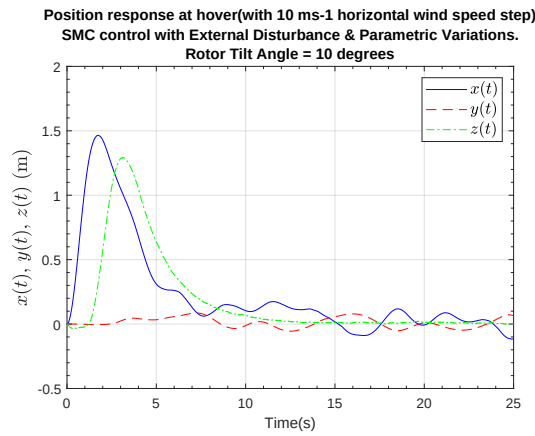


Figure F.18: Yaw angle response at hover without wind gust step

to F.36 shows the comparative attitude responses with 10 ms^{-1} wind gust step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

Figure F.19: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$ Figure F.20: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$ Figure F.21: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

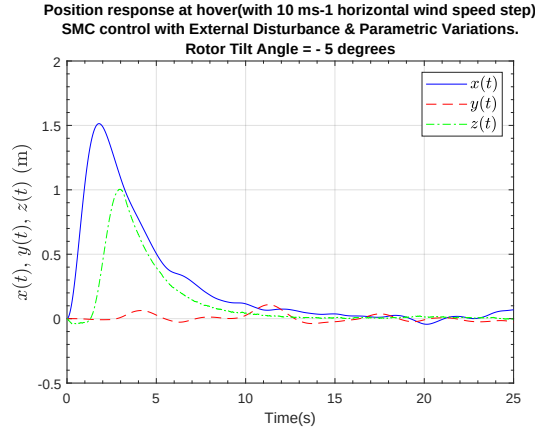


Figure F.22: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

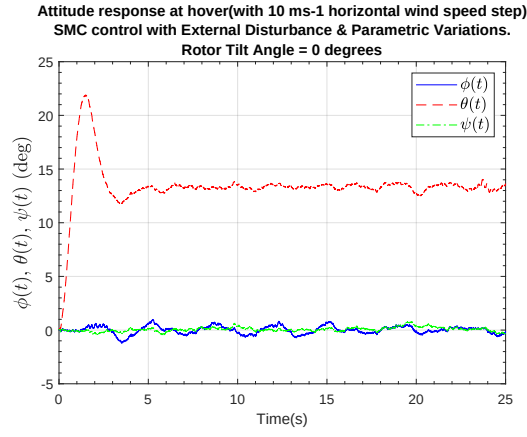


Figure F.23: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

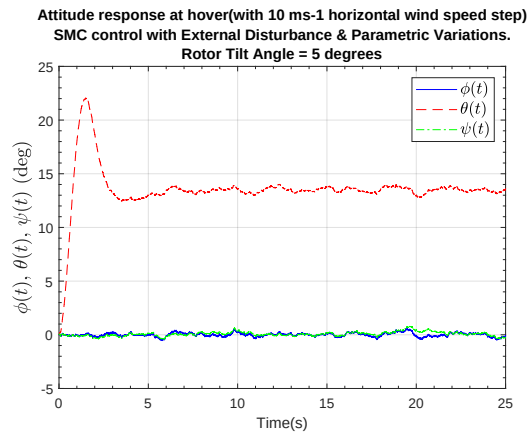
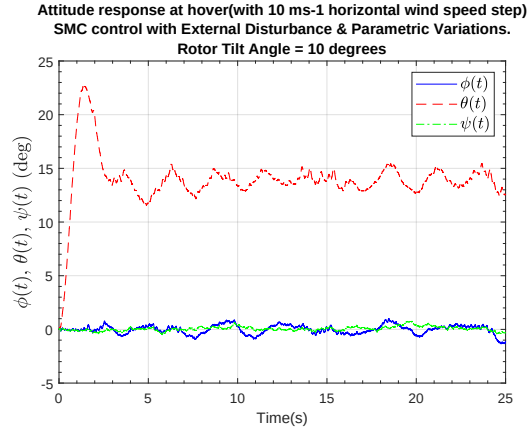
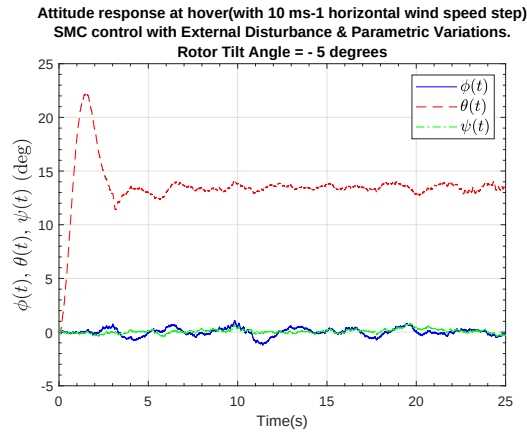
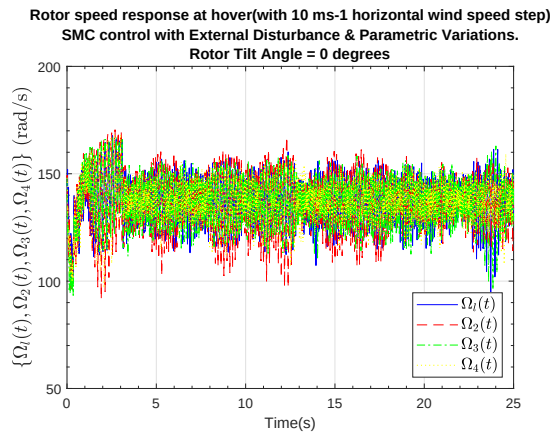


Figure F.24: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

Figure F.25: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$ Figure F.26: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$ Figure F.27: Rotor speed response at hover (with 10 ms⁻¹ horizontal wind speed step). $\Upsilon = 0^\circ$

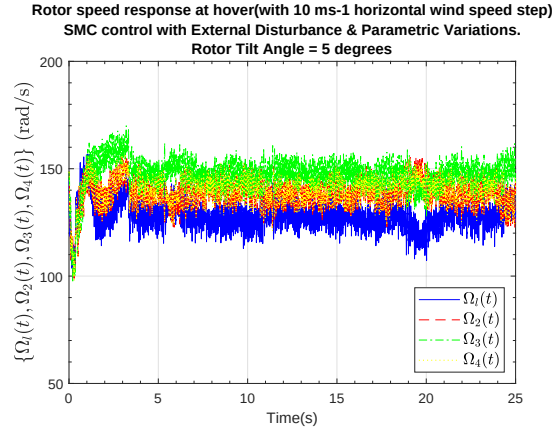


Figure F.28: Rotor speed response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

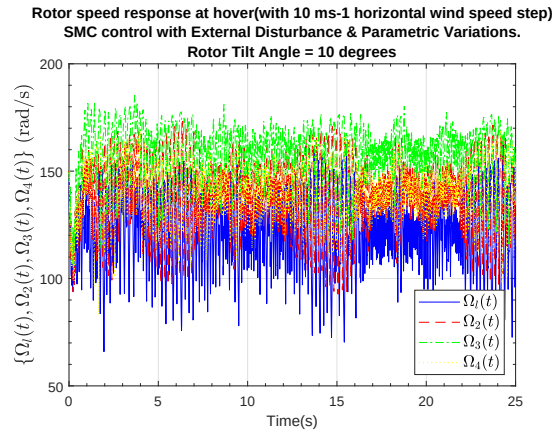


Figure F.29: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

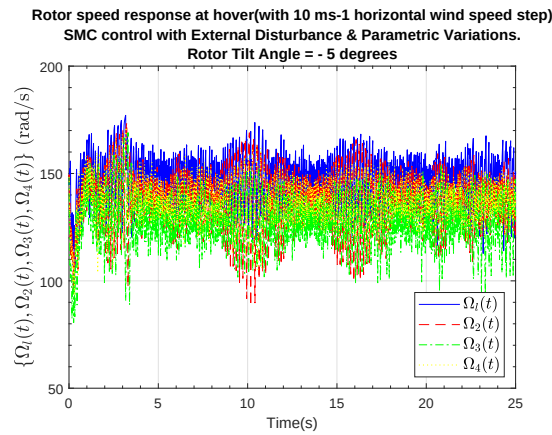
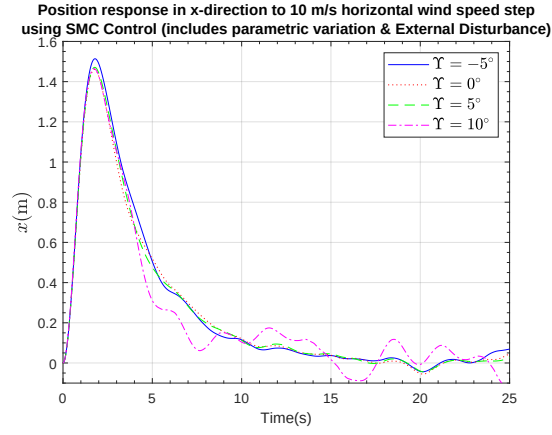
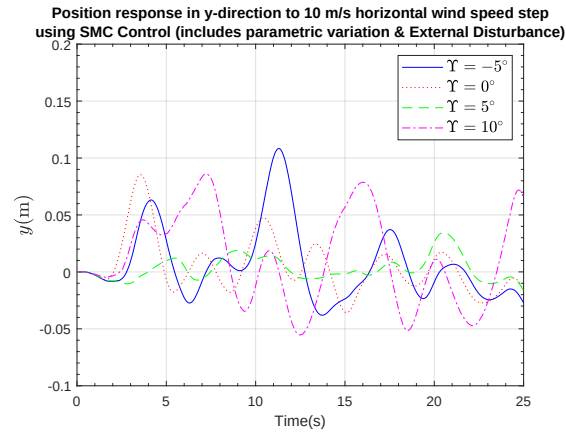
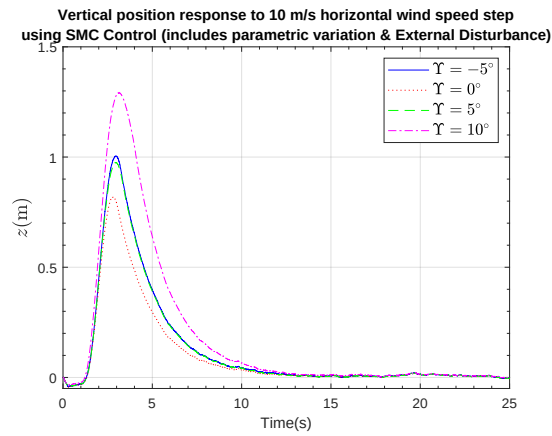
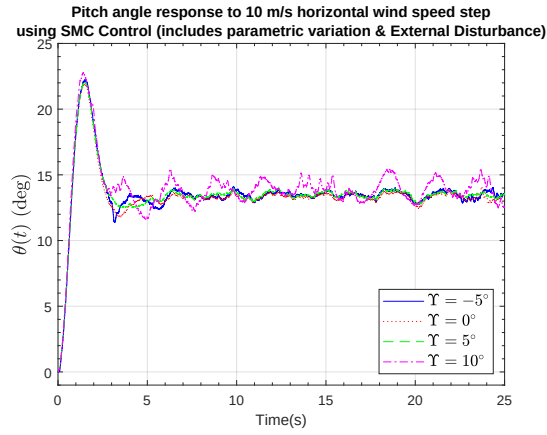
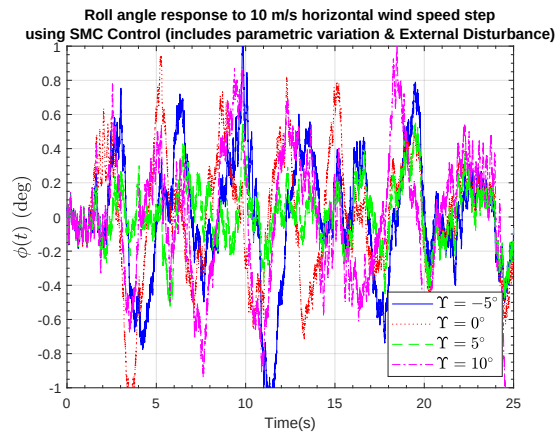
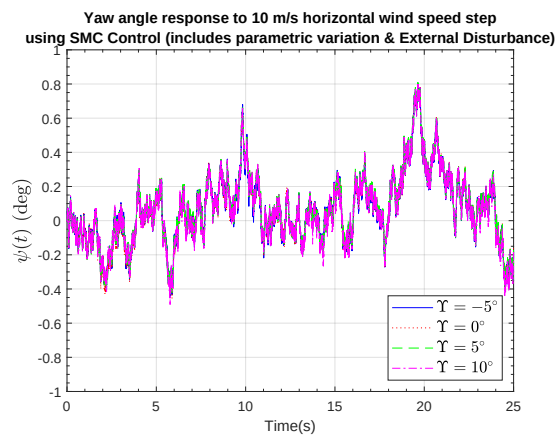


