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Advanced Computational Modelling and
Simulation of Transition to Turbulence in Separated
Suddenly-Expanded Channel Flows

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Abstract

The main scope of this PhD thesis is the analysis of unsteady laminar and transitional suddenly expanded flows. For this reason Implicit Large Eddy Simulation (ILES) approach was used in combination with high order, high resolution numerical methods. The numerical methods examined are a 2nd order Monotonic Upwind Scheme for Scalar Conservation Laws (MUSCL) with Van Leer limiter, a high order (3rd) interpolation and a 5th order Weighted Essentially Non-Oscillatory scheme (WENO).

First the numerical data for three low (steady state) Reynolds numbers and for two unsteady (in the form of primary frequencies) were compared to the experimental data and were found in good agreement. A grid convergence study was undertaken for two Reynolds numbers demonstrating grid convergence and justifying the selection of the grid. The three numerical methods were evaluated for two Reynolds numbers showing good agreement for Reynolds number 412 and discrepancies at Reynolds number 900 between MUSCL and WENO with the MUSCL demonstrating a very dissipative behavior.

The physical behavior of the flow in a wide range of Reynolds numbers were examined. For this range the flow behavior changed from steady to unsteady laminar and finally exhibiting localized transition to turbulence. The behavior of the main recirculation areas was described and the vortex shedding that occur there and how this change with the Reynolds number. The flow was observed to change from an unsteady quasi three dimensional flow at Reynolds number 412 to an increased transitional state with three dimensional vortical structures at Reynolds number 550.

Kinetic energy spectra were calculated for the aforementioned range of Reynolds numbers. The primary frequencies are increasing with Reynolds number as expected. The slopes that were calculated for the inertial subrange revealed a trend. As the Reynolds number is increasing the slopes are decreasing approaching the value given by Kolmogorov $-5/3$.

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Dedicated to my parents Georgios and Maria

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Nomenclature

$\langle x \rangle$ Time average value of variable x

AR Aspect Ratio

CBS Characteristic Based Scheme

CFD Computational Fluid Dynamics

CFL Courant-Friedrichs-Lewy

DNS Direct Numerical Simulation

ENO Essentially Non Oscillatory

ER Expansion Ratio

H3 Third Order Interpolation

ILES Implicit Large Eddy Simulation

LES Large-Eddy Simulation

M2 Second Order MUSCL

MUSCL Monotone Upstream-centered Schemes for Conservation Laws

RANS Reynolds-Averaged Numerical Simulation

Re Reynolds Number

rms Root Mean Square

SGS Sub-Grid Scale

TVD Total Variation Diminishing

W5 Fifth Order WENO

WENO Weighted Essentially Non-Oscillatory

Chapter 1

Introduction

1.1 States of Flow

The behavior of the fluid can be characterized as laminar, transitional or turbulent. The main tool to recognize those states is the Reynolds number. When the Reynolds number is sufficient low the flow becomes laminar which is described by a smooth steady flow. As the Reynolds number increases the flow start the transition to the turbulence and the flow is characterized by a increasing unsteadiness and randomness and given a high enough Reynolds number become turbulent.

The flow in order to change behavior from laminar to transitional to turbulent undergoes, depending to the specific geometry and conditions, a series of bifurcations. Bifurcation, in stability analysis, is the change in the number, or in the qualitative character, of the set of possible steady flows (or unsteady flows in dynamic equilibrium) as Reynolds number varies [2]. The values of the Reynolds numbers where a bifurcation of the flow occurs are called a bifurcation points. A common way to present the bifurcations for a flow is through the bifurcation diagram with the Reynolds number in one axis and a flow variable in the other like velocity. There are several types of bifurcation depending on the specific flow. Some of them are the saddle-node, transcritical, supercritical, Hopf, and pitchfork bifurcation. A specific flow can undergo different types of bifurcations as the Reynolds number varies.

As the Reynolds number is increasing the flow undergoes bifurcation and it becomes unsteady, transitional and in the end turbulent. Since the bifurcation and the transition are depended to the specific characteristics of the flow examined each flow has a different way to reach transition and turbulence. Nevertheless, the different routes to transition and turbulence can be split into some general categories.

- Subcritical instability. On this route a stable flow, steady, periodic or quasi-periodic, becomes unstable as Reynolds number increases slowly through a

critical value, and the flow then 'jumps' rapidly to a turbulent one which is not a continuous extension of the stable flow for smaller values of Reynolds numbers. Examples of this behavior are the Poiseuille flow in a pipe, plane Poiseuille flow, plane Couette flow and other channel flows. This is also called bypass transition [2].

- Ruelle-Takens-Newhouse route. This route was first mapped by Ruelle & Takens [3] and later revised by Newhouse et al. [4] by use of the theory of dynamical systems. Along this route, there is a succession of bifurcations as Reynolds number increases in which a steady flow may directly, or via other steady flows, become time-periodic, then quasi-periodic with two and then perhaps three or even four frequencies, until a turbulent flow occurs. Examples of this kind of transition are the Couette flow between rotating cylinders and Rayleigh-Benard convection.
- Period doubling. Sometimes turbulence, or chaos rather, ensues after a sequence of period doubling bifurcations as Reynolds number increases. Here a sequence of time-periodic flows occur at bifurcations, the period of one flow being twice the period of the previous one. This has been observed in Rayleigh-Benard convection in a tall box by Libchaber & Maurer [5].
- Intermittent transition. On this route, first mapped mathematically by Pomeau & Manneville [6], a periodic flow becomes unstable as Reynolds number increases through a critical value, the stable periodic flow first becoming unstable to disturbances of smaller and smaller finite amplitude.

After the Reynolds number is high enough the transitional phase ends and turbulence occur. Turbulent flow exhibits complex features and numerous different scale structures. The transition from laminar to turbulent flow occurs when the momentum exchange by molecular transport cannot compensate the rate of the transport due to macroscopic fluctuations in the flow velocity. Turbulence does not depend to a specific fluid, but can occur in every fluid flow. The characteristics of the turbulence do not depend to a specific fluid or the molecular properties of it. Turbulent flow has some common characteristic regardless of the flow and those characteristics serve as criteria to distinguish laminar and turbulent flows.

The most prominent characteristic of turbulence is randomness. Although, all turbulent flows share similar attributes also have unique characteristics which are depended to the boundary and initial conditions of the flow. This can be observed if we run an experiment twice with the same conditions and plot the velocities.

Then we can see that the two plots does not match exactly due to the randomness but at the same time the mean values of the velocity are the same. Due to that inherit randomness of the turbulence statistical methods are widely used when it is examined.

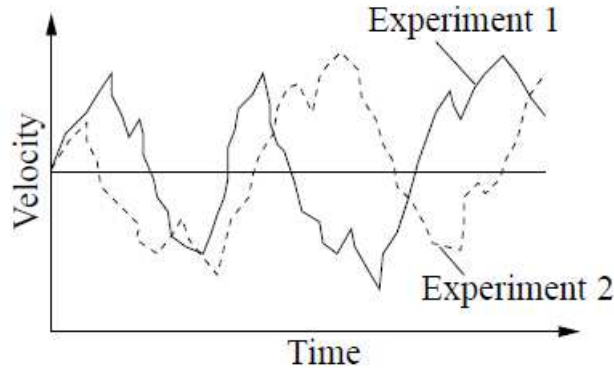


Figure 1.1: Two typical velocity histories obtained under identical experimental conditions Davinson [1]

Another aspect of the turbulence is the vorticity evolution and dissipation. So except of the random motion also coherent structures appear to the flow at random positions at space an time. Those structures are characterized by rotational movement and increased vorticity. The vortex dynamics is three dimensional phenomenon and the analysis of it is important to the investigation and prediction of turbulent flows. The dissipation occurs when those vortices brake down to smaller ones and this continues until the kinetic energy is dissipated into heat. Dissipation is a common characteristic of all turbulent flows and the transfer of energy between the largest and smaller scales can involve a large number of stages.

As discussed above, one of the characteristics of the turbulent flow is the coherent vortical structures and their behavior. The cascade of the eddies to smaller ones can be described by the Richardson's hypothesis [7]. This theory cannot be analytically proven but it has been confirmed from various experiments and simulations.

Kolmogorov managed to quantify the energy cascade and form the Kolmogorov hypothesis [8]. The assumptions on the physical behavior of the eddies in Kolmogorov's hypotheses enable calculation of the energy transfer rate and the scales in turbulent flows at high Reynolds numbers. Kolmogorov conclude that the anisotropy, of the larger scales due to the boundary conditions is diminishing as the results of

the transfer of energy to gradually smaller eddies. Thus, eventually the flow of the smallest scales becomes isotropic and those scales are not affected by the boundary conditions. As a result those scales are similar for all turbulent flows and can be described statistically. The statistics described the balance of energy between the inflow of energy from the larger scales and the heat dissipation due to viscous forces. Since the lifespan of the larger scales is large in comparison to the smaller ones the balance of energy flow can be achieved.

Therefore, the energy transfer rate is approximately equal to the heat dissipation rate φ and the molecular viscosity is used to the calculation of the viscous dissipation. Those two parameters are the main characteristics of the smallest scales of the flow, also known as the Kolmogorov scales. The Kolmogorov length, velocity and time-scales are defined as

$$\eta \sim (v^3/\epsilon)^{1/4}$$

$$u_\eta \sim (v\epsilon)^{1/4}$$

$$\tau_\eta \sim (v/\epsilon)^{1/2}$$

Using the viscous dissipation formula $\epsilon \sim u^3/l$ we can connect the Kolmogorov scales with the largest ones and we get

$$\eta/l \sim Re^{-3/4}$$

$$u_\eta/u \sim Re^{-1/4}$$

$$\tau_\eta/\tau \sim Re^{-1/2}$$

where is the flow Reynolds number. It is obvious that those ratios will almost always be less than 1 so the velocity and time scales of the smallest eddies will be smaller than those of the largest eddies, as stated before.

Since, the increasing of the Reynolds number will make the difference between the largest and the smallest scales becomes more prominent there will be a range of scales where will small compares to l and large compared to η . In this range the Reynolds number is sufficiently high that viscous effects can be neglected and the eddies will only transfer energy from the larger to the smaller scales. So, this region is only affected by one parameter the dissipation rate and there is no single length,

velocity or time scale for this range. Regardless that, we define a length scale for this region which is much smaller than the l and much larger than η . This length-scale, is known as the Taylor micro-scale and describes the mean spatial extent of the velocity gradients and is not related to any scales where dissipation takes an active part. Analyzing the energy balance between the energy transfer rate from the anisotropic, large scales and the viscous dissipation rate at the isotropic, small scales confirms that the Taylor micro-scale is an intermediate length-scale and it is given by

$$\lambda_\tau/l \sim Re^{-1/2}$$

$$\lambda_\tau/\eta \sim Re^{1/4}$$

Due to energy balance the energy flux has to remain constant through the range of intermediate length-scales because no energy is added by the boundary conditions and no energy is lost through viscous dissipation. The energy transfer rate for any given eddy-size is determined by the ratio of its characteristic velocity squared to its characteristic length, hence the velocity-scale, as well as the time-scale, decreases for progressively smaller length-scales.

In order to describe the distribution of kinetic energy over the scales observed in turbulent flows spectral space it is used. Fourier series are used to decompose the wavenumber k and the velocity field and the different length-scales are represented by their reciprocal value. The energy contained in a specific length-scale in Fourier space, $E(k)$, is the product of the corresponding velocity, $u(k)$, with its complex conjugate. For this reason, the spectral analysis gives a clear picture of the distribution of energy over the whole range of length-scales.

There are three categories that can be observed with the help of the kinetic energy spectrum. The energy containing range which includes the anisotropic large-scales (small wavenumbers) and is carrying most of the energy and it is responsible for the energy production process, the inertial subrange where the inertial forces are dominant and there is no production or dissipation of energy and the dissipation range where includes the Kolmogorov scales and the viscous dissipation mainly occurs. The inertial subrange and dissipation range contain the higher wavenumbers and are called universal equilibrium range.

In between the energy-containing and the dissipation range lies the inertial subrange. Here, in agreement with the concept of an energy cascade, the kinetic energy is transferred to progressively smaller scales. The extent of this range depends on the

flow as it becomes greater or smaller with increasing or decreasing flow Reynolds number, respectively. However, the single important parameter determining the statistics within the inertial subrange, the energy flux from large to small scales, is flow independent. Kolmogorov realized the consequences and used dimensional arguments to derive an analytical form for the energy spectrum in the inertial subrange given by

$$E(k) = C\epsilon^{2/3}k^{-5/3}$$

where C is a universal constant. This general power-law spectrum postulated by Kolmogorov has been confirmed by numerous experimental and numerical investigations and is considered a cornerstone in the analysis of turbulent flows.

The relations presented here strictly hold only for homogeneous, isotropic and statistically steady flows. However, it has been found that the estimates can still be applied even if the flow does deviate from its ideal state.

1.2 Separated and Suddenly Expanded Flows

A flow that can exhibit the flow states described before is a suddenly expanded one. In sudden expansion a series of bifurcation are taken place for low Reynolds numbers causing the breaking of symmetries and unsteadiness sooner than a simple channel flow. Moreover transition to turbulence is appearing locally to the flow for low Reynolds numbers making these kind of geometries both demanding and interesting to describe. A major characteristic of the suddenly expanded flows is the flow separation.

In fluid mechanics separation is defined when part of the flow has opposite direction from the main flow. There are two causes for separation of a fluid the geometric and the dynamic. Geometric separation occurs due to the geometry of the physical boundaries of the flow. Such an example can be a sharp edge of a rectangular pipe at sudden expansion or contraction. Due to the sharp edge and the physical discontinuity the fluid is forced to detach itself since the friction at the wall can not reduce the velocity fast enough. In order to define physical where the separation occurs we use the tangential flow velocity. When it changes direction in the boundary layer signifies the present of a recirculation zone and the separation. In the same way the position when the separation reattaches is signified from the change of the tangential velocity's sign to the opposite direction. An other way to identify the separation /reattachment is by the change of sign of the boundary wall shear stress. On the

other hand the dynamic separation is created by the pressure. When the fluid faces a high enough pressure in the flow direction for sufficient amount of time this could cause a change in the sign of the streamwise velocity and a separation region will occur.

Figure 1.2 illustrates an example of both geometrical and dynamic separation that occur during a sudden expansion. It is shown that the first two separation areas direct above and below the expansion are geometric separation areas and the third recirculation area is caused by the flow and the increased pressure gradient. The position of the point of separation is that point on the wall where the wall shear stress w becomes zero:

$$\tau_w = \mu \cdot \frac{\partial u}{\partial y} \Big|_w \quad (1.1)$$

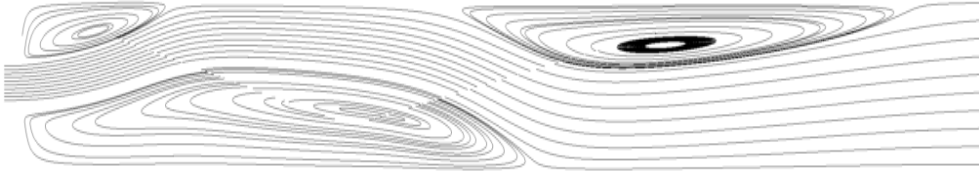


Figure 1.2: Geometrical and dynamic separation occurring at a sudden expansion geometry

Incompressible flow in sudden expansions is one of the classical examples in fluid mechanics which exhibit non-linear bifurcation phenomena.

Experimental studies by Durst et al. [9]; Chedron et al. [10] ; Fearn et al. [11] have shown that flows through suddenly-expanded geometries feature symmetric separation at low Reynolds numbers and beyond a certain Reynolds number exhibit bifurcation phenomena (instabilities) that are manifested as an asymmetric separation of the fluid flow. The critical Reynolds number for symmetry-breaking depends on the expansion ratio and upstream flow conditions. As the Reynolds number further increases the flow may encompass unsteadiness, three-dimensionality and chaos (Mullin and Cliffe [12]).

The early experimental studies by Durst et al. [9] were carried out for two different expansion ratios 1:2 and 1:3. Both flow geometries revealed similar flow

phenomena with instabilities appearing over a certain critical Reynolds number. Chedron et al. [10] demonstrated experimentally that a stable symmetric solution could only exist under a certain critical Reynolds number, beyond which the flow becomes unstable and asymmetric. An experimental and numerical study for a 1:3 sudden expansion was published by Fearn et al. [11] showing that the change from symmetric stable separation to an unstable asymmetric separation maybe due to a pitchfork symmetry-breaking bifurcation point. The experimental results of Fearn et al. [11] showed that the bifurcation diagram is disconnected due to small imperfections that are always present in an experimental setup. They observed no critical Reynolds number for symmetry-breaking and hence a symmetric state was never observed. Fearn et al. [11] attempted to numerically model the small imperfections observed in the experimental setup in order to try and account for the disconnection in the experimental bifurcation diagram. They repeated the numerical simulations with the downstream section of the grid shifted up by 0.05mm with respect to the axis of symmetry. This perturbation caused a disconnection of the bifurcation diagram as observed in the experiments. The size of the disconnection found experimentally was of the same order as the decoupling produced by perturbing the numerical problem. Linear stability analysis carried out by Shapira et al. [13] verified the experimental findings and obtained a good agreement with respect to the critical Reynolds number when compared to the work of Drikakis [14]. Fearn et al. [11] postulated that the transition to asymmetric flow was abrupt, which disagreed with the finding of Shapira et al. [13], who claimed that the transition is actually smooth. Numerical simulations carried out by Durst et al. [15] to validate previous experimental findings (Durst et al. [9]) found that the transition from symmetric to asymmetric separation caused by a pitchfork bifurcation is in fact smooth.

Computational studies based on high-order methods, (Drikakis [14]) were performed to numerically investigate the asymmetric flow structure at different Reynolds numbers, while continuation and Arnoldi-based iterative methods have been used by Alleborn et al. [16] to calculate the most unstable eigenmodes for steady flow in a symmetric channel and the bifurcation structure of the steady state solution of the flow. Drikakis [14] and Alleborn et al. [16] demonstrated that as the expansion ratio increases the critical Reynolds number decreases. Battaglia et al. [17] conducted a linear stability analysis and also performed numerical computations of steady flow through a suddenly expanded channel with various expansion ratios. They made use of bifurcation theory in order to determine numerically the bifurcation point and the results agreed with those of Drikakis [14] and Alleborn et al. [16]. Luo [18] numerically investigated symmetry-breaking of flow in 2-dimensional channels using

Lattice-Boltzmann methods. Their predictions of the critical Reynolds numbers for a suddenly expanded channel with an expansion ratio of 1:3 compared well with the studies of Fearn et al. [11] and Drikakis [14].

Goldstein et al. [19] carried out an experimental investigation of the laminar flow of air over a downstream-facing step. They observed that the laminar reattachment length cannot be expressed as a fixed number of step height as is the case for turbulent flow and provided an equation to predict the reattachment length. Goldstein et al. [19] also observed the presence of a secondary flow pattern in the separated zone. They found that when fluid from the boundary layer in the corner of the test section enters the separated zone at the step it recirculates in a spiral fashion to the geometry centerline and leaves the separated region near the reattachment point on the centerline plane.

Furthermore, Neofytou and Drikakis [20] have investigated flow instabilities in suddenly expanded channels for non-Newtonian fluids showing similar solutions to the Newtonian case, which, however, differ with respect to the critical values of Reynolds number where the symmetry-breaking bifurcation occurs. Simulations of suddenly expanded flows for power-law fluids were carried out by Manica and De Bartoli [21]. They found that power-law fluids (shear-thinning and shear thickening) behaved in a similar manner as Newtonian fluids. Critical Reynolds numbers, in which the solution becomes asymmetric, were found to be in close agreement for all cases. The effect of shear thinning and shear thickening was to increase and decrease, respectively, the Reynolds number upon where the third downstream separation bubble appears in comparison to the Newtonian case. Mizushima and Shiotani [22] used weakly nonlinear stability analysis to investigate the structural instability of the bifurcation for flow through a sudden expansion showing that the parameter range for weakly nonlinear stability analysis is limited to the vicinity of the critical point. Hawa and Rusak [23] and Rusak and Hawa [24] have also performed bifurcation analysis, linear stability and numerical simulations to study the dynamics of the flow through a sudden expansion. They showed that the flow instability is a result of the interaction of viscous dissipation, upstream convection induced by the asymmetric disturbances and downstream convection of perturbations by the symmetric base flow. Other studies which have been concerned with instabilities and bifurcation phenomena in similar geometries include the works of Sobey [25]; Sobey and Drazin [26]; Tsui and Wang [27] for two dimensional diffuser-like channel flows; Mizushima and Shiotani [28]; Mizushima et al. [29] for suddenly expanded and contracted channel and Mallinger and Drikakis [30] for three-dimensional flows in pipes with stenosis.

The majority of the numerical studies involving investigations into instabilities in suddenly expanded flows have been simulated in 2-dimensions with an infinite aspect ratio. The studies of Durst et al. [9]; Chedron et al. [10]; Fearn et al. [11]; Ouwa et al. [31] discussed above investigated various aspect ratio channels and found that the critical Reynolds number is dependent on the size of the aspect ratio. Chedron et al. [10] found that as aspect ratio was increased the critical Reynolds number decreased. This suggest that the side walls of the channel act to stabilize the flow and delay the onset of symmetry-breaking bifurcation.

A three-dimensional numerical study of bifurcation in sudden channel expansions was undertaken by Schreck and Schafer [32]. Their investigation focused on the three dimensional effects of a suddenly expanded channel for two different aspect ratios. They found that the critical Reynolds number at which symmetry breaking bifurcation occurs, increases as the aspect ratio decreases, hence confirming the trend observed by Chedron et al. [10]. Thiruvengadam et al. [33] simulated bifurcated 3-dimensional laminar forced convection in a plane symmetric suddenly expanded channel in order to illustrate how flow bifurcation effects temperature and heat transfer distributions at moderately low Reynolds numbers ($Re < 800$). They found that for a channel with an expansion ratio of 1:2 and an aspect ratio of 2 defined by the ratio of the spanwise length to the downstream channel height, the flow was steady and asymmetric in the transverse direction, but symmetric relative to the center width of the channel in the spanwise direction. Several papers by Chiang et al. [34, 35, 36] [37] have been published on three-dimensional flow through suddenly expanded or suddenly contracted channels. An in-depth study (Chiang et al. [34]) on the affect aspect-ratio has on the fluid flow characteristics showed that as the aspect ratio decreased the Reynolds number at which symmetry-breaking bifurcation occurs increases. This confirmed experimental observations made by Chedron et al. [10] that a decrease in the aspect ratio has a stabilizing effect on the subsequent fluid flow. Chiang et al. [35] found that there exists a critical aspect ratio for which the symmetry-breaking pitchfork bifurcation evolves with different symmetry-breaking orientations on the left and right sides of the channel in the spanwise direction. This second mode of bifurcation which occurs in the spanwise direction was found to be difficult to obtain due to the unstable flow symmetry at the spanwise symmetry plane.

More recently Battaglia and Papadopoulos [38] investigated the effects of three-dimensionality on low Reynolds number flows past a sudden expansion in a channel. The geometry of the channel investigated had an expansion ratio of 1:2 and an aspect ratio of 6 defined by the ratio of the spanwise length to the step height.

An experimental investigation using two-dimensional particle image velocimetry to visualize the fluid flow was carried out in tandem with two- and three-dimensional numerical simulations for Reynolds numbers in the range of 150-600. Battaglia and Papadopoulos [38] found that the two-dimensional simulations failed to capture the total expansion effect of the flow in comparison to the experimental results. The expansion effect of the flow is influenced by both geometric and hydrodynamic effects. They found that in order to correctly capture these expansion effects an effective expansion ratio, defined by the ratio of the downstream and upstream hydraulic diameters, hence taking into account both expansion and aspect ratios. Two-dimensional simulations using this effective expansion ratio were performed and compared very well with three-dimensional simulations and the experimental results. They also reported that the critical Reynolds number for symmetry-breaking bifurcation when using this effective expansion ratio for two-dimensional simulations is much closer to three-dimensional simulated results with a fixed aspect ratio and also to experimental results. As reported by various other authors, Battaglia and Papadopoulos [38] also comment on the stabilizing effect of the side wall proximity, in that the lower the aspect ratio the greater the influence the side walls have in stabilizing the flow and hence increasing the critical Reynolds number for symmetry-breaking bifurcation.

Sau [39, 40] studied three dimensional vortex dynamics and mass entrainment in both a three dimensional rectangular sudden expansion and also for a suddenly expanded and contracted channel. They focused on vortex generation by the use of rectangular-shaped protrusions into the flow. They found that the placement of these tabs could either stop or augment the axis switching mechanism. Yoshikawa et al [41] perform a DNS study to investigate the three-dimensional separated flow and heat transfer in a sudden-expansion rectangular channel of $H=4.0$ and $W=2.5$ at $Re=200$ to 1000. They found that the flow becomes unsteady over $Re=400$ and studied the behavior of the vortical structures and how the heat transfer affect them.

Latornell and Pollard [42] and Back and Roshke [43] have carried out experimental investigations into higher Reynolds number flows in the range $400 < Re < 1000$ based on the upstream channel height for axisymmetric sudden expansions. Latornell and Pollard show that the onset of shear layer instabilities is dependent on the inlet velocity profile. They also show that there is a linear relationship between the reattachment length of the shear layer and the inlet Reynolds number. This linear relationship is however dependent on the nature of the expansion inlet velocity profile. Latornell and Pollard [42] identified three different modes of laminar flow which can exist downstream of the expansion, depending on the inlet Reynolds number.

At low Reynolds number there exists an unconditionally stable mode characterized by a steady reattachment length and recirculation zone shape. The evolution of shear layer instability begins with the generation of sinusoidal waves in the shear layer, at higher Reynolds numbers. Small oscillations in the reattachment length occur due to the interaction of the waves in the shear layer and the wall. A further increase in Reynolds number causes a grossly unstable mode of flow to develop. Discrete vortices replace the shear layer waves in the vicinity of the reattachment point. This in turn causes large oscillations in the reattachment length.

Sheu et al [44] report a detailed analysis on vortical structure for a backward facing step at the transitional Reynolds number of 900. It was found that shear layer instabilities were amplified by the shedding-type instabilities and induced by the interaction of coherent structures with the sidewalls of the duct causing the flapping motion of interior shear layer. Also the relation between the Kelvin–Helmholtz instabilities and the evolution of them into lamda and hairpin vortices were presented. Mullin et al [45] performed an experimental investigation of the bifurcation phenomena in laminar and time-dependent flows through a sudden expansion in a circular pipe. Found the bifurcation point between the steady symmetric and steady asymmetric state. Also the asymmetric steady state which arises at the bifurcation point eventually gives way to time dependence in the form of bursts of periodic motion with further increase in flow rate. Vetel et al [46] carried out an experimental study of an axisymmetric constriction in the Reynolds number range $Re \sim 100-1100$. At $Re=400$ was found that the flow become unsteady with the periodic discharge of the unstable recirculation region induce alternating laminar and turbulent flow phases. Visualizations of isosurfaces of the swirling strength criterion reveal quasi-streamwise counter-rotating vortex pairs at the onset of instabilities, which transform into arches vortices. Later, hairpin-like vortices with varying sizes and asymmetry appear. Finally Furuichi et al [47] identified experimentally the Reynolds critical values for the laminar, transitional and turbulent flows in a axisymmetric sudden expansion. They found using the the zero-crossing point (where the flow changes its direction from negative to possitive) that the limits are $Re < 700$ for laminar, $700 < Re < 2,000$ for transition, and $Re > 2,000$ for turbulence.

1.3 Numerical approaches

A characteristic of a suddenly expanded geometry like the one that will be examined in this PhD study is that it can have for low Reynolds numbers transitional and turbulent regions except from laminar. For this reason a proper numerical approach

to turbulence need to be implemented.

In Computational Fluid Dynamics () are currently three major families of computational techniques: the Direct Numerical Simulation (), the Reynolds-Averaged Numerical Simulation () and the Large- Eddy Simulation (). Following the applicability, advantages and disadvantages of each technique will be presented as well as the Implicit Large Eddy Simulation () which is a branch of LES and the method used in this PhD will be presented.

1.3.1 Direct Numerical Simulation (DNS)

The DNS approach is the most accurate and the most cost demanding of the three techniques. It solves all the scales of turbulent motion of the problem so it does not have significant numerical errors. In order to achieve that it must solve all the scales of the motion must be represented to the computational grid, so the step at space and time must be smaller than the characteristic time and the characteristic length of the eddies of the Kolmogorov scale. Since the time of the simulation and the size of the domain are depended to the characteristic time and the characteristic length of the largest, energy-containing eddies, it is obvious that as the Reynolds number increases so the simulation becomes more cost demanding. So even with today's advances in computational power DNS applicability is limited to low Reynolds, simple geometry simulations.

The number of points required for a numerical simulation depends on the spatial resolution and the size of the flow field. Due to the fact that the DNS solve every scale and that the domain should have length at least a few time the characteristic length of the largest scale of the flow we can use the Kolmogorov hypothesis to calculate the size of the grid. For homogeneous turbulence in one direction the ratio of large to small length scale is proportional to $Re^{3/4}$. Thus in a three dimensional turbulent flow, which is always the case due to the three dimensional nature of the turbulence, the grid should have $3 * Re^{3/4} = Re^{9/4}$ points. Furthermore, we should also get into consideration, for the calculation of the total computational cost, the simulation time. It should be proportional to the characteristic time of the largest eddies, but at the same time the timestep should be small enough to resolve the smallest eddies. If we assume a velocity of one grid cell per time step from the above relations can be shown that the number of the time steps should be at least of the order of $Re^{3/4}$. Since the computational cost is of proportional to the number of grid points multiplied by the number time-steps then the total cost of the DNS is Re^3 . From that relation it is obvious the steep increase of the computational cost with the Reynolds number.

In the case of wall-bounded flows the computational cost is increased due to the

scaling laws in the near-wall region. From the Moin and Kim [48] study we see that a DNS of a transport airplane cruising at 250 m/s at an altitude of 10,000 meters would require approximately 10^{16} grid points to adequately resolve every length-scale. This is translated that even with today's computational power of 100 teraflops it will take decades for a single second of flight time something clearly unacceptable. However, in most engineering problems and applications the main point of interest is the mean flow and not the behavior of everyone of the smallest scales. So it is common to consecrate the effort of the simulation to the behavior of the largest scales and assume the behavior of the smallest with the use of statistical models. Two main techniques use this approach Reynolds-Averaged Numerical Simulations and Large-Eddy Simulations.

1.3.2 Reynolds-Averaged Numerical Simulation (RANS)

The Reynolds- Averaged Numerical Simulation is the most common technique used in engineering applications. The main idea behind the technique is the Reynolds decomposition of the variables of the Navier Stokes equations into mean and a fluctuating components.

$$u(x, t) = \overline{u(x, t)} + u'(x, t) \quad (1.2)$$

where $\bar{u}$ denotes the time average of u and u' the fluctuating part

The method of averaging is decided by the nature of the problem. This range from time average for a statistically steady flow, a spatial average for essentially two-dimensional flows, or an ensemble average for a family of similar flows.

Usually in engineering applications where the boundary conditions tend not to be time dependent like inlet conditions in internal flows or free-stream conditions in external flows time-averaging is preferred. Here, the resolved mean flow can be considered free of fluctuations and all the unsteadiness is contained in the unresolved turbulent scales that need to be modeled. Modeling is necessary because the averaging procedure introduces additional unknown terms in the governing equations that cannot be computed exactly from the mean flow variables. As a result, the averaged equations are always complemented by additional turbulence models that mimic the effects of the unsteady motions. Since RANS requires the least possible amount of resources it gained enormous popularity in steady-state computations where the turbulence is stationary, e.g. see Leschziner [49], Mallinger and Drikakis [30].

In unsteady flows, a time-scale associated with the organized unsteady motion exists and must be well separated from the time-scale of turbulent motion. Here,

the exact solution can be seen as the sum of three contributing terms: the time average, the conditional average of the coherent motion and the random fluctuation due to turbulent motion. However, very few unsteady flows are guaranteed to exhibit deterministic low-frequency motions that can, for example, be enforced externally through periodically changing inflow or free-stream conditions. If applicable, the conditional average in Unsteady Reynolds-Averaged Numerical Simulations (URANS) is therefore usually interpreted as a phase-averaged solution and the closure models are formally identical to the ones in steady-state computations.

The function of turbulence modeling in RANS is to devise approximations for the unknown correlations between mean flow and fluctuating component, the so-called Reynolds stresses, in order to close the system. The closure relations are based on combinations of known or determinable geometric parameters, flow scales and strains. However, these quantities are not able to completely represent the complex physical structures and interactions inherent to turbulent flows; hence it seems unlikely that any single model will successfully predict all types of turbulent flows with any degree of certainty. For this reason, numerous turbulence models have been developed over the past decades, all introducing a number of unknown coefficients. In order to adjust the models to particular flows, the unknowns are usually determined empirically by calibration against existing experimental and DNS data.

1.3.3 Large Eddy Simulation (LES)

The development of Large-Eddy Simulations is motivated by the limited applicability of DNS and RANS to turbulent flows and it can be regarded as an intermediate between both approaches with respect to accuracy and computational cost. In contrast to RANS, classical Large-Eddy Simulations model only the small-scale turbulent motions whereas the larger turbulent structures are directly resolved. Since the smaller structures are only slightly affected by the boundary conditions, they exhibit a more common character for different types of flows. Thus, the models employed in LES tend to be more universal and require fewer adjustments to the specific flow compared to a similar RANS model. On the one hand, the LES strategy provides superior accuracy, on the other hand, however, higher precision also comes at a higher computational cost than RANS.

Similar to the Direct Numerical Simulations, Large-Eddy Simulations provide a fully three-dimensional, time-dependent solution. As demonstrated previously, the grid requirements for DNS strongly depend on the smallest scales present in the flow, but most of the turbulent kinetic energy is contained in the larger structures. Therefore, while computing the large-scale dynamics of the flow directly, LES significantly

reduces the total computing time by modeling the less energetic, but computationally demanding, small scales. It is possible now to calculate more complex turbulent flow scenarios that are forbiddingly expensive for direct simulations without having to take a drastic accuracy penalty. Ideally, the computational cost of LES is independent of the Reynolds number, given that the reference length distinguishing the resolved, large scales from the modeled, small scales lies in the inertial sub-range and that no solid walls are present.

In classical LES all dependent variables of the governing equations are filtered with the use of a predefined filter. The filtering operation decomposes the flow field into the sum of a filtered, resolved component and a residual, subgrid-scale component. The spatial and temporal evolution of the filtered component representing the large scales is fully described by the filtered equations, and the unknown subgrid scale (SGS) stress tensor arising from the unresolved residual motions needs to be modeled. This system is commonly closed by more or less sophisticated SGS models that are primarily concerned with reflecting the dissipation of energy cascading down from the larger scales in a statistical sense.

