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Dynamic Multistatic Synthetic Aperture Radar Imaging: Resolving Motion via Differential Semblance and k -Space Diversity

Francis Watson¹  | Joshua Hellier¹  | Daniel Andre² 
¹Department of Mathematics, University of Manchester, Manchester, UK | ²Centre for Defence Engineering and Physical Science, Cranfield University, Defence Academy of the United Kingdom, Shrivenham, UK

Correspondence: Francis Watson (francis.watson@manchester.ac.uk)

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ABSTRACT

Moving targets are well known to appear displaced and defocused in synthetic aperture radar (SAR) imagery, complicating both detection and recognition—especially when displaced into cluttered regions, where signal-to-background ratio is reduced. Multistatic SAR offers the potential to extract rich scene information, including enhanced moving target imaging, yet fully exploiting this diversity remains a significant challenge. We present a novel multistatic SAR collection geometry and processing framework that enables moving target indication, approximate signature extraction, motion estimation, and high-fidelity image formation. By minimising differences in refocused signatures across multistatic channels via differential semblance optimisation, we estimate target velocity and use it to form joint multichannel reconstructions which suppress background clutter and enhance target features. The proposed collection geometry allows flexibility to balance spatial resolution and diversity in observation angles with motion sensitivity, and maximises the information content of multistatic data for a desired deployment aim. Our methodology is validated using laboratory-collected SAR data, and is readily applicable to UAV-based systems, offering practical flexibility in deployment for scenarios requiring both fine-resolution imaging and robust moving target detection.

1 | Introduction

Radar imaging provides a highly important remote sensing modality due to its all weather, day/night capability and long-range scene characterisation. Detecting and imaging moving objects is crucial in defence and security, but also has commercial applications like port monitoring and humanitarian uses such as disaster response planning. Yet imaging moving targets in an otherwise stationary scene—or even detecting their presence—presents a distinct algorithmic and radar system challenge. Synthetic aperture radar (SAR) processing typically defocuses, displaces or diminishes the response of moving

objects, depending on the specifics of the motion and radar parameters [1]. A defocused, reduced target response may make it challenging to detect, more so if it has also been displaced into a region of stronger background reflectivity.

In both SAR and inverse SAR (ISAR), image formation is straightforward when motion is known; the key challenge often lies in detecting (for SAR specifically) and estimating that motion. Multiple different approaches to the motion detection/estimation problem exist, including: interferometry; multistatic or multichannel ISAR approaches; and multi-look/sub-aperture processing. The main commonality between approaches is the

Abbreviations: DSO, differential semblance optimisation; ISAR, inverse synthetic aperture radar; PDHG, primal-dual hybrid gradient (algorithm); SAR, synthetic aperture radar.

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exploitation of coherence (or incoherence) between different observations, be it at different times or positions. Each approach naturally brings different compromises, such as ability to image versus detect, accurately geolocate, computational cost, or necessary system complexities.

SAR along-track interferometry (ATI) uses phase differences between SAR images formed at small time delays, from different antenna elements spaced along-track, in order to detect targets with a radial motion [2, 3] and provide moving target indication (MTI). As Budillon et al. [4] discuss, ATI requires a high pulse-repetition frequency (PRF) which limits swath width and therefore presents a challenge in large area surveillance, or alternatively by combining with scanning techniques which instead reduce resolution. Moreover, only the radial velocity component is determined.

Multistatic approaches have been proposed which form multidimensional SAR images resolved in both space and velocity [5–7]. This means a 2D SAR image becomes a 4D object, which might substantially increase computational cost. These works treat the scene as being made up either of isotropic point scatterers or by having a polarimetric scattering matrix, but a similar method might be developed for more general anisotropy to account for diversity of reflectivity observed with different observation angles. Each non-zero velocity slice of the multidimensional images is effectively a single ISAR image. Alternatively, one can attempt to extract the component of data associated with moving targets from the stationary clutter to form ISAR images, which is also aided by multistatic or multidimensional array data [8]. If the response from the moving target can effectively be filtered from the data, then producing a single 2D or volumetric image will have a substantially lower computational cost versus resolving all possible velocities—that is the 4D/6D images.

We also mention approaches which form multiple sequential SAR images using sub-apertures of data [9–12]. The extent of target motion is less in each subaperture, which reduces defocusing. Different displacements may be used to determine aspects of the target motion, and the sub-apertures may either be recombined to provide a single focused result or used as a SAR ‘video’. These approaches may be more challenging to apply for certain target motion. For example, with only a linear radial component the target will have the same displacement in each subaperture and will not appear defocused, so may go undetected. Large or nonlinear motion, or defocused targets hidden in strong clutter, may also be challenging.

We have previously explored different approaches to both moving target and multistatic SAR image formation and focussing. In Andre and Watson [13]; Andre et al. [14], we exploited coherence in crossing-path multistatic collection geometries to detect motion. Overlapping k -space support of the two channels in these multistatic collections enabled coherent subtraction; each channel sampling k -space support differently resulted in moving targets appearing in the coherent subtraction. In Huxley et al. [15] a differential semblance criterion was used to accurately estimate motion parameters in ISAR. This criterion took its minima at the correct target motion, similarly exploiting differing k -space sampling. We have also used

regularised reconstruction approaches to combine different channels or subsets of data. In Watson [16] a spatio-temporal regularisation term was employed both to estimate along-track velocity and combine SAR data across subapertures, negating motion-induced defocusing. In Watson et al. [17] we again used regularisation to support the combining of multistatic data across different channels and resolve additional ‘nuisance’ parameters. Finally, in Watson et al. [18] we provided a generalised regularisation theory for coherent imagery, demonstrating also how this may be used to combine information across different channels or subsets of data.

In this paper we present a novel method for detection/MTI and resolving moving targets in multistatic SAR which brings together elements of each of these previous approaches. Firstly, similar to our previous crossing-path SAR methodology [13, 14] we exploit a multistatic collection geometry which has an overlapping k -space support, but with the evolution of measurements through this support differing with each transmit/receive pair. Unlike our previous work on crossing-path SAR, this enables extraction of (approximate, displaced, defocused) target signatures for each channel, referred to as *incoherent features*, which are then used for velocity estimation. Velocity estimation is via a differential semblance method [15]. As with our previous work, this exploits the fact that incoherent feature images formed at the correct velocity are all aligned, whereas those formed with incorrect velocities are differently displaced or defocused. Finally, the extracted velocity is used to form images at the correct velocity in a sparsity-enforced reconstruction approach using the complete multistatic dataset. By combining information across channels via regularisation, the stationary background clutter is suppressed but the target features enhanced. Again, this has similarity to our previous work combining multiple subsets of SAR data via regularisation [16–18], but is applied to the multistatic multichannel case.

We validate the method using experimental data collected in a controlled laboratory environment. A five-channel multistatic dataset was acquired for a scene containing stationary clutter and a quarter-scale T64 tank model undergoing repeatable motion. Experiments were conducted across four orientations and six distinct target velocities, with the target deliberately displaced into clutter regions. The results demonstrate accurate moving target detection, velocity estimation, and fine-resolution image formation of moving targets.

The key novelties of this work are: (i) a multistatic SAR collection geometry enabling moving target extraction through overlapping k -space support, extending prior two-channel approaches; (ii) a new application of differential semblance optimisation to multistatic SAR data for accurate velocity estimation; (iii) a sparsity-regularised reconstruction framework that coherently integrates multistatic data to enhance moving target signatures while suppressing stationary clutter; and (iv) experimental validation using a controlled multichannel dataset with known target motion, demonstrating practical feasibility and robustness. The method is applicable to real-world scenarios such as UAV-based SAR systems, offering potential to design collections which more easily balance the need to generate fine-resolution images of both stationary and moving objects.

Our paper is laid out as follows. Section 2 introduces the data model and multistatic k -space coverage. Section 3 explains the SAR image formation processes used, both via backprojection and regularised inversion. Section 4 presents the steps of our algorithm: k -space trimming; forming coherent difference images; extracting incoherent feature information; estimating target motion; and forming dynamic multistatic images. Section 5 describes the experimental setup and data used to demonstrate, test and validate our algorithm in Section 6. Finally, we provide concluding remarks in Section 7, discussing its benefits as well as constraints.

2 | The Data Model and k -Space

2.1 | SAR Model and Fourier Interpretation

In SAR, a radar transmitter travels along a flight path $\gamma_T(s)$ with a receiver moving along (commonly co-located) $\gamma_R(s)$, with pulses emitted at particular slow-times s . The received data is most commonly modelled as each radar pulse having been reflected only once from the stationary scene $V(\mathbf{z})$ using a scalar wave equation, and with the antenna being stationary during the time of flight of the pulse, known as the *start-stop* approximation (or *stop-and-shoot*) [17, 19]. Under such assumptions, ignoring amplitude effects such as antenna radiation pattern and following demodulation/range compression, the received data d for a single transmitter/receiver antenna pair at slow-time s and angular frequency ω is given by

$$\begin{aligned} d_{\text{phd}}(s, \omega) &= \int \frac{e^{i\omega(R_T(s, \mathbf{z}) + R_R(s, \mathbf{z}) - (R_T(s, \mathbf{z}_{\text{ref}}) + R_R(s, \mathbf{z}_{\text{ref}})))/c_0}}{(4\pi)^2 R_T R_R} U(\mathbf{z}) d\mathbf{z} \\ &= \int \frac{e^{i\omega\Delta R(s, \mathbf{z})/c_0}}{(4\pi)^2 R_T R_R} U(\mathbf{z}) d\mathbf{z} \end{aligned} \quad (1)$$

where $\mathbf{z}_{\text{ref}}$ is a reference location (generally central to the scene) known as the scene reference or stable reference point (SRP), $R_T(s, \mathbf{z})$ and $R_R(s, \mathbf{z})$ are the range of the transmitter and receiver (respectively) to a position $\mathbf{z}$ at slow-time s ,

$$R_T(s, \mathbf{z}) = |\gamma_T(s) - \mathbf{z}|, \quad (2)$$

$$R_R(s, \mathbf{z}) = |\gamma_R(s) - \mathbf{z}|, \quad (3)$$

and c_0 is the speed of light. Further, we have defined the differential bistatic range ΔR to be

$$\Delta R(s, \mathbf{z}) := R_T(s, \mathbf{z}) + R_R(s, \mathbf{z}) - R_T(s, \mathbf{z}_{\text{ref}}) - R_R(s, \mathbf{z}_{\text{ref}}). \quad (4)$$

Where there are multiple transmitters or receivers in a multistatic system, we simply have multiple such subsets of data indexed by each transmit/receive pair.

To help in understanding the information content of U captured in the data model (1) it will help to linearise the range terms in the exponent. Let us denote $\Delta R_T := R_T(s, \mathbf{z}) - R_T(s, \mathbf{z}_{\text{ref}})$ and $\Delta R_R := R_R(s, \mathbf{z}) - R_R(s, \mathbf{z}_{\text{ref}})$, the differential one-way ranges from the transmitter and receiver (respectively) to a point $\mathbf{z} = \mathbf{z}_{\text{ref}} + \delta\mathbf{z}$. For each of these differential ranges, by taking

Taylor series about $\mathbf{z}_{\text{ref}}$ and truncating to first order we may approximate.

$$\Delta R_T \approx \hat{\mathbf{R}}_T \cdot \delta\mathbf{z}, \quad \text{if } |\Delta R_T(s, \mathbf{z})| \ll R_T(\mathbf{z}), \quad \text{and} \quad (5)$$

$$\Delta R_R \approx \hat{\mathbf{R}}_R \cdot \delta\mathbf{z}, \quad \text{if } |\Delta R_R(s, \mathbf{z})| \ll R_R(\mathbf{z}), \quad (6)$$

where $\hat{\mathbf{R}}_T$ and $\hat{\mathbf{R}}_R$ are the unit vectors in the directions $(\mathbf{z} - \gamma_T)$ and $(\mathbf{z} - \gamma_R)$, respectively. Under this approximation,

$$\Delta R = \delta\mathbf{z} \cdot (\hat{\mathbf{R}}_T + \hat{\mathbf{R}}_R), \quad (7)$$

and so we may write the Fourier integral operator (1) as

$$d_{\text{phd}}(s, \omega) = \int \frac{e^{i\mathbf{k} \cdot \delta\mathbf{z}}}{(4\pi)^2 R_T R_R} U(\mathbf{z}_{\text{ref}} + \delta\mathbf{z}) d\delta\mathbf{z} \quad (8)$$

where the spatial wavenumber vector $\mathbf{k}$ is given by

$$\mathbf{k} := \omega(\hat{\mathbf{R}}_T + \hat{\mathbf{R}}_R)/c_0. \quad (9)$$

Consider now a scene consisting of a single point scatterer at $\mathbf{z}_s$, $U(\mathbf{z}) = u\delta(\mathbf{z} - \mathbf{z}_s)$. The data due to this scatterer will be

$$d_{\text{phd}}(s, \omega) = u \frac{e^{i\mathbf{k}(s, \omega) \cdot (\mathbf{z}_s - \mathbf{z}_{\text{ref}})}}{(4\pi)^2 R_T(s, \mathbf{z}_s) R_R(s, \mathbf{z}_s)}. \quad (10)$$

Equation (10) shows that the data from a (bistatic) SAR system corresponds to a spatial Fourier transform of the scene U under the Born approximation [20]. Each position in the scene is mapped to a point $\mathbf{k} = \omega(\hat{\mathbf{R}}_T + \hat{\mathbf{R}}_R)/c_0$ in spatial frequency for each given datapoint parameterised by s and ω , with the full band-limited dataset circumscribing an annulus in k -space.

