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Nonlinear Analysis for Wing Rock System with Adaptive Control

Dongyang Li[1]
Cranfield University, Cranfield MK43 0AL, UK
Nanjing University of Science and Technology, Nanjing 210094, China

Dmitry Ignatyev[2] and Antonios Tsourdos[3]
Cranfield University, Cranfield MK43 0AL, UK

Zhongyuan Wang[4]
Nanjing University of Science and Technology, Nanjing 210094, China

Adaptive control has the potential to improve the performance and reliability of aircraft. However, the inherent nonlinearity of adaptive control causes difficulty in stability and robustness verification which is critical before entry into service. In this paper, the region of attraction (ROA) estimation using sum of squares (SOS) technique is explored for adaptive control systems. The problem of wing rock suppression by a model reference adaptive control (MRAC) serves as a benchmark example to demonstrate the effectiveness of the method. Considering the potential disturbance presented in measurement signals, the operative range of aircraft must be included in the ROA of the control system. Therefore, the ROA is evaluated using SOS polynomial optimization under various combinations of design parameters in the adaptive law, including adaptation rate and sigma modification values. This gives insight into the interactions of design parameters on the adaptive control performance.

I. Introduction

The potential of adaptive control in improving the performance and reliability of control systems attracts researchers and engineers from different disciplines. However, the verification and validation of the flight control system regarding modeling error, uncertainty, disturbance, and time delay is not an easy task due to the inherent nonlinearity of the problem as well as the lack of rigorous analysis tools for nonlinear systems[1–6]. The fact is that safety-critical flight systems require extensive validation before entry into service. Thus, the flight control community will benefit a lot from the development of the nonlinear analysis[6].

A significant metric for a flight control system is stability. Unlike the global stability of a linear system, it is usually high-demanding or, most of the time, impossible, to guarantee stability across the whole state space for a nonlinear system. In many practical applications, global stability is also unnecessary. The actual dynamical system usually operates in a specific range of the state space due to the restrictions from the physical properties of the system. Therefore, for the engineers, it will be enough to ensure that the system works normally inside the operative range. Then, system states will not escape from the allowable range and always go back to the desired steady-state that is designed as the equilibrium of the dynamical system. The region of attraction (ROA) for the specific equilibrium happens to mark the set of initial conditions from which a trajectory will always travel back to the corresponding

[1] PhD student, School of Energy and Power Engineering, Nanjing University of Science and Technology.
[2] Senior Research Fellow, School of Aerospace, Transport and Manufacturing, Cranfield University.
[3] Professor, School of Aerospace, Transport and Manufacturing, Cranfield University. AIAA Senior Member.
[4] Professor, School of Energy and Power Engineering, Nanjing University of Science and Technology.

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equilibrium. Thus, the designer will have to work out how to ensure the operative range is included in the ROA[7,8]. This then gives rise to the problem of determining the ROA of a nonlinear system.

However, it is hard to determine the exact ROA if not impossible[7–10]. Various kinds of methods dedicated to estimating ROA, which are roughly divided into analytical and computational methods[11,12]; or into Lyapunov-based methods and non-Lyapunov-based methods[13] regarding the theory behind the methods. The analytical methods can only apply to a limited kind of relatively simple nonlinear systems. As regards the computational ones, the Monte Carlo simulation is usually used as a supplement to the analysis. The numerical simulation takes plenty of time especially for a complex and high-dimensional system, but still the results cannot be taken deterministic. The indeterminism is rooted in the sampling initial conditions. It cannot be guaranteed that the simulation distinguishes all the divergent trajectories in the final set. Another kind of computational method, instead of doing exhausted numerical integrations, solves the problem formulated by matrix inequalities, such as the Lyapunov-based estimation[8,14,15]. The obtained results can be deterministic since it is derived analytically but solved computationally.

