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THE AERODYNAMIC CHARACTERISTICS
OF THE JET WING AND ITS
APPLICATION TO HIGH SPEED AIRCRAFT.

by

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and its Application to High Speed Aircraft

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Abstract

The slender wings and bodies suitable for supersonic flight have, in general, relatively poor aerodynamic characteristics especially at low speeds. In order to improve their performance the use of edge blowing has been explored. In this scheme high velocity air jets in the form of thin sheets are used to fix separation lines on the wing or body and to favourably influence the external stream. Thus the equivalent wing, called here the "Jet Wing", is composed of the wing-body itself plus the extended curved jet sheets which spring from its edges. A limited use of edge jets has been proposed in the "Jet Flap" concept but the efficiency of this device falls off with decreasing aspect-ratio and the problem of trimming could be severe as most of the increased lift is generated near the trailing edge.

At very low aspect-ratios a considerable part of the lift is contributed by the leading-edge vortices which dominate the flow field at moderate incidence. It follows therefore that leading edge blowing is particularly useful for small aspect-ratio wings and the trimming problem can be avoided by a suitable jet arrangement which does not disturb the conical nature of the flow. When sufficiently large auxiliary thrusts are available peripheral jet sheets can be deflected downwards close to the ground, the aircraft becoming a Ground-Effect-Machine, with substantial reductions in both take-off and landing speeds and distances.

The "Jet Wing" problem has been investigated both theoretically and experimentally with particular reference to blowing from the edges of a delta wing. With leading-edge blowing only, using engine compressor bleed, reductions of about fifteen knots in the minimum flying speed are obtained, and with peripheral jet sheets using auxiliary power landing and take-off distances can be halved.

Foreword

This thesis describes research work carried out in the Aerodynamics Department, The College of Aeronautics, Cranfield, between August 1958 and September 1961. The work was partly supported by a Ministry of Aviation Contract.

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Contents

1. Summary
2. List of Symbols
3. Introduction
 - 3.1 Summary of existing work.
 - 3.2 Practical deviations from theoretical flows.
 - 3.3 Methods for increasing lift at low speeds.
 - 3.4 S.T.O.L. using auxiliary power.
4. Theoretical discussion
 - 4.1 The unblown case.
 - 4.2 Extension of the theory to predict the effects of conical blowing in the plane of the wing.
 - 4.3 Conclusions.Appendices.
5. Model and Experimental Method
 - 5.1 Tests with undeflected blowing.
 - 5.2 Tests with downward deflected blowing.
6. Discussion of Results. Part I. Blowing in the plane of the wing.
 - 6.1 Introduction.
 - 6.2 Lift.
 - 6.3 Drag.
 - 6.4 Cross-wind-force.
 - 6.5 Pitching moments.
 - 6.6 Rolling moments.
 - 6.7 Yawing moments.
 - 6.8 Upper and Lower surface static pressures.
 - 6.9 Spanwise traverses using a five tube pitch-yawmeter.

7. Conclusions

Appendix.

References.

Graphs.

8. Discussion of Results. Part II. Downward deflected blowing.

8.1 Introduction

8.2 Phase I. Hovering.

8.3 Phase II. Ground run.

8.4 Phase III. The initial climb.

8.5 Phase IV. The climb without auxiliary power.

8.6 Estimation of take-off performance.

9. Conclusions. Downward deflected blowing.

Graphs.

1. Summary

The rapid rise of aircraft cruising speeds in recent years and the prospect of large supersonic aircraft within a decade, emphasises the need for an urgent and careful study of the proposed slender shapes, in particular of their poor low speed flight characteristics. In order to improve the low speed performance of these large highly swept wings the use of edge blowing is being explored. In this "Jet Wing" concept high velocity air jets in the form of thin sheets are used to fix separation lines on the wing or body and to favourably influence the external stream.

A theoretical and experimental study has been made to determine the effects of slot blowing from the leading edge of a 70° Cropped Delta Wing. The theoretical work includes the unblown case for which it predicts the spacial movement of the leading edge vortex with incidence quite well although the lift is overestimated as trailing edge effects are not included. A simple extension to the case with "conical" leading edge blowing in the plane of the wing gives increases in lift in good agreement with experiment for moderate incidence and small values of the blowing momentum coefficient C_μ . It is also shown that changing the angle of blowing relative to the leading edge, still in the plane of the wing, provides a useful additional thrust which can be directed backwards to assist take-off or forwards on landing, without seriously affecting the increased lift.

The experimental results include six-component force and moment measurements and the distribution of static pressure at four chordwise

stations. The tests show that leading edge blowing increases the lift at constant incidence by increasing the size and strength of the leading edge vortices. The vortex cores are moved upwards and outwards and the secondary separation eliminated.

At a typical landing incidence of 15° the lift magnification $\Delta C_L / C_\mu$ is about four for practical C_μ values of .03 giving a reduction in landing speed of about fifteen knots. C_μ here is based on the total momentum ejected at the leading edge, independent of ejection angle, and for landing at least the C_μ value of .03 could be supplied by bleeding the main engine compressors. With a suitable blowing arrangement there is no movement of the centre of pressure and hence no trimming penalty. This is particularly useful on tailless configurations.

An experimental investigation was also made of a more radical proposal to shorten landing and take-off distances. Blowing a jet sheet from all edges and using small flaps to deflect it downwards through at least 90° the aircraft becomes a Ground-Effect-Machine when very close to the ground and tests show that a suitable hover height of about four feet is achieved with an installed auxiliary thrust of one quarter of the all-up weight.

Take-off is accomplished in the normal manner using conventional thrust engines. The type of lift is changed from ground effect to normal aerodynamic lift by increasing incidence as soon as possible subject to a suitable trailing edge clearance. This procedure reduces ground erosion and avoids large nose down pitching moments which occur at low incidence and high speeds close to the ground. A height of

fifty feet is reached at a flying speed of only 160 knots compared with conventional take-off speeds of about 190 knots. The auxiliary thrust is reduced to zero and flaps retracted as flying speed with the clean aircraft is reached. Estimated distance to fifty feet on take-off is 3500 ft and for landing over a fifty feet screen 2900 ft.

2. List of Symbols

α	Geometric wing incidence
β	Angle of sideslip
ϵ	Wing semi-apex angle
θ	Jet sweep angle, see fig. 6
ϕ	Velocity potential
θ_1, ϕ_1	Angles in plane vortex sheet, see fig. 8f
γ	Vortex strength
μ	Jet momentum
a	Local wing semi-span
b	Wing span = 1.62 ft.
C_0	Root chord = 3.33 ft.
$\bar{c}$	Aerodynamic mean chord = $\frac{\int_{-\frac{b}{2}}^{+\frac{b}{2}} C^2 dy}{\int_{-\frac{b}{2}}^{+\frac{b}{2}} C dy} = 2.60 \text{ ft.}$
h	Ground clearance at zero incidence with 90° downward deflected blowing
h_1, l_1	Distances along wing leading edge, see fig. 8f
$\dot{m}_j$	Rate of jet mass flow
v_j	Final jet velocity assuming isentropic expansion to free stream static pressure
q_0, p_0	Mainstream dynamic pressure and static pressure
p	Static pressure
s	Wing semi-span

x', y', z'	Body axes for experimental work, see fig. 7
x, y, z	Body axes for theoretical work, see fig. 8a
$x_{C.P.}$	Distance of centre of pressure from apex
A, B	Real and imaginary parts of $\sqrt{\sigma^2 - a^2}$
A_R	Wing aspect ratio = 0.73
H	Total head measured in the leading edge vortex
K	Cotangent of leading edge swept angle ($\pm \epsilon$)
M_o	Mainstream mach number
R	Radius of curvature of vortex sheet in plane normal to main vortex axis
R_e	Reynolds number based on aerodynamic mean chord
S	Wing area = 3.60 sq.ft.
V	Mainstream velocity
C_p	Static pressure coefficient = $\frac{p-p_o}{q_o}$
C_μ	Blowing momentum coefficient (experimental) = $\frac{\mu}{q_o S} = \frac{m_j v_j}{q_o S}$
	see § 5.1
$C_{\mu theor.}$	Blowing momentum coefficient (theoretical) = $\frac{m_j / \text{unit length} \cdot v_j}{q_o a}$
	= $2 C_\mu$ for leading edge blowing only
C_L	Lift coefficient = $\frac{\text{Lift}}{q_o S}$
C_D	Drag coefficient = $\frac{\text{Drag}}{q_o S}$
C_c	Cross-wind force coefficient = $\frac{\text{Cross-wind force}}{q_o S}$
C_l	Rolling moment coefficient about x axis = $\frac{\text{Rolling moment}}{q_o S b}$

C_m^* Pitching moment coefficient about aerodynamic mean quarter
 chord $0.415 C_o = \frac{\text{Pitching moment}}{q_o S \bar{c}}$

C_n^* Yawing moment coefficient about $0.415 C_o$
 $= \frac{\text{Yawing moment}}{q_o S b}$

Suffices

- o indicates conditions at main vortex position
- 1 indicates conditions at small vortex position
- j indicates conditions pertaining to jet

Downward deflected blowing only

- μ Auxiliary thrust
- C_μ Blowing momentum coefficient = $\frac{\mu}{q_o S}$
- L_A Lift augmentation factor = $\frac{\text{Lift}}{\mu}$
- D_A $\frac{\text{Drag}}{\mu}$
- P_A $\frac{\text{Pitching moment}}{\bar{c} \mu}$
- V_A Speed of modified aircraft with downward deflected blowing
- D Drag of modified aircraft
- T Main engine thrust for modified aircraft = 112,000 lbs.
- v_c Rate of climb

3. Introduction

For the first thirty years of powered flight the top speed of aircraft, with the exception of a few racing machines, was low and the performance of straight relatively thick wings quite adequate. During the last war, however, fighters and experimental aircraft began to experience compressibility troubles at maximum speed and sweepback was suggested as a means of delaying the undesirable effects to higher Mach numbers.

For moderate angles of sweep and with the comparatively thick conventional wing sections which were in current use the effect of sweepback on lift at low speeds was small. As top speeds increased larger angles of sweepback became common, wing sections were much thinner and the trend to the thin, highly swept delta wing for high speed aircraft was established. As a direct result of the increased leading edge sweep aspect ratios were less than half the value of comparable aircraft only a decade before. This seriously reduced both the wing lift-curve-slope and the lift-to-drag ratio, the former leading to considerable increases in minimum flight speeds.

As a further complication the relatively sharp leading edges of the thinner wing sections gave rise to flow separation at the leading edge close to the tips at moderate incidence. With increasing incidence the separation spread rapidly inboard forming a leading edge vortex with substantial spanwise airflow on the wings and the resultant change of trim associated with the spanwise extent of the separated flow still further increased the difficulties of low-speed flight. Various

methods were tried which were designed to limit the extent of the separated region. Basically they fall into two categories: Firstly, attempts were made to suppress the separation entirely by the use of vortex generators near the leading edge and secondly, separations were permitted but their spanwise extent fixed by such devices as the "cut" in the leading edge, "the sawtooth" leading edge, and the boundary layer fence. While most of these aids enjoyed a measure of success they did not entirely resolve a rather unsatisfactory situation.

3.1. Summary of existing work

The difficulties encountered provided a stimulus to research and soon a considerable amount of information had accumulated on this topic. Observations made of the characteristic leading edge vortices and experimental determinations of forces and moments made it increasingly clear that the available theories relating to low aspect ratio wings and based on attached flow were inadequate. The existing theories of R.T. Jones (ref 1) and Krienes (ref 2) for low aspect ratio wings were based on potential (attached) flow solutions and although they predicted a reduction in lift curve slope with aspect ratio the slope was still linear and the drag zero.

The first experimental study of the flow over low aspect ratio swept wings with leading edge separations appears to have been made by Winter (ref 3) as early as 1936. He realised that the flow differed radically from that over high aspect ratio wings and refers to the characteristic "vortex braid". Other early studies were made on a

full-scale aircraft by Wilson & Lovell in 1947 (ref 4). It was not until 1952 however, that the experimental investigations of Roy (ref 5) enabled Legendre (ref 6) to postulate a theory which took account of the leading edge separations. Many experimental investigations followed and some of the more important contributions to date are listed in refs 7 - 17.

An important simplification introduced by Legendre on the basis of the experimental results was that of "conical flow", i.e., constant flow characteristics along rays through the apex. This means in fact that in planes normal to the wing, the "cross-flow plane", conditions are both dynamically and geometrically similar and the problem reduces to a two-dimensional one. The boundary conditions however are still those of the complete three-dimensional flow and equivalent two-dimensional conditions must be found.

Basically, Legendre's model is a flat plate delta wing with discrete line vortices emanating from the apex representing the leading edge vortices. The velocity potential ϕ is shown to satisfy Laplace's equation in two dimensions and suitable equivalent two-dimensional boundary conditions are derived. The flow is assumed to separate smoothly from the leading edge (Kutta-Joukowski condition) and this leads to the determination of incidence. It should be noted that this condition leads to the loss of leading edge suction and gives a finite drag since all pressures act normal to the wing surface. The drag is therefore $Lift \times \tan \alpha$.

Two conditions must be satisfied on the vortex sheet (I) It must

be a stream surface of the three dimensional flow and (II) there is no pressure difference across the sheet. Legendre chose the position of his vortex line such that it was a streamline of the three dimensional flow and hence had no force acting on it. Although this model gave the correct trends as regards lift and vortex position, the theory was unsatisfactory in many respects.

The solution gives rise to a logarithmic term in the potential function which gives a multi-valued solution. This difficulty was resolved after the publication of Legendres first paper by Adams (ref. 18) who pointed out that in order to obtain a single value for the lift a "cut" was necessary in the cross-flow plane representation in order to avoid circling the vortex more than once. Although this argument avoided a multiplicity of solutions, the change in pressure and velocity across the cut gave rise to an uncanceled force in the flow field and led to an ambiguity in the lift depending on whether or not this force was included. A further deficiency of this model was that although the vortex strength increased with distance downstream to satisfy the conical flow conditions there is nothing in the theory which provides for this growth.

In order to satisfy Kelvins theorem and to resolve the ambiguity of the lift results both Edwards (ref. 19) and Brown & Michael (ref. 20) have developed theories rather similar to Legendre's but interpreting the "cuts" as feeding vortex sheets originating at the leading edge and joined to the line vortices representing the leading edge vortices. The condition of constant fluid pressure is replaced by a

"no overall force" condition on the vortex system whereby the force arising from the "cut" is balanced by the Kutta-Joukowski lift due to the induced velocities at the vortex position.

The lift, pressure distribution and vortex position calculated from these theories are compared with experiment and show the correct trends in regions near the wing apex. The lift-curve slope is now non-linear giving greater lift and the inboard and upward movement of the vortices with incidence follows the experimentally observed trend. The vortex height at a given incidence is predicted quite well but the spanwise position is greatly in error, also the experimental lift values at low speeds are well below theoretical estimates partly due to the zero lift condition at the trailing edge.

The next contribution to the theory was made by Kuchemann (ref. 21) who reasoned that the height of the vortex above the wing was of secondary importance and considered the effect of a vorticity distribution concentrated in the plane of the wing inboard of the leading edge. The load distribution is such that there is a constant induced downwash with a discontinuity at a spanwise station corresponding to the position of the vortex core. Also the flow leaves the leading edge tangentially thus giving no load there. The loading can be calculated for a given value of the spanwise discontinuity but unfortunately its position is not given by the theory and an approximate value has to be postulated. The value chosen depends on the incidence α and gives rise to a lift term dependent on $\alpha^3/2$. The agreement with experiment is much better than would be expected of a theory which does

not include trailing edge effects.

A more comprehensive theory is that due to Mangler & Smith (refs 22, 23). With the usual approximations the problem reduces to a two-dimensional one and the equivalent forms of the boundary conditions are similar to Legendre's. Their model consists of a wing with a spiral core of vorticity (representing the rolled-up vortex sheets) above it joined to the leading edge by a curved vortex sheet. The problem is solved by using a transformation such that the wing becomes part of the imaginary axis, the vortex cores become images in the imaginary axis and the vortex sheets a smooth curve joining the vortex cores and passing through the origin. Unfortunately the shape of the sheet cannot be calculated and a circular arc is used in the calculations with the condition of zero pressure difference across the sheet applied at the leading edge and the mid-point of the sheet. The vorticity distribution on the sheet is assumed to be specified by the first three terms of a fourier series. This theory takes into account the effect of the vorticity in the sheet on the spanwise pressure distribution and is clearly an advance on previous solutions in which the streamwise vorticity was considered concentrated at one point. Seven equations are then obtained which specify the unknown quantities in terms of the non-dimensional parameter α/K and these are solved numerically.

Despite the additional complication of the method the results are only in fair agreement with experiment. The spanwise position of the vortex is predicted more accurately than by previous theories but the

height is lower than the experimental value. Values of normal force again exceed the experimental values.

The latest contributions to the theory have been made by Hall (ref. 24 & 26) and Earnshaw and Hall (ref. 25). The approach is fundamentally different from the previous theories in that only the vortex itself is considered and not in relation to a wing. There is no feeding vortex representation and vorticity is carried in the normal way along stream lines. The flow is rotational and a solution is found for Eulers equations for steady axially symmetric flow in terms of the stream function. The solution is divided into two parts, an outer solution in which viscosity is neglected and an inner solution in which viscosity is admitted but certain assumptions including boundary layer type approximations are necessary to obtain a solution. The two solutions are then matched at a suitable radius. The predicted axial and circumferential velocities and the pressures are in good agreement with experiment except very close to the axis. Since the agreement with experiment is good over the greater part of the vortex in which viscosity is neglected, it provides some justification for Mangler & Smith's model since it is unlikely that the small core region will affect the lift or vortex position to any great extent.

3.2 Practical deviations from theoretical flows

In order to obtain theoretical solutions to this problem it has been necessary to make assumptions which are justified only in certain regions and to neglect other observed phenomena. Although the existing

theories predict the experimental trends with varying degrees of accuracy the quantitative agreement between theory and experiment is poor when compared with other branches of the subject. The main discrepancies between theory and practice are now summarised and their relative importance indicated.

(1) Two assumptions made by all the existing theories are those of slenderness and conical flow. It has been established that they are fairly well justified for small aspect ratio wings in regions near the apex at moderate incidence but in subsonic flow at least the second condition must break down some distance upstream of the trailing edge in order to satisfy the zero load condition there. In supersonic flow the pressure rise at the trailing edge will take place through a shock wave but this in itself will probably distort the flow pattern upstream. In practice, at low speeds at least, conical flow rarely extends beyond about 60% of the root chord. At larger incidences "vortex breakdown" occurs, see for instance (refs. 12, 27, 28) and moves rapidly upstream with increasing incidence. Neglecting this trailing edge condition in the theory affects mainly the lift and is the prime cause of the large discrepancy between theory and experiment.

(2) The shape of the leading edge and its angle of sweep also influence the type of flow. As mentioned previously, for small angles of incidence, moderate angles of sweep and well rounded leading edges the flow may be completely attached and leading edge separation when it does occur may occupy only part of the span. Large angles of sweep and sharp leading edges, however, tend toward the theoretical limit,

i.e., attached flow only at zero incidence and Peckham (ref. 15) has shown that leading edge separations may occur at angles of incidence as low as 1° .

(3) Of necessity, viscosity is neglected except insofar as it is used to fix the primary separation line at the leading edge. It has been argued (see 3.1) that neglecting the small viscous core of the vortex is likely to have only second order effects.

A far more serious discrepancy due to viscosity is the secondary separation which occurs outboard of the main vortex when the spanwise surface flow encounters an adverse pressure gradient near the leading edge. This is manifest in the pressure distributions as a region of roughly constant pressure. Its main effect is to move the leading edge vortex inboard and the discrepancy between theory and experiment may be as much as 0.15 of the semi span for $\alpha = 25^\circ$.

Fig. 35 shows the spanwise variation of static pressure both for theory and experiment. The influence of the single line vortex gives a pressure peak under the vortex but it is very limited in spanwise extent compared with the real distribution. Somewhat fortuitously the larger pressure peak tends to balance its smaller spanwise extent and reasonably good agreement is obtained, for the lift only, in regions for which the theoretical approximations are valid, i.e., close to the apex. In the present tests for instance comparison is made between experimental lifts and the various theoretical predictions, making a qualitative allowance for the trailing edge effect. (see 4.1). With this modification better agreement is obtained with experiment,

the values of Brown & Michael falling very close to the experimental curve. Accurate prediction of vortex position and lift of the complete wing must, however, await the arrival of a much more comprehensive theory.

3.3 Methods for increasing lift at low speeds

Since the low speed regime, in particular the landing and take-off regimes, becomes critical for very low aspect ratio wings, especially for civil aircraft, a considerable research effort has been directed towards improving their characteristics.

One alleviating factor is the very existence of the leading edge vortex system and its contribution to the lift. In the present tests, for instance, the lift without blowing at $\alpha = 25^\circ$ is twice the value predicted by the R.T. Jones (ref. 1) attached flow theory. However, lift coefficients are still low compared with unswept wings and naturally attempts have been made to improve the low speed performance using methods that have proved successful on straight wings.

Several series of tests using trailing edge flaps both without (refs. 29-31) and with (refs. 32 & 33) blowing boundary layer control have been made and the work reported in ref. 33 has been extended to include the effect of jet flaps (ref. 34). All the tests resulted in increased lift at constant incidence but the large centre of pressure movements incurred may be difficult to trim, especially with tailless configurations. Fewer tests seem to have been made with leading edge modifications or controls (refs. 32, 35, 36) but here again any

appreciable increase in lift was accompanied by relatively large pitching moments.

A modification of the jet flap system, in which air was ejected from a spanwise slot at various chordwise positions in order to avoid large pitching moments, has been reported by Melbourne (ref. 37). Unfortunately, this device produces negative induced effects which reduce the total lift below the direct jet lift and would not appear to have a practical application.

A rather different approach (ref. 38) suggested that improvements in both lift and lift/drag ratio might be obtained by blowing a sheet of air from the swept leading edge in order to influence favourably the leading edge vortices. The two reported experiments to use this type of blowing were carried out on a low-aspect-ratio straight wing (ref. 39) and a 40° swept wing (ref. 40) with thin high velocity jet sheets issuing from their streamwise tips.

In both series of tests spanwise blowing resulted in increased lift. The effect on the unswept wing was to change the spanwise lift distribution from near elliptic to approximately constant loading as the ability of the jet sheet to support a pressure difference increased the loading near the tips. Also the trailing vortex was moved some distance outboard. With the swept wing spanwise blowing increased the size and strength of the leading edge vortex and moved it outboard. In this case also the increase in lift was due mainly to an increase in the loading of the outer sections of the wing. The increase in lift was obtained with little change in the longitud-

inal static stability over the greater part of the usable C_L range.

During 1957 some preliminary tests with blowing from the leading edge of a 70° swept delta wing were carried out at the Royal Aircraft Establishment, Farnborough (ref. 41). The shape of the leading edge slot was varied and it was concluded that the "triangular" slot, i.e., increasing in width linearly from the apex, was the most suitable. For the optimum blowing arrangement a lift magnification factor $\Delta C_L / C_{\mu}$ of 1.5 was obtained but the lift to drag ratio was reduced due to the increased drag caused by the partly forward facing jet.

The present series of tests was planned in order to extend this preliminary work on the Jet-Wing concept and provision was made for directed blowing both in the plane of the wing and normal to it. The cropped delta planform was chosen since it has a lower vortex drag at supersonic speeds for a given value of the slenderness parameter $\frac{(M_0^2 - 1)s}{C_0}$ and it was hoped that the inline tips would reduce the drag due to blowing and thus improve the lift to drag ratio. Trailing edge blowing was also incorporated to increase the thrust and to increase the lift at low speeds by increasing the loading near the trailing edge.

It is shown (see 4.2) that it is possible to maintain conical flow with edge blowing provided that the momentum ejected increases linearly with distance from the apex, thus confirming the R.A.E. results. Hence it is possible to increase lift without changing the trim, so that although the lift magnification factor $\Delta C_L / C_{\mu}$ is small compared with flap blowing for instance, no lift is lost as in

other systems by the subsequent trimming. This would be particularly useful on tailless aircraft where the amount of trimming possible is rather limited.

It would seem to be possible, by means of differential blowing, to exercise control both in roll and yaw without a corresponding change of trim. Similarly the static stability might be changed, and in particular, reductions in the magnitude of ℓ_y (rolling moment due to sideslip) at high lift coefficients would be very useful.

3.4. S.T.O.L. using auxiliary power

Up to this point edge blowing has been conceived as an aid to low-speed flight in which the jet sheet is supplied by bleeding the main engine compressors. The reduction in landing speed would be the same order as that resulting from the application of boundary layer control to trailing edge flaps on more conventional aircraft.

In order to achieve any spectacular decrease in landing and take-off speeds an entirely new approach to the problem is necessary. In 1958, after some preliminary tests on a circular Ground-Effect-Machine and an initial study of the possibilities of the present edge blowing delta model, it was decided that a scheme worth further study was one in which the conventional undercarriage was disposed of and the aircraft supported at low speeds and at rest by deflecting peripheral jet sheets downward through at least 90° , the aircraft thus becoming a ground-effect-machine at very small heights. Subsequently it became clear that similar proposals had been put forward by O.N.E.R.A., Avro

Aircraft Ltd., and the Vertol Aircraft Corporation (see ref. 42).

The proposed aircraft would have conventional large jets for propulsion and the air for the peripheral jet sheets would be supplied by auxiliary jet engines. The increase in weight and size due to accommodating these extra engines would no doubt lead to performance penalties but utilising the ground effect principle the number of engines needed would only be one quarter of the number necessary for full V.T.O.L. operation as proposed by Griffiths (ref. 43).

The concept of a V.T.O.L. all weather airliner is a very attractive one since it would seem to offer 100% reliable travel from city centre to city centre. However, with this type of aircraft two almost insuperable problems present themselves. Firstly noise. Present day subsonic jet airliners have available perhaps 60,000 lbs thrust at take-off and already they are creating considerable problems. Even Griffiths proposal for a small (125,000 lbs) airliner would roughly treble this thrust and although this might only raise the noise level by 3-5 db it seems that, at present, city centre operation for this type of aircraft is out of the question. The second problem is an economic one. Undoubtedly a large penalty must be paid for V.T.O.L. capability and not only the weight of the lift engines, their fuel and associated equipment must be considered but also the space occupied and its effect on the optimum shape of the aircraft for cruising.

There are, of course, alleviating factors to be considered. The size of airports can be reduced with a considerable saving in capital and maintenance costs, no time will be lost in ground operations and

manoeuvring, and the all weather guidance necessary with this type of aircraft will ensure almost 100% reliability in operation.

Although reductions in noise from the lifting engines can be expected from the use of high by-pass ratios, it seems unlikely, on the grounds of noise alone, that city centre to city centre operations by large aircraft will be possible in the foreseeable future. Thus it is likely that the present day trend of airports being situated on the edge of large cities will continue. Away from the city centres, of course, the noise problem is not so formidable and V.T.O.L. no longer necessary.

The S.T.O.L. concept is attractive on many grounds. The number of auxiliary engines is reduced by a factor of four using the proposed scheme thus reducing the noise problem although of all the difficulties this is likely to be the most severe. The smaller number of auxiliary engines, equipment and fuel will make smaller demands on space and thus reduce the penalty on cruising. Also it would be possible to design the aircraft for optimum cruise performance unencumbered by the off-design compromises necessary on conventional aircraft, leaving the auxiliary jet system to improve the low speed performance.

Furthermore the size of airports could be considerably reduced although not to the extent possible with V.T.O.L. Again a conventional undercarriage would not be needed although some means of support would be necessary to enable the aircraft to be moved without running the engines. The answer might be to use several small undercarriage legs because these need not be stressed for touch-down loads or alternatively

it might be possible for the aircraft to alight on a trolley thereby saving weight on the aircraft.

Although noise remains the major problem there are others to be considered in connection with the proposal. Runway erosion, for instance, will need careful thought. With high bypass ratio auxiliary engines the jet temperature will not be excessive but runways will have to be suitably surfaced particularly in the areas where the aircraft will be hovering. From the point of view of strength, however, requirements will be less severe since the weight of the aircraft is spread over a large area and pressures at any one point will be fairly small.

Stability and control will also provide many problems but it seems likely that experience gained on aircraft like the Short SC1 with its jet controls will be directly applicable to the larger aircraft. Finally it must not be forgotten that a considerable momentum drag penalty will be incurred by use of the lifting engines. To provide 85,000 lbs thrust will require a mass flow of the order of 50 slugs/sec giving a momentum drag of $50 V$ lbs i.e., 5000 lbs at only 100 ft/sec.

It has not been possible in the time available to study every aspect of this proposal. Three component balance measurements only were made with a small amount of flow visualisation using tufts. As far as possible the whole range of ground clearances was covered from hovering to free flight conditions but clearly much is left to do. Without a full design study on a particular configuration using this auxiliary lift system it is impossible to say whether or not the proposed

scheme is advantageous or even practicable. A great deal more information would be necessary before this could be attempted in particular with regard to stability and the effect of planform but it is hoped that the information presented here will be sufficient to encourage further thought and research on the subject.

4. Theoretical discussion

It is clear from the review of existing theories that a complete solution of the problem of a delta wing with leading edge separations presents some extraordinary difficulties. It would appear, however, that potential flow solutions, i.e., neglecting viscosity, will give a reasonable approximation to the real flow within the framework of the assumptions necessary to make the problem tractable.

In the present work the following assumptions have been made:

(I) The flow is conical and the aspect ratio small enough to permit slender wing approximations to be used.

(II) The leading edge is sharp and the flow separates smoothly there.

(III) The secondary separation and its effects are neglected. The apparent representation of the effects of the secondary separation on the pressure distribution must be regarded as fortuitous.

There are five boundary conditions which define the problem, the first two conditions are those appropriate to the attached flow solution, while the others pertain to the leading edge vortex system.

(a) The normal velocity on the surface of the wing is zero.

(b) All disturbances vanish at infinity where there exists a steady stream of velocity V .

(c) The fluid pressure is continuous everywhere in the unblown case, in particular there is no pressure difference across the vortex sheet. With blowing, however, the thin jet vortex sheet is able to support a pressure difference due to its momentum.

(d) The vortex sheets must be stream surfaces of the three-dimensional flow.

(e) The flow separates smoothly at the sharp leading edge.

The preceding conditions apply to both the blown and unblown case except where stated. The two cases are now considered separately.

4.1 The unblown case

Within the assumptions of slender wing theory the velocity potential ϕ satisfies the two-dimensional Laplace Equation (see for instance ref. 22)

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (1)$$

The axes are fixed in the wing with the origin at the apex, see fig. 8a.

The question of a suitable representation of the problem in the cross-flow (y, z) plane is now considered.

The flow without secondary separation is shown in fig. 8b. The flow separates smoothly at the leading edge and the resulting vortex sheet rolls up forming a spiral core into which most of the vorticity is concentrated. In common with most other theories this core of vorticity will be represented by a line vortex passing through the origin. Mangler & Smith (ref. 23) show that this representation is satisfactory at all but very small distances from the core.

Experimental measurements made at the centre of the vortex show that very high axial velocities are attained, in some cases as much as

three times free stream velocity (see ref. 16). The associated low pressures give rise to an apparent sink effect, Kuchemann (ref. 21) in particular considers that a source-sink distribution is necessary, but it seems clear from the available evidence that the only fluid involved is that moving down the core, i.e., axially, which originally formed the vortex sheets and is not associated with the removal of fluid from the cross-flow plane. This is a purely three-dimensional effect involving velocities normal to the cross-flow plane and does not require a sink representation. Evidence to support this is obtained from the theories of both Hall (ref. 26) and Mangler & Smith (refs. 22 and 23) where high axial velocities are obtained by considering the complete flow and ref. 23 shows a pattern of closed streamlines in the cross-flow plane which could not occur with a vortex-sink representation.

Both Edwards (ref. 19) and Brown & Michael (ref. 20) include feeding vortex sheets in their theories to satisfy Kelvins theorem but only in the theory of Mangler and Smith (refs. 22 & 23) is the effect of a curved feeding vortex sheet on the wing pressure distribution considered. The shape of the sheet and its vorticity distribution are not known a priori and with the assumptions and approximations necessary Mangler & Smith obtain an appreciable pressure difference across the sheet, except at the two points where the boundary condition is applied and this approach can only be regarded as a first order approximation to the problem. That it is an important consideration is clear from their results especially at higher incidence when there

is an appreciable part of the total vorticity in the sheet.

Ideally, neglecting the secondary separation, the situation in the cross-flow plane could be represented as in fig. 8c where the main core is represented by a point vortex and the sheet by a large number of point vortices distributed on a suitable curve. Mangler & Smith (ref. 23) concluded that the point vortex representation of the core was satisfactory at all but very small distances from the core and of course the vortex sheet is well represented by a distribution of point vortices whose distance apart tends to zero. Boundary conditions (c) and (d) would then be satisfied if a no force condition similar to Legendres was applied not only at the main vortex but at each of the point vortices on the sheet. Although in principle this could be done and the shape of the sheet found assuming a suitable strength distribution it is clearly a task of great magnitude with twice as many equations as vortices.

It is essential, however, to represent the effects of vorticity outside the main core and in the present analysis the vortex core is represented by a line vortex and considered to be fed by a vortex sheet originating at the leading edge and along which the appropriate boundary conditions are satisfied. At small incidence, at least, the effects of the transverse vorticity components on the wing pressure distribution will be small compared with the effects of streamwise vorticity and are neglected. From the point of view of representing the streamwise component of sheet vorticity in the cross-flow plane, it is assumed that, to first order at least, this can be done by

concentrating the whole of the streamwise vorticity in the sheet into a second line vortex and applying a no-force boundary condition in order to obtain its position. In order to obtain a unique answer suitable cuts must be made in the cross-flow plane joining the main vortex and the small vortex representing the sheet, to one another and to the leading edge. These cuts introduce uncanceled forces which are the result of neglecting the effects of the transverse vorticity components on the wing pressure distribution but for small incidence at least the effect on lift will be small. A posteriori justification for this is obtained by comparing lift values obtained from momentum considerations, which include the forces on the cuts, and the lift obtained from wing pressure distributions. Even at $\alpha = 15^\circ$ the lift force due to the cut is only 5% of the total lift.

The simplification of the present problem can be likened to the difference between the simple horseshoe vortex representation of a finite wing and a suitable vorticity distribution so that the representation proposed would seem to have a good chance of displaying the main features of the real flow.

An approximate streamline pattern of the proposed model for $\alpha/K \approx .1$ is shown in fig. 8d. This may be compared with fig. 14 of ref. 23. A stagnation point will exist somewhere between the two vortices and closed streamlines will encircle both point vortices. Outside the first few closed streamlines the flow pattern will be similar to that of Mangler and Smith (ref. 23).

Thus in the x plane and the transformed σ plane the model will be

as shown in fig. 8e.

With the convention $\vec{q} = -\nabla\phi$ the complex potential equation is:

$$W(x) = iVax + \frac{i\gamma_0}{2\pi} \log \frac{x-x_0}{x+\bar{x}_0} + \frac{i\gamma_1}{2\pi} \log \frac{x-x_1}{x+\bar{x}_1} \quad (2)$$

where x_0 , $-\bar{x}_0$, x_1 and $-\bar{x}_1$ are the positions of the four point vortices of strengths $\frac{\gamma_0}{2\pi}$, $-\frac{\gamma_0}{2\pi}$, $\frac{\gamma_1}{2\pi}$, $-\frac{\gamma_1}{2\pi}$ respectively.

Transforming to the σ plane by the transformation $x = \sqrt{\sigma^2 - a^2}$ we have:

$$W(\sigma) = iV\alpha \sqrt{\sigma^2 - a^2} + \frac{i\gamma_0}{2\pi} \log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_0^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) + \frac{i\gamma_1}{2\pi} \log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_1^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right) \quad (2')$$

Now the velocity is given by:

$$\frac{dW(\sigma)}{d\sigma} = -u + i v$$

We have $\sigma = y+iz$ and for convenience we write:

$$\begin{aligned} \sqrt{\sigma^2 - a^2} &= A+iB & \sigma_0 &= y_0+iz_0 \\ \sqrt{\sigma_0^2 - a^2} &= A_0+iB_0 & \sigma_1 &= y_1+iz_1 \\ \sqrt{\sigma_1^2 - a^2} &= A_1+iB_1 \end{aligned}$$

This gives (see Appendix I at end of chapter)

$$\begin{aligned} -u &= \frac{(By-Az)}{A^2+B^2} \left[V\alpha + \frac{\gamma_0}{2\pi} \left\{ \frac{(A-A_0)}{(A-A_0)^2+(B-B_0)^2} - \frac{(A+A_0)}{(A+A_0)^2+(B-B_0)^2} \right\} + \frac{\gamma_1}{2\pi} \times \right. \\ &\quad \left. \left\{ \frac{(A-A_1)}{(A-A_1)^2+(B-B_1)^2} - \frac{(A+A_1)}{(A+A_1)^2+(B-B_1)^2} \right\} \right] + \frac{(Ay+Bz)}{A^2+B^2} \left[\frac{\gamma_0}{2\pi} \left\{ \frac{(B-B_0)}{(A-A_0)^2+(B-B_0)^2} - \right. \right. \\ &\quad \left. \left. \frac{(B+B_0)}{(A+A_0)^2+(B+B_0)^2} \right\} + \frac{\gamma_1}{2\pi} \left\{ \frac{(B-B_1)}{(A-A_1)^2+(B-B_1)^2} - \frac{(B+B_1)}{(A+A_1)^2+(B+B_1)^2} \right\} \right] \end{aligned}$$

$$\begin{aligned}
v = \frac{(Ay+Bz)}{A^2+B^2} & \left[V\alpha + \frac{\gamma_0}{2\pi} \left\{ \frac{(A-A_0)}{(A-A_0)^2+(B-B_0)^2} - \frac{(A+A_0)}{(A+A_0)^2+(B-B_0)^2} \right\} + \frac{\gamma_1}{2\pi} \times \right. \\
& \left. \left\{ \frac{(A-A_1)}{(A-A_1)^2+(B-B_1)^2} - \frac{(A+A_1)}{(A+A_1)^2+(B-B_1)^2} \right\} - \frac{(By-Az)}{A^2+B^2} \left[\frac{\gamma_0}{2\pi} \left\{ \frac{(B-B_0)}{(A-A_0)^2+(B-B_0)^2} - \right. \right. \right. \\
& \left. \left. \frac{(B-B_0)}{(A+A_0)^2+(B-B_0)^2} \right\} + \frac{\gamma_1}{2\pi} \left\{ \frac{(B-B_1)}{(A-A_1)^2+(B-B_1)^2} - \frac{(B-B_1)}{(A+A_1)^2+(B-B_1)^2} \right\} \right] \quad (3)
\end{aligned}$$

To satisfy boundary condition (e) the Kutta-Joukowski condition is applied at the edge ($\sigma = \pm a$). For this we need the value of u and v as $\sigma \rightarrow a$.

Hence (see Appendix II at end of chapter)

$$\begin{aligned}
-u_a &= \frac{By}{A^2+B^2} \left[V\alpha - \frac{\gamma_0}{2\pi} \left(\frac{2A_0}{A_0^2+B_0^2} \right) - \frac{\gamma_1}{2\pi} \left(\frac{2A_1}{A_1^2+B_1^2} \right) \right] \quad (3') \\
V_a &= \frac{Ay}{A^2+B^2} \left[V\alpha - \frac{\gamma_0}{2\pi} \left(\frac{2A_0}{A_0^2+B_0^2} \right) - \frac{\gamma_1}{2\pi} \left(\frac{2A_1}{A_1^2+B_1^2} \right) \right]
\end{aligned}$$

Now for smooth separation at the leading edge $v = 0$.

$$\therefore V\alpha = \frac{\gamma_0}{2\pi} \cdot \frac{2A_0}{A_0^2+B_0^2} + \frac{\gamma_1}{2\pi} \cdot \frac{2A_1}{A_1^2+B_1^2} \quad (4)$$

Hence $u = 0$ also.

Thus the leading edge is a three-dimensional separation line as defined by Maskell (ref. 44).

Non-dimensionalising (4) we have:

$$\frac{\alpha}{\epsilon} = \frac{\gamma_0}{2\pi a V \epsilon} \times 2 \left\{ \frac{\frac{A_0}{a}}{\frac{A_0^2}{a^2} + \frac{B_0^2}{a^2}} + \frac{\gamma_1}{\gamma_0} \cdot \frac{\frac{A_1}{a}}{\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2}} \right\} \quad (4')$$

where ϵ is the wing semi-apex angle.

In order to satisfy the no-force boundary condition on the main vortex line it is necessary to calculate the velocity at (say) the right hand main vortex (σ_0) due to all other singularities in the cross-flow plane. For convenience a further complex potential $W_0(\sigma)$ is introduced where:

$$W_0(\sigma) = W(\sigma) - \frac{i\gamma_0}{2\pi} \log(\sigma - \sigma_0)$$

Thus (Appendix III, end of chapter)

$$\begin{aligned} -u_0 + iv_0 &= \frac{iV\alpha\sigma_0}{\sqrt{\sigma_0^2 - a^2}} - \frac{i\gamma_0}{2\pi} \cdot \frac{\sigma_0}{\sqrt{\sigma_0^2 - a^2}} \left(\frac{1}{\sqrt{\sigma_0^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) - \\ &\frac{i\gamma_0}{2\pi} \cdot \frac{a^2}{2\sigma_0(\sigma_0^2 - a^2)} + \frac{i\gamma_1}{2\pi} \cdot \frac{\sigma_0}{\sqrt{\sigma_0^2 - a^2}} \left(\frac{1}{\sqrt{\sigma_0^2 - a^2} - \sqrt{\sigma_1^2 - a^2}} - \frac{1}{\sqrt{\sigma_0^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right) \end{aligned} \quad (5)$$

The actual no-force condition is applied in a plane normal to the main vortex axis since velocities perpendicular to the axis must be considered and these include a component of the free stream. The component of the free stream is $-\frac{V\epsilon\sigma_0}{a}$ and the contribution from the cross-flow plane is $-u_0 + iv_0$. Thus for zero force:

$$u_o + iv_o = \frac{V\epsilon\sigma_o}{a}$$

$$\therefore -u_o + iv_o = -\frac{V\epsilon\bar{\sigma}_o}{a} \quad (6)$$

From equations (5) and (6) we have (see Appendix IV) rearranging and non-dimensionalising:

$$\begin{aligned} \frac{2\pi a V \epsilon}{y_o} &= \frac{-1}{\frac{y_o}{a} \left(\frac{A_o^2}{a^2} + \frac{B_o^2}{a^2} \right)} \times \left[\frac{2 \frac{A_o}{a} \left(\frac{B_o}{a} \frac{y_o}{a} - \frac{A_o}{a} \frac{z_o}{a} \right)}{\frac{A_o^2}{a^2} + \frac{B_o^2}{a^2}} - \right. \\ &\quad \frac{\frac{z_o}{a} \left(\frac{A_o^2}{a^2} - \frac{B_o^2}{a^2} \right) + 2 \frac{A_o}{a} \frac{B_o}{a} \frac{y_o}{a}}{2 \left(\frac{y_o^2}{a^2} + \frac{z_o^2}{a^2} \right) \left(\frac{A_o^2}{a^2} + \frac{B_o^2}{a^2} \right)} - \frac{\frac{B_o}{a} \frac{y_o}{a} - \frac{A_o}{a} \frac{z_o}{a}}{2 \frac{A_o}{a}} + \frac{y_1}{y_o} \times \\ &\quad \left\{ \frac{2 \frac{A_1}{a} \left(\frac{B_o}{a} \frac{y_o}{a} - \frac{A_o}{a} \frac{z_o}{a} \right)}{\left(\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2} \right)} + \frac{\left(\frac{A_o}{a} - \frac{A_1}{a} \right) \left(\frac{B_o}{a} \frac{y_o}{a} - \frac{A_o}{a} \frac{z_o}{a} \right) + \left(\frac{B_o}{a} - \frac{B_1}{a} \right) \left(\frac{A_o}{a} \frac{y_o}{a} + \frac{B_o}{a} \frac{z_o}{a} \right)}{\left(\frac{A_o}{a} - \frac{A_1}{a} \right)^2 + \left(\frac{B_o}{a} - \frac{B_1}{a} \right)^2} \right. \\ &\quad \left. - \frac{\left(\frac{A_o}{a} + \frac{A_1}{a} \right) \left(\frac{B_o}{a} \frac{y_o}{a} - \frac{A_o}{a} \frac{z_o}{a} \right) + \left(\frac{B_o}{a} - \frac{B_1}{a} \right) \left(\frac{A_o}{a} \frac{y_o}{a} + \frac{B_o}{a} \frac{z_o}{a} \right)}{\left(\frac{A_o}{a} + \frac{A_1}{a} \right)^2 + \left(\frac{B_o}{a} - \frac{B_1}{a} \right)^2} \right] \quad (7) \end{aligned}$$

$$\text{Also } \frac{2\pi a V \epsilon}{y_o} = \frac{1}{\frac{z_o}{a} \left(\frac{A_o^2}{a^2} + \frac{B_o^2}{a^2} \right)} \times \left[\frac{2 \frac{A_o}{a} \left(\frac{A_o}{a} \frac{y_o}{a} + \frac{B_o}{a} \frac{z_o}{a} \right)}{\left(\frac{A_o^2}{a^2} + \frac{B_o^2}{a^2} \right)} - \right.$$

$$\frac{\frac{y_o}{a} \left(\frac{A_o^2}{a^2} - \frac{B_o^2}{a^2} \right) - 2 \frac{A_o}{a} \frac{B_o}{a} \frac{z_o}{a}}{2 \left(\frac{y_o^2}{a^2} + \frac{z_o^2}{a^2} \right) \left(\frac{A_o^2}{a^2} + \frac{B_o^2}{a^2} \right)} - \frac{\frac{A_o}{a} \frac{y_o}{a} + \frac{B_o}{a} \frac{z_o}{a}}{2 \frac{A_o}{a}} +$$

$$\begin{aligned}
& + \frac{y_1}{y_0} \left\{ \frac{2 \frac{A_1}{a} \left(\frac{A_0}{a} \frac{y_0}{a} + \frac{B_0}{a} \frac{z_0}{a} \right)}{\left(\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2} \right)} + \frac{\left(\frac{A_0}{a} - \frac{A_1}{a} \right) \left(\frac{A_0}{a} \frac{y_0}{a} + \frac{B_0}{a} \frac{z_0}{a} \right) - \left(\frac{B_0}{a} - \frac{B_1}{a} \right) \left(\frac{B_0}{a} \frac{y_0}{a} - \frac{A_0}{a} \frac{z_0}{a} \right)}{\left(\frac{A_0}{a} - \frac{A_1}{a} \right)^2 + \left(\frac{B_0}{a} - \frac{B_1}{a} \right)^2} \right. \\
& \left. - \frac{\left(\frac{A_0}{a} + \frac{A_1}{a} \right) \left(\frac{A_0}{a} \frac{y_0}{a} + \frac{B_0}{a} \frac{z_0}{a} \right) - \left(\frac{B_0}{a} - \frac{B_1}{a} \right) \left(\frac{B_0}{a} \frac{y_0}{a} - \frac{A_0}{a} \frac{z_0}{a} \right)}{\left(\frac{A_0}{a} + \frac{A_1}{a} \right)^2 + \left(\frac{B_0}{a} - \frac{B_1}{a} \right)^2} \right\} \quad (8)
\end{aligned}$$

Two similar equations will be obtained by applying the no-force condition at σ_1 . This gives (Appendix V)

$$\begin{aligned}
\frac{2\pi a V_0}{y_0} &= \frac{-1}{\frac{y_1}{a} \left(\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2} \right)} \left[\frac{2 \frac{A_0}{a} \left(\frac{B_1}{a} \frac{y_1}{a} - \frac{A_1}{a} \frac{z_1}{a} \right)}{\left(\frac{A_0^2}{a^2} + \frac{B_0^2}{a^2} \right)} - \right. \\
&\frac{\left(\frac{A_0}{a} - \frac{A_1}{a} \right) \left(\frac{B_1}{a} \frac{y_1}{a} - \frac{A_1}{a} \frac{z_1}{a} \right) + \left(\frac{B_0}{a} - \frac{B_1}{a} \right) \left(\frac{A_1}{a} \frac{y_1}{a} + \frac{B_1}{a} \frac{z_1}{a} \right)}{\left(\frac{A_0}{a} - \frac{A_1}{a} \right)^2 + \left(\frac{B_0}{a} - \frac{B_1}{a} \right)^2} - \\
&\frac{\left(\frac{A_0}{a} + \frac{A_1}{a} \right) \left(\frac{B_1}{a} \frac{y_1}{a} - \frac{A_1}{a} \frac{z_1}{a} \right) - \left(\frac{B_0}{a} - \frac{B_1}{a} \right) \left(\frac{A_1}{a} \frac{y_1}{a} + \frac{B_1}{a} \frac{z_1}{a} \right)}{\left(\frac{A_0}{a} + \frac{A_1}{a} \right)^2 + \left(\frac{B_0}{a} - \frac{B_1}{a} \right)^2} \\
&\left. + \frac{y_1}{y_0} \left\{ \frac{2 \frac{A_1}{a} \left(\frac{B_1}{a} \frac{y_1}{a} - \frac{A_1}{a} \frac{z_1}{a} \right)}{\left(\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2} \right)} - \frac{\frac{z_1}{a} \left(\frac{A_1^2}{a^2} - \frac{B_1^2}{a^2} \right) + 2 \frac{A_1}{a} \frac{B_1}{a} \frac{y_1}{a}}{2 \left(\frac{y_1^2}{a^2} + \frac{z_1^2}{a^2} \right) \left(\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2} \right)} - \frac{\frac{B_1}{a} \frac{y_1}{a} - \frac{A_1}{a} \frac{z_1}{a}}{2 \frac{A_1}{a}} \right\} \right] \quad (9)
\end{aligned}$$

$$\begin{aligned}
\frac{2\pi a V \epsilon}{\gamma_0} &= \frac{1}{\frac{z_1}{a} \left(\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2} \right)} \left[2 \frac{A_0}{a} \left(\frac{A_1}{a} \frac{y_1}{a} + \frac{B_1}{a} \frac{z_1}{a} \right) - \right. \\
&\quad \frac{\left(\frac{A_0}{a} - \frac{A_1}{a} \right) \left(\frac{A_1}{a} \frac{y_1}{a} + \frac{B_1}{a} \frac{z_1}{a} \right) - \left(\frac{B_0}{a} - \frac{B_1}{a} \right) \left(\frac{B_1}{a} \frac{y_1}{a} - \frac{A_1}{a} \frac{z_1}{a} \right)}{\left(\frac{A_0}{a} - \frac{A_1}{a} \right)^2 + \left(\frac{B_0}{a} - \frac{B_1}{a} \right)^2} - \\
&\quad \frac{\left(\frac{A_0}{a} + \frac{A_1}{a} \right) \left(\frac{A_1}{a} \frac{y_1}{a} + \frac{B_1}{a} \frac{z_1}{a} \right) + \left(\frac{B_0}{a} - \frac{B_1}{a} \right) \left(\frac{B_1}{a} \frac{y_1}{a} - \frac{A_1}{a} \frac{z_1}{a} \right)}{\left(\frac{A_0}{a} + \frac{A_1}{a} \right)^2 + \left(\frac{B_0}{a} - \frac{B_1}{a} \right)^2} \\
&\quad + \frac{y_1}{\gamma_0} \left\{ \frac{2 \frac{A_1}{a} \left(\frac{A_1}{a} \frac{y_1}{a} + \frac{B_1}{a} \frac{z_1}{a} \right)}{\left(\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2} \right)} - \frac{\frac{A_1}{a} \frac{y_1}{a} + \frac{B_1}{a} \frac{z_1}{a}}{2 \frac{A_1}{a}} \right. \\
&\quad \left. \left. \frac{\frac{y_1}{a} \left(\frac{A_1^2}{a^2} - \frac{B_1^2}{a^2} \right) - 2 \frac{A_1}{a} \frac{B_1}{a} \frac{z_1}{a}}{2 \left(\frac{y_1^2}{a^2} + \frac{z_1^2}{a^2} \right) \left(\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2} \right)} \right\} \right] \quad (10)
\end{aligned}$$

Before these simultaneous equations can be solved it is necessary to obtain a value for $\frac{\gamma_1}{\gamma_0}$. Consider fig. 8f in which vorticity is assumed to be shed at a constant rate from the leading edge and transported to the core along vortex lines which describe constant angle helices on a conical vortex sheet. These assumptions are probably not greatly in error on the first half turn of the sheet. Consider now a vortex line originating at a point A on the leading edge and joining the core, at the station shown, at a point B. All the vorticity between the leading edge OC and the line OB is considered to be in the sheet but once it reaches OB it is in the core. If a

small element of vorticity leaves the leading edge at A and travels along the vortex line to B in time t , then at time t all the vortex elements which left the leading edge at the same time upstream of A will have joined the core and all those which left the leading edge downstream of A will be in the sheet. Thus in the steady state, since vorticity is assumed to be shed at a constant rate the ratio of vorticity in the sheet to that in the core is the ratio of leading edge length upstream of A to the length downstream.