When deriving the classical LES equations, it was implicitly assumed that differentiation commutes with the convolution, see Fureby and Tabor[50], Ghosal and Moin [51]. However, this is not valid at solid boundaries and for a variable filter kernel. The former requires reality to be modeled in a finite domain, which introduces commutation errors in the spatial derivatives. If the finite domain changes in time, additional errors in the temporal derivatives arise. A variable filter kernel could solve the problems at solid boundaries by reducing the filter width in order to resolve the small scales. This procedure violates the above commutation assumption and introduces new errors, which can be removed by correction terms. Yet, there are no available methods to tackle the correction terms and thus previous work, e.g. by Van der Ven [52], Vasilyev et al. [53], has been aiming at filters that can eliminate these terms.

Furthermore, like in all numerical approaches, errors in classical LES arise from the approximation of the variables on a finite basis and numerical discretisation. For discretising the governing equations, the differentiation operators are substituted by numerical approximations, which lead to dissipation and dispersion terms, see truncation error analysis in Anderson [54]. The dissipation terms are responsible for the numerical diffusion, especially near discontinuities, whereas the dispersion terms produce oscillations near discontinuities. Drawbacks of conventional LES also arise from SGS modeling, the possible masking of the SGS terms by the leading order truncation error and the difficulties in devising SGS models for complex high

Reynolds number wall-bounded flows.

1.3.4 Implicit Large Eddy Simulation (ILES)

The concept behind Implicit Large Eddy Simulation is that the equations are solved in their original form without any filtering (hence the commutation error can be dropped) or explicit modeling leaving the embedded numerical viscosity inherent to the numerical methods used to resolve the small scales. These methods are implicitly based on the hypothesis; The action of subgrid scales on the resolved scales is equivalent to a strictly dissipative action, as written by Sagaut [55]. Simulations using these methods most commonly use dissipation terms introduced in the framework of upwind schemes for the convection term. This diffusive term adapts itself to the nature of the local solution in order to obtain a solution that is both accurate and has some physical meaning. There are several types of numerical methods used in the context of ILES; Godunov (Yan and Knight [56]), PPM (Colella and Woodward [57]), TVD (Cousteix [58]), FCT (Boris and Book [59]), MPDATA (Margolin et al. [60, 61]) among others.

In order to further understand the ILES methodology it is important to outline the differences between standard LES, and ILES, with some referencing to DNS. In DNS the dissipation needed in order to maintain numerical stability is provided by the physical viscosity. As discussed in the previous subsection, in LES a SGS model is provided to the solver in order to represent the effects of the unresolved scales. An essential part of these SGS models are that they must provide sufficient dissipation to the flow otherwise the resulting build up of energy in the smallest resolved scales grows unbounded, until the numerical solution breaks down. The models and filtering of the equations in LES should ensure that the flow is smooth accordingly without having to worry about added dissipation from the numerical algorithm which should be kept to a minimum. In LES subgrid models are defined by applying physical theories of homogeneous isotropic turbulence in Kolmogorov's framework. These models are then coupled to the ideal (zero dissipation) Euler equations to provide a representation of reality. In the case of ILES the model and the numerics are merged together. The models have theoretical foundations in vanishing viscosity used in the selection of entropy satisfying weak solutions. High-resolution and non-linear stability is achieved via extensions to the numerics such as monotonicity, , TVB, and other such physical/mathematical principles. These extensions are a key to the ILES approach and allow at least second order of accuracy in smooth areas of the flow. Without these extensions the vanishing viscosity approach produces first-order results which are not considered as 'high-resolution'.

High-resolutions scheme have been built upon circumventing Godunov's theorem

(Godunov [62]) which stated that: if an advection scheme preserves the monotonicity of the solution it is at most first-order accurate. Non-linear discretisation of the advection equation allows higher orders of accuracy and forms the basis of high-resolution methods. The non-linearity of these methods differentiates them from classical techniques and guarantees numerical stability and physical results. It is this connection between the numerics and flow physics that makes these methods attractive in the simulation of a wide variety of flows. There has been an increasing amount of evidence to suggest that high-resolution methods have an embedded turbulence model (Boris et al. [63]; Drikakis [64]; Drikakis and Geurts [65]; Fureby and Grinstein [66, 67]; Linden et al. [68]; Margolin and Rider [69]; Margolin et al. [60]; Oran and Boris [70]; Porter et al. [71]; Youngs [72, 73, 74]). High-resolution methods have been accepted as powerful and efficient methods in simulating laminar flows and their extension to turbulent flows require no further changes to the numerical methods employed. The user does not need to decide whether the flow is laminar or turbulent leaving the numerics to make the decision themselves. This makes for simple and flexible numerical codes.

Modified Equation Analysis (MEA) can be used to shed light on the embedded similarities between standard LES and ILES. MEA derives the effective differential equation of a numerical algorithm as a basis to analyze the numerical algorithms behavior (Griffiths and Sanz-Serna [75]; Knoll et al. [76]; Warming and Hyett [77]). MEA can be applied to examine several LES models two of which (Smagorinsky and Bardina) will be now discussed (for a complete analysis see the text of Drikakis and Rider [78]). The general form of the numerical/modeled solution is:

$$U_t + \nabla \cdot E(U) = \nabla \cdot \tau(U), \quad (1.3)$$

where the subscript t represents the time derivative and (U) the subgrid scale stress. The left hand side is the idealized inviscid equation and the right hand side is the subgrid model. The model by Smagorinsky [79] can be defined by

$$\tau(U) \sim C \Delta^2 \|\nabla U\| \nabla U, \quad (1.4)$$

where C is a constant and Δ is the cell size. Bardina et al. [80] proposed a model based on filtering, which uses the difference in the subgrid term evaluated at two different filter sizes, and can be defined by

$$\tau(U) \sim \overline{E(\overline{U})} - \overline{\overline{E(U)}} \approx E''(U) \Delta^2 \nabla U \nabla U. \quad (1.5)$$

Evaluating the differential form of the Smagorinsky model in one spatial direction

is a simple process since the form can be directly used, giving

$$\tau(U) = C\Delta^2 |U_x| U_x \quad (1.6)$$

For the Bardina model which is based on filtering, MEA is needed to produce the differential forms. Using a box filter at 2Δ and 4Δ gives,

$$\tau(U) = C\Delta^2 (U_x)^2 \quad (1.7)$$

It should be noted that the constant C is different for each of the two models. Also the Smagorinsky model has been found to be explicitly dissipative whereas the Bardina model is unstable without additional dissipation. This additional dissipation is provided by adding the Smagorinsky model. Now it will be shown that ILES can produce the same effect naturally including the same differential terms as the self similar Bardina model, producing a mixed model through the nonlinear regularization associated with nonoscillatory differencing. Analysis of spatial errors in a one-dimensional high-resolution algorithm can be carried out by considering the following form,

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x} [E(U_{j+1/2}) - E(U_{j-1/2})] \quad (1.8)$$

A high-resolution Godunov method based on a reconstruction procedure employing limited slopes, $S_j(U)$, produces two edge values in each cell.

$$U_{j\pm 1/2;L/R} = U_j \pm \frac{1}{2} S_j \quad (1.9)$$

The two values at each node can be resolved by employing a linearised Riemann solver:

$$E(U_{j+1/2}) = \frac{1}{2} [E_{j+1/2;L} + E_{j+1/2;R}] - \frac{\|E'\|}{2} (U_{j+1/2;R} - U_{j+1/2;L}) \quad (1.10)$$

where $E' \equiv \frac{\partial E}{\partial U}$.

The effective subgrid stress $\tilde{\tau}$ from the general form of the modified equation from (1.3.6), at an order of Δ^2 is

$$\tilde{\tau}_2(U) = c_1 E'(U) U_{xx} + c_2 E''(U) (U_x)^2. \quad (1.11)$$

The subgrid stress τ_2 is a second order approximation and the constants c_1 and c_2 depending on the specific differencing scheme employed. Margolin and Rider [81]

have shown that the term $E''(U)(U_x)^2$ is a consequence of the conservation form and is not present if the differencing is not in conservation form. Furthermore, this term is identical to the leading order term for the self similar model with standard LES. Hence, with the use of MEA one can show that high-resolution methods used for solving ILES have embedded similar subgrid stresses as explicit models used for LES.

1.4 Aim and Objectives

From the previous section we can conclude that a lot of effort is being made for the development of numerical techniques properly describing transitional and turbulent flows. A suddenly expanded channel features a geometry that complex transitional and turbulent phenomena occur but at those Reynolds numbers the literature provides us with very few experimental and numerical cases.

The aim of this PhD project is to investigate unsteadiness, transition and localized turbulence in separated flows with the use of high order methods within the ILES framework.

The objectives of the thesis are:

- Implementation of high-order methods in the ILES framework.
- Assessment of the high order methods in flows featuring bifurcation and transition to turbulence.
- Investigation of the flow physics, structures and transitional properties for the suddenly expanded flows.

Chapter 2

Methodology

The following investigation is implemented by the solution of the set set of three-dimensional, unsteady, incompressible Navier Stokes equations. Although in nature every fluid is compressible, at low Mach number the rate of its compressibility is very small so is a correct approximation to consider and examine that fluid as incompressible. The framework utilize a block-structure finite volume technique in generalized curvilinear coordinate system with multiple reconstruction techniques to been examined.

In separated transitional and turbulent flows, the flow components fluctuate rapidly and high gradients are encountered frequently, thus a simple explicit time integration method is preferred. For the discretisation in space, a central difference scheme is employed for the viscous terms and a Godunov-type method is utilized for the advective flux derivatives. High resolution is achieved through the reconstruction step incorporating different variants of non-linear schemes. Furthermore, the code is fully parallelised following a domain decomposition approach. In order to provide a complete description of the numerical technique, all of the above components will be presented in this section.

2.1 Governing Equations

The physics of (Newtonian) fluid flow is governed by the Navier-Stokes equations. These equations can be solved by considering the coupled generalized conservation laws, continuity, momentum and energy equations.

The conservative form of the Navier-Stokes equations for a compressible flow is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \quad (2.1)$$

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u u) = -\nabla \cdot P \quad (2.2)$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e u) = -\nabla \cdot (P \cdot u) - \nabla \cdot q \quad (2.3)$$

where u , ρ , e , and q stand for the velocity components (u, v, w), density, total energy per unit volume and the heat flux, respectively. The tensor P for a Newtonian fluid is defined by

$$P = p(\rho, T)I + \frac{2}{3}\mu(\nabla \cdot u)I - \mu [(\nabla u) + (\nabla u)^T] \quad (2.4)$$

where $p(\rho, T)$ is the scalar pressure, I is a unit diagonal tensor, T is the temperature and μ is the dynamic viscosity coefficient. The above system of equations is completed by an equation of state. For a perfect gas the equation of state is given by: $p = \rho R T$, where R is the gas constant.

In the case of incompressible flows the Navier-Stokes equations are altered to the following form

$$\nabla \cdot u = 0 \quad (2.5)$$

$$\frac{\partial u}{\partial t} + \nabla \cdot (u u) = -\frac{1}{\rho} \nabla p + \mu \nabla^2 u \quad (2.6)$$

It should be noted that for this PhD study the energy equation is not solved for the incompressible flows.

A common form of the Navier Stokes equations is their non-dimensionilized form. Such alteration is useful since it allows a more direct comparison of the data between different simulation as well as experiments. For the non-dimensionalisation process characteristic reference values for length (l_c), velocity (u_c) and dynamic viscosity (μ_c) are used. For the incompressible Navier Stokes equations the non-dimensionilised variables are

$$t^* = \frac{t}{l_c/u_c}, \quad x^* = \frac{x}{l_c}, \quad y^* = \frac{y}{l_c}, \quad z^* = \frac{z}{l_c},$$

$$u^* = \frac{u}{u_c}, \quad v^* = \frac{v}{u_c}, \quad w^* = \frac{w}{u_c}, \quad p^* = \frac{p}{u_c^2}, \quad \mu^* = \frac{\mu}{\mu_c}$$

for the rest of the section for reasons of simplicity the superscript $*$ will be omitted.

2.1.1 Incompressibility Formulation

The range of flows modeled by the Incompressible Navier-Stokes equations encompasses a large number of industrially important applications for which the Mach number is less than 0.3. This includes use in the automobile industry, structural flows, and sub-sea applications. All of these applications require accurate numerical solutions to the Navier-Stokes equations that can be obtained in a realistic time scale using high performance computing facilities. However, the solution of the incompressible Navier-Stokes equations still presents a significant numerical challenge. The reason for this is that there is a lack of coupling between the velocity and the pressure fields. This means that the equations themselves provide no way of explicitly updating pressure as velocity is advanced.

Methods for solving the incompressible Navier-Stokes equations in primitive variables can be grouped into two broad categories. The first can be referred to as the pressure correction method approach. This approach is discussed in detail in the texts of Harlow and Welch [82]; Patanker [83]; Raithby and Scheider [84]. The distinguishing feature of this method is the use of a derived equation to determine the pressure. Typically, the momentum equations are solved for the velocity components independently. This produces linearised equations by using values lagged in iteration level for the other unknowns, including pressure. Since the velocity components have been obtained without the using the continuity equation as a constraint, a Poisson equation is usually developed for the pressure that will alter the velocity field in a direction such as to satisfy the continuity equation.

The second category is a coupled approach where the discretised conservation equations are solved, treating all dependent variables as simultaneous unknowns. The method referred to is called the artificial compressibility method (Chorin [85]; Kwak et al. [86]; Choi and Merkle [87]).

2.1.2 Artificial Compressibility

Methods for solving the compressible flow equations have attracted a significant interest resulting in a number of different methods being developed. Ideally one would like to be able to use existing compressible methods to solve the incompressible flow equations. This cannot be carried out directly due to the inherent mathematical character of the compressible equations. The compressible flow equations are hyperbolic which means that they have real characteristics on which signals travel at finite propagation speeds; this reflects the ability of compressible fluids to support sound waves. On the other hand, the incompressible flow equations have a mixed parabolic-elliptic character. Hence, in order to use numerical methods orig-

inally developed for solving the compressible flow equations, the character of the incompressible flow equations needs to be modified.

The difference in character between the compressible and incompressible flow equations lies in the lack of the time derivative term in the incompressible continuity equation. Hence the simplest way to give the incompressible equations a hyperbolic character is to include a time derivative in the continuity equation. The compressible continuity equation contains the time derivative of the density but since density is constant for the incompressible equations one cannot use this approach. Time derivatives of the velocity appear in the incompressible momentum equations and hence are not logical choices. That leaves the time derivative of the pressure as the clear choice.

The addition of a time derivative of the pressure to the incompressible continuity equation means that the equations being solved are no longer truly incompressible. A result of this is that the time history generated is no longer accurate and the extension of the equations to unsteady incompressible flow needs further consideration.

For steady flows, Chorin [85] introduced the auxiliary system of equations,

$$\frac{1}{\beta} \frac{\partial p}{\partial \tau} + \frac{\partial u_i}{\partial x_j} = 0 \quad (2.7)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i \partial u_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} \quad (2.8)$$

where β is the artificial compressibility parameter, u_i are the velocity components, p is the pressure, ρ is the density, ν the kinematic viscosity and t is the time. The indices $i, j = 1, 2, 3$ refer to the space coordinates x, y, z . For the case of steady flow $\tau = t$. The artificial compressibility parameter, which has dimensions of a velocity, is a disposable parameter, which enables the above system of equations to converge to a solution that satisfies the incompressibility condition as the steady state is approached (Chang and Kwak [86]). The value chosen for is a key parameter to the performance of this method. It is clear that the larger the value β of the more “incompressible” the equations become. This is an undesired effect leading to the inviscid terms of the equations becoming very steep numerically. The choice of the artificial compressibility parameter will be discussed further later in this section.

Equations 2.7 and 2.8 also have similarities with low Mach number compressible flow equations, hence the artificial compressibility parameter can be related to an artificial speed of sound

$$c = \sqrt{\beta} \quad (2.9)$$

The artificial compressibility approach transforms the incompressible flow equations to fully hyperbolic and hyperbolic-parabolic for inviscid and viscous flows, respectively. The artificial compressibility approach has been found to be less computationally expensive in comparison to solving the elliptic equations and has been used extensively by various researchers. (Temam [88]; Steger and Kutler [89]; Peyret and Taylor [90]; Chang and Kwak [86]; Choi and Merkle [87]; Rizzi and Eriksson).

Chorin [85] originally designed the artificial compressibility approach for steady flow problems because the solutions had to be iterated to time convergence for the artificial term to vanish. It has now been well documented (Merkle and Athavale [91]; Soh and Goodrich [92]; Rogers and Kwak [93]; Rogers et al. [94]; Breuer and Hanel [95]; Kim and Menon [96]; Drikakis [64] that by adding a pseudo-time derivative to the momentum equation the artificial compressibility approach can be extended to unsteady flows. The system of equations become

$$\frac{1}{\beta} \frac{\partial p}{\partial \tau} + \frac{\partial u_i}{\partial x_j} = 0 \quad (2.10)$$

$$\frac{\partial u_i}{\partial \tau} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial u_i}{\partial t} - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} \quad (2.11)$$

The above set of equations are iterated to pseudotime τ convergence where the divergence free flow field is satisfied at each real time step. This procedure is a dual-time stepping technique and can be commonly found in the solution of the compressible equations for both steady and unsteady flows.

The estimation of the artificial compressibility parameter β can affect the convergence in both steady and unsteady problems. The optimum value is problem dependent, although some authors have suggested an automatic procedure for choosing it. Chang and Kwak [86] derived a criterion for a simple channel flow in which they considered the interaction of upstream propagating waves with vorticity spreading. With the requirement that the upstream propagating waves must propagate much faster than the spread of vorticity the following criterion was derived,

$$\frac{\beta}{u_{ref}} \gg \left(1 + \frac{4L}{\delta Re}\right)^2 - 1 \quad (2.12)$$

where L is the length of the channel and δ is the half-width. The parameter u_{ref} is a reference velocity used in the calculation of the Reynolds number. A more complete derivation is given by Drikakis and Rider [78]. Chang and Kwak [86]

pointed out that the choice of the artificial compressibility parameter β , is more important for internal flows with respect to the rate of convergence.

With the artificial compressibility formulation the incompressible Navier-Stokes equations can be written with Cartesian coordinates in the following form

$$\frac{\partial U}{\partial \tau} = \frac{\partial E_V}{\partial x} + \frac{\partial F_V}{\partial y} + \frac{\partial G_V}{\partial z} - \frac{\partial E_I}{\partial x} - \frac{\partial F_I}{\partial y} - \frac{\partial G_I}{\partial z} \quad (2.13)$$

where,

$$U = (p/\beta, u, v, w)^T$$

$$E_I = (u, u^2 + p, uv, uw)^T$$

$$F_I = (v, uv, v^2 + p, vw)^T$$

$$G_I = (w, uw, vw, w^2 + p)^T$$

$$E_V = (0, \tau_{xx}, \tau_{xy}, \tau_{xz})^T$$

$$F_V = (0, \tau_{yx}, \tau_{yy}, \tau_{yz})^T$$

$$G_V = (0, \tau_{zx}, \tau_{zy}, \tau_{zz})^T$$

where β is the artificial compressibility parameter and τ_{ij} the viscous stresses. Since most problems require a grid that is fitted on a curved geometry the above equation will be transformed into a curvilinear system. The curvilinear system has $(\xi(x, y, z), \eta(x, y, z), \zeta(x, y, z))$ with the Jacobian transformation being $J = |\partial(x, y, z)/\partial(\xi, \eta, \zeta)|$. Then the incompressible Navier Stokes can be written in curvilinear coordinates as

$$\frac{\partial JU}{\partial \tau} = \frac{\partial E_V}{\partial \xi} + \frac{\partial F_V}{\partial \eta} + \frac{\partial G_V}{\partial \zeta} - \frac{\partial E_I}{\partial \xi} - \frac{\partial F_I}{\partial \eta} - \frac{\partial G_I}{\partial \zeta} \quad (2.14)$$

where,

$$E_I = J(E_I \frac{\partial \xi}{\partial x} + F_I \frac{\partial \xi}{\partial y} + G_I \frac{\partial \xi}{\partial z})$$

$$F_I = J(E_I \frac{\partial \eta}{\partial x} + F_I \frac{\partial \eta}{\partial y} + G_I \frac{\partial \eta}{\partial z})$$

$$G_I = J(E_I \frac{\partial \zeta}{\partial x} + F_I \frac{\partial \zeta}{\partial y} + G_I \frac{\partial \zeta}{\partial z})$$

$$E_V = J(E_V \frac{\partial \xi}{\partial x} + F_V \frac{\partial \xi}{\partial y} + G_V \frac{\partial \xi}{\partial z})$$

$$F_V = J(E_V \frac{\partial \eta}{\partial x} + F_V \frac{\partial \eta}{\partial y} + G_V \frac{\partial \eta}{\partial z})$$

$$G_V = J(E_V \frac{\partial \zeta}{\partial x} + F_V \frac{\partial \zeta}{\partial y} + G_V \frac{\partial \zeta}{\partial z})$$

2.2 Time Integration

2.2.1 Runge-Kutta Method

Runge-Kutta methods are widely used for integrating ODEs (Ordinary Differential Equations). Several variation of the Runge-Kutta methods exists with different order. The numerical code used in this study has implemented different Runge-Kutta orders ranging from first to fourth. The fourth order was used for the completion of these simulation despite the more complex formulation due to the increased convergence rate it provides. The fourth-order Runge-Kutta method, which is used in this study for the calculation of the time derivative for the inner time stepping, consists of four stages which are the following:

$$\left. \begin{aligned} \frac{U^1 - U^n}{h} &= \frac{1}{2} f(U^n, \tau^n) \\ \frac{U^2 - U^n}{h} &= \frac{1}{2} f(U^1, \tau^{n+1/2}) \\ \frac{U^2 - U^n}{h} &= f(U^2, \tau^{n+1/2}) \\ \frac{U^{n+1} - U^n}{h} &= \frac{1}{6} [f(U^n, \tau^n) + \\ &+ 2f(U^1, \tau^{n+1/2}) + 2f(U^2, \tau^{n+1/2}) + f(U^3, \tau^{n+1})] \end{aligned} \right\}. \quad (2.15)$$

2.2.2 CFL Number

For time-dependent flows, time marching in all computational cells has to be performed with the same global time step t , which, for a given Courant-Friedrichs-

Lewy () number, is defined as

$$\Delta t = \min \left(J \frac{CFL}{\max \left(|\lambda_0^\xi|, |\lambda_+^\xi|, |\lambda_-^\xi|, |\lambda_0^\eta|, |\lambda_+^\eta|, |\lambda_-^\eta|, |\lambda_0^\zeta|, |\lambda_+^\zeta|, |\lambda_-^\zeta| \right)} \right) \quad (2.16)$$

where J denotes the Jacobian determinant and λ are the eigenvalues associated with the advective fluxes E , F , G , respectively. The theoretical value of $CFL \leq 1$ leads to a stable integration in time. This condition simply states that the length of the time step is equal or less than it takes for the fastest acoustic wave to travel from one grid point to the next. However, this condition is necessary but not sufficient to ensure stability of the algorithm. Time marching methods satisfying a CFL condition may still lead to instabilities in the sense of permitting large errors or they may simply blow up. In practice, this behavior is commonly cured by lowering the CFL number until a stable solution is obtained. This applies to all Runge Kutta schemes.

2.3 High Resolution Methods

High-resolution methods employ a non-linear technique to control oscillations in the solution. High-resolution methods differ from linear methods, which use the same differencing stencil everywhere regardless of the local solution, by using the local solution to adapt the stencil used for the differencing and also to use the non-linearity to control oscillations. The inherent non-linearity of high-resolution methods means that even if the equations being solved are linear high-resolution methods are still nonlinear. This means that high-resolution methods are both a function of space and time as well as being dependent upon the nature of the local solution. Another property which must be satisfied in order for a numerical method to be classed as high-resolution is that the nonlinear principle used must remove spurious oscillations as well as allowing at least second-order accuracy in areas where the solution is smooth.

The above discussion leads to a more general definition of high-resolution. High resolution methods select the “best” technique for approximating the solution given the evidence provided by the local solution. Thus, high-resolution methods adapt themselves to their circumstances so that the solution obtained is accurate and has some physical meaning.

In 1959, Godunov suggested an approach for the numerical solution of fluid flows (Godunov [62]). In this work Godunov states that “There are no monotone, linear schemes for the linear advection equation of second or higher order of accu-

racy". This suggests that second order accuracy and monotonicity are contradictory requirements. Godunov's first approach involved solving a general flow field by implementing directly a numerical solution of the Euler equations written in partial differential equation form (discretised by the finite difference approach). Godunov suggested that exact solutions of the Euler equations for a local region of the flow be pieced together to synthesize the general flow field. The concept here is that you are constructing a general flow field from elements that are themselves solutions to the Euler equations in a local region of the flow. In order to construct the general flow field you are piecing together local solutions of a smaller problem, rather than visualizing a widely sweeping solution of the governing partial differential equations or integral equations over the whole space of the flow. The evolution of flow to the next time step results from the wave interactions originating at the boundaries between adjacent cells.

The key to circumventing Godunov's theorem lies in the assumption made by Godunov that the schemes are linear. Thus, in order to design schemes that are higher than first-order accurate and still preserve monotonicity, nonlinear methods need to be developed. The development of high-resolution methods needs to be carried out in a one-dimensional context due to the lack of adequate theory in multi-dimensions. It should be noted that even though a numerical method can be designed to be second order accurate for one-dimensional problems, its accuracy in multi-dimensions is not guaranteed to be second-order. High-order flux methods can be derived by using a finite difference approach where the dependent variables are point values. If the dependent variables are viewed as averages over a cell then a mean preserving high-order interpolation can produce high-order methods. Note, that the description of the extension to high-order methods is equivalent to that of those for linear problems. Many non-linear problems develop discontinuities and as such the solution is nominally first order accurate (Majda and Osher [97]).

Other methods of interest which have built upon high-resolution Godunov-type methods are: total variation diminishing (TVD), essentially nonoscillatory (ENO), total variation bounding (TVB) and weighted ENO (WENO). For further information of these numerical methods refer to the text of Drikakis and Rider [78] and Toro [98].

2.3.1 Characteristic Based Scheme

The Characteristics-based scheme is a Riemann solver which defines the variables along the characteristics as functions of their characteristic values. The method was firstly presented by Eberle [99] for the compressible Euler equations and was extended by Drikakis et al. [100] and Drikakis [64] to solve the incompressible

Navier-Stokes equations. For a more detailed explanation of the numerical scheme please refer to the work of Shapiro[101],[102] Drikakis et al. [100],[64] and Eberle [99].

We consider the artificially compressibility formulation of the Navier-Stokes equations with respect to the ξ -direction

$$\frac{\partial U}{\partial \tau} + \frac{\partial \xi}{\partial x} \frac{\partial E_I^c}{\partial \xi} + \frac{\partial \xi}{\partial y} \frac{\partial F_I^c}{\partial \xi} + \frac{\partial \xi}{\partial z} \frac{\partial G_I^c}{\partial \xi} = 0 \quad (2.17)$$

We divide the equation by $L = \sqrt{(\partial \xi / \partial x)^2 + (\partial \xi / \partial y)^2 + (\partial \xi / \partial z)^2}$ and introduce $\tilde{k} = (1/L)((\partial \xi / \partial k))$, where $k=x,y,z$ thus obtaining

$$\frac{1}{\beta L} \frac{\partial p}{\partial \tau} + \tilde{x} \frac{\partial u}{\partial \xi} + \tilde{y} \frac{\partial v}{\partial \xi} + \tilde{z} \frac{\partial w}{\partial \xi} = 0 \quad (2.18)$$

$$\frac{1}{L} \frac{\partial u}{\partial \tau} + (u\tilde{x} + v\tilde{y} + w\tilde{z}) \frac{\partial u}{\partial \xi} + \frac{\partial p}{\partial \xi} \tilde{x} = 0$$

$$\frac{1}{L} \frac{\partial v}{\partial \tau} + (u\tilde{x} + v\tilde{y} + w\tilde{z}) \frac{\partial v}{\partial \xi} + \frac{\partial p}{\partial \xi} \tilde{y} = 0$$

$$\frac{1}{L} \frac{\partial w}{\partial \tau} + (u\tilde{x} + v\tilde{y} + w\tilde{z}) \frac{\partial w}{\partial \xi} + \frac{\partial p}{\partial \xi} \tilde{z} = 0$$

The advective and viscous fluxes are discretised on the cell centers using the intercell values, e.g. the inviscid flux derivative in the ξ -direction is given by

$$\frac{\partial E_I}{\partial \xi} = \frac{(E_I)_{i+1/2,j,k} - (E_I)_{i-1/2,j,k}}{\Delta \xi} \quad (2.19)$$

The system of the conservative formulation can be written is matrix form as

$$\frac{1}{L} \frac{\partial U}{\partial \tau} + A \frac{1}{L} \frac{\partial U}{\partial \xi} = 0 \quad (2.20)$$

where A is

$$A = \begin{pmatrix} 0 & \tilde{x} & \tilde{y} & \tilde{z} \\ \tilde{x} & \lambda_0 & 0 & 0 \\ \tilde{y} & 0 & \lambda_0 & 0 \\ \tilde{z} & 0 & 0 & \lambda_0 \end{pmatrix} \quad (2.21)$$

where $\lambda_0 = u\tilde{x} + v\tilde{y} + w\tilde{z}$ denotes velocity normal to the cell face. The eigenvalues of this matrix are given by λ_0 and $\lambda_{\pm}$ where

$$\lambda_{\pm} = \lambda_{1,2} = \lambda_0 \frac{1}{2} \pm s \quad (2.22)$$

and the artificial speed of sound s is given by

$$s = \frac{\sqrt{\lambda_0^2 + 4\beta}}{2} \quad (2.23)$$

Following the same derivation procedure as for the original scheme [78, 100], we obtain the following formula for the reconstructed variables at the cell face $(\tilde{p}, \tilde{u}, \tilde{v}, \tilde{w})$:

$$\begin{aligned} \tilde{p} &= \frac{1}{\lambda_+ - \lambda_-} (p_- \lambda_+ - p_+ \lambda_- + \lambda_+ \lambda_- (R_+ - R_-)) \\ \tilde{u} &= u_0 + \frac{\tilde{x}}{\lambda_+ - \lambda_-} (p_+ - p_- - \lambda_+ R_+ + \lambda_- R_-) \\ \tilde{v} &= v_0 + \frac{\tilde{y}}{\lambda_+ - \lambda_-} (p_+ - p_- - \lambda_+ R_+ + \lambda_- R_-) \\ \tilde{w} &= w_0 + \frac{\tilde{z}}{\lambda_+ - \lambda_-} (p_+ - p_- - \lambda_+ R_+ + \lambda_- R_-) \end{aligned} \quad (2.24)$$

where R_+ and R_- are the auxiliary functions

$$\begin{aligned} R_+ &= \tilde{x}(u_0 - u_+) + \tilde{y}(v_0 - v_+) + \tilde{z}(w_0 - w_+) \\ R_- &= \tilde{x}(u_0 - u_-) + \tilde{y}(v_0 - v_-) + \tilde{z}(w_0 - w_-) \end{aligned} \quad (2.25)$$

The variables with the subscript $\pm$ are defined from the left and right states depending on the sign of the corresponding eigenvalue, i.e.

$$U_0 = \frac{U_L + U_R}{2} - \text{sign}(\lambda_0) \frac{U_R - U_L}{2} \quad (2.26)$$

$$U_- = U_R$$

$$U_+ = U_L$$

The above steps are also performed for the calculation of the advective fluxes in η and ζ directions. The discretised flux derivatives are then added (including the viscous fluxes in the case of the Navier-Stokes equations) and the system of the equations is integrated in time using a time integration scheme.

2.3.2 High Order Interpolation

In the high order interpolation the two states, left and right, for the intercell characteristic variables can be defined as follows

$$U_{i+1/2,L} = aU_i - bU_{i-1} + cU_{i+1} + dU_{i+2} \quad (2.27)$$

for the left state, and

$$U_{i+1/2,R} = aU_{i+1} - bU_{i+2} + cU_i + dU_{i-1} \quad (2.28)$$

for the right state. The coefficients a, b, c and d need to be determined.

By taking the derivative of the characteristic variable at the cell center for the case of a positive eigenvalue and developing all variables in a Taylor series expansion around the cell center i, yields

$$\left(\frac{\partial U}{\partial \xi}\right)_i = (a-b+c+d)U^{(1)} + [c-a+3(b+d)]U^{(2)} + [c+a+7(d-b)]U^{(3)} + [c-a+15(b+d)]U^{(4)} \quad (2.29)$$

Using the above schemes of different orders of accuracy can be derived. The formulation for the third order interpolation used in this study is presented below.

- Third-order scheme:

$$\begin{aligned} U_{i+1/2,R} &= \frac{1}{6}(5U_{i+1} - U_{i+2} + 2U_i) \\ U_{i+1/2,L} &= \frac{1}{6}(5U_i - U_{i-1} + 2U_{i+1}) \end{aligned} \quad (2.30)$$

The interpolation formula 2.30 can be used for calculating the characteristic variables p_l, u_l, v_l and w_l ($l = 0, 1, 2$) for each of the three eigenvalues.

2.3.3 MUSCL Scheme

The acronym stands for Monotone Upstream-centered Schemes for Conservation Laws, after the name of the first code applying this method as developed by van Leer [103]. To represent the numerical approximation of the solution as a piecewise constant is equivalent to a first-order spatial discretisation. Hence, a linear approximation of the solution on each cell would produce a second-order space discretisation, while a quadratic representation on each cell leads to a third-order spatial discretisation.

The standard MUSCL interpolation can be formulated as

$$U_{i+1/2}^L = U_i + \frac{1}{4} [(1-k)(U_i - U_{i-1}) + (1+k)(U_{i+1} - U_i)] \quad (2.31)$$

$$U_{i+1/2}^R = U_{i+1} - \frac{1}{4} [(1-k)(U_{i+2} - U_{i+1}) + (1+k)(U_{i+1} - U_i)] \quad (2.32)$$

This form of the extrapolation is symmetric about the interface $i + 1/2$, and k is a free parameter between -1 and 1. The interface values can be considered as resulting from a combination of backward and forward extrapolations. In particular $k = -1$ corresponds to a linear one-sided extrapolation at the interface between the averaged values at the two upstream cells i and $(i-1)$

$$\begin{aligned} U_{i+1/2}^L &= U_i + \frac{1}{2}(U_i - U_{i-1}) \\ U_{i+1/2}^R &= U_{i+1} - \frac{1}{2}(U_{i+2} - U_{i+1}) \end{aligned} \quad (2.33)$$

leading to a second order fully one-sided scheme.