Figure F.30: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

Figure F.31: Position response in x-direction at hover with 10 ms^{-1} horizontal wind gust step)Figure F.32: Position response in x-direction at hover with 10 ms^{-1} horizontal wind gust stepFigure F.33: Vertical response at hover with 10 ms^{-1} horizontal wind gust step

Figure F.34: Pitch angle response at hover with 10 ms^{-1} horizontal wind gust stepFigure F.35: Roll angle response at hover with 10 ms^{-1} horizontal wind gust stepFigure F.36: Yaw angle response with 10 ms^{-1} horizontal wind gust step

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APPENDIX G

Additional Simulation Results - Part 3

G.0.1 SMC for Altitude, Attitude and Position Control of Quadrotor at Hover without Wind Gust Step

Initially, the simulation results for SMC control of Altitude, Attitude and Position Control of quadrotor at hover without any wind gust in the presence of parametric variations and external disturbance is considered. The position response, the attitude response and the rotors speed response are studied.

Figures G.1, G.2, G.3 and G.4 show the simulation results for the position response at hover without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance as considered in Chapter 5.

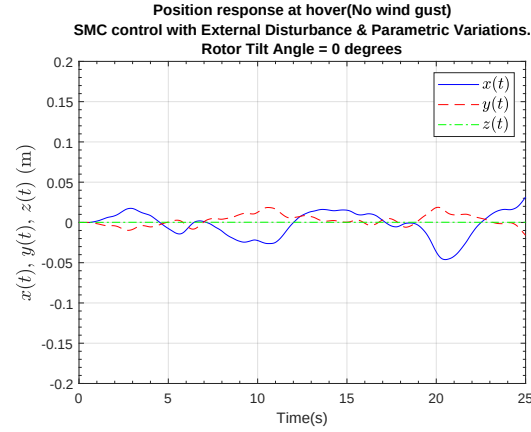
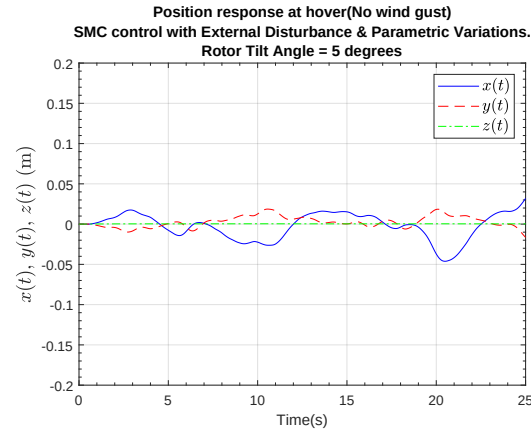
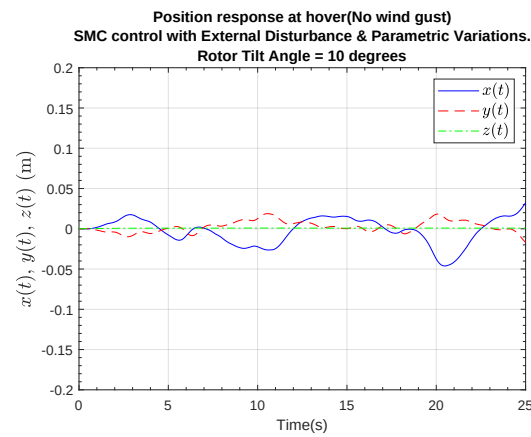
Figures G.5, G.6, G.7 and G.8 show the simulation results for the attitude response at hover without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance.

As can be seen from the simulation results, there is no significant differences in the position and attitude responses which gives an earlier indication that the SMC controller behaves in the same way for different rotor tilt angles which needs to be further explored in the case of taking wind gust into account.

Figures G.9, G.10, G.11 and G.12 show the simulation results for the rotor speed response at hover without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance. It can be seen from the rotor rotation rates, the rotors work with the same rates for different rotor tilt angles with a mean value around 150 rad/s.

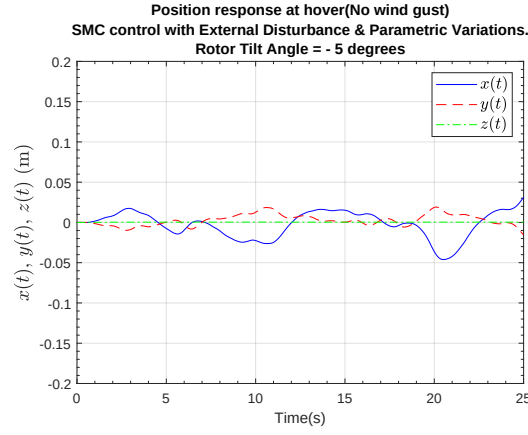
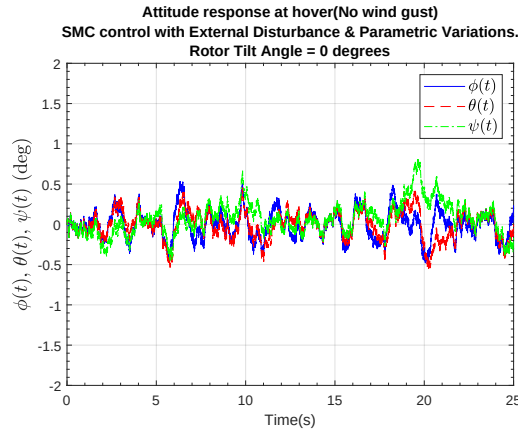
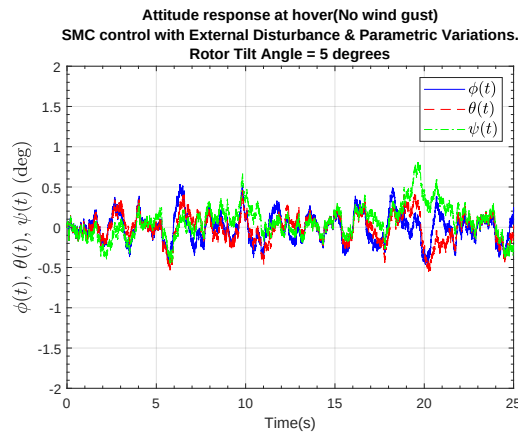
G.0.2 Comparison of SMC Control of Altitude and Attitude and Position Control of Quadrotor at Hover without any Wind Gust

Figure G.13 shows the comparative position response in x-direction, Figure G.14 shows the comparative position response in y-direction, Figure G.15 shows the comparative vertical response and Figures G.16 to G.18 shows the comparative attitude responses without any wind gust for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

Figure G.1: Position response at hover (no wind gust). $\Upsilon = 0^\circ$ Figure G.2: Position response at hover (no wind gust). $\Upsilon = 5^\circ$ Figure G.3: Position response at hover (no wind gust). $\Upsilon = 10^\circ$

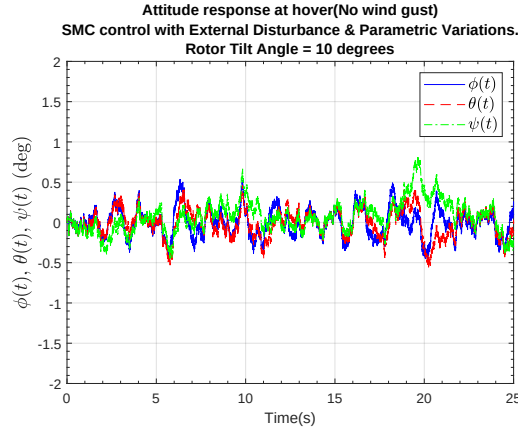
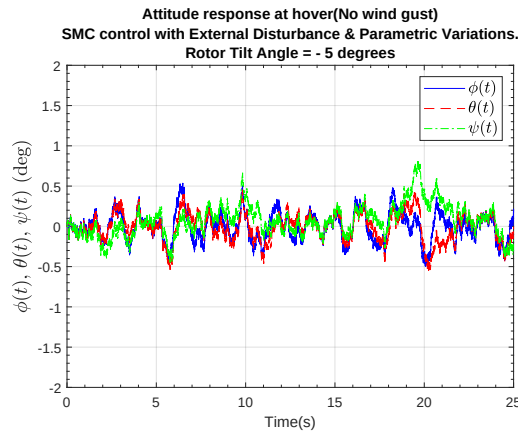
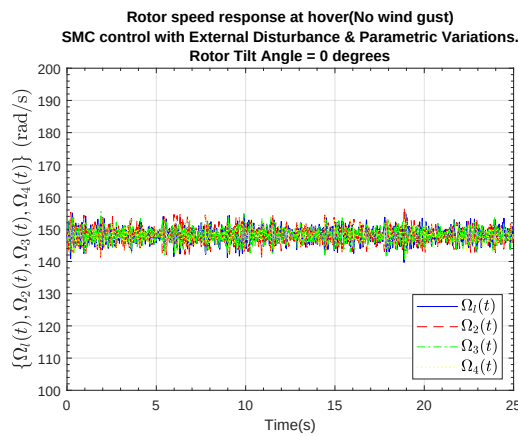
G.0.3 SMC Control of Altitude, Attitude and Position Control of Quadrotor at Hover with 10 ms^{-1} Wind Gust Step

