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Observer based incremental backstepping terminal sliding-mode control with learning rate for a multi-vectored propeller airship

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Abstract The problem of finite-time trajectory tracking control for a multivectored propeller airship with model parameter uncertainties and unknown disturbances is addressed in the paper. In order to obtain a fast transient response and finite time convergence without singularity, an incremental backstepping nonsingular terminal sliding mode controller is designed to track desired trajectory within finite time. To overcome the limitation of backstepping terminal sliding-mode control design being dependant on prior knowledge, a nonlinear disturbance-observer is designed to estimate the external observable disturbances. Meanwhile, to reduce system chattering when the tracking error reaches the sliding mode surface, an adaptive learning rate design is proposed for the terminal sliding mode controller such that the tracking error approaches the sliding mode surface at a low speed and so the system chattering is restrained. The closed-loop trajectory tracking control system is proved to be stable and finite time convergence by using Lyapunov theory. The results are compared with traditional backstepping sliding mode control and different learning rate based control design, and they demonstrate the averaged tracking error in altitude and pitch motion is reduced more than 30% by using the disturbance-observer based incremental backstepping terminal sliding mode controller for the multivectored propeller airship to execute a realistic trajectory tracking mission, even in the presence of aerodynamic coefficient uncertainties, unknown disturbances.

NOMENCLATURE

α, β	Angle of attack, side-slip angle in wind frame respectively.	$\mathbf{v} = [u \quad v \quad w]^T$	Airship speed in the body frame
α_γ, β	TSMC parameters in Eq.(50)	$\mathbf{v}_d$	Desired virtual control command.
C_h, C_Z and C_{my}	Aerodynamic force coefficients along the horizontal and z-axis, and moment coefficient along y- axis, respectively	$\boldsymbol{\omega} = [p \quad q \quad r]^T$	Airship angular rate in the body frame
h, ς	Reachability condition parameters in Eq.(52)	x_G, z_G	Coordinate position of CG alone x- and z-axis respectively.
I_i, I_{ia}	Airship moments and apparent moments of inertia along the body axis respectively ($i = x, y, z$),	$ x $	<u>Modulus</u> of vector x
I_{xz}, I_{xza}	Airship apparent product of inertia and products of inertia along xz-axis in $\mathbf{F}_b$ respectively.	$\ x\ = \sqrt{x^T x}$	Euclidean norm of vector x .
I_n	An $n \times n$ identity matrix	$\mathbf{z}_1$	Position and attitude tracking error vector
J_i	Sum of apparent moments of inertia and moments (or product) of inertia with $J_i = I_i + I_{ia}$ ($i = x, y, z, xz$ in $\mathbf{F}_b$).	$\mathbf{z}_2$	Speed tracking error vector
m, m_{ia}	Total mass and apparent masses of the airship respectively with $m_i = m_{ia} + m$, $i = x, y, z$.	$\boldsymbol{\eta} = [\phi \quad \theta \quad \psi]^T$	Airship attitude vector
$\bar{q} \in R$	Dynamic pressure	μ_i	Deflection angle of the i th propeller
$s(\cdot), c(\cdot)$	Sine and cosine functions respectively	$diag(a, b, c)$	Diagonal matrix with diagonal elements of a, b, c
s	Sliding mode surface	φ	Angle between the body x-axis and the horizontal component of the airspeed, see Fig.2.
S_{ref}, l_{ref}	Reference area and length of the airship respectively	ϕ_s	The sliding surface boundary layer parameter
$sgn(\cdot)$	Sign function	$\boldsymbol{\xi} = [x \quad y \quad z]^T$	Airship position in the inertial frame
T_i	The i th propeller thrust	λ, p, q	ITSMC parameters in Eq.(52)
$\mathbf{u}_d$	The desired thruster control inputs.		

Keywords: airship; non-singular terminal sliding mode control; nonlinear disturbance observer; learning rate; backstepping control

I. INTRODUCTION

Multi-vector propeller airships are lighter-than-air vehicles that have potential applications as earth observation, remote sensing and communication relay platforms. A flat peach shape for the stratospheric platform is proposed; this has smaller drag coefficient and higher lift efficiency than conventional airships and meets the requirement for the stratospheric environment [1]. However, the airship dynamics are

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inherently nonlinear. The center of gravity (CG) of the airship is often beneath the center of volume (CV), which is easily affected by air pressure and temperature, and the apparent mass and inertia of the airship are not constant [2]. Furthermore, the four vectored propellers which are used for attitude and position control are both coupled and saturate easily since the airship lacks aerodynamic control surfaces. Hence the control is a challenging problem.

Wen, Chen *et al.* [3] proposed a nonlinear model predictive control for station keeping of a multivectored propeller airship, but the approach requires significant online computation resource. Liu and Whidborne [4] proposed neural network adaptive backstepping method to deal with actuator fault and unknown disturbances of the multi-vectored propeller airship, but neural network need complicate calculation to approximate the nonlinear function.

The trajectory tracking problem of the stratospheric airship is also nonlinear, multivariable and with uncertainty. Traditional control design approaches such as gain scheduling are hard design successfully due to nonlinear dynamics [5, 6]. Hence many nonlinear control approaches have been considered, among which there are two well-known methods, that is, backstepping control [7] and sliding mode control [8]. The backstepping approach has been proposed robust control of nonlinear systems in the strict feedback form through construction of control Lyapunov functions (CLFs) [7]. Nonlinear backstepping design has been widely studied for various aerospace control problems, see [9, 10] for example.

SMC is the other well-known nonlinear control method, which can make the system first reach a desired manifold, then reach the equilibrium point by moving along the sliding manifold by means of a variable structure control law design [11, 12]. Because of the insensitivity to matched uncertainties, robustness to external perturbations, simplicity and ease of implementation, and its finite-time convergence, sliding mode control is widely used in aircraft control [12, 13]. Liu, Whidborne *et al.* used SMC to improve the adaptive capability of a nonlinear system [12]. Chen and Edwards [13] proposed a sliding mode online control allocation scheme for fault tolerant control of an aircraft. To improve the speed of the output responses, terminal sliding mode control (TSMC) has been proposed [14]. To overcomes the common singularity

problem in TSMC, Yu *et al.* proposed non-singular TSMC [15, 16]. Recently Yang proposed a time-specified TSMC for airships to improve their response speed [17].

SMC can, however, suffer from control chattering. To suppress this, Yang *et al.* proposed a neural network approximation-based sliding-mode approach for positioning control of an autonomous airship [18]. To exploit the advantages of both backstepping and SMC, a combination of adaptive backstepping and SMC has been proposed [19]. Vieira and Paiva *et al.* proposed a unified backstepping sliding mode framework for airship control design [20]. Van *et al.* proposed a backstepping nonsingular TSMC to improve robustness and fast transient responses for fault tolerant control of a PUMA560 robot [21]. However, these methods depend on an accurate model and so it is hard to deal with highly nonlinear airship dynamics with aerodynamic model uncertainties and unknown variable disturbances. Jia *et al.* proposed an integral backstepping sliding mode control for quadrotor helicopter with external uncertain disturbances [22]. Singh proposed a robust backstepping sliding mode method for the aircraft attitude and altitude control, and an adaptive neural network is used to approximate such an unknown non-linear function with symmetric barrier Lyapunov function to constrain the state [23]. However they are passive methods to process the unknown disturbances, which will produce some conversation.

Since an operating airship is subject to unknown disturbances such as wind, actuator friction and environmental noises, a Disturbance Observer-Based Control (DOBC) approach is introduced in this paper to enhance the disturbance attenuation ability and reduce the control conversation [24, 25]. Chen *et al.* proposed a backstepping sliding mode control and observer-based fault estimation for a quadrotor UAV [26], Liu and Whidborne have proposed a nonlinear disturbance observer to deal with bounded unknown disturbances [27], but these observers do not cope well with variable disturbances or actuator faults. Furthermore, motivated by the work of Yang *et al.* [17], Van *et al.* [21], Chen *et al.* [28] and Lu *et al.* [29], this paper mainly considers the trajectory tracking control of the multi-vectored propeller stratospheric airship. The main contributions are listed as follows:

- 1) A disturbance observer based incremental backstepping terminal sliding-mode control (DO-IBTSMC)

is proposed for trajectory tracking of a multi-vectored propeller stratospheric airship. The benefits of this approach are that it has the non-singular TSMC merits of high robustness and fast convergence, as well as backstepping control in terms of globally asymptotic stability. The IBTSMC does not depend on the accurate model and is robust for the disturbances and model uncertainties.

2) An adaptive learning rate is designed to avoid the tracking error to converge at an over-fast speed and to overcome the problem of system chattering when the error reaches the sliding mode surface.

3) A nonlinear disturbance observer and an adaptive observer are designed for coping with aerodynamic coefficient uncertainty and variable external bounded disturbances. Thus the limitation of the BTSMC dependent on prior knowledge of the bound value of the disturbance and uncertainties is overcome.

4) A weighted pseudo inversion method-based control allocation is embedded into the DO-IBTSMC design to find the optimal control inputs under actuator saturation constraints.

Simulation results show the proposed DO-IBTSMC method has better performance for trajectory tracking control for a multi-vectored propeller airship by comparing with common backstepping sliding mode method.

This paper is organized as follows. Section II gives the nonlinear dynamics model of the multivectored propeller stratospheric airship and presents the trajectory tracking problem. Section III proposes a DO-IBTSMC design, and stability is analyzed for the closed-loop tracking system. Simulations and performance of three scenarios with the DO-IBTSMC method are demonstrated in Section IV. Section V gives some conclusions.

II. DYNAMICS MODELING AND PROBLEM FORMULATION

This section describes the airship model for control law design. The multi-vectored propeller stratospheric

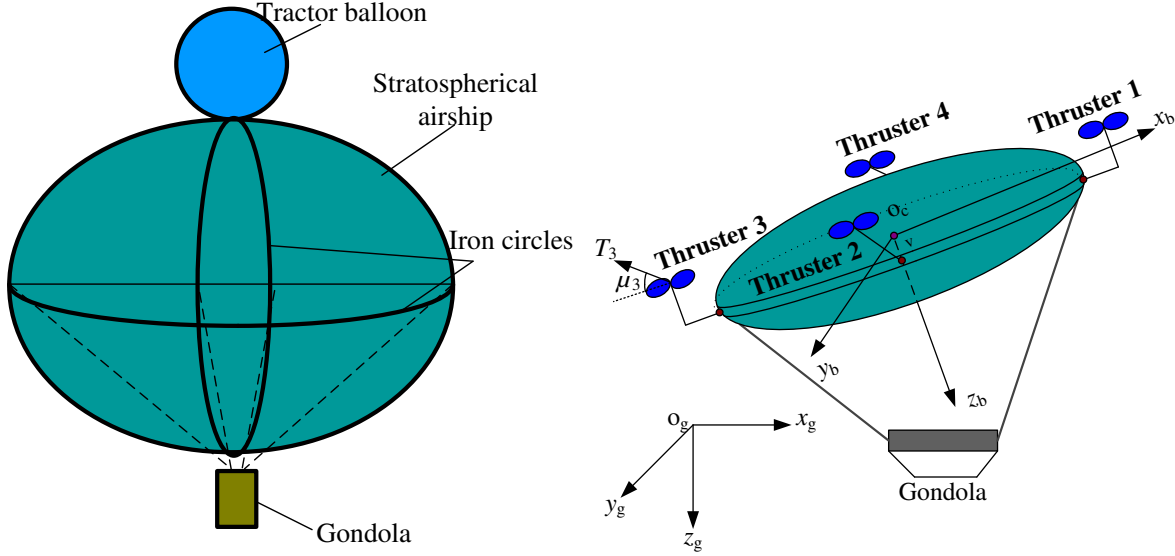


Fig. 1. Structure of the multi-vectored propeller airship

airship is shown in Fig.1. Due to its large volume, it's often drawn by a tractor balloon during the flight.

There are four vectored propellers located on the left, right, front and rear of the airship body and there are no aerodynamic control surfaces. The gondola below the airship envelope is equipped with the control system and payloads. It is assumed that the Earth is flat and non-rotating, so the ground frame is equal to the inertial frame.

A. Airship kinematics model

The kinematics model of the multi-vectored propeller airship's position and attitude are as follows [3, 4].

$$\dot{\mathbf{x}}_1(t) = \mathbf{T}\mathbf{x}_2(t) \quad (1)$$

where $\mathbf{x}_1 = [\boldsymbol{\xi}^T \quad \boldsymbol{\eta}^T]^T$, $\mathbf{x}_2 = [\boldsymbol{\nu}^T \quad \boldsymbol{\omega}^T]^T$. $\boldsymbol{\xi}$ and $\boldsymbol{\nu}$ denote airship position vector and speed vector respectively.

$\boldsymbol{\eta}$ and $\boldsymbol{\omega}$ denote airship attitude vector and angular rate vector respectively. The transition matrix

$$\mathbf{T} = \begin{bmatrix} \mathbf{R}(\boldsymbol{\eta}) & \mathbf{0} \\ \mathbf{0} & \mathbf{J}(\boldsymbol{\eta}) \end{bmatrix}, \quad (2)$$

The direction cosine matrix is

$$\mathbf{R}(\boldsymbol{\eta}) = \begin{pmatrix} c(\theta)c(\psi) & -c(\phi)s(\psi) + s(\phi)s(\theta)c(\psi) & s(\psi)s(\phi) + c(\phi)s(\theta)c(\psi) \\ c(\theta)s(\psi) & c(\phi)c(\psi) + s(\phi)s(\theta)s(\psi) & -s(\phi)c\psi + c(\phi)s(\theta)s(\psi) \\ -s(\theta) & s(\phi)c(\theta) & c(\theta)c(\phi) \end{pmatrix} \quad (3)$$

and the transformation matrix $\mathbf{J}$ is

$$\mathbf{J}(\boldsymbol{\eta}) = \begin{pmatrix} 1 & s(\phi)\tan\theta & c(\phi)\tan\theta \\ 0 & c(\phi) & -s(\phi) \\ 0 & s(\phi)\sec\theta & c(\phi)\sec\theta \end{pmatrix} \quad (4)$$

where $|\theta| < \pi/2$ is assumed to avoid the singularity of matrix because $\theta = \pm\pi/2$ is not likely to be encountered during practical operation of the airship.

B. Dynamics model of the multi-vectored propeller airship

The dynamics model of the multi-vectored propeller airship with consideration of an external disturbance $\mathbf{D}$ is established as follows [30-31]:

$$\mathbf{M}_a \begin{bmatrix} \dot{\mathbf{v}}(t) \\ \dot{\boldsymbol{\omega}}(t) \end{bmatrix} = \mathbf{F}_k + \mathbf{F}_{GB} + \mathbf{F}_A + \mathbf{F}_T + \mathbf{F}_D, \quad (5)$$

where subscripts A , T , k , GB and D denote aerodynamic force, thrusts, kinematics force, sum of gravity and buoyancy, disturbance force respectively. $\mathbf{M}_a$ denotes the generalized mass matrix as follows

$$\mathbf{M}_a = \begin{bmatrix} m_x & 0 & 0 & 0 & m \cdot z_G & 0 \\ * & m_y & 0 & -m \cdot z_G & 0 & m \cdot x_G \\ * & * & m_z & 0 & -m \cdot x_G & 0 \\ * & * & * & J_x & 0 & -J_{xz} \\ * & * & * & * & J_y & 0 \\ * & * & * & * & * & J_z \end{bmatrix}$$

where * denotes the symmetrical elements of the matrix.

