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Integrated NDI-based Controller and Incremental Control
Allocation for an eVTOL

Centre for Aeronautics, School of Aerospace, Transport and
Manufacturing
Aerospace Engineering

PhD
Academic Year: 2020–2024

Supervisor: Prof James F. Whidborne
Associate Supervisor: Dr Linghai Lu
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I declare that:

- the thesis submitted has been written by me alone.
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- that all quotations and references have been duly acknowledged according to the requirements of academic research.

I understand that to knowingly submit work in violation of the above statement will be considered by examiners as academic misconduct.

Dedication

I would like to dedicate this thesis to the almighty God, my lovely wife Ogochukwu, my sons Albert and Adrian, and my daughter Areli.

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation contains fewer than 65,000 words including appendices, bibliography, footnotes, tables and equations and has fewer than 150 figures.

Abstract

The concept of urban air mobility (UAM) has been proposed to address the transport problems of congestion and pollution in ever-growing urban conurbations. Mature fixed wing aircraft technologies have many constraints that limit their applicability for UAM hence piloted all-electric vertical take-off and landing (eVTOL) vehicles known as eVTOL ‘air-taxis’ provide a promising option being lighter, quieter, potentially having net zero emissions and with lower operating costs. Furthermore, their higher bandwidth thrust actuation allows for novel configurations. However, one challenge of eVTOL aircraft operation is their strong nonlinear flight dynamics since they operate over a wide range of operating envelope. One particularly challenging eVTOL configuration is the Lilium jet, a fixed-wing, over-actuated vectored thrust, canard light sports aircraft. However, its canard configuration can destabilize the short-period mode requiring fly-by-wire stability augmented flight control systems. A second challenge of this configuration is the coupling between the aerodynamic surface control and the vectored thrust makes control allocation (CA) during transition very difficult. A third challenge of this configuration is its high degree of over-actuation presents a challenging CA. Finally, a fourth challenge of the eVTOL configuration is that the vectored thrust results in nonlinear effector mapping preventing the direct use of classical CA approaches. In order to address the control challenges of the Lilium-style eVTOL, a novel full-envelope controller is presented which employs a generic control architecture applicable to piloted, semi-automatic and fully automated flight. The full-envelope controller consist of an aircraft-level controller which produces an overall control demand and a CA scheme which allocates the overall control demand to individual redundant effectors. The CA scheme also performs control error minimisation, control channel prioritization and control effort minimization. The aircraft-level controller adopts a modular approach, consisting of a main inner-loop nonlinear dynamic inversion (NDI) controller which cancels the bare airframe dynamics and an outer-loop proportional-integral (PI) linear controller which together addresses several of the eVTOL operation challenges. The inner-loop NDI controller is a velocity controller while the outer-loop PI controller is an attitude and navigation position controller which together are used for hover/low speed control and forward flight. This thesis proposes and initially conceptually tests three real-time CA approaches for transforming a nonlinear CA problem to a linear problem and then applies active set linear CA technique to address all the remaining challenges of the eVTOL configuration. This thesis fully verifies one of the CA approaches on a full six-degrees-of-freedom model version of the eVTOL aircraft. Simulation results clearly demonstrates that the controller has reasonable disturbance rejection, can guarantee stable flight in case of severe actuator saturation and has satisfactory compliance to typical mission tasks for this class of vehicle.

Keywords:

active set, actuator saturation, nonlinear effector mapping, vertical takeoff and landing (VTOL), air-taxi, urban air mobility (UAM), transition, over-actuated, virtual controls, full envelope, effector model, weighted least squares (WLS), optimal, high level controller, unified control framework, real-time, online, linear time varying (LTI) constrained optimisation, linear quadratic programming

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Table of Contents

Academic Integrity Declaration	i
Dedication	ii
Declaration	iii
Abstract	iv
Acknowledgements	vi
Table of Contents	vii
List of Figures	xii
List of Tables	xvii
Notations	xvii
Nomenclature	xviii
List of Abbreviations	xxii
1 Introduction	1
1.1 Background, Context and UAM Concept	1
1.2 Classification of eVTOLs	2
1.3 Lilium eVTOL, EDFs and VTOL Operation Review	4
1.4 Aim and Objectives	6
1.5 Vehicle Models Definition	7
1.6 Motivation and Contributions	7
1.6.1 Motivation	7
1.6.2 Contribution	9
1.7 Publications Resulting from Thesis	11
1.8 Organisation and Overview	12

2	Vehicle Modelling and Analysis	14
2.1	Chapter Contribution	14
2.2	6DoF Vehicle Mathematical Model	14
2.2.1	Overview of Vehicle Model 1	14
2.2.2	Flight Mechanics and Equation of Motion	15
2.3	Vehicle Dynamic Modelling	18
2.3.1	Aerodynamic Modelling Approach	19
2.3.2	Aerodynamic Model Blending	19
2.3.3	Hover Aerodynamic Model	21
2.3.4	Cruise Aerodynamic Model	22
2.3.5	Cruise Aerodynamic Model - Part 1	22
2.3.6	Combined Cruise Aerodynamic Model	29
2.3.7	Aircraft Effector Configuration	30
2.3.8	EDF Set Control Simplification	31
2.3.9	Simplified Effector Model	32
2.4	Dynamic Characteristics Analysis	34
2.4.1	Trimming	34
2.4.2	Linearization	36
2.4.3	Open loop Linear Analysis	37
2.4.4	Linearized Cruise Model - Vehicle Model 3	38
2.4.5	Analysis of Transmission Zeros and Minimum Phase Test	40
2.4.6	Closed Loop Analysis of Decoupled Linearised Model	41
2.4.7	Summary	41
3	Architecture and Controller Design	43
3.1	Chapter Contribution	43
3.2	NonLinear Control Techniques Review	43
3.3	NDI Literature Review	44
3.4	Control Architecture	46
3.4.1	Command Generator and Outerloop	47
3.4.2	Outer Loop Linear Controller	48
3.4.3	Control Limits and Flight Modes	51
3.5	Non Linear Dynamic Inversion	52
3.5.1	NDI Controller Problem formulation	52
3.5.2	NDI Propulsive and Aerodynamic Forces Decoupling	56
3.6	NDI Controller Concept Testing	56
3.6.1	Manoeuvre 1 - Vertical Take-off, Hover & Low Speed Flight, Vertical Landing	58
3.6.2	Manoeuvre 2 - VTO and heading control, Forward Transition	62
3.6.3	Manoeuvre 3 - Cruise, Reverse Transition and VL with Heading change	65
3.6.4	NDI Controller Testing Conclusions	68

4	Control Allocation Approaches	69
4.1	Challenges of the Vectored Thrust eVTOLs	69
4.2	Main Chapter Contributions	70
4.3	Background of Control Allocation	71
4.4	Vehicle Model 3	75
4.4.1	3DoF Mathematical Model	76
4.5	3DoF Control Design	77
4.5.1	3DoF Control Architecture	77
4.5.2	3DoF NDI Control Law	77
4.5.3	3DoF Outer-Loop Linear Controller	77
4.6	General Control Allocation	78
4.6.1	Control Allocation Formulation	78
4.6.2	Active Set Prioritised Constrained Linear Control Allocation	79
4.7	CA Approach 1	80
4.7.1	CA Approach 1 - Formulation of Real Controls	80
4.7.2	CA Approach 1 Functional Architecture	81
4.7.3	Computation of Unconstrained Controls	81
4.7.4	Transformation of real controls	82
4.7.5	Conversion of Actuator Limits from Polar to Cartesian	82
4.7.6	CA Approach 1 - Prioritised Linear CA	83
4.8	CA Approach 2	89
4.8.1	CA Approach 2 Drift and Correction	89
4.8.2	CA Approach 2 Effector Model Formulation	90
4.8.3	CA Approach 2 Functional Architecture	91
4.8.4	Control Allocation Incremental Constraints Formulation	93
4.8.5	CA Approach 2 - Derivation of LTV Effector Mapping	93
4.8.6	CA Approach 2 - Computation of Unconstrained Real Controls	94
4.8.7	CA Approach 2 - Correction of Drift	94
4.8.8	CA Approach 2 - Prioritised Incremental Linear CA	94
4.9	CA Approach 3	96
4.9.1	CA Approach 3 Effector Model Formulation	96
4.9.2	CA Approach 3 Functional Architecture	96
4.9.3	Approach 3 - Incremental Constraints Formulation	98
4.9.4	Approach 3 - Conversion of the Virtual Controls from Cartesian to Polar	98
4.9.5	CA Approach 3 - Derivation of LTV Effector Mapping	98
4.9.6	CA Approach 3 - Computation of Unconstrained Real Controls	99
4.9.7	CA Approach 3 - Correction of Drift	99
4.9.8	Approach 3 - Prioritised Incremental Linear CA	99
4.10	Simulation Results and Discussions	99
4.10.1	ACM 1 - VTO/VL with Pitch Control and Limited Control Authority	102
4.10.2	ACM 2 - VTO, Hover to High Forward Speed with Limited Control Authority	105
4.10.3	ACM 3 - Reverse Backward and Pitch Control with Limited Control Authority	108

4.11	Conclusions	111
5	Simulation and Verification	112
5.1	Controller Testing of Nonlinear Model	112
5.1.1	Control Architecture	112
5.1.2	6DoF Effector Model	112
5.1.3	CA Approach 2 - LTV Effector Mapping	116
5.1.4	Correction of Saturation Drift	118
5.1.5	CA Incremental Constraints Formulation	119
5.1.6	Prioritised Incremental Linear CA	119
5.2	Simulations Results and Discussions	120
5.2.1	Actuator Dynamics - RPM and Tilting Mechanism	123
5.2.2	Estimation of the EDF Thrust and Torque Coefficients	124
5.2.3	Manoeuvre 1 - Vertical Take-off, Hover & Low Speed Flight, Vertical Landing	127
5.2.4	Manoeuvre 2 - VTO and Heading Control, Forward Transition	132
5.2.5	Manoeuvre 3 - Criticality of CA: VTO and Heading Control, Forward Transition	137
5.2.6	Manoeuvre 4 - Cruise, Reverse Transition and VL with Heading change	139
5.2.7	Manoeuvre 5 - Cruise, Climb, Descend, Cruise	143
5.2.8	Manoeuvre 6 - Cruise and Coordinated Turn	147
5.2.9	Manoeuvre 7 - Criticality of CA: Roll Disturbance Rejection with or without Constrained CA	151
5.2.10	Manoeuvre 8 - Criticality of CA: Pitch Disturbance Rejection with or without Constrained CA	154
5.2.11	Manoeuvre 9 - Robustness to the Model Dependent Parameters: Error in Roll and Yaw Moment of Inertia and Mass	157
5.2.12	Manoeuvre 10 - Robustness to Model Dependent Parameters: Error in Drag and Lift Coefficient	161
5.2.13	Manoeuvre 11 - Criticality of CCA: Redundant EDF Sets Total Actu- ator Failures during VTO	169
6	Conclusions	175
6.1	Discussion and Conclusions	175
6.2	Further Work	179
6.3	Potential Impact	180
	References	181
A	Appendix 1	194
A.1	Cruise Model Data - Part A	194
A.2	Cruise Model Data - Part B	195
A.2.1	Cruise Aerodynamic Model - Part 2	195
A.3	LDI Controller Problem formulation	196

A.3.1	Zero Dynamics	198
A.3.2	Controlled Variables (CV)	199
A.4	Linear Controller Design	199
A.4.1	Linear Controller Control Architecture	199
A.4.2	6DoF Effector Model for Linear Controller	200
A.4.3	Linear Dynamic Inversion Controller	201
A.5	Acceleration Plots for Chapter 3	204
A.6	Acceleration Plots for Manoeuvres 1 - 5 of Chapter 5	207
B	Appendix 2	211
B.1	VTOL Certification Landscape	211
B.1.1	VTOL Certification Requirements	212
B.1.2	Applicable EASA VTOL Certification requirements	212
B.1.3	FAA UAM and AAM Certification Requirement Development	213

List of Figures

1.1	The Lilium Jet (Lilium, 2024)	3
2.1	Aircraft canard-planform showing EDF sets distribution	15
2.2	Aerodynamic Blending Coefficient	20
2.3	Lilium Plan View Strip Theory Discretisation (Moses, 2021)	23
2.4	Plan View of Lilium-style Fixed-Wing Over-actuated Vectored Thrust eVTOL with EDF Sets Distribution and Control Grouping	31
3.1	Generic Flight Controller Architecture for piloted, semi-automatic and auto- mated flight	46
3.2	Command Generator and Outer loop Controller	47
3.3	NDI Problem Formulation (Enenakpogbe et al., 2022)	52
3.4	Visual Representation - VTO, Hover Flight, VL	58
3.5	No Effector/CA - Manoeuvre 1: VTO, Hover Flight, VL - Attitude	59
3.6	No Effector/CA - Manoeuvre 1: VTO, Hover Flight, VL - Velocity	59
3.7	No Effector/CA - Manoeuvre 1: VTO, Hover Flight, VL - Virtual Controls . .	60
3.8	Visual Representation - Manoeuvre 2: VTO, Forward Transition	63
3.9	No Effector/CA - Manoeuvre 2: VTO, Forward Transition - Attitude	63
3.10	No Effector/CA - Manoeuvre 2: VTO, Forward Transition - Velocity	64
3.11	No Effector/CA - Manoeuvre 2: VTO, Forward Transition - Virtual Controls .	64
3.12	Visual Representation - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change	66
3.13	No Effector/CA - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change -Attitude	66
3.14	No Effector/CA - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change - Velocity	67
3.15	No Effector/CA - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change - Virtual Controls	67
4.1	General Linear Control Allocation Scheme	72
4.2	Vectored Thrust Actuator Limits Illustration.	75
4.3	3DoF motion showing aircraft state variables and virtual controls.	76
4.4	CA Approach 1 - 3DoF VTOL schematic showing effector variables.	80
4.5	CA Approach 1 Functional Architecture	86

4.6	Polar to Cartesian Plane Illustration	87
4.7	Combined position and rate cartesian limits illustration	88
4.8	3DoF VTOL schematic showing the effector variables for CA Approaches 2 and 3	90
4.9	CA Approach 2 Functional Architecture	92
4.10	CA Approach 3 Functional Architecture	97
4.11	VTO/VL with pitch control - State Variables	103
4.12	VTO/VL with Pitch Control - State Variable Error	103
4.13	VTO/VL with Pitch Control - Effector Commands	104
4.14	VTO/VL with Pitch Control - Control Limitation Status	104
4.15	VTO, Hover to High Forward Speed - State Variables	106
4.16	VTO, Hover to High Forward Speed - State Variable Error	106
4.17	VTO, Hover to High Forward Speed - Effector Commands	107
4.18	VTO, Hover to High Forward Speed - Control Limitation Status	107
4.19	Reverse Backward and Pitch Control - State Variables	109
4.20	Reverse Backward and Pitch Control - States Variables Error	109
4.21	Reverse Backward and Pitch Control - Effector Commands	110
4.22	Reverse Backward and Pitch Control - Control Limitation Status	110
5.1	NDI-Based Linear Flight Controller Architecture for piloted, semi-automatic and automated flight	112
5.2	Plan View of Lilium-style fixed-wing over-actuated vectored thrust eVTOL aircraft configuration showing EDF Sets distribution, control groupings and real controls	113
5.3	CA Approach 2 6DoF Functional Architecture	114
5.4	Thrust (T) and Tilting Mechanism (Γ) Actuators Dynamics	124
5.5	Manoeuvre 1 Visual Representation - VTO, Hover Flight, VL	128
5.6	VTO, Hover Flight, VL - Attitude	129
5.7	VTO, Hover Flight, VL - Velocity	129
5.8	VTO, Hover Flight, VL - Effector Commands	130
5.9	VTO, Hover Flight, VL - Control Limitation Status	130
5.10	Manoeuvre 2 3 Visual Representation: VTO/Right Turn, Forward Transition	133
5.11	VTO/Right Turn, Forward Transition - Attitude	133
5.12	VTO/Right Turn, Forward Transition - Velocity	134
5.13	VTO/Right Turn, Forward Transition - Effector Commands	134
5.14	VTO/Right Turn, Forward Transition - Control Limitation Status	135
5.15	Criticality of CCA: VTO/Right Turn, Forward Transition - Attitude	137
5.16	Criticality of CCA: VTO/Right Turn, Forward Transition - Velocity	138
5.17	Criticality of CCA: VTO/Right Turn, Forward Transition - Effector Commands	138
5.18	Manoeuvre 4 Visual Representation: Cruise, Reverse Transition and VL/Left Turn	140
5.19	Cruise, Reverse Transition and VL/Left Turn - Attitude	140
5.20	Cruise, Reverse Transition and VL/Left Turn - Velocity	141
5.21	Cruise, Reverse Transition and VL/Left Turn - Effector Commands	141

5.22	Cruise, Reverse Transition and VL/Left Turn - Control Limitation Status . . .	142
5.23	Manoeuvre 5 Visual Representation: Cruise, Climb, Descend	144
5.24	Cruise, Climb, Descend - Attitude	144
5.25	Cruise, Climb, Descend - Velocity	145
5.26	Cruise, Climb, Descend - Effector Commands	145
5.27	Cruise, Climb, Descend - Control Limitation Status	146
5.28	Manoeuvre 6 Visual Representation: Cruise and Coordinated Turn	148
5.29	Cruise and Coordinated Turn - Attitude	148
5.30	Cruise and Coordinated Turn - Velocity	149
5.31	Cruise and Coordinated Turn - Effector Commands	149
5.32	Cruise and Coordinated Turn - Control Limitation Status	150
5.33	Roll Moment Disturbance Rejection Characteristics at Hover with and without CCA - State Variables	152
5.34	Roll Moment Disturbance Rejection Characteristics at Hover with and without CCA - Effector Commands	152
5.35	Roll Moment Disturbance Rejection Characteristics at Hover with and with- out CCA, the CA Trade-off between Roll Moment Disturbance Rejection and Altitude Command Tracking	153
5.36	Pitch Moment Disturbance Rejection Characteristics at Hover with and with- out CCA - State Variables	155
5.37	Pitch Moment Disturbance Rejection Characteristics at Hover with and with- out CCA - Effector Commands	155
5.38	Pitch Moment Disturbance Rejection Characteristics at Hover with and with- out CCA, the CA Trade-off between Pitch Moment Disturbance Rejection and Altitude Command Tracking	156
5.39	Robustness to error in Roll Moment of Inertia - State Variables	158
5.40	Robustness to error in Roll Moment of Inertia - Effector Commands	158
5.41	Robustness to error in Yaw Moment of Inertia - State Variables	159
5.42	Robustness to error in Yaw Moment of Inertia - Effector Commands	159
5.43	Robustness to error in Mass - State Variables	160
5.44	Robustness to error in Mass - Effector Commands	160
5.45	Manoeuvre 10 Visual Representation - Hover, Forward Transition	161
5.46	Robustness to Error in Fuselage Aerodynamic Section Lift Coefficients - State Variables	163
5.47	Robustness to Error in Fuselage Aerodynamic Section Lift Coefficients - Ef- fector Commands	163
5.48	Robustness to Error in Fuselage Aerodynamic Section Drag Coefficients - States	164
5.49	Robustness to Error in Fuselage Aerodynamic Section Drag Coefficients - Ef- fector Commands	164
5.50	Robustness to Error in Aerofoils Aerodynamic Section Lift Coefficients - State Variables	165
5.51	Robustness to Error in Aerofoils Aerodynamic Section Lift Coefficients - Ef- fector Commands	165

5.52	Robustness to Error in Aerofoils Aerodynamic Section Drag Coefficients - State Variables	166
5.53	Robustness to Error in Aerofoils Aerodynamic Section Drag Coefficients - Effector Commands	166
5.54	Robustness to Error in Aerofoils with Fans Aerodynamic Section Lift Coefficients - State Variables	167
5.55	Robustness to Error in Aerofoils with Fans Aerodynamic Section Lift Coefficients - Effector Commands	167
5.56	Robustness to Error in Aerofoils with Fans Aerodynamic Section Drag Coefficients - State Variables	168
5.57	Robustness to Error in Aerofoils with Fans Aerodynamic Section Drag Coefficients - Effector Commands	168
5.58	Plan View of Lilium-style Fixed-Wing Over-actuated Vectored Thrust eVTOL showing Loss of EDF sets 1 and 10 Failure Configuration	170
5.59	Criticality of CCA: EDF Sets Actuator Failures During VTO - State Variables	170
5.60	Criticality of CCA: EDF Sets Actuator Failures During VTO - Effector Commands	171
5.61	Criticality of CCA: EDF Sets Actuator Failures During VTO - Trade-off due to EDF Set 1 loss	171
5.62	Criticality of CCA: EDF Sets Actuator Failures During VTO - Trade-off due to EDF Set 10 loss	172
A.1	Coupled Aerodynamic coefficients C_u , C_v , $C_{m/4}$ vs C_p^* (Moses, 2021; Welles, 2018). NOTE: flap angle Γ in this work is denoted by θ in the figure different from pitch angle	194
A.2	Longitudinal Aerodynamic Dynamic coefficients in Stability Axis (Suiçmez, 2021)	195
A.3	Lateral-Directional Aerodynamic Static coefficient in Stability Axis (Suiçmez, 2021)	196
A.4	Lateral-Directional Aerodynamic Dynamic coefficients in Stability Axis (Suiçmez, 2021)	196
A.5	LDI problem formulation	197
A.6	LDI-Based Linear Flight Controller Architecture for piloted, semi-automatic and automated flight	200
A.7	No Effector/CA - Manoeuvre 1: VTO, Hover Flight, VL - Acceleration . . .	205
A.8	No Effector/CA - Manoeuvre 2: VTO, Forward Transition - Acceleration . .	205
A.9	No Effector/CA - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change - Acceleration	206
A.10	VTO, Hover Flight, VL - Acceleration	207
A.11	VTO, Forward Transition - Acceleration	207
A.12	Criticality of CCA: VTO and Heading Control, Forward Transition - Acceleration	208
A.13	Criticality of CCA: VTO and Heading Control, Forward Transition - Control Limitation Status	208

A.14 Cruise, Reverse Transition and VL with Heading change - Acceleration . . .	209
A.15 Cruise, Climb, Descend - Acceleration	209
A.16 Cruise and Coordinated Turn - Acceleration	210

List of Tables

2.1	Aerodynamic Strip Categories and Type (Moses, 2021)	23
2.2	Component mass and cg location - modified relative to Moses (2021)	24
2.3	Moment of Inertia	24
2.4	Aerodynamic sections area and coordinates from nose	25
2.5	Effector Model Moment Arms from CG assuming NED	32
2.6	Operating Point of Cruise Trim - States	34
2.7	Operating Point of Cruise Trim - Control inputs	34
2.8	Operating Point of Cruise Trim - Outputs	35
2.9	Cruise trim linearized model aircraft modes	36
4.1	Control Allocation Approaches Comparison	71
4.2	Simulink Simulation Coefficients.	100
4.3	Actuator Physical Limits and Dynamics	100
4.4	Active Set CA Algorithm Input Constants	100
5.1	Effector Model Moment Arms Turn Directions and from CG assuming NED .	115
5.2	Active Set CA Algorithm Input Constants	119
5.3	Command Generator Filtering Simulation Coefficients.	121
5.4	Controller Simulation Coefficients.	122
5.5	Actuator Physical Limits and Dynamics	124
5.6	Some Parameters in Schuebeler EDF	125

Nomenclature

$*_a$	rear/aft rotor
$*_f$	front/fore rotor
$*_{1,2}, *_a$	front/canard left EDFs actual and control demands
$*_{3,4}, *_b$	front/canard right EDFs actual and control demands
$*_{5-8}, *_c$	rear/main wing left EDFs actual and control demands
$*_{9-12}, *_d$	rear/main wing right EDFs actual and control demands
α, β	angles of attack and side-slip angle
$\mathbf{B}_{as}$	active set row augmented weighted virtual control matrix
$\bar{*}$	maximum limit
$\mathbf{B}_e$	linear control effectiveness matrix
$\mathbf{B}_e^\dagger$	Moore-Penrose inverse of the linear control effectiveness matrix for general and 3DoF
$\mathbf{C}$	actuator rate constraints $\dot{\mathbf{u}}_{\min}$ and $\dot{\mathbf{u}}_{\max}$
$\mathbf{G}_e$	LTV control effectiveness matrix
$\mathbf{G}_e^\dagger$	Moore-Penrose inverse of the LTV control effectiveness matrix for general and 3DoF
$\mathbf{B}_{\text{inv}}$	Moore-Penrose inverse of the linear control effectiveness matrix for 6DoF
$\boldsymbol{\tau}$	actual generalised forces and moments
$\boldsymbol{\tau}_c$	virtual or overall aircraft control command
$\mathbf{U}$	actuator saturation constraints $\mathbf{u}_{\min}$ and $\mathbf{u}_{\max}$
$\mathbf{A}$	set of attainable moments
Δ	control change or increment from previous to current software iteration
δ	complementary filter gain used for drift correct in incremental CA Approaches

Δt	sampling period or time increment from previous to current software iteration
$\dot{*}$	maximum rate limit
$\dot{*}_{-}$	minimum rate limit
$\mathbf{e}$	NDI high-level CTL inner loop control error vector
$\mathbf{f}$	non-linear system plant state transition vector
$\mathbf{F}(\mathbf{x})$	product of (Jacobian $\frac{\partial \mathbf{h}}{\partial \mathbf{x}}$ of output matrix $\mathbf{h}$ with respect to $\mathbf{x}$) and $\mathbf{f}(\mathbf{x})$
$\mathbf{g}$	affine non-linear control control effectiveness vector
$\mathbf{G}(\mathbf{x})$	product of (Jacobian $\frac{\partial \mathbf{h}}{\partial \mathbf{x}}$ of output matrix $\mathbf{h}$ with respect to $\mathbf{x}$) and $\mathbf{u}$
Γ	effector tilt angle
γ	primary objective weighting factor ensuring $\mathbf{W}_u \ll \mathbf{W}_v$
Γ_0	actuator tilt angle at time $t = 0$ or previous software iteration
$\mathbf{G}_{inv}$	Moore-Penrose inverse of the LTV control effectiveness matrix for 6DoF
$\mathbf{h}$	output vector
$\mathbf{K}$	NDI outer loop (feedback) linear controller gain vector
$\mathbf{K}_{CA}$	control allocation
$\mathbf{K}_{CTL}$	high-level control law consisting of $\mathbf{K}_{out}$ and $\mathbf{K}_{inn}$
$\mathbf{K}_{out}$	high-level CTL outerloop linear controller
$\mathbf{K}_{inn}$	high-level CTL inner loop NDI controller
λ, ζ	flight path or glide angle, and course angle or heading angle
$\mathbf{I}_n$	the $n \times n$ identity matrix
$\mathbf{p}$	active set pertubation on the the real controls
ϕ, θ, ψ	Euler angles - roll, pitch and yaw angles
$\mathbf{q}$	active set row augmented weighted real control matrix
$\mathbf{r}$	high-level CTL vector of reference signals to be tracked
$\mathbf{s}$	slack variable representing the control error in all control channels and degrees of freedom
$\mathbf{u}$	vector of effector demands or real controls

$\mathbf{u}_p$	preferred actuator position
$\mathbf{u}_{cp}$	drift-compensated actuator position using complimentary filter
$\mathbf{u}_{op}$	optimal actuator position
$\mathbf{u}_{uc}$	unconstrained actuator position using Moore-Penrose inverse
$\mathbf{u}_{x,z}$	vector of real controls in Cartesian form (the x and z coordinate rotor thrust components)
$*$ —	minimum limit
$\mathbf{v}$	NDI high-level CTL outer loop controller output vector
$\mathbf{V}_b$	linear velocity vector of the body
$\boldsymbol{\omega}$	angular velocity vector of the body
$\boldsymbol{\omega}_r$	rotational speed of rotor
$\mathbf{W}_u$	actuator utilization or prioritization weight positive definite matrix
$\mathbf{W}_v$	weight matrix or control channel priority matrix
$\mathbf{x}$	vector of state variables
$\mathbf{y}$	NDI high-level CTL vector of outputs to be controlled
F_X, F_Y, F_Z	overall vehicle (virtual) propulsive thrust in the body x , y & z -directions
g	gravitational acceleration constant
h	output vector
i_{max}	maximum number of Active Set control allocation optimisation software iterations
I_{xx}, I_{yy}, I_{zz}	moment of inertia about the body x , y & z -axis
I_{xz}	product moment of inertia about the body $x - z$ plane
J	secondary objective cost function
L_a, M_a, N_a	net aerodynamic moment acting about the body x , y & z -direction
L_T, M_T, N_T	net vehicle pitching moment acting about the body x , y & z -direction
m	vehicle mass
pqr	angular rates
T	effector thrust

T_0 actuator thrust at time $t = 0$ or previous software iteration

T_X, T_Y, T_Z rotor thrust in the body x , y & z -directions

u, v, w velocities in the body x , y and z -axis direction

V Airspeed

X, Y, Z net aerodynamic force acting at the centre of gravity in the body x , y and z -axis direction

x, y, z vehicle position along the north, east and down respectively from starting point

List of Abbreviations

3DoF	Three degrees of freedom
6DoF	Six degrees of freedom
Abs	Absolute
AESCTE	Aerospace Science and Technology
An	Analytical
ANAC	National Civil Aviation Agency
AS	Active Set
CA	Control Allocation
CAA	Civil Aviation Authority
Cart	Cartesian
CCA	Constrained Control Allocation
CDS	Cranfield Defence and Security
CTL	Control Laws
Con	Constraints
EASA	European Union Aviation Safety Agency
EDF	Electric Ducted Fan
Eff	Effector
eVTOL	Electrical Vertical Take-Off and Landing
FAA	Federal Aviation Authority
Incr	Incremental
LTV	Linear Time Varying
Num	Numerical
Pol	Polar
SATM	School of Aerospace, Technology and Manufacturing
SEEA	School of Energy, Environment and Agrifoods
TCCA	Transport Canada Civil Aviation
UCA	Unconstrained Control Allocation
VTOL	Vertical Take-Off and Landing
WLS	Weighted Least Squares

Chapter 1

Introduction

1.1 Background, Context and UAM Concept

The United Nations projects global population to grow from about 7.8 billion today to 9.3 billion in 2050 and 10.1 billion billion in 2100 (Lee, 2011). Urban centres across the globe are struggling to come to terms with the rising vehicle numbers and the resulting congestion and associated air pollution, especially during peak traffic hours. There is a need for new urban transport solutions to overcome the problems of congestion (e.g. linking urban centers with suburbs and ease the traffic burden on city roads) and pollution in the ever-growing major urban conurbations. The concept of Urban Air Mobility (UAM) has been proposed to address this problem (Hill et al., 2020).

Fixed wing aircraft technologies, despite being mature and reliable, have many constraints, both infrastructural and operational, that limit their applicability for UAM. Piloted all-electric vertical takeoff and landing (eVTOL) vehicles also known as ‘air-taxis’ or ‘flying taxis’ provide a promising option (Hascaryo and Merret, 2020; Rehan et al., 2022) potentially having net zero emissions. Furthermore, electric propulsion is lighter, quieter, potentially lower operating costs and with higher bandwidth actuators than fuelled, allowing novel configurations.

But growth of flying taxis market faces several challenges (Littell, 2019; Ploetner et al., 2020; Straubinger et al., 2020). While the first flying taxi services may have human pilots, later replaced by remote-controlled or AI-powered autonomous vehicles, regulators around the world have been trying to get ahead of the commercialisation rush by creating standards and virtual sandboxes where developers can experiment. Aside from risks to passengers and people on the ground, air taxis could pose hazard for other aircraft. They could also become a target for hackers. The regulations in development will cover everything from vehicle safety, airworthiness and traffic control to noise pollution, operator certification and software security. Lombaerts et al. (2020a) discussed the development and evaluation of simplified Urban Air Mobility (UAM) operations concepts using representative aircraft models, controls, inceptors and displays.

Electric air taxis come in several shapes and sizes, and many look quite different from conventional fixed-wing aircraft. Electric motors replace jet engines, and vertical takeoff and landing (eVTOL) aircraft, designed to avoid the need for long runways, have rotating wings

and, in some cases, rotors in place of propellers. Driving this trend is a confluence of technologies, including autonomous vehicles such as drones and self-driving cars, more efficient batteries, developments in power generation systems, advanced manufacturing techniques, propulsive power becoming more distributed (both in terms of the propulsors themselves and the energy sources) and electrification in flight.

1.2 Classification of eVTOLs

The Vertical Flight Society (VFS), has categorized eVTOLs into the following classifications (eVTOL News; Tan, 2020; Vertical Flight Society):

- vectored thrust - an eVTOL aircraft that uses any of its thrusters for lift and cruise
- lift + cruise - completely independent thrusters used for cruise compared to for lift without any thrust vectoring
- wingless (multicopter) - no thruster for cruise – only for lift
- hover bikes/personal flying devices - these single-person eVTOL aircraft are considered to be in the general class of hover bikes or personal flying devices with the primary differentiation being that the pilot sits on a saddle or is standing, or something similar. Nearly all are multicopter-type wingless configurations
- electric Rotorcraft/helicopters - an eVTOL aircraft that utilizes a rotor, such as an electric helicopter or electric autogyro

Vectored thrust eVTOLs are supported in vertical takeoff and landing flight by the horizontally-mounted propulsion systems, directing thrust downwards to generate lift. Horizontal flight is supported primarily by lift generated from the wings, with some or all of the propulsion systems rotated to the required position to provide horizontal thrust for forward speed. Only vectored thrust eVTOL configurations will be discussed because it is the subject aircraft configuration of this doctoral research.

Vectored thrust eVTOLs can be further classified into two subcategories:

- Tilt-rotor
- Tilt-wing

Tilt-rotor eVTOLs rotate only the propulsion system(s) while tilt-wing eVTOLs rotate the entire wing together with the propulsion system(s). An example of a tilt-wing eVTOL is the Airbus A³ Vahana (Airbus, no date) while that of a tilt-rotor eVTOL is the Lilium Jet (Lilium, 2024) (Figure 1.1 in page 3). Tilt-wings are a challenging configuration for control, particularly in the transition phase between hover and cruise, where the aerodynamics may be in a post-stall condition.

According to Lilium, the Lilium jet aims to connect rural to urban areas and cities to cities with its competitively longer range due to its cruise efficiency (Lilium, 2024). The Airbus



Fig. 1.1 The Lilium Jet (Lilium, 2024)

A³ Vahana completed its last flight in December 2019 as a demonstrator, paving the way for a new eVTOL design for Airbus' UAM solution (Airbus, no date). It is also worth noting that the Lilium Jet uses ducted-fans for propulsion, while the A³ Vahana uses open-mounted propellers. These are two main types of propulsion systems in the vectored thrust eVTOL category.

However, eVTOL aircraft operation requires nonlinear or scheduled control since they operate over a wide range of operating envelope (hover, forward flight, forward transition from hover/low speed to cruise/forward flight, reverse transition from cruise to hover transition, post-stall operation, cruise, climb, descent, coordinated turn etc.). Their wide operating envelope require them to operate in several operating modes with a need to switch between flight controllers. Nonlinear control schemes which can handle nonlinear flight dynamics offer potential solutions in providing a unified-control approach valid in all flight modes without the need to perform predefined-gain scheduling (Postlethwaite et al., 2002, 2005; Prempain and Postlethwaite, 2008; Prempain et al., 2001, 2010; Tran et al., 2018).

Gress (2002) presented an analysis which shows that hover control of a two-propeller

autonomous VTOL aircraft is possible using two tilting (lateral and longitudinal) rotors. The pitch stability is increased by combining the opposed lateral tilting with the longitudinal tilting of the two rotors. The resulting oblique tilting appears to be effective and practical. A 6DoF model of the birotor aircraft was developed and a nonlinear controller (Backstepping-based tracking-control) which leads to a satisfactory control was synthesised.

To reduce cost and increase affordability a pre-requisite for eVTOL air-taxis business model is to have an aircraft that is relatively ‘easy-to-fly’ requiring minimal pilot training compared to that of commercial and military aircraft (Raab et al., 2018). This requires eVTOLs to provide a high degree of automation to reduce the pilot workload and cost of pilot training hence Fly-by-Wire (FBW) control systems also known as Indirect Flight Control Systems (IFCS) are needed. The Fly-by-Wire (FBW) control systems not only satisfy the primary flight objectives (stability, envelope protection, fault tolerance, robustness to both external and internal disturbances etc.) but also the secondary objectives (flight performance, efficiency, drag reduction, flying and handling qualities etc.). A trend in eVTOL development to address the challenge of a need to mitigate control complexity and available computing resources (Ducard and Allenspach, 2021b; Lombaerts et al., 2020a) is towards finding a unified-control approach valid for all flight modes (e.g. Ducard and Allenspach (2021a); Raab et al. (2018)) without the need to switch between flight controllers or to perform predefined-gain scheduling.

In order to ensure the required levels of reliability, safety and actuator/effector fault tolerance, eVTOLs are usually overactuated, i.e., equipped with more effectors than the degrees of freedom they control, thus introducing redundancy in case of effector failure (Littell, 2019; Marks et al., 2012; Ploetner et al., 2020; Straubinger et al., 2020). This over-actuation creates the challenge of allocating the overall control demand to individual redundant effectors known as control allocation (CA). CA provides an optimized allocation of control effectors for the various degrees of freedom under fault-free operation, optimization of actuator utilization (including secondary objectives like drag, wing load reduction etc.) with respect to varying flight phases and provide scope for reconfiguration in the event of a fault in the control effector.

1.3 Lilium eVTOL, EDFs and VTOL Operation Review

This section presents a brief literatures review on Lilium Jet VTOL aircraft, ducted fans used for distributed electric propulsion (DEP) systems and general VTOL operation in order to provide wider context to the choice of Lilium jet as this thesis’s subject aircraft inspiration. A comparison and assessment of different VTOL aircraft designs whether they are appropriate for urban and regional air mobility was performed by Nathen et al. (2021). The analysis showed that the Lilium jet is appropriate for both of these applications. It also details the distribution of the thirty-six electric ducted fans (EDFs) along the wings of the aircraft i.e. a distributed electric propulsion (DEP) arrangement. The canard wing holds twelve ducted fans whereas the main wing holds 24 of them.

Nathen et al. (2021) mentions that the engine configuration was not efficient in the hover mode due to the high disc loading, however this was justified by the fact that the aircraft would be in hover for a very small period of the total flight. Increasing the hover time reduces the endurance of the aircraft (Finger, 2018). Therefore, Lilium have an incentive to reduce the

hover time for the aircraft and prioritise cruise flight. Polaczyk et al. (2019) supports the idea that tilt rotor technology improves the range and horizontal speed of VTOL aircraft, therefore validating the Lilium jet design. Nathen et al. (2021) also explains that the ducted fans are more efficient than alternatives due to the duct surrounding the blade tips. This reduces the blade tip vortices which in turn reduces the drag resulting in a more efficient cruise flight. However, this also increases the required hover power since a higher disc loading is required.

Courtin et al. (2021) compared the performance of the Lilium with Electric Short Take Off and Landing (eSTOL) aircraft and determined that the performance difference between eSTOL and eVTOL was almost independent of whether the power plant was hybrid or electric. In both circumstances, the eSTOL design was found to have a payload capability between 1.8 and 1.9 times greater than an eVTOL.

Finger (2018) states that a typical L/D ratio is between 9 and 18, so while the rated 18.26 ratio may seem optimistic, it is very close to the typical range. The L/D ratio of the Lilium should also be larger than typical aircraft due to the fans increasing the air flow speed over the upper surface. The increased airflow will increase the lift performance and enable higher angles of attack due to boundary layer ingestion. Boundary layer ingestion will also reduce the drag which will further increase the L/D ratio (Hall and Crichton, 2007).

Krase et al. (1980) discusses how the range in power required is a significant problem for the aircraft design team. This is especially a problem when considering the importance of vertical landing where thrust control must be very precise. An additional complication to this problem is that the aircraft must be able to handle engine failure. However, Lilium intentionally reduced the impact of engine failure by using a large number of fans (Lilium, 2024). He further explains that for a ducted propeller to have the desired efficiency advantages, the gap between the blade tip and the duct must be less than 0.5% of the blade length.

Hall and Crichton (2007) showed that boundary layer ingestion reduces the drag of the airframe. This means that, assuming boundary layer ingestion can be achieved, the efficiency of the aircraft would improve which would increase the distance and flight time of the aircraft. Positioning the engines on the trailing edge of the upper surface of a lifting body is being incorporated in other aircraft concept designs such as the Airbus zero emission aircraft and the SAX-40. Both of these designs aim to improve the lift to drag ratio of the aircraft by reducing the decrease in lift caused by a turbulent boundary layer and the additional drag that the turbulent boundary flow causes.

Welles (2018) explores the interactions between aerodynamics and hybrid electric power system (HEPS) design and control for vertical takeoff and landing (VTOL) aircraft applications. Specifically, this research focuses on embedded distributed electric propulsion (DEP) systems, for which the aerodynamic forces and moments are inextricably linked to power input. Welles (2018) presents the computational fluid dynamics (CFD) aspect and wind tunnel testing of distributed lift and thrust aircrafts using the Lilium jet as a case study, and in other studies (Young et al., 2002). The wind tunnel experiment has been conducted based on a scaled model of the Lilium jet wing. The flow is assumed to be 2D (wing made infinite in the wind tunnel), the pressure distribution is assumed to be the same at all wing station and the stagnation pressure is taken at the outlet of the fans. The data acquired for one fan is assumed to be representative of each fan. This paper also identifies the different regimes of the jet according to the flap deflection.

The destabilising effects of canards were explored in Dong et al. (2019); Wibowo et al. (2018). Dong et al. (2019) investigates how the sideslip stability is affected by the use of canards, while Wibowo et al. (2018) investigates how the placement of canards affects the stability of the aircraft. The conclusion in Dong et al. (2019) is that the lateral stability of an aircraft is worsened when canards are included in the design. The tests were all performed at 12m/s and at angles 16° and 35° . The results showed that the sideslip effects became more unpredictable at higher angles of attack. However, a limitation of this project is that the only airspeed tested was 12m/s. This is a very low velocity that is only representative of take off and landing procedures, therefore these results are not applicable to cruise flight or manoeuvres. Wibowo et al. (2018) provides evidence that incorporating a canard increases the coefficient of lift at very high angles of attack ($\geq 25^\circ$). The tests also showed that increasing the distance between the canard and the main wing reduces the coefficient of lift. The geometry of the wing used in this project was a delta planform area, therefore the applications of this wing may not be appropriate to the Lilium Jet design.

In order to ensure a coherent flow of contributions, sections 3.3 and 4.3 will review some techniques and specific class of techniques proposed in the thesis to address the technical challenges. Particularly, section 3.3 in chapter 3 will present the literature review for nonlinear dynamic inversion (NDI) as a nonlinear control technique to overcome the nonlinear dynamics in VTOL operation (due to its wide operating flight envelope and couplings between aerodynamic and propulsive forces) by cancelling the nonlinear airframe dynamics. Section 4.3 in chapter 4 will present a brief literature review of control allocation techniques which can be used to address the challenge of over-actuation for eVTOLs and nonlinear effector mapping for this particular Lilium-Jet inspired vectored thrust VTOL.

1.4 Aim and Objectives

- The aim of the doctoral research is to design and test through simulation an automatic flight control system for a novel fixed-wing over-actuated vectored thrust eVTOL air vehicle
- In order to achieve the aim, the following objectives have been specified
 1. Assemble a representative model of the longitudinal and lateral-directional motion dynamics of the a novel fixed-wing over-actuated vectored thrust eVTOL air vehicle
 2. Derive the control strategy and architecture
 3. Design an aircraft-level controller (NDI) for the longitudinal and lateral-directional motion
 4. Verify the aircraft-level controller for the longitudinal and lateral-direction motion
 5. Design a control allocation scheme and verify using a simple planar VTOL model
 6. Simulate, test and verify the control scheme with the representative model

1.5 Vehicle Models Definition

In order to carry out open loop analysis and understand the open loop characteristics (open loop instability) of the main Lilium jet-inspired eVTOL vehicle, a linear version of the aircraft is produced at cruise trim conditions. Also, in order to test the NDI controller in isolation, a 6DOF model without any effector (real controls) but virtual controls (generalised forces and moments) is generated. Moreover, in order to test the control allocation approaches proposed in this thesis, while using pure controls knowledge and minimising any scalability or aerodynamic issues, a simplified 3DoF eVTOL model without any aerodynamics was developed. Once the NDI-based controller (without any control allocation) and the control allocation (using a 3DoF version) have been tested separately, the NDI controller and Control allocation are then integrated and verified on the full closer-to-industrial 6DoF Model of a lilim-style over-actuated fixed-wing vectored thrust eVTOL. A linear dynamic inversion (LDI) controller was also designed for the linear cruise model and is presented in Appendix 1. Hence, there are four vehicle models used in this thesis:

- **Vehicle Model 1 - main nonlinear 6DoF model** - This is a full closer-to-industrial 6DoF model of a Lilim-style over-actuated fixed-wing vectored thrust eVTOL used for full verification of the overall high-level NDI-based controller and prioritised control allocation techniques in chapter 5. Assembly of Model 1 is described in chapter 2 as a contribution of this study.
- **Vehicle Model 2 - linearized 6DoF Model** - This is a linearised version of Model 1 at cruise trim operating conditions used only for linear analysis of the open loop characteristics in chapter 2.
- **Vehicle Model 3 - Planar VTOL (PVTOL) 3DoF Model** - This is a 3DoF model of an over-actuated vectored thrust eVTOL, a planar VTOL Model without any aerodynamics. This model is described and only used in chapter 4 for conceptual testing of the three prioritised control allocation concepts.
- **Vehicle Model 4 - Virtual Control 6DoF Nonlinear Model** - This is Model 1 without effectors/actuators using virtual controls (generalised forces and moments). This Model 4 is used to test the NDI controller without any control allocation in chapter 3.

1.6 Motivation and Contributions

1.6.1 Motivation

One particularly challenging eVTOL configuration is the Lilium jet (Lilium), an early version of which is the 7-seater canard light sports aircraft with 36 ducted fans (six on each of the two front canards and twelve on each rear main wings). The electric ducted fans (EDFs) are located on top of the control surfaces, which provides vectored thrust simultaneously with aerodynamic control. The 7-seater Lilium jet is an VTOL electrically powered personal air vehicle, designed with both urban and regional air mobility in mind, which can cruise at up to

186 mph, have none of the three principal aerodynamic control surfaces, elevator, aileron and rudder present in conventional aircraft. For simplicity and weight reasons, the Lilium jet has no water cooling, no folding propellers, no gear boxes, no variable pitch, no rudder, no tails, but just tiltable electric jet engines (single-stage motors) (Lilium). It aims to connect rural to urban areas and cities to cities with its competitively longer range due to its cruise efficiency (Lilium). Lilium jet has its ducted fans integrated into the wings to reduce drag and optimize efficiency, while their ducted design provide noise shielding advantage over open rotors.

One of the control challenges of the Lilium jet is that its electric engines arrangement results in at least eight control effectors (four sets of rotors and four sets of tilt angles) to control six degrees of freedom (roll, pitch, yaw, surge, sway and heave). The eight control effectors to control six degrees of freedom results in an over-actuated (number of controls is greater than number of degrees of freedom) system presenting a challenge of allocating the overall aircraft-level control demands to the individual redundant effectors. Another control challenges of the Lilium jet being a vectored thrust tilt-rotor eVTOLs is the interaction and coupling between the aeromechanical and dynamic forces and moments makes control allocation during transition very difficult and potentially deplete the natural aerodynamic damping or stiffness tending to drive an aircraft unstable (Padfield and Lu, 2022). A third challenge of this eVTOL configuration is that its canard configuration can destabilize the short-period mode (Dong et al., 2019; Wibowo et al., 2018) requiring fly-by-wire (FBW) flight control systems. Moreover, a fourth challenge of the VTOL configuration is that the vectored thrust results in non-linear effector mapping (from the overall control demand to the individual effector demand) preventing the direct use of classical linear control allocation approaches and consequent computational infeasibility of performing non-linear programming in real time.

Finally, thrust control saturation is also a major challenge and a crucial constraint for vectored thrust eVTOL configuration with coupled control effectors where actuator limitation and limited control authority during normal and degraded operation and significant nonlinear characteristics in the flight dynamics over their wide range of operating envelope may lead to undesired behavior and even loss of control with a potential to result in deterioration of the controller's performance in specific flight conditions. For example, consider a 6DoF eVTOL with an unprioritized control allocation scheme. The desired combination of rolling, pitching and yawing moments, together with vertical and forward force may not be achievable due to maximum or minimum thrust limitations of the thruster and/or maximum deflection limits of the tilting mechanisms. In absence of an adequate (prioritised) control allocation algorithm, it is arbitrary which one (or combination) of the five objectives will be affected. However, for the flight stability of these over-actuated vehicles, it is crucial to apply the full required rolling, pitching and yawing moment before anything else (Lombaerts et al., 2020b, 2019; Suiçmez, 2021). Thereafter, the vertical force for altitude (Heave DoF) control comes in the second place, followed by the forward force for the airspeed (surge DoF) which is the least essential from a safety point of view. Therefore, a control allocation algorithm is needed that prioritises the control objectives of rotational channels over altitude, and subsequently over forward speed translational; channels, and that calculates the required thrust levels per rotor accordingly.

Control allocation for this degree of over actuation is well studied for unmanned air vehicles (UAV) e.g. drones etc. (Chen et al., 2019; Ducard and Allenspach, 2021a; Liu et al., 2021,

2018; Mimouni et al., 2024; Park et al., 2023; Santos and Bezerra, 2022; Wang et al., 2019b; Xia et al., 2020) but not for the manned air vehicles like the Lilium-style multi-rotor vectored thrust tilt-rotor eVTOL aircraft. This thesis proposes a real-time linear analytical control allocation which considers both actuator saturation and rate limits combined with classical NDI for full envelop control of a novel Lilium-style multi-rotor vectored thrust tilt-rotor eVTOL configuration. A quadratic programming error minimization linear control allocation approach incorporating both actuator saturation and rate constraints has been carefully chosen.

In the research literature, a combination of an NDI-based high level motion controller combined with a control allocation scheme under a unified framework and control architecture consisting of several flight control modes (requiring switching between them) has been applied for the full envelop control of several limited special class of eVTOL tilt-rotor configurations (Lombaerts et al., 2020b, 2019; Suiçmez, 2021).

1.6.2 Contribution

This thesis proposes a simple and unified control framework for full envelop control of a novel Lilium-style multi-rotor vectored thrust tilt-rotor eVTOL configuration. NDI (Enenakpogbe et al., 2022) is used as the high-level controller (aircraft-level controller) for the eVTOL (Lilium, 2024) configuration in this paper rather than incremental NDI (INDI) of Suiçmez (2021); Suiçmez and Kutay (2023) because although it has some drawbacks (poor disturbances rejection, poor robustness to model uncertainties, can only be applied to minimum phase systems), it is well established practical flight control design technique and has proven stability analysis.

A novel full-envelope controller for an over-actuated fixed-wing vectored thrust eVTOL aircraft is presented. It proposes a modular approach, a generic control architecture applicable to piloted, semi-automatic and fully-automated flight consisting of an aircraft-level controller (high-level motion controller) which produces an overall control demand and a control allocation scheme which allocates the overall control demand (virtual controls) to individual redundant effectors (real controls) for the overactuated eVTOL aircraft. The aircraft-level controller also adopts a modular approach, consists of a main inner-loop NDI controller which cancels the bare airframe dynamics and an outer-loop proportional-integral linear controller. The inner-loop classical NDI controller is a velocity controller while the outer-loop proportional-integral (PI) linear controller is an attitude and navigation controller. Both the inner-loop classical NDI and outer-loop PI linear controllers are together used to control hover/low speed and forward (cruise) flights.

The high-level (aircraft-level) controller addresses two (significant nonlinear characteristics in the flight dynamics and unstable open loop operation due to canard configuration) out of the three eVTOL operation challenge. The controller does not address the last challenge of coupling between the aeromechanical and dynamic forces and moments interaction because of the nature of classical NDI (see details in chapter 3) hence the aerodynamic model in chapter 2 is simplified to ensure a decoupled aeromechanical and dynamic forces and moments. Since the focus of this thesis is not aerodynamic modelling, the resolution of coupling between the aeromechanical and dynamic forces and moments for this kind of eVTOL requires significant effort, time and resources which is beyond the scope of this study. The approach proposed in section 4.8 of Chapter 4 can be developed further but the jacobians (differentials) of the

virtual forces with respect to the states and controls have to be computed offline numerically and stored onboard aircraft as complex 3D lookup tables as stated in Bordignon (1996) for the case of traditional aircraft control surfaces.

This thesis proposes three CA approaches for transforming a nonlinear control allocation problem to a linear problem so that classical linear control allocation approaches can then be used. The first approach transfers the non-linearity from the effector mapping to the computation of the actuator limits by formulating the individual effector demand (real controls) in cartesian form (decomposing its thrust components on the body axes) resulting in a Linear Effector mapping. It then converts the physical actuator limits from polar to cartesian form via a rigorous and analytical non-linear computation. The second approach transforms the non-linear effector mapping to a linear mapping via numerical linearisation of the non-linear effector mapping in real-time resulting in a linear time varying (LTV) effector mapping with the real controls formulated in incremental polar form. The third approach is similar to second approach above except an additional step which transforms the virtual controls from cartesian to polar before performing an analytical linearisation resulting in a different and more complicated LTV effector mapping with the advantage that the analytical linearisation is more accurate (compared to numerical linearisation) and more linear since the virtual control and the real controls are in similar polar form. Active set (AS) linear quadratic programming (QP) constrained optimisation algorithm with a weighted least squares (WLS) formulation was used as the classical CA approach.

The resulting CA schemes are novel real-time prioritised CA approaches which addresses the main challenges (limited control authority in some specific flight operation, non-linear effector mapping and over-actuation) of the particular eVTOL configuration. The CA allocates the overall control demand to the individual redundant effectors and also performs control error minimisation, control channel prioritization (prioritizes the rotational channels over the translational channels to adequately allocate the limited control authority in case of actuator saturation) and control effort minimization. The control allocation can degrade the flight performance by prioritizing safety of flight over flight performance and flying quality (FHQ) during actuator failures and saturation. The control allocation scheme addresses all the remaining three challenges of the particular eVTOL configuration (allocation of overall aircraft-level control demand to individual effectors due to over-actuation, limited or loss of control authority in the presence of actuator saturation in some specific flight operation and non-linear effector mapping).

The main contributions of this thesis are summarized as follows:

- It proposes a simple (fewer flight control modes and less switching between modes) and unified control framework with a functional architecture generic for piloted, semi-automatic and automated flight
- It proposes the application of NDI (Enenakpogbe et al., 2022) as the high-level (aircraft-level) controller for the Lilium-style (Lilium) vectored thrust eVTOL configuration. NDI is selected rather than INDI of Suiçmez (2021); Suiçmez and Kutay (2023) because it is well established and has proven stability analysis (Alam and Celikovsky, 2017; Morton et al., 1996).

- It proposes three real-time analytical optimal CA approaches (Enenakpogbe et al., 2025b) which consider both actuator saturation and rate limits using AS linear QP WLS formulation on a simple 3DoF Planar VTOL model with no aerodynamics. It then selects one the proposed CA approaches to control the challenging Lilium-style aircraft model. It will also build on Enenakpogbe et al. (2022) by including lateral-directional control channels and effector model of an over-actuated vectored thrust tilt rotor aircraft (Lilium) as shown in Figure 2.1.
- It proposes an assembly of a more suitable aerodynamic model from existing work (Moses, 2021; Suiçmez, 2021) for this novel vectored thrust tilt rotor eVTOL aircraft which is more complete than the individual aerodynamic models (Suiçmez, 2021) and (Moses, 2021).
- It proposes a blending coefficient which is a continuous differentiable hyperbolic tangent function for performing transition between hover and forward flight aerodynamic models

1.7 Publications Resulting from Thesis

The following are publications associated with this thesis:

1. Title - *Control of an eVTOL using Nonlinear Dynamic Inversion (Enenakpogbe et al., 2022)*.
 - Emmanuel Enenakpogbe, James F. Whidborne, and Linghai Lu. Control of an eVTOL using nonlinear dynamic inversion. In *2022 UKACC 13th International Conference on Control (CONTROL)*, pages 158-164. IEEE, 2022. doi:10.1109/Control55989.2022.9781449
 - *Context* - This is mainly chapters two and three of this thesis. It presents a a control architecture is general for piloted, semi-automatic and fully-automated flight and a NDI-based flight controller using virtual controls, generalised forces and moments for the longitudinal motion control of a VTOL aircraft
2. Title - *Control of an Over-Actuated Fixed-Wing Vectored Thrust eVTOL (Enenakpogbe et al., 2024a)*.
 - Emmanuel Enenakpogbe, James F. Whidborne, and Linghai Lu. Control of an Over-actuated Fixed-wing Vectored Thrust eVTOL. In *2024 UKACC 14th International Conference on Control (CONTROL)*, pages 315-316. IEEE, 2024. doi:10.1109/Control60310.2024.10531934
 - *Context* - This is an extended abstract of this thesis excluding chapter five. It presents a full envelope control of the Lilium-style VTOL using a near-to-industrial model showing and analysing of simulation results for forward transition

3. Title - ***Control Allocation Problem Transformation Approaches for Over-actuated Vectored Thrust VTOLs (Enenakpogbe et al., 2025b).***
 - Emmanuel Enenakpogbe, James F. Whidborne, and Linghai Lu. Control Allocation Problem Transformation Approaches for Over-actuated Vectored Thrust VTOLs. *Aerospace Science and Technology*, 161(2025), pp. 1-19, doi:10.1016/j.ast.2025.110145.
 - *Context* - This paper proposes and tests three control allocation approaches for transforming a nonlinear control allocation problem to a linear problem so that classical linear control allocation approaches can then be used
4. Title - ***Full Envelop Control of Over-Actuated Fixed-Wing Vectored Thrust eVTOL (Enenakpogbe et al., 2024b).***
 - Emmanuel Enenakpogbe, James F. Whidborne, and Linghai Lu. Full Envelope Control of Over-Actuated Fixed-Wing Vectored Thrust eVTOL. *Aerospace, MDPI*, 11(12):979, 2024. doi:10.3390/aerospace11120979
 - *Context* - This is the full paper version of Enenakpogbe et al. (2024a) and focusses on verification of control allocation Approach CA 1 from Chapter 4 assuming already designed control architecture and NDI controller. It presents a full envelope control of the Lilium-style VTOL using a near-to-industrial model showing and analysing of simulation results for a few selected critical manoeuvres
5. Title - ***Full Envelop NDI-based Control of an Over-actuated eVTOL using Incremental Control Allocation (Enenakpogbe et al., 2025a).***
 - Emmanuel Enenakpogbe, James F. Whidborne, and Linghai Lu. Full Envelop NDI-based Control of a Novel Over-actuated Vectored Thrust eVTOL using Incremental Control Allocation. *The Aeronautical Journal*, 2025. To be submitted in July 2025
 - *Context* - This is a summary of this thesis but selecting only Control Allocation Approach 2 (CA 2) from Chapter 4. It presents a full envelope control of the Lilium-style VTOL using a near-to-industrial model showing and analysing of simulation results for all relevant manoeuvres

1.8 Organisation and Overview

In this section, the organization of the thesis chapters are briefly summarised to provide a general overview about the thesis structure and chapters.

Chapter 2 details of the full eVTOL 6DoF near-to-industrial model (known as Vehicle Model 1 in section 1.5). of the linearised cruise model (Vehicle Model 2) was performed to observe the dynamic characteristics and behavior of the aircraft including observing the aircraft modes motivating the type and structure of the controller in the next chapter. Appendix

A also proposed an LDI-based linear controller for the linearised eVTOL Model (Vehicle Model 2).

Chapter 3 presents a simple and unified control framework and functional architecture generic for piloted, semi-automatic and automated flight. Next, it outlines the vehicle-independent part of the controller. Furthermore, it also presents the formulation of a classical NDI-based vehicle-dependent (Enenakpogbe et al., 2022) high-level inner-loop controller which addresses two out of the three eVTOL operation challenge. A 3DoF version of the controller structure and NDI-based nonlinear controller in chapter 3 are published in Enenakpogbe et al. (2022). Chapter 3 concludes with test results of the NDI controller in isolation on vehicle Model 4 (sections 1.5 and 3.6).

Chapter 4 discusses novel real-time prioritised CA approaches which addresses the main challenges of the particular eVTOL configuration. It proposes and present test results of three approaches for transforming a nonlinear CA problem to a linear problem so that classical linear CA approaches can then be used. It applies AS Linear QP constrained optimisation algorithm with a WLS formulation as the classical CA approach. This chapter uses Vehicle Model 3 in section 1.5 and 4.4) and same control architecture and classical NDI-based controller presented in Enenakpogbe et al. (2022). Chapter 4 has been published in the Aerospace Science and Technology (AESCTE) Journal (Enenakpogbe et al., 2025b).

Chapter 5 presents verification of the NDI controller (in chapter 3) and CA2 algorithm (in Chapter 4) via nonlinear simulation of the full eVTOL 6DoF near-to-industrial model (covered in Chapter 2). Several test cases are analysed that consider the wide-flight envelope of the vectored thrust eVTOL aircraft. In addition, the CA performance is tested for specific flight conditions (see sections 5.2.3 to 5.2.8). Disturbance rejection properties, robustness to errors in model dependent parameters, and effector failures are also analyzed (see sections 5.2.9 to 5.2.13).

Finally, Chapter 6 concludes with a summary of the main contributions of the thesis, a summary of the main findings of the thesis, a discussion of any limitations of the method and approach and recommends further work still to be carried out. Appendix A and ?? contains some important findings and results generated during the course of this study which are not the focus of this study.

Chapter 2

Vehicle Modelling and Analysis

2.1 Chapter Contribution

Although the aerodynamic modelling approach used in Moses (2021) is appropriate for the non-conventional aircraft, it is however a longitudinal-only (3DoF) model with only static derivatives. It has no dynamic derivatives and lateral-directional aerodynamic (static and dynamic) derivatives. Meanwhile, the model in Suiçmez (2021) though is a 6DoF model with both longitudinal and lateral-direction static and dynamic aerodynamic derivatives, the modelling approach used is that of a conventional aircraft which is not appropriate for this kind of eVTOL nonconventional aircraft. Hence, the main contribution of this chapter are:

- an assembly of a more representative aerodynamic model from existing work of Suiçmez (2021) and Moses (2021) for this novel vectored thrust tilt rotor eVTOL aircraft This model is known as Vehicle Model 1 as mentioned in section 1.5.
- derivation of a blending coefficient which is a continuous differentiable hyperbolic tangent function for performing transition between hover and forward flight aerodynamic models

2.2 6DoF Vehicle Mathematical Model

2.2.1 Overview of Vehicle Model 1

This is the derivation of Vehicle Model 1 (as mentioned in section 1.5) and it is based on the 7-seater Lilium Jet (Lilium), a canard light sports aircraft with 36 all-electric engines (six on each of the two front canards and twelve on each rear main wings). This results in eight control effectors to control 6DOF (roll, pitch, yaw, surge, sway and heave) resulting in an over-actuated system and consequently presents CA challenges previously discussed in section 1.6. The plan view of the Vehicle Model 1 is shown below in Figure 2.1.

The aircraft has a fixed-wing aerodynamic surface and twelve tilting EDFs distributed through the front canard and rear main wing sections of the aircraft (see Figure 2.1). The challenging part is estimating the aerodynamic and propulsion forces and moments acting on

the aircraft, especially during the transition region. High fidelity modeling of the transition dynamics is quite complex since tilting EDFs also generate aerodynamic forces/moments, and flow separation over the EDF surfaces might occur during the transition. Since the main focus and motivation of this work are the full envelope flight control of the unique eVTOL aircraft, the flight dynamics model is constructed considering the main effects at hover and high speed forward flight. During the transition, the dynamics are obtained by blending the hover and forward flight aerodynamic models based on the airspeed though in practice this introduces significant nonlinearity in its aerodynamic forces and moments due to the complex interaction between the incoming freestream and the propeller slipstream. However, the complex interaction between the incoming freestream and the propeller slipstream are challenging to accurately model resulting in some unmodelled dynamics and a low fidelity aerodynamic model.

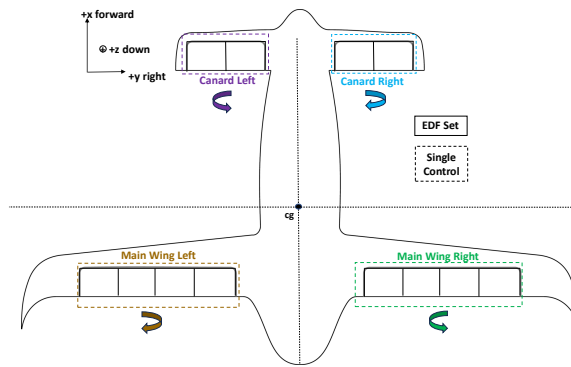


Fig. 2.1 Aircraft canard-planform showing EDF sets distribution

2.2.2 Flight Mechanics and Equation of Motion

Consider an air vehicle with pitch, roll, yaw, heave, surge and sway 6DOF. Assuming rigid body in still air, the dynamic equations for linear and angular accelerations (standard Newton-Euler equations) are:

$$\begin{bmatrix} \mathbf{F} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} m\mathbf{I}_3 & 0 \\ 0 & \mathbf{I}_b \end{bmatrix} \begin{bmatrix} \dot{\mathbf{V}}_b \\ \dot{\boldsymbol{\omega}} \end{bmatrix} + \begin{bmatrix} m\boldsymbol{\omega} \times \mathbf{V}_b \\ \boldsymbol{\omega} \times \mathbf{I}_b \boldsymbol{\omega} \end{bmatrix} \quad (2.1)$$

where $\mathbf{F}$ is the total force vector acting on the center of mass in the body axes and $\mathbf{M}$ is the total moment vector acting about the center of mass given by:

$$\mathbf{F} = \mathbf{F}_{aero} + \mathbf{F}_{prop} + \mathbf{F}_{grav} \quad (2.2)$$

$$\mathbf{M} = \mathbf{M}_{aero} + \mathbf{M}_{prop} \quad (2.3)$$

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} \mathbf{F}_{aero} + \mathbf{F}_{grav} \\ \mathbf{M}_{aero} \end{bmatrix} \quad (2.4)$$

$$\mathbf{G}(\mathbf{u}) = \begin{bmatrix} \mathbf{F}_{prop} \\ \mathbf{M}_{prop} \end{bmatrix} \quad (2.5)$$

The body frame translational rates are related to the earth frame rates by the kinematic equations: $\dot{\mathbf{X}} = R(\phi, \theta, \psi) \mathbf{V}_b$ where $\mathbf{X} = [x, y, z]^T$ is the position of the centre of gravity in the earth frame in the north, east, down convention, (ϕ, θ, ψ) is the standard aerospace Euler angle (Tait-Bryan) triple and

$$R(\phi, \theta, \psi) = \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{bmatrix} \quad (2.6)$$

is the Direction Cosine Matrix (DCM).

The equations of motion are then given by:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}(X + T_X) - g \sin \theta + rv - qw \\ \frac{1}{m}Y + g \sin \phi \cos \theta + pw - ru \\ \frac{1}{m}(Z + T_Z) + g \cos \theta + qu - pv \end{bmatrix}, \quad (2.7)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad (2.8)$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad (2.9)$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \mathbf{I}_b^{-1} \left\{ \begin{bmatrix} L \\ M \\ N \end{bmatrix} - \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right\} \quad (2.10)$$

The equations of motion in a scalar form are given by:

$$\begin{aligned} \dot{x} = & u(\cos(\theta) \cos(\psi)) + v(\sin(\phi) \sin(\theta) \cos(\psi) - \cos(\phi) \sin(\psi)) \\ & + w(\cos(\phi) \sin(\theta) \cos(\psi) + \sin(\phi) \sin(\psi)) \end{aligned} \quad (2.11)$$

$$\begin{aligned} \dot{y} = & u(\cos(\theta) \sin(\psi)) + v(\sin(\phi) \sin(\theta) \sin(\psi) + \cos(\phi) \cos(\psi)) \\ & + w(\cos(\phi) \sin(\theta) \sin(\psi) - \sin(\phi) \cos(\psi)) \end{aligned} \quad (2.12)$$

$$\dot{z} = -u \sin(\theta) + v \sin(\phi) \cos(\theta) + w \cos(\phi) \cos(\theta) \quad (2.13)$$

$$\dot{u} = \frac{1}{m}(X + T_X) - g \sin \theta + rv - qw \quad (2.14)$$

$$\dot{v} = \frac{1}{m}Y + g \sin(\phi) \cos(\theta) + pw - ru \quad (2.15)$$

$$\dot{w} = \frac{1}{m}(Z + T_Z) + g \cos \theta + qu - pv \quad (2.16)$$

$$\dot{\phi} = p + (q \sin(\phi) + r \cos(\phi)) \tan(\theta) \quad (2.17)$$

$$\dot{\theta} = q \cos(\phi) - r \sin(\phi) \quad (2.18)$$

$$\dot{\psi} = (q \sin(\phi) + r \cos(\phi)) \sec(\theta) \quad (2.19)$$

$$\dot{p} = \frac{I_{zz}}{I_D} [L_T + L_A + I_{xz}pq - (I_{zz} - I_{yy})qr] + \frac{I_{xz}}{I_D} [N_T + N_A - I_{xz}qr - (I_{yy} - I_{xx})pq] \quad (2.20)$$

$$\dot{q} = \frac{1}{I_{yy}} (M_A + M_T) \quad (2.21)$$

$$\dot{r} = \frac{I_{xz}}{I_D} [L_T + L_A + I_{xz}pq - (I_{zz} - I_{yy})qr] + \frac{I_{xx}}{I_D} [N_T + N_A - I_{xz}qr - (I_{yy} - I_{xx})pq] \quad (2.22)$$

where

- T_X and T_Y are the combined rotor thrusts in the body x and z axes
- m and g : vehicle mass and gravitational acceleration constant
- X, Y, Z are the net aerodynamic body-axis force
- $\mathbf{V}_b$ is the velocity vector of the center of mass
- $\boldsymbol{\omega}$ is the angular velocity body axis vector p, q, r
- L_A, M_A, N_A are the net aerodynamic moments
- L_T, M_T, N_T are the net rotor moments
- $\mathbf{I}_3$ is the 3×3 identity matrix
- I_{xx} is the moment of inertia about the x -body direction,
- I_{yy} is the moment of inertia about the y -body direction,
- I_{zz} is the moment of inertia about the z -body direction and
- I_{xz} is the moment of inertia about the xz -body direction
- $\mathbf{I}_b$ are the moments of inertia matrix about the CG given by $\mathbf{I}_b = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix}$
- $I_D = I_{xx}I_{zz} - I_{xz}^2$
- $\mathbf{I}_b^{-1}$ is the inverse moments of inertia matrix given by $\mathbf{I}_b^{-1} = \frac{1}{I_D} \begin{bmatrix} I_{zz} & 0 & I_{xz} \\ 0 & I_D & 0 \\ I_{xz} & 0 & I_{xx} \end{bmatrix}$

state variable vector $\mathbf{x}$ and virtual control vector $\boldsymbol{\tau}$:

$$\mathbf{x} = [x \ y \ z \ u \ v \ w \ \phi \ \theta \ \psi \ p \ q \ r]^T$$

$$\boldsymbol{\tau} = [T_X \ T_Z \ L_y \ M_y \ N_y]^T$$

In still air, the wind-axis variables are given by:

$$V_\infty = \sqrt{(u^2 + v^2 + w^2)} \quad (2.23)$$

$$\alpha = \text{atan2}(w, u) \quad (2.24)$$

$$\beta = \text{atan2}\left(v, \sqrt{(u^2 + w^2)}\right) \quad (2.25)$$

$$(2.26)$$

The navigation variables are given by:

$$\gamma = \text{atan2}\left(-\dot{z}, \sqrt{(\dot{x}^2 + \dot{y}^2)}\right) \quad (2.27)$$

$$\zeta = \text{atan2}(\dot{y}, \dot{x}) \quad (2.28)$$

$$(2.29)$$

where

- $\dot{x}$, $\dot{y}$ and $\dot{z}$ are the velocity on the flat-earth in the North, East and Down (NED) direction respectively
- γ is the flight path angle or climb angle which is angle that flight path of the aircraft centre of mass makes with the horizon along the vertical plane
- ζ is the course angle or course-over-ground which is angle between the direction of centre of mass and the True North along the horizontal plane or the bearing of the direction of the aircraft relative to true North.

2.3 Vehicle Dynamic Modelling

As mentioned in section 1.8, one of the main contributions of this work is an assembly of a more suitable and representative aerodynamic model from existing work Suiçmez (2021); Suiçmez and Kutay (2023) and Moses (2021) for this vectored thrust Tilt rotor eVTOL aircraft. The aerodynamic terms X , Y , Z , M_A , L_A and N_A from section 2.2 are dependent upon the airspeed and wing angles of attack, side slip, angular rates via the static and dynamic aerodynamic coefficients.

2.3.1 Aerodynamic Modelling Approach

At low speeds and hover ($u < 10\text{m/s}$ (u_{hovth})), the propulsive forces are dominant (aerodynamic control surfaces are ineffective with low control authority) while at high forward cruise speeds ($u > 20\text{m/s}$ (u_{cruth})) the aerodynamic forces are effective hence separate aerodynamic models have been formulated for both flight regimes.

The cruise aerodynamic model is derived from Moses (2021), Welles (2018), Suiçmez (2021) and Suiçmez and Kutay (2023) while the hover aerodynamic model is a flat plate resistance force model from Suiçmez (2021) and Suiçmez and Kutay (2023). The geometric information, moment of inertia, mass and effector distribution as indicated in Figure 2.4 are based on Moses (2021).

2.3.2 Aerodynamic Model Blending

In order to model the nonlinearity in the aerodynamics during transition between hover/low speed and forward flight/cruise and address numerical issues in computing α and β at low speed/hover in simulation, a blending coefficient is empirically derived for performing transition between hover and forward flight aerodynamic models rather than relying on dynamic pressure to perform the aerodynamics. Dynamic pressure is not used to perform the aerodynamics blending because of the differences between the hover/low speed and cruise aerodynamic characteristics of the eVTOL over its wide flight envelop.

Using a smooth, differentiable blending coefficient for transitioning between low-speed/hover and high-speed/cruise aerodynamic models offers several advantages over relying solely on dynamic pressures:

- **Continuity and Stability:** A smooth blending coefficient ensures a continuous transition between aerodynamic models, reducing abrupt changes that could destabilize the aircraft's control systems (Sequeira et al., 2006).
- **Predictability:** Differentiable blending coefficients provide predictable behavior across different flight regimes, making it easier to design and tune control algorithms (Baker et al., 2000).
- **Computational Efficiency:** Smooth transitions can be more computationally efficient, as they avoid the need for complex calculations associated with dynamic pressure changes (Grauer and Morelli, 2014).
- **Improved Accuracy:** By smoothly blending aerodynamic characteristics, the model can more accurately represent the aircraft's behavior across a wide range of speeds and conditions (Grauer and Morelli, 2014).

The blending coefficient for blending hover and cruise models to obtain a more suitable model is proposed in this work. This involves a derivation of a continuous differentiable hyperbolic tangent function K_b of u , u_{hov} and u_{cru} given by:

$$K_b = \frac{1}{2} \left[1 + \tanh \left(\frac{4u - 2(u_{cruth} + u_{hovth})}{u_{cruth} - u_{hovth}} \right) \right] \quad (2.30)$$

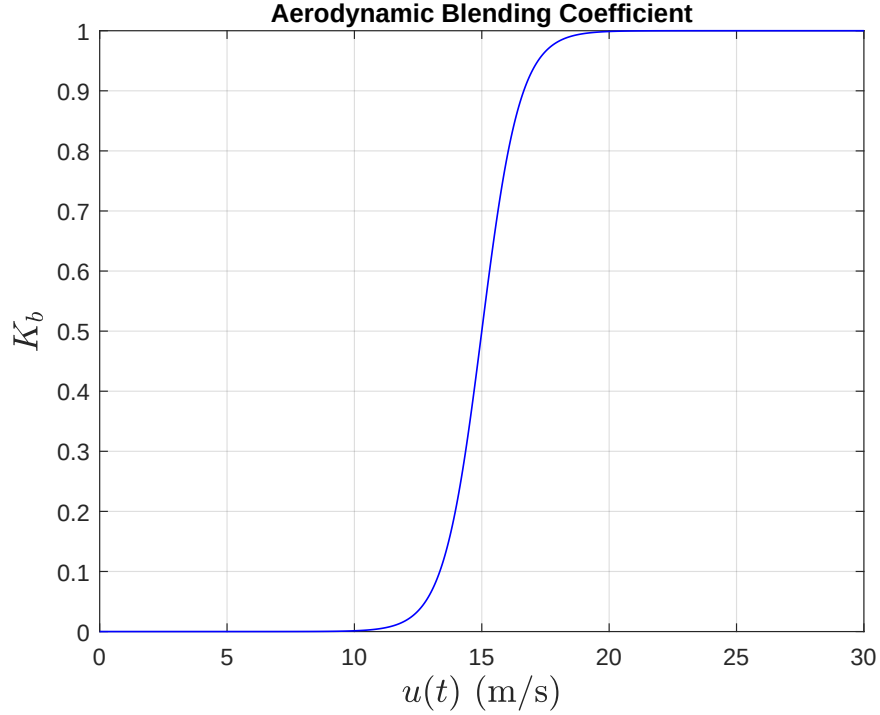


Fig. 2.2 Aerodynamic Blending Coefficient

The blending coefficient K_b is set to 0 for values of $u \leq u_{hovth}$, to 1 for $u \geq u_{cruth}$, to a value between 0 and 1 for $u_{hovth} < u < u_{cruth}$ is illustrated in Figure 2.2. $u_{hovth} = 12$ (m/s) and $u_{cruth} = 18$ (m/s).