Figures G.19, G.20, G.21 and G.22 show the simulation results for the position response at hover with a 10 ms^{-1} wind gust step in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and

Figure G.4: Position response at hover (no wind gust). $\Upsilon = -5^\circ$ Figure G.5: Attitude response at hover (no wind gust). $\Upsilon = 0^\circ$ Figure G.6: Attitude response at hover (no wind gust). $\Upsilon = 5^\circ$

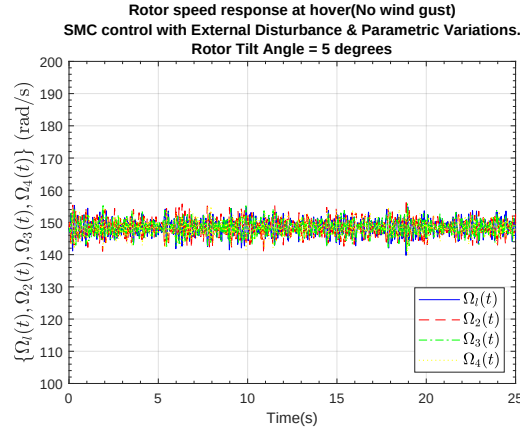
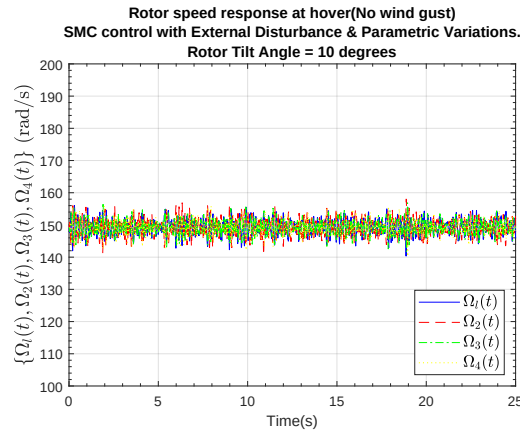
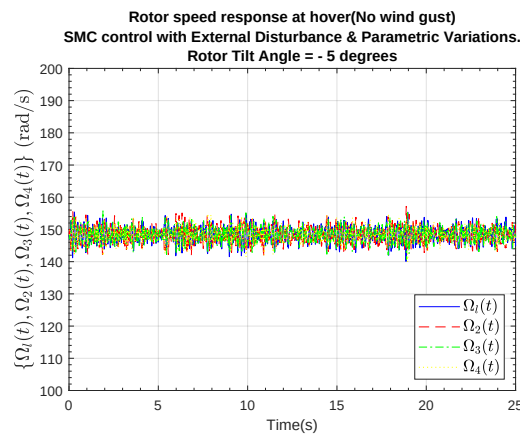
$\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance as considered in Chapter 5.

Figures G.23, G.24, G.25 and G.26 show the simulation results for the attitude response at hover with a 10 ms^{-1} wind gust step in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance.

Figure G.7: Attitude response at hover (no wind gust). $\Upsilon = 10^\circ$ Figure G.8: Attitude response at hover (no wind gust). $\Upsilon = -5^\circ$ Figure G.9: Rotor speed response at hover (no wind gust). $\Upsilon = 0^\circ$

Figures G.27, G.28, G.29 and G.30 show the simulation results for the rotor speed response at hover with a 10 ms^{-1} wind gust step in the x-direction for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$ respectively, in the presence of parametric uncertainty and external disturbance.

As can be seen from the simulation results, there is no significant differences in the position and attitude responses which indicates that the SMC controller behaves in the same way for different rotor tilt angles

Figure G.10: Rotor speed response at hover (no wind gust). $\Upsilon = 5^\circ$ Figure G.11: Rotor speed response at hover (no wind gust). $\Upsilon = 10^\circ$ Figure G.12: Rotor speed response at hover (no wind gust). $\Upsilon = -5^\circ$

even in the presence 10 ms^{-1} wind gust step, parametric uncertainty and external disturbances. From the position response for different rotor tilt angles, the maximum displacement for x is around 1.5 m and stabilises after 17 secs, the position response in y -direction remains almost similar to different rotor tilt angles but is sensitive to noise. The maximum displacement for $\Upsilon = 10^\circ$ is greater when compared to other tilt angles but they have the nearly the settling times of around 15 seconds. The attitude responses for different rotor tilt angles are almost the same.

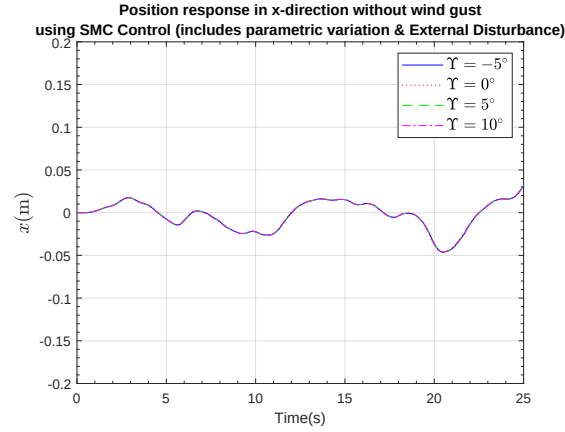


Figure G.13: Position response in x-direction at hover without wind gust

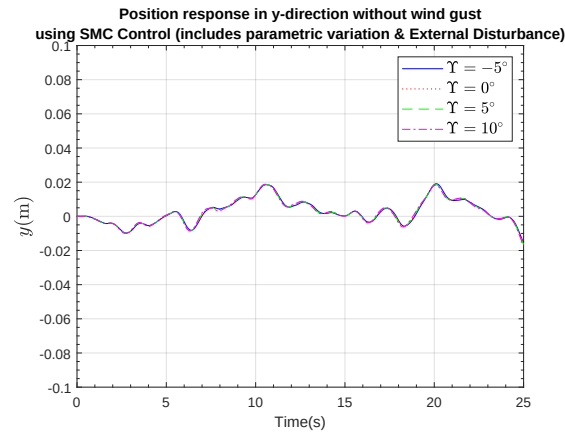


Figure G.14: Position response in y-direction at hover without wind gust

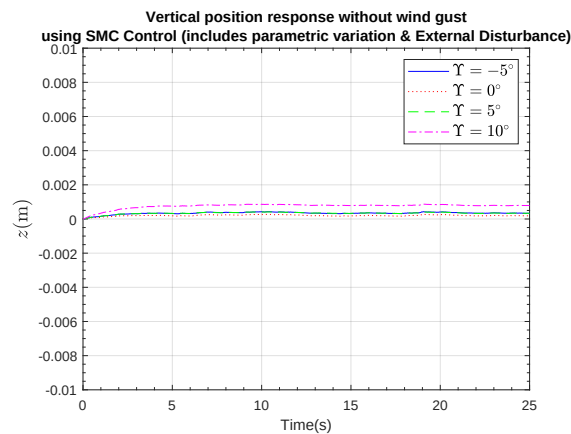


Figure G.15: Vertical response at hover without wind gust

G.0.4 Comparison of SMC Control of Altitude and Attitude and Position Control of Quadrotor at Hover with 10 ms^{-1} Wind Gust Step

Figure G.31 shows the comparative position response in x-direction, Figure G.32 shows the comparative position response in y-direction, Figure G.33 shows the comparative vertical response and Figures G.34