For $k = 0$ the interface value is approximated by a linear interpolation between one upstream and one downstream cell:

$$\begin{aligned} U_{i+1/2}^L &= U_i + \frac{1}{4}(U_{i+1} - U_{i-1}) \\ U_{i+1/2}^R &= U_{i+1} - \frac{1}{4}(U_{i+2} - U_i) \end{aligned} \quad (2.34)$$

It should be noted that when $k = 1$ the interface values are the arithmetic mean of the adjacent cell values and the upwind character is totally lost. This corresponds to a central scheme since there is no discontinuity at the cell interfaces. When $k = 1/3$ the MUSCL reconstruction is third-order accurate, however, this does not translate to third-order accuracy in space when the extrapolated variables are used to calculate the fluxes, instead the fluxes reduce to second order accuracy.

Second-order upwind schemes are naturally oscillatory around discontinuities and on there own are not stable enough to avoid over- and undershoots in the numerical solutions. The physical solution to the Euler and Navier-Stokes equations, however, do not seem to allow the appearance of new extrema in the evolution of the flow variables. This can be proven for one-dimensional flows. Therefore, the numerical generation of oscillations is due to the treatment of the second-order approximation, since first-order schemes are free of these over- and undershoots. The extension of Godunov's method from a piecewise constant representation of the state variables to a piecewise linear representation obtains second-order spatial accuracy. This can create problems in that the slope of the linear variation can cause oscillations due to large differences of the slope in one cell compared to the difference of adjacent

mean values.

In order to define a scheme which is non-oscillatory excessive large gradients should be avoided. One way to obtain a numerical scheme which is non-oscillatory is to use limiters. Limiters provide a non-linear correction factor and were initially introduced by van Leer [103] and separately by Boris and Book [59]. The idea behind limiters is to ensure that the interpolation procedure itself cannot produce any new extrema in the data at the cell interface, i.e. that the cell interface value must lie between the values of the neighboring cell.

The MUSCL scheme is not stable on it's own and hence a limiting method must be incorporated. Limiters are generally defined using the parameter r where;

$$r^L = \frac{U_{i+1} - U_i}{U_i - U_{i-1}} \quad r^R = \frac{U_{i+1} - U_i}{U_{i+2} - U_i} \quad (2.35)$$

A limiting function $\phi(r)$ is defined giving a symmetric limited MUSCL scheme in the form;

$$U_{i+1/2}^L = U_i + \frac{1}{4} \left[(1 - k)\phi(r^L)(U_i - U_{i-1}) + (1 + k)\phi\left(\frac{1}{r^L}\right)(U_{i+1} - U_i) \right] \quad (2.36)$$

$$U_{i+1/2}^R = U_{i+1} + \frac{1}{4} \left[(1 - k)\phi(r^R)(U_{i+2} - U_{i+1}) + (1 + k)\phi\left(\frac{1}{r^R}\right)(U_{i+1} - U_i) \right] \quad (2.37)$$

It is important to point out that for a limiting scheme to be high-resolution it must satisfy the TVD (total-variation-diminishing) concept. This will not be discussed here in detail and the reader is referred to the texts of Toro [98] and Drikakis and Rider [78]. A simple way to describe the TVD concept is to consider the equation below in conservation form

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = 0 \quad (2.38)$$

where $f = f(u)$. At any given point along the x axis, both u and its derivative, $\partial u / \partial x$ are known at time level n . An important property of the physical solutions is that $|\partial u / \partial x|$ integrated over the entire domain on the x axis does not increase with time. This integrated quantity is called the total variation, TV, given by

$$TV = \int \left| \frac{\partial u}{\partial x} \right| dx \quad (2.39)$$

Hence, for a physically proper solution, TV does not increase with time. In terms

of a numerical solution , where $\partial u/\partial x$ can be discretised by $(u_{i+1} - u_i)/\delta x$, the TV can be written as;

$$TV(u) \equiv \sum_i |u_{i+1} - u_i| \quad (2.40)$$

Equation 2.40 defines the total-variation in x of a discrete numerical solution. If $TV(u^{n+1})$ and $TV(u^n)$ represent Eq. 2.40 evaluated at time level $n + 1$ and n , respectively, and if

$$TV(u^{n+1}) \leq TV(u^n) \quad (2.41)$$

the numerical algorithm is said to be total-variation-diminishing (TVD). From the above discussion, if a numerical solution is to properly follow the physical behavior of a given flow field, then the scheme should be a TVD scheme

Baines and Sweby [104] outlined a TVD region for the design of slope limiters. They showed that for a negative r the TVD region is a single line $\phi = 0$ and for a positive r the TVD region lies between 0 and $\min\{\phi_L(r), \phi_R(r)\}$

Various limiters have been described in the literature of Toro [98] , namely: van Albada (VA) (van Albada et al.[104]); van Leer (VL) (van Leer [103]); Minbee (MB) (Harten [105]); Superbee (SB) (Roe [106]); and a 5th order limiter (KK5) by Kim and Kim [107] . The formula for the Van Leer limiter used in the current study is

$$\phi_{VL} = \frac{2r}{1 + r} \quad (2.42)$$

2.3.4 WENO Scheme

The Essentially Non-Oscillatory (ENO) scheme was first developed by Harten [105]. He tried to develop a high order method without the use of limiters like the MUSCL scheme. Since a high order polynomial is not stable in terms of oscillations he tried to tackle this with the use of multiple stencils using each time the one that was having the smoothest behavior avoiding in this way discontinuities.

methods, Balsara, Jiang and Liu [108, 109, 110], are an extension of the ENO ones. Instead of using a single stencil they combine all the available stencils creating a weighted average. The weights assigned to each stencil are determined from the individual smoothness. Using this technique they are reaching a higher order of accuracy, nominally $2r - 1$, with r being the number of the sample points. By the use of all the available stencils WENO reach a higher order and stability than the ENO schemes.

In this study a fifth order WENO scheme is utilized using stencils of three cells

in each side of the interface. If we consider a cell i the stencils for the left side of it will be

$$S_0 = (x_i, x_{i+1}, x_{i+2}), \quad S_1 = (x_{i-1}, x_i, x_{i+1}), \quad S_2 = (x_{i-2}, x_{i-1}, x_i).$$

The smoothness indicators for those stencils will be [109]

$$\beta_0 = \frac{13}{12}(u_i - 2u_{i+1} + u_{i+2})^2 + \frac{1}{4}(3u_i - 4u_{i+1} + u_{i+2})^2$$

$$\beta_0 = \frac{13}{12}(u_{i-1} - 2u_i + u_{i+1})^2 + \frac{1}{4}(u_{i-1} - u_{i+1})^2$$

$$\beta_0 = \frac{13}{12}(u_{i-2} - 2u_{i-1} + u_i)^2 + \frac{1}{4}(u_{i-2} - 4u_{i-1} + 3u_i)^2$$

The optimal weights d_i for the extrapolated value are provided by [109]

$$d_0 = \frac{3}{10}, \quad d_1 = \frac{3}{5}, \quad d_2 = \frac{1}{10}$$

from the above smoothness and weights the non-linear weights α and ω are defined

$$\alpha_0 = \frac{d_0}{(\beta_0 + \epsilon)^2}, \quad \alpha_1 = \frac{d_1}{(\beta_1 + \epsilon)^2}, \quad \alpha_2 = \frac{d_2}{(\beta_2 + \epsilon)^2}$$

and

$$\omega_0 = \frac{\alpha_0}{\alpha_0 + \alpha_1 + \alpha_2}, \quad \omega_1 = \frac{\alpha_1}{\alpha_0 + \alpha_1 + \alpha_2}, \quad \omega_2 = \frac{\alpha_2}{\alpha_0 + \alpha_1 + \alpha_2}$$

where ϵ is a very small number to avoid division by zero. Finally the convex polynomial is given by

$$u_{i-1/2} = \frac{1}{6}\omega_0(-u_{i+2} + 5u_{i+1} + 2u_i) + \frac{1}{6}\omega_1(-u_{i+1} + 5u_i + 2u_{i+1}) + \frac{1}{6}\omega_2(2u_{i-2} - 7u_{i-1} + 11u_i) \quad (2.43)$$

Similarly for the right side we get

$$u_{i+1/2} = \frac{1}{6}\omega_0(2u_{i-2} - 7u_{i-1} + 11u_i) + \frac{1}{6}\omega_1(-u_{i+2} + 5u_{i+1} + 2u_i) + \frac{1}{6}\omega_2(-u_{i+1} + 5u_i + 2u_{i+1}) \quad (2.44)$$

2.4 Domain Decomposition

There are many physical problems that exhibit multiple length and time scales in the form, for example, high velocity and temperature gradients, recirculating zones, and phase change fronts, as well as geometric complexities due to irregular shapes of the flow domain. There are many techniques available for generating complex grid geometries to handle such characteristics. These techniques include unstructured, hybrid, chimera, and structured multiblock just to name a few. In the field of Computational Fluid Dynamics (CFD), the methods used to calculate the flow field, and the flow characteristics themselves, place some rather stringent requirements on the computational grid. For most CFD applications, the structured multiblock technique is usually preferred over the others for its ability to both resolve the desired characteristics of the flow and provide for a fair amount of computational efficiency. The multiblock method is an approach which can break a complicated geometry into sub domains (blocks) with simple shapes. Structured grids can then be generated within each block independently. It is not always required for the grid lines to be continuous across the block interface. This is dependent on the numerical algorithm. There are certain advantages of using multiblock methods:

- Multiblock methods can reduce the topological complexity of a single structured grid system by employing several grid blocks, permitting each individual grid block to be generated independently so that both geometry and resolution in the boundary region can be treated more satisfactorily;
- More freedom is allowed in the generation of grid lines, since grid lines can be discontinuous across the block interface, and local grid refinement can be conducted more easily to accommodate different physical length scales present in different regions. More grid lines can be put in high gradient region without wasting computational resource in other zones.
- With structural grids used in each block, standard structured flow solvers can be used, which greatly obviate the needs of complicated data structure, book keeping and algorithm design.
- This approach provides a natural routine for parallel computing.

Chapter 3

Grid Convergence, Numerical Methods and Validation with the Experiment

3.1 Case Description

In the following sections the physical and numerical behavior of various high order methods and the validations of the simulations will be discussed. Simulations at several Reynolds numbers will be conducted ranging from $Re=50$ to $Re=900$. Those Reynolds numbers cover a wide range of physical behaviors starting from a steady symmetric flow for Reynolds number 50 to unsteady with local transitional behavior at $Re=900$. The geometry examined here, figure 3.1, is a suddenly expanded channel with expansion ratio $ER = H/h = 3$ similar with experiment of Fearn et al [11]. The characteristic length scale of the flow is the upper channel height $h = 1$ and the characteristic velocity is the U_{max} where is equal with the maximum velocity at the parabolic velocity profile at the inlet so the Reynolds number can be calculated as

$$Re = \frac{U_{max}h}{\nu}$$

where ν is the kinematic viscosity.

The dimensions of the geometry are for the inlet channel 3 step height in length 1 in height and 3 in width. The small length for the inlet channel is justified from the laminar inlet Reynolds number and the fully developed parabolic profile as boundary condition. For the main channel the length is 170 step heights, 3 in height and 3 in width. This grid is divided into 91 blocks (90 blocks for the main channel and one block for the inlet) for the purpose of parallel computations. It should also be noted

that the total number of blocks does not limit the flexibility of the simulations by forcing the computations to be conducted with 91 processors. The numerical code is parallelised in such a way that allows more blocks to be assigned to one processor if necessary thus the total number of blocks is only the upper limit in terms of number of processors.

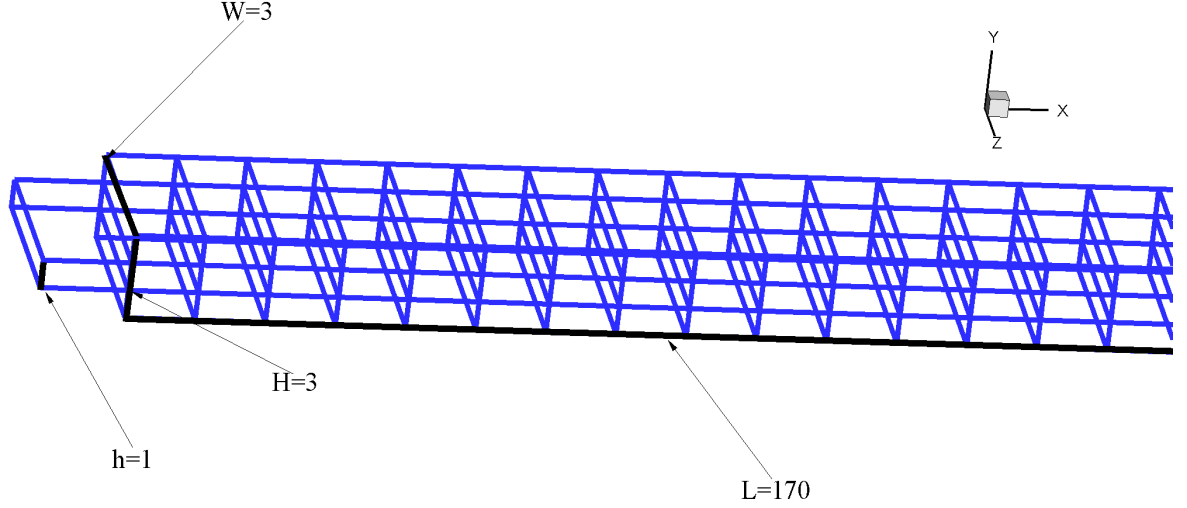


Figure 3.1: Sudden expansion geometry

This configuration had been chosen for the numerical and the physical study at the mid-plane in order for finer grids to be utilized and higher accuracy with respect to the existing computational capacity to be achieved. The spanwise dimension length was selected in order to allow the development of the 3D flow while keeping in mind the resource constraint.

3.2 Numerical tools

Since for the most part of the examined Reynolds numbers range the flow have time depended behavior mean flow statistics must be calculated. After the flow has been statistically developed the instantaneous values of the whole domain are used for each timestep to calculate the mean flow field of the domain. By comparing different time samples was found that a time sample of 300 non-dimensional time is sufficient long for the mean values to be statistically sample independent. With the use of those mean values also the mean velocities are also being calculated, figure 3.2. The mean and rms values are being dimensionless by the characteristic velocity of the flow u_{max} .

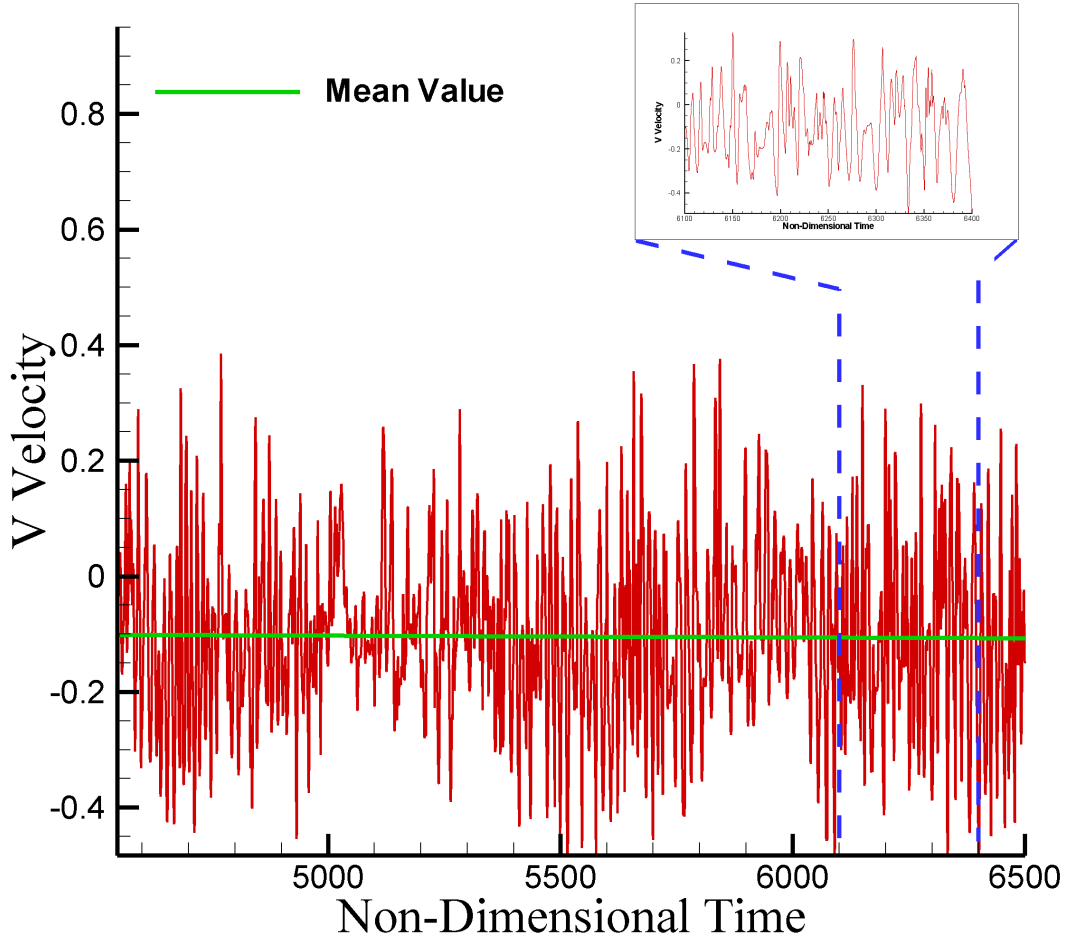


Figure 3.2: Averaging Window

This process provide, after the completion of the simulation, an accurate mean flow field for all the variables allowing the better understanding of the flow and providing a huge amount of data for further analysis. In case of separated flows where vortex areas appear a common characteristic of those flows that is measured is the length of those vortices. A way to accurately calculate the length is by examining the value of the streamwise velocity at the first cell from the wall and identify the changes in the sign signaling a separation/reattachment zone. This is the technique used in this study. Another feature of the mean flow is the mean position of the vortex cores. The calculation of the position was implemented with the use of the commercial software package Tecplot 360. This program uses the vorticity vectors to pinpoint the location of the vortex cores.

In order to obtain deeper insight into the distribution of kinetic energy among the length scales present in the flow, the one-dimensional energy spectrum $E(f)$ is employed, where f is the frequency. Details on the calculation of the energy spectrum can be found in the book of Hinze [111]. Kolmogorov [8] found that the kinetic energy spectrum for homogeneous and isotropic turbulence is proportional to $k^{-5/3}$ in the inertial subrange. This power law is widely used to prove the existence of a fully developed, turbulent flow. Although this flow has not homogeneous and isotropic turbulence and the Reynolds numbers examined here are not high enough to have turbulence the kinetic energy spectrum and the Kolmogorov criterion of $k^{-5/3}$ will be used to demonstrate the transitional nature of the flow and its evolution along with the Reynolds number as well as the physical behavior of the larger structures by the examination of the primary frequency.

For the better understanding of the physical behavior of the flow the coherent vortical structures need to be identified. We achieve that we the use of Q criterion. This method was developed by Hunt et al [112] and is essentially the second velocity gradient tensor ∇u . The Q criterion is calculated by

$$Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})$$

where $\Omega_{ij} = (\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i})/2$ and $S_{ij} = (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})/2$ are the antisymmetric and symmetric components of ∇u . From the formula can be concluded that the Q is the difference between the rotational and strain rate and the $Q > 0$ is indicating the present of a vortex area.

3.3 Simulation Parameters

For the completion of the following simulations a series of numerical techniques and parameters were utilized. For the dual time stepping required for the incompressible set of equations a 4th order Runge-Kutta methods was used for the inner time stepping to induce the incompressibility effect, (pseudo time). The evolution in real time was implemented by second order backward differencing (outer time stepping) . The time step is adaptive and was calculated with the use of the CFL parameter before each time advancement. Generally, as high is the value of the CFL the faster the time advancement providing it abides with the condition $CFL \leq 1$ but for the reason of the numerical stability of the simulation a smaller value is chosen. In the bulk of the simulation the value of the CFL is 0.7. In a similar way for the inner time stepping the CFL is 0.5 for the calculation of the inner time step used to converge to the incompressibility limit. A lower value than the real time CFL was selected to ensure the numerical stability due to the reduce stability region of the 4th order Runge-Kutta method. Another important aspect of the simulations is the artificially compressibility parameter β . Also the value of this parameter, despite some approximations for specific cases [78], is case depended. For the conducted simulations a value of β is found to be a sufficient compromise in terms of convergence rate. The convergence rate used as incompressibility criterion for all variables is 10^{-3} . For the simulations in total three different reconstruction methods were examined. Those methods are a second order MUSCL scheme with the use of the van Leer limiter condition, a third order interpolation and a fifth order WENO scheme.

3.4 Boundary Conditions

Since the numerical methods examined require the use of ghost (fictitious) cells at the boundaries three cells were added in each boundary surface to accommodate the calculation of the fluxes at the boundary conditions. The boundary conditions used in those simulations are for the inlet a parabolic profile for the streamwise velocity and second order extrapolation for the pressure,

$$u(y) = 1 - \left(\frac{y - (y_{max}/2)}{y_{max}/2} \right)^2,$$

$$v = 0,$$

$$w = 0,$$

$$\frac{\partial^2 P}{\partial x^2} = 0$$

The upper and lower surfaces are walls so a no-slip boundary condition is applied. The non-slip boundary condition is formulated on the fictitious cells by applying for the normal component of the velocity minus values and using extrapolation for the other two components and the pressure.

For $i=0$ to $np-1$ (number of fictitious cells)

$$u_i = -u_{2*np-i-1}$$

$$v_i = 2 * v_{i+1} - v_{i+2}$$

$$w_i = 2 * w_{i+1} - w_{i+2}$$

$$P_i = P_{np-i-1}$$

For the outlet a fix value is given to the pressure and extrapolation is applied to the velocities. Finally for the spanwise boundaries a periodic condition is applied. Also due to the fact that multiple blocks are present in the geometry an interchange boundary was applied to facilitate the exchange of information between the different blocks. At the appendix A a comparison between periodic and wall boundary conditions in the spanwise direction for the mid-plane can be found presenting the effects the existence of side walls have on the flow.

3.5 Grids

Grid	Dimensions	Grid Size	Δx (at $x < 50$)	Δy	Δz
Coarse Grid	600X60X20	720,000	0.116	0.05	0.15
Medium Grid	800X80X20	1,280,000	0.085	0.0375	0.15
Fine Grid	1000X100X20	2,000,000	0.07	0.03	0.15

Table 3.1: Table of Grid dimensions

Three different grids were generated for the simulations in order to conduct a grid convergence study. The grid was generated using a custom made code written in FORTRAN computing language. For the wall normal and spanwise direction grid with uniform spacing was used. The absence of clustering in the wall normal direction near the wall does not have a detrimental effect in the capturing of the wall boundary layer since the grid in this direction is very fine for those Reynolds number, as can be seen in table 3.1, and the transitional effects that appear are more prominent at the middle of the channel, where high resolution is needed, and not near the wall region. At the x direction clustering was applied with Δx being constant for 50 step heights downstream where most of the flow phenomena occur and then clustering is implemented in order to reduce the computational cost of the simulations and provide denser grid near the expansion area.

The X coordinates are calculated from the following formulas

$$\begin{cases} \text{for } X \leq 50 & X_i = X_{i-1} + \Delta x \\ \text{for } X > 50 & X_i = (X_i - 50)^{\log(170-50)/\log(70-50)} + 50 \end{cases} \quad (3.1)$$

The dimensions of the three grids are given by the table 3.1

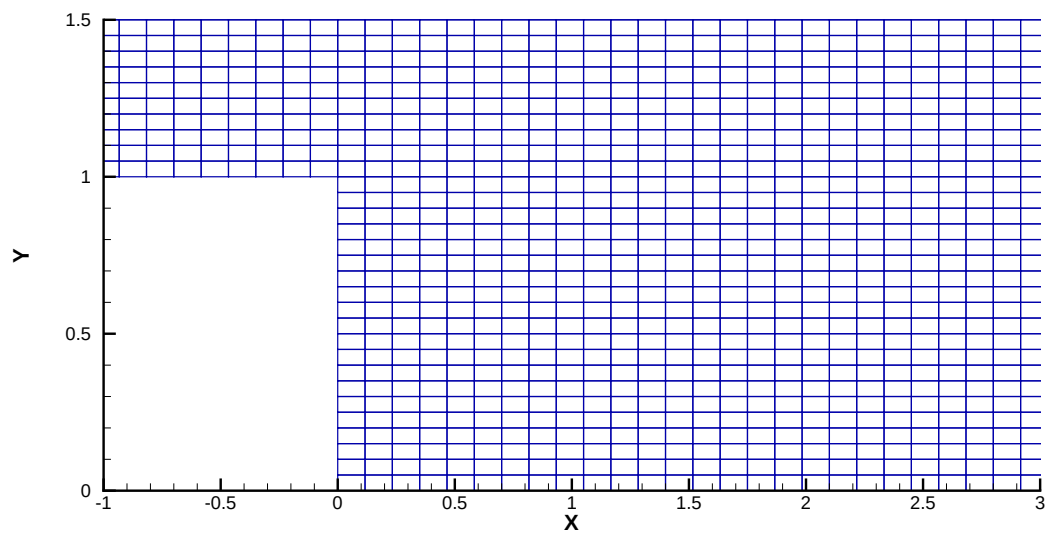


Figure 3.3: Coarse grid

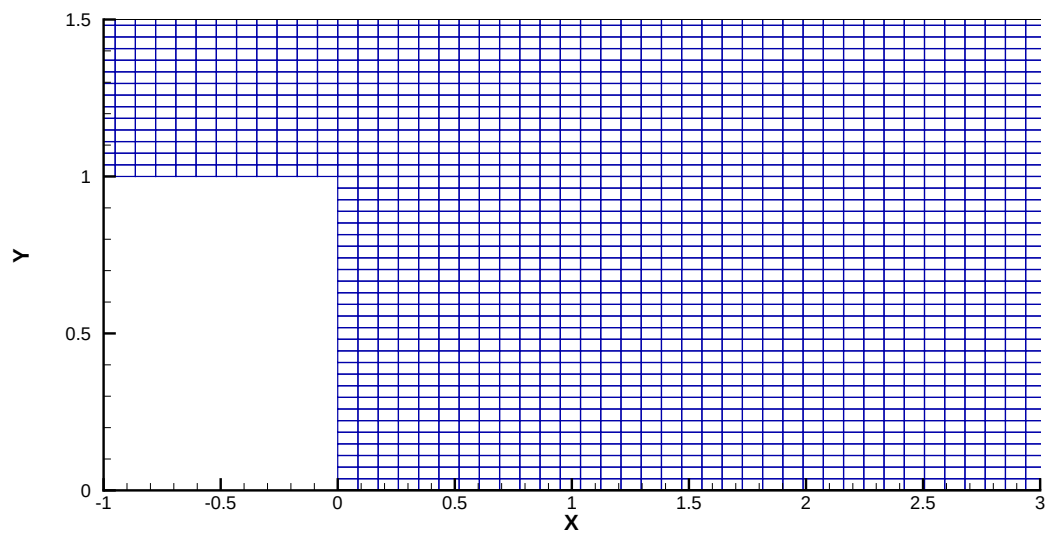


Figure 3.4: Medium grid

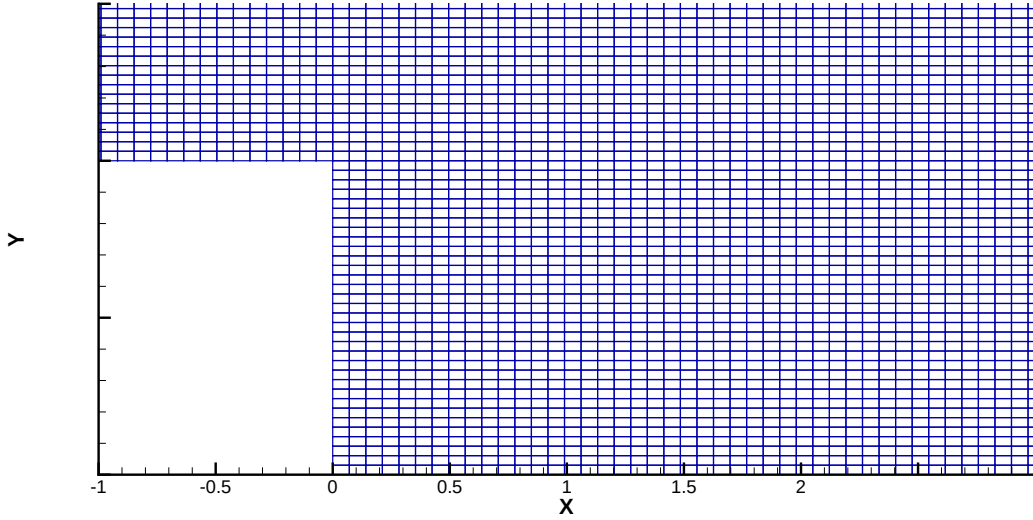


Figure 3.5: Fine grid

3.6 Validation with the Experiment

For the initial validation with the available experimental data the medium grid will be used in combination the fifth order WENO reconstruction scheme. This combination will also be used for the bulk of the simulation. In the next sections a grid convergence study and numerical study for two Reynolds numbers will also be conducted to justify this choice.

At low Reynolds numbers the fluid flow was found to separate symmetrically with equal sized vortices attached to the upper and lower walls as expected. As Reynolds number was increased the separation regions increased in size and upon exceeding the critical Reynolds number for symmetry-breaking bifurcation the flow became unstable with an instability manifested as an asymmetric separation of the flow. Although the solution is regarded to be unstable the flow remained steady.

Fearn et al. [11] provide experimental data in the form of mean velocities values along Y lines at various positions for low Reynolds number flows and in the form of primary frequencies at higher Reynolds numbers. This data has been compared to the present results and are presented in Fig. (3.7)(3.9)(3.11). For $Re = 50$, figure 3.6, the flow is shown to separate symmetrically and agrees well with the experimental data from Fearn et al. [11]. The size of the recirculation zones is also captured well by the numerical simulation and can be seen from the region of

flow reversal represented by the negative flow velocity. The flow has regained the fully developed parabolic profile at a downstream position of 10 step heights which corresponds well with the experimental data. As the flow separates asymmetrically at $Re = 120$ the profile shows that at downstream positions of 1.25 and 2.5 the velocity is negative close to both the lower and upper wall since both positions lie in the region of the recirculation zones. As we move further down the channel at a position of 5 step heights the velocity profile is positive at the lower wall but negative at the upper wall. This shows that the flow has separated asymmetrically with a larger recirculation vortex situated at the upper wall (see Fig. 3.8). Due to the asymmetric separation, the flow takes longer to regain the fully developed symmetric profile which is achieved at a downstream position of 20 step heights. The percentage difference between the computed results and the experimental data is within 5%. Figure (3.10) shows the velocity profile corresponding to $Re = 280$. The main feature of the flow at this Reynolds number is the third recirculation region situated at the top wall downstream from the small upstream recirculation region. The third recirculation area is shown at a downstream position of 20 step heights by a positive velocity at the lower wall and negative velocity at the upper wall. The velocity profiles at downstream positions of 5 and 10 step heights show an inconsistency when compared to the experimental data. This is primarily due to the difference in size and shape of the separation vortex at the lower wall obtained with the computations compared to the experiments. A fully developed profile is attained at a downstream position of 40 step heights.