The resultant aerodynamic forces and moments are given by:

$$\mathbf{F}_{aero} = K_b \mathbf{F}_{cru} + (1 - K_b) \mathbf{F}_{hov} \quad (2.31)$$

$$\mathbf{M}_{aero} = K_b \mathbf{M}_{cru} + (1 - K_b) \mathbf{M}_{hov} \quad (2.32)$$

where

$\mathbf{F}_{hov}$ is the aerodynamic force vector $[X_{hov} Y_{hov} Z_{hov}]^T$ for the hover model

$\mathbf{F}_{cru}$ is the aerodynamic force vector $[X_{cru} Y_{cru} Z_{cru}]^T$ for the cruise model

$\mathbf{M}_{hov}$ is the aerodynamic moment vector $[L_{A_{hov}} M_{A_{hov}} N_{A_{hov}}]^T$ for the hover model

$\mathbf{M}_{cru}$ is the aerodynamic moment vector $[L_{A_{cru}} M_{A_{CRU}} N_{A_{cru}}]^T$ for the cruise model

$\mathbf{F}_{aero}$ is the resultant blended aerodynamic force vector

$\mathbf{M}_{aero}$ is the resultant blended aerodynamic moment vector

2.3.3 Hover Aerodynamic Model

The hover aerodynamic model is a resistance drag force model from Suiçmez (2021) and Suiçmez and Kutay (2023) because at hover and low speed flight, a reasonable assumption is that the resistance drag force along the translational axes (horizontal, vertical, and sideward motion) is the dominant aerodynamic effect. The lift force and aerodynamic moments are considered negligible ($M_{hov} = 0$) and not modeled at low-speed flight since the maximum speed is restricted to 10m/s for horizontal motion and 5m/s for vertical motion. Although ground effect is also relevant for the hover flight, but it is not modeled for the preliminary modeling since it requires more detailed and complex aerodynamic modeling.

The drag is estimated as a resistance force for each direction. It is based on a flat-plate for y and z directions, and a circular cylinder for x direction since the aircraft is assumed circular in the aircraft is assumed to be circular in the x direction. The drag forces aerodynamic drag forces are given by;

$$\mathbf{F}_{hov} = \begin{bmatrix} -\frac{1}{2}(\text{sgn } u) \rho u^2 S_x C_{d,x} \\ -\frac{1}{2}(\text{sgn } v) \rho v^2 S_y C_{d,y} \\ -\frac{1}{2}(\text{sgn } w) \rho w^2 S_z C_{d,z} \end{bmatrix} \quad (2.33)$$

where

- $\text{sgn } u$, $\text{sgn } v$ and $\text{sgn } w$ are *sign* functions which extract the signs of u , v and w respectively based on their direction or polarity. The *sign* function returns a value of 1 for positive numbers, 0 for zero and -1 for negative numbers
- $C_{d,y}$ and $C_{d,z}$ are the drag coefficients for y and z directions respectively for a rectangular plate and equal to 1.2 (Suiçmez, 2021; Suiçmez and Kutay, 2023)
- $C_{d,x}$ is the drag coefficient for x direction for a circular cylinder, which is equal to 0.74 (Suiçmez, 2021; Suiçmez and Kutay, 2023)
- Based on Suiçmez (2021) and Suiçmez and Kutay (2023), S_x , S_y and S_z are the effective frontal areas calculated roughly for x, y and z direction given by:

$$S_x = \frac{\pi h_{fus}^2}{4} \cong 3m^2 \quad (2.34)$$

$$S_y = \ell_{fus} h_{fus} \cong 8m^2 \quad (2.35)$$

$$S_z = \ell_{fus} h_{fus} + S \cong 10m^2 \quad (2.36)$$

where

- $h_{fus}=2m$ is the mean fuselage height (Suiçmez, 2021; Suiçmez and Kutay, 2023)
- $\ell_{fus}=4m$ is the fuselage length (Suiçmez, 2021; Suiçmez and Kutay, 2023)
- $S=2.7m^2$ is the wing reference area (Suiçmez, 2021; Suiçmez and Kutay, 2023)

2.3.4 Cruise Aerodynamic Model

The longitudinal static aerodynamic derivatives of the cruise model are obtained from Moses (2021); Welles (2018) using an appropriate approach for this type of VTOL configuration, a computational fluid dynamics-based aerodynamic modelling method known as boundary layer ingestion. The cruise model longitudinal dynamic derivatives, lateral-directional static derivatives and lateral-directional dynamic aerodynamic derivatives are based on Suiçmez (2021); Suiçmez and Kutay (2023). The first part of the sub-section (section 2.3.5) presents the aerodynamic forces and moments and propulsive forces and moments (effector model) contributed by the longitudinal static aerodynamic derivatives in the work of Moses (2021) which is different from the conventional aerodynamics derivations in traditional aircrafts. The second part of this section (section A.2.1) lists the longitudinal dynamic aerodynamic derivatives and lateral-directional aerodynamic derivatives (static and dynamic) from Suiçmez (2021); Suiçmez and Kutay (2023) based on conventional aerodynamics derivations in traditional aircrafts. The propulsive forces and moments (effector model) are also derived.

2.3.5 Cruise Aerodynamic Model - Part 1

Details of the longitudinal static derivatives aerodynamic Modelling can found in Moses (2021).

The dynamic pressure is given by:

$$\bar{q} = \frac{\rho V^2}{2} \quad (2.37)$$

where:

- V is the total airspeed over the wetted wing surface due to boundary flow ingestion of the EDF

Using strip theory discretisation, the aerodynamic coefficients for the aircraft was determined for four components:

- wings with fans
- wings without fans
- wing tips
- fuselage

The aerodynamic effects of the landing gears were ignored due to their complexity. Details of the aerodynamic modelling of each components can be found in Moses (2021). The discretisation of the plan view into various aerodynamic components is shown in Figure 2.3. Table 2.1 shows each aerodynamic component type. The mass and cg locations (using the aircraft nose and NED convention) of the various components used to compute the overall vehicle CG,

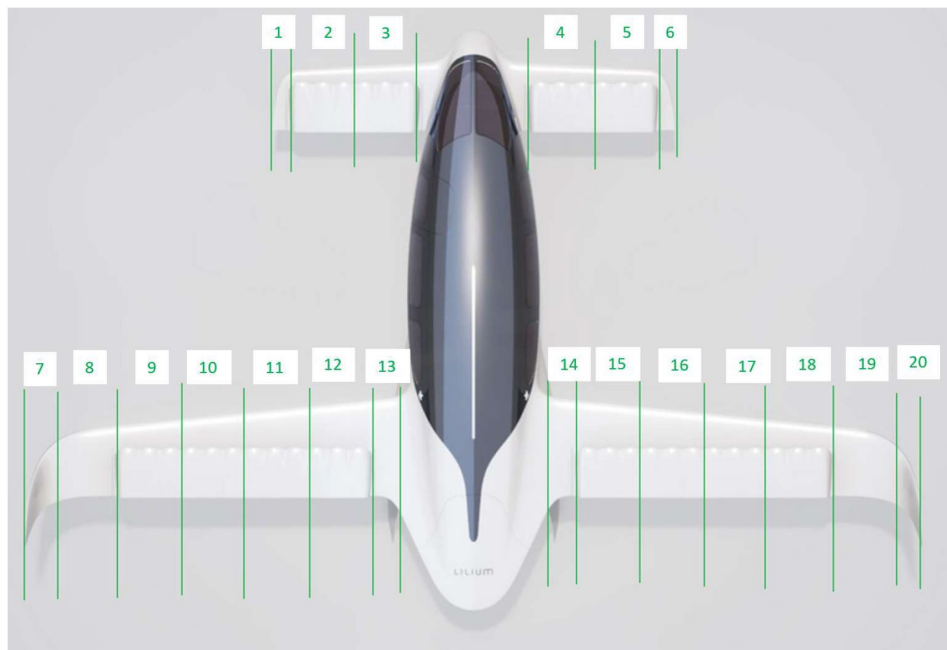


Fig. 2.3 Lilium Plan View Strip Theory Discretisation (Moses, 2021)

Table 2.1 Aerodynamic Strip Categories and Type (Moses, 2021)

Number	Value	Number	Value
1	Aerofoil	11	Aerofoil with a fan
2	Aerofoil with a fan	12	Aerofoil with a fan
3	Aerofoil with a fan	13	Aerofoil
4	Aerofoil with a fan	14	Aerofoil
5	Aerofoil with a fan	15	Aerofoil with a fan
6	Aerofoil	16	Aerofoil with a fan
7	Wing tip	17	Aerofoil with a fan
8	Aerofoil	18	Aerofoil with a fan
9	Aerofoil with a fan	19	Aerofoil
10	Aerofoil with a fan	20	Wing tip
21	Fuselage		

moment of inertia and moment arms of the aerodynamic and EDF fans were slightly modified relative to Table 4 in Moses (2021) as shown in Table 2.2. Using the values in Table 2.2, the moment of inertia matrix $\mathbf{I}_b$ terms are calculated and given in Table 2.3:

The wetted area and coordinates (using the aircraft nose and NED convention) of the various aerodynamic surfaces strips listed in Table 2.1 that are used to compute the aerodynamic forces using strip theory are shown in Table 2.4.

Table 2.2 Component mass and cg location - modified relative to Moses (2021)

Component	Mass (kg)	Distance from Nose (NED)		
		x(m)	y(m)	z(m)
Fuselage	608.8	-3.890	0	-0.154
Main Wing Left	152.2	-5.867	-3.111	-0.6670
Main Engine Left	152.2	-6.044	-3.111	0.8210
Canard Left	76.1	-1.422	-1.600	0
Canard Engine Left	76.1	-1.600	-1.778	-0.1540
Main Wing Right	152.2	-5.867	3.111	-0.6670
Main Engine Right	152.2	-6.044	3.111	-0.8210
Canard Right	76.1	-1.422	1.600	0
Canard Engine Right	76.1	-1.600	1.778	-0.1540
Pilot	100.0	-1.600	0	-0.2050
Passenger 1	100.0	-3.120	-0.400	-0.2050
Passenger 2	100.0	-3.120	0.400	-0.2050
Passenger 3	100.0	4.080	-0.400	-0.2050
Passenger 4	100.0	4.080	0.400	-0.2050
Passenger 5	100.0	-4.960	-0.360	-0.2050
Passenger 6	100.0	-4.960	0.360	-0.2050
Battery	953.0	-6.481	0	0.5100
Overall CG	3175.0	-4.7944	0	-0.0716

Table 2.3 Moment of Inertia

Element	Inertia(kg m ²)
I_{xx}	7489
I_{yy}	82625
I_{zz}	88843
I_{xz}	484
I_{xy}	0
I_{yz}	0

Table 2.4 Aerodynamic sections area and coordinates from nose

A/C loc	Number	Area (m ²)	Coordinates from Nose (NED)		
			x(m)	y(m)	z(m)
canard left tip	1	0.140	-1.099	-2.958	0
canard left engine tip	2	0.816	-0.961	-2.366	0
canard left engine root	3	0.876	-0.938	-1.368	0
canard right engine root	4	0.876	-0.938	1.368	0
canard right engine tip	5	0.816	-0.961	2.366	0
canard right tip	6	0.140	-1.099	2.958	0
wing left tip	7	0.224	6.408	-6.655	-1.026
wing left	8	0.841	-6.023	-6.032	-0.667
wing left engine 1 tip	9	1.004	-5.934	-5.041	-0.667
wing left engine 2 tip	10	1.120	-5.890	-4.049	-0.667
wing left engine 1 root	11	1.202	-5.840	-3.066	-0.667
wing left engine 2 root	12	1.243	-5.787	-2.067	-0.667
wing left root	13	0.570	-5.740	-1.360	-0.667
wing right root	14	0.570	-5.740	1.360	-0.667
wing right engine 1 root	15	1.243	-5.787	2.067	-0.667
wing right engine 2 root	16	1.202	-5.840	3.066	-0.667
wing right engine 1 tip	17	1.120	-5.890	4.049	-0.667
wing right engine 2 tip	18	1.004	-5.934	5.041	-0.667
wing right	19	0.841	-6.023	6.032	-0.667
wing right tip	20	0.224	-6.408	6.655	-1.026
fuselage	21	16.397	-5.400	0	-0.154

2.3.5.1 Wings without Fans/EDFs, Wing Tips and Fuselage

Moses (2021) selected the NACA 23015 aerofoil for the ‘wing without fans/EDFs’ (aerodynamic sections 8, 13, 14, 19) component, NACA 0015 aerofoil for the wing tips (aerodynamic sections 1, 6, 7, 20) and the NACA 66(3)-418 aerofoil for the fuselage (aerodynamic section 21).

2.3.5.2 Wings with Fans/EDFs Effector Model

The pressure coefficient which is function of EDF thrust RPM/thrust and airspeed is given by:

$$C_p^* = \frac{P}{\bar{q}VA} \quad (2.38)$$

where:

- A is the total EDF set cross-sectional area $= nA_d$
- P is the EDF set total power

The nonlinear longitudinal static derivatives which are three-dimensional lookup table are given by:

$$C_u = f_{C_u}(C_p^*, \Gamma, \alpha) \quad (2.39)$$

$$C_v = f_{C_v}(C_p^*, \Gamma, \alpha) \quad (2.40)$$

$$C_{m/4} = f_{C_{m/4}}(C_p^*, \Gamma, \alpha) \quad (2.41)$$

where:

- C_u is the coupled horizontal (drag) aerodynamic coefficient due to coupled propulsive EDF thrust and aerodynamic forces
- C_v is the coupled vertical (lift) aerodynamic coefficient due to coupled propulsive EDF thrust and aerodynamic forces
- $C_{m/4}$ is the coupled pitch aerodynamic coefficient due to coupled propulsive EDF thrust and aerodynamic forces
- α is the angle of attack
- Γ is the flap angle of EDF

The coupled aerodynamic coefficients C_u , C_v , $C_{m/4}$ are shown in Figure A.1.

2.3.5.3 Customisation of Existing eVTOL Longitudinal Dynamic Model

As discussed in chapter 1, the controller does not address the last challenge of coupling between the aeromechanical and propulsive forces and moments hence the aerodynamic coefficients in equation (2.39), (2.40) and (2.41) of Moses (2021); Welles (2018) is customised to fit the scope of this study. The customisation involves decoupling the aeromechanical (propulsive) and dynamic (aerodynamic) forces and moments. Due to the significant amount of effort, time and resources (requires tools and software, personnels of industrial scale) required to resolve the coupling between the aerodynamic and propulsive forces and moments for this class of VTOL, it is beyond the scope of this study.

Clearly, the nonlinear longitudinal static derivatives in equations (2.39), (2.40) and (2.41) consists of coupled aerodynamic and propulsive components (via a nonlinear function due to pressure coefficient ($C_p^* = \frac{P}{qVA}$)). The aerodynamic and propulsive coupling is due to the boundary layer ingestion phenomenon of the EDF fans mounted on wings since P and Γ are the propulsive real controls while V , q and α are aerodynamic quantities.

The following assumptions are made to decouple the aeromechanical (propulsive) and dynamic (aerodynamic) forces and moments of aerodynamic sections 2 – 5, 9 – 12, 15 – 18 in the controller and simulation models, and as will be seen later in Chapter 3 lend the control of this class of vehicle suitable to classical NDI and ensure that the effector model is invertible (inverse effector model can be obtained) for the purpose of control allocation.

The EDF RPM/thrust/power is set to $P = 0$ in (2.38) (and $C_p^* = 0$) and the tilt/flap angle is set to a fixed angle of 0 ($\Gamma = 0$) so that the propulsive component will be equal to 0, resulting in only an aerodynamic component as follows:

$$C_u^A = f_{C_u}(0, 0, \alpha) = f_{C_u}(\alpha) \quad (2.42)$$

$$C_v^A = f_{C_v}(0, 0, \alpha) = f_{C_v}(\alpha) \quad (2.43)$$

$$C_{m/4}^A = f_{C_{m/4}}(0, 0, \alpha) = f_{C_{m/4}}(\alpha) \quad (2.44)$$

$$\mathbf{F}_{DW} = \bar{q} S C_u^A \quad (2.45)$$

$$\mathbf{F}_{LW} = \bar{q} S C_v^A \quad (2.46)$$

$$\mathbf{M}_{m/4} = q S \bar{c} C_{m/4}^A \quad (2.47)$$

where:

- $\mathbf{F}_{LW}$ is the aerodynamic lift force (assumed decoupled from propulsive forces)
- $\mathbf{F}_{DW}$ is the aerodynamic drag force (assumed decoupled from propulsive forces)
- $\mathbf{M}_{m/4}$ is the aerodynamic pitching moment (assumed decoupled from propulsive forces)

It is assumed that:

- the aerodynamic and propulsive side force is zero since the EDFs don't generate forces in the y-direction

- The rolling/lateral moment of the EDF is zero
- The aerodynamic directional moment is zero
- There is a net directional torque (moment) due to the EDF fan dependent of EDF rotation speed and direction. It is intended as will be seen later that the counter rotation of opposite EDFs will cancel out any directional torque at steady state to prevent net yaw.

2.3.5.4 Decoupled and Simplified Wings with Fans/EDFs Propulsive Forces

The propulsive EDF thrust and power are then given by the invertible functions:

$$T = C_T \rho \omega^2 \quad (2.48)$$

$$Q = C_Q T \quad (2.49)$$

$$P = \left(\frac{T^3}{2\rho n A_d \eta^2} \right)^{1/2} \quad (2.50)$$

where:

- $A = n A_d$ is the total EDF set cross-sectional area
- C_T is the thrust coefficient
- C_Q is the torque coefficient
- ω is the EDF set fan rotational speed (rads^{-1})
- P is the EDF set total power
- T is the EDF set total thrust
- Q is the torque due to EDF set total thrust
- A_d EDF cross-section area
- η EDF set efficiency

2.3.5.5 Longitudinal Aerodynamic Lift Forces

The vector body-axis generalised forces and moments/virtual controls (cross product of forces and moment arms) due to the aerodynamic lift forces are given by the following effector model equations:

$$Z_{LB} = F_{LW} \cos \alpha \quad (2.51)$$

$$X_{LB} = F_{LW} \sin \alpha \quad (2.52)$$

$$L_L = Z_{LB} \Delta y \quad (2.53)$$

$$M_L = (Z_{LB} \Delta x) + (X_{LB} \Delta z) \quad (2.54)$$

$$N_L = X_{LB} \Delta y \quad (2.55)$$

2.3.5.6 Longitudinal Aerodynamic Drag Forces

The vector of body-axis generalised forces and moments/virtual controls (cross product of forces and moment arms) due to the aerodynamic drag forces are given by the following effector model equations:

$$Z_{DB} = -F_{DW} \sin \alpha \quad (2.56)$$

$$X_{DB} = -F_{DW} \cos \alpha \quad (2.57)$$

$$L_D = -(Z_{DB} \Delta y) \quad (2.58)$$

$$M_D = -((Z_{DB} \Delta x) + (X_{DB} \Delta z)) \quad (2.59)$$

$$N_D = X_{DB} \Delta y \quad (2.60)$$

2.3.5.7 Longitudinal Aerodynamic Moments

The vector of body-axis generalised pitching moments/virtual controls (cross product of forces and moment arms) due to the aerodynamic lift forces is given by the following effector model equation: total body axis aerodynamic force is given by:

$$F_{AX} = X_{LB} + X_{DB} - W \sin \theta \quad (2.61)$$

total body axis aerodynamic force is given by:

$$F_{AZS} = Z_{LB} + Z_{DB} + W \cos \theta \quad (2.62)$$

total aerodynamic rolling moment due to static coefficients is given by:

$$L_{AS} = L_L + L_D \quad (2.63)$$

total aerodynamic pitching moment due to static coefficients is given by:

$$M_{AS} = M_L + M_D + M_{m/4} \quad (2.64)$$

total aerodynamic yawing moment due to static coefficients is given by:

$$N_{AS} = N_L + N_D \quad (2.65)$$

2.3.6 Combined Cruise Aerodynamic Model

The combined cruise aerodynamic forces and moments due to part 1 (section 2.3.5) and part 2 (section A.2.1) are given by:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} F_{AX} \\ F_{AYS} + F_{AYD} \\ F_{AZS} + F_{AZD} \end{bmatrix} \quad (2.66)$$

$$\begin{bmatrix} L_A \\ M_A \\ N_A \end{bmatrix} = \begin{bmatrix} L_{AS} + L_{AD} \\ M_{m/4} + M_{AD} \\ N_{AS} + N_{AD} \end{bmatrix} \quad (2.67)$$

where:

- F_{AX} is the total aerodynamic forces in the body x-axis
- F_{AYS} and F_{AYD} are the aerodynamic forces in the body y-axis due to static and dynamic coefficients respectively
- F_{AZS} and F_{AZD} are the aerodynamic forces in the body z-axis due to static and dynamic coefficients respectively
- L_{AS} and L_{AD} are the aerodynamic rolling moments about the body x-axis due to static and dynamic coefficients respectively
- $M_{m/4}$ and M_{AD} are the aerodynamic pitching moments about the body y-axis due to static and dynamic coefficients respectively
- N_{AS} and N_{AD} are the aerodynamic yawing moments about the body z-axis due to static and dynamic coefficients respectively

and F_{AYS} , F_{AYD} , F_{AZD} , L_{AS} , L_{AD} , M_{AD} , N_{AS} and N_{AD} are computed based on traditional aerodynamic forces and moments. The aerodynamic contributions defined by means of their dimensionless coefficients are as follows:

$$\begin{aligned}
 F_{AYS} &= \bar{q}S C_{yb,sta} \\
 F_{AYD} &= \bar{q}S C_{yb,dyn} \\
 F_{AZD} &= \bar{q}S C_{zb,dyn} \\
 L_{AS} &= \bar{q}Sb C_{lb,sta} \\
 L_{AD} &= \bar{q}Sb C_{lb,dyn} \\
 M_{AD} &= \bar{q}S\bar{c} C_{mb,dyn} \\
 N_{AS} &= \bar{q}Sb C_{nb,sta} \\
 N_{AD} &= \bar{q}Sb C_{nb,dyn}
 \end{aligned} \tag{2.68}$$

2.3.7 Aircraft Effector Configuration

As mentioned in Chapter 1 (referring to Figure 1.1), the 7-seater canard light sports Lilium Jet Lilium aircraft has 36 ducted fans (six on each of the two front canards and twelve on each rear main wings). The aircraft configuration showing the EDF sets, arrangement and their distribution is shown in Figure 2.4.

- *Each EDF set consist of three EDFs controlled to the same (ganged) rotational/thrust, turn direction and tilt angles.*
- *All EDF set on same side of canard or main wing has the same turn directions*
- *EDF sets of left and right of canard or main wing has opposite turn directions to ensure zero rolling moment in wing-level flight*

- *EDF sets of front (canard) and aft (main wing) has opposite turn directions to ensure zero yawing moment banking/rolling*

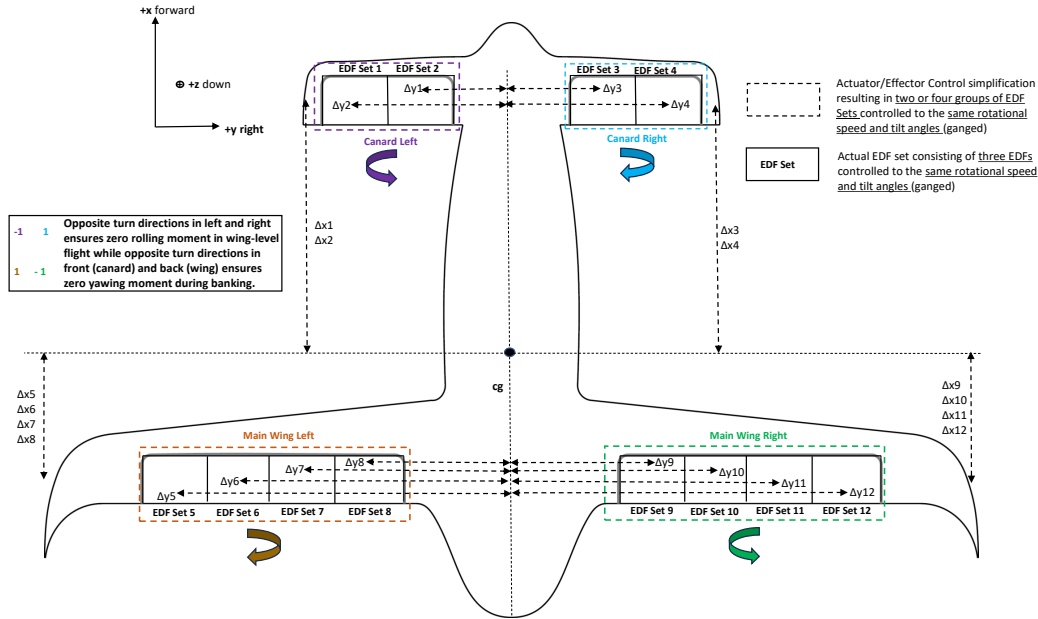


Fig. 2.4 Plan View of Lilium-style Fixed-Wing Over-actuated Vectored Thrust eVTOL with EDF Sets Distribution and Control Grouping

Based on the centre of gravity computed in section 2.2 and the aerodynamic coordinates in section 2.4, the moment arms of the EDF set (i.e. wings/aerofoil with fans) distribution shown in Figure 2.4 which are aerodynamic surface number 2, 3 for canard left, number 4, 5 for canard right, number 9, 10, 11, 12 for wing left and number 15, 16, 17, 18 for wing right is shown in Table 2.5. The moment arms for all the aerodynamic-only surfaces (strip 1, 6, 7, 8, 13, 14, 19, 20, 21) in Table 2.4 are computed by subtracting the CG for their coordinates as in Table 2.5 for the effectors.

NOTE: It is assumed that the effector strips (aerofoil with fans) no longer provide coupled aerodynamic and propulsive forces hence are now separate aerofoils and fans (EDFs).

2.3.8 EDF Set Control Simplification

The following control simplifications were made:

- *Each EDF set on same side of canard or main wing are ganged together and controlled to the same (ganged) rotational/thrust, turn direction and tilt angles.*
- *The control simplification means that there are no redundant independently controlled effectors*
- *The control simplification means that only total failure (not partial) of EDF sets can be accommodated. e.g if a total failure of EDF set 2 of left canard is detected, the control*

Table 2.5 Effector Model Moment Arms from CG assuming NED

A/C loc	Strip No.	EDF Set	Effector Moment Arms from CG (NED)		
			$\Delta x(m)$	$\Delta y(m)$	$\Delta z(m)$
canard left engine tip	2	Set 1	3.8334	-2.3660	0.0716
canard left engine root	3	Set 2	3.8564	-1.3680	0.0716
canard right engine root	4	Set 3	3.8564	1.3680	0.0716
canard right engine tip	5	Set 4	3.8334	2.3660	0.0716
wing left engine 1 tip	9	Set 5	-1.1396	-5.0410	-0.5951
wing left engine 2 tip	10	Set 6	-1.0956	-4.0490	-0.5951
wing left engine 1 root	11	Set 7	-1.0456	-3.0660	-0.5951
wing left engine 2 root	12	Set 8	-0.9926	-2.0670	-0.5951
wing right engine 1 root	15	Set 9	-0.9926	2.0670	-0.5951
wing right engine 2 root	16	Set 10	-1.0456	3.0660	-0.5951
wing right engine 1 tip	17	Set 11	-1.0956	4.0490	-0.5951
wing right engine 2 tip	18	Set 12	-1.1396	5.0410	-0.5951

allocation should be able to reconfigure the control effectiveness matrix (refer to chapter 5) to compensate loss in EDF Set 2 contribution by increasing the rpm/thrust of EDF Set 1

2.3.9 Simplified Effector Model

The body-axis effector model (without torque induced roll and yaw moments) is given by:

$$\mathbf{F}_x = \mathbf{T} \cos \Gamma \quad (2.69)$$

$$\mathbf{F}_z = \mathbf{T} \sin \Gamma \quad (2.70)$$

$$\mathbf{L}_T = \mathbf{T}_z \Delta \mathbf{x} \quad (2.71)$$

$$\mathbf{M}_T = (\mathbf{T}_z \Delta \mathbf{x}) + (\mathbf{T}_x \Delta \mathbf{z}) \quad (2.72)$$

$$\mathbf{N}_T = \mathbf{T}_x \Delta \mathbf{y} \quad (2.73)$$

Since the thrust T is proportional to the EDF speed of rotation ω_r , the real controls are given by:

$$\mathbf{u} = \begin{bmatrix} \omega_{r1} \\ \vdots \\ \omega_{rn} \\ \Gamma_1 \\ \vdots \\ \Gamma_n \end{bmatrix} \rightarrow \begin{bmatrix} T_1 \\ \vdots \\ T_n \\ \Gamma_1 \\ \vdots \\ \Gamma_n \end{bmatrix} \quad (2.74)$$

where:

$$\mathbf{T} = [T_1 \dots T_n] \quad (2.75)$$

$$\mathbf{T}_x = [T_{x1} \dots T_{xn}] \quad (2.76)$$

$$\mathbf{T}_z = [T_{z1} \dots T_{zn}] \quad (2.77)$$

$$\cos \mathbf{\Gamma} = \begin{bmatrix} \cos \Gamma_1 \\ \vdots \\ \cos \Gamma_n \end{bmatrix} \quad (2.78)$$

$$\sin \mathbf{\Gamma} = \begin{bmatrix} \sin \Gamma_1 \\ \vdots \\ \sin \Gamma_n \end{bmatrix} \quad (2.79)$$

$$\Delta \mathbf{x} = \begin{bmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{bmatrix} \quad (2.80)$$

$$\Delta \mathbf{y} = \begin{bmatrix} \Delta y_1 \\ \vdots \\ \Delta y_n \end{bmatrix} \quad (2.81)$$

$$\Delta \mathbf{z} = \begin{bmatrix} \Delta z_1 \\ \vdots \\ \Delta z_n \end{bmatrix} \quad (2.82)$$

n is the number of independently controlled EDF set

The virtual control vector $\boldsymbol{\tau}_c$ is given by:

$$\boldsymbol{\tau}_c = \begin{bmatrix} F_X \\ F_Z \\ L_T \\ M_T \\ N_T \end{bmatrix} \quad (2.83)$$

The effector model mapping within the vehicle model is given by:

$$\mathbf{u} \mapsto \boldsymbol{\tau} = \begin{bmatrix} \mathbf{F}_{prop} \\ \mathbf{M}_{prop} \end{bmatrix} \quad (2.84)$$

The inverse effector model mapping within the control allocation is given by:

$$\boldsymbol{\tau}_c \mapsto \mathbf{u} \quad (2.85)$$

2.4 Dynamic Characteristics Analysis

In order to control the dynamic system in section 2.2, we need to first understand its dynamic behaviour by performing a dynamic analysis. A linear analysis of the linearized model is performed to understand the system characteristics and predict its dynamic behaviour. The open loop dynamics and stability characteristics analysis of the open-loop air-vehicle is conducted through the linearization of the unaugmented model at cruise trim conditions. Due to the degree of over-actuation of the air vehicle, the trim solutions are not unique hence significant effort is required to obtain a single trim condition. As a consequence, only the longitudinal modes are excited in this work with only one cruise trim conditions with an investigation of the long-period (phugoid) and short-period longitudinal aircraft modes. To obtain the input trim for the longitudinal motion, left and right controls input are controlled to same inputs.

2.4.1 Trimming

The vehicle was trimmed for straight and level cruise flight condition with no wind using the MATLAB Simulink[®] Model Linearizer App and Steady State Manager App. The actual trim results are shown in Tables 2.6, 2.7 and 2.8. The trim point was verified via applying the control input trim values and simulated for 30s.

Table 2.6 Operating Point of Cruise Trim - States

States x	Actual Value x_{trim}	Actual derivative $\dot{x}_{trim}$
ϕ	0°	$0^\circ s^{-1}$
θ	2.837°	$0^\circ s^{-1}$
ψ	0°	$0^\circ s^{-1}$
p	$0^\circ/s$	$0^\circ s^{-2}$
q	$0^\circ/s$	$0^\circ s^{-2}$
r	$0^\circ/s$	$0^\circ s^{-2}$
u_b	77.7745 m/s	0 ms^{-2}
v_b	0 m/s	0 ms^{-2}
w_b	3.855 m/s	0 ms^{-2}
x_N	0 m	77.87 m/s
y_E	0 m	0 m/s
z_E	-3000 m	0 m/s

Table 2.7 Operating Point of Cruise Trim - Control inputs

Control Inputs u	Actual Value u_{trim}
Canard Thrust T_{canard}	89.8191 N
Main Wing Thrust T_{wing}	149.6018 N
Canard Tilt Angle Γ_{canard}	28°
Main Wing Tilt Angle Γ_{wing}	0.0013229°

Table 2.8 Operating Point of Cruise Trim - Outputs

Ouputs y	Actual Value y_{trim}
AoA α	2.837°
Pitch rate q	$0^\circ/s$
True Airspeed V	77.87 m/s
Flight Path Angle λ	0°
Side Slip β	0°

2.4.2 Linearization

Using the operating point obtained in previous section 2.4.1, the model is linearized using the MATLAB[®] Simulink[®] ‘Linear Analysis’ Tool. Ignoring the navigation state variables (ψ , x_e and y_e) the aircraft modes of the remaining state variables obtained using modal analysis are as follows:

Table 2.9 Cruise trim linearized model aircraft modes

Mode name	Pole	Damp	Freq rad/s	Time const s	Stability	Osc
Roll Mode	-1.2316	1	1.232	0.812	Stable	Non-osc
Spiral	0.124	-1	0.124	-8.06	Unstable	Non-osc
Dutch Roll	$-0.0891 \pm 0.127i$	0.575	0.155	11.2	Stable	Osc
Short-Period	$-1.29 \pm 0.522i$	0.927	1.39	0.776	Stable	Osc
Phugoid	$0.0169 \pm 0.184i$	0.0914	0.185	-59.1	Unstable	Osc
Heave Mode	-2.33×10^{-14}	1	2.33×10^{-14}	4.29×10^{13}	Stable	Non-osc

The remaining nine state variables $\mathbf{x}_{lin}$ is given by:

$$\mathbf{x}_{lin} = [\phi \quad \theta \quad p \quad q \quad r \quad u \quad v \quad w \quad z]^T \quad (2.86)$$

$$\mathbf{A}_{trim} = \begin{bmatrix} -1.232 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.288 & 0.5218 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.5218 & -1.288 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.08907 & 0.1269 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1269 & -0.08907 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.124 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0003017 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01693 & 0.1843 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1843 & 0.01693 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.87)$$

$$\mathbf{B}_{trm} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.0000514 & -0.0000514 & 0.0029 & 0.0029 \\ 0.000243 & 0.000243 & -0.0130 & -0.0130 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0002991 & 0.0002991 & 0.0011 & 0.0011 \\ -0.0002934 & -0.0002934 & -0.00085 & -0.00085 \\ -0.0001494 & -0.0001494 & -0.00027 & -0.00027 \\ 0 & 0 & 0 & 0 \\ 1.378 & 1.378 & -1.293 & -1.293 \\ 8.289 & 8.289 & -1.947 & -1.947 \\ 0.03655 & 0.03655 & -0.02025 & -0.02025 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.3195 & -0.3195 & 0.1235 & 0.1235 \\ -0.8423 & -0.8423 & 0.3211 & 0.3211 \\ -0.01695 & -0.01695 & 0.005529 & 0.005529 \end{bmatrix} \quad (2.88)$$

$$\mathbf{C}_{trm} = \begin{bmatrix} -0.00008 & 0.00871 & -0.001912 & 0 & 0 & 0 \\ -0.000099 & 0.01141 & 0.002004 & 1 & 0 & 0 \\ 0.001145 & -0.08292 & -0.05656 & -0.0007794 & -0.0005556 & 0.0007063 \\ 0.0001605 & -0.01577 & -0.002504 & -0.000014 & 0 & 0 \\ -0.0122 & 0 & 0 & 0.001438 & 0.00035 & -0.001538 \\ -0.0006038 & -0.000779 & 0 & & & \\ 0.006876 & 0.007623 & 0 & & & \\ 2.395 & -1.589 & -0.5675 & & & \\ -0.03701 & -0.03998 & 0 & & & \\ 0 & 0 & 0 & & & \end{bmatrix} \quad (2.89)$$

A linear dynamic inversion (LDI) based controller was designed for the linearized aircraft model around the trim cruise conditions, details of which are covered in Appendix A since it is not the focus of this work.

2.4.3 Open loop Linear Analysis

As can be seen from Table 2.9 of Section 2.4.2, the spiral and phugoid modes have their poles on the positive real axis (unstable), resulting in the aircraft being open loop unstable and hence a closed loop controller for flight control is required.

Furthermore, inspecting of state space matrices in equations (2.87), (2.88), (2.89), (2.96), the controllability and observability matrices (not shown here) reveal respectively that the linearized aircraft is controllable and observable.

Since the linearized model is valid only for one cruise trim conditions, to obtain a linear controller which covers the entire aircraft envelop, several trim points and linearized models need to be generated and gain scheduling performed to cover the entire envelop resulting significant amount of effort and time hence out of scope of this work. More detailed open loop analysis of the linearised model using frequency response (e.g. Nyquist criterion) and time domain analysis (e.g. root locus) can be used to analyse the open loop response of each control channel or degree of freedom with respect to each control inputs at trim conditions to gain deeper insight into the system behaviour*.

Due to the open-loop unstable characteristics of the aircraft (confirmed at cruise conditions) coupled with the non-linearities in the aerodynamics, coupling with the controls as well as non-linear wide operating envelop of the eVTOL, a non-linear controller is proposed which is covered in the next section.

As will be discussed later in sections 3.3, 3.5.1.1 and 3.5.1.3 of chapter 3, dynamic inversion (LDI or NDI) can only be applied to minimum phase system because inversion of non-minimum phase system will result in instability due to their unstable transition (internal) zeros (Hauser et al., 1992). We will now check if there are any unstable transition zeros in our open loop eVTOL linear cruise model (Model 2).

2.4.4 Linearized Cruise Model - Vehicle Model 3

As will be seen later in equation (3.53) of Chapter 3, using output $\mathbf{y}$ as:

$$\mathbf{y}_{DI} = \begin{bmatrix} p \\ q \\ r \\ u \\ w \end{bmatrix} \quad (2.90)$$

Rearranging the state variables as

$$\mathbf{x} = \begin{bmatrix} x \\ z \\ u \\ w \\ \theta \\ q \\ y \\ v \\ \phi \\ \psi \\ p \\ r \end{bmatrix} \quad (2.91)$$

*This activity would form a good undergraduate or master's degree project such as the analysis covered in Moses (2021)

have been cancelled by the internal poles due to the dynamic inversion. To check the transmission zeros, the states variables are stripped out.

2.4.6 Closed Loop Analysis of Decoupled Linearised Model

A set of linear controllers (e.g. PID) can be designed for each control channels or degree of freedom using the characteristics of the open loop linearized model (Model 2) but this not the focus of this work. [†].

A non-minimum phase test of the zero dynamics of the closed loop linearized cruise system model using dynamic inversion controller will be conducted in section 3.5.1.3 of Chapter 3.

2.4.7 Summary

In summary, this chapter assembles an eVTOL vehicle model based on the Lilium Jet, a 7-seater canard light sports over-actuated aircraft with 36 all-electric engines (six on each of the two front canards and twelve on each rear main wings) resulting in eight control effectors to control six (roll, pitch, yaw, surge, sway and heave) DoF which presents a CA problem. The plan view of the vehicle is shown in Figure 2.1.

As discussed in section 2.2.1, the aircraft has a fixed-wing aerodynamic surface and twelve tilting EDFs distributed through the front canard and rear main wing sections of the aircraft (see Figure 2.1). A main challenge is estimating the aerodynamic and propulsion forces and moments acting on the aircraft, especially during the transition region. High fidelity modeling of the transition dynamics is quite complex since tilting EDFs also generate aerodynamic forces/moments, and flow separation over the EDF surfaces might occur during the transition. Hence the flight dynamics model is constructed considering the main effects at hover and high speed forward flight. During the transition, the dynamics are obtained by blending the hover and forward flight aerodynamic models based on the airspeed though in practice this introduces significant nonlinearity in its aerodynamic forces and moments but the complex interaction between the incoming freestream and the propeller slipstream are hard to accurately model hence not modelled resulting in some unmodelled dynamics and a low fidelity aerodynamic model.

This chapter assembles a more suitable aircraft models (Vehicle Model 1 and Vehicle Model 2) from existing work for this novel vectored thrust Tilt rotor eVTOL aircraft which is more complete than the individual aerodynamic models. The model has three parts: the first part is a hover model, the second part covers the static longitudinal aerodynamics while the third part covers the dynamic longitudinal derivatives and lateral-directional derivatives all based on existing work. The second and third parts are integrated to form the cruise model. A blending coefficient which is a continuous differentiable hyperbolic tangent function was derived and used to perform transition between hover and forward flight by blending hover and cruise model. In order to understand the open loop characteristics and aircraft models of the full non-linear model (Vehicle Model 1), a linearization was performed at cruise trim

[†]This activity would form a good undergraduate or master's degree project such as covered in Moses (2021)

conditions to give a linear cruise model (known as Vehicle Model 2). Open loop analysis revealed that the model is open loop unstable since the spiral and phugoid modes have their poles on the positive real axis (unstable), and the analysis (the controllability and observability matrices) of the linearized aircraft model (at cruise trim condition) reveal respectively that the linearized aircraft is controllable and observable, hence a closed loop flight controller is required. Analysis of the open loop cruise linearized model (Vehicle Model 2) of the aircraft show that it has no transmission zeros hence a minimum phase system making it compatible with an NDI-based controller. To enable the aircraft to be controlled by classical NDI (see in Chapter 3) and resolved by the proposed control allocation (see in Chapter 4), a customisation of the 3-D lookup combined aerodynamic and propulsive longitudinal model from existing work was performed so as to decouple the aerodynamic and propulsive forces resulting in an affine form. A simple thrust (propulsive) model and effector mapping was then derived for use later in the control allocation (see Chapter 4).

Chapter 3

Architecture and Controller Design

3.1 Chapter Contribution

A survey of the control techniques being used by aviation industry to design flight control laws for fixed-wing aircraft are presented in Balas (2003). As mentioned in Chapter 1.8, one of the main contributions of this work is a simple and unified control framework with a functional architecture that is generic for piloted, semi-automatic and automated flight. Another contribution of this chapter is the formulation of an NDI-based high-level/aircraft-level controller (Enenakpogbe et al., 2022) for the Lilium-style vectored thrust eVTOL configuration (Lilium) in this thesis.

The high-level motion controller addresses two (significant nonlinear characteristics in the flight dynamics and unstable open loop phugoid dynamics due to the canard configuration) out of the three eVTOL operation challenges listed in section 1.6 of Chapter 1.8. The controller does not address the last challenge of coupling between the aeromechanical and dynamic forces and moments interaction because of the nature of classical NDI hence the aerodynamic model in the chapter 2 is simplified to ensure a decoupled aeromechanical and dynamic forces and moments.

3.2 NonLinear Control Techniques Review

Some techniques for compensating nonlinear effects and nonlinear control design are:

- *Gain Scheduling* - Design several controllers at various operating regions or trim points and then schedule or switch the controllers and parameter in the different operating regions using a scheduling variable (e.g. air speed in the case of flight control), Auto-tuning controllers etc. (Postlethwaite et al., 2002, 2005; Prempain and Postlethwaite, 2008; Prempain et al., 2001, 2010; Tran et al., 2018)
- *Nonlinear Function Approximations* - using fuzzy logic and neural networks (Bosworth, 2008; Bosworth and Williams-Hayes, 2007; Hanson et al., 2011; Huang et al., 1992; Johnson and Calise, 2000; Kaneshige et al., 2000)

- *Adaptive Control* - Self Tuning Controllers (STC), Model Reference Adaptive Control (MRAC), and Nonlinear Adaptive Control (Başak and Prempan, 2015, 2017; Johnson and Calise, 2000; Mahdianfar and Prempan, 2016; Schaefer et al., 2011)
- *Bang-bang Control* - Switching between operating limits also known as time-optimal control (Casas, 2012; Enenakpogbe, 2011; Felgenhauer, 2003; Ledzewicz and Schättler, 2002; Maurer and Osmolovskii, 2004; Seyde et al., 2021; Zandvliet et al., 2007)
- *Variable Structure Control* - Sliding mode control (Casas, 2012; Koshkouei et al., 2005; Shtessel et al., 2014; Utkin and Vadim, 2004; Yu et al., 2020)
- *Backstepping Design* - design control input in an iterative way by moving down the different virtual control until the control input is designed (Fang et al., 2018; Farrell et al., 2009; Harkegard, 2003a; Madani and Benallegue, 2006; Vaidyanathan and Azar, 2021; Zhou et al., 2008)
- *Forward Control* - an iterative design working in way opposite to backstepping (Astrom et al., 2001; Benachour et al., 2013; Huang et al., 2016; Krstic, 2004; Mazenc et al., 2003; Ouendi et al., 2019).
- *Semiglobal Control Design* - achieving stability in arbitrarily large regions, often by using high gain control (Boker and Khalil, 2016; Fan et al., 2014; Li et al., 2024; Lin, 2019; Yang and Lin, 2014; Zhai and Du, 2013; Zhang et al., 2015, 2023).
- *Feedback Linearisation* - By using nonlinear control laws like classical nonlinear dynamic inversion (NDI), incremental nonlinear dynamic inversion (INDI), hybrid incremental nonlinear dynamic inversion (Hybrid INDI) and extended nonlinear dynamic inversion (Extended NDI). Since the focus of this research is NDI-based controller, feedback linearization techniques are reviewed within the context of NDI in section 3.3.

3.3 NDI Literature Review

Classical nonlinear dynamic inversion (NDI) is a well-established nonlinear control approach which has been widely applied in various forms (Alam and Celikovsky, 2017; Bacon and Ostroff, 2000; Bugajski et al., 1990; Enns et al., 1994; Georgie and Valasek, 2003; Gregory, 1999; Hameduddin and Bajodah, 2012; Holzapfel and Sachs, 2004; Ito et al., 2002; Lane and Stengel, 1988; Lombaerts and Looye, 2011; Miller, 2011a,b; Smith and Berry, 2000; Smith, 1994, 1998; Snell et al., 1992; Spaulding et al., 2005; Valasek and Georgie, 2001). Although NDI can deal with the nonlinear characteristics in the motion of an aircraft, a drawback is its poor performance in the presence of external disturbances and model uncertainties because it relies on an accurate model of the controlled system. Hence NDI has been combined with robust controllers to deal with these disturbances and uncertainties (Adams and Banda, 1993a,b; Adams et al., 1994; Yang and Morales, 2021). Another drawback of classical NDI is that it is applicable to system with minimum phase characteristics because inversion of an unstable

transition zeros (non-minimum phase NMP system) will result in instability (Stevens et al., 2015).

A flight control strategy called incremental nonlinear dynamic inversion which uses properties of general mechanical systems and nonlinear dynamic inversion by feeding back angular accelerations was presented by Sieberling et al. (2010), Morton et al. (1996), Pollack and Van Kampen (2023), Reiner et al. (1996), Di Francesco and Mattei (2016), Grondman et al. (2018), Pfeifle and Fichter (2021), Chen et al. (2020), Yang et al. (2020), Salahuddin et al. (2024), Steffensen et al. (2023), Smeur et al. (2018), van Ekeren et al. (2018), Wang et al. (2019a), Zhang et al. (2018), Zhao et al. and Schneider et al. (2024). It uses the relationship between thrust increment and angular acceleration increment (obtained through Taylor series expansion) and angular acceleration feedback. Theoretically, feedback of angular accelerations eliminates sensitivity to model mismatch, greatly increasing the robust performance of the system compared with conventional nonlinear dynamic inversion. However, angular accelerations are not readily available because they usually cannot be measured directly using a sensor. Angular rates are obtained as derivatives of the angular velocity state variable which suffer from feedthrough of sensor noise (are susceptible to high-frequency noise resulting in reduced robustness against high-frequency dynamic uncertainty due to its high-gain nature) thereby introducing complexity in the system. This complexity arises from the need to introduce a tracking differentiator, an observer or a low-pass filter which in turn introduces a new error (a delay due to low-pass filtering) in the systems. This delay affects the angular rate feedback causing time synchronization problems in the control system caused by the relative timing or phase between the sensor-based output derivative estimate and the input feedback signal. Inadequate consideration of this effect may lead to severe stability issues, which implies an inherent lack of a-priori robustness guarantees.

A hybrid approach which aims to compromise between the sensor-based and the model-based variants in an effort to improve overall robustness is called hybrid INDI (Kim et al., 2021; Kumtepe et al., 2022). In order to further mitigate NDI robustness issue and INDI stability issue, the hybrid INDI approach fuses the system model (OBM) and sensor measurement of the angular acceleration using a complementary filter to generate an estimate of the angular acceleration of the system such that both complement each other in the frequency domain. The estimation responds rapidly to the system input due to the on-board model (NDI), whereas adequate accuracy in the low-to-medium frequency range is maintained by the sensor measurement (INDI). The resulting control law is found to retain good performance in case of model mismatches and measurement delays. It was shown that hybrid INDI effectively introduces a scaled and filtered inversion error compensation signal that corrects the the on-board model (OBM) prediction of the NDI. Moreover, the concept of hybrid INDI enables a visible decoupling of flying quality and robustness design goals from the bare airframe dynamics in an IMF (implicit model-following) context. However, no stability guarantees can be provided.

Another nonlinear control approach which is the focus of future research is an extended INDI control concept which commands the time derivatives of the pseudocontrols (virtual controls) instead of the increments allowing for the cancellation of first order actuator dynamics (Raab et al., 2019; Smeur et al., 2016). It enhances the inherent advantages of INDI by consideration of control effector dynamics and saturations. First and second order control effectors including absolute and rate saturations can be taken into account enabling a physically

transparent way of designing pseudo control loop and outer loops. It also reduces cross couplings in different channels in the presence of control effectors with dissimilar bandwidths and system orders. In case of redundant control effectors or “over-actuation”, the control effectors are utilized depending on their bandwidth. However, since the approach is INDI based, no stability guarantees can be provided.

3.4 Control Architecture

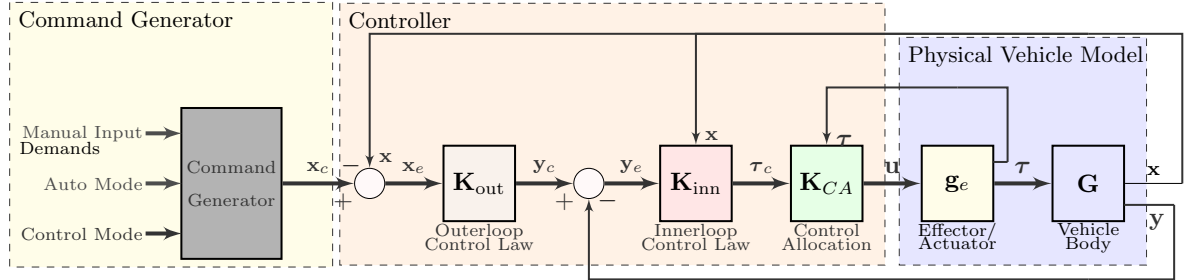


Fig. 3.1 Generic Flight Controller Architecture for piloted, semi-automatic and automated flight

A unified control architecture which can be applied to piloted, semi-automatic and automated flight consisting of a command generator, a controller and the physical vehicle (represented by a simulation Vehicle Model 1) it controls. The architecture is shown in Figure 3.1. Typical command generator performs reference input computation (e.g. as presented in Bhardwaj et al. (2018)), scaling and limitation (e.g. flight envelope protection e.g. as presented in Lombaerts et al. (2010)), flight mode selection (automode or manual mode), control mode switching and pilot command filtering.

The controller comprises a high-level (aircraft-level) controller and the control allocation. The higher-level controller consists of an outer-loop controller and an inner loop velocity controller. A control mode switch is used to switch between velocity and position control. In velocity control mode the references are airspeed V_T , flight path angle λ , roll rate p , pitch rate q and yaw rate r which are controlled directly by the inner-loop velocity controller. For position control mode, the references are lateral (north) position x , altitude h , east position y , roll attitude ϕ , pitch attitude θ and yaw ψ .

The control allocation scheme $\mathbf{K}_{CA}$ ensures that the forces and moments vector, $\boldsymbol{\tau}$, that characterizes the physical model given by equation (3.50) is equal to the virtual control demands output, $\boldsymbol{\tau}_c$, produced by the high-level control law (outer linear controller and inner loop NDI controller) $\mathbf{K}_{CTL}$. The physical vehicle model consists of an effector/actuator model $\mathbf{g}_e$, thus normally would include low-level actuator controller (e.g. servo controllers), and the virtual controls (generalized forces and moments) $\mathbf{G}$. The inverse effector model $\mathbf{K}_{CA}$ (i.e. control allocation) using the effector/actuator model $\mathbf{g}_e$ will be covered in Chapter 4.

In this thesis, two dynamic inversion-based velocity controllers are designed. First, in section 3.6, the NDI-based controller will be tested conceptually without an effector or control allocation using Vehicle Model 4 while in Chapter 5, the same NDI-based velocity controller is used to fully verify the eVTOL model (Vehicle Model 1) and architecture. In Appendix A

a linear dynamic inversion (LDI) controller based on the linearized (trimmed at cruise flight condition in section 2.4.2) Vehicle Model 2 is designed as the inner loop velocity controller. Testing of the validity of the controller around the cruise trim conditions is trivial hence not covered in this thesis. The physical vehicle model consists of an effector/actuator model, thus normally would include low-level actuator controller (e.g. servo controllers), and the physical model with virtual controls. For the remainder of the chapter, it is assumed that a perfect control allocation scheme is in operation, so that the virtual control demands output, τ_c , that is produced by the inner-loop controller, $\mathbf{K}_{inn}$ is equal to the forces and moments vector, τ , that characterizes the physical model give by (3.50).

Based on NASA Pilot-Automation-Interaction (PAI) framework (Feary et al., 2023; Kaneshige et al., 2023) which characterises an aircraft real-time functional state automation dimensions, the proposed control architecture in this thesis is PAI-3 (Manoeuvre Command) - Autopilot with specified linkable manoeuvres.

3.4.1 Command Generator and Outerloop

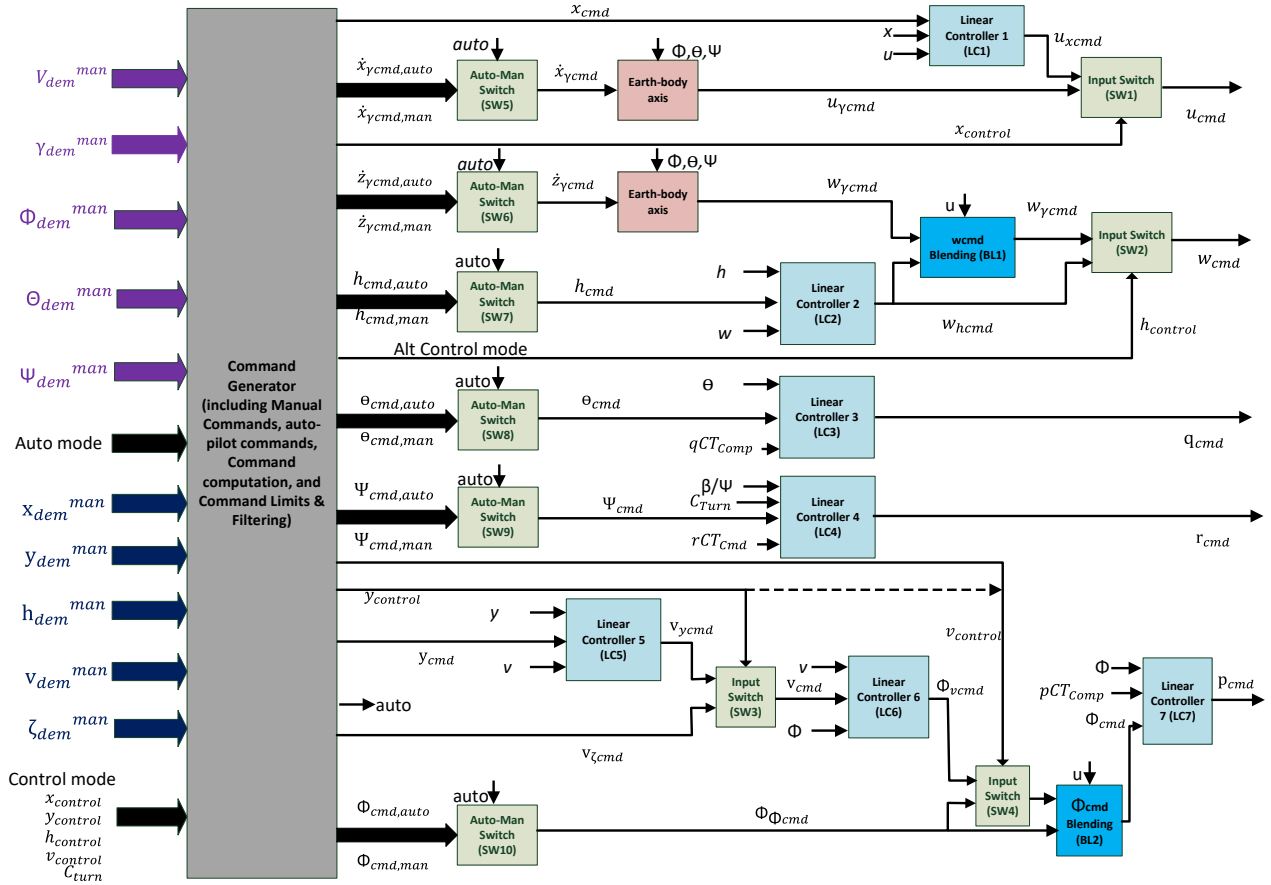


Fig. 3.2 Command Generator and Outer loop Controller

The Command Generator and Outerloop controllers are shown in Figure 3.2. The Command Generator consists of:

- Manual and auto-pilot commands
- Command limits & filtering
- Command computation
- Manual-auto mode switching
- Position control switching
- Outerloop linear controller
- Control mode switching
- Command blending between hover and forward flight using equation 2.30 in section 2.3.2 and a blending equation (same structure as equation 2.31 in in section 2.3.2) given by:

$$\mathbf{x}_{cmd} = K_b \mathbf{x}_{cmd,cru} + (1 - K_b) \mathbf{x}_{cmd,hov} \quad (3.1)$$

The outerloop controller is airframe independent since it does not consider the airframe aerodynamics

3.4.2 Outer Loop Linear Controller

Based on the terminology in Durham (2013), to control the uncontrollable states variables, the controllable states variables can be used as the controls resulting in a cascaded loop consisting of the inner controllable state variables (u, w, p, q, r) velocity loop and an outer loop $(x, y, z, v, \phi, \theta, \psi)$. Since eVTOLs do not directly generate thrust in the y -direction, there is no NDI inner loop side velocity control. Referring to Figure 3.2, the roll control is used to control sway resulting in a cascaded inner (side motion velocity v) and outer loop (side displacement y) used as command for the bank angle ϕ outer loop. A simple diagonal proportional-integral is used for the outerloop position control and proportional linear controller is used for the outer loop attitude control. There are cascaded loops in the outer loop eg. from $(x, h/z)$ to (u, w) control (inner/slave loop) using $(x_{control}, h_{control})$ (master/outer loop), from y to v control using $y_{control}$, from v to ϕ using $v_{control}$. The innermost outer loop input commands $\mathbf{x}_c$ as shown in Figures 3.2 and 3.1 are given by;

$$\mathbf{x}_c = \begin{bmatrix} u_{cmd} \\ w_{cmd} \\ \phi_{cmd} \\ \theta_{cmd} \\ \psi_{cmd} \end{bmatrix} \quad (3.2)$$

The outer-loop linear controller consists of the following:

- Translational position (x) controller - PI Linear Controller 1 (LC1)

- Sway position (y) Linear Controller 5 ($LC5$) via side velocity (v) P Linear Controller 6 ($LC6$) and ϕ P Linear Controller 7 ($LC7$)
- Altitude ($h = -z$) controller - PI Linear Controller 2 ($LC2$)
- Course angle (ζ) controller via side velocity (v) P Linear Controller 6 ($LC6$) and ϕ P Linear Controller 7 ($LC7$)
- Bank angle (ϕ) P Linear Controller 7 ($LC7$)
- Pitch attitude (θ) P Linear Controller 3 ($LC3$)
- Side-slip angle (β) and heading (ψ) P Linear Controller 4 ($LC4$)
- Side velocity (v) controller ($LC6$) via ϕ P Linear Controller 7 ($LC7$)

Position control mode ($x_{control}$, $y_{control}$), altitude control mode ($h_{control}$) and sway velocity control mode ($v_{control}$) are used only for low ground speed in the presence of external disturbances and hover. Actuator saturation can result in integral windup due to the integrators (PI controller) in the outer loops (e.g. altitude loop). If the PI controller control causes the actuator limiter to saturate, then the altitude loop gain becomes zero, and there is no control, thus acting open loop. The PI controller will now be semi-stable and will “wind-up”, that is integrate the error with no correction, which can cause a major problem and would require anti-windup protection scheme. In order to reduce the likelihood of integral windup and the need for anti-windup protection scheme (e.g. (Prempain et al., 2009; Turner et al., 2020)) due to the integrators in the outer loops (e.g. altitude loop) and actuator magnitude limits, magnitude and rate limits (see Table 5.3) are imposed on the input commands within the pilot generator. The outer-loop controllers ($LC1$ - $LC7$) parameters are provided in Table 5.4 and their equations are given as:

$$LC1 : u_{xcmd} = K_{xp}(x_{cmd} - x) + K_{xi} \left[\int (x_{cmd} - x) + x_0 \right] + K_{xd}(\dot{x}_{cmd} - \dot{x}) \quad (3.3)$$

$$LC2 : w_{hcmd} = K_{hp}(h_{cmd} - h) + K_{hi} \left[\int (h_{cmd} - h) + h_0 \right] + K_{hd}(\dot{h}_{cmd} - \dot{h}) \quad (3.4)$$

$$LC3 : q_{\theta cmd} = K_{\theta p}(\theta_{cmd} - \theta) + K_{\theta i} \left[\int (\theta_{cmd} - \theta) + \theta_0 \right] + K_{\theta d}(\dot{\theta}_{cmd} - \dot{\theta}) \quad (3.5)$$

$$LC4 : r_{\psi cmd} = K_{\psi p}(\psi_{cmd} - \psi) + K_{\psi i} \left[\int (\psi_{cmd} - \psi) + \psi_0 \right] + K_{\psi d}(\dot{\psi}_{cmd} - \dot{\psi}) \quad (3.6)$$

$$LC5 : v_{ycmd} = K_{yp}(y_{cmd} - y) + K_{yi} \left[\int (y_{cmd} - y) + y_0 \right] + K_{yd}(\dot{y}_{cmd} - \dot{y}) \quad (3.7)$$

$$LC6 : \phi_{vcmd} = K_{vp}(v_{cmd} - v) + K_{vi} \left[\int (v_{cmd} - v) + v_0 \right] + K_{vd}(\dot{v}_{cmd} - \dot{v}) \quad (3.8)$$

$$LC7 : p_{\phi cmd} = K_{\phi p}(\phi_{cmd} - \phi) + K_{\phi i} \left[\int (\phi_{cmd} - \phi) + \phi_0 \right] + K_{\phi d}(\dot{\phi}_{cmd} - \dot{\phi}) \quad (3.9)$$

$$p_{cmd} = p_{\phi cmd} + pCT_{Comp} \quad (3.10)$$

$$q_{cmd} = q_{\theta cmd} + qCT_{Comp} \quad (3.11)$$

$$r_{cmd} = r_{\psi cmd} \parallel rCT_{Cmd}, \text{ using } CT_{Turn} \quad (3.12)$$

where $\parallel$ means a selection of only one out of the two demands based on the value of the switching control input e.g. rCT_{Cmd} in equation (3.12), and for a coordinated turn of bank angle demand (ϕ_{Cturn}), the turn radius (R_{Cturn}), yaw ψ (turn) rate ($\dot{\psi}_{Cturn}$), roll rate turn compensation (pCT_{Comp}), pitch rate turn compensation (qCT_{Comp}) and body-axis yaw rate (turn) command (rCT_{Cmd}) are given by:

$$R_{Cturn} = \frac{V_{\infty}^2}{g \tan \phi_{Cturn}} \quad (3.13)$$

$$\dot{\psi}_{CturnCmd} = \frac{g \tan \phi_{Cturn}}{V_{\infty}} \quad (3.14)$$

$$pCT_{Comp} = \frac{g}{V_{\infty}} \tan \phi_{Cturn} \sin \theta_{Cturn} \quad (3.15)$$

$$qCT_{Comp} = \frac{g}{V_{\infty}} \sin \phi_{Cturn} \tan \phi_{Cturn} \cos \theta_{Cturn} \quad (3.16)$$

$$rCT_{Cmd} = \frac{g}{V_{\infty}} \sin \phi_{Cturn} \cos \theta_{Cturn} \quad (3.17)$$

The various switches (SW1-SW4) for switching between various modes and automanual switches (SW5-SW10) for switching between manual and automatic (autopilot) commands are as follows:

$$SW1 : u_{cmd} = u_{xcmd} \parallel u_{\gamma cmd}, \text{ using } x_{control} \quad (3.18)$$

$$SW2 : w_{cmd} = w_{hcmd} \parallel w_{\gamma cmd}, \text{ using } h_{control} \quad (3.19)$$

$$SW3 : v_{cmd} = v_{ycmd} \parallel v_{\zeta cmd}, \text{ using } y_{control} \quad (3.20)$$

$$SW4 : \phi_{cmd} = \phi_{\phi cmd} \parallel \phi_{v cmd}, \text{ using } v_{control} \quad (3.21)$$

$$SW5 : u_{\gamma cmd} = u_{\gamma man} \parallel u_{\gamma auto}, \text{ using } Auto \quad (3.22)$$

$$SW6 : w_{\gamma cmd} = w_{\gamma man} \parallel w_{\gamma auto}, \text{ using } Auto \quad (3.23)$$

$$SW7 : h_{cmd} = h_{cmd,man} \parallel h_{cmd,auto}, \text{ using } Auto \quad (3.24)$$

$$SW8 : \theta_{cmd} = \theta_{cmd,man} \parallel \theta_{cmd,auto}, \text{ using } Auto \quad (3.25)$$

$$SW9 : \psi_{cmd} = \psi_{cmd,man} \parallel \psi_{cmd,auto}, \text{ using } Auto \quad (3.26)$$

$$SW10 : \phi_{cmd} = \phi_{cmd,man} \parallel \phi_{cmd,auto}, \text{ using } Auto \quad (3.27)$$

where $\parallel$ means a selection of only one out of the two demands based on the value of the switching control input e.g. Switch 1 (SW1) in equation (3.18) if $x_{control} = 1$, $u_{cmd} = u_{xcmd}$ else ($x_{control} \neq 1$) $u_{cmd} = u_{\gamma cmd}$, similarly for the remaining switches defined by equations (3.19) - (3.27).

From Figure 3.2:

- V_{cmd} is the commanded airspeed
- γ_{cmd} is the commanded flight path angle
- ζ_{cmd} is the commanded track or course angle

$$\begin{bmatrix} \dot{x}_{\gamma cmd} \\ \dot{y}_{\zeta cmd} \\ \dot{z}_{\gamma cmd} \end{bmatrix} = V_{cmd} \begin{bmatrix} c_{\gamma} \\ c_{\gamma} t_{\zeta} \\ -s_{\gamma} \end{bmatrix} \quad (3.28)$$

$$\begin{bmatrix} u_{\gamma cmd} \\ v_{\zeta cmd} \\ w_{\gamma cmd} \end{bmatrix} = \begin{bmatrix} c_{\theta} c_{\psi} & c_{\theta} s_{\psi} & -s_{\theta} \\ s_{\phi} s_{\theta} c_{\psi} - c_{\phi} s_{\psi} & s_{\phi} s_{\theta} s_{\psi} + c_{\phi} c_{\psi} & s_{\phi} c_{\theta} \\ c_{\phi} s_{\theta} c_{\psi} + s_{\phi} s_{\psi} & c_{\phi} s_{\theta} s_{\psi} - s_{\phi} c_{\psi} & c_{\phi} c_{\theta} \end{bmatrix} \begin{bmatrix} \dot{x}_{\gamma cmd} \\ \dot{y}_{\zeta cmd} \\ \dot{z}_{\gamma cmd} \end{bmatrix} \quad (3.29)$$

For wing-level non-turning flight,

$$\alpha_{cmd} = \theta_{cmd} - \gamma_{cmd} \quad (3.30)$$

For level, turning, skidding (sideslipping) flight,

$$\beta_{cmd} = \psi_{cmd} - \zeta_{cmd} \quad (3.31)$$

$$(3.32)$$

For level, turning, non-skidding (non-sideslipping) flight,

$$\psi_{cmd} = \zeta_{cmd} \quad (3.33)$$

3.4.3 Control Limits and Flight Modes

The various flight regime modes are:

- *Vertical TakeOff and Landing (VTOL)*: $h_{control}$ mode, h control, $V = 0$, $\theta = 0$, $\phi = 0$, $\psi = 0$
- *Climb, Descend, Approach and Flare (CDAF)*: V control, λ control, θ control, $\phi = 0$, $\psi = 0$
- *Cruise (CRU)*: V control, λ control, θ control, $\phi = 0$, $\psi = 0$
- *Hover (HOV) & Position control*: $x_{control}$ mode, $y_{control}$ mode, $z_{control}$ mode, x control, y control, z control, $\theta = 0$, $\psi = 0$
- *Station Keeping (SK)*: $x_{control}$ mode, $y_{control}$ mode, $z_{control}$ mode, x hold, y hold and z hold, $\theta = 0$, $\psi = 0$
- *Forward Transition (FT)*: move from *Hover (HOV) & Position control* to *Cruise (CRU)*
- *Reverse Transition (BT)*: move from *Cruise (CRU)* to *Hover (HOV) & Position control*
- *Banking (ROL)*: V control, λ control, ϕ control, θ control, ψ control, $h_{control}$ mode
- *Coordinated Turn (CTURN)*: V control, λ control, ϕ control, θ control, ψ control

3.5 Non Linear Dynamic Inversion

3.5.1 NDI Controller Problem formulation

One main contribution of this research is an application of NDI (Enenakpogbe et al., 2022) as the 6DOF high-level controller (aircraft-level controller) for the Lilium-style multi-rotor vectored thrust tilt-rotor eVTOL (Lilium, 2024) configuration rather than INDI of Suiçmez (2021). Based on the exposition in Stevens et al. (2015), Enenakpogbe et al. (2022), Lane and Stengel (1988), Smith and Berry (2000), Smith (1998), Ito et al. (2002), Georgie and Valasek (2003), Valasek and Georgie (2001), Miller (2011b), Miller (2011a), Holzapfel and Sachs (2004), Gregory (1999), Bugajski et al. (1990); Smith (1994), Enns et al. (1994), Hameduddin and Bajodah (2012), Snell et al. (1992), Spaulding et al. (2005), Bacon and Ostroff (2000), Lombaerts and Looye (2011) and Alam and Celikovsky (2017), in this section the NDI approach is outlined. Consider a plant model with general state equations given by:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\boldsymbol{\tau} \quad (3.34)$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}) \quad (3.35)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the vector of state variables, $\boldsymbol{\tau} \in \mathbb{R}^m$ is the (virtual) control and $\mathbf{y}(t) \in \mathbb{R}^m$ is the vector of outputs to be controlled.

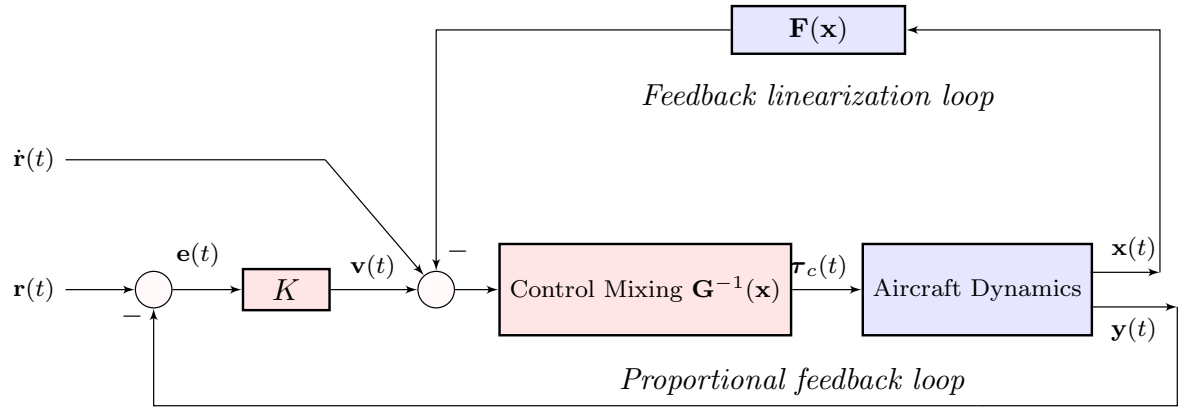


Fig. 3.3 NDI Problem Formulation (Enenakpogbe et al., 2022)

The error is given by:

$$\mathbf{e}(t) = \mathbf{r}(t) - \mathbf{y}(t) \quad (3.36)$$

where $\mathbf{r}(t) \in \mathbb{R}^m$ is the vector of reference signals to be tracked. Differentiating the output gives

$$\dot{\mathbf{y}} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \dot{\mathbf{x}} \quad (3.37)$$

$$= \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) + \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \mathbf{g}(\mathbf{x}) \boldsymbol{\tau} \quad (3.38)$$

Defining

$$\mathbf{F}(\mathbf{x}) := \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}), \quad \mathbf{G}(\mathbf{x}) := \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \mathbf{g}(\mathbf{x}) \quad (3.39)$$

we get

$$\dot{\mathbf{y}} = \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\boldsymbol{\tau} \quad (3.40)$$

Rearranging, assuming that the function matrix $\mathbf{G}(\mathbf{x})$ is always nonsingular, we get the control input to be

$$\boldsymbol{\tau} = \mathbf{G}^{-1}(\mathbf{x}) [-\mathbf{F}(\mathbf{x}) + \dot{\mathbf{r}} + \mathbf{v}] \quad (3.41)$$

where $\mathbf{v}$ is the output of the outer loop linear controller such that $\mathbf{v} = \mathbf{K}\mathbf{e}$ if the controller is a proportional gain matrix, $\mathbf{K} \in \mathbb{R}^{m \times m}$. Substituting for $\boldsymbol{\tau}$ in (3.38) gives

$$\dot{\mathbf{y}} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) + \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \mathbf{g}(\mathbf{x}) \mathbf{G}^{-1}(\mathbf{x}) [-\mathbf{F}(\mathbf{x}) + \dot{\mathbf{r}} + \mathbf{v}] \quad (3.42)$$

$$= \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x}) \mathbf{G}^{-1}(\mathbf{x}) [-\mathbf{F}(\mathbf{x}) + \dot{\mathbf{r}} + \mathbf{v}] \quad (3.43)$$

$$= \dot{\mathbf{r}} + \mathbf{v} \quad (3.44)$$

From (3.36), the error dynamics is given by:

$$\dot{\mathbf{e}} = \dot{\mathbf{r}} - \dot{\mathbf{y}} \quad (3.45)$$

$$= -\mathbf{v} \quad (3.46)$$

This is a first order system, so any linear control technique can be used to choose $\mathbf{K}$ to stabilize the error so that $\dot{\mathbf{e}} = -\mathbf{K}\mathbf{e}$. Thus the total controller is

$$\mathbf{v} = \mathbf{K}\mathbf{e} \quad (3.47)$$

$$\boldsymbol{\tau} = \mathbf{G}^{-1}(\mathbf{x}) [-\mathbf{F}(\mathbf{x}) + \dot{\mathbf{r}} + \mathbf{v}] \quad (3.48)$$

The state variable vector $\mathbf{x}$ and virtual control vector $\boldsymbol{\tau}$ are respectively given by:

$$\mathbf{x} = [x \ y \ z \ u \ v \ w \ \phi \ \theta \ \psi \ p \ q \ r]^T \quad (3.49)$$

$$\boldsymbol{\tau} = [T_X \ T_Z \ L_T \ M_T \ N_T]^T \quad (3.50)$$

The controller structure is shown in Figure 3.3 and consists of the linear output feedback loop (3.47) and the state feedback linearisation loop (3.48) (which assumes all the states are observable). The feedback linearisation loop ensures that the system from $\mathbf{v}(t)$ to $\mathbf{y}(t)$ appears like a linear system with poles at the origin, thus simplifying the outer tracking loop design. The velocity feedforward term $\dot{\mathbf{r}}$ improves the tracking accuracy of the closed-loop system. This signal is generated either by the pilot inceptor for manned operation or the guidance system for automated flight.

Since the controller in Figure 3.3 contains a model of the aircraft dynamics (3.39), the nonlinear functions of the aircraft must be known (i.e $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ in equation (3.48) are equal to $\mathbf{F}(\mathbf{x})$ in equation (2.4) and $\mathbf{G}(\mathbf{x})$ in equation (2.5)) in order to implement it. Hence NDI has been combined with robust controllers to deal with these disturbances and uncertainties (Adams and Banda, 1993a,b; Adams et al., 1994; Balas et al., 1992; Snell and Stout, 1996; Spaulding et al., 2005; Yang and Morales, 2021). For the control of the novel fixed-wing vectored thrust eVTOL aircraft, other robust control schemes such as H_∞ loop shaping

control (Başak and Prempain, 2015; Postlethwaite et al., 2005; Prempain and Dzeukou, 2005; Prempain and Postlethwaite, 2005; Prempain et al., 2001), linear parameter varying control (Ameho and Prempain, 2011; Prempain et al., 2009), switched fault tolerant control (Başak and Prempain, 2017), structural singular value control (Enns, 1990), iterative optimization-based control (Livet et al., 1995) could be combined with NDI to improve NDI's robustness to model mismatches.