Let the control input be $\mathbf{U} = \mathbf{F}_T$, and then (5) can be rewritten as

$$\dot{\mathbf{x}}_2(t) = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + \mathbf{g}_2 \mathbf{U}(t) + \mathbf{D}, \quad (6)$$

where $\mathbf{f}_2 = \mathbf{M}_a^{-1} \cdot (\mathbf{F}_k + \mathbf{F}_{GB} + \mathbf{F}_A)$, $\mathbf{g}_2 = \mathbf{M}_a^{-1}$. Since there exist winds, environmental and sun-radiation noise for the stratospheric airship, then the model disturbance $\mathbf{D}$ is introduced, $\mathbf{D} \in \mathbb{R}^6$ denotes a unknown disturbance vector. Suppose that the disturbance is generated by a linear exogenous system,

$$\begin{cases} \dot{\zeta}_d(t) = W_d \zeta_d(t) \\ D(t) = V_d \zeta_d(t) \end{cases} \quad (7)$$

where $\zeta_d \in R^p$, W_d , V_d are matrices with corresponding dimensions, and this model represents a wide range of periodic disturbances.

Airship aerodynamic force, kinematics force, sum force of gravity and buoyancy, and their moments on the right hand side of (6) are as in [3, 4].

The airship aerodynamic forces and moments on the right hand side of (5) can be expressed as follows [1],

$$\begin{cases} \mathbf{F}_A = \bar{q} S_{ref} [C_h \cos \varphi & C_h \sin \varphi & C_z]^T \\ \boldsymbol{\tau}_A = \bar{q} S_{ref} l_{ref} [-C_{my} \cos \varphi & C_{my} \sin \varphi & 0]^T \end{cases} \quad (8)$$

where φ denotes the angle between the body x-axis and the horizontal component of the airspeed, see

Fig.2, the reference area S_{ref} and length l_{ref} of the airship satisfy $S_{ref} = \bar{V}^{2/3}$, $l_{ref} = \bar{V}^{1/3}$.

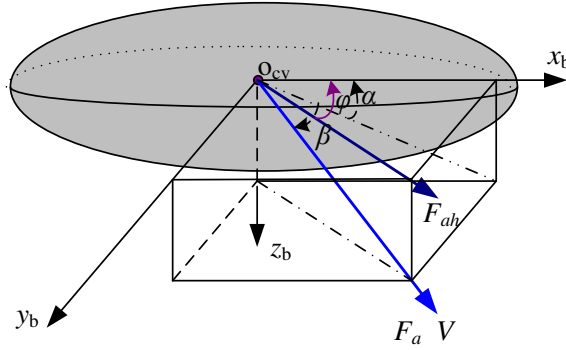


Fig. 2. Vector graph of aerodynamic forces

The kinematics force vector of the airship is

$$\mathbf{F}_k = \begin{bmatrix} -m_z wq + m_y vr - m_{z_G} pr + m_{x_G} (r^2 + q^2) \\ -m_x ur + m_z wp - m_{z_G} qr - m_{x_G} pq \\ -m_y vp + m_x uq + m_{z_G} (p^2 + q^2) - m_{x_G} pr \\ (J_y - J_z) qr + J_{xz} pq + m_{z_G} (ru - pw) \\ (J_z - J_x) pr + J_{xz} (r^2 - p^2) - m_{z_G} (qw - rv) + m_{x_G} (pv - qu) \\ (J_x - J_y) qp - J_{xz} qr - m_{x_G} (ru - pw) \end{bmatrix} \quad (9)$$

Assume the center of buoyancy is the center of volume, and the sum force of gravity and buoyancy in the body-fixed frame $\mathbf{F}_b$ can be described as

$$F_{GB} = \begin{bmatrix} f_{GBx} \\ f_{GBy} \\ f_{GBz} \\ n_{GBx} \\ n_{GBy} \\ n_{GBz} \end{bmatrix} = \begin{bmatrix} (G-B) \cdot \mathbf{R}(3,1) \\ (G-B) \cdot \mathbf{R}(3,2) \\ (G-B) \cdot \mathbf{R}(3,3) \\ -G \cdot z_G \cdot \mathbf{R}(3,2) \\ G \cdot z_G \cdot \mathbf{R}(3,1) - G \cdot x_G \cdot \mathbf{R}(3,2) \\ G \cdot x_G \cdot \mathbf{R}(3,2) \end{bmatrix} \quad (10)$$

where $\mathbf{R}(i, j)$ denotes the element in the i^{th} line and the j^{th} column of the direction cosine matrix $\mathbf{R}(\boldsymbol{\eta})$.

To describe the vectored thrust, a vectored thrust frame is established as shown in Fig.3. The vectored propeller can rotate the vectored axis at a range of $[-180^\circ, 180^\circ]$, in this paper the thrust T is presented in 2-dimensional variant instead of 3-dimensional one, that is, each vector thrust is decomposed into two orthogonal force in the vectored rotation plane, thus the thrusts and their moments in the body frame can be easily described by each vector angle μ_i and their thrust T_i , so the i^{th} propeller thrust T_i can be decomposed into two orthogonal forces T_{ih} and T_{iv} , see Fig.3, with

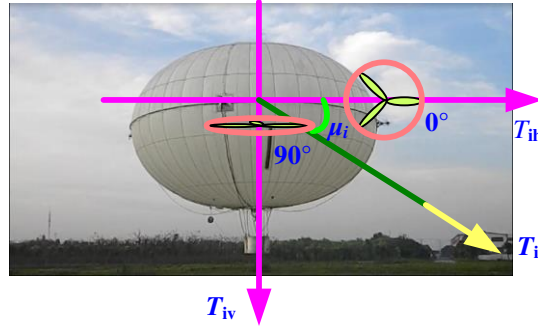


Fig. 3. Vectored thrust frame and its deflection angle

$$\begin{cases} T_{ih} = T_i \cos \mu_i \\ T_{iv} = -T_i \sin \mu_i \end{cases}, (i=1, 2, 3, 4). \quad (11)$$

Due to the symmetric configuration of the four thrusters, see Fig.4,

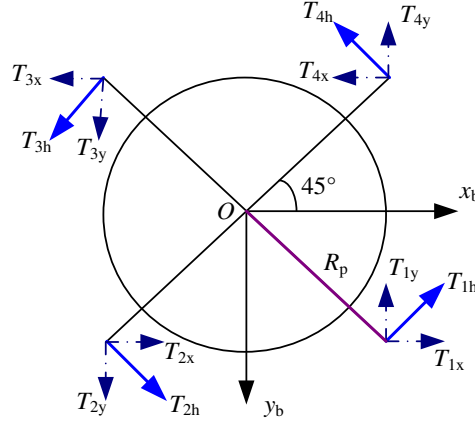


Fig. 4. Decomposition of horizontal thrust in the body-frame

the forces along the body axis are

$$\begin{cases} T_{ix} = T_{ih} \cos \frac{\pi}{4} \\ T_{iy} = T_{iv} \sin \frac{\pi}{4} \end{cases} \quad (12)$$

and the vectored thrusts and their moments are

$$U = \begin{bmatrix} \mathbf{F}_T \\ \boldsymbol{\tau}_T \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}}_{\mathbf{B}_u} \underbrace{\begin{bmatrix} \mathbf{T}_h \\ \mathbf{T}_v \end{bmatrix}}_{\mathbf{u}} \quad (13)$$

where $\mathbf{F}_T = [F_{Tx} \ F_{Ty} \ F_{Tz}]^T$, $\boldsymbol{\tau}_T = [\tau_{Tx} \ \tau_{Ty} \ \tau_{Tz}]^T$, $\mathbf{T}_h = [T_{1h} \ T_{2h} \ T_{3h} \ T_{4h}]^T$, $\mathbf{T}_v = [T_{1v} \ T_{2v} \ T_{3v} \ T_{4v}]^T$, $\mathbf{B}_u$ is the control transformation matrix,

$$\mathbf{B}_{11} = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{B}_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \mathbf{B}_{21} = R_p \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 \end{bmatrix}, \mathbf{B}_{22} = \frac{\sqrt{2}}{2} R_p \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$\mathbf{u}$ represents thrust forces, and R_p denotes the distance from CV of the airship to the propeller.

C. The trajectory tracking control problem

Consider the airship models of (1) and (6). Let $x_{1d}(t) : [0, \infty) \rightarrow R^6$ be a given attainable and sufficiently smooth time-varying reference trajectory with its time-derivatives $(\dot{x}_{1d}(t), \ddot{x}_{1d}(t))$ bounded. The control task is to design a trajectory tracking controller such that the closed-loop system meets following

requirements:

1) The trajectory tracking error dynamics are asymptotically stable under specified model uncertainty and unknown bounded disturbances;

2) The output trajectory x_1 is steered towards a given reference trajectory x_{1d} within finite time.

III. DO-IBTSMC TRAJECTORY TRACKING DESIGN

In this section, a nonlinear disturbance observer is given, and a DO-IBTSMC design is proposed for trajectory tracking control.

A. Nonlinear disturbance observer design

To the best knowledge of the authors, there is no available device so far to measure accurately the wind field information in stratosphere in real time. Hence a disturbance observer is designed as follows according to the airship motion,

$$\begin{cases} \dot{z}_d(t) = (W_d - l(x_2)V_d)z_d(t) + W_d p(x_2) - l(x_2)(V_d p(x_2) + f_2(x_1, x_2) + g_2 U(t)) \\ \hat{\varsigma}_d(t) = z_d(t) + p(x_2) \\ \hat{D}(t) = V_d \hat{\varsigma}_d(t) \end{cases} \quad (14)$$

where $\hat{D}$ is estimated disturbance, (W_d, V_d) is assumed to be observable. $l(x_2)$ is the vector of observer gains to be tuned for performance, z_d is the internal state vector, and $p(x_2)$ is nonlinear function to be designed. The nonlinear disturbance observer (NDOB) gain $l(x)$ is determined by

$$l(x_2) = \frac{\partial p(x_2)}{\partial x_2} \quad (15)$$

Integrating $l(x_2)$ with respect to the airship state x

yields

$$p(x_2) = L_d \text{Lie}_f^{r-1} h(x_2) \quad (16)$$

where L_d is a gain vector, Lie_f denotes a Lie derivative of function $f(x)$, and r is the relative degree. Select

$$h(x_2) = x_2.$$

It has been shown that the estimation $\hat{D}$ of the NDOB approaches the disturbance $D(t)$ exponentially if the observer gain $l(x)$ is chosen such that (14) is global exponentially stable for all $x \in R^n$ [24]. The estimation of harmonic disturbance, $\hat{D}$, yielded by the DO of (14) approaches disturbance, D , is global exponentially stable if the following inequalities are satisfied [25]

$$V = \frac{1}{2\gamma_d} d^T d > 0, \quad \dot{V} < 0 \quad (17)$$

where the nonlinear disturbance estimation error or unobservable disturbance is $d = D - \hat{D}$. Hence the disturbance compensator can be designed as

$$u_d(t) = -g_2^{-1} \hat{D}(t) \quad (18)$$

By using the nonlinear disturbance observer, $\hat{D}$ will approach D , so the disturbance can be attenuated through the disturbance compensator (18) and the system (6) can be transformed as follows

$$\begin{aligned} \dot{x}_2(t) &= f_2(x_1, x_2) + g_2 U(t) + D \\ &= f_2(x_1, x_2) + g_2 (U_{BTSMC}(t) + u_d(t)) + D(t) \\ &= f_2(x) + g_2(x) U_{BTSMC}(t) + d. \end{aligned} \quad (19)$$

B. BTSMC based trajectory tracking control

Lemma 1 [21]. Consider the dynamics described as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = M^{-1}(x_1)u + f(x_1, x_2) + \Delta \\ y = x_1 \end{cases} \quad (20)$$

where $f(x_1, x_2)$ denotes the lumped known component, Δ denote the lumped unknown component in the system dynamics. Select an integral nonsingular fast terminal sliding mode surface as follows

$$s = \int (e + k_1 e^\lambda + k_2 \dot{e}^{p/q}) dt \quad (21)$$

where $e = x_1 - x_{1d}$, $k_1 = \text{diag}(k_{11}, k_{12}, \dots, k_{1n}) \in R^{n \times n}$, $k_2 = \text{diag}(k_{21}, k_{22}, \dots, k_{2n}) \in R^{n \times n}$ are positive definite matrices,

p and q are positive odd numbers satisfying the relation $1 < p/q < 2$ and $\lambda > p/q$. When the sliding mode surface (21) converges to zero, i.e., $e + k_1 e^\lambda + k_2 \dot{e}^{p/q} = 0$, the defined finite time t_c that is taken to travel from $e(t_r) \neq 0$ to $e(t_r + t_c) = 0$ is given by

$$t_c = \frac{\frac{p}{q} |x_1(t_r)|^{1-\frac{q}{p}}}{k_1 (\frac{p}{q} - 1)} \cdot \Omega \left(\frac{p-1}{p}, \frac{p-1}{(\lambda-1)\frac{p}{q}}; 1 + \frac{p-1}{(\lambda-1)\frac{p}{q}}; -k_1 |x_1(t_r)|^{\lambda-1} \right) \quad (22)$$

where Ω denotes Gauss's hyper-geometric function [32].

This section is to design the backstepping sliding mode controller. The objective is to make the system output x_1 converge to the desired value vector x_{1d} . Now consider the kinematics (1), (2) and dynamics (6), the trajectory controller is derived in two steps.

Step 1(Backstepping for the variable z_1).

The tracking error vector of trajectory is defined as

$$z_1 = x_1 - x_{1d} \quad (23)$$

and its derivative is

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_{1d} = T x_2 - \dot{x}_{1d} \quad (24)$$

Since the desired behavior for $\dot{x}_1$ can still be chosen, it can be considered as the “virtual” control input. In the backstepping design this desired dynamic behavior is called the stabilizing function. Define the stabilizing function $\alpha_1(x)$, as the virtual speed vector of $\dot{x}_{1r}$,

$$\alpha_1(x) = \dot{x}_{1d} - K_1 z_1, \quad (25)$$

where $\dot{x}_{1r}$ represents the reference or virtual speed vector, $K_1 > 0$ is often chosen as a diagonal matrix to simplify the design, i.e., $K_1 = \text{diag}(k_{11}, \dots, k_{16})$, k_{1i} ($i = 1, 2, \dots, 6$) is constant value.

