

CRANFIELD UNIVERSITY

JAMES P NORMAN

**A LASER-BASED MULTILATERATION SYSTEM FOR
MEASUREMENT OF METRE-SCALE LOW-SLOPE
FREEFORM NON-SPECULAR SURFACES**

SCHOOL OF AEROSPACE, TRANSPORT, AND
MANUFACTURING

PhD

Academic Year: 2018 - 2019

Supervisors: Dr C. Giusca & Dr X. Tonnellier

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Abstract

The manufacture of metre-scale mirror segments for telescopes requires measurement with low uncertainty to enable accurate form correction. The uncertainty of this measurement has a direct effect upon the time to manufacture these segments; it is therefore critical, to the viability of large optic manufacture, that a method for low uncertainty measurement of these surfaces is developed and evaluated. The state-of-the-art system for measurement of metre-scale surfaces has a length measurement uncertainty of $1.2\text{ }\mu\text{m}$ over 1 m . Multilateration is a method for determining Cartesian coordinates of measured positions utilising range displacement measuring stations. A four laser tracker multilateration system has been proposed and tested with the aim to determine whether a system can measure the specified surfaces with measurement uncertainties below $1\text{ }\mu\text{m}$. Influence factors that affect the multilateration input parameters have been identified and utilised in Monte Carlo simulation of the multilateration system to estimate the uncertainty associated with the coordinate measurement. A small flat optic ($0.2\text{ m} \times 0.2\text{ m}$) was measured with the multilateration measurement setup to have z-coordinates with a standard deviation, $\sigma_z = 0.25\text{ }\mu\text{m}$, and a large flat optic ($0.4\text{ m} \times 0.4\text{ m}$) was measured with the multilateration measurement setup to have z-coordinates with a standard deviation, $\sigma_{\Delta z} = 0.73\text{ }\mu\text{m}$. The 10×10 point multilateration measurement of the large optic was repeated 10 times; the 100 points on the surface have a square root mean variance, $\sigma_z = 0.46\text{ }\mu\text{m}$. Monte Carlo simulations indicate the independence of measurement area and measurement noise for the experimental setup tested. It is concluded that a laser-based multilateration system can measure a metre-scale optic with z-coordinate measurement uncertainty below $1\text{ }\mu\text{m}$. A 15.5 h measurement of the larger flat was carried out to determine the effect of time dependent parameters on measurement uncertainty. The measurement solution had a standard deviation of $1.88\text{ }\mu\text{m}$: a factor of approximately 2.4 times the equivalent short-term multilateration measurement solution. Simulations have shown how this measurement drift is strongly related to temperature change in the local environment of the measurement setup. This has been confirmed by long-term measurements of laser tracker stationary SMR.

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Abbreviations

ADM	Absolute distance meters
CMM	Coordinate measurement machine
CGH	Computer-generated hologram
E-ELT	European Extremely Large Telescope
FSI	Frequency scanning interferometry
GPS	Global Positioning System
GUM	Guide to the expression of uncertainty in measurement
IFM	Interferometry
JWST	James Webb Space Telescope
MSF	Mid-spatial frequency
MC	Monte Carlo
MCA	Monte Carlo analysis
MCM	Monte Carlo method
MPE	Maximum permissible error
MRR	Material removal rate
NIF	National Ignition Facility
NIST	National Institute of Standards and Technology
NPL	National Physical Laboratory
PTB	Physikalisch-Technische Bundesanstalt
pdf	probability density function
PSI	Phase-shifting interferometry
SAP	Swing arm profilometer
SMR	Spherically Mounted Reflector
SCE	Super CatEye Reflector

TMT	Thirty Meter Telescope
RI	Refractive index
RMS	Root-mean-squared
RoC	Radius of Curvature
RRR	Red-Ring Reflector

Nomenclature

A	Positive constant (manufacturer)	μm
b	parameter for SMR sphericity error	m
B	Positive constant (manufacturer)	μm
c	speed of light	ms^{-1}
d	distance	m
$d_{i,j}$	measured range displacement	m
$d_{0,j}$	interferometric deadpath	m
D	measured distance/displacement	m
$E_{\text{L,MPE}}$	Maximum permissible error	μm
f	fringe order	—
$F(x)$	function dependent on x	
h_S	laser tracker height	m
i	measured position	—
I	spatial light intensity	—
j	measuring station position	—
$\mathbf{J}$	Jacobian matrix	
k	Coverage factor	—
K	Positive constant (manufacturer)	—
L	Length measurement	m

m	total number of measuring station positions	—
n	Refractive index	—
n	total number of measured positions	—
n_{samples}	number of samples for Monte Carlo simulation	—
r	radius	m
$R_{i,j}$	range measured from station j to position i	m
u_0	constant first term of range uncertainty	m
u_{dp}	deadpath/ADM measurement uncertainty	m
u_L	Length measurement uncertainty	m
$w_{i,j}$	residual of equation solution	m
x_i	x-axis coordinate of a measured position i	m
$\vec{X}$	coordinate vector	
y_i	y-axis coordinate of a measured position i	m
z_i	z-axis coordinate of a measured position i	m
x_p	x-axis coordinate of a point p	m
y_p	y-axis coordinate of a point p	m
z_p	z-axis coordinate of a point p	m