3.5.1.1 Zero Dynamics

Substituting the control input (3.48) into the state equations (3.34), the full closed-loop dynamics is given by:

$$\dot{\mathbf{x}} = \left[\mathbf{I} - \mathbf{g}(\mathbf{x})\mathbf{G}^{-1}(\mathbf{x})\frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right] \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{G}^{-1}(\mathbf{x})[\dot{\mathbf{r}} + \mathbf{v}] \quad (3.51)$$

where $\mathbf{I}$ denotes the identity matrix. To ensure that the system is stable, the *zero dynamics* equation (3.45) must be stable. The zero dynamics is the dynamics of the system when the auxiliary input $\mathbf{v}(t)$ is selected to give an output $\mathbf{y}(t)$ equal to zero. Since $\mathbf{y}(t) = 0$, then $\dot{\mathbf{y}}(t) = 0$, such that $\mathbf{v}(t) = -\dot{\mathbf{r}}(t)$ which when substituted into (3.51) yields the *zero dynamics* equation given by:

$$\dot{\mathbf{x}} = \left[\mathbf{I} - \mathbf{g}(\mathbf{x})\mathbf{G}^{-1}(\mathbf{x})\frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right] \mathbf{f}(\mathbf{x}) \quad (3.52)$$

The poles of the zero dynamics (equation (3.52)) of the linearized cruise model contain both p poles (error dynamic equation (3.45) poles at the origin which are guaranteed stable by the choice of $\mathbf{v} = \mathbf{K}\mathbf{e}$) and $n - p$ poles which are unobservable selecting the output $\mathbf{y}$ (may not be stable and cannot be moved using the linear dynamic inversion). If some of unobservable modes or internal zeros are non-minimum phase (NMP), then the closed-loop system designed by the non-linear dynamic inversion will be unstable.

3.5.1.2 Controlled Variables (CV)

A necessary and sufficient condition for a successful NDI design is the selection of the *controlled variables (CV)* ($\mathbf{y} = \mathbf{h}(\mathbf{x})$) that will make the *zero dynamics* of the linearized model stable. For this research, the CV was tested for suitability on a linearized version of the error dynamics at a cruise conditions using MATLAB simulation to confirm that the *zero dynamics* state converge to zero from various initial conditions, as suggested by Stevens et al. (2015).

Since the state equations for $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{v}$, $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ have no control terms, they are referred to as *unobservable states since they are not included in the error dynamics* while the remaining states $\dot{u}$, $\dot{w}$, $\dot{p}$, $\dot{q}$ and $\dot{r}$ are called *controllable states**. Equations (2.8) and (2.9) are called the *complementary equations* and (2.7) and (2.10) the *controlled equations* (Durham, 2013). A simple rule is to ensure that the selected CV are at least a function of all the controllable states.

*As distinct from the *controllable modes*, an entirely different concept that derives from Kalman's well-known controllability test

For this research, the selected CV, $\mathbf{y} = \mathbf{h}(\mathbf{x})$ for the first five (surge, heave, roll, pitch and yaw) degrees of freedom are given by:

$$\mathbf{y} = (u, w, p, q, r)^T \quad (3.53)$$

Since eVTOLs do not generate propulsive thrust in the y-direction, the sixth degree of freedom (sway) is achieved by rolling so the controlled variable for the sway axis is $\mathbf{y} = p$.

3.5.1.3 NDI Controller Stability Considerations

Highly maneuverable jet aircraft, such as the V/STOL Harrier, belong to an important class of a slightly NMP nonlinear systems. The NMP character of aircraft is a result of the small body forces that are produced in the process of generating body moments. NDI, which is an input-output feedback linearisation control technique has a major shortcoming of inability to deal with non-minimum phase (NMP) nonlinear systems because of the unstable internal zero dynamics (Hauser et al., 1992).

Firstly, due to the canard and tailless eVTOL configuration (see Figure 2.4), vectored thrust effector configuration, decoupled control channels (equation (3.53)), the way in which the control come into the dynamic equations (equations (2.14), (2.16), (2.20), (2.21), (2.22) and Table 2.5) of the eVTOL aircraft discussed in this thesis, the way the aircraft is flown (see sections 3.6 and 5.2 e.g pitch using the canard to increase AoA and lift in order to climb without initial loss of altitude for the tailless eVTOL aircraft instead of NMP behaviour for traditional aircraft of pitching by lowering the tail resulting in initial altitude loss before climbing), the aircraft does not exhibit NMP behaviour hence no instability concerns for NDI applications.

Finally, due to the intentional choice of the classical NDI controller CV and output $\mathbf{y}$, the error dynamics observable state variables u, w, p, q and r are all linear or LTV functions of the derivatives of the unobservable modes or states variables ($n - p$ poles) which are x, z, ϕ, θ and ψ hence stabilizing the observable modes ensures the stability of the unobservable mode. Note that on this VTOL aircraft, heading β, v and y are indirectly controlled by ϕ (roll to sway) due to the effector configuration as discussed in section 4.5.3.

A NMP test is conducted on the linearized eVTOL model closed loop system based on LDI discussed in section A.4.3 by finding the eigen values (roots or poles) of the zero dynamics from equation (3.52) given by

$$\mathbf{A}_Z = \left[\mathbf{I} + \mathbf{B}_{cru}(\mathbf{C}_{cru}\mathbf{B}_{cru})^{-1}\mathbf{C}_{cru} \right] \mathbf{A}_{cru} \quad (3.54)$$

where:

$$(\mathbf{C}_{cru}\mathbf{B}_{cru})^{-1} = (\mathbf{C}_{cru}\mathbf{B}_{cru})^T ((\mathbf{C}_{cru}\mathbf{B}_{cru})(\mathbf{C}_{cru}\mathbf{B}_{cru})^T)^{-1} \quad (3.55)$$

The MATLAB code used which computes the eigen values of the zero dynamics based on equation (3.54) (equivalent to using the MATLAB[®] function *tzero* function in section 2.4.5) is given below:

```
I=eye(12);
CB=C_cru*B_cru;
```

```

CBT=CB' ;
CBS=CB*CBT;
CBIn=zeros(8,5);
CBInv=CBT/CBS;
Az=(I+(B_cru*CBInv*C_cru))*A_cru;
ZeroDynP=eig(Az);

```

The roots of the closed-loop zero dynamics are $(-2.2775 \times 10^4, -0.0005 \times 10^4, -0.0001 \times 10^4, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ which shows there are no unstable (in the RHP) zeros since all the numerical zero are on the Left Hand Plane (LHP) hence again the closed loop system using an LDI based controller is a minimum phase. The small values of the zero dynamics of the linearized model are a numerical feature because they have been cancelled by the internal poles due to the dynamic inversion. To check the zeros, the states variables are stripped out.

3.5.2 NDI Propulsive and Aerodynamic Forces Decoupling

From the plant and classical NDI equations 3.34, 3.39 and 3.41, the following affine conditions must be satisfied:

- The aerodynamic forces and moments ($f(\mathbf{x})$) must be separated and decoupled from the propulsive forces and moments/effector model ($\mathbf{g}(\mathbf{x})$) such that equation 3.34 is in affine form
- ($\mathbf{G}(\mathbf{x})$) must be invertible (i.e. $\mathbf{G}^{-1}$ must be singular and tractable)
- Any non-linearity in ($\mathbf{F}(\mathbf{x})$) can be fed back into the classical NDI controller as in equation 3.41

However, the method presented in Hovakimyan et al. (2008) for non-affine systems can be investigated to resolve the propulsive and aerodynamic forces decoupling, but this is beyond the scope of this study.

3.6 NDI Controller Concept Testing

In order to test the NDI controller in isolation, we make the following Model assumptions on Model 1 assumptions to produce Model 4 described in section 1.5):

- There are no individual propulsive forces and moments/effector model ($\mathbf{G}(\mathbf{x})$), but only generalised forces and moments knowns as virtual controls ($\boldsymbol{\tau}_c$ and $\boldsymbol{\tau}$), and decoupled aerodynamic forces and moments ($\mathbf{F}(\mathbf{x})$)
- since there no effectors, then there are no actuator limits.
- there is no need for control allocation scheme

To illustrate the results, four plots are used for each test scenario. First and second plot gives the aircraft state response and commands. The third plot shows the commanded accelerations and actual acceleration while the fourth plot shows the commanded and actual virtual controls (generalised forces and moments at the overall aircraft-level).

The constants (aircraft aerodynamics, pilot command limits, NDI and outer-loop controller parameters) used for the model are given in 2.4, 2.3, 5.3 in Chapter 5 and 5.4 in Chapter 5.

Three selected critical aircraft control manoeuvres are investigated on a simplified version of Vehicle Model 1 (Vehicle Model 4). These aircraft control manoeuvres (ACM) are devised to cover the aircraft envelope and are not obtained from any external source. The flight test manoeuvres are also chosen to test the ability of the NDI controller to control the aircraft and ensuring stable flight by cancelling the nonlinear bare airframe dynamics hence requiring no gain scheduling despite the eVTOL vehicles wide operating envelop and nonlinear flight dynamics. The first and second ACMs (Manoeuvre 1 and Manoeuvre 2) visually represented in Figures 3.4 and 3.8 aim to test how the NDI controller ensures satisfactory tracking of states during a Vertical Take-off, Hover & Low Speed Flight, Vertical Landing scenarios, and forward transition from Hover to high forward speed. The third ACM (Manoeuvres 3) represented in Figure 3.12 aims to test how the NDI controller ensures satisfactory tracking of the states when the aircraft transitions from high forward speed to Hover. The three selected ACMs are detailed below.

3.6.1 Manoeuvre 1 - Vertical Take-off, Hover & Low Speed Flight, Vertical Landing

First, the controller is tested for hover & low speed flight conditions. The test case is explained as following. Some commands are applied simultaneously to observe the controller's response during the coupled dynamics.

- At 3s, 3000m, a Vertical take-off (VTO) altitude command ($h_{cmd}=3020m$) is applied at constant climb rate of 2m/s, and the controller tracks the corresponding vertical velocity command (w_{cmd}). Similarly, at 72s vertical landing (VL) command is applied.
- Between 20s and 43s, forward and backward motion is tested via applying velocity command in the body x-direction u_{cmd} (via airspeed V_{cmd} and flight path angle γ_{cmd} commands).
- Between 35s and 72s, positive and negative side velocity is commanded (v_{cmd}) to achieve side-ward translational motion via generating corresponding roll angle command (ϕ_{cmd}).
- Between 28s and 72s, positive and negative pitch attitude is commanded via generating corresponding pitch angle command (θ_{cmd}).
- Between 65s and 75s, aircraft heading is changed by commanding yaw angle command (ψ_{cmd}).

A visual representation of manoeuvre 1 is shown in Figure 3.4.

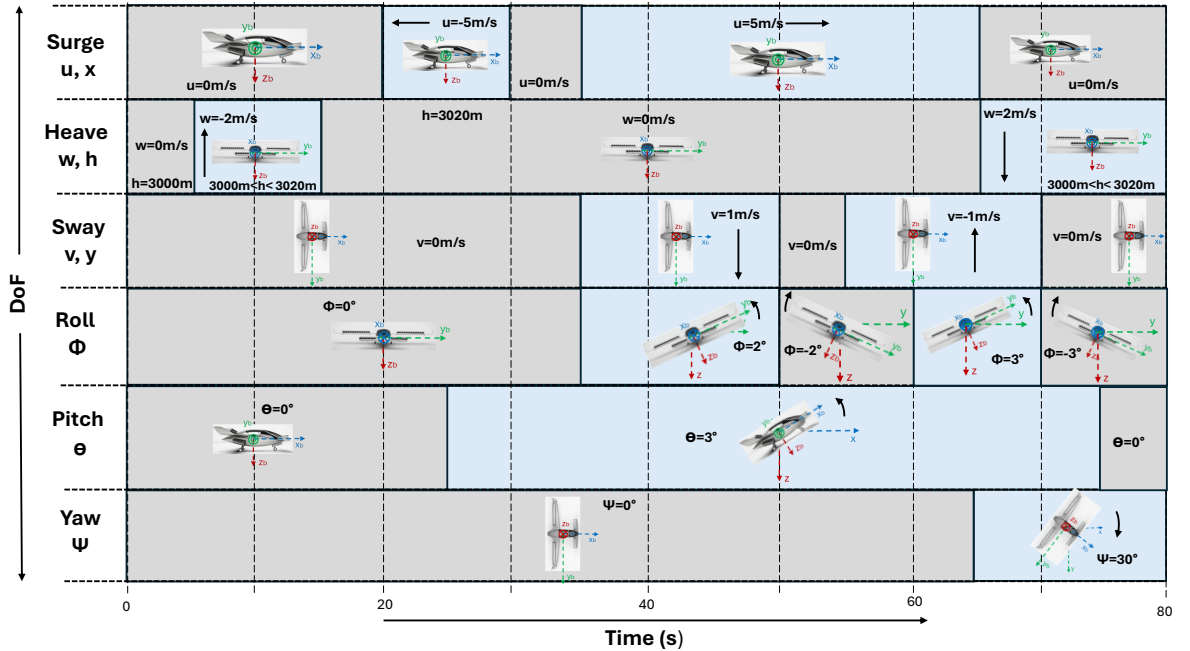


Fig. 3.4 Visual Representation - VTO, Hover Flight, VL

The state variable, state variable rates and virtual controls are shown in Figures 3.5 - 3.7.

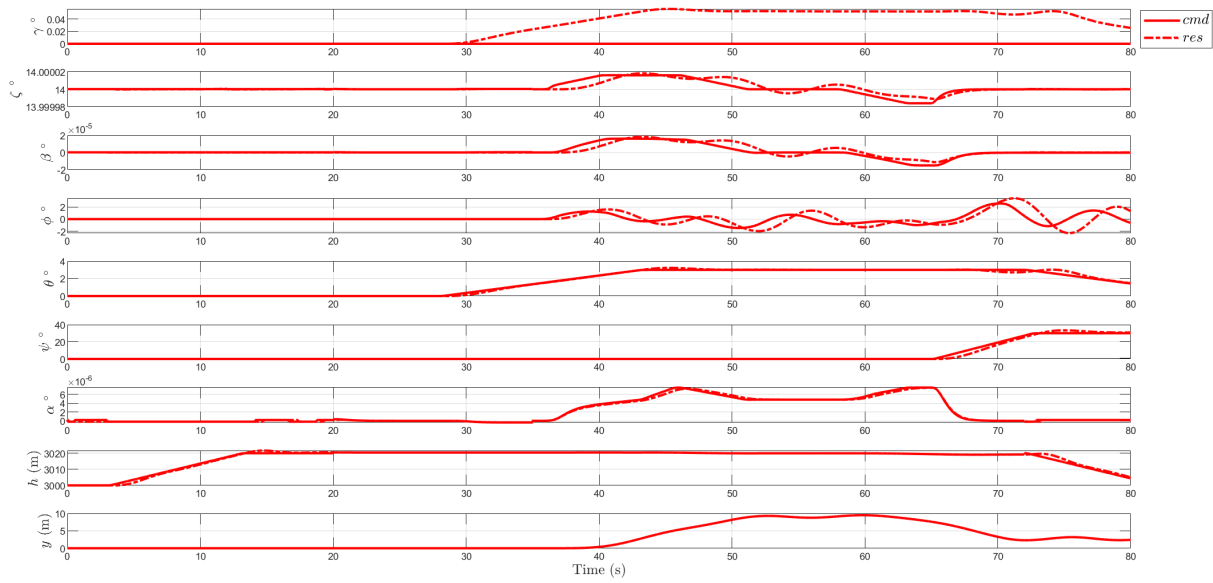


Fig. 3.5 No Effector/CA - Manoeuvre 1: VTO, Hover Flight, VL - Attitude

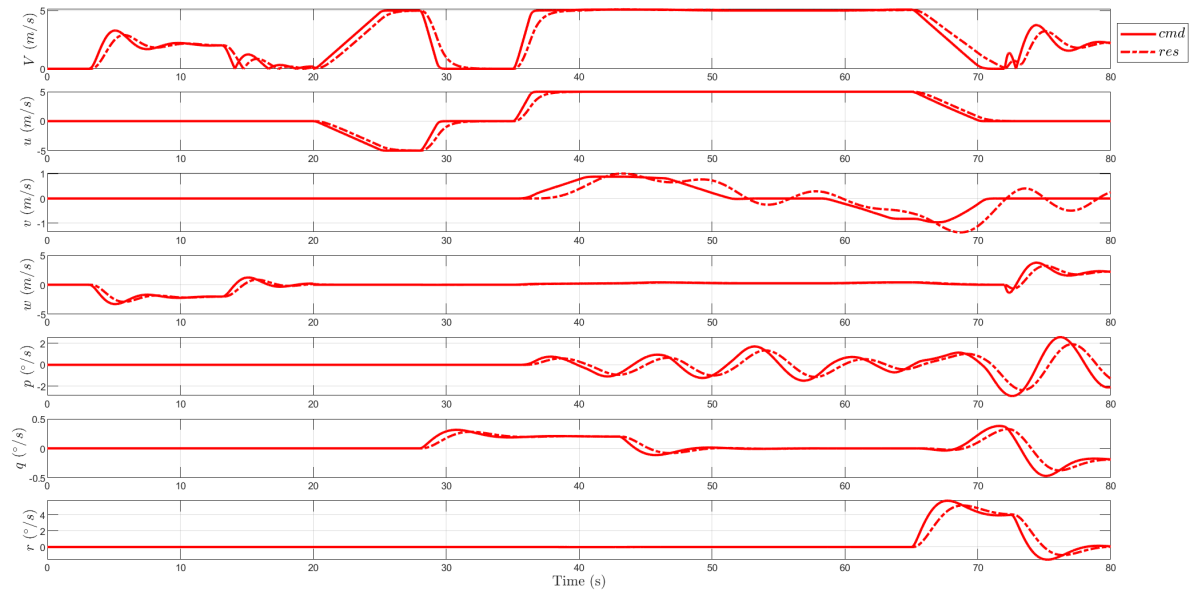


Fig. 3.6 No Effector/CA - Manoeuvre 1: VTO, Hover Flight, VL - Velocity

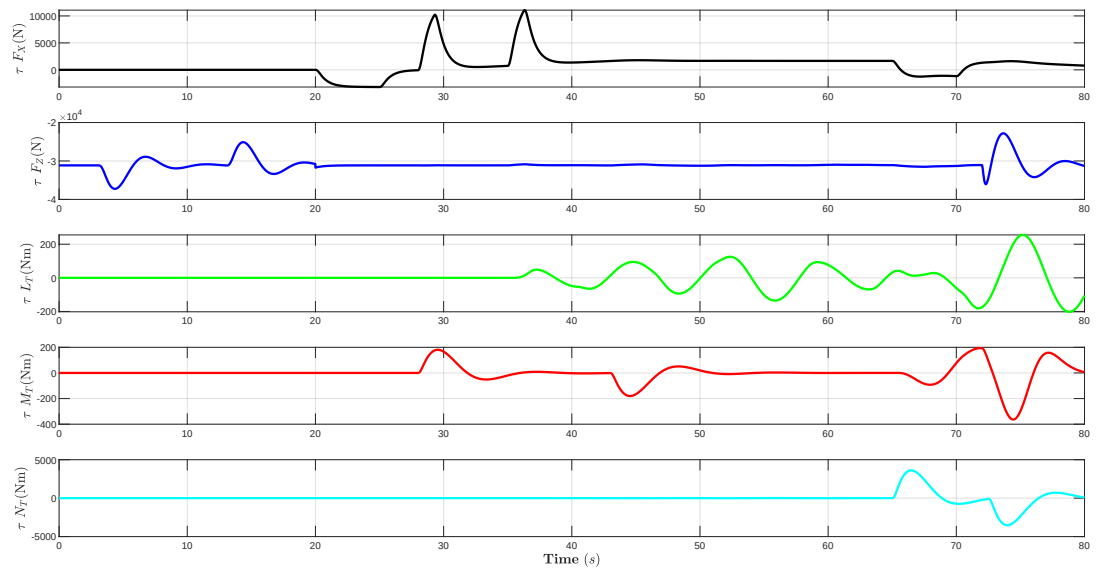


Fig. 3.7 No Effector/CA - Manoeuvre 1: VTO, Hover Flight, VL - Virtual Controls

1. Analyses of the Altitude Response: According to Figure 3.5, 20m altitude increase is commanded at 3s, and the aircraft tracks the corresponding vertical speed command (w_{cmd}). In between 3s and 75s, several commands are applied in other channels. The aircraft keeps the 3020m altitude during the forward, side-ward motions and also the heading change. Note that the command generator applies approximately $2(m/s^2) \approx 0.2g$ maximum vertical acceleration command (Figure A.7). According to Figure A.7, w_{cmd} is tracked well and the aircraft holds altitude (h) while other commands cause disruptive effects on the vertical channel. The virtual control inputs τ_c are shown in Figure 3.7 where only F_Z changes for the VTO. *NOTE: There is a misleading nuisance spike seen in $\dot{w}_{cmd}$ at time stamp 20s of Figure A.7. This is not the actual $\dot{w}_{cmd}$ since NDI is a velocity controller (not like INDI which is a rate controller). The $\dot{w}_{cmd}$ (and the other rate commands) displayed in Figure A.7 is a derivative (implemented using MATLAB time derivate block) of the w_{cmd} . Only the w_{cmd} generated by the w outerloop is used as a reference control input into the NDI controller. The w_{cmd} switches (SW2 in equation (3.19)) between that due to hover control (altitude, low speed and VTO/VL) w_{hcmd} and airspeed/flight-path angle control $w_{\gamma cmd}$ whenever there is a transition from hover to forward speed as seen at $t = 20s$. Due to the sudden change in the value of w_{cmd} , the derivate of the signal ($\dot{w}_{cmd}$) can be appear as a spike (exagerated) depending on the previous state of the two feeding loops considering the sampling rate. This is a well known feature of rates which are prone to noise. Since the commanded rates shown in Figure A.7 including $\dot{w}_{cmd}$ are not actual commands, they can be removed from the plot because they are misleading. The $\dot{w}$ does not show the spike hence confirming that the spike in $\dot{w}_{cmd}$ is just a nuisance.*

2. Analyses of the Forward and Backward Motion: Translational motion in the body x direction is obtained via commanding body x -velocity (u_{cmd}) via airspeed V_{cmd} and flight path angle (fpa) γ_{cmd} commands. The same command is also used to perform the transition easily and also to control the angle of attack indirectly in forward flight. Based on Figure 3.6, $\pm 5m/s$ forward velocity command is applied between 20s and 40s, the state derivative ($\dot{u}$) is shown in Figure (A.7). To accelerate in forward direction, it is expected to increase forward thrust F_X and increase pitching moment M_T . Similarly, to accelerate in reverse direction, it is expected to increase forward thrust F_X and reduce pitching moment M_T between 20s and 40s in the virtual control inputs τ_c shown in Figure 3.7.

3. Analyses of the Side Motion: To perform sway motion at low speeds, positive and negative side velocity commanded between 35s and 75s. As previously mentioned, EDFs do not generate side force directly. Therefore, thrust direction is changed via applying a navigational bank angle command (ϕ_{cmd}) at low speeds based on Equation 3.8. As shown in Figure 3.6, v_{cmd} and the corresponding (ϕ_{cmd}) are tracked well. There exists a small delay in tracking of the side velocity command. This result is expected since side velocity is not directly controlled and result of change in thrust vector direction. Linear navigational controller gain (K_{vp}) can be increased to reduce the delay in side velocity command tracking if necessary (Table 5.4). Roll rate tracking performance of the controller is given Figure 3.5. Commdated roll rate p_{cmd} is tracked accurately. To change the roll angle at low speeds, virtual controls F_z , L_T , M_T and N_T are changed in opposite manner. The required change in thrust is very small to achieve the

desired roll rates. On the other hand, to keep the altitude during the side-motion thrust must be increased significantly since thrust vector is not aligned with the gravity. Therefore, the asymmetric change in thrust between the right and left sections cannot be observed directly in rolling and yawing moments L_T and N_T respectively of Figure 3.7 since it illustrates the total thrust command. No pitching moment changes M_T as seen.

4. Analyses of the Heading Response: To change the aircraft heading, $\pm 30^\circ$ yaw angle is commanded between 65s and 72s (Figure 3.5). The NDI controller tracks the required body yaw rate (r) to achieve the desired heading. According to Figure 3.5 and A.7, yaw rate tracking performance is satisfactory. To change the aircraft heading at hover, there is a commanded yawing moment N_T . This result can be seen between 65s and 75s of the virtual control plot (Figure 3.7) when the negative heading command is applied. It is observed that an approximately 3000N change in the yawing moment is enough to achieve the desired heading command accompanied with some coupling leading to small changes in pitching (M_T) and yawing moment (L_T). Since changes in pitching moment M_T are very small, additional thrust F_X and F_Z required to keep the altitude is very small.

3.6.2 Manoeuvre 2 - VTO and heading control, Forward Transition

Transition from hover to forward flight is one of the most critical flight regime regarding the flight control due the coupled vectored thrust control and strong nonlinear dynamics during the transition (see Section 2.3.2). Note that nonlinear dynamic behaviour occur in the aerodynamic model during the transition due to the blending between the hover and forward flight models explained in equations 2.30, 2.31 and 2.32.

Recall from Table 2.6 of Section 2.4.1 that the aircraft is trimmed at cruise condition with 77.87m/s airspeed and 2.84° degree angle of attack.

- At 5s, 2980m, a vertical take-off (VTO) altitude command ($h_{cmd} = 3000\text{m}$) is applied at constant climb rate of 2m/s, and the controller tracks the corresponding vertical velocity command (w_{cmd}).
- At 8s, aircraft heading is changed by commanding yaw angle command (ψ_{cmd}) of $\psi=30^\circ$ to change the aircraft heading while climbing vertically and then then heading returned back to $\psi=0^\circ$
- Between 20s and 70s, forward transition is commanded by simultaneously applying velocity command in the x and z direction body-axis u_{cmd} , w_{cmd} (via airspeed V_{cmd} and flight path angle fpa γ_{cmd} commands) and AoA α . An airspeed speed V of 77.8m/s was commanded at an acceleration of 4m s^{-2} , fpa γ of 0° and pitch attitude θ of 2.84° at a constant rate of $\pm 0.01\text{ rad s}^{-1}$ was commanded over a 17.5s duration.

A visual representation of manoeuvre 2 is shown in Figure 3.8.

The state variable, state variable rates and virtual controls are shown in Figures 3.9 - 3.11.

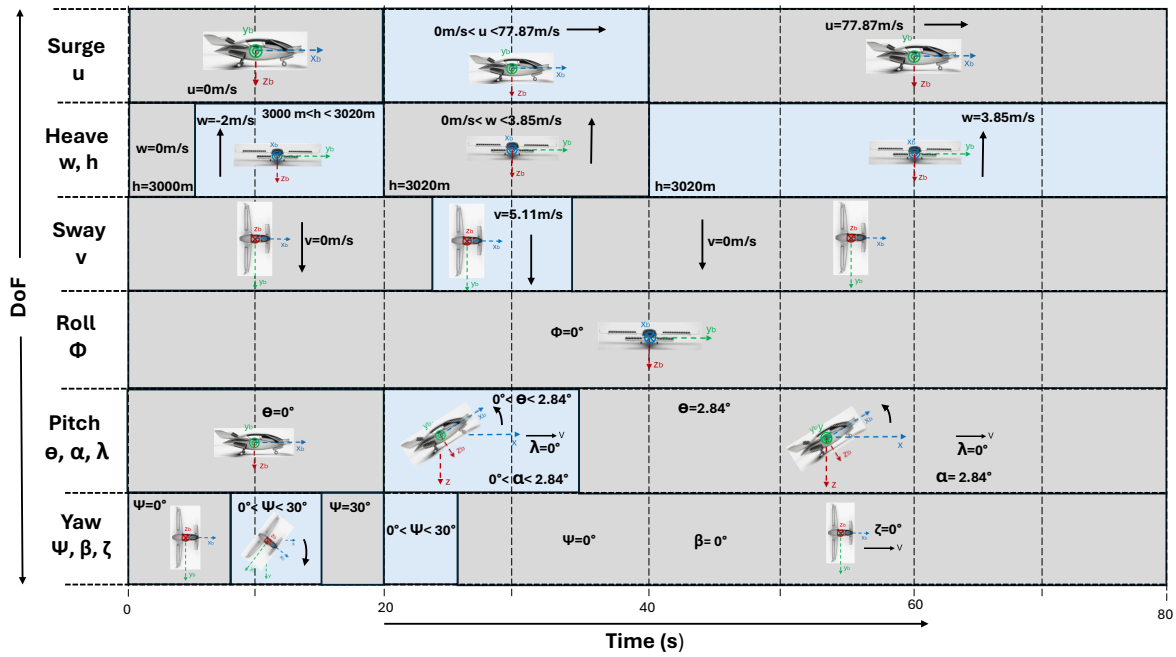


Fig. 3.8 Visual Representation - Manoeuvre 2: VTO, Forward Transition

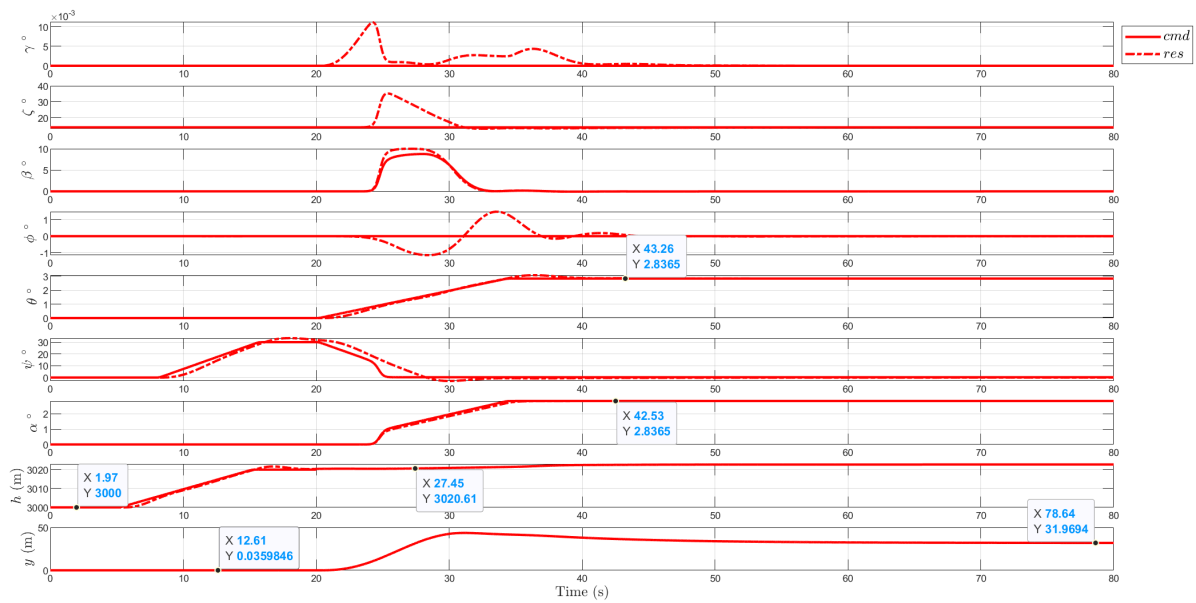


Fig. 3.9 No Effector/CA - Manoeuvre 2: VTO, Forward Transition - Attitude

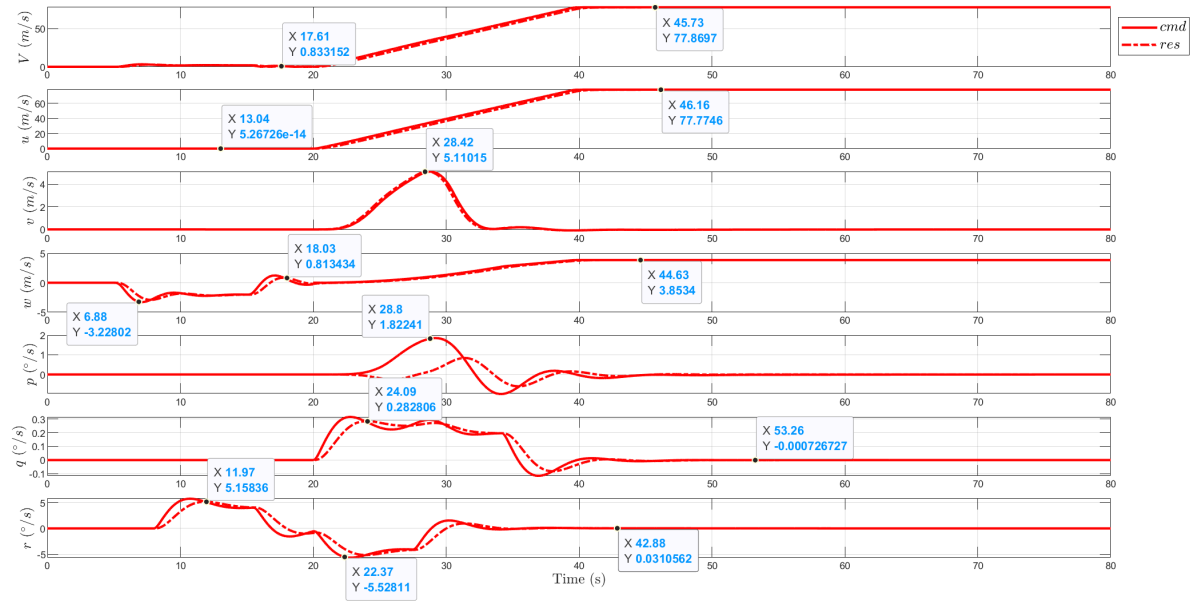


Fig. 3.10 No Effector/CA - Manoeuvre 2: VTO, Forward Transition - Velocity

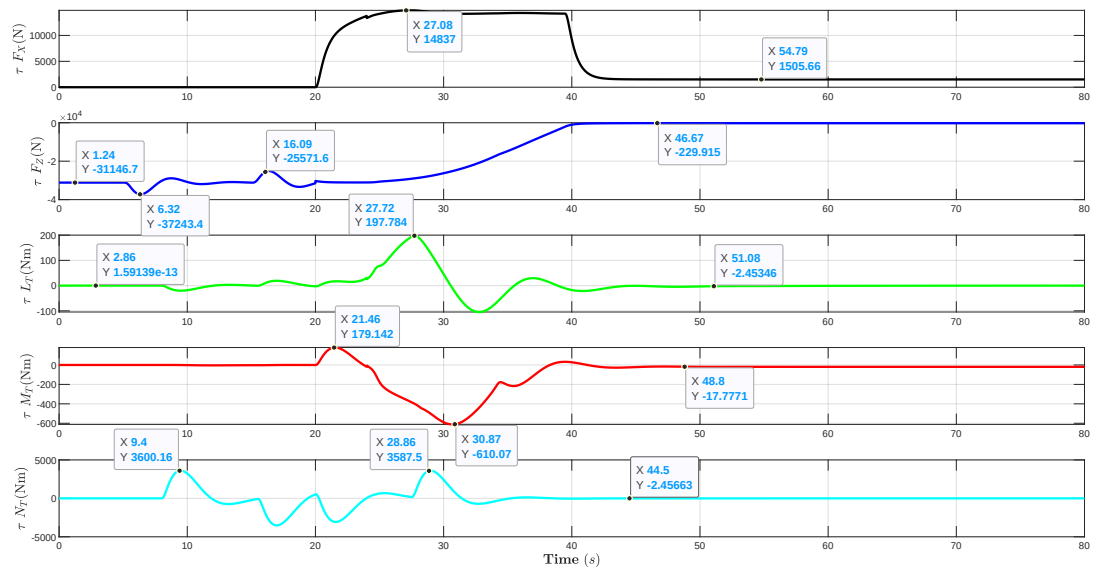


Fig. 3.11 No Effector/CA - Manoeuvre 2: VTO, Forward Transition - Virtual Controls

Analysis of Manoeuvre 2 Results *During the forward transition both the Airspeed (V) and AoA (α) are commanded simultaneously such that cruise trim AoA α of 2.84° is reached before cruise trim airspeed.*

Figure 3.9 and 3.10 shows the state response of the forward transition. The aircraft climb to 3000m at first and commands heading change of initially 30° and back to 0° before accelerating to 77.8m/s after 39s with an approximate acceleration of 2m/s^2 which can be seen on the Figure A.8 $\dot{u}$ plot.

The NDI controller is designed to control the body angular and translational velocities, see section 2.3.5.3 and Equations 3.47, 3.48, 3.53 and 3.49. Therefore, it is possible to control the angle of attack and flight path angle indirectly using the relations given in Equations 3.28, 3.29 and 3.30. To achieve 2.84° trim angle of attack, combined 2.84° , θ_{cmd} , 0° γ_{cmd} and 77.8m/s V_{cmd} are applied at 20s (see Figure 3.9). Desired trim values are reached without any loss the altitude (See Figure 3.9), trim states are achieved with very small errors such as: $\alpha_{trim} = 2.85^\circ$, $w_{trim} = 3.85\text{m/s}$, $u_{trim} = 77.77\text{m/s}$, $\beta_{trim} \cong 0^\circ$, $\gamma_{trim} \cong 0^\circ$, compared to the trim data given in Table 2.6. Except F_X , at the end of the forward transition, the values of the remaining virtual controls F_Z , L_T , M_T and N_T decreases towards zero (see Figure 3.11) for cruise trim conditions (wing level, level flight) since the aircraft weight is now supported by the increased aerodynamic lift.

3.6.3 Manoeuvre 3 - Cruise, Reverse Transition and VL with Heading change

Transition from high speed cruise flight to low speed hover flight referred to as 'Reverse Transition' is also a critical flight regime regarding the flight control due the coupled vectored thrust control and strong nonlinear dynamics during the transition (see Section 2.3.2). Note that nonlinear dynamic behaviour occur in the aerodynamic model during the reverse transition due to the blending between the forward flight and hover models explained in equations 2.30, 2.31 and 2.32.

The reverse transition test starts at the cruise condition with $h = 3000\text{m}$, 2.84° degree angle of attack and 77.87m/s airspeed. During the reverse transition both the Airspeed (V) and AoA (α) are commanded simultaneously such that the trim AoA α of 2.84° was reduced to 0° before cruise trim airspeed decreased from 77.8m/s to hover speed of 0m/s. At 92s, 30° left-turn is commanded by commanding yaw angle command (ψ_{cmd}) of $\psi = -30^\circ$ to change the aircraft heading while landing from $h = 3000\text{m}$ to $h = 2977\text{m}$.

The state variable, state variable rates and virtual controls are shown in Figures 3.13 - 3.15.

A visual representation of manoeuvre 3 is shown in Figure 3.12.

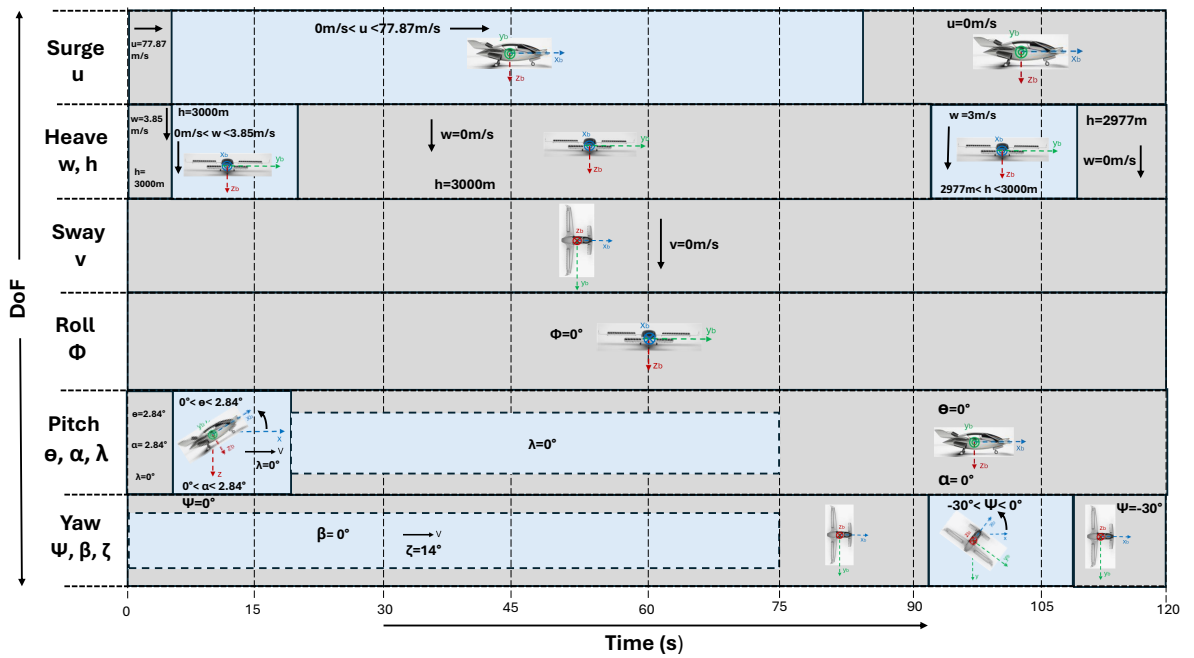


Fig. 3.12 Visual Representation - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change

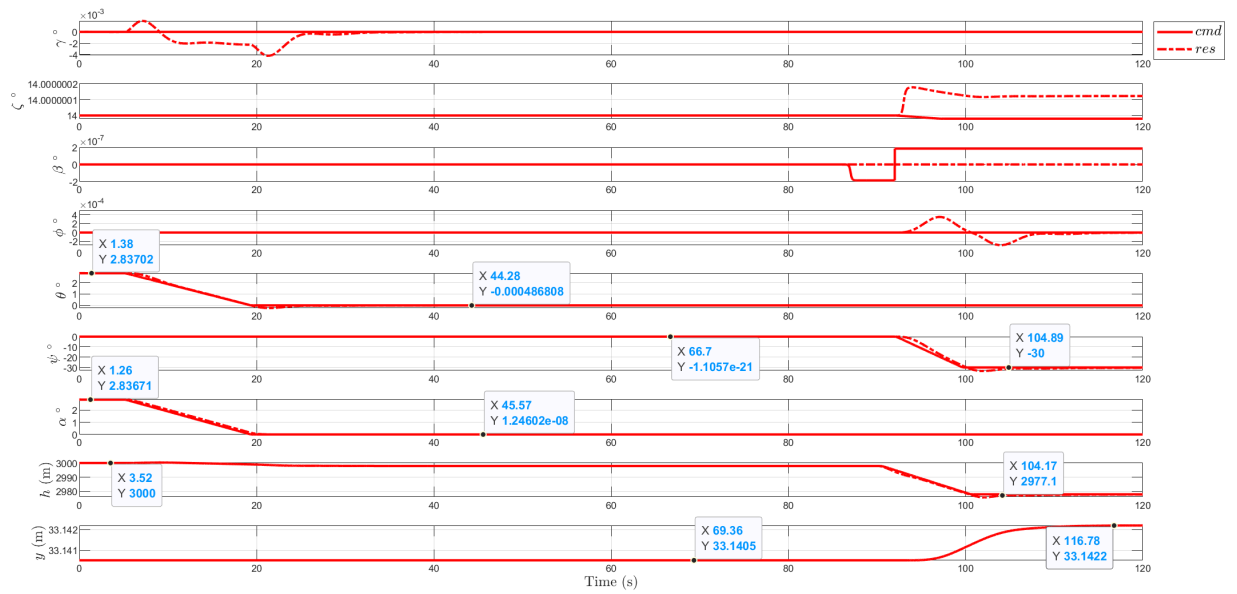


Fig. 3.13 No Effector/CA - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change -Attitude

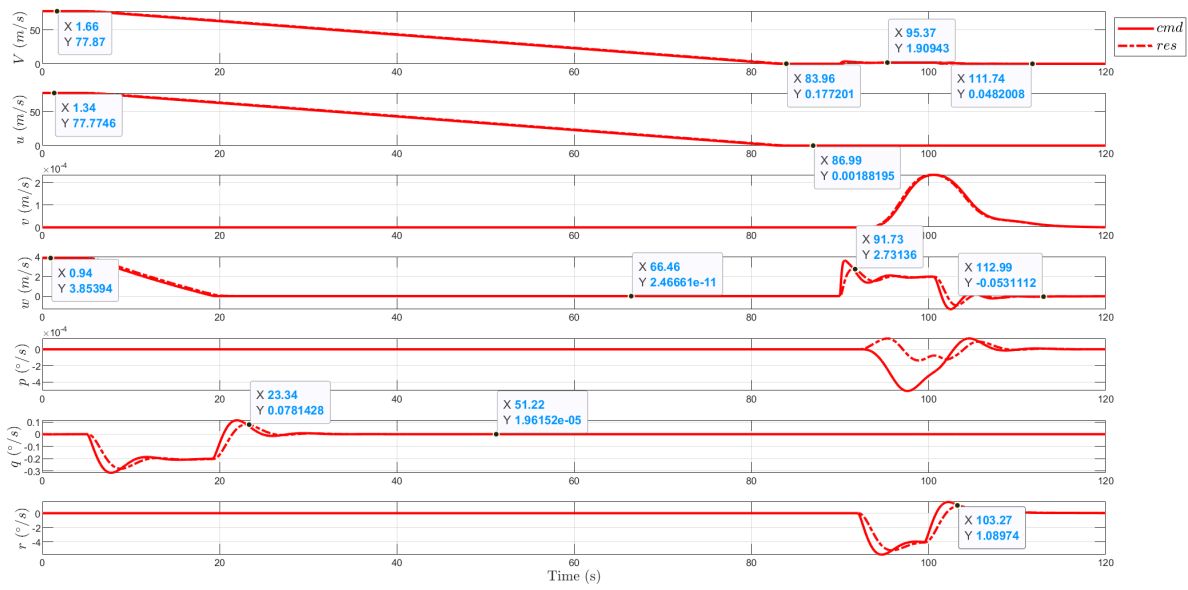


Fig. 3.14 No Effector/CA - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change - Velocity

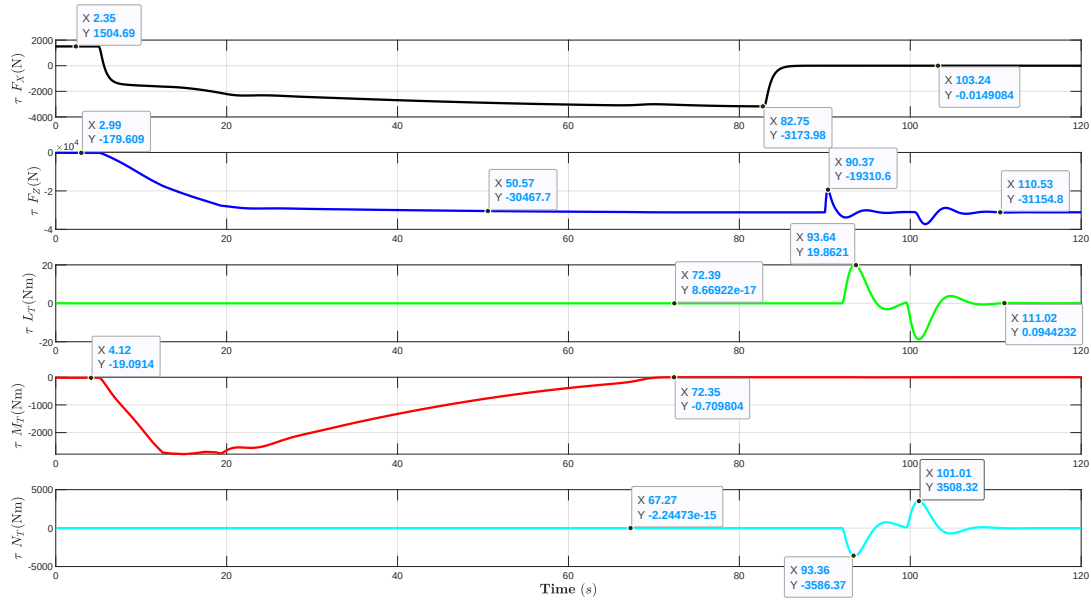


Fig. 3.15 No Effector/CA - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change - Virtual Controls

Analysis of Manoeuvre 3 Results As expected, during the reverse transition, the values of virtual controls F_Z , L_T , M_T and N_T increases or decreased away from zero while F_X reduced towards zero (see Figure 3.15) as hover conditions is reached since the aircraft weight is now fully supported by the propulsive virtual control F_Z due to reduction in lift generation by the wing as a result of decreasing airspeed. At $t = 82$ s, aircraft reaches hover condition (see Figure 3.15 where F_X is approximately zero). At $t = 82$ s, $h = 2980$ m command (VL) and ($\psi = -30^\circ$) heading changes were simultaneously applied and the aircraft lands vertically at $t = 102$ s and completed the (-30°) heading change at $t = 100$ s.

In general, NDI controller performed well during the reverse transition and vertical landing as the states tracking is satisfactory.

3.6.4 NDI Controller Testing Conclusions

From the test results, the NDI controller performance is satisfactory as the state variables and accelerations are tracked well during all three manoeuvres considering the wide flight envelope of the eVTOL and nonlinear dynamic behaviour during transition between hover and forward flight due to the blending of the hover and cruise aerodynamic models. The next chapter 4 will now consider effector models and hence present control allocation for this unique eVTOL air vehicle class.

Chapter 4

Control Allocation Approaches

4.1 Challenges of the Vectored Thrust eVTOLs

As discussed in Chapter 1, the Liliu-style vectored thrust eVTOLs have several challenges:

1. The electric engines arrangement of the Liliu-style multi-rotor vectored thrust tilt-rotor aircraft results in eight control effectors to control six (roll, pitch, yaw, surge, sway and heave) degrees of freedom resulting in a +2 degree of over-actuation consequently presenting a challenging ‘control allocation’.
2. The vectored thrust results in non-linear effector mapping (control effectiveness mapping) preventing the direct use of classical linear control allocation approaches and consequent computational infeasibility of performing non-linear programming in real time.
3. Thrust control saturation may lead to undesired behavior, deterioration of the controller performance and even loss of control in specific flight conditions due to the limited control authority during normal and degraded operation (e.g. actuator limitation or failures). Consider for example a 3DoF VTOL, with an unprioritized control allocation scheme, it may be that the desired combination of pitching moments together with the vertical and forward force is not achievable due to maximum or minimum thrust limitations of the thruster and/or due the maximum deflection limits of the tilting mechanisms. In the absence of an adequate (prioritised) control allocation algorithm, it is arbitrary which one (or combination) of the three objectives will be affected. However, for the flight stability of these over-actuated vectored thrust VTOLs, since the open loop phugoid dynamics is unstable, it is crucial to apply the full required pitching moment before anything else (Lombaerts et al., 2020b, 2019; Suiçmez, 2021). Thereafter, the vertical force for altitude (Heave DoF) control comes in the second place, followed by the forward force for the airspeed (surge DoF) which is the least critical from a safety point of view. Therefore, a control allocation algorithm is needed that prioritizes the control objectives of pitching moment over altitude and vertical speed, and subsequently over forward speed, and that calculates the required thrust levels per rotor accordingly.

The rest of this chapter is structured as follows: the 3DoF equations of motion and vehicle dynamic model, a summary of the control architecture, high-level control laws, and general

control allocation concepts and the proposed classical/linear control allocation technique are presented in sections 4.4, 4.5 and 4.6 respectively. Control allocation approach 1 is presented in section 4.7, Control allocation approach 2 is presented in section 4.8 while Control Allocation Approach 3 is presented in section 4.9. The nonlinear control simulations results of some selected manoeuvres are presented and analysed in section 4.10. In the final section 4.11, the proposed CA solutions are discussed as well as any limitations and future work.

4.2 Main Chapter Contributions

In order to avoid the online computational burden of nonlinear programming, this chapter proposes three approaches for transforming a nonlinear control allocation problem as in equation 4.5 to a linear problem as in equation 4.1 so that classical linear control allocation approaches reviewed above in (Johansen and Fossen, 2013) can then be used. Active set linear quadratic programming (QP) constrained optimisation algorithm was used as the classical control allocation approach. Some Active set QP formulations are weighted least squares (WLS), sequential least squares and minimal least squares.

This work considers a weighted least squares formulation (weighted, bounded least-squares problem) with a quadratic cost function and a corresponding quadratic optimization problem, solves the problem by making a trade off between the actual input commands $\mathbf{u}$ and the virtual commands $\boldsymbol{\tau}$, a solution of which can be found in a straightforward way in Harkegard (2002); Härkegrard (2004a); Petersen and Bodson (2005a,b). The resulting control allocation schemes are three novel real-time prioritised control allocation approaches which addresses the main challenges (limited control authority in some specific flight operation, non-linear effector mapping and over-actuation) of the particular VTOL configuration.

It then verifies the approaches using the control architecture and NDI high-level controller (aircraft-level controller) proposed in Enenakpogbe et al. (2022) but on a 3DoF planar VTOL version of the over-actuated vectored thrust canard-planform VTOL 6DoF Lilium model shown in Figure 2.1. A simple 3DoF planar VTOL model with no aerodynamics is chosen to focus this study on control allocation and so that the model can be analyzed using first level control knowledge.

The main contributions of this chapter are summarized as follows:

- proposes a novel approach for transforming the nonlinear control allocation problem to a linear one by transferring the non-linearity from the effector mapping to the computation of the actuator limits but formulates the individual effector demand (real controls) in cartesian form (decomposing its thrust components on the body axes) resulting in a linear effector mapping. It retains the overall control demands (virtual control) in cartesian form. The required computation of the actuator limits which converts the physical actuator limits from polar to cartesian form is however non-linear and complex. It proposes a more rigorous and analytical solution to the problem of conversion of the actuator limits from polar to cartesian, over the heuristic solution provided by Enenakpogbe et al. (2024a); Suiçmez (2021). This control allocation approach will be known as CA Approach 1 or extended thrust formulation CA thereafter.

- proposes a novel approach for transforming the nonlinear control allocation problem to a linear one by transforming the effector mapping from non-linear to linear via linearisation of the non-linear effector mapping in real-time resulting in a linear time varying (LTV) effector mapping (control effectiveness matrix - Jacobian of τ with respect to $\mathbf{u}$) with the real controls formulated in incremental polar form. This control allocation approach will be known as CA Approach 2.
- proposes another approach for transforming the nonlinear control allocation problem to linear one which is similar to CA Approach 2 above except an extra step which transforms the virtual controls from cartesian to polar before differentiation and linearisation resulting in a different and more complicated LTV effector mapping. This control allocation approach will be known as CA Approach 3.
- compares the performance of all three control allocation approaches (CA Approach 1, CA Approach 2 and CA Approach 3) for three unusual flight manoeuvres (aircraft control manoeuvres) which may be different from how an aircraft is normally flown. The choice of the three selected manoeuvres is to enable analysis of the CA performance during limited control authority due to actuator saturation in the absence of any aerodynamic lift and drag being a planar VTOL. The selected aircraft control manoeuvres are:
 - VTO/VL with pitch control and limited control authority.
 - VTO, Hover to high forward speed with limited control authority.
 - Reverse and pitch control with limited control authority

Table 4.1 below shows a summary comparison of the three control allocation approaches features:

Table 4.1 Control Allocation Approaches Comparison

CA Approaches	τ_c form	$\mathbf{u}$ form	Eff. Map	Eff. Const.	CA	Coupled Thrust
Approach ₁	Abs Cart	Abs Cart	linear	Pol to Cart	AS - Abs	No
Approach ₂	Incr Cart	Incr Pol	Num., LTV	Abs to Incr	AS - Incr	Yes
Approach ₃	Incr Pol	Incr Pol	Ana., LTV	Abs to Incr	AS - Incr	+Approach 2 Yes

4.3 Background of Control Allocation

There are several linear and nonlinear CA approaches and techniques proposed for motion control of over-actuated mechanical systems (Durham et al., 2017; Fossen and Johansen, 2006; Johansen and Fossen, 2013). Some CA techniques for over-actuated unmanned aerial vehicles

(UAVs) and drones are studied in Chen et al. (2019); Ducard and Allenspach (2021a); Liu et al. (2021); Mpanza et al. (2021); Park et al. (2023); Santos and Bezerra (2022); Wang et al. (2019b); Xia et al. (2020). CA approaches for linear and nonlinear effector models are reviewed in Johansen and Fossen (2013), and include unconstrained linear CA approaches, constrained linear CA approaches and constrained nonlinear CA.

Several unconstrained linear CA approaches are reviewed in Durham et al. (2017), and include Generalized inverses or pseudoinverses, Moore-Penrose pseudo-inverse and SVD-based Generalised inverses whereby no actuator constraints are applied and the effector mapping is linear. Meanwhile, constrained Linear control allocation approaches which applies effector constraints but only for systems with linear effector mapping are reviewed in Johansen and Fossen (2013) include Redistributed pseudo-inverse, cascaded generalised inverse, daisy chaining, direct control allocation, simplex numerical linear programming, active set (AS) numerical quadratic programming (QP) method, interior point numerical QP method, iterative fixed-point numeric QP method, multi-parametric quadratic programming, binary search trees, lattice representations, decomposition approach for master and slave problems, an optimization-based control allocation method integrated with a parameter estimation scheme, fault tolerant optimization-based control allocation, dynamic control allocation approach, model predictive control optimisation-based control approach etc.

Control allocation approaches for nonlinear effector models are reviewed in Johansen and Fossen (2013) and include nonlinear programming for control allocation, sequential quadratic programming, mixed-integer linear programming, Lyapunov-design based nonlinear optimization control allocation method, direct nonlinear allocation etc. Optimal solutions of linear programming (LPs) are found at the vertices of the feasible set thereby favouring the use of a smaller number of effectors, while methods based on a quadratic cost function and ∞ -norm tends to use all effectors but to a smaller degree (Bodson, 2002).

Consider a control allocation scheme and linear effector mapping with an architecture shown in Figure 4.1:

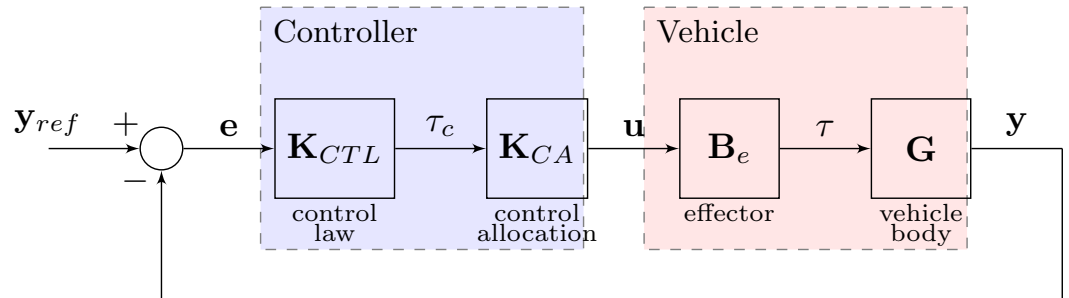


Fig. 4.1 General Linear Control Allocation Scheme

The control allocation scheme $\mathbf{K}_{CA}$ in Figure 4.1 ensures that the forces and moments vector, $\boldsymbol{\tau}$, that characterizes the physical model given by equation (4.1) is equal to the virtual control demands output, $\boldsymbol{\tau}_c$, produced by the high-level control law (outer linear controller and inner loop NDI controller) $\mathbf{K}_{CTL}$. The physical vehicle model consists of an effector/actuator model $\mathbf{B}_e$, thus normally would include low-level actuator controller (e.g. servo controllers), and the virtual controls (generalized forces and moments) $\mathbf{G}$. The focus of this chapter is on the inverse effector model $\mathbf{K}_{CA}$ (i.e. control allocation) using the effector/actuator model $\mathbf{B}_e$

and generalized forces and moments $\mathbf{G}$ which forms the vehicle with an assumption that $\mathbf{K}_{CTL}$ is already designed.

The control allocation $\mathbf{K}_{CA}$ can be formulated by the following problem:

Problem 1. *Given an effector mapping*

$$\boldsymbol{\tau}(t) = \mathbf{B}_e \mathbf{u}(t) \quad (4.1)$$

determine the effector demand vector $\mathbf{u}(t)$ such that

$$\boldsymbol{\tau}(t) = \boldsymbol{\tau}_c(t) \quad (4.2)$$

subject to

$$\underline{\mathbf{u}} \leq \mathbf{u}(t) \leq \bar{\mathbf{u}} \quad (4.3)$$

$$|\dot{\mathbf{u}}(t)| \leq \dot{\bar{\mathbf{u}}} \quad (4.4)$$

Less common is when there is a non-linear mapping relating $\mathbf{u}$ and $\boldsymbol{\tau}$. That is, we have the effector mapping given in problem 2

Problem 2. *Given an effector mapping*

$$\boldsymbol{\tau}(t) = \mathbf{g}_e(\mathbf{u}(t)) \quad (4.5)$$

determine the effector demand vector $\mathbf{u}(t)$ such that

$$\boldsymbol{\tau}(t) = \boldsymbol{\tau}_c(t) \quad (4.6)$$

subject to

$$\underline{\mathbf{u}} \leq \mathbf{u}(t) \leq \bar{\mathbf{u}} \quad (4.7)$$

$$|\dot{\mathbf{u}}(t)| \leq \dot{\bar{\mathbf{u}}} \quad (4.8)$$

This often arises when there are vectored thrust mechanisms (which can be commanded to produce thrust of varying amplitude and direction in two or three degrees of freedom) in the aircraft, such as tilt-rotors, tilt-wings and some of the various eVTOL configurations that are currently attracting research and investor interest (Enenakpogbe et al., 2024b; Kang et al., 2023). Nonlinear control allocation approaches (including nonlinear programming) reviewed in Johansen and Fossen (2013) can be used to solve the nonlinear problem with nonlinear effector mapping as in equation 4.5 while linear control allocation approaches can be used to solve the optimisation problem in problem 4.1-4.3.

The parameterisation of the control input, i.e. the choice of elements of the $\mathbf{u}$ vectors in the model, is sometimes crucial. It is usually desirable to choose the effector mapping such that it has a linear form (such as Yang et al. (2024)) as in equation 4.1 and thus avoiding more complex nonlinear control allocation methods. It is also desirable to transform a nonlinear control allocation problem (such as the nonlinear effector mapping for eVTOL with vectored

thrust effectors) into a linear one such that linear classical control allocation approaches can then be used.

The parameterisation of the control input leading to a linear effector mapping for vector thrust devices (with nonlinear effector mapping) can be achieved either by (1) an extended thrust formulation (Lindfors, 1993; Sørdaalen, 1997) decomposing its thrust components on the body axes and treating them as elements of the control input $\mathbf{u}$ as used in Control Allocation Approach 1 (CA1) later proposed in this work or (2) parameterising the thrust by its amplitude and angles as used in Control allocation approaches 2 (CA2) and 3 (CA3) later proposed in this work. However, the extended thrust formulation results in a more complicated formulation of actuator saturation and rate limit constraints in the control allocation formulation e.g. Johansen et al. (2008). The parameterisation of the thrust by its amplitude and angles (formulation of controls in polar form) requires linearisation of the nonlinear effector mapping to give a linear time varying (LTV) effector mapping such as a simple mechanism known as discrete time allocation (DTA) suggested by Harkegard (2003b) and proposed by Bordignon (1996); Doman and Oppenheimer (2002); Matamoros and deVisser (2018) for traditional aircraft control surfaces (not VTOL aircraft). The DTA transforms a non-linear problem to a linear problem by time discretization (numerical linearisation) of small enough steps (performing a first order Taylor expansion of the mapping at that point in the control space at each time step) of the effector mapping. Also, when sensor-based NDI (INDI) controller is used with incremental virtual control ($\Delta \tau_c$), to ensure that the real controls are also incremental ($\Delta \mathbf{u}$), incremental control allocation has been used even with a linear effector mapping (De Heer et al., 2023; Suiçmez and Kutay, 2023; Zhao et al.).

Linear control allocation approaches (e.g. Active set methods, interior point Quadratic programming etc. (Harkegard, 2002; Nocedal and Wright, 1999; Petersen and Bodson, 2005a,b)) have lower online computational burden and complexity (Spjøtvold and Johansen, 2009) than nonlinear control allocation approaches employing nonlinear programming. The use of linear control allocation approaches also simplifies the requirements for fault tolerance and reconfigurability.

Amongst the several linear and nonlinear control allocation approaches and techniques proposed for motion control of over-actuated mechanical systems (Durham et al., 2017; Fossen and Johansen, 2006; Johansen and Fossen, 2013), some of the control allocation scheme address prioritisation, such as linear programming and quadratic programming while other do not address prioritisation (e.g. ganging, redistributed pseudo-inverse, direct control allocation). Active set linear control allocation methods (Fossen and Johansen, 2006; Harkegard, 2002; Nocedal and Wright, 1999; Petersen and Bodson, 2005a,b) for Quadratic programming (QP) addresses control channel prioritisation and are iterative methods, whereby at each iteration they improve their guess of the optimal active set. Interior point method (Harkegard, 2002; Nocedal and Wright, 1999; Petersen and Bodson, 2005b) for quadratic programming on the other hand though also addresses control channel prioritisation, replaces the inequality constraints with a barrier function that prevents the solution from going into the infeasible region.

Although both active set and interior point QP CA methods determine the optimal solution in a finite number of iterations, the active set methods tend to perform well in control allocation problems (Harkegard, 2002), while interior point methods have their advantage for

larger-scale problems (Petersen and Bodson, 2005b). Active set methods have the advantage that their initialisation can use the previous sample (known as warm/hot start), which is often a good guess for the optimal solution at the current sample. This generally reduces the number of iterations needed to find the optimal solution. Interior-point methods on the other hand are generally initialized with points near the centre of the feasible region and will always need a minimum number of iterations in order to converge due to the need to reduce the barrier function penalty in several steps. Warm start procedures are therefore difficult to implement for interior-point methods (Fossen and Johansen, 2006). Quadratic Programming (QP) problems are usually solved using active set methods or interior point methods (Harkegard, 2002; Petersen and Bodson, 2005a,b).

4.4 Vehicle Model 3

The vehicle model is *Vehicle Model 3 - Planar VTOL (PVTOL) 3DoF Model* mentioned in Section 1.5 of Chapter 1. This is a 3DoF Model of an over-actuated canard-planform vectored thrust eVTOL, a planar VTOL model without any aerodynamics whose 6DoF version is shown in Figure 2.1.

Multirotor models are relatively complicated, hence some insight can be obtained by restricting the vehicle motion to the vertical plane. The vehicle then has just three degrees-of-freedom, so the models are simple and can be analysed using first level control knowledge. Hence a 3DoF planar VTOL model similar to the VTOL aircraft studies (Fantoni and Palomino, 2013; Whidborne and Cooke, 2017; Whidborne et al., 2022) is chosen.

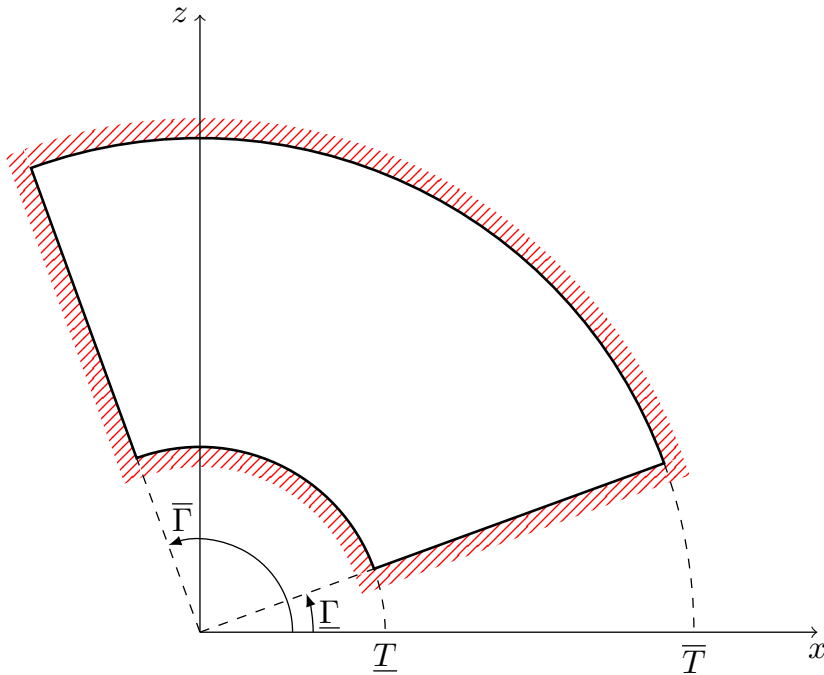


Fig. 4.2 Vectored Thrust Actuator Limits Illustration.

The vectored thrust actuator (thrust T and tilt angle Γ) limits are illustrated in Figure 4.2

which only shows the actuator position limits for simplicity. The unshaded regions are the set of admission controls U constrained by the effector limits. The physical/polar position and rate limits of the actuator thrust and tilting mechanism are as follows:

- *Actuator Thrust position limit:* $\bar{T}, \underline{T}$

$$\underline{T} \leq T \leq \bar{T}$$

- *Actuator Thrust rate limit:* $\dot{\bar{T}}, \dot{\underline{T}}$

$$\dot{\underline{T}} \leq \dot{T} \leq \dot{\bar{T}}$$

- *Tilting mechanism position limits:* $\bar{\Gamma}, \underline{\Gamma}$

$$\underline{\Gamma} \leq \Gamma \leq \bar{\Gamma}$$

- *Tilting mechanism rate limits:* $\dot{\bar{\Gamma}}, \dot{\underline{\Gamma}}$

$$\dot{\underline{\Gamma}} \leq \dot{\Gamma} \leq \dot{\bar{\Gamma}}$$

4.4.1 3DoF Mathematical Model

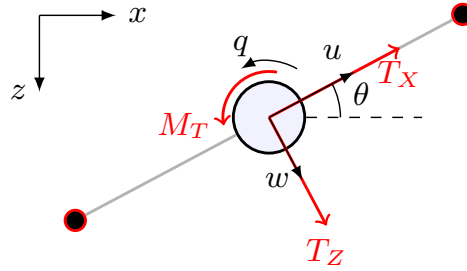


Fig. 4.3 3DoF motion showing aircraft state variables and virtual controls.

Considering 3DoF longitudinal motion, surge, heave and surge, the equations of motion are:

$$\dot{x} = u \cos \theta + w \sin \theta \quad (4.9)$$

$$\dot{z} = w \cos \theta - u \sin \theta \quad (4.10)$$

$$\dot{\theta} = q \quad (4.11)$$

$$\dot{u} = \frac{1}{m} F_X - g \sin \theta - qw \quad (4.12)$$

$$\dot{w} = \frac{1}{m} F_Z + g \cos \theta + qu \quad (4.13)$$

$$\dot{q} = \frac{1}{I_{yy}} M_T \quad (4.14)$$

The 3DoF schematic is shown in Figure 4.3. The longitudinal state variable vector $\mathbf{x}$ and virtual control vector $\boldsymbol{\tau}$ are respectively given by:

$$\mathbf{x} = [\theta \quad q \quad x \quad z \quad u \quad w]^T \quad (4.15)$$

$$\boldsymbol{\tau} = [F_X \quad F_Z \quad M_T]^T \quad (4.16)$$

4.5 3DoF Control Design

The controller structure/architecture (Figure 5.1) and the high-level controller (Classical NDI) is that already discussed in Chapter 3 and also presented in Enenakpogbe et al. (2022) for a 3DoF Model.

4.5.1 3DoF Control Architecture

In velocity control mode the outer loop references (input demands) are airspeed V_{cmd} and flight path angle λ_{cmd} (produces u_{cmd} and w_{cmd}), and pitch attitude θ_{cmd} (outputs pitch rate command q_{cmd}). For position control mode, the input demands are lateral position x_{cmd} , altitude h_{cmd} (acts as master loop to a slave innermost outer loop producing u_{cmd} and w_{cmd} respectively) and pitch attitude θ_{cmd} . Output of the outer loop, body-axis velocity commands u_{cmd} , w_{cmd} and pitch rate command q_{cmd} are feed as input demands to the NDI inner loop controller which directly controls the body-axis velocities u , w and pitch rate q .

4.5.2 3DoF NDI Control Law

The controller structure is shown in Figure 3.3, same structure as in section 3.5.1 and consists of the linear output feedback loop equations (3.47) and the state feedback linearisation loop (with linear controller $\mathbf{K}$ in equation (3.48)) which assumes all the states are observable. The feedback linearisation loop ensures that the system from $\mathbf{v}(t)$ to $\mathbf{y}(t)$ appears like a linear system with poles at the origin, thus simplifying the outer tracking loop design. The velocity feedforward term $\dot{\mathbf{r}}$ improves the tracking accuracy of the closed-loop system. This signal is generated either by the pilot inceptor for manned operation or the guidance system for automated flight.

Since the controller in Figure 3.3 contains a model of the aircraft dynamics (equation (3.39)), the nonlinear functions of the aircraft must be known in order to implement it. The classical NDI inner loop is airframe-dependent, uses an onboard (OBM) model of aircraft to invert and cancel the airframe dynamics such that the system appears as an integrator. The outer NDI loop is airframe-independent and can also be used for flying and handling quality control design. Refer to Enenakpogbe et al. (2022) and Chapter 3 for classical NDI controller details to verify the various control allocation approaches proposed in this chapter.

For this chapter, the selected controlled variable (CV), $\mathbf{y} = \mathbf{h}(\mathbf{x})$ for 3DOF (surge, heave, pitch) degrees of freedom are given by:

$$\mathbf{y} = (u, w, q)^T \quad (4.17)$$

4.5.3 3DoF Outer-Loop Linear Controller

Based on the terminology in Durham (2013), to control the uncontrollable states variables, the controllable states variables can be used as the controls resulting in a cascaded loop consisting of the inner controllable state variables (u, w, q) velocity loop (section 3.5.1) and an outer uncontrollable state variables (x, z, θ) position loop. A simple diagonal proportional (P) linear

controller is used for the outer loop control. For this 3DoF example, the outer-loop linear controller $\mathbf{K}_{\text{OUT}}$ in 5.1 (contained in $\mathbf{K}$ in Figure 4.1) consists of the following:

- Altitude ($h = -z$) controller - PI Controller
- Pitch attitude (θ) controller - P Controller
- Position (x) controller - PI Controller

4.6 General Control Allocation

In the classical problem (Durham et al., 2017) there is a known linear relationship between $\mathbf{u}$ and $\boldsymbol{\tau}$, that is $\boldsymbol{\tau}_c = \mathbf{B}_e \mathbf{u}$ where $\boldsymbol{\tau}_c$ is the virtual control input in the system model, usually in the form of a vector of generalized forces and moments and $\mathbf{B}_e$ is the linear effector mapping or control effectiveness matrix. The problem is then to determine the effector/actuator demand vector $\mathbf{u}$ such that $\boldsymbol{\tau} = \boldsymbol{\tau}_c$ with the constraint $\boldsymbol{\tau} = \mathbf{B}_e \mathbf{u}$. The scheme is illustrated in Figure 4.1.

Given ℓ as the number of real controls (effectors) and p as the number of degrees of freedom or control channels, then if $\ell = p$ then the solution is trivially given by $\mathbf{u} = \mathbf{B}_e^{-1} \boldsymbol{\tau}_c$ (for example the quadrotor control problem) under the assumption that $\mathbf{B}_e$ is full rank and that no effector constraints are breached. If $\ell > p$ then multiple solutions exist, for example $\mathbf{u} = \mathbf{B}_e^\dagger \boldsymbol{\tau}_c$ where $[\cdot]^\dagger$ represents the Moore-Penrose inverse of the matrix. Hence the problem is often extended to account for some other requirements (e.g. minimum power) as well as to take account of various constraints. The constraints are for example saturation constraints on $\mathbf{u}$ and slew rate constraints on $|\dot{\mathbf{u}}|$.

4.6.1 Control Allocation Formulation

As mentioned in the previous section, minimizing the control error ($s = \boldsymbol{\tau}_c - \mathbf{B}_e$) known as a primary objective or equality constraints, imposing additional physical constraints in form of effector/actuator saturation and rate limits, and introduction of secondary objective(s) to the control allocation algorithm ensuring a unique solution results in the following control allocation formulation:

$$\begin{aligned} \min_{\mathbf{u} \in \mathbb{R}^p, \mathbf{s} \in \mathbb{R}^m} \quad & (\|\mathbf{W}_v \mathbf{s}\| + J(\mathbf{x}, \mathbf{u}, t)) \text{ s.t.} \\ & \boldsymbol{\tau} - \mathbf{B}_e(\mathbf{u}, \mathbf{x}, t) = \mathbf{s}, \mathbf{u} \in \mathbf{U}, \mathbf{u} = \mathbf{u}_i + \cdot \mathbf{u}, \cdot \mathbf{u} \in \mathbf{C} \end{aligned} \quad (4.18)$$

where

- $\mathbf{s}$ is the slack variable representing the control error in all m control channels and degrees of freedom
- $\mathbf{W}_v$ is some weight matrix or control channel priority matrix $\mathbb{R}^{m \times m}$ that prioritizes the requirement that should be satisfied in case the commanded virtual control input $\boldsymbol{\tau}_c$ cannot be achieved due to actuator constraints or failure (i.e. $\boldsymbol{\tau}_c \neq \boldsymbol{\tau}$)

- $\mathbf{U}$ are the set of actuator saturation limit constraints
- $\mathbf{C}$ are the set actuator rate limit constraints
- J is the secondary objective cost function which is usually some norms. An example of generic cost function often used as secondary objectives are:

$$J(\mathbf{x}, \mathbf{u}, t) = \frac{1}{2}(\mathbf{u} - \mathbf{u}_p)^T \mathbf{W}_u (\mathbf{u} - \mathbf{u}_p) \quad (4.19)$$

$$J(\mathbf{x}, \mathbf{u}, t) = \|\mathbf{W}_u \mathbf{u}\| \quad (4.20)$$

where:

- Weighting $\mathbf{W}_u$ is actuator utilization or prioritization weight positive definite matrix $\mathbb{R}^{p \times p}$
- $\mathbf{u}_p$ is preferred actuator position

$\mathbf{W}_u$ is intentionally chosen small compared to $\mathbf{W}_v$ (i.e. $\mathbf{W}_u \ll \mathbf{W}_v$) to reflect the fact that J represents an objective that is secondary to the primary objective of minimizing the slack variables weighted by $\mathbf{W}_v$. The number of secondary objectives $J(\mathbf{x}, \mathbf{u}, t)$ can be more than one such as that studied in Beck (2002).