Many nonlinear systems, including the adaptive system considered in this paper, are governed by polynomial dynamics. The Sum of squares (SOS) technique has the ability to solve polynomial inequalities. SOS polynomial optimization has been used previously to analyze the performance and robustness of systems described by polynomial dynamics. Computational algorithms have been developed not only for estimating ROA but also reachability sets, input-output gains, robustness with respect to uncertainty, and time-delay margin[10,16]. Applying the SOS technique for ROA estimation, this paper aims to show the ability of the techniques to analyze the stability and robustness verification for a nonlinear adaptive control system.

Wing rock is a well-known lateral-directional instability that occurs in aircraft of varying configurations and aspect ratios at a moderate to high angle of attack. The accurate modeling and control have been explored by many authors[17,18]. Therefore it serves as a classical benchmark example for the demonstration of various control methodologies[19]. In this paper, wing rock augmented with a classical model reference adaptive control (MRAC) is taken as a demonstration of the ability of the SOS technique in the nonlinear analysis of adaptive control.

The SOS techniques show great promise for ROA estimation[20] and are then modified and improved in order to gain a better approximation[12,15,21–23]. It has been applied successfully to a four-state aircraft control system for the investigation of the falling-leaf mode[24], and an eight-state aircraft model for the investigation of the spin mode[25]. A simple procedure is used in [26] for estimating the ROA of an adaptive control system. Although the SOS results are conservative, they provide rigorous proof of the stability characteristics of nonlinear systems. In its current form, the solver for the SOS optimization problem is sensitive to the dynamics of the system, such as the dimension of the system and the highest order of the polynomials. But it stands great promising in the nonlinear analysis if pursued further[6].

The paper is organized as follows. Firstly, the wing rock dynamics and the classical adaptive controller is described in Sec. III. In Sec. IV, the algorithms for ROA estimation using SOS polynomial optimization are presented. Sec. V summarizes the results of ROA analysis for an adaptive control system with respect to adaptation gain and sigma modification parameter. Finally, the paper concludes in Sec. VI.

II. Wing Rock Dynamics and Adaptive Control

A. Wing Rock Model

The model of wing rock, taken from [17], considering a flat, thin wing constrained such that it is free only to roll about its x-axis, is described as

$$\ddot{\phi} = (\rho U_\infty{}^2 Sb / 2I_{xx} C_l) + d_0 u \tag{1}$$

where ϕ is the roll angle and the overdot denotes time derivative. d_0 is the control effectiveness element and the input signal u acts as part of the angular acceleration. The roll-moment coefficient is written as

$$C_l = a_1\phi + a_2\dot{\phi} + a_3\phi^3 + a_4\phi^2\dot{\phi} + a_5\phi\dot{\phi}^2 \tag{2}$$

The aerodynamic parameters a_i are nonlinear functions of the angle of attack which have been presented in [17]

Defining $x = (x_1, x_2)^T = (\phi, \dot{\phi})^T$, the wing rock system can be cast as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + d_0 u \end{cases} \tag{3}$$

where $x \in \mathbb{R}^2$, the Euclidean space of dimension of two,

$$f(x) = b^T h(x)$$

and

$$b = [b_1, b_2, b_3, b_4, b_5]^T, b_i = (\rho U_\infty{}^2 Sb / 2I_{xx}) a_i \quad i = 1,...,5$$
$$h(x) = [x_1,\ x_2,\ x_1{}^3,\ x_1{}^2 x_2,\ x_1 x_2{}^2]^T$$

B. Model Reference Adaptive Control

A model reference adaptive controller is applied to suppress the wing rock motion. The reference model carrying the desired response is given by

$$\dot{x}_m = A_m x_m \tag{4}$$

where $x_m = [x_{m1}, x_{m2}]^T$, $\varsigma > 0$, $\omega_n > 0$ and

$$A_m = [0, 1; -\omega_n{}^2, -2\varsigma\omega_n] \tag{5}$$

The reference signal is chosen as $x_m(t) = 0$ in this study. The desired damping ς and natural frequency ω_n are 0.707 and 0.5, respectively.

