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DESIGN OF AN INTERACTIVE FINITE ELEMENT COMPUTER PACKAGE FOR THE
ANALYSIS OF THE RIDE OF A GENERALISED OFF-ROAD VEHICLE

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ABSTRACT:

A versatile computer package to evaluate ground vehicle ride comfort has been designed. Wheeled, tracked, and semitracked vehicles have been modelled. The emphasis has been placed upon the tracked-vehicle suspension systems with the nearly "standard" torsion bar arrangement operating with either telescopic or rotary-vane dampers. Substantial investigation of the four-bar linkage, associated with the rotary-vane damper, has been carried out and is presented here. The package includes linear, nonlinear, (2-D), and (3-D) vehicle models.

A new technique employing the finite element method (FEM) for suspension system modelling has been developed.

Several methods of numerical integration are available in the package. A new efficient scheme based on a hyperoscillatory Hermitian weighted residual of the second order for the solution of the system of differential equations which simulate the vehicle, has been derived. Whenever necessary the error developed during the integration process was limited. The graphic display facility of digital computers has been used extensively in the present code. As an example, the damped eigenvalue problem has been solved graphically. The graphical solution is simple, direct, and reliable compared to the analytical solution.

The terrain configuration might be of either harmonic or arbitrary shape function.

Two tolerance criteria have been employed for ride comfort evaluation, namely, the absorbed power criterion and the international standard organization ISO 2631 criterion. The two criteria are fully integrated into the package and operate automatically.

All individual subroutines have been tested successfully for the purpose intended. Complete sectors have been validated by running various case studies. The results obtained from the package agree reasonably well with the analytical or experimental results.

The package allows updating in response to new vehicle and vehicle-terrain technology and can be run on both time-sharing or batch processing computer machines. The main package structure is independent of computer type.

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NOTATION

A	Amplitude of a harmonic wave.
C	Damping coefficient of the main damper.
CT	Equivalent damping coefficient of a tyre.
$F(t)$	Forcing function of time.
g	Gravitational acceleration.
I_x	Vehicle moment of inertia about X-axis.
I_y	Vehicle moment of inertia about Y-axis.
K	Stiffness coefficient of the main spring.
KB_i	Stiffness coefficient of bump stop number i .
K_s	Stiffness coefficient of a segment of a tyre.
KT	Equivalent stiffness coefficient of a tyre.
l	Roadwheel arm length.
l_2	Distance between torsion bar centre and point of attachment of the damper to the road wheel arm.
m_i	Unsprung mass of wheel assembly number i .
M_s	Vehicle sprung mass.
$M(t)$	External moment about X-axis, function of time.
$M(t)$	External moment about Y-axis, function of time.
N	Number of wheels of a vehicle.
S_i	Distance between subsequent wheels i and $i-1$.
ST	Static to bump stop stroke.
t_i	time node number i .
V	Vehicle velocity.
W, W', W''	Displacement, velocity, and acceleration of a point on the main frame.
W_1, W_1', W_1''	Displacement, velocity, and acceleration of the centre of a wheel.
W_2, W_2'	Displacement and velocity of point of contact of a tyre and the terrain.
X, Y, Z	Vehicle fixed-ground parallel coordinates.
X_d, Y_d, Z_d	Coordinates of the point of attachment of a damper to the main frame.
X_1, Y_1, Z_1	Vehicle input data coordinates.
X_2, Y_2, Z_2	Vehicle coordinates.
X_3, Y_3, Z_3	Ground-vehicle coordinates.

NOTATION CONT'D

μ	Coulmb friction coefficient.
θ	Pitch angle.
ϕ	Roll angle.
λ	Transmission ratio of a Four-bar linkage.
η_i	Force-deflection angle of the i^{th} segment spring of a tyre.

Some other specific notations have been defined at the point in the text where they are used.

CHAPTER 1

INTRODUCTION

INTRODUCTION:

Vehicle dynamics is the study, prediction, measurement, and analysis of the motion of vehicles. For automotive vehicles this study is customarily subdivided into the three areas of performance, handling, and ride. The performance motion of a vehicle refers to its ability to accelerate, decelerate, or travel at a constant velocity in response to driver commands. The handling motion of a vehicle is the response to steering commands or environmental inputs that affect directional control. The ride motion is the vibratory activity of the vehicle, which is excited by road irregularities.

On relatively smooth roads the division of vehicle dynamics into the areas of ride, handling, and performance is justified, but frequently the interaction caused by rough surface conditions precludes separation into these three categories. The handling and performance motions of a vehicle will, in general, be greatly degraded on rough road surfaces. It is, however, meaningful to consider the ride motion in the absence of any control inputs.

This work concentrates on off-road vehicle ride.

The higher power/weight ratios of off-road (especially military) vehicles has increased potential cross-country speed and placed greater demands on vehicle suspension systems. This has led to the need for a means of predicting and measuring suspension performance, and to study the effect of specified design changes on cross-country performance.

Starting from the system model and applying the basic laws of Newton and Euler or Lagrange's equations for setting up the equations of motion is a formal procedure but cumbersome, tedious, and susceptible to errors. Also any change in vehicle type or components or even slight modelling variations, may result in very extensive new derivations and, in consequence, new programming activities. Thus useful and expensive time elapses until new and reliable results can be obtained. Computeraided formalisms represent an adequate solution of the problem.

The development of complex vehicle system formalisms has been reviewed in [1]*.

In 1965 and 1966 formalisms were developed for the numerical computerised derivation of equations of motion for an n-body satellite as applied in space[2,3]. The difficulties faced in applying this formalism to vehicle engineering are the kinematics of closed chains and nonholonomic constraints rarely found in space. Those difficulties were overcome in 1976[4]. The general purpose vehicle system dynamics software based on multibody formalisms is reviewed in [5]. A useful review covering most of the presently available advanced multibody-system programs is given in [6].

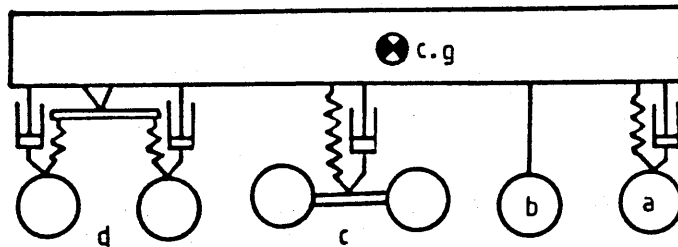
Generally three forms of digital computer code might be considered in the area of vehicle dynamics. First is a rigidly structured computer code that is written to carry out a single class of application using a well defined set of components. Such a computer code can be made quite efficient and transparent to the user, but most likely be limited in its capability for expansion. This approach generally leads to a computer code of maximum efficiency but minimum flexibility. Examples of this form are found in [7,8,9]. A second form is a computer code organisation which employs a formulation that defines a rigidly structured system of equations. This type of computer code constructs contributing equations using standard components from a library of elements used in formulating dynamic models. This form allows the user to derive and incorporate equations governing nonstandard effects and to enter FORTRAN descriptions of such equations and array dimension modifications that are required in order to assemble and solve the system of equations of motion. This approach has the desirable feature of allowing a high degree of efficiency, but at the same time provides a capability for the experienced user to incorporate nonstandard effects for special purpose subsystem descriptions as they are encountered in applications, as well as solution routines. The drawback of such an approach is that a relatively high level of sophistication on the part of the user is required in creating user supplied routines that are needed for special purpose applications. Typical examples of this form are NEWEUL and NUBEMM, whose summaries have been given in [5].

*Number(s) in brackets designate reference(s) at end of thesis.

A third alternative is to adopt a formulation and construct a computer code that is modular in nature, with certain classes of functions carried out in standard and nonstandard modules in the digital computer code. This approach maximises the flexibility of the software system for expansion of capability to incorporate effects that may not have been envisioned when the code was originally constructed. Further, basic functions within the code can be organised in such a way that the level of sophistication that is required of the user, in creating a new module to represent some effect or a special purpose subsystem, is minimal. Examples of this form are programs ADAMS (Automated Dynamic Analysis of Mechanical Systems), MEDYNA (MEchanical DYNamic Analysis), and DADS (Dynamic Analysis and Design Systems). Only the program MEDYNA includes vehicle ride comfort evaluation. An essential feature of MEDYNA is the availability of specific vehicle models describing the primary lift and guidance suspension mechanics for wheel/rail, and magnetically levitated systems, but it only deals to a limited extent with road vehicles with pneumatic tyres.

At this point it is perhaps desirable to briefly describe the most important and powerful vehicle dynamics package available, namely the Nato Reference Mobility Model (NRMM). This description is based upon reports [10,11,12,13] published before the model became officially NRMM and reports [14,15] after that time. The first generation model was completed in July 1971 and was designated the AMC-71 ground mobility model. The AMC-71 ride dynamics module represents the heart of that model. That module was programmed for four specific vehicles (only two tracked and two wheeled). Nuttal [10] concluded that the model is incomplete in some aspects, lacks reliability in others, and has not yet been fully validated in some parts or as a whole. A series of controlled field tests and studies using AMC-71 revealed a number of areas where the model needed immediate improvement. It was understood at the initiation of the model that a more general ride dynamics module would be requested before the model would be suitable for general use. Therefore, the development of a single computer program that would readily accommodate any type of rigid-framed, wheeled, or tracked, vehicle was included as a part of a proposed five-year plan.

This computer program, which will be described below, has been developed and is referred to as the Army Mobility Model AMM-75. It is perhaps worth mentioning that AMM-75 forms the basis of NRMM. The model is restricted to rigid-frame vehicles and two-dimensional (2-D) motion. The types of suspension system which can be handled by the model are illustrated in Fig(1).



Fig(1) Types of suspension system in AMC-75 model.

Those types are:

- a) Independent suspension,
- b) Unsprung assemblies,
- c) Walking beams,
- ,and d) Bogies.

In fact the last named type of suspension system was used extensively on early tracked vehicles but is now obsolete [16]. It has been replaced with the nearly "standard" arrangement of transverse torsion bars directly operated by longitudinal wheel arms. The type of damper used with such an arrangement is either telescopic or rotary-vane. The army mobility model did not account for the above widely used type of suspension system and this can be considered as a serious deficiency. Other shortcomings of this model are:

a) Program dimensions and printout format allow a maximum of eight wheels.

b) The ratio of the vehicle length to the minimum average input profile spacing must be >100 mm.

c) All tabular inputs are limited to 400 values.

The objective of the present work is to design a versatile computer-aided design package for off-road vehicle ride comfort analysis with emphasis placed on tracked vehicles.

The off-road operation differs from on-road operation in essentially four ways:

a) In off-road travel the path is somewhat more freely selected by the driver.

b) The surface bearing capacity and traction characteristics are more variable off-road than on, and frequently are very low.

c) The surface will generally be much rougher in terms of minor geometric anomalies and major slopes.

d) The selected path will usually be strewn with obstacles of various sizes and shapes.

The present package will be capable of:

a) Analysis of expected or actual terrain configuration.

b) Study of new vehicle configurations for stated missions and terrains.

c) Assessment of the operational capabilities of new vehicle designs.

d) As part of a more comprehensive study, to make comparative evaluations among a number of operational or mechanical concepts, or even vehicles.

The above capabilities have been achieved by the simulation of various types of vehicle with the proper selection of vehicle-terrain interaction, and, the use of the most probable vehicle field of action as an excitation to the simulation model. The output from the package will take one or more of the following forms:

a) Determination of natural frequencies and the display of the mode shapes of the system either analytically or graphically.

b) Determination of the resonance frequencies of the system either analytically or graphically.

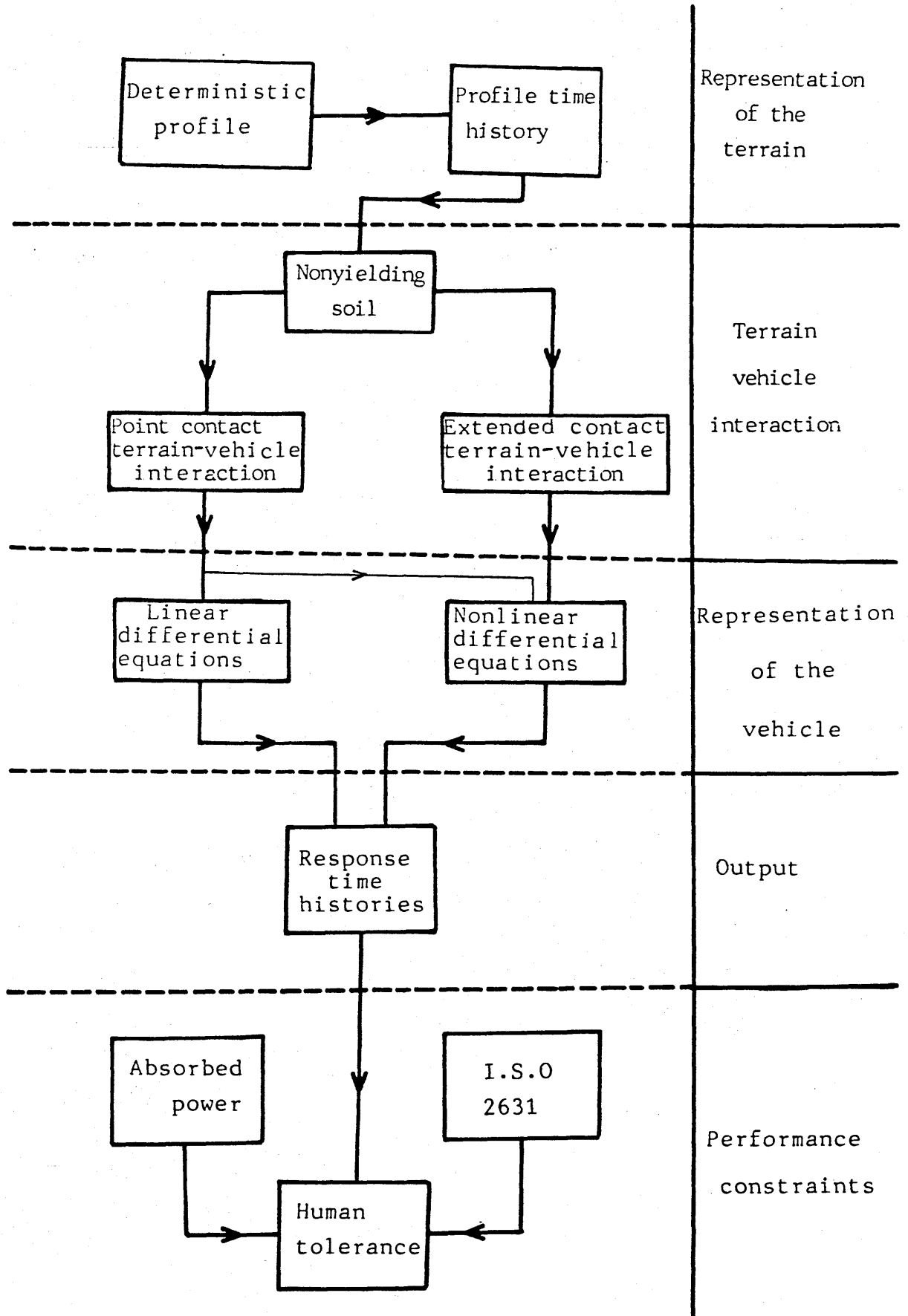
c) Steady state response analysis and graphical display of displacement-, velocity-, and, acceleration-time curves.

d) Time response of the vehicle system to arbitrary terrain surface using several solution techniques.

e) Ride evaluation by two methods.

Fig(2) shows an outline block diagram meets the above requirements.

(6)



Fig(2)The ride dynamic model.

In this chapter the objectives of the present work are laid out and the means of achieving these objectives are discussed. Computer packages are also reviewed. Later in the present chapter the state-of-the-art of ride dynamics will be reviewed.

In Chapter 2 vehicle simulation methods were reviewed, and various types of vehicle were simulated. Emphasis was placed on tracked vehicle simulation where little published work was discovered or access to the work done was restricted. This latter restriction arises because most of tracked vehicle use is military. A new and reliable technique using the finite element method for vehicle simulation has been arrived at.

In Chapter 3 various types of vehicle-terrain interaction were reviewed. The justification of the types of interaction selected to be included in the present code were discussed. Types of terrain excitation are investigated.

In Chapter 4 analysis of the general linear dynamic equation which simulates the vehicle has been presented. This analysis includes a study of the system features such as the natural frequencies, resonance frequencies, and the response to harmonic excitation. The graphical display facility of digital computers has been used extensively in the analysis.

In Chapter 5 most of the solution methods were studied. Several suitable methods of solution were included. A general trend adopted through the design of the present code was the inclusion of more than one method of performing a specific task, whenever possible, as long as there is no firm published evidence that one particular method is adequate for carrying out the task. In order to keep this thesis to a reasonable size only sufficient mathematical derivation has been included to allow for computer implementation. For further derivation the references listed at the appropriate stage should be consulted. Only for original work has the full derivation been included. At all times simplicity and ease of use have been kept to the forefront. A new method of initial value problem solution, namely, the Hermitian hyperoscillatory weighted residual of the second order has been developed.

In Chapter 6 the ride performance constraints have been reviewed and two, widely used, constraints were included.

In Chapter 7 verification of various sections of the code either separately, or together with other sections, has been carried out. Further results have been presented to illustrate the capabilities of the package.

In Chapter 8 conclusions and recommended future work have been presented.

Due to the wide range of subjects included in this work, a literature review for each subject will be included in the chapter related to that subject. However an overall review of the state-of-the-art of the ride comfort aspect of vehicle dynamics is given below.

1.1 STATE-OF-THE-ART:

Any simulation of a vehicle system requires the establishment of a model i.e the generation of physical or mathematical models which behave analogously (or at least in a similar manner) to the real vehicle.

The central problem of ride analysis is the representation of the vehicle. An accurate mathematical model that accounts for structural flexing is too unwieldy for practical applications, so the use of rigid-body mechanics has become the standard approach. Vehicles are represented as a collection of masses, springs, and damping elements. These give rise to sets of differential equations that describe motions throughout the vehicle model. Nonlinearities, inevitable in the vehicle structure, arise from the physics of suspension components, limitations on suspension travel, large pitch and roll motions, etc. The spring and damper assemblies connecting the unsprung mass to the main frame describe the suspension compliance, while the spring and damper assemblies below the unsprung masses serve to describe the tyre compliance. The mass elements are assumed to be rigid bodies. The spring elements possess elasticity and are assumed to be of negligible mass. The spring-force-deformation relationship may be either linear or nonlinear. The damping elements have neither mass nor elasticity. There are two common types of damping which occur in vehicle suspensions. One type is Coulomb damping, sometimes called frictional

damping, in which the damping force is a function of the normal force between the sliding bodies as well as the material involved (as in leaf springs). The other type, which is used most often in modelling, is that which is characterized by dashpot action in which the damping force depends on the relative velocity of the deformation. Some recent vehicle models have included both types of damping in the suspension elements, but generally the gross damping in suspension is lumped in the latter type.

The relations of spring force-deflection and damper force-velocity may be in the form of tables, polynomials, or piecewise linear segments.

The number of degrees of freedom of any system is determined by the number of independent coordinates required to describe the configuration of each mass of the system. Generally, three coordinates are required to describe the configuration of the sprung mass (bounce, pitch and roll). The axles are permitted to have motion in only the vertical direction, thus requiring a single coordinate to describe each axle configuration.

A vehicle operating off-road encounters two types of ground roughness. One type consists of discrete obstacles to which the driver tends to give individual attention. The other type consists of continual, more-or-less gradual, variations in elevation. Terrain profiles used in vehicle ride analysis may be in either deterministic or statistical form. Selection of the form to be used is dictated by the desired accuracy of the output and by the details of available terrain data. A deterministic profile can be used as recorded in the form of a deterministic displacement-distance function to excite a vehicle model passing over it. As an alternate to the deterministic approach, a terrain profile statistically as a power spectral density curve, of mean square amplitude per unit bandwidth against spatial frequency can be used. For a valid physical or subjective assessment of ride comfort, the vehicle must travel over a surface that can be accurately specified. The disadvantage of using actual field surface

is that they change with varying soil and climatic conditions and repeated travel over them. Therefore a nonyielding test surface is more desirable providing it can generate similar (or worse) vibrational characteristics on the vehicle and driver, as would the actual terrain.

Most of the work involving mathematical modelling of vehicle dynamic analysis uses a common representation of the vehicle traction element. When this element is a pneumatic tyre, a spring and viscous damper are used to represent tyre compliance. For tracked vehicles the track is represented by a set of springs simulating the track compliance whilst the roadwheel is modelled in the same way as would be used for a pneumatic tyre. The terrain displaces the traction element, which, in turn, transmits vertical forces to the vehicle. The interface of terrain and the traction element is represented as a single point contact. Such representation should yield generally satisfactory results where the terrain and traction element can also be represented by a patch contact which results in improved accuracy.

There are several quantitative ride comfort evaluation criteria, two of which are widely used in the literature, namely, International Standard Organization ISO 2631, and the American Absorbed Power A.P.

The majority of vehicle dynamics simulations are restricted to travel along a straight path over a rigid surface at constant speed. However, a few models have been developed which do include the features of variable speeds and yielding surfaces.

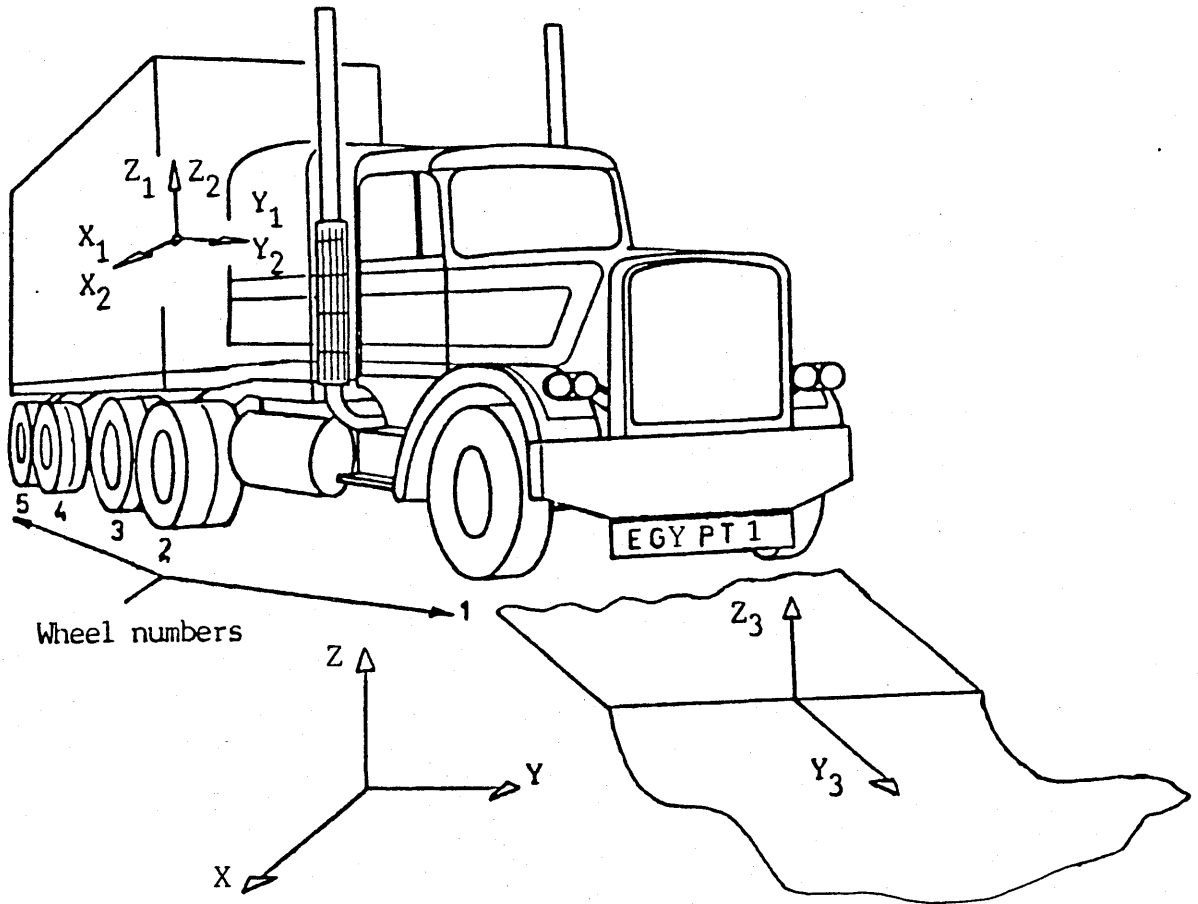
The derivation of the equations of motion of any system first requires the establishment of a coordinate system and an appropriate sign convention for each degree of freedom.

CHAPTER 2

VEHICLE-TERRAIN SYSTEM SIMULATION

2.1 COORDINATE SYSTEMS:

Generally the choice of coordinate system will not affect the equations of motion in any way. However, kinematic and Newton-Euler equations, as well as d'Alembert's principle, may be greatly simplified by the choice of a proper coordinate system. Four separate coordinate systems Fig(3) are used in the present simulation and are summarised as follows:



Fig(3) Different types of coordinate system.

1-Vehicle input data coordinate system:

This coordinate system is based on the vehicle centre of gravity (c.g.) when the vehicle is resting on a hard, flat surface and facing towards the right of the observer. The X -axis and the Y -axis are parallel to the ground surface while the Z -axis is perpendicular to it. The axes of this system are represented by the axes X_1, Y_1, Z_1 in Fig(3).

2-Vehicle Coordinate System:

This coordinate system is based on the vehicle c.g and moves with the vehicle. In fact, the initial positions of the axes are the same as those used for the vehicle input data coordinate system. The axes of this system are represented by the axes X_2, Y_2, Z_2 in Fig(3).

3-Ground Vehicle Coordinate System:

This coordinate system remains fixed to the ground and is based on the starting point of the surface profile of the required terrain intended as excitation force. This coordinate system is represented by the axes X_3, Y_3, Z_3 in Fig(3).

4-Vehicle Fixed-Ground Parallel Coordinate System:

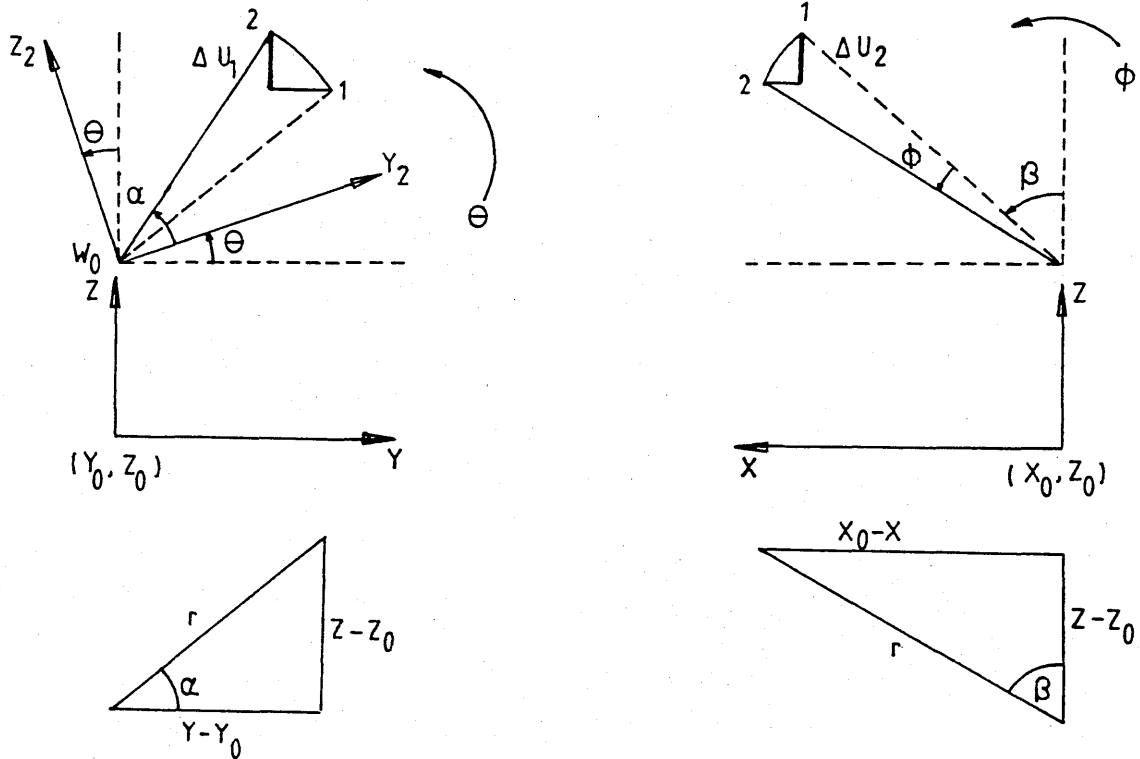
This coordinate system is based on the vehicle c.g and moves with the vehicle, however it remains parallel to the ground fixed coordinate system. Initially it coincides with the vehicle coordinate system when the vehicle is at rest on hard, flat ground. This coordinate system is represented by the axes X, Y, Z in Fig(3).

The Y-axis is always the longitudinal axis of the vehicle or is parallel to it. The Z-axis points upwards. For a vehicle with the front on the right hand side of the observer, the Y-axis points to the front, the X-axis points out in the direction of the observer, and the Z-axis points upwards. The tyres or the roadwheels are numbered from the vehicle front on the side facing the observer for all of the roadwheels on this side, then from the front wheel on the furthest side to the end roadwheel. The right hand rule applies for the positive directions of pitch and roll with respect to X-axis and Y-axis respectively.

For immediate evaluation of the response results it is preferable to fix the vehicle whilst moving the terrain surface with a velocity equivalent to the vehicle speed, and to fix the coordinate system number 4 at the vehicle c.g at time $t=0$. In this case the coordinate system number 1 and number 3 will be used for input data of the vehicle and the terrain surface irregularity respectively. Coordinate

system number 2 represents the c.g vibration with respect to coordinate system number 4.

The relation between the coordinates of different parallel systems (systems number 1,2, and 4) is quite simple and can be easily deduced by adding or subtracting constants represent the two systems concerned. The relation between the coordinates of systems number 2 and number 4 is deduced below. The motion of the c.g is restricted to the Z direction, rotation about X-axis (pitch), and rotation about Y-axis (roll). In fact, axes fixed in relation to the ground whose origin coordinates are (X_0, Y_0, Z_0) or the coordinate system number 4 can be employed.



Fig(4) Effect of rotation about the X and Y axes.

Effect of rotation about X-axis (see Fig.4):

$$\Delta U_1 = r \sin (\theta + \alpha) - r \sin \alpha$$

(14)

$$\Delta U_1 = r (\sin \theta \cos \alpha + \cos \theta \sin \alpha) - r \sin \alpha$$

but

$$r \sin \alpha = Z - Z_0$$

$$r \cos \alpha = Y - Y_0$$

$$\Delta U_1 = (Y - Y_0) \sin \theta + (Z - Z_0) \cos \theta - (Z - Z_0)$$

$$= (Y - Y_0) \sin \theta - (1 - \cos \theta) (Z - Z_0)$$

Considering small deflections then,

$$\sin \theta = \theta \quad , \text{ and}$$

$$\cos \theta = 1$$

$$\Delta U_1 = (Y - Y_0) \theta$$

Effect of rotation about Y-axis:

$$\Delta U_2 = r \cos (\phi + \beta) - r \cos \beta$$

$$= r \cos \phi \cos \beta - r \sin \phi \sin \beta - r \cos \beta$$

$$= - r \sin \phi \sin \beta - (1 - \cos \phi) r \cos \beta$$

but

$$r \cos \beta = Z - Z_0$$

$$r \sin \beta = X - X_0 \quad \text{then,}$$

$$\Delta U_2 = - (X - X_0) \sin \phi - (Z - Z_0) (1 - \cos \phi)$$

$$= (X - X_0) \sin \phi - (Z - Z_0) (1 - \cos \phi)$$

effect of the c.g bounce W_0 , pitch θ , and roll ϕ :

(15)

$$W = W_0 + (Y - Y_0) \sin \theta + (X_0 - X) \sin \phi \\ - (Z - Z_0) (2 - \cos \theta - \cos \phi) \quad \dots(2.1)$$

Equation (2.1) gives the contribution to the vertical displacement W_0 due to the pitch and roll. The velocity W' is :

$$W' = W_0' + [(Y - Y_0) \cos \theta + (Z - Z_0) \sin \theta] \theta' \\ + [(X_0 - X) \cos \phi + (Z - Z_0) \sin \phi] \phi' \quad \dots(2.2)$$

and the acceleration W'' is :

$$W'' = W_0'' + [(Y - Y_0) \cos \theta - (Z - Z_0) \sin \theta] \theta'' \\ - [(Z - Z_0) \cos \theta + (Y - Y_0) \sin \theta] \theta' \\ + [(X_0 - X) \cos \phi - (Z - Z_0) \sin \phi] \phi'' \\ - [(Z - Z_0) \cos \phi - (X_0 - X) \sin \phi] \phi' \quad \dots(2.3)$$

For the special case of small deflections,

$$W = W_0 + (Y - Y_0) \theta + (X_0 - X) \phi \quad \dots(2.4)$$

$$W' = W_0' + (Y - Y_0) \theta' + (X_0 - X) \phi' \quad \dots(2.5)$$

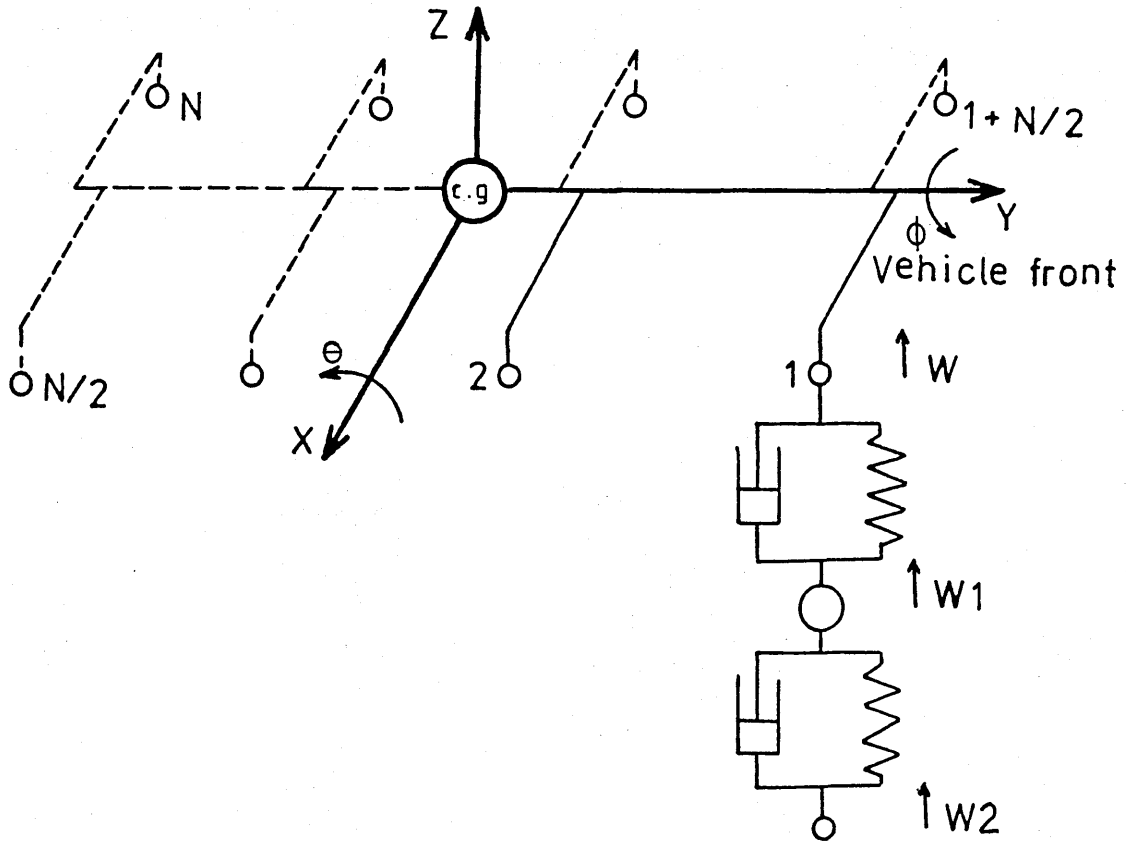
$$W'' = W_0'' + (Y - Y_0) \theta'' + (X_0 - X) \phi'' \quad \dots(2.6)$$

2.2 GENERAL WHEELED VEHICLE RIDE MODEL IN COMPACT FORM:

The model intended is a general model applicable to those vehicles with independent wheel suspension provided the following assumptions hold:

- 1-The body of the vehicle is only subjected to small deflections.
- 2-The terrain surface is rigid.
- 3-The spring element masses are negligible.

4-The damping elements have neither mass nor elasticity.



Fig(5) Ride model of a wheeled vehicle.

Referring to Fig(5) the vehicle sprung mass M_s is concentrated at the c.g and its motion is restricted in the vertical direction (bounce Z), the rotation about the X-axis (pitch θ), and the rotation about the Y-axis (roll ϕ). The sprung mass moments of inertia about the X-, and the Y-axes are I_x , and I_y respectively. The unsprung wheel assembly mass (m) is allowed to move in the vertical direction only with displacement (W_1). The vertical displacement of the common point of the vehicle frame and the wheel assembly is (W) and the rigid terrain vertical elevation is (W_2). The stiffness of the main spring is (K) and the damping coefficient of the main damper is (C). The coefficient of the spring representing tyre stiffness is (K_T) and the damping coefficient of the damper representing tyre damping is (C_T).

(17)

Applying Newton's law and d'Alembert's principle, then

i) The differential equation of motion of the vehicle body in the Z-direction is as follows:

$$F(t) = M_s \ddot{z} + \sum [C_r (\dot{W}_r - \dot{Wl}_r) + K_r (W_r - Wl_r)] \quad \dots(2.7)$$

where

$F(t)$ is the external force applied to the c.g vertically,

r is the wheel assembly number, $r=1,2,\dots,N$,

N is the number of the wheel assemblies.

For simplicity the summation symbol $\sum_{r=1}^N$ will be written as $\sum$.

Substituting W_r , $\dot{W}_r$ from equations (2.4) and (2.5) into equation (2.7) and arranging terms, then

$$\begin{aligned} F(t) = & M_s \ddot{W}_0 + \dot{W}_0 \sum C_r + \theta' \sum C_r (Y_r - Y_0) \\ & + \phi' \sum C_r (X_0 - X_r) + W_0 \sum K_r \\ & + \theta \sum K_r (Y_r - Y_0) + \phi \sum K_r (X_0 - X_r) \\ & - \sum [C_r \dot{Wl}_r + K_r Wl_r] \end{aligned} \quad \dots(2.8)$$

ii) Considering the pitch motion, then

$$\begin{aligned} M_x(t) = & I_x \ddot{\theta} + \sum (Y_r - Y_0) [C_r (\dot{W}_r - \dot{Wl}_r) \\ & + K_r (W_r - Wl_r)] \end{aligned} \quad \dots(2.9)$$

where

$M_x(t)$ is the external torque applied to the c.g in the Y-Z plane,

Substituting, again, W_r and $\dot{W}_r$ from equations (2.4) and (2.5) into equation (2.9) and arranging terms, then

(18)

$$\begin{aligned}
M_x(t) = I_x \ddot{\theta} + W_0' \sum (Y_r - Y_0) C_r + \dot{\theta} \sum (Y_r - Y_0)^2 C_r \\
+ \dot{\phi} \sum (Y_r - Y_0) C_r + W_0 \sum (Y_r - Y_0) K_r + \\
+ \dot{\theta} \sum (Y_r - Y_0)^2 K_r + \dot{\phi} \sum (Y_r - Y_0) (X_0 - X_r) K_r \\
- \sum (Y_r - Y_0) (C_r W_{1r}' + K_r W_{1r}) \quad \dots(2.10)
\end{aligned}$$

iii) Considering the roll motion, then

$$\begin{aligned}
M_y(t) = I_y \ddot{\phi} + \sum (X_0 - X_r) [C_r (W_r' - W_{1r}') \\
+ K_r (W_r - W_{1r})] \quad \dots(2.11)
\end{aligned}$$

where

$M_y(t)$ is the external force applied to the c.g in the X-Z plane.
Substituting W_r , and W_r' in equation (2.11) and arranging terms then,

$$\begin{aligned}
M_y(t) = I_y \ddot{\phi} + W_0' \sum (X_0 - X_r) C_r + \dot{\phi} \sum (X_0 - X_r)^2 C_r + \\
\dot{\theta} \sum (Y_r - Y_0) (X_0 - X_r) C_r + W_0 \sum (X_0 - X_r) K_r + \\
\dot{\phi} \sum (X_0 - X_r)^2 K_r + \dot{\theta} \sum (X_0 - X_r) (Y_r - Y_0) K_r \\
- \sum (X_0 - X_r) (C_r W_{1r}' + K_r W_{1r}) \quad \dots(2.12)
\end{aligned}$$

iv) The differential equation of the r^{th} wheel assembly in the Z-direction is as follows:

$$\begin{aligned}
m_r W_{1r}'' + C_r (W_{1r}' - W_r') + K_r (W_{1r} - W_r) + CT_r (W_{1r}' - W_{2r}') + \\
KT_r (W_{1r} - W_{2r}) = 0 \quad \dots(2.13)
\end{aligned}$$

Eliminating W_r and W_r' from equation (2.13), then

(19)

$$\begin{aligned} m_r \ddot{w}_r + (C_r + C_r^T) \dot{w}_r + (K_r + K_r^T) w_r - C_r \dot{w}_0 \\ - \dot{\theta} (Y_r - Y_0) C_r - \dot{\phi} (X_0 - X_r) C_r - K_r w_0 - \theta (Y_r - Y_0) K_r \\ - \phi (X_0 - X_r) K_r = C_r^T \dot{w}_2 + K_r^T w_2 \end{aligned} \quad \dots (2.14)$$

The equations (2.8), (2.10), and (2.14) form $N+3$ differential equations. The differential equation for the whole suspension system can be written in the matrix form as follows:

$$\underline{M} \ddot{\underline{q}} + \underline{C} \dot{\underline{q}} + \underline{K} \underline{q} = \underline{F}(t) \quad \dots (2.15)$$

where

$\underline{M}$ is the square mass matrix of order $(N+3) \times (N+3)$,
 $\underline{C}$ is the square damping matrix of order $(N+3) \times (N+3)$,
 $\underline{K}$ is the square stiffness matrix of order $(N+3) \times (N+3)$,
 $\underline{q}$, $\dot{\underline{q}}$, $\ddot{\underline{q}}$ are the displacement, velocity and acceleration vectors respectively each being of order $N+3$,
 $\underline{F}(t)$ is the force vector and is of order $N+3$.

The displacement vector is

$$\underline{q} = \{ w_0, \theta, \phi, w_{l1}, w_{l2}, \dots, \dots, w_{lN} \}$$

The velocity vector is

$$\dot{\underline{q}} = \{ \dot{w}_0, \dot{\theta}, \dot{\phi}, \dot{w}_{l1}, \dot{w}_{l2}, \dots, \dots, \dot{w}_{lN} \}$$

and the acceleration vector is

$$\ddot{\underline{q}} = \{ \ddot{w}_0, \ddot{\theta}, \ddot{\phi}, \ddot{w}_{l1}, \ddot{w}_{l2}, \dots, \dots, \ddot{w}_{lN} \}$$

Equations (2.8), (2.10), (2.12), and (2.14) apply for the small deflection case of the vehicle body. With the large deflection is to be taken into consideration, equations w_r , $\dot{w}_r$ (2.1 and 2.2) should be substituted into equations (2.7), (2.9), (2.11), and (2.13).

The resulting equations are as follows:

$$\begin{aligned}
 \text{i) } F(t) = & M_s W_0'' + W_0' \left[C_r + \theta' \right] C_r H_r + \phi' \left[C_r G_r + \right. \\
 & W_0 \left[K_r + \sin \theta \left[(Y_r - Y_0) K_r + \sin \phi \left[(X_0 - X_r) K_r \right. \right. \\
 & \left. \left. - \left[K_r E_r - \left[(C_r W_{l_r}' + K_r W_{l_r}) + M_s g \right] \right] \right] \right] \quad \dots (2.16)
 \end{aligned}$$

where

$$E_r = (2 - \cos \theta - \cos \phi) (Z_r - Z_0) ,$$

$$G_r = (X_0 - X_r) \cos \phi - (Z_r - Z_0) \sin \phi ,$$

$$H_r = (Y_r - Y_0) \cos \theta - (Z_r - Z_0) \sin \theta , \text{ and}$$

g is the gravitational acceleration.

$$\begin{aligned}
 \text{ii) } M_x(t) = & I_x \theta'' + W_0' \left[(Y_r - Y_0) C_r + \theta' \left[(Y_r - Y_0) C_r H_r \right. \right. \\
 & \left. \left. + \phi' \left[(Y_r - Y_0) C_r G_r + W_0 \left[(Y_r - Y_0) K_r \right. \right. \right. \right. \\
 & \left. \left. \left. + \sin \theta \left[(Y_r - Y_0)^2 K_r \right. \right. \right. \right. \\
 & \left. \left. \left. + \sin \phi \left[(Y_r - Y_0) (X_0 - X_r) K_r \right. \right. \right. \right. \\
 & \left. \left. \left. - \left[(Y_r - Y_0) \left[(C_r W_{l_r}' + K_r W_{l_r}) + K_r E_r \right] \right] \right] \right] \quad \dots (2.17)
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } M_y(t) = & I_y \phi'' + W_0' \left[(X_0 - X_r) C_r + \theta' \left[(X_0 - X_r) H_r C_r \right. \right. \\
 & \left. \left. + \phi' \left[(X_0 - X_r) G_r C_r + W_0 \left[(X_0 - X_r) K_r + \right. \right. \right. \right. \\
 & \left. \left. \left. + \sin \phi \left[(X_0 - X_r)^2 K_r \right. \right. \right. \right. \\
 & \left. \left. \left. + \sin \theta \left[(X_0 - X_r) (Y_r - Y_0) K_r \right. \right. \right. \right. \\
 & \left. \left. \left. - \left[(X_0 - X_r) \left[(C_r W_{l_r}' + K_r W_{l_r}) + K_r E_r \right] \right] \right] \right] \quad \dots (2.18)
 \end{aligned}$$

(21)

$$\begin{aligned}
\text{iv) } m_r W_l'' + (C_r + CT_r) W_l' + (K_r + KT_r) W_l & \\
- C_r (W_0' + H_r \theta' + G_r \phi') & \\
- K_r [W_0 + (Y_r - Y_0) \sin \theta & \\
+ (X_0 - X_r) \sin \phi - E_r] = CT_r W_2' + KT_r W_2 - m_r g &
\end{aligned}$$

The set (a) of differential equations (2.8), (2.10), (2.12), and (2.14) and the set (b) (2.16) to (2.19) are applicable as long as the following condition is true:

$$W_r - W_l + ST_r \leq 0$$

where ST is the static to bump stop travel.

If the preceding condition is false, the bump stop is brought into action in order to attenuate heavy vehicle bottoming. A set (c) of differential equations accounting for bump stop condition is given as follows:

i) The first equation is

$$\begin{aligned}
F(t) = M_s W_0'' + W_0' [C_r + \theta'] [C_r H_r + \phi'] [C_r G_r + & \\
+ W_0 [(K_r + KB_r) & \\
+ \sin \phi [(X_0 - X_r) (K_r + KB_r) & \\
+ \sin \theta [(Y_r - Y_0) (K_r + KB_r) & \\
- [C_r W_l' + (K_r + KB_r) W_l] & \\
- [(K_r + KB_r) E_r + M_s g + KB_r ST_r] ..(2.20) &
\end{aligned}$$

ii) The second equation is

(22)

$$\begin{aligned}
M_x(t) = & I_x \ddot{\theta} + W_0' \int (Y_r - Y_0) C_r + \theta' \int (Y_r - Y_0) C_r H_r + \\
& \phi' \int (Y_r - Y_0) C_r G_r + W_0 \int (Y_r - Y_0) (K_r + KB_r) \\
& + \sin \theta \int (Y_r - Y_0)^2 (K_r + KB_r) + \\
& + \sin \phi \int (Y_r - Y_0) (X_0 - X_r) (K_r + KB_r) \\
& - \int (Y_r - Y_0) [(C_r W_{1r}' + (K_r + KB_r) W_{1r}) \\
& + (K_r + KB_r) E_r] + \int KB_r ST_r \quad \dots (2.21)
\end{aligned}$$

iii) The third equation is

$$\begin{aligned}
M_y(t) = & I_y \ddot{\phi} + W_0' \int (X_0 - X_r) C_r + \theta' \int (X_0 - X_r) H_r C_r + \\
& \phi' \int (X_0 - X_r) G_r C_r + W_0 \int (X_0 - X_r) (K_r + KB_r) + \\
& \sin \phi \int (X_0 - X_r)^2 (K_r + KB_r) + \\
& \sin \theta \int (X_0 - X_r) (Y_r - Y_0) (K_r + KB_r) \\
& - \int (X_0 - X_r) [(C_r W_{1r}' + (K_r + KB_r) W_{1r}) + \\
& (K_r + KB_r) E_r] + \int KB_r ST_r \quad \dots (2.22)
\end{aligned}$$

iv) The r^{th} equation is

$$\begin{aligned}
m_r W_{1r}'' + (C_r + CT_r) W_{1r}' + (K_r + KT_r + KB_r) W_{1r} \\
- C_r (W_0' + H_r \phi' + G_r \theta') - (K_r + KB_r) \\
[W_0 + (Y_r - Y_0) \sin \theta + (X_0 - X_r) \sin \phi - E_r] \\
+ KB_r ST_r = CT_r W_{2r}' + KT_r W_{2r} - m_r g \quad \dots (2.23)
\end{aligned}$$

The set of equations (a) represents a linear model provided that all of the coefficients are constants i.e linear springs and viscous dampers are used. On the other hand if the small deflection assumption is violated, or the bump stop is brought into action, or nonlinear springs and/or dampers are used, the model becomes nonlinear.

2.3 SIMULATION OF SUSPENSION SYSTEMS BY THE FINITE ELEMENT METHOD (FEM)^{*}

In the general area of vehicle dynamics the FEM has been used in the main for stress analysis of vehicle frames and/or vehicle operator cabinets or automotive components [18,19,20,21]. In these examples the vehicle frames, cabinets, and components, considered either as continuum structures and/or elastic beams and plates, were discretised into multinode elements and a standard algorithm employed for the assembly and solution of the equations for the whole structure.

There are no examples in the literature of the application of FEM concepts to the formalism of a multibody system consisting of elastic components (as springs and torsion bars), damping components (as shock absorbers and hydrogas units), and rigid or elastic bodies in any combination of these components.

In reference [5] a review of the state-of-the-art of general purpose, vehicle-system-dynamics software was given. In this review all of the problems considered except one were based on multibody formalism. The exception was a program which is known as APLDYN and which was based on rigid bodies as 'finite element' elastic beams. The multibody formalism by the 'vector-network' method as explained in [22] is a combination of vector dynamics and some concepts of graph theory. The multibody formalism explained in [5] is based on Newton's and Euler's equations for each body and d'Alembert's or Jourdain's principle for the complete system. The computer model can be either in numerical form as in [22] or in symbolic form as in [23] and [1]. In reference [6] it was concluded that the overwhelming success of finite element structural codes indicates that systematic formulations are possible in these areas, which can provide a guide for the development of a similar set of techniques and computer codes for mechanical system dynamics.

^{*}The idea explained in this section published in [17].

In reference [5] it is concluded that general purpose software based on multibody formalism would play the same role for mechanical systems, especially vehicle systems, as FEM plays for elastic structures. However, because of the popularity of the FEM and its advanced capabilities, it is obviously advantageous to develop a formalism of suspension systems based on this method.

The wheeled vehicle simulated in item (2.2) is employed as an example in order to illustrate the present technique. The basic formalism and all useful derivations are explained through the standard steps of FEM which are summarised as follows:

1) Discretization of the Domain:

In the standard procedure of the FEM, the domain is usually a continuum or body of solid, liquid or gaseous matter which is represented as an assemblage of subdivisions called finite elements. In vehicle dynamics the domain is the vehicle body and all of the suspension components. The vehicle body is considered as a concentrated mass with inertias about particular axes. The suspension component locations, configurations, and characteristics should be defined. The discretization of the suspension system should be a matter of optimisation of simplicity, time, and effort needed to set up a model. For example considering each spring, damper and mass as an element appears to be simple but in fact leads to a complicated assembly of the final equations. For the case of the suspension system configurations the domain can be divided into the following elements:

a) Element type (0)

This element represents the body, the hull or the main frame of the vehicle, as either an elastic or a rigid body. For the case of a three-dimensional ride model, a study of the bounce, pitch and roll motions of the vehicle body is of interest. Considering, for simplicity, a rigid-main frame, the mass M_s , the moment of inertia about the X-axis I_x , the moment of inertia about the Y-axis I_y , and the c.g coordinates should define the main frame .

b) Element type (1)

This element represents the wheel assembly. The wheel assembly consists of the wheel and the suspension components which join the wheel to the main frame. This element varies extensively from one type of vehicle to another. Fig(6) illustrates several widely-used forms of this element. Element 1(a) is a typical wheeled vehicle suspension system element. Element type 1(b) is a torsion bar arrangement of the suspension system which is a widely used tank suspension arrangement [24]. Element 1(c) is a torsion bar arrangement with rotary vane damper as employed in many types of vehicle. Arbitrary configurations of this element can be dealt with according to the actual type of suspension system employed in the vehicle.

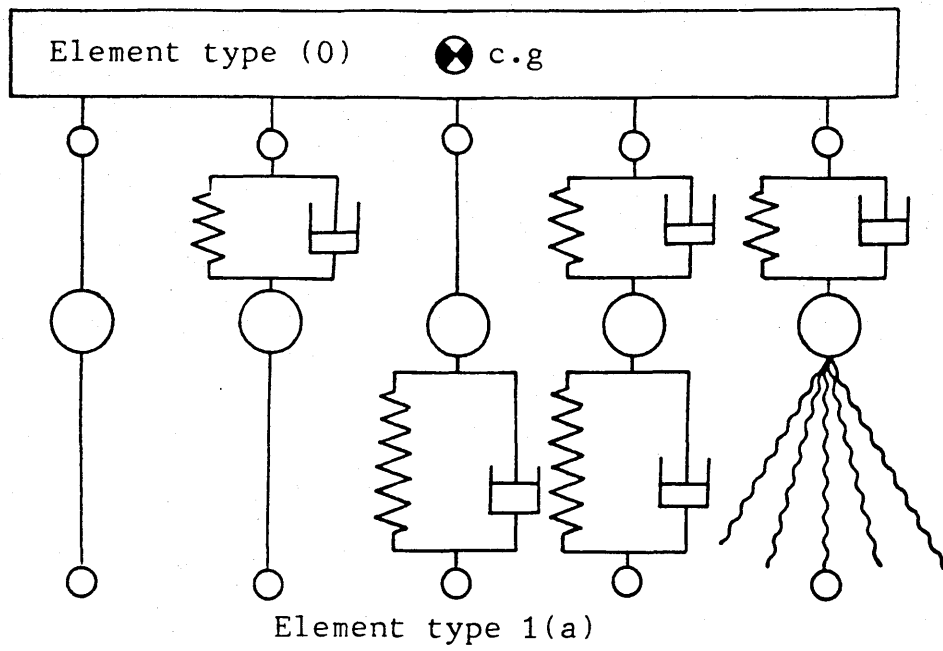
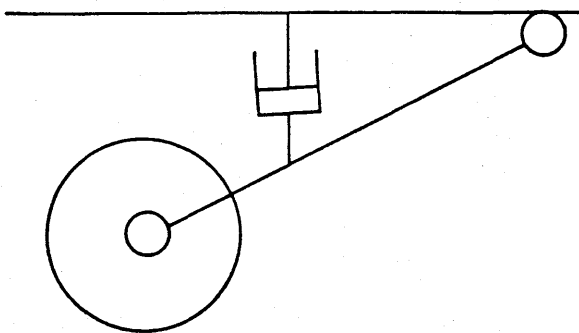
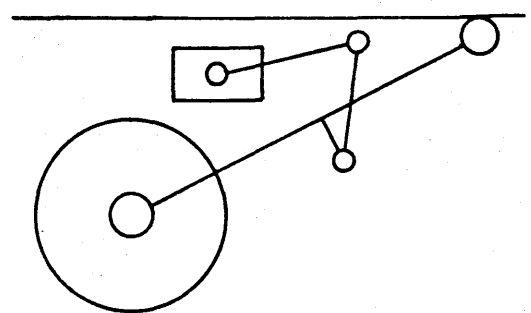
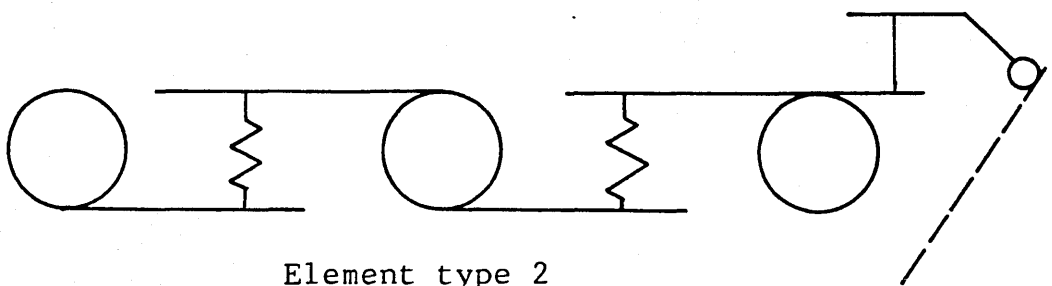
In fact each wheel assembly element represents a family of different forms of that element. For example, the element 1(a) may be of the following forms (see Fig 6):

- Rigid wheel and unsprung assembly.
- Rigid wheel and sprung assembly with or without a damper.
- Elastic tyre and unsprung assembly.
- Elastic tyre with or without damping effect and sprung assembly with or without a damper.

The tyre terrain interaction is considered either a point contact or batch contact.

c) Element type (2)

This element represents the track of tracked or semi-tracked vehicles. Past experiments have shown that tracks have a significant influence on the ride of a vehicle [24]. As a result of a compromise involving model complexity, adequate description of the significant motion, and the time and the cost of computer simulations, a track model was used that would be as simple as possible and yet afford suitable simulations of cross country vibrations [25]. The track compliance is represented by interconnecting linear springs between adjacent roadwheels, and a massless feeler that is connected to the



Fig(6)Various types of element.

front roadwheel by a stiff spring. The stiffness of the spring can be determined approximately by observing photographs of the vehicle in different positions on an obstacle, or more accurately by use of potentiometers interconnected between roadwheels and connected to the main frame. In the former case the stiffness is determined by estimation of the influence of the displacement of a particular roadwheel on the displacement of the adjacent roadwheel. With the approximate mass of each roadwheel assembly and their displacements relative to each other and the main frame known, an appropriate spring constant could be determined.