Thus, see fig. 8f.

$$\frac{y_1}{y_0} = \frac{AC}{OA} = \frac{h_1 - l_1}{l_1} = \frac{h_1}{l_1} - 1 \quad (11)$$

To obtain l_1 it is necessary to postulate the shape and extent of the sheet and also the helix angle of the vortex lines. Appeal is made to experiment so that a simple form may be obtained for these quantities without departing too far from physical reality.

Since the secondary separation has been neglected it is suggested that the postulated sheet shape should correspond more closely to the case with blowing (provided C_μ is small) where the secondary separation is eliminated, than to the unblown case where the presence of the secondary separation distorts the profile. The shape chosen was the simplest possible i.e. a semi-circle, since it was not clear, a priori, how critical the shape would be and it was felt that the first shot should at least have the merit of simplicity. The theoretical and experimental (Line of maximum total head from fig. 38b. $\alpha = 10^\circ$ $C_\mu = 0.048$) shapes are compared in fig. 8g. It was also assumed that the

sheet joined the core at B on a diameter through the vortex centre and the wing tip.

Some idea of the helix angle was obtained from the velocity traverses made in the present work although their accuracy was somewhat limited (see § 5.1). Apparently the only other measurement made is given in ref. 16 in which the angle of the vortex line to a plane normal to the vortex core is about 40° . The present experimental work indicated a slightly higher value and for the unblown theory the helix angle has been taken as 45° . This value has the added merit that a very simple form is obtained for the value $\frac{V_t}{V_G}$.

In order to find the ratio h/l , it is more convenient to consider conditions in a plane than on a solid surface. If we consider the vortex sheet in fig. 8f to be "unwrapped", it forms a segment of a circle and the helix line chosen will become some curve in the plane always at 45° to the generators (radii), fig. 8h. To find the equation of this curve take axes x, y with origin at the centre of the circular arc with one side along the x axis fig. 8h. At any point (x, y) on the curve it will cut a generator $y = mx$ at 45° .

Let the equation of the curve be $y = f(x)$ then $f'(x) = \frac{dy}{dx} = \tan \theta_1$ where $\theta_1 = 135^\circ + \tan^{-1} m$

$$\therefore \tan \theta_1 = \frac{dy}{dx} = \frac{\tan 135^\circ + m}{1 - m \tan 135^\circ} = \frac{m - 1}{m + 1} = \frac{y/x - 1}{y/x + 1}$$

which is a homogeneous linear equation.

The equation of the curve is (see Appendix VI)

$$h_1 = \sqrt{x^2 + y^2} e^{\tan^{-1} y/x} \quad (12)$$

On $y = x \tan \phi_1$

$$h_1 = x \sec \phi_1 e^{\phi_1} = l_1 e^{\phi_1}$$

$$\therefore \frac{h_1}{l_1} = e^{\phi_1}$$

From equation (11) $\frac{y_1}{y_0} = \frac{h_1}{l_1} - 1 = e^{\phi_1} - 1 = \phi_1$, to 1st order in ϕ_1

$$\therefore \frac{y_1}{y_0} = \phi_1 = \frac{\pi |\sigma_0 - a|}{h_1} \approx \pi \epsilon \left| \frac{\sigma_0}{a} - 1 \right| = \pi \epsilon \sqrt{\left(1 - \frac{y_0}{a}\right)^2 + \frac{z_0^2}{a^2}} \quad (13)$$

Using this value the solution of the four simultaneous equations (7) - (10) is possible in principle. Clearly there is no analytic solution and it was necessary to adopt a numerical solution.

One of the unknowns is chosen, say $\frac{y_0}{a}$, and $\frac{z_0}{a}$, $\frac{y_1}{a}$, $\frac{z_1}{a}$ are permuted until the four values of $\frac{2\pi a V \epsilon}{y_0}$ coincide. With the very large number of possible permutations, solutions are best obtained using a digital computer. An Autocode programme was prepared for Pegasus which, assuming $\frac{y_0}{a}$ fixed, would permute three values of each of the other variables $\frac{z_0}{a}$, $\frac{y_1}{a}$, $\frac{z_1}{a}$. Thus twenty-seven values of $\frac{2\pi a V \epsilon}{y_0}$ were obtained from each data tape in only about ten minutes of machine time. As each set of answers was obtained it was hoped that definite trends would become apparent which would assist the iteration process. This seemed preferable to writing an iteration programme for the machine since the relative importance of the variables was not known nor was the size of step suitable for iteration.

It would seem possible, intuitively, to predict roughly the paths of the vortices with increasing incidence. The effect, in the Mangler

and Smith solution, of adding vorticity outboard of the main vortex core was to move the core inboard for a given height relative to the position estimated by Brown & Michael. This is the correct trend and it was hoped that the present analysis would give similar results. Also, for a real physical interpretation, the point vortex representing the sheet will be situated between the main vortex and the tip and probably closer to the surface as in fig. 8d. Mangler & Smith's vorticity distribution on the sheet suggests that the "centre of gravity" of the vorticity distribution is not likely to move far from $\frac{y}{a} = 1$ but that its height may well be comparable with the height of the main vortex.

A value of $\frac{y_0}{a} = 0.95$ was chosen for a first attempt at a solution. It seemed desirable to start fairly close to the tip as the range of the variables would then be smaller. For this value of $\frac{y_0}{a}$ the range of the other variables was as follows:

$$\begin{aligned} 0 < \frac{z_0}{a} &\leq 0.05 && (\text{Brown \& Michael value } 0.035) \\ 0.94 &\leq \frac{y_1}{a} \leq 0.9995 \\ 0 < \frac{z_1}{a} &\leq 0.12 \end{aligned}$$

In all about two hundred different combinations were tried but in no case was the desired result achieved or even approached. It was possible to obtain any three values of $\frac{2\pi a V_\infty}{\gamma_0}$ approximately equal but the fourth was always much lower. Similar results were obtained for $\frac{y_0}{a}$ as low as 0.80... It was concluded that, for some unknown reason, no physically real solution to the problem exists.

In order to obtain a solution it was necessary to relax the boundary

conditions somewhat. In other solutions forces have been found to act both on the line vortices and on the sheet but this is considered acceptable provided the component forces balance and there is no overall force in the system. If this reasoning is applied to the present problem the smallest relaxation of the original boundary conditions is to permit forces to act on the vortices i.e., main vortex and vortex representing the sheet, provided they are equal and opposite and provided also that they act only in the horizontal plane and not in the lift direction.

Thus instead of equation 6 and its equivalent for the smaller vortex we have:

$$\begin{aligned} u_0 &= \frac{y_0}{a} \cdot V \cdot \epsilon. \\ u_1 &= \frac{y_1}{a} \cdot V \cdot \epsilon. \\ \left(v_0 - \frac{z_0}{a} V \epsilon \right) \rho \gamma_0 &+ \left(v_1 - \frac{z_1}{a} V \epsilon \right) \rho \gamma_1 = 0 \end{aligned} \quad \begin{array}{l}) \\) \\) \end{array} \quad (14)$$

Only a small modification to the computer programme was necessary with the new boundary conditions to enable suitable values of $\frac{y_0}{a}$, $\frac{z_0}{a}$, $\frac{y_1}{a}$ and $\frac{z_1}{a}$ to be found.

This seemingly small relaxation of the boundary conditions gave rise to an apparently infinite number of combinations which satisfied equations 14. For a given value of $\frac{y_0}{a}$ it was possible to find many combinations of $\frac{z_0}{a}$, $\frac{y_1}{a}$, $\frac{z_1}{a}$ and $\frac{2\pi a V \epsilon}{\gamma_0}$ satisfying the boundary conditions. The results for $\frac{y_0}{a} = 0.95$ are plotted in figs. 9a and 9b. In fig. 9a the effective velocities at the respective vortex cores - $\left(\frac{v_0}{V \epsilon} - \frac{z_0}{a} \right)$ and $\frac{y_1}{y_0} \left(\frac{v_1}{V \epsilon} - \frac{z_1}{a} \right)$ are plotted against $\frac{y_1}{a}$, the spanwise position of the smaller

vortex, for various values of $\frac{z_0}{a}$. The common value of the velocity is plotted against $\frac{z_0}{a}$ in fig. 9b. This seems to support the conclusion already reached that the only solution in the regions considered is the trivial one $y_0 = 0$.

In order to choose a suitable answer from the multitude available consider the trends shown in figs. 9a and 9b. Although there is no resultant force at any point on the curve shown in fig. 9b it is clearly undesirable that the component forces should be very large and should preferably be of small order compared with the total lift. This condition sets an approximate upper limit to the value of $\frac{z_0}{a}$.

Another condition could probably be obtained from consideration of the stability of the two vortices but no simple stability criteria could be obtained from the equations. Consideration of the numerical results showed that if one vortex was displaced a small amount the forces induced on it were stabilizing in that they tended to return the displaced vortex to its original position but forces were induced on the fixed vortex and were destabilizing since they tended to move the vortex away from its original position. Thus it is not possible to assess the stability of the system but clearly any movements which introduce appreciable forces are undesirable. From figs. 9a and 9b it is clear that the out-of-balance forces increase with decreasing vortex height, thus setting an approximate lower limit to the value of $\frac{z_0}{a}$.

Also from fig. 9a it is seen that the out-of-balance forces due to displacement become very small for $\frac{z_0}{a} = 0.0204$ which (fig. 9b) is just

before the very steep rise of the curve. Thus in this particular case ($\frac{y_0}{a} = 0.95$) a value of $\frac{z_0}{a}$ of 0.0204 satisfies the two conditions as closely as possible and has been taken as the criterion for obtaining values of $\frac{z_0}{a}$ for other values of $\frac{y_0}{a}$. The exact point at which the two conditions are best observed is not very clearly defined and is, to some small extent, arbitrary but the possible variation on lift and vortex position is acceptably small.

The spacial variation of the positions of the vortices and the change of height with incidence is shown in fig. 36. The agreement between theory and experiment for the spacial variation of the main vortex is good except at small incidence where there is possibly some experimental uncertainty. The predicted height is somewhat above actual values which appear to lie between the present theory and that of Brown & Michael. As expected the position of the smaller vortex is outboard of, and lower than, the main vortex.

The lift and pressure distribution have been calculated from expressions similar to those used by Brown & Michael ref. 20, and are given in appendices VII and VIII. Direct comparison of theoretical and experimental pressure distributions is possible since it is expected that conical flow conditions obtain near the apex in the real flow. With regard to lift comparisons however it was observed earlier (§ 3.2) that theory will always overestimate the lift since no account is taken of the Kutta-Joukowski condition at the trailing-edge. It is very difficult to estimate this effect in general but with the particular cropped delta configuration tested the problem is somewhat easier.

In the theoretical analyses abrupt changes may occur in the loading of the wing. For instance, the R.T. Jones attached flow theory shows that the linear lift falls suddenly to zero if the sweep of the leading edge becomes 90° and assumptions of conical flow mean that the loading increases right up to the trailing edge. In practice of course sharp changes are smoothed out by viscous effects and the zero load condition at the trailing edge ensures a decreasing load over the rear part of the wing.

Undoubtedly the majority of the lift on the present model is produced on the forward tapered part of the wing where flow conditions are approximately conical but the satisfaction of the no load condition at the trailing edge results in the "spreading" of the lift from the highly loaded forward sections on to the more lightly loaded rear sections.

The centre of area of the present wing is at $0.64 c_o$ and with conical loading this would be the position of the centre of pressure. In fact the position is at $0.52 c_o$ and this together with the observations above on theoretical models, suggests that a fairer comparison will be obtained between theoretical and experimental values of lift for the present model, if a theoretical lift distribution is assumed in which the forward swept part of the wing produces lift in accordance with conical flow dictates and the rest of the wing (with streamwise edges) is unloaded. In this case the centre of pressure would be at $0.45 c_o$ and the "spreading" of lift occurring in practice would account for the rearward movement of the centre of pressure to the experimental

value of $0.52 c_o$.

As usual the experimental lift coefficients have been calculated using the total wing area but in the comparisons made here all the theoretical lift coefficients are one half of their calculated value following the above argument.

The present experimental values of lift are compared with one half of the present theoretical values in fig. 11. Also shown are one half of the theoretical values of Kuchemann, Brown & Michael and Mangler & Smith. Even with the proposed basis of comparison the present theoretical work overestimates the lift by nearly 20% at the higher incidences while the values of Brown & Michael are greater by only 5%. Mangler & Smith predict lower values while Kuchemann's semi-empirical results agree better with experiment if compared on the basis of equal wing areas. The theoretical and experimental results all tend to one half of the R.T. Jones value near zero incidence. Although agreement between theory and experiment is improved by making some allowance for the trailing edge effect the results are still empirical and the present basis of agreement can only be used with composite planforms.

Comparison of theoretical and experimental pressure distributions is made in fig. 35. The theories of Brown & Michael and Mangler & Smith have rather sharp narrow peaks well outboard of the experimental position. Actually the experimental case with blowing is much closer to these theoretical predictions since blowing increases the vortex strength at a given incidence and eliminates the secondary separation.

With the present theoretical results it is interesting to note that although the predicted peak is inboard of the experimental one the general shape is in much better agreement. The peak is no longer narrow and outboard of it there is a region of more constant pressure.

This agreement is rather fortuitous since the model chosen ignores the secondary separation .

As with the results of Mangler and Smith the addition of vorticity outside of the main vortex core, representing the sheet, moves the core inwards at a given height relative to the Brown and Michael values. With the present assumptions the proportion of vorticity in the sheet to vorticity in the core is about 0.6 for $\alpha/K = 1.0$, compared with Mangler and Smith's value of 0.2. Since the two models are roughly comparable better agreement might have been expected but it was not possible to test the effect of varying the basic assumptions and the difference cannot be attributed to a particular effect.

4.2. Extension of the theory to predict the effects of conical blowing in the plane of the wing.

It seems clear from the results of pressure plotting and flow visualisation (see § 6.8) that the flow field is changed in detail rather than fundamentally when a thin sheet of high velocity air is discharged in the plane of the wing from a swept edge. Blowing increases the strength of the vortex sheet shed from the edge but the subsequent rolling up of the vortex sheet to form the characteristic spiral changes but little. The most obvious change in the flow pattern from

the unblown case is the elimination of the secondary separation due to jet entrainment. Thus the real flow with blowing corresponds more closely to the theoretical (unblown) model in which the secondary separation is neglected.

With blowing the flow will still separate smoothly at the leading edge with the condition $v = 0$ but clearly we cannot have $u = 0$. However, most of the current theories allow violations of the boundary conditions in small regions provided they are satisfied overall and it seems unlikely that this will have a serious effect. Conical flow is maintained with blowing provided the momentum ejected increases linearly with distance downstream from a zero value at the apex.

Boundary condition (c) is changed with the application of blowing. In general there will be a pressure difference across the jet sheet in order to balance the radial component of momentum.

From ref. 45 we have for a thin two-dimensional jet sheet:

$$\frac{d\gamma}{dS} = \frac{\mu'}{\rho V R} \quad (15)$$

where γ is the local strength of the sheet, S the arc length, μ' the jet momentum, and R the radius of curvature. Conditions will be approximately two-dimensional in a plane normal to the vortex sheet and since the radius of curvature R is assumed constant in this plane and both μ' and R are proportional to x , $\frac{d\gamma}{dS}$ is constant.

If γ_{0j} is the strength of the main vortex due to blowing we have:

$$\frac{d\gamma_{0j}}{dS} = \frac{\mu' \cos\theta}{\rho V R} \quad \text{where } \mu' \cos\theta \text{ is the momentum in the cross-flow plane and the other component } \mu' \sin\theta \text{ is}$$

assumed to show up as direct jet thrust. R is also measured in the cross-flow plane.

Both with and without blowing, vorticity is assumed to be fed into the core at a constant rate and removed along the axis downstream at the same rate such that a steady state is reached. Thus the rate of increase of vorticity in the core with distance downstream is equal to the rate of change of strength along the feeding sheet in the plane normal to the vortex axis, $\frac{dy_{oj}}{ds}$. It should be noted that this was tacitly assumed by Brown & Michael.

$$\therefore y_{oj} = \int_0^x \frac{\mu \cos \theta}{\epsilon VR} ds$$

$$\therefore y_{oj} = \frac{x \mu \cos \theta}{\epsilon VR}$$

$$\therefore \frac{y_{oj}}{2\pi a V \epsilon} = \frac{C_{\mu} \cos \theta}{4\pi \epsilon^2 B/a} = \frac{C_{\mu} \cos \theta}{4\pi \gamma_1 \gamma_0} \quad (16)$$

$$\text{Where } C_{\mu} = \frac{\mu}{\frac{1}{2} \rho V^2 a}$$

It should be noted that the theoretical value of C_{μ} is twice the experimental value since, as with lift, only the swept part of the wing is considered in the theory.

Thus for a given set of conditions it is possible to calculate the increase in vortex strength due to blowing.

The general effect of blowing on the lift and movement of the vortices, apart from the removal of the secondary separation at very small C_{μ} , is typical of an increase of incidence and in order to estimate its effects it is considered equivalent to a change of incidence. Increased

vorticity, with blowing, will be fed into the main vortex and the effects of the jet sheet and the normal vortex sheet will be intermingled in practice. Also the helix angle of the vortex lines will depend, initially at least, upon the angle of blowing. It would be extremely difficult to represent this intermingling in the theory so the effects will be considered separately.

The total lift on the wing with blowing is composed of a linear part and a non-linear part, with the non-linear part further sub-divided into the separate effects of blowing and incidence. Now since blowing primarily affects the strength of the leading edge vortices it may be considered to affect only the non-linear part of the lift. Therefore at a given incidence with blowing we have:

$$\frac{\gamma_0}{2\pi a V_\infty} = \frac{\gamma_0 (C\mu = 0)}{2\pi a V_\infty} + \frac{\gamma_{0j}}{2 a V_\infty} \quad (17)$$

Hence we may infer that the non-linear lift on a wing with blowing at a given incidence is the same as that on an unblown wing at a greater incidence.

To find the change of incidence due to blowing the procedure is to fix $C\mu$ and to choose a suitable value of incidence α_1 , say. $\frac{\gamma_{0j}}{2\pi a V_\infty}$ is then calculated from equation 16 using the value for $\frac{\gamma_1}{\gamma_0}$ at α_1 . $\frac{\gamma_1}{\gamma_0}$ is found from equation 13 using fig. 36. Subtracting the value of $\frac{\gamma_{0j}}{2\pi a V_\infty}$ from $\frac{\gamma_0}{2\pi a V_\infty}$ at α_1 (found from fig. 10) gives another value of $\frac{\gamma_0}{2\pi a V_\infty}$ which corresponds to an incidence, α_2 say, from fig. 10. Then the non-linear lift at α_2 with blowing, is equal to the non-linear lift at α_1 without blowing. To find the total lift at α_2 with blowing it

is necessary to add the linear lift at α_2 since this is not affected by the blowing.

A comparison is made in figs 12 and 13 between the experimental results with blowing for jet sweep angles $\theta = 0^\circ$ and 50° and the corresponding theoretical values calculated by the above method. The quantitative agreement is good only for small incidence and C_μ but the increasing deviation between theory and experiment with increasing α and C_μ is due largely to the poor agreement between the unblown theoretical and experimental results. For example, with $\theta = 0^\circ$ and $\alpha \approx 15^\circ$ the incidence change due to blowing for experimental C_μ values of 0.048 and 0.096 is 2.75° and 4.5° respectively, the corresponding theoretical values being 2.25° and 4.75° .

Owing to the various assumptions made for the case with blowing it is clear that good agreement between theory and experiment is likely only if C_μ values are low and the incidence high i.e., where the contribution due to blowing is not too large compared with the unblown lift. In practice of course this will be the case as landing incidences are about 15° and C_μ values unlikely to exceed 0.03 and given the unblown lift values and vortex positions, the increase in lift due to blowing can be calculated with reasonable accuracy.

4.3 Conclusions

An attempt has been made to predict theoretically the lift and position of the leading edge vortex of slender delta wings without blowing. A simple extension of the theory enables the increase in lift

due to "conical" leading edge blowing to be assessed correctly.

The use of a second line vortex to represent the vorticity outside the core was successful in that the spacial variation of the main vortex with incidence is predicted with reasonable accuracy. The theoretical estimate of vortex height at a given incidence is somewhat greater than the experimental values, which fall roughly midway between the present theoretical results and those of Brown & Michael. Sectional values of lift are also overestimated and as with any theory neglecting trailing edge effects the total lift is much greater than the experimental lift results. Even when an allowance is made for trailing edge effects (see §4.1) the present theory still gives results about 20% greater than experiment.

The pressure distribution obtained from the present theory is in better general agreement with experiment than previous theories although the appearance of a roughly constant pressure region outboard of the main vortex must be regarded as fortuitous since the effects of the secondary separation are not included.

It was not possible to test the effect of varying some of the assumptions made although it would be useful to know how sensitive are the final answers to a possible variation. For instance, although the present model is roughly comparable to that of Mangler & Smith, the ratio of vorticity in the sheet to that in the core is about 0.6 for $\alpha/K \approx 1$ while Mangler and Smith's value is only 0.2. The helix angle of the vortex lines may well play an important part in determining this ratio.

Again, the assumed shape of sheet is not entirely satisfactory, bearing in mind that the semi-circle chosen is closer to the case with blowing than without. However, it is likely that the shape of the sheet is one of the less important factors in determining the vorticity ratio and it is questionable whether sophisticated sheet shapes are advantageous in such a relatively crude theory.

Finally it is desirable to have a more rigorous condition to fix the height of the main vortex at a given spanwise position. Working with the original equations is difficult but possibly some simplification could be devised along the lines of Brown & Michael's (ref. 20) second order approximation which enables them to obtain an analytic solution to two rather similar equations close to the leading edge.

Appendix I

Equation 2' is differentiated to find a general expression for the complex velocity.

$$\begin{aligned}
 -u+iv = \frac{dW(\sigma)}{d\sigma} &= \frac{iV_0\sigma}{\sqrt{\sigma^2-a^2}} + \frac{iy_0}{2\pi} \cdot \frac{\sigma}{\sqrt{\sigma^2-a^2}} \left(\frac{1}{\sqrt{\sigma^2-a^2} - \sqrt{\sigma_0^2-a^2}} - \frac{1}{\sqrt{\sigma^2-a^2} + \sqrt{\sigma_0^2-a^2}} \right) \\
 &+ \frac{iy_1}{2\pi} \cdot \frac{\sigma}{\sqrt{\sigma^2-a^2}} \left(\frac{1}{\sqrt{\sigma^2-a^2} - \sqrt{\sigma_1^2-a^2}} - \frac{1}{\sqrt{\sigma^2-a^2} + \sqrt{\sigma_1^2-a^2}} \right)
 \end{aligned}$$

and with $\sigma = y+iz$

$$\sqrt{\sigma^2-a^2} = A+iB$$

$$\sqrt{\sigma_0^2-a^2} = A_0+iB_0$$

$$\sqrt{\sigma_1^2-a^2} = A_1+iB_1$$

$$\begin{aligned}
 -u+iv = \frac{i(y+iz)}{A+iB} &\left[V_0 + \frac{y_0}{2\pi} \left(\frac{1}{A+iB - A_0-iB_0} - \frac{1}{A+iB + A_0-iB_0} \right) \right. \\
 &\left. + \frac{y_1}{2\pi} \left(\frac{1}{A+iB - A_1-iB_1} - \frac{1}{A+iB + A_1-iB_1} \right) \right]
 \end{aligned}$$

$$\text{Now } \frac{1}{(A-A_0)+i(B-B_0)} - \frac{1}{(A+A_0) + i(B-B_0)}$$

$$= \frac{(A-A_0) - i(B-B_0)}{(A-A_0)^2 + (B-B_0)^2} - \frac{(A+A_0) - i(B-B_0)}{(A+A_0)^2 + (B-B_0)^2}$$

$$\text{Also } \frac{i(y+iz)}{A+iB} = \frac{i(y+iz)(A-iB)}{(A^2 + B^2)} = \frac{(By-Az) + i(Ay+Bz)}{(A^2 + B^2)}$$

$$\begin{aligned}
\therefore -u+iv &= \frac{(By-Az)+i(Ay+Bz)}{(A^2+B^2)} \left[v\alpha + \frac{\gamma_0}{2\pi} \left(\frac{(A-A_0)-i(B-B_0)}{(A-A_0)^2+(B-B_0)^2} - \frac{(A+A_0)-i(B-B_0)}{(A+A_0)^2+(B-B_0)^2} \right) \right. \\
&\quad \left. + \frac{\gamma_1}{2\pi} \left(\frac{(A-A_1)-i(B-B_1)}{(A-A_1)^2+(B-B_1)^2} - \frac{(A+A_1)-i(B-B_1)}{(A+A_1)^2+(B-B_1)^2} \right) \right] \\
&= \frac{(By-Az)+i(Ay+Bz)}{(A^2+B^2)} \left[v\alpha + \frac{\gamma_0}{2\pi} \left\{ \frac{(A-A_0)}{(A-A_0)^2+(B-B_0)^2} - \frac{(A+A_0)}{(A+A_0)^2+(B-B_0)^2} \right\} \right. \\
&\quad \left. + \frac{\gamma_1}{2\pi} \left\{ \frac{(A-A_1)}{(A-A_1)^2+(B-B_1)^2} - \frac{(A+A_1)}{(A+A_1)^2+(B-B_1)^2} \right\} \right] + \frac{(Ay+Bz)-i(By-Az)}{A^2+B^2} \times \\
&\quad \left[\frac{\gamma_0}{2\pi} \left\{ \frac{(B-B_0)}{(A-A_0)^2+(B-B_0)^2} - \frac{(B-B_0)}{(A+A_0)^2+(B-B_0)^2} \right\} + \frac{\gamma_1}{2\pi} \left\{ \frac{(B-B_1)}{(A-A_1)^2+(B-B_1)^2} - \right. \right. \\
&\quad \left. \left. \frac{(B-B_1)}{(A+A_1)^2+(B-B_1)^2} \right\} \right]
\end{aligned}$$

Appendix II

An expression is found for the velocity at the leading edge.

Clearly A & $B \rightarrow 0$ as $y \rightarrow \pm a$, $z \rightarrow 0$.

Also A_0, A_1, B_0, B_1 are constant at constant incidence.

Consider the leading edge $y = a$ and let $y - a = r \cos \theta'$ and $z = r \sin \theta'$.

$$A + iB = \sqrt{\sigma^2 - a^2} = \sqrt{y^2 - z^2 - a^2 + 2iyz}$$

$$\begin{aligned} \therefore A^2 - B^2 + 2AiB &= y^2 - z^2 - a^2 + 2iyz \\ &= a^2 + 2ar \cos \theta' + r^2 \cos^2 \theta' - r^2 \sin^2 \theta' - a^2 + 2ir \sin \theta' (a + r \cos \theta') \end{aligned}$$

$$\therefore A^2 - B^2 = r(r \cos 2\theta' + 2a \cos \theta') \quad AB = r \sin \theta' (a + r \cos \theta')$$

$$\therefore A^4 - r(r \cos 2\theta' + 2a \cos \theta') A^2 - r^2 \sin^2 \theta' (a + r \cos \theta')^2 = 0$$

$$\begin{aligned} \therefore A^2 &= \frac{r(r \cos 2\theta' + 2a \cos \theta') + \sqrt{r^2 (r^2 \cos^2 2\theta' + 4ar \cos \theta' \cos 2\theta' + 4a^2 \cos^2 \theta') + 4r^2 \sin^2 \theta' (a + r \cos \theta')^2}}{2} \\ &= \frac{r^2 (\cos^2 \theta' - \sin^2 \theta') + 2ar \cos \theta' + \sqrt{4a^2 r^2 + o(r^3)}}{2} \end{aligned}$$

$\rightarrow ar(1 + \cos \theta')$ as $r \rightarrow 0$ retaining only the highest order terms

$$\therefore A \rightarrow \sqrt{ar(1 + \cos \theta')} \rightarrow 0 \text{ as } \sqrt{r}$$

$$B \rightarrow \frac{r \sin \theta' (a + r \cos \theta')}{\sqrt{ar(1 + \cos \theta')}} \rightarrow \sqrt{\frac{ar}{1 + \cos \theta'}} \rightarrow 0 \text{ as } \sqrt{r}$$

$$\text{Consider } \frac{A_y}{A^2 + B^2} = O\left(\frac{\sqrt{r} \cdot a}{r}\right) \rightarrow \infty \text{ as } \frac{1}{\sqrt{r}} \text{ as } \sigma \rightarrow a$$

$$\frac{B_z}{A^2 + B^2} = O\left(\frac{\sqrt{r} \cdot r}{r}\right) \rightarrow 0 \text{ as } \sqrt{r} \text{ as } \sigma \rightarrow a.$$

Now let $\sigma \rightarrow a$ and retain only the highest order terms

$$-u+iv = \frac{1}{A^2+B^2} \left[B_y \left\{ V\alpha - \frac{Y_0}{2\pi} \cdot \frac{2A_0}{A_0^2+B_0^2} - \frac{Y_1 2A_1}{2\pi A_1^2+B_1^2} \right\} \right. \\ \left. + iA_y \left\{ V\alpha - \frac{Y_0}{2\pi} \cdot \frac{2A_0}{A_0^2+B_0^2} - \frac{Y_1}{2\pi} \cdot \frac{2A_1}{A_1^2+B_1^2} \right\} \right]$$

Appendix III

An expression is found for the velocity at the point σ_0

$$\begin{aligned}
 -u_x + iv_y &= \frac{dW_0(\sigma)}{d\sigma} = \frac{dW}{d\sigma} - \frac{i\gamma_0/2\pi}{\sigma - \sigma_0} \\
 &= \frac{iV\alpha\sigma}{\sqrt{\sigma^2 - a^2}} + \frac{i\gamma_0}{2\pi} \cdot \frac{\sigma}{\sqrt{\sigma^2 - a^2}} \left(\frac{1}{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_0^2 - a^2}} - \frac{1}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) \\
 &\quad - \frac{i\gamma_0}{2\pi} \cdot \frac{1}{\sigma - \sigma_0} + \frac{i\gamma_0}{2\pi} \cdot \frac{\sigma}{\sqrt{\sigma^2 - a^2}} \left(\frac{1}{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_1^2 - a^2}} - \frac{1}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right)
 \end{aligned}$$

as $\sigma \rightarrow \sigma_0$

$$\begin{aligned}
 \frac{dW_0}{d\sigma} &= \frac{iV\alpha\sigma_0}{\sqrt{\sigma_0^2 - a^2}} - \frac{i\gamma_0}{2\pi} \cdot \frac{\sigma_0}{\sqrt{\sigma_0^2 - a^2}} \left(\frac{1}{\sqrt{\sigma_0^2 - a^2} - \sqrt{\sigma_0^2 - a^2}} + \frac{i\gamma_0}{2\pi} \times \right. \\
 &\quad \left. \left(\frac{\sigma}{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_0^2 - a^2}} - \frac{1}{\sigma - \sigma_0} \right) \right)_{\sigma \rightarrow \sigma_0}
 \end{aligned}$$

$$+ \frac{i\gamma_0}{2\pi} \cdot \frac{\sigma_0}{\sqrt{\sigma_0^2 - a^2}} \left(\frac{1}{\sqrt{\sigma_0^2 - a^2} - \sqrt{\sigma_1^2 - a^2}} - \frac{1}{\sqrt{\sigma_0^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right)$$

$$\text{Consider } \left(\frac{\sigma}{\sqrt{\sigma_0^2 - a^2} - \sqrt{\sigma^2 - a^2}} - \frac{1}{\sigma - \sigma_0} \right)_{\sigma \rightarrow \sigma_0}$$

$$= \left(\frac{a^2 - \sigma\sigma_0 + \sqrt{(\sigma^2 - a^2)(\sigma_0^2 - a^2)}}{(\sigma - \sigma_0)(\sigma^2 - a^2 - \sqrt{(\sigma^2 - a^2)(\sigma_0^2 - a^2)})} \right)_{\sigma \rightarrow \sigma_0}$$

$$= \left(\frac{\sigma \sqrt{\sigma_0^2 - a^2}}{(\sigma^2 - a^2 - \sqrt{(\sigma^2 - a^2)(\sigma_0^2 - a^2)}) + (\sigma - \sigma_0)(2\pi - \frac{\sigma}{\sqrt{\sigma^2 - a^2}} \sqrt{\sigma_0^2 - a^2})} \right)_{\sigma \rightarrow \sigma_0}$$

using L'hospitals rule.

$$= \left(\frac{\sqrt{\sigma_0^2 - a^2} \left(\frac{\sigma^2}{\sqrt{\sigma^2 - a^2}} - \sqrt{\sigma^2 - a^2} \right)}{\left(2\sigma - \sigma \frac{\sigma_0^2 - a^2}{\sigma^2 - a^2} \right) + \left(2\sigma - \sigma \frac{\sigma_0^2 - a^2}{\sigma^2 - a^2} \right) + (\sigma - \sigma_0) \left(2 - \sqrt{\sigma_0^2 - a^2} \left(\frac{-a^2}{(\sigma^2 - a^2)^{3/2}} \right) \right)} \right) \sigma_+ \sigma_0$$

using L'hospitals rule.

$$= \left(\frac{\frac{-a^2}{\sigma^2 - a^2}}{\frac{\sigma}{\sigma + \sigma_0}} \right) \sigma_+ \sigma_0 = \frac{-a^2}{2\sigma_0(\sigma_0^2 - a^2)}$$

Appendix IV

Two expressions are found for the main vortex strength.

Substituting for V_a from equ (4) in eqs. (5) and (6) we have:

$$-V_a \frac{\gamma_0}{a} = \frac{2i(y_0+izo)}{A_0+iB_0} \left\{ \frac{\frac{\gamma_0 A_0}{2\pi}}{A_0^2+B_0^2} + \frac{\frac{\gamma_0 A_1}{2\pi}}{A_1^2+B_1^2} \right\} - \frac{i \frac{\gamma_0}{2\pi} a^2}{2(y_0+izo)(A_0^2-B_0^2+2iA_0B_0)}$$

$$- \frac{i \frac{\gamma_0}{2\pi}}{A_0+iB_0} \frac{(y_0+izo)}{2A_0} + \frac{i \gamma_0}{2\pi} \frac{y_0+izo}{A_0+iB_0} \left\{ \frac{1}{A_0+iB_0-A_1-iB_1} - \frac{1}{A_0+iB_0+A_1-iB_1} \right\}$$

$$(a) \frac{i(y_0+izo)}{A_0+iB_0} = \frac{(-Z_0+iy_0)(A_0-iB_0)}{A_0^2+B_0^2} = \frac{Boyo-Aozo+i(A_0yo+Bozo)}{A_0^2+B_0^2}$$

$$(b) \frac{i}{(y_0+izo)(A_0^2-B_0^2+2iA_0B_0)} = \frac{i(y_0-izo)(A_0^2-B_0^2-2iA_0B_0)}{(y_0^2+z_0^2)(A_0^2+B_0^2)^2} =$$

$$\frac{zo(A_0^2-B_0^2)+2A_0Boyo+iy_0(A_0^2-B_0^2)-2iA_0Bozo}{(y_0^2+z_0^2)(A_0^2+B_0^2)^2}$$

$$(c) \frac{i(y_0+izo)}{A_0+iB_0} \left\{ \frac{1}{A_0+iB_0-A_1-iB_1} - \frac{1}{A_0+iB_0+A_1-iB_1} \right\}$$

$$= \frac{(Boyo-Aozo)+i(A_0yo+Bozo)}{A_0^2+B_0^2} \left\{ \frac{(A_0-A_1)-i(B_0-B_1)}{(A_0-A_1)^2+(B_0-B_1)^2} - \frac{(A_0+A_1)-i(B_0-B_1)}{(A_0+A_1)^2+(B_0-B_1)^2} \right\}$$

$$= \left[\frac{(A_0-A_1)(Boyo-Aozo)+(B_0-B_1)(A_0yo+Bozo)}{(A_0^2+B_0^2)[(A_0-A_1)^2+(B_0-B_1)^2]} - \right.$$

$$\left. \frac{(A_0+A_1)(Boyo-Aozo)+(B_0-B_1)(A_0yo+Bozo)}{(A_0^2+B_0^2)[(A_0+A_1)^2+(B_0-B_1)^2]} \right]$$

$$+ i \left[\frac{(A_0-A_1)(A_0yo+Bozo)-(B_0-B_1)(Boyo-Aozo)}{(A_0^2+B_0^2)[(A_0-A_1)^2+(B_0-B_1)^2]} - \right.$$

$$\left. \frac{(A_0+A_1)(A_0yo+Bozo)-(B_0-B_1)(Boyo-Aozo)}{(A_0^2+B_0^2)[(A_0+A_1)^2+(B_0-B_1)^2]} \right]$$

$$\therefore -V_{\epsilon y_0} = \frac{2(Boyo-Aozo)}{Ao^2+Bo^2} \left\{ \frac{\gamma_0 Ao}{2\pi} + \frac{\gamma_1 A_1}{2\pi} \right\} -$$

$$\frac{\gamma_0}{2\pi} \cdot \frac{(Boyo-Aozo)}{Ao^2+Bo^2} - \frac{\gamma_0 z_0^2}{2} \cdot \frac{z_0(A_o^2 - B_o^2) + 2A_o B_o y_0}{(y_o^2 + z_o^2)(A_o^2 + B_o^2)^2}$$

$$+ \frac{\gamma_1}{2\pi} \left[\frac{(Ao-A_1)(Boyo-Aozo) + (Bo-B_1)(Aoyo+Bozo)}{(Ao^2+Bo^2)[(Ao-A_1)^2 + (Bo-B_1)^2]} - \right.$$

$$\left. \frac{(Ao+A_1)(Boyo-Aozo) + (Bo-B_1)(Aoyo+Bozo)}{(Ao^2+Bo^2)[(Ao+A_1)^2 + (Bo-B_1)^2]} \right]$$

$$V_{\epsilon} \frac{z_0}{a} = \frac{2(Aoyo+Bozo)}{Ao^2+Bo^2} \left\{ \frac{\gamma_0 Ao}{2\pi} + \frac{\gamma_1 A_1}{2\pi} \right\} - \frac{\gamma_0 z_0^2}{2} \cdot \frac{y_0(A_o^2 - B_o^2) - 2A_o B_o z_0}{(y_o^2 + z_o^2)(A_o^2 + B_o^2)^2}$$

$$- \frac{\gamma_0}{2\pi} \frac{(Aoyo+Bozo)}{Ao^2 + Bo^2}$$

$$+ \frac{\gamma_1}{2\pi} \left[\frac{(Ao-A_1)(Aoyo+Bozo) - (Bo-B_1)(Boyo-Aozo)}{(Ao^2+Bo^2)[(Ao-A_1)^2 + (Bo-B_1)^2]} - \right.$$

$$\left. \frac{(Ao+A_1)(Aoyo+Bozo) - (Bo-B_1)(Boyo-Aozo)}{(Ao^2+Bo^2)[(Ao+A_1)^2 + (Bo-B_1)^2]} \right]$$

Appendix V

Expressions are found for the velocity and vortex strength at the point σ_1

$$\begin{aligned} \frac{dW_1(\sigma)}{d\sigma} &= \frac{dW(\sigma)}{d\sigma} - \frac{i\gamma_1}{2\pi} \cdot \frac{1}{\sigma - \sigma_1} \\ &= iV\alpha \frac{\sigma}{\sqrt{\sigma^2 - a^2}} + \frac{i\gamma_0}{2\pi} \frac{\sigma}{\sqrt{\sigma^2 - a^2}} \left(\frac{1}{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_0^2 - a^2}} - \frac{1}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) \\ &\quad + \frac{i\gamma_1}{2\pi} \left\{ \frac{\sigma}{\sqrt{\sigma^2 - a^2}} \left(\frac{1}{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_1^2 - a^2}} - \frac{1}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right) - \frac{1}{\sigma - \sigma_1} \right\} \\ -u_1 + iv_1 &= \frac{dW_1(\sigma)}{d\sigma} \Big|_{\sigma=\sigma_1} = iV\alpha \frac{\sigma_1}{\sqrt{\sigma_1^2 - a^2}} + \frac{i\gamma_0}{2\pi} \frac{\sigma_1}{\sqrt{\sigma_1^2 - a^2}} \times \\ &\quad \left(\frac{1}{\sqrt{\sigma_1^2 - a^2} - \sqrt{\sigma_0^2 - a^2}} - \frac{1}{\sqrt{\sigma_1^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) \\ &\quad - \frac{i\gamma_1}{2\pi} \cdot \frac{\sigma_1}{\sqrt{\sigma_1^2 - a^2}} \left(\frac{1}{\sqrt{\sigma_1^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right) - \frac{i\gamma_1}{2\pi} \times \\ &\quad \frac{a^2}{2\sigma_1(\sigma_1^2 - a^2)} \end{aligned}$$

The last term was obtained in appendix III.

$$\begin{aligned} \therefore -V_\infty \left(\frac{y_1}{a} - \frac{iz_1}{a} \right) &= \frac{iV\alpha(y_1 + iz_1)}{A_1 + iB_1} \\ &\quad + \frac{i\gamma_0}{2\pi} \cdot \frac{(y_1 + iz_1)}{A_1 + iB_1} \left(\frac{1}{A_1 + iB_1 - A_0 - iB_0} - \frac{1}{A_1 + iB_1 + A_0 - iB_0} \right) \\ &\quad - \frac{i\gamma_1}{2\pi} \left(\frac{y_1 + iz_1}{2A_1(A_1 + iB_1)} + \frac{a^2}{2(y_1 + iz_1)(A_1 + iB_1)^2} \right) \end{aligned}$$

$$\text{Now } \frac{i(y_1 + iz_1)}{A_1 + iB_1} = \frac{(-Z_1 + iy_1)(A_1 - iB_1)}{A_1^2 + B_1^2} = \frac{(B_1 y_1 - A_1 z_1) + i(A_1 y_1 + B_1 z_1)}{A_1^2 + B_1^2}$$

$$\frac{1}{A_1 + iB_1 - A_0 - iB_0} = \frac{(A_1 - A_0) - i(B_1 - B_0)}{(A_1 - A_0)^2 + (B_1 - B_0)^2}$$

$$\frac{1}{A_1 + iB_1 + A_0 - iB_0} = \frac{(A_1 + A_0) - i(B_1 - B_0)}{(A_1 + A_0)^2 + (B_1 - B_0)^2}$$

$$\begin{aligned} \frac{i a^2}{2(y_1 + iz_1)(A_1 + iB_1)^2} &= \frac{i(y_1 - iz_1)(A_1^2 - B_1^2 - 2i A_1 B_1)}{2(y_1^2 + z_1^2)(A_1^2 + B_1^2)^2} \\ &= \frac{z_1(A_1^2 - B_1^2) + 2A_1 B_1 y_1 - 2i A_1 B_1 z_1 + i x_1(A_1^2 - B_1^2)}{2(y_1^2 + z_1^2)(A_1^2 + B_1^2)^2} \end{aligned}$$

Separating real and imaginary parts we have:

$$\begin{aligned} -\frac{V_e y_1}{a} &= \frac{B_1 y_1 - A_1 z_1}{A_1^2 + B_1^2} \left\{ \frac{y_0}{2\pi} \cdot \frac{2A_0}{A_0^2 + B_0^2} + \frac{y_1}{2\pi} \cdot \frac{2A_1}{A_1^2 + B_1^2} \right\} \\ &+ \frac{y_0}{2\pi} \left[\frac{(A_1 - A_0)(B_1 y_1 - A_1 z_1) + (B_1 - B_0)(A_1 y_1 + B_1 z_1)}{(A_1^2 + B_1^2) \{ (A_1 - A_0)^2 + (B_1 - B_0)^2 \}} \right. \\ &\quad \left. \frac{(A_1 + A_0)(B_1 y_1 - A_1 z_1) + (B_1 - B_0)(A_1 y_1 + B_1 z_1)}{(A_1^2 + B_1^2) \{ (A_1 + A_0)^2 + (B_1 - B_0)^2 \}} \right] \\ &- \frac{y_1}{2\pi} \left[\frac{z_1(A_1^2 - B_1^2) + 2A_1 B_1 y_1}{2(y_1^2 + z_1^2)(A_1^2 + B_1^2)^2} + \frac{(B_1 y_1 - A_1 z_1)}{2A_1(A_1^2 + B_1^2)} \right] \end{aligned}$$

$$\begin{aligned} \text{and } V_e \frac{z_1}{a} &= \frac{(A_1 y_1 + B_1 z_1)}{A_1^2 + B_1^2} \left\{ \frac{y_0}{2\pi} \cdot \frac{2A_0}{A_0^2 + B_0^2} + \frac{y_1}{2\pi} \cdot \frac{2A_1}{A_1^2 + B_1^2} \right\} \\ &- \frac{y_0}{2\pi} \left[\frac{(B_1 - B_0)(B_1 y_1 - A_1 z_1) - (A_1 - A_0)(A_1 y_1 + B_1 z_1)}{(A_1^2 + B_1^2) \{ (A_1 - A_0)^2 + (B_1 - B_0)^2 \}} \right. \\ &\quad \left. - \frac{(B_1 + B_0)(B_1 y_1 - A_1 z_1) - (A_1 + A_0)(A_1 y_1 + B_1 z_1)}{(A_1^2 + B_1^2) \{ (A_1 + A_0)^2 + (B_1 - B_0)^2 \}} \right] \end{aligned}$$

$$\frac{(B_1 - B_0)(D_1 y_1 - A_1 z_1) - (A_1 + A_0)(A_1 y_1 + B_1 z_1)}{(A_1^2 + B_1^2) \left\{ (A_1 + A_0)^2 + (B_1 - B_0)^2 \right\}}$$

$$- \frac{y_1}{2\pi} \left[\frac{(A_1 y_1 + B_1 z_1)}{2A_1(A_1^2 + B_1^2)} + \frac{y_1(A_1^2 - B_1^2) - 2A_1 B_1 z_1}{2(y_1^2 + z_1^2)(A_1^2 + B_1^2)^2} \right]$$

$$\therefore \frac{-2\pi V a \epsilon}{y_0} \cdot \frac{y_1}{a} = \frac{\frac{A_0}{2a}}{\frac{A_0^2 + B_0^2}{a^2}} \cdot \frac{\frac{B_1 \cdot y_1}{a} \frac{A_1 \cdot z_1}{a}}{\frac{A_1^2}{a^2} + \frac{B_1^2}{a^2}} -$$

$$\left[\frac{\left(\frac{A_1 - A_0}{a} \right) \left(\frac{B_1 \cdot y_1}{a} - \frac{A_1 \cdot z_1}{a} \right) + \left(\frac{B_1 - B_0}{a} \right) \left(\frac{A_1 \cdot y_1}{a} + \frac{B_1 \cdot z_1}{a} \right)}{\left(\frac{A_1^2 + B_1^2}{a^2} \right) \left\{ \left(\frac{A_1 - A_0}{a} \right)^2 + \left(\frac{B_1 - B_0}{a} \right)^2 \right\}} - \right.$$

$$\left. \frac{\left(\frac{A_1 + A_0}{a} \right) \left(\frac{B_1 \cdot y_1}{a} - \frac{A_1 \cdot z_1}{a} \right) + \left(\frac{B_1 - B_0}{a} \right) \left(\frac{A_1 \cdot y_1}{a} + \frac{B_1 \cdot z_1}{a} \right)}{\left(\frac{A_1^2 + B_1^2}{a^2} \right) \left\{ \left(\frac{A_1 + A_0}{a} \right)^2 + \left(\frac{B_1 - B_0}{a} \right)^2 \right\}} \right]$$

$$- \frac{y_1}{y_0} \left[\frac{2 \frac{A_1}{a} \left(\frac{B_1 \cdot y_1}{a} - \frac{A_1 \cdot z_1}{a} \right)}{\left(\frac{A_1^2 + B_1^2}{a^2} \right)^2} - \frac{\frac{z_1}{a} \left(\frac{A_1^2 - B_1^2}{a^2} \right) + 2 \frac{A_1 \cdot B_1 \cdot y_1}{a^2}}{2 \left(\frac{y_1^2 + z_1^2}{a^2} \right) \left(\frac{A_1^2 + B_1^2}{a^2} \right)^2} - \frac{\frac{B_1 \cdot y_1}{a} \frac{A_1 \cdot z_1}{a}}{2 \frac{A_1}{a} \left(\frac{A_1^2 + B_1^2}{a^2} \right)} \right] = 0$$

$$\frac{2\pi V a \epsilon}{y_0} \cdot \frac{z_1}{a} = \frac{2 \frac{A_0}{a} \left(\frac{A_1 \cdot y_1}{a} + \frac{B_1 \cdot z_1}{a} \right)}{\left(\frac{A_0^2 + B_0^2}{a^2} \right) \left(\frac{A_1^2 + B_1^2}{a^2} \right)} + \frac{\left(\frac{B_1 - B_0}{a} \right) \left(\frac{B_1 \cdot y_1}{a} - \frac{A_1 \cdot z_1}{a} \right) - \left(\frac{A_1 - A_0}{a} \right) \left(\frac{A_1 \cdot y_1}{a} + \frac{B_1 \cdot z_1}{a} \right)}{\left(\frac{A_1^2 + B_1^2}{a^2} \right) \left\{ \left(\frac{A_1 - A_0}{a} \right)^2 + \left(\frac{B_1 - B_0}{a} \right)^2 \right\}}$$

$$- \frac{\left(\frac{B_1 - B_0}{a} \right) \left(\frac{B_1 \cdot y_1}{a} - \frac{A_1 \cdot z_1}{a} \right) - \left(\frac{A_1 + A_0}{a} \right) \left(\frac{A_1 \cdot y_1}{a} + \frac{B_1 \cdot z_1}{a} \right)}{\left(\frac{A_1^2 + B_1^2}{a^2} \right) \left\{ \left(\frac{A_1 + A_0}{a} \right)^2 + \left(\frac{B_1 - B_0}{a} \right)^2 \right\}}$$

$$- \frac{y_1}{y_0} \left[\frac{2 \frac{A_1}{a} \left(\frac{A_1 \cdot y_1}{a} + \frac{B_1 \cdot z_1}{a} \right)}{\left(\frac{A_1^2 + B_1^2}{a^2} \right)^2} - \frac{\frac{A_1 \cdot y_1}{a} \frac{B_1 \cdot z_1}{a}}{2 \frac{A_1}{a} \left(\frac{A_1^2 + B_1^2}{a^2} \right)} - \frac{\frac{y_1}{a} \left(\frac{A_1^2 - B_1^2}{a^2} \right) - 2 \frac{A_1 \cdot B_1 \cdot z_1}{a^2}}{2 \left(\frac{y_1^2 + z_1^2}{a^2} \right) \left(\frac{A_1^2 + B_1^2}{a^2} \right)^2} \right] = 0$$

Theoretical Work Appendix VI

Solution of the equation

$$\frac{dy}{dx} = \frac{\frac{y}{x} - 1}{\frac{y}{x} + 1}$$

$$\text{Let } v = \frac{y}{x} \therefore y = vx \quad \therefore \frac{dy}{dx} = v + \frac{xdv}{dx}$$

$$\therefore \frac{xdv}{dx} + v = \frac{v-1}{v+1}$$

$$\therefore \frac{xdv}{dx} = \frac{v-1}{v+1} - \frac{v^2+v}{v+1} = -\frac{1+v^2}{v+1}$$

$$\therefore \frac{dx}{x} + \frac{dv(1+v)}{1+v^2} = 0$$

$$\therefore \log x + \tan^{-1} v + \frac{1}{2} \log (1+v^2) = \log K_1 \text{ (const)}$$

$$\therefore x e^{\tan^{-1} \frac{y}{x}} \sqrt{1 + \frac{y^2}{x^2}} = K_1$$

$$\therefore \sqrt{x^2 + y^2} e^{\tan^{-1} \frac{y}{x}} = K_1$$

$$y = 0 \text{ if } x = h, \quad \therefore K_1 = h,$$

$$\therefore \sqrt{x^2 + y^2} e^{\tan^{-1} \frac{y}{x}} = h,$$

Appendix VII

Calculation of lift.