Now construct a candidate Lyapunov function V_1 , which is required positive definite around the desired position as follows:

$$V_1 = \frac{1}{2} \mathbf{z}_1^T \mathbf{z}_1 \quad (26)$$

and its derivative is

$$\dot{V}_1 = \mathbf{z}_1^T \dot{\mathbf{z}}_1. \quad (27)$$

Step 2 (Backstepping for the variable z_2). Because there exists a dynamic error between the speed $\dot{\mathbf{x}}_1$ and its desired behavior of $\dot{\mathbf{x}}_{1d}$, the speed tracking error vector for the position and attitude dynamics is defined as

$$\mathbf{z}_2 = \dot{\mathbf{x}}_1 - \alpha_1(x_1) = \mathbf{x}_2 - \alpha_1(x_1) \quad (28)$$

and its derivative is

$$\dot{\mathbf{z}}_2 = \ddot{\mathbf{x}}_1 - \ddot{\mathbf{x}}_{1d} + K_1 \dot{\mathbf{z}}_1. \quad (29)$$

Substituting (25) and (28) into (24) yields

$$\dot{\mathbf{z}}_1 = \mathbf{z}_2 + \alpha_1(x_1) - \dot{\mathbf{x}}_{1d} = \mathbf{z}_2 - K_1 \mathbf{z}_1 \quad (30)$$

that is,

$$\mathbf{z}_2 = \dot{\mathbf{z}}_1 + K_1 \mathbf{z}_1. \quad (31)$$

The derivative of V_1 by (26) and (30) is then

$$\dot{V}_1 = \mathbf{z}_1^T \dot{\mathbf{z}}_1 = \mathbf{z}_1^T (\mathbf{z}_2 - K_1 \mathbf{z}_1) = -\mathbf{z}_1^T K_1 \mathbf{z}_1 + \mathbf{z}_1^T \mathbf{z}_2 \quad (32)$$

Differentiating (31) and substituting (23) yields

$$\dot{\mathbf{z}}_2 = \ddot{\mathbf{x}}_1 - \ddot{\mathbf{x}}_{1d} + K_1 \dot{\mathbf{z}}_1. \quad (33)$$

From (25) and (30), the derivative of the stabilizing function $\alpha_1(x)$ is

$$\dot{\alpha}_1(x_1) = \ddot{\mathbf{x}}_{1d} - K_1 (\mathbf{z}_2 - K_1 \mathbf{z}_1) = \ddot{\mathbf{x}}_{1d} - K_1 \mathbf{z}_2 + K_1^2 \mathbf{z}_1. \quad (34)$$

To obtain a fast transient response and finite time convergence without singularity, a non-singular terminal sliding mode (NTSM) surface or manifold is designed by using the fractional order derivative as follows [21]

$$\mathbf{s} = \mathbf{z}_1 + \alpha_\gamma \mathbf{z}_1^\lambda + \frac{1}{\beta} \mathbf{z}_2^{p/q} = \mathbf{z}_1 + \alpha_\gamma |\mathbf{z}_1|^\lambda \text{sgn}(\mathbf{z}_1) + \frac{1}{\beta} |\mathbf{z}_2|^{p/q} \text{sgn}(\mathbf{z}_2) \quad (35)$$

where the SMC parameter $\lambda > p/q$, $s = [s_1 \ \cdots \ s_6]^T \in R^6$ denotes sliding mode surface, $1 < p/q < 2$, $\beta > 0$.

To improve system tracking performances, the coefficient α_γ is defined as

$$\alpha_\gamma = \alpha_{\gamma 0} + \alpha_{\gamma 1} |\dot{z}_1|, \quad (36)$$

where $\alpha_{\gamma 1}$ is learning rate, which is defined as $\alpha_{\gamma 1} = b^{|z_1|} - 1$, $\alpha_{\gamma 0}$ is a positive constant, $b > 0$ and $b \neq 1$.

Remark 1 The variable coefficient α_γ is introduced to adjust the speed that the output tracking errors converge to zero. As the output tracking error z_1 are relative large at the initial stage, the rate of change of the tracking error $\dot{z}_1$ is also large due to the effect of the sliding mode surface. The learning rate changes at an exponential rate with the rate of change of z_1 , so that the coefficient α_γ rapidly changes with the change of the tracking error z_1 , thus causing the tracking error to converge at a fast speed. However, the relative high tracking error rate will result in the problem of system chattering when the error reaches the sliding mode surface. The learning rate will follow the change with the tracking error rate, making the coefficient α_γ change into a small number, so that the tracking error approaches the sliding mode surface at a low speed, and the system chattering has been restrained.

The derivative of sliding mode surface s is then

$$\dot{s} = \dot{z}_1 + \alpha_\gamma \lambda |z_1|^{\lambda-1} \cdot \dot{z}_1 + \frac{1}{\beta} \cdot \frac{p}{q} |z_2|^{(p/q-1)} \cdot \dot{z}_2 = \dot{z}_1 + \alpha_\gamma \lambda |z_1|^{\lambda-1} \cdot \dot{z}_1 + \frac{1}{\beta} \cdot \frac{p}{q} |z_2|^{(p/q-1)} (\ddot{x}_1 - \ddot{x}_{1d} + K_1 \dot{z}_1) \quad (37)$$

The second candidate CLF is constructed as

$$V_2 = V_1 + \frac{1}{2} s^T s + \frac{1}{2\gamma_d} \tilde{d}^T \tilde{d}, \quad (38)$$

where γ_d is a positive constant that determines the convergence speed of the estimation. Differentiating (38) yields

$$\dot{V}_2 = \dot{z}_1^T \dot{z}_1 + s^T \dot{s} + \frac{1}{\gamma_d} \tilde{d}^T \dot{\tilde{d}}. \quad (39)$$

With the assumption that the unobservable disturbance d is an unknown slowly varying disturbance, the

derivative of the estimated parameter error is $\dot{\tilde{\mathbf{d}}} = \dot{\hat{\mathbf{d}}} - \dot{\mathbf{d}} \approx \dot{\hat{\mathbf{d}}}$, where $\hat{\mathbf{d}}$ and $\tilde{\mathbf{d}}$ are the estimated parameter vector of $\mathbf{d}$ and the associated estimated parameter error vector respectively. Substituting (29), (30), (32), (34) and (35) into (39) yields

$$\begin{aligned}
\dot{V}_2 &= \mathbf{z}_1^T \dot{\mathbf{z}}_1 + \mathbf{s}^T \dot{\mathbf{s}} + \frac{1}{\gamma_d} \tilde{\mathbf{d}}^T \dot{\tilde{\mathbf{d}}} \\
&= -\mathbf{z}_1^T K_1 \mathbf{z}_1 + \mathbf{z}_1^T \mathbf{z}_2 + \mathbf{s}^T \left(\dot{\mathbf{z}}_1 + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1}) \cdot \dot{\mathbf{z}}_1 + \frac{1}{\beta} \cdot \frac{p}{q} |\mathbf{z}_2|^{(p/q-1)} (\ddot{\mathbf{x}}_1 - \ddot{\mathbf{x}}_{1d} + K_1 \dot{\mathbf{z}}_1) \right) + \frac{1}{\gamma_d} \tilde{\mathbf{d}}^T \dot{\tilde{\mathbf{d}}} \\
&= -\mathbf{z}_1^T K_1 \mathbf{z}_1 + \mathbf{z}_1^T \mathbf{z}_2 + \mathbf{s}^T \left((\mathbf{z}_2 - K_1 \mathbf{z}_1) (1 + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) + \frac{1}{\beta} \cdot \frac{p}{q} |\mathbf{z}_2|^{(p/q-1)} (\ddot{\mathbf{x}}_1 - \ddot{\mathbf{x}}_{1d} + K_1 \dot{\mathbf{z}}_1) \right) + \frac{1}{\gamma_d} \tilde{\mathbf{d}}^T \dot{\tilde{\mathbf{d}}} \\
&= -\mathbf{z}_1^T K_1 \mathbf{z}_1 + \mathbf{z}_1^T \mathbf{z}_2 + \mathbf{s}^T \left((\mathbf{z}_2 - K_1 \mathbf{z}_1) (1 + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) + \frac{1}{\beta} \cdot \frac{p}{q} |\mathbf{z}_2|^{(p/q-1)} (Tf_2 + Tg_2 \mathbf{U}_{BSMC} + T\mathbf{d} + \dot{\mathbf{T}}\mathbf{x}_2 + K_1 \dot{\mathbf{z}}_1 - \ddot{\mathbf{x}}_{1d}) \right) + \frac{1}{\gamma_d} \tilde{\mathbf{d}}^T \dot{\tilde{\mathbf{d}}} \\
&= \mathbf{z}_1^T (-K_1 \mathbf{z}_1 + \mathbf{z}_2) + (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) (\mathbf{s}^T \mathbf{z}_2 - K_1 \mathbf{z}_1^T \mathbf{s}) + \mathbf{s}^T \frac{1}{\beta} \cdot \frac{p}{q} |\mathbf{z}_2|^{(p/q-1)} (Tf_2 + Tg_2 \mathbf{U}_{BSMC} + \dot{\mathbf{T}}\mathbf{x}_2 + K_1 \dot{\mathbf{z}}_1 - \ddot{\mathbf{x}}_{1d} + T\mathbf{d}) + \frac{1}{\gamma_d} \tilde{\mathbf{d}}^T \dot{\tilde{\mathbf{d}}}
\end{aligned} \tag{40}$$

A desired control input is selected as follows to make (40) semi-negative definite,

$$\mathbf{U}_{BSMC} = (Tg_2)^{-1} \left(-\beta \cdot \frac{q}{p} (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) |\mathbf{z}_2|^{(2-p/q)} - K_1 \dot{\mathbf{z}}_1 - Tf_2 - \dot{\mathbf{T}}\mathbf{x}_2 + \ddot{\mathbf{x}}_{1d} - T\hat{\mathbf{d}} - (hs + \varsigma \text{sgn}(\mathbf{s})) \right) \tag{41}$$

Substituting (41) in (40) gives

$$\dot{V}_2 = -\mathbf{z}_1^T K_1 \mathbf{z}_1 + \mathbf{z}_1^T \mathbf{z}_2 + (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) (-K_1 \mathbf{z}_1^T \mathbf{s}) + \mathbf{s}^T \frac{1}{\beta} \cdot \frac{p}{q} |\mathbf{z}_2|^{(p/q-1)} (T(\mathbf{d} - \hat{\mathbf{d}}) - (hs + \varsigma \text{sgn}(\mathbf{s}))) + \frac{1}{\gamma_d} \tilde{\mathbf{d}}^T \dot{\tilde{\mathbf{d}}} \tag{42}$$

Because

$$\begin{aligned}
&\mathbf{z}_1^T (-K_1 \mathbf{z}_1 + \mathbf{z}_2) - (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) K_1 \mathbf{z}_1^T \mathbf{s} = -K_1 \mathbf{z}_1^T \mathbf{z}_1 + \mathbf{z}_1^T \mathbf{z}_2 - (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) K_1 \mathbf{z}_1^T \left(\mathbf{z}_1 + \alpha_\gamma \mathbf{z}_1^\lambda + \frac{1}{\beta} \mathbf{z}_2^{p/q} \right) \\
&= -K_1 \mathbf{z}_1^T \mathbf{z}_1 + \mathbf{z}_1^T \mathbf{z}_2 - (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) K_1 \mathbf{z}_1^T \mathbf{z}_1 - (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) \alpha_\gamma K_1 \mathbf{z}_1^T \mathbf{z}_1^\lambda - \frac{1}{\beta} (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) K_1 \mathbf{z}_1^T \mathbf{z}_2^{p/q} \\
&= -\left(2\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1}) \right) K_1 \mathbf{z}_1^T \mathbf{z}_1 - (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) \alpha_\gamma K_1 |\mathbf{z}_1|^{\lambda-1} \mathbf{z}_1^T \mathbf{z}_1 + \mathbf{z}_1^T \mathbf{z}_2 - \frac{1}{\beta} (\mathbf{I}_n + \alpha_\gamma \lambda \text{diag}(|\mathbf{z}_{1,i}|^{\lambda-1})) K_1 |\mathbf{z}_2|^{p/q-1} \mathbf{z}_1^T \mathbf{z}_2
\end{aligned}$$

$$= -\left(2K_1 + \alpha_\gamma(\lambda + 1)K_1 \text{diag}(|z_{1,i}|^{\lambda-1}) + \alpha_\gamma^2 \lambda K_1 \text{diag}(|z_{1,i}|^{2\lambda-2})\right) z_1^T z_1 + z_1^T z_2 - \frac{1}{\beta} \left(I_n + \alpha_\gamma \lambda \text{diag}(|z_{1,i}|^{\lambda-1})\right) K_1 \cdot \text{diag}(|z_{2,i}|^{p/q-1}) z_1^T z_2 \quad (43)$$

and define positive definite matrix

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ -I_n & O_n \end{bmatrix} \quad (44)$$

where $Q_{11} = 2K_1 + \alpha_\gamma(\lambda + 1)K_1 \text{diag}(|z_{1,i}|^{\lambda-1}) + \alpha_\gamma^2 \lambda K_1 \text{diag}(|z_{1,i}|^{2\lambda-2})$, $Q_{12} = \frac{1}{\beta} (I_n + \alpha_\gamma \lambda \text{diag}(|z_{1,i}|^{\lambda-1})) K_1 \cdot \text{diag}(|z_2|^{p/q-1})$. Then it yields

$$z_1^T (-K_1 z_1 + z_2) - (I_n + \alpha_\gamma \lambda |z_1|^{\lambda-1}) K_1 z_1^T s = - \begin{bmatrix} z_1^T & z_2^T \end{bmatrix} Q \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad (45)$$

Substituting (45) and $s^T \tilde{d} = \tilde{d}^T s$ into (42) yields

$$\begin{aligned} \dot{V}_2 &= - \begin{bmatrix} z_1^T & z_2^T \end{bmatrix} Q \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + s^T \frac{1}{\beta} \cdot \frac{p}{q} |z_2|^{(p/q-1)} \left(T(\mathbf{d} - \hat{\mathbf{d}}) - (hs + \varsigma \text{sgn}(s)) \right) + \frac{1}{\gamma_d} \tilde{\mathbf{d}}^T \dot{\hat{\mathbf{d}}} \\ &= - \begin{bmatrix} z_1^T & z_2^T \end{bmatrix} Q \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - s^T \frac{1}{\beta} \cdot \frac{p}{q} |z_2|^{(p/q-1)} (hs + \varsigma \text{sgn}(s)) - s^T \frac{1}{\beta} \cdot \frac{p}{q} |z_2|^{(p/q-1)} T \tilde{\mathbf{d}} + \frac{1}{\gamma_d} \tilde{\mathbf{d}}^T \dot{\hat{\mathbf{d}}} \\ &= - \begin{bmatrix} z_1^T & z_2^T \end{bmatrix} Q \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - s^T \frac{1}{\beta} \cdot \frac{p}{q} |z_2|^{(p/q-1)} (hs + \varsigma \text{sgn}(s)) + \tilde{\mathbf{d}}^T \left(- \left(\frac{1}{\beta} \cdot \frac{p}{q} |z_2|^{(p/q-1)} T \right)^T s + \frac{1}{\gamma_d} \dot{\hat{\mathbf{d}}} \right) \end{aligned} \quad (46)$$

Choose the update law as:

$$\dot{\hat{\mathbf{d}}} = \frac{1}{\beta} \cdot \frac{p}{q} \gamma_d \left(|z_2|^{(p/q-1)} T \right)^T s \quad (47)$$