2) Selection of Proper Displacement Model:

This step implies a proper selection of coordinate systems and the deduction of the formulae depicting the relationship between them in order to eliminate the coordinates of the holonomic constraints, which in turn minimise the size of the system of differential equations. This step is explained in item (2.1).

3) Derivation of the Element Characteristic Equations:

Generally the dynamic equation of any element is in the form:

$$\underline{M} \underline{q}_\ell'' + \underline{C} \underline{q}_\ell' + \underline{K} \underline{q}_\ell = \underline{F}_\ell \quad \dots (2.24)$$

where

$\underline{M}$, $\underline{C}$, $\underline{K}$ are the mass, damping, and stiffness matrices,

$\underline{q}_\ell''$, $\underline{q}_\ell'$, $\underline{q}_\ell$ are the local acceleration, velocity, and displacement vectors respectively, and

$\underline{F}_\ell$ is the local force vector

The order of the square matrices is the number of nodes (number of degrees of freedom). The order of the vectors is also the number of nodes.

i) Characteristic equations of the main frame (element type 0):

For the simple case of a rigid body with a concentrated mass at the c.g, a three-node element can be used to simulate the main frame motion in bounce, pitch, and roll. The non-zero elements of the 3X3 matrices of mass, damping, and stiffness are:

$$M_0(1,1) = M_s$$

$$M_0(2,2) = I_x$$

$$M_0(3,3) = I_y$$

The displacement vector of the centre of gravity is:

$$q_0 = [w_0 , \theta , \phi]$$

and the velocity vector is:

$$\dot{q}_0 = [\dot{w}_0 , \dot{\theta} , \dot{\phi}]$$

and the acceleration vector is:

$$\ddot{q}_0 = [\ddot{w}_0 , \ddot{\theta} , \ddot{\phi}]$$

Finally the equations of motion of the main frame are:

$$\underline{M}_0 \ddot{q}_0 + \underline{C}_0 \dot{q}_0 + \underline{K}_0 q_0 = \underline{F}_0$$

where

$$\underline{F}_0 = [F_z , M_x , M_y] ,$$

F_z is the external vertical force, and

M_x, M_y are the external moments about the X-axis and Y-axis respectively.

Where the flexibility of the main frame is to be taken into consideration, the frame can be discretised into a number of structural elements; beams, plates, and shells. The equations for such elements can be found in the literature [26,27].

ii) Characteristic equations of the element type 1(a):

This element is connected to the main frame at the connection point (X, Y, Z) from the global coordinate system number 4. The forces required to form the global dynamic equation are:

a) The vertical force arising at the connection point of the suspension assembly with the main frame is given as follows (see Fig.5 for notation)

$$(F_z)_1 = C (W' - Wl') + K (W - Wl)$$

But for a rigid main frame and small deflections

$$W = W_0 + (Y - Y_0) \theta + (X_0 - X) \phi$$

and

$$W' = W_0' + (Y - Y_0) \theta' + (X_0 - X) \phi'$$

then

$$\begin{aligned} (F_z)_1 = & C [W_0' + (Y - Y_0) \theta' + (X_0 - X) \phi'] + \\ & K [W_0 + (Y - Y_0) \theta + (X_0 - X) \phi] - \\ & C Wl' - K Wl \end{aligned} \quad \dots (2.25)$$

b) The moment of $(F_z)_1$ about the X-axis is:

$$M_x = (Y - Y_0) [C (W' - Wl') + K (W - Wl)]$$

(30)

by substituting W and W' , then

$$\begin{aligned}
 M_x = & C (Y - Y_0) W_0' + C (Y - Y_0)^2 \theta' + \\
 & C (Y - Y_0) (X_0 - X) \phi' + K (Y - Y_0) W_0 + \\
 & K (Y - Y_0)^2 \theta + K (X_0 - X) (Y - Y_0) \phi \\
 & - K (Y - Y_0) W_1 - C (Y - Y_0) W_1' \quad \dots (2.26)
 \end{aligned}$$

c) The moment about the Y-axis is

$$M_y = (X_0 - X) [C (W' - W_1') + K (W - W_1)]$$

substituting W and W' then,

$$\begin{aligned}
 M_y = & C (X_0 - X) W_0' + C (X_0 - X) (Y - Y_0) \theta' + \\
 & C (X_0 - X)^2 \phi' + K (X_0 - X) W_0 + \\
 & K (X_0 - X) (Y - Y_0) + K (X_0 - X)^2 \phi - \\
 & C (X_0 - X) W_1' - K (X_0 - X) W_1 \quad \dots (2.27)
 \end{aligned}$$

d) The force affecting the wheel center $(F_z)_2$ is

$$(F_z)_2 = C (W_1' - W') + K (W_1 - W) +$$

$$CT (W_1' - W_2') + KT (W_1 - W_2)$$

Eliminating W and W' then,

$$\begin{aligned}
 (F_z)_2 = & (C + CT) W_1' + (K + KT) W_1 - C W_0' - CT W_2' \\
 & C (Y - Y_0) \theta' - C (X_0 - X) \phi' - K W_0 \\
 & K (Y - Y_0) \theta - K (X_0 - X) \phi - KT W_2 \quad \dots (2.28)
 \end{aligned}$$

e) Accounting for the nonholonomic constraints:

$$(F_z)_3 = C (W_1' - W_2') + KT (W_1 - W_2) \quad \dots (2.29)$$

iii) Characteristic equations of elements type 1(b) and 1(c):

The torsion bar is the elastic component of these two elements. The damper of the element type 1(b) is directly connected with the roadwheel arm. The damper of the element type 1(c) is joined to the roadwheel arm through a two-bar linkage. A procedure similar to that followed for the deduction of the characteristic equations of the element type 1(a) should be followed in order to obtain the characteristic equations of each element. These equations are shown later in item (2.5) because the damper linkages must be explained and it would appear desirable to avoid interruption of the explanation of the present technique.

4) Assemblage of Element Equations:

Since the suspension system is composed of several elements, the mass, stiffness, and damping matrices and loading vector are to be assembled in a meaningful manner to form the overall equilibrium equations of the system as indicated in the following formula

$$\underline{M} \underline{\ddot{Z}} + \underline{C} \underline{\dot{Z}} + \underline{K} \underline{Z} = \underline{F} \quad \dots (2.30)$$

Similar procedures to those explained in [27] or [28] can be employed to obtain the general dynamic equations of the suspension system (2.30).

5) Solution of The Global System of Equations:

Once the equations of motion are formalized a solution can be obtained using simulation language integrating routines such as ACSL [29], or by using Runge-Kutta methods of different order (with constant time step or variable time step and error control) [30], or

the Lagrangian weighted residual method [26], or the Hermitian weighted residual method [32], or the method developed in item (4.5) or any other suitable method.

2.4 TYPES OF DAMPER LINKAGE:

The damper linkage is that mechanism which joins the unsprung mass of the wheel assembly to the sprung mass of the vehicle. For vehicle simulation, the damping characteristics of a damper depends upon the way the linkage joins the unsprung mass to the sprung mass, and whether the performance test has been carried out separately or as a part of the unsprung mass assembly for the same damper. There are three widely used types of linkage.

a) Co-linear damper linkage:

This is the most popular arrangement for wheeled vehicles (see Fig 7 a). For the case of viscous damping, the damping force value depends only on the wheel velocity. The shock absorber in this arrangement is of the telescopic type.

b) Inclined damper linkage:

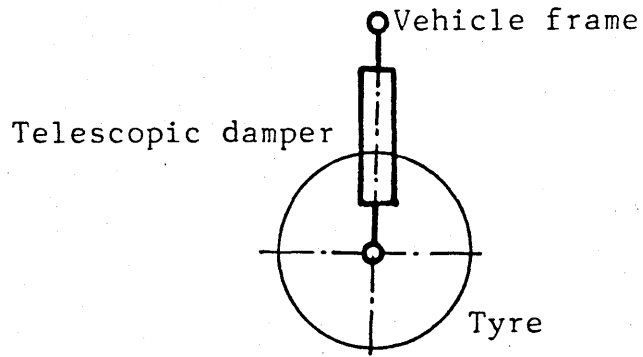
This arrangement is similar to that employed with the co-linear linkage except that the damper axis inclines at an angle to the vertical direction (see Fig 7 b and c). In this case, the effective vertical damping force depends on both wheel velocity and position. To show the change in that force due to different wheel positions, consider the upper and the lower positions of the wheel.

$$F_1 = C V \cos \alpha_1 \quad \text{and} \quad F_2 = C V \cos \alpha_2$$

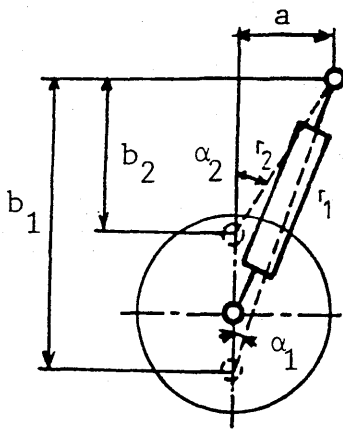
where

C is the damping coefficient,

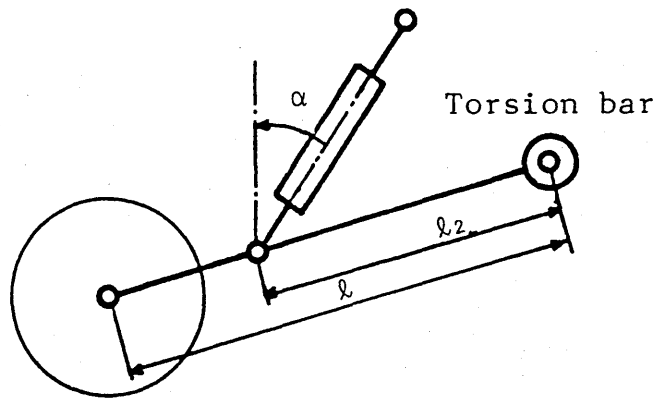
V is the wheel velocity, and



(a) Colinear arrangement

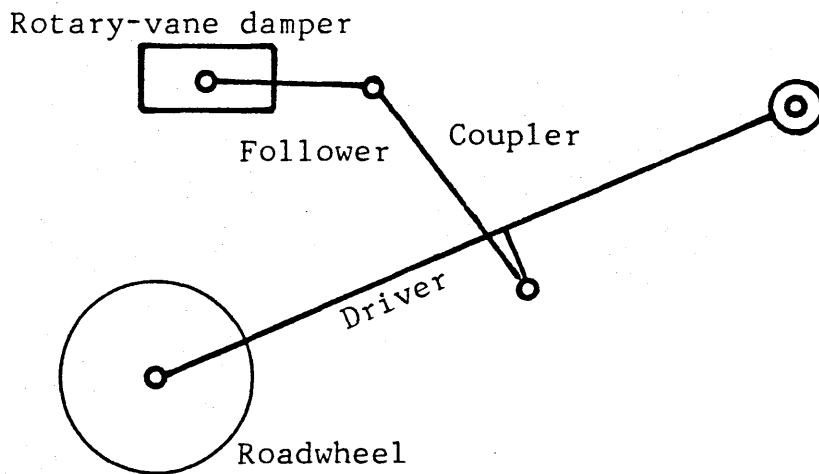


(b)



(c)

Inclined arrangement



(d) Four-bar linkage

Fig(7)Types of shock absorber linkage.

α_1, α_2 are the inclination angles of the damper at the extreme positions of the wheel.

$$\begin{aligned} \frac{F_2}{F_1} &= \frac{\cos \alpha_2}{\cos \alpha_1} = \frac{b_2}{r_2} \frac{r_1}{b_1} \\ &= \gamma \left\{ \frac{\xi^2 + 1}{\xi + \gamma^2} \right\}^{\frac{1}{2}} \end{aligned}$$

where

$$\gamma = b_2 / b_1, \quad \xi = a / b_1$$

For the co-linear case $a = 0$, and $F_1 = F_2$

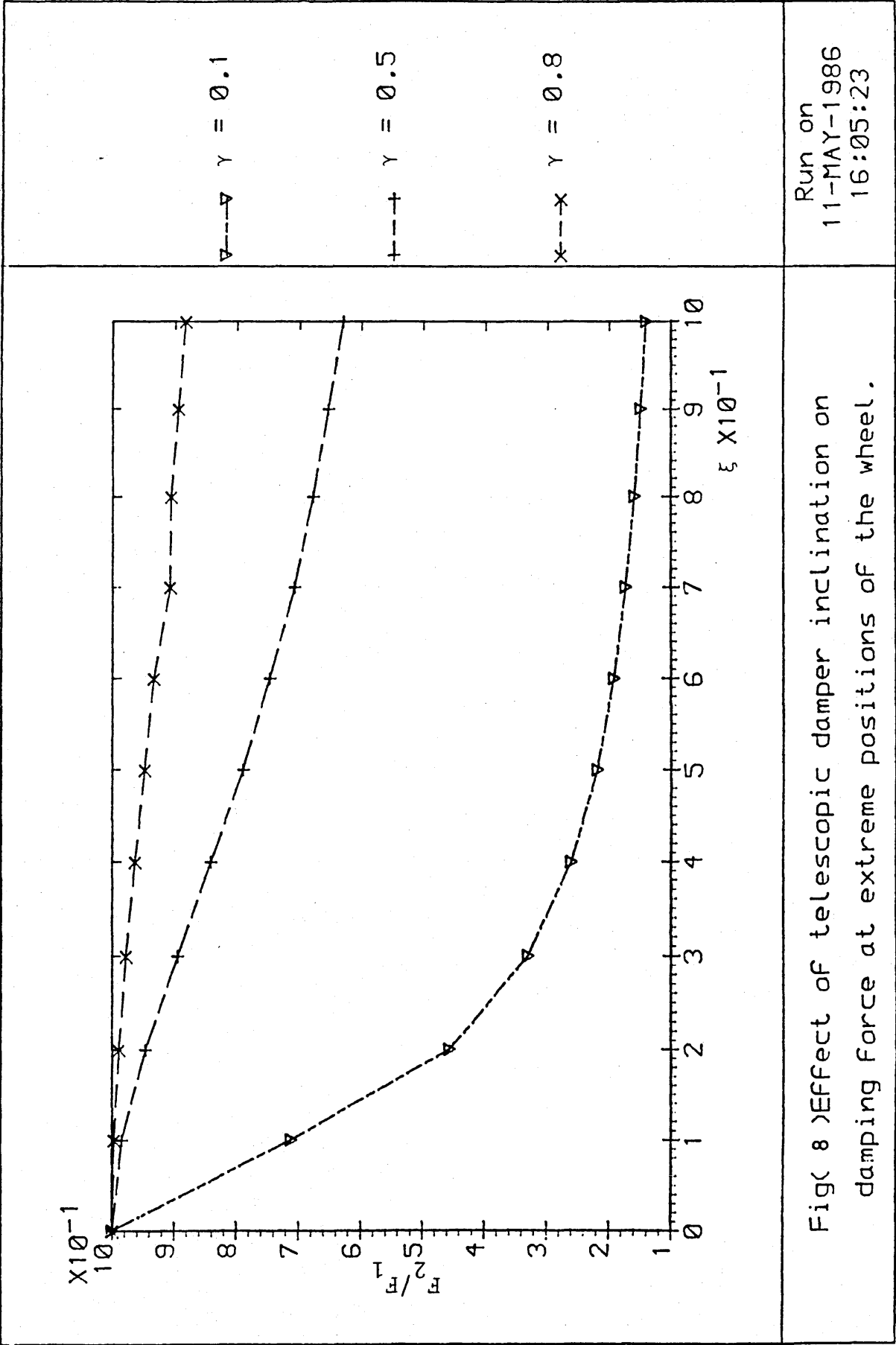
Investigation of various values of γ (from 0.1 to 1) and ξ (from 0 to 1) was carried out. The results are shown in Fig(8).

It is clear from the figure that the damping force increases as the value of γ increases. When the values of $\gamma = 0.8$ and $\xi = 1$ ($\alpha_1 = 45^\circ$) the value of the effective damping force is 12% less than the damping force which can be developed by the shock absorber at the extreme position of the wheel. That reduction increases rapidly with the increase of the value of ξ at lower values of γ . To keep that reduction below 12%, the value of ξ should be ≤ 1 at $\gamma = 0.8$, or the value of ξ should be ≤ 0.3 if $\gamma = 0.5$. The value of ξ decreases rapidly to less than 0.03 if $\gamma = 0.1$. The ratio F_2/F_1 indicates the efficiency of the employment of the damping force developed by the shock absorber in the vertical direction.

Hence two conditions should be taken into consideration in order to keep the reduction in the damping force to a reasonable level:

- to keep the value of a as small as possible (≈ 0), and
- to keep the value of γ as big as possible (≈ 1).

The preceding investigation applies to Fig(7)b. For Fig(7)c there is a new factor arising from the rotation of the road-wheel arm. To study this new factor assume that the damper is in the vertical position in the static equilibrium state of the wheel assembly (i.e. $\xi = 0$).



Fig(8)Effect of telescopic damper inclination on damping force at extreme positions of the wheel.

Referring to Fig(9), then

$$\begin{aligned}
 e &= 2 r^2 - 2 r^2 \cos \alpha_1 \\
 &= b^2 + b_1^2 - 2 b b_1 \cos \alpha_2 , \\
 \cos \alpha_2 &= (b^2 + b_1^2 - 2 r^2 \cos \alpha_1) / 2 b b_1 \quad \dots(2.31)
 \end{aligned}$$

$$\text{and } b^2 = r_1^2 + r^2 - 2 r r_1 \cos \alpha ,$$

$$b_1^2 = r_1^2 + r^2 - 2 r r_1 \cos(\alpha + \alpha_1)$$

The left hand side of equation (2.31) includes only α_2 and the right hand side is a nonlinear function of α_1 . Assume:

$$r_1 = 0.6 \text{ m} , \quad \alpha = 60^\circ$$

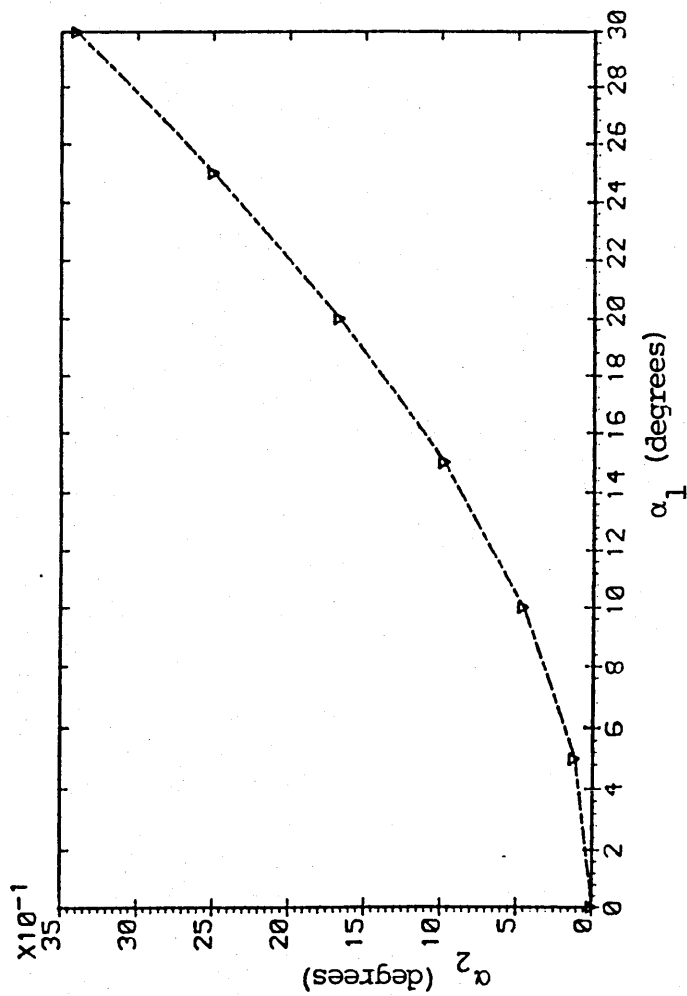
$$r = 0.3 \text{ m} , \text{ and maximum of } \alpha_1 = 30^\circ$$

Furthermore, let the damper be perpendicular to the roadwheel arm at the static equilibrium position. The roadwheel arm motion will be $\pm$ maximum of α_1 . The deflection of the roadwheel arm from the static equilibrium position by an angle α_1 causes a deviation of the damper from the vertical position by an angle α_2 .

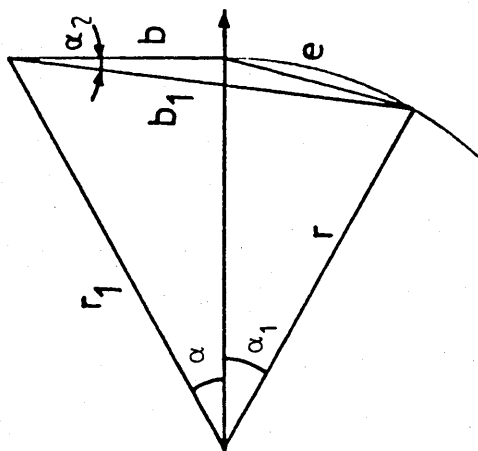
Fig(10) shows the plot of α_2 against α_1 . The value of α_2 increases as the value of α_1 increases. However the effect of such deviation at its maximum value ($\alpha_2 = 3.4^\circ$) upon the decrease in the damping force is small (0.0018).

c) Four-bar linkage:

This type of linkage is that usually associated with the rotary-vane damper. The roadwheel arm represents the driver of the linkage, the damper arm represents the follower, and the rod connecting those two arms represents the coupler. The fourth link is represented by the

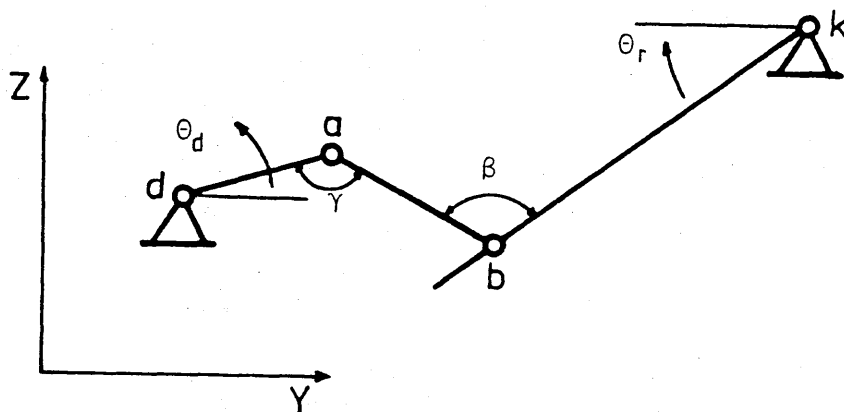


Fig(10)Effect of roadwheel arm rotation on the vertical inclination of the telescopic damper.



Fig(9)Effect of roadwheel arm rotation on the vertical damping force.

rigid main frame. The whole linkage lies in the Y-Z plane. Some particular cases of the mechanism are explained in [32]. A short explanation is given in [33]. More study and general considerations are explained in [20]. A summary of an extensive investigation of the linkage and its use in suspension systems, follows. A computer program has been designed to study the effect of the linkage geometry on the damping performance of the damper. The results of such a program will be given in the results chapter later.



Fig(11) Kinematics of the four-bar linkage.

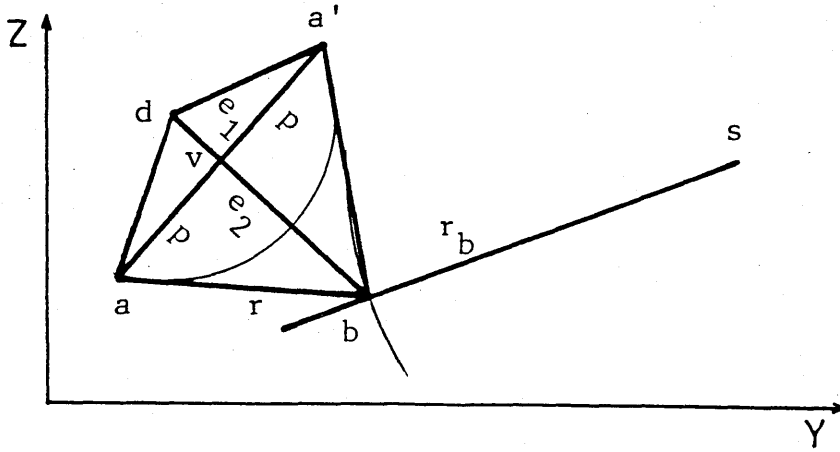
The mechanical advantage of the four bar linkage λ is defined as the ratio of input torque caused by the terrain irregularity to the output torque developed by the damper. The value of λ changes as the mechanism moves and it is proportional to $\sin \gamma$ and to $1/\sin \beta$. If the angle $\beta = 0$, the value of $\lambda = \infty$, and the mechanism is said to be in the toggle position. This position never occurs in practice because of the limits of the motion of the driver link. From experience the value of the angle γ should be kept below 50° in order to produce a reasonable value of λ . It is perhaps worthwhile mentioning that the values of β and γ depend on the choice of the driver and the follower and some mechanisms have no toggle positions.

Referring to Fig(11) and using the elementary rules in [35] the following equation can be derived:

(39)

$$\omega_d = \frac{(Y_b - Y_s)(Z_b - Z_a) - (Y_a - Y_b)(Z_s - Z_b)}{(Y_a - Y_d)(Z_b - Z_a) - (Y_a - Y_b)(Z_d - Z_a)} \omega_r \quad \dots (2.32)$$

Given the four link dimensions and the position of the point b, the two possible positions of the point a can be derived in the following manner:



Fig(12) Solution of four-bar linkage displacement.

(1) Equation of straight line db is

$$Z = m Y + c_1$$

where

$$m = (Z_b - Z_d) / (Y_b - Y_d)$$

and

$$c_1 = Z_d - m Y_d$$

(2) Coordinates of point v:

$$Y_v = (-P_2 \pm \sqrt{P_2^2 - 4 P_1 A}) / 2 P_1$$

and

$$Z_v = m Y_v + c_1$$

where

$$P_1 = m^2 + 1, \quad ,$$

(40)

$$P_2 = 2 m c_1 - 2 Y_d - 2 m Z_d ,$$

$$A = c_1^2 - 2 c_1 Z_d + Z_d^2 + Y_d^2 - e_1^2 ,$$

$$h = \sqrt{(Z_b - Z_d)^2 + (Y_b - Y_d)^2} ,$$

$$e_2 = (r^2 + h^2 - r_a^2) / 2 h , \text{ and}$$

$$e_1 = h - e_2$$

The choice of positive or negative sign in the Y_v equation depends upon which gives the shortest distance between the resulting point and the point b.

(3) Equation of the straight line $a a'$:

$$Z = m_1 Y + c_2$$

where

$$m_1 = -1 / m , \text{ and}$$

$$c_2 = Z_v - m_1 Y_v$$

(4) Results:

The coordinates of the required point a or a' will be the intersection points of the straight line $a a'$ and the circle whose centre is at d and whose radius is r_a .

$$Y_a = (-u_2 \pm \sqrt{u_2^2 - 4 u_1 u_3}) / 2 u_1 , \text{ and}$$

..(2.33)

$$Z_a = m_1 Y_a + c_2$$

where

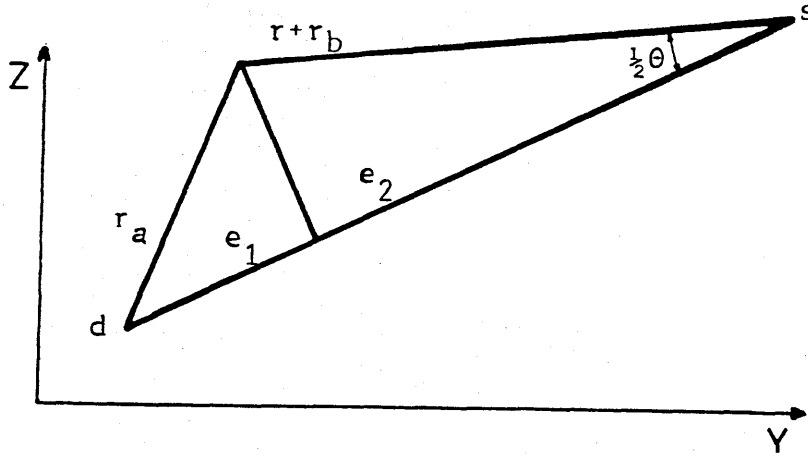
$$u_1 = m_1^2 + 1 ,$$

(41)

$$u_2 = 2 (m_1 c_2 - m_1 z_d - Y_d)$$

$$u_3 = c_2^2 - 2 c_2 z_c - r_a^2 + z_c^2$$

Again the choice of coordinates for equations (2.33) will depend on the specific suspension system. The applicable rule is that the developed damping torque will damp the vibration not amplify it.



Fig(13) Working angle of the four-bar linkage.

The geometrical dimensions of the linkage might limit the motion of the driver. The working angle θ of the driver is that angle between the maximum upper position of the driver (at which the driver and the coupler are on the same straight line) and the minimum lower position (see Fig 13).

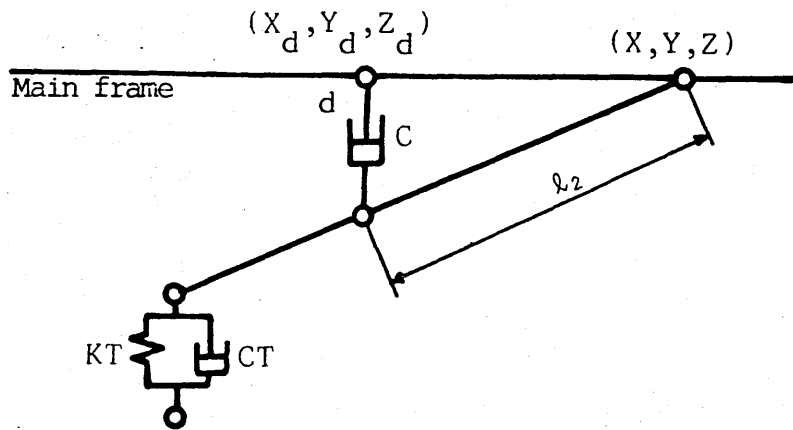
$$\theta = 2 \cos^{-1} (e_1/r) \quad \dots (2.34)$$

where

$$e_1 = (r^2 - r_a^2 + h^2) / 2 h$$

2.5 CHARACTERISTIC EQUATIONS OF THE ELEMENT TYPE 1(b):

This element represents most closely the "standard" system for springing and wheel location on modern military tracked vehicles[25]. It consists of a transverse torsion bar directly operated by a longitudinal wheel arm (see Fig 14). One end of the damper is fixed to the hull while the other is attached somewhere between the wheel centre and the torsion bar centre.



Fig(14) Model of an element type 1(b).

Proceeding in the same way as for the case of element type 1(a), then

a) Force $(F_z)_1$

$$\begin{aligned}
 (F_z)_1 = & u K W_0 + v C W_0' - u K W_1 - v C W_1' + u K (Y - Y_0) \theta + \\
 & C [(Y_d - Y_0) + (v - 1)(Y - Y_0)] \theta' + \\
 & C [(X_0 - X_d) - (1 - v)(X_0 - X)] \phi' + \\
 & u K (X_0 - X) \phi
 \end{aligned}$$

where

$$u = \ell^{-2}, \quad \text{and} \quad v = \ell_2 / \ell$$

b) Torque M_x :

$$\begin{aligned} M_x = & (Y_d - Y_0) [(Y_d - Y_0) + (v - 1)(Y - Y_0)] C \Theta' + \\ & (Y_d - Y_0) [(X_0 - X_d) + (v - 1)(X_0 - X)] C \Phi' + \\ & u (Y - Y_0)^2 K \Theta + u (Y - Y_0)(X_0 - X) K \Phi + \\ & v (Y_d - Y_0) C W_0' + u (Y - Y_0) K W_1 - v (Y_d - Y_0) C W_1' \end{aligned}$$

c) Torque M_y :

$$\begin{aligned} M_y = & (X_0 - X_d) [(X_0 - X_d) + (v - 1)(X_0 - X)] C \Phi' + \\ & u (X_0 - X)^2 K \Phi + v (X_0 - X_d) C W_0' + u (X_0 - X) K W_0 + \\ & (X_0 - X_d) [(Y_d - Y_0) + (v - 1)(Y - Y_0)] C \Theta' + \\ & u (X_0 - X)(Y - Y_0) K \Theta - v (X_0 - X_d) C W_1' - \\ & u (X_0 - X) K W_1 \end{aligned}$$

d) $(F_z)_2$

$$\begin{aligned} (F_z)_2 = & (u K + K_T) W_1 + (C_T - v^2 C) W_1' - \\ & u K W_0 - v^2 C W_0' - u (Y - Y_0) K \Theta - \\ & v [(Y_d - Y_0) + (v - 1)(Y - Y_0)] C \Theta' - \\ & v [(X_0 - X_d) + (v - 1)(X_0 - X)] C \Phi' - \\ & u (X_0 - X) K \Phi - K_T W_2 - C_T W_2' \end{aligned}$$

e) $(F_z)_3$:

$$(F_z)_3 = CT (W_1' - W_2') + KT (W_1 - W_2)$$

2.6 CHARACTERISTIC EQUATIONS OF THE ELEMENT TYPE 1(c):

This element differs from the element type 1(b) in the type of damper provided. This damper is a rotary-vane damper which is connected to the roadwheel arm by a two-bar linkage. Following the same notations and steps as for the element type 1(b), then,

$$\begin{aligned} \text{a) } (F_z)_1 &= (m + n) C W_0' + [m (Y_d - Y_0) + n (Y - Y_0)] C \theta' \\ &+ [m (X_0 - X_d) + n (X_0 - X)] C \phi' + \\ &u K W_0 + u (Y - Y_0) K \theta + u (X_0 - X) K \phi - \\ &u K W_1 - n C W_1' \end{aligned}$$

where

$$m = 1/r_a^2, \text{ and}$$

$$n = \lambda / (\ell r_a)$$

$$\begin{aligned} \text{b) } M_X &= (p + p_1) C W_0' + [p (Y_d - Y_0) + p_1 (Y - Y_0)] C \theta' \\ &+ [p (X_0 - X_d) + p_1 (X_0 - X)] C \phi' \\ &+ u (Y - Y_0) K W_0 + u (Y - Y_0)^2 K \theta \\ &+ u (X_0 - X) (Y - Y_0) K \phi - u (Y - Y_0) K W_1 \\ &- p_1 (Y_d - Y_0) C W_1' \end{aligned}$$

where

$$p = 1/a \quad \text{and} \quad p_1 = \lambda/\ell$$

$$\begin{aligned}
 c) M_y = & (m+n)(X_0 - X_d) C W_0' + u(X_0 - X) K W_0 \\
 & + (X_0 - X_d) [m(Y_d - Y_0) + n(Y - Y_0)] C \theta' \\
 & + (X_0 - X_d) [m(X_0 - X_d) + n(X_0 - X)] C \phi' \\
 & + u(X_0 - X)(Y - Y_0) K \theta + u(X_0 - X)^2 K \phi \\
 & - u(X_0 - X) K W_1 - p_1(X_0 - X_d) C W_1'
 \end{aligned}$$

$$\begin{aligned}
 d) (F_z)_2 = & (KT + uK) W_1 + p_1(p + \lambda) C W_0' \\
 & + p_1[p(Y_d - Y_0) + p_1(Y - Y_0)] C \theta' \\
 & + p_1[p(X_0 - X_d) + p_1(X_0 - X)] C \phi' \\
 & - u(Y - Y_0) K \theta - u(X_0 - X) K \phi + CT W_1'
 \end{aligned}$$

$$e) (F_z)_3 = CT(W_1' - W_2') + KT(W_1 - W_2)$$

2.6-1 SPECIAL CASE :

For the element type 1(c), the damping characteristics of the element might be given in the form of the damping force at the wheel centre versus a wheel centre velocity curve or table. In this case the characteristic equations should be:

$$\begin{aligned}
 a) (F_z)_1 = & (H_1 + H_2) W_0' - H_1 W_1' - u K W_1 \\
 & + [H_1(Y - Y_0) + H_2(Y_d - Y_0)] \theta' \\
 & + [H_1(X_0 - X) + H_2(X_0 - X_d)] \phi' \\
 & + u[W_0 + (Y - Y_0) \theta + (X_0 - X) \phi] K
 \end{aligned}$$

where

$$H_1 = \frac{\ell C}{r_a \lambda} \quad , \quad H_2 = \left\{ \frac{\ell}{r_a \lambda} \right\}^2 C$$

$$\begin{aligned} \text{b) } M_x = & (Y_d - Y_0) [(H_1 + H_2) W_0' - H_1 W_1'] - u (Y - Y_0) K W_1 \\ & + (Y_d - Y_0) [H_1 (Y - Y_0) + H_2 (Y_d - Y_0)] \Theta' \\ & + (Y_d - Y_0) [H_1 (X_0 - X) + H_2 (X_0 - X_d)] \Phi' \\ & + u (Y - Y_0) [W_0 + (Y - Y_0) \Theta + (X_0 - X) \Phi] K \end{aligned}$$

$$\begin{aligned} \text{c) } M_y = & (X_0 - X_d) [(H_1 + H_2) W_0' - H_1 W_1'] - u (X_0 - X) K W_1 \\ & + (X_0 - X_d) [H_1 (Y - Y_0) + H_2 (Y_d - Y_0)] \Theta' \\ & + (X_0 - X_d) [H_1 (X_0 - X) + H_2 (X_0 - X_d)] \Phi' \\ & + u (X_0 - X) [W_0 + (Y - Y_0) \Theta + (X_0 - X) \Phi] K \end{aligned}$$

$$\begin{aligned} \text{d) } (F_z)_2 = & (u K + K T) W_1 - u K W_0 - u (Y - Y_0) K \Theta - \\ & u (X_0 - X) K \Phi + (C T - E H_1) W_1' + \\ & E (H_1 + H_2) W_0' + E [H_1 (Y - Y_0) + H_2 (Y_d - Y_0)] \Theta' \\ & + E [H_1 (X_0 - X) + H_2 (X_0 - X_d)] \Phi' \end{aligned}$$

where

$$E = r_a \lambda / \ell$$

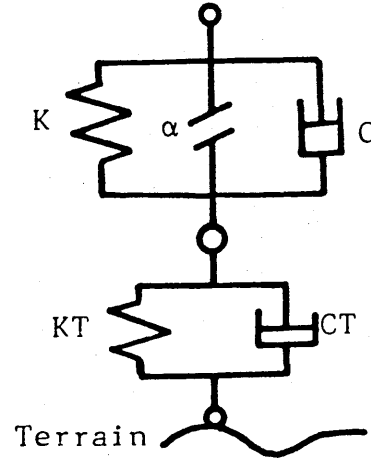
$$\text{e) } (F_z)_3 = C T (W_1' - W_2') + K T (W_1 - W_2)$$

2.7 THE CHARACTERISTIC EQUATIONS OF THE ELEMENT TYPE 1(d):

This element represents the hydrogas unit used in recent times in the suspension system of vehicles (see Fig 15). Coulomb friction has a significant effect on the performance of the unit.

(47)

$$\begin{aligned}
 \text{a) } (F_z)_1 &= C W_0' + (Y - Y_0) C \theta' \\
 &+ (1 + \alpha) [(X_0 - X) C \phi' \\
 &+ K W_0 + K (Y - Y_0) \theta \\
 &+ K (X_0 - X) \phi - K W_1] \\
 &- C W_1'
 \end{aligned}$$



where

Fig(15)Element type

α is the Coulomb friction coefficient.

1(d) model.

$$\begin{aligned}
 \text{b) } M_x &= (Y - Y_0) [C W_0' + (Y - Y_0) C \theta' - C W_1'] \\
 &+ (1 + \alpha) (Y - Y_0) [(X_0 - X) C \phi' + K W_0 \\
 &+ K (Y - Y_0) \theta + K (X_0 - X) \phi - K W_1] \\
 \text{c) } M_y &= (X_0 - X) [C W_0' + (Y - Y_0) C \theta' - C W_1'] \\
 &+ (1 + \alpha) (X_0 - X) [(X_0 - X) C \phi' + K W_0 \\
 &+ K (Y - Y_0) \theta + K (X_0 - X) \phi - K W_1]
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } (F_z)_2 &= (C + CT) W_1' + [(1 + \alpha) K + KT] W_1 \\
 &- C W_0' - (Y - Y_0) C \theta' - (X_0 - X) C \phi' \\
 &- (1 + \alpha) K [W_0 + (Y - Y_0) \theta + (X_0 - X) \phi] \\
 &- KT W_2 - CT W_2'
 \end{aligned}$$

$$\text{e) } (F_z)_3 = CT (W_1' - W_2') + KT (W_1 - W_2)$$

2.8 SIMULATION OF SEMI-TRACKED VEHICLES:

In this case the vehicle under consideration comprises the 1(a), 1(b), and 2 types of element. The front two wheel assemblies are the same as for element type 1(a) while the remaining wheel assemblies are of type 1(b) with the track included. This arrangement has excellent controllability for crosscountry operation. Fortunately there is no need to derive a new set of differential equations in order to simulate this type of vehicle because the suspension system consists of elements whose characteristic equations have already been derived. All that is required is for the assembler to recognise the type of element needed in the assembly. This is one of the advantages of employing the FEM technique in the field of vehicle dynamics.

2.9 TYPES OF NONLINEARITIES:

There are three types of nonlinearity which can arise through derivation of the characteristic equations.

a) Large Deflection Nonlinearity

This nonlinearity derives mainly from large deflection consideration of the vehicle pitch and roll. Hence substituting equations (2.1-2.3) in the main set of differential equations (as 2.7, 2.9, 2.11, and 2.13) produce the final set of differential equations.

b) Stiffness and Damping Nonlinearity:

This nonlinearity originates from the nonlinear performance of springs and dampers usually found in practice, and in analytical terms means that the stiffness and damping matrices have dependant variables as deflection and velocity. There is no difference between linear and nonlinear form of the final set of the differential equations.

c) Bump Stop Nonlinearity:

The function of the bump stop is to attenuate the bottoming of the vehicle by restoring some or all its energy in the elastic form. The bump stop can consist of either rubber or steel springs, and its

action begins when the following relation applies:

$$W_a + ST - Wl < 0$$

where

W_a is the displacement of point a on the main frame which lies directly above the wheel centre,

ST travel from the initial position to the bump stop

In this case the bump stop spring assists the assembly main spring. Necessary alterations to the set of differential equations should be carried out in order to include this nonlinearity. For example the equation (2.8) should be:

$$\begin{aligned} F(t) - \sum KB_r (ST_r - Z_r) = M_s W_0'' + W_0' \sum C_r + \theta' \sum C_r (Y_r - Y_0) \\ + \phi' \sum C_r (X_0 - X_r) + W_0 \sum (K_r + KB_r) + \\ + \theta \sum (K_r + KB_r) (Y_r - Y_0) - \sum C_r Wl_r' \\ + \phi \sum (K_r + KB_r) (X_0 - X_r) - \sum (K_r + KB_r) Wl_r \end{aligned}$$

where

KB is the stiffness coefficient of the bump stop.

2.10 TWO-DIMENSIONAL VEHICLE SIMULATION (2-D):

This is the case in which the vehicle vibration is considered in the Y-Z plane. It is a special case of all the three-dimensional (3-D) vehicle simulation in the previous sections. To derive the set of differential equations representing the 2-D simulation from the 3-D condition, it is simply necessary to cancel the third equation and all of those terms which include ϕ .

CHAPTER 3

VEHICLE-TERRAIN INTERACTION

Vehicle-terrain interaction can be completely defined by a proper selection of the tyre model, provided that the terrain surface is assumed to be rigid.

3.1 TYRE MODELS:

No mathematical model of a tyre has yet demonstrated accuracy in predicting both frequencies and amplitude in all directions. The fundamental reason is that tyre behaviour, both on-road and off-road, is not sufficiently understood. For this reason a brief review of available tyre models is given below in order to justify the selection of appropriate alternatives for the ride model.

Crolla [34] and Captain [35] reviewed the different tyre models as follows:

a) The Point Contact Tyre Model (Fig 16-a):

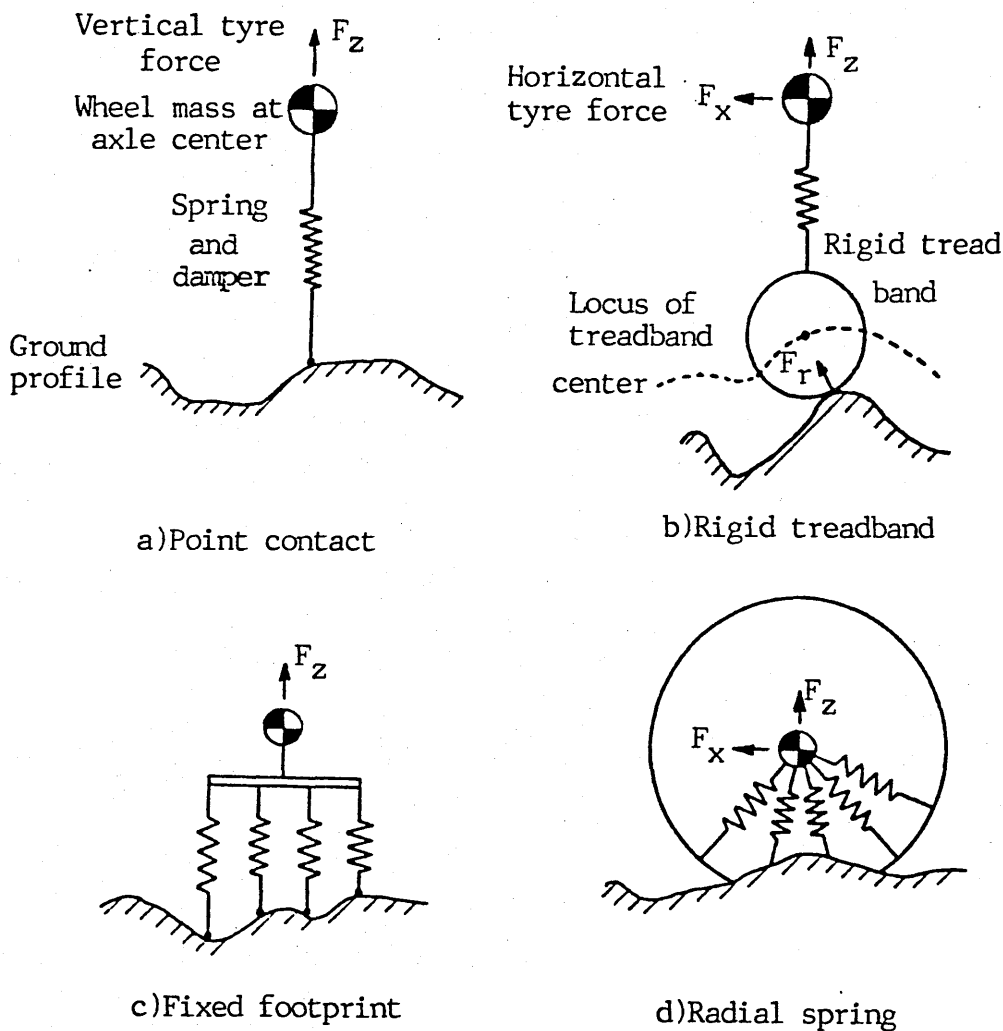
The point contact model is represented by a parallel spring-dashpot combination that transmits the support force from the terrain to the vehicle and contacts the ground through a point follower. Dynamic support forces occur due to deflection of the spring and the dashpot caused by the motion of the wheel relative to the terrain. The tyre mass is concentrated at the wheel centre, and the terrain follower is free to leave the ground to simulate wheel hop.

This model has been used by the authors of references [12,36-39] and many others, for simplicity. No published evidence is available concerning the limits of the model validity, either through verification with experimental data or by comparison with more sophisticated analytical predictions. This model overestimates the vertical force [34,35]. However, the model can be used at low frequency.

b) Rigid Tread Band Tyre Model (Fig 16-b):

This model is similar to the point contact model, with the modification that the point follower is replaced by a roller follower of diameter equal to the tyre diameter. This model is equivalent to a point contact model operating over the modified profile described by

the centre of the rigid wheel (tread band) as it rolls over the real profile. For gradual changes in profile elevation, filtering becomes significant, and the point contact and the rigid tread band models give equivalent results. This model allows for wheel hop and overestimates the transmitted tyre forces. This might be because the model is unable to simulate fully the dominant footprint characteristics of a real tyre.



Fig(16) Various tyre models.

c) Fixed Footprint Tyre Model (Fig 16-c):

The fixed footprint tyre model interacts with the terrain through a footprint of constant size, independent of the tyre deflection. In fact, with linear tyre characteristics this model is equivalent to a

point contact model in which the local terrain elevation is replaced by the average terrain elevation taken across the footprint length.

d) Adaptive Footprint Tyre Model (Fig 16-d):

The adaptive footprint tyre model consists of a flexible tread band inflated by internal pressure and linked to the wheel centre by angularly distributed stiffness and damping. This model was described by Lessen [40] and developed by Captain [35] and referred to as a segmented wheel. The model performs well over the widest range of frequencies.

e) Other Tyre Models:

- (1) A model in which the tyre forces and rolling torque are assumed to be functions of radial deflections, tangential slip and rolling speed, with the vertical and longitudinal forces coupled by the effect of tyre damping and variation in effective rolling radius.
- (2) A model in the frequency domain by estimating or measuring the tyre spatial frequency response with respect to ground input.
- (3) Finite element model which deals with the tyre as a thin, damped, elastic shell.

3.1-1 DISCUSSION:

For a modified input terrain profile, the point contact tyre model is equivalent either to the rigid tread band tyre model or fixed footprint tyre model. This equivalence is independent of the tyre linear or nonlinear characteristics. The point contact tyre model is equivalent to the adaptive footprint tyre model for the case of linear tyre characteristics. The point contact tyre model is the most simple of all. The segmented wheel concept used in the adaptive footprint model allows for realistic modelling of the effects of the tyre geometry and the distribution of forces in the tyre-terrain contact patch [11].

3.2 THE ADAPTIVE FOOTPRINT TYRE MODEL:

When the pneumatic tyre is represented by many spring elements, the displacement forcing function for one spring element need not be the same as that for the neighbouring spring. This feature permits small size terrain features with abrupt changes in slope to be 'enveloped' by the model. As each spring element is deflected by its displacement forcing function, the spring force produced and transmitted to the vehicle axle is determined. The magnitude of this force, together with knowledge of the location of the spring element, permits a force vector to be computed and resolved into horizontal and vertical components. By summing the simultaneous force contribution of all the spring elements, the total horizontal and vertical axle forces can be obtained. These forces are used as inputs to the mathematical model that represents the vehicle dynamic characteristics.

It was anticipated that a model composed of only spring elements would be adequate for high inflation pressure and low speeds; for low inflation pressures and high speeds, the absence of damping elements would reduce the models performance.

Simply stated the physical reality of a tyre rotating over stationary, segmented active region traversed by a moving terrain profile. This approach is a valid representation of a tyre dynamic analysis of vehicles on nonyielding terrain [35].

3.2-1 DETERMINATION OF SEGMENTAL SPRING COEFFICIENTS K_S :

The segments spring coefficients are the same for all segments. To calculate the value of K_S proceed as follows:

a) Select a suitable 'active region' angle α (see Fig 17-a).

In reality the model active region varies with time just as does the tyre contact area. The active region was assumed constant in size and should be large enough to include most anticipated fluctuations in contact area. Furthermore, select the number of segments n .

b) Obtain an effective radial deflection Δ corresponding to load F_1 from a load-deflection test of the tyre. In fact any point on the load-deflection curve of the tyre can be chosen with no essential change in

the development of the model.

c) calculate K_s from the following equation:

$$K_s = \frac{F_1}{\sum_{i=1}^n \Delta_i \cos \eta_i} \quad \dots (3.1)$$

where

Δ_i is the effective radial deflection of i^{th} segment,
 η_i is the force deflection angle of i^{th} segment, and

referring to Fig(16)b

$$\Delta_i = R (\cos \eta_i - 1) + \Delta > 0 \quad , \quad 0 \leq \eta_i \leq \frac{1}{2} \alpha, \quad \dots (3.2)$$

$$\Delta_i = 0 \quad \text{for} \quad \eta_i > \frac{1}{2} \alpha$$

c) Calculate the total vertical force F_z as follows:

$$F_z = K_s \sum_{i=1}^n W_{2i} \quad \dots (3.3)$$

Let the equivalent point contact model tyre stiffness coefficient be K_T , then

$$K_T = F_z / W_{20}$$

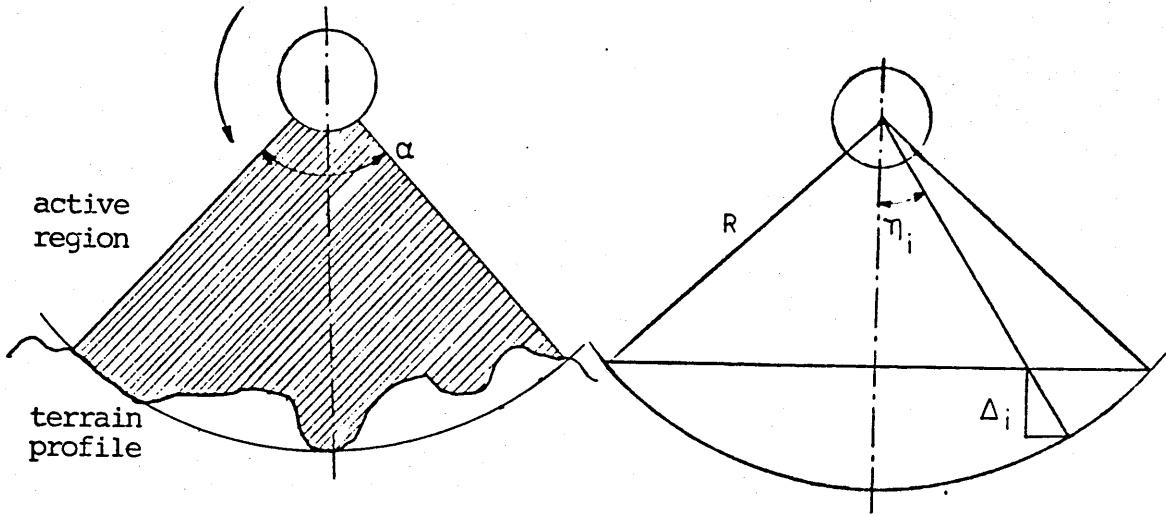
where W_{20} is the terrain elevation below the wheel centre.

3.3 FORCE FUNCTIONS:

Vehicle vibration is excited by terrain surface roughness which can be represented as a series of elevations Z_3 at corresponding Y_3 i.e

$$Z_3 = f(Y_3).$$

By introducing the vehicle velocity V , Z_3 becomes a function of time t also i.e



Fig(17) Extended contact tyre model.

$$Z_3 = f(Y_3, t)$$

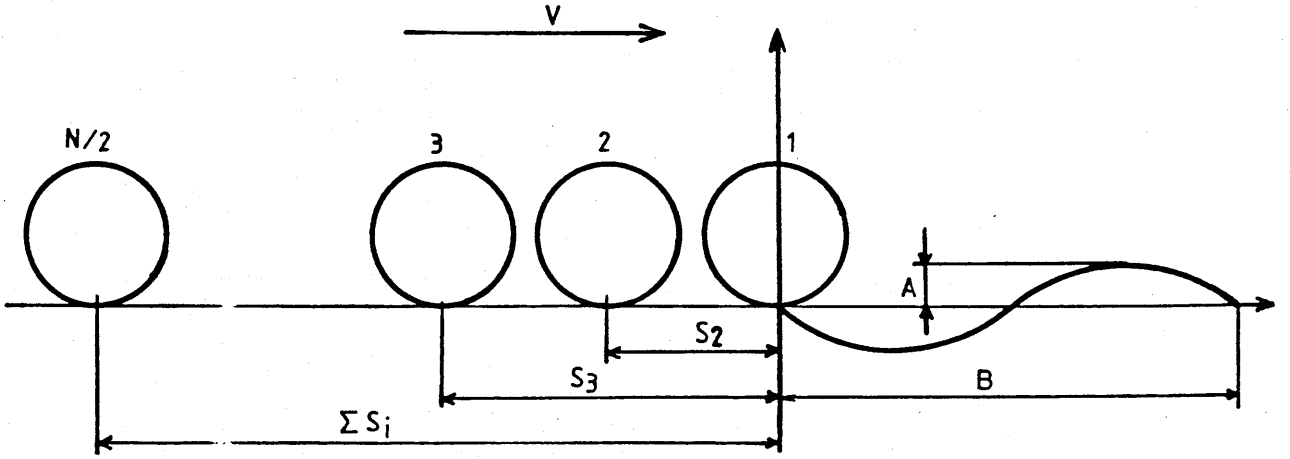
The terrain elevation can be measured at equal distances along the Y_3 -axis or unequal distances according to the shape of the terrain surface and/or data available. An interpolation formula should be used for the calculation of the elevation of the terrain at any point Y_3 not included in the data table given. It is simple to write any function in a form of a table but the reverse might not be true. Once the function Z_3 is determined, the velocity Z_3' can be derived as the first derivative.

The force functions resulting from the terrain are calculated using the values of Z_3 , KT , Z_3' , and CT .

3.3-1 HARMONIC FORCE FUNCTION:

This is a sine wave terrain surface profile found in many rigid test courses of vehicles. For example in reference [26] Maclaurin described several sine wave test courses with 0.1, 0.2, and 0.2 m amplitudes, and 4.5, 7, 12 m wave lengths respectively. Sometimes, a particular sine wave test course is used for a quick evaluation of the ride comfort of tanks. It is generally found that if a battle tank

cannot clear the second sinewave course (0.2 m amplitude and 7 m wave length)of its pitch resonant speed without bottoming on idler or suspension then its off-road performance is likely to be limited [26]. The resonance speed caused by a certain sine wave course can be employed for the calculation of the pitch moment of inertia of heavy vehicles [26].



Fig(18) Sine wave force function

From Fig(18) the right front wheel lies on the starting position of the force function at time $t = 0$ sec i.e $S_1 = 0$. The distance between the wheel number i and the wheel $i-1$ is S_i . After time t , the vehicle will have moved a distance $d = V t$ with respect to the terrain i.e all wheels will be shifted by the same distance d . To obtain the elevations of the terrain which produce the displacement of wheels proceed as follows:

(1) Calculate

$$Y_{2i} = d - \sum_{j=1}^i S_j, \quad i=1,2,\dots,N/2 \quad \dots (2.4)$$

(2) Calculate

$$Z_{2i} = A \sin (2 \pi Y_{2i} / B) \quad , \quad Y_{2i} > 0$$

(3) Derive

$$Z'_{2i} = 2 \pi (A/B) \cos (2 \pi Y_{2i} / B) \quad , \quad Y_{2i} > 0$$

(4) Given the damping and elastic characteristics of the tyre, the force function can be simply calculated as the right hand side of

equations (2.14), (2.19), and (2.23). The same procedure holds for the left side of the vehicle.