The wing rock control problem is formed under the assumption on the function $f(x)$: suppose that in Eq. (3) the parameters $b_i, i = 1,...,5$ and d_0 are unknown, but the sign of d_0 is known.
Defining the tracking error by

$$\dot{e} = A_m e + d_m[(b + b_m)^T h(x) + d_0 u] \tag{6}$$

where $d_m = [0, 1]^T$ and $b_m = [\omega_n{}^2, 2\varsigma\omega_n, 0, 0, 0]^T$. Then the control law is chosen as

$$u = -\theta^T h(x) \tag{7}$$

where $\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T$ is a vector of adaptation parameters to enable the control input to cancel the system uncertainty. Then the adaptation law governing the behavior of parameter vector θ is defined as

$$\dot{\theta} = \text{sgn}(d_0)\Gamma(e^T P d_m) h(x) - \kappa_R \Gamma\theta \tag{8}$$

where $d_m = [0, 1]^T$; the tracking error e represents the difference between the state x and the reference signal x_m. The adaptation gain $\Gamma = \text{diag}(\Gamma_i), i = 1,...,5$ with $\Gamma_i > 0$ and sigma modification gain κ_R are two important parameters to tune. The adaptation gain determines how quickly θ evolves, and the sigma modification gain helps to prevent the parameter drift by ensuring θ bounded, which adds robustness to the system. P is the positive definite solution of the Lyapunov equation

$$A^T P + PA = -Q,\ Q > 0 \tag{9}$$

where $A = A_m$. The stability analysis is given in [27,28].

C. Disturbed Adaptive Control

Before entry into service, the implementation of controllers must be verified and validated under certain circumstances including modeling error, time delay, uncertainty, and disturbance. In terms of disturbance, a common scenario where the signal measurement process inevitably interferes is considered. The disturbance is then introduced during the measurement of roll angle ϕ and angular acceleration $\dot{\phi}$. Here, it is denoted as d_d and takes a quadratic form

$$d_d = \mu x_2{}^2 \tag{10}$$

where the constant μ indicates the level of the disturbance.

The disturbed measurement signals are written as

$$x_d = [x_{1d}, x_{2d}]^T = [x_1 + d_d, x_2 + d_d]^T \tag{11}$$

which is used to construct control input signal

$$u_d = -\theta^T h_d(x_d) \tag{12}$$

where u_d is the actual signal sent to the executing agency, and h_d is the relevant five-vector function. In this case, the actual adaptation law applied to the wing rock system becomes

$$\dot{\theta} = -\text{sgn}(d_0)\Gamma(e^T P d_m) \cdot h_d(x_d) - \kappa_R \Gamma\theta \tag{13}$$

and the error dynamics

$$\dot{e} = A_m e + d_m\{d_0 \tilde{\theta} h_d(x_d) + (b - b_m)[h(x) - h_d(x_d)]\} \tag{14}$$

It shows that the control input cannot cancel the part of the uncertainty so that the system cannot guarantee asymptotical stability. It indicates that the disturbance affects the original stability of the system. Thus, the information of the region of attraction for the origin is necessary to evaluate whether the operative range is still included in the stable range, which helps to provide confidence for the implementation of the controller.

The aerodynamic parameters for an angle of attack of 25 deg [27] are used in this paper for the demonstration of the occurrence and suppression of wing rock. The dimensionless time $t^* = (4U_\infty / b)t$ as in [27] is also introduced. Parameters in Eq. (3) are listed below with t^* as an independent variable rather than t

