

CRANFIELD UNIVERSITY

ESTELA BRAGADO ALDANA

ASSESSMENT OF NON-ELLIPTIC LIFT DISTRIBUTIONS
ON SPAN-EXTENDED WING DESIGN

SCHOOL OF AEROSPACE, TRANSPORT AND
MANUFACTURING
Aerospace Engineering
Dynamic, Simulation and Control Group

PhD

Academic Year: 2018-2022

Supervisors: Dr. Atif Riaz; Dr. James F. Whidborne
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Abstract

In the pursuit of reducing the environmental impact of aviation, novel aircraft concepts and technologies are receiving increased attention from researchers and manufacturers. Amongst the solutions expected to improve aerodynamic efficiency, aircraft with high aspect ratio wings are being regarded as a promising solution, despite the numerous hindrances derived from such a configuration. Furthermore, the current socio-economic prospect has exacerbated the need to address the multi-disciplinary nature of the conceptual design process, in which traditional methods are becoming less reliable for the modelling of the envisioned novel configurations and technologies.

To address some of these challenges, an elegant analytical approach promising improvements in the aero-structural efficiency and flight dynamic characteristics of wings has gained the interest of researchers around the world. This design approach proposes to remove the fixed-span constraint and instead prescribe structural requirements. This yields a set of non-elliptic lift distributions with theoretically improved aero-structural efficiency and lateral-directional flight dynamic characteristics. However, conclusions on the actual benefits and practical implications of the application of this theory remain unsettled. This thesis investigates some of the research questions posed by such an approach applied to the conceptual design of high aspect ratio wings. To do so, it provides a multidisciplinary physics-based design environment that integrates in-house developed and existing computational models within a set-based design approach. This allows for the analysis of feasible solutions with regard to overall performance improvement whilst shedding light on the relevant trade-offs.

The proposed design and analysis approach yields span-extended configurations for which aerodynamic efficiency is improved through span extension and the consequent growth in wing structural weight is reduced as a result of the applied non-elliptic spanloads. The presentation of alternative figures of merit to comparatively assess the performance of the designs provides further insight than the use of other traditional metrics such as the lift-to-drag ratio. This yields several span-extended configurations without penalties in performance, following the methodology and metrics employed in the proposed design and analysis process. Furthermore, the semi-analytical approach to proverse yaw enables to identify the unconventional behaviour of induced

drag during aileron deflection on wings with the selected non-elliptic spanloads. Additionally, it highlights that attainment of these solutions are highly dependent on flight condition, aileron sizing and location within a given spanload, and the magnitude of deflection. Overall, this thesis contributes to a further insight on the aero-structural trade-offs and proverse yaw characteristics derived from the use of such a design approach. This can facilitate the identification of the determining contributors and compromises to be made at early stages of the design, amplifying the designer's control over the design and decision-making processes, while delaying critical decisions and enhancing the optimisation process with more informed drivers.

Keywords

Conceptual Wing Design; Non-Elliptic Lift Distributions; High Aspect Ratio Wings; Aero-Structural Analysis; Mission Performance; Proverse Yaw.

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List of Abbreviations

ADG	Airplane Design Group
AVL	Athena Vortex Lattice
BSLD	Bell-Shaped Lift Distribution
BWB	Blended-Wing-Body
CAD	Computer-Aided Design
CFD	Computational Fluid Dynamics
CS	Certification Specification
DLM	Double-Lattice Model
DOE	Design of Experiments
EASA	European Union Aviation Safety Agency
ESDU	Engineering Sciences Data Unit
FAA)	Federal Aviation Agency
FEM	Finite Element Methods
HARW	High Aspect Ratio Wing
ICAO	International Civil Aviation Organization
MDO	Multidisciplinary Design Optimisation
MTOW	Maximum Take-Off Weight
NACA	National Advisory Committee for Aeronautics
NASA	National Aeronautics and Space Administration
NELD	Non-Elliptic Lift Distribution
RANS	Reynolds-Averaged Navier–Stokes

CHAPTER 0. LIST OF ABBREVIATIONS

SBD	Set-Based Design
SUGAR	Subsonic Ultra Green Aircraft Research
TBW	Truss Braced Wings
TLAR	Top-Level Aircraft Requirements
TOGW	Take-Off Gross Weight
TSFC	Thrust-Specific Fuel Consumption
VLM	Vortex Lattice Method

Nomenclature

α	Angle-of-attack, [°]
α	Wing incidence angle-of-attack, [rad]
α_0	Zero lift angle-of-attack, [°]
α_i	Induced angle-of-attack, [rad]
α_T	Aerofoil geometric twist angle, [rad]
α_{0L}	Aerofoil camber angle-of-attack, [rad]
$\mathcal{R}$	Aspect ratio
β	Prandtl–Glauert compressibility factor; Sideslip angle, [°]
δ_a	Aileron deflection, [°]
Γ	Circulation, [m ² /s]
Λ	Sweep angle, [°]
λ	Taper ratio
$\mathbf{F}$	Force vector
$\mathbf{r}$	Vector of distance between vortex segment, [m]
$\mathbf{V}$	Matrix of velocities, [m/s]

NOMENCLATURE

ϕ	Bank angle, [°]
ψ	Heading angle, [°]
ρ	Air density, [kg/m ³]
θ	Trigonometric spanwise coordinate, [rad]
ξ	Non-dimensional Cartesian spanwise coordinate
b	Span, [m]
c	Chord, [m]
C_d	Local spanwise drag coefficient
C_L	Lift coefficient
C_l	Local lift coefficient
C_{D_i}	Induced drag coefficient
C_{l_α}	Lift curve slope, [1/rad]
$C_{l_{\delta_a}}$	Rolling moment due to aileron deflection coefficient
C_{l_p}	Rolling moment due to roll rate coefficient
$C_{n_{\delta_a}}$	Yawing moment due to aileron deflection coefficient
D	Drag force, [N]
D_i	Drag force, [N]
E	Young's modulus, [Pa]
e	Span efficiency factor
F_s	Shear force, [N]

G	Fourier coefficient
g	Gravity acceleration, [m ² /s]
h	Altitude, [m]
I_L	Moment of inertia of the lift, [Nm ²]
K_s	Structural proportionality coefficient
L	Lift, [N]
l	Lift per unit span, [N/m]
M_I	Integrated bending moment, [Nm ²]
M_T	Torsion, [Nm]
$M_{X_{root}}$	Root bending moment, [Nm]
n	Load factor
p	Roll rate, [rad/s]
R	Range, [m]; Radius of gyration of the lift distribution, [m]
r	Yaw rate, [rad/s]
r_b	Ratio of aileron span over wing span]
r_f	Ratio of aileron chord over wing chord]
r_m	Non-dimensional midpoint location of the aileron along the semispan]
r_s	Ratio of span extension
r_0	Ratio of ()
S	Reference area, [m]

NOMENCLATURE

s	Semispan, [m]
T	Torque, [N]; Thrust, [N]
t	Thickness, [m]
t/c	Aerofoil thickness-to-chord ratio
u	Longitudinal velocity, [m/s]
V	Airspeed, [m/s]
v	Lateral velocity, [m/s]
W	Weight, [N]
w	Vertical velocity, [m/s]
w_i	Induced downwash, [m/s]
W_s	Structural weight, [N]
y	Cartesian spanwise coordinate, [m]
y_C	Centroid of the lift distribution, [m]

Chapter 1

Introduction

The study of wing design has been central to the work of aeronautical engineers from the earliest days of human flight. The theories and methods developed have been consolidated through the engineering excellence reflected in the safety standards of aviation. However, the current socio-economic prospect has fuelled the prevailing need for a technological future that is in closer harmony with the environment yet accommodating to our ever-growing demands. The consequential strive for more efficient flight has led many to challenge a diversity of conventionalities in civil aircraft design. These range from the venture into novel aircraft configurations to the redefinition of the design methods employed to devise such products. The developing computational capabilities and technologies keep providing new grounds in which to exploit these aspirations. Progress is being achieved through incremental improvements in the various aerospace disciplines. Nevertheless, this ongoing process is far from accomplished and requires the aerospace community to continue advancing towards the goal of cleaner skies.

1.1 Today's Problems in Wing Design

Today, the civil aerospace sector is predominantly focused on increasing efficiency to reduce fuel consumption and emissions. Researchers and manufacturers from all participating engineering disciplines are relentlessly putting forward numerous potential solutions to address this difficult task. For instance, radical changes are being targeted in the field of propulsion, where hydrogen and electric powered engines promise reduced greenhouse emissions. This, alongside rising fuel prices, increased demand for regional air travel and the emergence of urban air mobility are boosting the resurgence of propeller-driven aircraft and distributed propulsion. Aircraft with longer wingspans will access more airports with the implementation of folding wingtips. Moreover, new designs could significantly increase performance, fly further,

and reduce noise through morphing technology. Meanwhile, a flock of aircraft flying in formation could be the next fashion patterning the skies, in a more nature-inspired blended wing body design. Improved material properties with faster manufacturing would accelerate any of the above proposals altogether.

Some of these solutions are rather more tangible than others; they can be developed and implemented relatively fast, they bring acceptable trade-offs against the penalties that arise, and do not require major redefinition from current designs. But it is precisely these established designs that have, to a certain extent, plateaued in terms of the efficiency they can provide. Such solutions tend to yield small increases in efficiency. Nevertheless, they also bring further insight on the tackled problem, and can increase the understanding on why a solution is better than the previous one. Such solutions accumulate in the collective knowledge and might eventually lead to a major contribution. However, the pressing need to reduce the environmental impact of aviation might require a more radical revision of the current status in order to bring that step increase in efficient, green flight. This has further encouraged the industry to pursue a flying revolution that might have otherwise remained loitered, since it is not only dependant on the technical and technological capabilities; major factors arise from safety uncertainty and certification requirements, as well as the huge economic investment it demands. Hence, it is imperative to keep contributing to this progression in all possible ways.

Inescapably, the field of aerodynamics remains a core aspect of this ensemble of opportunities. In the view of the author, this discipline might not be directly responsible for the transformations that are envisioned for future aircraft; this task would be mainly adopted by propulsion and materials sciences. Nevertheless, it is a core enabler for any type of configuration, and it still carries many prospects for improvement. Here, the sustained contribution towards increasing efficiency mainly focuses on drag reduction, as repeated estimations predict that a 10% reduction in drag could result in fuel savings corresponding to approximately 25% of the operational cost [8]. Uncountable research streams diverge into different methods that investigate the air-flow around an aircraft in order to further understand and then control it, in order to reduce drag forces exerted from viscous and compressibility effects, which are critical to civil aircraft flying at transonic speeds, or simply from the motion of large air masses displaced in exchange for lift.

This thesis places its focus on the latter: induced drag. A significant number of aircraft concepts that have been recently presented for future airliners revolve around the idea of induced drag reduction. Figure 1.1 shows some examples of such design concepts. These aircraft are characterised by having high aspect ratio wings (HARWs), and their link to reduced induced drag is found at the core of aerodynamic theory; Equation 1.1 shows one of the fundamental derivations for an incompressible potential



Figure 1.1: Recent HARW concepts. From left to right: Airbus and SAS Scandinavian Airlines E-Thrust concept [1]; Boeing and NASA's Subsonic Ultra Green Aircraft Research (SUGAR) Freeze [2]; Airbus ZEROe Turbopan [3]; Airbus AlbatrossONE [4]

flow [5]

$$C_{D_i} = \frac{C_L^2}{\pi \mathcal{A} e} \quad (1.1)$$

which states that the induced drag coefficient, C_{D_i} , shows a quadratic relation to the lift coefficient, C_L , and is inversely proportional to the wing aspect ratio, $\mathcal{A}$, and the span efficiency factor, e . Hence, increasing the value of these two latter parameters will reduce induced drag for the same lift coefficient. HARWs have been extensively investigated and established as a near-term achievable solution to increase aerodynamic efficiency, despite the numerous drawbacks derived from such a wing design; for instance, higher flexibility, increased structural weight, or operational limitations. Today, a similar 'span extension' solution can be seen on wings with winglets, the coming folding wingtips, and in elegantly designed wing twist distributions, often obtained through state-of-the-art multidisciplinary design optimisation (MDO) processes. Looking again at Equation 1.1, the span efficiency factor is the term that characterises the aerodynamic efficiency of a given lift distribution. In today's conceptual wing design processes, the underlying principle for optimal aerodynamic efficiency is still based on Prandtl's early lifting-line theory [6], where he derived that the optimal solution for minimum induced drag of a lifting surface was given by an elliptic lift distribution and planform. Prandtl demonstrated that no other planar configuration could be more aerodynamically efficient, yielding a span efficiency factor of 1. However, this term will also have an impact on the structural weight of the wing, as it is effectively linked to the spanwise position of centre of lift. Here, the centre of lift must be moved inboard to reduce structural loads at the wing root as well as avoid stalling wingtips. This results in more bell-shaped spanloads with lower span efficiency factors. Nevertheless, the fact is that the majority of aircraft do not implement elliptic wing planforms nor lift distributions, since they bring enough practical disadvantages to outweigh the benefits gained through the optimal aerodynamic efficiency; for instance, high manufacturing complexity or the high risk of a full wing stall.

It is well known that optimising for aerodynamic efficiency in isolation will not produce an optimal aircraft due to the multidisciplinary character of wing design. The

trade-offs exposed above are just a few examples of the myriad of compromises that must be considered. As highlighted, two of the most critical disciplines involved in wing design are aerodynamics and structures. These disciplines are highly coupled, and in practice, the requirements for minimum induced drag and required structural strength are opposed, therefore constraining the possibility of increasing the wing aspect ratio without weight or aeroelastic penalties. Even today, conceptual design and multidisciplinary design optimisation (MDO) tools approach these disciplines independently until a compromise is reached. Moreover, solutions are often bounded to a pre-conceived idea of the shape of an optimal spanload, to which the structure's jig shape is tailored so that the deflected wing yields the desired distribution under a 1g load. Whilst this approach to aero-structural design has been adequate for the design of classic swept-back wings, it is proving to be a limiting factor in the unconventional measures required to make the leap to green aviation. Therefore, in more advanced configurations that lack accumulated historical data or require multiple design points as part of the mission, the complexity of aero-structural interactions can result in sub-optimal designs. Furthermore, more efficient structures and materials have led to increased flexibility of wings, which exhibit a range of undesirable aeroelastic phenomena, such as flutter or divergence, and which can only be suppressed efficiently if aero-structural interactions are considered at the early design stages. All things considered, there is a calling for improved design and optimization approaches to correctly model these coupled physics. Aero-structural wing design problems have been of interest to academia and industry over the past decades, and are still a widely researched topic today that has progressively become more consequential.

The conceptual design stage is getting ever more complex in attempts to de-risk potential solutions. Nevertheless, it is still necessary to put forward high-level design parameters that will be fixed and passed onto the preliminary design stage. Unplanned factors and decisions may also be introduced at later stages, with significant development implications that lead to trade-offs between aerodynamics, structures and systems. The conceptual design stage traditionally employs a range of empirical and low-fidelity models to reduce computational cost. Such models consist of simpler representations of the geometrical characteristics and physical properties of the system, which is therefore described with a reduced number of parameters under simplified assumptions. Such characteristics make them particularly suitable to rapidly evaluate and explore design concepts, without requiring the detailed information and resources needed for a more complex and accurate model. However, the need for novel airframe configurations makes the conventional semi-empirical methods unsuitable, due to the lack of historical data as well as the limitations posed by traditional assumptions. The need for coupled analysis approaches to properly identify the associated multidisciplinary trade-offs has increased the consideration of higher-fidelity methods at early

stages of the design process, such as computational fluid dynamics (CFD) or finite element methods (FEM). These methods employ numerical modelling techniques that take into account more complex physical phenomena and more detailed information about the model, at the cost of becoming computationally expensive and limiting the design space, thus making the original baseline methodology inefficient. Next generation design methods must therefore be robust enough to provide these capabilities whilst increasing confidence at early stages of the design phase.

This unavoidable approach results in a sub-optimal product, the subsystems of which will have had to sacrifice efficiency to adapt to the dominant factors. At the same time the unprecedented demands posed by the current economic, geo-political and societal circumstances create a need for fast results and advanced methods to design, produce and maintain aircraft systems and components. To meet the demands of the above ambitious targets, deeper changes are required in the concept, the methodology and the technology.

1.2 An Alternative Approach

There are several challenges to be addressed in wing design for induced drag minimisation. As exposed, many of them arise from the necessary integration of the various airframe disciplines. A comprehensive approach to conceptual design has proven to be beneficial for later stages, as these are addressed with less uncertainty and more understanding of the interdisciplinary trade-offs.

In particular, a community of researchers and designers have undertaken this task through the reconsideration of the underlying theoretical principles of conceptual wing design, in order to minimise induced drag and advance towards a holistic flight model. This endeavour has led to the re-inspection of an alternative approach first published by Prandtl in 1933 [7], where he proposed to remove the fundamental constraint of fixed span, established in the lifting line theory, and instead prescribe wing bending moment. By doing so, he synthesised both aerodynamic and structural efficiency into a single theory, for which all solutions would reduce induced drag whilst maintaining the same structural weight, and would be given by non-elliptic lift distributions (NELDs). The optimal solution for minimum induced drag would provide an 11% reduction of this force with a 22% span extension as compared to the ideal elliptic spanload, and would be described by a bell-shaped lift distribution (BSLD). Researchers are now trying to integrate Prandtl's 1933 approach in wing design frameworks so that traditional assumptions, such as the elliptic spanload being the optimal, do not constrain their proposed concept. Furthermore, experimental research on the application of this design approach to flying wings has unveiled the potential to

remove the adverse yaw characteristics inherent to conventional swept tapered wings.

Up to this point, the scarce research carried out in this field seems to promise significant improvements in aero-structural efficiency and flight dynamic characteristics. However, conclusions on the actual benefits of the application of this theory remain unsettled. The available research has mainly focused on the analytic extension of the theory, or on the independent impact of its application on various disciplines involved in the conceptual design stage, and mainly considering low-speed flight conditions. These results are based on classical theories and reduced order models that carry many assumptions and simplifications. However, the simplicity and beauty of the foundational theories is a practical and robust approximation to rapidly provide sufficiently accurate characteristics at the early stages of the design phase. These methods can consider most of the critical flight phenomena that will settle the top-level requirements of a design. Hence, these methods are critical to obtain foundational insight while keeping this design stage rapid and efficient. From this perspective, in order to confront novel configurations it is beneficial to rely on physics-based methods built upon reduced order models rather than historical data that only remains valid for traditional aircraft configurations.

Prandtl's alternative wing design approach presents an opportunity to address the current socio-economic demands, but there are still many questions left to answer regarding the practicality and efficacy of this design philosophy. In this sense, it is still necessary to assess the impact of such an approach at an aircraft level. Here, the use of physics-based methods would enable to identify the propagated contributors of such wing designs and use them to improve decision-making in the design process. This presents several research opportunities, with questions such as:

- Is it possible to design more aero-structurally efficient wings for civil aircraft under the principles of NELD? What is the impact at a vehicle performance level?
- What actual benefits can be attained under the design constraints established by industry at the conceptual stage?
- How do the design contributors propagate throughout the disciplines involved? Can they enhance information on the design trade-offs at early stages of the design process?

For such an assessment, a broader range of variables must be considered compared to those in the original theoretical derivation. These variables are crucial, even in the conceptual design phase, and their significance could outweigh the anticipated advantages. For instance, the increased aspect ratio raises questions on the impact of non-structural weights, critical load cases, and aeroelastic characteristics. Similarly,

the corresponding the twist may require examination of its effect on compressibility, parasitic drag, stall characteristics, and the choice of the aerofoil. Furthermore, it would be useful to quantify the uncertainties related to the aero-structural efficiency derived under this design approach. Overall, the principles of NELDs align with the future vision for aviation, and it is in this context that the characteristics of NELD presented above can become beneficial. This lays down the motivation for the research presented in this thesis, which aims to address some of the challenges posed by this approach.

1.3 Thesis Aim and Objectives

The aim of this research is to evaluate the synthesis of wing aerodynamic and structural requirements under a single design theory for the conceptual design of high aspect ratio wings. This is done as a means to improve overall vehicle performance and enlighten the relevant complexities of a multidisciplinary design process.

In attempting this aim, the following objectives have been proposed:

1. Develop a reduced-order wing shape design methodology that produces the geometric parameters necessary for a wing to satisfy prescribed aerodynamic and structural requirements. Within this, integrate and compare existing theoretical approaches that remove the constraint on span for aero-structural efficiency.
2. Compose a multidisciplinary physics-based design framework for the conceptual design stage to produce a set-based design space for the analysis of aircraft with high aspect ratio wings under prescribed aero-structural requirements. Formulate this environment to enable a state-of-the-art approach to conceptual design and minimise the use of statistical and historical data.
3. Develop a methodology to assess the impact of wings designed under the assumptions described above on aircraft lateral-directional flight dynamics with a focus on proverse yaw.
4. Identify the capabilities, advantages and drawbacks originating from the application of the proposed design process to improve aircraft efficiency and enhance decision-making within the conceptual design stage. Conduct comparative aircraft performance analyses of the produced configurations relative to a conventional airliner, to establish the basis for validation, and interpret the results through the derivation of trends and metrics.

1.3.1 Research Scope and Assumptions

The scope of this research has been confined to a particular context and set of variables in order to give clarity to the study outcomes, better identify trends in a well-known scenario, and be able to increase the confidence in the methodology. The items to highlight are listed below:

- The evaluated aircraft conforms to the conventional tube and low-wing configuration, with proposed modifications limited to the wing's geometry. In reality, any such alterations to the wing are likely to have a significant impact on other components and characteristics of the aircraft, including but not limited to the tail and longitudinal center of gravity location.
- The engine design and integration remain constant across all the assessed aircraft configurations, with thrust and body axis alignment. Nonetheless, in practice, the proposed wing modifications would necessitate an engine location reassessment, and alterations in the aircraft's operational empty weight would require a re-evaluation of the engine's power capabilities.
- The design of the aerofoil is selected out of existing designs and will be fixed across configurations. The design of flaps and secondary control surfaces is not considered, but instead included through their aerodynamic contributions.
- Secondary load cases are included, considering critical manoeuvres, gusts and continuous turbulence scenarios.
- Aeroelastic constraints are not considered to drive the structural sizing, which will be instead determined mainly by structural integrity requirements. Such a limitation can result in increased weight, reduced performance, increased development time and cost, and safety concerns. Further discussion can be found in Chapter 2.
- The structural sizing process does not take into account the impact of weight reduction resulting from fuel consumption on the sizing of the wing. This can lead to over-dimensioned wings, which in turn may impose analogous limitations to those identified above.
- The aerodynamic analysis employs linear models. Therefore, non-linear transonic flow characteristics are approximated.
- The profiles defining the aircraft mission are two-dimensional and remain constant across all the assessed aircraft configurations.

- The selection and impact of airframe material are not explored in the design and analysis process. It is fixed for all configurations and only one material will be used for the complete airframe.
- Structural fatigue and other dynamic effects are excluded, which would further impact the sizing of the wing structure.
- The assessment process does not encompass manufacturing considerations, which are subject to potential alterations from the anticipated twisting and extension of the wing. Such modifications could increase the complexity of the manufacturing process, as well as cause detrimental impact on the internal space allowances of the wing.

The author acknowledges that the excluded constituents also play a critical role in the design of wings and minimisation of drag and weight. The rationale behind the exclusion of these constituents is motivated by the simplification of the analysis methods, thereby reducing the number of variables and model complexity. Despite the potential reduction in the accuracy of the results, this approach has the ability to conduct rapid analyses, which is preferable during the preliminary stages of the design process for exploring the wide range of possible designs. Furthermore, the comparative nature of the proposed assessment will allow for a relative normalisation of the results, where any systematic error derived from simplified assumptions and methodology limitations will still allow for meaningful comparisons.

1.4 Thesis Outline

The chapters within this thesis describe the work developed in attempting the aim and objectives of this research.

Chapter 2 presents a broad literature review of the theoretical basis of the approaches to structurally constrained non-elliptic lift distributions, the developments carried out in this area and their relevance to the future of aviation, as well as the impact on the conceptual design process. The general overview of the analysis framework being proposed to carry out this research is discussed in Chapter 3. A description of the multi-disciplinary workflow adopted for the analysis of configurations with NELDs is provided, along with the presentation the baseline case-study and its relevant data. The selected theoretical approaches that remove the constraint on span for aero-structural efficiency are assessed in Chapter 4 and integrated into the wing shape design methodology, the development of which is presented and discussed as a comparative ground to evaluate the analytical aero-structural properties of the computed wing designs with extended initial conditions. The adopted approach for the

consecutive initial sizing of the structure is found in Chapter 5, along with the selection of critical load cases that will drive the sizing. In Chapter 6, the impact of the trade-off between the aerodynamic and structural characteristics brought by NELDs wings is investigated here through the mission analysis, with inspection of performance and top-level aircraft requirements. Also, a discussion encompassing the previous findings is presented from a multidisciplinary perspective. The lateral-directional flight dynamic characteristics of NELDs under proverse yaw considerations are evaluated in Chapter 7 through the proposed methodology. Finally, Chapter 8 summarises the content of the thesis, highlighting the key findings and general conclusions of the work carried out. A list of further work is proposed.

Chapter 2

Literature Review

This chapter takes the reader through a set of developments and milestones that have led to the state-of-the-art of NELDs and HARW design in conceptual design. Specifically, it places the focus on the trade-offs originating between aerodynamic efficiency and weight, and presents a brief collection of some historical approaches that addressed this in juxtaposition with the modern methods for conceptual design. Prevailing limitations in concert with the vision for the future of aviation will instigate the need for the research presented on the following chapters.

2.1 The Foundations: A Brief Epic on Ludwig Prandtl

It is widely acknowledged that Prandtl's work in fluid dynamics has shaped modern wing theory, and is now so fundamental to conceptual aircraft design that it can be found at the core of almost all modern wing design tools.

Living in a time of discoveries, such as Planck's quantum theory or Einstein's theory of relativity, Prandtl was not left behind, but became one of the most important representatives in the field of fluid mechanics. He developed, led and collaborated to produce a vast body of work that has become the foundation of many scientific areas. This work would be often conceived in concert with some of his over 80 doctorate students [19], many of whom would also succeed and become notable scientists in their fields¹. Prandtl revolutionised aerodynamics by providing the first account of the notion of a boundary layer, introducing the no-slip condition at the surface to explain large velocity gradients within the layer and separation of the fluid from the surfaces. Using the recent development of the circulation theory of lift, he derived the thin-aerofoil theory and later provided guidelines to account for compressibility effects.

¹To illustrate a modest list of some of Prandtl's notable students: A. Betz, T. von Kármán, A. Busemann, C. Tietjens, D. Küchemann, M. Munk, H. Multhopp, H. Schlichting, W. Tollmien, P. Blasius, T. Meyer, etc.

He also made pioneering contributions in the area of supersonic flows, providing the means to calculate the properties of oblique shocks and expansion waves, or presenting a robust method for the design of supersonic nozzles. Interestingly, his initial field of research was actually not fluid dynamics, but solid mechanics, hence he also played a part in the development of structural mechanics and even applied flow physics to meteorology [20,21].

Lifting-line theory [6] is among the many successful theorems he conceived; the focus of this research lies on this and its use in conceptual design. This method rapidly became the epitome of wing design for the prediction of the aerodynamic properties of a three-dimensional finite wing, and it is still used as the underlying model in many of the current tools and standards since it was published in 1918 [22]. This was the first formal mathematical model of a wing's lifting properties, and was a product of the work carried out by Prandtl in collaboration with Betz [23] and Munk [24]. However, it could be argued that this could not have been achieved without Lanchester's earlier work, published in 1907 [25]. Described by von Kármán as a multisided man and full of imagination [26], Lanchester was inspired by the fluid mechanics of nature, and published his observations for application to wing design. In his work, he already recognised the existence of asymmetric vortices being shed from the wingtips of a finite wing in a three-dimensional flow, which would dissipate energy into the wake. This has led to historical controversy on the origins of lifting line theory and thin aerofoil theory [26], since many of the key ideas behind it had already been exposed by Lanchester². In the recent years, further credit has been given to Lanchester by many authors, who have included his name on the theory's headline. Be that as it may, this theory is built upon the works of Kutta-Joukowski [27,28], Biot-Savart [29], Helmholtz [30] and Munk [24]. Altogether, these play a crucial part in the theoretical and mathematical foundations behind the aerodynamics of the aircraft we see today.

2.1.1 Impact of the Lifting Lifting Line Theory

Prandtl's lifting line theory has remained the baseline tool for the analysis of finite wings and the calculation of lift and induced drag, becoming the standard in aircraft conceptual design.

Lifting-line theory defines a finite wing as a bound vortex filament consisting of an infinite number of horseshoe vortices shedding downstream, as depicted in Figure 2.1. This approximation captures the main aerodynamic features of a three-dimensional lifting surface. Detail on the mathematical derivations is provided in Section 4.1.1.

The importance of the lifting line theory can be elevated to be one of the most essen-

²Prandtl was, in fact, present at one of Lanchester's talks in Göttingen, where both exchanged thoughts through an interpreter on these ideas [26].

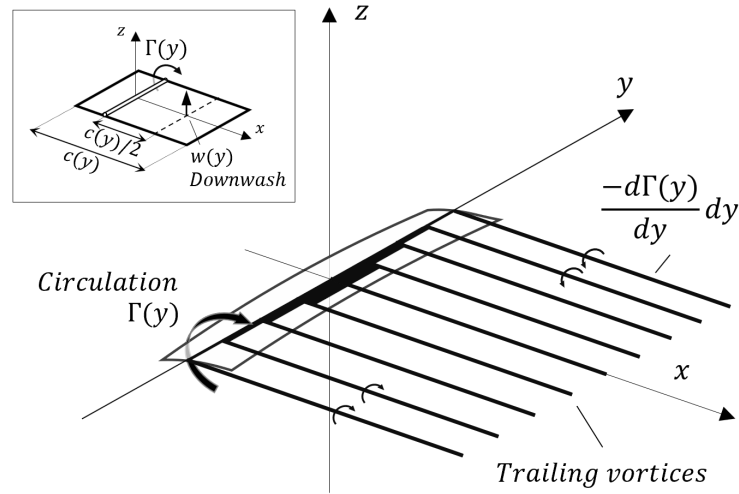


Figure 2.1: Horseshoe vortices representation of a lifting line model. Reproduction from reference [5].

tial contributions to aeronautics. Precisely, the definition of induced drag paved the way for making rational design calculations for aircraft. This included the possibility to resolve the effect of aspect ratio, to understand its correlation to lift, and unveil the interaction between multiple lifting surfaces due to the effect of stagger. Furthermore, the discovery of the elliptic circulation distribution as the solution for minimum induced drag and the ideal wing became a standard against which the efficiency of a design could be evaluated. Moreover, by deriving the span efficiency factor and determining its optimal value of unity, a clear and unified target was established for interpreting the previous experimental data. [31,32].

Prandtl confirmed the validity of his work experimentally in 1915 [6]. The left-hand plot in Figure 2.2 shows measured drag forces against lift on wings of the same cross-section but different aspect ratio. Assuming the induced drag to be described by Equation 1.1, it should be possible to collapse the results onto a single, universal curve. Prandtl did this by using Equation 1.1 to predict the polar for $\mathcal{R} = 5$ from the polars for other aspect ratios. The results, shown in the right-hand plot, are convincing, noting that the aspect ratios are not especially large. Yet, a review of Prandtl's early work in this theory shows the difficulty he faced when trying to link spanwise circulation to wing downwash distribution using the Biot-Savart law³, which describes the velocity induced by a vortex filament. Furthermore, the lifting line

³Von Kármán looks back at Prandtl's effort to find the mathematical sleight of hand that would give him the solution: "I am calculating these damned vortices and can't get a reasonable result for the induced drag. I tried to make the lift suddenly drop to zero at the wing tips, but the induced velocity becomes infinite. All right, I thought, Nature does not like such a discontinuity, so I made the lift increase linearly with the distance from the wing tip. That did not work either. This distribution of lift also does not produce finite induced velocity at the tip." [26]

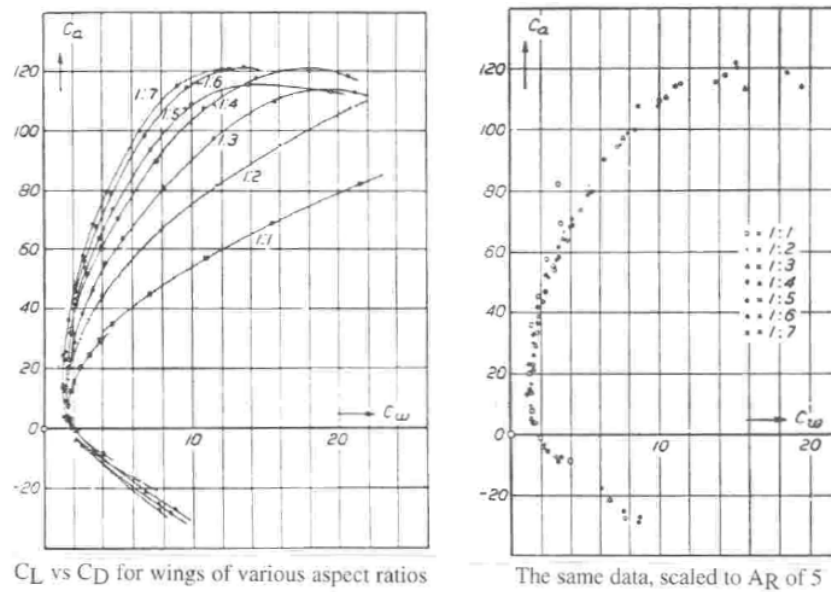


Figure 2.2: Prandtl's results [6] for the lift (C_L) and drag (C_D) of finite wings. Extracted from Reference [6].

theory is formulated in terms of easy-handling integral equations, which are typically solved through collocation methods consisting of assuming a finite sine series for the circulation distribution, where the Fourier series coefficients are determined by imposing the lifting-line equation to be satisfied at a number of spanwise locations. The simplicity of the resolution, along with the rapid estimation and low computational cost it demands, builds on to the fact that it is still extensively used in the conceptual design of wings. This fact is most remarkable given the considerable number of assumptions, which lead to as many limitations. The wing is modelled as an unswept lifting surface of relatively high aspect ratio, with no dihedral, and immersed in a steady, incompressible, inviscid, irrotational flow at small angles of attack. The method does not consider airfoil thickness and computes an infinite downwash at the wingtips.

For this and other conceptual and practical considerations, the profound relevance of lifting line theory has led many to extend the methodology in order to relax the limiting assumptions upon which it is based and thus increase its range of application. However, thorough description of the extensions made towards the generalisation of the theory would be lengthy and is not directly relevant to this work; but it is worth mentioning some of the capabilities available that provide a satisfactory level of accuracy for conceptual design stages. Many methods are available to deal with the classical lifting-line equation [33–35]. Additionally, it can now be applied for the study of curved, swept or other arbitrary-shaped wings [36–40]; it can consider unsteady flow conditions [41–44] and integrate nonlinear data [45–47]; or include compressibility effects [48–50].

The legacy of the lifting line theory expanded along with the increase of computational capabilities. Hence, numerical methods stemmed from the lifting line representation, to lifting surface methods. Here, a lifting surface is modelled with a lattice of horseshoe vortices, where the lift is not concentrated along the quarter-chord of the wing anymore, therefore allowing chordwise variation of vorticity and yielding the vortex lattice method [51] (VLM). The number and distribution of horseshoe vortices will correspond to the required resolution of the pressure distribution, and which will affect the accuracy of the resulting aerodynamic coefficients.

2.2 Theoretical Shift to Aero-Structural Efficiency

Despite the accomplishments Prandtl achieved with his aerodynamic model, he continued exploring the mathematics behind his method, the equations of which could yield diverse solutions. However, the design concepts that Prandtl and his students wanted to predict always considered a wing with fixed span. With this constraint, the elliptic curve would always be the optimal spanwise loading.

This constraint was practical when considering, for instance, operational limitations. However, he still believed it unseemly to consider the elliptic spanload as the only optimum. Therefore, Prandtl reformulated the problem for a given design flight condition: knowing that increasing aspect ratio reduces induced drag, he enquired whether it was possible to extend the wing while minimising the increase in structural weight. To do so, the integrated bending moment due to spanload was prescribed as the parameter driving structural weight, while the span was allowed to extend and the wing area remained fixed.

In 1933 Prandtl presented the generated family of curves that would yield the spanload for minimum induced drag and structural weight at each span extension. While the half ellipse still provided the ideal aerodynamic characteristics for the baseline wing, the other solutions would yield NELDs, of which the optimal curve would have a bell-shaped form⁴. The latter provided an 11% induced drag reduction with a 22.5% span extension, therefore minimising induced drag while maintaining the same structural weight as its equivalent wing with elliptic loading.

The results of this approach are shown in Figure 2.3: curves *a* show the well-known elliptic circulation and constant downwash distributions, which agree with the earlier solution derived from the lifting line theory, as there is no span extension. Curves *b* show an intermediate case given by an arbitrary span extension. Curves *c* show the

⁴Hereafter, the author refers to the bell-shaped lift distribution as the optimal non-elliptic curve for minimum induced drag and structural weight corresponding to each of the theories. The term NELDs will be used to designate any of the solutions from the general family of curves derived from this approach.

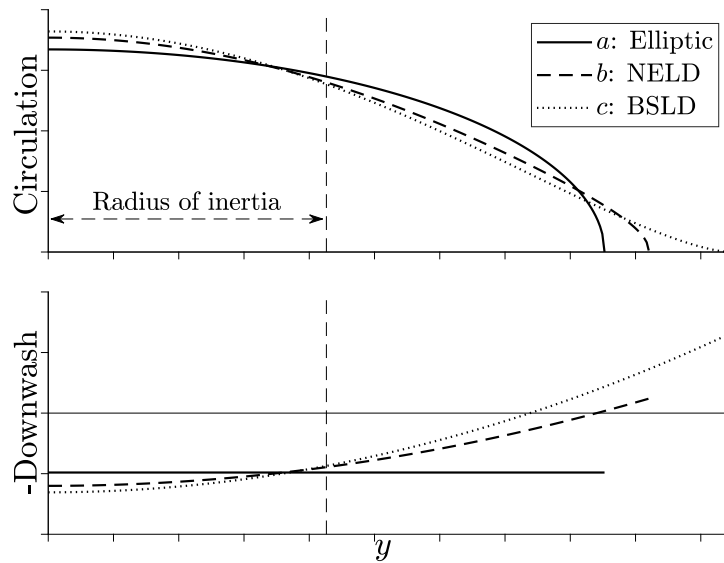


Figure 2.3: Reproduction of Prandtl's 1933 solutions for circulation and downwash distributions [7].

optimal BSLD. With this, Prandtl elegantly found an alternative wing design approach that would suit both operational and structural design requirements at a conceptual design stage. The solution could be derived analytically and would provide minimum induced drag for a given design point.

The ideal elliptic solution had been acclaimed for years by the time this wing design approach was released and, at the time, there was a greater interest in the design of robust aircraft rather than in the accomplishment of optimally coupled aero-structural designs. Moreover, the year 1933 was a fruitful time for Prandtl, who published five other papers [31], while Göttingen was immersed in times of turmoil⁵. It could be deduced that these facts contributed to the paper in question never being translated officially into English, and therefore hindering its dissemination. This might be the reason why Jones published a similar solution to the same problem in 1950 [52] without citing Prandtl's publication. A confidential translation from the Aeronautical Research Committee was produced in 1935. Oddly enough, it is only recently that an official English translation of Prandtl's 1933 paper was published by Hunsaker and Phillips [53].

2.2.1 Other Classical Approaches to NELDs

The trade-off between drag and structural weight is of major importance in wing design, and many others tried to derive similar theoretical models to address this. In

⁵A whole chapter is dedicated to the adversities that Prandtl faced in the year 1933 in one of his bibliographies [32].

1950, Jones [52, 54] supported a design philosophy for practical wing design driven by structural requirements, and addressed it independently by prescribing the wing root bending moment. On a later revision on induced drag prediction published in 1979 [54], Jones expressed concern about the simplicity and limited accuracy of wing and wake models used at that time. Nevertheless, after almost three decades, he still recognised the benefits that the principles of NELDs can provide in terms of structural efficiency, referencing Prandtl's 1933 work and acknowledging the work of Klein and Viswanathan.

Klein and Viswanathan also explored the added value that NELDs can provide in terms of aero-structural efficiency. Their work insisted on the importance of incorporating structural considerations in the aerodynamic design of wings as a way of coupling both design limitations into a single theory. They first derived a solution providing minimum induced drag for a prescribed wing-root bending moment [55] instead of the integrated bending moment. Interestingly, this proposal was built upon Prandtl's 1933 paper alone and with no reference Jones' work, and could be applied to higher aspect ratio wings with relatively small sweep. Later, their concern on the lack of research effort aimed at induced drag minimisation took them to expand on their previous work. They updated the structural boundary conditions to include a more precise approximation of the wing structural weight [56, 57], in which the constraints were now determined by the integrated bending moment and shear force distributions.

In 1979, DeYoung [58] was inspired by the advance in both structural design and computation, which could open opportunities to realised unconventional aircraft configurations. With the widespread interest in performance, it was the right juncture to provide the means for designing under structural and performance constraint conditions. He collected all the preceding work and derived solutions for spanwise loading with minimum induced drag subjected to constraints on the wing bending moment fixed at an arbitrary span position. His methodology expanded the range of applications to include non-planar wings and consider multi-wing aircraft. Hence, the design philosophy of wings with NELDs could be applied on wings within ground effect, in formation flying or banked, and several planform shapes such as closed wings, wings with winglets, or cruciform wings, and the approach could be useful for the design of truss-braced wings.

The prior work exposed presents a rich history of proposed methodologies focused on optimising lift distributions for minimum induced drag and optimum structural weight. The works from Prandtl's 1933 paper to DeYoung's approach in 1979 can be now considered classical theories in this field, which are set apart by the variations on the constraint used to account for structural weight. The literature emphasises that the optimum wing lift distribution is a conditional problem, the solution of which is not

unique, but attached to the assumptions required by the design criteria adopted. All the solutions favour an inboard shift of the concentrated lift load relative to the elliptic spanload to allow larger span extensions and reduce induced drag. Table 2.2.1 lists the optimum induced drags achieved by these solutions relative to the elliptic standard. The mathematical derivations relevant to this research are presented in Sections 4.1.2 and 4.1.3.

Table 2.1: Comparison of span and drag ratios derived from the application of structural constraints over an elliptically loaded wing.

Reference	Constraint	Induced drag	Span
Prandtl 1933	Integrated bending moment	0.89	1.22
Jones 1950	Root bending moment	0.85	1.15
Klein and Viswanathan 1975	Integrated bending moment and shear force	0.93	1.16
DeYoung 1979	Bending moment at arbitrary location	0.90	1.07

These early theories for aero-structural wing design are insightful and elegant approximations that can be employed as a prelude to the conceptual design phase. However, analytical consideration of all the potential variables would be an extremely difficult task, if ever possible. To solve the problem, the classical formulations must therefore assume several simplifications to derive an analytical solution.

2.2.2 Extensions and Limitations

Further work has been carried out to extend the above theories in the combined field of induced drag and structural weight minimisation to include additional design and mission considerations in the early stages of the design. Approaches to this problem include closed-form solutions, semi-analytical methods as well as numerical optimisation.

Besides the premises carried by prior theories, one of the main limitations of these early approaches arises from the determination of wing loads. In all cases, the structural constraint is set at the same flight condition at which drag is minimised: cruise flight. Nevertheless, structural strength requirements must be met at all regions of the flight envelope to ensure structural integrity. Not only it is a critical design driver, but its omission can also bring dramatic induced drag increase due to the sensitivities of the spanload to off-design conditions. The large washout needed to yield the traditional non-elliptic spanloads penalises performance during high load manoeuvres, since twist has a direct impact on lift coefficient. Similarly, to select a critical load condition as the design point will also increase drag considerably during cruise. These considerations have been addressed by many via extensions of the theories presented above. A more practical approach would allow the minimisation of induced drag at a

different lift coefficient at which the bending moment must be prescribed [59] in order to reduce off-design penalties while alleviating wing structural loads during high load manoeuvres [10,60].

Moreover, the choice of aerodynamic and structural models alters the trade-off between span extension and wing structural weight, and impacts the proportionality coefficient between bending moment and weight [61]. The structural models applied in the early works are crude approximations, where a spar with constant section is the only supporting structure and any non-structural weight contributions are disregarded. This is a useful approximation when employed within the frameworks of fast and manageable conceptual design tools. However, the spar is not the only structural component of the wing, and it usually carries a significant portion of the aircraft's total weight, such as the engines, fuel, and its system components. Taking this into account will affect the bending moment distribution and consequently, the optimal spanload. However, more accurate models for wing weight can be embedded into an analytical approach. For a simple beam, the local integrated bending moment divided by the section thickness [62] can capture the variation of material properties carried along the span due to changes in chord or thickness, which would affect the wing box strength and mass distribution [63]. The root bending moment can be a misleading indicator when winglets are being considered, because they can be designed with no apparent effect, or even negative contributions, to the root bending moment. Furthermore, it does not consider any changes in chord length, which will induce higher bending weight on the structure [64].

The inclusion of spanwise aerofoil thickness as a variable provides further insight on various off-design considerations. It is a determining factor in the relation between weight and bending moment, and in turn is a function of the wing span and area, becoming a crucial aspect when total drag is considered. Hence, when thickness-to-chord ratio is taken into account to also minimise parasitic or compressibility drag, it will result in a decreasing thickness-to-chord towards the outboard wing, which must be limited to avoid unreasonably small thicknesses. At the same time, larger thickness-to-chord will increase the maximum lift coefficient, allowing a reduction in wing area and therefore profile drag [65]. If the thickness-to-chord is held fixed, chord extension would allow for a larger structural box that would increase the wing structural strength while reducing section lift coefficients, and in turn reduce compressibility drag. Compromise becomes evident and more complex. Tapering the wing will provide higher reductions in total drag but will simultaneously yield an exacerbated stall performance due to higher lift coefficients at the wingtips [64]. Profile drag is susceptible to section lift coefficient, and becomes more significant as speed increases. Including this term into the minimisation objective prevents high section lift coefficients and therefore precludes the wingtip from having very small chords [66].

The use of NELDs might face further limitations in transonic flight regimes. Here, compressibility effects are unavoidable and aircraft designers invest great effort to address such phenomena. Existing methods focus on cross section and planform design to increase the speed at which drag divergence occurs while simultaneously weakening the shock strength. These include supercritical aerofoils and wings, laminar flow wings, bodies shaped under the Whitcomb area rule, and, of course, the established wing sweep. Wing warp is another design approach that is gaining interest due to the growing MDO capabilities [67,68]. Here, twist and camber distributions have proved to be significantly more critical than spanwise thickness; and these parameters are key features of wings with NELDs. These designs require a non-linear downwash that is commonly achieved through a bell-shaped twist distribution, yielding a washout that can be notably higher than that found on conventional wings. Such a characteristic wing twist can also have a negative impact in total drag and override the benefits achieved through span extension [69]. Alternatively, the same effect could be achieved through careful design of camber distribution. However, these requirements could lead to an increased likelihood of shock waves and drag onset at both design and off-design conditions.

Wing area is another variable that will compromise the results, depending on whether it is fixed or not, and depending on whether we refer to the projected or the wetted area. Many studies prescribe the projected area when the objective is to minimise induced drag alone. Instead, total drag minimisation would intuitively lead to a reduction of the total area and in turn reduce the area-dependant structural weight, although consideration of other limitations such as stall speed would neglect this kind of solution [61,66]. Alternatively, one could prescribe the wing loading as a critical performance requirement [13], yielding an alternative set of design compromises [10]. In any case, increased aspect ratio will be a consequent result of any of the discussed optimisation paths. This will cause the total wing weight to increase and lead again to a trade-off between the optimal wing span and area [70]. Moreover, the resulting high aspect ratio together with the required non-linear twist distribution increases the risk of encountering adverse aeroelastic phenomena [71], which has been assessed through its impact on wing deformation [72] and with consideration of aeroelastic feedback on the bending moment constraints [61,64,65,70].

Emphasis must be kept on span extension to achieve aero-structural optimisation under the assumptions of the above methodologies. Past work [73–75] that did not take into account a span extension did not see significant improvements in performance. Weight savings can be achieved on a fixed wing by exchanging an elliptic spanload for a NELD as the load is shifted inboard. A wing with fixed span that shifts the load inboard to reduce bending moment will always see an increase in drag: the high circulation at the root required to match the necessary amount of lift will develop higher induced

drag due to the quadratic relationship between these two variables. At the same time, the wing twist necessary to produce such spanload might increase the parasitic drag. Hence, without a span extension, weight savings will be rapidly overcome by drag penalties as the spanload deviates from the elliptic and the span efficiency factor reduces. In practise, however, it is not always feasible to extend the span on an existing aircraft. When operational aspects are considered, weight reduction at expense of induced drag becomes a debatable trade-off. The optimal solution for such a problem would be given by the spanload that provides the desired structural weight reduction with the smallest increase in drag. But this would not necessarily improve the aircraft's performance. This approach to aero-structural efficiency does not essentially represent Prandtl 1933 proposal.

Alternatively, such an approach could be chosen if the design trade-off is to be made within the traditional vision of wing equivalence. ESDU methods [76,77] define that two wings will be equivalent if they have equal exposed wing areas, tip chords and fixed gross spans. But this definition arises from the fact that majority of available methods to predict wing aerodynamic properties are predominantly based on tapered planforms. Thus, this guideline fulfils the need to make allowances for shape variations, such as cranks, and its implementation is done through the geometric representation of such wings in their equivalent tapered form. However, it could be argued that this traditional definition, which is only a function of geometric parameters, is not suitable in the context of NELD. Instead, it appears more sensible to define wing equivalence in relation to the parameters that are to be compared. NELD wing designs are aerodynamically and structurally equivalent because they yield the same prescribed aerodynamic and structural properties⁶. Wing equivalence might therefore become a malleable criterion that is shaped to provide the appropriate comparison required in each analysis⁷, but should always be properly and clearly defined to avoid confusion or inaccuracy.

Now, assuming that span extension remains inherent to the analysis, aeroelasticity is another important factor that will impact the design of wings with NELD. The resulting wings with high aspect ratio and non-linear twist can be more prone to adverse aeroelastic phenomena. There are simplified approximations that provide insight into

⁶From now on, the author will term this concept as aero-structural equivalence.

⁷We could argue that there is further more to the diversity and flexibility of the concept of equivalent wing. The concept of similitude is widely used in aircraft design to relate the measurements from one system to describe the behaviour of another similar system. The choice of similitude requirements depends on the specific observations of interest, which may call for geometric, kinematic, or dynamic characteristics. For instance, the application of similitude through dimensional analysis enables the scaling of aerodynamic properties for wind tunnel testing is another form of equating two aircraft that are geometrically different but must remain equal physically. Additionally, dynamic and aeroelastic equivalence can be also attained in terms of dynamic scaling principles via the Buckingham Π theorem [78].

the probable outcome of this scenario, observing the impact on wing deformation and aeroelastic feedback on the bending moment constraints. For instance, if maximum deflection is prescribed in a rectangular wing, together with a fixed lift, total weight and wing loading, the resulting optimal spanload is closer to the semi-ellipse than to the bell-shaped curve [10]. Aeroelastic effects become more apparent at higher subsonic speeds, which induce the need for swept wings. Consideration of sweep as a variable at such conditions should then demand the inclusion of aeroelastic feedback in the analysis [64]. The elastic twist will have an impact on the bending moment, which in turn will modify the permissible span extension [61]. The reduction in effective angle of attack due to wing deflection could be compensated through area increase to achieve the target optimal spanload [72]. Moreover, these effects will be magnified proportionally at high load conditions. Examination of the static aeroelasticity characteristics might bring more critical limitations than other structural constraints, but it can also provide significant weight savings due to alleviation of the outboard wing [65]. For the same reason, skin gauges could be reduced when regarded as another structural limitation of an elastic wing [70]⁸.

Admittedly, the impact of non-aerodynamic and off-design considerations in the design of efficient wings is extensive. The importance of constraining further degrees of freedom is becoming definite from past design studies, which have included aerofoil thickness, chord and twist distributions, as well as sweep and other operational design variables. Significant changes in the optimal shape can be introduced by considerations of aeroelasticity effects, high lift performance, fuel distribution, and landing gear integration. Further research has been carried out in this context to include additional mission and loads considerations in the early stage of the design. These include the addition of constraints such as chord distribution and wing deflection [10, 72], designing washout-optimised wings [79] or multiple leading and trailing edge control surfaces [80] on aeroelastic wings. Other research has focused on addressing the significant nose-down pitching moment induced by high subsonic speeds aerofoils, instead of compensating with the tailplane inefficiently [81]. Furthermore, researchers have incorporated viscous drag [82], and investigated on the minimisation of induced drag during turn [83–85]. An analytical approach for trimming and performance coupling at different stages of the flight mission [82] was also developed. Finally, the reduction of wing weight during flight due to fuel consumption [60] has also been considered.

⁸Another observation found in the literature points out the potential benefits of forward swept wings in terms of aero-structural efficiency. However, this type of wings will not be favoured by the aeroelastic bending and again the trade-off between the advantages and disadvantages of such unconventional designs will come into play. The reader is forwarded to the references [62] and [61] for a further discussion in this topic.

Discussion of this wide variety of approaches and compromises exhibits the futility of presenting a unique optimal solution to the problem of aero-structural efficiency; or in fact, to any multivariable problem. Conversely, the broad spectrum of answers substantiates the removal of conventions subject to the elliptic solution to obtain further optimal designs. These methodologies can provide a more efficient baseline upon which to build, which will have consolidated further critical aspects of wing design rather than just induced drag. Any of the cited references provide methods that allow the inspection of the intertwined dependencies and sensitivities between each of the variables considered. Analytical methods will elegantly clarify the trends of the main trade-offs, while numerical optimisation will cope with the relative assessment of even more degrees of freedom.

2.2.3 Planar vs. Non-planar Span Extension

The compromises exposed in Section 2.2.2 have been likewise discussed extensively in the design of wings with winglets. Wingtip devices provide the means to achieve similar aerodynamic improvements to those gained by increasing the wing aspect ratio, but also entail many of the trade-offs discussed above [8, 86–90]. As is well known, the criteria to meet minimum induced drag and minimum structural weight are in contradiction with each other. This becomes even more evident in the design of winglets, since they are usually devised as an addendum to the wing, as opposed to the integral re-design approach used in the above theories. For the purpose of this discussion, it is necessary to differentiate between: tip devices designed to be affixed to an existing wing, either (1) in-plane or (2) out-of-plane, and (3) span extensions designed in the form of an equivalent extended wing. Hence, the the discussion that follows will be comparing these three different elements.

It was Whitcomb’s 1976 publication [91] that set the benchmark for the development and implementation of wingtip devices that we see today, specifically in commercial aircraft. In particular, winglets, a term coined by Whitcomb himself and inspired by soaring vultures and eagles [92], provide an increase of 6 to 9% in overall lift-to-drag ratio, and reduced induced drag by approximately 20%, according to Whitcomb’s report. In response to the 1973 oil crisis, such benefits translated into the necessary economic relief that the aerospace industry was seeking due to the consequent reduction in fuel burn⁹. Whitcomb realised that the crossflow around the wingtip could be manipulated to reduce the energy dissipated at that region, while increasing the

⁹Nowadays, it is possible to quantify the benefits brought by the implementation of winglets. Taking winglet technology of Aviation Partners Boeing as a case study, the implementation of their designs amounts to savings of more than 10 billion gallons of jet fuel and has suppressed the emission of 105 million tons of carbon dioxide into the atmosphere. These numbers were announced as of September 2019 and conglomerate the savings achieved since their first winglet design was patented in 1994 [93].

effective aspect ratio of the wing. Such manipulation would effectively reduce the strength of the vortex and the resulting drag force. Moreover, winglets appeared to be a more structurally optimal solution than simply extending the wingspan horizontally. Generally, horizontal tip extensions can provide higher reduction in induced drag compared to winglets of similar size. However, tip extensions will also induce higher loads on the wing structure, requiring further structural reinforcement, in addition to the potential operational limitations. In this sense, Whitcomb quantified that for the same penalty in wing root bending moment, tip extensions would yield half of the induced drag reduction that could be achieved with a winglet [91]. Whitcomb's results provided the fundamentals for the design of a device that would require almost as much engineering effort as the wing itself. In any case, careful design of twist and camber is necessary to extract significant improvements from the integration of this device [94–98].

The spanload carried by the basic wing will greatly impact the effectiveness of a retrofitted device. If a given wing already carries a fairly efficient load distribution, only the implementation of very large wingtip elements (measuring up to 60% of the span) would be able to provide significant improvements in induced drag, which would be overthrown by the associated increase in weight and wetted area. Winglets further enhance the performance of wings with highly loaded outboard sections [8,91]. On the other hand, a basic wing with increased washout will see small differences in induced drag decrements between vertical and planar extensions [97]. This type of baseline wing design is currently found in commercial transport aircraft to reduce bending loads at the expense of induced drag increase. Such a configuration is closer to the geometry of a wing with non-elliptic spanload. Yet, affixing a wingtip device will always bring more structural penalties than designing the extension as an integral part of the wing.

Jones [89,90] extended Prandtl's 1933 theory to include non-planar span extensions and compared the resulting benefits to those achieved by in-plane extensions. Hereby, a winglet whose height is 20% of the semispan can achieve reductions in drag of approximately 10% without increasing either the horizontal span or the structural weight of the wing, through the assumption of NELDs principles. The same reduction can be achieved by a 10% planar span extension. This design approach thus yields no real difference between equivalent wings with planar or vertical extensions in terms of aero-structural efficiency. The main compromises will appear through other design or operational considerations, such as the impact on the pitching moment, the dihedral effect, aeroelastic implications, or other forms of drag [99].

Consideration of off-design conditions can, however, establish preferences. A more recent study [63] shows that if the bending moment is constrained at its design limit load, winglets appear more advantageous than tip extensions. In this case, the design

method handled a more accurate weight model for the optimisation of the new wing planform, where the dihedral of the tip section was allowed to vary. To reduce the number of degrees of freedom of such multi-variable problem, the definition of the wing twist was limited to five sections and varied linearly between each section.

Later investigations [8, 66, 100–103], some of which were carried-out with higher-fidelity methods, agree with Jones' [89] results that winglets have essentially the same performance potential as planar span extensions in terms of aero-structural efficiency when the baseline structural weight is constrained in an all-new wing design. This can be observed in Figure 2.4, which shows the resulting aero-structural balance of various planforms optimised in ensemble with various wingtip devices using Boeing's WINGOP [8, 70] design tool. According to this case study, tip feathers were the most advantageous design. Proposed biomimetic wingtips require thorough understanding of the conditions and purpose of the mirrored element. Simply splitting the outboard wing might not be sufficient to reshape the spanload, but with careful design, a split wingtip can achieve the same induced drag reduction as a standard winglet with a smaller additional wetted area. On the other hand, the raked wingtip, which features a planar extension, has shown to be up to 1.5 times [104] more efficient than a conventional winglet for aircraft such as the Boeing 767-400ER, the Boeing 777-200LR/300ER/Freighter or the Boeing 777X. One of the shared aspects of these aircraft is that they are designed for ultra-long range. Raked tips yield superior performance at cruise, where an ultra-long mission can compensate for the additional weight that a folding mechanism would bring. Other non-planar tip devices are more efficient in climb and therefore are more commonly found in short range mission aircraft.

A good aerodynamic design practice to avoid large increases in profile drag appears more critical in the development of winglets than in planar tip extensions. Early designs would yield a higher interference and profile drag rise than would reduce induced drag. Some of the current designs have blended the intersection between basic wing and wingtip device to achieve a smooth transition that reduces the chances of flow separation [93] in exchange for some loss of induced drag reduction. Standard design practice, which is essentially represented by retrofitting, requires an increase in wetted area that directly increases profile drag. This can be addressed for all-new designs by fixing the wetted area of the entire wing configuration. Such a constraint would keep the profile drag and area dependent weight roughly constant. As discussed in Section 2.2.2, the resulting smaller chords will yield higher bending weight. The integration of a winglet under these considerations would also imply a reduction in the projected area, yielding in turn a negative impact on maximum lift coefficient and stall characteristics. On the other hand, fixing the planform area would have the opposite effect. Hence, keeping either parameter constant will not necessarily simplify the identification of the contrasts between planar and non-planar wing extensions [64].

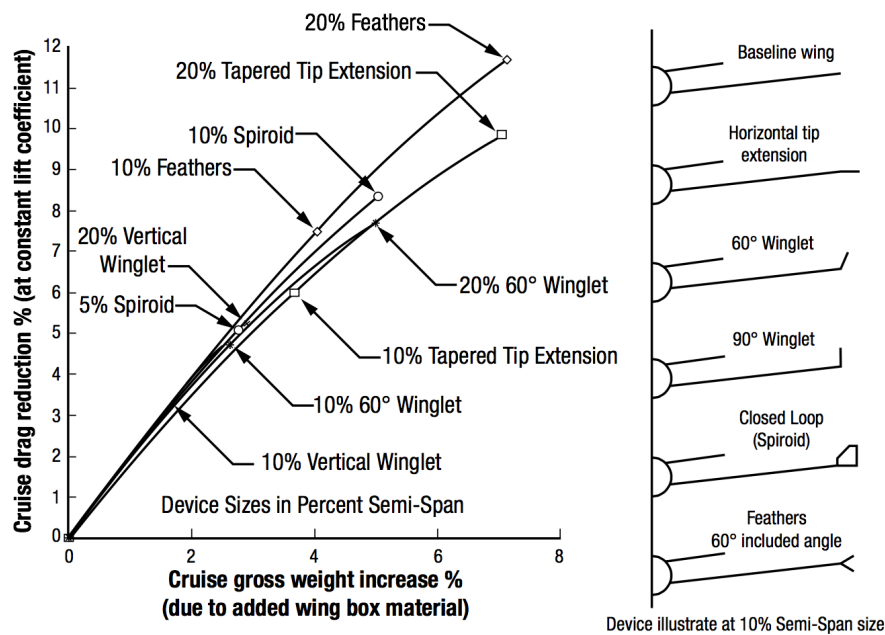


Figure 2.4: Comparison of different wingtip devices for the API blended winglets on 737NG, derived from CFD-based drag predictions and flight tests. Obtained from Reference [8]

Again, design for performance becomes a trade-off between aerodynamic and structural features, which will in turn determine the compromise between weight and efficiency, along with handling characteristics. Thus, the advantage of the equivalent extended wing lies in the fact that induced drag is reduced while minimising structural penalties. The value of wingtip technologies excels in their use as retrofitting devices, retaining the main aerodynamic and handling characteristics while avoiding span increase or the whole redesign of the wing. As discussed, the type of wingtip device that will best suit a given aircraft will depend on various factors such as the spanload carried by the baseline wing, airfield gate or manoeuvring limitations, the convenience for wing structure modifications, as well as the type of design mission of the aircraft. Moreover, span cannot be increased arbitrarily, neither in-plane or out-of-plane, detached of any penalty, although there could be cases in which the fuel savings derived from the reduction in induced drag could neutralise the weight increase. Generally, to bear these and other concessions in mind is a fundamental principle of any design map, in which the necessary prioritisation of any of these options will tip the balance towards either a conventional result or an innovation. This will become even more apparent with the implementation of artificial intelligence technologies, for which the designer will become primarily a decision maker.

2.3 The Challenges of Induced Drag

2.3.1 Definition and Methods

Any finite wing that is generating an aerodynamic force has a flight-direction component associated with it as a result of the induced velocity of the trailing vortices [20]. This force is equivalent to the work done on the flow, which appears as the kinetic energy of the trailing vortex system [105]. This is the means through which momentum is transferred to the fluid [106] as a reversible phenomenon. This equals the rate of increase of downwash momentum, which, in turn, is equivalent to the downward impulse in a unit section of two-dimensional flow downstream [107].

There are many definitions for this force, and a few variants of its denomination: drag due to lift, vortex drag, or simply *induced drag*¹⁰. In the same way, there exist many methods to estimate its magnitude, and just as many others to minimise its presence.

Generally, drag forces manifest themselves by altering the balance of momentum and pressure, while conservation of energy is achieved by an increase in heat and kinetic energy. The latter is particularly specific to induced drag, which is associated with the motion of large air masses surrounding the wing and characterised by a downwards motion perpendicular to the direction of flight. This phenomenon can be treated in both the near-field and far-field planes. The near-field requires surface pressure integration, and the far-field approach necessitates wake contour integration in the Trefftz plane [109]. Each of the methods carry their corresponding assumptions and limitations. The problem is exacerbated by the fact that any attempt of validation requires a separation of total drag into its subcomponents, impossible to do with current flight test techniques.

Drag prediction is a challenging task. Theoretical derivations differ in their physical conception but work accordingly within theoretical fluid mechanics, and have been used in the analysis of planar and non-planar lifting surfaces [110–112]. Analytical methods, however, do not always address the accuracy of their predictions [113–115], although various methodologies have achieved improved agreement [116, 117], while also considering higher order terms neglected in the classical theories [118]. Earlier methods for approximate solutions still show favourable predictions when compared to later procedures developed for accurate computation of minimum induced drag [87, 94, 119]. These discrepancies suggest that previous theories assumed excessively simplified boundary conditions. Lifting surface theories such as the vortex-lattice

¹⁰This is how Munk [108] coined the portion of thrust that would develop into downwash to keep the aircraft up. Prandtl justified it through the similarity of the phenomena happening on the wings to the physics around electric conductors [6]

method include chordwise and spanwise effects, and panel methods also consider wing thickness. However, induced drag predictions from these latter methodologies are usually sensitive to discretisation [120–123].

Although immensely brief, this overview on induced drag intends to recall the complex phenomena that we are trying to address. The methods and tools available have been successful in its estimation, but there exists a persistent strive to find the right mathematical approach to translate such physical events into an expression that allows further understanding and manipulation. For more detailed discussions on induced drag, the reader is directed to the references [124–130] .

2.3.2 Induced Drag in NELDs

Countless efforts have been made to improve aircraft performance through induced drag reduction. Nowadays, most solutions deviate the spanload from the elliptic ideal, which suggests that our baseline theory is not the most efficient in real flight. Recent trade-off concepts have led to incremental but successful gains in performance, including planform optimisation, highly non-planar surfaces, and diverse tip devices. Still, this undeniably leads to compromise, either penalising on weight, increasing systems complexity, or reducing internal airframe space, and it is to the judgement of the designer to determine which sacrifice will be less detrimental to satisfy the mission specifications. Nevertheless, non-elliptic spanloads seem to promise a more favourable deal in the trade between induced drag and weight.

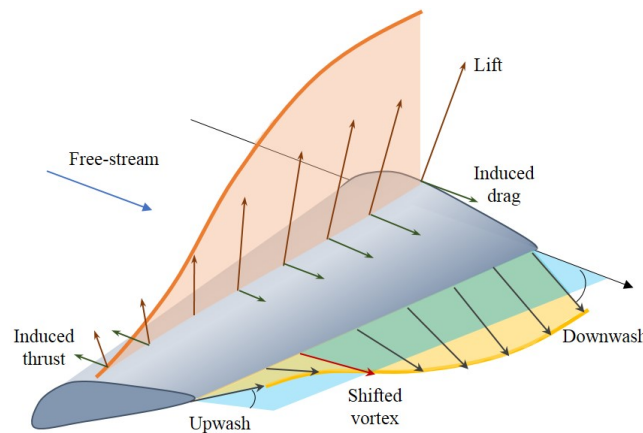


Figure 2.5: Representation of the aerodynamic forces on a wing with non-elliptic lift distribution. Reproduced from Reference [9].

The optimal bell-shaped curve is associated with a span extension, which reduces induced drag. To relieve the increased structural loads derived from the extended span, the centre of lift is shifted towards the root. Effectively, the lift is redistributed to be locally high at the inboard wing and have lightly loaded tips. Here, upwash is

developed, generating local negative induced drag, or induced thrust. But induced thrust, as many experts say, is one of those things that one would rather sweep under the carpet. The division between the concepts of drag and thrust may vary with the reference frame or context used, and such a divergence is typically subjected to theoretical simplifications. The projection of aerodynamic forces into the flight direction is classified as drag, and excludes thrust forces produced by the engine. Within this framework, negative induced drag could be understood as a propulsive self-induced aerodynamic force, and could be represented by the sailboat analogy used in the context of winglets. This remarkably popular definition of induced thrust depicts the winglet as a sail that takes advantage of the distorted inflow at its tip to produce lift, in the same manner as a sailboat navigates through the sea. The net resultant force will sometimes have a forward component in the reference frame of the moving body. This is occasionally identified as thrust. Although it can be a matter of nomenclature convention used to simplify the explanation of this phenomena, it is just lift and drag forces generated locally on a portion of a lifting surface. But it is misleading to state that winglets use the energy from the wingtip vortex to reduce induced drag by generating thrust. A winglet will not simply reduce the strength of the wingtip vortex, its presence will alter the three-dimensional flow-field at the wingtip as projected on the Trefftz plane and will redistribute the whole spanload so that its expended kinetic energy is reduced. Drag can be regarded as the contribution from the dissipative effects of the flow, and induced thrust could be assigned as a contribution to propulsive forces. However, fuel burn will be reduced not due to the apparent thrust generated by the winglets, but because the modified spanload will expend less energy. Winglets will combine an appropriate toe angle, aerofoil camber and height for the wing to develop optimal loads. Likewise, wings with NELDs redistribute the three-dimensional flow-field by means of a careful wing twist design. A similar outcome is therefore obtained, where apparent thrust can be found at the outboard wing. Horten [131] explained it as the induced propulsion at the wingtips shown by bell-shaped lift distributions, which is favourable with aileron deflection. Bowers [9] describes it as the resultant force locally tilted upstream as a consequence of the induced upwash, represented in Figure 2.5.

The treatment of total forces intuitively simplifies the isolation of local contributions, where negative induced drag will be subtracted from the total drag and never added to the engine thrust. However, even this distinction is penalised by, for instance, the presence of spurious drag in high-fidelity methods, and the classification of drag-producing mechanisms becomes even more complex [132–134]. Furthermore, this problematic dichotomy can reach the fundamental definition of induced drag, as given by Equation 2.1. Thus, arisen from dissatisfaction with the underlying theory, some have tried to demonstrate or disprove mathematically the existence of negative

induced drag. Spalart [135] raised essential questions on the analysis and evaluation of induced drag as defined in the Trefftz plane due to the paradoxical negative squared term of the perturbation velocity u^2 along the wake in the following equation:

$$D_i = \frac{1}{2}\rho_\infty \int_{S_{far}} (v^2 + w^2 - u^2) dS \quad (2.1)$$

whereas intuitively, the integral for induced drag should contain the expression for kinetic energy given by the perturbation velocities (u, v, w) . Such a definition appears to allow the prediction of negative induced drag, since the integrand is not positive-definite, and thus the perturbation along the wake could overcome the velocities perpendicular to it. Moreover, this expression adds complexity to the frequently used formulation that only considers the perturbation of kinetic energy within the plane perpendicular to the free-stream

$$D_i = \frac{1}{2}\rho_\infty \int_{S_{far}} (v^2 + w^2) dS \quad (2.2)$$

This simplification is derived from the assumption that the trailing vortices far downstream cannot induce velocities along their own axis, which now follows the free-stream direction, and thus any parallel velocity perturbation must fade. Spalart provides a rigorous assertion that removes the ambiguity in the negative sign of u^2 , while deriving a mathematical proof for consistent positivity of induced drag within a set of idealised but generally valid assumptions. Yet, he believes that a concise mathematical and physical description of induced drag is still unconcluded. Alternatively, Drela [136] presents a control volume approach in which drag and thrust forces can be regarded indistinguishably. The analysis focuses on the evaluation of mechanical power and kinetic energy of the compressible viscous flow around an aircraft, as opposed to Spalart's force-based analysis. Drela's approach shows that the negative u^2 term in Equation 2.1 originates from the pressure work done on the fluid crossing the Trefftz plane. This contribution is negative and twice as large as the wake axial and transverse kinetic energy terms. Altogether, this conforms the three components of the energy of the potential flow region of the Trefftz plane, which reduces to the classical expression for induced drag in Equation 2.1 multiplied by the airspeed.

The complex formulations can be derived into simple and desirable results, for which physics can be justified through reasoned language, but which do not fully comply with the mathematical grounds in which they are established. This was the case of a particular approach to Prandtl's small-perturbation wing theory from a force-impulse balance perspective [107, 137–139]. Here, a particularly provocative outcome resulted in the static pressure at the Trefftz plane being greater than the atmospheric

pressure, which would produce small forward increments of velocity in the wake. Thereof, concepts such as forward flow of energy or wake buckling were derived. Although it is counter-intuitive for such a phenomenon to occur given the association of induced drag with kinetic energy, it is possible to find regions of positive perturbation pressure behind a lifting wing. However, a combination of an unsuitable assumptions had led to this surprising interpretation of conditions downstream [135, 140, 141]. It is clear that any appearance of negative induced drag is usually received with curiosity and scepticism, as it ignites the need to give a reason for and resolve this oddity, regardless of how desirable its existence may be.

2.3.3 Wing Wake, Downwash and Upwash

The above discussion conveys the need for accurate prediction of downwash and upwash distributions to estimate the aerodynamic characteristics of a finite wing. Furthermore, the problem of induced drag minimisation could be posed as the manipulation of the flow-field around the wing, since induced drag can be viewed as a by-product of the downwash associated with the vortical wake.

The downwash distribution induced by NELDs is determined by the degree of the functions that compose the problem conditions, which is quadratic for an integrated bending moment constraint and linear if the root bending moment is prescribed. The gradient of the bell-shaped circulation distributions approaches zero at the wingtip, which features a weak circulation strength, and sheds stronger vortices inboard. With this, it can be interpreted that the downwash at the vicinity of the wing root can overcome the downwash at the wing tip, transforming it into upwash [9, 142]. Conversely, the gradient of an elliptic spanload approaches infinity at the tip to satisfy a null circulation, leading to an immediate formation of strong wingtip vortices. Figure 2.6 illustrates the contrast between both downwash distributions. The wake characteristics of wings with NELDs could be observed recently at NASA after a set of flight tests carried out under the direction of Bowers [9, 143]¹¹. The inboard vortex roll-up and the outboard upwash were captured during flight by streamers attach on the trailing edge of the flying wing. The fundamental design approach followed Prandtl's 1933 theory and was inspired by the Horten aircraft and aimed to gather experimental data on the performance and handling qualities of such configurations.

Induced downwash is expressed as the vortex strength induced by the trailing vortex sheet. Thus, variations of the self-induced velocity profile behind the wing will dictate the roll-up behaviour of the wake vortex sheet. A recent comparative study [144] was performed on the wake characteristics of elliptic and NELD configu-

¹¹Albion Bowers introduced the *NELD* terminology to his work, as a means of identify the aerosturally constrained wing design approach.