3.3-2 ARBITRARY FORCE FUNCTION:

The arbitrary terrain profile is usually associated with the crosscountry operation of off-road vehicles. However, there are rigid surface simulations of that yielded soil terrain. That simulation is considered as a true rigid surface filtered from high frequency components to account for the effect of wheel size and/or the presence of a track.

For terrain surface data given in a form of a table of elevations Z_{2i} corresponding to the coordinate Y_{2i} , The procedure employed to obtain the force function is as follows:

(1) Calculate Y_{2i} as in equation (3.4).

(2) Generally the value of corresponding Z_{2i} should be interpolated using an interpolation formula such as the three point Lagrangian polynomial. Those three points are (Y_{21}, Z_{21}) , (Y_{22}, Z_{22}) , and (Y_{23}, Z_{23}) Where

$$Y_{21} < Y_{2i} < Y_{23} \quad \text{then,}$$

$$\begin{aligned} Z_{2i} = & A1 (Y_{2i} - Y_{22})(Y_{2i} - Y_{23}) \\ & + A2 (Y_{2i} - Y_{21})(Y_{2i} - Y_{23}) \\ & + A3 (Y_{2i} - Y_{21})(Y_{2i} - Y_{22}) \end{aligned} \quad \dots (3.5)$$

where

$$A1 = \frac{Z_{21}}{(Y_{21} - Y_{22})(Y_{21} - Y_{23})} ,$$

$$A2 = \frac{Z_{22}}{(Y_{22} - Y_{21})(Y_{22} - Y_{23})}$$

(58)

,and

$$A3 = \frac{z_{31}}{(y_{23} - y_{21})(y_{23} - y_{22})}$$

(3) Differentiating equation (3.5) to get the tangential velocity $(z'_{2i})_t$, then

$$(z'_{2i})_t = 2(A1 + A2 + A3)y_{2i} - (A2 + A3)y_{21} - \\ (A1 + A3)y_{22} - (A1 + A2)y_{23}$$

and the vertical velocity is

$$z'_{2i} = (z'_{2i})_t \cdot \sin \eta \quad \dots (3.6)$$

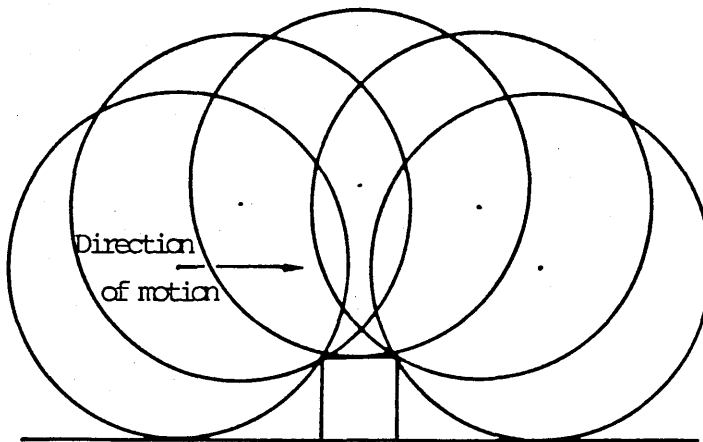
where

$$\eta = \tan^{-1} (z'_{2i})_t$$

(4) Again calculation of the force function can be simply carried out by substitution in the right hand side of equations (2.14), (2.19), and (2.23).

3.3-3 EFFECT OF WHEEL DIMENSION ON TERRAIN SURFACE:

The wheel dimension can be accounted for in the vehicle simulation or by modifying the terrain surface to correct for the wheel diameter. The latter is preferable because the whole terrain surface is modified once, whilst the former needs the modification to be made at each integration step for every wheel. An example of how to arrive at the displacement of centre of a wheel passing over a square block is shown in Fig(19).



609 mm. Diameter wheel passing 152X152 mm. block.



Displacement of centre of wheel passing over block.



Computer approximation displacement of wheel.

Fig(19)Modification of a square bump for computer use.

CHAPTER 4

LINEAR ANALYSIS OF SUSPENSION SYSTEMS

4.1 INTRODUCTORY SURVEY:

After the governing equations of motion have been derived, the engineer or the analyst is still faced with the problem of obtaining a solution of the differential equations and the initial conditions. This solution enables him to understand the features and response of the system, hence to realise the goal of the study. The word 'solution' means one or more or all of the following:

a) Eigenvalue solution and determination of the natural frequencies and mode shapes of the suspension system.

b) Calculation of the resonance frequencies of the damped system.

c) Steady-state response analysis i.e response of the vehicle to harmonic excitation.

d) Transient response analysis of the vehicle to arbitrary excitation.

The first two parts (a and b) are solvable for linear or instantly nonlinear systems. For the steady-state response (c), a simple method is available for the solution of the linear system with harmonic excitation. The solution of parts (a), (b), and (c) will illustrate the characteristics of the system at the design stage. The transient response analysis (d) can only be based on the ride evaluation. It is employed for linear- or nonlinear- systems analysis.

The general dynamic equations (2.15) which simulate the vehicle, are a set of second order differential equations which are employed for the solution of parts (a), (b), and (c). In most mathematics texts such as [30,41,42] the form (2.15) is written as:

$$\underline{\ddot{Z}} = \underline{M}^{-1} [\underline{F}(t) - \underline{C} \underline{\dot{Z}} - \underline{K} \underline{Z}] \quad \dots(4.1)$$

or

$$\underline{\ddot{Z}} = f(t, \underline{Z}, \underline{\dot{Z}})$$

where

the vector $\underline{Z}$ of order N

Furthermore, each second order differential equation can be reduced to two first order differential equations by introducing a new variable $\underline{ZI}$ as follows:

$$\underline{ZI} = \underline{Z}', \text{ then}$$

$$\underline{ZI}' = \underline{M}^{-1} [\underline{F}(t) - \underline{C} \underline{ZI} - \underline{K} \underline{Z}]$$

where

$$f(t, \underline{Z}, \underline{Z}') = \begin{bmatrix} \underline{ZI} \\ \underline{ZI}' \end{bmatrix} \quad \dots (4.2)$$

In this way the number of second order differential equations represented by (4.1) is doubled and transformed to first order differential equations. This reduction is desirable because it is simpler to deal with a system of first order differential equations, than with second order equations. Also the solution of the reduced system is explained in more detail in the literature from the point of view of time step size, associated errors, and stability. The part (d) of the solution may use the form (4.1) or the form (4.2) of the general dynamic system of equations.

Since these equations are very complex, the prospect of obtaining closed form solutions is remote. With the availability of analog and digital computers, engineers began to use the computer with the available numerical integration methods in order to solve the equations of motion. Such solution, however, involves a substantial amount of manpower for patching circuit boards for analog computation or writing FORTRAN programs to define the differential equations that are to be solved by numerical integration techniques. Because of the tremendous capability of digital computers nowadays, in both the speed of performance and storage capacity, it is now mostly used in vehicle dynamic analysis.

Generally, the explanations given in this chapter are restricted

to the digital computer use. For detailed mathematical derivations, the interested reader should consult the references listed at each section of the solution explanation.

4.2 EIGENVALUE PROBLEM:

Sometimes this problem is referred to as the free vibration problem i.e the solution of the undamped system without external excitation. In this case the equation (2.15) becomes

$$\underline{M} \underline{\ddot{Z}} + \underline{K} \underline{Z} = \underline{0} \quad \text{..(4.3)}$$

The solution of the problem is explained in the literature, for example [42,43]. In reference [43] a brief explanation is given of eigenvalue and eigenvector solutions using an iterative method for the calculation of the largest and the lowest eigenvalues. In reference [42] a detailed explanation is given of the solutions considered in [43], plus the calculation of the eigenvalue near a given frequency and a flowchart for computer use with an iterative method. In FEM books [26,27] the solution of the problem was given. In reference [26] the solution of the 'standard' eigenvalue was given in brief. An extensive explanation to the solution of the problem was given in references [27], [44], and [45].

4.2-1 SOLUTION OF THE EIGENVALUE PROBLEM:

The problem is defined as the solution of the following equation

$$[\underline{K} - \omega^2 \underline{M}] \underline{Z} = \underline{0} \quad \text{..(4.4)}$$

where

ω^2 is the eigenvalue , and

$\underline{Z}$ is the eigenvector.

The square root of the eigenvalue ω^2 is the natural frequency of the system ω . The number of natural frequencies equals the number of

degrees of freedom of the system, and there is an eigen vector $\underline{Z}$ for each natural frequency.

In most computer-based methods of solution, the eigenvalue problem is first converted into the form of a 'standard' eigenvalue problem, which can be stated as

$$[\underline{H} - \omega^2 \underline{I}] \underline{Z}^* = \underline{0} \quad \dots(4.5)$$

where

$$\underline{H} = (\underline{M}_{\underline{U}}^T)^{-1} \underline{K} \underline{M}_{\underline{U}}^{-1} ,$$

$$\underline{Z}^* = \underline{M}_{\underline{U}} \underline{Z} ,$$

$\underline{I}$ is a unit matrix ,

T is a superscript indicates a matrix transpose, and

U is a subscript indicates an upper triangular matrix using the Choleski decomposition method.

Matrices $\underline{H}$, $\underline{M}$, and $\underline{K}$ are square symmetric matrices of order $N+3$ and are definite positive.

There are two methods of solution, namely, the transformation methods such as Jacobi and Givens and Householder schemes, and iterative methods such as the power method. These methods are used successfully in the field of FEM and any of them will satisfy our needs because the number of degrees of freedom of the problem under investigation is very small compared to that encountered in the FEM field.

4.2-2 COMPUTING THE LARGEST EIGENVALUE BY THE POWER METHOD:

Step(1) Choose an initial vector $\underline{Z}_0$

Step(2) Generate a sequence of vectors $\underline{Z}_i$ from

$$\underline{Z}_i = \underline{H} \underline{Z}_{i-1}$$

(64)

Step(3) Calculate the ratio $\frac{\underline{z}_i}{\underline{z}_{i-1}}$

Step(4) Repeat steps 2 and 3 until two consequent values of the ratio are the same, or very close. This ratio is the eigenvalue and the vector $\underline{z}_i$ is the eigenvector $\underline{z}^{(1)}$ at the largest eigenvalue.

4.2-3 COMPUTING THE REMAINING EIGENVALUES:

Step(5) Calculate

$$\underline{H}^* = \underline{z}^{(1)} \underline{r}_1^T$$

where

$\underline{r}_1^T$ is the transpose of the first row of the matrix $\underline{H}$.

Step(6) Calculate

$$\underline{H} - \underline{H}^*$$

The resulting matrix is of order $(N+3) \times (N+3)$. By eliminating the first row and the first column a matrix $\underline{H}_2$ of order $(N+2) \times (N+2)$ is produced.

Step(7) Repeat the procedure explained in item 4.2-2 to obtain the largest eigen value for the matrix $\underline{H}_2$.

Step(8) Repeat steps 5, 6, and 7 to obtain all of the remaining eigenvalues.

In the event of failure of the Choleski decomposition of the mass matrix $\underline{M}$ to an upper triangular matrix $\underline{M}_U$, the conclusion is that this matrix is not positive definite and there is one or more negative eigenvalues. To cope with this new condition a small modification to equation (4.4) must be made as follows:

$$[\underline{K}_s - \omega^2 \underline{M}] \underline{Z} = 0 \quad \dots (4.6)$$

where

$$\underline{K}_s = \underline{K} + s \underline{M} \quad , \text{ and an estimation to } s \text{ is}$$

(65)

$$s = s_a \frac{\sum_{i=1}^{N+3} K_{ii}}{\sum_{i=1}^{N+3} M_{ii}}$$

The value of s should be positive and the value of s_a lies between zero and one. The algorithm will result in shifted eigenvalues ω_{si} . The original eigenvalues will be:

$$\omega^2 = \omega_{si}^2 - s$$

4.3 RESONANCE FREQUENCY ANALYSIS:

Occasionally, this analysis is known as the free response or damped dynamic eigenvalue analysis.

The system of differential equations to be solved is

$$\underline{M} \underline{\ddot{Z}} + \underline{C} \underline{\dot{Z}} + \underline{K} \underline{Z} = \underline{0} \quad \dots(4.7)$$

Assume the solution in the form

$$\underline{Z} = \underline{A} e^{j\omega t}$$

where

$$j = \sqrt{-1}$$

Substituting $\underline{Z}$, $\underline{\dot{Z}}$, and $\underline{\ddot{Z}}$ in equation (4.7), then

$$[-\Omega^2 \underline{M} + j \Omega \underline{C} + \underline{K}] \underline{A} = \underline{0} \quad \dots(4.8)$$

The solution to equation (4.8) is obtained by finding the complex roots, Ω_r , which make the real part of the determinant $R(\Omega)$ vanish.

$$\underline{R}(\Omega) = [-\Omega^2 \underline{M} + j \Omega \underline{C} + \underline{K}] \quad \dots(4.9)$$

where $R(\Omega)$ is the real part of the residual function.

A solution to this problem is given in vibration text references [46,47] for single degree of freedom systems, and in reference [48] for one and two DOF systems. In reference [49], the solution was extended to multidegree of freedom systems but in brief. In reference [43] the eigenvalue problem solution for a given range of frequency using the determinant search method was explained. The principle of the search method can be employed for the solution of the free response problem since the detection of resonance frequencies in a given range is required. Naturally, modification of the search technique was necessary in order to cope with the complex roots of equation (4.7), in addition to the adoption of a suitable iteration method.

4.3-1 DETERMINATION OF RESONANCE FREQUENCIES:

There are two types of solution employed for determining resonance frequencies, namely, analytical and graphical solutions. In fact the graphical solution is complementary to the analytical solution. The graphical solution is a sufficiently accurate technique, as well as being a direct, and simple method to use. However, it does require a graphical computer terminal.

a) Analytical solution algorithm:

Given the search range, begin with Ω_s , end value Ω_e and increment $\Delta \Omega$, then

Step(1) Calculate

$$R_1 = R(\Omega_s),$$

$$R_2 = R(\Omega_s + \Delta \Omega)$$

Step(2) Check for change of the residual function sign:

$$R_1 R_2 < 0$$

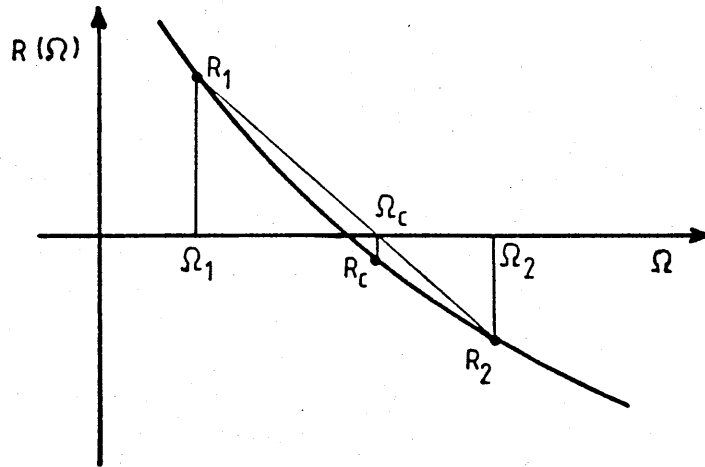
Step(3) There are three possible answers to step 2 as follows:

- (a) $R_1 R_2 = 0$ which means that either R_1 or R_2 is a root (unlikely to be the case for the first estimate).

In Fig(20)a, R_2 in the range 1, R_1 in the range 2 are resonance frequencies.

- (b) $R_1 R_2 > 0$ which means that both R_1 and R_2 have the same sign and there is no root between Ω_s and $\Omega_s + \Delta\Omega$, as is the case for ranges 1, 2, and 4 in Fig(20)b unless the axis is tangential to the residual function curve as in range 3.

- (c) $R_1 R_2 < 0$ which means that there is definitely a root in between Ω_s and $\Omega_s + \Delta\Omega$ as shown in Fig(20)c in ranges 1 and 2. A false position iteration method can be used to obtain the value of Ω at which $R(\Omega) = 0$ as follows (see Fig 21):



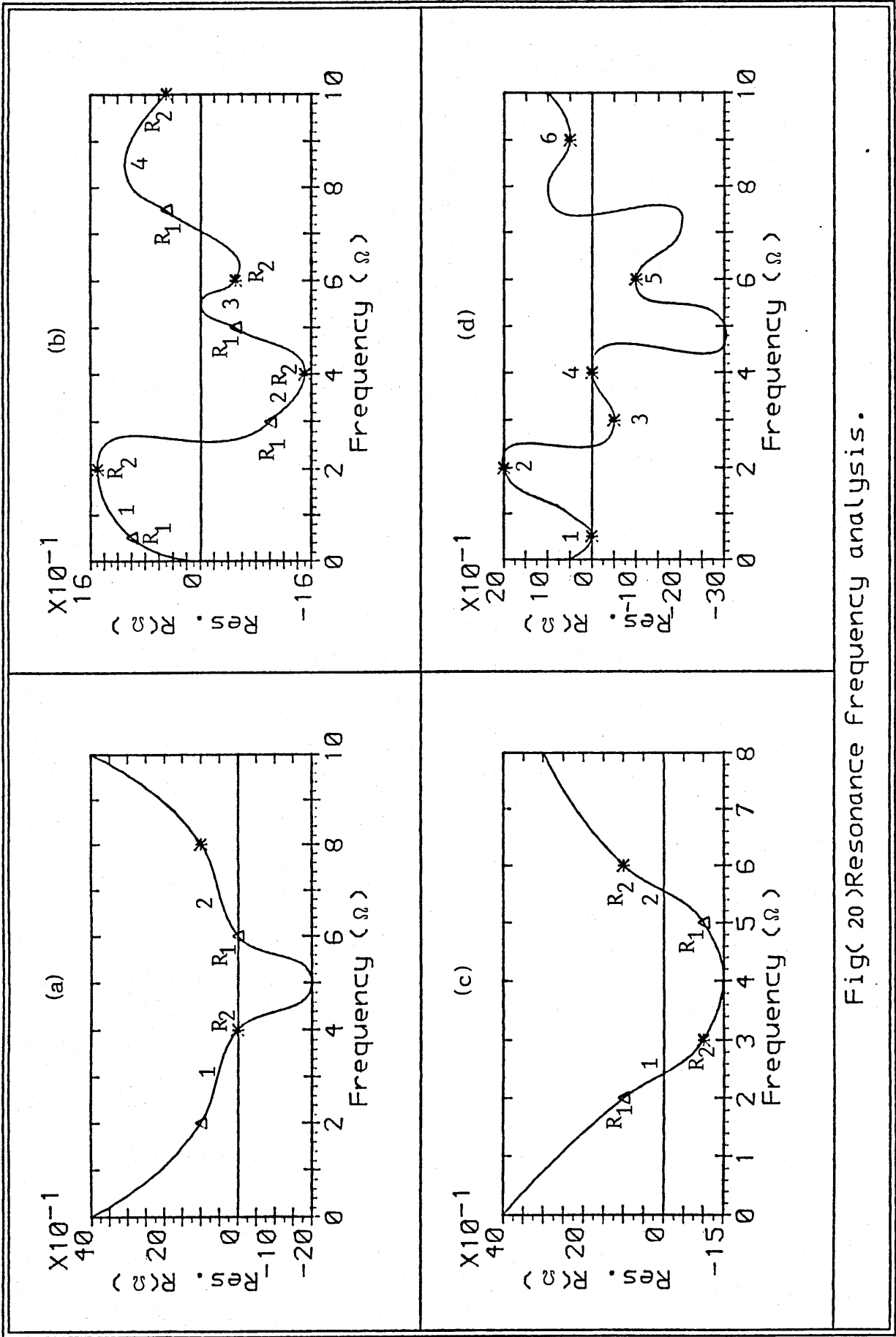
Fig(21) False position iteration method.

$$(1) \Omega_c = \Omega_1 - \frac{R_1 (\Omega_2 - \Omega_1)}{R_2 - R_1}$$

(2) Calculate $R_c = R(\Omega_c)$

(3) Check $|R_c| < \text{error}$.

(4) If $|R_c| > \text{error}$, consider the new Ω_c and the frequency corresponding to the residual of opposite sign to R_c and repeat from step (1). In this case $R_1 = R_1$ or R_c and $R_2 = R_c$ or R_2 respectively.



Fig(20)Resonance frequency analysis.

Step(4) Search in the next $\Delta\Omega$ where

$$\Omega_1 = \Omega_s + \Delta\Omega, \text{ and}$$

$$\Omega_2 = \Omega_1 + \Delta\Omega$$

Repeat from step(1) $R_1, R_2, \dots$ to the final frequency.

b) Graphical solution:

If the analytical solution fails to detect the roots of the function $R(\Omega)$ at which the Ω axis is tangential to the curve, (see Fig 20-b range 3), a simple plot of the curve of $R(\Omega)$ versus Ω will show the points of intersection of the horizontal axis Ω with the residual function, which are the required roots. However, because of the order of magnitude of $R(\Omega)$, the graph might not indicate clearly the roots for a large range of frequency. Although Fig(22) shows a resonance frequency at 4.2 rad./sec. and Fig(23) shows another resonance frequency at 7.5 rad./sec. Fig(24) is obscure and one cannot determine the resonance frequencies easily. For that reason only the immediate area of the resonance frequency is displayed, but the facility remains for displaying the whole range of frequency under consideration.

The conditions of resonance frequency are:

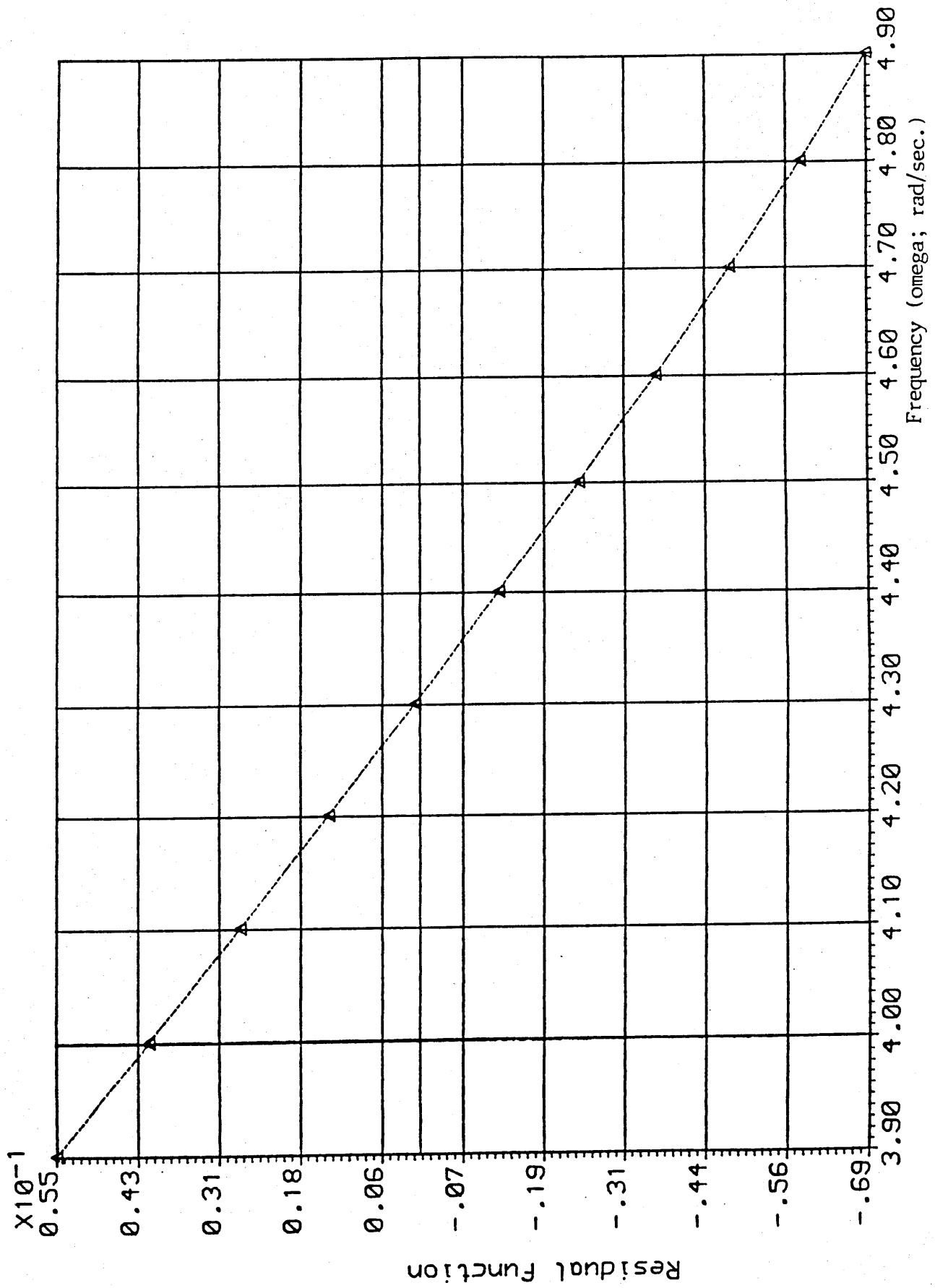
$$(1) R_1 R_2 < 0, \text{ and}$$

$$(2) (R_3 - R_2)(R_2 - R_1) < 0 \quad \dots(4.10)$$

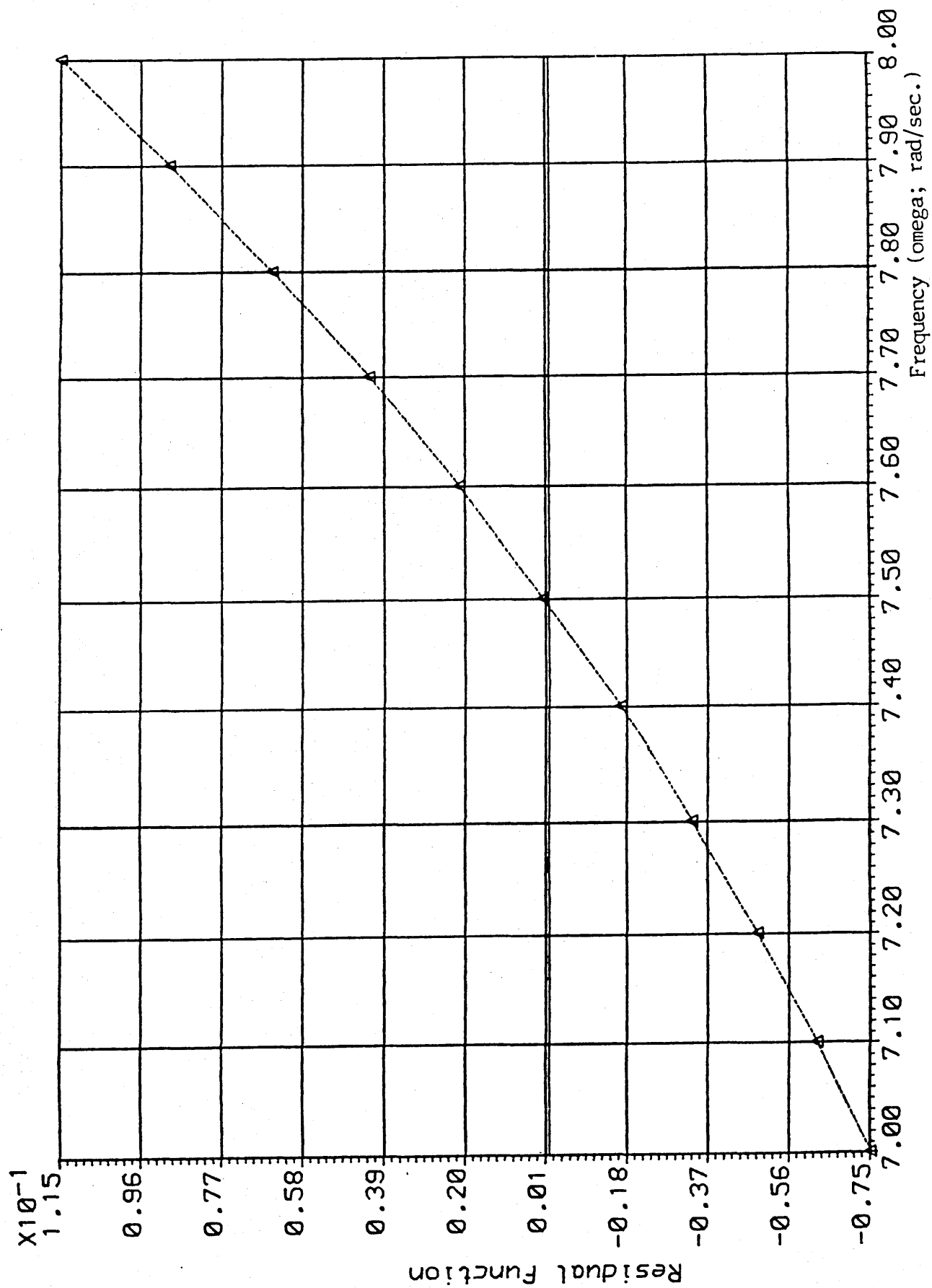
The above condition refers to the maximum and the minimum points (1-6) in Fig(20)d. Only two of the six points are required roots (1 and 4). To limit the search to these two points the following additional condition must be satisfied.

$$|R_2| \leq R_{\min}, \quad 0 > R_{\min} \leq 0.1$$

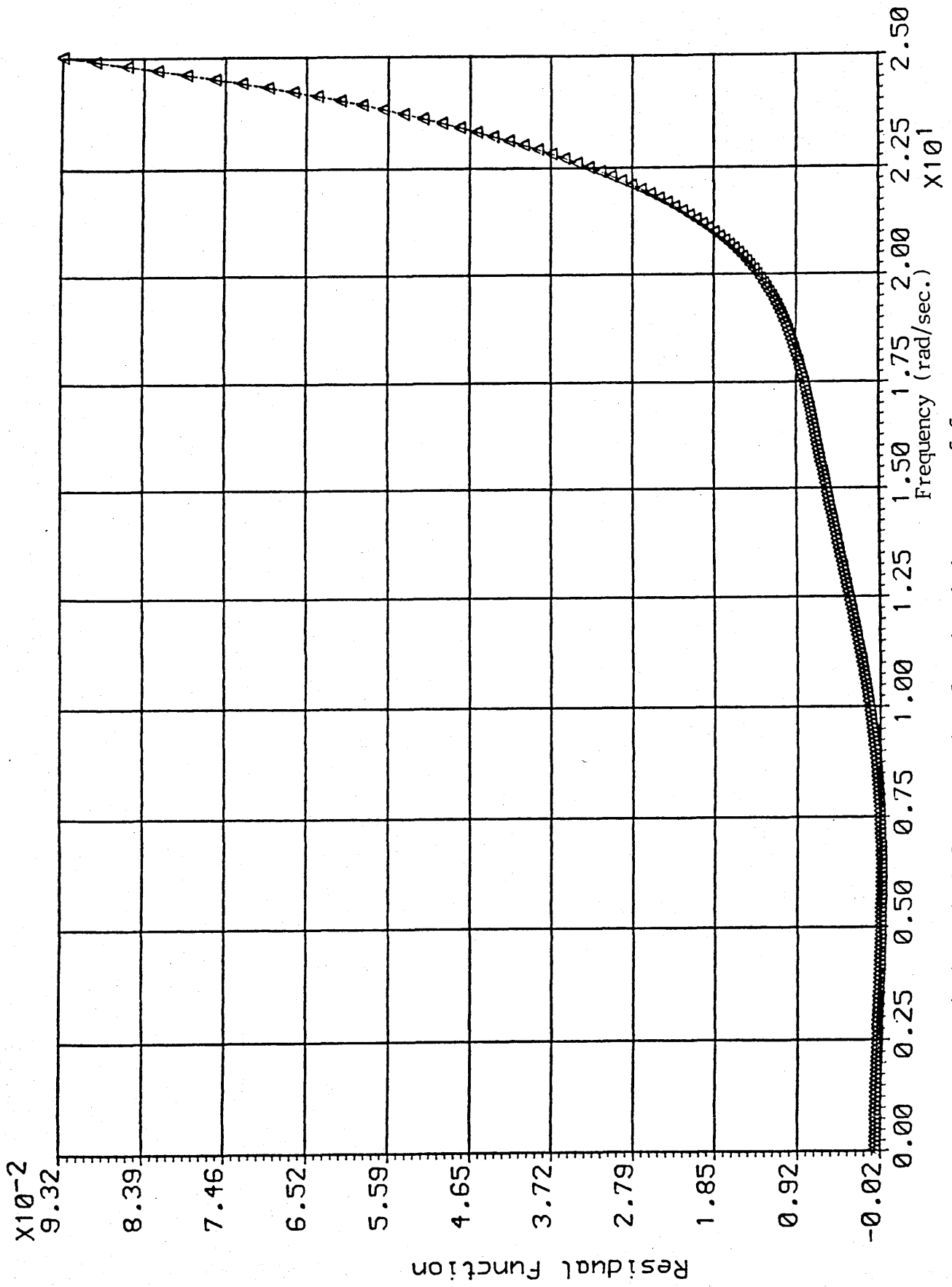
Finally, after following the above procedure the graph will definitely show the required root.



Fig(22) Residual function plot near resonance frequency of 4.3 rad/sec.



Fig(23)Residual function plot near a resonance frequency of 7.5 rad/sec.



Fig(24)Residual function plot for the whole range of frequency.

4.4 STEADY STATE RESPONSE:

The steady state response, is sometimes called the forced periodic response. In this section we are concerned with steady state general solutions which take no account of the initial conditions of the system or of the non-periodic form of the forcing term. The problem reduces to discovering the solution to the following system of equations:

$$\underline{M} \underline{Z}'' + \underline{C} \underline{Z}' + \underline{K} \underline{Z} = \underline{F}(t) \quad \dots (4.11)$$

where

$\underline{F}(t)$ is the periodic function of period T .

From Fourier analysis, this function can be expressed as:

$$\underline{F}_r = (\underline{a}_r \cos r \omega t + \underline{b}_r \sin r \omega t)$$

where

$$\omega = 2 \pi / T \quad ,$$

$$\underline{a}_r = \frac{2}{T} \int_0^T \underline{F}(t) \cos r \omega t \, dt \quad , \text{ and}$$

$$\underline{b}_r = \frac{2}{T} \int_0^T \underline{F}(t) \sin r \omega t \, dt$$

Then,

$$\underline{F}(t) = \frac{1}{2} a_0 + \sum_{r=1}^{\infty} \underline{F}_r$$

For linear systems, each $\underline{F}_r$ can be applied to the system separately and the final result can be obtained by superposition.

Hence, it is required to solve the following systems:

$$\underline{M} \underline{Z}_r'' + \underline{C} \underline{Z}_r' + \underline{K} \underline{Z}_r = \underline{F}_r \quad \dots (4.12)$$

where

(74)

$$r = 0, 1, 2, \dots$$

and the final answer is given by:

$$\underline{z}(t) = \sum_{r=0}^{\infty} \underline{z}_r(t)$$

4.4-1 SOLUTION OF STEADY STATE RESPONSE:

A suitable method of solution employing complex variable algebra follows. The forcing function can be expressed in the form of complex variables, thus

$$\begin{aligned}\underline{F}_r &= \underline{A}_r \cos r \omega t + \underline{B}_r \sin r \omega t \\ &= \text{Real part of } [(\underline{A}_r + j \underline{B}_r) e^{j r \omega t}] \\ &= \text{Real part of } [\underline{\tilde{F}}_r e^{j r \omega t}]\end{aligned}$$

where

$$\underline{\tilde{F}}_r = \underline{A}_r + j \underline{B}_r$$

Consider the r^{th} system of equations (4.12), a solution of this system can be assumed as follows:

$$\begin{aligned}\underline{z}_r(t) &= \underline{\alpha}_r \cos r \omega t + \underline{\beta}_r \sin r \omega t \\ &= \text{Real part of } [\underline{\tilde{z}}_r e^{j r \omega t}]\end{aligned}$$

where

$$\underline{\tilde{z}}_r = \underline{\alpha}_r + j \underline{\beta}_r$$

Hence,

$$\begin{aligned}\underline{z}_r'(t) &= -r \omega (\underline{\alpha}_r \sin r \omega t - \underline{\beta}_r \cos r \omega t) \\ &= \text{Real part of } [-j r \omega \underline{\tilde{z}}_r e^{j r \omega t}], \text{ and} \\ \underline{z}_r''(t) &= - (r \omega)^2 (\underline{\alpha}_r \cos r \omega t + \underline{\beta}_r \sin r \omega t) \\ &= \text{Real part of } [- (r \omega)^2 \underline{\tilde{z}}_r e^{j r \omega t}]\end{aligned}$$

(75)

Substituting in the differential equation (4.12), it can be shown that

$$\text{Real part of } [\{ - (r\omega)^2 \underline{M} - j (r\omega) \underline{C} + \underline{K} \} \underline{\tilde{Z}}_r e^{j\omega t}] =$$
$$\text{Real part of } (\underline{\tilde{F}}_r e^{j\omega t})$$

The solution of the above system of equations is equivalent to the solution of the following system:

$$[- (r\omega)^2 \underline{M} - j (r\omega) \underline{C} + \underline{K}] \underline{\tilde{Z}}_r = \underline{\tilde{F}}_r$$

which represents a system of simultaneous complex equations. This system can be solved by many solvers such as the Gauss elimination method.

4.4-2 FORCED HARMONIC RESPONSE:

This is a special case of the forced periodic response. The forcing function vector is in the form of a sine wave. The number of harmonics r in this case is one. Following the same procedure as in section 4.4-1,

$$\underline{M} \underline{\ddot{Z}} + \underline{C} \underline{\dot{Z}} + \underline{K} \underline{Z} = \underline{F}_1 \quad \dots (4.13)$$

where

$$\underline{F}_1 = \text{Real part of } [\underline{\tilde{F}}_1 e^{j\omega t}] , \text{ and}$$

$$\underline{\tilde{F}}_1 = jB$$

The solution of the system of equations (4.13) is assumed to be

$$\underline{Z}(t) = \text{Real part of } [\underline{\tilde{Z}} e^{j\omega t}]$$

CHAPTER 5

TRANSIENT RESPONSE ANALYSIS

5.1 REVIEW:

This chapter concerns the solution of the general dynamic system of equations (2.15) with arbitrary forcing function $\underline{F}(t)$. Two factors determine the forcing function, the tyre characteristics and the terrain surface roughness.

Fortunately, a number of well-tested and efficient computer subprograms are available for the solution of the system of differential equations (2.15). One major class of such programs is based on the so called 'one-step' or 'direct' methods such as the explicit Runge-Kutta methods. These methods have the advantage of being self-starting, are free of iterative processes, and readily permit arbitrary time increments to successive times where results are needed. A second major class of numerical integration routines belong to the so called 'predictor-corrector' or 'multistep methods'.

The main advantage of these iterative procedures over the one-step procedures is that they permit more direct control over the size of the numerical errors developed. However, most predictor-corrector methods suffer from the drawbacks of starting difficulties, and inability to change the time step size once started.

There are many one-step methods with which the system of equations (2.15) could be solved for the case of general excitation. Euler's method is the simplest of all but its accuracy might be limited. Taylor's method is not practical unless the function $\underline{Z}$ in equation (5.1) is so simple that many of its derivatives vanish. Implicit Runge-Kutta methods need to solve a system of nonlinear equations for each step. Explicit Runge-Kutta methods are widely quoted in the literature for solving the general dynamic system of differential equations. Those methods are direct, self-starting, free from iterative procedure in addition to the capability of control of time step size and developed error [30,42]. However, the stability of those methods are conditional and there is no available procedure for selecting the order of the method to be used. To allow for the instability of a particular solution method, various methods have to be included in one

package. Weighted residual methods have produced good results in the area of FEM.

5.2 RUNGE-KUTTA-NYSTROM METHOD:

Runge-Kutta methods can be used for solving the first-order system of differential equations (4.2) with order one or more. However, the low-order methods produce less accurate results as compared to higher-order methods. The error values decrease as the time step size h diminishes or the order of the method increases. If the number of evaluations of the function f for two different order methods is the same, the higher-order methods are economical for obtaining high accuracy with small step size [30]. An algorithm based on the Runge-Kutta-Nystrom formula of fourth order is given below.

For known initial conditions $\underline{Z}_0$ and $\underline{Z}'_0$ at time t_0 , calculate,

$$\underline{L}_1 = \frac{1}{2} h^2 f(\underline{t}_0, \underline{Z}_0) ,$$

$$\underline{L}_2 = \frac{1}{2} h^2 f(\underline{t}_0 + \frac{2}{5} h, \underline{Z}_0 + \frac{2}{5} h \underline{Z}'_0 + \frac{4}{25} \underline{L}_1) ,$$

$$\underline{L}_3 = \frac{1}{2} h^2 f(\underline{t}_0 + \frac{2}{3} h, \underline{Z}_0 + \frac{2}{3} h \underline{Z}'_0 + \frac{4}{9} \underline{L}_1)$$

,and

$$\underline{L}_4 = \frac{1}{2} h^2 f[\underline{t}_0 + \frac{4}{5} h, \underline{Z}_0 + \frac{4}{5} h \underline{Z}'_0 + \frac{8}{25} (\underline{L}_1 + \underline{L}_2)] ,$$

The final results are:

$$\underline{Z} = \underline{Z}_0 + h \underline{Z}'_0 + \frac{1}{96} (23 \underline{L}_1 + 75 \underline{L}_2 - 27 \underline{L}_3 + 25 \underline{L}_4) ,$$

$$\underline{Z}' = \underline{Z}'_0 + \frac{1}{96h} (23 \underline{L}_1 + 125 \underline{L}_2 - 81 \underline{L}_3 + 125 \underline{L}_4)$$

,and

$$\underline{Z}'' = f(\underline{t}_0 + h, \underline{Z}, \underline{Z}') \quad \dots (5.1)$$

5.3 RUNGE-KUTTA (R.K) METHOD WITH VARIABLE TIME STEP SIZE AND CONTROLLED ERROR:

In a general purpose program it is necessary to estimate the local truncation error from a knowledge of the numerical solution only, and to use this information to control the time step size. A technique called 'step doubling' is applicable to all methods of solution but is usually restricted to the Runge-Kutta methods because probably there are more suitable techniques for the other methods [30]. For the same order of method used, each basic step of size h is repeated twice, once as two steps of size $\frac{1}{2}h$ and once as one step of size h . The difference between the two results is a measure of the error at that step. A calculated relative error which is smaller than the required error bound, leads to double the time step size, whilst a greater relative error leads to the rejection of the whole step results and a repeat of the process using a given minimum step size. The following algorithm is substituted for the one explained in section (5.2).

Step(1) Using procedures explained in the last section, calculate the vectors $\underline{Z}$ and $\underline{Z}'$ with time step size h .

Step(2) Repeat step(1) twice to calculate $\underline{Z}^*$ and $\underline{Z}'^*$ with two time steps of size $\frac{1}{2}h$.

Step(3) Calculate the relative errors $\underline{Er}$:

$$\underline{Er}_1 = | \underline{Z}^* - \underline{Z} | / 15 \quad , \quad \underline{Er}_2 = | \underline{Z}'^* - \underline{Z}' | / 15$$

and the maximum value $Er_{\max}$ of the relative errors

$$\underline{Er}_3 = \frac{\underline{Er}_1}{\epsilon \underline{Z}_{\max}} \quad , \quad \underline{Er}_4 = \frac{\underline{Er}_2}{\epsilon \underline{Z}'_{\max}}$$

where ϵ is the error test constant (selected by the user)

Step(4) According to the value of $Er_{\max}$ proceed as follows:

a) If $Er_{\max} = 0$, double the time step size for the next time step

b) If $0 < Er_{\max} < 1$ decrease the time step size as

$$h_{\text{new}} = h Er_{\max}^{-0.198} \quad \text{for the next time step.}$$

c) If $Er_{\max} > 1$, reject the results and use the minimum step size $h_{\min}$ provided by the user and repeat the calculations for the same step.

5.4 LAGRANGIAN-WEIGHTED RESIDUAL (L.W.R) METHOD:

Generally the weighted residual methods have been widely used with considerable success in the field of FEM. This could be because of the availability of many weighting functions which makes the method suitable for the solution of a broad range of problems. This method is directly applicable to the general dynamic system of equations (4.1).

Let $f_1 = f(t_1, Z, Z')$

$$f_2 = f(t_2, Z, Z')$$

$$f_3 = f(t_3, Z, Z'), \text{ and}$$

$$t_3 - t_2 = t_2 - t_1 = h$$

We can write

$$f(t, Z, Z') = N_1 f_1 + N_2 f_2 + N_3 f_3$$

and use an intrinsic time coordinates ξ such that:

$$\xi = \frac{t - t_1}{t_3 - t_1}, \text{ then}$$

$$N_1(\xi) = (1 - \xi)(1 - 2\xi),$$

$$N_2(\xi) = 4\xi(1 - \xi), \text{ and}$$

$$N_3(\xi) = -\xi(1 - 2\xi)$$

The same rule can be applied to any nodal vector defined at three nodes, hence

$$\underline{F} = N_1 \underline{F}_1 + N_2 \underline{F}_2 + N_3 \underline{F}_3$$

where

$$\underline{F}_i = \underline{F}(t_i) \quad , \quad i = 1, 2, 3, \text{ and}$$

the displacement vector

$$\underline{Z}(t) = N_1 \underline{Z}_{(1)} + N_2 \underline{Z}_{(2)} + N_3 \underline{Z}_{(3)} \quad ,$$

$$\underline{Z}'(t) = \frac{\partial}{\partial t} \underline{Z}(t) = \frac{\partial}{\partial \xi} \underline{Z}(t) \frac{\partial \xi}{\partial t}$$

$$= \frac{1}{t_3 - t_1} \frac{\partial \underline{Z}}{\partial \xi}$$

$$= \frac{1}{2h} (N_1' \underline{Z}_{(1)} + N_2' \underline{Z}_{(2)} + N_3' \underline{Z}_{(3)})$$

where

$$N_i' = \frac{d N_i}{d \xi}$$

Similarly

$$\underline{Z}''(t) = \left(\frac{1}{2h} \right)^2 (N_1'' \underline{Z}_{(1)} + N_2'' \underline{Z}_{(2)} + N_3'' \underline{Z}_{(3)})$$

where

$$N_i'' = \frac{d^2 N_i}{d \xi^2}$$

The vector of residuals $R(\xi)$ is:

$$\underline{R}(\xi) = \underline{M} \underline{Z}'' + \underline{C} \underline{Z}' + \underline{K} \underline{Z} - \underline{F}$$

$$= \left(\frac{1}{2h} \right)^2 \underline{M} (N_1'' \underline{Z}_{(1)} + N_2'' \underline{Z}_{(2)} + N_3'' \underline{Z}_{(3)})$$

$$+ \frac{1}{2h} \underline{C} (N_1' \underline{Z}_{(1)} + N_2' \underline{Z}_{(2)} + N_3' \underline{Z}_{(3)})$$

$$+ \underline{K} (N_1 \underline{Z}_{(1)} + N_2 \underline{Z}_{(2)} + N_3 \underline{Z}_{(3)})$$

$$- (N_1 \underline{F}_1 + N_2 \underline{F}_2 + N_3 \underline{F}_3)$$

Rearranging terms:

$$\begin{aligned} \underline{R}(\xi) &= \underline{P}_1 \underline{Z}_{(1)} + \underline{P}_2 \underline{Z}_{(2)} + \underline{P}_3 \underline{Z}_{(3)} \\ &\quad - (N_1 \underline{F}_1 + N_2 \underline{F}_2 + N_3 \underline{F}_3) \quad \dots (5.2) \end{aligned}$$

where

$$\underline{P}_i = -\frac{N_i''}{4h^2} \underline{M} + \frac{N_i'}{2h} \underline{C} + N_i \underline{K}, \quad i = 1, 2, 3$$

Using the standard weighted residual method:

$$\int_0^1 W(\xi) \underline{R}(\xi) d\xi = 0 \quad \dots (5.3)$$

and substituting from equation (5.2) into equation (5.3), then

$$\underline{Q}_1 \underline{Z}_{(1)} + \underline{Q}_2 \underline{Z}_{(2)} + \underline{Q}_3 \underline{Z}_{(3)} = \underline{F} \quad \dots (5.4)$$

where

$$\begin{aligned} \underline{Q}_i &= \int_0^1 W(\xi) \underline{P}_i d\xi \\ &= \left(\frac{1}{2h}\right)^2 \alpha_i \underline{M} + \frac{1}{2h} \beta_i \underline{C} + \gamma_i \underline{K} \end{aligned}$$

where

$$\begin{aligned} \alpha_i &= \int_0^1 W(\xi) N_i'' d\xi, \\ \beta_i &= \int_0^1 W(\xi) N_i' d\xi, \\ \gamma_i &= \int_0^1 W(\xi) N_i d\xi, \text{ and} \\ \underline{F} &= \int_0^1 W(\xi) (N_1 \underline{F}_1 + N_2 \underline{F}_2 + N_3 \underline{F}_3) d\xi \\ &= \gamma_1 \underline{F}_1 + \gamma_2 \underline{F}_2 + \gamma_3 \underline{F}_3 \end{aligned}$$

For the special case of Galerkin $W(\xi) = N_3$ it can be proved that

i	1	2	3
α_i	1	-2	1
β_i	0.5	-2	1.5
γ_i	-0.2	0.5	0.8

For a time marching scheme the equation (5.4) is the basis of the three node recurrence formula at nodes $i-1$, i , and $i+1$, hence

$$Q_3 \underline{Z}_{(i+1)} = \underline{F} - Q_1 \underline{Z}_{(i-1)} - Q_2 \underline{Z}_{(i)} \quad \dots (5.5)$$

In the recurrence formula (5.4), two subsequent values of $\underline{Z}$ are employed to calculate the following value of $\underline{Z}$. Once $\underline{Z}_{(i+1)}$ is known, $\underline{Z}'_{(i+1)}$ and $\underline{Z}''_{(i+1)}$ can be calculated from the following formula at time $t = t_{i+1}$ ($\xi = 1$)

$$\underline{Z}'_{(i+1)} = \frac{1}{2h} (\underline{Z}_{(i-1)} - 4 \underline{Z}_{(i)} + 3 \underline{Z}_{(i+1)}) \quad \dots (5.6)$$

$$\underline{Z}''_{(i+1)} = \frac{1}{h^2} (\underline{Z}_{(i-1)} - 2 \underline{Z}_{(i)} + \underline{Z}_{(i+1)}) \quad \dots (5.7)$$

At the start of the scheme, assume a fictitious node at t_{-1} , then

$$\underline{Z}(t) = N_1 \underline{Z}_{-1} + N_2 \underline{Z}_0 + N_3 \underline{Z}_1 \quad , \text{and}$$

$$\begin{aligned} \underline{Z}'(t) &= \frac{1}{2h} (N'_1 \underline{Z}_{-1} + N'_2 \underline{Z}_0 + N'_3 \underline{Z}_1) \\ &= \frac{1}{2h} [(-3 + 4 \xi) \underline{Z}_{-1} + 4 (1 - 2 \xi) \underline{Z}_0 \\ &\quad + (-1 + 4 \xi) \underline{Z}_1] \end{aligned}$$

For $\xi = 0.5$

$$\underline{Z}'_0 = \underline{Z}'(0.5) = \frac{1}{2h} (- \underline{Z}_{-1} + \underline{Z}_1)$$

which gives

$$\underline{z}_{-1} = \underline{z}_1 - 2 h \underline{z}'_0$$

Substitute $\underline{z}_{-1}$ in the recurrence formula (5.4), then

$$\begin{aligned} \underline{Q}_3 \underline{z}_1 &= \underline{F} - \underline{Q}_1 \underline{z}_{-1} - \underline{Q}_2 \underline{z}_0 \\ &= \underline{F} - \underline{Q}_1 (\underline{z}_1 - 2 h \underline{z}'_0) - \underline{Q}_2 \underline{z}_0 \end{aligned}$$

Thus

$$(\underline{Q}_1 + \underline{Q}_3) \underline{z}_1 = \underline{F} + 2 h \underline{Q}_1 \underline{z}'_0 - \underline{Q}_2 \underline{z}_0 \quad \dots (5.8)$$

which is a system of equations in $\underline{z}_1$.

In brief, given $\underline{z}_0$ and $\underline{z}'_0$, use equation (5.8) to start, and equation (5.4) after that.

In reference [26] there are eight types of weighting functions $W(\xi)$ calculated for the derivation of the values of α_i , β_i , and γ_i . Those types are given in appendix(1).

5.5 HERMITIAN-WEIGHTED RESIDUAL (H.W.R) METHOD:

In the field of vehicle dynamics, results in the form of an acceleration-time response are of great importance particularly in the area of ride comfort. The interpolation polynomial used for the case of the L.W.R solution method is based on the displacement vector. The velocity and acceleration vectors are calculated as functions of the displacement vector. Runge-Kutta methods, and the H.W.R of the first order, employ the displacement and the velocity vectors as the basis for the interpolation polynomials used in the solution. The acceleration vector is then calculated as a function of the displacement and the velocity vectors. The H.W.R of the first order is explained in reference [31]. The hyperoscillatory H.W.R of the second order might improve the accuracy by the inclusion of the highest order derivative

implied in the general dynamic system of equations (2.15), in the interpolation polynomial.

A general two node interpolation function is

$$\underline{z}(t) = \sum_{i=1}^2 \sum_{j=0}^2 H_{ji}(t) \underline{f}_i^j \quad \dots (5.9)$$

where the upper limit of j is the highest derivative to be considered.