$$b_1 = -0.02012844, \quad b_2 = 0.01051916$$

$$b_3 = 0.02596236, \quad b_4 = -0.1273338, \quad b_5 = 0.5197074$$

$$d_0 = 1$$

In Fig. 1, the wing rock phenomenon occurs at a smaller initial condition which shows as a limit cycle on the $\phi - \dot{\phi}$ plane. But the aircraft loses stability at larger conditions. For the convenience of explanation, the disturbance level μ is specified to take the value of 0.1. Simulation of the response is similar to that of the undisturbed control system for the two smaller initial conditions as shown in Fig. 2, where the wing rock phenomena are successfully suppressed. This corresponds to the equilibrium analysis of the closed-loop system (13) and (14), which indicates that the origin is locally asymptotically stable with eigenvalues -0.3535 + 0.3536i. However, random trajectory simulation shows that the roll angle and angular acceleration go into infinity under some initial conditions, for example, the largest initial condition in Fig. 2. Therefore, the ROA needs to be evaluated.

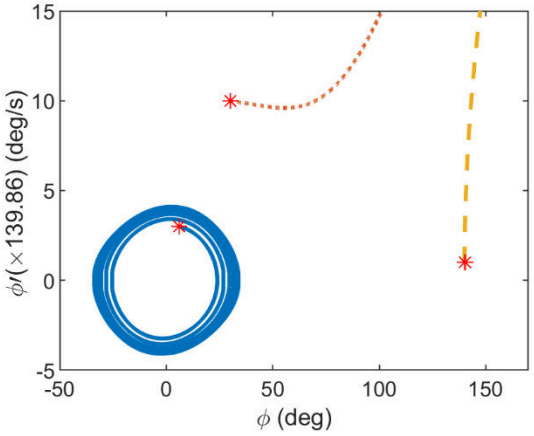
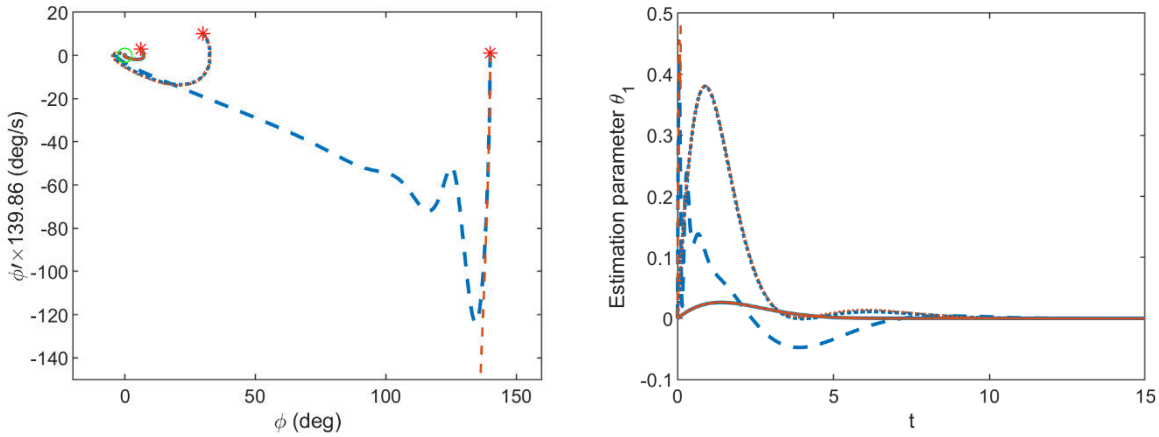


Fig. 1 Phase portrait for the open-loop wing rock system



a) b)

Fig. 2 Phase portrait for adaptive wing rock system with (red) and without disturbance (blue) measurement error

III. Region of Attraction Estimation

A. Lyapunov Theory

Consider an autonomous nonlinear polynomial system of the form

$$\dot{x} = F(x), \quad x(0) = x_0 \tag{15}$$

with $F : \mathbb{R}^n \to \mathbb{R}^n$ and $x \in \mathbb{R}^n$. Without loss of generality, we assume that the origin is an asymptotically stable equilibrium point such that $f(0) = 0$. The ROA, a set of initial conditions whose trajectories will always converge back to the origin, can be defined as

$$\Omega := \left\{ x_0 \in \mathbb{R}^n : \text{If } x(0) = x_0 \text{ then } \lim_{t \to \infty} x(t) = 0 \right\} \tag{16}$$

Based on the proved lemma which follows the direct Lyapunov theory that specifies a sublevel set of a Lyapunov function as an estimation of the ROA for an asymptotically stable equilibrium point, the compact set Ω is estimated.