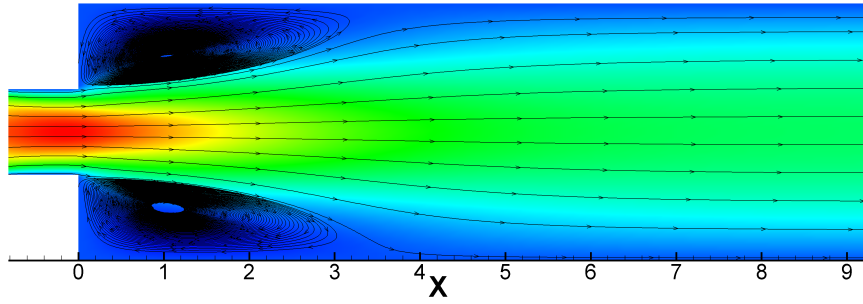


Figure 3.6: Streamlines of velocity at Re 50 in the mid-plane at Z direction

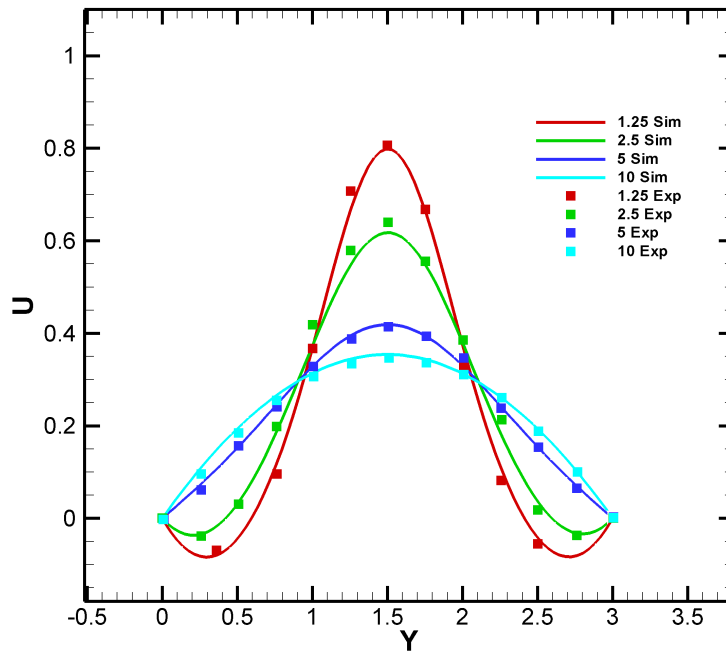


Figure 3.7: Comparison of the U velocities between simulation and experiment at Re=50 in the mid-plane at Z direction

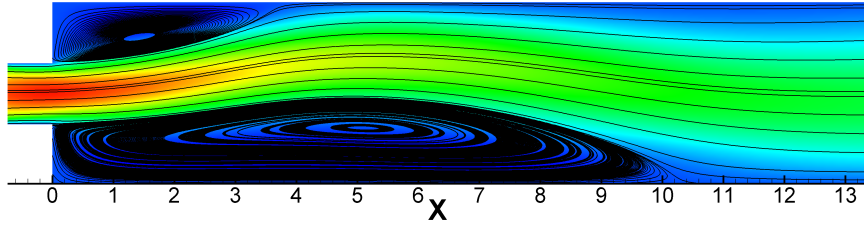


Figure 3.8: Streamlines of velocity at $Re=120$ in the mid-plane at Z direction

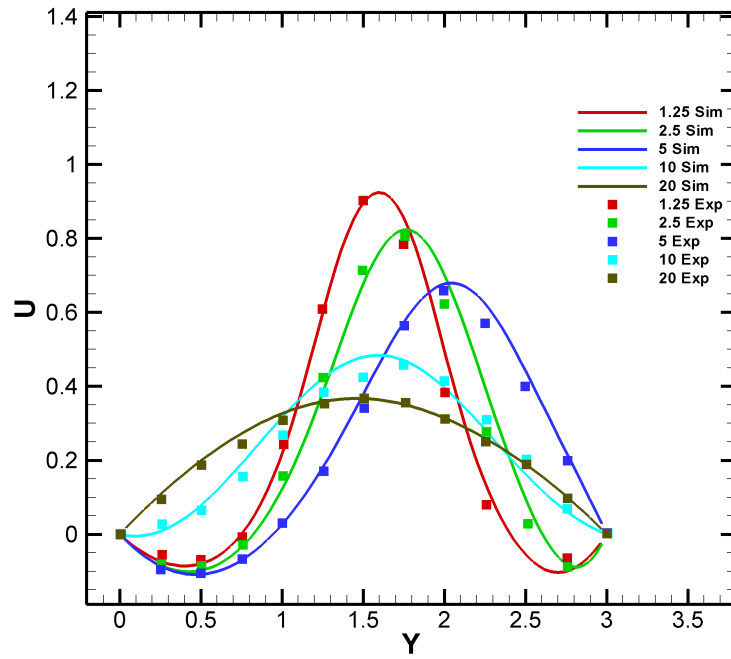


Figure 3.9: Comparison of the U velocities between simulation and experiment at $Re=120$ in the mid-plane at Z direction

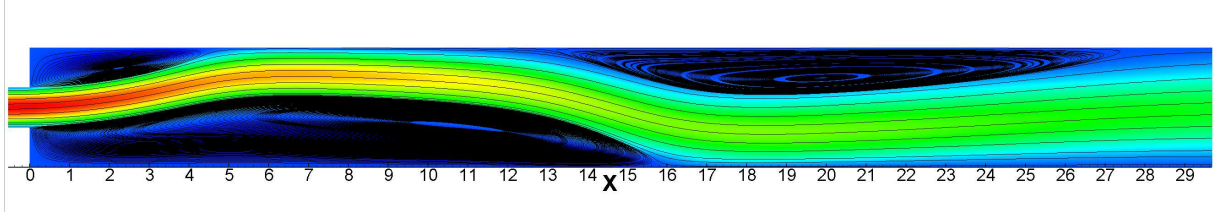


Figure 3.10: Streamlines of velocity at Re280 in the mid-plane at Z direction

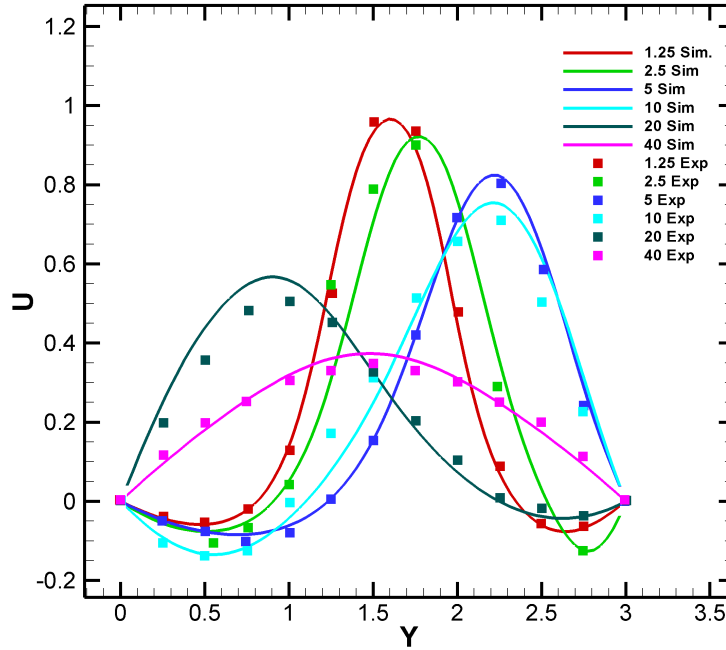


Figure 3.11: Comparison of the U velocities between simulation and experiment at Re=280 in the mid-plane at Z direction

As the Reynolds number exceeds 300 the flow becomes time-dependent. At those Reynolds numbers the three recirculation areas described before are still evident but their length vary with time. Moreover the main characteristic of that region is the shedding of vortices from the third recirculation area. The frequency, amplitude and position of the shedding vary with Reynolds number.

At Reynolds number 350, just a little over the time-dependence limit, the shedding of the vortices occur at the end of the third recirculation area, which end approximately 30 step heights downstream. At this Reynolds number, so close to

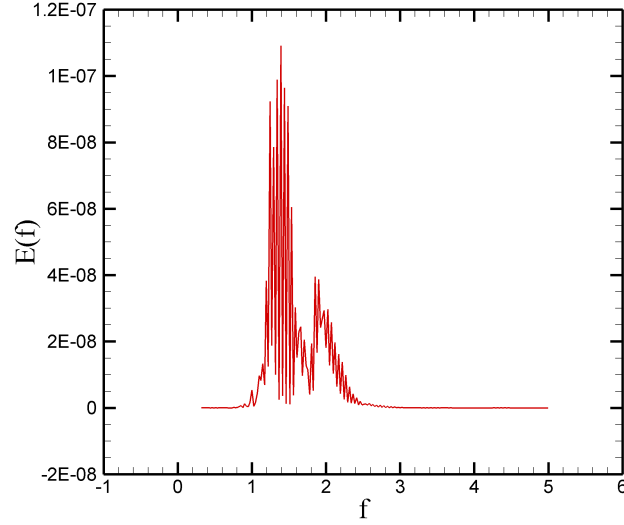


Figure 3.12: Primary frequency of the kinetic energy at Reynolds 350

the onset of the unsteadiness, the amplitude of the vortices tend to be small. With the rise of the Reynolds number to 412 the position of the shedding change from the end of the third recirculation area to the middle of it. Also the amplitude to the vortices is now more pronounced. At the next chapter the physical behavior of the flow, both in mean values and time dependent terms will be discussed in more detail.

For those aforementioned Reynolds numbers Fearn [11] provide us with experimental results, mainly in the form of kinetic energy spectrum and primary frequencies. Primary frequency of the velocity is a adequate measure of the unsteady behavior of the flow since can capture the shedding of the vortices. In order to compare the experimental data and the ones obtained from the numerical simulations the latter need to be re-dimensionilised. For the re-dimensionilisation process the experimental data for the maximum inlet velocity and the length of the step height was used. The experimental step height length is equal to 0.004 meters and the maximum inlet velocity for Reynolds 412 is 0.105 meters per second. For Reynolds 350 Fearn [11] gives a primary frequency equals to 1.3 ± 0.1 Hz a number that is in accordance with the numerical results giving a frequency of 1.4 Hz as can be seen in figure 3.12. For Reynolds 412 the experiment gives a primary frequency of 1.7 ± 0.1 Hz. Also in this case the numerical results are in good agreement with a frequency of 1.7 Hz as it is shown in figure 3.13.

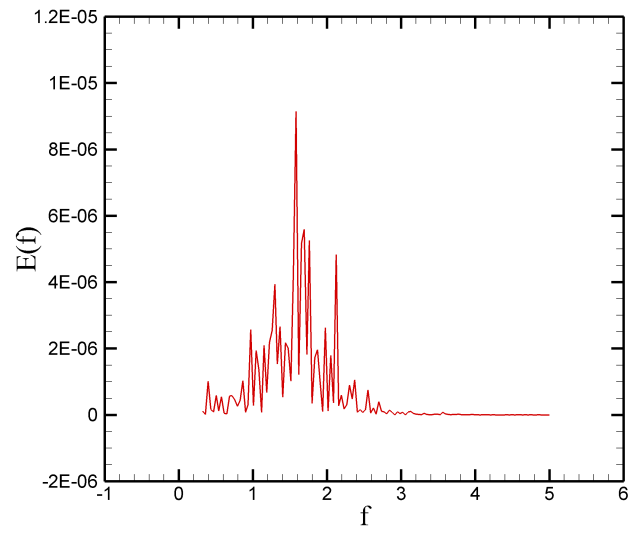


Figure 3.13: Primary frequency of the kinetic energy at Reynolds 412

3.7 Grid Convergence Study

In this section a comparison between the three grids previously generated will be conducted in order to justify the selection of the medium grid for the bulk of the simulations. The study was conducted in two unsteady Reynolds numbers 412 and 900. At Re 412 the flow has unsteady laminar characteristics and also experimental data are provided by Fearn et al [11] and at Re 900 the flow has a clearly transitional chaotic behavior and is the highest Reynolds number examined. Those are the reasons for the selection of these two Reynolds numbers for the grid convergence study. The flow will be examined with the help of velocity streamlines comparing the vortex sizes between the grids and with the use of lines at the X direction comparing mean velocities and rms of velocities. Also primary frequencies will be used for the comparison and at Re 412 those will be compared with the experimental data in the form of primary frequencies provided from Fearn [11]. The positions of those lines are characteristic of the flow and are positioned at $X=0.5$, $X=1.5$ and $X=2.5$, figure 3.14. The first line is crossing the large recirculation area near the lower wall of the channel. The second line is positioned in the middle of the channel and the third near the top wall crossing the two recirculation areas located there. The WENO fifth reconstruction method was used in the simulation of those cases having the higher accuracy from the methods examined. The comparison of the numerical methods will be presented in the next subsection.

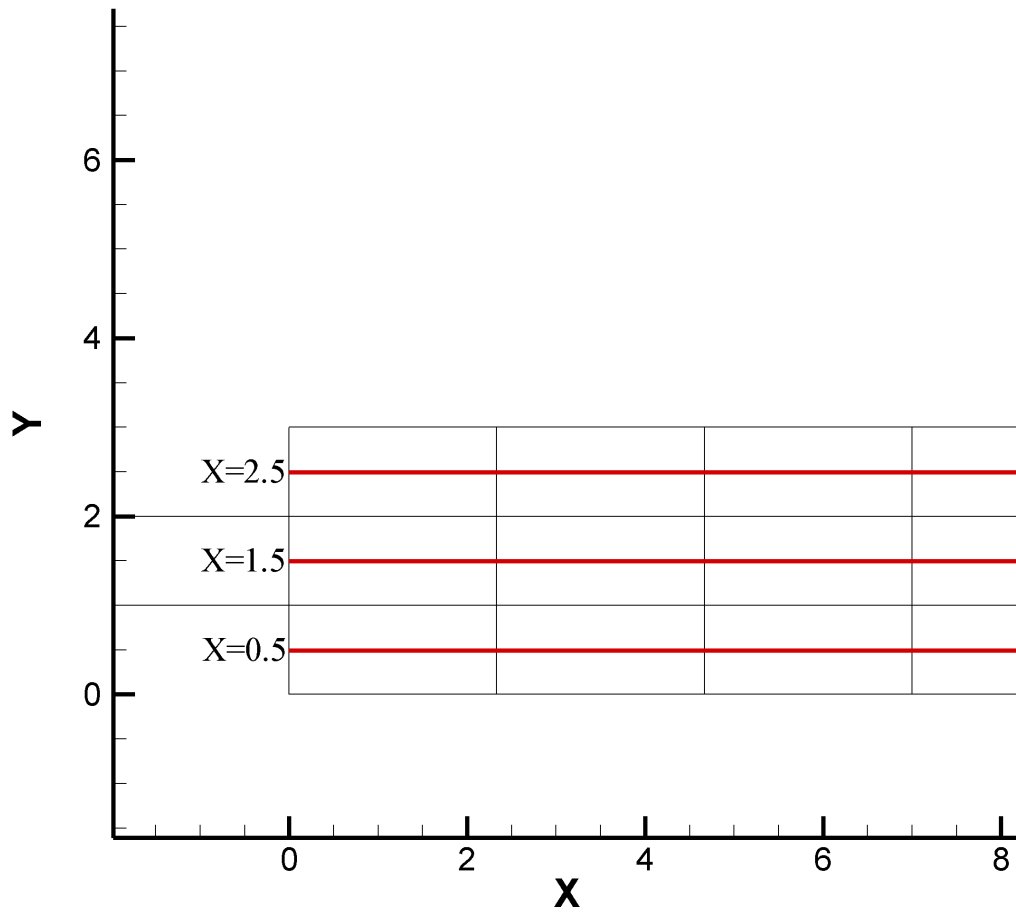
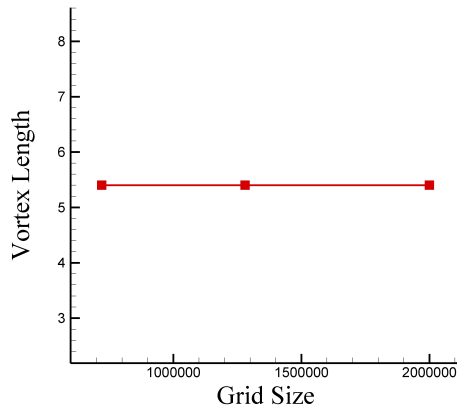


Figure 3.14: Lines along the domain

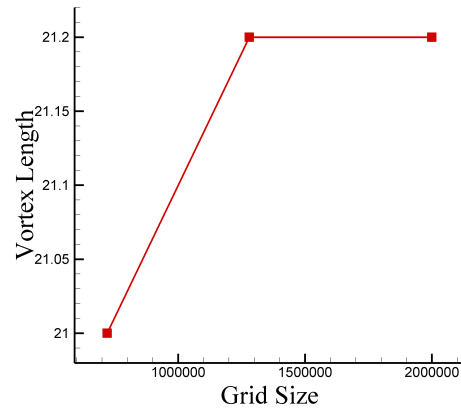
3.7.1 Re 412

At this Reynolds number the flow is unsteady with main characteristics the existence of three major vortex areas which are stable enough in time to be captured by the mean flow field. In terms of the unsteady behavior the main characteristic is the shedding of the vortices that occur near the end of the third recirculation area. As figure 3.16, 3.15 and table 3.2 show, the length of the first recirculation area is not affected by the grid size as expected since it is located in one of the most steady areas of the flow at the expansion and away of the vortex shedding at the third

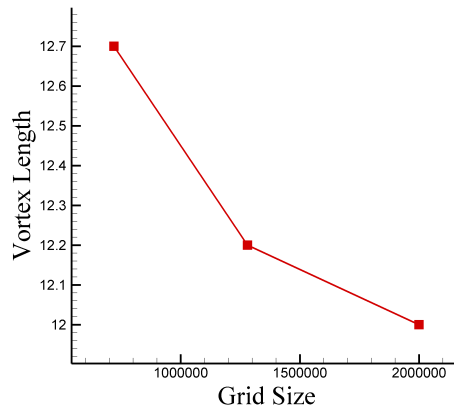
recirculation area. The length of the second vortex area has also minimal change between the grids with less than 1% difference between the coarse and the medium and fine grids. The difference between the grids can be more clearly be seen in the length of the third vortex area where the difference between the coarse and the medium, 5% , is more pronounced than the difference between the medium and the fine, 2%. The fact that the area that is most affected by the grid size is the third recirculation area is not surprising since as a dynamic separation feature is more prone to variations as the different grids provide different resolutions and also due to the vortex shedding occurring there the vortex length variates more in time than the other vortex features of the flow. Overall the differences between the three grids in terms of vortical lengths are below 5% and not significant enough.



(a) First vortex area

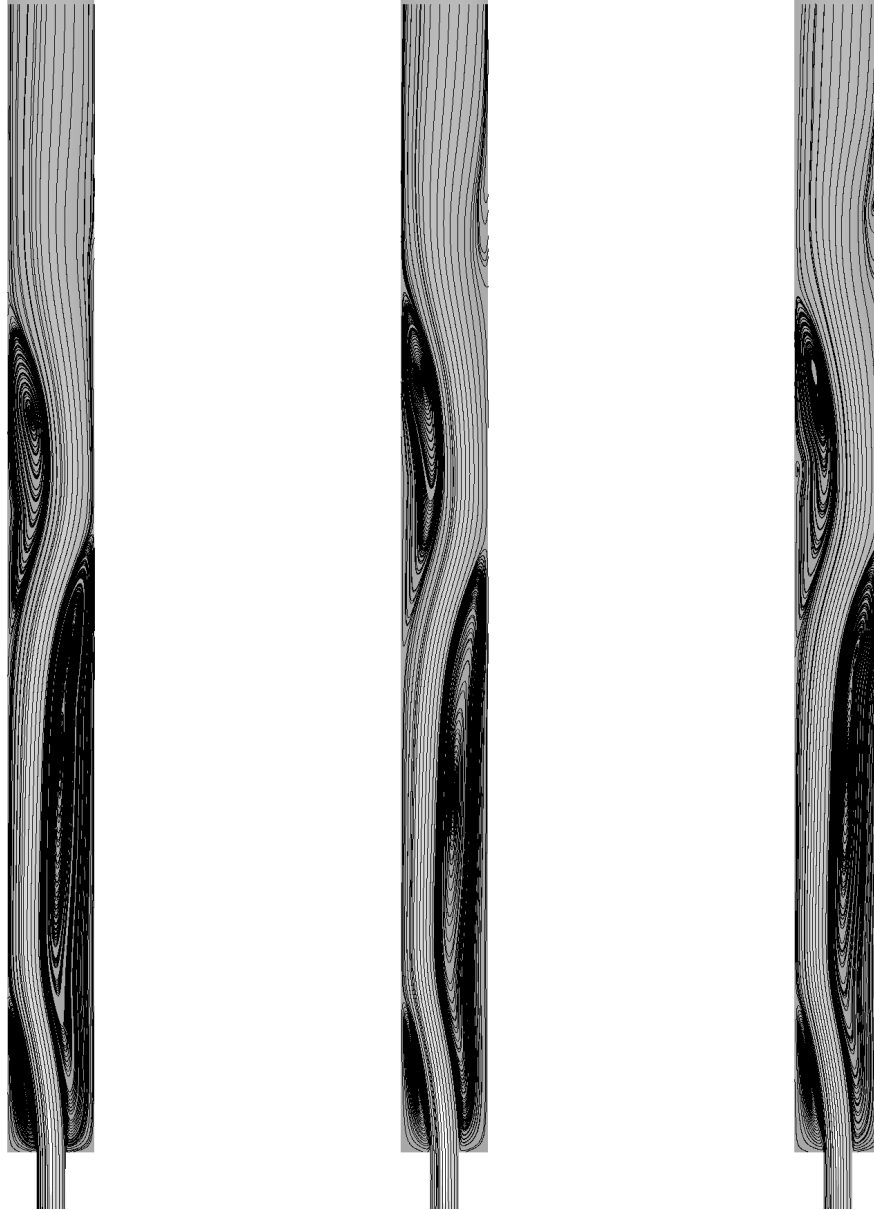


(b) Second vortex area



(c) Third vortex area

Figure 3.15: Evolution of the vortex sizes with the increase of the grid resolution for Re 412



(a) Streamlines of velocity at mid-plane of the coarse grid (b) Streamlines of velocity at mid-plane of the medium grid (c) Streamlines of velocity at mid-plane of the fine grid

Figure 3.16: Comparison of the vortex sizes between the three grids at Re 412

The mean values of streamwise and wall normal velocities as well as the U and V rms are presented in the figures 3.18 and 3.19. The first line at $X = 0.5$ is near the lower wall and is crossing the large recirculation area. In terms of the U and V mean velocities we can see that up until 20h downstream the three grids have essentially

Grid	First vortex	Second vortex	Third vortex
Coarse	5.4 h	21.0 h	12.7 h
Medium	5.4 h	21.2 h	12.2 h
Fine	5.4 h	21.2 h	12.0 h

Table 3.2: Vortex sizes for the three grids at Re 412

the same values for the velocities. This area includes the whole lower recirculation zone and in this Reynolds number the behavior of the large recirculation area is not significantly changing with time. At 30h downstream and up to 50h there is a small difference between the grids with the medium and fine grid are a lot closer than the coarse. This is the area after the third recirculation where the vortices released area traveling downstream and is the area of the highest unsteadiness in the domain.

The rms of velocities up to 20 step heights, the end of the lower vortex, for both U and V rms are very close for all the grids and have small values indicating a more stable flow at the large recirculation area. From 20h to 40h, where the maximum value of RMS is reached, the medium and fine grid are in better agreement than the coarse grid but the trend is the same for all the grids. The maximum value is at about 35h after the end of the third vortex area. After 40h a discrepancy is appearing since the U and V rms of the medium grid return to zero slower than the other two grids although the trend for all the grids are the same. This could be explained since this is the area where due to the shedding of the vortices from the third recirculation are traveling making it very sensitive to grid effects and also after 50h the uniform grid spacing up to that point is changing to a clustered one affecting the accuracy of the grids in this region.

The second line is positioned in the middle of the channel. Examining the mean velocities the grid convergence is apparent with very small differences (below 1%) at the U velocity and at V velocity component. The only discrepancy is a small difference (about 10%) at the values between the coarse grid and the other two at about 30h downstream. This is the area at the end of the third vortex. Examining the behavior of the rms of velocities at U rms the agreement for the three grids is very good for the whole length of the line and with the medium and fine grid values being closer than the coarse reaching the maximum value at 35h at the end of third vortex area. Similarly the V rms has its peak value at 35h after the third vortex but the same discrepancy as before is evident after 35h for the medium grid.

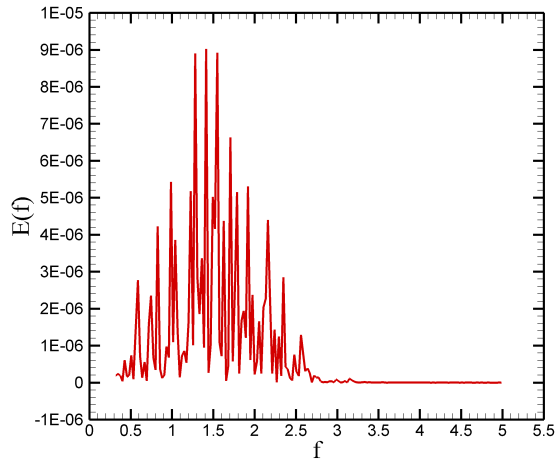
Finally the third line is crossing the two recirculation areas at the top wall as well as the area where the vortices that are shedded from the third recirculation area

Reynolds Number	Coarse Grid	Medium Grid	Fine Grid
412	1.5 Hz	1.66 Hz	1.72 Hz

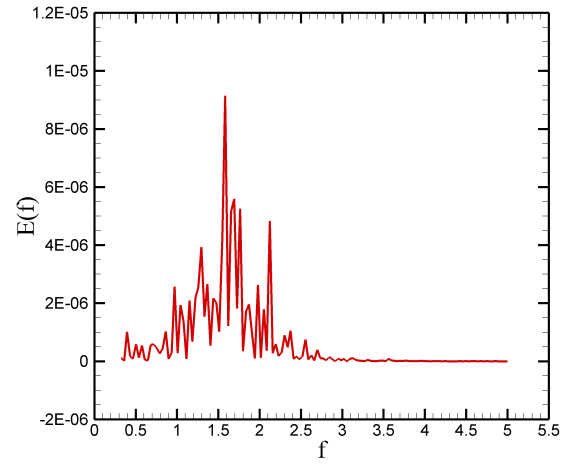
Table 3.3: Primary frequencies

are traveling downstream. The behavior of the flow at this line is similar to the two previous ones. The fine and medium grid are in good agreement in terms of mean velocities with the coarse one having some discrepancies after 30h downstream. Up to 6h, at the first vortex, the values between the grids are identical confirming the same size of those vortices reported before. Differences are present between the coarse grid and the other two from 25h and afterward indicating the difference in third vortex size. Regarding the U and V rms also a similar behavior arise the peak values is at 35h after the third vortex with the discrepancy between the medium grid and the other two after 40h.

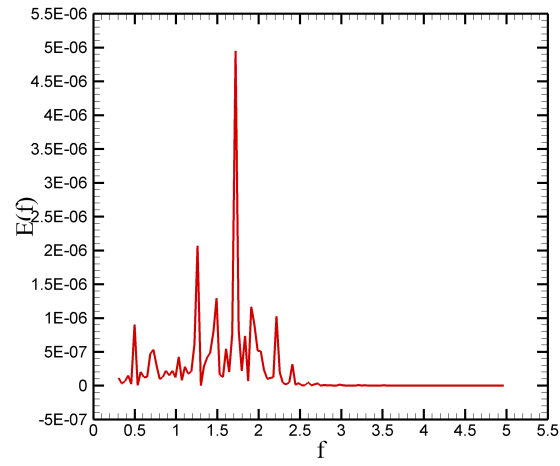
Next a comparison will be presented between the three grids and the experimental results of Fearn et al [11]. The experiment of Fearn et al for this Reynolds number provide data for comparison in the form of primary frequencies. As it was mentioned before the primary frequency for this Reynolds number from the experimental data is 1.7 Hz. Table 3.3 and Figure 3.17 present the primary frequencies captured from the three grids. It can be seen that the values for the medium and fine grid are very close to the value provided where the coarse grid under evaluate this value although is still close to it. This provide another evidence regarding the suitability of the medium and fine grid compared to the coarse one.



(a) Primary frequency of the kinetic energy of the coarse grid

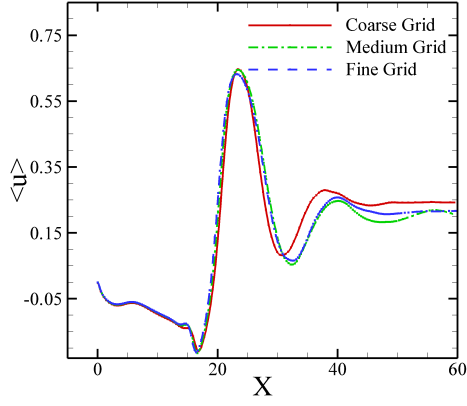


(b) Primary frequency of the kinetic energy of the medium grid

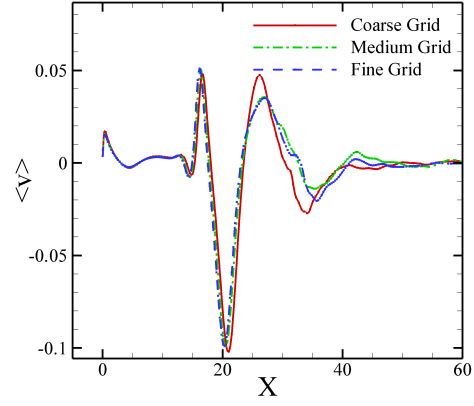


(c) Primary frequency of the kinetic energy of the fine grid

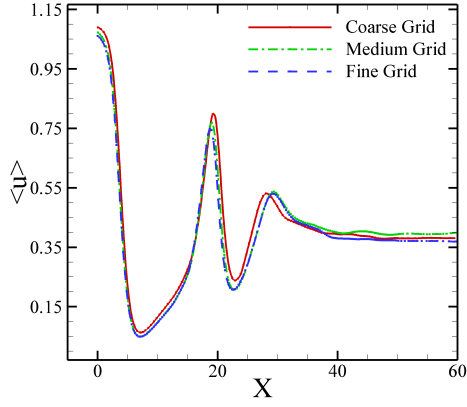
Figure 3.17: Comparison of the primary frequencies between the three grids at 22h downstream



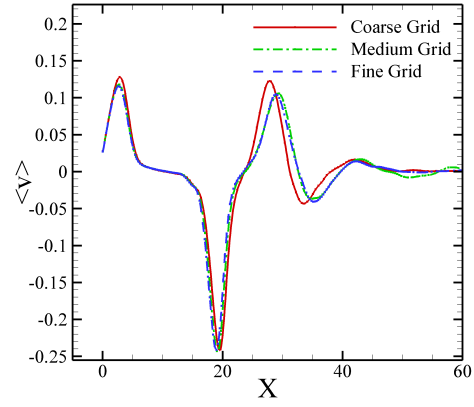
(a) streamwise velocity at $Y = 0.5h$



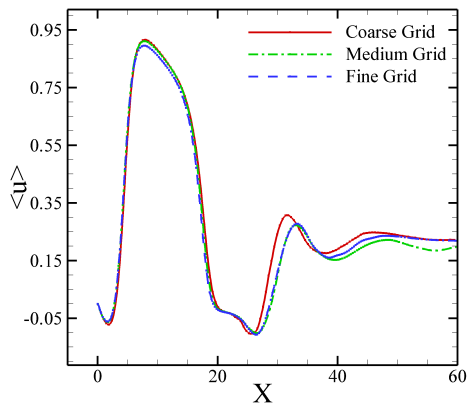
(b) wall normal velocity at $Y = 0.5h$



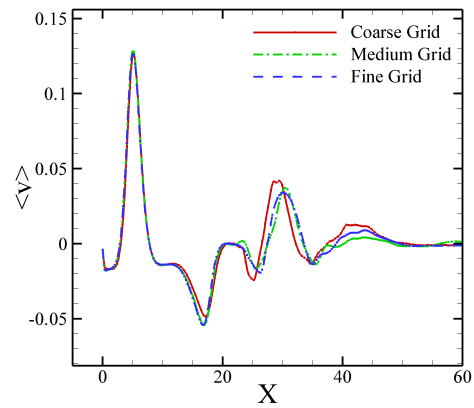
(c) streamwise velocity at $Y = 1.5h$



(d) wall normal velocity at $Y = 1.5h$

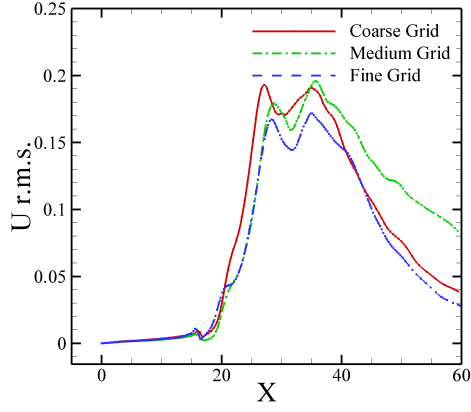


(e) streamwise velocity at $Y = 2.5h$

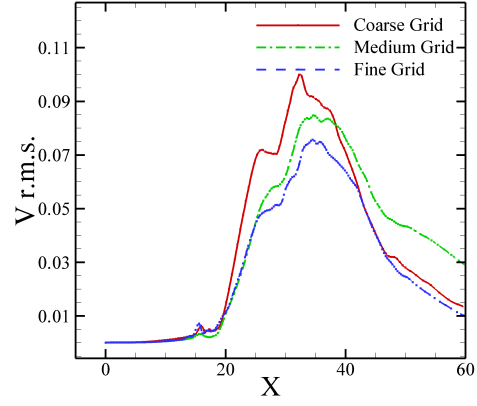


(f) wall normal velocity at $Y = 2.5h$

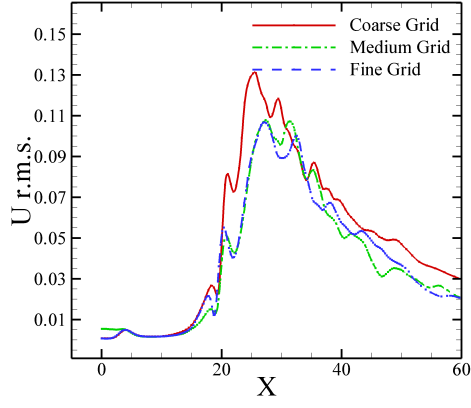
Figure 3.18: Velocity profiles at various positions in flow direction for $Re\ 412$



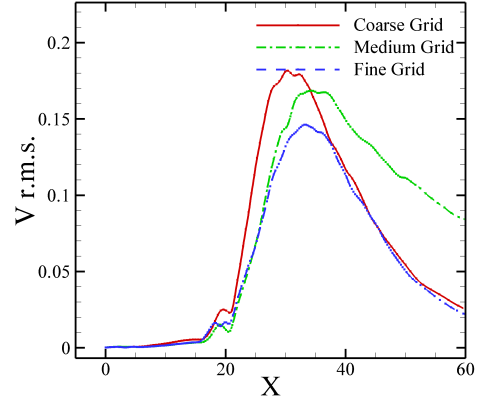
(a) U rms at $Y = 0.5h$



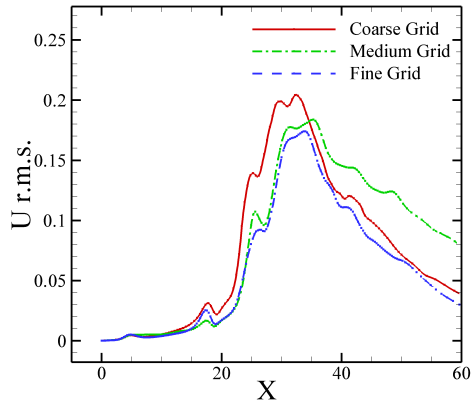
(b) V rms at $Y = 0.5h$



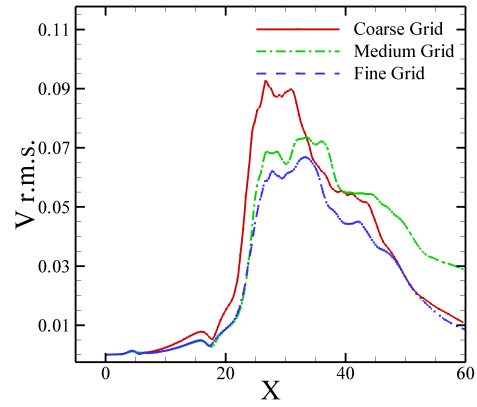
(c) U rms at $Y = 1.5h$



(d) V rms at $Y = 1.5h$



(e) U rms at $Y = 2.5h$



(f) V rms at $Y = 2.5h$

Figure 3.19: Rms of velocity at various positions in flow direction for $Re\ 412$

Grid	First vortex	Second vortex
Coarse	6.3	14.5
Medium	6.5	15.2
Fine	6.6	15.4

Table 3.4: Vortex sizes for the three grids at Re 900

3.7.2 Re 900

At this Reynold number ,which is the highest examined, the behavior of the flow has changed considerably. As the figures of the streamlines of the velocity show two distinctive vortex structures appear. The third recirculation area evident at Reynold 412 no longer clearly appearing and the large recirculation area at the lower wall has retreated upstream near the expansion. This is due to the fact that the flow has become increasingly unsteady and that the shedding of the vortices now occur at the end of the large recirculation area. Those factors caused the third vortex area to be not clearly evident at the mean flow field. A comparison between the velocity streamlines of the three grids reveal the grid convergence between the three grids. This can be seen in table 3.4 and at figures 3.21 and 3.20. The size of the first vortex is increasing with the grid resolution where grid converge is present since the rate of the increase is slower as the grid resolution is increasing. The second vortex has the same behavior with the difference in the size between the coarse and medium grids being a lot higher than the one between the medium and the fine.