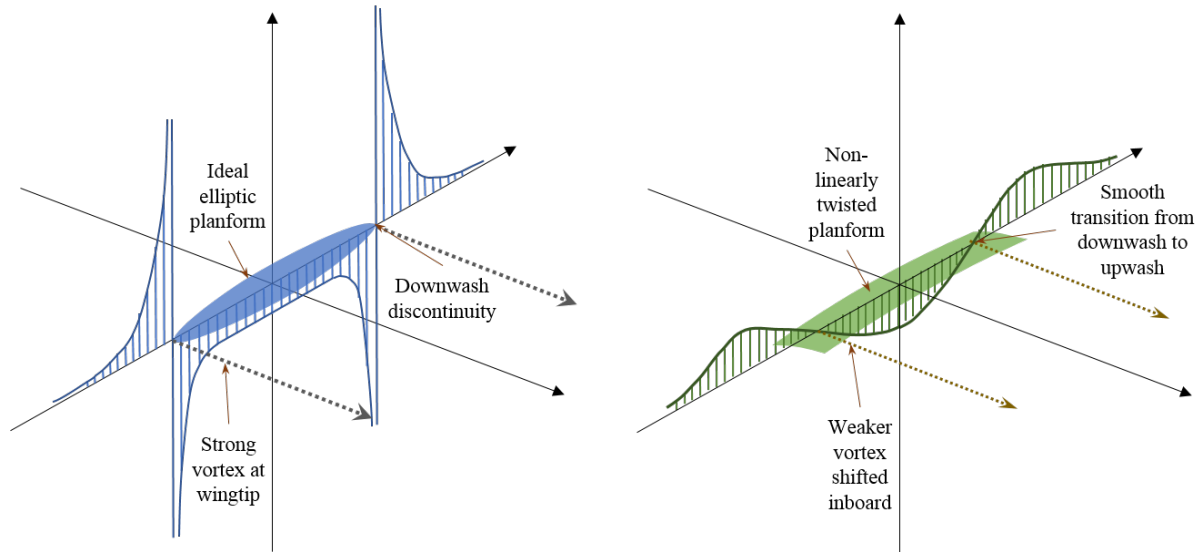


Figure 2.6: Diagram comparing the downwash induced by an elliptic circulation distribution (left) and a bell-shaped circulation distribution (right). Reproduced from Reference [9].

rations, with the use of a RANS approach at the near-field and free-wake simulations at the far-field. The results show an unconventional wake development for the aerodynamically optimised wings, which agree with past studies on wakes shed by similar spanloads [145–147]. Firstly, Prandtl and Jones’ configurations featured a substantial delay in wake roll-up (approximately 6 and 12 times farther respectively, compared to an equivalent elliptic spanload). Moreover, the roll-up rate behaved correspondingly to the quadratic and linear downwash distribution characteristic of each configuration. Secondly, the trailing vortices of the non-elliptic configurations did not develop at the wingtips but were formed further inboard. Thirdly, the vortex span was smaller in the wake of the non-elliptic spanloads. However, it was identified that the separation of the vortex cores corresponded to the distance between the maxima and minima of the shed vortex strength, multiplied by the ratio of the vortex span to the wingspan of the elliptic configuration. The vortex mitigation achieved by means of non-elliptic spanloads can challenge existing methods for passive alleviation of the vortex roll-up process. These results could be beneficial in the context of formation flying, and may address other operational challenges such as decreasing minimum time intervals between aircraft at take-off and landing or reducing noise pollution.

2.3.4 Measuring Efficiency

In the search for the spanload for minimum induced drag, it is customary to quantify the aerodynamic efficiency of a given lift distribution in terms of the span efficiency factor e . In lifting line theory, it is derived from the expression for induced drag, given

by:

$$D_i = \frac{2L^2}{\pi b^2 \rho_\infty V_\infty^2} \left[1 + \sum_{n=2}^{\infty} \frac{n G_n^2}{G_1^2} \right] = \frac{2L^2}{\pi b^2 \rho_\infty V_\infty^2} [1 + \delta] = \frac{2L^2}{\pi e b^2 \rho_\infty V_\infty^2} \quad (2.3)$$

for a fixed wing area, and is obtained from the general Fourier series formulation for the circulation distribution. Hence, the expression for e becomes:

$$e = \frac{1}{1 + \delta} = \frac{1}{1 + \sum_{n=2}^{\infty} \frac{n G_n^2}{G_1^2}} \quad (2.4)$$

Here, the inviscid span efficiency factor measures the departure of a given spanload from the elliptic solution [148], as a result of the contribution of the additional non-zero Fourier coefficients G_n to the circulation distribution. Within the classical lifting line theory framework, the elliptic spanload is defined by only the first Fourier coefficient G_1 , and $G_n = 0$ for $n > 1$, thus yielding the minimum value $\delta = 0$. Any other solution will yield $G_n \neq 0$, which deviates the spanload from the elliptic curve and will necessarily yield $\delta > 0$ (or $e < 1$) and will increase induced drag. This criterion initiated the search for wing configurations other than the elliptic planform that could provide the closest value to $e = 1$. Glauert [105] showed that a straight tapered wing with a taper ratio of $\lambda = 0.40$ would be the most advantageous. Nevertheless, as theory and technology were developed along with the endeavour to increase efficiency, many unconventional designs emerged that included highly non-planar configurations. The analysis of such wings required a more generalised expression for the span efficiency factor to account for arbitrary geometries. This could be derived through Munk's optimal normalwash solution, which was obtained as proportional to the cosine of the dihedral angle. This led to a wide range of configurations that provided $e > 1$, which included wings with an extensive variety of tip forms, box wings, V-wings, etc [87, 149–151]. It was possible to obtain values of span efficiency higher than the theoretical maximum, despite the contradiction with the original formulation. However, these favourable results would be overcome, among other factors, by the increase in profile drag or weight.

Considering that weight is a cardinal determinant for efficient flight, it seems rather inconvenient that it is not integrated in one of the fundamental metrics for efficiency. Spanloads such as the bell-shaped distribution provide reductions in induced drag by outweighing their reduced aerodynamic efficiency with a higher aspect ratio. Yet, this does not reflect that weight has not theoretically increased with span extension, fact that could be considered more efficient from an aircraft performance standpoint. The classical expression for span efficiency factor e is a result of the specific constraints defined for a problem with a particular set of variables: a straight wing with a given fixed lift and fixed span; and it conveys how much a spanload will vary from the

optimal solution of this particular problem. However, when the lifting line is extended to consider structural weight, the resulting expression for induced drag is now different from Equation 2.3, and it will vary for each modification of the assumed structural constraints.

Phillips and Hunsaker [10] put this approach into practice to demonstrate the significance of such considerations. The corresponding analytic expressions for induced drag are derived, together with their pertaining efficiency factors as a function of Fourier coefficients. With this, Prandtl's 1933 optimal circulation distribution, shown in Equation 4.32 and having $\mu = -\Gamma_2/\Gamma_0 = 1$, can be rewritten in trigonometric coordinates as:

$$\Gamma(\theta) = 2bV_\infty G_1 \left[\sin \theta - \frac{1}{3} \sin 3\theta \right] \quad (2.5)$$

Comparing Equation 2.5 to the generalised Fourier series expansion for circulation [6], and defining $B_n = G_n/G_1$, the Fourier coefficients for $n > 1$ are now $B_3 = -1/3$ and $B_{n>3} = 0$. Introducing B_3 into Equation 2.4, which assumes fixed gross lift and span, reveals a lower span efficiency factor than that of the ideal elliptic spanload. This result does not indicate that the drag will be smaller due to the extended span. A more suitable criterion could therefore be to have the highest value of the product b^2e , following Equation 2.3. Yet, this measure would not consider structural equivalence. To do so, structural parameters must be present in the derivation of the expression for aero-structural efficiency.

Nonetheless, it must be borne in mind that the choice of constraints remains arbitrary and bound to the assumed critical design points, as to encompass all the structural variables is, again, an unattainable task that demands compromise within theoretical analysis. Furthermore, this information may not yet be known at the conceptual design stages, and would therefore bring an approximate and iterated solution that would carry uncertainty until it is estimated at the detailed design phase.

With this, one could fix weight and maximum stress on the structure due to limit loads on a rectangular wing, and assume the integrated bending moment to be proportional to the weight so that the proportionality factor is constant. The integrated bending moment could be defined from its dependency on the aerodynamic loads as well as the weight distribution, which could be a function of the wing structural weight, the total aircraft weight and the specified maximum load factors. From these assumptions it derives that the wing structural weight is a function of B_3 only, and that this coefficient can be traded with span to keep the weight to a minimum while reducing induced drag. The corresponding expression for induced drag would yield:

$$D_i = \frac{\kappa_g W_r}{16\pi\rho_\infty V_\infty^2} \frac{W^2}{K_s W_s} (1 + B_3 + 3B_3^2 + 3B_3^3) \quad (2.6)$$

In this Equation, κ_g is the weight distribution coefficient, W_r portion of of the aircraft net weight carried at the wing root, and W_s is the total weight of the wing structure required to support the wing bending moment distribution. Now, because all Fourier coefficients contribute to induced drag, this study assumed that $B_n = 0$ for $n \neq 3$. Thus, the expression for induced drag is composed of a constant term, which is a function of various weight parameters, including the aircraft's total weight W , the proportionality factor K_s and dynamic pressure, multiplied by a new efficiency function that depends solely on B_3 . Now, if values of B_3 that yield any negative circulation are likewise excluded, the minimum of Equation 2.6 corresponds to Prandtl's 1933 solution, and can be visualised in Figure 2.7.

As aforementioned, this is just one arbitrary solution subjected to some arbitrary set of constraints. If, instead of the above conditions, the weight, maximum stress, and wing loading were to be fixed for a same structural model and weight distribution, the resulting induced drag would now be a function of:

$$D_i = f\left(\left(1 + 3B_3^2\right)(1 + B_3)^{2/3}\right) \quad (2.7)$$

The coefficients that minimise the above expression are now $B_3 = -3/8 + \sqrt{9/64 - 1/12}$ and $B_n = 0$ for $n \neq 3$, the trend of which is shown in Figure 2.8. A final design approach presented by Phillips and Hunsaker [10] evaluates maximum deflection constraints under the previous structural and weight assumptions. The corresponding expression for induced drag as a function of B_3 is shown in Equation 2.8, and its trend is illustrated in Figure 2.9.

$$D_i = f\left(\left(1 + 3B_3^2\right)(1 + B_3)^{1/3}\right) \quad (2.8)$$

In this case, the solution for minimum induced drag is given by $B_3 = -3/7 + \sqrt{9/49 - 1/21}$ and $B_n = 0$ for $n \neq 3$.

Figures 2.7 to 2.9 show the variation of the induced drag efficiency factor as the spanload changes with the third Fourier coefficient. These results manifest how each particular problem yields its own optimal solution in accordance with the chosen set of constraints. Moreover, the above expressions for induced drag are independent of the span, which has become a function of the prescribed parameters and the spanload, specified by B_3 . Hence, when structural considerations are incorporated in the lifting line theory, the trade-off between span and weight is balanced through B_3 . For a more extensive discussion of this approach the reader is directed to Reference [10].

The derivation of alternative efficiency factors can provide significant insight into the aero-structural sensitivities of a particular design problem at a conceptual design level. Such an analysis allows the quantification of the trade-offs between aerodynamic and structural coupling independently of the span. In other cases, it could further

include considerations such as coupling between stability and performance [152].

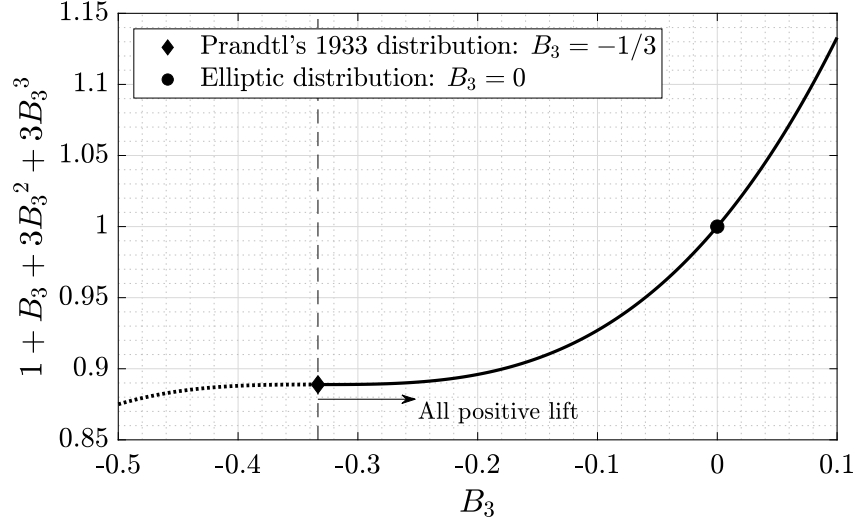


Figure 2.7: Induced drag efficiency factor from Equation 2.6 as a function of B_3 on a rectangular wing with fixed weight, maximum stress, and constant K_s . Reproduced from Reference [10].

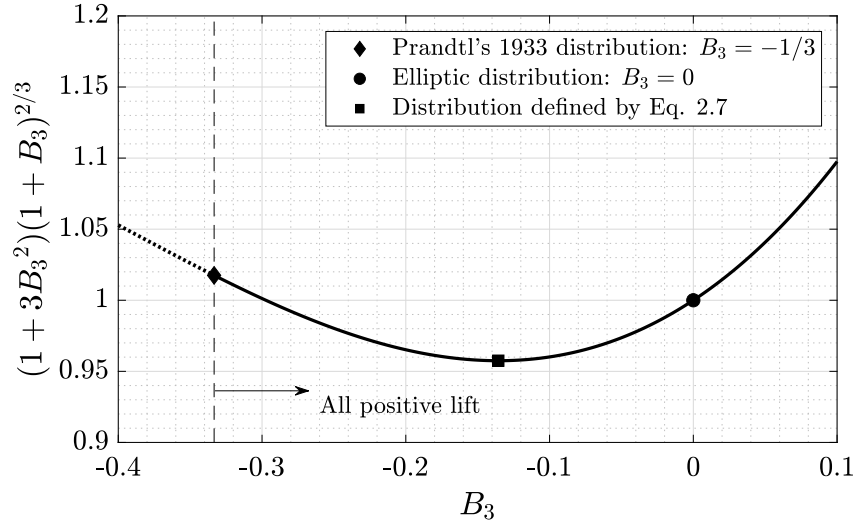


Figure 2.8: Induced drag efficiency factor from Equation 2.7 as a function of B_3 . Extracted from Reference [10].

Efficiency metrics serve as a means to provide absolute comparisons in relation to a defined set of parameters. However, there are many other factors that are neglected, and the comparison is often subjected to the optimal solution of a specific variational problem. Therefore, it would be appropriate to consider the use of further comparative indicators for wings designed under the same underlying considerations. However, an overall comparison of efficiency might require alternative approaches. In such cases, the lift-to-drag ratio L/D is widely chosen in the juxtaposition of overall

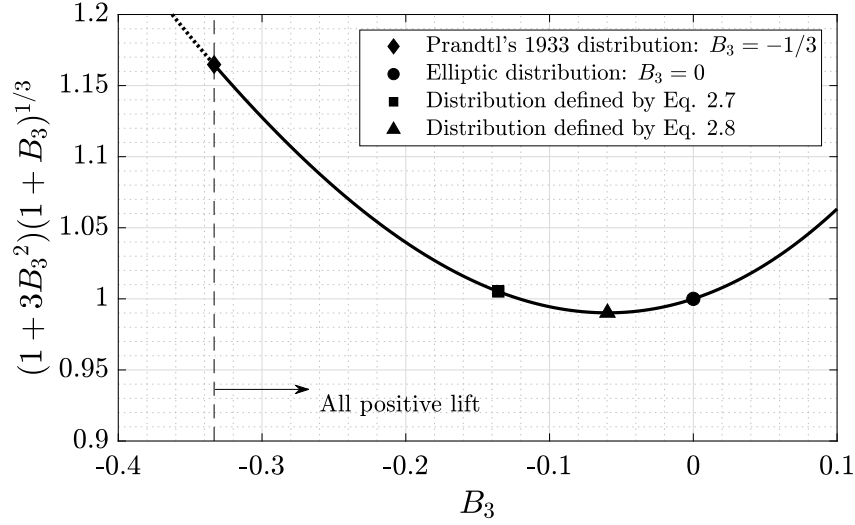


Figure 2.9: Induced drag efficiency factor from Equation 2.8 as a function of B_3 . Extracted from Reference [10].

aerodynamic performance of any aircraft configuration. Its extensive use is due to the simple concept that it carries, in which the invariable goal of achieving the most lift with the least drag can be quantified at any flight condition. Furthermore, it is directly linked with the Breguet range equation, which interconnects the main parameters that define a mission. However, this metric does not explicitly incorporate the trade-off between induced drag and weight. Hence, there is still the need to define a metric that captures aero-structural efficiency as a distinct parameter.

Another classical metric for efficiency is given by the lift-to-drag ratio L/D (or alternatively ML/D) and it is widely chosen for the estimation and comparison of overall aerodynamic performance of any aircraft. Its extensive use is due to the simple concept that it carries, in which the invariable goal of achieving the most lift with the least drag is quantified. Moreover, it is also directly linked with the Breguet range equation¹², which shows the relation between the main parameters that define a mission as follows [13]:

$$R = \frac{V}{TSFC} \frac{L}{D} \log_e \left(\frac{W_0}{W_F} \right) \quad (2.9)$$

The above equation depicts the role of each these pivotal parameters to best achieve the mission. Hence, for a maximum range R we want to fly with minimum thrust-

¹²The original Breguet range equation was devised for a propeller-driven aircraft and first appeared in the literature in 1920 [153]. Coffin, the author of this first publication, provided the complete derivation of the range and endurance formulae without any reference to Breguet. It was not until four years later that the range equation was credited to Breguet [154]. Further research was performed by Anderson to conclude that Coffin's work could be original and that there is no substantial evidence to assign the range equation to Breguet [155]

specific fuel consumption $TSFC$, at a maximum L/D , with the highest initial to end weight ratio W_0/W_F , which is ideally obtained with the largest fuel weight, at the specific speed V of the range, usually being the primary cruise leg. This practical formula is generally accurate within 10% to 20% [155].

Despite the convenience of the L/D and its direct implication in the prominent Breguet range equations, this metric is not free from caveats. In the context of the Breguet equation, we have seen that it is intuitively required to have a maximum L/D . However, it must be regarded that this is sometimes achieved by flying with a higher weight, and therefore increasing the drag, which is contrary to the essence of aerodynamic or structural efficiency. Furthermore, the L/D must be measured at different flight stages for an appropriate comparison among different aircraft, since it is not unique throughout the flight unlike other parameter such as the span or the maximum take-off weight. Most importantly, it does not involve the trade-off between induced drag and weight. Hence, there is still the need to define a metric that captures aero-structural efficiency as a single parameter. For instance, Guynn [156] recently proposed an alternative efficiency parameter derived from the Breguet range equation that could measure aero-structural efficiency at an overall aircraft level. Termed 'effective lift-to-drag ratio' and aimed as a communication tool, this metric measure efficiency through the impact of the wing weight contribution on performance. Thus, assuming a zero-weight wing on an idealised aircraft model, the effective lift-to-drag ratio is simplified to:

$$\left(\frac{L}{D}\right)^E = \frac{L}{D} \left(1 - \frac{W_{wing}}{W_{TO}}\right) \quad (2.10)$$

This expression represents how the increase in L/D due to span extension should not be offset by the decrease of $\left(1 - W_{wing}/W_{TO}\right)$ due to the corresponding increase in wing weight. By means of simulation-based validation, the metric provided agreeing results between the analytical expression and the simulated aircraft models.

Such efficiency parameters are necessary for guidance. To provide a nominal vision of the trade-off between aerodynamic and structural performance, or more specifically, to quickly quantify the compromise between induced drag and weight.

2.4 Flying with Non-elliptic Lift Distributions

Prandtl viewed non-elliptic lift distribution as a solution for minimum induced drag and structural weight. However, this aerodynamic problem can provide other benefits for the wing in terms of handling qualities. As it has been mentioned previously, an

additional benefit derived from the circulation characteristics of non-elliptic distributions is the possibility to address adverse yaw. It is a natural response of the aircraft to aileron deflection, during which an asymmetric lift distribution is developed to roll into the desired direction and the consequent drag differential creates a yawing moment in the opposite direction, as portrayed in Figure 2.10. This phenomenon has become an established nuisance that is usually solved by rudder input. The use of non-elliptic lift distributions implies a high reduction in the use of additional drag to counteract the adverse effect, which in turn is favourable to the direction of the turn. With this, other stages of the flight mission could become more efficient too, added to the increased performance in cruise.

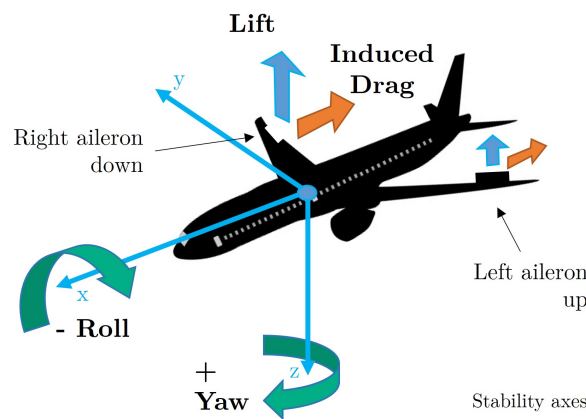


Figure 2.10: Diagram of forces acting on an adverse yaw scenario.

2.4.1 Early Efforts of the Hortens

Not much experimental research has been carried out on wings design under the NELD approach. The pioneers in this field are the Horten brothers, whose strong advocacy of flying wings prevails in their legacy of enigmatic aircraft designs. Turbulent times had ignited the struggle for aeronautical innovation, and the attractive possibilities that flying wings had unveiled was enough justification for them to dive into the intricacies of such unconventional designs [157,158]¹³. The Hortens devised their aircraft through intuition rather than on a theoretical basis. However, they had studied the

¹³At the time, both the Horten brothers in Germany and Northrop in the United States were challenging the standards of the time through their flying wings. Both faced many stability-related adversities in their designs, greatly due to the lack of inherent yaw stability brought by the tube and wing configuration. In the early days, the vibration of the propellers often amplified instabilities, where Northrop ran into a lot of stability issues with the XB-35 and XB-49 prototypes. This problem was mostly fixed when turbine engines were used instead and with the introduction of fly-by-wire control, which led to the design of the B-2 Spirit. But even before that, the Horten flying wings were achieving significant improvements in stability through their unusual control surfaces devised under the philosophy of NELDs [159]. For this reason, the discussion in this Section focuses specifically on their work.

fundamental theories and were interested in approximate methods¹⁴. The challenge was addressed under the terms of handling qualities by exploring planforms with careful non-linear wing twist distributions. To do so, they used the Horten H.II [160] to assesses the aerodynamic characteristics of various circulation distributions, which included various trigonometric curves, as well as the normal and the elliptic distribution for a given lift, fixed span and fixed wing area. The non-elliptic configurations would show lightly-loaded wingtips, particularly the bell-shaped distribution, which was especially beneficial for tailless aircraft to prevent wingtip stall. Nevertheless, other trigonometric curves provided either higher induced drag or poor handling qualities that demanded great effort from experienced pilots.

Thus, the anticipated spanload, coined by them as ‘bell-shaped’, provided a downwash distribution that would smoothly become upwash at the outboard wing while the lift remained positive. With this, it was possible to produce induced thrust at the wingtips. Furthermore, it could be used to manipulate the aircraft’s yawing moment to make a coordinated turn without the need of any vertical surface. In effect, a combination of the upwash induced at the outboard wing together with an appropriate aileron sizing could yield a drag differential that would cause a favourable yawing moment (proverse yaw). Following this design philosophy, the Hortens conducted experimental research and observed the behaviour of wool tufts along the wing, which pointed in the desired flight direction during turns. With this, the previously required rudder correction would be reduced or even become unnecessary, and so the structure of vertical stabiliser could be significantly reduced. However, the achievement of proverse yaw was critically dependant on the position of the centre of gravity, and these tests were also used to determine its correct position. Flying wings are strongly affected by the location of their centre of gravity, which must closely correspond to the centre of lift in the longitudinal direction to prevent large moments being induced in a flow separation scenario. Herein, the bell-shaped spanload became especially beneficial, since it would readily satisfy this requirement. Moreover, the consequent inboard shift of the centre of lift would improve lateral-directional characteristics [149]. During the Horten H.III project, variable washout was implemented by operating several control surfaces at different deflection angles. In this way, the necessary lift distribution could be attained more accurately. This design vision allowed the Hortens to claim that the aileron and rudder could be completely independent and that it was possible to achieve proverse yaw.

¹⁴Reimar Horten praised Multhopp’s and Lippisch’s early dissertations, which, according to him, were never recognised due to missing fulfilment of formalities in the curriculum. In this unpublished work, Lippisch mentioned the solution which Horten would later entitle as the bell distribution [131]

2.4.2 Proverse Yaw Implications

For the Hortens the elimination of adverse yaw became one of the primary justifications of adopting the bell-shaped lift distribution. Adverse yaw is usually solved with the aid of flight control systems that couple aileron and rudder deflections. By making use of additional drag, the manoeuvre becomes an aerodynamically inefficient exercise. Still, this energy expense has not been considered significant enough for engineers to invest time and effort. However, various amending devices have been conceived to reduce the magnitude of adverse yawing moments, often at the expense of higher induced drag and only effective within a limited range of the flight envelope. Here, the trade-offs arise from the attached mechanical complexity, cost increase and weight penalty [161]. Along with flying wings, adverse yaw can be particularly augmented in aircraft with high aspect ratio wings. Even though the longer moment arm reduces the magnitude of the loads required to roll, a full rudder deflection is sometimes still insufficient to compensate for adverse yaw.

Theoretical research on spanloads that minimise drag during turn are scarce. However, the potential benefits of NELDs on aircraft handling qualities has drawn inquisitive minds towards the investigation of theoretical approaches for synthesised roll control and adverse yaw elimination. Nickel [83] used lifting line theory to investigate the existence of load distributions that could yield null yawing moment and minimum induced drag for any prescribed lift and rolling moment. This theory was tested experimentally with the construction of the Falter 1, an ultra-light glider with no vertical surfaces or drag rudders. Nickel wanted to prove that the motion around the vertical axis could be controlled by an adequate distribution of induced drag caused solely by the deflection of horizontal control surfaces. The design, shown in Figure 2.11 had two independent ailerons on each semi-span that would be covered with a single fabric to produce a morphing effect. Such design enabled the pilot to hold control around the three axes, where turn initiation would yield no adverse yaw and steady sideslip would be achieved without any opposing moments¹⁵.

¹⁵Nickel was a mathematician and flight enthusiast that worked for the Hortens in the design of some of their early flying wings, built his own models and was later employed as an aircraft designer in Argentina [162]. But even with his vast experience in tailless aircraft, he clearly disagreed with the idea of the bell-shaped lift distribution being as favourable as it is claimed to be. Nickel [11] asserted that *"The use of the bell-shaped lift distribution for flying wings is not recommended"*. The reasons for this clear statement come from the resulting performance and handling qualities of the Horten flying wings. These aircraft did not attain the expected proverse yaw for which they had been designed, with pilots like himself experiencing disturbing dynamics. While this is discordant with other assessments reporting satisfactory handling characteristics [149,163], and the popularity of the bell-shaped design philosophy has sprawled among RC wing modellers, it is true that the Horten models had very poor performance in terms of efficiency [164], and this was even admitted by R. Horten. However, he was not particularly interested in improving aerodynamic efficiency and did not take into consideration wing equivalence. A flying wing generally provides higher performance than a conventional aircraft due to the removal of most of the non-lifting surfaces, and thus the Horten wings were directed at addressing adverse yaw.

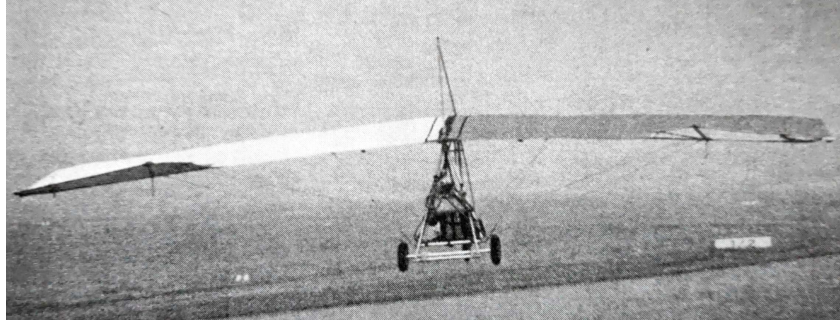


Figure 2.11: Nickel and Schmidt's Falter 1 [11].

Hunsaker et al. have recently carried out theoretical research in this field, building upon Prandtl's lifting-line theory and following a similar approach as that of Nickel. Specifically, the control of lateral-directional flying characteristics was addressed through wing twist design, in which elliptic and non-elliptic loadings were contrasted in the context of pure rolling motion. With this, the theory explored constraints on turn initiation, steady rolling rate and transition [85]. Analytic solutions confirm that a symmetric non-elliptic spanload must be superposed to the baseline lift distribution to achieve proverse yaw, and can be implemented either in the clean configuration or through trailing edge deflection. Moreover, this work concluded that no antisymmetric spanwise loads that can be added to an elliptic spanload to produce proverse yaw [166], as derived from this analytical approach. Secondly, for each asymmetric contribution with minimum drag there exists a symmetric load that will also yield minimum induced drag [53]. Third, the induced drag present during roll will always be higher than the induced drag of the clean configuration. As discussed in previous sections, any non-elliptic lift contribution to a fixed wingspan will increase induced drag. Hence, the more elliptic is the symmetric contribution to the roll, the more aerodynamically efficient the manoeuvre will be for a fixed wing. On the other hand, the more non-elliptic, the more roll power and proverse yaw it will have [167].

These solutions are effectively applicable on wings equipped with morphing technologies that enable a smooth and continuous twist distribution. On the other hand, the use of discrete ailerons can yield a set of solutions that satisfy the elimination of adverse yaw for a given spanload. Hence, an aileron design methodology was developed to provide insight into the impact of aileron sizing on the induced drag distribution to provide proverse yaw [168]. This preliminary study assessed the lateral-directional static stability characteristics of a conventional sailplane. The baseline configuration carried an elliptic lift distribution and was compared to modified models, in which

Moreover, later evaluation of the performance of Horten wings was done after restoration because the wings were damaged during the war, modifying key parameters such as the centre of gravity that could impact negatively the results [165].

equivalent wings with NELDs were retrofitted. In each case, the ailerons had been sized under the constraints of roll authority and proverse yaw. Therefore, the stability and control derivative that quantifies yawing moment due to aileron deflection was regarded as the metric for proverse yaw. The methodology provided a design space of aileron geometries for the NELD configurations that satisfied these conditions, whereas no solutions were obtained for the conventional elliptic wing. It is generally believed that the cause of proverse yaw is brought by induced negative drag increasing with downwards aileron deflection [9]. This study showed that this is true for small deflections. However, larger aileron deflections increase drag in both up-going and down-going motions, contrary to the induced drag differential observed in any conventional wing. Instead, the additional drag contribution in each semi-span is almost identical, being marginally bigger on the wing inside of the turn and consequently producing a favourable yawing moment. Moreover, the increase in induced drag per area was found to be lower in relation to that given by the elliptic case.

Proverse yaw was recently observed at NASA during a large series of flight tests performed with the PRANDTL-D flying wing [9], shown in Figure 2.12. Here, Bowers and his team observed improved flying characteristics and coordinated flight over a broad range of flight conditions. Onboard instrumentation data showed that both rolling and yawing motions were headed in the same direction during turn initiation. With this, they defend that these designs can attain induced thrust and proverse yaw [169], improving efficiency over a wider region of the low-subsonic flight envelope.

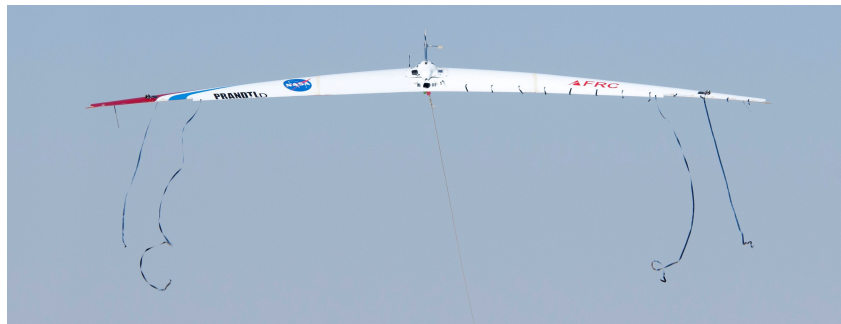


Figure 2.12: NASA's Prandtl-D No. 2 during launch [9].

2.4.3 Removal of the Vertical Stabiliser

The above theoretical and experimental results together with the bell-shaped spanload suggest at least certain wings with specific mission characteristics enable total dynamic control with very reduced projected vertical control surfaces or even no rudder [170, 171]. Would this theory still be applicable under the design and flight conditions of civil transport aircraft? The development of the flight control system and fly-by-

wire technologies around the 1970s made a significant impact in the controllability of tailless aircraft, where early attempts had generally failed because of poor handling qualities that led to unsafe flight characteristics. Nevertheless, the apparent non-linearity required for the control surfaces to produce a suitable roll load distribution that eliminates adverse yaw adds complexity to the design and dynamics, and might seem more appropriate for the design of morphing wings [172].

There has always been a motivation to remove vertical tails from aircraft as seen in the increased interest in blended-wing-body aircraft, given that our fundamental stimulus is driven by the bird flight model. However, a bird's wing has high morphing and flexibility capabilities, and their feathers can be controlled to perform the function of all control surfaces and high-lift devices concerning dynamic and static control and trim. Furthermore, smaller-sized flying surfaces demand smaller aerodynamic damping capabilities [173]. To date, however, there is only one operational aircraft that has no vertical surfaces, this is the Northrop Grumman B-2 Spirit [174,175].

There are several difficulties preventing blended-wing aircraft from success. As aforementioned, flying wings are inherently aerodynamically unstable, although several methods exist to address this [161]. The fuselage cross-section needs to be wide enough to be usable, which can increase drag and structural complexity. In addition, control and stability is a challenge to overcome. On the other hand, blended-wing-body aircraft are often considered theoretically as one of the most aerodynamically efficient layouts and lighter than a traditional fixed-wing design. Another advantage of tailless aircraft is the savings gained in drag and weight. The penalties [13,176] associated with the vertical tail can range from 5 to 10% in drag, and can add around 25 to 50 kg/m² of projected side area. Furthermore, a main benefit that concerns the removal of the vertical tail for military purposes is the significant impact in the electromagnetic signature due to the reduced radar cross section.

Still, vertical tails compensate these losses due to their high reliability in providing directional control and stability and yaw damping. Moreover, they are the safest protection method to maintain a straight flight in a crosswind landing or, more critically, a one-engine-out scenario, which usually drives its design criteria and sizing [177] on any conventional tube and wing aircraft with wing mounted engines. Vertical tail design is directly linked to the aircraft's directional stability and control, which are always coupled with the lateral dynamics. Certainly, the elimination of the vertical surfaces will lead to unconventional approaches to address safety, performance, and flight qualities. Furthermore, the aircraft designed to fly without a vertical tail should be inherently more directionally stable. The potential drag and weight savings during cruise could be cancelled by the complexity of alternative devices required to address directional characteristics. During turn, even the use of a non-elliptic lift distribution will necessarily increase drag when proverse yaw is attained. However, traditional methods for

the estimation of lateral-directional stability derivatives predominantly rely on standard semi-empirical formulations [178–184] that are not entirely adequate [185] for this approach and pose limitations on the departure from conventional designs. With this, it could be advantageous to seek alternative methods to investigate the optimal compromise between proverse yaw, induced drag and weight derived from the trade-off between spanload and rudder sizing.

2.5 Current Trends in Efficient Wing Design

The established desire to develop unprecedented civil aircraft is becoming more conceivable and is present in the vision for the near future generations of aircraft [186–188]. Aircraft configuration is regarded as a critical factor that will redefine the way in which the current technological challenges are addressed. So far, airliners have been restricted to the conventional cantilever tube-and-wing configuration due to several limitations on structural and operational aspects, which have hindered revolutionary changes in aviation.

The increasing interest in alternative configurations, such as high aspect ratio wings or blended wing body aircraft [159,189], aligns with the constant strive to reach a more organic flight model, in which efficient bird-like flight and manoeuvring are possible without artificial requirements such as additional drag or a vertical stabilisers.

Other examples that seek efficiency include the double-bubble lifting fuselage, truss-braced wing, or configurations with over-wing mounted nacelles making use of boundary layer ingestion. Furthermore, advances in alternative propulsion systems expands the list through concepts adapted to electric, hybrid-electric, or distributed propulsion architectures. Solutions to the problem of time-varying lift distributions have become more relevant as materials, manufacturing and control systems technologies are developed toward the removal of discrete control surfaces. Morphing wings enable key wing shape parameters to be changed to produce optimal spanloads and alleviate loads at a wide range of flight conditions [190–192]. Many configurations [193–195] have been experimentally tested and show performance and handling qualities improvements while minimising airflow separation. Moreover, part of this future vision is targeting sub-transonic cruise speeds in order to accommodate the impending green propulsive technology. This, together with the prominent appeal towards urban air mobility, could represent the new generation of aerospace vehicles that meet both the environment and society's long-term needs.

In the subject of drag reduction, the ever-growing concern is now aligned with the rising need to revolutionise the almost depleted existing approaches, which are reaching stagnating levels of improvement. Thus, the prediction of turbulent flows remains

a key focus, for which the increasing availability of computational power is becoming a critical contributor. High fidelity and efficient computational tools are required to understand, model, and control complex three-dimensional unsteady viscous flows at transonic speeds. This would not only bring advances in aerodynamic design, but also the possibility to certify such concepts through analysis alone. Consequently, wind tunnel testing could be dedicated to the development of validation databases or specific phenomena-based testing. Along these lines, the development of active and passive flow control devices could make the above possible, for laminar flow wing designs, control of the boundary layer characteristics, for adaptive wings in transonic speeds, enhanced by the use of composites in wing structure, which facilitates the achievement of new aerodynamic design [196,197].

The development of improved aerospace materials is a determining factor to any of the above potential solutions, and has seen outstanding progress in recent years. Innovative composite materials and alloys are developed with superior mechanical performance. However, current aerospace materials still face major challenges that hinder the evolution of airframe shape and structural design. Thus, there is great attention on the research of enhancing mechanical properties, corrosion resistance, efficient and flexible design, manufacturing and joining issues, all at a reduced life cycle cost [198–200].

At the core of this roadmap towards the future aircraft lies a crucial aspect that involves all the previously mentioned aspects and, if not addressed adequately, can impede progress in each of them. This aspect pertains to the design tools employed. Currently, design methods have been refined to converge towards conventional configurations that have been exploited over the years. The industry possesses abundant amounts of empirical and experimental data, obtained from its own aircraft families, that enrich and solidify the analysis of any redesign they embark on, provided it keeps a resemblance to the configurations from which that information was extracted. Nevertheless, the exposed future vision makes it increasingly necessary to develop more flexible design methods that can handle any type of configuration and integrate the impact of new technologies. This includes the strong eagerness to reduce the amount of experimental testing required for validation and certification, and to be able to substitute most of it with simulation. Additionally, it is necessary to provide satisfactory fidelity to ensure the model captures the required features and behaviour to reach the specific design goals at this early stage, while providing large analysis capabilities without being time-consuming and computationally expensive. On the other hand, new designs as well as new design methodologies will need experimental flight testing to ensure their validation and then certification. This requires large investments, blind to a certain degree, to make a reality the re-establishment of the industry within a new mentality with new opportunities. This shift is happening, in a way, thanks to the

surge in entrusting big ideas to small businesses. The drawback is that this somewhat convoluted path will need much more time to accumulate the same knowledge as the large established companies, as well as to gain the trust of the safety and certification authorities. At the same time, there is a risk that the expertise developed within each of these design niches becomes isolated and shielded from the aviation community, obstructing the much needed collaboration and therefore hindering progress.

2.5.1 The Approach to Conceptual Design

The change in methodology advocated above will take different shapes depending on what stage of the design phase it concerns. The interest of this research lies on the conceptual design stage, which is already under transformation. In brief, it started from the use of analytical methods to predict the key parameters in each discipline, and has evolved into a vast design space where the interaction between variables can be evaluated through their sensitivity and uncertainty, and for which thousands of design points can be discarded through numerical optimisation. However, the novel concepts being pursued present many challenges to legacy methods traditionally employed in conceptual design, where the fundamental assumptions that provided simplicity and speed now become limiting factors.

Thus, conceptual wing design is already a multidisciplinary task, as it is clearly understood that to obtain viable wing planform geometries via MDO, it is imperative to include multiple disciplines together with realistic constraints. Still, new concepts are becoming increasingly coupled across multiple disciplines, brought greatly by propulsion-airframe integration with technologies such as boundary layer ingestion or distributed propulsion. Moreover, most of the life cost of a design is incurred during the decisions made in the conceptual design phase, because an appropriate early configuration will avoid costly re-design and corrections at later stages [201,202]. This is illustrated in Figure 2.13, a graph that is gaining popularity due to its depiction of the request for change in the design approach to improve the process and its effectiveness as well as reduce its costs.

Contemporary civil aircraft conceptual design tools make extensive use of semi-empirical approaches, which consist of analytically-based equations derived from a theoretical foundation that are then adjusted using statistical correlations from historical data. This can be combined with further experimental data, when available, to feed approximated models of the various disciplines involved. Nevertheless, such approaches are not reliable for the prediction of novel configurations and technologies, which represent the outliers that cannot be fitted into the curve. It is widely accepted that novel aircraft concepts being proposed deviate significantly enough from the empirical databases for the methods to become unreliable to predict weights,

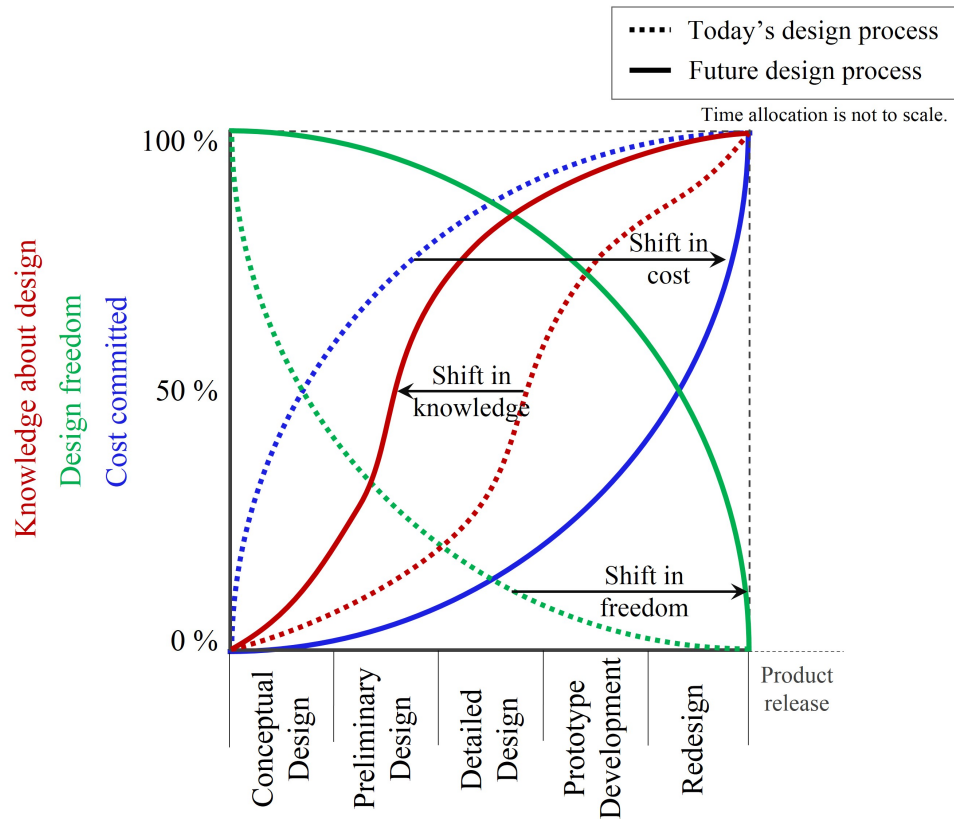


Figure 2.13: Evolution of the design freedom, the cost of design change and the knowledge about the design along the project timeline. Reproduced from reference [12].

aerodynamics, propulsion, or performance accurately. Even with the rise of artificial intelligence, the optimality of the end result will be only as good or innovative as the set of data provided. Hence, the obtained trends and correlations between variables are limited to conventional designs, while a significant part of the airframe information is effectively absent at this stage. Therefore, physics-based models are becoming essential to be able to simulate any configuration with sufficient accuracy.

With this, trade-offs being assessed nowadays are heavily linked to the level of model fidelity required at this stage. High fidelity points are expensive and limit the number of design points that can be assessed, but can comprise and build a more precise description of the aircraft and yield highly accurate results. Lower fidelity models can be used with less preamble, but their range of applicability can be more limited, and sometimes disregard the impact of interconnected variables. Lower fidelity methods can be used to build knowledge, whether it being the fundamental correlation between a reduced number of variables, or the top-level characteristics of a system. As an alternative path, the implementation of meta-modelling capacities is increasing, in order to approximate high fidelity models, simplifying parts of the overall process

and reducing computational cost of solving MDO problems. MDO formulation is also being revised, proposing multi-stage problems, including uncertainty derived from the design process or enhancing its human-machine interface. Here, the increasing capabilities of MDO have escalated the amount of data processed, which has increased the risk of losing control in the decision-making process, as well as reducing the possibilities to feed judgement and creativity to the design.

Altogether, the approach to the renovation of the conceptual design stage should be able to expand the design freedom and control, while improving the quality of the results and becoming a more efficient and robust process. Extensive discussion of these challenges can be found in the following references [203–215].

2.5.2 Design Methods and Tools

The increase of fidelity in aircraft design and analysis tools for the conceptual design stage enables a more flexible but multi-disciplinary approach earlier in the design process for more complex vehicle geometries. This is beneficial so as to have a more insight into the synergies between aerodynamics, structures, propulsion, etc, and be able to exploit them for the design of unconventional configurations that challenge the current understanding of such correlations. However, most contemporary approaches still rely on independent analyses of the disciplines, and are tailored to conventional aircraft layouts and systems, which in part is done simply because of industrial practice habit and readiness. This has already raised questions against the effectiveness of such a design philosophy, which does not facilitate the numerous multidisciplinary couplings to be solved, and presents many challenges for the traditional methods and legacy sizing and synthesis tools to be applied to novel concepts.

Particularly, aero-structural optimisation deals with the correlations found between steady aerodynamics and solid mechanics. Traditionally, the design variables are described by aerodynamic shape and structural parameters of the lifting surfaces to maximise performance at a single flight condition. Nevertheless, this may result in designs with unacceptable off-design performance, which calls for multipoint design optimisation with constraints and objective functions that commonly include interdependent structural integrity and aircraft performance variables. For instance, range or fuel burn are common targets of the objective functions, which depend on the aircraft's weight and aerodynamic efficiency. But as aforementioned, the conditions to maximise aerodynamic efficiency and minimise structural weight effectively conflict with each other. Moreover, the aerodynamic wing shape desired at any point of the flight envelope is iteratively dependent on the structural stiffness, as aerodynamic loads deform the jig shape, which in turn change the airflow around the wing, changing the loads on the wing again until structural and aerodynamic loads balance. The aerodynamic

wing shape strongly affects the accuracy of the aerodynamic loads calculation, which has a significant impact on its structural sizing and thus the final mass.

Weakly-coupled approaches often used at the conceptual design stage neglect the dependency of the structural deformations on aerodynamic shape changes during the optimisation process. Moreover, numerous methods assume simplified shear force and bending moment distributions to model the structure, or approximate the lift distribution to an elliptic or Schrenk spanload for the calculation of the aerodynamic loads and mass estimation. These approaches have revealed important inaccuracies, but provide computationally cheap and reliable simulation for conventional designs. However, the objective of having more efficient and lighter structures, together with the course following the use of high aspect ratio solutions to reduce drag result in highly flexible wings that show significant deflections in operation. A weak coupling approach would prevent such aeroelastic requirements to be included in the analysis, which would be passed on to the next design stage. This exacerbates the need to include tighter aero-structural coupling and to increase the use of non-linear and higher-fidelity solvers. Furthermore, an appropriate aero-structural integration will provide better mass estimation of the airframe structures. Similarly, current mass prediction methods are heavily reliant on empirical approaches, and there is a lack of reliable and accessible physics-based methods that can encompass novel configurations and technologies, for which mass and inertia historical data is insufficient or non-existent [216].

The above limitations are slowly being ameliorated within the academic context, where several alternative methodologies for the analysis of aero-structural interactions are being proposed. Here, the trade-off lies between the fidelity and accuracy of the structural and aerodynamic models, as well as on the coupling methods employed [217–219]. The introduction of higher-fidelity models into the conceptual design stage requires the capability of rapid optimisation with respect to large numbers of variables. Efforts to make this process increasingly cost-effective are being put on gradient-based optimisation algorithms together with coupled-adjoint methods to compute the required fully-differentiated aeroelastic formulation efficiently [220–222].

Furthermore, methods that integrate the internal wing structure properties will see improvement in the estimation of the spanwise moment of inertia and stress characteristics, which will impact the aeroelastic and flight dynamic response of the aircraft model. Finite element solvers can compute structural displacements and stresses of the wing structures, but present challenges in the manipulation of thousands of degrees of freedom, or the significant details required on the wing geometry, for which the reduction to a beam model representation becomes beneficial.

To compute external aerodynamic loads, models that can capture very complicated geometries are preferred. However, some of them, such as VLM and panel methods [223], do so at the expense of ignoring important flow phenomena such as

separation, skin-friction drag, wave drag, and transonic shocks. Common solutions to this particular problem couple the formulation with boundary layer equations to include viscosity effects, or incorporate corrections for compressibility effects. Other solvers choose to provide highly accurate solutions by including Euler and Reynolds-averaged Navier-Stokes equations while sacrificing on computational cost and time. These strongly-coupled approaches require more complex aeroelastic formulation, and they are slowly being introduced in the early design optimisation process [224,225].

The coupling of other disciplines is also proving ever more critical. The interaction between airframe and propulsion technologies in novel aircraft concepts also requires innovative analysis methods that are appropriate for conceptual design but provide higher fidelity information. Furthermore, the general aircraft sizing process requires the ability to estimate the propulsive energy requirements over the course of a defined mission. These energy requirements are represented by fuel consumption for conventional power-plants, on which the majority of mission analysis tools and methods are based. However, many unconventional configurations being proposed feature hybrid-electric and fully electric propulsion systems with strong aero-propulsive and engine-airframe interactions that will require alternative approaches for mission performance analysis.

Several computational models that follow the above vision have been developed to achieve optimised aircraft designs through a multi-fidelity approach of the conceptual design stage. SUAVE [226] is an aircraft design tool that addresses some of the issues above by augmenting relevant empirical correlations with physics-based methods, allowing the design of unconventional configurations. OpenAeroStruct [227,228] is a low-fidelity tool that couples VLM with a 6-degrees-of-freedom three-dimensional beam model to allow rapid aero-structural design of lifting surfaces, and provides MDO capabilities through NASA's OpenMDAO [229] framework. CEASIOM [230] is a design environment envisioned for complex design and optimisation workflows of conventional and unconventional configurations. This framework provides several independent tools for various disciplines in aircraft design, including aerodynamics, structures, mass and the analysis of flight dynamic characteristics, performed in SDSA [231], for the design of flight control systems. CEASIOM provides a quasi-analytical structural analysis and sizing methodology named NeoCASS [232] that supports aeroelastic problem formulation and solution. ZONAIR [233] integrates aeroelasticity and aerothermoelasticity effects in the calculation of flight loads via a panel method with viscosity correction. FlightStream [234,235] is a rapid fully viscous-coupled non-linear flow solver with analysis capabilities in transonic unsteady flows for vehicles with a wide range of unconventional geometries. Further developing tools are focusing on incorporating aerodynamic and structural geometric non-linearities [236], including adaptive wings with morphing [237] or load allevi-

ation technologies [238], detailed consideration of composite structures' properties together with the impact of electric propulsion on aero-structural design [239], as well as the mission level analysis of unconventional aircraft concepts and architectures that include electric and hybrid-electric propulsion systems [240, 241]. The approach to the integration of different existing tools to create a more complete multidisciplinary framework remains on the rise. There is a decent availability of state-of-the-art, specialised and open source computational models, and thus many researchers wish to exploit those capabilities by bringing together many of these tools to consolidate them into a single design engine that can contain as many critical disciplines as possible at these early stages [242, 243]. Further discussion on established and new state-of-the-art design tools that address a number of the presented limitations have been compiled in Reference [244].

2.5.3 HARW Aircraft

Based on the technological state-of-the-art, many trade space analyses and system architecture assessments are identifying very high aspect wings as a near-term achievable solution to reduce fuel consumption [245–249]. Remarkably, the acclaim stems from several research areas, such as the field of laminar wing design and active flow control, or the investigations in new aerospace materials. These configurations have now been extensively investigated and the potential benefits they can bring have been established. These designs take benefit directly from the relation shown in Equation 1.1, by which the induced drag of the wing reduces proportionally to the increase in aspect ratio, or quadratically with just the extension of span.

This alone could result in tremendous increases in aerodynamic efficiency. Nevertheless, there is still large uncertainty directly related to aero-structural, aeroelastic and control trade-offs that prevent the assertion of the feasibility of HARW concepts. Many of these challenges have not been thoroughly addressed, but have proven to initiate a large number of snowballing effects that call for trade-off analyses, compromising on the expected improvements by adding design sacrifices. To start, such designs are prone to larger bending moments, which demands reinforcement of some form. One solution to this problem could be to reinforce the structural wingbox by adding more material weight. However, this solution has a limited extent of application, as the wing cannot become substantially heavier without bringing overwhelming penalties.

Once this tipping point is reached, an alternative solution could consist of including external reinforcement, which is becoming a popular fix to mitigate structural strength requirements and alleviate aeroelastic effects caused by the increased flexibility and weight of the wing. If this design path is taken, yet another major design change must be commonly introduced, in which the wing is now arranged at the top of the fuselage

so that the struts can be attached at the bottom of the fuselage for support, as shown previously in Figure 1.1. Again, new trade-offs appear, inasmuch as this solution will still add weight through both brace and wingbox reinforcements. Drag will also increase, partly due to the new external non-lifting surfaces, and will potentially add interference drag near the wing-brace attachment when lifting struts are in place. Furthermore, design complexity will be exacerbated in the landing gear storage and system, as well as in the fuel system architecture. An additional disadvantage of this airframe layout appears during the consideration of safety requirements for vertical crash and ditching scenarios. On a low-wing configuration, fuselage and wing touch the ground almost simultaneously, so the weight of the airframe is distributed across both components. In contrast, a high-wing aircraft will have only the fuselage to carry weights and inertial loads. Hence, it is highly probable that additional reinforcement would be required for the fuselage, further increasing weight.

Bird strike is another safety concern derived from the strut-braced layout that might result in increased weight. Certification requirements specify several strike scenarios at which the airframe integrity must be maintained. At present, it is unclear how this requirement will be imposed on strut-braced wing, but to meet current specifications would demand greater reinforcement of these elements, thus penalising on weight.

Many of the already proposed HARW designs target lower cruise speeds, and therefore have small wing sweep angles. In such cases, the gust load alleviation benefits due to bending-torsion coupling inherent to swept-back wings would be minimal. In addition, longer wings yield a higher moment of inertia of the aircraft, which reduce roll manoeuvrability and aileron effectiveness. To improve these unfavourable characteristics, the use of spoilers can increase manoeuvrability at low-speeds near the ground where high controllability is required due to gusts, and add controllability at high-speeds where aileron effectiveness decreases due to the presence of shocks. The difference here is that a greater number of these devices may be needed when compared to conventional aircraft of the same category, thus increasing weight and system complexity.

HARWs are also highly flexible and therefore become especially susceptible to non-linear aeroelastic instabilities. Accurate models of this phenomena have not been achieved yet, so its understanding and prediction are limited. To avoid such instabilities, the wing needs to be stiffened, again penalising with the addition of structural weight. Further compromises to be considered due to the increased flexibility could be caused by wingtip strike due to gust encounter during ground roll, bumps on taxiways, or excessive sink rate at touchdown.

The combination of a high span with the use of supercritical aerofoils can aggravate the variations in longitudinal trim due to the aft-shifting of the aerodynamic center. This could result in an increase the trimming load required from the horizontal tail and

consequently higher trim drag during cruise. Other likely changes in wing geometry would further trade-offs, such as chords becoming smaller in exchange for smaller maximum lift coefficients and necessity for wingbox reinforcement, or increasing wing area along with higher parasitic drag and more material required [250–252].

Operational limitations are another disadvantage of this type of configuration. Such long wingspans make the designs incompatible with existing airport taxiways and gates. Furthermore, the category that the aircraft is defined with would need to be changed¹⁶. This could result in a significant reduction in the number of airports that can accommodate them, leading to a decreased demand for production.

A technological diversion from this problem is brought by folding wingtips. This device is designed to be folded up when the aircraft is on the stand or taxiing in order to reduce the operational space required, and is deflected down for flight, either to be locked or to enable the ability to flap for load alleviation. Again, this will bring weight penalties, as the structure will still need to be reinforced due to the elongated span, while the folding mechanism will add more mass. Moreover, this mechanism will not only add system complexity, but also increase safety risks, since the failure of the device on one side could create force imbalance and unstable handling of the aircraft.

The distinctive features of HARWs develop into a challenging task for the analysis of such wing designs. Accurate predictions of static and dynamic behaviours through the conventional linear modelling approaches become unreliable. Here, it is critical to capture the non-linear geometric effects on aerodynamic lift to compute the resulting shear forces and moments and accurately estimate the flight mechanics and aircraft performance. Recent studies are providing alternative modelling approaches that investigate the effects of including such geometric non-linearities into the analysis of HARWs using beam formulations that have highlighted the strong impact on structural dynamics [254,255].

The multi-disciplinary properties of these configurations have not been subjected to sufficient flight tests, which has not allowed the establishment of useful correlations with wind tunnel or CFD models. Hence, there is a gap in knowledge that cannot be covered with existing empirical databases and cannot be suitably modelled with traditional design tools at the conceptual design stage. Additional limitations and design compromises stem from aircraft with HARWs, but will not be discussed here any further for conciseness and the lesser relevance to the research being presented.

¹⁶In this case, element 2 of the ICAO Aerodrome Reference Code [253] would be altered, which designates a letter depending on the wingspan of the aircraft.

2.6 Conclusions

So far, some of the challenges being faced by the aviation industry have been presented. Specifically, the difficulties exposed concern the environmental and socio-economic need to reduce greenhouse emissions, for which numerous potential solutions are being put forward in each of the disciplines involved in aviation. In particular, the discussion here has centred on the approaches focusing on the reduction of induced drag, and especially on the ones making use of the fundamental relation between induced drag and aspect ratio to propose novel concepts with HARWs. However, the multi-disciplinary nature of aircraft design has also been highlighted, and with it the penalties that derive from this approach to address these challenges. Out of the many disadvantages that arise from the use of HARW configurations to increase aerodynamic efficiency, this research revolves around the compromises that it creates on the structural requirements since the optimisation of one is usually detrimental to the other. But alongside the technical obstacles caused by the coupling physical phenomena, this prospective answer to the aviation revolution accentuates the limitations of today's design methods to adapt to the required innovation, provide a comprehensive inclusion of the multi-disciplinary trade-offs, and enhance design freedom and decision-making, which are imperative traits to be able to satisfy this vision. With this, aero-structural design outcomes are usually a result of iterative design concessions that stem from the orthodox ideal solutions established for each discipline. The work carried out in the field of NELDs opens a wide range of research opportunities that can bring beneficial contributions to the requirements for the design of efficient wings, in which penalties are reduced and improvements are achieved without major increases in design complexity.

Chapter 3

General Overview of the Methodological Approach

This chapter provides an overall presentation of the design environment that has been developed to enable the execution of this research. A description of the conceptual design framework for the analysis of configurations with NELDs is provided, along with the workflow devised for the given inputs and outputs of the simulation. The physics-based methods employed at each stage of the process are introduced, whilst a more extensive description of the technical methodology is provided in Chapters 4 and 5. An overview of the main geometric and performance characteristics of the baseline aircraft is also presented, as well as the approach taken on the use of this data.

3.1 Conceptual Design Analysis Framework

This study aims to explore the relationship between increasing aerodynamic efficiency and reducing structural weight by simultaneously manipulating the lift distribution and wingspan. The study investigates the adoption of NELDs as a premise for efficient aero-structural design of HARWs, as opposed to conventional design methods which require compromising aerodynamic and structural efficiency independently. The attainment of such goals, which have been explicitly outlined in Section 1.3, has been brought about through the development of a multi-fidelity and interdisciplinary design environment that allows the conceptual stage analysis of civil aircraft under the principles of NELD. The framework integrates physics-based methods for more accurate modelling and synthesis of several aircraft design disciplines already in the early design phase, intending to provide extensive analysis of aircraft performance, whilst induced drag is reduced through span extension and the consequent weight increase is minimised as a result of NELDs.

Figure 3.1 shows a typical workflow followed in the conceptual design stage that

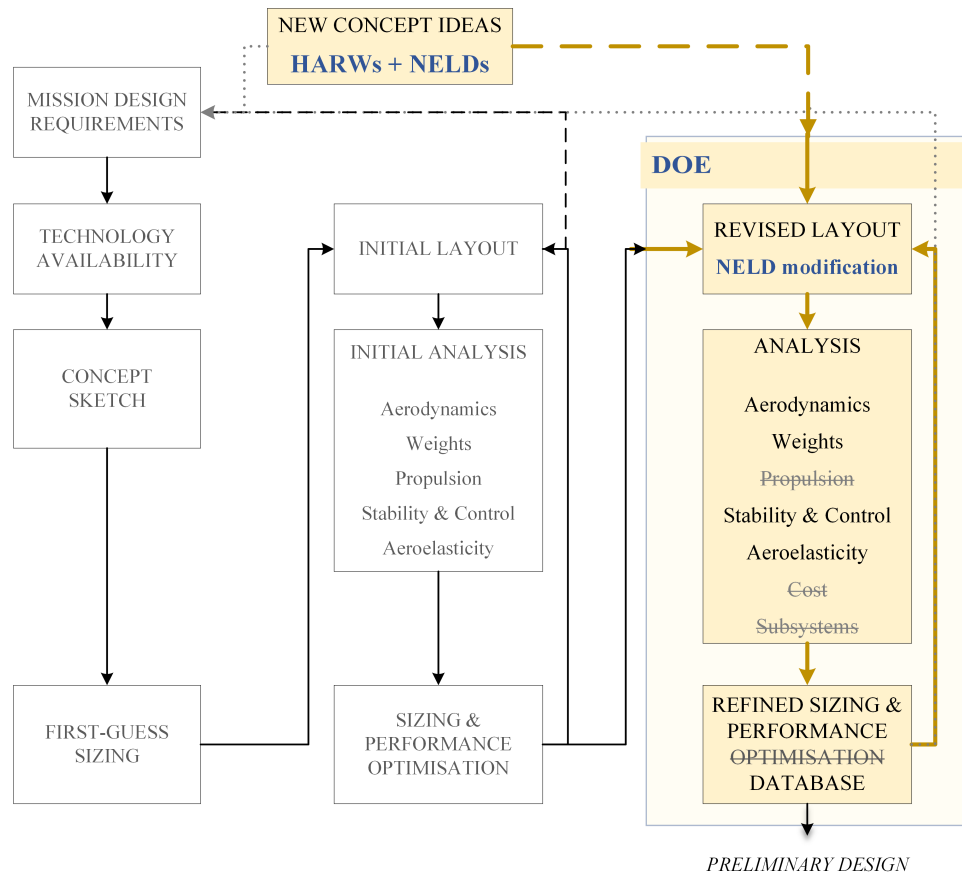


Figure 3.1: Modified workflow of a typical conceptual design process. Adapted from reference [13].

has been complemented and modified to illustrate the alternative process that has been proposed for the developed framework. Because the purpose of this research is not to provide an optimal HARW design but to evaluate the impact of the modification of wing lift distribution to efficiently increase the aspect ratio, there are several steps of this task sequence that are assumed as peripheral to the principal assessment. These blocks are shown in white and start with the definition of novel concepts. The aircraft envisioned here are given by the design of HARWs via the NELD theoretical approach. However, rather than studying the impact of these concepts on a clean-sheet design, the evaluation is performed on the re-designed wing of an existing aircraft. This reduces the number of factors that interfere with the principal focus of interest, converging the design space towards the high-priority set of variables. For this reason, the innovation block is fed later in the work process as a revision of the existing design. Hence, mission and top-level aircraft requirements are acquired from the selected baseline aircraft, against which the impact of the NELD wing modification will be compared and assessed. More details on the selection of the baseline aircraft model are discussed in Section 3.2. With this, the technology can be assumed to be at the most recent stage

in its development and incorporating all features available on the baseline aircraft, and the concept therefore remains akin to the selected baseline configuration. The first-guess sizing is obtained from the available information on the baseline model, and the initial layout analysis and optimisation are assumed to conclude as the baseline configuration. Now, the baseline model is revised via the re-design of its wing under various NELD approaches introduced in Section 2.2, yielding a wide design space of wings with increasing aspect ratio and corresponding twist distributions. This workflow has been highlighted in yellow and is the centre of interest of this research. Here, the multi-disciplinary database produced from the analysis and sizing of each configuration is used to extract the trends derived from the application of NELDs for HARWs and provide insight into the trade-offs of such designs. Consequently, rather than optimising for the “ideal” solution, the methodology involves utilising a design of experiments (DOE) approach to enhance comprehension of the evolution of key parameters when fundamental aero-structural variables are altered. This enables the elucidation of the underlying causes of one design’s superiority over another. Such a framework will also enable to determine whether this approach produces designs that yield at least the same degree of performance as the baseline aircraft. For these reasons, the resulting sizing and performance data are not fed back to re-define the mission design requirements. Instead, the feedback has only a comparative purpose in order to evaluate the impact and construct the trends of NELD modification.

3.1.1 Proposed Analysis Process

The design and analysis environment developed for this research aims to assess the impact caused by the modification of the baseline wing via NELDs on several fundamental aspects of the aircraft. The main focus lies in the aero-structural trade-offs, but the literature suggested that a more comprehensive treatment of the problem would promise to unveil additional by-products of such a design approach. Therefore, the optimistic opinions on the capabilities of these spanloads to improve later-directional flight dynamics inspired the diversification of the analysis approach. Moreover, the examination of overall performance at a mission level provides a more holistic evaluation of the related compromises. As it has been emphasised, a superlative outcome of a part does not imply a most favourable design as a whole. Hence, the multi-disciplinary character of this assessment aligns with the increasing necessity of the conceptual design stage to encompass as many dependencies as possible to enhance the completeness of the solution and reduce expensive compromises at later stages.

To enable the analyses described above and produce the desired data, the framework has been devised to integrate several multi-fidelity modules that communicate with each other. Generally, each module is dedicated to a specific discipline, the re-

sults of which are then transferred to the subsequent module, either in an iterative or a sequential manner. The analysis methods employed must be flexible and have minimal reliance on empirical data. Thus, the classical approaches for the prediction of aircraft characteristics at the conceptual design stage are avoided, such as those in references [13, 148, 176], and instead physics-based methods have been prioritised. With this, all the necessary data is produced to then be evaluated in a comparative approach and interpreted to extract the relevant trends and metrics, both interdependent and dissociated ones. Such a framework, the workflow of which is shown in Figure 3.2, provides a comprehensive insight into the fundamental principles governing wing conceptual design under NELD design philosophy. Given that the interest lies on the specific impact of varying the wing lift distribution on the aero-structural interactions, the number of design variables has been limited to reduce the influence of irrelevant design parameters on the analysis. The effect of such modifications on mission performance and flight dynamic characteristics is considered too. Hence, the main parameters that will be studied are associated and interact with wing shape, induced drag and structural weight. Furthermore, the proposed framework pursues the representation of results to enhance the interpretation of trends. This is somewhat contrasting to the present fashion of emerging conceptual design tools, which put a high value on the visual generation of the configuration and user interface for an interactive design of aircraft geometry. However, the purpose of the research presented here is to evaluate the behaviour of the design space, while the aircraft configuration being envisioned does not depart from conventional layouts.

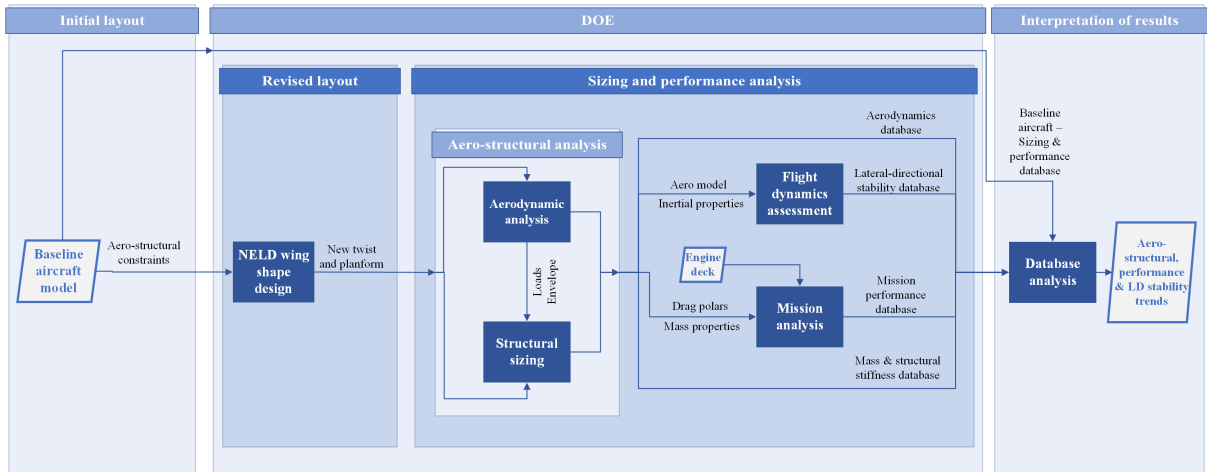


Figure 3.2: Flowchart of the design methodology followed.

The following lines provide an outline of the design and analysis process that has been developed, as well as a brief description of the methodology employed within the various framework components. A detailed exposition of the technical approach taken to redefine the layout has been allocated to Chapter 4. The methods employed

to perform the structural sizing and its results are exposed in Chapter 5. The performance analysis is presented in Chapter 6, along with the overall comparative analysis of the database. The flight dynamics assessment has been given a dedicated discussion in Chapter 7.

3.1.2 Initial Layout

The initial layout is completed by the use of a conventional and well-documented single-aisle airliner as the baseline aircraft, described in Section 3.2, which facilitates the validation and verification of results against available data. This approach directly provides a database from which to extract the aero-structural constraints necessary for the revised NELD design. Thus, an initial sizing via constraint analysis is not required, since the top-level aircraft requirements (TLARs) are given and define the minimum requisites that the new designs must satisfy. It is important to note that rather than the accuracy of the baseline data, it is the trends derived from the modification of the baseline model through NELD which expose the impact of this design approach. Thus, normalisation is the basis of the comparative approach. The baseline configuration is subjected to the same analysis process and methods that will be employed on the revised NELD layouts in order to compute the necessary benchmark data under equal fidelity and limitations.

3.1.3 Revised Layouts

The simulation starts with the modification of the baseline wing, for which an aero-structurally synthesised wing shape design methodology has been developed to enable the design of HARWs under the design principles of NELDs. Here, the aerodynamic and structural design constraints corresponding to each of the NELD theories are derived from the baseline aircraft and fixed, along with other wing planform and mission parameters, which are being discussed in this Section and have been listed in Appendix A. A reduced-order aerodynamic model based on the lifting line theory is implemented to compute the geometric parameters of a wing that provides the desired lift distribution. This target spanload is derived from NELD theory so as to satisfy the aero-structural requirements that have been prescribed for the wing. Thus, all wing geometries are computed under the principles of aero-structural equivalence. Numerous wing configurations can be obtained with this approach, depending on the span extension applied and the aero-structural constraints chosen according to the theory on which they are based. The methodology also provides an initial analysis of the aerodynamic and structural characteristics of the designed wings. This is done purely based on aerodynamic loads over an idealised beam representation of the wing at the design flight condition. These numerical results correspond to the analytical

solution of the problem.

3.1.4 Aerodynamic Analysis

For each wing geometry that has been computed, a second aerodynamic analysis is performed under a higher-fidelity approach, where VLM is employed to model the complete lifting surfaces layout, consisting of the wing and empennage, to obtain aerodynamic forces and moments of the full configuration. This approach allows the trimming of the aircraft to yield the aerodynamic spanloads at the chosen manoeuvre flight conditions. The loads envelope is calculated for the principal structural components of the airframe, these being the wing, empennage and fuselage, and is then used iteratively to size the structure. The computation of trimmed drag polars for low and high-speed conditions is also performed here, for which the contribution of profile drag is superimposed and added independently. The aerodynamic database is further enhanced with the estimation of force and moment derivatives with respect to angles, rotations and controls.

3.1.5 Structural Sizing

A structural analysis is performed to size the main structural components of each new configuration. The fidelity of the methods employed in this module has been increased as compared to the ones used in the NELD theory in order to provide the level of accuracy that is adequate for modern industry practices at the conceptual design stage. The structural model and analysis is based on beam theory, which enables consideration of further structural constraints such as torsion, aerofoil thickness and minimum skin gauge allowances, to discard infeasible solutions that do not satisfy the chosen structural integrity requirements. The analysis is performed to calculate the minimum amount of material under structural integrity requirements, which have been set to satisfy ultimate static buckling and strength. This enables the estimation of structural weight, as well as the calculation of stresses and displacements, to be directly driven by approximate physics phenomena instead of being confined to an unrepresentative database. These include a more precise approximation of the aircraft shape, as well as the sizing being performed under the load envelope derived from a selection of critical manoeuvre load cases. Material properties are also considered, although normalised across all configurations. Moreover, a distributed account of the loads enhances this by considering local conditions. However, the analysis of such a model based solely on fundamental principles would only give an estimate of structural weight. Hence, non-structural weights, such as systems or payload masses, are included and have been estimated from correlation to the baseline aircraft.

3.1.6 Aero-structural Analysis and Sizing

The methodology adopted here for structural sizing coupled with the impact of aerodynamic loads relies on the classic approach based on quasi-steady VLM. This accommodates the theoretical models employed in the modules described above for the prediction of aerodynamic and structural properties, and enables fast trim calculations for the aircraft under the assumption of small structural displacements. As aforementioned, at this early phase the conceptual sizing process presented for minimum weight is assessed by considering structural integrity requirements. However, the design constraints adopted here do not necessarily yield the global optimal solution for minimum structural weight.

3.1.7 Mission Analysis

The mission analysis module uses the resulting weight breakdown and the computed drag polars to predict the performance of every resulting configuration. The analysis is based on energy considerations, a low-order method that is fairly established for the evaluation of fixed-wing aircraft. This allows for rapid evaluation and optimisation of energy on each mission segment without being restrictive to this type of configuration, as it might be for other novel concepts. The mission profile has been equally defined for all aircraft configurations, and is optimised for maximum range. The propulsion system has been kept equal throughout all the configurations, enabling the normalisation of propulsive data across all models. The engine model has been integrated via a series of lookup tables that provides power and thrust characteristics for the different segments of the mission profile. Other considerations such as the analysis of engine emissions or noise footprint have not been included. This module returns the effect caused by NELD wing modification on the performance of the aircraft as a whole. Its impact on range and other TLARs is studied to unveil the effect of such parameters to the resulting aero-structural trade-offs.

3.1.8 Flight Dynamics Assessment

The assessment of lateral-directional flight characteristics of NELD wings for proverse yaw has been addressed through an aileron design methodology developed to provide roll authority and nullify adverse yaw. This problem has been formulated semi-analytically to provide insight into the consequences of deflecting an aileron placed on a region of negative induced drag. To do so, the wing shape design methodology has been extended to include the analysis of the relevant stability derivatives under the assumption of the above requirements, providing a set of wing-and-aileron geometries that satisfy the above conditions. Here, the design space is enlarged and

modified to look into the flight dynamic response due to coupled wing shape and aileron design. The results yielded by this approach enable to extract correlations between the lift distribution, aileron sizing, and the achievement of proverse yaw.

3.1.9 Interpretation of Results

The simulation of each design point through the analysis process described above has constructed the databases for aerodynamic loads, structural sizing, mass breakdown, performance and lateral-directional flight dynamics. The revised layout designs can now be compared against the baseline aircraft model in order to unveil the interdisciplinary impact caused by the modification of the wing via NELD. The framework has been developed under the terms of DOE, in such a way as to provide a systematic and efficient method that enables the study of the correlations between key input variables as well as the interdependence of the resulting output parameters. This enables approximate modelling of the behaviour of the response as a function of these factors in the form of trends and metrics. The set-based design (SBD) approach adopted throughout the analysis process enables keeping the computed design points until this stage and delay the elimination of potentially poor solutions until the complete mission analysis has been performed.

3.2 Baseline Aircraft Model

The applicability of NELD theory is being evaluated here as an underlying principle for the design of transonic wings for civil aircraft. Particularly, the assumptions and derivation of this approach lead to HARWs, which are being regarded as a potential step towards cleaner aviation. Majority of the research done on the design of wings with NELDs was performed on configurations that did not have a specific mission or did not present a realistic context in which to set the case study. It is difficult to extract the actual impact of this design approach from such cases, where the performance outcome cannot be put side by side with that of existing aircraft.