From ref. 20 we have:

$$\begin{aligned}
 \text{Lift} &= \text{Real part} - \left[\rho V \int_C w_x \frac{d\sigma}{dx} dx \right] \\
 &= \text{Real part} - \left[\rho V \int_C \left\{ iV\alpha x + \frac{i\gamma_0}{2\pi} \log \frac{x-x_0}{x+\bar{x}_0} + \frac{i\gamma_1}{2\pi} \log \frac{x-x_1}{x+\bar{x}_1} \right\} \right. \\
 &\quad \left. \times \frac{x dx}{\sqrt{x^2+a^2}} \right] \\
 &= \rho V^2 \alpha \pi a^2 + \rho V \gamma_0 \frac{(x_0 + \bar{x}_0)}{a} + \rho V \gamma_1 \frac{(x_1 + \bar{x}_1)}{a}
 \end{aligned}$$

$$\begin{aligned}
 C_L &= \frac{L}{\frac{1}{2} \rho V^2 a x} = 2\pi\alpha\epsilon + \frac{2\gamma_0\epsilon}{Va} \frac{(x_0 + \bar{x}_0)}{a} + \frac{2\gamma_1\epsilon}{Va} \cdot \frac{(x_1 + \bar{x}_1)}{a} \\
 &= 2\pi\alpha\epsilon + \frac{4\gamma_0\epsilon}{Va} \cdot \frac{A_0}{a} + \frac{4\gamma_1\epsilon}{Va} \cdot \frac{A_1}{a} \\
 &= 2\pi\alpha\epsilon + 8\pi\epsilon^2 \cdot \frac{\gamma_0}{2\pi a V \epsilon} \cdot \left\{ \frac{A_0}{a} + \frac{\gamma_1}{\gamma_0} \cdot \frac{A_1}{a} \right\}
 \end{aligned}$$

Appendix VIII

Calculation of pressure distribution.

From ref. 20 we have for the surface pressures

$$\frac{\Delta p}{q} = + \frac{\infty}{V} \left(\frac{\partial W}{\partial a} \right)_{z=0} + a^2 - \frac{1}{V^2} \left(\frac{\partial W}{\partial \sigma} \right)_{z=0}^2 \quad \text{Note the change in size of the first term since the velocity sign convention is different from ref. 20.}$$

$$\text{Now } W(\sigma) = iV\alpha \sqrt{\sigma^2 - a^2} + \frac{i\gamma_0}{2\pi} \log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_0^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) + \frac{i\gamma_1}{2\pi} \log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_1^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right)$$

$$\frac{\partial W}{\partial \sigma} = \frac{iV\alpha\sigma}{\sqrt{\sigma^2 - a^2}} + \frac{i\gamma_0}{2\pi} \frac{\sigma}{\sqrt{\sigma^2 - a^2}} \left(\frac{1}{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_0^2 - a^2}} - \frac{1}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) + \frac{i\gamma_1}{2\pi} \frac{\sigma}{\sqrt{\sigma^2 - a^2}} \left(\frac{1}{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_1^2 - a^2}} - \frac{1}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right)$$

$$\sigma = y + iz, (\sigma)_{z=0} = y$$

$$\therefore \left(\frac{\partial W}{\partial \sigma} \right)_{z=0} = \frac{iV\alpha y}{\sqrt{y^2 - a^2}} + \frac{i\gamma_0}{2\pi} \cdot \frac{y}{\sqrt{y^2 - a^2}} \left(\frac{1}{\sqrt{y^2 - a^2} - \sqrt{\sigma_0^2 - a^2}} - \frac{1}{\sqrt{y^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right)$$

$$\text{Let } \sqrt{y^2 - a^2} = ic + \frac{i\gamma_1}{2\pi} \cdot \frac{y}{\sqrt{y^2 - a^2}} \left(\frac{1}{\sqrt{y^2 - a^2} - \sqrt{\sigma_1^2 - a^2}} - \frac{1}{\sqrt{y^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right)$$

$$\therefore \left(\frac{\partial W}{\partial \sigma} \right)_{z=0} = \frac{y}{c} \left[V\alpha - \frac{\gamma_0}{2\pi} \left(\frac{1}{A_0 + i(B_0 - c)} + \frac{1}{A_0 - i(B_0 - c)} \right) \right]$$

$$- \frac{\gamma_1}{2\pi} \left(\frac{1}{A_1 + i(B_1 - c)} + \frac{1}{A_1 - i(B_1 - c)} \right)$$

$$= \frac{y}{c} \left[V\alpha - \frac{\gamma_0}{2\pi} \left(\frac{2A_0}{A_0^2 + (B_0 - c)^2} \right) - \frac{\gamma_1}{2\pi} \left(\frac{2A_1}{A_1^2 + (B_1 - c)^2} \right) \right]$$



$$W(\sigma) = iV\alpha \sqrt{\sigma^2 - a^2} + \frac{i\gamma_0}{2\pi} \log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_0^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) \\ + \frac{i\gamma_1}{2\pi} \log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_1^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right)$$

$$\frac{\partial W(\sigma)}{\partial a} = -\frac{iaV\alpha}{\sqrt{\sigma^2 - a^2}} + \frac{i\gamma_0}{2\pi} \cdot \frac{a}{\sqrt{\sigma^2 - a^2} \sqrt{\sigma_0^2 - a^2}} + \frac{i\gamma_0}{2\pi} \cdot \frac{a}{\sqrt{\sigma^2 - a^2} \sqrt{\sigma_0^2 - a^2}} +$$

$$\frac{i\gamma_1}{2\pi} \cdot \frac{a}{\sqrt{\sigma^2 - a^2} \sqrt{\sigma_1^2 - a^2}} + \frac{i\gamma_1}{2\pi} \cdot \frac{a}{\sqrt{\sigma^2 - a^2} \sqrt{\sigma_1^2 - a^2}} + \frac{i\gamma_0}{2\pi a} \times$$

$$\left[\log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_0^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) + \frac{\gamma_1}{\gamma_0} \log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_1^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right) \right]$$

since γ_0 is dependent on x and therefore on a .

$$= \frac{-ia}{\sqrt{\sigma^2 - a^2}} \left[V\alpha - \frac{\gamma_0}{2\pi} \left(\frac{1}{\sqrt{\sigma_0^2 - a^2}} + \frac{1}{\sqrt{\sigma_0^2 - a^2}} - \frac{\gamma_1}{2\pi} \left(\frac{1}{\sqrt{\sigma_1^2 - a^2}} + \frac{1}{\sqrt{\sigma_1^2 - a^2}} \right) \right) \right] \\ + \frac{i\gamma_0}{2\pi a} \left[\log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_0^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_0^2 - a^2}} \right) + \frac{\gamma_1}{\gamma_0} \log \left(\frac{\sqrt{\sigma^2 - a^2} - \sqrt{\sigma_1^2 - a^2}}{\sqrt{\sigma^2 - a^2} + \sqrt{\sigma_1^2 - a^2}} \right) \right]$$

By equation 4 the first term is identically zero.

$$\left(\frac{\partial W}{\partial a} \right)_{(z=0)} = \frac{i\gamma_0}{2\pi} \left[\log \left(\frac{-A_0 - i(B_0 - c)}{A_0 - i(B_0 - c)} \right) + \frac{\gamma_1}{\gamma_0} \log \left(\frac{-A_1 - i(B_1 - c)}{A_1 - i(B_1 - c)} \right) \right] \\ = \frac{\gamma_0}{2\pi a} \left[\frac{2 \frac{A_0}{a} \left(\frac{B_0}{a} - \frac{c}{a} \right)}{\frac{A_0^2}{a^2} + \left(\frac{B_0}{a} - \frac{c}{a} \right)^2} + \frac{\gamma_1}{\gamma_0} \left\{ \frac{2 \frac{A_1}{a} \left(\frac{B_1}{a} - \frac{c}{a} \right)}{\frac{A_1^2}{a^2} + \left(\frac{B_1}{a} - \frac{c}{a} \right)^2} \right\} \right]$$

$$\therefore \frac{\Delta_p}{q} = \frac{\gamma_0}{2\pi a V\epsilon} \cdot 2\epsilon^2 \left[\frac{2 \frac{A_0}{a} \left(\frac{B_0}{a} - \frac{c}{a} \right)}{\frac{A_0^2}{a^2} + \left(\frac{B_0}{a} - \frac{c}{a} \right)^2} + \frac{\gamma_1}{\gamma_0} \frac{2 \frac{A_1}{a} \left(\frac{B_1}{a} - \frac{c}{a} \right)}{\frac{A_1^2}{a^2} + \left(\frac{B_1}{a} - \frac{c}{a} \right)^2} \right] + \alpha^2 -$$

$$- \frac{\epsilon^2 y_a^2}{\frac{c^2}{a^2}} \left[\frac{\alpha}{\epsilon} - \frac{y_0}{2\pi a V \epsilon} \left(\frac{\frac{2A_0}{a}}{\frac{A_0^2}{a^2} + \left(\frac{B_0 - c}{a}\right)^2} + \frac{y_1}{y_0} \cdot \frac{\frac{2A_1}{a}}{\frac{A_1^2}{a^2} + \left(\frac{B_1 - c}{a}\right)^2} \right) \right]^2$$

5. Model and Experimental Method

5.1 Tests with undeflected blowing.

The model is shown mounted in the wind tunnel in figure 1. It is a 70° swept delta wing with cropped tips, of chord equal to one third of the root chord, and has an aspect ratio of 0.73. The main body is a hollow gunmetal casting and detachable brass edges form a continuous blowing slot round the periphery (of constant width 0.040 ins.) except for a small region near the apex. The model is of rhombic cross-section, the total edge angle on both leading edge and tips is 20° whilst the trailing edge angle is 15° .

Pressure plotting stations were located at $0.33 c_o$, $0.49 c_o$, $0.63 c_o$ and $0.87 c_o$ from the apex and consisted of two continuous tubes each spanning half of the wing. Thirty-six static pressure holes (.020" dia.) were drilled at each station to enable the span-wise pressure distribution to be accurately described.

The tests were made in the College of Aeronautics 8 ft x 6 ft low speed wind tunnel with the model supported on a Warden type six-component balance. The velocity calibration was made using two pitot-static tubes, one on the tunnel centre line three feet upstream of the model and the other in the plane of the model at the pivot point, mid-way between the tunnel centre line and the wall. At zero incidence and yaw there was a two per cent increase in tunnel speed at the second pitot. About one half per cent may be ascribed to solid blockage and the rest to boundary layer growth on the tunnel walls. Tunnel speed was kept constant using a Betz manometer

connected to static pressure tapings on either side of the contraction i.e. the dynamic head through the working section was kept constant. The effects of incidence (up to 25°) sideslip (up to 15°) and blowing ($C_d \approx .3$) were small, a change of wind speed of only about one half per cent was recorded on either pitot-static tube due to any or all of these factors. Thus in calculating coefficients a constant velocity as given by the second pitot-static tube has been assumed. A similar procedure was adopted in the ground effect tests and this automatically takes into account the increase in speed due to the blockage of the ground plate.

High pressure air was fed to the model at the balance virtual centre through the hollow support strut, and constraints were kept to a minimum by use of a flexible circular ring-main feed see fig. 4. A rotary seal at the centre enabled the model to be yawed for the stability tests. The pressure in the system was controlled by a Hale Hamilton R.L.6 reducing valve using an L.15 Controller, and could be maintained constant to within ± 0.25 p.s.i. Pressures were recorded on Budenberg Standard Test Gauges.

The rate of mass flow of air to the model, m_j , was measured using orifice plates in the main feed pipe, there was of course no loss of air as with an air bearing system. The pressure distribution in the slot was measured with a total head tube and the theoretical jet velocity calculated assuming isentropic expansion to free stream pressure. In the first series of tests with blowing from all edges with an open slot the variation of velocity along the slot makes the calculation

of C_{μ} in the normal way impossible. An average constant velocity V_j , giving the same total rate of mass flow, was computed from the velocity distribution. C_{μ} was then defined as $C_{\mu} = \frac{m_j V_j}{\rho_0 S}$. With directed blowing from the leading edge only the total momentum was obtained from graphs such as fig. 15 and C_{μ} calculated as above. Where possible C_{μ} was obtained from balance measurements also and was within 2% of the value obtained from the slot pressure distributions.

As noted previously both tapered and directed blowing are desirable and this has been achieved in the present model using thin perspex slips in the slot, see fig. 6. Some preliminary tests with directed blowing showed that in order to obtain true direction it was necessary to break up the jet into a large number of small jets each capable of individual direction. This was accomplished on the present model with saw-cuts in the 0.5" wide perspex slips, each .020" wide with .020" spacing, cut at the appropriate angle to the leading edge. By varying the depth of the saw cut it was possible to vary the momentum ejected and thus obtain an approximately linear increase in momentum from the apex to the leading edge-wing-tip junction, see fig. 15.

In the first series of tests with blowing from all edges six-component balance measurements were made both with and without the model yawed. The forces have been referred to wind axes and the moments to body axes, see fig. 7. Conversion of the moments from wind axes to body axes was made using the following formulae:

$$C_1 = C'_1 \cos \beta \cos \alpha - \frac{\bar{c}}{b} C'_m \sin \beta \cos \alpha - C'_n \sin \alpha$$

$$C_m = C'_m \cos \beta + \frac{b}{\bar{c}} C'_1 \sin \beta$$

$$C_n = C'_n \cos \alpha + C'_1 \cos \beta \sin \alpha - \frac{\bar{c}}{b} C'_m \sin \beta \sin \alpha$$

where the dashes denote measured moments relative to wind axes.

Pitching and Yawing moments are referred to the aerodynamic mean quarter chord point by the formulae:

$$C_m^* = C_m - \frac{a^*}{\bar{c}} \left[C_L \cos \alpha + C_D \cos \beta \sin \alpha - C_c \sin \beta \sin \alpha \right]$$

$$C_n^* = C_n + \frac{a^*}{b} \left[C_c \cos \beta + C_D \sin \beta \cos \alpha - C_L \sin \alpha \sin \beta \right]$$

where a^* is the distance from the pivot point to the aerodynamic quarter-chord point.

In tests with blowing the high pressure air caused the flexible ring main to distort slightly and this induced additional forces and moments. To correct for these changes the model was removed periodically and a calibrator (see fig. 2) was attached to the balance strut. The calibrator consisted of a short length of pipe feeding air to the adjustable gap between two flanges. The gap was set such that with any given model configuration the rate of mass flow was the same for a fixed control pressure. Since the air was emitted radially from the virtual centre of the balance the only forces and moments present should be those due to the distortion of the ring main. In order to allow for the slight inaccuracies in manufacture the balance measurements were taken with the calibrator in a fixed position and then rotated through 180° . The average of the two sets of readings was taken to be the balance constraint correction

due to blowing. These corrections, for a control pressure of 30 p.s.i. gauge at the orifice plates ($C_{p_a} = .278$) with the whole slot open are given below, coefficients based on $U_o = 100$ ft/sec:

Lift	-.41 lbs	C_L	-.009
Drag	-.09 lbs	C_D	-.002
Cross Wind force	+.63 lbs	C_c	+.015
Rolling moment	+1.91 ft.lbs.	C_l'	+.027
Pitching moment	+.46 ft.lbs	C_m'	+.004
Yawing moment	+2.10 ft.lbs	C_n'	+.030

The moments here are referred to the balance virtual centre and wind axes.

Overall accuracy of coefficients (moments referred to body axes.

See List of Symbols).

C_L	$\pm .002$
C_D	$\pm .0002$
C_c	$\pm .002$
C_l	$\pm .002$
C_m^*	$\pm .001$
C_n^*	$\pm .002$

Wind tunnel constraint corrections have not been applied to the present results since no suitable corrections are available but conventional corrections are small, $\Delta\alpha = 0.5^\circ$ for $\alpha = 25^\circ$ and $\Delta C_D = 0.009$ for $C_D = 0.45$. Corrections have been applied to C_D C_m^* C_n^* to allow for the drag of the incidence wire and the small exposed part of the main feed pipe.

In order to explore the vortex in greater detail a five tube pitch-yawmeter, fig. 3, was used which enabled traverses to be made in a spanwise plane. The angular movements on the head were 360° in yaw but only $\pm 20^\circ$ in pitch. The head was similar to the five-tube probe described in ref. 48. The apex angle was 70° and the outside diameter 0.125".

To estimate the helix angle of the vortex lines the head was offset initially about 20° so that measurements up to 40° in pitch were possible. Some slight movement of the head occurred as yaw was altered and it is doubtful if the flow angle could be measured to better than $\pm 2^\circ$.

5.2 Model and Experimental Method

Tests with downward deflected blowing

The model was tested with the jet sheet deflected downwards through two angles, 30° and 90° , see fig. 5. In order to deflect the jet 30° a new set of edges was used with the jet emerging from the edge, but for the 90° deflected case a small wooden flap was attached to the lip of the 30° drooped edge, see fig. 5.

The ground was represented by a large wooden plate eight feet square and two inches thick stiffened by "L" shaped steel supports to ensure flatness. The plate had an elliptic leading edge and a chamfered trailing edge and was set at zero incidence relative to the tunnel stream. The plate projected forward about two feet ahead of the wing apex and could be positioned vertically by means of screw

jacks.

The displacement thickness of the boundary layer on the plate was approximately 0.3" at the pivot point, i.e., about one fifth of the minimum ground clearance at zero incidence. It is not known what effect this had on the results but it does not seem likely that the results would be seriously modified.

Balance measurements were made in all the tests of lift, drag and pitching moment for zero sideslip only. The height of the model above the ground plate was varied and tests were made with deflected blowing out of ground effect. A range of incidence was covered at each ground height, three blowing pressures and the unblown case were tested and the tunnel speed varied from zero to 100 ft/sec in 20 ft/sec increments.

6. Discussion of Results. Part I. Blowing in the plane of the wing.

6.1 Introduction

As a preliminary to the main wind tunnel programme a short series of tests was carried out, both with and without blowing at all edges, in order to provide some basic information on both the rig and the effects of blowing.

Wind speed was varied between 50 ft/sec and 200 ft/sec with no appreciable Reynolds number effect, hence the tests without blowing were made at the maximum speed of 200 ft/sec to obtain the greatest possible accuracy but tests with blowing were made at lower wind speeds in order to achieve a large range of C_{μ} values. A few tests were made at a wind speed of 50 ft/sec giving a maximum C_{μ} value of 1.05, but balance readings were less accurate at this low speed and most of the tests with blowing were made at 100 ft/sec when the maximum value of C_{μ} was 0.28.

Comparative tests were also made without blowing (a) with the slot open and (b) sealed to prevent any airflow through the slot. In general sealing the slot had little effect on the balance readings and the only appreciable changes were observed at the highest incidence on the rolling and yawing moments.

Measurement of balance constraints due to blowing using the calibrator proved to be a simple matter and experience showed that it was necessary to recalibrate only after the model had been yawed since friction in the sliding seal changed the constraints somewhat. Even for this case only the yawing and rolling moment corrections were

changed appreciably.

In order to use edge blowing as a control it would be necessary to use some form of asymmetric blowing. Four asymmetric blowing configurations were tested as follows:

- (1) Blowing from one tip only
- (2) Blowing from one tip and the whole trailing edge
- (3) Blowing from one leading edge and tip only
- (4) Blowing from one leading edge, tip and the whole trailing edge.

These tests were designed to show the effects of asymmetric blowing and also the effect of using a trailing edge jet sheet in conjunction with blowing from swept and streamwise edges. Six-component balance measurements were made for all blowing configurations for sideslip angles of -5° , 0° and $+5^\circ$. As the test results for the above cases were not dissimilar only the results with tip blowing are presented here. The effect of trailing edge blowing was small although the amount of air ejected from the trailing edge was somewhat lower than anticipated due to losses in the system.

Although the preliminary tests and the series of tests with blowing at all edges from an open slot were essential to obtain a better understanding of the effects of edge blowing, it soon became clear that more efficient ways of using the air were possible. Fig. 18 shows clearly that only for small C_{μ} values (< 0.2) and moderate incidences is there any lift magnification ($\Delta C_L > C_{\mu}$) and all further effort was concentrated on the low C_{μ} range.

6.2 Lift

The jet sheet originating at the leading edges and tips rolls up to form the leading edge vortices in a manner similar to the rolling up of the free vortex sheets without blowing, although the pressure boundary condition is changed since the jet sheet can now support a pressure difference. This rolling up of the jet sheet has occurred in all the reported tests using slot blowing from streamwise tips and is a stable type of flow. A sharp change in lift and pitching moment occurs however on passing through zero incidence when the vortex so formed moves from one surface to the other. Without blowing of course the flow is completely attached at zero incidence even with sharp leading edges but with blowing there is a vortex sheet at zero incidence and since the flow is apparently stable only when the jet sheet has rolled up there is an abrupt change when the wing passes through zero incidence. The effect can be seen in figs. 14a and 14b where both the lift and pitching moment exhibit discontinuities near $\alpha = 0$. There was no sudden change in the other forces and moments in this region.

In the tests near zero incidence with leading edge blowing it was possible to find an incidence at which the jet sheets oscillated from one surface to the other producing a sinusoidal lift. The frequency of the oscillation was about one cycle per second and is thought to be associated with downwash lag in the following manner. Only a very small positive incidence ($< 0.25^\circ$) is sufficient to ensure that the jet sheet rolls up over the wing upper surface, thus producing

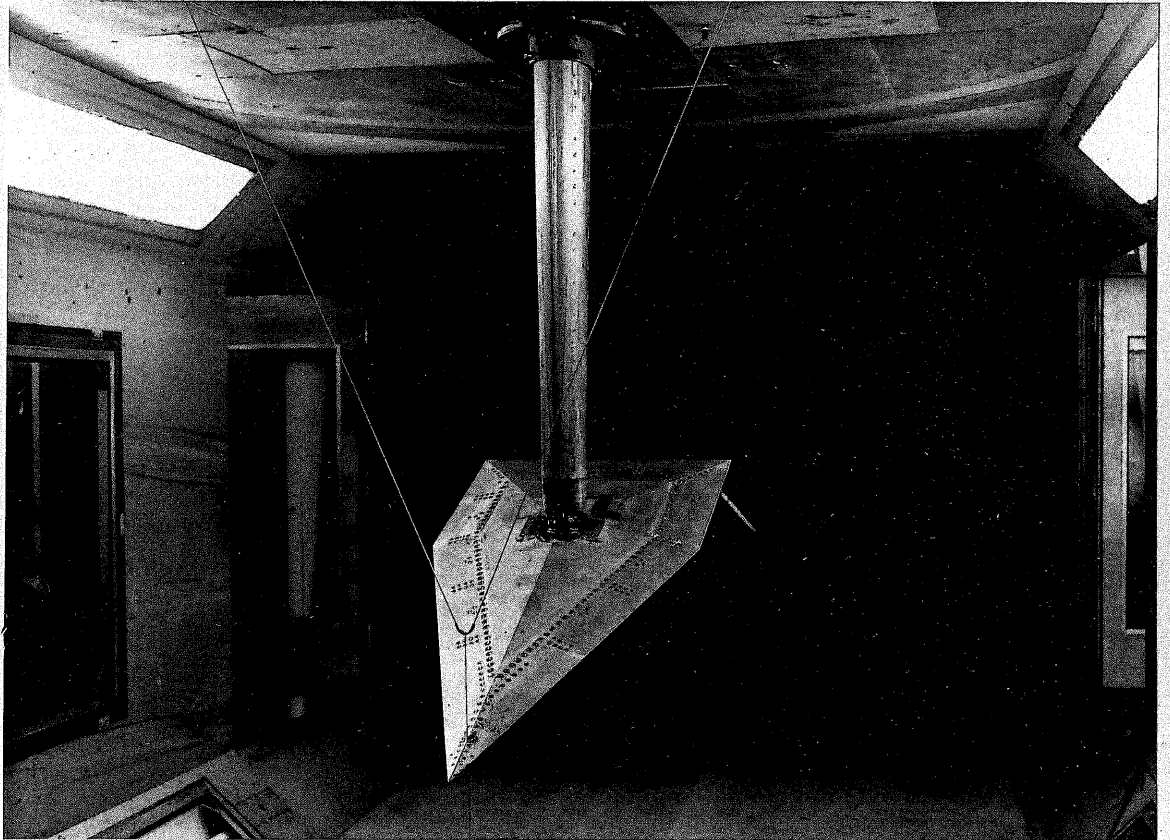


FIG. 1. MODEL MOUNTED IN WIND TUNNEL.

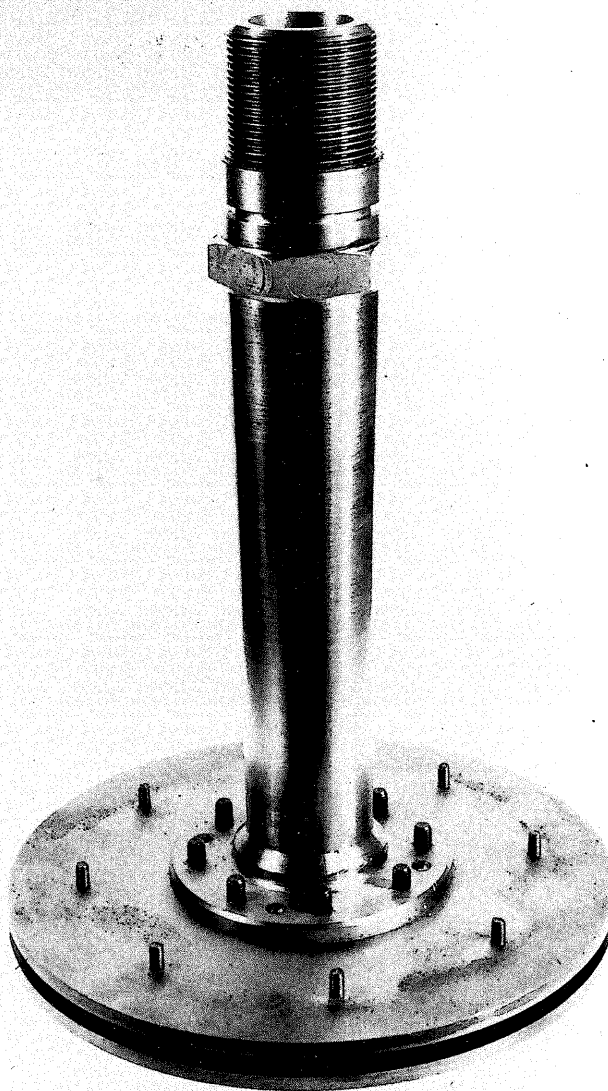


FIG. 2. THE CALIBRATOR.

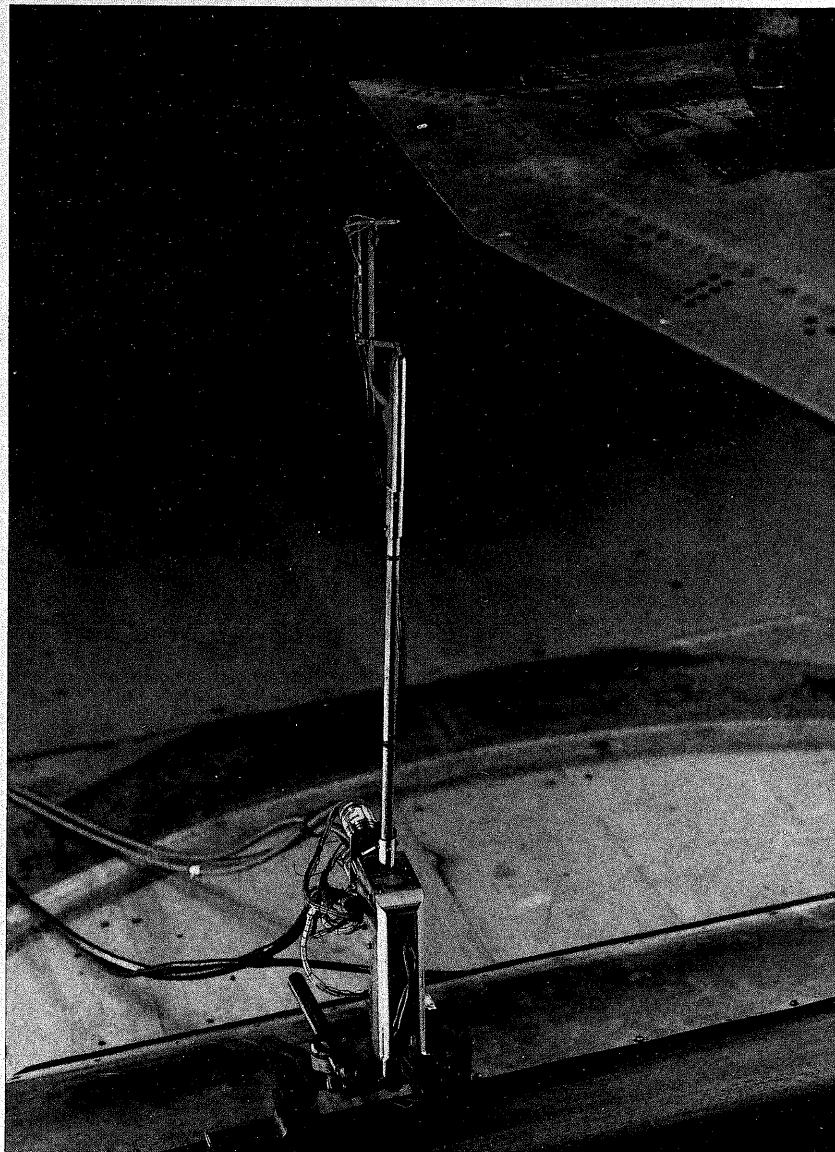


FIG. 3. FIVE TUBE PITCH-YAWMETER.

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(for values of C_{μ} used in the present tests) values of C_L of 0.02 or greater i.e., at least four times the lift produced by the wing without blowing at the same incidence. This lift is sufficient to produce a downwash on the wing of at least 0.5° ($\epsilon \approx \frac{C_L}{\pi AR}$) which is greater than the geometric incidence. Consequently the wing is now at an effective incidence of -0.25° , sufficient to induce the vortices to move to the other surface. Thus an oscillatory motion is set up which would be undesirable in an aircraft although it seems unlikely that this condition would be approached in practice.

In order to calculate C_{μ} values the total head of the jet was measured at fairly closely spaced intervals along the slot. In fig. 15 the jet momentum is plotted against distance along the slot. The theoretical analysis showed that to keep the flow conical with blowing the variation should be a linear one. Two cases are compared, the momentum distribution along the swept leading edge when blowing from all edges, and with leading edge blowing only using the edge slots of fig. 6 to taper and direct the jet. The use of the edge slots enabled a rather better distribution to be achieved in the critical region near the apex despite the larger values of momentum. It should be mentioned however that the pressure distribution inside the model was far from uniform and it was not possible to achieve an ideal momentum distribution.

Some lift-incidence curves are shown in fig. 16. Without blowing the curve exhibits the usual non-linear slope associated with wings having leading edge separations although near zero incidence it

follows closely the R.T. Jones value for attached flow. At $\alpha = 25^\circ$ the non-linear lift is equal to the linear contribution. Sideslip angles of up to 5° had no effect on lift.

Also shown in fig. 16 is the effect of three different edge blowing configurations. In all cases the effect of blowing was to increase lift at constant incidence. Since blowing increases the size and strength of the leading edge vortices it increases mainly the non-linear lift but its effect is felt over most of the upper surface. (see § 6.8). Of the three cases shown the smallest increase of lift was obtained with blowing from one wing-tip at a C_{μ} of .121. Only at the highest incidence tested was the lift magnification ($\frac{\Delta C_L}{C_{\mu}}$) greater than one. With tip blowing only the flow appeared to be quite steady and although no pressure plotting measurements or flow visualisation were made it is thought likely that the normal leading edge vortex without blowing from the forward part of the wing and the jet induced tip vortex coalesce rather than form two separate vortices which sometimes occur in tests with leading edge extensions. Slightly better results were obtained with blowing from all edges at the same C_{μ} but it became clear from a study of the pressure distributions that edge blowing was most effective when used in a region where a powerful vortex system was already in existence, and that the lift contribution from the rear part of the wing was comparatively small both with and without blowing from all edges. When blowing from one tip at a C_{μ} of 0.12 however, the amount of air ejected in a short region of the edge was very large ($C_{\mu} \approx 1.0$ on a

slot length basis) and the effect on lift was much greater.

The largest increase in lift was obtained blowing normal to the leading edge using the edge slots to give a reasonable blowing distribution. The C_{μ} value plotted here is lower than the two previous cases, 0.096, but ΔC_L 's of 0.24 are obtained at $\alpha = 20^\circ$ and 25° giving a value of $\frac{\Delta C_L}{C_{\mu}} \approx 2.5$.

Fig. 17 shows the effect of varying the angle of ejection of the jet relative to the normal to the leading edge in the plane of the wing. Maximum lift is obtained when blowing normal to the leading edge but there is only a small loss if the jet is directed as much as 50° back from this. It may seem surprising at first sight that the lift is so little affected by the angle of ejection but the velocity of the jet is greatly reduced in a short distance by mixing and the helix angle of the streamlines rolling up to form the vortex are probably little different from the unblown case independent of the blowing velocity and angle of ejection. Fig. 38b shows that a velocity of about 900 ft/sec at the slot is reduced to 110 ft/sec. after only half a turn. This reduction in velocity is somewhat lower than for a comparable two-dimensional turbulent jet.

Fig. 18 shows the variation of lift coefficient C_L with blowing momentum coefficient C_{μ} , at constant incidence. The increase in lift is similar in form to that obtained with a wing having blowing boundary layer control on a deflected flap i.e., an initial steep rise in lift followed by a more gradual increase. In this case however the increase is due rather to slow changes in the flow pattern without

essentially changing its form, rather than the large changes which occur when flow separation over the deflected flaps is suppressed. The initial increase in lift is due to an increase in the vortex strength while the vortex core remains over the wing. Tuft observations, however, show that for much larger values of C_{μ} the vortex core moves off the wing surface, for instance at a C_{μ} of 1.0 and 10° incidence the sweep of the vortex core is about 65° compared with a leading edge sweep of 70° , with a corresponding drop in the rate of increase of lift with increase of C_{μ} .

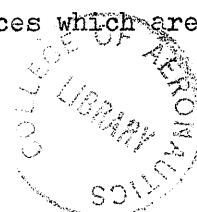
Again, the results obtained with leading edge blowing only are much better than those with blowing from all edges at a given value of C_{μ} . For practical purposes the values of C_{μ} will be limited to less than 0.1 and in this region the lift magnification $\frac{\Delta C_L}{C_{\mu}}$ reaches reasonably high values for moderate incidences. For instance, at a typical landing incidence of 15° , a C_{μ} of 0.03 gives a ΔC_L of 0.12, a ratio of four. While this is small compared with the untrimmed values obtained with blowing over flaps it must be remembered that with a suitable blowing configuration there will be no movement of the centre of pressure due to blowing and hence no trimming penalty. With a C_{μ} of 0.0275 which might just be achieved by bleeding the main engine compressors the increase in C_L would give a reduction in landing speed of about 15 knots, (see Appendix) at least as good as any figure achieved by flap blowing and with no control problems. Also with increasing leading-edge sweep and the resulting smaller lift coefficients the gains due to blowing become relatively larger.

6.3. Drag

Drag is plotted against lift in figs 19 and 20 for both the unblown case and for various blowing configurations. The effect on drag of sealing the slot in the unblown case, and of sideslip between $\pm 5^\circ$ both with and without blowing, was negligible.

It has been observed in some tests e.g., ref. 46, that there is a dip in the drag curve near zero lift due to the formation of areas of laminar flow. No such effect has been observed in the present tests and the minimum drag value of 0.011 is sufficiently close to the estimated turbulent value (skin-friction + leading edge form drag) of 0.0092 to indicate that the flow was turbulent at all times although no special effort was made to fix transition.

Fig. 19 shows the effect on drag of blowing at all edges and blowing from the wing-tip only. With blowing at all edges there was an appreciable increase in the value of drag at zero lift. Ideally, with the constant width open slot configuration tested (excluding the small region near the apex) and with constant slot pressure there would have been no thrust on the model due to internal pressure forces. However, due to losses in the system the trailing-edge jet pressure was rather low giving a net drag at zero lift. The drag value of 0.028 obtained at zero lift was reduced to 0.008 when allowance was made for the wind-off jet drag. Thus there is a slight reduction in drag at small incidence due to the aerodynamic effect of edge blowing, presumably due to the lower pressures induced on the forward facing surfaces by the leading edge vortices which are



stronger than for the unblown case at the same incidence. At the higher incidences the reduction in drag for a given lift with blowing is due almost entirely to the lower induced drag.

With blowing from the wing-tip only a small amount of thrust recovery was obtained but did not exceed 10% of the theoretical C_{μ} value. At higher incidence the drag was close to that obtained with blowing from all edges at the same value of C_{μ} . The drag was slightly greater than the corrected value with blowing at all edges.

Fig. 20 shows the effect on drag of leading edge blowing only. At low incidence the forward facing jet ($\theta = 0^\circ$) increases the drag as might be expected but somewhat less ($\Delta C_D = .003 - .005$) than the direct thrust component, due to the increased suction on the forward facing surfaces mentioned above. With the rearward facing jet about 90% of the direct jet thrust is recovered at small incidence. Again at high incidence considerable reductions in drag are achieved due to lower induced drag values.

Fig. 21 gives lift-to-drag ratios both with and without leading edge blowing. The maximum value obtained without blowing is 6.5 at a C_L of 0.13 and agrees fairly well with the delta wing results of ref. 15. With blowing the lift-to-drag ratio must include the effect of the jet momentum and this is done by calculating the value $\frac{C_L}{C_D + C_{jx}}$. The C_D values include any direct jet thrust, and, assuming no losses, if the jet were directed rearwards with zero deflection the value of $\frac{C_L}{C_D + C_{jx}}$ would be the same as that obtained without blowing. Any alternative ways of using the jet thrust will vary both C_L and C_D

but must take into account the fact that C_{μ} has been subtracted from the total thrust (by engine bleed) and hence effectively added to the drag. The maximum value of $\frac{C_L}{C_D + C_{\mu}}$ is much lower than $(\frac{C_L}{C_D})_{\max}$ without blowing and occurs at a higher incidence. It is also reduced by decreasing θ and increasing C_{μ} . Thus it does not seem feasible to use this device to improve lift-to-drag ratios for the cruise case.

With a flat plate delta wing having leading edge separations (no nose suction) the induced drag will be $L \tan \alpha$ and the corresponding lift-to-drag ratio (excluding skin friction) is $\cot \alpha$. This is also plotted in fig. 21 using the lift values obtained without blowing. Above a C_L of 0.35 in the unblown case the lift-to-drag ratio exceeds $\cot \alpha$ due to the suction forces acting on the forward facing surfaces. With blowing however this only occurs for much higher values of lift.

6.4 Cross-wind force

Cross-wind force is plotted against the sideslip angle β in fig. 22. For the range of sideslip tested ($\beta = -5^\circ, 0^\circ, +5^\circ$) the variation is apparently linear although no tests were made at intermediate points. The results of ref. 15 suggest that the variation will be linear between $\beta = \pm 5^\circ$ for the incidence range tested. The results with blowing from all edges differ only slightly from the unblown case, apart from a small force at zero sideslip due to asymmetries in the blowing, and are not presented here.

With blowing from the tip only (starboard) there is a large cross-wind force at zero sideslip due to the jet momentum in the cross-wind

direction. This value is close to the theoretical momentum at small incidence and provides a useful check on the method of estimating $C_{y'}$. The graph is appreciably asymmetric with respect to zero sideslip since the tip jet rolls up to form a strong vortex and the cross-wind force due to the resultant increased suctions on that section of the wing oppose the jet reaction.

The value of $\frac{\partial C_c}{\partial \beta}$ was hardly affected by any of the blowing configurations tested and it seems unlikely that this type of blowing i.e., with edge jets, will cause any changes apart from producing a cross-wind force with asymmetric blowing configurations.

6.5 Pitching moments

The pitching moment about the aerodynamic mean quarter-chord point is plotted against lift in figs. 23 and 24. Without blowing the curve is straight up to a C_L of 0.5 ($\alpha = 15^\circ$) but is slightly non-linear for higher incidence, the wing becoming slightly more stable in pitch. As with the force measurements sealing the slot and sideslipping up to 5° had very little effect.

In all tests with blowing there was a sudden change of pitching moment near zero incidence, this can be clearly seen in fig. 14b, and is caused by the vortex moving from one surface to the other. Away from zero incidence however the flow pattern was quite stable and the overall effect of the blowing was to change the pitching moment at zero lift $C_{m_0}^*$. In the tests with blowing from all edges and from the wing tip only, the aerodynamic centre moved back slightly at high incidence

i.e., the wing became more stable in pitch. With leading edge blowing only the wing was slightly less stable both at low and high incidence but little changed at moderate incidence.

The movement of the centre of pressure is shown in fig. 25. Without blowing there is little movement above a C_L of 0.4 while for C_L values as low as 0.10 the centre of pressure moves forward only about $0.02C_o$. The centre of pressure is rather far forward on this model, since the centre of area is at $.64C_o$. This is due mainly to the cropped delta planform. Tests on a flat plate 70° true delta wing ref. 11 gives the centre of pressure position at $0.57C_o$ and it can be seen from a comparison of the pressure distributions that the rear part of the wing is more heavily loaded in the case of the true delta wing. Theoretically of course the rear part of the wing with streamwise edges would contribute no linear lift and this combined with the Kutta-Joukowski condition at the subsonic trailing edge means that the lift from the rear part of the section would be small and the centre of pressure well forward. Fig. 31 shows the large decrease in lift at the rear part of the wing.

In tests with blowing from all edges and with leading edge blowing only the centre of pressure moved only slightly at the higher incidences but at lower incidence, where the jet induced vortex contributed a large part of the lift, conditions were rather critical and the centre of pressure moved forward rapidly. The main difficulty in obtaining the correct blowing distribution was due to the non-uniform pressure inside the model but an improvement was effected for the case $\Theta = 50^\circ$ with

leading edge blowing only and this resulted in a change in the centre of pressure of not more than $0.01C_0$ over the whole incidence range. It is fairly certain that a suitable blowing configuration can be found which will give no movement of the centre of pressure over the whole incidence range.

With tip blowing only the centre of pressure moves aft due to the increased load near the tip. Above a C_L of 0.5 the movement is small but below this value it moves aft with increasing rapidity since at small incidence the major part of the lift is due to the action of the tip jet.

6.6 Rolling moments

In fig. 26 the rolling moment about the model centre line is plotted against sideslip angle β and the corresponding slopes $\ell_v (= \frac{\partial C_l}{\partial \beta})$ are plotted against lift in fig. 27. The variation of C_l appears to be linear within the range $-5^\circ \leq \beta \leq 5^\circ$ and this is supported by evidence from ref. 15.

The rolling moments induced on a delta wing in sideslip are due to an asymmetric pattern of the leading edge vortices see fig. 32. The vortex on the advancing edge remains tightly rolled but that on the retreating edge becomes more diffuse and weaker. Thus for negative sideslip there is a positive rolling moment i.e. ℓ_v is negative and increases in magnitude with increase of incidence due to the increased non-linear lift. These trends are clearly shown in figs. 26 and 27. Only at the highest incidence is there an appreciable difference due

to sealing the slot.

Without blowing the rolling moment due to sideslip is caused mainly by a weakening of one vortex relative to the other. With blowing, however, the rolling up of the leading edge vortices is partly controlled by the jet sheet, which is comparatively unaffected by sideslip, and would resist this deformation of the flow pattern. This would tend to reduce the magnitude of C_L and hence ℓ_v at a given incidence. From the results, however, it appears that this is only true at higher incidence. Fig. 27 shows that ℓ_v is reduced with blowing from all edges above a CL of 0.5.

Also shown is the effect of blowing from one wing tip only. The increased vortex strength and the consequent increase in lift produces substantial rolling moments at zero sideslip and should be capable of trimming 4° of sideslip up to high incidence with no loss of efficiency. The value of ℓ_v is hardly changed from the unblown case.

6.7 Yawing moments.

Yawing moments about the aerodynamic mean quarter chord are plotted against sideslip angle β in fig. 28 and the slopes $n_v (= \frac{\partial C_n}{\partial \beta})$ are plotted against lift in fig. 29. Again the variation was linear in the range of sideslip tested.

Without blowing the asymmetric pressure distribution due to sideslip (see § 6.6) will produce a yawing moment on a wing with thickness due to the pressure differential on the side area. The effect of sealing the slot is small at low and moderate incidence but becomes

appreciable above $\alpha = 15^\circ$. Results with the slot sealed are given here.

Due to the forward position of the axis about which the moments are measured negative sideslip produces a positive yawing moment. This is increased above 15° incidence with the slot open because air can flow forward inside the slot due to the chordwise pressure gradient. This effect will be greater on the advancing edge due to the larger pressure gradient and will tend to increase the yawing moment.

The yawing moment is changed very little with blowing from all edges at a given incidence but n_v is reduced for a given lift. $C_{n_0}^*$ has a non-zero value given by the yawing moment produced by the direct jet thrust which in turn depends on the blowing configuration. True asymmetric blowing configurations, such as blowing from one wing tip, produce large values of $C_{n_0}^*$ but with blowing from all edges the non-zero value of $C_{n_0}^*$ is due to irregularities in the slot pressure distribution.

6.8 Upper and Lower surface static pressures

The spanwise variation of static pressure on both upper and lower surfaces was measured at four chordwise stations, $0.33 c_o$, $0.49 c_o$, $0.63 c_o$ and $0.87 c_o$ for sideslip angles β of -5° , 0° , $+5^\circ$. Without blowing the range of incidence was $\alpha = 2^\circ$, 5° , 10° , 15° , 20° , 25° and with blowing from all edges and from the leading edge only $\alpha = 5^\circ$, 10° , 15° . Measurements were made at two values of C_μ and for $\theta = 0^\circ$, 20° , 50° with leading edge blowing only. The results are shown in figs. 30 to 35. The pressures on the starboard wing are on the left in the

figures, i.e., the wing is viewed from the stream direction.

Without blowing, figs. 30 to 32, the pressure distributions are typical of wings with sharp highly swept leading edges. The suction peak which occurs beneath the vortex core can be clearly seen, particularly at the more forward stations, and small suction peaks are evident at angles of incidence as low as two degrees. At higher incidence the secondary separation, due to the adverse pressure gradient near the leading edge is manifest by a region of roughly constant pressure outboard of the main vortex core. Reasonable agreement is obtained with the results of ref. 11 in which surface static pressures were measured on a 70° true delta wing.

Fig. 30 shows the variation in the static pressure distribution with incidence at $0.33C_0$, the inward movement and increase in vortex strength can be clearly seen. Fig. 31 shows the chordwise variation of static pressure without blowing at constant incidence. The flow may be said to remain conical in form back to at least 65% of the root chord i.e., to the leading edge - tip junction. Although there is an appreciable fall-off in the peak suction values the main vortex and secondary separation are still clearly defined. At the last station ($0.87C_0$) however it is clear that there has been a considerable fall-off in lift even allowing for the increased span. With the present model the reason is two-fold. Firstly, according to the R.T. Jones attached flow theory the part of the wing with streamwise edges would produce no lift (i.e. no linear lift) and secondly in subsonic flow the Kutta-Joukowski condition requires that the lift should fall to

zero at the trailing edge. In practice of course there is no clearly defined region of no lift (and in any case tip separations occur producing lift) and the result is a slow fall-off in lift in the forward regions where the flow is approximately conical in form and a much more rapid fall-off over the rear part of the wing. The results of ref. 11 show that the fall-off in lift is less with a true delta wing than with the present model.

Fig. 32 shows the effect on the pressure distribution of five degrees of sideslip. The shape of the curves indicates that the flow pattern has not been radically changed. The vortex on the advancing edge is stronger and has moved slightly inboard, while the vortex on the retreating edge is weaker and has moved slightly outboard. Clearly this is in part due to the effective change of the leading edge sweep. From theoretical considerations the vortex strength will increase and also the lift on the advancing edge (i.e., with decreasing sweep) but it might be expected that the vortex would move outboard since α/K is reduced at constant incidence. Also the change in the value of the minimum pressure is somewhat less than would be expected from the available theories. Ref. 14 shows that at much larger angles of sideslip ($\geq 15^\circ$) the vortex on the retreating leading edge has moved well outboard and the secondary separation cannot be detected. Integration of the pressure distribution for $\beta = 0^\circ$ and -5° at $\alpha = 15^\circ$ shows that there is only about 1% difference in the total lift thus confirming the balance measurements.

With edge blowing the leading edge vortices are increased in

strength due to the higher rate at which vorticity is shed from the leading edges and in size due to the change in the pressure boundary condition. The increase in vortex strength is shown by the larger suction peaks obtained on the wing upper surface with blowing, see fig. 33. The increase in vortex size results from the ability of the jet sheet to support a pressure difference. Initially, at least, the boundary condition will be of the type applicable to thin curved jets i.e.,

$$\Delta p = \frac{\mu}{R} \quad (1) \quad \text{ref. 45.}$$

where μ is the jet momentum and R the jet radius of curvature assuming approximately two-dimensional conditions in the normal plane to the curve. This may be re-written:

$$-C_p = \frac{C_\mu S}{R} \quad \text{or} \quad -C_p \propto \frac{C_\mu}{R} \quad (2)$$

As mentioned previously, limitations on the traversing gear prevented accurate measurements of static pressure in regions where the pitch of the flow was large and it was not possible to obtain reliable values of the pressure change across the jet sheet. However, the value of $-C_p$ close to the edge may be taken as indicative of the trend. In fig. 33 the static pressure distribution is shown with blowing from the leading edge only at a C_μ of 0.096. Also shown for comparison is the corresponding curve for $C_\mu = 0.048$ at $.33C_0$. This comparison is typical and shows that although C_μ is doubled the value of $-C_p$ only increases by 25%. Thus from equation 2 the value of R must increase.

This increase is confirmed by tuft observations.