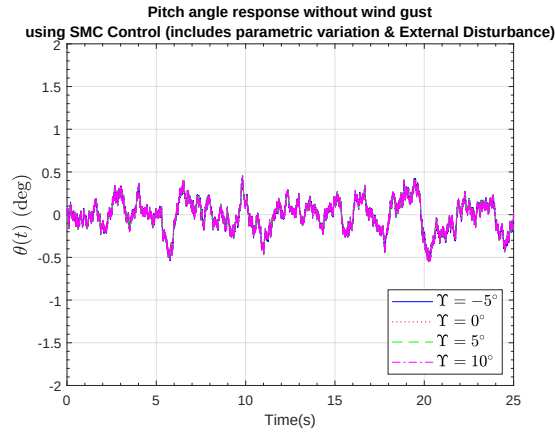


Figure G.16: Pitch angle response at hover without wind gust

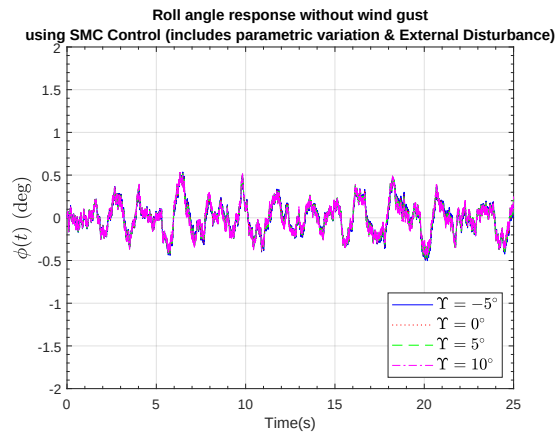


Figure G.17: Roll angle response at hover without wind gust

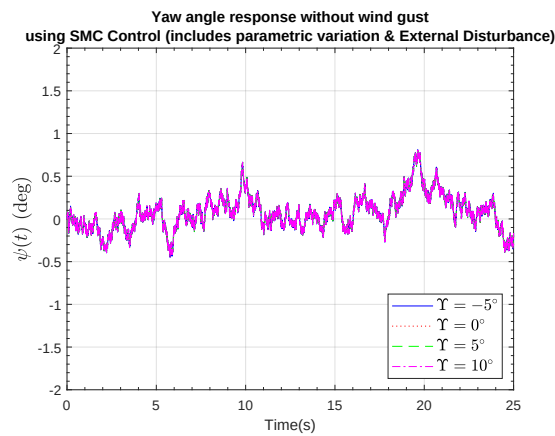
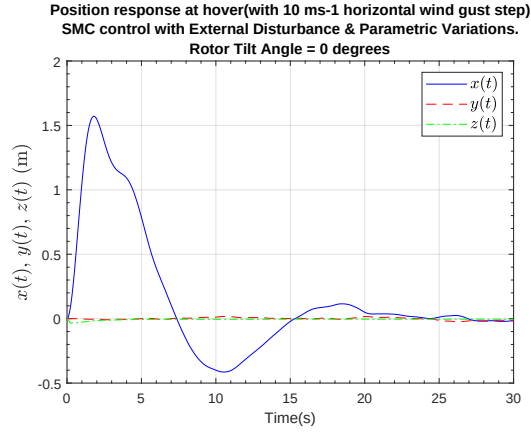
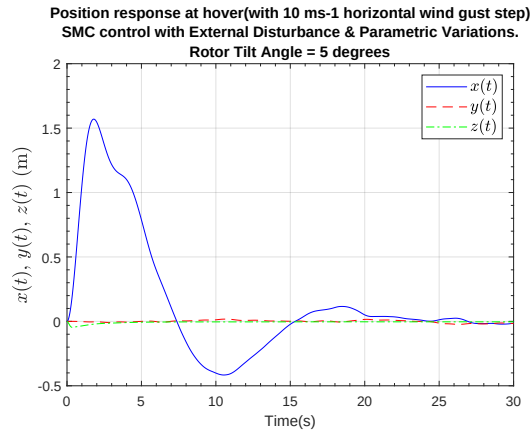
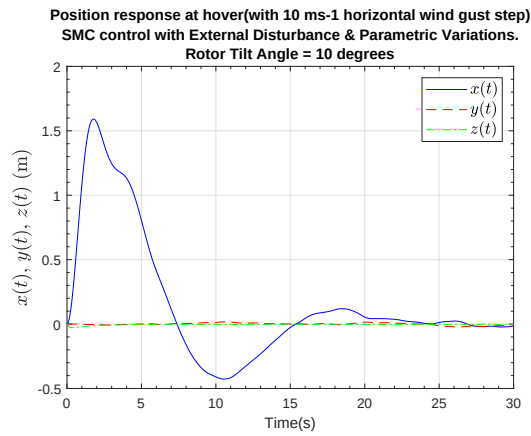


Figure G.18: Yaw angle response at hover without wind gust

to G.36 shows the comparative attitude responses with 10 ms^{-1} wind gust step for different rotor tilt angles, $\Upsilon = 0^\circ$, $\Upsilon = 5^\circ$, $\Upsilon = 10^\circ$ and $\Upsilon = -5^\circ$, respectively.

Figure G.19: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$ Figure G.20: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$ Figure G.21: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

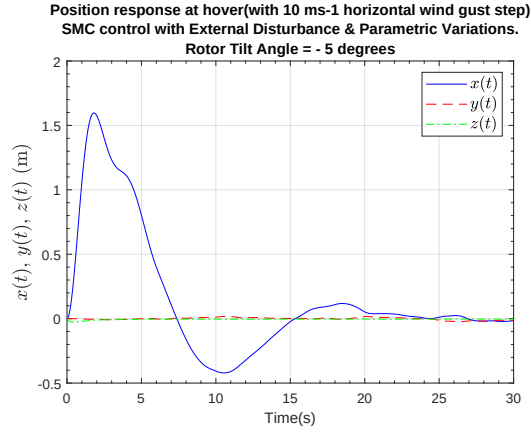


Figure G.22: Position response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

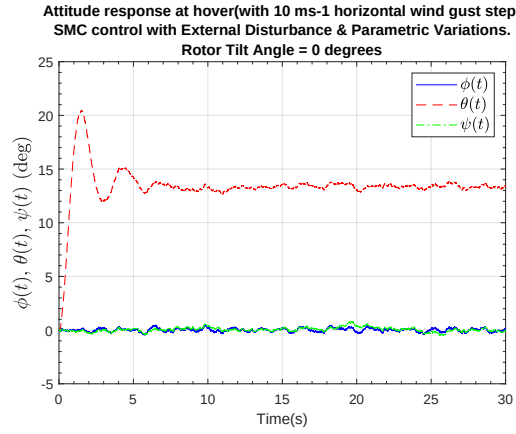


Figure G.23: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

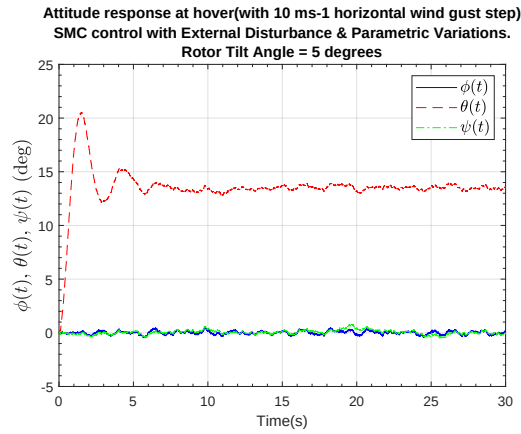
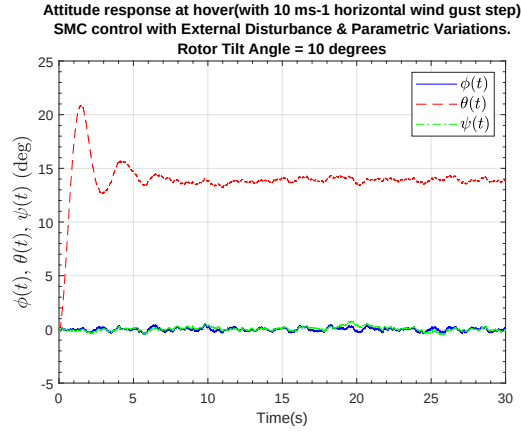
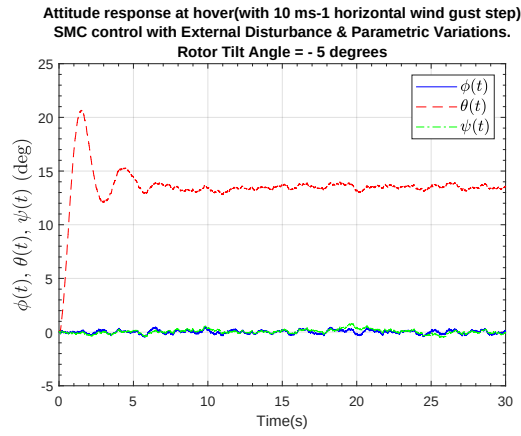
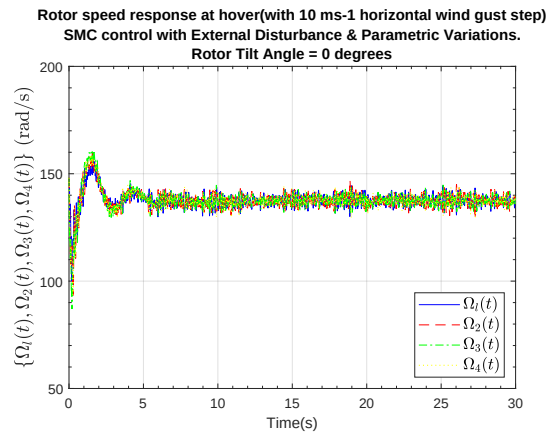


Figure G.24: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

Figure G.25: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$ Figure G.26: Attitude response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$ Figure G.27: Rotor speed response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 0^\circ$

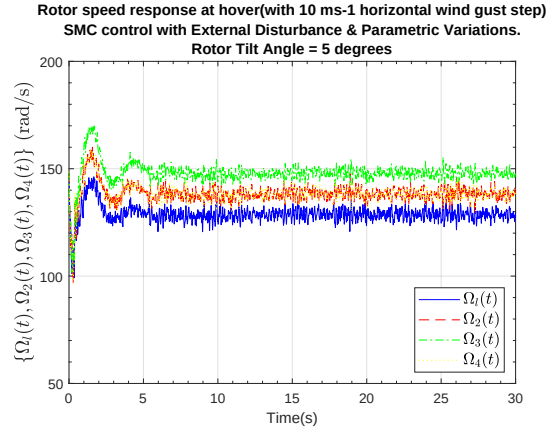


Figure G.28: Rotor speed response at hover (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 5^\circ$