The configurations being proposed here maintain a conventional layout consisting of tube-and-wing, low cantilever wing, single-aisle, and twin-engine aircraft with engines mounted under the wing. Such an approach does not follow the commonly derived HARW configuration consisting of a brace-reinforced high-wing, in order to avoid yet further major design trade-offs discussed earlier in Section 2.5.3. The fact of maintaining a traditional airliner layout facilitates concentrating the effort towards the design of the wing, while enabling the assessment of the feasibility of avoiding the need for reinforcement or folding mechanisms. Hence, the Airbus A320 [14] model has been selected as the aircraft from which the baseline model will be constructed

for this study. This established airliner provides a suitable basis for comparison, for which a vast amount of data is publicly available. Nevertheless, the A320 is not exactly replicated here, but it is adopted to create a generic baseline that closely represents a civil transport aircraft in operation today.

3.2.1 Performance and TLARs

There are several airliner variants in service within the A320 family in service. The baseline model that has been constructed for this thesis has a maximum take-off weight of 73,500 kg and seats 180 passengers in a single-class configuration. It has a design range of 4,300 km and cruises at Mach 0.78 at 37,000 ft. The controls surfaces defined include two independent outboard ailerons for roll control, a trimmable tailplane and elevator in the pitch-axis, a single rudder for yawing motion, and trailing and leading edge flaps extending 70% of the span.

Additional performance specifications and top-level aircraft requirements are needed for the estimation of loads operational envelope or mission analysis, and haven been listed in Table A.3. The determination of the maximum lift coefficient $C_{L_{max}}$ is a critical but difficult task, and the reduced-order aerodynamic models that will be employed here do not have the capabilities to predict it. In this study, the value of $C_{L_{max}}$ has been obtained from the literature [256] and remains a constant parameter across configurations. The approach speed V_{app} is the main parameter necessary to correctly analyse the landing performance [257]. Furthermore, the mass-breakdown of primary and secondary elements is shown in Table A.2 [14]. It must be noted that the values of the non-structural components remain fixed in all revised models¹.

3.2.2 Geometrical Layout

The configuration and main dimensions of the baseline model are displayed in Figure 3.3, through a side, top and front view of the represented aircraft. The revised layouts modified via NELD wings maintain many of these parameters fixed and unvaried throughout the redesign and analysis. These include the dimensions of the fuselage, the horizontal stabiliser and the fin, as well as the positions of all lifting surfaces and the sizing of the control surfaces and deflections. Further geometric properties have been listed in Table A.3 and Figure A.1.

¹Any element presented with an asterisk (*) is considered a variable, and thus will be re-calculated in the analysis of each modification of the wing.

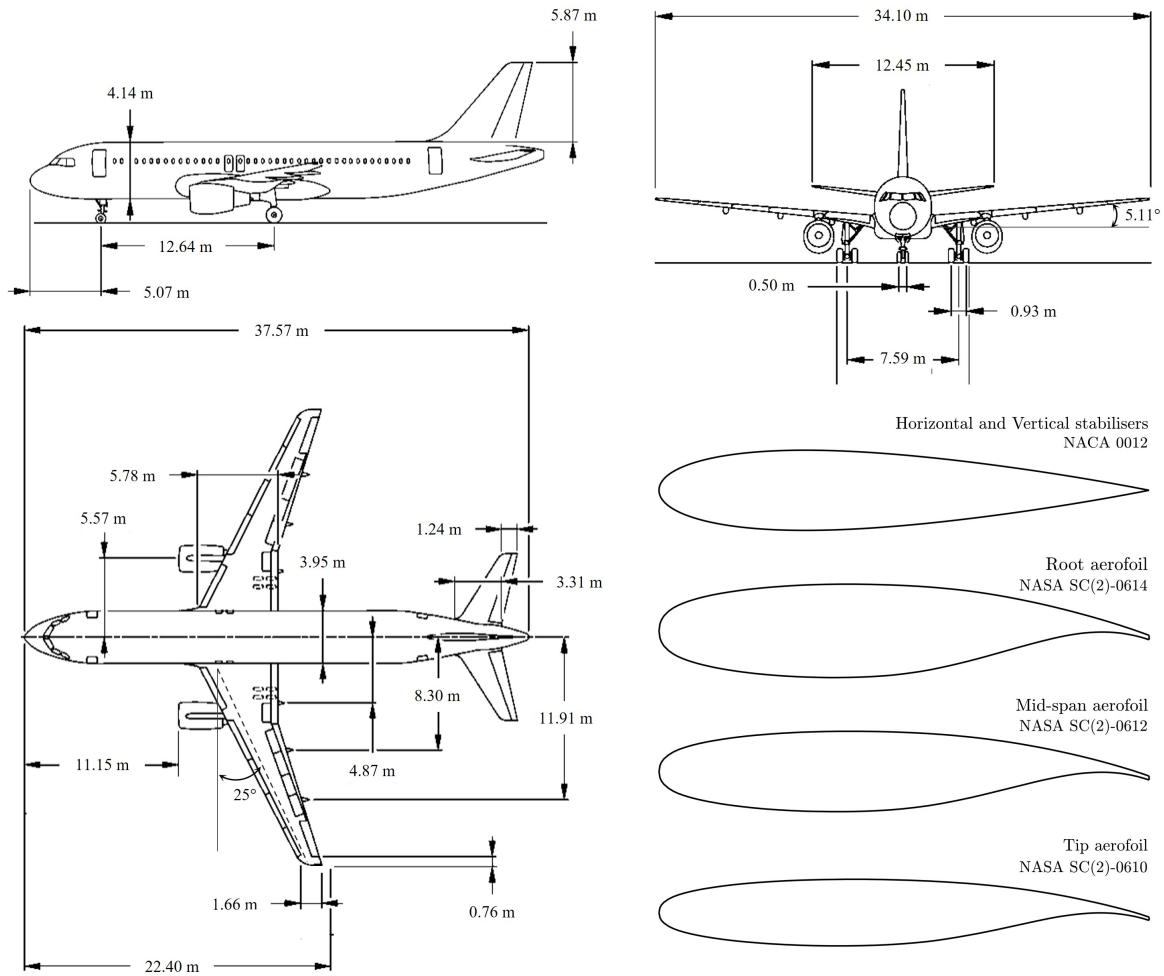


Figure 3.3: Baseline A320-like configuration. Adapted from reference [14].

3.2.3 A Brief Discussion on Aerofoil Design and Selection

Aerofoil design and selection are fundamental aspects of aerodynamics. Aerofoils have a significant impact on the performance and efficiency of a lifting surface, as well as on its stability and structural design. It is common practice for designers and manufacturers to develop customised aerofoils to meet the requirements of the particular aircraft. In such way, tailored performance in specific flight regimes can be prioritised. This praxis has earned large investments and its furtive product is frequently treasured by its owners. As critical as this is, the scope of this thesis excludes the investigation of the design and impact of aerofoils on wings with NELDs. The particularities of such spanloads could even demand an independent work of research by itself. For this reason, the selection of aerofoils for the baseline and subsequent NELD-modified models has been addressed from a perspective of simplicity and commonality, introducing established aerofoils whose characteristics have been extensively validated. Moreover, the aerodynamic analyses carried out in this thesis are limited to linear solvers, which

will not include stall or separation characteristics.

An airliner cruising at transonic speeds will have regions where the local airflow is supersonic, which can induce flow separation, shock waves and a high increase in drag. The initial velocity peak at the leading edge carries increased momentum and is more likely to transition to an adverse pressure gradient as speed decelerates on the upper surfaces, which can cause separation or stall. However, such momentum can be manipulated through the geometry, which must translate into a pressure distribution that reduces flow acceleration and still generates the required lift. For this reason, civil aircraft that fly at high subsonic flow regimes have shifted towards the use of supercritical aerofoils. The current predominance of this type of wing cross-section has been gained through the direct aerodynamic and structural benefits yielded by the design. The use of supercritical aerofoils is widespread in modern airliners and suits the current case study. The central concept behind such aerofoils is to delay the speed at which compressibility drag becomes significant by increasing the design critical and divergence Mach numbers. To do so, the flattened upper surface reduces the flow acceleration so that a weaker shock is generated closer to the trailing edge, while the highly cambered aft section generates further lift. The resulting aft-loaded wings bring disadvantages that will not be discussed in this work, but can be reviewed in references [258,259].

The selection of aerofoils adopted along the wing span was made accordingly, ensuring to provide the determined thickness-to-chord ratio distribution and estimated theoretical optimal lift coefficient at cruise flight. The thickness-to-chord has been linearly distributed across the span corresponding to the normalised spanwise locations determined in Table A.4 and shown in Figure A.1 along with other distributed aerofoil properties. These aerofoils and their corresponding characteristics have been applied as such in all wing modifications. The NASA SC(2) series [260] cambered supercritical profiles provided the expected characteristics and were designated as the wing aerofoils in all configurations.

The aerofoil selected for the tailplane and fin is the NACA 0012 symmetric profile. No camber is typically preferred for the empennage for improved stability and increased range of operation. The empennage is assumed to have a uniform spanwise thickness-to-chord ratio of 12%. All aerofoils used in this study are shown in Figure 3.3.

3.2.4 Design Flight Conditions

The design flight condition dictates several design and flight requirements of the aircraft. Moreover, the aerodynamic characteristics of the aircraft at cruise will be determinant of the overall mission performance. Hence, the definition of the aircraft

design lift coefficient for cruise is a careful and iterative process that often targets the maximum lift-to-drag ratio. This flight point is ideally reached when the aircraft is flying at the speed for minimum drag. In such cases, the exploration of drag polars with integration of viscous and compressible effects will provide a clear vision of the flight point yielding maximum lift-to-drag characteristics. However, several other considerations might shift the final value of the lift coefficient at which the aircraft will fly at cruise from the optimal operational cruise point, resonating on both wing planform parameters and desired mission requirements.

Here, the design flight condition parameters for the baseline aircraft have been prescribed and are shown in Table A.1. The design cruise speed has been determined according to reference data and typical cruise speeds for this aircraft type and category, as well as to stand within the valid range of the employed compressibility correction for the prediction of aerodynamic properties. More on the correction for compressibility effects is discussed in Section 4.1.1, but it is worth mentioning at this stage that an underestimation of the coefficients is expected under the methodology employed, due to its limitations later discussed in Section 4.1.1. However, the proposed comparative approach enables a systemic underestimation that is bounded such that comparisons amongst designs can be made.

Bearing the above factors in mind, and given that the overall configuration of the aircraft being simulated follows a conventional layout, the estimation of the design lift coefficient was refined through the application of an empirical relation [261] that is directly dependant on wing planform. This formulation, shown in Equation 3.1 is convenient here since the only component that is changing in the designs is the wing. The data correlation from which the equation was derived stems primarily from configurations that operate in the transonic regime and includes supercritical and advanced technology wings.

$$C_{L_{Design}} = (0.029 + 0.1843\mathcal{R}) \cos \Lambda_{c/4} \frac{1 + (t/c)_{mean}/10}{\sqrt{\mathcal{R}}} \quad (3.1)$$

3.2.5 Power-plant

The aircraft has been equipped with two high-bypass turbofan engines [262] able to produce 150 kN of thrust each at sea-level, which are representative of the propulsive performance given by Pratt & Whitney PW1100G or the LEAP-1A from CFM International. The engine model has been implemented in the form of a look-up table.

3.3 Conclusions

An overview of the methodological approach proposed in this thesis has been presented. The approach evaluates the impact of adopting an alternative design philosophy for HARW conceptual design based on the use of NELD theoretical assumptions that balances aerodynamic and structural requirements from first principles. The assessment is performed within an environment that integrates physics-based multidisciplinary and multi-fidelity models into modern conceptual design practises. The development of such a design framework enables extensive analysis of aircraft performance, whilst induced drag is reduced through span extension and the consequent weight increase is minimised as a result of NELDs. The resulting configurations are analysed to create a database from which to interpret and extract trends that enhance decision-making related to this type of wing.

The design environment has been devised as a modular framework to provide the capability of carrying out independent analyses of the designs for each of the incorporated disciplines, as well as to provide a holistic examination of the airframe. With this, the set-based approach allows to reflect the variation of wing spanload on the balancing of aero-structural efficiency, the attainment of proverse yaw coupled with aileron sizing, and the mission performance outcomes. To achieve this, several control variables have been presented that must be held constant to prevent external factors from affecting the results.

To reinforce the body of work that has been completed so far, which is largely theoretical, the proposed methodology has been deployed on a case study to generate a further in-depth and multifaceted understanding of the design of wings with NELD in a more realistic context. To do so, the Airbus A320 has been referenced as the archetype from which to build the generic baseline airliner model.

Overall, this initial framework complies with the vision for design capabilities with increased flexibility, modularity, discipline interaction and theoretical foundations in order to address the challenges posed by the ever-growing complexity of aircraft.

Chapter 4

Wing Shape Design Methodology

The wing shape design methodology described below has been envisioned as a tool to compute the geometry of the NELD wing that will be analysed against the baseline model. Furthermore, it has been devised to provide insight on the fundamental aspects of aero-structural equivalence theories, and the correlations they yield between induced drag and weight. To do so, lifting line theory is utilized at the core of this design tool. The relevance of this aerodynamic model has been explained in Section 2.1.1. This method has the capability of producing the necessary geometric and aerodynamic information of the desired NELD wings at the early conceptual design stage, but more importantly, it enables an elegant analysis of the aero-structural characteristics of lifting surfaces through the analytical comparison of key design variables. Therefore, the approach taken here follows a numerical implementation of the analytically derived methodology via MATLAB [263].

Various of the NELD theoretical approaches mentioned in Section 2.2 have been included in this study to provide a comprehensive juxtaposition of their impact on aero-structural characteristics. The theoretical basis of each of the NELD approaches employed will be expanded on in the following sections.

This stage of the design process obeys most of the fundamental assumptions adopted by lifting line theory. Nevertheless, the approach has been extended to account for some first order effects of compressibility that would be encountered at the chosen design flight condition. Spanwise variations in wing geometry, such as chord length or aerofoil thickness-to-chord ratios, along with the effects of sweep are also incorporated in the analysis. It should be noted that the presence of the fuselage has been ignored, for simplicity and to avoid altering the essential characteristics that are wished to be observed.

4.1 Theoretical Foundations

4.1.1 Lifting Line Theory

Lifting line theory defines a finite wing as a bound vortex filament consisting of an infinite number of horseshoe vortices that are shed downstream, as seen in Figure 2.1. This model predicts the main aerodynamic features of a lifting surface in a three-dimensional flow despite the considerable number of simplifications derived from the assumption of a potential flow, which is inviscid, incompressible and irrotational, and can only capture small disturbances. Moreover, the lifting line approximation becomes increasingly applicable as the ratio of the wing's span to its chord gets bigger. In this case, it is anticipated that the flow at a given spanwise station becomes closer to its two-dimensional equivalent as the wing aspect ratio is increased towards infinity.

Now, each bound vortex segment is placed at the quarter-chord line of the lifting surface and will have a local circulation $\Gamma(y)$, and is accompanied by a trailing vortex sheet of strength $-d\Gamma(y)/dy$, which allows the model to satisfy elemental fluid dynamics principles. Specifically, the Kutta condition¹ [27] is ensured by the positioning of the bound vortex and the fact that a three-dimensional flow will now have a spanwise component. The presence of spanwise pressure gradients, due to pressure continuity at the wingtips, causes corresponding spanwise cross-flows. Therefore, the requirement of the flow to avoid going around the trailing edge does not necessarily imply a stagnation point, but the need to specify a singular inviscid flow solution that is physically relevant while invoking viscous effects is still fulfilled. Furthermore, the cross-flows result in the shedding of a sheet of streamwise vorticity at the trailing edge, with strength determined by the spanwise variation of wing circulation. Thus, Helmholtz's first² and second³ laws [30] are also satisfied by the shedding and continuity of the trailing vortices described by the horseshoe model. Moreover, the flowfield associated with the trailing vorticity that is now modelled will provide further information on the aerodynamic drag force, since the work done by the vortical wake and its associated kinetic energy are effectively linked to induced drag. In addition, note that the vortex sheet is shown as remaining plane and aligned with the free-stream, not rolling up as in practice. This approximation is necessary to make the model tractable, and is justified on the basis that the vorticity nearest the wing has the greatest influence on

¹The Kutta condition states that the fluid moving around a body with a sharp trailing edge will leave the body at that point smoothly and will hold a stagnation point. By causing this condition to exist it is possible to find a solution to the infinite number of possible circulations that would satisfy the zero normal flow boundary condition.

²Helmholtz's first law states that vortex lines move with the flow and thus any free vorticity in a steady flow must have its vector aligned with the local flow velocity.

³Helmholtz's second law states that vortex lines cannot start or end in the fluid, and thus requires continuity of vorticity.

it. It becomes less tenable as the flow velocities depart further from the free-stream value and thus the loading on the wing increases. The lifting-line model is therefore most applicable to lightly-loaded wings at small angles of attack. The theory is then developed assuming that this restriction applies.

In order to compute the aerodynamic forces of a lifting surface it is necessary to find the variable circulation along the bound vortex on the wing. To do so, it is useful to start with the computation of the local force at a given section, which is assumed locally two-dimensional. Thus, it is known from this aerofoil theory that the lift coefficient can be expressed as

$$C_l(y) = C_{l_\alpha}(\alpha(y) - \alpha_0(y)) \quad (4.1)$$

where $\alpha(y)$ now represents the section angle-of-attack and is described by the difference between the wing incidence α and the locally induced angle-of-attack α_i as

$$\alpha(y) = \alpha - \alpha_i(y) \quad (4.2)$$

and the zero-lift angle $\alpha_0(y)$ as expressed here represents the contribution of both camber and twist of the section. These are rewritten respectively as

$$\alpha_0(y) = \alpha_{0L}(y) - \alpha_T(y) \quad (4.3)$$

The relation between local lift per unit span and the corresponding section lift coefficient is given by

$$l(y) = \frac{1}{2}\rho V_\infty^2 c(y) C_l(y) = \frac{1}{2}\rho V_\infty^2 c(y) C_{l_\alpha}(y) (\alpha - \alpha_i(y) - \alpha_{0L}(y) + \alpha_T(y)) \quad (4.4)$$

Here, the Kutta–Joukowski theorem is applied to relate the lift to circulation as

$$l(y) = \rho V_\infty \Gamma(y) \quad (4.5)$$

Equating the above expressions yields the lifting line equation

$$\frac{-2\Gamma(y)}{C_{l_\alpha}(y)c(y)V} - \alpha_i(y) + \alpha + \alpha_T(y) - \alpha_{0L} = 0 \quad (4.6)$$

representing the contribution of the angles viewed by the section to the local force, respectively: the effective angle-of-attack, the induced angle-of-attack, wing incidence angle α , geometric twist α_T and zero-lift angle-of-attack α_{0L} . Assuming a small angle approximation, the above formulation could be expressed in terms of the normal

velocity components induced by the wing's bound and wake vortices and of the free-stream flow. This shows that the boundary condition that requires no flow crossing the wing surface is satisfied. It is necessary now to calculate the induced angle-of-attack, which can be obtained through the application of the Biot-Savart law [5], and describes the velocity induced by a vortex filament at a given point as inversely proportional to the square of the distance to the filament. The collocation point is situated at the three-quarter point, and the assumption of the lifting surface having a large aspect ratio is now introduced to yield

$$\alpha_i(y) = \frac{1}{4\pi V_\infty} \int_{-s}^s \frac{d\Gamma(y_0)}{dy} \frac{dy_0}{y - y_0} \quad (4.7)$$

With this, the complete lifting line integro-differential equation can be expressed as

$$\frac{-2\Gamma(y)}{C_{l_\alpha}(y)c(y)V_\infty} - \frac{1}{4\pi V_\infty} \int_{-s}^s \frac{d\Gamma(y_0)}{dy} \frac{dy_0}{y - y_0} + \alpha + \alpha_T(y) - \alpha_{0L}(y) = 0 \quad (4.8)$$

With this, if the geometrical characteristics of a lifting surface are given, $\Gamma(y)$ is the only variable that remains unknown, and Equation 4.8 can be solved and used to obtain the circulation distribution.

In his work, Prandtl highlights [6] the difficulty of finding a suitable analytical expression for the spanwise circulation $\Gamma(y)$ from which a reasonable induced downwash distribution $w_i(y)$ would be obtained after solving the singular integral term given by the Biot-Savart law. The desired solution was initially found in the equation of a half-ellipse as follows

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{y}{s}\right)^2} \quad (4.9)$$

With this, the circulation distribution $\Gamma(y)$ at each spanwise position y would be determined by the maximum circulation at the root Γ_0 and the wing semi-span s . This solution is still of special interest, since the elliptic lift distribution yielded the minimum induced drag for a given wing, as will be shown shortly.

First, to generally solve the singular integral found in Equation 4.8 when $y = y_0$, it is necessary to evaluate it using Cauchy's principal value⁴ and Glauert's integral⁵ resolution. To do so, it is required to transform the reference system into trigonometric

⁴Cauchy principal value is a method for assigning values to certain improper integrals which would otherwise be undefined, and depends on the type of singularity in the integrand.

⁵Glauert's solution is used to obtain the principal value of the integral after the transformation of the variables into trigonometric coordinates. It is frequently used in many small-disturbance flow applications and has the form $PV \int_0^\pi \frac{\cos n\theta_0}{\cos\theta_0 - \cos\theta} d\theta_0 = \frac{\pi \sin n\theta}{\sin\theta}$ for $n = 0, 1, 2, \dots$ [264].

coordinates via

$$\begin{aligned} y &= s \cos \theta \\ dy &= -s \sin \theta \end{aligned} \quad (4.10)$$

This method was very useful during pre-computer era, as it allows to easily resolve the integro-differential term. Nevertheless, it remains of interest due to the insight that it gives on the relation between the spanload and the aerodynamic forces. Now, applying this transformation into Equation 4.9 yields

$$\Gamma(\theta) = \Gamma_0 \sin \theta \quad (4.11)$$

and then Equation 4.7 becomes

$$w_i(\theta) = \frac{-\Gamma_0}{2\pi b} \int_0^\pi \frac{\cos \theta_0}{\cos \theta_0 - \cos \theta} d\theta_0 = \frac{-\Gamma_0}{2b} \quad (4.12)$$

where the principal value of the integral has been obtained through Glauert's integral, yielding the elegant solution of a constant induced downwash.

Integration of the Kutta–Joukowski theorem in Equation 4.5 yield the three-dimensional expression for lift force

$$L(y) = \rho V_\infty \int_{-s}^s \Gamma(y) dy \quad (4.13)$$

while the induced drag can be obtained by rotating the two-dimensional lift vector by the induced flow, assuming small angles, as

$$D_i(y) = \rho V_\infty \int_{-s}^s \alpha_i(y) \Gamma(y) dy \quad (4.14)$$

The solution of Equations 4.13 and 4.14 is divided by dynamic pressure and lifting surface area to obtain the lift and induced drag coefficients, which for the elliptic lift distribution become

$$C_L = \frac{\pi b \Gamma_0}{2S V_\infty} \quad (4.15)$$

$$C_{D_i} = \frac{\pi \Gamma_0^2}{4S V_\infty^2} = \frac{C_L^2}{\pi \mathcal{R}} \quad (4.16)$$

Although this result is specific to the elliptic lift distribution, it already exhibits two features that are true of induced drag in general: (1) it increases as the square of the lift; (2) it decreases with increasing wing aspect ratio. The latter observation explains why flight vehicles that prioritise low values of induced drag tend to have high aspect ratio wings. Unfortunately and as aforementioned, the inconvenient realities of structural design limit the wing designer's ability to increase aspect ratio without penalties.

Another interesting result of the three-dimensional solutions involving the wing aspect ratio is that the reduction in lift towards the wing tips also corresponds to a drop in the lift-curve slope compared to the two-dimensional value. From Equations 4.15 and 4.1 it is possible to show that

$$C_L = \frac{2\pi}{1 + 2/\mathcal{R}}(\alpha - \alpha_0) \quad (4.17)$$

where the lift-curve slope is thus reduced by a factor of $(1 + 2/\mathcal{R})^{-1}$. Once again, the aerodynamic desirability of a high aspect ratio can be seen.

The value of the elliptic circulation distribution can be seen when it is compared to the general solution for the spanwise circulation distribution. This can be also expressed in terms of a trigonometric expansion

$$\Gamma(\theta) = 2bV_\infty \sum_{n=1}^{\infty} G_n \sin n\theta \quad (4.18)$$

for which $n = \text{odd}$ for a symmetric spanload. Substituting Equation 4.18 and its derivative into Equation 4.8 leads to the lifting line equation in trigonometric coordinates

$$\frac{-4b}{C_{l_\alpha}(\theta)c(\theta)} \sum_{n=1}^{\infty} G_n \sin n\theta - \sum_{n=1}^{\infty} nG_n \frac{\sin n\theta}{\sin \theta} + \alpha + \alpha_T - \alpha_{0L} = 0 \quad (4.19)$$

Equation 4.19 can be rearranged into a set of linear equations with N_{G_n} unknowns G_n in the following way

$$\sum_{n=1, \text{odd}}^{\infty} \left[\frac{-4b}{C_{l_\alpha}(\theta)c(\theta)} + \frac{n}{\sin \theta} \right] G_n \sin n\theta = \alpha + \alpha_T - \alpha_{0L} \quad (4.20)$$

The wing can be numerically discretised to result in an over-determined set of equations that can be numerically solved in a least squares sense. Once the Fourier G_n terms that describe the circulation distribution are known, it is possible to calculate the aerodynamic loads. Equation 4.5 can be rewritten in dimensionless form and transformed into trigonometric coordinates to derive the local lift coefficient as

$$C_l(y) = \frac{2\Gamma(y)}{V_\infty c(y)} \quad (4.21)$$

which in trigonometric coordinates becomes

$$C_l(\theta) = \frac{4b}{c(\theta)} \sum_{n=1}^{\infty} G_n \sin n\theta \quad (4.22)$$

and the wing lift coefficient can be obtained through the spanwise integration of Equations 4.21 or 4.22 yielding

$$C_L = \int_{-s}^s \frac{C_l(y)c(y)}{S} dy = \frac{2b^2}{S} \int_0^\pi \sum_{n=1}^{\infty} G_n \sin n\theta \sin \theta d\theta = \pi AR G_1 \quad (4.23)$$

Solution of Equation 4.23 derives in the lift coefficient being solely dependent on the first Fourier coefficient G_1 , which conveys that only the corresponding curve out of all the n superimposed distributions will contribute to the lift. The section induced drag coefficient follows from Equations 4.14 and 4.19

$$C_{d_i}(\theta) = C_l(\theta)\alpha_i(\theta) = \frac{4b}{c(\theta)} \sum_{n=1}^{\infty} G_n \sin n\theta \sum_{k=1}^{\infty} k G_k \frac{\sin k\theta}{\sin \theta} \quad (4.24)$$

Integrating the above equation and contrasting with Equation 4.23 yields the induced drag coefficient

$$C_{D_i} = \frac{C_L^2}{\pi AR} \left[1 + \sum_{n=2}^{\infty} \frac{n G_n^2}{G_1^2} \right] = (C_{D_i})_{elliptic} (1 + \delta) \quad (4.25)$$

As aforementioned in Chapter 1, Equation 4.25 corresponds to Equation 1.1, and demonstrates the essential relation that justifies the design of HARWs for induced drag reduction, which decreases reciprocally with increasing aspect ratio. Further inspection of Equation 4.25 yields additional dependencies that are fundamental for the basis of conceptual design: (1) the induced drag coefficient increases quadratically with the lift coefficient; and (2) since δ is always positive, it shows that the elliptic lift distribution is associated with the minimum induced drag. This can be observed by comparing Equation 4.25 to the corresponding solution for the elliptic case shown in Equation 4.16, which yields $G_n = 0$ for all $n > 1$.

The well-known span efficiency factor e can be now derived from Equation 4.25 as

$$e = \frac{1}{1 + \delta} \quad (4.26)$$

yielding the fundamental expression for the induced drag coefficient, shown in Equation 1.1

$$C_{D_i} = \frac{C_L^2}{e\pi AR} \quad (4.27)$$

The span efficiency factor can be used to measure the departure of a general spanload from the elliptic distribution in terms of aerodynamic efficiency. This parameter is of great interest in this research and will be further discussed.

The essential formulations and conclusions derived from the lifting line theory

have been now explained. In the interest of conciseness, the author directs the reader to Reference [5] for the detailed theoretical derivations.

The Finite Sweptback Wing

Some of the exposed limitations require further treatment of the theory in order to expand its applicability. In terms of geometry, the lifting line theory cannot directly resolve swept wings as it will exhibit some important differences compared to the unswept case. Once the quarter-chord line is allowed to sweep backwards or forwards, the downwash on the bound vortex includes an infinite self-induced contribution. This implies that at least some account of the finite chord must be taken for swept-back wings. As mentioned in Section 2.1.1, the lifting line theory has been extended to consider a wider range of wing geometries through methods such as Blackwell's or VLM, which are now easily resolved numerically. However, it is of interest in this section for the analysis to remain as analytic as possible.

A simplified approach considers the two-dimensional problem of an infinite wing. In a purely inviscid fluid, it is only the perpendicular component that determines its flow field. To analyse the flow, it is necessary to work in the plane perpendicular to the leading edge. The wing can be kept in its reference plane, and thus the incidence is represented by an oncoming flow at angle α , while the streamwise component is resolved into components parallel and perpendicular to the leading edge. The problem is shown in Figure 4.1. Then, according to thin-aerofoil theory, the two-dimensional

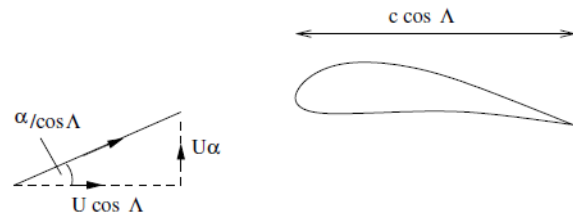


Figure 4.1: Oncoming flow to a swept wing in the plane perpendicular to the leading edge.

lift coefficient becomes

$$C_L = C_{L_\alpha}(\alpha - \alpha_0)\cos\Lambda \quad (4.28)$$

which shows that sweep reduces the lift-curve slope of an infinite wing by a factor of $\cos\Lambda$. This approach can be extended to incorporate the lumped-parameter representation of the two-dimensional wing, where the collocation points lie on a swept three-quarter chord line, centred between the horseshoe trailing vortices. The contribution of each bound vortex element to the lift is then $\rho V_\infty \Gamma \cos\Lambda$ per unit length.

There are some further considerations that limit the application of this approach. When introducing viscous effects, it cannot be expected that the two-dimensionality of the potential field to continue into the boundary layer, nor the boundary layer be purely two-dimensional in the streamwise plane, as it is now subjected to spanwise pressure gradients. If the path taken by a fluid particle inside and outside the boundary layer are compared, the particle inside will experience a greater curvature in its path due to the lower speeds in the boundary layer. This tends to thicken the boundary layer there in comparison to an unswept wing, as the flow has been retarded for longer. With this, swept wings end up to have worse drag and stall characteristics than their unswept counterparts.

Compressibility Effects

One of the main limitations of aerodynamic reduced order models based on potential flow solutions is that they assume an incompressible flow. However, density variations in the flow become more significant as the Mach number increases and the accuracy of the governing equations reduces. Such methods used in conceptual design therefore require to correct the initial incompressible results in order to consider small effects of compressibility when applied to high subsonic flight conditions. The loads are resolved in incompressible flow and the results are transformed to the compressible domain. Several similarity rules between compressible and incompressible flows exist, where the range of speeds at which the corrections can be applied vary with the order of the relation, but they are generally applicable within the speed region up to the onset of transonic effects, which extends up to $M < 0.6$. For wing configurations with sweep, the validity of such transformations must be applied to the wing-perpendicular Mach number $M_{\perp} = M \cos(\Lambda)$. Since $M_{\perp} < M$, swept wings can be modelled up to higher Mach values than unswept configurations.

A well established compressibility correction is the Prandtl-Glauert transformation [49], which can be applied to the section lift coefficient as

$$C_{l_{M>0}} = \frac{C_{l_{M=0}}}{\beta} \quad (4.29)$$

where

$$\beta = \sqrt{1 - M^2} \quad (4.30)$$

This correction provides a straightforward application but underestimates the coefficients at velocities just below the transonic region. At Mach numbers between 0.6 and 0.7 where transonic flow is likely to appear, the application of Prandtl-Glauert transformation will be suspect but with acceptable margins of error for the conceptual design

stage. Above this speed, transonic effects are certain and this correction becomes exceedingly unreliable. Moreover, most methods were developed for two-dimensional flows and they require special attention when utilised in three-dimensional flows, as they will not describe the induced drag accurately [265, 266]. Other higher-order methods require manipulation of the pressure coefficient and can model the non-linear aspects of the flow, providing a more accurate representation of the compressible flow-field, but increasing the complexity of the analysis.

The limitations just exposed will affect the results provided and have been taken into consideration. Regarding the planform, because the approximation used does not fully capture the spanload variation due to sweep, its impact on the consequent structural load will be predicted with less accuracy. Furthermore, the inherent structural issues of swept designs mean that the wing will also be heavier. These issues will depart the results from the analytical solutions provided by NELD theory, since they assumed straight wings. In terms of flow condition, the use of potential flow methods to calculate the aerodynamic characteristics of a wing at the high subsonic region will provide underpredicted values of induced drag. In further analysis within the set-based design context, these errors must be approximated and the margins should be wide enough such that a decision to remove a design point from consideration would not result in discarding a promising solution. Because the basis of this study is to provide comparative analysis, majority of these systematic errors may be neglected.

4.1.2 Prandtl's Theory for Aero-structural Equivalence

The elliptic circulation distribution obtained for minimum induced drag was just the first order term of a series equation, which had been presented as [6]

$$\Gamma(\xi) = \sqrt{1 - \xi^2}(\Gamma_0 + \Gamma_2 \xi^2 + \Gamma_4 \xi^4 + \dots) \quad (4.31)$$

where $\xi = y/s$. Alternatively, $\xi_{base} = y/s_{base}$ will represent the span of a given configuration normalised against the baseline span. This formulation describes a multi-variable problem that could yield many different solutions. However, the design concepts that Prandtl was previously evaluating always considered a prescribed wing. With this constraint, the elliptic curve would always be the optimal spanwise loading.

His perseverance took him to an alternative solution, presented in 1933 [7], which removed the span constraint and instead considered structural limitations. Prandtl expanded Equation 4.9 according to Equation 4.31 to account for second order circulation terms, becoming

$$\Gamma(y) = [\Gamma_0 + \Gamma_2 \xi^2] \sqrt{1 - \xi^2} \quad (4.32)$$

which would associate with a quadratic downwash distribution necessary to satisfy the problem conditions.

Let us expand on the derivations to show the mathematical implications of the above problem statement. Prandtl assumed the structural weight W_s of the wing to be proportional to the integrated bending moment $M_I(y)$ at each location as

$$W_s(y) = \frac{1}{K_s} M_I(y) = \frac{1}{K_s} \rho_\infty V_\infty \int_y^s \Gamma(y_0) (y_0 - y) dy_0 \quad (4.33)$$

with ρ_∞ representing the free-stream density. Here, the spar was assumed to have a constant section, which implies a rectangular planform and a constant proportionality factor K_s along the span. The prescribed structural weight is computed through the integration of the bending moment $M_I(y)$, which can be expressed in terms of the aerodynamic load as the moment of inertia of the lift

$$I_L = LR^2 = \rho_\infty V_\infty \int_{-s}^s \Gamma(y) y^2 dy \quad (4.34)$$

where R is radius of gyration of the lift distribution and L is the total lift. Betz derived the respective auxiliary constraints by applying a variation $\delta\Gamma$ to an auxiliary wing placed far downstream. Here, the streamwise induced velocities are negligible, and thus the wing is only subjected to a downwash given by $2w$. If the required circulation must satisfy $\Gamma(y)$, the addition of $\delta\Gamma$ will cause no variation on the induced drag D_i , complying with Helmholtz's theorem. With this, the expression for induced drag became

$$D_i = \frac{L^2}{8\pi\rho_\infty V_\infty^2 R^2} \frac{(1 - \mu/2)(1 - \mu/2 + \mu^2/4)}{(1 - \mu/4)^3} \quad (4.35)$$

where $\mu = -\Gamma_2/\Gamma_0$. From here, Prandtl's factor for aero-structural efficiency can be presented in terms of both nominal superimposed circulations and Fourier coefficients, from Equations 4.35 and 2.6 respectively

$$e_{W,P33} = \frac{(1 - \mu/2)(1 - \mu/2 + \mu^2/4)}{(1 - \mu/4)^3} = (1 + B_3 + 3B_3^2 + 3B_3^3) \quad (4.36)$$

where, to recall, $B_n = G_n/G_1$. This yields the tabulated results presented in the right-most column in Figure 4.2. Finally, the span b becomes a function of the spanload in the form of

$$b = 4R \sqrt{\frac{1 - \mu/4}{1 - \mu/2}} \quad (4.37)$$

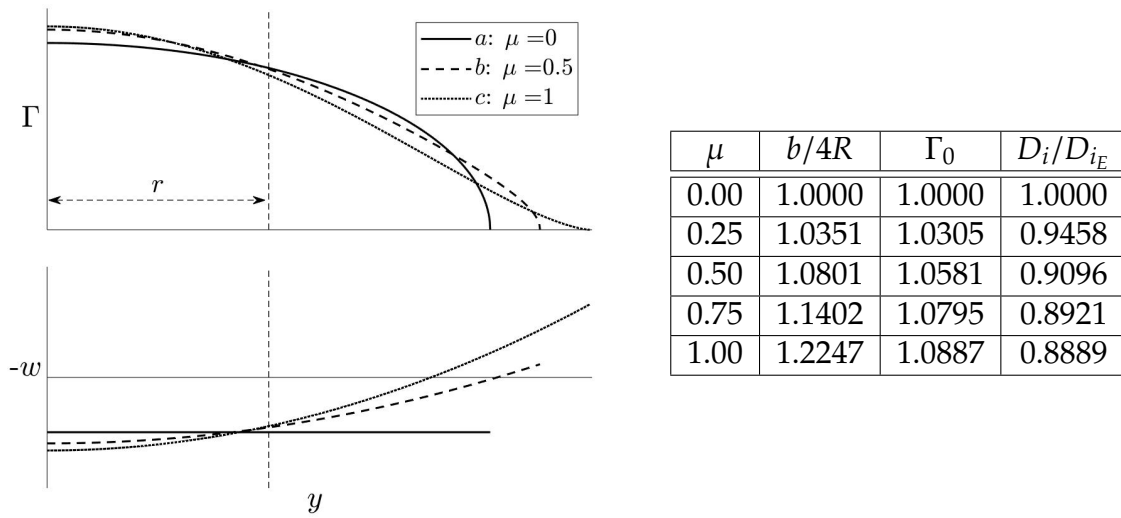


Figure 4.2: Reproduction of Prandtl's 1933 solutions [7]. Curves for circulation and downwash distributions (left). Tabulated span, circulation and induced drag factors for different values of μ (right).

The results of this approach are shown in Figure 4.2: curves *a* show the well-known elliptic circulation and constant downwash distributions, which agree with the earlier solution derived from the lifting line theory, as there is no span extension; curves *b* show an intermediate case given by an arbitrary span extension; curves *c* show the optimal bell-shaped distribution, having the same lift and integrated bending moment as the elliptic case but providing the aforementioned 11% reduction in induced drag with a 22.5% longer span. Prandtl examines this “optimal” point as a function of μ . This parameter expresses the ratio of the second linearly superposed circulation, driven by the coefficient Γ_2 , over the constant term in the expansion, which determines the elliptic spanload. Later it will be shown how the second term is directly linked to the structural weight [10]. Thus, when $\mu = 0$ the solution is elliptic and no additional circulation is superimposed. As μ increases, the second term re-shapes the original curve by superposition, while induced drag decreases with increasing span at a fixed radius of inertia R . However, for $\mu > 1$ the resulting circulation distributions start yielding negative values from the wingtip, which is not physically consistent nor aerodynamically efficient. The optimal point is therefore given by the curve that has a null gradient at the wingtips, corresponding to $\mu = 1$. With this, Prandtl elegantly found an alternative wing design approach that would suit both operational and structural design requirements at a conceptual design stage. The solution could be derived analytically and would provide minimum induced drag for a given design point.

4.1.3 Later Approaches to Aero-structural Efficiency

Many have tried to derive similar theoretical models for minimum induced drag and optimal structural weight. In 1950, Jones [52, 54] supported a design philosophy for practical wing design driven by structural requirements, and addressed it independently by prescribing the wing root bending moment $M_{X_{root}}$, given by

$$M_{X_{root}} = \rho_{\infty} V_{\infty} \int_0^s \Gamma(y) y dy \quad (4.38)$$

To reach Jones solution, Munk's mutual drag theorem [24] was assumed to find an additional distribution that varies the loading without increasing induced drag. As previously, far-field wake assumptions were applied to determine the downwash through potential theory. Herein, the downwash is defined by a first order polynomial to satisfy the boundary conditions of this approach, as opposed to the quadratic variation demanded by Prandtl's 1933 proposal.

Prandtl [7] had defined the radius of inertia of the lift distribution and held this parameter fixed and equal to that of the elliptic loading. Unlike Prandtl, Jones determines the circulation as a function of the non-dimensional spanwise position of the spanload centroid over the semi-span $\overline{y_C}$. This parameter needs to vary with span extension for the circulation to redistribute the loading. However, what remains fixed and equal to the baseline elliptic spanload is the dimensional centroid $y_C = \overline{y_C} s$. The loading distribution for a given lift and root bending moment can be expressed as a function of the non-dimensional centroid as

$$\Gamma(y) = \frac{L}{2\rho_{\infty} V_{\infty} s} \left(\frac{12}{\pi} - 6\overline{y_C} \right) \frac{\sqrt{s^2 - y^2}}{s} + \left(18\overline{y_C} - \frac{24}{\pi} \right) \frac{y^2}{s^2} \cos^{-1} \frac{s}{|y|} \quad (4.39)$$

and the expression for minimum induced drag yields

$$D_i = \frac{L^2}{\pi \rho_{\infty} V_{\infty} 4s^2} \left(\frac{9}{2} \pi^2 \overline{y_C}^2 - 12\pi \overline{y_C} + 9 \right) \quad (4.40)$$

With this, Jones' aero-structural efficiency factor is determined by the non-dimensional centroid of the load distribution as

$$e_{W,J50} = \frac{1}{s^2} \left(\frac{9}{2} \pi^2 \overline{y_C}^2 - 12\pi \overline{y_C} + 9 \right) \quad (4.41)$$

This result is of especial interest in the design of tailless aircraft [149], for which the location of the centre of lift is extremely important to achieve trim.

With this, Jones had obtained another set of non-elliptic spanloads that provided minimum induced drag and equivalent structural weight for any span extension. The

optimal solution was now determined pragmatically, given that the rate of induced drag reduction with span increase is insignificant for $1.15 < r_s < 1.33$, and further extensions will again lead to negative circulation at the wingtips. Here, r_s designates the ratio of span extension given by the span of the extended wing over the span of the baseline elliptic planform as $r_s = b/b_E$. The upper limit of this span extension range can be found by computing the first and second derivatives of Equation 4.40. The root of the first derivative is found at $r_s = 1.33$, and, like Prandtl's solution, it will not be a global minima, but an inflection point. Thus, the optimal point can be set at the lower limit, where Equation 4.39 yields a non-elliptic spanload that reduces induced drag by 15% with a 15% increase in span. If the span remains fixed, the resulting circulation curve agrees with the solution of an elliptic spanload, and any other solution will hold the same root bending moment as the elliptic case.

Klen and Viswanathan's work insists on the importance of incorporating structural considerations in aerodynamic design as a way of coupling both design limitations into a single theory. Initially, they derived a solution providing minimum induced drag for a prescribed wing-root bending moment [55], as defined in Equation 4.38. As Jones had shown, the downwash was required to vary linearly along the span. The circulation distribution they obtained was as follows

$$\Gamma(y) = r_s \left[\sqrt{1 - \left(\frac{y}{s}\right)^2} \left(2C_1 + 2\frac{C_2}{\pi} \right) - \frac{C_2}{\pi} \left(\frac{y}{s}\right)^2 \log \frac{1 - \sqrt{1 - \left(\frac{y}{s}\right)^2}}{1 + \sqrt{1 - \left(\frac{y}{s}\right)^2}} \right] \quad (4.42)$$

where the constants C_1 and C_2 are a function of the span ratio r_s [55].

Although Klein and Viswanathan employ a different approach to obtain the circulation distribution in Equation 4.42, this can be rewritten in terms of the inverse hyperbolic cosine to yield Jones' solution in Equation 4.39, and therefore yield the same load and induced drag behaviour with span extension, given that the boundary conditions of the problem are the same. In this case, Klein and Viswanathan defined the optimal point at the upper bound of r_s , where a reduction of 16% in induced drag is achieved with a 33% span extension. This result shows the negligible reduction in induced drag compared to span increase that Jones had observed previously.

Later, Klein and Viswanathan expanded the above theory to include more precise approximation of the wing structural weight [56,57], in which the structural boundary conditions were now determined by the integrated bending moment M_l and shear

force F_s distributions, expressed as

$$\begin{aligned}\int_0^s F_s(y)dy &= \rho_\infty V_\infty \int_0^s \int_y^s \Gamma(y_0)dy_0dy \\ \int_0^s M_I(y)dy &= \rho_\infty V_\infty \int_0^s \int_y^s \Gamma(y_0)(y_0 - y)dy_0dy\end{aligned}\quad (4.43)$$

which now required a quadratic downwash distribution. Altogether, the circulation distribution derived from these assumptions follows

$$\Gamma(y) = \frac{s}{s_E} \left[\sqrt{1 - \left(\frac{y}{s}\right)^2} \left(2C_1 + 2\frac{C_2}{\pi} + C_3\right) - \frac{C_2}{\pi} \left(\frac{y}{s}\right)^2 \log \frac{1 - \sqrt{1 - \left(\frac{y}{s}\right)^2}}{1 + \sqrt{1 - \left(\frac{y}{s}\right)^2}} - \frac{2}{3}C_3 \sqrt{\left(1 - \left(\frac{y}{s}\right)^2\right)^3} \right] \quad (4.44)$$

where $C_1 = C_2 = C_3 = f(r_s)$, and the ratio of induced drag over the elliptic case is expressed as

$$e_{W,KV75} = \frac{4}{r_s^6} \left[9r_s^4 - 40r_s^3 + \frac{145}{2} r_s^2 - 60r_s + \frac{75}{4} \right] \quad (4.45)$$

As per their publication, the minimization of Equation 4.45 would lead to a NELD exhibiting a 7% decrease in induced drag and a 16% increase in span. The obtained solution presents such a reduction in induced drag resulting from a 14% extension in span.

4.1.4 Theoretical Design for Proverse Yaw

The Horten brothers pondered what form of lift distribution should the H.II pursue. They had been wondering about this solution during the flight testing of the H.I in 1934, after realising that the yawing moment due to aileron deflection had been a determining factor for its controllability, which was one of the main concerns for improvement in flying wings. To make use the induced upwash, they applied a twist distribution that would yield the desired spanwise circulation to address adverse yaw, shown as

$$\Gamma(\theta) = \frac{8}{3\pi} \frac{L}{\rho V_\infty s} \sin^3 \theta \quad (4.46)$$

However, the disadvantage that the H.II brought was that it showed 33% higher induced drag compared to a wing with elliptic distribution and the same span. From here, they derived that a minimum of 16% span extension was needed in order to match the induced drag of an elliptic wing with the same design flight conditions and a same wing area.

Although the Hortens had realised that their method did not provide competent

efficiency, it was the first step on the determination of the potential advantages on flight dynamic characteristics of non-elliptic lift distributions. Indeed, they regarded this solution as especially beneficial to the flight characteristics of flying wings due to the shifting of the centre of lift they provided. Recalling from Section 2.4.1, balance and static horizontal trim of a flying wing requires that the centre of gravity corresponds with the longitudinal position of the centre of lift. Close to the centroid is where flow separation should first take place so that the locally decreased lift induces minimal moment around the centre of gravity. The bell-shaped distribution enhanced this characteristic whilst protecting the wing from wingtip stall at any flight condition and for any wing taper. In comparison, the elliptical lift distribution has its centre of lift at approximately $y/s = 0.42$ of the semispan and the maximum local lift coefficient occurs at $y/s = 0.88$. Preferably, the centroid of the bell-shaped spanload is found at $y/s = 0.33$ with a maximum local lift coefficient happening at $y/s = 0.37$, which proves to be more satisfactory in the fulfilment of the above demand.

4.2 Proposed Approach to NELD Wing Shape Design

The above theoretical derivations have been arranged into a single design process to create equivalent wings under the principles of NELD and obtain the semi-analytical results of their geometric, aerodynamic and structural properties. The process conceived to achieve this is presented in Figure 4.3.

The lifting line theory is the basis for the calculation of equivalent NELD wing designs. To start, it is necessary to derive the required aerodynamic and structural characteristics from the baseline wing. Using the corresponding data from Table A.3 and Figure 3.3 for the planform characteristics, Figure A.1 for the wing twist distribution, and Table A.4 for the aerofoil characteristics, Equation 4.20 is used, setting a null wing angle-of-attack $\alpha = 0$, to obtain the Fourier coefficients G_n for n , where n represents a large number of superimposed sinusoidal functions. As specified in Section 4.1.1, Equation 4.20 can be imposed to be satisfied at each value of θ to result in an over-determined system of equations. In this case, the wing has been discretised from tip to tip into $N_\theta = 301$ sections to reduce numerical errors while maintaining the time and size of the computation efficient.

Now, Equation 4.13 is employed to compute the lift force of the baseline wing at the design condition listed in Table 4.1. This result becomes the prescribed lift required to define the circulation distributions for NELDs. Similarly, Equation 4.23 is used to compute the wing design C_L . The induced drag force and coefficient are also calculated with Equations 4.14

The structural requirements derived from the aerodynamic loads can be also com-

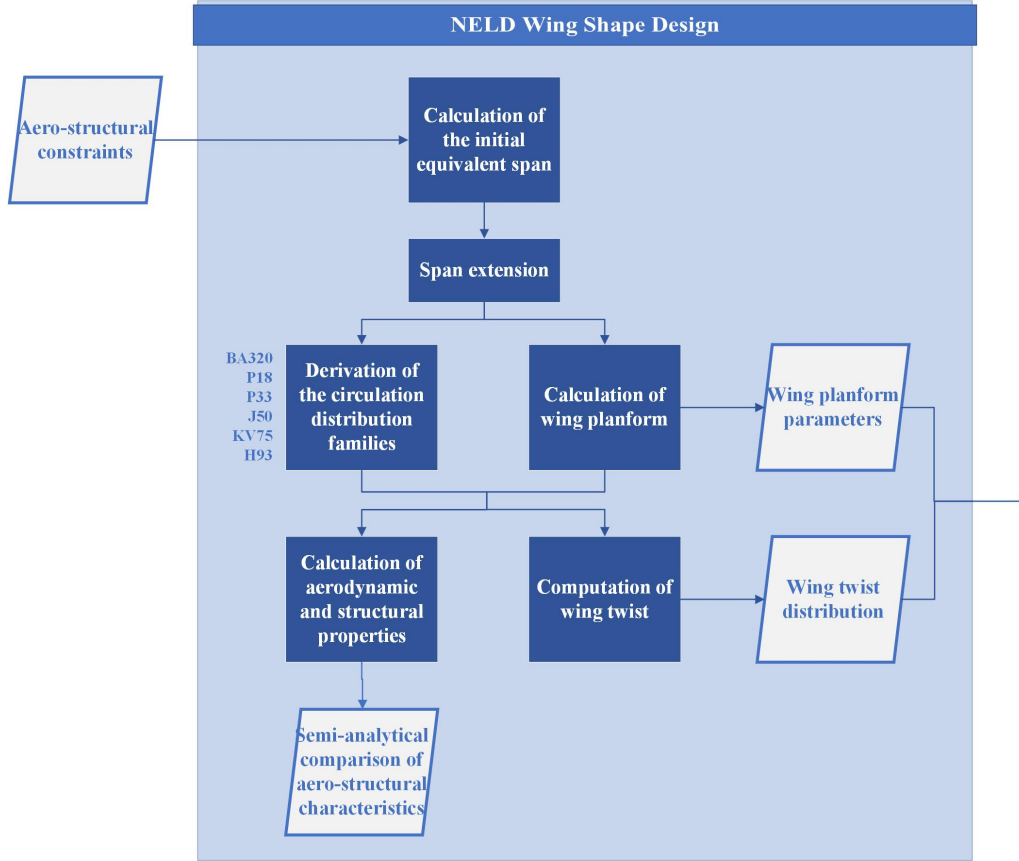


Figure 4.3: Workflow for the proposed NELD wing shape design methodology.

puted, with the use of Equations 4.33, 4.34 and 4.38 to calculate the structural weight, the moment of inertia of the lift and the root bending moment respectively. The results are shown in Table 4.1, where the incompressible values have been listed here.

Table 4.1: Aerodynamic and structural forces and moments of the baseline wing at the design condition without compressibility correction.

Lift, L	595.50 kN
Induced drag, D_i	12.35 kN
Lift coefficient, C_L	0.5267
Induced drag coefficient, C_{D_i}	0.0109
Integrated bending moment, M_I	37,149 kNm ²
Root bending moment, $M_{X_{root}}$	1,919 kNm

Now that the required aerodynamic and structural constraints are known, it is necessary to derive the aero-structurally equivalent wing from which the NELD span extension will depart. The NELD theories define their baseline wing as the one corresponding to the elliptic spanload. That is, one of the solutions of the family of curves obtained always yields the equation for the half-ellipse. This result is of great analytical interest, since it is the reference solution for minimum drag for a fixed span. However,

the need for non-elliptic load distribution against an elliptic spanload is well established in today's aircraft, as discussed in Chapter 2, and its practical implications and trade-offs are well known to designers and manufacturers. Thus, comparison against an elliptic spanload will not provide any new information about the aero-structural benefits achieved by the theoretical approach of NELD theories. The majority of analytical and numerical studies found in existing literature have mainly adopted this methodology, where the elliptic spanload described the baseline wing. Instead, this research proposes a further applied and pragmatic focus, and therefore uses existing aircraft as the benchmark against which to evaluate NELD theory as an alternative conceptual design philosophy that will further enlighten aero-structural trade-offs to enable HARW designs with reduced structural penalties. To do so, it is necessary to consider an additional step to the design process to provide a meaningful basis for comparison through the application of NELD theory. The approach taken here consists on producing a wing with elliptic loading that is equivalent to the baseline model. Now, referring back to the discussion on wing equivalence in Section 2.2.2, the equivalent baseline NELD has not been computed geometrically, but according to the aero-structural design constraints that are being prescribed, namely, the wing lift at the design condition and the corresponding structural constraint. In this work, several circulation distributions from the various NELD theories are explored, which include Prandtl's 1933 approach (P33), Jones' 1950 work (J50), Klein and Viswanathan reformulation from 1975 (KV75), as well as Horten's proposal to address adverse yaw (H93). A summary of their main theoretical characteristics is presented in Table 4.2.

To start, the span for a wing with elliptic loading that will yield the same integrated bending moment as the baseline model can be derived from Equations 4.32, 4.13 and 4.34, and imposing $\mu = 1$, to yield

$$b_{ibm_e} = \sqrt{\frac{16I_L}{L}} \quad (4.47)$$

Similarly, Equations 4.39, 4.13 and 4.38 can be used to compute the span of a wing with elliptic loading that will have the same root bending moment as the baseline model, provided that the non-dimensional centroid corresponding to an elliptic distribution equals $4/(3\pi)$, yielding

$$b_{rbm_e} = 3\pi \frac{M_{X_{root}}}{L} \quad (4.48)$$

From the above equations it derives that a wing with equivalent elliptic loading requires a reduction in span of $b_{ibm} = 0.9273b_{BA320}$ to yield the same integrated bending moment as the baseline wing, and $b_{rbm} = 0.8924b_{BA320}$ to match the root bending moment. To satisfy both integrated bending moment and integrated shear force simultaneously, both Equations 4.47 and 4.48 must converge to the same value. However,

this will also be dependant on the design lift constraint. Given the particular baseline aircraft and design flight condition, this would only be satisfied for an equivalent wing with an initial span of $b_{ibm\&rbm_e} = 0.9637b_{BA320}$ and a reduction in the lift of 7.6%. Alternatively, the baseline wing properties and flight condition parameters could be iteratively revised until Equations 4.47 and 4.48 agreed. Nevertheless, this could yield other design penalties in trim or mission requirements. For the purpose of this study, it was decided to maintain the same design lift for all configurations. Consequently, only the integrated bending moment was fixed for the configurations derived from the P18, P33, KV75 and H93 family curves, while the J50 designs were prescribed for the root bending moment. Choosing the root bending moment for the KV75 family, which is equivalent to the integrated shear force at the root, would yield significantly lower reductions in induced drag. Alternatively, if no intermediate transformation to a baseline elliptic spanload is performed, both integrated shear and bending moment may be retained. Derivation and solution of these approach is shown in Appendix B.1.

With this, the span must be extended in order to observe the impact of NELD aerostuctural equivalence⁶. It is now possible to lay out the range of the key independent variable that will be changed. Thus, the span will now vary from the corresponding initial span computed from Equation 4.47 or 4.48, up to the span increase given by a reference HARW aircraft concept, which is given by the Subsonic Ultra Green Aircraft Research [245–247] (SUGAR), where revolutionary concepts are being constructed by Boeing and NASA⁷. The limits for span extension are presented below as the ratio of the modified wing span to the span of the baseline BA320 model

$$\begin{aligned} \text{For equivalent integrated bending moment: } r_s &= 0.9273 \dots 1.5191 \\ \text{For equivalent root bending moment: } r_s &= 0.8924 \dots 1.5191 \end{aligned} \quad (4.49)$$

The circulation distributions for each family of curves can be now obtained. To do so, the lifting line theory is used in its inverse design approach. The derivations shown in Section 4.1.1 are mainly concerned with solving the lifting-line equation for a prescribed wing geometry, and from here obtain the corresponding Fourier coefficients that will describe its circulation distribution. Alternatively, however, one can specify $\Gamma(y)$ and evaluate the downwash angle to find a geometry that yields such a circulation and estimate its aerodynamic characteristics. This process would correspond to the example given with the elliptic spanload,

To do so, Equations 4.32, 4.42, 4.44 and 4.46 respectively representing the NELD families are written in trigonometric coordinates, and Equation 4.18 is solved for G_n

⁶To recall, the wing area remains fixed in this study and therefore the aspect ratio varies only as a function of the squared span.

⁷In collaboration with Virginia Tech and Georgia Institute of Technology.

as an overdetermined system of equations. Then, the lifting line equation simply specifies the relationship between chord, camber and twist distributions that must be satisfied. The formulations corresponding to the various theoretical approaches have been summarised in Table 4.2 in Fourier series form. As previously demonstrated, it can be seen that the elliptic spanload only has one non-zero Fourier coefficient, and that the Prandtl and Horten families are constructed with two superimposed curves, while the remaining configurations only converge to a negligible error when a higher order is reached. The spanwise circulations of the various NELD families have been plotted for the complete set of span extensions against the baseline spanload and can be found in Figure B.1.

Table 4.2: Summary of circulation distributions investigated.

Author	Label	Formulation $\Gamma(\theta)/2bV_\infty$	Characteristic constraint	Ideal Solution
Prandtl Elliptic	P18	$G_1 \sin n\theta$	Span	Reference
Prandtl NELD	P33	$G_1 \sin n\theta + G_3 \sin 3\theta$	Integrated bending moment	$D_i = -11.1\%$ $b = +22.5\%$
Jones	J50	$\sum_{n_{odd}}^{n=150} G_n \sin n\theta$	Root bending moment	$D_i = -15.0\%$ $b = +15.0\%$
Klein and Viswanathan	KV75	$\sum_{n_{odd}}^{n=150} G_n \sin n\theta$	Integrated bending moment and shear force	$D_i = -7.0\%$ $b = +14.0\%$
Horten	H93	$G_1 \sin n\theta + G_3 \sin 3\theta$	Proverse yaw	$D_i = -5.1\%$ $b = +18.5\%$
Baseline A320	BA320	$\sum_{n_{odd}}^{n=150} G_n \sin n\theta$	-	Benchmark

Figure 4.4 shows the baseline circulation distribution at the design flight condition compared against the equivalent ideal circulations for each NELD theoretical approach. Here, the circulation is presented normalised against the value of the circulation at the root, while the spanwise axis has been normalised against the corresponding semispan. Each of the corresponding spanloads maintain the structural constraint and the design lift that has been prescribed. Comparing the resulting span and induced drag values to those shown in Table 4.2, which show the span increase from the equivalent wing with elliptic spanload, there is a further improvement on both the span extension required and induced drag reduction. The reason for requiring smaller extensions stems from the NELD modifications being applied from the smaller equivalent wingspan values computed from Equations 4.47 and 4.48. On the other hand, the correlation to induced drag reduction must be analysed independently, and is given by the fact that the baseline configuration does not have an elliptic spanload, but carries an inboard-shifted centre of lift. Figure 4.4 also includes the equivalent elliptic spanload for fixed integrated bending moment.

From here, the spanwise aerodynamic and structural loads can be calculated using

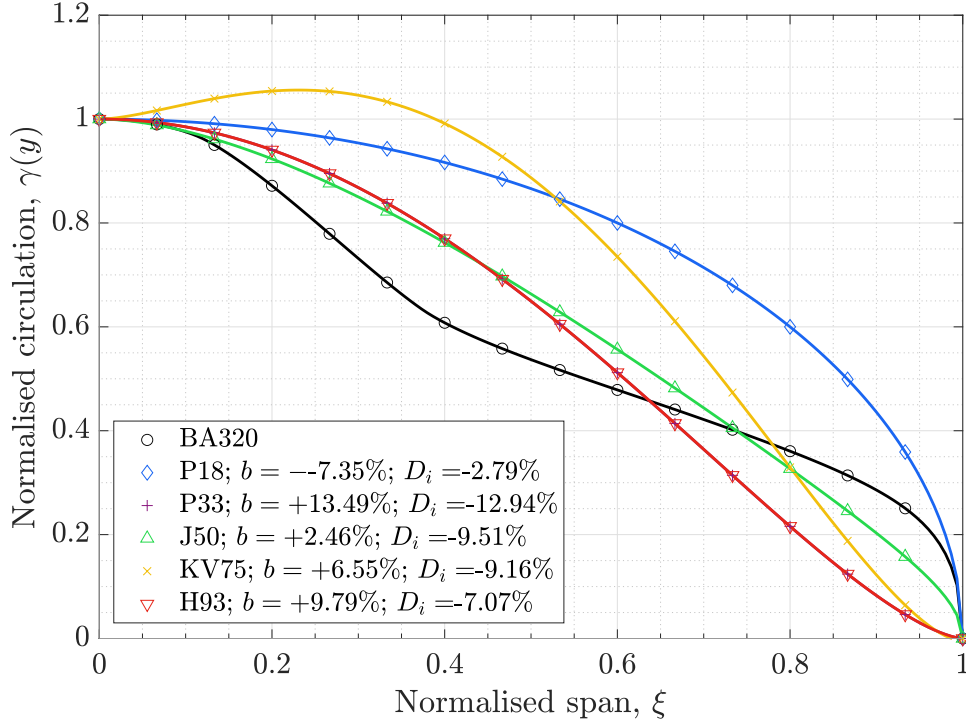


Figure 4.4: Comparison of ideal NELD circulation distributions against the baseline wing, its equivalent elliptic spanload.

the equations derived in Section 4.1.1. The evolution of the local lift coefficient distribution can be observed in Figure B.2 as the spanload changes with span extension. It is shown how the lift coefficient becomes negative for the NELD configurations once the span extension is above the upper bounds derived in Section 4.1.1. As the negative values of lift coefficient become higher, the spanwise position where the induced drag coefficient becomes negative is displaced inboard, as shown in Figure B.3, which eventually leads to positive induced drag being generated at the tip again. The consequent impact on shear force and bending moment over the semispan can be seen in Figures B.4 and B.5 respectively.

Now, with the range of span extension values that will be analysed given in Equation 4.49, it is also possible to compute the planform geometries for the new wings. The chord distribution for the ideal analytical case are shown in Figure 4.5. The local chord has been normalised against the carry-through chord at the wing-fuselage intersection of the baseline A320 wing, presented in Figure 3.3. Baseline and NELD wing families share a fixed value of wing area found in Table A.3.

Figure 4.6 shows the variation of aero-structural characteristics with span extension for each family of curves as a ratio to the BA320 values. The BA320 spanload has also been adapted for wing geometries with increasing span and fixed lift. Similarly, the trends of a wing with elliptic circulation P18 have also been included for comparison.

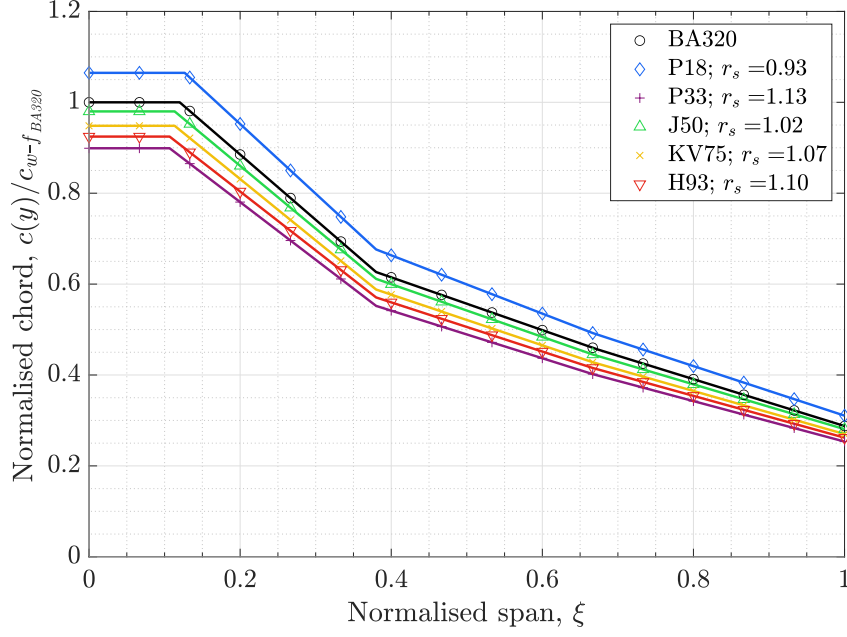


Figure 4.5: Comparison of computed chord distributions between ideal NELD wing, baseline configuration.

Starting here, it can be seen that, as expected, the induced drag ratio varies inversely proportional to the aspect ratio, or in this case, to the squared span, and the span efficiency factor as: $D_i \propto 1/Re = 1/b^2e$. It can be seen that the trends of the families that do not remain fixed to a structural constraint as the span is extended show greater gradients in both drag reduction and structural load increase. Conversely, the NELD families shift the position of the non-dimensional centre of lift to sustain the structural requirement, which will penalise on the span efficiency factor to reduce the benefits achieved in induced drag reduction. A quick inspection of these trends clearly exposes the known trade-offs between aerodynamic efficiency and dramatic structural penalties. It must be noticed that the configurations obtained for span extensions falling in corresponding range of dotted lines are physically inconsistent and inappropriate in practice, since the spanwise circulations will yield negative values, which will start developing at the wingtips. The design point corresponding to the analytical ideal solution shown in Figure 4.4 has been highlighted with a pentagram marker.

While the BA320 and P18 trends are naturally disadvantageous outside their design point, the H93 spanload provides a range of beneficial configurations that yield improved aero-structural characteristics found for a 6.7% to 13.5% span extension from the baseline. This translates to a span extension from the P18 design point of about 15% to 19.5%, which agrees with Horten’s developments explained in Section 4.1.3. In the case of the KV75 family, the decision of trading a fixed integrated shear force for a sustained design lift is observed here through a constant 3.9% increase in root

bending moment. The P33 family sees a increase in root bending moment that ranges between the 1% and 3.9%. The spanloads for root bending moment constraint are analytically more beneficial as they provide a general decrease in structural weight from the baseline wing, going between the 7.4% to the 1.1%⁸.

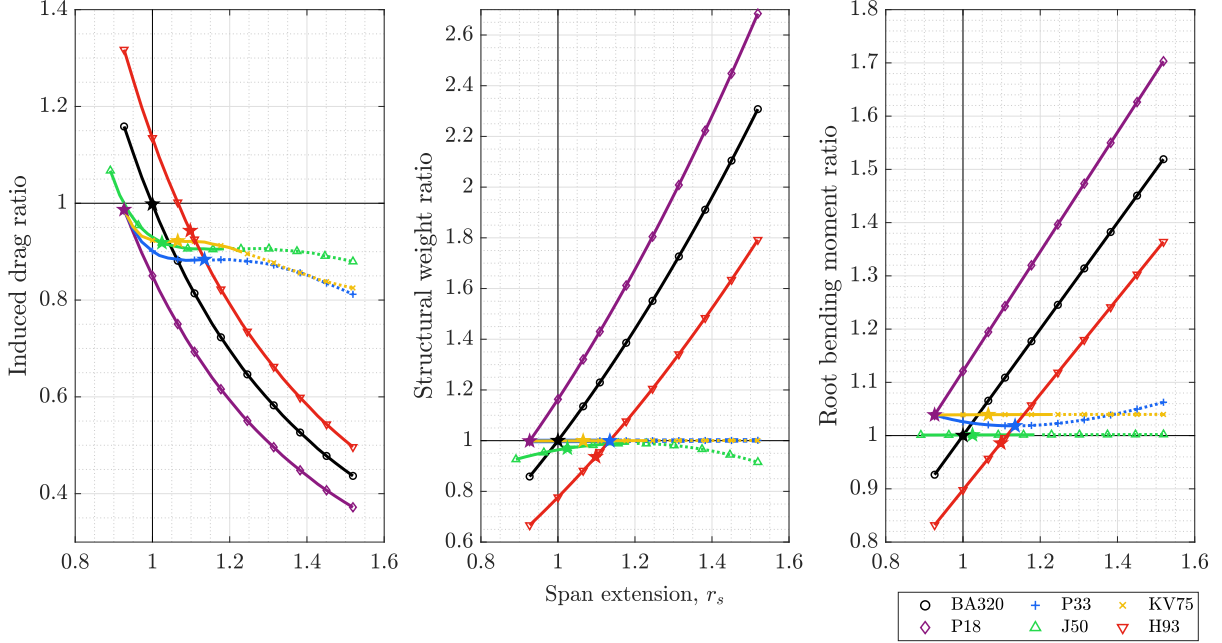


Figure 4.6: Variation of aero-structural characteristics with span extension.

To further dissect the results in Figure 4.6, the trends can be examined as a function of three variables. Firstly, the span efficiency factor is presented by combining Equations 4.25 and 4.26 to yield:

$$e = \frac{1}{1 + \sum_{n=3}^{\infty} \frac{nG_n^2}{G_1^2}} \quad (4.50)$$

Secondly, the centre of lift is expressed as a function of the circulation distribution and presented in its non-dimensional form as

$$\frac{\overline{y_C}}{s} = \frac{1}{s} \frac{\int_{-s}^s y \Gamma(y) dy}{\int_{-s}^s \Gamma(y) dy} = \frac{\int_{\pi/2}^0 \cos \theta \sin \theta \Gamma(\theta) d\theta}{\int_{\pi/2}^0 \sin \theta \Gamma(\theta) d\theta} \quad (4.51)$$

Solution of the above equation results in a summation of two terms, in which one is a series of Fourier coefficients and the other one is the non-dimensional value of the

⁸Alternatively to this analysis, if the NELD trends shown in Figure 4.6 were to be normalised against the P18 design point, the resulting variations would be equal to the outcomes discussed in Sections 4.1.2 and 4.1.3, and the analytical ideal design would correspond the that shown in Table 4.2.

centroid for an elliptic distribution:

$$\overline{y_C} = (\overline{y_C})_e + f(G_n) = \frac{4}{3\pi} - \left[\frac{4}{\pi G_1} \sum_{m=2}^{\infty} \frac{G_{n=2m-1}}{(-1)^{1+m}(-3-4m+4m^2)} \right]. \quad (4.52)$$

The third variable is given by the spanwise radius of inertia of the lift distribution, presented below normalised to the semispan, which becomes solely dependant on the first two Fourier coefficients, giving

$$R = \left[\frac{\rho V}{sL} \int_{-s}^s \Gamma(y) y^2 dy \right]^{1/2} = \frac{Vb^2}{4} \sqrt{\frac{\pi \rho}{2L} (G_1 + G_3)}. \quad (4.53)$$

The evolution of these three variables with span extension is presented in Figure 4.7, where the corresponding non-dimensional results have been plotted on the left graph, whilst the dimensional form, multiplying by the semispan, is shown on the right. Here, the span efficiency has been divided by a factor of 10 for a better graphical presentation. One of the key aspects of NELD families is illustrated here, where it can be seen that maintaining a fixed non-dimensional arm for the lift will result in a constant value of span efficiency factor. It has been already seen that this results in major decreases in induced drag but dramatic penalties in structural loads. Instead, retaining the dimensional value of the lift arm virtually constant with increasing span will penalise the span efficiency but minimise the compromise on structural loads.

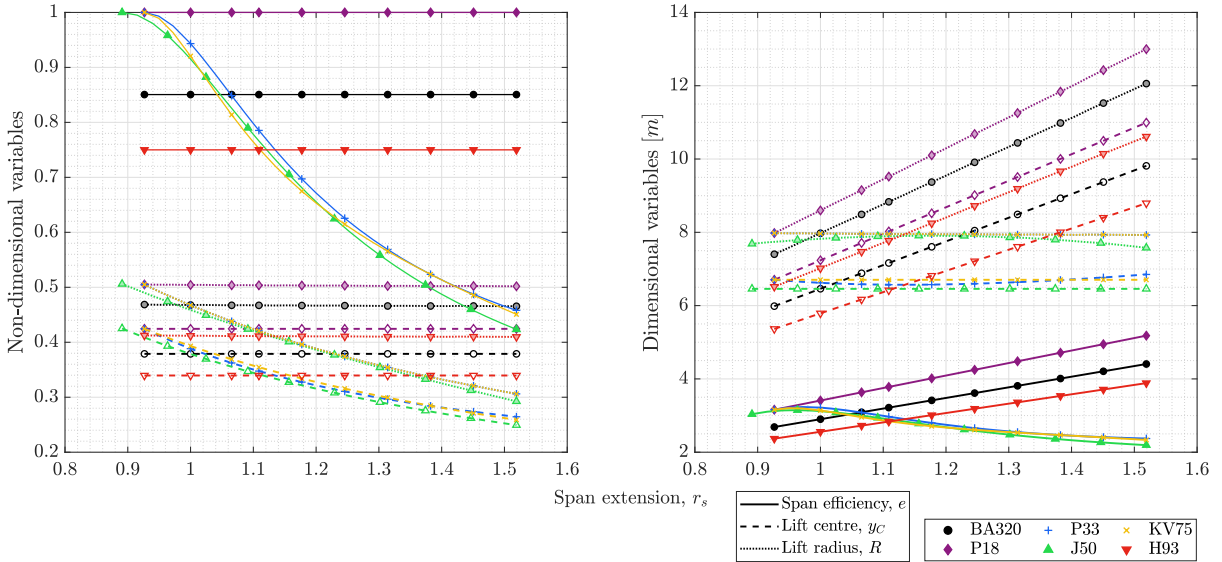


Figure 4.7: Variation of aero-structural parameters with span extension.

Alternatively, the correlation between these three variables is depicted in Figure 4.8 for the non-dimensional values of centroid and radius of inertia of the lift. The variation of the span efficiency factor with the spanwise distance of the lift arm shows

the trade-offs between aerodynamic efficiency and structural penalties. For ideal aero-structural characteristics, the design point should be found at the top left region of the design space. The NELD allow the design to move within the design constraints.