In figure 33 also comparison is made between the first series of tests with a constant width slot and blowing from all edges at a C_{μ} of 0.123, and tapered blowing from the leading edge only for a C_{μ} of 0.096 and $\Theta = 0^\circ$. There is clearly an improvement in the chordwise variation of static pressure back to about $0.65C_0$, the effect of blowing on the streamwise edges is small ($\frac{x}{c_0} = 0.87$) except close to the edge. That this does not improve the movement of the centre of pressure fig. 25 is due to the higher lifts induced on the forward part of the wing. Clearly as C_{μ} increases the discrepancy between the forward blown part of the wing and the unblown rear part will increase and it might be desirable to increase the extent of the leading edge blowing. This last quantity, however, will depend on planform, C_{μ} and to a lesser extent on Θ so that the optimum value for each particular configuration would have to be determined separately.

Another consequence of the blowing is to move the vortex cores outboard, for example (see fig. 35), at $\alpha = 15^\circ$ the spanwise movement due to blowing is about $0.12S$. The spanwise position of the vortex varies both with incidence and blowing but at a given incidence the application of sufficient blowing will move the vortex right off the wing surface.

The spanwise movement of the vortex core due to blowing is accomplished in two stages. Firstly, at a fairly low value of C_{μ} (≈ 0.01) the entrainment effect of the jet is sufficient to eliminate the secondary separation. A characteristic feature of the upper surface

pressure distributions with blowing is the absence of the roughly constant pressure region outboard of the main vortex. Without blowing, the air flowing outwards under the vortex core encounters an adverse pressure gradient near the leading edge (free stream pressure is reached at the vortex sheet) and this causes the cross-flow to separate and move away in a chordwise direction. With blowing the cross-flow is entrained by the jet-sheet and the steady removal of air by the sink effect of the jet prevents a build-up of air near the leading edge. Thus the cross-flow can negotiate this region without separating despite the larger adverse pressure gradient. Clearly then, the initial movement of the vortex core is due to the removal of the secondary separation by the entrainment effect of the jet. For moderate increases in C_{μ} the spanwise position of the vortex core is little changed but for much larger values of C_{μ} , however, eqn. 2 indicates that the radius of the jet sheet must also increase considerably and eventually the vortex core moves off the wing surface.

The effect of sideslip on the pressure distribution with leading edge blowing is shown in fig. 34. Also shown is the variation with ϕ . The variation of lift with ϕ is less than suggested by the results given here, see fig. 17. The low values for $\phi = 50^\circ$ are probably due to local variations in the blowing distribution at the leading edge. The change in flow pattern due to sideslip is basically the same as for the unblown case and blowing does not reduce the rolling moments as might have been expected (see § 6.6).

A comparison of both the blown and unblown results is made with

the present theory and also with the theories of Brown & Michael ref. 20 and Mangler & Smith, ref. 23, for $\alpha = 15^\circ$, in fig. 35. Clearly the blown case corresponds more closely to the last two theories than the unblown case. The spanwise position of the vortex changes little with increased blowing, at least up to a C_{μ} of 0.1, and the theory of Mangler and Smith ($C_{\mu} = 0$) predicts this position very accurately. Brown & Michael's results are also more closely allied to the blown case than the unblown case. The present theory shows better general agreement although the values of $-C_p$ are still too high but emphasises the need for a line vortex representation of the secondary separation.

Although the blowing case cannot strictly be compared with the unblown theoretical model, it is interesting to note that while removal of the secondary separation moves the main vortex core outboard, the area of the wing affected by the main vortex is little changed from the unblown case with secondary separation and is much larger than that predicted theoretically by refs. 20 & 23. In the present theory a much larger extent of the wing is affected by the vortices and the reason for this would appear to be that the proportion of vorticity outside of the core is much larger than the value obtained by Mangler & Smith.

A comparison of theoretical and experimental vortex core positions is made in figs. 36 and 37. In the present work these have been obtained both from traverses using a five-tube head and from tuft-grid measurements with a check on the spanwise position from the upper

surface pressure distributions. Vortex positions measured by other investigators, refs. 11 and 47, are included for comparison.

The experimental values of vortex height agree fairly well (fig. 36) and a mean line through the experimental points falls roughly midway between the present theoretical curve and that due to Brown and Michael. The agreement between the experimental points as regards spanwise position is less satisfactory, although the trend of the points given in ref. 47 is rather unusual. The present experimental values and those of ref. 11 show good agreement with the new theory whilst the other theoretical results underestimate the inward movement of the main vortex to a considerable degree.

The blown and unblown cases are compared in fig. 37. At small incidences the vortex strength and height with blowing greatly exceed those without blowing but for $\alpha/K > 1$ the difference is smaller. This is due to the relative amounts of vorticity supplied by the blowing and by the normal leading edge separation without blowing. The height of the vortex core is also influenced to some extent by the increased resistance to rolling up of the jet sheet. This latter also contributes to the spanwise movement of the vortex although most of it at small C_{μ} is due to the removal of the secondary separation by jet entrainment.

6.9 Spanwise traverses using a five-tube pitch-yawmeter

The traversing gear used for measurements in the leading edge vortex is shown in fig. 3. The apparatus was used initially to locate the vortex core as a check on the tuft observations. There was a

certain amount of flexibility on the head and small sideways movements could occur especially when the yaw angles changed sign. The height of the vortex measured by this method is considered reliable but the spanwise position has a possible variation of $\pm .01s$. In general the agreement between tufts and probe was good.

In order to display the fundamental differences between the structure of the vortex with and without edge blowing, traverses were made through the complete vortex structure at 10° incidence and zero sideslip for values of C_{μ} of zero and 0.048 at $0.5C_o$. The results are given in figs. 38a and 38b in which the quantity $\frac{H-p_o}{q_o}$ is plotted against spanwise position. This gives a good indication of the losses occurring and shows the structure very well.

Fig. 38a shows the case without blowing. The extent of the leading edge vortex is defined by the value 1.0 of the variable. The vortex core can be clearly seen and also the region of secondary separation. The minimum value of $\frac{H-p_o}{q_o}$ recorded was $-.79$ which indicates a high axial velocity although the actual value of velocity was not calculated owing to the uncertain measurement of static pressure in the core.

Fig. 38b shows the case with blowing. Clearly a much larger extent of the flow is affected and it is possible to trace the effect of the jet for almost a full turn of the sheet. The vortex height has increased and the core is located further outboard. The secondary separation has been eliminated. The axial velocities in the core have also increased, the maximum value of $\frac{H-p_o}{q_o}$ is now -2.5 indicating an increase of velocity of the order of two over the unblown case.

7. Conclusions

Slot blowing from the edges of highly swept wings has been suggested as a means of improving their low-speed characteristics and low-speed wind tunnel tests have been made in order to investigate the effects of edge blowing on a particular planform, in this case a 70° Cropped Delta Wing.

The main conclusions are as follows:

1. Blowing from the leading edge and tips increases the lift by increasing the size and strength of the leading edge vortices, i.e., the increase in lift is due mainly to an increase in the non-linear contribution.
2. The plane jet sheets issuing from the swept leading edges or streamwise tips roll up to form vortices giving stable flow patterns with the wing at incidence. Close to zero incidence, however, the jet sheet oscillates from one surface to the other due to a downwash lag effect, producing a sinusoidal lift. This effect occurs only for incidences of less than 0.25° and seems unlikely to be a practical limitation.
3. In the first set of tests with blowing from an open slot at all edges the blowing distribution was not entirely satisfactory. The use of small perspex wafers suitably grooved enabled both the blowing distribution and direction to be controlled and in one case at least the lift was increased with virtually no change in the position of the centre of pressure.

4. With tapered leading edge blowing only, the lift magnification $\Delta C_L / C_{\mu}$ at a typical landing incidence of 15° is at least three up to values of C_{μ} of 0.05.
5. Tests were made both with and without blowing at wind speeds of 50 ft/sec to 200 ft/sec (R_e of 0.8×10^6 to 3.2×10^6) and no appreciable Reynolds number effect was observed.
6. In all the cases tested sideslip up to $\pm 5^\circ$ had no appreciable effect on lift, drag, or pitching moment.
7. Surface static pressure distributions were recorded both without blowing, and with blowing from all edges and from the leading edge only, for a range of incidence at four chordwise stations. Results without blowing are in general agreement with other work and with blowing the following effects are observed:
 - (a) Edge blowing increases the peak suction at all chordwise stations but the effect is more pronounced at the forward stations with the wing at incidence where a powerful leading edge vortex already exists without blowing.
 - (b) Blowing moves the vortex core upwards and outwards at constant incidence. The spanwise movement is considerable, for instance at $\alpha = 25^\circ$ the vortex moves from 0.65 to 0.80 of the semi-span.
 - (c) Without blowing, the secondary separation can be seen as a region of roughly constant pressure outboard of the main suction peak but is eliminated by the entrainment effect of the jet with blowing.
 - (d) The importance of the secondary separation is shown by the good agreement for the spanwise position of the vortex between the results

with blowing (no secondary separation) and the theory of Mangler & Smith ($C_{\mu} = 0$, secondary separation neglected).

8. Tests have been made with various forms of asymmetric blowing and the results with slot blowing from one wing tip only are presented here. Appreciable gains in lift are still achieved, for example at $\alpha = 25^{\circ}$ with a C_{μ} of 0.12 the value of C_L is 1.14 compared with a value of 1.16 obtained with blowing from all edges. In this case, however, tip blowing results in a nose-down pitching moment as most of the increased lift is generated on the rear part of the wing. This type of asymmetric blowing produces large rolling moments and it would seem possible to trim up to 4° of sideslip by this means.

9. Calculations in the Appendix show that on landing a maximum C_{μ} value of 0.0275 can be achieved using compressor bleed giving a reduction in landing speed of about fifteen knots. This is roughly the same reduction that is achieved on more conventional aircraft using blowing boundary layer control on trailing edge flaps. With a suitable blowing arrangement there will be no movement of the centre of pressure making this method particularly attractive for tailless aircraft.

Appendix

In order to calculate the maximum available C_{μ} and its effect on lift and reduction in landing speed the following assumptions were made:

Total thrust 100,000 lbs.

Engine mass flow 1850 lbs/sec.

Assuming 12 $\frac{1}{2}$ % compressor bleed for blowing air = 232 lbs/sec = 7.25 slugs/sec. Further, assume bleed air at pressure ratio of 3:1 and maximum expanded velocity 1300 ft/sec.

Wing area = 6000 sq ft.

Touch-down speed without blowing 145 knots.

Landing incidence 15°, at which $C_L = 0.50$ without blowing

To reduce landing speed to 130 knots $C_L = 0.50 \left(\frac{145}{130} \right)^2 = 0.62$

At 130 knots $C_{\mu} = \frac{7.25 \times 1300}{57.3 \times 6000} = 0.0275$

∴ $C_L = .615$ (see fig. 18)

Thus the reduction in landing speed is approximately 15 knots.

This air must be ducted from the engines along the leading edge and the duct area will be, assuming gas temperature of 150°C

Volume flow $\hat{=} 7.25 \times \frac{423}{273} \times \frac{1}{.002378 \times 3} = 1580$ cu.ft/sec.

With a maximum duct velocity of 250 ft/sec, this gives a duct area of about six square feet or three square feet per side, maximum. Naturally the area will decrease towards the nose since the volume flow decreases. Thus the ducting will compose a maximum of about 5% of the cross section area. It is not possible to say what effect this

will have on performance but it will certainly be much less than with some schemes e.g., leading edge extensions, and would seem to be a practical proposition.

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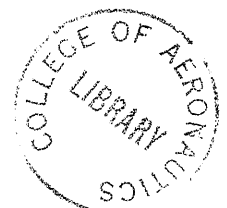
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Harris, T.M. wing with peripheral jet. College of
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Merrick, V.K. of minimum-field aircraft. Eighth Anglo-
American Aeronautical Conference. Sept. 1961.

FIGS. 4, 5.

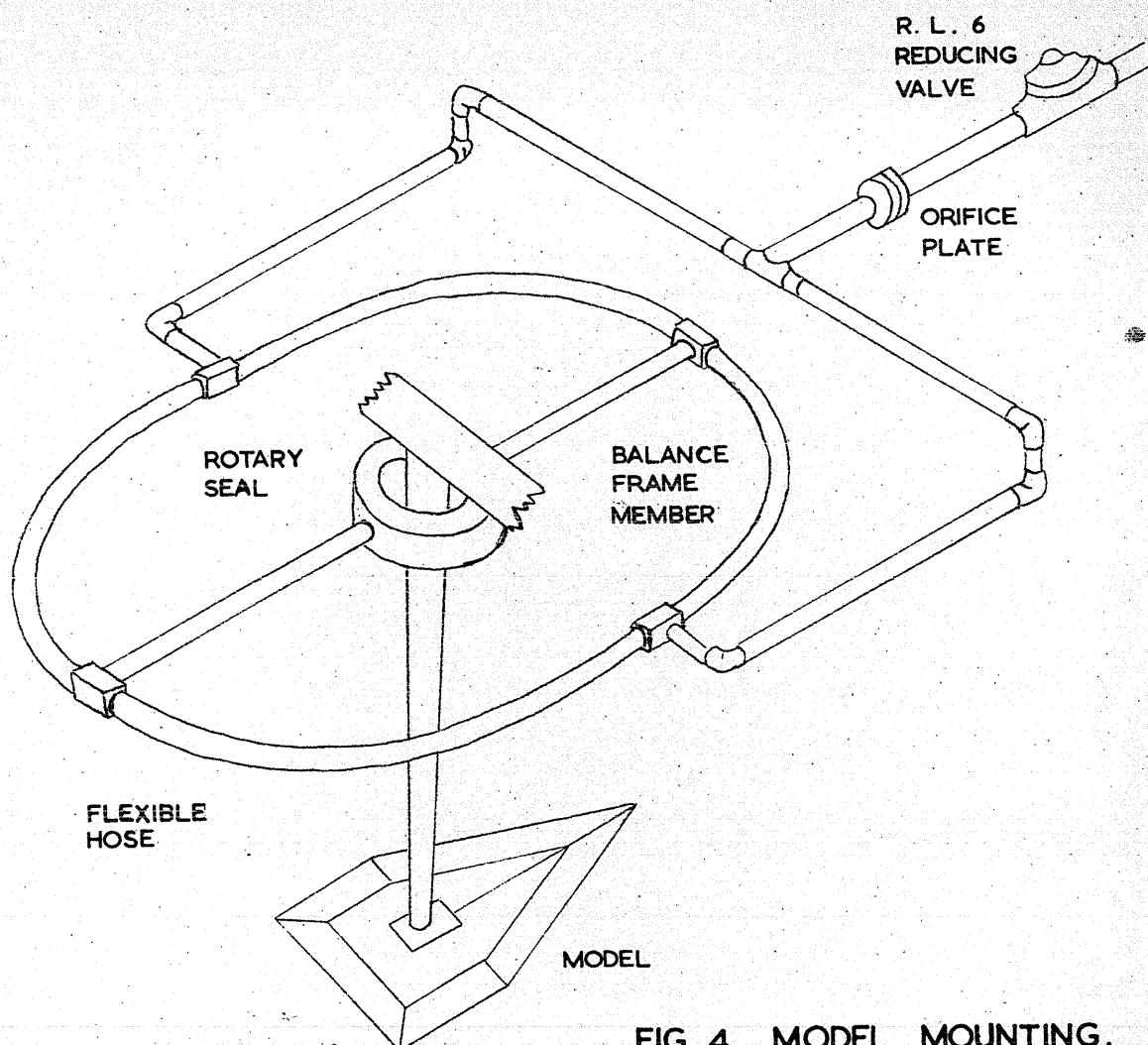


FIG. 4. MODEL MOUNTING.

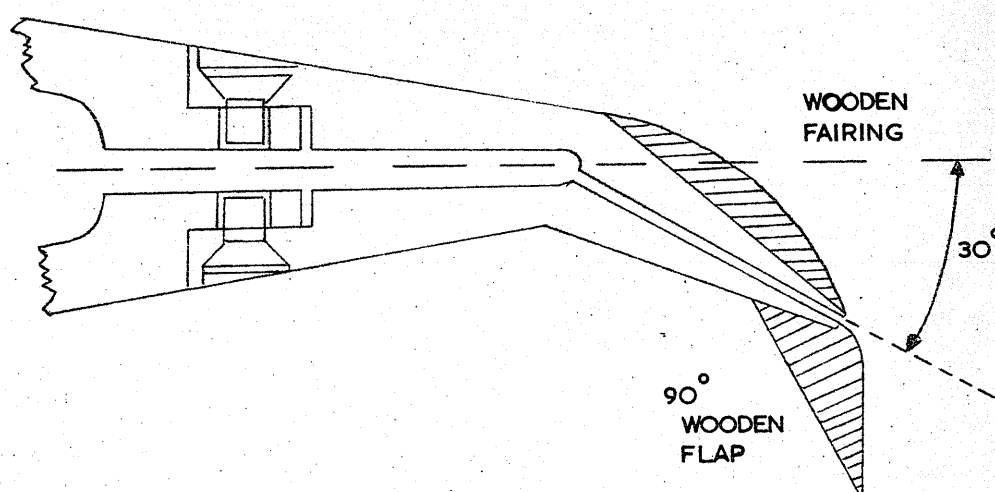
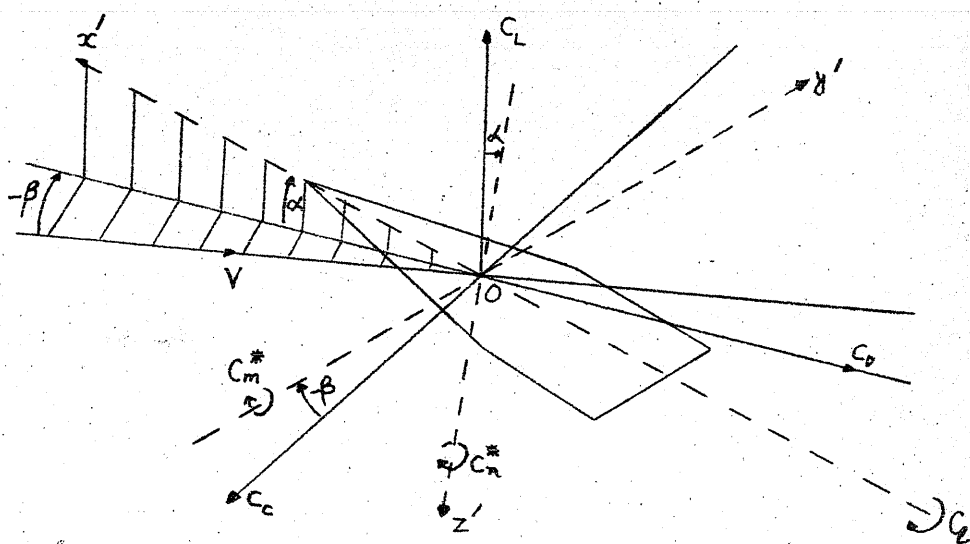


FIG. 5.
30° DROOPED EDGE.

A diagram illustrating a wedge-shaped assembly of Perspex slips. The main part of the diagram shows a large, rectangular block with a wavy top edge and a dashed line indicating a cut. A section of the block is shown in cross-section, revealing multiple layers of Perspex slips. The top surface of this section is labeled "PERSPEX SLIPS" with an arrow. An angle θ is indicated between the top surface and the side of the block. To the right, a smaller, more detailed view shows a series of vertical Perspex slips stacked on top of each other, forming a wedge shape.

EDGE SLOTS FOR TAPERING AND DIRECTING JET.



FORCE AND MOMENT AXES SYSTEM.

FIGS. 8a, 8b.

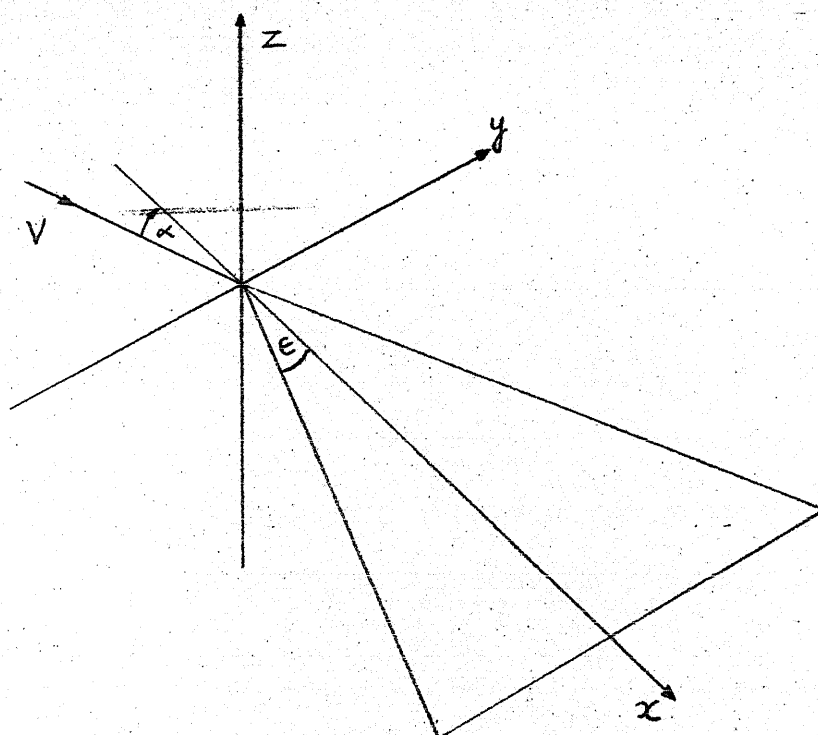


FIG. 8a.
SYSTEM OF AXES FOR THEORETICAL MODEL.

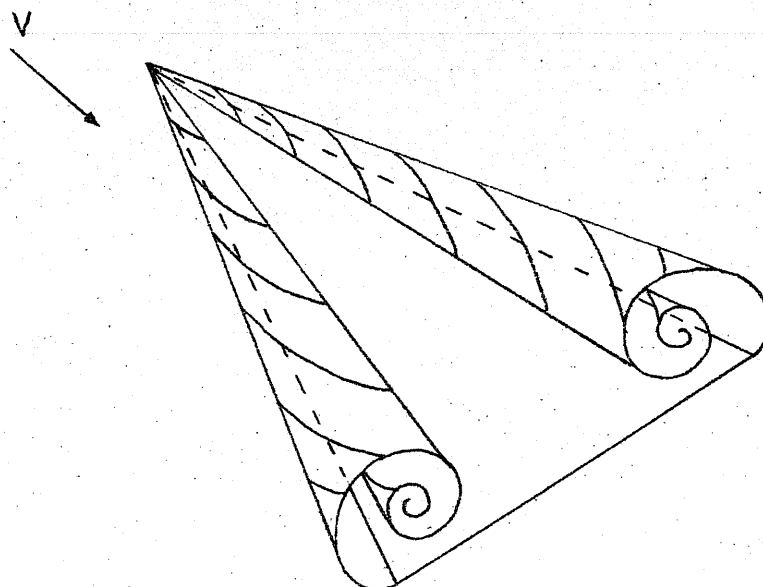


FIG. 8b.
VORTEX SYSTEM WITHOUT SECONDARY SEPARATION.

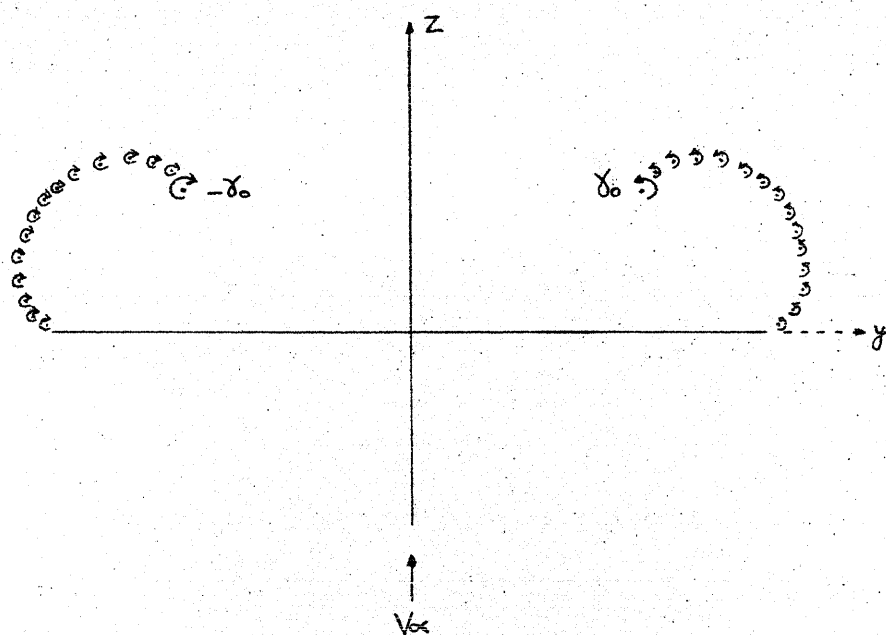


FIG. 8c.
IDEALISED MATHEMATICAL MODEL.

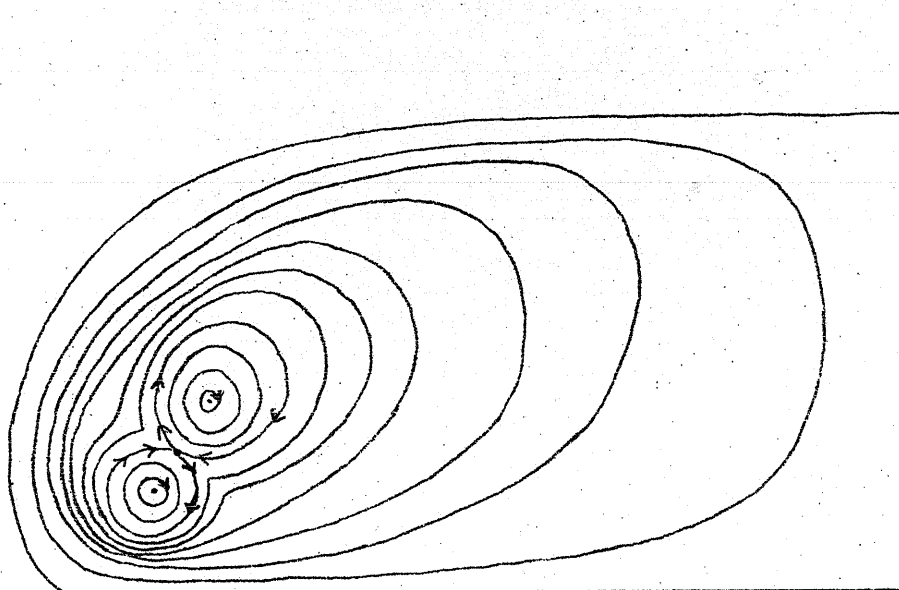


FIG. 8d.
APPROXIMATE STREAMLINE PATTERN OF PROPOSED MODEL. $\alpha/K \approx 1$

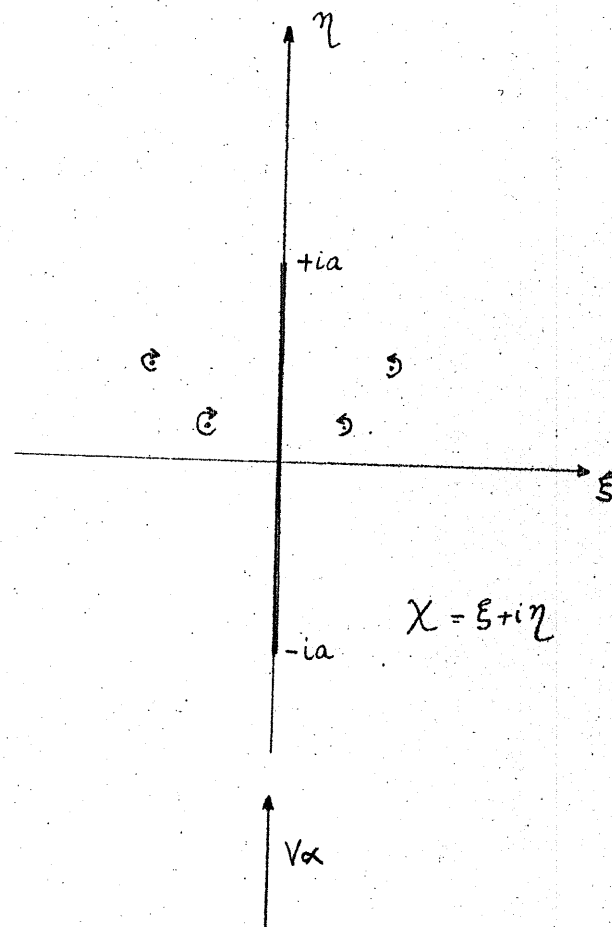
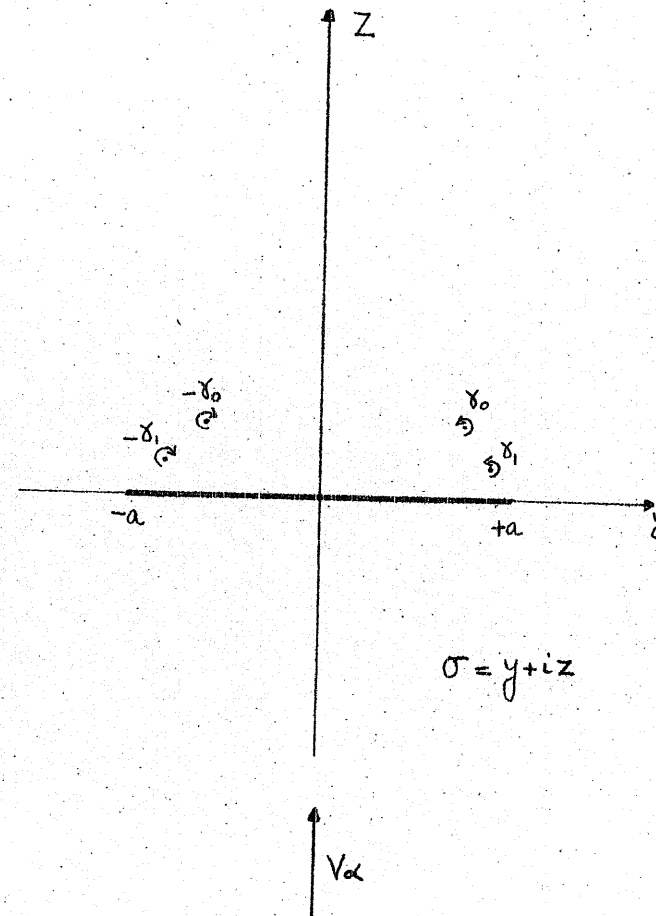


FIG. 8e.

MATHEMATICAL REPRESENTATION IN CROSS-FLOW PLANE.

FIG. 8e.

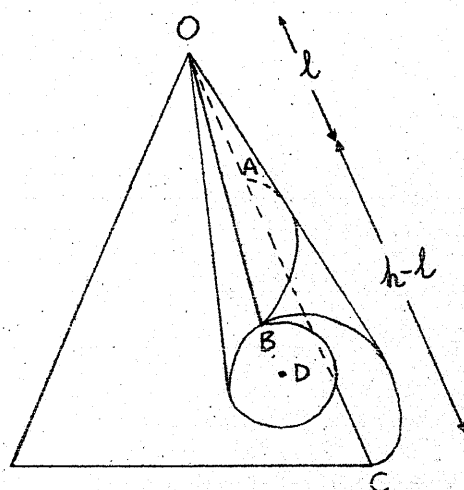


FIG. 8f.

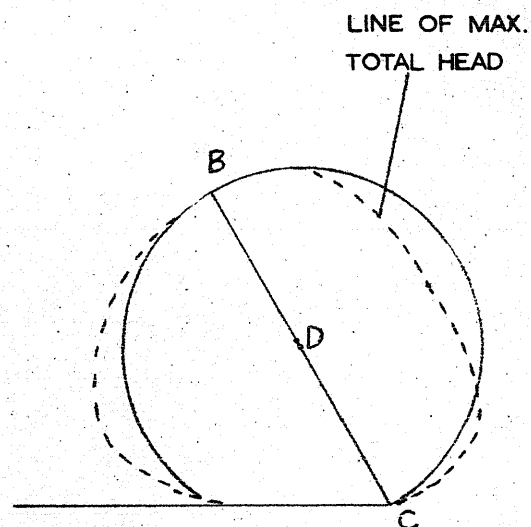


FIG. 8g.

REPRESENTATION OF THE VORTEX SHEET.

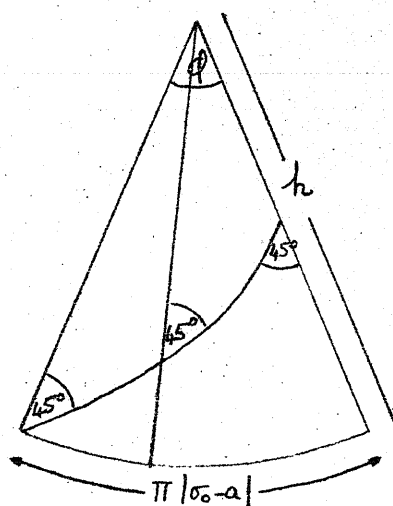
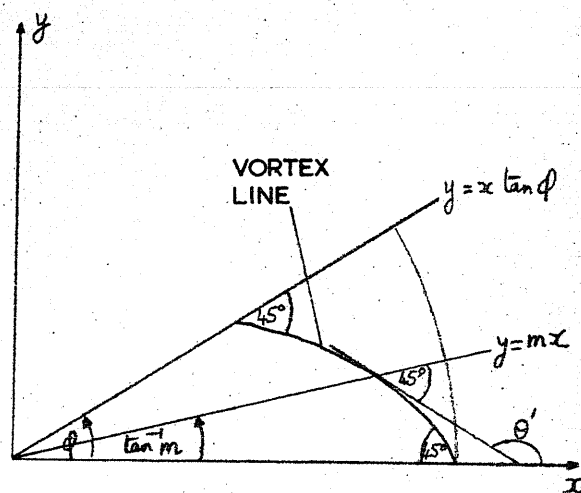


FIG. 8h.

UNWRAPPING THE VORTEX SHEET.



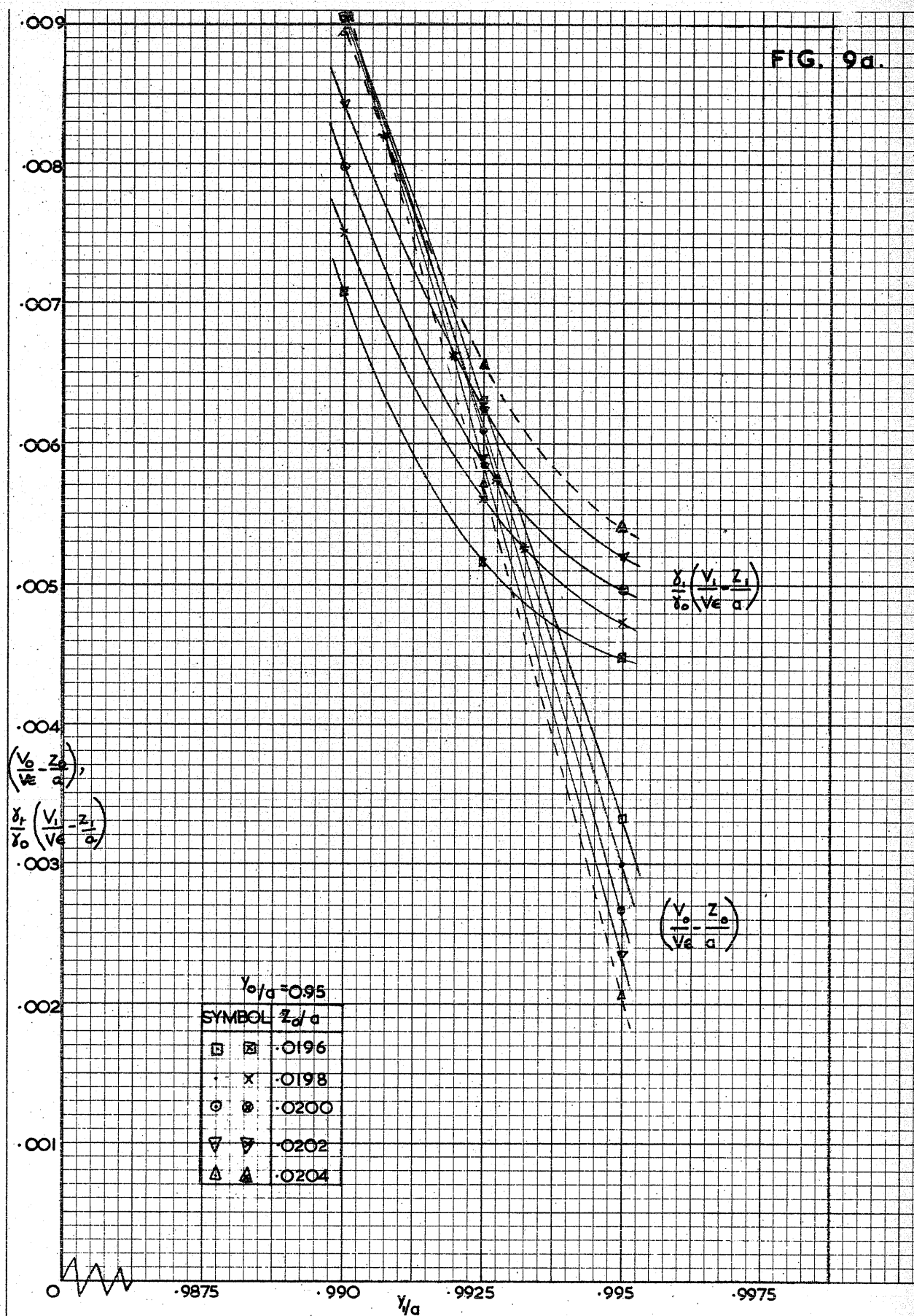


FIG. 9a.

VARIATION OF VELOCITIES AT σ_0 AND σ_1 WITH y_1/a .

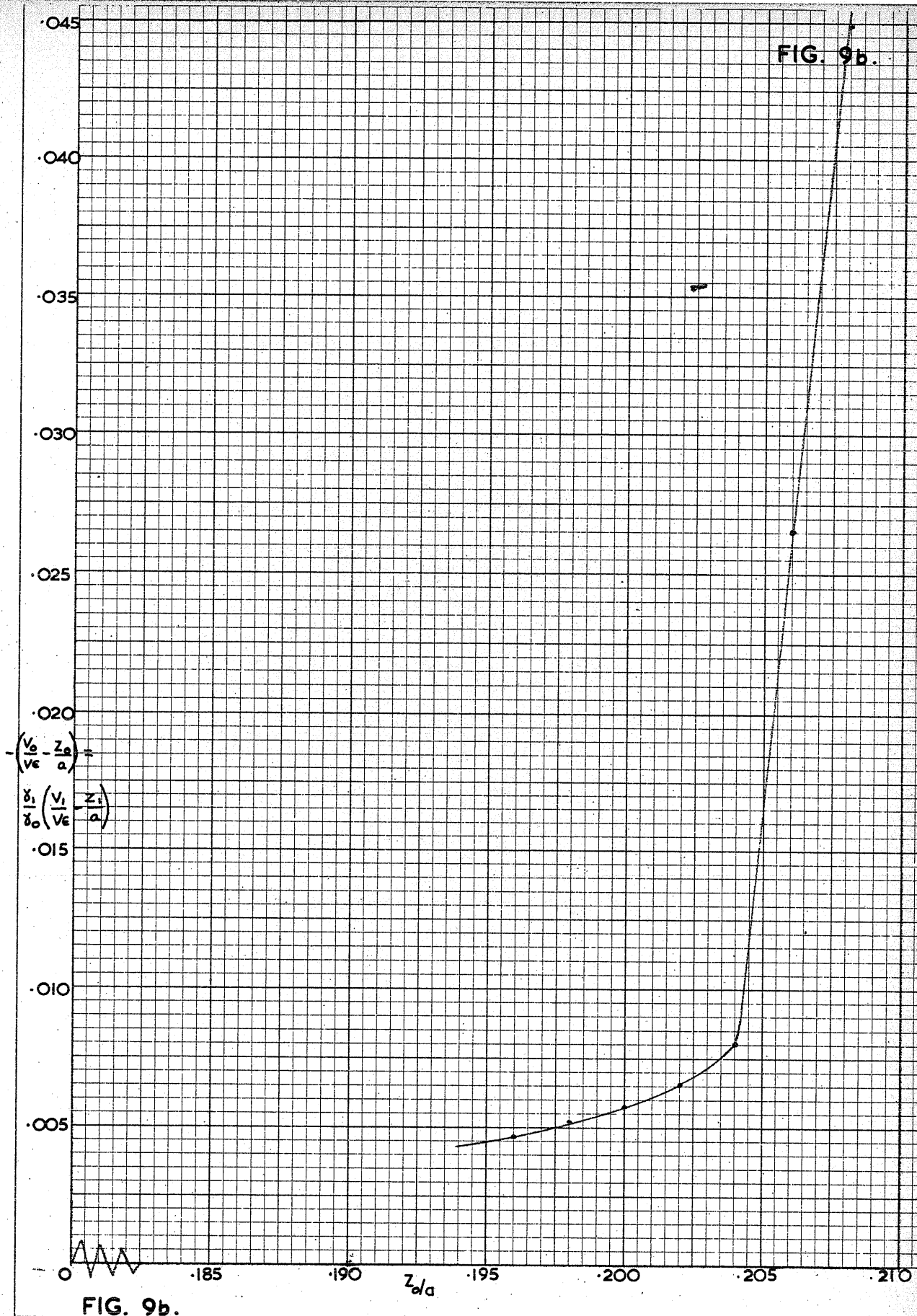


FIG. 9b.

VARIATION OF EQUILIBRIUM VELOCITIES AT σ_0 AND σ_1 WITH Z_0/a .

FIG. 10.

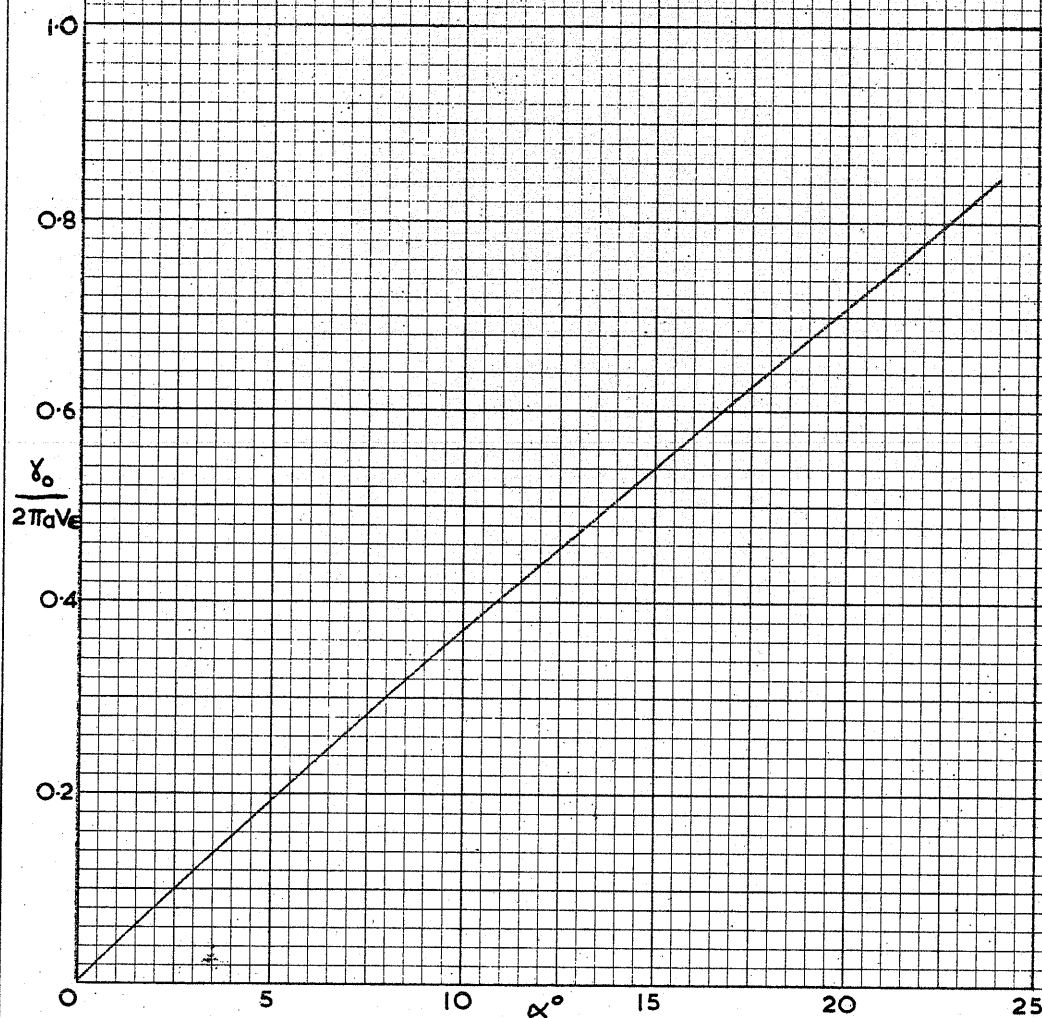
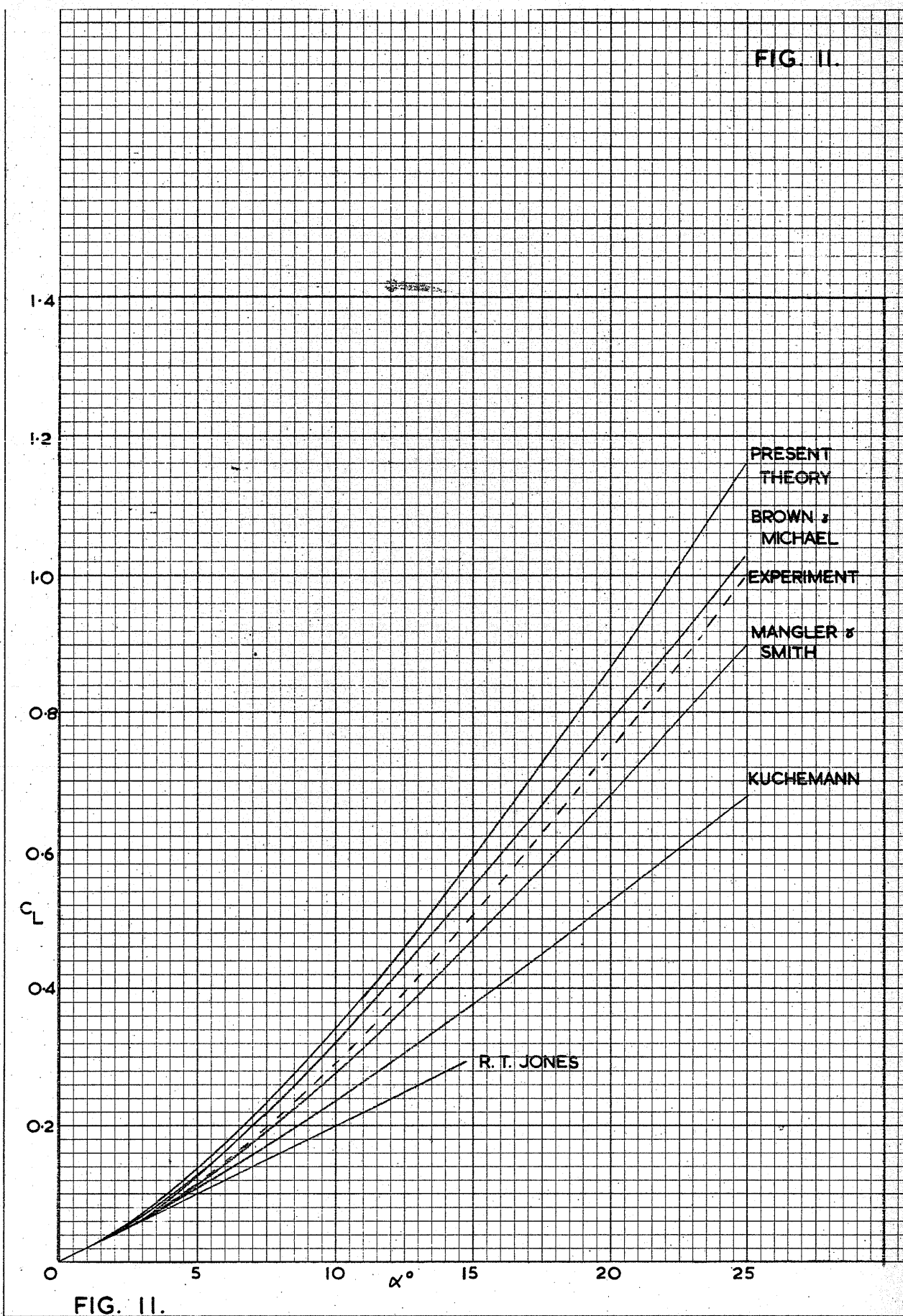


FIG. 10.

VARIATION OF MAIN VORTEX STRENGTH WITH INCIDENCE. $C_{\mu} = 0$.



COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS.

FIG. 12.

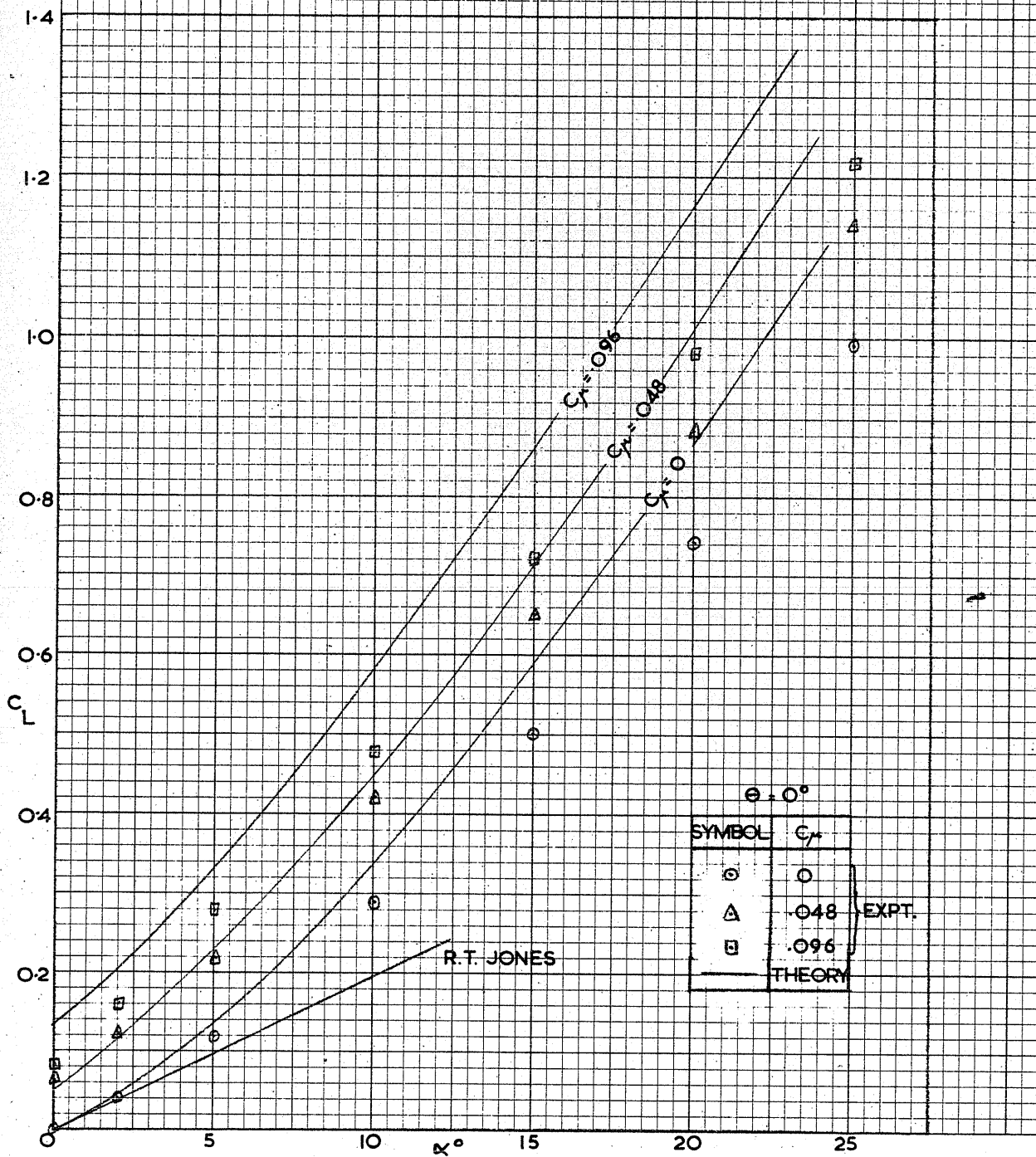


FIG. 12.