If $h > 0, \varsigma > 0$, then

$$\begin{aligned} \dot{V}_2 &= - \begin{bmatrix} z_1^T & z_2^T \end{bmatrix} Q \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - s^T \frac{1}{\beta} \cdot \frac{p}{q} |z_2|^{(p/q-1)} (hs + \varsigma \text{sgn}(s)) \\ &= - \begin{bmatrix} z_1^T & z_2^T \end{bmatrix} Q \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \frac{1}{\beta} \cdot \frac{p}{q} |z_2|^{(p/q-1)} (hs^2 + \varsigma |s|) \\ &\leq \sum_{i=1}^6 -\varsigma \rho(z_2) |s_i| \end{aligned} \quad (48)$$

where $\rho(z_2) = \inf_{1 \leq i \leq 6} \left\{ \frac{1}{\beta} \cdot \frac{p}{q} |z_{2,i}|^{(p/q-1)} \right\}$. Since p and q are odd numbers and $1 < p/q < 2$, there exist two possible cases: 1) $|z_{2,i}|^{(p/q-1)} > 0$ when $z_{2,i} \neq 0$; 2) $|z_{2,i}|^{(p/q-1)} = 0$ when $z_{2,i} = 0$. Consider sliding mode surface (35), when $s_i \neq 0$, two different cases may occur: 1) $z_{2,i} \neq 0$ and 2) $z_{2,i} = 0$. When $s_i \neq 0$ and $z_{2,i} \neq 0$, we get $\rho(z_2) > 0$ and $\dot{V}_2 < 0$. Therefore, the closed-loop system is stable According to the Lyapunov stability theory when $z_{2,i} \neq 0$; the system will move fast to the sliding mode $s_i = 0$ within finite time. When $s_i \neq 0$ and $z_{2,i} = 0$, but $z_{1,i} \neq 0$, by substituting (41) into Eq.(33), we get

$$\begin{aligned} \dot{z}_2 &= \ddot{x}_1 - \ddot{x}_{1d} + K_1 \dot{z}_1 \\ &= T f_2 + T g_2 U_{BSMC} + T d + \dot{T} x_2 - \ddot{x}_{1d} + K_1 \dot{z}_1 \\ &= T(d - \hat{d}) - (hs + \varsigma \operatorname{sgn}(s)) \end{aligned} \quad (49)$$

Since d can be approximated by $\hat{d}$, the effect of the term $T(d - \hat{d})$ can be neglected. Now considering (49) when $s_i > 0$, we have $\dot{z}_{2,i} < -\varsigma$ and when $s_i < 0$, we have $\dot{z}_{2,i} > \varsigma$. It means that $z_{2,i}$ is not an attractor and the system will not always keep staying on the points ($z_{1,i} \neq 0$ and $z_{2,i} = 0$). It also means that there exists a vicinity of $z_{2,i} = 0$ such that for a small $\delta > 0$ and $|z_{2,i}| < \delta$, there are $|z_{2,i}| \leq -\varsigma$ for $s_i > 0$ and $\dot{z}_{2,i} \geq \varsigma$ for $s_i < 0$, respectively. Therefore, the crossing of the trajectory from the boundary of the vicinity $z_{2,i} = \delta$ to $z_{2,i} = -\delta$ for $s_i > 0$, and from $z_{2,i} = -\delta$ to $z_{2,i} = \delta$ for $s_i < 0$ occurs in finite time. For other regions where $|z_{2,i}| > \delta$ the switching line $s_i = 0$ can be reached in the finite time because we have $|z_{2,i}| \leq -\varsigma$ for $s_i > 0$ and $\dot{z}_{2,i} \geq \varsigma$ for $s_i < 0$. Therefore, it is concluded that the system will converge to $s_i = 0$ from everywhere in the phase plane. When the sliding surface (35) converges to zero, we have

$$z_1 + \alpha_\gamma z_1^\lambda + \frac{1}{\beta} z_2^{p/q} = z_1 + \alpha_\gamma z_1^\lambda + \frac{1}{\beta} (\dot{z}_1 + K_1 z_1)^{p/q} = z_1 + \left(\alpha_\gamma z_1^\lambda + \frac{1}{\beta} K_1 z_1^{p/q} \right) + \frac{1}{\beta} \dot{z}_1^{p/q} = 0, \quad (50)$$

where z_1 is the terminal attractor of the system. By using Lemma 1, the defined finite time t_c that is taken to

travel from $z_1(t_r) \neq 0$ to $z_1(t_r + t_c) = 0$ as [21].

The robustness of such a backstepping design can be improved by introducing its incremental counterpart, using the implicit approach with the recursive control law,

$$U_{BTSMC} = U_{BTSMC} + U_0 \quad (51)$$

where U_0 denotes the control input at the last time step according to incremental nonlinear design [29], which can be calculated using (13) via the last step thrust T_i and vector angle μ_i .

According to the backstepping control law (41) and considering the approximate dynamics around the current reference state for the dynamics $\dot{x}_2 \approx \dot{x}_{20} + G_2 U_{BTSMC}$, the backstepping control (41) can be rewritten in incremental form as,

$$U_{BTSMC} = (Tg_2)^{-1} \left(-\beta \cdot \frac{q}{p} \left(I_n + \alpha_\gamma \lambda \text{diag}(|z_{1,i}|^{\lambda-1}) \right) |z_2|^{(2-p/q)} - K_1 \dot{z}_1 - T \begin{bmatrix} \dot{v}_0^T & \dot{\omega}_0^T \end{bmatrix}^T - \dot{T}x_2 + \ddot{x}_{1d} - T\hat{d} - (hs + \varsigma \text{sgn}(s)) \right). \quad (52)$$

The output $\dot{x}_{20}$ represents the current estimated angular acceleration, which is measured by differentiating the angular rates. The approximation function from x_{20} to derivative $\dot{x}_{20}$ is

$$H_\omega(s) = \frac{s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \quad (53)$$

where ζ denotes the filter damping ratio, ω_n denotes the filter natural frequency.

Remark 2 In order to effectively eliminate the chattering phenomenon of reachability condition $\dot{s} = -hs - \varsigma \text{sgn}(s)$ in (41) in the sliding mode control, a continuous function is used to define practical sliding surface dynamics using the hyperbolic tangent function[33],

$$\dot{s} = -hs - \varsigma \tanh(s / \phi_s), \quad (54)$$

where ϕ_s is the sliding surface boundary layer parameter that is used to retain continuity of control as the motion trajectories cross the sliding surface and so prevent chattering. Thus high frequency chattering of the control is prevented by using the boundary layer.

Therefore the control law (41) with the hyperbolic tangent function (54) will eliminate the chattering phenomenon and ensure the finite-time convergence of the system state to the equilibrium. From the above

analysis, the structure of the DO-IBTSMC controller can be designed as Fig.5. The structure of the DO-IBTSMC design includes an incremental BTSMC, a nonlinear disturbance observer, a multi-vectorored propeller airship dynamics and a control allocation module.

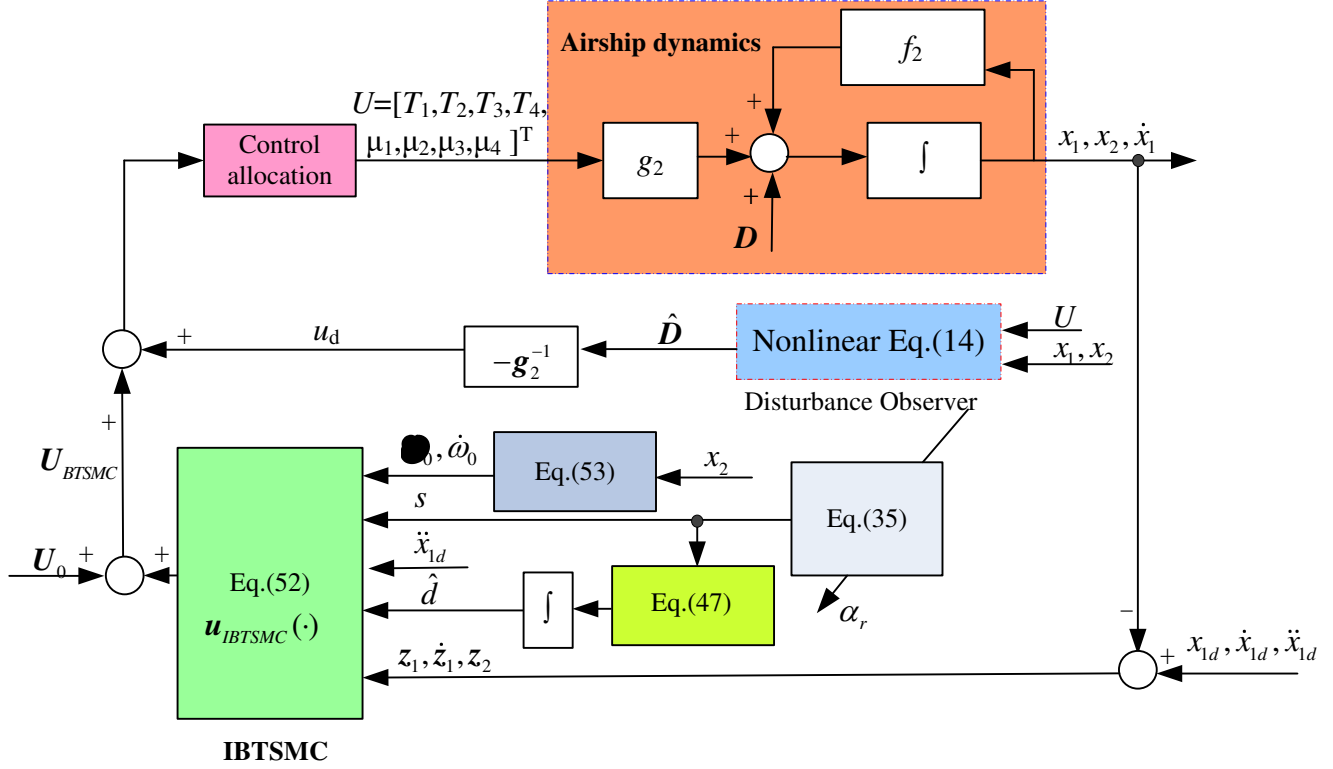


Fig.5 block diagram of the DO-IBTSMC controller

So the desired control (52) can be implemented as following information flowchart:

$$\begin{aligned}
 & \left. \begin{array}{l} x_1 \\ x_{1d} \end{array} \right\} \xrightarrow{\text{Eq.(23)}} z_1 \\
 & \left. \begin{array}{l} \dot{x}_1 \\ \dot{x}_{1d} \end{array} \right\} \xrightarrow{\text{Eq.(24)}} \dot{z}_1 \\
 & \left. \begin{array}{l} z_1 \\ \dot{z}_1 \end{array} \right\} \xrightarrow{\text{Eq.(31)}} z_2; \quad \left. \begin{array}{l} z_1 \\ z_2 \end{array} \right\} \xrightarrow{\text{Eq.(35)}} s; \\
 & s \xrightarrow{\text{Eq.(47)}} \hat{d} \rightarrow \int \hat{d} dt \rightarrow \hat{d}; s \rightarrow u_{smc} = -(hs + \varsigma \text{sgn}(s)); \quad x_1, x_2, U \xrightarrow{\text{Eq.(14)}} z_d, \hat{s}_d, \hat{D}; \\
 & \left. \begin{array}{l} x_2 \\ \dot{x}_2 \end{array} \right\} \xrightarrow{\text{Eq.(53)}} [\dot{v}_0 \quad \dot{\omega}_0]; \quad \left. \begin{array}{l} z_1 \quad z_2 \quad \dot{z}_1 \quad \ddot{x}_{1d} \quad \dot{v}_0 \quad \dot{\omega}_0 \quad \hat{d} \quad u_{smc} \\ \eta \quad \text{Eq.(2)} \quad T, \dot{T}; \end{array} \right\} \xrightarrow{\text{Eq.(52)}} U_{IBTSMC}. \quad (55)
 \end{aligned}$$

To obtain the practical control surfaces and thruster inputs, a control allocation problem is presented to

solve (13) with actuator saturation constraints, i.e.,

$$U = \begin{bmatrix} \mathbf{f}_{CP} \\ \boldsymbol{\tau}_{CP} \end{bmatrix} = B\mathbf{u} \quad , \quad (56)$$

s.t.

$$\max(u(t-T_s) + \dot{u}_{\min} T_s, u_{\min}) = \underline{u} \leq u \leq \bar{u} = \min(u(t-T_s) + \dot{u}_{\max} T_s, u_{\max}) \quad , \quad (57)$$

where $u_{\min}$, $u_{\max}$, $\dot{u}_{\min}$ and $\dot{u}_{\max}$ denote lower and upper bounds of the actuator position and rotation rate, respectively. $u(t-T_s)$ denotes control input at previous T_s , $\dot{u}_{\min} T_s$ denotes incremental variable. $\underline{u}$ and $\bar{u}$ are lower and upper bounds of the actuator inputs. T_s denotes sample period or motion period. The objective of the control allocation is

$$\min_{\mathbf{u}} J = \|\mathbf{W}_u(\mathbf{u}_a - \mathbf{u}_d)\| \quad (58)$$

s.t.

$$\mathbf{U} = \mathbf{v}_d = \mathbf{B}_u \mathbf{u}_a \quad (59)$$

where $\mathbf{W}_u \in R^{m \times m}$ is symmetric positive definite matrix. The control allocation problem of (58) and (59) can be transformed into a constrained optimization problem by using a weighted pseudo inverse (WPI) method [4]. Through the WPI control allocation algorithm, the control allocation component can be implemented and incorporated into the DO-IBTSMC scheme. When the optimal solution $\mathbf{u}^*$ is found, the practical control input signals, including thrusts and tilt angles, are obtained.

IV. SIMULATION RESULTS AND ANALYSIS

In this section three scenarios are presented that demonstrate the proposed DO-IBTSMC performance. The physical parameters of the unmanned airship are listed as follows [3, 4],

$m=72$ kg, $m_{11}=10.8147$ kg, $m_{22}=m_{11}$, $m_{33}=38.9521$ kg, $m_{44}=19.3024$ kg, $m_{55}=m_{44}$, $m_{66}=0$, $I_x=409.4482$ kg·m², $I_y=I_x$, $I_z=34.591$ kg·m², $I_{xz}=0$, $S_{\text{ref}}=10.7\text{m}^2$, $\bar{V}=35\text{m}^3$, $l_{\text{ref}}=3.27$ m, $R_p=2.81$ m, $[x_G, y_G, z_G]=[0, 0, 2]$ (m), total length $L_0=8.37\text{m}$, and diameter $D_0=5.11\text{m}$.

The aerodynamic parameters of C_h , C_z and C_{my} are shown in Table 1, which are transformed from the wind frame to the body frame.