4.6.2 Active Set Prioritised Constrained Linear Control Allocation

The MATLAB[®] toolbox QCAT (Härkegard, 2004b) QP-based control allocation, an implementation of the methods in Harkegard (2002); Petersen and Bodson (2005b) was customised for use in this work.

The active set algorithm solves the bounded and equality constrained least squares problem by solving a sequence of equality constrained problems. In each step, some of the inequalities are regarded as equality constraints, and form the working set $\mathbf{W}$, while the remaining inequality constraints are disregarded. The control allocation method uses a dynamic model where the resulting control distribution depends not only on the current control virtual demand but also on the distribution in the previous sampling instant due to the actuator rate limit constraints.

As previously discussed in section 4.3, active set linear control allocation methods (Fossen and Johansen, 2006; Harkegard, 2002; Nocedal and Wright, 1999; Petersen and Bodson, 2005a,b) for Quadratic programming (QP) addresses control channel prioritisation using the value of $\mathbf{W}_v$ in equation 4.80 to overcome thrust control saturation, a major challenge and crucial constraint for the vectored thrust eVTOL configuration. Thrust control saturation due to actuator saturation can potentially lead to undesired behavior, deterioration of the controller performance and even loss of control in specific flight conditions. The loss of control can occur during normal and degraded operation (e.g. actuator limitation or failures) due to limited control authority. Other parameters for the prioritised constrained linear optimisation Active Set algorithm are listed below in Table 4.4 with detailed explanation presented in section 4.10.

4.7 CA Approach 1 - Extended Thrust Formulation

One of the main contribution of this work is the design of a novel control allocation architecture and approach for transforming the nonlinear control allocation problem for vectored thrust VTOL with nonlinear effector mapping to a linear problem by transferring the non-linearity from the effector mapping to the computation of the actuator limits as stated in Chapter 1 known as CA Approach 1 or extended thrust formulation CA. It retains the overall control demands (virtual control) in cartesian form but formulates the individual effector demand (real controls) in cartesian form (decomposing its thrust components on the body axes) resulting in a linear effector mapping allowing the use of classical linear control allocation techniques with prioritisation. The required computation of the actuator limits which converts the physical actuator limits from polar to cartesian form is however non-linear and complex offering a more rigorous and analytical solution to the problem of conversion of the actuator limits from polar to cartesian, an improvement over Enenakpogbe et al. (2024a); Suiçmez (2021); Suiçmez and Kutay (2023) whereby the solution is heuristic in nature. This control approach offers a real-time analytical optimal control allocation (i.e., does not utilize dedicated non-linear optimization software) which considers both actuator saturation and rate limits using active set prioritised linear QP weighted least squares (WLS) formulation as the linear control allocation algorithm. The computation of the actuator rate limits in cartesian form involves discretization (sampling over small time steps) of the actuator positions to ensure linearization, and considering the physical (polar) actuator limits (position and rates), the previous cycle value of the actuator positions in cartesian form and polar forms.

4.7.1 CA Approach 1 - Formulation of Real Controls

The 3DoF aircraft model showing the effector model is shown below:

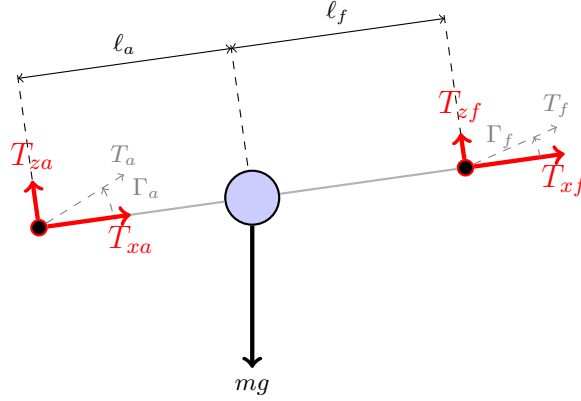


Fig. 4.4 CA Approach 1 - 3DoF VTOL schematic showing effector variables.

The effectors are a canard (fore) and main wing (aft) pair of decoupled thrust components along the body x and z axes T_{xf} and T_{xa} , T_{zf} and T_{za} respectively as shown in Figure 4.4. Formulating the vector of real controls in cartesian form as:

$$\mathbf{u}_{x,z} = [T_{xf} \quad T_{xa} \quad T_{zf} \quad T_{za}]^T.$$

$$(4.21)$$

and the generalised forces and moments (commanded virtual controls τ) as:

$$\tau_c = [F_X \quad F_Z \quad M_T]^T \quad (4.22)$$

The effector mapping is given by:

$$\begin{bmatrix} F_X \\ F_Z \\ M_T \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & \ell_c & \ell_w \end{bmatrix} \begin{bmatrix} T_{xf} \\ T_{xa} \\ T_{zf} \\ T_{za} \end{bmatrix} \quad (4.23)$$

Such that the effector model equation is of the form in equation 4.1 and is given below:

$$\tau_c = \mathbf{B}_e \mathbf{u}_{x,z} \quad (4.24)$$

where

$$\mathbf{B}_e = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & \ell_f & \ell_a \end{bmatrix}. \quad (4.25)$$

and ℓ_f and ℓ_a are the front and aft moment arms.

4.7.2 CA Approach 1 Functional Architecture

The control allocation architecture is shown in Figure 4.5 and consist of four parts:

- *Step 1 - Formulation of real controls in cartesian form resulting in a linear effector mapping*
- *Step 2 - Transformation of previous cycle real controls from cartesian to polar form*
- *Step 3 - Conversion of the actuator constraints from polar to cartesian*
- *Step 4 - Application of prioritised linear control allocation technique*

4.7.3 Computation of Unconstrained Controls

The unconstrained real control is given by the Moore-Penrose pseudo-inverse:

$$\mathbf{u}_{x,z} = \mathbf{B}_e^\dagger \tau_c \quad (4.26)$$

where:

$$\mathbf{B}_e^\dagger = \mathbf{B}_e^T (\mathbf{B}_e \mathbf{B}_e^T)^{-1} \quad (4.27)$$

where

- $[\cdot]^\dagger$ represents the Moore-Penrose inverse of the matrix
- $\mathbf{u}_{x,z}$ is the vector of real controls in Cartesian form (the x- and z- coordinate rotor thrust components).

4.7.4 Transformation of Previous Cycle Real Controls from Cartesian to Polar form

The previous cycle real controls value in polar form is given by:

$$T_0 = \sqrt{(T_{x0}^2 + T_{z0}^2)} \quad (4.28)$$

$$\Gamma_0 = \text{atan2}(T_{z0}, T_{x0}) \quad (4.29)$$

4.7.5 Conversion of Actuator Limits from Polar to Cartesian

Given a vectored thrust actuator which can produce thrust T which can be oriented at an angle Γ (rad) to the body x-axis within the body x-z plane subject to position and rate limit shown in Figure 4.2 in Section 4.10. Decoupling the actuator thrust (T (N) oriented at an angle Γ (rad)) into body x and z components assuming north east up (NEU) convention, we have:

$$T_x = T \cos(\Gamma) \quad (4.30)$$

$$T_z = T \sin(\Gamma) \quad (4.31)$$

Linearizing with small perturbations we get the Jacobian matrix:

$$\begin{bmatrix} \Delta T_x \\ \Delta T_z \end{bmatrix} = \begin{bmatrix} \cos \Gamma_0 & -T_0 \sin \Gamma_0 \\ \sin \Gamma_0 & T_0 \cos \Gamma_0 \end{bmatrix} \begin{bmatrix} \Delta T \\ \Delta \Gamma \end{bmatrix} \quad (4.32)$$

Expanding equations 4.32 gives:

$$\Delta T_x = \Delta T \cos \Gamma_0 - \Delta \Gamma T_0 \sin \Gamma_0 \quad (4.33)$$

$$\Delta T_z = \Delta T \sin \Gamma_0 + \Delta \Gamma T_0 \cos \Gamma_0 \quad (4.34)$$

Considering inputting all the combinations of minimum and maximum thrust (T) and tilt angle (Γ) limits (Figure 4.2 of Section 4.4) into equations 4.33, 4.34 and:

$$T_x = T_{x0} + \Delta T_x \quad (4.35)$$

$$T_z = T_{z0} + \Delta T_z \quad (4.36)$$

then the combined position and rate actuator limits in cartesian form illustrated in Figure 4.7 are given by:

$$\underline{T}_x = \min(T_{x\underline{T}\underline{\Gamma}}, T_{x\underline{T}\overline{\Gamma}}, T_{x\overline{T}\underline{\Gamma}}, T_{x\overline{T}\overline{\Gamma}}) \quad (4.37)$$

$$\overline{T}_x = \max(T_{x\underline{T}\underline{\Gamma}}, T_{x\underline{T}\overline{\Gamma}}, T_{x\overline{T}\underline{\Gamma}}, T_{x\overline{T}\overline{\Gamma}}) \quad (4.38)$$

$$\underline{T}_z = \min(T_{z\underline{T}\underline{\Gamma}}, T_{z\underline{T}\overline{\Gamma}}, T_{z\overline{T}\underline{\Gamma}}, T_{z\overline{T}\overline{\Gamma}}) \quad (4.39)$$

$$\overline{T}_z = \max(T_{z\underline{T}\underline{\Gamma}}, T_{z\underline{T}\overline{\Gamma}}, T_{z\overline{T}\underline{\Gamma}}, T_{z\overline{T}\overline{\Gamma}}) \quad (4.40)$$

where

$$T_{xT\underline{\Gamma}} = T_{x0} + \underline{\Delta T} \cos \Gamma_0 - \underline{\Delta \Gamma} T_0 \sin \Gamma_0 \quad (4.41)$$

$$T_{xT\overline{\Gamma}} = T_{x0} + \underline{\Delta T} \cos \Gamma_0 - \overline{\Delta \Gamma} T_0 \sin \Gamma_0 \quad (4.42)$$

$$T_{x\overline{T}\underline{\Gamma}} = T_{x0} + \overline{\Delta T} \cos \Gamma_0 - \underline{\Delta \Gamma} T_0 \sin \Gamma_0 \quad (4.43)$$

$$T_{x\overline{T}\overline{\Gamma}} = T_{x0} + \overline{\Delta T} \cos \Gamma_0 - \overline{\Delta \Gamma} T_0 \sin \Gamma_0 \quad (4.44)$$

$$T_{zT\underline{\Gamma}} = T_{z0} + \underline{\Delta T} \sin \Gamma_0 + \underline{\Delta \Gamma} T_0 \cos \Gamma_0 \quad (4.45)$$

$$T_{zT\overline{\Gamma}} = T_{z0} + \underline{\Delta T} \sin \Gamma_0 + \overline{\Delta \Gamma} T_0 \cos \Gamma_0 \quad (4.46)$$

$$T_{z\overline{T}\underline{\Gamma}} = T_{z0} + \overline{\Delta T} \sin \Gamma_0 + \underline{\Delta \Gamma} T_0 \cos \Gamma_0 \quad (4.47)$$

$$T_{z\overline{T}\overline{\Gamma}} = T_{z0} + \overline{\Delta T} \sin \Gamma_0 + \overline{\Delta \Gamma} T_0 \cos \Gamma_0 \quad (4.48)$$

and where the incremental physical/polar actuator limits are given by;

$$\overline{\Delta T} = \min((\overline{T} - T_0), (\dot{\overline{T}} \Delta t)) \quad (4.49)$$

$$\underline{\Delta T} = \max((\underline{T} - T_0), (\dot{\underline{T}} \Delta t)) \quad (4.50)$$

$$\overline{\Delta \Gamma} = \min((\overline{\Gamma} - \Gamma_0), (\dot{\overline{\Gamma}} \Delta t)) \quad (4.51)$$

$$\underline{\Delta \Gamma} = \max((\underline{\Gamma} - \Gamma_0), (\dot{\underline{\Gamma}} \Delta t)) \quad (4.52)$$

where Δt is the discretization time period of the control allocation module, T_0 and Γ_0 are respectively the thrust and actuator tilt angle at the initial or previous time step, $\overline{T}$, $\underline{T}$, $\dot{\overline{T}}$, $\dot{\underline{T}}$ are respectively the physical thrust saturation and rate limits, and $\overline{\Gamma}$, $\underline{\Gamma}$, $\dot{\overline{\Gamma}}$, $\dot{\underline{\Gamma}}$ are respectively the physical tilting mechanism actuator angle saturation and rate limits.

4.7.6 CA Approach 1 - Prioritised Linear CA

To solve the CA problem in equation 4.18 following the active set described in Harkegard (2002), the problem is first formulated in weighted least squares form given by:

$$\mathbf{u}_{op} = \arg \min_{\mathbf{u} \leq \mathbf{u} \leq \overline{\mathbf{u}}} \|\mathbf{W}_u(\mathbf{u} - \mathbf{u}_p)\|^2 + \gamma \|\mathbf{W}_v(\mathbf{B}_e \mathbf{u} - \boldsymbol{\tau})\|^2 \quad (4.53)$$

The cost function is incorporated in **Algorithm 1** given below:

The Active Set algorithm(**Algorithm 2**) is presented below.

Algorithm 1 - CA Approach 1 WLS Formulation for Active Set (Harkegard, 2002)

1. Let $\mathbf{u}^0$ and $\mathcal{W}$ be the optimal solution and working set respectively from the previous time step.

2. Rewrite the cost function as:

$$\|\mathbf{W}_u(\mathbf{u} - \mathbf{u}_p)\|^2 + \gamma\|\mathbf{W}_v(\mathbf{B}_e\mathbf{u} - \boldsymbol{\tau})\|^2 = \left\| \begin{pmatrix} \gamma\mathbf{W}_v\mathbf{B}_e \\ \mathbf{W}_u \end{pmatrix} \mathbf{u} - \begin{pmatrix} \gamma\mathbf{W}_v\boldsymbol{\tau} \\ \mathbf{W}_u\mathbf{u}_p \end{pmatrix} \right\|^2$$

3. and solve

$$\mathbf{u}_{op} = \arg \min_{\mathbf{u} \leq \mathbf{u} \leq \bar{\mathbf{u}}} \|\mathbf{A}_{as}\mathbf{u} - \mathbf{q}\|$$

using *Algorithm 2* where:

$$\mathbf{A}_{as} = \begin{pmatrix} \gamma\mathbf{W}_v\mathbf{B}_e \\ \mathbf{W}_u \end{pmatrix} \quad (4.54)$$

$$\mathbf{q} = \begin{pmatrix} \gamma\mathbf{W}_v\boldsymbol{\tau} \\ \mathbf{W}_u\mathbf{u}_p \end{pmatrix} \quad (4.55)$$

4. NOTE: The weighted rows are ordered first in $\mathbf{A}_{as}$ (row augmented weighted virtual control matrix) and $\mathbf{q}$ (row augmented weighted real control matrix) to avoid numerical problems.
-

Algorithm 2 - CA Approach 1 Active Set Algorithm (Harkegard, 2002)

1. Let $\mathbf{u}^0$ be a feasible starting point. A point is feasible if it satisfies both the control constraint ($\mathbf{B}_e \mathbf{u} = \mathbf{v}$) and actuator constraints ($\underline{\mathbf{u}} \leq \mathbf{u} \leq \bar{\mathbf{u}}$).
2. Let the working set $\mathbf{W}$ contain (a subset of) the active inequality constraints at $\mathbf{u}^0$
3. **for** $k = 0, 1, 2, \dots, N - 1$ **do**
4. Given $\mathbf{u}^k$ the value of $\mathbf{u}$ at the k^{th} iteration, find the optimal perturbation $\mathbf{p}$, considering the constraints in the working set $\mathcal{W}$ as equality constraints and disregarding the remaining inequality constraints. Solve

$$\begin{aligned}
 & \min_{\mathbf{p}} \|\mathbf{A}(\mathbf{u}_k + \mathbf{p}) - \mathbf{q}\| \\
 & \mathbf{B}_e \mathbf{p} = 0 \\
 & \mathbf{p}_i = 0, i \in \mathbf{W}
 \end{aligned} \tag{4.56}$$

where $\mathbf{A}_{as}$ and $\mathbf{q}$ are as defined in equations 4.54 and 4.55

5. **if** $\mathbf{u}^k + \mathbf{p}$ is feasible **then**
 6. Set $\mathbf{u}^{k+1} = \mathbf{u}^k + \mathbf{p}$ and compute the Lagrange multipliers, $\begin{pmatrix} \boldsymbol{\mu} \\ \boldsymbol{\lambda} \end{pmatrix}$, where $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ are associated with the the control constraint ($\mathbf{B}_e \mathbf{u} = \mathbf{v}$) and actuator constraints ($\underline{\mathbf{u}} \leq \mathbf{u} \leq \bar{\mathbf{u}}$) respectively
 7. **if** all elements of $\boldsymbol{\lambda} \geq 0$ **then**
 8. $\mathbf{u}_{op} = \mathbf{u}^{k+1}$ is the optimal solution to equation 4.56
 9. **else**
 Remove the constraint associated with the most negative $\boldsymbol{\lambda}$ from the working set
 10. **else**
 Determine the maximal step length α such that $\mathbf{u}^{k+1} = \mathbf{u}^k + \alpha \mathbf{p}$ is feasible.
 Add the primary bounding constraint to the working set.
 11. **end**
-

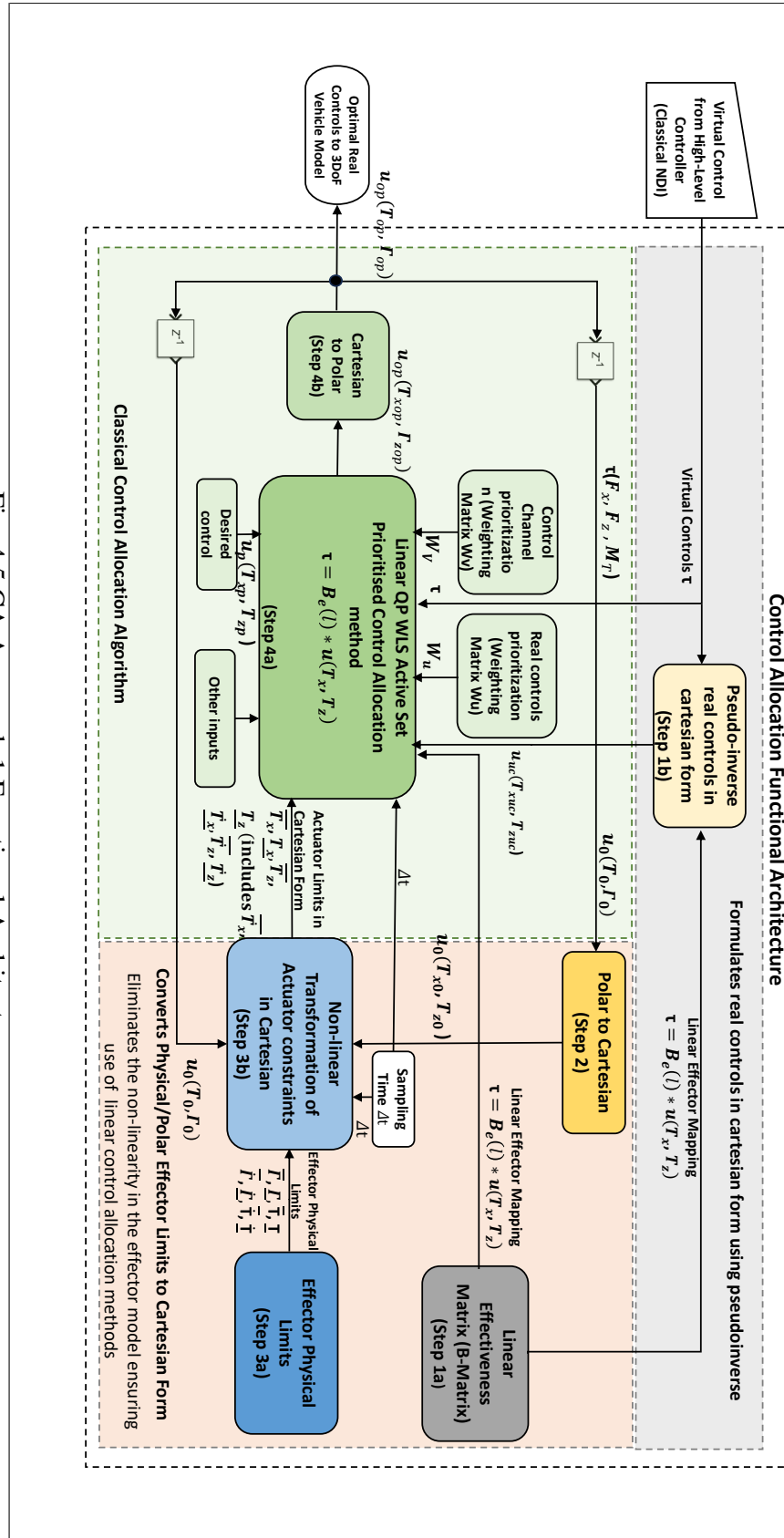


Fig. 4.5 CA Approach 1 Functional Architecture

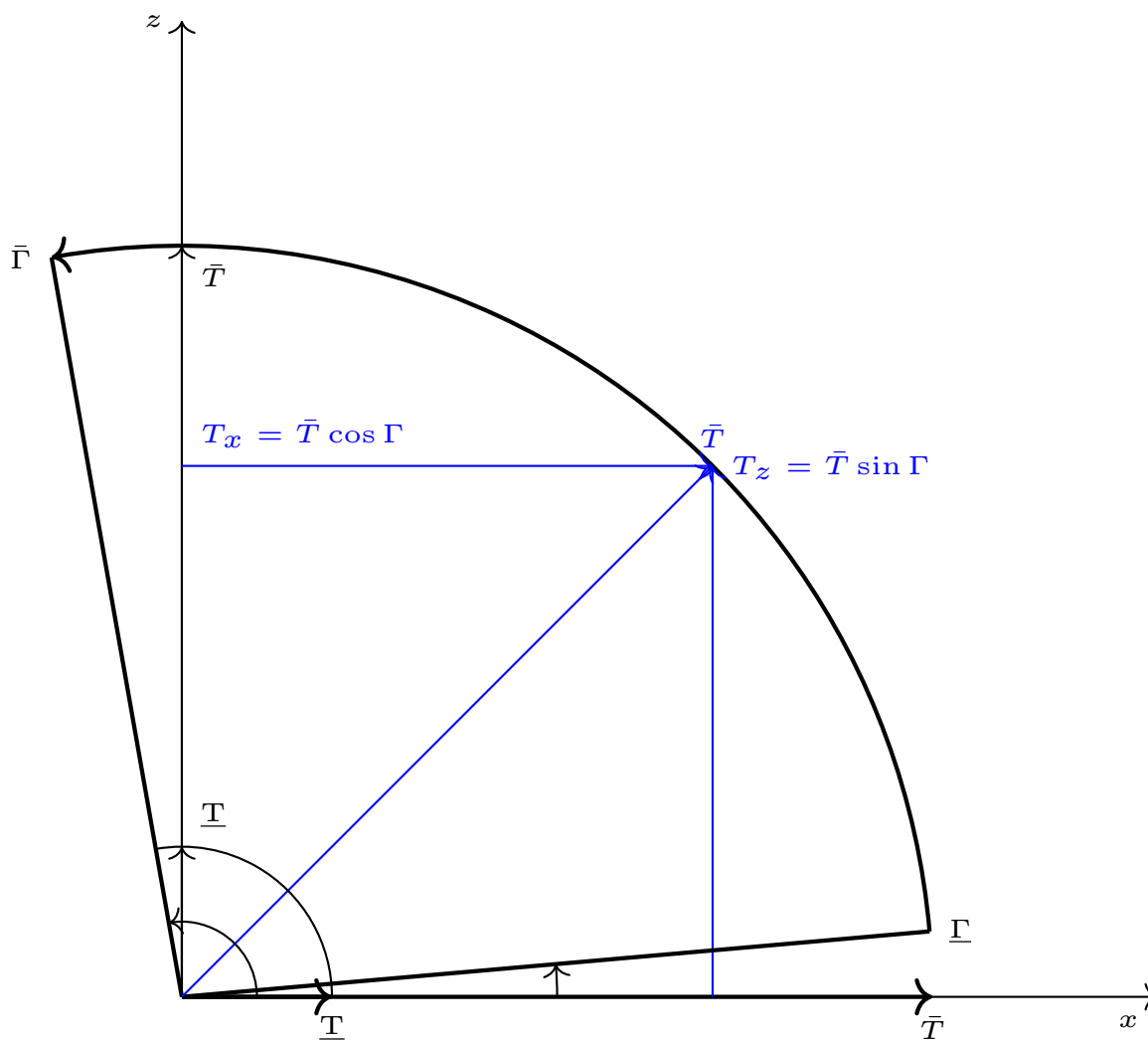


Fig. 4.6 Polar to Cartesian Plane Illustration

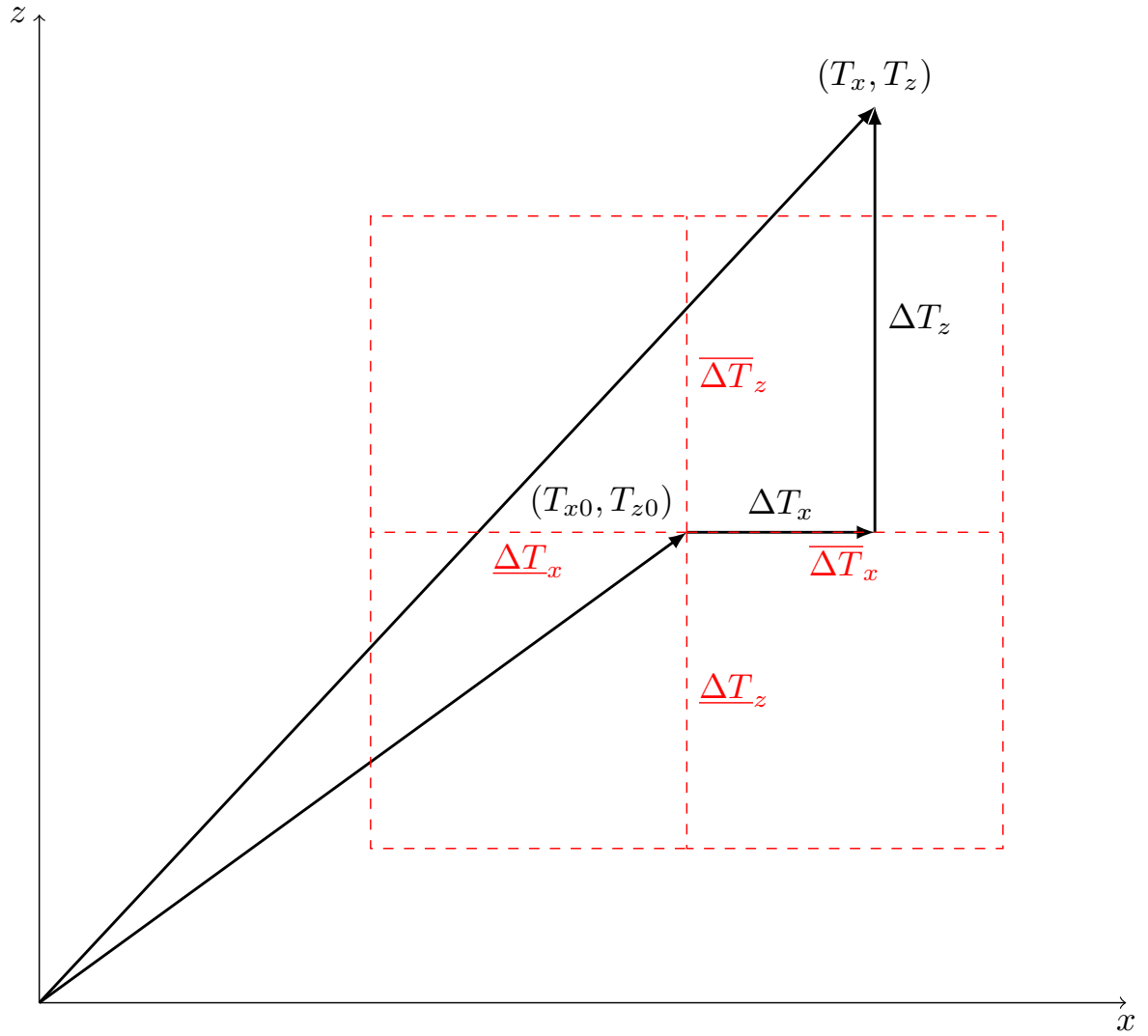


Fig. 4.7 Combined position and rate cartesian limits illustration

4.8 CA Approach 2

As stated in section 1, a second contribution of this work is the proposal of a second approach (Control Allocation Approach 2) which is a simple mechanism to transform a non-linear problems to linear problems similar to that suggested by Harkegard (2003b) and in the work of Bordignon (1996); Doman and Oppenheimer (2002); Matamoros and deVisser (2018); Pfeifle and Fichter (2023) for traditional flight control surfaces. This is done by a time discretization (numerical linearisation) of small enough steps to ensure the linearization step is accurate. At each time-step, a first order Taylor expansion of the mapping at that point in the control space (in a similar manner to an Extended Kalman Filter (Grewal and Andrews, 2014, pp. 374). Classical control allocation can then be applied and the constraints satisfied. The resulting point in the effector space is tracked and the process repeated at each time-step. Again the scheme relies on the feedback control of the system to account for uncertainty in the linearization and effector mappings. Sometimes the effector forces and moments depend on the system state $\mathbf{x}(t) \in \mathbb{R}^n$ as well as the effector controls.

Although for this system, the rotor thrust for a particular rotor rotational speed depends on the air density which in turn is dependent on altitude (an aircraft state), for simplicity and since the thrust (rather than the rotation speed or RPM) is used as the real control changes in air density over the altitude range of the VTOL are ignored. In reality, for this kind of VTOL configuration where there are aerodynamic couplings with the propulsive forces such as that studied in Moses (2021) where the total 3DoF/longitudinal virtual forces/generalised forces and moment (net forward thrust/drag, lift and pitching moment) are a complex (3D) function of the controls $\mathbf{u}$ (fan power/RPM, tilt angle Γ) and states $\mathbf{x}$ (AoA α and airspeed V), equation 4.60 of control allocation approach 2 can be used but the jacobians (differentials) of the virtual forces with respect to the states (α , V) and controls (RPM, Γ) have to be computed offline numerically and stored onboard aircraft as complex 3D lookup tables as stated in Bordignon (1996) for the case of traditional aircraft control surfaces.

The functional architecture is shown in figure 4.9. It converts the overall control demands (virtual control) from absolute cartesian to incremental cartesian form before the numerical linearisation of the nonlinear effector mapping to a linear time varying (LTV) effector mapping. Since the real controls are formulated in polar form it requires only a transformation of the actuator limits from absolute to incremental polar form. It offers a real-time analytical optimal control allocation which considers both actuator saturation and rate limits using active set priorotised linear QP WLS formulation as the linear control allocation algorithm.

4.8.1 CA Approach 2 Drift and Correction

. The virtual controls input are discretised (incremental, with small time steps taken) to enable dynamic computation of the LTV effector mapping with an incremental polar real controls output from the algorithm before being reintegrated again to give the current controls output. However, since the virtual controls input to the CA algorithm are discretised, this scheme is prone to drift in the presence of noise in either the differentiated virtual controls or the CA incremental output. Also, it is prone to drift because the CA algorithm computes only the change in control and unable to track the reference due to the control allocation being open

loop in terms of system states despite being closed loop in terms of the effector positions because the current control is a function of the previous actuator positions.

To correct the drift, similar approaches used in Kalman filter (KF) and Extended Kalman Filter (EKF) (Grewal and Andrews, 2014, pp. 374) is proposed to update the estimate/prediction using measurement from another source which does not drift (often a slower and less noiser source). A complementary filter (with static gain) is proposed unlike the KF and EKF where the Kalman Gain is dynamic and varies depending on errors in estimation and measurement. The choice of a simple complementary filter is made because the characteristics of the noise which is due to differences (unable to track changes) in unconstrained and constrained control allocation outputs (changes in real controls) during actuator saturation is not zero-mean gaussian - a major prerequisite/assumptions for use of Kalman Filter. The main incremental CA outputs is the estimate/prediction while the differentiated non-incremental (absolute) CA output sources is selected as the measurement/update. Without this drift correction via a complementary filter, the CA output will drift after actuator saturation resulting in a steady state error in the system states in a similar fashion to insufficient integration action in the control laws.

4.8.2 CA Approach 2 Effector Model Formulation

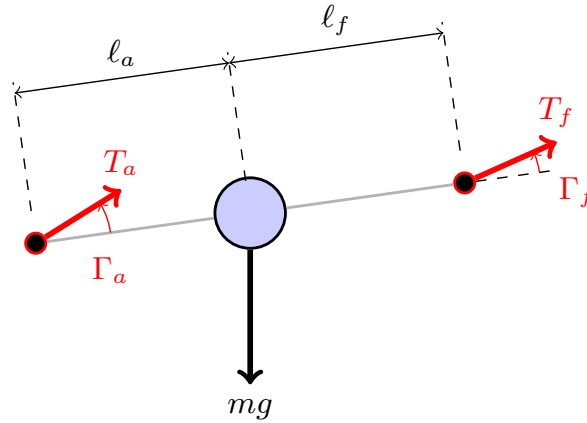


Fig. 4.8 3DoF VTOL schematic showing the effector variables for CA Approaches 2 and 3

The effectors (real controls) in polar form are fore and aft pair of vectored thrusters with thrust T_f and T_a acting at angles Γ_f and Γ_a respectively as shown in Figure 4.8.

The control forces and moment (overall control demand or virtual controls) are thus given by

$$T_X = T_f \cos \Gamma_f + T_a \cos \Gamma_a, \quad (4.57)$$

$$T_Z = -T_f \sin \Gamma_f - T_a \sin \Gamma_a, \quad (4.58)$$

$$M_T = -T_f \ell_f \sin \Gamma_f + T_a \ell_a \sin \Gamma_a, \quad (4.59)$$

where ℓ_f and ℓ_a are the front and aft thrusters moment arms respectively.

Defining the virtual forces and moment control vector as $\mathbf{\tau}_c = [F_X, F_Z, M_T]^T$ and the

real control vector as $\mathbf{u} = [T_f, T_a, \Gamma_f, \Gamma_a]^T$ we get the nonlinear mapping $\boldsymbol{\tau}_c(t) = \mathbf{g}_e(\mathbf{u}(t))$ defined by equations (4.57)–(4.59).

4.8.3 CA Approach 2 Functional Architecture

The control allocation architecture is shown in Figure 4.9 and consist of five steps:

- *Step 1 - Computation of the actuator incremental limits at each time step*
- *Step 2 - Linearization of the nonlinear effector mapping to linear-time-varying effector mapping*
- *Step 3 - Discretisation of virtual controls and computation of unconstrained real controls*
- *Step 4 - Application of classical constrained linear control allocation technique*
- *Step 5 - Correction of drift using complementary filter*

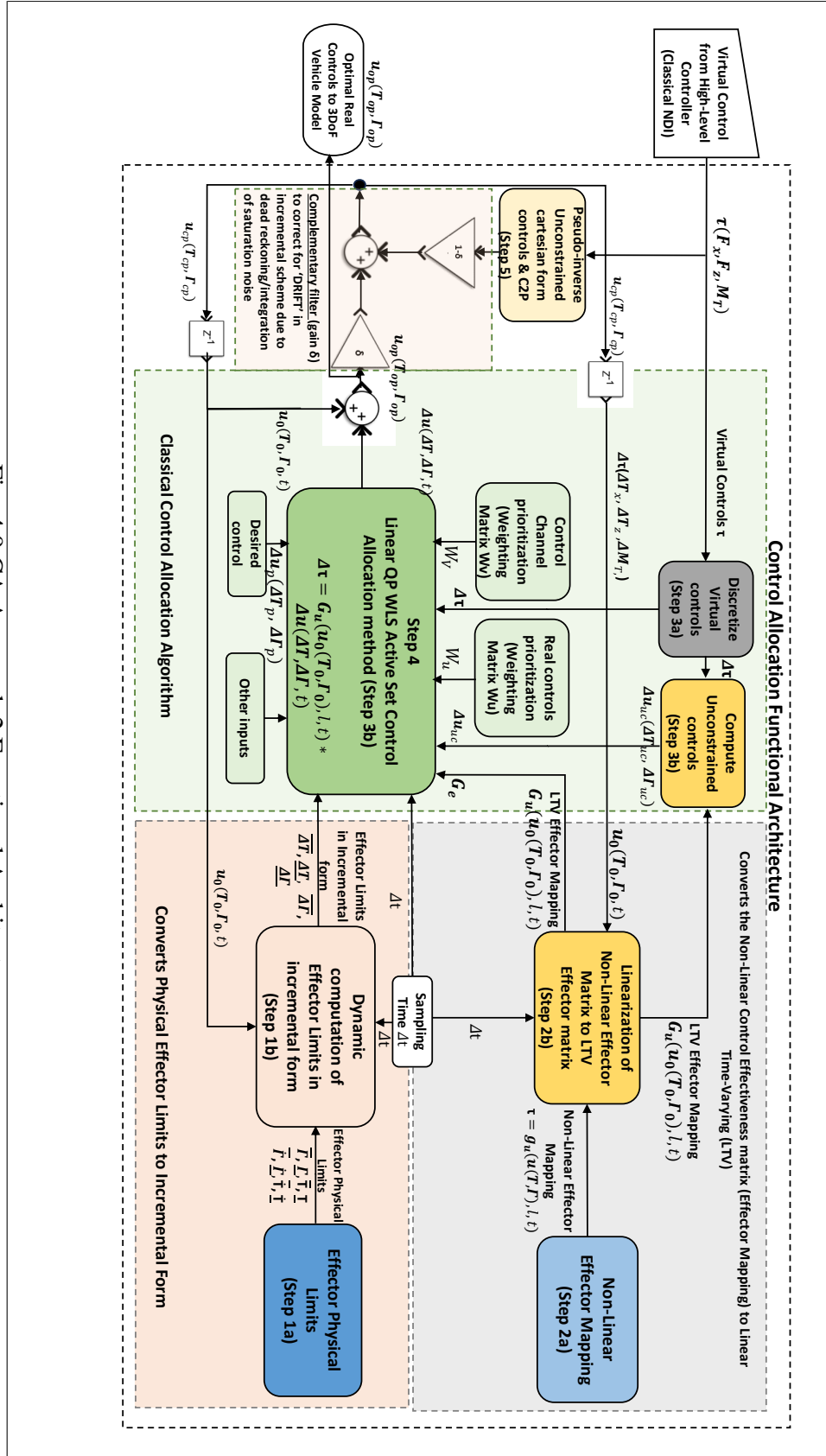


Fig. 4.9 CA Approach 2 Functional Architecture

4.8.4 Control Allocation Incremental Constraints Formulation

The incremental actuator limits ($\bar{T}, \underline{T}, \dot{\bar{T}}, \underline{\dot{T}}, \bar{\Gamma}, \underline{\Gamma}, \dot{\bar{\Gamma}}, \underline{\dot{\Gamma}}$) are given by equations 4.49 - 4.52 in section 4.7.5.

4.8.5 CA Approach 2 - Derivation of LTV Effector Mapping

Given the nonlinear effector mapping given in equation 4.5, by means of a time discretisation we can approximate the problem to a sequence of linear control allocation problems. The proposed solution is given by the following steps:

1. Sample the signals $\boldsymbol{\tau}(t)$, $\mathbf{x}(t)$, $\mathbf{u}(t)$, $\boldsymbol{\tau}_c(t)$ with a sample time of Δt so that $\boldsymbol{\tau}_k = \boldsymbol{\tau}(k\Delta t)$, $\mathbf{x}_k = \mathbf{x}(k\Delta t)$, $\mathbf{u}_k = \mathbf{u}(k\Delta t)$, $\boldsymbol{\tau}_{ck} = \boldsymbol{\tau}_c(k\Delta t)$ for $k = 0, 1, 2, \dots$, etc
2. For each sample point, k , let $\Delta\boldsymbol{\tau}(k\Delta t) = \boldsymbol{\tau}(k\Delta t + \Delta t) - \boldsymbol{\tau}(k\Delta t)$ etc
3. Given $\boldsymbol{\tau}(k\Delta t) = \mathbf{g}_e(\mathbf{x}(k\Delta t), \mathbf{u}(k\Delta t))$, and dropping the notational dependence on $k\Delta t$, we get the Taylor's expansion

$$\mathbf{g}_e(\mathbf{x} + \Delta\mathbf{x}, \mathbf{u} + \Delta\mathbf{u}) = \mathbf{g}_e(\mathbf{x}, \mathbf{u}) + \frac{\partial \mathbf{g}_e}{\partial \mathbf{x}} \Delta\mathbf{x} + \frac{\partial \mathbf{g}_e}{\partial \mathbf{u}} \Delta\mathbf{u} + \text{higher order terms} \quad (4.60)$$

Ignoring second and higher order terms gives

$$\Delta\boldsymbol{\tau} = \mathbf{G}_x \Delta\mathbf{x} + \mathbf{G}_u \Delta\mathbf{u} \quad (4.61)$$

where

$$\mathbf{G}_x(k\Delta t) = \left[\frac{\partial \mathbf{g}_e(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} \right]_{\substack{\mathbf{x}=\mathbf{x}(k\Delta t) \\ \mathbf{u}=\mathbf{u}(k\Delta t)}} \quad \mathbf{G}_u(k\Delta t) = \left[\frac{\partial \mathbf{g}_e(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}} \right]_{\substack{\mathbf{x}=\mathbf{x}(k\Delta t) \\ \mathbf{u}=\mathbf{u}(k\Delta t)}} \quad (4.62)$$

4. We now have a linear control allocation equation

$$\mathbf{G}_u \Delta\mathbf{u} = \Delta\boldsymbol{\tau} - \mathbf{G}_x \Delta\mathbf{x} \quad (4.63)$$

which, along with magnitude and slew rate constraints can be solved at each time step k using classical control allocation methods.

The gradient matrix $\mathbf{G}_u$ is given by

$$\mathbf{G}_u = \left[\frac{\partial \mathbf{g}_e(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}} \right]_{\mathbf{u}=\mathbf{u}(k\Delta t)} \quad (4.64)$$

$$= \begin{bmatrix} \cos \Gamma_f & \cos \Gamma_a & -T_f \sin \Gamma_f & -T_a \sin \Gamma_a \\ -\sin \Gamma_f & -\sin \Gamma_a & -T_f \cos \Gamma_f & -T_a \cos \Gamma_a \\ -\ell_f \sin \Gamma_f & \ell_a \sin \Gamma_a & -\ell_f T_f \cos \Gamma_f & \ell_a T_a \cos \Gamma_a \end{bmatrix}. \quad (4.65)$$

Note that for this system, $\mathbf{G}_x = 0$ since the aerodynamics and their couplings with propulsive forces are not considered, unlike traditional aircraft control surfaces (e.g. rudder, aileron, elevators etc.) where the control effectiveness depends not only on the control deflections but also on the states variables (e.g. airspeed V , AoA or theta for wing level, level flight etc).

4.8.6 CA Approach 2 - Computation of Unconstrained Real Controls

The unconstrained real control is given by the Moore-Penrose pseudo-inverse:

$$\mathbf{u}_{uc} = \mathbf{G}_u^\dagger \boldsymbol{\tau}_c \quad (4.66)$$

where:

$$\mathbf{G}_u^\dagger = \mathbf{G}_u(\mathbf{x}, t)^T (\mathbf{G}_u(\mathbf{x}, t) \mathbf{G}_u(\mathbf{x}, t)^T)^{-1} \quad (4.67)$$

4.8.7 CA Approach 2 - Correction of Drift

The complementary filter is shown in step 5 of Figure 4.9. The drift correction update is done using the unconstrained controls in cartesian form (of CA Approach 1) in equations 4.26 to 4.27 and then transformed from cartesian form to compatible polar form using equation 4.28 with a gain set to 0.95 resulting in only 5% of the update source and 95% of the incremental scheme constituting the final control which is subsequently constrained by the incremental actuator limits in the classical CA algorithm (Active Set). Explanation of the drift and correction has been provided in section 4.8.1.

4.8.8 CA Approach 2 - Prioritised Incremental Linear CA

To solve the CA problem in equation 4.18, the absolute values of virtual and real controls were changed to incremental (Δ) value. Following the active set described in Harkegard (2002), the problem is first formulated in weighted least squares form below:

$$\Delta \mathbf{u}_{op} = \arg \min_{\Delta \mathbf{u} \leq \Delta \mathbf{u} \leq \overline{\Delta \mathbf{u}}} \left\| \mathbf{W}_u (\Delta \mathbf{u} - \Delta \mathbf{u}_p) \right\|^2 + \gamma \left\| \mathbf{W}_v (\mathbf{G}_u \Delta \mathbf{u} - \Delta \boldsymbol{\tau}) \right\|^2 \quad (4.68)$$

The cost function is incorporated in *Algorithm 3* given below as follows:

Algorithm 3 - Active Set Algorithm for Weighted Least Squares (WLS) (Harkegard, 2002)

1. Let $\mathbf{u}^0$ and $\mathcal{W}$ be the optimal solution and working set respectively from the previous sampling instant.

2. Rewrite the cost function as:

$$\left\| \mathbf{W}_u (\Delta \mathbf{u} - \Delta \mathbf{u}_p) \right\|^2 + \gamma \left\| \mathbf{W}_v (\mathbf{G}_u \Delta \mathbf{u} - \Delta \boldsymbol{\tau}) \right\|^2 = \left\| \begin{pmatrix} \gamma \mathbf{W}_v \mathbf{G}_u \\ \mathbf{W}_u \end{pmatrix} \Delta \mathbf{u} - \begin{pmatrix} \gamma \mathbf{W}_v \Delta \boldsymbol{\tau} \\ \mathbf{W}_u \Delta \mathbf{u}_p \end{pmatrix} \right\|^2$$

3. and solve

$$\Delta \mathbf{u}_{op} = \arg \min_{\Delta \mathbf{u} \leq \Delta \mathbf{u} \leq \overline{\Delta \mathbf{u}}} \left\| \mathbf{A}_{as} \Delta \mathbf{u} - \mathbf{q} \right\|^2$$

using *Algorithm 4* where $\mathbf{A}_{as}$ and $\mathbf{q}$ are as defined in equations 4.54 and 4.55.

Algorithm 4 - Active Set Algorithm (Harkegard, 2002)

1. Let $\Delta \mathbf{u}^0$ be a feasible starting point. A point is feasible if it satisfies both the control constraint ($\mathbf{G}_e \Delta \mathbf{u} = \Delta \boldsymbol{\tau}$) and actuator constraints ($\underline{\Delta \mathbf{u}} \leq \Delta \mathbf{u} \leq \overline{\Delta \mathbf{u}}$).
2. Let the working set $\mathcal{W}$ contain (a subset of) the active inequality constraints at $\mathbf{u}^0$
3. **for** $k = 0, 1, 2, \dots, N - 1$ **do**
4. Given $\Delta \mathbf{u}^k$ the value of $\Delta \mathbf{u}$ at the k^{th} step, find the optimal perturbation $\mathbf{p}$, considering the constraints in the working set as equality constraints and disregarding the remaining inequality constraints. Solve

$$\begin{aligned}
 & \min_{\mathbf{p}} \|\mathbf{A}_{as}(\Delta \mathbf{u}_k + \mathbf{p}) - \mathbf{G}_u\| \\
 & \mathbf{G}_u \mathbf{p} = 0 \\
 & \mathbf{p}_i = 0, i \in \mathbf{W}
 \end{aligned} \tag{4.69}$$

where $\mathbf{A}_{as}$ and $\mathbf{q}$ are as defined in equations 4.54 and 4.55

5. **if** $\Delta \mathbf{u}^k + \mathbf{p}$ is feasible **then**
 6. Set $\Delta \mathbf{u}^{k+1} = \Delta \mathbf{u}^k + \mathbf{q}$ and compute the Lagrange multipliers, $\begin{pmatrix} \boldsymbol{\mu} \\ \boldsymbol{\lambda} \end{pmatrix}$, where $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ are associated with the the control constraint ($\mathbf{G}_u \Delta \mathbf{u} = \Delta \boldsymbol{\tau}$) and actuator constraints ($\underline{\Delta \mathbf{u}} \leq \Delta \mathbf{u} \leq \overline{\Delta \mathbf{u}}$) respectively
 7. **if** all elements of $\boldsymbol{\lambda} \geq 0$ **then**
 8. $\Delta \mathbf{u}_{op} = \Delta \mathbf{u}^{k+1}$ is the optimal solution to equation 4.69.
 9. **else**
 10. Remove the constraint associated with the most negative λ from the working set
 11. **else**
 Determine the maximal step length α such that α such that $\Delta \mathbf{u}^{k+1} = \Delta \mathbf{u}^k + \alpha \mathbf{p}$ is feasible. Add the primary bounding constraint to the working set.
 12. **end**
-

4.9 CA Approach 3

The final contribution of this chapter is CA 3 which is similar to CA 2 except an extra step which transforms the virtual controls from cartesian to polar before differentiation and linearisation resulting in a different and more complicated LTV Effector mapping (Jacobian of τ with respect to u). The other aspects (incremental actuator constraints and AS WLS Prioritised constrained CA) are the same as can be seen from the functional architectures in Figures 4.5 and 4.9. Also, the linearisation is analytical which is more accurate than numeric linearisation for the case of Approach 2. The transformation of the virtual control to same polar form as the real controls and actuator limits has the advantage that the resulting effector mapping though more complex, is more linear in the real controls and actuator limits.

It offers a real-time analytical optimal control allocation which considers both actuator saturation and rate limits using active set prioritised linear QP WLS formulation as the linear CA algorithm. The CA also relies on the feedback control of the system to account for uncertainty in the effector mappings.

4.9.1 CA Approach 3 Effector Model Formulation

The 3DoF aircraft model showing the effector model is the same as in Figure 4.8 and the effector mapping same as in equations 4.57 - 4.59.

4.9.2 CA Approach 3 Functional Architecture

The control allocation architecture is shown in Figure 4.10 and consist of five steps:

- *Step 1 - Conversion of the actuator constraints from absolute to incremental limits*
- *Step 2 - Linearization of the Non-Linear Effector mapping to Linear-Time-Varying Effector Mapping*
- *Step 3 - Conversion of virtual controls from cartesian to polar, discretisation of virtual controls and computation of unconstrained real controls*
- *Step 4 - Application of classical constrained linear control allocation technique*
- *Step 5 - Correction of drift using complementary filter*

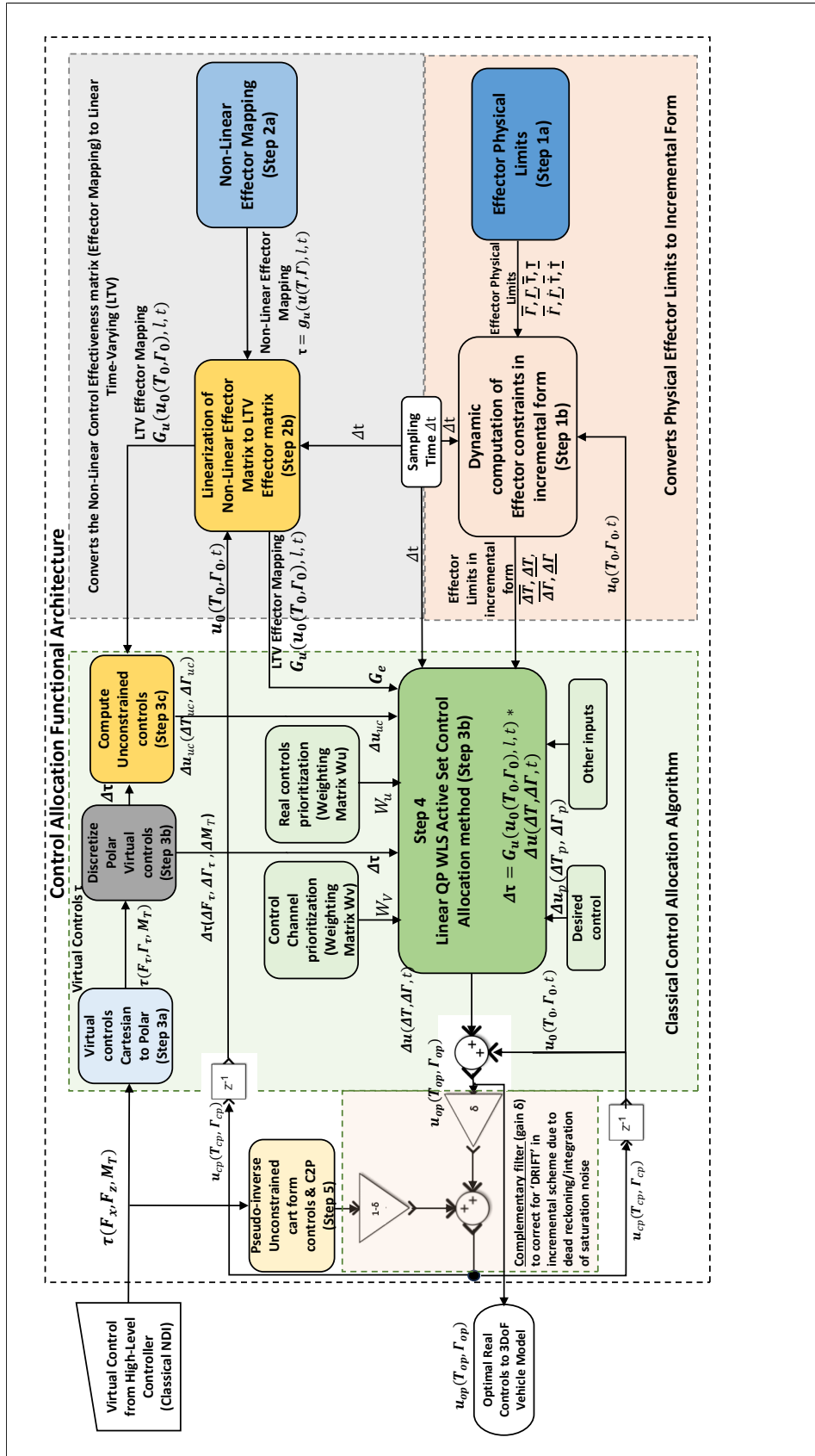


Fig. 4.10 CA Approach 3 Functional Architecture

4.9.3 Approach 3 - Incremental Constraints Formulation

Again, the incremental actuator limits are given by equations 4.49 - 4.52 in section 4.7.5.

4.9.4 Approach 3 - Conversion of the Virtual Controls from Cartesian to Polar

Following on from the effector model formulation in section 4.9.1, the relationship between the virtual controls in the polar body axes and the real controls are then:

$$F_\tau \sin \Gamma_\tau = T_f \sin \Gamma_f + T_a \sin \Gamma_a \quad (4.70)$$

$$F_\tau \cos \Gamma_\tau = T_f \cos \Gamma_f + T_a \cos \Gamma_a \quad (4.71)$$

giving

$$F_\tau = \sqrt{(T_a \cos \Gamma_a + T_f \cos \Gamma_f)^2 + (T_a \sin \Gamma_a + T_f \sin \Gamma_f)^2}, \quad (4.72)$$

$$\Gamma_\tau = \text{atan2}(T_a \sin \Gamma_a + T_f \sin \Gamma_f, T_a \cos \Gamma_a + T_f \cos \Gamma_f), \quad (4.73)$$

$$M_T = \ell_f T_f \sin \Gamma_f - \ell_a T_a \sin \Gamma_a. \quad (4.74)$$

4.9.5 CA Approach 3 - Derivation of LTV Effector Mapping

In this CA approach 3, a simple mechanism to transform such non-linear problems to linear problems is proposed. This is done by analytical linearization by taking small time steps to ensure the control change is small and the linearization step is accurate. Classical control allocation can then be applied and the constraints satisfied. The resulting point in the effector space is tracked and the process repeated at each time-step. Again the scheme relies on the feedback control of the system to account for uncertainty in the linearization and effector mappings.

Linearizing equations 4.72 - 4.74 with small perturbations, we get a linear control allocation equation

$$\Delta \mathbf{\tau} = \mathbf{G}_u \Delta \mathbf{u} \quad (4.75)$$

which, along with magnitude and slew rate constraints can be solved at each time step k using classical control allocation methods.

- where the gradient matrix $\mathbf{G}_u$ is given by:

$$\mathbf{G}_u = \left[\frac{\partial \mathbf{g}_e(\mathbf{u})}{\partial \mathbf{u}} \right]_{\mathbf{u}=\mathbf{u}(kT)} \quad (4.76)$$

$$= \begin{bmatrix} (T_f + T_a \cos \Gamma_{\tau 2})d & (T_a + T_f \cos \Gamma_{\tau 2})d & T_a T_f \sin \Gamma_{\tau 2}d \\ -T_a \sin \Gamma_{\tau 2}d^2 & T_f \sin \Gamma_{\tau 2}d^2 & T_f (T_f + T_a \cos \Gamma_{\tau 2})d^2 \\ \ell_f \sin \Gamma_f & -\ell_a \sin \Gamma_a & \ell_f T_f \cos \Gamma_f \\ -T_a T_f \sin \Gamma_{\tau 2}d & T_a (T_a + T_f \cos \Gamma_{\tau 2})d^2 & -\ell_a T_a \cos \Gamma_a \end{bmatrix} \quad (4.77)$$

$$d = \frac{1}{(T_a^2 + 2T_a T_f \cos \bar{\Gamma} + T_f^2)^{1/2}} \quad (4.78)$$

$$\bar{\Gamma} = \Gamma_a - \Gamma_f \quad (4.79)$$

4.9.6 CA Approach 3 - Computation of Unconstrained Real Controls

The unconstrained real control is as given in equation 4.66 and 4.67 for CA Approach 2.

4.9.7 CA Approach 3 - Correction of Drift

The drift correction update is shown in step 5 of figure 4.10 and is same as that in CA Approach 2 in section 4.8.7.

4.9.8 Approach 3 - Prioritised Incremental Linear CA

Refer to section 4.8.8 for algorithms 3 and 4 presented in CA Approach 2.

4.10 Simulation Results and Discussions

The vehicle model and NDI controller are coded in Simulink and MATLAB simulation software. All the simulations are conducted in MATLAB Simulink with Software version R2021b on a personal computer of Intel Core i9-8950H 2.90GHz CPU and 32GB RAM. The MATLAB simulation was run at a fixed step sample time time of 10ms (100Hz).

The constants (outer-loop controller parameters, pilot command limits and control limits) used for the model are given in Table 4.2 with an assumption of noise-free measurements. The actuator constraints and dynamics are presented in Table 4.3. Also, the inputs constants into the prioritised constrained linear optimisation Active Set algorithm are listed in Table 4.4.

$\mathbf{W}_v$ and $\mathbf{W}_u$ are:

$$\mathbf{W}_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 10 \end{bmatrix} \quad (4.80)$$

$$\mathbf{W}_u = \mathbf{I}_4 \quad (4.81)$$

Table 4.2 Simulink Simulation Coefficients.

Quantity or Parameters	Value
Total mass, m	1.0 kg
Gravitational constant, g	9.81 m s ⁻²
Moment of inertia in the pitch axis, I_{yy}	2 kg m s ²
Inner loop NDI u tracking loop P gain, K_{up}	1.5 N m ⁻¹ s
Inner loop NDI w tracking loop P gain, K_{wp}	1.5 N m ⁻¹ s
Inner loop NDI q loop P gain, K_{qp}	1.5 N m rad ⁻¹ s
Outer loop altitude control P gain, K_{hp}	1.5 N m rad ⁻¹
Outer loop altitude control integral gain, K_{hi}	0.2 N m rad ⁻¹
Outer loop attitude control P gain, $K_{\theta p}$	1.5 N m rad ⁻¹
Outer loop position hold loop P gain, K_{xp}	1 N m ⁻¹
Outer loop lateral position hold loop integral gain, K_{xi}	0.005 N m ⁻¹
Canard moment arm (ℓ_c)	1 m
Main wing moment arm (ℓ_w)	1 m
Total airspeed pilot command upper limit on r_u	8g m s ⁻¹
Total airSpeed pilot command lower limit on r_u	-5 m s ⁻¹
Total airspeed pilot command rate limit on r_u	± 4 m s ⁻²
Flight path angle pilot command upper limit $\bar{\gamma}$	4 ° s ⁻¹
Flight path angle pilot command lower limit $\underline{\gamma}$	-2 ° s ⁻¹
Pitch attitude pilot command limit on r_θ	$\pm \pi/18$ rad
Pilot command rate limits on pitch angle rate $\dot{r}_q$	± 0.01 rad s ⁻¹
Altitude pilot command upper limit on r_h	3000 m
Altitude pilot command lower limit on r_h	-1000 m
Altitude pilot command upper rate limit on r_h	5 m/s
Altitude pilot command lower rate limit on r_h	-4 m/s

Table 4.3 Actuator Physical Limits and Dynamics

Actuator control input	ω_n	ζ	Minimum	Maximum	Rate Limit
Thrust for all EDFs	25rad/s	1	0N	5.7N	± 4 N/s
Γ for canard EDFs	20rad/s	1	-30°	100°	$\pm 30^\circ$ /s
Γ for wing EDFs	20rad/s	1	0°	100°	$\pm 30^\circ$ /s

Table 4.4 Active Set CA Algorithm Input Constants

Input constant name	Symbol	Type	Order	Value
Linear effectiveness matrix	$\mathbf{B}_u$ or $\mathbf{G}_u$	Matrix	$\mathbb{R}^{3 \times 4}$	See Eqn. 4.64
Virtual control weighting matrix	$\mathbf{W}_v$	Matrix	$\mathbb{R}^{3 \times 3}$	See Eqn. 4.80
Control weighting matrix	$\mathbf{W}_u$	Matrix	$\mathbb{R}^{4 \times 4}$	See Eqn. 4.81
Primary objective weighting factor	γ	scalar	$\mathbb{R}$	10 ⁶
Maximum number of iterations	i_{max}	scalar	$\mathbb{R}$	10
WLS Algorithm Simulation Period	Δt	scalar	$\mathbb{R}$	10 ⁻²

As discussed in Section 4.6.2, the values of $\mathbf{W}_v$ in equation 4.80 are chosen so that pitch rate q has the highest priority, followed by body z-axis velocity w before body x-axis velocity u . The rotational channel (pitch rate q) is prioritised over translational channels (heave w and surge u) to maintain stable and safe flight during limited control authority due to actuator saturation (rate or position) because rotational channels tracking is more critical in ensuring stable flight compared to translation channel tracking for VTOLs. Similarly, for the translational channels, the body z-axis velocity w tracking error minimisation is more critical in maintaining stable flight than body x-axis velocity u error tracking. Furthermore, during limited control authority where all remaining control authority allocated to the pitch rate is sufficient, this could potentially only result in degraded performance (poor u and w tracking) but safe and stable flight would be maintained since rotational channels has the tendency to cause instability more than translational channels.

Although the effector configuration used throughout this chapter is over-actuated (four real control variables to control only three degrees of freedom), $\mathbf{W}_u$ is set to the identity matrix so no penalties are applied on any of the real controls because stable flight can not be guaranteed if any of the front and rear thrust T and tilting mechanism Γ is lost.

Moreover, considering the real time implementation, the maximum number of CA software iterations $CA_{iter} (i_{max})$ is set to 15 to ensure that the CA algorithm is constrained to find the optimal solution within a short time. Moreover, the CA algorithm does not cause discontinuities/sudden jumps in the actuator positions since it always finds an optimal solution even while considering both actuator slew rate and position limits which is crucial for the real time implementation.

Furthermore, the γ is set to a high value of 10^6 to ensure that the primary objective of control error minimisation with weighting matrix $\mathbf{W}_v$ is properly distinguished from secondary objective of control effort minimisation with weighting matrix of $\mathbf{W}_u$.

Finally, the working set $\mathcal{W}$ of the Active Set CA algorithm is vector whose elements indicates which of the effectors are saturated or not and is explained as follows: $\mathcal{W} = [W1 \ W2 \ W3 \ W4]^T$ whether the control allocation is active or not, explained as follows:

- $W* = 0$ means no actuator saturation, $W* = -1$ means actuator saturation at lower limit, $W* = 1$ means actuator saturation at upper limit
- $W1$ means fore EDF thrust body x-axis component for CA1, means fore actuator EDFs thrust/rpm for CA2 and CA3
- $W2$ means aft EDF thrust body x-axis component for CA1, means aft actuator EDFs thrust/rpm for CA2 and CA3
- $W3$ means fore EDF thrust body z-axis component for CA1, means fore actuator EDFs tilt angle for CA2 and CA3
- $W4$ means aft EDF thrust body z-axis component for CA1, means aft actuator EDFs tilt angle for CA2 and CA3

In order to test the effectiveness of the control allocation approaches, three selected critical aircraft control manoeuvres are investigated considering the longitudinal motion of the

planar VTOL. These aircraft control manoeuvres (ACM) are not typical realistic flight test manoeuvres (FTM) but chosen to test the ability of the CA to allocate remaining control authority to higher priority and critical control channels in order to maintain operational safety and stability during certain flight conditions while there is limited control authority due to actuator saturation. The actuator (thrust and tilting mechanism) limits (position and rates) are intentionally set tighter than in reality so that they are violated thereby activating the active set linear CA algorithm at some point.

The first and second ACMs (ACM 1 and ACM 2) aim to test how the control allocation responds when maximum actuator thrust limits is reached during a vertical take-off (VTO)/Vertical Landing (VL) scenarios with pitch control, and hover to high forward speed. The third ACM (ACM 3) aims to test how the control allocation responds when maximum actuator tilt angle limits is reached during a reverse manoeuvre with pitch control. Another way of testing the CA response when maximum actuator tilt angle limits are reached is by performing a deceleration from high forward speed to hover. One way of testing the CA response to minimum actuator tilt angle limits violation will be performing a transition from hover to cruise (forward transition) on a full aerodynamic aircraft when the wing tilt angles can tilt close to its lower limit. The three selected ACMs and results are detailed below.

4.10.1 ACM 1 - VTO/VL with Pitch Control and Limited Control Authority

The manoeuvre starts at hover altitude of $h = 1000\text{m}$. At time $t = 5\text{s}$, a VTO altitude of 1050m at constant climb rate of 4m/s is commanded for 12.5s . Simultaneously, (at same $t = 5\text{s}$), a pitch attitude θ of 5° at a constant rate of $\pm 0.01 \text{ rad s}^{-1}$ is commanded. At time $t = 45\text{s}$, A VL hover altitude of 1000m is commanded at constant descend rate of 2m/s for 25s . Simultaneously, (at same $t = 45\text{s}$), a pitch attitude (θ) of 0° at a constant rate of $\pm 0.01 \text{ rad s}^{-1}$ is commanded. The commanded and actual states, commanded and actual effector positions, virtual controls, control authority (actuator limitation) status, state tracking error as well as virtual control tracking errors are shown for all three control allocation approaches in Figures 4.11 - 4.14.

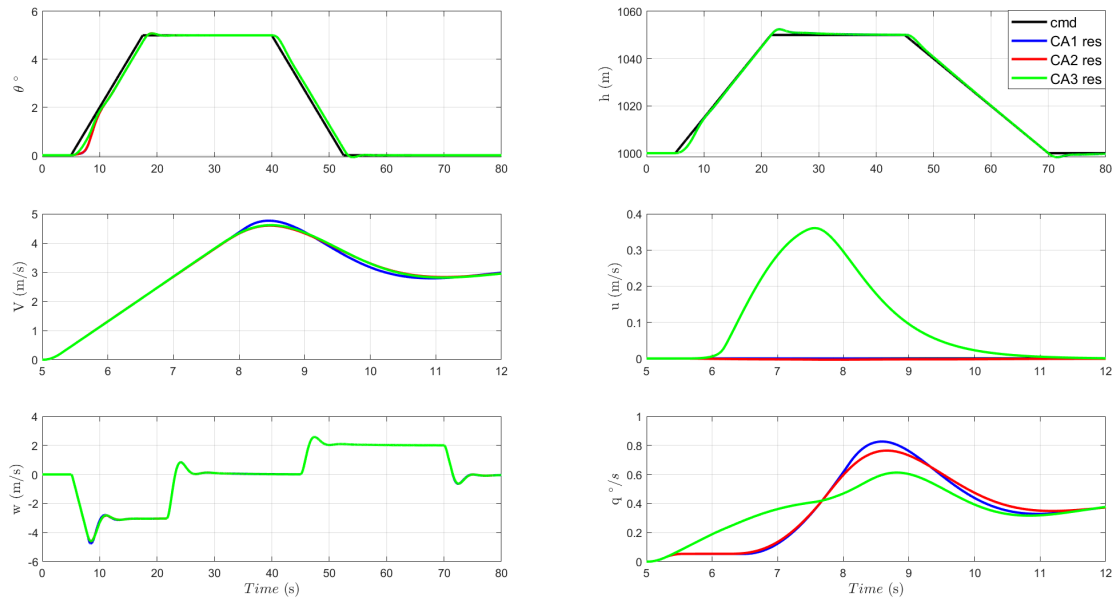


Fig. 4.11 VTO/VL with pitch control - State Variables

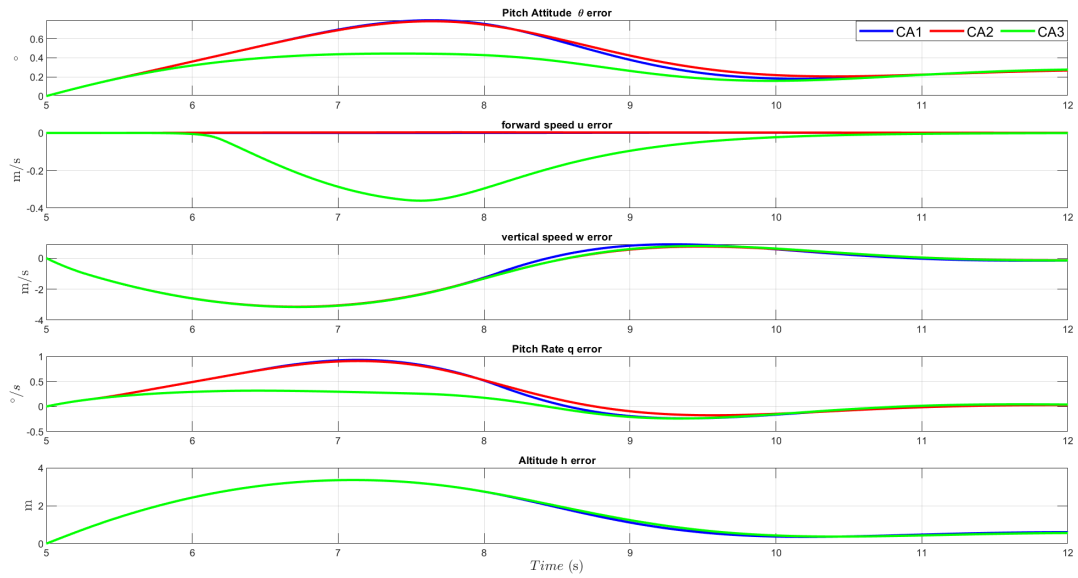


Fig. 4.12 VTO/VL with Pitch Control - State Variable Error

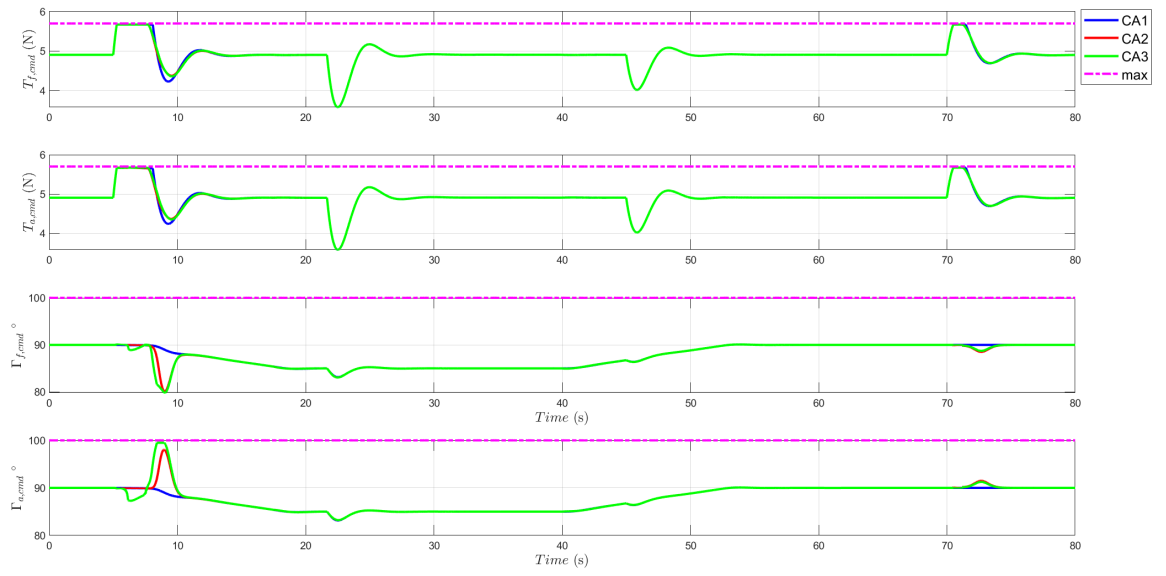


Fig. 4.13 VTO/VL with Pitch Control - Effector Commands

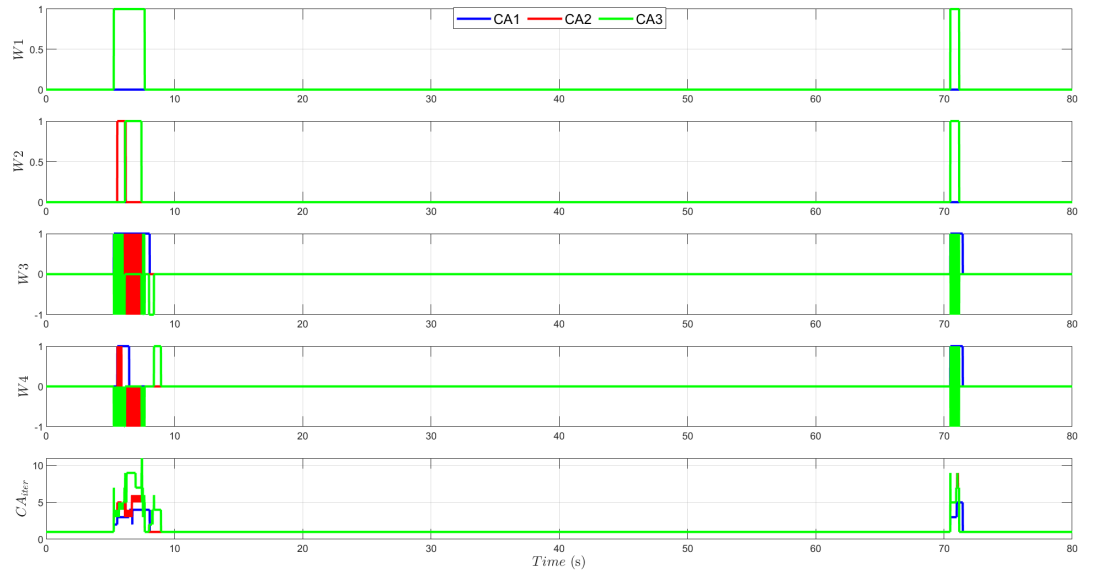


Fig. 4.14 VTO/VL with Pitch Control - Control Limitation Status

Analysis of ACM 1 Results During the initial phase of the VTO, as can be seen from Figure 4.13, the high-level controller outputs violated the maximum thrust limit ($W1$ and $W2$ for CA2 and CA3) and tilt angle rates ($W3$ and $W4$ for CA2 and CA3) limits between 5s and 7s resulting the CAs being active (see Figure 4.14) imposing actuator constraints limitations on the CA output which prevent violation of the physical actuator constraints. While the CAs were active due to actuator thrust saturation, the CAs allocates the limited control authority to the highest priority rotational channel pitch rate/pitching moment (see equation 4.80 and discussion in section 4.6.2) resulting in lower q error compared to u and w as can be seen from Figures 4.11 and 4.12. In terms of comparison of the three CA schemes, they perform similarly except that CA2 and CA3 (green) additionally imposed a minimum thrust rate limitation at time 22s at the end of the VTO command (figure 4.14). Also, CA1 seems to converge faster (fewer number of CA software iterations CA_{iter}) than CA2 and CA3 for the same manoeuvre (Figure 4.14).

The performances for all three CA schemes are also investigated considering the real time implementation. According to Figure 4.14, the maximum CA software iterations CA_{iter} are below 10 when the CA is active. Therefore, the CA algorithm finds the optimal solution very quickly. Moreover, the CA algorithm does not cause discontinuities/sudden jumps in the actuator positions which is crucial for real time implementation because it was able to find optimal solution subject to both actuator slew rate and position limits. In the absence of any control authority limitation, the control allocation correctly allocates the overall control demand from the high-level controller to the individual control effectors/actuators resulting in accurate tracking of pilot commands.