COMPARISON OF THEORETICAL BLOWING RESULTS WITH EXPERIMENT.

FIG. 13.

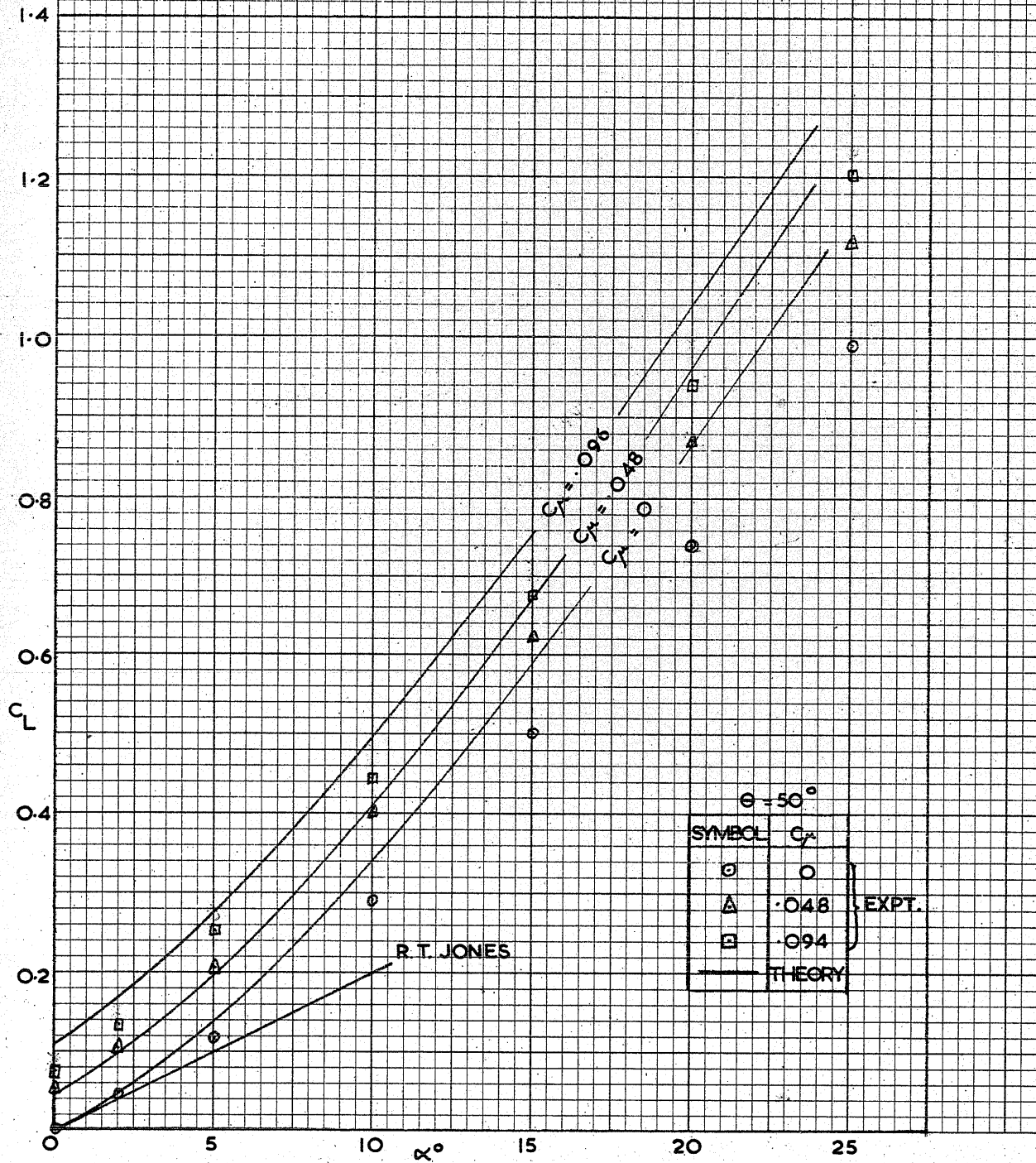
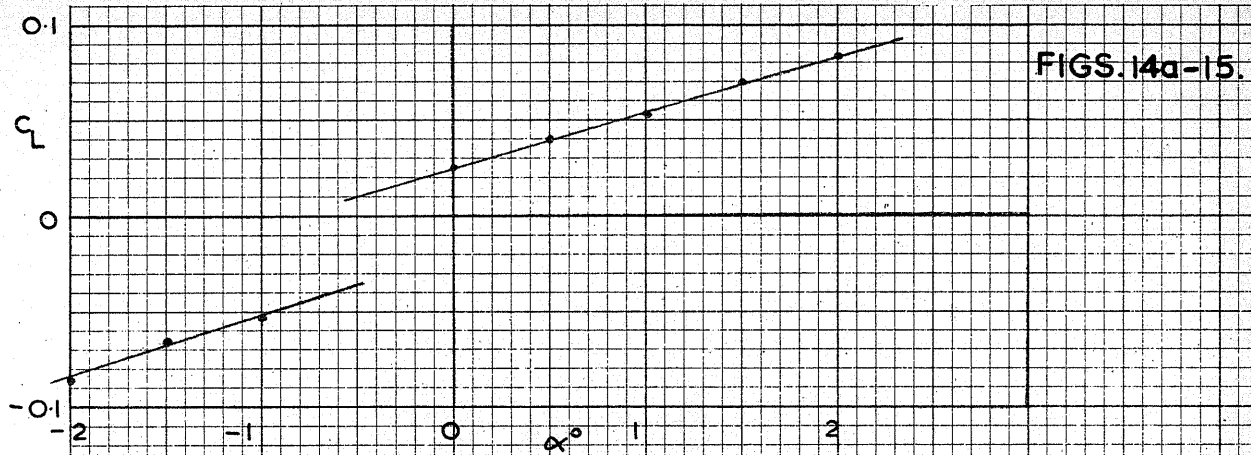
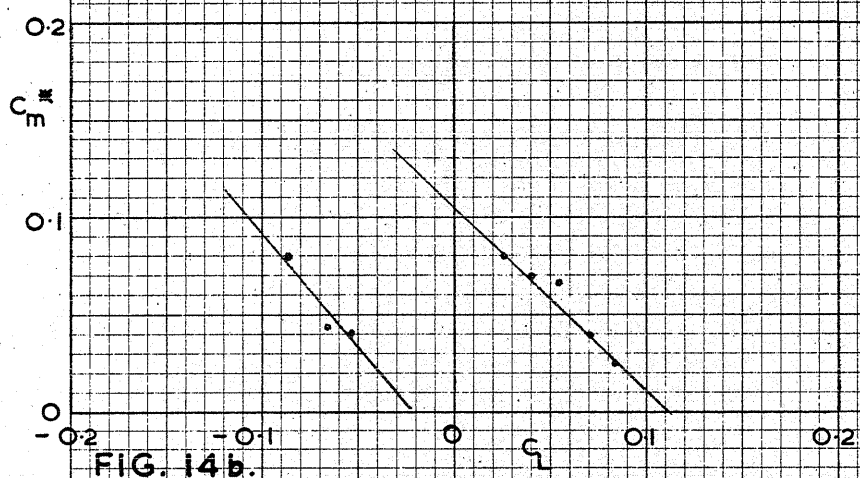


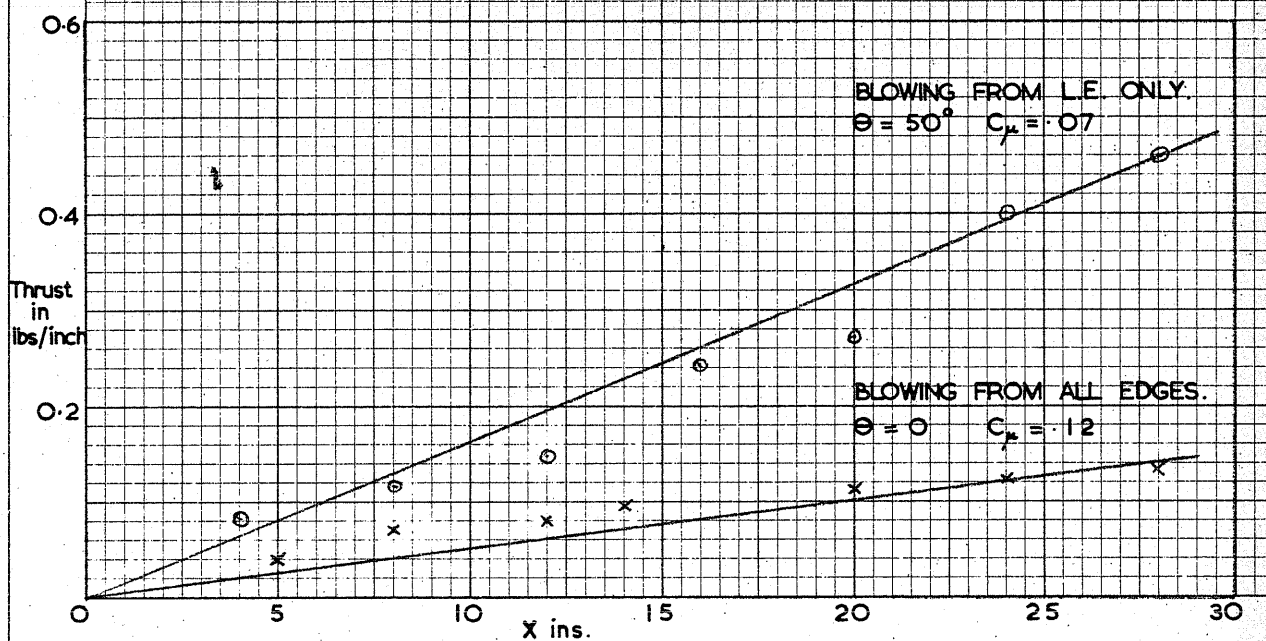
FIG. 13.

$\theta = 50^\circ$

COMPARISON OF THEORETICAL BLOWING RESULTS WITH EXPERIMENT.



JET SHEET INSTABILITY NEAR ZERO INCIDENCE, BLOWING FROM ALL EDGES.



BLOWING MOMENTUM DISTRIBUTION ALONG LEADING EDGE.

FIG. 16.

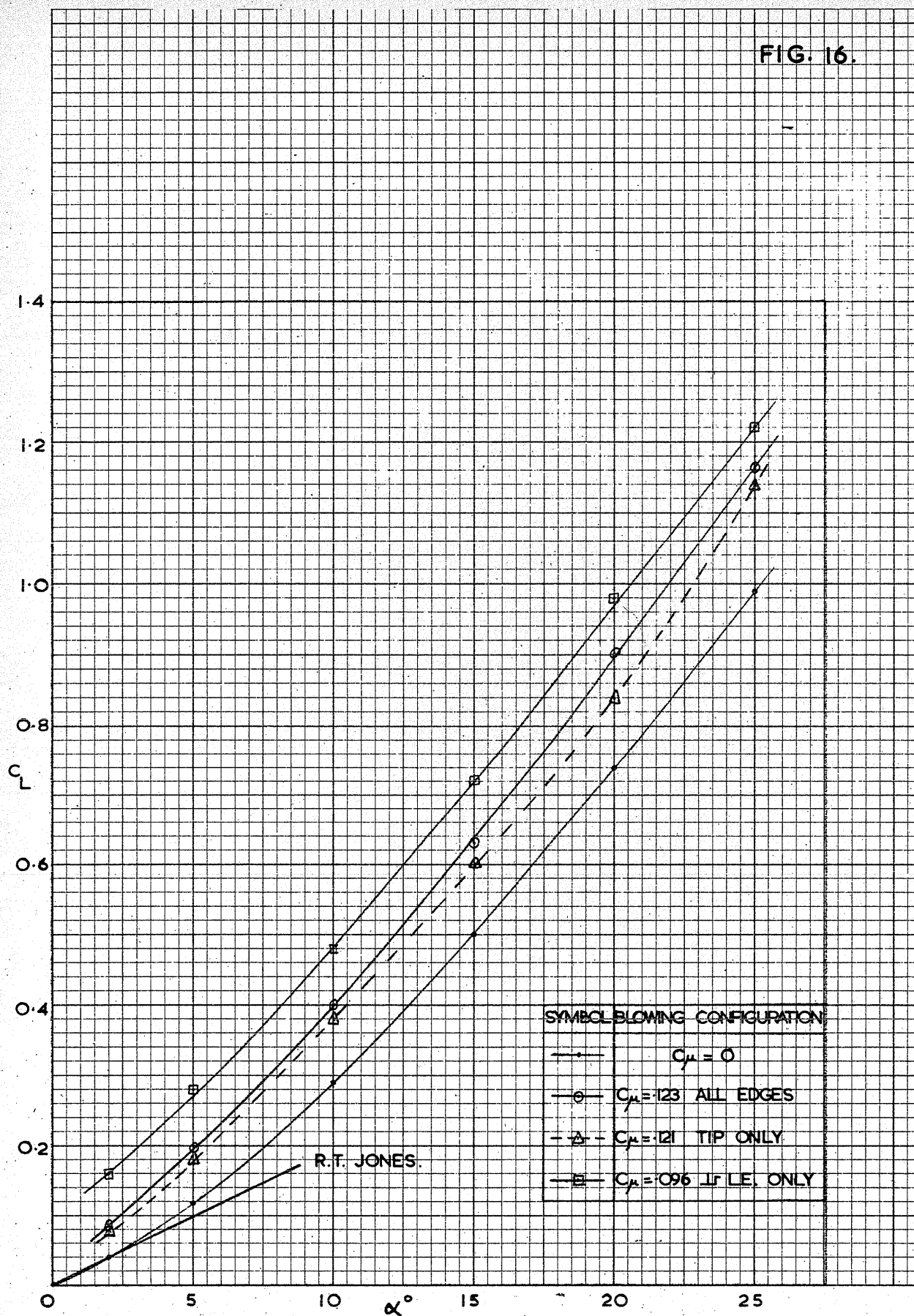


FIG. 16.

COMPARISON OF BLOWING CASES FOR $C_\mu \approx 0.1$

FIG. 17.

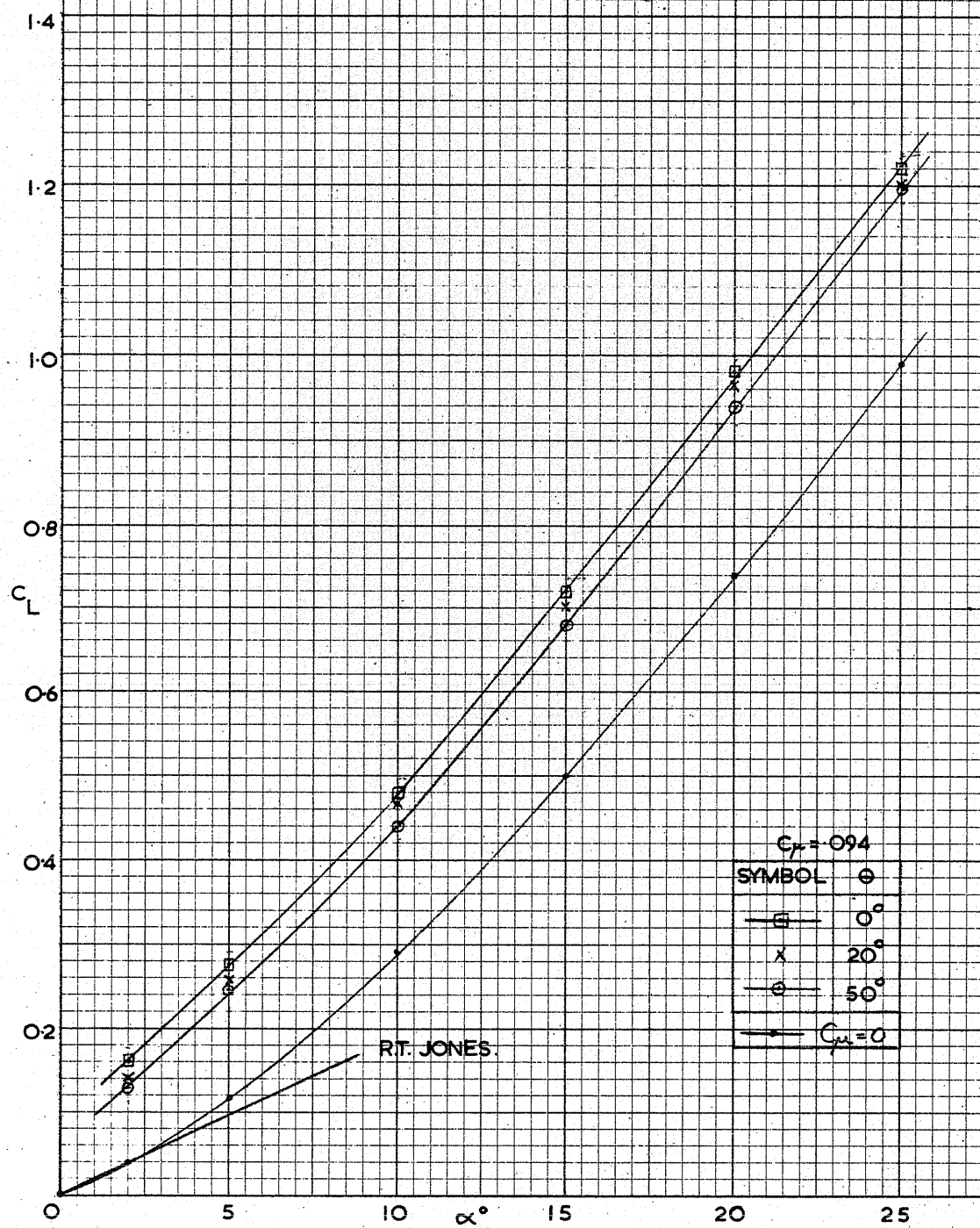


FIG. 17.

VARIATION OF LIFT WITH JET SWEEP ANGLE θ . BLOWING FROM L.E. ONLY.

FIG. 18.

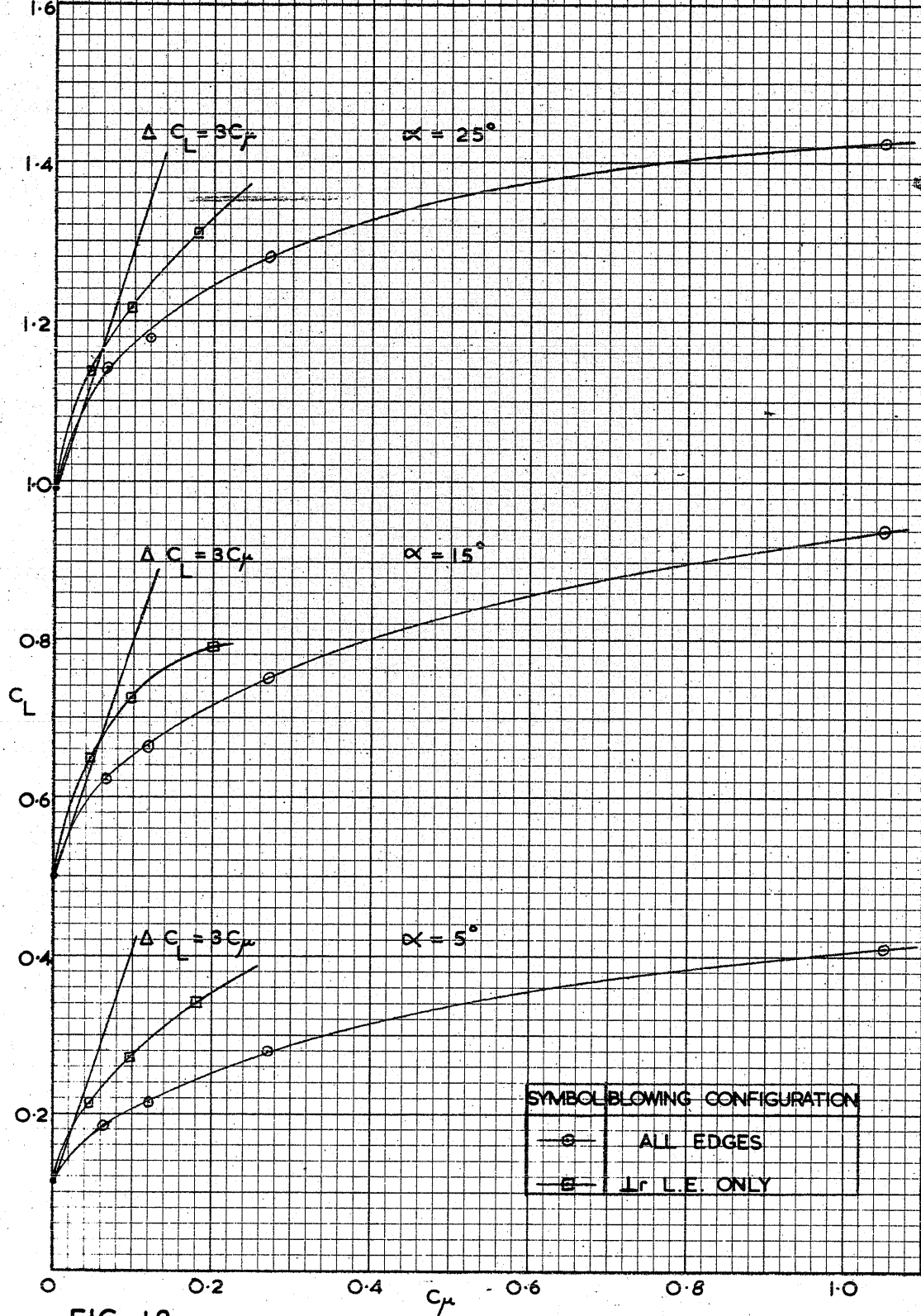


FIG. 18.

VARIATION OF LIFT WITH MOMENTUM COEFFICIENT.

FIGS. 19, 20.

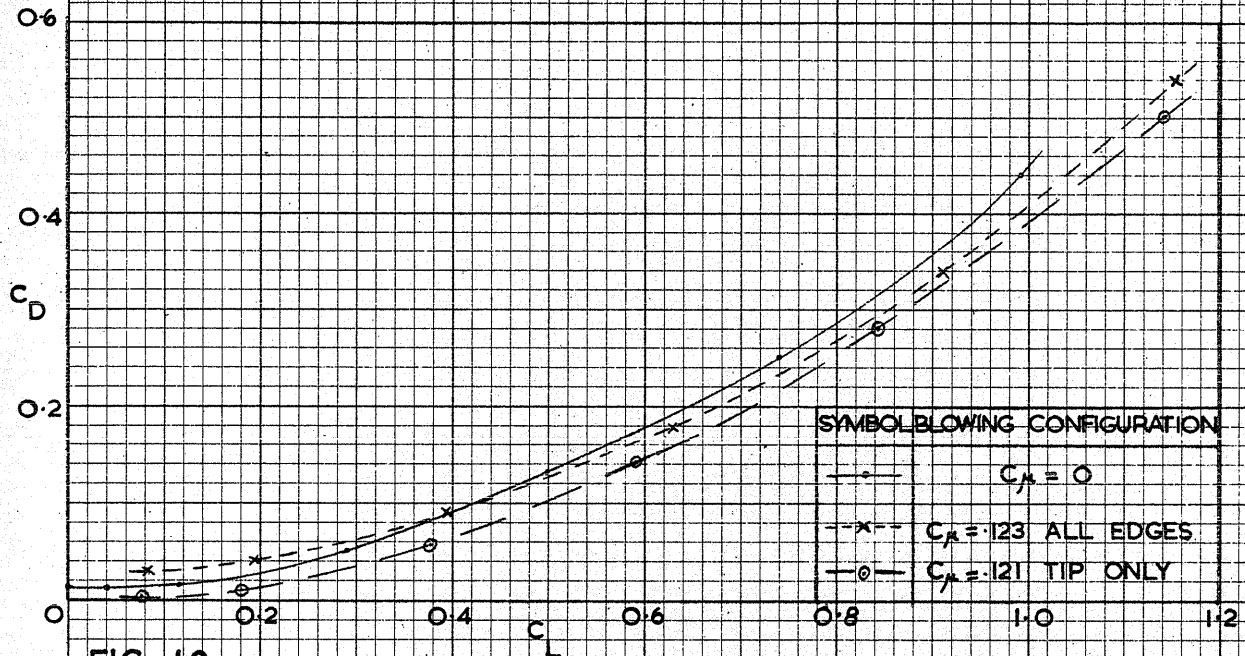


FIG. 19.

VARIATION OF DRAG WITH LIFT.

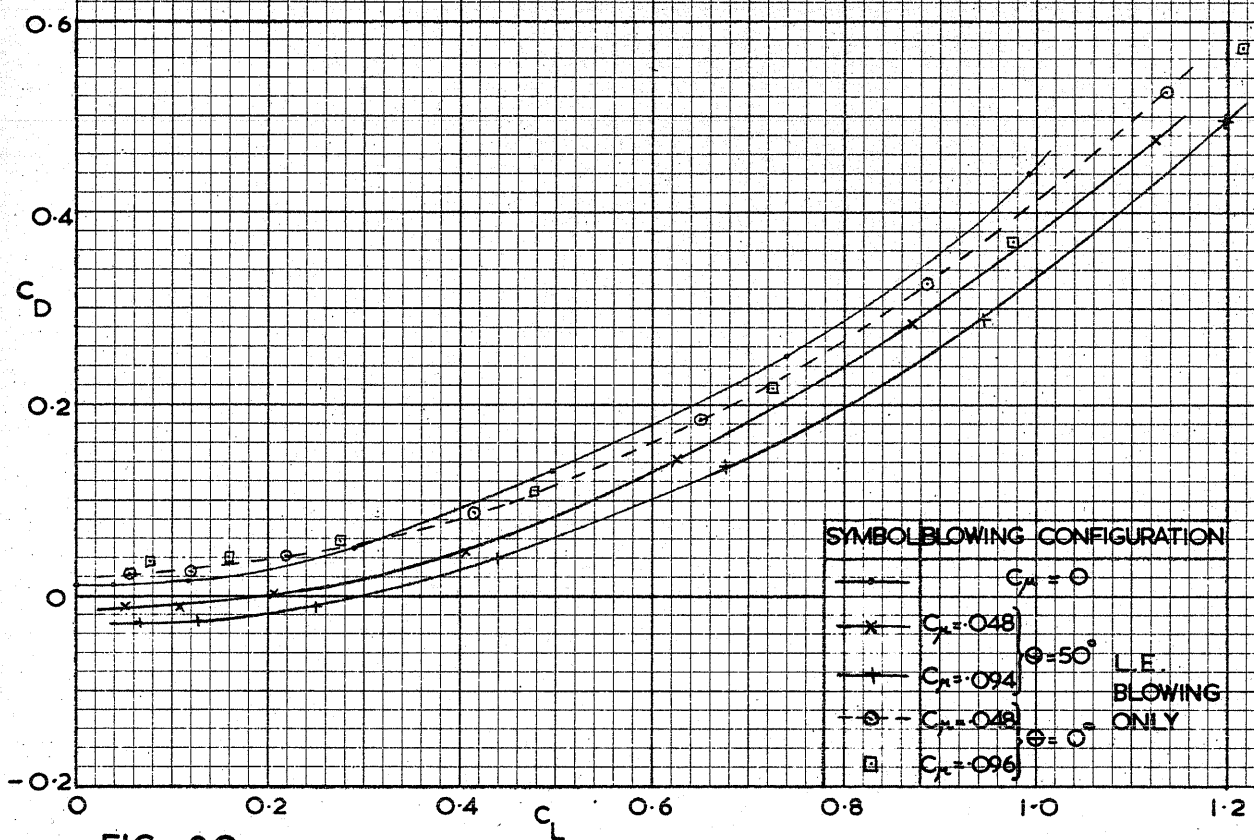


FIG. 20.

VARIATION OF DRAG WITH LIFT.

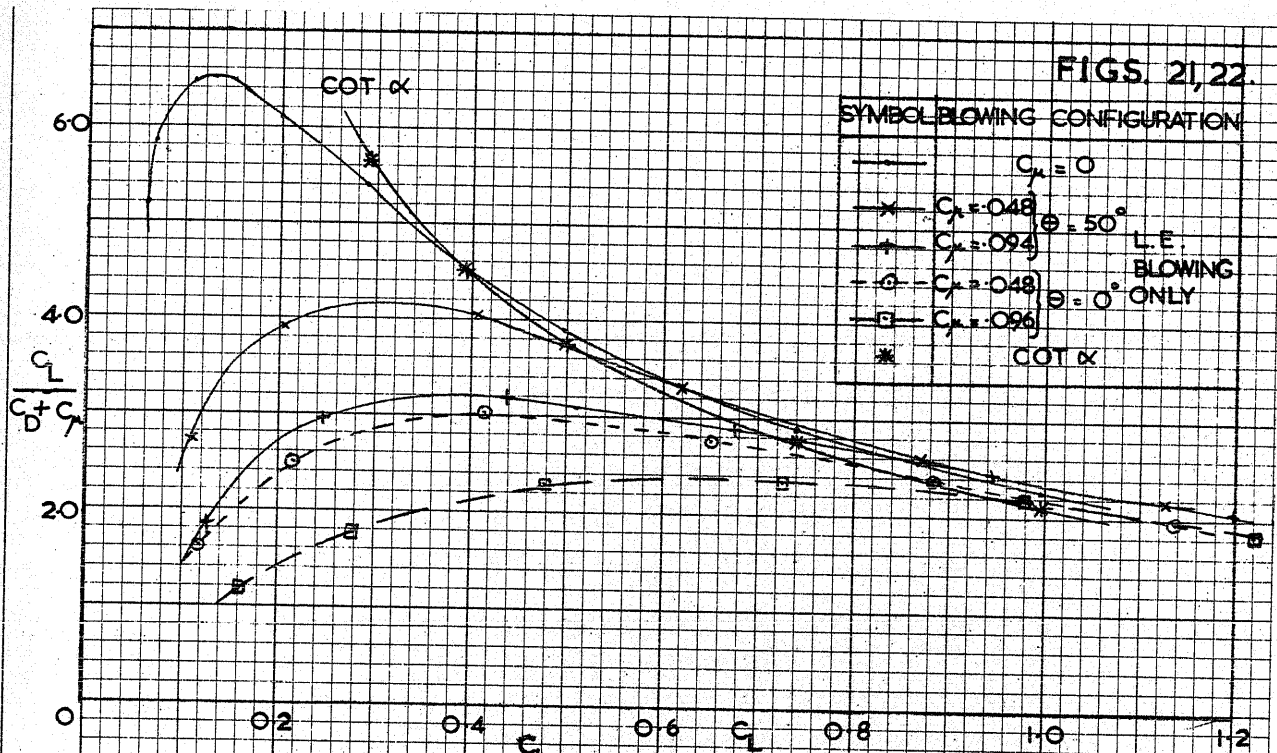


FIG. 21. VARIATION OF $\frac{C_L}{C_D + C_{\mu}}$ WITH LIFT FOR VARIOUS BLOWING CONFIGURATIONS.

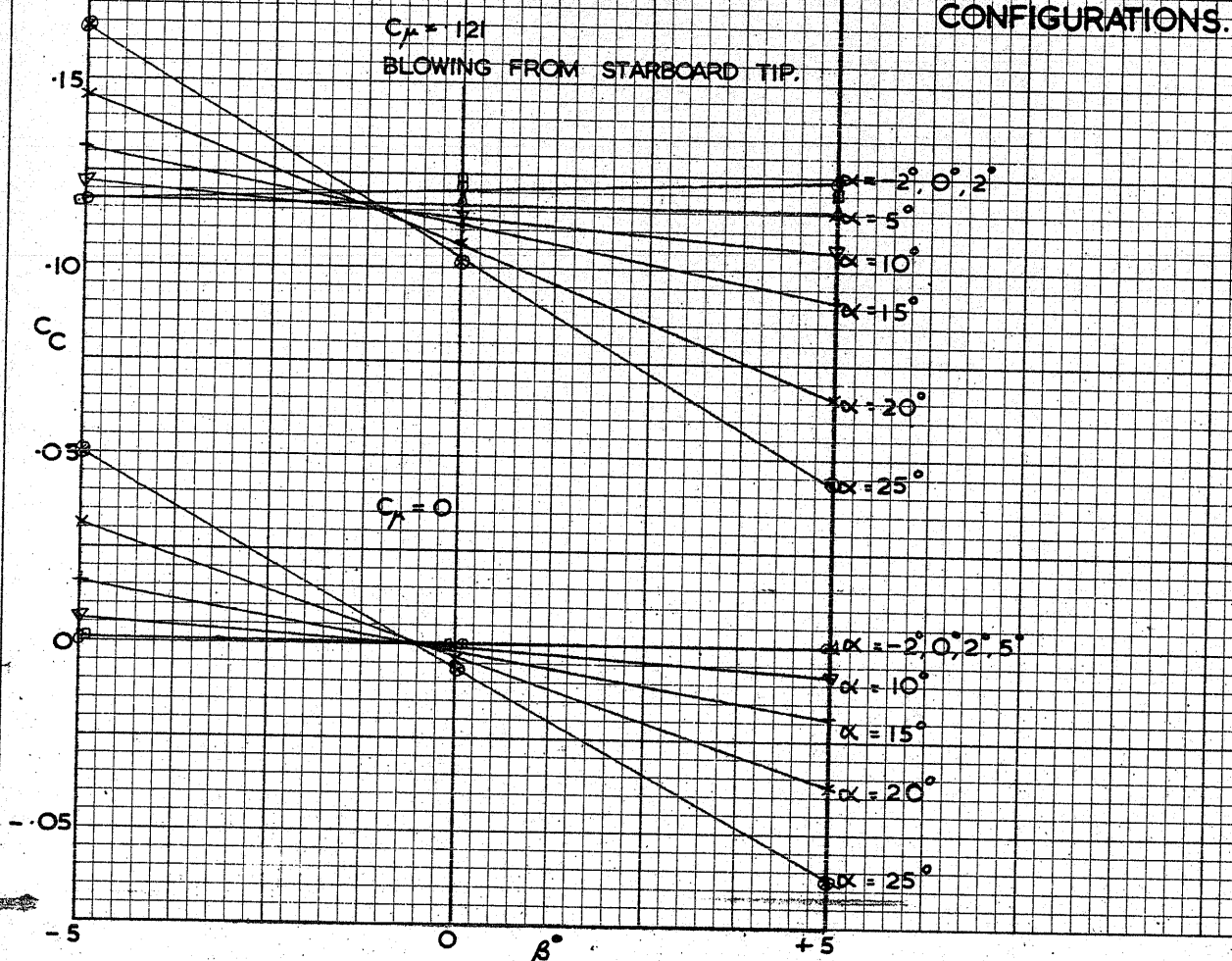
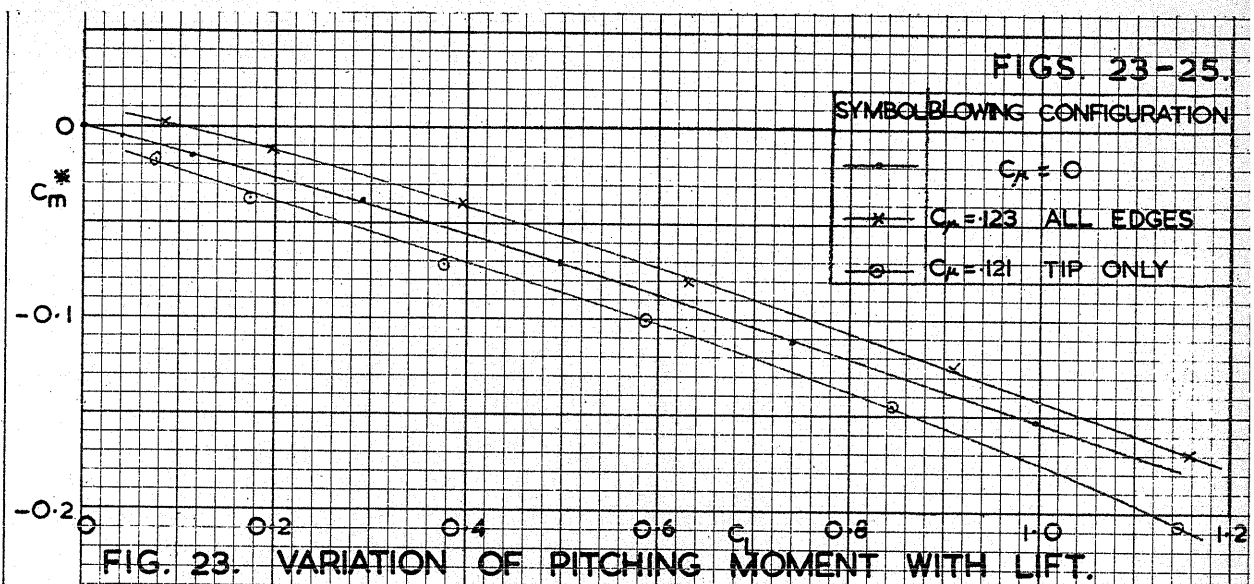
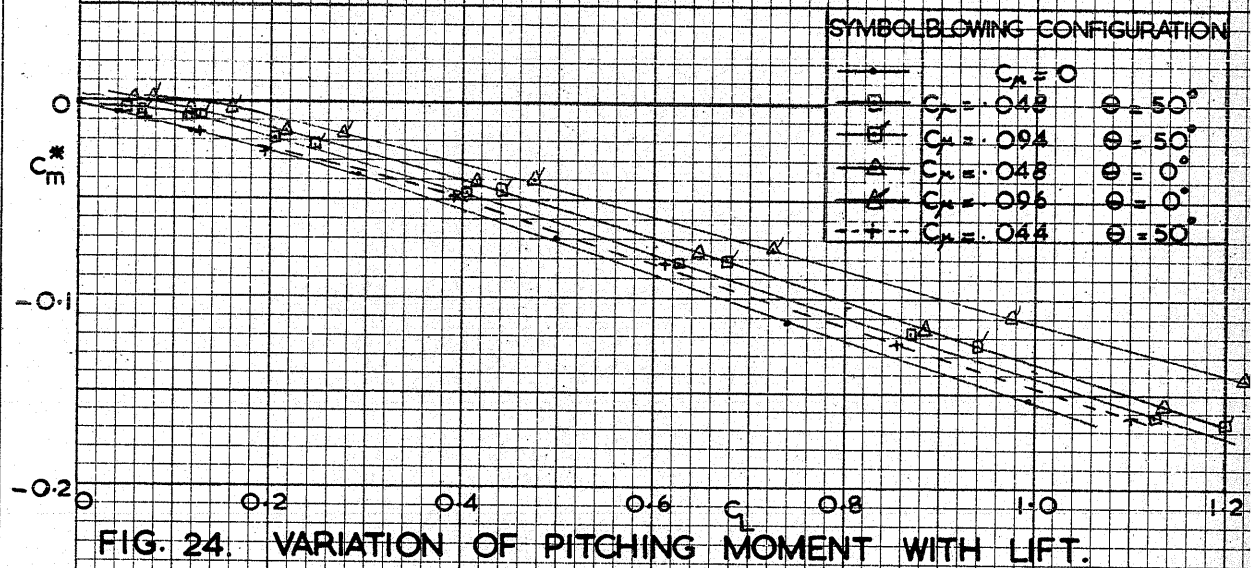
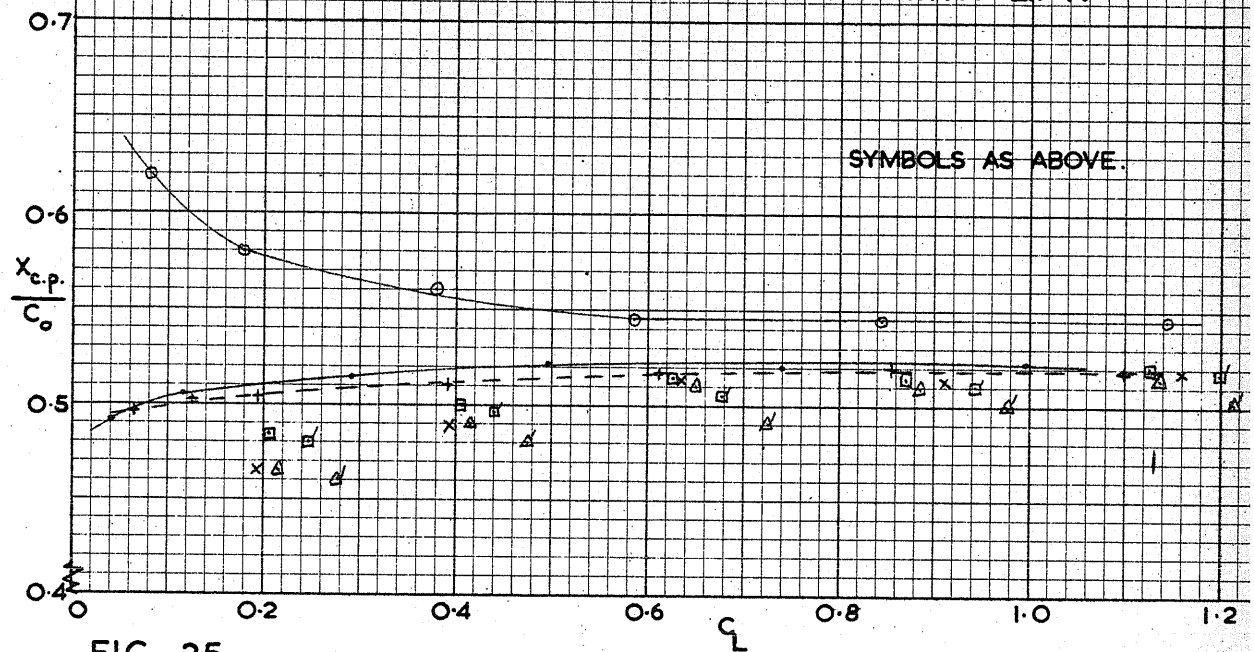
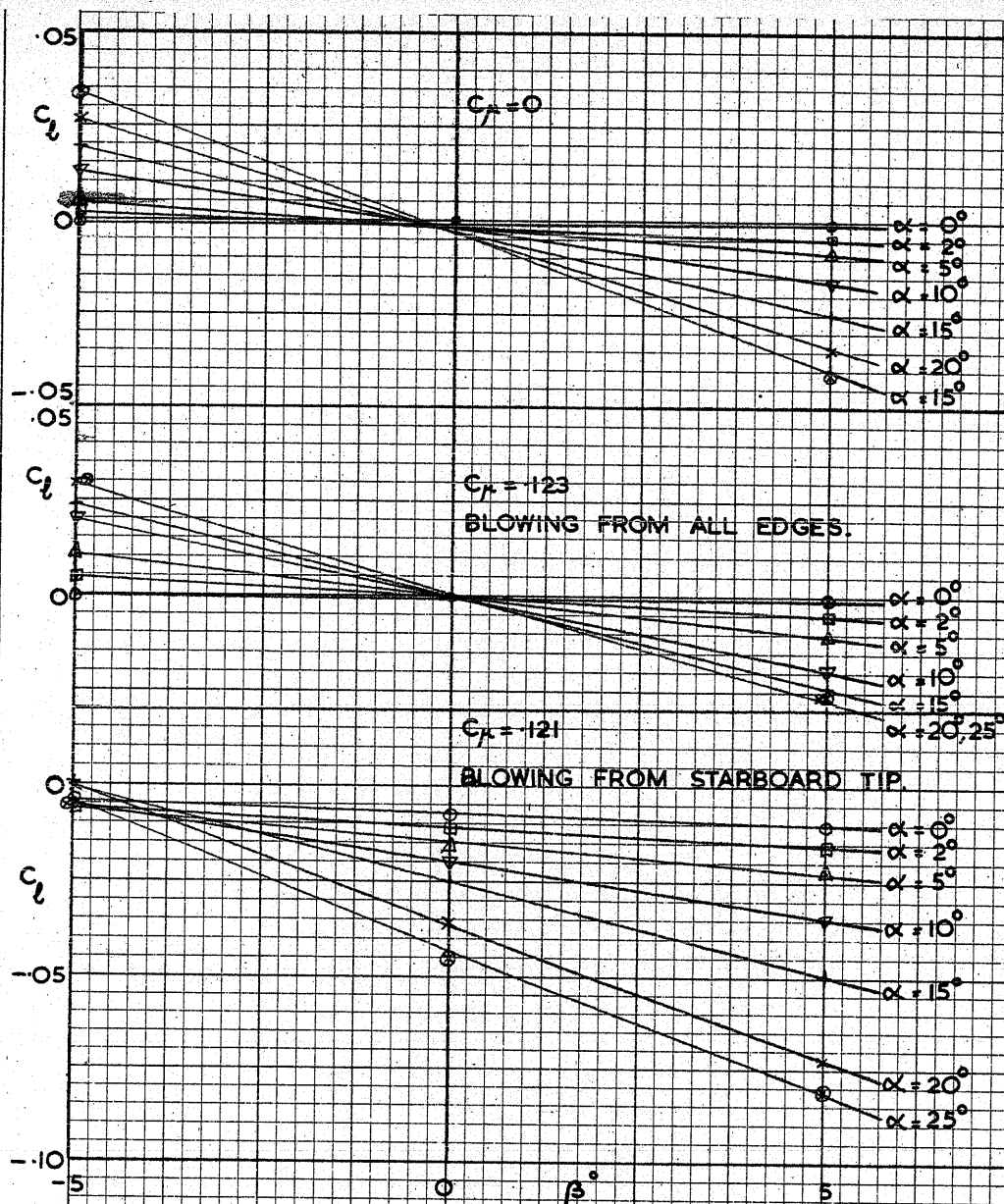
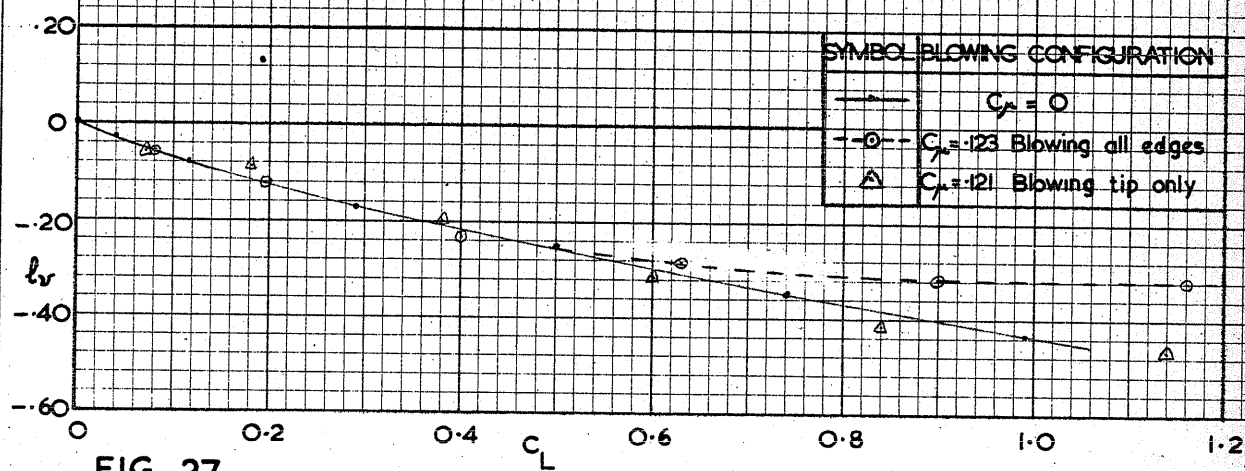


FIG. 22.

VARIATION OF CROSS WIND FORCE WITH SIDESLIP ANGLE β .



MOVEMENT OF CENTRE OF PRESSURE WITH LIFT.



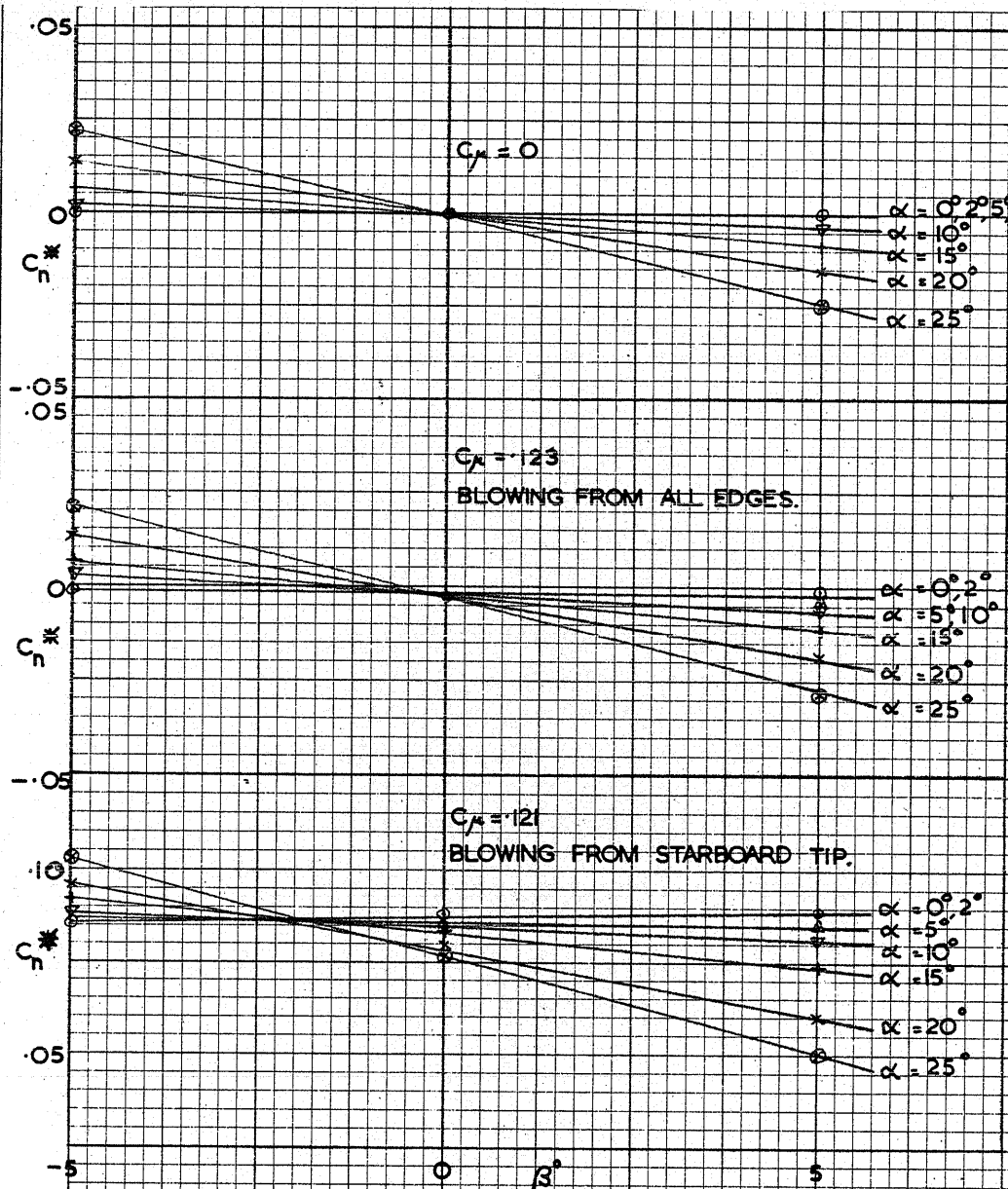


FIG. 28.