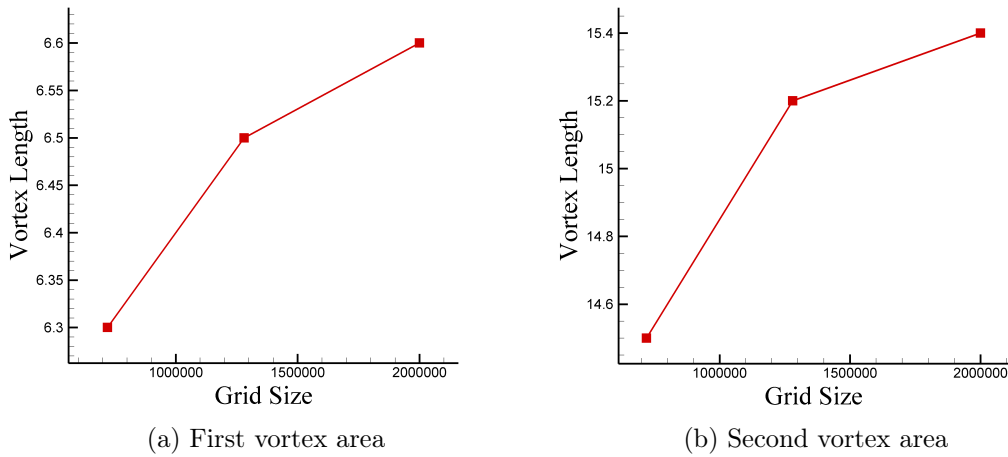
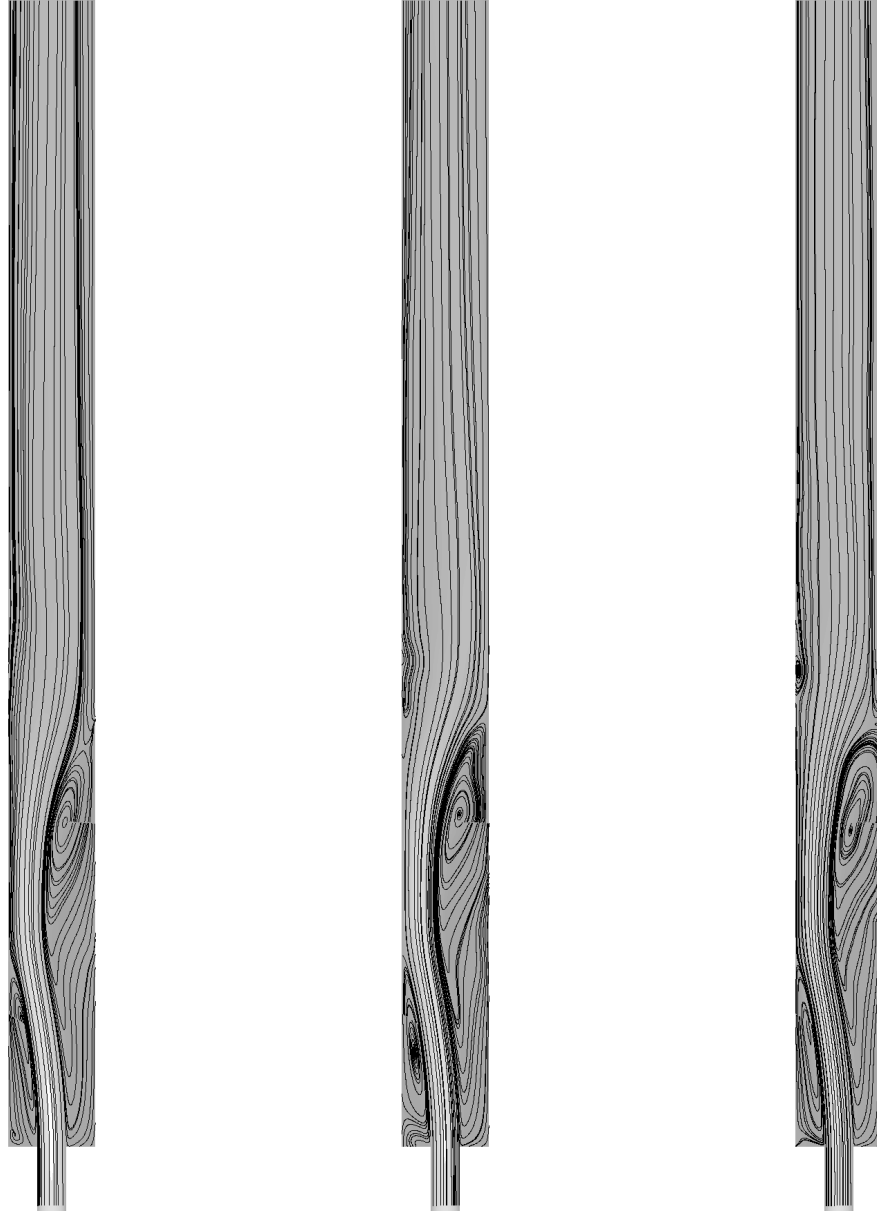


Figure 3.20: Evolution of the vortex sizes with the increase of the grid resolution for Re 900

The first observation from the velocity lines, figure 3.22, 3.23, is that the flow has moved upstream nearer the expansion. Here at the first line across the lower wall it can be seen more clearly than the previous Reynolds number the grid convergence. Although the velocities for all the grids are close and all have the same trend the medium and fine grid are closer than the coarse. The main difference in the values at approximately $16h$ at the end of the large vortex where the difference in the vortex length between the grids can be seen. The same can be derived also for the rms of velocities. The peak now are approximately at $15h$, from $30h$ at $Re\ 412$, at the end of the lower recirculation zone where the area with the highest unsteadiness is located. All three grids have the same maximum value and trend with the coarse one being in less agreement with the other two and the rms of the coarse grid are also shifted upstream as a result of the smaller recirculation area. Also the slower decrement in the value of the medium grid found at $Re\ 412$ is no longer present.



(a) Streamlines of velocity at mid-plane of the coarse grid (b) Streamlines of velocity at mid-plane of the medium grid (c) Streamlines of velocity at mid-plane of the fine grid

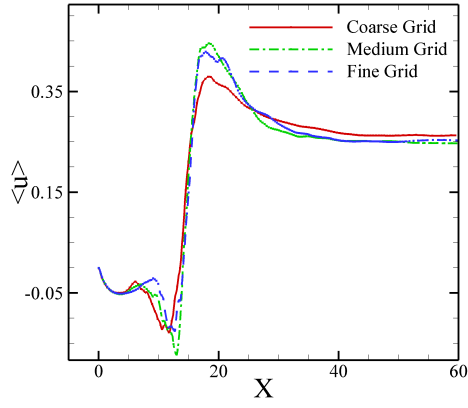
Figure 3.21: Comparison of the vortex sizes between the three grids at $Re\ 900$

For the line in the middle of the channel can be drawn the same conclusions. The trend is the same for all the grids but it is clear a convergence between the medium and the fine grid with the more pronounced differences between 15 and 30 step heights downstream near the end of the large recirculation area. Also the rms of velocity has the same peak value at 15h and the coarse grid is shifted upstream. Similar is the behavior of the grids at the upper wall area. The main difference is at 20h where

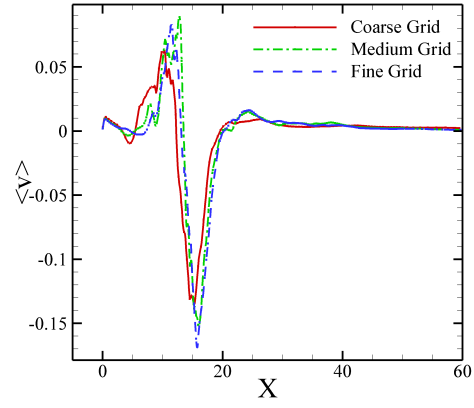
the medium and fine grid have an more intermittent behavior in terms of V velocity and rms of V velocity. This is the area where the third recirculation zone would appear and this behavior is hinting to the existence of a small vortex area there that its existence is affected by the grid resolution. This behavior is not captured from the coarse grid.

After the comparisons between the three grids at two different Reynolds numbers some conclusions can be drawn regarding the grid choice. The analysis of the results of the three grids in those two Reynolds numbers present that although all grids are close in terms of behavior there is clear convergence between them with the medium grid being closer to the fine than the coarse. This is also confirmed by the examination of the vortex sizes at the two Reynolds numbers and the comparison of the primary frequencies at Re 412 and the experimental results of Fearn et al [11].

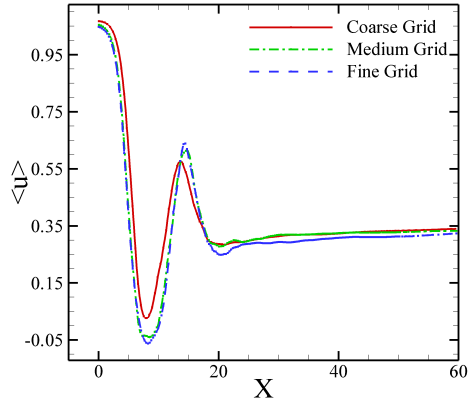
For Re 412 the comparison between the grid sizes shown that although for the first two vortex areas the difference between the grids is very small, 1%, for the third one a difference of 5% exists between the coarse grid and the other two. Also the comparison between the X lines for the three grids revealed a better agreement between the medium and fine grid. At Re 900 the difference of the vortex sizes is smaller between the medium and fine than the coarse. Furthermore the X lines presented a close agreement between the medium and the fine grid where the coarse one presented discrepancies near the end of the large recirculation area. For those reasons the use of medium grid for the bulk of the simulations is justified since it provide an acceptable compromise between accuracy and computational resources.



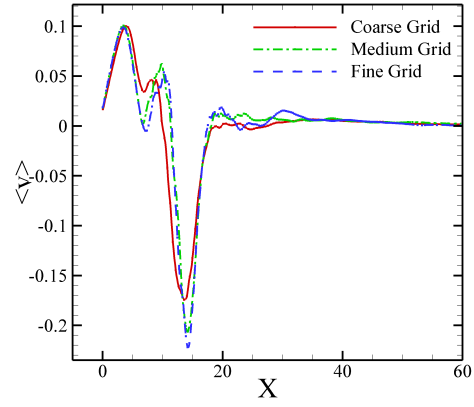
(a) streamwise velocity at $Y = 0.5h$



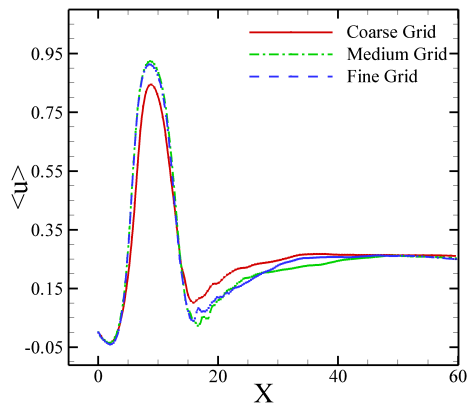
(b) wall normal velocity at $Y = 0.5h$



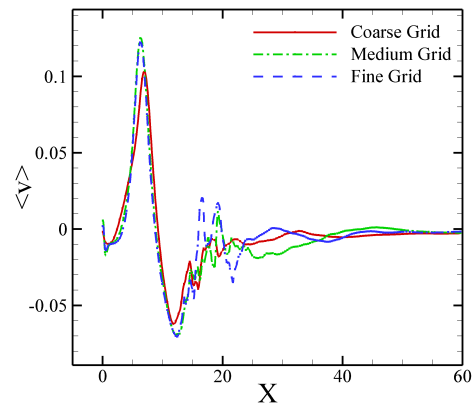
(c) streamwise velocity at $Y = 1.5h$



(d) wall normal velocity at $Y = 1.5h$

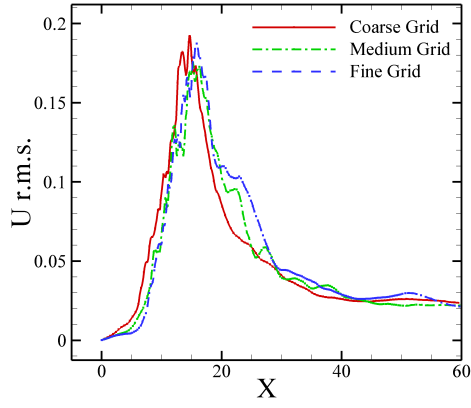


(e) streamwise velocity at $Y = 2.5h$

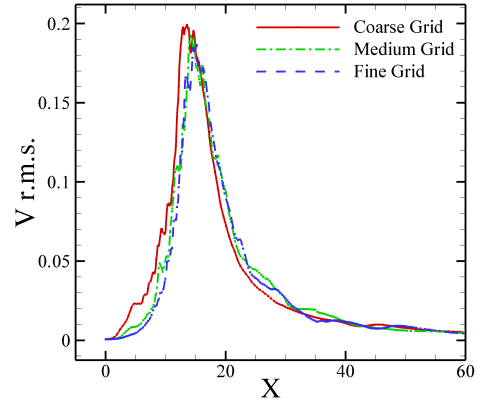


(f) wall normal velocity at $Y = 2.5h$

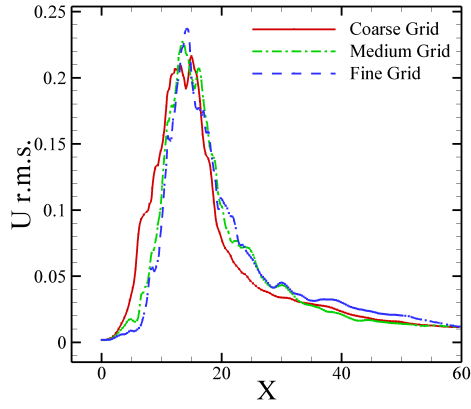
Figure 3.22: Velocity profiles at various positions in flow direction at $Re\ 900$



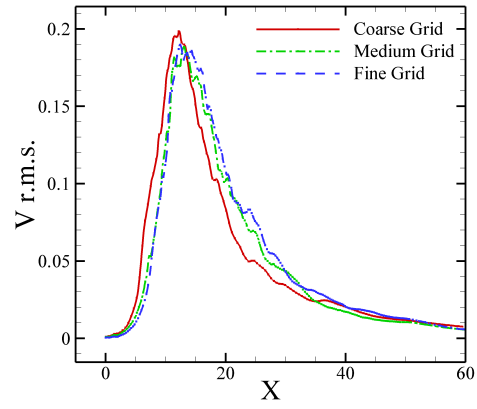
(a) U rms at $Y = 0.5h$



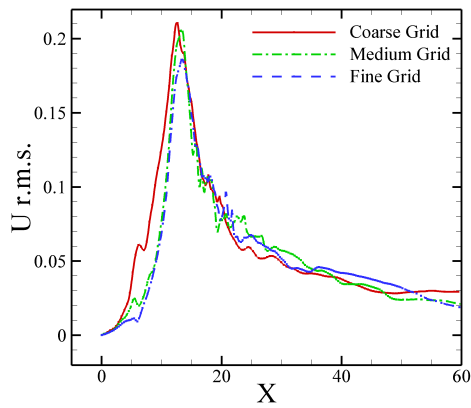
(b) V rms at $Y = 0.5h$



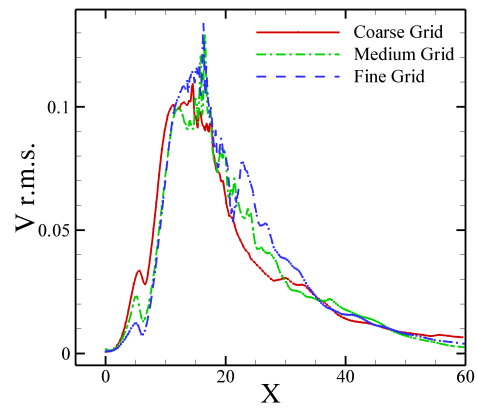
(c) U rms at $Y = 1.5h$



(d) V rms at $Y = 1.5h$



(e) U rms at $Y = 2.5h$



(f) V rms at $Y = 2.5h$

Figure 3.23: Rms of velocity at various positions in flow direction at $Re\ 900$

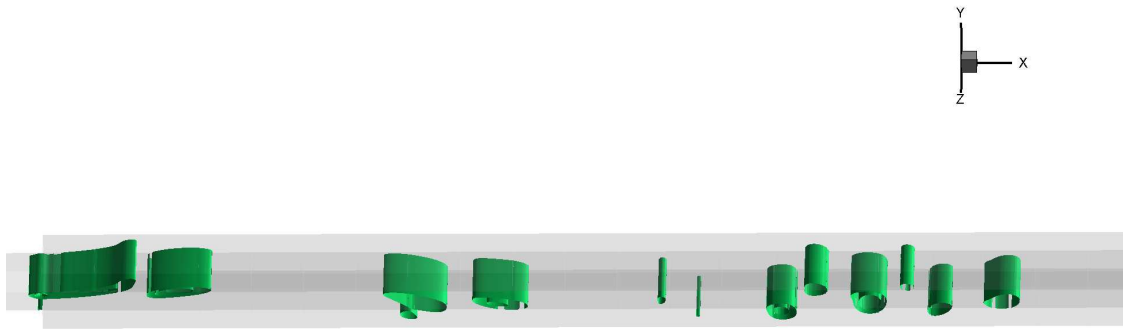
3.8 Numerical Analysis

The sudden expansion geometry at this Reynolds number range is suited to test the behavior of the ILES technique with different high order numerical schemes. At this range due to the fact that the flow is unsteady and has both laminar and transitional regions is a suitable testing ground to evaluate the characteristics and the performance of various methods. In this case three different reconstruction methods were tested, their description can be found at Chapter 2. A high order interpolation method (3rd order), a TVD method (2nd order MUSCL with Van Leer limiter) and a non TVD method (5th order WENO).

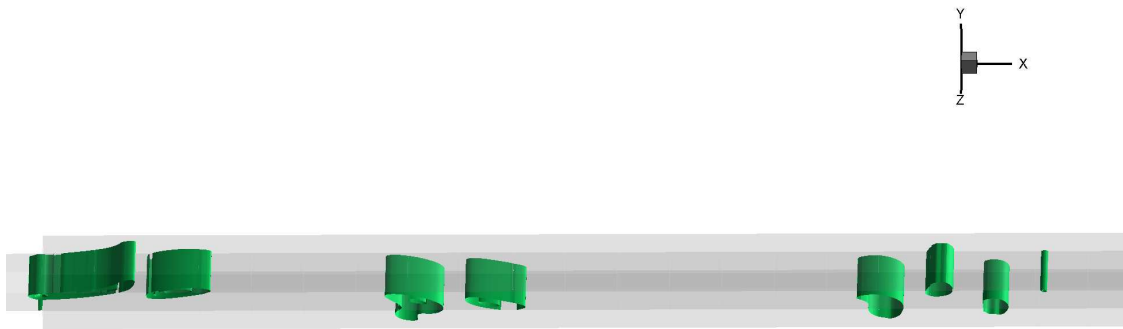
The three numerical methods will be evaluated at two different Reynolds numbers, Re 412 and Re 900, for the same reasons explained at the introduction of the grid convergence study. Three X lines, as before, will be used for the comparison of mean and root-mean-square velocities between the methods. Moreover, experimental data in the form of primary frequencies and also Q criterion isosurfaces will be discussed to further our understanding of the numerical methods performance.

3.8.1 Re 412

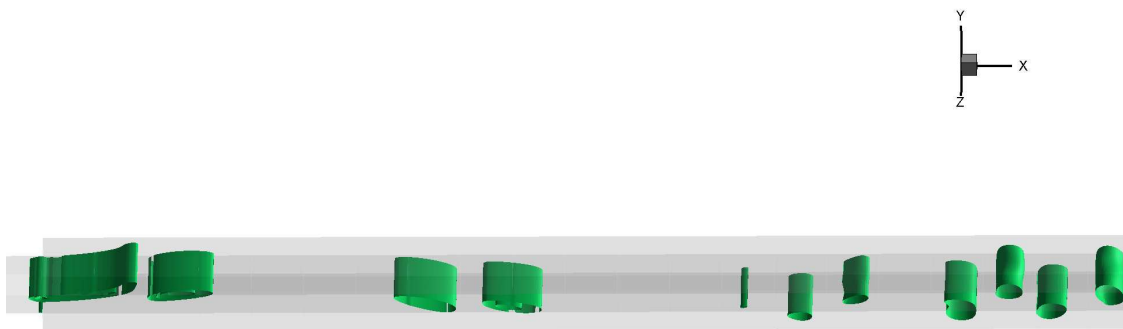
As it was mentioned in the previous subsection the flow at this Reynolds number has essentially an unsteady laminar behavior after the expansion with three major vortex areas and with a regular vortex shedding occurring near the middle of the third recirculation area. Figures 3.26 and 3.27 present the mean velocities and the mean square root of velocities along the geometry. Examining the velocities near the lower wall area we find that the three different numerical methods produce essentially the same results with the W5 having a slightly higher peak at $x=25h$ and some small discrepancies between the schemes at 25h and 50h. At the center and the upper wall the velocities have also a similar behavior with the velocity being the same for all the schemes with the only exception of slight irregularities between 25 and 50 step heights downstream at the vortex shedding area. Those discrepancies are mostly occurring at and after the vortex shedding area as expected since in this area the flow has the most intermittent behavior.



(a) Isosurfaces of Q criterion of the high order interpolation



(b) Isosurfaces of Q criterion of the second order MUSCL



(c) Isosurfaces of Q criterion of the fifth order WENO

Figure 3.24: Comparison of the three numerical methods with the use of Q criterion isosurfaces

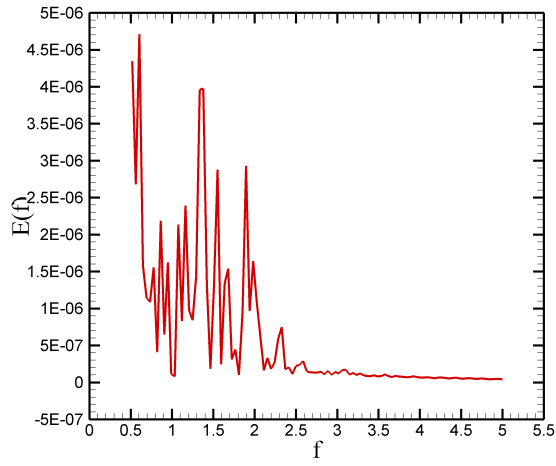
Reynolds Number	H3	M2	W5
412	1.35	1.61	1.66

Table 3.5: Primary frequencies

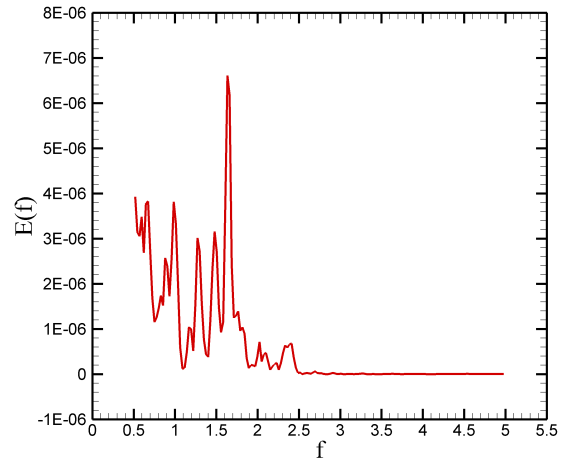
The rms velocities provide us with a more clear view of the schemes behavior. There in all the positions the rms values are essentially the same up to 25h downstream and the although all the numerical methods follow the same trend a difference in the values exist. About 30 step heights downstream, where the vortex shedding phenomenon is more intense, at all three positions examined the rms of velocities reach their maximum value. After that point the W5 have a higher value followed from the H3 interpolation with the M2 having the lower rms. That behavior could be attributed to the difference in the dissipation of the methods due to the different order and formulation of them. The W5 due to the higher order of the method and the nature of the weighted essentially non oscillatory schemes is producing less dissipation and is more oscillatory than the other two schemes especially the second order MUSCL. This in turn causes the high fluctuations from the vortices that are created at the third recirculation area after 30h downstream to be more intense in W5 and the disturbances to “die out” with a reduced rate further downstream.

Comparing the primary frequencies of the three methods with the experimental results from Fearn et al [11] show that the W5 and M2 manage to capture the primary frequency provided by the experimental data where the H3 fails to do, table 3.5 and figure 3.25. This is in accordance with the data from the X lines examined previously showing not a significant difference between the three numerical methods at this Reynolds number.

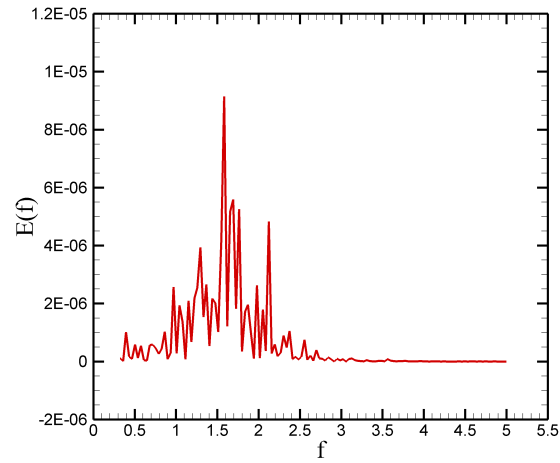
Figure 3.24 present the Q criterion of velocity of the three methods at $Q=0.02$. This level of Q criterion is used in this text unless stated otherwise. It was chosen so it would be possible to capture vortical structures even to the lowest Reynolds numbers examined. Structures exist near the expansion where the first vortex is located and at the end of the second vortex. Those structures are identical for the three methods. Further downstream the vortical structures that are appearing are indicating the existence of the vortex shedding at the third recirculation area and those vortices traveling downstream. This comparison does not produce any significant



(a) Primary frequency of the kinetic energy of the third-order interpolation



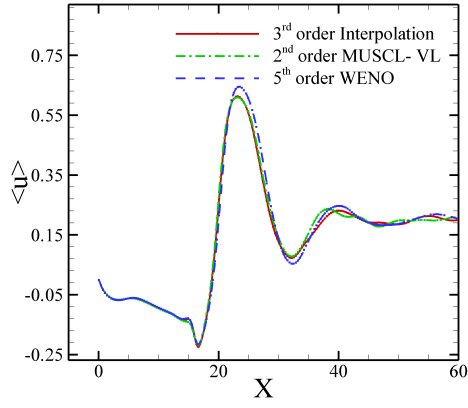
(b) Primary frequency of the kinetic energy of the second order MUSCL



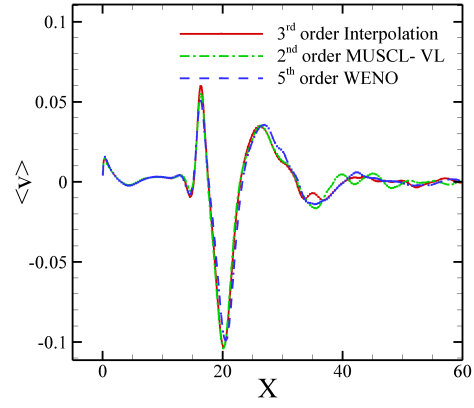
(c) Primary frequency of the kinetic energy of the fifth order WENO

Figure 3.25: Comparison of the primary frequencies between the three numerical methods at 22h downstream

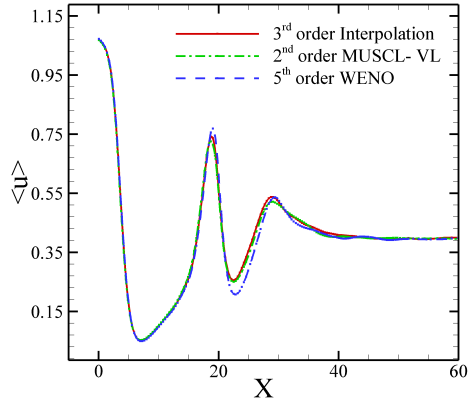
differences between the schemes in turn of capturing the vortical scales of the flow. Although the H3 and W5 seem to have captured more vortical scales downstream than the M2 this could be also due to the fact that the Q criterion isosurfaces are representing instantaneous values. Regarding the physical characteristics of those vortical structures it is obvious that they have a quasi three dimensional behavior with small variations in the spanwise direction. This behavior is indicating that the flow in this Reynolds number is primarily unsteady with early onset of transition.



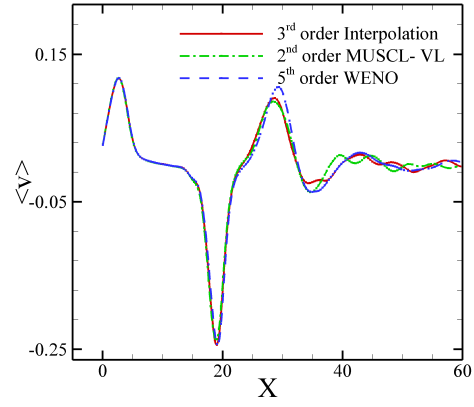
(a) streamwise velocity at $Y = 0.5h$



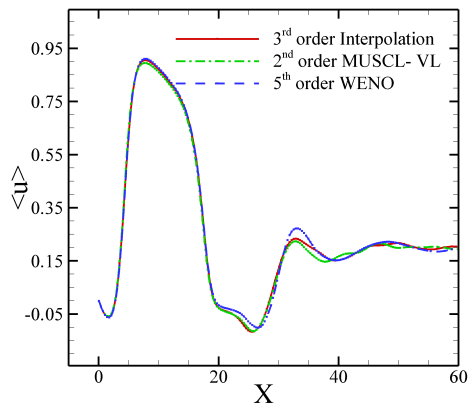
(b) wall normal velocity at $Y = 0.5h$



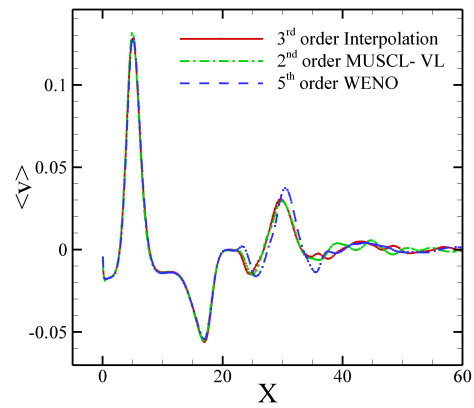
(c) streamwise velocity at $Y = 1.5h$



(d) wall normal velocity at $Y = 1.5h$

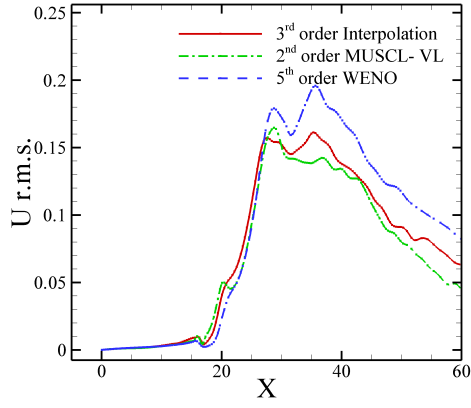


(e) streamwise velocity at $Y = 2.5h$

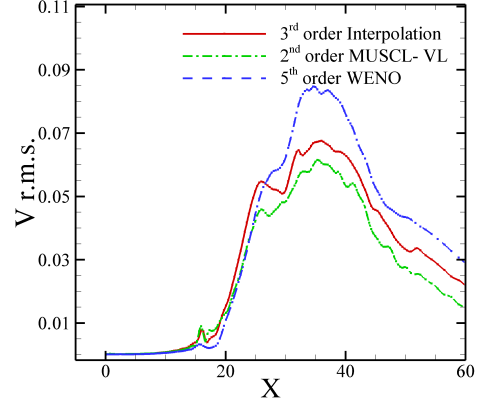


(f) wall normal velocity at $Y = 2.5h$

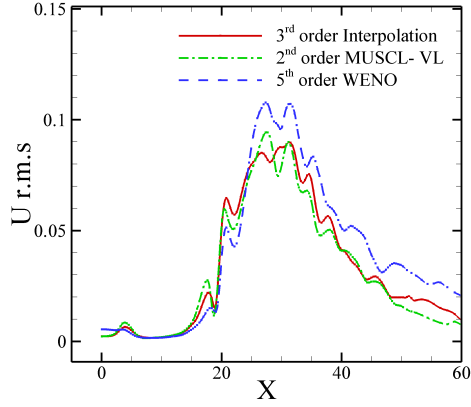
Figure 3.26: Velocity profiles at various positions in flow direction for $Re\ 412$



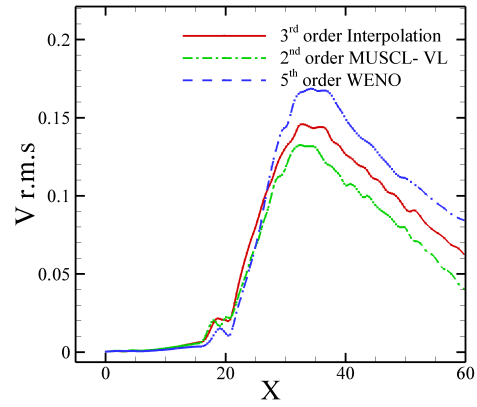
(a) U rms at $Y = 0.5h$



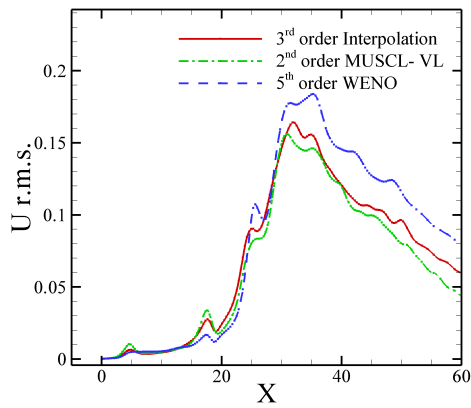
(b) V rms at $Y = 0.5h$



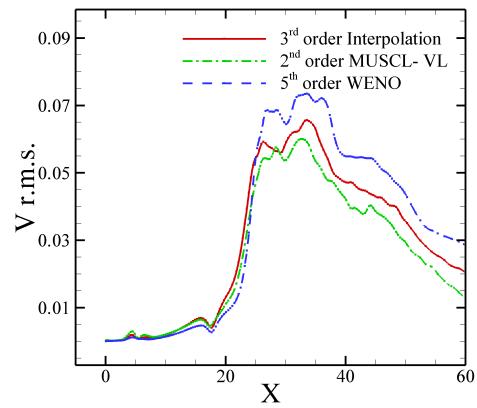
(c) U rms at $Y = 1.5h$



(d) V rms at $Y = 1.5h$



(e) U rms at $Y = 2.5h$



(f) V rms at $Y = 2.5h$

Figure 3.27: Rms of velocity at various positions in flow direction for $Re\ 412$

3.8.2 Re 900

As the Reynolds number reaches 900 the flow is having a more clear transitional behavior. Here the third recirculation area is far less prominent, has more interim behavior and the area with the most unsteady behavior is located near the end and after the large recirculation area at the lower wall. Figure 3.29 and 3.30 present the mean velocities and the fluctuations of the flow. From the mean velocities in the form of X lines it is obvious that the numerical scheme plays a much more prominent role in this Reynolds number than the previous one.