4.10.2 ACM 2 - VTO, Hover to High Forward Speed with Limited Control Authority

The manoeuvre starts at hover altitude of $h = 1000\text{m}$. At time $t = 5\text{s}$, a VTO altitude of 1020m at constant climb rate of 4m/s is commanded for 5s. Simultaneously, (at same $t = 5\text{s}$), a pitch attitude θ of 3.85° at a constant rate of $\pm 0.01 \text{ rad s}^{-1}$ is commanded. At time $t = 25\text{s}$, an airspeed speed V of 70m/s is then commanded at an acceleration of 4m s^{-2} and fpa γ of 0° over a 17.5s duration. During the increase in forward speed, the altitude of $h = 1020\text{m}$ was within $\pm 0.5\text{m}$. In this manoeuvre, since there is no aerodynamic lift and drag, this is no transition hence the tilt angles are still high due to the rotor providing both lift and forward acceleration forces. The commanded and actual states, commanded and actual effector positions, virtual controls, control authority (actuator limitation) status, state tracking error as well as virtual control tracking errors are shown for all three control allocation approaches from Figures 4.15 - 4.18.

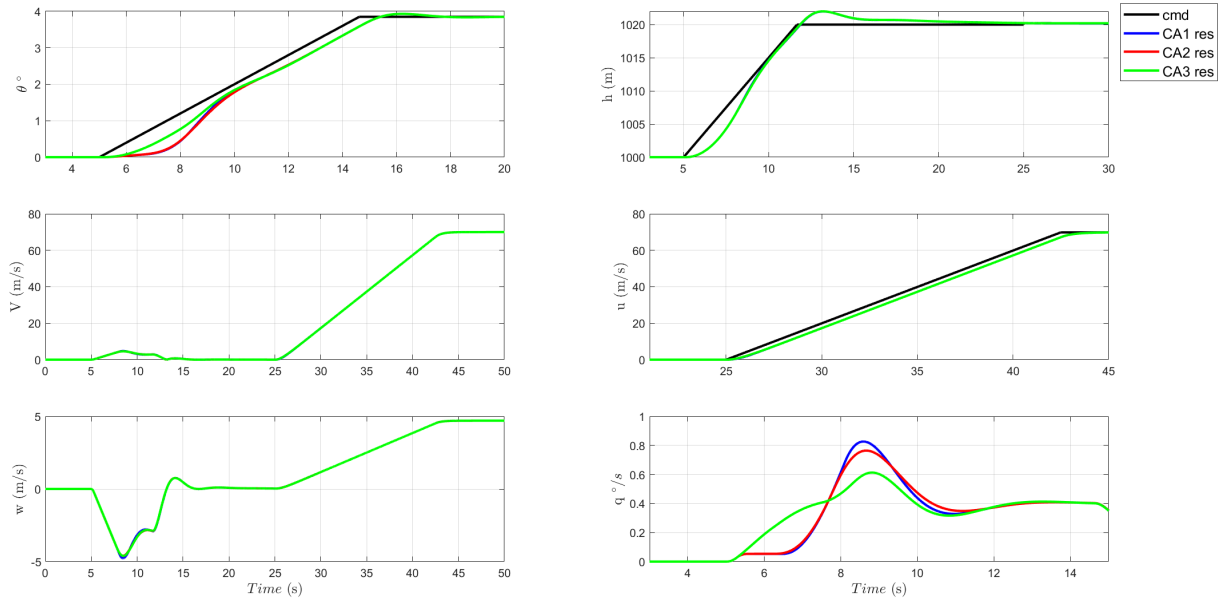


Fig. 4.15 VTO, Hover to High Forward Speed - State Variables

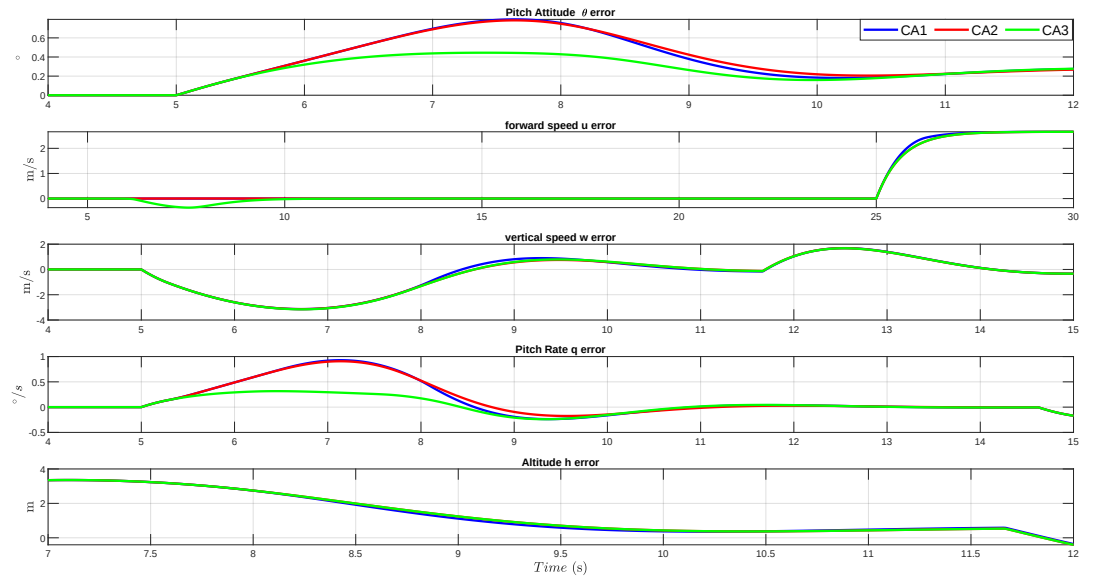


Fig. 4.16 VTO, Hover to High Forward Speed - State Variable Error

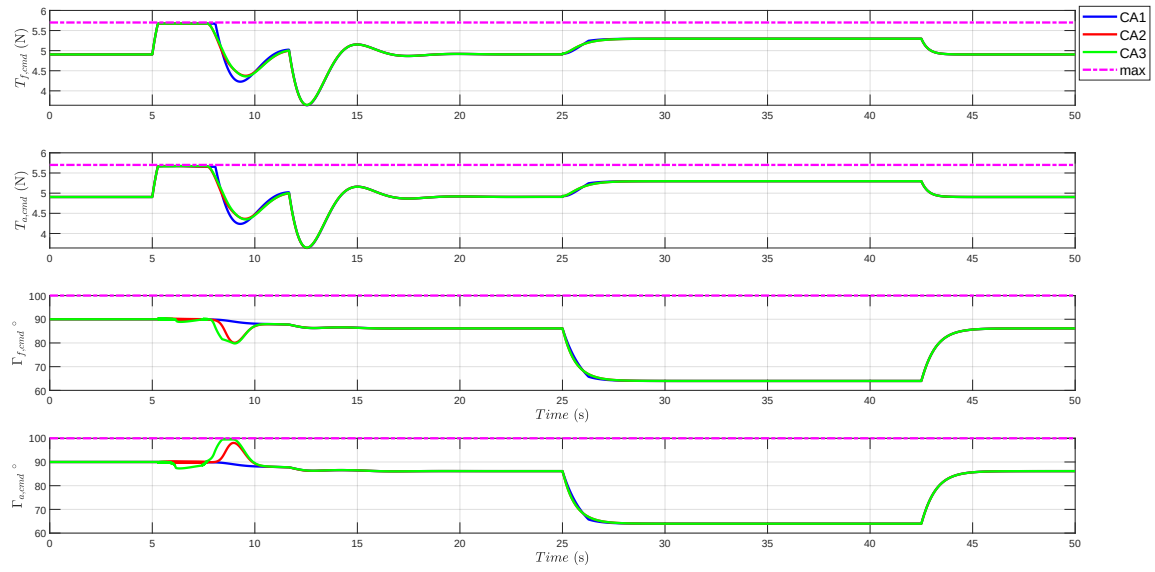


Fig. 4.17 VTO, Hover to High Forward Speed - Effector Commands

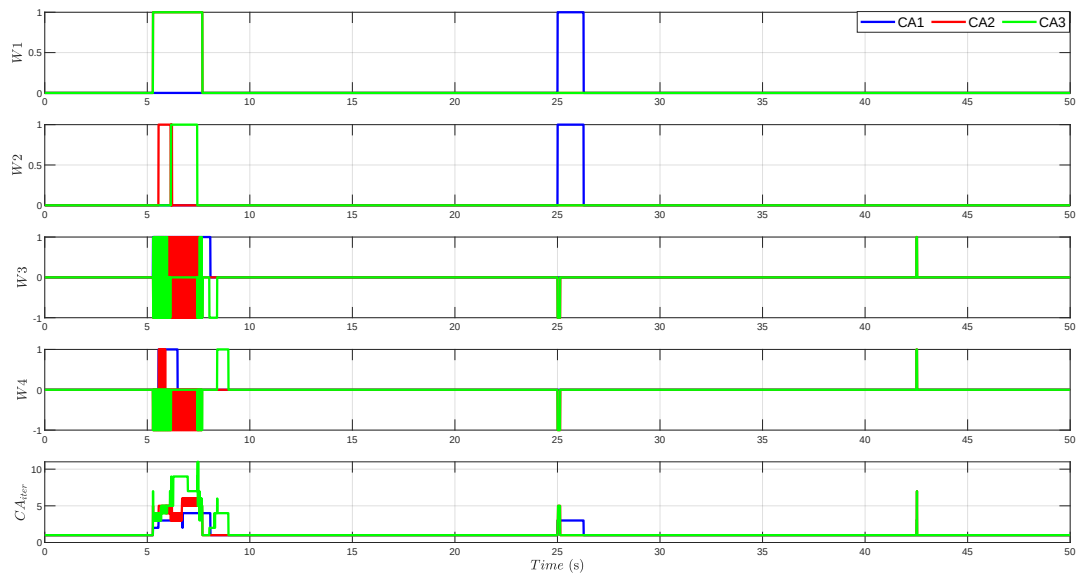


Fig. 4.18 VTO, Hover to High Forward Speed - Control Limitation Status

Analysis of ACM 2 Results During the initial phase of the flight transition, as can be seen from Figure 4.17, the high-level controller outputs violated the maximum thrust limit ($W1$ and $W2$) and tilt angle rates ($W3$ and $W4$) limits between 5s and 7s resulting in the CAs being active (see Figure 4.18) imposing actuator constraints limitations on the CA output which prevent violation of the physical actuator constraints. Again, while the CAs were active due to actuator thrust saturation, the CAs allocates the limited control authority to the highest priority rotational channel pitch rate/pitching moment (see equation 4.80 and discussion in section 4.6.2) resulting in lower q error compared to u and w as can be seen from Figures 4.15 and 4.16. Also, between 5s and 7s there was a 10° dip in the front rotor tilt angle and corresponding 10° increase in the aft rotor tilt angle for both CA2 and CA3 which was not experienced for CA1.

In terms of comparison of the three CA schemes, they perform similarly except that CA1 additionally imposed a maximum body x-axis thrust limitation between time 25s and 26s at the end of the VTO command (Figure 4.18). CA3 also momentarily imposed tilt angle rates ($W3$ and $W4$) limits at time 42.5s. Again CA1 seems to converge faster (lower number of CA software iterations CA_{iter}) than CA2 and CA3 for same manoeuvre (4.18). The performances for all three CA schemes are also investigated considering the real time implementation. According to Figure 4.18, the maximum CA software iterations CA_{iter} are below 12 when the CA is active. Therefore, the CA algorithm finds the optimal solution very quickly. Again, the CA algorithm does not cause discontinuities/sudden jumps in the actuator positions which is crucial for real time implementation.

4.10.3 ACM 3 - Reverse Backward and Pitch Control with Limited Control Authority

The manoeuvre start at hover altitude of $h = 1000\text{m}$. At time $t = 5\text{s}$, a reverse at -5m/s was commanded. Simultaneously, (at same $t = 5\text{s}$), a pitch attitude θ of 5° at a constant rate of $\pm 0.01 \text{ rad s}^{-1}$ was commanded. At time $t = 20\text{s}$, the pitch attitude θ of 5° was then cancelled (commanded to θ of 0°) at a constant rate of $\pm 0.01 \text{ rad s}^{-1}$. The commanded and actual states, commanded and actual effector positions, virtual controls, control authority (actuator limitation) status, state tracking error as well as virtual control tracking errors are shown for all three control allocation approaches in Figures 4.19 - 4.22.

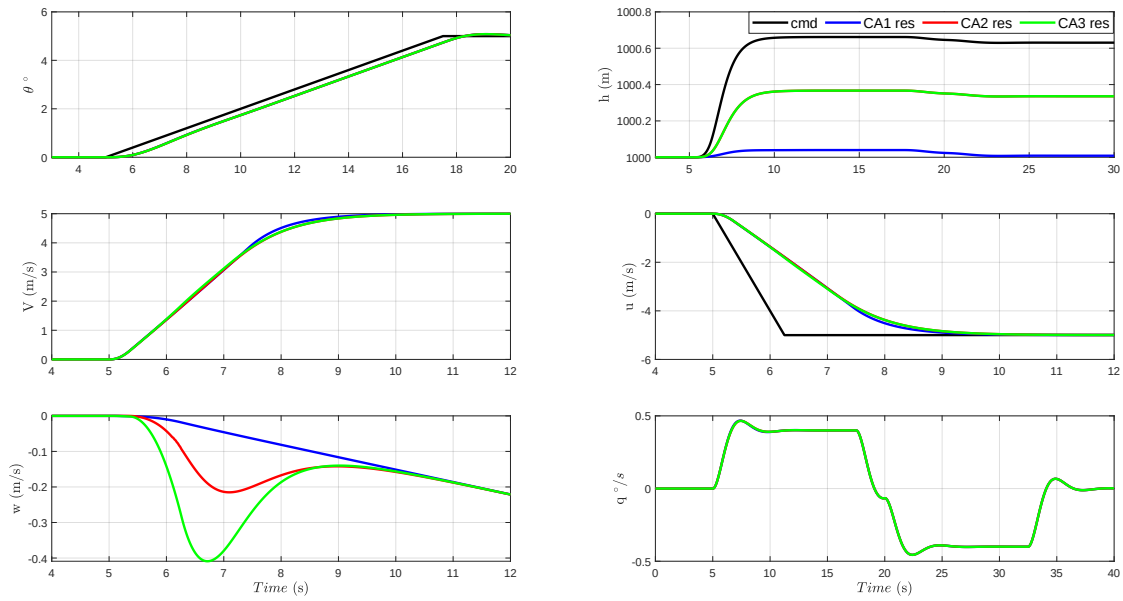


Fig. 4.19 Reverse Backward and Pitch Control - State Variables

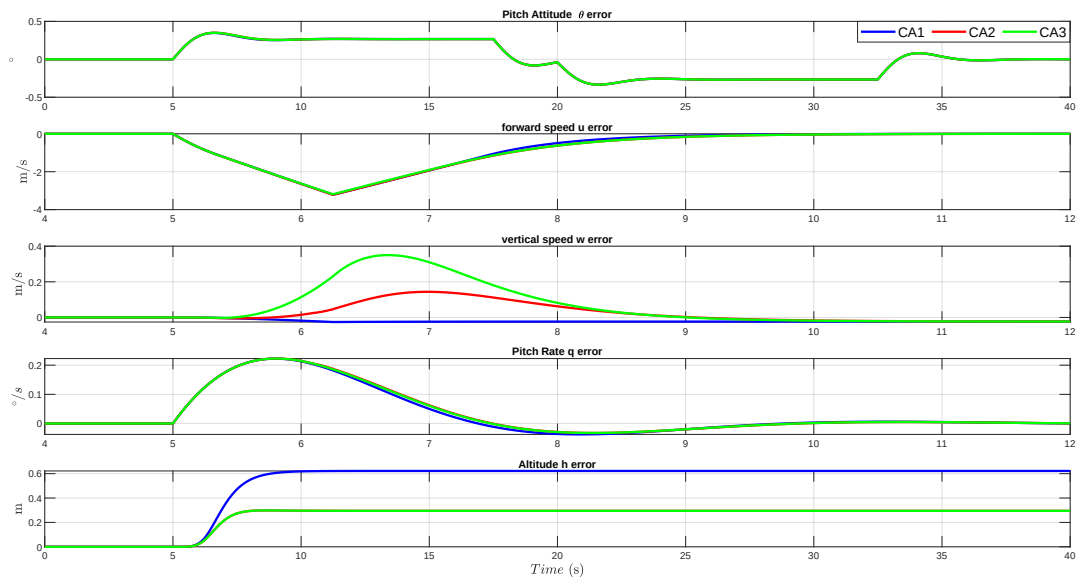


Fig. 4.20 Reverse Backward and Pitch Control - States Variables Error

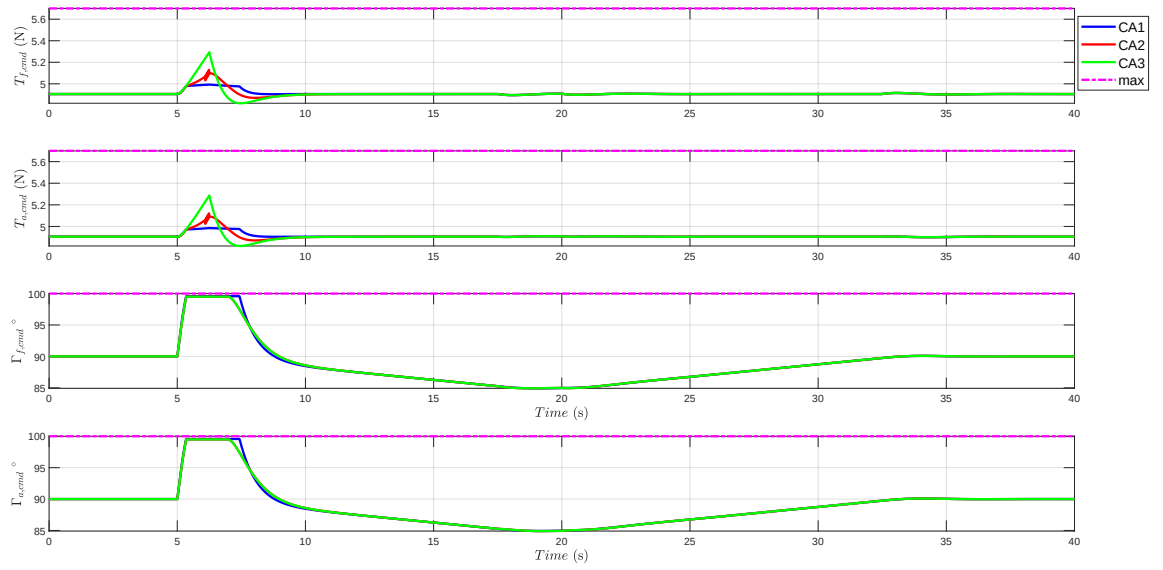


Fig. 4.21 Reverse Backward and Pitch Control - Effector Commands

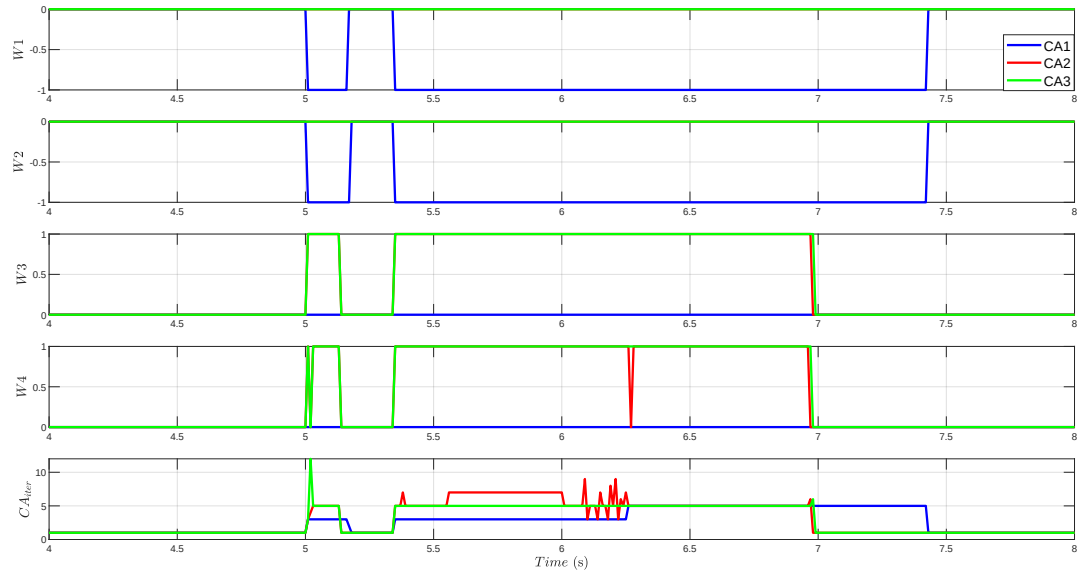


Fig. 4.22 Reverse Backward and Pitch Control - Control Limitation Status

Analysis of ACM 3 Results During the initial phase of the reverse speed manoeuvre, as can be seen from Figure 4.21, the high-level controller outputs violated the tilt angle rates ($W3$ and $W4$) limits for CA2 and CA3 (and body x-axis horizontal thrust $u_{x,f}$ and $u_{x,a}$ lower limit in CA1) between 5s and 7.5s resulting the CAs being active (see Figure 4.22) imposing actuator constraints limitations on the CA output which prevent violation of the physical actuator constraints. Again, while the CAs were active due to actuator tilting mechanism saturation, the CAs allocates the limited control authority to the highest priority rotational channel pitch rate/pitching moment (see equation 4.80 and discussion in section 4.6.2) resulting in lower q error compared to u and w as can be seen from Figures 4.19 and 4.20. Also, between 5s and 7.5s there was a 25% increase in both rotor thrust for both CA2 and CA3 compared to only 8% increase for CA1. Again CA1 seems to converge faster (lower number of CA software iterations CA_{iter}) than CA2 and CA3 for same manoeuvre (Figure 4.22). The performances for all three CA schemes are also investigated considering the real time implementation. According to Figure 4.22, the maximum CA software iterations CA_{iter} are below 13 when the CA is active. Therefore, the CA algorithm finds the optimal solution very quickly. Again, the CA algorithm does not cause discontinuities/sudden jumps in the actuator positions which is crucial for real time implementation

4.11 Conclusions

The results demonstrate that the control allocation can affectively allocate the overall control demand to the individual effectors and also impose effector constraints due to actuator saturation. The results also demonstrate the effectiveness of the proposed control allocation schemes to allocate remaining control authority to higher priority and critical control channels in order to maintain operational safety and stability during certain flight conditions where there is limited control authority. All three control allocation schemes have similar performance but CA1 generally converges faster than CA2 and CA3. CA1 and CA3 will be more suitable for few actuators due to the complex nonlinear transformation of the effector constraints (position and rate) from polar to cartesian and the more complex LTV Effector mapping respectively. CA2 will be easier than CA1 and CA3 to implement for VTOL configurations with many redundant effectors due to its simpler LTV effector mapping (compared to CA3) and simpler transformation of effector limits from absolute to incremental. Full 6DoF verification of CA1 is contained in Enenakpogbe et al. (2024b), while full 6DoF verification of CA2 is presented in Chapter 5.

Chapter 5

Simulation and Verification

5.1 Controller Testing of Nonlinear Model

As a recommended further work in the previous chapter Chapter 4, in order to verify the proposed CA Approach 2, the complete high-level controller consisting of an outer-loop linear controller and NDI-based controller (designed in Chapter 3) outer loop controller is used to control the full 6DoF eVTOL model (Model 1) designed in Chapter 2. The control architecture in Figure 3.1 is modified to use NDI as the inner loop controller as shown in Figure 5.1

5.1.1 Control Architecture

The Control Allocation Approach 2 (CA2) discussed in chapter 4 on the 6DoF aircraft is presented in Figure 5.3.

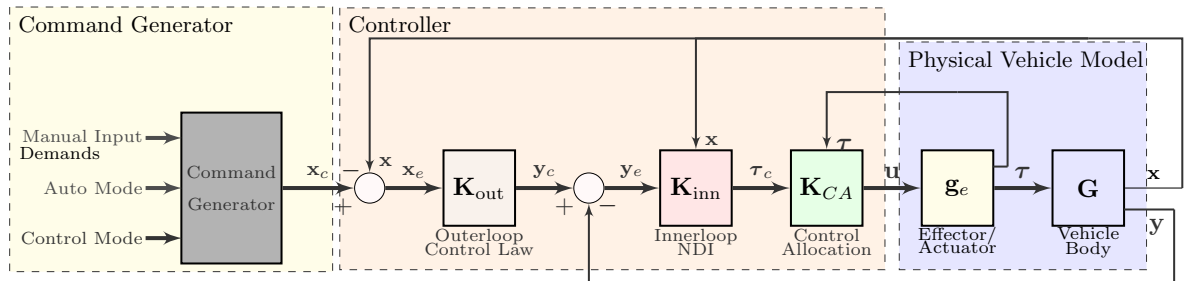


Fig. 5.1 NDI-Based Linear Flight Controller Architecture for piloted, semi-automatic and automated flight

5.1.2 6DoF Effector Model

A plan view of Lilium-style eVTOL aircraft configuration showing EDF sets distribution, control groupings and real controls is shown below in Figure 5.2 with more details than Figure 2.4 of Chapter 2.

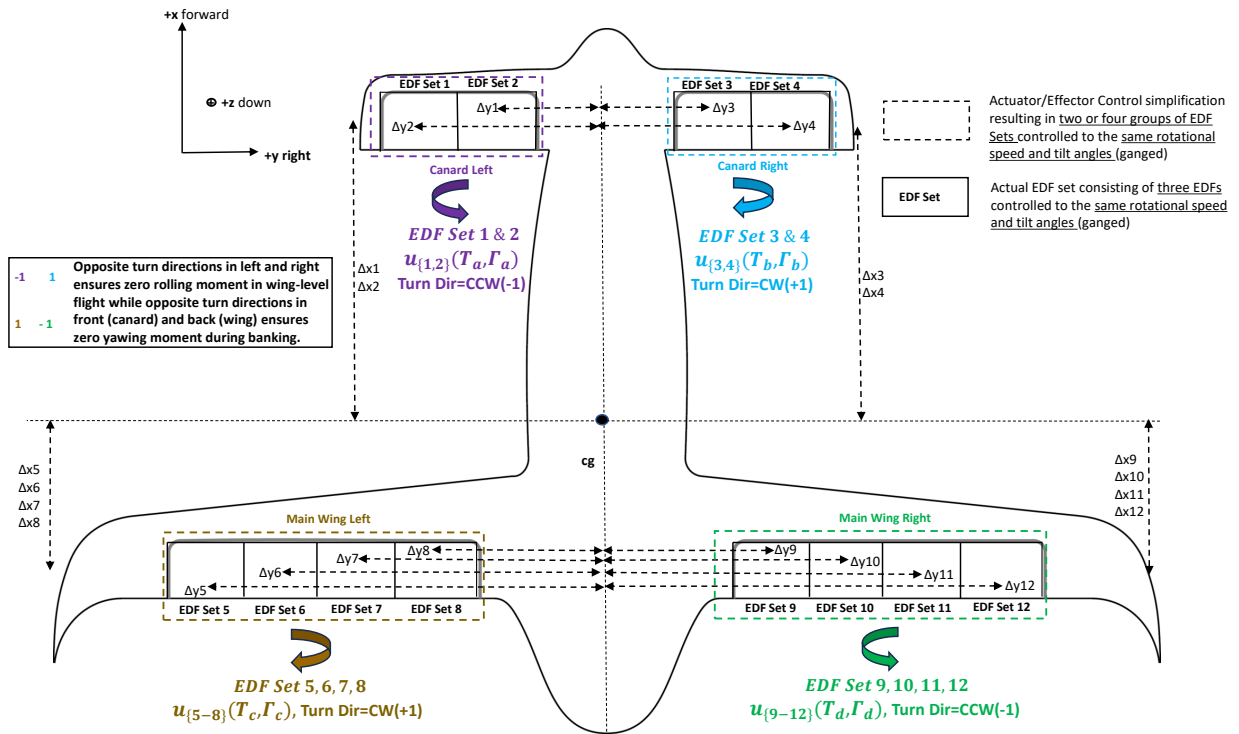


Fig. 5.2 Plan View of Lilium-style fixed-wing over-actuated vectored thrust eVTOL aircraft configuration showing EDF Sets distribution, control groupings and real controls

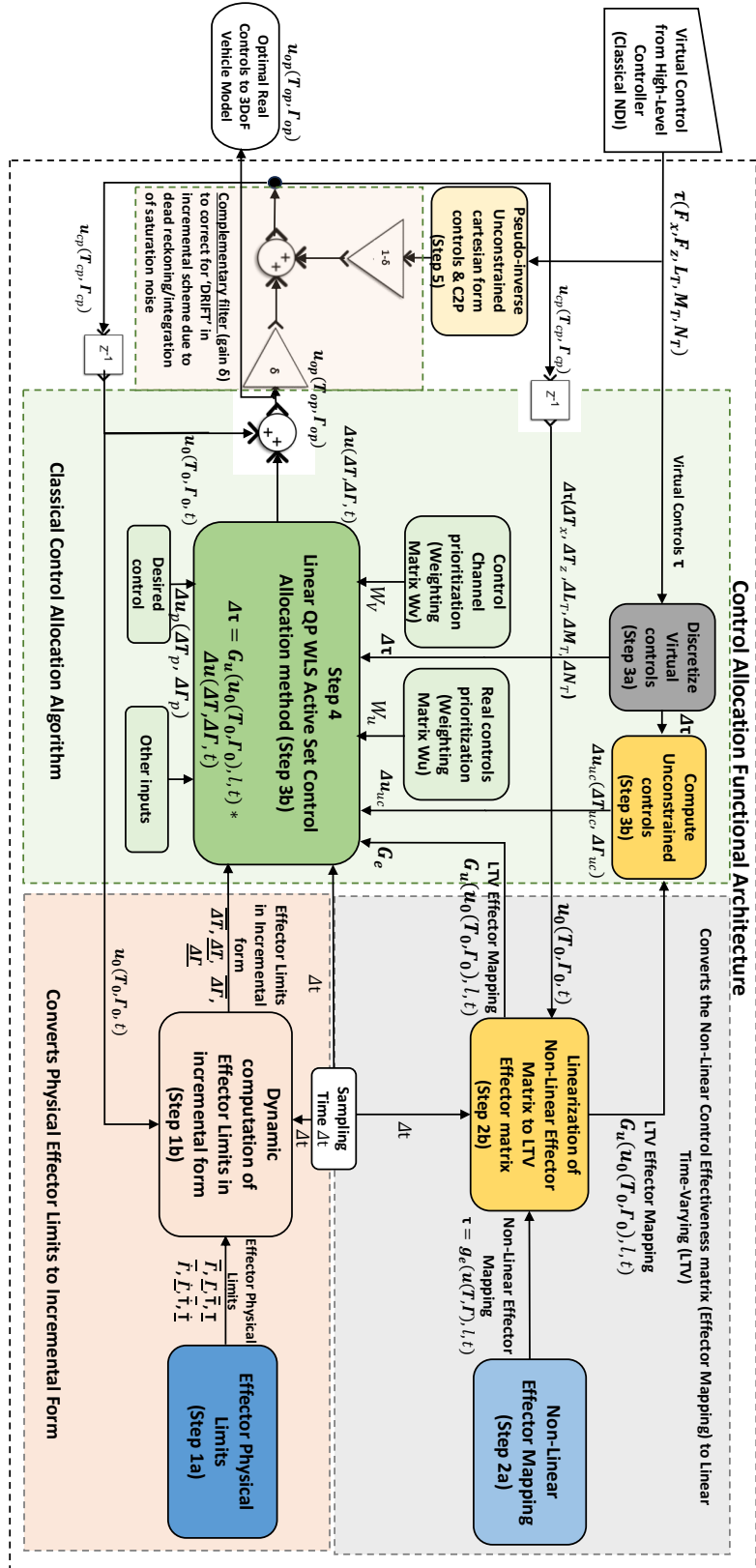


Fig. 5.3 CA Approach 2 6DoF Functional Architecture

Table 5.1 shows the turn directions of the various EDF sets in Figure 2.4

Table 5.1 Effector Model Moment Arms Turn Directions and from CG assuming NED

A/C loc	EDF Set	Turn Dir (td)	Effector Moment Arms from CG (NED)		
			$\Delta x(m)$	$\Delta y(m)$	$\Delta z(m)$
canard left engine tip	Set 1	CCW(-1)	3.8334	-2.3660	0.0716
canard left engine root	Set 2	CCW(-1)	3.8564	-1.3680	0.0716
canard right engine root	Set 3	CW(+1)	3.8564	1.3680	0.0716
canard right engine tip	Set 4	CW(+1)	3.8334	2.3660	0.0716
wing left engine 1 tip	Set 5	CW(+1)	-1.1396	-5.0410	-0.5951
wing left engine 2 tip	Set 6	CW(+1)	-1.0956	-4.0490	-0.5951
wing left engine 1 root	Set 7	CW(+1)	-1.0456	-3.0660	-0.5951
wing left engine 2 root	Set 8	CW(+1)	-0.9926	-2.0670	-0.5951
wing right engine 1 root	Set 9	CCW(-1)	-0.9926	2.0670	-0.5951
wing right engine 2 root	Set 10	CCW(-1)	-1.0456	3.0660	-0.5951
wing right engine 1 tip	Set 11	CCW(-1)	-1.0956	4.0490	-0.5951
wing right engine 2 tip	Set 12	CCW(-1)	-1.1396	5.0410	-0.5951

Based on the Torque equations in section 2.49, the rolling and yawing torque moments generated due to the turn directions of the EDF sets are given by:

$$L_{TQ} = C_Q T_x t d \quad (5.1)$$

$$N_{TQ} = C_Q T_z t d \quad (5.2)$$

where:

$$T_x = T \cos \Gamma \quad (5.3)$$

$$T_z = T \sin \Gamma \quad (5.4)$$

based on equations 2.69 and 2.70 and $t d = +1$ for clockwise direction (CW) and $t d = -1$ for counterclockwise (CCW) direction.

5.1.3 CA Approach 2 - LTV Effector Mapping

As mentioned in equation 4.63 of section 5.1.3, ignoring the effector mapping dependence on system states variables, the effector mapping is given by:

$$\Delta \boldsymbol{\tau}_c = \mathbf{G}_u \Delta \mathbf{u} \quad (5.5)$$

Using the simplified control and EDF sets distribution illustrated in the Lilium Jet eVTOL aircraft configuration shown in Figure 5.2, since EDFs do not produce forces in the Y- direction (i.e. Y propulsive = 0) and reference direction of the vertical force from the NDI is opposite that in the vehicle model (i.e. $T_Z = -Z$), the generalised forces and moments (commanded virtual controls $\boldsymbol{\tau}$) is given by:

$$\boldsymbol{\tau} = \begin{bmatrix} F_X \\ F_Z \\ L_T \\ M_T \\ N_T \end{bmatrix} = \mathbf{g}_e \quad (5.6)$$

where:

$$F_X = t_{x1} T_a \cos \Gamma_a + t_{x2} T_b \cos \Gamma_b + t_{x3} T_c \cos \Gamma_c + t_{x4} T_d \cos \Gamma_d \quad (5.7)$$

$$F_Z = t_{z1} T_a \sin \Gamma_a + t_{z2} T_b \sin \Gamma_b + t_{z3} T_c \sin \Gamma_c + t_{z4} T_d \sin \Gamma_d \quad (5.8)$$

$$L_T = L_1 T_a \cos \Gamma_a + L_2 T_b \cos \Gamma_b + L_3 T_c \cos \Gamma_c + L_4 T_d \cos \Gamma_d + L_5 T_a \sin \Gamma_a \\ + L_6 T_b \sin \Gamma_b + L_7 T_c \sin \Gamma_c + L_8 T_d \sin \Gamma_d \quad (5.9)$$

$$M_T = M_1 T_a \cos \Gamma_a + M_2 T_b \cos \Gamma_b + M_3 T_c \cos \Gamma_c + M_4 T_d \cos \Gamma_d + M_5 T_a \sin \Gamma_a \\ + M_6 T_b \sin \Gamma_b + M_7 T_c \sin \Gamma_c + M_8 T_d \sin \Gamma_d \quad (5.10)$$

$$N_T = N_1 T_a \cos \Gamma_a + N_2 T_b \cos \Gamma_b + N_3 T_c \cos \Gamma_c + N_4 T_d \cos \Gamma_d + N_5 T_a \sin \Gamma_a \\ + N_6 T_b \sin \Gamma_b + N_7 T_c \sin \Gamma_c + N_8 T_d \sin \Gamma_d \quad (5.11)$$

The gradient matrix $\mathbf{G}_u$ is given by

$$\mathbf{G}_u = \begin{bmatrix}
t_{x1} \cos \Gamma_a & t_{x2} \cos \Gamma_b & t_{x3} \cos \Gamma_c & t_{x4} \cos \Gamma_d \\
t_{z1} \sin \Gamma_a & t_{z2} \sin \Gamma_b & t_{z3} \sin \Gamma_c & t_{z4} \sin \Gamma_d \\
L_1 \cos \Gamma_a + L_5 \sin \Gamma_a & L_2 \cos \Gamma_b + L_6 \sin \Gamma_b & L_3 \cos \Gamma_c + L_7 \sin \Gamma_c & L_4 \cos \Gamma_d + L_8 \sin \Gamma_d \\
M_1 \cos \Gamma_a + M_5 \sin \Gamma_a & M_2 \cos \Gamma_b + M_6 \sin \Gamma_b & M_3 \cos \Gamma_c + M_7 \sin \Gamma_c & M_4 \cos \Gamma_d + M_8 \sin \Gamma_d \\
N_1 \cos \Gamma_a + N_5 \sin \Gamma_a & N_2 \cos \Gamma_b + N_6 \sin \Gamma_b & N_3 \cos \Gamma_c + N_7 \sin \Gamma_c & N_4 \cos \Gamma_d + N_8 \sin \Gamma_d \\
-t_{x1} T_a \sin \Gamma_a & -t_{x2} T_b \sin \Gamma_b & -t_{x3} T_c \sin \Gamma_c & \\
t_{z1} T_a \cos \Gamma_a & t_{z2} T_b \cos \Gamma_b & t_{z3} T_c \cos \Gamma_c & \\
L_5 T_a \cos \Gamma_a - L_1 T_a \sin \Gamma_a & L_6 T_b \cos \Gamma_b - L_2 T_b \sin \Gamma_b & L_7 T_c \cos \Gamma_c - L_3 T_c \sin \Gamma_c & \\
M_5 T_a \cos \Gamma_a - M_1 T_a \sin \Gamma_a & M_6 T_b \cos \Gamma_b - M_2 T_b \sin \Gamma_b & M_7 T_c \cos \Gamma_c - M_3 T_c \sin \Gamma_c & \\
N_5 T_a \cos \Gamma_a - N_1 T_a \sin \Gamma_a & N_6 T_b \cos \Gamma_b - N_2 T_b \sin \Gamma_b & N_7 T_c \cos \Gamma_c - N_3 T_c \sin \Gamma_c & \\
-t_{x4} T_d \sin \Gamma_d & & & \\
t_{z4} T_d \cos \Gamma_d & & & \\
L_8 T_d \cos \Gamma_d - L_4 T_d \sin \Gamma_d & & & \\
M_8 T_d \cos \Gamma_d - M_4 T_d \sin \Gamma_d & & & \\
N_8 T_d \cos \Gamma_d - N_4 T_d \sin \Gamma_d & & &
\end{bmatrix} \quad (5.12)$$

$$\Delta \tau_c = [\Delta F_X \quad \Delta F_Z \quad \Delta L_T \quad \Delta M_T \quad \Delta N_T]^T \quad (5.13)$$

$$\Delta \mathbf{u} = \mathbf{G}_{\text{inv}}(\mathbf{x}, t) \Delta \tau_c = [\Delta T_a \quad \Delta T_b \quad \Delta T_c \quad \Delta T_d \quad \Delta \Gamma_a \quad \Delta \Gamma_b \quad \Delta \Gamma_c \quad \Delta \Gamma_d]^T \quad (5.14)$$

with terms

- $t_{x1} = 2, t_{x2} = 2, t_{x3} = 4, t_{x4} = 4, t_{z1} = -2, t_{z2} = -2, t_{z3} = -4, t_{z4} = -4$
- $QDir* = C_Q t d*, * = 1, 2, 3, 4$
- $L_1 = 2 QDir1, L_2 = 2 QDir2, L_3 = 4 QDir3, L_4 = 4 QDir4, N_5 = -2 QDir1, N_6 = -2 QDir2, N_7 = -4 QDir3, N_8 = -4 QDir4$
- $L_5 = -(\Delta y_1 + \Delta y_2), L_6 = -(\Delta y_3 + \Delta y_4), L_7 = -(\Delta y_5 + \Delta y_6 + \Delta y_7 + \Delta y_8), L_8 = -(\Delta y_9 + \Delta y_{10} + \Delta y_{11} + \Delta y_{12})$
- $M_1 = (\Delta z_1 + \Delta z_2), M_2 = (\Delta z_3 + \Delta z_4), M_3 = (\Delta z_5 + \Delta z_6 + \Delta z_7 + \Delta z_8), M_4 = (\Delta z_9 + \Delta z_{10} + \Delta z_{11} + \Delta z_{12})$
- $M_5 = (\Delta x_1 + \Delta x_2), M_6 = (\Delta x_3 + \Delta x_4), M_7 = (\Delta x_5 + \Delta x_6 + \Delta x_7 + \Delta x_8), M_8 = (\Delta x_9 + \Delta x_{10} + \Delta x_{11} + \Delta x_{12})$
- $N_1 = -(\Delta y_1 + \Delta y_2), N_2 = -(\Delta y_3 + \Delta y_4), N_3 = -(\Delta y_5 + \Delta y_6 + \Delta y_7 + \Delta y_8), N_4 = -(\Delta y_9 + \Delta y_{10} + \Delta y_{11} + \Delta y_{12})$

5.1.4 Correction of Saturation Drift

The complementary filter is shown in step 5 of figure 5.3. Without this drift correction via a complementary filter, the CA output will drift after actuator saturation resulting in a steady state error in the system states in a similar fashion to lack of integration action in the control laws. As presented in sections 4.8.1 and 4.8.7, the update source which is the unconstrained controls of CA Approach 1 $\mathbf{u}_{x,z}$ in equation 4.26 to 4.27 is then transformed from cartesian form to compatible polar form using equation 4.28 with a gain set to 0.95 resulting in only 5% of the update source and 95% of the incremental scheme constituting the final control which is subsequently constrained by the incremental actuator limits in the classical CA algorithm (active set). The linear effector mapping $\mathbf{B}_{\text{inv}}(\Delta x, \Delta y, \Delta z) \tau_c$ is given by:

$$\tau_c = [F_X \ F_Z \ L_T \ M_T \ N_T]^T = \mathbf{B}_e(\Delta x, \Delta y, \Delta z) \mathbf{u}_{x,z} \quad (5.15)$$

where:

$$\mathbf{B}_e(\Delta x, \Delta y, \Delta z) = \begin{bmatrix} t_{x1} & t_{x2} & t_{x3} & t_{x4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_{z1} & t_{z2} & t_{z3} & t_{z4} \\ L_1 & L_2 & L_3 & L_4 & L_5 & L_6 & L_7 & L_8 \\ M_1 & M_2 & M_3 & M_4 & M_5 & M_6 & M_7 & M_8 \\ N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 \end{bmatrix} \quad (5.16)$$

$$\mathbf{u}_{x,z} = \mathbf{B}_{\text{inv}}(\mathbf{x}, t) \tau_c = \begin{bmatrix} T_{x1,2} & T_{x3,4} & T_{x5-8} & T_{x9-12} \\ T_{z1,2} & T_{z3,4} & T_{z5-8} & T_{z9-12} \end{bmatrix}^T \quad (5.17)$$

which in compact form becomes

$$\begin{bmatrix} F_X \\ F_Z \\ L_T \\ M_T \\ N_T \end{bmatrix} = \begin{bmatrix} t_{x1} & \cdots & t_{x4} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & t_{z1} & \cdots & t_{z4} \\ L_1 & \cdots & L_4 & L_5 & \cdots & L_8 \\ M_1 & \cdots & M_4 & M_5 & \cdots & M_8 \\ N_1 & \cdots & N_4 & N_5 & \cdots & N_8 \end{bmatrix} \begin{bmatrix} T_{xa} \\ \vdots \\ T_{xd} \\ T_{za} \\ \vdots \\ T_{zd} \end{bmatrix} \quad (5.18)$$

where:

$$\begin{bmatrix} T_{xa} \\ T_{xb} \\ T_{xc} \\ T_{xd} \\ T_{za} \\ T_{zb} \\ T_{zc} \\ T_{zd} \end{bmatrix} = \begin{bmatrix} T_{x1,2} \\ T_{x3,4} \\ T_{x5-8} \\ T_{x9-12} \\ T_{z1,2} \\ T_{z3,4} \\ T_{z5-8} \\ T_{z9-12} \end{bmatrix} \quad (5.19)$$

where:

- τ_c is the commanded virtual control (control effect and generalised forces)
- $\mathbf{u}_{x,z}$ is the unconstrained real controls in Cartesian form $[\mathbf{u}_x, \mathbf{u}_z]$
- $\mathbf{B}(\Delta x, \Delta y, \Delta z)$ is the effector model which only depends on basic geometry lever arms information as shown in Figure 5.2

5.1.5 CA Incremental Constraints Formulation

The incremental actuator limits for each actuator ($\bar{T}, \underline{T}, \dot{\bar{T}}, \dot{\underline{T}}, \bar{\Gamma}, \underline{\Gamma}, \dot{\bar{\Gamma}}, \dot{\underline{\Gamma}}$) are given by equations 4.49 - 4.52 in section 4.7.5.

5.1.6 Prioritised Incremental Linear CA

The inputs constants into the prioritised constrained linear optimisation Active Set algorithm (section 4.8.8) are listed in Table 5.2.

Table 5.2 Active Set CA Algorithm Input Constants

Input Constant name	Symbol	Type	Order	Value
Linear Effectiveness Matrix	$\mathbf{G}_e$	<i>Matrix</i>	$\mathbb{R}^{5 \times 8}$	See Eqn. 4.64
Virtual control weighting matrix	$\mathbf{W}_v$	<i>Matrix</i>	$\mathbb{R}^{5 \times 5}$	See Eqn. 5.20
Control weighting matrix	$\mathbf{W}_u$	<i>Matrix</i>	$\mathbb{R}^{8 \times 8}$	See Eqn. 5.21
Primary objective weighting factor	γ	<i>scalar</i>	$\mathbb{R}$	10^6
Maximum number of iterations	i_{max}	<i>scalar</i>	$\mathbb{R}$	10
WLS Algorithm Simulation Period	Δt	<i>scalar</i>	$\mathbb{R}$	10^{-2}

The weighting matrices are:

$$\mathbf{W}_v = \begin{bmatrix} 50 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 1000 \end{bmatrix} \quad (5.20)$$

$$\mathbf{W}_u = \mathbf{I}_8 \quad (5.21)$$

where $\mathbf{I}_8$ denotes the 8×8 identity matrix

The value of $\mathbf{W}_v$ in equation 5.20 means that the rotational channels (roll rate p , pitch rate q and yaw rate r) are prioritised over translational channels (heave w and surge u) to maintain stable and safe flight during limited control authority due to actuator saturation (rate or position) because rotational channels tracking is more critical in ensuring stable flight compared to translation channel tracking for VTOLs. Recall in Chapter 2 that the phugoid and spiral mode have unstable dynamics in open loop. Similarly, for the translational channels, the body z-axis velocity w tracking error minimisation is more critical in maintaining stable flight than body x-axis velocity u error tracking. Furthermore, during limited control authority where

all remaining control authority is allocated to the rotational channels, this will only result in degraded performance (poor u and w tracking) but safe and stable flight would be maintained.

Although the effector configuration used throughout this chapter is over-actuated (eight real controls to control directly only five degrees of freedom), there is however no redundancy because both left and right canard and wing EDFs are required to maintain stable flight. Hence $\mathbf{W}_u$ is set to an identity matrix of size 8 with no penalties applied on any of the real controls.

Furthermore, considering the real time implementation, the maximum CA software iterations i_{max} is set to 50 to ensure that the CA algorithm is allowed to find the optimal solution within a short time. Moreover, the CA algorithm does not cause discontinuities/sudden jumps in the actuator positions which is crucial for the real time implementation.

Finally, the λ is set to a high value of 10^6 to ensure that the primary objective of control error minimisation with weighting matrix $\mathbf{W}_v$ is properly distinguished from secondary objective of control effort minimisation with weighting matrix $\mathbf{W}_u$.

Verification of the NDI controller and CA algorithm via nonlinear simulation model will be given in the next section. Several test cases are analysed considering the wide-flight envelope of the air vectored thrust eVTOL. In addition, the CA performance is tested for specific flight conditions (see Sections 5.2.3 to 5.2.8). Disturbance rejection properties, robustness to errors in model dependent parameters, and effector failures are also analyzed (see Sections 5.2.9 to 5.2.13). It is observed that the role of CA is critical in case of severe actuator saturation (i.e., limited control authority) as already demonstrated for the 3DoF concept testing in Chapter 4.

5.2 Simulations Results and Discussions

- Simulation Software: The vehicle model and NDI controller are coded in Simulink.
- Software version: All the simulations are conducted in MATLAB[®] Simulink[®] with version R2021b on a personal computer of Intel Core i9-8950H 2.90GHz CPU and 32GB RAM.
- MATLAB/Simulink simulation: The simulation was run at a fixed step sample time time of 10ms (100Hz).

The constants (aircraft aerodynamics, pilot command limits, NDI and outer-loop controller parameters and effector constraints) used for the model are given in Tables ??, 5.3, 5.4 and 5.5. It is assumed that the measurements are noise-free.

Table 5.3 Command Generator Filtering Simulation Coefficients.

Parameter	Value
Total airspeed pilot command lower limit $\underline{V}_{cmd}$	-5 m s^{-1}
Total airspeed pilot command upper limit $\overline{V}_{cmd}$	80 m s^{-1}
Total airSpeed pilot command upper rate limit $\dot{\overline{V}}_{cmd}$	2 m s^{-2}
Total airSpeed pilot command lower rate limit $\dot{\underline{V}}_{cmd}$	-1 m s^{-2}
Flight path angle pilot command upper limit $\overline{\gamma}_{cmd}$	5° s^{-1}
Flight path angle pilot command lower limit $\underline{\gamma}_{cmd}$	-5° s^{-1}
Flight path angle pilot command upper rate limit $\dot{\overline{\gamma}}_{cmd}$	0.2° s^{-2}
Flight path angle pilot command lower rate limit $\dot{\underline{\gamma}}_{cmd}$	$-0.1^\circ \text{ s}^{-2}$
Altitude pilot command lower limit $\underline{h}_{cmd}$	-2000m
Altitude pilot command upper limit $\overline{h}_{cmd}$	4000m
Altitude pilot command rate limit $\dot{\overline{h}}_{cmd}$	$\pm 2 \text{ m s}^{-1}$
North (translational) position pilot command rate limit $\dot{\underline{x}}_{cmd}$	$\pm 2 \text{ m s}^{-1}$
East (lateral) position pilot command rate limit $\dot{\underline{y}}_{cmd}$	$\pm 0.2 \text{ m s}^{-1}$
Heading pilot command upper limit $\overline{\zeta}_{cmd}$	5° s^{-1}
Heading pilot command lower limit $\underline{\zeta}_{cmd}$	-5° s^{-1}
Heading pilot command rate limit $\dot{\overline{\zeta}}_{cmd}$	$\pm 1^\circ \text{ s}^{-2}$
Bank angle pilot command upper limit $\overline{\phi}_{cmd}$	45° s^{-1}
Bank angle pilot command lower limit $\underline{\phi}_{cmd}$	-5° s^{-1}
Bank angle pilot command rate limit $\dot{\overline{\phi}}_{cmd}$	$\pm 4^\circ \text{ s}^{-2}$
Pitch attitude pilot command upper limit $\overline{\theta}_{cmd}$	10° s^{-1}
Pitch attitude pilot command lower limit $\underline{\theta}_{cmd}$	-10° s^{-1}
Pitch attitude pilot command rate limit $\dot{\overline{\theta}}_{cmd}$	$\pm 0.2^\circ \text{ s}^{-2}$
Yaw angle upper limit $\overline{\psi}_{cmd}$	10° s^{-1}
Yaw angle lower limit $\underline{\psi}_{cmd}$	-10° s^{-1}
Yaw angle rate limit $\dot{\overline{\psi}}_{cmd}$	$\pm 0.2^\circ \text{ s}^{-2}$

Table 5.4 Controller Simulation Coefficients.

Parameter	Value
Outer loop translational position hold loop P-gain, K_{xp}	1 N m ⁻¹
Outer loop translational position hold loop I-gain, K_{xi}	0.005 N m ⁻¹
Outer loop translational position hold loop D-gain, K_{xd}	0 N m ⁻¹
Outer loop side position control P-gain, K_{yp}	1 N m ⁻¹
Outer loop side position control I-gain, K_{yi}	0.01 N m ⁻¹
Outer loop side position control D-gain, K_{yd}	0 N m ⁻¹
Outer loop sltitude control P-gain, K_{hp}	2.5 N m rad ⁻¹
Outer loop sltitude control I-gain, K_{hi}	0.01 N m rad ⁻¹
Outer loop altitude control D-gain, K_{hd}	0 N m rad ⁻¹
Outer loop bank angle control P-gain, $K_{\phi p}$	1 N m rad ⁻¹
Outer loop bank angle control P-gain, $K_{\phi i}$	0 N m rad ⁻¹
Outer loop bank angle control D-gain, $K_{\phi d}$	0 N m rad ⁻¹
Outer loop attitude control P-gain, $K_{\theta p}$	1 N m rad ⁻¹
Outer loop attitude control P-gain, $K_{\theta i}$	0 N m rad ⁻¹
Outer loop attitude control D-gain, $K_{\theta d}$	0 N m rad ⁻¹
Outer loop turn angle control P-gain, $K_{\psi p}$	1 N m rad ⁻¹
Outer loop turn angle control P-gain, $K_{\psi i}$	0 N m rad ⁻¹
Outer loop turn angle control D-gain, $K_{\psi d}$	0 N m rad ⁻¹
Outer loop side motion speed control P-gain, K_{vp}	1 N m rad ⁻¹
Outer loop side motion speed control P-gain, K_{vi}	0 N m rad ⁻¹
Outer loop side motion speed control D-gain, K_{vd}	0 N m rad ⁻¹
Inner loop NDI u tracking loop P-gain, K_{up}	1.5 N m ⁻¹ s
Inner loop NDI w tracking loop P-gain, K_{wp}	2.5 N m ⁻¹ s
Inner loop NDI p loop P-gain, K_{pp}	1 N m rad ⁻¹ s
Inner loop NDI q loop P-gain, K_{qp}	1 N m rad ⁻¹ s
Inner loop NDI r loop P-gain, K_{rp}	1 N m rad ⁻¹ s

5.2.1 Actuator Dynamics - RPM and Tilting Mechanism

There is no exact reference value for the tilting dynamics of EDFs modeling. Hence, assumptions are made considering the small size and mass of EDFs. Based on the data sheet given in Table 5.6, each EDF has 3.4 kg mass and 20 cm fan diameter. Therefore, it is reasonable to assume that the hinge moments to rotate EDF sets will be very small. Then, tilting dynamics is modelled as a 2nd order transfer function with a natural frequency of 20 rad/s and a damping ratio of 1. Moreover, the rate limit of the tilting mechanism is taken as $\pm 60 \text{ deg/s}$. Minimum and maximum position limits of the tilt angles are considered separately for the EDFs on the front (canard) and rear (wing) sections. The wing EDFs are deflected between 0deg and 120deg. The minimum tilt angle is taken 0deg since EDFs are on the trailing edge of the wing. It is noted that EDFs are parallel to the wing surface when the tilt angle is 0deg. For the EDFs on the canards, there is no fixed surface, so that the minimum limit is extended to -30 deg , whereas the maximum limit is the same as wing EDFs (i.e., 120deg).

The thrust (rpm) and tilting mechanisms actuator dynamics consisting of a 2nd order transfer functions T_{2tf} and Γ_{2tf} , position and rate limits given in Table 5.5 are illustrated in Figure 5.4. The 2nd order transfer function are given as:

$$T_{2tf} = \frac{1}{(s/\omega_{nT})^2 + 2\zeta_T(s/\omega_{nT}) + 1} \quad (5.22)$$

$$\Gamma_{2tf} = \frac{1}{(s/\omega_{n\Gamma})^2 + 2\zeta_\Gamma(s/\omega_{n\Gamma}) + 1} \quad (5.23)$$

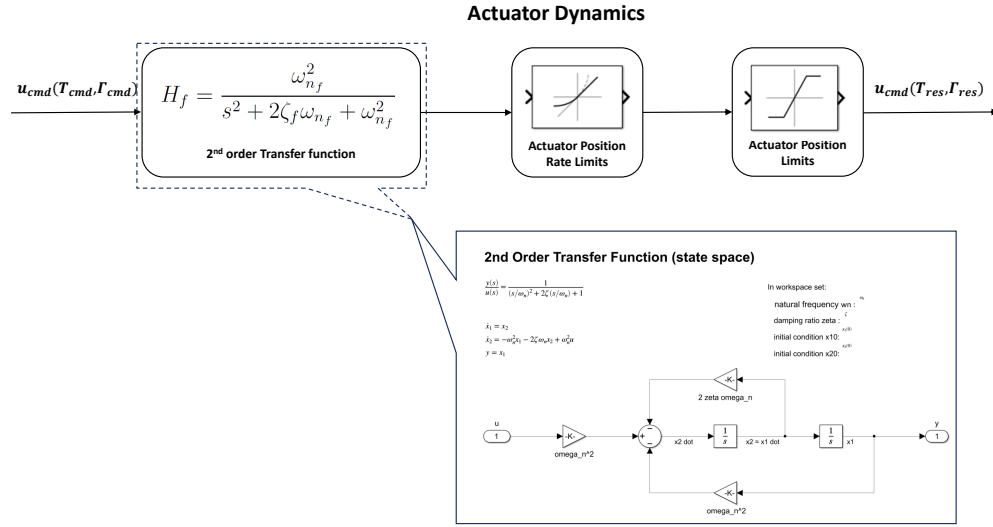
Fig. 5.4 Thrust (T) and Tilting Mechanism (Γ) Actuators Dynamics

Table 5.5 Actuator Physical Limits and Dynamics

Actuator control input	ω_n	ζ	Minimum	Maximum	Rate Limit
Thrust for canard EDFs	25rad/s	1	0N	3000N	$\pm 2000\text{N/s}$
Thrust for wing EDFs	25rad/s	1	0N	4000N	$\pm 3000\text{N/s}$
Γ for canard EDFs	20rad/s	1	-30°	120°	$\pm 60^\circ/\text{s}$
Γ for wing EDFs	20rad/s	1	0°	120°	$\pm 60^\circ/\text{s}$

5.2.2 Estimation of the EDF Thrust and Torque Coefficients

Nathen et al. (2021) estimates that the power required at cruise is 224kW hence each of the individual 36 fans needs to provide 6.2kW of power during cruise. This paper also provides an equation for hover power:

$$P = \left(\frac{T^3}{2\rho n A_d \eta^2} \right)^{1/2} \quad (5.24)$$

where A is the Total EDF set cross-sectional area $= n/A_d$, ω is the EDF Set Fan rotational speed (rads^{-1}), P is the EDF Set total Power (Watt), T is the EDF Set total Thrust (N), A_d EDF cross-section area, η EDF set efficiency. An off-the-shelf EDF product, named Schübeler HST-215 in SCHUEBELER (2000)(2020) was used for this study. A similar EDF is used in a NASA-founded DEP aircraft project (Freeman and Klunk, 2018). An approximation by up to third order polynomials by Suiçmez (2021) was used to model the thrust, torque, and power characteristics of the Schübeler EDF, datasheet SCHUEBELER (2000, 2020) and it is seen that using an average C_T (Thrust coefficient) and C_Q (Torque coefficient) value gives thrust and torque characteristics accurate enough. The simple thrust and torque models in Suiçmez (2021) are given by invertible functions:

$$T = C_T \omega^2 \quad (5.25)$$

$$Q = C_Q T \quad (5.26)$$

where

- C_T is the thrust coefficient
- C_Q is the torque coefficient

Table 5.6 Some Parameters in Schuebeler EDF

Maximum Torque	10Nm
Maximum Power	15.6kW
Mass	3.5kg
Diameter	21cm

In this section, the unified controller is verified for several test cases via nonlinear simulation model build in MATLAB[®] Simulink[®]. Flight dynamics model generated in Chapter 2 is used to represent the open-loop flight dynamics.

First, the unified controller is tested considering the full flight envelope of the eVTOL given as follows: vertical take-off, low speed hover flight, transition to forward flight, climb, cruise flight, descent, transition to hover flight and vertical landing.

Secondly, to show the criticality of the CA in case of limited control authority (i.e. the actuator saturation), disturbance rejection tests are done injecting severe wind disturbance to the model. To analyze the performance of the CA design, tests are performed disabling the classical CA block (see Figure 5.3) in the loop by using the unconstrained controls. When the CA is disabled, the pure NDI law given in Equations 3.48, 3.47, 4.66 and 4.67 are used in the controller. Also, when the CA algorithm is not active, only the simple pseudo-inverse approach in equations 4.66 and 4.67 is applied for the allocation, and the actuator saturation related problems are not resolved effectively. Simulations show that the CA plays a vital role to handle saturation-related stability problems (Section 5.2.5).

Third, the NDI controller sensitivity to model parameters is analysed. As mentioned in Chapter 3, the classical NDI approach is significantly dependent on accuracy of model (including the mass, inertia and aerodynamic coefficients). To verify this, transition from hover to forward flight tests are performed applying significant error in the lift and drag coefficients of the aerodynamic sections. It is observed that the NDI controller is not fully robust to modeling errors, this is as expected. Based on the results, reasonable errors in mass and inertia can be handled by the controller. Therefore, accurate modeling of the mass and inertia is not required to have desired controller response. However, the controller is not robust to aerodynamic modelling errors.

Finally, the criticality of the CCA is also tested during actuator failure. To do this, two EDF sets were made to become inoperational (total loss of thrust during VTO while some EDFs set are close to the limits. The CCA maintained a stable flight by reallocating the limited control authority to both critical control channels and operational EDF sets while considering their saturation limits.

To illustrate the results, five plots are used for each test scenario. The first and second plot gives the aircraft state variable response and commands. Third plot illustrates the actuator positions including both the direct control allocation output (commanded effector positions) with subscript "cmd" and actual effector positions states considering the actuator dynamics with subscript "res" (see equations 5.22 and 5.23 and Figure 5.4). The fourth plot shows the commanded accelerations and actual acceleration while the fifth plot shows

$$\mathcal{W} = \{W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8\}$$

whether the control allocation is active or not, explained as follows:

- $W_* = 0$ means no actuator saturation
- $W_* = -1$ means actuator saturation at lower limit
- $W_* = 1$ means actuator saturation at upper limit
- W_1 means canard left actuator EDFs thrust/rpm
- W_2 means canard right actuator EDFs thrust/rpm
- W_3 means wing left actuator EDFs thrust/rpm
- W_4 means wing right actuator EDFs thrust/rpm
- W_5 means canard left actuator EDFs tilt angle
- W_6 means canard right actuator EDFs tilt angle
- W_7 means wing left actuator EDFs tilt angle
- W_8 means wing right actuator EDFs tilt angle

Note that simulations are performed applying different types of commands discussed in the command generation function described in Sections 3.4.1 and 3.4.2 and Figure 3.2. To observe the CA algorithm's computational properties, the 'CA Iteration Number' is also given in the plots. If the "CA Iteration Number" is above 1, then the CA algorithm overwrites the NDI controller output to find the optimum allocation iteratively considering the actuator limits and channel prioritization described in Section 5.1.6.

5.2.3 Manoeuvre 1 - Vertical Take-off, Hover & Low Speed Flight, Vertical Landing

First, the controller is tested for hover & low speed flight conditions. The test case is explained as following. Some commands are applied simultaneously to observe the controller's response during the coupled dynamics.

- At 3s, 3000m, a Vertical take-off (VTO) altitude command ($h_{cmd}=3020\text{m}$) is applied at constant climb rate of 2m/s, and the controller tracks the corresponding vertical velocity command (w_{cmd}). Similarly, at 72s vertical landing (VL) command is applied.
- Between 20s and 43s, forward and backward motion is tested via applying velocity command in the body x-direction u_{cmd} (via airspeed V_{cmd} and flight path angle γ_{cmd} commands).
- Between 35s and 72s, positive and negative side velocity is commanded (v_{cmd}) to achieve side-ward translational motion via generating corresponding roll angle command (ϕ_{cmd}).
- Between 28s and 72s, positive and negative pitch attitude is commanded via generating corresponding pitch angle command (θ_{cmd}).
- Between 65s and 75s, aircraft heading is changed by commanding yaw angle command (ψ_{cmd}).

A visual representation of manoeuvre 1 is shown in Figure 5.5.

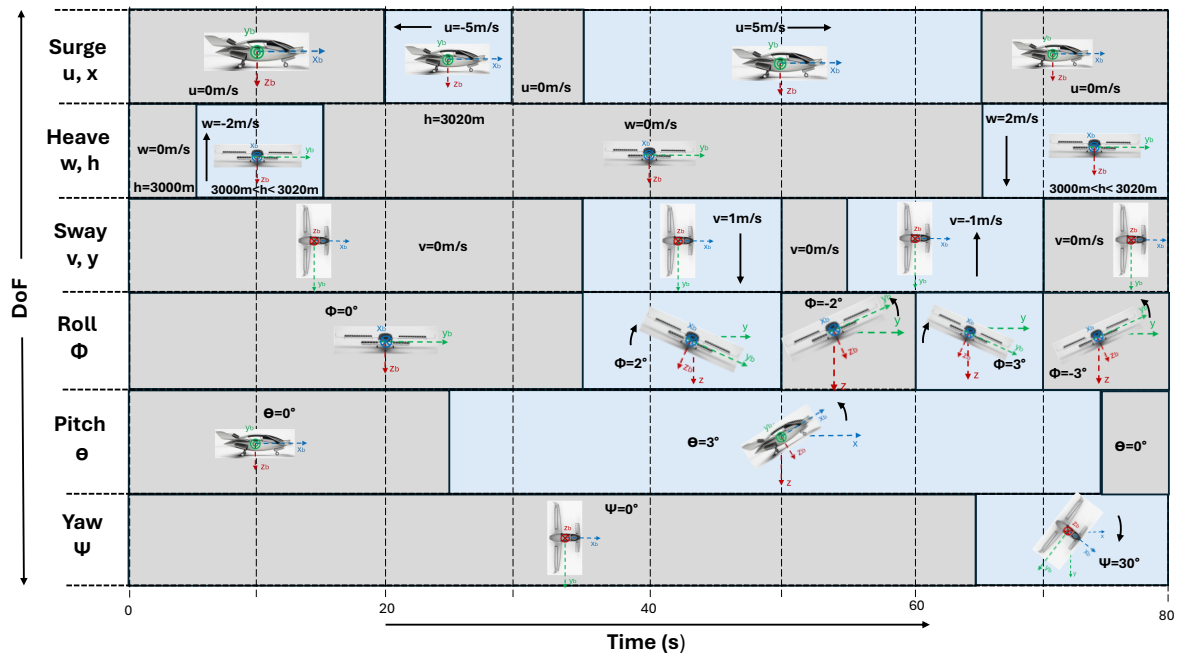


Fig. 5.5 Manoeuvre 1 Visual Representation - VTO, Hover Flight, VL

The commanded and actual states, commanded and actual effector positions and control authority (actuator limitation) status are shown in Figures 5.6 - 5.9.

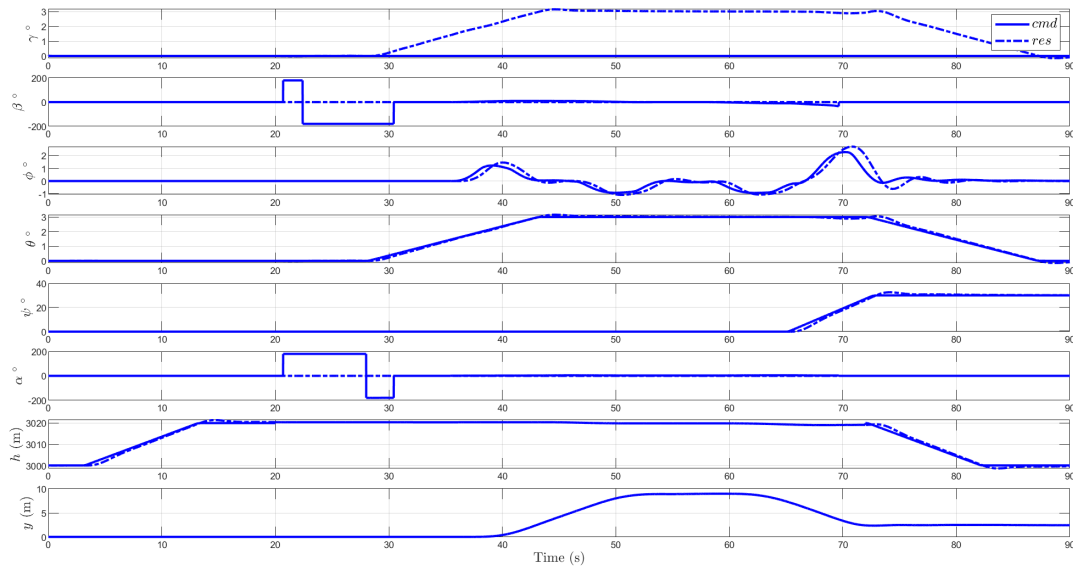


Fig. 5.6 VTO, Hover Flight, VL - Attitude

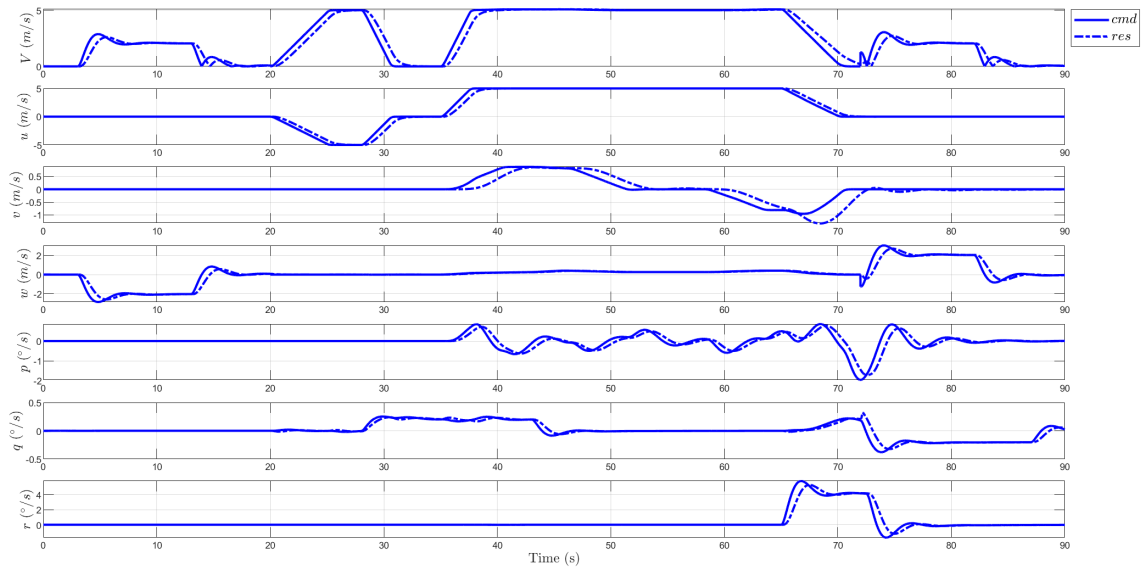


Fig. 5.7 VTO, Hover Flight, VL - Velocity

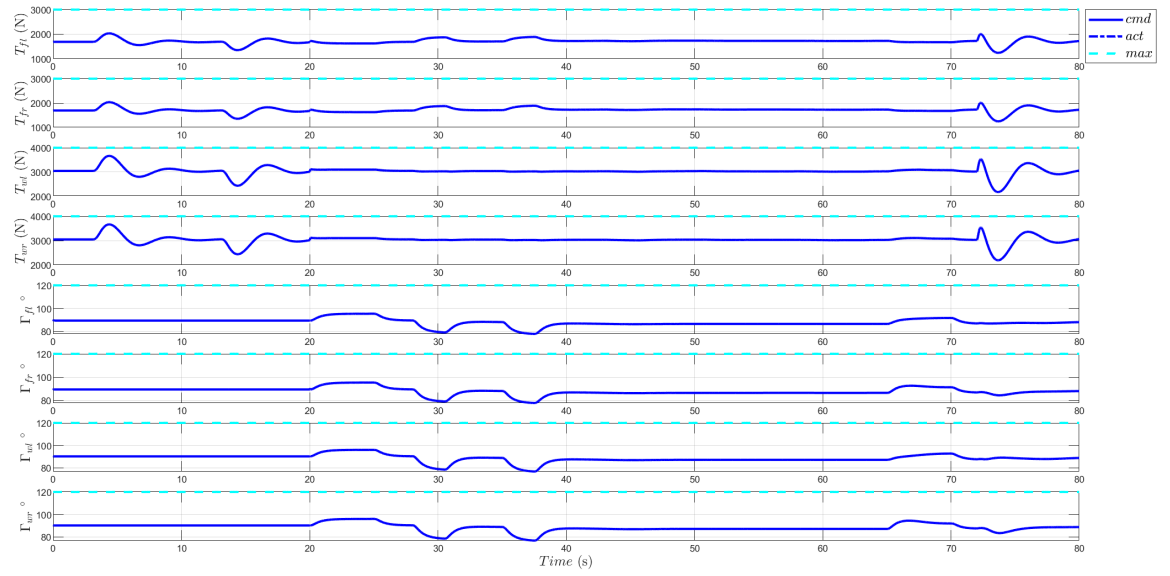


Fig. 5.8 VTO, Hover Flight, VL - Effector Commands

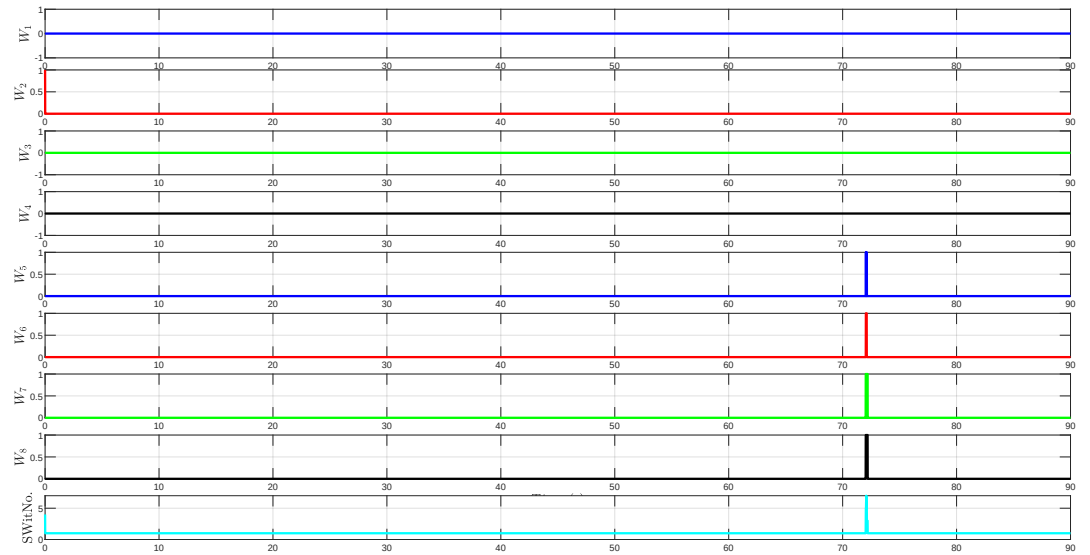


Fig. 5.9 VTO, Hover Flight, VL - Control Limitation Status

1. Analyses of the Altitude Response: According to Figure 5.6, 20m altitude increase is commanded at 3s, and the aircraft tracks the corresponding vertical speed command (w_{cmd}). In between 3s and 75s, several commands are applied in other channels. The aircraft keeps the 3020m altitude during the forward, side-ward motions and also the heading change. Note that the command generator applies approximately $2(m/s^2) \approx 0.2g$ maximum vertical acceleration command (Figure A.10). According to Figure A.10, w_{cmd} is tracked well and the aircraft holds altitude (h) while other commands cause disruptive effects on the vertical channel. Regarding the control inputs, it is expected to increase only thrust and not changing tilt angles to gain altitude. Results given in Figure 5.8 is consistent with the expectation. Thrust levels total (two EDF sets of 400N each) increase around 800N on the canard EDFs and a total (four EDF sets of 600N each) of 2400N on the wing EDFs to gain altitude. $\frac{1}{3}$ ratio in thrust increment between the canard and wing EDFs is to achieve zero pitch disturbance while gaining altitude since x-lever arms(Δx) of the front and wing EDFs has an approximate ratio of $\frac{1}{3}$ as given in Table 2.5.

2. Analyses of the Forward and Backward Motion: Translational motion in the body x direction is obtained via commanding body x -velocity (u_{cmd}) via airspeed V_{cmd} and flight path angle γ_{cmd} commands. The same command is also used to perform the transition easily and also to control the angle of attack indirectly in forward flight. Based on Figure 5.7, $\pm 5m/s$ forward velocity command is applied between 20s and 40s, the state derivative ($\dot{u}$) is shown in Figure (A.10). To accelerate in forward direction, it is expected to tilt EDFs in negative direction to use some of the thrust for forward acceleration. Similarly, to decelerate, tilt angles need to increase. Note that at hover tilt angles are 90° . These two outcomes can be seen on the actuator positions given in Figure 5.8 between 20s and 40s.