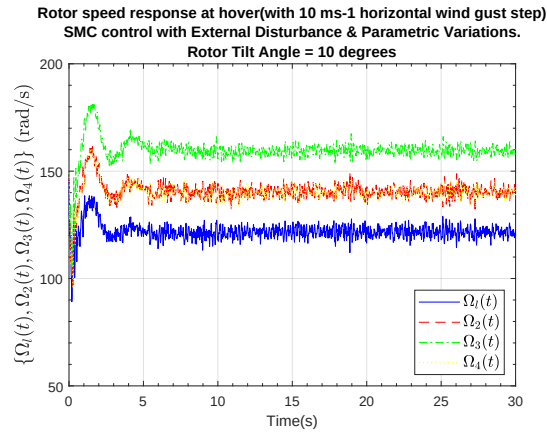


Figure G.29: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

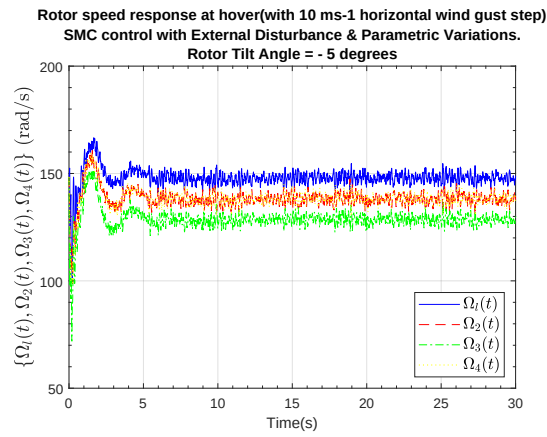
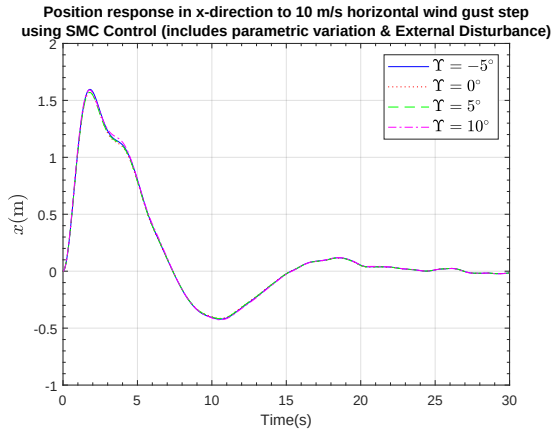
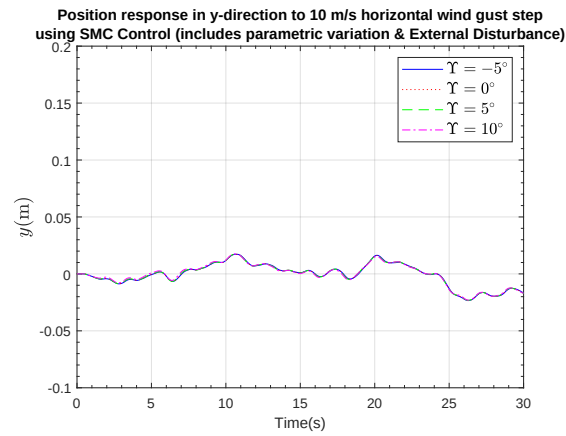
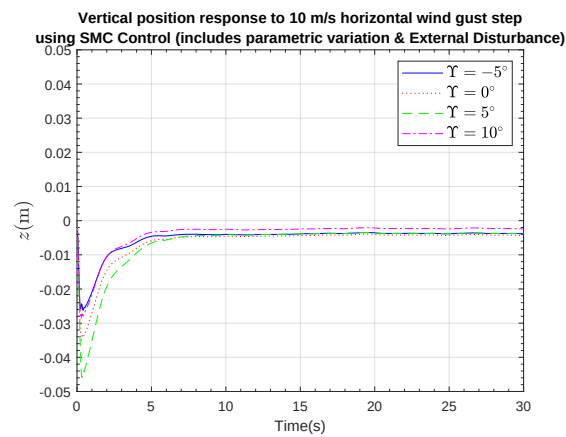
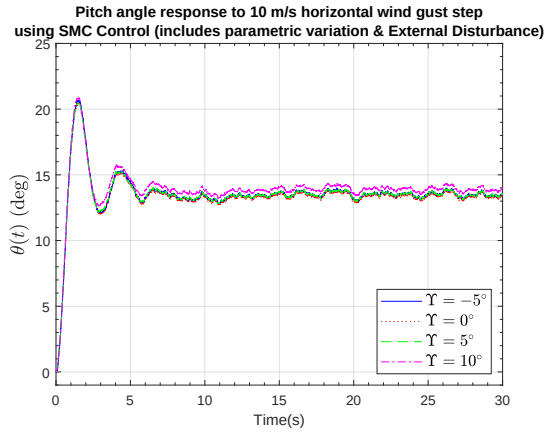
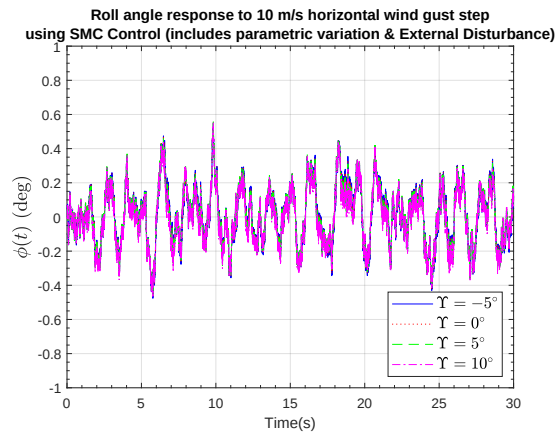
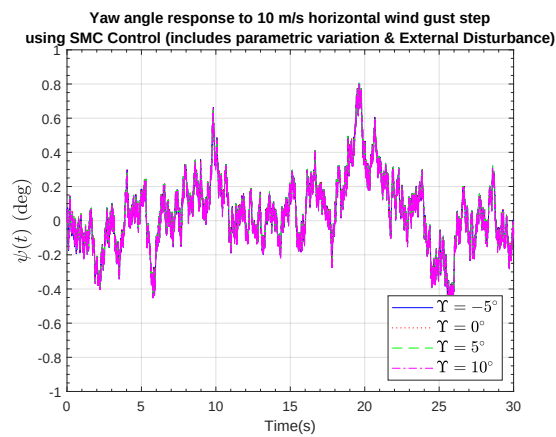


Figure G.30: Rotor speed response to 10 m position step demand both in x and y direction and 20 m altitude demand (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = -5^\circ$

Figure G.31: Position response in x-direction at hover with 10 ms^{-1} horizontal wind gust step)Figure G.32: Position response in x-direction at hover with 10 ms^{-1} horizontal wind gust stepFigure G.33: Vertical response at hover with 10 ms^{-1} horizontal wind gust step

Figure G.34: Pitch angle response at hover with 10 ms^{-1} horizontal wind gust stepFigure G.35: Roll angle response at hover with 10 ms^{-1} horizontal wind gust stepFigure G.36: Yaw angle response with 10 ms^{-1} horizontal wind gust step

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APPENDIX H

Additional Simulation Results - Part 4

H.0.1 SMC Control of Altitude, Attitude and Position Control of Quadrotor for altitude and position demands with 10 ms^{-1} Wind Gust Step without parametric uncertainty or noise for $\Upsilon = 10^\circ$ angle

Figures H.1, H.2 and H.3 show the simulation results for the position, attitude and rotor speed responses for altitude and position demands with a 10 ms^{-1} wind gust step in the x-direction for $\Upsilon = 10^\circ$ rotor tilt angle in the absence of parametric uncertainty and external disturbance.

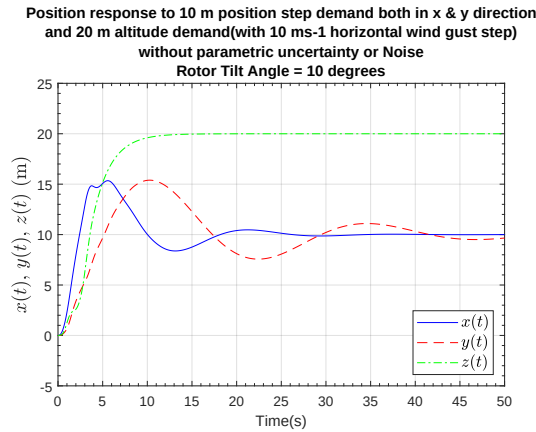


Figure H.1: Position response for altitude and position demands (with 10 ms^{-1} horizontal wind gust step). $\Upsilon = 10^\circ$

H.0.2 SMC Control of Altitude, Attitude and Position Control of Quadrotor for altitude and position demands with 10 ms^{-1} Wind Gust Step with Parametric Uncertainty only and no Noise for $\Upsilon = 10^\circ$ angle

Figures H.4, H.5 and H.6 show the simulation results for the position, attitude and rotor speed responses for altitude and position demands with a 10 ms^{-1} wind gust step in the x-direction for $\Upsilon = 10^\circ$ rotor tilt angle in the presence of parametric uncertainty but no noise.

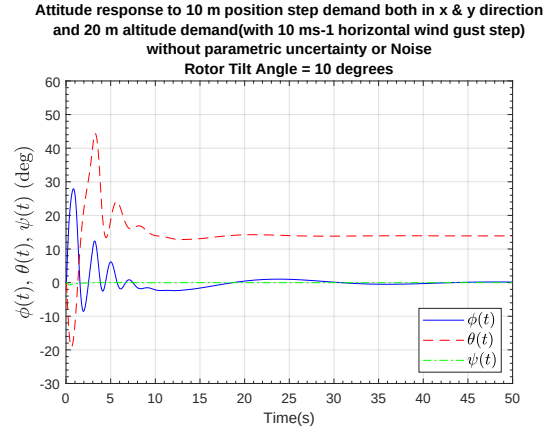


Figure H.2: Attitude response for altitude and position demands (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

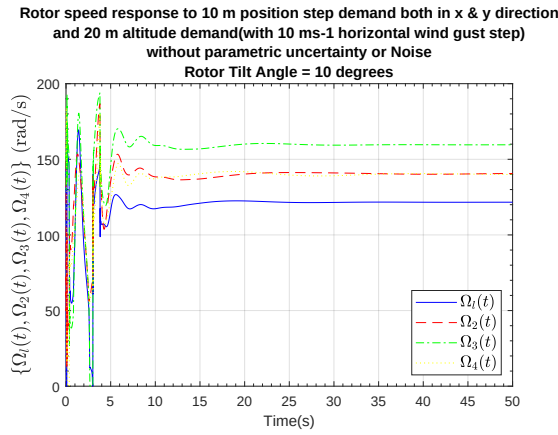


Figure H.3: Rotor speed response for altitude and position demands (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

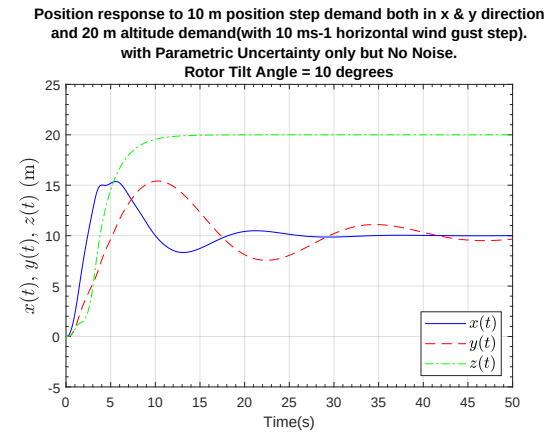


Figure H.4: Position response for altitude and position demands (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

H.0.3 SMC Control of Altitude, Attitude and Position Control of Quadrotor for altitude and position demands with 10 ms⁻¹ Wind Gust Step with Noise only and without Parametric Uncertainty for $\Upsilon = 10^\circ$ angle

Figures H.7, H.8 and H.9 show the simulation results for the position, attitude and rotor speed responses for altitude and position demands with a 10 ms⁻¹ wind gust step in the x-direction for $\Upsilon = 10^\circ$ rotor tilt

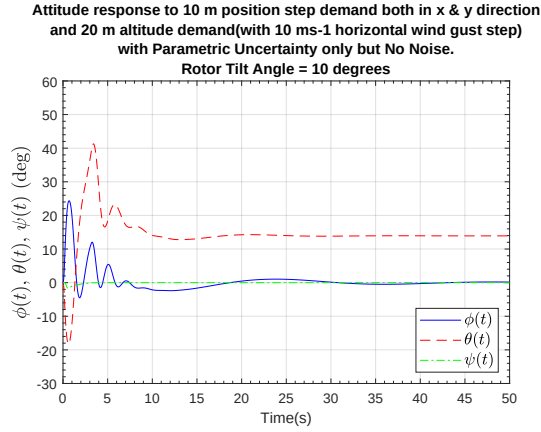


Figure H.5: Attitude response for altitude and position demands (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

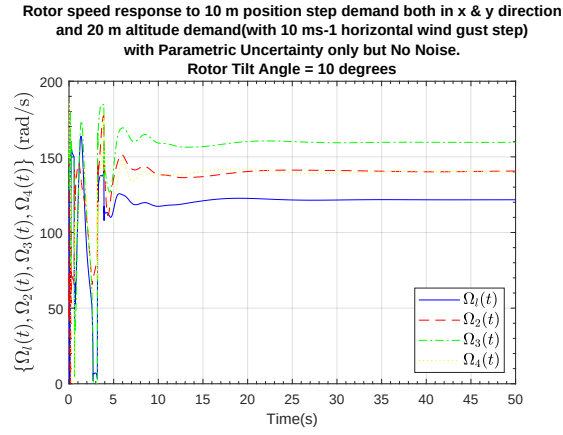


Figure H.6: Rotor speed response for altitude and position demands (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

angle in the presence of noise only but without parametric uncertainty.