VARIATION OF YAWING MOMENT WITH SIDESLIP ANGLE β .

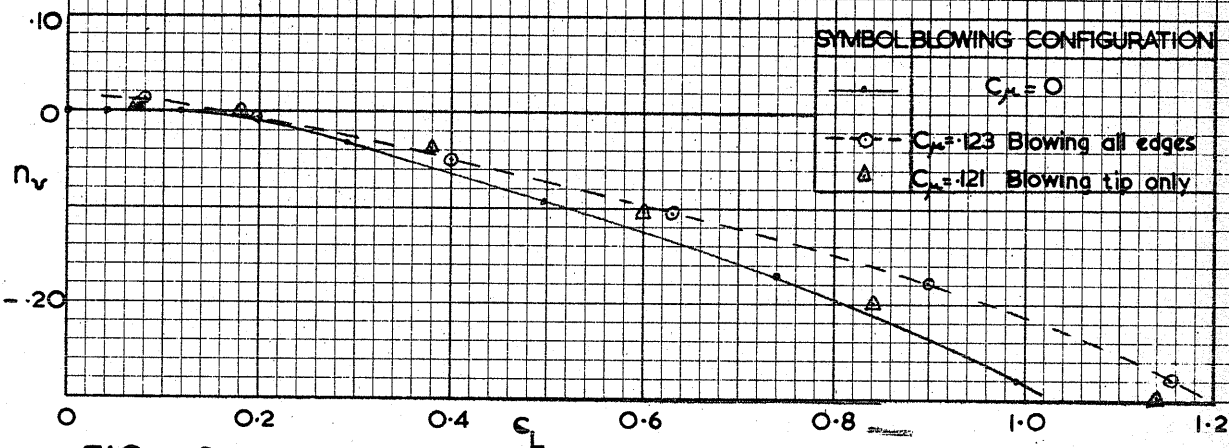


FIG. 29.

VARIATION OF $n_v \left(\equiv \frac{\partial C_n^*}{\partial \beta} \right)$ WITH LIFT.

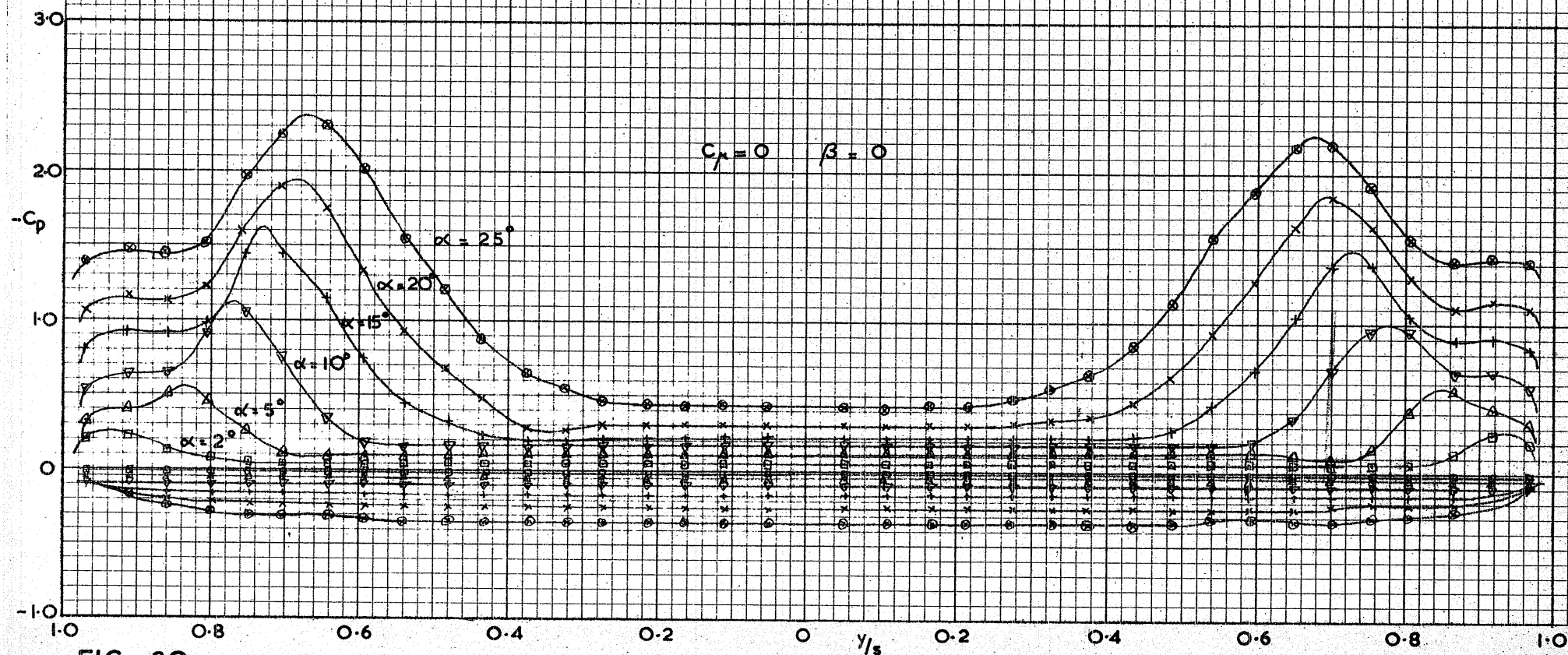


FIG. 30.

SPANWISE VARIATION OF UPPER AND LOWER SURFACE STATIC PRESSURE DISTRIBUTIONS. $x/C_0 = .49 \quad \beta = 0.$

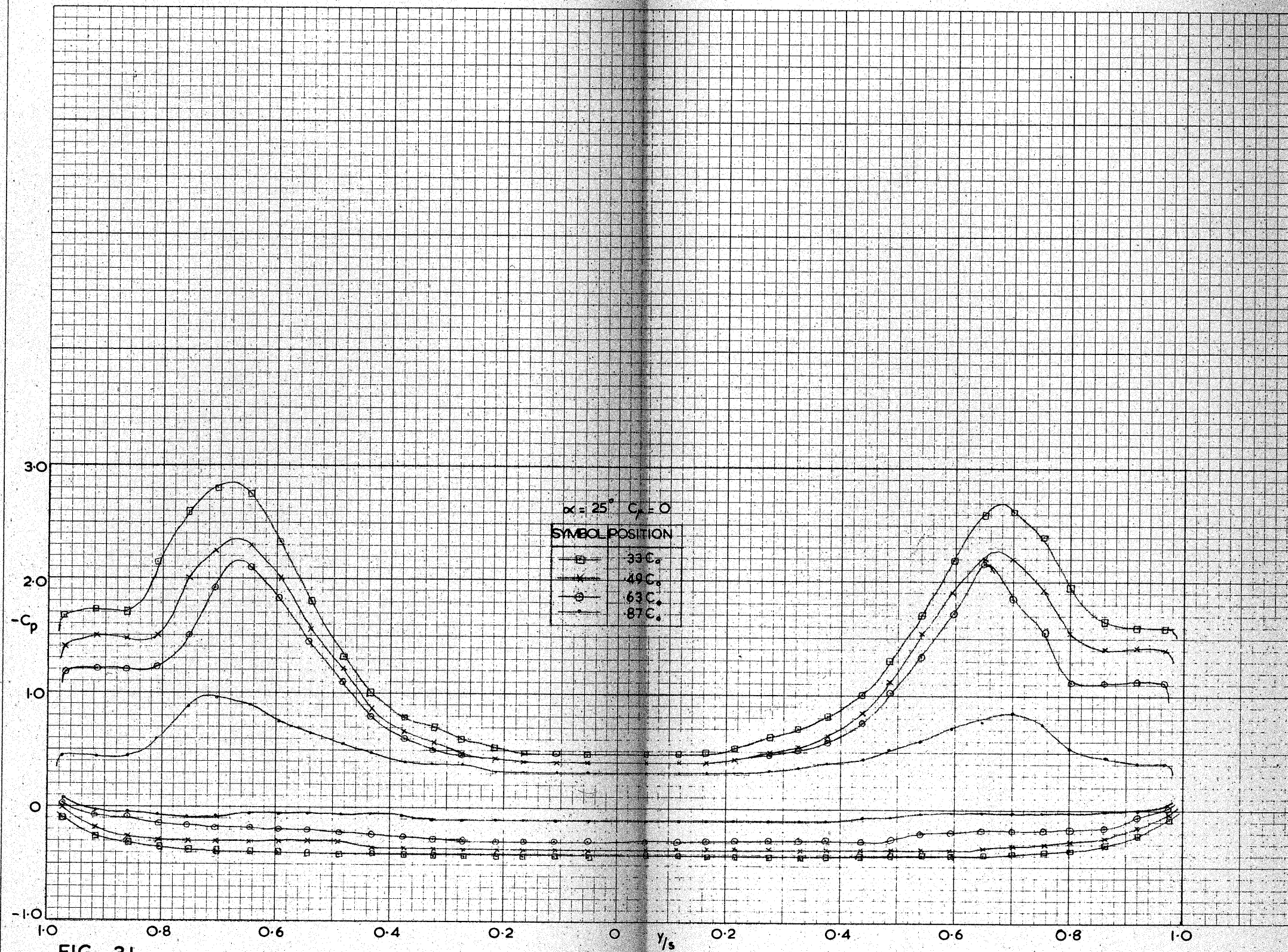


FIG. 31.

VARIATION OF SPANWISE STATIC PRESSURE DISTRIBUTIONS WITH x . $\beta = 0$.

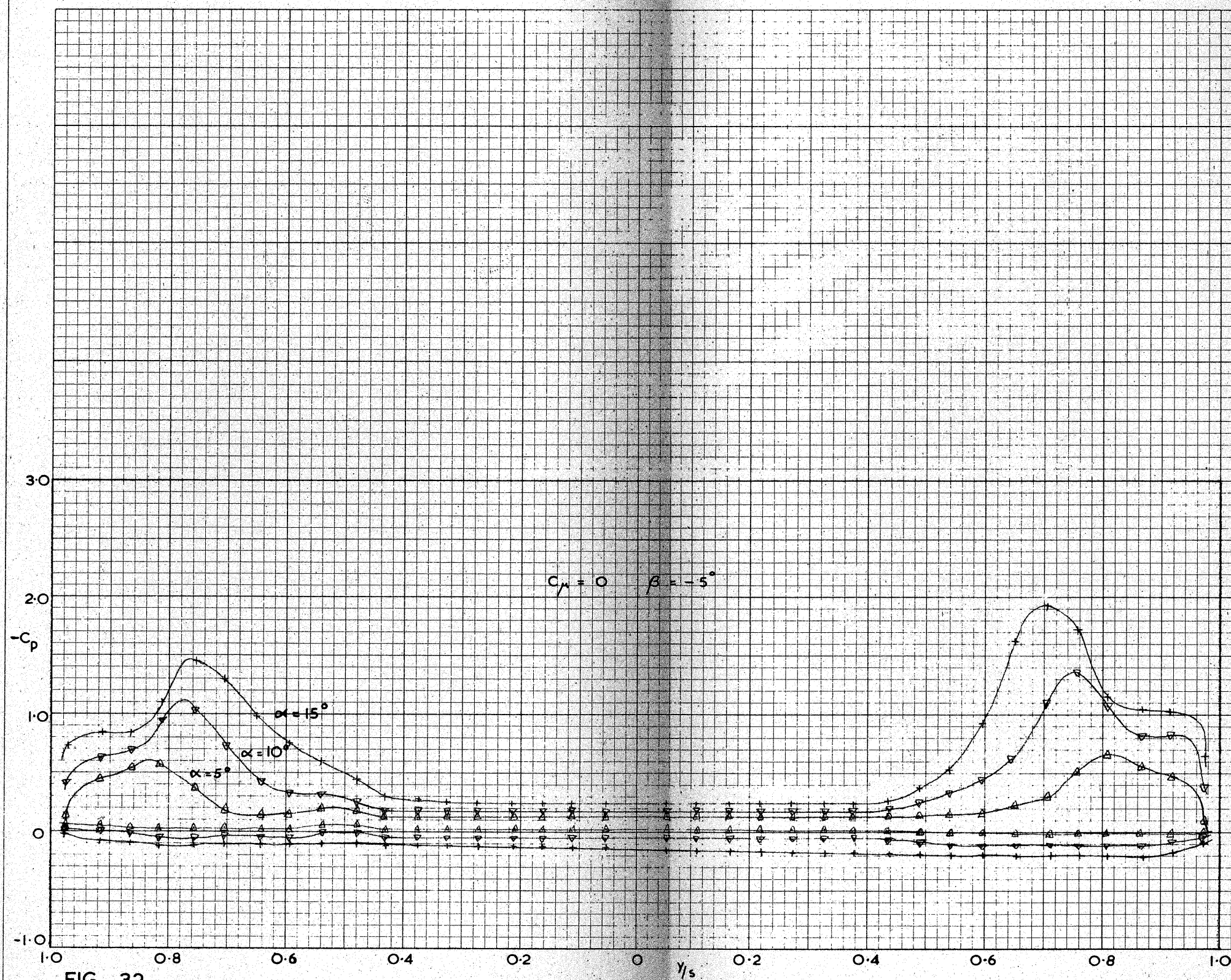


FIG. 32.

SPANWISE VARIATION OF UPPER AND LOWER SURFACE STATIC PRESSURE DISTRIBUTIONS. $x/c_o = .49$. $\beta = -5^\circ$

FIG. 33.

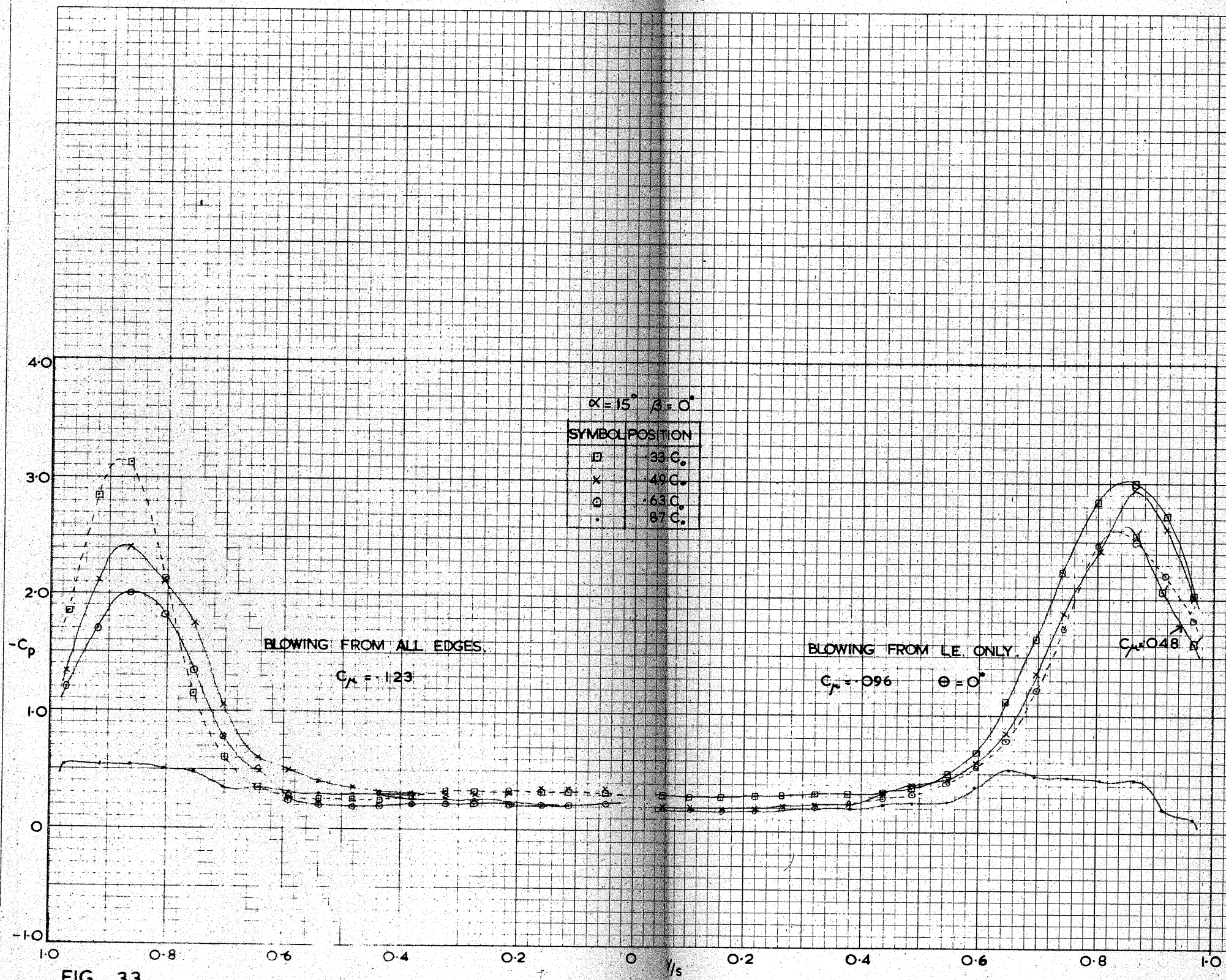


FIG. 33.

VARIATION OF UPPER SURFACE STATIC PRESSURE DISTRIBUTIONS WITH x . $\beta = 0$.

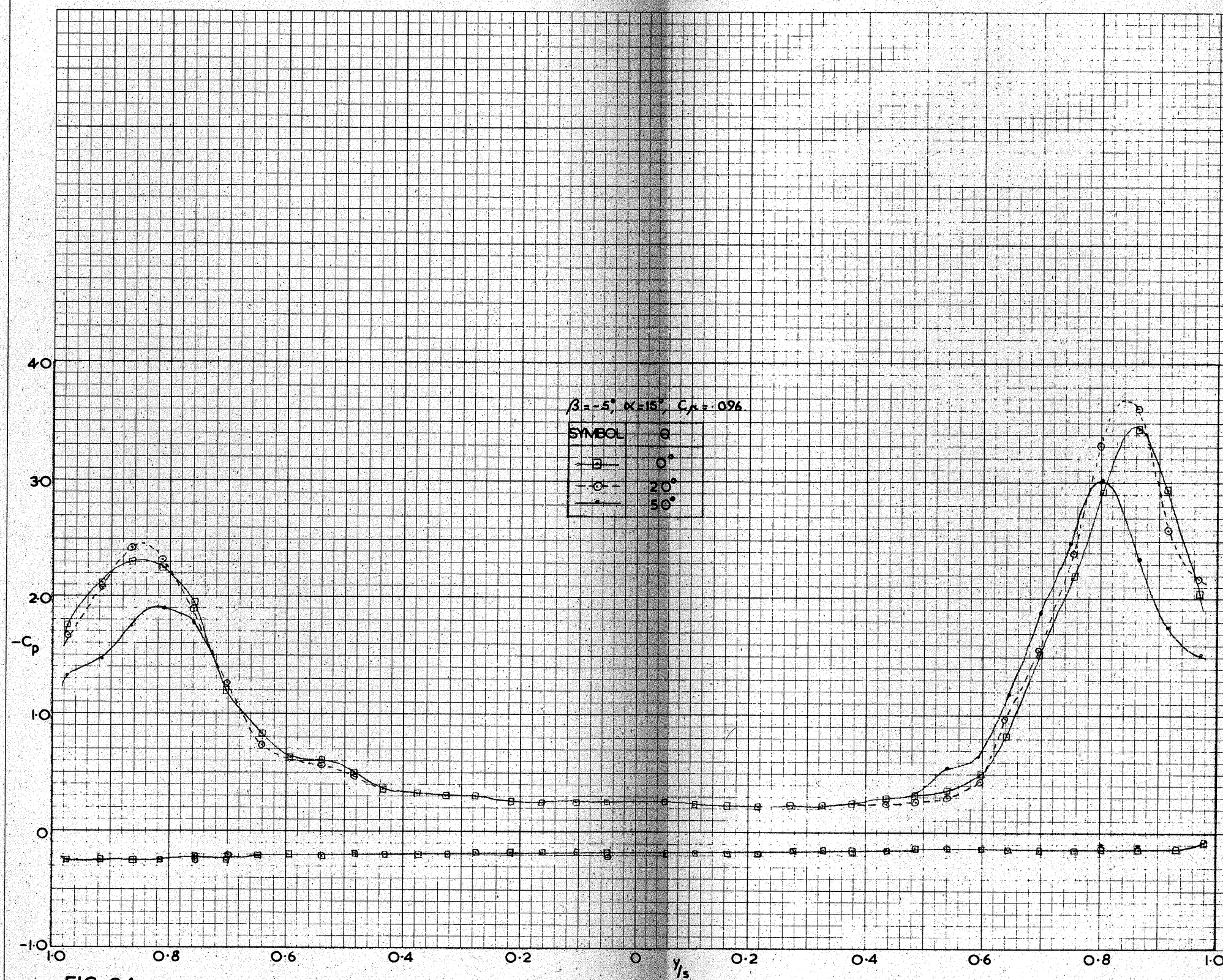


FIG. 34.

VARIATION OF UPPER AND LOWER SURFACE STATIC PRESSURE DISTRIBUTIONS WITH JET SWEEP ANGLE θ . $x/C_o = 0.49$, $\beta = -5^\circ$

FIG. 35.

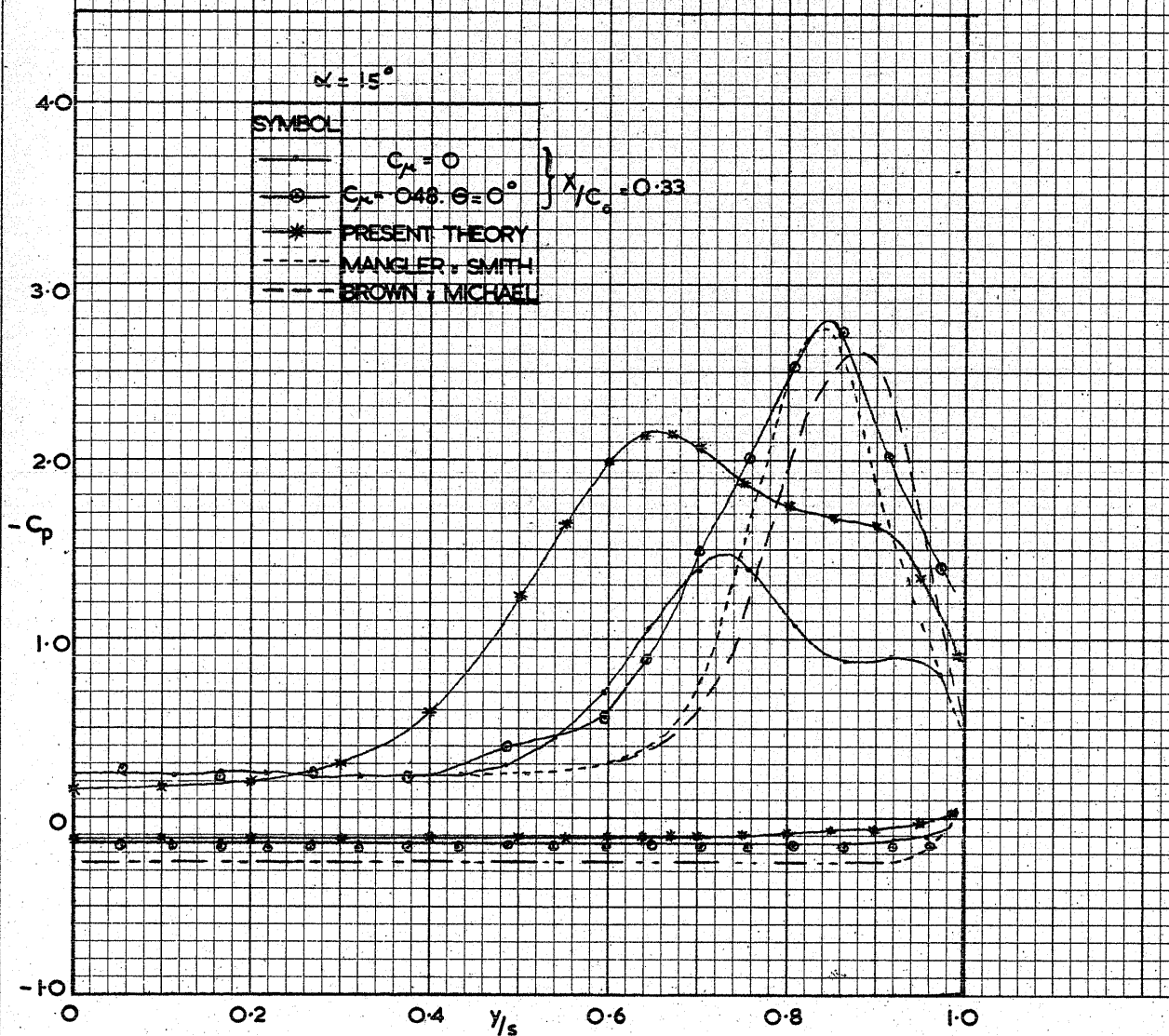
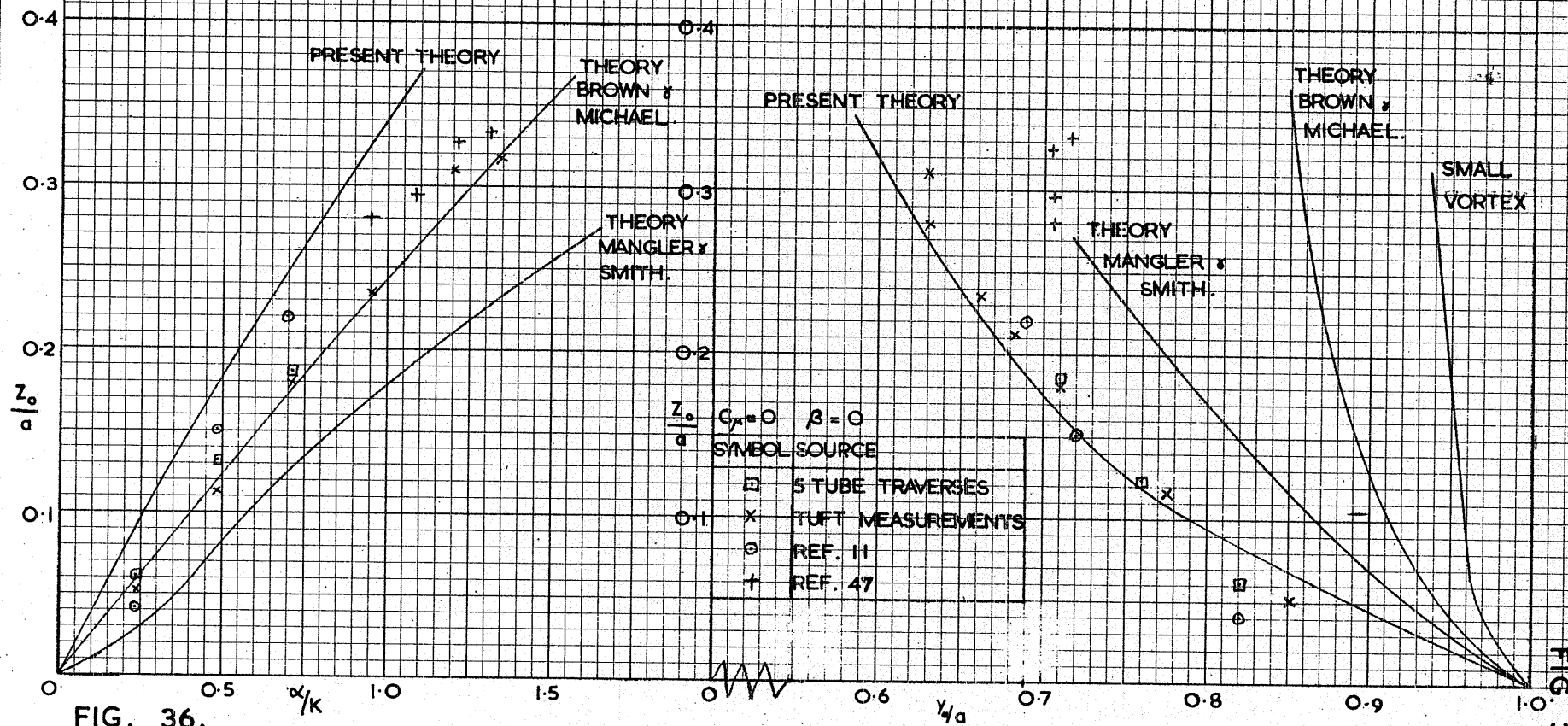
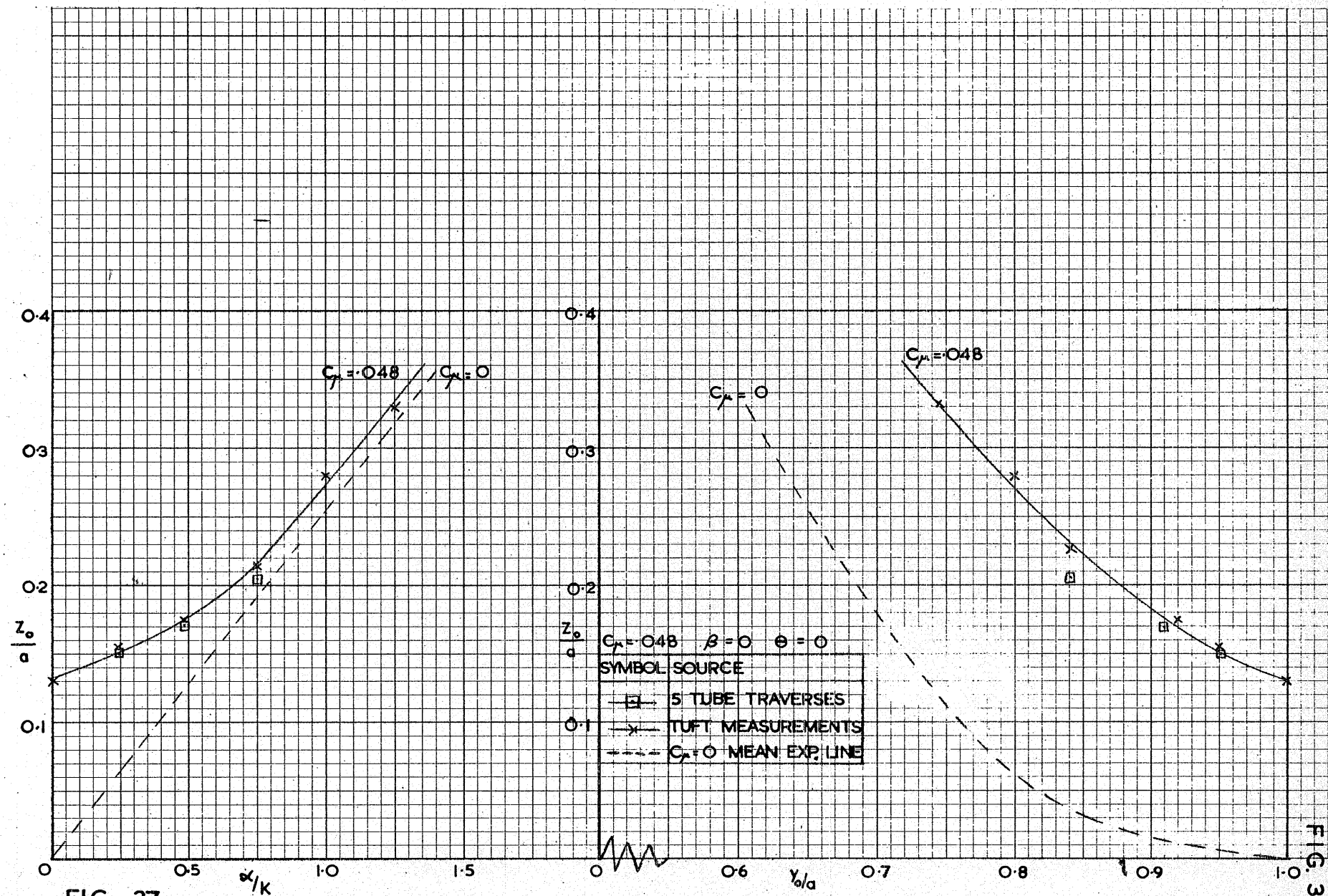


FIG. 35.

SURFACE STATIC PRESSURE DISTRIBUTIONS. COMPARISON WITH THEORY.





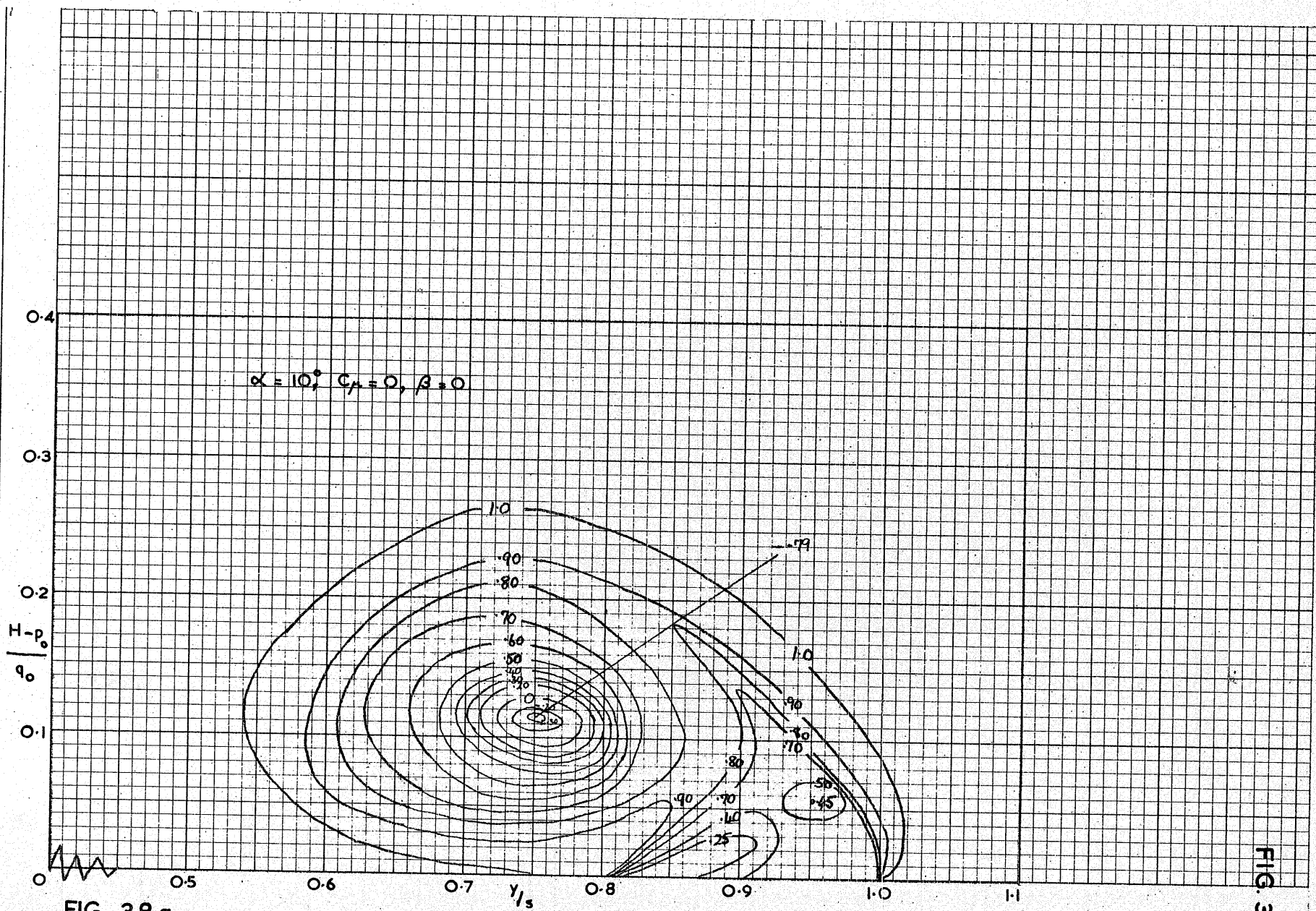


FIG. 38a.

VARIATION OF TOTAL HEAD THROUGH VORTEX. $C_\mu = 0$. $x/C_0 = 0.50$.

FIG. 38a.

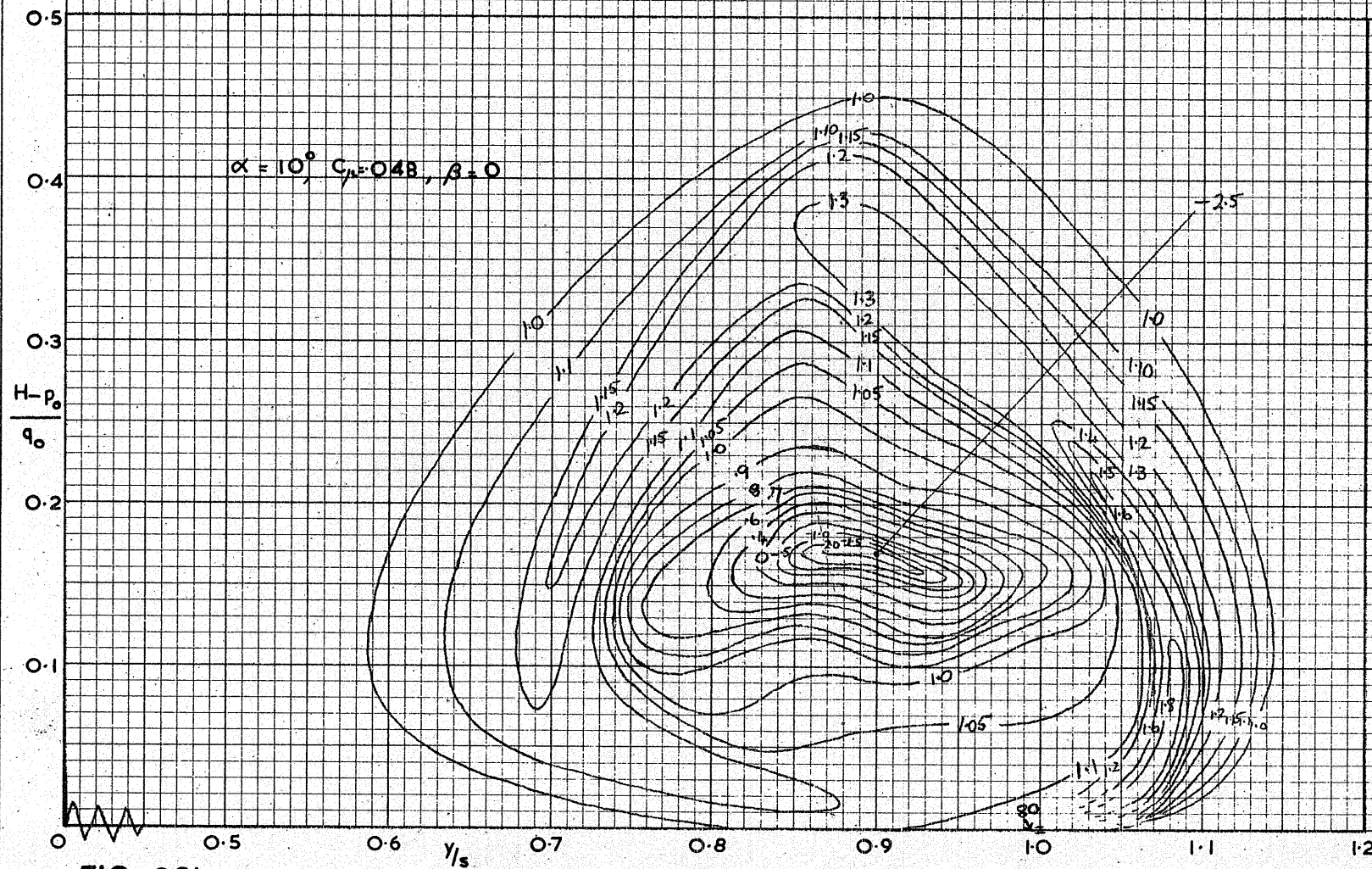


FIG. 38b.

VARIATION OF TOTAL HEAD THROUGH VORTEX. $C_{\mu} = 0.48, \theta = 0^\circ, X/C_0 = 0.50.$

8 Discussion of results. Part II Downward deflected blowing.

8.1 Introduction

To date, there has been very little published work in this field. Various schemes have been suggested (ref. 42) but not substantiated with experimental evidence, the only results available being the mainly two-dimensional work of O.N.E.R.A. Thus there is no accumulated experience on which to base a series of tests.

The main variables are incidence, wind speed, ground clearance and momentum coefficient C_{μ} for the downward deflected jet. It was necessary to establish whether or not C_{μ} was a unique parameter for this type of test since the blowing air available was insufficient to produce scale values of the auxiliary deflected thrust. Previous experience, however, at the two ends of the speed and height scale which it was proposed to cover, suggested that this would be so.

The range of variables covered was as follows:

Incidence: -3° to the maximum possible incidence, or 20° whichever is the smaller.

Wind speed: 0, 20, 40, 60, 80, 100 ft/sec for $h/c_0 = 0.025$ with a smaller range as h/c_0 increased.

Ground clearance: $h/c_0 = 0.025, 0.075, 0.15, \infty$ (no ground plate).

C_{μ} 0 - 7.5 approximately for $h/c_0 = 0.025$ and 0 - 0.28 for $h/c_0 = \infty$

Auxiliary thrust: 3.4 lbs, 7.6 lbs, 12.9 lbs.

In order to reduce the possible number of tests a tentative flight plan was devised, so that the effort and time available could be used to the best effect. Intuitively, the flight path would seem to take

the following form at take-off. Firstly, there will be the hover phase, in which the aircraft is supported at rest by the peripheral jet sheets supplied by the auxiliary engines. Secondly, there is the ground acceleration phase, in which the thrust is supplied by the main engines, but the major part of the lift is due to ground effect. Thirdly, the climb phase, in which height is gained much more rapidly than in phase two, with conventional wing lift now predominating. Towards the end of this phase the amount of blowing is gradually reduced to zero and finally the flaps are returned to the undeflected position, leaving the aircraft clean for the final cruise condition. It is likely that the landing procedure will be roughly the reverse of take-off but some changes in attitude may be necessary to assist with braking. It was not possible to study the effect of varying the angle of blowing in the plane of the jet but clearly this is a powerful method of increasing the thrust on take-off, or increasing the drag for landing. With the very large auxiliary thrust available, a deflection of only ten degrees gives nearly 15,000 lbs of horizontal thrust while having only a small effect on the lift.

It was not clear, initially, which combination of flap deflection and blowing would prove the best. The jet sheet could be deflected downwards 30° using the cranked edges shown in Fig. 5 of Part I, and some tests were made using this arrangement. Close to the ground, ($h/co = 0.075$) at small incidence, it was found that the jet sheet spread out over the ground plate and eventually rolled up well away from the wing. At a certain critical incidence ($\approx 6^\circ$) however, for wind speeds

above 50 ft/sec, the flow pattern changed very suddenly and the jet sheet rolled up above the wing upper surface, just as it did with blowing in the plane of the wing away from the ground. When this occurred, the lift was roughly doubled and the centre of pressure moved forward almost 10% of the root chord, from about $0.75 C_o$ to $0.65 C_o$. This change occurred, although less violently, at all ground clearances including tests without a ground plate. Away from the ground, the change in lift was very small and the movement of the centre of pressure only about 1% of the root chord.

It was concluded from these tests that raising the flaps with the jet on was undesirable and a better procedure would be to reduce the jet thrust to zero, with the flaps deflected sufficiently to prevent rolling up occurring over the wing, and then to raise the flaps for cruising.

Lift magnifications close to the ground with only 30° deflected blowing were small and did not exceed one at $h/c_o = 0.075$ (the smallest clearance tested) until speeds of about 50 ft/sec at five degrees incidence were reached. Ninety degrees deflection was considered the minimum necessary for efficient hovering and in order to deflect the jets through this angle, flaps of one inch chord were fitted to the undersurface of the wing at the edge, to turn the jet by Coanda Effect. See fig. 5 of Part I.

Some preliminary work carried out on the Coanda Effect out of ground effect, suggests that a minimum flap length of only five times the slot width is sufficient to turn the jet sheet through at least 90° ,

so, if this ratio holds in ground effect the flaps used on the model are too large, since an aircraft would presumably use the smallest flap possible to avoid mechanical complications. The effect of this on the model is to give optimistic lift results away from the ground owing to the camber effect but close to the ground the larger flaps will not change the lift appreciably.

Ground clearance h is measured from the lower edge of the flap to the ground at zero incidence but this is equal to the distance between the lowest part of the model and the ground and so is representative of full scale clearance. The large trailing edge flap reflects in fig. 2, which shows trailing edge clearance at a given incidence. With a representative flap, of course, this clearance would be much larger.

With the jet deflected 90° no sharp changes in either lift or pitching moment due to blowing, were observed and this arrangement was considered suitable for the proposed tests. It was not possible to make tests at any other jet angle and the results given here refer only to the 90° deflected case blowing normal to the slot.

In order to fix ideas we consider two aircraft. A "basic" aircraft, representative of a supersonic ($M \approx 2-2.5$) airliner with an all-up weight of 300,000 lbs, a wing area of 6000 sq. ft. and a maximum thrust at sea level of 100,000 lbs, and a "modified" aircraft of roughly the same capacity but with an installed auxiliary thrust of 85,000 lbs for S.T.O.L.

The specific weight of jet-lift engines is at present approaching

0.1 lb/lb lift (0.11 for the Rolls-Royce R.B.108) and values as low as 0.06 are envisaged (ref. 49). Thus in order to provide an auxiliary thrust of one-quarter of the new all-up weight, (335,000 lbs) the dry weight of the lift engines will be about 8,500 lbs. Fuel and associated equipment are likely to weigh as much again, bringing the installed weight of the lift engines to 17,000 lbs, or 20% of the auxiliary thrust. A similar estimate of the installed weight is obtained from ref. 50. The weight of fuel for this type of aircraft is approximately one-half of the total weight, (see for instance ref. 51) so that the increased fuel weight is another 17,000 lbs, bringing the new all-up weight to (in round figures) 335,000 lbs.

In order to keep the thrust/weight ratio constant larger main engines will be required, giving 112,000 lbs thrust but it is assumed that the increased weight will be offset by the use of a much lighter undercarriage. It is further assumed that the wing areas of the two aircraft are the same, thus increasing the wing loading from 50 lbs/sq.ft. on the basic aircraft, to 56 lb/sq.ft. on the modified aircraft. With the proposed type of assisted take-off the effect of this increase in wing loading on the low speed performance is very small and even under cruise conditions the reduction in the lift/drag ratio can be offset to some extent by reducing the cruise height. A more serious consequence of installing the lift engines and peripheral ducting, is the resultant increase in cross section area and its effect on the cruise but only a careful design study of a practical installation could give an accurate estimate of the possible reduction in cruise performance.

The experimental results are now discussed and an attempt is made to assess the improvement in take-off and landing performance with the modified aircraft. The results are discussed in four parts, each referring to a particular phase of the take-off and for convenience the graphs corresponding to each phase have been grouped together.

The variation of C_{μ} (auxiliary thrust), for the modified aircraft, with wind speed V_A is shown in fig. 1. The experimental values of C_{μ} are shown, together with the range covered at a given height above the ground. Minimum flying speed in the "clean" condition at 15° incidence on take-off is 306 ft/sec or 181 knots, so that the speed range is adequately covered by the experimental results.

For reasons of safety and to avoid severe ground erosion it is desirable that the ground clearance should be as large as possible. This is limited by the ratio of auxiliary thrust to weight and in order to minimise erosion, it is necessary to raise the incidence as soon as possible to increase the lift and hence the ground clearance. As with conventional take-off it will be necessary to gather speed before increasing incidence and also to maintain an adequate trailing-edge clearance. This has been set somewhat arbitrarily at about five feet, $h/c_o = 0.025$, i.e., the aircraft must reach this height before increasing incidence and thereafter the trailing edge clearance must be at least five feet (see fig. 2 and § 8.3).

8.2 Phase I Hovering.

Tests were made at zero wind speed for three blowing pressures and varying heights. Variation of the lift augmentation factor L_A with non-dimensional height h/c_o is plotted in fig. 3a. Values obtained with the cropped delta wing are compared with some circular wing results obtained with the same rig (ref 52) and with inviscid theory. The curves are typical of ground effect tests at zero wind speed and show the reduction in lift of the elongated planform compared with the circular wing. (See also pp 263 and 361 of ref. 42). The experimental points fall well below the inviscid theoretical curve for the delta wing, because of jet mixing and the vortex system below the model caused by jet entrainment. The reason for the slight hump in the lift curve at $h/c_o \approx 0.15$ for the delta results is not known, although again it is typical of experimental measurements. Poisson-Quinton in ref. 42 claims that it is due to the interaction of the two lower surface vortices when they completely fill the space below the model but his theoretical values are not in good agreement with experiment.

The "scaled" value of auxiliary thrust for the model is about 24 lbs, but the maximum value available for the present tests was only 12.9 lbs. However, the results obtained with thrusts of 3.4, 7.6 and 12.9 lbs collapse on to a single curve. With no ground ($h/c_o = \infty$) the values of L_A fall well below unity owing to lower surface suction due to jet entrainment. Fig. 3b shows that the variation of L_A with incidence at the lowest height tested, $h/c_o = 0.025$, is small.

In fig. 3c the non-dimensional drag D_A is plotted against incidence for $h/Co = 0.025$. The experimental curves are displaced from the origin and show a considerable drag at zero incidence. This is due to large pressure losses in the model ducts leading to the trailing edge slot, which produce a pressure difference between the forward and rear parts of the model, giving a net drag. The theoretical non-dimensional drag is $L_A \tan \alpha$ but here comparison is made using the experimental value of L_A of 3.0. Agreement is quite good, allowing for the net drag at $\alpha = 0^\circ$, with a slight fall-off in experimental values at the highest incidence.

The non-dimensional pitching moment P_A about $0.64 Co$ is plotted against incidence in fig. 3d. Between -1° and $+2^\circ$ the model is stable and outside these limits unstable. This is similar to the results obtained for the S.R.N.I. Hovercraft (Ref. 42). Static instability on this type of machine is due to an increase in the strength of the trailing edge vortex, beyond a certain (positive) incidence, which sucks the trailing edge on to the ground. Movement of the centre of pressure is shown in fig. 3e. The maximum movement is only 3% of the root chord. Note that for this configuration the centre of area is at $0.64 Co$.

The non-uniform pressure which produces a drag at zero incidence must also affect the pitching moment but no simple correction is possible in the case of the pitching moment and none has been applied. Comparison between various ground heights is probably valid and it may be assumed that in a practical application non-uniform blowing could be

used to position the centre of pressure correctly relative to the centre of area (see § 8.6).

8.3 Phase II Ground run

It will not be possible to increase incidence until a certain height is reached, from safety considerations and to avoid too much ground erosion. (See § 8.1). However, lift increases with forward speed even at zero incidence since the camber effect produces lift from the upper surface and the lower surface lift is scarcely affected initially.

Fig. 4a shows lift plotted against momentum coefficient C_μ for $h/co = 0.025$. Results were obtained over a range of speeds and blowing pressures and the results collapse on to a single curve, indicating that C_μ is a unifying parameter for this type of test. Only values of lift for $\alpha = 0$ are given here since lift in the range $-2^\circ \leq \alpha \leq +3^\circ$ changes little up to about 200 ft/sec and also at this small ground clearance it is not practicable to have the wing at incidence. Values of C_μ up to eight were obtained in the test but are irrelevant to the present discussion.