3. Analyses of the Side Motion: To perform side-ward motion at low speeds, positive and negative side velocity commanded between 7s and 25s. As previously mentioned, EDFs do not generate side force directly. Therefore, thrust direction is changed via applying a navigational bank angle command (ϕ_{cmd}) at low speeds based on Equation 3.8. As shown in Figure 5.7, v_{cmd} and the corresponding (ϕ_{cmd}) are tracked well. There exist a small delay in tracking of the side velocity command. This result is expected since side velocity is not directly controlled and result of change in thrust vector direction. Linear navigational controller gain (K_{vp}) can be increased to reduce the delay in side velocity command tracking if necessary (Table 5.4). Roll rate tracking performance of the controller is given Figure 5.6. Commanded roll rate p_{cmd} is tracked accurately. To change the roll angle at low speeds, thrust levels are changed in opposite manner on the right and left sections of the aircraft and no change in the tilt angles are expected. The required change in thrust is very small to achieve the desired roll rates. On the other hand, to keep the altitude during the side-motion thrust must be increased significantly since thrust vector is not aligned with the gravity. Therefore, the asymmetric change in thrust between the right and left sections cannot be observed directly in Figure 5.8 since it illustrates the total thrust command. Tilt angles are kept same during the side-motion as expected.

4. Analyses of the Heading Response: To change the aircraft heading, $\pm 30^\circ$ yaw angle is commanded between 65s and 72s (Figure 5.6). The NDI controller tracks the required body yaw rate (r) to achieve the desired heading. According to Figure 5.6 and A.10, yaw rate tracking performance is satisfactory. To change the aircraft heading at hover, tilt angles of the right and left EDFs change asymmetrically. This result can be seen at 70s seconds of actuator position plot (Figure 5.8) when the negative heading command is applied. It is observed that approximately $2^\circ - 3^\circ$ degree changes on the EDFs are enough to achieve the desired heading command, and tilt angle changes on the front EDFs are much more smaller. Since change in tilt angles are very small, additional thrust required to keep the altitude is very small.

5.2.4 Manoeuvre 2 - VTO and Heading Control, Forward Transition

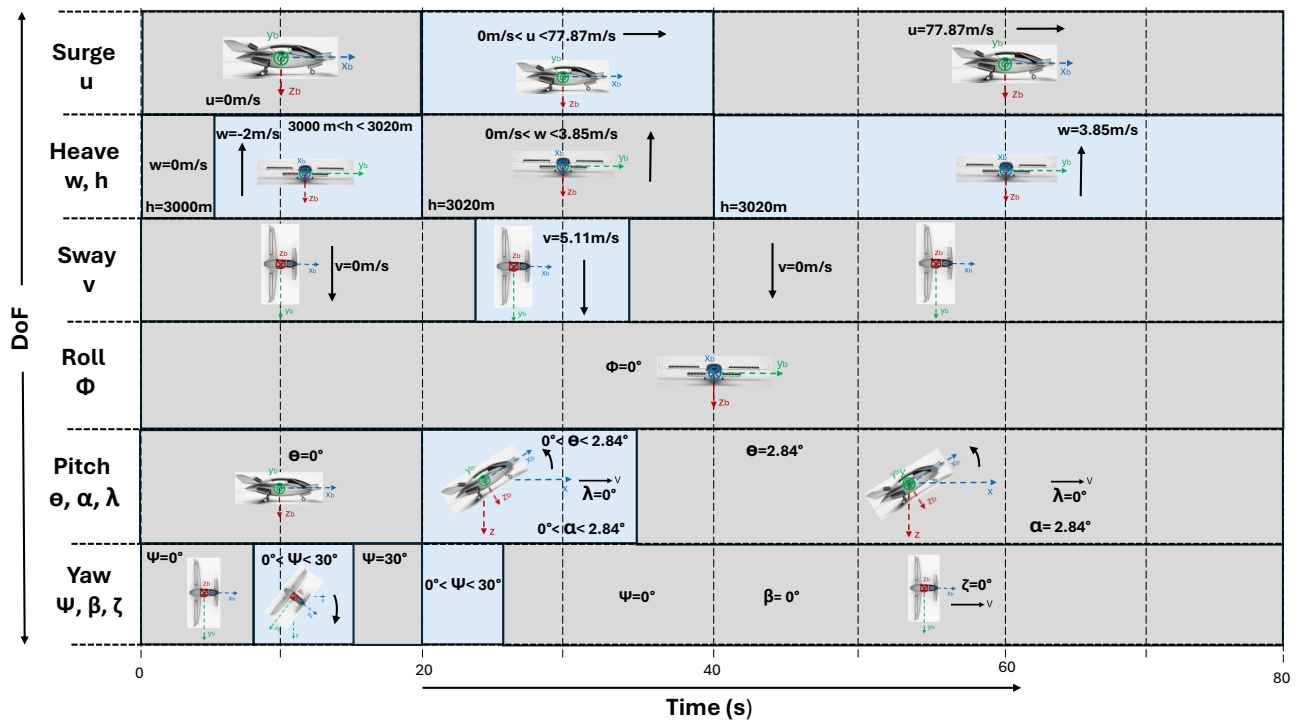
Transition from hover to forward flight is one of the most critical flight regime regarding the flight control due the coupled vectored thrust control and strong nonlinear dynamics during the transition (see Section 2.3.2). Note that nonlinear dynamic behaviour occur in the aerodynamic model during the transition due to the blending between the hover and forward flight models explained in equations 2.30, 2.31 and 2.32.

Recall that the aircraft is trimmed at cruise condition with 77.87m/s airspeed and 2.84° degree angle of attack. Detailed trim data including the trim actuator states for cruise condition are given in Table 2.6 in Section 2.4.1. Actuator positions are given in terms of thrust and tilt angle in Table 2.7 and to be consistent with the NDI structure and actuator position using the unconstrained control allocation.

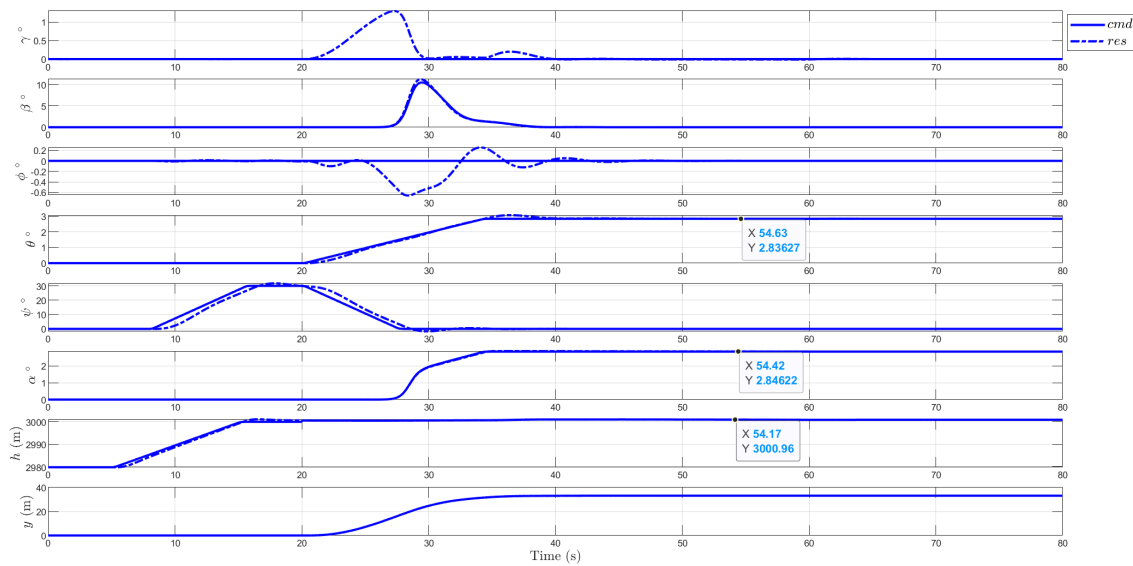
EDFs on both canard and wing sections are tilted forward to accelerate the aircraft. Thrust is also adjusted to keep both pitch attitude and altitude at desired level while accelerating the aircraft to high speed forward flight. According to the trim data at 77.77m/s, the canard EDFs generate total of $\approx 180\text{N}$ (2×89.8191 for EDF Sets 1/2 and EDF Sets 3/4) at both left and right sections and tilt angle is around 28° (0.52 rad); whereas, the wing EDFs generate total of $\approx 600\text{N}$ (4×149.6018 for EDF Sets 5 – 8 and EDF Sets 9 – 12) at both left and right sections with tilt angle of approximately 0.0013229° .

- At 5s, 2980m, a Vertical take-off (VTO) altitude command ($h_{cmd} = 3000\text{m}$) is applied at constant climb rate of 2m/s, and the controller tracks the corresponding vertical velocity command (w_{cmd}).
- At 8s, aircraft heading is changed by commanding yaw angle command (ψ_{cmd}) of $\psi=30^\circ$ to change the aircraft heading while climbing vertically and then then heading returned back to $\psi=0^\circ$
- Between 20s and 70s, forward transition is commanded by simultaneously applying velocity command in the x and z direction body-axis u_{cmd} , w_{cmd} (via airspeed V_{cmd} and flight path angle γ_{cmd} commands) and AoA α . An airspeed speed V of 77.8m/s was commanded at an acceleration of 4m s^{-2} , fpa γ of 0° and pitch attitude θ of 2.84° at a constant rate of $\pm 0.01 \text{ rad s}^{-1}$ was commanded over a 17.5s duration.

A visual representation of manoeuvre 2 is shown in Figure 5.10.



The commanded and actual states, commanded and actual effector positions and control authority (actuator limitation) status are shown in Figures 5.11 - 5.14.



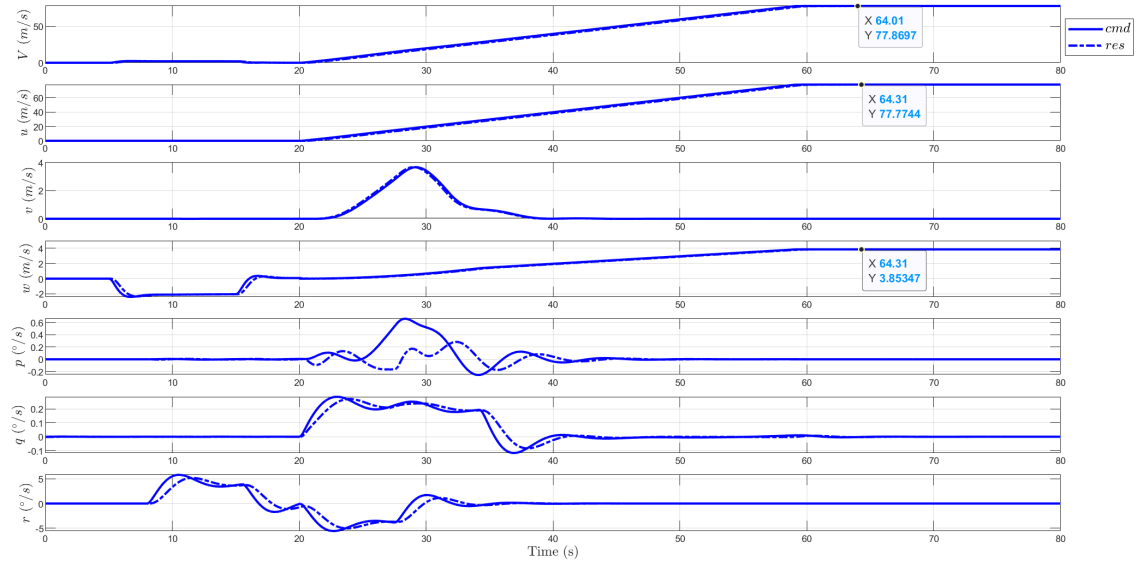


Fig. 5.12 VTO/Right Turn, Forward Transition - Velocity

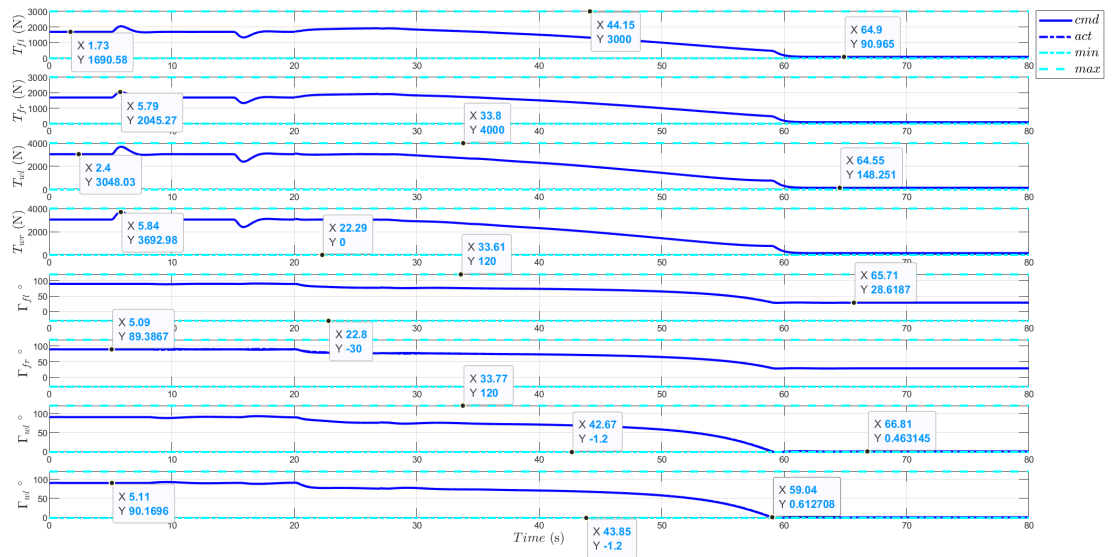


Fig. 5.13 VTO/Right Turn, Forward Transition - Effector Commands

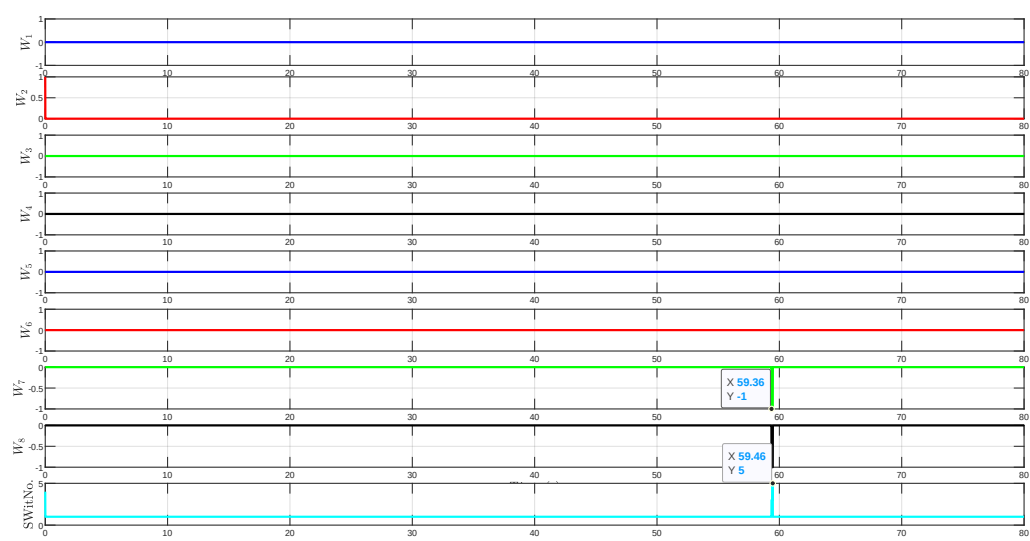


Fig. 5.14 VTO/Right Turn, Forward Transition - Control Limitation Status

Analysis of Manoeuvre 2 Results *During the forward transition both the Airspeed (V) and AoA (α) are commanded simultaneously such that cruise trim AoA α of 2.84° is reached before cruise trim airspeed ensuring very low tilt angles in order to reduce rotor-induced drag (resulting in high power consumption) that occurs at high airspeed and high actuator tilt angles. This is different from the forward transition performed in Suiçmez (2021) and Suiçmez and Kutay (2023) where cruise trim speed was first commanded before trim AoA was commanded resulting in a period of time in between of high rotor-induced drag and high power consumption due to high tilt angle at high forward speed.*

Figure 5.11 and 5.12 show the state response of the forward transition. The aircraft climb to 3000m at first and commands heading change of initially 30° and back to 0° before accelerating to 77.77m/s after 39s with an approximate acceleration of $2m/s^2$ which can be seen on Figure A.11 $\dot{u}$ plot.

According to the actuator position plot given in Figure 5.13, both front and wing section EDF tilt angle decrease after 20s to provide thrust for the forward acceleration. During the forward acceleration, thrust levels decreased by $\approx 95\%$ (Canard EDF Set from 3048N to 149N, Wing EDF Set from 3048N to 149N i.e below 10% (canard 5.4% and main wing 4.89%) compared to hover on both front and wing EDFs since the lift is primarily provided by the wings and canards leaving the EDF to only provide forward thrust. Thrust decreases at the wing EDFs are higher compared to the canard EDFs. This is expected since with increasing airspeed the lift generated by the wings increases and generates positive pitch moment. To balance the lift generated pitch moment, the front EDFs lost less thrust (generate more thrust) compared to the wing EDFs from Hover to Cruise.

The NDI controller is designed to control the body angular and translational velocities, see equations 3.47, 3.48, 3.53 and 3.49. Therefore, it is possible to control the angle of attack and flight path angle indirectly using the relations given in Equations 3.28, 3.29 and 3.30. To achieve 2.84° trim angle of attack, combined 2.84° , θ_{cmd} , 0° γ_{cmd} and 77.8m/s V_{cmd} are applied at 20s (see Figure 5.11). Desired trim values are reached without any loss of altitude (See Figure 5.11), trim states are achieved with very small errors such as: $\alpha_{trim} = 2.85^\circ$, $w_{trim} = 3.85m/s$, $u_{trim} = 77.77m/s$, $\beta_{trim} \approx 0^\circ$, $\gamma_{trim} \approx 0^\circ$, compared to the trim data given in Table 2.6.

It is critical to also analyze the actuator positions at the cruise trim condition, and to check consistency with the linearized model cruise trim data given in Table 2.6. Based on Figure 5.13, thrust values at trim are 91N (89.819N at cruise trim) for the front EDFs, and 148N (149.60N at cruise trim) for the wing EDFs. Moreover, tilt angles are 28.62° (28° at cruise trim) for the front EDFs, and 0.46° (0.0013229° at cruise trim) for the wing EDFs. Although there exist small differences in the trim actuator positions compared to the linearized model trim data given in Table 2.6, the trend is the same. It is expected to have minor discrepancy in the trim actuator positions since the nonlinear simulation model includes additional effects (e.g., actuator rates, dynamics, delays, etc.) that are not represented in the linearized trim data.

Performance of the CA approach and Active Set algorithm is also analyzed. The final plots of Figures A.11 and 5.14 show the CA iteration number, more explicitly whether the CA algorithm is actively working or not. At the end of the forward transition, as can be seen from Figure 5.13, the unconstrained CA (Moore Penrose pseudo-inverse) output ($\Delta \mathbf{u}_{uc}$ in Figure 5.3) violated the minimum tilt angle limit of the wing EDF (see W_7 and W_8) limits (-1.2°)

at 59s resulting in the Active Set Constrained (ASC) CA algorithm being active (see Figure 5.14) imposing actuator constraints limitations on the CA output which prevents violation of the physical actuator constraints.

The CA performance is also investigated considering the real time implementation. According to the last plots of Figure A.11 and Figure 5.14, the maximum “CA Iteration Number” are below 10 when the ASC CA is active. Also, Figure 5.14 shows that minimum tilt angles limits were violated for the wing EDFs - see the 7th(W_7) and 8th(W_8) plots. Therefore, the CA algorithm finds the optimal solution quickly. Moreover, the CA algorithm does not cause discontinuities/sudden jumps in the actuator states which is crucial for the real time implementation

5.2.5 Manoeuvre 3 - Criticality of CA: VTO and Heading Control, Forward Transition

To analyze the criticality of the active set classical constrained CA (CCA) design, same forward transition is performed disabling the classical CCA block (see Figure 5.3) in the loop by using the unconstrained controls. When the CCA is disabled, the pure NDI law given in equations 3.48, 3.47, 4.66 and 4.67 and the simple pseudo-inverse unconstrained control allocation (UCA) approach in equations 4.66 and 4.67 is applied for the allocation, and the actuator saturation related problems are not resolved effectively. Simulations results in Figures 5.15 - A.12 show that the CA plays a vital role to handle saturation related stability problems (Section 5.2.5). A visual representation of manoeuvre 3 is same as that of manoeuvre 2 shown in Figure 5.10.

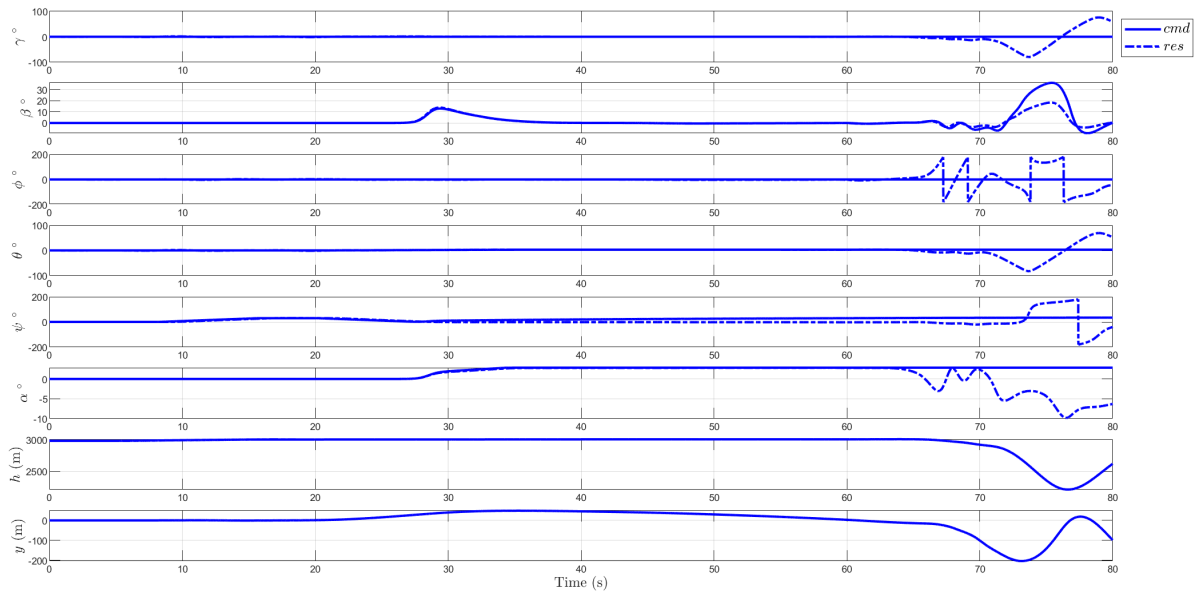


Fig. 5.15 Criticality of CCA: VTO/Right Turn, Forward Transition - Attitude

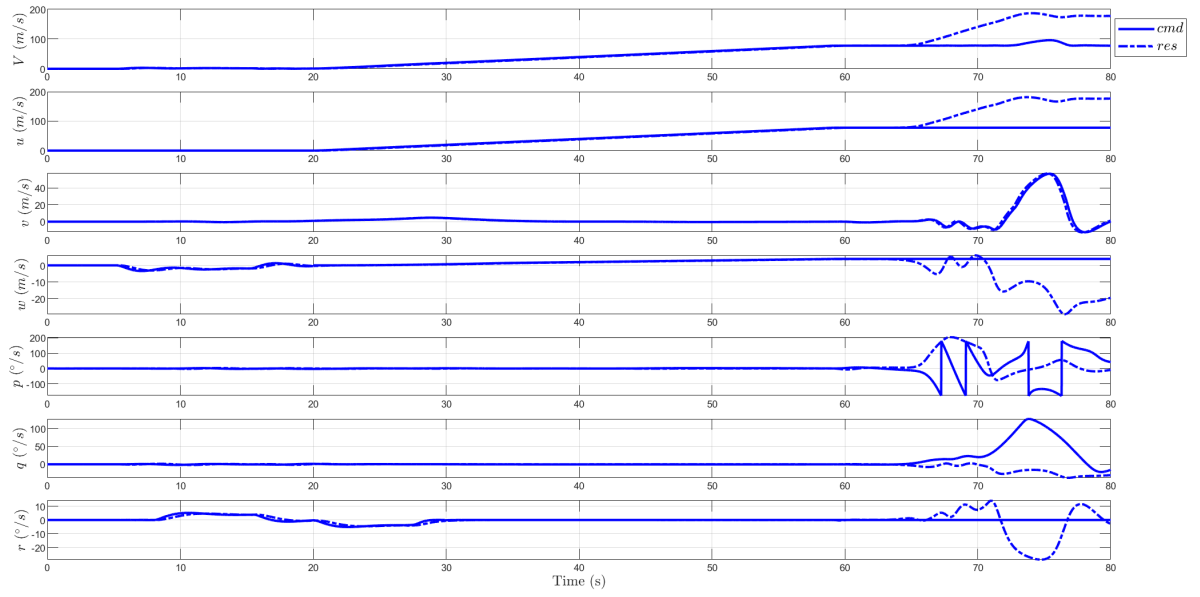


Fig. 5.16 Criticality of CCA: VTO/Right Turn, Forward Transition - Velocity

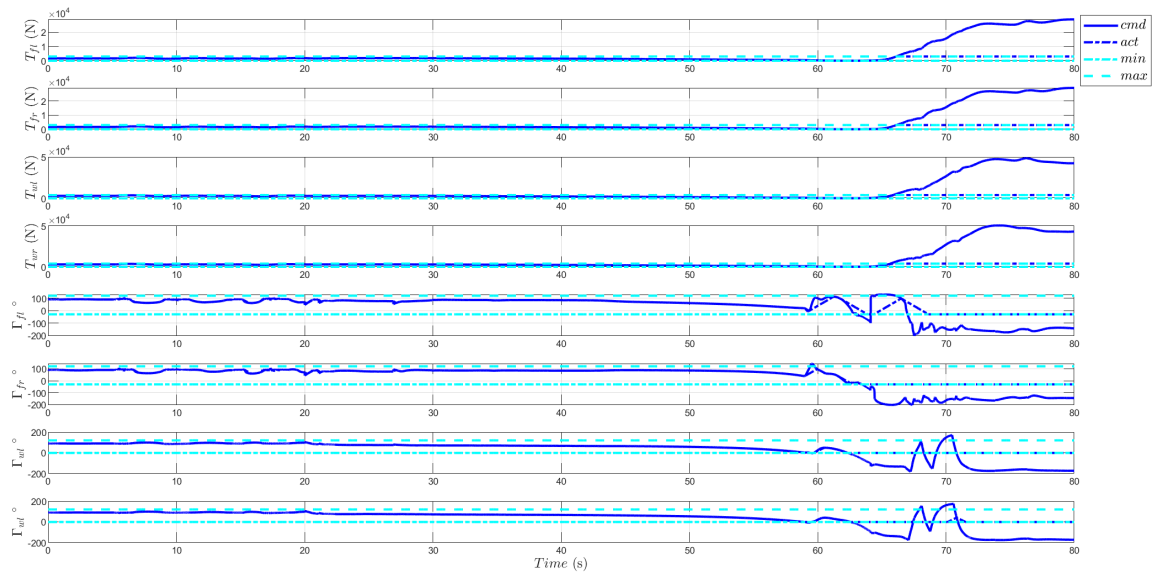


Fig. 5.17 Criticality of CCA: VTO/Right Turn, Forward Transition - Effector Commands

Analysis of Manoeuvre 3 Results Based on the results, actuator saturation are not observed up to 59s, and the psedo-inverse unconstrained controls performance is satisfactory. However, the unconstrained controls (UCA) are out of actuator limits after 59s (Figure 5.17). Then, actuator commands generated by the pseudo-inverse can not be achieved in reality due to the position limits, and control is lost since the psedo-inverse unconstrained controls becomes ineffective. As described in Section 5.2.4, the same transition test case is performed successfully when the CCA algorithm is active. *To conclude, the Active Set Classical CCA has a vital role regarding the proper allocation of the limited control authority. Criticality of the CCA will be also discussed thoroughly in disturbance rejection test case (Section 5.2.9)*

5.2.6 Manoeuvre 4 - Cruise, Reverse Transition and VL with Heading change

Transition from high speed cruise flight to low speed hover flight refered to as ‘Reverse Transition’ is also a critical flight regime regarding the flight control due the coupled vectored thrust control and strong nonlinear dynamics during the transition (see Section 2.3.2). Note that nonlinear dynamic behavior occur in the aerodynamic model during the reverse transition due to the blending between the forward flight and hover models explained in equations 2.30, 2.31 and 2.32.

The reverse transition test starts at the cruise condition with $h = 3000\text{m}$, 2.84° degree angle of attack and 77.87m/s airspeed. During the reverse transition both the Airspeed (V) and AoA (α) are commanded simultaneously such that the trim AoA α of 2.84° was reduced to 0° before cruise trim airspeed decreased from 77.87m/s to hover speed of 0m/s ensuring the duration of high low tilt angles is minmised in order to reduce rotor-induced drag (resulting in high power consumption) that occurs at high airspeed and high actuator tilt angles. At 92s, 30° left-turn is commanded by commanding yaw angle command (ψ_{cmd}) of $\psi = -30^\circ$ to change the aircraft heading while landing from $h = 3000\text{m}$ to $h = 2977\text{m}$. *This is different from the forward transition performed in Suiçmez (2021) and Suiçmez and Kutay (2023) where the cruise trim AoA (α) was first commanded to 0° before the cruise trim speed was first commanded before decreased from 77.87m/s to hover speed of 0m/s resulting in a period of time in between of high rotor-induced drag and high power consumption due to high tilt angle at high forward speed.*

A visual representation of manoeuvre 4 is shown in Figure 5.18.

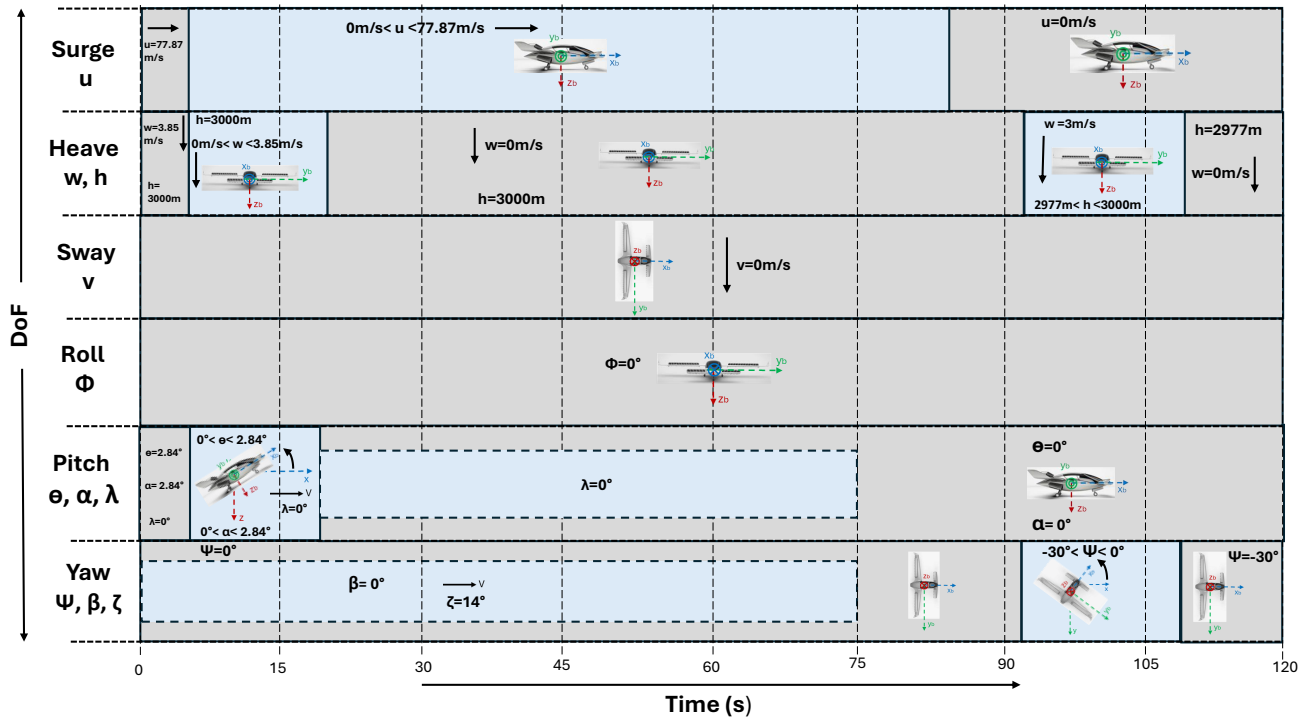


Fig. 5.18 Manoeuvre 4 Visual Representation: Cruise, Reverse Transition and VL/Left Turn

The commanded and actual states, commanded and actual effector positions and control authority (actuator limitation) status are shown in Figures 5.19 - 5.22.

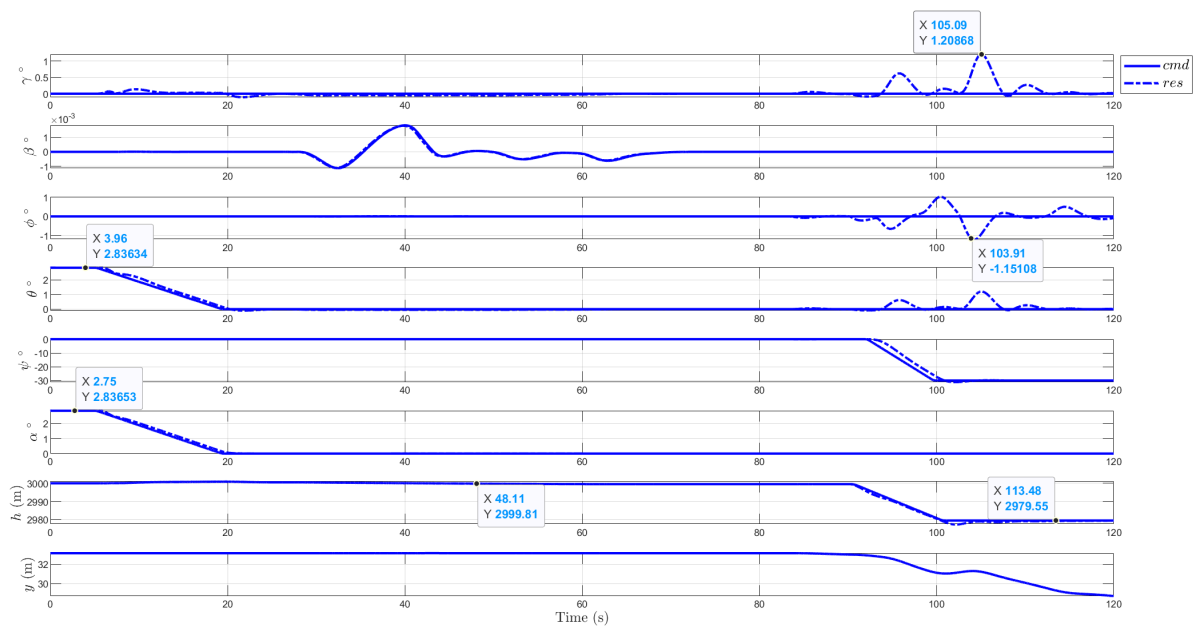


Fig. 5.19 Cruise, Reverse Transition and VL/Left Turn - Attitude

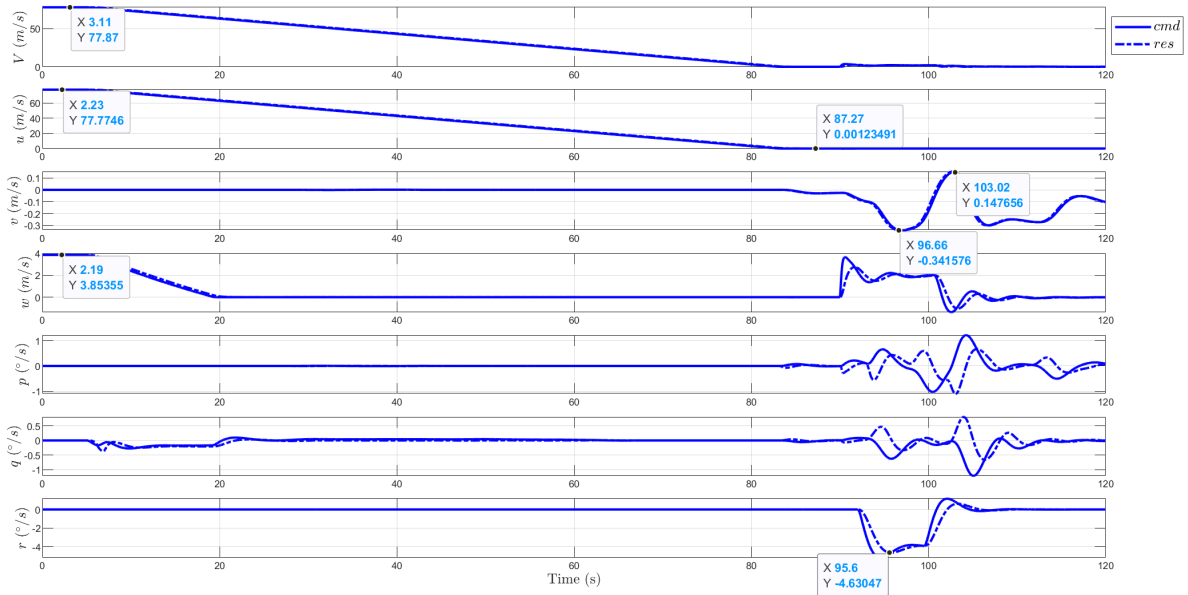


Fig. 5.20 Cruise, Reverse Transition and VL/Left Turn - Velocity

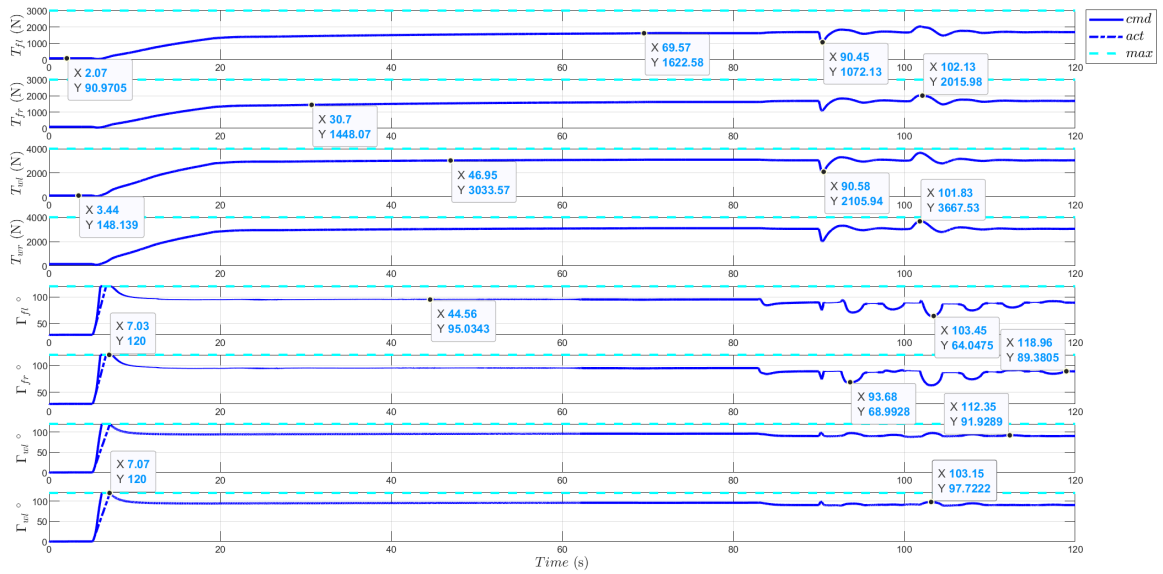


Fig. 5.21 Cruise, Reverse Transition and VL/Left Turn - Effector Commands

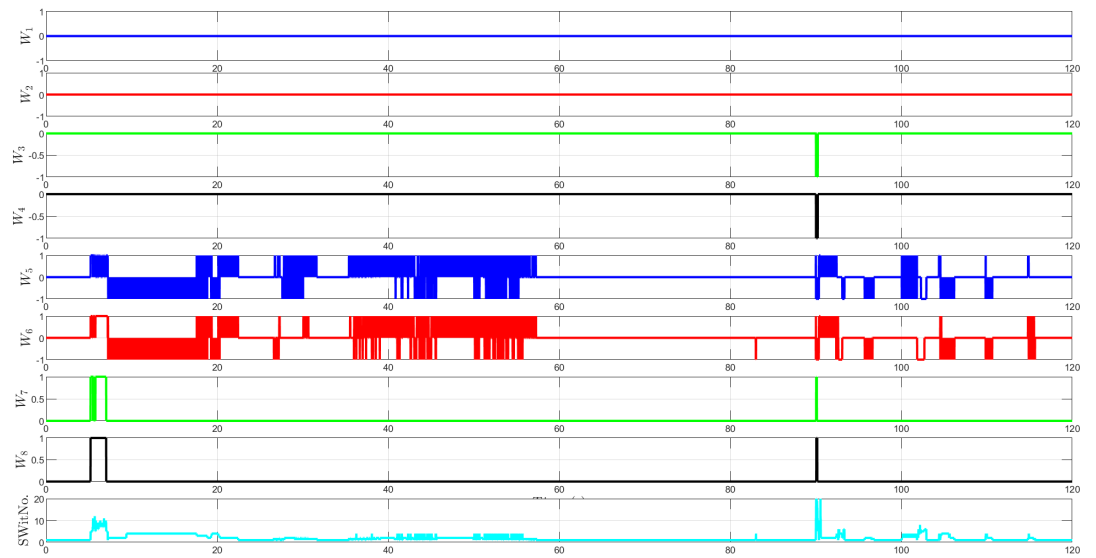


Fig. 5.22 Cruise, Reverse Transition and VL/Left Turn - Control Limitation Status

Analysis of Manoeuvre 4 Results As expected, tilt angles and rpm/thrust increase during the reverse transition since lift generated by the wing reduces with the decreasing airspeed. At $t = 82s$, the aircraft reaches the hover condition with 90° tilt angles and hover trim thrust values (Figure 5.21). At $t = 82s$, $h = 2980m$ command (VL) and a heading change ($\psi = -30^\circ$) were simultaneously applied and the aircraft lands vertically at $t = 102s$ and completed the (-30°) heading change at $t = 100s$.

This differs from the forward transition performed in Suiçmez (2021); Suiçmez and Kutay (2023) where the cruise trim AoA (α) was first commanded to 0° before the cruise trim speed was first commanded before decreased from $77.87m/s$ to hover speed of $0m/s$ resulting in a period of time in between of high rotor-induced drag and high power consumption due to high tilt angle at high forward speed.

During the initial phase of the reverse transition, as can be seen from Figure 5.21, the high-level controller outputs violated all the tilt angle position ($W_1 - W_7$) limits between $5s$ and $7.5s$ resulting in the CAs being active (see Figure 5.22) imposing actuator constraints limitations on the CA output which prevent violation of the physical actuator constraints. Also, the canard left and right tilt angle rates (W_5 and W_6) limits were violated several times as can be seen from Figures A.14 and 5.22.

In general, controller performance is satisfactory during the reverse transition and vertical landing as the maximum number of control allocation iteration is 20 which is satisfactory for real-time safety critical software applications.

5.2.7 Manoeuvre 5 - Cruise, Climb, Descend, Cruise

Climb and descent maneuvers are tested at cruise condition. The commanded and actual states, commanded and actual effector positions and control authority (actuator limitation) status are shown in Figures 5.24 - 5.27. Beginning at cruise altitude $3000m$, first, 4° flight path angle commands (γ_{cmd}) are applied between $5s$ and $26s$ during which the aircraft climbed $200m$ (to $3200m$). Then -2° flight path angle commands (γ_{cmd}) are applied between $26s$ and $130s$ during which the aircraft descended back to cruise altitude ($3000m$) and then continues in cruise. The climb FPA (γ) of 4° is set by a combination of pitch attitude ($\theta_{cmd} = 6.3672^\circ$) while maintaining cruise AoA ($\alpha_{cmd} = 2.3672^\circ$) using equation 3.30 while the descent FPA (γ) of -2° is set by a combination of pitch attitude ($\theta_{cmd} = 0^\circ$) while setting an AoA ($\alpha_{cmd} = 2^\circ$). This manoeuvre though not a typical manoeuvre is used in emergencies to clear obstacle along eVTOL air-taxis flight path and since it shows a climb and descent manoeuvres at cruise conditions performed within a short time window is suitable for performance analysis in flight simulations.

A visual representation of manoeuvre 5 is shown in Figure 5.23.

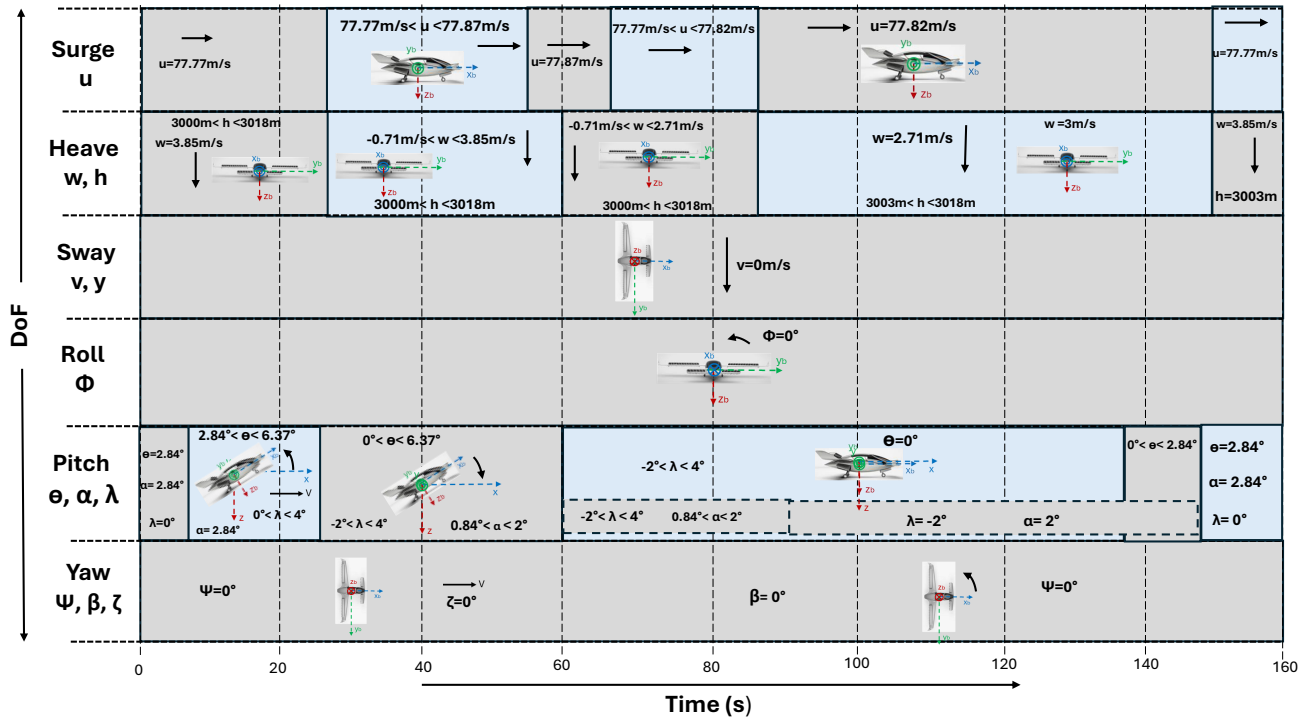


Fig. 5.23 Manoeuvre 5 Visual Representation: Cruise, Climb, Descend

The commanded and actual states, commanded and actual effector positions and control authority (actuator limitation) status are shown in Figures 5.24 - 5.27.

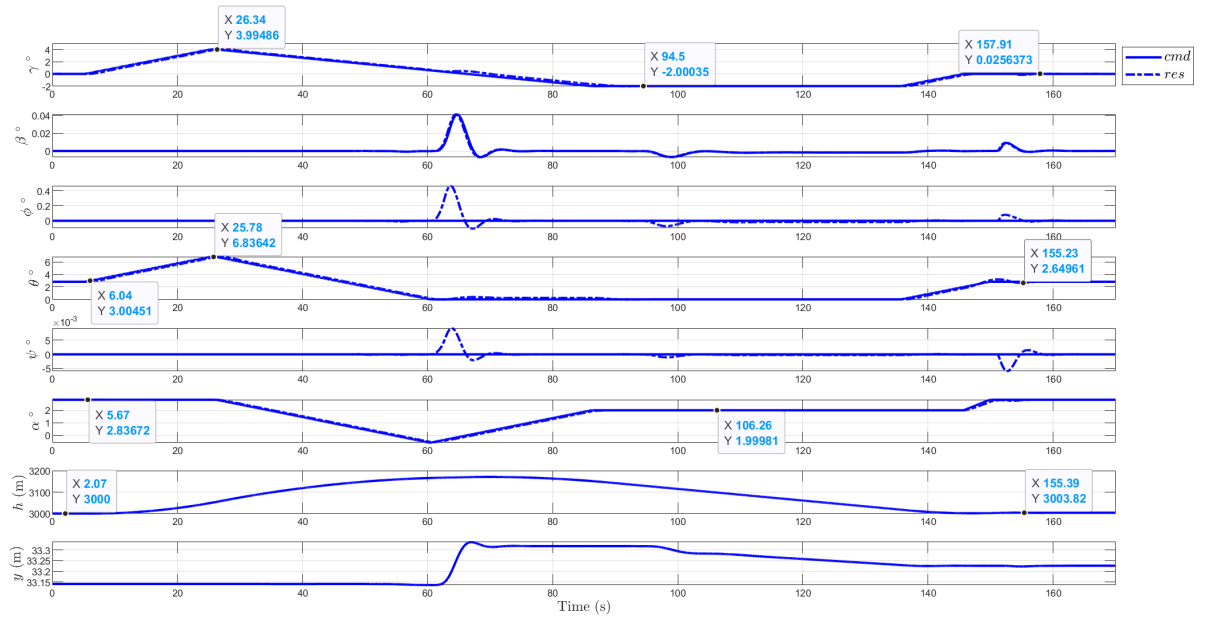


Fig. 5.24 Cruise, Climb, Descend - Attitude

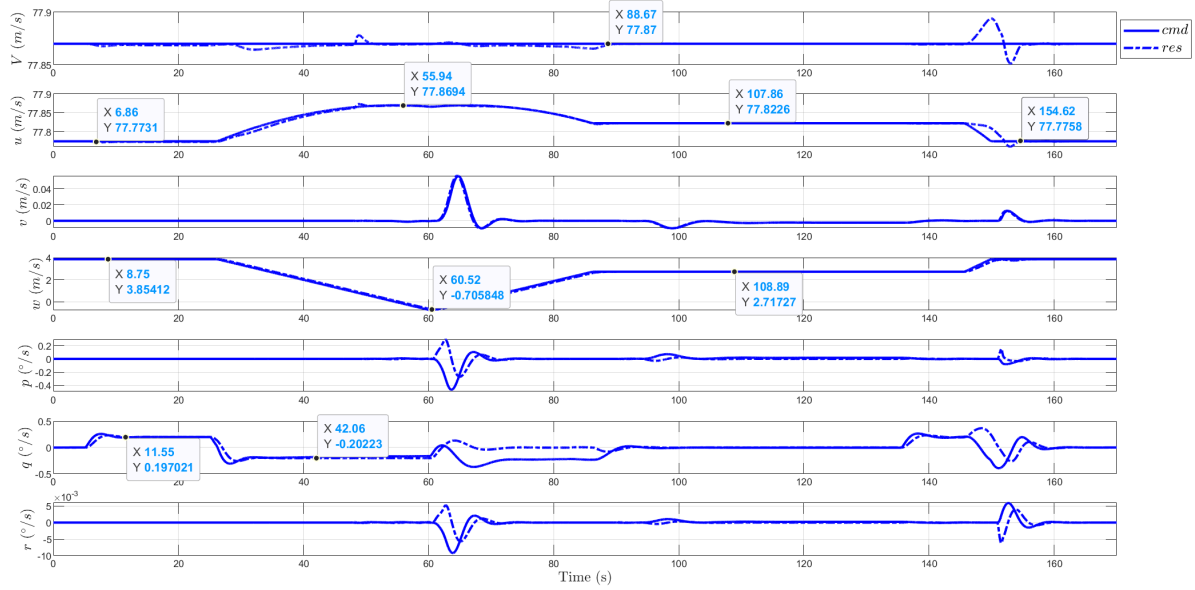


Fig. 5.25 Cruise, Climb, Descend - Velocity

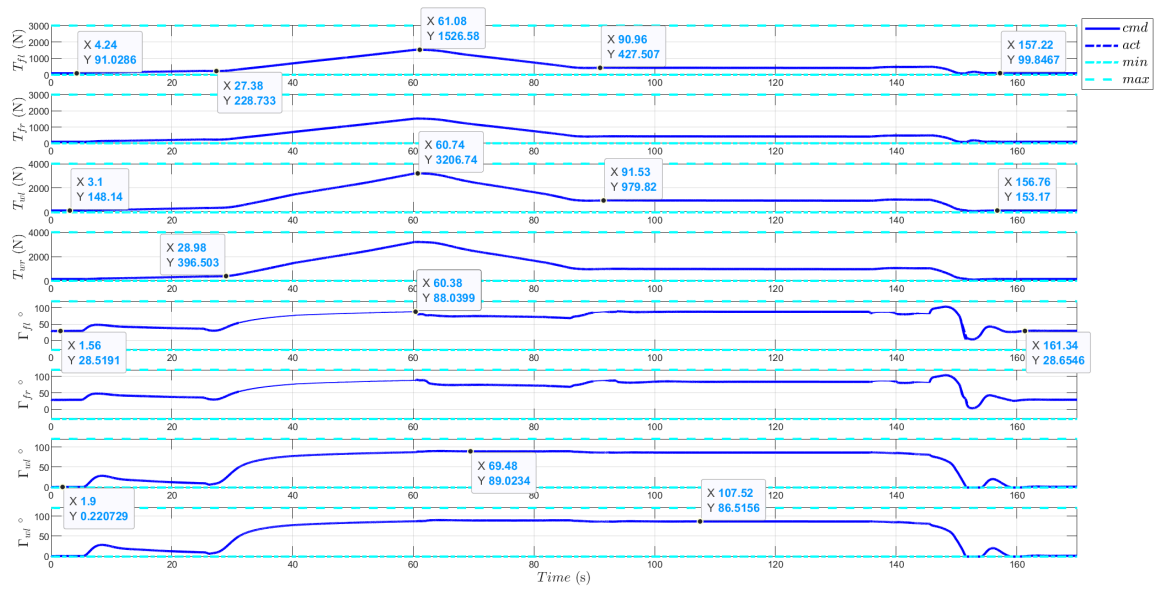


Fig. 5.26 Cruise, Climb, Descend - Effector Commands

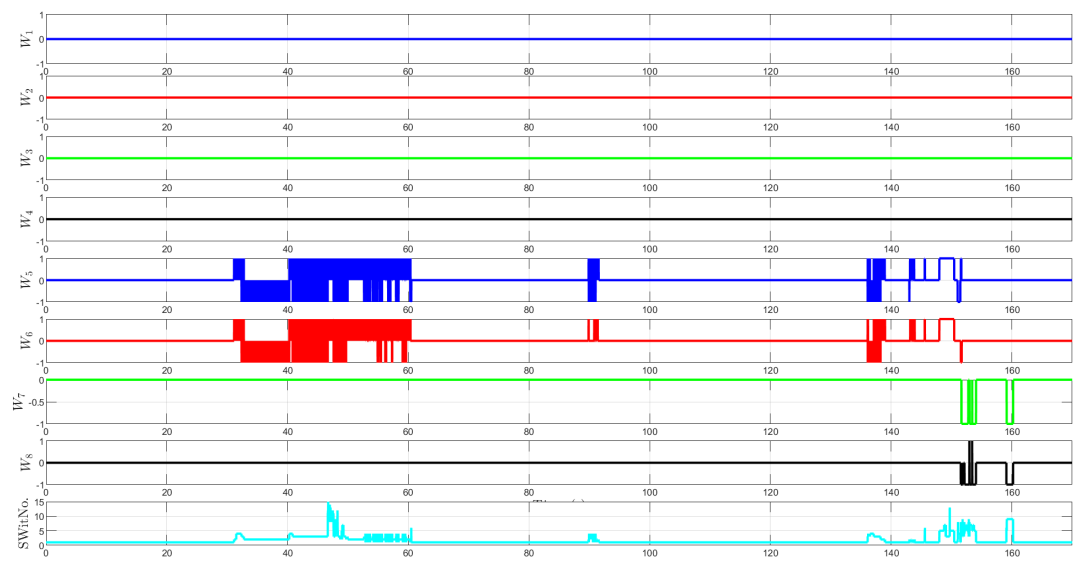


Fig. 5.27 Cruise, Climb, Descend - Control Limitation Status

Analysis of Manoeuvre 5 Results The Controller performance is satisfactory during the climb and descent maneuvers. According to the actuator position plot given in Figure 5.26, the CA algorithm becomes active several times during the climb and descent manoeuvres. Lower position tilt angle limit and tilt angle rate limits are reached numerous times during the maneuver (see Figures 5.26, A.15 and 5.27). Note that the maximum iteration number of the CA algorithm is fewer than 20, and the actuator commands are continuous (Figures A.15 and 5.27).

5.2.8 Manoeuvre 6 - Cruise and Coordinated Turn

The coordinated turn maneuvers is tested at cruise condition. The commanded and actual states, commanded and actual effector positions and control authority (actuator limitation) status are shown in Figures 5.29 - 5.32. Beginning at cruise altitude 3000m, forward speed 77.77m/s alongs 0° flight path angle commands (γ_{cmd}), a coordinated turn is performed between 5s and 40s by applying 30° roll angle command (see Figures 5.29 and Figure 5.30) and the corresponding turn radius and yaw rate command are given in equations 3.13 and 3.14 respectively.

The roll rate, pitch rate and yaw rate compensation are given in equations 3.15, 3.16, 3.17. To reduce the altitude loss behavior during the turn, the aircraft can be trimmed for coordinated turn (level flight, turning, non-skidding flight) at cruise using a trim bank angle, corresponding turn rate and turn radius ensuring equilibrium state (balance of horizontal and vertical force).

A visual representation of manoeuvre 6 is shown in Figure 5.28.

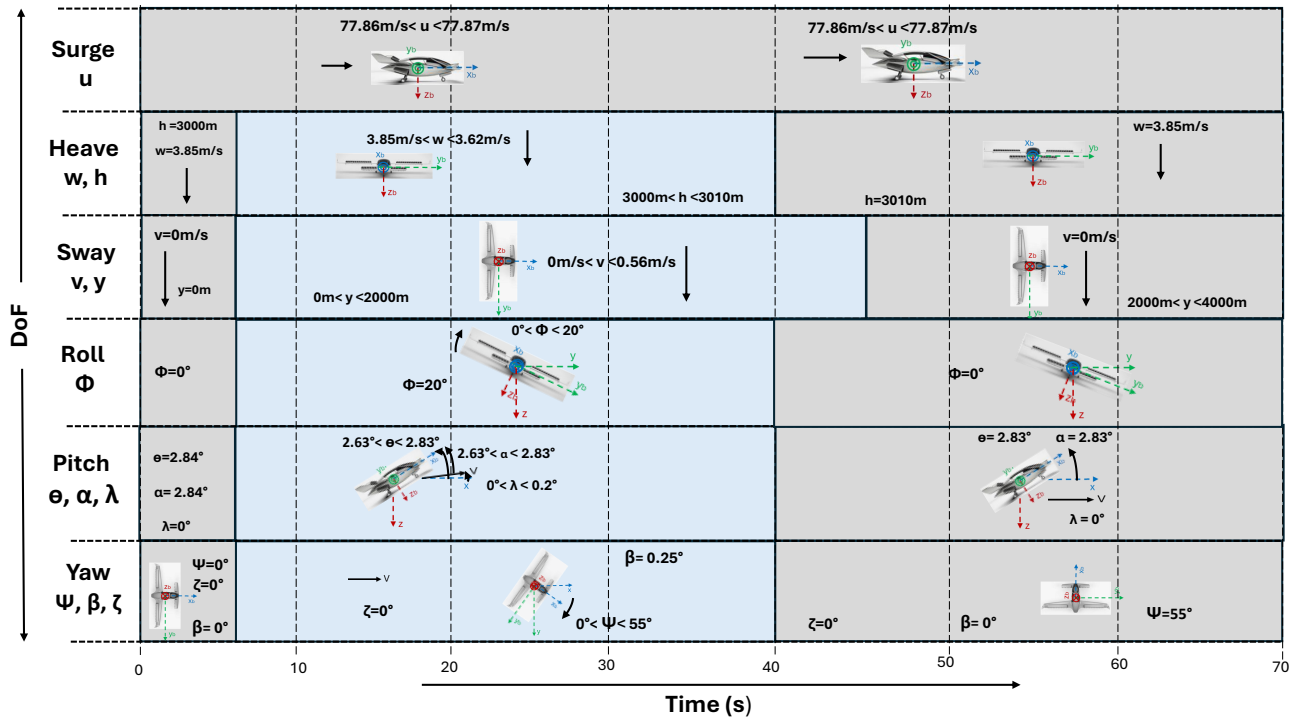


Fig. 5.28 Manoeuvre 6 Visual Representation: Cruise and Coordinated Turn

The commanded and actual states, commanded and actual effector positions and control authority (actuator limitation) status are shown in Figures 5.29 - 5.32.

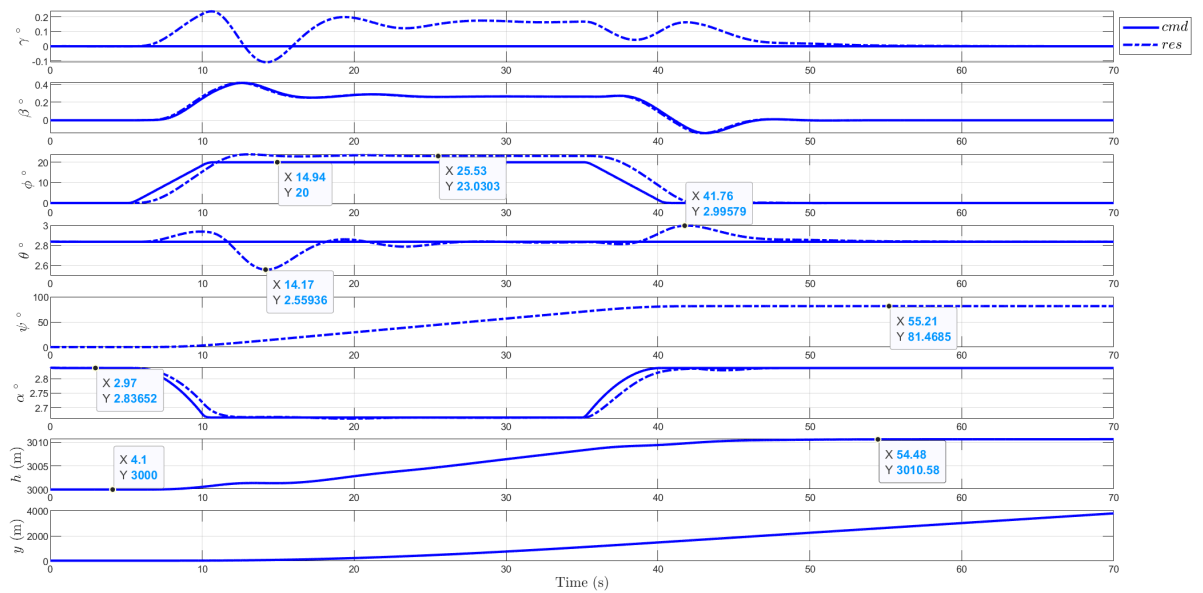


Fig. 5.29 Cruise and Coordinated Turn - Attitude

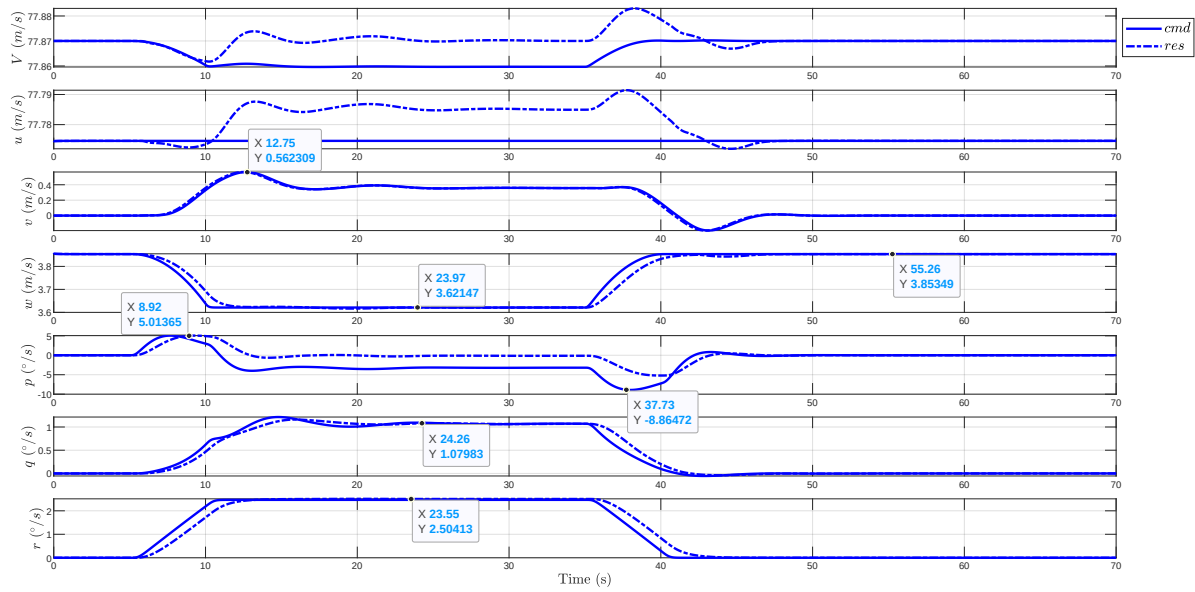


Fig. 5.30 Cruise and Coordinated Turn - Velocity

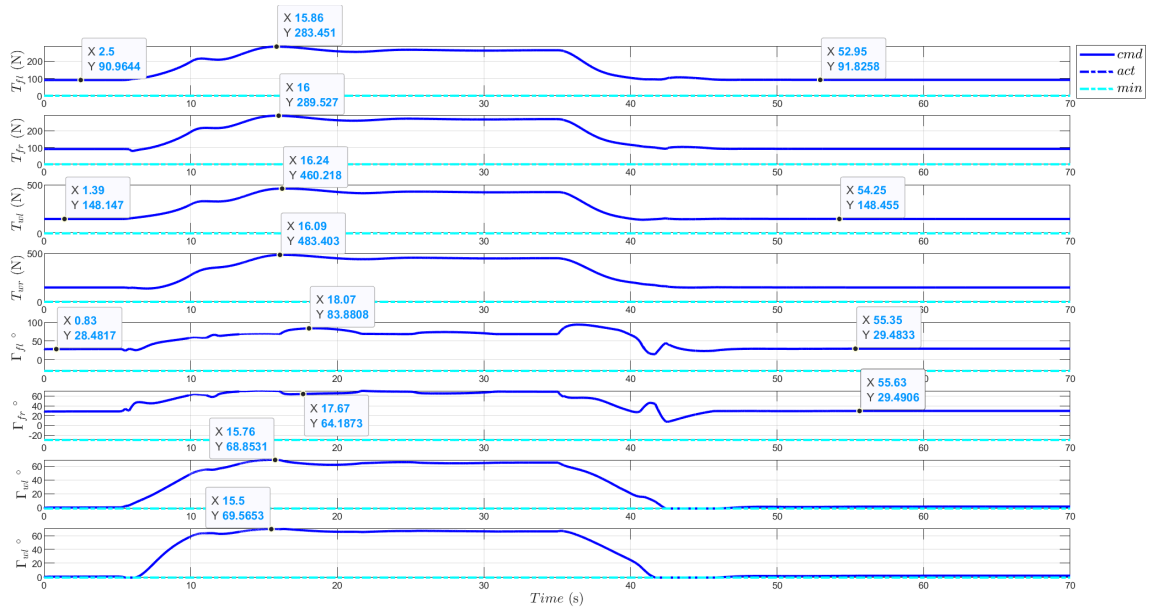


Fig. 5.31 Cruise and Coordinated Turn - Effector Commands

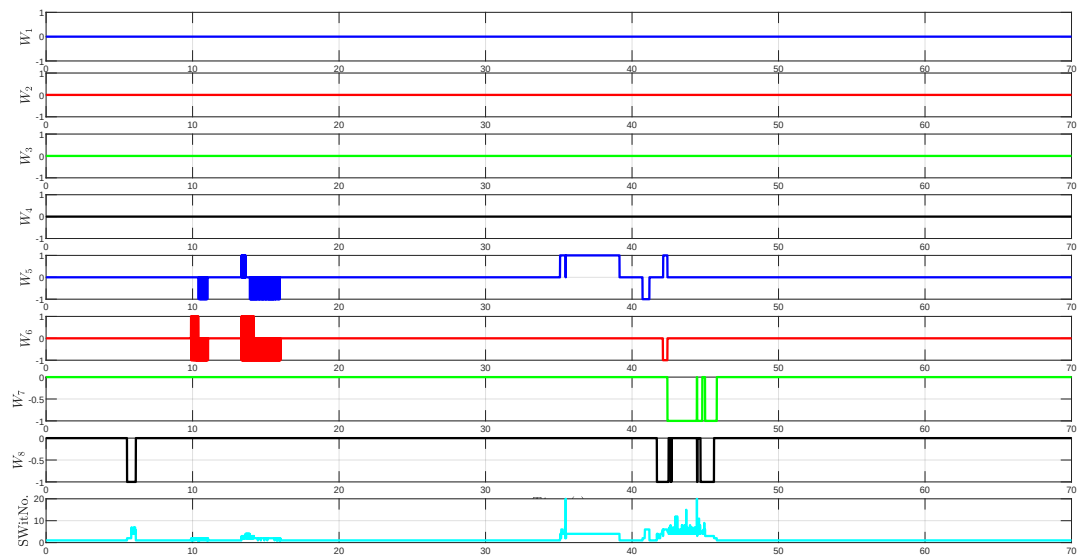


Fig. 5.32 Cruise and Coordinated Turn - Control Limitation Status

Analysis of Manoeuvre 6 Results To maintain altitude during coordinated turn (level turn), lift must be increased via increasing AoA (α) by increasing pitch (based on pitch rate turn compensation in equation 3.16) assuming cruise airspeed is maintained which in turn will increase drag (i.e. since drag increases with lift). To compensate for the increase in drag, thrust must be increased. Figure 5.31 shows that all EDFs increased in thrust and tilt angle during the turn to compensate for reduction in aerodynamic lift and increase in aerodynamic drag. Since it was right turn ($\phi_{Cturn} = 20^\circ$), the left EDFs (canard from 90N to 289N, wing from 148N to 483N) experienced higher thrust increase compared to the right EDFs (canard from 90N to 283N, wing from 148N to 460N). However, differential increase in tilt angle between the canard right EDFs (canard from $\approx 28^\circ$ to $\approx 64^\circ$) and left EDFs (canard from $\approx 28^\circ$ to $\approx 84^\circ$). The stall speed in turn is also increased but the bank angle is limited.

The yaw rate compensation (equation 3.17) during turn suppressed adverse yaw effects ensuring a non-skidding non side-slipping turn. The roll rate compensations (equation 3.15) ensures sufficient lift due to banking. The CA prioritizes rotational channels over the translational channels to satisfy stable flight in case of actuator saturation. This prioritization order can be seen in Figure 5.31. Translational commands (u_{cmd} and w_{cmd}) are not properly tracked (Figure 5.30 show small increase) when the CA prioritizes the rotational channels to make sure that the rotational commands are tracked accurately hence the aircraft gained altitude (increased from 3000m to 3011m) during the turns (altitude response after 50s seconds) in Figure 5.29) as evident from $\approx 0.2^\circ$ increase in flight path angle γ during the turn. Although $\theta_{Cturn} \approx 0^\circ$ and $\psi_{Cturn} \approx 0^\circ$ was as expected during a coordinated turn (see Figure A.16), it was not a perfect non-skidding, non-slidding and level turn as small side-slipping ($\beta \approx 0.2^\circ$ in Figure 5.29) and skidding ($v \approx 0.4\text{m/s}$ in Figure 5.30).

The performance of the CA is also investigated considering the real time implementation. According to Figure 5.32, the maximum CA software iterations CA_{iter} is 20 when the CA is active. Therefore, the CA algorithm finds the optimal solution quickly. Again, the CA algorithm does not cause discontinuities/sudden jumps in the actuator positions which is crucial for the real time implementation since it considers the actuator position and rate limits.

5.2.9 Manoeuvre 7 - Criticality of CA: Roll Disturbance Rejection with or without Constrained CA

To analyze the disturbance rejection characteristics of the controller (NDI + CCA), severe roll moment disturbance is injected to the simulation model at hover condition. To observe the disturbance rejection characteristics while control authority is limited, a vertical Take-off command (i.e, a negative vertical velocity command, w_{cmd}) is also applied during the roll moment disturbance (Figure 5.33).

The aim is to observe the effectiveness of the constrained CA (CCA) algorithm while control authority is limited. Therefore, the same test case is run when the CCA algorithm is enabled ("CCA ON") and disabled ("CCA OFF, UCA ON"). Simulations results in Figures 5.33 - 5.35 show the results for two cases and demonstrated how the CCA plays a vital role to handle saturation related stability problems.

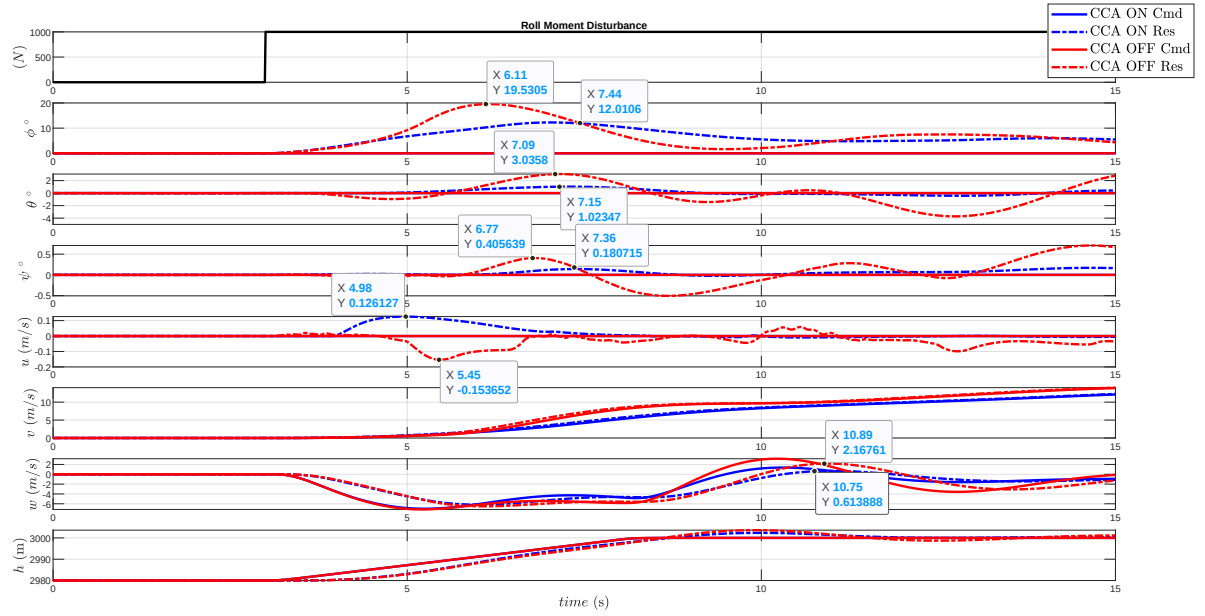


Fig. 5.33 Roll Moment Disturbance Rejection Characteristics at Hover with and without CCA - State Variables

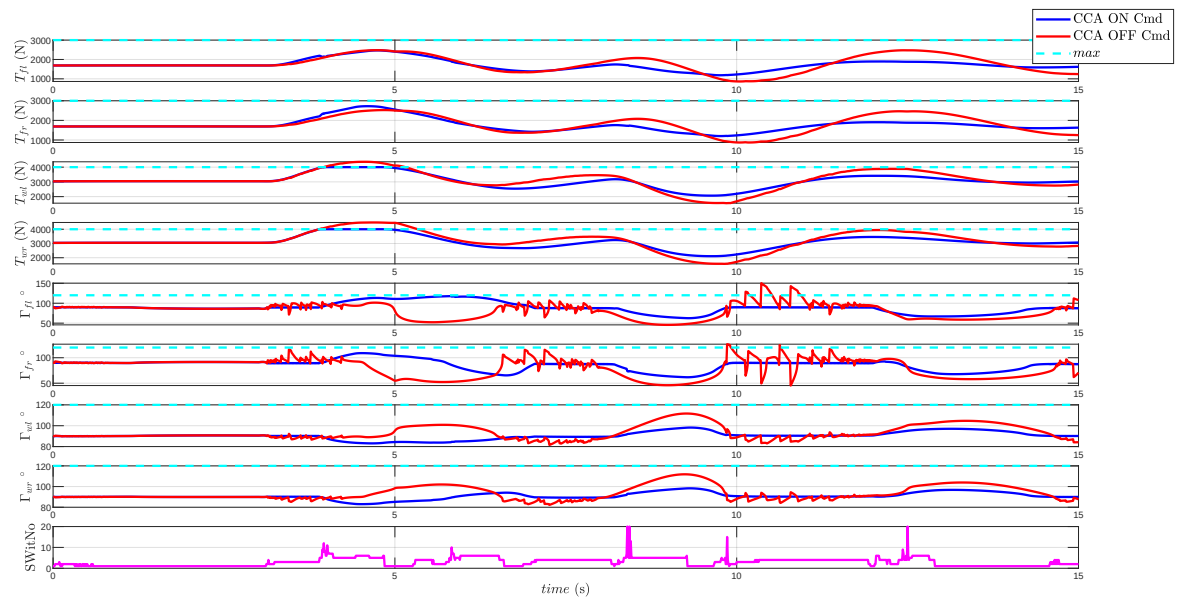


Fig. 5.34 Roll Moment Disturbance Rejection Characteristics at Hover with and without CCA - Effector Commands

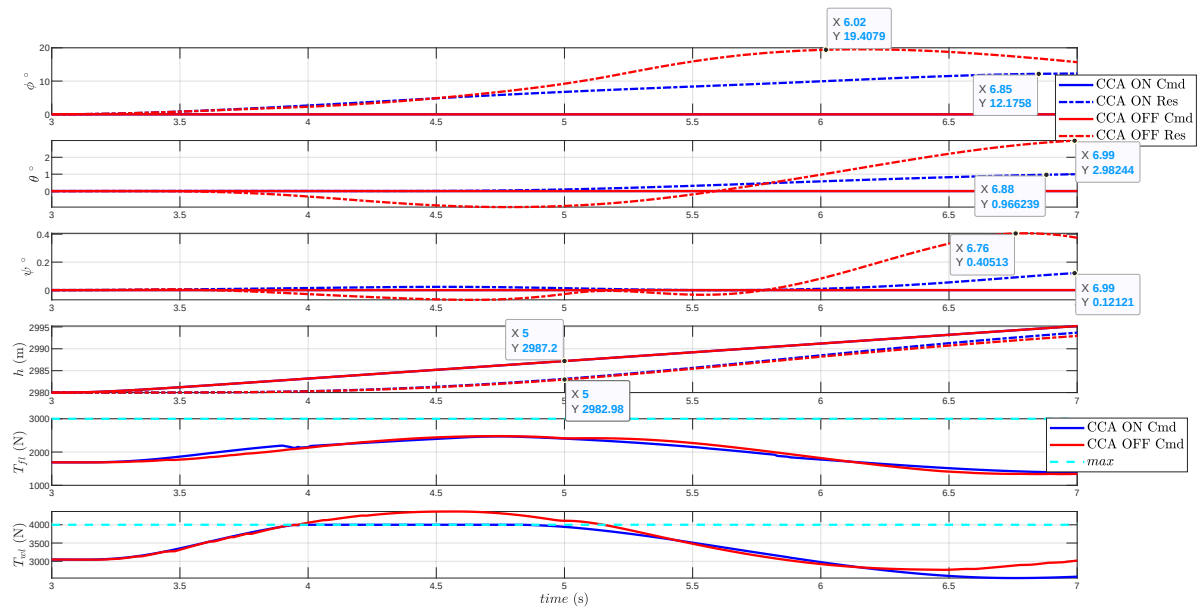


Fig. 5.35 Roll Moment Disturbance Rejection Characteristics at Hover with and without CCA, the CA Trade-off between Roll Moment Disturbance Rejection and Altitude Command Tracking

Analysis of Manoeuvre 7 Results First, the results when the CCA algorithm is not active are analyzed.