The Born approximation model of the data (1) is the basis for the majority of SAR image formation algorithms, and in particular this relationship to the Fourier transform (8) provides the basis for many efficient algorithms such as the polar format algorithm [20]. The SAR data model may be further refined to incorporate other real effects on the data, for example higher-order terms from the Taylor approximation (6) or effects due to platform motion [1], subsequently improving image formation.

The Fourier interpretation relies on several key assumptions: that the scene is stationary, that scatterers are isotropic, and that measurements are perfectly aligned in space and time. Under these conditions, measurements from different geometries or times should be coherent when mapped to the same k -space location. In practise, deviations from these assumptions—such as target/scene motion, anisotropic scattering, or platform navigation errors—introduce incoherence between measurements. These sources of incoherence become particularly significant in multistatic configurations, where multiple transmitter-receiver pairs observe the scene from different geometries.

2.2 | Multistatic Effects and Incoherence

Unlike monostatic and bistatic SAR, multistatic systems involve multiple bistatic pairs that may sample overlapping regions of

k -space at different times and from different angles. Indeed, there is more than one configuration of frequency, transmitter and receiver position which captures the same measurement in k -space for any given point in the scene. This is illustrated in Figure 1.

If all the assumptions in Equation (8) were correct, as well as our measurement and position accuracy being perfect, then these coincident k -space measurements would be the same. In general however, there will be a differing component between coincidental measurements. This can occur, for example, due to measurement or position, navigation and timing (PNT) accuracy, causing incoherence throughout the system. For incoherence due to position errors we are effectively mapping to the wrong location in k -space, since we do not know the R_T and R_S precisely enough in the same (relative) frame of reference.

It is perhaps more interesting to consider when this is due to modelling errors present in the data model (1), rather than some form of measurement or instrumentation accuracy. Recall that two of the key assumptions are that the scene is stationary, and that it is made of isotropic scatterers. That is, it doesn't matter when we observe V , and it looks the same from each direction. In reality, the observed reflectivity of objects in the scene depends on both the transmitter and receiver direction—that is, it is anisotropic—though for small changes in observation angle isotropy is a reasonable approximation. Equally, if we observe the same position in k -space at different slow-times, then any moving targets will result in these measurements being different.

3 | SAR Image Formation

In this paper we form SAR images in two ways: backprojection [21], and regularised reconstruction [22]. The data model (1) set out in Section 2.1 is continuous, but both the data collected and images formed are discrete. Data has inherent sampling of pulses which are emitted at specific times, and images have discrete pixel locations. So before we describe these image formation processes, we first need to discretise this data model.

3.1 | Discretisation

The scene U is commonly modelled as being comprised of point scatterers of strength u_j and at locations $\mathbf{z}_j, j = 1, \dots, n$, so that

$$U(\mathbf{z}) := \sum_{j=1}^n \delta(\mathbf{z} - \mathbf{z}_j) u_j. \quad (11)$$

For each pulse emitted at a particular slow time, the data is digitised at a discrete set of frequencies. We can therefore write the i^{th} datapoint as being due to the i^{th} pulse-frequency pair (s_i, ω_i) ,

$$d_i := d_{\text{phd}}(s_i, \omega_i), \quad i = 1, \dots, m. \quad (12)$$

In this notation, assuming each pulse has the same sampling, m will be the product of the number of pulses and number of frequency samples. That is, some coincident s_i and ω_i are

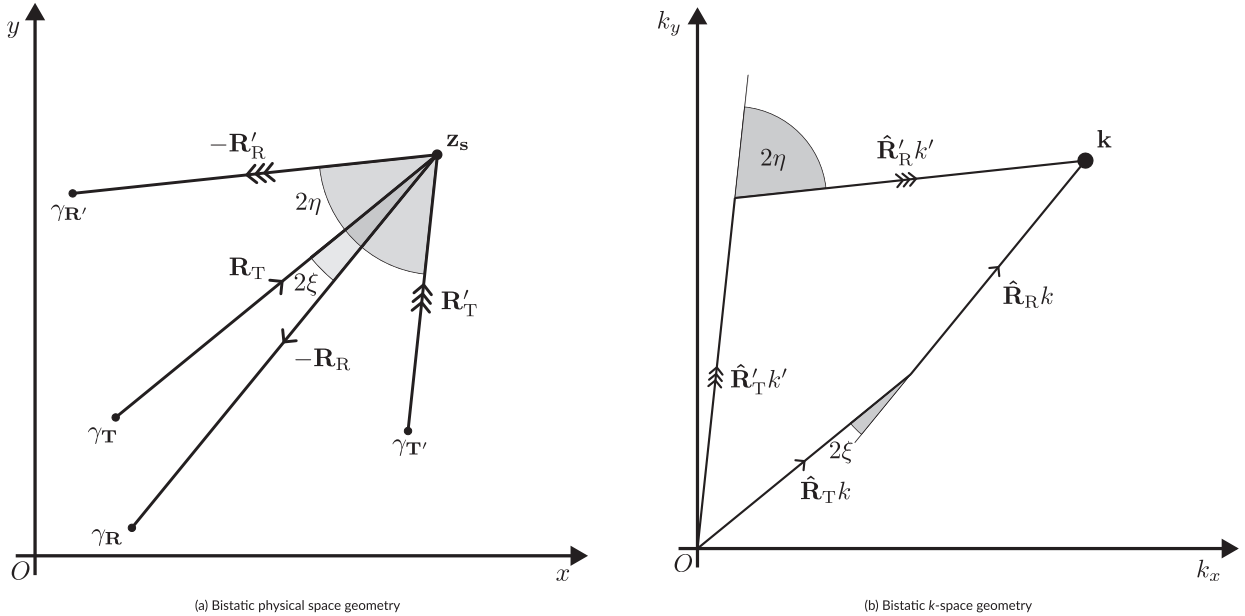


FIGURE 1 | Geometry of two different bistatic radar configurations which collect data mapping to the same position in k -space regarding a particular spatial scattering point $\mathbf{z}_s$, (a) in real space and (b) in k -space. The first system has a bistatic opening angle of 2ξ , while the second (primed) system has a bistatic angle of 2η . Note that as we adjust the bistatic angle, the frequencies $\omega = c_0 k$ and $\omega' = c_0 k'$ used to probe the same k -point $\mathbf{k}$ shift, such that $\omega \cos 2\xi = \omega' \cos 2\eta$. (a) Bistatic physical space geometry. (b) Bistatic k -space geometry.

assumed, but each pair is unique. This same notation would also allow for each pulse to have a different frequency sampling.

With this discretisation, Equation (1) becomes

$$d_i = \sum_{j=1}^n a(s_i, \omega_i, \mathbf{z}_j) e^{i\omega_i \Delta R(s_i, \mathbf{z}_j)/c_0} u_j, \quad (13)$$

where we have absorbed the range-dependent amplitude loss terms into a . While other amplitude effects such as radiation patterns were ignored in Equation (1), they can be included in this term. Often the differences in range and radiation-dependent amplitude over a scene size can be negligible compared with the overall dynamic range of the received data, and it is acceptable to take a to be constant.

Equation (13) may be succinctly written in matrix form as

$$\mathbf{d} = \mathbf{A}\mathbf{u}, \quad (14)$$

where $\mathbf{d} := [d_1, \dots, d_m]^T$, $\mathbf{u} := [u_1, \dots, u_n]^T$, and the matrix elements are

$$A_{ij} := a(s_i, \omega_i, \mathbf{z}_j) e^{i\omega_i \Delta R(s_i, \mathbf{z}_j)/c_0}. \quad (15)$$

While Equation (14) writes the model as a matrix-vector product, this matrix is never formed or stored in memory, and (14) may be implemented efficiently making use of Fast Fourier Transform (FFT) algorithms (Appendix [18]).

We also use the notation $A(\mathbf{v})$, meaning the discrete forward operator for targets moving with velocity $\mathbf{v}$. Applying $A(\mathbf{v})$ is equivalent to applying A with the change the collection geometry $\gamma(s) \rightarrow \gamma(s) - s\mathbf{v}$ and $\mathbf{z}_{\text{ref}} \rightarrow \mathbf{z}_{\text{ref}} - s\mathbf{v}$.

The discrete model (13) allows us to describe image formation from discrete sampled data.

3.2 | Forming Discrete Images

SAR imagery can be formed by ‘backprojecting’ the data—meaning to send each datapoint back to the isochrone (isorange) at which reflection occurred. This is achieved by applying the adjoint of A to the data,

$$\mathbf{I} = \mathbf{A}^* \mathbf{d}, \quad (16)$$

where $*$ is understood to mean conjugate transpose. As with forward projection (14), the operation (16) may be implemented without forming or storing the matrix or its adjoint and make use of efficient FFT implementations.

Equivalently, $\mathbf{A}^*(\mathbf{v})\mathbf{d}$ forms the SAR backprojection image focused at velocity $\mathbf{v}$ (or the ISAR image).

We also make use of regularised least-squares reconstruction techniques to form images. That is, to solve optimisation problems of the form

$$\mathbf{I}_{\text{sol}} := \underset{\mathbf{I}}{\operatorname{argmin}} \left\| \mathbf{A}\mathbf{I} - \mathbf{d} \right\|_2^2 + \lambda R(\mathbf{I}), \quad (17)$$

where R is a regularisation function used to enforce prior knowledge about the scene and enforce stability, and the regularisation parameter $\lambda > 0$ balancing the fit to data versus regularisation [22]. As an example, R might enforce sparsity via an L_1 norm, as is common in compressed sensing [23].

Equation (17) may be solved with any suitable optimisation procedure, the particularities usually dictated by the choice of regularisation function. For example, if it is non-smooth (such as L_1 norm) then a suitable non-smooth optimisation method is required. We use the CCPi Core Imaging Library (CIL) [24–26] which provides an open-source framework for reconstruction problems of the form (17) including suitable regularisation functions and optimisation methods.

A subtlety in applying reconstruction methods is that the data first needs to be filtered of features due to objects which are outside the region to be imaged. This can occur where a wide area has been illuminated but we only wish to image a smaller sub-scene, or due to sidelobes of bright scatterers located outside the scene. If the data has not been suitably filtered then Equation (17) will not be able to fit the data which is not in the range of the operator A (it cannot place objects where they need to be), and we cannot guarantee the optimisation problem will converge.

Another consideration is that the image $\mathbf{I}$ also needs to be sufficiently sampled at least as fine as the range- and cross-range resolutions. This is unlike backprojection (16), in which we may (if we wish) arbitrarily place pixels in any locations and back-project onto them, provided the *data* is at least Nyquist sampled. Since optimisation methods used to solve Equation (17) will invariably involve both back- and forward projecting (i.e., applications of both A and A^*), we require that both the *data* and image are Nyquist sampled to avoid cycle skipping in both projections.

4 | Algorithm Formulation

In order to gain information about motion in the scene from SAR data, we use a multistatic configuration in which different bistatic pairs observe the same positions in k -space at different slow times. As with previous work for ISAR Huxley et al. [15], this is achieved by having one transmitter and multiple receivers moving at different speeds, such that each bistatic pair ‘sweeps’ k -space at different rates.