To deduce the polynomials $H_{ji}(t)$, use the property

$$\frac{d^r H_{ji}^{(2)}}{dt^r} (t_p) = \delta_{ip} \delta_{jr} \quad \dots (5.10)$$

for $p=1,2$ and $j,r=0,1,2$

where

δ_{mn} is the kronecker delta having the property that

$$\begin{aligned} \delta_{mn} &= 0 & \text{for } m \neq n, \\ &= 1 & \text{for } m = n \end{aligned}$$

(2) Equation (5.10) gives six conditions for each of the polynomials $H_{ji}(t)$ and hence will be a fifth degree polynomial.

$$H_{01}^{(2)} = a_1 + a_2 t + a_3 t^2 + a_4 t^3 + a_5 t^4 + a_6 t^5$$

Since,

$$\begin{aligned} H_{01}^{(2)}(t_1 = 0) &= 1, & H_{01}^{(2)}(t_2) &= 0, \\ \frac{dH_{01}^{(2)}}{dt}(t_1 = 0) &= 0, & \frac{dH_{01}^{(2)}}{dt}(t_2) &= 0, \\ \frac{d^2 H_{01}^{(2)}}{dt^2}(t_1 = 0) &= 0, & \text{and } \frac{d^2 H_{01}^{(2)}}{dt^2}(t_2) &= 0 \end{aligned}$$

then,

$$a_1 = 1, \quad a_2 = 0, \quad a_3 = 0,$$

$$a_4 = -\frac{10}{t_2^3}, \quad a_5 = \frac{15}{t_2^4}, \quad \text{and} \quad a_6 = -\frac{6}{t_2^5}.$$

Thus the polynomial $H_{01}^{(2)}$ is given by

$$H_{01}^{(2)} = \frac{1}{t_2^5} (t^5 - 10 t_2^2 t^3 + 15 t_2 t^4 + 15 t_2 t^4 - 6 t^5) \quad \dots (5.11)$$

For the intrinsic time coordinates

$$H_{01}^{(2)} = 1 - 10 \xi^3 + 15 \xi^4 - 6 \xi^5$$

Similarly the remaining $H_{ji}^{(2)}$ are

$$H_{02}^{(2)} = 10 \xi^3 - 15 \xi^4 + 6 \xi^5 \quad \dots (5.12)$$

$$H_{11}^{(2)} = (\xi - 6 \xi^3 + 8 \xi^4 - 3 \xi^5) h,$$

$$H_{12}^{(2)} = (-4 \xi^3 + 7 \xi^4 - 3 \xi^5) h,$$

$$H_{21}^{(2)} = (\xi^2 - 3 \xi^3 + 3 \xi^4 - \xi^5) \frac{h^2}{2}, \quad \text{and}$$

$$H_{22}^{(2)} = (\xi^3 - 2 \xi^4 - \xi^5) \frac{h^2}{2},$$

The nodal force vector at any time can be expressed by the following Lagrangian interpolation formula:

$$\underline{F}(t) = (1 - \xi) \underline{F}_1 + \xi \underline{F}_2$$

Hence, the residual function can be expressed as follows:

$$\begin{aligned} \underline{R}(\xi) = & \underline{A}_1 \underline{Z}_{(1)} + \underline{A}_2 \underline{Z}_{(2)} + \underline{B}_1 \underline{Z}'_{(1)} + \underline{B}_2 \underline{Z}'_{(2)} \\ & + \underline{D}_1'' \underline{Z}_{(1)} + \underline{D}_2'' \underline{Z}_{(2)} - \underline{F} \end{aligned}$$

where

$$\underline{A}_i = \frac{1}{h^2} H_{0i}'' \underline{M} + \frac{1}{h} H_{0i}' \underline{C} + H_{0i} \underline{K},$$

$$\underline{B}_i = \frac{1}{h^2} H_{1i}'' \underline{M} + \frac{1}{h} H_{1i}' \underline{C} + H_{1i} \underline{K},$$

$$\underline{D}_i = \frac{1}{h^2} H_{2i}'' \underline{M} + \frac{1}{h} H_{2i}' \underline{C} + H_{2i} \underline{K}, \text{ and } i=1,2$$

Applying the Galerkin weighted-residual approach, then,

$$\int_0^1 H_{02}(\xi) R(\xi) d\xi = 0 \quad \dots (5.13)$$

$$\int_0^1 H_{12}(\xi) R(\xi) d\xi = 0 \quad \dots (5.14)$$

$$\int_0^1 H_{22}(\xi) R(\xi) d\xi = 0 \quad \dots (5.15)$$

Substituting H_{02} and $R(\xi)$ in (5.13), then

$$\begin{aligned} \int_0^1 (10\xi^3 - 15\xi^4 + 6\xi^5) (\underline{A}_1 \underline{Z}_{(1)} + \underline{A}_2 \underline{Z}_{(2)} + \underline{B}_1 \underline{Z}_{(1)}' \\ + \underline{B}_2 \underline{Z}_{(2)}' + \underline{D}_1 \underline{Z}_{(1)}'' + \underline{D}_2 \underline{Z}_{(2)}'' - \underline{F}) d\xi = 0 \quad \dots (5.16) \end{aligned}$$

Integrating the term which contains $\underline{Z}_1$ then,

$$\begin{aligned} \underline{Z}_1 \int_0^1 (10\xi^3 - 15\xi^4 + 6\xi^5) \left[\frac{1}{h^2} (-60\xi + 180\xi^2 - 120\xi^3) \underline{M} \right. \\ \left. + \frac{1}{h} (-30\xi^2 + 60\xi^3 - 30\xi^4) \underline{C} \right. \\ \left. + (1 - 10\xi^3 + 15\xi^4 - 6\xi^5) \underline{K} \right] d\xi \\ = \left(\frac{10}{7h^2} \underline{M} - \frac{1}{2h} \underline{C} + \frac{25}{231} \underline{K} \right) \underline{Z}_1 \end{aligned}$$

Completing the integration of equation (5.13) and proceeding in the same manner with equations (4.14) and (4.15), the final equations are:

$$\underline{R}_{11} \underline{Z}_1 + \underline{S}_{11} \underline{Z}_2 + \underline{R}_{12} \underline{Z}_1' + \underline{S}_{12} \underline{Z}_2' + \underline{R}_{13} \underline{Z}_1'' + \underline{S}_{13} \underline{Z}_2'' = \underline{F}_{(1)} \quad \dots (5.17)$$

$$\underline{R}_{21} \underline{Z}_1 + \underline{S}_{21} \underline{Z}_2 + \underline{R}_{22} \underline{Z}'_1 + \underline{S}_{22} \underline{Z}'_2 + \underline{R}_{23} \underline{Z}''_1 + \underline{S}_{23} \underline{Z}''_2 = \underline{F}_{(2)} \quad \dots (5.18)$$

$$\underline{R}_{31} \underline{Z}_1 + \underline{S}_{31} \underline{Z}_2 + \underline{R}_{32} \underline{Z}'_1 + \underline{S}_{32} \underline{Z}'_2 + \underline{R}_{33} \underline{Z}''_1 + \underline{S}_{33} \underline{Z}''_2 = \underline{F}_{(3)} \quad \dots (5.19)$$

where

$$\underline{R}_{11} = \frac{10}{7 h^2} \underline{M} - \frac{1}{2h} \underline{C} + \frac{25}{231} \underline{K} \quad ,$$

$$\underline{S}_{11} = \frac{-10}{7 h^2} \underline{M} - \frac{1}{2h} \underline{C} + \frac{905}{2310} \underline{K} \quad ,$$

$$\underline{R}_{12} = \frac{3}{14 h} \underline{M} - \frac{11}{84} \underline{C} + \frac{151}{4620} h \underline{K} \quad ,$$

$$\underline{S}_{12} = \frac{17}{14} \underline{M} - \frac{11}{84} \underline{C} + \frac{311}{4620} h \underline{K} \quad ,$$

$$\underline{R}_{13} = \frac{1}{84} \underline{M} - \frac{1}{84} h \underline{C} + \frac{181}{55440} h^2 \underline{K} \quad ,$$

$$\underline{S}_{13} = -\frac{37}{84} \underline{M} - \frac{1}{84} h \underline{C} + \frac{281}{55440} h^2 \underline{K} \quad ,$$

$$\underline{F}_{(1)} = \frac{1}{7} \underline{F}_1 + \frac{5}{14} \underline{F}_2 \quad ,$$

$$\underline{R}_{21} = \left(-\frac{3}{14} \underline{M} + \frac{11}{84 h} \underline{C} - \frac{151}{4620} \underline{K} \right) h \quad ,$$

$$\underline{S}_{21} = \left(\frac{3}{14} \underline{M} - \frac{11}{84 h} \underline{C} + \frac{311}{4620} \underline{K} \right) h \quad ,$$

$$\underline{R}_{22} = \left(\frac{1}{70} \underline{M} + \frac{13}{420} \underline{C} - \frac{19}{1980} h \underline{K} \right) h \quad ,$$

$$\underline{S}_{22} = \left(-\frac{8}{35} \underline{M} + \frac{52}{3465} h \underline{K} \right) h \quad ,$$

$$\underline{R}_{23} = \left(\frac{1}{210} \underline{M} + \frac{13}{5040} h \underline{C} - \frac{13}{13860} h^2 \underline{K} \right) h \quad ,$$

$$\underline{S}_{23} = \left(\frac{1}{60} \underline{M} + \frac{1}{1008} h \underline{C} - \frac{23}{18480} h^2 \underline{K} \right) h \quad ,$$

$$\underline{F}_{(2)} = h \left(-\frac{4}{105} \underline{F}_1 - \frac{13}{210} \underline{F}_2 \right) \quad ,$$

$$\underline{R}_{31} = \left(\frac{1}{42 h^2} \underline{M} - \frac{1}{42 h} \underline{C} + \frac{181}{27720} \underline{K} \right) \frac{h^2}{2} \quad ,$$

$$\underline{S}_{31} = \left(\frac{-1}{42 h^2} \underline{M} + \frac{1}{42 h} \underline{C} + \frac{281}{27720} \underline{K} \right) \frac{h^2}{2} ,$$

$$\underline{R}_{32} = \left(\frac{-1}{105 h} \underline{M} - \frac{143}{27720} \underline{C} + \frac{13}{6930} h \underline{K} \right) \frac{h^2}{2} ,$$

$$\underline{S}_{32} = \left(\frac{1}{30 h} \underline{M} - \frac{1}{504} \underline{C} + \frac{23}{9240} h \underline{K} \right) \frac{h^2}{2} ,$$

$$\underline{R}_{33} = \left(\frac{-1}{630} \underline{M} - \frac{1}{2520} h \underline{C} + \frac{1}{5544} h^2 \underline{K} \right) \frac{h^2}{2} ,$$

$$\underline{S}_{33} = \left(\frac{-1}{315} \underline{M} + \frac{1}{4620} h^2 \underline{K} \right) \frac{h^2}{2} ,$$

, and

$$\underline{F}_{(3)} = \frac{h^2}{2} \left(\frac{1}{140} \underline{F}_1 + \frac{1}{105} \underline{F}_2 \right)$$

The matrices representing the equations (5.17-5.19) can be written in the following form:

$$\begin{bmatrix} \underline{S}_{11} & \underline{S}_{12} & \underline{S}_{13} \\ \underline{S}_{21} & \underline{S}_{22} & \underline{S}_{23} \\ \underline{S}_{31} & \underline{S}_{32} & \underline{S}_{33} \end{bmatrix} \begin{bmatrix} \underline{Z}_{(2)} \\ \underline{Z}'_{(2)} \\ \underline{Z}''_{(2)} \end{bmatrix} = \begin{bmatrix} \underline{F}_{(1)} \\ \underline{F}_{(2)} \\ \underline{F}_{(3)} \end{bmatrix} - \begin{bmatrix} \underline{R}_{11} & \underline{R}_{12} & \underline{R}_{13} \\ \underline{R}_{21} & \underline{R}_{22} & \underline{R}_{23} \\ \underline{R}_{31} & \underline{R}_{32} & \underline{R}_{33} \end{bmatrix} \begin{bmatrix} \underline{Z}_{(1)} \\ \underline{Z}'_{(1)} \\ \underline{Z}''_{(1)} \end{bmatrix} \quad (5.20)$$

The solution of the system of equations (5.20) gives the displacement, velocity, and acceleration vectors $\underline{Z}_{(2)}$, $\underline{Z}'_{(2)}$, and $\underline{Z}''_{(2)}$ directly without any starting difficulty.

Because of the huge task of calculating the coefficients included in the large R and S matrices, a computer program using the Gauss quadrature formula (10 points) was used to check the values of all coefficients.

5.6 NONLINEAR TIME RESPONSE:

The types of nonlinearities usually found in the simulation of suspension systems are:

1-Nonlinear characteristics and/or blow-off function of the main damper.

2-Nonlinear characteristics of the main spring or torsion bar.

3-Nonlinearity originating from the bump-stop action.

4-Nonlinear characteristics of the tyre or the road wheel.

5-Nonlinearity resulting from large deflection considerations.

6-Nonlinearity arising from wheel-ground separation.

From the point of view of solution methods, the types of nonlinearity are subdivided into two groups. First group includes the nonlinearity produced by the large deflection assumption plus any combination of the remaining -mentioned above- nonlinearities. The second group includes any type of nonlinearity except that resulting from large deflections.

Accounting for the second group of nonlinearities, the form of the general dynamic equations simulating the suspension system, remain the same as (2.15), only except that one or more of the $\underline{C}$ and $\underline{K}$ matrices, and the $\underline{F}$ vector are nonlinear, hence

$$\underline{M} \underline{\ddot{z}} + \underline{C}(\underline{z}') \underline{\dot{z}} + \underline{K}(\underline{z}) \underline{z} = \underline{F}(t) \quad \dots (5.21)$$

Indeed the full integration of the previous equation can be carried out numerically and, in general, iteration within each time step appears to be required. Further, at each time step a different set of equations will require solution, thus making procedure costly. These iterative methods are not recommended by Zeinkiewicz [26]. An alternative procedure is the extrapolation of the various values of $\underline{M}$, $\underline{C}$, $\underline{K}$, or $\underline{F}$ from previously established steps. Such extrapolation leads to more stable solutions as indeed, within each interval, the usual linearised stability obtains. Further, instability which may sometimes arise solely from iteration is avoided.

With the large deflection assumption to be taken into consideration, the general dynamic equation will include harmonic terms and the matrix form is no longer suitable. A suitable form is

$$\underline{\ddot{z}} = \underline{f}(t, \underline{z}, \underline{z}') \quad \dots (5.22)$$

(90)

where

f is a nonlinear function.

An example of the form (5.22) can be derived from equation (2.7) by using equations (2.1-2.3) and the result is:

$$\begin{aligned} \ddot{z} = & M^{-1} [F(t) - W_0' \sum C_r - \theta' \sum C_r B1_r - \phi' \sum C_r B2_r \\ & - W_0 \sum K_r - \sin \theta \sum K_r (Y_r - Y_0) - \sin \phi \sum K_r (X_0 - X_r) \\ & + \sum K_r B3_r + \sum C_r W1_r' + \sum K_r W1_r + g] \quad \dots (5.22) \end{aligned}$$

where

$$B1_r = (Y_r - Y_0) \cos \phi - (Z_r - Z_0) \sin \phi ,$$

$$B2_r = (X_0 - X_r) \cos \theta - (Z_r - Z_0) \sin \theta , \text{ and}$$

$$B3_r = (2 - \cos \theta - \cos \phi) (Z_r - Z_0)$$

A similar procedure can be followed for the derivation of the remaining equations. The full set of equations are given in section (5.7-1).

5.7 SETTING OF INITIAL CONDITIONS BY NEWTON-RAPHSON METHOD.

Before the discussion of any method of solution, the determination of the initial displacement vector of the system is necessary.

It is required to solve the following system of nonlinear algebraic equations:

$$\underline{K} \underline{Z}_0 = \underline{F}_0 \quad \dots (5.23)$$

where

$\underline{K}$ is the nonlinear stiffness matrix,

(91)

$\underline{Z}_0$ is the vector of initial displacements ,
 $\underline{F}_0$ is the force vector, and

$$\underline{F}_0 = [M_s g, 0, 0, m_i g]^T, \quad i=1,2,\dots,N$$

Verbally, the gravitational forces caused by the sprung and the unsprung masses are in equilibrium with the forces developed by the elastic elements of the system. For linear systems, the $\underline{K}$ matrix terms are constant, and the solution of equation (5.23) is quite simple.

For nonlinear systems an iteration should be performed in order to reach the equilibrium state of the system as follows:

Express the set of nonlinear equations in the form:

$$\underline{K} \underline{Z}_0 - \underline{F}_0 = \underline{0} \quad ,$$

Denote the starting values of the vector $\underline{Z}_0$ by $\underline{Z}_0^*$, furthermore, introduce a vector $\underline{V}$ such that:

$$\underline{V}^* = \underline{K} \underline{Z}_0^* - \underline{F}_0$$

The vector $\underline{V}^* \neq \underline{0}$ otherwise $\underline{Z}_0^*$ is the exact required solution. Let

$$(\underline{Z}_0)_{\text{new}} = \underline{Z}_0^* + \underline{\Delta Z}$$

such that

$$(\underline{V})_{\text{new}} = \underline{V}^* + \underline{\Delta V} = \underline{0} \quad \dots (5.24)$$

Using Taylor series,

$$(\underline{V})_{\text{new}} = \underline{V}^* + \left(\frac{\partial \underline{V}}{\partial \underline{Z}_0} \right)^* \underline{\Delta Z}_0$$

From equation (5.24)

(92)

$$-\underline{V}^* = \left(\frac{\partial \underline{V}}{\partial \underline{Z}_0} \right)^* \underline{\Delta Z}_0 \quad \dots (5.25)$$

The system of equations (5.25) represents N+3 equations in $\underline{\Delta Z}_0$.
Generally, after r iterations, solve

$$\underline{U}_r (\underline{\Delta Z}_0)_{r+1} = -\underline{V}_r$$

To obtain the value of $\underline{Z}_0$ at the iteration number r+1

$$(\underline{Z}_0)_{r+1} = (\underline{Z}_0)_r + (\underline{\Delta Z}_0)_{r+1}$$

The matrix $\underline{U}$ is as follows

$$\underline{U}_r = \begin{bmatrix} \frac{\partial V_1}{\partial W_0} & \frac{\partial V_1}{\partial \theta} & \frac{\partial V_1}{\partial \phi} & \frac{\partial V_1}{\partial W_{11}} & \dots & \frac{\partial V_1}{\partial W_{1N}} \\ \frac{\partial V_2}{\partial W_0} & \frac{\partial V_2}{\partial \theta} & \frac{\partial V_2}{\partial \phi} & \frac{\partial V_2}{\partial W_{11}} & \dots & \frac{\partial V_2}{\partial W_{1N}} \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot_N \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot \\ \frac{\partial V_{N+3}}{\partial W_0} & \cdot & \cdot & \cdot & \dots & \frac{\partial V_{N+3}}{\partial W_{1N+3}} \end{bmatrix}$$

All the partial derivative values at $(\underline{Z}_0)_r$ and $\underline{V}_r$.
Iteration continues until

Maximum of $(\underline{\Delta Z}_0)_r < \text{a certain permissible error.}$

5.7-1 INITIAL CONDITIONS FOR A NONLINEAR MODEL OF A CAR:

The application of the procedure explained in section (5.7) as,
for example, applied to a 3-D car model undergoing large deflection
follows:

The nonlinear equations to be solved are:

$$\begin{aligned} \text{i) } W_0 \int K_r + \sin \theta \int K_r (X_0 - X_r) + \sin \phi \int K_r (Y_r - Y_0) \\ - \int K_r W_{1r} - \int K_r B_{3r} + M_s g = 0 \end{aligned}$$

where $\sum$ indicates $\sum_{r=1}^N$

$$\begin{aligned} \text{ii)} \quad W_0 \sum K_r (Y_r - Y_0) + \sin \theta \sum K_r (X_0 - X_r) (Y_r - Y_0) + \\ \sin \phi \sum K_r (Y_r - Y_0)^2 - \sum K_r W_{1r} (Y_r - Y_0) - \\ \sum K_r B_{3r} (Y_r - Y_0) = 0 \end{aligned}$$

$$\begin{aligned} \text{iii)} \quad W_0 \sum K_r (X_0 - X_r) + \sin \theta \sum K_r (X_0 - X_r)^2 + \\ \sin \phi \sum K_r (X_0 - X_r) (Y_r - Y_0) - \\ \sum K_r W_{1r} (X_0 - X_r) - \sum K_r B_{3r} (X_0 - X_r) = 0 \end{aligned}$$

$$\begin{aligned} \text{iv)} \quad -K_r W_0 - K_r (X_0 - X_r) \sin \theta - K_r (Y_r - Y_0) \sin \phi + \\ B_{3r} K_r + (K_r + K_{T_r}) W_{1r} + m_r g = 0 \end{aligned}$$

The number of the above equations consists of the first three equations plus N equations represented by the equation (iv).

The terms of the $\underline{U}$ matrix are

$$\frac{\partial V_1}{\partial W_0} = \sum K_r$$

$$\frac{\partial V_1}{\partial \theta} = \cos \theta \sum K_r (X_0 - X_r) - \sin \theta \sum (Z_r - Z_0) K_r$$

$$\frac{\partial V_1}{\partial \phi} = \cos \phi \sum K_r (Y_r - Y_0) - \sin \phi \sum (Z_r - Z_0) K_r$$

$$\frac{\partial V_1}{\partial W_{1r}} = -K_r, \quad r = 1, 2, 3, \dots, N$$

$$\frac{\partial V_2}{\partial W_0} = \sum K_r (Y_r - Y_0)$$

$$\frac{\partial V_2}{\partial \theta} = \cos \theta \sum K_r (X_0 - X_r) (Y_r - Y_0) - \sin \theta \sum (Z_r - Z_0) (Y_r - Y_0) K_r$$

$$\frac{\partial V_2}{\partial \phi} = \cos \phi \sum K_r (Y_r - Y_0)^2 - \sin \phi \sum (Z_r - Z_0) (Y_r - Y_0) K_r$$

$$\frac{\partial V_2}{\partial W1_r} = - K_r (Y_r - Y_0)$$

$$\frac{\partial V_3}{\partial W} = \int K_r (X_0 - X_r)$$

$$\frac{\partial V_3}{\partial \theta} = \cos \theta \int K_r (X_0 - X_r)^2 - \sin \theta \int (Z_r - Z_0) (X_0 - X_r) K_r$$

$$\frac{\partial V_3}{\partial \phi} = \cos \phi \int K_r (X_0 - X_r) (Y_r - Y_0) - \sin \phi \int (Z_r - Z_0) (X_0 - X_r) K_r$$

$$\frac{\partial V_3}{\partial W1_r} = - K_r (X_0 - X_r)$$

$$\frac{\partial V_r}{\partial W_0} = - K_r$$

$$\frac{\partial V_r}{\partial \theta} = - K_r (X_0 - X_r) \cos \theta + K_r (Z_r - Z_0) \sin \theta$$

$$\frac{\partial V_r}{\partial \phi} = - K_r (Y_r - Y_0) \cos \phi + K_r (Z_r - Z_0) \sin \phi$$

$$\frac{\partial V_r}{\partial W1_r} = K_r + K T_r$$

The algorithm of the solution is:

Step(1) Assume $\underline{Z}_0^*$ to be the small deflection solution to minimise the number of iterations.

Step(2) Using the values of the vector $\underline{Z}_0^*$ calculate the $\underline{U}$ matrix.

Step(3) Solve the system of equations:

$$\underline{U} \underline{\Delta Z}_0 = - \underline{F}_0$$

Step(4) Calculate

$$(\underline{Z}_0)_1 = \underline{Z}_0^* + \underline{\Delta Z}_0$$

Step(5) Check if

$$a) \frac{\text{absolute maximum value of } |\underline{V}|}{\text{absolute maximum value in } \underline{K}} \leq 10^{-6}$$

$$\text{or } b) \frac{\text{absolute maximum value of } \underline{\Delta Z}_0}{\text{absolute maximum value of } \underline{Z}_0} \leq 10^{-5} : 10^{-6}$$

then $(\underline{Z}_0)_1$ is the required initial displacement vector.

Step(6) If the conditions in step(5) are not satisfied repeat from step(2) again, using the recent values of the vector $(\underline{Z}_0)_1$ as starting values.

The above procedure is applicable to all types of suspension system explained in chapter one.

5.8 NONLINEAR HERMITIAN WEIGHTED RESIDUAL SOLUTION:

This method is applicable to the first group of nonlinearities. The step doubling technique explained in section (5.3) can be employed to solve the nonlinear system of equations of the form (5.21). It is perhaps worth mentioning that the $\underline{C}$ and $\underline{K}$ matrices are assumed to be the same at the start and at the end of the time step (linearised). Updating these matrices at the start of each time step, in accordance with the nonlinear characteristics of the suspension elements, can now be carried out.

5.9 NONLINEAR R.K. SOLUTION:

This method is applicable to those cases which include one or more of the nonlinearity groups. The nonlinear system of differential equations solvable by this method is on the form (5.22). To obtain the solution at the end of a specified time increment, in terms of the conditions at the starting point, the following algorithm can be applied.

Step(0) Initial conditions:

At time $t=0$, the following vectors are assumed to be known

$$\underline{Z}_0, \underline{Z}_0', \text{ and } \underline{F}_0$$

Step(1) Assuming that $t_1 = t_0$,

(96)

$$\underline{Z}_1 = \underline{Z}_0 , \quad \underline{Z}'_1 = \underline{Z}'_0 , \quad \underline{F}_1 = \underline{F}_0 ,$$

$$\underline{K}_1 = \underline{K}(\underline{Z}_{(1)}) , \quad \text{and} \quad \underline{C} = \underline{C}(\underline{Z}'_{(1)})$$

the following equations can be solved

$$\underline{Z}''_{(1)} = \underline{f}(t_0, \underline{Z}_{(1)}, \underline{Z}'_{(1)})$$

Define

$$\underline{G}_1 = h \underline{Z}''_{(1)}$$

Step(2) Calculate the following vectors

$$\underline{Z}_{(2)} = \underline{Z}_0 + h \left(\underline{Z}'_0 + \frac{1}{4} \underline{G}_1 \right)$$

$$\underline{Z}'_{(2)} = \underline{Z}'_0 + \frac{1}{2} \underline{G}_1$$

$$\underline{F}_2 = \underline{F}(t_2) , \quad t_2 = t_0 + \frac{1}{2} h$$

$$\underline{K}_2 = \underline{K}(\underline{Z}_{(2)})$$

$$\underline{C}_2 = \underline{C}(\underline{Z}_{(2)})$$

and solve the equations:

$$\underline{Z}''_{(2)} = \underline{f}(t_2, \underline{Z}_{(2)}, \underline{Z}'_{(2)})$$

and

$$\underline{G}_2 = h \underline{Z}''_{(2)}$$

Step(3) As step(2), with

$$\underline{Z}_{(3)} = \underline{Z}_0 + \frac{1}{2} h \left(\underline{Z}'_0 + \frac{1}{4} \underline{G}_1 \right)$$

$$\underline{Z}'_{(3)} = \underline{Z}'_0 + \frac{1}{2} \underline{G}_2$$

$$t_3 = t_0 + \frac{1}{2}h$$

$$\underline{K}_3 = \underline{K}(\underline{Z}_{(3)})$$

$$\underline{C}_3 = \underline{C}(\underline{Z}'_{(3)})$$

Step(4) As before with

$$\underline{Z}_{(4)} = \underline{Z}_0 + h \left(\underline{Z}'_0 + \frac{1}{2}\underline{G}_3 \right)$$

$$\underline{Z}'_{(4)} = \underline{Z}'_0 + \underline{G}_3$$

$$t_4 = t_0 + h$$

$$\underline{K}_4 = \underline{K}(\underline{Z}_{(4)})$$

$$\underline{C}_4 = \underline{C}(\underline{Z}'_{(4)})$$

Final answers

$$\underline{Z} = \underline{Z}_0 + h \left[\underline{Z}'_0 + \frac{1}{6} (\underline{G}_1 + \underline{G}_2 + \underline{G}_3) \right],$$

$$\underline{Z}' = \underline{Z}'_0 + \frac{1}{6} (\underline{G}_1 + 2\underline{G}_2 + 2\underline{G}_3 + \underline{G}_4), \text{ and}$$

$$\underline{Z}'' = \underline{f}(t_4, \underline{Z}, \underline{Z}')$$

5.10 "STIFF" ORDINARY DIFFERENTIAL EQUATION (ODE) SOLVER:

The definition 'stiffness', as used in this section, is not the same as that used in structural mechanics (including elsewhere in this investigation). In the conventionally accepted sense, the 'stiffer' the matrix $\underline{K}$, the more interactions between the components, implying typically, a more stable system. However a stiff system, in the conventional sense is not necessarily simulated by a stiff ODE [50]. Unfortunately, there does not appear to be consensus between analysts on

what exactly constitutes 'stiffness' in this new sense. Aiken [51] defined the stiffness as a 'stubborn' numerical method behaviour where differential equations are involved. Shampine [52] referred to the stiffness of an ODE when there were severe restrictions on time step size as employed by the classical solver.

Usually the simulation of suspension systems produces a nonstiff set of linear, or nonlinear, ODE, but occasionally that set might be of the kind 'stiff'. Furthermore, the nonstiff set of ODE might change its nature to one of stiff ODE, during the solution by a nonstiff solver, due to the presence of various nonlinearities, as discussed in [53].

The forcing function vector $\underline{F}$ can make accuracy an important consideration for any component of vector $\underline{Z}$, including the stiff ones, at any time in the transient, and the numerical stability of all components is essential. The use of a stiff ODE solver is advisable in the event of failure of classical solvers. This advice agrees with the recommendation of Aiken [50]. Switching between nonstiff and stiff solvers is possible by salvaging the results from just before the solver failure and using them as the initial values for the stiff solver. The adaptive methods can be used as stiff solvers. The adaptive methods are the numerical methods which contains arbitrary parameters so as to tailor the numerical methods to fit the particular problems. This approach is used to stabilise the numerical methods. The implicit Runge-Kutta methods follow the approach.

5.10-1 IMPLICIT RUNGE-KUTTA-TEARNOR SOLVER [41]:

This method is applicable to a set of ODE of the form (5.2). The time nodes and the procedure are similar to those given in section (5.9).

The final solution is given^o by the following equation.

(99)

$$\begin{aligned}
\underline{Z}_{i+1} = \underline{Z}_i + & - (\underline{G}_1 + 2 \underline{G}_2 + 2 \underline{G}_3 + \underline{G}_4) \\
& - (\text{ph})^2 [(\underline{G}_2 - \underline{G}_3) Q_3 \\
& + (\underline{G}_1 - 4 \underline{G}_2 + 2 \underline{G}_3 + \underline{G}_4) Q_4 \\
& - 4 (\underline{G}_1 - \underline{G}_2 - \underline{G}_3 + \underline{G}_4) Q_5] \quad \dots (5.26)
\end{aligned}$$

where

$$Q_3 = \frac{e^{-\text{ph}} - \frac{1}{2} (\text{ph})^2 + \text{ph} - 1}{(-\text{ph})^3} ,$$

$$Q_{j+1} = \frac{Q_j - (1/j)}{-\text{ph}} , \quad j= 3,4 , \text{and}$$

$$\text{ph} = -2 \left(\frac{G_3 - G_2}{G_2 - G_1} \right)$$

The first term in (5.26) is due to the fourth order Runge-Kutta method and the additional term is due to the fifth order, and higher, in h . It is seen that when the equation (5.26) is used to integrate over the interval where ph is small, the result will be identical with the Runge-Kutta method. If ph is large, a condition where the Runge-Kutta method is known to be unstable, then equation (5.26) produces a superior solution.

CHAPTER 6

HUMAN TOLERANCE

6.1 INTRODUCTION:

When a vehicle is operated under severe environmental conditions, the weakest link in the performance of the vehicle system is usually the human operator. This has been especially true in recent years because of the increase in allowable stress levels of vehicle components. As a clear example, the off-road vehicle speed is limited by the operator tolerance to the vibration severity level. Perhaps no area of human factors had proved more difficult to quantify than has the matter of human vibration tolerance to vehicle ride vibration. This is because -on one hand- the vibration severity as well as the human response are complicated nonlinear functions of amplitude, frequency, and time of vibration. On the other hand, it is customary to speak of a specific vibration as being comfortable, unpleasantly severe, or intolerable which means different things to different people. The laws of mechanics allow us to write differential equations accurately describing the response of vehicle subsystems to any control or disturbance input function. There are no comparable quantitative laws describing the influence of the resultant vehicle motions on the physiological or, especially, psychological functions of the operator and other vehicle occupants. For that reason, all present tolerance criteria have used experimental data for the creation of their standards.

An experimental technique employed in evaluating vehicle ride quality is the 'jury rating' method. In this procedure, a group of practical experts assign numbers to their subjective impressions, and these values are used as a classification of the vibration severity. However, because of the lack of an absolute calibration reference, jury ratings do not constitute a suitable basis for objective performance specifications, requirements or standards [54].

6.2 INTERNATIONAL STANDARD ISO 2631:

This standard is titled 'Guide for the evaluation of human exposure to whole body vibration'. It is the result of some 10 years of

work by ISO/TC 108/SC4 which is the 'Human exposure to mechanical vibration and shock subcommittee' of the 'Mechanical vibration and shock committee'. The document has been in circulation for about 5 years, first as a draft, and then as a standard since 1972. The first edition of the standard issued as ISO 2631-1974 in 1974. ISO member bodies of 22 countries participated in the development of this standard and only two countries voted disapproval on technical grounds. The first edition has been modified and republished as the second edition ISO 2631-1978 in 1978 [56]. The second edition had been approved by the member bodies of 19 countries, and only two country member bodies have expressed disapproval of the document on technical grounds. Two amendments were published in 1982 [57,58]. Because of participation by experts from many countries, the ISO 2631 standard has generated much attention and interest on the part of governmental bodies and private industry faced with possible compliance to the exposure levels set forth. It is necessary to carefully read and study all the qualifiers in the document prior to applying it to a vibration exposure situation. The main thrust of the standard can be summarised as follows:

1-In the case of seated persons such as vehicle operators, seat/buttock interface in the buttock-to-head(a_z), back-to-chest (a_y), and right-to-left (a_x) direction are the parameters of importance.

2-The 1/3 octave intensities in units of m/s^2 are plotted versus frequency.

3-For a_z , minimum tolerance is in the 4 to 8 Hz range. For a_x and a_y minimum tolerance is below 2 Hz.

4-The standard sets forth three different exposure criteria each of which are presented as dependent on exposure time. These recommended exposure boundaries are termed:

- a) Exposure limits from the health and safety aspect,
- b) Fatigue-decreased proficiency boundaries, and,
- c) Comfort boundaries.

Fig(25) shows the format of the boundary curves for the a_z direction. Where the primary concern is to maintain the working efficiency of a vehicle driver or a machine operator working in vibration, the 'fatigue-decreased proficiency boundary' would be used

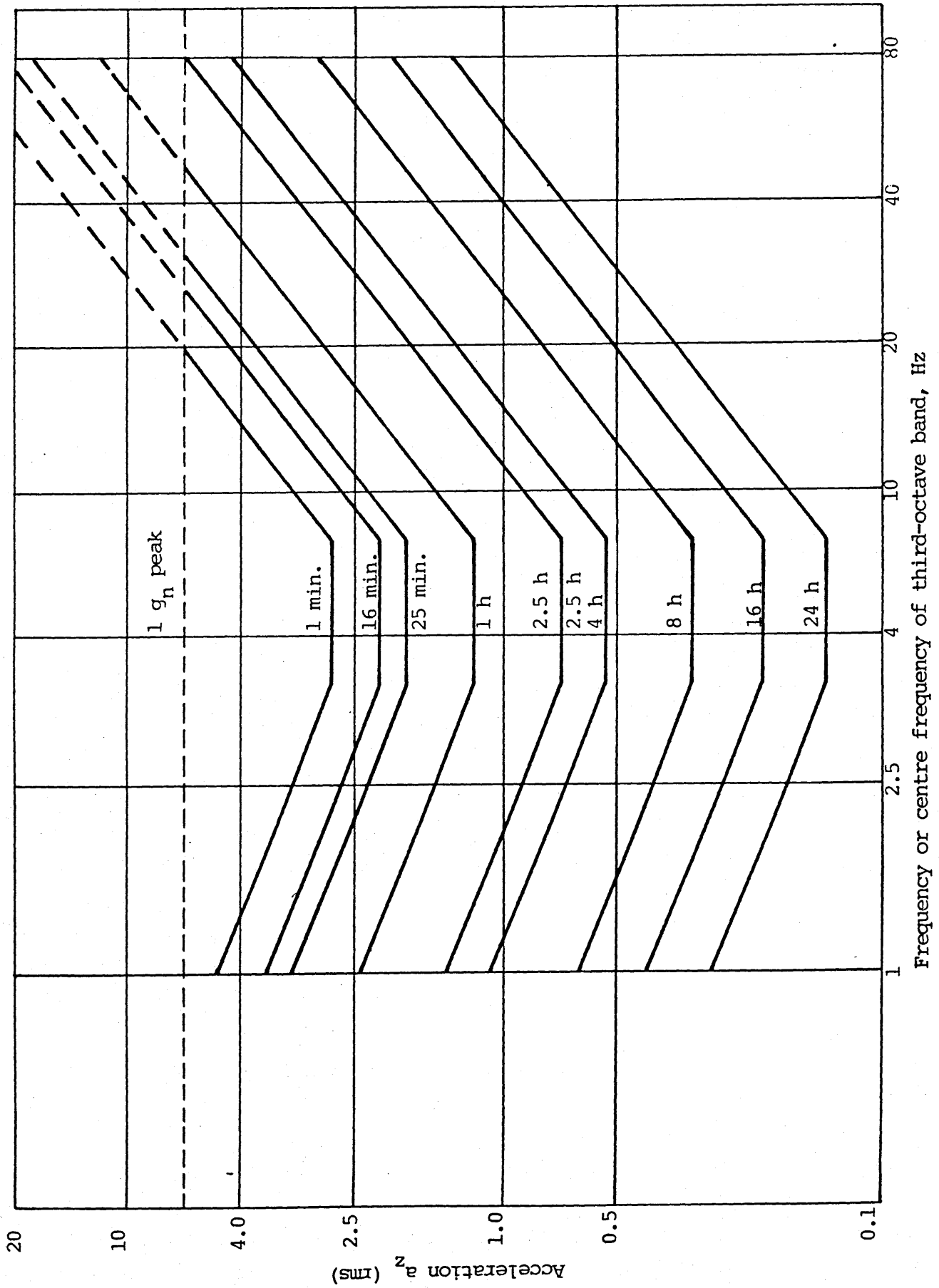


Fig (25) Fatigue decreased proficiency boundaries for a_z acceleration.

as the guiding limit in laying down vibration specifications or in carrying out vibration control measures, while, in the design of passenger accommodations, the 'reduced comfort boundary' should be considered.

5-The standard is applicable only to situations involving people in normal health.

6-The limits given in the standard should be regarded as very tentative for the case of vibrations having high (more than 6) crest factors (ratio of maximum peak acceleration to root mean square value).

7-If vibrations occur in more than one direction simultaneously (multiaxis vibration), the corresponding limits apply separately to each vectorial component in the three axes.

8-The overall vibration signal for the frequency range 1 to 80 Hz may be weighted with an electronic network. In most practical cases the difference between the detailed third-octave band boundary evaluation method and the weighted overall vibration measurement method is small.

9-Tolerance varies considerably with age and sex.

As might be expected, many questions and concerns can be raised about this document and its application. Some of these have been discussed in reference [59], and can be summarised as follows:

1-That part of the standard which refers to the techniques of measurement and analysis of vibration affecting the body, is incomplete.

2-The duration of vibration exposure has less effect on human performance and comfort than is implied in the standard.

3-No quantification to individual variations in response to vibration has been attempted.

In the original standard (1978), there were no limits given for frequencies less than 1 Hz. Addendum 2 (1982) covers vibration transmitted to the body in the frequency range 0.1 to 0.63 Hz. There is some evidence that motion sickness rarely occurs in the frequency range 0.63 to 1 Hz. Lack of data confines the recommendations specifically to Z-axis vibration, and 'severe discomfort boundaries' only. The boundaries are given to exposure times 30 min., 2 h, and tentatively

to 8 h. Further conarguments are discussed in references [60,61].

The positive aspects of this worldwide standard, irrespective of detailed absolute validity considerations, are given in [62]. Some points of note are:

1-The standard does represent perhaps the best technical and judgmental compromises that could be arrived at by a panel of experts possessing state-of-the-art knowledge in this area.

2-The authors of the document readily admit that it is meant to be a guide only and special interpretations may be required for some applications.

3-The document standardises the analysis and reporting of vibration exposure data.

4-The standard certainly merits application as one of the assessment tools during any quantitative study of vehicle ride.

6.3 ABSORBED POWER CRITERION:

The term absorbed power (A.P) implies that the quantity is a measure of the average rate at which energy is dissipated by the body as a result of the complex, damped, elastic properties of the anatomy. The absorbed power parameter was introduced by Lee and Pradko [36] who have mainly applied it to off-road surface vehicles. In the time domain, the parameter is defined as:

$$P_{av}(T) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T F(t) V(t) dt$$

where

P_{av} is the average absorbed power,
 $F(t), V(t)$ are input force and velocity respectively.

and, in the frequency domain [64] as:

$$P_{av} = \sum_{i=0}^j K_i A_i^2$$

where

K_i is a function of frequency ,

A_i is the rms acceleration at each frequency, and
 j is the number of frequencies.

As a scalar quantity, the A.P. can be summed in complex multiple-degree-of-freedom systems to determine human response. Tables of K_i for the vertical, fore-aft, and side-to-side vibrations are included in [64]. Further development has been done by Lins and Dugoff [54]. They analysed the large body of experimental data reported in [65] to empirically derive transfer functions describing the vibration response characteristics of an 'average' young (28 years) male of 150 lb seated weight. Fig(26) shows the derived frequency response characteristics for computing absorbed power in the vertical, fore-aft, and side-to-side directions. The transfer functions corresponding to the plotted curves are given in Table 1 of the appendix.

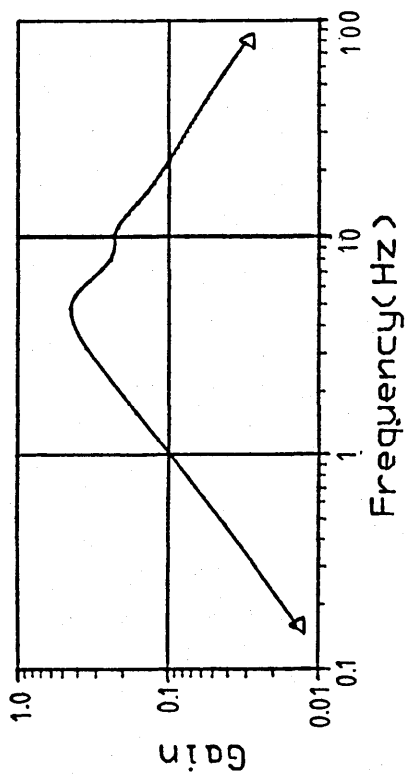
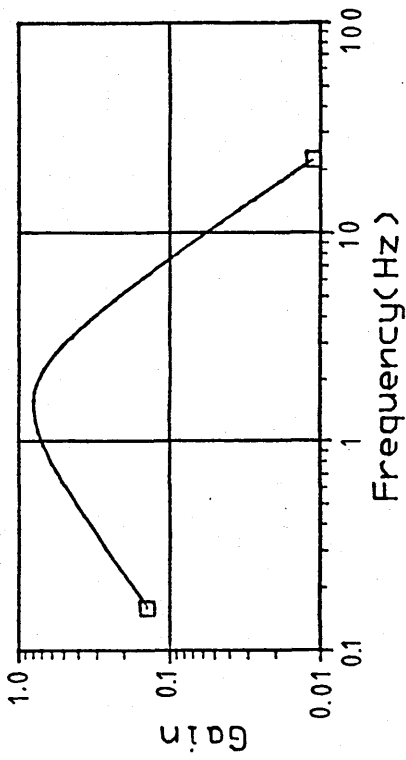
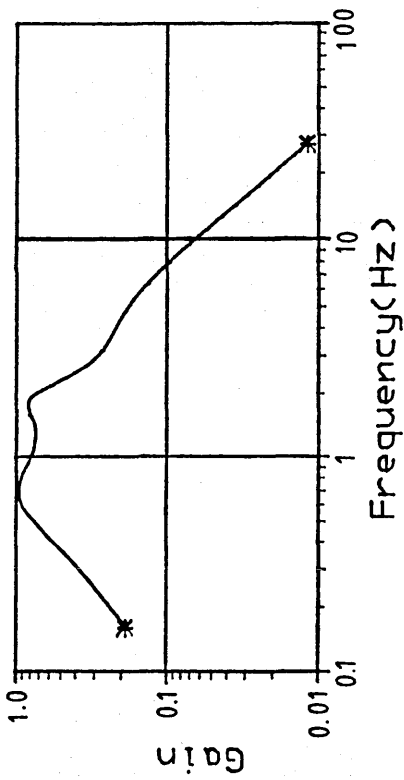
An instrument consisting of electronic circuits, to compute absorbed power in three directions, has been developed by Lee and Lins [66].

6.4 THE K FACTOR:

The K factor criterion is being actively applied in Germany for approval or rejection of agricultural tractor seats. The K value can be expressed as a function of frequency and either rms acceleration, velocity, or displacement. According to the value of K, the vibration severity is subdivided into nine ranges A, B, C, D, E, F, G, H, AND J respectively. Differentiation between these ranges is made in words as not, just, easily, strongly or very strongly noticeable.

6.5 DISCUSSION:

The two widely used criterion are ISO and A.P. Although they use different techniques, there are many points of agreement and some differences between the two criteria. Some points of note are:



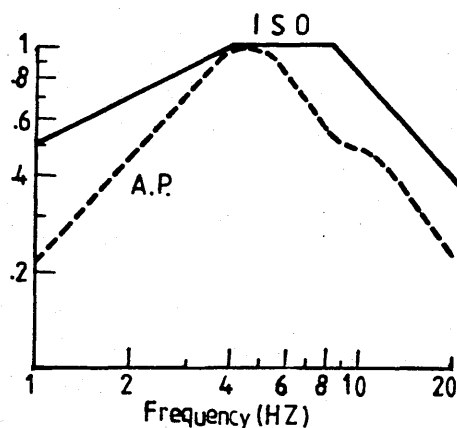
△ Vertical direction (Z-axis)

□ Fore-aft direction (Y-axis)

* Side-to-side direc. (X-axis)

Fig(26) Human response characteristics.

1-The most sensitive range for vertical vibration is 4 to 8 Hz in both criteria. A quite good comparison between ISO and A.P is given in reference [68], and shown in Fig(27) with slight alteration.



Fig(27) ISO weighted overall rms versus absorbed power.

2-Both criteria are applicable to healthy people. Whenever there is a selection within a population such as a military group; or a specific activity like reading, eating .. etc; or a special function to be performed like fire-on-the-move of military vehicles, the limits given by both criteria should be changed. As an example, Murphy [69] found that initial evaluations of absorbed power measurements at the gunner's position of the M60 tank indicated that firing-on-the-move was almost totally ineffective at absorbed power levels beyond about 2 or 3 watts, which is about one third to one half of the 6-watt tolerance limit.

3-Both criteria recognise the difference between an individuals response to vibration. However, no quantification of such a difference with respect to sex and age is made.

4-The vibration severity can be expressed with one number by both criteria, in the frequency domain. Moreover, the A.P criterion can be used in the time domain.

5-The A.P criterion discriminates between fore-aft and side-to-side directions, but in ISO the limits are the same.

6-The limits given by ISO for vibration at less than 1 Hz are in one direction(Z) and incomplete, unlike the vibration range 1 to 80Hz.

7-To set tolerance or comfort limits to a vibration one must also define the environment. For example, a vibration in an automobile may be annoying, uncomfortable, or even intolerable, but this same vibration in a truck may be termed 'comfortable'.

8-No limits are given for the case of shocks by either criteria.

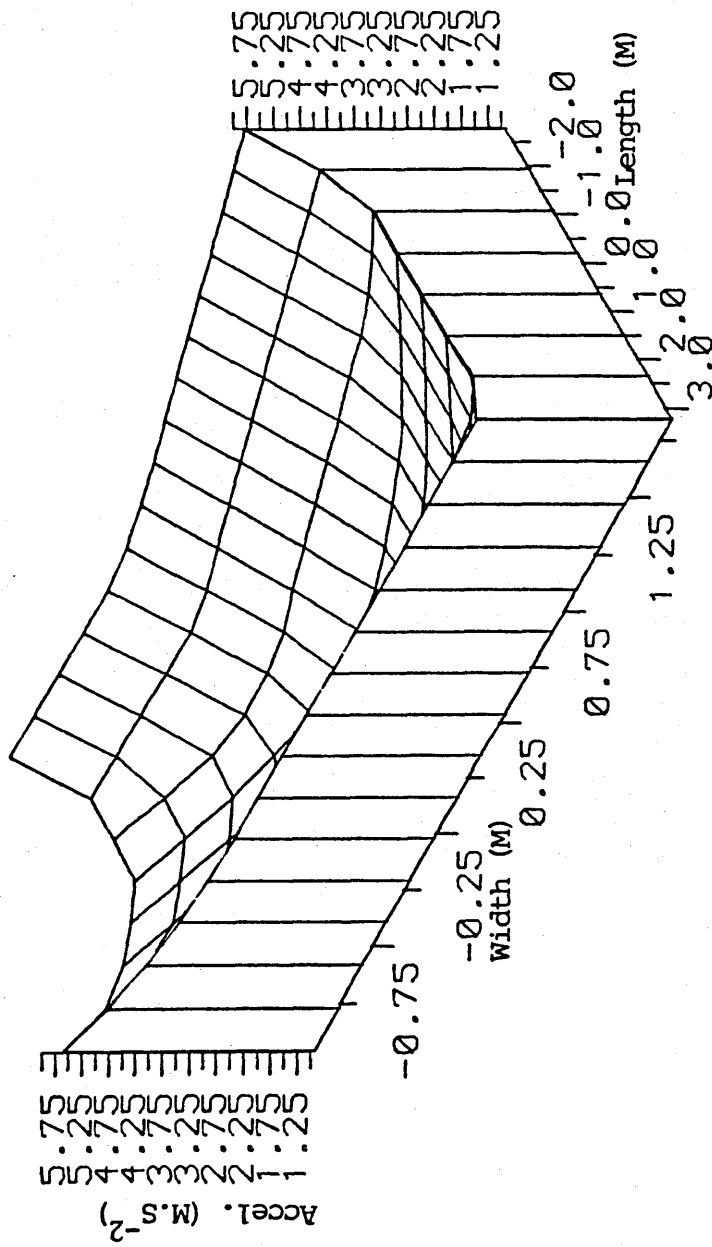
Crolla and Maclaurin [70] referred to a revision of ISO 2631 titled 'Guide to the evaluation of human exposure to whole-body mechanical vibration and repetitive shock'. The primary purpose of this draft revision is to define methods for quantifying vibration and repetitive shock in relation to:

- (a) degraded health;
- (b) impaired activities (effect on vision and hand control);
- (c) impaired comfort (and perception);
- (d) motion sickness (vertical, oscillation between 0.05 and 0.5 Hz).

To include both standards in the present code, the time-response results of the vehicle c.g should be used to calculate the time-response at the location of the operator/occupant, and, transform the latter to the frequency domain.

6.6 DRIVER DYNAMICS:

For a rigid vehicle body, once the time response of the c.g is known, the time response of any location of the body can be determined. This location can be of the driver or the operator or the occupant or any member of the vehicle crew. The information of the acceleration values at the nodes of a grid of the vehicle body, is of great importance to the designer in order to select the optimum location for the occupants. This information can be in the form of a diagram as in Fig(28), where the vehicle body is divided into squares of 0.3 m side length, and the acceleration value is plotted at each node in the vertical direction. It turns out that the optimal comfort with minimal acceleration appears to be in the centre of gravity of the body. This conclusion agrees with that arrived at by Schiehlen [71]



Fig(28) Acceleration plot of a tank body.

as an analysis of numerical results published by Kreuzer and Rill [72].

The link between the seat and the vehicle body can be rigid, or elastic. In the latter case, a damper can be used to attenuate input vibration to the operator.

6.6-1 RIGID SEAT LINK:

It is required to determine the operator/occupant acceleration as a function of the vehicle c.g vibration descriptors. First, the driver velocity vector $\vec{V}_d$ will be expressed in terms of c.g velocity vector $\vec{V}_c$, then, an integration to $\vec{V}_d$ produces acceleration vector $\vec{a}_d$.

$$\vec{V}_d = \vec{\omega}_d \times \vec{R}_d + \vec{V}_c \quad \dots (6.1)$$

where

ω_d is the angular velocity of the driver,
 R_d is the radius of the driver location ,
 and $\times$ stands for cross product.

$$\begin{aligned} \vec{\omega}_d \times \vec{R}_d &= \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ \theta' & \phi' & 0 \\ X_d & Y_d & Z_d \end{bmatrix} \\ &= Z_d \phi' \underline{i} - Z_d \theta' \underline{j} + (Y_d \theta' - X_d \phi') \underline{k} \quad \dots (6.2) \end{aligned}$$

where

$\underline{i}$, $\underline{j}$, and $\underline{k}$ are unit vectors in the X, Y, and Z directions respectively.

$$\vec{a}_d = \vec{V}'_d = \vec{\omega}_d \times \vec{R}'_d + \vec{\omega}'_d \times \vec{R}_d + \vec{V}'_c \quad \dots (6.3)$$

(111)

but,

$$\vec{V}_d = \vec{R}_d',$$

$$\vec{a}_c = \vec{V}_c' = w_0'' \underline{k}$$

$$\vec{\omega}_d \times \vec{V}_d = \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ \theta' & \phi' & 0 \\ V_{Xd} & V_{Yd} & V_{Zd} \end{bmatrix}$$

$$= V_{Zd} \phi' \underline{i} - V_{Zd} \theta' \underline{j} + (V_{Yd} \theta' - V_{Xd} \phi') \underline{k} \quad \dots (6.4)$$

where

V_{Xd} , V_{Yd} , and V_{Zd} are the driver's velocity components in X, Y, and Z directions.

$$\vec{\omega}_d' \times \vec{R}_d = \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ \theta'' & \phi'' & 0 \\ X_d & Y_d & Z_d \end{bmatrix}$$

$$= Z_d \phi'' \underline{i} - Z_d \theta'' \underline{j} + (Y_d \theta'' - X_d \phi'') \underline{k} \quad \dots (6.5)$$

Substituting equations (6.2), (6.4), and (6.5) into equation (6.3), and arranging terms, then

$$\begin{aligned} \vec{a}_d &= [(Y_d \theta' - X_d \phi' + w_0') \phi' + Z_d \phi''] \underline{i} \\ &\quad - [(Y_d \theta' - X_d \phi' + w_0') \theta' + Z_d \theta''] \underline{j} \\ &\quad - [X_d \phi'' - Y_d \theta'' + Z_d \theta'^2 + Z_d \phi'^2 - w_0''] \underline{k} \quad \dots (6.6) \end{aligned}$$

Substituting equation (6.2) into equation (6.1), then

$$\vec{V}_d = z_d \phi' \underline{i} - z_d \theta' \underline{j} + (y_d \theta' - x_d \phi' + w_0') \underline{k} \quad \dots (6.7)$$

$$\vec{S}_d = z_d \phi \underline{i} - z_d \theta \underline{j} + (y_d \theta - x_d \phi + w_0) \underline{k} \quad \dots (6.8)$$

6.6-2 ELASTIC SEAT LINK:

If the designer cannot avoid locating one or more of vehicle-crew seats at positions of the vehicle body where there is high vibration intensity, elastic seat links should be used. Obviously, the function of an elastic link is to attenuate the vibration-severity input to the person. An example of a seat suspension can be found in reference [56]. An alternative to a seat suspension is the suspension of the whole cabin as is the case with some types of farm tractors [62].

The differential equation of motion of a person of mass M_d (see Fig 28) is:

$$M_d Z'' + C_d Z' + K_d Z = C_d Z_d' + K_d Z_d \quad \dots (6.9)$$

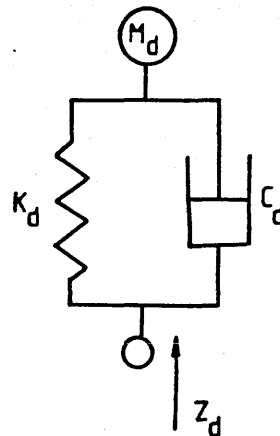
where

C_d is the damping coefficient of the damper,

K_d is the stiffness coefficient of the spring, and

z_d, z_d' are the excitation displacement and velocity.

Equation (6.9) is a nonhomogeneous, second order differential equation. This equation can be solved by any of the methods discussed in chapter 5.



Fig(28) Seat suspension model.

If the side-to-side or fore-to-aft vibration is of concern, similar differential equation can be derived.

6.7 FAST FOURIER TRANSFORM (FFT):

FFT is an efficient way of producing a correlation of a waveform with sinewaves whose frequencies are integer multiples of the frequency corresponding to the time duration of the wave. The main function of the FFT is the transformation of the input function from the time domain to the frequency domain. In this way, the acceleration function of time $Z''(t)$ can be transformed to a function of frequency $Z''(\omega)$ as follows:

$$Z''(\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} Z''(t) dt \quad \dots (6.9)$$

In reference [42], a definition of the FFT is given. In reference [47], several examples to illustrate the philosophy behind the transform are given. In reference [73], the applications of the FFT in the field of measurement systems is introduced. Random vibration analysis employing the FFT has been included in [74,75]. Computer implementation of the FFT has been included in [76].

It is not intended to explain the FFT technique here because it is well documented in the literature, but some points of note and findings of application of this technique in the field of vehicle dynamics are given below.

The popular form of FFT is applied on a number of data points equal to 2^n ($n = 0, 1, 2, \dots$). When the number of the data points available is less than the closest 2^n number, the difference should be added. The simplest way to achieve this is to add points of zero ordinates, but this could distort the transform results.

An important point to be taken into consideration is the necessity to avoid 'jumps' in function particularly at the end of

the time duration, because FFT approximates the function except at the discontinuity points [76]. This problem is referred to as 'the leakage'.

To obtain the required number of data points necessary to perform FFT, and at the same time to avoid leakage, the available number of data points should be interpolated. One point of zero ordinate should be added at the end of the time interval. Fortunately, the starting point has always zero acceleration. To show this in analytical terms, let the available number of data points be n_a , and the closest larger value of 2^n be n_c . Moreover, the original time increment is h . The new time increment for the interpolation process is :

$$h_n = \frac{(n_a - 1)}{(n_c - 2)} h$$

The value of acceleration a_n at the time t lying between two consequent times t_1 and t_2 , of acceleration values a_1 and a_2 respectively, is

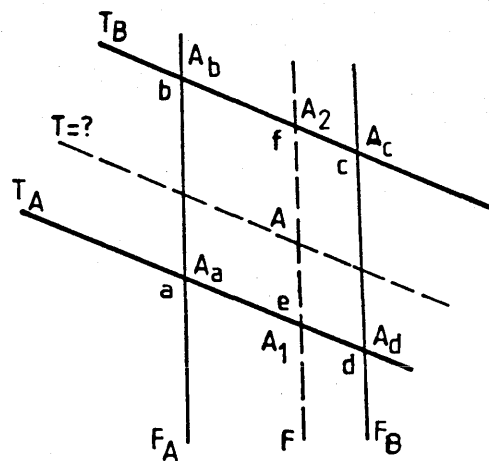
$$a_n = a_1 + \frac{a_2 - a_1}{t_2 - t_1} (t - t_1)$$

6.8 ISO INTERPOLATION:

The exposure time limits are given in the standard from 1 minute to 24 hours with irregular acceleration, and time steps.

Referring to Fig(29), if the rms acceleration A and frequency F are given in such a way that

$$A_d < A < A_b,$$



Fig(29) ISO interpolation.

and

$$F_A < F < F_B ,$$

an interpolation to the standard is necessary in order to obtain the corresponding exposure time T.

A linear interpolation will be accurate at the specified limits of the standard, and approximate between them. An interpolation process including frequency, acceleration, and time interpolations, can be carried out as follows:

(1) Frequency interpolation

Equation of straight line ad is

$$AC = m_1 FR + c_1$$

where

AC, FR are the acceleration and the frequency variables,

$$m_1 = (A_d - A_a) / (F_B - F_A) , \text{ and}$$

$$c_1 = A_a - m_1 F_a \quad \text{or} \quad c_1 = A_d - m_1 F_B$$

then,

$$A_1 = m_1 F + c_1 \quad \dots (6.10)$$

(2) Acceleration interpolation

Similar to frequency interpolation, then

$$A_2 = m_2 F + c_2 \quad \dots (6.11)$$

where

(116)

$$m_2 = (A_C - A_B) / (F_B - F_A) \quad , \text{ and}$$

$$c_2 = A_b - m_2 F_B$$

(3) Time interpolation

$$T = m_3 A + c_3 \quad \dots (6.12)$$

where

$$m_3 = (T_B - T_A) / (A_2 - A_1) \quad , \text{ and}$$

$$c_3 = T_A - m_3 A_1$$

6.9 ABSORBED POWER COMPUTER ALGORITHM:

For a vibration wave defined by frequency F and acceleration A , the following steps should be followed to calculate the average absorbed power P_a :

1- Calculate the rms acceleration A_{rms} as follows:

a) For sinewave of peak-to-peak amplitude $2A$

$$A_{rms} = 0.707 A$$

b) For a tabular form of acceleration values A_i at unequal time intervals δ_i

$$(A_{rms})^2 = \frac{\sum_{i=1}^n A_i^2 \delta_i}{\sum_{i=1}^n \delta_i}$$

For the special case of equal time intervals

(117)

$$(A_{rms})^2 = \sum_{i=1}^n A_i^2 / n$$

where

n is the number of data points.

c) For a continuous wave form

$$A_{rms} = \frac{1}{T} \int_0^T A(t) dt$$

where

T is the time duration of the wave.

2- Calculate the rms acceleration value in feet/sec² (multiply the value in M/S² by 3.2808399)

3- Obtain the angular frequency

$$\omega = 2 \pi F$$

4- Employing the empirical formula for the given direction of vibration (appendix 2), calculate the gain G(ω) corresponding to ω .

5- Finally,

$$P_a = [A_{rms} G(\omega)]^2$$

CHAPTER 7

VERIFICATIONS & RESULTS AND DISCUSSIONS

7.1 PROBLEMS ASSOCIATED WITH VEHICLE MODELLING:

It is probably useful -prior to model verification- to highlight the difficulties which frequently arise in the field of vehicle dynamics.

One of the main problems is the unavailability of accurately measured vehicle parameter values. Many authors have referred to this problem, as explained in the references [11,12,24,69]. For example, the linear force-deflection characteristics of the torsion bar reflected from strain-gauge measurements within a laboratory, differ significantly from the true suspension response [11]. Sometimes, the required piece of data is not available or difficult to measure for a variety of reasons. The Challenger tank pitch moment of inertia has not been measured but instead has been derived from calculations of mass distribution and measured resonance speeds on a sine wave terrain profile [24].

The other main problem which frequently causes significant differences between the model prediction and vehicle measured response, is due to the limited frequency range of the model as compared with the actual vehicle. The contribution of the higher frequencies transmitted by the engine, flexural modes of the vehicle frame, and the transducer peculiarities among others, can significantly affect the vehicle response. These high frequency oscillations are negligible, however, when considering ride evaluation and should be filtered from the measurements when making comparison with model response [12]. This is because models employing the concept of rigid body mechanics are an assemblage of lumped masses, springs, and dashpots that have discrete principal modes of vibration, usually not exceeding 20-25 cycles per seconds.