Recall that when the constrained CA (CCA) is disabled, the pure NDI law given in Equations 3.48, 3.47 and the simple pseudo-inverse unconstrained CA (UCA) approach in equations 4.66 and 4.67 is applied for the allocation, and the actuator saturation related and control channel prioritisation problems are not resolved effectively.

Based on Figure 5.33, at 3s pilot gives climb command at hover and just after that a strong roll moment disturbance occurs. Then, the pseudo-inverse UCA increase thrust at right EDFs and decrease thrust at the left EDFs to reject the roll moment disturbance. However, all EDFs are already close to the maximum thrust limit due to the climb command. Additional thrust command generated by the UCA to reject the roll moment disturbance is not effective since the actuators are saturated (Figure 5.34). Then, roll moment disturbance cause large deviations on the roll and pitch angles while the actuators are saturated (Figure 5.33, red lines-CCA OFF).

When the same test is applied with the CCA enabled, it is seen that the CA prioritizes the rotational channels over the translational channels (i.e., prioritizes the rejection of roll disturbance over the tracking of climb command), and consider the actuator limits in the optimum allocation. As a result, disturbance rejection characteristics are improved significantly thanks to the proper allocation of the limited control authority (Figure 5.33, blue lines-CCA ON). The trade-off is the worse tracking performance of the climb command. This is an expected result since the CCA makes sure that rotational commands are tracked well as a first objective. Moreover, according to Figure 5.35, the performance loss on the altitude tracking when the CA is active is not significant. To conclude, the CA works very well and provides good disturbance rejection characteristics in case of limited control authority.

5.2.10 Manoeuvre 8 - Criticality of CA: Pitch Disturbance Rejection with or without Constrained CA

To analyze the disturbance rejection characteristics of the controller (NDI+CCA), severe pitch moment disturbance is injected to the simulation model at hover condition. To observe the disturbance rejection characteristics while control authority is limited, a vertical take-off command (i.e, a negative vertical velocity command, w_{cmd}) is also applied during the pitch moment disturbance (Figure 5.36). The aim is to observe the effectiveness of the CCA algorithm while control authority is limited. Therefore, the same test case is run when the CCA algorithm is enabled ("CCA ON") and disabled ("CCA OFF, UCA ON"). Simulations results in Figures 5.36 - 5.38 show the results for two cases and demonstrated how the CA plays a vital role to handle saturation related stability problems.

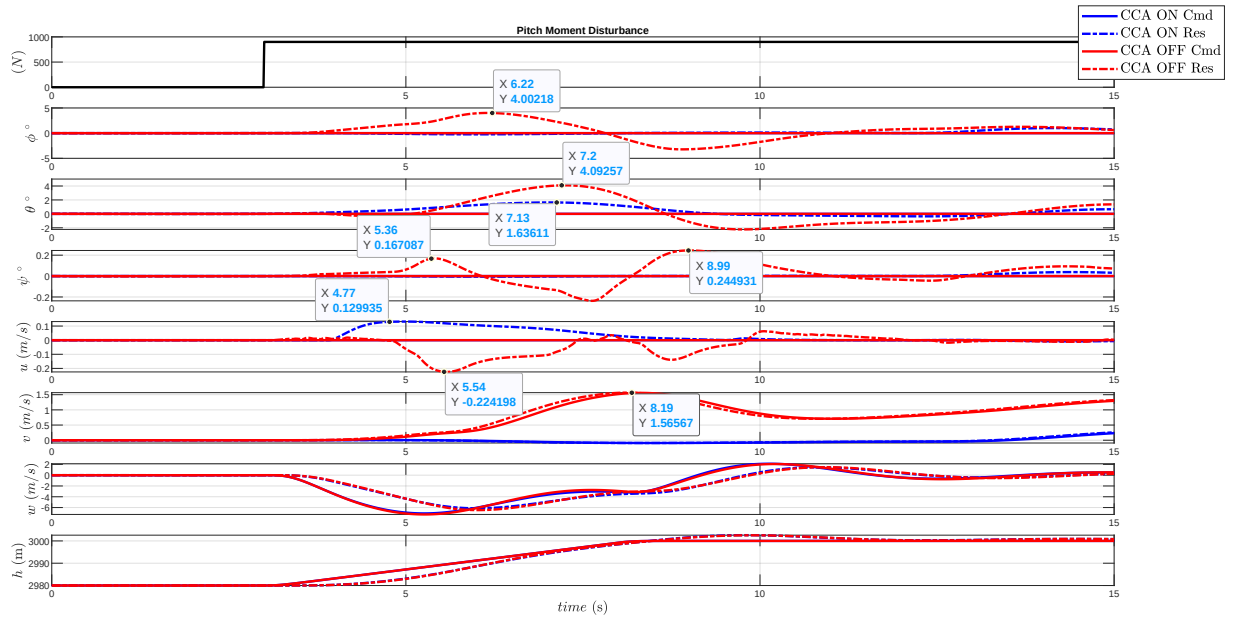


Fig. 5.36 Pitch Moment Disturbance Rejection Characteristics at Hover with and without CCA - State Variables

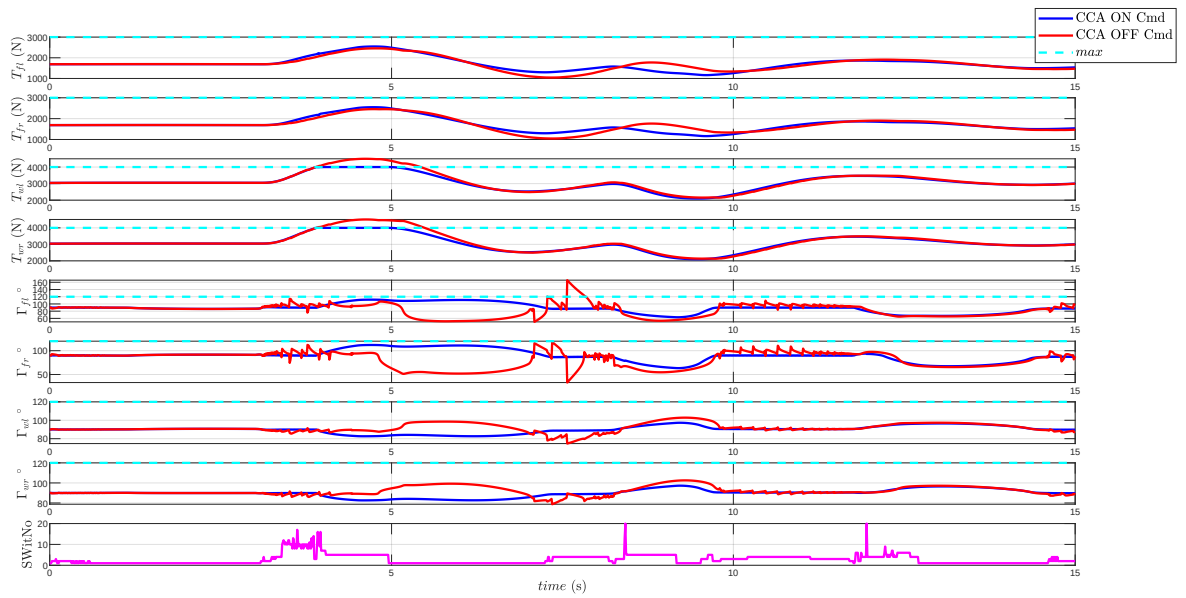


Fig. 5.37 Pitch Moment Disturbance Rejection Characteristics at Hover with and without CCA - Effector Commands

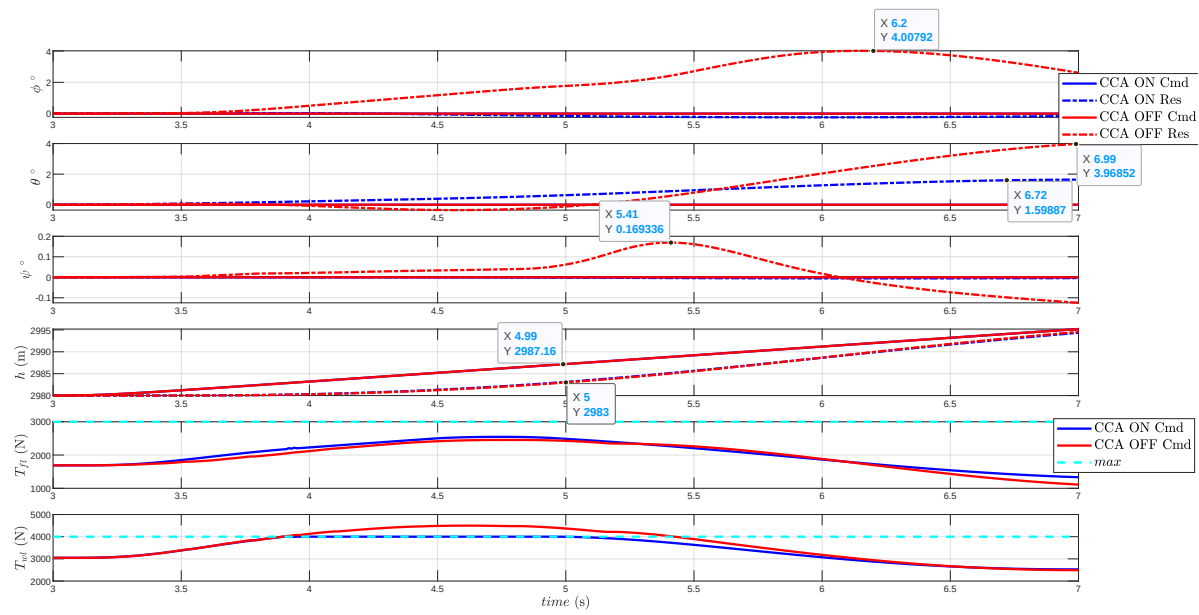


Fig. 5.38 Pitch Moment Disturbance Rejection Characteristics at Hover with and without CCA, the CA Trade-off between Pitch Moment Disturbance Rejection and Altitude Command Tracking

Analysis of Manoeuvre 8 Results First, the results when the CCA algorithm is not active are analyzed. Recall that when the CCA is disabled, the pure NDI law given in Equations 3.48, 3.47 and the simple pseudo-inverse unconstrained CA (UCA) approach in equations 4.66 and 4.67 is applied for the allocation, and the actuator saturation related and control channel prioritisation problems are not resolved effectively.

Based on Figure 5.36, at 3s pilot gives climb command at hover and just after that a strong pitch moment disturbance occurs. Then, the pseudo-inverse UCA increase thrust at right EDFs and decrease thrust at the left EDFs to reject the roll moment disturbance. However, all EDFs are already close to the maximum thrust limit due to the climb command. Additional thrust command generated by the UCA to reject the pitch moment disturbance is not effective since the actuators are saturated (Figure 5.37). Then, pitch moment disturbance cause large deviations on the roll and pitch angles while the actuators are saturated (Figure 5.36, red lines-CCA OFF).

When the same test is applied with the CCA enabled, it is seen that the CA prioritizes the rotational channels over the translational channels (i.e., prioritizes the rejection of roll disturbance over the tracking of climb command), and consider the actuator limits in the optimum allocation. As a result, disturbance rejection characteristics are improved significantly thanks to the proper allocation of the limited control authority (Figure 5.36, blue lines-CCA ON). The trade-off is the worse tracking performance of the climb command. This is an expected result since the CCA makes sure that rotational commands are tracked well as a first objective. Moreover, according to Figure 5.38, the performance loss on the altitude tracking when the CA is active is not significant.

In conclusion, the CA performed satisfactorily and provides good disturbance rejection characteristics in case of limited control authority. This is despite NDI drawback of poor performance in the presence of external disturbances and model uncertainties because it relies on an accurate model of the controlled system.

5.2.11 Manoeuvre 9 - Robustness to the Model Dependent Parameters: Error in Roll and Yaw Moment of Inertia and Mass

Recall in section 3.5.1 that the NDI controller in Figure 3.3 with equations 3.48, 3.47 contains a model of the aircraft dynamics (3.39). In this section, robustness to the error in model dependent parameters is analysed. Particularly, robustness to roll and yaw moment of inertias, and mass will be tested.

The following robustness tests are performed to analyse the sensitivity to the modeling errors in mass moment of inertia, yaw moment of inertia, and mass. Erroneous mass and roll/yaw moment of inertia values are applied up to the values that result in unstable response. All of tests are performed at hover trim condition. Results are given in Figures 5.39, 5.40, 5.41, 5.42, 5.43 and 5.44.

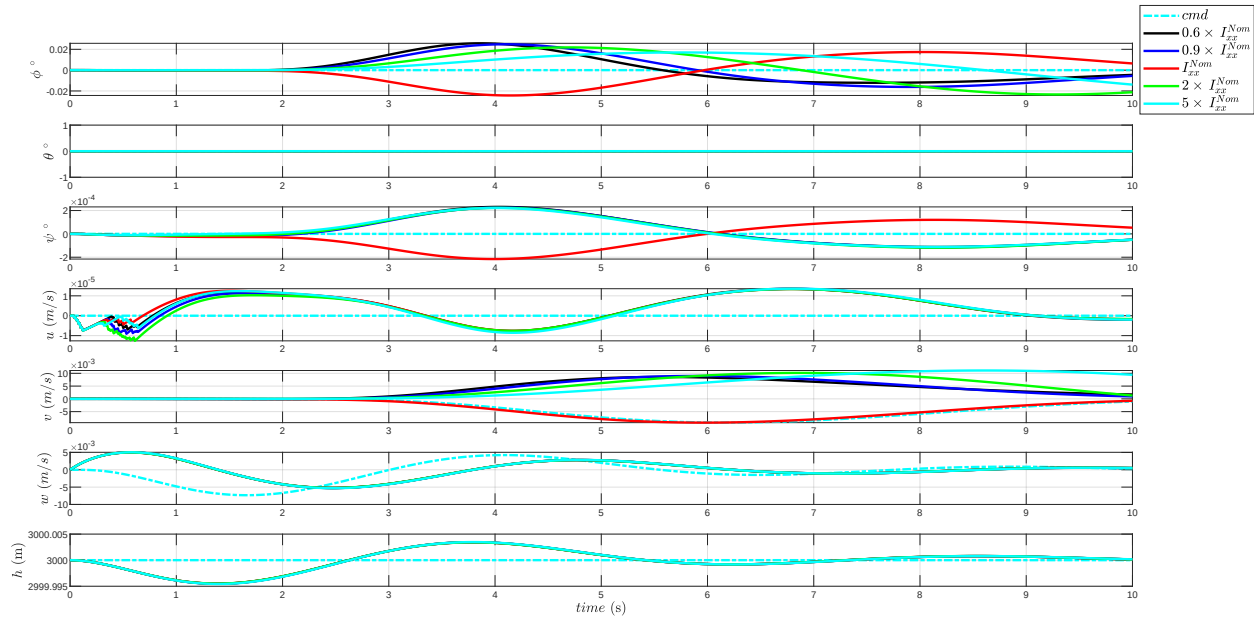


Fig. 5.39 Robustness to error in Roll Moment of Inertia - State Variables

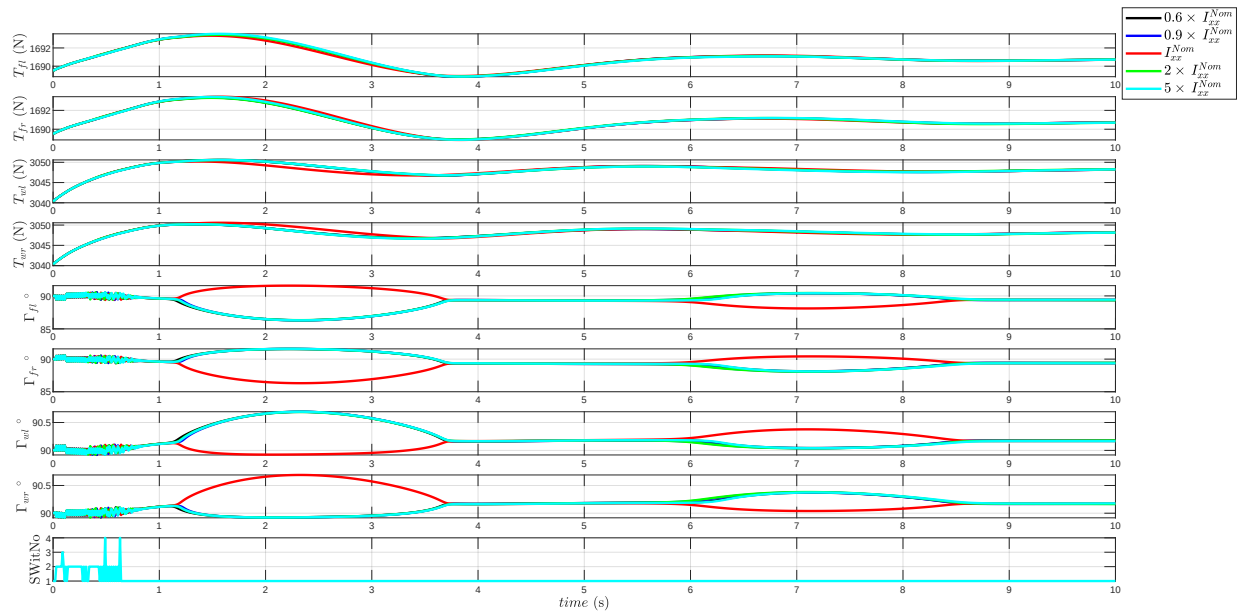


Fig. 5.40 Robustness to error in Roll Moment of Inertia - Effector Commands

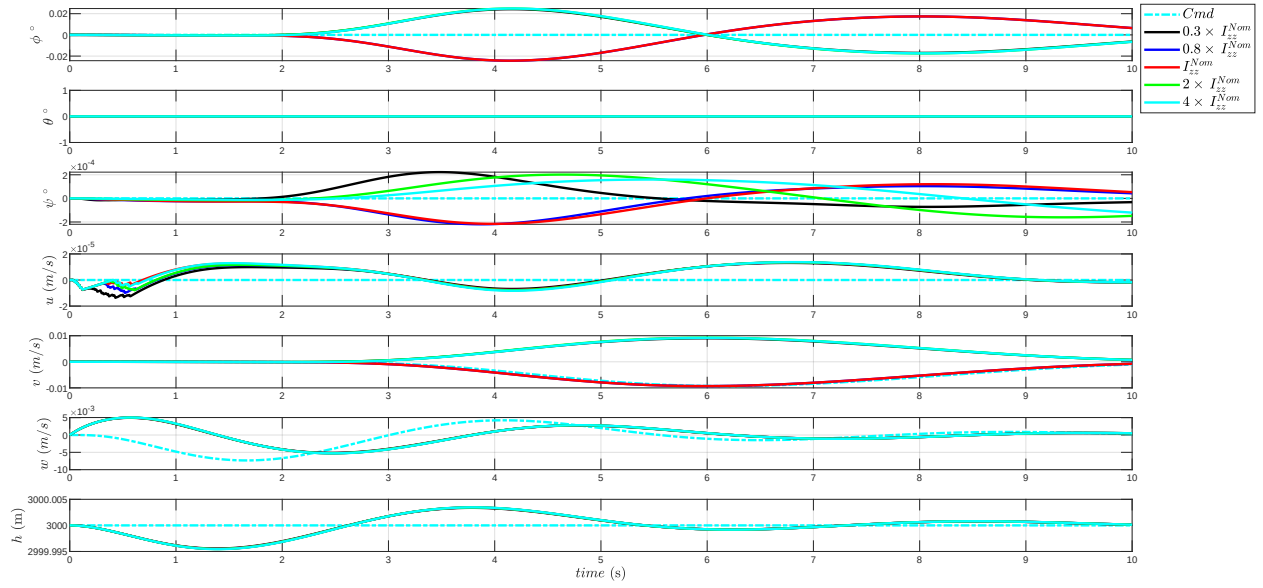


Fig. 5.41 Robustness to error in Yaw Moment of Inertia - State Variables

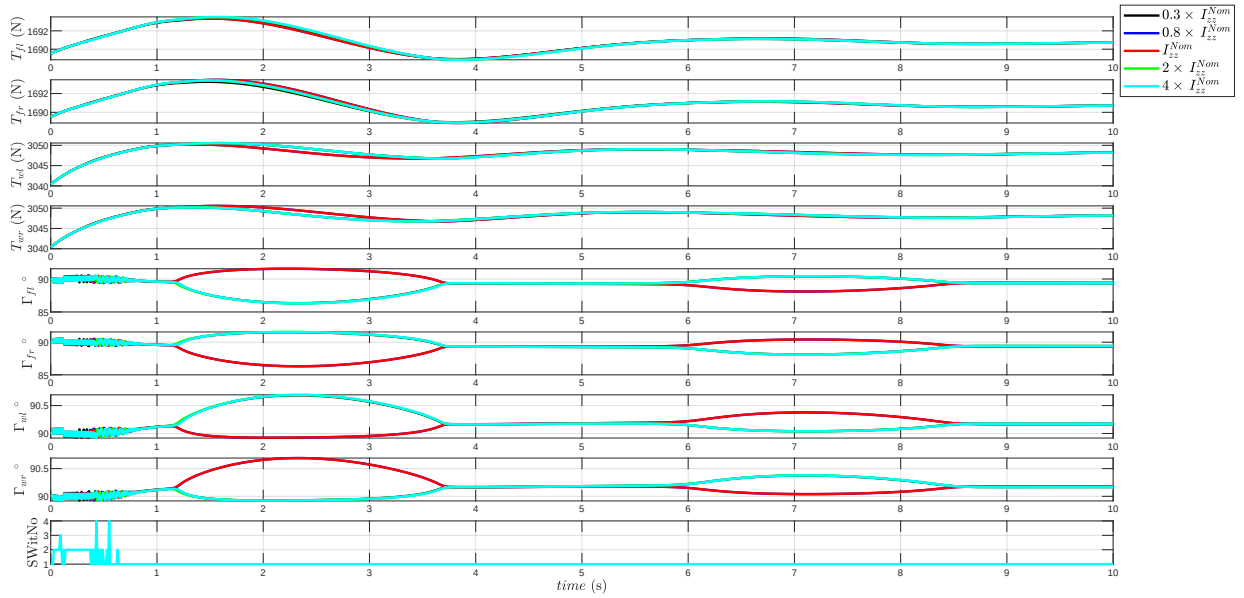


Fig. 5.42 Robustness to error in Yaw Moment of Inertia - Effector Commands

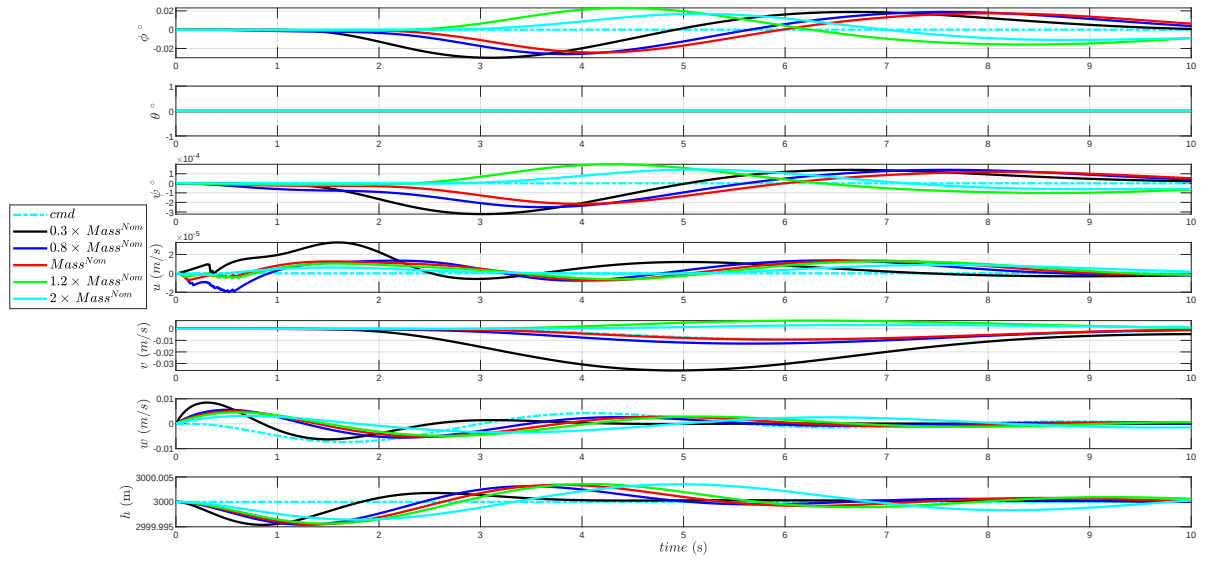


Fig. 5.43 Robustness to error in Mass - State Variables

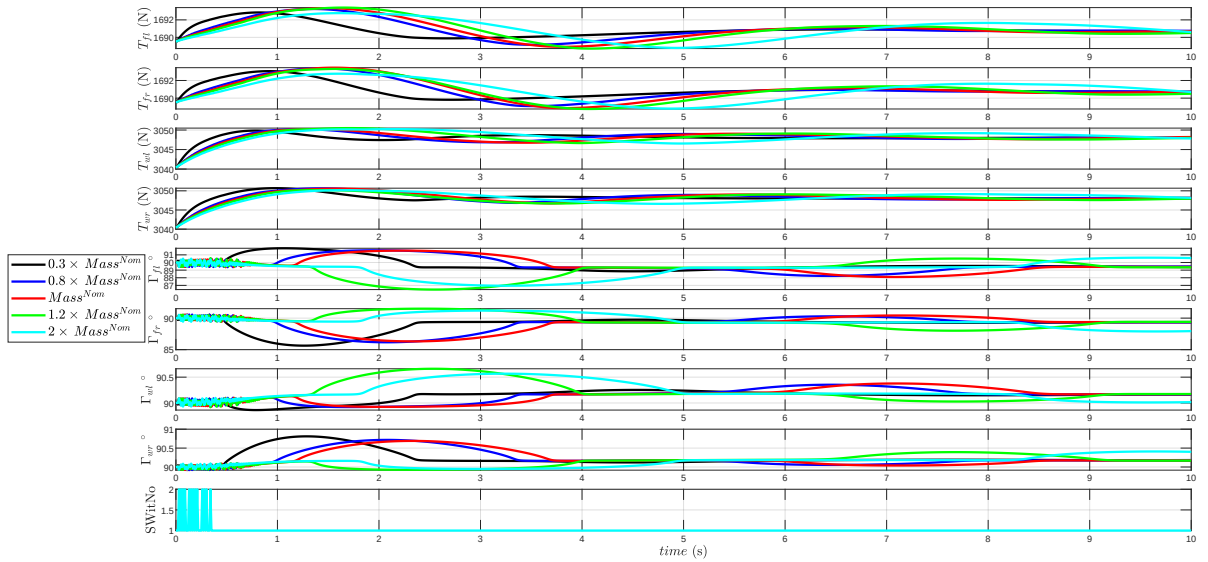


Fig. 5.44 Robustness to error in Mass - Effector Commands

Analysis of Manoeuvre 9 Results Based on the results given in 5.39 - 5.44, 40% decrease and 400% increase in the roll moment of inertia are tolerated by the controller (Figures 5.39, 5.40). Similarly, the controller can handle 70% decrease and 300% increase in the yaw moment of inertia (Figures 5.41, 5.42). In Figures 5.43, 5.44, results of 20% error in mass are given. As expected, the controller increase and decrease thrust accordingly when higher and lower mass values are applied, and stability is not effected at all.

To conclude, the NDI controller has an acceptable level of robustness against the error in model dependent parameters (i.e., the mass and inertia) considering that only very large modeling errors cause instability.

5.2.12 Manoeuvre 10 - Robustness to Model Dependent Parameters: Error in Drag and Lift Coefficient

Recall in section 3.5.1 that the NDI controller in Figure 3.3 with Equations 3.48, 3.47 contains a model of the aircraft dynamics (3.39). In this section, robustness to the error in model dependent parameters in particular to aerodynamic coefficients is analysed.

A visual representation of manoeuvre 10 is shown in Figure 5.45.

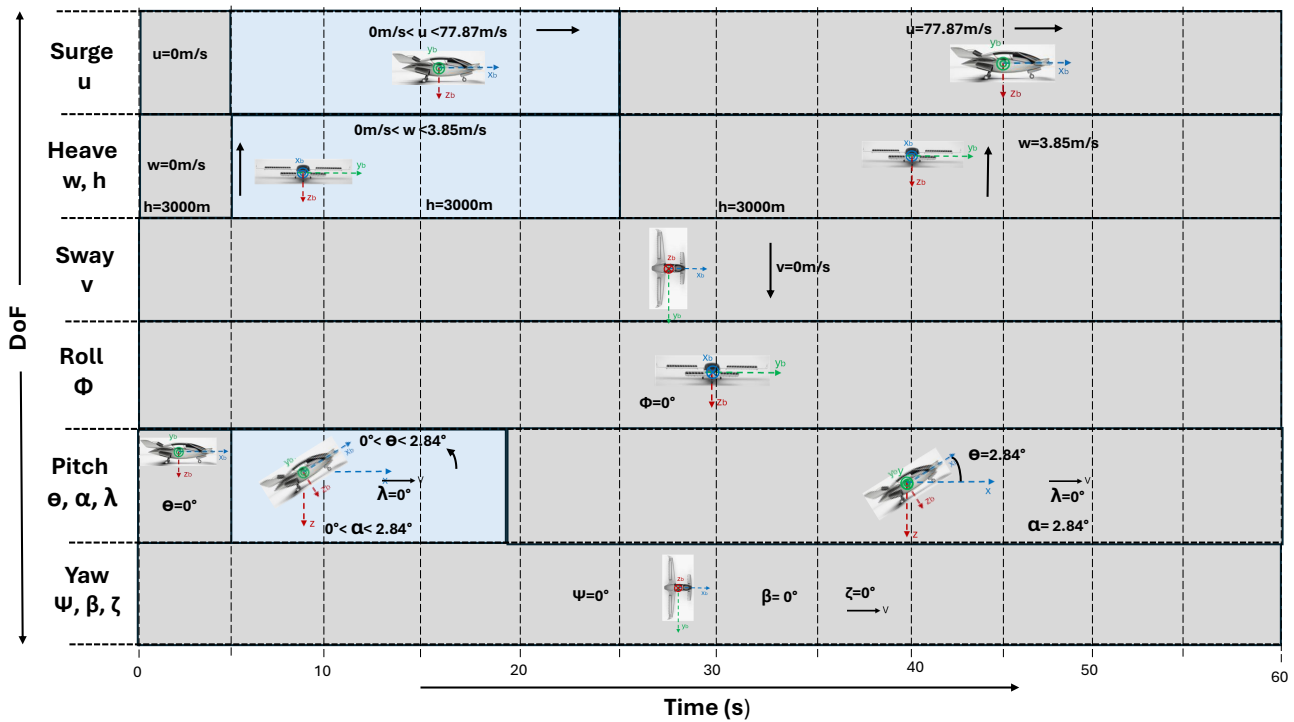


Fig. 5.45 Manoeuvre 10 Visual Representation - Hover, Forward Transition

To assess the NDI controller's inherent susceptibility and vulnerability to modeling errors, the forward transition (transition from hover to cruise, with a visual representation is shown in Figure 5.45) test is performed by multiplying the lift and drag coefficients of three key aerodynamic sections (Fuselage, Aerofoils, and Aerofoils with Fans, see Table 2.1) by 0.8, 0.9, 1.1 and 1.2. The errors in aerodynamic coefficients is not compensated by the NDI controller. The NDI controller dependence and over-reliance (low tolerance on modelling errors) on accuracy of the vehicle model are analysed comparing the simulation results for the nominal coefficients and erroneous coefficient cases for each aerodynamic sections. Results are given in Figures 5.46, 5.47, 5.48, 5.49, 5.50 and 5.51, 5.52, 5.53, 5.54, 5.55, 5.56 and 5.57 where:

- *FusLft* stand for fuselage aerodynamic section lift coefficient
- *FusDrg* stand for fuselage aerodynamic section drag coefficient
- *WngLft* stand for Aerofoils aerodynamic section lift coefficient
- *WngDrg* stand for Aerofoils aerodynamic section drag coefficient
- *FanLft* stand for Aerofoils with fans aerodynamic section lift coefficient
- *FanDrg* stand for Aerofoils with fans aerodynamic section drag coefficient

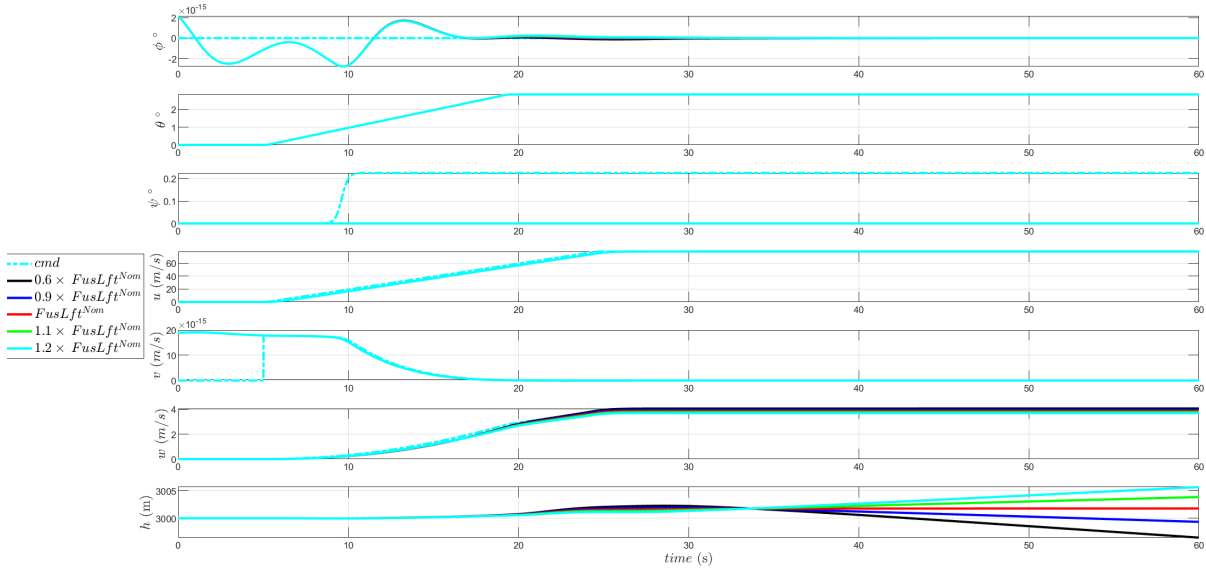


Fig. 5.46 Robustness to Error in Fuselage Aerodynamic Section Lift Coefficients - State Variables

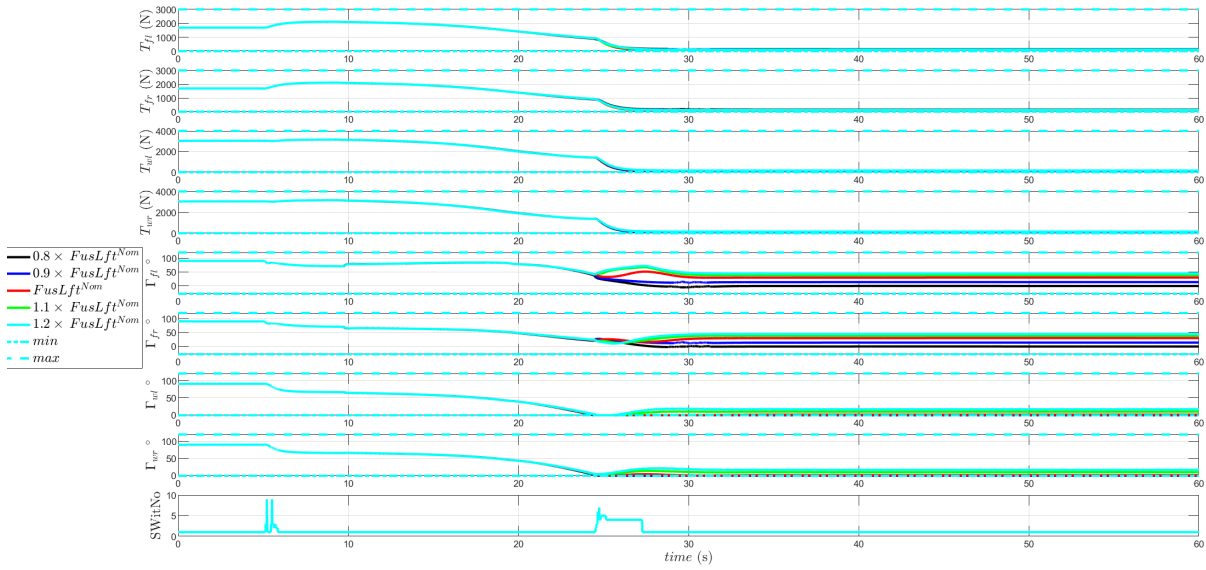


Fig. 5.47 Robustness to Error in Fuselage Aerodynamic Section Lift Coefficients - Effector Commands

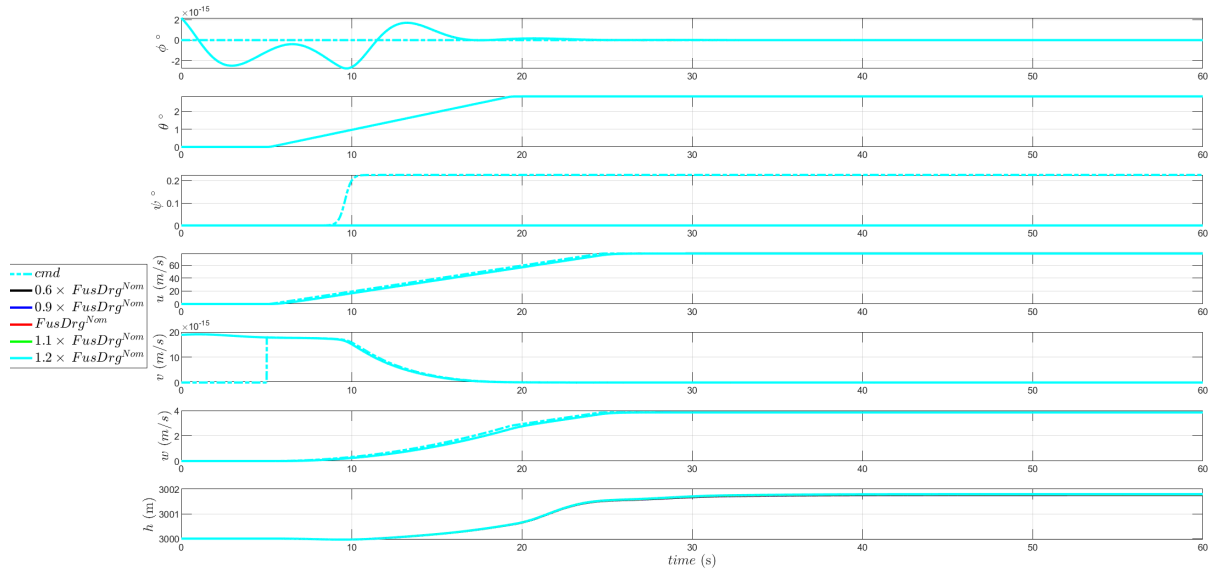


Fig. 5.48 Robustness to Error in Fuselage Aerodynamic Section Drag Coefficients - States

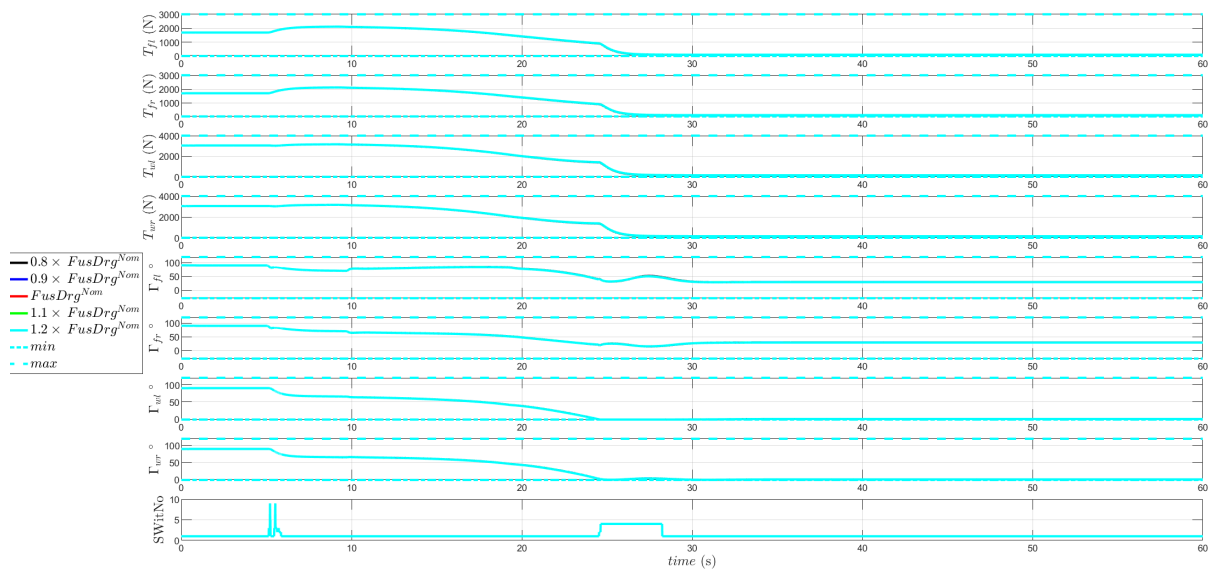


Fig. 5.49 Robustness to Error in Fuselage Aerodynamic Section Drag Coefficients - Effector Commands

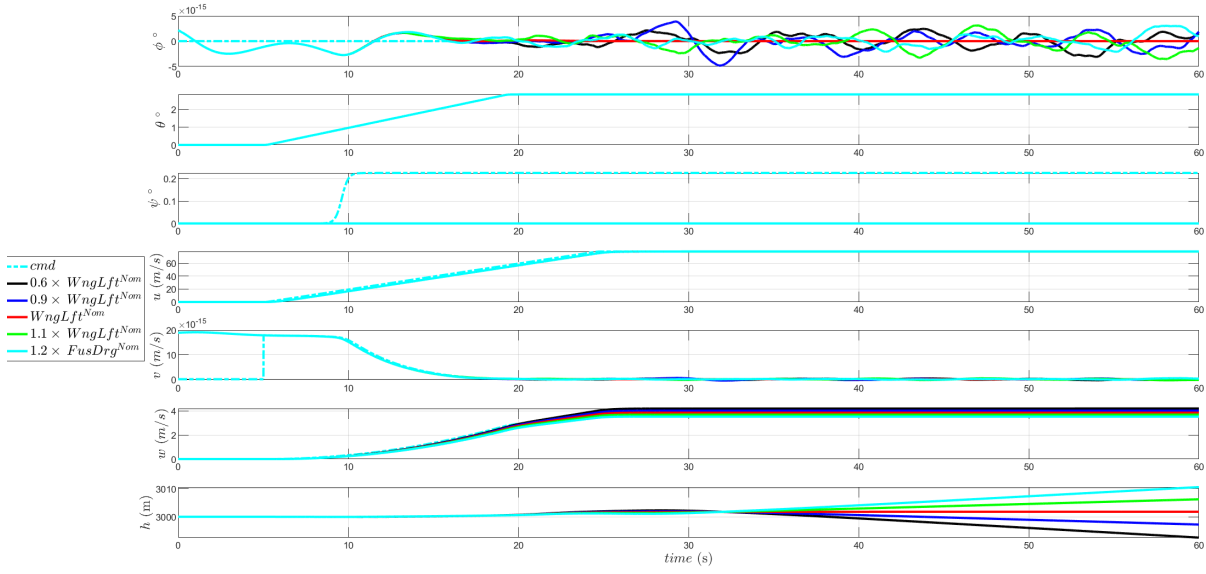


Fig. 5.50 Robustness to Error in Aerofoils Aerodynamic Section Lift Coefficients - State Variables

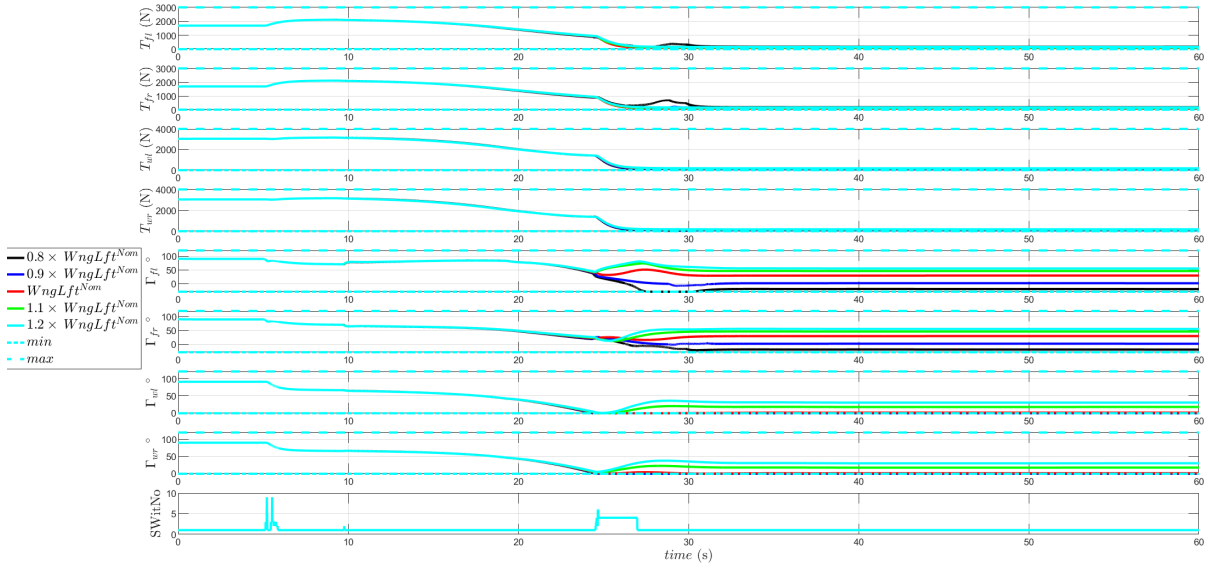


Fig. 5.51 Robustness to Error in Aerofoils Aerodynamic Section Lift Coefficients - Effector Commands

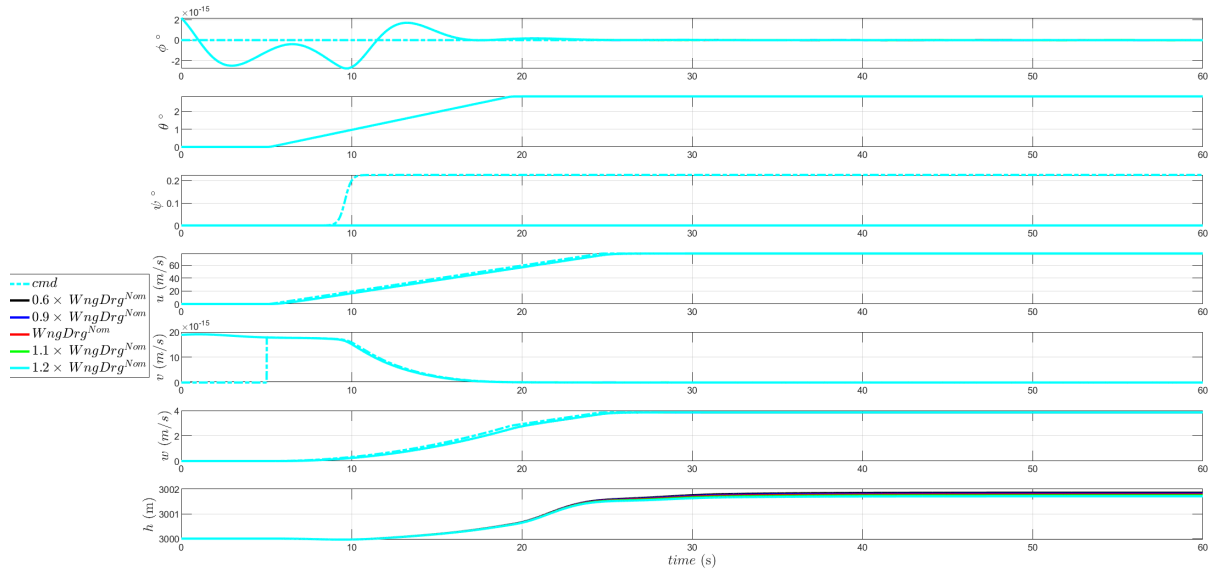


Fig. 5.52 Robustness to Error in Aerofoils Aerodynamic Section Drag Coefficients - State Variables

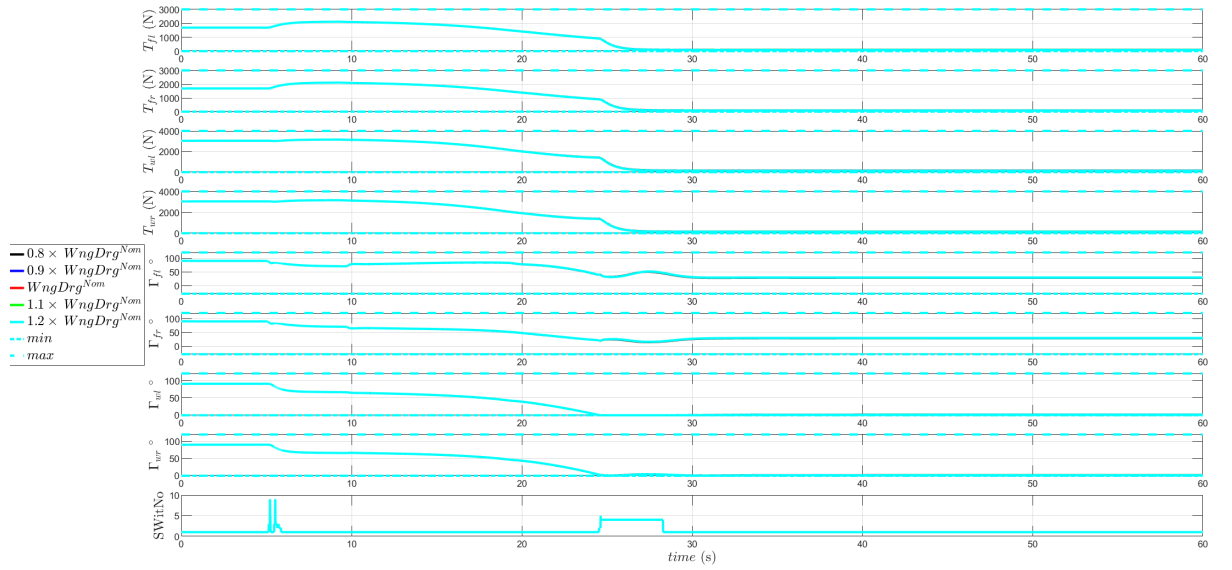


Fig. 5.53 Robustness to Error in Aerofoils Aerodynamic Section Drag Coefficients - Effector Commands

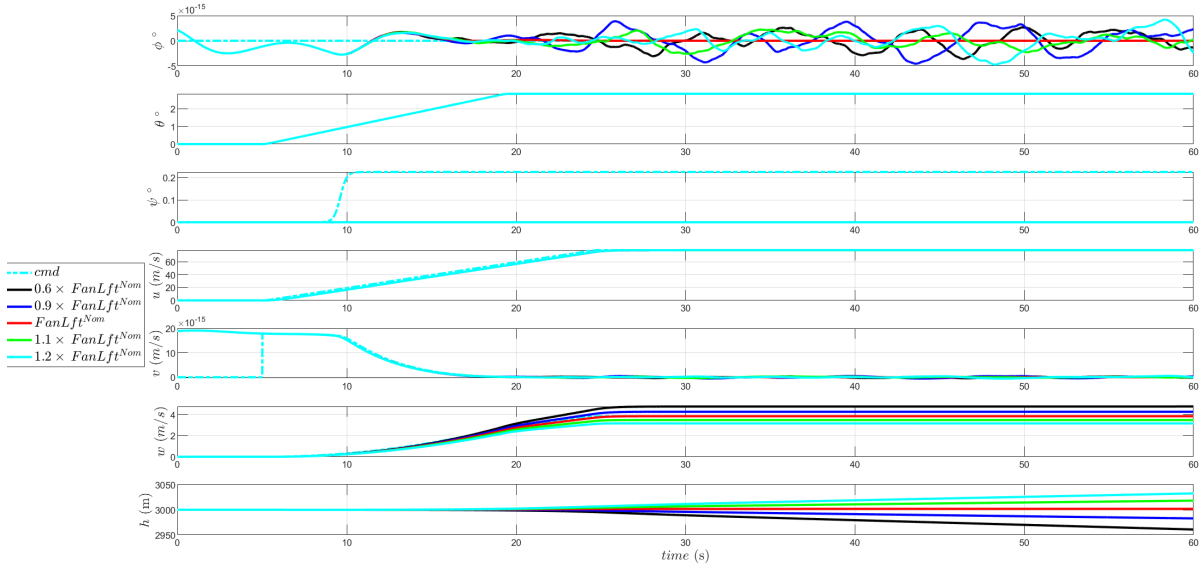


Fig. 5.54 Robustness to Error in Aerofoils with Fans Aerodynamic Section Lift Coefficients - State Variables

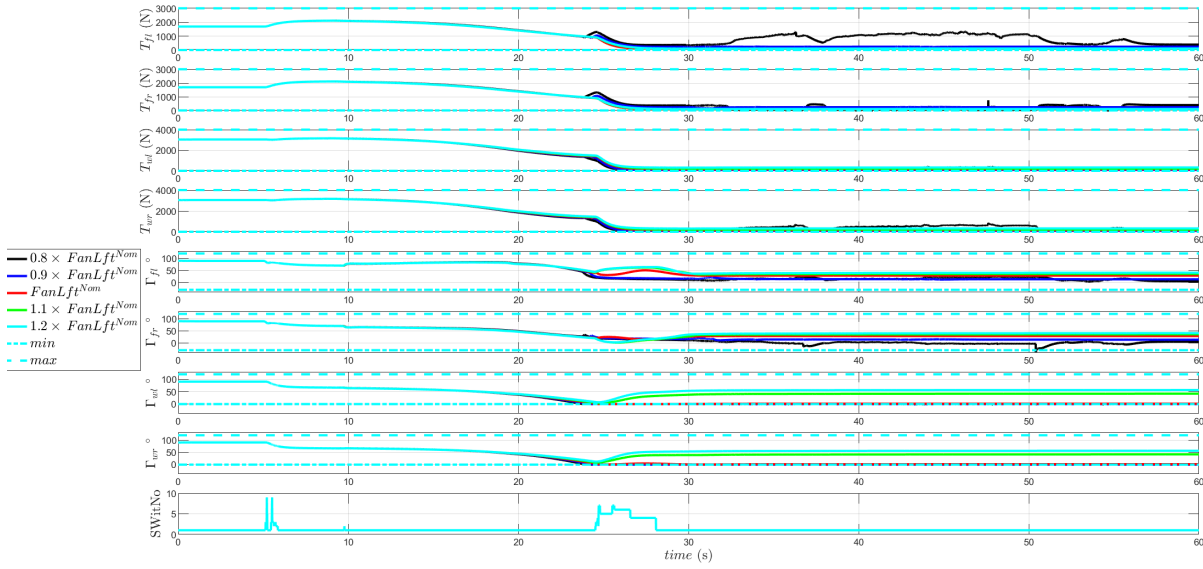


Fig. 5.55 Robustness to Error in Aerofoils with Fans Aerodynamic Section Lift Coefficients - Effector Commands

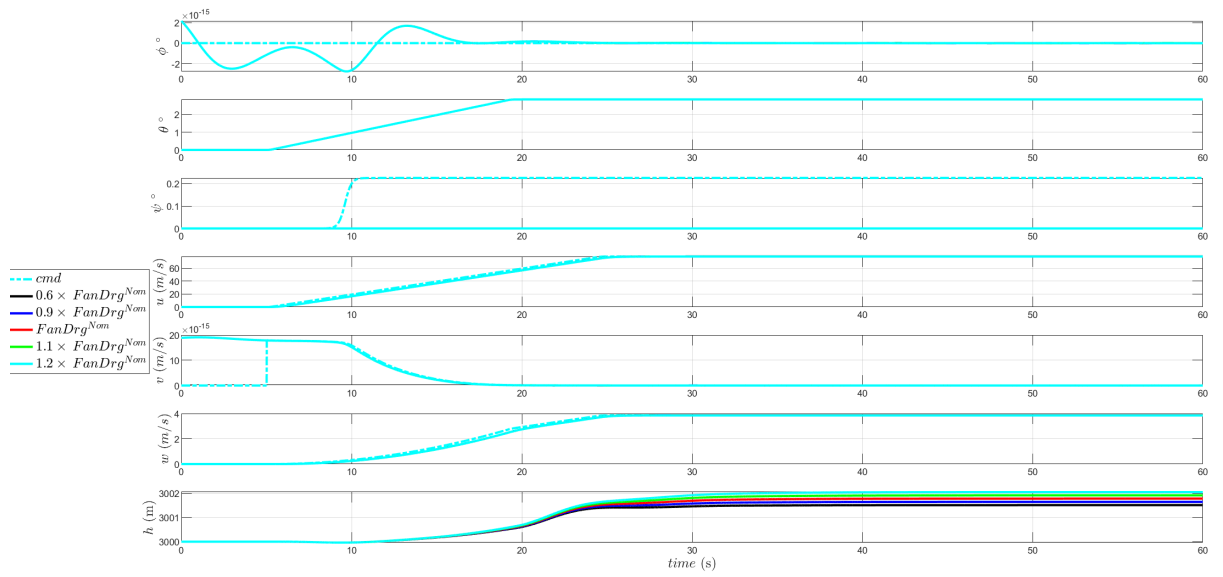


Fig. 5.56 Robustness to Error in Aerofoils with Fans Aerodynamic Section Drag Coefficients - State Variables

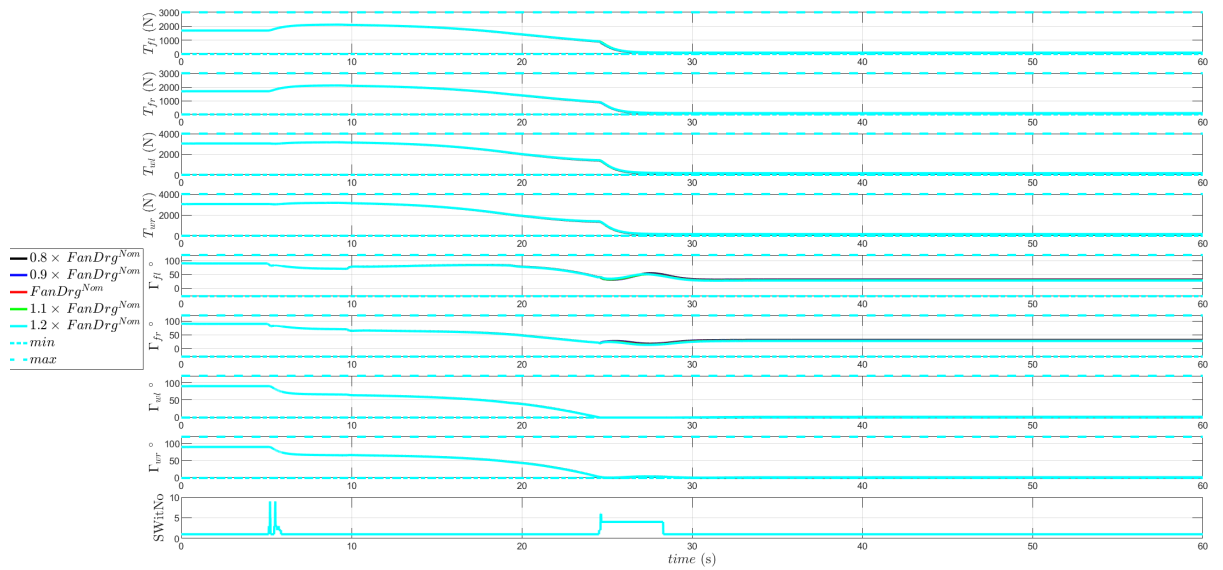


Fig. 5.57 Robustness to Error in Aerofoils with Fans Aerodynamic Section Drag Coefficients - Effector Commands

Analysis of Manoeuvre 10 Results Based on Figures 5.46 - 5.57, the NDI controller performance degrades slightly but remain stable for up to 20% error in aerodynamic coefficients as can be seen for the states and effector commands for lift coefficients (see 5.46, 5.47, 5.50 and 5.51, 5.54, 5.55). The results for the errors in the drag coefficients (Figures 5.48, 5.49, 5.52, 5.53, 5.56 and 5.57) seems to be gives the false impression of the NDI rubustness. This is because the norminal drag coefficients (and forces) are generally very small hence up to 20% error are insignificant to cause noticeable degradation to the NDI controller performance.

In conclusion, the NDI controller is robust against up to 20% aerodynamic modeling errors despite its reliance on the accuracy of the vehicle modelling. However, in reality, aerodynamic modelling mismatches could be more than 20% and at some point as the model mismatch increase can result in instability if NDI controller is used to control an aircraft. This is why NDI should be combined with robust controllers to deal with these aerodynamic modelling errors Adams et al. (1994); Yang and Morales (2021)

5.2.13 Manoeuvre 11 - Criticality of CCA: Redundant EDF Sets Total Actuator Failures during VTO

To demonstrate the actuator fault tolerance and reconfigurability of the CCA in improving the safety and reliability of the over-actuated redundant actuators (not redundant in the controls), a mutiple EDF sets total failures are simulated during VTO. The aim is to observe the effectiveness of the CCA algorithm while control authority is limited. Therefore, the same test case is run when the CCA algorithm is enabled “CCA ON”) and disabled (“CCA OFF, UCA ON”). Figure 5.58 shows the configuration of the multiple EDF sets total failure (complete lost of thrust) consisting of total failures of EDF Set 1 (canard left engine tip) and 10 (wing right engine 2 root) from Table 5.1. The failed EDF sets are marked with a red 'X'. Simulations results in Figures 5.59 - 5.62 show that the CA plays a vital role to handle saturation related stability problems (Section 5.2.5) and reconfiguring the control effectiveness matrix based on available EDF set.

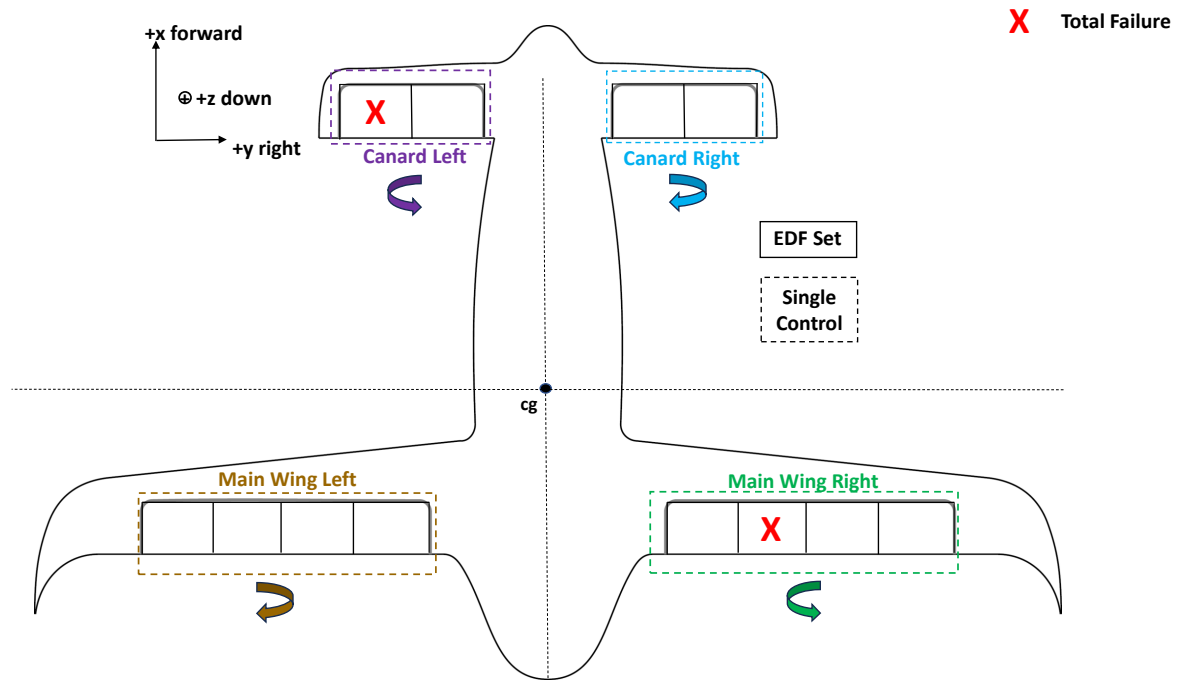


Fig. 5.58 Plan View of Lilium-style Fixed-Wing Over-actuated Vectored Thrust eVTOL showing Loss of EDF sets 1 and 10 Failure Configuration

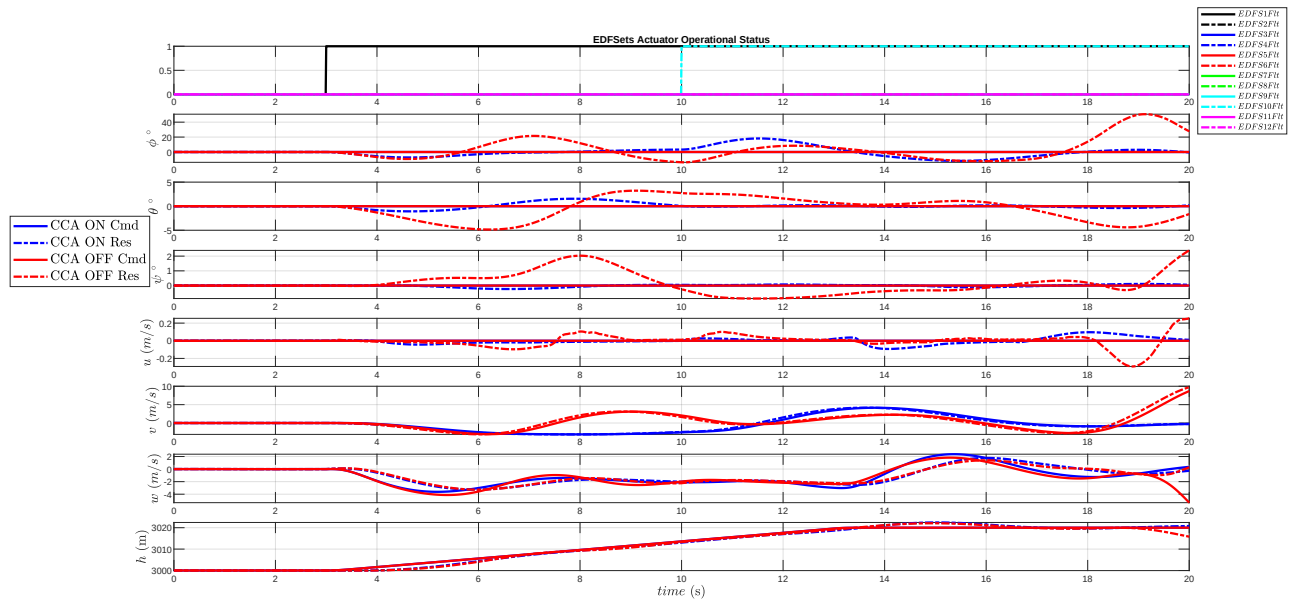


Fig. 5.59 Criticality of CCA: EDF Sets Actuator Failures During VTO - State Variables

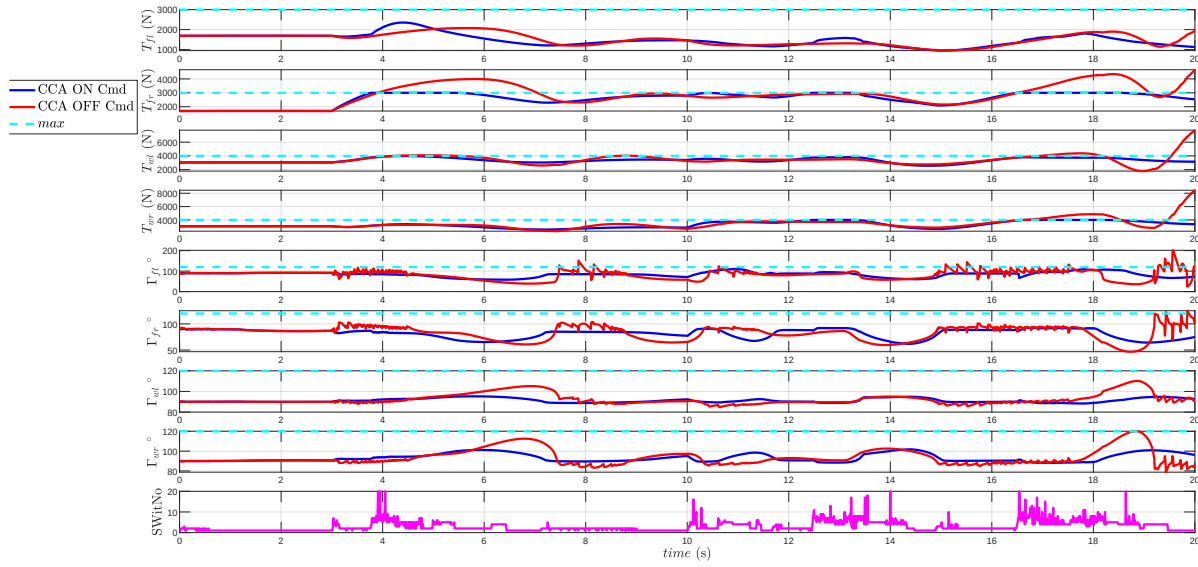


Fig. 5.60 Criticality of CCA: EDF Sets Actuator Failures During VTO - Effector Commands

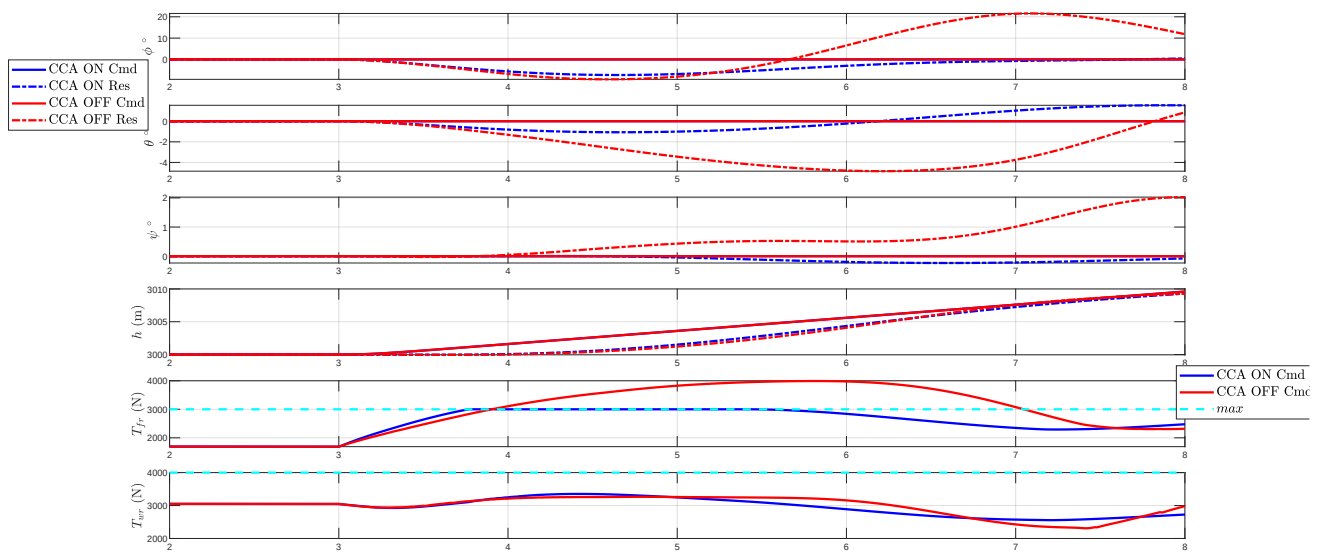


Fig. 5.61 Criticality of CCA: EDF Sets Actuator Failures During VTO - Trade-off due to EDF Set 1 loss

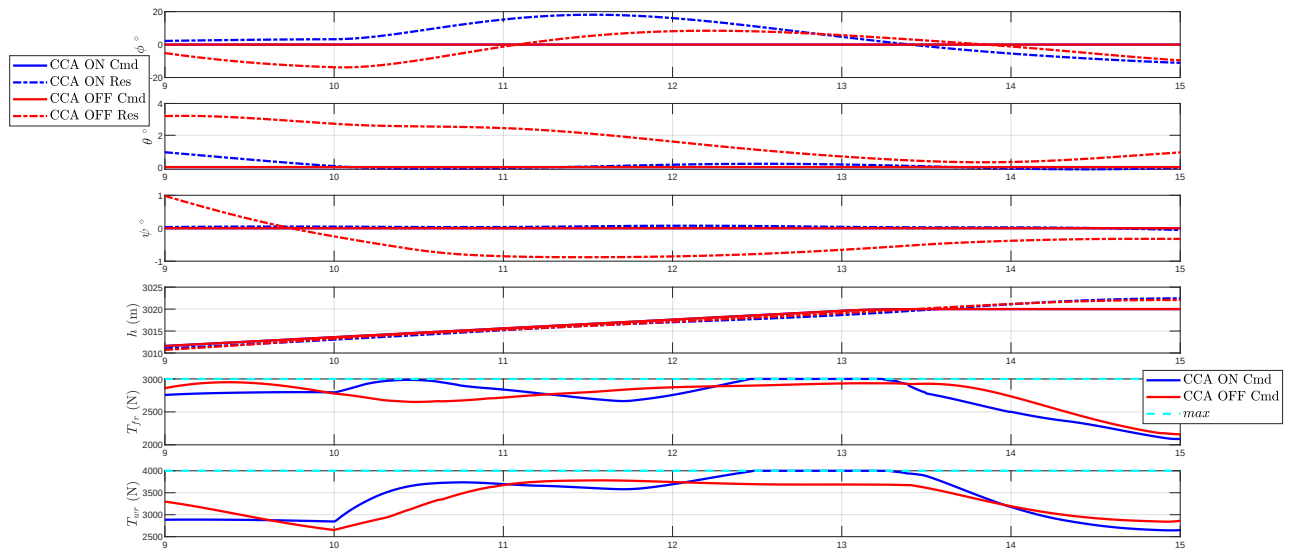


Fig. 5.62 Criticality of CCA: EDF Sets Actuator Failures During VTO - Trade-off due to EDF Set 10 loss

Analysis of Manoeuvre 11 Results First, the results when the CCA algorithm is not active are analyzed.

