

CRANFIELD INSTITUTE OF TECHNOLOGY

FLUID ENGINEERING UNIT

PhD THESIS

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A THEORETICAL AND EXPERIMENTAL INVESTIGATION OF  
EROSION PRONE AREAS ON THE BLADE SURFACES OF A  
CENTRIFUGAL IMPELLER HANDLING GRANULAR SOLIDS

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TO MY PARENTS

## ABSTRACT

The erosion prone areas on the blade surfaces of a centrifugal impeller handling granular solids have been investigated both theoretically and experimentally.

The objective of the theoretical study is to develop software to predict the erosion prone areas on the blade surfaces of a slurry handling impeller. The basic concept in the theoretical study has been to find the particle trajectories inside the impeller. The governing equations of the motion of particles within the impeller are solved in a stepwise manner with time for a given flow field. The flow field is calculated by solving the conservation equations for the flow, using a finite difference method.

The number density of the particles in the cross-section of the pipe through which the particles enter the impeller is calculated by applying a discretization procedure to the insitu concentration profile of the particles.

The damage to the surface caused by impacting particles is estimated by calculating the velocity and the angle at which the particles impact on a surface. The material property of the particle or the surface does not explicitly enter into calculations. It is shown that for a given combination of particles and surface material, the material property of the particles and the surface can be isolated. In this way a relative measure of the depth of volume loss at different positions of the surface can be calculated. The surface can then be divided into a number of erosion prone areas.

The model is tested by undertaking a number of experiments using a commercially available slurry pump which handled a mixture of fine pea gravels in water.

The predicted erosion pattern suggested the area near the leading edge to be the most erosion prone zone. The agreement with the experimental results in this respect was satisfactory. The extent of the erosion zone on the blade surface as suggested by the analytical model was in close agreement with the experimental values. The analytical model also suggested that the major part of the erosion areas would lie near the back shroud of the impeller. This was in close agreement with the experimental results.

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# NOTATIONS

|                        |                                                       |         |
|------------------------|-------------------------------------------------------|---------|
| $A$                    | projected area of a particle                          | $(m^2)$ |
| $A_i, A_j$             | area of $i$ th and $j$ th element on a surface        | $(m^2)$ |
| $A_n$                  | acceleration number                                   |         |
| $a$                    | exponent as defined in eqn. 4.7                       |         |
| $\underline{a}$        | acceleration vector                                   |         |
| $a_c$                  | area of a cracked surface in brittle erosion          |         |
| $a_i, a_{i+1}$         | area of annulus rings                                 | $(m^2)$ |
| $a_i$                  | area of an element in a pipe section                  | $(m^2)$ |
| $a_0, a_1, \dots, a_4$ | coefficients of polynomial, eqn. 5.4                  |         |
| $B$                    | integration factor                                    |         |
| $b$                    | exponent as defined in eqn. 4.7                       |         |
| $b_0, b_1, \dots, b_5$ | coefficients of polynomial, eqn. 5.6b                 |         |
| $C$                    | coefficient defined in eqn. 2.1.11                    |         |
| $C_a, C_h$             | coefficients defined in eqn. 2.1.10a                  |         |
| $C_D$                  | drag coefficient                                      |         |
| $C_L$                  | lift coefficient                                      |         |
| $C_T$                  | total resistance coefficient                          |         |
| $c_d$                  | insitu discharge concentration, volume fraction       |         |
| $c_t$                  | insitu transport concentration, volume fraction       |         |
| $\bar{c}_d$            | mean discharge concentration, volume fraction         |         |
| $\bar{c}_t$            | mean transport concentration, volume fraction         |         |
| $c_{t_{z_0}}$          | insitu concentration at level $z_0$ , volume fraction |         |

|                 |                                                             |                     |
|-----------------|-------------------------------------------------------------|---------------------|
| $c_0, c_1, c_4$ | coefficients of polynomial, eqn. 5.12b                      |                     |
| $D_f$           | time derivative in an inertial frame                        | (s <sup>-1</sup> )  |
| $D_i$           | impeller outside diameter                                   | (m)                 |
| $D_r$           | time derivative in a rotating frame                         | (s <sup>-1</sup> )  |
| $d$             | diameter of a particle/solid sphere                         | (m)                 |
| $\bar{d}$       | weighted particle size                                      | (m)                 |
| $d_n$           | diameter of particles in size fraction n                    | (m)                 |
| $d_{50}$        | opening of mesh passing 50% of solids by mass               | (mm)                |
| $E_p, E_s$      | modulus of elasticity of particle and surface, respectively | (Nm <sup>-2</sup> ) |
| $E_t$           | energy of turbulence                                        | (Nm)                |
| $E_v$           | energy in viscous damping                                   | (Nm)                |
| $e$             | restitution coefficient                                     |                     |
| $\hat{e}$       | unit vector                                                 |                     |
| $\hat{e}_g$     | unit vector in the direction of gravity                     |                     |
| $F$             | body force on a fluid particle                              | (N)                 |
| $F_B$           | submerged weight of a particle                              | (N)                 |
| $F_C$           | function of particle size and slurry concentration          |                     |
| $F_D$           | drag force                                                  | (N)                 |
| $F_e$           | friction force                                              | (N)                 |
| $F_L$           | lift force                                                  | (N)                 |
| $F_n$           | vertical force on a particle during cutting                 | (N)                 |
| $F_t$           | tangential force on a particle during cutting               | (N)                 |
| $F_S$           | instantaneous force on a sphere                             | (N)                 |
| $f_d$           | coefficient of dynamic friction                             |                     |

|                    |                                           |                               |
|--------------------|-------------------------------------------|-------------------------------|
| $f_s$              | factor defined in eqn. 2.6.2b             |                               |
| $f_1$              | function defined in eqn. 2.1.10a          |                               |
| $f_2$              | function defined in eqn. 2.1.11b          |                               |
| $f_3$              | function defined in eqn. 2.1.12b          |                               |
| $f_4$              | function defined in eqn. 2.1.13a          |                               |
| $f_5$              | function defined in eqn. 2.3.15a          |                               |
| $f_6$              | function defined in eqn. 2.3.15b          |                               |
| $f_7$              | function defined in eqn. 2.3.15c          |                               |
| $f_8$              | function defined in eqn. 4.1a             |                               |
| $g$                | acceleration due to gravity               | $(\text{ms}^{-2})$            |
| $H_o$              | total enthalpy                            | $(\text{m}^2\text{s}^{-2})$   |
| $H_p$              | pump head                                 | $(\text{m})$                  |
| $\bar{H}$          | head coefficient, $gH_p/\omega^2 D_i^2$   |                               |
| $H_v$              | Vickers hardness                          |                               |
| $H_z$              | angular momentum                          | $(\text{kgm}^2\text{s}^{-1})$ |
| $h$                | depth of water                            | $(\text{m})$                  |
| $h_{f_m}$          | friction head loss in slurry manometer    | $(\text{m})$                  |
| $h_i$              | indentation depth                         | $(\text{m})$                  |
| $h_{f_1}, h_{f_2}$ | friction head losses in counterflow meter | $(\text{m})$                  |
| $h_1, h_2, h_m$    | heights in air-over water manometer       | $(\text{m})$                  |
| $I$                | rothalpy $(H_o - \omega r u_\theta)$      | $(\text{m}^2\text{s}^{-2})$   |
| $i, j$             | element numbers on a blade surface        |                               |
| $\underline{K}$    | curl of fluid velocity                    |                               |
| $k$                | Kármán's universal constant               |                               |
| $k_1$              | constant defined in eqn. 4.1b             |                               |
| $k_2$              | constant defined in eqn. 4.3a             |                               |
| $k_3$              | constant defined in eqn. 4.3b             |                               |

|                 |                                                                          |      |
|-----------------|--------------------------------------------------------------------------|------|
| $k_4$           | constant defined in eqn. 4.7                                             |      |
| $k'_1, k'_2$    | constants defined in eqn.'s 6.7a and 6.7b, resp.                         |      |
| $L_C$           | distance between piezometric rings in counterflow meter                  | (m)  |
| $L_M$           | distance between pressure tappings in slurry manometer                   | (m)  |
| $l$             | depth of crater in an element                                            | (m)  |
| $l_i, l_j$      | depth of crater in ith and jth element                                   | (m)  |
| $l_t$           | depth of contact                                                         | (m)  |
| $M$             | mass of a particle flux                                                  | (kg) |
| $m_a, m_b$      | mass of particle a and b, resp.                                          | (kg) |
| $m_f$           | mass of fluid displaced by a solid particle                              | (kg) |
| $m_w$           | mass of water displaced by a solid particle                              | (kg) |
| $m_p$           | mass of a particle                                                       | (kg) |
| $\underline{N}$ | force vector on a fluid particle                                         |      |
| $n, N$          | coordinates in local orthogonal system n-N-S defined on a stream surface |      |
| $N_d$           | contact force between two colliding bodies during deformation            | (N)  |
| $N_f$           | contact force between two colliding bodies                               | (N)  |
| $N_r$           | contact force between two colliding bodies during restoration            | (N)  |
| $n$             | coordinate in local orthogonal system n-N-S defined on a stream surface  |      |
| $\hat{n}$       | unit vector in n-direction                                               |      |
| $n_b$           | number of blades of impeller                                             |      |



|                        |                                                                  |               |
|------------------------|------------------------------------------------------------------|---------------|
| $n_c$                  | number of coats of paint                                         |               |
| $n_e$                  | number of particles passing through an element in a pipe section |               |
| $n_i, n_{i+1}$         | number of elements in annulus rings                              |               |
| $n_s$                  | number of particles in the sample                                |               |
| $n_t$                  | total number of size fractions of particles                      |               |
| $P$                    | field property                                                   |               |
| $P_n$                  | weight function of solid particles of size fraction $n$          |               |
| $P_1, P_2, \dots, P_4$ | field property at the nodes of an element                        |               |
| $p$                    | thermodynamic pressure                                           |               |
| $P_f$                  | plastic flow stress                                              | $(Nm^{-2})$   |
| $Q$                    | volume flow rate                                                 | $(m^3s^{-1})$ |
| $\bar{Q}$              | flow coefficient                                                 |               |
| $q_1$                  | function defined in eqn. 2.4.8                                   |               |
| $q_2$                  | function defined in eqn. 2.4.13                                  |               |
| $q_3$                  | function defined in eqn. 2.4.17                                  |               |
| $q_4$                  | function defined in eqn. 2.7.4                                   |               |
| $q_5$                  | function defined in eqn. 5.3c                                    |               |
| $q_6$                  | function defined in eqn. 5.6a                                    |               |
| $q_7$                  | function defined in eqn. 5.12                                    |               |
| $Re$                   | Reynolds number                                                  |               |
| $R_w, R_m$             | load cell readings for water and mixture, resp.                  |               |
| $r$                    | coordinate in a system of axes $r-\theta-z$                      |               |
| $\underline{r}$        | position vector                                                  |               |
| $r_b$                  | radial position of a particle at entry to a blade channel        | $(m)$         |
| $r_c$                  | radial distance of centroid of an element                        | $(m)$         |

|                 |                                                                            |                     |
|-----------------|----------------------------------------------------------------------------|---------------------|
| $r_i$           | inner radius of (i+1)th element                                            | (m)                 |
| $r_{i+1}$       | outer radius of (i+1)th element                                            | (m)                 |
| $r'$            | radial distance in a pipe section                                          | (m)                 |
| $S$             | coordinate in local orthogonal system<br>n-N-S defined on a stream surface |                     |
| $S_h$           | shape factor                                                               |                     |
| $S_s$           | solids specific gravity                                                    |                     |
| $s$             | distance on a blade surface                                                | (m)                 |
| $\hat{s}$       | unit vector in s-direction                                                 |                     |
| $s'$            | distance on a blade surface from<br>l.e. to t.e.                           | (m)                 |
| $s_x, s_y, s_z$ | components of $\hat{s}$ in coordinate<br>directions                        |                     |
| $t$             | time                                                                       | (s)                 |
| $t_c$           | total time of contact between two<br>colliding bodies                      | (s)                 |
| $t_d$           | total time required for deformation                                        | (s)                 |
| $t_s$           | time of free fall                                                          | (s)                 |
| $\bar{t}_s$     | mean time of free fall                                                     | (s)                 |
| $t'_1, t'_2$    | time as defined in eqn.'s 2.6.6<br>and 2.6.7, resp.                        | (s)                 |
| $U$             | free stream velocity                                                       | (ms <sup>-1</sup> ) |
| $u$             | flow velocity                                                              | (ms <sup>-1</sup> ) |
| $u_n$           | quantity defined in eqn. 2.7.25                                            |                     |
| $u_s$           | quantity defined in eqn. 2.7.23                                            |                     |
| $u'$            | component of fluctuating velocity in<br>axial direction                    | (ms <sup>-1</sup> ) |
| $u^*$           | friction velocity                                                          | (ms <sup>-1</sup> ) |
| $V_B$           | units of volume removed in brittle<br>erosion                              |                     |

|               |                                                                         |             |
|---------------|-------------------------------------------------------------------------|-------------|
| $V_C$         | units of volume removed by cutting wear                                 |             |
| $V_{C_{max}}$ | units of volume removed by cutting at angle of maximum erosion          |             |
| $V_D$         | units of volume removed by deformation wear                             |             |
| $V_f$         | units of volume loss, defined in eqn. 4.5                               |             |
| $V_L$         | limit deposit velocity                                                  | $(ms^{-1})$ |
| $V_T$         | units of volume removed by combined cutting and deformation wear        |             |
| $V_O$         | common velocity during a collision                                      | $(ms^{-1})$ |
| $v$           | velocity of a particle/solid sphere                                     | $(ms^{-1})$ |
| $v_e$         | velocity of collision at which elastic limit of the surface is exceeded | $(ms^{-1})$ |
| $v_m$         | mixture velocity                                                        | $(ms^{-1})$ |
| $\bar{v}_m$   | mean mixture velocity                                                   | $(ms^{-1})$ |
| $v_n$         | component of particle velocity normal to a surface                      | $(ms^{-1})$ |
| $v_p$         | particle velocity at point of contact with a solid wall                 | $(ms^{-1})$ |
| $v_s$         | component of particle velocity parallel to a surface                    | $(ms^{-1})$ |
| $v_{so}$      | solids velocity in the mixture                                          | $(ms^{-1})$ |
| $v'$          | particle velocity with respect to a rotating system                     | $(ms^{-1})$ |
| $v^*$         | normalised particle velocity                                            |             |
| $w$           | settling velocity                                                       | $(ms^{-1})$ |
| $\bar{X}$     | distance of centroid of an element from pipe centre in x-direction      | (m)         |
| $x$           | coordinate in a system of axes x-y-z                                    |             |
| $x_t$         | length of crater                                                        | (m)         |
| $W$           | fluid velocity in a rotating frame                                      | $(ms^{-1})$ |

|              |                                                       |     |
|--------------|-------------------------------------------------------|-----|
| $x', y', z'$ | cartesian orthogonal rotating system                  |     |
| $Y$          | distance from pipe bottom normalised by pipe diameter |     |
| $\bar{Y}$    | quantity defined in eqn. 3.3a                         |     |
| $y$          | coordinate in a system of axes x-y-z                  |     |
| $y_t$        | depth of cut                                          | (m) |
| $Z$          | distance from bottom of a channel                     | (m) |
| $z$          | coordinate in a system of axes x-y-z                  |     |

### Greek Symbols

|                        |                                                                               |        |
|------------------------|-------------------------------------------------------------------------------|--------|
| $\alpha$               | angle of impact                                                               | (deg.) |
| $\alpha'$              | angular separation between blades                                             | (deg.) |
| $\alpha_a, \alpha_b$   | approach angle of mass a and b, resp.                                         | (deg.) |
| $\alpha'_a, \alpha'_b$ | rebound angle of mass a and b, resp.                                          | (deg.) |
| $\alpha_0$             | limiting angle of incidence                                                   | (deg.) |
| $\alpha_{\max}$        | angle of impact at maximum erosion                                            | (deg.) |
| $\beta$                | ratio of turbulent diffusion coefficients                                     |        |
| $\beta'$               | relative angular position of a particle at entry to a blade channel           | (deg.) |
| $\gamma$               | angle of an elementary distance on a blade p.s. with the tangential direction | (deg.) |
| $\epsilon$             | random number                                                                 |        |
| $\delta$               | angle subtended by an element at the centre of the pipe                       | (deg.) |
| $\delta l_T$           | total thickness of a painted layer                                            | (m)    |
| $\xi$                  | intrinsic coordinate                                                          |        |
| $\eta$                 | intrinsic coordinate                                                          |        |
| $\eta_p$               | pump efficiency                                                               |        |
| $\theta$               | coordinate in a system of axes r- $\theta$ -z                                 |        |

|              |                                                                     |                                                                        |
|--------------|---------------------------------------------------------------------|------------------------------------------------------------------------|
| $\theta_p$   | angular position of rotating reference axis in an inertial system   | (deg.)                                                                 |
| $\lambda$    | angularity factor                                                   |                                                                        |
| $\lambda'$   | geometry angle                                                      | (deg.)                                                                 |
| $\mu$        | dynamic viscosity of fluid                                          | (Nsm <sup>-2</sup> )                                                   |
| $\mu_p$      | Poisson's ratio of particles                                        |                                                                        |
| $\mu_s$      | Poisson's ratio of a surface                                        |                                                                        |
| $\mu'$       | geometry angle                                                      | (deg.)                                                                 |
| $\nu$        | kinematic viscosity of fluid                                        | (m <sup>2</sup> s <sup>-1</sup> )                                      |
| $\epsilon_c$ | turbulent diffusion coefficients for solids                         | (m <sup>2</sup> s <sup>-1</sup> )<br>(m <sup>2</sup> s <sup>-1</sup> ) |
| $\epsilon_m$ | turbulent diffusion coefficient for momentum                        | (m <sup>2</sup> s <sup>-1</sup> )                                      |
| $\rho$       | density of fluid                                                    | (kgm <sup>-3</sup> )                                                   |
| $\rho_w$     | density of water                                                    | (kgm <sup>-3</sup> )                                                   |
| $\rho_{so}$  | density of solids                                                   | (kgm <sup>-3</sup> )                                                   |
| $\rho_m$     | density of mixture                                                  | (kgm <sup>-3</sup> )                                                   |
| $\tau$       | aerodynamic response time                                           | (s)                                                                    |
| $\phi_C$     | energy needed to remove unit volume of material by cutting wear     |                                                                        |
| $\phi_D$     | energy needed to remove unit volume of material by deformation wear |                                                                        |
| $\psi$       | stream function                                                     | (kgs <sup>-1</sup> )                                                   |
| $\Omega$     | angular speed of a sphere                                           | (rad s <sup>-1</sup> )                                                 |
| $\omega$     | angular speed of pump                                               | (rad s <sup>-1</sup> )                                                 |
| $\sigma_r$   | maximum tensile stress in a radial direction                        | (Nm <sup>-2</sup> )                                                    |

### Subscripts

(unless defined otherwise)

|       |                                                                               |
|-------|-------------------------------------------------------------------------------|
| a,b   | pertaining to particles a and b, resp.                                        |
| f     | pertaining to fluid                                                           |
| m     | pertaining to a mixture of solids in water                                    |
| p     | pertaining to a particle                                                      |
| r,θ,z | pertaining to components in cylindrical polar coordinate directions           |
| s     | pertaining to a surface                                                       |
| w     | pertaining to water                                                           |
| x,y,z | pertaining to components in cartesian orthogonal coordinate directions        |
| p/f   | pertaining to particle condition in an inertial frame of axes                 |
| p/r   | pertaining to particle condition in a rotating frame of axes                  |
| r/f   | pertaining to condition of a rotating frame with respect to an inertial frame |
| o     | referring to an initial condition                                             |
| 1,2   | referring to conditions before and after an impact, resp.                     |

### Subsubscripts

(unless stated otherwise)

|       |                                                                        |
|-------|------------------------------------------------------------------------|
| i     | pertaining to ith element                                              |
| j     | pertaining to jth element                                              |
| x,y,z | pertaining to components in cartesian orthogonal coordinate directions |
| o     | referring to an initial condition                                      |
| 1,2   | referring to conditions before and after an impact, resp.              |

Abbreviations

|        |                         |
|--------|-------------------------|
| °C     | degree centigrade       |
| deg.   | degrees                 |
| eqn.   | equation                |
| HB     | Brinell Hardness Number |
| l.e.   | leading edge            |
| max    | maximum                 |
| no.    | number                  |
| p.s.   | pressure surface        |
| resp.  | respectively            |
| r.p.m. | rotations per minute    |
| s.s.   | suction surface         |
| sec.   | seconds                 |
| t.e.   | trailing edge           |

## CHAPTER 1

### INTRODUCTION

#### 1.1 General

Since time immemorial, nature has been transporting slurries with both air and water as the carrier medium. The formation of the continental shelves off our coastline is an example; the rivers empty abraded land surface and deposit it into bays and gulfs. As the erosion process takes place, the fines are suspended in water which flows in streams and eventually are deposited where the velocity of the flow reduces sufficiently to allow the fines to settle out. This example illustrates rather simply how the formation of slurries allows the transport of solids by a liquid medium.

The simplest and most economical means for transporting large volume of liquid is the pipeline. For this reason at least, the idea of transporting solids in liquid suspension merits some consideration. The movement of liquids in pipelines necessarily requires the use of pumps. As early as 1893 Andrews presented a small-size equipment at the Chicago World Fair in which a coal dust-water mixture was pumped through a pipeline, (Ref. 1).

Since the days of Andrews, the pumping of solids through pipelines has proliferated both in size and character. These range from relatively simple in-plant loops to long distance transport of minerals and concentrates. Much publicity has been given in studies involving transportation of coal from mines directly to furnaces by means of pipelines, see Ref. 2. Needless to say, several installations around the world would testify to the practicality of this mode of transport. A good account of these installations already in operation as well as those in the pipelines may be found in Ref. 3.



The main criteria in the selection of slurry pumps, either centrifugal or positive displacement type, are the discharge pressure of the system and the size of the particles. For low pressure applications, Colley (Ref. 4) suggests the upper limit of the discharge pressure for use of centrifugal pumps as 6000 kPa. For high pressure applications positive displacement pumps are used although they are restricted to pumping slurries which carry suspended solids of size not larger than 3 to 6 mm (Ref. 5). The suction and delivery valves of this type of pump cannot operate satisfactorily, if the size of suspended hard particles in the slurry is large enough to interfere with the proper closing of the valves on their seats and more than normal valve lift is required for the passage of the solids.

Because of the limited head, the use of centrifugal pumps is restricted to short haul services, for example, in-plant transportation of slurries in mining, cement and power generation. In long-distance pipeline systems, centrifugal pumps are usually used as charge pumps, providing the required suction pressure for the mainline reciprocating pumps, as well as pumping slurry through bypass loops for online monitoring of slurry quality.

## 1.2 Centrifugal Pumps in Slurry Service

The presence of solids in liquid suspension effects the performance of centrifugal pumps. When handling such mixtures, the hydraulic performance of the pump changes from its clear water characteristics. In the long term also if the suspended solids are abrasive in nature, they may cause excessive surface damage in the wetted parts of the pump; this in turn impairs the hydraulic performance of the pump. The economic life of the machine is thereby reduced.

Conventional centrifugal pumps operate with high efficiency when handling clear liquids. If suspended solids are present in the liquid, a drop in efficiency is expected. The magnitude of this drop in efficiency depends on the following factors :-

- (i) concentration of solids in the mixture
- (ii) relative density of the solids
- (iii) shape, size and distribution of solid particles
- (iv) viscosity of the mixture

A comprehensive review of the subject has been prepared by the author for the British Hydromechanics Research Association and published as a Technical Note (Ref. 6). Hence, this subject will not be dealt in here any further. One important conclusion of the review worthy of a mention here is that the best efficiency point occurs at the same flowrate both for water and slurries.

### 1.3 Abrasive Wear in Solids Handling Centrifugal Pumps

Wilson (Ref. 7) defined 'abrasive wear' as the surface damage caused by the dynamic action of particulate matter impinging or grazing on the wearing surfaces. This mechanism of wear, therefore, should be distinguished from other modes of wear, e.g., corrosion involving electro-chemical action, and cavitation resulting from collapse of vapour bubbles. Wilson made the following classification of abrasive wear :-

- (i) erosion abrasion
- (ii) grazing abrasion
- (iii) grinding abrasion

Because of the importance of this mode of wear in relation to the surface damage caused by the particulate solids in centrifugal pumps, they are briefly discussed here.

#### 1.3.1 Erosion Abrasion

This type of wear is mostly due to small

particles. Depending upon the angle of impact of the particles onto the target material, this wear may occur in any of the following three modes :-

(i) Deformation Wear :

This occurs when the hard suspended solids of the slurry impinge perpendicularly on a surface, Fig. 1a. The surface layers will be plastically deformed if the impact stress generated on the surface exceeds the elastic limit of the material. After initial work hardening, a crack initiates within the deformed surface layer parallel to the contact area. Eventually, part of the surface flakes off.

(ii) Cutting Wear :

This occurs if the particles strike a surface at very shallow angles, almost parallel to the surface, as shown in fig. 1b. The surface will be torn off if the resulting stress exceeds the ultimate shear strength of the surface.

(iii) Oblique Wear :

This occurs as the solid particles strike a surface at an oblique angle as shown in fig. 1c. This is a case when a surface fails due to combined action of surface cracking and tearing. Hence, it is a case where deformation and cutting occur simultaneously.

### 1.3.2 Gauging Abrasion

This occurs when coarse particles impinge on the target material with sufficient force to produce high impact stresses, resulting in the tearing of sizeable pieces from the wear surfaces. For dredging and gravel handling systems, this type of damage may be significant.

### 1.3.3 Grinding Abrasion

Here the fine abrasive particles are subjected

to crushing between two rubbing surfaces. High stresses are involved and wear occurs by scratching, local plastic flow and micro-cracking.

#### 1.3.4 Previous Work

Although there is quite extensive literature on the fundamentals of wear, the number of publications concerned with abrasive wear in centrifugal pumps are quite limited. Most of the literature originates from service experience. Laboratory tests on pump wear are reported from only a few places. Theoretical studies of the problem are very rare. In the following paragraphs, these work will be briefly discussed.

##### 1.3.4.1 Service Experience and Laboratory Tests

Worster and Denny (Ref. 8) in 1955 reported their service experience with solids handling centrifugal pumps. Significant improvement in wear life of the pumps made of abrasion resistant steel were obtained by rubber lining the impeller. The types of solids used were gravel and coal of sizes upto  $\frac{3}{4}$  inch and 2 inches, respectively.

Anon (Ref. 9) considered the relative advantage between rubber-lined and all metal pumps. He noted that all-metal pumps, in general, do better with large size particles than rubber lined ones. Based on the manufacturer's data, the maximum particle size was reported to be 2 inches for rubber lined pumps. For all metal pumps the maximum particle size was stated to be  $\frac{1}{2}$  inch less than the discharge bore.

Franz (Ref. 10) highlighted the interdependence of the pump material and speed upon the wear qualities of a pump. He suggested that a standard cast iron bronze fitted pump of open impeller type will result in acceptable wear qualities if the pump is run at relatively low speeds. For more severe applications, he suggested an alloy of cast

iron with varying amounts of Chromium which would increase the Brinell hardness to 350 - 450. However, as this kind of cast iron could only be used in specially constructed pumps, as it is not machineable and has to be ground, he suggested as an alternative impeller material an alloy of cast iron with Nickel.

The interdependence of the material property and the speed of the pump on the wear life of the pump was also pointed out by Antunes et al (Ref. 11). They noted that wear of ductile material varies roughly as the cube of velocity below 30 m/s, and as the square of the velocity above this. The velocity exponent might be higher for brittle materials. Hence, where the application involves high heads, a judicious selection of materials is needed. Use of materials like rubber and ceramics, under those circumstances might prove prohibitive. They also stressed the importance of maintaining good hydraulic design to avoid separation and formation of vortices as these cause localised wear.

Welte (Ref. 12) studied the problem of wear in centrifugal dredge pumps and identified three areas of the impeller prone to high wear. These were :-

- (i) vane inlet edges
- (ii) vane exit
- (iii) impeller sides at suction end

Welte noted that the energy transfer from the impeller to the particle laden mixture starts at the vane inlet edges. Because the solids mostly do not follow the fluid streamlines, the vane inlet edges were damaged by the impacting solids, larger solids causing more damage. The vane tips showed high wear where the vanes joined the impeller sides. Welte argued that it was due to the solids on the pressure side of the vane which accumulated in the transition zone between the vanes and the impeller sides thereby increasing the local wear rate. The wear at the impeller sides at suction end was considered to be primarily due to leakage

of mixture between the volute and the suction through the end clearance space.

Warring (Ref. 13) studied the effect of hydraulic design on the wear pattern of the impeller. By referring to the Simon group of pumps, Warring showed that the formation of localised leakage paths because of wear of the tongue of the volute can be considerably improved by careful redesigning of the impeller and the volute. Similar discussion on the subject was also given by Arnstein (Ref. 14).

Want (Ref. 15) in a recent article maintained that the wear pattern can be significantly affected by the size of the particle. For coarse particles ( $d_{50} = 0.25$  mm, characteristic particle size  $d_{50}$  is defined in section 3.3.1), the worn surfaces were observed to be very smooth. Presence of shallow scallops were found with intermediate particle size ( $d_{50} = 0.10$  mm). For fine particles, the worn surfaces of impellers consisted of small deep sharp scallops. Wiedenroth's (Ref. 16,17) wear tests on a small dredge pump impeller also showed dependence of wear pattern on grain size.

The difficult task of selecting pump materials for different service conditions was discussed by Wilson (Ref. 7). He classified service conditions as gauging abrasion, grinding abrasion, cutting abrasion or erosion abrasion. Service conditions where gauging abrasion may occur (e.g., in dredge and heavy duty gravel pumps), Wilson noted that the toughness of the material was important; an austenitic Manganese steel was suggested. Where grinding abrasion might occur (e.g., in sand pumps) materials with high shear and compressive strengths were required. Toughness of the material might be sacrificed to gain hardness. Use of Tungsten carbide composites and martensitic white irons such as 15/3 alloy (15% Chrome, 3% Molybdenum) and Ni-hard (4% Nickel, 2% Chrome with reduced carbon content for toughness) was reported. In case of erosion abrasion

occurring at low impact, the hardness of the material was cited as the most significant property. Both martensite white irons such as 15/3 alloy and Ni-hard which provide high hardness (in the range of 550 to 700) were recommended. On the use of rubber lined impeller, Wilson noted that "the ability of rubber when vulcanised to deform elastically under impact makes it ideally suitable to resist erosion abrasion", provided the head is not high, the particles are small not sharp. There is a critical velocity (the exact magnitude depends upon the type of the rubber) at which, if exceeded, the particle will cut rubber. Wilson also noted that natural gum rubber excels over all others in resistance to erosion abrasion if it is not subjected to attack from oils and if the operating temperature is below the vulcanisation temperature (138 °C to 177 °C).

In addition to the work cited above originating mostly from service experience, laboratory pump wear tests were reported from a number of sources (Ref.'s 16, 17, 18, 19). The work of Wong et al (Ref. 18) is the most important. Results of experiments on a number of promising pump materials were reported. The materials included structural alloys, hard metal, hard facing, and coatings. The experiments were done in two stages. First, the candidate pump materials were eroded by coal slurry under well controlled laboratory conditions; later these results were compared with field exposure tests in a coal slurry pump. It was noted that sintered tungsten carbide inserted in highly localised wear areas increased the pump overall wear life. These areas were volute cutwater, impeller blade leading edge, etc. Wong et al noted that local wear patterns might vary for any given pump design, particularly for the impeller blades, depending on the size and specific gravity of the solids passing through the passage.

Truscott (Ref. 19) reported the abrasive wear study with centrifugal pumps of same geometry, but different blade materials. The blade materials comprised, amongst

others, two grades of Ni-hard, and coats of rubber and polyurethane on a cast iron substrate. By comparing the material loss data with that from a reference blade, a relative wear life index of the materials was provided. This quantitative form of material loss data is a very useful contribution to the field of erosion study since it is very rare. (Some recent contribution was made by Faddick et al, Ref. 20). Truscott (Ref. 19) reported increase in wear rate with concentration between 1500 and 1800 r.p.m., but no significant change was noticed between 1800 to 2000 r.p.m.. It was suggested that this pattern was due to the flow regime entering the pump which changed significantly over the whole range of test speeds. Wear patterns of all the blades lay in areas near the leading edge and pressure surface of the blade. No change in wear pattern with increasing speed was observed, unlike Wiedenroth (Ref. 17).

Systematic approaches to the problem of pump wear as affected by hydraulic design were reported by Bak (Ref. 21), Zarzycki (Ref. 22) and Herbach (Ref. 23). Bak's (Ref. 21) work on pumps of different design using a wide variety of materials showed that conventional 'bladed' design is susceptible to higher wear rates than the 'unchokable' channel type. (The unchokable channel type is a design in some solid handling centrifugal pumps where the impeller vane passages do not contain contractions between suction and delivery in order to prevent the trapping of solids in the passage and the accompanying detrimental effects.) Similar findings were reported by Zarzycki (Ref. 22). Both reported maximum wear near the leading and trailing edge of the blades. While the wear at the trailing edge of the blade surface does not seem to be well focussed in literature, at least the work of Herbach (Ref. 23) suggested that by making some adjustment in the blade outlet angle an improvement in the wear life of the pump was possible.



#### 1.3.4.2 Theoretical Studies

The only theoretical work available on wear in pumps was by Bergeron (Ref. 24). It was assumed in Bergeron's work that :-

- (i) a particle having arrived at a solid wall of the pump would slide on it
- (ii) the damage done to a surface is a combination of impact and rubbing action of the particles onto the surface
- (iii) acceleration and velocity of the particles at the wall were proportional to the acceleration and velocity of water

Under the above assumptions, Bergeron suggested that wear would be directly proportional to :-

- (i) the difference in density between the solids and the liquid
- (ii) acceleration of the main flow
- (iii) mean velocity of the flow
- (iv) coefficient of sliding friction between the solids and the liquid
- (v) thickness of the particle layer sliding on the surface

The limitation of Bergeron's (Ref. 24) work is that it provided means only to compare wear in two pumps; given test results from a pump handling a certain mixture, the probable amount of wear in a similar pump operating at a hydraulically similar condition but with a different mixture can be calculated.

#### 1.4 Summary of the Previous Work, Plan of the Present Work

Summarising the discussion on the previous work, the following points can be made :-

- (i) In granular solids handling centrifugal pumps, the general tendency on the part of the

manufacturers has been to increase the hardness of the surface of the blades and the areas in the wetted surface of the impeller which require particular attention for protection against abrasive erosion.

- (ii) The effect of the mixture condition on the erosion of the impeller surfaces has not been investigated so far.
- (iii) There is no analytical model which can identify the areas of erosion in a blade surface as a function of the mixture condition.

As the erosion of a surface is due to the dynamic action of the particles, the analysis of the problem of erosion is, therefore, related to the analysis of the motion of particles effecting this kind of erosion. It is in this context that the present plan of work was initiated. The motion of the particles are traced individually in the flow field of the impeller. The effect of intergranular collisions on the motion of the particles is ignored. The objective was to explore if the calculation of individual particle trajectories inside an impeller can adequately represent the general direction of motion of a collection of particles. In other words, the trajectory history of a collection of particles would be considered representative of the motion of all the solid particles in the mixture. The concentration of solids in the mixture does not explicitly appear in the calculation process, but in an implicit form the concentration at the inlet is used in determining the number of particles for which the trajectory calculations are to be performed. The ability of the particles to erode a surface is known by calculating the velocity and the angle at which the particles impact on a surface. These, together with the knowledge of the density of impacts of particles at different places in a surface form a basis for classifying a blade surface of an impeller into a number of erosion prone areas.

On the experimental side, a commercially available slurry impeller was put to handle a mixture of fine gravels in water, in a test rig. A light coat of paint of different colour was applied onto the wetted surface of the impeller. By observing the colour contrast of the eroded painted surface, the areas of the painted surface which were more prone to erosion were identified. In this way a comparison between theory and experiment was made.

#### 1.5 Layout of the Thesis

The thesis has been organised into the following chapters :-

Chapter 1 gives an introduction to the objective of the work, reviews the previous work on centrifugal slurry pumps and presents the methodology of the present work.

Chapter 2 contains analysis of forces on solid particles, the governing equations of motion for the particle and the flow, and the method of solution of the equations.

Chapter 3 gives the background information and the method adopted to determine particle distribution at an upstream position of the impeller.

Chapter 4 presents the erosion model developed in this study.

Chapter 5 presents the results obtained by using the software developed in this study, the trajectory calculations of the particles and the predicted erosion patterns on the blade surfaces of an impeller.

Chapter 6 describes the experimental apparatus and the techniques used to find the erosion on the blade surfaces of a centrifugal slurry impeller, makes comparison between the observed patterns and the predicted ones.

Chapter 7 summarises the present study and makes recommendations for future work.

## CHAPTER 2

### ANALYSIS OF PARTICLE MOTION

#### 2.1 Introduction

The motion of a solid particle is governed by various forces all of which, but gravity, are variant in both space and time. In this chapter these forces are discussed and the governing equations of the motion of particles are formulated. These equations are solved for a given flow field inside the impeller. The flow field is calculated by solving the governing equations of the flow which is discussed in detail. The method to calculate the rebound characteristics of a particle as the particle hits and bounces off a wetted surface of an impeller are given. The steps involved in developing a computer code for calculating the motion of particles in the computed flow field of an impeller are presented.

#### 2.2 Forces on a Solid Particle

These forces are :-

- (i) Gravitational pull
- (ii) Drag due to flow around the particle
  - (a) form drag
  - (b) skin drag
- (iii) Lift due to fluid-particle interaction
  - (a) lift due to particle rotation
    - spin drift
  - (b) lift due to fluid shearing
    - shear drift
- (iv) Forces arising due to particle collision upon a wall
  - (a) solid friction and wall reaction
  - (b) elastic rebound

Items (ii) and (iii) have been covered in this section and

item (iv) in section 2.7. A comprehensive discussion of the subject has been made by the author in the form of a technical report and may be found in Ref. 25.

### 2.2.1 Drag due to Fluid

This force acts on a particle in the direction of fluid flow as a result of the exchange of momentum between the fluid and the particle. It is conventionally considered to be the summation of two components — skin-drag and form-drag. The former is a result of the fluid friction on the surface of the particle, the latter (which depends on the shape or form of the boundary) is due to the pressure gradient around the solid body and hence is very much related to the flow regimes around and behind the body (the drag being considered here is the profile drag in contrast to the induced drag which may occur on a body producing a lift force). Investigations concerning this drag force are complicated due to a variety of reasons. These are mainly:-

- (i) unsteady motion of the particle and/or the fluid
- (ii) shear in the flow
- (iii) spin of the particle
- (iv) fluid turbulence
- (v) surface roughness of the particle, etc.

First the flow field around the solid body, which is considered spherical for the sake of simplicity is examined. The effects due to items (i) to (v) are then presented individually.

#### 2.2.1.1 Flow around a Solid Sphere

The various flow regimes which develop around a spherical particle depend on the Reynolds number of the particle,

$$Re_p = \frac{\rho v d}{\mu} \quad (2.1.1)$$

In the case of uniform laminar flow around a sphere, for  $Re_p < 0.1$ , the viscous effects dominate over the inertia effects. The slow viscous fluid moves around the sphere and closes the flow on the rear surface of the body. The flow is symmetrical upstream and downstream, and the boundary layer on the sphere surface is laminar. Such a flow regime is widely known as the 'creeping flow' regime.

As the  $Re_p$  is increased beyond 0.1, the upstream-downstream symmetry tends to disappear and an adverse pressure gradient is developed along the boundary layer at the back of the sphere. With increasing  $Re_p$ , flow separation takes place on the surface of the sphere and the fluid moves away from the sphere before reaching the rear point of symmetry and a stationary vortex ring composed of a pair of attached eddies with an axial symmetry is developed (Ref. 26). This separation of boundary layer begins at the downstream stagnation point (Ref. 27) and moves forward as  $Re_p$  increases and the ring grows in size. The exact value of  $Re_p$  at which the first vortex ring appears thereby ushering in a wake behind the sphere is a matter of strong controversy. Although it is more likely to be formed at  $Re_p = 10$ , investigations by Williams (Ref. 28), Nissi et al (Ref. 29) suggested that this value could be from as low as 8.14 to as high as 720. Because of the force between the fluid and the sphere, momentum is extracted from the fluid. The rate of momentum transport downstream of the sphere is, therefore, smaller than that in front of it.

With further increase of  $Re_p$ , the separation point moves upstream while the vortex ring continues to grow in size and at  $Re_p \approx 1000$ , a stable separation point at about  $80^\circ$  from front stagnation point is achieved (Ref. 27). Because of the shift of the separation point from rear to the forward part of the sphere, the pressure drag takes precedence to skin friction.

At  $Re_p \approx 130$ , the vortex ring becomes unsteady and begins to oscillate although all imposed conditions are being held steady. For a turbulence free unbounded flow at  $Re_p \approx 500$  (Ref. 30), the attached ring breaks and moves away from the sphere to form a trail of vortices behind the body surrounded by the separated boundary layer. The value of  $Re_p$  associated with this change of flow regime is referred to as the lower critical Reynolds number. Nemenyi (Ref. 31) observed this phenomenon at  $Re_p \approx 200$ , Lunnon (Ref. 32) at  $Re_p \approx 1000$ , while Massey (Ref. 27) suggested a range of  $Re_p$  between 200 to 2000. Massey (Ref. 27) also noted that this vortex shedding phenomenon did not follow a definite frequency, although Schmiedel (Ref. 33) observed such a periodicity for a limited range of  $Re_p$  (500-1000, as reported in Ref. 30) and Winny (Ref. 34) for a different  $Re_p$  range (2000-8000). On the effect of this periodicity on drag, Lunnon (Ref. 35) remarked that there would also be periodic increase and decrease of drag. This observation would probably explain the fluctuation of the fall velocities of a sphere as observed by Liebster (Ref. 36) and reported in Ref. 30.

As  $Re_p$  increases the boundary layer of the sphere undergoes a process of transition from laminar to turbulent. This begins at  $Re_p \approx 10.0$  and is normally assumed to extend upto  $Re_p$  20,000 (Ref. 37).

The separation considered at the beginning of this discussion refers to the case of a laminar separation. However, following this and with increasing  $Re_p$ , the transition point in the separated boundary layer gradually approaches the separation point. At  $Re_p \approx 3 \times 10^5$  (Ref. 27), the boundary layer becomes fully turbulent before separation and as a result the separation is delayed and moves downstream to some rear point on the sphere surface. This condition is referred to as the higher critical Reynolds number (this value is reduced if there is turbulence in the free stream). The consequence of a delayed separation is a narrower wake; Streeter (Ref. 38) remarked that the

high momentum transfer in the turbulent boundary layer delayed separation so that the wake was substantially reduced resulting in a total drag on the sphere less than half that occurring in the case of the laminar boundary layer separation.

At higher  $Re_p$ , the boundary layer becomes fully turbulent, the wake narrower and statistically stable (Ref. 37). In such a situation, the pressure drag is small and it is the skin friction which mainly contributes to the total drag.

The various flow regimes around and behind the sphere discussed above correspond to a free stream which is unbounded, laminar and uniform. Although in a turbulent free stream, the general pattern of the flow regimes is likely to be similar, the corresponding Reynolds number relating to different flow regimes around the sphere will be different.

#### 2.2.1.2 Calculation of Drag : Theoretical Investigations

Because of the complexity of the problem associated with the fluid resistance, theoretical investigations cannot generate any simple formula that would relate this resistance with different flow regimes occurring around the sphere.

For many years, considerable attention has been given to the theory of viscous flow around a sphere under different boundary conditions. Because of the non-linearity in the general Navier-Stokes equation due to the acceleration term, most of the theoretical approaches assumed a very low Reynolds number flow. In this regime the Navier-Stokes equation may be simplified to a linear, fourth-order partial differential equation by dropping the inertia term. Stokes (Ref. 39) first used this simplified form of the equation of motion to solve for a slow moving sphere in an unbounded stagnant fluid (creeping motion). The original



Navier-Stokes equation reduced to :-

$$\text{grad } p = \nu \nabla^2 u \quad (2.1.2)$$

yielded the following expression for drag :-

$$F_D = 3\pi d \mu u \quad (2.1.3)$$

Skin-friction constituted two-thirds of this force and pressure gradient, one third. Dimensional considerations (Ref. 40) can show that the co-efficient of drag,  $C_D$  is related to the particle Reynolds number,  $Re_p$ . Stokes' solution gave :-

$$C_D = \frac{24}{Re_p} \quad (2.1.4)$$

The drop of the inertial term, however, severely restricted the use of the Stokes' solution for drag to a very low value of  $Re_p$ . Experimental evidence suggested that this expression predicted the drag accurately only for  $Re_p < 0.1$ . Later Oseen (Ref. 41) attempted to incorporate the inertia term in the solution of the Navier-Stokes equation by considering the motion of a sphere in an unbounded uniformly moving fluid. In this case the simplified Navier-Stokes equation became :-

$$\underline{u} \cdot \text{grad } u = -\frac{1}{\rho} p + \nu \nabla^2 u \quad (2.1.5)$$

Clearly, Oseen considered the inertia term only in that region of the flow around the sphere where the velocity was approaching the free stream value. The solution of this equation produced the following expression for the drag co-efficient :-

$$C_D = \frac{24}{Re_p} \left( 1 + \frac{3}{16} Re_p \right) \quad (2.1.6)$$

Oseen's value of  $C_D$  agreed well with the experimental values for  $Re_p < 1$ . For higher  $Re_p$ , however, investigations

by Pearcy et al (Ref. 42) suggested that Oseen's solution did not predict any wake formation.

#### 2.2.1.3 Calculation of Drag : Experimental Investigations

The drag of a single solid sphere moving steadily relative to a fluid is represented as a plot of  $C_D$  vs  $Re_p$ , first suggested by Rayleigh (Ref. 43). Several investigations in the past produced results which deviated considerably from the standard drag curve of today. Principal causes of these deviations were due to :-

- (i) unsteadiness in the motion of either the solid or the fluid or both
- (ii) incursions of the wall effect
- (iii) supporting problems of the spheres in the fluid stream
- (iv) presence of turbulence in the flow

According to Ref. 37, it was Wieselsberger (Ref. 44) who first published values of the drag coefficients for a wide spectrum of Reynolds number ( $7 \times 10^2 - 7.7 \times 10^5$ ) which form part of the present day standard drag curve.

Wieselsberger's results were reproduced by Lunnon (Ref. 35) through a series of experiments with spheres of various densities and diameters. Lunnon (Ref. 35) furthermore, proved that the drag coefficient would depend on the relative velocity of the fluid with respect to the particle and not the absolute velocity of the fluid stream.

Lapple and Shepherd (Ref. 45) in 1940 collected the available data and produced a graph of the drag coefficient as a function of  $Re_p$  which forms the standard drag curve of today. Various empirical expressions were suggested to find  $C_D$  analytically from known values of  $Re_p$  by fitting in different parts of the drag curve (see Ref. 25). Recently, Morsi and Alexander (Ref. 46) produced a series of expressions for  $C_D$  as a function of

$Re_p$  in the range of  $0.1 < Re_p < 5 \times 10^4$ .

#### 2.2.1.4 Calculation of Drag in Unsteady Motions

This section is devoted to the dynamic force exerted by a viscous fluid on a sphere when the relative velocity between the two changes with time.

Most of the information in this field was obtained through experiments. Theoretical investigations are limited in number and cover only the low Reynolds number regime where the magnitude of the inertia term in the general Navier-Stokes equation is sufficiently low as compared to the viscous term and may be dropped.

When a particle is accelerating in the fluid, it experiences drag forces which are additional to that given by Stokes' law. (equation 2.1.3).

In 1888 Basset (Ref. 47) presented his theoretical studies on the rectilinear motion of a slowly moving accelerating sphere in an infinite stagnant viscous fluid (for a discussion of Basset's work see Ref. 69). By ignoring the convective terms of the Navier-Stokes equation, Basset showed that the sphere would experience a time dependent force  $F_S$  given as follows:-

$$\begin{aligned}
 F_S &= \frac{\pi d^3}{6} \rho_p \frac{dv}{dt} \\
 &= \underbrace{3\pi\mu dv}_{\text{I}} + \underbrace{\frac{1}{2}m_f \frac{dv}{dt}}_{\text{II}} + \underbrace{\frac{3}{2}d^2 \sqrt{\pi\rho\mu} \int_0^t \frac{\frac{dv}{dt'}}{\sqrt{t-t'}} dt'}_{\text{III}} \quad (2.1.8)
 \end{aligned}$$

Term I represents the force required to accelerate the sphere as given by Stokes' law (equation 2.1.3). Term II is

an account of the resistance of the fluid to acceleration of the sphere;  $m_f$  is the mass of the fluid displaced by the sphere and is equal to  $\pi d^3 \rho / 6$ . Term II does not contain viscosity and hence is the force given by an inviscid-irrotational flow theory. Term III is the most significant part in Basset's equation. It shows that the resistance force on the sphere depends not only on its instantaneous values of velocity and acceleration, but also on the entire history of the sphere acceleration. The integral term, therefore, represents the effect of the previous acceleration upon the force at the present time,  $t$ .

For the case when the fluid motion was also unsteady, Tchen (Ref. 48) showed that the motion of the sphere could be analysed from the following expression :-

$$\begin{aligned}
 \frac{\pi d^3}{6} \rho_p \frac{dv}{dt} &= \underbrace{3\pi\mu d(\underline{u} - \underline{v})}_{\text{I}} + \underbrace{\frac{\pi d^3}{6} \rho \frac{du}{dt}}_{\text{II}} + \underbrace{\frac{\pi d^3}{12} \rho \left[ \frac{du}{dt} - \frac{dv}{dt} \right]}_{\text{III}} + \\
 &+ \underbrace{\frac{3}{2} d^2 \sqrt{\pi \rho \mu} \int_0^t \frac{\frac{du}{dt'} - \frac{dv}{dt'}}{\sqrt{t - t'}} dt'}_{\text{IV}} \quad (2.1.8)
 \end{aligned}$$

Acceleration of the fluid leads to a pressure gradient in the vicinity of the particle and hence an additional force, term II.

Subsequently, Hjelmfelt and Mockros (Ref. 49) showed that the above equation could be solved in the case of a turbulent free stream under the following assumptions :-

- (i) the turbulence was homogeneous
- (ii) the particles were much smaller than the microscale of turbulence
- (iii) Stokes' drag law would apply
- (iv) there was no particle-particle interaction
- (v) each particle was always surrounded by the

same fluid molecules

Item (iv) requires that solutions which produce large relative motion between the particle and the fluid are not acceptable.

The theoretical work outlined above is valid only for low values of  $Re_p$  (creeping flow). Hence, experimental work (discussed below) remains the only possible way to the understanding of the problem.

Most of the experimental observations were carried out to investigate if the Basset's equation (Ref. 47) for creeping motion ( $Re_p < 0.1$ ) also applied for the higher values of  $Re_p$  and if not, to determine the co-efficients of the terms in the acceleration equation given earlier. The main problem encountered by different investigators was to devise a sophisticated system which would continuously monitor the particle position with time precisely, so that the second derivatives of the distance traversed by the particle could be obtained with sufficient accuracy.

Most of the researchers up to now have confined themselves to the determination of the 'added mass' co-efficient for spheres oscillating in liquids (see Ref. 25). Most of them either did not consider the history term of the Basset equation (2.1.8) and those who did assumed that the theoretical value of this term given by Basset (Ref. 47) also applied to their case. Odar and Hamilton (Ref. 50) tried to overcome this problem by introducing empirical co-efficients into the three terms of equation 2.1.8. The co-efficients were obtained from a series of experiments with a sphere oscillating with finite amplitude in a viscous fluid. If  $C_D$ ,  $C_a$ ,  $C_h$  respectively indicate the co-efficient of the steady-state drag, added-mass and history integral terms, then from equation 2.1.8 :-

$$\frac{\pi d^3}{6} \rho_p \frac{dv}{dt} = C_D \rho \pi \frac{d^2}{8} v^2 + C_a m_f \frac{dv}{dt} + C_h \frac{d^2}{4} \sqrt{\pi \rho \mu} \int_0^t \frac{\frac{dv}{dt'}}{\sqrt{t-t'}} dt' \quad \text{---- (2.1.10a)}$$

where :-

$$C_D = f_1(Re_p, \text{ as given by the standard drag curve}) \quad (2.1.10b)$$

$$C_a = 1.05 - \frac{0.066}{A_n^2 + 0.12} \quad (2.1.10c)$$

$$C_h = 2.88 + \frac{3.12}{(A_n + 1)^3} \quad (2.1.10d)$$

and  $A_n$  is the acceleration number defined as the ratio of the convective acceleration to the local acceleration term of the Navier-Stokes equation.

As equation 2.1.8 was derived by ignoring the convective terms,  $A_n = 0$ , hence  $C_a = 0.5$ ,  $C_h = 6$ , which were also the theoretical values of the coefficients given by Basset (Ref. 47). In Odar and Hamilton's (Ref. 50) work, the value of  $Re_p$  did not exceed 62.

It was concluded that the resistance data could be best correlated by using a single term with an empirical coefficient,  $C$ . The fluid resistance,  $F_s$  then became :-

$$F_S = C \frac{1}{8} \rho |v|v \pi d^2 \quad (2.1.11a)$$

where :-

$$C = f_2(Re_p, A_n) \quad (2.1.11b)$$

Very recently Karanfilian and Kotas (Ref. 51) reported their experiments with a sphere subjected to a simple harmonic motion in an otherwise undisturbed liquid at high Reynolds number. It was observed that a Basset type equation (2.1.8) could not be used to correlate their test results.

A different approach was taken by Bugliarello (Ref. 52) whose work is reported in Ref. 51. The concept of a total resistance coefficient,  $C_T$  was introduced. The force was given by :-

$$F_S = C_T \frac{1}{8} \rho |v|v \pi d^2 \quad (2.1.12a)$$

where :-

$$C_T = f_3\{vd/v, \dot{v}d/v^2, v/(gd)^{\frac{1}{2}}, \text{ geometry of the body}\} \quad (2.1.12b)$$

This resulted in a better correlation between the theoretical and experimental values than the added mass coefficient. Bugliarello's data also suggested that the effect of acceleration on drag was considerably reduced with increasing  $Re_p$ .

These various independent approaches have complicated the problem rather than ease it and very little has emerged conclusively. However, the following points can be made:-

- (i) It appears that the 'added mass' concept is unable to predict the unsteady-state drag at high  $Re_p$ . The co-efficients of the added mass and the history integral terms change with the acceleration number and this variation has not been clearly established for a wide range of  $Re_p$ .
- (ii) Use of the total drag co-efficient appears to be very practical although there is insufficient data to establish its validity.

#### 2.2.1.5 Calculation of Drag in Rotating Spheres

There is limited information regarding the effect of the circulation induced by the rotation of the sphere on the steady state drag co-efficients.

Garstang (Ref. 53) showed analytically that for low Reynolds number, an identical expression to Stokes' drag force could be used although the sphere has both translational and rotational motion. A similar conclusion could be derived from the work of Drazin (Ref. 54). Rubinow and Keller (Ref. 55) investigated the low Reynolds number flow about a spinning sphere moving in a viscous fluid and they too reached the same conclusion.

Maccoll (Ref. 56) first measured the drag force on a 15.24 cm diameter sphere rotating at speeds upto about 1800 r.p.m. in an air stream having velocities upto 10.36 m/s. Plots of drag coefficient,  $C_D$  against the ratio of the equatorial speed of the sphere to the air speed were provided. At  $Re_p \approx 10^4$ ,  $C_D$  was a function of speed ratio. At higher values of  $Re_p$ , the effect due to sphere rotation on the observed  $C_D$  was found to be small. Wind tunnel experiments by Davies (Ref. 57) produced a general pattern



of the drag co-efficients similar to Maccoll's (Ref. 56).

Experiments by Barkla and Auchterlonie (Ref. 58) with rotating spheres at Reynolds number between  $1.5 \times 10^3$  to  $3 \times 10^3$  showed that the drag co-efficient differed considerably from those given by Maccoll (Ref. 56) dropping slightly with increasing spin and did not rise to a near steady state value as observed by Maccoll.

In conclusion it can be said that if the Reynolds number is small, the effect due to particle rotation on the drag co-efficient is negligible. With increasing Reynolds number there appears to be some effect although it is very small. Finally, there is no study on the effect of rotation on drag for rectilinearly accelerating sphere.

#### 2.2.1.6 Calculation of Drag in Spheres with a Rough Surface

In 1916 Pannell (Ref. 59) reported wind tunnel results on a sphere roughned with sand. The results were compared with those obtained from a smooth varnished sphere of the same dimension. The tests were carried out for  $0.32 \times 10^5 < Re_p < 1.5 \times 10^5$  and indicated that the resistance of the rough sphere over the whole range was less than that for the smooth sphere at the corresponding Reynolds number. For  $Re_p \geq 1.5 \times 10^5$ , the resistance of the roughned sphere sharply increased and at  $Re_p = 1.5 \times 10^5$ , a value close to that for the smooth sphere was reached. For  $Re_p > 1.58 \times 10^5$ , no comparison was possible due to lack of data.

Bacon and Reid (Ref. 60) also showed that even very fine protuberances (as well as the supporting wires) on certain parts of a sphere had a very large effect on drag characteristics.

#### 2.2.1.7 Calculation of Drag in a Turbulent Flow

Both theoretical and experimental work in this field are limited. The basis of the theoretical analysis is that if the particle is large compared with the scale of turbulence, then the main effect of the turbulence on the particle will be to increase its flow resistance, and the particle will, at most, tend to follow the slow large scale turbulent motion of the fluid. Conversely, if the particles are very small compared with the micro-scale of turbulence, then its motion will be governed by the fluctuating components of the fluid and the resistance will be entirely viscous in nature (Ref. 61).

The first extensive study on the motion of a small spherical particle in a turbulent fluid was made by Tchen (Ref. 48). Tchen's equation of motion for the particle in a turbulent fluid, that was both homogeneous and stable, was based on the assumption that the same fluid molecules surrounded the particle throughout its motion. In a real situation this was highly unlikely since the real fluid undergoes considerable stretching and distortion and in this process the same fluid elements would not contain the same particle. The above assumption also imposed the restriction that the particle should be almost neutrally buoyant in the fluid to avoid overshooting. All these confined the analysis to the case when  $Re_p < 1$ . Extension of Tchen's work came from Hjelmfelt and Mockros (Ref. 49), but this was limited to  $Re_p < 1$ . The essence of their work was that the turbulent characteristics of one phase could be derived from those of the other.

A semi-theoretical treatment of the problem emerged from the work of Torobin and Gauvin (Ref. 62). They showed that the critical Reynolds number of the particle at which the transition of the sphere surface boundary layer took place could be related to the relative turbulence intensity of the solid-gas system. Torobin

and Gauvin argued that the energy of turbulence,  $E_t$  that each elementary volume of fluid carried with it into the boundary layer of a sphere (that was still laminar) was subjected to a decaying action due to the viscous damping energy,  $E_v$  originating at the surface of the sphere. At the transition point, the ratio of these two energy terms, which tended to cancel each other, was related to the Reynolds number of the particle and the relative turbulence intensity,  $\sqrt{u'^2}/v$  as follows:-

$$(E_t/E_v)_c = f_4\{(Re_p)_c (\sqrt{u'^2}/v)^2\} \quad (2.1.13a)$$

or :-

$$(Re_p)_c (\sqrt{u'^2}/v)^2 = \text{constant} \quad (2.1.13b)$$

for a given system of solids-gas flow; subscript c refers to the condition at which the steeply sloped section of the constant intensity curve on a  $C_D$  vs  $Re_p$  plot intersects the value of  $C_D = 0.3$ .  $u'$  is the velocity fluctuation of the fluid in the axial direction. Torobin and Gauvin thus related the transition phenomenon with the relative intensity of turbulence. However, the value of the constant, as it was a function of the system, could only be found from a plot of  $(Re_p)_c$  and  $(\sqrt{u'^2}/v)^2$ , based on the experimental data. Although the constant was given as 45, its value is not universal.

Further conclusions that can be drawn from their work are :-

- (i) At low relative turbulence intensities (<10%), the standard drag curve values could be used to prescribe the fluid resistance even though there was turbulence in the free stream.

- (ii) As the relative intensity increased, there was some increase though small, of the  $C_D$  values with  $Re_p$ . This increase was due to vorticity diffusion from the free stream into the wake.
- (iii) At higher relative intensities (>35%), the drag first gradually dropped from the standard  $C_D$  values and then sharply upswung (as  $Re_p$  increased) and attained a value very closely given by the standard drag curve at the corresponding  $Re_p$ . The sharp drop corresponded to the boundary layer transition augmented by the free stream turbulence.
- (iv) There was a systematic regression of the critical Reynolds number as the intensity of turbulence increased.
- (v) The above observations were true irrespective of the position of the particle in the wind tunnel, where the particle was moving freely, either accelerating or decelerating, thereby suggesting that the acceleration effect as discussed in section 2.2.1.4 for laminar flows would not hold if there was turbulence in the free stream.

#### 2.2.2 Lift Due to Fluid-Particle Interaction

These forces are :-

- (i) spin induced circulation around the body
- (ii) flow induced circulation (i.e., lift due to the presence of a velocity gradient in the flow)
- (iii) fluid turbulence

When a spinning body is travelling through a fluid so that its axis of rotation forms an angle with the trajectory, the body will experience a lift force acting in a direction perpendicular to the plane containing the flight path and the axis of rotation.

Maccoll (Ref. 56) first provided evidence of the

lift force acting on a sphere which was spinning on an axis normal to the direction of the air stream flowing in a channel. Introducing a co-efficient of lift,  $C_L$  as:-

$$C_L = \frac{F_L}{\rho \cdot d^2 \cdot U^2} \quad (2.1.14)$$

Maccoll noted that the plot of  $C_L$  versus the speed ratio (equatorial speed to the air speed) showed negative values of the lift co-efficient for small values of the speed ratio (up to about 0.5). Thereon the value of  $C_L$  increased to about 0.18 at the speed ratio of 2 and remained constant thereafter. In the critical Reynolds number region, the sphere rotation enhanced the boundary layer transition on one side of the sphere while on the other side transition was delayed. An asymmetric wake structure resulting therein caused the negative lift.

Davies (Ref. 57) also noted the existence of a negative lift force in a later work with, among others, the smooth golf balls dropped in a horizontal wind stream.

In contrast to a spinning body in a uniform flow, Tollert (Ref. 63) pointed out that circulation could develop with a translating sphere either by the rotation of the sphere itself or due to the asymmetric streaming around the non-rotating sphere.

The first explanation on the phenomena of particle migration in pipe flows was given by Young (Ref. 64). He noted that in the vertical glass tubes, containing spheres which were heavier than the fluid, the particles tended to migrate towards the axis during the upward flow of the mixture. An annulus of the clear fluid near the wall and surrounding the core of particles was clearly identified. Conversely, in a downward flow, the particles showed the tendency to collect near the wall.

of the tube. Also, in turbulent flows the particles showed little radial migration, apparently because the turbulent eddies offset the motion due to lift force.

Radial migration of particles in pipe flows also was reported in Ref.'s 65 and 66. In Ref. 68, it was shown that particles' spinning was responsible for their migration in pipe flows.

Rubinow and Keller (Ref. 55) studied the motion of a spinning sphere and provided the following expression for the spin induced lift :-

$$\underline{F}_L = \frac{1}{8} \pi d^3 \rho \underline{\Omega} \times \underline{v} [1 + O(Re_p)] \quad (2.1.15)$$

which was derived from the inertia considerations near the sphere surface, seemingly neglected in all previous work. Their expression was valid for small values of  $Re_p$  only.

Saffman (Ref. 67) made an analytical study of the lift on a spherical particle due to fluid shear. Considering the motion of a sphere relative to an unbounded simple shear flow, Saffman suggested an expression for a lift force, which in vector form became :-

$$\underline{F}_L = 20.3 \rho d^2 (v/K)^{\frac{1}{2}} \underline{K} \times (\underline{v} - \underline{U}) \quad (2.1.16)$$

where,  $\underline{K}$  was the curl of the fluid velocity (in simple shear flow the velocity gradient normal to the flow direction). The sphere was assumed to rotate at a rate equal to the local velocity of the fluid particles and spin induced circulation was zero. Saffman tried unsuccessfully to extend his work to the case of Poiseuille flows. For further discussion on flow induced lift, see Ref. 25.

Although the discussion so far concerned the lift force arising only due to circulation, the possibility of the production of some kind of lift due to fluid turbulence also was recognised by some investigators. It was suggested that in a turbulent flow, the mechanical effect of the lift force due to circulation would be superimposed on the particle response to free stream turbulence. A useful discussion on the subject is given in Ref. 68.

### 2.2.3 Summary of Discussion of Forces on a Spherical Particle

The mechanism of momentum transfer between the fluid and a spherical solid particle has been discussed in the previous paragraphs. It is shown that the observed drag on a solid sphere can be different from the steady state value calculated using the standard drag curve for a number of reasons. These are the unsteady motion of the fluid relative to the particle, the presence of turbulence in the free stream, etc.

It is recognised that because of the unsteadiness in the motion of the particle relative to the fluid, the particle experiences drag forces which are additional to that given by the steady state drag laws. Analytical models are available to indicate that the additional forces are due to the resistance of the fluid to acceleration of the sphere, the acceleration of the flow itself. These analytical models were developed by dropping the convective acceleration terms while solving the Navier-Stokes equation of the flow. Hence, these models are "obviously not valid when the convective acceleration terms become important" (Ref. 69). Thus, for high  $Re_p$  ( $>62$ ), unsteady drag cannot be calculated analytically. Also, the concept of total resistance coefficient in calculating unsteady drag although looks very practical, needs to be fully explored.

On the effect of turbulence, it is seen that

the presence of free stream turbulence may cause an early transition of the boundary layer on the surface, thereby ushering in a reduction in the drag coefficient. The physical insight into the effect of turbulence seem to be reasonably understood, although the task of quantifying the effect is still a formidable one.

Analytical models for lift due to spin of the particle or the fluid shear can give an estimate of the lift only at very low Reynolds number. An estimate of the lift force calculated from equation 2.1.16 was found to be less than other force terms (see section 2.3.2) by an order of magnitude,  $10^{-1}$  or less. Hence this force was neglected.

Summarising, it can be stated that while it is possible to conjecture as to the influence of turbulence or unsteady effects, accounts for these effects must await further work. In view of these conclusions, only the steady state drag and the gravitational forces are considered in the formulation of the governing equations of motion of particles in the present work.

### 2.3 Equations of Motion of Particles

In this section the equations of motion of individual particles passing through a centrifugal impeller are given. The equations are written assuming that the particles are spherical. The geometry of the impeller used in the present work (see figure 6.2a) is such that a particle, after it has entered the impeller does not immediately see a blade channel, but has to traverse some distance before it enters the channel. In view of the above, the equations of motion of the particles are written in two frame of axes. In the first case, the equations are written in an inertial coordinate system and applied to particle motion between the inlet of the impeller and just before the leading edge. The solution of this set of equations gives the absolute trajectory of the particles.



In the second case, the equations are written in a rotating coordinate system and applied to the motion of the particles in the blade channel between the leading edge and the exit of the impeller. The solution of this set of equations gives the trajectory of the particles relative to a blade surface.

### 2.3.1 Equations in an Inertial Coordinate System

The equation of motion of a spherical solid particle of mass  $m_p$  moving in water with a velocity  $\underline{v}$  can be given in an inertial frame of axes from Newton's second law of motion as follows :-

$$m_p \frac{D\underline{v}}{Dt} = \underline{F}_D + \underline{F}_B \quad (2.3.1)$$

where :-

$\underline{F}_D$  = drag force vector due to the relative motion between the particle and water

$\underline{F}_B$  = body force vector due to the difference in density between the solid particle and water.

The drag force is calculated as the product of the drag coefficient and the dynamic pressure of the relative flow times the projected area of the particle (Ref. 41). In vectorial form this can be written as follows :-

$$\underline{F}_D = \frac{1}{2} C_D \rho_w |\underline{u} - \underline{v}| (\underline{u} - \underline{v}) A \quad (2.3.2)$$

$C_D$  is given as a function of the Reynolds number of the particle which is defined in terms of the relative velocity between the particle and water as follows:-

$$Re_p = \frac{\rho_w |\underline{u} - \underline{v}| d}{\mu} \quad (2.3.3)$$

The body force term can be written as follows :-

$$\underline{F}_B = \frac{1}{6} \pi d^3 (\rho_p - \rho_w) g \hat{e}_g \quad (2.3.4)$$

The components of equation 2.3.1 in cylindrical coordinate system can be written as follows (see Appendix A) :-

r - direction :

$$m_p (\ddot{r} - r\dot{\theta}^2) = \frac{1}{2} C_D \rho_w A (u_r - v_r) |\underline{u} - \underline{v}| - (m_p - m_w) g \sin\theta \quad (2.3.5a)$$

$\theta$  - direction :

$$m_p (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = \frac{1}{2} C_D \rho_w A (u_\theta - v_\theta) |\underline{u} - \underline{v}| - (m_p - m_w) g \cos\theta \quad (2.3.5b)$$

z - direction :

$$m_p \ddot{z} = \frac{1}{2} C_D \rho_w A (u_z - v_z) |\underline{u} - \underline{v}| \quad (2.3.5c)$$

where, overdot indicates time derivatives. See fig. 2.1 for definition of the coordinate frame. Rearranging terms of equation 2.3.5a :-

$$\dot{v}_r = \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_r - v_r) |\underline{u} - \underline{v}| -$$

$$- \left[ 1 - \frac{\rho_w}{\rho_p} \right] g \sin\theta + \frac{v_\theta^2}{r} \quad (2.3.6a)$$

$$\ddot{r} = \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_r - v_r) |\underline{u} - \underline{v}| - \left[ 1 - \frac{\rho_w}{\rho_p} \right] g \sin\theta + \frac{v_\theta^2}{r} \quad (2.3.6b)$$

Rearranging terms of equation 2.3.5b :-

$$\dot{v}_\theta = \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_\theta - v_\theta) |\underline{u} - \underline{v}| - \left[ 1 - \frac{\rho_w}{\rho_p} \right] g \cos\theta - \frac{v_r v_\theta}{r} \quad (2.3.7a)$$

$$\ddot{\theta} = \frac{3}{4} \frac{1}{r} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_\theta - v_\theta) |\underline{u} - \underline{v}| - \frac{1}{r} \left[ 1 - \frac{\rho_w}{\rho_p} \right] g \cos\theta - 2 \frac{v_r v_\theta}{r^2} \quad (2.3.7b)$$

Rearranging terms of equation 2.3.5c :-

$$\dot{v}_z = \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_z - v_z) |\underline{u} - \underline{v}| \quad (2.3.8a)$$

$$\ddot{z} = \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_z - v_z) |\underline{u} - \underline{v}| \quad (2.3.8b)$$

Integration of equations 2.3.6a, 2.3.7a and 2.3.8a will give the velocity of the particle in the coordinate directions. Integrating equations 2.3.6b, 2.3.7b and 2.3.8b, the position of the particle can be calculated. The equations are nonlinear. Hence these are solved numerically. This is discussed in section 2.3.3.

### 2.3.2 Equations in a Rotating Coordinate System

The coordinate system has its origin coinciding with the origin of the inertial system defined in section 2.3.1, but rotates about z-axis (fig. 2.1) with a constant angular speed equal to the rotational speed of the pump. The equation of motion of a particle written in this coordinate system can be written as follows :-

$$m_p \frac{D\mathbf{v}}{Dt} = \mathbf{F}_D + \mathbf{F}_B - m_p \mathbf{2} \underline{\omega} \times \underline{v} - m_p \underline{\omega} \times (\underline{\omega} \times \underline{r}) \quad (2.3.9)$$

I          II                  III                  IV

III and IV are inertia terms which appear in the above equation due to the rotation of the coordinate system.  $2\underline{\omega} \times \underline{v}$  is the coriolis and  $\underline{\omega} \times (\underline{\omega} \times \underline{r})$  is the centripetal acceleration. See Appendix A for further details.

The components of equation 2.3.9 in cylindrical polar coordinate system can be written as follows :-

r direction :

$$m_p \{ \ddot{r} - r(\dot{\theta} + \omega)^2 \} = \frac{1}{2} C_D \rho_w A (u_r - v_r) |\underline{u} - \underline{v}|$$

$$- (m_p - m_w) g \sin \theta_p \quad (2.3.10a)$$

$\theta$  - direction :

$$m_p \{ r\ddot{\theta} + 2\dot{r}(\dot{\theta} + \omega) \} = \frac{1}{2} C_D \rho_w A (u_\theta - v_\theta) |\underline{u} - \underline{v}|$$

$$- (m_p - m_w) g \cos \theta_p \quad (2.3.10b)$$

z - direction :

$$m_p = \frac{1}{2} C_D \rho_w A (u_z - v_z) |\underline{u} - \underline{v}| \quad (2.3.10c)$$

where :-

$$\theta_p = \theta + \omega t \quad (2.3.11)$$

The  $\theta = 0$  axis of the rotating system of coordinates coincides with the trailing edge of a blade, see fig. 2.2. If  $v'_\theta$  is the tangential component of particle velocity in the rotating coordinate system, then rearranging terms of equation 2.3.10a :-

$$\begin{aligned} \dot{v}_r &= \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_r - v_r) |\underline{u} - \underline{v}| - \left(1 - \frac{\rho_w}{\rho_p}\right) g \sin \theta_p + \\ &+ r \left( \frac{v'_\theta}{r} + \omega \right)^2 \end{aligned} \quad (2.3.12a)$$

$$\begin{aligned} \ddot{r} &= \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_r - v_r) |\underline{u} - \underline{v}| - \left(1 - \frac{\rho_w}{\rho_p}\right) g \sin \theta_p + \\ &+ r \left( \frac{v'_\theta}{r} + \omega \right)^2 \end{aligned} \quad (2.3.12b)$$

Rearranging terms of equation 2.3.10b :-

$$\begin{aligned} \dot{v}_\theta &= \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_\theta - v_\theta) |\underline{u} - \underline{v}| - \left(1 - \frac{\rho_w}{\rho_p}\right) g \cos \theta_p - \\ &- \left( \frac{v_r}{r} \frac{v'_\theta}{r} + 2 v_r \omega \right) \end{aligned} \quad (2.3.13a)$$

$$\ddot{\theta} = \frac{3}{4} \frac{1}{r} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_\theta - v_\theta) |\underline{u} - \underline{v}| -$$

$$- \frac{1}{r} \left(1 - \frac{\rho_w}{\rho_p}\right) g \cos\theta_p - 2 \frac{v_r}{r} \left[ \frac{v_\theta}{r} + \omega \right] \quad (2.3.13b)$$

Rearranging terms of equation 2.3.10c :-

$$\dot{v}_z = \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_z - v_z) |\underline{u} - \underline{v}| \quad (2.3.14a)$$

$$\ddot{z} = \frac{3}{4} \frac{C_D}{d} \frac{\rho_w}{\rho_p} (u_z - v_z) |\underline{u} - \underline{v}| \quad (2.3.14b)$$

The last term in equations 2.3.12a and 2.3.13a is the centrifugal and coriolis acceleration, respectively (Ref. 70). The components of the particle velocity with respect to the rotating system can be calculated by integrating equations 2.3.12a, 2.3.13a and 2.3.14a. The position of the particle in the blade channel of the impeller relative to a surface is calculated by integrating equations 2.3.12b, 2.3.13b and 2.3.14b. The equations are nonlinear. Hence they are solved numerically. This will be discussed in the following section.