Sometimes, the practical and experimental difficulties limit the useful conclusions of a test program. For example, in reference [19], it is concluded that difficulties reduced the effectiveness of the test program to the extent that no conclusive results were achieved.

In reference [25], severe mechanical problems reduced the 3-D model to a 2-D model, besides, cancelling part of the test program. In reference [69], due to the wet, slippery conditions, the tanks seldom were able to attain speeds that would produce ride-limiting conditions, and the relationships were based primarily on extrapolations.

Another problem of equal importance concerns model verification—that is, once a model is implemented, how to validate it to ensure that it is representative of the desired vehicle. Even 'tuning' a model to match the actual vehicle's response to a given terrain or obstacle configuration only produces a model verification for that particular terrain or obstacle configuration.

Tyre data is a particular problem for off-road operation [78]. Comprehensive measurements of tyre vibration characteristics in the vertical, longitudinal, and lateral directions have been reported for the non-rolling case. But measurements under rolling or tractive conditions are very limited. In part, this is due to the size of typical off-road tyres, e.g tractor rear tyres, which will simply not fit the many test rigs developed for road vehicle tyres. Rolling stiffnesses are known to be of the order 20-30% lower than non-rolling measurements, but since the tyres are the major suspension element of unsprung vehicles (as for example for farm tractors), accuracy of the tyre data is crucial.

7.2 RIDE-MODEL OUTPUT:

In order to quantify human response to a particular type of vibration, the vibration magnitude, frequency, position, direction, and duration should be defined. The output will be either ride performance or ride quality. The ride performance is based on the vehicle speed as the dependent variable. It is concerned with the speed at which the tolerance criterion reaches a tolerance limit. The ride quality is based on the parameter used to quantify the vibration severity (e.g A.P) as the dependent variable. It is concerned with the

limit (e.g A.P) at selected speed. Murphy et al [79] described a satisfactory form of output. This form is referred to as the Terrain-Vehicle-Speed (TVS) graph which depicts vibration severity as a function of terrain roughness and vehicle speed. It is quite simple to use these graphs to assess ride quality at different speeds, in addition to ride performance for a given type of terrain. In practice, a course consisting of discrete obstacles can be used in order to evaluate the shock performance of a vehicle, and a cross-country or harmonic course in order to evaluate ride.

It is recommended that more emphasis be placed on evaluating the dynamic response levels at the more common operational speeds than on speeds which the human frame can barely tolerate [69].

7.3 MODEL VERIFICATIONS:

Model verifications are the final and most important phase of package design. Every possible method has been tried in order to ensure that various sectors of the package are working properly. This has been achieved by testing all subprograms separately, and with remaining subprograms forming the whole sector. Test results of individual subprograms were compared with either analytical results or results from the literature. A whole sector has been verified by performing a run for a case study and comparing the results with the experimental results. Two principal case studies, among others, have been considered in order to verify the results obtained from the present code.

The first of these case studies is a vehicle provided with the suspension element type 1(a). The data and the response results for the vehicle are taken from reference [16]. A 2-D model of the vehicle has been simulated by analog computer and the response of the model to a 0.15 m square bump excitation at vehicle speeds of 8 km/h and 24 km/h has been presented in the reference. The square bump was modified as shown in Fig(19) to account for wheel diameter.

The second case study is a fully tracked vehicle called "Scorpion" provided with the suspension element type 1(c). The Scorpion tank was excited by a sine wave of 0.05 m amplitude and 4.5 m wavelength. The maximum acceleration, as detected at the driver location, and the vehicle speed, were recorded. Another type of excitation to the Scorpion tank was a bump. The bump dimensions and the experimental response results are taken from reference [80]. These response results were the peak accelerations of the front of the vehicle and peak pitch rates at different vehicle speeds.

Next, verification of all main subprograms and programs will be explained in brief in the order of their presentation in the previous chapters.

7.3-1 VEHICLE SIMULATIONS:

The vehicle simulation produces a set of differential equations such as the set represented by (2.15). The matrices $\underline{M}$, $\underline{C}$, and $\underline{K}$ are the numerical coefficients of the variable vectors $\underline{Z}''$, $\underline{Z}'$ and $\underline{Z}$ respectively. Fortunately, the present code includes two methods of vehicle simulation (compact form and FEM). Every vehicle type was simulated once using the compact form method and once using the FEM. The results of both simulations were found to be exactly the same. As an example of the difficulty to be encountered when comparing the two simulations, the Scorpion tank model produces 3 matrices of 169 elements each. Comparison of these matrices with their equivalents and repetition of the whole process for every vehicle type are tedious and time consuming. The preferable alternative is to use the resonance frequency program to determine the resonance frequencies in a given range, and compare them for the two methods of simulation. In this way the speed of the computer is employed to verify the simulation results. The results will be one or two numbers for each simulation, which makes for a very simple comparison. A point to note is that the resonance frequency program employs the values included in the $\underline{M}$, $\underline{C}$, and $\underline{K}$ matrices as the basis of the determination of the resonance frequencies.

7.3-2 THE ADAPTIVE FOOTPRINT TYRE MODEL:

A separate test program was designed to verify the results obtained from the adaptive footprint tyre model subroutine. Several runs were performed with different active region angles, number of segments, and terrain profile. Run results were compared with results obtained analytically. They were found to be identical.

Another test was carried out in order to verify the equivalence of the adaptive footprint tyre model and the point contact tyre model for the case of linear tyre characteristics. A Scorpion tank model excited by a bump, was used for that purpose. The results from both tyre models were found to be the same.

7.3-3 EIGENVALUE PROBLEM:

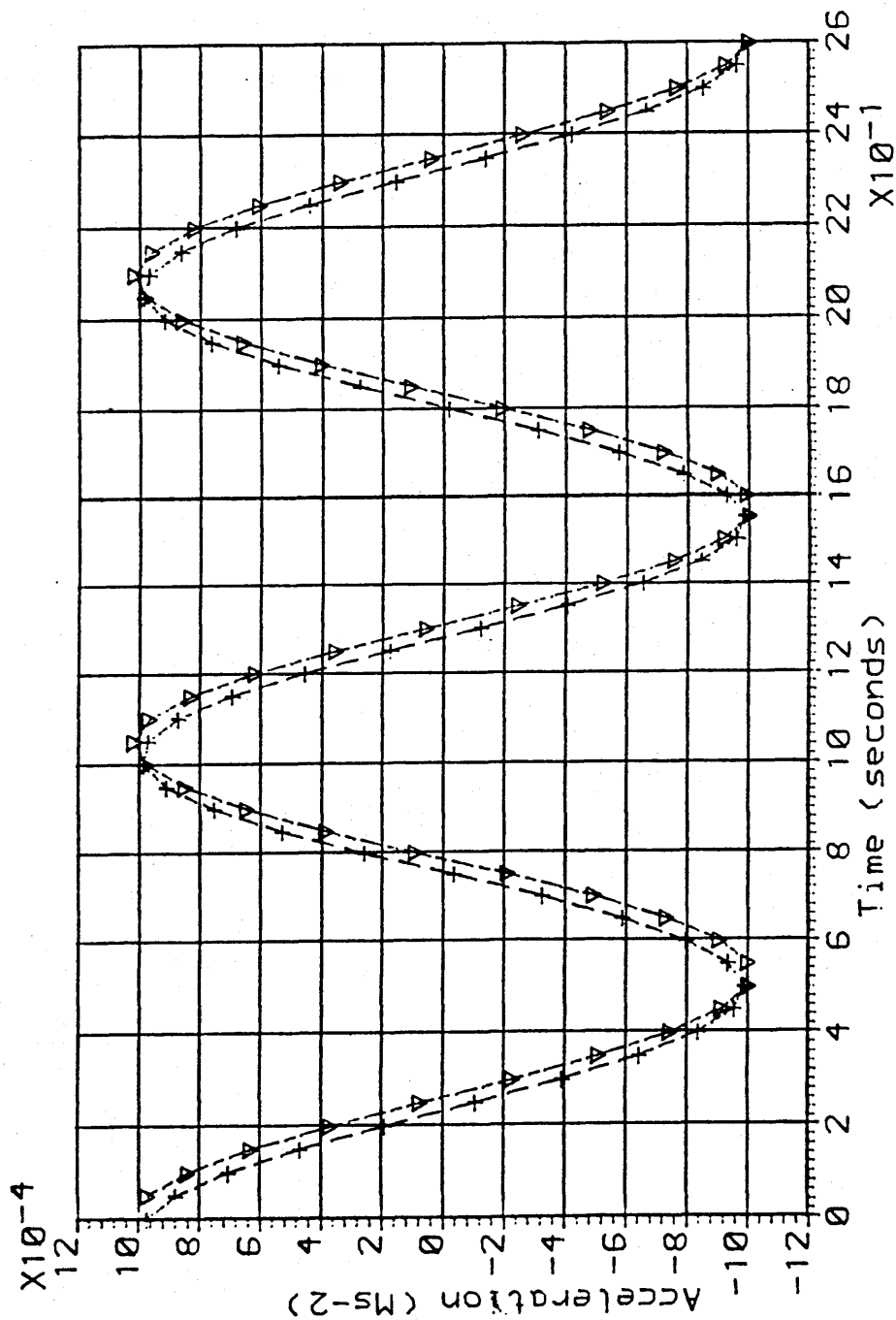
A problem run to determine the eigenvalues of the first case study has been carried out. The seven resulting natural frequencies of the system (1.2, 1.6, 33.3, 33.3, 33.2, 33.4 and 33.2 c/s) agree reasonably well with those found in reference [16] (1.2, 1.6, 32.95, 32.95, 32.95, 32.95, and 32.95 c/s).

7.3-4 STEADY STATE RESPONSE:

The analytical solution for a one-DOF system (similar to that shown in Fig 29) has been employed in order to verify the results from the steady state program. The system consists of a mass of 100 kg, spring of stiffness 50 kg/s^2 , and a damper of coefficient 30 kg/s.

The system was excited by a sine wave of 0.1 m amplitude and of angular frequency 6 rad/s.

Fig(30) shows the results given by both the analytical solution and the steady state program. Both results are seen to be of about the same response wave amplitude and wavelength.



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Fig(30)Comparison of Steady State response program
results and analytical solution.

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7.3-5 TIME RESPONSE:

The one-DOF system discussed in section 7.3-4 was employed in this section to verify the numerical integration methods included in the present code. The results obtained from the analytical solution agree very well with those obtained from various programs.

Fig(31), Fig(32), and Fig(33) show a comparison of both sets of results for hyperoscillatory Hermitian weighted residual of second order (displacement, velocity and acceleration). Fig(34) and Fig(35) show the comparison for Runge-Kutta and Lagrangian Weighted Residual respectively. Further results will be presented later in this chapter.

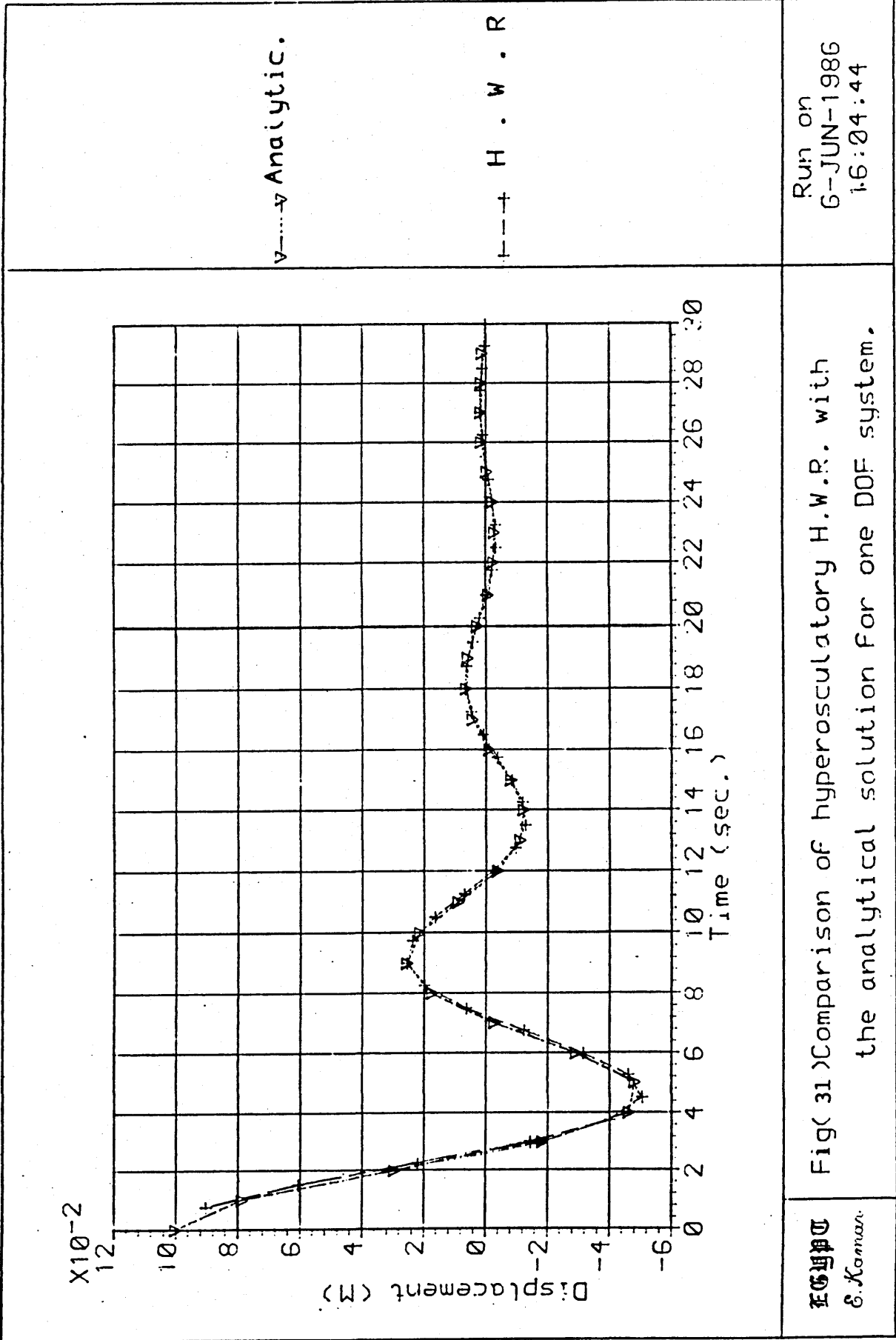
7.3-6 ISO 2631:

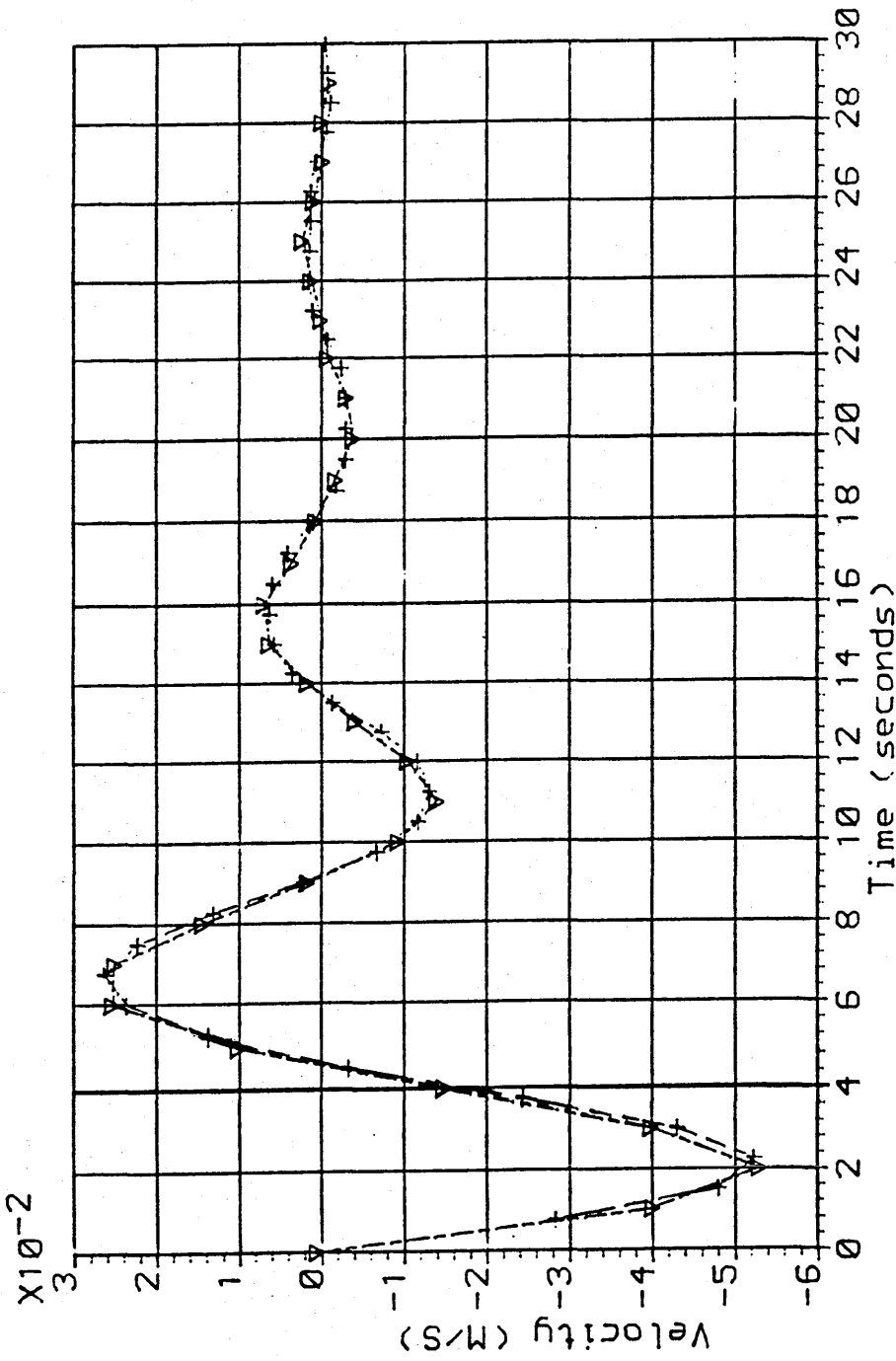
A separate master program was designed in order to test the subprogram based on the ISO standard. This program allows an input data of frequency and rms acceleration, and output of allowable working time. Many values from the tables available in the ISO standard, and many others taken from interpolations, were used as input data to the program. The output was compared with the limits available in the standard and the limits calculated analytically. This comparison showed reasonable agreement between these results.

7.3-7 ABSORBED POWER:

Murphy [11] presented two examples, with their analytical solutions, using the tables in [64]. The experimental results are given in reference [54]. The same examples were solved employing the A.P subprogram included in the present code. The results in watts are:

	Reference [11]	Present code	Measured A.P
Example 1	3.82	3.95	4.05
Example 2	178	172.3	160





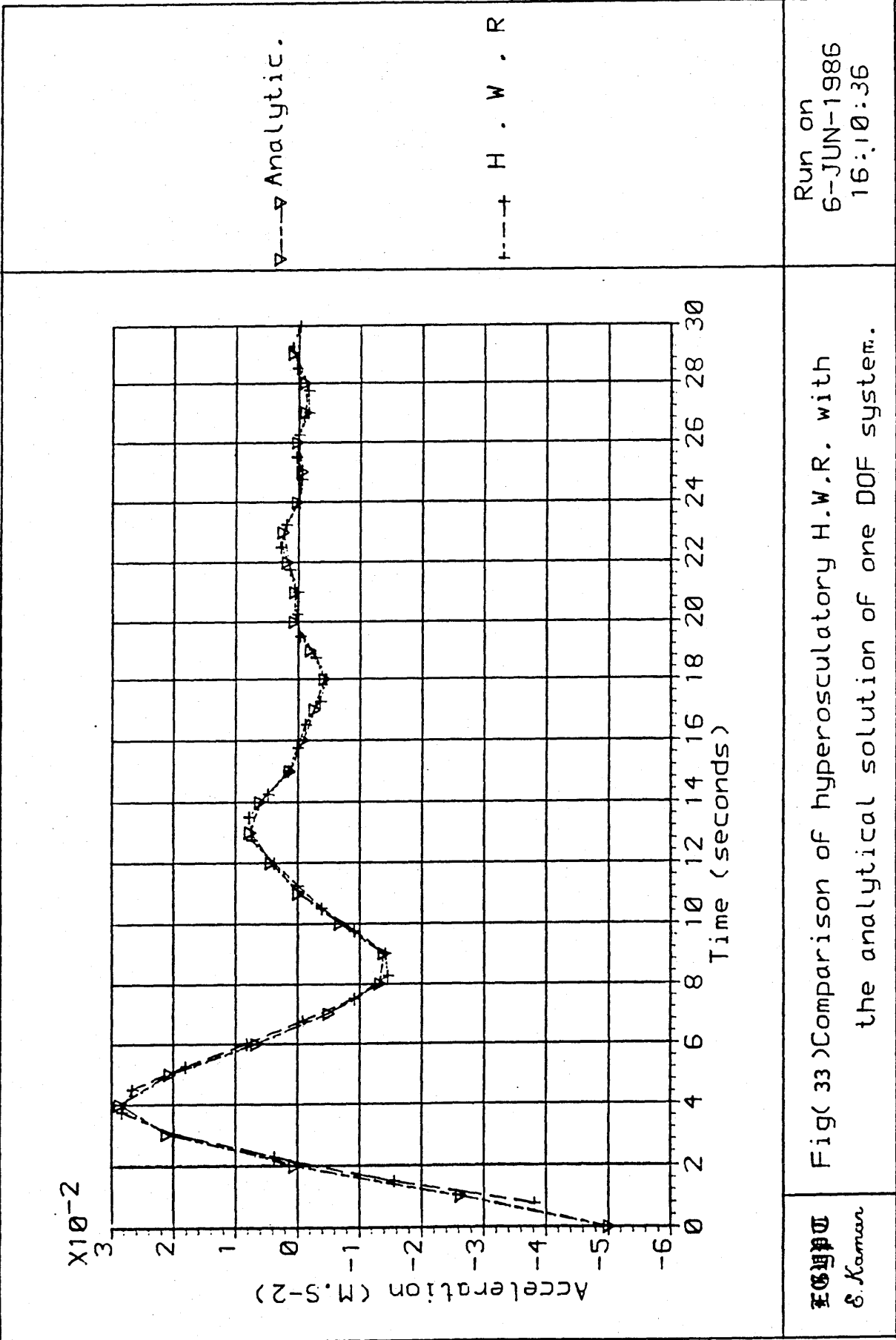
---+ Analytic.

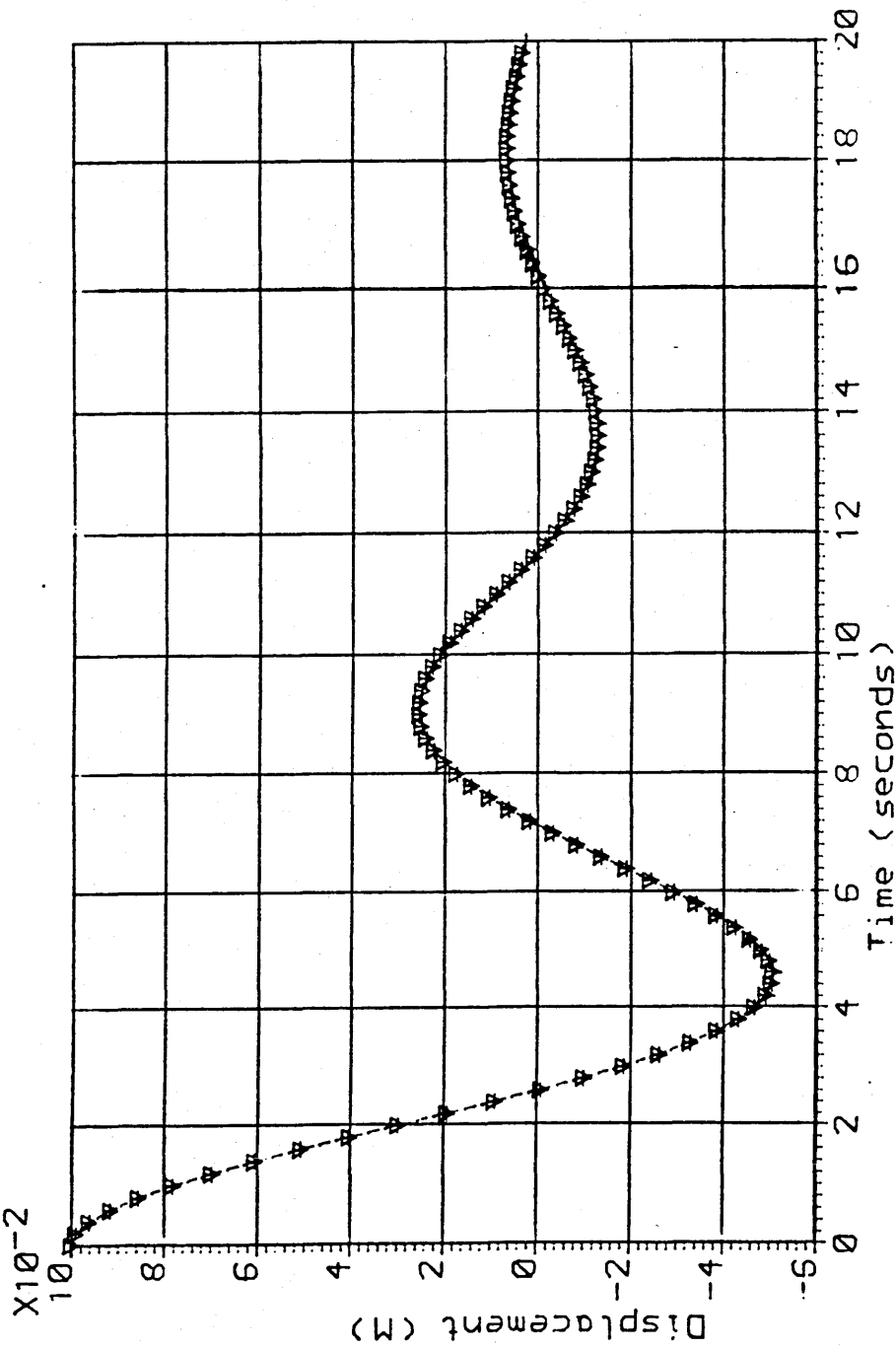
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Fig(32)Comparison of hyperosculatory H.W.R. with
the analytical solution of one DOF system.

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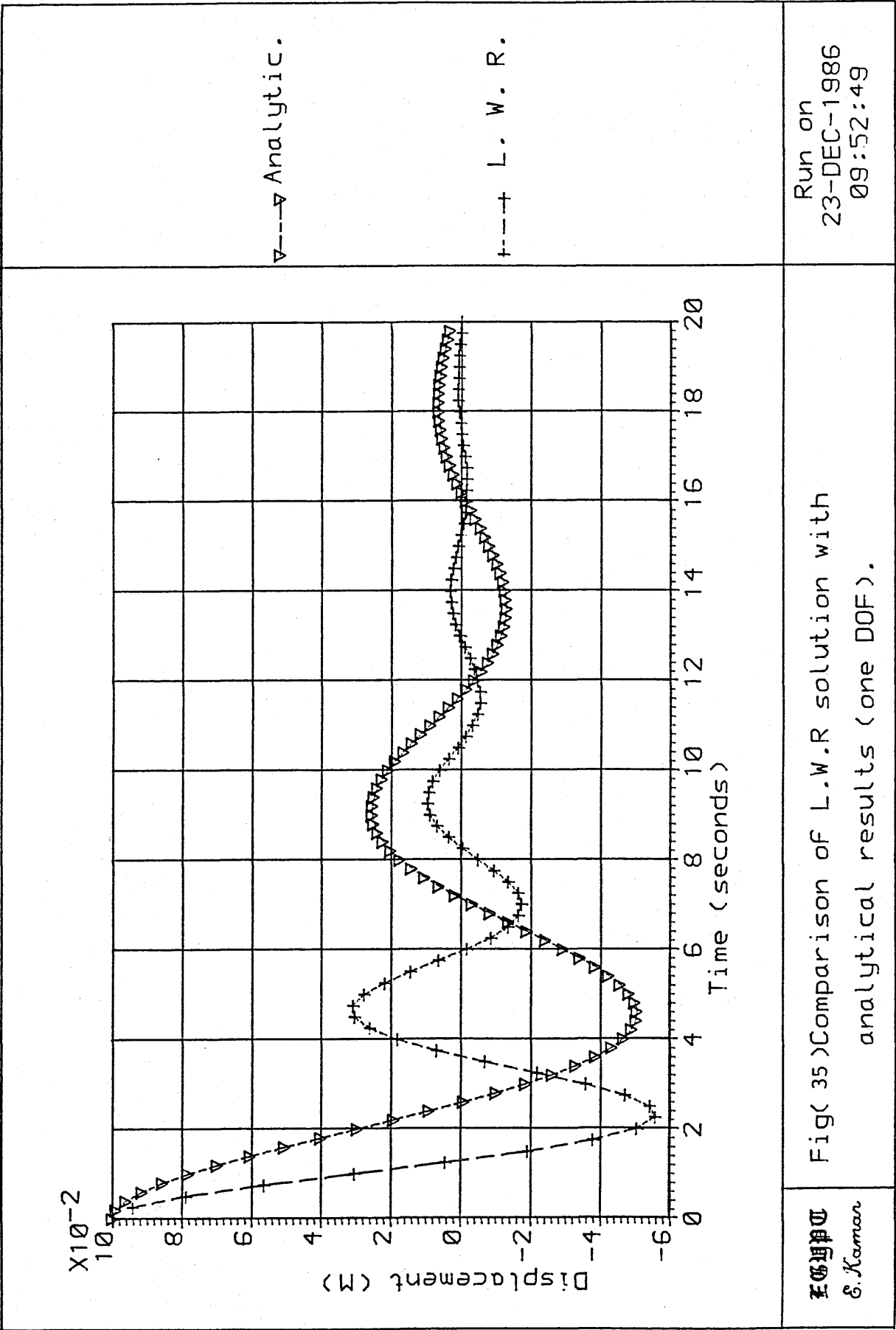
▽---▽ Analytic.

+---+ R. K.

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Fig(34)Comparison of R.K solution with analytical results (one DOF).

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7.4 CASE STUDIES:

The case study number 1 has been employed for verification of the 2-D linear car model and time response solution methods.

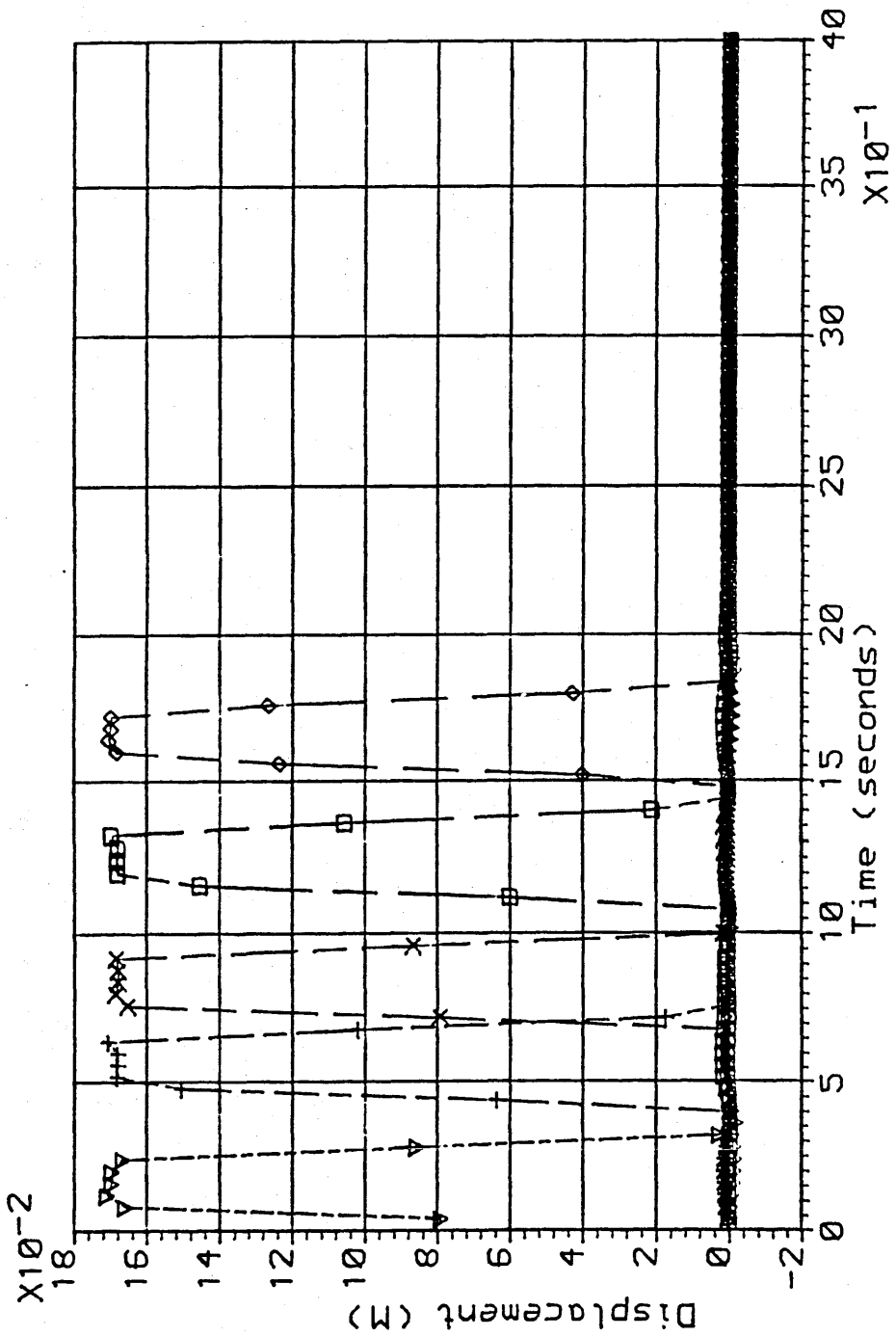
The data and results given in reference [16] are, unfortunately, in imperial units. A facility to deal with these units has been included in the present code. However, the results related to this case study are presented in I.S.O units. Moreover, although blow-off of the main damper has been taken into consideration in the analog simulation, the time response results arrived at by the present code and in the reference are compared.

Fig(36) shows the wheel centre displacements of the five tyres and indicates that all tyres follow the bump without any tyre-ground separation. Fig(37) shows the car c.g velocity results obtained by the L.W.R and H.W.R solution methods and the analog solution at a car speed of 2.2 M/S. The positive and negative peak velocity values are quite close. Fig(38) shows the pitch angle again determined from the three solution methods for a car speed of 2.2 M/S. The resulting graphs agree quite well in both absolute value and distribution.

For a car speed of 6.7 M/S, Figures (39-41) show comparisons of the results obtained by either L.W.R OR H.W.R with the analog results. It is clear from these figures that the results are comparable.

A single precision calculation has been employed for all of the above results. Employing a double precision calculation did not produce any marked difference for the case of the H.W.R solution as shown in Fig(42), whilst it led to a slight difference for the case of the L.W.R solution as shown in Fig(43) and Fig(44).

The case study number 2 (the Scorpion tank) has been employed to verify linear and nonlinear 2-D and 3-D simulations, as well as nonlinear time response solutions.

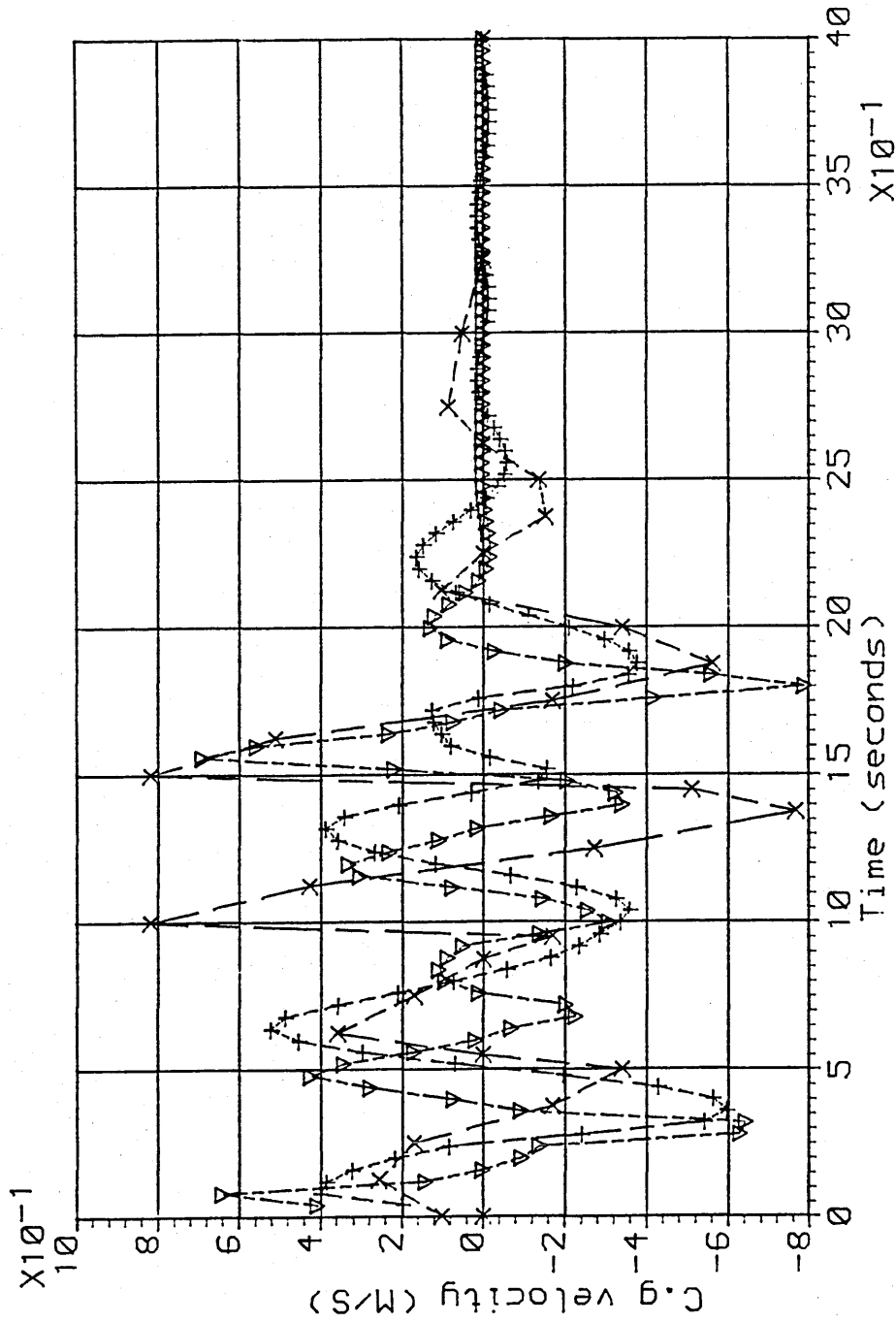


Δ --- Δ Tyre no 1
 $+$ --- $+$ Tyre no 2
 $\times$ --- $\times$ Tyre no 3
 $\square$ --- $\square$ Tyre no 4
 $\diamond$ --- $\diamond$ Tyre no 5

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Fig(36) Wheel centre displacements of a linear car
model excited by a bump ($V=2.2$ M/S).

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L.W.R.

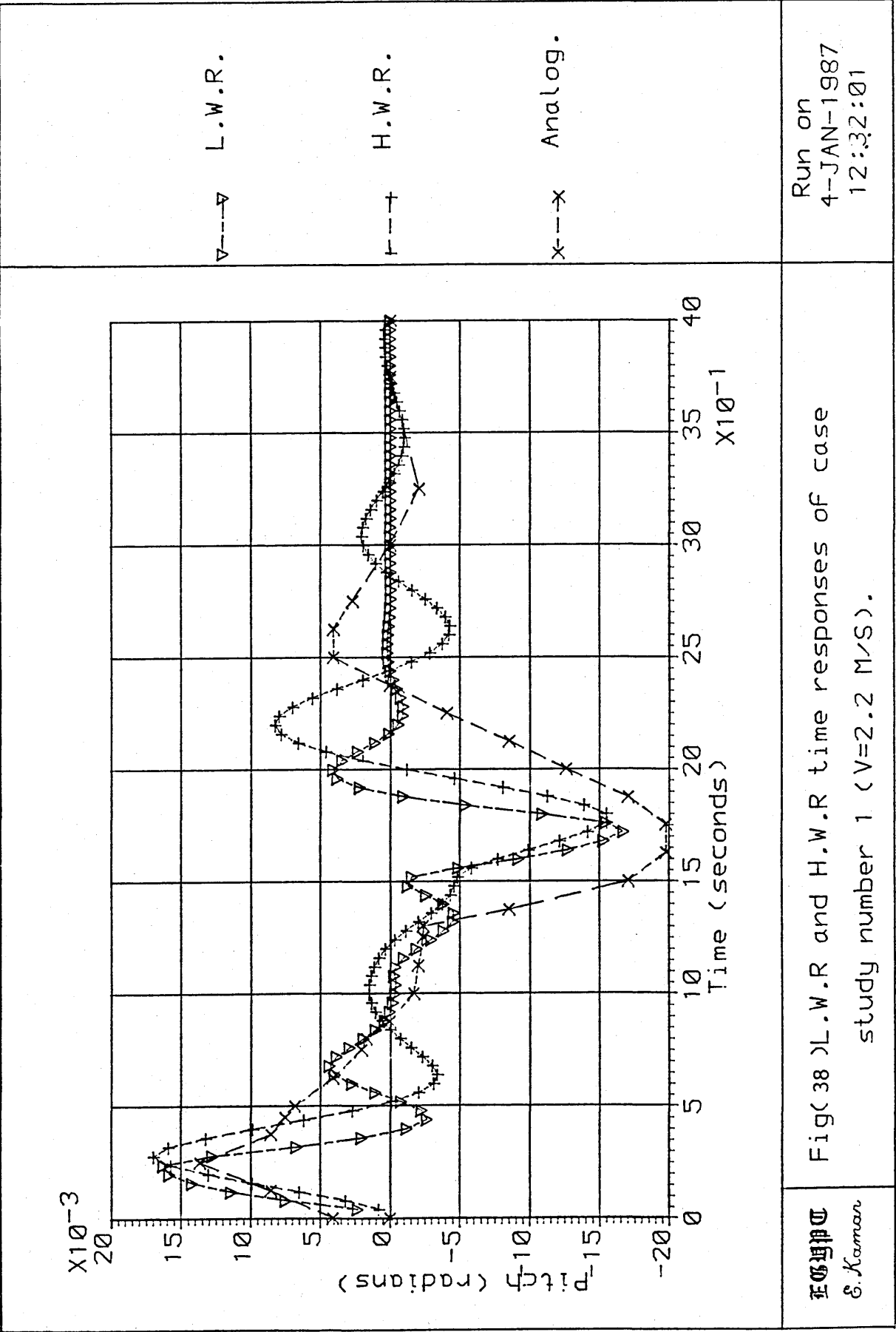
H.W.R.

Analog.

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Fig(37) L.W.R and H.W.R time responses of case
study number 1 ($V=2.2$ M/S).

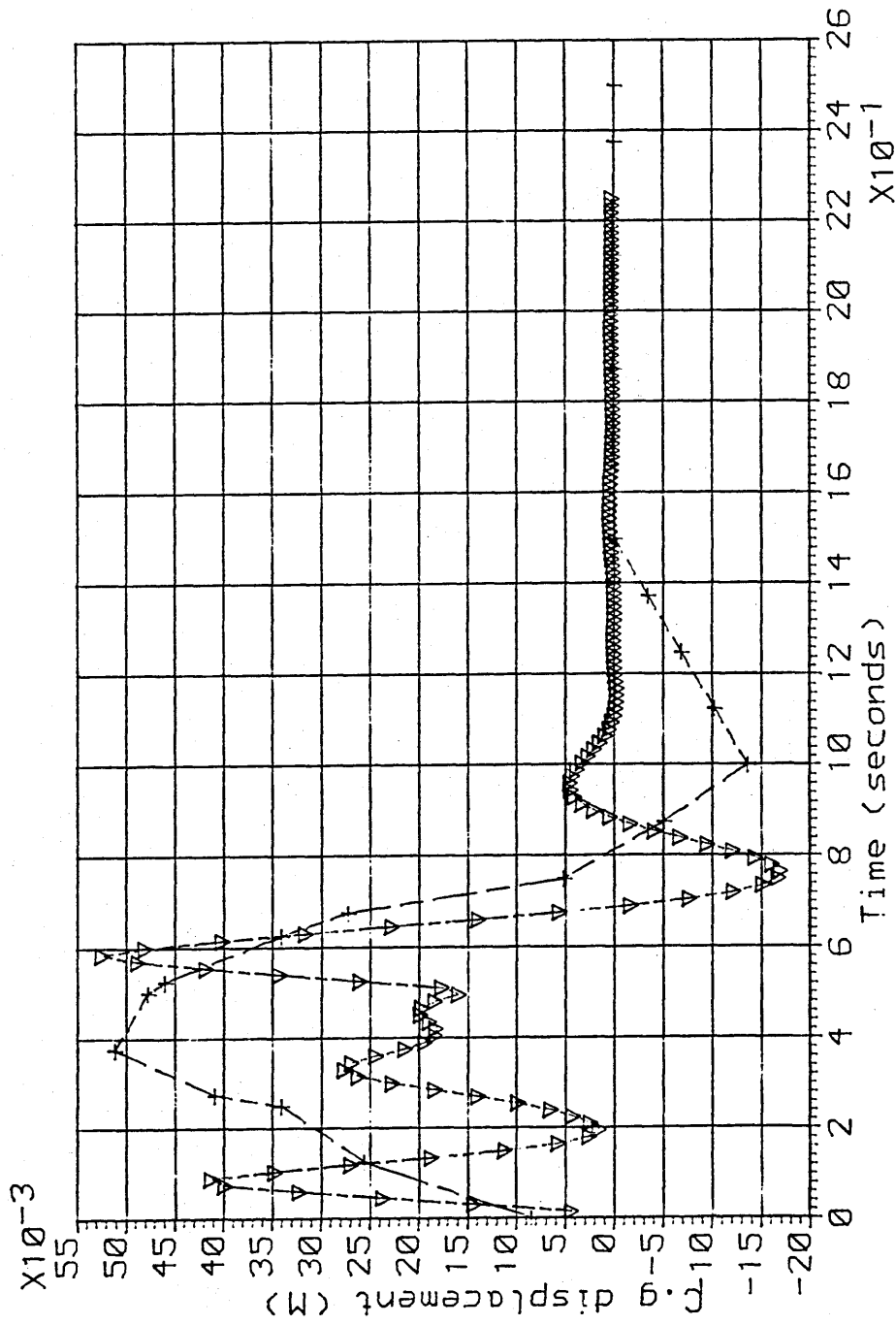
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Fig(38) L.W.R and H.W.R time responses of case study number 1 ($V=2.2$ M/S).

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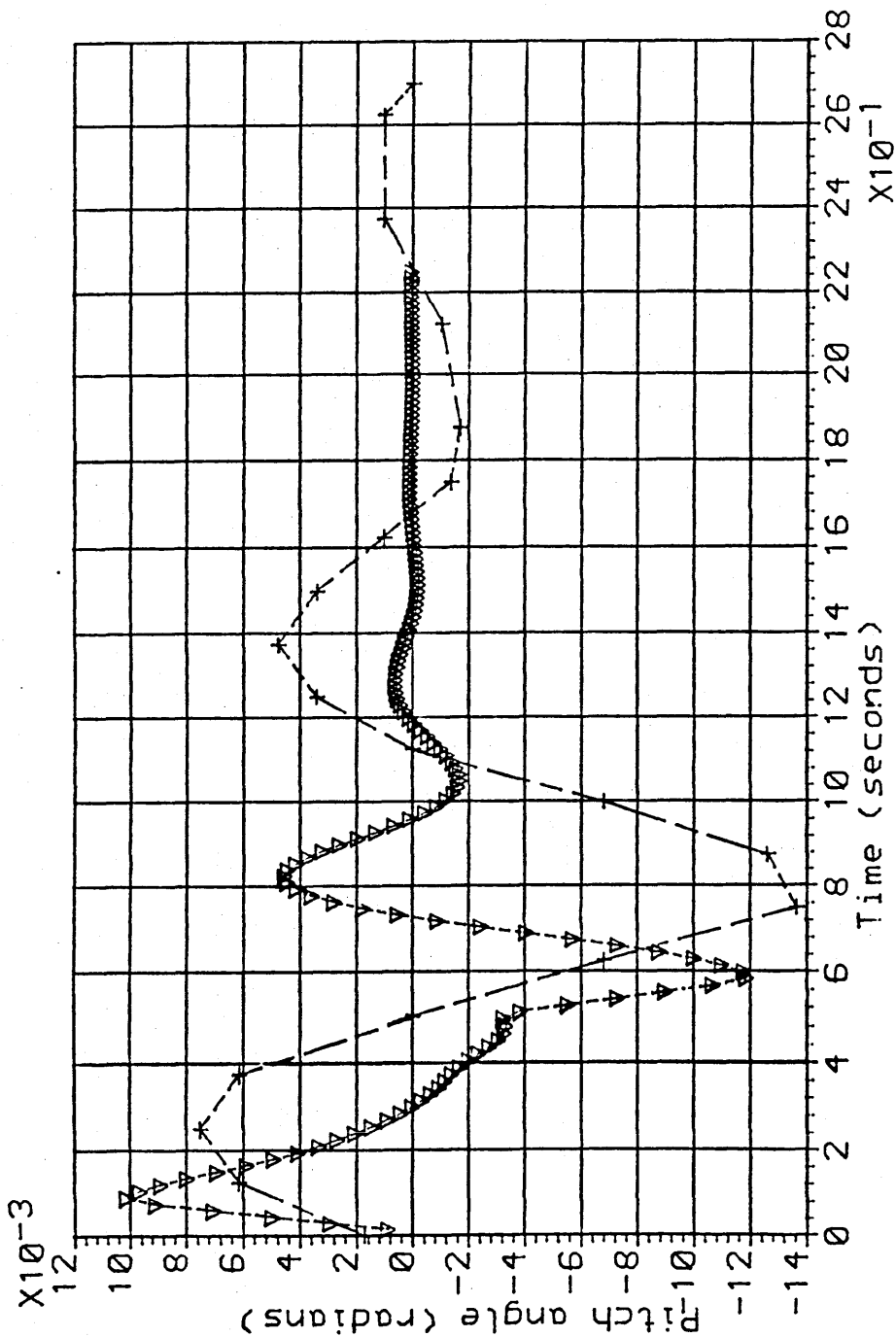
▽---▽ L.W.R.

+---+ Analog.

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Fig(39) L.W.R time response of case study number 1
(V=6.7 M/S).

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S. Kamar



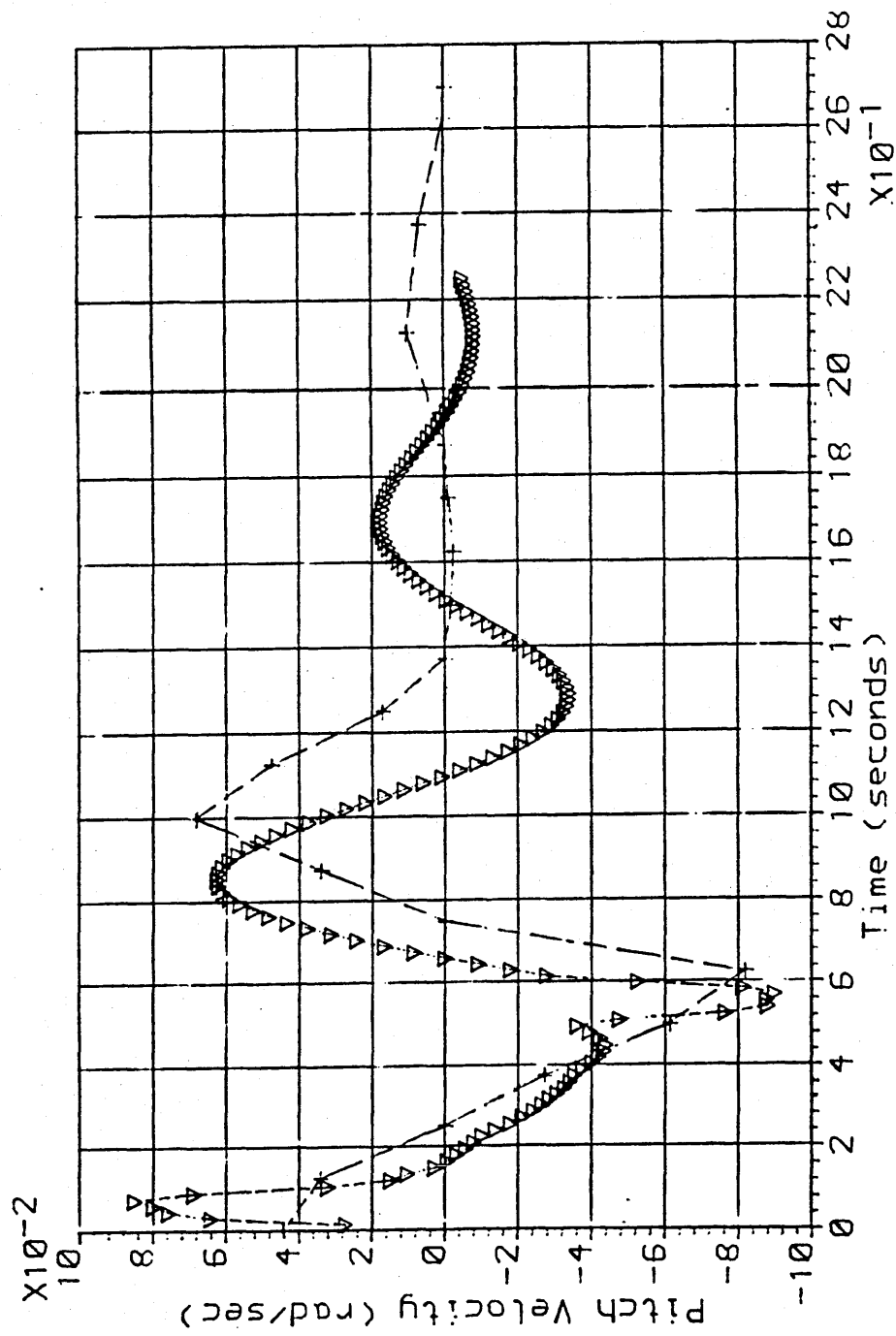
L.W.R.

Analog.

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Fig(40) L.W.R time response of case study number 1
(V=6.7 M/S).