Recall that when the CCA is disabled, the pure NDI law given in Equations 3.48, 3.47, 4.66 and 4.67 and the simple pseudo-inverse UCA approach in equations 4.66 and 4.67 is applied for the allocation, and the actuator saturation related and control channel prioritisation problems are not resolved effectively.

Based on Figure 5.33, at 3s pilot gives climb command at hover. Simultaneously EDF set 1 (canard left engine tip) was lost (became inoperational) and 2s after that at 5s EDF set 10 was also lost. When EDF set 1 was lost, the pseudo-inverse UCA increase thrust at the remaining canard left EDF set and to minimise any rotational channel tracking error, the UCA tries to increase the canard right EDFs and wing left EDFs. However, the increase in thrust demands on the canard right and wing left EDFs drives them to the maximum thrust limit. Additional thrust command generated by the UCA is not effective since the actuators are saturated (Figure 5.60). Then, loss of EDF set 1 cause large deviations on the roll and significant disturbance in pitch and yaw angles, while the canard right (EDF sets 3 and 4) and wing left (EDFs set 5 – 8) actuators are saturated (Figure 5.59, red lines-CCA OFF) resulting in instability and loss of control.

When the same test is applied with the CCA enabled, it is seen that the CCA prioritizes the rotational channels over the translational channels (i.e., prioritizes the rejection of attitude tracking error over the tracking of climb command), and consider the actuator limits in the optimum allocation. As a result, EDF set 1 fault accommodation are improved significantly thanks to the proper allocation of the limited control authority (Figure 5.59, blue lines-CCA ON). The trade-off is the worse tracking performance of the climb command. This is an expected result since the CCA makes sure that rotational commands are tracked well as a first objective. Moreover, according to Figure 5.61, the performance loss on the altitude tracking when the CA is active is not significant. Between 16.5s and 18.5s, the canard right and wing EDFs run at its maximum thrust limit due to loss of EDF Set 1 and 10 but stable flight is maintained.

When EDF set 10 was lost at 10s, the pseudo-inverse UCA increase thrust at the remaining three wing right EDF set and to minimise any rotational channel tracking error. However, the increase in thrust demands on the wing right EDFs drives it close to the maximum thrust limit. Additional thrust command generated by the UCA is not effective since the actuators are saturated (Figure 5.60). Then, EDF set 10 loss cause large deviations on the roll, and significant disturbance in pitch and yaw angles while the wing right actuators are saturated (Figure 5.59, red lines-CCA OFF) resulting in instability and loss of control.

When the same test is applied with the CCA enabled, it is seen that the CCA prioritizes the rotational channels over the translational channels (i.e., prioritizes the rejection of attitude tracking error over the tracking of climb command) and consider the actuator limits in the optimum allocation. As a result, EDF set 10 fault accommodation are improved significantly (Figure 5.59, blue lines-CCA ON).

The trade-off is the worse tracking performance of the climb command. This is an expected result since the CCA makes sure that rotational commands are properly tracked as a first objective. Moreover, according to Figure 5.62, the performance loss on the altitude tracking when the CA is active is not significant. Beyond 16.5s, all remaining wing EDFs run at their

maximum thrust limit due to loss of EDF Set 1 and 10.

In conclusion, the CCA works well to handle saturation related stability problems (Section 5.2.5) and reconfiguring the control effectiveness matrix based on available EDF set in case of limited control authority thereby improving the safety and reliability of the over-actuated redundant actuators (not redundant in the controls).

Chapter 6

Conclusions

6.1 Discussion and Conclusions

The concept of urban air mobility (UAM) has been proposed to address the transport problems of congestion and pollution in ever-growing urban conurbations. Mature fixed wing aircraft technologies have many constraints that limit their applicability for UAM hence piloted eVTOL vehicles known as eVTOL ‘air-taxis’ provide a promising option being lighter, quieter, potentially having net zero emissions and with lower operating costs. Furthermore, their higher bandwidth thrust actuation allows for novel configurations. However, one challenge of eVTOL aircraft operation is their strong nonlinear flight dynamic characteristics since they operate over a wide range of operating envelope.

One particularly challenging eVTOL configuration is the 7-seater Lilium jet, a fixed-wing, over-actuated vectored thrust, canard light sports aircraft, a tilt-rotor eVTOL air-taxi with distributed electric propulsion (DEP) through the front and wing sections of the air-taxi. The EDFs tilts to provide required thrust at hover, overcome the drag at high speed forward flight, and perform transition between the hover and forward flight. The fixed-wing eVTOL Air-taxi has longer range and endurance compared to the eVTOLs without a wing surface thanks to the aerodynamically efficient lift generation at high speed flight via fixed-wing surface. The eVTOL air-taxi use vectored thrust to achieve full envelope flight control, and do not have conventional stability/control surfaces such as aileron, elevator, rudder, vertical/horizontal tail. This configuration makes the fixed-wing eVTOL air-taxi unique compared to the other concepts studied in the literature. Absence of conventional control/stability surfaces has many advantages (e.g., improved aerodynamic efficiency at cruise, design flexibility, simplicity, better sizing-weight savings, etc.). The challenges of this Lilium-style eVTOL configuration are presented in section 1.6.1

Motivated by the challenges of the Lilium-style eVTOL, a novel over-actuated fixed-wing vectored thrust tilt-rotor eVTOL air-taxi with DEP concept is studied in this thesis (section 2.2.1). The eVTOL air-taxi studied has a novel configuration, and the flight dynamics model has been derived from existing work available in the literature (section 2.3). It is particularly challenging to estimate the aerodynamic and propulsion forces and moments acting on the aircraft during the transition region. High fidelity modeling of the transition dynamics is quite

complex since tilting EDFs also generate aerodynamic forces/moments and flow separation over the EDF surfaces might occur during the transition. Hence the flight dynamics model is constructed considering the main effects at hover and high speed forward flight (section 2.3.1).

To model the transition dynamics, hover (section 2.3.3) and forward flight models (sections 2.3.4, 2.3.5 and A.2.1) are simply blended using the airspeed as the scheduling variable and a blending coefficient which is a continuous differentiable hyperbolic tangent function is used (section 2.3.2). Although in practice this introduces significant nonlinearity in its aerodynamic forces and moments but the complex interaction between the incoming freestream and the propeller slipstream are challenging to accurately model hence not modelled resulting in some unmodelled dynamics and a low fidelity aerodynamic model. Although some of the nonlinear effects are not modelled, the flight dynamics model (sections 2.3, 2.3.1 and 2.3.2) has strong nonlinear dynamic behaviour during the transition regions due to the blending of the hover and cruise models, and coupled vectored thrust control.

More suitable aircraft models (Vehicle Model 1 and Vehicle Model 2) are assembled from existing work for this novel vectored thrust tilt rotor eVTOL aircraft which is more complete than the individual aerodynamic models (section 2.3). The model has three parts; the first part is a hover model (section 2.3.3) while the second (section 2.3.5) and third (section A.2.1) parts are integrated to form the cruise model (section 2.3.4). The second part covers the static longitudinal aerodynamics while the third part covers the dynamic longitudinal derivatives and lateral-directional derivatives. A blending coefficient which is a continuous differentiable hyperbolic tangent function was derived and used to perform transition between hover and forward flight by blending hover and cruise model previously described aerodynamic models. In order to understand the open loop characteristics and aircraft models of the full non-linear model (Vehicle Model 1), a linearization was performed at cruise trim conditions (see section 2.4.1) to give a linear cruise model known as Vehicle Model 2 (section 2.4.4).

Open loop analysis (section 2.4.7) revealed that the model is open loop unstable since the spiral and phugoid modes have their poles on the positive real axis (unstable), and the analysis (the controllability and observability matrices) of the linearized aircraft model (at cruise trim condition) reveal respectively that the linearized aircraft is controllable and observable, hence a closed loop flight controller is required (section 2.4.3). Analysis of the open loop cruise model (Vehicle Model 2) of the aircraft show that it has no transition zeros hence it is a minimum phase system confirming it is compatible with NDI-based controller (section 2.4.5). To enable the aircraft to be controlled by classical NDI (section 3.5.1) and resolved by the proposed control allocation (section 4.8), a customisation of the 3-D lookup combined aerodynamic and propulsive longitudinal model from existing work was performed so as to decouple the aerodynamic and propulsive forces resulting in an affine form (section 2.3.5.3). A simple thrust (propulsive) model and effector mapping was then derived (section 2.3.9) for use later in the control allocation (see Chapter 4). Hence, the coupling between the aerodynamic surface control and the vectored thrust is not modelled. This achieves the first objective of this thesis presented in section 1.4 of Chapter 1

In order to address the control challenges of the Lilium-style eVTOL, a novel full-envelope controller is presented which employs a simple and unified control framework with a functional architecture generic for piloted, semi-automatic and fully automated flight consisting of an aircraft-level controller which produces an overall control demand and a CA scheme

which allocates the overall control demand to individual redundant effectors, performs control error minimisation, control channel prioritization and control effort minimization (section 3.4). The aircraft-level controller adopts a modular approach, consisting of a main inner-loop NDI controller (section 3.5.1), an outer-loop proportional-integral linear (PI) controller (section 3.4.2) and a command generator (section 3.4.1) which together addresses several of the eVTOL operation challenges. The inner-loop NDI controller (sections 3.5.1 and 3.5.2) is a velocity controller which cancels the nonlinear bare airframe dynamics by inversion, the outer-loop PI controller is an attitude and navigation position controller which together are used for hover/low speed control and forward flight, while the command generator generates all the reference commands compatible with piloted, semi-automatic and fully automated flight. Several flight modes are used to control the eVTOL over its wide envelop (section 3.4.3).

Three selected critical aircraft control manoeuvres are investigated on a simplified version of Vehicle Model 1 (section 3.6) without effectors (Vehicle Model 4). These aircraft control manoeuvres (ACM) are typical realistic flight test manoeuvres chosen to test the ability of the NDI controller to control the aircraft and ensuring stable flight by cancelling the nonlinear bare airframe dynamics hence requiring no gain scheduling despite the eVTOL vehicles wide operating envelop and nonlinear flight dynamics. From the test results, the NDI controller performance is satisfactory as the state variables and accelerations are tracked well during all three manoeuvres considering the wide flight envelop of the eVTOL and nonlinear dynamic behaviour during transition between hover and forward flight due to the blending of the hover and cruise aerodynamic models. This achieves the second, third and fourth objectives of this thesis presented in section 1.4 of Chapter 1.

Since the studied eVTOL air-taxi is over-actuated and has vectored thrust control, allocation of the limited control authority properly to guarantee stable flight in severe flight conditions referred as “Control Allocation (CA)” in general is more challenging hence proper allocation of the limited control authority is very critical and must be solved carefully as part of the controller design (see section 4.6). To allocate the limited control authority in case of actuator saturation, the mapping between the overall aircraft control demand τ_c (virtual controls or generalised forces and moments) and the real control controls $\mathbf{u}$ (individual effectors/actuators) known as effector mapping need to be formulated (see section 4.6.1).

This mapping is more straightforward for the aircraft with conventional control surfaces because it is linear and the aircraft is not over-actuated. However, for the studied vectored thrust eVTOL air-taxi, the vectored thrust control is used and, CA problem becomes more complicated due to the nonlinear effector mapping and strong couplings. Since linear CA approaches have lower online computational and complexity burden, and has simpler requirements for fault tolerance and reconfigurability than nonlinear CA approaches (see brief CA literature review section 4.3), this thesis proposed three real-time CA approaches (sections 4.7, 4.8 and 4.9) for transforming a nonlinear CA problem to a linear problem and then applies active set linear CA technique to address all the remaining challenges of the eVTOL configuration (section 4.6.2).

The first approach transfers the non-linearity from the effector mapping to the computation of the actuator limits by formulating the real controls in cartesian form and then converts the physical actuator limits from polar to cartesian form (section 4.7). The second approach transforms the non-linear effector mapping to a linear mapping via numerical linearisation of

the non-linear effector mapping in real-time (section 4.8). The third approach is similar to second approach except an extra step which transforms the virtual controls from cartesian to polar before performing an analytical linearisation resulting in a different and more complicated linear effector mapping (section 4.9). The approaches are then initially conceptually tested using three selected flight test manoeuvres on a planar VTOL (Vehicle Model 3) with no aerodynamics (sections 4.4, 4.4.1, 4.5, 4.5.1, 4.5.2 and 4.5.3 of 4).

In order to test the effectiveness of the control allocation approaches, three selected critical aircraft control manoeuvres are investigated considering the longitudinal motion of the planar VTOL (section 4.10). The results demonstrate that the control allocation can affectively allocate the overall control demand to the individual effectors and also impose effector constraints due to actuator saturation (section 4.11). All three control allocation schemes have similar performance but CA1 generally converges faster than CA2 and CA3. CA1 and CA3 will be more suitable for few actuators due to the complex nonlinear transformation of the effector constraints (position and rate) from polar to cartesian and the more complex LTV Effector mapping respectively. CA2 will be easier than CA1 and CA3 to implement for VTOL configurations with many redundant effectors due to its simpler LTV effector mapping (compared to CA3) and simpler transformation of effector limits from absolute to incremental. The results demonstrate the effectiveness of the proposed control allocation schemes to allocate remaining control authority to higher priority and critical control channels in order to maintain operational safety and stability during certain flight conditions while there is limited control authority. This achieves the fifth objective of this thesis presented in section 1.4 of Chapter 1.

To verify the simple and unified flight controller (NDI high-level controller and control allocation), several tests are performed on a representative full 6DoF flight dynamic model of the eVTOL aircraft via nonlinear simulation model build in MATLAB[®] Simulink[®]. First, the unified controller tracking and stability performance are tested at different flight conditions considering the wide flight envelope of the eVTOL in sections 5.2.3, 5.2.4, 5.2.6, 5.2.7 and 5.2.8.

Secondly, to show the criticality of the CA in case of limited control authority (i.e. the actuator saturation), the CCA design is tested via performing a forward transition with and without the CCA (section 5.2.5). The CCA design is also test by applying severe disturbances that limits the control authority and cause severe actuator saturation. Disturbance rejection tests are done injecting severe wind disturbance to the model (sections 5.2.9 and 5.2.10).

Thirdly, the NDI controller robustness to aerodynamic errors in flight dynamics model is tested via applying significant modelling error in flight dynamics model in the controller compared to the physical vehicle model. To verify this, transition from hover to forward flight tests are performed applying significant error in the lift and drag coefficients (of several aerodynamic sections) in the controller in which the system degraded in performance but remained stable in some cases (section 5.2.12). Moreover, robustness to the model dependent parameters (i.e., the mass and inertia) are also tested to analyse the sensitivity of the controller considering that the mass and inertia are some of the model information required for the NDI controller (section 5.2.11). Based on the simulation results, reasonable errors in mass and inertia can be handled by the controller (NDI controller has an acceptable level of robustness against the error in model dependent parameters) hence, accurate modeling of the mass and inertia is not required to have desired controller response. As expected, it is observed that the

NDI controller is not fully robust to modeling errors. Finally, the criticality of the CCA is also tested during actuator failure (section 5.2.13) where it was shown that the CCA maintained a stable flight by reallocating the limited control authority to both critical control channels and operational EDF sets while considering their saturation limits. This achieves the last (sixth) objective of this thesis presented in section 1.4 of Chapter 1.

In conclusion and summarizing the contribution of thesis presented in section 1.6.2, a simple and unified flight nonlinear controller is designed and verified for the novel fixed-wing eVTOL air-taxi considering the full envelope flight control challenges. Based on the several verification tests, the unified controller addresses three of the control challenges of the studied eVTOL (canard configuration destabilizing the short-period mode resulting in open-loop instability, high degree of over-actuation, vectored thrust results in nonlinear effector mapping preventing the direct use of classical CA approaches) effectively and shows satisfactory performance. The results demonstrate that NDI-based control can naturally handle eVTOL's over a wide range of operating envelope removing the need for gain scheduling leading to improved levels of performance over conventional flight controller designs developed that uses linearizing approximations. The results also demonstrates the tracking capabilities of the NDI scheme and potentially the operational safety during transition, the controller reasonable disturbance rejection, stable flight guarantee in case of severe actuator saturation and satisfactory compliance to typical mission tasks for this class of vehicle. This fully achieves the aim of this thesis presented in section 1.4 of Chapter 1.

6.2 Further Work

The next stage of this research work will be to further improve the proposed control scheme to achieve required flying and handling performance for this kind of vehicle by including a reference model. Secondly, the constrained control allocation (CCA) approach proposed in equation 4.60 can be developed further (but the differentials of the virtual forces with respect to the states and controls have to be computed offline numerically and stored onboard aircraft as complex 3D lookup tables as stated in Bordignon (1996) for the case of traditional aircraft control surfaces) to resolve the unresolved challenge of the studied vectored thrust eVTOL air-taxi, the coupling between the aeromechanical and dynamic forces and moments, such that state dependent effector models (e.g. traditional flight control surfaces like ailerons, rudders, flaps, slats, elevators etc. where the control effectiveness depends on airspeed) should also be tested.

Also, robust control techniques such as H_∞ loop shaping control (Başak and Prempain, 2015; Postlethwaite et al., 2005; Prempain and Dzeukou, 2005; Prempain and Postlethwaite, 2005; Prempain et al., 2001), linear parameter varying control (Ameho and Prempain, 2011; Prempain et al., 2009), switched fault tolerant control (Başak and Prempain, 2017), structural singular value control (Enns, 1990), iterative optimization-based control (Livet et al., 1995) could be investigated for their potential of being applied to improve the adaptability of the control allocation scheme to actuator faults or effector model mismatches.

Full verification of CA3 (refer to Chapter 4) should be carried out on a full 6DoF model similar to the full verification of CA1 in Enenakpogbe et al. (2024b) which is not part of

this thesis. More representative mission tasks elements for flying and handling quality performance assessments should be tested and simulated in order to show that the aircraft is easy-to-fly. Moreover, pilot-in-the-loop simulation trials could be conducted to evaluate the full envelope control design performance and handling qualities. Furthermore, other than the controller study, high fidelity flight dynamics modeling for the unique air-taxi can be studied focusing on the aerodynamics and propulsion system modeling similar to Kristofer et al. (2018); Millidere et al. (2023); Moses (2021); Welles (2018).

6.3 Potential Impact

The simple and unified flight control scheme consisting of an NDI-based controller, incremental control allocation transformation technique (CA 2) and active set linear CA scheme proposed and verified in the thesis can be applied to real-life industrial vectored thrust eVTOL (e.g. the Lilium Jet) once the following improvements are made to the controller:

- include a reference model to the controller (high-level motion controller and control allocation) in order to achieve required flying and handling performance for this kind of vehicle
- Due to classical NDI's inherent poor performance in the presence of external disturbances and model uncertainties because it relies on an accurate model of the controlled system, the NDI controller need to combined with robust controllers to deal with these disturbances and uncertainties (e.g. Adams and Banda (1993a,b); Adams et al. (1994); Alam et al. (2016); Yang and Morales (2021)).
- the constrained control allocation (CCA) approach proposed in equation 4.60 can be developed further (but the differentials of the virtual forces with respect to the states and controls have to be computed offline numerically and stored onboard aircraft as complex 3D lookup tables as stated in Bordinon (1996) for the case of traditional aircraft control surfaces) to resolve the unresolved challenge of the studied vectored thrust eVTOL air-taxi, the coupling between the aeromechanical and dynamic forces and moments, such as state dependent effector models (e.g. traditional flight control surfaces like ailerons, rudders, flaps, slats, elevators etc. where the control effectiveness depends on airspeed)
- techniques such as those proposed in Başak and Prempain (2015, 2017); Bosworth (2008); Bosworth and Williams-Hayes (2007); Hanson et al. (2011); Huang et al. (1992); Johnson and Calise (2000); Kaneshige et al. (2000); Mahdianfar and Prempain (2016) could be investigated for their potential of being applied to improve the adaptability of the control allocation scheme to actuator faults or effector model mismatches

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Appendix A

Appendix 1

A.1 Cruise Model Data - Part A

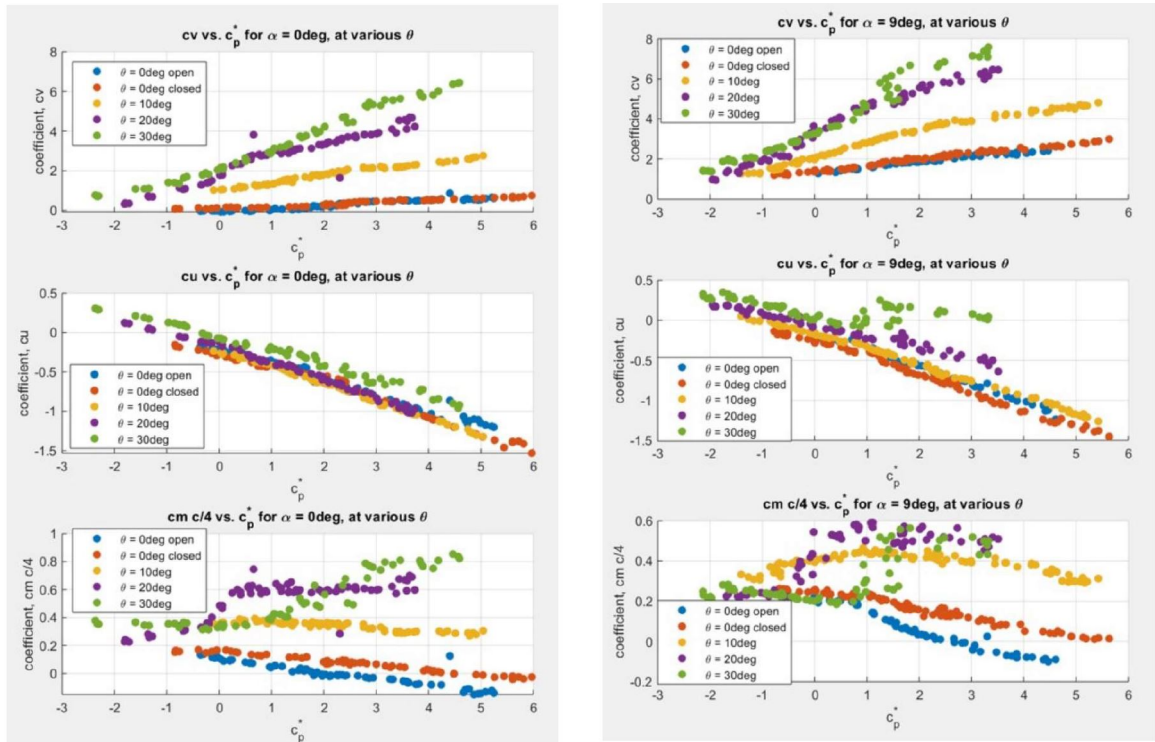


Fig. A.1 Coupled Aerodynamic coefficients C_u , C_v , $C_{m/4}$ vs C_p^* (Moses, 2021; Welles, 2018).
NOTE: flap angle Γ in this work is denoted by θ in the figure different from pitch angle

A.2 Cruise Model Data - Part B

A.2.1 Cruise Aerodynamic Model - Part 2

This section summarises the derivations of Aerodynamic forces and moments as derived in the work of Suiçmez (2021); Suiçmez and Kutay (2023) as a contribution of the longitudinal dynamic aerodynamic derivatives and as well as the static lateral-directional derivatives using conventional aerodynamic derivates for traditional aircraft. Refer to Suiçmez (2021) and Suiçmez and Kutay (2023) for details of the aerodynamic modelling. Suiçmez (2021) and Suiçmez and Kutay (2023) used USAF (2021), compatible with "MATLAB" software MATLAB (2021) to build the 6DOF simulation model of the aircraft being a very fast and efficient method with sufficiently accurate enough results for building preliminary flight dynamics models of several aircraft (Blake, 1985). The "United States Air Force (USAF) Stability and Control Digital DATCOM" (USAF, 2021) is a computer program that estimates static stability, control, and dynamic derivative characteristics of fixed-wing aircraft using the methods contained in USAF Stability and Control DATCOM. The longitudinal dynamic aerodynamic derivatives, the static lateral-directional derivatives (static and dynamic) are plotted in Figures A.2, A.3 and A.4 with the values given in the stability axis and hence are converted from stability axis ($C_{y,sta}$, $C_{y,dyn}$, $C_{z,dyn}$, $C_{l,sta}$, $C_{l,dyn}$, $C_{m,dyn}$, $C_{n,sta}$ and $C_{n,dyn}$) to the body axis ($C_{yb,sta}$, $C_{yb,dyn}$, $C_{zb,dyn}$, $C_{lb,sta}$, $C_{lb,dyn}$, $C_{mb,dyn}$, $C_{nb,sta}$ and $C_{nb,dyn}$) by multiplying with the transformation matrix given by

$$T_{bs} = \begin{bmatrix} c_\alpha & 0 & -s_\alpha \\ 0 & 1 & 0 \\ s_\alpha & 0 & c_\alpha \end{bmatrix} \quad (A.1)$$

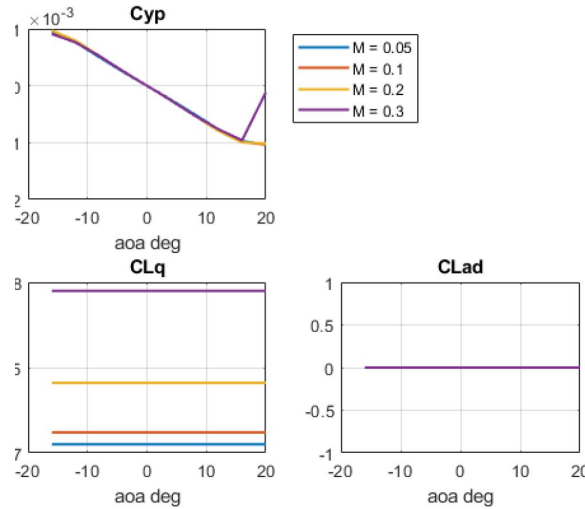


Fig. A.2 Longitudinal Aerodynamic Dynamic coefficients in Stability Axis (Suiçmez, 2021)

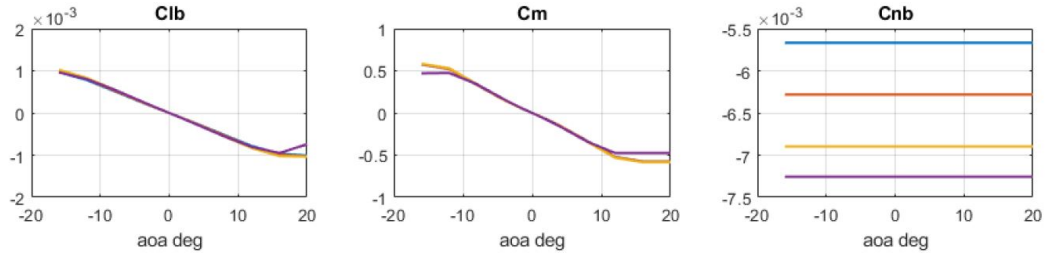


Fig. A.3 Lateral-Directional Aerodynamic Static coefficient in Stability Axis (Suiçmez, 2021)

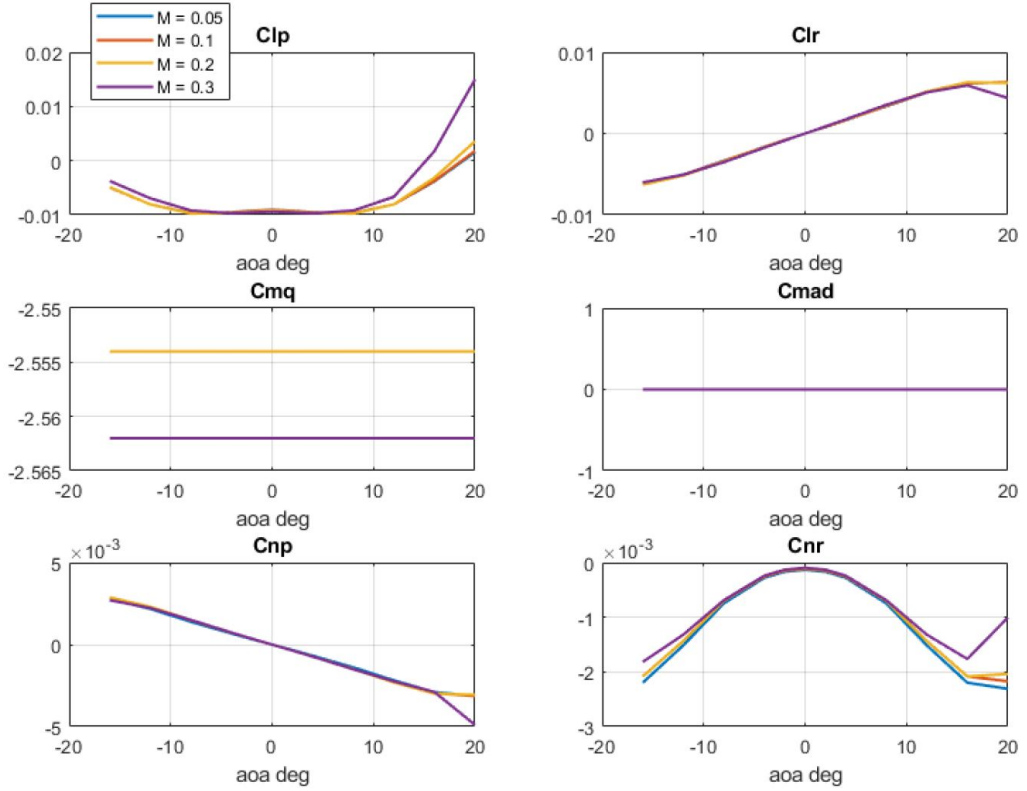


Fig. A.4 Lateral-Directional Aerodynamic Dynamic coefficients in Stability Axis (Suiçmez, 2021)

A.3 LDI Controller Problem formulation

One main contribution of this paper is an application of LDI (Enenakpogbe et al., 2022) as a cruise trimmed flight linear controller for the Lilium-style multi-rotor vectored thrust tilt-rotor eVTOL (Lilium) configuration. Based on the exposition in Stevens et al. (2015), in this section the LDI approach is outlined. Consider a plant model with general state equations given by:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad (\text{A.2})$$

$$\mathbf{y} = \mathbf{Cx} \quad (\text{A.3})$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the vector of state variables, $\mathbf{u} \in \mathbb{R}^m$ is the (virtual) control and $\mathbf{y}(t) \in \mathbb{R}^m$ is the vector of outputs to be controlled.

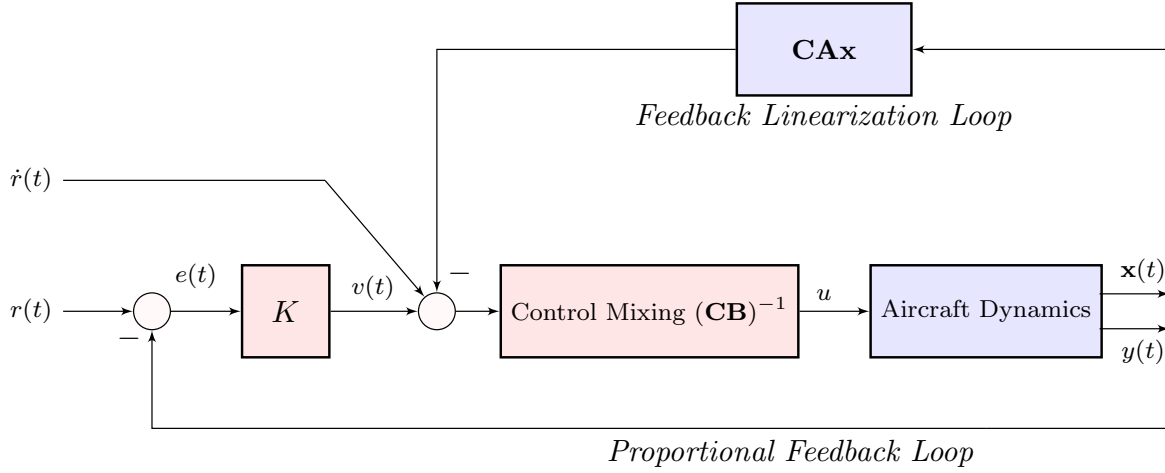


Fig. A.5 LDI problem formulation

The error is given by:

$$\mathbf{e}(t) = \mathbf{r}(t) - \mathbf{y}(t) \quad (\text{A.4})$$

where $\mathbf{r}(t) \in \mathbb{R}^m$ is the vector of reference signals to be tracked. Differentiating the output gives

$$\dot{\mathbf{y}} = \mathbf{C}\dot{\mathbf{x}} = \mathbf{C}\mathbf{B}\mathbf{u} + \mathbf{C}\mathbf{A}\mathbf{x} + \mathbf{C}\mathbf{B}\mathbf{u} \quad (\text{A.5})$$

Defining an auxilliary input $\mathbf{v}(t)$ by

$$\mathbf{v} = \mathbf{C}\mathbf{A}\mathbf{x} + \mathbf{C}\mathbf{B}\mathbf{u} - \dot{\mathbf{r}} \quad (\text{A.6})$$

where $\mathbf{v}$ is the output of the outer loop linear controller such that $\mathbf{v} = \mathbf{K}\mathbf{e}$ if the controller is a proportional gain matrix, $\mathbf{K} \in \mathbb{R}^{m \times m}$. Assuming that the function matrix $\mathbf{CAB}$ is always nonsingular, we get

$$\mathbf{u} = (\mathbf{CB})^{-1}(\dot{\mathbf{r}} - \mathbf{C}\mathbf{A}\mathbf{x} + \mathbf{v}) \quad (\text{A.7})$$

Rearranging and substituting equation for $\mathbf{u}$ in equation A.5 yields

$$\dot{\mathbf{y}} := \mathbf{C}\mathbf{A}\mathbf{x} + \mathbf{C}\mathbf{B} \left[(\mathbf{CB})^{-1}(\dot{\mathbf{r}} - \mathbf{C}\mathbf{A}\mathbf{x} + \mathbf{v}) \right] := \mathbf{C}\mathbf{A}\mathbf{x} + \dot{\mathbf{r}} - \mathbf{C}\mathbf{A}\mathbf{x} + \mathbf{v} \quad (\text{A.8})$$

or, the error dynamics is given by:

$$\dot{\mathbf{e}} = -\mathbf{v} \quad (\text{A.9})$$

It is a first order system, so any linear control technique can be used to choose $\mathbf{K}$ to stabilize the error so that $\dot{\mathbf{e}} = -\mathbf{K}\mathbf{e}$. Due to the way $\mathbf{v}(t)$ was defined in equation A.6, System equation A.9 is the *error dynamics* which has a very simple form - an integrator with p poles at $s = 0$, hence it is very easy to select $\mathbf{v}(t)$ to stabilize the error system.

Although, several linear system control design techniques (e.g. LQR/LTR etc), a simple proportional controller can be used such that:

$$\mathbf{v} = \mathbf{K}\mathbf{e} \quad (\text{A.10})$$

Then, the closed-loop error dynamics is given by:

$$\dot{\mathbf{e}} = -\mathbf{K}\mathbf{e} \quad (\text{A.11})$$

which is a stable system as long as gain matrix $\mathbf{K}$ is positive definite. $\mathbf{K}$ is a diagonal matrix to keep the control channels in the outer loop decoupled. The value of $\mathbf{K}$ could be selected to meet flying and handling qualities requirements.

The overall Linear Dynamic Inversion control input is given by:

$$\boldsymbol{\tau} = (\mathbf{CB})^{-1}(\dot{\mathbf{r}} + \mathbf{K}\mathbf{e} - \mathbf{CAx}) \quad (\text{A.12})$$

Thus the total controller is:

$$\mathbf{v} = \mathbf{K}\mathbf{e} \quad (\text{A.13})$$

$$\boldsymbol{\tau} = (\mathbf{CB})^{-1}(\dot{\mathbf{r}} + \mathbf{K}\mathbf{e} - \mathbf{CAx}) \quad (\text{A.14})$$

The state variable vector $\mathbf{x}$ and virtual control vector $\boldsymbol{\tau}$ are respectively given by:

$$\mathbf{x} = [x \ y \ z \ u \ v \ w \ \phi \ \theta \ \psi \ p \ q \ r]^T \quad (\text{A.15})$$

$$\boldsymbol{\tau} = [F_X \ F_Z \ L_y \ M_y \ N_y]^T \quad (\text{A.16})$$

The controller structure is shown in Figure A.5 and consists of the linear output feedback loop (A.13) and the state feedback linearisation loop (A.14) (which assumes all the states are observable). The feedback linearisation loop ensures that the system from $\mathbf{v}(t)$ to $\mathbf{y}(t)$ appears like an integrator (linear system with poles at the origin), thus simplifying the outer tracking loop design. The velocity feedforward term $\dot{\mathbf{r}}$ improves the tracking accuracy of the closed-loop system. This signal is generated either by the pilot inceptor for manned operation or the guidance system for automated flight.

Since the controller in Figure A.5 contains a model of the aircraft dynamics (3.39), the nonlinear functions of the aircraft must be known in order to implement it.

A.3.1 Zero Dynamics

Substituting the control input (A.14) into the state equations (A.2), the full closed-loop dynamics is given by:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}(\mathbf{CB})^{-1}\mathbf{C}(\dot{\mathbf{r}} - \mathbf{CAx} + \mathbf{v}) = [\mathbf{I} + \mathbf{B}(\mathbf{CB})^{-1}\mathbf{C}]\mathbf{Ax} + \mathbf{B}(\mathbf{CB})^{-1}(\dot{\mathbf{r}} + \mathbf{v}) \quad (\text{A.17})$$

where $\mathbf{I}$ denotes the identity matrix. To ensure that the system is stable, the *zero dynamics* must be stable. The zero dynamics is the dynamics of the system when the auxiliary input $\mathbf{v}(t)$ is selected to give an output $\mathbf{y}(t)$ equal to zero. Since $\mathbf{y}(t) = 0$, then $\dot{\mathbf{y}}(t) = 0$, such that $\mathbf{v}(t) = -\dot{\mathbf{r}}(t)$ which when substituted into (A.17) yields the *linear error dynamics* equation given by:

$$\dot{\mathbf{x}} = [\mathbf{I} + \mathbf{B}(\mathbf{CB})^{-1}\mathbf{C}]\mathbf{Ax} = \mathbf{A}_Z\mathbf{x} \quad (\text{A.18})$$

where:

$$\mathbf{A}_Z = [\mathbf{I} + \mathbf{B}(\mathbf{CB})^{-1}\mathbf{C}] \mathbf{A} \quad (\text{A.19})$$

The poles of Zero dynamics $\mathbf{A}_Z$ contain both p poles (Error Dynamic poles at the origin) which are guaranteed stable by the choice of $\mathbf{v} = \mathbf{K}\mathbf{e}$ and $n - p$ poles which are unobservable selecting the output $\mathbf{y}$, may not be stable and cannot be moved using the linear dynamic inversion. If some of unobservable modes or internal zeros are non-minimum phase (NMP), then the closed-loop system designed by the linear dynamic inversion will be unstable.

Fortunately, for aircraft system, the error dynamics observable state variables u , w , p , q and r are all linear or LTV functions of the derivatives of the unobservable modes or states variables ($n - p$ poles) which are x , z , ϕ , θ and ψ hence stabilizing the observable modes ensures the stability of the unobservable mode. Note that on this VTOL aircraft, heading β and v are indirectly controlled by ϕ (roll to sway) due to the effector configuration as discussed in section 4.5.3.

A.3.2 Controlled Variables (CV)

A necessary and sufficient condition for a successful NDI design is the selection of the *controlled variables (CV)* ($\mathbf{y} = \mathbf{h}(\mathbf{x})$) that will make the *zero dynamics* stable.

Since the state equations for $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ have no control terms, they are referred to as *uncontrollable states* while the remaining states $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{p}$, $\dot{q}$ and $\dot{r}$ are called *controllable states**. Equations (2.8) and (2.9) are called the *complimentary equations* and (2.7) and (2.10) the *controlled equations* Durham (2013). A simple rule is to ensure that the selected CV are at least a function of all the controllable states. For this work, the selected CV, $\mathbf{y} = \mathbf{h}(\mathbf{x})$ for the first five (surge, heave, roll, pitch and yaw) degrees of freedom are given by $\mathbf{y} = (u, w, p, q, r)^T$. Since eVTOLs do not generate propulsive thrust in the y -direction, the sixth degree of freedom (sway) is achieved by rolling so the controlled variable for the sway axis is p .

However, the proposed linear velocity controller will be valid only for the *Cruise (CRU)* mode.

A.4 Linear Controller Design

In order to design the linear controller (LDI) the control architecture in Figure 3.1 is modified to use LDI as the inner loop controller as shown in Figure A.6

A.4.1 Linear Controller Control Architecture

An LDI-based (Stevens et al., 2015) linear controller using the linearized (trimmed at cruise flight condition in section 2.4.2) 6DoF vehicle model will be the inner loop velocity controller.

*As distinct from the *controllable modes*, an entirely different concept that derives from Kalman's well-known controllability test.

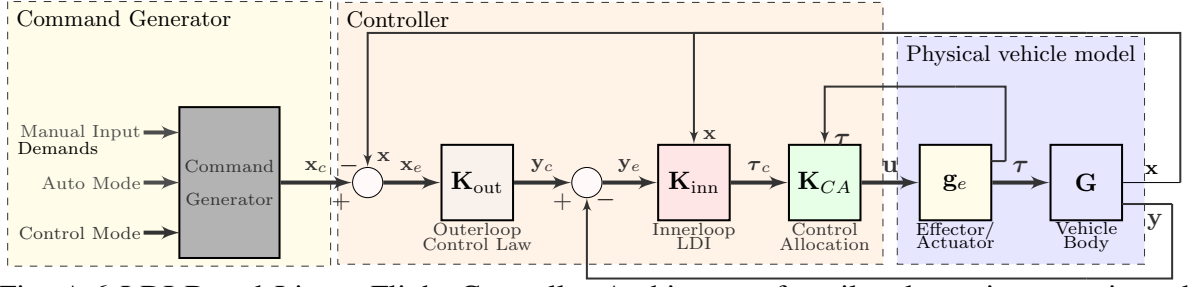


Fig. A.6 LDI-Based Linear Flight Controller Architecture for piloted, semi-automatic and automated flight

A.4.2 6DoF Effector Model for Linear Controller

Using the real control formulation in CA1 to be discussed in chapter 5 on the 6DoF aircraft in Figure 2.4 of chapter 2

Using the simplified control and EDF sets distribution illustrated in the Lilium Jet eVTOL aircraft configuration shown in Figure 2.4, since EDFs don't produce forces in the Y- direction (i.e. $Y_{propulsive} = 0$) and reference direction of the vertical force from the NDI is opposite that in the vehicle model (i.e. $T_z = -Z$), the generalised forces and moments (commanded virtual controls τ) is given by: Such that linear effector model equation is of the form:

$$\tau_c = \mathbf{B}(\Delta x, \Delta y, \Delta z) \mathbf{u}_{x,z} \quad (\text{A.20})$$

where

$$\tau_c = [F_X \ F_Z \ L_T \ M_T \ N_T]^T \quad (\text{A.21})$$

$$\mathbf{B}(\Delta x, \Delta y, \Delta z) = \begin{bmatrix} 2 & 2 & 4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Delta z_{1,2} & \Delta z_{3,4} & \Delta z_{5,6,7,8} & \Delta z_{9,10,11,12} \\ -\Delta y_{1,2} & -\Delta y_{3,4} & -\Delta y_{5,6,7,8} & -\Delta y_{9,10,11,12} \\ 0 & 0 & 0 & 0 \\ -2 & -2 & -4 & -4 \\ -\Delta y_{1,2} & -\Delta y_{3,4} & -\Delta y_{5,6,7,8} & -\Delta y_{9,10,11,12} \\ \Delta x_{1,2} & \Delta x_{3,4} & \Delta x_{5,6,7,8} & \Delta x_{9,10,11,12} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{A.22})$$

$$\mathbf{u}_{x,z} = \begin{bmatrix} T_{x1,2} & T_{x3,4} & T_{x5,6,7,8} & T_{x9,10,11,12} \\ T_{z1,2} & T_{z3,4} & T_{z5,6,7,8} & T_{z9,10,11,12} \end{bmatrix}^T \quad (\text{A.23})$$

which in compact form becomes

$$\begin{bmatrix} F_X \\ F_Z \\ L_T \\ M_T \\ N_T \end{bmatrix} = \begin{bmatrix} 2 & \cdots & 4 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -2 & \cdots & -4 \\ 0 & \cdots & 0 & -\Delta y_{1,2} & \cdots & \Delta y_{9,10,11,12} \\ \Delta z_{1,2} & \cdots & \Delta z_{9,10,11,12} & \Delta x_{1,2} & \cdots & \Delta x_{9,10,11,12} \\ -\Delta y_{1,2} & \cdots & -\Delta y_{9,10,11,12} & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} T_{x1,2} \\ \vdots \\ T_{x9,10,11,12} \\ T_{z1,2} \\ \vdots \\ T_{z9,10,11,12} \end{bmatrix} \quad (\text{A.24})$$

where τ_c is the commanded virtual control (control effect and generalised forces), $\mathbf{u}_{x,z}$ is the unconstrained real controls in Cartesian form $[\mathbf{u}_x, \mathbf{u}_z]$, $\mathbf{B}(\Delta x, \Delta y, \Delta z)$ is the effector model which is only depends on basic geometry lever arms information as shown in Figure 2.4 The real control is given by:

$$\mathbf{u}_{x,z} = \mathbf{B}_{\text{inv}}(\Delta x, \Delta y, \Delta z) \tau_c \quad (\text{A.25})$$

where:

$$\mathbf{B}_{\text{inv}} = \mathbf{B}(\Delta x, \Delta y, \Delta z)^T (\mathbf{B}(\Delta x, \Delta y, \Delta z) \mathbf{B}(\Delta x, \Delta y, \Delta z)^T)^{-1} \quad (\text{A.26})$$

$\mathbf{u}_{x,z}$ is the unconstrained real controls in Cartesian form - the x- and z- coordinate rotor thrust components.

A.4.3 Linear Dynamic Inversion Controller

Adding the navigations states ψ , x_e and y_e to the $\mathbf{A}$ and $\mathbf{B}$ in equations 2.87 and 2.88 the full state transition matrices ($\mathbf{A}$ and $\mathbf{B}$) based on the states order given in Table 2.6 considering the dominant modes and unexcited states (navigation states) of the longitudinal motion at cruise trim operating conditions is given by:

$$\mathbf{A} = \begin{bmatrix} -1.232 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.288 & 0 & 0.5218 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.5218 & 0 & -1.288 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.08907 & 0.1269 \\ 0 & 0 & 0 & 0 & -0.1269 & -0.08907 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0003017 & 0 & 0 & 0 & 0 \\
0.124 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.01693 & 0.1843 & 0 & 0 & 0 \\
0 & -0.1843 & 0.01693 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad (A.27)$$

From section A.3.2, the controlled variables and outputs for the LDI controller are

$$\mathbf{y} = \mathbf{h}(\mathbf{x}) = \begin{bmatrix} u \\ w \\ p \\ q \\ r \end{bmatrix} \quad (A.28)$$

Hence the $\mathbf{C}_{LDI}$ matrix ($\mathbf{h}$) is modified to:

$$\mathbf{C} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad (A.29)$$

Separating the left and right the control inputs in Table 2.7, the $\mathbf{B}$ matrix based on thrust (T) and tilt angle (Γ)

$$\mathbf{u}_{T,\Gamma} = \begin{bmatrix} T_{C,L} & T_{W,L} & \Gamma_{C,L} & \Gamma_{W,L} \\ T_{C,R} & T_{W,R} & \Gamma_{C,R} & \Gamma_{W,R} \end{bmatrix}^T \quad (A.30)$$

$$\mathbf{B}_{T,\Gamma} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.0000514 & -0.002886 & 1.378 & -1.293 \\ 0 & 0 & 0 & 0 \\ 0.000243 & -0.01301 & 8.289 & -1.947 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0002991 & 0.001146 & -0.3195 & 0.1235 \\ -0.0002934 & 0.0008533 & -0.8423 & 0.3211 \\ -0.0001494 & -0.0002704 & -0.01695 & 0.005529 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0000514 & -0.002886 & 1.378 & -1.293 \\ 0 & 0 & 0 & 0 \\ 0.000243 & -0.01301 & 8.289 & -1.947 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0002991 & 0.001146 & -0.3195 & 0.1235 \\ -0.0002934 & 0.0008533 & -0.8423 & 0.3211 \\ -0.0001494 & -0.0002704 & -0.01695 & 0.005529 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{A.31})$$

Transforming inner loop LDI output from real control input $\mathbf{u}_{T_x, T_z} = [T_{x,CL} \ T_{x,CR} \ T_{x,WL} \ T_{x,WR} \ T_{z,CL} \ T_{z,CR} \ T_{z,WL} \ T_{z,WR}]^T$ to virtual controls input $\boldsymbol{\tau}_c = [F_X \ F_Z \ L_T \ M_T \ N_T]^T$ in line with the effector mapping in section A.4.2, the transformed $\mathbf{B}$ matrix based on the decoupled thrust components along the body x and z axes (T_x and T_z respectively) $\mathbf{B}_{T_x, T_z}$ is given by:

$$\mathbf{B}_{T_x, T_z} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.0000514 & -0.0000514 & 0.0029 & 0.0029 \\ 0 & 0 & 0 & 0 \\ 0.000243 & 0.000243 & 0.0130 & 0.0130 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.0002991 & 0.0002991 & 0.0011 & 0.0011 \\ -0.0002934 & -0.0002934 & -0.00085 & -0.00085 \\ -0.0001494 & -0.0001494 & -0.00027 & -0.00027 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 \\
-0.0000012 & -0.0000012 & 0.000065 & 0.000065 \\
0 & 0 & 0 & 0 \\
0.000035 & 0.000035 & 0.00044 & 0.00044 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0.0000017 & 0.0000017 & 0.0000025 & 0.0000025 \\
-0.0000043 & -0.0000043 & -0.0000048 & -0.0000048 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \quad (A.32)$$

while the transformed $\mathbf{B}$ matrix based on virtual controls $\boldsymbol{\tau}_c$ is then given by:

$$\mathbf{B} = \mathbf{B}_{T_x, T_z} \mathbf{B}_{\text{inv}}(\Delta x, \Delta y, \Delta z) \quad (A.33)$$

where:

$$\mathbf{B}_{\text{inv}}(\Delta x, \Delta y, \Delta z) = \mathbf{B}_e(\Delta x, \Delta y, \Delta z)^T (\mathbf{B}_e(\Delta x, \Delta y, \Delta z) \mathbf{B}_e(\Delta x, \Delta y, \Delta z)^T)^{-1} \quad (A.34)$$

Testing of the linear LDI based controller using either virtual controls or real controls (even at cruise trim conditions) are not performed in this thesis because it is trivial and would not add much to the open loop analysis already performed in Chapter 2.

A.5 Acceleration Plots for Chapter 3

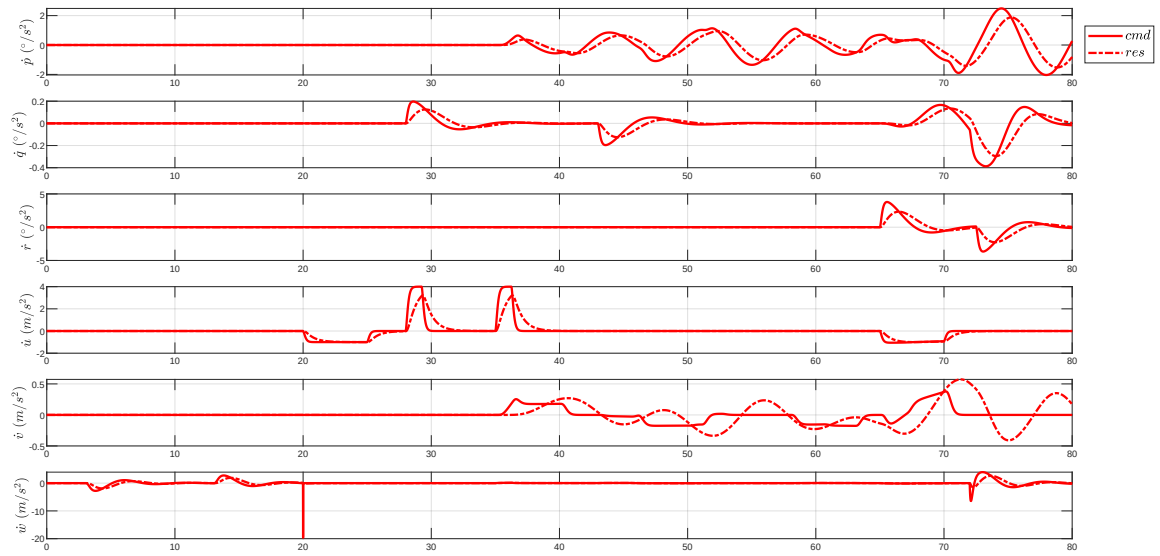


Fig. A.7 No Effector/CA - Manoeuvre 1: VTO, Hover Flight, VL - Acceleration

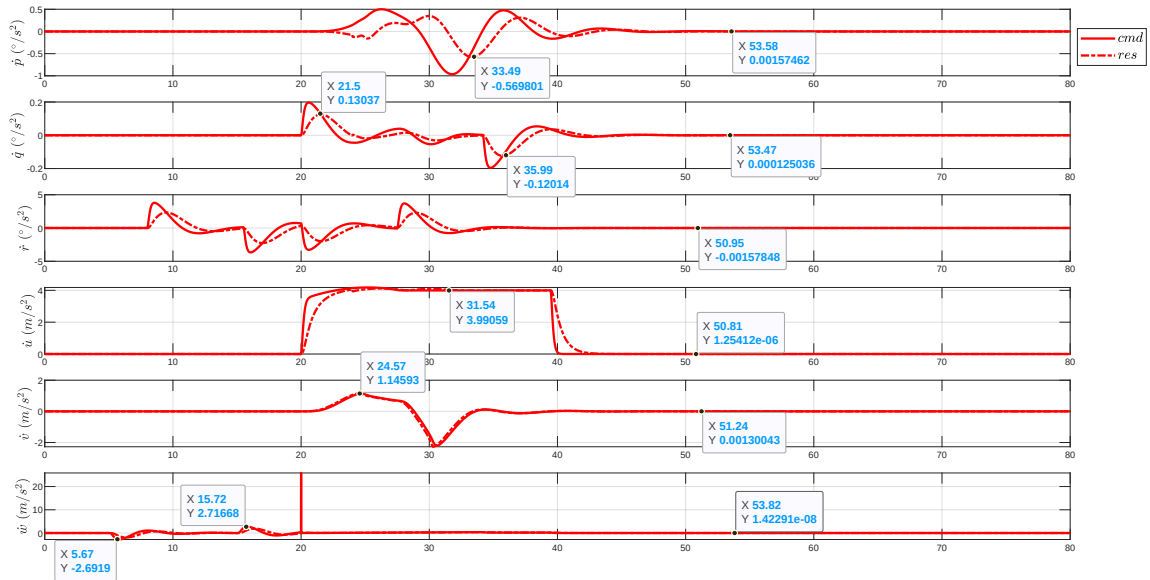


Fig. A.8 No Effector/CA - Manoeuvre 2: VTO, Forward Transition - Acceleration

NOTE: There is a misleading nuisance spike seen in $\dot{w}_{cmd}$ at time stamp 20s of Figure A.8. This is not the actual $\dot{w}_{cmd}$ since NDI is a velocity controller (not like INDI which is a rate controller). The $\dot{w}_{cmd}$ (and the other rate commands) displayed in Figure A.8 is a derivative (implemented using MATLAB time derivative block) of the w_{cmd} . Only the w_{cmd} generated by the w outerloop is used a reference control input into the NDI controller. The w_{cmd} switches (SW2 in equation (3.19)) between that due to hover control (altitude, low speed and VTO/VL) w_{hcmd} and airspeed/flight-path angle control $w_{\gamma cmd}$ whenever there is a transition from hover to forward speed as seen at $t = 20s$. Due to the sudden change in the value of w_{cmd} , the derivate of the signal ($\dot{w}_{cmd}$) can be appear as a spike (exaggerated) depending on the previous state of the two feeding loops considering the sampling rate. This is a well known feature of rates which are prone to noise. Since the commanded rates shown in Figure A.8 including $\dot{w}_{cmd}$ are not actual commands, they can be removed from the plot because they are misleading. The w does not show the spike hence confirming that the spike in $\dot{w}_{cmd}$ is just a nuisance.

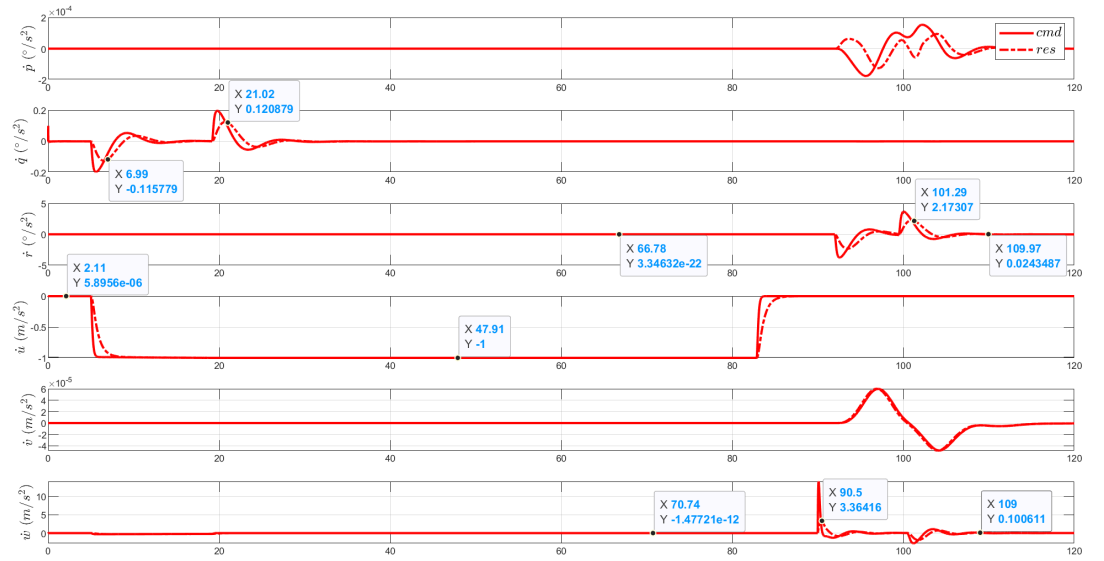


Fig. A.9 No Effector/CA - Manoeuvre 3: Cruise, Reverse Transition and VL with Heading change - Acceleration

A.6 Acceleration Plots for Manoeuvres 1 - 5 of Chapter 5

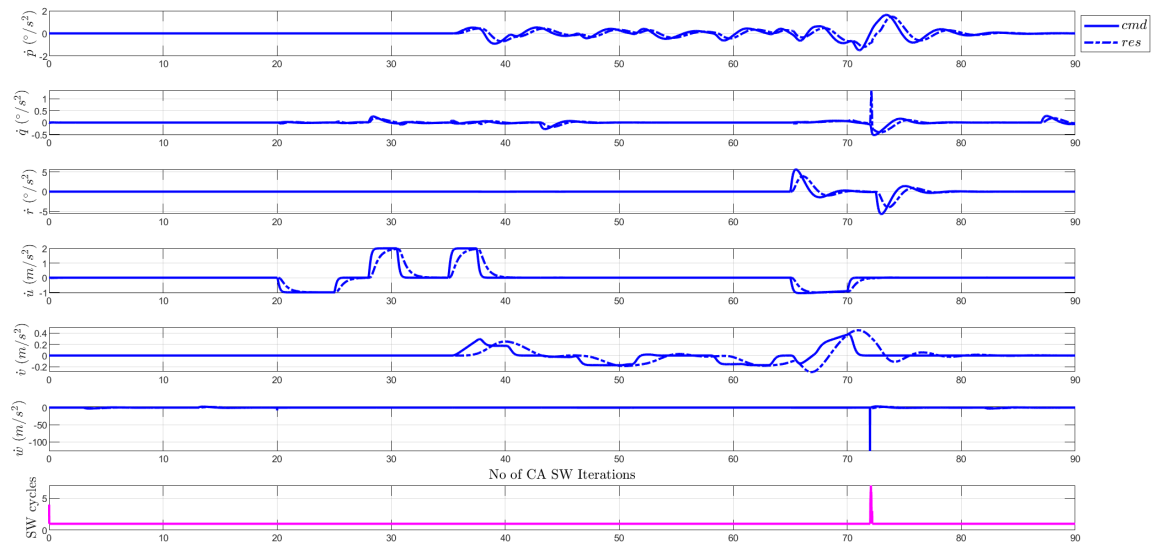


Fig. A.10 VTO, Hover Flight, VL - Acceleration

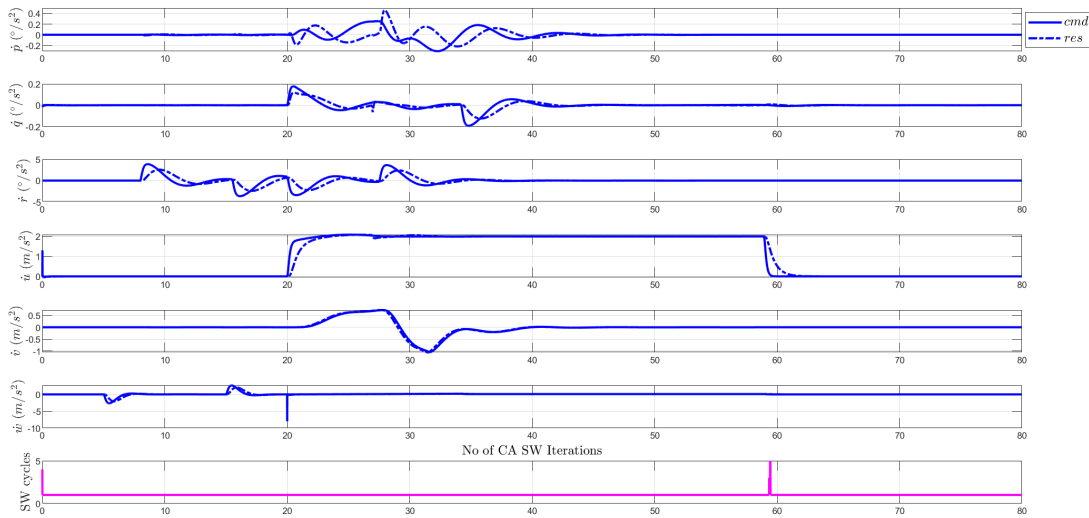


Fig. A.11 VTO, Forward Transition - Acceleration

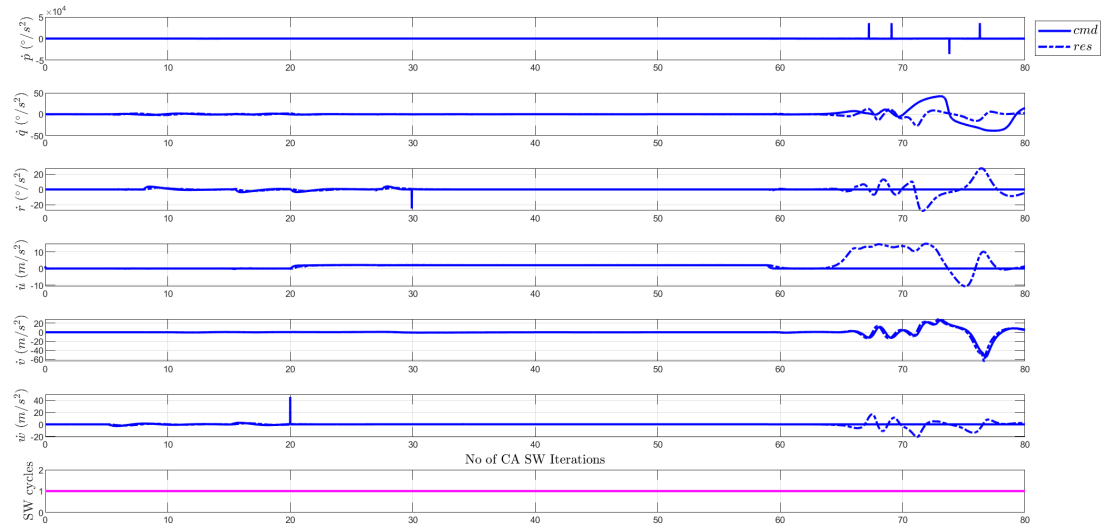


Fig. A.12 Criticality of CCA: VTO and Heading Control, Forward Transition - Acceleration

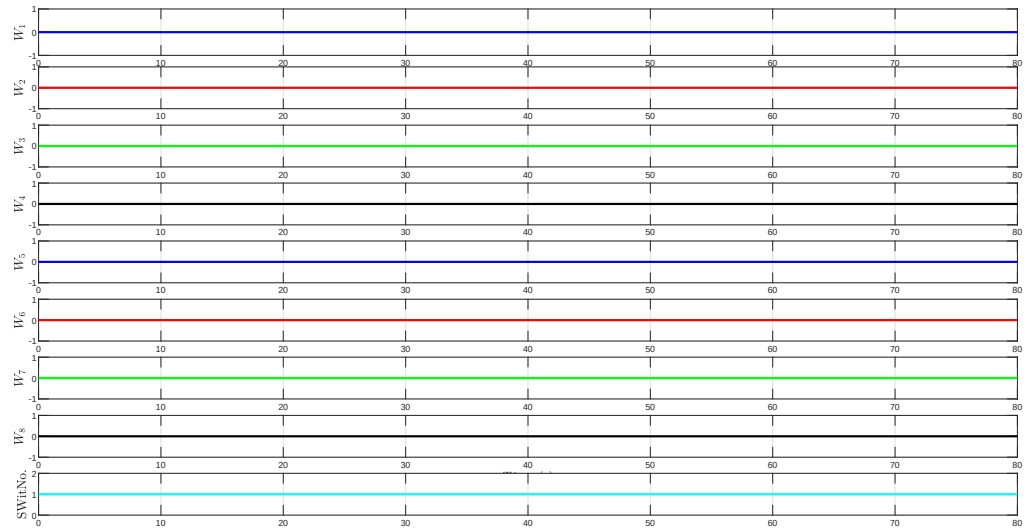


Fig. A.13 Criticality of CCA: VTO and Heading Control, Forward Transition - Control Limitation Status

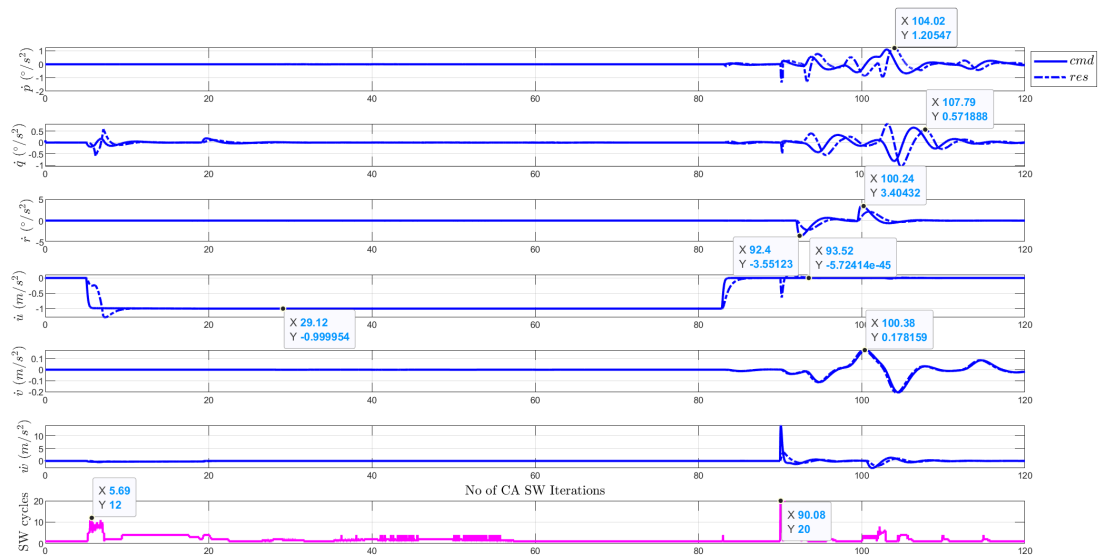


Fig. A.14 Cruise, Reverse Transition and VL with Heading change - Acceleration

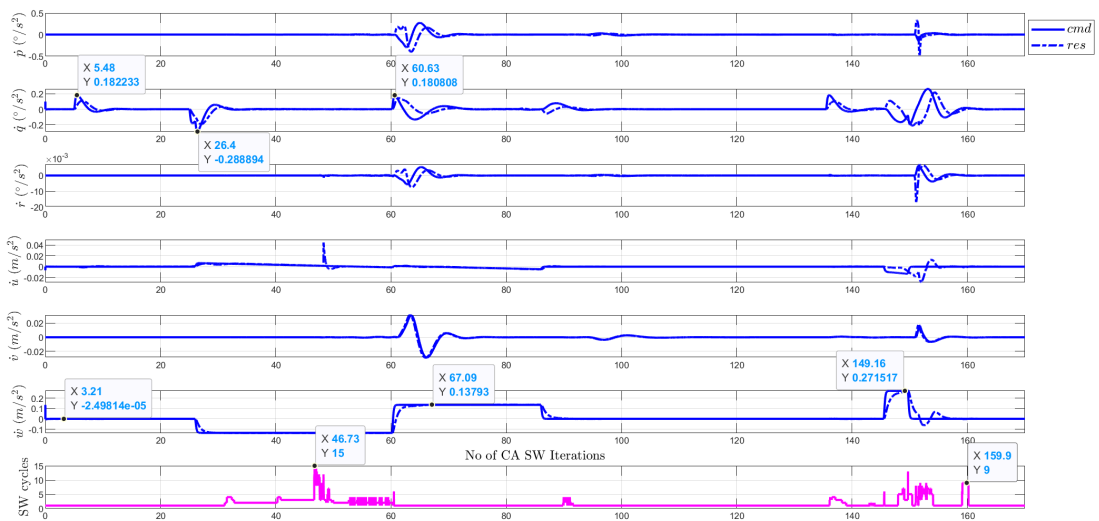


Fig. A.15 Cruise, Climb, Descend - Acceleration

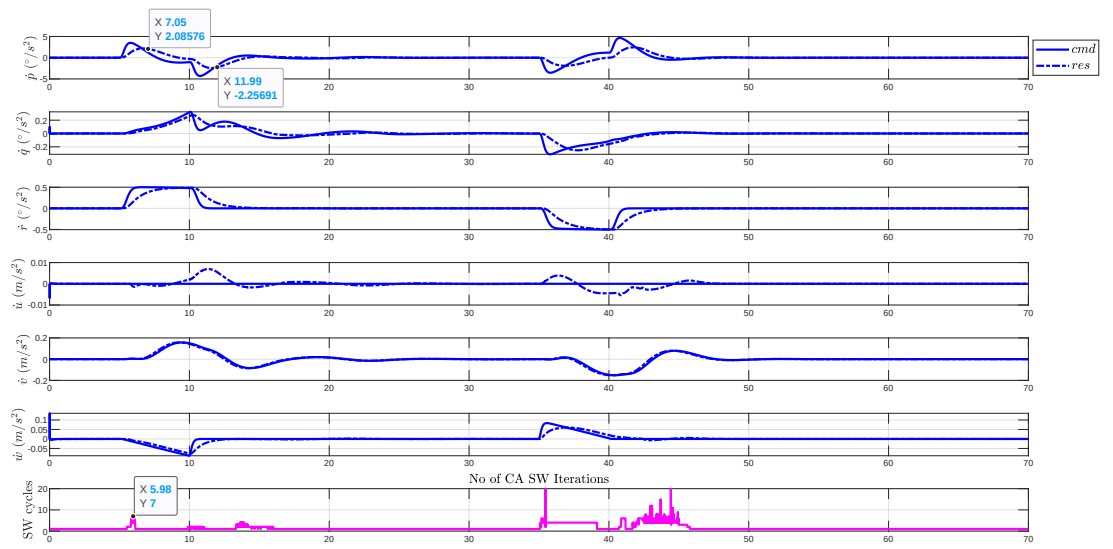


Fig. A.16 Cruise and Coordinated Turn - Acceleration

Appendix B

Appendix 2

B.1 VTOL Certification Landscape

As the feasibility of urban air mobility (UAM) service using eVTOL aircraft increases due to aircraft electrification, distributed propulsion, and artificial intelligence technologies, the U.S. and European aeronautical societies have been improving their certification system and regulations for the type certification of eVTOLs. The improved certification system is expected to be ready in the near future, after the European Union Aviation Safety Agency (EASA) proposed the eVTOL Special Condition, SC-VTOL in 2019. However, the current certification system is still insufficient to properly respond to eVTOL certifications (Gil et al., 2020; LimDaejin and YeeKwanjung, 2021). Also, certification, though an important aspect of the eVTOL life-cycle is not covered in this thesis as it is not the focus of this study. Flying and handling qualities consideration are not addressed in this thesis.

As at the time of writing this thesis, there are several designs of vectored thrust eVTOLs, lift + cruise eVTOLs, wingless (multicopter) eVTOLs, hover bikes/personal flying devices eVTOLs and electric rotorcraft/helicopters eVTOLs and the numbers of designs are still growing (eVTOL News, 2025).

Finally, the U.S. and European aeronautical societies have been improving their certification system and regulations for the type certification of eVTOL. The improved certification system is expected to be ready in the near future after the European Union Aviation Safety Agency (EASA) proposed the VTOL Special Condition, SC-VTOL in 2019. However, the current certification system is still insufficient to properly respond to eVTOL certifications (Gil et al., 2020; LimDaejin and YeeKwanjung, 2021).

The VTOL aircraft certification requirements are under development by the various Certification Authorities comprising of FAA (US), EASA (EU), CAA (UK), ANAC (Brazil), and TCCA (Canada) etc. A summary of the applicable VTOL certification landscape are presented for two main aircraft certification authorities, namely EASA (EU) and FAA (US).

To cover certification aspect of this eVTOL vehicle, an awareness of the applicable cardinal VTOL aircraft certification requirements being developed by two main regulatory authorities (EASA and FAA) as at the time of writing this thesis is required hence an outline is provided in the next section.

B.1.1 VTOL Certification Requirements

The Flying Qualities specification 'Level of Suitability' for VTOLs is expected to be Level 1 (Satisfactory) which means 'Flying qualities' clearly adequate for the mission Flight Phase. Desired performance is achievable with no more than minimal pilot compensation'.

To achieve this, the control architecture and flight control system should ensure no or negligible coupling between the degrees of freedom during steady state and transient manoeuvres.

B.1.2 Applicable EASA VTOL Certification requirements

The EASA VTOL aircraft certification requirements are specified as 'Special Condition VTOL' (EASA SC-VTOL) and 'Acceptable Means of Compliance Special Condition VTOL' (EASA MOC SC-VTOL) documents summarising the requirements and detailing the means of demonstrating compliance to the requirements respectively. As at the time of writing this paper, below are VTOL certification specification documents issued to date from earliest to latest:

- SPECIAL CONDITION Vertical Take-Off and Landing (VTOL) Aircraft - Issue 1 (Proposed) (EASA SC-VTOL-01) EASA (2018)
- SPECIAL CONDITION Vertical Take-Off and Landing (VTOL) Aircraft - Issue 1 (EASA SC-VTOL-01) (EASA, 2019)
- Proposed Means of Compliance with the Special Condition VTOL - Issue 1 (EASA MOC SC-VTOL-01) (EASA, 2020)
- Means of Compliance with the Special Condition VTOL - Issue 2 (EASA MOC SC-VTOL-02) (EASA, 2021a)
- Second Publication of Proposed Means of Compliance with the Special Condition VTOL - Issue 1 (EASA MOC-2 SC-VTOL-01) (EASA, 2021b)
- Third Publication of Proposed Means of Compliance with the Special Condition VTOL - Issue 1 (EASA MOC-3 SC-VTOL-01) (EASA, 2022b)
- Second Publication of Proposed Means of Compliance with the Special Condition VTOL - Issue 2 (EASA MOC-2 SC-VTOL-02) (EASA, 2022a)

Considering the scope of this work, below are some applicable requirements (SC-VTOL) and Means of compliances (MOC SC-VTOL):

- 2105 Performance Data
- 2110 Flight Envelope
- 2115 Take-off Performance
- 2120 Climb requirements
- 2130 Landing

- 2135 Controllability
- 2140 Control Forces
- 2145 Flying Qualities
- 2150 Stall characteristics and stall warning
- 2270 Emergency landing conditions
- 2300 Flight Control System
- 2405 Thrust/Lift Control Systems
- 2425 Thrust/Lift Operational characteristics

B.1.3 FAA UAM and AAM Certification Requirement Development

The FAA VTOL certification specification is still at early developmental stage compared to EASA. The development of VTOL certification is under Urban Air Mobility (UAM) and Advanced Air Mobility (AAM). As at the time of writing this thesis, some key early concept documents for VTOL certification development are listed below:

- FAA FHQ MIL-STD-1797 Class VI
- FAA UAM Concept of Operations - Issue 1 (FAA, 2020)
- FAA Presentation on Vertiport Design - Issue 1 (FAA, 2022c)
- FAA Memorandum on Vertiport Design - Draft (FAA, 2022b)
- FAA Advanced Aviation Advisory Committee (AAAC) Public eBook - Issue 1 (FAA, 2022a)