Table 1 Aerodynamic coefficients of the multi-vectored propeller airship

Parameter $\alpha(^{\circ})$	C_h	C_z	C_{my}
0	0.1388	0	0
9	0.1051	-0.1615	0.1846
18	0.1062	-0.4492	0.3391
27	0.1644	-0.6873	0.4435
36	0.0081	-0.8402	0.5045
45	-0.1286	-0.8694	0.5640
54	-0.2663	-1.1755	0.4605
63	-0.4804	-1.4046	0.3521
72	-0.2020	-0.8802	0.2336
81	-0.0681	-0.8351	0.1086
90	0.0045	-0.8306	-0.0008

The constant aerodynamic control coefficients of the unmanned airship are as follows

$$X_{\dot{u}}=-10.8147, \quad Y_{\dot{v}}=X_{\dot{u}}, \quad Z_{\dot{w}}=-38.9521, \quad L_{\dot{p}}=-19.302, \quad M_{\dot{q}}=-19.3024, \quad N_{\dot{r}}=0.$$

In the simulation example, the initial position of the airship is $\xi_0=[0, 0, 0]^T$ (m), the initial velocity in $\mathbf{F}_b$ is $v_0=[0,0,0]^T$ (m/s), initial attitude is $\eta_0=[0, 0, 0]^T$, the initial angular velocity is $\omega_0=[0, 0, 0]^T$.

To validate the trajectory tracking control performance of the DO-IBTSMC design, the three studied scenarios are as follows.

Scenario I: Trajectory tracking control under unknown bounded disturbances.

In this case the external unknown bounded disturbance is

$$D(t)=2.5\sin(0.05t)\times[1 \quad 1 \quad 1 \quad 0.02 \quad 0.02 \quad 0.01]^T.$$

$$(\text{unit: m/s}^2 \text{ for acceleration, rad/s}^2 \text{ for angular acceleration}) \quad (60)$$

The trim inputs of forces and moments for the airship are $U_{\text{trim}}=1.0\text{e}2*[0.24431, \quad -0.00085, \quad -3.7237, \quad 0.00687, \quad 1.5222, \quad 0.00032]^T$, (unit: N for forces, Nm for moments) and the parameters of the DO-

IBTSMC and IBTSMC controllers for scenario I are designed as follows,

$$\begin{aligned}
 K_1 &= \text{diag}(0.025, 0.025, 0.025, 0.0005, 0.006, 0.03), h = \text{diag}(3, 3, 3, 0.1, 2.5, 6), \\
 \varsigma &= \text{diag}(15, 15, 15, 50, 35, 10), \phi_s = 0.4, p=5, q=3, \alpha_r = 10 + (3^{|z_1|} - 1)|\dot{z}_1|, \\
 \beta &= \text{diag}(1, 1, 1, 0.5, 0.5, 0.3), \quad \lambda = 1.8 * \text{diag}(1, 1, 1, 1, 1, 1), \gamma_d = 1.
 \end{aligned} \tag{61}$$

The disturbance parameters of (7) are

$$W_d = \begin{bmatrix} 0 & 0.05 \\ -0.05 & 0 \end{bmatrix}, V_d = \begin{bmatrix} 2 & 0 \end{bmatrix}, \tag{62}$$

The associated DO gains of (15) are designed as follows

$$l(u, \dot{u}) = \begin{bmatrix} 2.5 & 0.17 \end{bmatrix}^T, \quad l(v, \dot{v}) = l(w, \dot{w}) = l(u, \dot{u}), \quad l(p, \dot{p}) = \begin{bmatrix} 0.4 & 0.03 \end{bmatrix}^T, \quad l(q, \dot{q}) = l(p, \dot{p}), \quad l(r, \dot{r}) = \begin{bmatrix} 2.5 & 0.17 \end{bmatrix}^T, \tag{63}$$

and because maximum thrust is 110N and the maximum vector deflection angle is 180deg, the weighted matrix for control allocation is selected as $W_u = \text{diag}(1/110, 1/110, 1/110, 1/110, 1/\pi, 1/\pi, 1/\pi, 1/\pi)$. The reference trajectories and the desired attitude vector are a series of square wave signals along the x , y and z -axis. For an easier comparison, the averaged tracking error is defined as

$$E = \sqrt{\frac{1}{N} \sum_{k=1}^N (\|e(k)\|^2)} \tag{64}$$

where N is the number of simulation steps, and the elapsed time (ET) is the time that MATLAB has used to complete the 10-s simulation time. This may not reflect the true computational burden of the controllers, but it can be used to provide a general idea about the comparison in computational time among the controllers.

The simulation results of trajectory and attitude tracking control are shown in Figs 6 and 7 and Table 2.

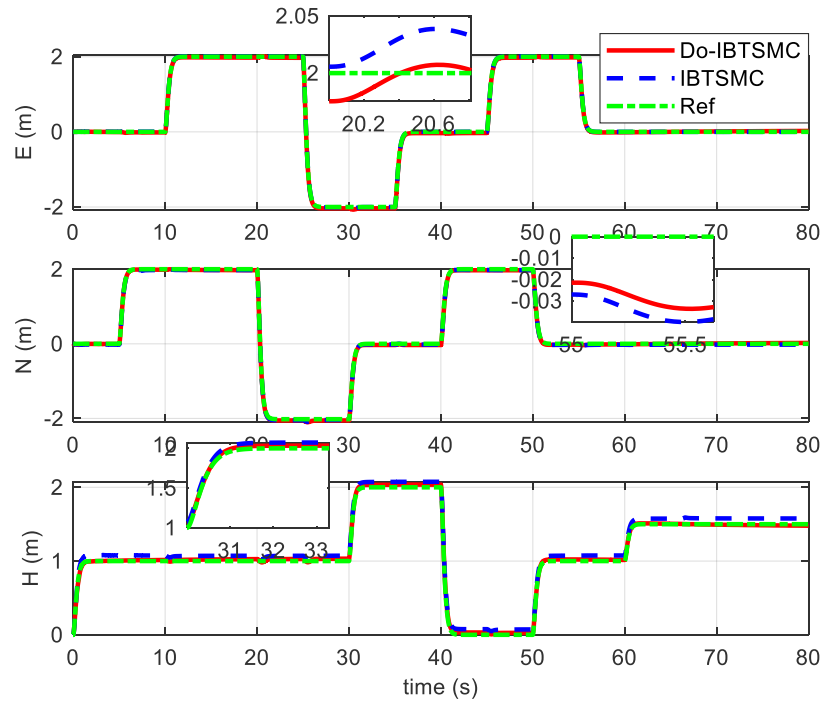


Fig. 6. Position tracking responses under unknown disturbances

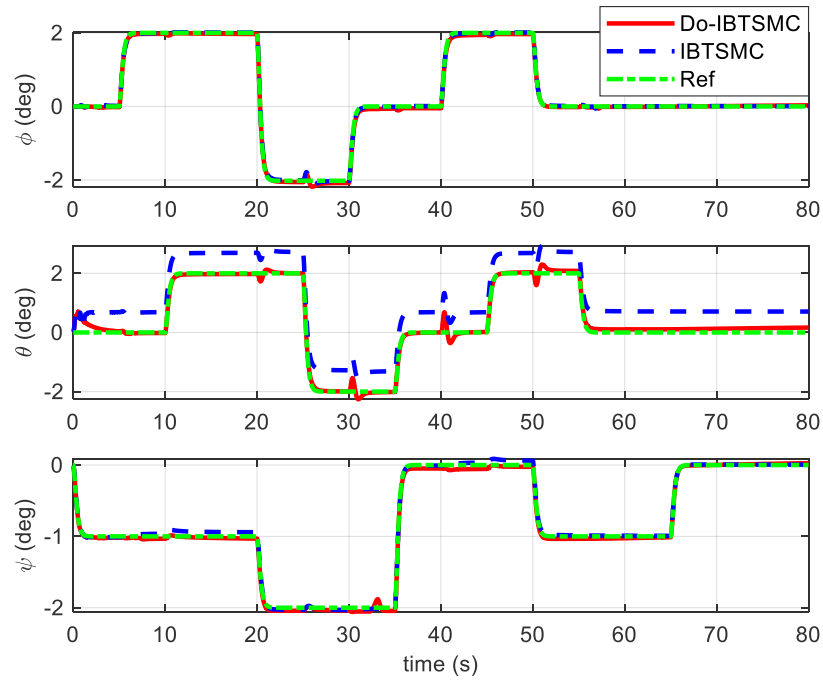


Fig. 7. Attitude angles tracking responses under unknown disturbances

Table 2 Tracking errors and ET of the airship under the input of the controller

Controller Error	$E_E(m)$	$E_N(m)$	$E_H(m)$	$E_\phi(rad)$	$E_\theta(rad)$	$E_\psi(rad)$	$ET(s)$
DO- IBTSMC	0.6402	1.9636	1.6545	0.0501	0.2571	0.0437	354.1460
IBTSMC	1.8003	2.0729	6.2095	0.0637	0.9952	0.0463	204.8385

From Figs 6 and 7 and Table 2, it is clear that the reference trajectory and attitudes have been accurately tracked by using DO-IBTSMC. The DO-IBTSMC position responses have smaller overshoots and shorter transient times. Compared with the responses of IBTSMC without the nonlinear disturbance observer, there are smaller steady tracking errors of positions and attitudes for DO-IBTSMC. In Fig 7, the difference for roll angle performance is small, this reason is the trim input for roll moment is small (0.687Nm), while there are great differences in pitch motion, because of its large trim input (152.22Nm), which will affect the performances of the IBTSMC, whose averaged tracking error E_θ is more about four times than that of the DO-IBTSMC. Hence the DO-IBTSMC can reduce the unknown disturbances affected by disturbance-observer compensation, but the elapsed time (ET) of the DO-IBTSMC is more than that of IBTSMC due to the DO, see Table 2.

To further illustrate our method in trajectory tracking control, a DO-BTSMC and a backstepping sliding mode control (BSMC) design from [12] is used for comparison. The sliding mode surface of BSMC is designed as

$$s = \lambda_1 z_1 + z_2 \quad (65)$$

and the BSMC controller is designed as follows

$$U_{BSMC} = (Tg_2)^{-1} \left(-(K_1 + \lambda_1) \dot{z}_1 - Tf_2 - \dot{T}x_2 + \ddot{x}_{1d} - T\hat{d} - (hs + \varsigma \operatorname{sgn}(s)) \right) \quad (66)$$

The parameters of the DO-BSMC controller is designed as

$$\begin{aligned} K_1 &= \operatorname{diag}(7.5, 7.5, 7.5, 1.5, 1.5, 1.5), h = \operatorname{diag}(4, 4, 4, 5, 5, 5), \\ \varsigma &= \operatorname{diag}(15, 15, 15, 50, 40, 10), \lambda_1 = 0.5 \times \operatorname{diag}(1, 1, 1, 1, 1, 1). \end{aligned} \quad (67)$$

and the parameters of the DO-BTSMC controller is designed as

$$\begin{aligned} K_1 &= \operatorname{diag}(5, 5, 5, 0.05, 0.8, 2.5), h = \operatorname{diag}(3, 3, 3, 0.1, 2.5, 6), \\ \varsigma &= \operatorname{diag}(15, 15, 15, 50, 20, 8), \end{aligned} \quad (68)$$

The other parameters are the same as the DO-IBTSMC. The reference trajectories and the desired attitude vector are the same as before. The simulation results of trajectory and attitude tracking control are shown in

Figs 8 and 9.

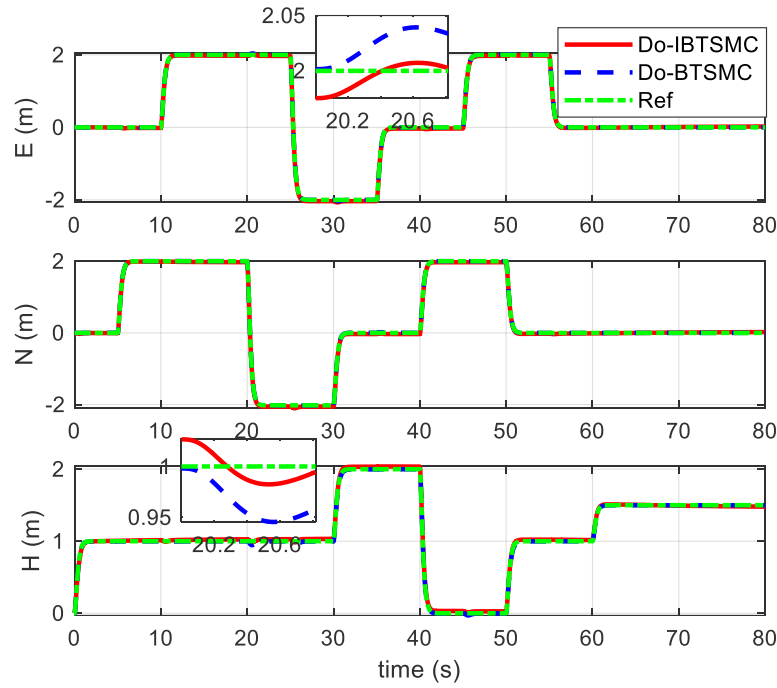


Fig. 8. Position tracking responses under unknown disturbances

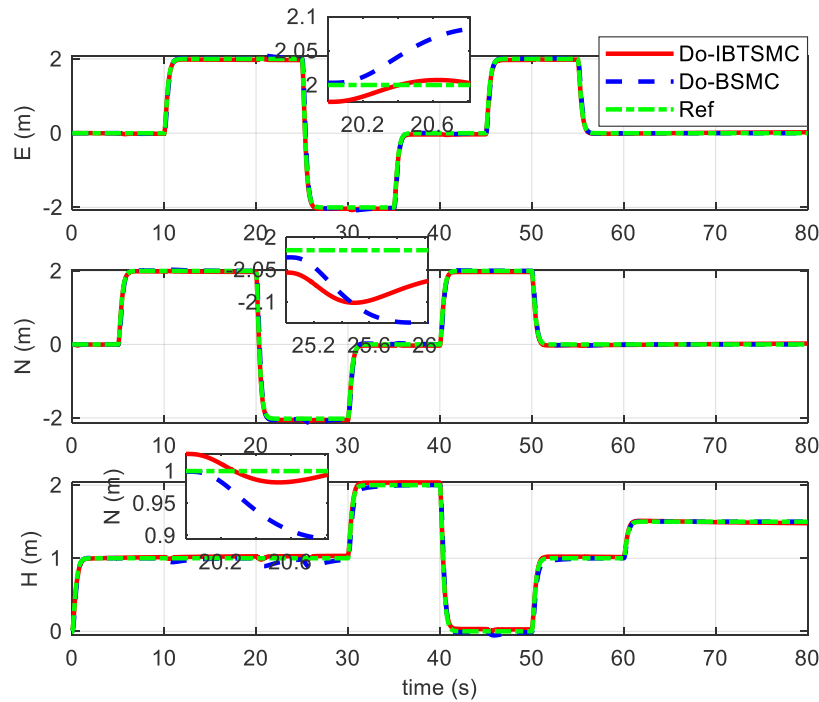


Fig.9. Attitude angles tracking responses under unknown disturbances

From Figs 8 and 9, it is clear that the reference trajectory and attitudes have been accurately tracked by using DO-IBTSMC and DO-BSMC. The DO-IBTSMC position and attitude responses have smaller overshoots and shorter transient times, the tracking errors converge rapidly and the steady state errors are

smaller than DO-BSMC. This demonstrates is better tracking performances for the proposed DO-IBTSMC.