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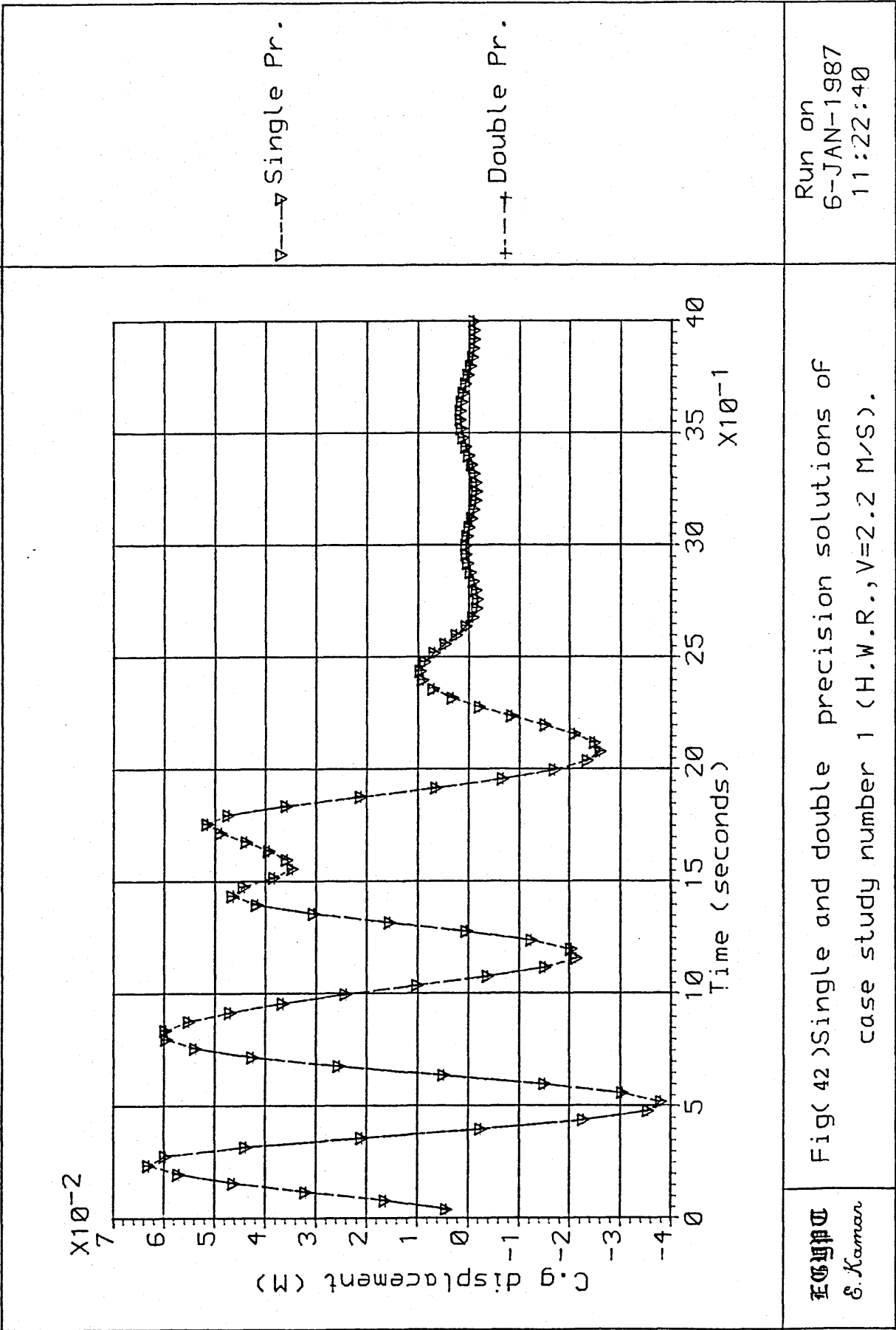


H.W.R.
△
Analog.
+

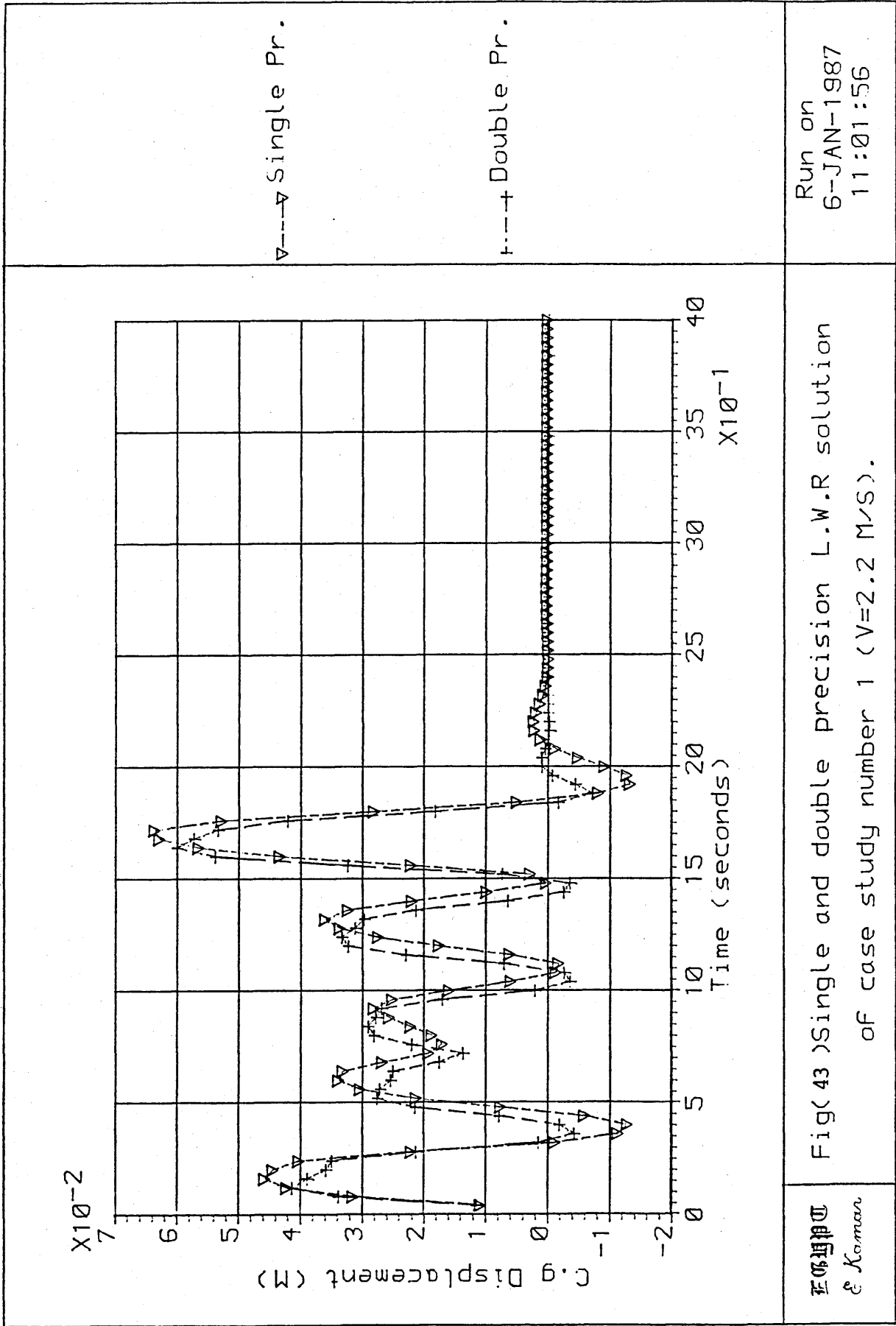
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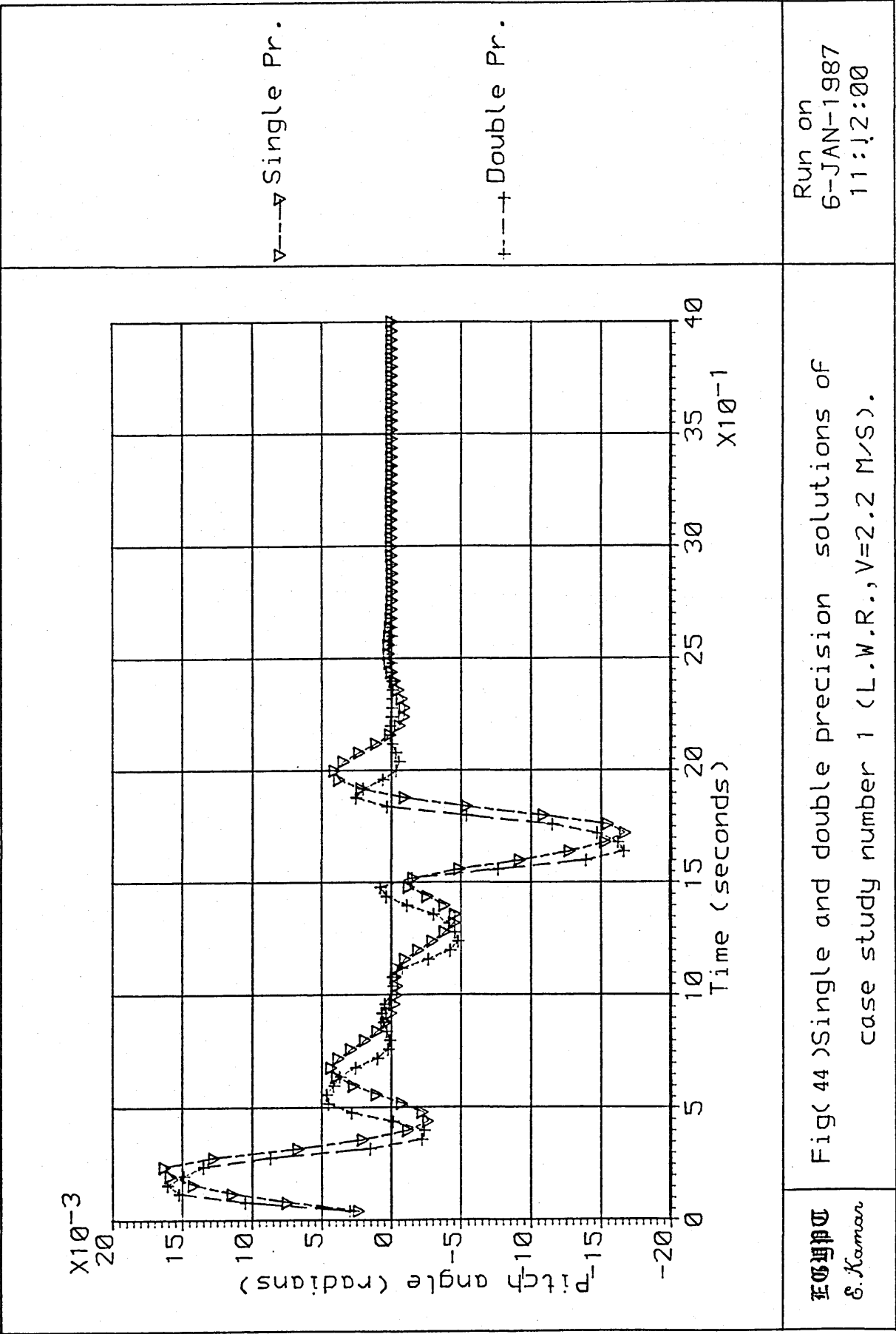
Fig 41) H.W.R time response of case study number 1
(V=6.7 M/S).

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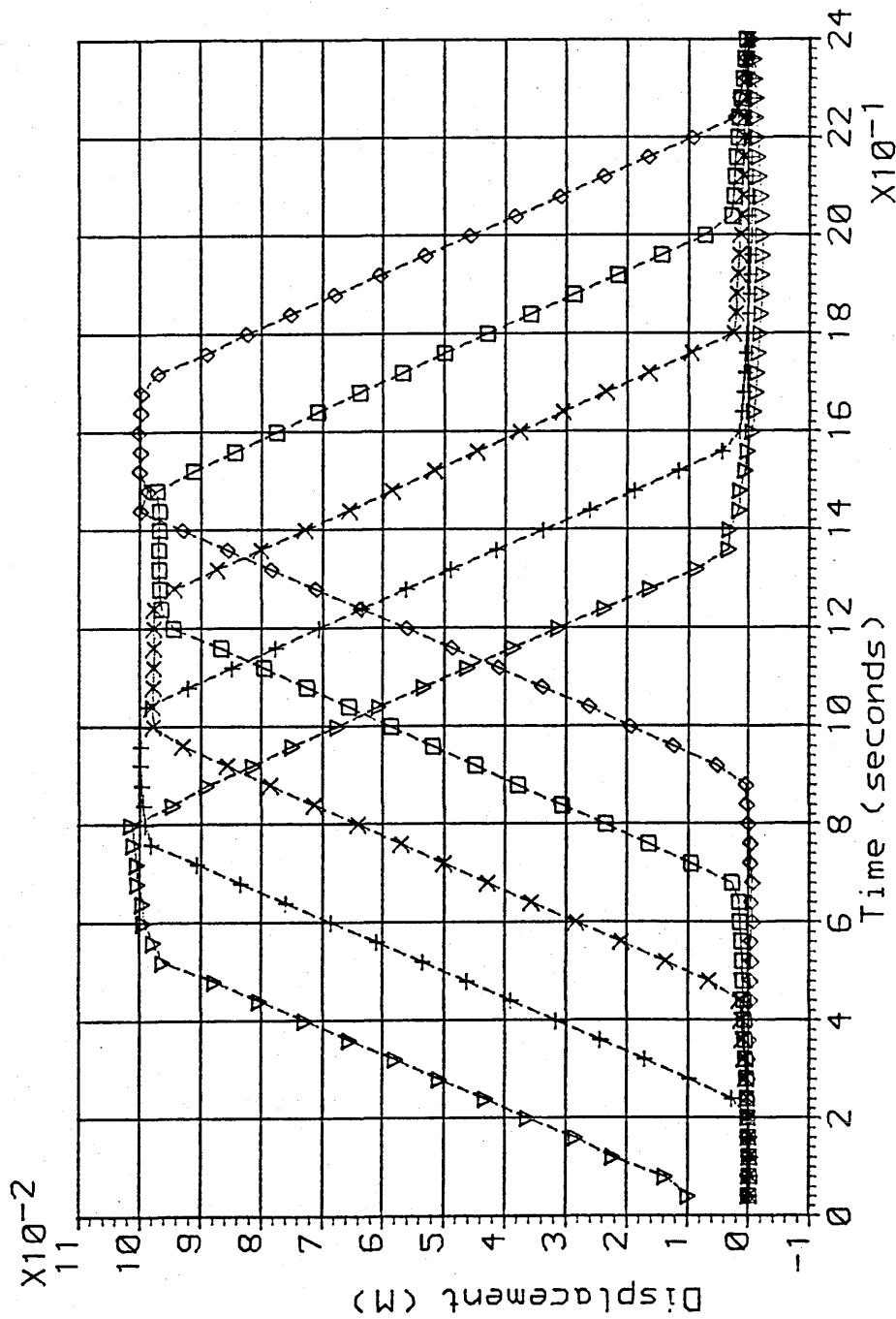
A 2-D and a 3-D linear model has been excited by a bump and the results are compared with the experimental results. Fig(45) shows the displacement of the wheel centres of the tank. The maximum displacement of all wheels lies within the maximum bump height (0.1 m). Fig(46) shows the effect of vehicle speed on the excitation for the same bump. The vehicle speed is found to affect the amplitude of vibration as shown in Fig(47) and Fig(48). The positive and negative peak accelerations of the tank front which result from bump excitation of the 2-D and 3-D linear models are compared with the experimental results in Fig(49) and Fig(50). As linear models the results are acceptable. The limiting speeds according to A.P (6 watts limit) and I.S.O 2631 (8 hours limit) tolerance criteria, as shown in Fig(51), are 8.5 M/S and 4.7 M/S respectively.

The time response results of a 'nonlinear' tank model excited by a bump are compared with the experimental results in Fig(52) and Fig(53). The results agree reasonably well with the experimental results.

A sine wave excitation of 0.05 m amplitude and 4.5 m wave length has been employed to test the steady state response program. Moreover, the same wave has been tabulated, as the case of arbitrary excitation, and employed to excite the nonlinear tank model. The comparisons of the steady state response and time response with the experimental results are shown in Fig(54) and Fig(55). The steady state response overestimates the vibration whilst the results of the nonlinear model are quite acceptable.

7.5 FOUR-BAR LINKAGE STUDY RESULTS:

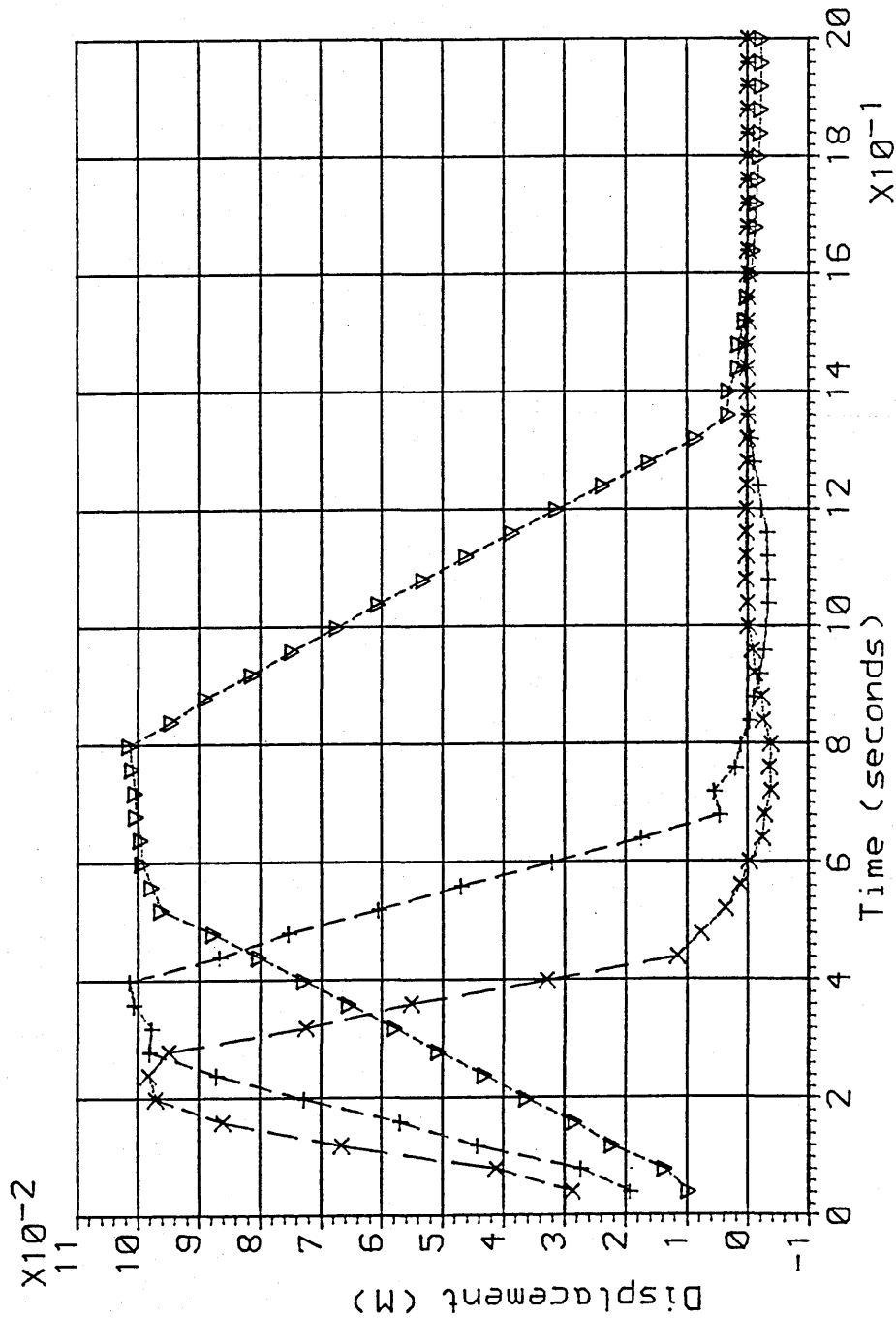
There is no published work related to the use of four-bar linkage in automotive vehicles. However, experts of ground research organizations in the U.S.A and the Federal Republic of Germany met in order to exchange information concerning the increase of the cross-country speed of heavy vehicles. At this meeting the importance of



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Fig(45) Displacement of wheel centres of a Scorpion
tank linear model excited by a bump.

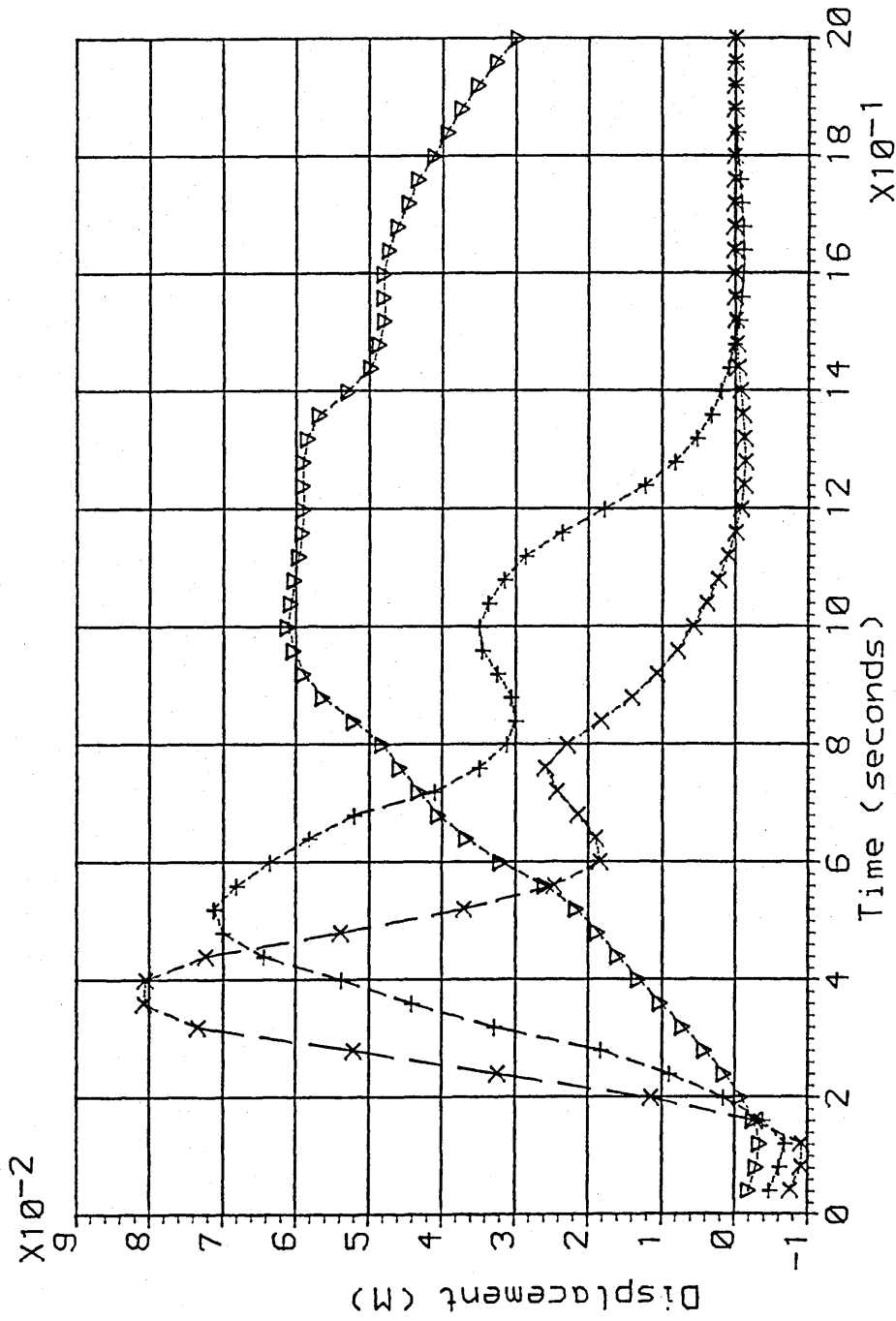
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Fig(46)Displacement of First wheel centre of a Scorpion tank linear model excited by a bump.

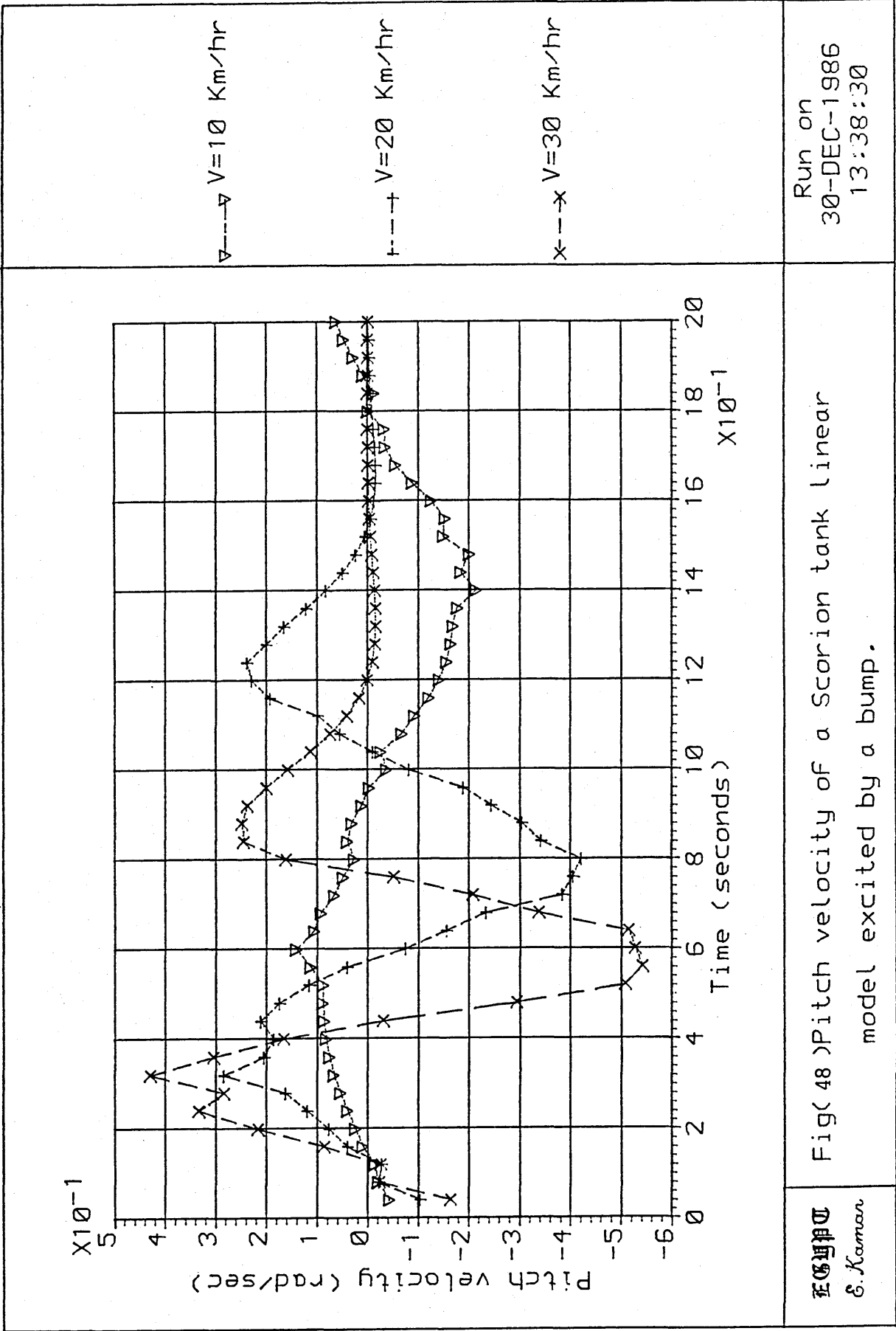
Б.Камар



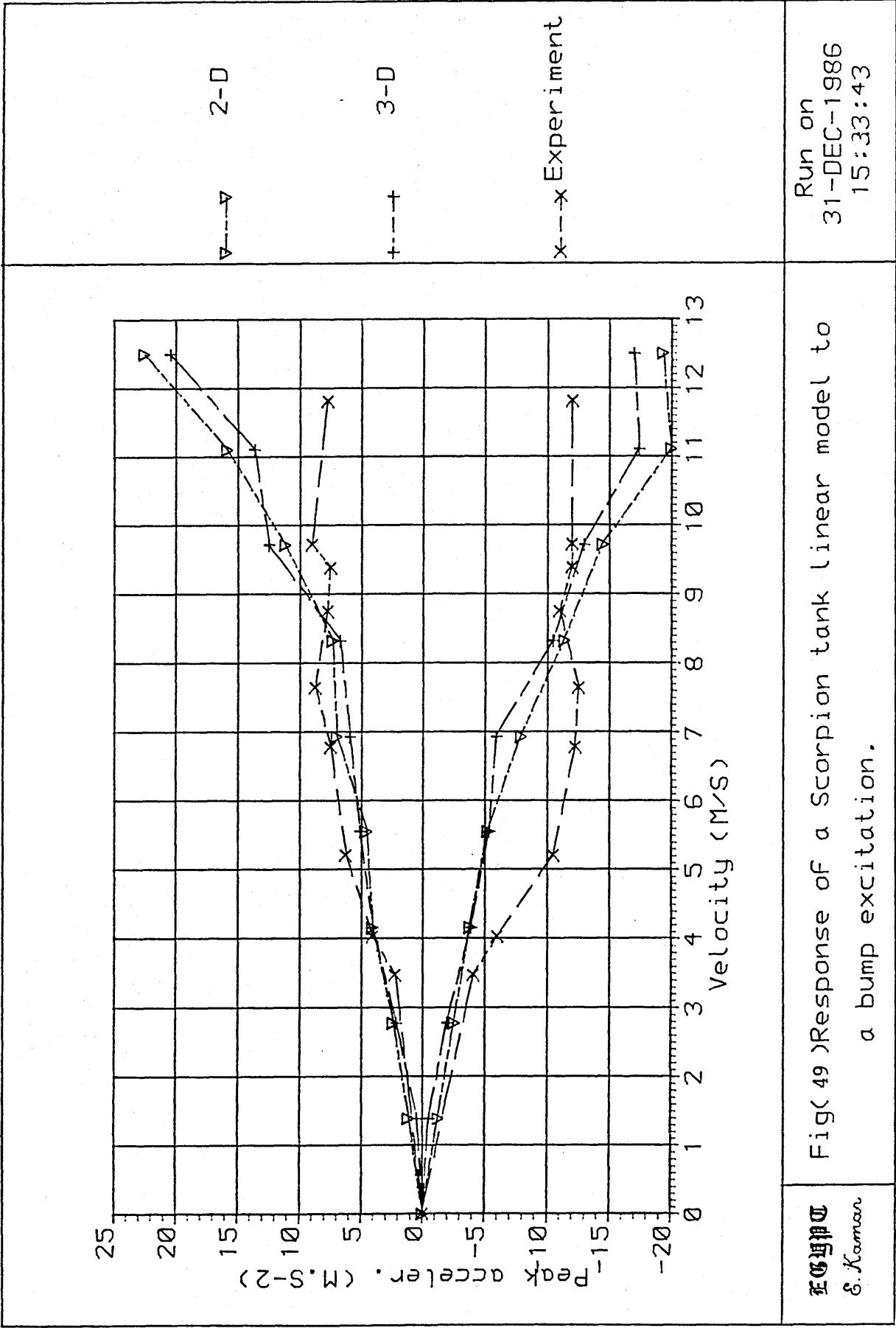
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Fig(47) Displacement of the c.g of a Scorpion tank
linear model excited by a bump.

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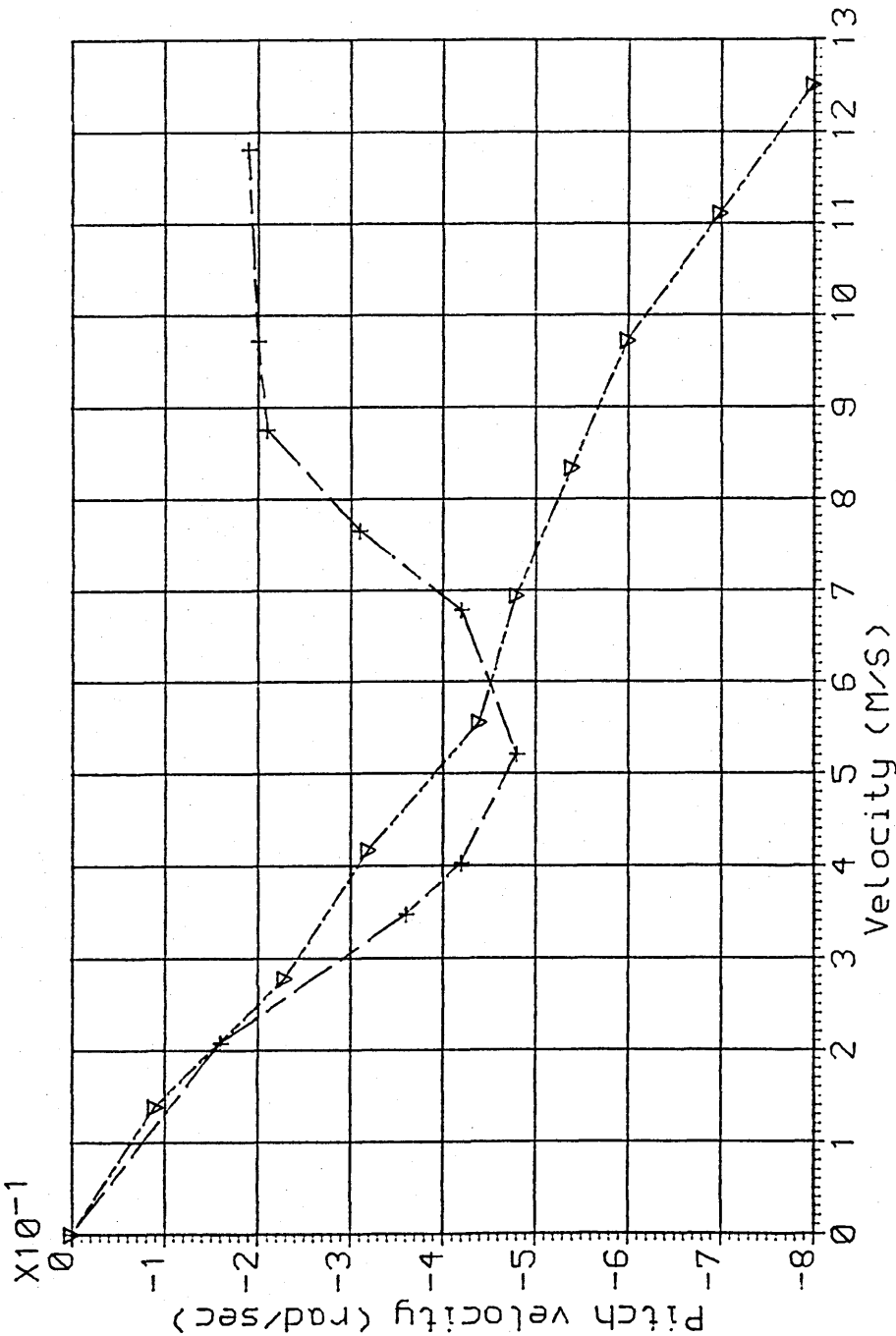


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Fig(49)Response of a Scorpion tank linear model to a bump excitation.

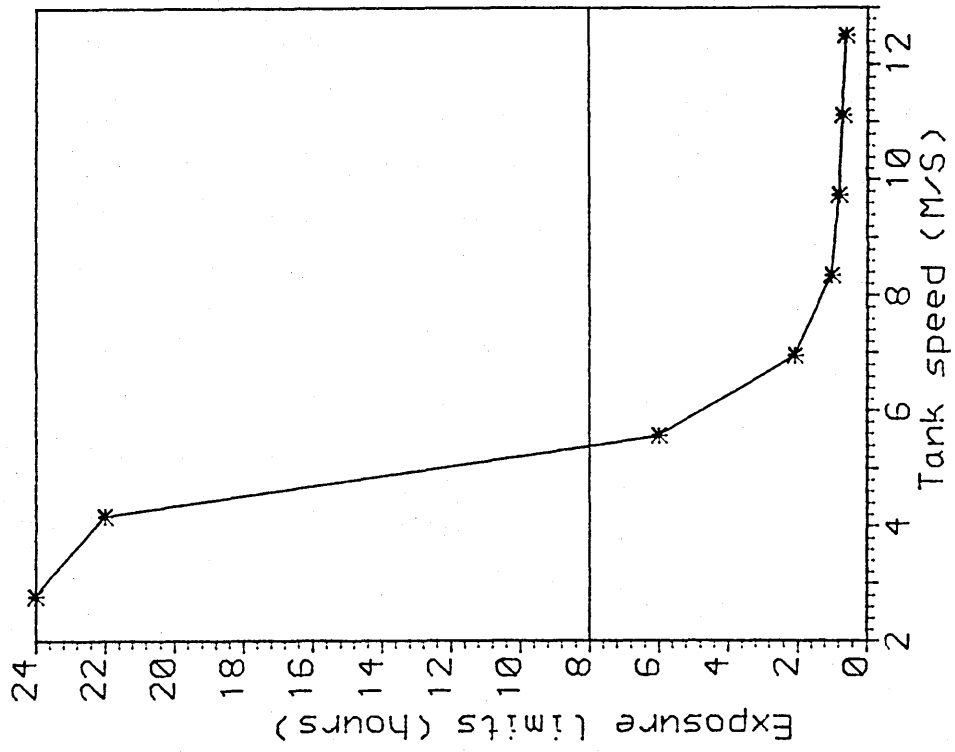
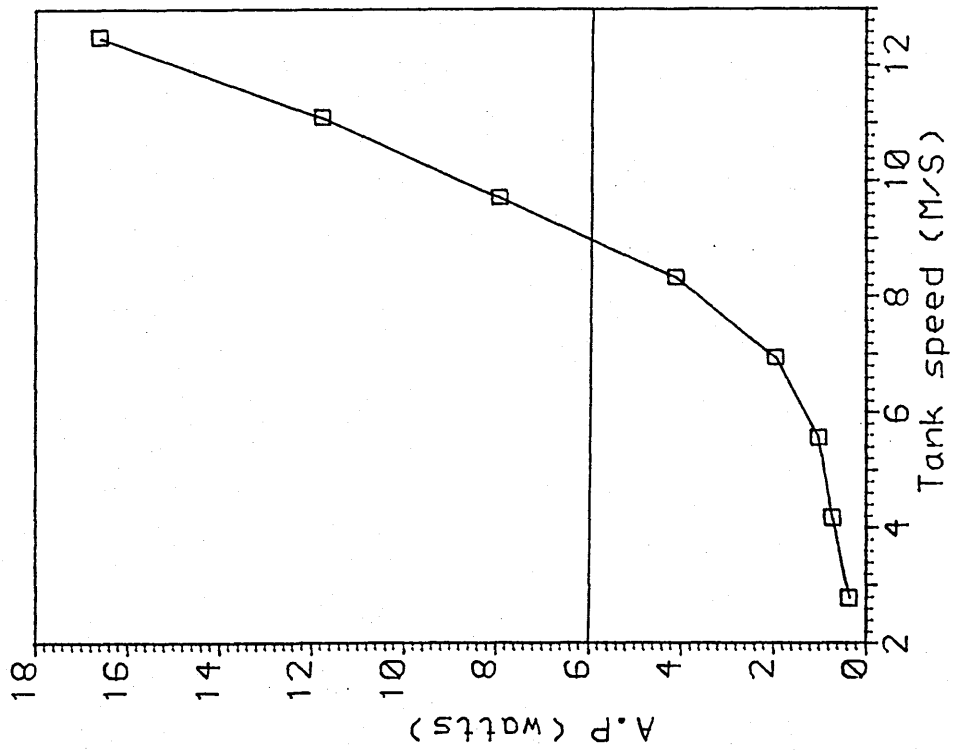
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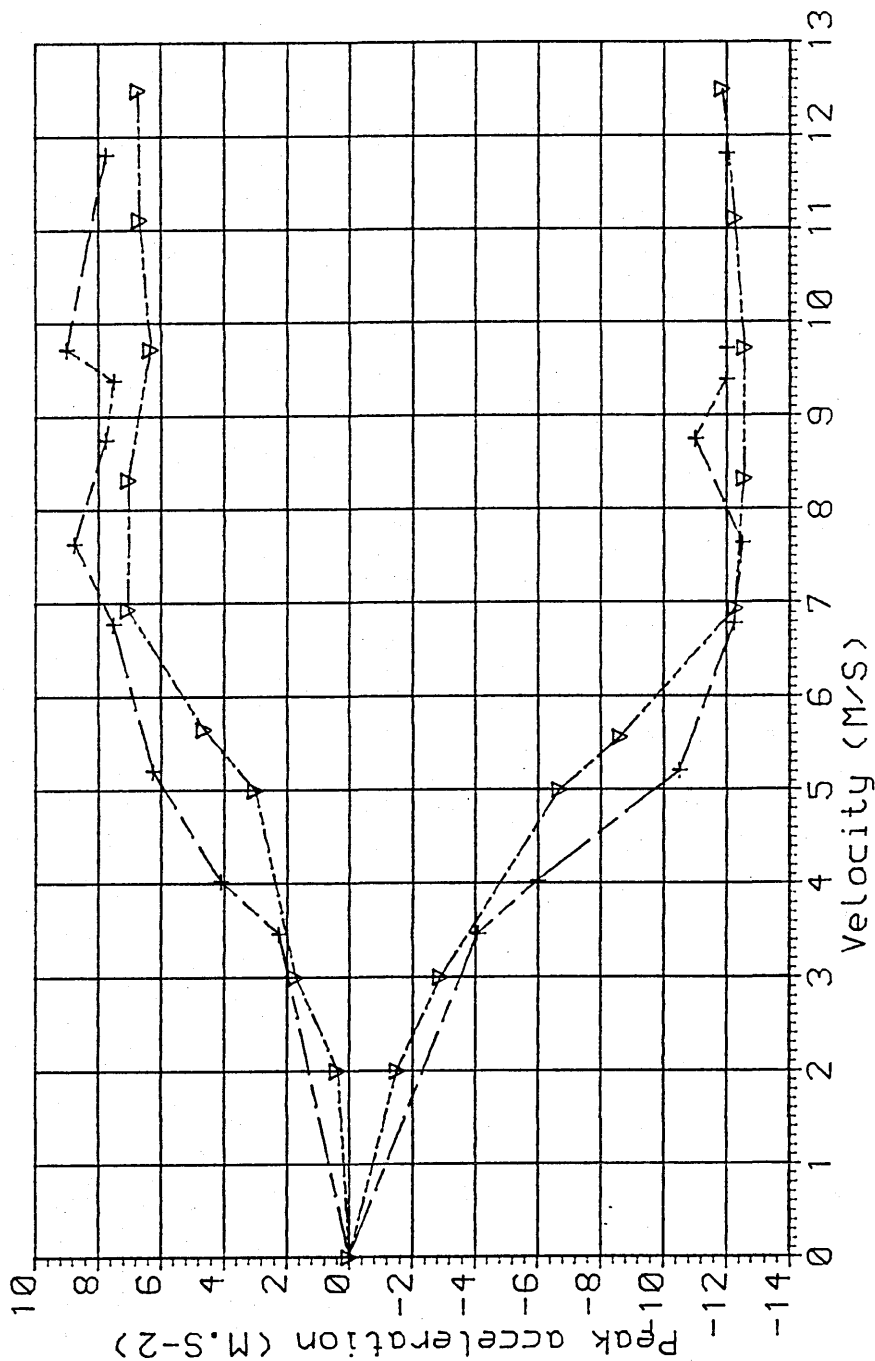
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Fig(50)Peak pitch velocity of a Scorpion tank
linear model excited by a bump.

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Fig(51) A.P and I.S.O limits for a tank model excited by a bump.



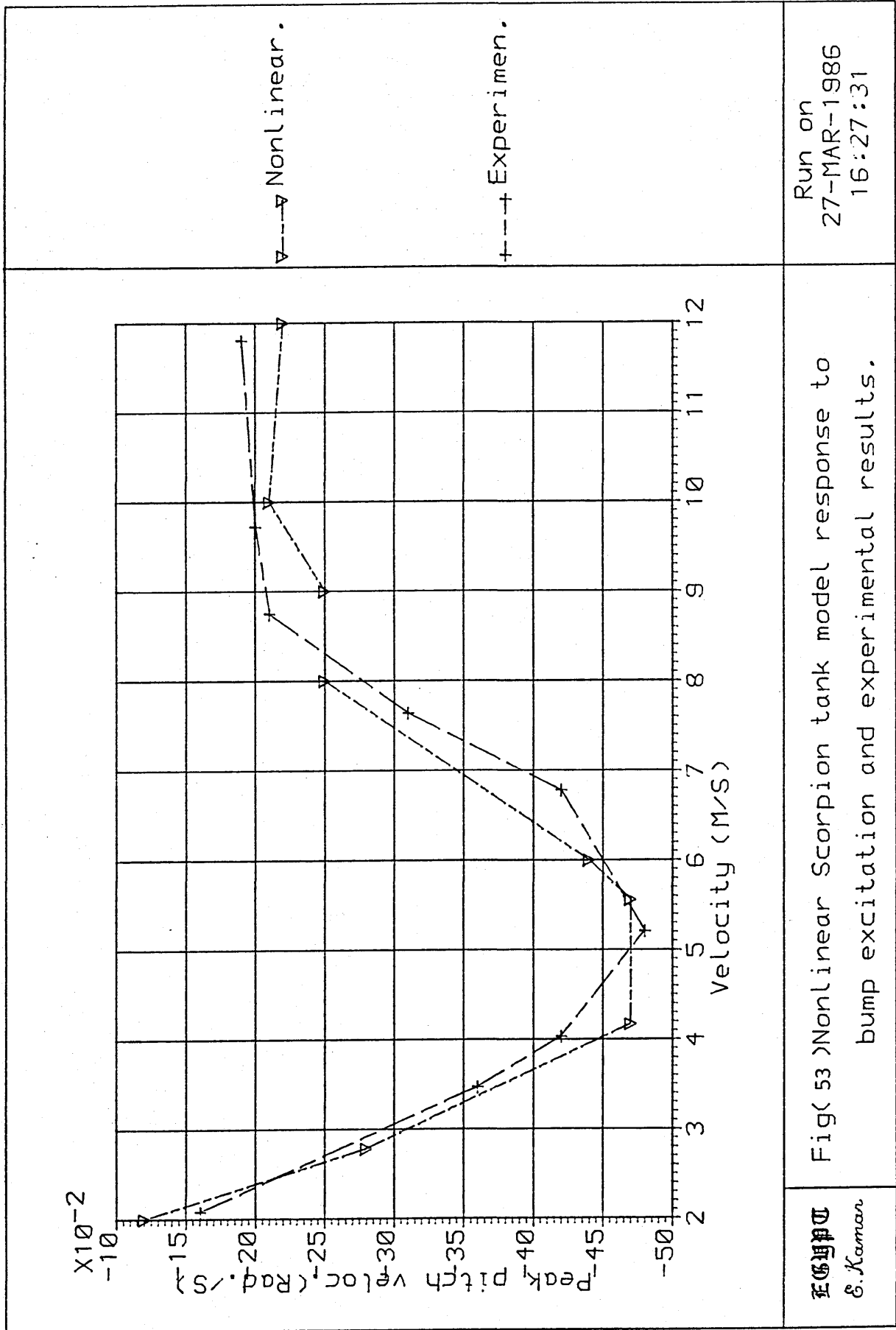
▽---▽ Nonlinear.

+---+ Experimen.

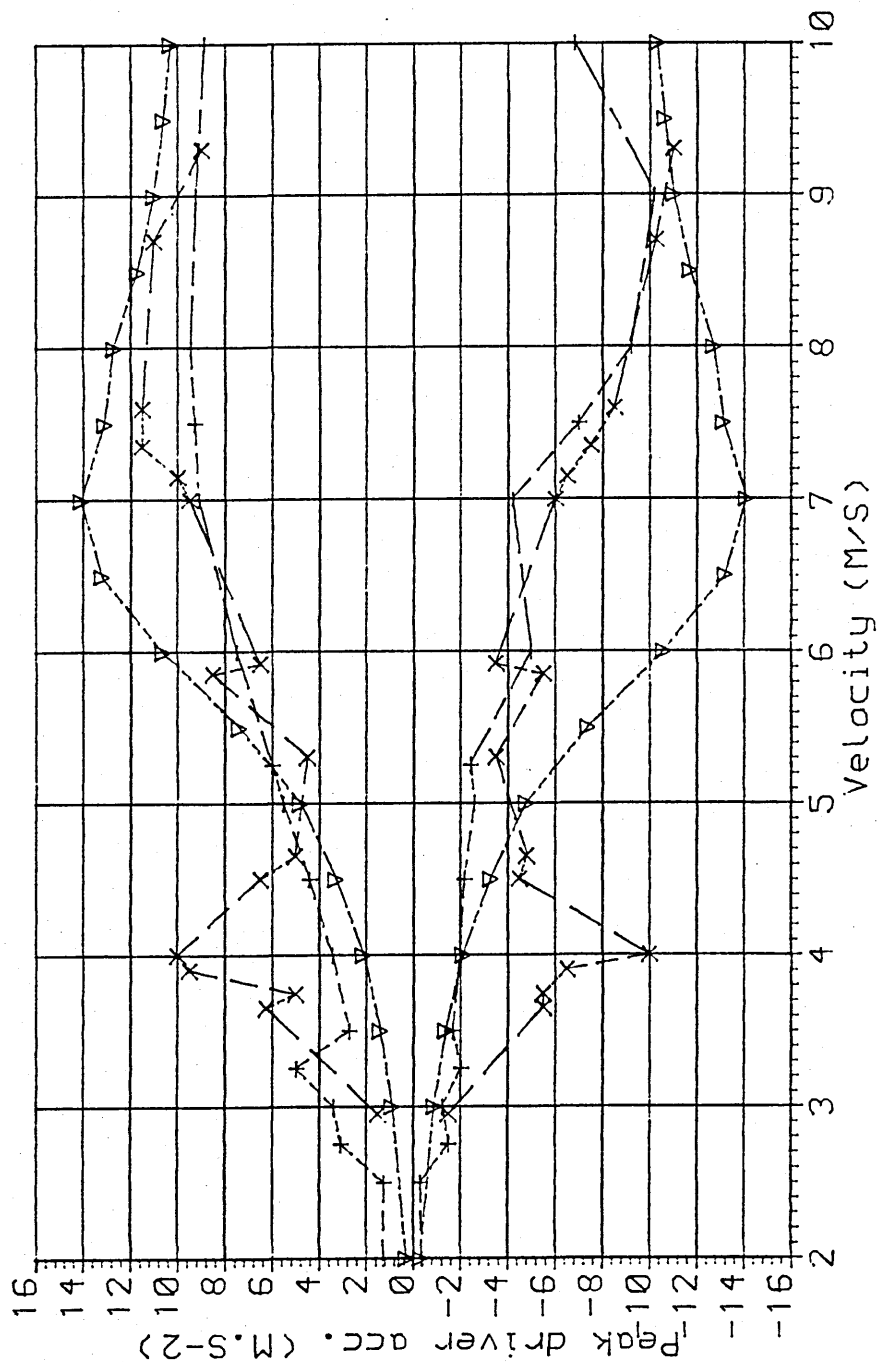
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Fig(52) Nonlinear Scorpion tank model response to
bump excitation and experimental results.

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△---△ Linear.

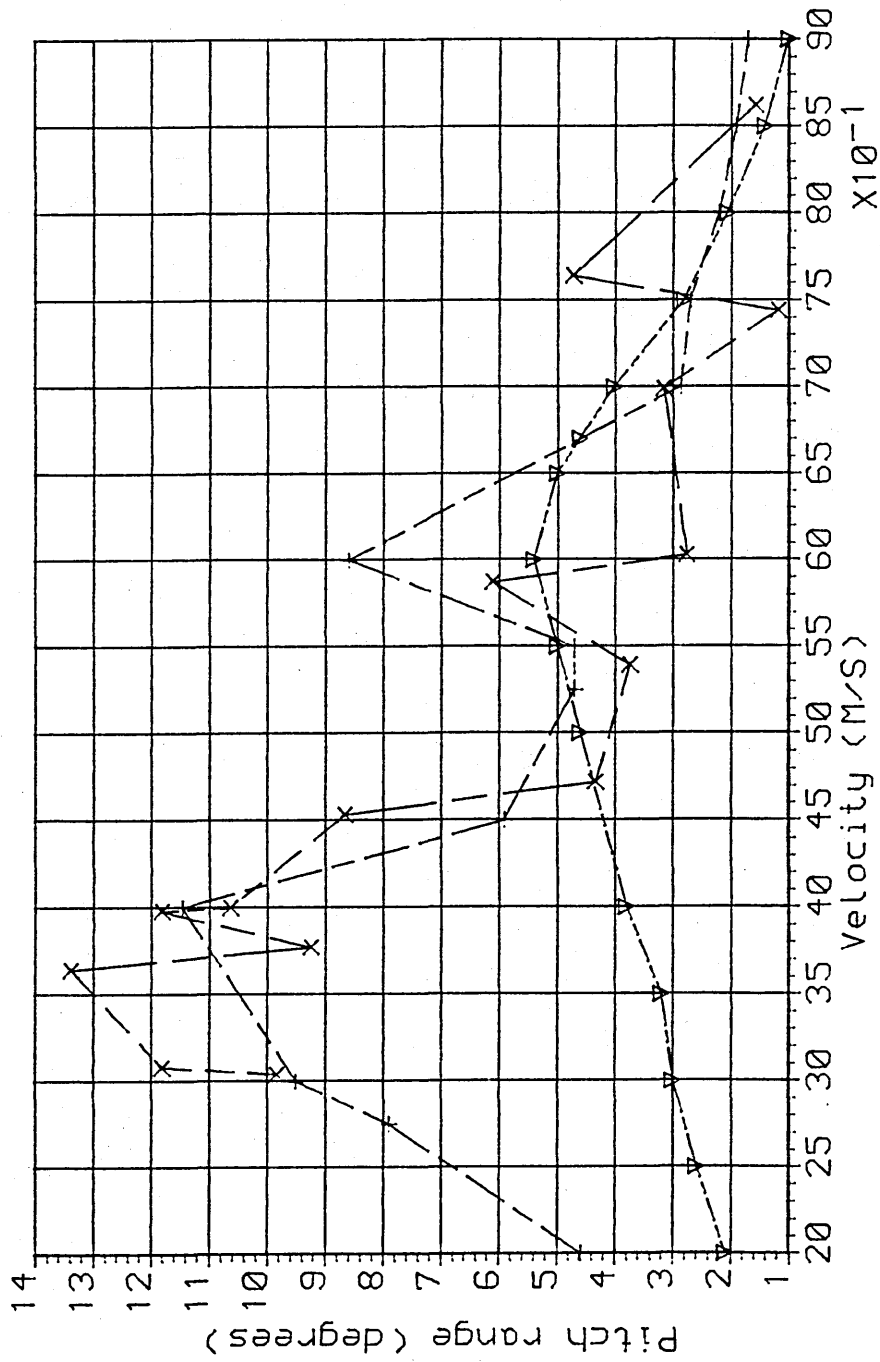
+---+ Nonlinear.

x---x Experimen.

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Fig(54) Comparison of linear and nonlinear tank models response with the experimental results.

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△---△ Linear.

+---+ Nonlinear.

x---x Experimen.

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Fig(55)Comparison of linear and nonlinear tank models response with the experimental results.

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rotary-vane dampers was emphasised [81].

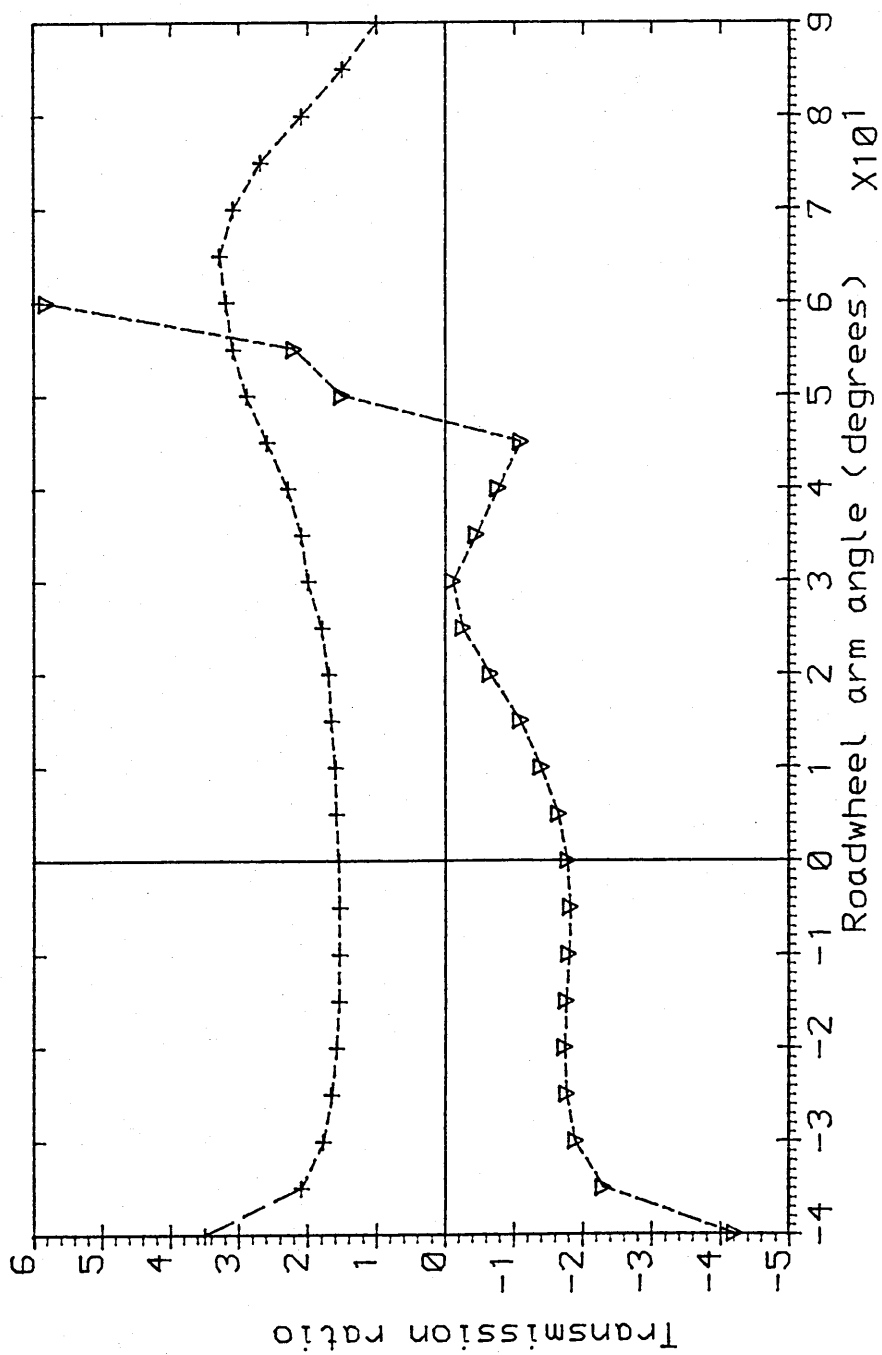
Siorek [82] presented the U.S.A view and concluded that for tracked vehicles their approach has been to develop the rotary damper. This unit with its increased fluid volume and controlled force generation capabilities has shown the ability to run cooler while producing proper damping over an extended range and for long periods of time. This approach largely has cleared up the problem for the development of new vehicles which will replace the current fleet.

Fig(56) shows the relation between the transmission ratio and roadwheel arm angle for the rear and front dampers of the Scorpion tank. The rear damper linkage allows an angle of the roadwheel arm from about -40° to $+60^\circ$ only due to geometry consideration. The bump stop, and the track, limit that angle to be within the range -25° to 21° . For this range of angle the transmission ratio is approximately constant. The two curves show that the transmission ratio varies considerably at the range extremes. This feature can be employed to increase the damping force at extreme positions of the roadwheel arm (similar to the blow-off function of the damper).

The damper arm length affects the transmission ratio as shown in Fig(57). Five arm lengths are considered in the roadwheel arm angle range from -30° to 30° . All cases have the same transmission ratio λ at θ_r of 10° . The negative part of the curves are bounded by an increasing curve ($r=152$ mm) at the bottom and a decreasing curve at the top ($r=232$ mm). The positive parts of all the curves are increasing functions.

The position of the roadwheel arm where the coupler link is attached, affects greatly the ratio λ around $\theta_r=0$. This effect decreases as the angle θ_r grows in the negative or positive direction as shown in Fig(58).

The coupler link length r_{AB} has a reduced and varying effect on the ratio λ as shown in Fig(59).