### 2.3.3 Solution of Equation of Motion of Particles

In sections 2.3.1 and 2.3.2, the equations of the motion of a solid particle suspended in water are given. Two sets of equations are provided. One (equations 2.3.6 to 2.3.8) is in an inertial system for particle motion before the particle has entered a blade channel of the impeller. The other (equations 2.3.12 to 2.3.14) is in a rotating system when the particle is inside the blade channel. Both sets of equations are time dependent. Hence their solution will provide time distance history of the movement of the particle.

In general the equations can be written in the following form :-

$$\ddot{r} = f_5(r, \theta, z, \dot{r}, \dot{\theta}, \dot{z}) \quad (2.3.15a)$$

$$\ddot{\theta} = f_6(r, \theta, z, \dot{r}, \dot{\theta}, \dot{z}) \quad (2.3.15b)$$

$$\ddot{z} = f_7(r, \theta, z, \dot{r}, \dot{\theta}, \dot{z}) \quad (2.3.15c)$$

They are solved numerically by using a Taylor series expansion for particle position and velocity.

Let the coordinates of the particle at time  $t$  be given as  $r_t, \theta_t, z_t$ . After an increment of time  $\Delta t$ , the position of the particle along the coordinate directions are :-

$$r_{t+\Delta t} = r_t + \Delta t \dot{r}_t + \frac{(\Delta t)^2}{2} \ddot{r}_t + \text{higher order terms} \quad \text{--- (2.3.16a)}$$

$$\theta_{t+\Delta t} = \theta_t + \Delta t \dot{\theta}_t + \frac{(\Delta t)^2}{2} \ddot{\theta}_t + \text{higher order terms} \quad \text{--- (2.3.16b)}$$

$$z_{t+\Delta t} = z_t + \Delta t \dot{z}_t + \frac{(\Delta t)^2}{2} \ddot{z}_t + \text{higher order terms} \quad \text{--- (2.3.16c)}$$

To be able to evaluate the right hand side of equations 2.3.16a to 2.3.16c, it is necessary that the acceleration and the position of the particles at time  $t$  should be known. The information on particle acceleration at time  $t$  can be obtained from equations 2.3.15a to 2.3.15c provided the particle velocity at time  $t$  is also known. Hence, the following initial conditions for particle velocity must be specified :-

$$\dot{r}(t_0) = \dot{r}_0 \quad (2.3.17a)$$

$$\dot{\theta}(t_0) = \dot{\theta}_0 \quad (2.3.17b)$$

$$\dot{z}(t_0) = \dot{z}_0 \quad (2.3.17c)$$

Equations 2.3.17 may be substituted in equations 2.3.15 to get the initial accelerations,  $\ddot{r}_0$ ,  $\ddot{\theta}_0$ ,  $\ddot{z}_0$  at time  $t = t_0$ .

Also, by writing the Taylor series expansion for the velocity of the particle, the velocities along the coordinate directions are obtained as follows :-

$$\dot{r}_{t+\Delta t} = \dot{r}_t + \Delta t \ddot{r}_t + \text{higher order terms} \quad (2.3.18a)$$

$$\dot{\theta}_{t+\Delta t} = \dot{\theta}_t + \Delta t \ddot{\theta}_t + \text{higher order terms} \quad (2.3.18b)$$

$$\dot{z}_{t+\Delta t} = \dot{z}_t + \Delta t \ddot{z}_t + \text{higher order terms} \quad (2.3.18c)$$

where, the information for acceleration and velocity at instant  $t$  are obtained from equations 2.3.15a to 2.3.15c and 2.3.17a to 2.3.17c, respectively.

Hence, by using equations 2.3.16a to 2.3.16c, 2.3.17a to 2.3.17c and 2.3.18a to 2.3.18c, it is possible to integrate equations 2.3.15a to 2.3.15c in a stepwise manner with time and obtain the history of the movement of particles inside an impeller. The integration continues until the particle collides with a solid wall of the impeller. The velocity with which the particle bounces off a solid wall depends on a number of factors which are discussed in section 2.7. Once this velocity is known, the solution of the equations of motion is continued till another collision occurs or the particle leaves the impeller.



#### 2.3.4 Estimation of Errors

It may be noted that the error in calculating the position of the particle along the coordinate directions from 2.3.16a to 2.3.16c is of the order of  $(\Delta t)^3$ . An error of order  $(\Delta t)^2$  is associated with the calculation of the velocities from equations 2.3.18a to 2.3.18c.

#### 2.3.5 Calculation of Drag Coefficient

The drag coefficient defined in equation 2.3.2 is calculated as a function of the Reynolds number of the particle which is defined by equation 2.3.3. Morsi and Alexander's (Ref. 46) analytical fit of the standard drag curve for spherical particle is used in the present work. The following expressions are used :-

$$C_D = 24/Re_p \quad \text{for } Re_p < 0.1 \quad (2.3.19a)$$

$$C_D = 22.73/Re_p + 0.0903/Re_p^2 + 3.69 \quad \text{for } 0.1 < Re_p < 1.0$$

---- (2.3.19b)

$$C_D = 29.1667/Re_p - 3.8889/Re_p^2 + 1.222 \quad \text{for } 1.0 < Re_p < 10$$

--- (2.3.19c)

$$C_D = 46.5/Re_p - 116.67/Re_p^2 + 0.6167 \quad \text{for } 10 < Re_p < 100$$

---- (2.3.19d)

$$C_D = 98.33/Re_p - 2778/Re_p^2 + 0.3644 \quad \text{for } 100 < Re_p < 1000$$

--- (2.3.19e)

$$C_D = 148.62/Re_p + 4.75 \times 10^4 / Re_p^2 + 0.357 \quad \text{for } 1000 < Re_p < 5000$$

--- (2.3.19f)

$$C_D = -490.546/Re_p + 57.87 \times 10^4 / Re_p^2 + 0.46 \quad \text{for } 5000 < Re_p < 10000$$

--- (2.3.19g)

## 2.4 Computation of Flow Field in the Impeller :

### Theory and Method of Solution

In section 2.3 the equations of the motion of particles are given. One of the forces acting on the particles is the fluid dynamic drag which occurs because of the difference in velocity between the particle and the fluid. To obtain the velocity of the fluid phase, the governing equations of the flow through an impeller should be solved. The method to obtain such a solution is discussed in the following paragraphs.

#### 2.4.1 Governing Equations for the Motion of the Fluid Phase

It is assumed that the flow is steady, inviscid and incompressible. The basic equations governing the flow within an impeller are derived from the principles of conservation of mass and momentum. The mathematical form of the equations in a system of cylindrical coordinates rotating with the impeller at a constant angular velocity are as follows :-

continuity :

$$\frac{\partial}{\partial r} (\rho_w r W_r) + \frac{\partial}{\partial \theta} (\rho_w W_\theta) + \frac{\partial}{\partial z} (\rho_w r W_z) = 0 \quad (2.4.1)$$

momentum :

$$\frac{W_\theta}{r} \left[ \frac{\partial}{\partial r} (r u_\theta) - \frac{\partial W_r}{\partial \theta} \right] - W_z \left[ \frac{\partial W_r}{\partial z} - \frac{\partial W_z}{\partial r} \right] = \frac{\partial I}{\partial r} \quad (2.4.1a)$$

$$\frac{W_r}{r} \left[ \frac{\partial}{\partial r} (r u_\theta) - \frac{\partial W_r}{\partial \theta} \right] - \frac{W_z}{r} \left[ \frac{\partial W_z}{\partial \theta} - \frac{\partial}{\partial z} (r u_\theta) \right] = \frac{1}{r} \frac{\partial I}{\partial \theta} \quad (2.4.1b)$$

$$\frac{W_r}{r} \left[ \frac{\partial W_r}{\partial z} - \frac{\partial W_z}{\partial r} \right] - \frac{W_\theta}{r} \left[ \frac{\partial W_z}{\partial \theta} - \frac{\partial}{\partial z} (r u_\theta) \right] = \frac{\partial I}{\partial z} \quad (2.4.1c)$$

Equations 2.4.1 and 2.4.2a to 2.4.2c form a set of four differential equations with four unknowns — the three velocity components,  $w_r$ ,  $w_\theta$ ,  $w_z$  and the rothalpy of the fluid. A method of solving the above system of equations will be discussed below.

#### 2.4.2 The Method of Solution

Full three dimensional solution of equations 2.4.1 and 2.4.2a to 2.4.2c are rarely attempted since it requires very fast computers with a very large storage. To overcome this problem, Wu (Ref. 71) proposed a method of solution in which an appropriate combination of mathematically two dimensional flows on two different kinds of relative stream surfaces provides a three dimensional solution of the flow. This is well known as the general through flow theory for turbomachine flows. By definition, a stream surface is a surface such that the velocity vector at each point in the surface is tangent to the surface. Inspired by Wu's work, Marsh (Ref. 72) presented a Matrix through flow analysis to solve the flow of an inviscid fluid in one of the stream surfaces. The Marsh Matrix method was modified to calculate viscous flows in a centrifugal compressor by Goulas (Ref. 73). For the calculation of the flow pattern in the present work, the author has <sup>a</sup>lent heavily on the work of Ref. 73.

#### 2.4.3 Flow on a Stream Surface

By definition, the following relation must be satisfied on a stream surface :-

$$\underline{W} \cdot \hat{n} = 0 \quad (2.4.3)$$

where,  $\hat{n}$  is normal to the stream surface. Let n-N-S be a local orthogonal system defined on the stream surface. The S-axis is in the direction of the local velocity and

N-axis is normal to it, but is tangent to the surface. The third axis,  $n$ , is normal to the surface. Hence, the body force which must exist for the flow to follow the stream surface must be parallel to  $n$ . Hence, the equations of the motion of fluid particles written in the local orthogonal system  $n$ - $N$ - $S$  will have the following characteristics :-

- (i) that the equations of motion for  $S$  and  $N$  directions do not contain a component of the body force  $\underline{F}$  and may be used to calculate the flow pattern
- (ii) that the equation of motion in the  $n$ -direction can be replaced by the geometric condition (equation 2.4.3)

Hence, the governing equations for the fluid particles on a stream surface become :-

continuity :

$$\nabla \cdot (\rho \underline{W}) = 0 \quad (2.4.4)$$

$N$ -direction equation of motion :

$$\underline{N} \cdot \underline{W} \times (\nabla \times \underline{u}) = \underline{N} \cdot \nabla I \quad (2.4.5)$$

$S$ -direction equation of motion :

$$\underline{W} \cdot \underline{W} \times (\nabla \times \underline{u}) = \underline{W} \cdot \nabla I \quad (2.4.6)$$

Geometry condition :

$$\underline{W} \cdot \underline{F} = 0 \quad (2.4.7)$$

The analytical steps involved in the solution are :-

- (i) to express the equations of motion (equations 2.4.2a to 2.4.2c) in terms of special derivatives taken on the stream surface. As the equations are to be solved on a prescribed stream surface, the ordinary partial derivatives are replaced by the special derivatives taken on the stream surface (Ref. 71).
- (ii) to form the N-direction equation of motion by taking the dot product of the equations derived in item (i) with the vector  $\underline{F} \times \underline{W}$  which lies in the N-direction
- (iii) to introduce a stream function which satisfies continuity and express the N-direction equation of motion as derived in item (ii) in terms of this stream function

The final form of the equation in the N-direction is of the following form :-

$$\nabla^2 \psi = q_1(r, \theta, z) \quad (2.4.8)$$

which may be solved by adopting an iterative scheme. The above procedure was used to obtain the equation of motion on a stream surface extending from hub to shroud. Details of the stream surface are given in the following section.

#### 2.4.3.1 Description of the Stream Surface

The stream surface chosen in the present study for the computation of the fluid flow has the following properties :-

- (i) the surface extends from hub to shroud in the blade channel of the impeller
- (ii) outside the channel, the surface is extended to an upstream position. This allowed the computation of flow to be made in the passage outside the blade channel, between impeller

inlet and the position upto the leading edge. The projection on a meridional plane of this stream surface is shown in fig. 2.3. It may be seen in the figure that some adjustment of the boundary of the stream surface has been made near the eye and the nose of the impeller. The boundary of the stream surface in these areas slightly deviates from the physical boundary of the system. This is done to avoid stagnation points at the eye and the nose of the impeller. If the boundary of the stream surface coincides with the physical boundary, then an isentropic solution will result in excessive velocity gradient in the areas near the eye and the nose of the impeller which is not realistic. Hence some adjustment in the boundary of the stream surface in these areas was necessary.

#### 2.4.3.2 Selection of Coordinates to Express Partial Derivatives on the Stream Surface

The body force which must exist for the flow to follow the stream surface bears the following relationship with the normal to the stream surface :-

$$\hat{n} \times \underline{F} = 0 \quad (2.4.9)$$

The largest component of the body force will be parallel to the largest component of  $\hat{n}$ . For mathematical reasons detailed in Ref. 73, the two coordinates chosen to express a stream function should be such that the largest component of  $\hat{n}$  must be in the direction of the third coordinate.

The stream surface described in section 2.4.3.1 lies on a meridional plane from impeller inlet upto the entry to a blade channel. The largest component of  $\hat{n}$  is  $n_\theta$  and  $r$ - $z$  will be the coordinates chosen. In the blade channel of the impeller the stream surface extends from hub to shroud, the largest component of  $\hat{n}$  is  $n_r$  and hence

z- $\theta$  will be the coordinate system chosen.

#### 2.4.3.3 Stream Surface Expressed in r-z Coordinates

This case has been discussed in Ref. 73. Only the basic equations are given here.

The stream surface is defined by two geometry angles  $\lambda'$  and  $\mu'$  which are related to the normal to the surface as follows :-

$$\tan \lambda' = \frac{n_r}{n_\theta} \quad (2.4.10a)$$

$$\tan \mu' = \frac{n_z}{n_\theta} \quad (2.4.10b)$$

A stream function which satisfies continuity may be defined for flow on this surface as follows :-

$$\overline{\frac{\partial \psi}{\partial r}} = \rho_w B r u_z \quad (2.4.11a)$$

$$\overline{\frac{\partial \psi}{\partial z}} = -\rho_w B r u_r \quad (2.4.11b)$$

where the overbar indicates special derivatives. The term B is an integration factor and its value is an indication of the finite thickness of a thin stream sheet whose mean surface is the stream surface considered here.

The equation of motion for the N-direction expressed as an equation for the stream function  $\psi$  is :-

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{\partial^2 \psi}{\partial z^2} = u_z \frac{\partial}{\partial r} (\rho_w B r) - u_r \frac{\partial}{\partial z} (\rho_w B r) + (\rho_w B r)^2 \frac{dI}{d\psi} +$$

$$+ \rho_w B \{ \tan \mu' \frac{\partial}{\partial r} (ru_\theta) - \tan \lambda' \frac{\partial}{\partial z} (ru_\theta) \} \quad (2.4.12)$$

This is a non-linear equation of the form :-

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{\partial^2 \psi}{\partial z^2} = q_2(r, z) \quad (2.4.13)$$

which is solved by successive iterations. From an initial distribution of stream function,  $q_2(r, z)$  is calculated which is then used to solve for  $\psi$ .  $q_2(r, z)$  is recalculated and the process continues till the difference in  $\psi$  between two successive iterations converges to a preset value (Ref. 73).

#### 2.4.3.4 Stream Surface Expressed in z- $\theta$ Coordinate

This case has been discussed in Ref. 73. Only the basic equations are given here.

The geometry angles  $\lambda'$  and  $\mu'$  are related to the normal to the stream surface as follows :-

$$\tan \lambda' = \frac{n_z}{n_r} \quad (2.4.14a)$$

$$\tan \mu' = \frac{n_\theta}{n_r} \quad (2.4.14b)$$

The stream function which satisfies continuity is introduced as follows :-

$$\frac{\partial \psi}{\partial z} = -B \rho_w W_\theta \quad (2.4.15a)$$



$$\frac{\bar{\partial}\psi}{r\partial\theta} = B\rho_w W_z \quad (2.4.15b)$$

The equation of motion for the N-direction expressed as an equation for the stream function  $\psi$  is :-

$$\begin{aligned} \frac{1}{r^2} \frac{\bar{\partial}^2 \psi}{\partial \theta^2} + \frac{\bar{\partial}^2 \psi}{\partial z^2} = W_z \frac{1}{r} \frac{\bar{\partial}}{\partial \theta} (\rho_w B) - W_\theta \frac{\bar{\partial}}{\partial z} (\rho_w B) + (\rho_w B)^2 \frac{dI}{d\psi} + \\ + \rho_w B \left( \frac{1}{r} \frac{\bar{\partial} W}{\partial \theta} r \tan \lambda' - \frac{\bar{\partial} W}{\partial z} r \tan \mu' \right) - \rho_w B \left( \frac{W_\theta}{r} + 2\omega \right) \tan \lambda' \end{aligned}$$

--- (2.4.16)

This is a non-linear equation of the form :-

$$\frac{1}{r^2} \frac{\bar{\partial}^2 \psi}{\partial \theta^2} + \frac{\bar{\partial}^2 \psi}{\partial z^2} = q_3(z, \theta) \quad (2.4.17)$$

which is solved by successive iterations.

#### 2.4.3.5 Finite Difference Approximations of the Partial Derivatives : Definition of the Grid

Equations 2.4.12 and 2.4.16 are non-linear and they do not have any analytical solutions. Hence, they need to be solved numerically. To do so a grid is formed on the stream surface. This grid is shown in fig. 2.3. At each point of the grid the partial derivatives of equations 2.4.12 and 2.4.16 are replaced by equivalent algebraic expressions. Following the method as outlined

in Ref. 73, the algebraic expressions are obtained from finite difference approximations of the partial derivatives. Details of the procedure are given in Ref. 73 and hence this subject will not be dealt in here any further.

#### 2.4.3.6 Matching of the Equations on the Stream Surface

In section 2.4.3.1 the description of the stream surface employed for the calculation of the flow pattern in the impeller used in the present work is given. It is also stated that two different sets of equations are required to be solved for this stream surface (section 2.4.3.2), equation 2.4.13 for the calculation of flow pattern from impeller inlet upto the entry to a blade channel and equation 2.4.16 for the calculation of flow pattern within a blade channel. Hence, there is a line across which the change of coordinates will take place. Along the transition line, the following conditions have to be satisfied.:-

- (i) the velocity must be independent of the system of coordinates chosen
- (ii) the mass flow across the transition line must be conserved

To satisfy conditions (i) and (ii), it is necessary that the mass flow obtained from the meridional solution (r-z coordinates) is used to define stream function for z- $\theta$  set of coordinates. This is done as follows. If at any point on the transition line  $W_\theta$ ,  $W_z$  be the velocity components calculated from the meridional solution, then for flows in the blade channel, a stream function can be introduced as follows :-

$$\frac{\partial \psi}{\partial z} = -W_\theta \rho_w B \quad (2.4.15a)$$

$$\frac{1}{r} \frac{\partial \psi}{\partial \theta} = W_z \rho_w B \quad (2.4.15b)$$

At the boundary of the stream surface (either along the hub or the shroud),  $\psi$  is constant since there is no mass flow across the boundary. Let  $\psi = 0$ , along a boundary of the stream surface. A point of the transition line lies on this boundary. At any point on this line :-

$$\psi = - \int W_{\theta} \rho_w B \, dz + \int W_z \rho_w B r \, d\theta \quad (2.4.18)$$

Knowing  $W_{\theta}$ ,  $W_z$  from the meridional solution, the above integral can be evaluated numerically. In this way the value of the stream function at all points on the transition line can be obtained, and later used as the initial distribution for the stream surface downstream of the transition line.

## 2.5 Location of a Particle in the Computed Flow Field

In the previous section, the grid defined for the calculation of the flow field inside the impeller has been described. The equations of motion of a particle are solved in this flow field. In order to be able to calculate the fluid dynamic drag on the particle, the velocity of flow at the position of the particle should be known. As the velocity information of the flow is given only at each node of the grid, it is necessary that :-

- (i) the element containing the particle is correctly identified
- (ii) the fluid properties (e.g., velocity) given at the nodes of the element are appropriately weighted at the position of the particle, as the position of the particle in an element may not coincide with a node

The method adopted in the present work for locating the element containing the particle is based on the definition

of grid shown in fig. 2.3. The following steps are involved:-

- (i) For particles outside the blade channel  
(between stations 1 to 11, fig. 2.3) : First, the angle of inclination of every column (whose number is the same as that of the station) with the inlet section of the grid (represented by station 1, fig. 2.3) is calculated upto and including the leading edge. The angle of inclination that the position of a particle defines with the inlet section is then calculated and compared with those calculated for each column. This allows identification of two successive columns within which the particle lies. The next step is to calculate the slope of the rows between two columns. Calculation then can be performed to find the radial position of a point in a row corresponding to the axial position of the particle. By comparing this radial position with that of the particle, two successive rows between which the particle lies and hence the element containing the particle can be identified.
- (ii) Particle position inside the blade channel :  
The columns of the grid here are straight lines parallel to the axis of the impeller (see fig. 2.3). The position of the particle between two successive columns is identified by comparing the radial position of the particle with those of the columns. The element containing the particle is then identified by comparing the axial position of the rows with that of the particle.

The method of calculating the fluid properties inside an element is given in the following paragraphs.

### 2.5.1 Weighted Field Property inside an Element

An element in the grid with its node defined in a physical coordinate system (cartesian orthogonal,  $x, y$ ) is shown in fig. 2.4a. The position of the particle inside the element is given as  $x_0, y_0$ . The field properties are given at the four nodes of the element. The sides of the element are straight lines.

Let  $\zeta, \eta$  define an intrinsic coordinate system. The four nodes of the element transformed from the physical to the intrinsic system are shown in fig. 2.4b. Let  $(\zeta, \eta)$  define the position of the particle in the intrinsic system. It is required to find the value of  $\zeta, \eta$  which when transformed to the physical system  $x, y$  will give the true position of the particle inside the element, i.e.,  $x_0, y_0$ . Let  $(x_1, y_1), (x_2, y_2), (x_3, y_3)$  and  $(x_4, y_4)$  be the coordinates of the four nodes of the element in the physical system. Performing an isoparametric transformation of  $\zeta$  and  $\eta$  into  $x$  and  $y$  using bivariate Lagrangian interpolation formula, the following relations are obtained :-

$$x = (1 - \zeta)(1 - \eta)x_1 + \zeta(1 - \eta)x_2 + \zeta\eta x_3 + (1 - \zeta)\eta x_4$$

---(2.5.1a)

$$y = (1 - \zeta)(1 - \eta)y_1 + \zeta(1 - \eta)y_2 + \zeta\eta y_3 + (1 - \zeta)\eta y_4$$

---(2.5.1b)

$x$  and  $y$  calculated from equations 2.5.1a and 2.5.1b, respectively, for assumed values of  $\zeta, \eta$  of the particle in the intrinsic system is compared with the actual values  $x_0, y_0$ . Values of  $\zeta, \eta$  are then corrected and the process is repeated until the difference between the calculated and the actual value of  $x$  and  $y$  become negligible. Having

correctly located the particle in the intrinsic system, the field property,  $P$  at the position of the particle can be calculated from the following relationship :-

$$P(\zeta, \eta) = (1 - \zeta)(1 - \eta)P_1 + \zeta(1 - \eta)P_2 + \zeta\eta P_3 + (1 - \zeta)\eta P_4$$

---(2.5.2)

## 2.6 Criterion for a Particle to see a Leading Edge

When a particle enters a blade channel, the angular position of the particle relative to the pressure side of the channel is arbitrary. The probability of a particle hitting a blade surface at areas near the leading edge depends upon :-

- (i) the velocity at which the particle enters the channel
- (ii) the ability of the particle to negotiate the velocity field in the channel
- (iii) the rotational speed of the blade

In this section a semi-theoretical argument is presented to define the conditions in which a particle can hit these areas.

The ability of a particle to negotiate a new flow field is manifested in the magnitude of its aerodynamic response time. Considering, for the present case, that the motion of the particle is controlled by inertia and drag only, then its equation of motion is (Ref. 41) :-

$$\frac{dv}{dt} = \frac{18\mu}{\rho_p d^2} (u - v) f_s \quad (2.6.1)$$

The term  $f_s$  is introduced to account for the change in the drag coefficient from that given in the Stokes' Regime (see section 2.2.1.2) and hence :-

$$f_s = \text{Re}_p \frac{C_D}{24} \quad (2.6.2a)$$

From Clift et al (Ref. 74) :-

$$f_s = 1 + 0.15 \text{Re}_p^{.687} \quad (2.6.2b)$$

The aerodynamic response time of the particle, therefore, is :-

$$\tau = \frac{\rho_p d^2}{18 \mu f_s} \quad (2.6.3)$$

Let  $v_r$  be the radial velocity of the particle as the particle enters a blade channel. It is assumed that the particle will retain this velocity for a period of time equal to its aerodynamic response time. The radial displacement of the particle within this time, therefore, is  $v_r \tau$ .

Also, let  $n_b$  be the number of blades of the impeller so that the angular separation between blades,  $\alpha'$  is given as :-

$$\alpha' = \frac{2\pi}{n_b} \quad (2.6.4)$$

As the particle enters a blade channel, its angular

position relative to the pressure side of the blade channel,  $\beta'$  may be given by any value between 0 to  $\alpha'$ . Angle  $\beta'$  in relation to angle  $\alpha'$  is shown in fig. 2.5. Introducing a random number  $\epsilon$  which is real and whose value lies between 0 and 1, the relative angular position of the particle at any instant as it enters a blade channel is given as :-

$$\beta' = \frac{2\pi\epsilon}{n_b} \quad (2.6.5)$$

Also, let  $s$  be a characteristic distance on the blade pressure surface measured from the leading edge downstream, on a  $z$ -plane (constant axial position), fig. 2.6. This distance is shown as length  $ab$  in fig. 2.6. If  $s$  is small and  $\gamma$  is the angle that the length  $s$  forms with the tangent at the leading edge (fig. 2.6), then the projection of this distance onto a radial plane is  $s \cdot \sin \gamma$ . The time,  $t_1'$  taken by the particle to move radially by  $s \cdot \sin \gamma$  is :-

$$t_1' = \frac{s \sin \gamma}{v_r} \quad (2.6.6)$$

The time taken by the leading edge of the blade to rotate by an angle  $\beta'$  (equation 2.6.5) and be on the radial plane in which the particle lies is :-

$$t_2' = \frac{2\pi\epsilon}{n_b \omega} \quad (2.6.7)$$

Hence, the particle will hit the pressure side of the blade within length  $s$ , if :-



$$t_2' < t_1' \quad (2.6.8a)$$

Substituting equations 2.6.6 and 2.6.7 into the inequality (2.6.8a) :-

$$v_r \leq \frac{s \sin \gamma}{2\pi \epsilon} n_b \omega \quad (2.6.8b)$$

Therefore, a particle will be considered as hitting an area in the immediate vicinity of the blade leading edge if :-

- (i) the radial displacement corresponding to the aerodynamic response time is such that, -

$$v_r \tau > s \sin \gamma \quad (2.6.9a)$$

(ii)  $v_r \leq \frac{s \sin \gamma}{2\pi \epsilon} n_b \omega \quad (2.6.9b)$

Application of these criteria in calculating particle trajectories inside an impeller is discussed in section 5.1.1.2.2.1.

## 2.7 Calculation of Rebound Characteristics of a Particle

When a solid particle moves in a fluid stream, it does not in general follow the fluid streamline due to its higher inertia. In passing through an impeller, the particle tends to collide with the impeller blades or the shrouds. As a result of a collision, the velocity of the particle changes, its course is modified. This change in velocity must be known in order that the solution of the equations of motion of the particle may be continued till the particle leaves the impeller.

In this section the analytical methods to calculate the rebound characteristics of a particle bouncing off a solid wall will be discussed. It is assumed that the particle is spherical.

A segment of surface at the impeller wall is shown shaded in fig. 2.7. A particle approaches this

surface with a velocity  $v_1$  and impinges at point 0.  $v_{n_1}$  is the component of  $v_1$  in a direction normal to the surface.  $v_{s_1}$  is the component of  $v_1$  in a direction which is tangential to the surface at point 0. Hence :-

$$\underline{v}_1 = \underline{v}_{n_1} + \underline{v}_{s_1} \quad (2.7.1)$$

Let a cartesian orthogonal coordinate system  $x-y-z$  be defined at 0. Hence :-

$$\underline{v}_1 = \underline{v}_{x_1} + \underline{v}_{y_1} \quad (2.7.2)$$

$\hat{n}$  is a unit vector along the normal to the surface at point 0. The direction of this normal sees the fluid. The surface is defined as follows :-

$$q_4(x, y, z) \quad (2.7.3)$$

The component of  $\hat{n}$  in coordinate directions, therefore, can be calculated from the following geometric relationship :-

$$\begin{aligned} \frac{n_x}{\frac{\partial q_4}{\partial x}} &= \frac{n_y}{\frac{\partial q_4}{\partial y}} = \frac{n_z}{\frac{\partial q_4}{\partial z}} = \\ &= \frac{1}{\left[ \left[ \frac{\partial q_4}{\partial x} \right]^2 + \left[ \frac{\partial q_4}{\partial y} \right]^2 + \left[ \frac{\partial q_4}{\partial z} \right]^2 \right]^{1/2}} \end{aligned} \quad (2.7.4)$$

It follows that :-

$$v_{n_1} = \underline{v}_1 \cdot \hat{n} \quad (2.7.5a)$$

or,

$$v_{n_1} = v_{x_1} n_x + v_{y_1} n_y + v_{z_1} n_z \quad (2.7.5b)$$

$\hat{s}$  is a unit vector in the direction of  $v_{s_1}$ .  
 $\hat{s}$  bears following relationship with  $\hat{n}$  and  $\underline{v}_1$  :-

$$n_x s_x + n_y s_y + n_z s_z = 0 \quad (2.7.6)$$

$$s_x^2 + s_y^2 + s_z^2 = 1 \quad (2.7.7)$$

$$(\hat{n} \times \underline{v}_1) \cdot \hat{s} = 0 \quad (2.7.8a)$$

$$\begin{array}{ccc} s_x (n_y v_{z_1} - n_z v_{y_1}) & + & s_y (n_z v_{x_1} - n_x v_{z_1}) + \\ \text{I} & & \text{II} \end{array}$$

$$+ s_z (n_x v_{y_1} - n_y v_{x_1}) = 0 \quad (2.7.8b)$$

III

For the sake of brevity, the coefficients of  $s_x$ ,  $s_y$  and  $s_z$  in equation 2.7.8b shall be referred to as I, II and III, respectively. Multiplying equation 2.7.6 by I/ $n_x$  and subtracting it from equation 2.7.8b, an expression for  $s_y$

may be obtained as follows :-

$$s_y = -s_z \frac{III - I \frac{n_z}{n_x}}{II - I \frac{n_y}{n_x}} \quad (2.7.9a)$$

Substituting  $s_y$  into equation 2.7.8b, an expression for  $s_x$  can be written as follows :-

$$s_x = \frac{s_z}{I} \left[ II \frac{III - I \frac{n_z}{n_x}}{II - I \frac{n_y}{n_x}} - III \right] \quad (2.7.9b)$$

Substituting  $s_x$ ,  $s_y$  into equation 2.7.7 and rearranging terms, an expression for  $s_z$  can be obtained as follows :-

$$s_z = \left[ 1 + \left[ \frac{III - I \frac{n_z}{n_x}}{II - I \frac{n_y}{n_x}} \right]^2 + \left[ II \frac{III - I \frac{n_z}{n_x}}{II - I \frac{n_y}{n_x}} - III \right] I^{-2} \right]^{-\frac{1}{2}} \quad --- (2.7.9c)$$

The value of  $s_x$  and  $s_y$  can be obtained by substituting  $s_z$  into equations 2.7.9a and 2.7.9b, respectively. Thus, knowing  $s_x$ ,  $s_y$  and  $s_z$ , the value of  $v_{s_1}$  can be calculated from the following relationship :-

$$v_{s_1} = s_x v_{x_1} + s_y v_{y_1} + s_z v_{z_1} \quad (2.7.10)$$

If  $v_{n_2}$  and  $v_{s_2}$  are the normal and the tangential component of the rebound velocity  $v_2$ , then :-

$$\underline{v}_2 = \underline{v}_{n_2} + \underline{v}_{s_2} \quad (2.7.11)$$

Procedure to calculate  $v_{n_2}$  and  $v_{s_2}$  are detailed in the following paragraphs.

### 2.7.1 Calculation of Normal Component of Rebound Velocity

An expression for the normal component of rebound velocity can be formulated by considering the physical phenomena involved in a collision process. An impacting particle generates relatively large contact forces which act in a very short interval of time. During this interval the contact area grows rapidly to a peak value as deformation increases and then reduces to zero during the restoration period. Considering for example two particles  $m_a$  and  $m_b$  which are on a collision course to each other. Their velocities are  $v_a$  and  $v_b$ , respectively. The position of the particles before, during and after a collision is shown in fig. 2.8. Let  $N_f$  be the contact force on mass  $m_a$  at any instant during contact and hence on mass  $m_b$  it is  $-N_f$ , fig. 2.8d. It is assumed that any forces other than the internal contact forces which may act on the particles during an impact are small and produce negligible impulse in comparison with those associated with the impact forces.

If  $e$  is the coefficient of restitution defined as the ratio of the impulse during the period of restoration to the impulse during the period of deformation and if  $N_r$  and  $N_d$  are the contact forces during

these two periods, respectively, then from the impulse-momentum equation for mass  $m_a$  :-

$$e = \frac{\int_{t_d}^{t_c} N_r dt}{\int_0^{t_d} N_d dt} = \frac{m_a (v_a' \sin \alpha_a' - V_o)}{m_a (V_o + v_a \sin \alpha)} \quad (2.7.12a)$$

where,  $V_o$  is the common velocity of the two particles in the direction of  $N_f$  at the instant of transition from deformation to restoration. The velocity changes are considered positive in the direction of the contact force on the masses. Similarly for mass  $m_b$  :-

$$e = \frac{m_b (v_b' \sin \alpha_b' + V_o)}{m_b (v_b \sin \alpha_b - V_o)} \quad (2.7.12b)$$

Eliminating  $V_o$  between equations 2.7.12a and 2.7.12b, the following relation for the coefficient of restitution can be obtained :-

$$e = \frac{v_a' \sin \alpha_a' + v_b' \sin \alpha_b'}{v_a \sin \alpha_a + v_b \sin \alpha_b} \quad (2.7.13)$$

or,

$$e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}$$

where, the velocity components are measured in the direction of the impact forces. A value of  $e = 1$  indicates that the ability of two particles to restore equals their tendency to deform. This is a case of an elastic impact. On the other hand,  $e = 0$  indicates that the particles cling to each other as they collide and hence it is a case of a plastic impact. In reality, for all impact conditions the value of  $e$  lies between 0 and 1.

Now considering that a particle hits a solid wall of an impeller, from analogy to the example cited above :-

$$v_{n_1} = -v_a \sin \alpha \quad (2.7.14a)$$

$$v_{n_2} = v_a' \sin \alpha_a' \quad (2.7.14b)$$

hence :-

$$v_{n_2} = -e v_{n_1} \quad (2.7.14c)$$

since :-

$$v_b = 0, \quad v_b' = 0 \quad (2.7.14d)$$

as the quantities  $v_b$  and  $v_b'$  refer to the solid wall of the impeller.

The calculation of the tangential component of rebound velocity is discussed in the following section.

### 2.7.2 Calculation of Tangential Component of Rebound Velocity

A particle approaches a wall with velocity :-

$$\underline{v}_1 = \underline{v}_{s_1} + \underline{v}_{n_1} \quad (2.7.15)$$

After a short period of time, the particle leaves the wall with a velocity :-

$$\underline{v}_2 = \underline{v}_{s_2} + \underline{v}_{n_2} \quad (2.7.16)$$

In the short interval which the particle resides at the wall, its motion is governed by the wall reaction and the friction force at the wall. Two situations are possible :-

- (i) The particle hits the wall and the impulse of the friction at the wall brings the point of contact of the particle with the wall to rest before the particle leaves the wall as a result of wall reaction on it. This is referred to as zero slip condition.
- (ii) The particle hits the wall and the impulse of the friction at the wall is not enough to bring the point of contact of the particle with the wall to rest. In this case the particle slips on the wall before leaving as a result of the wall reaction on it. This is referred to as non-zero slip condition.

While in either case, the normal component of the rebound velocity  $v_{n_2}$  can be obtained from equation 2.7.14c, the calculation<sup>2</sup> of the tangential velocity  $v_{s_2}$  will depend on the zero or non-zero slip condition of the particle at the wall. This is discussed in the following paragraphs.



### 2.7.2.1 Zero Slip Between the Particle and the Solid Wall

When the particle reaches the wall, it tends to slip on the wall and if the impulse of the friction force on it is sufficiently high, the particle will, almost immediately following an impact, leave the wall. During this infinitesimal time interval the particle slips before its velocity at the contact point P (fig. 2.9) is brought to zero. Hence :-

$$\int_{t_1}^{t_2} F_e dt = \int_{t_1}^{t_2} f_d N_f dt \quad (2.7.17)$$

The residence time of the particle at the wall,  $\Delta t = t_2 - t_1$ , is so small that the whole process may be considered to have taken place instantaneously. Equation 2.7.17 is derived on the assumption that the motion of the particle during the collision process is restrained to the plane defined by the normal to the wall,  $\hat{n}$  and the initial velocity of the particle,  $v_{s_1}$ . In figure 2.9, the motion of the particle in this plane is shown. Of the characteristics shown in the figure, those subscripted 1 refer to the condition when the particle hits the wall and those subscripted 2 refer to the condition when the particle is about to leave the wall.

The right hand side of equation 2.7.17 indicates the maximum value of the impulse of the friction force which is obtainable to cause this kind of motion.

Considering that the particle is about to leave the wall, the conservation of angular momentum about point P gives :-

$$\frac{dH_z}{dt} = 0 \quad (2.7.18)$$

where,  $H_z$  is the angular momentum of the particle about point P. Hence :-

$$I_p \Omega_2 + \frac{1}{2} m_p d v_{s_2} = - I_p \Omega_1 + \frac{1}{2} m_p d v_{s_1} \quad (2.7.19a)$$

The moment of inertia of the particle about its centroid is :-

$$I_p = \frac{1}{10} m_p d^2 \quad (2.7.19b)$$

Hence, equation 2.7.19a becomes :-

$$v_{s_2} = - \frac{1}{7} d \Omega_1 + \frac{5}{7} v_{s_1} \quad (2.7.20)$$

Thus by knowing the initial tangential velocity and the angular speed of the particle, the tangential speed at which the particle bounces off a solid wall can be calculated. In equation 2.7.20, the term  $d \Omega_1$  is less than other terms by an order of magnitude of  $10^{-1}$ . Dropping  $d \Omega_1$ , equation 2.7.20 becomes :-

$$v_{s_2} = \frac{5}{7} v_{s_1} \quad (2.7.21)$$

### 2.7.2.2 Non-Zero Slip Between the Particle and the Solid Wall

In this case the particle slips on the wall before leaving as a result of the wall reaction on it. The velocity of the particle at the point of contact with the wall is :-

$$v_p = v_{s1} + \frac{1}{2} d \Omega_1 \quad (2.7.22)$$

The friction force acts in a direction opposite to  $u_s$ , where  $u_s$  is defined as follows :-

$$u_s = \frac{v_{s1} + \frac{d \Omega_1}{2}}{\left| v_{s1} + \frac{d \Omega_1}{2} \right|} \quad (2.7.23)$$

The momentum increase in the direction, tangential to the plane of contact, is :-

$$m_p (v_{s2} - v_{s1}) = - \int_{t_1}^{t_2} u_s F_e dt \quad (2.7.24)$$

where,  $F_e$  is the absolute value of the friction force. Similarly, a term  $u_n$  can be defined as :-

$$u_n = \frac{v_{n2} - v_{n1}}{\left| v_{n2} - v_{n1} \right|} \quad (2.7.25)$$

The momentum 'increase' in the direction normal to the surface is :-

$$m_p (v_{n_2} - v_{n_1}) = u_n \int_{t_1}^{t_2} N_f dt \quad (2.7.26)$$

Assuming that the friction factor  $f_d$  is independent of time, equation 2.7.26 becomes :-

$$f_d \frac{m_p}{u_n} (v_{n_2} - v_{n_1}) = \int_{t_1}^{t_2} F_e dt \quad (2.7.27)$$

Taking into account equation 2.7.24, an expression for the tangential component of the rebound velocity is derived as :-

$$v_{s_2} = v_{s_1} + f_d \frac{u_s}{u_n} (v_{n_1} - v_{n_2}) \quad (2.7.28)$$

### 2.7.2.3 Condition for Slip on the Wall

In order that the zero slip condition is satisfied, the impulse of the friction acting on the particle should be sufficiently high for the particle to be brought to rest at the point of contact with the wall. Hence, the following inequality must be satisfied :-

$$\int F_e dt \leq \int f_d N dt \quad (2.7.29)$$

The momentum 'increase' in the tangential direction, equation 2.7.24 which also holds for the present situation, gives :-

$$\int F_e dt = - m_p \frac{v_{s2} - v_{s1}}{u_s} \quad (2.7.30)$$

The momentum 'increase' in the normal direction, equation 2.7.26, also gives :-

$$f_d \frac{m_p}{u_n} (v_{n2} - v_{n1}) = f_d \int_{t_1}^{t_2} N dt \quad (2.7.31)$$

Substituting both the equations (2.7.30 and 2.7.31) into the inequality (2.7.29) and taking into account the relation for  $v_{s2}$  given by equation 2.7.20, the following condition for non-slip on the wall must be satisfied :-

$$\frac{22}{7u_s} (v_{s1} + \frac{d \Omega_1}{2}) \leq - \frac{f_d}{u_n} (1 + e) v_{n1} \quad (2.7.32)$$

### 2.7.3 Final Form of the Rebound Velocity

In section 2.7.1 the equation to find the normal component of the rebound velocity is given. The equation to find the tangential component of the rebound velocity under different wall conditions is formulated in section 2.7.2. In order that the solution of the equations of the motion of particles can be continued after a particle has collided with a solid wall, the components of  $v_{n_2}$  and  $v_{s_2}$  in coordinate directions need to be calculated. To do so, the following relationships are used :-

$$v_{x_2} = n_x v_{n_2} + s_x v_{s_2} \quad (2.7.33a)$$

$$v_{y_2} = n_y v_{n_2} + s_y v_{s_2} \quad (2.7.33b)$$

$$v_{z_2} = n_z v_{n_2} + s_z v_{s_2} \quad (2.7.33c)$$

### 2.8 Development of a Computer Code for Calculation of Trajectory of Particles in an Impeller

To obtain the trajectory of a particle in an impeller, the flow field of the impeller is first calculated using the method detailed in section 2.4. Then the equations of motion of the particle are solved in this flow field. Equations 2.3.6a to 2.3.8b or 2.3.12a to 2.3.14b are integrated for several time steps starting from the inlet until the particle exits from the impeller. After every time step the position and the velocity of the particle are calculated and used as the initial conditions for the next time step. The following steps are involved :-

- (i) The particle is released into the flow at known velocity and position. Information on these

- quantities is given in section 3.1.1.
- (ii) Position of the particle in the computed flow is calculated using methods detailed in section 2.5. The weighted field property at the position of the particle is calculated from equation 2.5.3.
  - (iii) With the initial velocity of the particle known, the velocity of the particle relative to the flow is calculated.
  - (iv) From the size of the particle, and density and absolute viscosity of water, the Reynolds number of the particle is calculated using equation 2.3.3.
  - (v) Corresponding to the Reynolds number of the particle, the drag coefficient is calculated using the analytical fit of the standard drag curve (section 2.3.5).
  - (vi) A time step is then chosen and equations 2.3.6a to 2.3.8b are integrated over this time step to find new velocity and position of the particle (section 2.3.3).

Steps (ii) to (vi) are repeated until the particle reaches a solid wall or enters a blade channel. If the particle hits a solid wall, its rebound characteristics are calculated analytically using methods detailed in section 2.7.

Solution of the equations of motion of the particles are continued once the particle rebounds from the wall with new initial conditions. Steps (ii) to (vi) are then repeated.

When the particle enters a blade channel, equations 2.3.12a to 2.3.14b are integrated instead of equations 2.3.6a to 2.3.8b.

The integration stops when the particle leaves

the blade channel.

The whole method is programmed in FORTRAN 77 for a DEC VAX/11-780 machine. The program makes use of a set of library subroutines (CALCOMP) for an automatic plotting of the trajectory of one or more particles so that a clear view of this may be obtained following each run of the program.



## CHAPTER 3

### PARTICLE CONCENTRATION UPSTREAM OF THE IMPELLER

#### 3.1 Introduction

In the previous chapter the equations governing the motion of particles passing through an impeller are given. These equations are solved numerically for given initial conditions of the position and the dynamic characteristics of the particles at the inlet to the flow field.

The particles enter the impeller through a pipe. The distribution of the particles in a cross-section of the pipe determines the initial position of the particles in the flow field of the impeller in which the motion of the particles is traced. This flow field is not uniform throughout the passage of the impeller. For that reason alone, the path taken by individual particles inside the impeller will depend upon the position at which the particles enter the flow field.

Also, the velocity of the particles in the cross-section of the pipe may not be uniform. It will be seen later that this velocity bears some relation to the manner in which the particles are distributed in the cross-section of the pipe.

It is most probable that the particles are not distributed uniformly in the cross-section of the pipe. In that situation the number of particles passing through different elementary areas of the same size in the cross-section of the pipe will be different. In calculating the motion of the particles passing through the impeller, it is not possible to perform these calculations for each and every individual particle because of the large number

of particles involved. The entire cross-section of the pipe is divided into a number of elementary areas and the motion of a single particle passing through each area is considered. In this way the motion of one particle inside the impeller is considered representative of all the particles passing through the same elementary area. During the passage through the impeller, the representative particle may impact on a blade surface or any other wetted part of the impeller. The total number of impacts on an area will be given as the summation of the number of impacts due to each and every such particle multiplied by the number of particles whose motion is represented by this particle. Information on the number of impacts is necessary to estimate the damage caused to a surface by the particles.

It can therefore be said that to calculate the motion of particles inside an impeller, the distribution of the particles in the pipe should be known. A measure of this distribution can be obtained if the profile of the concentration of the particles in the cross-section of the pipe can be found. In this chapter the analytical methods available to find the concentration of solids in a mixture with water is briefly discussed in the context of the present work, the limitation of the methods are reviewed and the concentration profiles assumed for the present work are given. A method to find the distribution of particles from a known concentration profile is also discussed.

### 3.2 Theoretical Prediction of Concentration

The particles are in suspension when the entire motion of the solid particles is such that they are surrounded by fluid. Owing to the weight of the particles, there is a tendency for settling, which however may be counter balanced by the irregular motion of the fluid, i.e., turbulent eddies. Thus the hydraulic conditions of a stream determine when the solids will be in suspension.

The classical model for the distribution of solid particles in a steady uniform flow was given by O'Brien (Ref. 75). This model is based on the assumption that at any elevation from the bottom of an open channel, there is no net transport of mass, the gravitational settling is balanced by particle diffusion due to turbulence. Hence, O'Brien proposed that the concentration,  $c_t$ , of solid particles at a distance,  $Z$ , measured vertically from the bottom of the channel (fig. 3.1) may be given as follows :-

$$\xi_c \frac{dc_t}{dt} + w c_t = 0 \quad (3.1)$$

where,  $w$  is the settling velocity of a single particle defined as the velocity at which the particle settles under gravity in a quiescent liquid with zero acceleration (Ref. 76). Equation 3.1 was integrated by assuming that :-

- (i) the shear stress of the flow was linearly distributed between the open surface and the bottom of the channel
- (ii) the flow velocity was given by a logarithmic profile

The following expression was given :-

$$\frac{c_t}{c_{t_{Z_0}}} = \left( \frac{h - Z}{Z} \cdot \frac{Z_0}{h - Z_0} \right)^{\bar{Y}} \quad (3.2)$$

Equation 3.2 gives concentration at a level  $Z$  provided that the concentration at level  $Z_0$  is known. The exponent  $\bar{Y}$  is related to the settling velocity of the solids  $w$ , Kármán's universal constant  $k$ , and the friction velocity of the flow  $u^*$ , as follows :-

$$\bar{Y} = \frac{w}{k \beta u^*} \quad (3.3a)$$

$\beta$  is the ratio of the turbulence diffusion coefficients of the solids and the momentum of the flow; hence :-

$$\beta = \xi_c / \xi_m \quad (3.3b)$$

The work of O'Brien (Ref. 75) has been discussed in some detail because it was found that both in the laboratory and the field, his model describes the concentration profile quite satisfactorily. For further details, see Ref. 77.

The first attempt to examine the behaviour of suspension in closed channels was made by Ismail (Ref. 78). He conducted a series of experiments and showed that equation 3.2 is applicable to flow in a rectangular channel if  $h$  is taken to be the point of maximum velocity. As the suspended solids tend to skew the velocity distribution (Ref. 78), the aforementioned point does not necessarily correspond to the channel centreline.

The assumption of a logarithmic velocity profile in O'Brien's model (Ref. 75) means that the distribution of  $\xi_c$  has a normal parabolic form. Ismail showed that in a rectangular duct  $\xi_c$  follows a parabolic form only at the outer two-thirds of the channel but has a constant value at the middle third. For the region where  $\xi_c$  is constant, Ismail showed that equation 3.2 can be integrated to find the relative concentration  $c_t / c_{t_{z_0}}$  as follows :-

$$c_t / c_{t_{z_0}} = \exp\{ - w(z - z_0) / \xi_c \} \quad (3.4)$$

For the region where  $\xi_c$  has the parabolic distribution, it was shown that equation 3.2 gives good agreement with experimental data.

The applicability of a one dimensional distribution model to the case of a circular pipe was partially made by Wasp et al (Ref. 79). Using Ismail's (Ref. 78) data for the concentration of solids at points located at 8% from the top or the bottom surface, they derived the following relationship :-

$$\log (c_t/c_{t_{z_0}}) = -1.8\bar{Y} \quad (3.3)$$

Verification of this relationship was carried out on a 12-inch pipeline transporting coal slurries. Samples were withdrawn at 1-inch from the top and from the bottom of the pipe and also from the centreline of the pipe. Comparison between observed values of  $c_t/c_{t_{z_0}}$  and those given by equation 3.3 showed excellent agreement. No attempt was made to measure the concentration at other points along the vertical diameter of the pipe.

### 3.2.1 Limitation of the Theoretical Studies

A critical examination of the applicability of a convection-diffusion model to explain the suspension of particles in closed channel flows was made by Shook et al (Ref. 80). They conducted a series of experiments with particles of different settling velocity and arrived at the conclusion that suspension of solid particles can be considered as due to turbulence only when the concentration of solid particles is low, the particle diameter is small ( $<0.2$  mm) and the ratio of the settling velocity of the particles to the friction velocity of the flow is less than 0.2. When these conditions are not satisfied, a significant deviation was observed between the concentration profiles obtained experimentally and those given by the solution of equation 3.1. It was noted that the ability of the particles to remain in suspension at high concentrations is not fully attributable to turbulence. The interaction between particles resulting in a dispersive

stress on the particles, as postulated by Bagnold (Ref. 81), had to be considered. This dispersive stress occurs normal to the direction in which the solid particles were sheared. Shook et al observed that the suspension of particles at high concentration and velocity gradient was partially due to Bagnold's (Ref. 80) dispersive stress. The suspension increased with particle diameter and velocity. Because of the complexity in calculating Bagnold's dispersive stress, mainly because the velocity gradient of the solids enter into the analysis (Ref. 81), Shook et al concluded that any prediction of concentration profiles for large particles, where the dispersive stress (Ref. 81) is significant in the suspension mechanism, is not possible. The limiting value of the particle size was noted to be 0.2 mm. In this context, it may be mentioned here that Ismail (Ref. 78) succeeded in using the convection-diffusion model to explain his experimental results because both the particle size and concentration were small (maximum 0.18 mm and 4.3% by weight, respectively).

Also, the work of Hsu et al (Ref. 82) suggested that a convection-diffusion model provides an accurate prediction of concentration of small solid particles in the cross-section of a circular pipe. Close agreement between theory and experiment was reported for a mean particle size of 0.16 mm and the maximum value of the ratio of the settling velocity of the particles to the friction velocity of the flow of 0.184.

### 3.3 Experimental Information

In the preceeding sections the importance of defining particle position upstream of an impeller has been discussed and the limitations in using an analytical method to obtain this distribution has been cited. The size of the solid particles of the present work is not small (for detail see section 6.3.1) prohibiting any analytical method to find their distribution in the inlet

pipe. Hence, some experimental information on their distribution was necessary. The procedures adopted to fulfill this requirement are detailed in this section.

In hydraulic transport of solids, the settling tendency of solids provides a basis for classifying slurries into the following two broad classes :-

(i) Non-settling type :

These slurries are made up of solids of colloidal dimensions (diameter less than 0.01 mm for sand). The settling velocity of the particles is generally less than 0.1 mm/s. They are called so because the solids do not settle out when the pipeline carrying this type of slurries is stopped. The particles are maintained in suspension by Brownian movement (see Ref. 41 on Brownian motion). The solid and the liquid phases do not behave separately and the mixture is homogeneous even in laminar flow. There is virtually no concentration gradient when transporting non-settling slurries, the solids are fairly uniformly distributed over the pipe cross-section.

(ii) Settling type :

These slurries are made up of solids (diameter >0.15 mm for sand) whose settling velocity is 1.5 mm/s or more. The solids settle out quickly unless the suspension mechanism (e.g., fluid turbulence, Bagnold's dispersive stress - Ref. 81) is strong. The solid and liquid phases are separate and the liquid properties remain unaltered by the presence of solids. In a quiescent state the solid and liquid phases separate readily. Flowing slurries of this type are characterised principally by a concentration gradient increasing towards the bottom of the pipe.

Lying in between these two types of slurries is the slow settling slurries of particles having settling velocity generally less than 1 mm/s (diameter 0.03 mm for sand). These particles settle slowly en masse in a quiescent state of the mixture. Further detail on slurry classification by settling velocity of solids may be found in Ref. 83.

The slurry used in the present work is of the settling type (solids settling velocity 18.56 cm/s, see section 6.3.3). All the slurry characteristics in the discussion which follows are referred to this type of slurries only.

The solids will remain in full suspension in the mixture if the flow velocity is sufficiently high. A measure of the limiting value of this velocity which just keeps all the particles fully suspended is the limit deposit velocity. According to Lazarus (Ref. 83), this velocity corresponds to "the transition from the moving bed flow to fully suspended flow"; a moving bed flow is a "suspended flow with a moving bed, i.e., a bed of solids moving at a uniform rate in the lower part of the pipe". Newitt et al (Ref. 68) considers Bagnold's dispersive stress (Ref. 81) to be largely responsible for the suspension of particles in this regime. Flows of this type mean large gradient of concentration in the cross-section of the pipe with "regions of high concentration at the bottom of the pipe", Ref. 68. Additionally, because of the presence of a gradient of concentration in the cross-section of the pipe, a local slip velocity exists between the solid and the liquid phase of the mixture, Ref. 84.

The distribution of concentration and slip velocity of solids are not possible to be determined analytically as the size of the solids in settling slurries is higher than the limiting value (see section 3.2.1) of 0.2 mm. Hence, experimental data are required.



Literature on simultaneous measurement of insitu concentration (for definition see Appendix B) and slip velocity of solids is very limited. Durand (Ref. 85) reported measurements to show the difference between an asymmetric suspension and a moving bed flow pattern exhibited by solids of same specific gravity but different sizes. Measurements were made on sand slurries transported through a 150 mm diameter pipeline. Two sizes were considered, fine sand of diameter 0.18 mm and coarse sand of diameter 2.04 mm. For fine particles, the observed velocity profiles of the flow were nearly symmetric while the concentration profiles (probably insitu discharge, for definition see Appendix B) were asymmetric, the latter varied in asymmetry with changes in mean flow velocity. The generation of an asymmetric suspension is due to weakening of the suspension mechanism, either through a reduction in the mixture velocity or the intensity of turbulence, and the settling tendency of the particle causing more of the particles to occupy the lower half of the pipe than the upper one. For coarse particles, a moving bed pattern with asymmetry in both velocity and concentration profiles was noticed. In contrast to fine particles, for coarse sands the velocity profiles retained their asymmetry with increasing velocity, an upward shift of the maximum velocity point from the pipe axis was noticed as the flow velocity was increased. No explanation has so far been given concerning this trend in the velocity profile.

A more comprehensive set of data, including insitu concentration and slip velocity of particles, covering solids of different size and specific gravity, was given by Newitt et al (Ref. 68). The following solids were used :-

- |       |             |                                     |
|-------|-------------|-------------------------------------|
| (i)   | zircon sand | (S.G. 4.60, mean diameter 0.116 mm) |
| (ii)  | fine sand   | (S.G. 2.65, mean diameter 0.182 mm) |
| (iii) | coarse sand | (S.G. 2.66, mean diameter 0.699 mm) |
| (iv)  | gravel      | (S.G. 2.67, mean diameter 1.547 mm) |

(v) perspex (S.G. 1.17, mean diameter 1.854)

The solids were transported through a pipe of 25.4 mm diameter.

Fine sand, zircon sand and perspex were fully suspended in the fluid in an asymmetric manner whereas gravel and coarse sand formed moving bed near the bottom of the pipe. The velocity profiles for water carrying coarse sand and gravel were given. An upward shift of the maximum point from the pipe axis was noticed as the mean discharge concentration in the pipe was increased. Mean velocities of liquid and of mixture both decreased in coarse sand and gravel slurries as the mean discharge concentration was raised at a constant hydraulic gradient. Also, the mean slip velocity between the solids and water progressively increased with mean discharge concentration. The effect of concentration on the velocities mentioned above was found to be negligibly small for perspex mixtures, a small effect was noticed in fine sand and zircon sand mixtures. Profiles of insitu discharge concentrations (for definition see Appendix B) for both coarse sand and gravel showed that the concentration towards the bottom of the pipe was substantially constant where the bed existed. Above the bed, the concentration fell rapidly with height. Profiles of insitu transport concentration calculated from measured insitu discharge concentration and water velocity were given for coarse sand and gravel slurries. The difference in two concentrations was substantial where the bed existed and decreased rapidly above the bed (the transport concentration was always greater than the discharge concentration). Calculated insitu slip velocity between the solids and the liquid (calculated from measured insitu discharge concentration and water velocity) showed that the slip was significant in coarse sand and gravel slurries, but its variation in the cross-section of the pipe was small.

The work of Newitt et al (Ref. 68) has been discussed in some detail for two reasons :-

- (i) In the experimental part of the present work the concentration profile of the solids in the pipe could not be measured. The work of Newitt et al thus provided a basis for estimating a concentration profile appropriate to the present work, as will be indicated later.
- (ii) Information on slip velocity appropriate to the present work was also obtained from the same source.

In the following section these points will be discussed.

### 3.3.1 The Concentration Profile at Impeller Inlet

In order to be able to derive information from data provided by Newitt et al (Ref. 68) so that the concentration profile, appropriate to the conditions of the experimental part of the present work may be obtained, a comparison is made of the characteristics of the solid particles used in the present work with those reported by Newitt et al.

The following properties of the particles used in two cases were considered for comparison :-

- (i) size of the particles
- (ii) specific gravity of the particles
- (iii) settling velocity of the particles

The particles used in the present work are fine pea gravel, detailed description of which is given in section 6.3. The size of these particles was distributed in a range. An analysis of the particle size distribution was carried out by vibrating a sample in B.S. 410 test sieves, results are given in Appendix D and shown diagrammatically in fig. 3.2 (details of sieve analysis are given in section 6.3.1). To make a comparison with the particle size cited in Newitt et al's work, it was necessary to

define a characteristic size of the particles used in the present work.

A number of methods are detailed in the literature to define a characteristic size of the particles ( a useful discussion is given in Ref. 76). Wiedenroth (Ref. 86), Sellgren (Ref. 87) advocated strongly for a characteristic particle size based on the weight function of the solid particles. According to the method, if  $P_n$  is the weight function of solid particles of diameter  $d_n$ , then the characteristic particle size may be taken to be the weighted value,  $\bar{d}$  of the particles as follows :-

$$\bar{d} = \frac{\sum_{n=1}^{n_t} P_n d_n}{\sum_{n=1}^{n_t} P_n} \quad (3.4)$$

where,  $n_t$  is the number of size fractions.

Also, in the literature a very frequently used characteristic particle size, in particular in the field of hydraulic transport of solids, is what is known as  $d_{50}$  size of the solids. According to the proponents (Ref. 88) of this size, an opening of the sieve in mm passing 50% of the solids by mass is a good representation of the size of the particles which is distributed in a range.

In the present work the characteristic particle size considered was the  $d_{50}$  size of the particles. There was no apriori justification for a preference to a  $d_{50}$  size to characterise the size of the particles. As it turned out later, this way of defining a characteristic size of the particles was quite fruitful.

In Appendix D, the results of the sieve analysis of the particles used in the present work are given. The  $d_{50}$  size of the particles obtained directly from the plot of this analysis (fig. 3.2) is 1.8 mm.

The other properties of the particle found from

measurements are the following :-

specific gravity - 2.659

settling velocity - 18.56 cm/s

Detail of the experimental procedures for measuring specific gravity is given in section 6.3.2, and settling velocity in section 6.3.3. The results are given in Appendices E and F, respectively.

Examination of the characteristics of the particles given in Newitt et al's (Ref. 68) work indicate that the characteristics of the gravel used in their work closely resembled those of the particles used in the present work. The comparison of the characteristics stands as follows :-

|                   | <u>Present Work</u> | <u>Newitt et al's</u> |
|-------------------|---------------------|-----------------------|
| Solids type       | Gravel              | Gravel                |
| Specific gravity  | 2.659               | 2.650                 |
| Particle size     | 1.80                | 1.63                  |
| $d_{50}$ (mm)     |                     |                       |
| Settling velocity | 18.56               | 21.18                 |
| (cm/s)            |                     |                       |

Encouraged by this close similarity in particle characteristics, the next step was to compare the flow conditions which existed during the course of these two sets of experiment (i.e., present work and that of Ref. 68). Two situations were considered :-

- (i) that corresponding to the first set of experiment in the present work in which mean discharge concentration of the solids issuing from the pipe was 3.362%
- (ii) that corresponding to the second set of experiment in the present work in which mean discharge concentration of the solids was 11.653%

The comparison is detailed below.

### 3.3.1.1 Case 1 : Mean Discharge Concentration of 3.362%

Detail of the procedures and important aspects pertaining to this set of experiment are given in section 6.7.2. Hence, only the basic information appropriate to the discussion in this section will be given here.

The following basic information pertaining to flow conditions of the mixture have been considered :-

- (i) Mean discharge concentration,  $\bar{c}_d = 3.362\%$   
(mean or time averaged discharge concentration)
- (ii) Mean mixture velocity,  $\bar{v}_m = 1.426 \text{ m/s}$   
(mean or time averaged flow, averaged over the cross-section of the pipe)
- (iii) Limit deposit velocity of the solids in the mixture (Ref. 85),  
 $V_L = F_C \sqrt{2gD(S_s - 1)} = 2.456 \text{ m/s}$   
( $F_C$  is a function of particle size and concentration for particle size up to 2.2 mm, see fig. 7 in Ref. 85)

The values will now be compared with those given by Newitt et al (Ref. 68).

For gravel slurries, information on insitu discharge concentration distribution are given in Ref. 68 for a number of mean discharge concentrations in the range of 2.05% to 11.55%. For comparison with the present case, insitu discharge concentration profile corresponding to  $\bar{c}_d = 3.77\%$  has been considered. Conditions of the flow pertaining to this concentration and those of the present work are given below :-

|                                           | <u>Present Work</u> | <u>Newitt et al's</u> |
|-------------------------------------------|---------------------|-----------------------|
| Mean discharge concentration, $\bar{c}_d$ | 3.362%              | 3.770%                |

|                                             | <u>Present Work</u> | <u>Newitt et al's</u> |
|---------------------------------------------|---------------------|-----------------------|
| Mean mixture<br>velocity, $\bar{v}_m$ (m/s) | 1.426               | not available         |
| Limit deposit<br>velocity, $V_L$ (m/s)      | 2.456               | 1.225                 |
| $\bar{v}_m/V_L$                             | 0.582               | not available         |

Given the close similarity in values of mean discharge concentration in two cases, it is argued that the distribution of the insitu discharge concentration in the cross-section of the pipe will also be similar in the two cases provided that the velocity ratios  $\bar{v}_m/V_L$  are close to each other, since  $\bar{v}_m/V_L$  is a measure of the way in which the particles are suspended in a pipeline. Unfortunately, there are no information on  $\bar{v}_m/V_L$  in Newitt et al's work. Hence, other characteristics of the transport of mixture in the cases considered need to be investigated before any similarity in the distribution of the insitu discharge concentration in the two cases can be assumed.

In the present work, the mean mixture velocity was less than the limit deposit velocity. Therefore, the formation of a bed of particles along the bottom of the pipe should be considered. Depending upon the magnitude of the mean mixture velocity in relation to the limit deposit velocity, a moving bed flow regime or a stationary bed might occur. Motion of the particles observed through a perspex viewing section located in a horizontal section of the test rig showed that in the bottom part of the pipe there was no sign of a stationary bed formation but only a moving bed of particles of almost constant thickness.

Newitt et al reported that for gravel slurries, "regions of high concentration at the bottom of the pipe, corresponding to a moving bed" was noticed.

In view of the manner in which the mixture was transported in the present work and in Ref. 68, and that the difference in the mean discharge concentration in the two cases was small, it was assumed that the insitu discharge concentration distribution given for  $\bar{c}_d=3.77\%$  in Ref. 68 would be used in the present work for  $\bar{c}_d=3.362\%$ . The value of the insitu discharge concentration at various positions in the cross-section of the pipe (position measured as the distance from the bottom of the pipe normalised by the pipe diameter, Y) was obtained from fig. 9 in Ref. 68 for use in the present work.

This only resolves part of the problem since information on insitu slip velocity is also required. In Ref. 68, at  $\bar{c}_d=3.77\%$ , only insitu discharge concentration was given and no information was provided on the distribution of insitu slip velocity. To overcome this problem the following procedures were used.

In Ref. 68 the insitu slip velocity profile for  $\bar{c}_d=10\%$  was given. As was noted by Newitt et al, the insitu discharge concentration "profiles (given for a range of  $\bar{c}_d$ ) were similar" in shape. In view of this similarity, it was assumed that the distribution of the insitu slip velocity would retain the same profile at all values of  $\bar{c}_d$ . There is, definitely, no physical reasoning which can be put forward to justify that similarity of the insitu discharge concentration profiles at different  $\bar{c}_d$  also implies a similarity of the insitu slip velocity distribution profiles. This is an assumption which had to be made. In further persuance of this assumption, the following relation was considered to be valid :-

$$\left[ \frac{\bar{v}_m - v}{\bar{v}_m} \right]_{\bar{c}_d=3.77\%} = \left[ \frac{\bar{v}_m - v}{\bar{v}_m} \right]_{\bar{c}_d=10\%} \quad (3.5)$$

i.e., the insitu velocity lag of the solids with respect



to the mean mixture velocity and normalised by the mean mixture velocity is same at two mean discharge concentrations of 10% and 3.77%. The value of the right hand side of equation 3.5 was calculated from fig. 28B, Ref. 68 for various positions in the cross-section of the pipe and used in the present work for which  $\bar{c}_d$  was equal to 3.362%.

Equation 3.5, thus, provided a basis for calculating the velocity of the particles at the entry to the impeller. More details on this are given in section 5.1.1.2.1.2. Profiles of insitu discharge concentration and insitu normalised velocity lag considered for the analytical part of the present work (corresponding to the experimental condition detailed above, case 1) are given in fig.'s 3.3 and 3.4, respectively.

### 3.3.1.2 Case 2 : Mean Discharge Concentration of 11.653%

Details of the procedures followed and the important aspects pertaining to this set of experiments are given in section 6.7.2; here only the basic informations appropriate to the discussion in this section are given.

The following basic information pertaining to flow conditions of the mixture have been considered.

- (i) Mean mixture velocity,  $\bar{v}_m = 1.497 \text{ m/s}$
- (ii) Limit deposit velocity,  $V_L = 2.456 \text{ m/s}$

In order to be able to estimate an insitu discharge concentration profile appropriate to the conditions of the present work, information provided in Ref. 68 was examined. Profiles of insitu discharge concentration corresponding to a number of mean discharge concentrations are given in this reference, as has been mentioned earlier. One of these distributions correspond to  $\bar{c}_d = 11.55\%$ . To check if this distribution can be considered to be representative of the distribution in the present work, a comparison of the conditions of flow

of the mixtures in two cases was made as follows :-

|                                           | <u>Present Work</u> | <u>Newitt et al's</u> |
|-------------------------------------------|---------------------|-----------------------|
| Mean discharge concentration, $\bar{c}_d$ | 11.653%             | 11.550%               |
| Mean mixture velocity, $\bar{v}_m$ (m/s)  | 1.497               | not available         |
| Limit deposit velocity, $V_L$ (m/s)       | 2.456               | 1.225                 |
| $\bar{v}_m/V_L$                           | 0.609               | not available         |

As the ratio of  $\bar{v}_m/V_L$  cannot be compared in two cases, the conditions of transport of mixture was then examined.

In the present case, the mean mixture velocity was less than the limit deposit velocity. Hence, the formation of a bed in the bottom of the pipe was considered. No stationary deposit of solid particles in the bottom of the pipe was evident when the flow was observed through a perspex viewing section of the test rig. A thick layer of moving particles near the bottom of the pipe was detectable. Also, in Ref. 68 the existence of a moving bed for transport of gravel slurries at all values of  $\bar{c}_d$  was mentioned. In view of this similarity in the manner in which the mixture was transported in both cases and that the value of  $\bar{c}_d$  in two cases were very close to each other, it was assumed that the insitu discharge concentration profile corresponding to  $\bar{c}_d=11.653\%$  in the present work was similar to that reported in Ref. 68 for  $\bar{c}_d=11.55\%$ .

Also, by following the arguments made in the preceeding section, it is assumed that the normalised insitu velocity lag between the solids and the mixture is given by the following relationship :-

$$\left[ \frac{\bar{v}_m - v}{\bar{v}_m} \right]_{\bar{c}_d=11.55\%} = \left[ \frac{\bar{v}_m - v}{\bar{v}_m} \right]_{\bar{c}_d=10\%} \quad (3.6)$$

At various positions in the cross-section of the pipe, the value of insitu discharge concentration was obtained from fig. 9, Ref. 68. This is shown in fig. 3.3. The right hand side of equation 3.6 has already been calculated for case 1, (section 3.3.1.1) and used in the present case for  $\bar{c}_d=11.653\%$ .

### 3.3.2 Discretization of the Concentration Profile

In section 3.3.1.1 and 3.3.1.2 the insitu concentration distributions assumed for the present work are given. In this section the methods to find the distribution of the particles in the cross-section of the pipe by knowing the insitu discharge concentration profile are given. The need for providing this distribution for the calculation of the motion of the particles in an impeller has been discussed in section 3.1 and hence will not be repeated here. The following procedures are involved.

The cross-section of the pipe is divided into a number of elements. To achieve this the section is first divided into a number of annulii of equal radial thickness. Each annulus is divided into a number of elements in such a way that the area of each element in the cross-section of the pipe is constant. The following relationship is used :-

$$n_{i+1} = n_i \frac{a_{i+1}}{a_i} \quad (3.7a)$$

$$n_{i+1} = n_i \frac{r_{i+2}^2 - r_{i+1}^2}{r_{i+1}^2 - r_i^2} \quad (3.7b)$$

where,  $n_{i+1}$  is the number of elements in (i+1)th annulus whose outer radius is  $r_{i+2}$  and inner radius is  $r_{i+1}$ , see fig. 3.5.

The next step is to find the space occupied by the particles in each elementary area. To determine this it is necessary to know the value of the insitu transport concentration at an elementary area. The difference between insitu transport and insitu discharge concentration in most part of the cross-section of the pipe is negligibly small (Ref. 68) and only near the bottom of the pipe some difference existed. In the present work, this difference is ignored and the value of the insitu transport concentration is taken to be equal to that of the insitu discharge concentration. As the area of each element is small, it is assumed that the value of this concentration at the centroid of each element applies all over the element. The centroid of each element is determined from the following relationship (for detail see Appendix C) :-

$$r_c = \frac{4}{3} \frac{\sin \frac{\delta}{2}}{\delta} \left( r_i + r_{i+1} - \frac{1}{\frac{1}{r_i} + \frac{1}{r_{i+1}}} \right) \quad (3.8)$$

where :-

$r_c$  = distance measured from the axis of the pipe to the centroid of an element located in (i+1)th annulus, fig. 3.6

$\delta$  = angle subtended by the element at the centre of the pipe

For  $\delta \leq 10$  deg., equation 3.8 reduces to the following form :-

$$r_c = \frac{2}{3} \left( r_i + r_{i+1} - \frac{1}{\frac{1}{r_i} + \frac{1}{r_{i+1}}} \right) \quad (3.9)$$

The value of the concentration at the centroid is calculated by knowing the vertical distance of the centroid from the bottom of the pipe (normalised by the diameter of the pipe) and from the best fit curve of the

insitu discharge concentration (see section 5.1.1.2.1.3).

The number of particles,  $n_e$  passing through each element is obtained from the product of the concentration at the centroid of the element and the area of the element divided by the projected area of a particle. The projected area of a particle is calculated on the basis of the characteristic size of the particle. The following relationship is used to find  $n_e$  :-

$$n_e = \frac{4 c_d a_i'}{\pi d_{50}^2} \quad (3.10)$$

Out of  $n_e$  number of particles passing through an element, only the motion of a single particle released from the centroid of the element is traced inside an impeller. The motion of this particle is considered representative of all the particles passing through that element. In section 5.1.1.2.1.3 the result of this discretization process (discretizing a concentration profile) has been discussed.