This particular multistatic configuration could be achieved in a practical setting with uncrewed air vehicles (UAVs)/drones. An equivalent data k -space coverage could also be achieved by having each receiver positioned at a different range but flying with the same velocity, resulting in the line of sight to scene centre moving at a different angular speed. Figure 2 depicts such an example UAV configuration.

As in the previous section, this collection geometry should provide information about motion in the scene, as well as more generally information about scattering phenomenology (such as

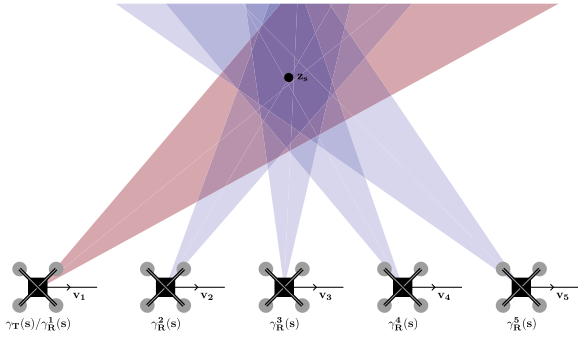


FIGURE 2 | Example multistatic UAV configuration with one transmitter and multiple receivers, with receivers moving at different velocities such that the phase history data from each bistatic pair ‘sweeps’ k -space at different rates. Red indicates a transmitted beam, blue indicates a receiving beam. This would provide data suitable for our algorithm.

anisotropy) by way of being multistatic. Our process to extract and image moving target data is summarised as follows:

1. k -space support trimming
2. Form coherent difference images
3. Extract incoherent image features and data
4. Determine target motion via differential semblance
5. Form dynamic multistatic reconstruction images

Figure 3 provides a flow diagram summarising these steps.

4.1 | k -Space Trimming

Our proposed collection geometry results in a different k -space coverage for each transmit/receive channel of data, so it is necessary to trim the data in k -space to a common spatial frequency region before coherent subtraction. Otherwise, the subtracted complex images may be dominated by differences in resolution, or more specifically, point-spread function, rather than differences due to motion or other physical effects. Otherwise, the effectiveness of complex subtraction would be limited even for parts of the data which are coherent between different bistatic pairs. k -space trimming is also required for the differential semblance criteria introduced in Section 4.4, as discussed in Huxley et al. [15].

For our experimental setup, we trim to the largest quadrilateral fitting into the intersection of all k -space supports, as calculated for a specific point in the scene. Calculating the position in k -space for each data-point, we simply set the value to 0 if it lies outside this bounding box. Denoting by d_{phd}^i the i th transmit/receive channel of data, and Q this largest common quadrilateral in k -space, the trimmed data is therefore

$$\tilde{d}_{\text{phd}}^i(s, \omega) = \begin{cases} d_{\text{phd}}^i & \text{if } k(s, \omega) \in Q \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

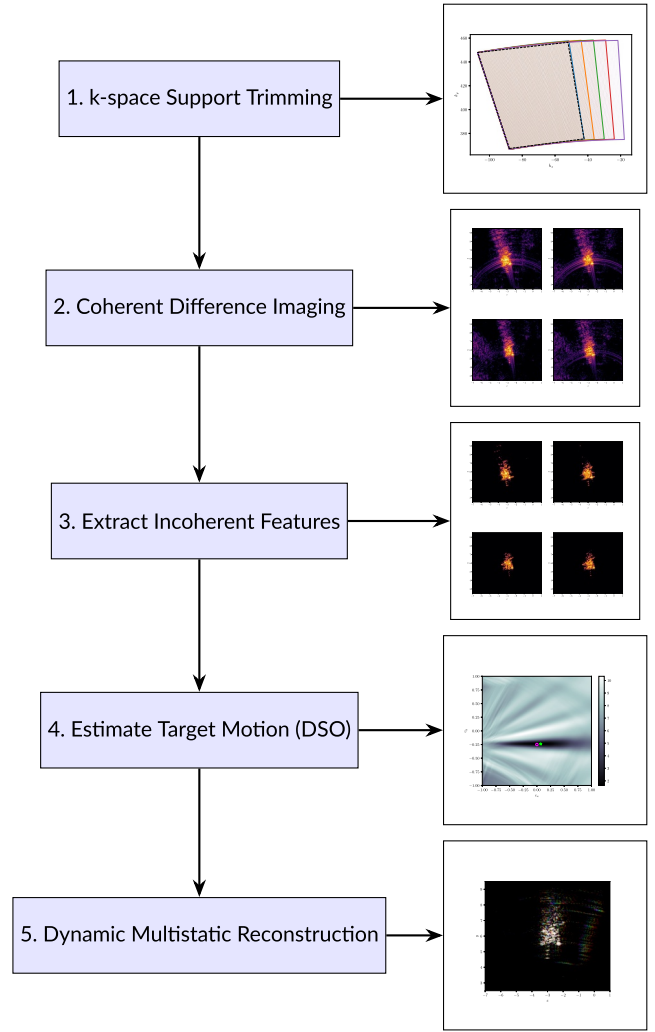


FIGURE 3 | Flow diagram of the moving target multistatic SAR image formation algorithm.

Figure 4 shows the k -space coverage for our experimental setup, with the resulting trimming highlighted by a dashed black line.

As in Equation (9), the points in k -space each measurement maps to, differs for each point in the scene, as this map is based on a Taylor expansion. Therefore, such a trimming becomes inaccurate away from this point in the scene. Spatially-varying incoherence trimming is possible [27], which would improve our results—particularly in the near-field environment of our laboratory data collection. Using the simpler method here presents a worst case for demonstration purposes.

4.2 | Coherent Subtraction

We form SAR imagery by backprojecting the k -space trimmed data. Because each transmit/receive channel now has a different number of datapoints which have been trimmed, normalisation is required. We normalise by the l_2 norm of the images,

$$\mathbf{I}^i = \mathbf{A}^* \mathbf{d}^i, \quad \hat{\mathbf{I}}^i := \mathbf{I}^i / \|\mathbf{I}^i\|_2. \quad (19)$$

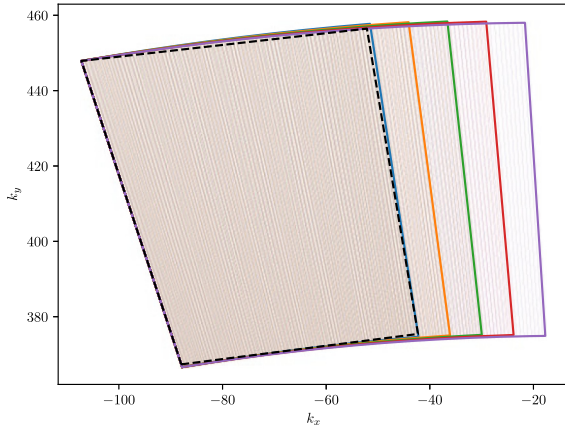


FIGURE 4 | Example k -space coverage resulting from our proposed multistatic collection geometry, calculated for a single reference point in the scene. The coverage of each transmit/receive channel is shown in a different coloured outline, with the fastest moving receiver providing the greatest cross-range coverage (i.e., finest resolution). The k -space trimming used is shown by the dashed black line, being the largest quadrilateral fitting into the coverage of each channel.

We then take adjacent difference images of the n channels, plus an additional difference.

$$\mathbf{b}^i = \begin{cases} \hat{\mathbf{I}}^{i+1} - \hat{\mathbf{I}}^i, & 1 \leq i \leq n-1, \\ \hat{\mathbf{I}}^i - \hat{\mathbf{I}}^{i-2}, & i = n, \end{cases} \quad (20)$$

$$:= \mathbf{D}\hat{\mathbf{I}}. \quad (21)$$

In the experimental results presented later we have $n = 5$, but in general require $n \geq 3$.

4.3 | Extract Incoherent Feature Images

Given n difference images $\mathbf{b}^i$, the task is to retrieve (an approximation to) the features only present in each individual image, denoted by $\mathbf{c}^i$, $1 \leq i \leq n$. We refer to these $\mathbf{c}^i$ as the ‘incoherent feature images’, which are obtained in a least-squares sense by solving

$$\tilde{\mathbf{c}} := \underset{\mathbf{c}}{\operatorname{argmin}} \left\| \mathbf{D}\mathbf{c} - \mathbf{b} \right\|_2^2, \quad \tilde{\mathbf{c}} = [\mathbf{c}^1, \dots, \mathbf{c}^n], \quad (22)$$

which has the general solution $\tilde{\mathbf{c}} = \mathbf{D}^\dagger \hat{\mathbf{I}} := (\mathbf{D}^* \mathbf{D})^{-1} \mathbf{D}^* \hat{\mathbf{I}}$ yielding minimum norm.

The difference matrix $\mathbf{D}$ clearly has a nullspace consisting of components common to all $\mathbf{c}^i$: if this were not the case then we would expect to recover from Equation (22) the original SAR imagery $\hat{\mathbf{I}}$. The inclusion of the additional difference in Equation (21) means that the nullspace contains only such components common to every $\mathbf{c}^i$.

Thus, the resulting $\mathbf{c}^i$ may have a common component if this results in a reduced norm of $\tilde{\mathbf{c}}$. The $\mathbf{c}$ are not used as final target imagery but only to determine target velocity so any

residual component is not problematic, noting also that any residual common component must be smaller in magnitude than the extracted moving target signature (else it would increase the norm of the solution). This approach could be developed further by use of regularisation.

To prepare for velocity estimation, we first forward project the incoherent feature images back into the data domain to produce incoherent feature data $\tilde{\mathbf{d}}$,

$$\tilde{\mathbf{d}} := \mathbf{A}\tilde{\mathbf{c}}. \quad (23)$$

The collection geometry ensures that any moving targets will have a different displacement in each transmit/receive channel, ensuring that moving targets appear in the incoherent features images $\tilde{\mathbf{c}}$ and data $\tilde{\mathbf{d}}$. These may also contain information about the differences due to observation angles, if these differences are significant enough.

4.4 | Determine Target Motion

To determine target velocity from $\tilde{\mathbf{d}}$ we use a differential semblance criteria: that is, an objective based on the difference between channel images, taking its minimum at the correct target velocity. For this we form images at different target velocities before taking the differences, giving the Differential Semblance Optimisation (DSO) scheme [15].

$$\mathbf{v}_{\text{sol}} = \underset{\mathbf{v}}{\operatorname{argmin}} \left\| \frac{\partial \tilde{\mathbf{I}}(\mathbf{v})}{\partial h} \right\|_2^2, \quad (24)$$

$$\tilde{\mathbf{I}}(\mathbf{v}) := \mathbf{A}^*(\mathbf{v})\tilde{\mathbf{d}} \quad (25)$$

here, $\tilde{\mathbf{I}}(\mathbf{v}) := [\tilde{\mathbf{I}}_1, \dots, \tilde{\mathbf{I}}_n]$ are the backprojection images for each transmit/receive channel formed at a particular target velocity $\mathbf{v}$. The partial derivative with respect to channel h is approximated by finite differences of the $\tilde{\mathbf{I}}_h$, asserting also that these channels are ordered by size of bistatic angle. The optimisation (24) may be carried out with any suitable routine, such as Gaussian Process Regressor in previous work [15].

Note that if there is no moving target present, then the incoherent feature data is only due to changes in reflectivity with geometry. These should generally be approximately co-located, particularly where bistatic angle is only gradually varying such as in our collection geometry, and so Equation (24) would be expected to return $\mathbf{v}_{\text{sol}} = \mathbf{0}$.

Our algorithm could be further enhanced if necessary with an additional detection step to further screen incoherent feature data which does not contain moving targets. This could be achieved by using a simple tracker on the incoherent feature images $\tilde{\mathbf{c}}$, ensuring there is a gradual change of target position through each channel. The displacements themselves could further provide an initial estimation of target velocity to seed the optimisation procedure (24). For our experimental results no such processing was necessary.

4.5 | Form Final Images

With target velocity determined, we return to the full dataset to form multistatic imagery. This is again formed at $\mathbf{v}_{\text{sol}}$, however we now use a regularisation (or *compressed sensing*) approach to suppress stationary clutter and bring out moving parts of the scene:

$$\mathbf{I}_{\text{sol}} = \underset{\mathbf{I}}{\operatorname{argmin}} \left\| A(\mathbf{v}_{\text{sol}})\mathbf{I} - \mathbf{d} \right\|_2^2 + \lambda_1 \left\| D_h \mathbf{I} \right\|_1 + \lambda_2 \left\| \mathbf{I} \right\|_1, \quad (26)$$

where D_h is the discrete first difference operator between neighbouring channels. To solve Equation (26), we use the Primal Dual Three Operator Splitting (PD3O) algorithm [28] available in the CIL library [26]. PD3O solves minimisation problems involving the sum of three functions $f(\mathbf{x}) + g(\mathbf{x}) + h(K\mathbf{x})$, where f is smooth, g and h are convex (with h potentially composite), and K a bounded linear operator.