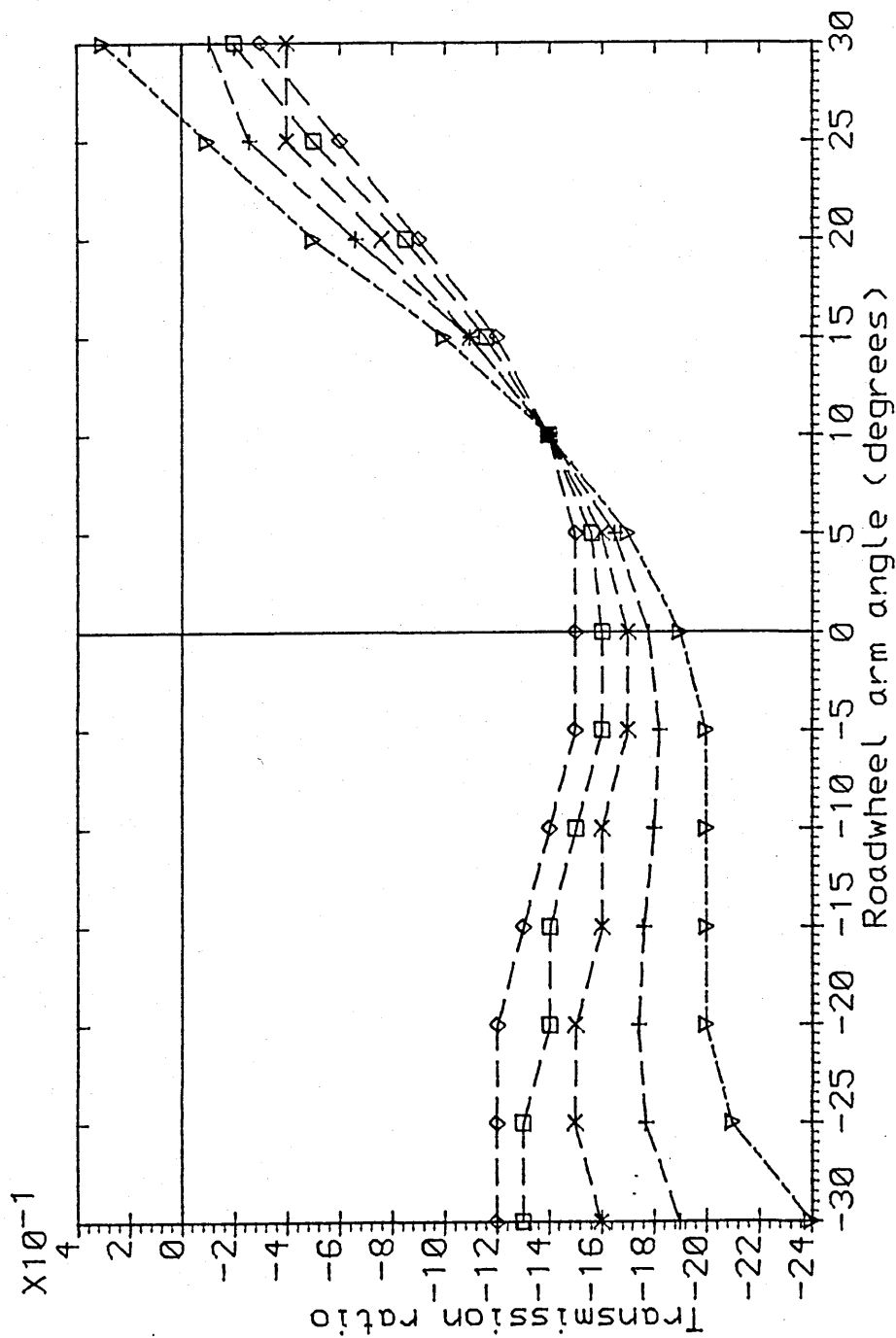


▽---▽ Rear damp.

+---+ Front one

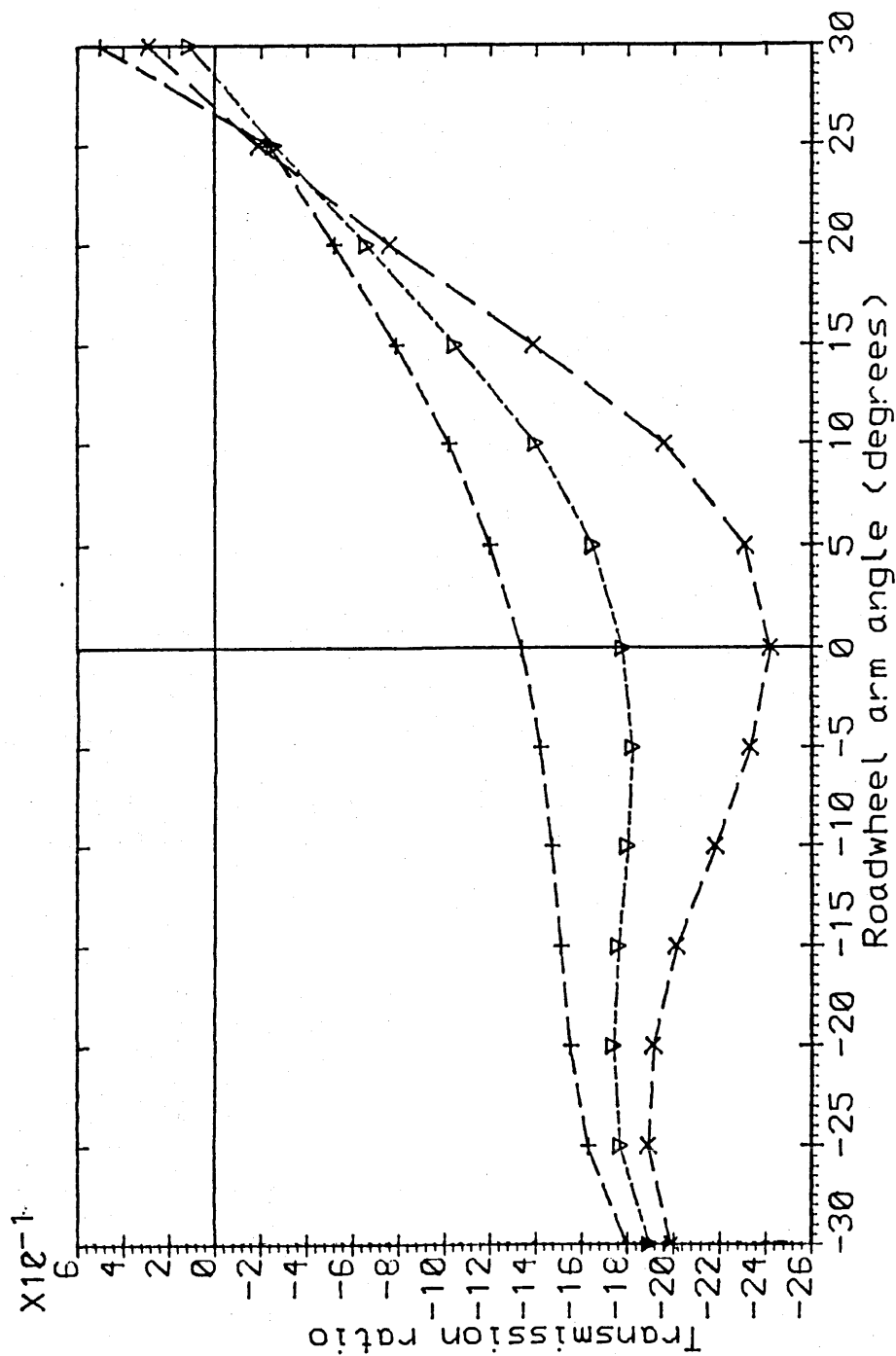
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Fig(56)Comparison of transmission ratios of rear
and front dampers of a Scorpion tank.



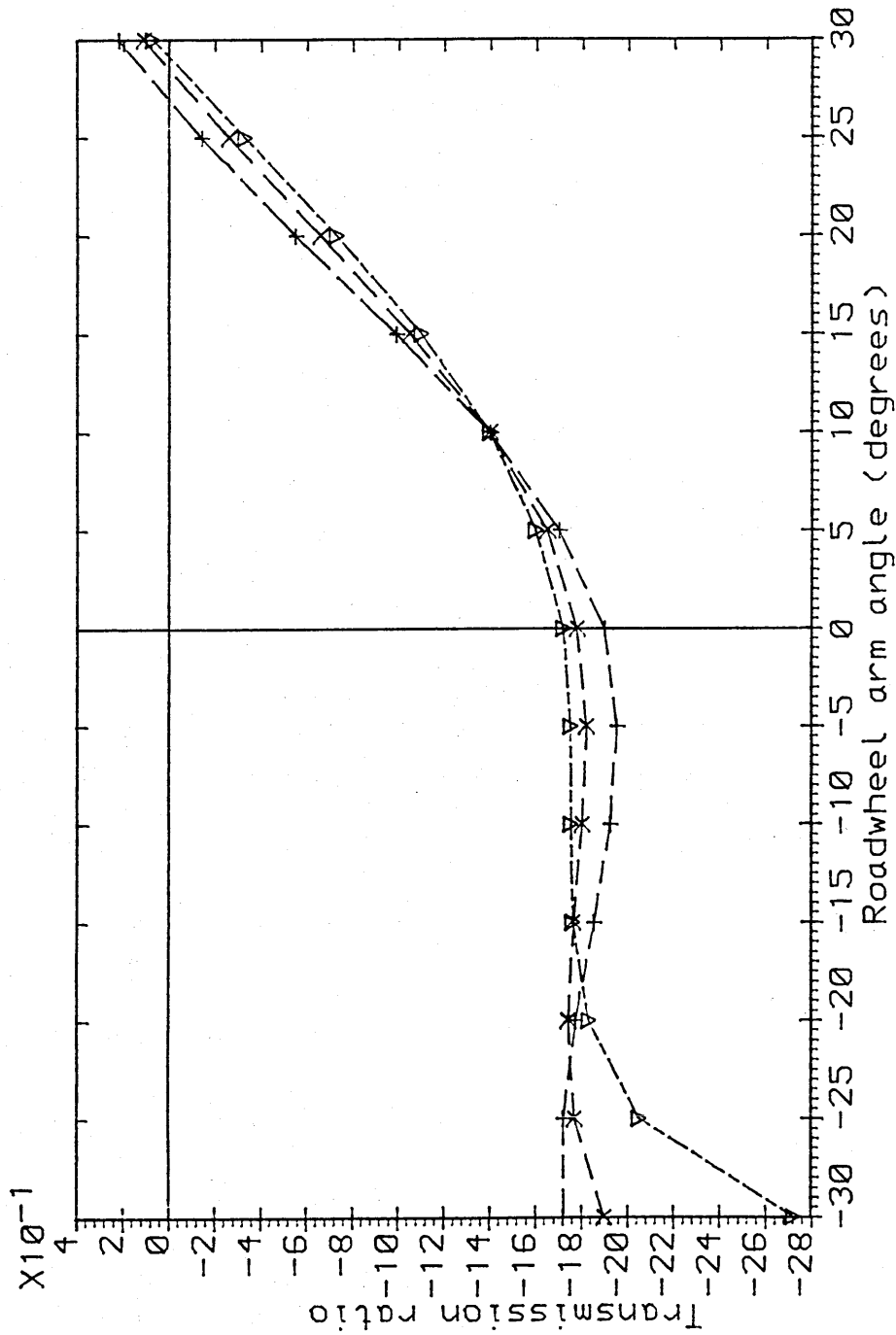
Fig(57) Effect of damper arm length on transmission ratio.

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Fig(58)Effect of radius r_B on transmission ratio.

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$\nabla \cdots \nabla r_{AB} = 188$ mm

$\circ \cdots \circ r_{AB} = 228$ mm

$\times \cdots \times r_{AB} = 268$ mm

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Fig(59)Effect of coupler link length on the transmission ratio.

The displacement of attachment point of the damper c and the position of the torsion bar in the Z -direction affects the values of the ratio λ more than the displacement in the Y -direction as shown in Fig(60).

The range of θ_r and corresponding λ can be selected according to the geometry of the undercarriage of the vehicle and the required damping force. Controllability of the developed damping force is possible by changing the dimensions of the four-bar linkage. This is a flexible tool in the hand of the designer.

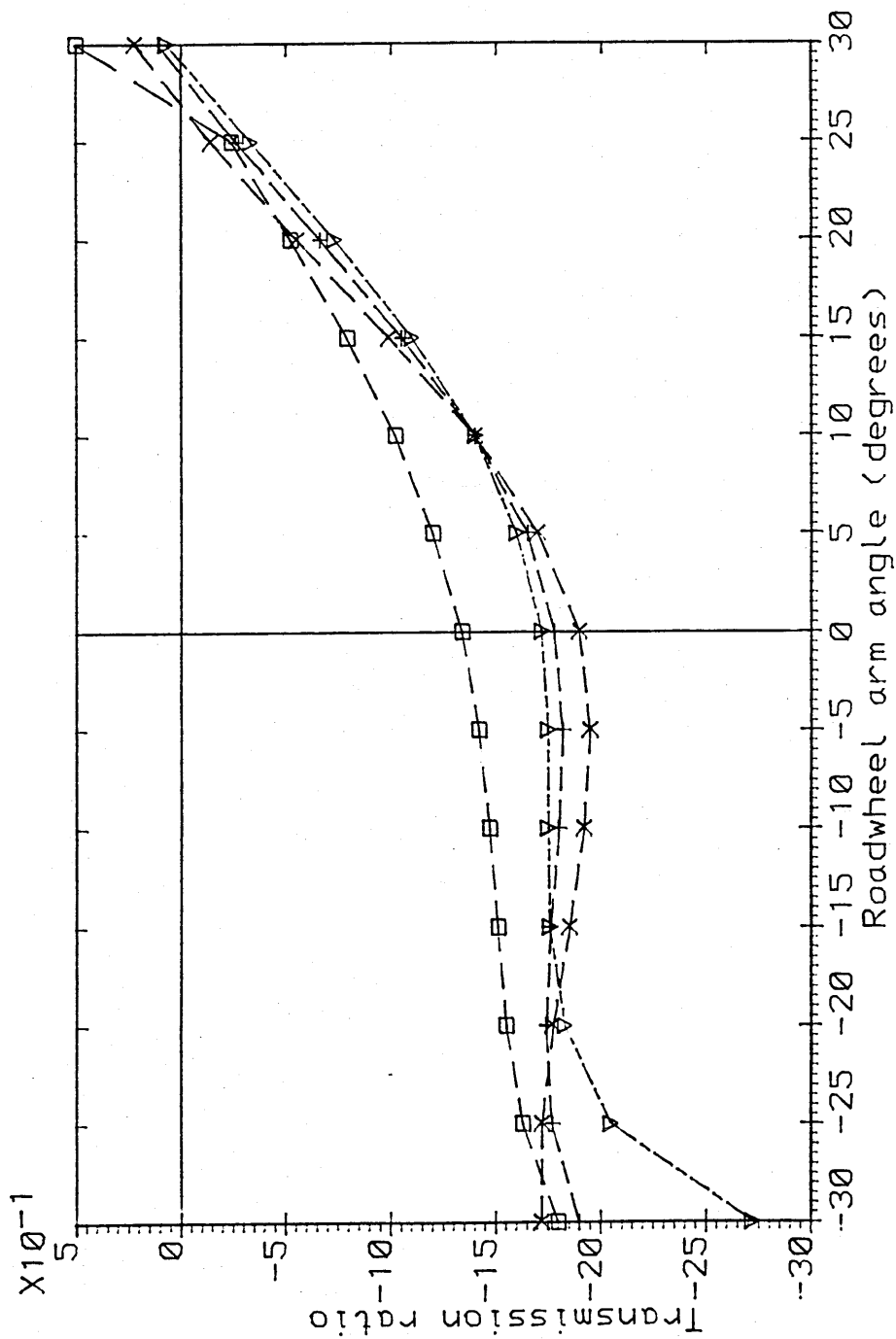
7.6 GENERAL DISCUSSION:

In this section, some important highlights will be raised in relation to vehicle dynamics, and some useful findings which have arisen during the use of the present code will be discussed.

7.6-1 3-D OR 2-D MODEL ?:

Many authors have employed 2-D modelling either because of simplicity or because of difficulties which may have arise during the experimentation used to verify the 3-D model [69], or because of the limitation of analogue computer size [37]. The use of a 2-D model reduces the number of degrees of freedom necessary to about half of those required for a 3-D model. An accepted justification for employing 2-D modelling is the symmetry of the suspension assemblies with respect to the centre of gravity of the body. In this case, care should be taken during experimental verification in order to avoid roll movement of the vehicle.

Unfortunately almost all relevant vehicles have unsymmetric suspension systems, for example in a tank there is a shift between left and right torsion bars, in addition to the displacement of the c.g towards the X and/or Y axis.



Fig(60)Effect of torsion bar location with respect to damper location (Y_d, Z_d) on transmission ratio.

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A typical tank with an asymmetric suspension system has been analysed with the aid of the present code using both 2-D and 3-D models, and the results due to excitation by a harmonic wave (amplitude 0.1 m and wave length 12 m) are shown in Fig(61) and Fig(62). The peak c.g acceleration of the 3-D model is about 90% of that of the 2-D model. The displacement of the front roadwheel centre determined from the 3-D model is about 73% of that produced by the 2-D model.

Fig(63) shows the peak positive and negative driver accelerations for the same tank, as obtained from the 2-D and 3-D models, as a function of the tank velocity for bump excitation. The two curves indicate that the velocity has significant effect on the peak driver acceleration value. The allowable tank velocity for crossing the bump for the 3-D model is shown from Fig(64) to be about 14% higher than that for the 2-D model if the six watt absorbed power criterion is applied.

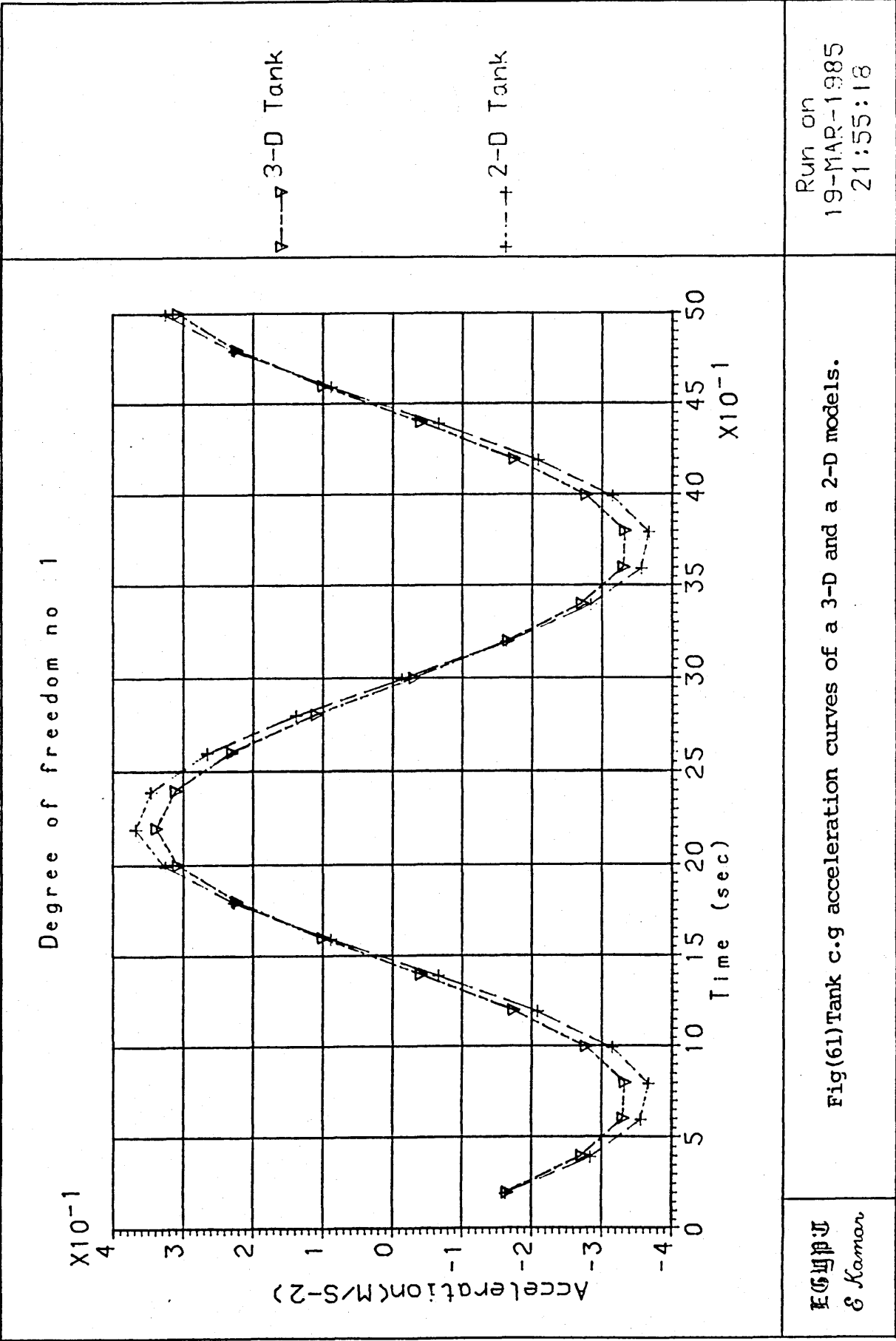
7.6-2 TANK SUSPENSION MODEL APPROXIMATION:

In most commercial packages, the transverse torsion bar suspension system is usually approximated to a vertical spring, for example, as shown in NRMM. Although this approximation is available in the present code when using the element type 1(a), the actual arrangement of a torsion bar can be accurately modelled using element types 1(b) and 1(c).

The vertical spring approximation leads to an underestimation of the response acceleration in some sections of the tank velocity range, whilst overestimating the response in other sections of the range as shown in Fig(65). The tank model allows an approximately 13% higher bump crossing velocity than the car approximation model, as illustrated by Fig(66).

7.6-3 THE EFFECT OF NON-CONSTANT TORSION BAR STIFFNESS:

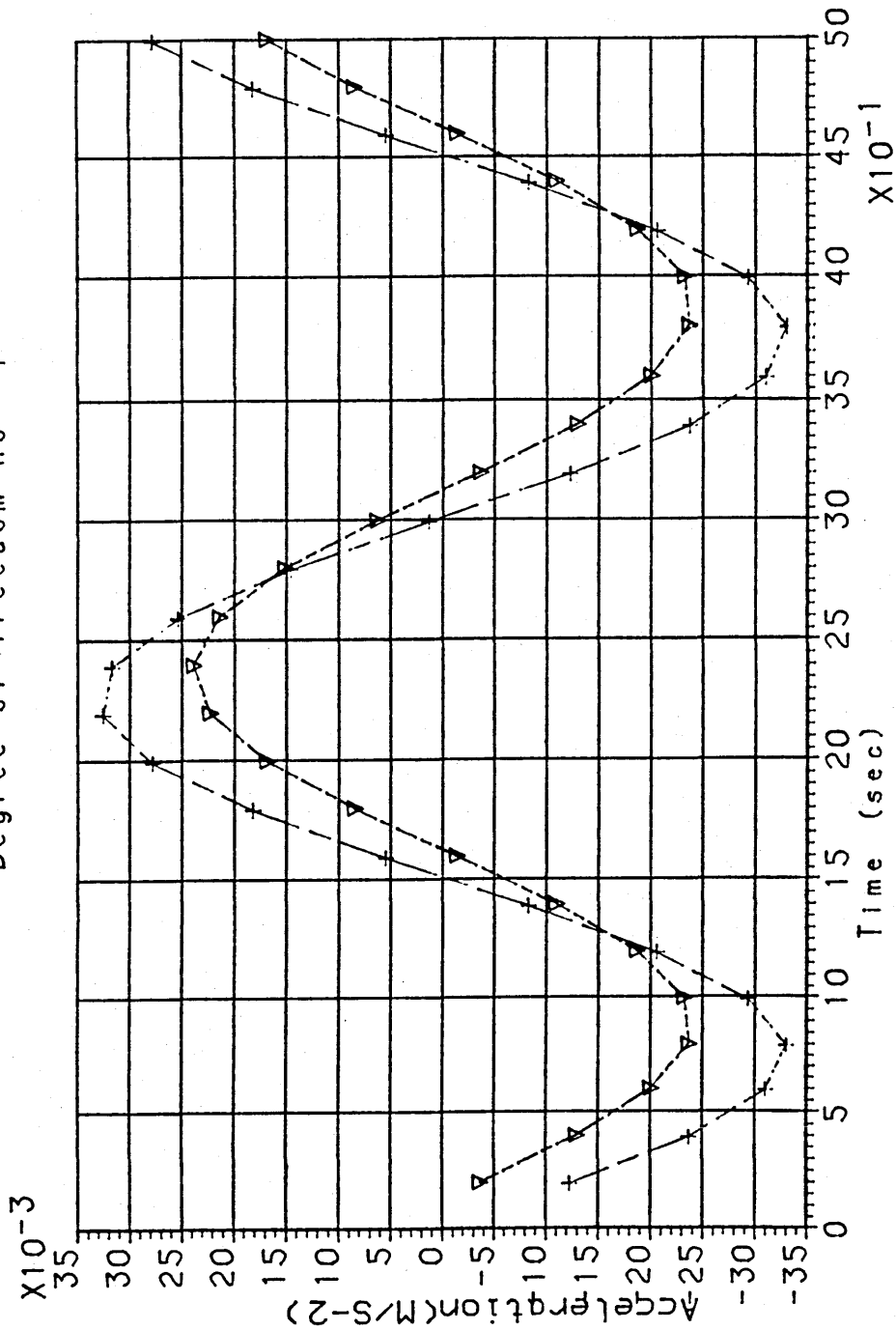
Fig(67) shows the displacement of the roadwheel centres for a Scorpion tank which was excited by a harmonic wave. The front and rear



Fig(61) Tank c.g acceleration curves of a 3-D and a 2-D models.

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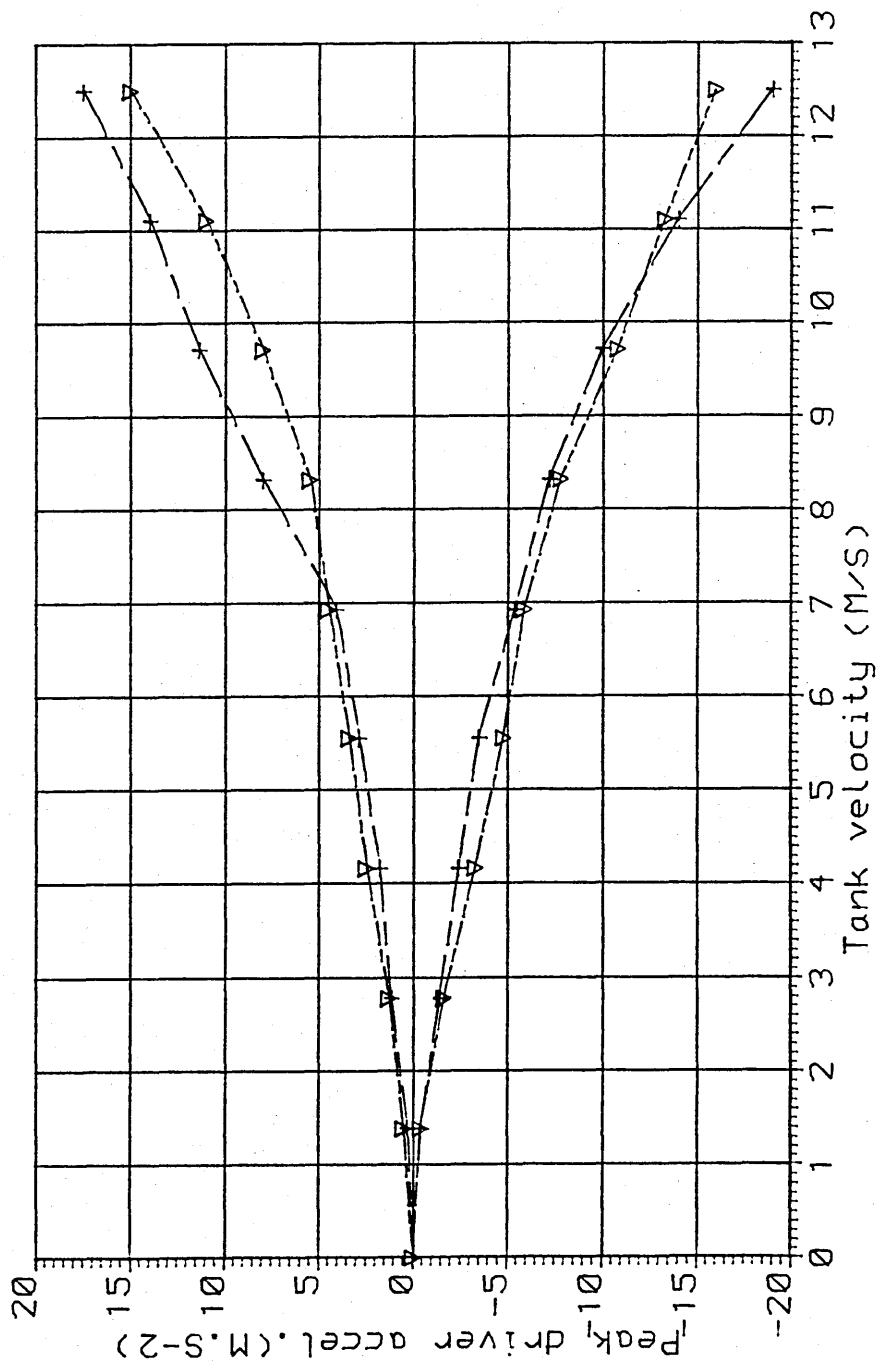
Degree of freedom no 4



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Fig(62) Front roadwheel centre displacement curves of 3-D and 2-D vehicle models.

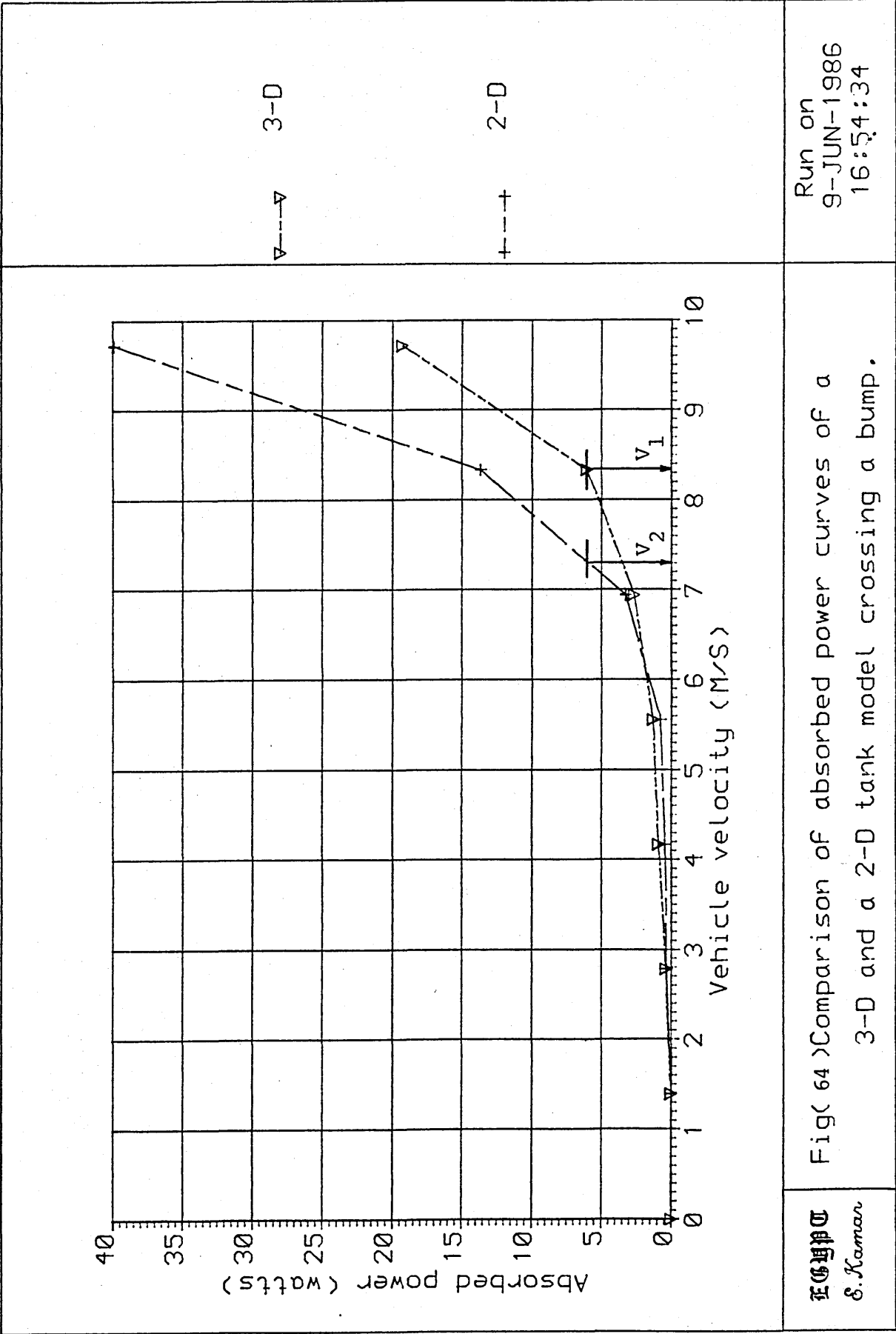
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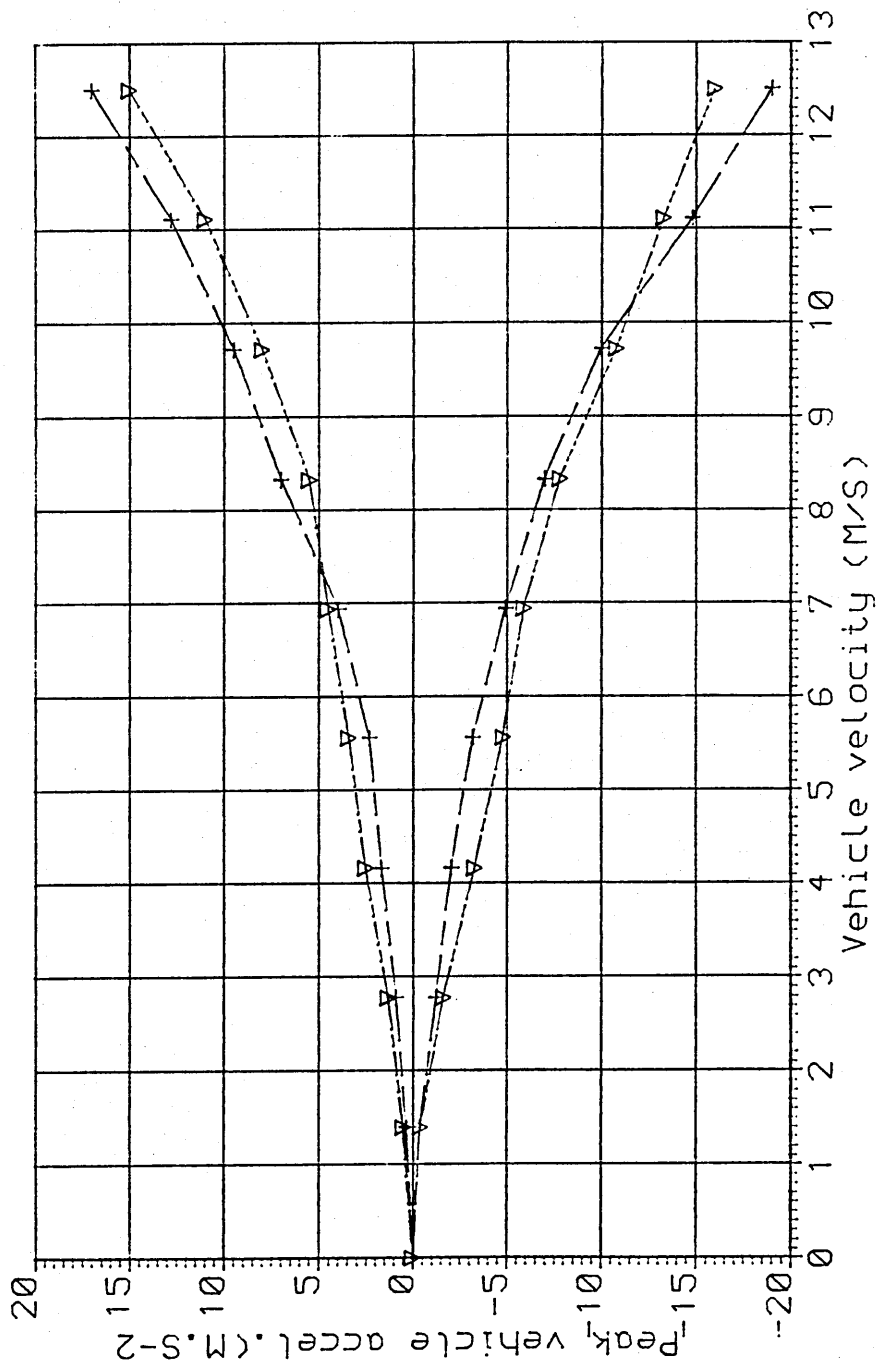


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Fig(63) Comparison of 3-D and 2-D model results of a Scorpion tank crossing a bump.

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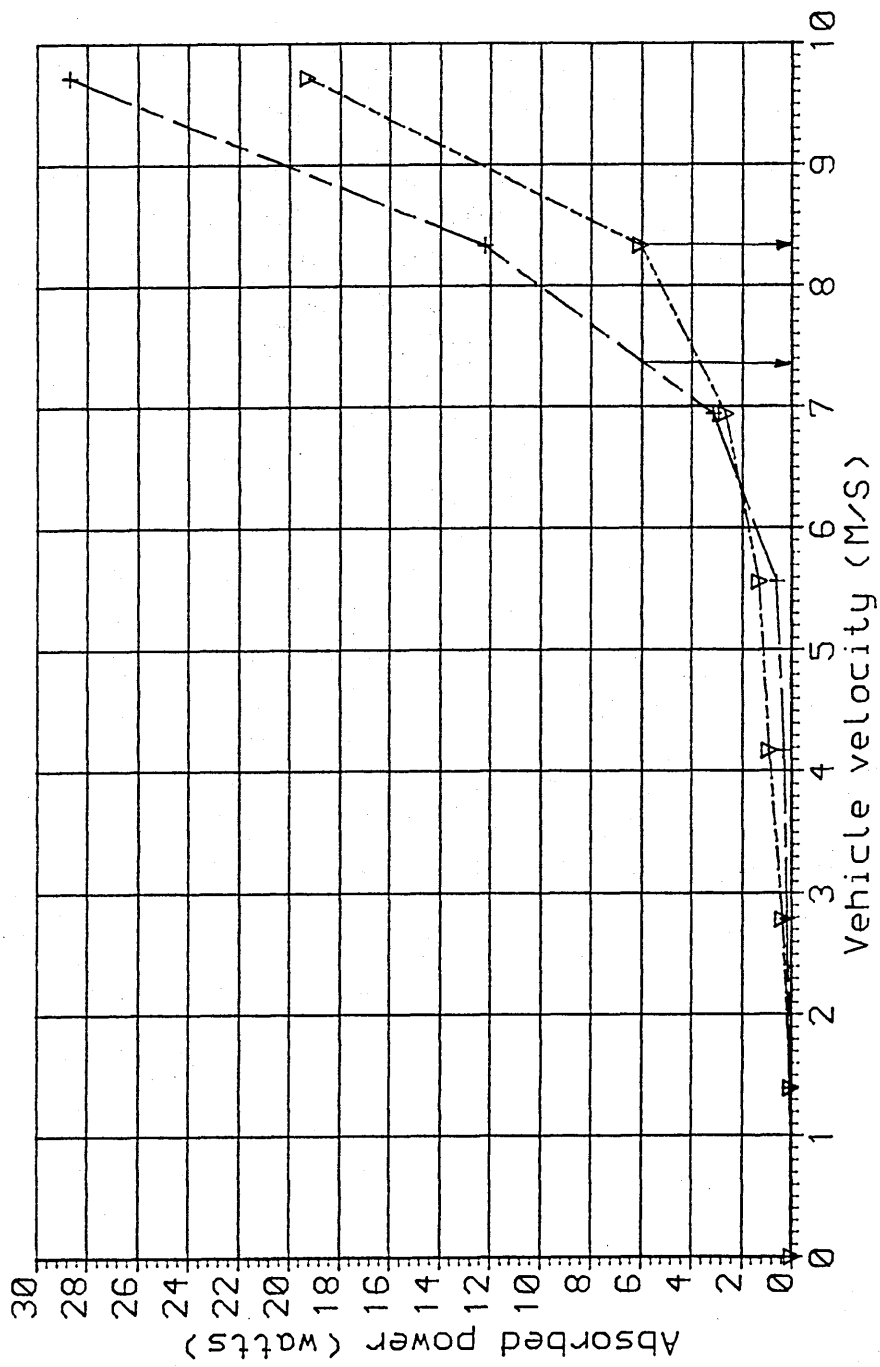
3-D tank

3-D car

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Fig(65)Comparison of responses of a tank model and its approximate model to bump excitation.

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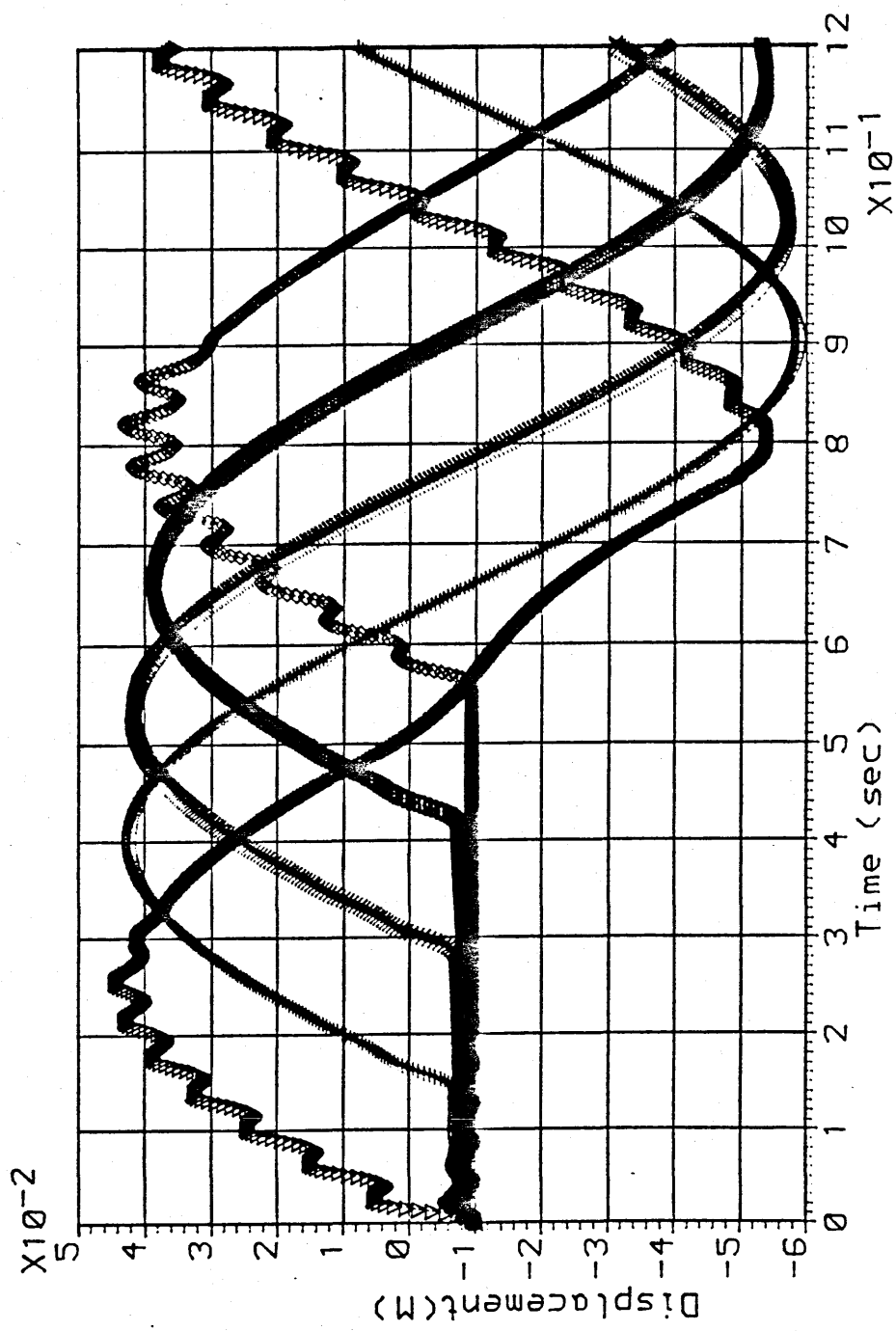
▽---▽ Tank

+---+ Car

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Fig(66) Absorbed power curves of a tank model and an equivalent car model crossing a bump.

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S. Kamar



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14-MAR-1986
19:08:06

Fig(67) Displacement of roadwheel centres of nonlinear 3-D tank model excited by a harmonic wave.

S. Kumar

roadwheel centres move upwards in a stepwise motion, unlike the intermediate roadwheels. All roadwheel centres follow the excitation wave when moving downwards. A model of the tank provided with stiffer torsion bars on the front and rear suspension assemblies has been excited by means of a harmonic wave. The ride performance of the tank improved (that is, both the driver acceleration and the vehicle pitch decreased) as shown in Fig(68) and Fig(69) respectively. Fig(70) shows that at vehicle velocities of more than 8 M/S the pitch velocity decreases greatly.

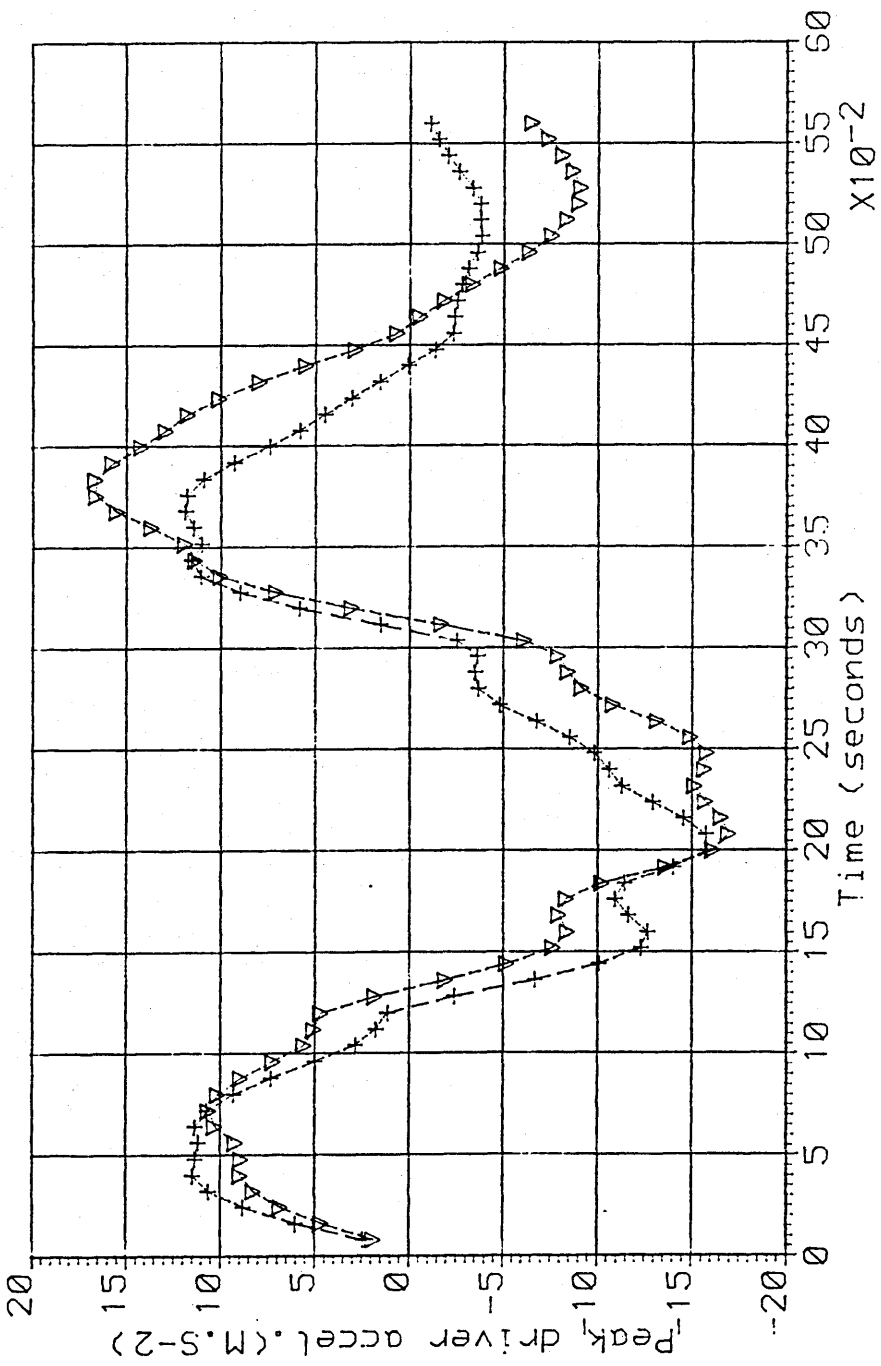
The drawback of using torsion bars of different stiffnesses in order to improve vehicle ride, is the increase in the number of torsion bar types necessary for a particular vehicle.

7.6-4 FOUR-BAR LINKAGE TRANSMISSION RATIO:

The value of transmission ratio of the four-bar linkage of a rotary-vane damper affects the ride performance of vehicles. For a Scorpion tank, excited by a harmonic wave, a significant decrease of the c.g acceleration can be observed at a transmission ratio of 1.25 whilst there is no change in acceleration at ratios of 1.667 and 2.0 as shown in Fig(71).

7.6-5 TRACK MODEL:

The inclusion of the track in the vehicle model leads to reduced peak acceleration values as shown in Fig(72). This reduction increases as the vehicle velocity increases. The estimated stiffness coefficients of springs representing the track have an effect on peak acceleration values. The stiffer are the springs the better is the ride performance as shown in Fig(73). The track effect increases the allowable operating vehicle velocity as shown in Fig(74).

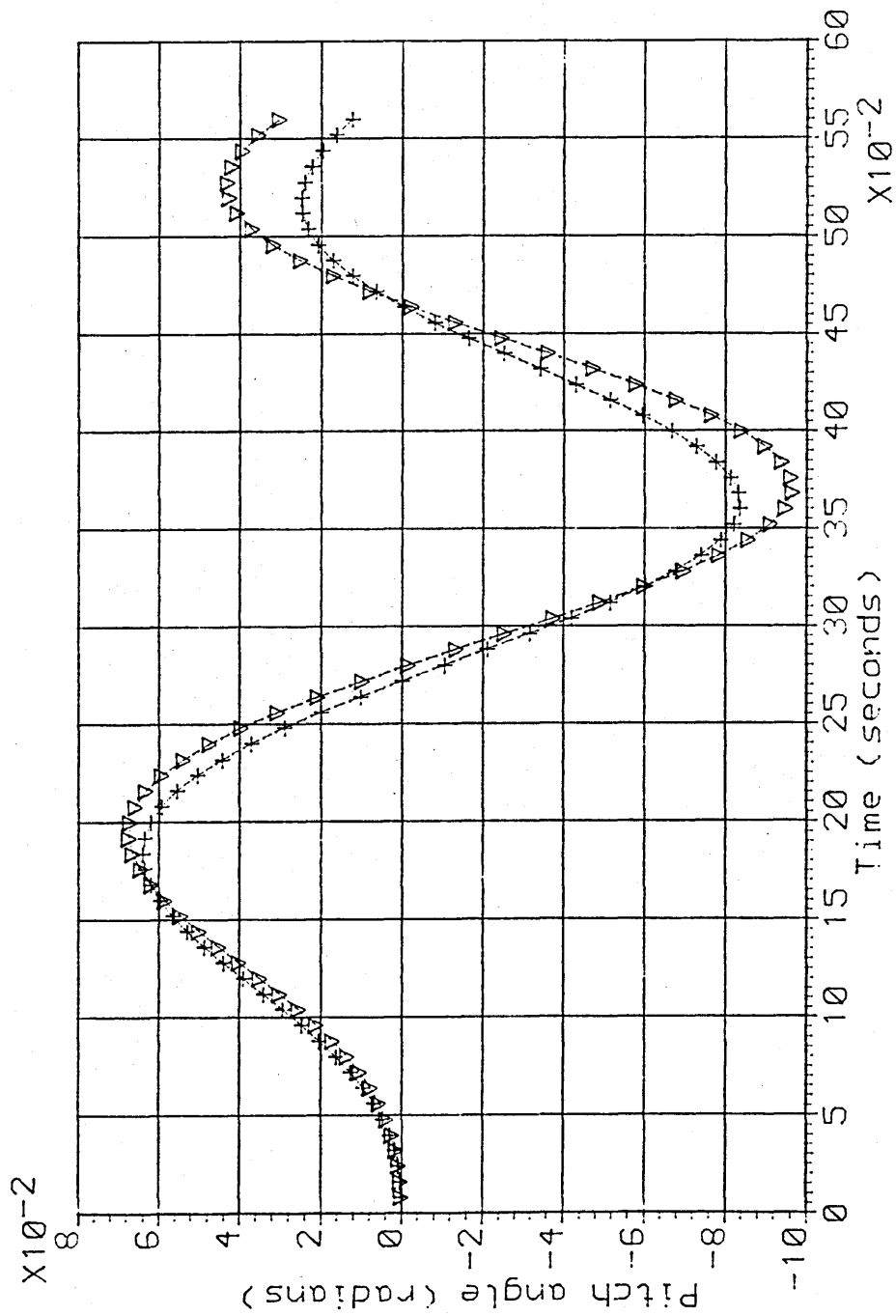


∇ --- ∇ constant.
 $+$ --- $+$ different.

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Fig(68)Comparison of responses of tank models with constant and with different torsion bars stiffness.

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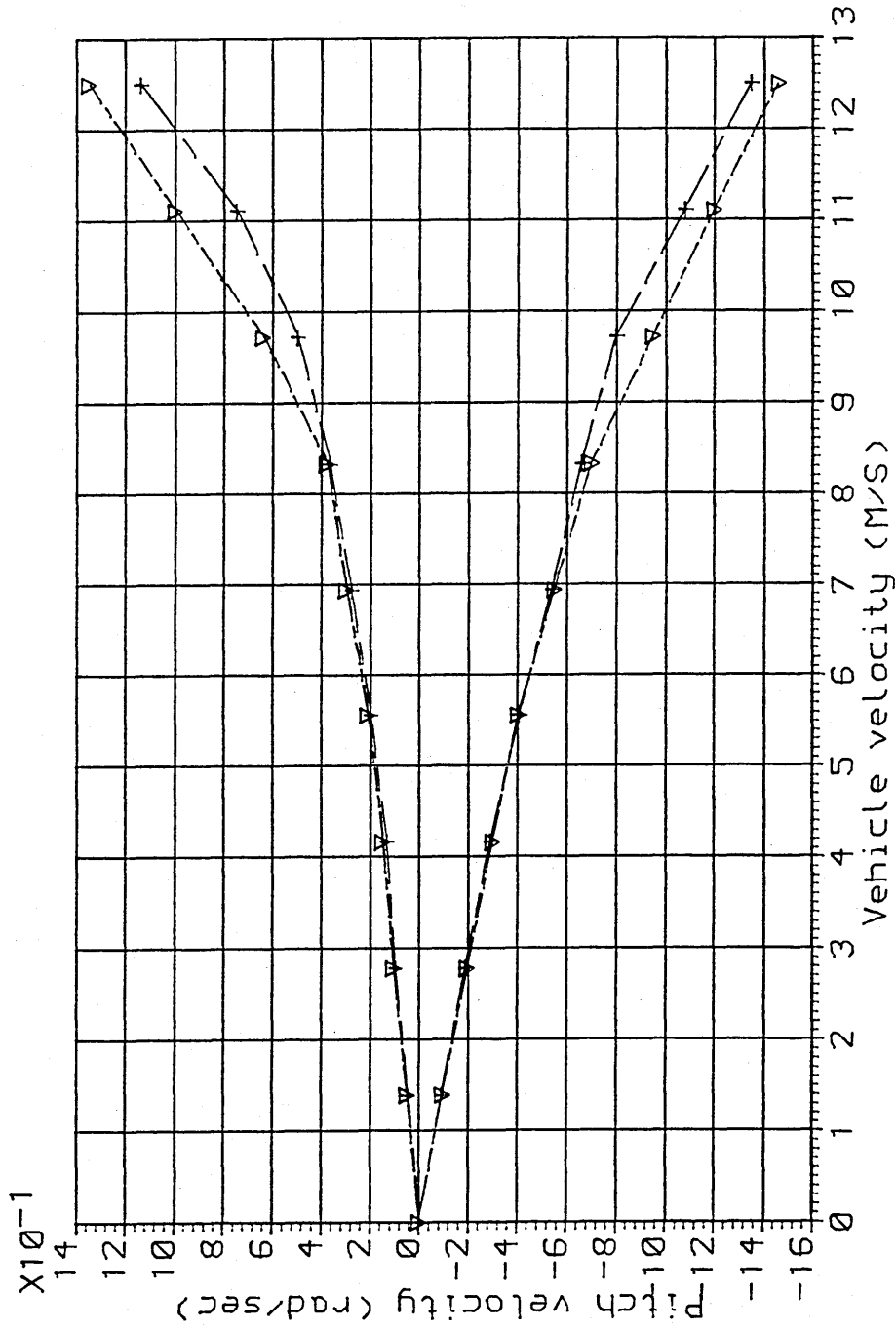
∇ ----- ∇ constant.

+---+ different.

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Fig(69)Comparison of responses of tank models with constant and with different torsion bars stiffness.

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ச. கமலா



∇ --- ∇ constant.

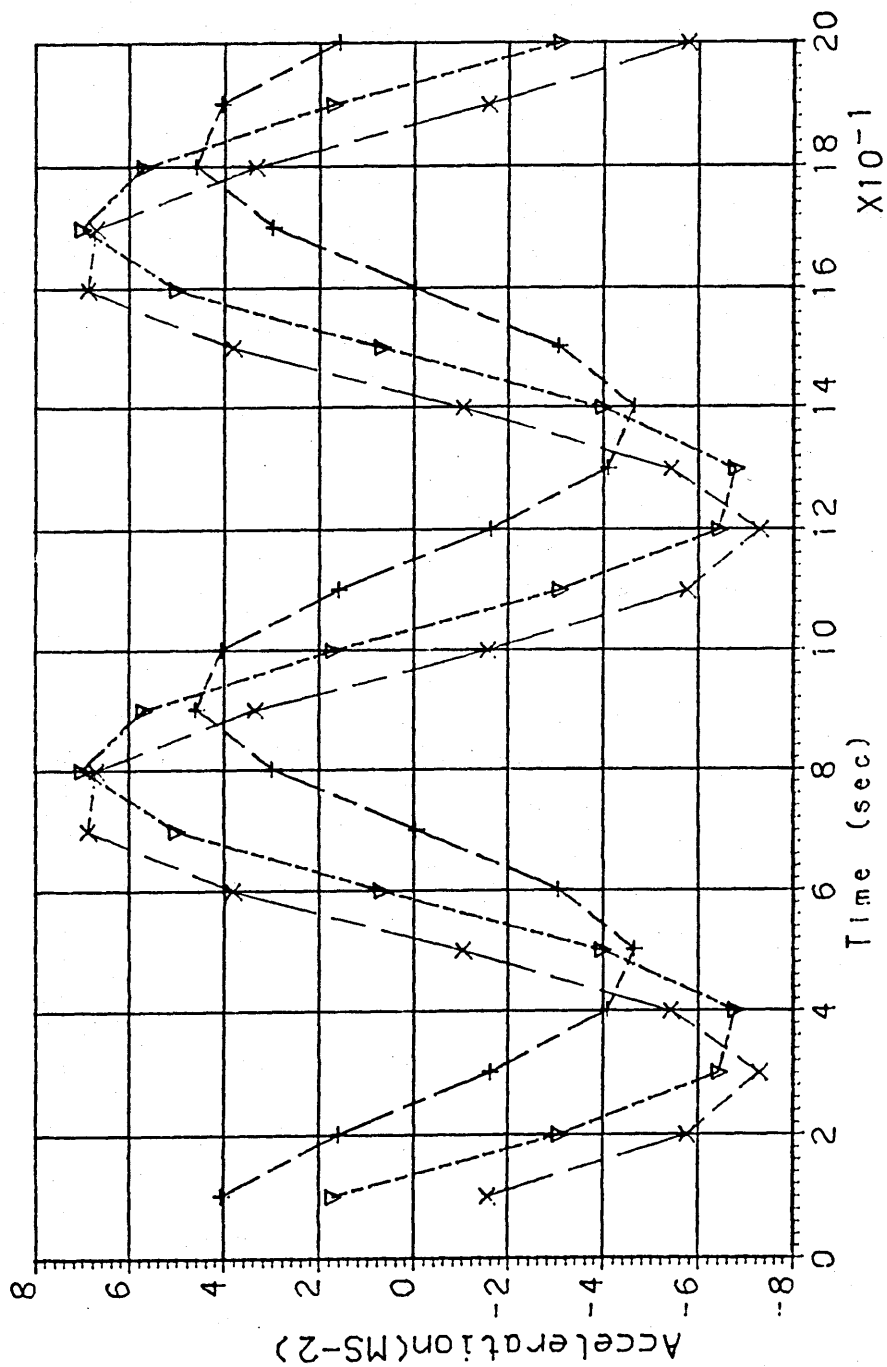
+---+ different.