## CHAPTER 4

### A MODEL TO PREDICT POTENTIAL EROSION AREAS ON A SURFACE DUE TO PARTICULATE IMPACT

#### 4.1 Introduction

The problem of surface damage caused by the impingement of solid particles entrained in a fluid stream is widely known. According to Finnie (Ref. 89), an understanding of erosion may be divided into two major parts :-

- (i) "the first part involves a determination from the fluid flow conditions of the number, direction and velocity of the particles striking the surface. With such information available,
- (ii) the second part of the problem is a calculation of the amount of surface material removed".

Earlier in chapter 2 the method to obtain information on the motion of particles inside an impeller has been given. The dynamic characteristics of the particles as they strike a wetted segment of the impeller may be known by following the procedures as laid down in that chapter.

In this chapter detail of a model that is developed to predict erosion prone areas on a surface will be given. This model uses information on the dynamic characteristics of the particles as the particles impact on a surface. The manner in which a surface is eroded by solid particles is an important part in any erosion study. Hence, this subject will be discussed in some detail. The discussion is made in the light of previous work and forms the basis of the model developed.

All the previous work have been done in strictly

dry conditions. Although the condition inside an impeller is wet, it is thought that the basic mechanisms in which material is removed from a surface are similar whether or not material is removed in a wet or dry condition.

#### 4.2 Analysis of Erosion due to Solid Particles

In studying the manner in which impacting solid particles erode a surface, the usual procedure is to classify the behaviour of the surface as either ideally ductile or ideally brittle. The ductile type, typical of most metals, is characterised by maximum erosion occurring at an impingement angle between zero and 90 degrees, usually 20-30 degrees (Ref. 90). The brittle type, typical of ceramics, is characterised by the erosion rate increasing with increasing impingement angle to a maximum rate at 90 degrees (Ref. 90), fig. 4.1. By classifying the materials, models of the particle-surface interaction may then be postulated and theoretical predictions of erosion resistance made. The influence of basic material properties on erosion resistance is often evident from these predictions.

##### 4.2.1 Ductile Erosion

Materials are characterised as ductile when their erosion is associated with large plastic strains.

The first fundamental analysis of a ductile erosion process was given by Finnie (Ref. 89). The analysis considered the motion of a solid particle cutting into a target material. The erosion of a ductile material was predicted as a function of impact angle and velocity of a solid particle and material constants.

The approach in Finnie's analysis was to write the equation of motion of a single particle. It was assumed that the target material was perfectly elastic and that the resistance to particle motion by the surface of the

material was equal to the plastic flow stress times the projected area in the direction of motion. The volume of material removed from the surface was the amount swept out by the particle as it moved into and out of the surface. The whole erosion process was considered to be due to pure cutting of the material. It was further assumed that the particle was sharp and rigid, only one face of it was in touch with the surface and the particle stopped cutting when either the motion of the particle in the direction of the surface had ceased or the particle had left the surface, see fig. 4.2. The following expression for the volume of material removed by a single abrasive particle was given :-

$$V_C = k_1 \frac{1}{p_f} m_p v^2 f_8(\alpha) \quad (4.1a)$$

where,

$$k_1 = \frac{l_t}{y_t} \frac{F_n}{F_t} \quad (4.1b)$$

At the angle of maximum erosion, equation 4.1a became :-

$$V_{C_{\max}} = 0.075 \frac{m_p}{p_f} \frac{v^2}{2} \quad (4.1c)$$

Finnie compared the predicted loss of volume given by equation 4.1a with data obtained from tests using a large number of grains. It was noticed that the predicted variation of material removal with angle for a single grain was similar to the experimental data for erosion by large number of grains. The agreement was poor at high impact angles ( $\alpha \geq 45^\circ$ ).

Finnie's theoretical work provided the following



information :-

- (i) At low impact angles ( $\alpha \leq 45^\circ$ ), the ductile erosion may be considered as due to cutting action of the particles.
- (ii) In cutting wear, material removal by plastic flow action can describe observed experimental data. The plastic flow stress of the material may be considered as a measure of its resistivity against ductile erosion, since from equation 4.1c,

$$V_{C_{\max}} \propto \frac{1}{P_f} \quad (4.1d)$$

- (iii) At higher angles ( $\alpha > 45^\circ$ ), the failure of the ductile material cannot solely be attributed to cutting action; some other mechanism need to be considered.

Another useful contribution to the subject was made by Bitter (Ref. 91). His theory was not specifically addressed to ductile materials but covered both brittle and ductile solids with the same equations. He considered erosion to consist of two simultaneous processes, cutting wear caused by the component of particle motion parallel to the surface, and one of shock or deformation wear caused by the normal component. Hence, for ductile metals at low angles, cutting wear would dominate; at high angles deformation wear would become significant. His postulation of ductile mode of erosion at low impact angle was similar to that of Finnie (Ref. 89). It is the deformation wear which accounts for the erosion of ductile materials at angle of attack normal to the surface and which is not accounted for in Finnie's (Ref. 89) analysis.

The fundamental difference in the two approaches may be somewhat explained by referring to the work of

De Haller (Ref. 92) as cited in Ref. 91. De Haller made microscopic examination of the surfaces damaged by solid particle impingement at both shallow and large angles. In the first case a less deformed scratched surface was observed (result of cutting action, Ref. 89). In the second case a flattened, highly deformed surface was noticed, although the same materials were used on both occasions. This suggested that in the same material solid particles could cause cutting wear or deformation wear depending upon the angle of attack.

In building his deformation wear model, Bitter (Ref. 91) considered the motion of a spherical particle which attacked a flat surface at a normal incidence. As a result of the impact, stress concentrations were set up in the surface. From Davies's (Ref. 93) work it was showed that the maximum concentration of stress occurred at the centre of the contact area and at a depth of about half the radius of the projection of the contact area on the surface. Assuming that as a result of the collision the elastic limit of the surface was exceeded, Bitter (Ref. 91) argued that plastic deformation occurred at the position of maximum stress. When the stress became equal to the strength of the material, the surface could no longer be plastically deformed. With increasing load a crack formed within the deformed surface layer, parallel to the contact area, resulting in the removal of a flake from the surface. Based on the plastic-elastic contact stress calculations and balances between the energy needed to deform the surface elastically and the kinetic energy of the colliding particle, the following expression for the material loss due to many particles of the same material hitting the surface normally or at a small angle was derived :-

$$V_D = 1/2 M(v \sin \alpha - v_e)^2 / \phi_D \quad (4.2)$$

The deformation wear factor  $\phi_D$  was defined as the amount of energy needed to remove one unit volume of material by deformation wear. The model, therefore, suggested that the normal component of kinetic energy of the impacting particles was absorbed in the target surface and accounted for deformation wear. Equation 4.2 was valid for  $v \sin \alpha \geq v_e$ . At lower values of  $v \sin \alpha$  no deformation wear occurred as the collision was purely elastic. The model, therefore, provided the idea of a limiting normal component of particle velocity below which no erosion took place. It was shown that this limiting value was dependent on particle shape.

To explain cutting wear, Bitter reasoned that the kinetic energy of the particles parallel to the surface was associated with cutting wear. For large angles of attack, the particles came to rest on the surface and the total parallel component of kinetic energy contributed to cutting wear. For small angles of attack, however, the particles swept into the surface and finally left with a residual component of kinetic energy in a direction parallel to the surface. In both cases the particles were pressed out of the body by the elastic reaction forces originating from elastically and plastically deformed segments of the surface.

The following relations were given for material loss due to cutting wear :-

for  $\alpha \leq \alpha_0$ ,

$$V_C = \frac{2 M k_2 (v \sin \alpha - v_e)^2}{\sqrt{v \sin \alpha}} \left[ v \cos \alpha - \frac{k_2 (v \sin \alpha - v_e)^2}{\sqrt{v \sin \alpha}} \phi_C \right]$$

--- (4.3a)

for  $\alpha \geq \alpha_0$ ,

$$V_C = \frac{M \left[ v^2 \cos^2 \alpha - k_3 \sqrt{(v \sin \alpha - v_e)^3} \right]}{2 \phi_C} \quad (4.3b)$$

$\alpha_0$  was defined as the angle of impact at which the horizontal component of the velocity of the particle had just become zero when the particle left the surface. Constants  $k_2$  and  $k_3$  defined the hardness of the eroded surface and the plastic-elastic behaviour of the particle and the surface, respectively. The cutting wear factor,  $\phi_C$ , was defined as the amount of energy needed to remove a unit volume of material by cutting wear. Evidently, at  $\alpha = \alpha_0$ , equations 4.3a and 4.3b both predicted the same material loss. Total erosion was, therefore, given as the summation of material loss due to deformation and cutting, as given by equations 4.2 and 4.3a and 4.3b.

Neilson and Gilchrist (Ref. 94) agreed with Bitter's (Ref. 91) idea that the kinetic energy corresponding to the component of particle velocity normal to the surface was responsible for deformation wear while the cutting wear was due to the kinetic energy corresponding to the component of particle velocity parallel to the surface. Equations given were of the following form :-

for  $\alpha \leq \alpha_0$ ,

$$V_T = \frac{M (v^2 \cos^2 \alpha - v_s^2)}{2 \phi_C} + \frac{M (v \sin \alpha - v_e)^2}{2 \phi_D} \quad (4.4a)$$

for  $\alpha \geq \alpha_0$ ,

$$V_T = \frac{M v^2 \cos^2 \alpha}{2 \phi_C} + \frac{M (v \sin \alpha - v_e)^2}{2 \phi_D} \quad (4.4b)$$

In some ways these equations may be considered as modifications of Bitter's (Ref. 91) equations (equations 4.3a and 4.3b).

Another approach to particle induced erosion from the materials science point of view was made by Smeltzer and his co-workers (Ref. 90). Finnie's (Ref. 89) analytical model (equation 4.1) suggested that the erosive strength of a material was inversely proportional to the plastic flow strength of the surface. Hence, appreciable difference in yield (minimum flow) strengths between the annealed and fully treated conditions for the surface should produce correspondingly marked differences in the measured erosion factors. Smeltzer et (Ref. 90) carried out experiments with two steels and Aluminium alloy and found that in the fully treated conditions these materials had room temperature strengths approximately 3-4 times higher than in the annealed condition. Under the circumstances, Finnie's (Ref. 89) model predicted the erosion of the weaker annealed surface 3-5 times more than the heat treated surface. However, the experiment of Smeltzer et al showed that the differences between the annealed and fully heat treated conditions were less than 5%. When the erosion data were related to the melting temperature of the target materials it was noticed that erosion loss decreased systematically with increasing melting temperature. Smeltzer et al suggested that local melting during impact and attachment of surface material to the impacting particles produced erosion. Some support to this viewpoint was given in Ref. 95.

Another model for erosion of ductile materials at normal angle of attack was suggested by Sheldon and Kanhere (Ref. 96). While at this angle of impact Bitter (Ref. 91) postulated a deformation theory based on subsurface cracking of the deformed surface, Sheldon and Kanhere took a different view of the problem. The basic hypothesis of their work was that the material removed by an impacting particle had flowed out around the sides of the cavity by the advancing particle until the displaced material was strained sufficiently to break off. The amount of material removed, therefore, bore some relation to the depth of penetration of the particle. The resistance of the material was calculated from its static hardness value. Based on experimental data on relative penetration depth of the particle (defined as the ratio of penetration depth to the radius of the particle) they proposed the following relationship for material removal per particle,  $V_f$  :-

$$V_f \propto d^3 v^3 \sqrt{(\rho_p H_v)^3} \quad (4.5)$$

Single particles (roughly 2500 $\mu$ ) were forced to impinge on an Aluminium surface. The material removal action was shown to have the same characteristics as multiple-particle erosion of surfaces by particles larger than 100 $\mu$ . Expression 4.5 based on the impact of a single spherical particle therefore gave an estimate of the multiparticle erosion of a surface. Another important result was that a velocity exponent of three was predicted rather than the two given by previous theories (Ref.'s 89, 91). This velocity exponent agreed more closely with the experimental results of the authors.

Another important contribution to the understanding of erosion of ductile materials was made by Hutchings (Ref. 97). Based on single-impact experiments,

Hutchings identified three possible mechanisms for removal of material at oblique impact angles -- plowing, cutting and plowing, and cutting only. The result of this three types of erosion is given in fig. 4.3. In the plowing type erosion was due to the action of a spherical particle or the rounded surface of an irregular particle causing the material to be extruded above the surface, fig. 4.3a. The lip at the exit end of the crater was pushed up in front of the particle and folded over onto the undisturbed surface of the metal. Erosion occurred at the more vulnerable raised end where material was removed by subsequent particles. In the case of impacts by square plate shaped particles, two quite different types of erosion behaviour existed. When the surface was struck by the particles edge-on, a rake angle was defined (see fig. 4.3 for definition). At a given oblique angle, a critical rake angle existed. For particles impacting at rake angles less than the critical value, it was noticed that the particle rebounded from the surface with appreciable rotational velocity in the forward direction. The indentation formed by this type of impact showed a triangular shape, fig. 4.3. At the exit end of the indentation, the material was pushed to form a large lip. This lip was removed by subsequent impacts. On the other hand at rake angles greater than the critical value, the particle on an impact showed backward rotation resulting in a perfect machining action (cutting out and completely removing the chip of metal). Hutchings concluded that the critical rake angle was normally such that impacts of randomly oriented plates at an oblique angle mostly caused combined cutting and plowing rather than pure cutting only, as envisaged in the theories of Finnie (Ref. 89), Bitter (Ref. 91) and Neilson et al (Ref. 94).

From the brief review made above it is evident that a variety of opinions exists on the mechanism or mechanisms of material removal in ductile erosion. Theories based on cutting action of the particles impacting at low

angles predict maximum erosion correctly (Ref. 89). Material science approach in explaining ductile erosion by considering the melting temperature of the surface (Ref. 90) has not been conclusively investigated. At normal or high angles of impact, erosion process involving an accumulation of plastic deformation in the surface of the target is still the dominant theory. An important addition to the theory will be the number of plastic strain cycles needed to remove a wear fragment (see e.g., Ref. 98). It was suggested in Ref. 99 that a low cycle fatigue phenomenon could be responsible for subsurface cracking at high impact angles, observed in two types of steel at a depth of approximately  $20\mu$  or less.

In the following section the factors that may affect erosion in ductile materials are briefly discussed.

#### 4.2.1.1 Parameters of Significance in Ductile Erosion

The following parameters are considered :-

- (i) Angle of impact
- (ii) Velocity of particle at impact
- (iii) Rotation of particle at impact
- (iv) Size of particle
- (v) Properties of the surface
- (vi) Strength of particle
- (vii) Concentration of particles

##### 4.2.1.1.1 Angle of Impact

Important points are :-

- (i) At low impact angle erosion, metal removal occurs by a cutting or displacing process. Theories developed (Ref.'s 89, 91) on the basis of this mechanism show that the motion of the particle during the cutting process is significant. This motion depends on the velocity of the particle parallel to the surface. For a



given system, the angle of impact, therefore, is an important factor.

- (ii) For each particle-surface combination, a critical angle exists at which the material removal is maximum (Ref. 89) and this has been proved experimentally (Ref.'s 89, 90).
- (iii) At normal or high angle of impact, the material removal mechanism is different from that at shallow angles. Accumulated plastic deformation, work hardening and cracking of the subsurface layer normally characterise this type of erosion (Ref.'s 98, 99).
- (iv) At oblique angles, a combination of cutting and deformation wear has also been postulated (Ref.'s 91, 94).

#### 4.2.1.1.2 Velocity of Particle at Impact

Important points are :-

- (i) Most theories are developed by considering a balance between the kinetic energy of the particle and the work expended in cutting or deformation of the surface. Theories predict volume loss to be proportional to the square of velocity. The agreement with experiment in this respect was reasonable.
- (ii) It was also noted that for both spherical and angular particles, the angle for maximum erosion was not influenced by the velocity of the particle (Ref. 89).
- (iii) A threshold particle velocity was postulated by some (Ref.'s 91, 94) below which the stresses induced by an impact are elastic and no damage takes place.

- (iv) The velocity of the particles may have an indirect effect if the particles strike a surface at a high velocity and break. In such a situation Goodwin et al (Ref. 100) noted that a two tier erosion process might take place in which the particles would produce some erosion and the fragments, some additional damage. On this basis, the erosion at normal or near normal incidence was attributed to the radial motion of the fragmented particles in the surface.
- (v) Both Sheldon et al (Ref. 96) and Hutchings (Ref. 98) showed analytically that at normal incidence, volume removal varies with a cubic power of velocity. No fragmentation of the particles was considered. This value of velocity exponent compares quite well with the observed value as reported in Ref. 101.
- (vi) At low angle of impact, the value of the velocity exponent lies in the range of 2.3 to 2.4, Ref. 102.

#### 4.2.1.1.3 Rotation of Particle at Impact

The effects of rotation of particles during an impact are as follows :-

- (i) If the particle is rotating when it strikes a surface, then the edge of the particle in contact with the surface will have a velocity component parallel to the surface. This means that at normal incidence and depending on the magnitude of rotation, the particle may induce a cutting type erosion. It was shown in Ref. 104 that as a result of the rotation of the particle maximum erosion occurred at an angle which was greater than that produced with zero rotation.

- (ii) Hutchings (Ref. 97) noted from experimental observations that the direction of rotation of the particle dictated the cutting process in ductile erosion at oblique impacts; a cutting and plowing mechanism was generally associated with the forward rotation of the particle while a backward rotation caused pure cutting with complete removal of material without any plowing.

#### 4.2.1.1.4 Size of Particle

Effects of particle size on erosion of a surface are as follows :-

- (i) The theoretical models (equations 4.1a, 4.2, 4.3a and 4.3b) suggest that volume removal is proportional to the total mass of the eroding particles and the size of a particle does not influence the volume removal.
- (ii) However, the experimental observations suggest that the volume removed by a given mass of abrasive grains is independent of particle size for particles larger than 100 $\mu$  (Ref. 105). For particles below this size, the erosion process becomes less and less efficient as the particle size is decreased. A variety of opinions exists as regards this size effect. Some argue that fragmentation of larger particles lead to more efficient cutting than with smaller particles (Ref. 105). A more fundamental answer to this question is given in Ref. 106. Indentation tests made with tiny needles and examined under scanning electron microscope revealed that a threefold increase in hardness relative to the conventional tests (Ref. 106) occurred. In other words, a true physical size effect was possible. However, size effect is still very much an

intriguing aspect of erosion and a critical examination of the views expressed until recently is made in Ref. 107.

#### 4.2.1.1.5 Properties of the Surface

Important points are :-

- (i) In ductile erosion, the mechanism of metal removal by the particle is considered analogous to a single tool metal cutting operation (micromilling), Ref. 89. It is shown in one theory (Ref. 89) that resistance of a surface to erosion is inversely proportional to the plastic flow stress of the surface. Drawing on metal cutting experience, it was shown in Ref. 108 that this stress may be considered to be equal to Vickers hardness when cutting annealed metals.

#### 4.2.1.1.6 Strength of Particles

Important points are :-

- (i) When the hardness of the abrasive is greater than that of the surface, it is unlikely that variation in the abrasive hardness will be important.
- (ii) For fragile particles, it is shown in Ref. 100 that thresholds on particle size and velocity exist. If the particles incur a substantial degree of fragmentation, then erosion is enhanced which "may be physically caused by secondary erosion due to the particle fragments tending to scour the surface by a radial wash which removes small chips of material", Ref. 100. Experimental evidence reported in Ref. 109 lend support to this viewpoint.

#### 4.2.1.1.7 Concentration of Particles

Considering the weight impacting unit area as a measure of particle concentration, it was noticed that concentration has some effect on erosion (Ref.'s 90, 105), increased erosion with decreased concentration was noticed. No explanation has been reported so far.

#### 4.2.2 Brittle Erosion

In this type of erosion no plastic deformation is present. Material is removed by the propagation and intersection of cracks ahead of and around the particle. Materials exhibiting this type of erosion are known as brittle materials. A brittle erosion, therefore, takes place in two stages :-

- (i) initiation of a fracture
- (ii) propagation of the fracture resulting in the removal of the material

While some analytical model may be developed to predict initiation of fracture in brittle erosion, estimation of material removed by eroding particles in this type of erosion is quite complex. An energy approach which was proved quite fruitful in developing analytical models for ductile erosion (e. g., Ref.'s 89 and 91), does not hold much premise in brittle erosion. It has been shown from crushing studies on brittle materials that "... such an approach is not likely to prove fruitful because the essential energy requirement to produce a fresh surface is found to be a small and unpredictable factor of the total energy expended. Hence some accounting can be made for the balance of the energy expended, and unless the fragment size and surface area can be predicted in advance, it would be unrealistic to attempt to predict volume removal on an energy basis", Ref. 110. An alternative method is to find the depth to which an eroding particle penetrates a surface and then from a knowledge of the conditions in which the surface cracks ,

an estimate of the material removed may be made. In the following paragraphs two works will be briefly discussed. One concerns initiation of a crack and the other on the estimation of metal removal rate in a brittle erosion.

To predict initiation of a crack in a brittle material, Finnie (Ref. 89) proposed a simple model using the following assumptions :-

- (i) the abrasive particle was an inelastic sphere
- (ii) it hit the surface perpendicularly
- (iii) the surface remained elastic until fracture occurred

Maximum tensile stress in the surface occurred in a radial direction around the periphery of the contact area. This was given as follows :-

$$\sigma_r = 0.187 \rho_p^{1/5} v^{1/5} (1 - 2\mu_s) \left\{ (1 - \mu_p^2)/E_p + (1 - \mu_s^2)/E_s \right\}^{-4/5}$$

--- (4.6)

As the maximum stress occurred in the radial direction around the periphery of the contact area, the surface cracked around this periphery forming a ring crack. The ring flared out below the surface in the form of a mantle of a cone, fig. 4.4. Material was removed when these cones began to intersect with one another. Evidently, from equation 4.6, erosion resistance of brittle materials increased with decreasing elastic modulus and increasing Poisson's ratio.

To formulate a material removal rate in brittle erosion, Sheldon et al (Ref. 110) noted that for brittle materials due to presence of volume flaws, fracture stress was not constant but depended on the stressed volume. As such it was possible to describe fracture stress for a brittle material only in a statistical sense. It was assumed that an abrasive particle upon impacting on a plane surface at normal incidence caused an indentation of depth  $h_i$  which

corresponded to the point where surface had just fractured. Then by assuming a flaw distribution, an approximation for the surface area  $a_c$  of the cracked material was derived. It was further assumed that the volume of material removed per particle was proportional to the product  $a_c h_i$ . The following relation for the material removed per particle was given :-

$$V_B = k_4 (d/2)^a v^b \quad (4.7)$$

where, exponents  $a$  and  $b$  depended on the shape of the particle and the flaw distribution in the surface; quantity  $k_4$  involved material constants. The characteristic value of the exponent  $a$  was found to be 3, if the particle was angular and 3.6, if the particle was spherical. The value of the exponent  $b$  was found to be 2.4. Reasonable agreement with the experimental results was reported. Unlike a ductile erosion (item i, section 4.2.1.1.4) the dependence of material removal on particle size was clearly evident in equation 4.7. The velocity dependence was almost similar to that of a ductile erosion at low angle of impact, see item vi, section 4.2.1.1.2.

Because of the complexity of the problem, analytical models in brittle erosion are very rare. The tendency in brittle erosion studies, therefore, is to identify material erosion characteristics from experimentation. It should also be mentioned here that brittle materials also exhibit certain amount of ductile behaviour as was shown by Sheldon et al (Ref. 104), the ductile behaviour being inferred from both the rippled appearance of the eroded surface and the variation of volume removal with angle of impingement.

In the preceeding paragraphs, the mechanics of material removal by impinging solid particles have been discussed in detail. It is shown that the dynamic

characteristics of the particle play a very significant part in the material removal process. In the following section it is shown that these characteristics may form a basis for identifying erosionprone areas in an impeller subject to particulate erosion.

#### 4.3            Erosion Study with Paint as a Surface Coat

The impeller used in the present investigation was an all-metal Ni-hard construction (minimum hardness on the surface 550 HB, according to manufacturer). A fine pea gravel slurry was used for the experiments (see section 6.1). A running period of more than fifty hours would be required for this type of impeller before any noticeable wear pattern emerged on the blade surfaces (Ref. 111). The solids in the mixture could undergo attrition if used for such a long period of time as they were continuously circulated in the test rig (for details of the operation of the system, see section 6.5). Attrition of the solid particles was not considered in the analytical part of the present work. Hence, some compromise was required so that during the course of an experiment the particles would undergo little attrition while at the same time areas on a blade surface which were prone to erosion due to impact of particles could be identified. This was done by applying paints of different colour onto a blade surface. By observing areas on the blade surface where one or more coats of the paint was removed by the solid particles, an estimate of the erosion prone areas in a blade surface was made. It was assumed that the way in which the particles removed paint from a surface was similar to the mechanism in which a metallic surface was eroded by solid particles. An analytical expression to estimate the rate of removal of the paint from a surface was then considered in line with those discussed in the preceeding section, for a bare unpainted surface.

There is not much information available in the





$v_e$  could be known from experiments only (Ref. 94).

The calculation of the trajectory of solid particles inside the impeller suggested that the maximum angle of impact of a particle on a blade surface was 53 degrees (item vii, section 5.1.1.3.1). As the deformation wear occurred at normal or near normal incidence of particles on the surface, term II in equations 4.4a and 4.4b was dropped out.

Estimation of angle  $\alpha_0$  at which the particle left the surface at zero tangential velocity required experimental data (Ref. 94). The value of  $\alpha_0$  was close to  $\alpha_{max}$ , the angle at which maximum erosion occurred (Ref. 94). It was reported in Ref. 90 that for a ductile type erosion the angle for maximum erosion was in the range of 20-30 degrees. For the paint being considered here, it was assumed that the value of  $\alpha_{max}$  was in the same range. In the present analysis this value was taken to be 25 degrees. This assumption did not drastically affect the volume loss estimation as given by equation 4.4a or 4.4b. This was because the calculation of particle trajectories showed that after a few initial impacts, particles hit a blade surface at an angle of incidence lying in the range of 3-15 degrees. For these impacts, the criterion  $\alpha \leq \alpha_0$  was considered satisfied. For the initial impacts where the angle of incidence was above 25 degrees, the criterion  $\alpha \geq \alpha_0$  was assumed to be satisfied.

Estimation of  $\phi_C$  for the paint being considered was not possible without any experimental data (Ref. 94). To overcome this problem instead of an absolute estimate of volume loss,  $V_T$ , a relative estimate of volume loss was considered. This was done as follows. Both sides of equations 4.4a and 4.4b were multiplied by  $\phi_C$ . Equations 4.4a and 4.4b, therefore, became,

$$V_T \phi_C = 1/2 M (v^2 \cos^2 \alpha - v_s^2) \text{ for } \alpha \leq \alpha_0 \quad (4.8a)$$

$$V_T \phi_C = 1/2 M v^2 \cos^2 \alpha \quad \text{for } \alpha \geq \alpha_0 \quad (4.8b)$$

$\phi_C$  was a function of the particle type and the target surface and independent of the dynamic characteristics of the particle. Hence, in the present analysis the value of  $\phi_C$  was taken to be constant for the entire erosion process as neither the particles nor the surface changed in the present work. The term  $V_T \phi_C$  might be considered as the volume removed per unit of  $\phi_C$ . In this way equations 4.8a and 4.8b gave an estimate of the relative erosion of the surface.

The next step was to calculate  $v_s$ , the residual parallel component of particle velocity at low angle of impact. The magnitude of  $v_s$  depended on the velocity and the angle at which a particle reached a surface, the resistance force acting on the particle as the particle cut into the surface. Hence, apart from the initial dynamic characteristics of the particle, the magnitude of  $v_s$  was also dependent on the material constants of the surface. In section 2.7.2, a method to calculate the tangential component of the velocity of the particle as the particle was about to leave a surface after an impact was given. The method did not consider any penetration of the particle into the surface. In the present analysis  $v_s$  was calculated according to the method given in section 2.7.2. It is admitted that the actual  $v_s$  may be less than the calculated one.

The terms  $M$  and  $v$  in equations 4.8a and 4.8b represented the mass and the velocity, respectively, of a particle flux. It was assumed that all the particles in the flux were incident on the surface with the same velocity,  $v$ . The velocity of a particle impacting on a surface was calculated analytically (see section 2.8). The velocity of the particles was different from one another, the variation in velocity depended upon the

position on the blade surface where a particle hit the surface. In the present analysis,  $M$  and  $v$  referred to every individual particle.

From equations 4.8a and 4.8b, it was, therefore, possible to obtain a relative measure of the loss of volume from a surface defined as  $V_T \phi_C$ . From knowledge of  $V_T \phi_C$  at different parts of a surface, an estimate of the erosion levels at the surface was made. In the following paragraphs it is shown how this idea was exploited to classify erosionprone areas of a surface.

#### 43.1 Classification of a Surface into Erosion Prone Areas

In the preceeding paragraphs it was shown that a relative measure of the volume loss from a painted surface due to each impacting particle was possible using equation 4.8a or 4.8b. This was given as  $V_T \phi_C$ . A value of  $V_T \phi_C$  was associated with every impact. As particles hit different parts of a surface with different dynamic characteristics, the value of  $V_T \phi_C$  calculated at corresponding parts were also different. This observation meant that different levels of erosion in a surface could be considered.

Let the surface be divided into a number of elements. Let the area of an element be  $A_i$ . Corresponding to each impact of a particle within an element the value of  $V_T \phi_C$  was calculated. Let  $(V_T \phi_C)_i$  be the summation of  $V_T \phi_C$  calculated at element  $i$  for a number of impacts within this element. Particles caused removal of material from this element. Let the loss of volume in the element be  $V_{T_i}$ . By assuming that the area of an element was small, the following relation was written :-

$$V_{T_i} = A_i \ell_i \quad (4.9)$$

where,  $\ell_i$  was the depth of the crater produced in  $i$ th element due to loss of volume caused by impacting particles. At some other element,  $j$ , if the depth of crater was  $\ell_j$ , then :-

$$\ell_j/\ell_i = V_{T_j}/V_{T_i} \cdot A_i/A_j \quad (4.10)$$

and using equation 4.8 :-

$$\ell_j/\ell_i = \{(V_{T_j} \phi_C)/(V_{T_i} \phi_C)\} A_i/A_j \quad (4.11)$$

The novelty of equation 4.11 was that it provided a basis for comparing the depth of the craters formed in two different elements. If the dynamic characteristics of the particles impacting in these elements were known, then from equation 4.8 and 4.11, the ratio of the depth of the craters in the elements could be estimated. Additionally, if the depth of the crater in one element was known, that in the other element could be calculated from equation 4.11.

It was mentioned in section 4.3 that a number of coats of paint of different colour was applied to blade surfaces of the impeller used in the present work. If  $\delta\ell_T$  was the total thickness of the paint layer on the surface and  $n_C$  was the number of coats of paint, then assuming that the thickness of each coat of paint was equal to one another :-

$$\Delta\ell = \delta\ell_T/n_C \quad (4.12)$$

where,  $\Delta\ell$  was the thickness of a coat of paint. The depth of crater of an element was in the range of 0 to  $\delta\ell_T$ . An element having a crater of zero depth implied that there was no erosion in that element.

A linear classification of the erosion of different elements in the painted surface was then made. An element in which the depth of crater,  $\ell$ , did not exceed  $\Delta\ell$ , the erosion level in that element was designated as wear class 1. In an element where bare metal was exposed at the crater, erosion level in that element was designated as wear class  $(n_c + 1)$ . The classification therefore was as follows :-

$$\left. \begin{array}{ll} \ell < \Delta\ell & \text{wear class : 1} \\ \ell < 2 \Delta\ell & \text{wear class : 2} \\ \text{---} & \text{---} \\ \ell < n_c \Delta\ell & \text{wear class : } n_c \\ \ell = \delta\ell_T & \text{wear class : } n_c + 1 \end{array} \right\} \quad (4.13)$$

In order that the proposed classification procedure proved effective, the absolute value of the crater depth at the elements had to be known. The only way to achieve this was to know the crater depth in at least one element in the surface. The crater depth in any other element then was obtained from equation 4.11.

If in an element on the surface the erosion was such that the bare metal underneath the paint was exposed, the depth of the crater in that element was  $\delta\ell_T$ . From the dynamic characteristics of the particles impacting within this element, the relative volume loss from the element was calculated using equation 4.8a or 4.8b. The depth of crater in any other element was obtained by calculating relative volume loss from equation 4.8a or 4.8b and comparing this loss with that of the element where bare metal was exposed at the crater. Equation 4.11 was used. Once the depth of the crater was known, the expression 4.13 was used to designate a wear class to the element.

Since the maximum depth of a crater was the thickness of the painted layer, this classification procedure did not consider any impact that took place in the surface once the bare metal had been exposed at an element. In reality, it was highly unlikely that the bare metal was exposed in more than one places in the blade surface at the same instant. Hence, maximum wear class in this simulated wear pattern occurred in only one element. Erosion level designated as wear class  $n_c$  or less could occur in a number of elements of the surface. This was because the variation in the depth of the craters in such cases was anywhere within  $\Delta l$ . In contrast, erosion level designated as  $(n_c+1)$  implied a depth of crater of  $\delta l_T$ , precisely.

An important step of this classification procedure was to divide the painted surface into a number of elements. This was by generating a mesh of elements on the surface (see section 5.1.1.4 for details). To find area of an element in the mesh, each element was divided into a number of triangles. Area of each triangle was then calculated from the following relationship :-

$$A_t = \text{abs.} \quad \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \quad (4.14)$$

where,  $x_m, y_m, m=1,2,3$  was the coordinate of the vertices of the triangle defined in a  $x,y$  coordinate frame.

## CHAPTER 5

### COMPUTER SIMULATION OF PARTICLE TRAJECTORY AND EROSION

#### PATTERN ON BLADE SURFACES OF AN IMPELLER

##### 5.1 Introduction

The various forces which act on the solid particles passing through an impeller have been discussed in chapter 2 and the governing equations of the motion of particles are given. The particles used in the present work, their characteristic size and specific gravity, and means to determine their position and dynamic characteristics at the inlet section of the computed flow field of the impeller are given in chapter 3. The task, therefore, is set to observe the motion of these particles as they impact on blade surfaces while passing through the impeller. An indication of the erosion prone areas on the surface will then be given on the basis of the erosion model proposed in chapter 4.

The problem is studied for two mean discharge concentrations of solids of 3.362% and 11.653%. The concentrations are the same with the ones studied experimentally (chapter 6). The flowrate of the mixture in two cases were as follows :-

- (i)  $Q = 11.616 \times 10^{-3} \text{ m}^3/\text{s}$  at  $\bar{c}_d = 3.362\%$
- (ii)  $Q = 12.198 \times 10^{-3} \text{ m}^3/\text{s}$  at  $\bar{c}_d = 11.653\%$

##### 5.1.1 Case 1 : Trajectory and Erosion Pattern at Mean Discharge Concentration of 3.362%

Particles are released into a flow field which is computed by ignoring the presence of solids in the mixture. That is to say, the single phase flowrate is considered to be the same as that of the mixture. The initial conditions and the description of the computed



flow field are given below.

#### 5.1.1.1 Computed Flow Field for Case 1

The initial conditions for the case considered are as follows :-

- (i) uniform axial velocity distribution; for a flowrate of  $11.616 \times 10^{-3} \text{ m}^3/\text{s}$  which means that :-

$$W_{z_o} = 1.426 \text{ m/s} \quad (5.1a)$$

- (ii) radial velocity equal to zero :-

$$W_{r_o} = 0 \quad (5.1b)$$

- (iii) circumferential velocity :-

$$W_{\theta_o} = - \omega r \quad (5.1c)$$

The isometric plot of the velocity profiles of the computed flow along different grid stations of the mesh, shown in fig. 2.3, are given in fig.'s 5.1a to 5.3b. For each velocity component, the figures are given to provide two angles of view (the surface generated by an isometric projection in one figure is rotated by 180 degrees in the second figure for the same velocity component). The numbers along the axis representing finite difference grid columns correspond to the station numbers given in fig. 2.3. The numbers along the axis representing grid rows correspond to discrete points along the inlet to the computed flow field, see fig. 2.3.

The flow enters the impeller gradually thereby causing an increase in radial velocity. The turning

is more near the front shroud of the impeller than at the back shroud (from station 5 onward; for station number see fig. 2.3), hence there is an almost linear increase of radial velocity from back to front shroud at any given station. The axial velocity, in contrast, shows a rapid fall downstream of the initial section (station 1, where it is maximum and uniform as per the boundary condition of the solution). In areas near the front shroud there is, however, some increase in axial velocity. This is because of the curvature effect and associated centrifugal force.

Downstream of the leading edge (station 11) the radial velocity gradually rises to a peak value and then decreases continuously upto the exit. The decrease is due to the diffusion of flow precipitated by the geometry of the impeller. The front and back shrouds of the impeller near the tip are flared outwardly, see fig. 2.3.

The flow continues to turn downstream of the leading edge and hence the axial velocity falls except in areas near the front shroud. Further downstream, the turning is almost complete and axial velocity reaches an absolute minimum value at station 19 (see fig. 2.3). Further down, the flow diffuses due to the flaring of the shrouds resulting in some increase in axial velocity at areas near the shrouds. Diffusion is less at the core of the channel and hence in this area axial velocity remains fairly uniform.

The circumferential velocity of the flow is zero upto the leading edge as no swirl at entry is specified (note that upto the leading edge the relative circumferential velocity is  $-\omega r$ ). After the leading edge, due to the diffusion of the main flow, the circumferential velocity decreases monotonically upto the exit point. The hub to shroud distribution of velocity at each column is almost uniform.

The flow field of the impeller is discussed in some detail as one of the contributing factors in particle motion (i. e., fluid dynamic drag) depends on this flow field. This point will be clarified by the calculation of trajectory of particles which is discussed below.

#### 5.1.1.2. Trajectory Calculations in the Computed Flow : Initial Conditions

In order to be able to perform trajectory calculations, certain information is required as regards to the characteristics of the particles and the boundary values of the calculations. This will be discussed in the following sections.

##### 5.1.1.2.1 Characteristics of the Particles

The following information about the particles are required :-

- (i) size, shape and specific gravity of the particles
- (ii) initial dynamic characteristics of the particles
- (iii) initial position of the particles

##### 5.1.1.2.1.1 Size, Shape and Specific Gravity of the Particles

The particles used in the present work are of different size and they are not spherical, see plate 1. It was possible to define a characteristic particle size as discussed in section 3.3.1. This characteristic size is 1.8 mm. The specific gravity of the solids was measured to be 2.659 (section 6.3.2). To allow for the increased drag experienced by a particle of irregular shape over that experienced by a solid sphere of the same mean projected area, an angularity factor  $\lambda$  is introduced (Ref. 113). The drag force calculated on the basis of the projected diameter of the particle will be multiplied by  $\lambda$  to obtain the drag force for an irregular particle of the same projected

diameter. In the present analysis the characteristic size of the particle will be substituted by the projected diameter of the particle. Angularity factor  $\lambda$  for a wide variety of materials is given by Zenz (Ref. 114) in terms of a shape factor  $S_h$  as follows :-

$$\lambda = \frac{\pi}{6 S_h} \quad (5.2)$$

For the present analysis the value of  $\lambda$  is assumed to be 2.

#### 5.1.1.2.1.2 Initial Dynamic Characteristics of the Particles

The equations of motion of the particles are solved in a stepwise manner with time for given initial position and dynamic characteristics of the particles (see section 2.3.3). The interaction between the particles and the fluid will depend on the difference in the dynamic characteristics of the particles and the fluid. The larger the difference, the more is the interaction and vice versa. However, besides the fluid dynamic forces, other forces also act on the particles (e.g., gravity). Thus if the particle does not respond fast to the fluid flow, then the path taken by the particle will depend on the initial difference in the dynamic characteristics of the particle and the fluid. As the equations of motion of the particles are solved in a given flow field, the initial dynamic characteristics of the particle will influence the path taken by the particle in the given flow field. Hence, it is imperative that the initial conditions be prescribed as close to reality as possible.

It is shown in section 3.3.1.1 that for the case being considered here, the axial velocity of the particle lags that of the flow at entry to the computed flow field and that an estimate of this lag may be obtained from the experimental evidence reported elsewhere (Ref. 68 ). This velocity lag is given in a nondimensional form as a function

of the particle position from the lowest point of the pipe (see section 3.3.1.1). It is assumed that this lag is constant along a horizontal section of the pipe. The axial velocity of the particle will be known from this velocity lag. The other components (radial and circumferential) of the particle velocity at entry to the flow are assumed to be zero. This means that :-

$$v_{r_o} = 0 \quad (5.3a)$$

$$v_{\theta_o} = 0 \quad (5.3b)$$

$$v_{z_o}^* = q_5(Y) \quad (5.3c)$$

where,

$v_{z_o}^*$  = normalised velocity lag,  $(v_m - v_{z_o})/v_m$

$v_m$  = mixture velocity at a distance  $Y$  from a horizontal plane tangential to the lowest point of the pipe

By fitting curve to the experimental data (Ref. 68), the following expression for function  $q_5$  is obtained :-

$$q_5 = a_o + a_1 Y + a_2 Y^2 + a_3 Y^3 + a_4 Y^4 \quad (5.4)$$

where,

$$a_o = 1.165999 \quad (5.5a)$$

$$a_1 = -7.424840 \quad (5.5b)$$

$$a_2 = 47.540690 \quad (5.5c)$$

$$a_3 = -142.497690 \quad (5.5d)$$

$$a_4 = 135.831411 \quad (5.5e)$$

In fig. 3.4, the normalised velocity is shown against normalised distance from the lowest point of the pipe.

#### 5.1.1.2.1.3 Initial Position of the Particles

Another important aspect in the trajectory calculations is the position of the release of the particles into the computed flow field. The importance of prescribing correctly this position is discussed in section 3.1 and hence will not be repeated here.

It is shown in section 3.3.1.1 that for the case being considered here, the particles are not uniformly distributed over the cross-section of the pipe, but tend to accumulate near the bottom of the pipe. The concentration of the particles in the cross-section of the pipe shows a gradient in the vertical direction increasing towards the bottom of the pipe. An estimate of this concentration was obtained from the experimental evidence reported in Ref. 68. By fitting a curve to the concentration profile given in Ref. 68, the following equation for the concentration distribution,  $c_d$ , along the vertical diameter of the pipe was obtained :-

$$c_d = q_6(Y) \quad (5.6a)$$

where,

$$q_6 = b_0 + b_1 Y + b_2 Y^2 + b_3 Y^3 + b_4 Y^4 + b_5 Y^5 \quad (5.6b)$$

$$b_0 = 0.17062 \quad (5.7a)$$

$$b_1 = -0.64846 \quad (5.7b)$$

$$b_2 = 3.39360 \quad (5.7c)$$

$$b_3 = -11.10060 \quad (5.7d)$$

$$b_4 = 15.05756 \quad (5.7e)$$

$$b_5 = -7.03526$$

(5.7f)

In fig. 3.3, the concentration distribution is shown as a function of Y.

It is assumed that the concentration is uniform in any horizontal section of the pipe. Based on this, a discretization procedure was adopted to generate the position and the number of particles in a vertical cross-section of the pipe (see section 3.3.2). This distribution is shown in fig. 5.4.

Having thus prescribed the characteristics of the particles, trajectory calculations of the particles may be accomplished once the boundary values of the calculations are known. This is discussed below.

#### 5.1.1.2.2 Boundary Values of the Calculations

As a particle moves through an impeller, its motion in the blade channel will be influenced by the position of the particle relative to a blade surface and the speed of the impeller. In addition, as the particle impacts on a solid wall of the impeller, its rebound velocity will depend on the restitution coefficient and the friction coefficient between the particle and the solid wall (see section 2.7). These conditions need to be specified. In the following paragraphs these conditions are discussed.

##### 5.1.1.2.2.1 Initial position of the Reference Blade of the Impeller

As the particle enters a blade channel, its trajectory is bounded by four solid walls of the channel. How early a particle will hit a blade surface will depend on the angular position of the particle relative to a blade surface. In the present analysis, two possibilities

are considered :-

- (i) The angular clearance of the particle relative to a certain position on the blade pressure surface is given at the moment a particle enters a blade channel (angle  $\beta'$  in fig. 2.5).
- (ii) The angular position of the particle is chosen randomly.

In item (i), one blade of the impeller is taken as the reference blade and given a certain angular position with respect to the coordinate frame in which the position of the particle at time  $t=0$  is prescribed (see fig. 2.2 for definition of the coordinate frame). The reference blade rotates at the speed of the impeller and the position of the reference blade as well as all other blades, therefore, is known at any instant of time. The position of the particle relative to the pressure surface of the blade channel is determined from the knowledge of the transit time of the particle to reach a blade channel after it has been released into the computed flow at station 1, fig. 2.3. By calculating the angular displacement of the pressure surface of the blade channel during this transit time and the angular position of the particle defined in the coordinate frame, the angular separation between the particle and the blade pressure surface,  $\beta'$  (fig. 2.5), may be calculated. For a given particle the transit time is fixed and with the blade position at  $t=0$  being preset, the angular position of the particle relative to a blade surface at entry is also fixed. The position of first impact by the particle on the blade pressure surface may be affected by this angular position of the blade pressure surface. This in turn means that the trajectory of the particle inside the channel may be affected. To check if the position of first impact on a blade pressure surface by a particle will depend on the blade position relative to the particle, a criterion has been defined in section 2.6. The criterion for a particle hitting a distance  $s$  from blade leading edge is :-



$$v_r \leq \frac{s \sin \gamma}{2 \pi \epsilon} n_b \omega \quad (2.6.9)$$

In the present work the impeller considered has five vanes and hence :-

$$n_b = 5 \quad (5.8a)$$

Also, from the geometry of the blade,  $\gamma$  is given as :-

$$\gamma = 63^\circ \quad (5.8b)$$

Substituting  $n_b$  and  $\gamma$  into expression 2.6.9 :-

$$v_r \leq 0.709 s \frac{\omega}{\epsilon} \quad (5.8c)$$

Considering, for example, the probability of a particle impacting on a blade pressure surface within a distance  $s=5$  mm to be  $\epsilon=0.5$ , and taking  $\omega=1050$  r.p.m., then :-

$$v_r \leq 0.78 \text{ m/s} \quad (5.8d)$$

This means that if the velocity of a particle is  $v_r \leq 0.78$  m/s, the probability of hitting the leading edge of the impeller is 0.5.

Some initial runs of the computer code (section 2.8) were made to see the effect of the criterion 5.8d on the motion of the particle. The particles were distributed at the inlet to the computed flow field according to fig. 5.4. A random data generating function was introduced for  $\epsilon$ . It was found that for all the particles, the value of

$v_r$  was less than the right hand side of expression 5.8c. This meant that the initial blade position did not affect the calculation of particle trajectories. In addition, the computer code was run at different preset values of the initial blade position. Again, no effect of the initial position was noticed. Hence, for all the runs in the present analysis, this initial position was set at  $\theta=0$  at  $t=0$ .

Also, the front and the back shroud of the impeller are axisymmetric in geometry. Hence, the angular position of the particle while entering a blade channel will not affect the possibility of the particle hitting either of these shrouds.

#### 5.1.1.2.2.2 Speed of the Impeller

The significance of the speed term can be assessed by referring to expression 5.8c. The probability of a particle hitting a blade pressure surface immediately after it has entered a blade channel increases as the speed of the impeller increases. For the present analysis, this speed is the same as the one at which the pump was run at a mean discharge concentration of 3.362% during the course of experiment (see section 6.7.2). Hence:-

$$\omega = 1050 \text{ r.p.m.} \quad (5.9)$$

#### 5.1.1.2.2.3 Restitution Coefficient

The particle loses momentum on impacting a solid wall of the impeller and an estimate of this momentum loss may be obtained from the ratio of the normal component of the rebound velocity to the normal component of the incident velocity (section 2.7). The value of this coefficient depends on :-

- (i) the material properties of the particle and the

- solid wall
- (ii) the texture of the particle and the solid wall

As no experimental data appropriate to the conditions of the present work were available, an estimation for the coefficient had to be made. For a strictly dry condition, Ottjes (Ref. 115) suggested that this coefficient is of the order of 0.6 for spherical particles although no mention of solids type or material properties of the solid was made. In the present analysis a number of initial runs of the computer code were carried out with different values of the restitution coefficient ranging from 0.3 to 0.6. Particle trajectories were observed to be affected very little by this variation of the restitution coefficient. The following value for the coefficient was used in the present analysis.:-

$$e = 0.3 \quad (5.10)$$

#### 5.1.1.2.2.4 Friction Coefficient

A particle on hitting a solid wall may continue to slip when it leaves a solid wall. Alternatively, its point of contact with the solid wall may be brought to rest as it is about to leave the wall (see section 2.7.2). The coefficient of dynamic friction between the particle and the solid wall is significant in determining the mode in which the particle will leave the wall (see expression 2.7.32). Rosenthal (Ref. 116) noted that for a lubricating surface, the friction coefficient may be as low as 0.1. From a few initial runs of the trajectory code in which the friction coefficient was varied between 0.1 to 0.3, very little variation in the trajectory of the particles were observed. For the present analysis, the following value was assumed for the dynamic friction factor :-

$$f_d = 0.2 \quad (5.11)$$

#### 5.1.1.3 Trajectory of Particles in the Computed Flow Field

In the preceeding paragraphs, the type of the particles used in the present analysis, the initial condition of the particles and the factors other than the external forces which might affect the motion of the particles in an impeller have been discussed. Calculation of particle trajectories for particles released at upstream locations of the computed flow field (station 1, fig. 2.3) may now be performed.

Of the distribution of particles given in fig. 5.4, four cases have been selected for investigation. These are :-

- (i) particles are released into the computed flow from the vertical diameter of the pipe
- (ii) particles are released near the bottom of the pipe
- (iii) particles are released near the centreline of the pipe
- (iv) particles are released halfway between the centreline and the bottom of the pipe

The above four cases are selected to investigate the effect of the initial condition of the particles on the path taken by the particles inside an impeller. The distribution of the particles for the cases are shown in fig.'s 5.5a to 5.5d. The geometry of the impeller as provided by the manufacturer is given in fig.'s 6.2a and 6.2b.

##### 5.1.1.3.1 Case 1a : Trajectory of Particles Released Along the Vertical Diameter of the Pipe

Five particles are released into the computed flow field from five different positions along the vertical

diameter of the pipe. The particles are numbered 1 to 5 in fig. 5.5a. Their normalised distance from the bottom of the pipe together with the velocity at which they are released into the flow field is given in table 5.1. The flow velocity at the inlet (station 1, fig. 2.3) is 1.426 m/s (section 5.1.1). Computer aided plots of the trajectory of the five particles in the impeller as projected on a meridional plane are given in fig. 5.6a. The abscissa of the plot coincides with the axis of the impeller and hence that of the pipe which feeds mixture into the impeller. The solid line drawn parallel to the axis about halfway between the axis and the exit of the impeller indicates the position of the leading edge. In fig. 5.6b, the trajectory of the same set of particles in a blade channel relative to the blades are shown as a projection on a z-plane. Although not all the particles enter the same blade channel, their trajectories are shown in a single channel since the geometry of all the channels are identical to one another.

Table 5.1

Initial Condition of the Particles : Case 1a

| Particle<br>no. | Normalised<br>distance<br>from pipe<br>bottom<br>(Y) | Initial<br>axial<br>velocity<br>$v_{z_0}$<br>(m/s) |
|-----------------|------------------------------------------------------|----------------------------------------------------|
| 1               | 0.05                                                 | 0.160                                              |
| 2               | 0.15                                                 | 0.420                                              |
| 3               | 0.25                                                 | 0.579                                              |
| 4               | 0.35                                                 | 0.967                                              |
| 5               | 0.44                                                 | 1.352                                              |

The following observations are made about the motion of the selected particles in the impeller :-

- (i) The distance from the bottom of the pipe of particles 1 and 2 as they are released into the computed flow lie within 25% of the pipe diameter. These particles enter a blade channel without hitting a solid wall of the impeller, see fig. 5.6a. It may, therefore, be concluded that particles which are released along or near the vertical diameter of the pipe and whose distance from the bottom of the pipe does not exceed 25% of the pipe diameter will enter a blade channel without any collision with a shroud.
- (ii) Particles 3, 4 and 5 enter the flow near the centreline of the pipe. Their distance from the centreline of the pipe is less than 25% of the pipe diameter. These particles hit and bounce off the back shroud of the impeller before they enter a blade channel, see fig. 5.6a. Hence, it may be inferred here that particles which are released along or near the vertical diameter of the pipe from these positions will hit the back shroud of the impeller before they enter a blade channel.
- (iii) Considering the collision free path of the particles in the passage before the leading edge, it may be noticed in fig. 5.6a that this path bends towards the blade channel, more so as the distance of a particle from the centreline of the pipe increases. This can be explained as follows. The velocity lag between a particle and the fluid at a position where the particle is released increases as the distance of the position from the centreline of the

pipe. This velocity lag decreases continuously after the particle has entered the computed flow field and traverses through the impeller. This is because of the fluid dynamic drag acting on the particle. The time taken by a particle to move a certain distance in the axial direction depends upon the axial velocity of the particle. During this period, under the action of the gravitational force, the particle is pushed in the vertical direction into the blade channel. The flow velocity in the path of the particles does not show any major variation as can be seen in fig.'s 5.1a to 5.3b. Comparing the trajectory of the particles 3 and 5 (fig. 5.6a), the ratio of the axial velocity of these two particles at entry to the computed flow is 2.32 (see table 5.1). The computed time-distance history of the particles suggests that after these particles have traversed a distance of 35 mm from their point of release, the ratio of their axial velocity is 1.08; the corresponding ratio of the transit time of the particles is 0.702 and the ratio of the vertical displacement of the particles in the specified distance is 0.283. Particle 5 has higher initial velocity than particle 3 and in the given computed flow field the initial velocity lag between the particle and the fluid will have considerable influence upon the trajectory of the particle in the impeller before the leading edge.

- (iv) Particles 3, 4 and 5 on hitting the back shroud lose momentum and bounce back with a reduced axial momentum in the opposite direction of the main flow. Particles cannot sustain this reverse momentum for a long time because of the drag of the flow. Hence, they cannot penetrate much into the main flow but are confined close to

the back shroud of the impeller.

(v)

As there is no swirl in the main flow, the absolute tangential velocity at which the particles enter a blade channel is very low. Relative to the blade leading edge this velocity is almost equal to  $-wr_b$ , where  $r_b$  is the radial position of the particle at entry to a blade channel. At this position the ratio of the radial to the absolute tangential velocity of the particle is of the order of  $10^{-1}$ . The ratio of the acceleration of the particle in the tangential direction to that in the radial direction is of the order of  $10^2$ . The acceleration of the particle in the tangential direction is entirely due to coriolis force which dominates the particle motion at this position. Any contribution by the centrifugal force to the particle motion in the radial direction is very small due to negligible absolute tangential velocity of the particle. Infact, the only significant force that pushes the particle in the radial direction is the fluid dynamic drag. Because of the large difference between the tangential and the radial component of particle velocity as well as acceleration, all the particles hit the pressure surface of the blade as soon as they have entered a blade channel, see fig. 5.6b.

(vi)

The particles enter a blade channel at a wide variety of angles relative to the pressure side of the blade. The angular separation between a particle and the pressure surface of a blade is maximum for particle number 4 (fig. 5.6b) and equals six times the angular separation between the same particle and the suction surface of the blade channel. In spite of this, neither



this nor any other of the selected particles escapes collision with the pressure surface of the blade within a short distance after entry to a blade channel. The initial blade position (section 5.1.1.2.2.1), therefore, has negligible effect on the trajectory of particles in the blade channel.

- (vii) As the particles reach the blade surface, they impact and rebound in an oscillating manner with decreasing amplitude. Their angle of incidence on first impact lies in the range of 49 to 53 degrees and decreases as the particle continues to impact on the blade pressure surface. After a number of impacts, this angle may be as low as 3 degrees. This decrease in the angle of impact may be due to two reasons. First, the centrifugal force acting on the particle gradually increases as the absolute tangential velocity of the particle slowly increases due to fluid dynamic drag. This centrifugal force combined with the fluid dynamic drag tends to push the particles in the radial direction. This means that the effect of coriolis force on the motion of the particle is gradually taken over by the centrifugal force. Consequently, as the particle is displaced more and more in the radial direction, its radial component of velocity becomes significant. Referring to the blade geometry shown in fig. 5.6b, the above observation means that the particle will tend to impact on the blade surface with a lower angle of incidence as the radial position of an impact on the blade surface increases. Secondly, the geometry of the blade itself is such that with increasing radius, the angle between the tangent drawn to the pressure surface at a point and the normal to the radius

vector at that point decreases. Hence, for a given velocity vector of the particle on the blade surface, an impact near the leading edge means a higher angle of incidence than one away from it.

- (viii) The magnitude of the displacement of the particles away from the surface between two consecutive impacts is quite low and in all the cases considered here, its maximum value is about 4 times the size of the particles. The main reason for this low value of displacement is the coriolis force which away from the blade surface causes the particle to decelerate and finally to collide with the blade.
- (ix) During the time in which the particle makes ballistic hops and jumps on the surface of the blade, its absolute tangential velocity increases. As a result, the centrifugal force acting on the particle gradually increases and soon dominates the radial component of the drag force. At a radial position of about 85 mm (tip diameter of the blade, 265 mm), the coriolis and centrifugal acceleration of the particle are almost equal, and greater than the acceleration due to other forces by an order of magnitude of  $10^1$  or more. Hence, the motion of the particle in this area is due to centrifugal and coriolis force only. These two forces are almost equal to each other. The resultant force vector on the particle directs it away from the blade surface. As a consequence, the particle will escape a collision with the blade surface.
- (X) From the position stated above, the coriolis

and the centrifugal force on the particle remain almost equal to each other. However, as the particle accelerates due to these forces, its radial and relative tangential velocity continuously increases. After a displacement of the particle in the radial direction by about 25 mm, the magnitude of the drag force in the radial and tangential direction become comparable to centrifugal and coriolis forces, respectively. The situation remains unchanged till the particle leaves the impeller. Also, for this part of the trajectory as the magnitude of the forces in the radial and the tangential direction remain unaltered, the direction of the motion of the particle remains unchanged as can be seen in fig. 5.6b.

(xi) Additionally, as the motion of the particle in the channel following an impact with the blade surface is controlled most of the time by the coriolis and centrifugal force (or a combination of these forces and the components of drag in the radial and tangential direction), the trajectory of the particles in the blade channel as projected on a meridional plane will be a radial line , see fig. 5.6a.

(xii) All the particles leave the impeller at a radial velocity about 5 times that of the flow and at an exit angle about 1.5 times the flow exit angle. The difference in density between the solids and the liquid causing a higher centrifugal force on the solid particles than on the liquid is responsible for the observed difference. Detail of the dynamic characteristics of the particles at exit from the impeller together with other salient features of the time-distance history of the particles is

given in Table 5.2.

#### 5.1.1.3.2 Trajectory of Particles Released Near the Bottom of the Pipe

In the previous section trajectory of five particles has been considered. Only one particle (number 1) was released into the flow field of the impeller close to the bottom part of the pipe. In order to be able to get a clearer picture of the trajectory of particles which pass near the bottom of the pipe, a number of particles are considered here.

The distribution of the particles in this case in the cross-section of the pipe before entering the flow field is given in fig. 5.5b. The particles are distributed about the vertical diameter of the pipe. The angular separation between two consecutive particles is approximately 19 degrees. The radial distance of a particle from the centreline of the pipe normalised by the pipe radius is 0.88. The position and the velocity of the particles as they enter the flow are given in Table 5.3. The particle number in the table corresponds to that given in fig. 5.5b. Computer aided plot of the trajectory of the particles as projected on a meridional plane of the impeller are given in fig. 5.7a and in a blade channel relative to the blades, in fig. 5.7b. The angular position of a particle as given in these figures is defined by the system of axes shown in fig. 2.1.

The following observations are made about the motion of the particles :-

- (i) The particles enter a blade channel without colliding with a solid wall. Therefore, it may be inferred that those particles which

Table 5.2

Some Aspects of the Motion of the Particles in the Impeller : Case 1a

| Particle number | Initial velocity (m/s) |                |          | Impaction zone          | Total number of impacts | Transit time from entry to exit (s) | Final velocity (m/s) |              |        |
|-----------------|------------------------|----------------|----------|-------------------------|-------------------------|-------------------------------------|----------------------|--------------|--------|
|                 | $v_{r0}$               | $v_{\theta 0}$ | $v_{z0}$ |                         |                         |                                     | $v_r$                | $v_{\theta}$ | $v_z$  |
| 1               | 0                      | 0              | 0.160    | blade p.s.              | 6                       | 0.0663                              | 3.431                | -4.561       | -0.237 |
| 2               | 0                      | 0              | 0.420    | blade p.s.              | 6                       | 0.0772                              | 3.385                | -4.595       | 0.127  |
| 3               | 0                      | 0              | 0.579    | back shroud, blade p.s. | 7                       | 0.1015                              | 3.423                | -4.588       | 0.281  |
| 4               | 0                      | 0              | 0.967    | back shroud, blade p.s. | 7                       | 0.1349                              | 3.382                | -4.593       | 0.274  |
| 5               | 0                      | 0              | 1.352    | back shroud, blade p.s. | 9                       | 0.1520                              | 3.430                | -4.586       | 0.205  |

Table 5.3

Initial Condition of the Particles : Case 1b

| Particle<br>no. | Normalised<br>distance<br>from pipe<br>bottom<br><br>(Y) | Initial<br>axial<br>velocity<br><br>$v_{z_0}$<br>(m/s) |
|-----------------|----------------------------------------------------------|--------------------------------------------------------|
| 6               | 0.15                                                     | 0.42                                                   |
| 7               | 0.08                                                     | 0.28                                                   |
| 8               | 0.05                                                     | 0.16                                                   |
| 9               | 0.06                                                     | 0.23                                                   |
| 10              | 0.12                                                     | 0.38                                                   |

are released near the wall of the pipe and whose distance from the bottom of the pipe do not exceed 15% of the pipe diameter will avoid any collision before entering a blade channel.

- (ii) The particles traverse almost through the same flow field, hence the effect of any variation of flow field on the trajectory of these particles before leading edge will be small. Of all the particles, particle number 6 travels the maximum distance in the axial direction before entering a blade channel, the distance in the same direction for particle number 8 is the minimum, fig. 5.7a. The former enters the computed flow at an axial velocity which is about 2.6 times that of the latter. Because of this variation in the initial axial momentum, these two particles after entering the blade

channel will impact on a blade surface at a slightly different axial position.

- (iii) Once inside the channel, the particles move very little in the radial direction before they collide with a blade surface, fig. 5.7b. This is because of the coriolis acceleration of the particle as discussed in item (v), section 5.1.1.3.1.
- (iv) The particles impact on the blade surface a number of times during which their absolute tangential velocity gradually increases resulting in an increase of the centrifugal force acting on the particles. At a radial position of about 85 mm, the magnitude of coriolis and centrifugal force on the particles are almost equal. From this position onward particles do not collide with a blade surface, for the reasons detailed in item (ix), section 5.1.1.3.1. It is worth observing here that the radial position on the blade surface at which the particles stop colliding with the blade is almost identical to the case discussed in the preceeding section. This is because the flow, for  $r=\text{constant}$ , is almost uniform along a station in the blade channel (see fig. 5.1a to 5.3b), except near the exit. As such, at a given station, the effect of any variation in the flow field on the motion of the particles is negligibly small. Furthermore, the magnitude of the coriolis or the centrifugal force on the particle do not vary at different points along a station where  $r=\text{constant}$ . Therefore, the motion of the particles in the blade channel at the same radial position will be almost identical regardless of axial position. The trajectory of the particles relative to the

blades as projected on a z-plane for particles released either along the vertical diameter of the pipe or near the bottom wall of the pipe will be similar to one another. Figures 5.6b and 5.7b will lend support to this observation.

Detail of the dynamic characteristics of the particles considered in the present case when they leave the impeller are given in Table 5.4. Other salient features of the time-distance history of the particles are also provided in the same table.

#### 5.1.1.3.3 Case 1c : Trajectory of Particles Released

##### Near the Centreline of the Pipe

Three particles are considered in this case. Their distance from the centreline of the pipe is within 30% of the radius of the pipe. Their distance from the bottom of the pipe,  $Y$ , is given in Table 5.5 together with the velocity at which they are released into the computed flow field of the impeller. Particle number in the table corresponds to that given in fig. 5.5c where the distribution of these particles in the cross-section of the pipe before being released into the computed flow field is given.

Table 5.5

#### Initial Condition of the Particles : Case 1c

| Particle<br>no. | Normalised<br>distance<br>from pipe<br>bottom<br><br>(Y) | Initial<br>axial<br>velocity<br><br>$v_{z_0}$<br>(m/s) |
|-----------------|----------------------------------------------------------|--------------------------------------------------------|
| 11              | 0.387                                                    | 1.142                                                  |
| 12              | 0.349                                                    | 1.001                                                  |
| 13              | 0.411                                                    | 1.243                                                  |



Table 5.4

Some Aspects of the Motion of the Particles in the Impeller : Case 1b

| Particle number | Initial velocity (m/s) |                |          | Impaction zone | Total number of impacts | Transit time from entry to exit (s) | Final velocity (m/s) |              |        |
|-----------------|------------------------|----------------|----------|----------------|-------------------------|-------------------------------------|----------------------|--------------|--------|
|                 | $v_{rO}$               | $v_{\theta O}$ | $v_{zO}$ |                |                         |                                     | $v_r$                | $v_{\theta}$ | $v_z$  |
| 6               | 0                      | 0              | 0.416    | blade p.s.     | 6                       | 0.0599                              | 3.402                | -4.590       | -0.142 |
| 7               | 0                      | 0              | 0.278    | blade p.s.     | 6                       | 0.0626                              | 3.400                | -4.586       | -0.191 |
| 8               | 0                      | 0              | 0.186    | blade p.s.     | 6                       | 0.0663                              | 3.431                | -4.561       | -0.237 |
| 9               | 0                      | 0              | 0.266    | blade p.s.     | 3                       | 0.0657                              | 3.391                | -4.570       | -0.146 |
| 10              | 0                      | 0              | 0.375    | blade p.s.     | 5                       | 0.0611                              | 3.460                | -4.556       | -0.056 |

In fig. 5.8a, the trajectory of the selected particles in the meridional plane of the impeller are given. The trajectories of the same particles relative to the blades in the blade channel are given in fig. 5.8b..

The following observations can be made on the motion of these particles :-

- (i) All the particles impact on the back shroud before entering a blade channel. Their radial displacement after being released into the computed flow field and the instant they hit the back shroud is small, for the reasons detailed in item (iii), section 5.1.1.3.1. The particles bounce off the shroud wall into the flow, but their penetration into the flow in the axial direction is small because of the axial drag of the flow continuously pushing the particles towards the back shroud.
- (ii) The motion of these particles in the passage before the leading edge is mainly due to fluid dynamic drag. The centrifugal acceleration is low as the tangential component of the particle velocity is quite low and is less than the other two components of the velocity by an order of  $10^{-1}$  or less. All the particles enter the blade channel near the back shroud of the impeller.
- (iii) The particles impact on the blade surface a number of times before exit from the impeller. The position on the blade surface where the particles make last contact with the blade is almost identical to that in the preceding two cases (case 1a and 1b) for reasons detailed in item (iv), section 5.1.1.3.2.
- (iv) The path of the particles near the exit show a

slight turning towards the axial direction.

Inside the blade channel these particles travel close to the back shroud which is flared near the exit. As a result, the axial momentum of flow in this area is higher than that in the centre of the channel. The drag force on the particle in the axial direction near the exit is comparable to the forces pushing the particles in the radial direction. Hence, near the exit the particle path slightly turns in the axial direction.

The important aspects of the motion of the particles in the impeller are given in Table 5.6.

5.1.1.3.4 Case 1d : Trajectory of Particles Released Halfway Between the Centreline and the Bottom of the Pipe

The position of these particles in a cross-section of the pipe at inlet to the computed flow field is given in fig. 5.5d. The initial condition of the particles are given in Table 5.7. Particle numbers in the table correspond to those given in fig. 5.5d. An added importance of this case is that as the bulk of the particles are distributed in this region of the pipe section (see fig. 5.4), the trajectory of these particles and the position at a wetted segment of the impeller where these particles impact will be indicative of the maximum erosion zone of the impeller.

In fig. 5.9a, the trajectory of the selected particles as projected on a meridional plane are shown. The trajectory of the same particles relative to blades in a blade channel are given in fig. 5.9b.

Table 5.6

Some Aspects of the Motion of the Particles in the Impeller : Case 1c

| Particle number | Initial velocity (m/s) |                |           | Impaction zone          | Total number of impacts | Transit time from entry to exit (s) | Final velocity (m/s) |              |       |
|-----------------|------------------------|----------------|-----------|-------------------------|-------------------------|-------------------------------------|----------------------|--------------|-------|
|                 | $v_{r_0}$              | $v_{\theta_0}$ | $v_{z_0}$ |                         |                         |                                     | $v_r$                | $v_{\theta}$ | $v_z$ |
| 11              | 0                      | 0              | 1.142     | back shroud, blade p.s. | 9                       | 0.134                               | 3.431                | -4.587       | 0.226 |
| 12              | 0                      | 0              | 1.001     | back shroud, blade p.s. | 7                       | 0.135                               | 3.382                | -4.593       | 0.274 |
| 13              | 0                      | 0              | 1.243     | back shroud, blade p.s. | 9                       | 0.135                               | 3.441                | -4.571       | 0.214 |

Table 5.7

Initial Condition of the Particles : Case 1d

| Particle no. | Normalised distance from pipe bottom (Y) | Initial axial velocity $v_{z_0}$ (m/s) |
|--------------|------------------------------------------|----------------------------------------|
| 14           | 0.334                                    | 0.898                                  |
| 15           | 0.334                                    | 0.898                                  |
| 16           | 0.269                                    | 0.647                                  |
| 17           | 0.155                                    | 0.420                                  |
| 18           | 0.269                                    | 0.647                                  |

The following observations are made about the motion of these particles :-

- (i) All the particles traverse through the blade channel close to the back shroud.
- (ii) Particles 14 and 15, whose trajectories coincide (fig. 5.9a) enter the flow field at a higher velocity than the other particles. Also, the vertical distance that these two particles have to traverse before they enter a blade channel is higher than the others. Particles 14 and 15, therefore, impact the back shroud of the impeller before entering a blade channel while the others escape any such impact.
- (iii) The distance from the bottom of the pipe is minimum for particle 17. It is released into the flow field with the minimum velocity (Table 5.7). Hence, this particle enters the blade channel with minimum traverse in the axial direction. Referring to figure 5.5d and observing the location of this particle in the

cross-section of the pipe, it may be concluded that the trajectory of this particle in the blade channel will be the limiting trajectory away from the back shroud so much so that the trajectory of all the particles released halfway between the centreline and the bottom of the pipe will lie between this limiting trajectory and the back shroud of the impeller.

- (iv) The particles impact on the blade surface a number of times before leaving the impeller, fig. 5.9b. The position of their last contact with the blade surface is approximately the same as the other three cases (case 1a, 1b and 1c) considered before, for the reasons detailed in item (iv), section 5.1.1.3.2.
- (v) The important aspects of the motion of these particles are given in Table 5.8.

#### 5.1.1.3.5 Summary of the Trajectory Calculations

The result of the trajectory calculations using a computer code developed as a part of the present work has been discussed in the preceding paragraphs. A number of cases have been considered to define the upstream condition of the particles. An indication of the motion of a particle released at any upstream location of the impeller may now be obtained by referring to the results of the four cases considered (case 1a, 1b, 1c and 1d). The general direction which the particles follow inside the impeller has been shown by providing a series of computer aided plots of particle trajectory which are output direct from the computer code. The information accumulated on the motion of the particle will now be used in the following paragraphs to predict the erosion prone areas on a blade surface.

Table 5.8

Some Aspects of the Motion of the Particles in the Impeller : Case 2

| Particle number | Initial velocity (m/s) |                |          | Impaction zone          | Total number of impacts | Transit time from entry to exit (s) | Final velocity (m/s) |              |       |
|-----------------|------------------------|----------------|----------|-------------------------|-------------------------|-------------------------------------|----------------------|--------------|-------|
|                 | $v_{rO}$               | $v_{\theta O}$ | $v_{zO}$ |                         |                         |                                     | $v_r$                | $v_{\theta}$ | $v_z$ |
| 14              | 0                      | 0              | 0.898    | back shroud, blade p.s. | 7                       | 0.112                               | 3.428                | -4.590       | 0.238 |
| 15              | 0                      | 0              | 0.898    | back shroud, blade p.s. | 7                       | 0.112                               | 3.432                | -4.585       | 0.240 |
| 16              | 0                      | 0              | 0.647    | blade p.s.              | 6                       | 0.081                               | 3.429                | -4.593       | 0.205 |
| 17              | 0                      | 0              | 0.420    | blade p.s.              | 6                       | 0.077                               | 3.385                | -4.595       | 0.127 |
| 18              | 0                      | 0              | 0.647    | blade p.s.              | 6                       | 0.082                               | 3.387                | -4.586       | 0.242 |

#### 5.1.1.4 Predicted Erosion Pattern on a Blade Surface

A particle in passing through an impeller may hit a blade surface or any other wetted part of the impeller. It arrives at a solid wall with certain momentum and upon an impact part of this momentum, which is lost, may contribute to the damage of the surface. For a given particle and the surface, the initial velocity of the particle at impact, the angle at which the particle strikes a surface are significant in any erosion process (see for example section 4.2.1.1).

For the concentration and the flow field being considered here, fig.'s 5.6a to 5.9a show that particles which enter the impeller from different parts of the cross-section of the pipe will hit a blade surface at distinctly different positions on the surface. Particles which enter the impeller near the bottom wall of the pipe, that feeds mixture into the impeller, will impact on the blade surface near the front shroud; particles which enter the impeller away from the bottom wall of the pipe will impact on the blade surface near the back shroud.

The position on the blade surface where each particle impacts must be correctly located so that for an elementary area on the blade surface, the number of impacting particles, the velocity and the angle of incidence of each impacting particle may be known. These information together with that on the area of the element will provide a basis for dividing a blade surface into a number of erosionprone areas as detailed in section 4.3.1.

To achieve the stated objective, the surface of a blade (pressure side or suction side) is divided into a number of elements. This is done by laying out the blade surface onto a two dimensional plane (e.g., plane of paper) and forming a grid on it. The following steps are involved.

- (i) To determine the profile equation of the surface from curve fitting of data on a blade geometry.



These data are obtained from the manufacturer's drawings.

- (ii) To integrate the profile equation along the surface of the blade to obtain the distance on the blade surface from leading edge to trailing edge.
- (iii) The geometry of the blade at the back and the front shroud are obtained from the manufacturer's drawings.
- (iv) From information obtained in item (ii) and (iii), the geometry of a blade surface as laid out on a plane of paper is obtained.
- (v) A grid is then formed as follows. Lines normal to the boundaries are drawn and equally spaced grid points are set on them. These lines formed columns of the grid. The rows are formed by joining similar points on each column. The shape of every element in the grid is that of a rhombus. The area of each element is calculated by drawing a diagonal across a rhombus and calculating area of each triangle on either side of the diagonal using equation (4.14). An effort is made to keep the density of the grid points on the blade surface as uniform as possible so that the variation in the area of the elements for most parts of the blade surface extending from the leading edge is small.