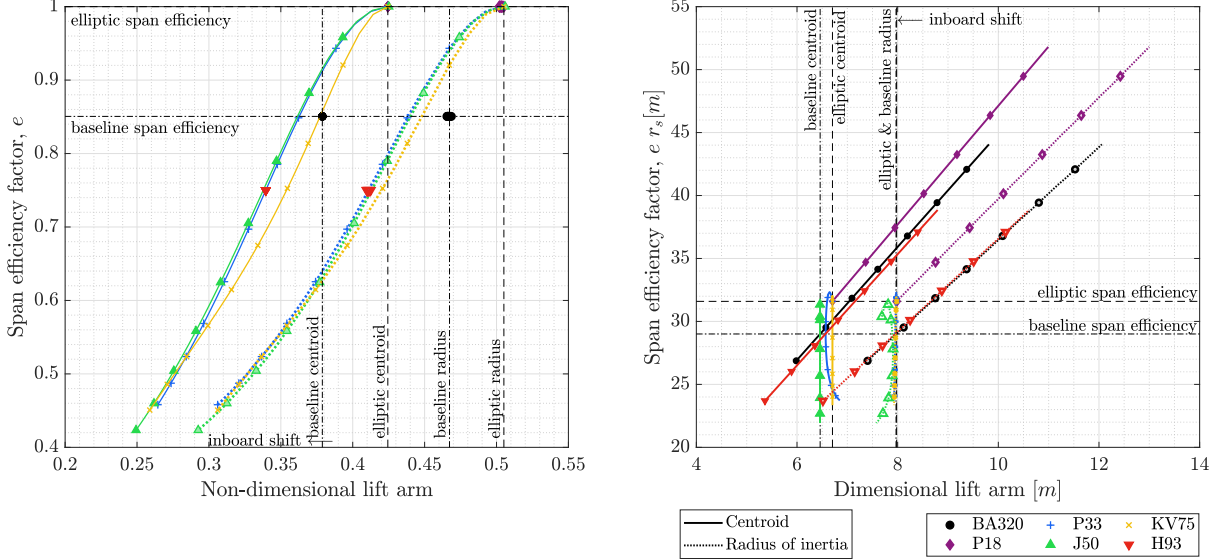


Figure 4.8: Variation of aero-structural parameters with span extension.

These three variables directly represent the above aero-structural trends through their correlation to span extension and the baseline case. Previously, the aero-structural efficiency factor for the P33 distributions was presented in Equation 4.36, representing the departure from a baseline elliptic spanload. Similarly, the corresponding aero-structural efficiency factors for J50 and KV75 were given by Equations 4.41 and 4.45. Nevertheless, it is the variation from the chosen baseline case that must be quantified here. To do so, three alternative formulations are derived to express the aero-structural trends as a function of spanload and span extension. To start, the reduction in induced drag can be expressed as inversely proportional to the span efficiency and the squared span extension when the design lift coefficient is the same, using Equations 4.27 and 4.50, assuming an equal C_L . However, the last term in the equation below is suited to compare wings that might not be aerodynamically equivalent:

$$r_{e,D_i} = \frac{(e)_{base}}{e r_s^2} \approx r_s^2 \frac{\sum_{n=1}^{\infty} n G_n^2}{\left(\sum_{n=1}^{\infty} n G_n^2\right)_{base}} \quad (4.54)$$

The root bending moment variation can be expressed as a function of the span extension and the centroid, which equates directly to a function dependant on the Fourier coefficient characteristics of each spanload. Therefore, from Equations 4.38 and 4.51:

$$r_{rbm} = \frac{M_{x_{root}}}{(M_{x_{root}})_{BA320}} = r_s^2 \frac{\overline{y_C}}{(\overline{y_C})_{BA320}} = r_s^3 \frac{g(G_n)}{(g(G_n))_{BA320}} \quad (4.55)$$

where

$$g(G_n) = \sum_{m=1}^{\infty} \frac{G_{n=2m-1}}{(-1)^{1+m}(-3-4m+4m^2)} \quad (4.56)$$

The parameter r_{rbm} establishes the relation between the impact of span extension and spanload on root bending moment. On the other hand, the expression for the structural weight or moment of inertia of the wing is determined by the radius of inertia. Thus, the corresponding ratio for the structural weight can be derived from Equation 4.33 as:

$$r_{ibm} = \frac{W_s}{(W_s)_{BA320}} = \frac{LR^2}{(LR^2)_{BA320}} = r_s^4 \frac{G_1 + G_3}{(G_1 + G_3)_{BA320}} \quad (4.57)$$

While induced drag and root bending moment will depend on all Fourier coefficients, the structural weight is determined by only the first two, G_1 and G_3 , which agrees with the work of Taylor [267] and Hunsaker [268]. The relation to span extension goes from second order for the induced drag ratio, third for root bending moment and fourth for the structural weight. These ratios exactly represent the trends of the aero-structural characteristics plotted in Figure 4.6.

Finally, the wing twist distribution can be also derived. After computing the Fourier coefficients for all configurations, Equation 4.19 can be rearranged to equate for α_T , where the chord distribution has been previously calculated as shown in Figure 4.5, and the section camber and lift curve slope values are obtained from Table A.4. An example of the resulting twist distributions is shown in Figure 4.9, where the configurations for the ideal analytical design point have been plotted.

4.2.1 Verification of Results

The wing aerodynamic model employed in this study has been derived from the theoretical approaches presented in Section 4.1.1. The developed methodology was implemented in MATLAB and the results were first compared to the analytical solutions provided in the relevant references. In this case, any resulting configuration was compared against the equivalent elliptically-loaded wing that was derived from the baseline layout. All the referenced theories relate the span and shape of the load curves to the elliptic loading in order to make the solution more comprehensive. Satisfactory agreement was achieved between the analytic results and the semi-numerical solution, where the minor differences stem from the implementation of numerical methods integration and derivation. To further determine that the wing shape design process is designed and developed as per the specified requirements, the resulting aerodynamic outcome of the MATLAB implementation of this model has been additionally verified by deploying the computed geometries in AVL [269] and comparing it against the

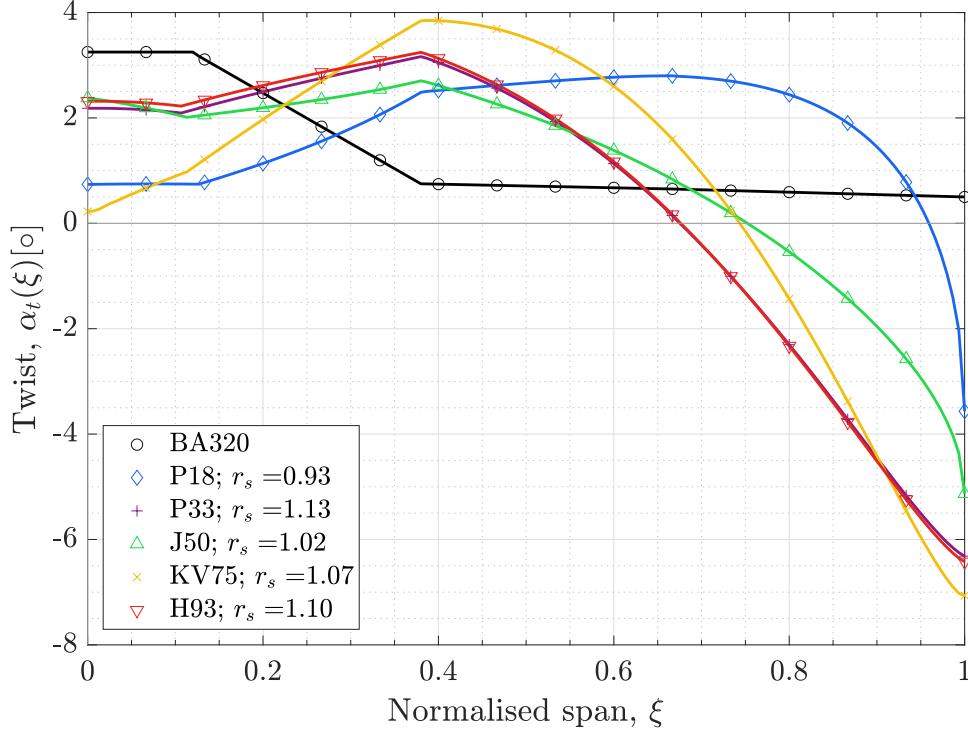


Figure 4.9: Comparison of ideal NELD twist distributions.

data produced. AVL is a reduced order aerodynamic solver that employs an extended vortex lattice model for the aerodynamic and flight-dynamic analysis of lifting surfaces. The verification analysis allows the estimation of the level of uncertainty that will be seen when numerical methods are used for the analysis. The errors against the analytical results are shown in Figure B.6. With this, it can be assumed that the wing shape design methodology presented here is a true and accurate representation of the original theory, and may be assumed to be valid within those bounds defined by each author respectively.

4.3 Conclusions

The methodology to create equivalent wing designs for a given lift and prescribed structural constraint has been presented in this section. The semi-analytical process proposed here is mainly founded on the lifting line theory. The essential derivations have been presented in Section 4.1.1, but a more comprehensive treatment of the theory can be found in References [270] and [5]. This work has been expanded by integrating several assumptions for wing aero-structural design introduced in the work of Prandtl (1933), Jones (1950), and Klein & Viswanathan (1975), as well as Horten's approach to address adverse yaw, to provide a revision of the foundational theories, and a comparison of the various approaches to different structural constraints.

This design approach has been applied to a baseline wing based on the A320 aircraft to form a set-based design space of NELD wing configurations. Here, the span extension factor is the principal independent variable, which yields as a response a set of spanload families and the consequent wing planform and twist distribution output. Prandtl-Glauert compressibility correction and local Mach number sweep correction have been incorporated to extend the application range of the theory. This process has been implemented into a single design framework in which the analytical derivations are solved numerically in MATLAB. The classical theories have been extended to include a non-elliptic spanload as the baseline configuration, from which semi-analytical metrics for aero-structural trends as a function of span extension. An initial comparative analysis of the aerodynamic and structural characteristics of the produced wing configurations has been presented. The trends for the extension of the baseline and elliptic spanloads with fixed design lift have been included too. The outcome has been verified against the analytical solutions provided in the corresponding references and reproduced in AVL with agreeing results.

Chapter 5

Aero-Structural Analysis

The wing shape design methodology presented above enables the integration of several independent structural constraints to create a set-based design space of aerodynamically efficient wing designs. However, this semi-analytical tool employs reduced-order low-fidelity methods to calculate the aerodynamic and structural properties required to derive the wing geometry. The lifting line theory only considers spanwise variation of vorticity over the lifting surfaces, while the structural loads are derived solely from the aerodynamic loading over a simple beam. Nevertheless, this methodology also provides the modest clarity of analytical approaches, allowing a more insightful derivation of trends and correlations.

Such an approach is extremely useful for the preparatory phases of the conceptual design stage, but it fails to cope with the increasing complexity and multifaceted character of the design space that defines the roadmap of this stage. Hence, the design framework is extended here to analyse the computed design space under efficient aerodynamic and structural reduced-order analysis methods with increased fidelity. This allows to break through some of the limitations assumed in the derivation of NELDs and the wing shape methodology, which will be discussed throughout the following sections.

This chapter discusses the theoretical foundations in which the proposed methodology is founded upon, as well as the techniques to estimate the initial mass of the structural components. The aerodynamic and structural models employed for the prediction of aerodynamic loads and the initial sizing of the main structural components of the entire airframe are presented, together with the approach taken to couple the models. The critical load cases are selected, which will drive the updated structural integrity requirements. This approach allows to investigate the effect of span-extended NELD wing designs on the structural mass estimation of the aircraft, for which the findings are presented and discussed.

5.1 Theoretical Foundations

A typical process followed to provide the sizing of the structural components of the airframe starts with the use of loads to determine shear force, bending moment and torsion distributions. Evaluation of these loads provides an initial sizing of the structural constituents. This may be done through the initial estimation of the structural characteristics and a consequent computational iterative process that converges into the final sizing of the members. This process requires the knowledge or the calculation of various aspects of the aircraft. Starting with the loads model, a comprehensive definition of the loads distribution and the concentrated loads is required, for which an initial geometrical definition of the main structural components must be done along with an initial materials selection, and finally assume any relevant stiffness criteria or life requirements specified for the airframe.

To construct such an aero-structural sizing environment in the contemporary conceptual design stage, a well-established approach assumes that the aerodynamics model may be represented by a vortex lattice, and the structural model can be given by a beam model. This presents an adequate premise, as discussed in Section 2.5.2. The aerodynamic and the structural models are coupled to transfer loads and displacements between them and construct the aero-structural model. This model is employed to evaluate internal and external loads and determine stiffness and mass properties that will drive the structure through the imposition of stress constraints for structural integrity and minimum mass.

5.1.1 Mass Estimation

To start the loads analysis and structural sizing processes, it is necessary to have an initial estimate of the overall and component mass for the configuration under consideration, as well as an approximation of its inertia matrix and centre of gravity. This task is usually carried out with empirical formulations and statistical databases, as the results are later refined through iterative processes within the structural sizing phase. However, an adequate prediction is still necessary for decision making and optimisation, and this is not a trivial task. Existing methods [13,176,271] are becoming less reliable, especially when unconventional layouts or advanced technologies and materials are investigated, due to the lack of existing historical data, the need for extensive correction coefficients and the increasing effort demanded.

The initial mass estimation is provided here by the semi-empirical method developed by Isikveren [272,273]. In his work, he proposes a combined approach that employs parametric description, which has been modified for major components, together with additional methods that increase fidelity by employing further parame-

ters specifically related to geometry and layout, applied loads and performance. The secondary mass group of components, also regarded here as constant non-structural masses, comprise elements such as high-lift devices, furnishings, or the various aircraft systems. The members of this group are assumed to be a function of passenger capacity via the definition of the fuselage size, engine used and aircraft classification.

The formulation for the wing is presented as an example and inspected in detail, as it will be later compared to the results derived from the analytical design approach and physics-based structural sizing methodology.

$$W_{wing} = K_{tech} C_{\alpha} K_C \left[\frac{W_G n_{ult} S_w \mathcal{R}^{k_{\beta}} (C_{\chi} + \lambda/2) \tau_s^{k_{\delta}}}{K_{t/c} \cos^{k_{\epsilon}} \Lambda_{c/4}} \right]^{k_{\phi}} \quad (5.1)$$

There are several groups of parameters in the above equation corresponding to different aspects of the wing. To start, the aircraft-type parameters are included in the K_{tech} linear factor, which is a proposed correction associated with the potential for weight reduction of posterior and therefore technologically advanced configurations. This parameter is based on the observed trends of a dataset of statistical information covering the last forty years. K_C is a design factor that depends on the aircraft configuration, such as wing placement, undercarriage arrangement or powerplant installation. The geometric properties of the wing are considered in through the wing area S_w , the aspect ratio $\mathcal{R}$, λ is the taper ratio, the sweep angle at the quarter-chord $\Lambda_{c/4}$, and a wing thickness factor $K_{t/c}$ expressed as a function of the mean thickness-to-chord ratio. Performance characteristics are given by the gross mass of the aircraft W_G and the ultimate load factor n_{ult} . The structural element in the formulation is τ_s , which is a measure of structural stiffness and is assumed to vary with dynamic pressure and limit loads, and is dependant on the design maximum operating speed. Finally, a set of statistical factors are given as $k_{\phi} = 0.656$ and $k_{\beta} = k_{\delta} = k_{\epsilon} = 1.5$, and associative constants given by $C_{\alpha} = 0.0328$ and $C_{\chi} = 1.1$. The coefficient C_{α} represents the departure from the original trend after updating the method with a more contemporary aircraft weights database.

In computing the principal moments of inertia, two common methods may be used at this stage that are a function of the information available. When the weights of the major constituents are known, a straightforward method [274] can be applied to obtain the inertia of each component about their respective centres. Alternatively, a more rudimentary approach can be taken when the input information is limited, where the aircraft is treated as a solid with constant density, and the radius of gyration in pitch, roll and yaw are correlated to the fuselage length and the wing span [13].

5.1.2 Loads Requirements and Computation

Loads are a fundamental input throughout the whole design process of an aircraft, from the establishment of design principles all the way to manufacturing and certification of the aircraft. The early design phase must already start with an estimation of external loads for the identification of the design solutions and assessment of the component weight objectives, which are then critical for stress calculations and structural design.

There are several types of structural analyses that will focus on different types of loads and their consequences on the design of a structure. This research will only consider static strength loads to size the structure, thus assuming the forces and moments to be stationary and constant throughout time at a given condition. Assessment of such loads ensures that the structure has adequate strength to withstand extreme and highly rare events of peak stress. Dynamic effects generating fatigue or long-term load-bearing criteria such as residual strength loads will not be considered on the structural sizing of the proposed design framework.

Strength requirements have been specified in terms of limit¹ loads, to be supported without permanent detrimental deformation nor losing operational capability, and ultimate² loads, which the structure must sustain without failure. The ultimate loads envelope therefore ensures that the structure is designed to withstand the most severe conditions to be encountered in the aircraft's lifetime. These extremely rare events are accounted for through the safety factor, which lowers the probability of structural failures well below the probability of meeting the limit loads condition. It must be also regarded that, just like the external loads, the mechanical properties of materials are of a statistical nature too. Hence, design values must be chosen such that the probability of a structure being overstressed due to the material capabilities is extremely remote.

Load cases

The scenarios considered here for static strength loads estimation have been defined according to the relevant airworthiness regulation criteria, in this case, to comply with EASA Certification Specification requirements for large transport aircraft (CS-25 [275]). In the stage of the design phase, it is required to evaluate the loading conditions that demand the highest static strength to ensure that the structural integrity of the aircraft is maintained at all corners of the corresponding operating envelope. Fortunately, these events are very rare, but each of them presents an array of unique features that could cause structural failure. Therefore, it is customary industry practise to evaluate thousands of load cases to that prescribe a complete set of detailed scenarios capturing as many events as possible. Alternatively, a tailored set of loads and design

¹The maximum loads anticipated on an aircraft during its service.

²The limit loads multiplied by a safety factor.

conditions have been selected for this study, corresponding to the assumed most critical conditions relevant to airframe sizing and to the main disciplines examined here. These manoeuvres are described in Table C.1, along with the flight condition at which they are assessed and the reference design requirement from CS-25.

With this, a loads model must be developed to simulate the prescribed design manoeuvres. This model will contain several external load groups, which are defined by the interaction of the aeroplane with the external environment. The groups that are being considered here are

$$\mathbf{F}_{\text{ext}} = \mathbf{F}_{\text{aero}} + \mathbf{F}_{\text{prop}} + \mathbf{F}_{\text{ground}} + \mathbf{F}_{\text{inertial}} \quad (5.2)$$

where the external loads can be composed of the aerodynamic, propulsive, ground and inertial loads respectively, and will have different contributions on the various airframe components.

Structural Model

It has been highlighted that the use of high-fidelity FEM solvers is avoided at the conceptual design stage due to the huge computational cost. With this in mind, a structure is commonly represented as the sum of various components made of beams that carry the local structural properties. Beams have become an essential element of the FEM for structural analysis, as they provide a balanced model for rapid and accurate results in this stage, compared to three-dimensional elements, while still capturing structural characteristics such as elastic deformation and three-dimensional dynamic response.

The traditional mathematical model to discretise a structure is developed using a beam model. The reference axes are usually set at the flexural axis, along which the shear centres are placed. The beams carry bending, shear, torsional and axial deformations, and can be divided into several elements, allowing the use of more refined meshes. The mechanical properties are generally estimated from the member section properties, represented in Figure 5.1, using classical structural analysis methods. The shear centre, the point at which a load applied parallel to the plane of the section will not induce torsion, is used as the reference for structural sizing against the section aerodynamic centre. This method is rapid and straightforward, but can be rather inaccurate. However, a higher accuracy can be achieved using finite element methods that can represent a box-like construction of the structure. Here, the model simulates bending and twisting of the entire structure, not the equivalent beam. This approach provides finer results, but it is more computationally demanding.

The structural model employed in this study is based on the Timoshenko beam theory. Unlike the simplified Euler-Bernoulli model, the Timoshenko beam model

accounts for the effects of shear distortion and rotational inertia, and does not require rotation continuity for the deflection. Furthermore, the cross-section does not have to be normal to the deformed axis of the beam, which can also be short and thick. This provides a model suitable for the above requirements and more tractable for numerical simulation [276]. However, such beam models are known to undergo shear locking, leaving the deformation considerably underestimated. This error is produced due to the linear nature of first order elements, which cannot accurately model the curvature of the element under bending, and a shear stress is introduced. An alternative solution to this problem is presented by Ghiringhelli et al. [15], which is based on a finite volume approach instead of the established variational principles. Their work proved the finite volume approximation to be shear lock free, and thus extended the formulation to be applicable to geometrically non-linear beams for multi-body formulations.

With the extension of the classical Timoshenko theory with Ghiringhelli's method, the internal cross-section of the wingbox can be defined to assess its properties and derive the structural mass and stiffness distributions along the lifting surface. These properties are then assigned to the beam elements that are placed along the elastic axis of the lifting surface. The wing structure is modelled as stiffened multi-web box beam to carry spanwise shear, bending and torsion. The carry-through structure is then defined to join the wing and the fuselage, and transfers the spanwise internal loads developed on the wing.

A typical rectangular wingbox section is illustrated in Figure 5.1, which will be represented by beam elements in the structural model. The model includes skin, stringers, and spars. For simplicity, it may be assumed that the upper and lower skin thicknesses t_{sk} are the same, whilst the front and rear spar have equal thickness t_{sp} too. The height b_{sp} approximated as the mean between the front and rear spars, which will result in a reduction factor of approximately 0.8 on the thickness-to-chord ratio. The stringers have a Z cross-section composed of three equivalent segments parametrised by the length of each segment, which is given by b_{st} , and their thickness is then $t_{st} = 0.12b_{st}$. The stringer pitch b_{ps} is kept constant, resulting in a spanwise variation of the number of stringers number according to the width of the box section c_{wb} . With this, the cross-section characteristics can be obtained and assigned to the equivalent beam model, and will include properties such as the torsional constant and the second moment of area.

Now, the FE model can be implemented, which will be displacement-based and given by a three-node finite volume beam, depicted in Figure 5.2, with linearised strains and curvature. A brief description is presented here; a detailed derivation can be found in the References [15, 250, 255, 277]. In this work, the equation for node equilibrium becomes

$$\mathbf{F}_{\text{ext}} = \mathbf{L}\mathbf{A}\mathbf{\Psi} \quad (5.3)$$

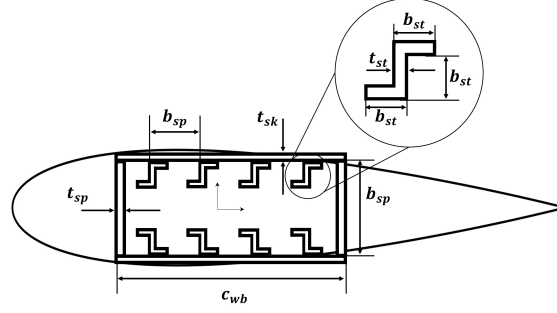


Figure 5.1: Schematic of a wingbox cross-section.

where $\mathbf{F}_{\text{ext}}$ is the vector for external loads, $\mathbf{L}$ is the matrix containing the moment arm that carries the loads from the evaluation points to the reference points, $\mathbf{A}$ contains the sectional stiffness matrices in the global axes and $\mathbf{\Psi}$ is the vector of strains and curvatures in the global axes, which are based on an intrinsic formulation of the beam. The displacements of the three geometric nodes at the evaluation points are recovered through the use of a quadratic shape function, which enables the evaluation of the deformations at these points. Also, the stiffness matrix is kept constant, so material non-linearity is not considered. On the other hand, geometric non-linearity is introduced through the transformation of the strains and curvatures into the local frame, while the internal loads are expressed in the global frame. This is done through the multiplication of the stiffness block matrices by a rotation matrix in the node equilibrium equation. Linearisation of Equation 5.3 with respect to the structural states derives in the structural stiffness matrix. If applied to a linear analysis, this is obtained along with the updated beam properties. For a non-linear analysis of the model, the matrix will be evaluated iteratively based on the deformation of the structure. With this, the finite element model used to describe the mass and stiffness properties of the aircraft is now ready.

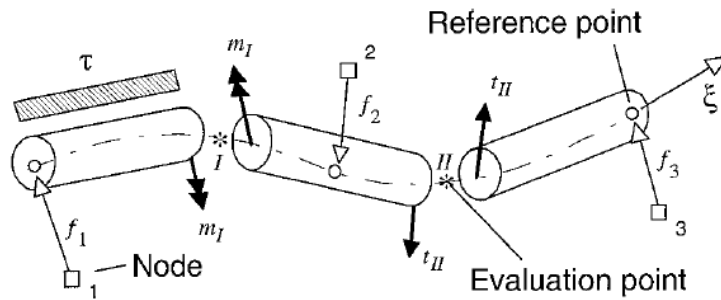


Figure 5.2: Schematic of the three-node finite volume beam (obtained from Reference [15]).

Aerodynamic Model

The lifting surfaces are considered here to be the wing, horizontal stabiliser and vertical stabiliser, while fuselage lift forces are assumed to be nominally zero in this study, which is an acceptable approximation for subsonic transports at this staged of the design. For this purpose, VLM seems an appropriate choice as the modelling method to derive the aerodynamic loads, especially since the lifting contribution of the fuselage is being neglected. Panel methods become more suitable if there are other type of blunt components, although it would still be more computationally expensive to run. As the intention is to keep the design options flexible for as long as possible during the study, the expected large design space will further benefit from faster computational times. As with the lifting line theory, this method follows the assumptions of a potential flow.

In the process of modelling the vortex lattice, the vortex rings have been distributed over the geometry of the lifting surfaces along the streamwise and chordwise direction, as seen in Figure 5.3. Their strength is determined such that the velocity at the centre of each vortex is tangent to the boundary of the local surface element. The shed wake is modelled as vortex filaments leaving the sharp trailing edges to infinity, hence consisting of only chordwise vortex lines. The strength is determined by enforcing the Kutta condition, ensuring the flow to leave the trailing edge smoothly. To solve this,

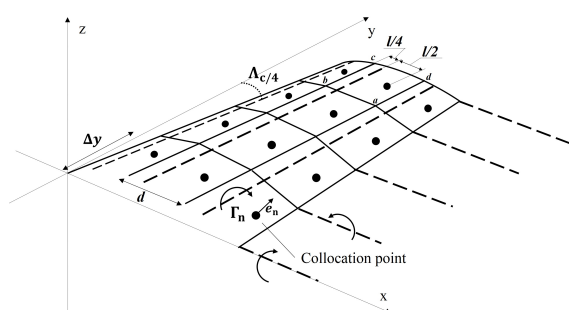


Figure 5.3: Schematic of a vortex lattice model of a thin lifting surface.

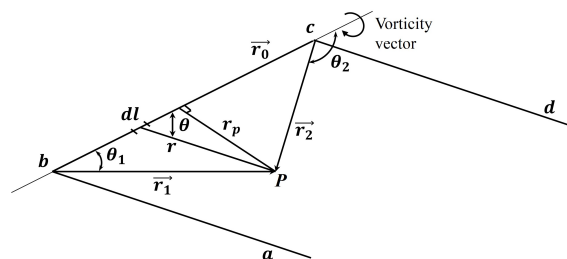


Figure 5.4: Schematic of the view angles and distances for the calculation of induced velocities of a finite-length vortex segment. Reproduced from Reference [5].

a finite number of horseshoe vortices of different strengths are superimposed on the lifting surface. A panel of chordwise length l has a horseshoe vortex $abcd$ of strength Γ_n placed such that the segment bc is at a distance $l/4$ from the leading edge and aligned with the local sweepback of the panel. A collocation point is placed on the centreline of the panel at a distance $3l/4$ from the leading edge. Moreover, the vortex panels are located along the mean camber surface of the wing. After this, a rigorous analysis

would model shed trailing vortices following a curved path. However, a linearised approach has been adopted for simplicity of the computation, in which the trailing vortices have been aligned parallel to the longitudinal body axis. The velocity induced at an arbitrary point $P(x, y, z)$ can be calculated from the Biot-Savart law by treating each of the vortex segments separately, as presented in Figure 5.4. The same process is followed for the entire lifting surface, which is built by a lattice of superimposed horseshoe vortices, each of different unknown strength. At every collocation point, the normal velocity induced by all the horseshoe vortices is obtained utilising the Biot-Savart law. The application of the flow-tangency boundary condition on all the collocation points results in a system of simultaneous equations that can be solved for the unknown strengths. With this, the circulation and the pressure differential between upper and lower surfaces can be obtained, which can be then integrated to yield the forces and moment of the lifting surface.

To calculate the velocity induced by the vortex rings, the Biot-Savart Law can be now applied to derive the generalised expression for each panel as

$$\mathbf{V}_{i_n} = \frac{\Gamma_n}{4\pi} \frac{\mathbf{r}_1 \times \mathbf{r}_2}{|\mathbf{r}_1 \times \mathbf{r}_2|^2} \left[\mathbf{r}_0 \cdot \left(\frac{\mathbf{r}_1}{r_1} - \frac{\mathbf{r}_2}{r_2} \right) \right] \quad (5.4)$$

where $\mathbf{r}_0$, $\mathbf{r}_1$ and $\mathbf{r}_2$ represent the distances between vortex segment ends and the arbitrary point $P(x, y, z)$, as shown in Figure 5.4, and can be applied to for any orientation of the vortices. Now the above equation can be used to calculate the total velocity induced at P by considering the summation of the influence of each of the segments in the horseshoe vortex. With this, the velocity induced at the m th control point by the vortex in the n th panel will be is given by

$$\mathbf{V}_{i_{m,n}} = \mathbf{c}_{m,n} \Gamma_n \quad (5.5)$$

where $\mathbf{c}_{m,n}$ is the normalwash influence coefficient matrix that depends on the geometry of the n th horseshoe vortex and its distance to the collocation point of the m th panel. Due to the linearity of the governing equation, the summation of the velocities induced by the total number of vortices N provides the expression for the total induced velocity at the m th collocation point, yielding

$$\mathbf{V}_{i_m} = \sum_{n=1}^N \mathbf{c}_{m,n} \Gamma_n \quad (5.6)$$

It is now necessary to compute the strengths of each vortex, which can be done with the application of the boundary condition that determines no flow across the surface,

yielding tangent flow at each collocation point. This ensures that the component of the induced velocity normal to the wing at this point balances with the normal component of the free-stream velocity. The total velocity at the collocation point is therefore given by

$$\mathbf{V}_m = \mathbf{V}_\infty + \mathbf{V}_{i_m} \quad (5.7)$$

where the free-stream velocity vector can be expressed as a function of the angle-of-attack and sideslip as

$$\mathbf{V}_\infty = V_\infty \begin{bmatrix} \cos \alpha \cos \beta \\ -\sin \beta \\ \sin \alpha \cos \beta \end{bmatrix} \quad (5.8)$$

Now, the trailing vortices parallel to the body x-axis are introduced, and the tangency requirement under small angles approximation can be expressed as

$$\mathbf{V}_m \cdot \mathbf{e}_m = \left(\mathbf{V}_\infty + \sum_{n=1}^N \mathbf{c}_{m,n} \Gamma_n \right) \cdot \mathbf{e}_m = 0 \quad (5.9)$$

where $\mathbf{e}_m$ is the vector normal to the surface at the m th collocation point. Now, the unknown circulation strengths required to satisfy the boundary condition can be obtained through the solution of the system of simultaneous equations shown below, that derives from the evaluation of the dot products above, where the new aerodynamic influence coefficient matrix is given by $a_{ij} = \mathbf{c}_{m,n} \cdot \mathbf{e}_m$, and the right hand side is given by free-stream contribution as $b_m = \mathbf{V}_\infty \cdot \mathbf{e}_m$:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & \ddots & & \dots \\ \vdots & & \ddots & \dots \\ a_{N1} & \dots & \dots & a_{NN} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_N \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} \quad (5.10)$$

which will demand the inversion of the aerodynamic influence coefficient matrix to solve for all vortex strengths. The total force and moment vectors about the origin are then computed by adding the contributions of all the forces on all the individual horseshoe vortices. To calculate the force on each panel, the Kutta–Joukowski theorem is applied to relate the lift to circulation of each panel as

$$\mathbf{F}_m = \rho \mathbf{V}_m \times \Gamma_m \Delta y_m \quad (5.11)$$

which represents the local flight conditions at the centre of the panel, multiplied by the vortex intensity vector given by the circulation strength and the spanwise length

of the panel. The total force is then

$$\mathbf{F}_m = \sum_{n=1}^N \mathbf{F}_m \quad (5.12)$$

and the total moment

$$\mathbf{M}_m = \sum_{n=1}^N \mathbf{F}_m \times \mathbf{r}_m \quad (5.13)$$

where $\mathbf{r}_m$ is the distance to the segment from the origin.

The VLM model presented above is a practical method for the fast prediction of aerodynamic properties of lifting surfaces with acceptable levels of accuracy. However, the coupling of such a method with the structural analysis of the airframe will only enable a rigid representation of the components. In order to increase the fidelity of the assessment of aero-structural interactions of HARW aircraft, it is necessary to capture the effects of flexibility by considering the effects of unsteady aerodynamics and structural dynamics. As discussed in Section 2.5.2, the choice of aerodynamic model to use here is critical, where the extension of VLM to unsteady double-lattice model (DLM) would be a common solution. But to avoid high calculation costs, this study adopts the method presented by Chuban [278], where VLM is modified to account for small elastic oscillations via the linearisation of expressions for the aerodynamic forces and changes in flow tangency boundary conditions caused by finite variation of normal vectors when finite values of angle-of-attack, sideslip, and controls setting are introduced. This approach is based on quasi-steady aerodynamics instead of unsteady, and allows for more accurate prediction of less conservative configurations. Moreover, this method deals with some of the limitations presented by the traditional DLM approach, in which usually the unsteady aerodynamic loads are independent of the reference trim solution. Here, in-plane motion of aerodynamic surfaces are neglected in the definition of the loads, which may give rise to considerable penalties and then require to include correction terms to improve the accuracy of the predictions. This method also introduces the capability for flutter analysis within this aerodynamic model.

Previously, the velocity at the centre of each panel e_m is usually described by Equation 5.7. Now, one more contribution is added to the velocity in the time domain to include the surface displacement velocity at the point $P(x, y, z)$ yielding

$$\mathbf{V}_m = \mathbf{V}_\infty + \mathbf{V}_i(m, t) - \dot{\mathbf{u}}(m, t) \quad (5.14)$$

This additional term stems from the surface displacement field at the point P , which is assumed to be known and given by the stationary finite displacement U^0 caused by an

initial deflection of controls or static structural deformation, the unsteady infinitesimal displacement u of the surface point.

$$U(P, t) = U^0(P) + u(P, t) \quad (5.15)$$

Solution of the boundary condition at the surface yields the induced velocity to be given by the sum of three terms:

$$\mathbf{V}_{i_m} = \sum_{n=1}^N \mathbf{b}_{m,n} \Gamma_n = \mathbf{B}_n^0 + \mathbf{B}_n^v + \mathbf{B}_n^\varphi \quad (5.16)$$

where $\mathbf{b}_{m,n}$ is the new aerodynamic influence coefficient matrix, and the vortex strengths can be decomposed in matrix notation as

$$\mathbf{\Gamma} = \mathbf{\Gamma}^0 + \mathbf{\Gamma}^v + \mathbf{\Gamma}^\varphi \quad (5.17)$$

Here, $\mathbf{\Gamma}^0$ characterises the vortex strengths as dependant of the steady boundary conditions of angle-of-attack, sideslip and the finite stationary displacements U_0 ; $\mathbf{\Gamma}^v$ describes the vortex strength contribution that is a function of the displacement velocities $\dot{u}$; $\mathbf{\Gamma}^\varphi$ considers the vortex strength as a function of the surface rotation during infinitesimal displacements.

Now from Equation 5.11, the aerodynamic force in each panel can be rewritten as

$$\mathbf{F}_m = \rho (\mathbf{V}_\infty + \mathbf{V}_i(m, t) - \dot{\mathbf{u}}(m, t)) \times \mathbf{y}_m \mathbf{\Gamma}_m \Delta y_m \quad (5.18)$$

where $\mathbf{y}_m$ is the vector length of the panel, and is a function of the rotation of the structure $\varphi(m, t)$, which in linearised form becomes

$$\mathbf{y}_m = \mathbf{y}_m^0 + \varphi(m, t) \times \mathbf{y}_m^0 \quad (5.19)$$

Substituting Equations 5.6, 5.17 and 5.19 on Equation 5.18 and neglecting second order terms yields the new expression for the generalised aerodynamic force applied to the vortex segment:

$$F_m = F_m^{00} + F_m^{0v} + F_m^{0\varphi} + F_m^{v0} + F_m^{\varphi0} \quad (5.20)$$

The terms in Equation 5.20 may be described as: F_m^{00} is the stationary force depending only on steady-state conditions and displacements of the surface; F_m^{0v} and F_m^{v0} describe the unsteady forces depending on angle-of-attack, sideslip and displacements, and are proportional to the displacement velocities; $F_m^{0\varphi}$ and $F_m^{\varphi0}$ contain the unsteady forces depending on angle-of-attack, sideslip and steady-state displacements that are proportional to the rotation angles due to the displacements. For further detail on the

derivation of the aerodynamic model used in this stage of the design and analysis framework presented here, the reader is taken to References [270, 278, 279]. The effects of compressibility have been integrated in the aerodynamic model as proposed by Melin et al. [266], in which the incompressible vortex strength at each panel is multiplied by the Prandtl-Glauert correction factor presented in Equation 4.30.

Trim Analysis and Loads Envelope

The aero-structural model has been set based on the initial conditions of the aircraft geometries and properties, and the sizing of the structure can be performed according to the prescribed manoeuvres. Thus, it is first necessary to obtain the critical shear, bending and torsion distributions over airframe structure that are generated by the external loads envelope. With the aerodynamic model and the selection of flight points at which the aircraft will be assessed, the aircraft is trimmed to determine the control and state of the aircraft under the given constraints, using conventional control surfaces that have been modelled through their aerodynamic contributions. Both flight and ground loads have been considered, which include standard manoeuvres such as pull-up at prescribed normal load, landing, gusts, or aileron maximum deflection. Specific directional manoeuvres have been considered according to CS-25 criteria to improve the accuracy of the vertical tail sizing. Altogether, these manoeuvres will determine the loads envelope that will be used for the structural sizing. Other scenarios involving more abnormal aircraft or atmospheric conditions have been excluded from the analysis.

Static or dynamic balance of forces and moment is achieved through the solution of the equilibrium equations of motion together with the definition of the specific manoeuvre constraints to provide a consistent trim problem [280], as seen in Table C.1. With this, the linear trim analysis of the aero-structural model described above starts with the linearisation of the aerodynamic forces and moments with respect to the trim variables in the aerodynamic system and with respect to the displacements in the structural system. Using a symmetric manoeuvre as an example, the determination of the required angle-of-attack α and elevator deflection δ_e to balance lift L and pitching moment M_q can be expressed as

$$\begin{bmatrix} \frac{\partial L}{\partial \alpha} & \frac{\partial L}{\partial \delta_e} \\ \frac{\partial M_q}{\partial \alpha} & \frac{\partial M_q}{\partial \delta_e} \end{bmatrix}^{-1} \begin{Bmatrix} \Delta L \\ \Delta M_q \end{Bmatrix} = \begin{Bmatrix} \Delta \alpha \\ \Delta \delta_e \end{Bmatrix} \quad (5.21)$$

where the partial derivative terms contain the effects of changes in the structural states v , given by the displacements and rotations along each axis. Now, the displacements

of the structure can be obtained through the solution of

$$\Delta v = \left[\frac{\partial F_{int}}{\partial v} - \frac{\partial F_{aero}}{\partial v} \right]^{-1} \left(\left[\frac{\partial F_{aero}}{\partial \alpha} \quad \frac{\partial F_{aero}}{\partial \delta_e} \right] \begin{Bmatrix} \Delta \alpha \\ \Delta \delta_e \end{Bmatrix} + F_{grav} \right) \quad (5.22)$$

where the derivatives with respect to the states represent the structural and aerodynamic stiffness matrices respectively and F_{grav} represents the gravitational terms. Now, the VLM can be updated with the displacements so that the internal and external loads are recovered through the structural and aerodynamic stiffness matrices. Further linearisation is applied by keeping any term related to the elastic forces constant.

To solve some of the manoeuvres listed in Table C.1, it is necessary to know the corresponding stability and control derivatives in order to solve the equations of motion. Classical methods to estimate these aircraft characteristics at the conceptual design stage range from semi-empirical methods largely based on geometric parameters [274, 281–284] to the application of strip theory and force balance to somewhat consider aerodynamic spanload shape [280]. As occurs with the other disciplines discussed in this work, this low-fidelity methods are inadequate for the analysis of unconventional aircraft. Increase in accuracy can be obtained through the use of higher fidelity linear aerodynamics tools, such as vortex lattice, double lattice or panel methods. However, these tools remain very sensitive to the geometric definition and discretisation of the mesh. With this, the relation between aerodynamic loads and structural deformation is achieved, alongside the definition of the control surfaces and flight attitude.

The remaining contributions to F_{ext} are considered as a function of the load factor. With this in mind, landing gear reactions and thrust are modelled as point forces at the fuselage and wing. The fuel is uniformly distributed along the wing, as a function of the internal volume of available. This is derived from the wingbox dimensions as well as chordwise camber distribution, which are determined from the local aerofoil. The inertial forces are given by the acceleration of the estimated mass of the structural component, as well as lumped masses allocated to the nodes in the beam model that represent the secondary and payload masses. Introducing fuel and inertial loads allows for consideration of bending moment relief. Fuselage loading must pay additional attention to the internal cabin pressure caused by pressure differential in passenger compartment, as well the longitudinal bending moments. From Equation 5.2, the general expressions for shear force, bending moment and torsion along the semispan will be composed of

$$F_S(y) = L_{aero} + L_{inertial} + L_{fuel} + L_{prop} + L_{lg} \quad (5.23)$$

$$M(y) = M_{aero} + M_{inertial} + M_{fuel} + M_{prop} + M_{lg} \quad (5.24)$$

$$M_T(y) = M_{T_{aero}} + M_{T_{inertial}} + M_{T_{fuel}} + M_{T_{prop}} + M_{T_{lg}} \quad (5.25)$$

To finalise, the most critical load found at each section of the airframe members is selected and multiplied by a safety factor to define the load envelope that will determine the structural sizing required to maintain integrity.

5.1.3 Aero-Structural Sizing

The aero-structural model presented allows to overcome many of the limitations encountered through the use of the proposed wing shape design methodology in Chapter 4. Namely, the major structural members of the airframe are modelled and sized under a tailored critical load envelope; local distributed conditions are evaluated along the longitudinal axis of the vehicle, and along the structural chord and span; additional contributions to the loads are included; material properties are considered in the analysis. Furthermore, the calculation of the structural weight will now integrate more realistic constraints regarding both weight and structural integrity, instead of being simply represented by a single structural parameter. Here, a method based on fundamental principles is now preferred for the estimation the load-bearing structure of the airframe. Such an approach is particularly useful at this stage due to the compromise it represents between the rapid analysis of component weights using empirical methods, and the computationally expensive analysis of the high order and fidelity finite-element method.

Starting with the sizing of the lifting surfaces, the multi-web beams wingbox presented in Figure 5.1 is generally driven by the simultaneous buckling of the covers due to local instability and the webs due to flexure induced crushing [16]. Under this assumption, the shear force and bending moment distributions along structural span are used for the sizing of webs and covers thicknesses. The carry-through structure must be designed to support the resulting bending, shear and torsion loads at the wing-fuselage intersection. With this, the weight of the structural box can be determined by calculating the minimum amount of material required to satisfy strength requirements as well as static buckling at each spanwise station. Additionally, the required shear material can be computed independently of buckling material. Finally, the total structural weight of the wing is computed as sum of the weights of all its structural elements.

The weight of the structural box may now be calculated as proposed by References [285] and [16]. First, the ratio between the volume of structural material to total wing-box volume of multi-web box beams with minimum weight is defined as solidity and

presented as

$$\varsigma = \varepsilon_1 \left(\frac{M}{Z_s t^2 E} \right)_2^\varepsilon \quad (5.26)$$

where M is the applied moment, t is the thickness, and E is the elastic modulus of the material. The constants ε_1 and ε_2 , and are dependant on the cover and web geometries selected, shown in Table 5.1. The parameter $Z_s(y) = c_s(y) \cos \Lambda_s(y)$ represents a dimension perpendicular to the structural semispan as seen in Figure 5.5 and can be found in Reference [285]. The above equation can now be expressed as a function of the weight of bending material per unit span W'_{BEND} as

$$\varsigma = \frac{W'_{BEND}(y)}{Z_s t \rho_s} \quad (5.27)$$

in which ρ_s is the density of the material. Equating the above formulations allows to resolve for the weight of bending material. Similarly, the weight of the shear material per unit span can be computed with

$$W'_{SHEAR} = \frac{\rho_s F_S}{\sigma_S} \quad (5.28)$$

where σ_S is the allowable shear stress. The optimum web spacing is presented with the expression

$$d_w = t \left[\frac{1 - 2\varepsilon_{2,c}}{(1 - \varepsilon_{2,c}) \sqrt{\varepsilon_{2,c} \varepsilon_{1,w}}} \left(\frac{M}{Z_s t^2 E} \right)^{\frac{2\varepsilon_{2,c}-3}{2\varepsilon_{2,c}}} \varepsilon_{1,c}^{\frac{3}{2\varepsilon_{2,c}}} \right]^{\frac{2\varepsilon_{2,c}-3}{4\varepsilon_{2,c}}} \quad (5.29)$$

in which the subscripts w and c refer to webs and covers, respectively, and the values of the factors can be found in Table 5.1. Lastly, the equivalent isotropic thicknesses of covers and webs are given by

$$t_c = d_w \left(\frac{M}{Z_s t E \varepsilon_{1,c} d_w} \right)^{\frac{1}{\varepsilon_{2,c}}} \quad (5.30)$$

$$t_w = t \sqrt{\left(\frac{M}{Z_s t^2 E} \right)^{2 - \frac{1}{\varepsilon_{2,c}}} \left(\frac{\varepsilon_{1,c} d_w}{t} \right)^{\frac{1}{\varepsilon_{2,c}}} \frac{2}{\varepsilon_{1,w}}} \quad (5.31)$$

Now, the weight of the wing-box that satisfies the above assumptions can be computed is

$$W_{BOX} = \frac{b_S}{N_S} \sum_{i=1}^{N_S} (W'_{BEND_i} + W'_{SHEAR_i}) \quad (5.32)$$

Table 5.1: Wing structural coefficients and exponents [16].

ε_1	ε_2	$\varepsilon_{1,c}$	e_c	$\varepsilon_{1,w}$
2.250	0.600	1.108	2.000	0.911

which represents the summation of the local weights along the number of spanwise elements N_S on the structural wingspan b_S .

The wing carry-through structure is designed to resist torsion as well as bending and shear loads. Therefore, the torsion material must be able to resist the twisting induced by the wing sweep. Moreover, only the longitudinal component of the bending moment will contribute to the loads carried. Defining the thickness and loads at the root denoted by the subscript $_0$, the solidity of the carry-through structure is similarly given by

$$\varsigma_{ct} = \varepsilon_1 \left(\frac{M_0 \cos \Lambda_S}{t_0^2 c_{root,S} E} \right)^{\varepsilon_2} \quad (5.33)$$

where $c_{root,S}$ is the chord of the wing-box at the wing-fuselage intersection, and therefore the chord of the carry-through structure. The bending material weight is then

$$W_{BEND_{ct}} = \rho_S \varsigma_{ct} c_{root,S} t_0 w_{ct} \quad (5.34)$$

where w_{ct} is the width of the carry-through structure across the fuselage. Now, the web spacing and web and cover thicknesses can be obtained as in Equations 5.29 to 5.31. The weight due to shear loads is now

$$W_{SHEAR_{ct}} = \frac{\rho_S F_{S_0}}{\sigma_S} w_{ct} \quad (5.35)$$

The torque on the carry-through structure is

$$T = M_0 \sin \Lambda_S \quad (5.36)$$

and the weight of the material required to support this load is expressed as

$$W_{TORSION_{ct}} = \frac{\rho_S T (t_0 + c_{root,S}) w_{ct}}{t_0 c_{root,S} \sigma_S} \quad (5.37)$$

Altogether, the summation of bending, shear and torsion weights provides the weight of the carry-through structure

$$W_{CT} = W_{BEND_{ct}} + W_{SHEAR_{ct}} + W_{TORSION_{ct}} \quad (5.38)$$

The fuselage sizing is performed in a similar fashion, but will also include the

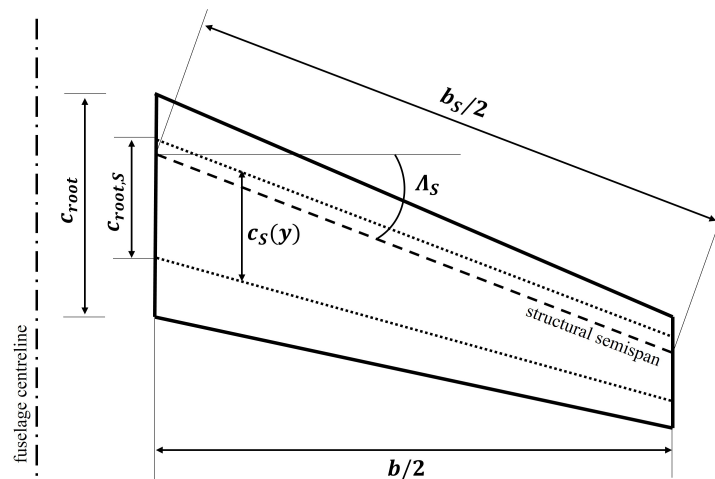


Figure 5.5: Schematic of the three-node finite volume beam. Reproduced from Reference [16].

loads due to cabin pressurisation. The above lines provide a brief derivation of the structural sizing process adopted in the analysis framework of this research, but for further detail, the reader is directed to References [16, 285–287]. It is important to note that following the general approach of this research, the optimal structural sizing will not be pursued. Instead, a pre-defined structural concept is adopted for all the analysed configurations, to which constraints on stiffness and minimum gage are imposed to avoid unfeasible solutions. This will not necessarily lead to the global optimum solution, but it will provide a consistent set-based design space of solutions for a first structural sizing based on minimum weight principles, where the resulting stiffness and inertia distributions will have been derived according to strength criteria on ultimate loads, and the sizing process will be consistent across all configurations.

5.2 Proposed Approach to Aero-structural Analysis

The theoretical models presented above conform the foundations for contemporary aero-structural design at the conceptual design stage. Many available and developing state-of-the-art computational models for the conceptual design of aircraft integrate these approaches, as discussed in Section 2.5.2, to more or less extent, due to the increased capabilities to provide a comprehensive multidisciplinary design environment with higher fidelity towards novel and unconventional aircraft concepts.

The task pursued in this research is to evaluate a set-based design space of aircraft configurations with HARWs designed under aero-structural efficiency principles using a DOE approach; it is not to derive a novel aero-structural design methodology for analysis and sizing. Hence, it was decided to employ existing and validated com-

putational models to facilitate the composition of the design framework needed to accomplish the proposed task. For this purpose, NeoCASS [232,277,287] provides the most suitable capabilities to develop and assess the aero-structural model. Other available tools rely significantly on parametric correlations, providing insufficient model fidelity, or mainly revolve around planform optimisation, which is not part of the proposed objective as the planform revision is fixed through the wing shape design methodology based upon NELD principles.

NeoCASS has been devised to embrace the multidisciplinary effects of the design of the airframe from the early conceptual design stage. It integrates physics-based methodologies that provide a satisfactory balance between accuracy and rapid analysis. Furthermore, its development in Matlab facilitates its integration and data transfer with the existing methodologies developed in this research, and provides a suitable modularity for the set-based approach targeted here. Generally, it agrees with the need to reduce the use of historical data-based formulations in favour of reduced-order physics-based methods to increase the fidelity of the analysis of unconventional designs. The following lines will provide more detail of NeoCASS' approach and how it has been adapted to this research, together with the presentation of the proposed methodology and the results obtained.

The process followed to complete the aero-structural analysis is shown in Figure 5.6. The framework uses NeoCASS to build the aero-structural model of the given aircraft layout and then evaluate the envelope of external and internal loads along the wingbox to determine the stresses, and with this obtain the required stiffness and inertial properties of the beam elements. Here, NeoCASS offers two different sizing paths. The first one performs the analysis over a rigid aircraft, where no aeroelastic constraints are considered. Referring to Figure 5.6, this option will ignore the loop containing the convergence of results and geometry update, and will follow the linear process to yield the aero-structural sizing results. This process can be used as the initial sizing that is then refined to satisfy aeroelastic constraints by iteratively accounting for structural flexibility. Non-linear trim would be performed on an aerodynamic model that is updated and combined to the non-linear beam model to consider geometric non-linear effects. The methodology proposed for the aero-structural analysis presented in Section 5.1.3 makes use of the former approach. The parametric study on span extension under NELDs is wrapped around these processes, in which inputs to this block are systematically changed in the form of a different wing designs.

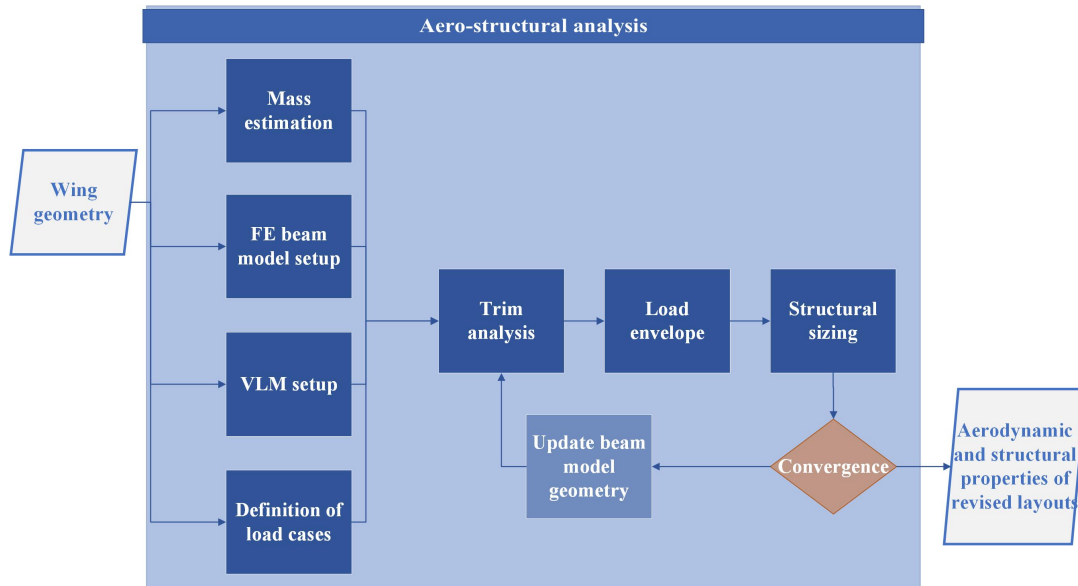


Figure 5.6: Flowchart of process followed for the aero-structural analysis.

5.2.1 Initialisation

To initialise the aero-structural model, NeoCASS requires an input file that contains geometric, mass and material properties, as well as the parametrisation of the numerical aerodynamic and structural model. The tool offers a user interface that employs CAD visualisation, where the initial definition of these characteristics can be done interactively.

In this research, an initial file was created in this manner to construct the BA320 baseline configuration. The data provided in Sections 3.2 and A was used to describe the aircraft geometry and provide initial component weights. Regarding the airframe material, a commonly used aluminium-zinc alloy [18] has been chosen in all cases, the properties of which are shown in Table 5.2. Even though manufacturers are quickly increasing the percentage of composite materials used in the airframe due to the wide variety of mechanical advantages and the technological progress being made on manufacturing techniques, it is not the aim of this study to examine the material used nor the impact of its selection. Again, the comparative basis of the results aims to normalise factors such as material properties in order to extract trends as a function of the designated critical variables that are being investigated here, such as the wingspan or the spanload.

The creation of a single input file defining the baseline configuration is sufficient for this study. This is because a simple routine has been developed within the framework so that the input file is automatically updated with the wing planform and twist parameters corresponding to each configuration in every analysis. Most of the initialisation parameters remain the same, such as initial mass guess, fuselage and tail

Table 5.2: Mechanical properties of the aluminium alloy Al-Zn 7075 [18].

Young's modulus, E	71.7 GPa
Shear modulus, G	26.9 GPa
Shear strength	331 MPa
Ultimate tensile strength	538 MPa
Poisson ratio, ν	0.33
Material density	2810 kg/m ³

geometry or aerofoil characteristics as highlighted in Chapter 3.

5.2.2 Initial Mass Estimation

With this, the initial estimation of the weight of the structural components is performed in NeoCASS through a semi-empirical approach. NeoCASS implements the methodology explained in Section 5.1.1 after some modification of the statistical coefficients to account for an updated database. Yet, the validity of the statistical formulations applied is not universal and therefore accuracy may vary between designs. Nevertheless, the main target of this particular analysis is to obtain a first reasonable estimate of the structural components to be later refined by the structural sizing process.

From all the available sizeable elements of the aircraft, which include structural members, systems, or payload, this study evaluates the impact of wing modification only on the structural components of the airframe, these being fuselage, wing, horizontal and vertical stabiliser. The weights of all the remaining the elements are kept fixed, including the payload and fuel. This allows the focus of the analysis to be put on the increase of the structural weight of the wing, and the corresponding effect on the other airframe structures, as well as the resulting range capabilities derived from the trade-off between the impact on maximum take-off weight (MTOW) and aerodynamic efficiency. From here, additional design conditions are derived for the analysis of the subsequent modified configurations:

1. Fuel and payload weights are kept constant across all configurations.
2. Fuel capacity across all configurations is assumed to remain the same. In reality, the internal volume for fuel in the wings would reduce due to the increasing aspect ratio.
3. The payload carried is assumed to be the maximum allowable with maximum fuel, and remains constant across all configurations.
4. The aero-structural sizing is performed at the single mass configuration described above.

5. The changing performance parameter will be the MTOW, due to the resizing of the structural components for each configuration.

The final MTOW is derived by means of an iterative method in which the fuel weight is prescribed as the control variable. This leads to the minimum MTOW that fulfils airframe strength requirements at a fixed payload. In this sense, the chosen mass configuration for the baseline aircraft is that of MTOW carrying maximum fuel, which allows for the maximum range attainable with MTOW. With this, approximations on the centre of gravity location and inertia matrix are also given using the method described in Reference [274].

Now, Equation 5.1 is rewritten as a function of the span extension for convenience of the study, yielding

$$W_{wing} = K_{tech} C_{\alpha} K_C \left[\frac{W_G n_{ult} S_w^{(1-k_{\beta})} r_s^{(2 k_{\beta})} b_{base}^{(2 k_{\beta})} (C_{\chi} + \lambda/2) \tau_s^{k_{\delta}}}{K_{t/c} \cos^{k_{\epsilon}} \Lambda_{c/4}} \right]^{k_{\phi}}. \quad (5.39)$$

According to this expression and the coefficient values provided in Section 5.1.1, this semi-empirical formulation predicts that the wing increases by a factor of $r_s^{1.968}$ with span extension, which yields less than 2% difference to assuming the increase to be proportional to the squared span. This relation is compared in Figure 5.14 to the metric extracted from the semi-analytical study and presented in Equation 4.57. It can be observed that the empirical method yields the exact same wing weight increase for all wing families. Looking into Equation 5.39, the formulation used to predict the wing weight does not weigh in any parameter linked to spanload. In fact, for each of the spanload families studied, the only variable that will change in Equation 5.39 is r_s , consequently resulting in the equal trends observed in Figure 5.14. Clearly, the structural constraint adopted in during the wing shape design process is not captured in the empirical estimation of the wing mass. If the spanload is not changed with span extension, it can be seen that the formulation used will increasingly overestimate the result as the centroid of the loading shifts outboard.

5.2.3 Aerodynamic and Structural Models Setup

NeoCASS has the capability to build the structural and aerodynamic meshes for each configuration based on the methodological approaches discussed in Section 5.1.

The aerodynamic model is built for the lifting surfaces using a rigid VLM as described above and adapted from KTH's aerodynamic solver Tornado [288]. The mesh is represented by a series of quadrilateral flat lifting panels and defined by span, twist, dihedral and sweep. Camber contribution on normalwash is defined through a prior

definition of the aerofoil distribution. The spanwise discretisation of the aerodynamic model was modified in order to enhance the tool's capability to define a finer wing twist distribution, and therefore capture the aerodynamic loading more precisely, overcoming NeoCASS' limitation to do so. It is critical aspect of this study to properly capture the variations in spanload shape, as well as the consequent shift in centre of lift and other aerodynamic properties.

The structural beam model is built from work in Reference [289], yielding a reduced-order low-fidelity analytical model that provides a first stiffness distribution. When defining the beam box properties, NeoCASS allows the user to specify the structural layout, providing various concepts for both fuselage and lifting surfaces that are characterised by the definition of the shell and stiffeners type, as well as the main structural design constraint. For this study, the fuselage structure has been defined by a Z-stiffened shell, longitudinal frames, and the minimum amount of structural material is considered as a compromise between critical buckling and pressure conditions. The wing-box of the lifting surfaces has been modelled with a truss stiffened cover and Z-stiffened webs. A beam model [15] is generated that carries the mechanical properties, as well as additional points along the wing-box and fuselage, which transfer and recover the stresses, and correctly locate them on the the elastic axis of the beam. Moreover, further nodes are modelled perpendicular to the beam axis that enable to visualise the rotational motion of the structure and enable the coupling to the aerodynamic model for the forces and displacements to be transferred.

The transfer of stiffness, inertial and aerodynamic properties between the models is of critical importance to obtain reliable results. This issue stems from the fact that the two meshes might have been discretised differently, and therefore an interface is required to exchange displacements and loads between structural and aerodynamic meshes. An interpolation scheme is used based on spatial coupling methods to do so. Aerodynamic loads are distributed along all the nodes, and the loads are then reduced to lumped forces and moments on the main set of nodes. Such an approach is taken to achieve a detailed description of the mass distribution to correctly define the inertial properties, which are particularly important during the trim analysis of the aircraft. For further details please refer to References [232,277,287]. Figure C.1 shows the resulting coupled meshes for the complete configurations of the ideal analytical design points of each family along with the baseline model.

5.2.4 Computation of Loads

The narrow down selection of critical manoeuvres for the sizing of structural components has been presented in Table C.1. The description of the manoeuvre is provided, along with the flight condition at which they are assessed, the main specifications, and

the reference design requirement from CS-25. Both symmetrical and asymmetrical manoeuvres have been specifically chosen to drive the sizing of the different structural components. These prescribed manoeuvres can be easily defined in NeoCASS through their trim file template, which can include any desired number of manoeuvres. Here, the manoeuvres is defined by constraining the aircraft's states and control surfaces as needed.

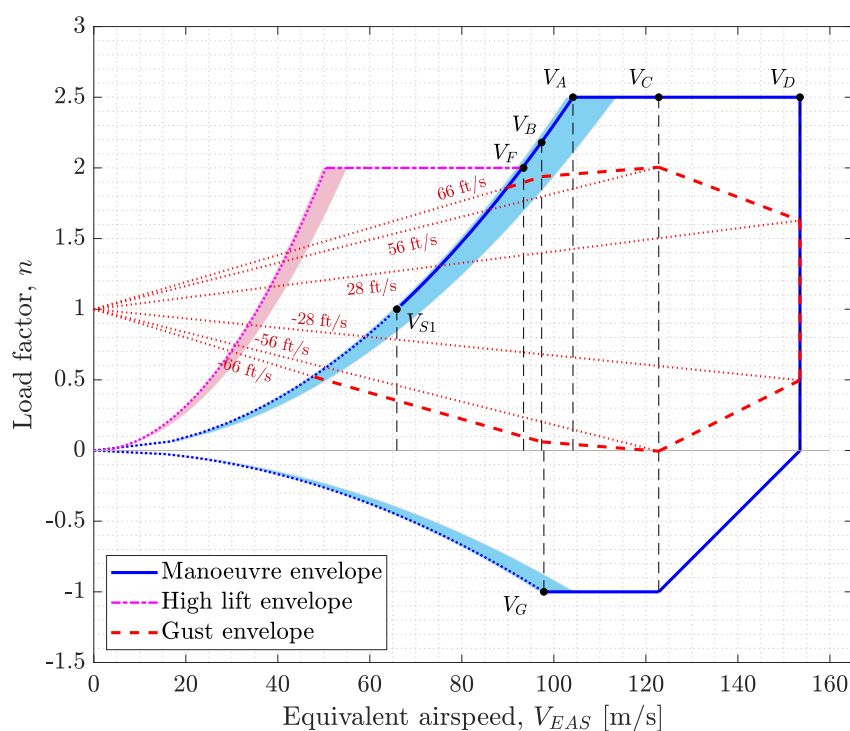


Figure 5.7: Design operational envelope.

The flight envelope shown in Figure 5.7 has been calculated according to the certification specification range of operational altitudes, the structural design speeds and the minimum and maximum prescribed load factors. It covers clean and high lift configuration as well as gust encounter. The blue shade seen in the graph represents the range of variation of the flight envelope due to wing modification. It can be observed that only the maximum and minimum lift coefficient limits of the envelope vary slightly, which is caused by the variation in the design stall speed V_{S1} due to the change in the initial aircraft mass estimation of each configuration, later discussed in Section 5.2.2. The design stall speed will then impact the values of the manoeuvring speeds for positive and negative loads, V_A and V_G respectively, as well as the design speed for maximum gust intensity V_B . Similarly, the high lift envelope will see a minor shift due to the change in mass of the different configurations. The design cruise speed V_C is a prescribed design constraint in the analysis of all configurations, and the dive speed V_D is derived from V_C .

With this, the manoeuvres can be now fully parametrised. A summary of the flight points assessed is presented in Figure 5.8, along with the ID of the manoeuvres that are analysed at that condition, which can be found in Table C.1.

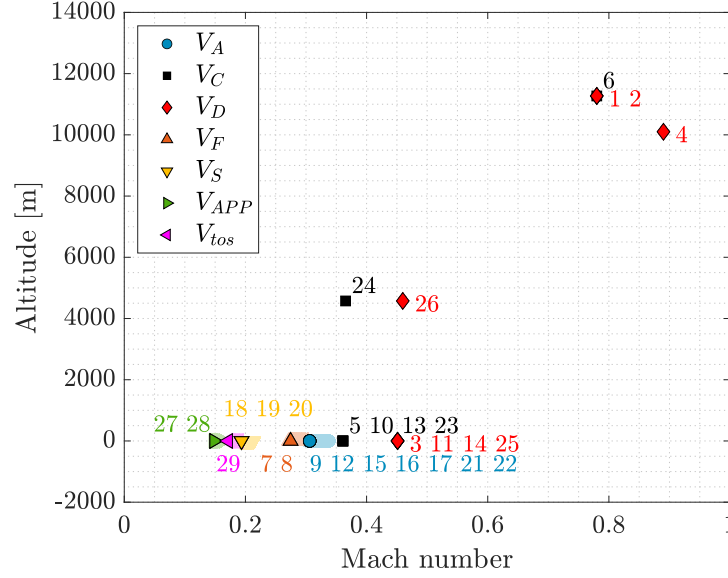


Figure 5.8: Flight points assessed and manoeuvre ID, , representing the manoeuvres listed in Table C.1.

A linear static trim analysis is performed to obtain the loads envelope and then size the structure. Here, NeoCASS is used to trim the rigid mesh of the unrestrained configuration. The wing loading is assumed to be transferred through the carry-through structure through two load vectors carrying the forces along the z-direction and torque. The lift load, center of pressure, inertia load, center of gravity, shear force, bending moment and torsion are computed at each spanwise station along the elastic line, which is placed in the middle of the wing box, from tip to wing-fuselage intersection.

The results of interest for this research mainly concern the wing loads envelope. Figures 5.9, C.2 and C.3 show the limit loads envelopes for spanwise bending, shear and torsions over the wing for every spanload family evaluated, thus the safety factor of 1.5 has not been included in the presentation of these results.

Starting with the inspection of the bending distributions, presented here, the first apparent difference to the analytical results obtained from the wing shape design methodology, shown in Section B, is that the behaviour of the bending distribution with span extension does not follow the same trends, in which the analytical predictions showed a minimisation of the bending loads increase with span extension. Furthermore, the magnitude of the baseline values is approximately 3 times higher than the analytical result. Certainly, the idealised assumptions and simplifications

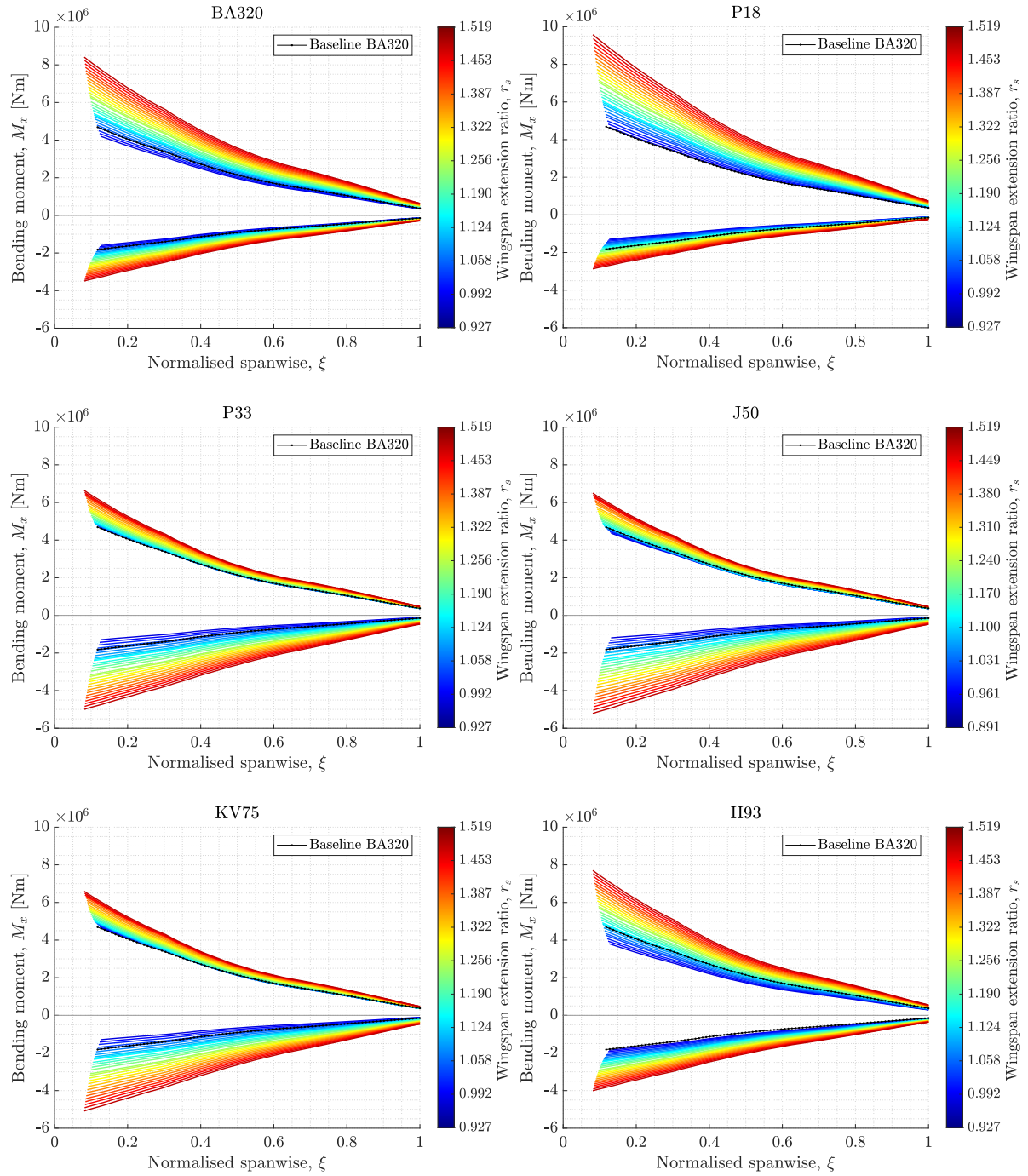


Figure 5.9: Comparison of wing bending moment distributions against the baseline configuration.

of the NELD theoretical approaches overlooked a wide range of factors that will impact the structural sizing. To reiterate some of the main differences: only the wing is previously simulated and so there is no full-aircraft trim analysis; the cruise design condition is the only driving manoeuvre; no mechanical properties nor structural integrity specifications are considered; only the aerodynamic loads were regarded. Therefore, behaviour and magnitude will be affected.

Let us dissect the contributions to the limit load envelope at the design condition first and at the same span than the baseline. Looking into Figure 5.10, the bending moment distributions due to the aerodynamic loads at cruise now closely agree with the results in Figure B.5. The NELD morphing families match the baseline bending moment as designed for, while the families with fixed lift distribution behave according to their relative positions of the centre of lift. As discussed, however, the structure is now being sized under the combined contribution from aerodynamic, inertial, fuel and engine loads, where the latter three will be providing bending moment relief. Moreover, the structural sizing will be further driven by high load factor manoeuvres, such as the 2.5g pull-up manoeuvre plotted in Figure 5.10. At this span extension, structural equivalence is also achieved at off-design conditions. Now, the results at the analytical ideal span extension are dissected too in Figure 5.10. While there is a better agreement across all spanload families on the bending moment due to aerodynamic loads developed at cruise as designed for, the envelope will see the impact of the off-design manoeuvres as well as the effect of span extension on inertial, fuel and engine loads. Going back to Figures 5.9, to C.3, this impact is significantly reduced by NELDs for positive loads, the negative load envelope sees a larger magnitude increment.

The overall behaviour of loads under NELD re-design is now presented as trends in Figure 5.12. The solid line depicts the results of the wing only, obtained at the design flight condition determined in the NELD theoretical approach. The analytical results have been plotted again in this figure for comparison. Starting with the ratio of induced drag against the baseline configuration, the methodology employed now provides more conservative results while still following similar trends to the ones predicted by the analytical theories. Conversely, the growth in aerodynamic efficiency given by maintaining a constant non-dimensional centre of lift as span is extended seems to slow down faster, whilst the NELD families show a later development of negative circulation due to the small positive angle of attack required to trim the aircraft for steady level flight at cruise.

The ratio of induced drag at the most critical manoeuvre is also presented. While the reduction in aerodynamic efficiency at off-design manoeuvres is only suffered in a small portion of the flight, the penalties on internal loads must be carried at all times, as the structure must be sized accordingly. The analytical approach generally overestimated the results of the integrated bending moment, while the opposite results for the root bending moment. Furthermore, the NELD approach for span extension is successful in maintaining the integrated bending moment to a minimum when only the aerodynamic loads are considered, to the point that this higher-fidelity methodology predicts more positive results for this parameter. Nevertheless, when further variables are considered, the benefits promised by the NELD approach are penalised. Thus looking at the trends derived from the loads envelope, now only the J50 and the H93 families

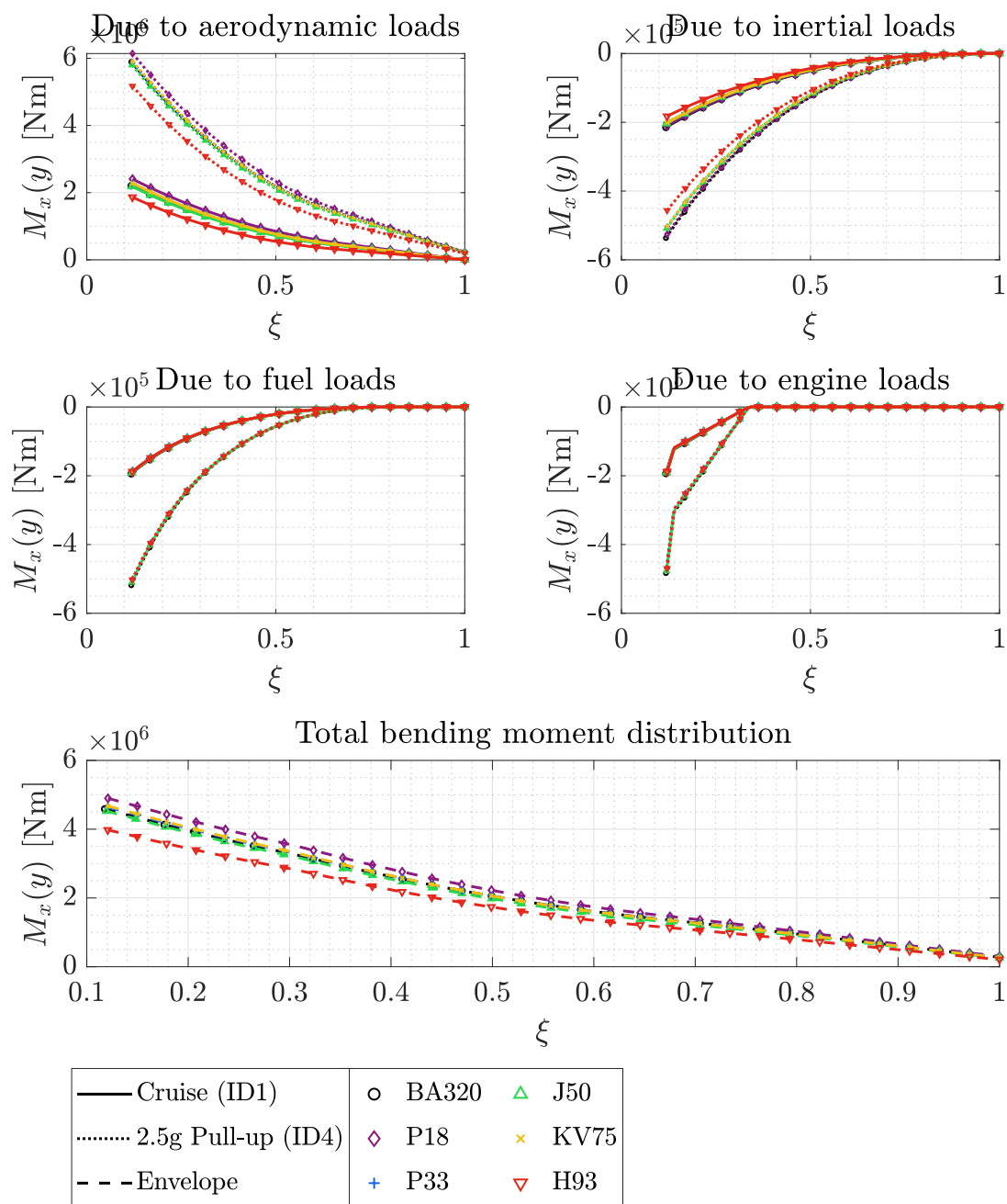


Figure 5.10: Contributions to the spanwise bending moment distribution at the design cruise condition for configurations at $r_s = 1$.

have design points at which induced drag and integrated or root bending moment are lower than the baseline values.