The components of this objective function say that the multistatic multichannel scenes $\mathbf{I}_{\text{sol}} := [\mathbf{I}_{\text{sol}}^1, \dots, \mathbf{I}_{\text{sol}}^n]$ formed at velocity $\mathbf{v}$ must best fit the full data $\mathbf{d}$, that (coherent) differences between each channel will be sparse, and that the images themselves are sparse. The balance of the first two terms are key to bringing out only moving targets: the data fitting term would cause stationary parts of the scene to appear differently displaced in each channel, but the second term will suppress these differences. Moreover, the term $\|D_h \mathbf{I}\|_1$ will promote coherent summation between channels, enhancing the signal from the moving target, but allow (sparse) differences between channels for example due to angle-dependent reflectivity. The third term further promotes overall sparsity, as we expect the moving target to be a smaller part of the scene.

A more general reconstruction formula could be considered. For example using group sparsity, that is $\|\cdot\|_{2,1}$ norms, might improve reconstructed target strength. Additionally, applying the channel-difference term incoherently, via $\|D_h |\mathbf{I}|\|_1$, might be beneficial if target reflectivity varies significantly across channels and we do not expect to be able to coherently combine. More general regularisation terms are of course possible.

One key benefit of this approach is that all of the multistatic data is used in forming the final image product. This allows for collection geometries to be designed which balance total k -space coverage for finer resolution versus a potentially smaller overlapping region used to detect moving targets and determine their velocity. In contrast, the incoherent feature data results from a reduced k -space support, limiting resolution, and (as discussed above) may not accurately have extracted exactly the data due to the target, and therefore cannot guarantee target signatures are accurately recovered.

5 | Experimental Setup

Our experimental setup in the GBSAR (ground-based SAR) laboratory [27] involves transmit and receive antennas on separate two-axis repositioners, connected to a Keysight N5225B two-port vector network analyser (VNA), which has proven to

be a low noise stable system suitable for SAR image formation and interferometry. The measured frequency range used was 9–11 GHz.

By repeating transmit locations we can effectively capture arbitrary (single transmit multiple receive) multistatic configurations. We could similarly capture arbitrary multi transmit/receive configurations, but have not done so for this work.

The antenna trajectories for the five different bistatic SAR geometries are presented in Figure 5. There is a single transmitter trajectory, in red, and five overlapping receiver trajectories, in blue, which traverse to their respective end points within the same time. Taking into account the scene centre, indicated by the black star, the corresponding equivalent monostatic positions are shown in magenta. The equivalent monostatic position is defined to be on the bisectrix, which is the line which passes through the scene centre and bisects the lines from the scene centre to the transmitter and to the receiver, at a range from the scene centre equal to the bistatic range to there. The antenna start positions are indicated by green crosses, whilst the end positions are indicated by appropriately coloured crosses. Important bistatic SAR parameters are presented in Table 1, showing the bistatic range, bistatic grazing angle, bistatic angle and bistatic aperture. The aperture value is the angle between the start and end bisectrix directions; the bistatic grazing angle is that of the mid aperture bisectrix, and the bistatic angle is that for the mid-aperture pulse. The particular apertures chosen for our experiment could also be referred to as a ‘multidynamic’ array [29].

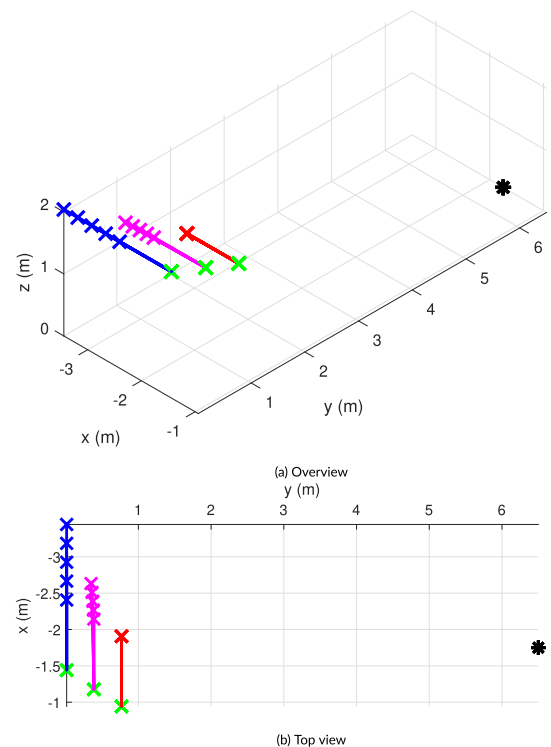


FIGURE 5 | 3D representations of the laboratory geometry for the five bistatic SAR collections, showing (a) an overview and (b) a top view.

TABLE 1 | SAR geometry parameters.

Bistatic geometry	Bistatic range (m)	Bistatic grazing angle (°)	Bistatic angle (°)	Bistatic aperture (°)
1	6.447	18.54	5.01	8.58
2	6.450	18.54	6.02	9.66
3	6.453	18.54	7.05	10.73
4	6.458	18.54	8.09	11.78
5	6.464	18.54	9.14	12.82

Note that the data collected is precisely ‘start-stop’, where antennas are stationary during each frequency sweep. This approach is commonly used in radar laboratories and is discussed in Cheney and Borden [19]; Corbett et al. [30]. The approach is valid for the cases where the moving target displacement between pulses is much smaller than the range resolution and where the motion speed is much less than that of light, which are the cases for the experiments reported here. For example, the scenario measured will involve an equivalent full-scale maximum target displacement of 3.2 m in 100 steps, giving step sizes of 3.2 cm, which are much smaller than the finest full-scale equivalent resolution of 28.8 cm.

The experiments were conducted so that the Nyquist criterion was satisfied, both in range and in cross-range. In the laboratory setting, the Nyquist sampling criterion in cross-range gives approximately $\Delta x = \lambda/2\theta$ where Δx is the aperture sample spacing, λ the centre wavelength and θ is the desired aliasing free angular extent across the scene. For our scene, we may choose $\theta = 1$ radian ($\approx 57.3^\circ$) so that we require $\Delta x \leq \lambda/2$. Thus for a centre frequency of 10 GHz we will require $\Delta x \leq 1.5$ cm. In the experiments we have kept the equivalent-monostatic displacements across the five bistatic trajectories to be less than this, varying between 0.96 and 1.48 cm.

It should be noted that the use of a single VNA linked via microwave cables to the transmitter and receiver antennas allows for exact bistatic synchronization, an issue which can be challenging to resolve in operational bistatic remote sensing systems.

The scene is shown in Figure 6. It consisted of an area of clutter made of gravel and rocks of various sizes and a quarter scale T62 tank model. The tank is placed on a fifth repositioner for accurately controlled motion calibrated with the antenna positions. By moving the target position with the transmit locations we can capture a surrogate for moving target SAR data. This is a slow-moving approximation: since the target does not move at all during a ‘pulse’ there is no Doppler shift of the reflected wave. Because of trade-offs between range and Doppler resolution in waveform design, for a fine range resolution SAR system this remains an acceptable surrogate.

Four different target and repositioner rail orientations were used with the target and rail oriented at 0° , 25.67° , 63.08° , and 90° from transmit antenna boresight, capturing 10 different target velocities, including stationary ground truth data for each orientation. These are summarised in Table 2 and the different

**FIGURE 6** | Experimental setup looking downrange of GBSAR system (above) and across/up-range (below).**TABLE 2** | Target motion experimental setups.

Experiment number	Orientation (°)	Distance traversed (m)	Displacement ^a (m)
1	0.0	0.250	[0.000, −0.250]
2	0.0	0.375	[0.000, −0.375]
3	0.0	0.0	[0.000, 0.000]
4	25.67	0.375	[−0.160, −0.340]
5	25.67	0.0	[0.000, 0.000]
6	63.08	0.375	[−0.334, −0.170]
7	63.08	0.750	[−0.669, −0.340]
8	63.08	0.0	[0.000, 0.000]
9	90.0	0.750	[−0.750, 0.000]
10	90.0	0.0	[0.000, 0.000]

^aEquivalent to velocity measured in m/1 unit time, where the total slow-time of the aperture is considered to take 1 unit time.

orientations shown in Figure 7. For experiments with a target velocity component in the range direction, it appears displaced into the clutter region in standard SAR imagery.

Note that because of the start-stop nature of data collection there is no intrinsic slow-time to the experiment, and therefore no intrinsic target velocity. One may choose any total collection time and scale transmit/receive antenna and target velocities accordingly. Stating the total collection time to be $\tau = 1$ unit

time, target velocity and total displacement are equivalent with units m. In the experimental results section, we therefore refer to target velocity and displacement interchangeably.

Regarding the use of the T62 tank model, we note that this comprised a convenient complex target we had at our disposal and that the approach presented is not specifically dedicated to tank detection, though having noted this point, the parameters chosen for the experiments can be seen to have relevance to a full-scale tank scenario.

As discussed, we are free to set a notional integration time for the purposes of connecting the experiment with a given scenario. If we choose 1 s as the integration time, we can assume that the motion of a full-scale massive target such as a tank should often be relatively stable during this short time. Our maximum equivalent full scale displacement was 3.2 m, so that given the proposed 1 s integration time, this would correspond to vehicle speeds of $3.2 \text{ m/s} = 11.5 \text{ km/hr} = 7.2 \text{ mile/hr}$, which we would not regard as unusual in realistic conditions.

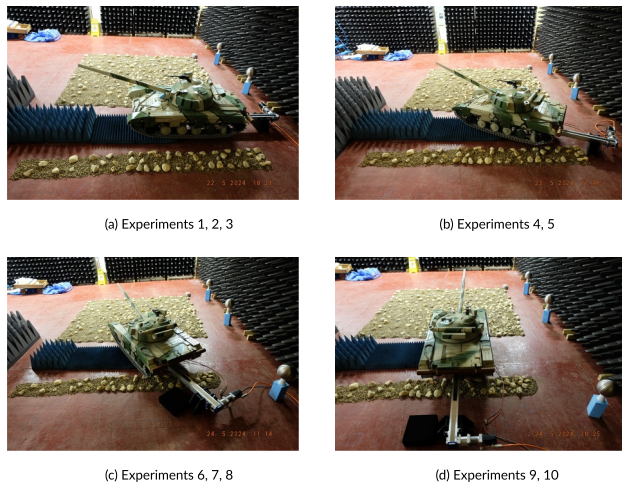


FIGURE 7 | Target and rail orientations for the 10 experiments. (a) Experiments 1, 2, 3. (b) Experiments 4, 5. (c) Experiments 6, 7, 8. (d) Experiments 9, 10.

6 | Experimental Results

Four experimental configurations were considered: with the target and rail oriented at 0° , 25.67° , 63.08° , and 90° from transmit antenna boresight. In each of these orientations, a baseline experiment captured the stationary target at the given orientation. The full list of experiments are captured in Table 2.

6.1 | Incoherent Feature Isolation

For each of these experiments, the standard backprojection image (with no motion compensation) of the pseudo-monostatic channel is shown in Figure 8, with (c), (e), (h) and (j) being for the data with stationary target. Table 3 also provides the mean coherence of the gravel clutter between SAR images formed by different bistatic channels, after k -space cropping, in experiment number 3, calculated using a 3×3 window. An average coherence between neighbouring bistatic channels of 0.934 shows a good level of coherence for coherent subtraction and processing. The lower coherence between channels 1 and five is expected due to the greater difference in bistatic angle [27].

Motion with a component in the range direction causes the expected observed misplacement of the target in the cross-range direction, with cross-range motion also defocusing [1]. In each case, the moving target results in worse sidelobes in the cross-range direction. The metallic target model has a substantively

TABLE 3 | Mean coherence between SAR images from different channels, calculated in the region of clutter $[-4, 5] \times [-3.2, 7]$ using a 3×3 pixel window, in experiment number 3.