In fig. 5.10, the grid formed on the pressure side of a blade is shown. Some of the elements of the grid are fully shaded. Particles which enter the impeller near the bottom wall of the pipe (case 1b, section 5.1.1.2) impact on the blade surface within these elements before leaving the impeller. Evidently, the impact of all the particles on the blade surface occur in areas near the leading edge and the front shroud. These areas comprise about 4% of the blade surface. After a distance of  $0.3s'$

from leading edge (where  $s$  is the distance measured on the blade surface from leading edge to trailing edge), the surface of the blade is free from any impact.

In fig. 5.11, the impact on the blade pressure surface due to particles considered in the case 1c (section 5.1.1.3.3) is shown. These particles enter the impeller near the centreline of the pipe. In contrast to the previous case (fig. 5.10), all the particles here impact on the blade surface near the back shroud. The elements within which some impact occur comprise 2.2% of the blade surface. There is no impact on the surface after a distance of  $0.32s$  from the leading edge. Also, the extent of the impact zone across a blade surface is low compared to that in the case 1b (fig. 5.10). This is because the trajectory of all the particles considered in the present case lie in close proximity to each other when entering the impeller, see fig. 5.8a. The motion of the particles across the blade channel is negligible for the reasons detailed in item (xi), section 5.1.1.3.1. Hence the axial thickness of the blade surface where the particles first collide with a blade surface will determine the thickness of the impact zone on the blade surface in the cross-stream direction.

In fig. 5.12, the impact on the blade pressure surface due to particles which enter the impeller halfway between the centreline and the lower part of the pipe (case 1d, section 5.1.1.3.4) is considered. Comparing the position of these selected particles in the cross-section of the pipe as they enter an impeller, fig. 5.5d and the distribution of all the particles given in fig. 5.4, it is evident that the areas in the blade surface where the selected particles impact will be indicative of the impact due to bulk of the particles entering the impeller. The selected particles impact on the blade surface near the leading edge and the back shroud. The zone of impact does not extend cross-stream from the back shroud by more than half the channel width. It comprises about 4% of the blade surface. There is no

impact on the blade surface after a distance of  $0.35s'$  from the leading edge.

The cases mentioned in the preceeding paragraphs (case 1b to 1d) have been treated separately so that the reasons for which the particles take preferential position inside a blade channel can be understood by referring to their trajectory as discussed in section 5.1.1.3.2 to 5.1.1.3.4. Together they give a fair indication of the area on the blade surface where most particles impact.

In fig. 5.13, the zone of impact on the same blade surface due to all the particles which are released from the pipe section (fig. 5.4) is given. This gives a first indication of the areas of the entire blade surface where erosion damage due to impacting particles may occur. Almost the entire area along the leading edge is affected. The zone of impact covers about 20% of the blade surface. Downstream of the leading edge, beyond a distance of  $0.43s'$  the blade surface is completely free from any impact.

In areas of the blade surface halfway between the back and the front shroud, there are no impacts beyond a distance of  $0.27s'$  from the leading edge. Also, areas very close to the back shroud do not experience any impact of particles after a distance of  $0.13s'$  from the leading edge.

What is not evident in fig. 5.13 is the fact that the number of impact occuring within the elements vary widely over the surface. Most of the impacts occur in areas near the leading edge because of the coriolis acceleration of the particles. Also, as the bulk of the particles enter the blade channel near the back shroud (as evident from the trajectory of particles shown in fig.'s 5.6a and 5.9a and the initial distribution of these particles given in fig.'s 5.5a and 5.5d), impact in areas near the back shroud are higher than in areas near the front shroud. Also, the number of impact in areas near the leading edge and halfway between the back and the front shroud are small compared

to those taking place near the shrouds.

In fig. 5.14, the classified erosionprone areas on the blade pressure surface is given. The classification is made by counting the number of impacts in each element, the velocity and the angle of impact of each particle, and calculating the characteristic erosion data using equation 4.8, and comparing the erosion data of the elements according to the procedure detailed in section 4.3.1.

The pattern clearly identifies two major areas on the blade surface which are prone to erosion. These are areas near the front shroud and the back shroud where the level of erosion range between 2 to 5. In areas, halfway between the back and the front shroud, the erosion level is low and equal to 1. Areas in this part of the surface experience fewer impacts than those near the shrouds and hence show up low level erosion (wear class 1).

Maximum erosion (wear class 5) occurs near the leading edge. This area lies very close to the back shroud and extends from the leading edge to a distance of  $0.05s'$  on the blade surface. It is to be noted here that according to the erosion model proposed in section 4.3.1, predicted erosion level of 5 can occur in only one element. However, as the characteristic erosion data in the fully shaded elements were within 96% of the maximum value, they are all shown equally erosion prone.

The next lower level erosion (wear class 4) is predicted mostly in areas near the back shroud after a distance of  $0.024s'$  and  $0.104s'$  from the leading edge. In the latter area, the number of impacts is lower than that in the maximum erosion zone near the leading edge, but particles impact here at much higher velocity than at the leading edge. Also, the angle of impact here is quite low. The combination of the high velocity and low impact angle contribute to a high level of erosion in this area, see equation 4.8. Although this observation is equally true for

particles hitting the areas far from the leading edge near the back shroud, erosion level in this zone is low (wear class  $\leq 3$ ), because of small number of impacts.

Near the front shroud and close to the leading edge, the erosion is also high and calculated wear class is 4. Erosion in this region is due to particles entering the impeller through the bottom section of the pipe, see section fig.'s 5.7a and 5.7b. These particles, after bouncing off the blade surface, impact further down and cause damage to the surface. However, the erosion level in this downstream zone is lower than that at the leading edge because the number of impact of particles is highest near the leading edge due to coriolis acceleration of the particles. Also, in contrast to the areas near the back shroud, erosion of the blade surface in this region may extend upto the edge of the front shroud.

The predicted erosion pattern also suggests that beyond a distance of  $0.027s'$  from the leading edge, the areas on the blade surface close to the back shroud is subjected to very little erosion (wear class = 1). At the leading edge halfway between the back and the front shroud, although particles impact as can be seen in fig. 5.13, the predicted erosion pattern does not show any high level erosion (wear class  $\leq 2$ ). The number of impacts in this zone is fewer than those occurring near the shrouds although the dynamic characteristics of a particle impacting in this region are of the same order of magnitude as in any other place near the leading edge. Particles impacting in this zone enter the impeller near the wall of the pipe (but not close to the pipe bottom), fig. 5.4. Evidently, the initial distribution of the particles in the cross-section of the pipe has an effect on the erosion pattern of the blade surface.

The predicted erosion also suggests that the pressure surface of the blade upto a distance of  $0.36s'$

from the leading edge is erosion prone. This limit is less in the central part of the blade surface where it is equal to 0.23s'.

This model does not indicate any erosion of the suction surface as the trajectory calculations of the particles in the present case do not suggest any impact of particles on this surface.

#### 5.1.2 Case 2 : Trajectory and Erosion Pattern at Mean Discharge Concentration of 11.653%

##### 5.1.2.1 The Flow Field in Case 2

The flowrate of the mixture in this case is  $12.198 \times 10^{-3} \text{ m}^3/\text{s}$ . The difference between this flowrate and that in the previous case (case 1, section 5.1.1) is  $0.582 \times 10^{-3} \text{ m}^3/\text{s}$ . Because of the nominal difference in these two flowrates, the computed flow field calculated for case 1 is used in the present case. This means that particle trajectories in this case are calculated in the flow field shown diagrammatically in fig.'s 5.1 to 5.3.

##### 5.1.2.2 Trajectory of Particles

The distribution of particles at the inlet of the impeller in the present case is obtained by curve fitting of the insitu discharge concentration profile of the particles in the cross-section of the pipe. This profile is shown in fig. 3.3. The equation of the curve obtained using the least square polynomial fit is :-

$$c_d = q_7(Y) \quad (5.12a)$$

where,

$$q_7 = c_0 + c_1Y + c_2Y^2 + c_3Y^3 + c_4Y^4 \quad (5.12b)$$

and,

$$c_0 = 0.35130 \quad (5.13a)$$

$$c_1 = -1.27093 \quad (5.13b)$$

$$c_2 = 5.80987 \quad (5.13c)$$

$$c_3 = -11.69528 \quad (5.13d)$$

$$c_4 = 7.17385 \quad (5.13e)$$

In fig. 5.15, the distribution of particles for the present case is shown. This distribution is obtained by dividing the cross-section of the pipe in the same way as in case 1, section 5.1.1.3.1.

Below the centreline of the pipe, the position of the particles in the cross-section of the pipe through which the particles enter the impeller are similar to the previous case, fig. 5.4. The trajectory of particles released from these positions are similar to those discussed in section 5.1.1.3. Hence, only the trajectory of few selective particles entering the impeller from positions above the centreline of the pipe is discussed here. The distribution of these particles in the cross-section of the pipe is shown in fig. 5.16. The initial conditions of these particles are given in Table 5.9. The particle numbers in the table correspond to those shown in fig. 5.16. In fig. 5.17a, the trajectory of these particles as projected in the meridional plane of the impeller is given, and the trajectory inside the blade channel is shown in fig. 5.17b.

The following observations are made on the motion of these selected particles in the impeller :-

- (i) Particles 1 and 5 are released near the wall of the pipe. The difference in radial position between these particles and the leading edge of the blade is small. Hence, these particles enter a blade channel without hitting the back shroud, see fig. 5.17a.

Table 5.9

Initial Condition of the Particles : Case 2

| Particle<br>no. | Normalised<br>distance<br>from pipe<br>bottom<br>(Y) | Initial<br>axial<br>velocity<br>$v_{z_0}$<br>(m/s) |
|-----------------|------------------------------------------------------|----------------------------------------------------|
| 1               | 0.523                                                | 1.426                                              |
| 2               | 0.624                                                | 1.426                                              |
| 3               | 0.651                                                | 1.426                                              |
| 4               | 0.624                                                | 1.426                                              |
| 5               | 0.523                                                | 1.426                                              |

(ii) Particles 2,3 and 4 hit the back shroud of the impeller before entering a blade channel, fig. 5.17a. Although these particles enter the flow field with the same axial velocity as those in item (i), the radial distance that these particles travel before entering a blade channel is, on the average, 5 times greater than for particles studied under item (i). The radial velocity of the flow in this region is low and only 0.35 times the initial axial velocity of the particles. Hence, the fluid dynamic drag cannot cause substantial deflection of the particle path in the radial direction. The particles, therefore, travel in the axial direction under the action of the initial axial momentum and hit the back shroud of the impeller.

(ii) Because of the coriolis acceleration, all the particles hit the pressure surface of a blade shortly after they have entered a blade channel, fig. 5.17b. Particles 1 and 5 hit the blade



surface near the centre of the blade while the rest hit the blade surface near the front shroud, fig. 5.17a. The trajectory of these particles inside the blade channel are dictated by the positions where these particles first impact on the blade surface (see item xi, section 5.1.1.3.1). As such, the trajectory of particles 1 and 5 in the blade channel will lie halfway between the shrouds. For similar reasons, the rest of the particles will pass through the blade channel near the back shroud. It can, therefore, be concluded that particles which enter the impeller from positions above the centreline of the pipe will hit the pressure surface of a blade between the centreline of the blade and the back shroud.

#### 5.1.2.3 Predicted Erosion Pattern on a Blade Surface

The particles, having the initial distribution shown in fig. 5.15, enter the impeller and hit the pressure surface of the blade in areas shown fully shaded in fig. 5.18. These areas extend from the leading edge upto a distance of  $0.4s'$  measured in the surface from the leading edge. Areas bordering the front shroud experience no impact after a distance of  $0.05s'$  from the leading edge. Also, areas on the blade surface halfway between the shrouds do not experience any impacts after a distance of  $0.23s'$  from the leading edge. The areas affected by the impact of particles comprise 36% of the blade surface.

In fig. 5.19, the areas on the blade pressure surface which are prone to erosion by the mixture considered in the present case is given. As in case 1 (section 5.1.1.4), the pattern clearly identifies two areas on the blade surface where erosion can occur. These are areas near the front and the back shroud. In the areas between these two zones, the erosion level is minimum (wear class = 1).

Also, most of the areas near the leading edge will be subjected to erosion. Downstream of the leading edge, higher order erosion (wear class  $\geq 2$ ) is predicted in areas extending upto a maximum distance of 0.23s' measured from the leading edge.

Maximum erosion occurs near the leading edge bordering the back shroud and extends upto a distance of 0.048s' from the leading edge. The pattern also suggests that areas near the back shroud are more prone to erosion than any other part of the blade surface.

Although particles impact in most of the areas lying within a distance of 0.23s' measured from the leading edge, the predicted erosion pattern suggests a concentration of erosion prone areas near the back shroud. This can be explained by referring to the trajectory of particles shown in fig.'s 5.8a, 5.9a and 5.17a, and considering the fact that these represent the trajectories of the bulk of the particles distributed in the cross-section of the pipe as shown in fig. 5.15.

The erosion near the front shroud is not so severe. This is because the number of impacts in this zone is low compared to that near the back shroud. Particles which impact on this area are only those which enter the impeller near the bottom of the pipe, see fig. 5.7a.

## CHAPTER 6

### EXPERIMENTAL APPARATUS

#### 6.1 Objective of the Experiment

The objective of the experimental work in the present study is to identify the erosion prone areas on the blade surfaces of a granular-solids-handling centrifugal pump. Due to impact by solid particles, the blade surfaces of an impeller are eroded. It is most probable that the loss of volume from the blade surfaces are different at different parts of a surface. This means that different parts of a surface are subjected to different levels of erosion by the solid particles. Some of the areas are more prone to erosion than the others.

One way of estimating the level of erosion on a surface is to measure the profile of the surface before and after an experiment and compare the two sets of measurement. The difference in two sets of measurement at various positions on the blade surface will indicate the volume loss at these positions and hence the areas of the surface which are more prone to erosion by particles.

The above procedure for identifying the erosion prone areas requires a measurable volume loss from the surface of the blade. Where this is not achieved, mainly due to short duration of an experiment, the identification of the erosion prone areas directly from surface profile measurement is not possible.

The material surface hardness of the impeller used in the present work was very high (minimum 550 HB), full details of the impeller are given in section 6.2. It was put to handle a mixture of fine pea gravels in water. With this type of mixture, the impeller needed to be run for a period of at least 50 hours (Ref. 111) before any

noticeable erosion pattern emerges on the surface of the blade. This might have caused in the mean time attrition of the particles which required to be avoided for reasons detailed in section 4.3. It was, therefore, decided to apply a light coat of paints of different colour onto the wetted surface of the impeller so that by observing the colour contrast of the eroded painted surface, the areas of the painted surface which are more prone to erosion are identified. The painted surface in this way is hydraulically similar to the actual blade surface although its resistance to erosion by particles is completely different from that of the blade.

The erosion pattern of the painted surface depends on the dynamic characteristics of the particles impacting on the surface. In the analytical part of the present work, these characteristics were calculated and the erosion pattern that could appear on a painted surface corresponding to these characteristics was given (see chapter 5). Experiments with a painted impeller, therefore, provide a basis for comparing a predicted erosion pattern with an actual one. This is the major objective of the experimental part of the present work. The effect of increasing solids concentration in the mixture on the erosion pattern of the painted surface has also been studied.

In the following paragraphs, the descriptions of the pump used in the present work, characteristics of the solids and the test rig used to conduct the experiments are given. This is followed by a discussion of the experimental results.

## 6.2      The Pump

The pump used in the present work was made by Simon-Warman Limited in Lancashire, type 4/3 DAH Series A. It was a centrifugal end-suction slurry pump with 102 mm suction, 75 mm discharge, and specific speed of 0.51. The

pump consists of a 5-bladed impeller and a volute type casing. The impeller was of a closed design with eye diameter of 102 mm and tip diameter of 265 mm. Both the impeller and the casing are made of a wear resistant Nihard. Minimum hardness on the surface, as specified by the manufacturer, was 550 HB. The pump was fitted with a centrifugal seal which required no flushing water thereby eliminating any slurry dilution. In addition it was fitted with a grease lubricated packed gland to stop leakage while the pump was stationary. A general arrangement of the pump is shown in fig. 6.1. The impeller is shown in plate 2 and detailed drawings of the same are given in fig.'s 6.2a and 6.2b.

There are certain features in the design of the impeller which are briefly discussed below :-

- (i) The shrouds of the impeller are outwardly flared near the tip. This is in contrast to any conventional design (e.g., fig. 6.3a) where the shrouds are almost flat and parallel.
- (ii) The outside of the impeller is fitted with expeller vanes.

The thinking behind these design modifications over a conventional design has been discussed in a number of papers, see for example Ref. 14. The reasoning briefly is as follows. In a conventional design, the flow is discharged from the impeller centrally into the casing, pushing the low-energy fluid from the casing boundary layer into the impeller along the sides of the casing. A double-spiral flow pattern, as shown in fig. 6.3a, may result. Part of the flow enters the side clearance, between the impeller and the casing, causing wear of the wetted surface. By providing flared shrouds, fig. 6.3b, the discharge from the impeller is directed outward against each side wall of the casing. A double-spiral flow directed inward into the impeller away from the sides of the casing is produced. Wear in the side spaces between the impeller and the casing is thereby reduced. In addition, the expeller vanes fitted

on the outside of the impeller, produce in the side spaces a head, opposing leakage, of the same order as the head produced by the main blades of the impeller.

### 6.3 The Solids

The solids used in the present work are fine pea gravels, supplied by Joseph Arnold and Sons Limited, Luton (supplier's specification -- fine pea gravel graded 8/16). A view of a sample of solids used is given in plate 1.

The following characteristics of the solids were measured :-

- (i) size distribution
- (ii) specific gravity
- (iii) settling velocity

Details of the measurement are given below.

#### 6.3.1 Size Distribution of the Solids

Size distribution analysis of the solids was made by taking a sample of solids of weight about 106 kg and dried in an oven for a period of 3 hours at 21.11 °C. The solids were sieved using ten B.S.410 sieves which were stacked in a vibrating machine (Endecott Test Sieve Shaker) for a period of 2 hours. The solids retained in each sieve were collected in a number of polythene containers. The weight of each container with and without the solids was measured in an electronic balance (Mettler H315, Mettler Instrumente AG, Zurich, Switzerland). Measurement of weight upto 4th decimal point in gm was obtained. From these measurements the weight of the solids retained by each sieve and hence cumulative weight and percentage of the solids passing through each sieve size were determined. The results are given in Appendix D. Plot of size analysis may be found in fig. 3.2. From this figure the  $d_{50}$  size of the particles was calculated to be 1.8 mm.

A further sieve analysis of the particles was made after the end of all experiments in the test rig. Results are given in Appendix D and plot of the analysis can be found in fig. 3.2. The  $d_{50}$  size of the solids was reduced to 1.71 mm. Hence, some degradation of the particles had taken place during the course of the experiment.

#### 6.3.2 Specific Gravity of the Solids

The specific gravity of the solids was measured by taking a small sample of solids in a gravity bottle (made of borosilicate, size 100 cm<sup>3</sup>, B.S.733). Three sets of measurements were taken.

The bottles were first carefully cleaned in tap-water and dried in an oven at a temperature of 21.11 °C for a period of 6 hours. These were then weighed in Mettler RK2000 electronic balance. The digital readout of the balance showed weight reading upto 2 decimal points in gm. Samples of solids were then taken in each bottle and the weight of the sample plus the bottle was measured in the balance. The bottles were then filled with tap-water. To remove air that might have been trapped in the space between the solids, the bottles were gently vibrated in an ultrasonic bath (Dawe Instruments, U. S. A.) for a period of 25 minutes. The bottles were then weighed in the balance. The solids were removed, the bottles carefully cleaned in tap-water, filled with water and weighed in the balance. From these measurements the specific gravity of the solids was determined. Detail of the measurement is given in Appendix E. The value of specific gravity of solids obtained from three sets of measurement were 2.656, 2.675 and 2.645. The mean value of the specific gravity of solids measured was, therefore, 2.659.

#### 6.3.3 Settling Velocity of the Solids

To measure the settling velocity of the solids,

a small sample of the solids was taken. Each particle in the sample was released to fall freely in water contained in a perspex cylinder. By timing the fall of individual particle for a certain distance and taking the mean of the readings for the sample, the settling velocity of the particles was determined.

The perspex cylinder used was 115.1 cm high. Its inside diameter was 11.6 cm. The ratio of the inside diameter to the characteristic particle size was 64.4, thereby eliminating any wall effect (Ref. 118). Two levels were marked on the face of the cylinder indicating a settling height of 90.2 cm. The upper mark was located about 10.0 cm below the top to permit the particles to acquire its terminal settling velocity before the start of the time of measurement. The lower level was marked 14.9 cm above the bottom to allow for accumulation of particles.

The cylinder was filled with water upto the edge of the open end. Each particle from the sample was released along the centreline of the cylinder. Each particle was held by a tweezer and slightly dipped into water before being released to avoid any entrainment of air by the particle should the particle be released above the surface of water. The particle fell by a distance of 10.0 cm before the time of free fall was recorded. In this distance first acceleration and then deceleration of the particle took place before the settling velocity was reached. A stop-watch was used to measure the time of fall between the upper and lower marks. The temperature was recorded to determine the viscosity of water. Details of the measurement are given in Appendix F.

The maximum value of time measured was 7.5 sec and the minimum was 3.4 sec. These two values are due to the smallest and largest size of the particles in the sample. Mean of all the measurements (arithmetic mean of 30 measurements) was 4.86 sec. Mean settling velocity was



18.56 cm/s. The Reynolds number of the particles in the sample, based on the settling velocity and characteristic particle size,  $d_{50}$ , was in the range of 212-468. Hence, the settling of the particles occurred in the transition regime, Ref. 76.

#### 6.4 System Layout

The general concept and layout of the test rig is shown in fig. 6.4 and plate 3.

The basic components of the test rig are :-

- (i) the pump-motor assembly
- (ii) a mixing hopper
- (iii) concentration meters
- (iv) a flowmeter

A brief description of the components is given below.

##### 6.4.1 Pump-Motor Assembly

The description of the pump used in the present work is given in section 6.2. The pump was driven by a 75 HP variable-speed D.C. motor. A thyristor controller enabled a continuous speed adjustment. The rotational speed of the pump was measured at the pump-motor coupling by a Radio Spares electromagnetic pick-up linked to a digital readout system. This instrument times the period of revolution and inverts the signal to give speed. A view of the pump-motor assembly can be found in plate 4.

##### 6.4.2 Mixing Hopper

The hopper acted as a store for the abrasive mixture when the rig was in use. It is cylindrical in the upper part and forms a conical shape in the bottom part. The nominal diameter of the cylindrical part of the hopper is 124 cm which reduces to 12.0 cm at the conical end of the hopper where it is connected to the suction line of the pump. The height of the cylindrical part is 123.0 cm and

the conical part is 93.0 cm. The way in which water or mixture may be drawn into the suction line of the rig is detailed in section 6.5.

#### 6.4.3 Concentration Meters

The rig is fitted with a number of meters to measure mean transport concentration and mean discharge concentration of the solids in the mixture. These are :-

- (i) counterflow meter
- (ii) slurry manometer
- (iii) pipe-weighing concentration meter
- (iv) weigh tank device

Meters (i) to (iii) measure mean transport concentration, and meter (iv) mean discharge concentration (for definitions of concentration terms, see Appendix B). Meters (i) and (ii) are pressure differential type and (iii) and (iv) are gravimetric type.

A brief description of the construction and operation of these meters is given below.

##### 6.4.3.1 Counterflow Meter

This device (fig. 6.5) measures the mean transport concentration in an inverted U-loop of pipe. The pressure difference between two pairs of piezometer ring tappings are measured using an air-over-water manometer. One pair of the rings is in the upward flowing limb of the U-loop and another pair in the downward flowing limb of the loop. The rings are apart by a distance of 1.53 m in both the limbs. The bore of the pipe is 67.44 mm. In one limb of the loop, the friction and buoyancy heads are in the same direction while in the other, these two heads are in opposite directions. The mean transport concentration can be measured from the following relationships (Ref. 119):-

For upward flow , the air-over water manometer reading is :-

$$h_1 = h_{f_1} + L_C \bar{c}_t (S_s - 1) \quad (6.1a)$$

For downward flow, the air-over water manometer reading is :-

$$h_2 = h_{f_2} - L_C \bar{c}_t (S_s - 1) \quad (6.1b)$$

Rearranging terms,

$$\bar{c}_t = \{(h_1 - h_{f_1}) - (h_2 - h_{f_2})\} / \{2 L_C (S_s - 1)\} \quad (6.2)$$

Equation 6.2 suggests that in order to be able to measure  $\bar{c}_t$ , the friction head loss due to the mixture in each limb of the U-loop should be known in advance. It is shown by Condolios et al (Ref. 120) that in a vertical pipe the friction head loss is not affected by the presence of solid particles. It is, therefore, assumed that friction head loss with a flowing slurry is given by clear water head loss at the same flowrate. Hence, the concentration meter is first calibrated for friction head loss on clear water at various flowrates. The calibration curve may subsequently be used to find friction head loss for flowing slurries at various flowrates.

#### 6.4.3.2 Slurry Manometer

This device (fig. 6.6) measures the mean transport concentration. The basic principle of measurement is similar to the counterflow meter. This instrument consists of a length of 64 mm bore perspex pipe mounted on an upward flowing part of the vertical loop of the rig. The pipe has two large bore inclined tappings 0.61 m apart. By measuring the head difference between the tappings using

an air-over water manometer, the mean transport concentration between the tappings can be calculated from the following relationship (Ref. 88) :-

$$\bar{c}_t = (h_m - h_{f_m}) / \{L_m (S_s - 1)\} \quad (6.3)$$

It is suggested in Ref. 120 that to account for the difference in settling velocity and flow velocity, the following expression can be used.

$$\bar{c}_t = (h_m - h_{f_m}) / \{(1 + w/\bar{v}_m) L_m (S_s - 1)\} \quad (6.4)$$

#### 6.4.3.3 Pipeline Weighing Device

This device (fig. 6.7) consists of two 3.0 m lengths of 75 mm bore steel pipe coupled together and to the rest of the pipeline by means of flexible rubber coupling. It is a gravimetric type concentration meter. The lengths of pipe are supported by counterweights placed in the middle of each length and a load cell (range 0-11 kg). The load cells themselves are coupled to a balancing bar pinned to a fixed member at the centre thus equalising the tensions in the load cells. The system is calibrated by placing dead weights at intervals along the pipes, first with empty pipes and then with the pipes full of water.

Let  $R_w$  be the load cell reading when the pipe is full of water. Then the load cell reading is zeroed. If  $R_m$  is the load cell reading when there is a mixture in the pipeline, then the density of the mixture can be obtained from a simple linear relationship as follows :-

$$\rho_m = \rho_w (1 + R_m/R_w) \quad (6.5)$$

$$\bar{c}_t = (\rho_m - 1)/(\rho_{so} - 1) \quad (6.5)$$

Further detail of the meter can be found in Ref.121.

#### 6.4.3.4 Weigh Tank Device

This device is used to determine the mean discharge concentration of the solids in the mixture. The flow in the test rig is taken to a weigh tank via a diverter valve for a short period of time. The shape of the weigh tank is similar to the mixing hopper, but much smaller in size. The bore of the cylindrical part of the tank is 60.1 cm and height is 45.0 cm. The bore of the conical part of the tank is 11.0 mm at the bottom. The height of the conical part is 44.4 cm.

The tank is suspended freely by three pressure gauge transducers which have a range from 0.0 to 226.8 kg. The transducer output is calibrated to give the weight of the sample in the tank directly as a readout on the control panel. An Endress and Hanser ultrasonic level sensor detects the free surface of the mixture in the tank. The sensor is calibrated to give the height of the mixture in the tank. The volume of the mixture can be calculated by knowing this height as the dimensions of the tank are known. The sample is discharged into the mixing hopper after all the measurements have been recorded, by opening the Linaflow pneumatically controlled valve fitted in the bottom of the tank. The operation of the diverter valve as well as the pneumatic valve is remotely controlled. General view of the weigh tank device is given in plate 5.

#### 6.4.4 Electromagnetic Flowmeter

A 65 mm nominal bore electromagnetic flowmeter mounted in a vertical section of the rig allowed online measurement of the flowrate through the rig. The flowmeter

brand name 'MAGFLO', is a point electrode type and is made in U. K. by Flow Metering Instruments Limited.

#### 6.5 Operation of the System

The slurry pump takes suction from the mixing hopper using either bottom suction (by opening valve 2, fig. 6.4) or top suction (by opening valve 4 and /or valve 3). The system has the facility of easy start-up by drawing clear water or a very weak mixture from the top of the mixing hopper by opening suction valve 4 only, and switching gradually to the more concentrated mixture by opening either top suction valve 3 or bottom suction valve 2. The pump discharges into a vertical loop in which a slurry manometer, a counterflow meter and an electro-magnetic flowmeter are mounted in succession. On return from the vertical loop, the flow may be taken to a horizontal piping system incorporating the pipeline weighing concentration meter. The flow then goes to the diverter valve and returns directly or via the weigh-tank assembly to the hopper. When it is not required to use the pipeweighing meter, valve 5 is closed and flow is taken through valve 17 to the diverter valve.

The slurry concentration may be altered by controlling the openings of the valves in the suction line while keeping the pump speed unchanged. The valves used in the rig are Saunders butterfly valves and 8" Saunders diaphragm valve - type KB. Except for adjusting the slurry concentration by regulating the opening of the valve in the rig, the entire operation can be remotely controlled from a control panel (plate 6) where digital readouts on pump-speed, motor power, mixture flowrate, pipeweighing concentration meter and weigh-tank device are given.

#### 6.6 Calibration Tests

Calibration tests were made on the following

devices :-

- (i) Weigh tank
- (ii) Ultrasonic level sensor
- (iii) Electromagnetic flowmeter
- (iv) Counterflow meter
- (v) Slurry manometer

#### 6.6.1 Weigh Tank Calibration

The weigh tank was supported by three strain gauge pressure transducers. Calibration was done to find the weight of the contents in the tank directly from transducer readouts. The dead weight of the empty tank plus the valve fitted to the bottom of the weigh tank was used as the zero and the load cell reading was zeroed by adjusting signal conditioning unit. Calibration weights in kg.'s upto 200 kg were then placed in the weigh tank, the range of the load cell was adjusted to read upto 200 kg. The weights were then removed and the zero readjusted, if required. The process was repeated six times until no zero-readjustment was necessary. Detail of the method can be found in Ref. 122.

#### 6.6.2 Ultrasonic Level Sensor Calibration

Calibration of the Endress and Hansor ultrasonic level sensor was made to find the height of the liquid in the weigh-tank and hence the volume of the liquid since the geometry of the tank was known. The tank was filled with water upto the top of the conical section of the tank (see fig. 6.4. and the device adjusted to zero. A known volume of water is then poured into the tank. The height of water in the tank is known from the dimension of the tank and the readout adjusted accordingly. The whole procedure was repeated a number of times, for details see Ref. 122.

#### 6.6.3 Electromagnetic Flowmeter Calibration

Calibration of the flowmeter for clean water

was carried out by diverting the flow into the weigh tank and timing the volume collected in the tank. The volume of the water in the tank was known from the height of water in the tank given by the ultrasonic level sensor. The sensor was calibrated prior to this test. For details of the flow-meter calibration, see Ref. 122.

#### 6.6.4 Counterflow Meter Calibration

Calibration tests of this meter were made to find out the friction loss ( $h_{f_1}$ ,  $h_{f_2}$ ) in the lengths of pipe between two pair of piezometric rings as a function of the square of the flow velocity,  $\bar{v}_m$ .

The rig was run on clean water and the flowrate measured by the electromagnetic flowmeter. The speed of the pump was changed to obtain a number of flowrates ranging from  $5.56 \times 10^{-3} \text{ m}^3/\text{s}$  to  $19.32 \times 10^{-3} \text{ m}^3/\text{s}$ . The flow velocity in the length of pipes between the piezometric rings of the counterflow meter (see 6.4.3.1) was obtained from these flowrates and the inside diameter of the pipe, which was 67.44 mm. The value of the flow velocity was in the range of 1.56 m/s to 5.41 m/s. The friction head loss between a pair of rings was measured in an air-over water manometer. The friction head loss was plotted against square of velocity to obtain the head loss factors,  $k'_1$  and  $k'_2$ , defined as follows :-

$$h_{f_1} = k'_1 \bar{v}_m^2 \quad (6.7a)$$

$$h_{f_2} = k'_2 \bar{v}_m^2 \quad (6.7b)$$

where, subscripts 1 and 2 refer to upward and downward flowing limbs, respectively of the counterflow meter. Coefficients  $k'_1$  and  $k'_2$  were calculated to be 0.0211 and 0.0191, respectively. The calibration curves are given



in fig. 6.8 and the experimental results for the calibration in Appendix G.

#### 6.6.5 Slurry Manometer Calibration

The calibration procedure is similar to the counterflow meter (section 6.6.4). In fact, calibration for the equipment was made at the same time when the counterflow meter was being calibrated. The operating conditions of the rig were, therefore, the same with the counterflow. The value of the coefficient  $k'_3$ , defined as :-

$$h_{f_m} = k'_3 \bar{v}_m^2 \quad (6.8)$$

was found to be 0.0149. The calibration curve is shown in fig. 6.9 and experimental results for calibration, in Appendix H.

#### 6.7 Experimental Procedure

The objective of the experimental work in the present study is discussed in section 6.1. In accordance with that two sets of experiment were conducted. These two sets differ from each other mainly in that the mean discharge concentration of the solids in the mixture was different to each other. The impeller was painted for reasons detailed in section 6.1. Basic steps in an experiment were as follows :-

- (i) Paint the impeller
- (ii) Run the rig
- (iii) Measure concentration
- (iv) Remove the impeller from the rig and observe erosion pattern on the blade surfaces

The steps are discussed in some detail in the following sections.

#### 6.7.1 Painting the Impeller

The wetted surface of the impeller was first carefully rubbed using a coarse grain silicon carbide paper to remove any rust from the surface. The surface was then cleaned using a degreasing fluid (no. 197-1005, Valentine Varnish and Lacquer Co. Limited, West Drayton) with cotton wool.

The next step was to apply a primer to the surface. High build etch primer, GP-2 100-50 (Valentine Varnish and Lacquer) was used.

The material used for painting the impeller was an automotive enamel brand name Valroc (Valentine Varnish and Lacquer), flash point 22-32 °C. The technical sales of the company was contacted for further information, but showed reluctance to release any technical detail concerning the paint. A paint hardener Valroc 08809 (Valentine Varnish and Lacquer) was mixed with the enamel at enamel to hardener ratio of approximately 4 : 1 as per instruction of the manufacturer. The mixture was then left for 30 minutes to allow the enamel to cure. The paint was then applied to the impeller surface. Care was taken to ensure that a uniform layer was applied on any part of the impeller. The paint was then allowed to dry for a period of 24 hours.

Four different colour layers were applied. The colour of the enamels in reverse order of application was as follows :-

- (i) golden yellow
- (ii) green
- (iii) orange
- (iv) electric blue

On completion of a run the remaining paint was removed using a water washable paint remover, NITROMORE (Wilcot Co. Ltd., Bristol).

#### 6.7.2 Operation of the Rig

Approximately  $0.9 \text{ m}^3$  of fine pea gravels (weight of gravels 1200 kg, specific gravity 2.659, assumed packing efficiency 0.5) were poured into the empty mixing hopper. The gravels filled almost 4/5th of the conical section of the hopper. The amount of gravels poured into the hopper had to be limited so that the suction valve 2 (see fig. 6.4) could be effectively used to regulate the flow of solids into the suction line from the mixing hopper. It was experienced in previous operations of the rig that the bottom of the conical section of the rig was blocked when the solids content of the rig was well above the top of the conical section of the hopper, and very little solids were drawn into the suction line through valve 2. To avoid this, only the conical section of the hopper was filled in with gravels. Water was then added to cover the clear water top suction.

The rig was started by drawing in a very weak mixture into the line through valve 4 while valves 2 and 3 were completely shut. The speed of the pump was gradually raised. It took about 1.5 minutes after the start of the pump before the mixture started circulating throughout the rig. This time interval was necessary to push the free surface of the mixture around the vertical loop of the rig (see fig. 6.4) as the pump speed was gradually increased. Once this condition was achieved, the speed of the pump was set at 1500 r.p.m.. The slurry suction valves 2 and 3 were gradually opened to increase the amount of solids drawn into the pipe. The mean discharge concentration of the solids in the mixture was measured by sampling the mixture via weigh tank diverter valve assembly. It took about 5 minutes before a mean discharge concentration of the solids in the mixture of 3.362% was achieved. The openings of the valves had to be carefully manipulated to reach this concentration (in fact, the rig was run previously employing the same impeller, not painted, to find the

appropriate valve openings that would give the desired concentrations). The mean discharge concentration and the mean transport concentration of the solids in the mixture was measured every 10 and 20 minutes, respectively. The mean transport concentration was measured by the counterflow and slurry manometers. The flowrate of the mixture was measured by an electromagnetic flowmeter. The results are given in Appendix J. The following were the experimental conditions for the first set of experiment (case 1).:-

- (i) Pump speed = 1050 r.p.m.
- (ii) Duration of experiment = 10 hours 30 minutes
- (iii) Mean discharge concentration,  $\bar{c}_d = 3.362\%$
- (iv) Mean transport concentration,  $\bar{c}_t = 3.034\%$   
(measured by counterflow meter)  
Mean transport concentration,  $\bar{c}_t = 3.661\%$   
(measured by slurry manometer)
- (v) Mean mixture flowrate,  $Q = 11.616 \times 10^{-3} \text{ m}^3/\text{s}$

The values of  $\bar{c}_d$ , and  $\bar{c}_t$  and  $Q$  are time averaged values. The duration of the experiment is considered to be the time between the opening of slurry suction valves 2 and 3, and opening of valve 4 when slurry suction valves 2 and 3 were closed. Usually it took about 5 minutes when most of the solids were collected in the hopper after the slurry suction valves had been closed. The mean discharge concentration measured after this time was 0.17%.

The performance characteristics of the pump when the pump was handling slurry could not be determined. The performance characteristics for clear water obtained by operating the pump in the present rig is reproduced in fig. 6.10 from Ref. 123. This figure does not show the best efficiency point as this was not reached because of the system resistance. From the manufacturer's information, the best efficiency point occurred at a flow coefficient  $(Q/\omega D_1^3)$  of  $13 \times 10^{-3}$ . It is reported in Ref. 88 that although the presence of solids suppressed the head produced by the pump, the best efficiency flowrate was close to the clear water value. At the initial stage of the present experiment,

only clear water was pumped at a flowrate of  $15.5 \times 10^{-3} \text{ m}^3/\text{s}$ . It was the maximum flow that was possible at the existent system resistance. The flow coefficient corresponding to this flowrate was  $9.703 \times 10^{-3}$ . The pump efficiency at this flowrate was calculated to be 64.7% from fig. 6.10. Also from extrapolation, the best efficiency of the pump at a coefficient of  $13 \times 10^{-3}$  was found to be 67.5%. Hence, when pumping water at the initial stage of the present experiment, the pump was operating at 95.85% of its best efficiency.

At  $\bar{c}_d = 3.362\%$ , the flowrate was  $11.616 \times 10^{-3} \text{ m}^3/\text{s}$  and corresponding flow coefficient was  $7.27 \times 10^{-3}$ . It is widely reported in literature that although in the presence of granular solids the efficiency of the pump falls below clear water values, the shape of the efficiency curve remains unchanged except near the zero flowrate (Ref. 6). Fig. 6.10 was used to find the difference between the maximum efficiency of the pump and the efficiency corresponding to a flow coefficient of  $7.27 \times 10^{-3}$ . The difference was 10.1%. This means that the pump was operating at 85.03% of its best efficiency.

After the experiment the impeller was removed from the rig, and the surfaces were cleaned using moist cotton wool. Solid particles eroded the painted surfaces of the blade at different magnitudes. Different zones of the surfaces showed up different colour. The boundary of each zone was traced onto a tracing paper and later reproduced in a 2-dimensional plane. More about this is discussed in the following section.

The total duration of the experiment had to be phased out into a number of sessions lasting 90 minutes each. The operation sequence in each session was to start the rig with a very weak mixture and gradually switch onto a mixture of desired concentration, and finally purging the rig with a very weak mixture before stopping the rig. The

casing of the pump was then removed and the blade surfaces were inspected. This phased out experiment was necessary to ensure that bare metal was exposed only at the minimum places on the blade surface, so that the erosion at other parts on the painted blade surface could be compared with these places, and an erosion gradient could be defined on the blade surface.

At the end of first set of experiment the paint was stripped off the impeller, and the impeller was cleaned, dried and repainted for the next experiment.

In the second set of experiment, the operating conditions were the following :-

- (i) Pump speed = 1050 r.p.m.
- (ii) Duration of experiment = 2 hours
- (iii) Mean discharge concentration,  $\bar{c}_d = 11.653\%$
- (iv) Mean transport concentration,  $\bar{c}_t = 10.990\%$   
(measured by counterflow meter)  
Mean transport concentration,  $\bar{c}_t = 13.136\%$   
(measured by slurry manometer)
- (v) Mean mixture flowrate,  $Q = 12.198 \times 10^{-3} \text{ m}^3/\text{s}$

The experimental procedure was similar to that detailed for first set of experiment. The flow coefficient of the pump when handling slurries was  $7.636 \times 10^{-3}$ . This meant that the pump was operating at 87.41% of its best efficiency.

## 6.8 Observed Erosion Patterns

### 6.8.1 Observed Erosion Patterns in Case 1 ( $\bar{c}_d = 3.362\%$ )

The following observations were made :-

- (i) Particles eroded a painted surface at different magnitudes on different parts of the surface. In some parts, the painted layers were completely eroded and bare metal was exposed. In other parts

erosion could not reach bare metal and as such different colours were visible on different parts of the surface.

- (ii) To indicate the gradient of erosion on the surface, a level was attached to the erosion of the surface as follows. Places on the surface, where the top layer of paint was not removed by the particles, were considered to have undergone minimum erosion and the erosion rating for these places was considered to be 1. In places where the second layer of paint was visible (implying that the first layer of paint of a different colour was completely eroded), the erosion rating was considered to be 2. In places where a painted surface was completely eroded and bare metal was exposed, the erosion rating was considered to be 5. The erosion pattern of the blade surfaces, both pressure side and suction side, are traced and reproduced in a 2-dimensional plane as shown in fig.'s 6.11a to 6.12e. (the impeller used had five blades and the blades were numbered 1 to 5; the surfaces were numbered accordingly). The wear classification given in the figures represents the erosion ratings discussed above. A view of the erosion pattern of some of the painted blade surface can be found in plates 7 to 10.
- (iii) The areas near the leading edge on both pressure and suction surfaces suffered the most severe erosion. See plate 11 for a general view. These areas extended from the leading edge downstream to a maximum distance of 5 mm, the width of these areas on the blade surface was between 13 to 26 mm.
- (iv) On a blade pressure surface, the eroded areas were not symmetric about the centreline of the blade surface and in most cases these were

close to the back shroud.

- (v) On all pressure surfaces there was no erosion after a distance of  $0.35s'$  from the leading edge of the blade.
- (vi) The area on a blade pressure surface bounded by the back shroud and the centreline of the blade surface suffered more severe erosion than that bounded by the centreline and the front shroud. Erosion types rated 3 and 4 mostly occurred in the former zone. In contrast, areas very close to the back or front shroud showed minimum erosion.
- (vii) Away from the leading edge and outside the zones indicated in item (iii), the central part of the blade surface showed low level erosion (type 2 or 1).
- (viii) It was also observed that the erosion patterns on either the pressure or suction surfaces for different channels were different to one another. It was mainly due to the following reasons. The impeller used in the present work was rough cast. The blade surfaces were not smooth and the profiles of the surface were not similar to one another. Hence despite the effort taken to provide a uniform thickness of the painted surface on all the blades, it was impossible to guarantee that all the surfaces were painted to the same thickness. As a result, surfaces eroded to the same depth showed up different patterns on different blades. In fig. 6.11f, the average erosion pattern of the blade pressure surfaces is given. The following steps were involved. The blade pressure surface was drawn onto a 2-dimensional plane and a grid formed on it similar to the manner detailed in section 4.3. Corresponding to the erosion pattern on five blade surfaces, the wear index for an element was calculated for each surface, added



and divided by five. For fine classification of the area of an element, some elements were divided into two triangles each.

- (ix) Erosion of the suction surfaces was very low and mostly confined to small areas near the leading edge of a blade.

#### 6.8.2 Observed Erosion Patterns in Case 2 ( $c_d = 11.653\%$ )

The traced out erosion patterns on the blade pressure surfaces in the present case are given in fig.'s 6.13a to 6.13e and the suction surfaces, in fig.'s 6.14a to 6.14e. The average erosion pattern on a blade pressure surface is given in fig. 6.13f.

The erosion pattern of a painted blade surface in this case was less spread than that in case 1 (section 6.8.1). This was due to the following reasons. That although the solids content in the mixture in this case was higher than in case 1, the duration of the experiment in the present case was 2 hours only while in case 1, the duration was 10.5 hours. The particles here eroded the painted surface to expose bare metal at a much faster rate than in case 1 and experiment was stopped as soon as bare metal was seen on any part of a painted surface. This allowed the erosion of different parts of the surface to be compared with the one where erosion was maximum, and a gradient of erosion to be defined on the surface. The philosophy underlying this experimental technique has been discussed briefly in section 6.7.2.

The erosion patterns in the present case showed strong similarity with the patterns observed in case 1 (fig.'s 6.11a to 6.11e) in terms of the distance away from the leading edge where erosion had taken place. In both cases erosion was high near the leading edge (see for example, fig.'s 6.11a and 6.13a). Away from the leading edge, erosion was high in both cases on areas located about 18 mm from the leading edge and 14mm from the back shroud. Areas on the blade surface damaged by particles were within 0.36s' in both cases.

Close similarity in erosion was also observed on all other blade surfaces. Since the speed of the pump was the same and the flowrates of the mixture were nearly equal

in both cases, an important conclusion that could be drawn from the experimental work in the present study was that the concentration of the particles would not significantly alter the erosion pattern on a blade surface provided the above two quantities were not changed.

## 6.9 Comparison Between the Predicted and the Observed Erosion Patterns

### 6.9.1 Comparison in Case 1

Fig. 5.14 shows the predicted classification of erosion on a blade pressure surface for the present case. The area on the blade surface where the particles impact are shown as fully shaded in fig. 5.13. Regardless of the gradient of erosion on the surface as indicated in fig. 5.14, fig. 5.13 suggests that the area on a blade pressure surface where particles impact as predicted by the analytical model were similar to those observed experimentally, fig. 6.11f. The extent of the damage on the blade surface measured from the leading edge downstream, fig. 6.11f, was very close to the extent of the impact area predicted in fig. 5.13. Hence, as a first step in erosion prediction the analytical model identifies with good accuracy the area on a blade pressure surface which is affected due to impact of particles.

In making a comparison of the predicted classified pattern (fig. 5.14) with any observed pattern, one basic assumption made in the development of the analytical model needs to be restated. The model assumes that maximum erosion occurs in only one elemental area on the surface and compares the erosion on other parts of the surface with the maximum erosion. In conducting an experiment in the present work it was impossible to ensure that the experiment was stopped as soon as bare metal had been exposed at any part of the surface. The fact that in the observed erosion pattern shown in fig. 6.11f, bare

metal was exposed at a number of places, indicates that the erosion on other parts of the surface had taken place after bare metal had already been exposed at some part of the surface. Hence, any direct comparison between the observed pattern and the predicted one is not possible. Nevertheless, the observed patterns on all the blade pressure surfaces (fig.'s 6.11a to 6.11e) suggest that erosion near the leading edge was very high. This is also predicted analytically in fig. 5.14 in which the values of the 'wear index' in areas near the leading edge range between 3 to 5. Also, fig.'s 6.11a to 6.11e suggest that the area close to and bordering on the back shroud had undergone very little erosion. This is clearly shown in the predicted pattern.

It is also predicted that the erosion on the blade surfaces would concentrate on areas near the back and the front shroud. Comparing with fig. 6.11f, some agreement between the observed pattern and the predicted one in this respect is clearly evident, except that near the front shroud this concentration of the erosion areas is closer to the shroud in the predicted pattern than is actually observed.

The predicted pattern indicates minimum erosion (wear class 1) on areas halfway between the shrouds. Agreement with the observed patterns in this respect can be found in fig.'s 6.11b to 6.11e.

Finally, the predicted pattern indicates that erosion (wear class  $\geq 2$ ) occurs on the blade surface upto a distance of 0.356' measured from the leading. In the observed pattern (fig. 6.11f) this distance was similar.

#### 6.9.2 Comparison in Case 2

In fig. 5.18 the areas on the blade surface where the particles impact in this case is shown as fully shaded. The predicted classification of erosion on the

blade pressure surface is shown in fig. 5.19.

Comparing fig. 5.18 with the observed erosion patterns shown in fig.'s 6.13a to 6.13e, it is evident that the areas on the blade pressure surface which were affected due to impact of particles was identified well by the analytical model.

In fig. 5.19 high erosion is predicted on areas near the leading edge and the back shroud. Agreement with the observed patterns in this respect is fair, see fig.'s 6.13b to 6.13e. Erosion near the front shroud is predicted on areas observed experimentally. Also, the predicted pattern indicates minimum erosion (wear class 1) on areas halfway between the shrouds. Agreement with the observed patterns in this respect can be seen in fig.'s 6.13a to 6.13e.

#### 6.9.3      Reasons for Discrepancy Between the Observed and the Predicted Erosion Patterns

The discrepancy between the observed and the predicted erosion patterns in cases 1 and 2 is mainly due to the following reasons :-

- (i)            The erosion model (section 4.3.1) applied to the case of a painted surface assumed that the surface would fail in a ductile mode, due to the cutting action of the grains. Such an idealised model could only give an estimate of the loss of volume from the surface.
- (ii)           The bouncing characteristics of the particles were determined analytically in the present work. This was a solution where no experimental data were available.
- (iii)           The distributions of particles at the inlet of the impeller was shown to have a significant effect on the motion of the particles inside a blade channel (chapter 5). In the present study these

distributions were assumed to be similar to those reported in Ref. 68. Some error in this respect was possible.

- (iv) Finally, the surfaces of the impeller were not smooth as the impeller was rough cast. In spite of the best effort, it was impossible to ensure a uniform thickness of the painted surface at all parts on the blade surfaces. This might have had some effect on the results.

## CHAPTER 7

### CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

#### 7.1 Conclusions

An analytical method to identify erosion prone areas on the blade surfaces of a centrifugal impeller handling granular solids has been presented in this work. The validity of the prediction is checked by undertaking a number of experiments using a commercially available slurry pump handling a mixture of fine pea gravels in water, in a test rig.

The basic concept of the analytical model is to determine the individual particle trajectories in a computed flow field inside the impeller. The flow field is calculated by solving the conservation equations for the flow on a meridional stream surface extending from hub to shroud of the impeller. The equations are solved using a finite difference method. The trajectory of the particles is calculated by solving the equations of motion of particles in a stepwise manner using a time marching method.

In passing through the impeller, the particles hit the blade surface of the impeller. Analytical models are presented to calculate the rebound characteristics of the particles bouncing off a surface following an impact. The damage to the surface caused by the individual particle is estimated by calculating the velocity and the angle at which a particle impacts on a surface. The material property of the particle or the surface does not explicitly enter into calculations. In implicit form, the material property of the surface is considered to determine the manner in which the particles erode a surface. The relationship between the volume loss from

the surface and the dynamic characteristics of the particles should be known. It is shown that for a given combination of the particles and the surface, the material property of the particles and the surface can be isolated. In this way a relative measure of the depth of volume loss at different parts of the surface can be calculated. The surface can then be divided into a number of erosion prone areas.

The model was applied to two different cases. In the first case the impeller handled a mixture of fine pea gravels in water of mean discharge concentration of 3.362% while in the second case the concentration was 11.653%. The size distribution of the solids was in a narrow range. As such the  $d_{50}$  size of the particles (1.8 mm) was considered to be the characteristic size for all the particles and the equations of motion of particles were solved for particles of this size only.

The initial distribution of the particles entering the impeller was determined by comparing the experimental conditions of the present work with those reported in the literature. The number density of the particles in the cross-section of the pipe through which the particles entered the impeller was determined by developing a discretization procedure. The method was applied to the insitu discharge concentration profile of the particles. In this way the number as well as the position of the particles in the cross-section of the pipe was determined. It was shown that the initial distribution of the particles in the pipe before the particles entered the impeller would be significant in determining the location of individual particles inside the impeller. To amplify the point, a number of cases was considered in which the particles entered the impeller through different positions in the cross-section of the pipe. It was shown that in the flow field considered, particles which entered the impeller through the bottom part of the pipe traversed near the front shroud of the



impeller. These particles hit a blade surface near the front shroud. In contrast, particles which entered the impeller near the axis of the pipe were observed to pass near the back shroud. Also, the bulk of the particles entered the impeller halfway between the centreline and the bottom of the pipe and these particles mostly impacted on the blade surface near the back shroud.

An analysis of forces acting on the particles showed that as the particles entered a blade channel, the coriolis acceleration of the particles was higher than the acceleration due to centrifugal force or fluid dynamic drag by an order of magnitude of  $10^2$ . As such all the particles hit the pressure surface of the blade as soon as they had entered a blade channel. They hit at an impact angle which was in the range of 49-53 degrees. The impact angle subsequently decreased as the centrifugal acceleration gradually became equally significant as the coriolis term and the motion of the particles in circumferential and the radial direction became of comparable magnitude. The position on the blade surface where the particles last impacted on the surface was found to be related to the profile of the blade. In this respect, the design of the blade was important in effecting the extent of erosion on the blade surface.

Predicted erosion pattern on the blade pressure surface showed that erosion would occur in areas near the leading edge and near the front and back shroud of the impeller. Maximum erosion would occur in an area close to the back shroud and extend a short distance downstream from the leading edge. This observation was made in the first case when the mean discharge concentration of the solids was 3.362% as well as in the second case when the mean discharge concentration was 11.653%.

The experimental erosion pattern was different for different blade surfaces mainly because the surfaces were not smooth and this contributed to the

variation in erosion pattern between blades. Apart from this, the extent of damage downstream of the leading edge for the pressure side of all the blade surfaces was within 38% of the length between the leading edge and the trailing edge of the blade. Also, areas near the leading edge on all surfaces showed maximum erosion.

The predicted erosion pattern suggested that the area near the leading edge to be most erosion prone. Maximum erosion was also observed experimentally in areas near the leading edge, but slightly away from the back shroud, unlike the predicted pattern. The extent of the erosion zone on the blade surface as suggested by the analytical model was in close agreement with the experimental values. The analytical model also suggested that the major part of the erosion area would lie near the back shroud of the impeller. This was in close agreement with the experimental results.

## 7.2        Recommendations for Further Work

The flow field in the present work is assumed to be axisymmetric and the flow is calculated on a meridional stream surface (hub to shroud). It is recommended that a quasi three dimensional solution based on a combination of flow calculations on a blade to blade and hub to shroud stream surfaces be made. Also, the flow field should be calculated by including the viscosity of flow in the governing equations. This may be important in cases where flow separation or secondary flow is expected. The inclusion of viscosity in the governing equations would allow investigations of the change in viscosity, caused due to presence of solid particles, on the flow field.

Trajectory calculations are greatly effected by the way in which the rebound characteristics of the particles following an impact are calculated analytically. It is suggested that wherever possible experimental values for these characteristics should be used.

The analytical model has been applied to two cases of different mean discharge concentration of solids in the mixture. It is suggested that the model should be applied to higher mean discharge concentrations and profile of the insitu transport concentration determined experimentally.

In the experimental part of the present work the impeller was run close to the design point. Slurry pumps do not always operate at the design point in service. Hence, at off-design, the areas on the blade surface which are prone to erosion, should be investigated. It is suggested that the impeller should be run in the rig at a number of off-design conditions and erosion prone areas on the surface be identified. On the analytical side, the calculation of flow field at off-design conditions would require that the shift of the stagnation point around the leading edge should be considered. The flow calculations do not consider this. It is an area where further work is necessary.

Finally, the analytical model should be applied to a hydraulically similar impeller and the results be compared with the present work. Similarity in erosion of hydraulically similar machines is an area where future work is recommended.

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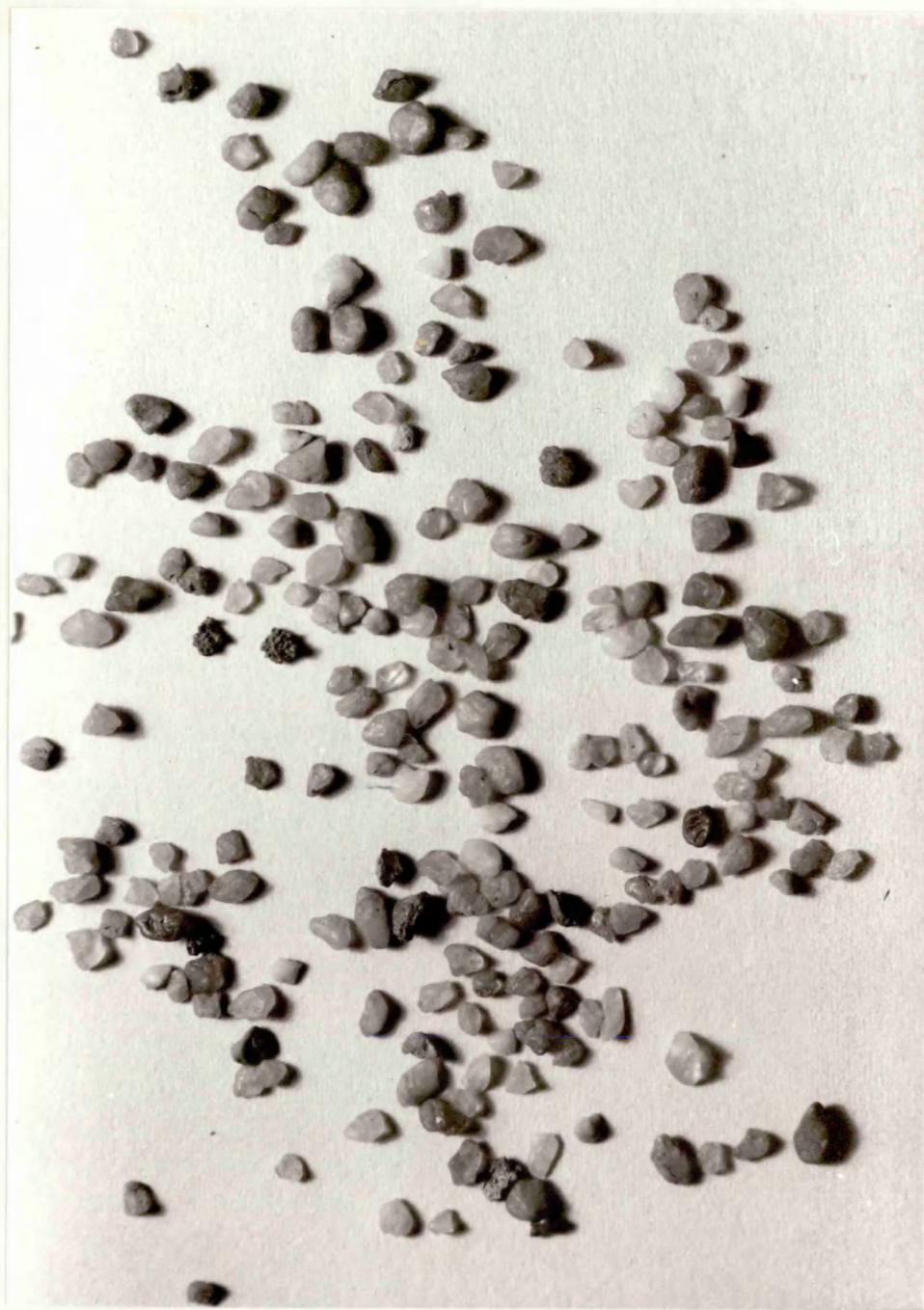


Plate 1

A view of the fine pea gravels used





Plate 2

A view of Simon Warman impeller



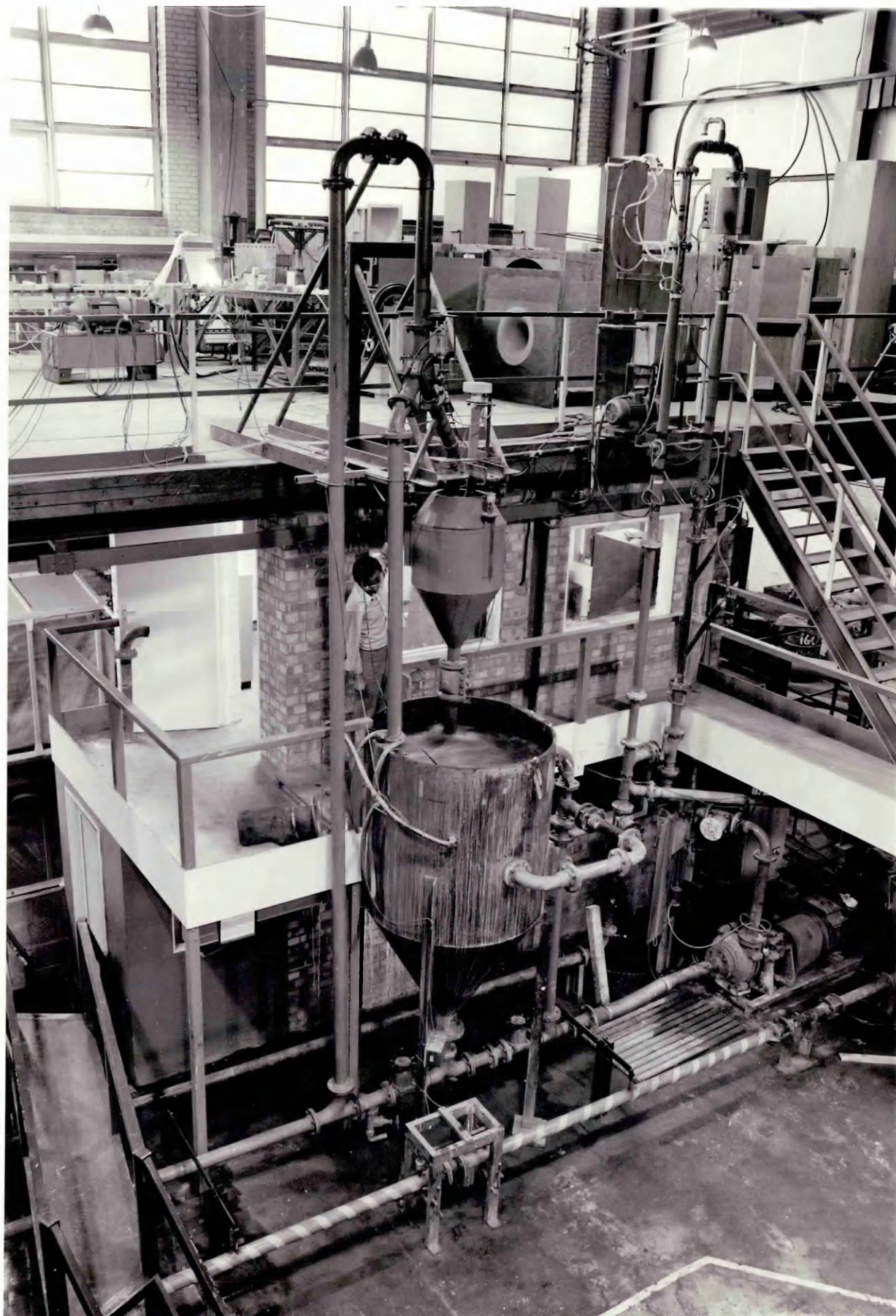


Plate 3  
General arrangement  
of the test rig



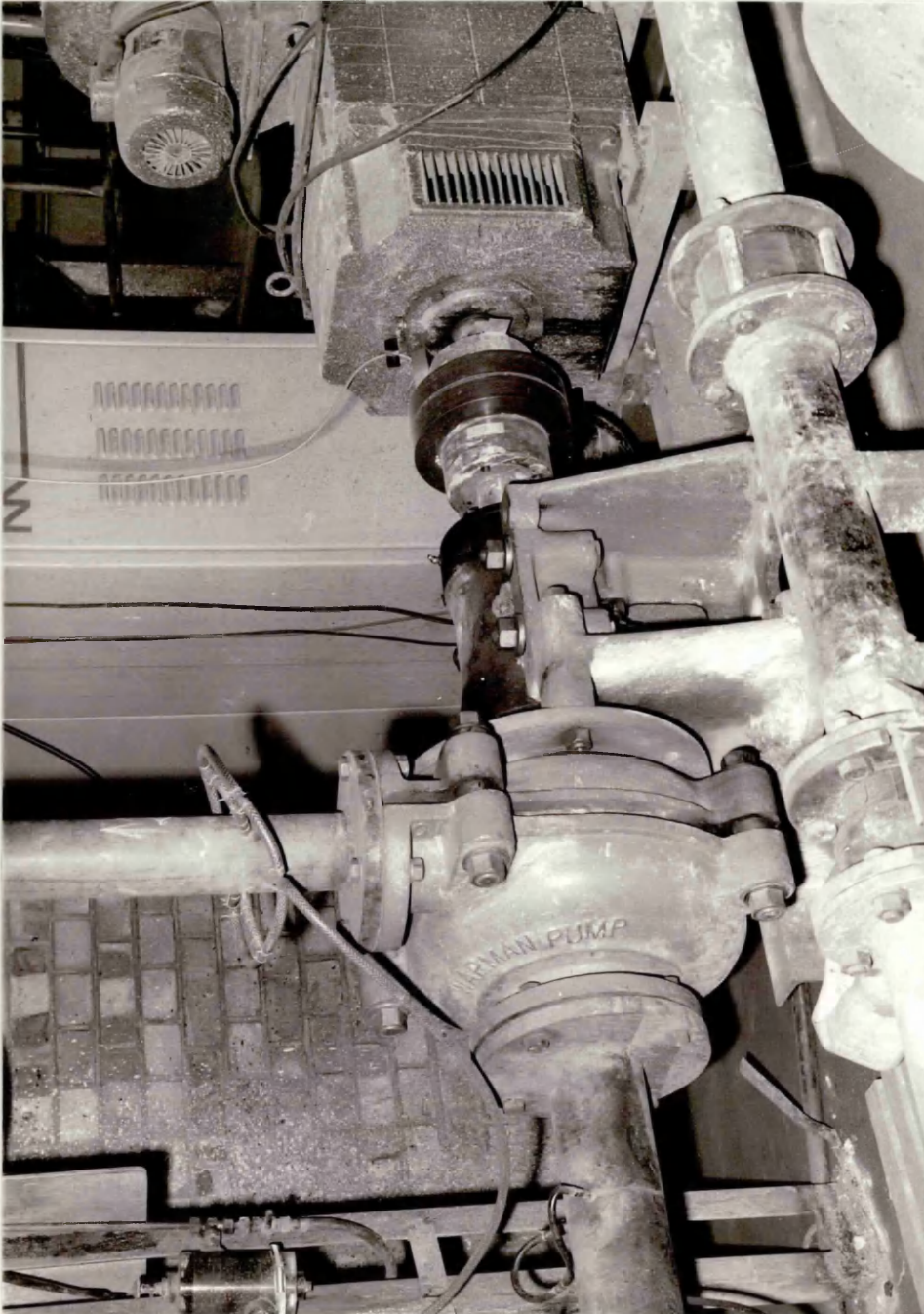


Plate 4  
General view of pump-motor assembly



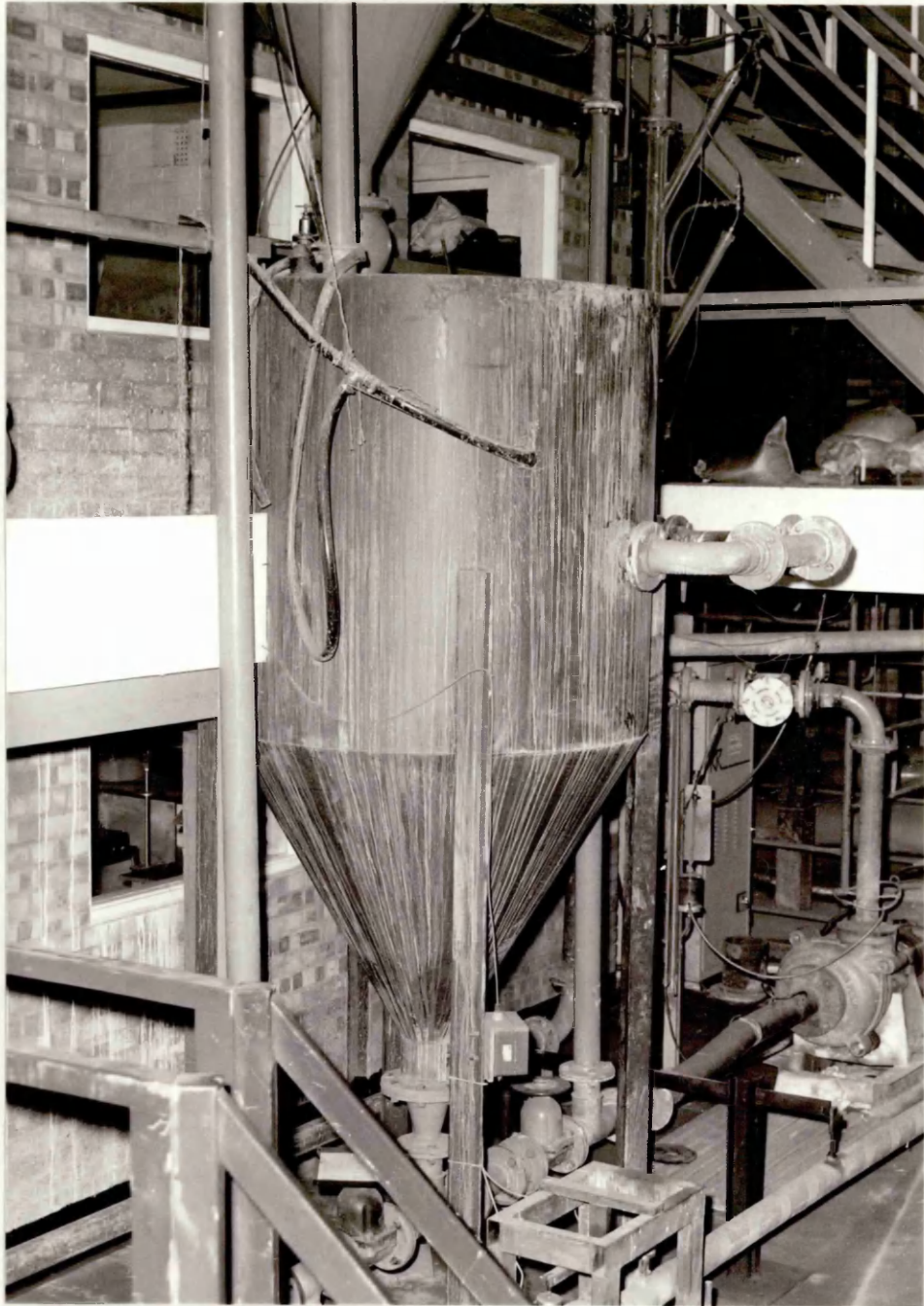


Plate 5

General view of the weigh tank device

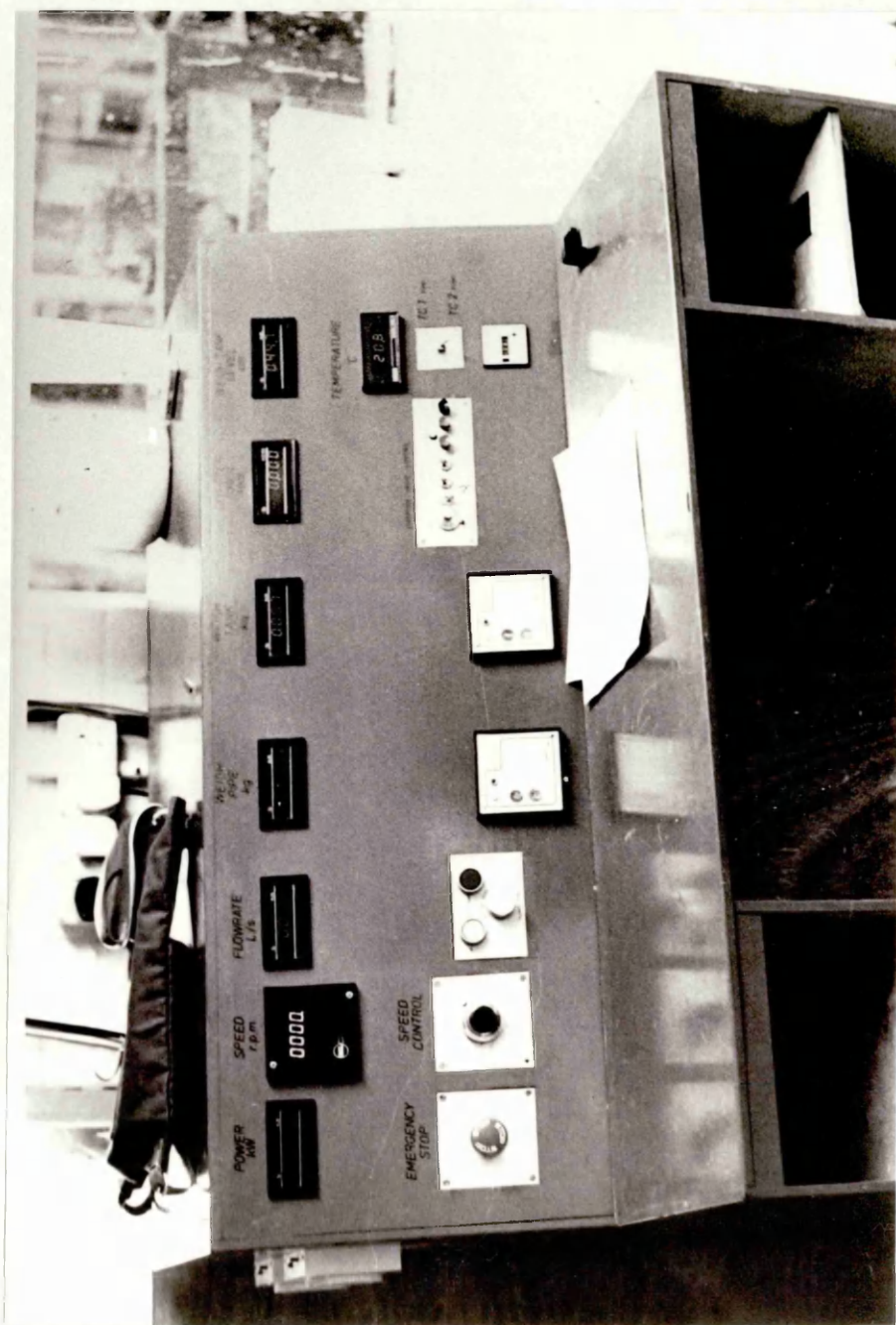


Plate 6  
General view of the control panel





Plate 7

A view of the erosion pattern on the painted  
blade pressure surface no. 1



Plate 8

A view of the erosion pattern on the painted  
blade pressure surface no. 3





Plate 9

A view of the erosion pattern on the painted  
blade pressure surface no. 5

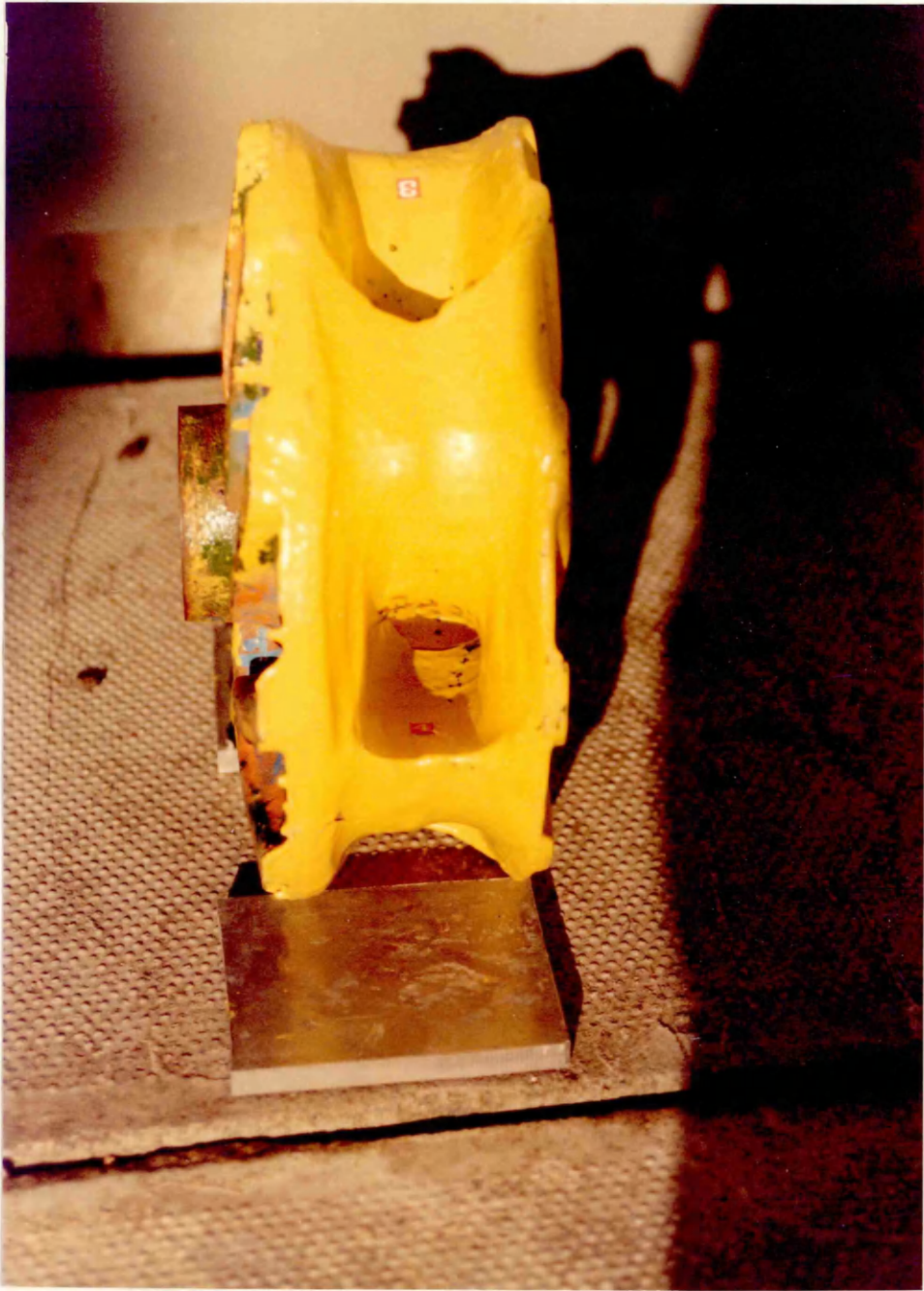


Plate 10

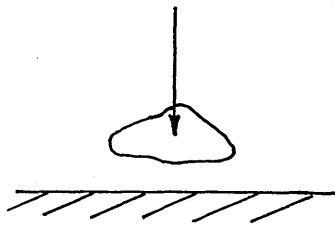
A view of the erosion pattern on the painted  
blade suction surface no. 5





Plate 11

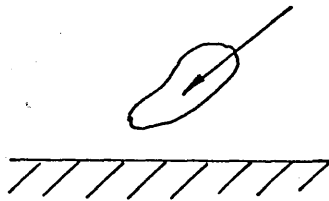
A view of the erosion pattern on the painted  
blade leading edge



(a) Deformation type



(b) Cutting type



(c) Oblique type

Fig. 1 Erosion abrasion types



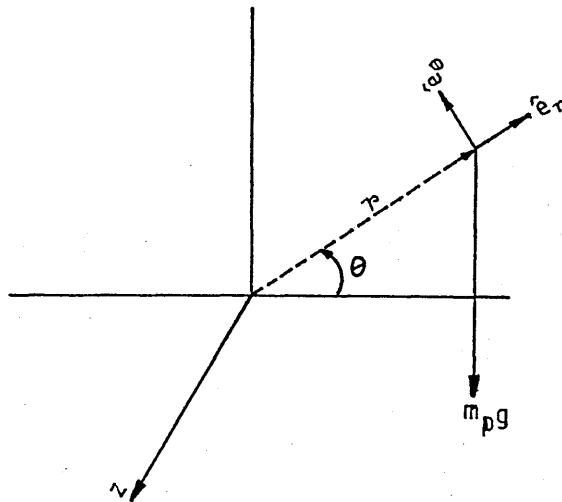


Fig. 2.1 Definition of coordinate system in the inertial frame.