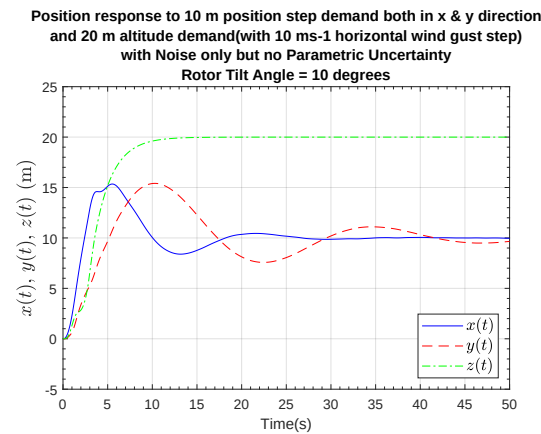


Figure H.7: Position response for altitude and position demands (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

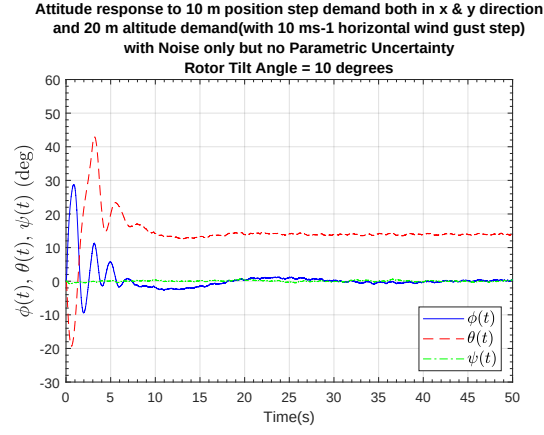


Figure H.8: Attitude response for altitude and position demands (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

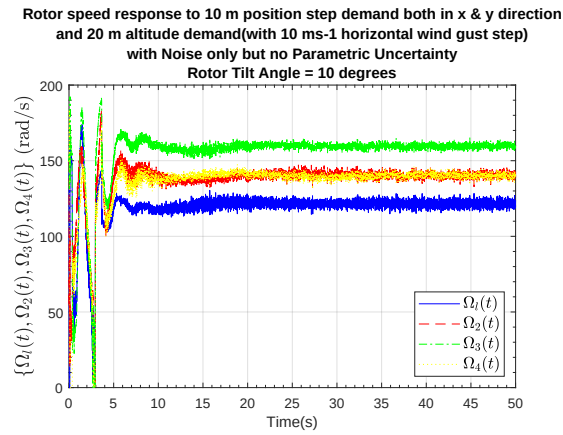


Figure H.9: Rotor speed response for altitude and position demands (with 10 ms⁻¹ horizontal wind gust step). $\Upsilon = 10^\circ$

Robust Sliding Mode Control of a Quadrotor

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Abstract—A sliding mode controller is proposed for the robust control of both the position and attitude of a quadrotor. The design is tested in simulation. The performance is compared with a proportional plus derivative controller. The robustness of the design is also tested for parametric uncertainty. The effect of disturbances is also investigated. It is found that the sliding mode controller provides good performance, robustness and disturbance rejection.

Index Terms—Sliding Mode Control; Robustness; Quadrotors; Nonlinear Control; VTOL

I. INTRODUCTION

Sliding Mode Control (SMC) is a particular form of Variable Structure Control (VSC) whereby the control signal varies discontinuously so as to force the system onto a particular manifold of the state space (including the origin) known as a sliding surface. Once on the surface, the control switches rapidly to both maintain the state on the surface and drive the state towards the origin.

As the name variable structure control suggests, the structure of the control is altered along the switching surfaces in state space with the result that certain desirable properties are achieved. Once the system state reaches the switching plane, the system response thereafter depends only on the gradients of the switching planes and remains insensitive to a class of disturbances and parameter variations. It has an attractive feature to keep the system insensitive to the uncertainties on the sliding surface.

VSC with sliding mode was first proposed in the early 1950s by Soviet researchers [1, p. 1]. In their work, the plant considered was a linear second-order system modelled in canonical form. Later on, VSC has developed into a general design method being examined for a wide spectrum of system types including nonlinear systems, MIMO systems, discrete-time models, large-scale and infinite-dimensional systems and stochastic systems. A comprehensive survey on VSC is presented by Hung et al. [2].

The control of a quadrotor is a challenging problem taking into account different practical issues such as parametric uncertainties, external disturbances, motor failures etc., while maintaining flight path tracking performance and stability over a wide range of operating conditions. SMC has been chosen as it is known for its robustness against unmodelled dynamics, parametric uncertainties and external disturbances [2].

There has been lot of interest in using sliding mode control for control of quadrotors and other VTOL vehicles. For example, sliding mode control for the altitude control a miniature radio control helicopter was presented by Sira-Ramirez et al. [3]. Xu and Ozguner [4] presented a new design method for the flight control of an autonomous quadrotor helicopter based on sliding mode control. A sliding mode control is proposed to stabilize a class of cascaded underactuated systems and is able to robustly stabilize the quadrotor helicopter and move it to any position with any yaw angle. Runcharoon and Srichatrapimuk [5] presented a controller based on sliding mode control for an autonomous quadrotor. The controller was split into two parts - the sliding mode controller was applied to the attitude of the system and a Proportional-Derivative (PD) type controller was applied to the altitude and horizontal position control. Alwi and Edwards [6] presented two fault tolerant control schemes based on a linear parameter varying system representation and utilizing a combination of sliding mode ideas and control allocation for an octorotor UAV in the presence of uncertainties and faults/failures.

This paper proposes SMC for position and attitude control of the quadrotor without the need of any observer in the presence of parametric uncertainty and external disturbances. A Proportional plus Derivative (PD) controller has also been designed and simulated as a baseline for comparison with SMC. Note that the propeller induced gyroscopic effects have also been considered in the model to improve the accuracy.

This paper is organised as follows. Section II develops the quadrotor model that is used for the design and subsequent simulation, Section III describes the design of the SMC for both attitude and altitude control of the quadrotor, Section IV deals with the design of the PD controller, Section V presents some simulation results of both PD and SMC controllers in the presence of parametric uncertainty and external disturbance, and Section VI provides the conclusion of the paper.

II. QUADROTOR MODEL

The general planform of a quadrotor has four propellers in a cross configuration driven by electric motors. An example vehicle is shown in Figure 1. The pair of rotors on the same axis rotate in the same direction, but one of the pairs spins clockwise while the other counter-clockwise. By varying the rotation speeds, moments can be generated about each axis. Moments about the horizontal axes are produced by varying



Fig. 1. Draganfly X-pro quadrotor (developed by Draganfly Innovations Inc)

the difference in the speeds of an axis pair, whilst the moment about the vertical axis is obtained from the difference in the drag torques between the clockwise and counter-clockwise rotating motors.

A lot of work has been done on the mathematical modelling of a quadrotor and the equations of motion are well established [7]–[13]. The dynamic model used in this work includes the gyroscopic effects resulting from both the rigid body rotation in space and the thrust generated by each propeller. It is derived using Euler-Langrange formalism and follows the development in Sharifi et al. [12] with the following assumptions:

- the structure is rigid and symmetric,
- the propellers are rigid,
- the rotor thrust and the drag are proportional to the square of the speed of the rotor,
- the rotor axes are parallel and lie in the z direction,
- ground effect is neglected,
- the inertia matrix is diagonal,
- the wind forces are not included,
- and the motor dynamics are ignored.

The axes system and forces are shown in Figure 2. The center of gravity is assumed to be at the origin of the body axis reference frame. The orientation of the quadrotor is given by (ϕ, θ, ψ) measured with respect to the Earth coordinate frame E.

The rotational and vertical accelerations are given by

$$\ddot{\phi} = \dot{\theta}\dot{\psi} \left(\frac{I_y - I_z}{I_x} \right) - \frac{J}{I_x} \dot{\theta}\omega + \frac{lU_1}{I_x} \quad (1)$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi} \left(\frac{I_z - I_x}{I_y} \right) + \frac{J}{I_y} \dot{\phi}\omega + \frac{lU_2}{I_y} \quad (2)$$

$$\ddot{\psi} = \dot{\phi}\dot{\theta} \left(\frac{I_x - I_y}{I_z} \right) + \frac{U_3}{I_z} \quad (3)$$

$$\ddot{Z} = -g + \cos \phi \cos \theta \frac{1}{m} U_4 \quad (4)$$

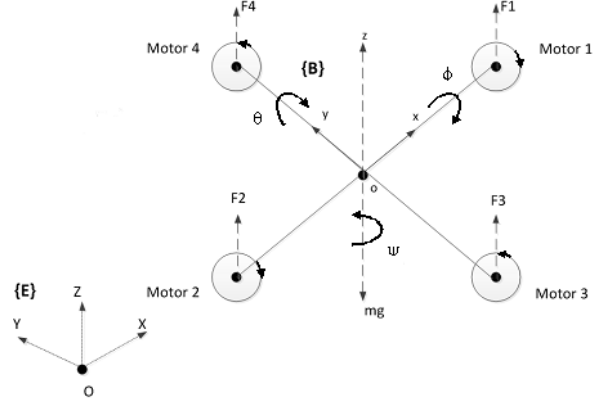


Fig. 2. Schematic model of a quadrotor

where g is gravitational field constant, l is the arm length, m is the mass, I_x, I_y, I_z are the moments of inertia about their respective axes, J is the rotor inertia, and $\omega = \omega_4 + \omega_3 - \omega_2 - \omega_1$ where $\omega_i, i = 1, \dots, 4$ is the speed of rotor, i .

The drag coefficients are negligible at low speeds, so they have been ignored. The system inputs U_1, U_2, U_3 and U_4 are defined as:

$$U_1 = b(\omega_4^2 - \omega_3^2) = F_4 - F_3 \quad (5)$$

$$U_2 = b(\omega_2^2 - \omega_1^2) = F_2 - F_1 \quad (6)$$

$$U_3 = d(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2) \quad (7)$$

$$U_4 = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) = \sum_{i=1}^4 F_i \quad (8)$$

where b and d are the thrust and drag factors respectively and the thrust force generated by rotor i is $F_i = b\omega_i^2$.