A quasi-static 2.5g pull-up manoeuvre under a given limit pitch rate is considered the most critical manoeuvre for sizing the wing. However, variation of span and spanload will develop critical loads differently, as seen in Figures C.4, C.5 and C.6. The manoeuvres driving the limit loads are shown here for each family of curves as span is extended. For the baseline and elliptic families, a standard 2.5g pull-up at the

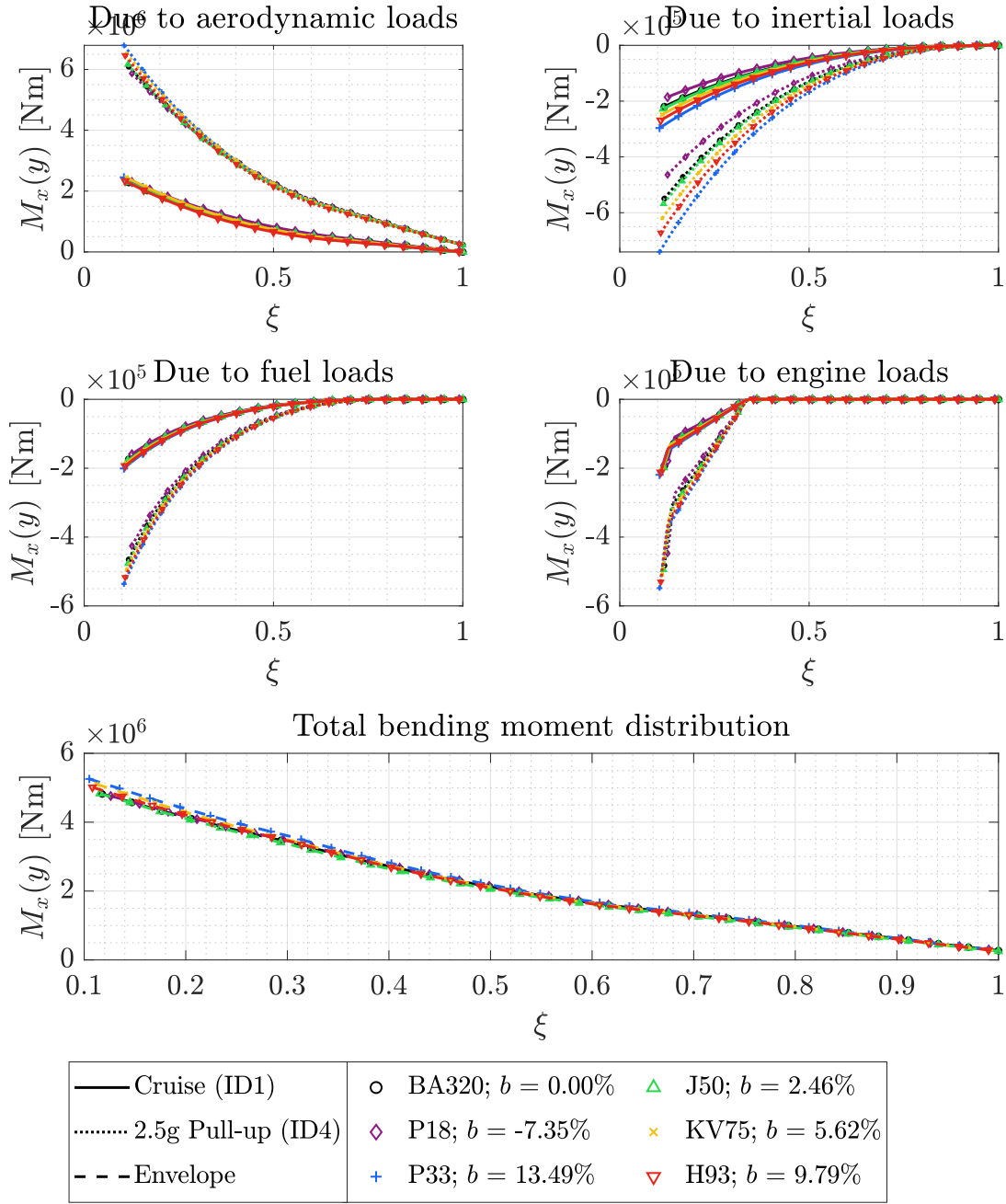


Figure 5.11: Contributions to the spanwise bending moment distribution at the design cruise condition for configurations at the analytical ideal span extension.

design dive speed and altitude is the driving structural manoeuvre for bending and shear, whereas a 2.5g pull-up at cruise will be mostly critical for NELDs. Moreover, a sudden elevator displacement at maximum deflection at the manoeuvre speed with a pitch acceleration for 2.5g becomes critical on the outer wing of NELD configurations at low span extensions, and is also driving bending loads of morphing NELD at larger spans. This manoeuvre also drives the critical load condition in spanwise torsion for all wing configurations. However, directional manoeuvres with sudden rudder

maximum deflection determine the loads around the wing-engine interface, while a pull-down to $-1g$ is critical at the wing-fuselage intersection.

Generally, off-design cases clearly impact the structural weight, although do not show a great penalty in overall aerodynamic efficiency at this stage, where only induced drag is being considered via a VLM model of the lifting surfaces. Here, the trim analysis will impact the total induced drag, while the total lift will iterate with the newly computed MTOW. However, the fuselage's contribution may still be assumed as negligible, given that the fuselage remains unchanged. However, the fuselage mass will increase due to the loads transferred from the wing. The load peak at the wing-fuselage intersection increases in the same manner as the the loads of over the wing does. This effectively results in additional material required to satisfy the structural requirements that translates into the small mass increase with span extension. For the purpose of this study, it is assumed here that the fuselage can be reinforced as needed without modification of its overall geometry.

Finally, the resulting spanwise variation of the torsional stiffness is shown in Figure 5.13. Now, the ability of the structural wingbox to resist torsion reduces with span extension. A more severe reduction can be observed on the NELD families that adapt the spanload to the span extension, due to the corresponding variation in wing twist. Nevertheless, observation of these results lead to the conclusion that the variation in torsional stiffness across the configurations is more closely linked to span extension than to the twist distribution. With this, as the span increases, the wings would be more susceptible to aeroelastic effects such as loss in aileron effectiveness, since the sizing constraints in the present study have not included aeroelastic considerations. Common solutions to improve the torsional stiffness include to increase skin thickness, which would have a detrimental impact on the computed structural weight of the wing.

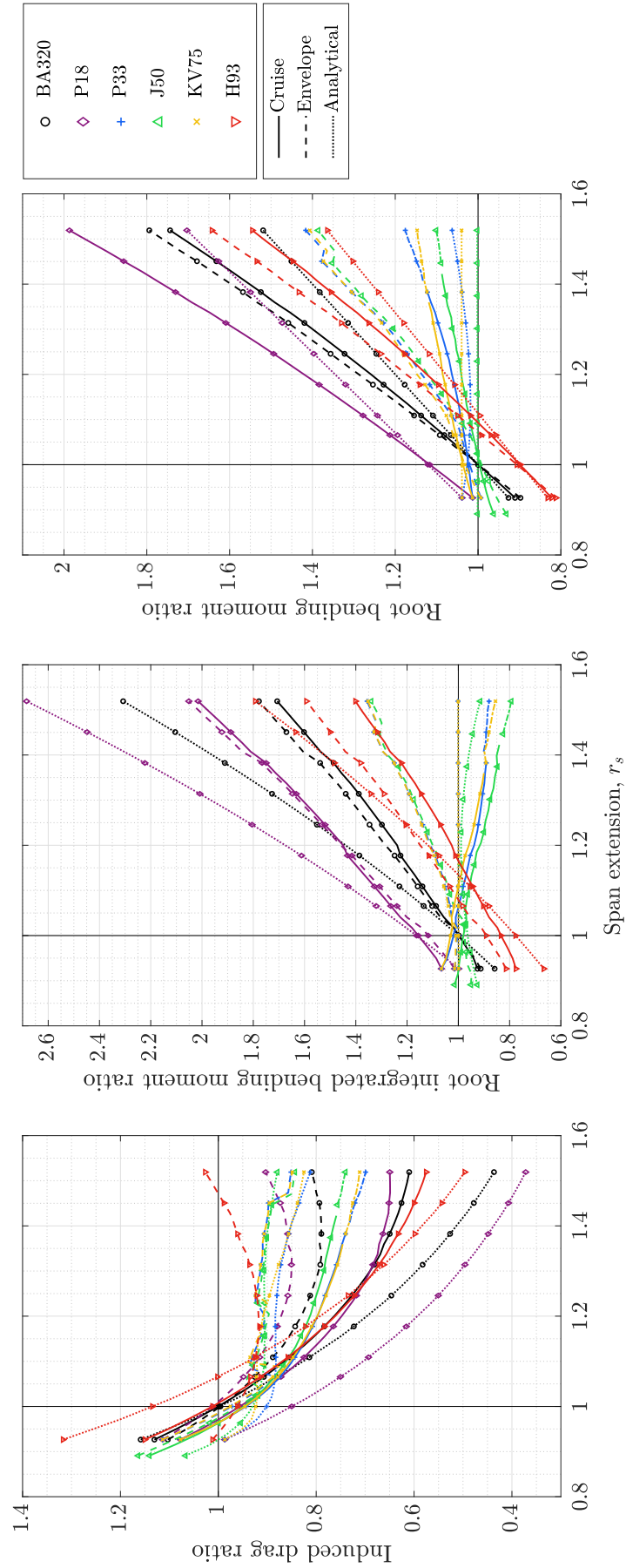


Figure 5.12: Aero-structural trends due to aerodynamic loads for span extension against the baseline configuration.

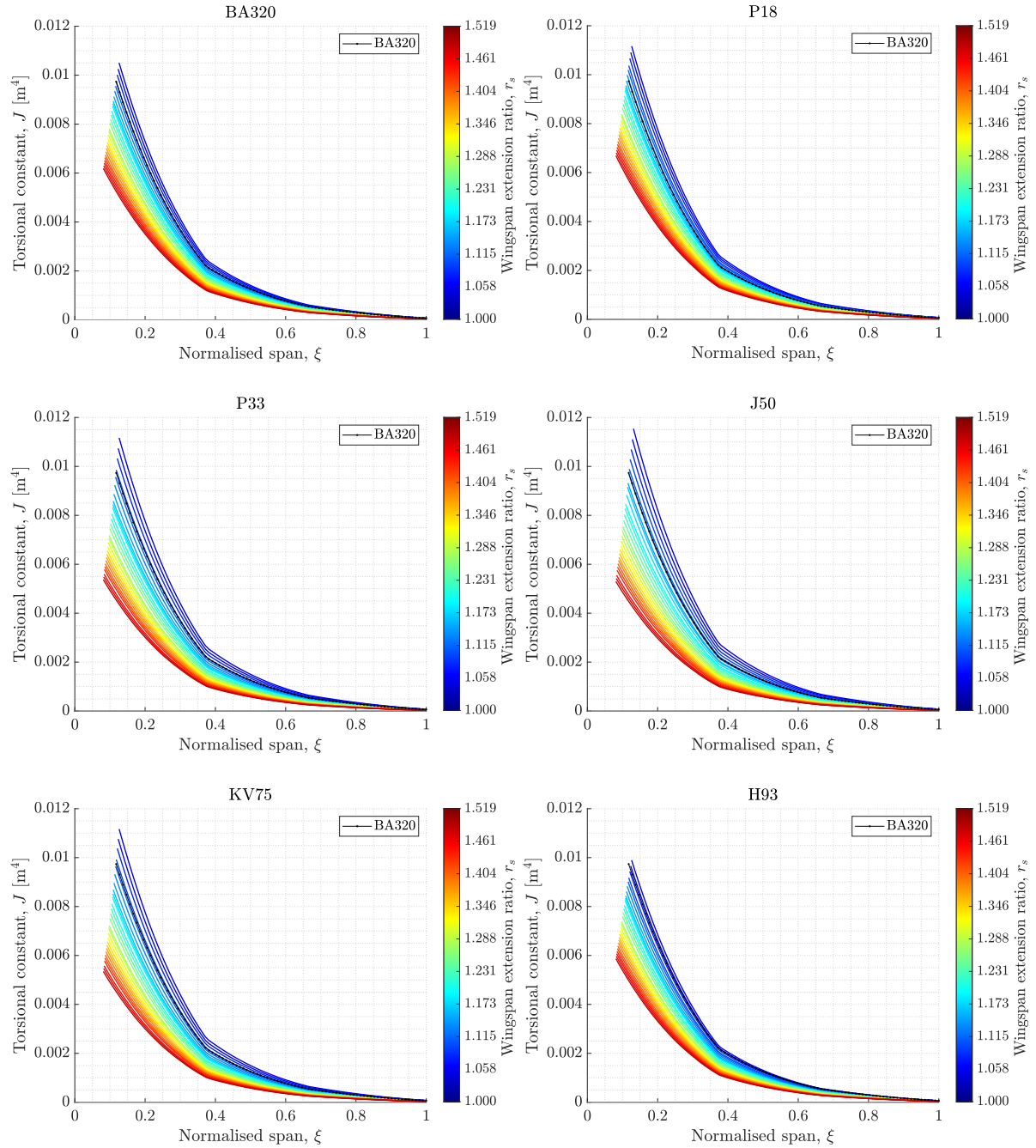


Figure 5.13: Comparison of the wing torsional stiffness distributions against the base-line configuration.

5.2.5 Structural Sizing Module

With the envelope of internal loads defined, the sizing is performed in NeoCASS for the rigid aircraft. An implementation of the linearised equations derived in Reference [15] is presented here based. This approach is beneficial because the internal loads only need to be evaluated at two discrete locations along the beam [255], shown as points *I* and *II* in Figure 5.2. At each station, the beam is evaluated under ultimate compressive and tensile strength, local and global buckling and minimum gage, adopting a method based on fundamental principles for the sizing of the load-bearing structural components [290].

To obtain the final MTOW with minimum material under the structural constraints specified, NeoCASS will start an iterative loop until convergence is reached. To do so, statistical analysis techniques are used to determine a relation between the predicted weight and the actual weight. This approach is assumed to be acceptable for this study, since the general layout of the aircraft being analysed can be considered conventional. However, concepts such as blended wing bodies or closed wings will be further penalised by these assumptions.

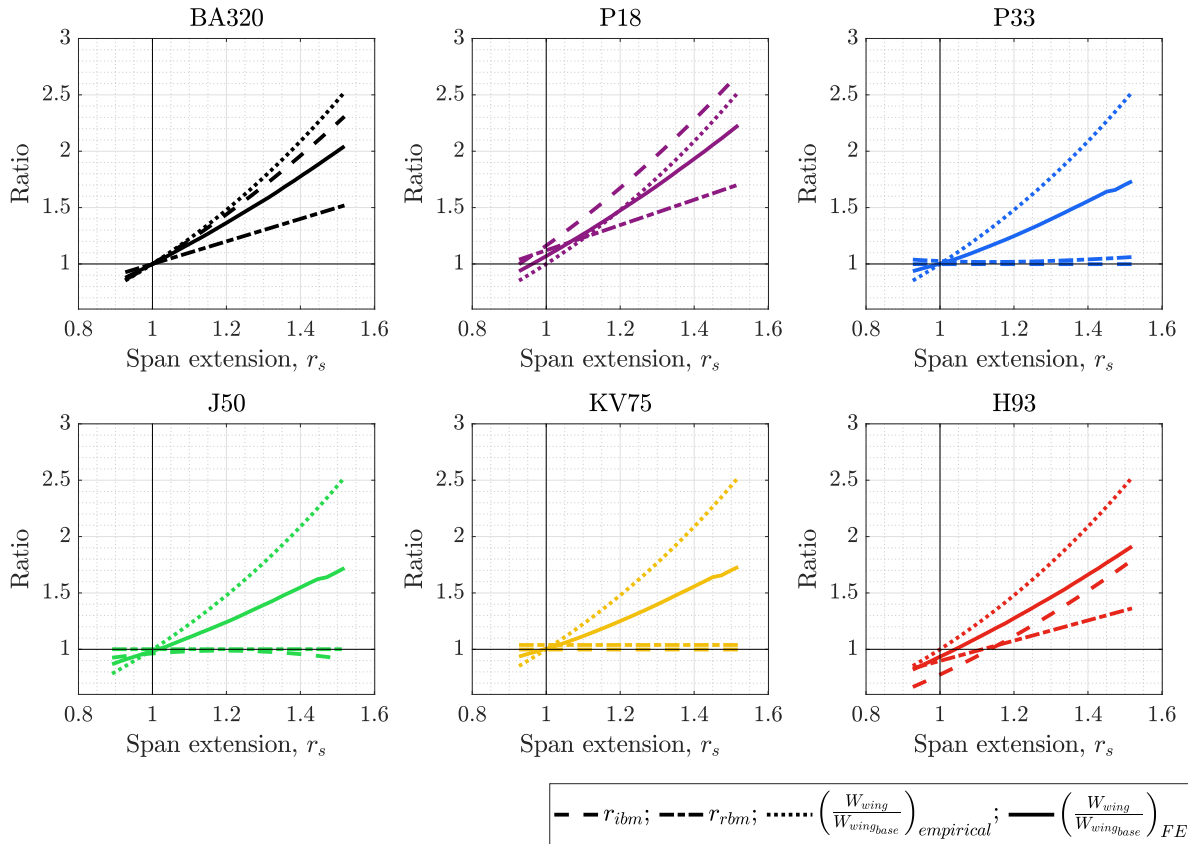


Figure 5.14: Variation of the wing weight with span extension as a ratio to the baseline configuration.

The wing weights resulting from this approach are presented in Figure 5.14, along with the prediction trends derived from both analytical and empirical methods employed, derived from Equations 4.55, 4.57 and 5.39 respectively. Inspection of the families with constant non-dimensional centroid reveals that the analytical ratio of integrated bending moment shows a vertical offset against the results from the current methodology that increases as the centroid is shifted outboard. Clearly, the estimation of wing weight through this parameter on the NELD families is erroneous. As is the root bending moment ratio, which will mainly underestimate the results significantly, except at small span extensions. The results yielded by the empirical formulation have been previously discussed, and show no link to the spanload characteristics.

Figure 5.15 shows the trends for weight increase of all the sized components, as well as the overall impact on MTOW, normalised against the baseline configuration. It is shown that while structural equivalence was not achieved through the use of NELDs, this wing design approach reduces the growth in wing weight increase as the span is extended by approximately 30% in all three cases. On the contrary, this gradient increases as the spanwise position of the centre of lift moves outboard, but this is avoided through the use of NELD. Consequently, a similar result is obtained for the trends in fuselage weight, the variation of which is solely affected in this study by the loads transferred from the wing. Looking into horizontal and vertical stabilisers, their weight is negligibly affected by the spanload variation. The impact on the aircraft inertia as the span is extended will contribute to the increase in the weight of the vertical stabiliser, while the horizontal stabiliser is affected by the backwards shift in centre of lift.

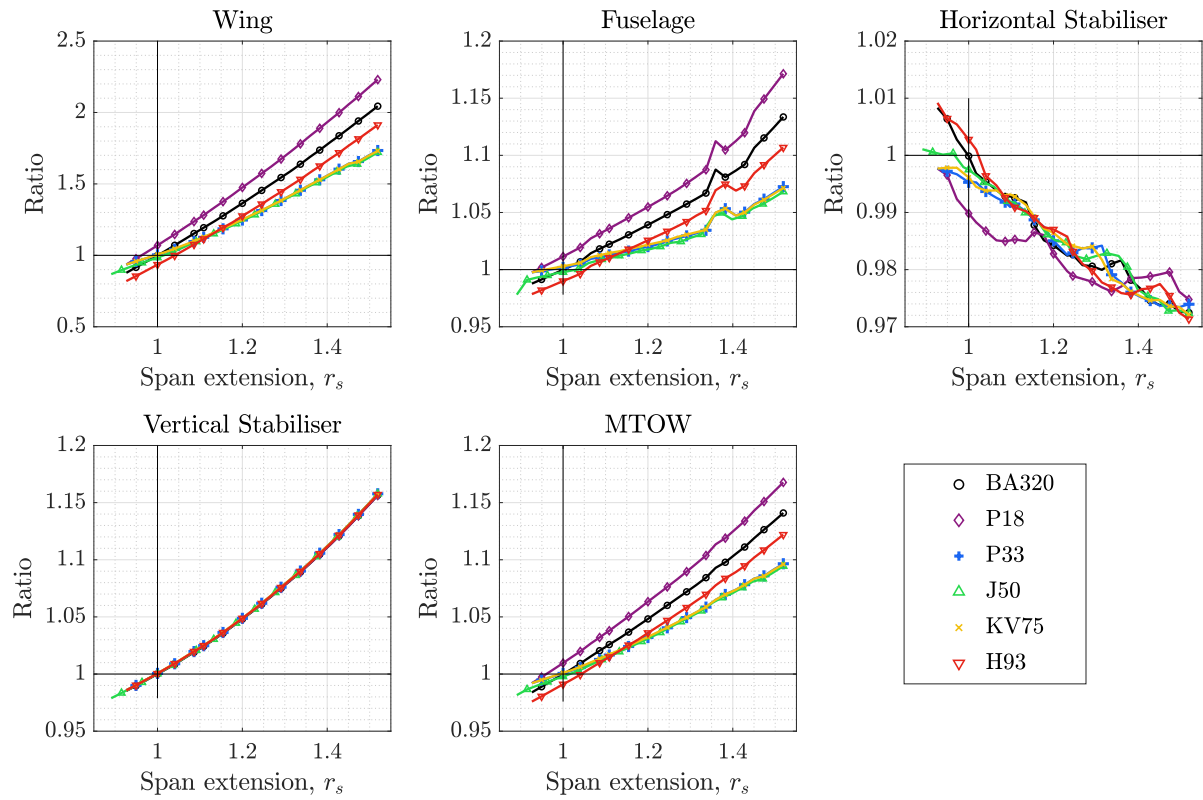


Figure 5.15: Weight variation of the structural components with span extension as a ratio to the baseline configuration.

5.3 Conclusions

The adopted approach to extend the aero-structural analysis of the computed configurations has been presented in this chapter. With this, the coupled VLM and beam element models provided in NeoCASS have been adapted and integrated into the framework to perform the structural sizing of the computed configurations under a broader number of critical load cases.

This has allowed to overcome some of the limitations assumed in the derivation of the wing shape design methodology presented in Chapter 4. For instance, the revised airframe layouts can now be assessed for several points in the flight envelope and based upon equilibrium manoeuvres loads, while the refined structural model includes more rigorous constraints and incorporates secondary weights and inertial loads. Altogether, this approach improves the accuracy of the design simulations while maintaining low processing costs. The final product is a design environment for aero-structural analysis based on physics-based computational methods. The results show that the idealised assumptions taken in the NELD classical theories cannot be sustained under higher fidelity considerations. Hence, the sought structural equivalence is not attained, but a decrease in the growth of structural mass required with span extension is achieved through the use of NELDs.

Furthermore, the approach followed enables later consideration of aeroelastic behaviour, performance, and stability and control of flexible wings within the conceptual design stage. In later studies, geometric non-linearities should be considered in order to enable the increasing necessity to predict static and dynamic behaviours of flexible aircraft at the conceptual design stage, especially if the configuration under consideration involves HARWs.

Chapter 6

Mission Performance Analysis

At this point of the research, the strongly coupled problem between aerodynamic efficiency and weight in HARWs has been raised and addressed through the analysis of NELDs and traditional wing designs. Nevertheless, it has also been reiterated that aircraft design encompasses multiple disciplines that are heavily interdependent. The ultimate goal, however, is to optimise an aircraft for a specified mission while trying to minimise or maximize a desired objective function. Hence, the culminating impact of the trade-off between the aerodynamic and structural characteristics brought by NELDs wings is investigated here through the mission performance analysis. The mission brings the combination of all the disciplines together, where the proposed metric of range over weight will be the figure of merit in this study to evaluate the final aircraft performance, as well as the feasibility of the approach taken towards aero-structural design and efficiency.

The methodological approach taken to complete this analysis is first presented from the theoretical aspects upon which the problem is founded, together with the assumptions taken to build the current method. Here, mission analysis methods that allow rapid analysis of conceptual aircraft have been favoured due to the beneficial flexibility and set-based design environment that they provide to perform trade-studies. Each design point carries the sized aircraft presented in Chapter 5, the wing shape design computed in Chapter 4 and the full aircraft layout built upon the baseline model. The aerodynamic database is extended to include a more comprehensive prediction of drag, for which low-order low-fidelity methods are employed to estimate the contribution of profile drag from all aircraft components, and with this build the required drag polars for all segments of the mission. Further aircraft characteristics will be defined as control variables and will be stated throughout this chapter. The proposed approach is later described along with the key findings derived from this particular analysis.

6.1 Theoretical Foundations and Research Assumptions

Mission analysis refers to the specification of a complete set of operational parameters and the weight and fuel planning that result in an aircraft completing its mission. Numerous parametric considerations are usually added, which may include mission profile optimisation, environmental impact or various commercial demands.

A number of established methods with differing levels of fidelity and applicability exist. Traditional analysis methods will estimate fuel weight or take-off gross weight (TOGW) through the assessment of the mission performance, using segment weight fractions that are either based on historical data or computed based on a limited set of parameters describing aerodynamics, propulsion, and flight condition. The weight fractions of the cruise and loiter segments are usually computed via the Breguet range or endurance equations. Each segment can be discretised into smaller time increments, where the variation of weight is tracked and then numerically integrated over the whole mission profile to obtain the conclusive vehicle performance. This process requires a significant database from the aerodynamics, propulsion, and weights characteristics at several flight conditions in order to estimate the energy requirements of the aircraft over the course of the defined mission. With an accurate database, traditional approaches work satisfactorily and rapidly for conventional propulsion systems powering established vehicle layouts. Nevertheless, these methods become unsuitable for the estimation of energy requirements of novel aircraft with hybrid-electric or fully-electric propulsion systems. Once again, yet another critical discipline of the conceptual design stage encounters fidelity penalties due to the past establishment of empirical and low-fidelity methods. Herein, this limitation will not have any impact of relevance, since the aircraft configurations being addressed in this analysis have a traditional tube-and-wing layout powered by fuel-burning propulsion.

Range is the quantity used in this analysis to characterise the performance of each design point. Following the discussion on the measure of efficiency in Section 2.3.4, range can represent the assembly of both efficiency and penalty contributions of an aircraft. The aircraft range is the distance covered in a mission. This includes a number of allowances that must be accounted, such as consideration of the fuel needed on the terminal phases, manoeuvring and fuel reserves. Within mission analysis, range can be treated in various ways, where it can be fixed to optimise weight, or maximised under a given configuration. The large number of free parameters involved here, such as Mach number, altitude, lift coefficient, angle-of-attack, gross weight, block fuel; and number of external constraints given by regulations, can yield a variety of trajectories with optimal and sub-optimal results. Range is generally estimated through the Breguet range equation, presented in Equation 2.9, which provides an integration of the conditions above across the cruise segment. Here, the optimal

trajectory will not be sought. Instead, a fixed set of constraints define a simple mission profile for which the range will be calculated as a consequence of the design flight conditions for each segment.

6.1.1 Energy-Based Method

At any point in time, the total energy of an aircraft is the sum of its potential and kinetic energy, and thus the measure of their exchange provides a reliable method to analyse and optimise mission profiles and trajectories. Energy-based mission analysis was first introduced by Rutowski [291] to address the limitations posed by the methods of the time, in which an optimum path for an aircraft was stated only through the change in altitude, and therefore speed was not considered. Instead, the energy method posed the problem in such a way that the significance of speed change to get to the altitude was now combined using an independent variable that combined both conditions. With this, the general aircraft performance problem was considered from the perspective of maintaining balance between the potential and kinetic energy, the energy dissipated against drag, and the energy derived from fuel.

The energy-based approach is a low-order method that models the aircraft as a point mass and analyses the complete mission from an energy balance perspective. This formulation simplifies the vehicle dynamics to a great extent, such as neglecting rotational effects, but can still compute trajectories with high accuracy and speed without the need to solve the aircraft equations of motion [240]. For this reason, this method has been extensively used over the past decades by both academia and industry, as it also becomes suitable for exploring large design spaces and performing trade studies.

The energy approach to mission performance can be started through the assumption that the energy source for propulsion is stored in the fuel. This stored energy is consumed and converted into propulsive work by the engine at some given efficiency η_0 . The energy delivered per fuel mass consumed can be expressed as the product of the overall power-plant efficiency and the fuel calorific value H_c . This energy is dissipated against drag forces and expended in increasing the total energy of the aircraft. Thus, the total energy of the aircraft may be expressed as the sum of its kinetic and potential energies as

$$E = Wh + \frac{WV^2}{2g} = W\left(h + \frac{V^2}{2g}\right) = W\left(\frac{E}{W}\right) \quad (6.1)$$

where W is now the aircraft gross weight, g is the acceleration due to gravity, and h is the altitude. Now, E/W represents the aircraft specific energy, or specific energy

height, and equates to the term

$$\left(\frac{E}{W}\right) = h + \frac{V^2}{2g} \quad (6.2)$$

Application of conservation of energy over a time differential dt yields

$$dE = \eta_0 H_c dW_{fuel} - \left(\frac{E}{W}\right) dW_{fuel} - D V dt \quad (6.3)$$

which describes the energy balance between the total energy change of the aircraft, the energy derived from the fuel consumption, and the energy dissipated by drag. Now, differentiation of Equation 6.1 yields

$$\begin{aligned} dE &= W d\left(\frac{E}{W}\right) + \left(\frac{E}{W}\right) dW \\ &= W d\left(\frac{E}{W}\right) - \left(\frac{E}{W}\right) dW_{fuel} \end{aligned} \quad (6.4)$$

given that $dW = dW_{fuel}$. Equating 6.3 and 6.3 becomes

$$W d\left(\frac{E}{W}\right) = \eta_0 H_c dW_{fuel} - D V dt \quad (6.5)$$

The energy balance relationship is expressed here in terms of the change in specific energy. As seen in Equation 6.2, the specific energy is a function only of the speed and altitude of the aircraft, as opposed to the total energy, which also depends on gross weight.

This general formulation represents the basic relationship from which most of the equations used in the calculation of mission performance can be derived, and it provides a common approach to several performance problems, such as climb and range, which are usually treated independently.

When applied to climb performance, the use of specific energy as the independent variable is particularly useful to solve common problems in this stage, such as minimum time and minimum fuel to climb. The energy balance approach to the range equation provides a clear isolation of the elements that determine aircraft range. The expression can be obtained through the definition of range as the product of the aircraft velocity by the time it remains in the air, such that $dR = V dt$, by introducing it in Equation 6.5 giving

$$dR = \frac{\eta_0 H_c}{D} dW_{fuel} - \frac{W}{D} d\left(\frac{E}{W}\right) \quad (6.6)$$

This expression can be integrated along with several assumptions. Firstly, force balance relations for cruise may be applied and assumed that lift-to-drag L/D and η_0 remain constant during this flight segment. Next, the product $\eta_0 H_c$ represents the useful work

of propulsion done per fuel unit, which can be expressed in terms of the thrust T and the fuel rate, and then rewritten as a function of the specific fuel consumption $TSFC$. With this, Equation 6.6 becomes

$$R = \frac{V}{TSFC} \frac{L}{D} \ln\left(\frac{W_0}{W_1}\right) - \frac{L}{D} \left[\left(\frac{E}{W}\right)_1 - \left(\frac{E}{W}\right)_0 \right] \quad (6.7)$$

where the subscripts $_0$ and $_1$ denote respectively the initial and final aircraft weight state at each segment. The first term in the above equation gives the well-known expression for range of a jet-powered aircraft as shown in Equation 2.9. The second term appears when there is a change in energy level during the cruise segment. The assumption of constant lift-to-drag during the cruise segment essentially implies a constant aircraft attitude, for which the dynamic pressure must decrease with the gross weight. This therefore results in a change in the energy level and will require the consideration of the additional term. With this, the second term may be expressed as a correction to the $V/TSFC$ ratio as

$$R = \left[\frac{V}{TSFC} - \frac{g}{R_0 T_0} \right] \frac{L}{D} \ln\left(\frac{W_0}{W_1}\right) \quad (6.8)$$

where R_0 and T_0 here correspond to the gas constant for air and the atmospheric temperature. With this, the range of each configuration can be now evaluated and compared. For further detailed derivations the reader is referred to References [291] and [13].

6.1.2 Mission Profile

With the aircraft's energy balance in place, it is possible now to determine the trajectories to be flown as specified by the desired mission profile. A mission profile establishes a scenario that determines weights, fuel, payload, range, speed, flight altitude, loiter and any other operational parameters specific to the type of aircraft that must be accomplished to complete its mission. A mission profile will also define the phases at which different events and environments occur, and records the time that is spent on each segment.

In this study, the climb segment will be determined by the minimum fuel-to-climb profile. At the cruise segment, the chosen trajectory will be determined by fixing the Mach number and flying at the optimum altitude for maximum range. For the evaluation of the cruise segment, steady flight conditions are assumed. This together with the assumptions previously made for the derivation of the energy-based mission analysis of constant specific fuel consumption and constant lift-to-drag lead to the condition that the lift coefficient must be held constant. As the aircraft becomes lighter due to fuel consumption, the aircraft needs to fly at a lower dynamic pressure.

Because velocity is also held constant in this mission profile, dynamic pressure will be only reduced by climbing, resulting in a flight path known as a cruise-climb, which maximises range. Alternatively, the range equation can be broken down into several cruise segments, for which the corresponding lift-to-drag as a function of weight are used. Generally, a cruise-climb profile is not performed by airliners to improve the control over air traffic, but it will still be applied here to obtain values of maximum range.

Now, to compute the optimal flight conditions for maximum range as the aircraft's weight changes, the weights and specific energy levels are discretised and optimised given the desired objective function, for which maximum range can be obtained by minimising the ratio of fuel-flow rate to the velocity. With this, the optimisation problem applied to each weight W_i can take the following form

$$\begin{aligned}
 &\text{minimise}_{h,V} \quad f_{obj} = f\left(\frac{1}{V} \frac{dW_{fuel}}{dt}\right) \\
 &\text{subject to} \quad T - D = 0 \\
 &\quad \quad \quad L - W_i = 0 \\
 &\quad \quad \quad h_{min} \leq h \leq h_{max} \\
 &\quad \quad \quad V_{min} \leq V \leq V_{max}
 \end{aligned} \tag{6.9}$$

6.1.3 Estimation of Drag Polars

Up to this point, the aerodynamic analyses carried out have only taken into consideration the impact of lift-induced drag through the use of inviscid and incompressible methodologies given by the lifting line and the vortex lattice models. To recall, the primary focus of this research lies on the trade-off between spanload manipulation and span extension to improve aero-structural efficiency by effectively reducing induced drag while minimising the increase on weight. But to provide a robust estimation of the lift-to-drag ratio and the final range of an aircraft it is also necessary to consider the remaining drag components, as they will have a significant contribution to the thrust required and the total energy balance.

Drag may be decomposed into several components arising from different generating sources, as depicted in Figure 6.1. In the present study, the additional drag contributions that are going to be included in the total drag estimation start with the profile drag C_{D_0} , which is composed of the drag caused by skin friction C_{D_f} , and pressure induced by viscous effects C_{D_p} . Wave drag C_{D_w} is going to be regarded too, and arises from the shock waves due to compressibility effects.

There are numerous available methods to predict these drag components at the conceptual design stage. For the calculation of profile drag, the two-dimensional

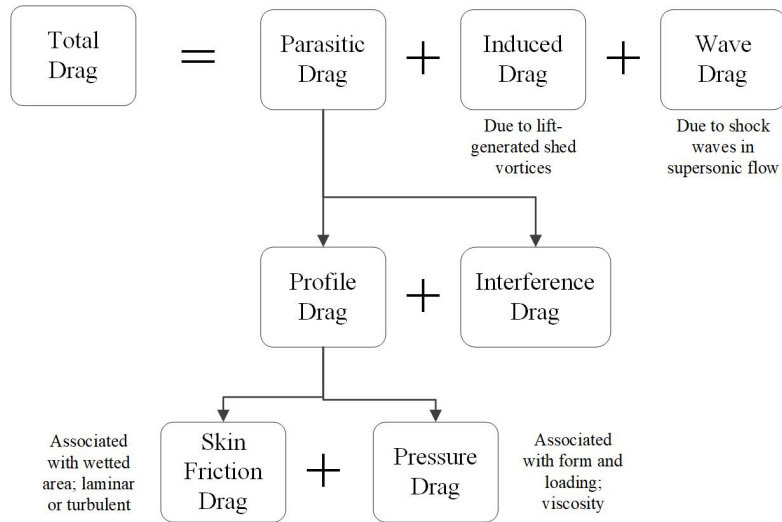


Figure 6.1: Diagram of drag breakdown.

viscous flow analysis is adopted, which couples an outer potential flow and an inner boundary layer model to obtain the aerofoil section contribution of the profile drag, composed of friction and pressure drags. The inviscid region may be described by the classic linear-vorticity stream function panel method corrected for compressibility, with the subsonic far-field being represented by vortex, source and doublet singularities placed at the aerofoil. The viscous boundary layer and wake regions are described by the momentum integral boundary layer formulation with a simplified free laminar-turbulent transition criterion. With this, the section profile drag can be determined from the wake momentum thickness far downstream, through its proportionality to the local energy dissipation rate due to viscous shear. The skin friction drag component can be derived from the resulting shear stress within the boundary layer, while the pressure drag component is the difference between the latter two. Further details on the calculation of profile drag through the two-dimensional coupling of potential flow and boundary layer can be found in References [5,292]. The drag build-up method [13] is an alternative empirical approximation to estimate the total parasitic drag contribution of each external component of the aircraft, since it includes both skin-friction and separation pressure drag as well as interference effects. Nevertheless, in this study the interference drag is assumed to be equal across all the simulated configurations and is therefore neglected. This simplification does not capture the impact that the change in twist distribution will have on the shed flow and consequently on the interference effect between the wing and the other components. Notwithstanding, the viscous-inviscid coupling methodology is preferred for the purpose of this research and the proposed framework. This approach allows the calculation of the parasitic drag polar at each chord section for local Reynolds and airspeed conditions. However, this low-order

approach will disregard the impact of three-dimensional crossflow instabilities.

The impact of profile drag increases with airspeed, becoming a significant contribution during transonic cruise. At this flight regime, local supersonic flow is assured to occur on the aircraft. A region of supersonic flow is terminated by a shock wave, which will slow down the flow to subsonic speeds but will simultaneously generate a drag onset, as discussed in Section 3.2.3. Since the design cruise speed of the aircraft configurations being analysed is found within the transonic regime, it is necessary to provide an estimate of the potential drag contribution due to transonic compressibility effects. To do so, an empirical method proposed by Lock [293] to derive drag rise due to transonic shock waves may be used and is given by

$$C_{D_w} = 20(M - M_{crit}) \quad (6.10)$$

for the airspeed range above critical Mach number $M \geq M_{crit}$. The critical Mach number can be obtained from the derivative of the drag rise formulation, yielding

$$M_{crit} = M_{DD} - \left(\frac{0.1}{80}\right)^{1/3} \quad (6.11)$$

where M_{DD} is the drag divergence Mach number, and can be computed as a function wing planform and design lift through the extended Korn equation [293,294] as

$$M_{DD} = \frac{\kappa_A}{\cos \Lambda} - \frac{t/c}{\cos^2 \Lambda} - \frac{\kappa_B C_L}{\cos^3 \Lambda} \quad (6.12)$$

in which κ_A is the aerofoil technology factor and can be found within the range of $\kappa_A = 0.87$ for NACA series aerofoils to $\kappa_A = 0.955$ for modern supercritical wing sections; $\kappa_B = -dM_{DD}/dC_L$ represents the effects of the aerodynamic lift on the divergence Mach number, and yield approximate values of $\kappa_B = 0.1$ – 0.14 for supercritical aerofoils to $\kappa_B = 0.2$ for aerofoils such as NACA 0012 used here for the empennage. With this, the wave drag contribution from the lifting surfaces can be calculated for the cruise segment.

For the low-speed segment of the mission profile, it is necessary to provide the corresponding drag polars with the addition of other relevant contributing factors. Here, high-lift control surfaces, aerodynamic brake devices and landing gear will be considered. To start, a flat plate approach [295,296] is used to estimate the drag increment due to flap deflection as

$$\Delta C_{D_{0flap}} = k_{flap} \left(\frac{c_{flap}}{c}\right)^{1.38} \frac{S_{flap}}{S_{wing}} \sin^2 \delta_f \quad (6.13)$$

for which $k_{flap} = 0.9$ for slotted flaps and the flap deflection δ_f corresponds to the take-

off or the landing configuration summarised in Table A.1. The drag increase due to spoiler deployment is given as $\Delta C_{D0,spoiler} = 0.05$ [297], while the contribution from the air-brakes is $\Delta C_{D0,air-brakes} = 0.04$ [298]. Finally, the increase in drag due to unretracted landing gear equals $\Delta C_{D0,landing\ gear} = 0.04$ [299].

The profile drag polars and drag increments derived from the above approaches can be now linearly added to the induced drag polars computed for several speeds and altitudes within the operation flight envelope shown in Figure 5.7. Altogether, the produced drag polars constitute the database that will characterise the aerodynamic behaviour of each configuration throughout the mission profile, and will be used to determine the aircraft performance.

6.2 Proposed approach

The theoretical approaches presented above provide the means to perform mission analyses that allow rapid assessment of the performance of conceptual aircraft with an appropriate degree of accuracy, whilst allowing to carry out a large number of feasibility and trade studies. Given the methodologies presented in Chapters 4 and 5 and Section 6.1 to compute the geometric, aerodynamic and structural characteristics of each configuration, together with the engine performance database discussed in Section 3.2.5, it is possible now to analyse each design point database under these approaches to assess the overall impact of the span and spanload under mission performance metrics. The workflow proposed to achieve such target is presented in Figure 6.2.

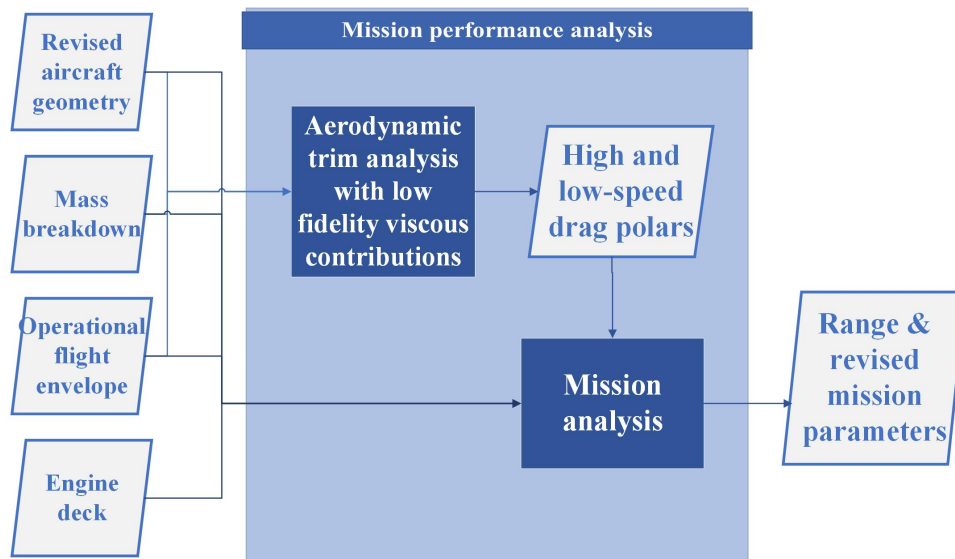


Figure 6.2: Proposed analysis process for the mission performance analysis.

6.2.1 Aerodynamic Analysis

As discussed in Section 2.5.2, there exist a wide number of design tools that adopt the methodologies presented above with different levels of fidelity. For the calculation of the complete set of aerodynamic data required to perform the mission analysis, a suite of legacy codes has been integrated into the framework under the criteria for simplicity of implementation and run-time minimisation. AVL [269] is used to compute the trimmed drag polars in potential flow. Compressibility is addressed in AVL using the Prandtl-Glauert correction, as discussed in Section 4.1.1. Furthermore, AVL enables the easy modelling of control surfaces and high-lift devices in order to obtain their contribution towards lift and drag as well as obtain the configuration's stability and control derivatives. Now, the inviscid data of the lifting surfaces is combined with the profile drag polars obtained from XFOIL [300], which is a tool for the design and analysis of subsonic aerofoils. Here, the profile drag is computed for each aerofoil at the corresponding flight conditions, and then added by linear superposition via interpolation and integration of the two-dimensional local drag distributions on each lifting surface. Finally, additional profile drag contributions for the fuselage and engine derived from historical data are directly added to the total component drag. AVL and XFOIL enable parallel batch processing via a straightforward communication with the Matlab-based framework through input and output formatted files. The polars are computed at a combination of flight speeds and altitudes for various angles-of-attack to create lookup tables that will be then treated as surrogate models for aerodynamics. The drag curves computed for the low-speed segments of the mission will include the flaps deflected and the additional contributions from landing gear and speed. The flight points are listed in Table 6.1. The lifting surfaces have been discretised to provide the same aerodynamic mesh as the one produced in NeoCASS' VLM model and shown in Figure C.1 and yielding 572 vortices. The relevant control surfaces have been modelled too, following the sizing provided in Table A.3.

Table 6.1: Range of flight points variables for the calculation of high and low-speed drag polars.

Mach, M_∞	0.10 to 0.82
Altitude, h	0 ft to 39,000 ft
Angle-of-attack, α	-2° to 14°

With this, the total drag polars at cruise steady level flight are shown in Figure 6.3, for the complete configurations and the wing only. The design cruise lift coefficient of each configuration has been marked for reference, and all the configurations are compared to the baseline model. Furthermore, let us recall that the NELD families will see negative circulation after a certain span extension is applied, as discussed in Section

4.1.1 and observed in Figure B.1. The results for these undesirable wing configurations are again plotted with dotted lines.

Looking into these results, most of the configurations move the curve to the left, reducing the the total drag for most lift coefficient values. It is seen that an inboard shift of the non-dimensional location of the centre of lift with span extension will provide a smaller improvement in drag characteristics, and will yield penalties at lower drag coefficients under approximately $C_L = 0.5$. The difference between the wing drag polar and the drag curves for the complete configuration shows that trim drag will exacerbate these penalties, since the contribution from non-lifting surfaces is here assumed constant with angle-of-attack.

The trends of the drag polars with span extensions for the different spanload families generally follow the same behaviour that was theoretically predicted for induced drag only, discussed in Chapter 3, and shown in Figure 4.6. The total drag is split into induced and parasitic in Figures D.1 and D.2 respectively, in order to provide a better insight of their contributions. Comparing both Figures, it is seen that majority of the impact in drag is derived from the variations in induced drag, while the profile drag remains almost constant across wing shapes except at higher lift coefficients, where the baseline configurations shows the lowest results. This is completely dependant on the given baseline wing planform and twist distribution, which in this case has coincidentally yielded the lowest profile drag. The general profile drag trends agree with the study by Abdel-Motaleb and Hunsaker [69], which shows that NELD wings with aerofoil cambers above 10% may slightly reduce the profile drag contribution as compared to an elliptic spanload. However, it is important to recall that a great limitation of the methods employed for this analysis is the exclusion of some transonic phenomena such as local shock waves and non-linear effects such as separation, which is partially accounted for in the form of local profile drag increase. Consideration of these effects will be a critical factor on the calculation of the actual aerodynamic efficiency across the flight envelope due to the large washout found in NELD wings. These results are similarly presented in the form of lift-to-drag ratio in Figure D.3, where majority of configurations show improvement at the design cruise condition and across the operating range of angles-of-attack.

The resulting induced drag variation is shown in Figure 6.4 against the baseline configuration, and is now compared to the analytical results obtained in Chapter 4. The graph on the left shows the total drag trends for the complete configurations trimmed at the design cruise condition. As expected, the improvement in drag is still achieved, but has reduced due to the consideration of profile and trim drag. The difference

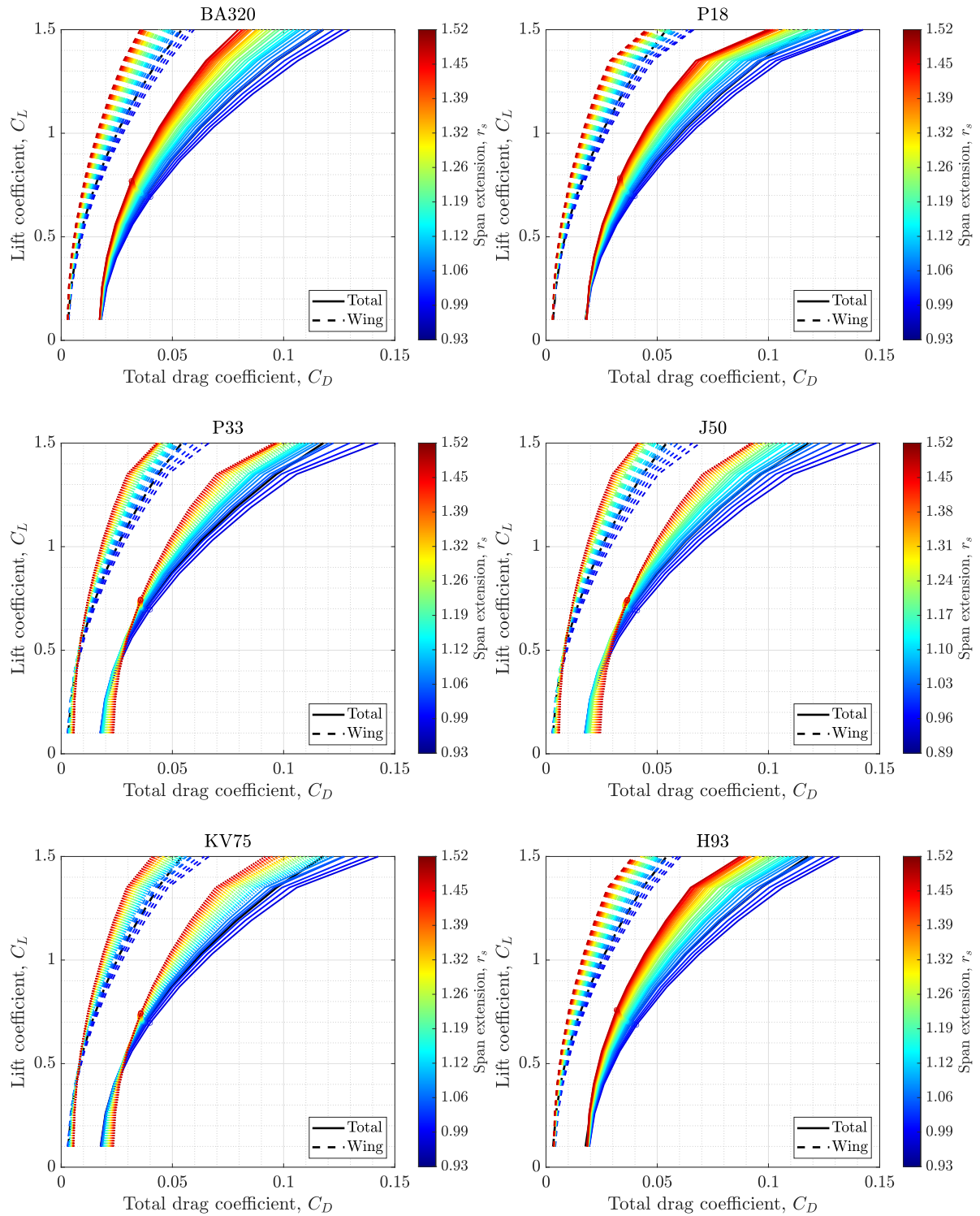


Figure 6.3: Total drag polar for steady level cruise flight.

between extending the span with a fixed centroid or an inboard-shifting centroid has diminished, showing higher penalties on the fixed spanload configurations. Furthermore, the impact on the trimmed condition required after the estimation of the aircraft mass can be observed in the further penalised results for the P18 family, where the

higher mass developed due to the outboard centre of lift eventually requires higher lift at the cruise flight condition.

Finally, the variation of lift-to-drag ratio over the baseline configuration is presented in Figure 6.5, which considers the change in gross weight through the design cruise lift coefficient. Rather than the drag force contribution, it is the lift-to-drag ratio that will have an impact on the range, as shown in Equation 6.8. When considering this metric, the difference in aerodynamic efficiency improvement between morphing and fixed spanload families starts to become significant above a 15% span extension, where the NELD families start to plateau and the fixed spanload families increase efficiency dramatically as theoretically predicted. Below that, increases in lift-drag ratio grow in a similar manner.

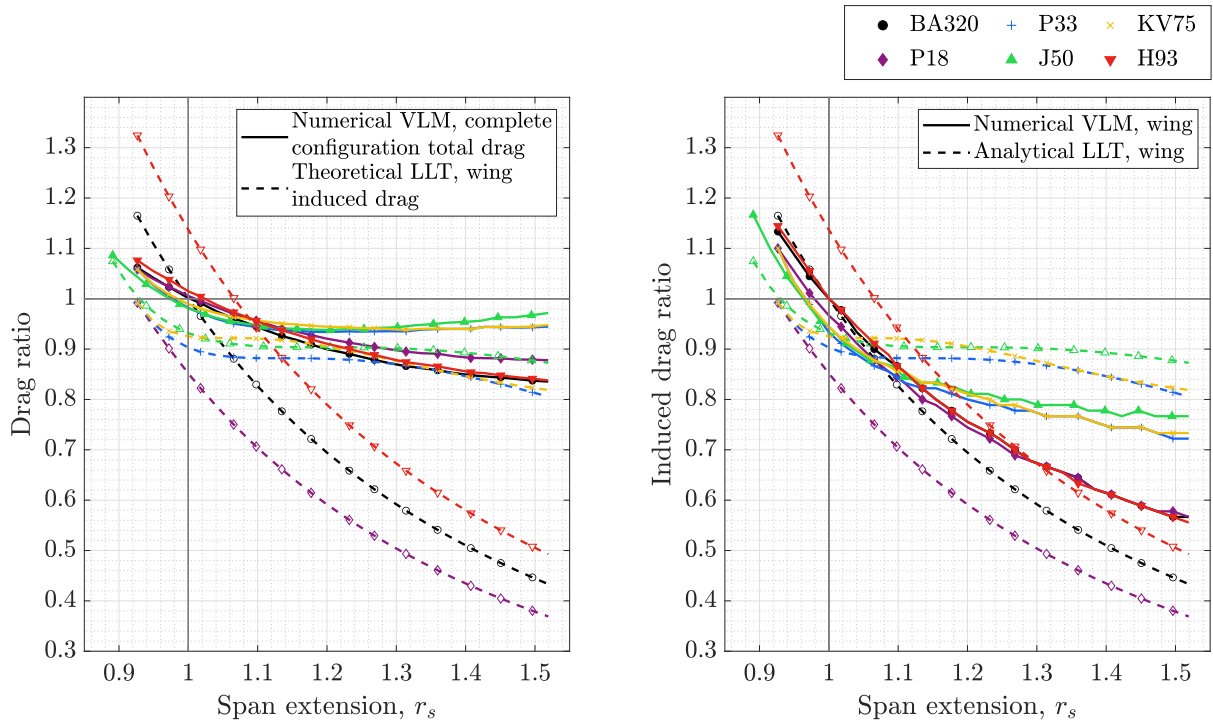


Figure 6.4: Comparison of the induced drag variation derived from theoretical and numerical approaches.

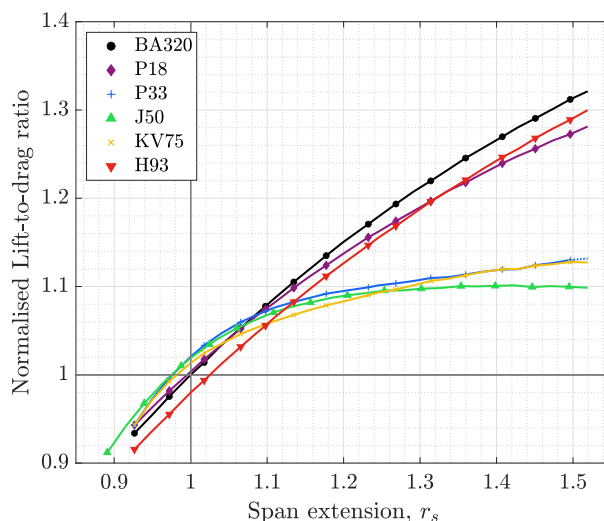


Figure 6.5: Lift-to-drag variation over baseline configuration.

6.2.2 Mission Analysis

For the calculation of mission performance, the Flight Optimization System (FLOPS) [301] has been integrated into the analysis framework. FLOPS is an aircraft design optimisation tool for the conceptual design stage developed for evaluating relatively advanced aircraft concepts. The program consists of several analysis modules that cover the estimation of multi-disciplinary aspects of the design, such as weights, aerodynamics, engine and propulsion data, mission performance or cost. The different computational models incorporated in FLOPS can be used as standalone tools. However, some of them carry a number of limitations that make it unsuitable for its use on unconventional aircraft designs or when higher fidelity is expected from the results. Particularly, the aerodynamic and weight estimation models are heavily based on reduced-order low-fidelity empirical and semi-empirical methods, and are therefore not used in this study. However, the mission analysis modules do provide an adequate level of fidelity given that the configurations being assessed are relatively traditional in terms of layout and power-plant. Regarding the other modules, the impact of NELD wing design on such disciplines is out of scope. FLOPS provides an interface that can be easily communicated and run in parallel within the Matlab environment of the proposed framework. Compared to other tools discussed in Section 2.5.2, FLOPS' approach to mission analysis carries a more extensive development of the energy-based method that allows a more comprehensive analysis of the mission. For instance, SUAVE will not allow to optimise climb and descent for minimum time or fuel. On the other hand, TASOPT will modify the aircraft to optimise it for the specified mission, which is inadequate to perform direct assessments of a configuration with a frozen design, as is the case in this study.

FLOPS uses the energy method described in Section 6.1.1 to determine the appropriate climb and descent trajectories, and steady level flight is used for the cruise segment. The fundamental assumptions adopted in its mission analysis principles include: the aerodynamic characteristics are solely a function of Mach number, altitude, and lift coefficient; the aircraft is modelled as a point mass with a single degree of freedom given by the energy altitude; weight reduces during the mission due to fuel consumption. With this and the input databases provided in the form of parameters for the weights, or look-up tables for aerodynamics and engine performance, a set of surrogate models will be created for the different mission segments. Now, the optimal trajectories can be determined under the chosen objectives for each segment.

In addition to the aerodynamic and propulsive input data presented in Sections 6.2.1 and 3.2.5 respectively, a set of other geometric and weight data parameters are determined for the analysis, which have been computed for each configuration within the analysis framework. The range is the design variable being computed, but reasonable initial guess is still necessary for the analysis, and is given by the reference design range of the baseline configuration shown in Table A.1, as it is the target minimum range that most configurations should be achieving. The mass configuration being used here is the MTOW. For consistency and comparability, cruise Mach number and maximum cruise altitude are fixed instead of being optimised. This will allow to compare the mission performance across configurations for the same flight conditions. For the take-off and landing analysis, the approach speed is provided for each configuration as a function of the stall speed computed in Section 5.2.4 as $V_{app} = 1.3V_{S1} + 20$ kts. As to engine performance, flight idle data generated internally in FLOPS with zero thrust and an extrapolated fuel flow.

The mission profile used for this study is simple and has been envisioned as such to emphasise the fundamental performance characteristics for each of the principal segments in a mission. Hence, it consists of

1. Start - taxi and take-off;
2. Climb - minimum fuel-to-climb;
3. Cruise - fixed $M_{cr} = 0.78$ and constant $C_{L_{cr}}$;
4. Descent - maximum L/D ;
5. End - land and taxi in.

No turns, accelerations nor hold segments are modelled in the mission. The climb profile optimisation is performed for minimum fuel-to-climb, in which the climb speed is limited to a maximum of 250 KCAS below 10,000 ft under the CS-25 certification specification requirement. The same constraint is applied for descent, which flown at

optimal lift-to-drag ratio. The trajectory optimisation routine in FLOPS will select the optimum altitude for each energy level. The cruise segment is calculated for a fixed Mach number, constant lift coefficient, obtained through the empirical formulation [301] below. The block fuel has been fixed in all cases. This includes the reserve fuel, which accounts for a hold time of 30 min, but no reserve mission to alternate airport is calculated. Figure 6.6 shows the diagram of such mission profile.

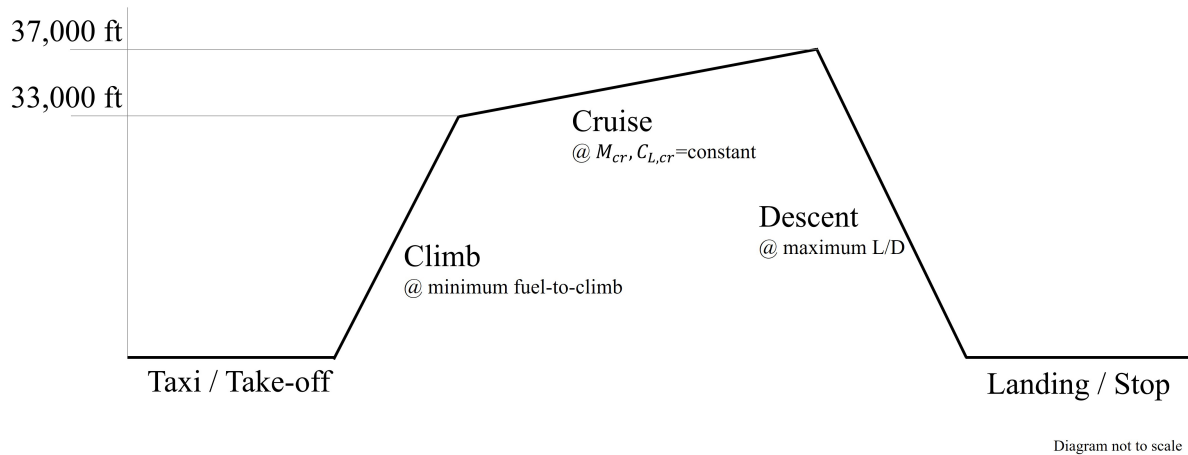


Figure 6.6: Representation of a typical cruise-climb mission profile.

With this, the analysis is set as a direct mission performance calculation for each individual configuration, in which the ramp weight is fixed and the range is calculated. The configurations must be able to fly the defined profile, for which the mission analysis will hold the design constraints. At each segment, a step integration technique is used to provide the values of the fuel burned, elapsed time, distance covered, and changes in speed and altitude. The climb segment is divided into energy steps, in which the combination of altitude and speed that optimises the objective for minimum fuel-to-climb is determined for each energy level. The cruise segment is calculated here from both ends of the mission, starting from the specified ramp weight to take-off, and integrating backwards in time from zero-fuel weight until conditions at cruise intersect by looking at the altitude and weight in the segment surrogate.

The resulting performance characteristics are shown below. In Figure 6.7, the evolution of several TLARs has been plotted against span extension, where the bound of acceptable values has been placed as the improvement from the results given by the baseline configuration. The range cannot be increased with span extension without eventually encountering penalties due the surpassing of the weight increase over the aerodynamic efficiency improvements achieved. The span-extended designs with fixed spanload maintain the growth in range up to higher spans values due to the increasing lift-to-drag ratio, but the range eventually plateaus for extensions above 40%. The NELD families see an increase in range only up to approximately a 10% of

span extension, after which they start yielding a gradual decrease in range as weight increases with span. Further discussion on this trade-off is later given with Figure 6.9.

Generally, increasing the wingspan of a given aircraft will provide a detrimental contribution to the top level aircraft requirements assessed, with the exception of the range. Nevertheless, the use of NELDs shows an analogous impact across most TLARs to that developed on the MTOW growth discussed in Section 5.2, in which the use of NELD for span extension becomes beneficial by reducing the penalties derived from span extension due to the carried trade-offs between aerodynamic and structural characteristics. This is achieved to such an extent that the trends deflect to provide minor improvements for the NELD configurations found at the lower bounds of span extension in the take-off field length, rate-of-climb and time-to-climb characteristics. Hence, the inwards shift of the centre of lift succeeds to reduce the variation from the benchmark value, whilst the extension of wingspan with a fixed centre of lift will offset the variation proportionally, thus being beneficial only according to the relative position to the baseline configuration.

Figure 6.8 shows the variation in performance characteristics in the form of parallel coordinates to visualise the sensitivity of wing modification on the various parameters. The data has been normalised against the baseline values. The paths show smaller peaks on the NELD families, representing the reduced impact on the TLARs enabled by this approach.

The design space between range and gross weight of the evaluated wing configurations is presented in Figure 6.9. Majority of the range increase achieved barely reaches 3% with up to 2% increase in weight. After that, the maximum range increase goes up to 5% necessitating high span extensions that cause increases in weight of over 10%. Optimal aircraft configurations would be found in the top left region of the design space, yielding higher range values for lower gross weights with extended spans. However, only the J50 family yield one such solution with an extended span of only 1.5%, a range increased by 1.6% and effectively the same weight as the baseline. However, the region in the design space that encompasses a 3% increase in range and weight shows a high density of solutions that could potentially provide higher performance than the baseline model with a low penalty risk.

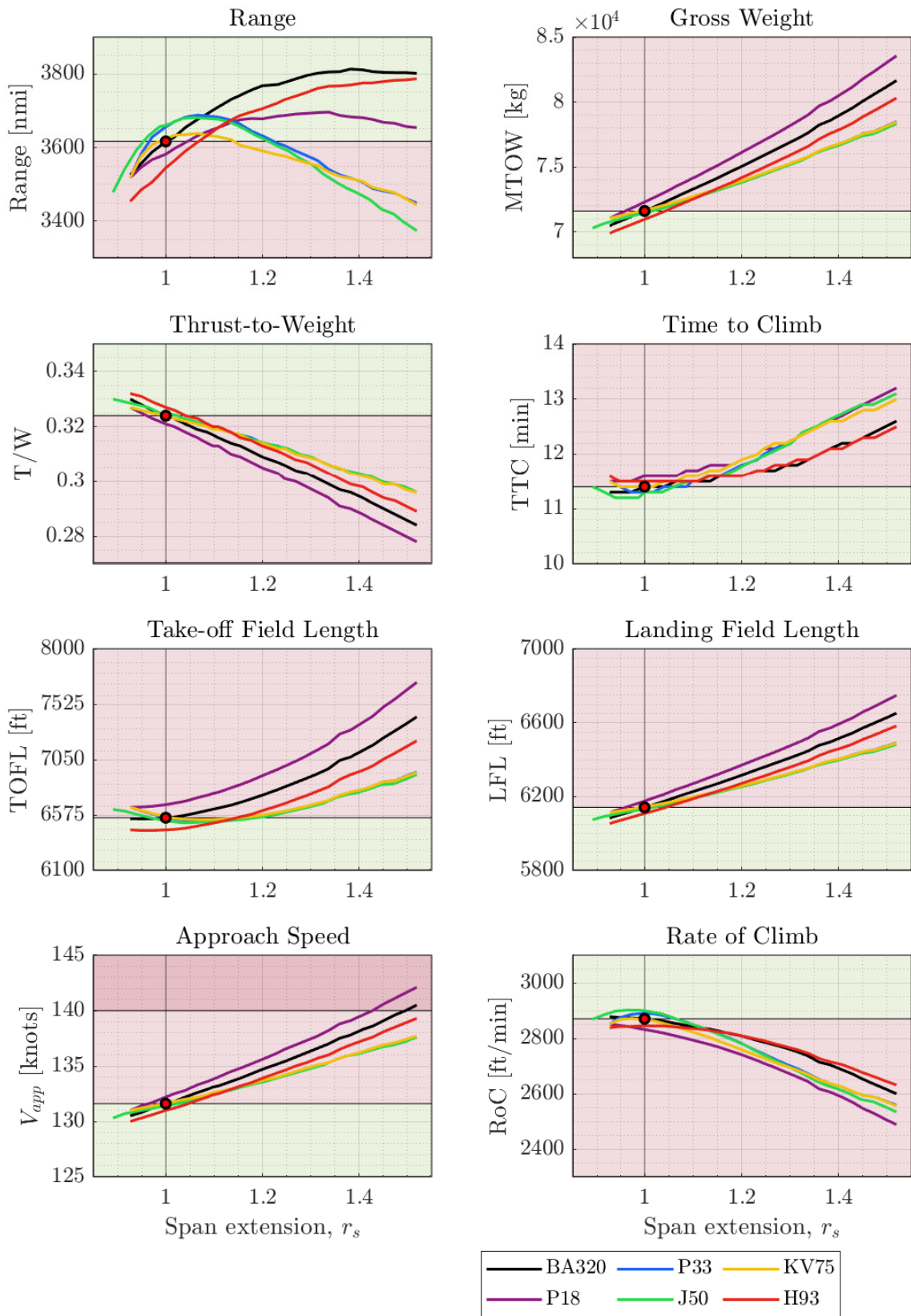


Figure 6.7: Trends for top level aircraft requirements with span extension.

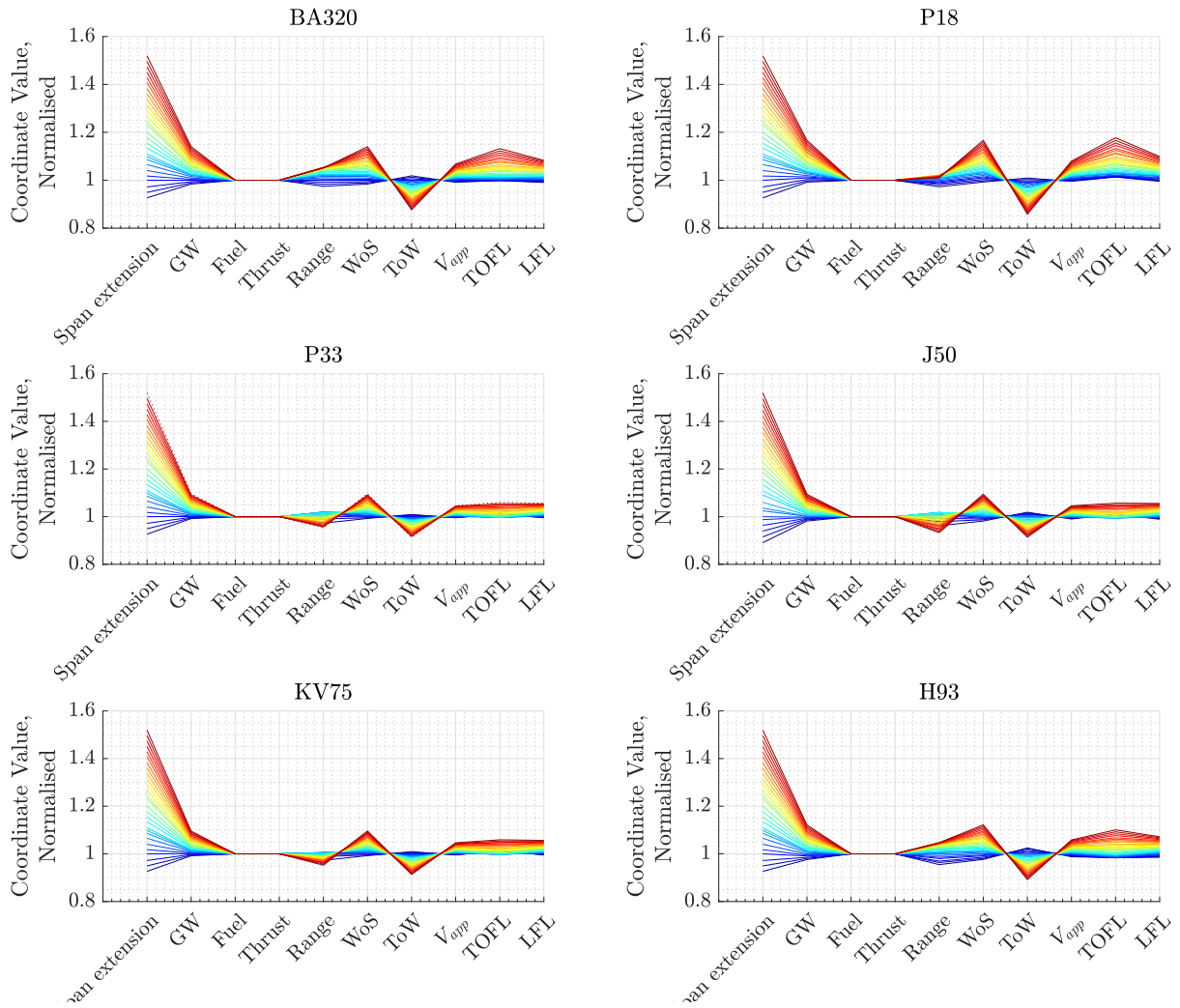


Figure 6.8: Parallel coordinates for top level aircraft requirements.

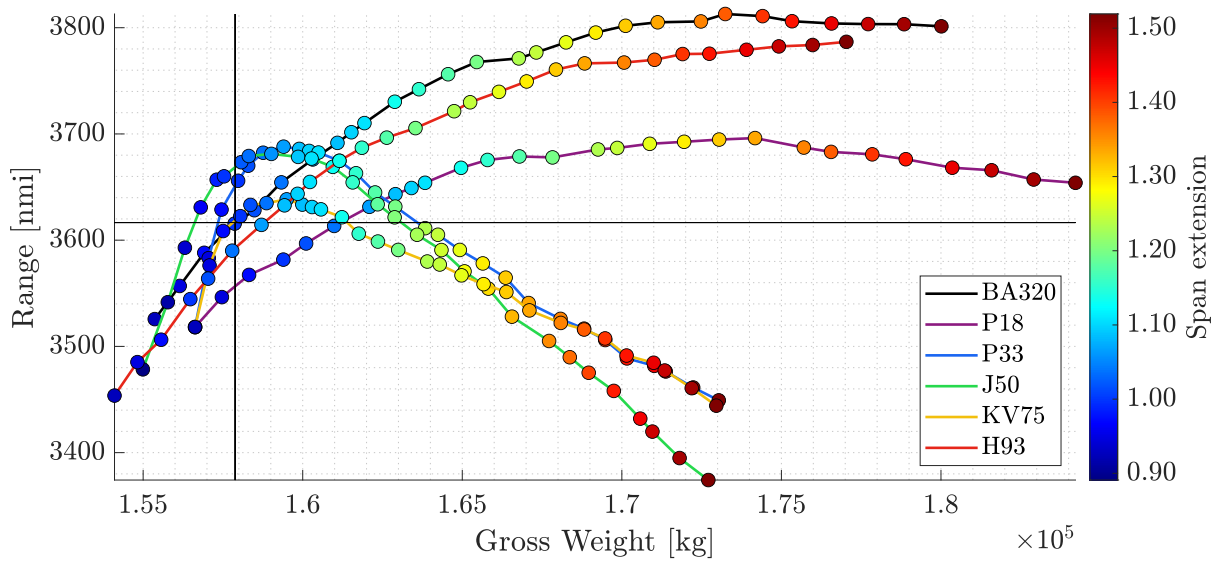


Figure 6.9: Range obtained for each weight design point.