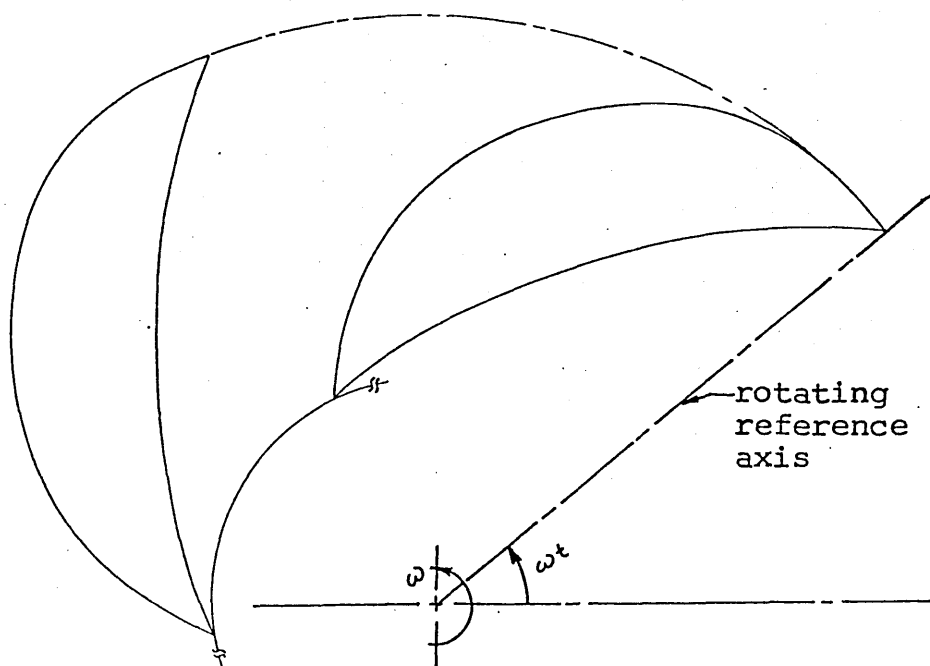
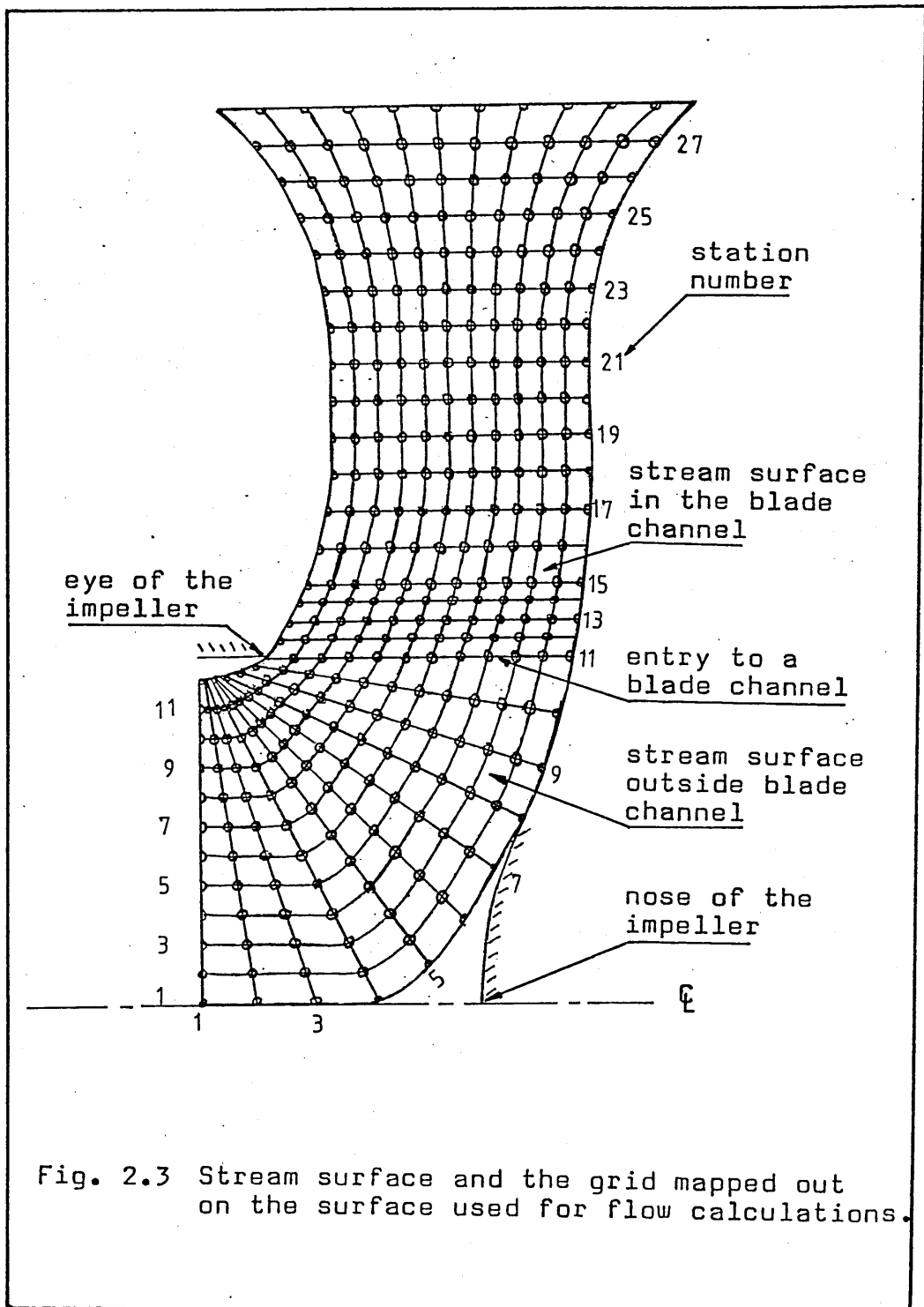


Fig. 2.2 Definition of rotating reference axis.



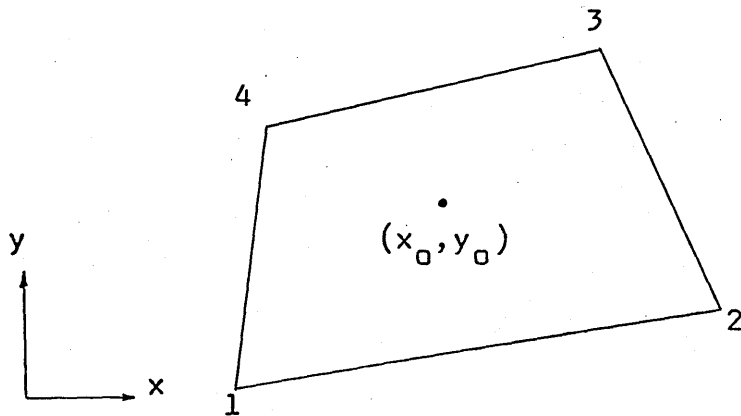


Fig. 2.4a Element defined in a physical coordinate system

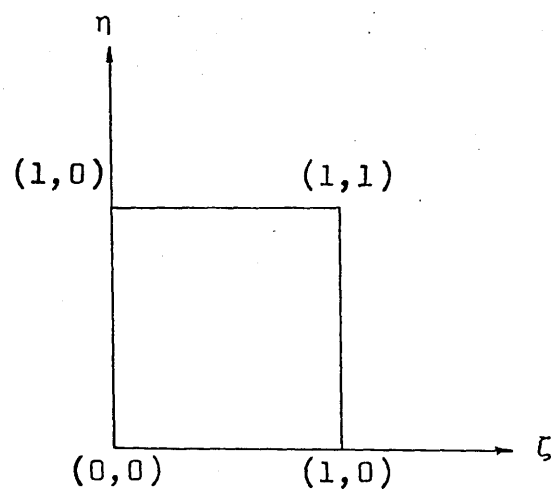
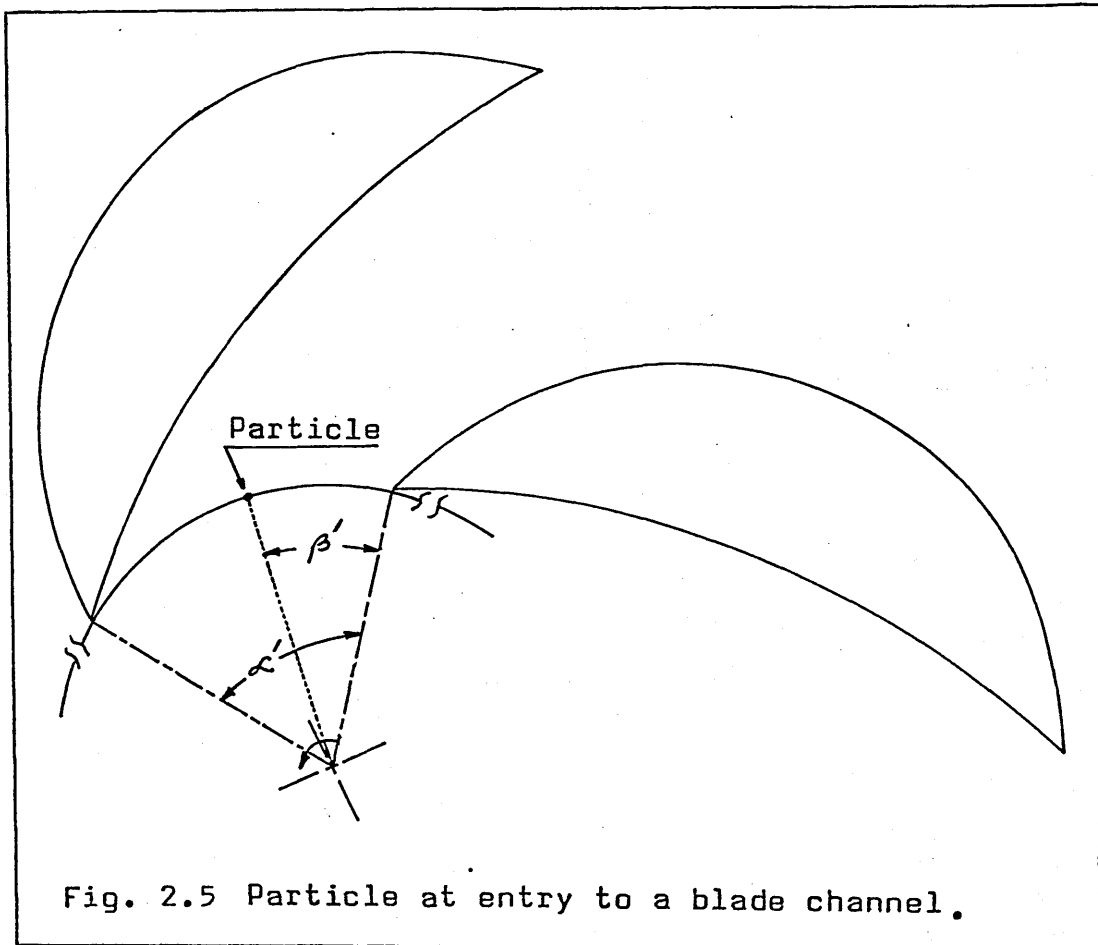
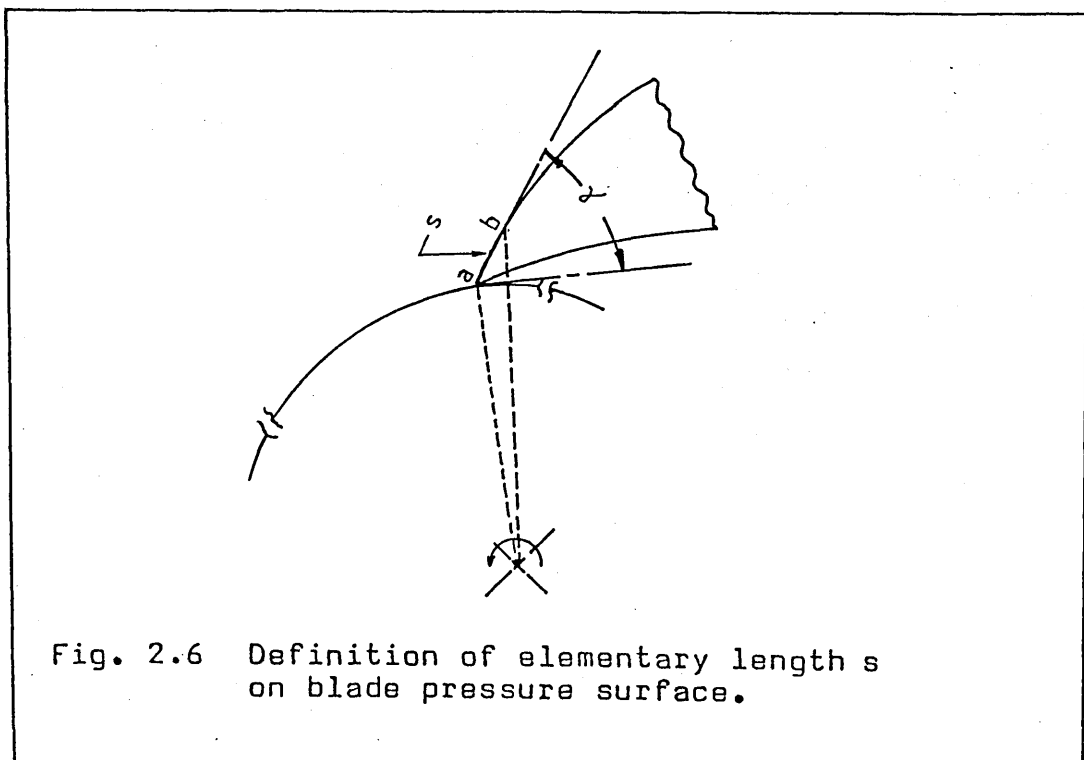
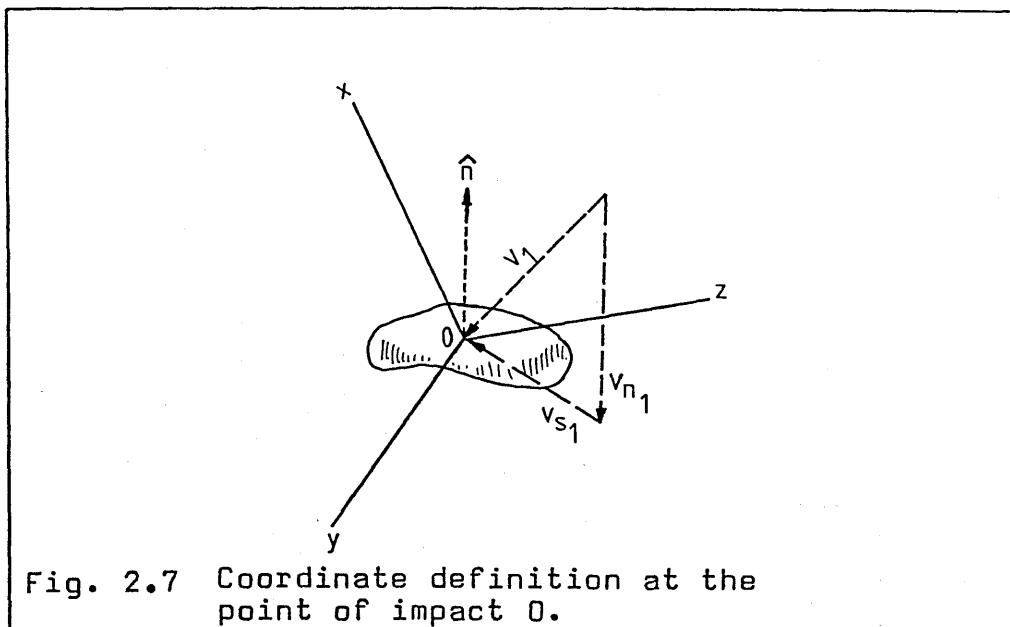
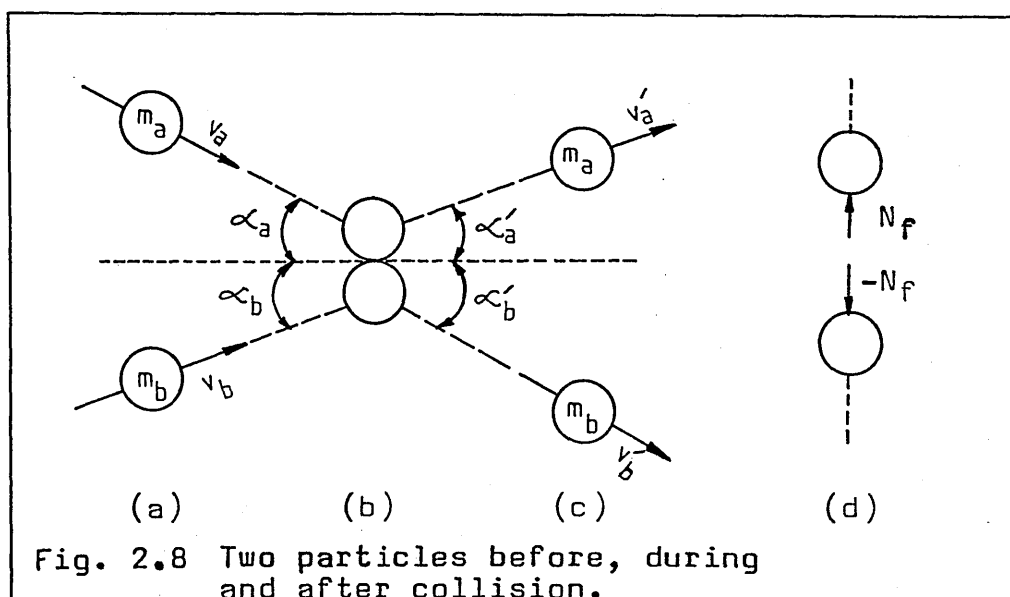
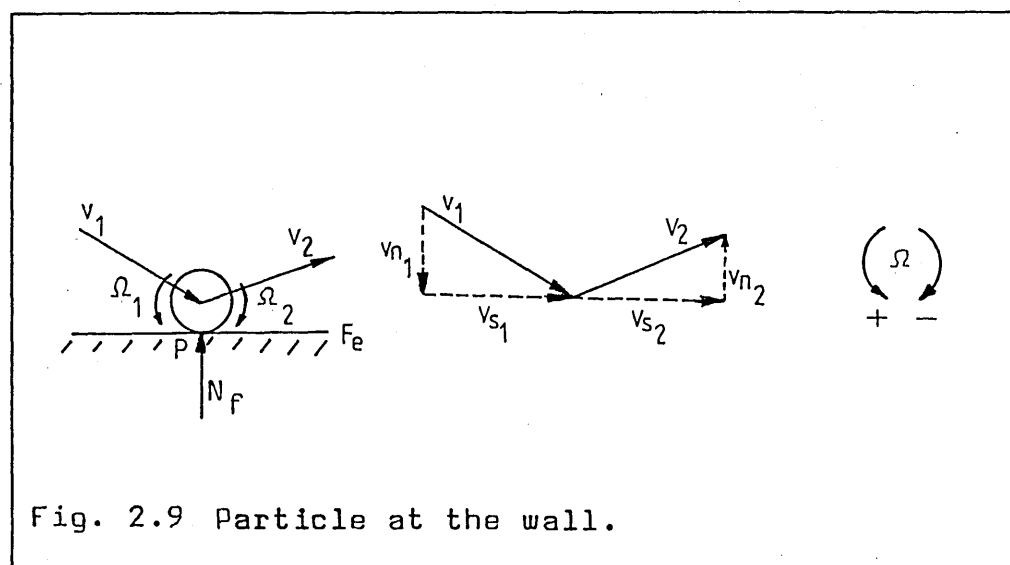


Fig. 2.4b Element defined in an intrinsic coordinate system





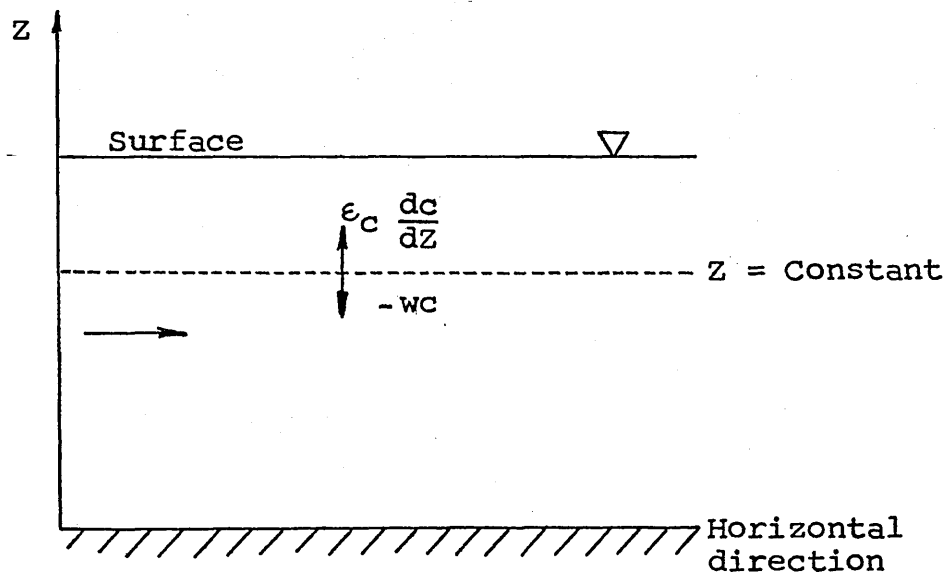


Fig. 3.1 Equilibrium due to settling and turbulence diffusion.

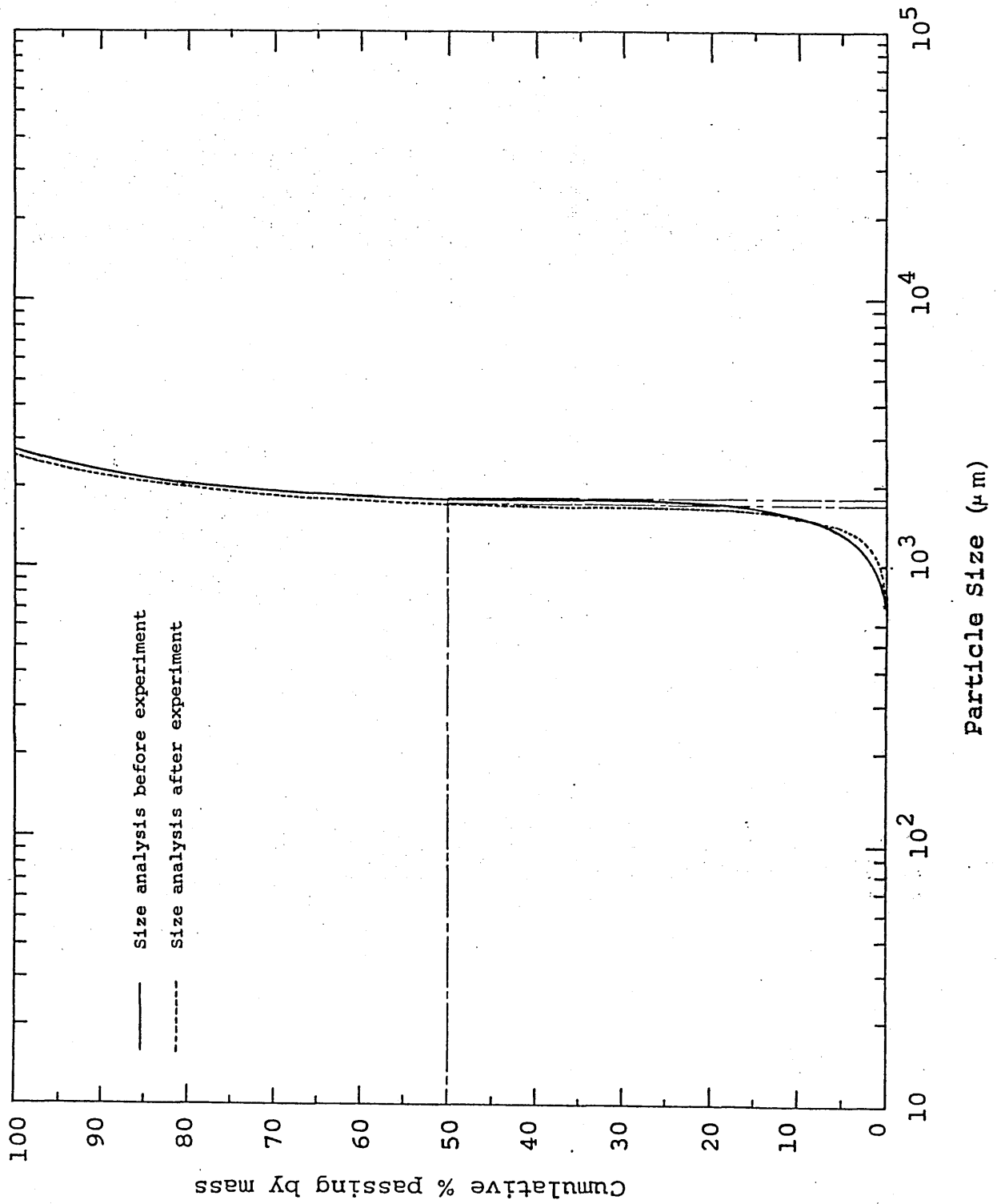


Fig. 3.2 Sieve analysis of fine pea gravels.

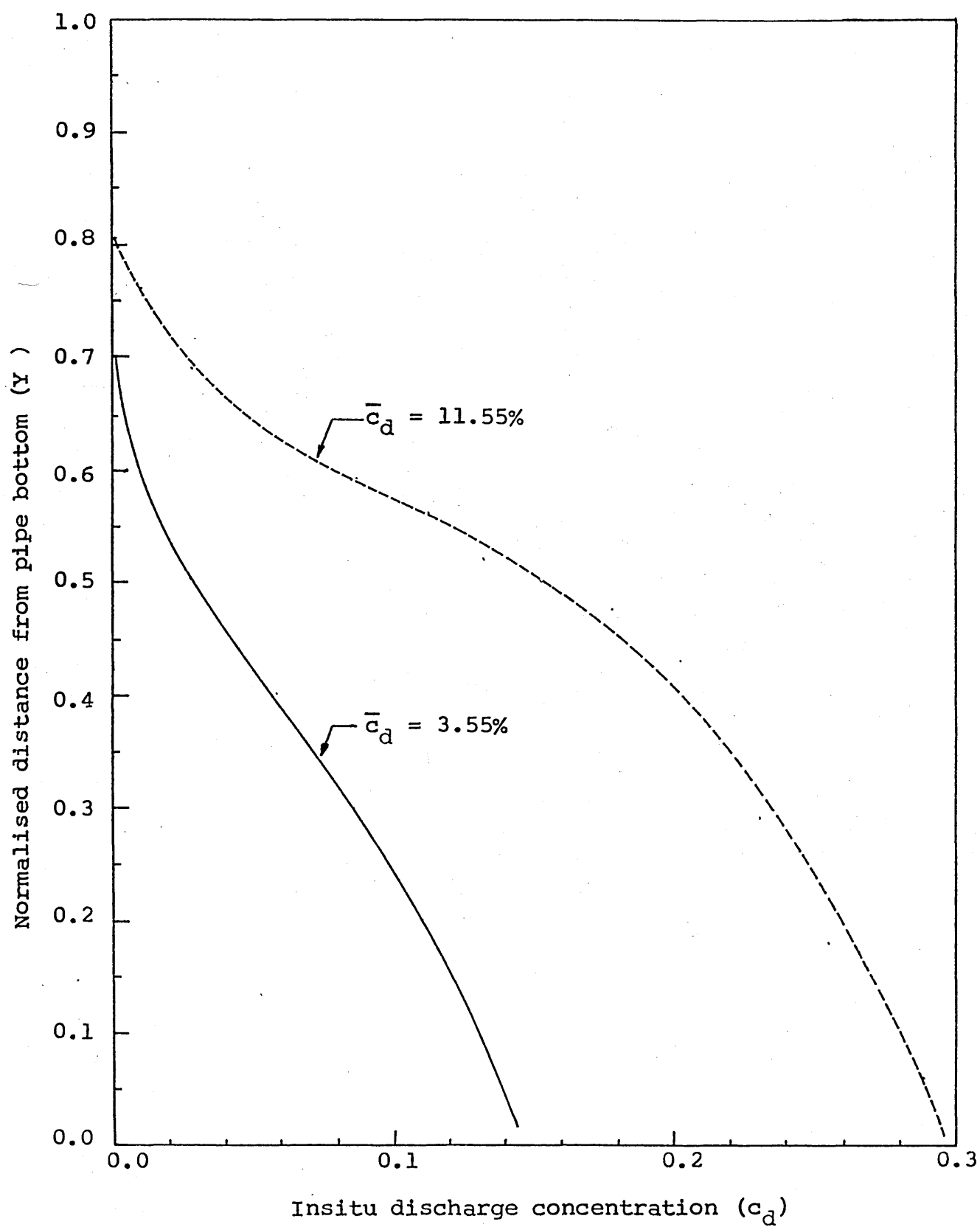


Fig. 3.3 Insitu discharge concentration along the vertical diameter of the pipe (Ref. 68)



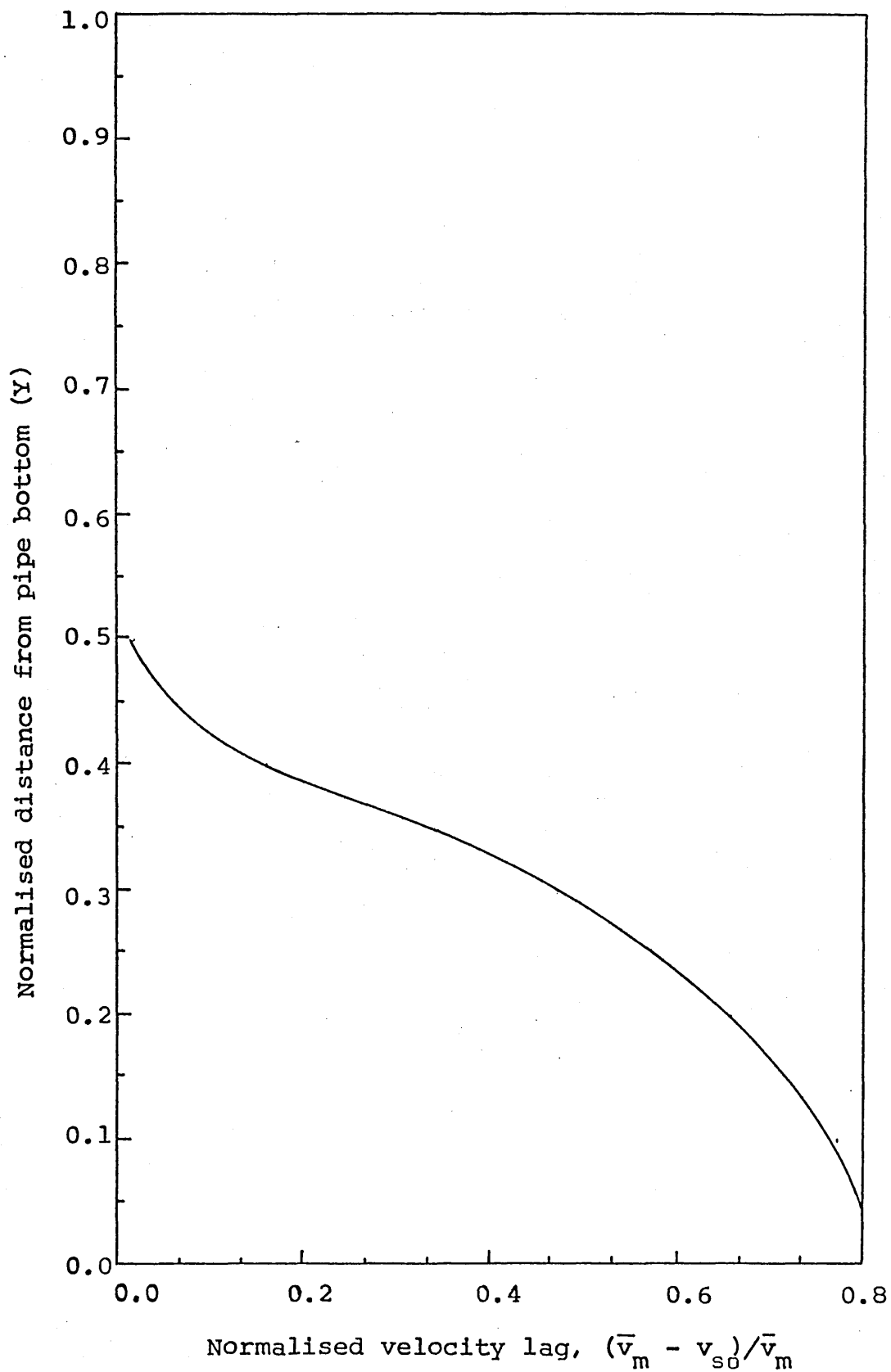


Fig. 3.4 Normalised velocity lag along the vertical diameter of the pipe (Ref. 68)

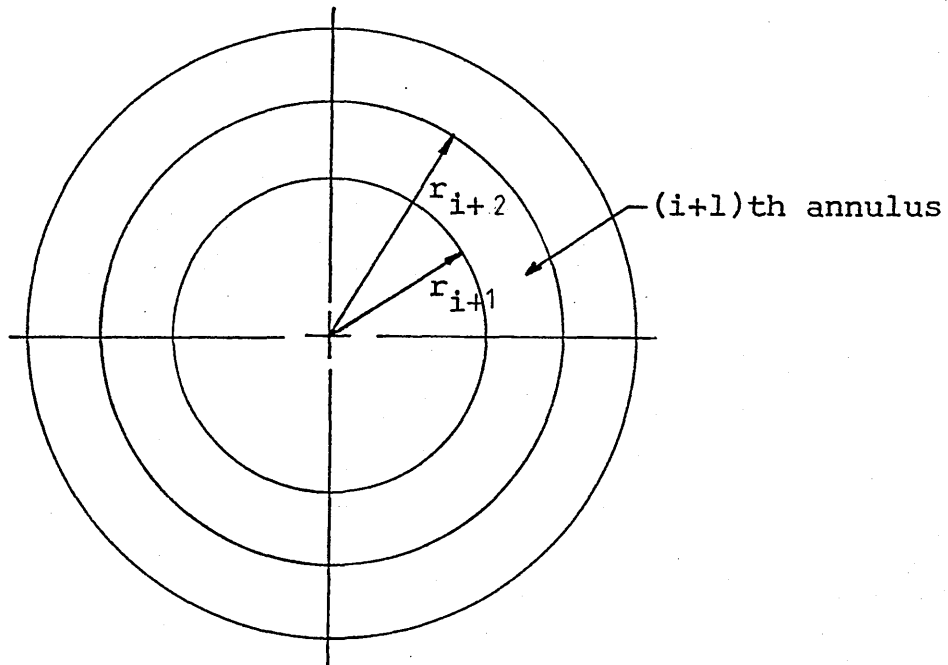


Fig. 3.5 Pipe cross-section divided into a number of annuli.

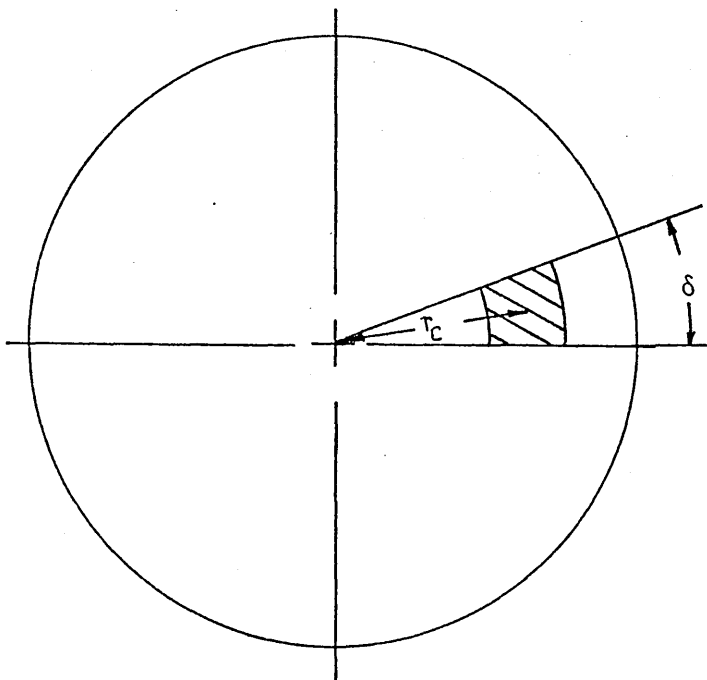


Fig. 3.6 Definition of centroid of an element.

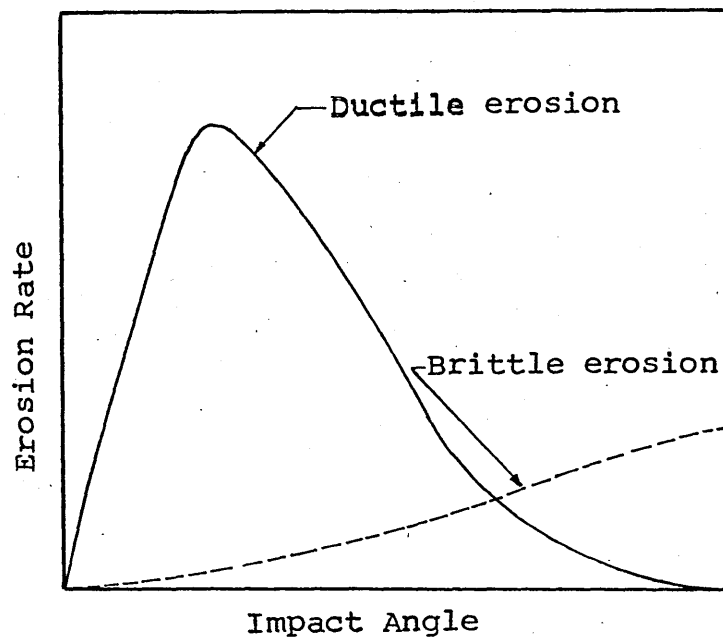


Fig. 4.1 Erosion models.

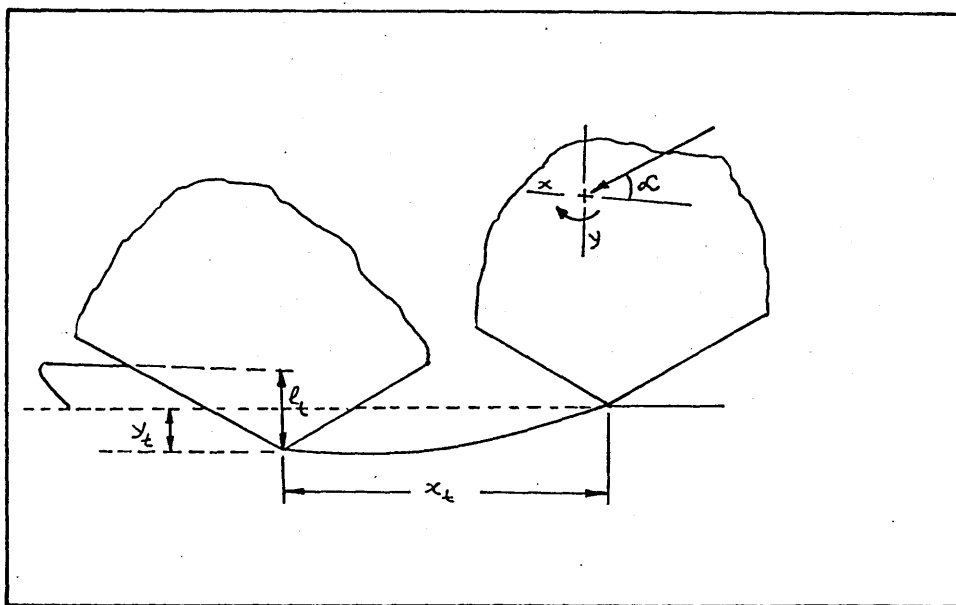


Fig. 4.2 Idealised two-dimensional model of a rigid grain cutting into a ductile metal.

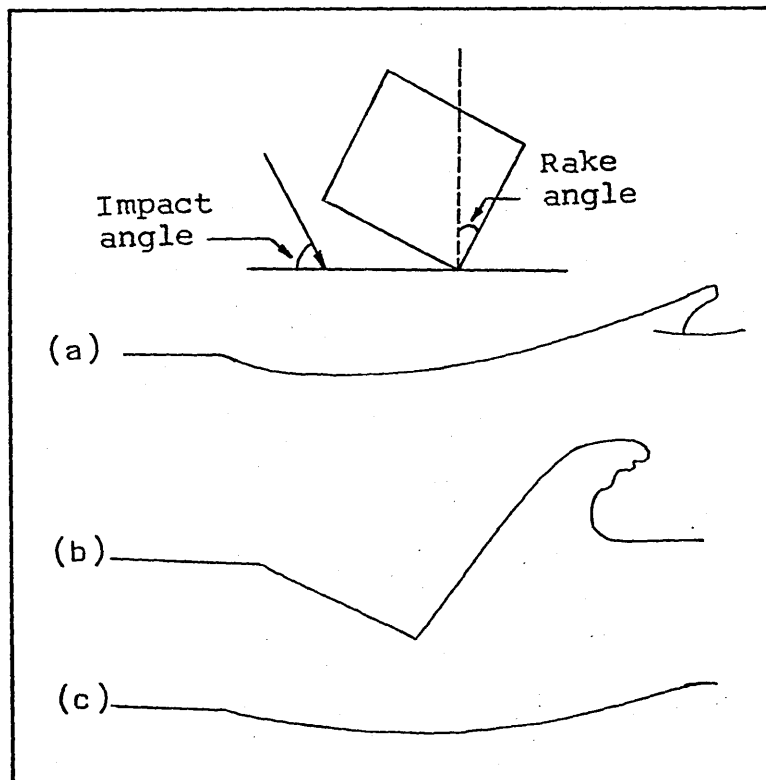


Fig. 4.3 (a) Plowing deformation,  
(b) cutting and plowing,  
(c) cutting only.

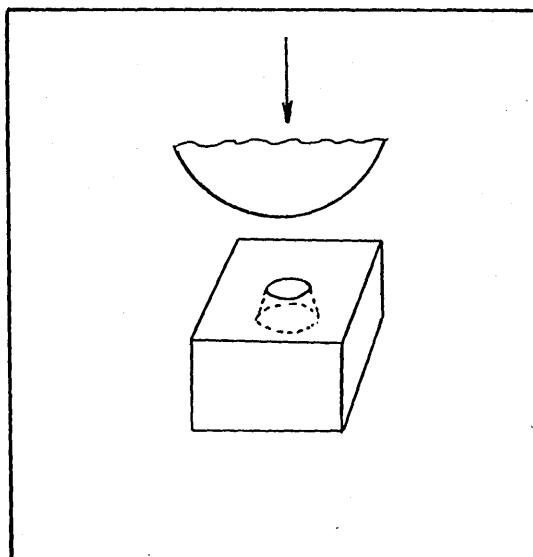
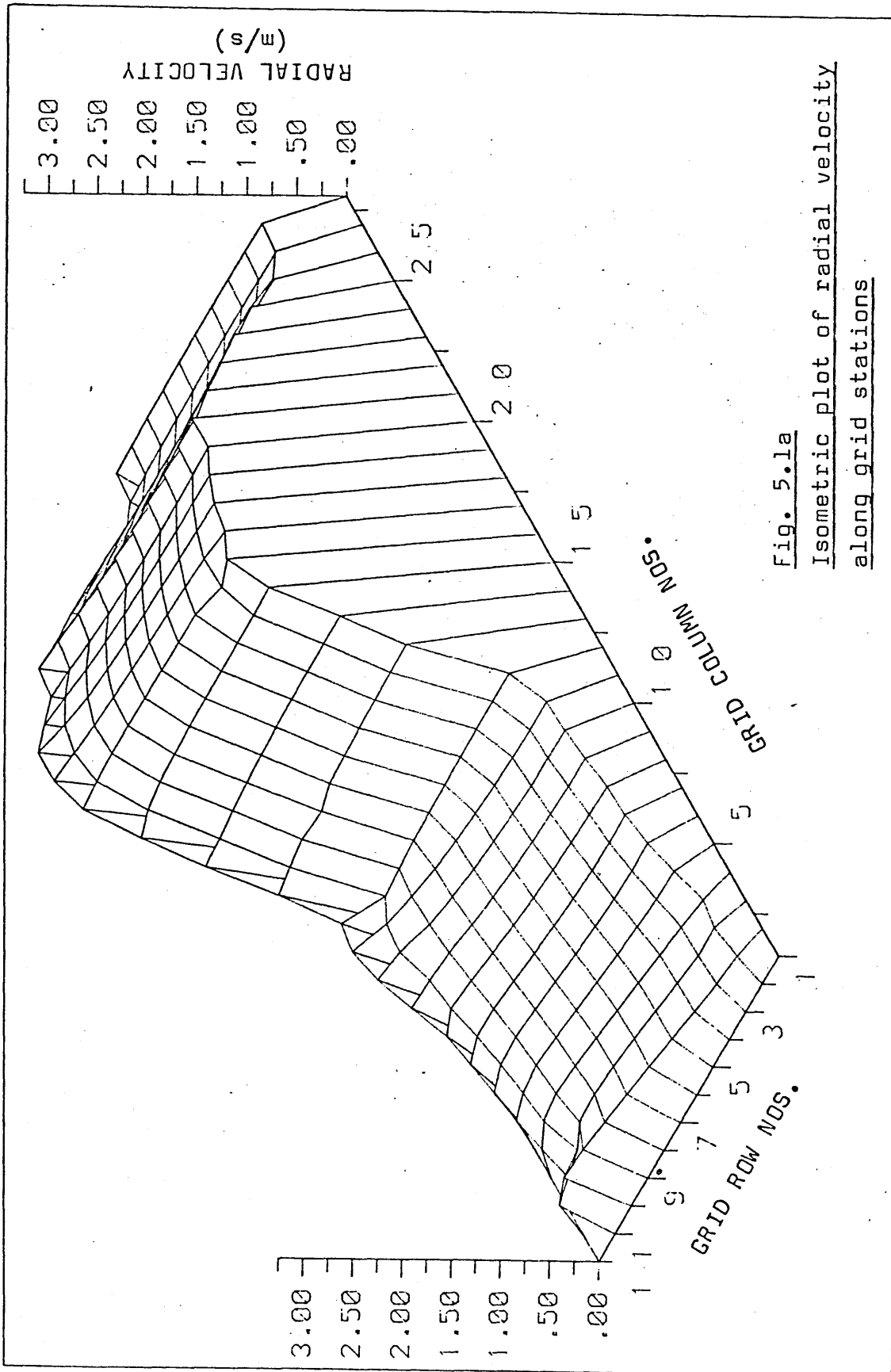
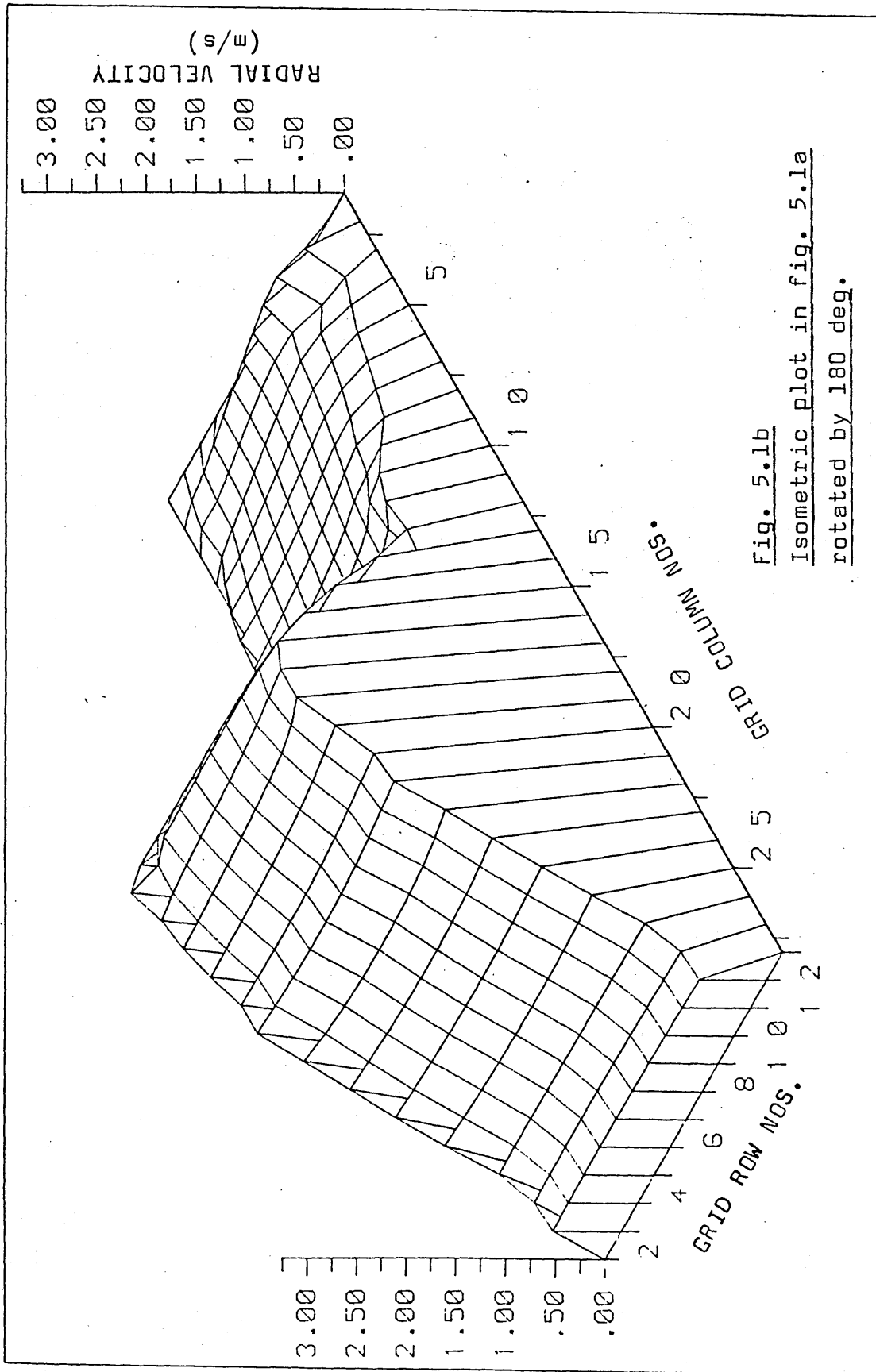


Fig. 4.4 Ring crack in brittle erosion.





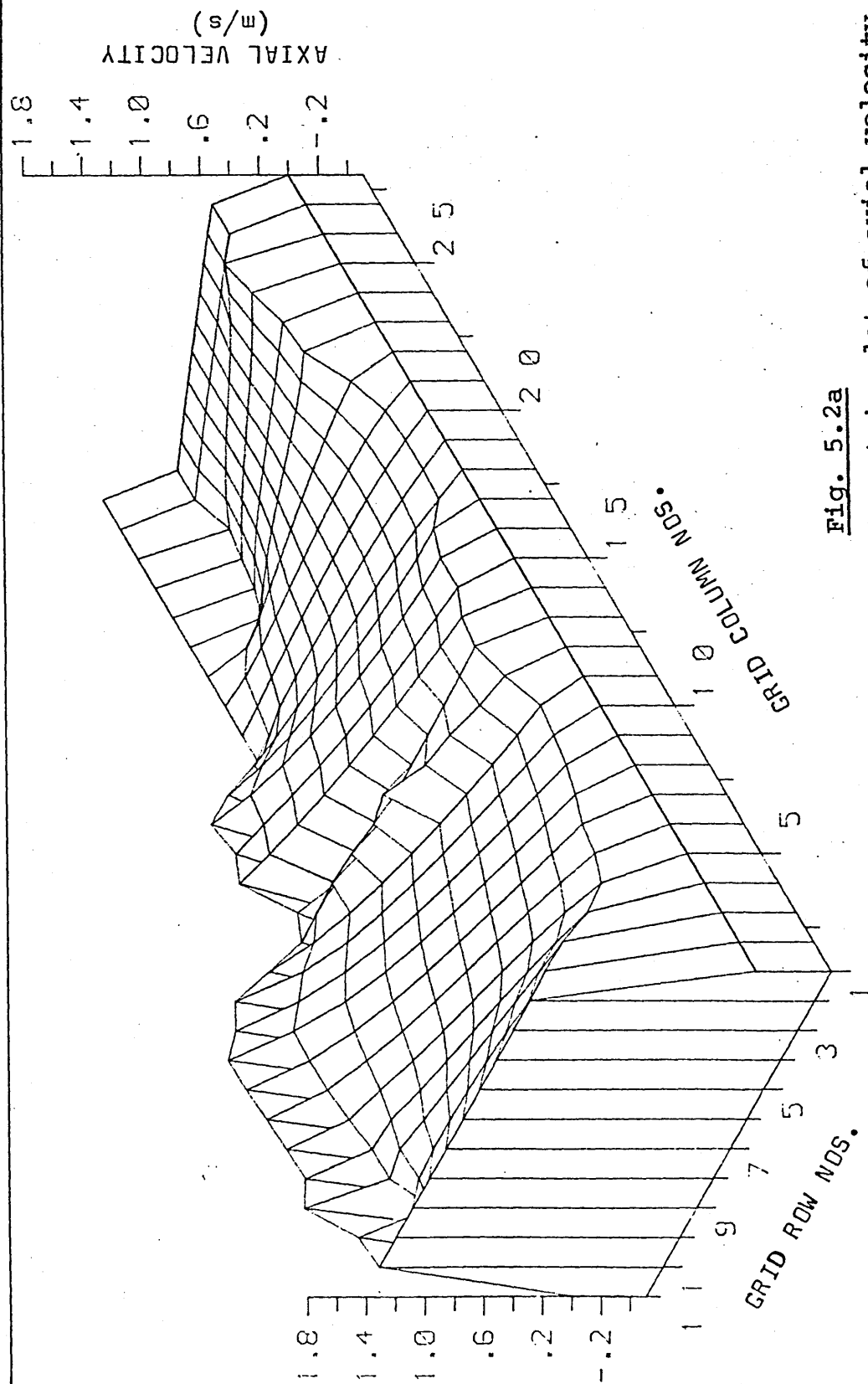


Fig. 5.2a  
Isometric plot of axial velocity  
along grid stations

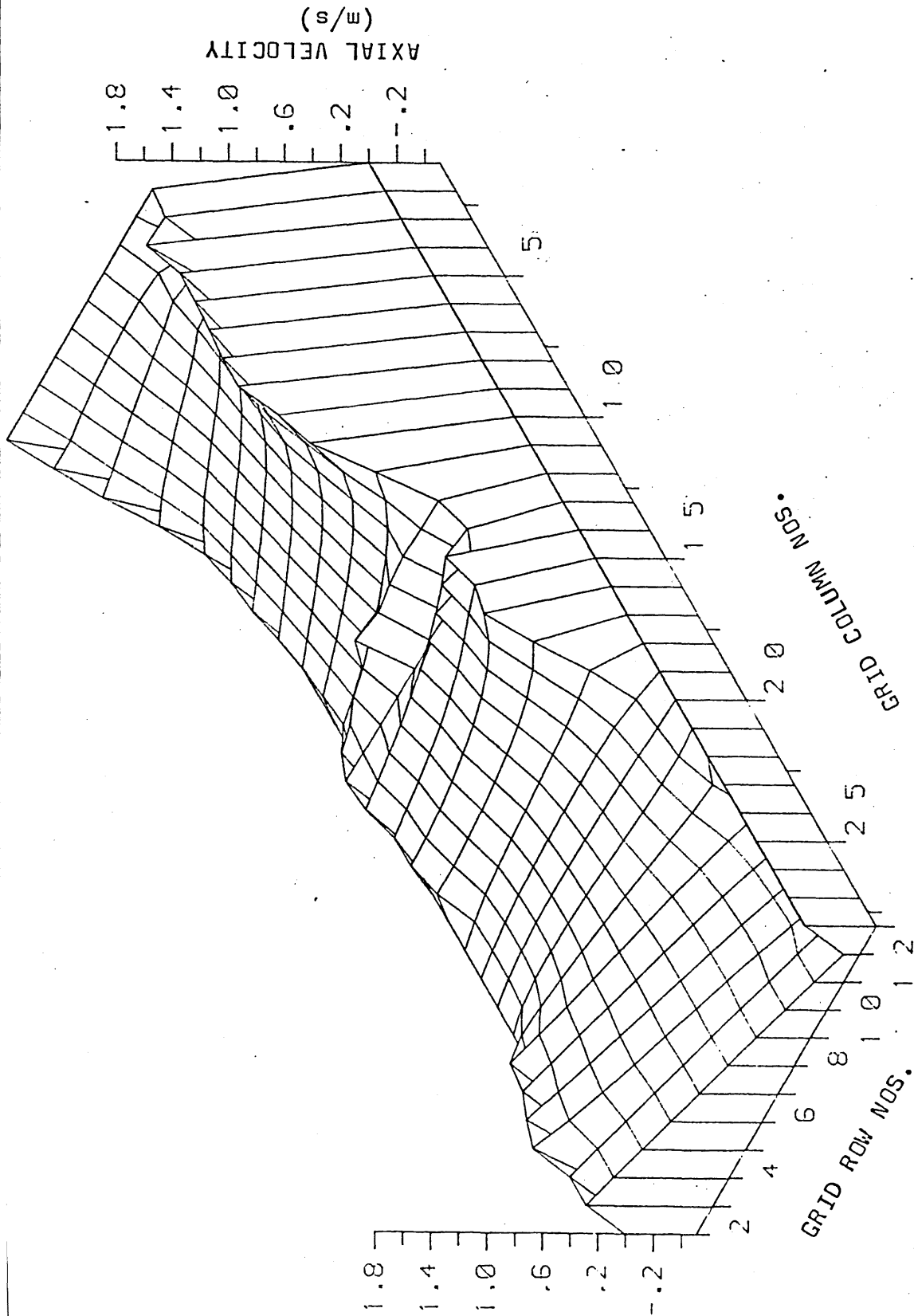


**Fig. 5.2b**  
**Isometric plot in fig. 5.2a**  
**rotated by 180 deg.**



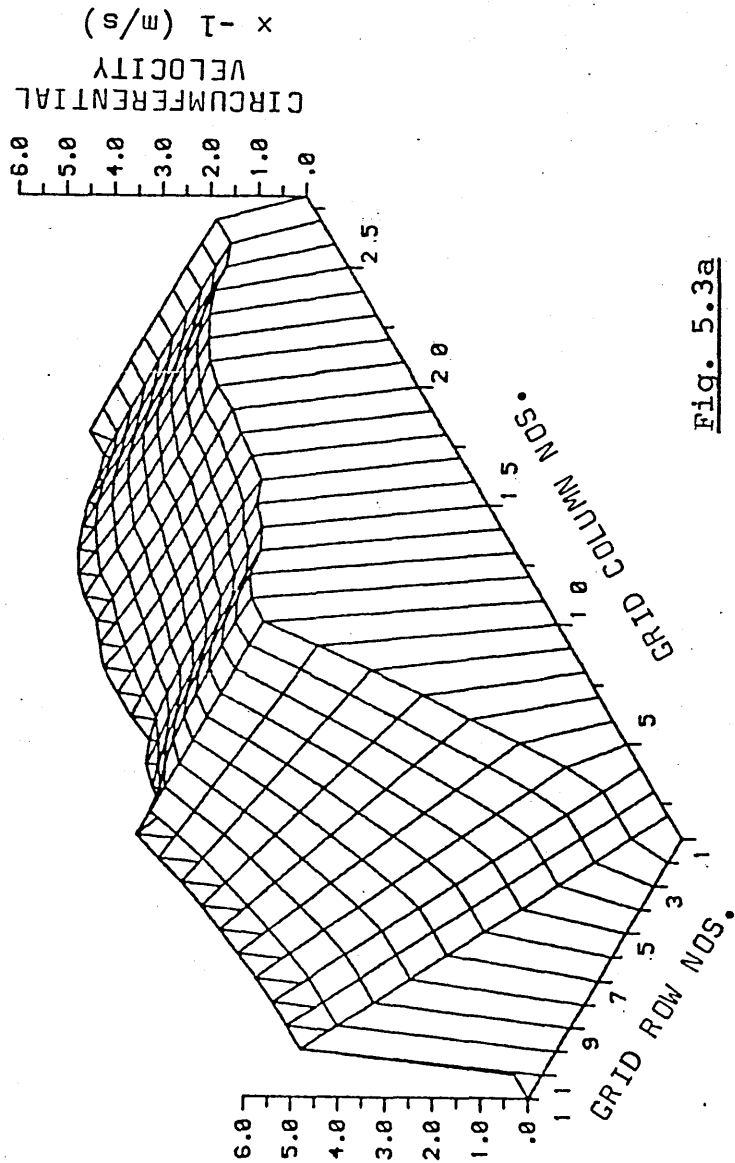


Fig. 5.3a  
Isometric plot of circumferential  
velocity along grid stations

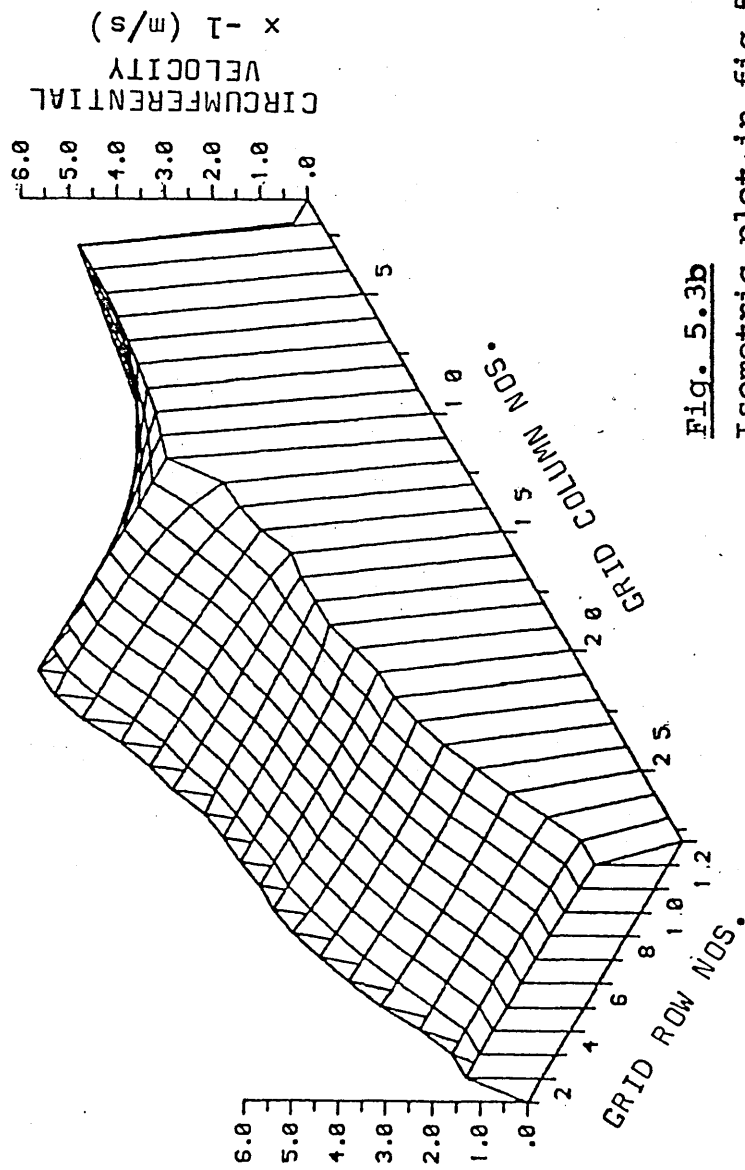


**Fig. 5.3b**  
Isometric plot in fig 5.3a  
rotated by 180 deg.

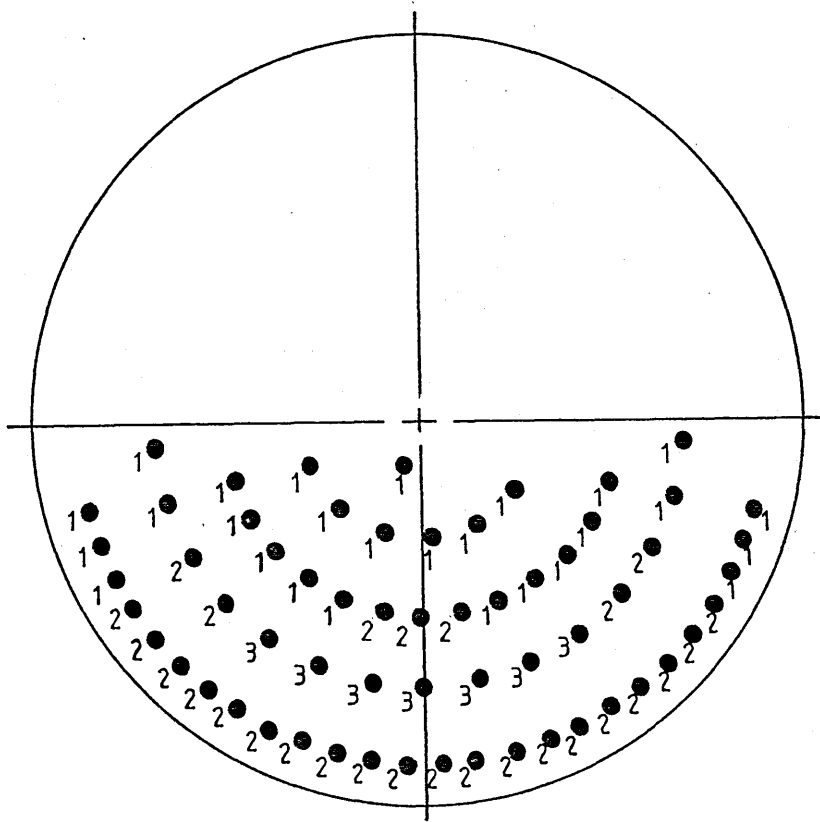
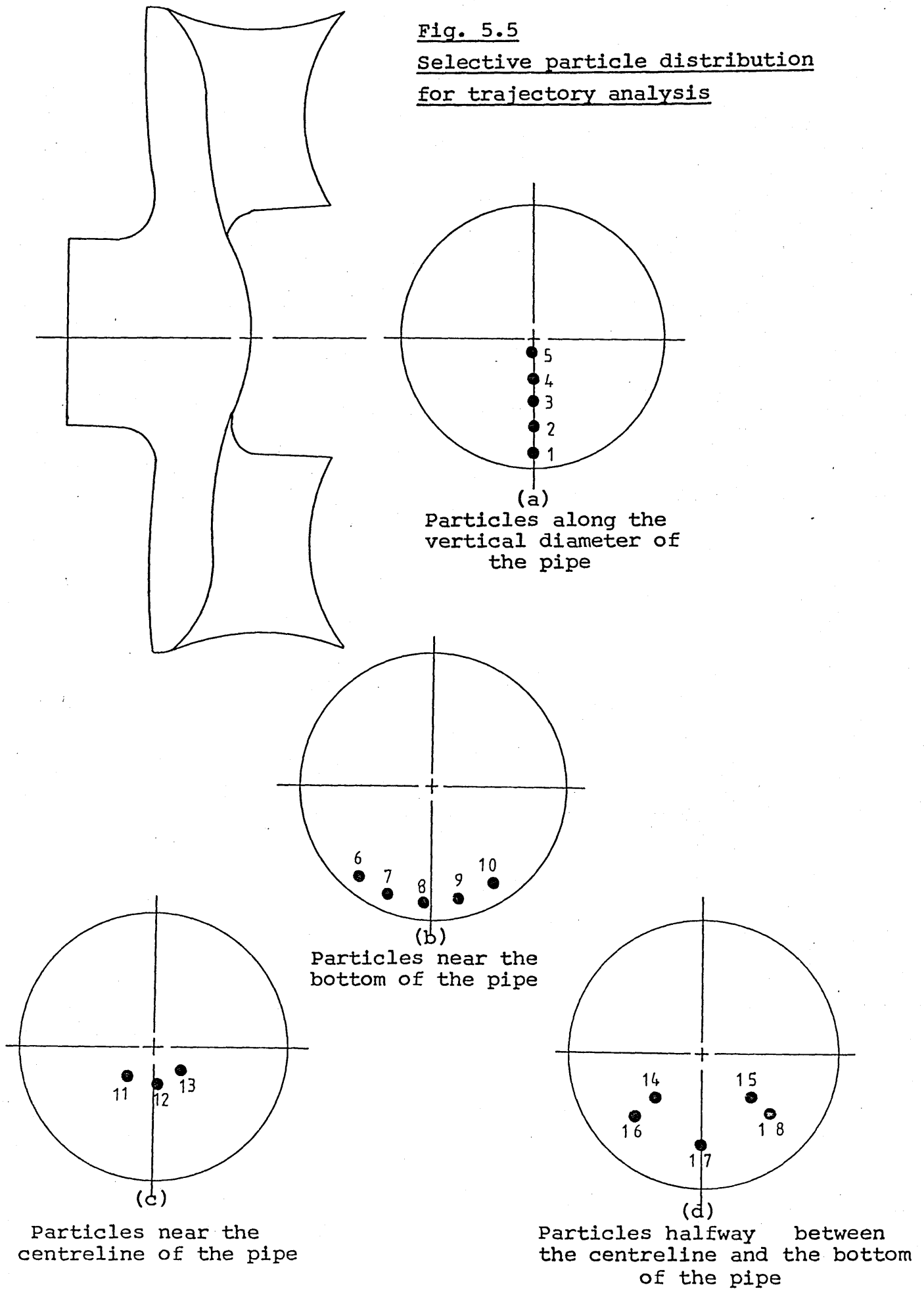


Fig. 5.4 Particle distribution in the cross-section of the pipe corresponding to  $\bar{c}_d=3.362\%$ , case 1.