Channel numbers	Mean coherence
1 and 2	0.915
2 and 3	0.947
3 and 4	0.929
4 and 5	0.944
1 and 5	0.875

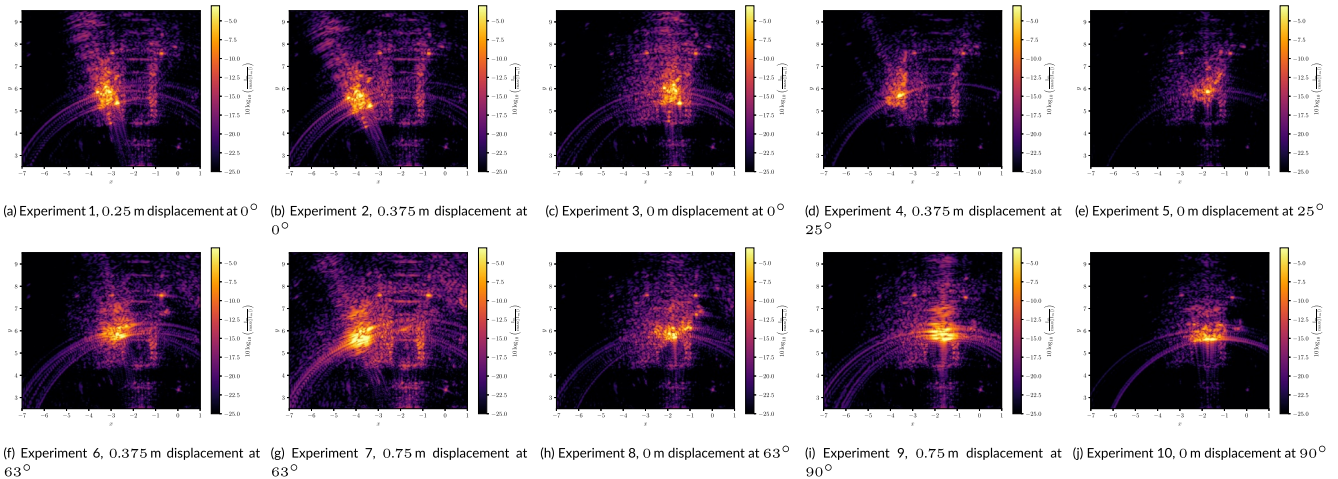


FIGURE 8 | Standard backprojection images for the pseudo-monostatic channel of each experiment.

stronger reflectivity than the background clutter arranged on the floor, so displacement into the clutter region has a limited effect on obscuring the target signature in the laboratory setting.

Nevertheless, in a real-world scenario it is possible for displaced objects to become hidden in clutter where reflectivities are at least more on par.

Figure 9 shows the intermittent processing results for Experiment 1 (0.25 m displacement at 0°). Figure 9a–e show the coherent difference images between channels. Figure 9f–j show the extracted incoherent feature images, and Figure 9k–o show the clipped incoherent features. The coherent subtractions remove background clutter moderately well despite the simple k -space cropping used which is only locally accurate to the scene centre in the near-field environment.

The incoherent feature images isolate the individual target relatively well, where the coherent differences each contain two copies of the tank. There is a constant component remaining throughout each image (i.e., a component in the nullspace of the difference operator), appearing as a shadow of the tank signature.

As discussed in Section 4.3, this is because it reduces the resulting norm overall. However, this ‘shadow’ is almost entirely removed in the clipped version, which shows individual tank signatures.

The measure of success here is not how well we accurately recover an individual signature, but how well we can use these to extract the correct velocities. Next we will see how well this allows for target velocity estimation.

6.2 | Velocity Estimation

To evaluate the accuracy of motion estimation, the differential semblance objective function (24) was applied across a grid of velocities for each experiment.

Figure 10 shows the forward-projected incoherent feature data in Experiment 1, which we observe retains the k -space trimming. This incoherent feature data is backprojected into scenes moving with velocities $(v_x, v_y) \in [-1, 1] \times [-1, 1]$, and the

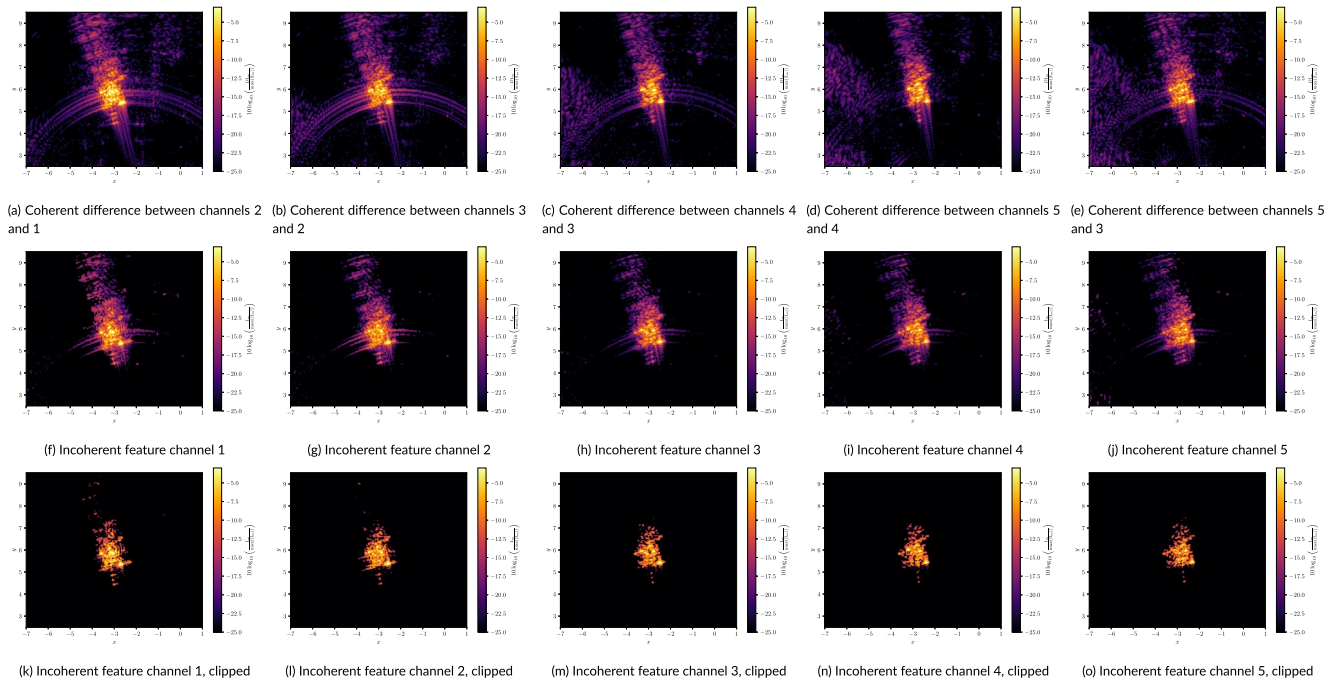


FIGURE 9 | Intermediate processing results from Experiment 1, 0.25 m motion at 0° to boresight. (a–e) are the magnitudes of coherent subtraction images, (f–j) are the extracted incoherent feature outputs of the least-squares solution, (k–o) are the incoherent feature images clipped to -15 dB from below used for velocity extraction.

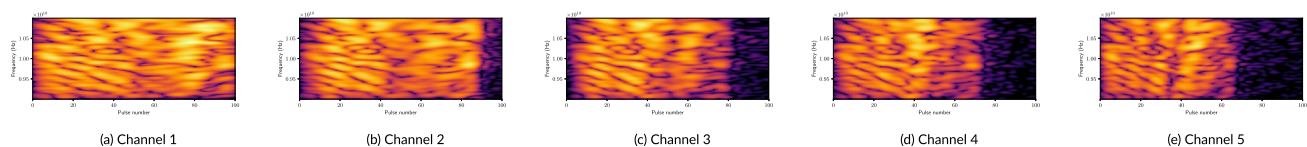


FIGURE 10 | Forward projected data from extracted incoherent features, for channel from Experiment 1, 0.25 m motion at 0° to boresight, in dB scale relative to maximum value.

differential semblance objective function Equation (24) evaluated at each. Figure 11 presents the differential semblance objective function evaluated on a velocity grid for each dynamic experiment. The minima (lime green stars) and true velocities (magenta circles) are indicated. We do not propose an exhaustive search as part of an algorithm, but have carried one out to understand the convexity and robustness of an optimisation approach.

We can see that in each case the minima lies in a long thin valley which is steep in the range direction, suggesting a degree of ill-posedness in the cross-range direction. This seems natural, as it is linear motion in the range-direction which causes target displacement. A collection geometry in which the angle of the flight path also gradually varies between channels might improve this, though this is challenging in our experimental lab setup.

We also see that the objective function is locally convex around the global minimum, with local minima away from this point. As with previous results this may necessitate use of a suitable (possibly Bayesian or global) optimisation scheme [15]. Away from the convex region containing the minimum we see the objective function is around an order of magnitude greater, suggesting it should be clear that the correct velocity has not been found (i.e., the different image channels have poor semblance). With this, any suitable optimisation scheme should readily be able to solve this low-dimensional problem. A study of ‘best’ optimisation schemes is beyond the scope here, so we use the brute-force result going forwards.

Table 4 provides these brute-force extracted target velocities for each of the dynamic experiments. Notwithstanding any ill-posedness, the optimal value in each case lies within only a few cm of the true displacement, which we generally consider negligible. In particular, the distance traversed in the range direction had an error of < 3 cm throughout all experiments, which is under half of the range-resolution (summarised later in Table 5). While there is greater error in the cross-range direction, as we expect from the discussion above, this is also less than the cross-range resolution throughout.

The DSO objective minima lying close to the ground truth value throughout, supports the previous observations in Section 6.1

TABLE 4 | Extracted target velocity.

Experiment number	Extracted displacement ^a (m)	Absolute error (m)
1	[0.07, -0.24]	[0.07, 0.01]
2	[0.03, -0.36]	[0.03, 0.02]
4	[-0.09, -0.33]	[0.07, 0.01]
6	[-0.32, -0.18]	[0.01, 0.01]
7	[-0.58, -0.37]	[0.09, 0.03]
9	[-0.60, -0.02]	[0.15, 0.02]

^aEquivalent to velocity measured in m/1 unit time, where the total slow-time of the aperture is considered to take 1 unit time.

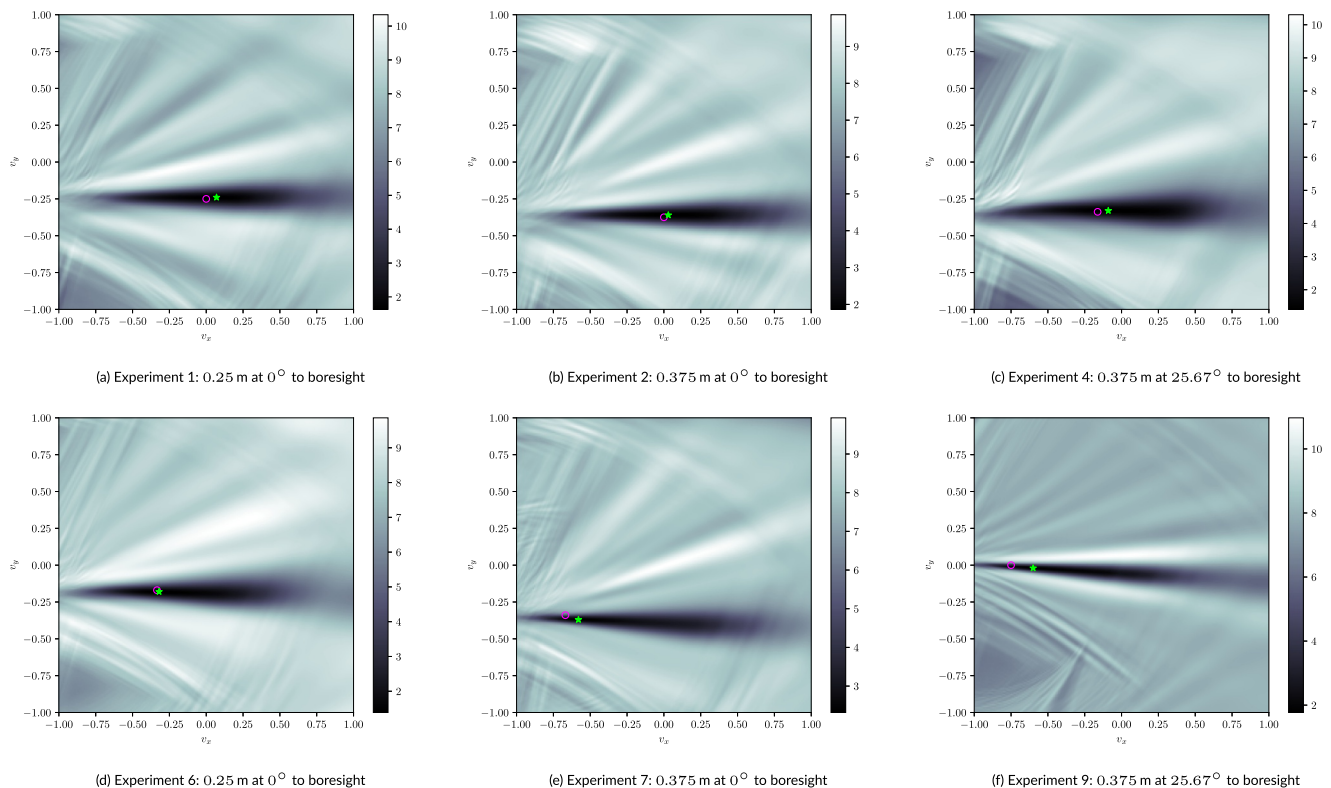


FIGURE 11 | Differential semblance objective function values against velocity (v_x, v_y) for each moving target dataset. The minimum of the objective function is marked by a lime green star, and the true velocity by a magenta circle.

that we are sufficiently well able to isolate moving target signatures, else we would not expect to estimate their velocities. We will later consider how closely the reconstructions at the extracted velocities match those at the ground truth, providing an indirect but practical measure of velocity estimation accuracy.