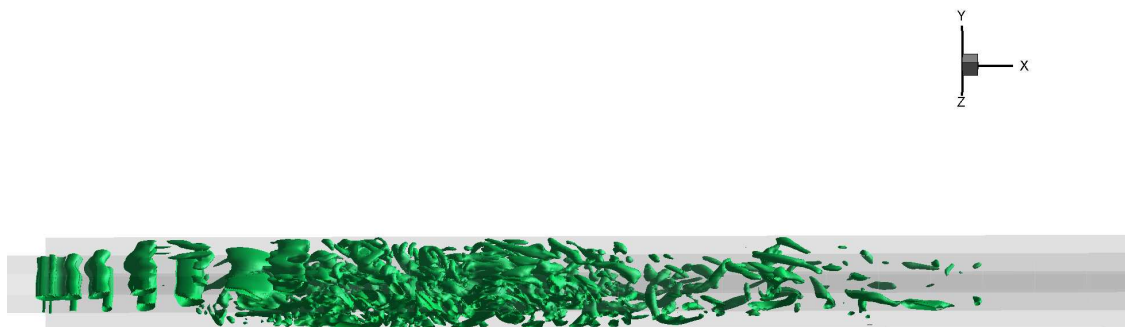
There is a clear difference here between the schemes even at the mean velocities values. Examining the X lines at the three positions it is made obvious that although the and the are generally in agreement the behavior diverge from the other two schemes. At the line close to the lower wall, that is crossing the large vortex, all methods keep the same basic trend with the W5 and the H3 differ mainly in the minimum and maximum values. On the other hand the M2 from 10h to 20h is displaced downstream both in U and V plots an indicating a larger recirculation area than the other two schemes. Examining the behavior of the flow at the other two locations we come to the same conclusion. The W5 and H3 are for most part in good agreement, the M2 despite having the same trend is displaced downstream. The rms of the velocities confirm this fact. The W5 and the H3 are very close and reach the same maximum value, at about 15h near the end of the large recirculation area, with the exception of the peak at the lower recirculation area where the H3 is having a higher U rms than the W5. The M2 having a somewhat lower value but more importantly is reaching this value further downstream than the other two methods. For this reason the rms of the M2 dissipate further downstream from the other two methods. This confirms the prediction of a larger recirculation area with the M2 since the highest rms intensity is reached at the end of the lower recirculation area.



(a) Isosurfaces of Q criterion of the high order interpolation



(b) Isosurfaces of Q criterion of the second order MUSCL

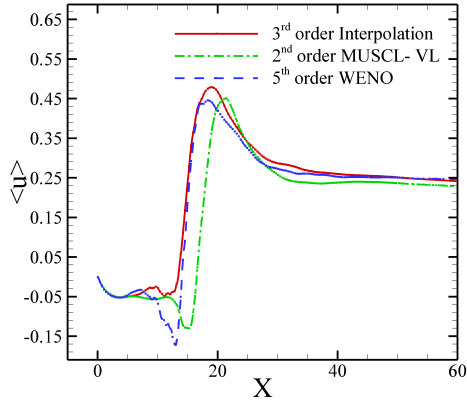


(c) Isosurfaces of Q criterion of the fifth order WENO

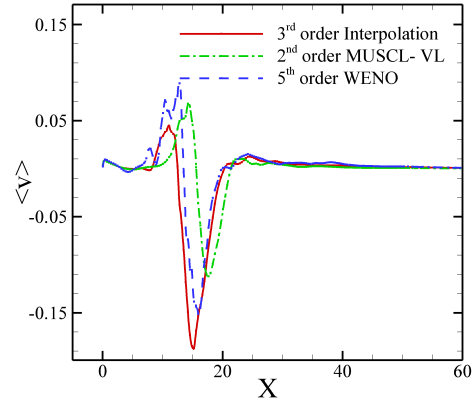
Figure 3.28: Comparison of the three numerical methods with the use of Q criterion isosurfaces

Figure 3.28 presents the Q criterion at $Q=0.02$ for the three methods underlining the scales present in the flow. The examination of the scales of the Q criterion shed a light at the behavior of the M2 at this Reynolds number. The Q criterion isosurfaces clearly show that the M2 managed to capture only the largest scales of the flow comparing to the H3 and W5 which captured more and smaller scales, especially the W5. This behavior could also be explained from the increased dissipation of the MUSCL schemes and the lower order of the method adding to the dissipation. Also the fact that the M2 present a less chaotic flow than the other two schemes explain the larger lower vortex since the main reason for the reduction of the length of the large vortex area is the unsteadiness of the flow at that area and the less chaotic flow of the M2 allowed the vortex to keep its form further downstream. The difference in the behavior between the schemes especially the M2 suggest the importance of the choice of the reconstruction method in capturing properly the transitional behavior of the sudden expansion geometry in low Reynolds numbers. In those Reynolds numbers the flow is complex one and the transitional behavior is localized and not present in the whole geometry due to the fact that is mainly caused from the expansion geometrical feature and the resulting behavior of the vortices rather than the Reynolds number in the channel. So the corresponding accuracy, order and dissipation of the numerical methods in combination with the ILES have an important role in capturing properly the transitional flows.

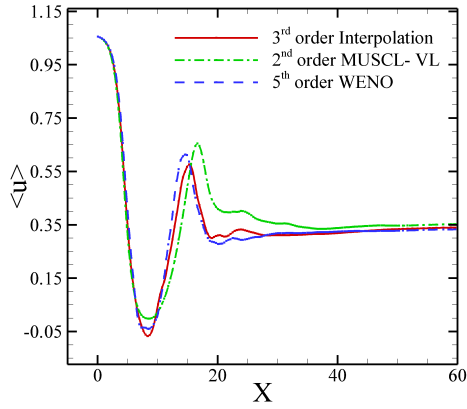
After the numerical analysis some conclusion can be reached regarding the selection of the numerical method. At Reynolds number 412 all three methods have similar behavior with the W5 having a higher rms value after 35h, as expected, followed by the H3 and the M2. The Q isosurfaces and the primary frequencies also did not produced an important difference between the schemes. At Reynolds 900 it became clear that the M2 deviated from the H2 and W5. The examination of the Q criterion isosurfaces revealed that the H2 and the W5 where in position to capture more and smaller structures than the M2 method. Although the lack of experimental data in Reynolds number 900 prevent us from being absolute certain of which method capture the flow better, the better consistency between the H2 and W5 and the higher order and resolution of W5 it make it the proper choice for the rest of the simulations.



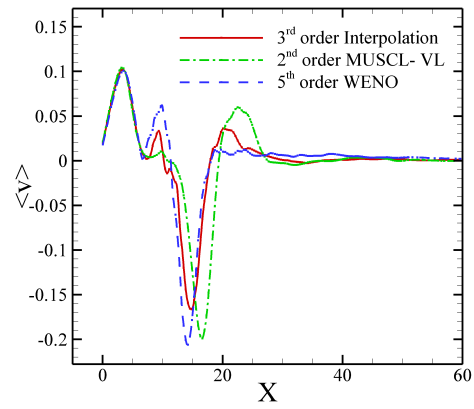
(a) streamwise velocity at $Y = 0.5h$



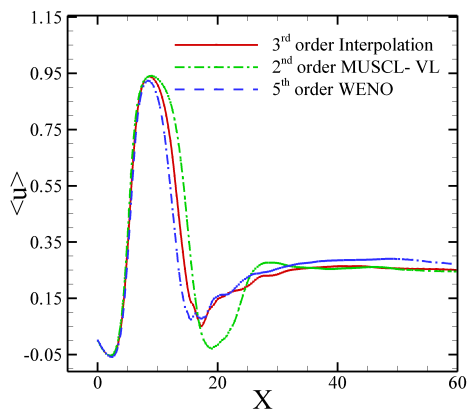
(b) wall normal velocity at $Y = 0.5h$



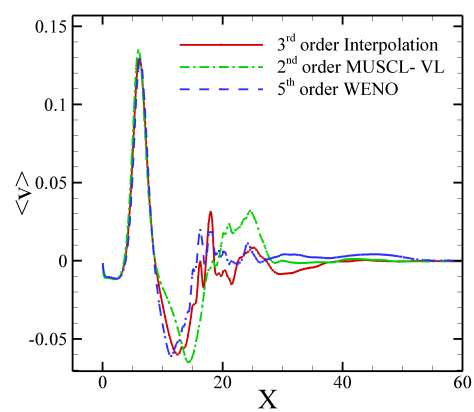
(c) streamwise velocity at $Y = 1.5h$



(d) wall normal velocity at $Y = 1.5h$

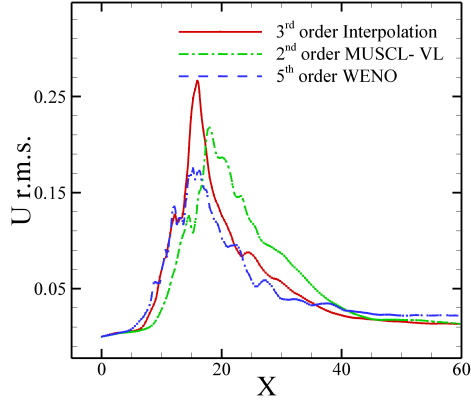


(e) streamwise velocity at $Y = 2.5h$

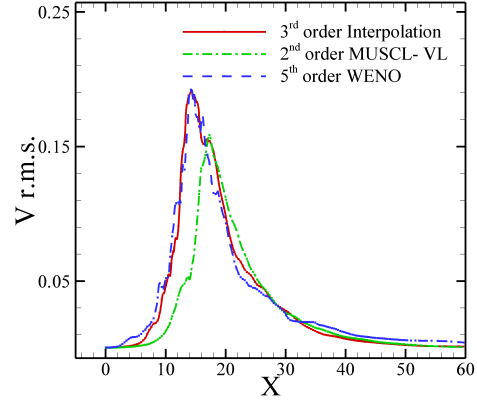


(f) wall normal velocity at $Y = 2.5h$

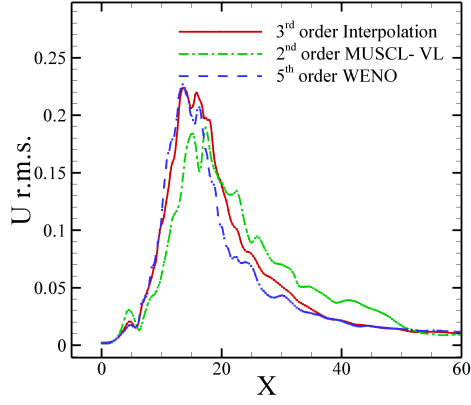
Figure 3.29: Velocity profiles at various positions in flow direction for $Re\ 900$



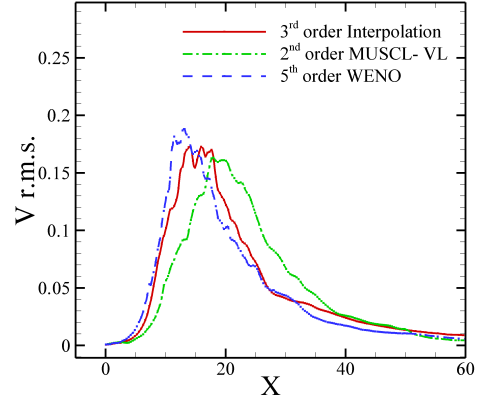
(a) U rms at $Y = 0.5h$



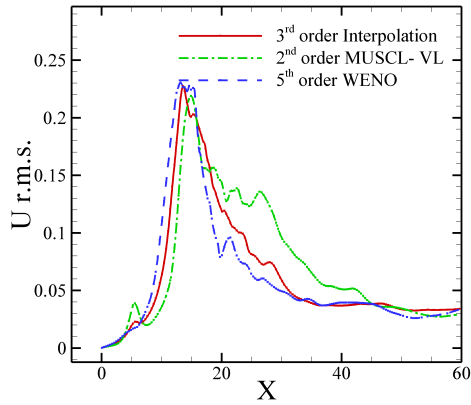
(b) V rms at $Y = 0.5h$



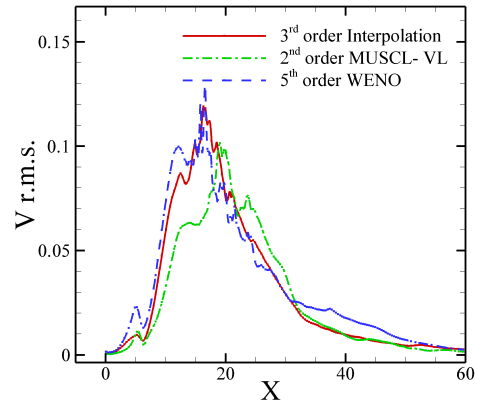
(c) U rms at $Y = 1.5h$



(d) V rms at $Y = 1.5h$



(e) U rms at $Y = 2.5h$



(f) V rms at $Y = 2.5h$

Figure 3.30: Rms of velocity at various positions in flow direction for $Re\ 900$

Chapter 4

Unsteady Behavior and Transition

4.1 Time averaged behavior

In previous chapter were described the physical characteristics of the flow for the stable flow region and for Reynolds 50 to 280. In this chapter the physical behavior of the mean flow will be presented for higher Reynolds numbers up to 900. First the mean flow characteristics will be analyzed with the help of streamline of velocity.

As described before from Reynolds number 50 to Reynolds number 280 the flow undergone series of instabilities resulting to a number of bifurcation of the flow and the breaking of its symmetries. First the flow had two symmetrical vortices at the two corners. Then the vortices started to grow asymmetrically and finally a third vortex appeared to the upper wall. Despite those changes the flow remained steady after each one of those bifurcations.

After the flow reached approximately Reynolds number 300 another bifurcation is taken place only this time the resulting state in an unsteady one. At Reynolds number 350 the flow has an unsteady behavior dominated from the existence of a shedding type instability creating coherent vortical structures from the end of the third recirculation area shear layer. The three vortex areas existing at Reynolds 280 are still present in the mean flow field. All three of those vortices increased their sizes further as it can be seen in table 4.1 and figure 4.4 with the third vortex reaching its maximum size during the scope of this study.

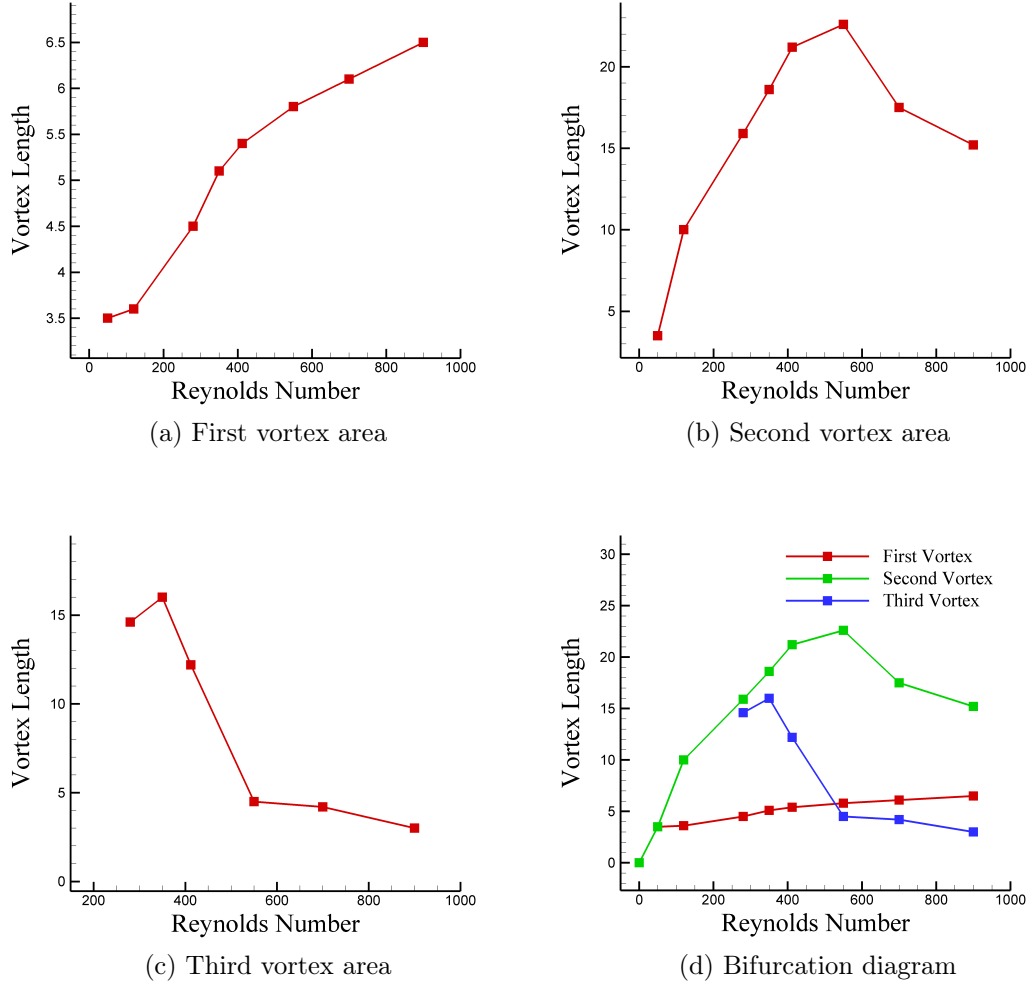


Figure 4.1: Evolution of the length of the vortex areas with the Reynolds number

At $Re\ 412$ the mean flow features become more complex, figure 4.5,. The first vortex area increases further in size. The second vortex area expands resulting to the relocation of the third vortex area further downstream although its size start to decrease. The reason of this decrement is the shedding instabilities where now are affecting almost the whole length of the third vortex area creating a more intense shedding of the vortices. This fact in turn cause the other characteristic of the flow which is the existence of a new small vortex area after the third one and at the opposite wall. This feature is explained by the unsteady behavior of the flow since due to the shedding of the vortices that occur at the third vortex area and their propagation downstream is causing a flapping of the main flow from one wall to the other and with the rise of the pressure gradient this is causing, a new vortex area

is created. This small recirculation area is not reported in the study of Fearn et al [11] but this could be explained by its highly intermittent behavior.

Reynolds Number	First vortex area	Second vortex area	Third vortex area
Re 50	3.5 h	3.5 h	-
Re 120	3.6 h	10.0 h	-
Re 280	4.5 h	15.9 h	14.6 h
Re 350	5.1 h	18.6 h	16.0 h
Re 412	5.4 h	21.2 h	12.2 h
Re 550	5.8 h	22.6 h	4.5 h
Re 700	6.1 h	17.5 h	4.2 h
Re 900	6.5 h	15.2 h	3.0 h

Table 4.1: Table of the length of the three primary vortex areas

Figure 4.6 present us the mean flow field at Reynolds number 550. In this Reynolds number the change behavior and shifts from mainly unsteady laminar behavior to an essentially transitional one. The shedding of the vortices is moved from the middle of the third recirculation area to the end of the larger one in the lower wall. This in turn and the increase of unsteadiness it inflicts to the region right after the large vortex area is one of the causes for the significant reduction of the size of the third recirculation area by almost $2/3$ of its previous size. The other two recirculation areas area also increased their size with the one in the lower wall reaching its maximum size in this Reynolds number. Also due to the increased unsteadiness the second recirculation area in the lower wall reported at the previous Reynolds is no longer present.

Moving to Re 700 the first characteristic of the flow that is observed is that for the first time the shear layer of the large recirculation area is retreating. This in turn moves the position of the third vortex area upstream, figure 4.7. The third vortex area also continues to lose size reaching 4.2 step heights from 16h at Re 350 with the first one still increase its size by 0.3 to 6.1h.

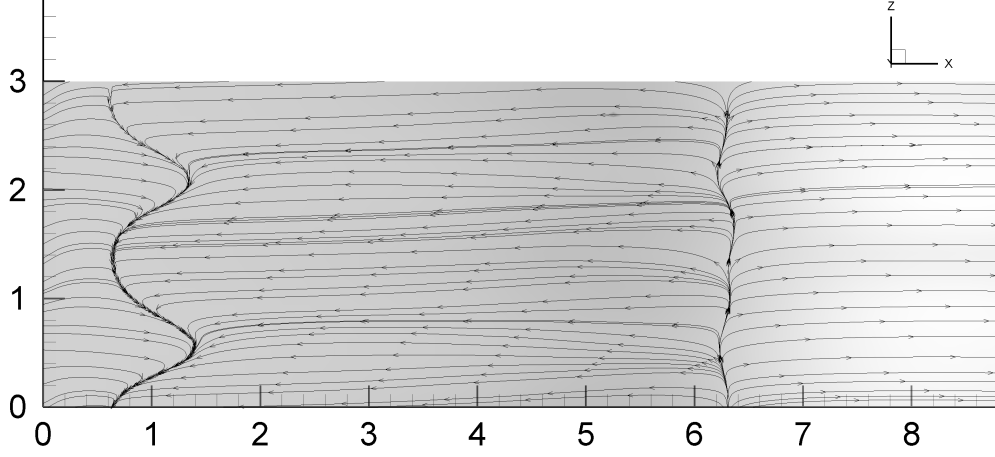
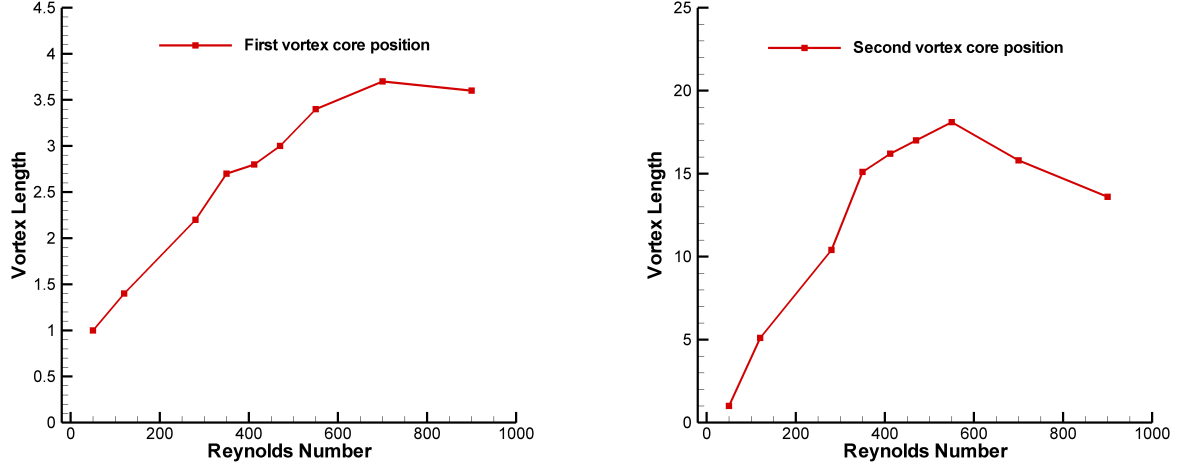


Figure 4.2: Limiting streamlines in the upper wall area $y=3$ presenting the separation/reattachment lines

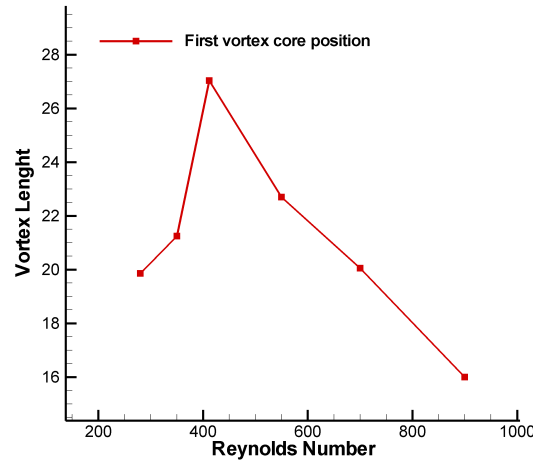
Finally at Reynolds 900, figure 4.8 the flow has its most unsteady behavior from the Reynolds examined in this study. The vortex shedding still is occurring near the end of the large vortex area but has now a very chaotic behavior. Due to that the third recirculation area has its minimum size of $3h$ and the large vortex area retreated to $15.2h$. The first recirculation area has increased its size to 6.5 and now at the right upper corner a new separation line is present. Figure 4.2 presents the limiting streamlines in the upper wall area. The reattachment line of the first vortex at $6h$ is present and also the new separation line at approximately $1h$ indicating the presence of a small recirculation area at the upper corner. The presence of this new vortex area as well as its twin at the lower corner is documented experimentally for higher Reynolds numbers, Casarsa et al [113], making its prediction for this relatively low Reynolds number an interesting fact.

Figure 4.1 summarizes the evolution of the length of the main vortex areas with Reynolds number. The size of the first recirculation area does not have a maximum value in the range of Reynolds number examined in this study and is increasing its length with the increment of Reynolds number. The second vortex area reaches its maximum value at Reynolds number 550. This coincided with the vortex shedding instabilities at the end of the large recirculation area. The existence of a maximum value and then the retreat of the length of the large recirculation area is also reported at the experimental study of Restivo et al [114]. A maximum value was also found for the third vortex area. The third recirculation reaches its maximum value at Reynolds number 350 which also coincides with the presence of vortex shedding instabilities at the third vortex. From Reynolds number 550 and afterwards the

vortex loses the greater part of its former size.



(a) Vortex core positions in space for different Reynolds numbers for the first vortex (b) Vortex core positions in space for different Reynolds numbers for the second vortex



(c) Vortex core positions in space for different Reynolds numbers for the third vortex

Figure 4.3: Vortex core positions diagrams for the main vortex areas

An other feature of the flow that can be extracted of the 2D slices is the vortex cores of the three main recirculation areas. The figures 4.3a, 4.3b, 4.3c and table 4.2 present the X, Y positions of the vortex cores in each of the three recirculation areas as function of the Reynolds number. The plot of the position of the first vortex core shows an steady increase with the Reynolds number up to Re 900 moving

downstream as the size of the vortex does and also closer to the wall. This trend is disrupted at Re 900 when the core marginally retreats upstream and moves away from the wall. This coincides with the appearance of the new separation line at the upper corner of the wall so it is reasonable to assume those two facts are related. Regarding the position of the second core it has the same behavior with the length of the recirculation area, the X position of the core is moving downstream as the length is increasing up to Re 550 and then as the shear layer start to regress the position of the core moves upstream. Similar conclusions can be drawn for the third area where it moves downstream up until Re 412 and then upstream until it is not longer clearly evident.

Reynolds Number	First vortex core	Second vortex core	Third vortex core
Re 50	1.0 2.3	1.0 0.6	-
Re 120	1.4 2.4	5.1 0.9	-
Re 280	2.2 2.4	10.4 1.0	19.8 2.2
Re 350	2.3 2.4	12.6 0.8	21.2 2.2
Re 412	3.0 2.5	17.0 0.8	27.0 2.1
Re 550	3.4 2.5	18.1 1.0	22.7 2.6
Re 700	3.6 2.5	15.8 0.8	18.1 2.8
Re 900	3.6 2.5	13.6 0.9	16.0 2.8

Table 4.2: Table of the coordinates of the three main vortex cores

The description of the mean flowfields for this range of Reynolds numbers presented the evolving behavior of the time averaged structures. The three main vortex areas while up to the time dependence threshold expanded their shear layer downstream eventually reversed that behavior. This change coincided with the vortex shedding at each recirculation area. This caused the third recirculation area to reach its maximum size at Reynolds 412 and for the second at Reynolds 550. The size of the third vortex area after Reynolds 412 decreased rapidly from 16h to 3h at Reynolds 900 indicating its disappearance with further increase of the Reynolds number. Also at Re 900 a new separation line appeared at the upper corner of the flow, a structure usually present at higher Reynolds numbers.

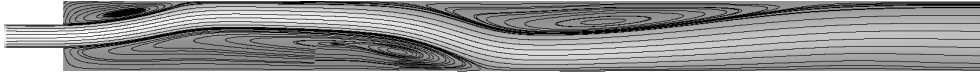


Figure 4.4: Streamlines of velocity at mid-plane at Re 350



Figure 4.5: Streamlines of velocity at mid-plane at Re 412

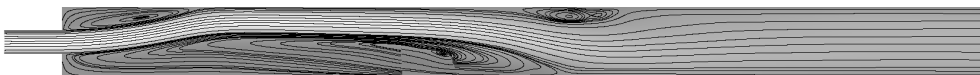


Figure 4.6: Streamlines of velocity at mid-plane at Re 550

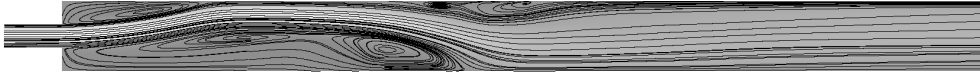


Figure 4.7: Streamlines of velocity at mid-plane at Re 700

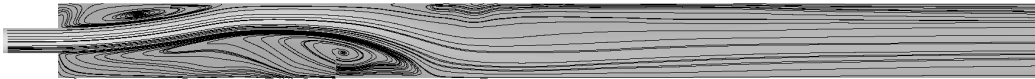


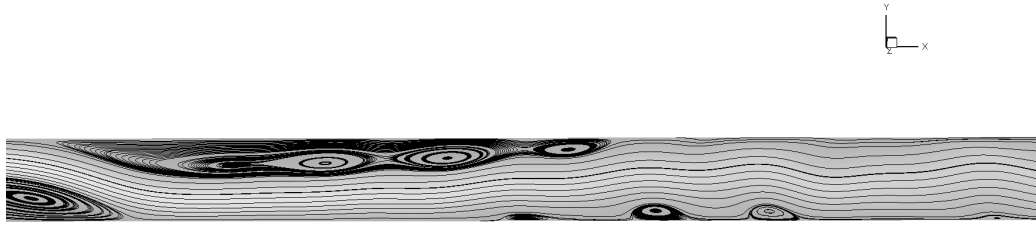
Figure 4.8: Streamlines of velocity at mid-plane at Re 900

4.2 Time dependence and transition to turbulence

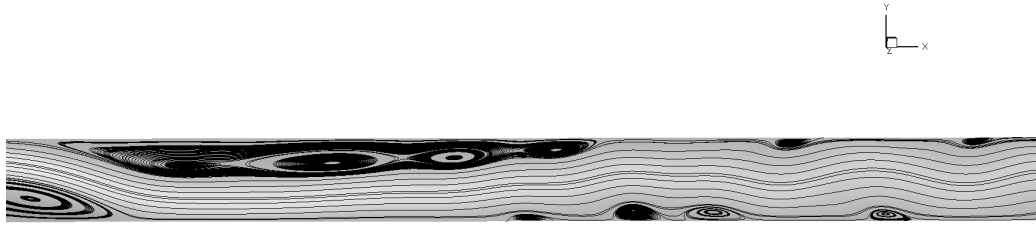
In the previous section the general characteristics of the mean flow field in its of the Reynolds numbers were presented. But of equal importance is the time-depended behavior of the flow and its phenomena. Although the analysis of the mean flow field can provide us with valuable data about the general attributes of the flow it cannot in detail demonstrate the time dependent behavior of an unsteady flow. For this reason an analysis of the instantaneous flow has been conducted. For this study streamlines of instantaneous velocities at several different time periods to illustrate the evolution of the flow with time as well as isosurfaces of Q criterion to demonstrate the transitional behavior of the flow.

For the Reynolds 350 the main time-depended feature of the flow is the vortex shedding occurring near the end of the third recirculation area. This feature is demonstrated at figure 4.9. It can be seen the how the vortices after its shedding from the end of the third recirculation area is traveling downstream. Moreover the flapping of the flow is causing the creation of vortices in the lower wall near the end of third recirculation area traveling also downstream. Inside the third vortex a series of Kelvin–Helmholtz (shear layer) instabilities are also evident. Those vortices are generated and traveling through the recirculation area where at the edge are causing the vortex shedding instabilities.

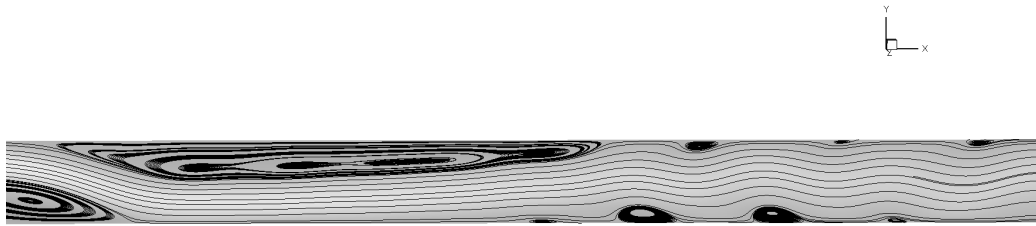
As the Reynolds number of the flow is increased to 412 the shedding of the vortices is more evident. This is occurring near 22 length downstream as Fearn [11] is predicted and is presented at figure 4.10. The instabilities in the third vortex area are now more intense creating a complex behavior inside the vortex area. The K-H vortices now are shedded not from the end of the recirculation area but from its middle. Moreover the vortices in upper wall, as they travel downstream, are causing the main flow to oscillate form one wall to the other. Due to the pressure gradients that oscillations apply to the flow, vortices are also appearing to the lower wall as well.



(a) $t=10$ non-dimensional time



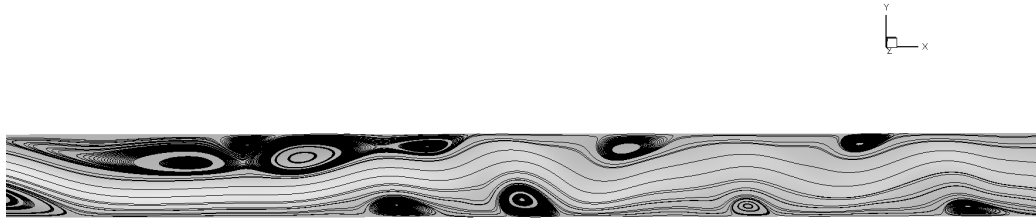
(b) $t=20$ non-dimensional time



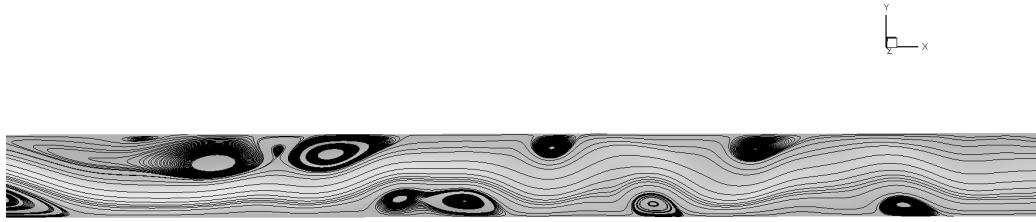
(c) $t=30$ non-dimensional time

Figure 4.9: Streamlines of velocity of the vortex shedding at the third recirculation area at $Re\ 350$

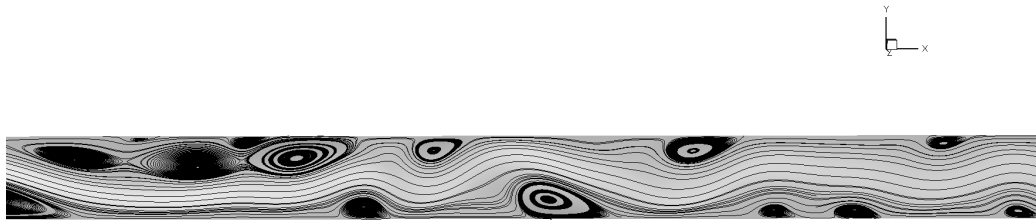
As the Reynolds number is increased to 550 it is clear that the flow demonstrates a more chaotic unsteady behavior and the vortex shedding takes place both at the upper and at the lower vortex regions. Adding to the fact, the third recirculation area has now a very intermittent behavior and the second recirculation area become more complex in turn with several K-H vortices inside it , figure 4.11. The disturbances to the third recirculation area from the shedding at the large lower vortex are evident explaining the great reduction of the third vortex size at this Reynolds number.