Fig. 5.5

Selective particle distribution  
for trajectory analysis



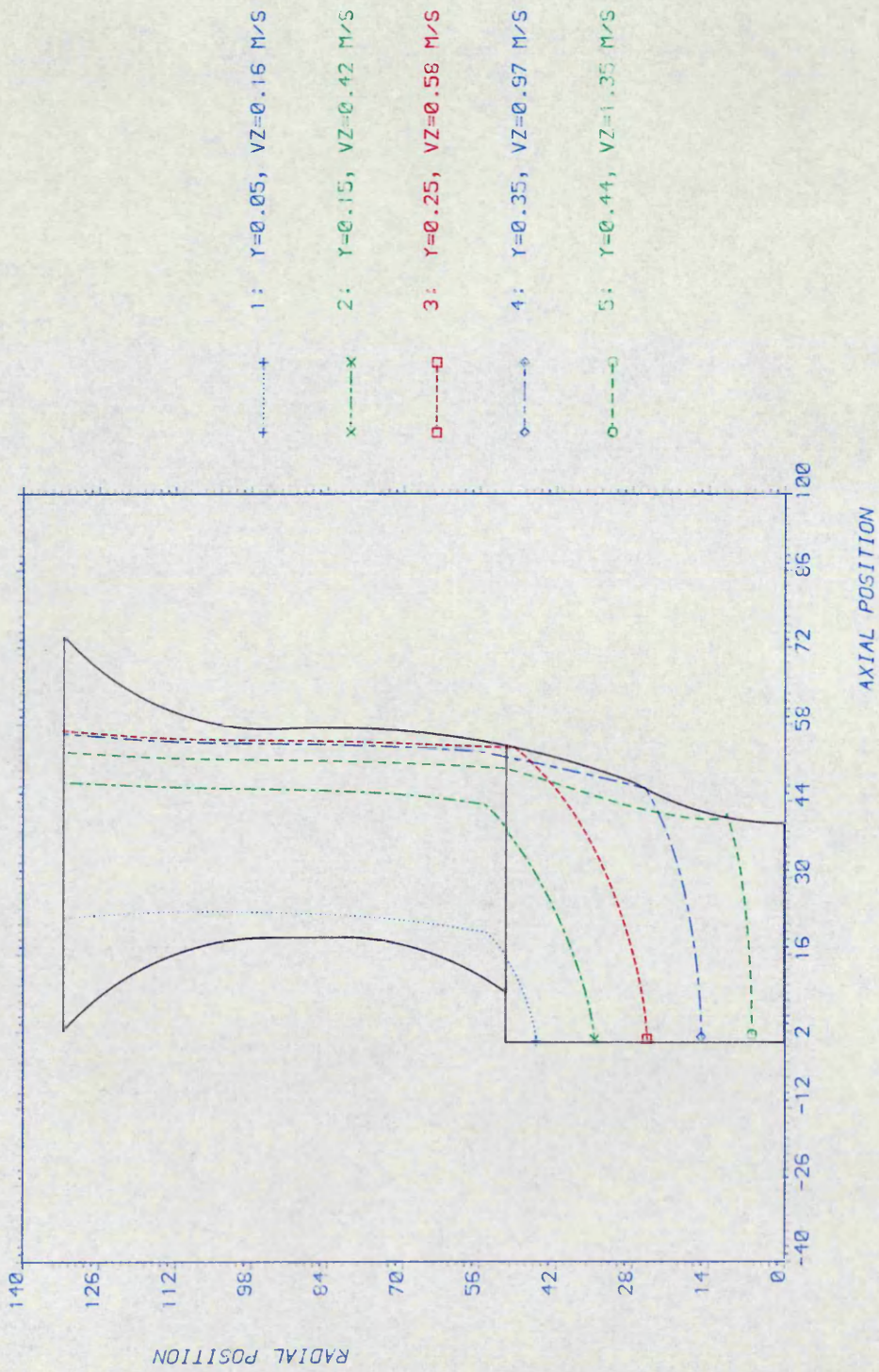


FIG. 5.6A TRAJECTORY OF PARTICLES IN THE IMPELLER FOR PARTICLES RELEASED ALONG THE VERTICAL DIAMETER OF THE PIPE.



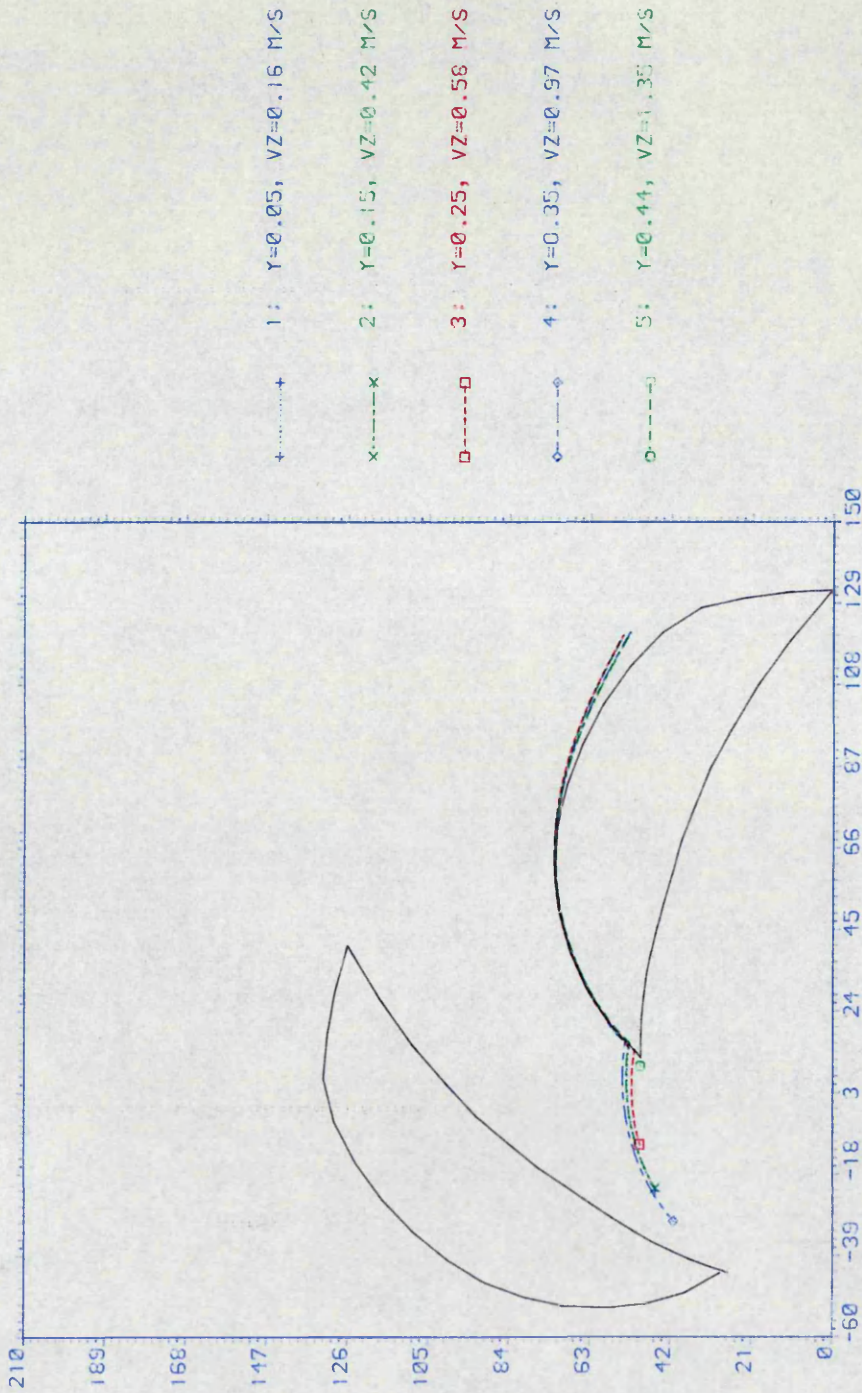


FIG. 5.6B TRAJECTORY OF PARTICLES IN THE BLADE CHANNEL FOR PARTICLES RELEASED ALONG THE VERTICAL DIAMETER OF THE PIPE.



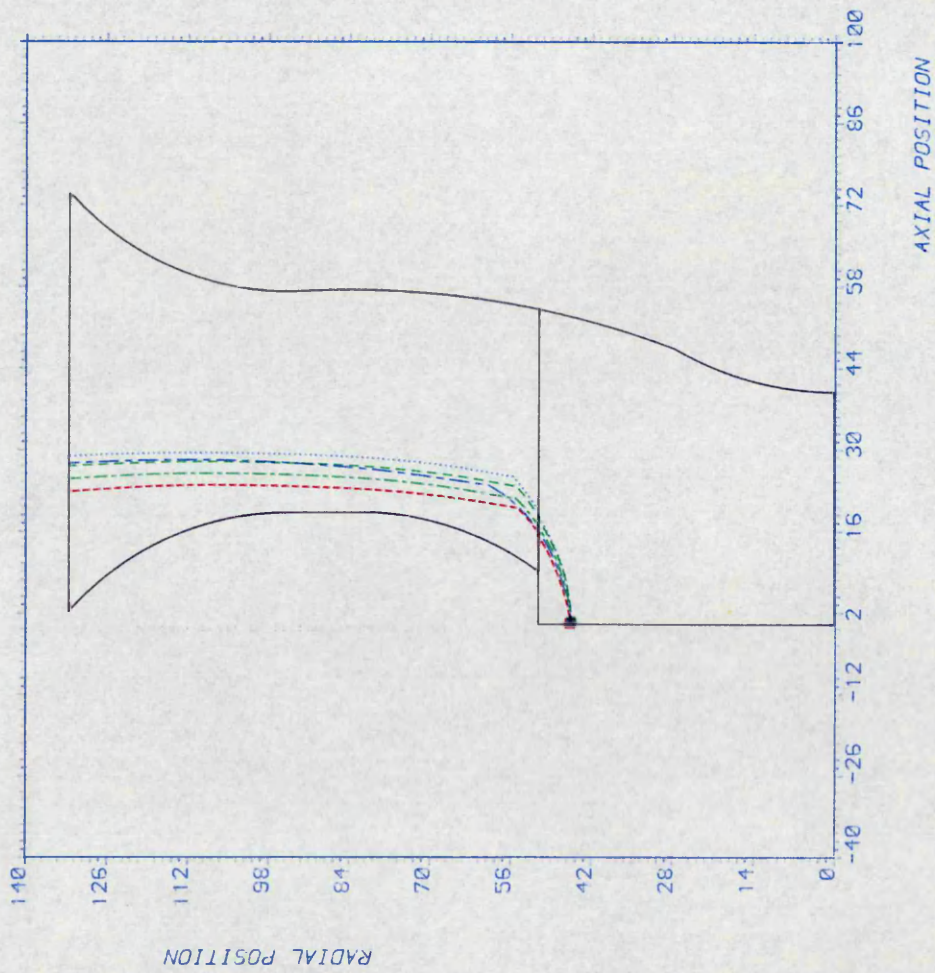
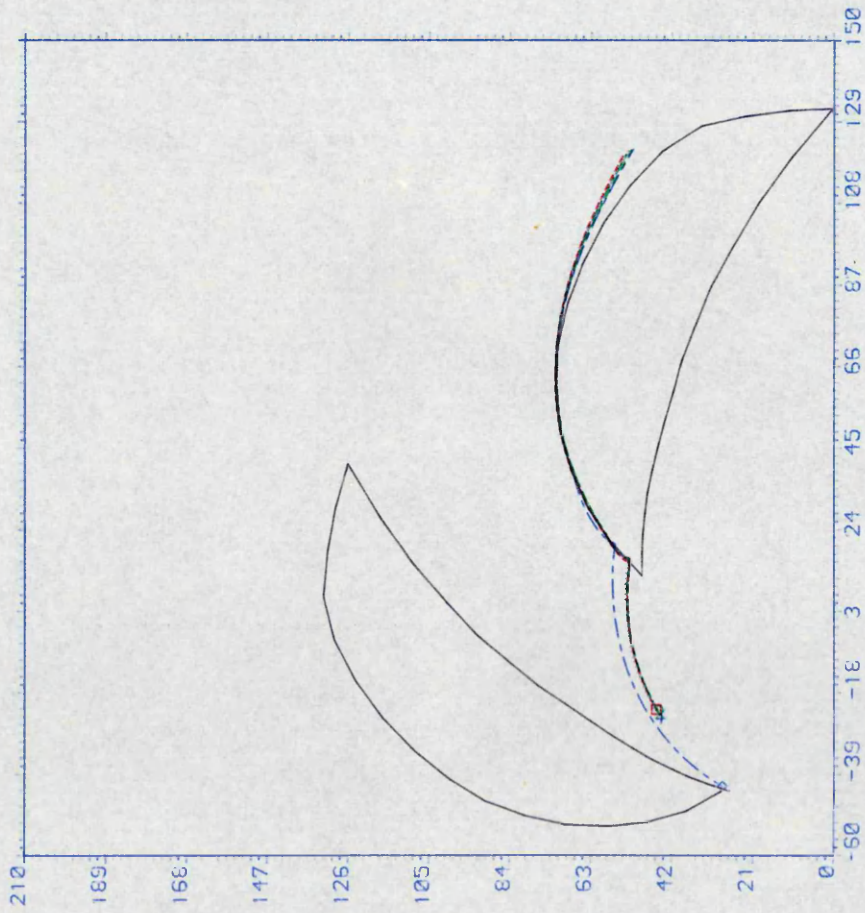


FIG. 5.7A TRAJECTORY OF PARTICLES IN THE IMPELLER FOR PARTICLES RELEASED NEAR THE BOTTOM OF THE PIPE.





- 6:  $Y=0.15$ ,  $VZ=0.42$  M/S
- 7:  $Y=0.08$ ,  $VZ=0.28$  M/S
- 8:  $Y=0.05$ ,  $VZ=0.16$  M/S
- 9:  $Y=0.06$ ,  $VZ=0.23$  M/S
- 10:  $Y=0.12$ ,  $VZ=0.38$  M/S

FIG. 5.7B TRAJECTORY OF PARTICLES IN THE BLADE CHANNEL FOR PARTICLES RELEASED NEAR THE BOTTOM OF THE PIPE.



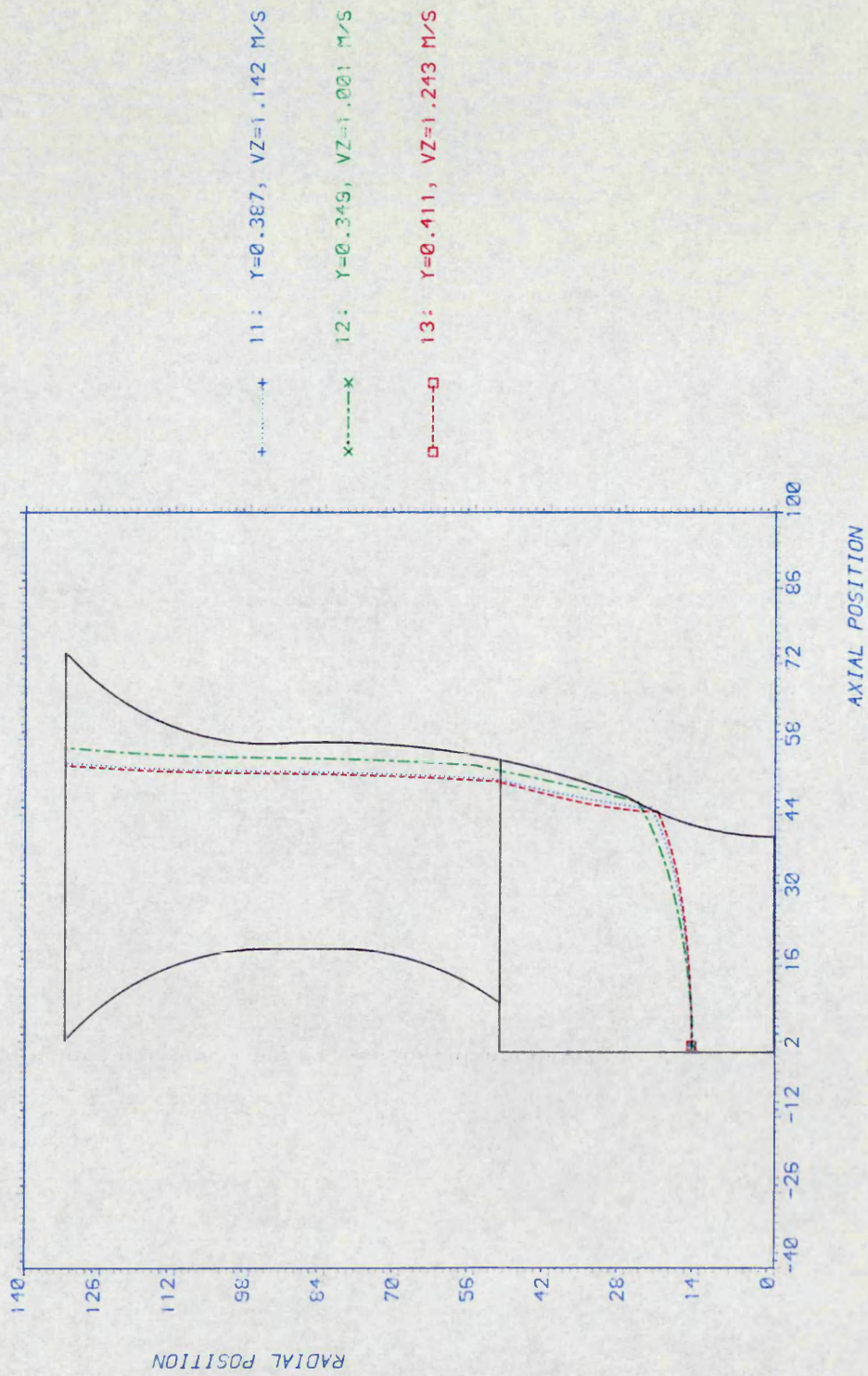
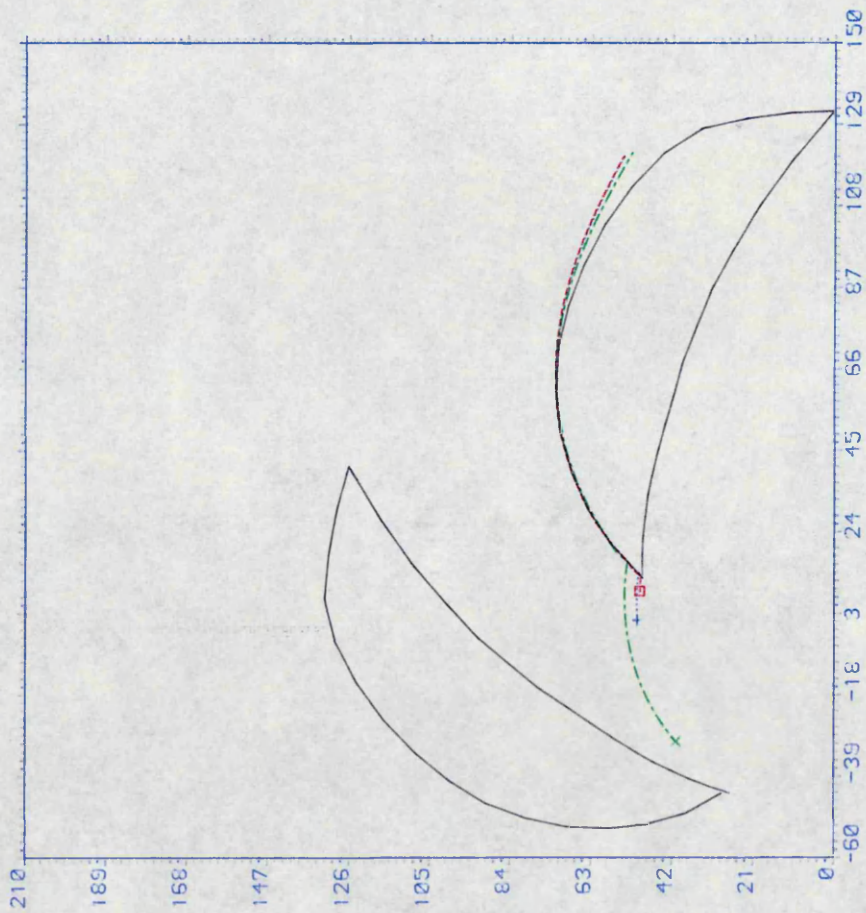


FIG. 5.8A TRAJECTORY OF PARTICLES IN THE IMPELLER FOR PARTICLES RELEASED NEAR THE CENTRELINE OF THE PIPE.





- +-----+ 11:  $Y=0.387$ ,  $VZ=1.142$  M/S
- x-----x 12:  $Y=0.349$ ,  $VZ=1.001$  M/S
- o-----o 13:  $Y=0.411$ ,  $VZ=1.243$  M/S

FIG. 5.8B TRAJECTORY OF PARTICLES IN THE BLADE CHANNEL FOR PARTICLES RELEASED NEAR THE CENTRELIN OF THE PIPE.



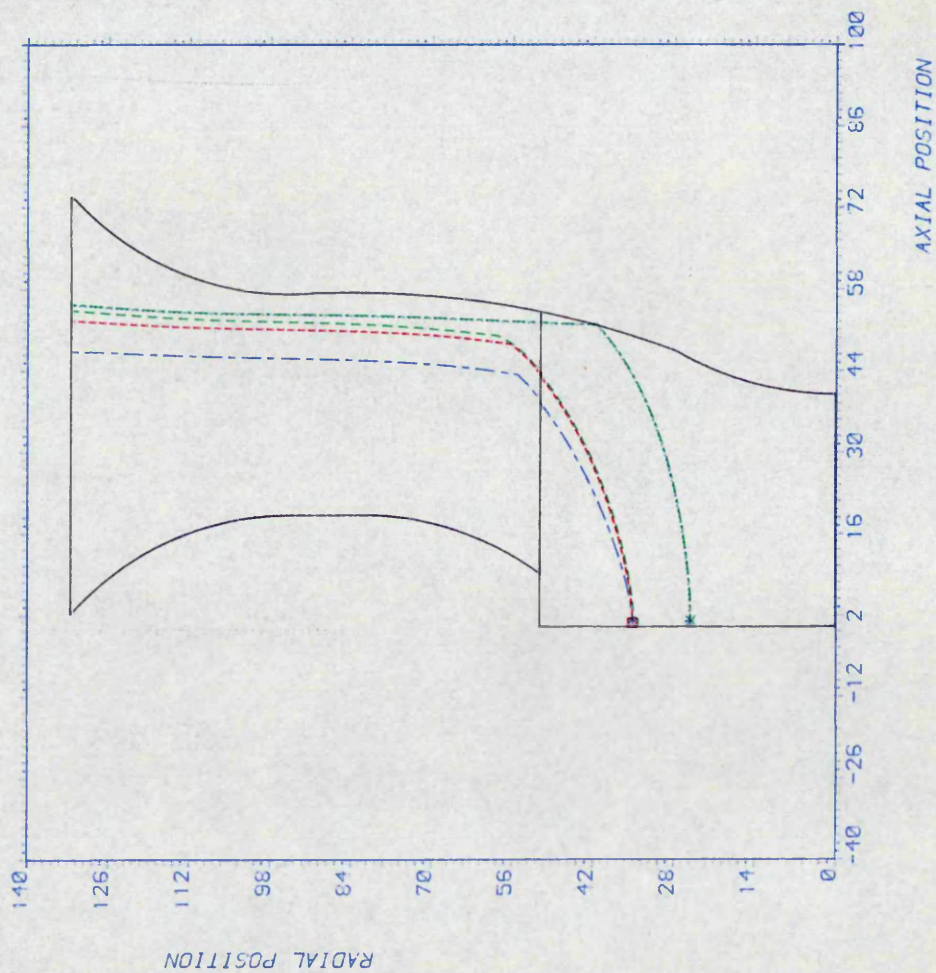
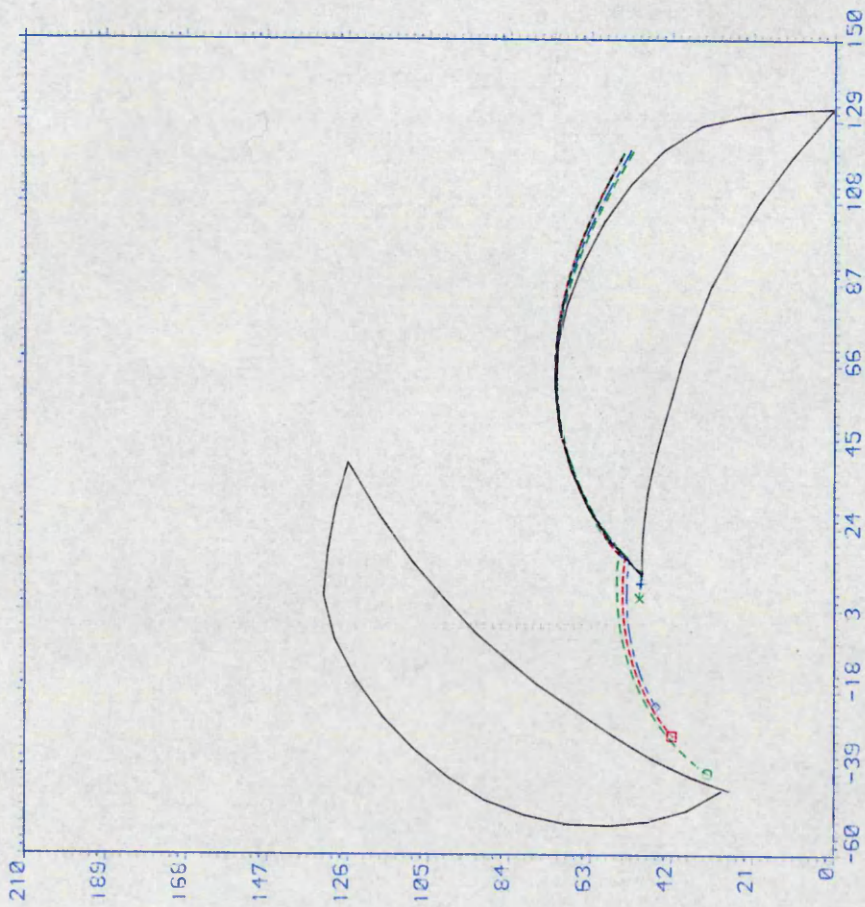


FIG. 5.9A TRAJECTORY OF PARTICLES. PARTICLES ARE RELEASED HALFWAY BETWEEN THE CENTRELINE AND THE BOTTOM OF THE PIPE.





14:  $Y=0.334$ ,  $VZ=0.698$  M/S

15:  $Y=0.334$ ,  $VZ=0.698$  M/S

16:  $Y=0.269$ ,  $VZ=0.647$  M/S

17:  $Y=0.155$ ,  $VZ=0.420$  M/S

18:  $Y=0.269$ ,  $VZ=0.647$  M/S

FIG. 5.9B TRAJECTORY OF PARTICLES. PARTICLES ARE RELEASED HALF-WAY BETWEEN THE CENTRELINE AND THE BOTTOM OF THE PIPE

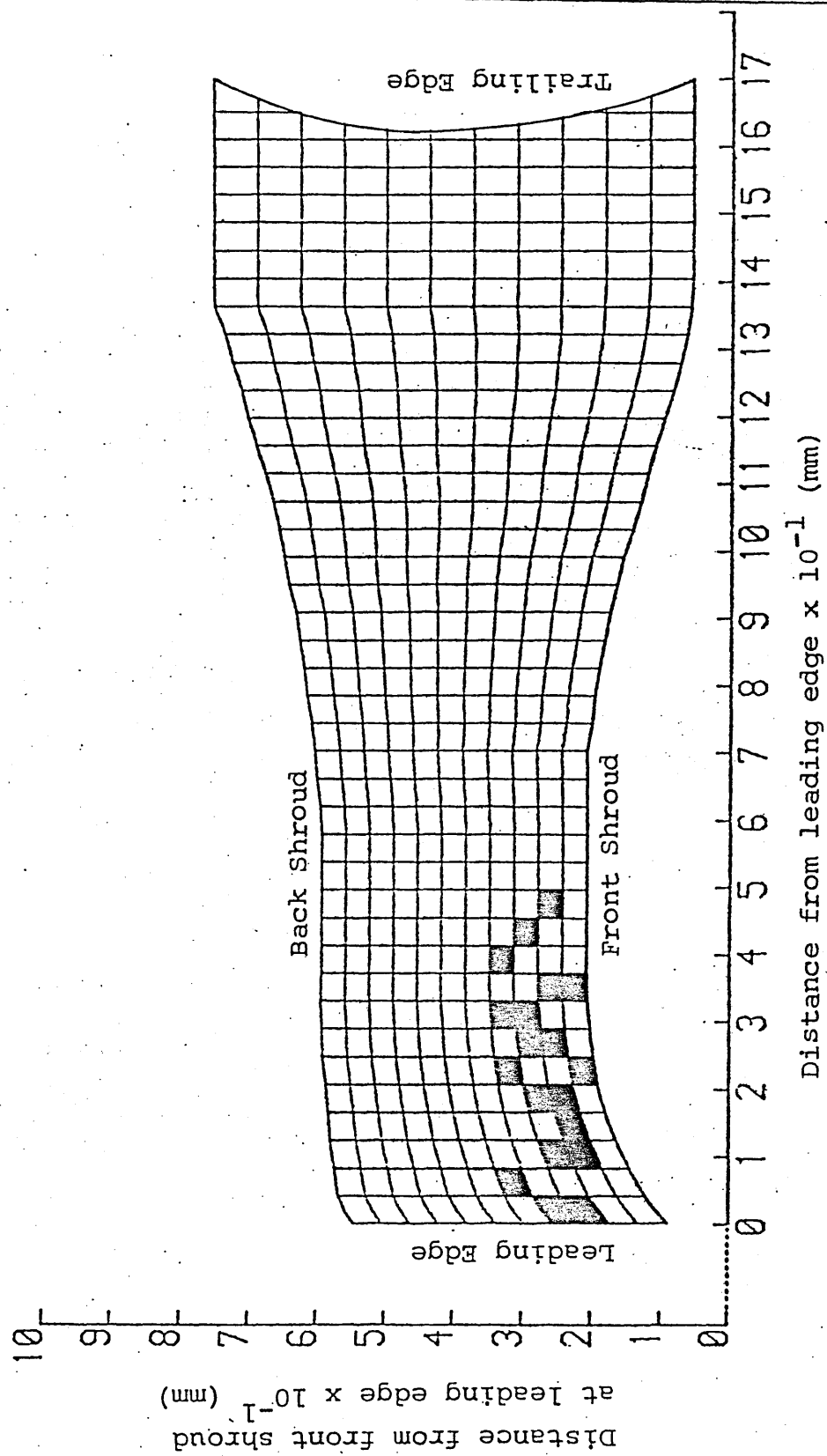


Fig. 5.10 Impactation Zone on Pressure Surface of a Blade : Case lb.

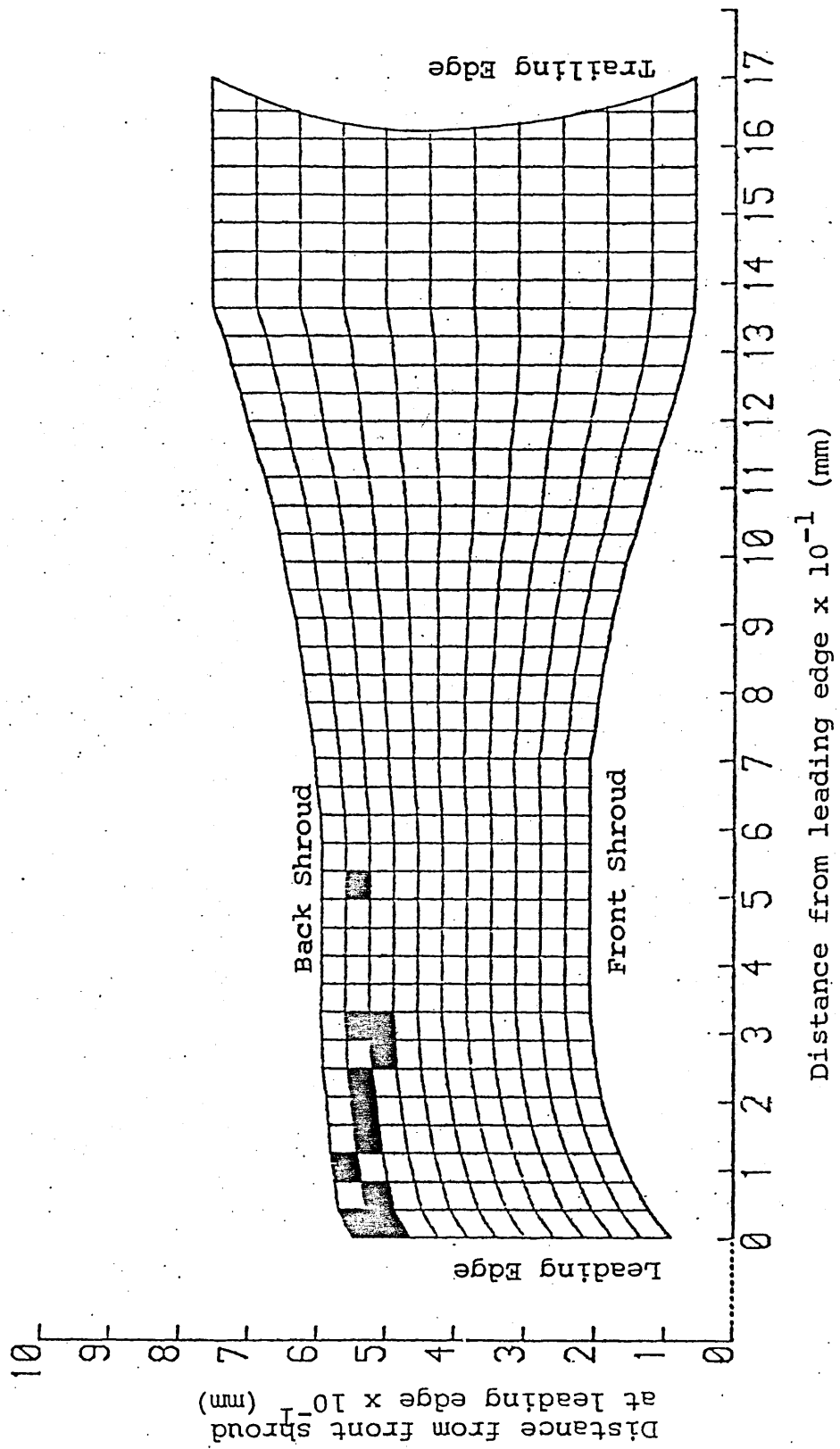


Fig. 5.11 Impactation Zone on Pressure Surface of a Blade : Case 1c.

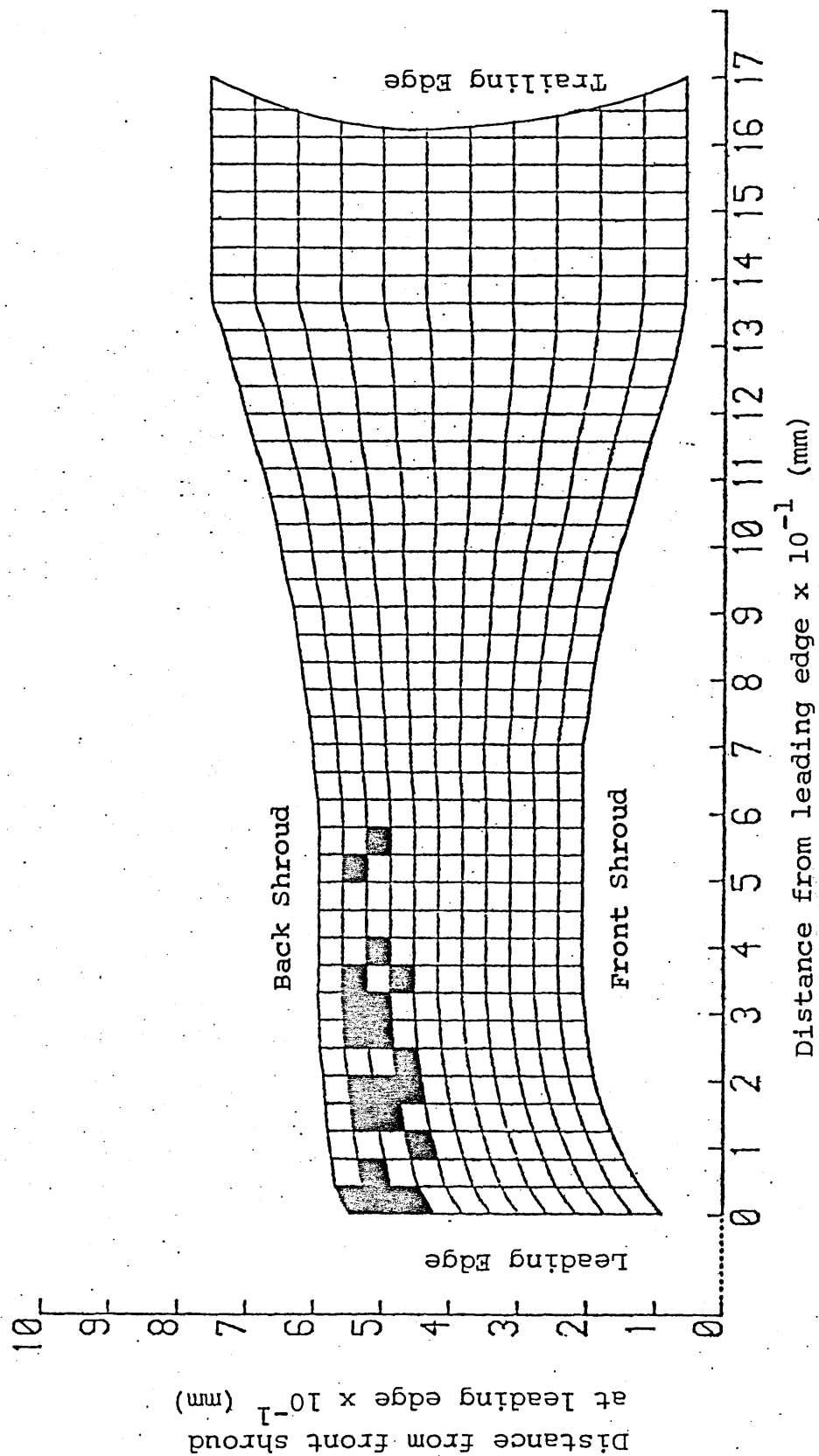


Fig. 5.12 Impaction Zone on Pressure Surface of a Blade : Case 1d.

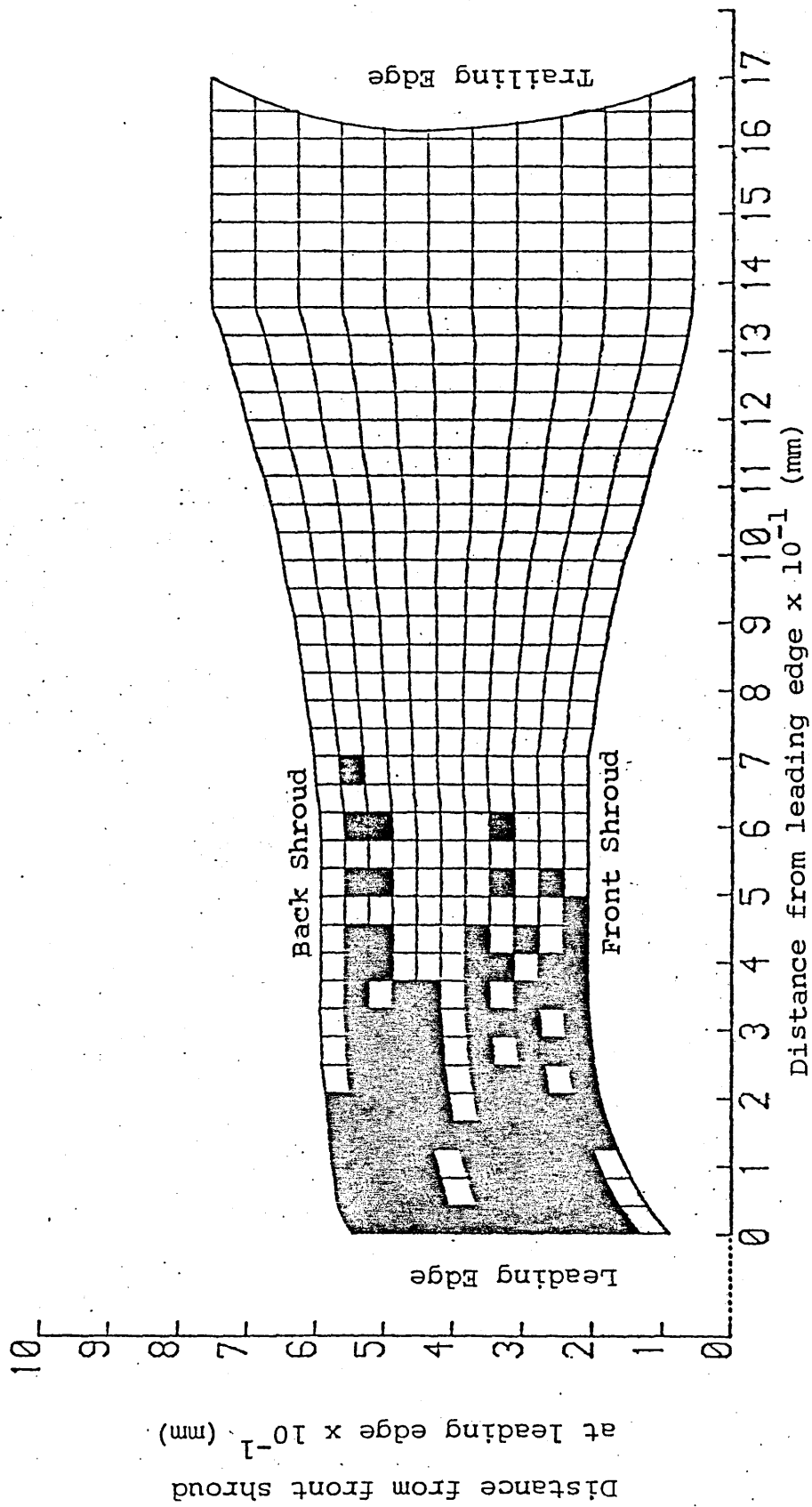


Fig. 5.13 Impaction zone on Pressure Surface of a Blade : Case 1.



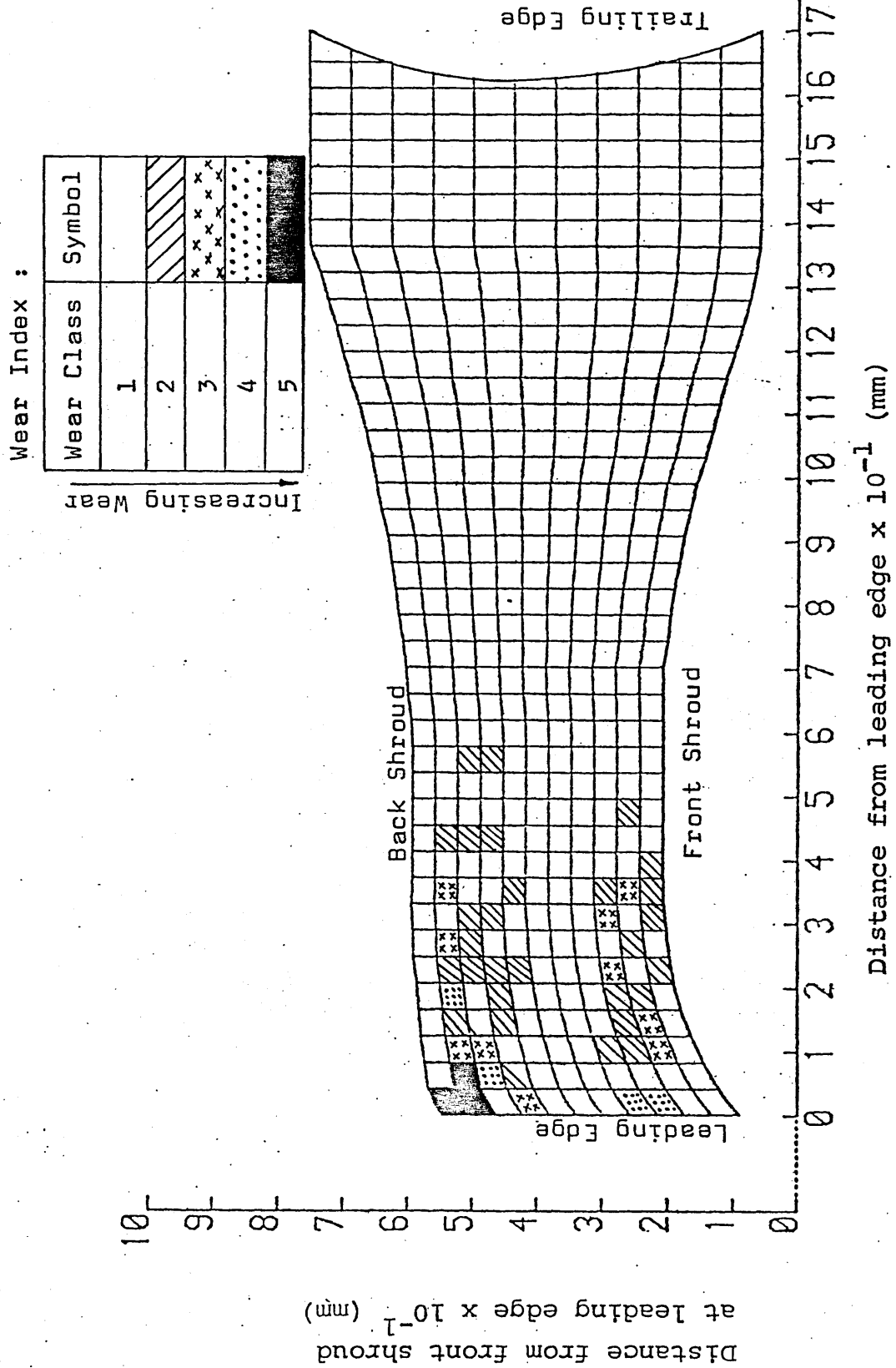


Fig. 5.14 Predicted erosion-prone areas on pressure surface of a blade , case 1.

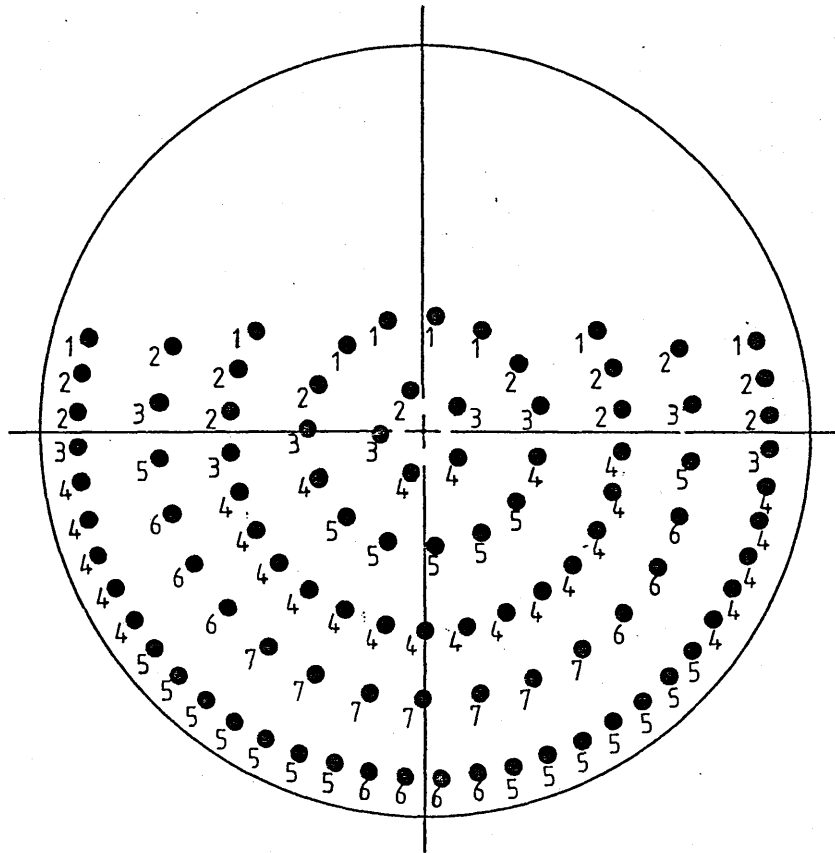


Fig. 5.15 Particle distribution in the cross-section of the pipe corresponding to  $\bar{c}_d=11.653\%$ , case 2.

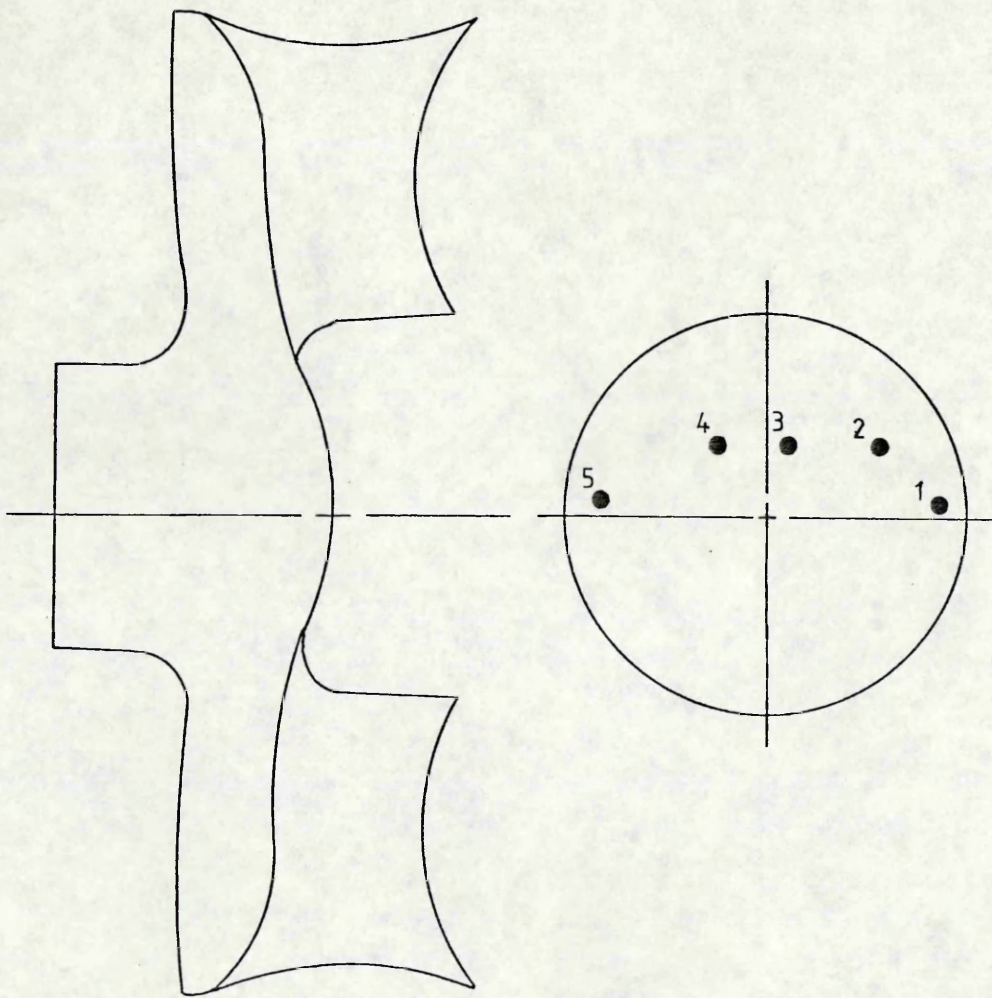


Fig. 5.16 Selective particle distribution for trajectory analysis, case 2



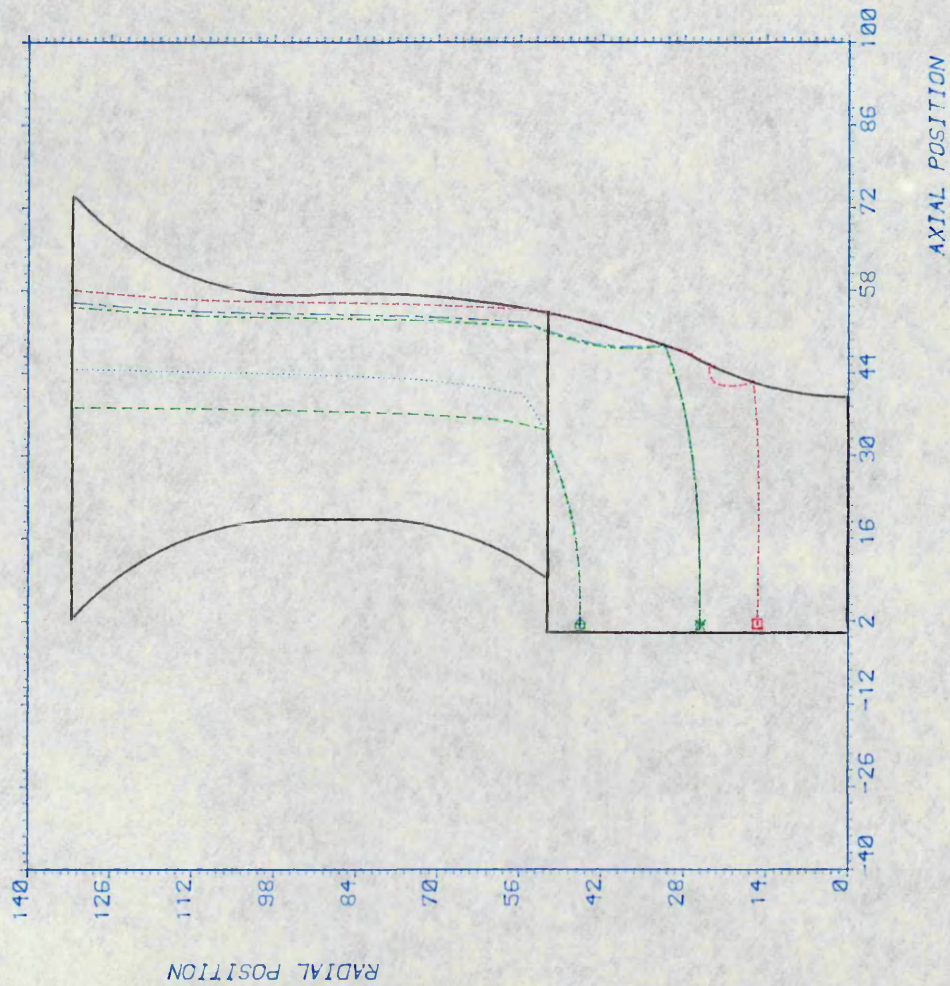
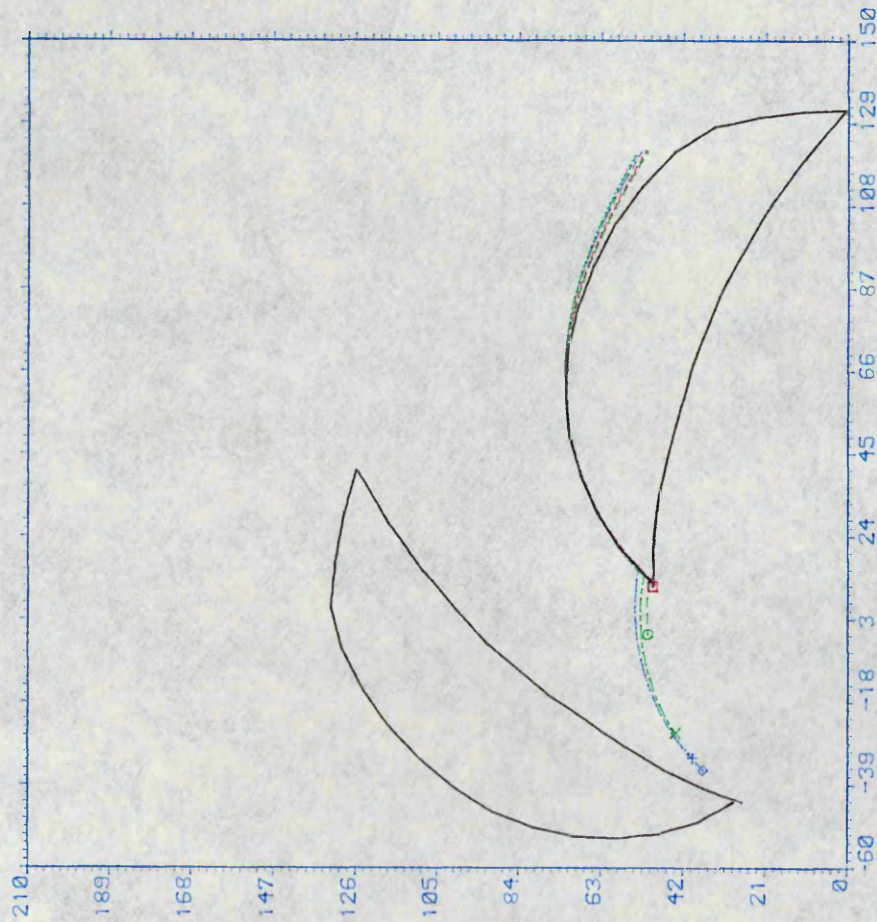


FIG. 5.17A TRAJECTORY OF PARTICLES IN THE IMPELLER FOR PARTICLES RELEASED ABOVE THE CENTRELINE OF THE PIPE.





- 1:  $\gamma = .523$ ,  $VZ = 1.426$  M/S
- 2:  $\gamma = .624$ ,  $VZ = 1.426$  M/S
- 3:  $\gamma = .651$ ,  $VZ = 1.426$  M/S
- 4:  $\gamma = .624$ ,  $VZ = 1.426$  M/S
- 5:  $\gamma = .523$ ,  $VZ = 1.426$  M/S

FIG. 5.17B TRAJECTORY OF PARTICLES IN THE BLADE CHANNEL FOR PARTICLES RELEASED ABOVE THE CENTRELINE OF THE PIPE.

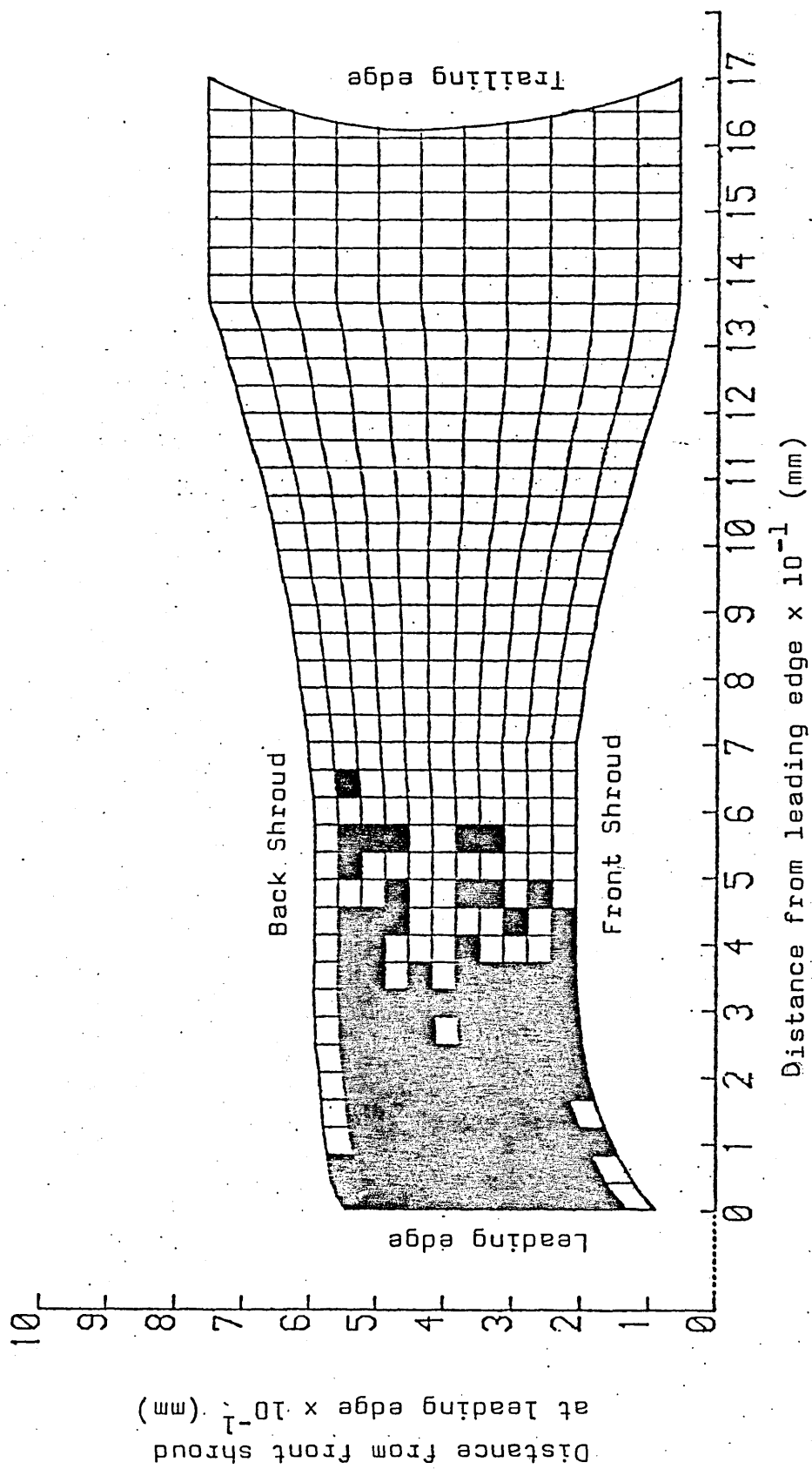
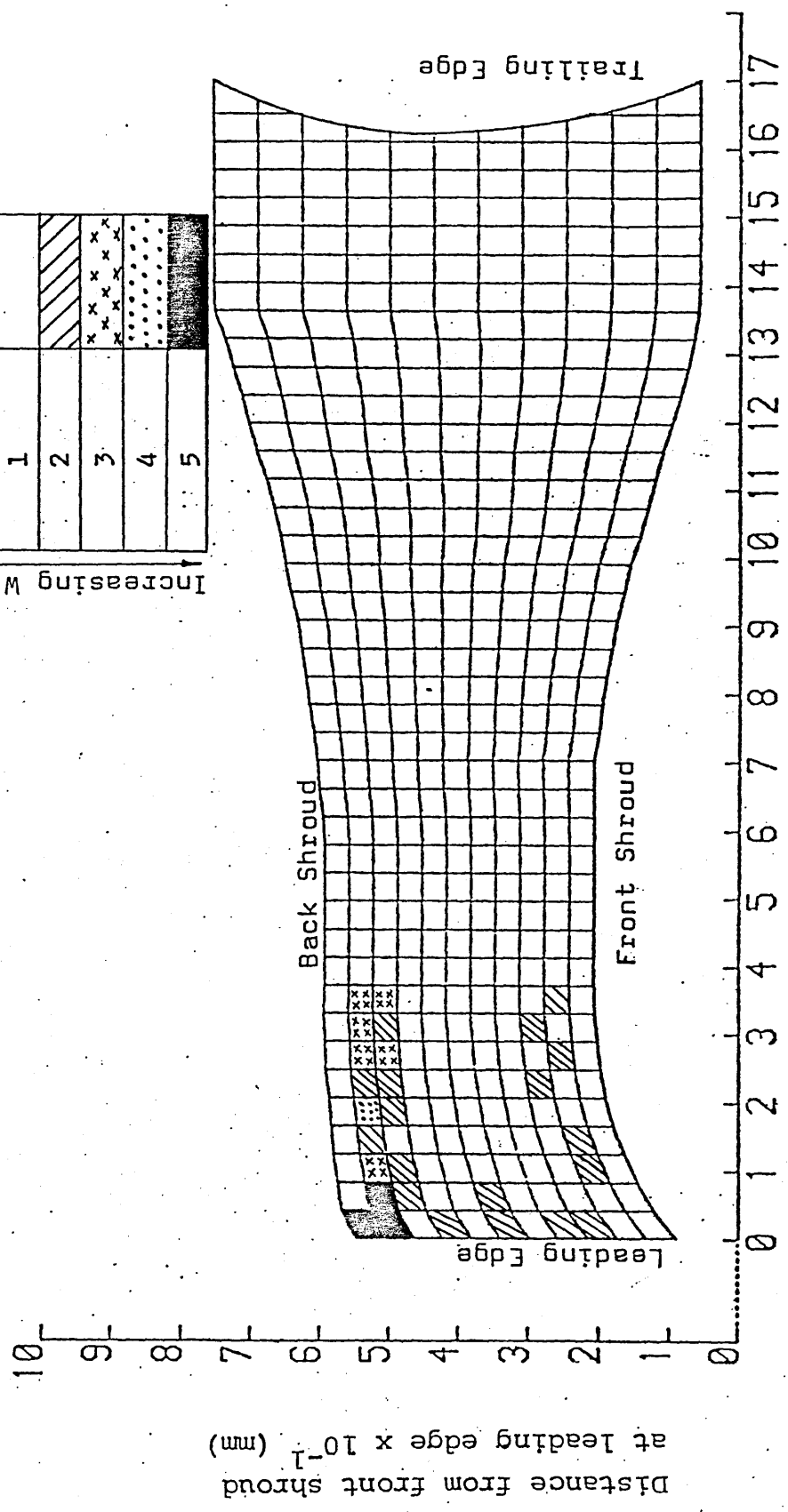


Fig. 5.18 Impact Zone on Pressure Surface of a Blade : Case 2

Wear Index :

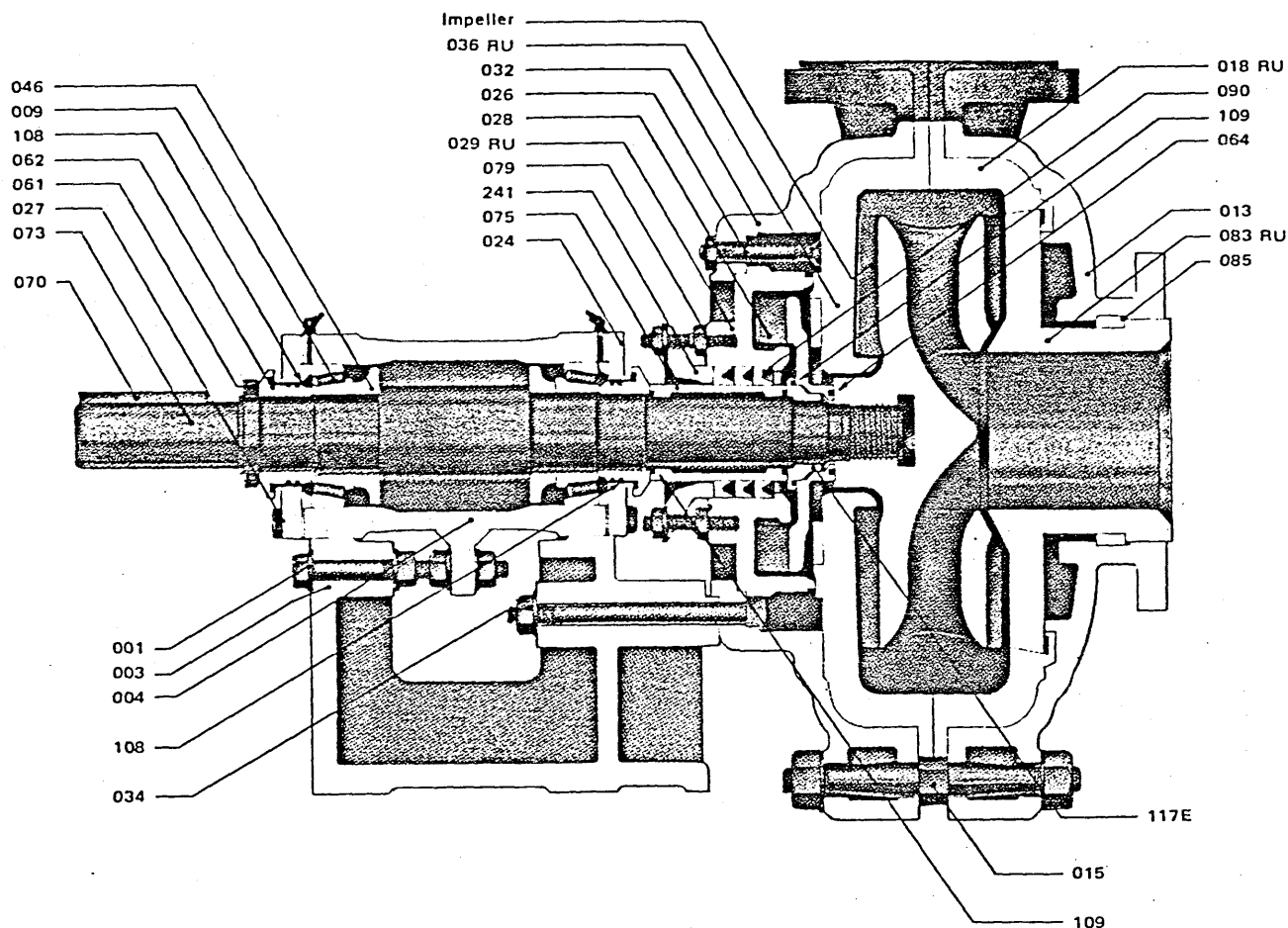
| Wear Class | Symbol |
|------------|--------|
| 1          |        |
| 2          |        |
| 3          | ///    |
| 4          | xxx    |
| 5          | •••    |

Increasing Wear ↑



Distance from leading edge  $\times 10^{-1}$  (mm)

Fig. 5.19 Predicted erosion-prone areas on pressure surface of a blade, case 2.