6.3 Discussion and Interpretation of Results

The work that has been presented throughout this thesis has led to the creation of a design space of aircraft designs, for which each point consists on the aerodynamic, structural and mission performance characteristics given by the corresponding span extension and spanload applied. This approach has been taken under a set-based design practise to explore the impact of extending the span under an aero-structural equivalence criteria dictated by the NELD theoretical approach. Thus, this chapter brings together the findings obtained in each disciplinary analysis and explores the trends and accumulated impact and trade-offs derived from the design principles adopted.

To recall, the final top-level methods adopted in the proposed analysis framework to obtain these results comprise: (1) VLM with Prandtl-Glauert compressibility correction and two-dimensional profile drag interpolation for the calculation of aerodynamic and flight dynamics properties of the wing and tail configurations; (2) linearised beam theory extended to a three-node finite volume approach for the calculation of load-bearing material and estimation of the main structural components mass; (3) energy-based approach for the calculation of mission performance of the complete configurations. The advantages and limitations of these methods and the computational models employed has been already presented throughout the above chapters. Here, the cruise condition is again closely studied since it is where the aircraft will spend most of the time during the mission.

To start the discussion of the findings provided throughout this thesis, Figure 6.10 is presented, which recollects the final values of the four main output variables investigated in this work for all the configurations. The parameters in Figure 4.6 are presented as a link to the theoretical foundations employed in Chapter 4, and the corresponding comparisons to the numerical counterpart has been shown and discussed in Figures 5.14 and 6.4.

The wing induced drag is the resulting value obtained at the steady level flight cruise condition, which is the design condition adopted in Chapter 4 for the design of the wing planform and twist distribution. Comparing the trends shown here against the induced drag ratios shown in Figure 4.6, it is observed that the use of higher fidelity methods and the analysis performed at the trim condition provide the same trends where the savings in induced drag are reduced by approximately 2% for the NELD families in the valid range of span extensions, and reach to be 15% lower for the fixed spanload families. The total drag for the entire lifting surfaces configuration at steady level cruise follows the same trend with span extension, where the reduction in drag is now expectedly smaller, and the divergence between the curves for NELD

and fixed spanload families has reduced. The plateauing reduction in drag is reached at approximately the same span extension as predicted by the theories, leading to a appropriate metric for a first estimate of how much the span can be extended with benefits in aerodynamic efficiency. With this, and together with the structural loads developed at off design conditions for the different configurations, as discussed and shown in Section 5.2.4, the wing weight is obtained. The trends of which show that the impact of shifting the lift centre inboard under the proposed approach achieves to reduce the growth in weight as the span is extended. For instance, a 10% span extension of the wing through NELD will increase the wing weight by 9.5% and decrease drag by 7%, whereas the mass of the wings with baseline and elliptic spanload at the same span extension will increase by 17% and 26%, with reductions in drag of only 7% and 5% respectively. However, aero-structural equivalence as defined for the NELD families used in this study is not achieved due to the added consideration of high load manoeuvres. For this aircraft category, the wing mass makes up approximately 10% of the MTOW [148]. After the sizing of all the main structural components performed here, the wing takes up between approximately 10% and 20% of the total mass. Alongside the mass variation of the other components shown in Figure 5.15, the MTOW increase represents the increase only on structural weight, and follows a trend proportional to the one shown by the wing mass. Finally, the above results culminate in the range characteristics, where the penalties of the plateau in aerodynamic efficiency along with the reduced mass increase show a reduced range of NELD configurations for which this performance variable can be improved. Similarly, the almost linear mass increase together with the quadratic decline of drag reduction eventually cause the range to stop increasing with further span extension, which is applicable to any type of spanload.

Several metrics for efficiency have been presented throughout this work, and are computed and compared in Figure 6.11. and normalised against the baseline value.

To start, the classical lift-to-drag ratio is included, which is found in the Breguet range equation and is sometimes presented as the product of the lift-over-drag with the Mach number to consider aircraft at a wider range of flight conditions. In this case, the design Mach speed is equal for all configurations so the lift-to-drag is simply presented. The effective lift-to-drag aims to include the impact on mass due to span extension by including the factor of wing mass over total mass in the formulation, where this factor will vary with each wing configuration. Accordingly, the impact of the increasing wing mass is seen with span extension through a decreasing gradient on the trend curve, thus providing a parameter that better integrates the aero-structural trade-offs. The effective span efficiency factor is representative of the wing aerodynamic efficiency with consideration of increase in efficiency due to span extension. Span efficiency is sometimes invoked to compare aircraft performance, but should remain

as a comparative factor for the wing, for which it can become an initial prediction metric for the impact on wing weight. Finally, the range to weight ratio R/W is proposed as an alternative performance metric, which incorporates the efficiency of the aerodynamics, mass and propulsion altogether, and may be used as a quantifier of the overall performance of the obtained solutions. The metrics have been rewritten below

$$\begin{aligned}
 \text{Lift-to-drag: } & \frac{L}{D} \\
 \text{Effective lift-to-drag: } & \left(\frac{L}{D}\right)^E = \frac{L}{D} \left(1 - \frac{W_{wing}}{W_{MTOW}}\right) \\
 \text{Effective span efficiency factor: } & e^E = \mathcal{R} e \\
 \text{Range-to-weight: } & \frac{R}{W}
 \end{aligned} \tag{6.14}$$

Now, the feasibility of the design of wings under the proposed NELD approach is assessed here through the use of the efficiency metrics presented above. As considered by the author, the most encompassing figure of merit to compare the improvements of the revised layouts over the baseline configuration is the range-to-weight ratio. It is seen now which solution provides a significantly relevant improvement on range with consideration of the penalty in weight due to span increase. The NELD configurations providing any benefits are the ones derived from Jones and Prandtl's approach, with increases in R/W with up to 10% span extension, yielding for the P33 family a maximum of 6.05% increase in range with a 4.05% span extension causing a weight increase of 0.56%, in the same manner for the J50 family, a maximum 5.30% increase in range can be achieved with a weight increase of 0.27% caused by a span increase of 3.59%. The baseline spanload itself can be extended up to approximately 13.49% in span without penalties in performance and increasing range by up to 3.29% and a weight 3.17% higher. The design points found at $\frac{R/W}{R/W_{base}} < 1$ will provide weight increases that will overthrow the gains in range when compared to the baseline configuration. With this, the design points providing the highest R/W are selected and compared. Table 6.3 summarises the main aero-structural characteristics and TLARs of interest.

Figure 6.12 compares the resulting wing lift coefficient distributions at steady level cruise and shows them against the theoretical ideal bell-shaped spanload analytically derived and shown in Figure 4.4 and Table 4.2, where each curve has been normalised against the root value for visual clarity. Similarly, Figure 6.13 plots the spanwise drag coefficient against the theoretical ideal distribution. Looking at Table 6.3, it is seen that the span extension that can be achieved without aero-structural penalties is now much smaller than predicted, but the method still provides a window for a beneficial extension of the span with no penalties on performance due to the aero-structural trade-offs. Some solutions such as the ones provided through the application of the

Table 6.2: Comparison of aero-structural characteristics for highest range-to-weight solutions.

Spanload family	Baseline	BA320	P18	P33	J50	KV75	H93
Span	34.1 m	4.05%	-2.79%	4.05%	3.59%	1.77%	10.88%
Non-dim. centre of lift	0.379	0.00%	4.55%	-0.72%	-1.33%	0.75%	-3.93%
MTOW	71215 kg	0.92%	0.28%	0.56%	0.27%	0.30%	1.49%
Wing weight	8515 kg	7.03%	1.76%	4.23%	2.27%	2.15%	11.46%
Wing induced drag	30980 N	-3.65%	2.19%	-6.57%	-5.84%	-4.38%	-8.76%
Lift-to-drag	24	1.05%	-1.36%	1.82%	1.73%	0.46%	1.06%
Range	3617 nmi	4.19%	-2.50%	6.05%	5.30%	3.08%	8.16%
Range-to-weight	94	0.12%	-1.64%	1.25%	1.46%	0.15%	-0.43%
Effective lift-to-drag	21	3.34%	-2.70%	5.53%	5.02%	2.82%	6.72%
Effective span efficiency	1.14	4.10%	-6.59%	12.41%	11.77%	7.57%	20.07%
Thrust-to-weight	0.324	-0.93%	-0.31%	-0.62%	-0.31%	-0.31%	-1.23%
Wing loading	119.8 kg/m ²	0.92%	0.33%	0.58%	0.33%	0.33%	1.50%
Time-to-climb	11.4 min	0.00%	0.88%	0.00%	-0.88%	0.00%	0.88%
Rate-of-climb	2972 ft/min	-3.73%	-4.42%	-3.05%	-2.83%	-3.65%	-4.65%
Take-off field length	6551 ft	0.32%	1.50%	-0.31%	-0.67%	-0.06%	-0.56%
Landing field length	6141 ft	0.54%	0.16%	0.33%	0.15%	0.18%	0.88%
Approach speed	132 kts	0.46%	0.08%	0.23%	0.08%	0.15%	0.68%

methodology would enable to maintain airport compatibility. However, for span extensions above 5.5%, the aircraft would imply a change in the element 2 category of the ICAO Aerodrome Reference Code, changing from C to D, and the group category defined by the Federal Aviation Agency (FAA) Airplane Design Group (ADG) would change from III to IV, which would reduce the airport compatibility.

Finally, the design space of solutions is presented in Figure 6.14 through the parameters given by the span extension r_s and the non-dimensional centre of lift $\bar{y}_C$, representing the spanload used at each span extension. The variation of the various performance and efficiency variables have been mapped within this design space to provide the coordinates of the evolution of the variable for a given combination of span extension and spanload. For visual clarity, the colormap has been created in such way that the coordinates that tend to blue represent the values tending towards optimal values, such as higher range or lower mass. Spanloads having its centroid further inboard than 30% of the semi-span should be avoided, since they yield a high risk of developing unwanted negative loads at the outboard wing. While it appears more clear that the centroid locations showing optimal values are found in the range of 30% to 40% of the semi-span, the optimal span extension varies according to the aero-structural trade-offs, where the range-to-weight parameter combines them within the range of 0% to 10%.

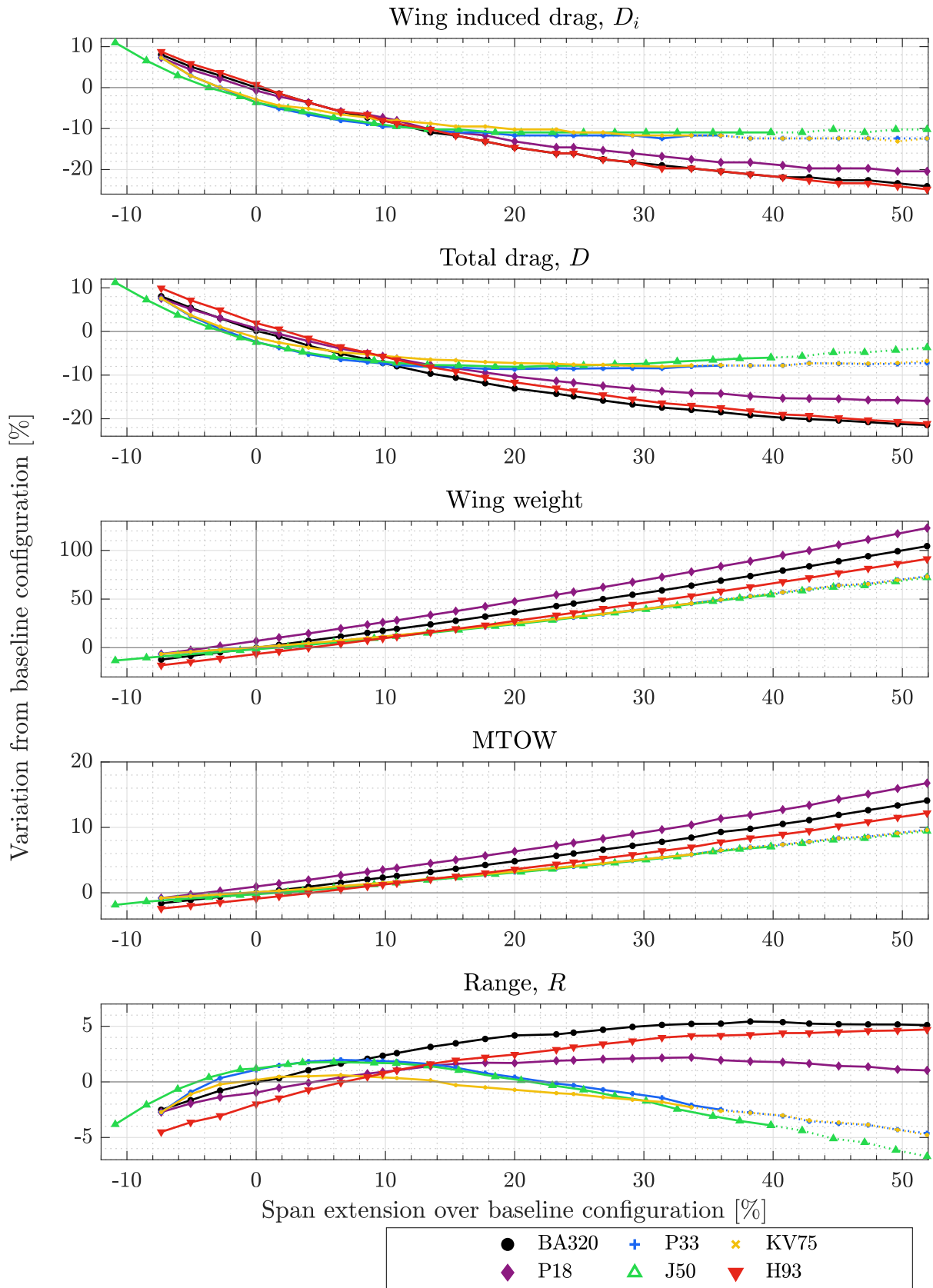


Figure 6.10: Comparison of aerodynamic, structural and performance top level parameters.

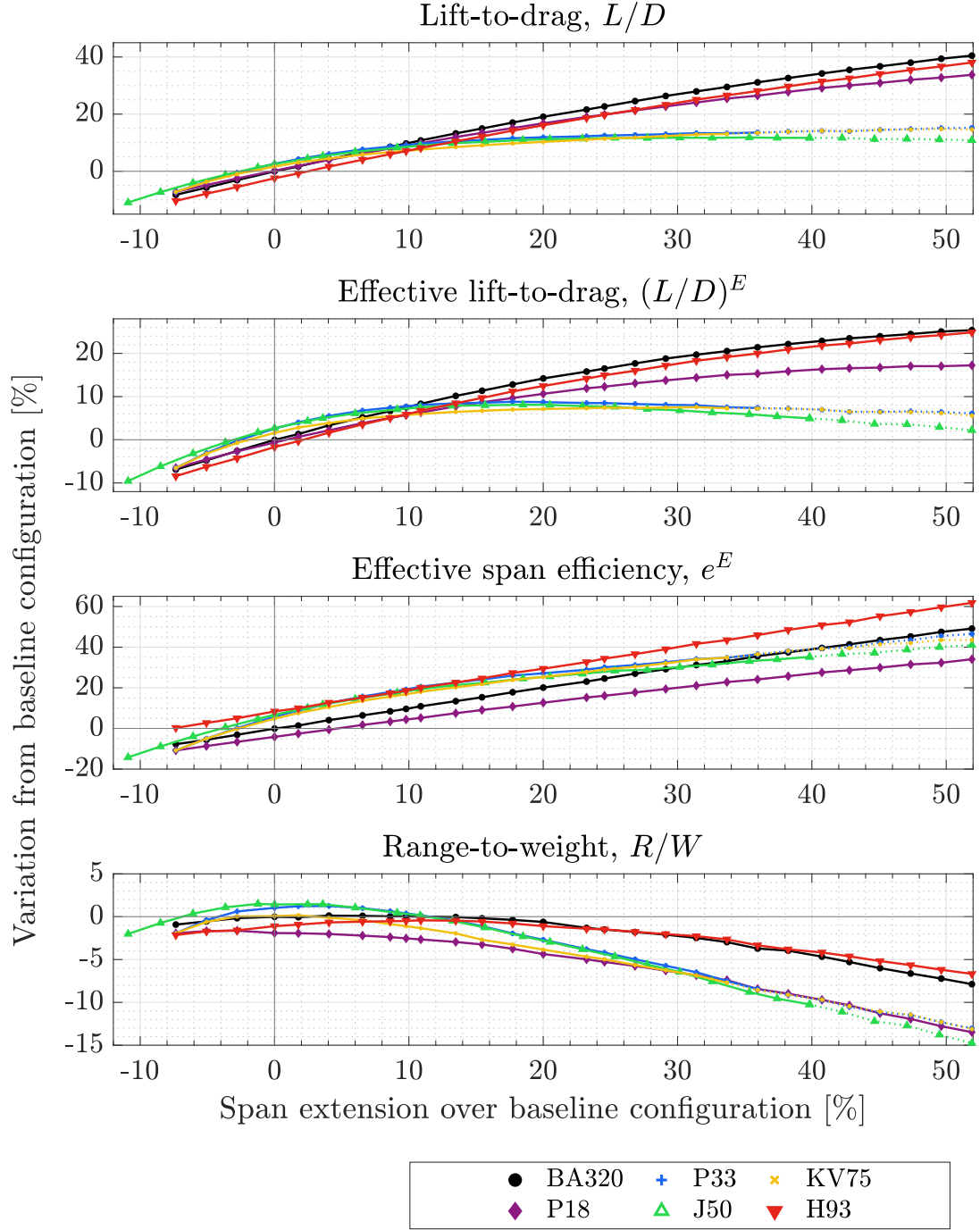


Figure 6.11: Comparison of efficiency and performance metrics.

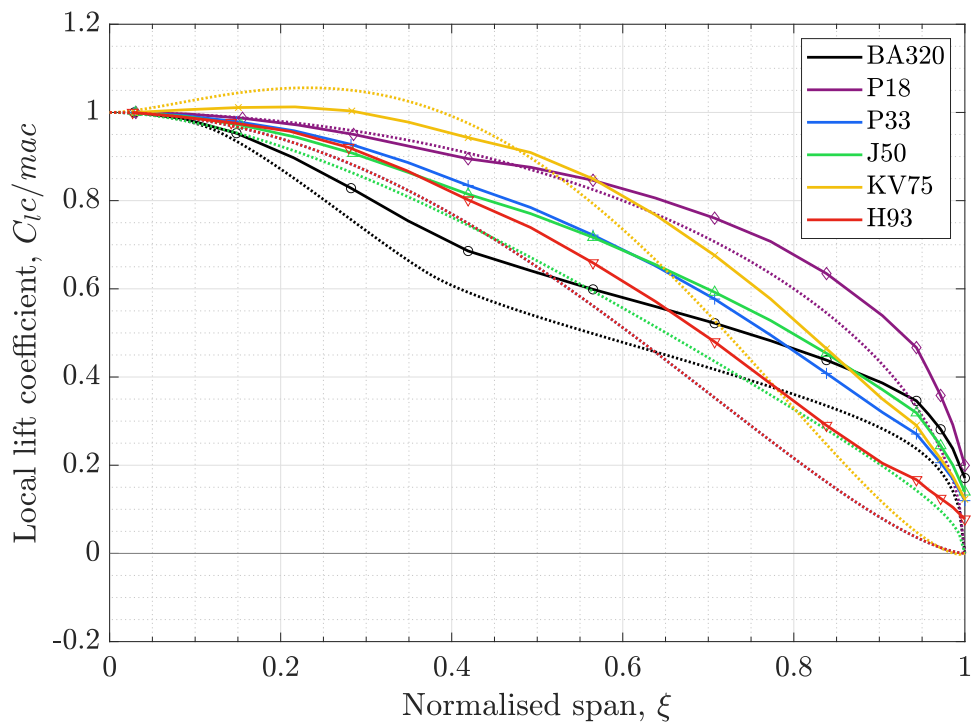


Figure 6.12: Comparison of the lift coefficient distributions of the highest R/W solutions for each spanload family.

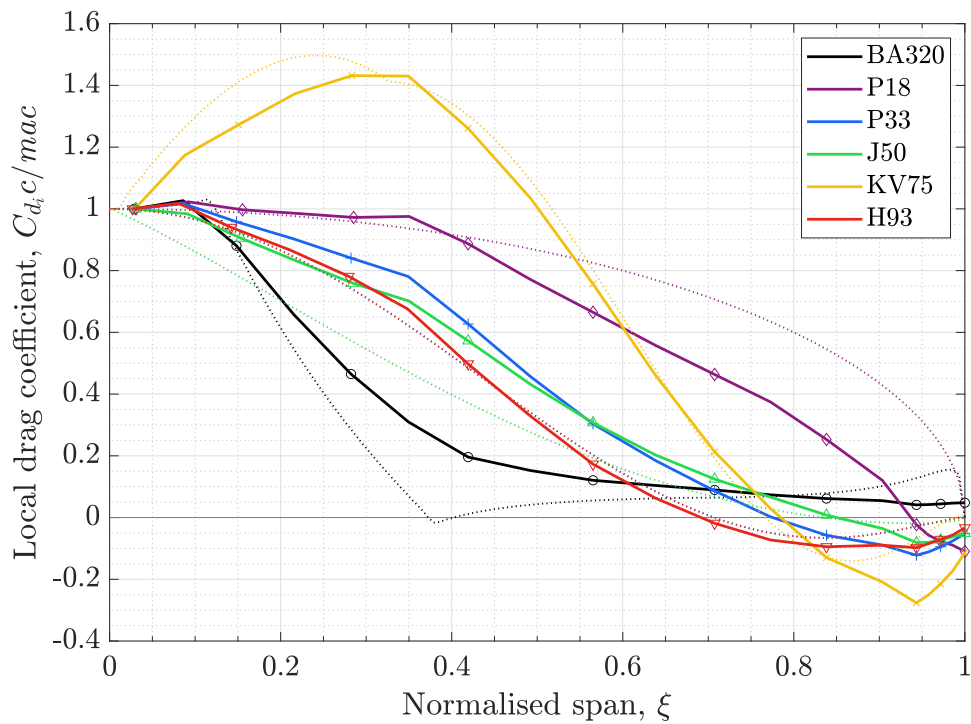


Figure 6.13: Comparison of the drag coefficient distributions of the highest R/W solutions for each spanload family.

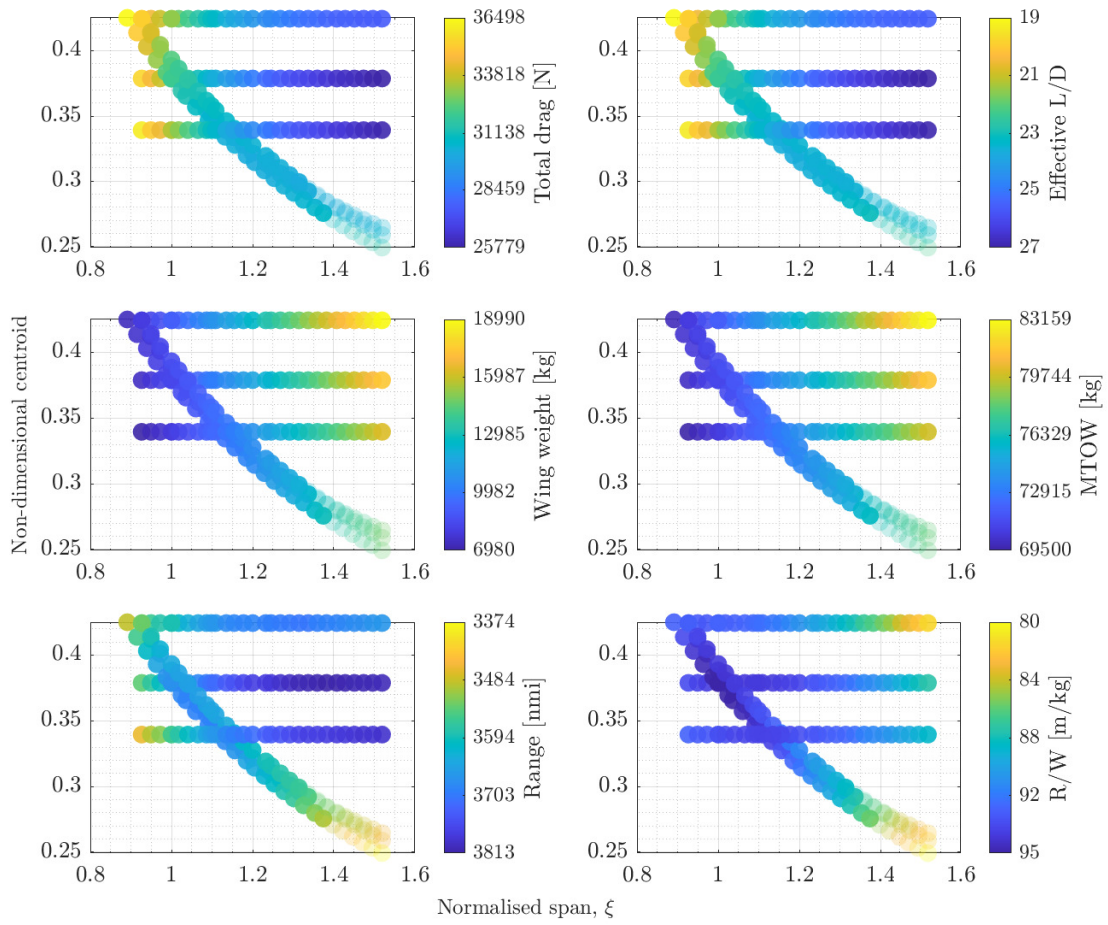


Figure 6.14: Design space of span extension and spanload for top level aerodynamic, structural and performance design outcomes.

6.4 Conclusions

Mission analysis plays a crucial role during the conceptual design of aircraft, and here it has been integrated to provide the grounds in which the aero-structural trade-offs come together, and hence provide synthesised trends and insight on the feasibility of the NELD wing design approach adopted, as compared to majority of the existing research.

Varied levels of complexity and fidelity can be achieved from the numerous mission analysis methodologies available. Here, the energy-based method has been employed through the integration of FLOPS into the analysis framework, allowing for a rapid assessment of the mission profile and performance parameters. The analysis has been performed from the assumption of a given fixed weight and block fuel quantity in order to maximise the range that can be attained. To do so, the induced drag polars for high and low-speed flight conditions have been computed through AVL and completed with empirical drag estimations of the non-lifting components of the airframe through the build-up method, and with sectional integration of the two-dimensional aerofoil profile drag polars.

Following the use of NELD to extend the span of a given configuration within the proposed analysis process, span-extended designs can be achieved that yield no penalties in performance. This is achieved by spanload families constrained by either the wing integrated bending moment or the root bending moment. Within this range of solutions, the majority of the design points will show increases in both range and gross weight within a beneficial ratio. Other TLARs will see penalties when compared to the baseline configurations, but are still found within acceptable values of airworthiness. The inclusion of the use of the range-to-weight output as a figure of merit provides further insight into the overall efficiency of the designs compared to other traditional metrics such as the lift-to-drag ratio.

Chapter 7

Flight Dynamics Assessment for Proverse Yaw

The challenge faced by aeronautical engineering today is to extract every gram of efficiency from the design being considered. Quite rightly, the majority of attention is focused on the aircraft in its cruise conditions where it spends most of its operational time. Little attention is paid to scenarios where the aircraft is manoeuvring and increases in drag are simply accepted [302]. In this chapter, the applicability of wings based on NELD design philosophy to address adverse yaw is explored, following the claims that seem to promise optimal coordinated turns, also referred to as proverse yaw. However, the caveat in the application of such wings to eliminate adverse yaw is that the consequent span load and extension requires the reconsideration of the aileron design parameters.

In the scenario of a lateral turn, adverse yaw is a typical response of the aircraft to aileron deflection caused by differential drag. Conventional aerospace solutions to this widely accepted nuisance make use of additional drag with the help of flight control systems, making a manoeuvre an aerodynamically inefficient exercise. Nevertheless, examination of bird flight in nature seemingly proves that manoeuvring flight is possible without the requirement of additional drag or even a vertical stabiliser. Past research briefly discussed in Section 2.4.3 puts forwards the practical and physical limitations of this vision applied to transonic transport aircraft. Nonetheless, this is the principle towards which wings with NELDs aim.

In this chapter, this aerodynamic and flight dynamic problem is approached under the principles of wing design as proposed in Chapter 4. The alternative solutions for minimum induced drag that combines aerodynamic and structural design constraints derive in spanloads that develop negative induced drag on the outboard of the wing. Here, the drag differential during turn becomes beneficial when the upwash is manipulated accordingly by an aileron designed for this purpose [83,84,303]. However,

because of the characteristic lightly loaded wingtips, the design could lead to insufficient roll authority, which must be also considered. Experimental research on the use of such spanload distributions to address adverse yaw has given positive results. The Horten brothers [131] independently explored this solution on many of their tailless aircraft models, achieving successful flights without vertical control surfaces. Recently, Bowers [9] has observed the phenomenon of proverse yaw in numerous flight tests using NASA's PRANDTL-D flying wing.

The aim of the work presented in this Chapter is to assess the benefits of wings with NELDs from a flight dynamic perspective, especially focusing on proverse yaw and drag minimisation during turn, together with the implications of aileron sizing at the conceptual design stage. The work presents a coupled analytical and numerical methodology for the assessment of aileron sizing for the minimisation of adverse yaw while providing the required roll control, for which a full factorial design of experiment method is employed for the creation of sets of aileron geometries. Under these objectives, the assessment of the lateral-directional characteristics of the revised configurations is presented and compared. The use of reduced order methods allows a rapid assessment of the flight dynamics and the capabilities of such configurations. The work carried out analytically focuses the effect of induced drag only on adverse yaw, while the extension to the numerical VLM approach includes the contribution of two-dimensional profile drag as applied in Chapter 6. The implications on the flight control system are not considered.

7.1 Problem Definition

The methodology developed in this work aims to derive a design criteria for an aileron geometry that provides roll authority and nullifies adverse yaw for a wing with non-elliptic lift distribution. The proposed problem is addressed through the assessment of the lateral-directional derivatives relevant in the design requirements for roll control and its secondary effects. Following these conditions, the output of the proposed problem will be the aileron chord ratio, the midpoint position of the aileron and the aileron span that satisfy both given objectives.

The design criteria applied to size the aileron for roll effectiveness and minimum adverse yaw has been based on two objectives: (1) Roll authority is given by the static characteristics of the wing during aileron deflection determined by certification requirements [275], in terms of a prescribed non-dimensional roll-rate; (2) adverse yaw is addressed analytically, where, following the standard right-hand stability axes, the yawing moment due to aileron deflection will be negative.

This problem has been formulated analytically to provide some insight into the

consequences of deflecting an aileron that is placed on a region of negative induced drag. Aileron deflection has been accounted for via two-dimensional plain flap theory. Lifting line theory has been subsequently used to derive the three-dimensional wing properties, combined with strip theory to obtain expressions for stability derivatives. The reason for this stems from the need to avoid the use of empirical approximations or other methods that will bypass the local effects on the induced drag distribution, since traditional methods tend to be based on the elliptic solution as the optimal for minimum induced drag [178, 179, 274]. Numerical implementation and computation of results has been made under a set-based design approach. At this stage, only the contributions from the wing have been considered in the aileron sizing methodology. Other assumptions and limitations of the proposed approach: include ignoring pilot stick force requirements as well as hinge moments and mass balance requirements; the contribution due to sideslip derivatives are ignored.

The definition of the problem is constrained by certification requirements, which allow the initial definition of the parameters. Roll control can be determined by the airworthiness specifications according to the EASA CS-25 [275]. The requirement for lateral and directional control is stated on item CS 25.147. Specifically, the acceptable means of compliance for roll capability with all engines operating states that it should be possible to roll the aeroplane from a steady 30° banked turn through an angle of 60° so as to reverse the direction of the turn in not more than 7 seconds. To accomplish this, the rudder may be used to the extent necessary to minimise sideslip. However, one of the objectives of this study is precisely to evaluate is the use of rudder may be reduced, with the additional assumption that there is no crosswind or inoperative engine to consider. These requirements must be satisfied at several points of the flight envelope. In this study, the initial flight condition at which the aircraft will be evaluated is at the manoeuvring speed V_A , defined in Figure 5.7, with flaps in en-route position, air brakes and landing gear retracted and trimmed from straight flight in these conditions, where the trimming controls will not be moved during the manoeuvre.

7.2 Methodological Approach and Results

The primary function of an aileron is the lateral-directional control of an aircraft, which is governed primarily through roll rate p and the consequent changes in heading. Lateral control research is mainly concerned with the design of control devices that provide enough effectiveness at all airspeeds within the flight envelope, as well as the ability to produce a required roll rate. Therefore, the requirements for satisfactory lateral control must be carefully specified. Among the various criteria proposed for specifying roll performance, analytical studies and pilot experience have indicated that the design

point based on the non-dimensional roll rate $pb/2V$, or helix angle, is most convenient to specify roll authority [304–307]. This parameter is independent of altitude for given aileron deflections and is also independent of aircraft mass properties for geometrically similar aeroplanes. From a design perspective, this means that the aileron size can be determined by the required steady state roll rate. In the case of subsonic aircraft such responsiveness must be assured at slow speeds with large deflections and will be determined by design requirements and certification specifications.

From the one degree-of-freedom equations of motion of a rigid aircraft, where it is assumed that the roll rate p is damped due to the local variation in angle-of-attack, the rolling motion caused by a given mean aileron deflection δ_a can be expressed as [308,309]

$$\frac{pb}{2V} = -\frac{C_{l_{\delta_a}}}{C_{l_p}} \delta_a \quad (7.1)$$

where $C_{l_{\delta_a}}$ is the rolling moment coefficient due to aileron deflection and C_{l_p} is the rolling moment coefficient due to roll rate. The conditions to determine the required roll performance are specified as aforementioned in CS-25 [275], where the aircraft must be able to reverse the direction of a turn with a bank of $\phi = 30^\circ$ in the opposite direction within 7s, at the manoeuvring airspeed V_A , and with maximum deflection $\delta_{a_{max}}$ allowed by the aileron [310]. The estimation the roll control derivative can be done by applying strip theory and assuming that the controller is a simple flap-like device, such that [280]

$$C_{l_{\delta_a}} = -\frac{2}{Sb} \int_{y_1}^{y_2} C_{l_f} c(y) y dy \quad (7.2)$$

where $c(y)$ is the wing chord at a given spanwise location y , for which the integration limits are defined by the location of the aileron inboard and outboard sections, designated by the locations y_1 and y_2 respectively along the semi-span. The lift coefficient due to trailing edge control surface deflection $C_{l_f} = \partial C_l / \partial \delta_f$ has been calculated through flat plate theory for a flap device. Considering a the flap-like aileron as a symmetric aerofoil and assuming a chordwise distribution of vorticity in the form of a series, the two-dimensional flap lift effectiveness can be computed as [311,312]

$$\frac{C_{l_f}}{C_{l_\alpha}} = k_{3D} \frac{2}{\pi} \left[\sin^{-1} \sqrt{\frac{c_f}{c}} + \sqrt{\frac{c_f}{c} \left(1 - \frac{c_f}{c} \right)} \right] \quad (7.3)$$

where C_{l_α} is the local lift curve slope, defined in Table A.4. The factor k_{3D} considers the losses due to three-dimensional effects and is taken here as $k_{3D} = 0.85$ from the literature, and c_f is the chord of the trailing edge control surface. The small perturbation approach used in this methodology is followed by the assumption that C_{l_f} remains in the linear range, which agrees with the resulting aerodynamic properties of ailerons observed

from experimental data [313].

The roll damping derivative is also calculated with strip theory and integrated along the wing span, yielding [280]

$$C_{l_p} = -\frac{1}{S_{ref}b^2} \int_{-s}^s (C_{l_\alpha}(y) + C_d(y)) c(y) y^2 dy \quad (7.4)$$

where $C_d(y)$ is the local spanwise drag coefficient, which sees a spanwise perturbation in incidence angle-of-attack approximated to $\alpha \cong py/V$ due to the roll rate.

Nonetheless, as mentioned earlier, one of the main concerns of this study is the secondary effects of lateral control due to aileron deflection, which will effect directional motion in the form of adverse yaw. This phenomenon can be described as the opposing moment caused by variation of the drag with aileron deflection. Estimates for this derivative are complex to predict and are usually obtained from empirical data or ESDU data items. However, such methods are based on the elliptic spanload as the ideal solution for minimum drag, which therefore overrules the unconventional behaviour of the drag in non-elliptic lift distributions, or essentially dependant on geometric and inertial characteristics that disregard the local effects of the wing spanload. Applying strip theory and assuming small perturbations, this can be captured as [280]

$$C_{n_{\delta_a wing}} = \frac{1}{S_{ref}b} \int_{-s}^s \frac{\partial C_d(y)}{\partial \delta_a} c(y) y dy \quad (7.5)$$

The following assumptions were made to obtain $\partial C_d(y)/\partial \delta_a$, where the drag varies with both profile and induced drag as

$$C_d(y) = C_{d_0}(y) + C_{d_i}(y) \quad (7.6)$$

where the profile drag variation is usually negligible [274]. The induced drag is generally expressed by [5,270]

$$C_{d_i}(y) = C_l(y) \alpha_i(y) \quad (7.7)$$

The minimum profile drag coefficient will also see an increment caused by control surface deflection

$$C_{d_0} = C_{d_0} + \Delta_f C_{d_0} \quad (7.8)$$

where $\Delta_f C_{d_0}$ can be approximated empirically if it is assumed that the increment is given only by the camber variation and the contribution from the load change is ignored. The chosen empirical approximation [17] accounts for aerofoil thickness t/c and type of flap. However, the drag increment is overpredicted when compared to

other methods such as ESDU 06014 [314]:

$$\Delta_f c_{d0} = \delta_1(c_f/c) \delta_2(\beta) \quad (7.9)$$

where the factors $\delta_1(c_f/c)$ and $\delta_2(\beta)$ are given in Figure 7.2. Therefore, the variation in

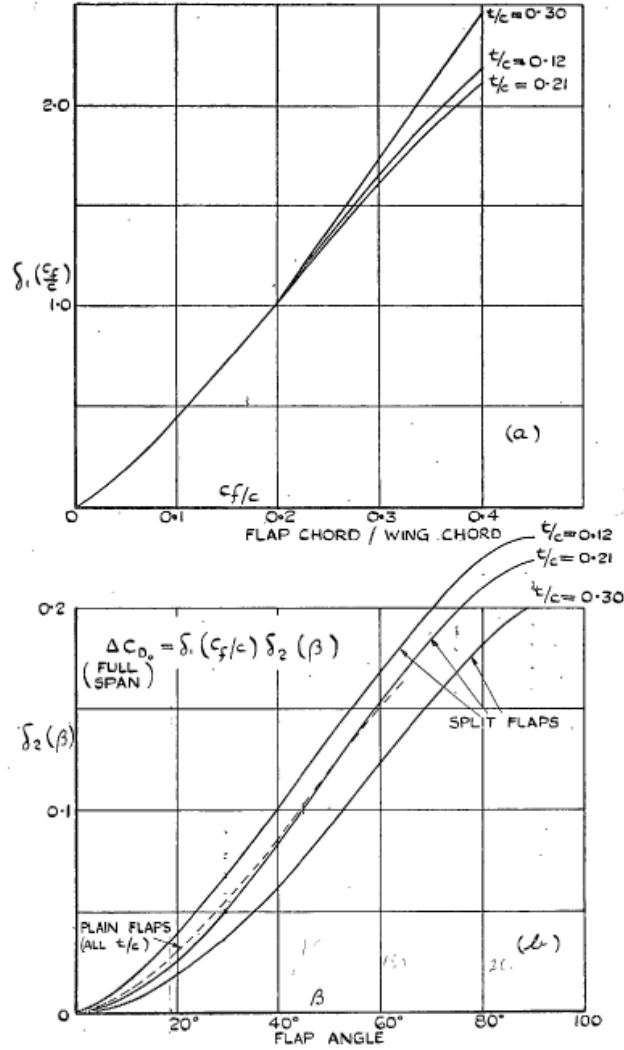


Figure 7.1: The functions $\delta_1(c_f/c)$ and $\delta_2(\beta)$ for split and plain flaps [17]

section drag due to aileron deflection becomes

$$\frac{\partial C_d}{\partial \delta_a} = C_l \frac{\partial \alpha_i}{\partial \delta_a} + \alpha_i C_{l_f} + \frac{\partial C_{d0}}{\partial \delta_a} \quad (7.10)$$

The lift distribution after control surface perturbation can be calculated with the use of Equation 7.3 as

$$C_l = C_{l_0} + C_{l_\alpha} \alpha + \frac{\partial C_l(y)}{\partial \delta_a} \delta_a = C_{l_{(clean)}} + C_{l_f} \delta_a \quad (7.11)$$

To obtain the induced angle of attack α_i , it is necessary to account for three-dimensional effects, derived from Biot-Savart's formula presented in Equation 4.7 and rewritten again here [315]

$$\alpha_i(y) = \frac{1}{4\pi V_\infty} \int_{-s}^s \frac{d\Gamma/dy}{y - y_0} dy_0 \quad (7.12)$$

where the circulation distribution $\Gamma(y)$ after aileron deflection can be obtained from the lift coefficient distribution calculated in Equation 7.11 as

$$\Gamma(y) = \frac{C_l(y) V_\infty c(y)}{2} \quad (7.13)$$

Now, it is possible to define the problem constraints, where the main objective is to find the minimum proverse yaw solution given the above conditions and obtain the design variables that will determine the aileron geometry, given by r_f , r_b and r_m , which remain unknown, and where $r_f = c_f/c$, r_b is the span of one aileron over the semispan and r_m is the midpoint location of the aileron along the semispan. From historical data [13, 148, 316, 317], we can extract the lower and upper boundaries for the design variables, which have been expanded for research purposes, as well as extract the maximum aileron deflection that will be assigned for this study. These are shown in Equation 7.14. Under the full factorial approach, five discrete levels have been applied to each design parameter for the findings that will be shown below, resulting in a set of 125 aileron designs, resulting in a set of total of 10,500 aircraft design solutions, which will be reduced under the criteria being laid down here.

$$\begin{aligned} 0.15 &\leq r_f \leq 0.50, \\ 0.15 &\leq r_b \leq 0.50, \\ 1 - r_{b_{ub}} + \frac{r_{b_{lb}}}{2} &\leq r_m \leq 1 - \frac{r_{b_{lb}}}{2}, \\ \delta_{a_{max}} &= \pm 25^\circ \end{aligned} \quad (7.14)$$

In order to ensure that the minimum roll authority required is achieved, the condition that must be satisfied is expressed from Equation 7.1 as

$$\frac{p_{req} b}{V_A} + \frac{C_{l_{\delta a}}}{C_{l_p}} \delta_a \geq 0 \quad (7.15)$$

where p_{req} is defined by the roll authority requirements defined in the certification specifications above, becoming

$$p_{req} = -\frac{2\phi}{t_{req}} = -\frac{60^\circ}{7s} \quad (7.16)$$

Proverse yaw is determined by the sign of $C_{n_{\delta_a}}$, where it is desired to have a yawing moment in the same direction as the rolling moment, as depicted in Figure 2.10. By definition and according to the standard right-hand stability axes [280] $C_{l_{\delta_a}} < 0$, which represents a negative rolling moment due to positive (downwards) starboard aileron deflection, and thus $C_{n_{\delta_a}}$ should be of the same sign

$$C_{n_{\delta_a}} < 0 \quad (7.17)$$

The decoupled process described above to achieve this is summarised in Figure 7.2.

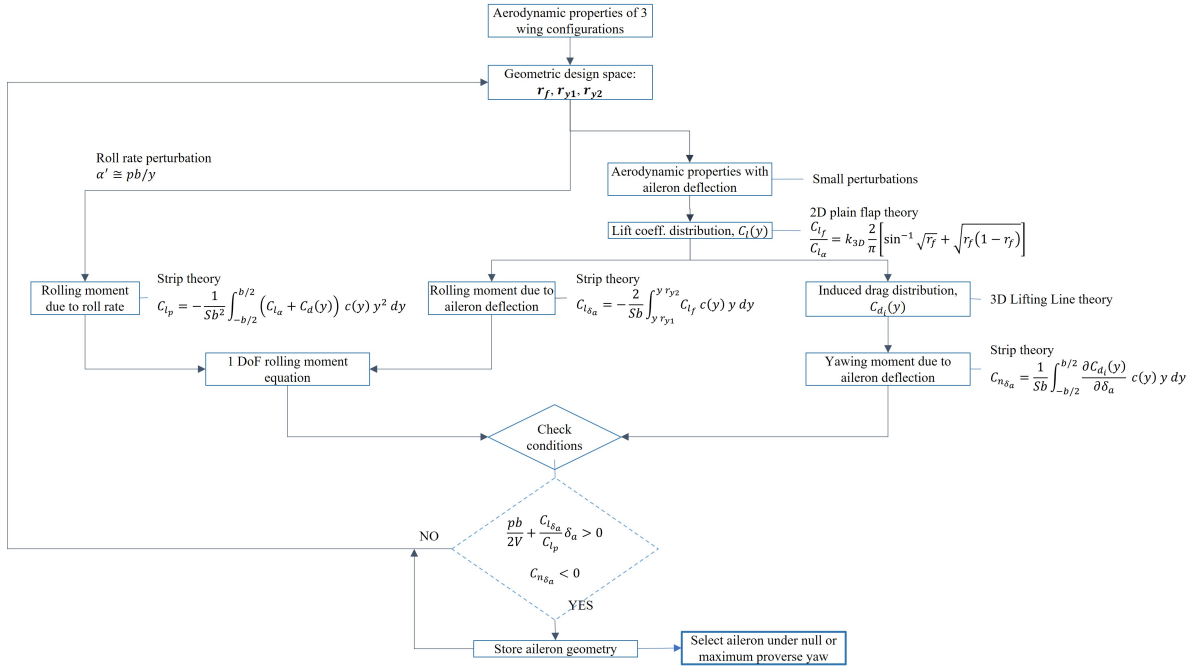


Figure 7.2: Work process of the analytical design methodology of aileron sizing for proverse yaw.

Out of all the solutions that satisfy the above constraints, the effect of having both minimum and maximum proverse yaw are explored. In other words, the solutions yielding maximum proverse yaw will show the largest negative values of $C_{n_{\delta_a}}$ and thus induced the largest yawing moment due to aileron deflection in the direction of the turn; the solutions for minimum proverse yaw or neutral yaw designate the configurations with approximately null yawing moment due to aileron deflection. Figure 7.5 shows the design space of solutions with span extension (please note that the markers have been marginally displaced around their values for visualisation purposes).

It can be seen that the NELD wings provide a larger number of solutions compared to the conventional spanloads, where the elliptic lift distribution does not show any wing and aileron design point in the surveyed design space that can satisfy both condi-

Table 7.1: Properties of wing and aileron configurations at CS-25 condition.

	BA320		P18		P33		J50		KV75		H93	
	max	min	max	min	max	min	max	min	max	min	max	min
r_s	1.000	0.927			1.135		1.025		1.056		1.098	
r_b	0.200	0.200	0.238	0.238	0.150	0.150	0.150	0.150	0.325	0.238	0.500	
r_m	0.850	0.850	0.838	0.750	0.925	0.838	0.925	0.838	0.838	0.838	0.750	
r_f	0.270	0.270	0.400	0.213	0.400	0.275	0.400	0.338	0.400	0.213		
$C_{n_{\delta a}}$ [deg ⁻¹]	1.40E-04	1.22E-04	-1.51E-04	-2.00E-06	-4.20E-05	-2.00E-06	-9.20E-05	-3.00E-06	-1.52E-04	-2.00E-06		
$pb/2V$	-3.63E-04	-3.40E-04	-4.00E-04	-4.00E-04	-3.60E-04	-3.60E-04	-3.70E-04	-3.80E-04	-3.90E-04	-3.90E-04		
C_n	6.60E-03	5.20E-03	5.00E-04	9.20E-04	1.30E-05	1.20E-03	7.00E-05	4.60E-03	6.10E-04	3.20E-03		
$C_{n_{\delta a}}$ [deg ⁻¹]	2.10E-05	3.20E-05	-9.20E-05	-1.10E-04	-2.18E-04	-1.59E-04	-1.18E-04	-4.90E-05	-9.30E-05	-5.20E-05		
$pb/2V$	-3.50E-04	-3.40E-04	-3.90E-04	-3.90E-04	-3.70E-04	-3.70E-04	-3.90E-04	-3.90E-04	-3.80E-04	-3.80E-04		
C_n	0.0026	0.0082	-5.20E-03	-4.30E-04	-5.90E-03	-6.30E-04	-4.10E-03	-1.30E-04	-6.00E-05	-4.00E-05		

tions for minimum roll authority and proverse yaw. For the NELD configurations, the spanload families with applied inboard shift of centre of lift show fewer solutions at the lower range of span extension, since here the lift distributions start morphing from the elliptic curve, and thus agree with the empty design space found for the P18 configurations. On the contrary, the BA320 and H93 families do provide solutions at all span extensions, since the spanload shape is kept constant throughout. Furthermore, the design points providing the highest proverse yaw values are mostly found for aileron sizing of shorter span and located beyond the 70% of the semispan where the upwash starts developing, and require the highest values of aileron chord. This agrees with the theoretical location of the inboard-shifted vortex in NELDs, discussed in Chapter 4. Recalling a previous work by the author [168], the induced drag of an elliptically loaded wing is traditionally reduced with downwards aileron deflection. In contrast, the induced drag over the wing of the NELD cases increases with the downwards deflection of the aileron. This increase brings a favourable yawing moment, and overall, a lower total drag per area increase when compared to the elliptic case.

The solutions were verified in AVL, where some of the above were simulated to compare their response to an aileron step input for equal roll rate demand. The baseline and elliptic configurations are here simulated with the baseline aileron sizing, which yields adverse $C_{n_{\delta a}}$. The NELD configurations chosen correspond to solutions yielding the maximum and minimum values in Figure 7.5. The sizing details are shown in Table 7.2. Looking at Figure 7.4, the overall lateral-directional response shows that the roll rate demand could not be achieved by the wing only configuration. However, a small roll rate can be induced through the positive aileron step input on the NELD families, while a rate of yaw in the opposite direction is developed simultaneously only on the KV75 solution. Thus, the wing starts banking into the desired direction and yawing in the opposite direction of turn. Eventually, directional contributions overtake the lateral effects and the wing rolls into the opposite side, settling in an opposing rate of roll, and increasing, but insignificant, yaw and roll into the opposite direction. Further investigation should be performed on the inadequate dynamics obtained on the wings.

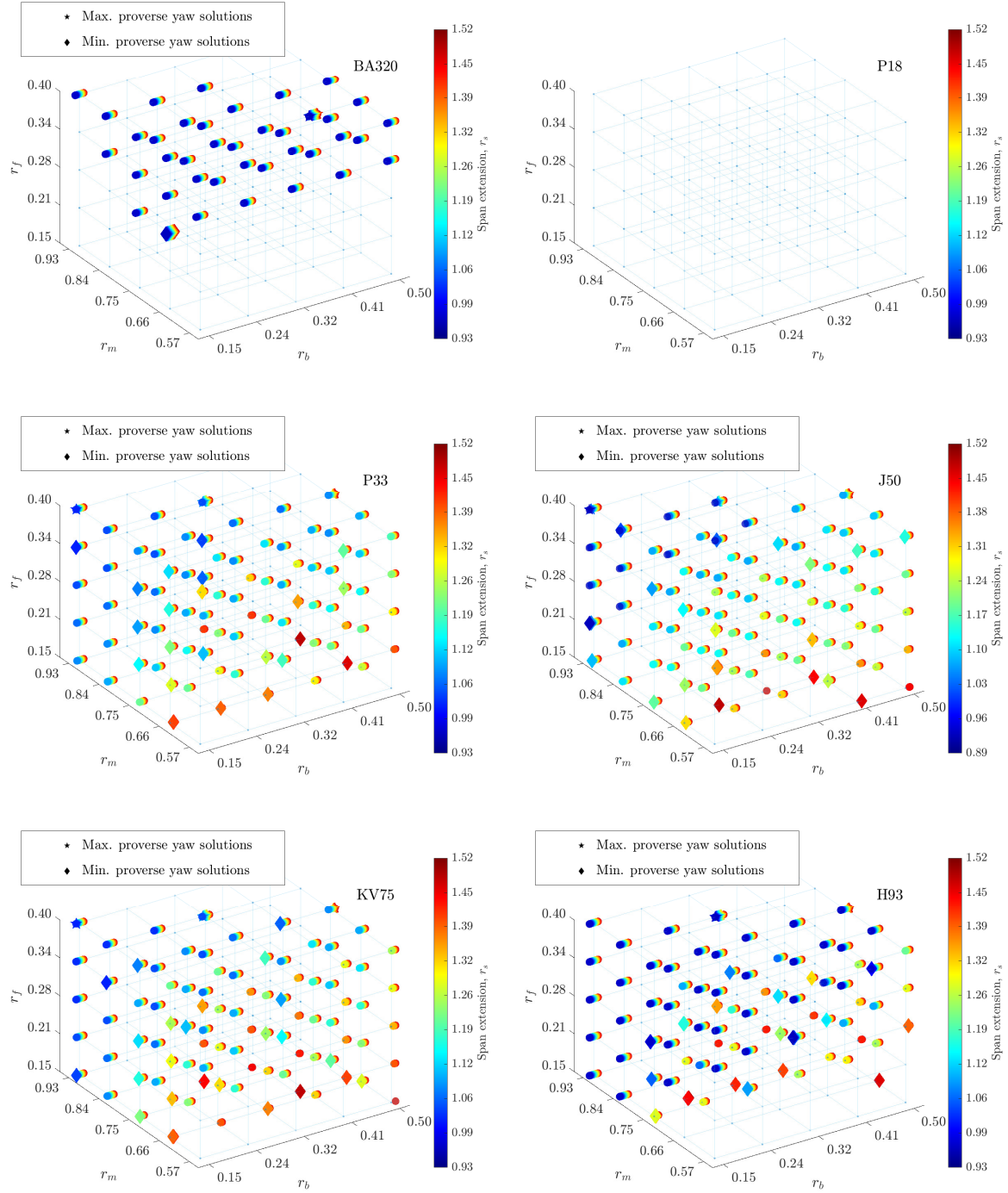


Figure 7.3: Design space of parametric study of aileron sizing for proverse yaw on wing only configurations, at $\phi = 30^\circ$ banked condition.

Nevertheless, the responses for the NELD wing configurations with maximum proverse yaw show differences to this response, which end up settling towards the desired direction. The KV75 configuration becomes the most favourable in this sense, and requires the smallest aileron deflection, which will develop less drag. This is a characteristic of the NELD wings, as shown by the aileron deflection ratio r_{δ_a} , which

describes the ratio of the aileron deflection required by the given configuration over the aileron deflection of the baseline configuration. The remaining NELD wings show a more neutral response, with highly reduced initial amplitude and absent or very minor divergence from the steady state after the aileron perturbation. The baseline and elliptic configurations, which provide adverse yaw solutions, generally shows the largest values of response to the step aileron input, as well as the highest divergence from the desired demand. Looking now into the responses of the NELD configurations with minimal $C_{n_{\delta_a}}$, it is observed that the reductions in proverse yaw strength has a detrimental effect, and eventually it is seen that having positive or neutral values of $C_{n_{\delta_a}}$ can yield equally adverse responses. Generally, the behaviour corresponds more closely to the yawing moment coefficient C_n as expected, as shown in Table 7.2 for the wing, rather than solely on $C_{n_{\delta_a}}$.

The above analysis explores the impact of the use of NELD on the drag differential caused due to aileron deflection, which is regarded as a main contributor to adverse yaw, and is done during turn initiation. This assessment can be useful for the design of flying wings and to provide further understanding on the benefits of this approach to manipulate the motion of the wing through induced drag. However, a complete aircraft configuration brings a larger set of contributors that will define its dynamic and static stability. As described by the equations of motion, these cannot be as simply decoupled in the analysis of the required roll manoeuvre described earlier, given that tail and fuselage will contribute to lateral and directional motions. With this, the design space was assessed using AVL to evaluate the configurations with the complete lifting surfaces layout under the criteria of roll authority and proverse total yawing moment. The results are shown in Figure 7.5, where a larger number of solutions that satisfy these new conditions are found for all configurations, but only for design points with wing span extensions larger than 15% in most cases. This points towards the results being more dependant to wing layout rather than the spanload it carries, as well as the contribution from the tail, which tend to directional stability. The fuselage has not been modelled here so its contribution has not been included.

The previously selected design points are simulated as a complete configuration against an aileron step input, and shown in Figure 7.6. Due to the additional and coupled contributors, the response of all configurations shows minimal differences, regardless of the value of $C_{n_{\delta_a}}$. Here, the J50 and H93 solutions are the configurations with maximum total proverse yaw, showing a slightly higher effectiveness than the others. Nevertheless, the aileron deflection required to achieve the same roll demand is generally higher, which may cause larger increases in drag, except for the H93 configuration, which shows the highest roll response to aileron among all cases examined.

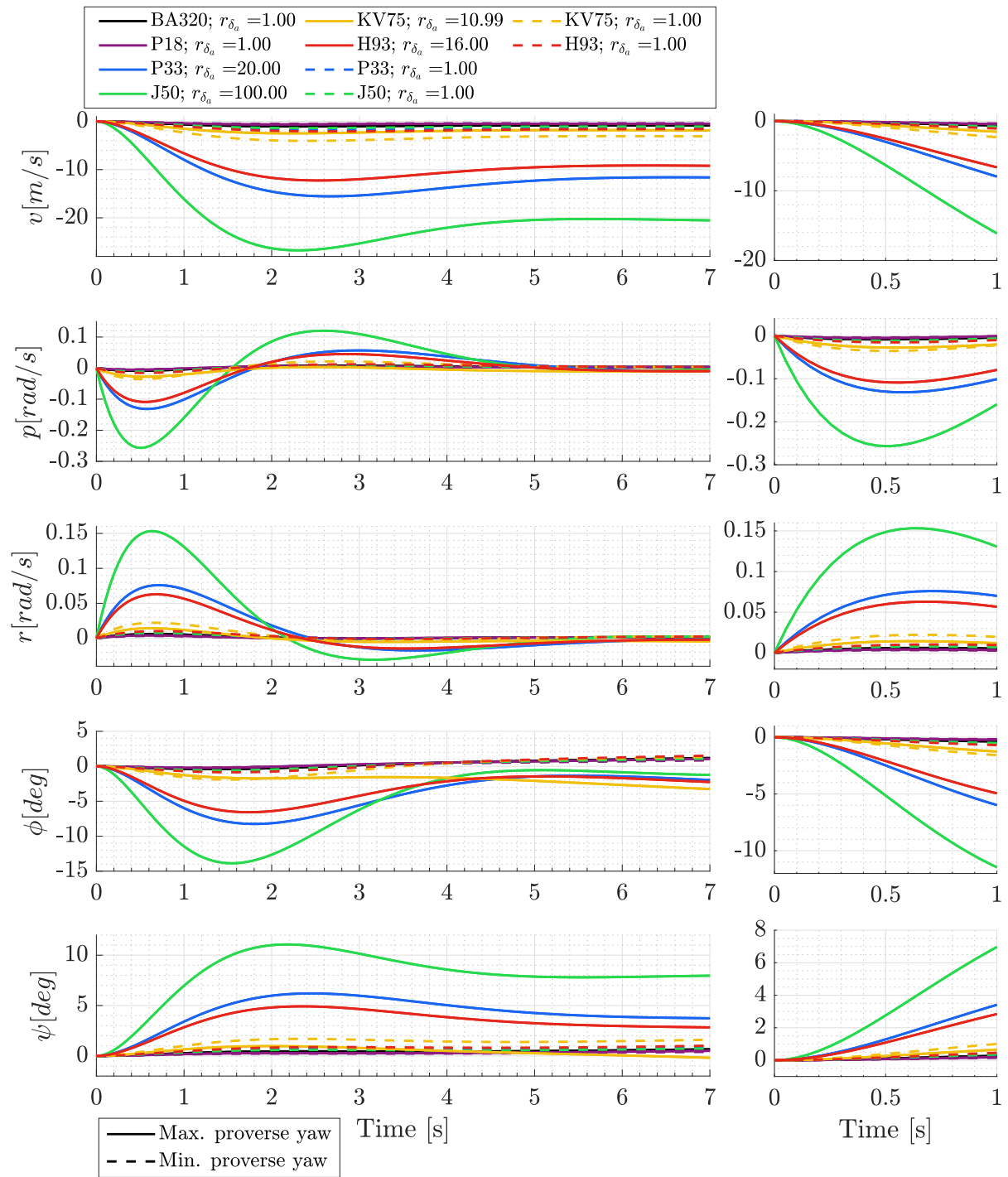


Figure 7.4: Lateral-directional response to aileron step on wing only configurations. Detail of the turn initiation response on the right.

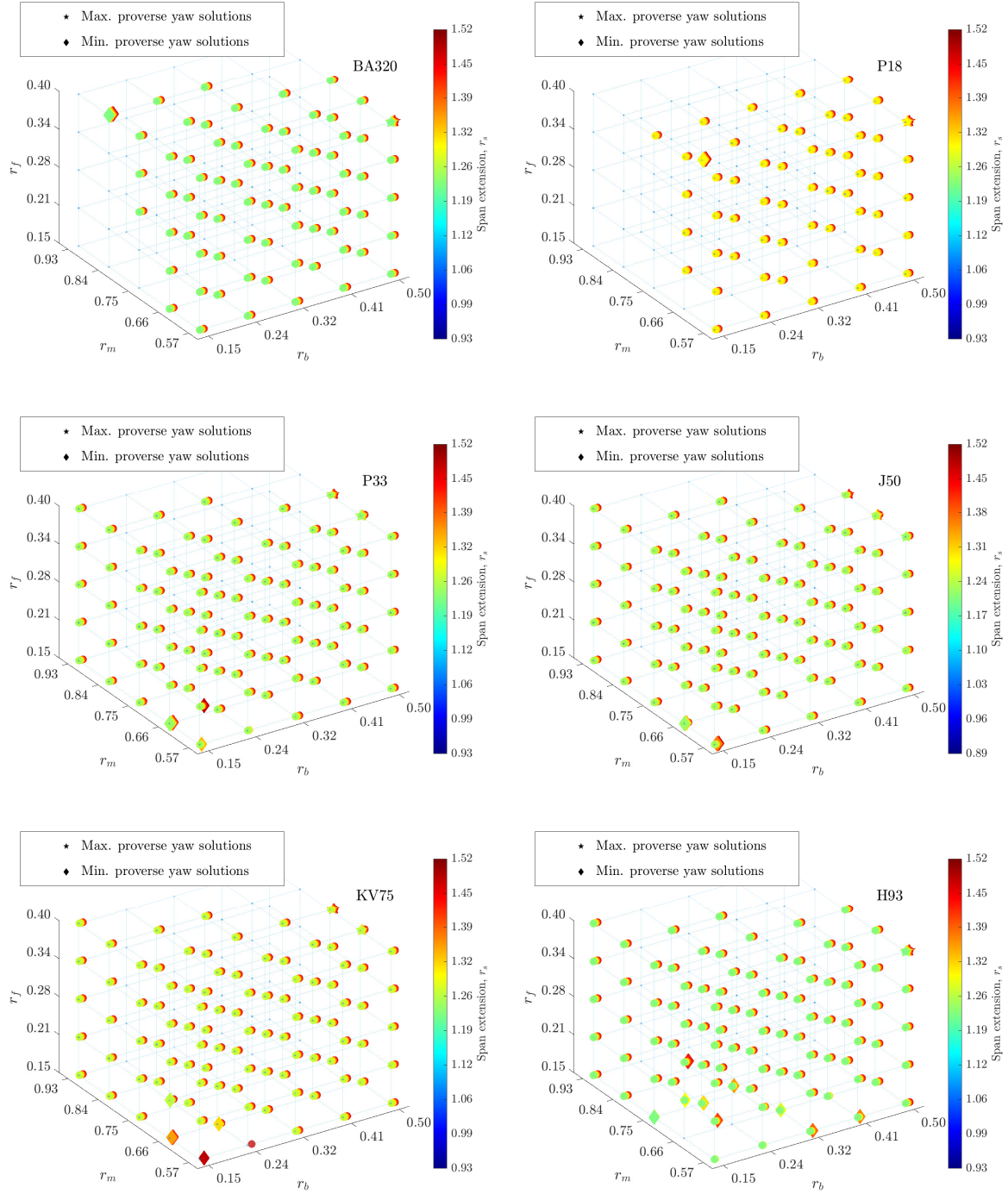


Figure 7.5: Design space of parametric study of aileron sizing for proverse yaw on complete lifting surfaces configurations.

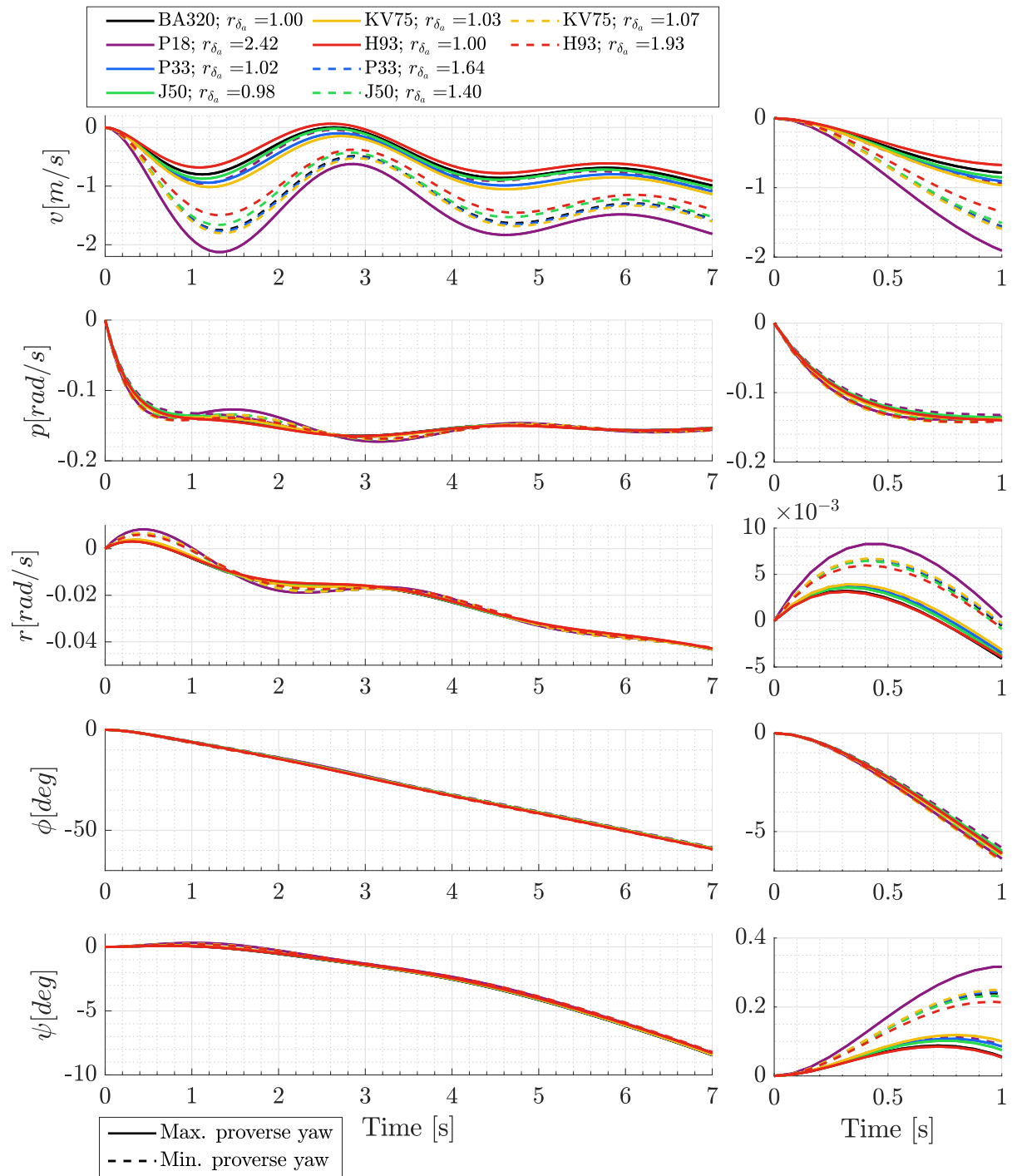


Figure 7.6: Lateral-directional response to aileron step on complete lifting surfaces configurations. Detail of the turn initiation response on the right.

7.3 Conclusions

This chapter has presented a study of aileron sizing for the attainment and impact of the minimisation of adverse yaw under roll authority requirements. The use of fundamental low-fidelity methods provided a fast and simple tool to demonstrate a set of solutions that could not have been considered with other traditional approaches, for which an underlying assumption is the optimality of the elliptic spanload for minimum induced drag.

It has been observed that the generation of proverse yaw on the wing is more easily achieved with a NELD, for which the more effective solutions are those providing the largest favourable $C_{n_{\delta a}}$. However, this response is greatly dependent on the flight condition, as well as on the placement and sizing of the ailerons within the downwash/upwash region along the span. The solutions satisfying the design requirements produce an inverse variation of induced drag that is observed on the wings with a down-going aileron, which satisfies the definition of the required drag differential to produce a favourable yawing moment due to aileron deflection. Furthermore, these solutions seem to provide less drag per degree of aileron deflection, but simultaneously require more deflection to achieve a demanded roll rate. This may overtake the initially thought benefits of a more aerodynamically efficient manoeuvre.

The majority of studies investigating proverse yaw focus on the wing only, in the same manner as the initial analytical methodology presented here. However, the assumptions and limitations of this aileron design method have been made evident through simulations of the complete aircraft configurations, where the capabilities of the wings to produce proverse yaw are surpassed by the contributions from the tail. Hence, the difficulties to analytically capture non-linear flight dynamic coupling terms within the aileron design process were exposed further into the manoeuvre, where the dynamics were found to be dominated by lateral-directional coupling effects. While it seems that the impact of wing proverse yaw for a complete configuration is negligible, further development of this research could be useful for the design of configurations such as flying wings of BWB or aircraft with morphing technologies.

Chapter 8

Conclusions and Further Work

The research work that has been presented in this thesis is summarised here. The general conclusions and key findings regarding the main aims and final state of research is collected and presented. More in depth technical conclusions, remarks or discussions can be found in each of the preceding chapters to which the interested reader is referred to.

8.1 Summary of Research

There are several challenges to be faced to reach today's vision for aviation to become an environmentally efficient and clean industry. Within the wide body of work and effort that is being put on wing design for this purpose, this research targeted the sparked interest on the fundamental relation between induced drag and aspect ratio. Numerous reasons have been identified as to why manufacturers are not yet operating aircraft with HARWs, with major drawbacks developing from the trade-off between aerodynamics and structures. Furthermore, the multi-disciplinary nature of the design process has also been highlighted, and with it the penalties that derive from both concept and design tools. To address some of these challenges, an elegant analytical approach promising improvements in aero-structural efficiency and flight dynamic characteristics has gained the interest of researchers around the world. This design philosophy, named here as the design of NELDs, elegantly proposed to remove the constraint on fixed span to instead prescribe structural requirements. However, conclusions on the actual benefits and practical implications of the application of this theory remain unsettled. A comprehensive discussion of the challenges, limitations and developments being made around this topic was presented in Chapters 1 and 2.

Altogether, this led to the aim of this thesis, which consists on the evaluation of the synthesis of wing aerodynamic and structural requirements under a single design theory for the conceptual design of high aspect ratio wings. This is done as a means

to improve overall vehicle performance and enlighten the relevant complexities of a multi-disciplinary design process. To reach this goal, the research has been classified into four objectives that address some of the major research questions revolving the aim.

The first objective was set to bring together the scattered knowledge about the alternative design approach to aero-structural efficiency, and provide a comparative ground to evaluate the theoretical properties of such wing designs applied to extended initial conditions. With this, the objective pursued to develop a reduced-order wing shape design methodology that produces the geometric parameters necessary for a wing to satisfy prescribed aerodynamic and structural requirements. Within this, integrate and compare existing theoretical approaches that remove the constraint on span for aero-structural efficiency. For this purpose, a comprehensive comparison of the classical theories for the synthesis of aerodynamic and structural constraints was presented. This was followed by a semi-analytical design methodology that was developed utilising the lifting line theory as the foundation to integrate several derivations for the design of NELDs, which include the work of Prandtl (1933), Jones (1950), Klein and Viswanathan (1975) and Horten (1993). This enabled compute the fundamental planform and twist characteristics that would satisfy the adopted design constraints and create a design space in which the corresponding trends could be identified against traditional approaches for span extension. Chapter 4 collects the work carried out on this objective.

Now, to provide a comprehensive evaluation of the feasibility and trade-offs of such wing designs and design approach, it was necessary to further extend the approach beyond several other limitations of the theoretical approach and the research available in the literature. Furthermore, it should be done in such a way that is in accordance with the contemporary demands of conceptual design practices and requirements. With this, the second objective was defined as to compose a multi-disciplinary physics-based design framework for the conceptual design stage to produce a design space for the analysis of aircraft with high aspect ratio wings under prescribed aero-structural requirements, where this environment should be formulated as to enable a state-of-the-art approach to conceptual design and minimise the use of statistical and historical data. This allowed to encompass a wider range of variables and considerations belonging to the different airframe disciplines, as well as to increase the relative fidelity of the analyses. The conceptual framework comprises various state-of-the-art validated computational models together with the in-house developed methodologies, integrated into a single design process with a set-based approach. The theoretical models thus employed include: VLM with Prandtl-Glauert compressibility correction; drag build-up method to estimate parasitic drag and two-dimensional profile drag integration; linear/non-linear beam element models with distributed and lumped

masses for efficient structural analysis; empirical mass formulations for initial estimation of primary and secondary components; extensive critical load cases specified by EASA; and mission performance based on energy considerations. This enables to culminate the accumulated aero-structural trade-offs of such novel configurations from the perspective of overall aircraft performance at a mission level. The physics-based tools included are: AVL for aerodynamics and flight dynamics analysis, NeoCASS for structural sizing, and FLOPS for mission analysis, and the main environment in which the framework is based on is Matlab. The general overview of this analysis framework was presented in Chapter 3, while the development the various modules and the corresponding findings were presented in Chapters 4 to 7.

The third objective was set to address the claimed potential of NELD spanloads to remove adverse yaw characteristics inherent to conventional swept tapered wings. Experimental research on the application of this design approach to flying wings unveiled promising results, but the general approach has focused on the derivation of closed form solutions for the low-speed wings. With this, it was proposed to develop a methodology to assess the impact of wings designed under the assumptions described above on aircraft lateral-directional flight dynamics with a focus on proverse yaw. This was pursued through an aileron design methodology dictated by roll authority and adverse yaw nullification, based on a full factorial design of experiments approach. Numerical analysis of the relevant stability derivatives was obtained through the use of the lifting line theory along with strip and aerofoil theory to provide a design space of aileron sizing that satisfy the above conditions. This was verified using AVL, and extended to include the impact on a wing and tail layout. This work has been presented in Chapter 7.

Finally, to provide more insight on the trade-offs derived from this wing design approach, the final objective was defined as to identify the capabilities, advantages and drawbacks originating from the application of the design process to improve aircraft efficiency and enhance decision making within the design stage. To establish the basis for validation, comparative analysis of the aircraft performance would be conducted for the produced configurations relative to a conventional airliner, and interpret the results through the derivation of trends and metrics. A systematic design space exploration was allowed through the use of a design of experiments methodology under a set-based design process, delaying the elimination of poorer design points until the end of the analysis process so as to avoid the use of conventional criteria used to quantify efficiency. Altogether, this approach enabled to reduce the impact of the errors derived from the limitations of the theories employed. With this, several of such results have been provided and discussed across the various chapters. Moreover, a case-study based on a well-known and documented single-aisle airliner was used to provide a realistic analysis setting and thus depart this research from the traditional

use of the ideal elliptic lift distribution as the benchmark for design.