6.3 | Image Reconstruction

For each of the dynamic experiments, we form reconstruction images using Equation (26) at the previously extracted optimal velocities. Figures 12–17 present comparative reconstruction results for each dynamic experiment. Each figure includes: (a)

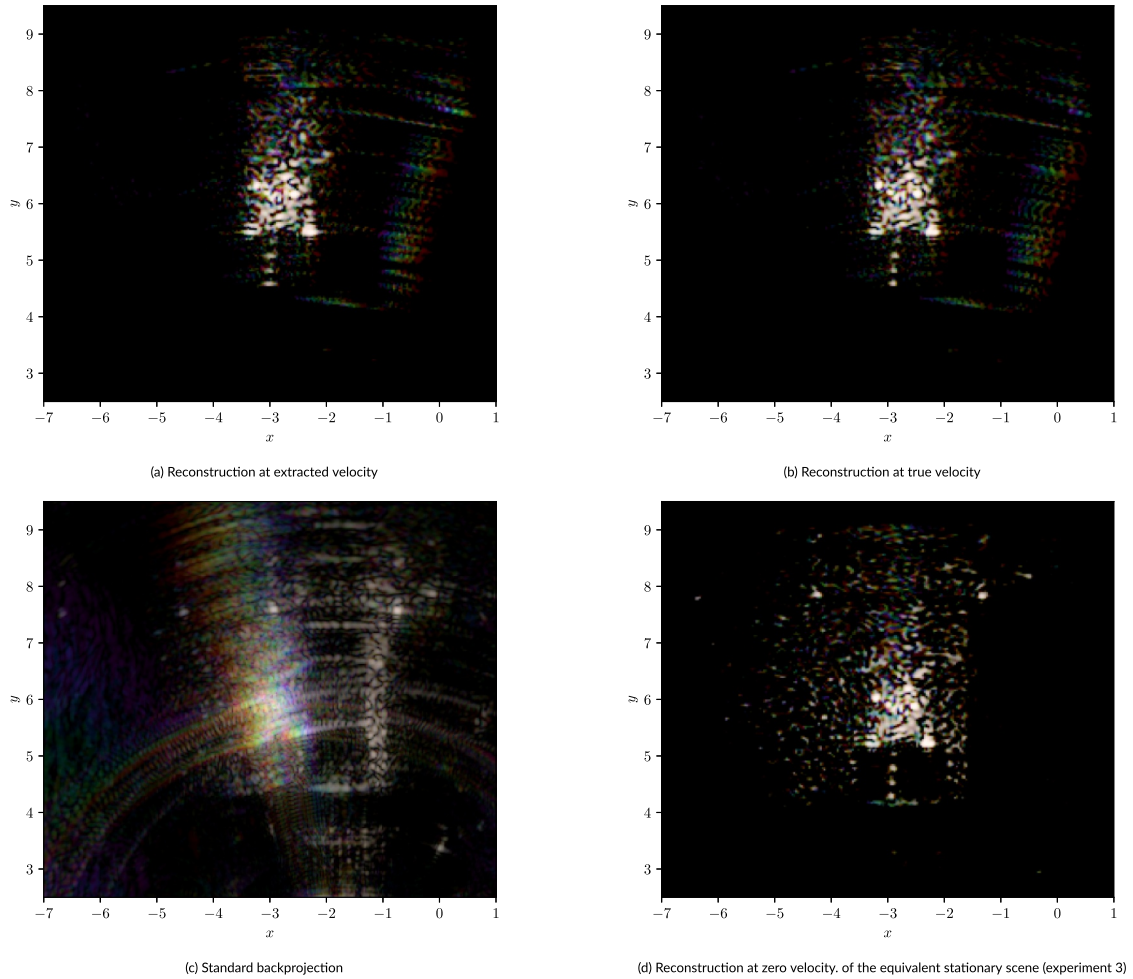


FIGURE 12 | Results from Experiment 1, 0.25 m motion at 0° to boresight, in dB scale relative to maximum value. (a) and (b) are reconstructed at the extracted velocity and at the (known) correct velocity, respectively. (c) Is the standard backprojection formed at zero velocity, and (d) is the reconstruction of the equivalent stationary scene (Experiment 3).

TABLE 5 | Theoretical resolution of experiments evaluated at the reference point $[-3, 6, 0]$ in units m.

Bistatic pair number	Resolution (m)					Equivalent resolution at full scale vehicle (m)					
	1	2	3	4	5	1	2	3	4	5	
Range resolution in the ground plane (slant plane)	0.079 (0.075)					0.316 (0.300)					
Equivalent ISAR cross-range resolution at given velocity	[0,0,0]	0.102	0.090	0.080	0.072	0.066	0.410	0.360	0.320	0.288	0.363
	[0, −0.25, 0]	0.107	0.093	0.819	0.073	0.066	0.428	0.371	0.328	0.293	0.265
	[0, −0.375, 0]	0.110	0.094	0.083	0.074	0.067	0.438	0.378	0.332	0.296	0.266
	[−0.16, −0.34, 0]	0.135	0.113	0.097	0.085	0.076	0.540	0.453	0.389	0.341	0.303
	[−0.33, −0.17, 0]	0.169	0.137	0.115	0.099	0.087	0.676	0.548	0.461	0.397	0.348
	[−0.67, −0.34, 0]	0.514	0.301	0.212	0.163	0.133	2.056	1.204	0.849	0.654	0.530
	[−0.75, 0, 0]	0.475	0.294	0.212	0.166	0.136	1.900	1.177	0.850	0.663	0.543

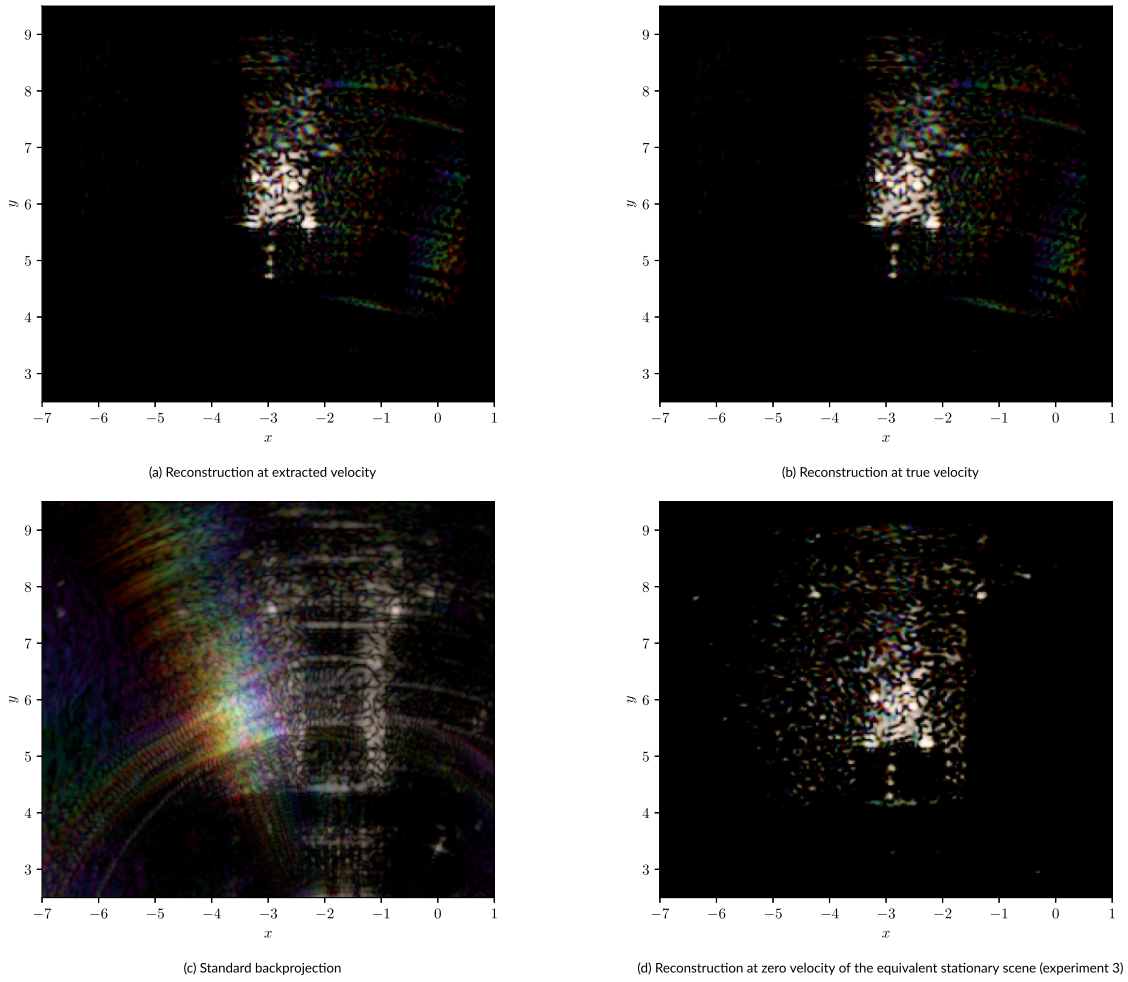


FIGURE 13 | Results from Experiment 2, 0.375 m motion at 0° to boresight, in dB scale relative to maximum value. (a) and (b) are reconstructed at the extracted velocity and at the (known) correct velocity, respectively. (c) Is the standard backprojection formed at zero velocity, and (d) is the reconstruction of the equivalent stationary scene (Experiment 3).

reconstruction at the extracted velocity, (b) reconstruction at the true velocity, (c) standard backprojection at zero velocity, and (d) reconstruction of the equivalent stationary scene for comparison. Each is presented in log scale and false colour, with each transmit/receive channel taking a different colour such that if the intensity is the same in each channel for a given pixel it will appear grey-scale. Reflectivity ≥ -3 dB relative to peak in each channel will appear white, and reflectivity ≤ -25 dB relative to peak in each channel will appear black.

For each of the experiments, the reconstruction strongly suppresses the stationary background, focussing the target and also strongly suppressing cross-range sidelobes. This is in stark contrast to the multistatic backprojection images.

Comparing Figures 12–17a,d, we observe the reconstructed signature also closely matches that of the equivalent stationary scenes. The differences here are driven by the presence of stationary background, as well as differences in cross-range resolution in the moving target. These effective resolutions are provided in Table 5.

Comparing also Figures 12–17a,b, we observe very little difference between the reconstructions at the extracted and ground

truth velocities. For the experiments at 0° orientation shown in Figures 12 and 13, we do observe the response from the barrel is marginally defocused when using the extracted velocity versus the ground truth. For the 63° oriented experiment shown in Figure 16(b), we see the (weak) responses from the barrel are not exactly co-located when using the ground truth velocity: that is, there is angle-dependent scattering phenomena (anisotropy). Presumably, aligning responses which appear at adjacent locations across channels contributed to a small error in velocity estimation. This emphasises our discussion on the causes of differences in measurements which are at the same k -space position.