From (5) to (8), it can be shown that:

$$\omega_1^2 = -\frac{U_2}{2b} + \frac{U_3}{4d} + \frac{U_4}{4b} \quad (9)$$

$$\omega_2^2 = \frac{U_2}{2b} + \frac{U_3}{4d} + \frac{U_4}{4b} \quad (10)$$

$$\omega_3^2 = -\frac{U_1}{2b} - \frac{U_3}{4d} + \frac{U_4}{4b} \quad (11)$$

$$\omega_4^2 = \frac{U_1}{2b} - \frac{U_3}{4d} + \frac{U_4}{4b} \quad (12)$$

Finally, the model can be represented in the generalized

TABLE I
VEHICLE PARAMETERS AND VARIABLES

Symbol	Definition	Value	Unit
l	arm length	0.25	m
m	mass	1.56	kg
I_x	inertia on x axis	0.04	kg.m ²
I_y	inertia on y axis	0.04	kg.m ²
I_z	inertia on z axis	0.08	kg.m ²
b	thrust coefficient	20×10^{-6}	N.s ²
d	drag coefficient	0.4×10^{-6}	N.m.s ²
J	rotor inertia	90×10^{-6}	kg.m ²
τ	actuator lag time constant	0.02	sec
ϕ	roll angle		rad
θ	pitch angle		rad
ψ	yaw angle		rad
ω_i	speed of rotor number i		rad.s ⁻¹

state space form, $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$, as:

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ (C_{x_7} S_{x_9} C_{x_{11}} + S_{x_7} S_{x_{11}}) . U_4 / m \\ x_4 \\ (C_{x_7} S_{x_9} S_{x_{11}} - S_{x_7} C_{x_{11}}) . U_4 / m \\ x_6 \\ -g + (C_{x_7} C_{x_9}) . U_4 / m \\ x_8 \\ (x_{12} x_{10} (I_y - I_z) - J x_{10} \omega + l U_1) / I_x \\ x_{10} \\ (x_{12} x_8 (I_z - I_x) - J x_8 \omega + l U_2) / I_y \\ x_{12} \\ (x_{10} x_8 (I_x - I_y) + U_3) / I_z \end{bmatrix} \quad (13)$$

where $\mathbf{u}^T = (U_1, U_2, U_3, U_4)$, $\mathbf{u} \in \mathbb{R}^4$ is the vector of input variables, $\mathbf{x}^T = (X, \dot{X}, Y, \dot{Y}, Z, \dot{Z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi})$, $\mathbf{x} \in \mathbb{R}^{12}$ is the vector of state variables and $S(\cdot)$ and $C(\cdot)$ represent $\sin(\cdot)$ and $\cos(\cdot)$ respectively [12].

The parameters and variables for the quadrotor are as listed in Table I and are taken from [15]. The system has four control inputs, U_1, U_2, U_3 and U_4 and six controlled outputs, X, Y, Z, ϕ, θ and ψ . U_1 is used to control the roll angle, U_2 controls the pitch angle, U_3 controls the yaw angle and U_4 determines the altitude of the vehicle. The values of U_i are determined from the rotor speed ω_i with an assumption of known thrust and drag coefficients b and d . The motor dynamics are ignored and it is assumed that the rotor speeds can be directly manipulated.

III. SLIDING MODE CONTROL

A sliding mode controller has also been designed to control the horizontal position, altitude and the rotational angles of the quadrotor.

Consider the roll angle dynamic equation:

$$\ddot{\phi} = \dot{\theta} \dot{\psi} \left(\frac{I_y - I_z}{I_x} \right) - \frac{J}{I_x} \dot{\theta} \omega + \frac{l U_1}{I_x} \quad (14)$$

where U_1 is the control input and ϕ is the output of interest. There are two steps to be followed in the design of sliding mode control. Firstly, for each output, ϕ, θ, ψ, Z , define a time-varying sliding surface, $S(t)$, in the state-space $\mathbb{R}^n$ by the scalar equation $s(\alpha, t) = 0$ where $\alpha = \phi, \theta, \psi$ or

Z . Secondly, select the feedback control law such that s^2 remains a Lyapunov-like function of the closed-loop system, despite the presence of model imprecision and disturbances and satisfying the necessary sliding condition, $s\dot{s} < 0$ [14].

The sliding surface equations are

$$s_\alpha = \dot{e}_\alpha + \lambda_\alpha e_\alpha \quad (15)$$

where where $\alpha = \phi, \theta, \psi$ or Z , e_α is the relevant tracking error and λ_α is a constant.

The Lyapunov functions can be assumed to be

$$V(s_\alpha) = \frac{1}{2} s_\alpha^2 \quad (16)$$

By choosing Lyapunov function candidate as $V = \frac{1}{2} s_\alpha^T s_\alpha$, its time derivative is negative definite, that is $\dot{V} = s_\alpha \dot{s}_\alpha < -\eta |s_\alpha|$ and

$$\dot{s}_\alpha = -f - u + \ddot{d} + \lambda_\alpha \dot{e}_\alpha \quad (17)$$

The control inputs are

$$U_1 = \frac{I_x}{l} \left[-\dot{\theta} \dot{\psi} \left(\frac{I_y - I_z}{I_x} \right) + \frac{J \dot{\theta} \omega}{I_x} + \ddot{\phi}_d + \lambda_\phi \dot{e}_\phi + k_\phi \text{sgn}(s_\phi) \right] \quad (18)$$

$$U_2 = \frac{I_y}{l} \left[-\dot{\phi} \dot{\psi} \left(\frac{I_z - I_x}{I_y} \right) - \frac{J \dot{\phi} \omega}{I_y} + \ddot{\theta}_d + \lambda_\theta \dot{e}_\theta + k_\theta \text{sgn}(s_\theta) \right] \quad (19)$$

$$U_3 = I_z \left[-\dot{\phi} \dot{\theta} \left(\frac{I_x - I_y}{I_z} \right) + \ddot{\psi}_d + \lambda_\psi \dot{e}_\psi + k_\psi \text{sgn}(s_\psi) \right] \quad (20)$$

$$U_4 = \frac{m}{\cos \phi \cos \psi} \left[g + \ddot{Z}_d + \lambda_Z \dot{e}_z + k_z \text{sgn}(s_z) \right] \quad (21)$$

where $\lambda_\phi, \lambda_\theta, \lambda_\psi, \lambda_Z$ and $k_\phi, k_\theta, k_\psi, k_Z$ are positive constant values and the signum function is defined as:

$$\text{sgn}(s) = \begin{cases} +1 & \text{if } s > 0 \\ 0 & \text{if } s = 0 \\ -1 & \text{if } s < 0 \end{cases} \quad (22)$$

IV. PD CONTROL

A PD controller was also designed to control the quadrotor's attitude and altitude. The design is based on those in previous work [8], [15], [16]. The controller is given by

$$U_1 = (\phi_d - \phi) k_{p\phi} + (\dot{\phi}_d - \dot{\phi}) k_{d\phi} \quad (23)$$

$$U_2 = (\phi_d - \theta) k_{p\theta} + (\dot{\phi}_d - \dot{\theta}) k_{d\theta} \quad (24)$$

$$U_3 = (\psi_d - \psi) k_{p\psi} + (\dot{\psi}_d - \dot{\psi}) k_{d\psi} \quad (25)$$

$$U_4 = m \frac{(Z_d - Z) k_{pz} + (\dot{Z}_d - \dot{Z}) k_{dz} + g}{\cos \phi \cos \theta} \quad (26)$$

The controller gains are from the work by [15] and are given by

$$\begin{aligned} k_{p\phi} &= 6.2, & k_{d\phi} &= 7.0, & k_{p\theta} &= 6.2, & k_{d\theta} &= 7.0, \\ k_{p\psi} &= 0.5, & k_{d\psi} &= 0.4, & k_{pz} &= 2.3, & k_{dz} &= 4.6. \end{aligned}$$

V. SIMULATION RESULTS

Simulations of the dynamic model (13) with both the SMC controller and the PD controller were performed using MATLAB/Simulink. In order to test the robustness to unmodelled uncertainty in the actuators, first-order lag dynamics were included for the rotor actuators. The time constant is given in Table I. The first test compares the step responses of the PD controller with the SMC controller for the nominal vehicle system for an initial step perturbation and a demand in the altitude. The second simulation tests the performance of the SMC controller for parametric uncertainty. The final simulation tests simultaneous parametric uncertainty and external disturbances.

The dynamic model of the quadrotor represented in (1) to (4), can be expressed as [17]:

$$\ddot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) + \mathbf{g}(\mathbf{y})\mathbf{u} \quad (27)$$

where $\mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) \in \mathbb{R}^4$ denotes an unknown, nonlinear vector function, $\mathbf{g}(\mathbf{y}) \in \mathbb{R}^{4 \times 4}$ is an uncertain input matrix, $\mathbf{u}(t) \in \mathbb{R}^4$ is a vector of control inputs and $\mathbf{y}(t) \in \mathbb{R}^4$ denotes the directly-actuated degrees-of-freedom state variables, $[\phi, \theta, \psi, Z]^T$, which is the vector of the state variables (x_7, x_9, x_{11}, x_5) .

For all the tests, the initial conditions are taken as:

$$\mathbf{y}(0) = [0.26 \quad 0.26 \quad 0.26 \quad 0]^T \quad (28)$$

and zero for the remaining state variables. The desired output vector is:

$$\mathbf{y}_d = [0 \quad 0 \quad 0 \quad 20]^T \quad (29)$$

The attitude demand is zero, that is hover. In other words, the quadrotor started with all 3 axes displaced by the same amount of 0.26 radians (15°) whilst an altitude increase of 20m was also demanded. Lateral position displacements on the x -axis and y -axis of 10m are also demanded.

A. Comparison of PD and SMC controller performance

For this test, the response to the initial condition displacement (28) and altitude demand (29) was tested without any uncertainties or external disturbances.

Figures 3 and 4 show the comparison of performance of the quadrotor with the PD and SMC controllers.

Figure 3 shows that the quadrotor with PD control returns to its desired attitude values in around 6 seconds, its desired position on x -axis and y -axis in about 5 seconds and the target altitude is achieved in about 7 seconds. With SMC Controller, the quadrotor reaches its designated attitude values in about 3 seconds, desired position on x -axis and y -axis in about 3 seconds and the target altitude is achieved in about 7 seconds.

Figure 4 shows the control efforts of the PD and SMC controller. For the moment controls, initially the control effort

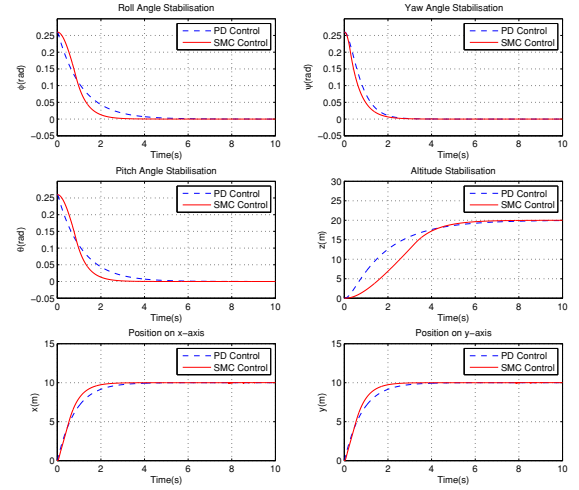


Fig. 3. Comparative simulation results with PD & SMC controllers

values are negative to correct for the attitude errors, followed by a positive demand to correct the subsequent lateral displacement errors.