8.2 Key Findings and Conclusions

The key findings and general conclusions of the work presented in this thesis are summarised here:

- The use of the selected NELD theoretical approach for the derivation of analytical trends to predict aero-structural trade-offs of the performance of the entire aircraft will exceedingly overpredict the benefits that can be achieved, and thus the claims found in the literature cannot be concurred with at a mission level. The analytical metrics extracted from NELD provide accurate trend predictions for the aerodynamic efficiency characteristics with span extension, although overestimated in magnitude by over 50%. Structural equivalence was not achieved through the use of NELDs, this wing design approach reduces the growth in wing weight increase as the span is extended by approximately 30% from a standard expansion of the baseline configuration. No significant differences were observed between the assumption of the various structural constraints selected to define wing NELD spanloads on the final estimation of the structural mass.
- The trends shown for the development of bending and shear loads show that the modification of the centre of lift with span extension will impact the structural load envelope in such way that the increase in the critical positive loads is reduced by approximately 30% while the negative bound of the structural loads envelopes becomes more critical as the span is extended, increasing by up to 70% when compared to the behaviour of wing extended with the same spanload. However, the dominance of positive load factor manoeuvres of the load cases selected in this study drive the sizing of the wing, which might not be representative of the actual trend of mass increase when a more comprehensive list of load cases is considered. The wing torsion distribution does not undergo any significant impact due to the use of NELD to extend the span.
- As expected, the addition of off-design and high load manoeuvres penalises the exclusion of such considerations during the definition of the spanload constraints on the structural sizing. Furthermore, the critical cases driving the sizing change as the spanload and span extension vary from wing to wing. Where engineering experience is extremely valuable in such a task, assumptions of the exclusion of load cases might not be recommended for this type of designs, as there is an increased risk of not meeting the design criteria for the complete set of design driving load cases.

- The extension of the evaluation of NELD up to the impact on mission performance, as compared to existing research, enables a more practical analysis of the trade-off between mass and aerodynamic efficiency. Hence, the inclusion of the use of the range-to-weight output as a figure of merit to comparatively assess the efficiency of the designs provides further insight than the use of other traditional metrics such as the lift-to-drag ratio.
- Generally, increasing the wingspan of a given aircraft will provide a detrimental contribution to the top level aircraft requirements assessed, with the exception of the range. Nevertheless, the use of NELD for span extension becomes beneficial in the impact of TLARs parameters by reducing the penalties due to the yielded trade-off between aerodynamic and structural characteristics. This is achieved to such an extent that the trends deflect to provide minor improvements for the NELD configurations found at the lower bounds of span extension in the take-off field length, rate-of-climb and time-to-climb characteristics.
- The use of NELD to extend the span of a given configuration allows up to an 11% of span extension without penalties in performance, following the methodology and metrics employed in the proposed design and analysis process. This is achieved by spanload families constrained by either the wing integrated bending moment or the root bending moment. Within these limits, both range and gross weight increase, and the penalised TLARs do so within acceptable values. Prescribing both structural constraints becomes more restrictive, allowing up to a 3% of span extension with no performance penalties but with negligible improvements.
- The semi-analytical approach to proverse yaw provided the means to identify the unconventional behaviour of induced drag during aileron deflection that generates a favourable yawing moment on NELD wings. The methodology highlighted too that attainment of these solutions is highly dependant on flight condition, aileron sizing and location within a given spanload, and the magnitude of deflection. The wing configurations showing proverse yaw were generally less responsive to aileron deflection, but achieved to yield the required roll rate demand and minimise adverse turning for the design points providing the highest differential of yaw due to aileron deflection. The impact of such a response on the complete wing-and-tail layouts was found to be negligible. According to these findings, the potential to decrease the lateral-directional coupling and complexity in flight control systems is suspect, but has not been investigated here. Furthermore, the proverse yaw capabilities of the wing are not sufficient to pursue the removal of the vertical tail in conventional tube-and-wing with

wing-mounted engine configurations. Instead, further development of these characteristics should be placed on aircraft types such as flying wings and BWB, as well as morphing technologies.

- The proposed methodology has enabled an extended multi-disciplinary and multi-fidelity design and analysis environment within a set-based design setting. With this, it has been possible to provide further insight on the practical impact of the aero-structural trade-offs derived from NELDs, and to develop pragmatic trends on the feasibility and limitations of such an approach. The integration of existing computational models has enabled to identify prevailing limitations within tools for conceptual design. These rightly aim at becoming larger design spaces with more flexibility in terms of airframe layout, power-plant type and integration, and several other considerations discussed throughout this work, which are critical for the design of novel and challenging aircraft. Nevertheless, many will assume a traditional spanload and will circumvent the importance of enabling an accurate definition of the wing twist. Even though it might not be possible to acquire this level of detail in the initial stages, the design process could be enhanced by parametrically studying the trade-off between span and centre of lift on earlier stages under design constraints based on NELDs, and reduce the uncertainty carried on to the next stage, while having a more informed scenario for the eventual MDO process.
- The work carried out presents an isolated analysis on the fundamental effect of the synthesised modification of span and spanload through the NELD relationship to provide a focused insight on its impact on vehicle performance. The use of NELD for the design of HARW can serve as a basis to create a design space of solutions that can potentially provide improvements in performance over a baseline configuration.
- The development of such a design framework enables extensive analysis of aircraft performance, whilst induced drag is reduced through span extension and the consequent weight increase is minimised as a result of NELDs.

8.3 Contribution to Knowledge

Overall, the contributions to knowledge which were identified and attributed to the work presented herein can be summarised as follows:

- A comprehensive assessment of the classical theories for the conceptual design of wings under combined aerodynamic and structural constraints, which have been

challenged through the analysis of its solutions through higher fidelity methods and constraints, and has been extended to quantify the derived trade-offs at a mission level applied to a pragmatic case-study. Altogether, this provides an alternative approach to span-extended wing design through the implementation of NELD theory into modern conceptual design practises.

- Following the claims of the beneficial use of NELDs for proverse yaw, a methodology based on fundamental low-fidelity methods was developed to compute and evaluate aileron geometries that can satisfy the minimisation of adverse yaw while ensuring that the minimum roll authority required by certification specifications is attained. This methodology has put forward that the generation of the drag differential characteristic of proverse yaw solutions is highly dependant on the downwash distribution, therefore on the specific trimmed/untrimmed flight conditions, as well as the aileron size and location along the spanload.
- The developed design and analysis framework provides multi-disciplinary, multi-fidelity, set-based design capabilities for the conceptual design of wings from the perspective of spanload as its main input. The framework is readily usable and can be expanded to other applications away from the NELD span extension. It brings together the integration and modification of existing and validated conceptual design tools, as well as in-house developed methodologies, and can be easily linked to other computational models. The environment enables independent analyses of the designs for each of the incorporated disciplines, as well as to provide a holistic examination of the airframe. This initial framework complies with the vision for design capabilities with increased flexibility, modularity, discipline interaction and theoretical foundations in order to address the challenges posed by the ever-growing complexity of aircraft.
- Several trends and metrics that define and quantify the impact of the use of NELD for span extension have been generated for the various disciplines involved. Such information provides novel insight on the aero-structural trade-offs and proverse yaw characteristics derived from the use of the NELD design philosophy. A discussion on the effectiveness of the derived and existing measures for vehicle performance has been provided, for which the derived range-to-weight seems to yield the most representative figure of merit to quantify performance improvements over a given baseline aircraft.

8.4 Impact

The strive for more efficient flight has led many to challenge a diversity of conventionalities in civil aircraft design. The research presented here has provided a design space in which the antagonism between aerodynamic and structural efficiency into synthesised and constructive design metrics. The analysis of impact of span extension through NELDs in an multi-disciplinary framework provides a holistic set of results at a mission level, as opposed to the disparate solutions currently available, and where traditional assumptions do not constrain the proposed concept. The relevant derivation of novel trends and metrics can enable the assimilation of the determining contributors and compromises to be made, amplifying the designer's control over the design and decision-making processes at the early design stage, while delaying critical decisions and enhancing the optimisation process with more informed drivers. Thus, a wider, physics-based, set-based design space that addresses the sensitivities of aero-structural trade-offs and its boundaries can promote the attainment of alternative configurations such as high aspect ratio wings with reduced penalties and improved vehicle performance. Furthermore, unconventional concepts such as blended-wing-bodies or morphing could benefit from the proposed approach to flight dynamics under the terms of proverse yaw to improve controllability and potentially minimise the need of vertical stabilisers. The proposed approach together with other research being made on the spanload requirements for proverse yaw could be combined when developing discrete control surfaces for near future designs. The developing morphing technologies could benefit from this approach in the design of continuous control surfaces in order to address the sizing and estimate the capabilities for proverse yaw potential at the early stages of the design. However, for conventional configurations, it appears to the author that this approach to achieve proverse yaw should remain secondary.

8.5 Further Work

A wide area of further work was identified throughout this research. The following list provides the most relevant limitations to address, questions that remained unattended, as well as paths of research interest to follow.

- Inclusion of other spanload families to create a wider trends map of the trade-offs between span extension and spanload centre of lift and the impact on the relevant design parameters. Expand the design of experiments approach to adopt further input variables related to planform.

- The impact of proverse yaw on flight control systems should be investigated, since the controls surfaces and high-lift devices have been modelled only through their aerodynamic contributions.
- Investigate the bounds within which proverse yaw is effective, focusing on flying wing or blended-wing-body configurations.
- The benefits of incorporating a flexibility considerations at the conceptual design stage has been highlighted in the literature review. Hence, coupled aero-structural analysis with the addition of aeroelastic constraints such as tip deflection and flutter would improve the structural weight estimates and further reveal span extension limitations. Within the proposed approach to aero-structural analysis, consideration geometric non-linearities would enable to predict static and dynamic behaviours of flexible wing at the conceptual design stage, which is of increasing necessity especially if the configuration under consideration involves HARWs.
- The methods and computational models employed rely on reduced-order, low-fidelity models, which carry many simplification that may reduce the accuracy of the results. Furthermore, the verification of results has relied heavily on the validation of the tools employed. Therefore, studies focused on uncertainty and validation should be performed to increase the robustness and accuracy of the proposed methodology and framework.
- Consideration of three-dimensional transonic flow effects should be integrated to a higher degree of fidelity for a more accurate prediction of its impact on the drag contribution.
- Even though this research has attempted to bring together state-of-the-art computational models that can confidently analyse unconventional configurations, there are still limitations to conventional tube-and-wing and traditional power-plant and propulsion systems. Further computational models that provide such capabilities should be integrated so that these analyses are readily available.
- Study the impact of span extended solutions on the compatibility of the designs to operate within current airports.
- The approach taken NELD wing modification assumes a direct retrofit on an existing fuselage. In practise this would affect the balance and longitudinal stability that has been left out of scope. The actual implications of this happening should be addressed, pointing out the limitations of the approach and presenting alternatives.

- The methodology and trends should be explored and verified on other type of airframe configurations, including traditional wide-body aircraft, and further unconventional designs, as well as expand the comparison against wings with winglets.
- Addition of an existing HARW aircraft concept, such as the SUGAR TBW being proposed by Boeing and NASA, to the comparative ground would set the benchmark for this type of configuration and for the classical design approaches employed on these concepts. This attempted comparison has not been presented due to the suspect results obtained in the structural sizing from the addition of the wing struts. A correct development of this analysis would be beneficial for the discussion of the impact of NELDs for span extension against the need to include structural support for the wing.

8.6 Dissemination of Research

Four conference papers relevant to the research presented here have been published and presented.

- E. Bragado Aldana, S. Fraysse, M. M. Lone, "Aerodynamic and flight dynamic study of non-elliptic wing lift distributions", in *AIAA SciTech Forum*, 2019. DOI: 10.2514/6.2019-0820. This paper puts forward the investigation on Prandtl wings to address adverse yaw and minimise induce drag, by coupling aerodynamic and flight dynamic design methodologies that utilise reduced-order aerodynamic tools.
- E. Bragado Aldana, A. P. Mendez, M. M. Lone, "Closer look at the flight dynamics of wings with non-elliptic lift distributions", in *AIAA SciTech Forum*, 2020. DOI: 10.2514/6.2020-0284. In this work, the lateral-directional characteristics of wings with non-elliptic lift distributions are studied, further focusing on proverse yaw and lateral-directional handling qualities.
- M. Carrizales, E. Bragado Aldana, M. M. Lone, "Non-elliptic wing lift distribution wings to decrease vertical tailplane size in commercial aircraft", in *AIAA SciTech Forum*, 2020. DOI: 10.2514/6.2019-0265. Contribution as a second author, provided the wing design methodology to aerodynamically address the aircraft's lateral-directional stability characteristics.
- E. Bragado Aldana, M. M. Lone, A. Riaz, "On wings with non-elliptic lift distributions", in *32nd Congress of the International Council of the Aeronautical Sciences*, 2021. 1st prize winner of the John McCarthy Student Awards. This work presents

a concise review of the theoretical basis of Prandtl's approach to non-elliptic lift distributions, the developments carried out in this area and their relevance to the future of aviation.

Appendix A

Data for the Baseline Aircraft Model

A.1 Performance Characteristics

Table A.1: Performance and TLARs of the baseline aircraft model.

Performance specification	Value	Description
Range*	4,300 km	Design range
C_L^*	0.5658	Design lift coefficient of the aircraft
L_{wing}		Design lift of the wing
Cruise altitude	37,000 ft	Design cruise altitude
M_c	0.78	Design cruise Mach number
M_{mo}	0.82	Maximum operating Mach
V_{mo}	350 kts	Maximum operating CAS
ICA	33,000 ft	Initial cruise altitude capability after take-off @ MTOW, ISA+10°C
TTC	23 min	Time-to-climb after take-off @ MTOW, ISA+10°C
Ceiling OEI	15,000 ft	One-engine-inoperative net ceiling
Service ceiling	39,000 ft	Maximum cruise altitude
TOFL	2,200 m	Take-off Field Length @ SL, ISA+15°C)
LFL	2,200 m	Landing distance limit @ MLW, ISA, SL, wet
V_{app}	145 kts	Approach speed: $V_{ls} + 5$ knots plus 1/3 the headwind / $V_{ls}=1.23 V_s$
Sink speed	200 ft/min	Sink speed at landing
Climb rate	1,600 ft/min	Rate of Climb
SSCG	2.4 %	Second segment climb gradient
MACG	2.5 %	Missed approach climb gradient
Thrust	115,000 N	Maximum installed take-off thrust per engine
T/W	0.32	Thrust to Weight ratio
W/S	600.5	Wing loading
$C_{L_{max, clean}}$	2.2	Maximum lift coefficient in clean configuration
$C_{L_{max, TO}}$	2.8	Maximum lift coefficient in take-off configuration
$C_{L_{max, LDG}}$	3	Maximum lift coefficient in landing configuration
$\delta_{f_{TO}}$	20°	Flap deflection in take-off configuration
$\delta_{f_{LDG}}$	39°	Flap deflection in landing configuration

A.2 Mass Properties

Table A.2: Mass breakdown of structural and secondary components of the baseline aircraft model.

Component	Mass [kg]
Wing*	9,000
Horizontal tail*	600
Vertical tail*	450
Fuselage*	8,500
Propulsion	7,000
Landing gear	2,350
Systems	5,400
Pilots	170
Interior	3,100
Passengers	16,000
Cargo	1,600
Fuel	18,700
Miscellaneous	200
Other Items	100
Fuel system	300
Flight control system	750
APU	200
Instruments	100
Avionics	750
Hydraulic pneumatic systems	1,100
Electrical	1,500
ECS anti-icing system	700
Furnishing	2,800
OEW*	39,556
MPW	22,500
MZFW*	60,500
MFW	18,728
MLW*	64,500
MRW*	73,900
MTOW*	73,500

A.3 Geometric Properties

Table A.3: Geometric properties of the lifting surfaces of the baseline aircraft model.

Component	Dimension	Value
Wing	Area, S_{wing}	122.40 m ²
	Aspect ratio*, R	9.50
	Taper ratio, λ	0.24
	Flaps chord ratio, c_{flap}/c	0.27
	Flaps total area, S_{flap}	21 m ²
	Flaps extension, s_{flap}/s	0.12-0.70
	Aileron chord ratio, $c_{aileron}/c$	0.27
	Aileron total area, $S_{aileron}$	2.80 m ²
	Aileron extension, $s_{aileron}/s$	0.75-0.95
	Aileron maximum deflection, $\delta_{a,max}$	$\pm 25^\circ$
	Slats total area, S_{slat}	12.55 m ²
	Spoilers total area, $S_{spoiler}$	8.64 m ²
	Air brakes total area, $S_{airbrake}$	2.45 m ²
Tail	Area	31 m ²
	Aspect ratio	5
	Taper ratio	0.33
	Quarter-chord sweep	28°
	Dihedral	6°
	Elevator chord ratio, $c_{elevator}/c$	0.30
	Elevator total area, $S_{elevator}$	7.50 m ²
	Elevator extension, $s_{elevator}/s$	0-1
Fin	Elevator maximum deflection, $\delta_{e,max}$	+15° / - 30°
	Area	21.50 m ²
	Aspect ratio	1.60
	Taper ratio	0.35
	Quarter-chord sweep	35°
	Rudder chord ratio, c_{rudder}/c	0.30
	Rudder total area, S_{rudder}	6.75 m ²
	Rudder extension, s_{rudder}/s	0-1
	Rudder maximum deflection, $\delta_{r,max}$	$\pm 30^\circ$

A.4 Aerofoil Properties

 Table A.4: Aerodynamic properties of wing aerofoils at Reynolds number 10⁶.

Airfoil	NASA SC(2)0614	NASA SC(2)0612	NASA SC(2)0610
Semispan location	Root to 11%	37%	67% to tip
Angle-of-attack at zero lift	-4.07°	-3.94°	-4.74°
Lift curve slope	6.665 rad	6.714 rad	6.133 rad
Drag at zero lift	0.0061	0.0062	-0.0119
Lift at zero angle-of-attack	0.4862	0.4615	0.4346
Maximum camber	2.10%	1.90%	1.80%
Design lift coefficient	0.6		

A.5 Spanwise Distribution of Wing Characteristics

All parameters shown here have been normalised by their corresponding value at the root, which can be found in either Figure 3.3, Table A.3 or Table A.4.

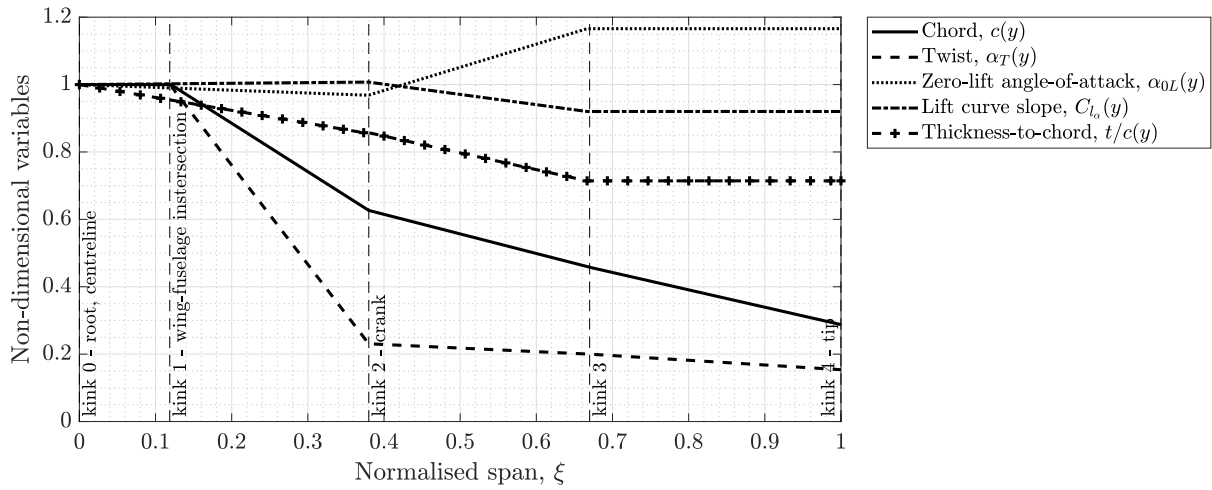


Figure A.1: Spanwise distribution of wing properties of the baseline configuration.

Appendix B

Aerodynamic and Structural Characteristics of the Revised Wing Layouts

B.1 Derivations for the Equivalent K&V75 family

From Klein and Viswanathan's 1975 paper [56, 57], the integral in Equation (1) is resolved. The indefinite solution is derived and shown here in Equation B.1, for which resolution along the non-dimensional semi-span yields Equation B.2.

$$\begin{aligned} \int \gamma(\xi) d\xi &= \int \sqrt{1-\xi^2} \left(2C_1 + \frac{2C_2}{\pi} + C_3 \right) - \frac{2C_2}{\pi} \xi^2 \ln \left(\frac{1-\sqrt{1-\xi^2}}{1+\sqrt{1-\xi^2}} \right) - \frac{2}{3} C_3 \sqrt{(1-\xi^2)^3} d\xi \\ &= \frac{1}{2\pi} \sqrt{1-\xi^2} \xi (2\pi C_1 + 2C_2 + \pi C_3) + \frac{1}{2\pi} \sin^{-1} \xi (2\pi C_1 + 2C_2 + \pi C_3) \\ &\quad - \frac{1}{3\pi} C_2 \sqrt{1-\xi^2} \xi - \frac{1}{3\pi} C_2 \xi^3 \ln \left(\frac{1-\sqrt{1-\xi^2}}{1+\sqrt{1-\xi^2}} \right) + \frac{1}{3\pi} C_2 \sin^{-1} \xi \\ &\quad - \frac{5C_3 \sqrt{(1-\xi^2)^3} \xi}{12(1-\xi^2)} - \frac{C_3 \sqrt{(1-\xi^2)^3} \sin^{-1} \xi}{4(1-\xi^2)^{3/2}} + \frac{C_3 \sqrt{(1-\xi^2)^3} \xi^3}{6(1-\xi^2)} + C \end{aligned} \tag{B.1}$$

$$\begin{aligned} \int_0^1 \gamma(\xi) d\xi &= \frac{1}{4} (\pi(2C_1 + C_3) + 2C_2) + \frac{C_2}{6} - \frac{\pi C_3}{8} \\ &= \frac{\pi C_1}{2} + \frac{2C_2}{3} + \frac{\pi C_3}{8} \end{aligned} \tag{B.2}$$

Similarly, the integrals in Equations (2) and (3) from Reference [56, 57] are resolved,

APPENDIX B. AERODYNAMIC AND STRUCTURAL CHARACTERISTICS OF THE REVISED WING LAYOUTS

yielding the definite integrals shown here in Equations B.4 and B.6.

$$\begin{aligned} \int \xi \gamma(\xi) d\xi &= \int \xi \left[\sqrt{1-\xi^2} \left(2C_1 + \frac{2C_2}{\pi} + C_3 \right) - \frac{2C_2}{\pi} \xi^2 \ln \left(\frac{1-\sqrt{1-\xi^2}}{1+\sqrt{1-\xi^2}} \right) - \frac{2}{3} C_3 \sqrt{(1-\xi^2)^3} \right] d\xi \\ &= \frac{-1}{60\pi} \left[(10(1-\xi^2)^{3/2} (2\pi(2C_1 + C_3) + 3C_2) + 30C_2 \sqrt{1-\xi^2} \right. \\ &\quad + 15C_2 \ln(1-\sqrt{1-\xi^2}) + 15C_2(\xi^2-1)(\xi^2+1) \ln \left(\frac{1-\sqrt{1-\xi^2}}{1+\sqrt{1-\xi^2}} \right) \\ &\quad \left. - 15C_2 \ln(1+\sqrt{1-\xi^2}) + 8\pi C_3(\xi^2-1) \sqrt{-(\xi^2-1)^3} \right] \end{aligned} \quad (B.3)$$

$$\int_0^1 \xi \gamma(\xi) d\xi = \frac{2C_1}{3} + \frac{C_2}{\pi} + \frac{C_3}{5} \quad (B.4)$$

$$\begin{aligned} \int \xi^2 \gamma(\xi) d\xi &= \int \xi^2 \left[\sqrt{1-\xi^2} \left(2C_1 + \frac{2C_2}{\pi} + C_3 \right) - \frac{2C_2}{\pi} \xi^2 \ln \left(\frac{1-\sqrt{1-\xi^2}}{1+\sqrt{1-\xi^2}} \right) - \frac{2}{3} C_3 \sqrt{(1-\xi^2)^3} \right] d\xi \\ &= \frac{1}{40\pi} \sqrt{1-\xi^2} \xi (5\pi(2\xi^2-1)(2C_1+C_3) + 16C_2(\xi^2-1)) \\ &\quad + \sin^{-1} \xi \left(\frac{C_1}{4} + \frac{2C_2}{5\pi} + \frac{C_3}{8} \right) - \frac{1}{5\pi} C_2 \xi^5 \ln \left(\frac{1-\sqrt{1-\xi^2}}{1+\sqrt{1-\xi^2}} \right) \\ &\quad - \frac{1}{24} C_3 \tan^{-1} \left(\frac{\xi \sqrt{-(\xi^2-1)^3}}{(\xi^2-1)^2} \right) + \frac{C_3 \xi (\xi^2-1)^2 (8\xi^4 - 14\xi^2 + 3)}{72 \sqrt{-(\xi^2-1)^3}} \end{aligned} \quad (B.5)$$

$$\begin{aligned} \int \xi^2 \gamma(\xi) d\xi &= \frac{1}{16} (\pi(2C_1 + C_3) + 2C_2) + \frac{3C_2}{40} - \frac{\pi C_3}{48} \\ &= \frac{\pi C_1}{8} + \frac{C_2}{5} + \frac{\pi C_3}{24} \end{aligned} \quad (B.6)$$

These equations are dimensionalised into lift force L , integrated shear force F and integrated bending moment M with the following constants, respectively:

$$\begin{aligned} C_{lift} &= 2\rho V^2 s^2 \\ C_{shear} &= 2\rho V^2 s^3 \\ C_{bending} &= \rho V^2 s^4 \end{aligned} \quad (B.7)$$

Assuming that the values for the lift force, integrated shear force and integrated bending moment of the baseline wing are known, the remaining unknowns, namely C_1 , C_2 and C_3 can be obtained by solving the simultaneous system of equations of the

form $\mathbf{A} \cdot \vec{x} = \vec{b}$

$$\begin{bmatrix} C_{lift} \frac{\pi}{2} & C_{lift} \frac{2}{3} & C_{lift} \frac{\pi}{8} \\ C_{shear} \frac{2}{3} & C_{shear} \frac{1}{\pi} & C_{shear} \frac{C_3}{5} \\ C_{bending} \frac{\pi}{8} & C_{bending} \frac{1}{5} & C_{bending} \frac{\pi}{24} \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} L \\ \int_0^s F(y) dy \\ \int_0^s M(y) dy \end{bmatrix} \quad (\text{B.8})$$

B.2 Circulation Distributions

Each spanload family is plotted against the spanwise circulation of the baseline configuration. Both axes are dimensional, where the spanwise axes has been normalised against the baseline semispan.

APPENDIX B. AERODYNAMIC AND STRUCTURAL CHARACTERISTICS OF THE REVISED WING LAYOUTS

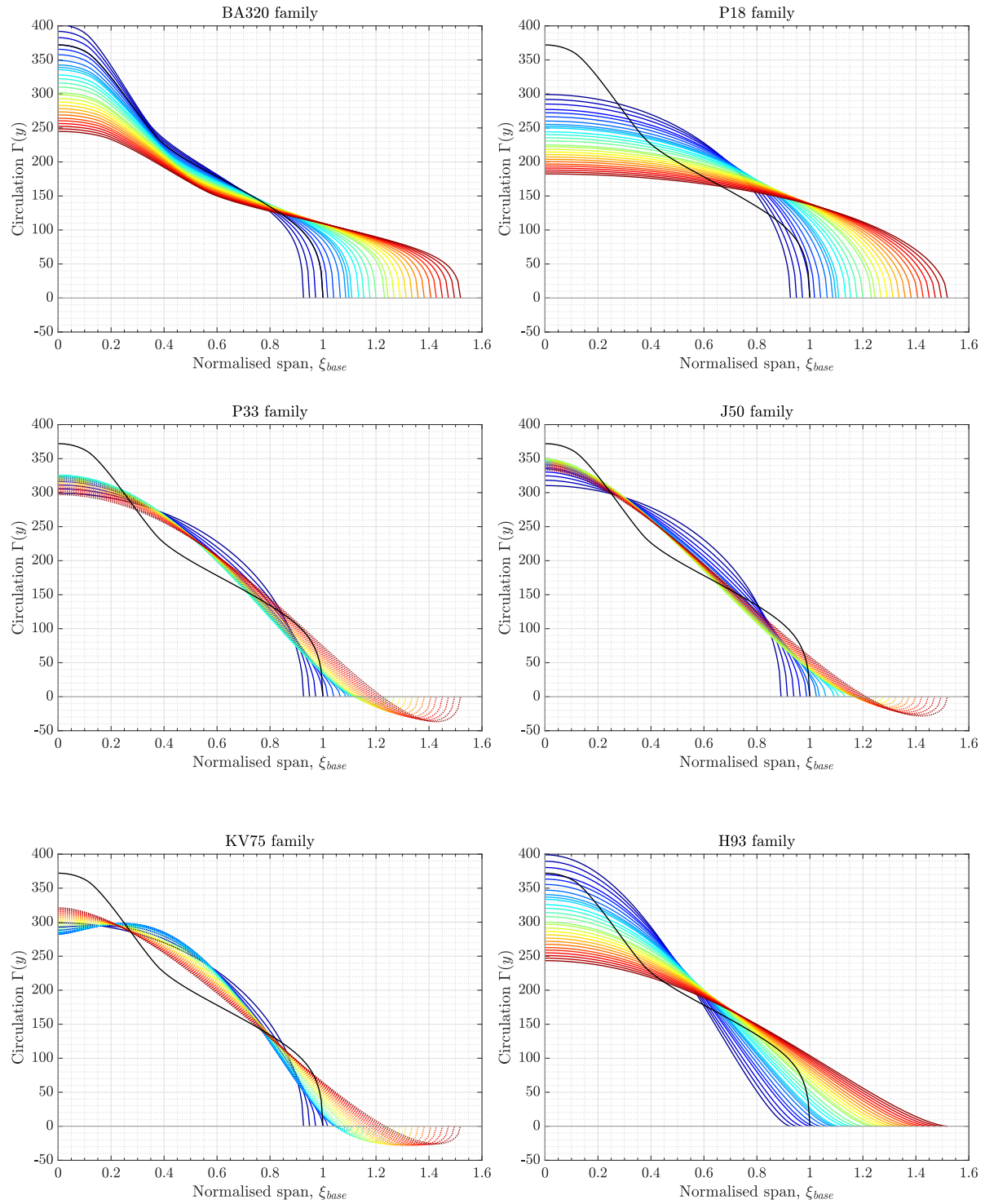


Figure B.1: Comparison of the computed circulation distributions against the baseline and reference HARW.

B.3 Lift Coefficient Distributions

The section lift coefficient is plotted for each spanload family against the respective distributions for baseline configuration. The lift coefficient is presented by its local value, where the non-dimensional $C_l(y)$ is multiplied by the local chord $c(y)$ and divided by the mean aerodynamic chord mac of the corresponding configuration. The spanwise axes has been normalised against the baseline semispan.

APPENDIX B. AERODYNAMIC AND STRUCTURAL CHARACTERISTICS OF THE REVISED WING LAYOUTS

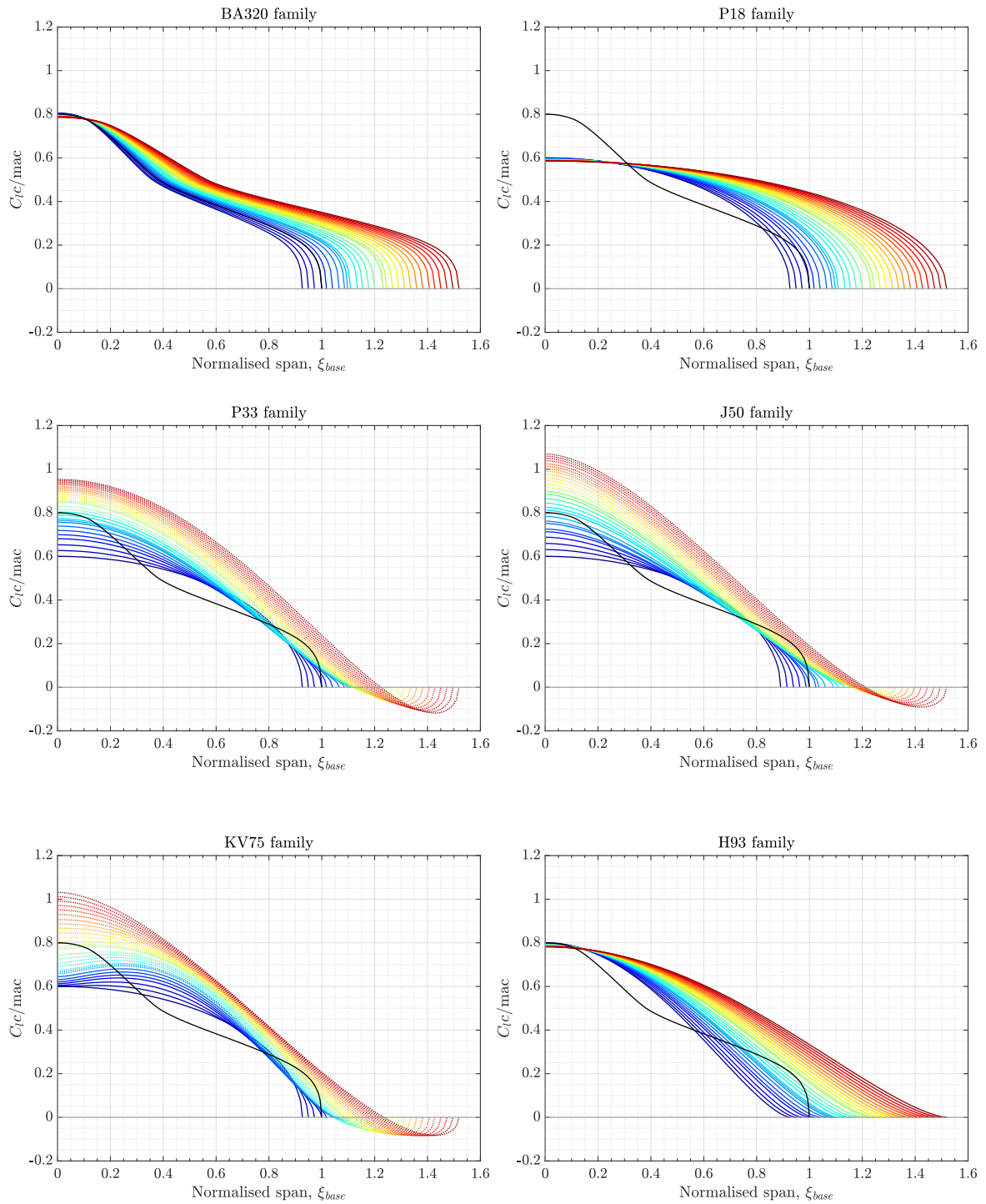


Figure B.2: Comparison of the computed local lift coefficient distributions against the baseline.

B.4 Induced Drag Coefficient Distributions

The section coefficient for induced drag is plotted for each spanload family and compared to the induced drag coefficient distribution of the baseline configuration. The local induced drag coefficient $C_{d_i}(y)$ has been multiplied by the local chord $c(y)$ and divided by the mean aerodynamic chord mac of the corresponding configuration. The spanwise axes has been normalised against the baseline semispan.

APPENDIX B. AERODYNAMIC AND STRUCTURAL CHARACTERISTICS OF THE REVISED WING LAYOUTS

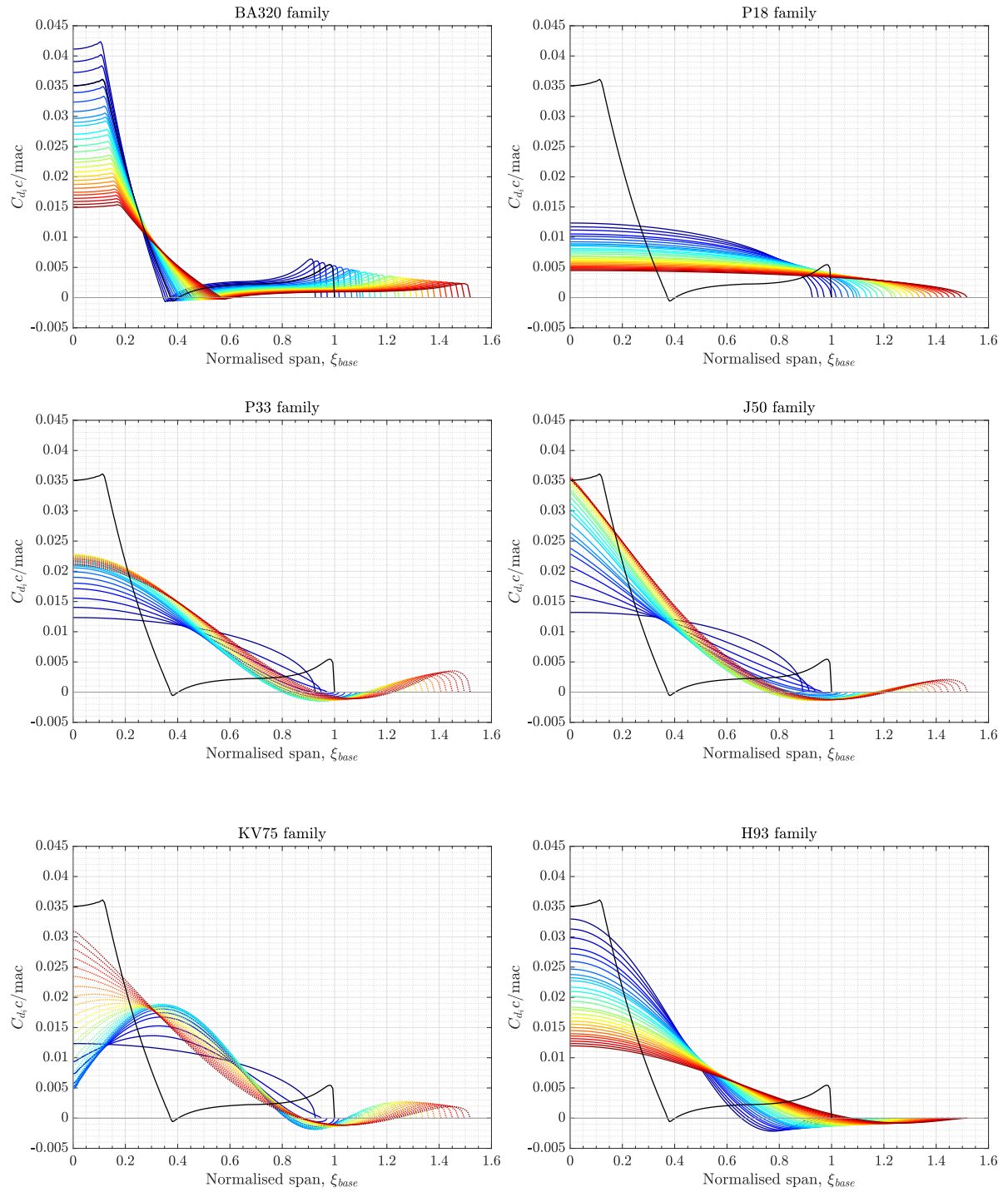


Figure B.3: Comparison of the computed local induced drag coefficient distributions against the baseline.

B.5 Shear Force Distributions

The shear force distribution due to the aerodynamic load is shown for each spanload family against the baseline configuration. The spanwise axes has been normalised against the baseline semispan.

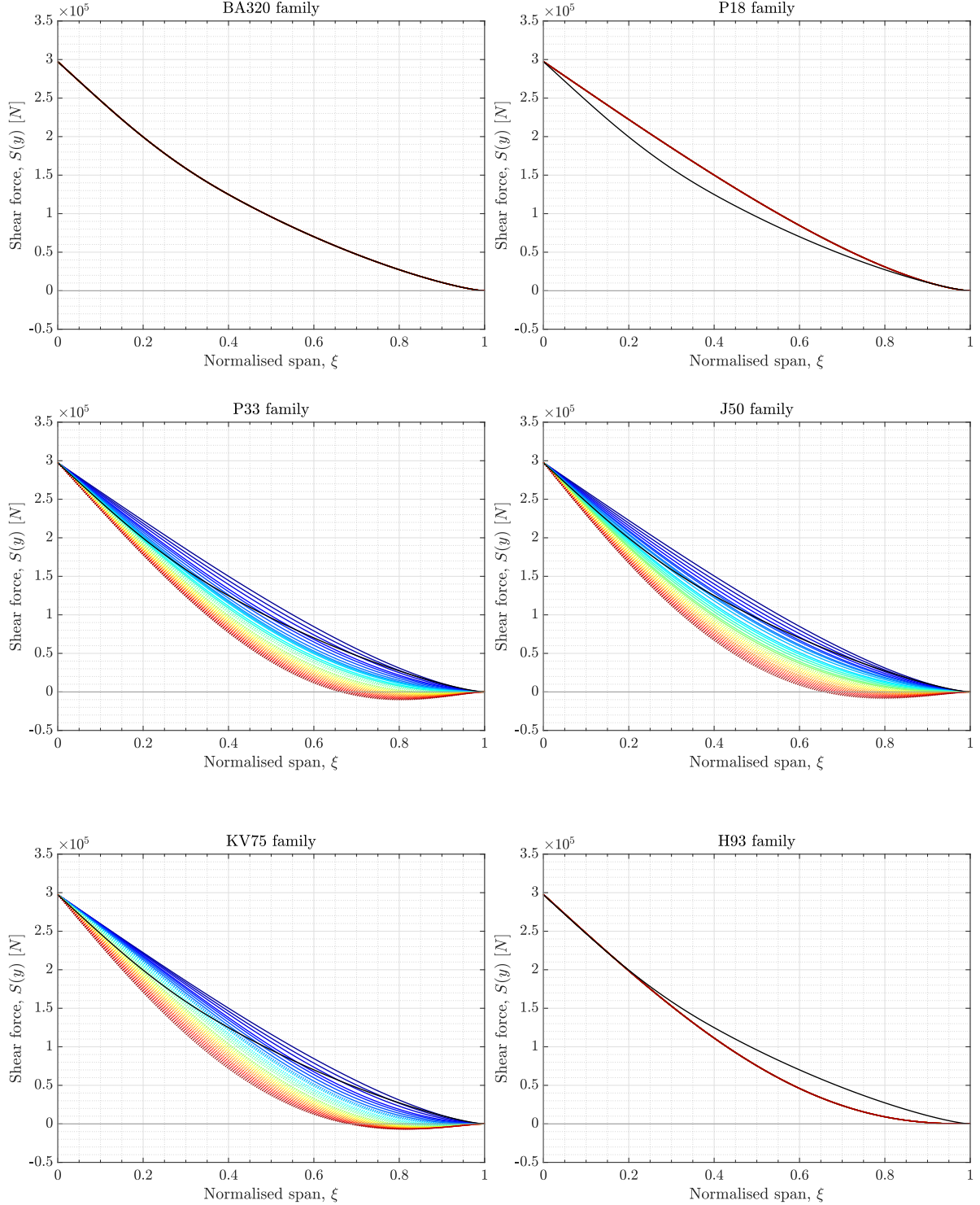


Figure B.4: Comparison of the computed shear force distributions against the baseline.

B.6 Bending Moment Distributions

The bending moment distribution due to the aerodynamic load is shown for each spanload family against the baseline configuration. The spanwise axes has been normalised against the baseline semispan.

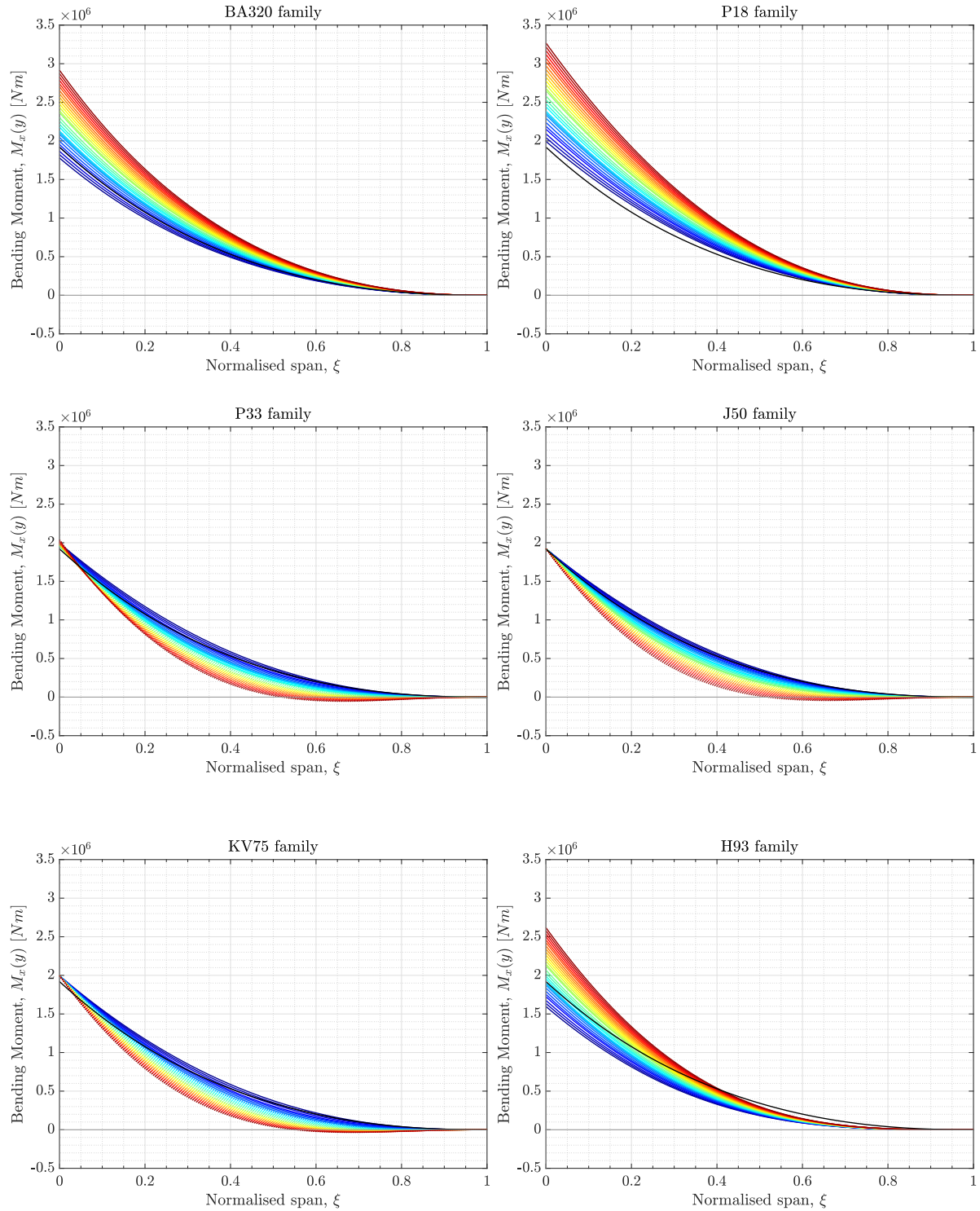


Figure B.5: Comparison of the bending moment distributions against the baseline.

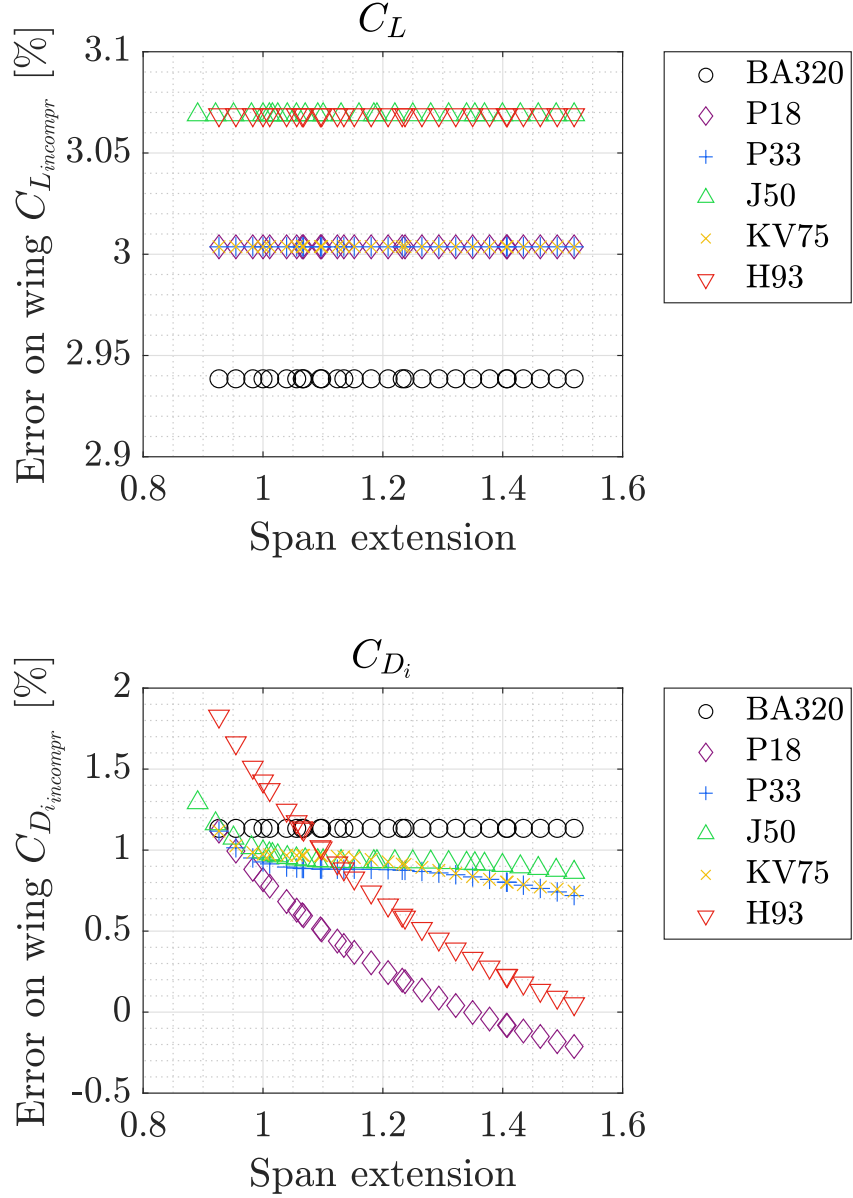


Figure B.6: Percentage error between analytical and numerical potential flow methods for the prediction of aerodynamic properties. Lift coefficient is shown on the top; induced drag coefficient is shown at the bottom.

Appendix C

Loads and Aero-structural Analysis

C.1 Manoeuvre Load Cases for Sizing

Table C.1: List of evaluated manoeuvres.

ID	Description	Mach	Altitude [ft]	Load factor	Other constraints	CS-25 reference
1	Steady level flight at cruise condition	0.78	37000	1	-	User-defined
2	Minimum load factor at V_C	0.78	37000	-1	Pitch rate	25.331(c)(2)
3	Maximum load factor at V_D	0.45	0	2.5	Pitch rate	25.331(c)(2)
4	Maximum load factor at V_D	0.89	33130	2.5	Pitch rate	25.331(c)(2)
5	Maximum load factor at V_C	0.36	0	2.5	Pitch rate	25.331(c)(2)
6	Maximum load factor at V_C	0.78	37000	2.5	Pitch rate	25.331(c)(2)
7	High lift at V_F and limit load factor	0.28	0	2	Pitch rate & Flap deflection	25.345(a)(1)
8	Vertical gust encounter at high lift at V_F	0.28	0	1	Gust velocity & Flap deflection	25.345(a)(2)
9	Sudden displacement of pitch control at V_A	0.31	0	1	Pitch acceleration & Elevator deflection	25.331(c)(1)
10	Sudden displacement of pitch control at V_C	0.36	0	1	Pitch acceleration & Elevator deflection	25.331(c)(1)
11	Sudden displacement of pitch control at V_D	0.45	0	1	Pitch acceleration & Elevator deflection	25.331(c)(1)
12	Sudden displacement of pitch control at V_A at maximum load factor	0.31	0	2.5	Pitch acceleration & Elevator deflection	25.331(c)(1)
13	Sudden displacement of pitch control at V_C at maximum load factor	0.36	0	2.5	Pitch acceleration & Elevator deflection	25.331(c)(1)
14	Sudden displacement of pitch control at V_D at maximum load factor	0.45	0	2.5	Pitch acceleration & Elevator deflection	25.331(c)(1)
15	Sudden displacement of yaw control at V_A	0.31	0	1	Rudder deflection	25.351(a)(1)
16	Yawing to overswing sideslip angle 22.5° at V_A from maximum rudder deflection	0.31	0	1	Sideslip angle & Rudder deflection	25.351(b)
17	Static sideslip at V_A	0.31	0	1	Sideslip angle	25.351(c)
18	Sudden displacement of yaw control at V_S	0.20	0	1	Rudder deflection	25.351(a)(1)
19	Yawing to overswing sideslip angle 22.5° at V_S from maximum rudder deflection	0.20	0	1	Rudder deflection	25.351(b)
20	Static sideslip at V_S	0.20	0	1	Rudder deflection	25.351(c)
21	Sudden displacement of roll control at V_A	0.31	0	1	Rudder deflection	25.349(a)(2)
22	Sudden displacement of roll control at V_A at 2/3 of maximum load factor	0.31	0	1.67	Pitch rate	25.349(a)(2)
23	Vertical gust encounter at V_C	0.36	0	1	Gust speed	25.341
24	Vertical gust encounter at V_C	0.37	15000	1	Gust speed	25.341
25	Vertical gust encounter at V_D	0.45	0	1	Gust speed	25.341
26	Vertical gust encounter at V_D	0.46	15000	1	Gust speed	25.341
27	Landing with a descent velocity of 10 fps at V_{FE}	0.15	0	0.8	Sink speed & Flap deflection	25.473(a)(2)
28	Landing with a descent velocity of 6 fps at V_{FE}	0.15	0	1	Sink speed & Flap deflection	25.473(a)(3)
29	Take-off with one-engine-inoperative	0.18	0	1	Flap deflection	25.121(c)

C.2 Aero-Structural Models

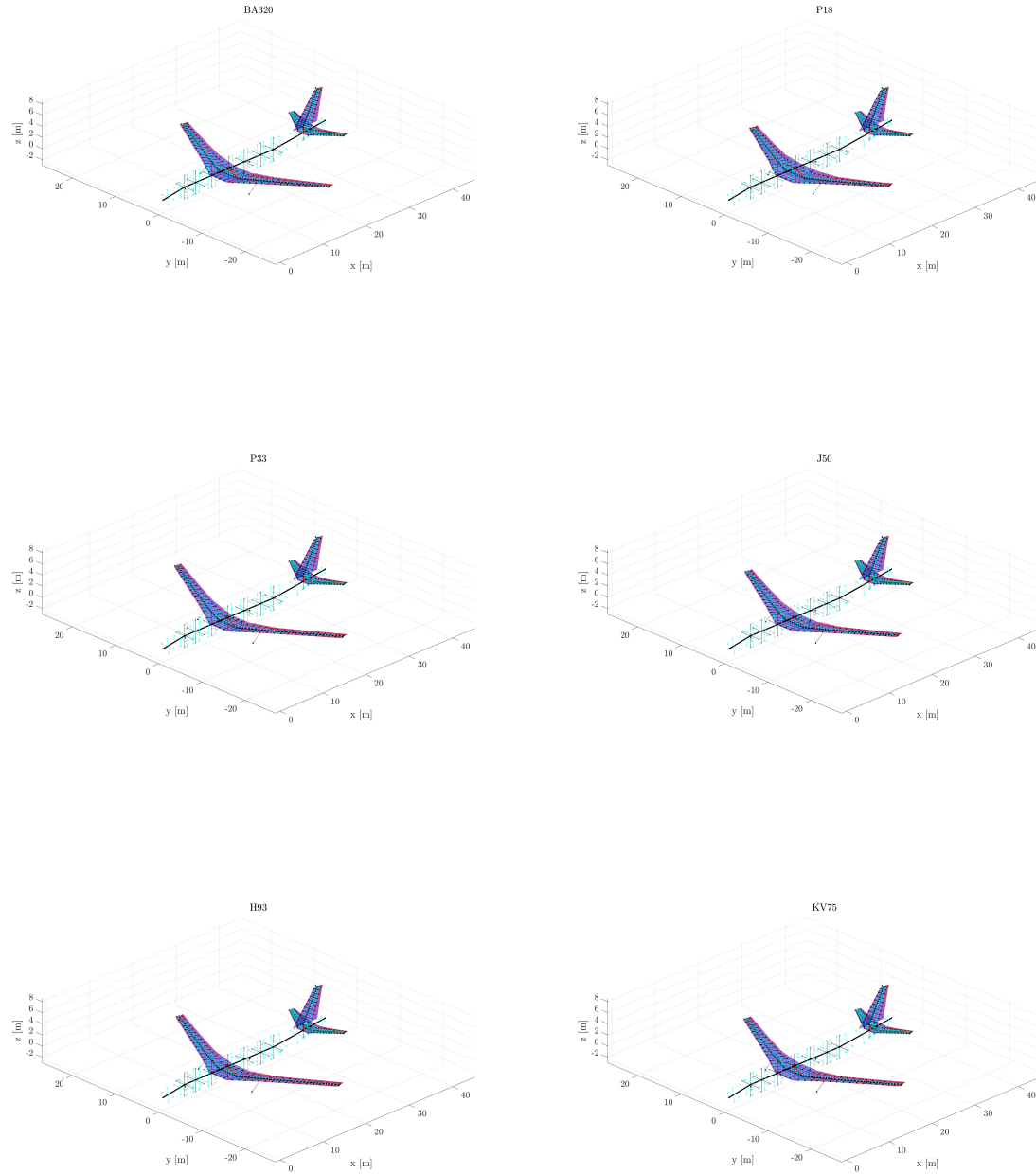


Figure C.1: Coupled structural and aerodynamic models.

C.3 Internal Load Distributions

C.3.1 Wing Shear Force Distribution

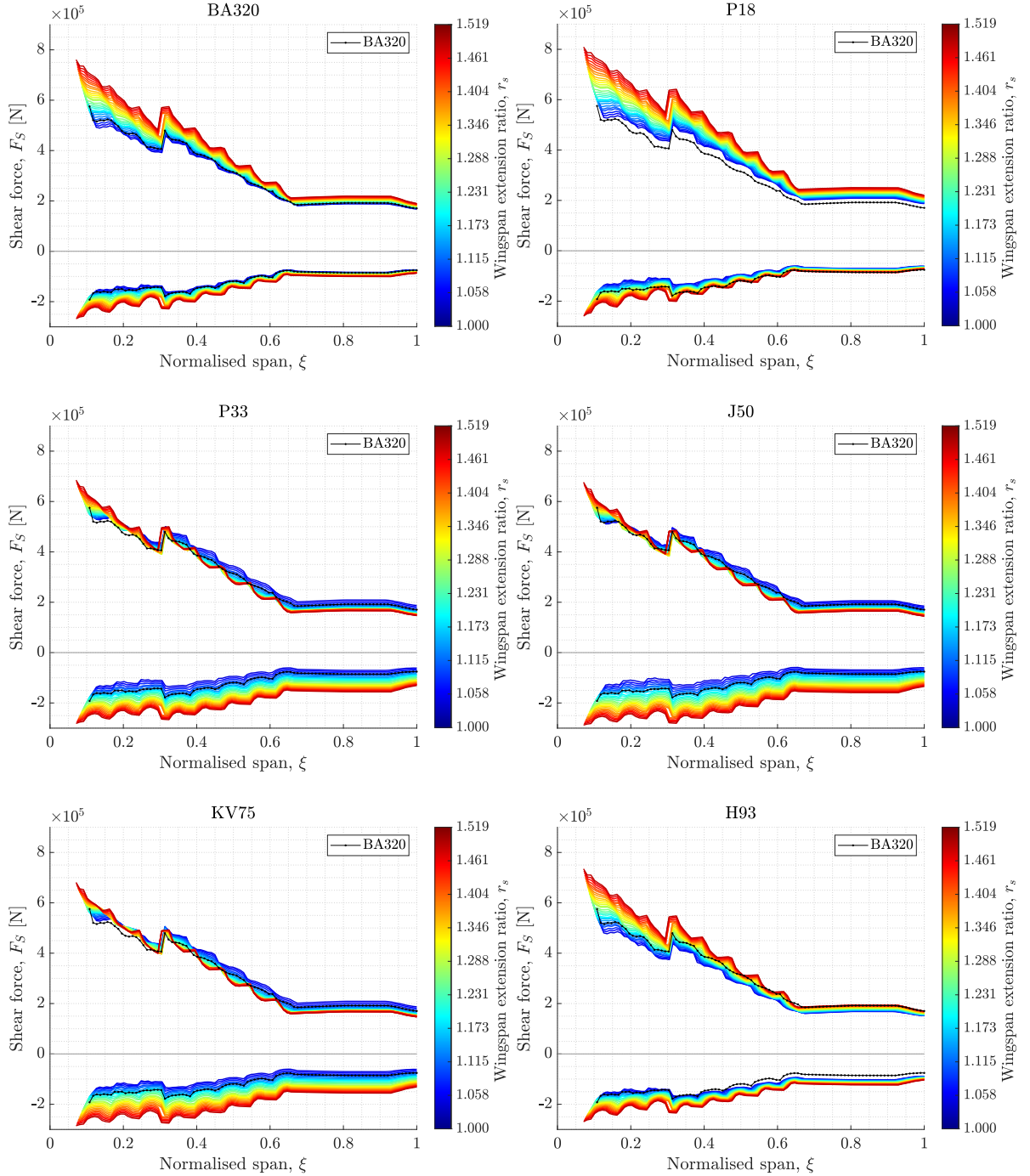


Figure C.2: Comparison of wing shear force distributions against the baseline configuration.

C.3.2 Wing Torsion Distribution

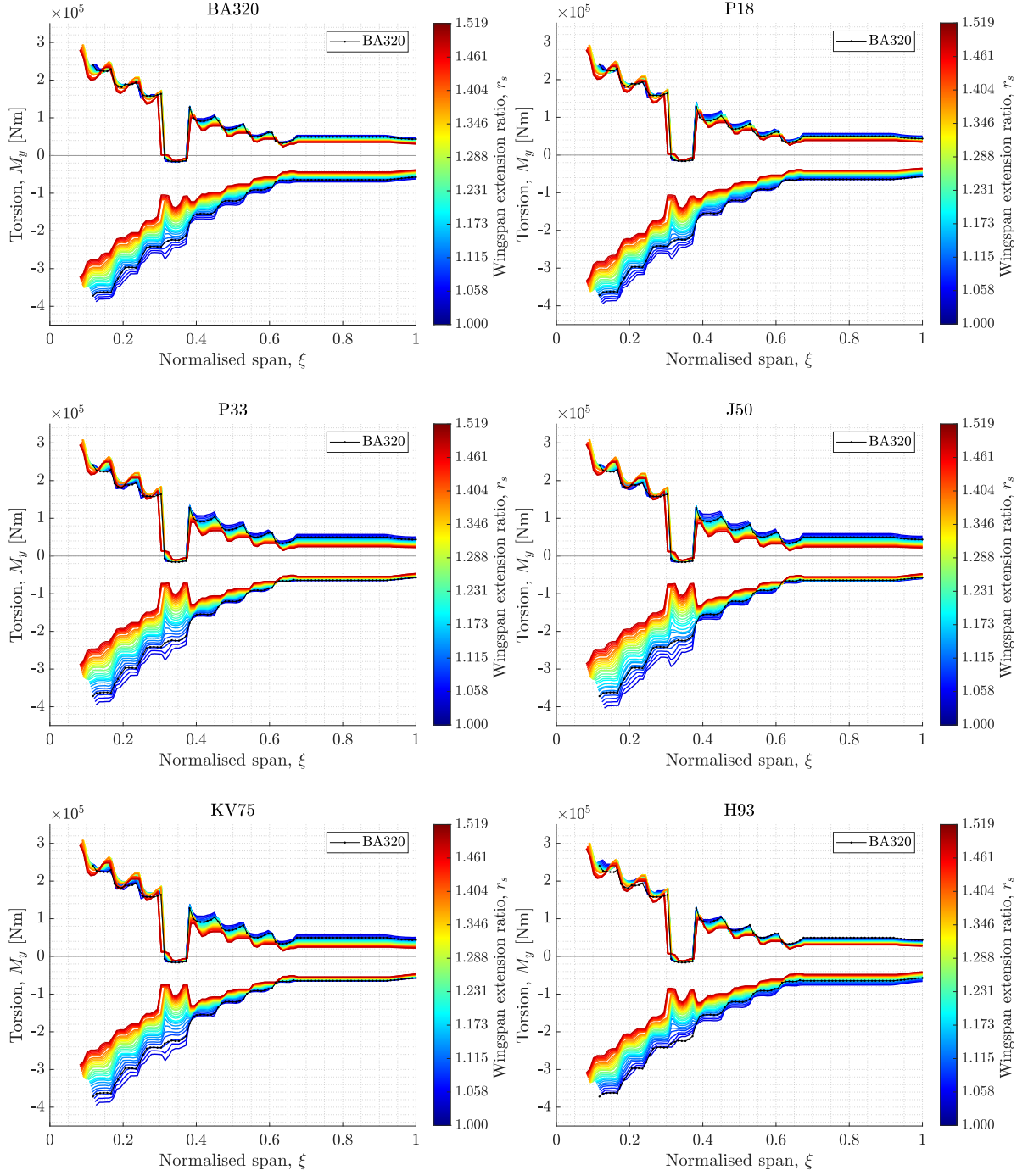


Figure C.3: Comparison of wing torsion distributions against the baseline configuration.

C.3.3 Critical Manoeuvres Driving Wing Bending Moment

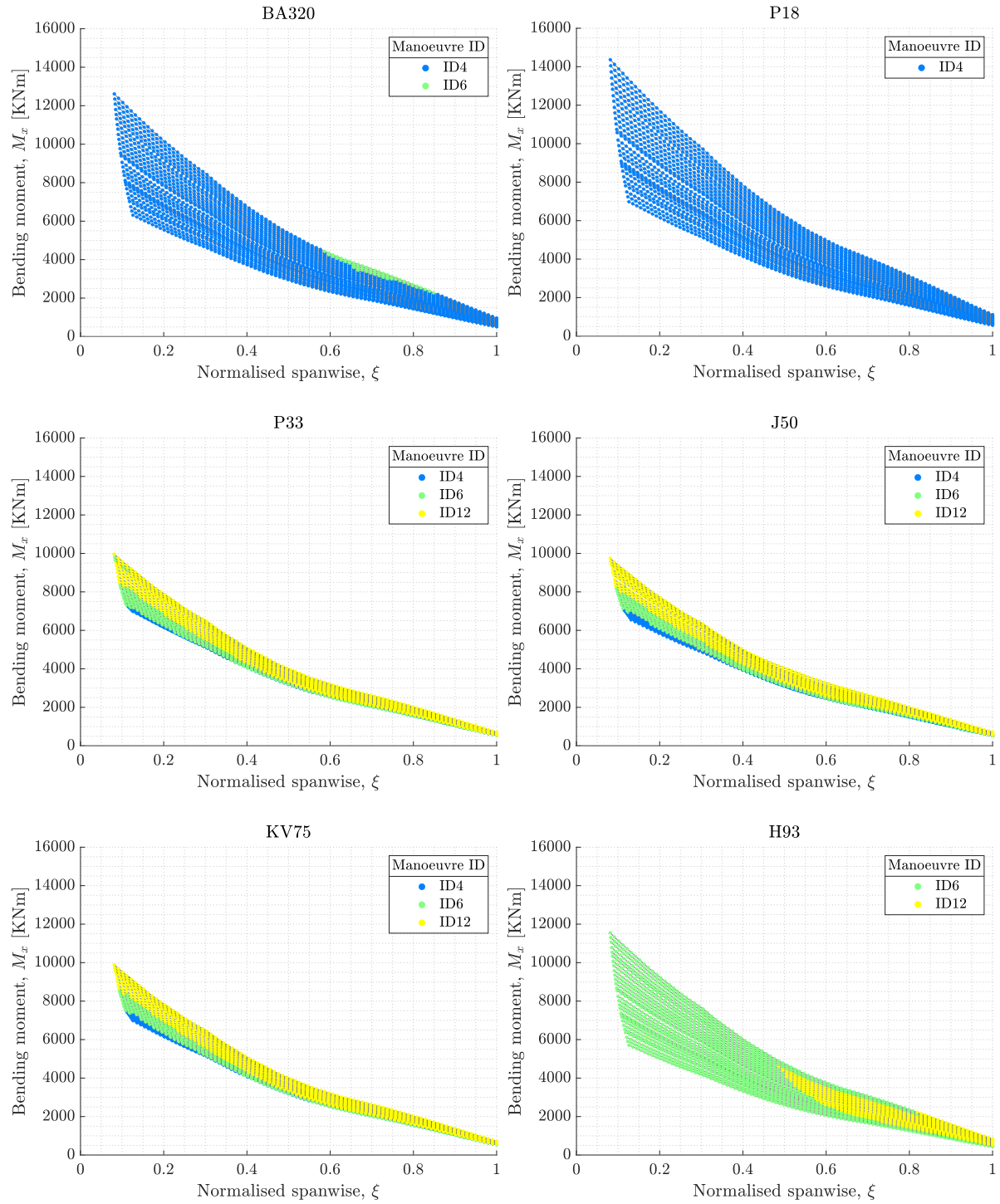


Figure C.4: Identification of critical manoeuvres driving wing bending moment.

C.3.4 Critical Manoeuvres Driving Wing Shear Force

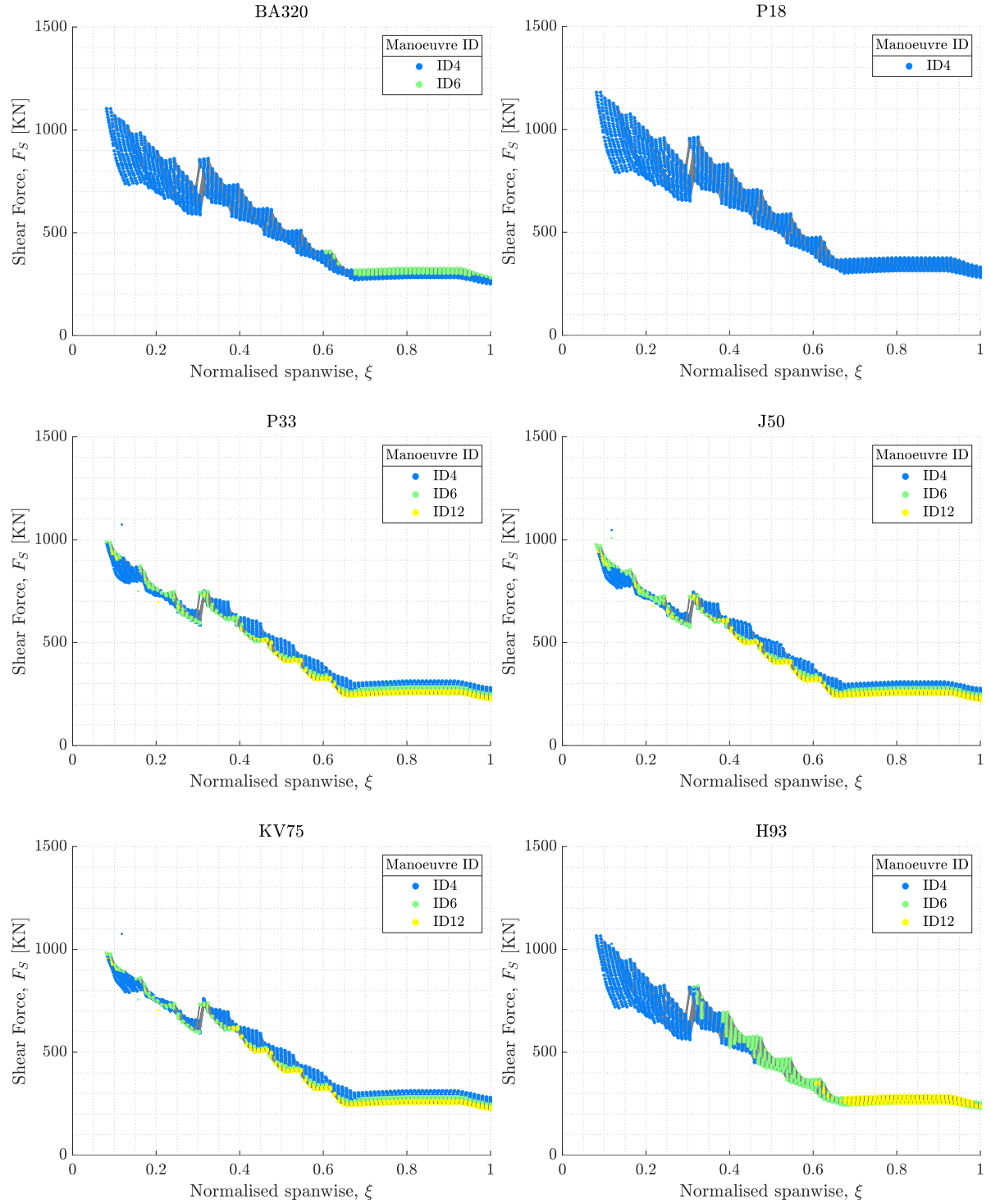


Figure C.5: Identification of critical manoeuvres driving wing shear force.

C.3.5 Critical Manoeuvres Driving Wing Torsion

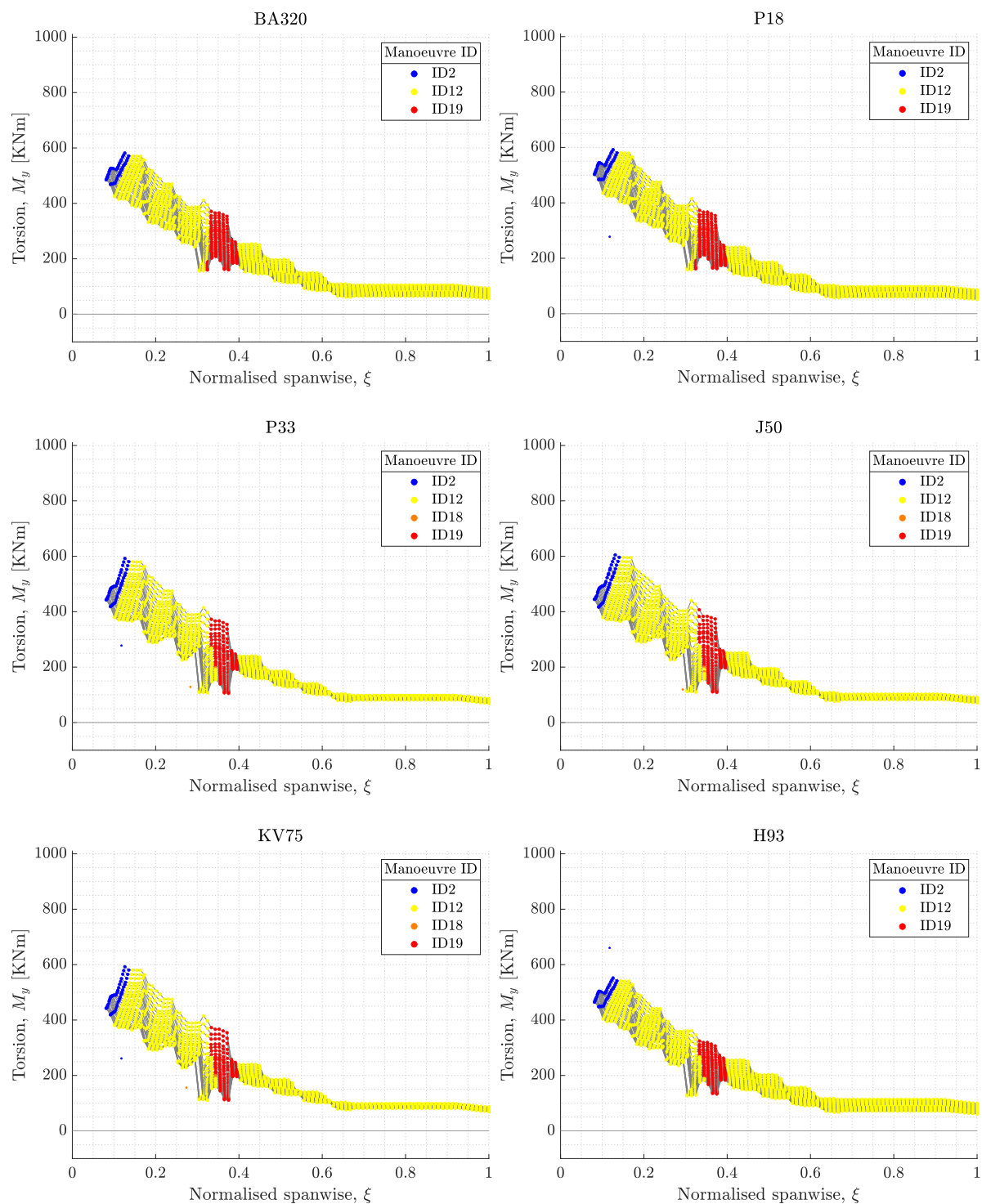


Figure C.6: Identification of critical manoeuvres driving wing torsion.