The overall close visual similarity between reconstructions at the extracted and ground truth velocities confirms the practical accuracy of velocity estimates.

As shown in Figures 14–16, for experiments carried out at 25° and 63° the weak responses from the barrel become unobservable in the reconstructed result. In contrast, these are unobservable not only in the backprojection images, but also in the reconstructions of the equivalent stationary scene, hidden amongst background clutter. This supports the case that our processing method will help identify moving objects obscured by clutter.

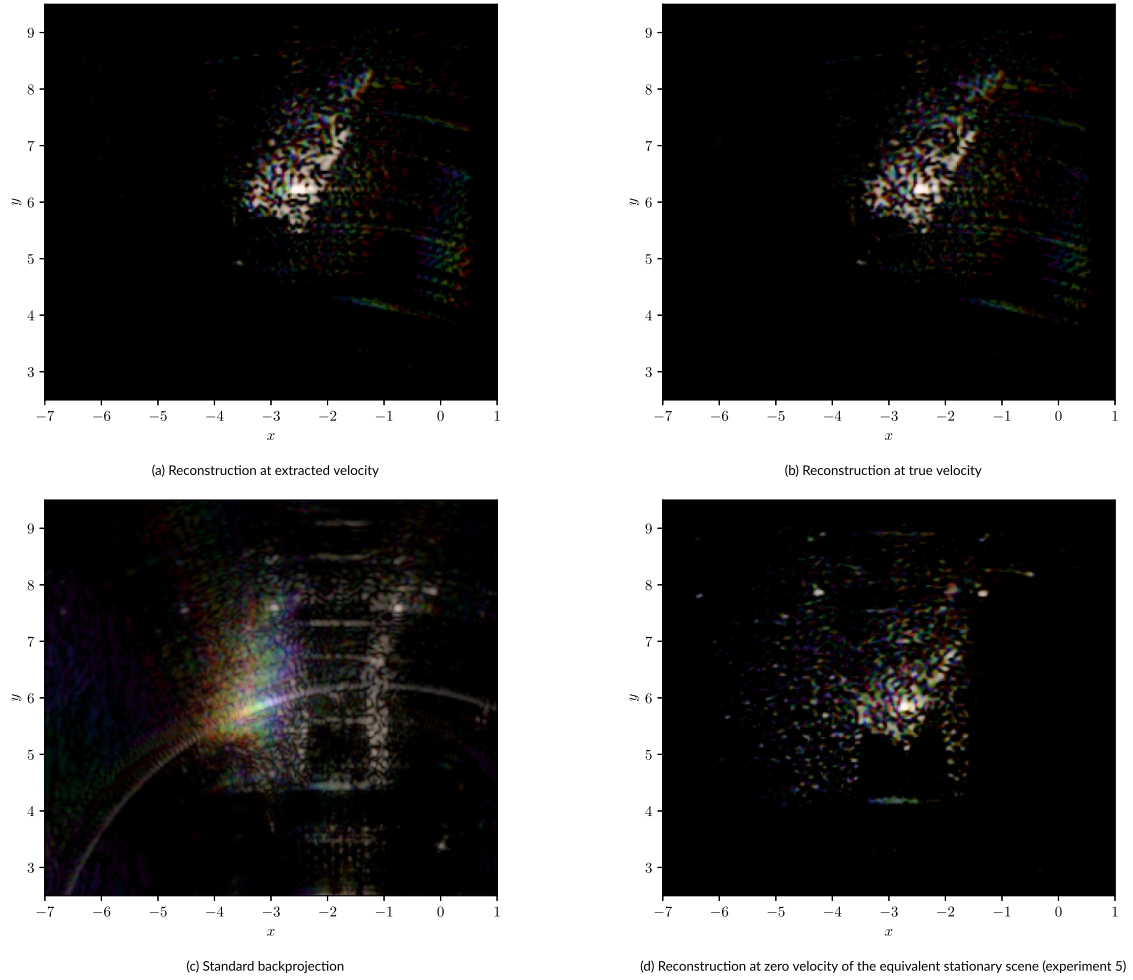


FIGURE 14 | Results from Experiment 4, 0.375 m motion at 25° to boresight, in dB scale relative to maximum value. (a and b) are reconstructed at the extracted velocity and at the (known) correct velocity, respectively. (c) Is the standard backprojection formed at zero velocity, and (d) is the reconstruction of the equivalent stationary scene (Experiment 5).

6.4 | Resolution and Imaging Trade-Offs

The different receiver channels naturally have different cross-range resolution owing to the varying aperture lengths. These resolutions are summarised in Table 5, calculated both for the stationary scene as well as the equivalent ISAR resolution for the moving targets. Unambiguous scene sizes also differ for each of the receiver channels due to the different distances travelled between pulses. These ambiguities are summarised in Table 6. Note that as resolution improves ambiguous ranges worsen—and vice versa—whether with target motion or receiver channel.

For experiments 7 and 9, the substantial cross-range motion reduced the resolution equivalent ISAR cross-range resolution of the target to approximately 0.5 m in the scene centre for the slowest-moving bistatic receivers. In the near-field laboratory environment this can be substantially worse still closer to the aperture, resulting also in significant sidelobes. This is particularly a problem for reconstruction methods such as (26): we are not only backprojecting data, but also attempting to fit it. When resolution and sidelobes become too large the result is trying to fit data due to objects potentially outside of the scene. It is also

possible for very large sidelobes to extend into the region close to the radar antennas in this near-field environment, a region which is not well sampled in cross-range. Because of this, for experiments 7 and 9 we use only the three fastest-moving receiver channels, which have a resolution amenable to performing a reconstruction.

We should ask if—or to what extent—this is a compromise of our proposed methodology. Had we arranged for the target to move in the opposite direction to the aperture then the equivalent ISAR resolution would have increased. However, the fastest-moving receivers may then have encountered cycle skipping. Whilst we could have increased sampling without practical limitation in the laboratory setting, where there is no sense of slow-time, in practical systems there are real system limitations for example due to time of flight of pulses and the aperture flight speed. In general, one could not expect to design a standard SAR mode which is equally well able to image stationary and arbitrarily moving targets. There is always some compromise and range of target velocities for which a particular aperture and sampling is capable of producing unambiguous images of adequate resolution.

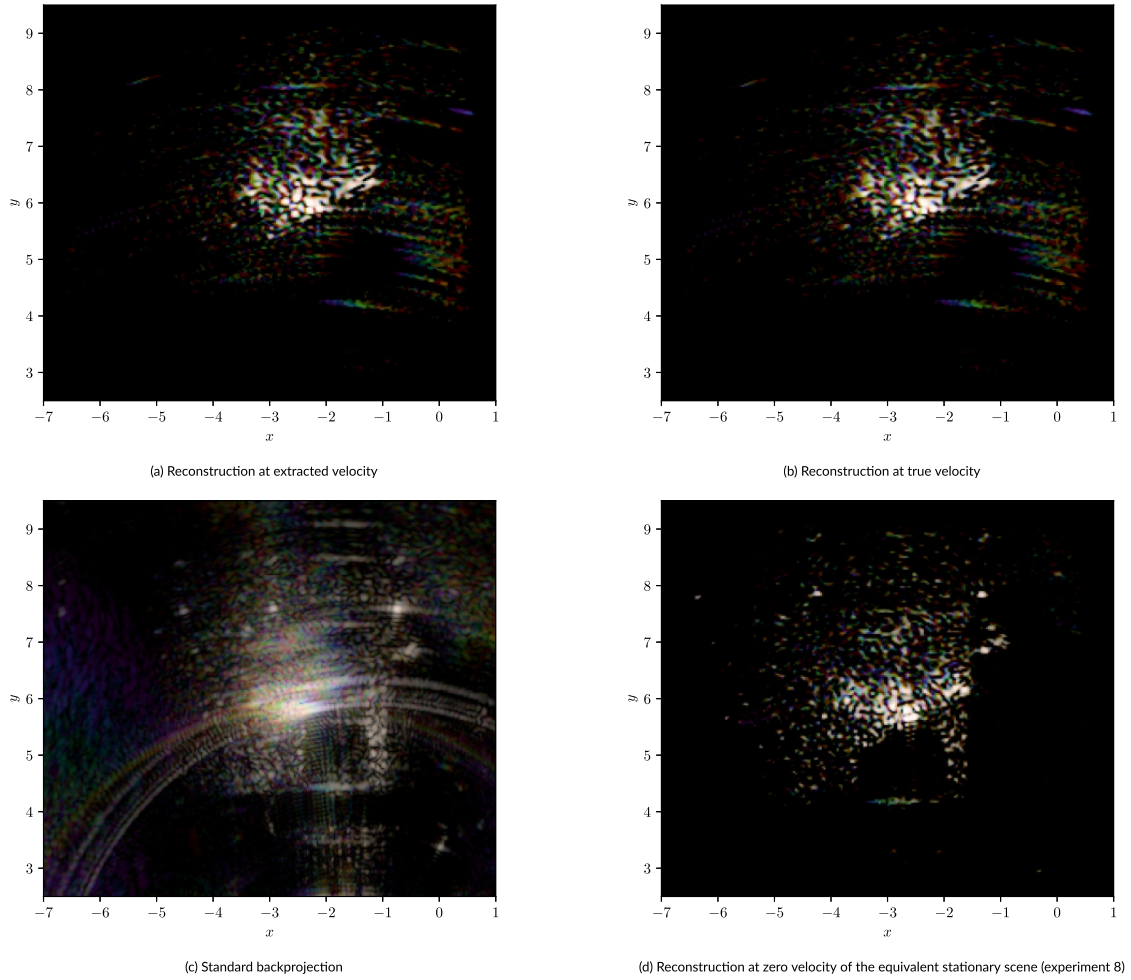


FIGURE 15 | Results from Experiment 6, 0.375 m motion at 63° to boresight, in dB scale relative to maximum value. (a and b) are reconstructed at the extracted velocity and at the (known) correct velocity, respectively. (c) Is the standard backprojection formed at zero velocity, and (d) is the reconstruction of the equivalent stationary scene (Experiment 8).

Because of the distribution of receiver velocities, our methodology provides a broader range of velocities which can be successfully imaged. The faster moving receivers provide increased resolution for targets moving in the same direction, and the slower moving receivers increase ambiguity extent for targets moving in the opposite direction. In a practical implementation, one might benefit from adaptively using a subset of channels appropriate for a given target velocity. We therefore consider this an overall benefit of our methodology, reducing the compromise inherent in radar system design.

6.5 | Summary and Discussion

Across all experiments, the proposed multistatic processing pipeline demonstrated consistent and accurate performance in isolating moving target signatures, estimating their velocities, and reconstructing focused imagery. The incoherent feature extraction reliably separated dynamic components from background clutter sufficiently well for use in our differential semblance criterion, with this criterion yielding velocity estimates within a few centimetres of ground truth. These estimates enabled high-fidelity reconstructions that closely visually

matched the signature in the equivalent stationary scenes, even in the presence of anisotropy and resolution variation.

The overall reconstruction performance was best for experiments which had a greater velocity component in the range direction and smaller component in the cross-range direction. This is natural for two reasons. Firstly, a range-rate component results in different target displacements in each channel image, providing the DSO criterion a better comparison than a different level of defocusing caused by cross-range motion alone. This is supported further by the shape of the cost function valleys previously observed in Figure 11. Secondly, cross-range motion in the same direction as the aperture also reduces target resolution.

The results confirm that the method is robust to moderate motion-induced defocusing and clutter interference, and that it effectively leverages multistatic diversity to enhance moving target detection and imaging.