- |       |                               |       |                           |
|-------|-------------------------------|-------|---------------------------|
| 001   | Adjusting Screw               | 062   | Labyrinth                 |
| 003   | Base                          | 064   | Impeller Sealing 'O' Ring |
| 004   | Bearing Housing               | 070   | Pulley Key                |
| 009   | Bearing                       | 073   | Shaft                     |
| 013   | Cover Plate                   | 075   | Shaft Sleeve              |
| 015   | Cover Plate Bolt              | 078   | Stuffing Box              |
| 018RU | Cover Plate Liner             | 079   | Gland Stud                |
| 024   | End Cover                     | 083   | Throatbush                |
| 025   |                               | 083RU | Throatbush                |
| 026   | Frame Plate Liner Stud        | 085   | Cotter                    |
| 027   | End Cover Set Screw           | 090   | Lip Seals                 |
| 028   | Expeller                      | 108   | Piston Ring               |
| 029RU | Expeller Ring                 | 109   | Shaft Spacer 'O' Ring     |
| 032   | Frame Plate                   | 110   | Volute Liner              |
| 034   | Frame Plate Bolt              | 111   | Packing Ring              |
| 036RU | Frame Plate Liner             | 117   | Shaft Spacer              |
| 040   | Frame Plate Liner Insert Bolt | 117E  | Shaft Spacer              |
| 041   | Frame Plate Liner Insert      | 118   | Lantern Restrictor        |
| 044   | Gland                         | 122   | Stuffing Box Seal         |
| 045   | Gland Bolt                    | 124   | Volute Liner Seal Ring    |
| 046   | Grease Retainer               | 132   | Discharge Joint Ring      |
| 060   | Joint Ring Intake             | 139   | Locating Clip             |
| 061   | Labyrinth Locknut             | 241   | Lip Seal Gland            |

Fig. 6.1 General arrangement of the Simon Warman pump



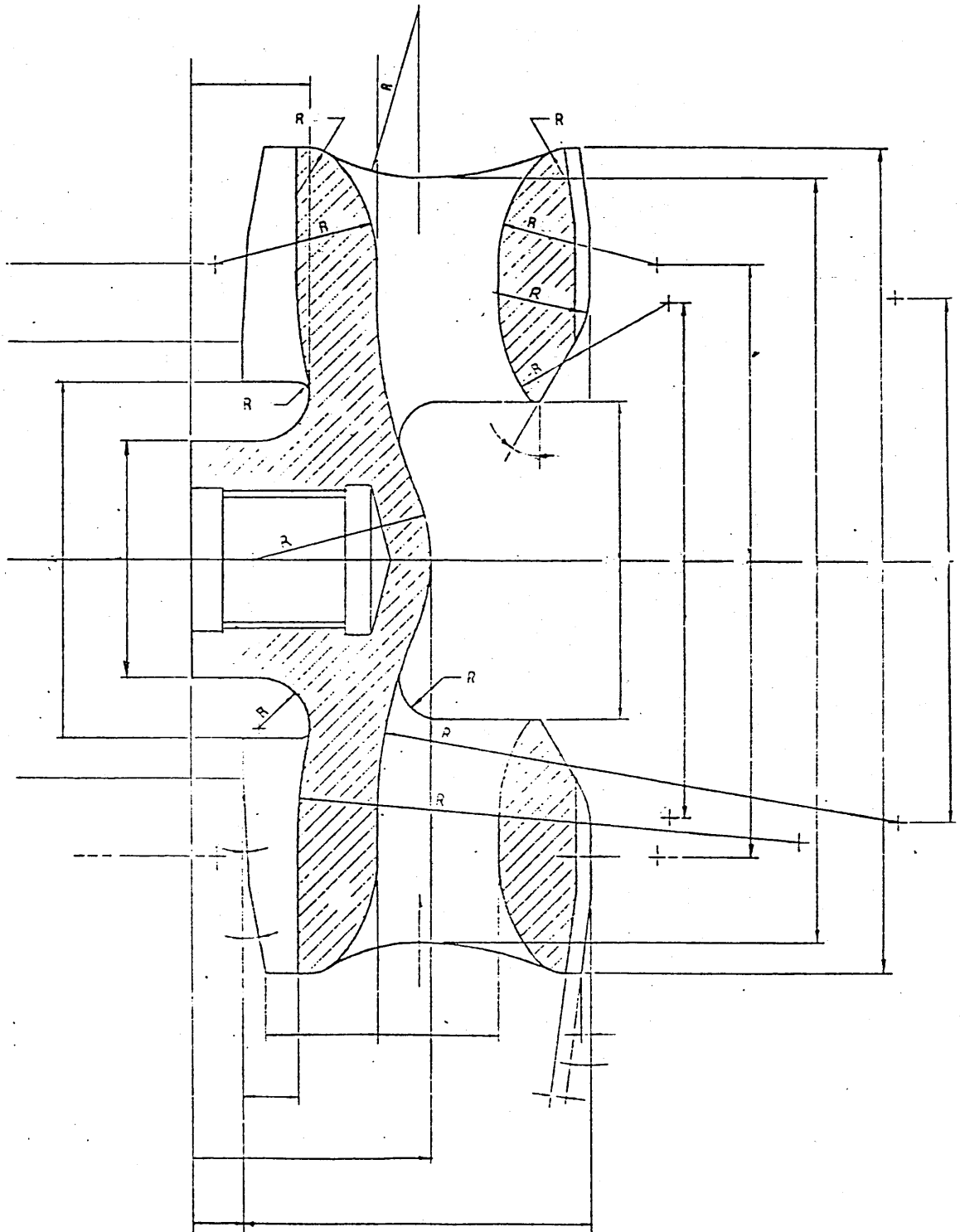


Fig. 6.2a Detailed drawing of the impeller used

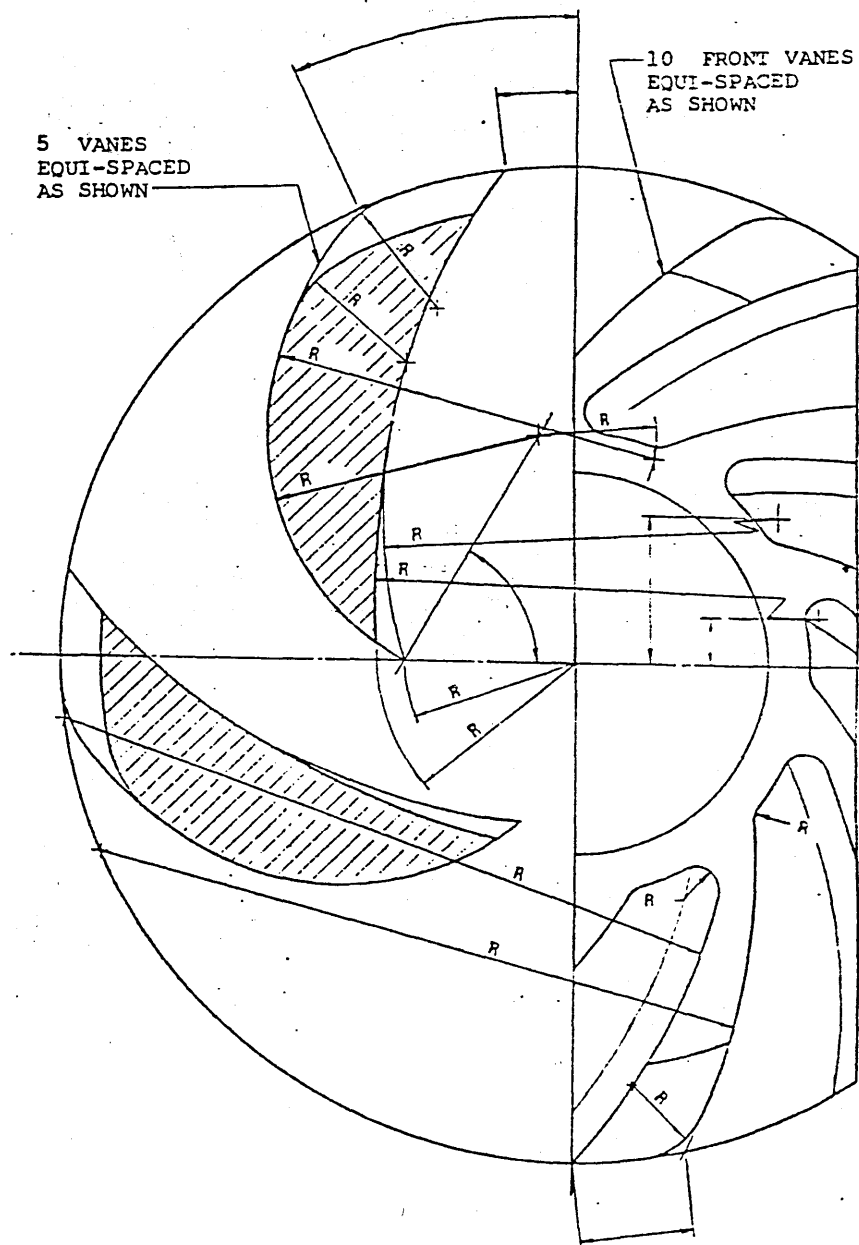


Fig. 6.2b Detailed drawing of the impeller used

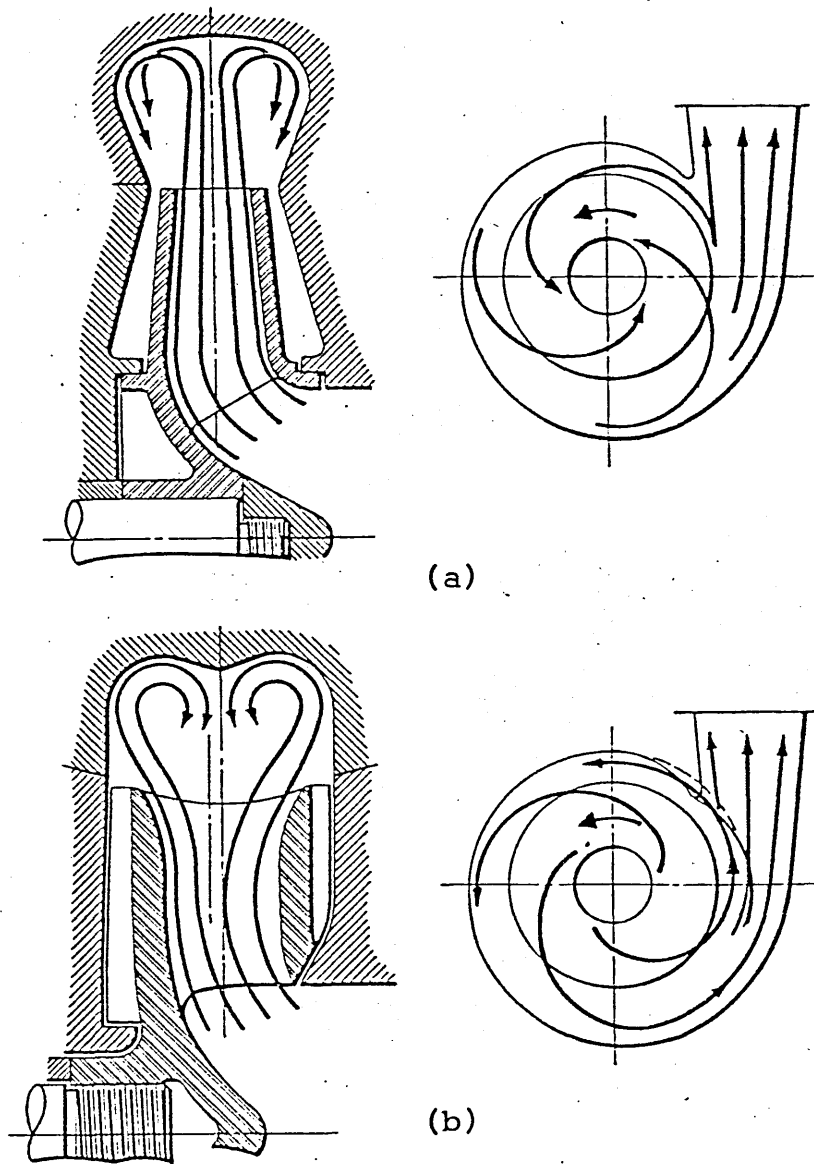


Fig. 6.3 (a) Flow pattern in a pump of conventional design (Ref. 14)  
(b) Flow pattern in a pump of Simon Warman design (Ref. 14)

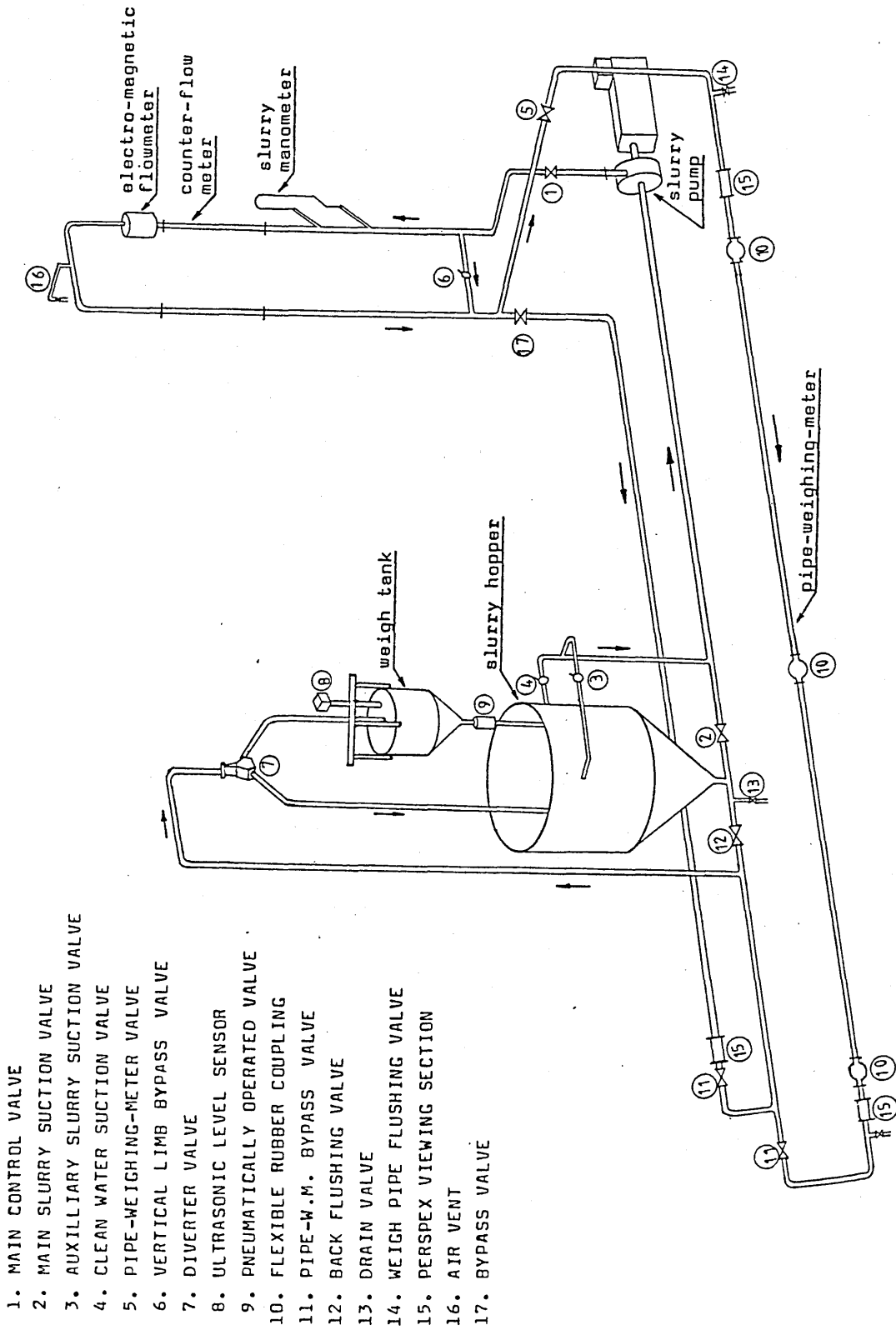


Fig. 6.4 Diagrammatic arrangement of the test rig layout.

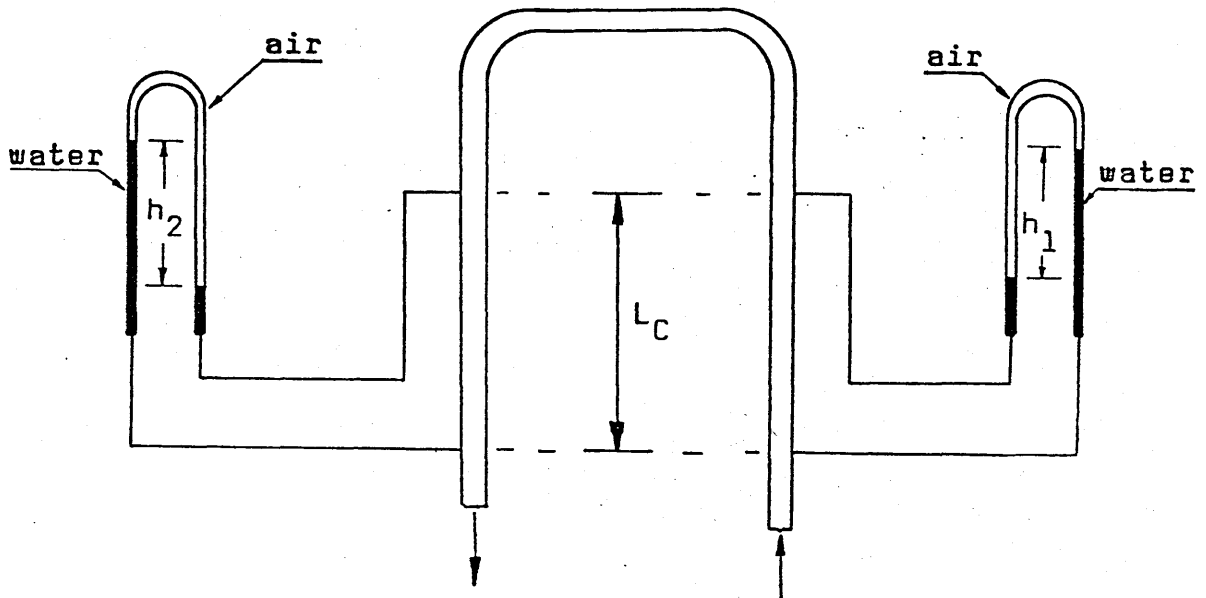


Fig. 6.5 Counterflow concentration meter

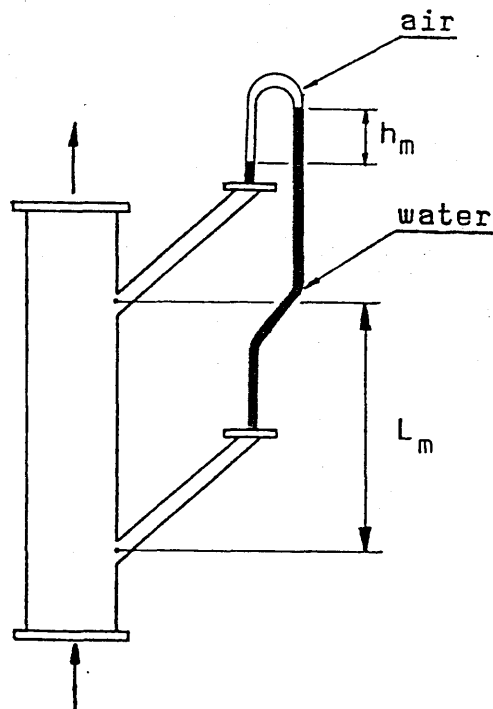
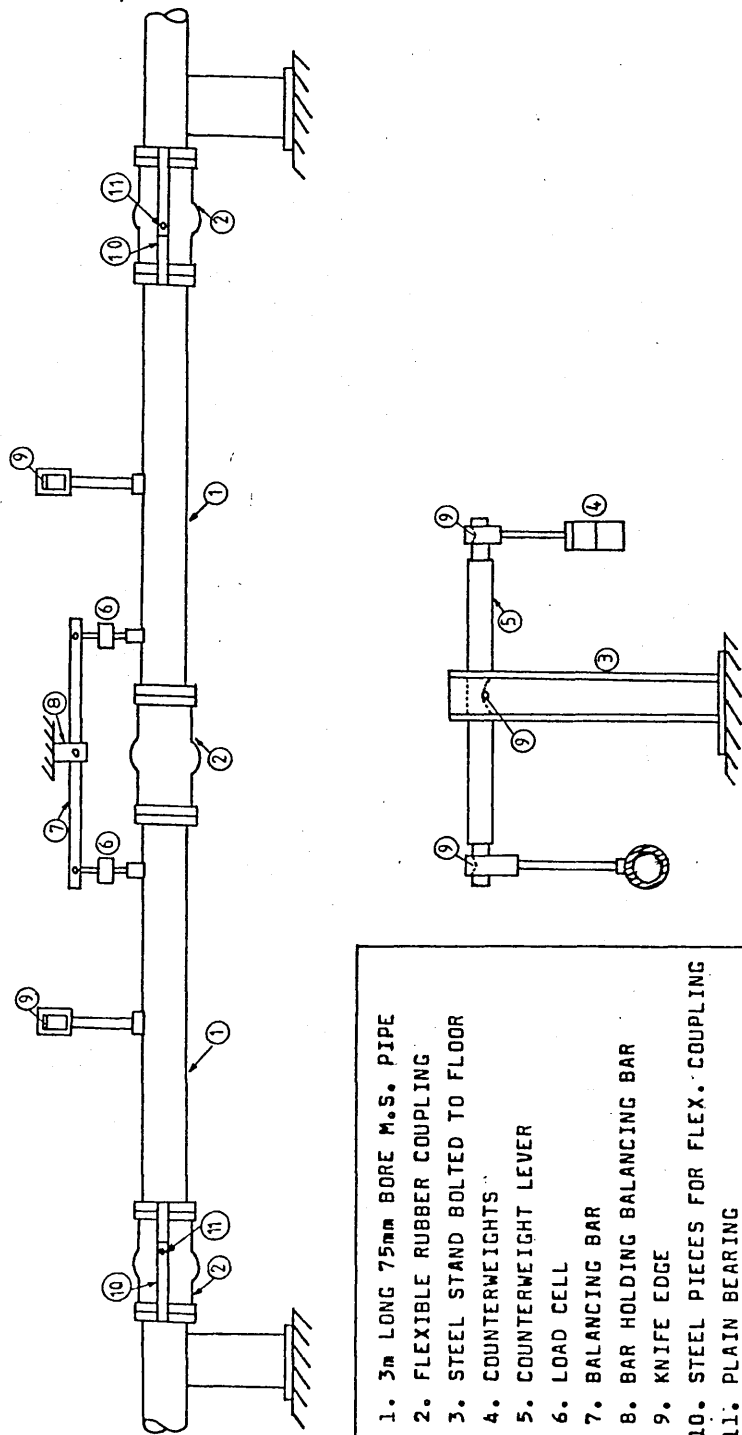


Fig. 6.6 Slurry manometer



1. 3m LONG 75mm BORE M.S. PIPE
2. FLEXIBLE RUBBER COUPLING
3. STEEL STAND BOLTED TO FLOOR
4. COUNTERWEIGHTS
5. COUNTERWEIGHT LEVER
6. LOAD CELL
7. BALANCING BAR
8. BAR HOLDING BALANCING BAR
9. KNIFE EDGE
10. STEEL PIECES FOR FLEX. COUPLING
11. PLAIN BEARING

Fig. 6.7 Pipe weighing concentration meter

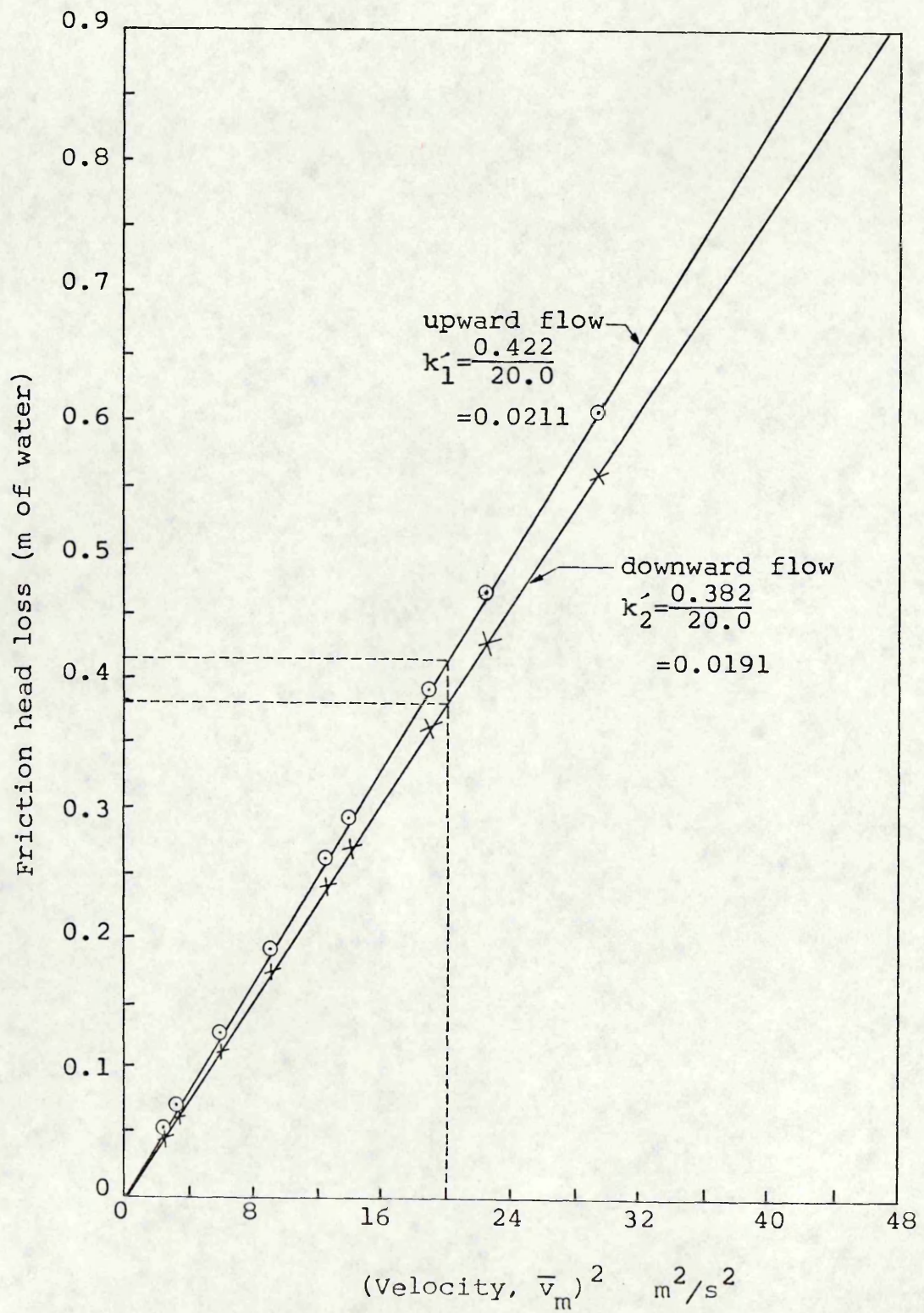


Fig. 6.8 Calibration curve for counterflow meter.



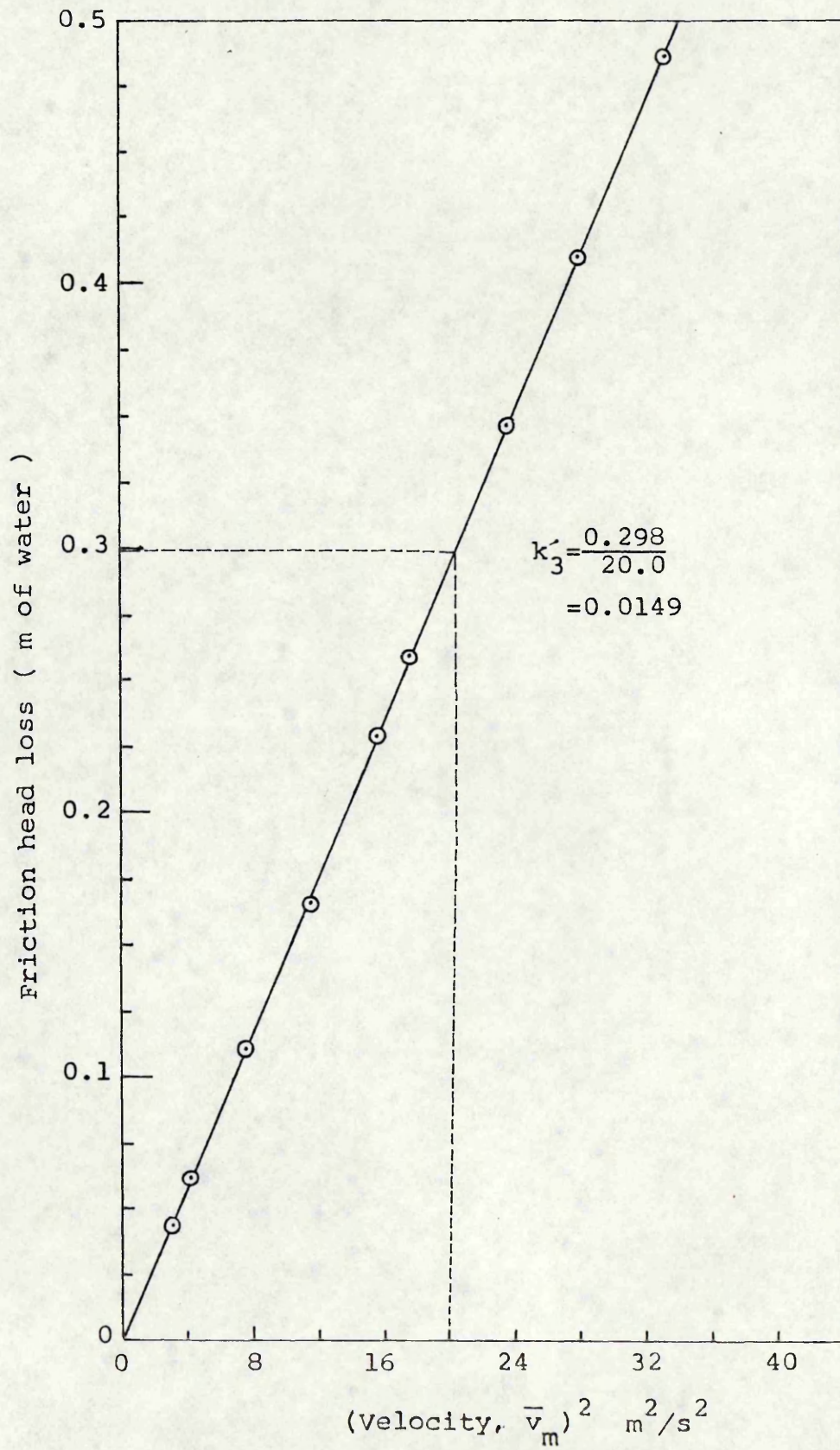


Fig. 6.9 Calibration curve for slurry manometer.



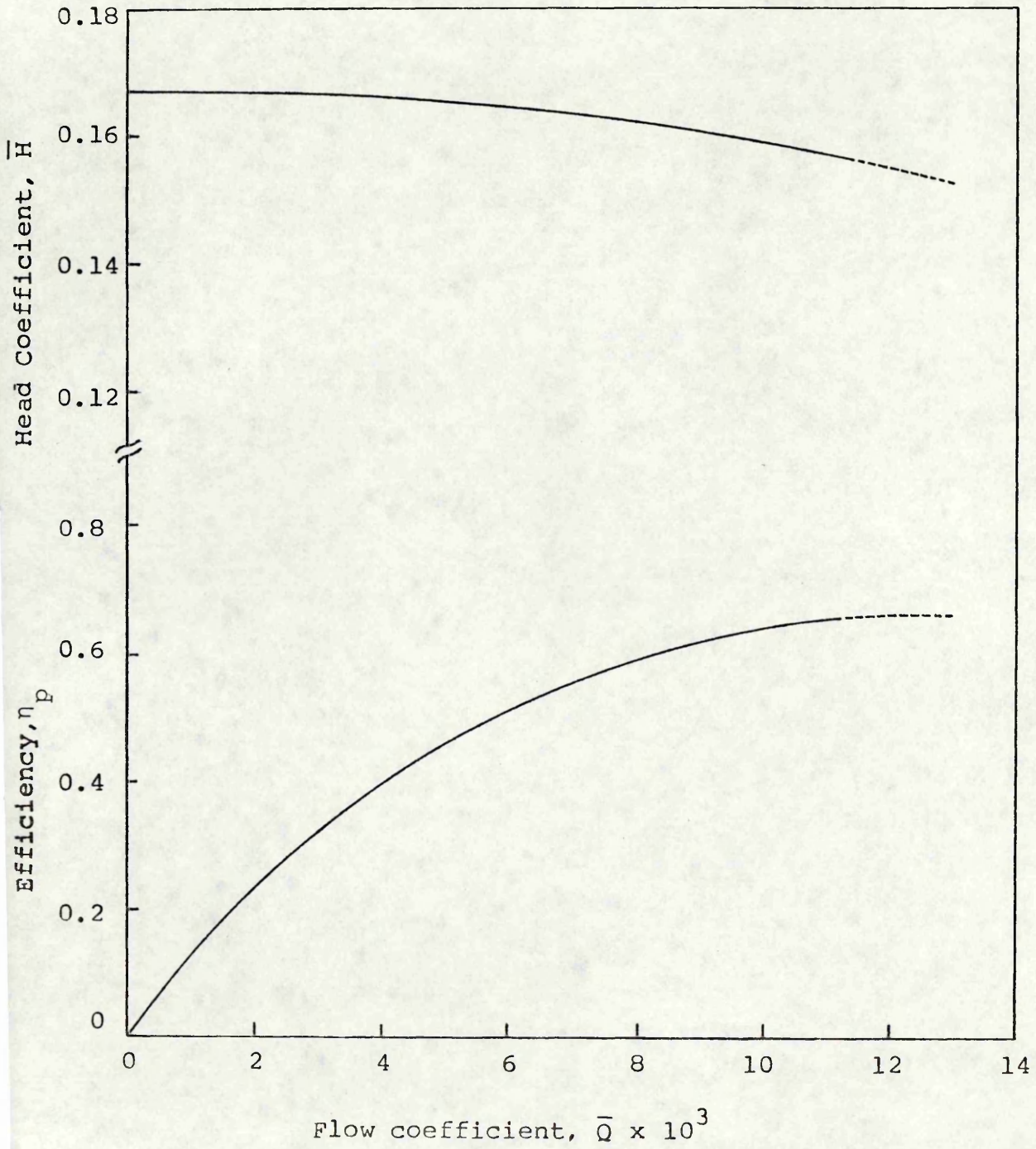


Fig.6.10 Clean water performance curves of the pump used.

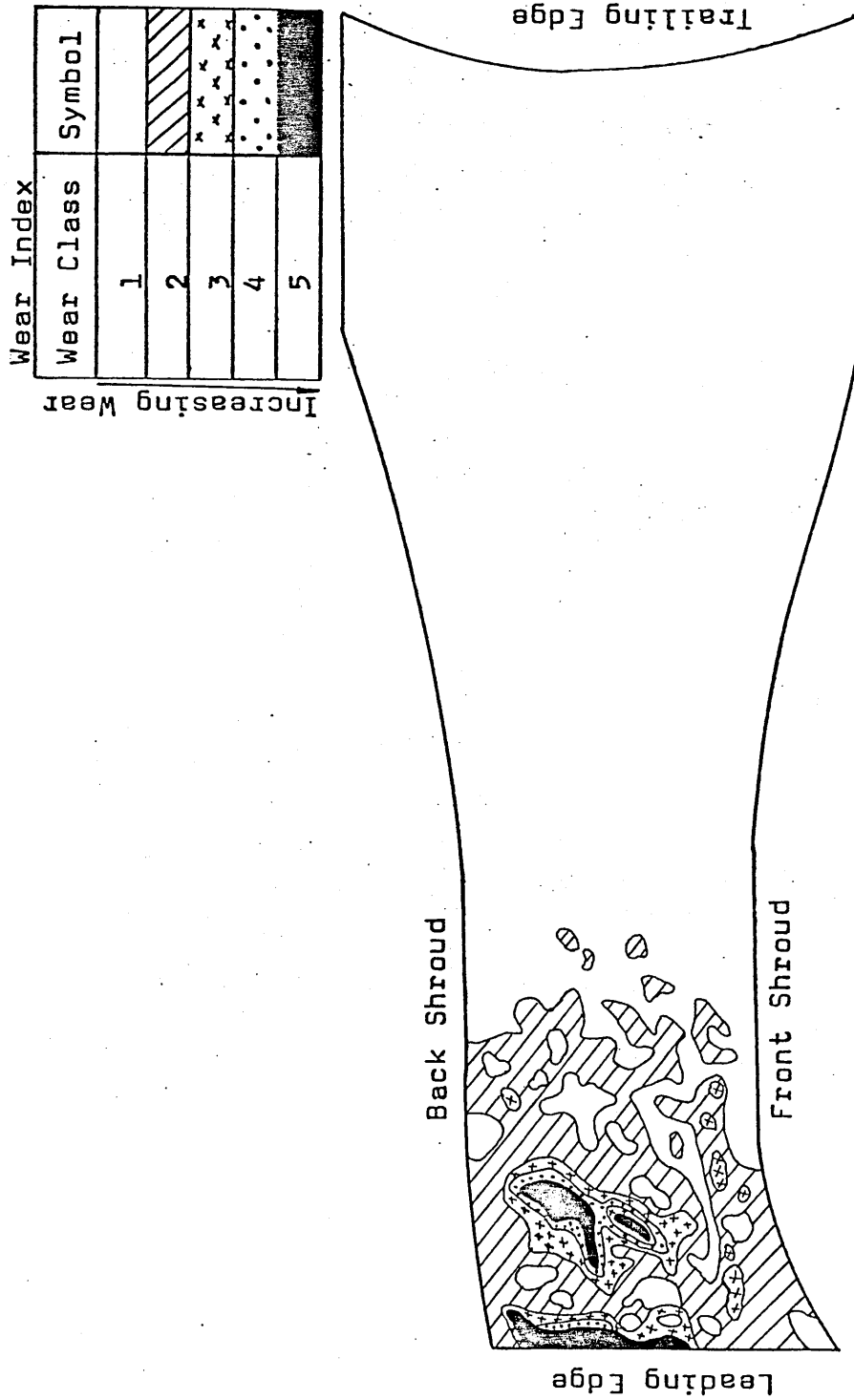


Fig. 6.11a Erosion pattern on blade pressure surface no. 1 (case 1).

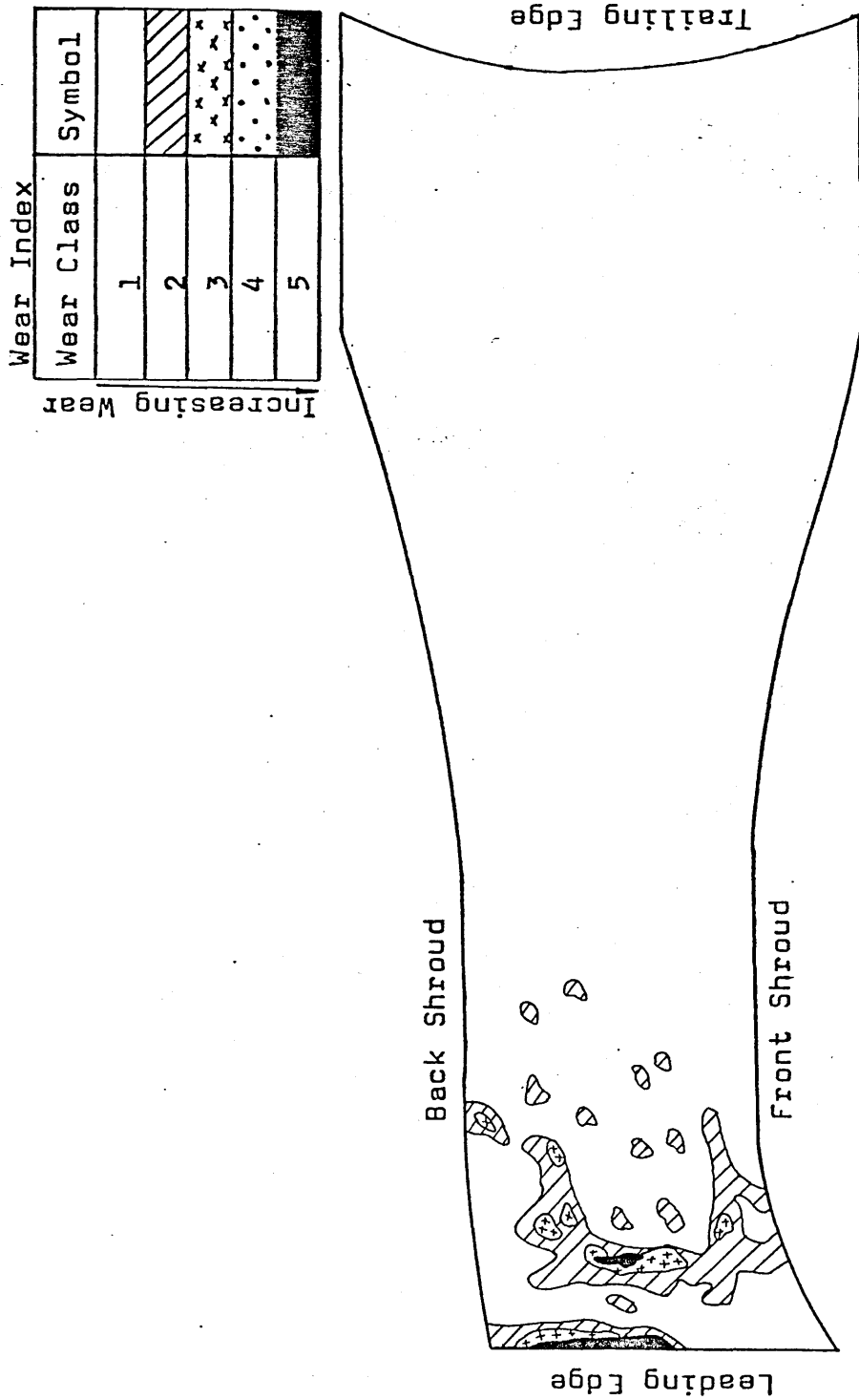


Fig. 6.11b Erosion pattern on blade pressure surface no. 2 (case 1).

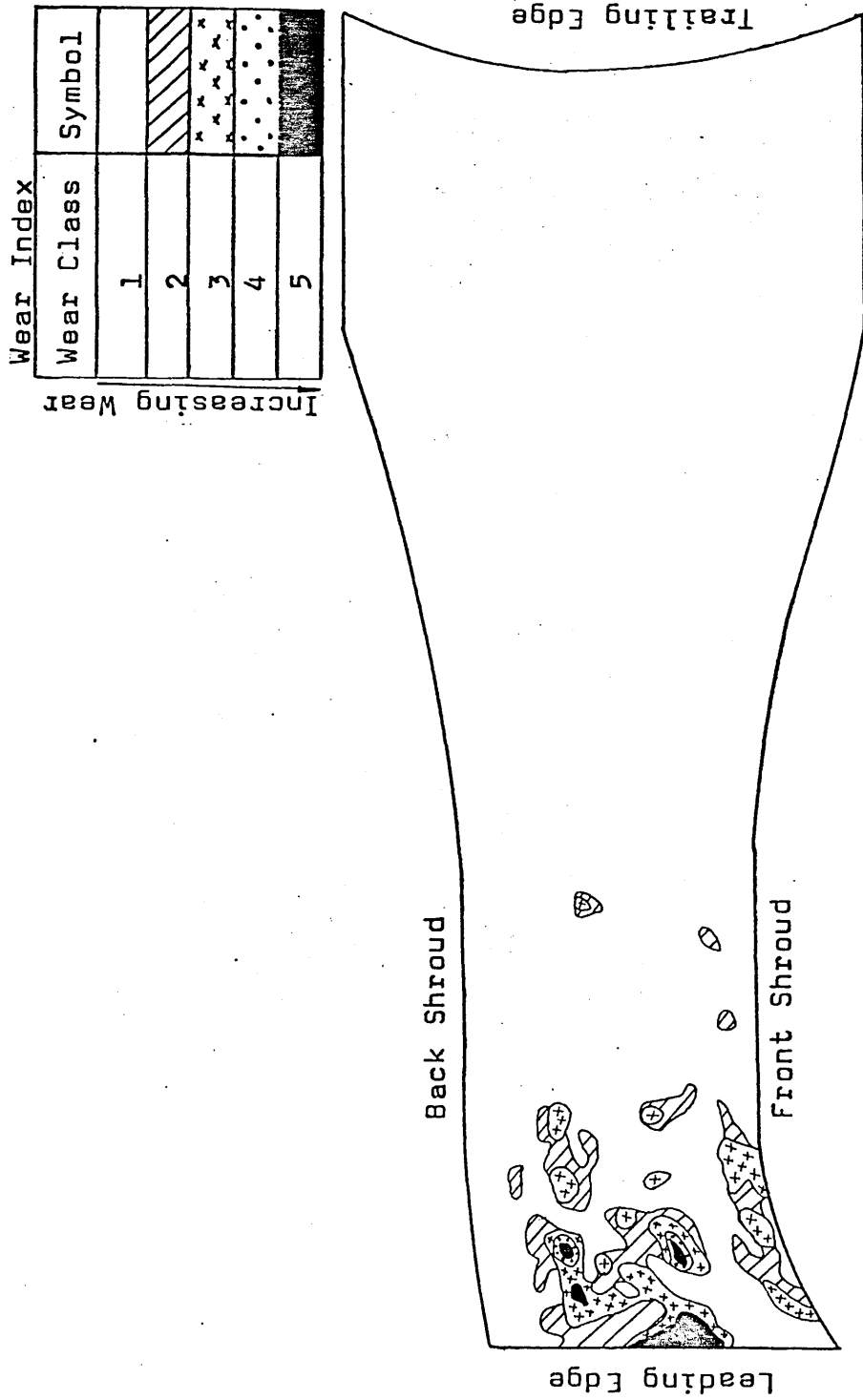


Fig. 6.11c Erosion pattern on blade pressure surface no. 3 (case 1).

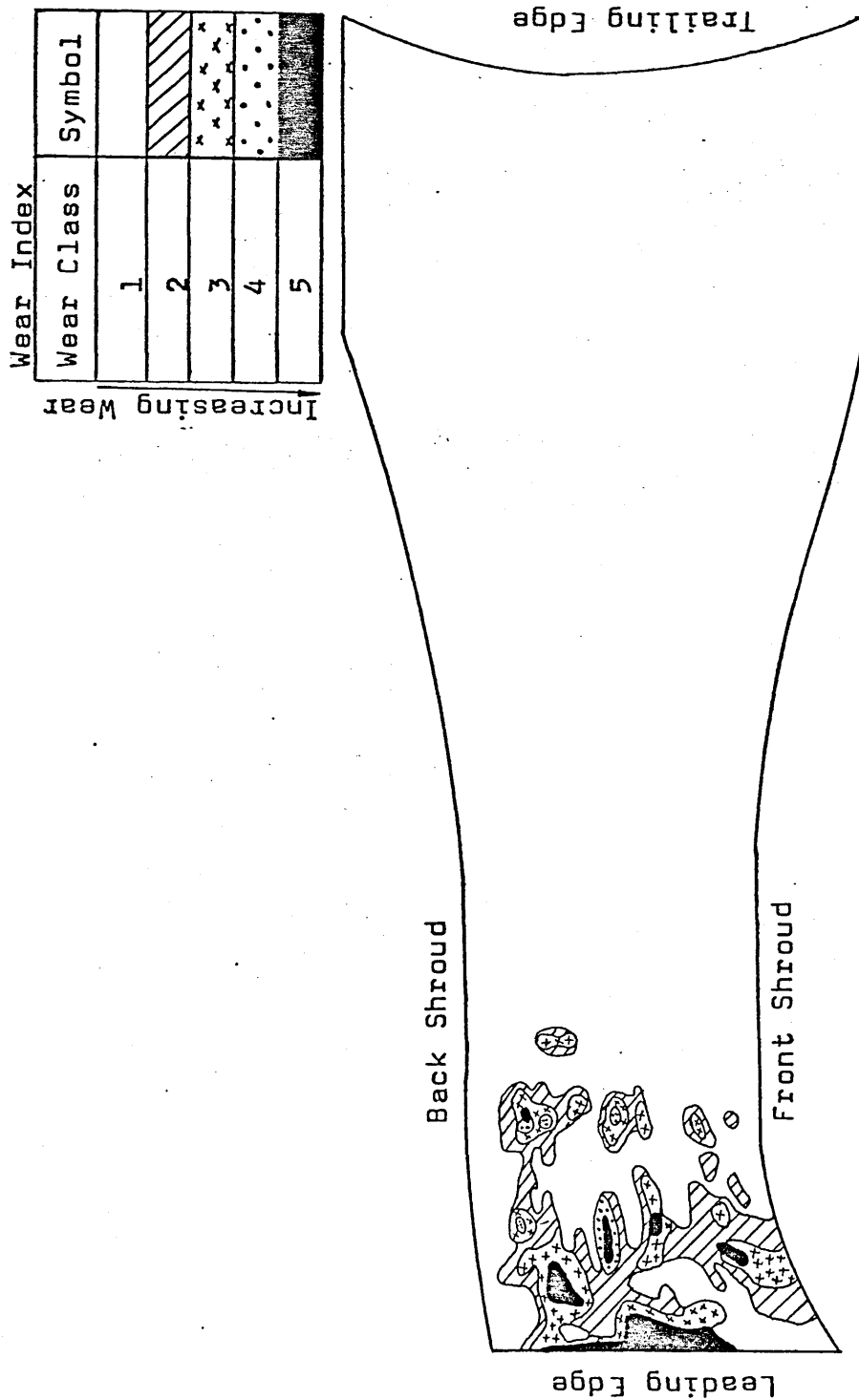


Fig. 6.11d Erosion pattern on blade pressure surface no. 4 (case 1).

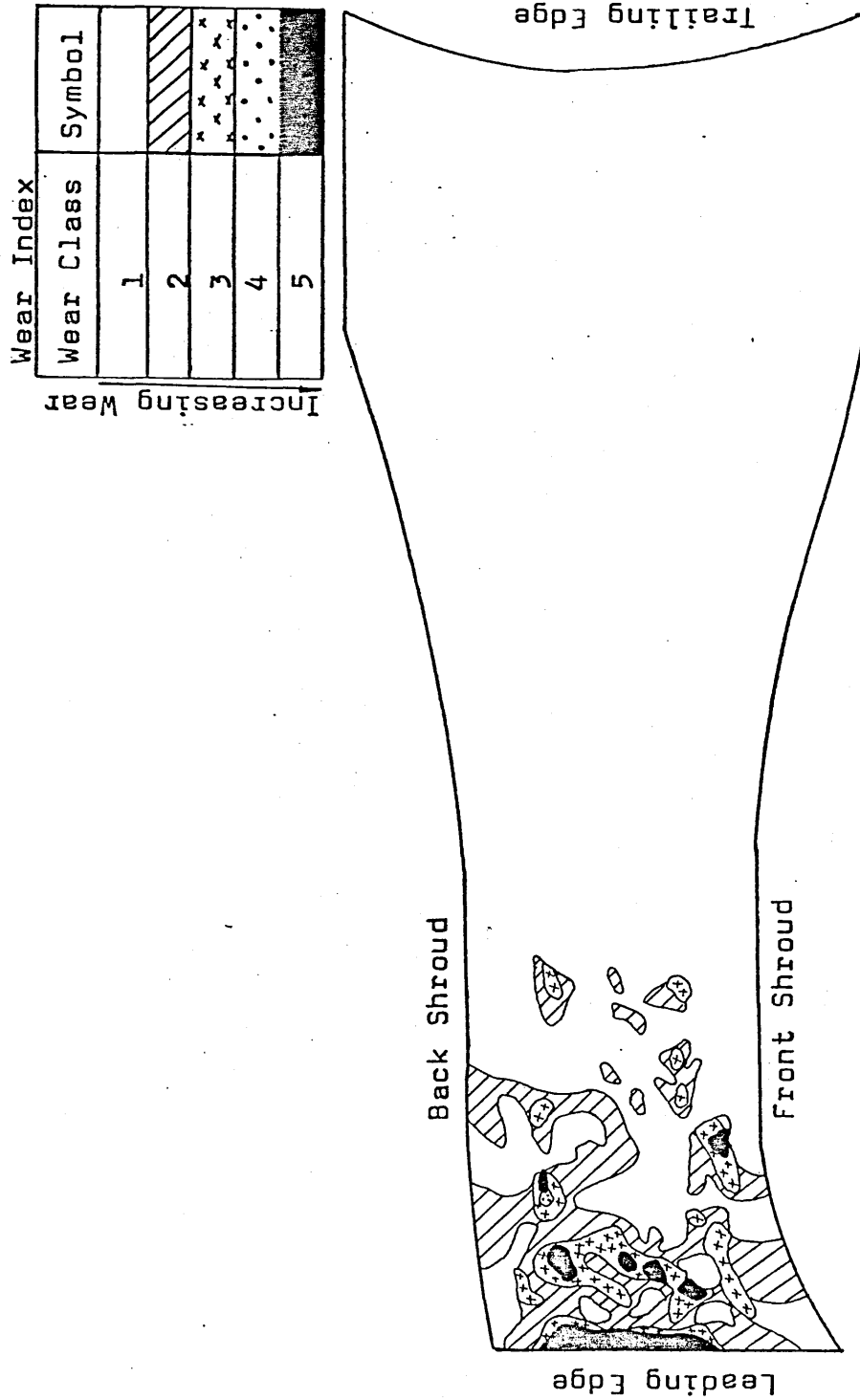


Fig. 6.11e Erosion pattern on blade pressure surface no. 5 (case 1).

Wear Index :

| Wear Class | Symbol |
|------------|--------|
| 1          |        |
| 2          |        |
| 3          |        |
| 4          |        |
| 5          |        |

Increasing Wear

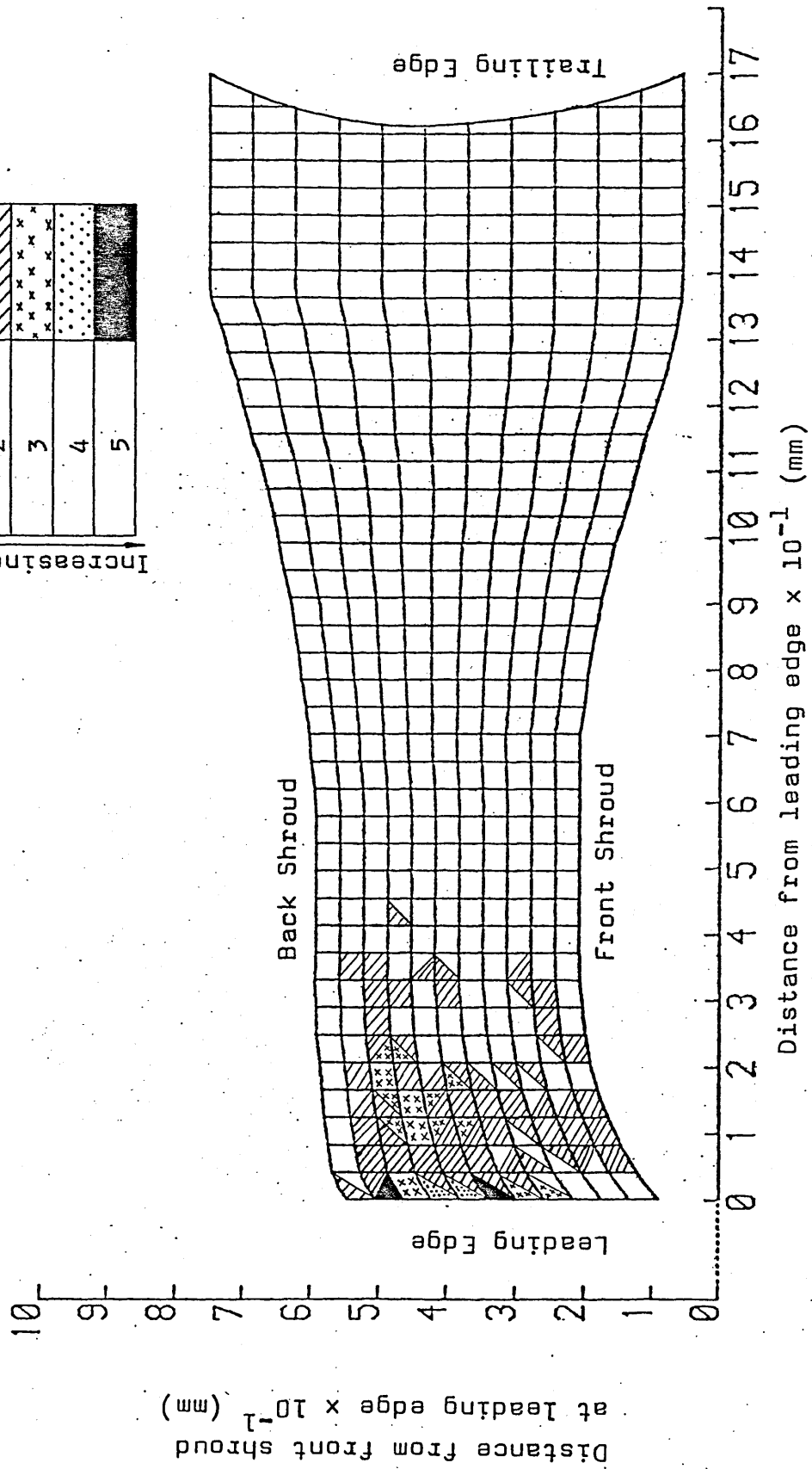


Fig. 6.11f Average erosion pattern on a blade pressure surface : Case 1.

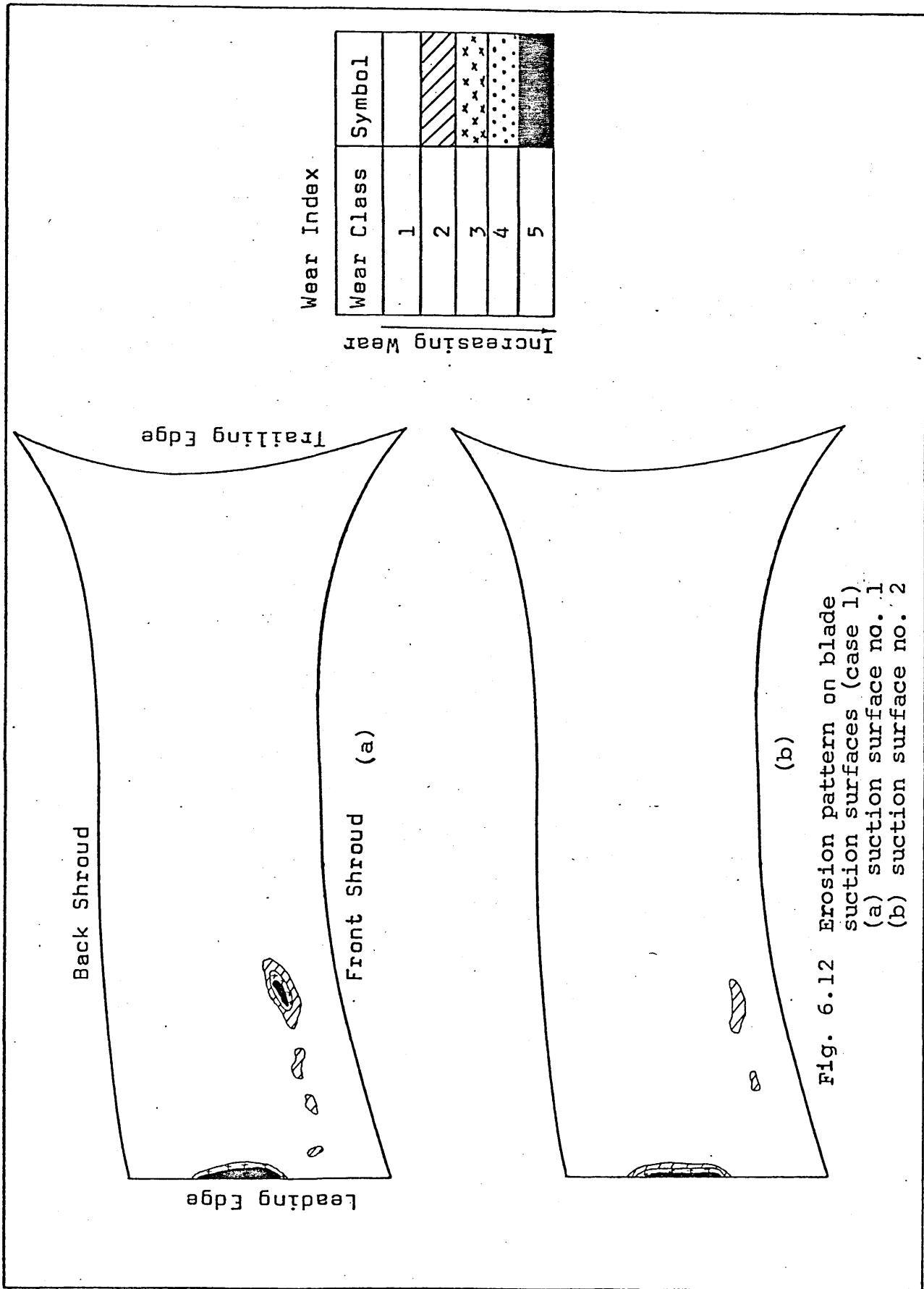


Fig. 6.12 Erosion pattern on blade suction surfaces (case 1)  
(a) suction surface no. 1  
(b) suction surface no. 2



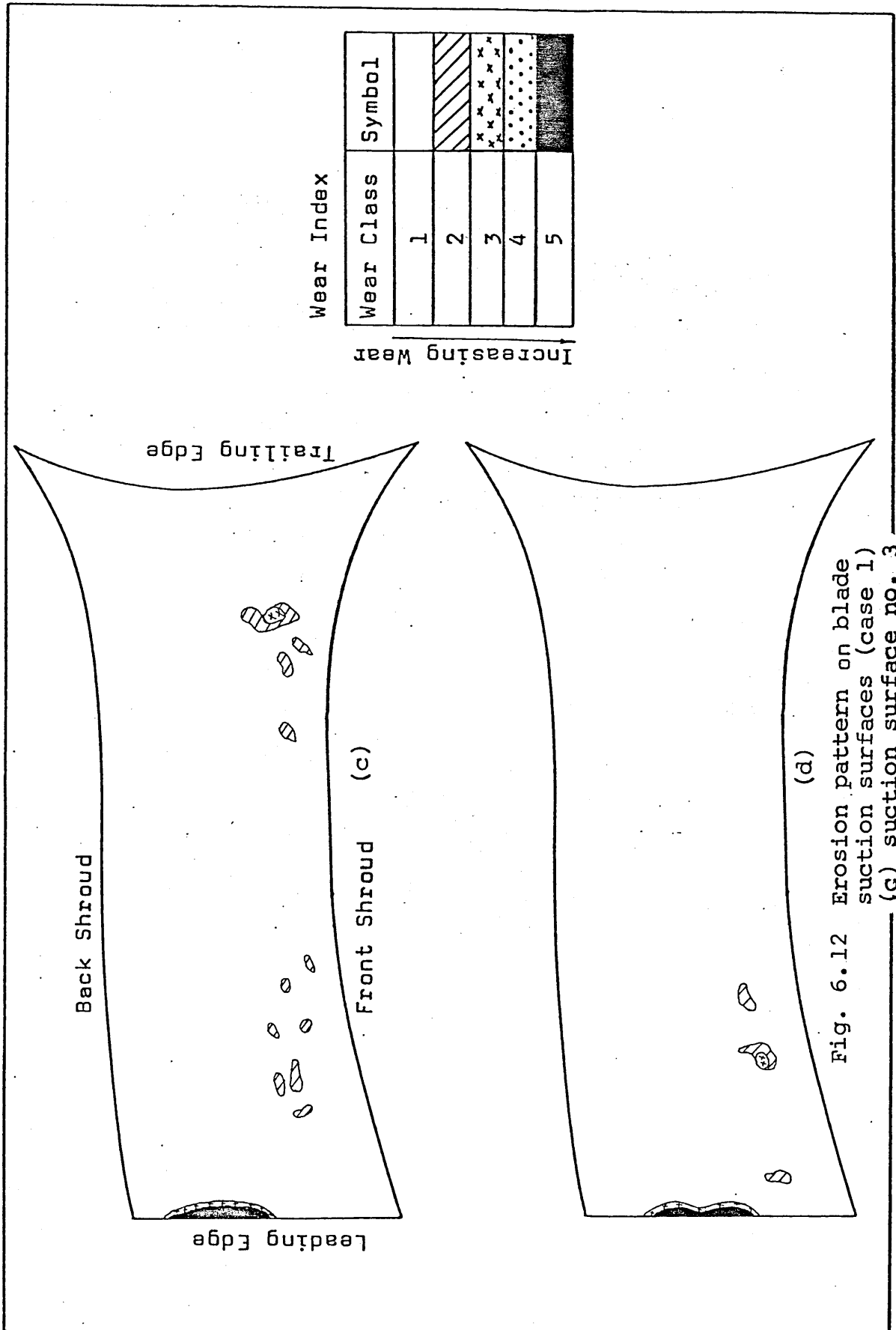


Fig. 6.12 Erosion pattern on blade suction surfaces (case 1)  
(c) suction surface no. 3  
(d) suction surface no. 4

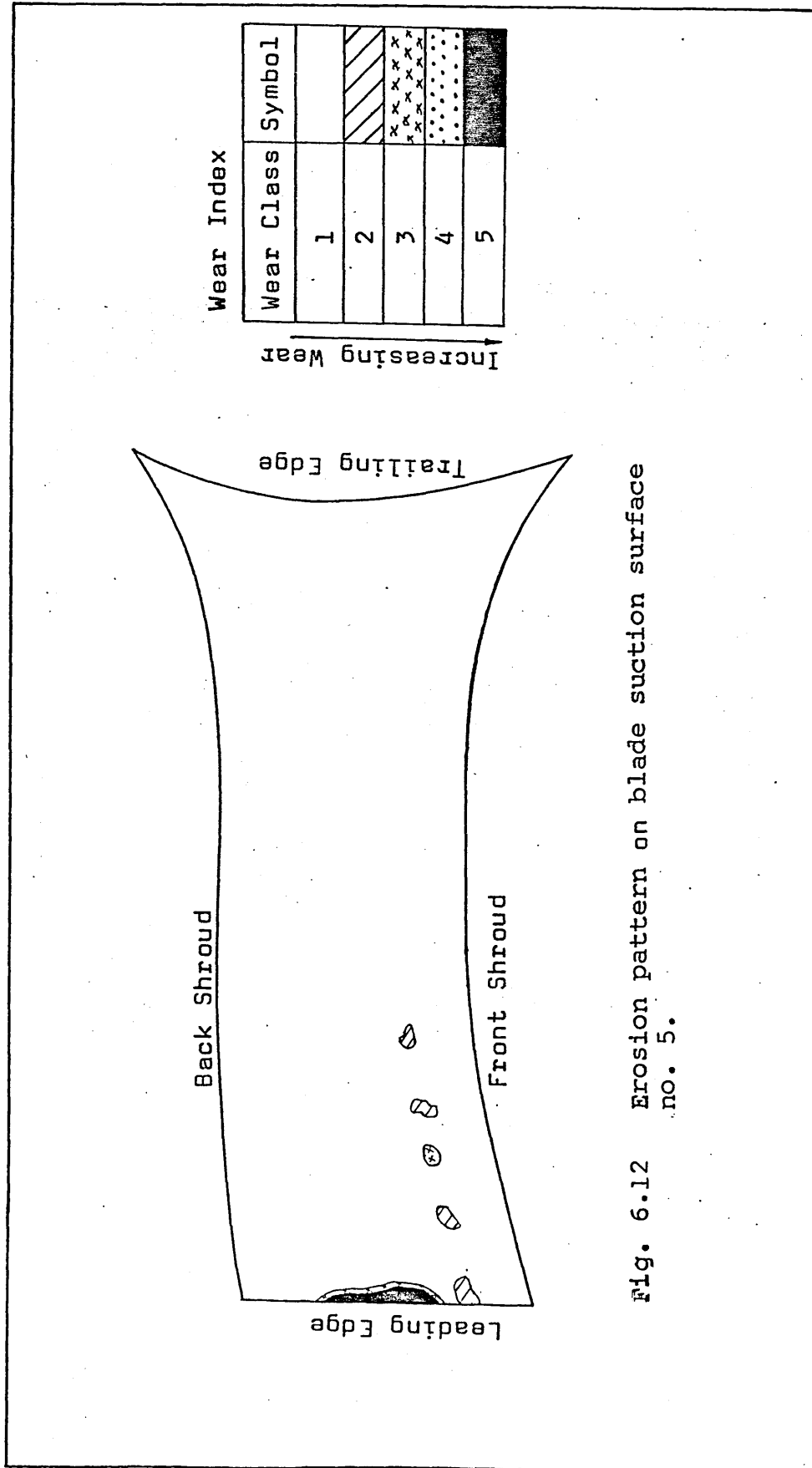


Fig. 6.12 Erosion pattern on blade suction surface no. 5.

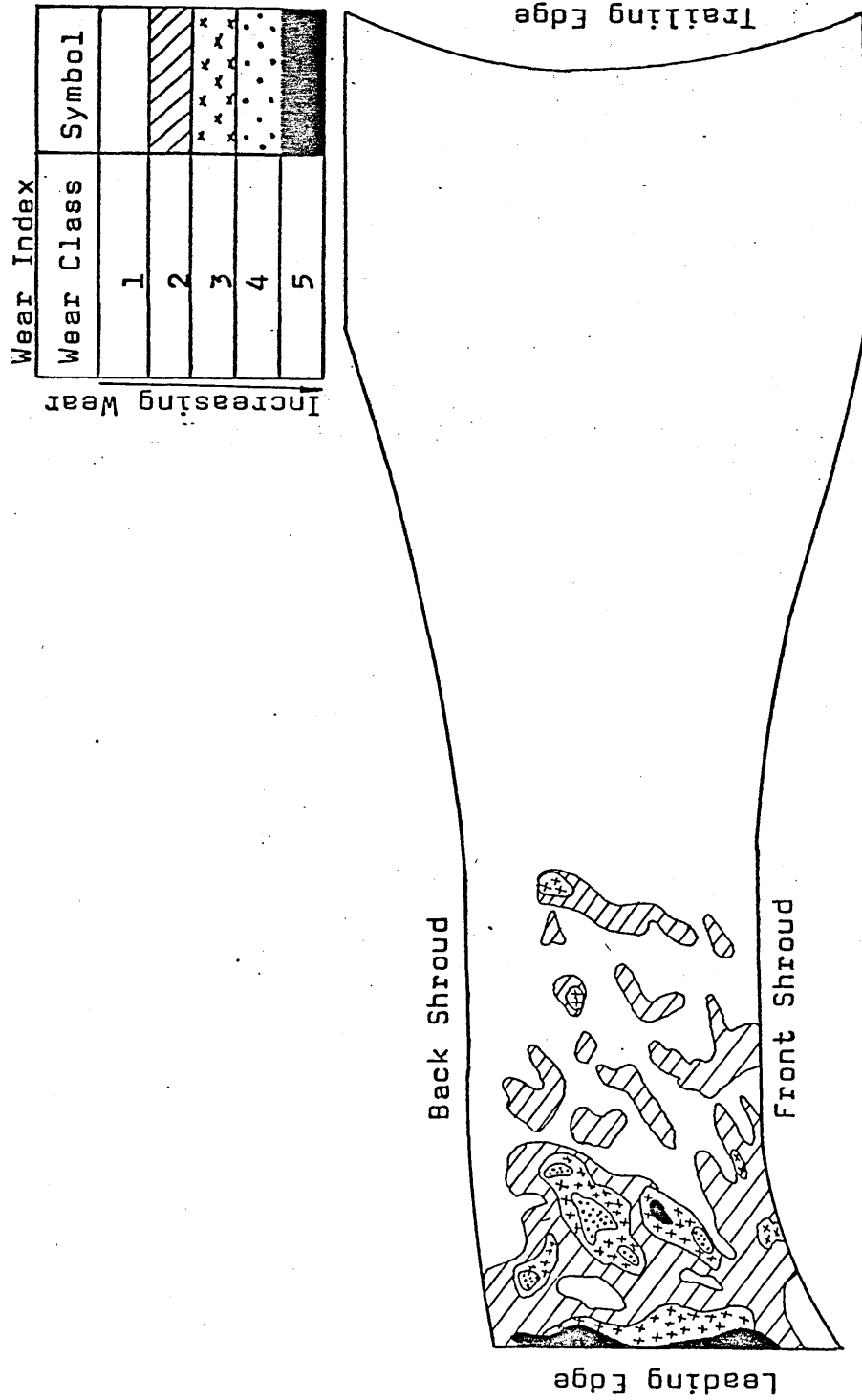


Fig. 6.13a Erosion pattern on blade pressure surface no. 1 (case 2).

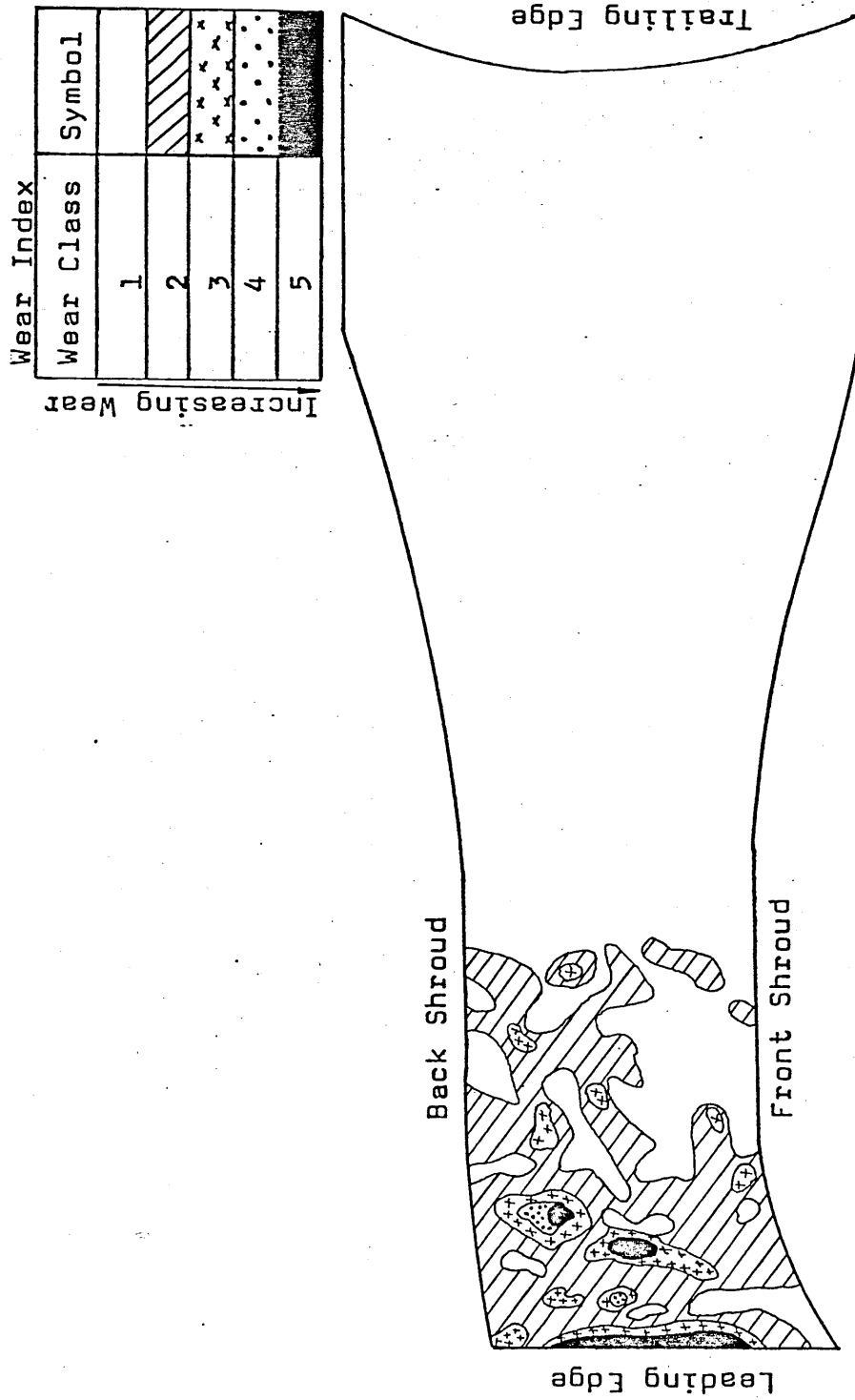


Fig. 6.13b Erosion pattern on blade pressure surface no. 2 (case 2).