Lemma 1 If there exist a continuously differentiable scalar function $V(x) : \mathbb{R}^n \to \mathbb{R}$ and a positive scalar $\gamma \in \mathbb{R}^+$, such that

$$V(x) > 0 \quad \forall x \neq 0 \text{ and } V(0) = 0 \tag{17}$$

$$\Omega_\gamma := \{ x : V(x) \leq \gamma \} \text{ is bounded} \tag{18}$$

$$\Omega_\gamma \subseteq \{ x : (\partial V / \partial x) F < 0 \} \cup \{0\} \tag{19}$$

then the origin is asymptotically stable and Ω_γ is a subset of the ROA. $V(x)$ is the so-called local Lyapunov function.

B. Sum of Squares (SOS) Techniques

Making use of the SOS polynomial, the hard problem of a multivariable polynomial being non-negative can be relaxed to being the sum of squares. Then the inequalities as well as set containment conditions in Eqs. (17)-(19) become tractable in the form of an SOS problem. Several toolboxes, such as SOSTOOLs, SOSOPTs, and solvers, such as SeDuMi[9], have been distributed and are freely available for solving such problems. For example, verifying a polynomial $s(x) \in \boldsymbol{R}(x)$ ($\boldsymbol{R}(x)$ represents the set of polynomials in $x \in \mathbb{R}^n$ with real coefficients) being an SOS polynomial is equivalent to checking the existence of a positive semidefinite matrix Q, such that

$$s(x) = Z^T(x) Q Z(x) \in \Sigma_n \tag{20}$$

where $Z(x)$ is the properly chosen vector of monomials and Σ_n denotes the set of SOS polynomial in $x \in \mathbb{R}^n$. This is the basic problem in SOS programming.

Besides, the well-known generalized S-procedure[14] is employed to provide sufficient conditions for the set containment condition.

Lemma 2 (Generalized S-procedure). Given polynomials $g_0(x),...,g_m(x) \in \boldsymbol{R}[x]$ and polynomials $s_1(x),...,s_m(x) \in \Sigma_n$, if

$$g_0(x) - \sum_{i=1}^{m} s_i(x) g_i(x) \geq 0 \tag{21}$$

then

$$\{x | g_1(x),..., g_m(x) \geq 0\} \subseteq \{x | g_0(x) \geq 0\} \tag{22}$$

Then we can approximate the set (16) by

$$\begin{aligned} &\max_{V \in \Sigma_n,\, s_1 \in \Sigma_n} \quad \gamma \\ &\text{Subject to}: -[(\partial V / \partial x)F + l_1] - (\gamma - V)s_1 \in \Sigma_n \end{aligned} \tag{23}$$

where l_1 is a small positive polynomial (typically $\varepsilon x^T x$ with some small real number ε) which is used to guarantee the derivative of V to be strictly negative. $s_1(x)$ is an SOS multiplier with a proper degree. According to [29], the degree can be chosen by

$$\begin{aligned} \max D(V \cdot s_1) &\geq \max D((\partial V / \partial x)F + l_1) \\ \min D(s_1) &\geq \min D((\partial V / \partial x)F + l_1) \end{aligned} \tag{24}$$

where $D(s_1)$ represents the degree of the polynomial s_1. A proper choice of s_1 is important to capture the feasible result of the problem described by Eq. (23) without introducing unnecessary computation burden. Because a higher degree of s_1 will introduce more decision variables.