7 | Conclusions

This work introduces a novel multistatic SAR processing framework that robustly detects and images moving targets,

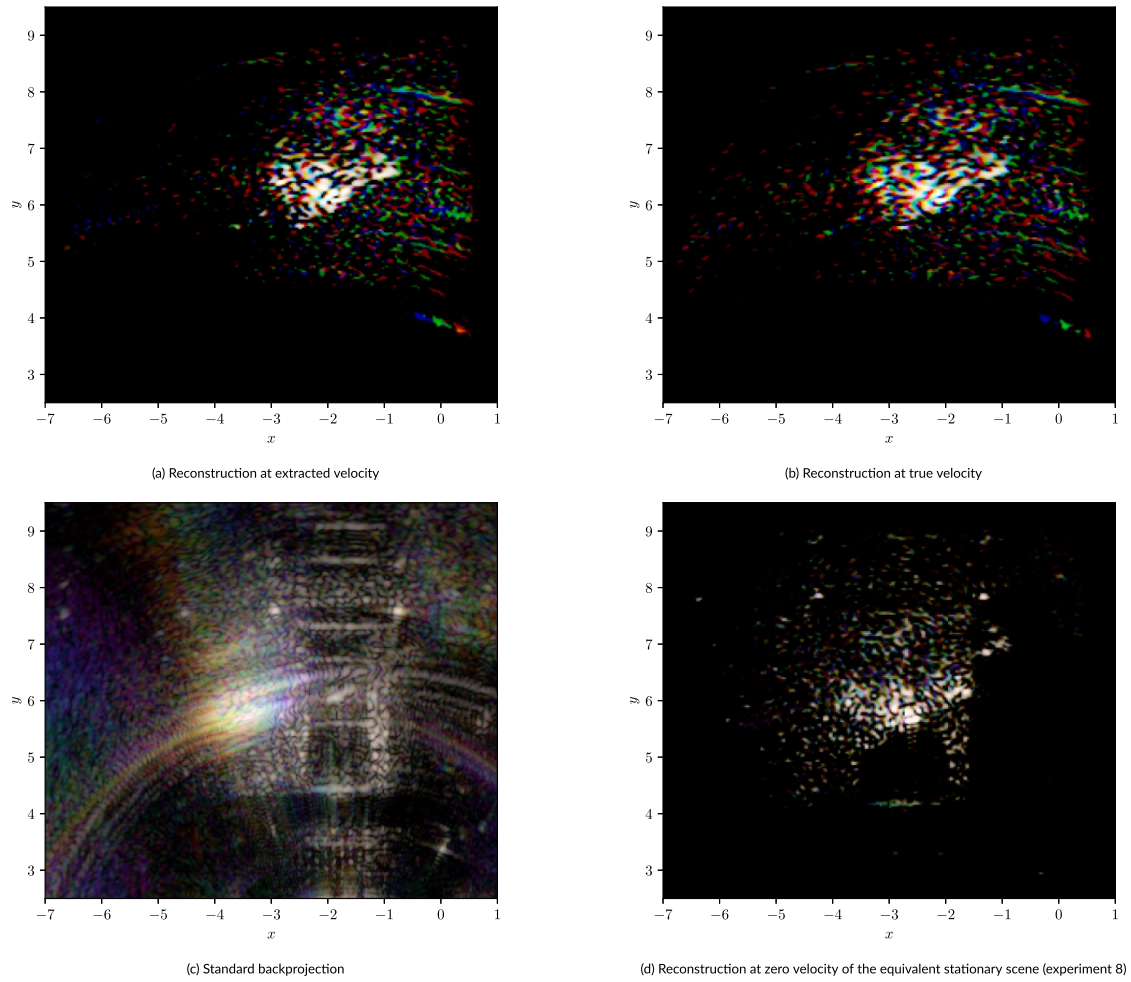


FIGURE 16 | Results from Experiment 7, 0.75 m motion at 63° to boresight, in dB scale relative to maximum value. (a and b) are reconstructed at the extracted velocity and at the (known) correct velocity, respectively. (c) Is the standard backprojection formed at zero velocity, and (d) is the reconstruction of the equivalent stationary scene (Experiment 8).

even in cluttered environments. It exploits a collection geometry which provides a region of overlapping k -space support that has been traversed at different rates by each receive channel in order to detect moving targets, extract approximate signatures (referred to as “incoherent target features”), and determine their velocities. Such a configuration could readily be realised with UAVs travelling at different speeds, or with same-speed platforms at different ranges. Estimated velocity is then used to form ISAR reconstructions enforcing sparsity and small changes between each channel. This has the effect of suppressing stationary background and enhancing target response, coherently combining data across channels where possible. While only the overlapping subset of data is used to determine motion, the full multistatic dataset is used to form final reconstructions. The full resolution of the collection is therefore utilised.

The method has been shown to be effective using experimental data collected in the GBSAR laboratory. With datasets covering 6 different target velocities across 4 different orientations, we found our method was able to extract moving target signatures, and that highly accurate velocity could be determined (to a total displacement less than a resolution cell). This is in spite of the more challenging near-field data

collection: our methodology makes use of far-field approximations. Results would be improved further by accounting for near-field effects, such as through use of spatially-varying k -space cropping.

We have shown that the differential semblance criteria used to extract velocity is robust and amenable to appropriate optimisation, with a distinct global minimum, through an exhaustive study. The scheme was more accurate in the range direction, since each channel observes a different target displacement in this case. Refinement of optimisation procedures was not considered further, since Bayesian optimisation has already been shown to be suitable elsewhere.

With accurate velocities extracted, we have shown the image formation method is well able to suppress stationary background clutter and produce well-focused target signatures. For each experiment, there was minimal difference between the images formed at estimated and true velocities, with target signatures also closely matching the image of the stationary target at the same orientation. The stationary target signatures were of course partially obscured by background clutter, and in some of our experiments we found the moving target

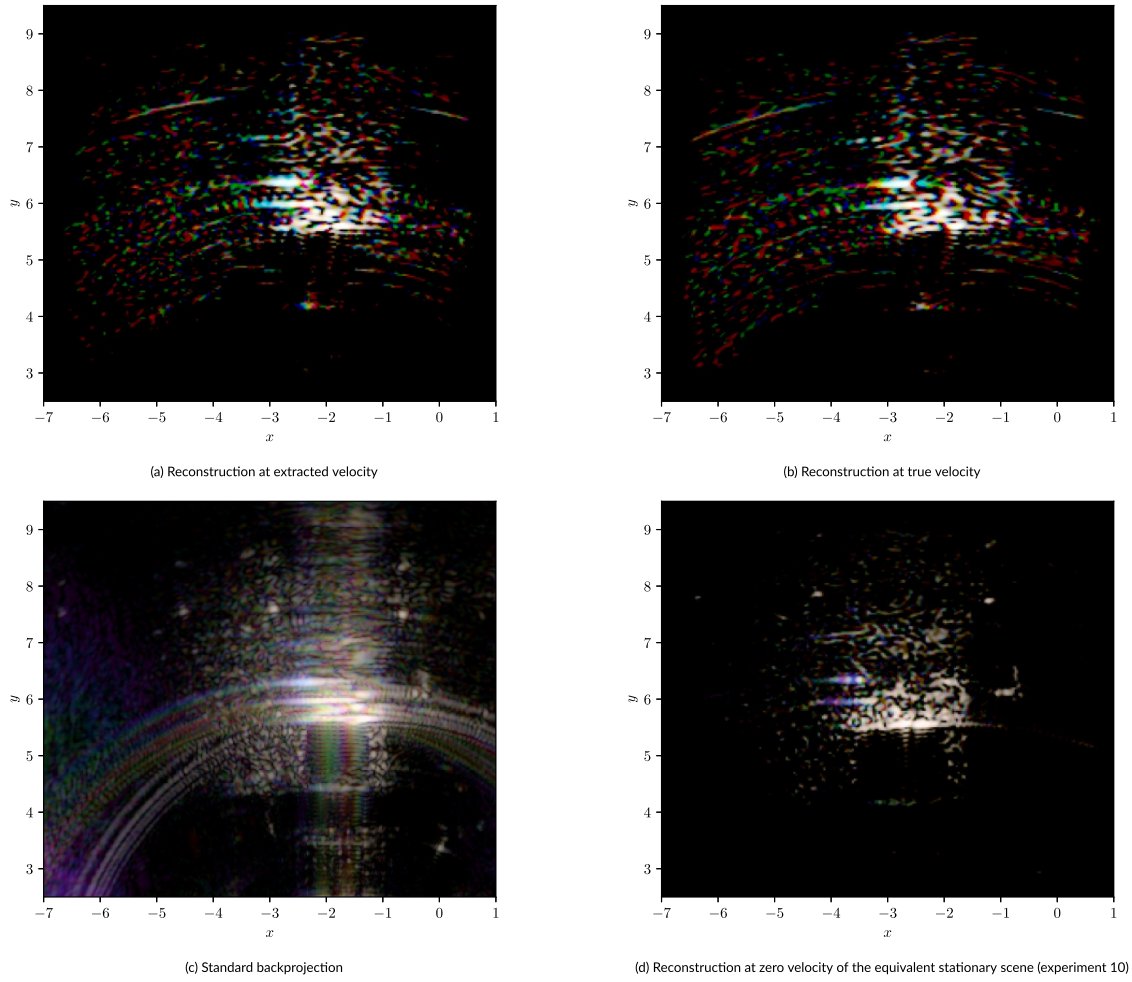


FIGURE 17 | Results from Experiment 9, 0.75 m motion at 90° to boresight, in dB scale relative to maximum value. (a and b) are reconstructed at the extracted velocity and at the (known) correct velocity, respectively. (c) Is the standard backprojection formed at zero velocity, and (d) is the reconstruction of the equivalent stationary scene (Experiment 10).

TABLE 6 | Unambiguous scene sizes of experiments evaluated at the reference point $[-3, 6, 0]$ in units m.

Bistatic pair number	Unambiguous range (m)					Equivalent unambiguous range at full scale (m)				
	1	2	3	4	5	1	2	3	4	5
Unambiguous range in the ground plane (slant plane)	79.1 (75.0)					316.6 (300.2)				
Equivalent ISAR unambiguous cross-range at given velocity (m)	[0,0,0]					8.9	7.8	6.9	6.3	5.7
	[0, -0.25, 0]					35.7	31.2	27.8	25.0	22.7
	[0, -0.375, 0]					8.9	7.7	6.8	6.1	5.5
	[-0.16, -0.34, 0]					35.6	30.8	27.1	24.3	22.0
	[-0.33, -0.17, 0]					8.9	7.7	6.7	6.0	5.4
	[-0.67, -0.34, 0]					44.4	37.1	31.8	27.8	24.7
	[-0.75, 0, 0]					11.1	9.3	7.6	7.0	6.2
						57.5	46.5	39.0	33.5	29.3
						43.1	25.1	17.6	13.5	10.9
						172.3	100.3	70.3	53.9	43.6
						42.3	26.1	18.7	14.6	11.9
						169.2	104.2	74.9	58.2	47.5

reconstructions were able to resolve parts of the target hidden amongst the clutter—such as the response from the barrel. This suggests our methodology will help to detect and recognise moving targets obscured by clutter. In contrast, standard SAR imagery produced with the same data results in defocused targets which are in cases partially obfuscated by background clutter.

We have noted the sampling flexibility our method provides for imaging moving targets: a range of receiver motions provide a range of effective k -space sampling at different target velocities. This means a wider range of potential target velocities will be properly sampled by a subset of channels. The methodology could also be enhanced in several ways for practical realisations and further flexibility.

Firstly, we should only require a smaller region of k -space coverage of each channel to be overlapping, which provides more flexibility in multistatic configurations. Greater total k -space coverage could therefore be enabled for finer resolution. This offers a pathway to design and deployment of UAV-based SAR systems which can flexibly trade off between resolution, coverage, and motion sensitivity. Secondly, having receivers also travel along (slightly) different trajectories may improve the ability to resolve velocity in the cross-range direction, since it will always have a range component in some of the channels. This is more challenging to achieve within the constraints of the two-axis repositioners of the GBSAR laboratory.

We note that it should be straightforward to extend our methodology to include more complex motion such as acceleration or rotation. These would appear as additional parameters in the differential semblance optimisation. Since such motion would result in differently stretched k -space coverage of each channel we expect the DSO approach to still be effective, but this remains to be tested.

Whilst this laboratory-based experiment has demonstrated the technique in a controlled environment, we hope that this demonstration will help encourage the development of multistatic SAR system technology to such a point that this can be tested in more realistic conditions. This includes integration of our technique as part of a wider processing chain, for example with (relative) motion compensation of individual platforms.

Author Contributions

Francis Watson: conceptualization, formal analysis, funding acquisition, investigation, methodology, software, writing – original draft. **Joshua Hellier:** investigation, software, writing – review and editing. **Daniel Andre:** conceptualization, data curation, funding acquisition, investigation, resources, writing – review and editing.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Watson, Francis; Hellier, Joshua; Andre, Daniel

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