In fig. 4b L_A is plotted against $1/C_\mu$. This shows the increase in lift with increase in the dynamic head of the stream, for convenience a speed scale is given also. The initial rise of L_A with speed is slow, indicating that the undersurface conditions are little changed from the hover case (see also ref. 52). At about 200 ft/sec, however, the lift falls due to a change in the path of the supporting jet sheet.

The increasing dynamic head of the oncoming stream eventually exceeds the pressure in the ground cushion and the jet sheet is forced to curve backwards under the model. (See ref. 52). This results in a considerable drop in pressure close to the edges affected, reducing the lift and giving a nose down pitching-moment.

The speed at which this drop in lift occurs can be predicted reasonably well by the following simple argument. Consider the pressure on the wing undersurface to be constant, say p_u , the total auxiliary thrust μ , wing area S and free stream speed V . Neglecting upper surface lift we have at zero incidence:

$$S \cdot p_u = L_A \cdot \mu \quad \therefore p_u = \frac{L_A \cdot \mu}{S}$$

The speed at which the dynamic head equals the undersurface pressure is $V_{crit.}$ and therefore:

$$\frac{1}{2} \rho V_{crit.}^2 = p_u = \frac{L_A \cdot \mu}{S}$$

$$\therefore C_{\mu crit.} = \frac{1}{L_A}$$

From fig. 3a $L_A = 3$ for $h/C_o = 0.025$ and $\therefore \frac{1}{C_{\mu crit.}} = 3$

This gives reasonable agreement with experiment in fig. 4b, for the lower incidences but at higher incidences the lower surface pressure is very non-uniform (ref. 52) and the simple method breaks down.

Drag is plotted against lift in fig. 4c. In order to simplify the presentation of results the wind-off drag due to blowing asymmetries has been subtracted from all the measured drag values. There is a

small kink in the curves for a C_L of about 1.3 corresponding to the bending back of the jet sheets. At negative incidence for large values of C_{μ} there is a positive thrust due to the forward inclination of the lift vector but this is reduced with increasing forward speed due to the rise in pressure and skin friction drag.

Fig. 4d shows the variation of pitching moment with lift. At low speeds the wing is stable in pitch, increasing incidence giving an increasing nose down pitching moment but note that here C_L is speed dependent and the graph cannot be interpreted in the usual way. For small C_{μ} values the stability is almost neutral and only for $C_{\mu} = 0$ is the slope unstable. The nose down pitching-moment above the critical speed can be seen although it is marked only at the higher incidences.

The forward movement of the centre of pressure is shown in fig. 4e. It is not severe below wind speeds of about 250 ft/sec and at this height it is not greatly affected by incidence. As the speed increases however, it is desirable to increase incidence to limit the centre of pressure travel.

8.4 Phase III The initial climb.

The next set of tests was carried out for $h/C_0 = 0.075$, equivalent to about fifteen feet altitude. The maximum incidence obtainable with a trailing edge clearance of $0.025 C_0$ is nine degrees but at this height incidence is unlikely to exceed five degrees on take-off giving a trailing-edge clearance of $0.05 C_0$ or approximately ten feet.

Fig. 5a shows the variation of lift with C_{μ} . The change of lift is linear up to a speed of about 180 ft/sec when the leading edge jet sheets are forced back resulting in a change of slope. The plot of L_A against $\frac{1}{C_{\mu}}$ in fig. 5b does not show the critical speed so clearly as for $h/C_0 = 0.025$ although the effect is probably smaller, as aerodynamic lift forms a greater part of the total lift. At ten degrees incidence and low speeds the lift drops below the value at lower incidences. This is due to the increase in strength of the vortex under the trailing edge as it approaches the ground, giving rise to considerable suction on the lower surface. Aerodynamic lift at this incidence is large and above 100 ft/sec the total lift is greater than at smaller incidence.

Drag is plotted against lift in fig. 5c, where the critical speed shows up as a change in slope of the curve. Large drag values are obtained at higher incidences and speeds owing to a combination of induced drag and forward facing thrust. The variation of pitching-moment and centre of pressure with lift is shown in figs. 5d and e. At $\alpha = 10^\circ$ the effect of the trailing edge approaching the ground is most marked.

Figs. 6a - 6e show results obtained at $h/C_0 = 0.15$. No additional comment is required except to observe that effects near the critical speed are smaller. Incidences up to the maximum of 15° are possible at this height.

8.5 Phase IV. The Climb without auxiliary power.

Results without ground and with no blowing are plotted in figs. 7a - 7d. C_L/α curves are shown in fig. 7a with the edges undeflected, with the outer 1.6" deflected 30° downwards and with the extra 1" flap deflected 90° downwards. The effect of the 30° deflection is small except near $\alpha = 0^\circ$ but the camber effect of the 90° flap is considerable even at $\alpha = 20^\circ$. As pointed out earlier the flaps are much larger than scale size and hence produce considerable lift. It should be noted that for the latter case the lift-curve slope is only slightly non-linear which indicates that leading edge vortices, if present at all, are very weak.

For comparison some lift results, with blowing, away from the ground with $C_{\mu} = 0.077$, are also given. At zero forward speed, fig. 3a, there is an adverse effect on lift away from the ground but at a reasonable forward speed the increase in lift is about twice the vertical thrust, showing that this type of blowing produces a favourable interference effect even with no ground.

Drag at a given lift is roughly the same as for the other two cases, fig. 7b, but the pitching moment increments and the associated centre of pressure movement, figs. 7c and d, are very large indeed and would probably not be acceptable.

8.6 Estimation of take-off performance.

With the information available it is not easy to make an accurate estimate of the improvement in take-off and landing using this form of

auxiliary power. The standard method of calculating take-off distances and climb speeds is not applicable, since from hovering onwards the total weight is supported by aerodynamic forces and also the lift during the initial climb is dependent on altitude until the ground effect contribution to lift is negligible.

Three further complications arise with the present model. Firstly, the size of flaps used to deflect the jet is much too large giving a camber effect with no ground which would not be present to the same extent in a practical application. Secondly, owing to the cropped delta shape, the centre of area is at $0.64 C_0$ and the centre of pressure at $0.52 C_0$ for moderate incidence out of ground effect. Finally, although a straight forward correction can be applied to the drag owing to asymmetries in the blowing, since with no asymmetries the drag is zero at $\alpha = 0$, no simple correction is possible with the pitching moments. With a constant blowing velocity at all edges and constant pressure over the bottom surface the centre of pressure can theoretically move from $0.64 C_0$ for $h = 0$ to $0.60 C_0$ for $h = \infty$.

In order to calculate the take-off and climb with peripheral downward deflected blowing the following assumptions have been made:

(I) Close to the ground $h/C_0 \leq 0.075$ the effect of the oversize flaps is likely to be small with the jet on and for this range it is assumed to have no effect. Further, the results for $h/C_0 = 0.15$ will not be used and the lift, drag and pitching moment are assumed to tend uniformly to the values obtained with the "clean" wing when the blowing is switched off and not to the values obtained with the 90° flaps.

(II) The shape of the present wing is not practicable for use on an airliner. An ogee wing with its centre of pressure closer to $0.65 C_o$, away from the ground, is more likely to be used. Thus the change in position of centre of pressure for a practical design is likely to be fairly small throughout the range from hovering, where the centre of pressure is reasonably close to the centre of area, to the cruise condition. It is assumed therefore that any pitching moment encountered on an aircraft can be controlled by either suitably placed round jets at low speeds, or normal aerodynamic controls at higher speeds. If necessary a pitching-moment could be built-in at low speeds by increasing the blowing at suitable points. This asymmetry in the blowing would not affect the lift but would cause some extra thrust or drag depending on where the increased blowing was applied.

(III) The aircraft is considered as a point mass subject to the usual equations of motion. The hovering condition for the modified aircraft, $L_A = 4$, is seen to occur at a value of $h/C_o = 0.02$ (fig. 3a) which corresponds to a full-scale hover height of about four feet. Incidence is taken to be zero until a speed of 180 ft/sec is reached when lift equals weight at $h/C_o = 0.025$. Thereafter incidence is increased until five degrees is reached at $h/C_o = 0.075$ where lift equals weight at 235 ft/sec. The auxiliary thrust is then assumed to be reduced linearly above a speed of 235 ft/sec reaching zero at 306 ft/sec. Although lift values for the assumed conditions are not available from experiment, fig. 7a shows that even out of ground effect there is still some lift magnification, $\frac{\Delta C_L}{C_{\mu}} \approx 2$, indicating that the changeover from ground

effect to aerodynamic lift is likely to be smooth, thus providing some justification for the assumptions made. The final stage is taken as flight at $\alpha = 15^\circ$ with $C_{\mu} = 0$ and flaps retracted at 306 ft/sec.

Two further points may be noted here:-

- (1) With such a large auxiliary thrust available a change of incidence is a powerful method of controlling horizontal thrust. Although lift is hardly changed close to the ground, a change of incidence of only one degree changes the horizontal thrust by about 6000 lbs.
- (2) Since on an aircraft the time spend close to the ground is small, the use of a second jet sheet to improve stability, as on the S.R.N.I. Hovercraft, is not justified. Low speed stability will have to be provided either by a round jet system, such as in the Short S.C.I., or by differential blowing with the peripheral jet sheets.

The times and distances from rest to fifty feet altitude and to minimum flying speed, on take-off, are now calculated on the basis of the above assumptions. With the further assumption that up to 50,000 lbs of reverse thrust is available, equivalent times and distances are found for landing.

If a_1 is the acceleration we have:

$$t = \int_{v_1}^{v_2} \frac{dv}{a_1} \quad \text{secs}$$

Also net thrust $(T - D)$ lbs. = $\frac{W}{g} a_1$. where W is the weight of the aircraft in lbs.

$$\therefore t = \frac{W}{g} \int_{v_1}^{v_2} \frac{dv}{(T-D)}$$

and the corresponding distance $S_{12} = \int_{t_1}^{t_2} v dt$ feet

These values are calculated using fig. 8a. See Table I.

The unknowns are the heights reached after $h/Co = 0.075$.

These are estimated as follows:

Climbing speed up to $h/Co = 0.025 \approx 0$ ft/sec.

Average climbing speed between $h/Co = 0.025$ & 0.075 is $\frac{10}{6.7} = 1.5$ ft/sec.

$\therefore$ Climbing speed at $h/Co = 0.075 \approx 3$ ft/sec.

For $h/Co = \infty$ the climbing speed v_c can be calculated from the usual formula.

$$v_c = \frac{V_A (T - D)}{W} \text{ ft/sec.}$$

giving a climbing speed of 22 ft/sec for $V_A = 306$ ft/sec.

Assuming a smooth increase in the rate of climb an estimate of the height can be obtained using fig. 8b. See Table I.

Table I

height	h/Co	time	distance
5 ft.	0.025	16.7 secs	1440 ft.
15 ft.	0.075	23.4 secs	2530 ft.
50 ft.	0.25	31.6 secs	3480 ft.
280 ft.	∞	43.5 secs	4750 ft.

The take-off distance to fifty feet altitude has been calculated for the basic aircraft (see §8.1) using the normal equations for the

ground run, (take-off speed = $1.05 V_{min}$) and then assuming a constant speed climb. Estimated distance from rest to fifty feet altitude is 8,300 feet. Thus take-off using peripheral downward deflected blowing shows a very considerable gain, reducing the take-off distance by more than one half.

However, the installation of 85,000 lbs additional thrust is bound to produce a considerable reduction in take-off distance and it is instructive to consider the effect of using this thrust in other ways. Two cases only were considered. First, the effect of installing an extra 85,000 lbs horizontal thrust. With assumptions similar to those used for the conventional take-off the distance from rest to fifty feet altitude is cut to only 2,200 ft. This method would be quite impractical on a civil aircraft, since initial accelerations of 0.65 g are involved, but might be suitable for military aircraft.

The second case considered was that in which the auxiliary engines were assumed to be mounted vertically, producing 85,000 lbs lift. The take-off speed for the modified aircraft was then reduced by 25 knots. This method reduced the distance from rest to fifty feet altitude to 6000 feet.

Sutcliffe and Merrick (ref. 53) have shown that an improvement in take-off performance is obtained if part of the total thrust available can be deflected downwards when close to take-off speed. If this were done with the first case considered here, take-off distance could be reduced below 2000 feet but this is not practical with civil aircraft.

On landing the gross weight and minimum flying speed, without

blowing, are greatly reduced to 170,000 lbs and 220 ft/sec respectively. Another consequence of the reduced weight is the increase in hover height, using full auxiliary thrust, to $h/Co = 0.05$. This is an advantage since it will permit the wing to be held at incidence when close to the ground thus increasing the braking force available. A considerable amount of reverse thrust from the main engines will also be required since the aircraft has no contact with the ground and hence no braking available as with a normal undercarriage.

The drag components and total drag are plotted in fig. 9a. In order to obtain values for $(D - T \cos \alpha)$ it has been assumed that the aircraft will approach using 25% of maximum thrust. Minimum flying speed is 220 ft/sec at $\alpha = 15^\circ$ at a height of about 330 ft. From this point onwards the main engine thrust is reduced by deflection until it is effectively zero at a height of 15 feet, ($h/Co = 0.075$) where the incidence is kept to the maximum of 9° to further increase the drag. Reverse thrust is then increased to give a maximum drag of 50,000 lbs (40% of gross thrust) and is assumed to stay constant down to a speed of 10 ft/sec, when it is then reduced to zero. The estimated rate of descent is plotted in fig. 9b with the approximate heights shown. Table II gives estimated times and distances for landing.

Table II

height	h/Co	time	distance
330 ft	∞	0	0
50 ft	0.25	16.3 secs	2100 ft.
15 ft	0.075	23.6 secs	3170 ft.
10 ft ($V_A = 0$)	0.05	50.6 secs	4990 ft.

Thus a considerable reduction in landing distance is also achieved.

Distance to rest, landing over a fifty foot screen, is only 2900 feet even using aerodynamic braking only.

9. Conclusions. Downward deflected blowing.

A limited investigation has been made on a 70° cropped Delta Wing into the effects of peripheral blowing on the landing and take-off performance. The effects of varying wind-speed, incidence, ground clearance and blowing pressure were measured but only with one jet deflection angle.

However, despite the fact that only one configuration (certainly not the best) was tested, simple calculations show that substantial reductions in landing and take-off distances could be achieved using deflected peripheral jets. Estimated distances from take-off and to rest over a 50 feet screen with an auxiliary thrust of one quarter of the all-up weight are about 3500 ft and 2900 ft respectively, or less than one half of the distance with conventional aircraft. Added to this is the fact that smooth runways are not needed and apart from a hovering pad only a rough grass surface is necessary.

The optimum downward deflection of the jet sheets is probably much greater than 90° and should not be difficult to achieve. If, in addition, it were possible to deflect the jets fore and aft through a small angle a considerable extra thrust would be available for acceleration or braking without greatly affecting the lift. In the case considered a ten degree deflection from the vertical in a fore and aft direction gives nearly 15,000 lbs of horizontal thrust.

The calculated times to normal flight indicate that it would not be necessary to run the lift engines for more than two minutes at either take-off or landing. Owing to the need to maintain a reasonable

hover height however, it would not appear possible to reduce the installed auxiliary thrust much below one quarter of the all-up weight unless very precise control was possible. It is possible that an optimum jet arrangement could reduce the figure slightly but fig. 3a shows that comparatively small increases in h/Co require large increases in auxiliary thrust and no spectacular reductions can be envisaged. On the other hand, of course, if smaller hover clearances prove acceptable substantial reductions in auxiliary thrust can be made.

The number of lift engines required is likely to be at least twenty and may even be as many as thirty. In the event of one engine failing on take-off it will probably be necessary to cut its opposite number to avoid large rolling moments. Thus it is possible to lose up to 10% of the auxiliary thrust. This would mean a reduction of hover height of about four inches and corresponding increases in take-off distance but losing two engines with this scheme is far less serious than with direct jet lift and even the loss of one quarter of the total thrust would not be catastrophic providing some sort of common ducting could be arranged which would avoid too large gaps in the jet sheet.

Naturally the use of such a large auxiliary thrust carries with it a considerable weight penalty but this is still quite small compared with the increased weight required to give full V.T.O.L. capability. In the present example the increased weight is about 11% of the total all-up weight without lift engines. With the present data it is not possible to estimate the economic penalty on the cruise imposed by the use of so many lift engines but this is offset in the overall economic

picture by considerable savings on capital and maintenance costs of airports. Also, very large airliners would no longer be tied to trips between major cities with huge airports and the ability to operate almost from fields would increase the versatility of large aircraft enormously.

On the question of noise it seems inevitable that S.T.O.L. aircraft will create more noise near the ground than conventional aircraft (3 - 5 db) since in this case at least the thrust will be doubled. However, the noise from the auxiliary jets can be minimised by the use of high by-pass ratios, which will also mean cooler jets, and the increase in performance plus the fact that the running time of the lift jets is small could even reduce the noise beyond about one mile from the starting point.

Summing up, it may be said that no large reduction in take-off and landing performance can be visualised without the addition of a considerable amount of auxiliary power. Leading-edge blowing using engine compressor bleed, described in Part I, offers reductions in landing speed of about fifteen knots for a comparatively small penalty. This system seems particularly attractive for tailless aircraft since no additional trimming is required but above C_{μ} values of 0.05 the lift increment $\frac{\Delta C_L}{C_{\mu}}$ falls off and the addition of auxiliary thrust with this scheme is hardly worth while.

Although the vertical take-off and landing airliner is a very attractive concept, the noise objection to city centre operation and the weight penalty of the lifting engines, severely limit its usefulness

in commercial operations. The form of assisted take-off and landing described here offers a very substantial increase in low speed performance with a very much smaller weight penalty making it more attractive economically.

From the present experimental evidence and calculations there seems good reason to hope that an aircraft designed along these lines would have great versatility, being able to operate from many parts of the world which are now taboo for large aircraft, and yet have an overall operating cost little, if any, greater than an equivalent conventional aircraft.

132

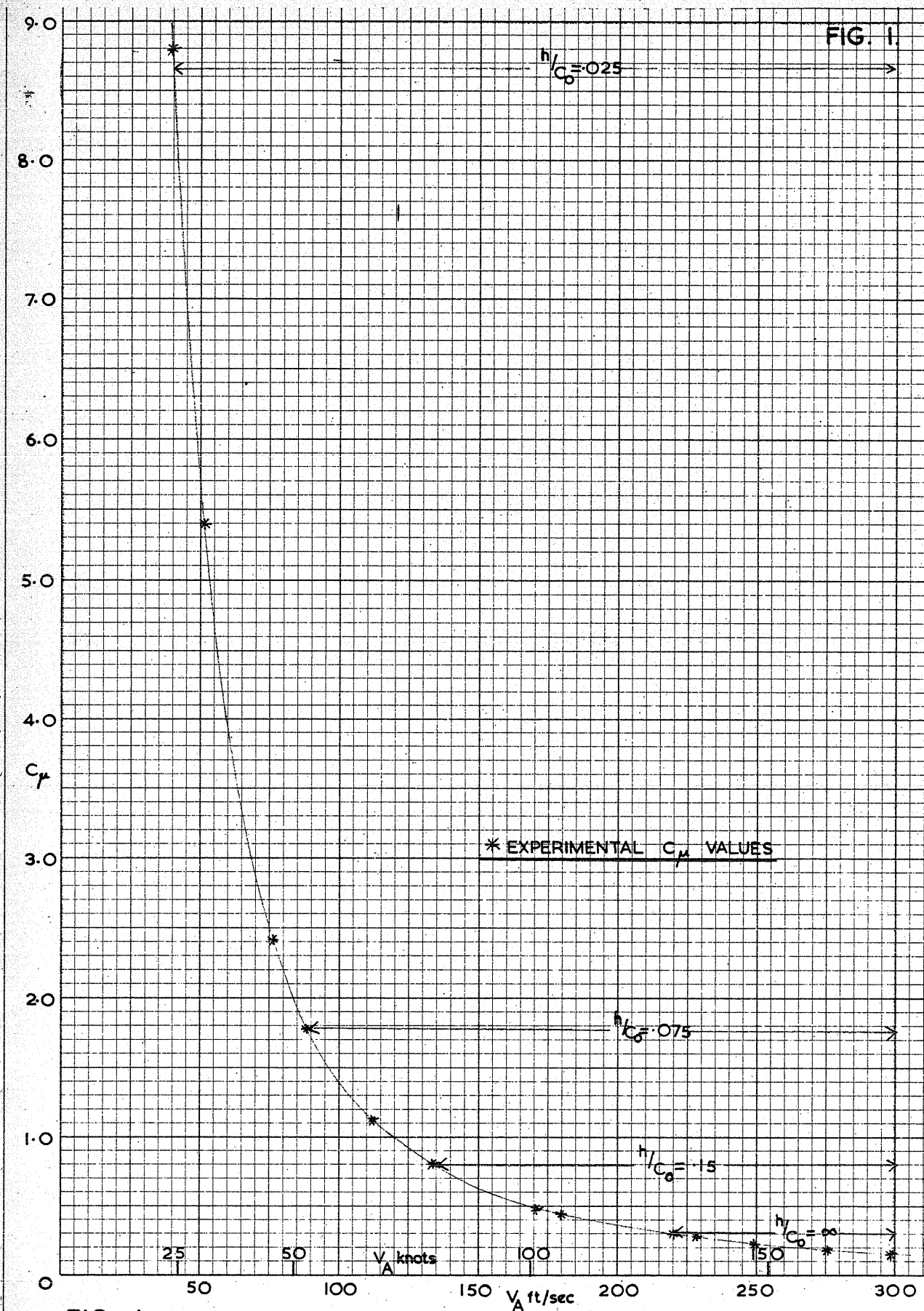


FIG. 1.
VARIATION OF C_μ WITH WIND SPEED FOR THE MODIFIED AIRCRAFT.

FIG. 2, 3a.

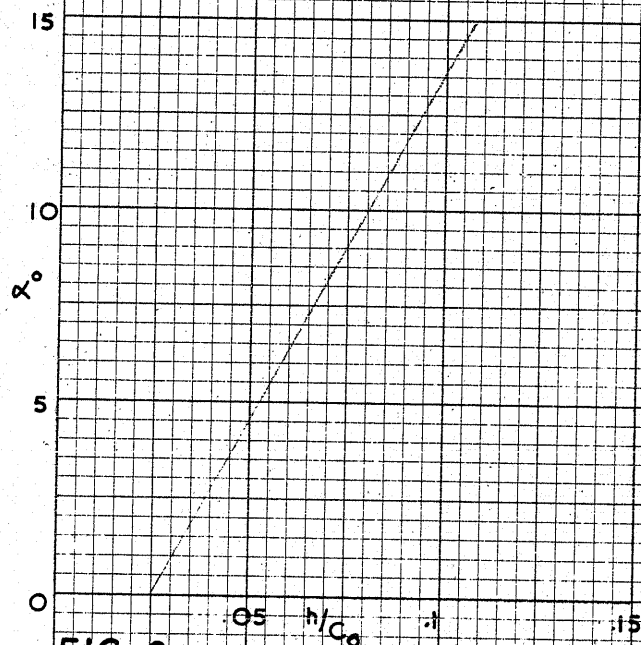


FIG. 2.
MAXIMUM α FOR $h/c_0 = 0.25$ AT TRAILING EDGE.

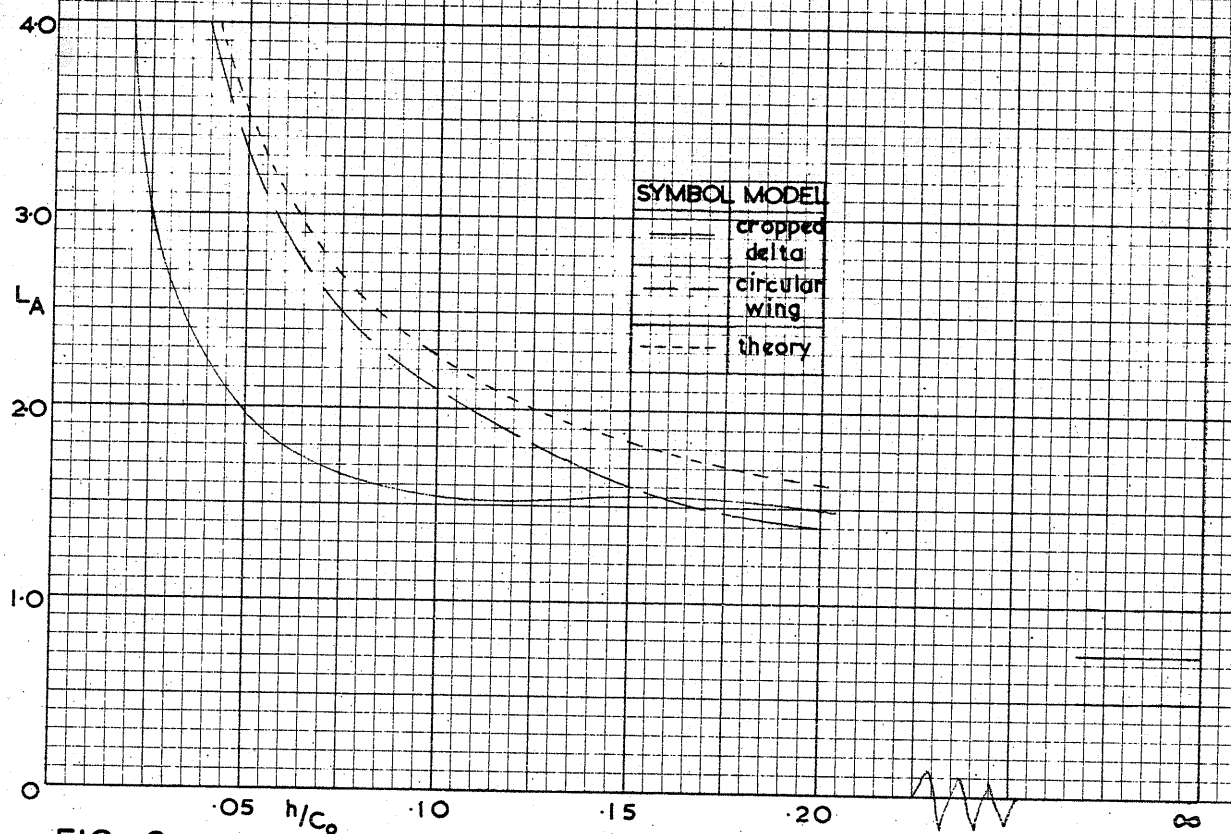


FIG. 3a.

VARIATION OF LIFT AUGMENTATION WITH HEIGHT. $V=0$.

FIGS. 3b-3e.

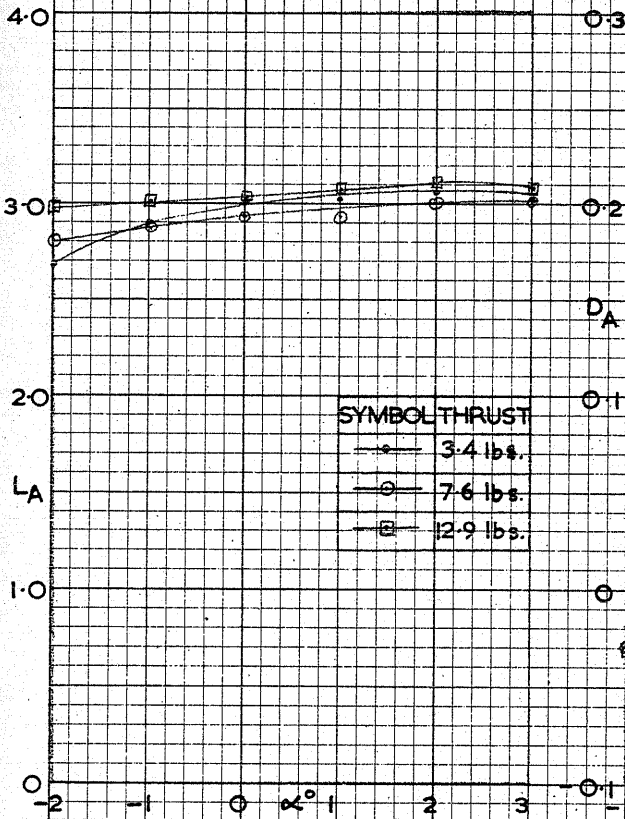


FIG. 3b.

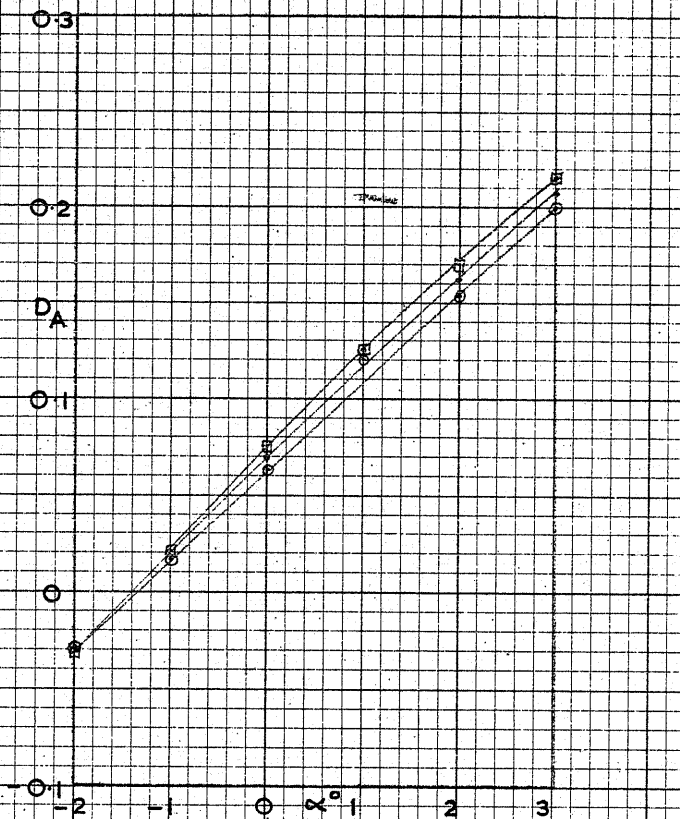


FIG. 3c.

VARIATION OF LIFT AND DRAG WITH INCIDENCE. $V=0$ $\eta/c_o=0.25$.

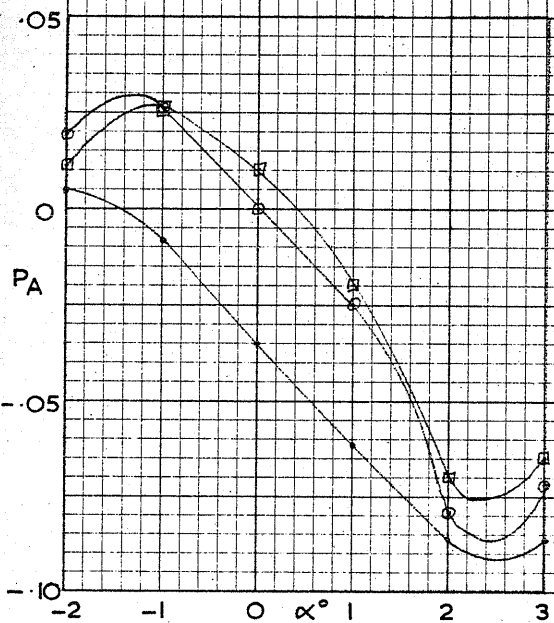


FIG. 3d.

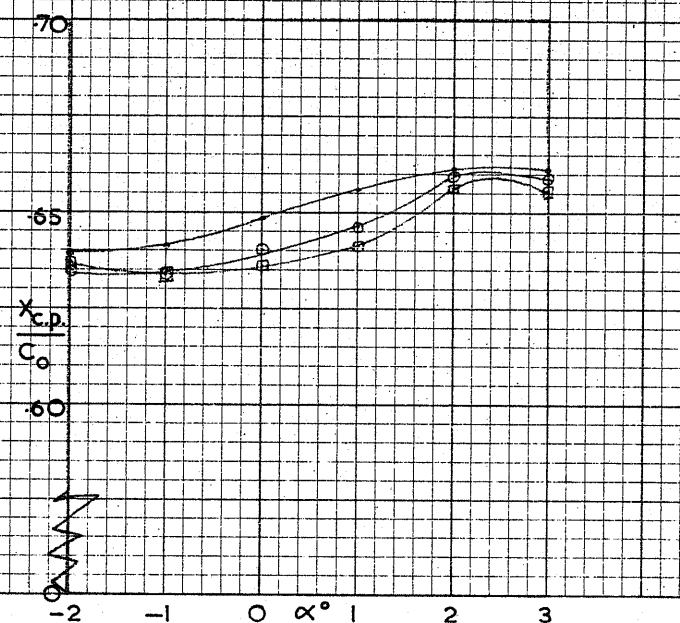


FIG. 3e.

VARIATION OF PITCHING MOMENT AND C.P. WITH INCIDENCE. $V=0$ $\eta/c_o=0.25$.

FIG. 4a.

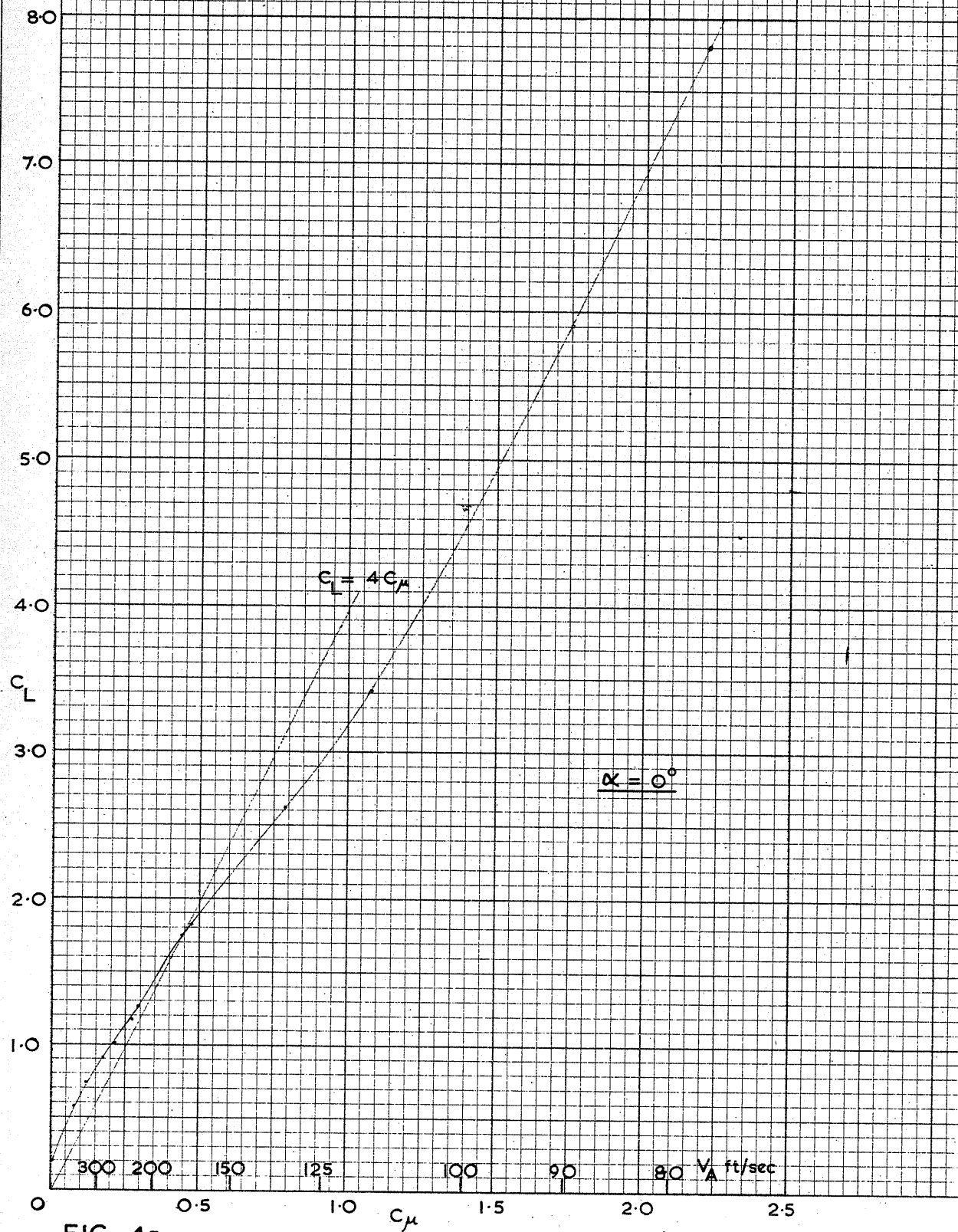


FIG. 4a.

VARIATION OF LIFT WITH MOMENTUM COEFFICIENT. $\eta/c_o = .025$

FIG. 4b.

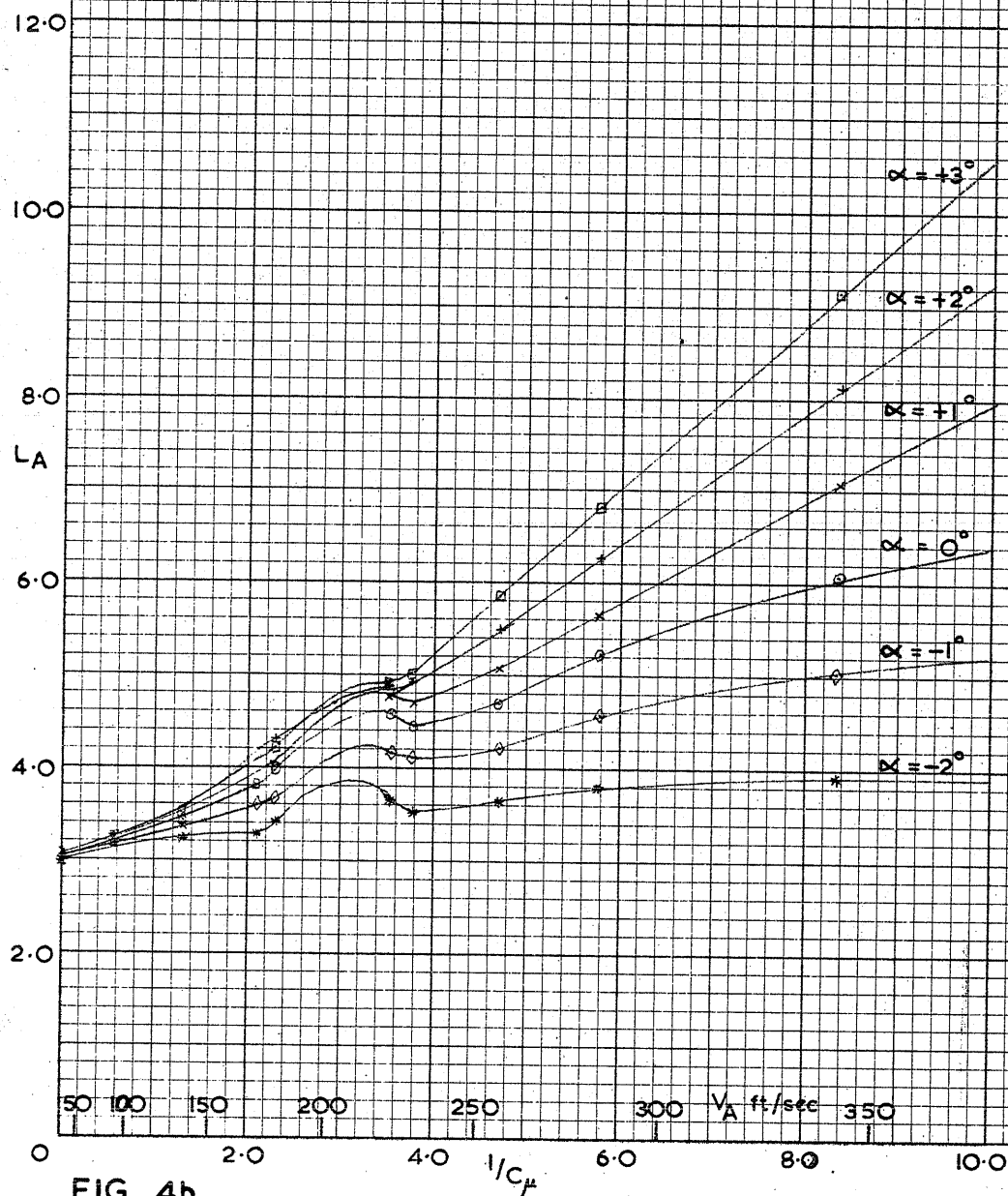


FIG. 4b.

VARIATION OF LIFT AUGMENTATION WITH $1/C_\mu$ (αq_0). $h/c_0 = 0.025$

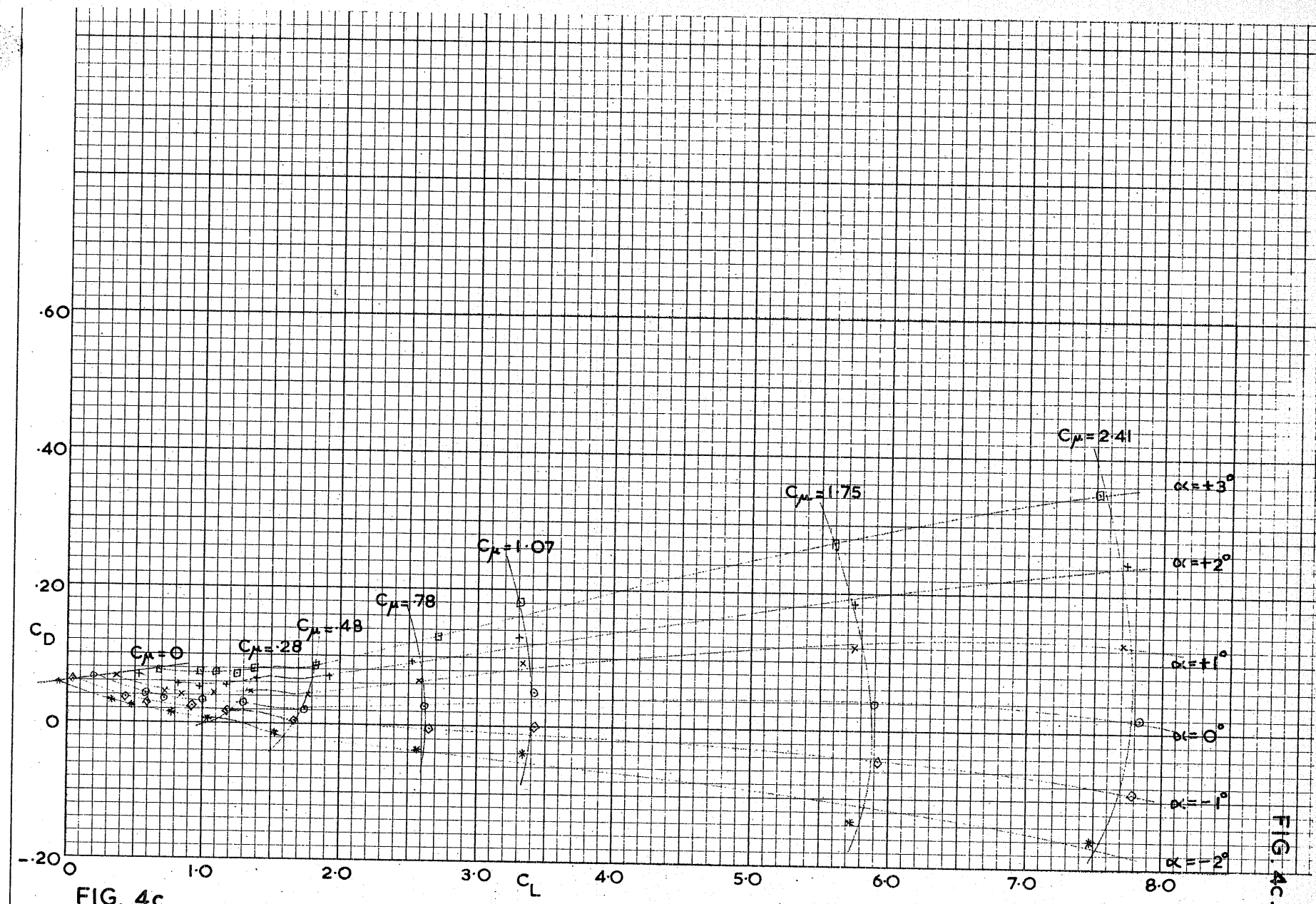


FIG. 4c.

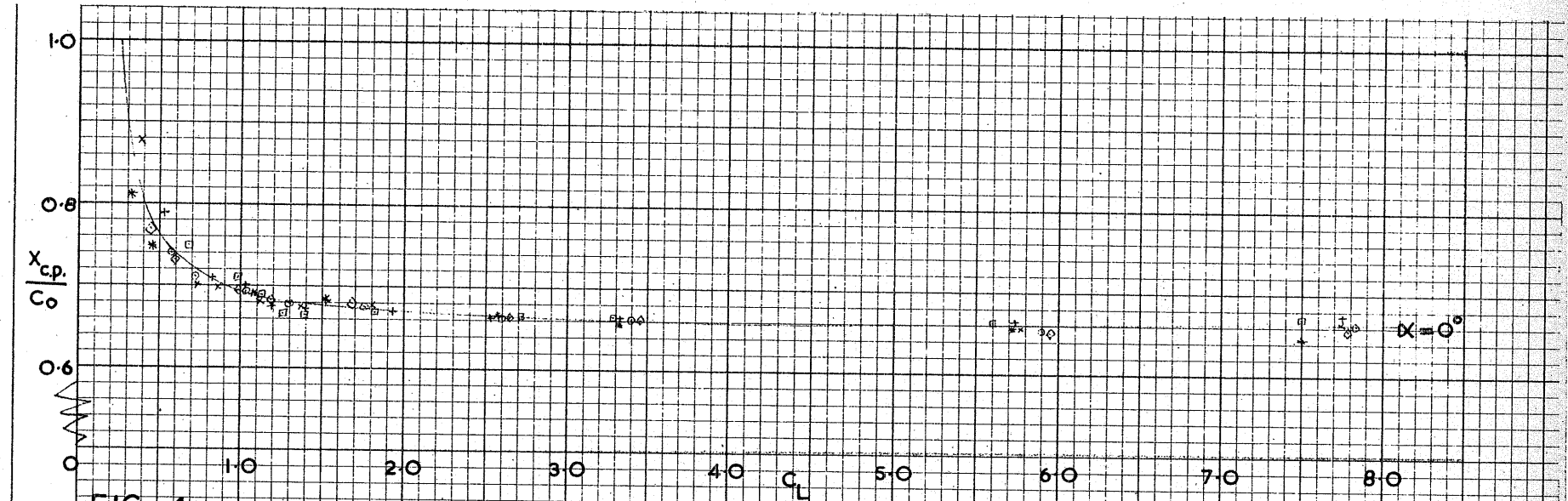


FIG. 4e.

MOVEMENT OF CENTRE OF PRESSURE WITH LIFT. $h/c_o = 0.25$

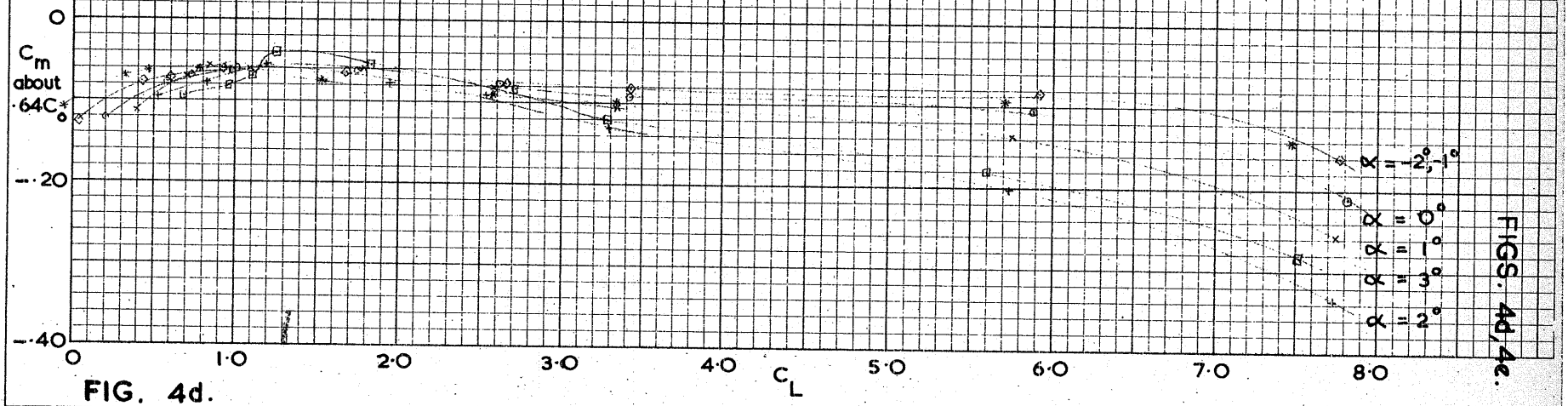


FIG. 4d.

VARIATION OF PITCHING MOMENT WITH LIFT. $h/c_o = 0.25$

FIG. 5a.

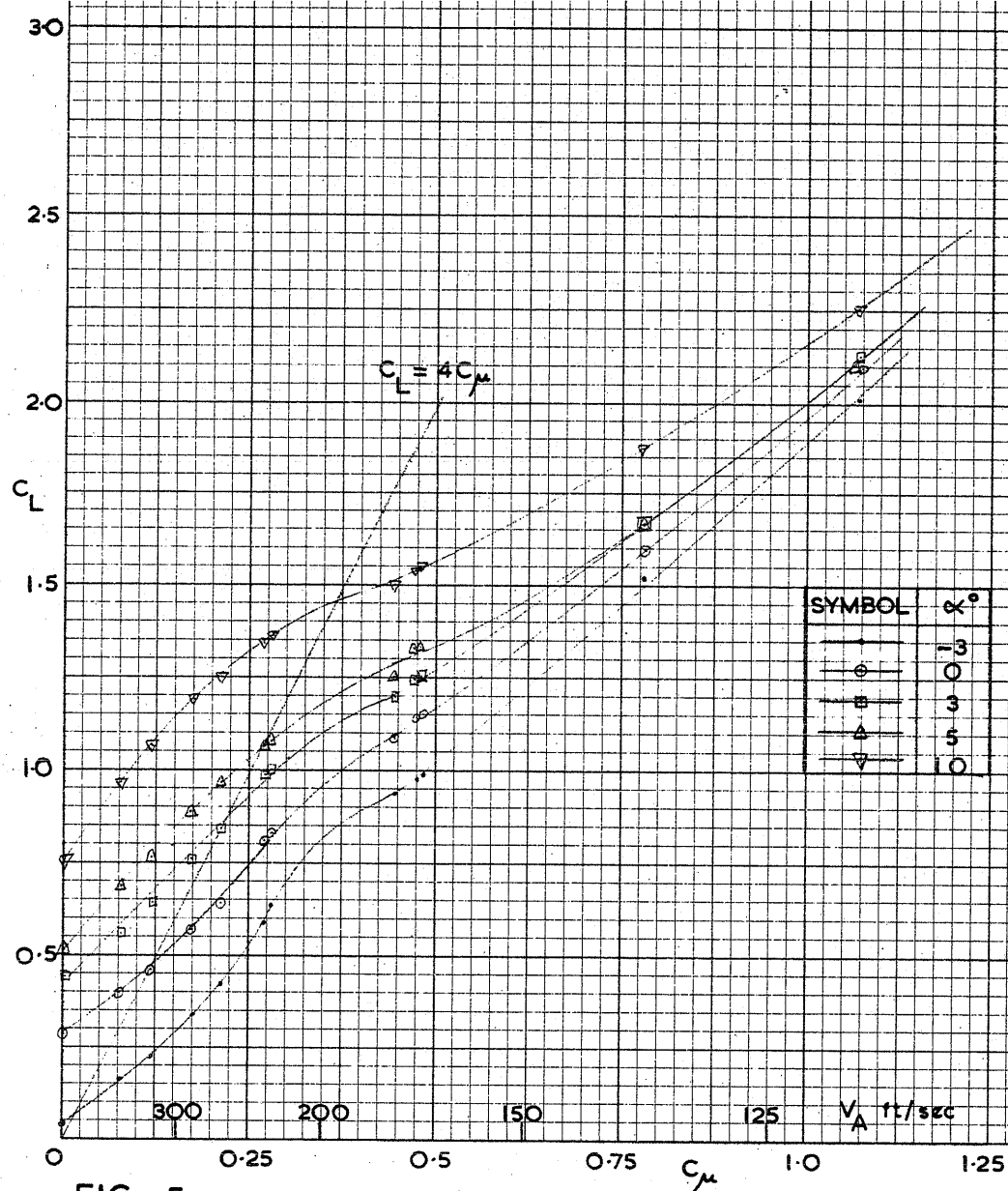


FIG. 5a.