Moreover, the problem (23) is bilinear because the decision variables in V and s_1 are coupled, and also the variable to be optimized γ is not affine. However, due to the fact that the linear solver has more advantages than the nonlinear ones such as PENMBI in YALMIP[30], (23) has to be modified to accommodate to linear solver used in this paper. It is decoupled by choosing a feasible Lyapunov function V in advance and keeping it fixed; the optimized γ is searched by bisection procedure.

A Lyapunov function can be constructed using its physical meaning for a specific problem. While a systematic way is employed here to generate an initial choice

$$V = x^T P x \tag{25}$$

where P is the solution of Lyapunov equation (9) with $A = (\partial F / \partial x)\big|_{x=0}$ that is assumed to be Hurwitz. In [15], it is explained why $Q = I$ is chosen but it is still a conservative choice since only the largest ball is provided. Other positive definite Q can also be customized to adapt to a specific shaped ROA when prior knowledge is available.

The optimized γ provides a lower bound for the ROA estimation. Then exhaustive Monte Carlo search is carried out using $\theta_i = 0\ (i = 1,...,5)$ to give an insight into the ROA on the $x_1 - x_2$ plane.

IV. Results

The parameter update law in the MRAC is tuned by adjusting the adaptation rate Γ and sigma modification κ_R. As mentioned before, the sigma modification is employed to ensure θ remains bounded. Generally, the system displays increased performance in tighter tracking and faster rise time as the adaptation rate increases. However, it has been shown that increasing Γ will otherwise bring a decrease in system robustness[1]. Therefore, it is necessary to gain more insights into the impact of these parameters on system performance. Especially with the introduced disturbance, how does the choice of these parameters influence the ROA of the control system? Should these parameters be redesigned to ensure the aircraft system operates normally under such a scenario? In the following parts, the problem is explored through ROA estimation of the disturbed adaptive system.

To initialize the bisection procedure for estimation, a first Lyapunov function is computed as

$$V_0 = x^T P x + \theta^T \Gamma^{-1} \theta \tag{26}$$

The lower bound for γ is set zero and an upper bound can be determined by trial and error. A level set is obtained when tolerance $\left|(\gamma_i - \gamma_{i-1}) / \gamma_{i-1}\right| \leq 1\mathrm{e}-3$ is reached.

A. Adaptation Rate

The effect of varying adaptation rates is explored by analyzing the ROA with respect to the origin of the disturbed adaptive system. In this case, the sigma modification value is held constant as 1 to isolate the effect of changing adaptation rate. The ROA variation trends are shown in Fig. 3.