The unknown observable disturbances are estimated via the DO as shown in Fig.10. The results show that the time-varying disturbances of unknown forces and moments have been precisely estimated.

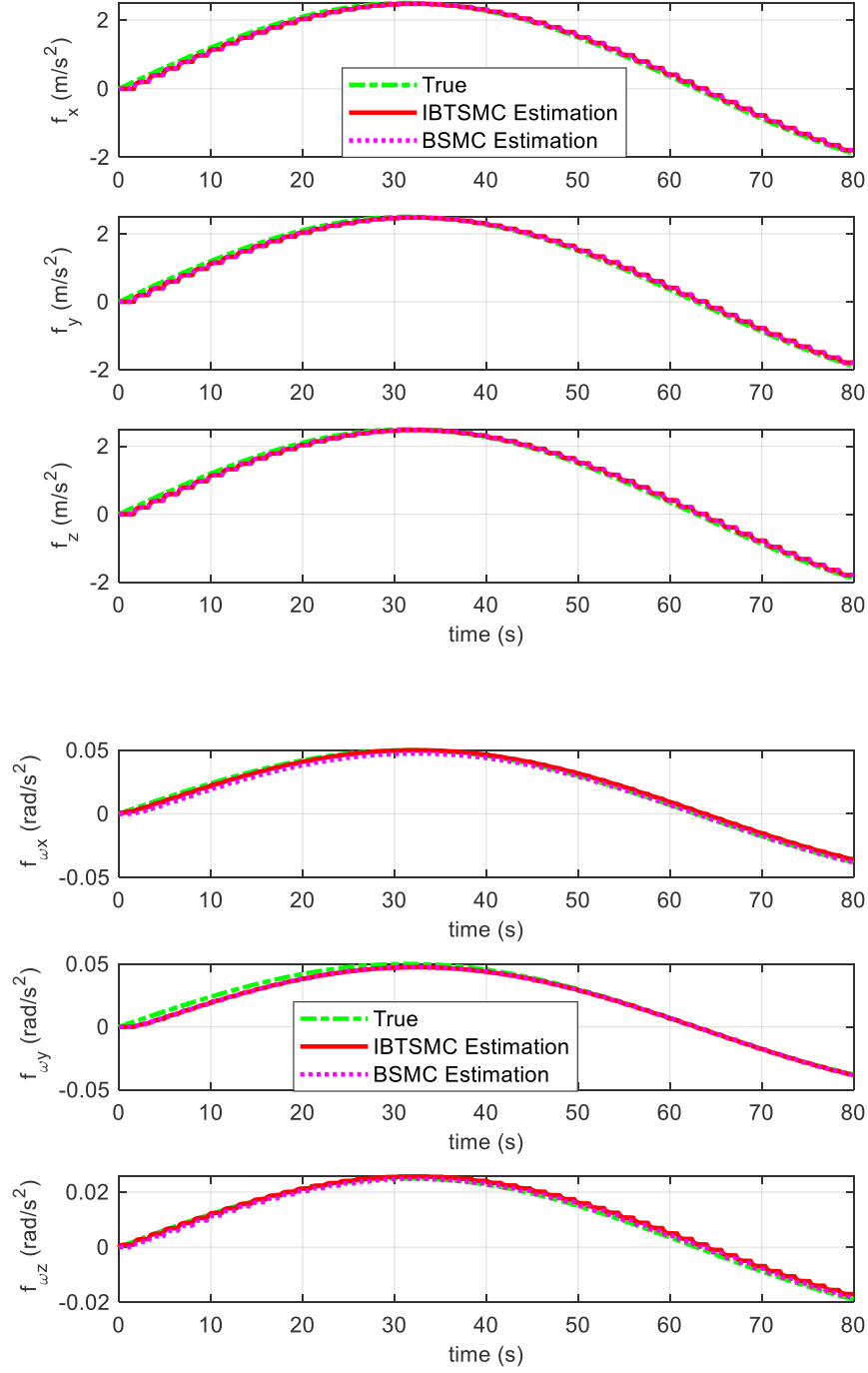


Fig. 10 Observable disturbances estimation

The unobservable disturbances have been estimated by the adaptive disturbance observer of (47). The

results are shown in Fig.11, from which it can be seen that the DO-BSMC disturbance estimation responses are larger than DO-IBTSMC; this shows that IBTSMC has more capability in rejecting unobservable disturbances. Furthermore the DO-IBTSMC steady state values are closer to their desired values of zero.

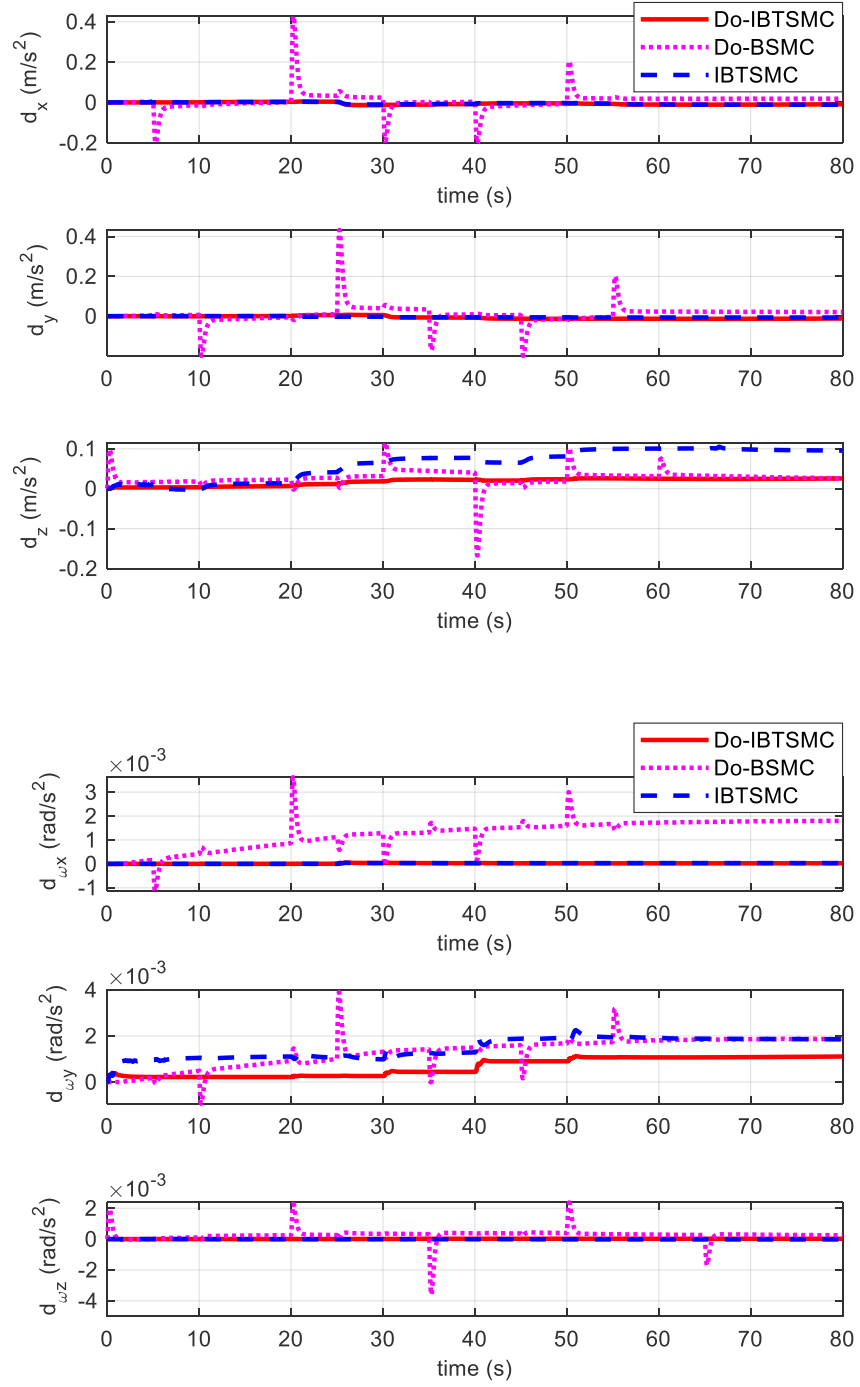


Fig. 11 Unobservable disturbances estimation

Scenario II: Trajectory tracking control under unknown disturbances with different learning rate.

To improve the tracking performances and restrain chattering, the variable coefficient α_γ is designed as (36), and different learning rates are designed to compare the tracking performances. Case1. Select $\alpha_{\gamma_0} = \{8, 10, 100\}$, $b = 3$, i.e., $\alpha_r = \alpha_{\gamma_0} + (3^{|z_1|} - 1)|\dot{z}_1|$. By using the proposed DO-IBTSMC design and the same design parameters as *Scenario I*, the simulation results of trajectory and attitude tracking control are shown in Figs 12 and 13.

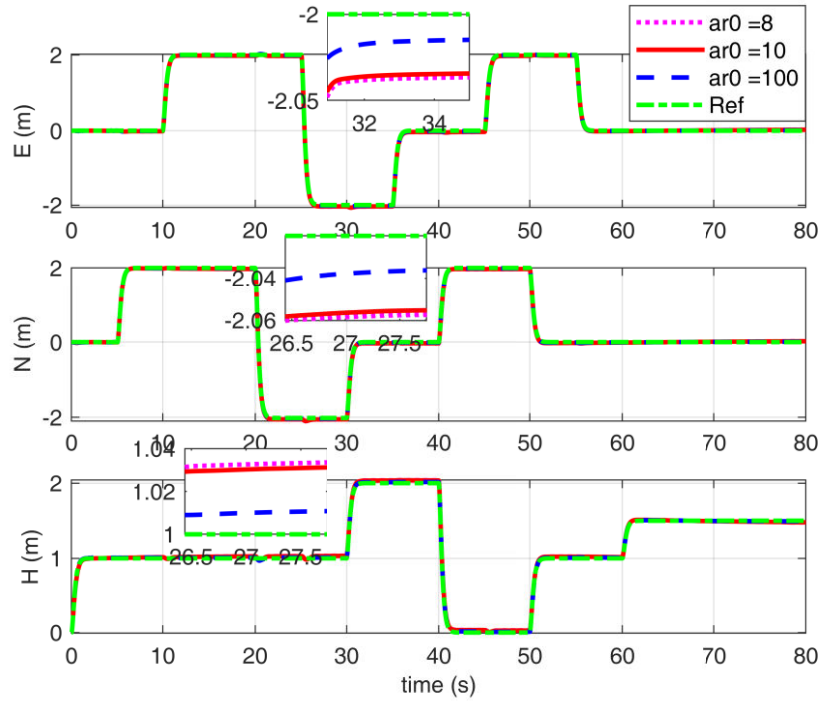


Fig. 12. Position tracking responses under with different learning rates

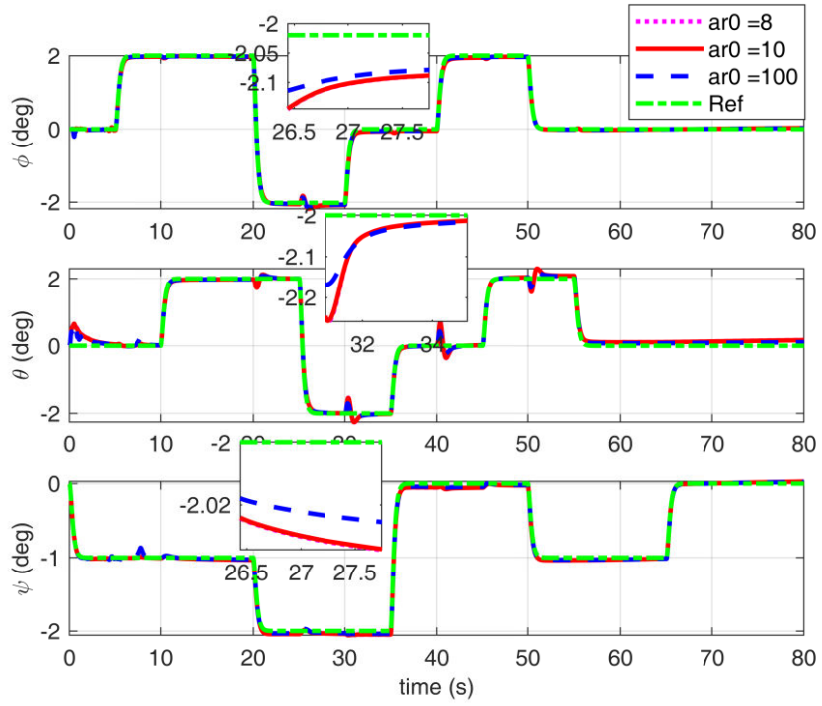


Fig. 13. Attitude angles tracking responses with different learning rates

Case2. Consider different learning rate, $\alpha_{\gamma_1} = b^{|z_1|} - 1$, where b is selected as $b = \{0.3, 3, 30\}$, $\alpha_{\gamma_0} = 10$, i.e., $\alpha_r = 10 + (b^{|z_1|} - 1)|\dot{z}_1|$. By using the proposed DO-IBTSMC design, the simulation results of trajectory and attitude tracking control are shown in Figs 14 and 15.