VARIATION OF LIFT WITH MOMENTUM COEFFICIENT. $\eta/c_o = 0.075$

FIG. 5b.

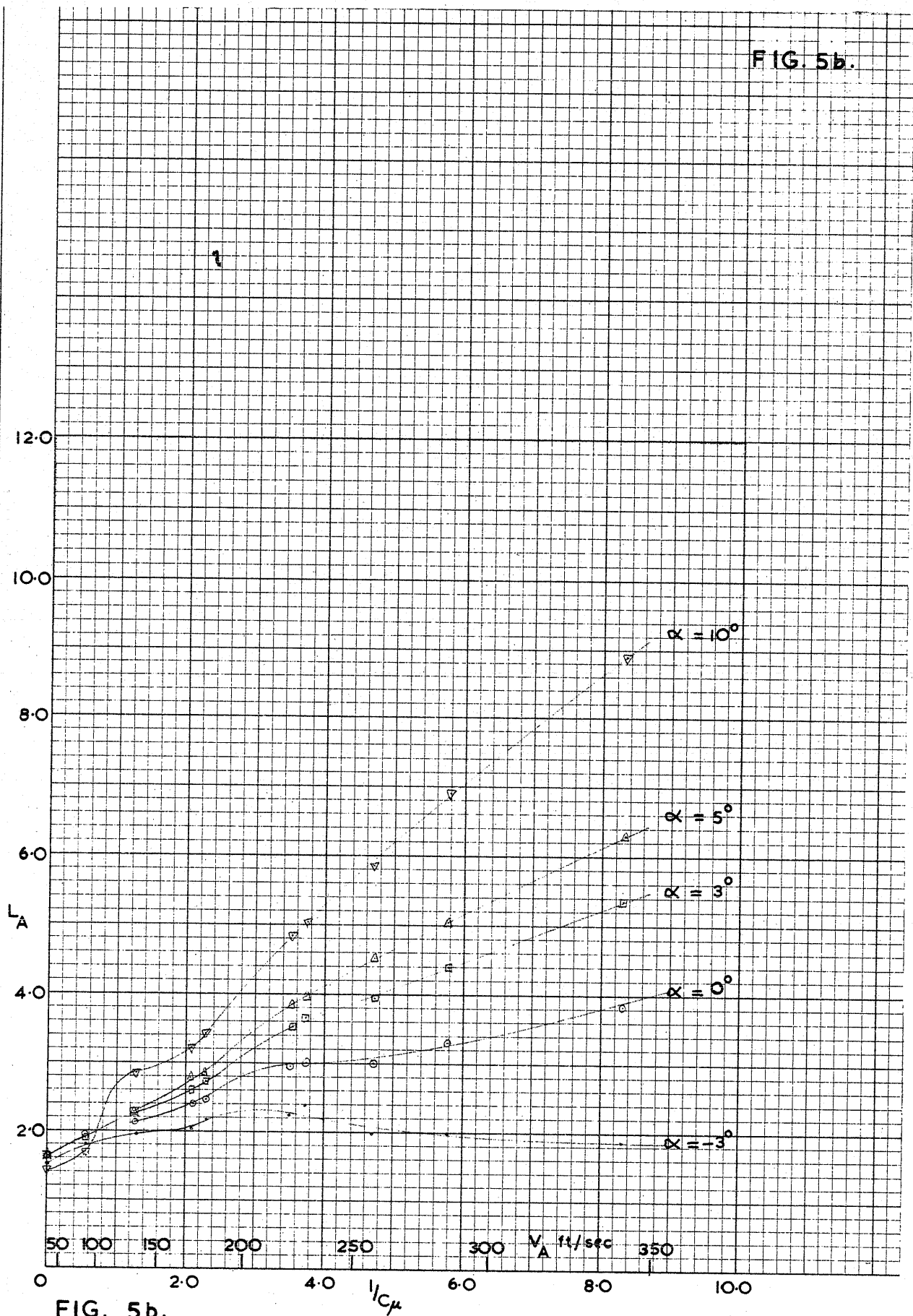


FIG. 5b.

VARIATION OF LIFT AUGMENTATION WITH $1/C_\mu (\infty q_0)$. $\eta/c_o = 0.075$

FIG. 5c.

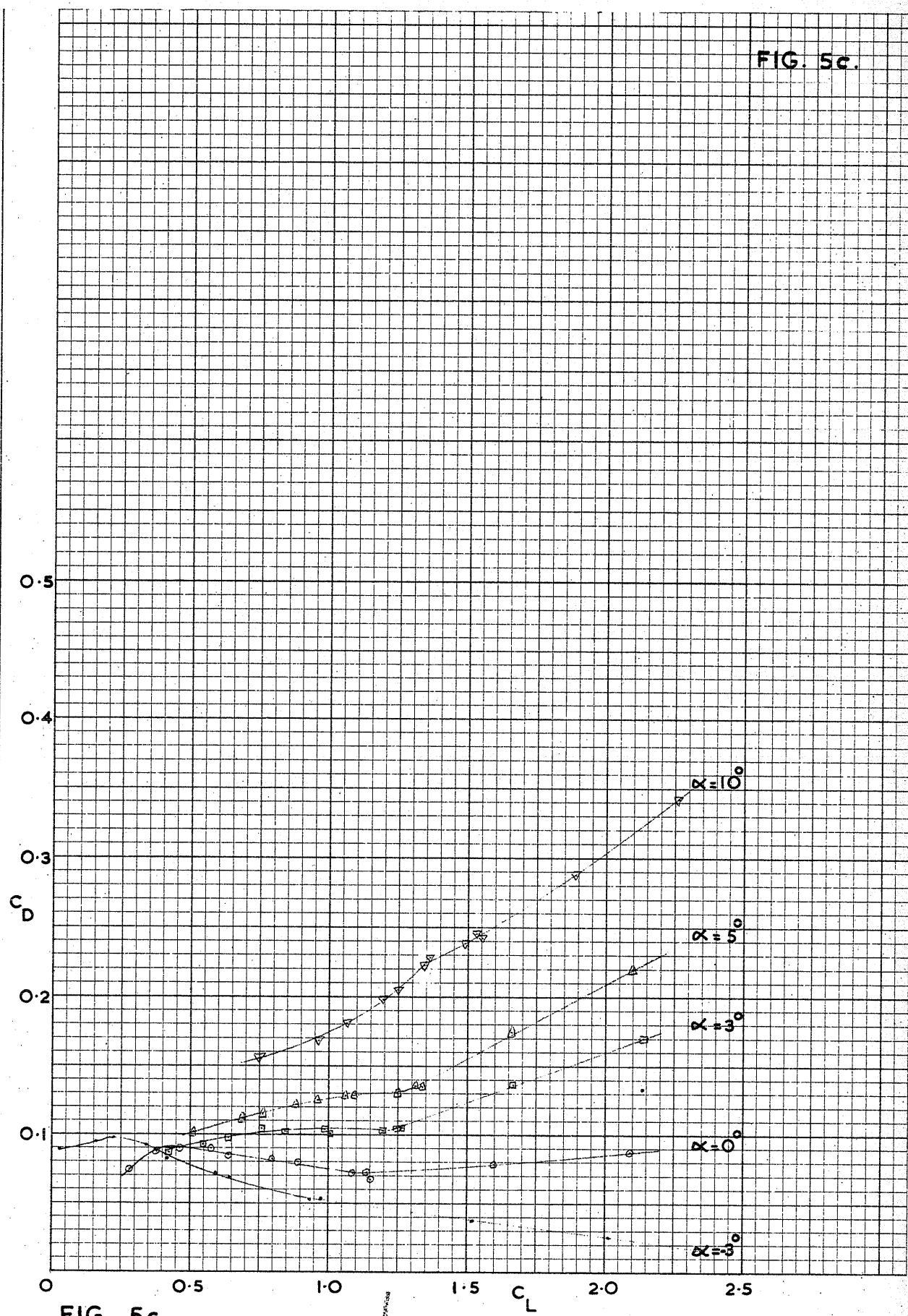


FIG. 5c.

VARIATION OF DRAG WITH LIFT. $h/c_o = 0.75$

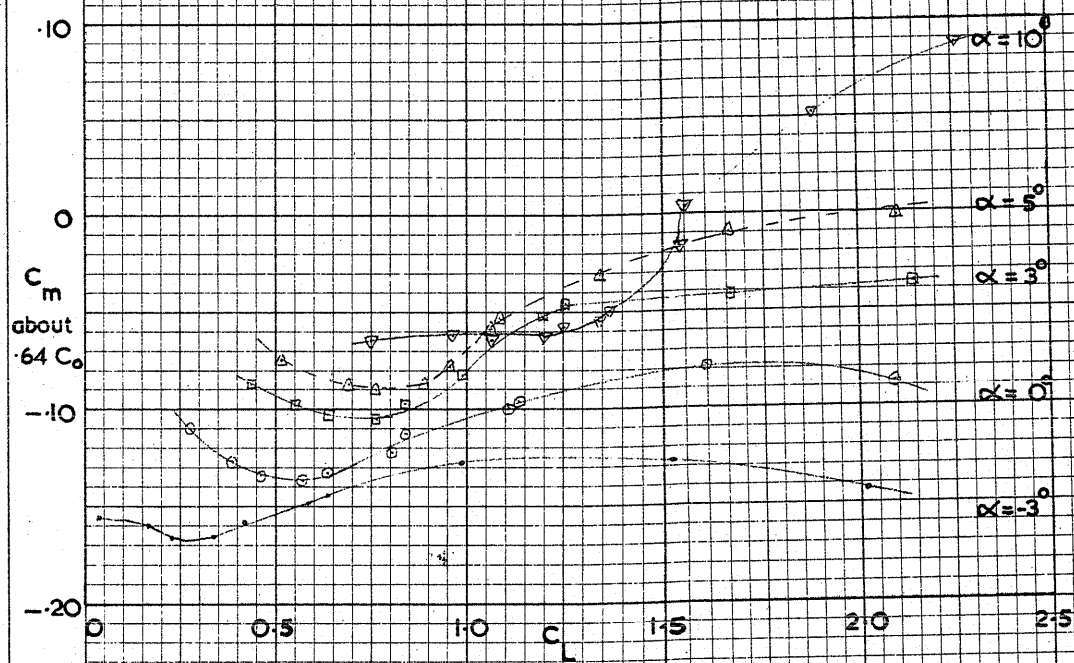


FIG. 5d.

VARIATION OF PITCHING MOMENT WITH LIFT. $h/c_o = .075$

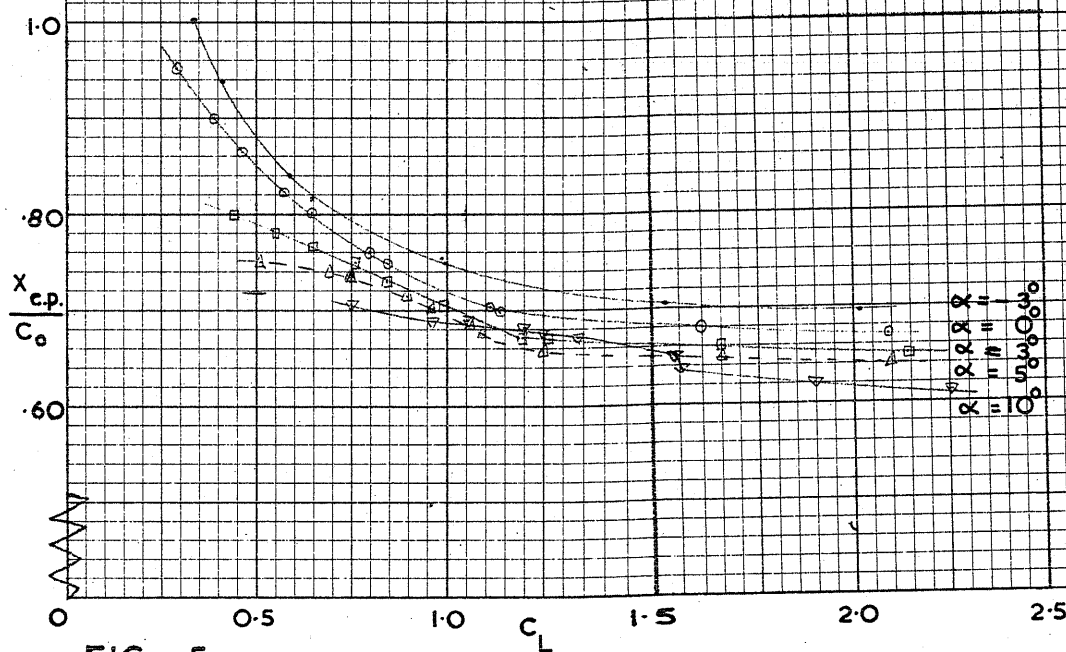


FIG. 5e.

VARIATION OF CENTRE OF PRESSURE WITH LIFT. $h/c_o = .075$

FIG. 6a.

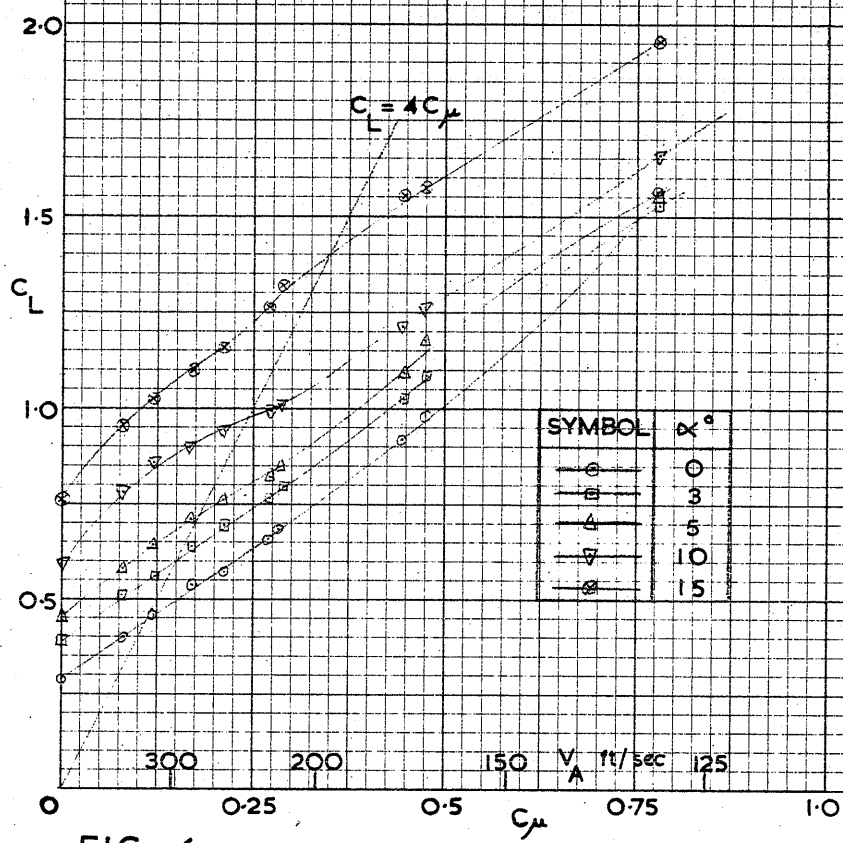


FIG. 6a.

VARIATION OF LIFT WITH MOMENTUM COEFFICIENT. $\eta/c_o = .15$

FIG. 6b.

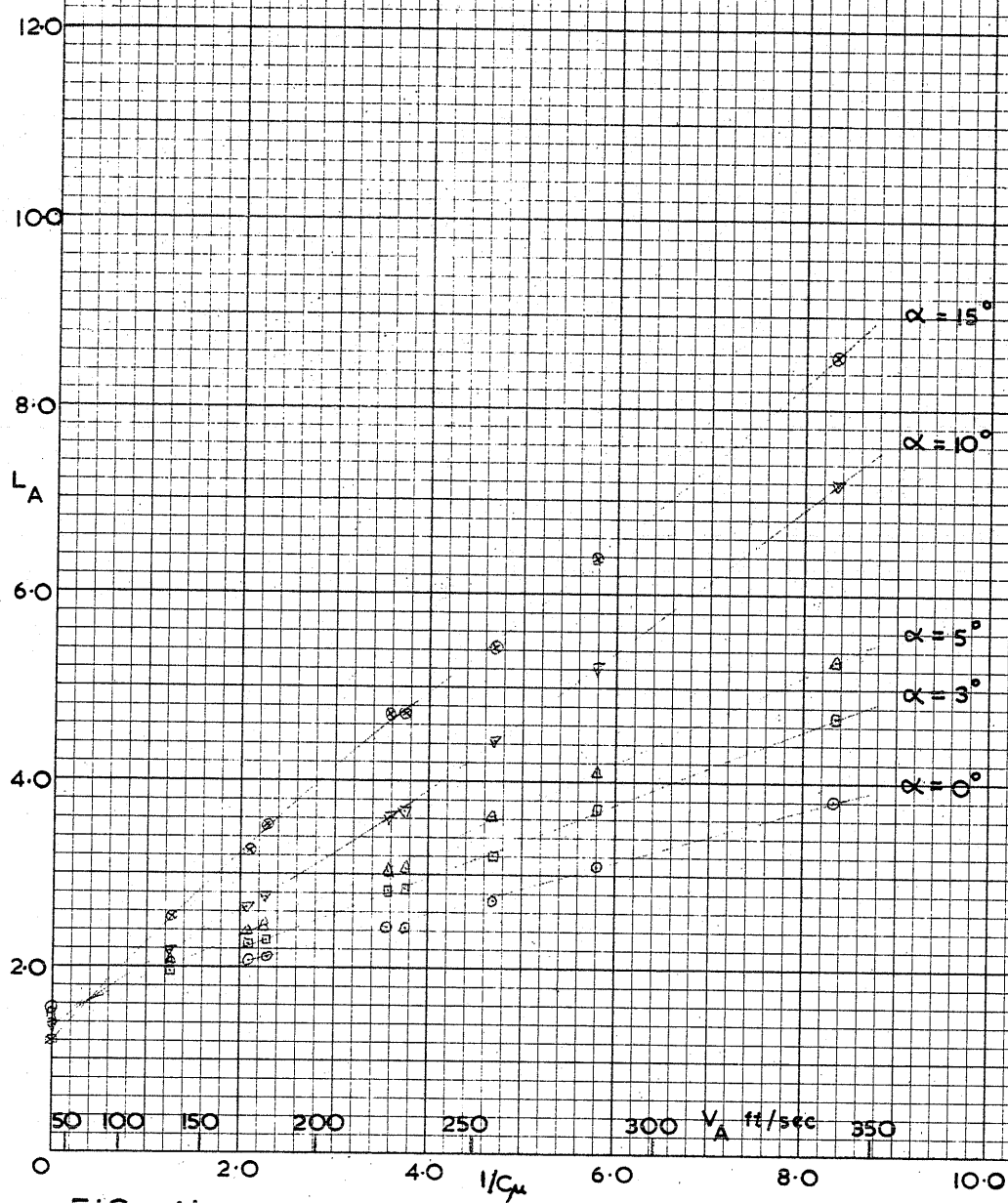


FIG. 6b.

VARIATION OF LIFT AUGMENTATION WITH $1/C_\mu (\propto q_0)$. $h/c_0 = .15$

FIG. 6c.

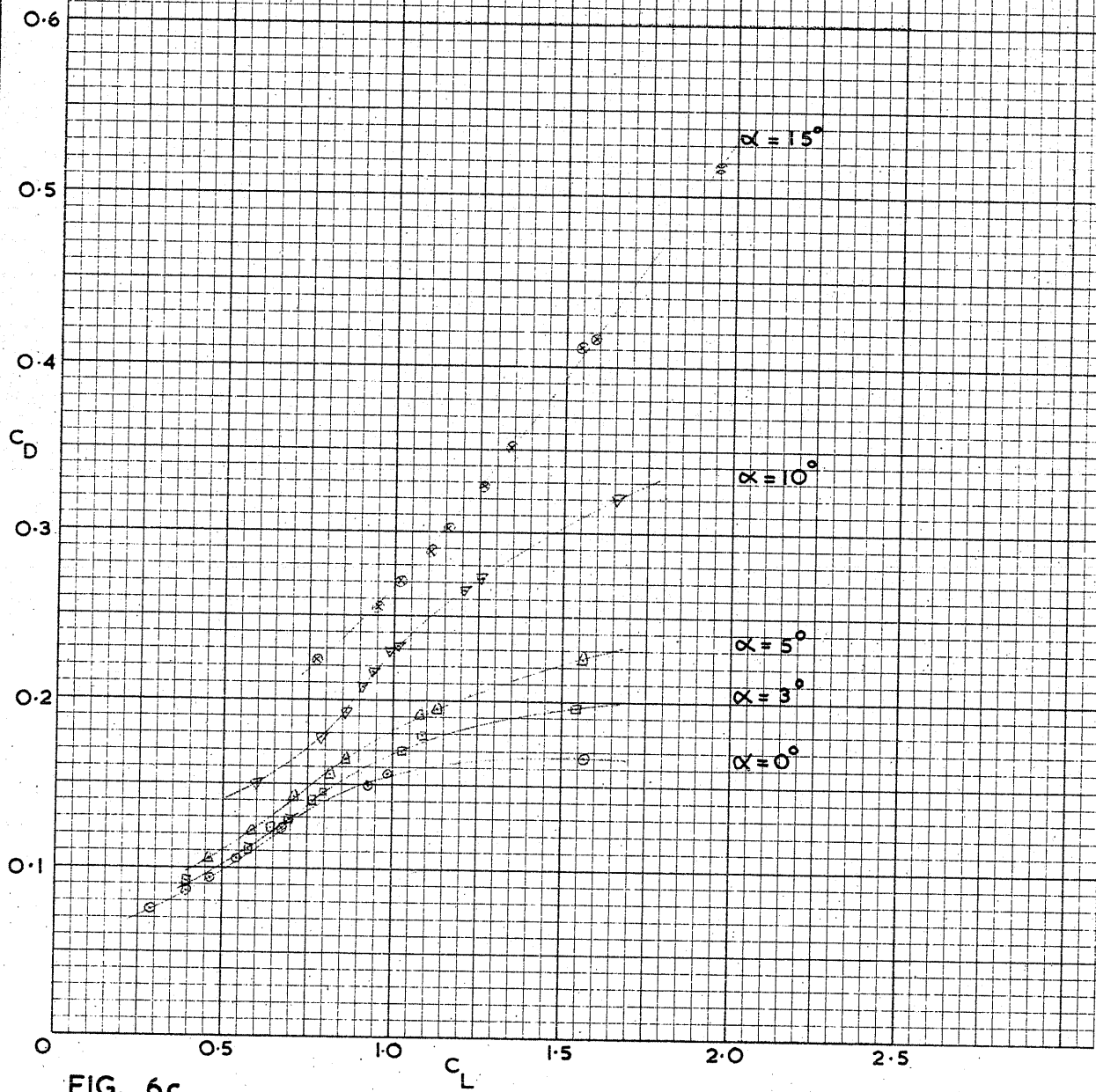


FIG. 6c.

VARIATION OF DRAG WITH LIFT. $\eta/c_o = .15$

FIGS. 6d, 6e.

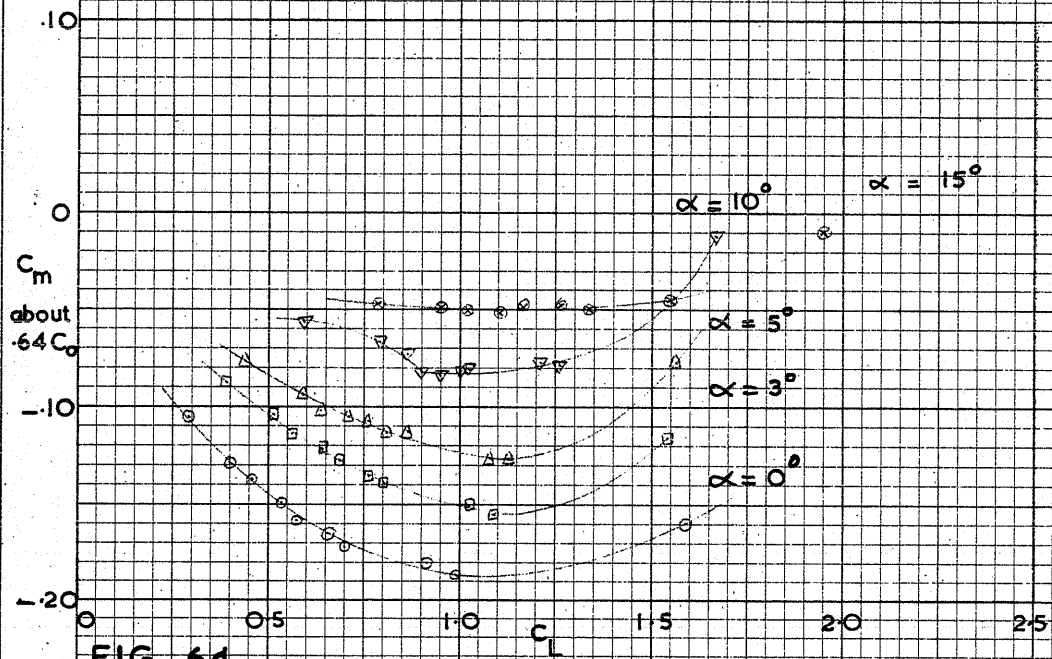


FIG. 6d.

VARIATION OF PITCHING MOMENT WITH LIFT. $h/c_0 = 0.15$

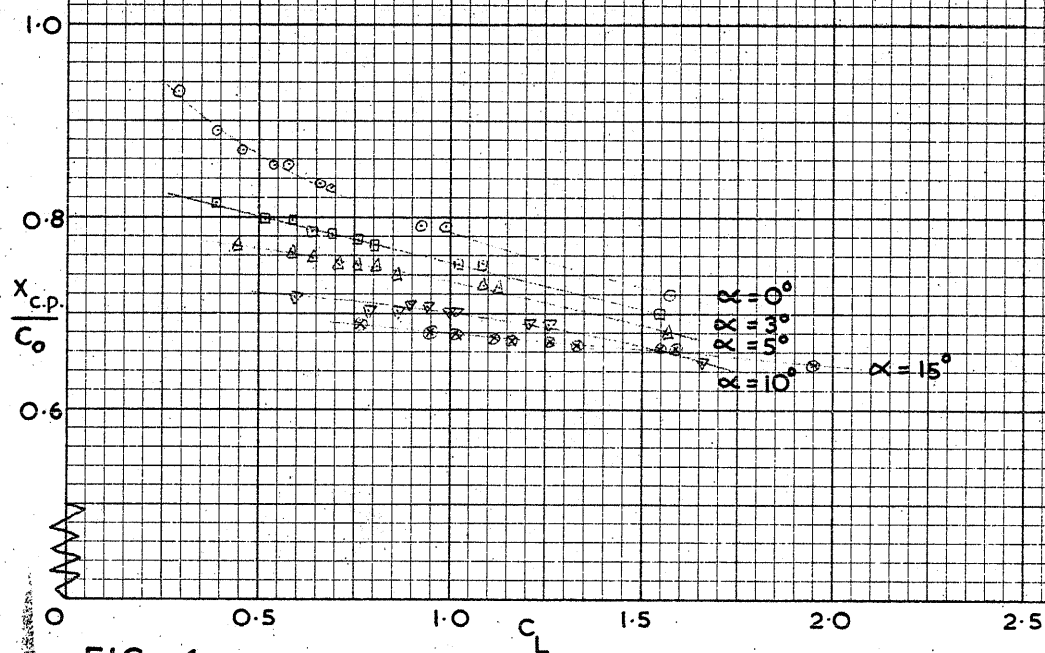


FIG. 6e.

MOVEMENT OF CENTRE OF PRESSURE WITH LIFT. $h/c_0 = 0.15$

FIG. 7a.

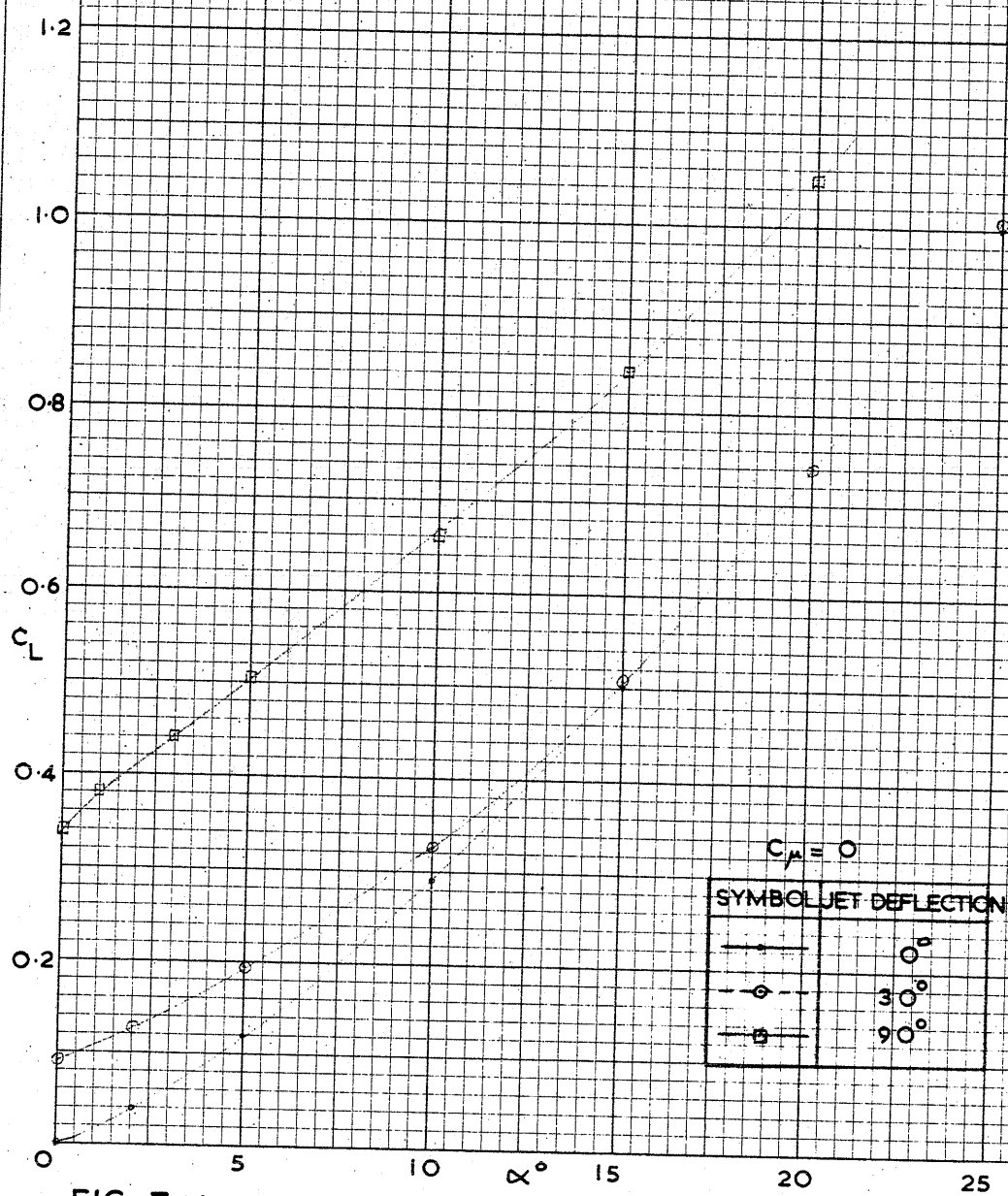


FIG. 7a.

VARIATION OF LIFT WITH INCIDENCE. $h/c_o = \infty$

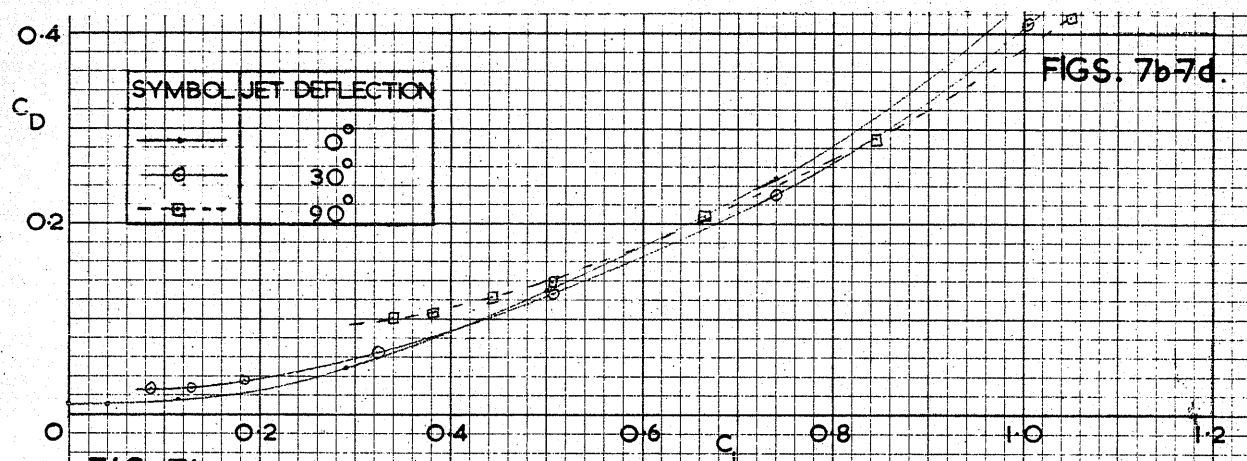


FIG. 7b.

VARIATION OF DRAG WITH LIFT. $C_{\mu} = 0$ $h/c_o = \infty$

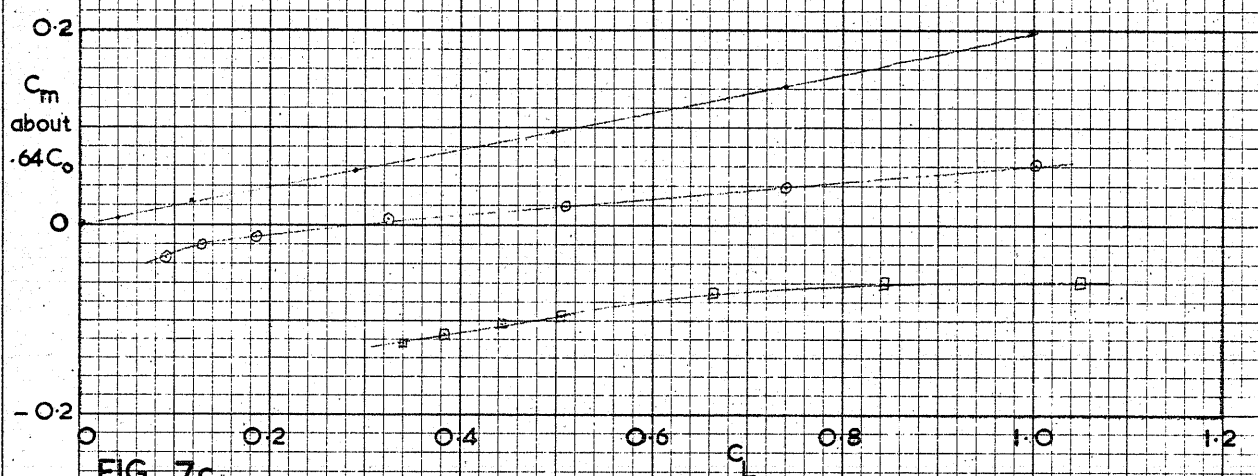


FIG. 7c.

VARIATION OF PITCHING MOMENT WITH LIFT. $C_{\mu} = 0$ $h/c_o = \infty$

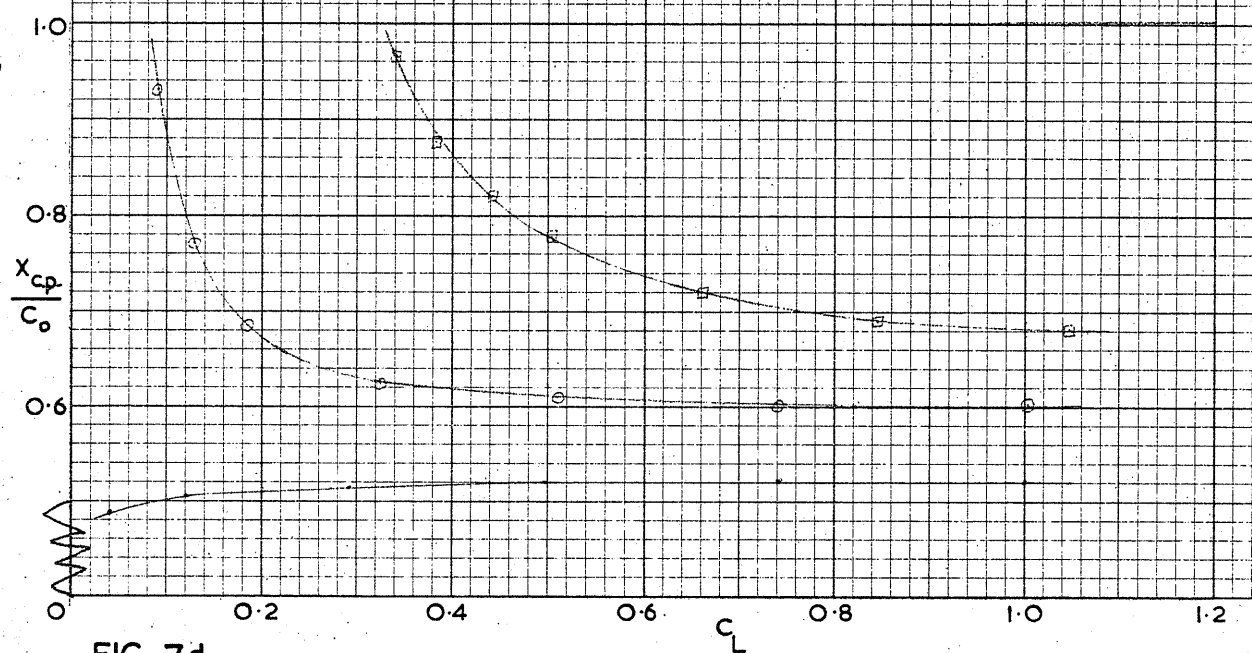


FIG. 7d.

MOVEMENT OF CENTRE OF PRESSURE WITH LIFT. $C_{\mu} = 0$ $h/c_o = \infty$

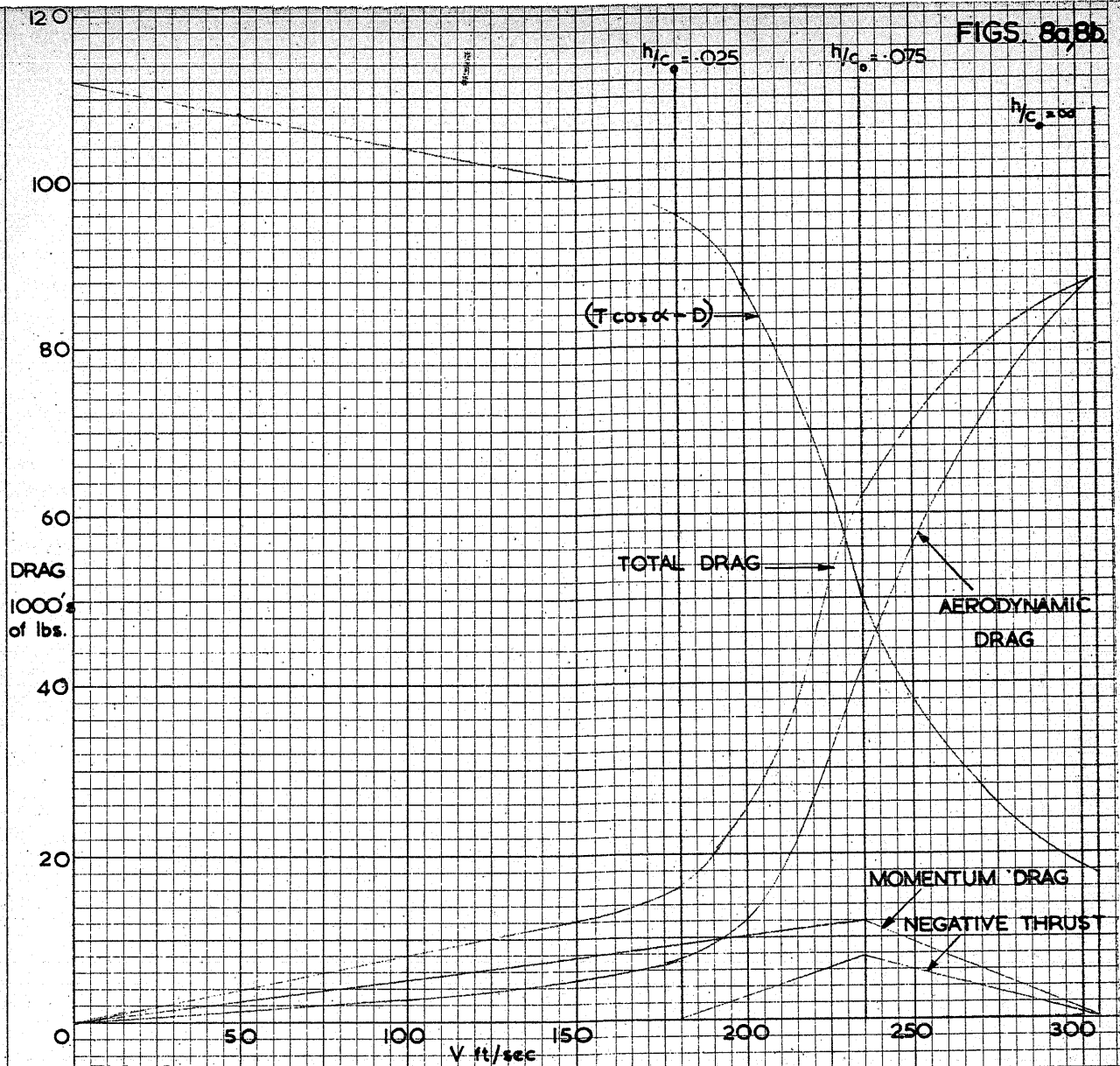


FIG. 8a.

VARIATION OF DRAG DURING TAKE - OFF.

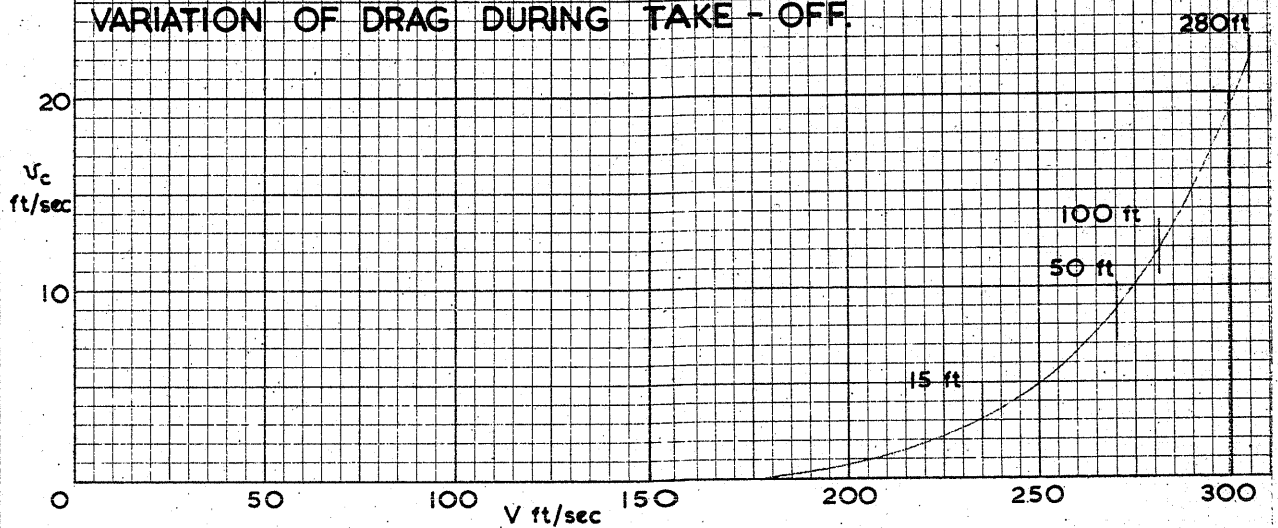


FIG. 8b.

RATE OF CLIMB AND HEIGHT DURING TAKE - OFF.

FIGS. 9a, 9b.

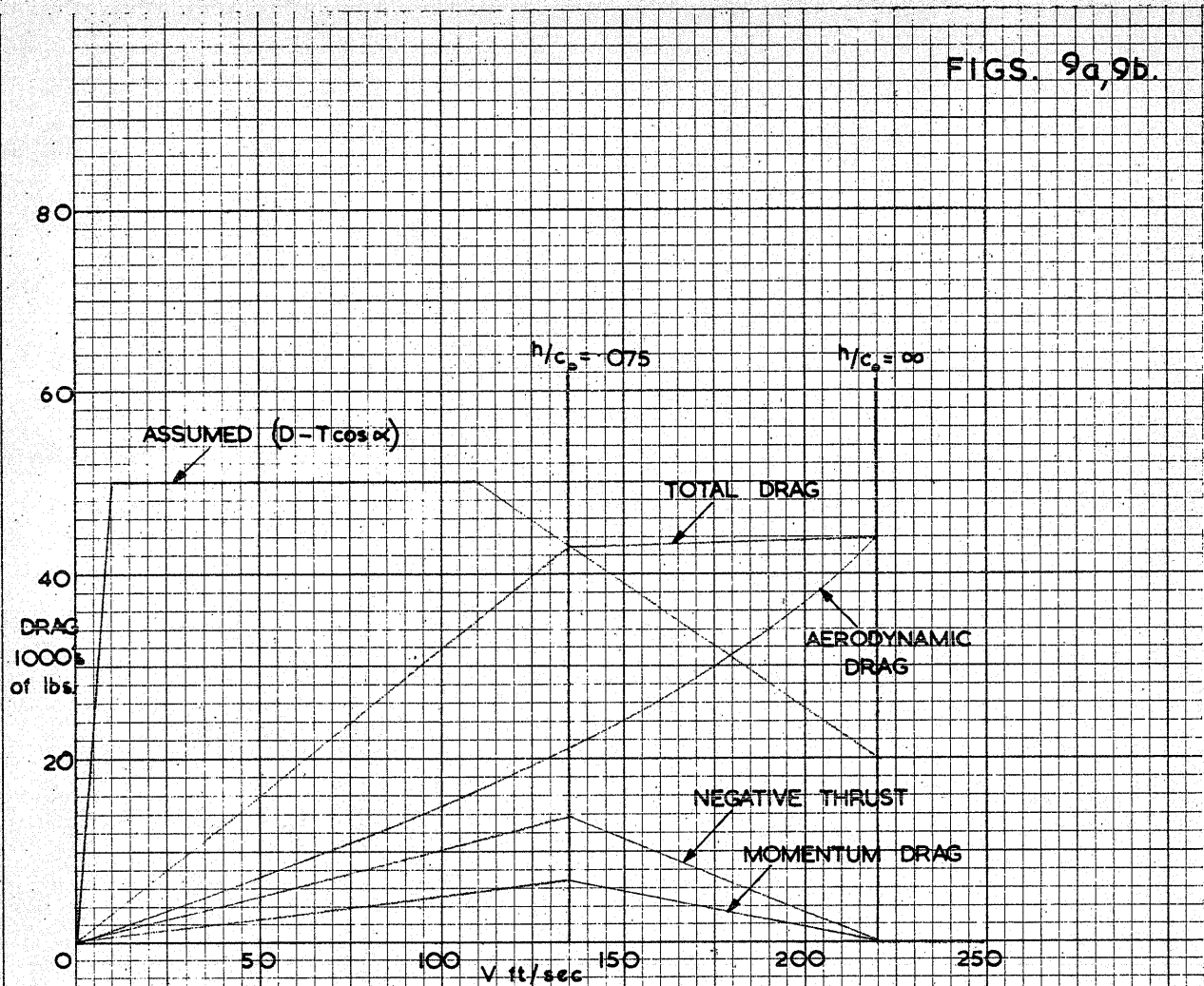


FIG. 9a.

VARIATION OF DRAG DURING LANDING.

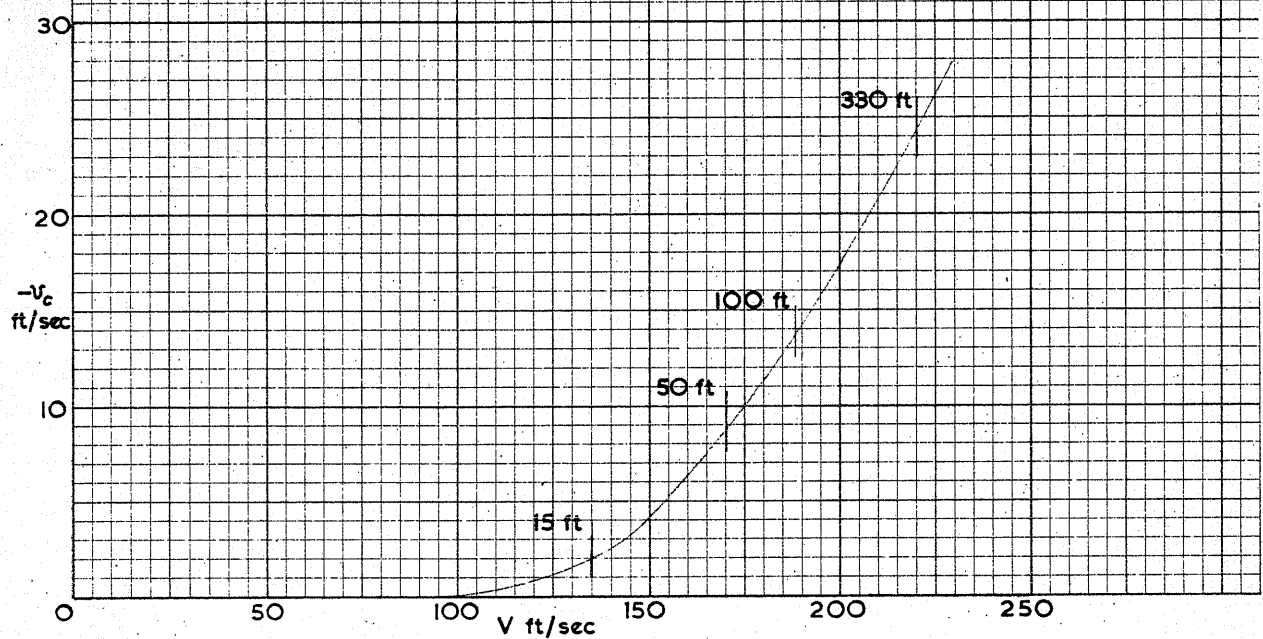


FIG. 9b.

RATE OF DESCENT AND HEIGHT DURING LANDING.