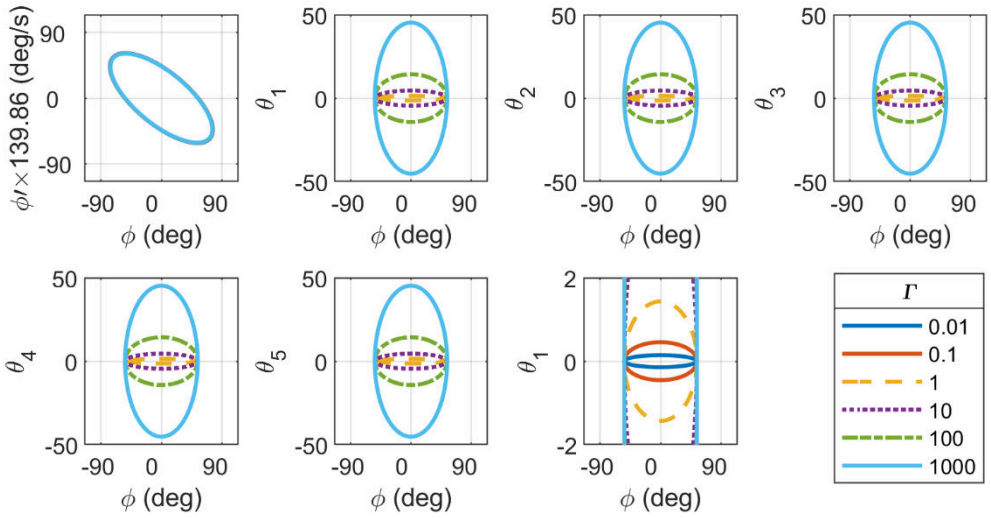


Fig. 3 ROA variation with adaptation rate

The results obtained by the SOS method give a lower bound of the actual ROA of the system. As shown in Fig. 3, the valid region on the $\phi-\dot{\phi}$ plane hardly changes with the variation of adaptation rate Γ, but the bounds for θ_i grow as the adaptation rate increases. This result is intuitive as an increasing Γ leads to a fast adaptation of the unknown parameters θ_i and brings them to a steady value quickly thus increasing the robustness of the system, which shows a larger ROA. It suggests that a larger adaptation rate can stabilize the system on a wider range of initial conditions.

B. Sigma Modification

The effect of varying sigma modification values on the MRAC closed-loop system under certain measurement errors is explored by analyzing the region of attraction around the origin.

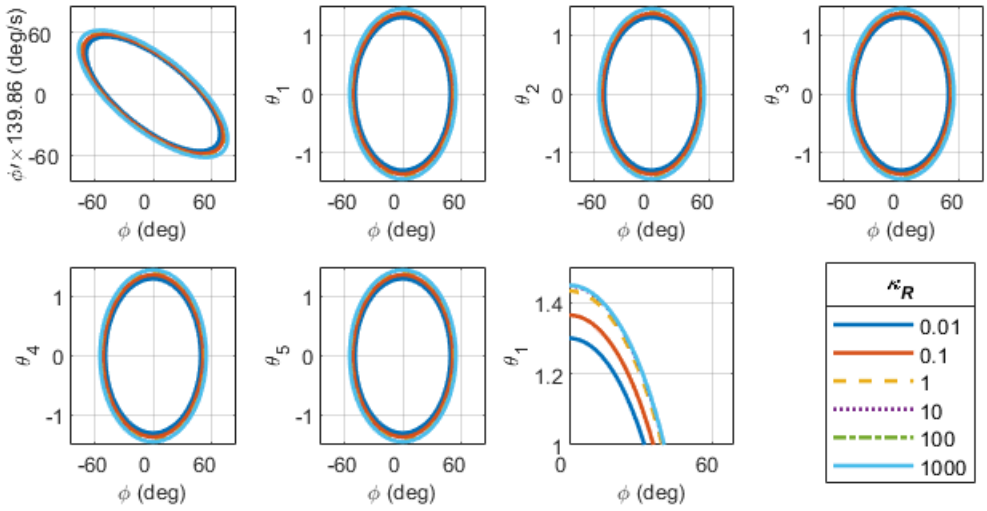


Fig. 4 ROA variation with sigma modification value

With the adaptation rate fixed at $\Gamma = I$, the ROA estimation with respect to various sigma modification values is shown in Fig. 4. The ROA expands slightly as κ_R increases. The sigma modification is used to prevent parameter

drift by bounding the evaluation of estimation parameters. Trajectory simulation shows that the larger the sigma modification value, the faster the convergence and the smaller the overshoot. This can bound the trajectories in a reasonable small range of variation thus is beneficial for the whole system, and correspondingly, can stabilize a larger range of initial conditions. The illustrative results by the SOS method in Fig. 4 reveal this characteristic.

V. Conclusion

The region of attraction(ROA) estimation using sum of squares(SOS) optimization technique is used for nonlinear analysis for the adaptive control system. The ROA is computed with respect to varying control design parameters, the adaptation rate and sigma modification values, and then their effects on ROA estimation are evaluated. Though conservative, the results are illustrative in revealing the nonlinear characteristics of the adaptive control system and in giving insights into the controller design. The SOS method provides guarantees on the certification process of control systems.

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Li, Dongyang; Ignatyev, Dmitry; Tsourdos, Antonios; Wang, Zhongyuan

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