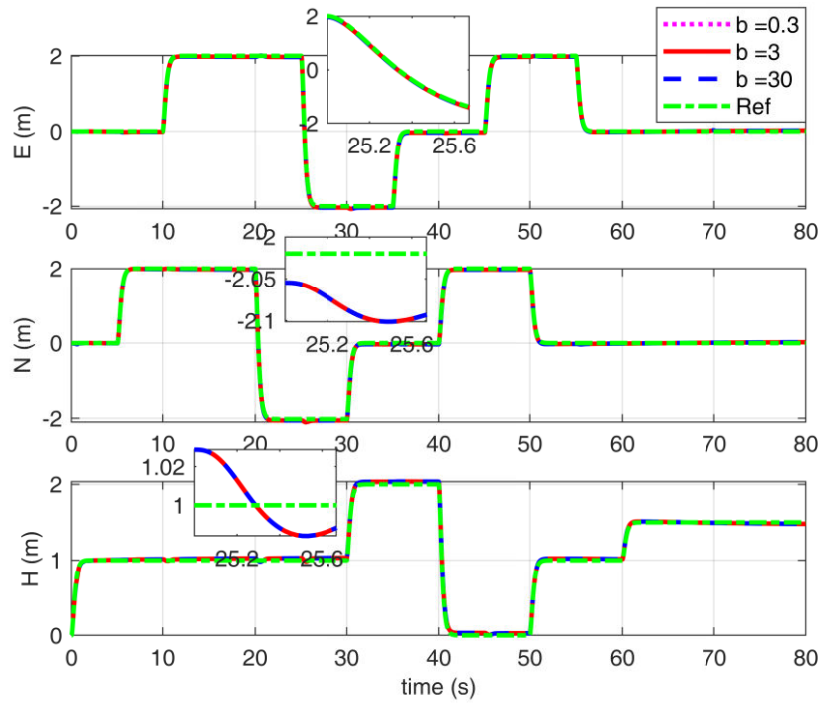


Fig. 14. Position tracking responses with different learning rates

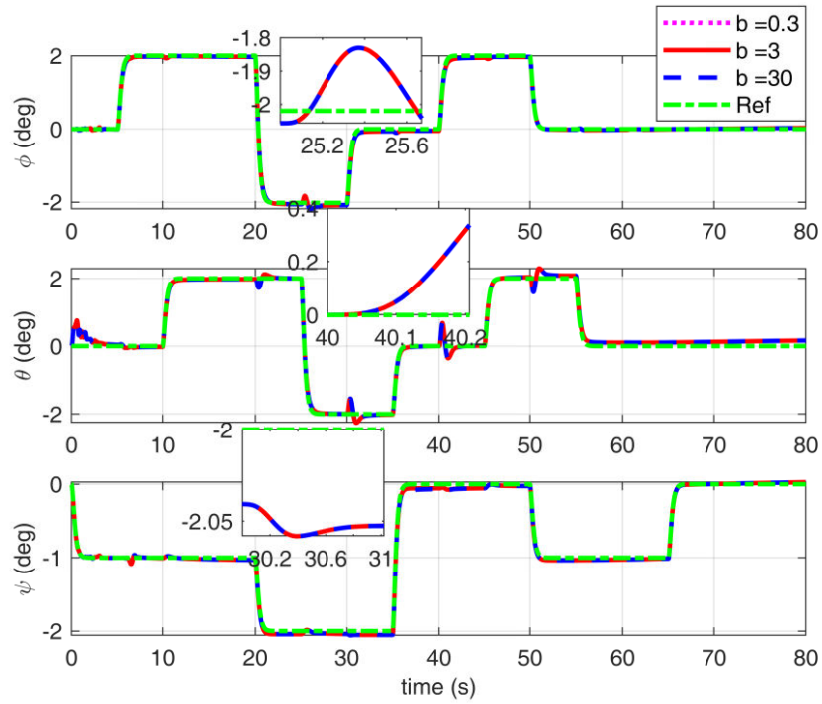


Fig. 15. Attitude angles tracking responses with different learning rates

The tracking errors are listed as Table 3.

Table 3 Tracking errors and ET of the airship under the input of the controller

Controller Error	$E_E(m)$	$E_N(m)$	$E_H(m)$	$E_\phi(rad)$	$E_\theta(rad)$	$E_\psi(rad)$	$ET(s)$
------------------	----------	----------	----------	---------------	-----------------	---------------	---------

$\alpha_{r0} = 8$	2.2043	2.3620	2.0545	0.0766	0.2979	0.0473	239.7733
$\alpha_{r0} = 10$	2.0716	2.2359	1.9366	0.0762	0.2930	0.0466	188.3364
$\alpha_{r0} = 100$	1.0476	1.1655	1.0286	0.0749	0.2130	0.0458	240.0545
$b = 0.3$	1.9123	2.0947	1.7463	0.0698	0.2443	0.0476	231.5199
$b = 3$	1.9121	2.0945	1.7462	0.0696	0.2441	0.0475	151.0156
$b = 30$	1.9120	2.0936	1.7460	0.0694	0.2440	0.0470	148.0612

From Figs 12 and 13, it is clear that the tracking performance is the worst for $\alpha_{\gamma 0} = 8$. With an increase of $\alpha_{\gamma 0}$ the tracking speed is gradually faster and steady state errors are gradually smaller. From Figs 14 and 15, it is clear that with an increase of b the tracking speed is gradually faster and steady state errors are gradually smaller, and the tracking performances are approximate for $b = 0.3, b = 3$ and $b = 30$, see Table 3.

Scenario III: Trajectory tracking control under parameter uncertainty and unknown disturbances.

This scenario considers model parameter uncertainty due to the aerodynamic derivatives varying with angle of attack (AOA). The aerodynamic coefficients are set as follows

$$\begin{bmatrix} C_{xt} \\ C_{yt} \\ C_{zt} \\ C_{mxt} \\ C_{myt} \\ C_{mzt} \end{bmatrix} = \begin{bmatrix} (1.3 + 0.4\alpha)C_x \\ (1.3 + 0.4\alpha)C_y \\ (1.3 - 0.4\alpha)C_z \\ (0.7 - 0.4\alpha)C_{mx} \\ (0.7 - 0.4\alpha)C_{my} \\ (0.7 - 0.4\alpha)C_{mz} \end{bmatrix}, \quad (69)$$

where subscript of t denotes a perturbation value. In this scenario the parameter uncertainty and external wind disturbances are considered simultaneously.

The parameters of the DO-IBTSMC controller for *Scenario III* are designed as follows

$$\begin{aligned} K_1 = \text{diag}(0.025, 0.025, 0.025, 0.0005, 0.006, 0.03), h = \text{diag}(3 \ 3 \ 3 \ 0.1 \ 2.5 \ 6), \phi_s = 0.4, p = 5, \\ q = 3, \alpha_r = 10 + (3^{|z_1|} - 1)|\dot{z}_1|, \varsigma = \text{diag}(15, 15, 15, 50, 35, 10), \beta = \text{diag}(1, 1, 1, 0.5, 0.5, 0.3), \lambda = 1.8 * \text{diag}(1, 1, 1, \\ 1, 1, 1), \gamma_d = 1. \end{aligned} \quad (70)$$

The reference trajectory is also the same as scenario I. By using the proposed algorithm, the simulation results

of trajectory and attitude tracking control are shown in Figs 16 and 17 and Table 4.

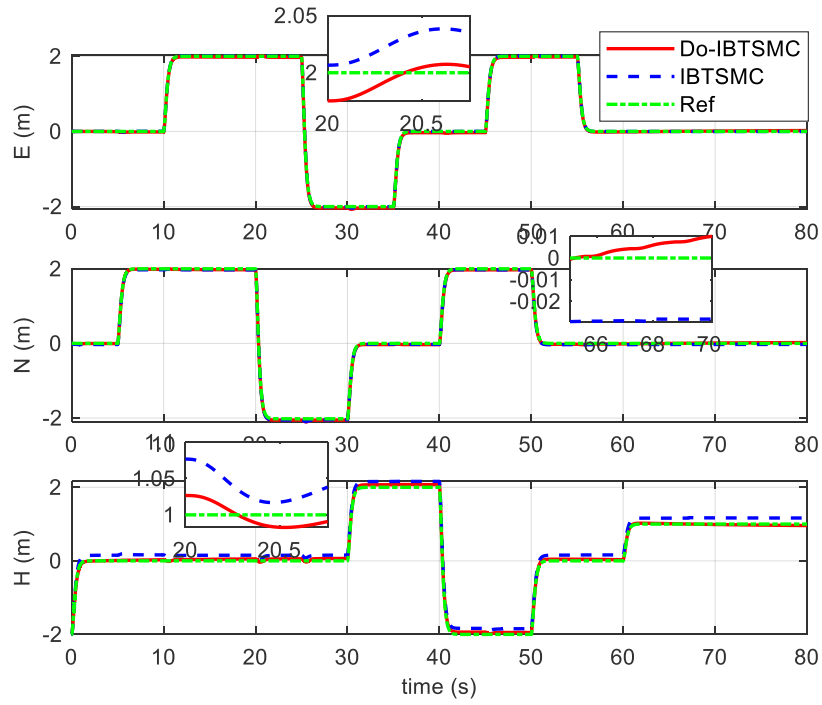


Fig.16. Positions tracking responses for *Scenario III* with model uncertainty

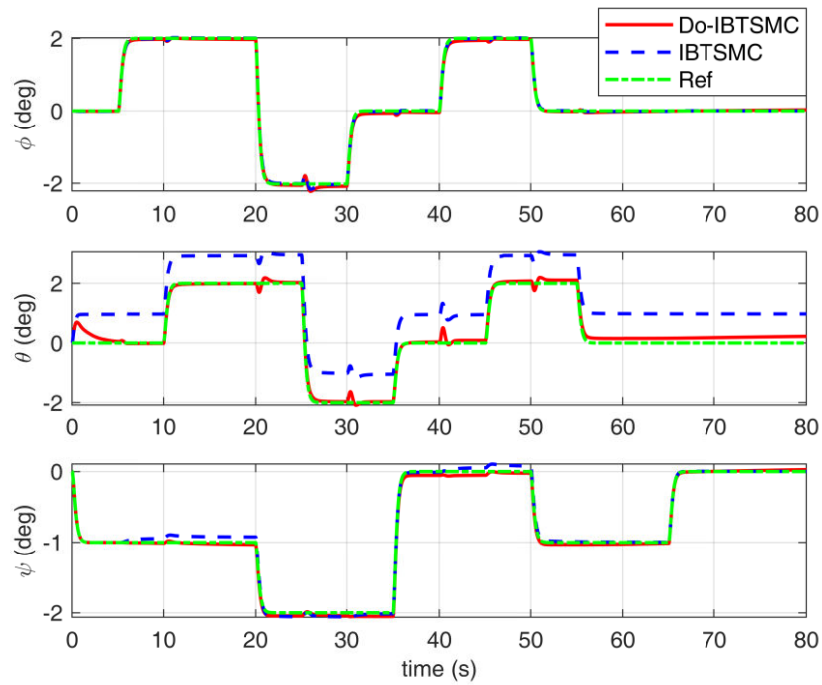


Fig.17. Euler angles tracking responses for *Scenario III* with model uncertainty

Table 4 Tracking errors and ET of the airship under the input of the controller

Controller Error	$E_E(m)$	$E_N(m)$	$E_H(m)$	$E_\phi(rad)$	$E_\theta(rad)$	$E_\psi(rad)$	$ET(s)$
DO- IBTSMC	1.9013	2.2403	2.0314	0.0707	0.9516	0.0434	193.0207
IBTSMC	1.9890	2.9766	6.7184	0.0759	1.7383	0.0702	105.0246
DO-BSMC	6.5532	3.2536	9.3065	0.0762	0.9895	0.0905	117.2025

The 3-D reference trajectory and its trajectory tracking responses are shown in Figs16 and 17 where it can be seen that desired positions are well tracked. Compared with the IBTSMC without the nonlinear disturbance observer, there are larger position and attitude tracking errors, especially for altitude and pitch responses, where the averaged tracking error E_H is only 2.0314 m for DO- IBTSMC while $E_H=6.7184$ m for IBTSMC, and E_θ is 0.9516 rad for DO- IBTSMC while $E_\theta=1.7383$ rad for IBTSMC, that is, the averaged tracking error in altitude and pitch motion is reduced more than 30%, see Table 4. This is because the DO-IBTSMC can reduce the unknown disturbances and uncertainties affected by disturbance-observer compensation.

The transitional speed and angular rate responses are shown in Figs18, 19 and 20. Because there is no control surface for the MVP airship, and the AOA and side-slip angle are controlled by the thrusters and the vectored angles, so it's easy to change the AOA and side-slip angle for the MVP airship. And it easily breaks up saturation limitation of AOA and side-slip when the thrusters and the vectored angles realize the flight task, so an envelope protect is embedded into the control loop to avoid saturation of the AOA and side-slip angle. It can be seen from Fig.18 that AOA and side-slip angle for IBTSMC with the maximum AOA $\alpha=30^\circ$ and $\beta=30^\circ$ is steady although it reaches their maximum or minimum values. Comparing with DO-BSMC, the IBTSMC is easier to be saturation due to its trimming input U_{trim} . Figs 19 and 20 show the transitional speed and angular rate responses of the DO-IBTSMC, DO-BSMC and IBTSMC, which shows the smaller overshoots of the DO-IBTSMC than the other controllers.

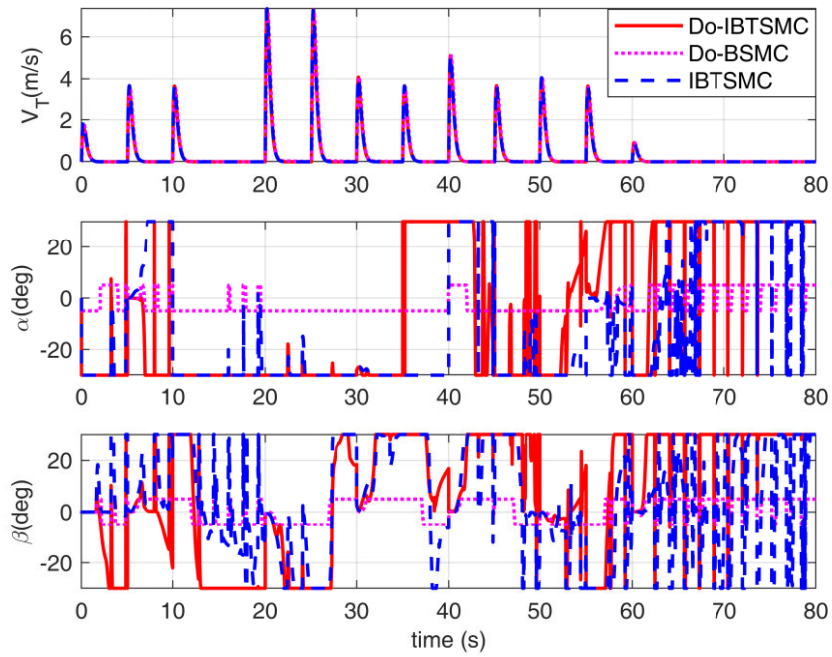


Fig. 18. Responses of the transitional speed for *Scenario III*

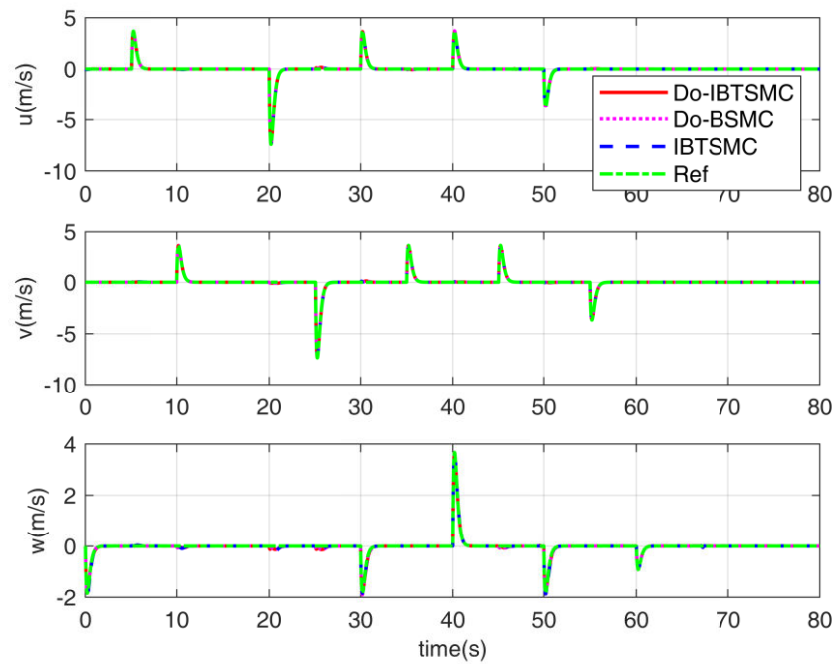


Fig. 19. Transitional speed responses for *Scenario III*

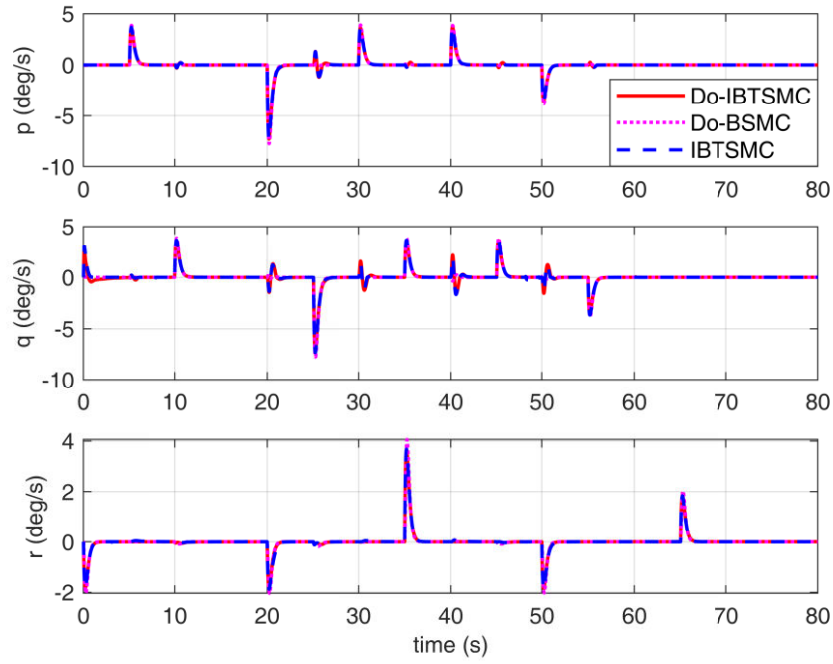


Fig. 20. Angular rate responses for *Scenario III*

The associated control inputs are shown in Figs 21 and 22.