(a) $t=10$ non-dimensional time



(b) $t=20$ non-dimensional time



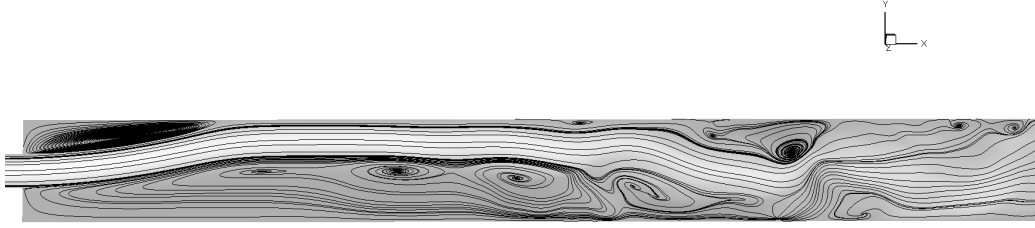
(c) $t=30$ non-dimensional time

Figure 4.10: Streamlines of velocity of the vortex shedding at the third recirculation area at Re 412

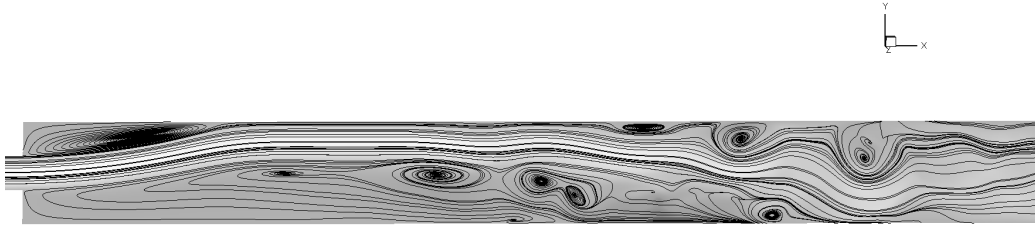
The same chaotic behavior is evident as the flow reach Reynolds 700. The flow at the largest recirculation area is clearly three dimensional with a number of smaller K-H vortices replacing the larger coherent ones found at lower Reynolds numbers, figure 4.13. The third vortex area keeps its small size and additional vortices are created in the vicinity of that area and are propagated downstream. On the other hand the first recirculation area still keeps an relatively stable behavior.

As the flow reach Re 900, 4.14, the chaotic behavior is more intense. This is an evidence of increased transitional behavior as smaller structures appear and then dissipate at the flow near the end of the second and third recirculation areas. The

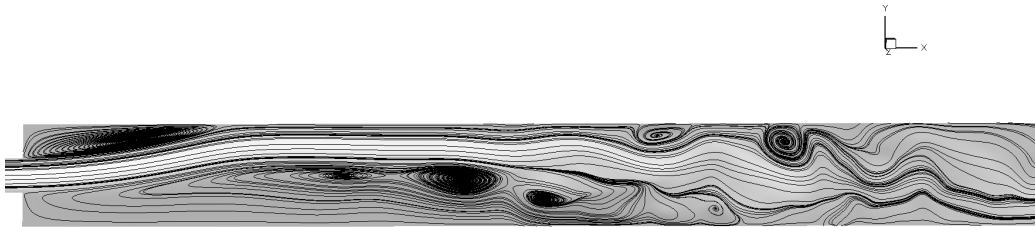
disturbances have propagated now to the whole length of the large vortex area with the form of K-H instabilities. Regarding the third recirculation area, in this Reynolds number, there are instances where is no longer clearly present in the flow. The first recirculation area changed also behavior with K-H instabilities appearing inside it and evidence of vortex shedding occurring at its end. Furthermore, the new separation area can be seen at the upper corner of the flow as described before.



(a) $t=10$ non-dimensional time



(b) $t=20$ non-dimensional time



(c) $t=30$ non-dimensional time

Figure 4.11: Streamlines of velocity of the vortex shedding near the third recirculation area at $Re\ 550$

The 2D slices can be used to analyze 2D characteristics of the flow but since the flow is three dimensional for the 3D characteristics isosurfaces of Q criterion will be

used. Figures 4.15a to 4.16b present the q criterion isosurfaces of the instantaneous flow for all the Reynolds numbers. For Reynolds numbers 350 and 412 clear quasi-three dimensional vortical structures are present in the flow as they are traveling downstream. When the flow reaches Reynolds number 550 the flow structures change dramatically with three dimensional vortical structures appearing in the flow after the second recirculation area demonstrating a chaotic behavior in comparison to the quasi three dimensional structures of the previous Reynolds numbers.

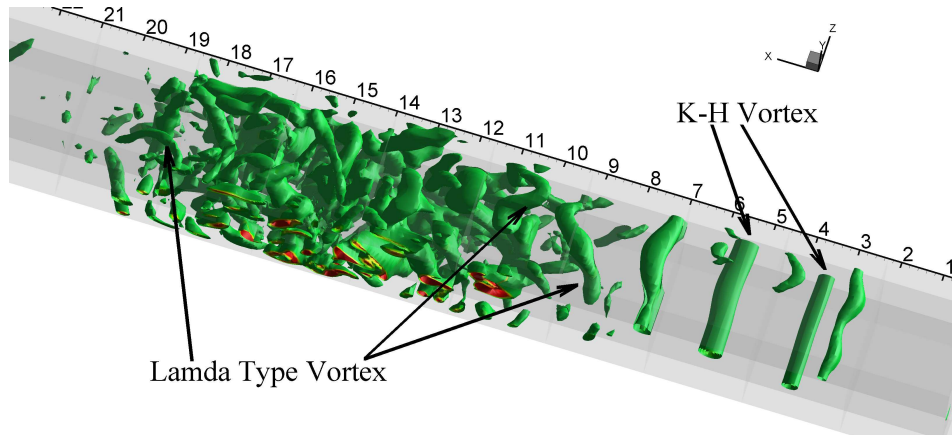
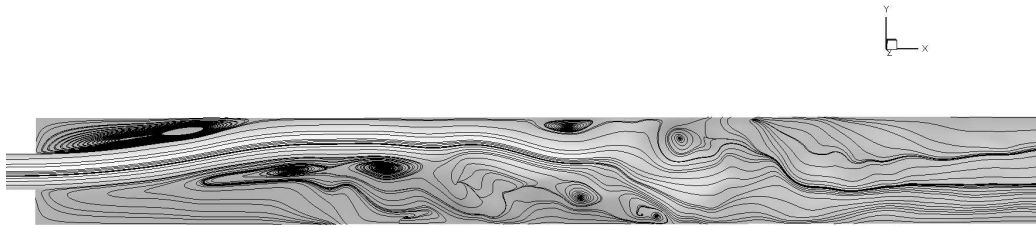


Figure 4.12: Vortical structures

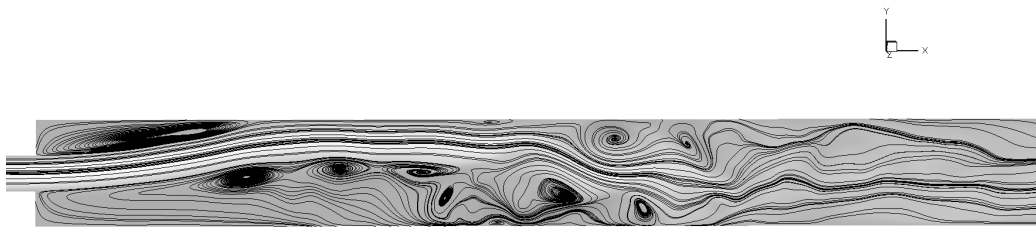
As the Reynolds number increases further the chaotic behavior become more pronounced with smaller vortical scales appearing as the flow reaches Reynolds 900. This can explain the decrement and finally the disappearance of the third recirculation area since this area of the flow is where the flow becomes more unsteady and therefore more complex affecting the stability and as the result the length of the vortex area. Also this behavior strongly suggests between Reynolds 412 and 550 the flow undergone another bifurcation and at the area of the recirculations and after it have now an essentially transitional behavior from an unsteady laminar at lower Reynolds numbers. The transition to that behavior will be further investigated in the next chapter with the help of the power spectrum analysis.

An closer examination at the coherent vortical structures at Reynolds 900, figure 4.12, can shade more light in the behavior of the flow at this Reynolds number. A higher level of Q criterion ($Q=0.25$) was used to illustrate only the larger coherent vortical structures. At the lower recirculation area near the expansion coherent K-H vortices can be seen. Those vortices show a quasi three dimensional behavior. As the flow is traveling downstream in the area of vortex shedding the vortical structures

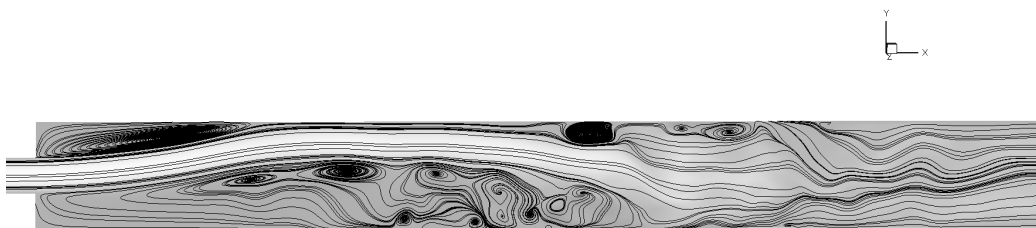
numerous vortical structures are appearing making the flow very complex. Those structures have a highly three dimensional distorted shape. Many of those vortical structures have streamwise or vertical direction that can change during the length of the structure. Among those structures lambda shape or horse-shoe shape structures can be identified. Those structures are usually created by the interaction of the side wall with the vortices that are traveling downstream in vortex shedding flow like sudden expansion and backward facing step. In this case ,since at the spanwise direction periodic boundary conditions instead of wall are applied, those structures seems to appear due to the interaction of the K-H vortices to the upper and lower walls as they are traveling downstream. This interaction is causing the center of the vortical structure to loose velocity and this in turn causes the vortex to elongate and take its characteristic shape.



(a) $t=10$ non-dimensional time



(b) $t=20$ non-dimensional time

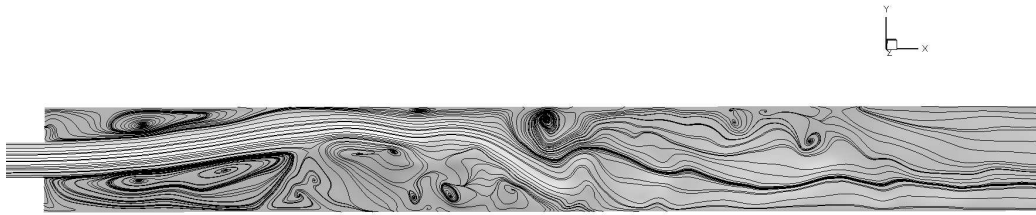


(c) $t=30$ non-dimensional time

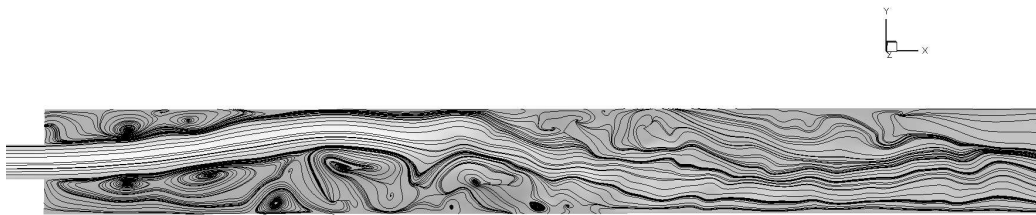
Figure 4.13: Streamlines of velocity of the vortex shedding near the third recirculation area at $Re\ 700$



(a) $t=10$ non-dimensional time

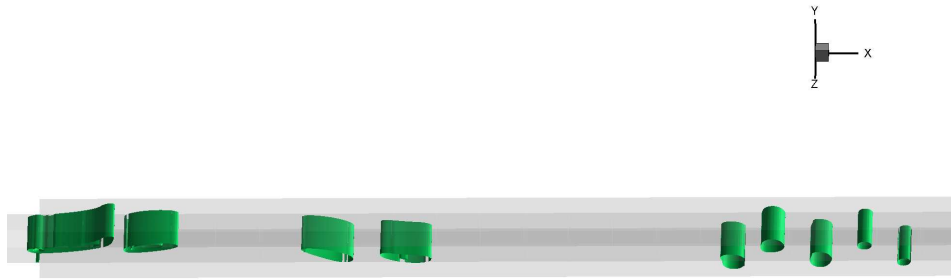


(b) $t=20$ non-dimensional time

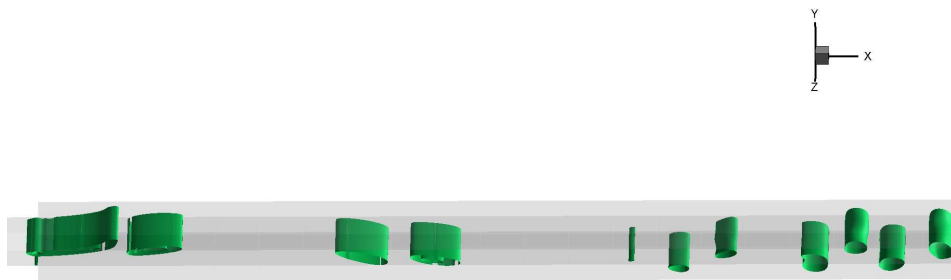


(c) $t=30$ non-dimensional time

Figure 4.14: Streamlines of velocity of the vortex shedding near the third recirculation area at $Re\ 900$



(a) Isosurfaces of Q criterion at $Re = 350$



(b) Isosurfaces of Q criterion at $Re = 412$

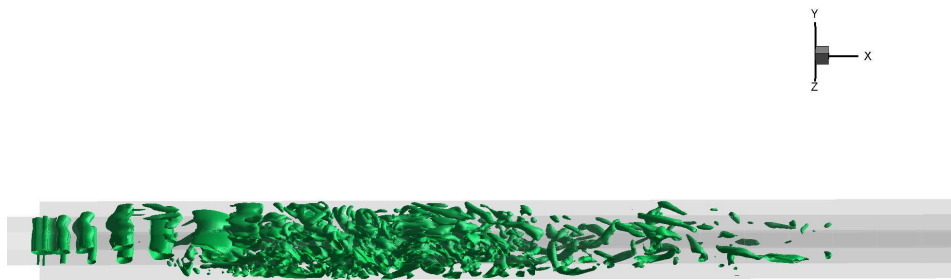


(c) Isosurfaces of Q criterion at $Re = 550$

Figure 4.15: Q criterion isosurfaces from $Re = 350$ to 550



(a) Isosurfaces of Q criterion at Re 700



(b) Isosurfaces of Q criterion at Re 900

Figure 4.16: Q criterion isosurfaces from Re 700 to 900

Chapter 5

Spectral Analysis

For the further understanding of the intermittent behavior of the flow a kinetic energy spectral study has been undertaken. Using Fast Fourier Transformation (FFT), for a time sample at different points in the flow for the Reynolds numbers examined during the previous chapters, the primary frequency and the slope of the spectrum have been calculated. In the terms of the primary frequency three locations in space was used. The positions of those locations are 10 h, 22 h and 40 h. Those locations were selected since the flow in its of their position behaves, depending the Reynolds number, in a different manner. The position of the first location is before the end of the large recirculation area, the second position is after the end of the first recirculation area and approximately at the middle of the third one and the third position is after the end of the third recirculation area.

For the better comparison with the experimental results of Fearn et al [11] and for consistency the non-dimensional time samples where dimensionilised using the maximum inlet velocity 0.105 m/s and the length of the step height 0.004 m for Reynolds number 412 from the experimental data provided by Fearn [11]. For the calculation of the slopes of the sub-ranges of the spectrum an upper frequency were

Reynolds Number\Position	10 h	22 h	40 h
Re 350	1.4	1.4	1.1
Re 412	1.5	1.7	0.9
Re 550	2.2	2.0	1.0
Re 700	2.4	1.9	-
Re 900	-	-	-

Table 5.1: Table of the primary frequencies at $Y= 1.5$ and different positions in the X direction for various Reynolds numbers

Reynolds Number\Position	0.5 h	1.5 h	2.5 h
Re 350	1.4	1.4	1.4
Re 412	1.6	1.7	1.7
Re 550	1.9	2.0	2.0
Re 700	1.9	1.9	2.0
Re 900	-	-	-

Table 5.2: Table of the primary frequencies at X=22 h and different positions in the Y direction for various Reynolds numbers

calculated. As presented before ,for a fully turbulent flow, the smallest viscous timescale is $\tau_\eta/\tau \sim Re^{-1/2}$ so the frequency of this viscous timescale will be $f_\eta/f \sim Re^{1/2}$. This upper frequency exist for a fully turbulent flow but can still provide an upper limit since if the flow is not fully turbulent the smallest viscous scale will be largest than the smallest turbulent viscous scale and consequently the higher frequency will be lower than the aforementioned upper limit.

First the behavior of the primary frequencies will be examined in the streamwise direction. The table 5.1 presents the primary frequencies in those position at the corresponding Reynolds numbers. For the first position, as being seen in figure 5.5, the primary frequency starts at a value of 1.4 and reach a value of 2.4 at Reynolds number 700. At Reynolds number 900 it has not a single identifiable primary frequency. Also the amplitude of the kinetic energy increases with the Reynolds number with the more significant increment to take place between Reynolds number 412 and 550. The reason for such an increase of almost two orders of magnitude is the relocation of the vortex shedding area from the third recirculation area to the end of the second one in the lower wall as presented in the physical analysis.

At 22 length scales downstream the primary frequency has the same trend as the previous position reaching the peak value of 2.0 at Re 550 ,figure 5.6, and 1.9 at Reynolds 700. Also there is not a single identifiable primary frequency at Reynolds number 900 . The differences in the values of those two positions are small for the low Reynolds numbers verifying the conclusion of Fearn [11] that the primary frequency does not change significantly with the measurement position. The amplitude rises with the Reynolds number until Re 550 and then start do decrease up to Reynolds number 900. This behavior of the amplitude also coincide with the change of the position of the vortex shedding area as well as the change of the position of the large vortex area shear layer moving both upstream.

Finally at the third position, figure 5.7, the primary frequency has a value of

about 1 Hz up until Re 550 and can not be discerned a specific one for higher Reynolds numbers. The amplitude reach its maximum value earlier at Re 550 and declined at higher values. The values of the primary frequencies for this position is not very close to the other positions upstream but this can be explained due to the distance of this position to the other two and to the area of the vortex generation causing this area to be affected not directly from the coherent vortical structures traveling downstream but from the disturbances its dissipation is causing.

Next the behavior of the primary frequencies will be examined for a fix X position and several vertical ones. For 22 step height downstream kinetic energy spectra were calculated at three position for all the Reynolds numbers. Table 5.2 and figure 5.4 provide the energy spectra. The examination of the primary frequencies show that there is not a substantial difference in terms of primary frequencies by changing position in the vertical direction. The maximum difference in the values between the positions is of the order of 0.1 Hz. The examination of the form of the energy spectra confirm that conclusion since the spectra for all the positions have the same trend for all the range of frequencies. On the other hand reveal that despite that there are some differences in the energy levels between positions. This is to be expected since this flow is not symmetric for those Reynolds numbers in the wall normal direction and thus different flow behavior affect the energy levels as moving across the vertical direction.

From the behavior of the primary frequencies of the velocity energy spectra some conclusion can be drawn. The values of the primary frequencies in the first two positions in the vortex area of the flow are close to each other. For the first two Reynolds number the highest frequency is predicted at 22 h downstream and for higher values the maximum measured frequency moves upstream to 10h. This is in accordance to the changing of the vortex shedding position from the third recirculation area to the large one at the lower wall. Changes in the vertical positions as measured at 22h downstream did not produce a significant change of the primary frequency although those measurement position covered different flow aspects. This means that the fluctuation that is captured in the form of the primary frequency affects the whole height of the channel as it travels downstream.

An other characteristic that can be extracted from the energy spectra plots is that for the lower Reynolds numbers ,depended on the position, the area of the primary frequency is clearly visible from the other frequencies due to the higher energy at this range. When the Reynolds number is increasing the area of the primary frequency start to be no longer so clearly visible until it can not be distinguished from the energy levels around it. This is the reason for the lack of primary frequencies in

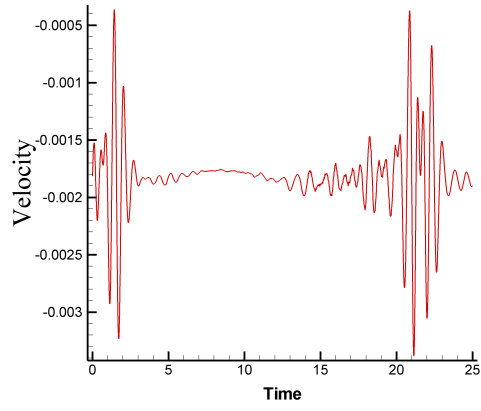
Reynolds Number \ Position	10 h		22 h		40 h	
Re 412	-	-	-	-	-4.41	-6.68
Re 550	-	-	-3.84	-5.68	-3.98	-7.95
Re 700	-	-	-3.1.	-7.5	-3.41	-8.45
Re 900	-3.74	-9.4	-2.68	-7.4	-3.4	-8.60

Table 5.3: Table of the spectral slopes at different positions and Reynolds numbers

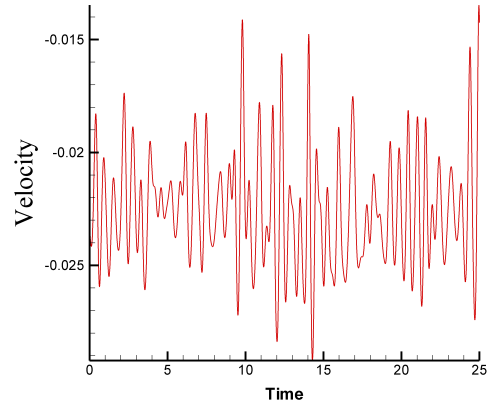
the highest Reynolds number. This is due to the increase of the chaotic behavior of the flow from Reynolds number 550 and above as it was discussed in the previous section. The clarity of the primary frequency at the lower Reynolds numbers is due to the vortex shedding of large coherent vortices that carried the majority of the energy that was transferred through the flow. As the flow reaches a more chaotic and three dimensional behavior the shedding have a more random character with different structures and scales transferring the energy causing a not discernible primary frequency. This fact is an extra evidence of the flow moving to increasingly transitional state.

Figures 5.1, 5.2, 5.3 present the change of the behavior of the velocity as the Reynolds number is increasing. For Reynolds 350 and 412 the velocity has an almost laminar behavior with some indications of the beginning of transition. As Reynolds number reach 550 the velocity has clearly a more chaotic profile a characteristic that is becoming more pronounced as the Reynolds number reaches 900. This confirms the results obtained in the form of Q criterion in the previous chapter that at Re 550 the flow has reached a clearly chaotic flow indicating an transitional to turbulence behavior. It should also be noted that each position change behavior at different Reynolds numbers with the positions further upstream keeping a less chaotic behavior until higher Reynolds numbers. This is to expected since the unsteady and transitional flow for this range of Reynolds numbers have localized characteristics and are depended on the specific flow behavior. Since from Re 550 and afterwords the flow has such a behavior the slopes of the kinetic energy spectrum will be analyzed for those Reynolds number for the further understanding of the flow behavior.

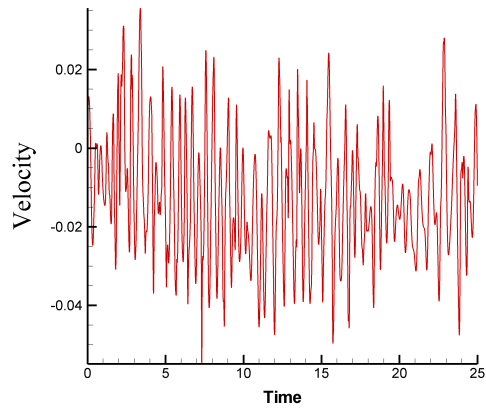
As its been discussed before although for channel flow for those Reynolds numbers the flow should be laminar the physics of it especially for the higher Reynolds



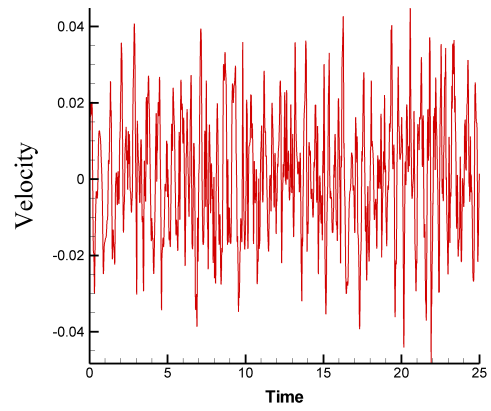
(a) Re 350



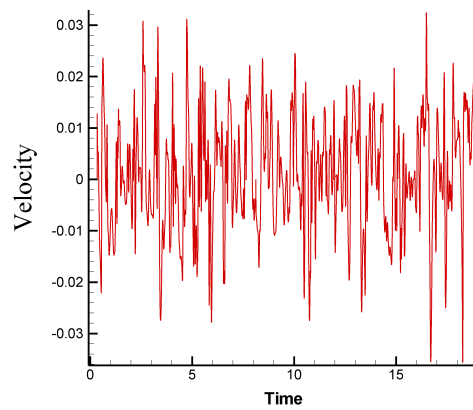
(b) Re 412



(c) Re 550



(d) Re 700



(e) Re 900

Figure 5.1: Time variation of V velocity at 22h for various Reynolds numbers

Reynolds Number\Position	0.5 h		1.5 h		2.5 h	
Re 550	-3.84	-5.37	-3.84	-5.68	-3.3	-6.95
Re 700	-3.11	-10.0	-3.1	-7.5	-2.94	-8.26
Re 900	-2.74	-7.64	-2.68	-7.4	-2.59	-6.5

Table 5.4: Table of the spectral slopes at different positions and Reynolds numbers

suggest otherwise. For this reason the kinetic energy slopes has been calculated. Kolmogorov has been stated that all homogeneous isotropic turbulent flows has its inertial subrange slope equals to $-5/3$. Although this is not homogeneous flow this criterion will be used, and for this reason the slopes where calculated, to judge if and how close the flow has reached to a transitional or turbulent state. The table 5.3 and 5.4 and the figures 5.5, 5.6, 5.7 show the slopes for the three positions in the streamwise direction and the three vertical position at 22h. The slopes of two subranges were calculated, where those two subranges could be identified, the inertial and the dissipation subrange.

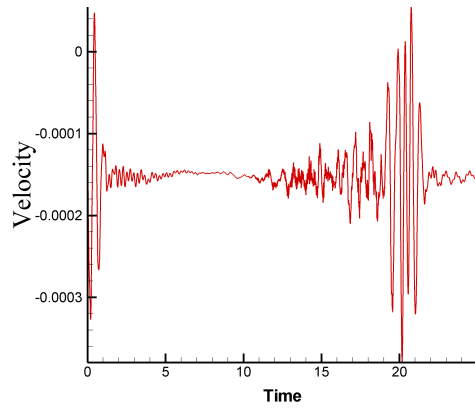
For Reynolds 350 the two slopes could not be clearly identified at any examined position. At the Reynolds 412 the two subranges were identified only at 40h downstream. This is expected since at this Reynolds number the flow has still a mainly laminar behavior for most of the upstream part as it can be deduced from the plots of the time signals and only further downstream where the vortices generated start to dissipate the flow reaches a more transitional behavior. At Reynolds 550 the slopes are recognizable further upstream as the flow reaches a more chaotic state. The inertial subrange slope decreased in 40h from -4.41 to -3.98 and now can be measured to -3.84 at 22h. Also the dissipation subrange become more steep at 40h downstream.

As the Reynolds number reaches 700 the same trend continues . At 22h the inertial subrange slope has decreased to -3.1 and at position 40h to -3.41. Regarding the dissipation range in both positions the corresponding slopes became steeper. Also there still not clearly identifiable inertial and dissipation subrange slopes at 10h upstream. Reaching Reynolds 900 the slopes can be intensified in all the examined positions. For the dissipation range they varies from -9.4 at 10h to -7.4 at 22h. The inertial slopes decreased further with position 22h having a slope of -2.68.

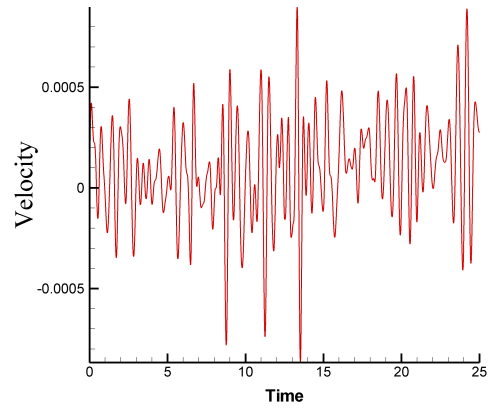
For the inertial subrange slope a definite trend is appearing. As the Reynolds number is increasing so the slope is approaching the $-5/3$. This seem to indicate that the inertial slope of a transitional signal with the increment of the Reynolds number is approaching a turbulent state in a smooth manner.

Examining the behavior of the slopes of the inertial subrange for the vertical direction at 22h the same trend is followed. The slopes for the three positions are very close with each other and are decreasing with the raise of the Reynolds number. Previously was shown that in terms of primary frequency the three positions had practically the same value there could be differences in the energy levels especially between the position at 2.5h and the other two. Here we can see that although the values of the slopes are very close still there is a pattern. The first two positions, 0.5h and 1.5h, have almost identical values where the third position have constantly a lower one an indication of a more transitional flow. This can be explained if we examine the location of the three positions in the flow. The first is near the lower wall and the second in the middle of the channel and are affected mainly from the large recirculation area. The third is located near the upper wall and is affected also from the third vortex area resulting to a slightly more chaotic flow. So different flow conditions in the vertical direction can cause a different behavior in terms of the kinetic energy spectra slopes.

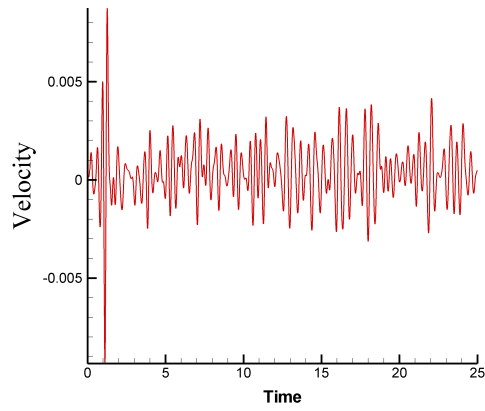
From the examination of those data some conclusion can be droned. It was shown that different parts of the flow are reaching to a transitional state in different Reynolds numbers showing the localized nature of the flow. Also the transition area will start further downstream and will propagate upstream as the Reynolds is increasing a fact in agreement with the physical behavior described in the previous chapter. Regarding the slopes it was shown that the inertial one is decreasing with the increment of the Reynolds number. In none of the examined position the slope reached the Kolmogorov slope of -1.667 but the overall behavior indicated that as the Reynolds number is increasing and the flow is reaching a turbulent state the kinetic energy spectra are reaching gradually those described by the Kolmogorov.