The attitude response for the SMC controller is much faster than the PD response, but uses about the same maximum control effort. The initial control effort for the collective thrust, U_4 , is much higher for the PD controller compared to the SMC controller. This is why the initial altitude response is faster for the PD controller. The maximum value of U_4 is more than 30 N for the PD controller, in practice this is likely to saturate the rotor actuator. The maximum value of U_4 for the SMC controller is about 26 N which is more reasonable.

A small amount of chatter can be observed in the control effort response for the SMC in Figure 4. This is of little concern for this application, because the rotor speed controllers use MOSFET switches along with Pulse Width Modulation (PWM) with frequencies above the bandwidth of the motors. In practice, the chatter will be at the PWM switching frequency.

B. SMC with parametric uncertainty

For this test, the response to the initial condition displacement (28) and altitude demand (29) was tested with the model including parametric uncertainties. From (1) to (4), functions $\mathbf{f}(\mathbf{y}, \dot{\mathbf{y}})$ and $\mathbf{g}(\mathbf{y})$ can be defined as:

$$\mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) = \begin{bmatrix} \left(\dot{\theta}\dot{\psi}(I_y - I_z) - J\dot{\theta}\omega \right) / I_x \\ \left(\dot{\phi}\dot{\psi}(I_z - I_x) + J\dot{\phi}\omega \right) / I_y \\ \left(\dot{\phi}\dot{\theta}(I_x - I_y) \right) / I_y \\ -g \end{bmatrix} \quad (30)$$

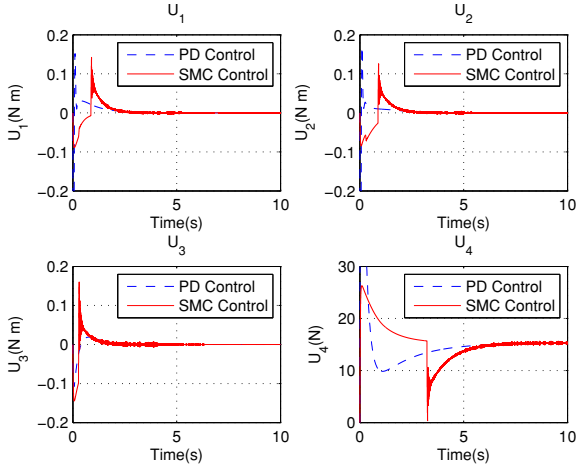


Fig. 4. Comparison of the PD & SMC control efforts

and

$$\mathbf{g}(\mathbf{y}) = \begin{bmatrix} l/I_x & 0 & 0 & 0 \\ 0 & l/I_y & 0 & 0 \\ 0 & 0 & l/I_z & 0 \\ 0 & 0 & 0 & \cos \theta \cos \phi / m \end{bmatrix} \quad (31)$$

Let p_1, p_2, p_3 and p_4 denote the uncertainty in the inertia, arm length and mass parameters, I_x, I_y, I_z, l and m as expressed by (31). The matrix $\mathbf{g}(\mathbf{y})$ can be written as:

$$\mathbf{g}(\mathbf{y}) = \mathbf{g}_0(\mathbf{y})\Delta\mathbf{g} \quad (32)$$

where $\mathbf{g}_0(\mathbf{y}) \in \mathbb{R}^{4 \times 4}$ is the known nominal matrix and $\Delta\mathbf{g} \in \mathbb{R}^{4 \times 4}$ is the uncertain constant matrix. The uncertain constant matrix, $\Delta\mathbf{g}$ can be defined as:

$$\Delta\mathbf{g} = \begin{bmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 \\ 0 & 0 & p_3 & 0 \\ 0 & 0 & 0 & p_4 \end{bmatrix} \quad (33)$$

For the simulations, the uncertain elements of $\Delta\mathbf{g}$ are selected as $p_1 = 0.5, p_2 = 0.7, p_3 = 0.6$ and $p_4 = 0.8$ which correspond to 20 – 50% parametric uncertainty in the inertia, arm length and mass parameters.

Figures 5 and 6 illustrate the performance of the quadrotor with the SMC controller in the presence of the parametric uncertainties given above. The initial conditions and altitude demands are as for the previous simulation in Section V-A. With the uncertainty, the SMC is still able to control the quadrotor for all three axes and quickly stabilize the attitude of the vehicle in 3 seconds, reach the target altitude of 20 metres in 7 seconds and reach the target position on x -axis and y -axis in 5 seconds. Note the greater amount of control effort required, this is because the lower values of the $l/I_{(\cdot)}$ uncertainty terms mean that greater moment forces are required to rotate the vehicle. Similarly, the increased mass means that great control effort in U_4 is required for the vehicle to achieve and hold the desired altitude.

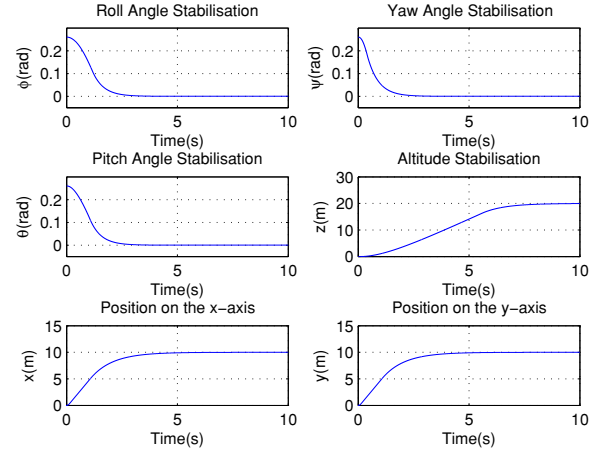


Fig. 5. Simulation results for sliding model control in presence of parametric uncertainty

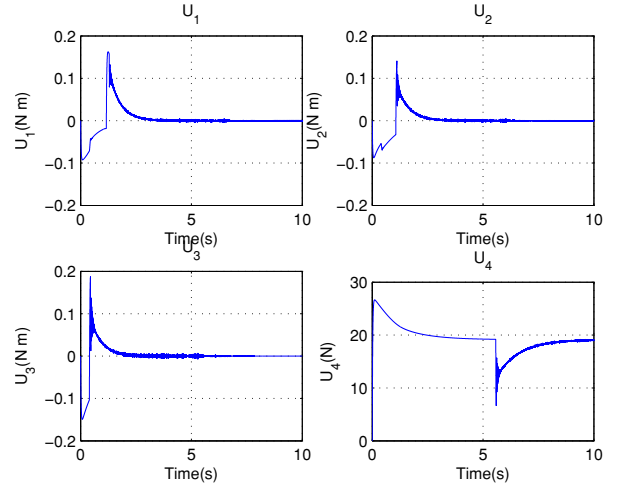


Fig. 6. Sliding mode control efforts with parametric uncertainty

C. SMC with parametric uncertainty and external disturbance

For this simulation test, the response to the initial condition displacement (28) and altitude demand (29) was tested with the model including parametric uncertainties and an external disturbances. The model (27) is augmented to [17]

$$\ddot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) + \mathbf{g}(\mathbf{y})\mathbf{u} + \mathbf{d}(t) \quad (34)$$

where $\mathbf{d}(t) \in \mathbb{R}^n$ is an unknown external disturbance vector. For this simulation the external disturbance, $\mathbf{d}(t)$, is taken as normal gaussian noise:

$$\mathbf{d}(t) = [\mathcal{N}(0, 0.5) \quad \mathcal{N}(0, 0.5) \quad \mathcal{N}(0, 0.5) \quad \mathcal{N}(0, 0.5)]^T \quad (35)$$

Figures 7 and 8 show the performance of the quadrotor with the SMC controller with parametric uncertainties in inertias and mass and external disturbance in the form of normal

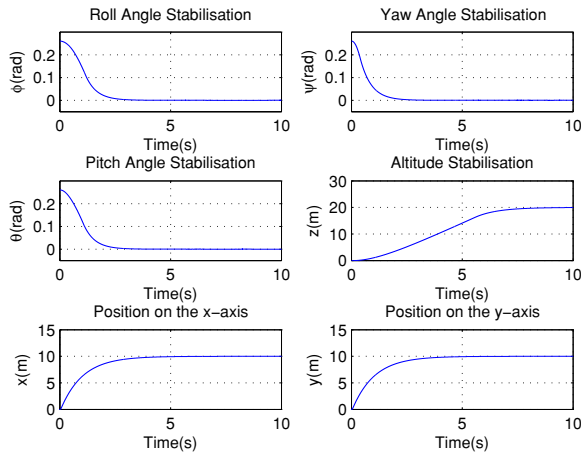


Fig. 7. Simulation results for sliding mode control in presence of parametric uncertainty and external disturbance

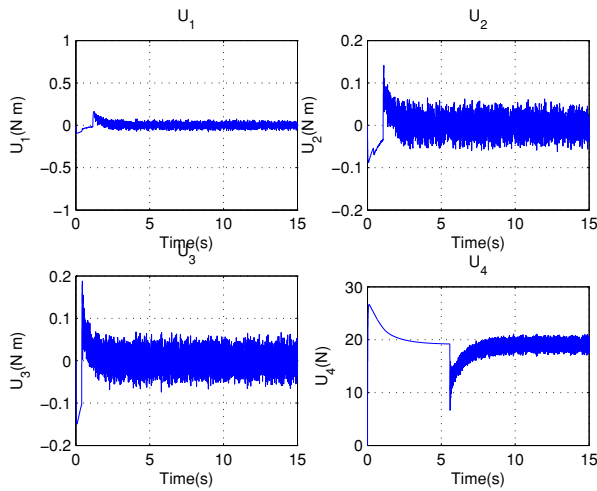


Fig. 8. Sliding mode control efforts in presence of parametric uncertainty and external disturbance

Gaussian noise. The noise disturbance is not visible in the outputs as seen in Figure 7. However, the controls have some sensitivity to the noise after 3 seconds. This is evident in the larger amount of chatter that can be observed.

VI. CONCLUSION

A robust sliding mode controller for a quadrotor in the presence of parametric uncertainties and external disturbances has been presented. A comparison of the simulation results of with a PD controllers is presented. The simulation results for both PD controller and SMC controller demonstrate that the quadrotor is able to stabilise itself at the desired position and altitude using both controllers. The overall performance of the controllers is seen to be satisfactory with the vehicle returning to a hover condition in a reasonable amount of time. The

SMC controller performs well in the presence of parametric uncertainties and external disturbance.

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