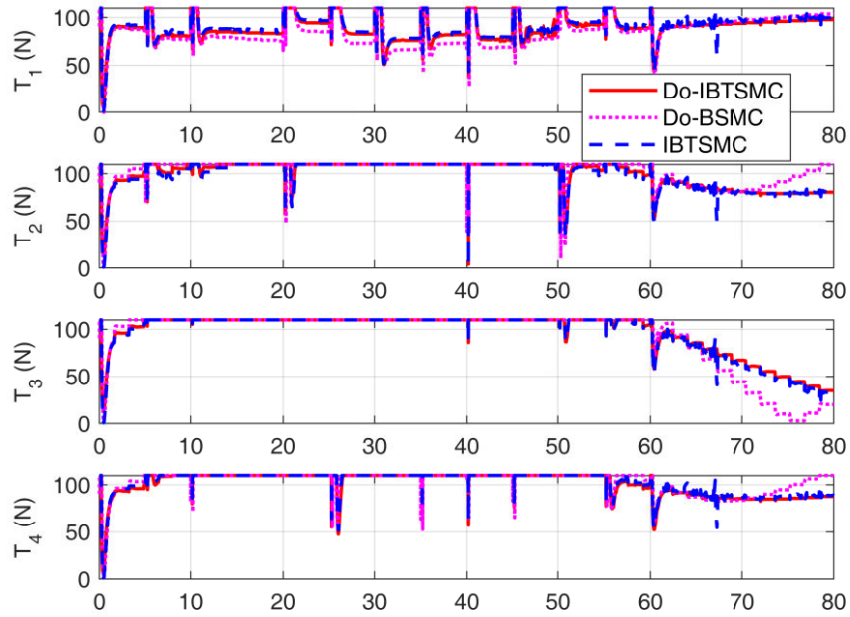


Fig. 21. Vectored thrust inputs for *Scenario III*

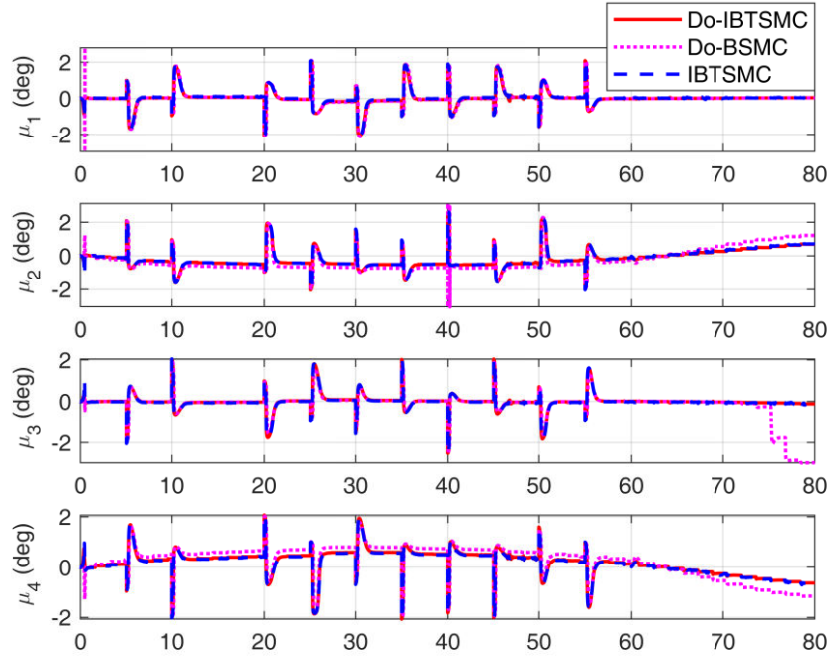


Fig. 22. Deflection angles of the vectored thruster for *Scenario III*

It can be seen that the thrust inputs and the deflection angles of the vectored thrusters of the DO-IBTSMC are close to those of the IBTSMC, but the inputs for the DO-BSMC are larger than those for the IBTSMC, this shows the robust performances of the IBTSMC. Again the control inputs have some oscillation after 60 sec because the forces and moments are required for the desired altitude and yaw commands when the airship flight in low airspeed. The disturbances and aerodynamic coefficient uncertainties are estimated, the results are consistent with Figs 10 and 11, which this shows that the IBTSMC has more capability in rejecting unobservable disturbances.

V. SUMMARY AND CONCLUSION

In this paper we propose the DO-IBTSMC approach with learning rate for multi-vectored propeller airship control. Based on a full 6-DOF nonlinear model, the trajectory tracking controller is designed. The developed controller stabilizes the attitude and position of the airship within finite-time convergence via backstepping fast nonsingular terminal sliding mode method. Furthermore, an adaptive learning rate is designed to improve system tracking performances and restrain system chattering. Meanwhile, a weighted pseudo inversion

method is applied to find the optimal thruster and control surface inputs under saturation constraints. Stability analysis shows that the closed-loop trajectory tracking error dynamics are globally exponentially stable. Three scenarios of the variable bounded disturbances and aerodynamic coefficient uncertainties were simulated and showed the proposed robustness. Compared with conventional BSMC and BTSMC controllers, the DO-IBTSMC achieves better trajectory tracking performances even though the airship is affected by parametric uncertainties and external bounded disturbances. Therefore, the effectiveness and availability of the DO- IBTSMC design are demonstrated. Autonomous trajectory generation for the stratospheric airship is an interesting work [34, 35], which will be studied for future works.

Acknowledgements

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References

- [1] L. Chen, H. Zhang and D.P. Duan, Control system design of a multi-vector thrust stratospheric airship, *Proc IMechE Part G: Journal of Aerospace Engineering*, 228(11),(2014) 2045–2054,
- [2] D. Han, X.L. Wang, L.Chen and D.P. Duan, Adaptive backstepping control for a multi-vector thrust stratospheric airship with thrust saturation in wind, *Proc IMechE Part G: Journal of Aerospace Engineering*, 230(1), (2016) 45–59
- [3] Y.B. Wen , Li Chen , K.K. Liang and D.P. Duan, Nonlinear MPC for a sensorless multi-vector propeller airship based on sliding mode observer with saturation, *Asian Journal of Control*, 21(1), (2019)248–263,
- [4] S.Q. Liu, J. F. Whidborne, Neural network adaptive backstepping fault tolerant control for unmanned airships with multi-vector thrusts, *Proc IMechE Part G: Journal of Aerospace Engineering*, vol.235, no.11, pp. 1–14, Sep. 2021
- [5] A. Moutinho, J. R. Azinheira, E.C. de Paiva, S.S. Bueno. Airship robust path tracking: a tutorial on airship modeling and gain scheduling control design. *Control Engineering Practice*. **50**, (2016) 22-36.
- [6] R. C. D. Valle, L. L. Menegaldo, A. M. Simões, Smoothly gain-scheduled control of a tri-turbofan airship, *Journal of Guidance, Control , and Dynamics*, 38(1), (2015),53-61
- [7] M. Krstic, I. Kanellakopoulos and P. Kokotovic, *Nonlinear and Adaptive Control Design*, ch2, pp.22-66, 1995
- [8] C. Edwards and S. Spurgeon, *Sliding Mode Control: Theory and Applications*. CRC Press, ch. 1 , 1998

- [9] S. Q. Liu, J. F. Whidborne, Observer-based incremental backstepping sliding mode fault-tolerant control for blended-wing-body aircrafts, *Neurocomputing*, no.464, pp.546-561. Aug.2021,
- [10] J. R. Azinheira, A. Moutinho, and E.C. de Paiva, Airship hover stabilization using a backstepping control approach, *Journal of Guidance, Control, and Dynamics*. **29**(4), (2006) 903-914.
- [11] Y. Shtessel, C. Edwards, L. Fridman, and A. Levant, *Sliding Mode Control and Observation*, 1st ed. Springer Science Business Media New York, ch. 2, 2015
- [12] S.Q. Liu, Y.J. Sang, J. F. Whidborne, Adaptive sliding-mode-backstepping trajectory tracking control of underactuated airships, *Aerospace Science and Technology*, 97, 105610, (2020)1-13
- [13] L. Chen, C. Edwards, H. Alwi and M. Sato, Flight evaluation of a sliding mode online control allocation scheme for fault tolerant control, *Automatica*, 114, 108829.(2020)1-8
- [14] Z. Man, A.P.Papilinski, H.R. Wu, A robust MIMO terminal sliding mode control for rigid robotic manipulators. *IEEE Trans. Automat. Contr.* 39(12),(1994)2464–2468,
- [15] Y. Feng, X. H. Yu and Z.H. Man, Non-singular terminal sliding mode control of rigid manipulators. *Automatica*, 38(12), (2002) 2159–2167.
- [16] S. H. Yu, X.H. Yu, S. Bijan and Z. H. Man, Continuous finite-time control for robotic manipulators with terminal sliding mode. *Automatica*, 41(11), (2005)1957–1964.
- [17] Y. Yang, A time-specified nonsingular terminal sliding mode control approach for trajectory tracking of robotic airships. *Nonlinear Dyn*, 92, (2018)1359–1367.
- [18] N. Yang, Y. Yan, Neural network approximation-based nonsingular terminal sliding mode control for trajectory tracking of robotic airships, *Aerospace Science and Technology*. 54, (2016)192–197
- [19] M. Rios-Bolivar, A. Zinober, and H. Sira-Ramirez, Dynamical adaptive sliding mode output tracking control of a class of nonlinear systems, *International Journal of Robust and Nonlinear Control*, 7, (1997) 387 – 405.
- [20] H. S. Vieira, E.C. de Paiva, S. K. Moriguchi, J. R. H. Carvalho, Unified backstepping sliding mode framework for airship control design, *IEEE Transactions on Aerospace and Electronic Systems*, 56(4), (2020)3246-3258
- [21] M. Van, M. Mavrovouniotis, and S. S. Ge. An Adaptive Backstepping Nonsingular Fast Terminal Sliding Mode Control for Robust Fault Tolerant Control of Robot Manipulators. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 49(7), (2019)1448-1458.
- [22] Z.Y. Jia, J.Q. Yu, Y.S. Mei, Y.B. Chen, Y.C. Shen, X.L. Ai, Integral backstepping sliding mode control for quadrotor helicopter under external uncertain disturbances, *Aerospace Science and Technology*, 68, (2017) 299–307

- [23] P. Singh, D.K. Giri, A. K. Ghosh, Robust backstepping sliding mode aircraft attitude and altitude control based on adaptive neural network using symmetric BLF, *Aerospace Science and Technology*, 126 (2022) 107653, pp.1-18
- [24] W. H. Chen, D. J. Ballance, P. J. Gawthrop, and J. O'Reilly, A nonlinear disturbance observer for robotic manipulators, *IEEE Transaction on Industrial Electronics*, 47(4), (2000) 932–938.
- [25] W. H. Chen, Disturbance observer based control for nonlinear systems, *IEEE/ASME Transactions on Mechatronics*, 9(4), (2004) 706-710.
- [26] F.Y. Chen, R.Q. Jiang, K.K. Zhang, B. Jiang, and G. Tao, Robust backstepping sliding-mode control and observer-based fault estimation for a quadrotor UAV, *IEEE Transactions on Industrial Electronics*, 63(8), (2016) 5044-5056
- [27] S.Q. Liu, J. F. Whidborne, L. He, Backstepping sliding-mode control of stratospheric airships using disturbance-observer, *Advances in Space Research*, 67(3), (2021) 1174-1187.
- [28] W. H. Chen, J. Yang, and L. Guo, Disturbance-observer-based control and related methods—an overview, *IEEE Transactions on Industrial Electronics*, 63(2), (2016) 1083-1095
- [29] P. Lu, E.J.V. Kampen, C.D. Visser and Q.P. Chu, Aircraft fault-tolerant trajectory control using incremental nonlinear dynamic inversion, *Control Engineering Practice*, 57, (2016) 126-141
- [30] S.B.V. Gomes. An investigation into the flight dynamics of airships with application to the YEZ-2A. Dissertation for the Doctoral Degree. Cranfield University, 1990.
- [31] G.H. Khoury, J.D. Gillett. *Airship technology*, Chapter 4, Cambridge University Press, Cambridge, England, U.K., 60-85, 2012
- [32] L. Yang and J. Y. Yang, Nonsingular fast terminal sliding-mode control for nonlinear dynamical systems, *International Journal of Robust and Nonlinear Control*, 21, (2011) 1865–1879
- [33] A.J. Healey. Multivariable sliding mode control for autonomous diving and steering of unmanned underwater vehicles. *IEEE Journal of Oceanic Engineering*. 18(3), (1993) 327-339.
- [34] A. Sandberg, and T. Sands. Autonomous trajectory generation algorithms for spacecraft slew maneuvers, *Aerospace*, 9(135), (2022) 1-22.
- [35] K. Raigoza, and T. Sands, Autonomous trajectory generation comparison for de-orbiting with multiple collision avoidance, *Sensors*, 22(7066), (2022) 1-17



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Observer based incremental backstepping terminal sliding-mode control with learning rate for a multi-vectored propeller airship

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