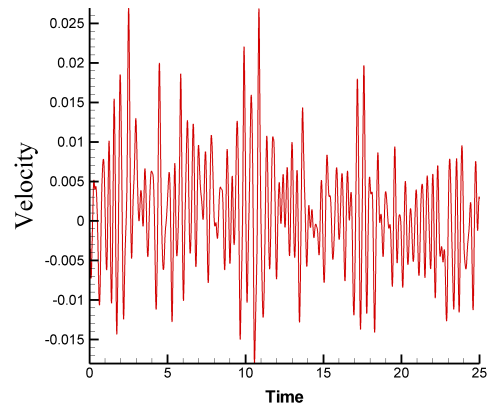
(a) Re 350



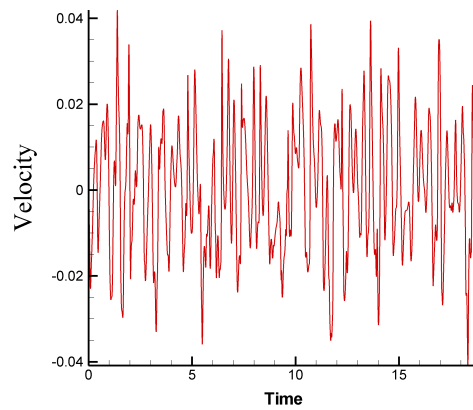
(b) Re 412



(c) Re 550

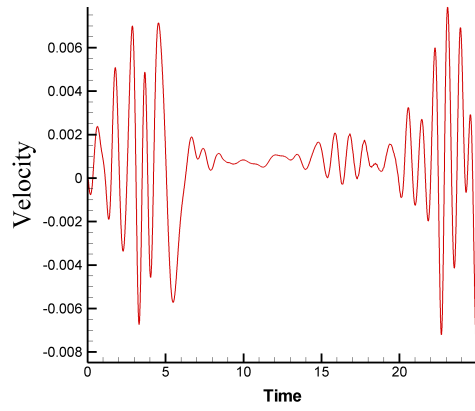


(d) Re 700

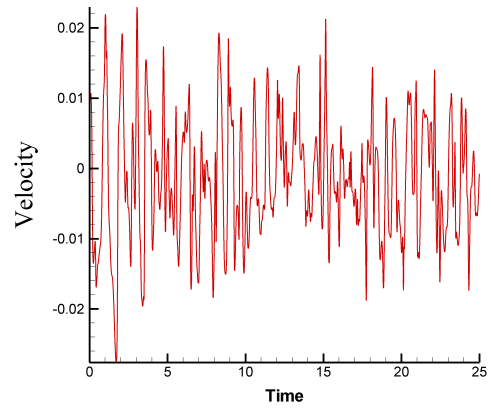


(e) Re 900

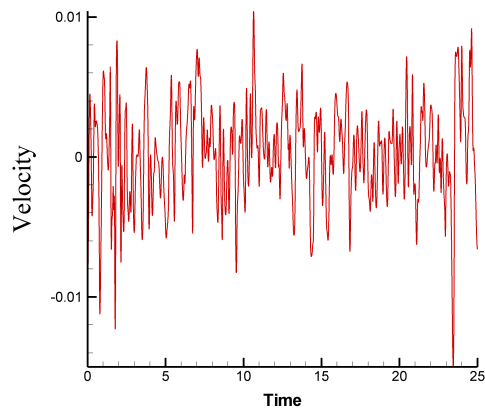
Figure 5.2: Time variation of V velocity at 10h for various Reynolds numbers



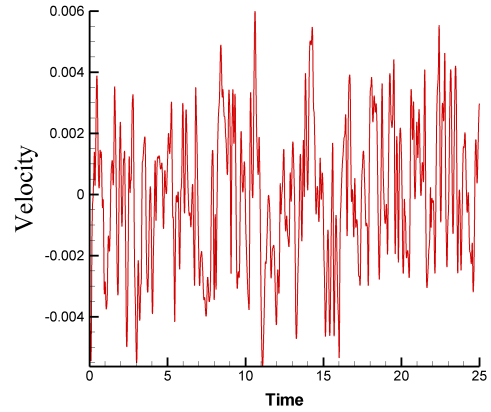
(a) Re 350



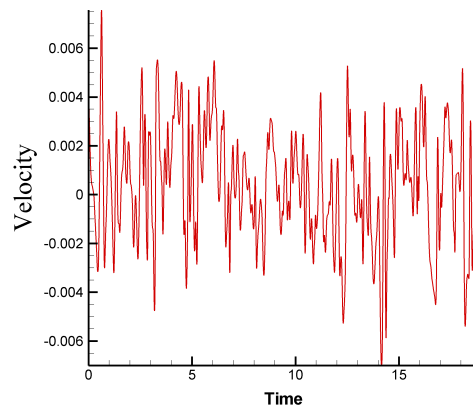
(b) Re 412



(c) Re 550

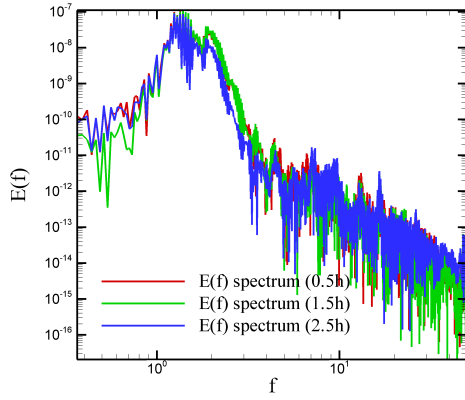


(d) Re 700

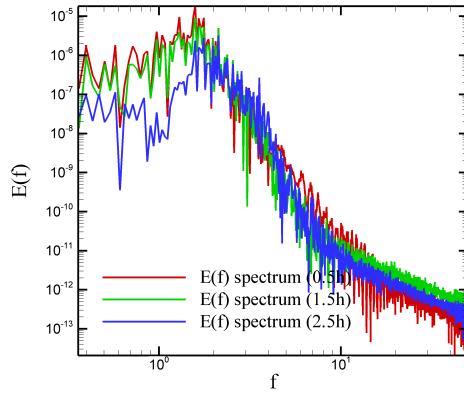


(e) Re 900

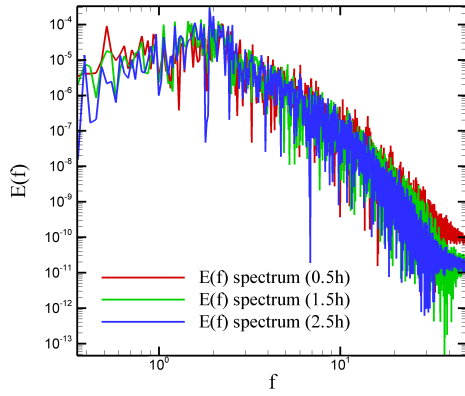
Figure 5.3: Time variation of V velocity at 40h for various Reynolds numbers



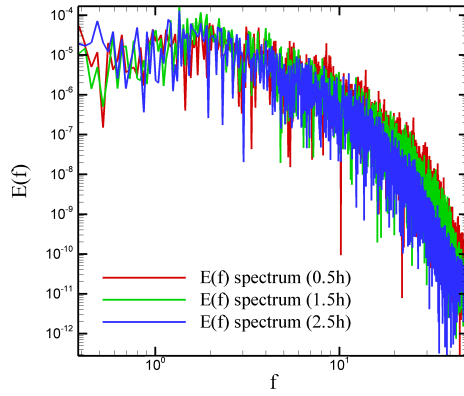
(a) Re 350



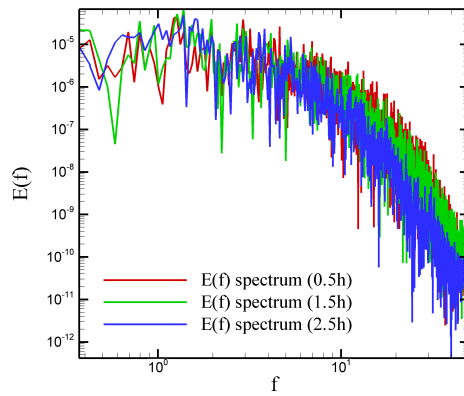
(b) Re 412



(c) Re 550

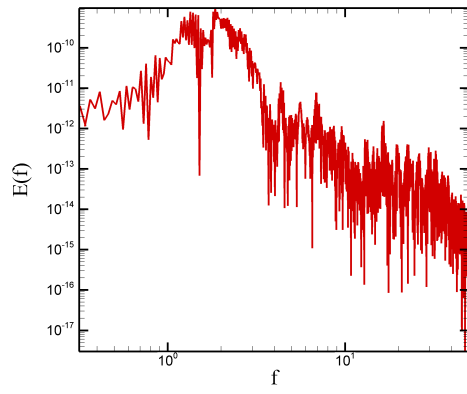


(d) Re 700

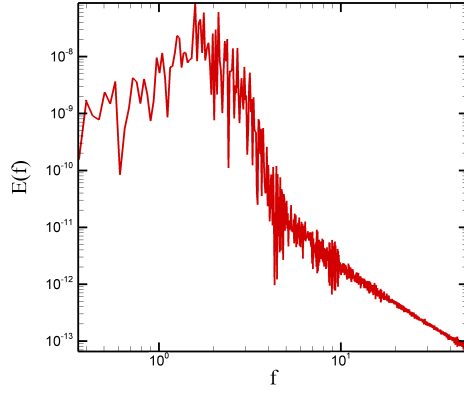


(e) Re 900

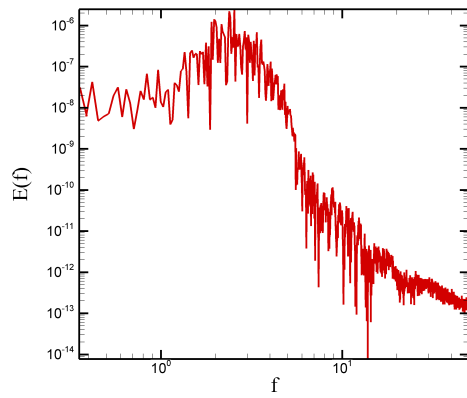
Figure 5.4: Kinetic energy spectra at 22h downstream for three different vertical positions and different Reynolds numbers



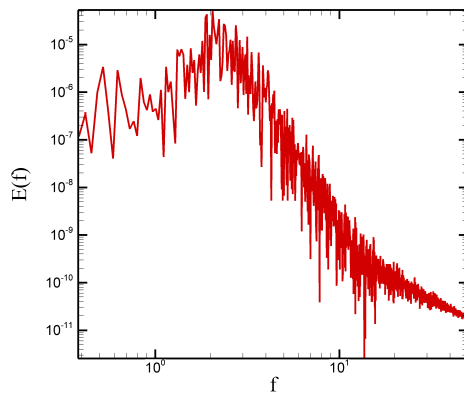
(a) Re 350



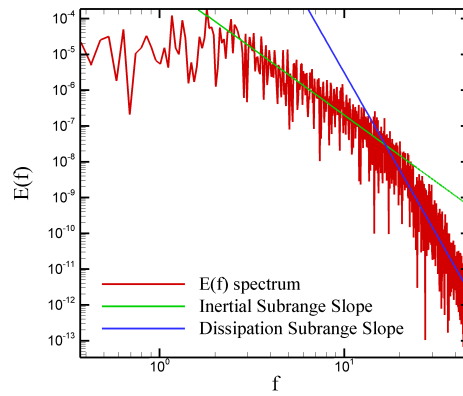
(b) Re 412



(c) Re 550

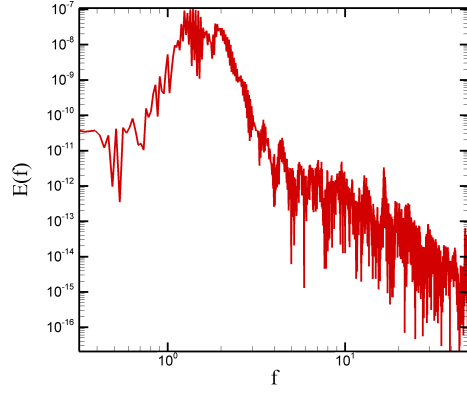


(d) Re 700

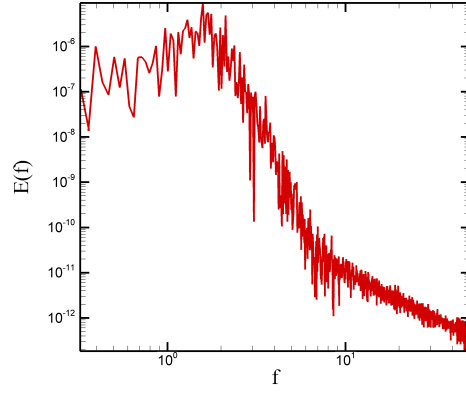


(e) Re 900

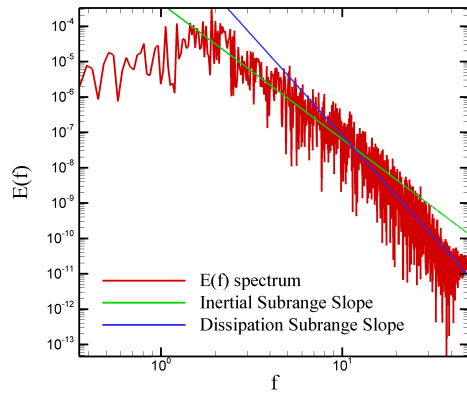
Figure 5.5: Kinetic energy spectrum slopes at 10 step heights downstream



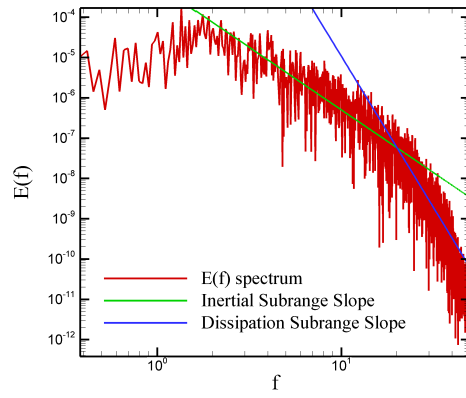
(a) Re 350



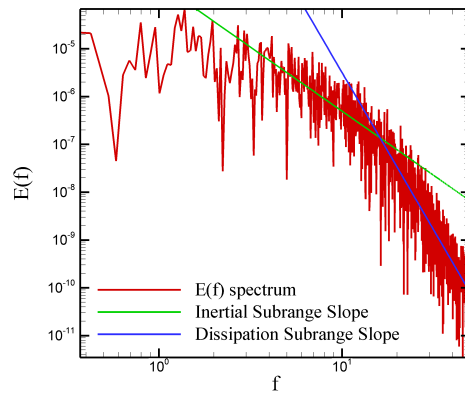
(b) Re 412



(c) Re 550

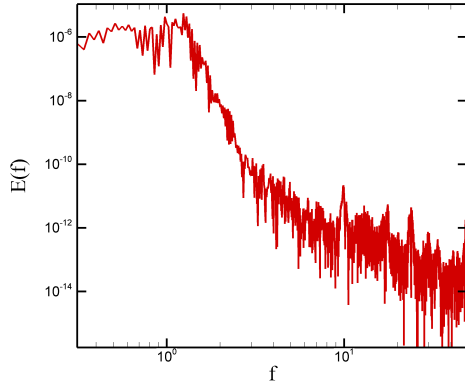


(d) Re 700

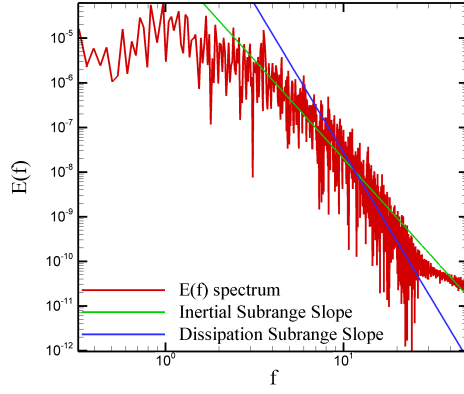


(e) Re 900

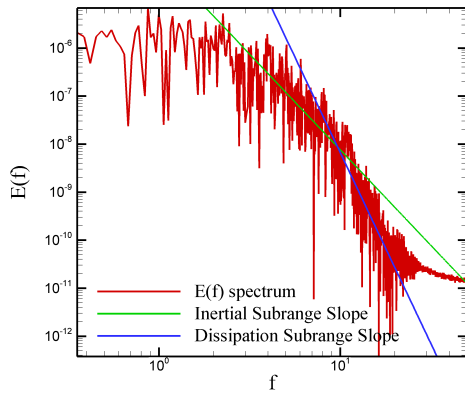
Figure 5.6: Kinetic energy spectrum slopes at 22 step heights downstream



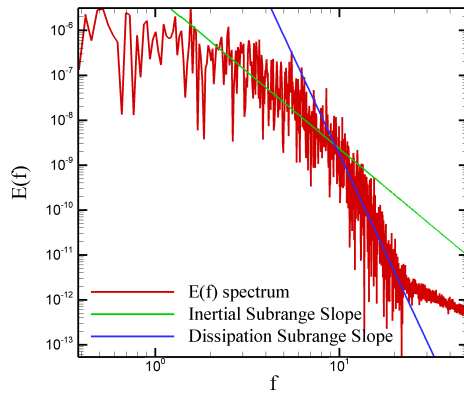
(a) Re 350



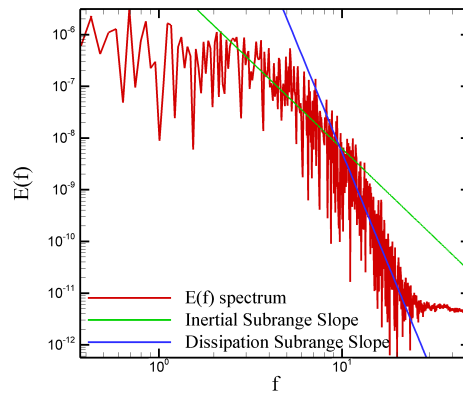
(b) Re 412



(c) Re 550



(d) Re 700



(e) Re 900

Figure 5.7: Kinetic energy spectrum slopes at 40 step heights downstream

Chapter 6

Conclusions

In the previous chapters a detailed analysis of a laminar and transitional suddenly expanded flow has been made. From this analysis a series of concluding remarks can be drawn.

- The comparison of different numerical methods revealed that the WENO scheme managed to capture the flow physics better than the MUSCL scheme. Especially at Reynolds number 900 the WENO scheme capture more and smaller scales of the flow than the MUSCL one. This is probably due to the difference in type of the methods and the higher resolution that the fifth order WENO scheme provides.
- The mean flow characteristics at various Reynolds numbers were studied. A bifurcation diagram of the three main vortex areas was created illustrating the evolution of those areas with the Reynolds number. An identification of a smaller vortical area in the upper left corner at Reynolds number 900 was made. This vortical area is present at high turbulent Reynolds numbers and was never before reported in this relatively low Reynolds number.
- The examination of the time depended behavior of the flow mapped the vortex shedding mechanism due to the presence of shear layer instabilities. This process lead to the shifting from an essentially quasi three dimensional behavior up to Reynolds 412 to a more chaotic fully three dimensional from Reynolds 550 and up. It was found that the occurrence of the shedding to a specific area coincides with the beginning of the reduction of the length of that recirculation area.
- Finally a quantitative velocity spectral study undertaken. The velocity time samples revealed clearly the presence of transitional bursts in lower Reynolds

numbers and the increased chaotic nature of the signal as the Reynolds number is increasing. The primary frequencies in several places in the flow increased with Reynolds number up to the point where the signal was too chaotic to be identified. The slopes of the inertial sub-range of the spectrum indicated a decrease in the slope, as the Reynolds number is increasing, gradually converging to the Kolmogorov's $K^{-5/3}$.

Chapter 7

Future Work

From the discussion of the conclusions a few suggestions of the future work can be made.

- Further investigation of the performance of the high order methods with the ILES is needed. Higher order of accuracy as well as different limiter functions should be used for the MUSCL scheme to verify whether the difference in the performance when compared to the WENO scheme is due to the different order of accuracy or the method itself.
- In terms of physical analysis higher Reynolds numbers should also be investigated up to the point when a fully turbulent occurs so the characteristics of the flow, both physical and spectral, can be analyzed and the processes that lead to turbulence can be fully studied. Also the effects of finite aspect ratio (side walls) should be investigated especially regarding the spectral behavior at the spanwise direction and near the side wall surfaces.

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Appendix A: Wall Effects

Case description

In the previous chapters were described in detail the physical behavior of the sudden expansion flow both in time averaged and time depended terms. It was discussed the evolution of the flow as the Reynolds numbers was increased the the flow exhibited first unsteady laminar and then transitional characteristics. In this chapter a full geometry will be used with added walls to the spanwise direction in order to examine the effects the side walls have to the flow at the middle of the channel.

Previously simulations with periodic conditions at the spanwise boundaries were conducted with a small aspect ratio for the bulk of the cases examining the flow at the middle of the channel for each Reynolds number limited by the availability of the computational capacity. In this case the geometry of the case has a full aspect ratio as can be seen at figure 7.1. The expansion ratio is 3 calculated from the main channel height $H=3$ divided by the step height $h=1$ and the aspect ratio is 8 calculated by the span of the channel $W=24$ divided by the channel height $H=3$. The length of the inlet is 5 and of the main channel is 170

The grid that was utilized has the same point distribution in the X-Y axis as the medium grid in the previous cases with added resolution in the Z axis having 800 points streamwise, 80 points in the height and 120 points in the spanwise direction in total 7,680,000. For the processes of parallilazation 91 blocks were created, 30 in the across the length, 3 along the height plus one for the inlet.

The Reynolds number for the spanwise wall case is 900 as the highest of the periodic case. 4th order Runge-Kutta method was used for the inner time stepping and first order backward differencing for the real one. The real time CFL is 0.7 and the inner time 0.5. The β for this case also is 1 and the incompressibility limit is 10^{-3} . Finally fifth order WENO scheme was used for the numerical reconstruction.

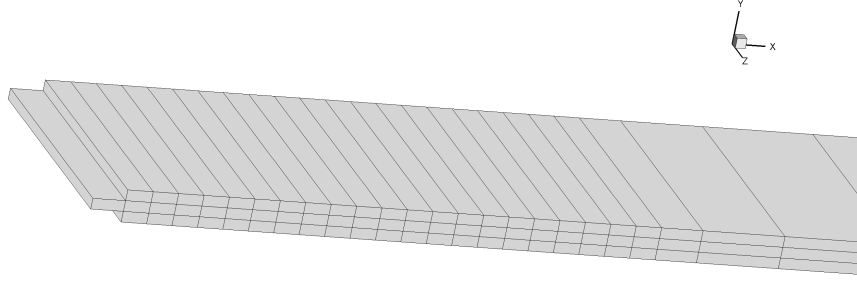


Figure 7.1: Geometry and blocking of the full aspect ratio sudden expansion case

Boundary conditions

The boundary conditions for the wall bounded case are for the X direction at the left side there is a parabolic velocity inlet and extrapolation for the pressure

$$u = 1 - \left(\frac{y - (y_{max}/2)}{y_{max}/2} \right)^2 * \left(\frac{y - (z_{max}/2)}{z_{max}/2} \right)^{10.9},$$

$$v = 0,$$

$$w = 0,$$

$$\frac{\partial^2 P}{\partial x^2} = 0$$

at the right X side an an outflow conditions is implemented with extrapolation for the velocities and a fix value for the pressure

$$\frac{\partial^2 u}{\partial x^2} = 0,$$

$$\frac{\partial^2 v}{\partial x^2} = 0,$$

$$\frac{\partial^2 w}{\partial x^2} = 0,$$

$$P = 1$$

For the walls at the Y and Z axis a non slip boundary condition were used.

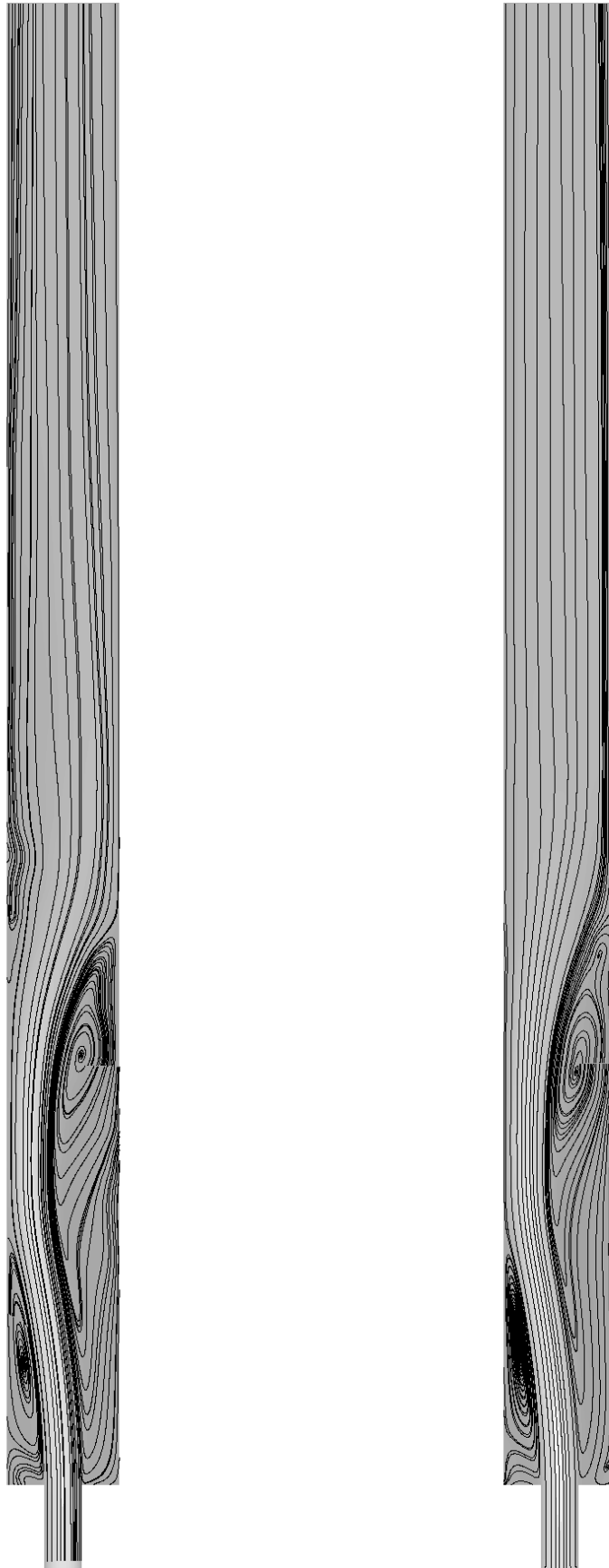
Case	First vortex	Second vortex
Periodic	15.2	6.5
Wall bounded	15.5	6.5

Table 7.1: Comparison of the length of the main vortices between Periodic and wall bounded case

Comparison between periodic and wall bounded case

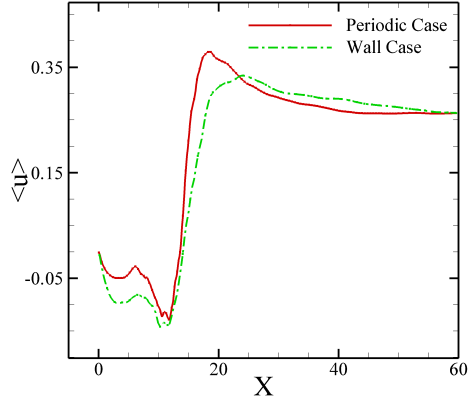
For the verification of the results obtained from the wall bounded case a comparison between the periodic case of medium grid for Re 900 with WENO scheme and the middle plane of the wall bounded case were compared. With the help of the streamlines at the mean flow of the wall bounded case as it can be seen in figure 7.2 two main recirculation areas are evident. This is in agreement with the periodic case. The comparison between the length of the two main vortices, table 7.1, demonstrates the close resemblance between the two flows.

The velocity lines plotted, figure 7.3, at the X (streamwise) direction at three different positions of Y (0.5,1.5,2.5) are used to demonstrate in more detail the effects to the flow from the introduction of the side walls and the finite aspect ratio. At position $y=0.5$, the mid-section crossing the large recirculation area, the flow between the two cases is very similar for the u velocity following the same trend with the main difference being the amplitude at 15 to 20 length after the end of the vortex area. This difference in the peak values is more pronounced in the v velocity although the trend and the general behavior is sufficient. The stresses for the same line are also in good agreement exhibiting as well the same difference at the maximum value between 15h and 20h. It should also be noted that the periodic case have the peak values for the velocities and the stresses. At the center of the channel ($y=1.5$) the difference between the two cases are even smaller where in case for the velocities being the deviation at the peak values at 15 step heights downstream at the end of the large recirculation area. In the case of the stresses the trend and the peak values are in agreement hinting to a very similar flow. In the upper section of the channel a similar behavior arise as before with the wall case being smoother at the 15-20 length area. This is due to the stabilizing effects the walls are having to the flow since at the periodic case small vortices appear intermittently where the third recirculation area was something not being observed at the wall bounded case.

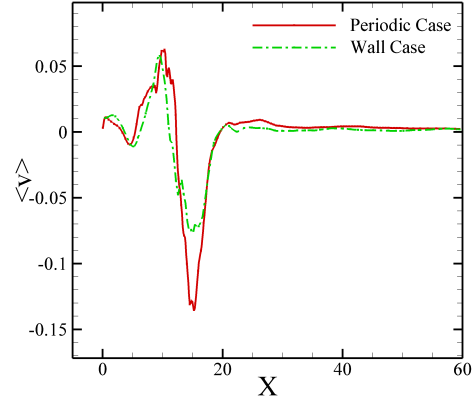


(a) Streamlines of velocity with periodic bound-ary in Z direction at Re 900 (b) Streamlines of velocity with wall boundary in Z direction at Re 900

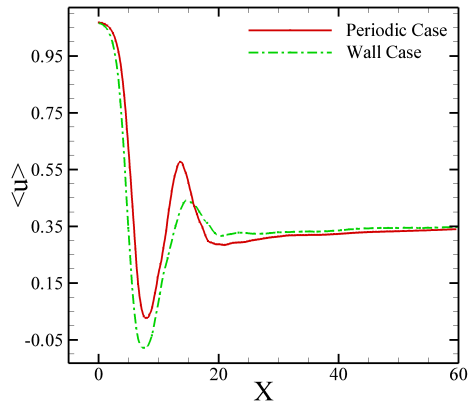
Figure 7.2: Comparison of the streamlines of the velocity between periodic and wall bounded case at the mean plane



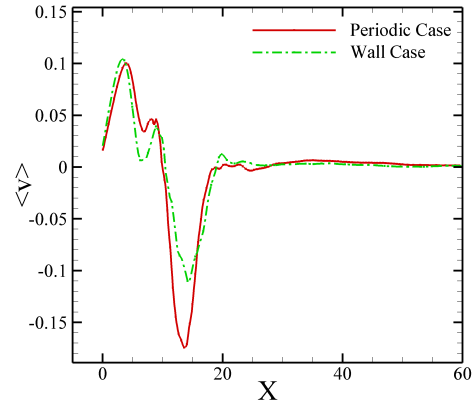
(a) streamwise velocity at $Y = 0.5h$



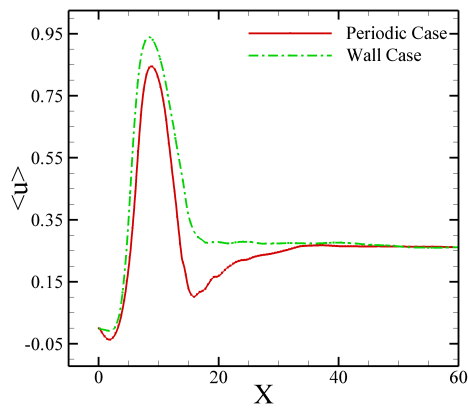
(b) wall normal velocity at $Y = 0.5h$



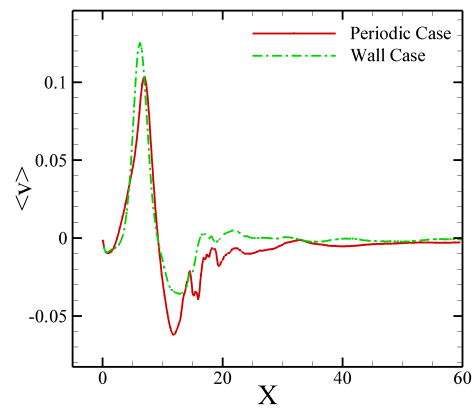
(c) streamwise velocity at $Y = 1.5h$



(d) wall normal velocity at $Y = 1.5h$

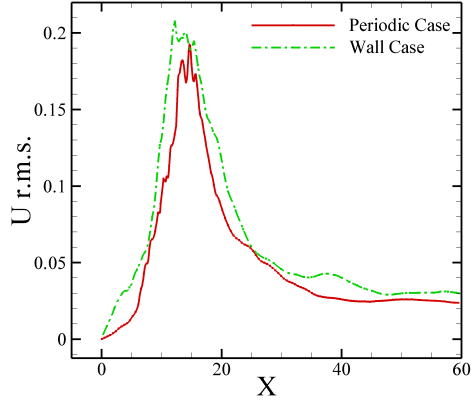


(e) streamwise velocity at $Y = 2.5h$

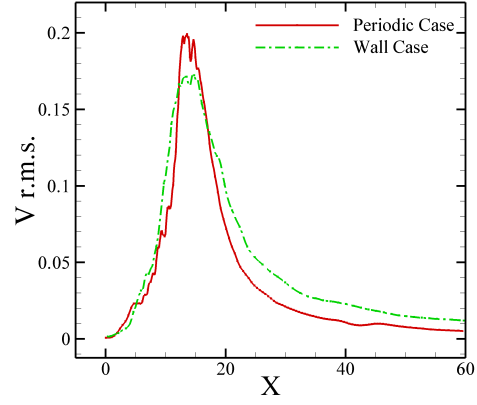


(f) wall normal velocity at $Y = 2.5h$

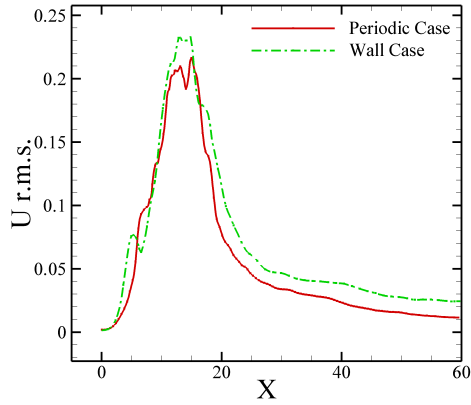
Figure 7.3: Comparison in terms of velocities of X lines between wall bounded and periodic cases



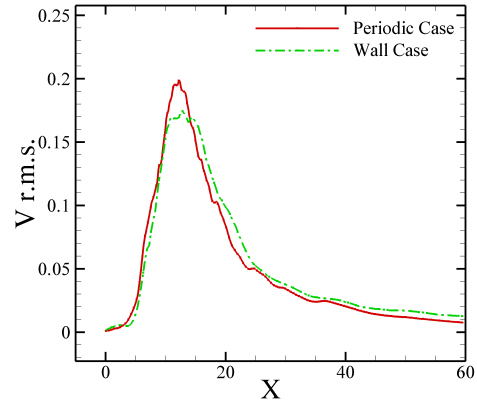
(a) u normal stress at $Y = 0.5h$



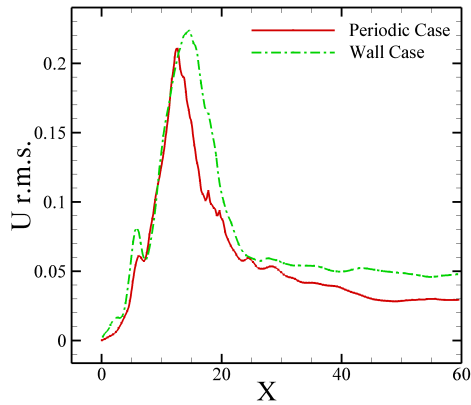
(b) v normal stress at $Y = 0.5h$



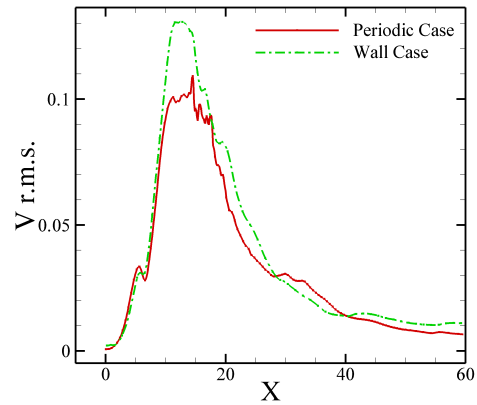
(c) u normal stress at $Y = 1.5h$



(d) v normal stress at $Y = 1.5h$



(e) u normal stress at $Y = 2.5h$



(f) v normal stress at $Y = 2.5h$

Figure 7.4: Comparison in terms of stresses of X lines between wall bounded and periodic cases