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Fig(70)Peak pitch velocity curves of tank models
with constant or different torsion bars stiffness.

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▽---▽ Ratio=1.667

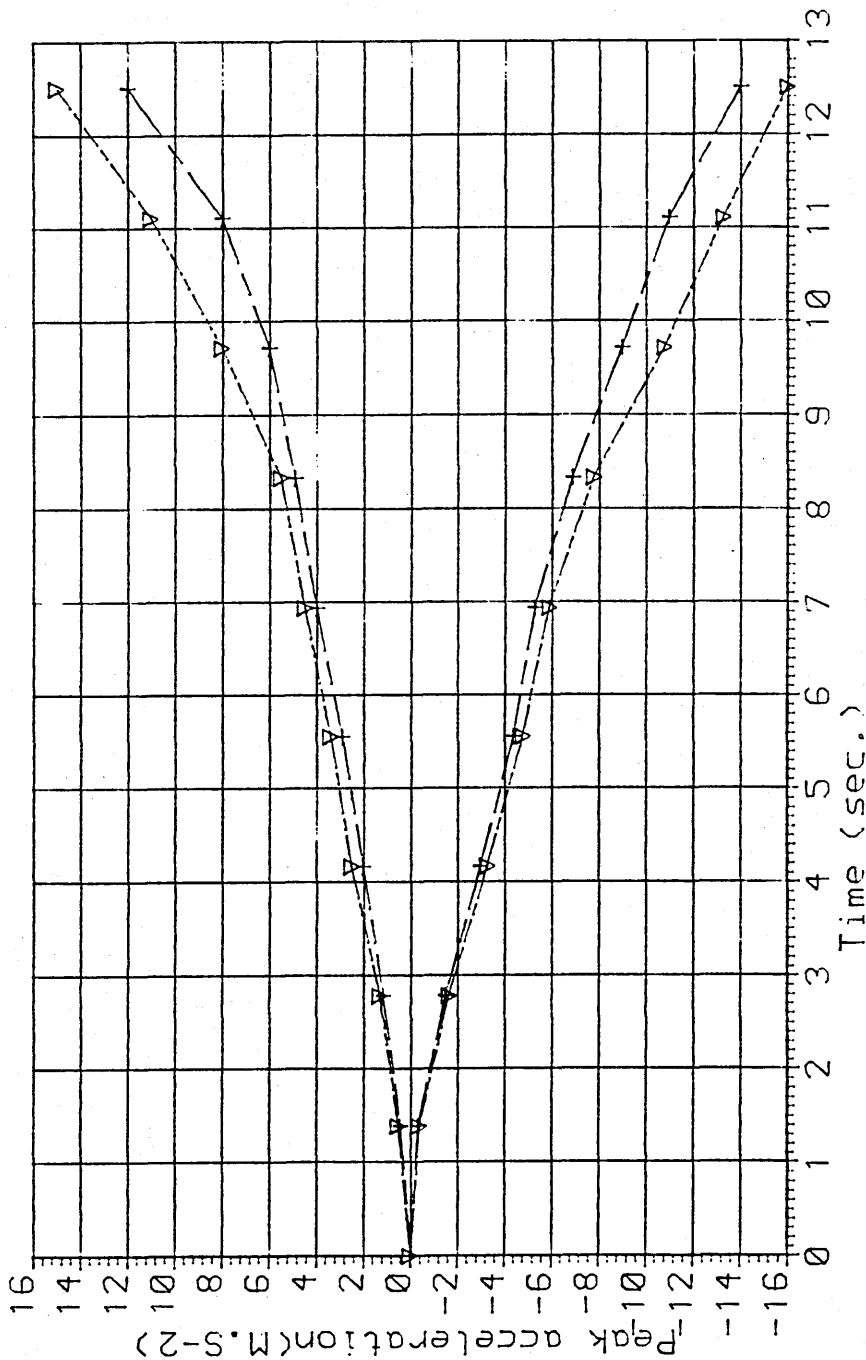
+---+ Ratio=1.25

x---x Ratio=2.0

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Fig(71)Effect of transmission ratio of a four-bar linkage on
c.g acceleration of a tank excited by a harmonic wave.

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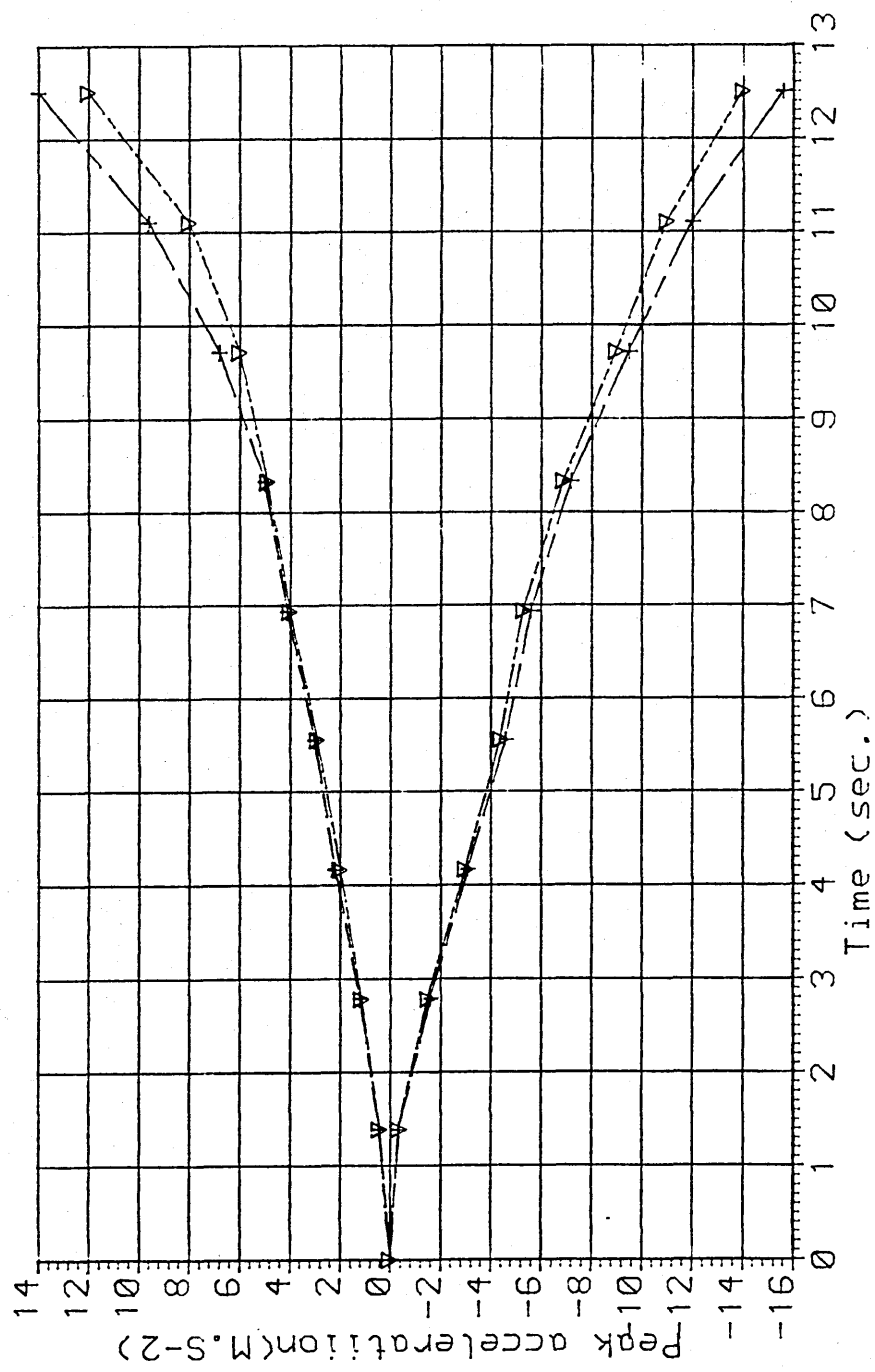
△---△ No track

+---+ With track

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26-MAY-1986
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Fig(72)Comparison of responses of a linear model considering and neglecting the track effect.

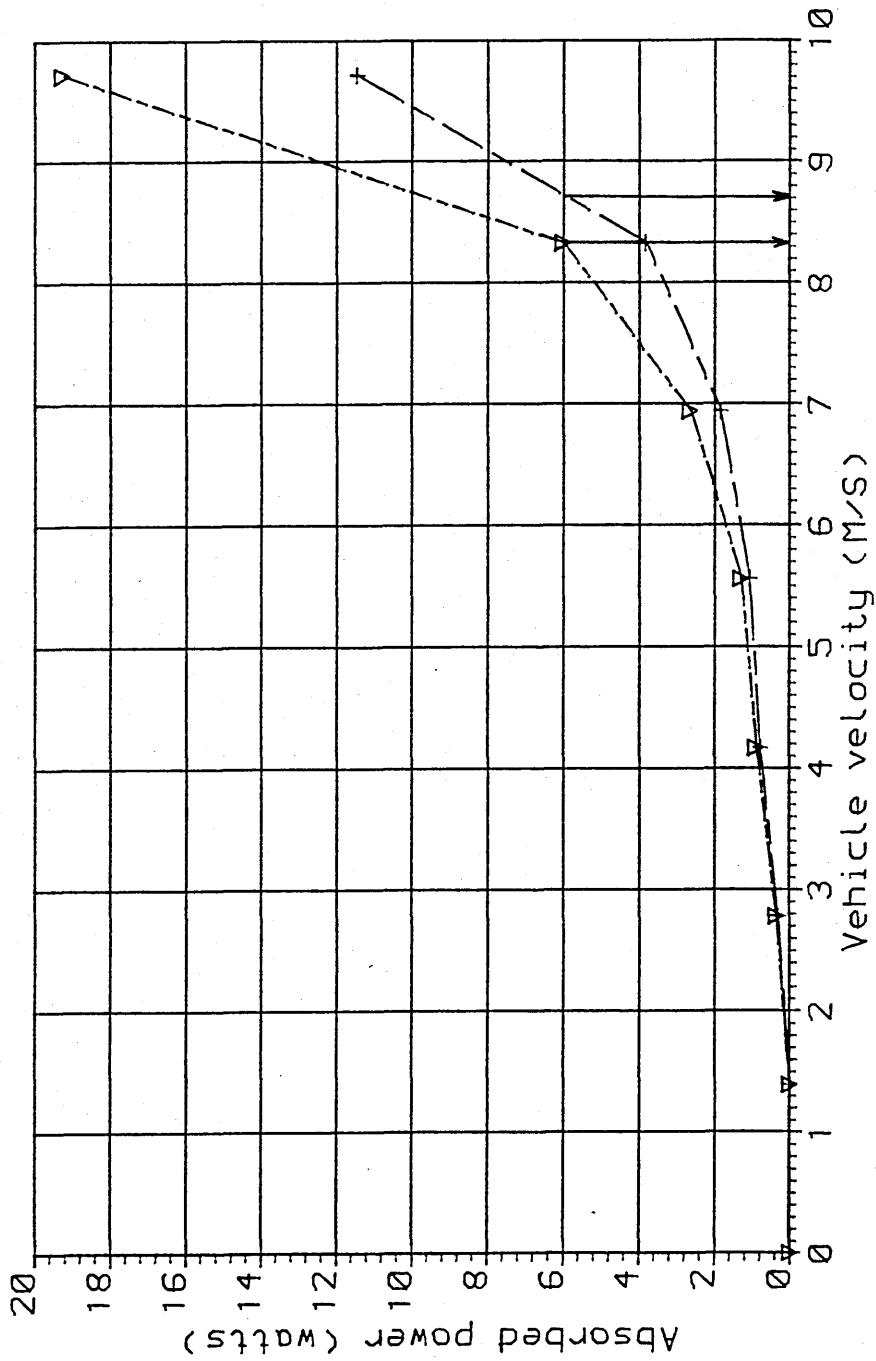
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Fig(73) Effect of stiffness of springs representing
the track on tank response to bump excitation.

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▽---▽ No track.

+---+ With track.

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Fig(74)Comparison of absorbed power curves of tank models considering or neglecting the track effect.

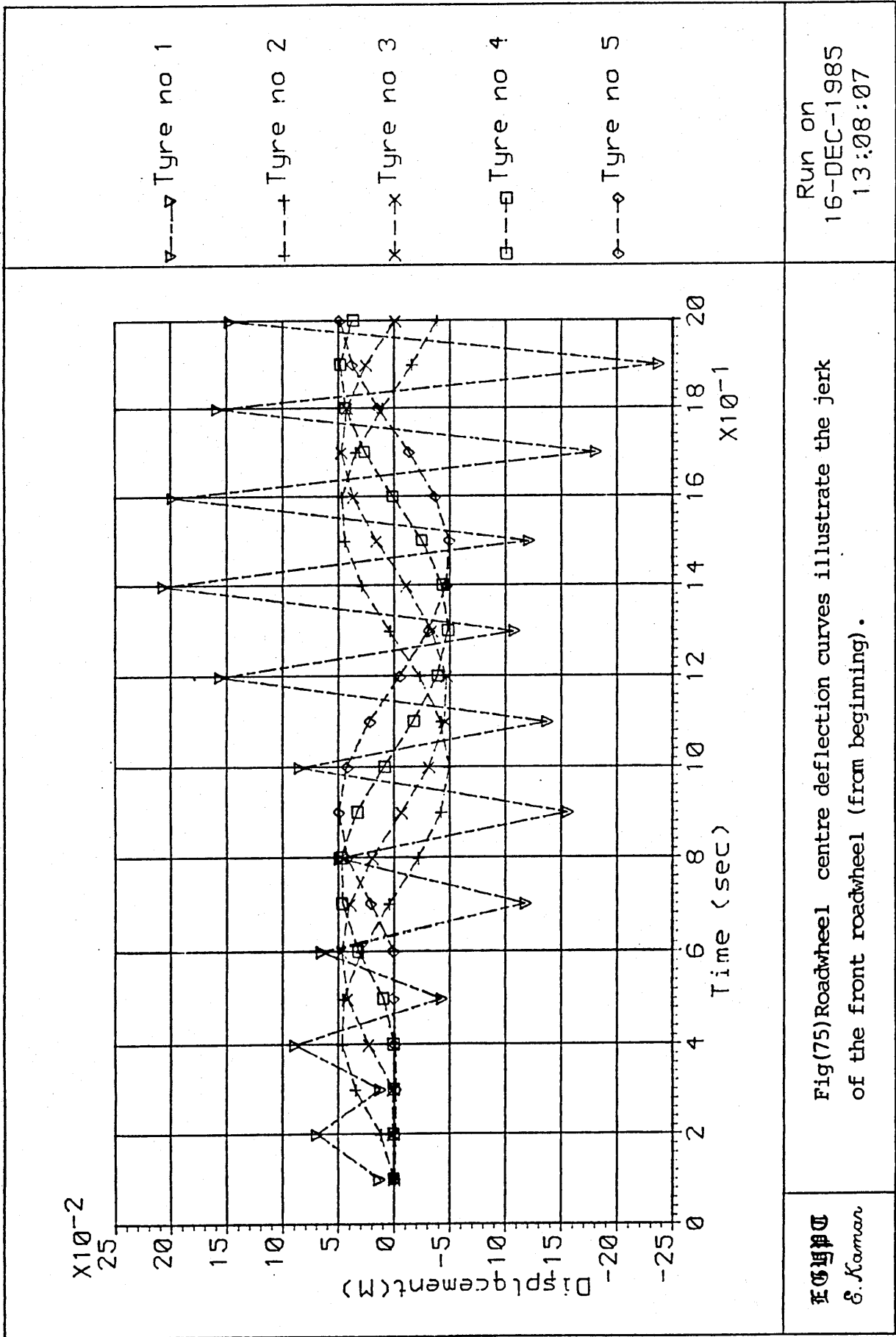
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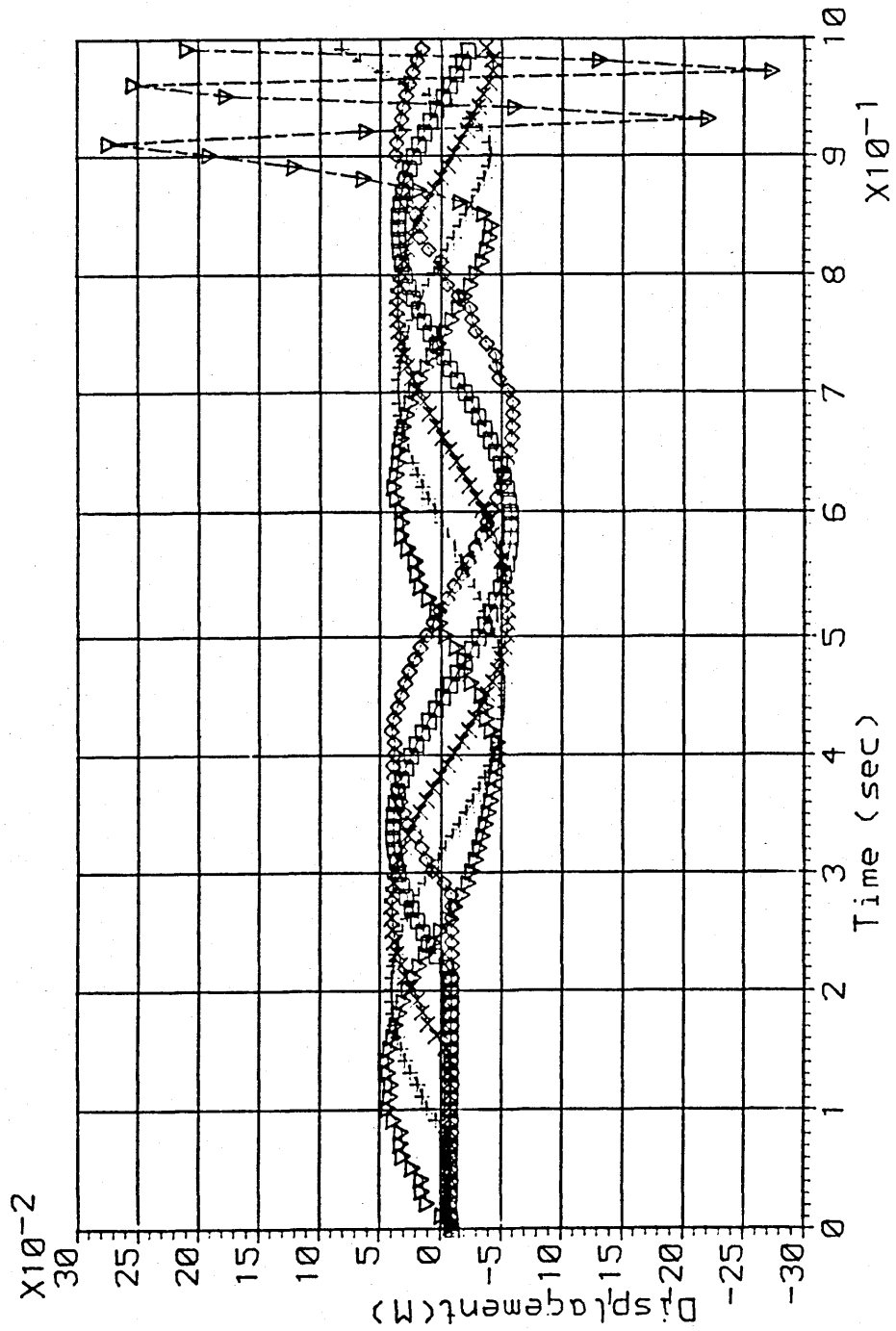
7.6-6 NUMERICAL STABILITY:

A particular time integration method may fail to solve a specific problem due to numerical instability. For the case of suspension systems, such a failure may occur after a considerable number of successful time steps indicating that the governing equations of the system have become 'stiff', and that a stiff integrator should be used. For the example of a Scorpion tank, excited by means of a harmonic wave, an indication of the complete failure of the integration scheme is the 'jerk' of the front wheel as shown in Fig(75). This jerk may occur some time after the start of the analysis as indicated in Fig(76). One of the reasons for this instability is the sensitivity of numerical integration methods to the size of the time-step. This is clearly demonstrated in Fig(77) for wheel centre displacements and in Fig(78) for vehicle c.g displacement. When the numerical method is stable for a particular problem, the effect of step size is insignificant as illustrated in Fig(79).

The wheel-ground separation and/or bump stop action (see Fig.80) cause no problem provided that they are within given limits.

The use of double precision arithmetic gives better accuracy as shown in Fig(81). Hence, it is advisable to use double precision calculations in all time response runs particularly for nonlinear cases which require a very small time-step size and a large number of steps. It has been found that VAX computers cannot recognise values smaller than $2.9802322 \text{ E-}8$ for the case of single precision and $6.9388939039072284 \text{ E-}18$ for the case of double precision.





▽---▽ TYRE NO 1

+---+ TYRE NO 2

x---x TYRE NO 3

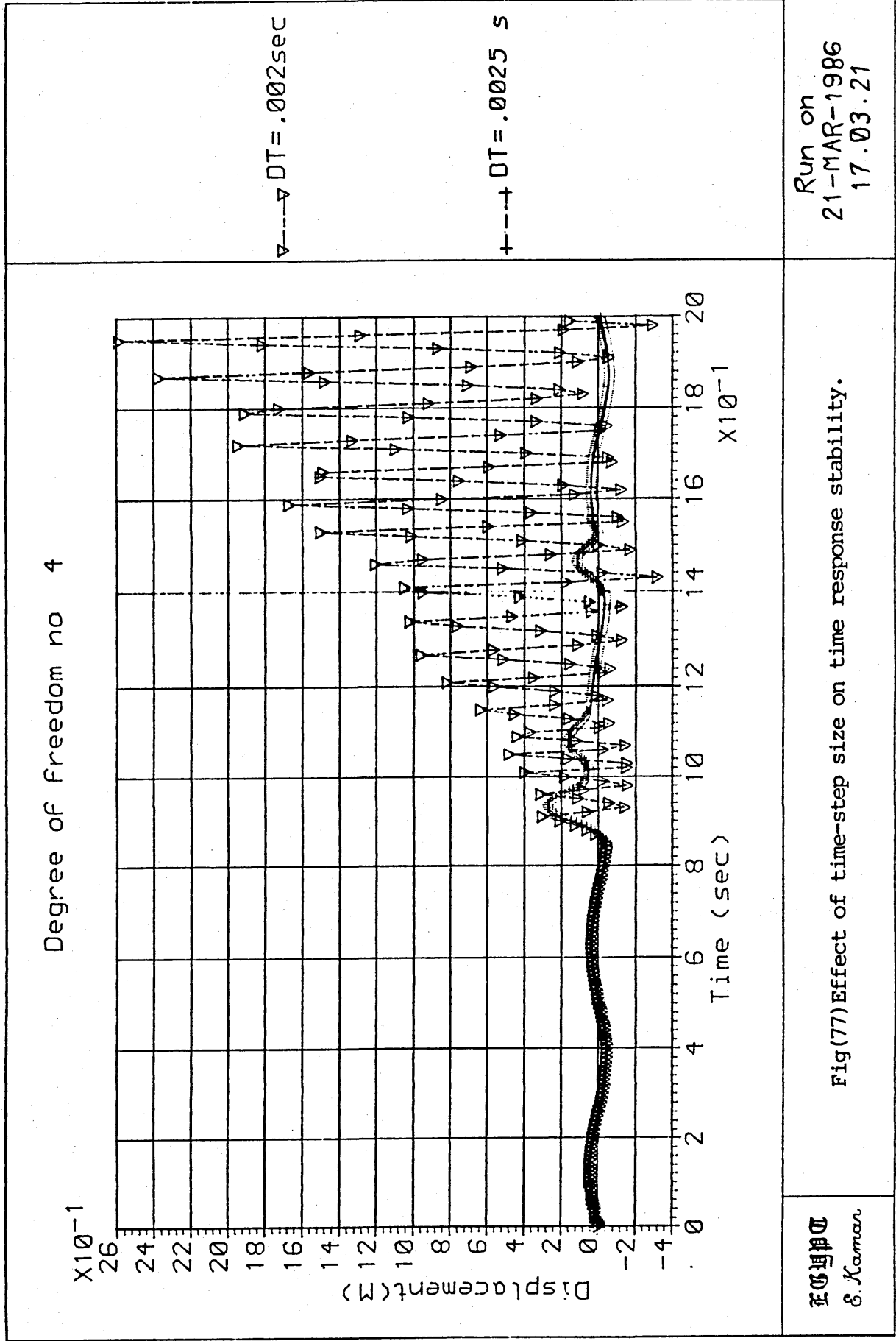
□---□ TYRE NO 4

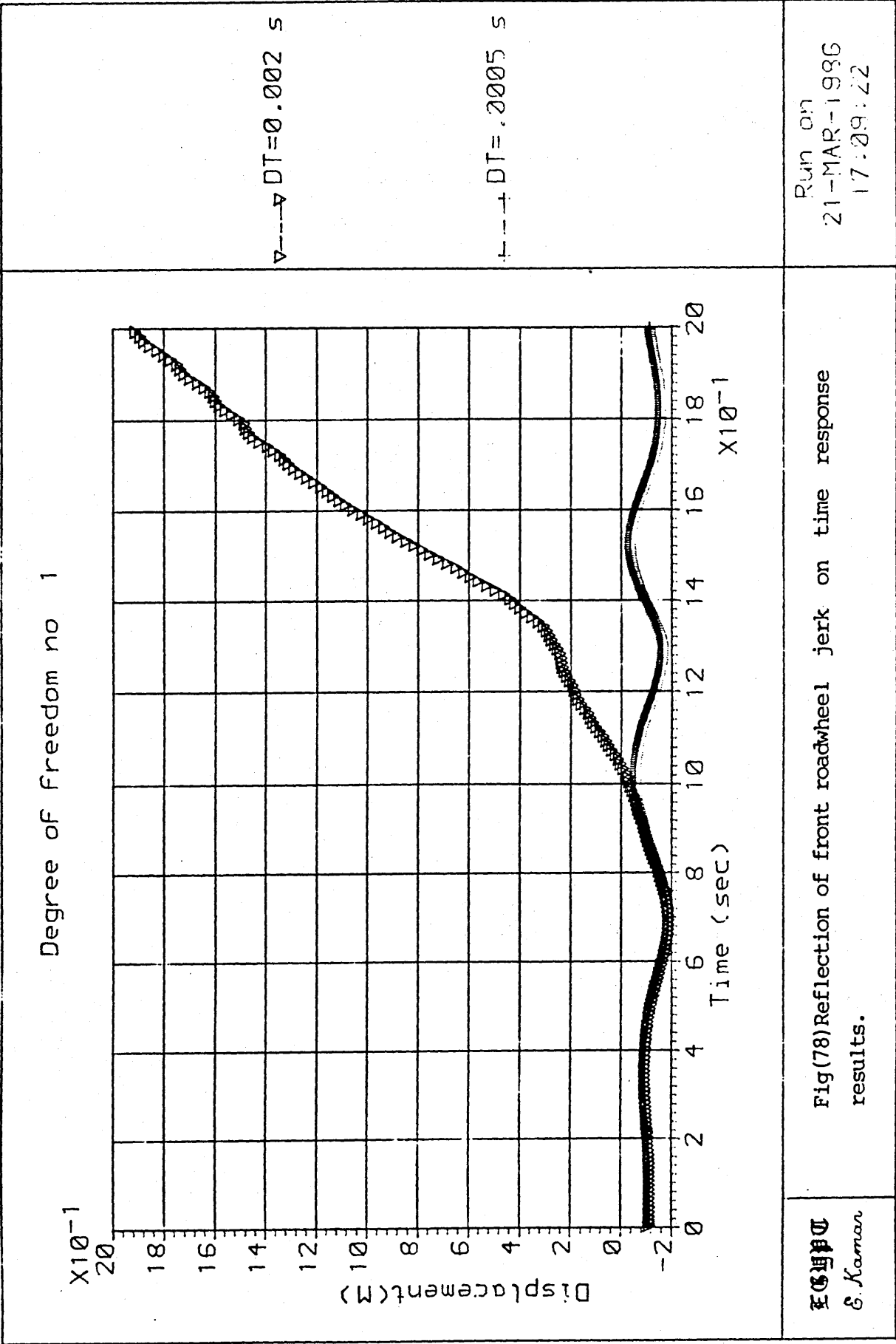
○---○ TYRE NO 5

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16:34:22

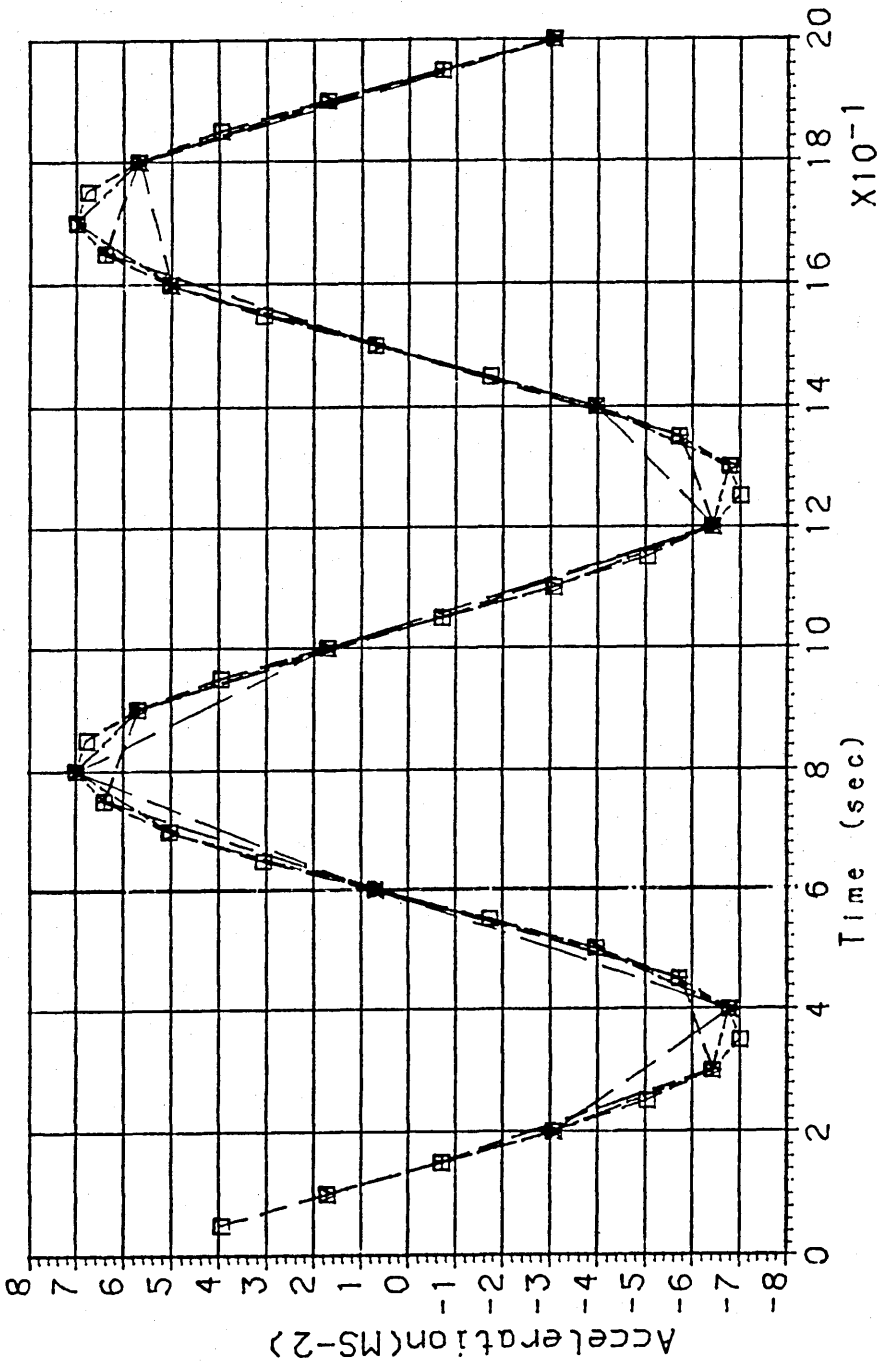
Fig(76) Roadwheel centre deflection curves show the jerk of the front roadwheel (after 0.85 sec.).

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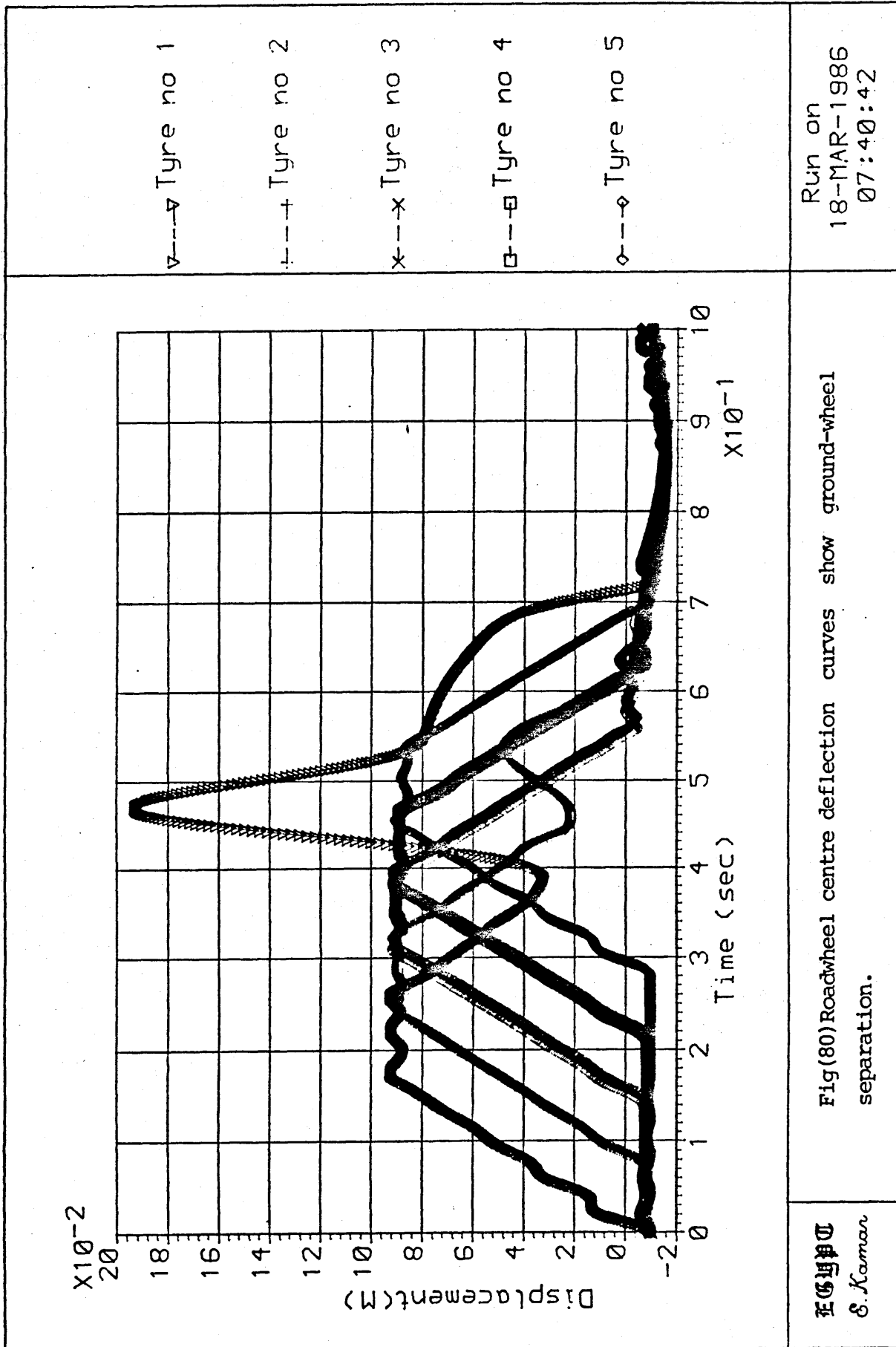
Degree of freedom no 14



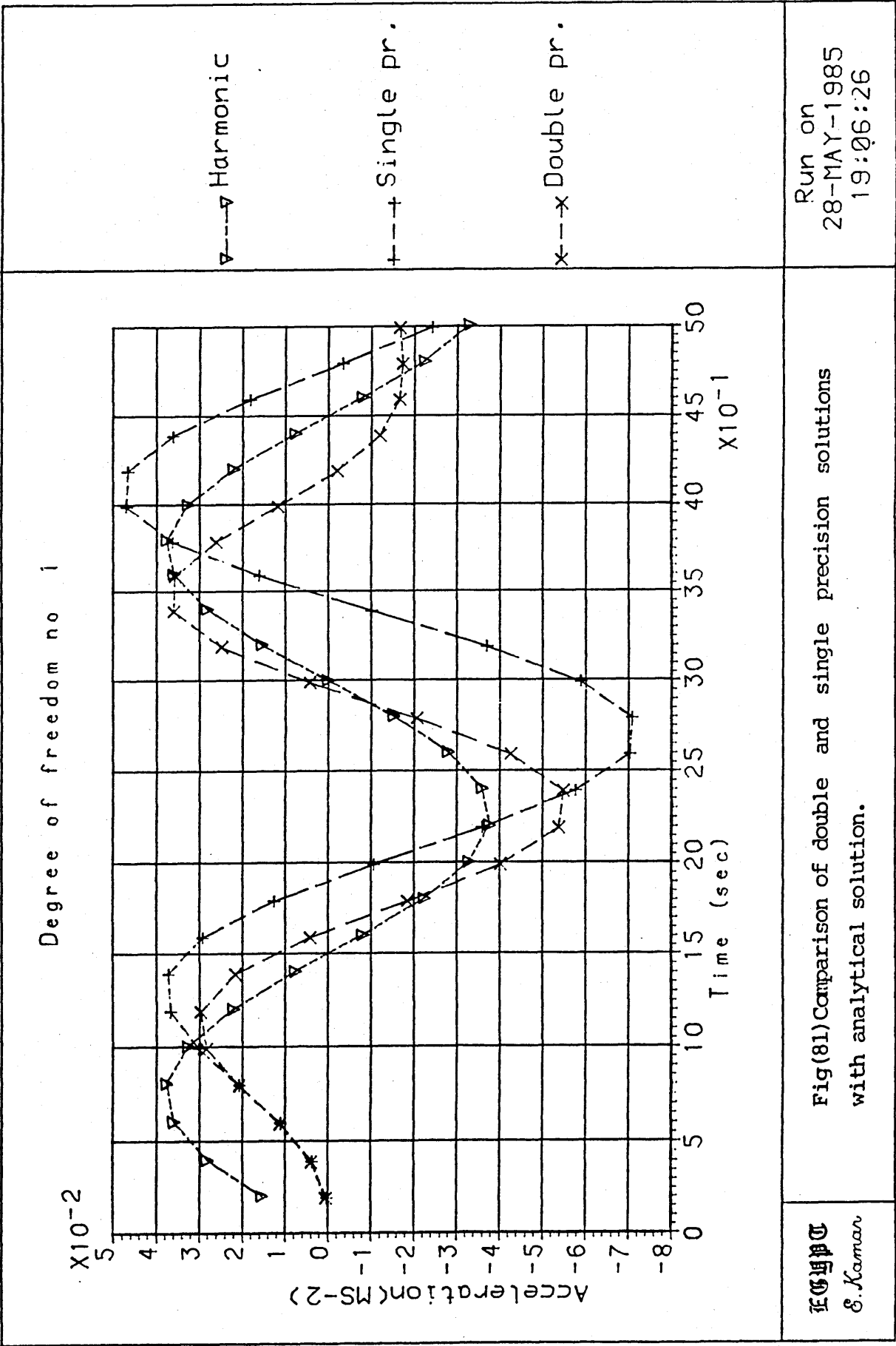
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Fig(79) Insignificant effect of time-step size for a stable integration method.

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CHAPTER 8

CONCLUSIONS

CONCLUSIONS:

The object of this work was to produce a computer package with which it would be possible to analyse the ride performance of an off-road vehicle, and this has clearly been achieved.

During the validation of the package many useful conclusions have been found and are summarised as follows:

(i) Three-dimensional vehicle modelling may be essential, particularly for the case of vehicles with asymmetric suspension systems. The present code offers the facility for three-dimensional analysis.

(ii) The approximated transverse torsion bar model, used in most packages, may lead to inaccurate ride evaluation. In the present code, the actual arrangement of the transverse torsion bar, either with a telescopic or with a rotary-vane damper, can be modelled.

(iii) Using torsion bars of different stiffness in the same vehicle may improve the vehicle ride. The present package offers the opportunity for the designer to optimise the torsion bar stiffnesses through changes in minor data parameters.

(iv) For the case of a four-bar linkage, the transmission ratio may have a significant effect on the damping characteristics of the suspension system. In commercial packages, such a linkage is usually approximated into an equivalent vertical linkage for all of the range of the variable transmission ratio. In the present system, the actual linkage is modelled and the effect of the transmission ratio can be utilised by the designer in order to optimise the linkage dimensions.

(v) The inclusion of the track in the vehicle model will improve the accuracy of the simulation results.

(vi) It has been concluded that the governing equations for a typical off-road vehicle could be, or could turn into, stiff

equations which require an adaptive method tailored for their solution. The present package contains a stiff solver which can deal with such a numerical instability.

The present work was based not only on the available literature but also on original work, some of which has already been published [17]. In addition to that work already mentioned, the following are some original contributions:

(i) A simple, efficient, and reliable method for suspension system simulation employing the finite element technique, has been developed.

(ii) An efficient numerical integrator for the solution of the general dynamic equations simulating the suspension system, has been derived (hyperoscillatory Hermitian weighted residual of the second order). The integrator has proved to be almost completely stable and has improved the accuracy of results obtained.

8.1 RECOMMENDATIONS FOR FUTURE WORK:

There are some useful points which could strengthen, and widen, the applications of the present code, and which can be summarised as follows:

(i) Adding 'obstacle' crossing to the types of terrain considered.

(ii) Simulation of articulated vehicles and trailers.

(iii) Track model improvements.

(iv) Evaluation of vibration, in the form of 'shocks'.

(v) Extending the present code to produce a mobility model.

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Appendix 1Values of β , and γ

No.	1	2	3	4	5	6	7	8
β	0	0	1	1/6	1/10	4/5	1/12	1/4
γ	-1/2	1/2	1/2	1/2	1/2	3/2	1/2	1/2

$$\begin{aligned}
 \alpha_1 &= 1 & , \alpha_2 &= -2 & , \alpha_3 &= 1 & , \\
 \beta_1 &= -1 + \gamma & , \beta_2 &= 1 - 2\gamma & , \beta_3 &= \gamma & , \\
 \gamma_1 &= \frac{1}{2} + \beta - \gamma & , \gamma_2 &= \frac{1}{2} - 2\beta + \gamma & , \gamma_3 &= \beta & ,
 \end{aligned}$$

Table 1

Human response transfer functions

$$\text{Vertical : } \frac{15.453S(S+5)(S^2+28.3S+2800)(S^2+105S+7570)}{(S+6)(S^2+29.8S+1000)(S^2+39.1S+3800)(S^2+125S+5180)}$$

$$\text{Fore-aft : } \frac{209S(S+110)}{(S^2+17.6S+125)(S^2+110S+1300)}$$

$$\text{Side-to-side: } \frac{478S(S+130)(S^2+11.2S+60)(S^2+14.2S+260)}{(S^2+3.33S+17)(S^2+5.5S+140)(S^2+44S+900)(S^2+255S+2500)}$$

Appendix 2PROGRAMMING SYSTEM:

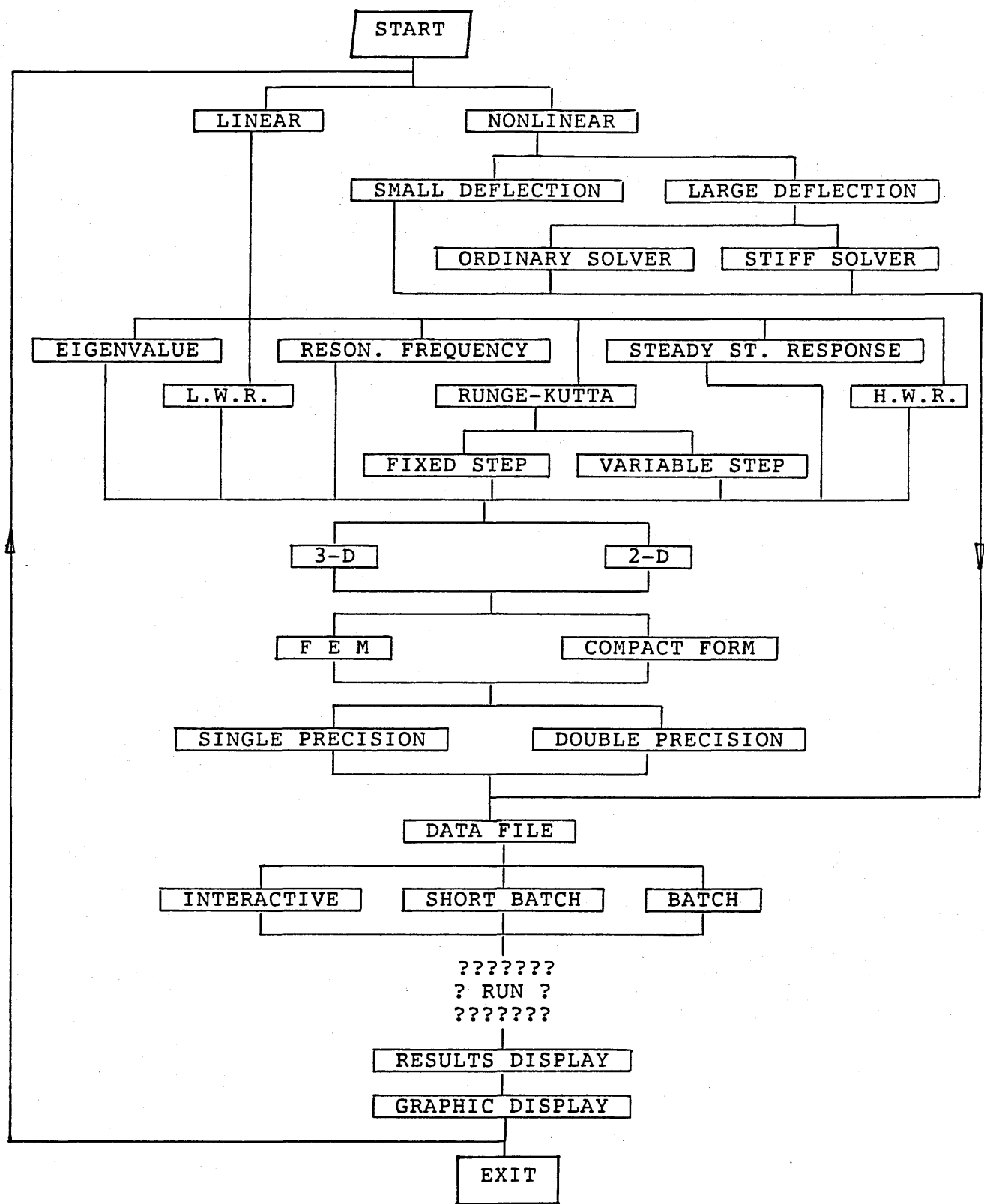
The present code is entitled "Ride Dynamics of Ground Vehicles" (RDGV lies in 18825 lines, 15289 of which are FORTRAN statements and 2536 comment lines, in addition to 1819 VAX command lines. The computer memory sizes for FORTRAN programs, image files, and executable files are 1107, 1229, and 6777 blocks respectively. The command file occupies 105 blocks only. A big saving in the computer memory space required for RDGV, can be achieved by keeping the image files only and creating an executable image of the intended case only prior to each run. This facility is available in the present code at VAX computers.

A2.1 PACKAGE STRUCTURE:

The main package structure is almost independent of computer type because of standard FORTRAN implementation. Only, the computer command files would need to be modified for the package to suit any other computer than a VAX computer.

The structure of the master command file is shown in Fig(82). Once the command file is activated, the first menu will be displayed. There is no possibility of accepting input data error whatsoever because the program will accept only the correct parameters suitable for the present menu before displaying the following menu.

The user is able to select any pass from the START to the EXIT shown in Fig(82) according to his particular problem. For example, the following pass: START, LINEAR, RUNGE-KUTTA, VARIABLE STEP, 3-D, COMPACT FORM, DOUBLE PRECISION, DATA FILE, INTERACTIVE, RUN, RESULTS DISPLAY, GRAPHIC DISPLAY, and EXIT or START again for a new job. For every pass, the computer will select the appropriate FORTRAN files and link the relevant modules.



Fig(82)Master command file structure.

The 'RUN' of the relevant program does require the control of interactive parameters. An example of such parameters are shown in Fig(83). Again there is no possibility of any input error. The pass for the case to be solved, must be selected correctly in order to obtain the required results.

Even for batch jobs (usually recommended for nonlinear problems), the computer accepts the interactive-FORTRAN-control parameters and stores them in a file before the submission of the job. This file is necessary to the computer before batch jobs can be run.

The basic facilities of the package can be summarised as follows:

a) Vehicle Simulation

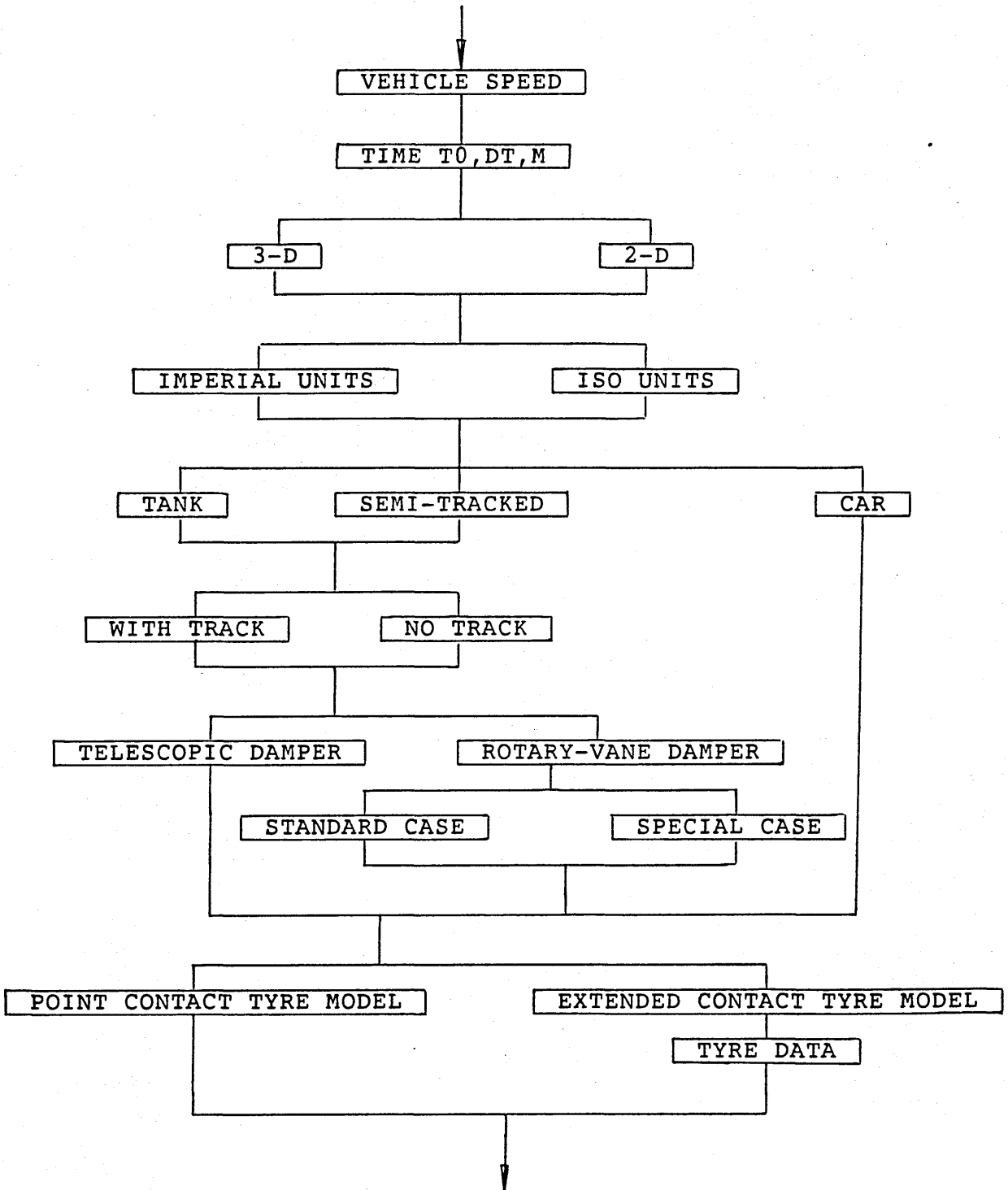
Most of the widely used types of suspension system have been modelled in the present code. These types are used for tanks, cars, tractors, semi-tracked vehicles, earth moving machines, lorries,.....etc. The suspension system assemblies and elements which have been modelled are:

- (1) A spring and a damper colinear with the wheel centre.
- (2) A transverse torsion bar directly operated by a longitudinal wheel arm plus a telescopic damper.
- (3) A transverse torsion bar directly operated by a longitudinal wheel arm plus a rotary-van damper.
- (4) The track.
- (5) Hydrogas units.

Both linear and nonlinear modelling have been considered. Types of nonlinearities investigated are:

- (1) Nonlinear characteristics and/of blow-off function of the main damper.
- (2) Nonlinear characteristics of the main spring or torsion bar.
- (3) Nonlinearity originating from the bump stop action.
- (4) Nonlinear characteristics of the tyre or the roadwheel.
- (5) Nonlinearity resulting from large deflection consideration.

(199)



FIG(83)Example of FORTRAN control parameters.

(6) Nonlinearity arising from wheel-ground separation.

A simple, efficient, and reliable method of suspension system simulation by the FEM technique has been developed.

b) Vehicle-Terrain Interaction:

The simplest and most accurate tyre models have been included in the present code, namely, the point contact tyre model and the extended contact (adaptive footprint) tyre model.

For both linear and nonlinear analysis where the terrain configuration is arbitrary it can be described with the aid of a table of elevations and coordinates.

Linear analysis can also be carried out for harmonic terrain configurations where such configurations can be considered to take the form of a sine wave of known amplitude and wave length.

c) Time Response

The package includes various numerical integration subroutines suitable for the solution of linear and nonlinear systems of second order differential equations. For a linear system of differential equations, these methods are:

- (1) Steady-state solution (response to harmonic excitation).
- (2) Runge-Kutta-Nystrom method.
- (3) Runge-Kutta method with variable time step and controlled error.
- (4) Lagrangian-weighted residual method.
- (5) Hermitian-weighted residual method.

For nonlinear time response, the methods of solution are:

- (1) Hermitian-weighted residual method.
- (2) Runge-Kutta method.
- (3) Stiff solver, namely, the Implicit Runge-Kutta-Turnor method.

The Hermitian-weighted residual method has been developed herein to include the highest derivative which appears in the system of differential equations, which is convenient for vehicle dynamic analysis.

The driver, or vehicle crew, dynamics has been added to the package as an option to the user.

d) Human Tolerance

Two widely used criteria have been employed to assess the ride performance or ride quality of vehicles. These two criteria are absorbed power and ISO 2631.

A2.2 PROGRAMMING SYSTEM EVALUATION:

The RDGV system has satisfied most of the requirements for an automatic code. Such requirements, and those which have been achieved, can be summarised as follows:

a) VRSATILITY

The RDGV system is a very versatile programming system. Not only does it contain a large variety of suspension types, but it also has the capability for allowing selection of different criteria and integrators.

b) MODULARITY

The system has been divided into a large number of modules. The modules which are relevant to the given job are the only ones which are linked together in the executable program.

c) CONTROLLABILITY

Control and execution of the system is straightforward. A command generator has been created for use in VAX computers. The generator reads the user data, generates the proper VAX commands, which assign the required files, and links the relevant system modules.

d) INPUT DATA

The system generates the maximum possible amount of data and there is also a logical assumption of default data whenever necessary.

e) OUTPUT DATA

Tabular and graphical representation of results is available. There is also facility to print selective results in order to avoid too large and unnecessary output.

f) ERROR DIAGNOSIS

The system is able to detect some of the basic probable errors and it sends back a clearly understandable message.

g) RELIABILITY

Necessary and sufficient test runs have been carried out on the system. The results have been compared either with theoretical solutions or with results from the literature or with experimental results. The system has proved to be very reliable.

h) SELF SUFFICIENCY

The system is almost self sufficient. It has its own mathematical library and numerical integrators.

A2.3 RANGE OF APPLICATIONS:

The package can be used for the ride analysis of a generalised off-road vehicle. The output can be obtained in the following forms:

a) Determination of natural frequencies and the display of the mode shapes either analytically or graphically.

b) Determination of the resonance frequencies of the suspension systems either analytically or graphically.

c) Steady-state response analysis and graphical display of displacement-, velocity-, acceleration-time curves.

d) Time response of the vehicle system to arbitrary terrain surface using several solution techniques.

e) Ride evaluation by absorbed power and ISO 2631 criteria.