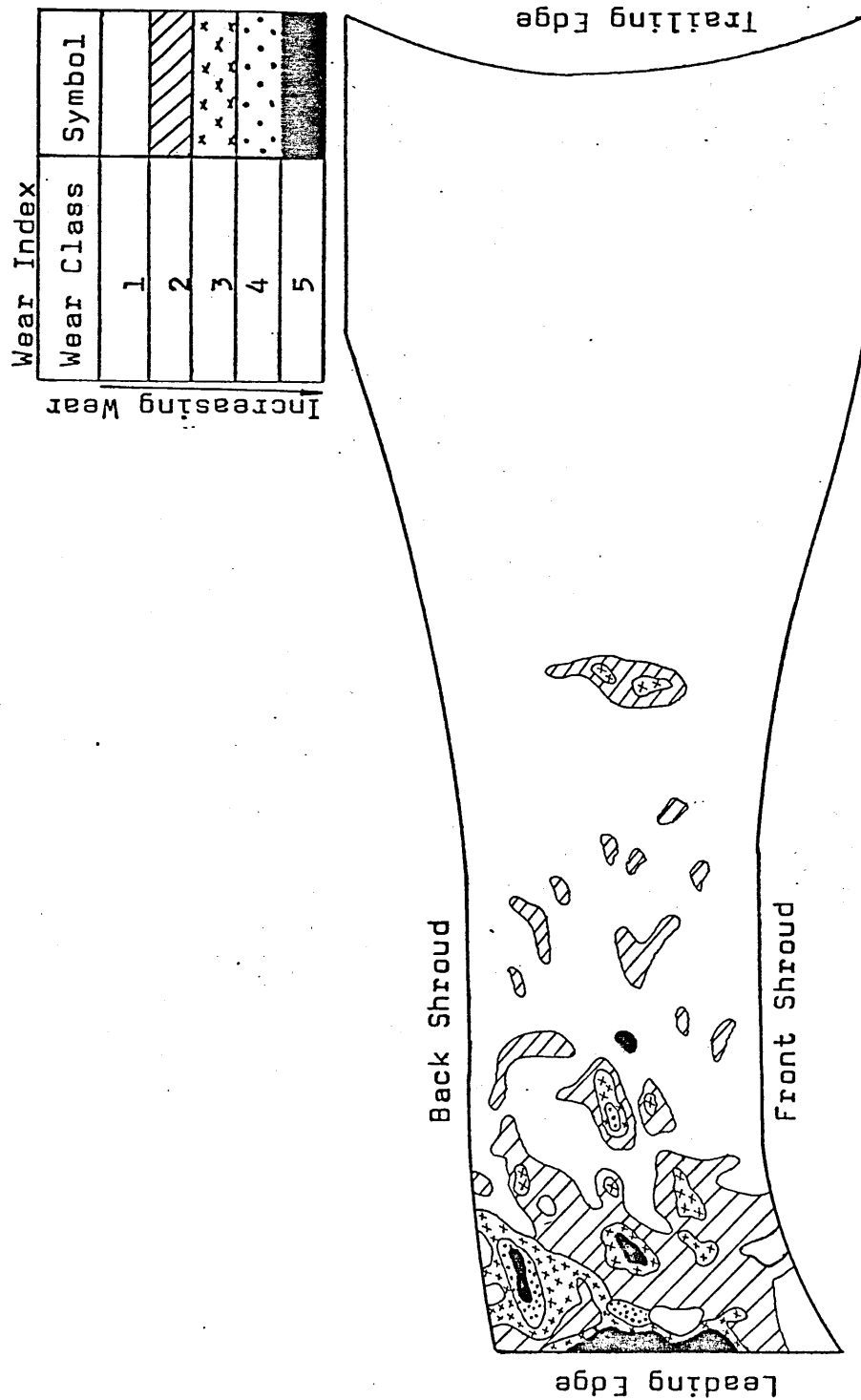


Fig. 6.13c Erosion pattern on blade pressure surface no. 3 (case 2).

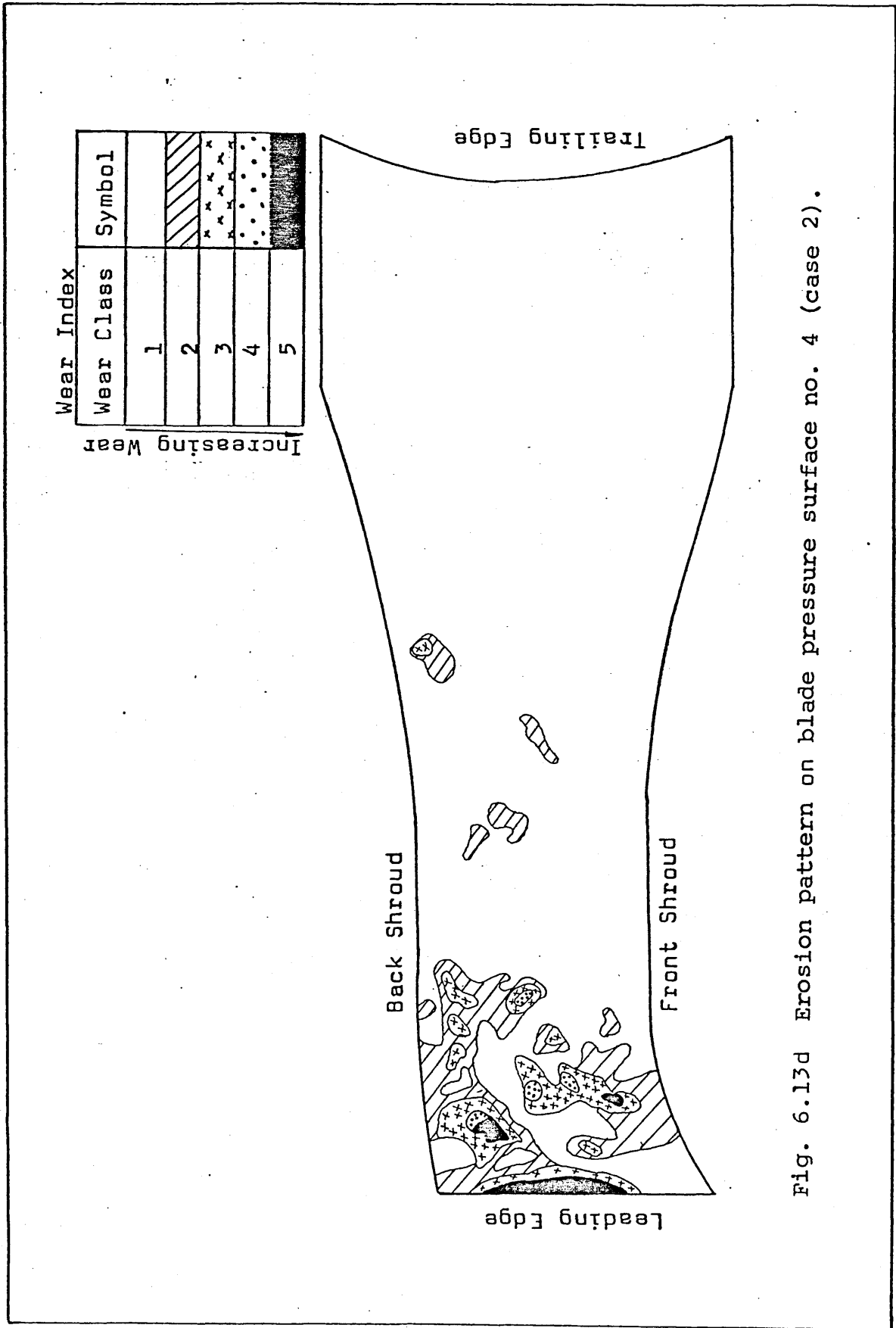


Fig. 6.13d Erosion pattern on blade pressure surface no. 4 (case 2).

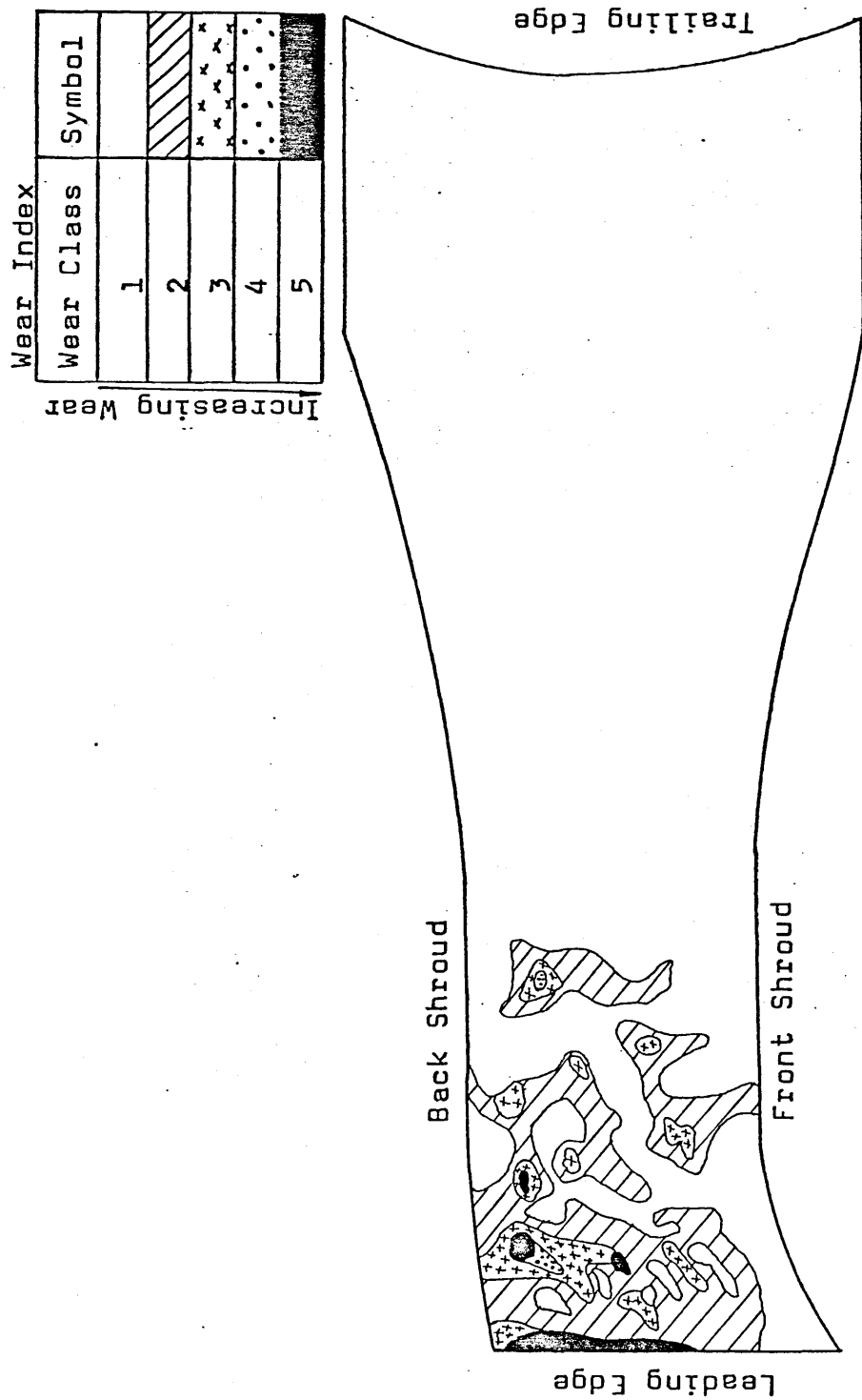


Fig. 6.13e Erosion pattern on blade pressure surface no. 5 (case 2).

Wear Index :

| Wear Class | Symbol |
|------------|--------|
| 1          |        |
| 2          |        |
| 3          |        |
| 4          |        |
| 5          |        |

Increasing Wear

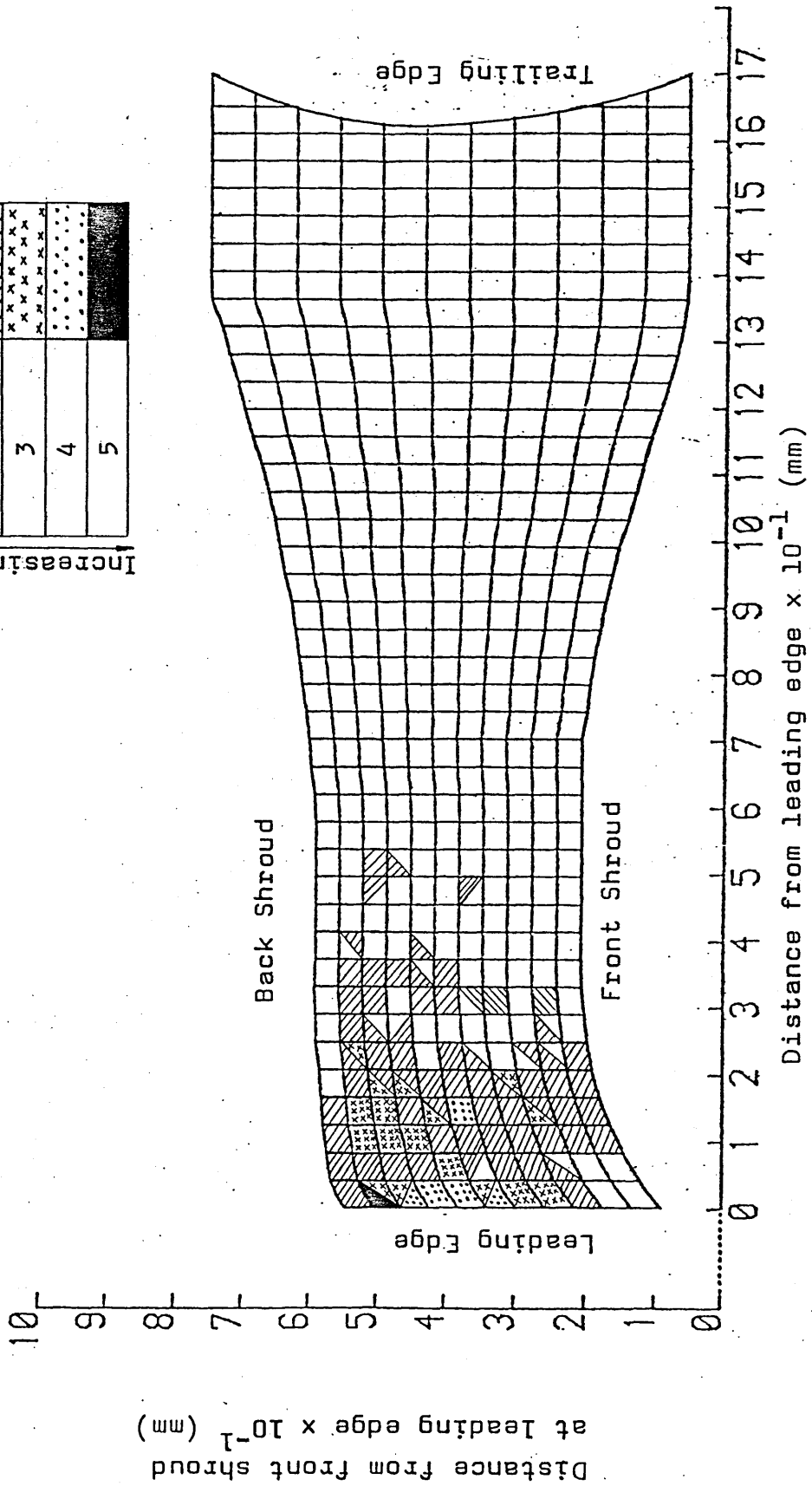
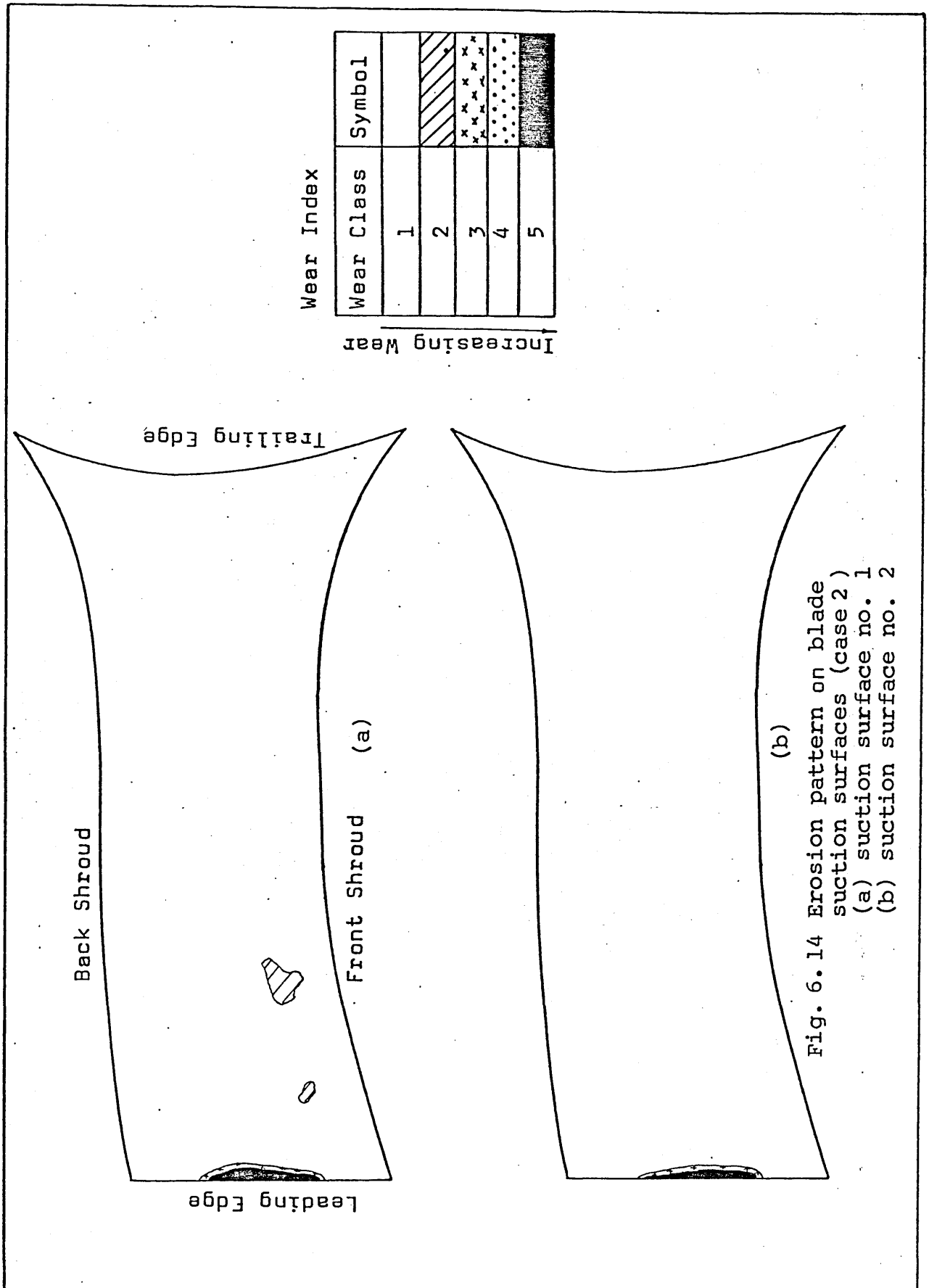


Fig. 6.13f Average erosion pattern on a blade pressure surface : Case 2.





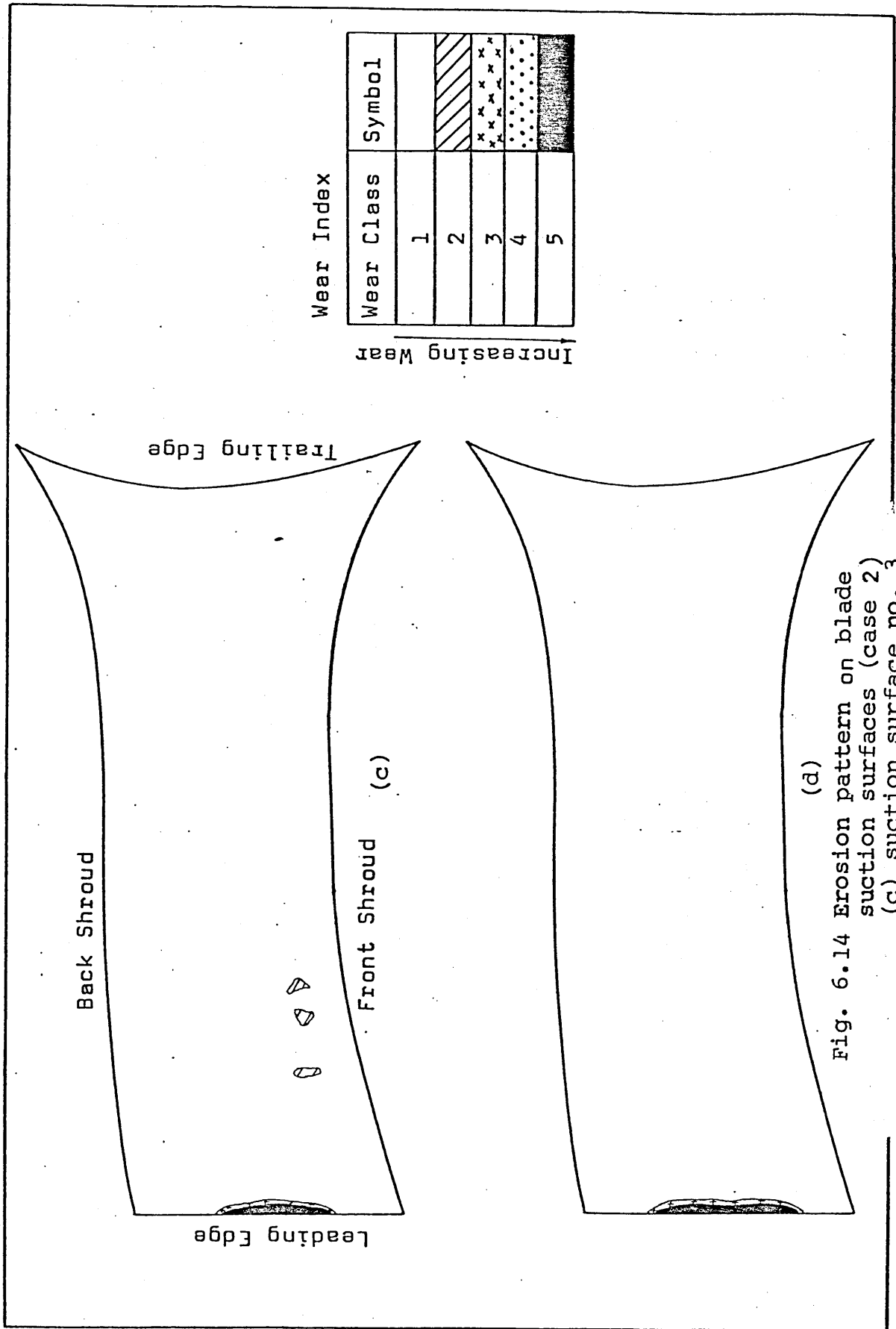


Fig. 6.14 Erosion pattern on blade suction surfaces (case 2)  
 (c) suction surface no. 3  
 (d) suction surface no. 4

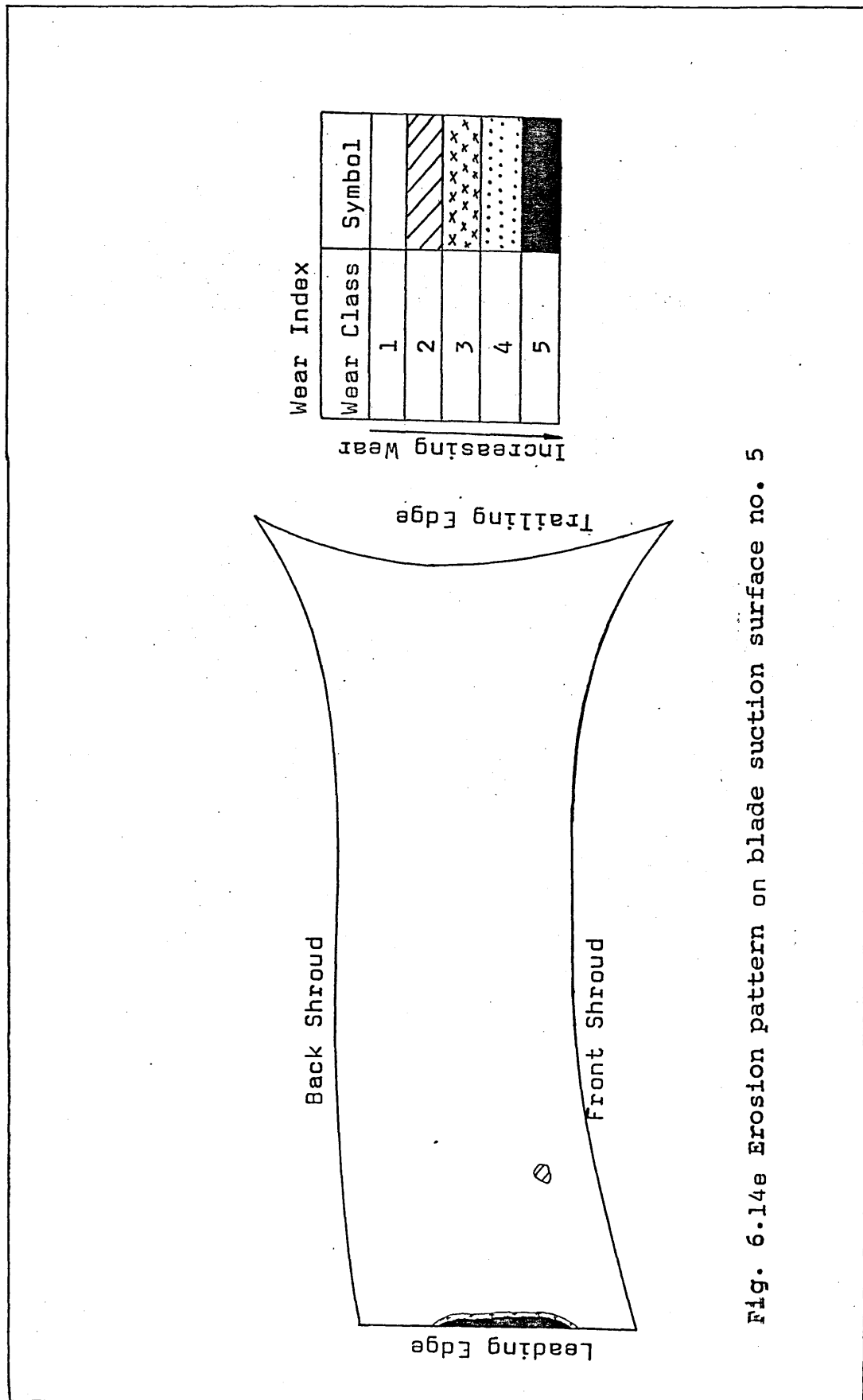


Fig. 6.14e Erosion pattern on blade suction surface no. 5

## APPENDIX A

### EQUATIONS OF MOTION OF PARTICLES IN A CYLINDRICAL POLAR COORDINATE SYSTEM

#### A.1 Inertial System

The equation of motion of a particle in an inertial coordinate system can be written from Newton's second law of motion as follows:-

$$m_p \underline{a}_{p/f} = \underline{F}_S \quad (A - 1)$$

where,  $a_{p/f}$  is the acceleration of the particle in the inertial system and  $F_S$  is the resultant of all the external forces acting on the particle.

In an inertial cartesian coordinate system x-y-z, the acceleration of the particle in the coordinate directions can be written as follows :-

$$m_p \ddot{x} = F_{S_x} \quad (A - 2a)$$

$$m_p \ddot{y} = F_{S_y} \quad (A - 2b)$$

$$m_p \ddot{z} = F_{S_z} \quad (A - 2c)$$

In an inertial cylindrical polar coordinate system r- $\theta$ -z, the equation of motion of the particle can be formed from transformation of the acceleration terms in equations (A - 2a) to (A - 2c), using the following geometric relationship :-

$$x = r \cos\theta \quad (A - 3a)$$

$$y = r \sin\theta \quad (A - 3b)$$

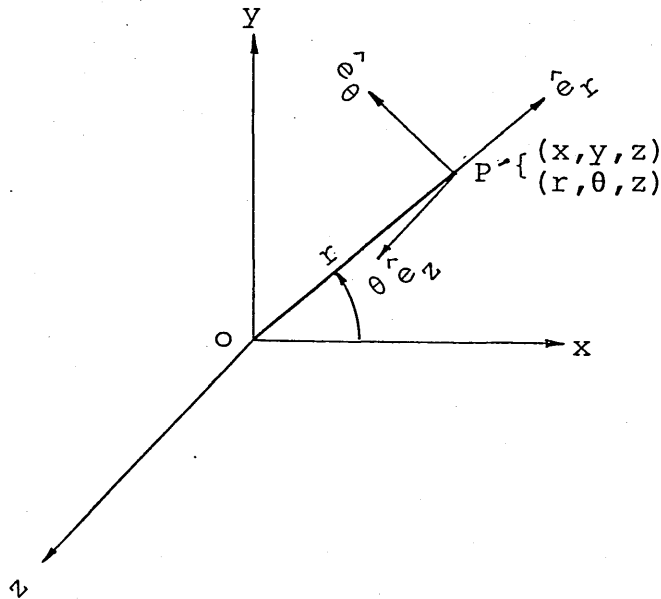


Fig. A1 Particle position in two coordinate systems;  $\hat{e}_r$ ,  $\hat{e}_\theta$ ,  $\hat{e}_z$  are unit vectors in positive  $r$ ,  $\theta$  and  $z$  directions, respectively.

Taking second derivative of equations (A - 3a) and (A - 3b) with respect to time :-

$$\ddot{x} = \ddot{r} \cos\theta - 2 \dot{r} \dot{\theta} \sin\theta - r \cos\theta \ddot{\theta}^2 - r \sin\theta \ddot{\theta} \quad (A - 4a)$$

$$\ddot{y} = \ddot{r} \sin\theta + 2 \dot{r} \dot{\theta} \cos\theta - r \sin\theta \ddot{\theta}^2 + r \cos\theta \ddot{\theta} \quad (A - 4b)$$

Also, if  $F_{S_r}$  and  $F_{S_\theta}$  are the components of the force vector  $\underline{F}_S$  in positive  $r$  and  $\theta$  directions, respectively, then :-

$$\underline{F}_S = F_{S_r} \hat{e}_r + F_{S_\theta} \hat{e}_\theta \quad (A - 5)$$

Taking components of  $\underline{F}_S$  in x and y directions :-

$$F_{S_x} = F_{S_r} \cos\theta - F_{S_\theta} \sin\theta \quad (\text{A} - 6a)$$

$$F_{S_y} = F_{S_r} \sin\theta + F_{S_\theta} \cos\theta \quad (\text{A} - 6B)$$

From equations (A - 2a), (A - 2b), (A - 4a), (A - 4b) and (A - 6a), (A - 6b), it follows that :-

$$\begin{aligned} m_p (\ddot{r} \cos\theta - 2 \dot{r} \dot{\theta} \sin\theta - r \cos\theta \ddot{\theta}^2 - r \sin\theta \ddot{\theta}) = \\ = F_{S_r} \cos\theta - F_{S_\theta} \sin\theta \end{aligned} \quad (\text{A} - 7a)$$

$$\begin{aligned} m_p (\ddot{r} \sin\theta + 2 \dot{r} \dot{\theta} \cos\theta - r \sin\theta \ddot{\theta}^2 - r \cos\theta \ddot{\theta}) = \\ = F_{S_r} \sin\theta + F_{S_\theta} \cos\theta \end{aligned} \quad (\text{A} - 7b)$$

since the above equations are valid for all values of  $\theta$ ,  
putting  $\theta = 0$  :-

$$m_p (\ddot{r} - r \dot{\theta}^2) = F_{S_r} \quad (\text{A} - 8a)$$

$$m_p (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) = F_{S_\theta} \quad (\text{A} - 8b)$$

## A.2 Rotating System

An inertial coordinate system x-y-z and a

rotating coordinate system  $x'-y'-z'$ , both having the same origin  $O$  are shown in fig. A2. The position vector  $\underline{r}$  of a particle is also shown in the same figure.

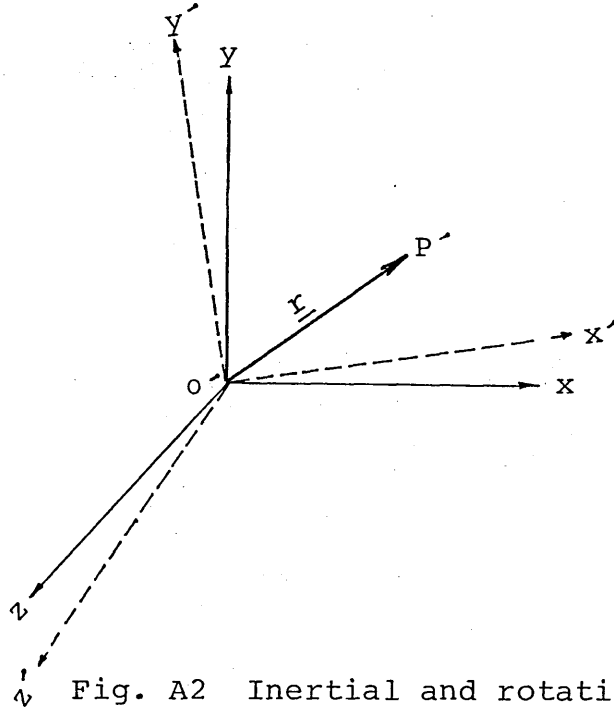


Fig. A2 Inertial and rotating systems

The velocity of the particle in the inertial system can be related to the rotating system as follows :-

$$D_f \underline{r} = (D_r + \underline{\omega}' \times ) \underline{r} \quad (A -9)$$

where,  $\underline{\omega}'$  is the angular velocity vector of the rotating system and the operators  $D_f$  and  $D_r$  have the following meaning :-

$$D_f = \left. \frac{d}{dt} \right|_f \quad D_r = \left. \frac{d}{dt} \right|_r$$

and are introduced to distinguish between time derivatives in inertial and rotating systems (subscripts f for a fixed or inertial system and r for a rotating system). From equation (A - 9), the acceleration of the particle in the two systems can be related as follows :-

$$D_f^2 \underline{r} = D_f (D_f \underline{r})$$

$$= (D_r + \underline{\omega}' \times) (D_r \underline{r} + \underline{\omega}' \times \underline{r})$$

$$= D_r^2 \underline{r} + (D_r \underline{\omega}') \times \underline{r} + 2 \underline{\omega}' \times D_r \underline{r} + \underline{\omega}' \times (\underline{\omega}' \times \underline{r}) \quad (A - 10a)$$

or :-

$$\underline{a}_{p/f} = \underline{a}_{p/r} + \underline{a}_{r/f} \quad (A - 10b)$$

where :-

$$\underline{a}_{p/f} = D_f^2 \underline{r} \quad (A - 11a)$$

$$\underline{a}_{p/r} = D_r^2 \underline{r}$$

$$\underline{a}_{r/f} = (D_r \underline{\omega}') \times \underline{r} + 2 \underline{\omega}' \times \underline{v}' + \underline{\omega}' \times (\underline{\omega}' \times \underline{r}) \quad (A - 11c)$$

where,  $\underline{v}'$  is the velocity of the particle in the rotating system. If  $\underline{\omega}'$  is constant, then :-

$$\underline{a}_{r/f} = 2 \underline{\omega}' \times \underline{v}' + \underline{\omega}' \times (\underline{\omega}' \times \underline{r}) \quad (A - 11)$$



Comparing with equation (A - 1) and bearing in mind that Newton's law is valid for an inertial system only, the equation of motion of the particle in the rotating system can be written as follows :-

$$m_p \underline{a}_{p/r} = \underline{F}_S - 2 m_p (\underline{\omega}' \times \underline{v}') - m_p \underline{\omega}' \times (\underline{\omega}' \times \underline{r}) \quad (A - 12)$$

Considering equations (A - 8a), (A - 8b) and (A - 2c), the momentum equations of the particle in the rotating coordinate system, defined in r-θ-z, can be written as follows :-

r - momentum :

$$m_p (\ddot{r} - r \dot{\theta}^2) = F_{S_r} - \{2 m_p \underline{\omega}' \times \underline{v}' + m_p \underline{\omega}' \times (\underline{\omega}' \times \underline{r})\} \cdot \hat{e}_r$$

$$= F_{S_r} + 2 m_p \omega_z v_\theta' + m_p \omega_z^2 r \quad (A - 13a)$$

Similarly :-

θ - momentum :

$$m_p (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) = F_{S_\theta} - 2 m_p \omega_z v_r' \quad (A - 13b)$$

z - momentum :-

$$m_p \ddot{z} = F_{S_z} \quad (A - 13c)$$

If the angular speed of the rotating system is completely

defined by its scalar component  $\omega'_z$ , then :-

$$\underline{\omega'} = \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_z \\ 0 & 0 & \omega'_z \end{vmatrix} \quad (\text{A} - 14)$$

Also :-

$$\underline{v'_r} = \dot{r} \quad (\text{A} - 15a)$$

$$\underline{v'_\theta} = r \dot{\theta} \quad (\text{A} - 15b)$$

Writting the scalar value of the angular speed as  $\omega'$  (instead of  $\omega'_z$ ), the final form of the momentum equations for the particle can be written as follows :-

$$m_p \{ \ddot{r} - r (\dot{\theta} + \omega')^2 \} = F_{S_r} \quad (\text{A} - 16a)$$

$$m_p \{ r \ddot{\theta} + 2 \dot{r} (\dot{\theta} + \omega') \} = F_{S_\theta} \quad (\text{A} - 16b)$$

$$m_p \ddot{z} = F_{S_z} \quad (\text{A} - 16c)$$

## APPENDIX B

### DEFINITIONS ON CONCENTRATION

The transference of energy from liquid to the particles during hydraulic conveying is effected by the drag forces which are set up as the result of differences in the velocities of the liquid and the particles. Because of slip, particles move at a lower velocity than the liquid, and consequently the concentration of particles in the liquid discharged from the end of the pipeline is less than the average value existing at a cross-section within the pipe. Two definitions of concentration, both by volume fraction, can, therefore, be considered :-

- (i) Discharge concentration
- (ii) Transport concentration

Discharge concentration is the ratio of the volume flow rate of solids to the volume flow rate of the mixture and in steady conditions is the same at all sections along a pipeline.

Transport concentration is the ratio of the volume occupied by solids to the volume occupied by the mixture in some fixed section of the pipeline.

The discharge concentration is a measure of the throughput of the pipeline while transport concentration is a measure of the pipeline blockage. By a material balance on the water and the solids, the discharge concentration,  $\bar{c}_d$  can be related to the transport concentration,  $\bar{c}_t$  as follows :-

$$(1 - \bar{c}_d) \bar{v}_m = (1 - \bar{c}_t) v_w \quad (B - 1)$$

$$\bar{c}_d \bar{v}_m = \bar{c}_t v_{so} \quad (B - 2)$$

From equation (B - 1) :-

$$\bar{c}_t = 1 - (1 - \bar{c}_d) \frac{\bar{v}_m}{v_w} \quad (B - 3)$$

Substituting  $\bar{c}_t$  from equation (B - 3) into equation (B - 2) and rearranging terms, the slip velocity between the liquid and the solid phase of the mixture is :-

$$v_w - v_{so} = v_w \left[ 1 - \frac{\bar{c}_d \bar{v}_m}{1 - (1 - \bar{c}_d) \frac{\bar{v}_m}{v_w}} \right] \quad (B - 4)$$

In heterogeneous flow,  $v_w > \bar{v}_m$ , hence,  $(v_w - v_{so}) \neq 0$ . Because of this non-equal velocities of the components of the mixture,  $c_t$  and  $c_d$  are different at a section of the pipeline.

Quantities  $\bar{c}_t$  and  $\bar{c}_d$  which are averaged over a cross-section of the pipe, can be considered in macroscopic forms, at an elementary area of the pipe section. These quantities then are referred to as insitu transport and insitu discharge concentrations, respectively, and equation (B - 1) to (B - 4) can be used to relate them.

## APPENDIX C

### DETERMINATION OF CENTROID OF AN ELEMENT IN A PIPE SECTION

The enlarged view of an element in an annulus of the cross-section of the pipe is shown in fig. C1.

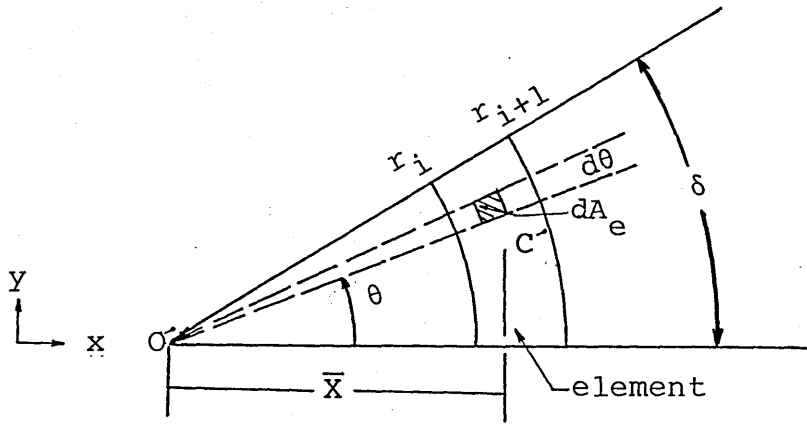


Fig. C1 Enlarged view of an element

$C'$  is the centroid of this element,  $O'$  is the centre of the pipe. Angle subtended by the element at the centre of the pipe is  $\delta$ .  $r_i$  is the inner radius of the element,  $r_{i+1}$  is the outer radius.  $dA_e$  is a differential element.

If  $r_c$  is the radial distance between centroid,  $C'$  and the centre of the pipe and  $r'$  is the radial distance between the centroid of the differential element and the centre of the pipe, then :-

$$\bar{X} = r_c \cos \frac{\delta}{2} \quad (C - 1)$$

where,  $\bar{X}$  is the distance of the centroid of the element from the centre of the pipe in the  $x$ -direction. The area of

the element is :-

$$a_i' = \int_{r_i}^{r_{i+1}} \int_0^{\delta} r' dr' d\theta$$

$$= \frac{\delta}{2} (r_{i+1}^2 - r_i^2) \quad (C - 2)$$

Also :-

$$\bar{X} a_i' = \int_{r_i}^{r_{i+1}} \int_0^{\delta} r'^2 \cos\theta dr' d\theta$$

$$= \frac{1}{3} (r_{i+1}^3 - r_i^3) \sin\delta \quad (C - 3)$$

Substituting for  $a_i'$  from equation (C - 2) into equation (C - 3) :-

$$\bar{X} \frac{\delta}{2} (r_{i+1}^2 - r_i^2) = \frac{1}{3} (r_{i+1}^3 - r_i^3) \sin\delta \quad (C - 4)$$

or :-

$$\bar{X} = \frac{2}{3} \frac{r_{i+1}^3 - r_i^3}{r_{i+1}^2 - r_i^2} \frac{\sin\delta}{\delta} \quad (C - 5)$$

Substituting for  $\bar{X}$  from equation (C - 1) into equation (C - 5) :-

$$r_c = \frac{2}{3} \frac{r_{i+1}^3 - r_i^3}{r_{i+1}^3 - r_i^3} \frac{\sin \delta}{\delta \cos \frac{\delta}{2}} \quad (C - 6a)$$

Rearranging terms, equation (C - 6a) becomes :-

$$r_c = \frac{4}{3} \frac{\sin \frac{\delta}{2}}{\delta} \left[ r_i + r_{i+1} - \frac{1}{\frac{1}{r_i} + \frac{1}{r_{i+1}}} \right] \quad (C - 6b)$$

APPENDIX D

SIZE ANALYSIS OF FINE PEA GRAVELS : EXPERIMENTAL RESULTS

Size Analysis Before Commencement of Experiment

| Sieve size<br>( $\mu\text{m}$ ) | Weight of solids<br>retained by<br>the sieve<br>(gm) | Cumulative weight<br>of the solids<br>passing through<br>the sieve size<br>(gm) | Cumulative<br>% passing<br>by mass |
|---------------------------------|------------------------------------------------------|---------------------------------------------------------------------------------|------------------------------------|
| 2411                            | 50.0331                                              | 1013.9004                                                                       | 95.2973                            |
| 2000                            | 174.6959                                             | 839.2045                                                                        | 78.8775                            |
| 1204                            | 810.4306                                             | 28.7739                                                                         | 2.7044                             |
| 853                             | 24.3840                                              | 4.3899                                                                          | 0.4126                             |
| 790                             | 1.2063                                               | 3.1836                                                                          | 0.2992                             |
| 599                             | 1.9799                                               | 1.2037                                                                          | 0.1131                             |
| 295                             | 0.7961                                               | 0.4076                                                                          | 0.0383                             |
| 152                             | 0.1074                                               | 0.3002                                                                          | 0.0282                             |
| 90                              | 0.0411                                               | 0.2591                                                                          | 0.0243                             |
| 76                              | 0.0884                                               | 0.1707                                                                          | 0.0160                             |
| -76                             | 0.1707                                               |                                                                                 |                                    |



APPENDIX D (cont.'d)

SIZE ANALYSIS OF FINE PEA GRAVELS : EXPERIMENTAL RESULTS

Size Analysis at the End of Experiment

| Sieve size<br>( $\mu\text{m}$ ) | Weight of solids retained by the sieve<br>(gm) | Cumulative weight of the solids passing through the sieve size<br>(gm) | Cumulative % passing by mass |
|---------------------------------|------------------------------------------------|------------------------------------------------------------------------|------------------------------|
| 2411                            | 43.3156                                        | 1533.5777                                                              | 97.2531                      |
| 2000                            | 252.6437                                       | 1280.9341                                                              | 81.2315                      |
| 1204                            | 1248.8980                                      | 32.0361                                                                | 2.0316                       |
| 853                             | 27.1445                                        | 4.8915                                                                 | 0.3102                       |
| 790                             | 0.4748                                         | 4.4168                                                                 | 0.2801                       |
| 599                             | 2.5876                                         | 1.8292                                                                 | 0.1160                       |
| 295                             | 1.2142                                         | 0.6149                                                                 | 0.0390                       |
| 152                             | 0.2049                                         | 0.4099                                                                 | 0.0260                       |
| 90                              | 0.0979                                         | 0.3122                                                                 | 0.0198                       |
| 76                              | 0.0394                                         | 0.2728                                                                 | 0.0173                       |
| -76                             | 0.2728                                         |                                                                        |                              |

APPENDIX E

MEASUREMENT OF SPECIFIC GRAVITY OF FINE PEA GRAVELS :

EXPERIMENTAL RESULTS

|                                                               | Sample<br>no. 1 | Sample<br>no. 2 | Sample<br>no. 3 |
|---------------------------------------------------------------|-----------------|-----------------|-----------------|
| (i) Weight of empty density<br>bottle, kg.                    | 47.48           | 38.54           | 36.83           |
| (ii) Weight of bottle +<br>solids, kg.                        | 57.07           | 49.32           | 48.18           |
| (iii) Weight of bottle +<br>solids + water, kg.               | 152.24          | 147.01          | 145.42          |
| (iv) Weight of bottle +<br>water, kg.                         | 146.26          | 140.26          | 138.36          |
| (v) Weight of solids (ii - i),<br>kg.                         | 9.59            | 10.78           | 11.35           |
| (vi) Weight of water in iii,<br>(iii - ii), kg.               | 95.17           | 97.69           | 97.24           |
| (vii) Weight of water in iv,<br>(iv - i), kg.                 | 98.78           | 101.72          | 101.53          |
| (viii) Weight of water displaced<br>by solids (vii - vi), kg. | 3.61            | 4.03            | 4.29            |
| (ix) Solids specific gravity<br>(v/viii)                      | 2.656           | 2.675           | 2.645           |

Mean value of the measured specific gravity = 2.659

## APPENDIX F

### MEASUREMENT OF SETTLING VELOCITY OF SOLIDS :

#### EXPERIMENTAL RESULTS

Measurements taken to find the settling velocity of particles whose size distribution is given in Appendix D and shown diagrammatically in fig. 3.2 are detailed here.

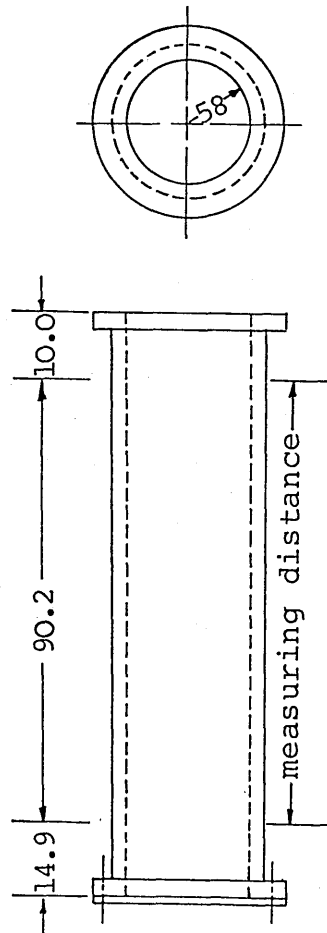


Fig. F1 Perspex cylinder  
(dimensions in cm)

A particle was chosen at random from the sample and released along the centre line of a perspex cylinder

whose dimensions are given in fig. F1. The free fall of the particle was timed over 90.2 cm depth in water.

| Particle no. | Time of free fall $t_s$ (s) | Particle no. | Time of free fall $t_s$ (s) | Particle no. | Time of free fall $t_s$ (s) |
|--------------|-----------------------------|--------------|-----------------------------|--------------|-----------------------------|
| 1            | 4.4                         | 11           | 5.4                         | 21           | 4.7                         |
| 2            | 4.0                         | 12           | 4.8                         | 22           | 3.4                         |
| 3            | 7.5                         | 13           | 5.6                         | 23           | 5.9                         |
| 4            | 3.7                         | 14           | 6.3                         | 24           | 3.8                         |
| 5            | 4.3                         | 15           | 5.3                         | 25           | 7.3                         |
| 6            | 5.1                         | 16           | 3.6                         | 26           | 4.9                         |
| 7            | 4.2                         | 17           | 4.4                         | 27           | 4.7                         |
| 8            | 3.8                         | 18           | 4.7                         | 28           | 4.5                         |
| 9            | 6.4                         | 19           | 5.4                         | 29           | 3.9                         |
| 10           | 4.0                         | 20           | 5.5                         | 30           | 6.3                         |

Mean time of free fall,  $\bar{t}_s = 4.86$  seconds.

Mean settling velocity,  $w = 18.56$  cm/s.

APPENDIX G

CALIBRATION OF COUNTERFLOW METER : EXPERIMENTAL RESULTS

| $Q \times 10^3$<br>(m <sup>3</sup> /s) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>(m <sup>2</sup> /s <sup>2</sup> ) | $h_{f_1}^*$<br>(m) | $h_{f_2}^*$<br>(m) |
|----------------------------------------|----------------------|----------------------------------------------------|--------------------|--------------------|
| 5.56                                   | 1.56                 | 2.42                                               | 0.0525             | 0.0520             |
| 6.49                                   | 1.82                 | 3.30                                               | 0.0690             | 0.0655             |
| 8.78                                   | 2.46                 | 6.04                                               | 0.1250             | 0.1150             |
| 10.77                                  | 3.01                 | 9.01                                               | 0.1905             | 0.1750             |
| 12.64                                  | 3.54                 | 12.52                                              | 0.2605             | 0.2405             |
| 13.41                                  | 3.75                 | 14.09                                              | 0.2925             | 0.2690             |
| 15.53                                  | 4.35                 | 18.90                                              | 0.3915             | 0.3611             |
| 16.92                                  | 4.73                 | 22.43                                              | 0.4661             | 0.4285             |
| 19.32                                  | 5.41                 | 29.25                                              | 0.6065             | 0.5612             |

\* head in m of water

APPENDIX H

CALIBRATION OF SLURRY MANOMETER : EXPERIMENTAL RESULTS

| $Q \times 10^3$<br>(m <sup>3</sup> /s) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>(m <sup>2</sup> /s <sup>2</sup> ) | $h_{f_m}^*$<br>(m) |
|----------------------------------------|----------------------|----------------------------------------------------|--------------------|
| 5.56                                   | 1.73                 | 2.99                                               | 0.043              |
| 6.49                                   | 2.02                 | 4.08                                               | 0.061              |
| 8.78                                   | 2.73                 | 7.45                                               | 0.110              |
| 10.77                                  | 3.35                 | 11.22                                              | 0.166              |
| 12.64                                  | 3.93                 | 15.44                                              | 0.229              |
| 13.41                                  | 4.17                 | 17.39                                              | 0.259              |
| 15.53                                  | 4.83                 | 23.32                                              | 0.347              |
| 16.92                                  | 5.26                 | 27.66                                              | 0.410              |
| 18.49                                  | 5.75                 | 33.06                                              | 0.485              |

\* head in m of water

# APPENDIX J

## MEASUREMENT OF CONCENTRATION : EXPERIMENTAL RESULTS, CASE 1

For details of the operating conditions see section 6.7.2. The mean discharge concentration of the mixture was measured by sampling the mixture once every 10 minutes, via the weigh tank and diverter valve assembly. The mean transport concentration was measured by a counterflow meter as well as a slurry manometer, once every 20 minutes. The flow rate of the mixture was measured by an electromagnetic flowmeter.

Measurement of Mean Discharge Concentration

| Sample no. | $Q \times 10^3$<br><br>(m <sup>3</sup> /s) | Mixture conditions |                    |
|------------|--------------------------------------------|--------------------|--------------------|
|            |                                            | Specific gravity   | $\bar{c}_d$<br>(%) |
| 1          | 11.70                                      | 1.0564             | 3.412              |
| 2          | 11.75                                      | 1.0562             | 3.401              |
| 3          | 11.65                                      | 1.0561             | 3.390              |
| 4          | 11.85                                      | 1.0560             | 3.385              |
| 5          | 11.80                                      | 1.0561             | 3.393              |
| 6          | 11.85                                      | 1.0561             | 3.390              |
| 7          | 11.60                                      | 1.0567             | 3.430              |
| 8          | 11.62                                      | 1.0564             | 3.412              |
| 9          | 11.57                                      | 1.0561             | 3.393              |
| 10         | 11.70                                      | 1.0641             | 3.872              |

Measurement of Mean Discharge Concentration (cont.'d)

| Sample no. | $Q \times 10^3$<br><br>( $m^3/s$ ) | Mixture conditions |                    |
|------------|------------------------------------|--------------------|--------------------|
|            |                                    | Specific gravity   | $\bar{c}_d$<br>(%) |
| 11         | 11.63                              | 1.0564             | 3.412              |
| 12         | 11.58                              | 1.0564             | 3.409              |
| 13         | 11.70                              | 1.0566             | 3.421              |
| 14         | 11.64                              | 1.0531             | 3.212              |
| 15         | 11.43                              | 1.0564             | 3.410              |
| 16         | 11.62                              | 1.0571             | 3.453              |
| 17         | 11.49                              | 1.0561             | 3.392              |
| 18         | 11.41                              | 1.0565             | 3.417              |
| 19         | 11.45                              | 1.0565             | 3.414              |
| 20         | 11.65                              | 1.0549             | 3.321              |
| 21         | 11.70                              | 1.0554             | 3.351              |
| 22         | 11.70                              | 1.0567             | 3.432              |
| 23         | 11.78                              | 1.0532             | 3.215              |
| 24         | 11.61                              | 1.0548             | 3.315              |
| 25         | 11.71                              | 1.0532             | 3.216              |
| 26         | 11.47                              | 1.0564             | 3.412              |
| 27         | 11.65                              | 1.0547             | 3.310              |
| 28         | 11.64                              | 1.0513             | 3.103              |



Measurement of Mean Discharge Concentration (cont.'d)

| Sample<br>no. | $Q \times 10^3$<br><br>(m <sup>3</sup> /s) | Mixture conditions  |                        |
|---------------|--------------------------------------------|---------------------|------------------------|
|               |                                            | Specific<br>gravity | $\bar{c}_d$<br><br>(%) |
| 29            | 11.81                                      | 1.0548              | 3.313                  |
| 30            | 11.62                                      | 1.0564              | 3.412                  |
| 31            | 11.63                                      | 1.0565              | 3.415                  |
| 32            | 11.50                                      | 1.0561              | 3.393                  |
| 33            | 11.45                                      | 1.0557              | 3.371                  |
| 34            | 11.53                                      | 1.0549              | 3.321                  |
| 35            | 11.83                                      | 1.0538              | 3.253                  |
| 36            | 11.80                                      | 1.0528              | 3.193                  |
| 37            | 11.55                                      | 1.0532              | 3.217                  |
| 38            | 11.45                                      | 1.0548              | 3.315                  |
| 39            | 11.62                                      | 1.0558              | 3.373                  |
| 40            | 11.55                                      | 1.0564              | 3.413                  |
| 41            | 11.61                                      | 1.0565              | 3.417                  |
| 42            | 11.70                                      | 1.0549              | 3.321                  |
| 43            | 11.63                                      | 1.0532              | 3.218                  |
| 44            | 11.65                                      | 1.0548              | 3.312                  |
| 45            | 11.70                                      | 1.0548              | 3.312                  |

Measurement of Mean Discharge Concentration (cont.'d)

| Sample no. | $Q \times 10^3$<br>(m <sup>3</sup> /s) | Mixture conditions |                    |
|------------|----------------------------------------|--------------------|--------------------|
|            |                                        | Specific gravity   | $\bar{c}_d$<br>(%) |
| 46         | 11.62                                  | 1.0548             | 3.315              |
| 47         | 11.63                                  | 1.0549             | 3.323              |
| 48         | 11.69                                  | 1.0532             | 3.219              |
| 49         | 11.63                                  | 1.0553             | 3.343              |
| 50         | 11.64                                  | 1.0554             | 3.351              |
| 51         | 11.60                                  | 1.0564             | 3.413              |
| 52         | 11.55                                  | 1.0548             | 3.317              |
| 53         | 11.65                                  | 1.0561             | 3.392              |
| 54         | 11.60                                  | 1.0555             | 3.359              |
| 55         | 11.55                                  | 1.0565             | 3.415              |
| 56         | 11.45                                  | 1.0581             | 3.512              |
| 57         | 11.50                                  | 1.0548             | 3.314              |
| 58         | 11.40                                  | 1.0565             | 3.414              |
| 59         | 11.45                                  | 1.0564             | 3.412              |
| 60         | 11.46                                  | 1.0555             | 3.358              |

Time averaged  $\bar{c}_d = 3.362\%$

Time averaged  $Q = 11.616 \times 10^{-3} \text{ m}^3/\text{s}$

APPENDIX J (cont.'d)

Measurement of Mean Transport Concentration by Counterflow Meter

| Sample no. | $Q \times 10^3$<br>(m <sup>3</sup> /s) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>(m <sup>2</sup> /s <sup>2</sup> ) | $h_{f1}^{*1}$<br>(m) | $h_{f2}^{*1}$<br>(m) | $h_1^{*2}$<br>(m) | $h_2^{*2}$<br>(m) | $\bar{c}_t$<br>(%) |
|------------|----------------------------------------|----------------------|----------------------------------------------------|----------------------|----------------------|-------------------|-------------------|--------------------|
| 1          | 11.75                                  | 3.29                 | 10.82                                              | 0.224                | 0.206                | 0.255             | 0.082             | 3.061              |
| 2          | 11.85                                  | 3.32                 | 11.01                                              | 0.228                | 0.208                | 0.257             | 0.083             | 3.061              |
| 3          | 11.85                                  | 3.32                 | 11.01                                              | 0.228                | 0.208                | 0.256             | 0.083             | 3.051              |
| 4          | 11.62                                  | 3.25                 | 10.58                                              | 0.219                | 0.202                | 0.255             | 0.082             | 3.040              |
| 5          | 11.70                                  | 3.27                 | 10.73                                              | 0.221                | 0.203                | 0.283             | 0.083             | 3.593              |
| 6          | 11.58                                  | 3.24                 | 10.51                                              | 0.216                | 0.200                | 0.255             | 0.082             | 3.061              |
| 7          | 11.64                                  | 3.26                 | 10.62                                              | 0.220                | 0.203                | 0.247             | 0.084             | 3.100              |
| 8          | 11.62                                  | 3.25                 | 10.54                                              | 0.219                | 0.202                | 0.253             | 0.082             | 3.041              |
| 9          | 11.41                                  | 3.19                 | 10.20                                              | 0.212                | 0.196                | 0.255             | 0.083             | 3.080              |
| 10         | 11.65                                  | 3.26                 | 10.63                                              | 0.220                | 0.203                | 0.250             | 0.082             | 2.982              |

\*<sub>1</sub> as defined in equations 6.1a, 6.1b and 6.7a, 6.7b.

\*<sub>2</sub> see equations 6.1a, 6.1b and fig. 6.7; error in reading  $\pm 0.0015$  m.

All heads in m of water.

Measurement of Mean Transport Concentration by counterflow Meter (cont.'d)

| Sample no. | $Q \times 10^3$<br>( $m^3/s$ ) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>( $m^2/s^2$ ) | $h_{f1}$<br>(m) | $h_{f2}$<br>(m) | $h_1$<br>(m) | $h_2$<br>(m) | $\bar{c}_t$<br>(%) |
|------------|--------------------------------|----------------------|--------------------------------|-----------------|-----------------|--------------|--------------|--------------------|
| 11         | 11.70                          | 3.27                 | 10.73                          | 0.221           | 0.203           | 0.257        | 0.083        | 3.080              |
| 12         | 11.61                          | 3.25                 | 10.56                          | 0.219           | 0.202           | 0.249        | 0.082        | 2.962              |
| 13         | 11.47                          | 3.21                 | 10.31                          | 0.213           | 0.191           | 0.255        | 0.083        | 2.962              |
| 14         | 11.64                          | 3.26                 | 10.62                          | 0.220           | 0.203           | 0.246        | 0.082        | 2.903              |
| 15         | 11.62                          | 3.25                 | 10.58                          | 0.219           | 0.202           | 0.254        | 0.082        | 3.021              |
| 16         | 11.50                          | 3.22                 | 10.37                          | 0.215           | 0.198           | 0.253        | 0.082        | 3.041              |
| 17         | 11.53                          | 3.23                 | 10.42                          | 0.215           | 0.198           | 0.251        | 0.083        | 2.982              |
| 18         | 11.80                          | 3.30                 | 10.91                          | 0.226           | 0.206           | 0.250        | 0.083        | 2.903              |
| 19         | 11.45                          | 3.21                 | 10.28                          | 0.213           | 0.191           | 0.254        | 0.082        | 2.962              |
| 20         | 11.55                          | 3.23                 | 10.46                          | 0.215           | 0.198           | 0.255        | 0.082        | 3.080              |

Measurement of Mean Transport Concentration by counterflow Meter (cont.'d)

| Sample no. | $Q \times 10^3$<br>(m <sup>3</sup> /s) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>(m <sup>2</sup> /s <sup>2</sup> ) | $h_{f1}$<br>(m) | $h_{f2}$<br>(m) | $h_1$<br>(m) | $h_2$<br>(m) | $\bar{c}_t$<br>(%) |
|------------|----------------------------------------|----------------------|----------------------------------------------------|-----------------|-----------------|--------------|--------------|--------------------|
| 21         | 11.70                                  | 3.27                 | 10.73                                              | 0.221           | 0.203           | 0.252        | 0.083        | 2.982              |
| 22         | 11.65                                  | 3.26                 | 10.64                                              | 0.220           | 0.203           | 0.249        | 0.082        | 2.962              |
| 23         | 11.62                                  | 3.25                 | 10.58                                              | 0.219           | 0.202           | 0.249        | 0.083        | 2.942              |
| 24         | 11.69                                  | 3.27                 | 10.71                                              | 0.221           | 0.203           | 0.248        | 0.084        | 2.883              |
| 25         | 11.64                                  | 3.26                 | 10.62                                              | 0.220           | 0.203           | 0.251        | 0.082        | 3.001              |
| 26         | 11.55                                  | 3.23                 | 10.46                                              | 0.215           | 0.198           | 0.249        | 0.082        | 2.962              |
| 27         | 11.60                                  | 3.25                 | 10.55                                              | 0.219           | 0.202           | 0.250        | 0.080        | 3.021              |
| 28         | 11.45                                  | 3.21                 | 10.28                                              | 0.213           | 0.191           | 0.260        | 0.080        | 3.119              |
| 29         | 11.40                                  | 3.19                 | 10.19                                              | 0.212           | 0.190           | 0.259        | 0.081        | 3.080              |
| 30         | 11.46                                  | 3.21                 | 10.29                                              | 0.213           | 0.191           | 0.261        | 0.082        | 3.100              |

APPENDIX J (cont.'d)

Measurement of Mean Transport Concentration by Slurry Manometer

| Sample no. | $Q \times 10^3$<br>( $m^3/s$ ) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>( $m^2/s^2$ ) | $w/\bar{v}_m$ | $h_{fm}^{*1}$<br>(m) | $h_m^{*2}$<br>(m) | $\bar{c}_t$<br>(%) |
|------------|--------------------------------|----------------------|--------------------------------|---------------|----------------------|-------------------|--------------------|
| 1          | 11.75                          | 3.65                 | 13.34                          | 0.055         | 0.194                | 0.234             | 3.759              |
| 2          | 11.85                          | 3.68                 | 13.56                          | 0.054         | 0.201                | 0.240             | 3.665              |
| 3          | 11.85                          | 3.68                 | 13.56                          | 0.054         | 0.201                | 0.241             | 3.759              |
| 4          | 11.62                          | 3.61                 | 13.04                          | 0.055         | 0.192                | 0.232             | 3.755              |
| 5          | 11.70                          | 3.64                 | 13.23                          | 0.055         | 0.193                | 0.238             | 4.225              |
| 6          | 11.58                          | 3.59                 | 12.96                          | 0.055         | 0.191                | 0.230             | 3.662              |
| 7          | 11.64                          | 3.62                 | 13.09                          | 0.055         | 0.190                | 0.227             | 3.474              |
| 8          | 11.62                          | 3.61                 | 13.05                          | 0.055         | 0.190                | 0.230             | 3.755              |

\*<sub>1</sub> as defined in equations 6.3 and 6.8.

\*<sub>2</sub> see equation 6.3 and fig. 6.8; error in reading  $\pm 0.0015$  m.  
All heads in m of water.

Measurement of Mean Transport Concentration by Slurry Manometer (cont. 'd)

| Sample<br>no. | $Q \times 10^3$<br>( $m^3/s$ ) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>( $m^2/s^2$ ) | $w/\bar{v}_m$ | $h_{f_m}$<br>(m) | $h_m$<br>(m) | $\bar{c}_t$<br>(%) |
|---------------|--------------------------------|----------------------|--------------------------------|---------------|------------------|--------------|--------------------|
| 9             | 11.41                          | 3.55                 | 12.58                          | 0.056         | 0.185            | 0.224        | 3.658              |
| 10            | 11.65                          | 3.62                 | 13.11                          | 0.055         | 0.192            | 0.231        | 3.662              |
| 11            | 11.70                          | 3.64                 | 13.23                          | 0.055         | 0.193            | 0.233        | 3.755              |
| 12            | 11.61                          | 3.61                 | 13.02                          | 0.055         | 0.190            | 0.229        | 3.662              |
| 13            | 11.47                          | 3.56                 | 12.71                          | 0.056         | 0.186            | 0.226        | 3.752              |
| 14            | 11.64                          | 3.62                 | 13.09                          | 0.055         | 0.190            | 0.226        | 3.380              |
| 15            | 11.62                          | 3.61                 | 13.05                          | 0.055         | 0.190            | 0.229        | 3.662              |
| 16            | 11.50                          | 3.57                 | 12.78                          | 0.056         | 0.186            | 0.225        | 3.658              |
| 17            | 11.53                          | 3.58                 | 12.84                          | 0.056         | 0.188            | 0.226        | 3.564              |
| 18            | 11.80                          | 3.66                 | 13.45                          | 0.055         | 0.194            | 0.231        | 3.474              |
| 19            | 11.45                          | 3.56                 | 12.69                          | 0.56          | 0.185            | 0.223        | 3.564              |

Measurement of Mean Transport Concentration by Slurry Manometer (cont.'d)

| Sample no. | $Q \times 10^3$<br>( $m^3/s$ ) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>( $m^2/s^2$ ) | $w/\bar{v}_m$ | $h_{fm}$<br>(m) | $h_m$<br>(m) | $\bar{c}_t$<br>(%) |
|------------|--------------------------------|----------------------|--------------------------------|---------------|-----------------|--------------|--------------------|
| 20         | 11.55                          | 3.59                 | 12.89                          | 0.056         | 0.188           | 0.227        | 3.658              |
| 21         | 11.70                          | 3.63                 | 13.23                          | 0.055         | 0.193           | 0.233        | 3.755              |
| 22         | 11.65                          | 3.62                 | 13.11                          | 0.055         | 0.192           | 0.230        | 3.567              |
| 23         | 11.62                          | 3.61                 | 13.05                          | 0.055         | 0.190           | 0.228        | 3.567              |
| 24         | 11.69                          | 3.63                 | 13.20                          | 0.055         | 0.193           | 0.229        | 3.380              |
| 25         | 11.64                          | 3.62                 | 13.09                          | 0.055         | 0.190           | 0.228        | 3.568              |
| 26         | 11.55                          | 3.59                 | 12.89                          | 0.056         | 0.188           | 0.226        | 3.564              |
| 27         | 11.60                          | 3.60                 | 13.00                          | 0.056         | 0.190           | 0.229        | 3.658              |
| 28         | 11.45                          | 3.56                 | 12.67                          | 0.056         | 0.185           | 0.226        | 3.846              |
| 29         | 11.40                          | 3.54                 | 12.56                          | 0.056         | 0.185           | 0.225        | 3.752              |
| 30         | 11.46                          | 3.56                 | 12.69                          | 0.056         | 0.185           | 0.224        | 3.658              |



APPENDIX K

MEASUREMENT OF CONCENTRATION : EXPERIMENTAL RESULTS, CASE 2

For details of the operating conditions see section 6.7.2. The mean discharge concentration of the mixture was measured by sampling the mixture once every 5 minutes, via the weigh tank and diverter valve assembly. The mean transport concentration was measured by a counterflow meter as well as a slurry manometer, once every 10 minutes. The flow rate of the mixture was measured by an electromagnetic flowmeter.

Measurement of Mean Discharge Concentration

| Sample no. | $Q \times 10^3$<br><br>(m <sup>3</sup> /s) | Mixture conditions |                    |
|------------|--------------------------------------------|--------------------|--------------------|
|            |                                            | Specific gravity   | $\bar{c}_d$<br>(%) |
| 1          | 12.15                                      | 1.1994             | 12.051             |
| 2          | 12.25                                      | 1.1991             | 12.032             |
| 3          | 12.31                                      | 1.1975             | 11.935             |
| 4          | 12.28                                      | 1.1975             | 11.951             |
| 5          | 12.23                                      | 1.1983             | 11.982             |
| 6          | 11.95                                      | 1.1988             | 12.013             |
| 7          | 12.31                                      | 1.1982             | 11.978             |
| 8          | 12.23                                      | 1.1978             | 11.953             |
| 9          | 12.41                                      | 1.1973             | 11.922             |
| 10         | 12.35                                      | 1.1984             | 11.993             |

Measurement of Mean Discharge Concentration (cont.'d)

| Sample no. | $Q \times 10^3$<br><br>( $m^3/s$ ) | Mixture conditions |                    |
|------------|------------------------------------|--------------------|--------------------|
|            |                                    | Specific gravity   | $\bar{c}_d$<br>(%) |
| 11         | 12.15                              | 1.1998             | 12.075             |
| 12         | 11.85                              | 1.1988             | 12.014             |
| 13         | 11.95                              | 1.1981             | 11.971             |
| 14         | 12.21                              | 1.1983             | 11.983             |
| 15         | 12.25                              | 1.1984             | 11.990             |
| 16         | 12.23                              | 1.1991             | 12.032             |
| 17         | 12.35                              | 1.1983             | 11.987             |
| 18         | 12.10                              | 1.1982             | 11.976             |
| 19         | 12.15                              | 1.1987             | 12.012             |
| 20         | 12.25                              | 1.1971             | 11.911             |

Time averaged  $\bar{c}_d = 11.653\%$

Time averaged  $Q = 12.198 \times 10^{-3} m^3/s$

APPENDIX K (cont. 'd)

Measurement of Mean Transport Concentration by Counterflow Meter

| Sample no. | $Q \times 10^3$<br>( $m^3/s$ ) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>( $m^2/s^2$ ) | $h_{f1}$<br>(m) | $h_{f2}$<br>(m) | $h_1^*$<br>(m) | $h_2^*$<br>(m) | $\bar{c}_t$<br>(%) |
|------------|--------------------------------|----------------------|--------------------------------|-----------------|-----------------|----------------|----------------|--------------------|
| 1          | 12.25                          | 3.43                 | 11.76                          | 0.248           | 0.226           | 0.608          | 0.029          | 11.001             |
| 2          | 12.28                          | 3.44                 | 11.82                          | 0.248           | 0.226           | 0.608          | 0.029          | 11.001             |
| 3          | 11.95                          | 3.35                 | 11.19                          | 0.233           | 0.213           | 0.593          | 0.031          | 10.700             |
| 4          | 12.23                          | 3.42                 | 11.72                          | 0.248           | 0.226           | 0.608          | 0.029          | 11.000             |
| 5          | 12.35                          | 3.46                 | 11.96                          | 0.250           | 0.231           | 0.597          | 0.028          | 10.860             |
| 6          | 11.85                          | 3.32                 | 11.01                          | 0.228           | 0.213           | 0.594          | 0.032          | 10.801             |
| 7          | 12.21                          | 3.48                 | 11.68                          | 0.241           | 0.226           | 0.604          | 0.029          | 11.662             |
| 8          | 12.23                          | 3.42                 | 11.72                          | 0.248           | 0.226           | 0.605          | 0.028          | 10.959             |
| 9          | 12.10                          | 3.39                 | 11.47                          | 0.238           | 0.221           | 0.603          | 0.032          | 10.939             |
| 10         | 12.25                          | 3.43                 | 11.76                          | 0.248           | 0.226           | 0.607          | 0.029          | 10.979             |

\* error in reading  $\pm 0.0015$  m.

All heads in m of water.

APPENDIX K (cont.'d)  
Measurement of Mean Transport Concentration by Slurry Manometer

| Sample no. | $Q \times 10^3$<br>( $m^3/s$ ) | $\bar{v}_m$<br>(m/s) | $\bar{v}_m^2$<br>( $m^2/s^2$ ) | $w/\bar{v}_m$ | $h_{fm}$<br>(m) | $h_m^*$<br>(m) | $\bar{c}_t$<br>(%) |
|------------|--------------------------------|----------------------|--------------------------------|---------------|-----------------|----------------|--------------------|
| 1          | 12.25                          | 3.81                 | 14.50                          | 0.052         | 0.213           | 0.353          | 13.182             |
| 2          | 12.28                          | 3.82                 | 14.57                          | 0.052         | 0.213           | 0.352          | 13.087             |
| 3          | 11.95                          | 3.71                 | 13.80                          | 0.053         | 0.201           | 0.341          | 13.169             |
| 4          | 12.23                          | 3.80                 | 14.45                          | 0.053         | 0.212           | 0.351          | 13.075             |
| 5          | 12.35                          | 3.84                 | 14.73                          | 0.052         | 0.213           | 0.353          | 13.182             |
| 6          | 11.85                          | 3.68                 | 13.57                          | 0.054         | 0.201           | 0.341          | 13.157             |
| 7          | 12.21                          | 3.79                 | 14.40                          | 0.052         | 0.210           | 0.349          | 13.087             |
| 8          | 12.23                          | 3.80                 | 14.45                          | 0.052         | 0.212           | 0.352          | 13.182             |
| 9          | 12.10                          | 3.76                 | 14.14                          | 0.053         | 0.207           | 0.347          | 13.169             |
| 10         | 12.25                          | 3.80                 | 14.50                          | 0.053         | 0.213           | 0.352          | 13.075             |

\* error in reading  $\pm 0.0015$  m.

All heads in m of water.