Appendix D

Performance Analysis

D.1 Induced Drag Polar Curves

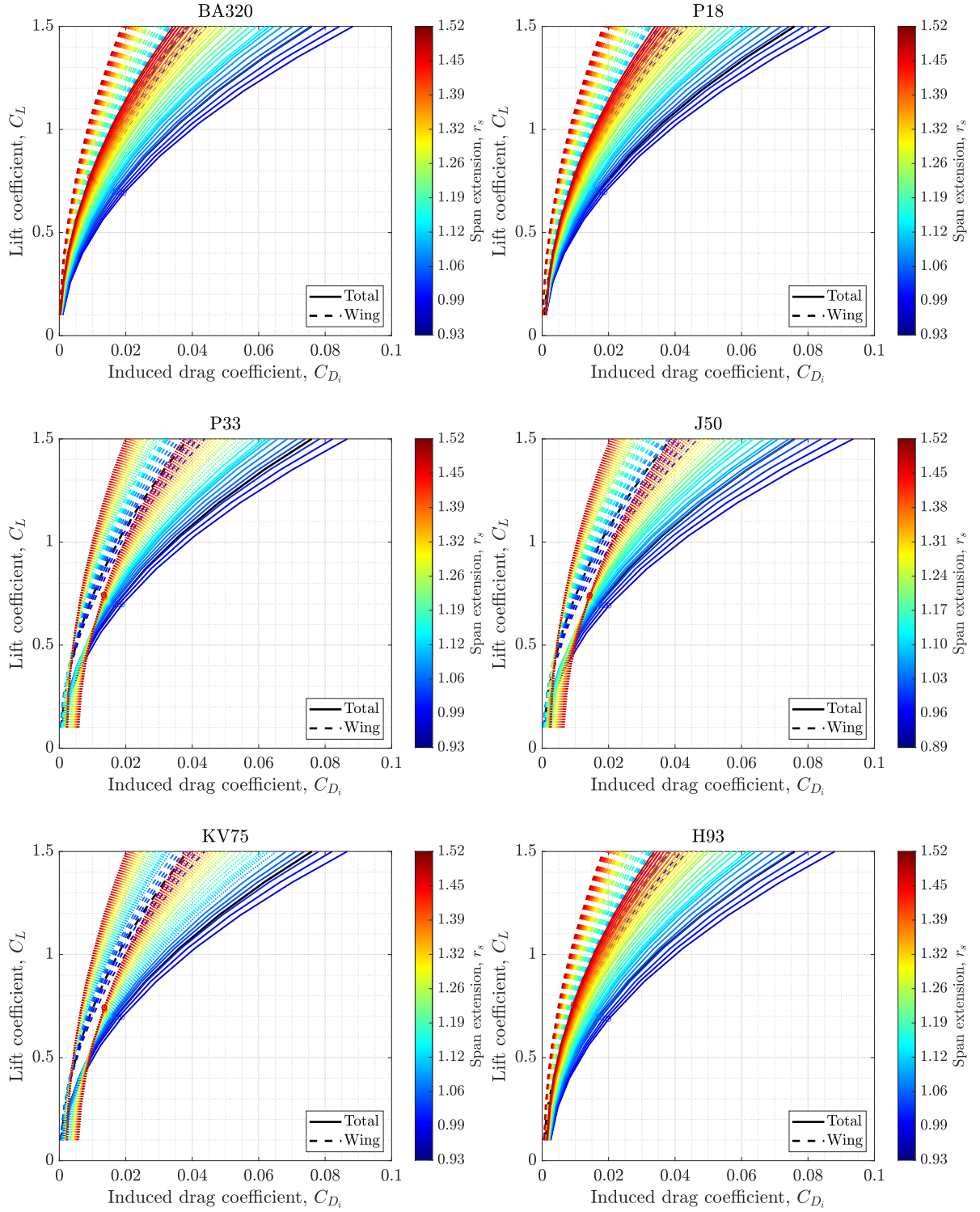


Figure D.1: Induced drag polar for steady level cruise flight.

D.2 Profile Drag Polar Curves

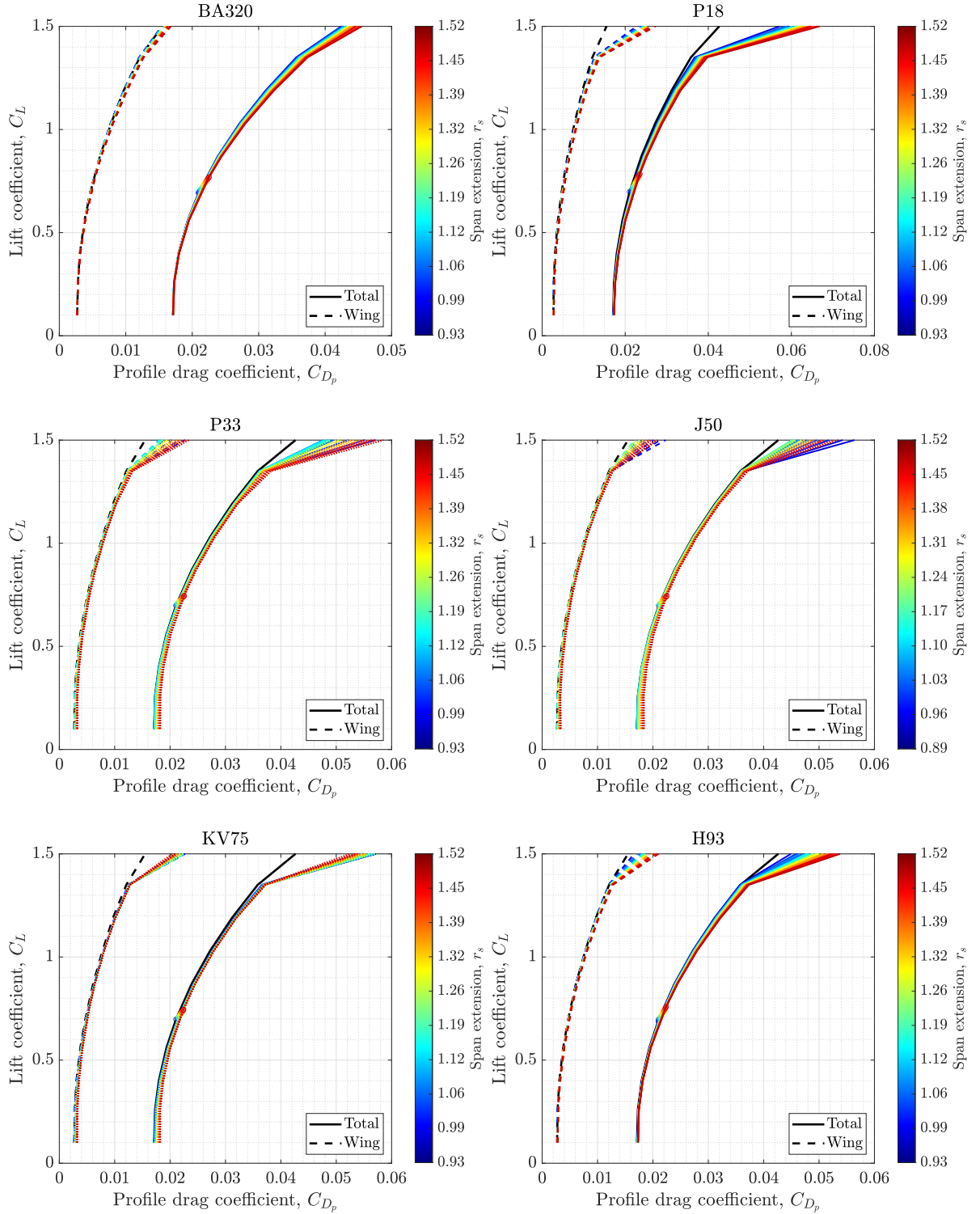


Figure D.2: Profile drag polar for steady level cruise flight.

D.3 Lift-to-Drag Ratio Curves

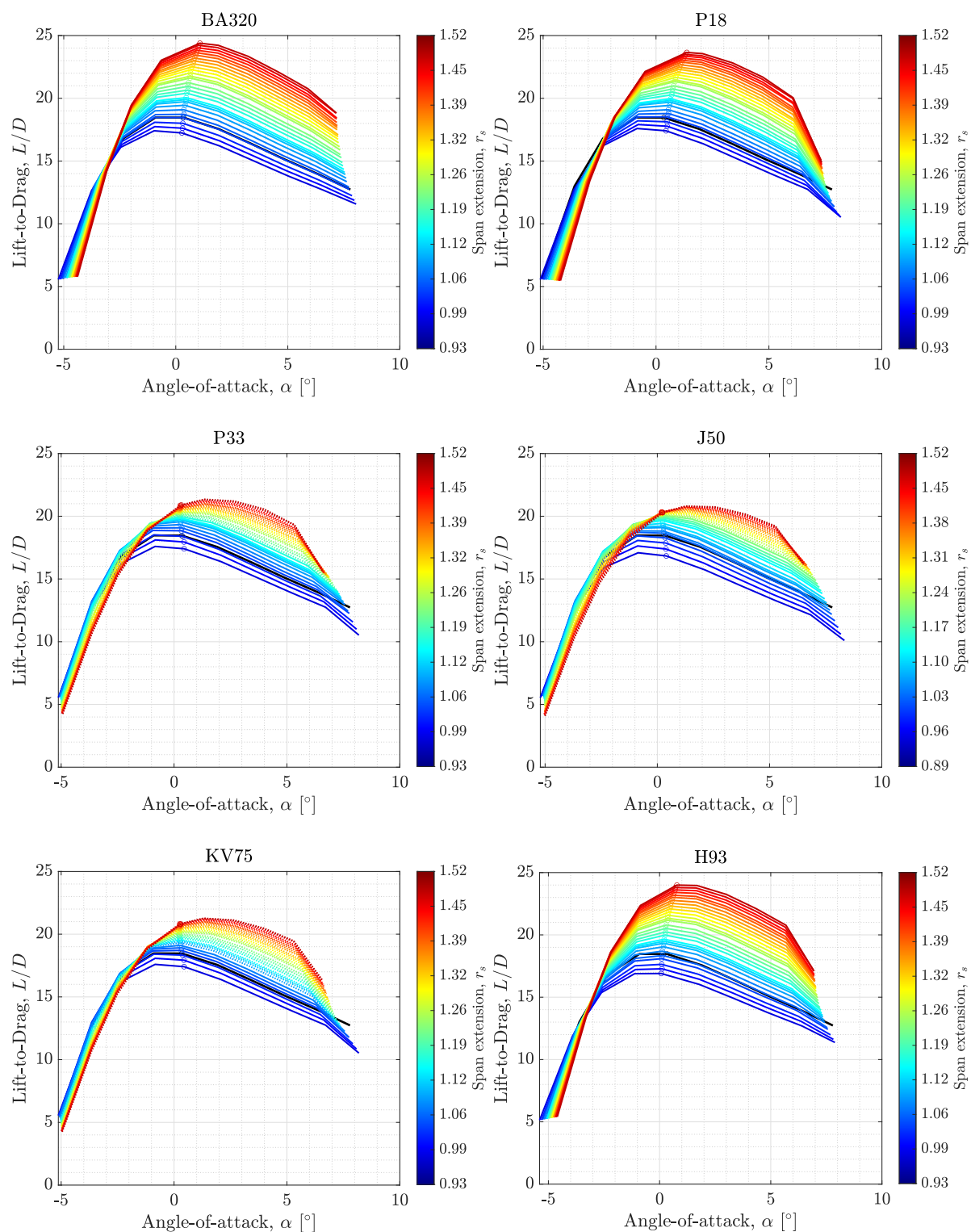


Figure D.3: Lift-to-drag ratio at cruise flight condition.

References

- [1] Airbus, “Airbus and SAS Scandinavian Airlines sign hybrid and electric aircraft research agreement,” 2019.
- [2] Boeing, “How sweet the future of aviation,” 2017.
- [3] Airbus, “Airbus increases its UK innovation footprint to develop new hydrogen technologies,” 2022.
- [4] Airbus, “A revolutionary approach to aircraft wing design,” 2021.
- [5] J. Katz and A. Plotkin, *Low-Speed Aerodynamics*. Cambridge: Cambridge University Press, 2nd ed., 2001.
- [6] L. Prandtl and A. Betz, “NACA-TR-116: Applications of Modern Hydrodynamics to Aeronautics,” tech. rep., National Advisory Committee for Aeronautics, Goettingen, Germany, 1923.
- [7] L. Prandtl, “Über tragflügel kleinsten induzierten Widerstandes,” *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, vol. 24, pp. 556–561, 1933.
- [8] D. McLean, “Wingtip Devices: What They Do and How They Do It,” in *Boeing Performance and Flight Operations Engineering Conference*, (Seattle, WA, United States), p. 20, Boeing - Aerodynamics, 2005.
- [9] A. H. Bowers, O. J. Murillo, B. Eslinger, and C. Gelzer, “On Wings of the Minimum Induced Drag: Spanload Implications for Aircraft and Birds,” tech. rep., NASA Armstrong Flight Research Center, Edwards, 2016.
- [10] W. F. Phillips, D. F. Hunsaker, and J. J. Joo, “Minimizing Induced Drag with Lift Distribution and Wingspan,” *Journal of Aircraft*, vol. 56, pp. 431–441, mar 2019.
- [11] K. L. E. Nickel and M. Wohlfahrt, *Tailless aircraft in theory and practice*. Michigan: Edward Arnold, 1994.
- [12] S. Briceno, B. Laughlin, and D. N. Mavris, “A Virtual Simulation Platform for the Design, Testing, and Verification of Unmanned Aerial Vehicle Designs,” in *Simulia Community Conference*, p. 16, 2014.
- [13] D. Raymer, *Aircraft Design: A Conceptual Approach, Sixth Edition*. Washington, DC: American Institute of Aeronautics and Astronautics, Inc., sep 2018.

REFERENCES

- [14] AIRBUS S.A.S., “A320 Aircraft Characteristics, Airport and Maintenance Planning,” tech. rep., Airbus Technical Data Support and Services, 2005.
- [15] G. L. Ghiringhelli, P. Masarati, and P. Mantegazza, “Multibody Implementation of Finite Volume C Beams,” *AIAA Journal*, vol. 38, pp. 131–138, jan 2000.
- [16] M. D. Ardema, M. C. Chambers, A. P. Patron, A. S. Hahn, H. Miura, and M. D. Moore, “Analytical Fuselage and Wing Weight Estimation of Transport Aircraft,” tech. rep., National Aeronautics and Space Administration, 1996.
- [17] A. Young, “The Aerodynamic Characteristics of Flaps,” tech. rep., Aeronautical Research Council, London, 1947.
- [18] MatWeb, “Aluminum 7075-T6 Datasheet,” 2022.
- [19] K. Oswatitsch and K. Wieghardt, “Ludwig Prandtl and His Kaiser-Wilhelm-Institut,” *Annual Review of Fluid Mechanics*, vol. 19, pp. 1–26, jan 1987.
- [20] J. D. Anderson, *A history of aerodynamics and its impact on flying machines*. Cambridge University Press, 1997.
- [21] J. D. Anderson, “Brief History of the Early Development of Theoretical and Experimental Fluid Dynamics,” in *Encyclopedia of Aerospace Engineering*, John Wiley & Sons, Ltd, dec 2010.
- [22] L. Prandtl, “Tragflügel Theorie,” 1918.
- [23] A. Betz, “Untersuchungen von Tragflächen mit verwundenen und nach rückwärts gerichteten,” in *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, ch. 5, pp. 237–239, München: R. Oldenbourg, 1914.
- [24] M. M. Munk, “NACA-TR-121: The Minimum Induced Drag of Aerofoils,” tech. rep., National Advisory Committee for Aeronautics, Germany, dec 1923.
- [25] F. W. Lanchester, *Aerodynamics, Constituting the First Volume of a Complete Work on Aerial Flight*. London: A. Constable & Co. Ltd., 1st ed., 1907.
- [26] T. von Karman, *Aerodynamics: selected topics in the light of their historical development*. Michigan: Cornell University Press, 1954.
- [27] W. M. Kutta, “Über eine mit den Grundlagen des Flugproblems in Beziehung stehende zweidimensionale Strömung,” tech. rep., Königlich Bayerischen Akademie der Wissenschaften, Munich, 1910.
- [28] N. Y. Zhukovsky, “On the adjunct vortices (in Russian),” tech. rep., Transactions of the Physical Section, Moscow, 1907.
- [29] J. Maxwell, “On physical lines of force,” *Philosophical Magazine*, vol. Fourth Ser, p. 62, feb 1861.

-
- [30] H. Helmholtz, "Über Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen," *Journal für die reine und angewandte Mathematik*, vol. 55, pp. 25–55, 1858.
- [31] A. Busemann, "Ludwig Prandtl, 1875-1953," *Biographical Memoirs of Fellows of the Royal Society*, vol. 5, pp. 193–205, 1960.
- [32] J. Vogel-Prandtl, *Ludwig Prandtl : A Personal Biography Drawn from Memories and Correspondence*. Universitätsverlag Göttingen, 2014.
- [33] B. Thwaites, "Wing Theory. A. Robinson and J. A. Laurmann. Cambridge Aeronautical Series. Cambridge University Press, 1956. 569 pp. Diagrams. 75s.," *The Journal of the Royal Aeronautical Society*, vol. 61, no. 554, pp. 136–136, 1957.
- [34] F. Ursell and B. Thwaites, "Incompressible Aerodynamics," *Journal of Fluid Mechanics*, vol. 11, no. 1, pp. 158–160, 1961.
- [35] H. T. Schlichting, *Boundary Layer Theory*. New York: McGraw-Hill, 7th ed., 1980.
- [36] R. T. Jones, "NACA-TN-817: Correction of the lifting-line theory for the effect of the chord," tech. rep., National Advisory Committee for Aeronautics, Washington, DC, 1941.
- [37] J. Weissinger, "NACA-TM-1120: The Lift Distribution of Swept-Back Wings," tech. rep., National Advisory Committee for Aeronautics, Langley Aeronautical Lab, 1947.
- [38] J. DeYoung and C. W. Harper, "NACA-TR-921: Theoretical symmetric span loading at subsonic speeds for wings having arbitrary plan form," tech. rep., National Advisory Committee for Aeronautics, Ames Aeronautical Lab, 1948.
- [39] J.-L. Guermond, "A generalized lifting-line theory for curved and swept wings," *Journal of Fluid Mechanics*, vol. 211, pp. 497–513, feb 1990.
- [40] W. F. Phillips and D. O. Snyder, "Modern Adaptation of Prandtl's Classic Lifting-Line Theory," *Journal of Aircraft*, vol. 37, pp. 662–670, jul 2000.
- [41] P. J. Phlips, R. A. East, and N. H. Pratt, "An unsteady lifting line theory of flapping wings with application to the forward flight of birds," *Journal of Fluid Mechanics*, vol. 112, pp. 97–125, nov 1981.
- [42] A. R. Ahmadi and S. E. Widnall, "Unsteady lifting-line theory as a singular perturbation problem," *Journal of Fluid Mechanics*, vol. 153, pp. 59–81, apr 1985.
- [43] P. D. Sclavounos, "An unsteady lifting-line theory," *Journal of Engineering Mathematics*, vol. 21, no. 3, pp. 201–226, 1987.
- [44] T. von Karman and H.-S. Tsien, "Lifting-Line Theory for a Wing in Non-uniform Flow," *Quarterly of Applied Mathematics*, vol. 3, no. 1, pp. 1–11, 1945.

REFERENCES

- [45] J. C. Sivells and R. H. Neely, "Method for calculating wing characteristics by lifting-line theory using nonlinear section lift data," tech. rep., National Advisory Committee for Aeronautics, Washington, DC, 1947.
- [46] C. E. Lan and M. H. Fasse, "Applications of an improved nonlinear lifting-line theory," *Journal of Aircraft*, vol. 14, pp. 404–407, apr 1977.
- [47] H. Karali, G. Inalhan, M. Umut Demirezen, and M. Adil Yukselen, "A new non-linear lifting line method for aerodynamic analysis and deep learning modeling of small unmanned aerial vehicles," *International Journal of Micro Air Vehicles*, vol. 13, 2021.
- [48] L. P. Cook and J. D. Cole, "Lifting Line Theory for Transonic Flow," *SIAM Journal on Applied Mathematics*, vol. 35, no. 2, pp. 209–228, 1978.
- [49] H. Glauert, "The effect of compressibility on the lift of an aerofoil," in *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 118, pp. 113–119, 1928.
- [50] R. Jacobs, H. Ran, M. Kirby, and D. Mavris, "Extension of a Modern Lifting-Line Method to Transonic Speeds and Application to Multiple-Lifting-Surface Configurations," in *30th AIAA Applied Aerodynamics Conference*, (Reston, Virginia), p. 13, American Institute of Aeronautics and Astronautics, jun 2012.
- [51] V. M. Falkner, "The scope and accuracy of vortex lattice theory," tech. rep., Aeronautical Research Council, 1949.
- [52] R. T. Jones, "NACA-TN-2249: The spanwise distribution of lift for minimum induced drag of wings having a given lift and a given bending moment," tech. rep., NASA Ames Research Center, Moffett Field, CA, United States, dec 1950.
- [53] D. F. Hunsaker and W. Phillips, "Ludwig Prandtl's 1933 Paper Concerning Wings for Minimum Induced Drag, Translation and Commentary," in *AIAA Scitech 2020 Forum*, (Orlando, FL), p. 12, American Institute of Aeronautics and Astronautics, jan 2020.
- [54] R. T. Jones, "Minimizing induced drag," *Soaring and Motorgliding*, vol. 43, no. 10, pp. 26–29, 1979.
- [55] A. Klein and S. P. Viswanathan, "Minimum induced drag of wings with given lift and root-bending moment," *Zeitschrift für angewandte Mathematik und Physik ZAMP*, vol. 24, pp. 886–892, nov 1973.
- [56] A. Klein and S. P. Viswanathan, "Approximate solution of minimum induced drag of wings with given structural weight," *Journal of Aircraft*, vol. 12, pp. 124–126, feb 1975.
- [57] A. Klein and S. P. Viswanathan, "Errata: Approximate Solution for Minimum Induced Drag of Wings with Given Structural Weight," *Journal of Aircraft*, vol. 12, pp. 756–756, sep 1975.

-
- [58] J. DeYoung, "Minimization Theory of Induced Drag Subject to Constraint Condition," tech. rep., National Aeronautics and Space Administration, Langley Research Center, 1979.
- [59] D. J. Pate and B. J. German, "Lift Distributions for Minimum Induced Drag with Generalized Bending Moment Constraints," *Journal of Aircraft*, vol. 50, pp. 936–946, may 2013.
- [60] D. F. Hunsaker, W. F. Phillips, and J. J. Joo, "Aerodynamic Shape Optimization of Morphing Wings at Multiple Flight Conditions," in *55th AIAA Aerospace Sciences Meeting*, (Reston, Virginia), p. 14, American Institute of Aeronautics and Astronautics, 2017.
- [61] T. McGeer, "Wing design for minimum drag with practical constraints," *Journal of Aircraft*, vol. 21, pp. 879–886, nov 1984.
- [62] G. Lobert, "Spanwise lift distribution of forward-and aft-swept wings in comparison to the optimum distribution form," *Journal of Aircraft*, vol. 18, pp. 496–498, jun 1981.
- [63] S. A. Ning and I. Kroo, "Tip Extensions, Winglets, and C-wings: Conceptual Design and Optimization," in *26th AIAA Applied Aerodynamics Conference*, (Reston, Virginia), p. 28, American Institute of Aeronautics and Astronautics, aug 2008.
- [64] S. A. Ning and I. Kroo, "Multidisciplinary Considerations in the Design of Wings and Wing Tip Devices," *Journal of Aircraft*, vol. 47, pp. 534–543, mar 2010.
- [65] S. Wakayama and I. Kroo, "Subsonic wing planform design using multidisciplinary optimization," *Journal of Aircraft*, vol. 32, pp. 746–753, jul 1995.
- [66] I. Kroo, "Trim Drag, Tail Sizing, and Soaring Performance," *Technical Soaring*, vol. 8, no. 4, pp. 127–137, 1984.
- [67] D. M. Bushnell, "Shock Wave Drag Reduction," *Annual Review of Fluid Mechanics*, vol. 36, no. 1, pp. 81–96, 2004.
- [68] J. Wu, L. Liu, and T. Liu, "Fundamental theories of aerodynamic force in viscous and compressible complex flows," *Progress in Aerospace Sciences*, vol. 99, pp. 27–63, 2018.
- [69] S. A. Abdel-Motaleb, J. D. Taylor, D. F. Hunsaker, and C. Coopmans, "Comparison of Induced and Parasitic Drag on Wings with Minimum Induced Drag," in *AIAA Scitech 2019 Forum*, (Reston, Virginia), p. 20, American Institute of Aeronautics and Astronautics, jan 2019.
- [70] A. Craig and D. McLean, "Spanload optimization for strength designed lifting surfaces," in *6th Applied Aerodynamics Conference*, (Reston, Virginia), p. 9, American Institute of Aeronautics and Astronautics, jun 1988.

REFERENCES

- [71] M. Castellani, J. E. Cooper, and Y. Lemmens, "Flight Loads Prediction of High Aspect Ratio Wing Aircraft Using Multibody Dynamics," *International Journal of Aerospace Engineering*, vol. 2016, pp. 1–13, 2016.
- [72] T. Sugimoto, "Wing design for hanggliders having minimum induced drag," *Journal of Aircraft*, vol. 29, pp. 730–731, jul 1992.
- [73] R. T. Haftka, "Optimization of flexible wing structures subject to strength and induced drag constraints," *AIAA Journal*, vol. 15, no. 8, pp. 1101–1106, 1977.
- [74] S. Iglesias and W. Mason, "Optimum spanloads incorporating wing structural weight," in *1st AIAA, Aircraft, Technology Integration, and Operations Forum*, (Reston, Virginia), p. 11, American Institute of Aeronautics and Astronautics, oct 2001.
- [75] T. T. Takahashi, "Optimum Transverse Span Loading for Subsonic Transport Category Aircraft," *Journal of Aircraft*, vol. 49, pp. 262–274, jan 2012.
- [76] IHS ESDU, "ESDU 76003: Geometrical properties of cranked and straight-tapered wing planforms," tech. rep., ESDU. The Royal Aeronautical Society, 2012.
- [77] IHS ESDU, "ESDU 10023: Computer program for calculation of equivalent straight-tapered wing planform," tech. rep., ESDU. The Royal Aeronautical Society, 2016.
- [78] S. Y. Yusuf, D. Hayes, A. Pontillo, M. A. Carrizales, G. X. Dussart, and M. M. Lone, "Aeroelastic Scaling for Flexible High Aspect Ratio Wings," in *AIAA Scitech 2019 Forum*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, 2019.
- [79] W. F. Phillips, "Lifting-Line Analysis for Twisted Wings and Washout-Optimized Wings," *Journal of Aircraft*, vol. 41, pp. 128–136, jan 2004.
- [80] D. Eller and S. Heinze, "Approach to Induced Drag Reduction with Experimental Evaluation," *Journal of Aircraft*, vol. 42, pp. 1478–1485, nov 2005.
- [81] J. L. Lundry, "Minimum swept-wing induced drag with constraints on lift and pitching moment," *Journal of Aircraft*, vol. 4, pp. 73–74, jan 1967.
- [82] K. Rokhsaz, "Effect of viscous drag on optimum spanwise lift distribution," *Journal of Aircraft*, vol. 30, pp. 152–154, jan 1993.
- [83] K. L. E. Nickel, "Minimal drag of wings with prescribed lift, roll moment and yaw moment," tech. rep., Madison Mathematics Research Center, University of Wisconsin, Madison, 1983.
- [84] W. F. Phillips, N. R. Alley, and W. D. Goodrich, "Lifting-Line Analysis of Roll Control and Variable Twist," *Journal of Aircraft*, vol. 41, pp. 1169–1176, sep 2004.

-
- [85] D. F. Hunsaker, Z. S. Montgomery, and J. J. Joo, "Lifting-Line Analysis of Wing Twist to Minimize Induced Drag During Pure Rolling Motion," in *AIAA Scitech 2019 Forum*, (San Diego, California), p. 24, American Institute of Aeronautics and Astronautics, jan 2019.
- [86] S. F. Hoerner, "Aerodynamic Shape of the Wing Tips," tech. rep., U.S. Air Force, Air Materiel Command, Wright-Patterson Air Force Base, Dayton, Ohio, 1949.
- [87] C. D. J. Cone, "The theory of induced lift and minimum induced drag of nonplanar lifting systems," tech. rep., NASA Langley Research Center, Hampton, VA United States, 1962.
- [88] P. E. Hemke, "Drag of Wings with End Plates," tech. rep., National Advisory Committee for Aeronautics. Langley Aeronautical Lab, Langley Field, VA, United States, 1928.
- [89] R. T. Jones, "Effect of Winglets on the Induced Drag of Ideal Wing Shapes," tech. rep., National Aeronautics and Space Administration, Ames Research Center, Moffett Field, CA, 1980.
- [90] R. T. Jones, "Improving the efficiency of smaller transport aircraft," tech. rep., NASA Ames Research Center, Moffett Field, CA, United States, 1984.
- [91] R. T. Whitcomb, "A design approach and selected wind tunnel results at high subsonic speeds for wing-tip mounted winglets," tech. rep., NASA Langley Research Center, Hampton, VA, United States, 1976.
- [92] J. R. Hansen, *The bird is on the wing: aerodynamics and the progress of the American airplane*. University of Michigan: Texas A&M University Press, 2004.
- [93] National Aeronautics and Space Administration, "Winglets Save Billions of Dollars in Fuel Costs," *Spinoff 2010*, p. 2, 2010.
- [94] W. Mangler, "NACA-TM-856: The lift distribution of wings with end plates," tech. rep., National Advisory Committee for Aeronautics; Washington, DC, United States, Germany, 1938.
- [95] J. Weber, "Theoretical Load Distribution on a Wing with Vertical Plates," tech. rep., H.M. Stationery Office. Aeronautical Research Council, 1954.
- [96] J. L. Lundry, "A Numerical Solution for the Minimum Induced Drag, and the Corresponding Loading, of Nonplanar Wings," tech. rep., McDonnell-Douglas Corp., Long Beach, CA, United States, 1968.
- [97] H. H. Heyson, G. D. Riebe, and C. L. Fulton, "Theoretical parametric study of the relative advantages of winglets and wing-tip extensions," tech. rep., NASA Langley Research Center, Hampton, VA, United States, 1977.
- [98] T. Chen and J. Katz, "Induced Drag of High-Aspect Ratio Wings," in *42nd AIAA Aerospace Sciences Meeting and Exhibit*, (Reston, Virigina), p. 7, American Institute of Aeronautics and Astronautics, jan 2004.

REFERENCES

- [99] S. Gudmundsson, *General Aviation Aircraft Design: Applied Methods and Procedures*. Butterworth-Heinemann, 1 ed., 2013.
- [100] K. Asai, "Theoretical considerations in the aerodynamic effectiveness of winglets," *Journal of Aircraft*, vol. 22, pp. 635–637, jul 1985.
- [101] H. Zimmer, *The Aerodynamic Optimization of Wings at Subsonic Speeds and the Influence of Wingtip Design*. Translated into english by the national aeronautics and space administration, University of Stuttgart, 1987.
- [102] J. G. Verstraeten and R. Slingerland, "Drag Characteristics for Optimally Span-Loaded Planar, Wingletted, and C Wings," *Journal of Aircraft*, vol. 46, pp. 962–971, may 2009.
- [103] M. Andrejašič and G. Veble, "Shape Optimization of Nonplanar Lifting Surfaces and Planar–Nonplanar Break Points," *Journal of Aircraft*, vol. 50, pp. 798–806, may 2013.
- [104] R. Faye, R. Laprete, and M. Winter, "Blended winglets for improved airplane performance," *Aero Boeing*, vol. 17, no. 1, pp. 16–31, 2002.
- [105] H. Glauert, *The Elements of Aerofoil and Airscrew Theory*. Cambridge: Cambridge University Press, 1983.
- [106] S. F. Hoerner, *Fluid-dynamic drag: practical information on aerodynamic drag and hydrodynamic resistance*. New Jersey: Hoerner Fluid Dynamics, 2nd ed., 1965.
- [107] W. R. Sears, "On Calculation of Induced Drag and Conditions Downstream of a Lifting Wing," *Journal of Aircraft*, vol. 11, pp. 191–192, mar 1974.
- [108] M. M. Munk, "My Early Aerodynamic Research - Thoughts and Memories," *Annual Review of Fluid Mechanics*, vol. 13, pp. 1–8, jan 1981.
- [109] E. Trefftz, "Ein Gegenstück zum ritzschen Verfahren," in *2nd International Conference of Applied Mechanics*, (Zurich), pp. 131–137, 1926.
- [110] T. P. Kalman, J. P. Giesing, and W. P. Rodden, "Spanwise distribution of induced drag in subsonic flow by the vortexlattice method," *Journal of Aircraft*, vol. 7, pp. 574–576, nov 1970.
- [111] C. E. Lan, "A Quasi-Vortex-Lattice Method in Thin Wing Theory," *Journal of Aircraft*, vol. 11, pp. 518–527, sep 1974.
- [112] J. A. J. Blackwell, "Numerical method to calculate the induced drag or optimum loading for arbitrary non-planar aircraft," in *NASA Langley Research Center: Vortex-Lattice Utilization* (National Aeronautics and Space Administration, ed.), (Langley Field, VA, United States), pp. 49–70, Lockheed-Georgia Co., 1976.
- [113] R. T. Medan, "improvements to the Kernel Function Method of Steady Subsonic Lifting Surface Theory," tech. rep., NASA Ames Research Center, Moffett Field, 1974.

-
- [114] S. Wagner, "On the singularity method of subsonic lifting- surface theory," *Journal of Aircraft*, vol. 6, pp. 549–558, nov 1969.
- [115] H. T. Wang, "Comprehensive evaluation of six thin-wing lifting surface computer programs," tech. rep., Naval Ship Research and Development Center, Bethesda Md., 1974.
- [116] M. Ichikawa, "Analytical expression of induced drag for a finite elliptic wing," *Journal of Aircraft*, vol. 33, pp. 632–634, may 1996.
- [117] T. Kida, "A Theoretical Treatment of Lifting Surface Theory of an Elliptic Wing," *ZAMM - Zeitschrift für Angewandte Mathematik und Mechanik*, vol. 60, no. 12, pp. 645–651, 1980.
- [118] R. Eppler, "Induced drag and winglets," *Aerospace Science and Technology*, vol. 1, pp. 3–15, jan 1997.
- [119] J. L. Lundry and P. B. S. Lissaman, "A numerical solution for the minimum induced drag of nonplanar wings," *Journal of Aircraft*, vol. 5, pp. 17–21, jan 1968.
- [120] F. R. DeJarnette, "Arrangement of Vortex Lattices on Subsonic Wings," in *NASA Langley Research Center: Vortex-Lattice Utilization* (North Carolina State University, ed.), (Langley Field, VA, United States), pp. 301–323, National Aeronautics and Space Administration, 1976.
- [121] S. Ando and A. Ichikawa, "Quadrature formulas for chordwise integrals of lifting surface theories," *AIAA Journal*, vol. 21, pp. 466–467, mar 1983.
- [122] S. C. Smith and I. M. Kroo, "Computation of induced drag for elliptical and crescent-shaped wings," *Journal of Aircraft*, vol. 30, pp. 446–452, jul 1993.
- [123] L. Demasi, "Induced Drag Minimization: A Variational Approach Using the Acceleration Potential," *Journal of Aircraft*, vol. 43, pp. 669–680, may 2006.
- [124] E. Kita and N. Kamiya, "Trefftz method: an overview," *Advances in Engineering Software*, vol. 24, pp. 3–12, jan 1995.
- [125] I. Kroo and S. Smith, "The Computation of Induced Drag with Nonplanar and Deformed Wakes," in *Aerospace Technology Conference and Exposition*, (Long Beach, CA, United States), p. 11, sep 1990.
- [126] J. Nathman, "Induced Drag of High-Aspect Ratio Wings," in *43rd AIAA Aerospace Sciences Meeting and Exhibit*, (Reston, Virginia), p. 8, American Institute of Aeronautics and Astronautics, jan 2005.
- [127] R. T. Jones, *Wing Theory*. Princeton, New Jersey: Princeton University Press, 1990.
- [128] D. W. F. Standingford, *Optimal lifting surfaces, including end plates, ground effect & thickness*. PhD thesis, University of Adelaide, 1997.

REFERENCES

- [129] M. Gariépy, B. Malouin, J.-Y. Trépanier, and É. Laurendeau, "Far-Field Drag Decomposition Applied to the Drag Prediction Workshop 5 Cases," *Journal of Aircraft*, vol. 50, pp. 1822–1831, nov 2013.
- [130] J. Hoefling, J. C. Schirra, A. Spohr, and D. Schäfer, "Induced drag computation with wake model schemes for highly non-planar lifting systems," in *Deutscher Luft- und Raumfahrtkongress*, (Stuttgart, Germany), p. 11, 2013.
- [131] R. Horten and P. F. Selinger, *Nurflügel*. Germany: Weishaupt, 1987.
- [132] S. Esquieu, "Reliable Drag Extraction from Numerical Solutions: Elimination of Spurious Drag," in *Computational Uncertainty in Military Vehicle Design* (ONERA, ed.), p. 16, NATO; RTO, 2007.
- [133] D. Destarac, "Spurious Far-Field-Boundary Induced Drag in Two-Dimensional Flow Simulations," *Journal of Aircraft*, vol. 48, pp. 1444–1455, jul 2011.
- [134] J. Reneaux, V. Brunet, S. Esquieu, M. Meunier, and S. Mouton, "Recent achievements in numerical simulation for aircraft power-plant configurations," *The Aeronautical Journal*, vol. 117, pp. 213–231, feb 2013.
- [135] P. R. Spalart, "On the far wake and induced drag of aircraft," *Journal of Fluid Mechanics*, vol. 603, pp. 413–430, may 2008.
- [136] M. Drela, "Power Balance in Aerodynamic Flows," *AIAA Journal*, vol. 47, no. 7, pp. 1761–1771, 2009.
- [137] W. R. Sears, "Comment on 'Induced drag and the ideal wake of a lifting wing'," *Journal of Aircraft*, vol. 27, pp. 191–191, feb 1990.
- [138] C. W. McCutchen, "Induced drag and the ideal wake of a lifting wing," *Journal of Aircraft*, vol. 26, pp. 489–493, may 1989.
- [139] C. W. McCutchen, "Author's Reply," *Journal of Aircraft*, vol. 27, no. 2, pp. 191–192, 1990.
- [140] D. McLean, *Understanding Aerodynamics*. Chichester, UK: John Wiley & Sons, Ltd, nov 2012.
- [141] G. Schouten, "Momentum, pressure, and energy in the Trefftz-plane," *Journal of Aircraft*, vol. 32, pp. 943–954, sep 1995.
- [142] V. Bembrekar, A. Rasane, A. Jadhav, O. Vaishnav, and S. Mirdude, "Design of a Wing with Bell-shaped Span-load using VLM Method," *International Journal of Engineering Trends and Technology*, vol. 59, pp. 182–185, may 2018.
- [143] A. H. Bowers and E. Uden, "Wing design that creates a bell shaped span load," 2016.
- [144] P. Ranjan and P. J. Ansell, "Computational Analysis of Vortex Wakes Without Near-Field Rollup Characteristics," *Journal of Aircraft*, vol. 55, no. 5, pp. 2008–2021, 2018.

-
- [145] V. J. Rossow, "Theoretical Study of Lift-Generated Vortex Wakes Designed to Avoid Rollup," *AIAA Journal*, vol. 13, pp. 476–484, apr 1975.
- [146] D. L. Ciffone and K. L. Orloff, "Far-Field Wake-Vortex Characteristics of Wings," *Journal of Aircraft*, vol. 12, pp. 464–470, may 1975.
- [147] V. R. Corsiglia, V. J. Rossow, and D. L. Ciffone, "Experimental Study of the Effect of Span Loading on Aircraft Wakes," *Journal of Aircraft*, vol. 13, pp. 968–973, dec 1976.
- [148] E. Torenbeek, *Synthesis of Subsonic Airplane Design*. Springer Netherlands, 1st ed., 1982.
- [149] I. Kroo, "Drag due to Lift: Concepts for Prediction and Reduction," *Annual Review of Fluid Mechanics*, vol. 33, pp. 587–617, jan 2001.
- [150] J. DeYoung, "Induced drag ideal efficiency factor of arbitrary lateral-vertical wing forms," tech. rep., National Aeronautics and Space Administration, Washington, United States, 1980.
- [151] J. S. Letcher, "V-Wings and Diamond Ring-Wings of Minimum Induced Drag," *Journal of Aircraft*, vol. 9, pp. 605–607, aug 1972.
- [152] M. Scherrer, N. Quendez, and C. Döll, "An analytical approach to trimming and performance coupling on flying wings," in *24th International Congress of the Aeronautical Sciences*, (Yokohama), p. 11, ICAS, 2017.
- [153] J. G. Coffin, "A Study of Airplane Ranges and Useful Loads," tech. rep., National Advisory Committee for Aeronautics, 1920.
- [154] W. S. Diehl, "Reliable formulae for estimating airplane performance and the effects of changes in weight, wing area, or power," tech. rep., Bureau of Aeronautics (Navy), Washington, DC, United States, 1924.
- [155] J. D. Anderson, *Introduction to Flight*. Mc Graw Hill, 8 ed., 2016.
- [156] M. D. Guynn, "Effective L/D: A Theoretical Approach to the Measurement of Aero-Structural Efficiency in Aircraft Design," in *15th AIAA Aviation Conference*, (Dallas, TX; United States), p. 14, American Institute of Aeronautics and Astronautics, 2015.
- [157] R. Horten and P. F. Selinger, *Nurflugel: die Geschichte der Horten-Flugzeuge 1933-1960*. Graz, Germany: Weishaupt, 1983.
- [158] A. Shepelev and H. Ottens, *Horten Ho 229 - Spirit of Thuringia: The Horten All-Wing Jet Fighter*. Ian Allan Printing Ltd, 2006.
- [159] R. M. Wood and S. X. S. Bauer, "Flying Wings / Flying Fuselages 39th AIAA Aerospace Sciences Meeting & Exhibit," Tech. Rep. January, NASA Langley Research Center, Hampton, VA United States, 2001.

REFERENCES

- [160] R. Storck, *Flying Wings: Die historische Entwicklung der Nurflügelflugzeuge der Welt*. Bonn, Germany: Bernard U. Graefe Verlag, 2002.
- [161] R. Colgren and R. Loschke, "To Tail or Two Tails? - The Effective Design and Modeling of Yaw Control Devices," in *AIAA Modeling and Simulation Technologies Conference and Exhibit*, (Reston, Virginia), p. 13, American Institute of Aeronautics and Astronautics, aug 2002.
- [162] P. Williams, "Dr. Karl Nickel Horten Interviews," tech. rep., National Air and Space Museum Archives, Smithsonian Institution, 2019.
- [163] D. Gyorgyfalvy, "Performance analysis of the Horten IV Flying Wing," in *8th OSTIV Congress*, (Cologne, Germany), p. 38, Aerophysics Department, Mississippi State University, 1960.
- [164] Central Air Documents Office, "TR-46-45: Horten Tailless Aircraft," tech. rep., Wright-Patterson Air Force Base, Dayton, Ohio, 1950.
- [165] K. G. Wilkinson, *The Horten tailless aircraft*. Farnborough Hants: Royal Aircraft Establishment, 1945.
- [166] D. F. Hunsaker, Z. S. Montgomery, and J. J. Joo, "Adverse-Yaw Control During Roll for a Class of Optimal Lift Distributions," *AIAA Journal*, pp. 1–12, may 2020.
- [167] M. R. Ahmed, M. M. Abdelrahman, G. M. ElBayoumi, and M. M. ElNomrossy, "Optimal wing twist distribution for roll control of MAVs," *The Aeronautical Journal*, vol. 115, pp. 641–649, oct 2011.
- [168] E. Bragado Aldana and M. M. Lone, "Closer look at the flight dynamics of wings with non-elliptic lift distributions," in *AIAA Scitech 2020 Forum*, (Orlando, FL), p. 13, American Institute of Aeronautics and Astronautics, jan 2020.
- [169] L. J. Newton, "Stability and Control Derivative Estimation for the Bell-Shaped Lift Distribution," in *AIAA Scitech 2019 Forum*, (Reston, Virginia), p. 9, American Institute of Aeronautics and Astronautics, jan 2019.
- [170] M. A. Carrizales, M. M. Lone, and E. Bragado Aldana, "Non-elliptic wing lift distribution wings to decrease vertical tailplane size in commercial aircraft," in *AIAA Scitech 2019 Forum*, (Reston, Virginia), p. 13, American Institute of Aeronautics and Astronautics, jan 2019.
- [171] K. Hainline, J. Richter, and R. K. Agarwal, "Vehicle Design Study of a Straight Flying-Wing with Bell Shaped Spanload," in *AIAA Scitech 2020 Forum*, (Reston, Virginia), p. 45, American Institute of Aeronautics and Astronautics, jan 2020.
- [172] R. Guiler and W. Huebsch, "Development and Testing of a Wing Morphing Mechanism for the Control of a Swept Wing Tailless Aircraft," in *Aerospace Technology Conference and Exposition*, (United States), p. 12, SAE International, oct 2005.

-
- [173] G. Sachs, "Why Birds and Miniscale Airplanes Need No Vertical Tail," *Journal of Aircraft*, vol. 44, pp. 1159–1167, jul 2007.
- [174] H. Grellmann, "B-2 aerodynamic design," in *Aerospace Engineering Conference and Show*, (Reston, Virigina), p. 5, American Institute of Aeronautics and Astronautics, feb 1990.
- [175] W. Flanagan, "Network Centric Development and Flight Testing: The B-2 Experience," in *2005 U.S. Air Force T&E Days*, (Reston, Virigina), p. 10, American Institute of Aeronautics and Astronautics, dec 2005.
- [176] J. Roskam, *Airplane Design Part I: Preliminary sizing of airplanes*. DARcorporation, 1985.
- [177] J. Roskam, *Airplane Design Part II: Preliminary configuration design and integration of the propulsion system*. DARcorporation, 1985.
- [178] IHS ESDU, "ESDU 88013: Rolling moment derivative , L_{ξ} for plain ailerons at subsonic speeds," Tech. Rep. August, ESDU. The Royal Aeronautical Society, 1988.
- [179] IHS ESDU, "ESDU 88029 - Yawing moment coefficient for plain ailerons at subsonic speeds," Tech. Rep. November, ESDU. The Royal Aeronautical Society, 2006.
- [180] IHS ESDU, "ESDU 83006: Contribution of fin to sideforce, yawing moment and rolling moment derivatives due to rate of roll, $(Y_p)_F$, $(N_p)_F$, $(L_p)_F$, in the presence of body, wing and tailplane," tech. rep., ESDU. The Royal Aeronautical Society, 1985.
- [181] IHS ESDU, "ESDU 82017: Contribution of fin to sideforce, yawing moment and rolling moment derivatives due to rate of yaw, $(Y_r)_F$, $(N_r)_F$, $(L_r)_F$," tech. rep., ESDU. The Royal Aeronautical Society, 1983.
- [182] IHS ESDU, "ESDU 82010: Contribution of fin to sideforce, yawing moment and rolling moment derivatives due to sideslip, $(Y_v)_F$, $(N_v)_F$, $(L_v)_F$, in the presence of body, wing and tailplane," tech. rep., ESDU. The Royal Aeronautical Society, 2017.
- [183] IHS ESDU, "ESDU Controls 01.01.01: Lift-Curve Slope for Single Fin and Rudder," tech. rep., ESDU. The Royal Aeronautical Society, 1982.
- [184] IHS ESDU, "ESDU 87008: Rudder sideforce, yawing moment and rolling moment control derivatives at low speeds: Y_{ζ} , N_{ζ} and L_{ζ} ," tech. rep., ESDU. The Royal Aeronautical Society, 2014.
- [185] D. Ciliberti, P. Della Vecchia, F. Nicolosi, and A. De Marco, "Aircraft directional stability and vertical tail design: A review of semi-empirical methods," *Progress in Aerospace Sciences*, vol. 95, pp. 140–172, nov 2017.

REFERENCES

- [186] E. Greitzer, P. Bonnefoy, E. D. I. R. Blanco, C. Dorbian, M. Drela, D. Hall, R. Hansman, J. Hileman, R. Liebeck, J. Lovegren, P. Mody, J. Pertuze, S. Sato, Z. Spakovszky, and C. Tan, "N+3 Aircraft Concept Designs and Trade Studies, Final Report. Volume 1," tech. rep., National Aeronautics and Space Administration, Glenn Research Center, 2010.
- [187] S. Bruner, S. Baber, C. Harris, N. Caldwell, P. Keding, K. Rahrig, L. Pho, and R. Wlezian, "NASA N+3 Subsonic Fixed Wing Silent Efficient Low-Emissions Commercial Transport (SELECT) Vehicle Study. Revision A," tech. rep., Northrop Grumman Corp., 2010.
- [188] M. M. D'Angelo, J. Gallman, V. Johnson, E. Garcia, J. Tai, and R. Young, "N+3 Small Commercial Efficient and Quiet Transportation for Year 2030-2035," tech. rep., General Electric Aircraft Engines, NASA Glenn Research Center, 2010.
- [189] Airbus, "Airbus reveals new zero-emission concept aircraft," 2020.
- [190] R. Vos, Z. Gürdal, and M. Abdalla, "Mechanism for Warp-Controlled Twist of a Morphing Wing," *Journal of Aircraft*, vol. 47, pp. 450–457, mar 2010.
- [191] R. Vos and R. Barrett, "Mechanics of pressure-adaptive honeycomb and its application to wing morphing," *Smart Materials and Structures*, vol. 20, no. 9, p. 094010, 2011.
- [192] J. J. Joo, C. R. Marks, L. Zientarski, and A. J. Culler, "Variable Camber Compliant Wing - Design," in *23rd AIAA/AHS Adaptive Structures Conference*, American Institute of Aeronautics and Astronautics, jan 2015.
- [193] J. N. Kudva, "Overview of the DARPA Smart Wing Project," *Journal of Intelligent Material Systems and Structures*, vol. 15, no. 4, pp. 261–267, 2004.
- [194] J. W. Smith, W. P. Lock, and G. A. Payne, "Variable-camber systems integration and operational performance of the AFTI/F-111 mission adaptive wing," tech. rep., National Aeronautics and Space Administration, Dryden Flight Research Facility, Edwards, California, 1992.
- [195] General Dynamics, "The Mission Adaptive Wing," *Aircraft Engineering and Aerospace Technology*, vol. 58, no. 1, pp. 9–14, 1986.
- [196] A. Kumar and J. N. Hefner, "Future challenges and opportunities in aerodynamics," *The Aeronautical Journal*, vol. 104, no. 1038, pp. 365–374, 2000.
- [197] S. L. Chernyshev, S. V. Lyapunov, and A. V. Wolkov, "Modern problems of aircraft aerodynamics," *Advances in Aerodynamics*, vol. 1, no. 1, p. 7, 2019.
- [198] X. Zhang, Y. Chen, and J. Hu, "Recent advances in the development of aerospace materials," *Progress in Aerospace Sciences*, vol. 97, pp. 22–34, 2018.
- [199] A. T. Kermanidis, "Aircraft Aluminum Alloys: Applications and Future Trends," in *Revolutionizing Aircraft Materials and Processes*, pp. 21–55, Springer International Publishing, 2020.

-
- [200] J. Kupski and S. Teixeira de Freitas, "Design of adhesively bonded lap joints with laminated CFRP adherends: Review, challenges and new opportunities for aerospace structures," *Composite Structures*, vol. 268, 2021.
- [201] A. R. Mileham, G. C. Currie, A. W. Miles, and D. T. Bradford, "A Parametric Approach to Cost Estimating at the Conceptual Stage of Design," *Journal of Engineering Design*, vol. 4, no. 2, pp. 117–125, 1993.
- [202] D. D. Pasquale, D. Gore, M. Savill, T. Kipouros, and C. Holden, "Cost Modelling for Aircraft in a Multi-Disciplinary Design Context," in *Advances in Manufacturing Technology XXX - Volume III*, pp. 471 – 476, IOS Press, 2016.
- [203] S. Wakayama, *Lifting Surface Design Using Multidisciplinary Optimization*. PhD thesis, Stanford University, 1995.
- [204] J. Sobieszczanski-Sobieski and R. T. Haftka, "Multidisciplinary aerospace design optimization: survey of recent developments," *Structural Optimization*, vol. 14, pp. 1–23, aug 1997.
- [205] M. Guenov, P. Fantini, L. Balachandran, J. Maginot, M. Padulo, and M. Nunez, "Multidisciplinary Design Optimization Framework for the Pre Design Stage," *Journal of Intelligent & Robotic Systems*, vol. 59, pp. 223–240, sep 2010.
- [206] S. Defoort, M. Balesdent, P. Klotz, P. Schmollgruber, J. Morio, J. Hermetz, C. Blondeau, G. Carrier, and N. Bérend, "Multidisciplinary Aerospace System Design: Principles, Issues and Onera Experience," *Aerospace Lab*, vol. Multidisci, no. 4, p. 15, 2012.
- [207] T. Lefebvre, M. Balesdent, N. Bartoli, L. Brevault, S. Defoort, R. Lafage, and P. Schmollgruber, "MDO advances for aircraft design in ONERA," in *4th SCAD Symposium*, p. 27, Onera, 2014.
- [208] D. L. Clark, D. L. Allison, H. Bae, and E. Forster, "Metamodeling for Effectiveness Based Aircraft Design Under Uncertainty," in *Multidisciplinary Analysis and Optimization Conference*, American Institute of Aeronautics and Astronautics, 2018.
- [209] A. Jungo, M. Zhang, J. B. Vos, and A. Rizzi, "Benchmarking New CEASIOM with CPACS adoption for aerodynamic analysis and flight simulation," *Aircraft Engineering and Aerospace Technology*, vol. 90, pp. 613–626, may 2018.
- [210] X. Du and L. Leifsson, "Optimum aerodynamic shape design under uncertainty by utility theory and metamodeling," *Aerospace Science and Technology*, vol. 95, 2019.
- [211] R. Shi, T. Long, N. Ye, Y. Wu, Z. Wei, and Z. Liu, "Metamodel-based multidisciplinary design optimization methods for aerospace system," *Astrodynamic*, vol. 5, no. 3, pp. 185–215, 2021.

REFERENCES

- [212] G. L. Mura, A. Riaz, M. Guenov, A. Molina-Cristobal, X. Chen, S. Sharma, K. Nicolls, and C. Lynas, "Early Stage Design of High-Lift Devices with System and Manufacturing Constraints," in *AIAA Scitech 2019 Forum*, (Orlando, FL), American Institute of Aeronautics and Astronautics, jan 2019.
- [213] M. D. Guenov, A. Riaz, Y. H. Bile, A. Molina-Cristobal, and A. S. Heerden, "Computational framework for interactive architecting of complex systems," *Systems Engineering*, vol. 23, pp. 350–365, may 2020.
- [214] S. G. Kontogiannis, J. Demange, A. M. Savill, and T. Kipouros, "A comparison study of two multifidelity methods for aerodynamic optimization," *Aerospace Science and Technology*, vol. 97, p. 105592, feb 2020.
- [215] S. G. Kontogiannis and M. A. Savill, "A generalized methodology for multidisciplinary design optimization using surrogate modelling and multifidelity analysis," *Optimization and Engineering*, may 2020.
- [216] O. Dababneh and T. Kipouros, "A review of aircraft wing mass estimation methods," *Aerospace Science and Technology*, vol. 72, pp. 256–266, jan 2018.
- [217] J. R. Martins, J. J. Alonso, and J. Reuther, "Aero-Structural Wing Design Optimization Using High-Fidelity Sensitivity Analysis," tech. rep., National Aeronautics and Space Administration, Ames Research Center, 2001.
- [218] D. E. Raveh, Y. Levy, and M. Karpel, "Structural Optimization Using Computational Aerodynamics," *AIAA Journal*, vol. 38, no. 10, pp. 1974–1982, 2000.
- [219] I. R. Chittick and J. R. R. A. Martins, "An asymmetric suboptimization approach to aerostructural optimization," *Optimization and Engineering*, vol. 10, no. 1, pp. 133–152, 2009.
- [220] J. R. R. A. Martins and J. T. Hwang, "Review and Unification of Methods for Computing Derivatives of Multidisciplinary Computational Models," *AIAA Journal*, vol. 51, pp. 2582–2599, nov 2013.
- [221] G. K. W. Kenway, G. J. Kennedy, and J. R. R. A. Martins, "Scalable Parallel Approach for High-Fidelity Steady-State Aeroelastic Analysis and Adjoint Derivative Computations," *AIAA Journal*, vol. 52, pp. 935–951, may 2014.
- [222] R. P. Liem, G. K. W. Kenway, and J. R. R. A. Martins, "Multimission Aircraft Fuel-Burn Minimization via Multipoint Aerostructural Optimization," *AIAA Journal*, vol. 53, pp. 104–122, jan 2015.
- [223] L. L. Erickson, "Panel Methods: An Introduction," tech. rep., NASA Ames Research Centre, 1990.
- [224] J. R. Martins, J. J. Alonso, and J. J. Reuther, "A Coupled-Adjoint Sensitivity Analysis Method for High-Fidelity Aero-Structural Design," *Optimization and Engineering*, vol. 6, no. 1, pp. 33–62, 2005.

- [225] M. Carini, C. Blondeau, N. Fabbiane, M. Meheut, M. Abu-Zurayk, J. M. Feldwisch, C. Ilic, and A. Merle, "Towards industrial aero-structural aircraft optimization via coupled-adjoint derivatives," in *AIAA AVIATION 2021 FORUM*, (Reston, Virginia), p. 20, American Institute of Aeronautics and Astronautics, 2021.
- [226] E. M. Botero, A. Wendorff, T. MacDonald, A. Variyar, J. M. Vegh, T. W. Lukaczyk, J. J. Alonso, T. H. Orra, and C. Ilario da Silva, "SUAVE: An Open-Source Environment for Conceptual Vehicle Design and Optimization," in *54th AIAA Aerospace Sciences Meeting*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, jan 2016.
- [227] J. P. Jasa, J. T. Hwang, and J. R. R. A. Martins, "Open-source coupled aerostructural optimization using Python," *Structural and Multidisciplinary Optimization*, vol. 57, pp. 1815–1827, apr 2018.
- [228] J. Jasa, J. Hwang, and J. S. Gray, "OpenAeroStruct," 2018.
- [229] J. S. Gray, J. T. Hwang, J. R. R. A. Martins, K. T. Moore, and B. A. Naylor, "OpenMDAO: an open-source framework for multidisciplinary design, analysis, and optimization," *Structural and Multidisciplinary Optimization*, vol. 59, pp. 1075–1104, apr 2019.
- [230] M. Ghoreyshi, K. Badcock, and M. Woodgate, "Integration of Multi-Fidelity Methods for Generating an Aerodynamic Model for Flight Simulation," in *46th AIAA Aerospace Sciences Meeting and Exhibit*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, jan 2008.
- [231] T. Goetzendorf-Grabowski, D. Mieszalski, and E. Marcinkiewicz, "Stability Analysis in Conceptual Design Using SDSA Tool," in *AIAA Atmospheric Flight Mechanics Conference*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, aug 2010.
- [232] L. Cavagna, S. Ricci, and L. Travaglini, "NeoCASS: An integrated tool for structural sizing, aeroelastic analysis and MDO at conceptual design level," *Progress in Aerospace Sciences*, vol. 47, no. 8, pp. 621–635, 2011.
- [233] P. C. Chen, D. Sarhaddi, and D. D. Liu, "Transonic-Aerodynamic-Influence-Coefficient Approach for Aeroelastic and MDO Applications," *Journal of Aircraft*, vol. 37, pp. 85–94, jan 2000.
- [234] V. Ahuja and R. Hartfield, "Predicting the Aero Loads Behind a Propeller in the Presence of a Wing Using Flightstream," in *15th AIAA Aviation Technology, Integration, and Operations Conference*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, jun 2015.
- [235] V. Ahuja, I. Chakraborty, and R. J. Hartfield, "Aero-Propulsive Analysis for Contemporary Conceptual Design," in *AIAA Aviation 2019 Forum*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, jun 2019.

REFERENCES

- [236] R. Bombardieri, R. Sanchez, R. Cavallaro, and N. R. Gauger, *Towards an Open-Source Framework for Aero-Structural Design and Optimization Within the SU2 Suite*. Springer, Cham, 2019.
- [237] G. E. Fujiwara, N. T. Nguyen, E. Livne, and M. B. Bragg, "Aerostructural Design Optimization of a Flexible Wing Aircraft with Continuous Morphing Trailing Edge," in *2018 Multidisciplinary Analysis and Optimization Conference*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, jun 2018.
- [238] T. F. Wunderlich, S. Dähne, L. Reimer, and A. Schuster, "Global aero-structural design optimization of composite wings with active manoeuvre load alleviation," *CEAS Aeronautical Journal*, vol. 13, pp. 639–662, jul 2022.
- [239] P. Ranjan, G. Das, and K. James, "A Large-Scale Aero-Structural Optimization Framework for Electric Aircraft Design," in *AIAA Propulsion and Energy 2021 Forum*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, aug 2021.
- [240] F. M. Capristan and J. Welstead, "LEAPS: An Initial Assessment Towards a Multi-Order Approach to Air Vehicle Mission Analysis," in *18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, jun 2017.
- [241] F. M. Capristan, D. Caldwell, R. Condotta, and B. Petty, "Aircraft Analysis Using the Layered and Extensible Aircraft Performance System (LEAPS)," tech. rep., National Aeronautics and Space Administration, Langley Research Center, USA, 2020.
- [242] A. T. Isikveren, S. Ziemer, G. Stenz, and M. Hornung, "Holistic Granular Programming: A Novel Approach for Modeling Aircraft," in *SAE AeroTech Congress and Exhibition*, SAE Technical Papers, oct 2011.
- [243] V. R. C. Munjulury, P. Berry, and P. Krus, "RAPID – Robust Aircraft Parametric Interactive Design: A Knowledge Based Aircraft Conceptual Design Tool," in *The International Conference of the European Aerospace Societies*, 4th CEAS Air & Space COnference, 2013.
- [244] J. R. Welstead, D. Caldwell, R. Condotta, and N. Monroe, "An Overview of the Layered and Extensible Aircraft Performance System (LEAPS) Development," in *2018 AIAA Aerospace Sciences Meeting*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, jan 2018.
- [245] R. K. Kapania, J. A. Schetz, W. Mallik, M. C. Segee, and R. Gupta, "Multidisciplinary Design Optimization and Cruise Mach Number Study of Truss-Braced Wing Aircraft," tech. rep., Boeing Research and Technology, NASA Langley Research Center, 2018.
- [246] M. K. Bradley and C. K. Droney, "Subsonic Ultra Green Aircraft Research," tech. rep., Boeing Research and Technology, NASA Langley Research Center, 2011.

-
- [247] M. K. Bradley, C. K. Droney, and T. J. Allen, "Subsonic Ultra Green Aircraft Research. Phase II - Volume I; Truss Braced Wing Design Exploration," tech. rep., NASA Langley Research Center, 2015.
- [248] G. Carrier, O. Atinault, S. Dequand, Hantrais-Gervois, J.-L. Liauzun, B. Paluch, A.-M. Rodde, and C. Toussaint, "Investigation Of A Strut-Braced Wing Configuration For Future Commercial Transport," in *28th International Congress of the Aeronautical Sciences*, p. 16, Onera, 2012.
- [249] Airbus, "The albatross is inspiring tomorrow's aircraft wings," 2019.
- [250] C. Howcroft, D. Calderon, L. Lambert, M. Castellani, J. E. Cooper, M. H. Lowenberg, and S. Neild, "Aeroelastic Modelling of Highly Flexible Wings," in *15th Dynamics Specialists Conference*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, jan 2016.
- [251] A. Eaton, *Nonlinear dynamics of high-aspect-ratio wings using numerical continuation*. PhD thesis, University of Bristol, 2019.
- [252] M. Sohst, J. Lobo do Vale, F. Afonso, and A. Suleman, "Optimization and comparison of strut-braced and high aspect ratio wing aircraft configurations including flutter analysis with geometric non-linearities," *Aerospace Science and Technology*, vol. 124, p. 107531, may 2022.
- [253] ICAO, "Annex 14 - Aerodromes - Volume I - Aerodromes Design and Operations," tech. rep., International Civil Aviation Organisation, 2022.
- [254] W. Su and C. E. S. Cesnik, "Nonlinear Aeroelasticity of a Very Flexible Blended-Wing-Body Aircraft," *Journal of Aircraft*, vol. 47, pp. 1539–1553, sep 2010.
- [255] D. E. Calderon, J. E. Cooper, M. Lowenberg, S. A. Neild, and E. B. Coetzee, "Sizing High-Aspect-Ratio Wings with a Geometrically Nonlinear Beam Model," *Journal of Aircraft*, vol. 56, pp. 1455–1470, jul 2019.
- [256] J. R. Wedderspoon, "The high lift development of the A320 aircraft," in *International Council of the Aeronautical Sciences*, ICAS; AIAA, 1986.
- [257] A. S. A. S., "A318/A319/A320/A321 Performance training manual," tech. rep., AIRBUS Training & Flight Operations Support and Services, Blagnac, France, 2005.
- [258] R. T. Whitcomb, "Review of NASA supercritical airfoils," in *International Council of the Aeronautical Sciences Congress*, p. 11, 1974.
- [259] D. Nixon, *Transonic Aerodynamics*. American Institute of Aeronautics and Astronautics, jan 1982.
- [260] C. D. Harris, "NASA Supercritical Airfoils: A Matrix of Family-Related Airfoils," tech. rep., NASA Langley Research Center, Hampton, VA, United States, 1990.
- [261] R. C. Feagin and W. D. J. Morrison, "Delta method, an empirical drag buildup technique," tech. rep., Lockheed California Company, 1978.

REFERENCES

- [262] T. e solutions, “Pacelab APD and SysArc,” 2022.
- [263] The MathWorks Inc., “MATLAB,” 2021.
- [264] Y.-M. Scolan and J. Louis, “On some integrals of Glauert’s type,” *Applied Mathematics Letters*, vol. 18, pp. 631–633, jun 2005.
- [265] H. Norstrud, “A correction for compressible subsonic planar flow,” *Journal of Aircraft*, vol. 8, no. 2, pp. 123–125, 1971.
- [266] T. Melin, A. T. Isikveren, and M. I. Friswell, “Induced-Drag Compressibility Correction for Three-Dimensional Vortex-Lattice Methods,” *Journal of Aircraft*, vol. 47, no. 4, pp. 1458–1460, 2010.
- [267] J. D. Taylor and D. F. Hunsaker, “Minimum Induced Drag for Tapered Wings Including Structural Constraints,” *Journal of Aircraft*, vol. 57, pp. 782–786, jul 2020.
- [268] D. F. Hunsaker, W. F. Phillips, and J. J. Joo, “Designing Wings with Fixed Twist for Minimum Induced Drag,” in *55th AIAA Aerospace Sciences Meeting*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, jan 2017.
- [269] M. Drela and H. Youngren, “AVL,” 2017.
- [270] J. D. Anderson, *Fundamentals of Aerodynamics*. McGraw-Hill Education, sixth ed., 2007.
- [271] E. Torenbeek, “Development and application of a comprehensive, design-sensitive weight prediction method for wing structures of transport category aircraft,” tech. rep., Delft University of Technology, 1992.
- [272] A. T. Isikveren, “Methodology for Conceptual Design and Optimisation of Transport Aircraft,” in *21st International Congress of the Aeronautical Sciences*, (Melbourne, Austalia), 1998.
- [273] A. T. Isikveren, *Quasi-Analytical Modelling and Optimisation Techniques for Transport Aircraft Design*. PhD thesis, Royal Institute of Technology (KTH), 2002.
- [274] R. Finck and D. Hoak, “USAF Stability and Control DATCOM,” tech. rep., United States Air Force, United States, 1978.
- [275] EASA, “Certification Specifications and Acceptable Means of Compliance for Large Aeroplanes (CS-25 Amendment 27),” tech. rep., European Union Aviation Safety Agency, 2021.
- [276] B. H. Tongue, *Principles of Vibration*. Oxford University Press, 2nd ed., 2001.
- [277] L. Cavagna, S. Ricci, and L. Riccobene, “A Fast Tool for Structural Sizing, Aeroelastic Analysis and Optimization in Aircraft Conceptual Design,” in *50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, (Reston, Virigina), American Institute of Aeronautics and Astronautics, 2009.

-
- [278] V. D. Chuban, "Influence of Angle of Attack and Stabilizer Deflection on T Empennage Flutter," *Journal of Aircraft*, vol. 42, pp. 264–268, jan 2005.
- [279] J. J. Bertin and R. M. Cummings, *Aerodynamics for Engineers*. Cambridge University Press, 6th ed., aug 2021.
- [280] M. Cook, *Flight Dynamics Principles*. Oxford: Elsevier Ltd., 3rd ed., oct 2012.
- [281] J. Roskam, *Methods for Estimating Stability and Control Derivatives of Conventional Subsonic Airplanes*. University of Virginia, 1971.
- [282] IHS ESDU, "ESDU 13005 - Information on the use of Data Items on the longitudinal stability of aircraft," tech. rep., ESDU. The Royal Aeronautical Society, 2015.
- [283] IHS ESDU, "ESDU AERO A.07.01.00 - Information on the use of Data Items on yawing moment and sideforce derivatives of an aircraft," tech. rep., ESDU. The Royal Aeronautical Society, 2015.
- [284] IHS ESDU, "ESDU AERO A.06.01.00 - Information on the use of Data Items on rolling moment derivatives of an aircraft," tech. rep., ESDU. The Royal Aeronautical Society, 2015.
- [285] R. F. Crawford and A. B. Burns, "Strength, efficiency, and design data for beryllium structures," tech. rep., Lockheed Missiles and Space Co. Inc., Sunnyvale, CA, USA, 1962.
- [286] G. Gerard, "Optimum structural design concepts for aerospace vehicles," *Journal of Spacecraft and Rockets*, vol. 3, pp. 5–18, jan 1966.
- [287] L. Cavagna, "NeoCASS - Next generation Conceptual Aero Structural Sizing," tech. rep., Politecnico di Milano, 2013.
- [288] T. Melin, "TORNADO 1.0," 2001.
- [289] M. D. Ardema, M. C. Chambers, A. P. Patron, A. S. Hahn, H. Miura, and M. D. Moore, "Fuselage and wing weight of transport aircraft," *SAE Technical Papers*, no. May 1996, 1996.
- [290] P. W. Jansen, R. E. Perez, and J. R. R. A. Martins, "Aerostructural Optimization of Nonplanar Lifting Surfaces," *Journal of Aircraft*, vol. 47, pp. 1490–1503, sep 2010.
- [291] E. S. Rutowski, "Energy Approach to the General Aircraft Performance Problem," *Journal of the Aeronautical Sciences*, vol. 21, no. 3, pp. 187–195, 1954.
- [292] M. Drela and M. Giles, "ISES - A two-dimensional viscous aerodynamic design and analysis code," in *25th AIAA Aerospace Sciences Meeting*, (Reston, Virginia), American Institute of Aeronautics and Astronautics, mar 1987.
- [293] B. Malone and W. H. Mason, "Multidisciplinary optimization in aircraft design using analytic technology models," *Journal of Aircraft*, vol. 32, pp. 431–438, mar 1995.

REFERENCES

- [294] A. Filippone, *Advanced Aircraft Flight Performance*. Cambridge University Press, dec 2012.
- [295] B. W. McCormick, *Aerodynamics, Aeronautics, and Flight Mechanics*. New York: Wiley, 2 ed., 1995.
- [296] J. Roskam, *Methods for Estimating Drag Polars of Subsonic Airplanes*. Roskam Aviation and Engineering Corporation, 1971.
- [297] H. Sadettin Hamut, R. S. El-Emam, M. Aydin, and I. Dincer, “Effects of rear spoilers on ground vehicle aerodynamic drag,” *International Journal of Numerical Methods for Heat & Fluid Flow*, vol. 24, pp. 627–642, apr 2014.
- [298] J. Rosenow, T. Sachwitz, S. Kamo, G. Chen, and H. Fricke, “Aircraft-Type-Specific Impact of Speed Brakes on Lift and Drag,” *Aerospace*, vol. 9, p. 263, may 2022.
- [299] IHS ESDU, “ESDU 79015: Undercarriage drag prediction methods,” tech. rep., ESDU. The Royal Aeronautical Society, 2000.
- [300] M. Drela and H. Youngren, “XFOIL,” 2013.
- [301] Arnie McCullers, “Flight Optimization System (FLOPS),” 2011.
- [302] J. S. Duncan, *Pilot’s Handbook of Aeronautical Knowledge*. Oklahoma City: Federal Aviation Administration, 2nd ed., 2016.
- [303] Z. S. Montgomery, D. F. Hunsaker, and J. J. Joo, “A Methodology for Roll Control of Morphing Aircraft,” in *AIAA Scitech 2019 Forum*, (Reston, Virginia), p. 18, American Institute of Aeronautics and Astronautics, jan 2019.
- [304] T. A. Toll, “NACA-TR-868: Summary of lateral-control research,” tech. rep., Langley Aeronautical Lab; National Advisory Committee for Aeronautics, Langley Field, VA, United States, 1947.
- [305] F. E. Weick and R. T. Jones, “NACA-TR-605: Resume and analysis of NACA lateral control research,” tech. rep., National Advisory Committee for Aeronautics, Langley Aeronautical Laboratory, 1937.
- [306] C. D. Perkins and R. E. Hage, *Airplane Performance, Stability and Control*. University of Michigan: Wiley, 1949.
- [307] S. Gudmundsson, *General Aviation Aircraft Design: Applied Methods and Procedures*. Elsevier Science, 2013.
- [308] J. Roskam, *Airplane Flight Dynamics and Automatic Flight Controls, Part 1*. United States: DARcorporation, 1998.
- [309] E. Seckel, *Stability and Control of Airplanes and Helicopters*. Elsevier, 1 ed., 1964.
- [310] J. Roskam, *Airplane Design Part VII: Determination of Stability, Control and Performance Characteristics*. University of Kansas: DARcorporation, 1st ed., 1985.

- [311] H. Glauert, "Theoretical Relationships for an Aerofoil with Hinged Flap," tech. rep., Aeronautical Research Committee, London, 1927.
- [312] R. F. Stengel, *Flight Dynamics*. New Jersey, United States: Princeton University Press, 2004.
- [313] W. Lekto and W. B. Kemp, "Wind-Tunnel Tests of Ailerons at Various Speeds III - Ailerons of 0.2 airfoil chord and true Contour with 0.35-aileron-chord airfoil," tech. rep., National Advisory Committee for Aeronautics, Washington, 1947.
- [314] IHS ESDU, "ESDU 06014: Zero-lift drag coefficient increment due to full-span plain flaps," tech. rep., Royal Aeronautical Society, 2006.
- [315] J. D. Anderson, *Fundamentals of aerodynamics*. McGraw Hill, 5th ed., 2010.
- [316] D. Howe, *Aircraft Conceptual Design Synthesis*. Chichester, UK: John Wiley & Sons, Ltd, oct 2000.
- [317] M. H. Sadraey, *Aircraft Design*. Chichester, UK: John Wiley & Sons, Ltd, 1st ed., oct 2012.