α_{xx}	coefficient of thermal expansion of material xx	—
Δh	height change	m
ΔL	length change	m
ΔR	measured range change	m
ΔT	temperature change	K
Δz	difference in z	m
θ	horizontal angle	
λ	Wavelength	m
σ_z	a standard deviation of z-coordinates	m
σ_z	square root mean variance of z-coordinates	m
$\sigma_{\Delta z}$	a standard deviation of the difference in z-coordinates	m
ν	light frequency	s ⁻¹
ϕ	vertical angle	
φ	light wavefront phase	—

Glossary of terms

This glossary follows the definitions as detailed within: International Vocabulary of Metrology – Basic and General Concepts and Associated Terms (VIM 3rd edition) JCGM 200:2012 [1].

measurand A quantity intended to be measured.

measurement method A method of measurement, generic description of a logical organization of operations used in a measurement.

measurement procedure Detailed description of a measurement according to one or more measurement principles and to a given measurement method, based on a measurement model and including any calculation to obtain a measurement result.

measurement result A result of measurement. A set of quantity values being attributed to a measurand together with any other available relevant information. A measurement result is generally expressed as a single measured quantity value and a measurement uncertainty. In the previous edition of the VIM, measurement result was defined as a value attributed to a measurand and explained to mean an indication, or an uncorrected result, or a corrected result, according to the context.

measured quantity value A value of a measured quantity measured value. A quantity value representing a measurement result.

true quantity value A true value of a quantity. A true value. A quantity value consistent with the definition of a quantity.

measurement accuracy Accuracy of measurement. Accuracy. Closeness of agreement between a measured quantity value and a true quantity value of a measurand.

measurement precision Precision. Closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions.

measurement error Error of measurement. Error measured quantity value minus a reference quantity value. The concept of ‘measurement error’ can be used both: when there is a single reference quantity value to refer to — e.g. from a calibration with a measurement standard that has negligible measurement uncertainty — in which case the measurement error is known; and if a measurand is supposed to be represented by a unique true quantity value — or a set of true quantity values — of negligible range, in which case the measurement error is not known. Measurement error should not be confused with production error or mistake.

systematic measurement error Systematic error of measurement. Systematic error. The component of measurement error that in replicate measurements remains constant or varies in a predictable manner.

random measurement error Random error of measurement. Random error. The component of measurement error that in replicate measurements varies in an unpredictable manner.

repeatability condition of measurement Repeatability condition. Condition of measurement, out of a set of conditions that includes the same measurement procedure, same operators, same measuring system, same operating conditions and same location, and replicate measurements on the same or similar objects over a short period of time

measurement repeatability Repeatability. Measurement precision under a set of repeatability conditions of measurement.

reproducibility condition of measurement Reproducibility condition. Condition of measurement, out of a set of conditions that includes different locations, operators, measuring systems, and replicate measurements on the same or similar objects.

measurement reproducibility Reproducibility. Measurement precision under reproducibility conditions of measurement.

measurement uncertainty Uncertainty of measurement. Uncertainty. A non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used.

Type A evaluation of measurement uncertainty Type A evaluation. An evaluation of a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions.

Type B evaluation of measurement uncertainty Type B evaluation. An evaluation of a component of measurement uncertainty determined by means other than a Type A evaluation of measurement uncertainty.

standard measurement uncertainty Standard uncertainty of measurement. Standard uncertainty. Measurement uncertainty expressed as a standard deviation.

expanded measurement uncertainty Expanded uncertainty. A product of a combined standard measurement uncertainty and a factor larger than the number one.

coverage factor A number larger than one by which a combined standard measurement uncertainty is multiplied to obtain an expanded measurement uncertainty.

calibration An operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication.

metrological traceability Property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty.

verification Provision of objective evidence that a given item fulfils specified requirements, such as manufacture specifications.

measurement model A model of measurement model. Mathematical relation among all quantities known to be involved in a measurement.

input quantity in a measurement model Input quantity quantity that must be measured, or a quantity, the value of which can be otherwise obtained, in order to calculate a measured quantity value of a measurand.

measuring system A set of one or more measuring instruments and often other devices, including any reagent and supply, assembled and adapted to give information used to generate measured quantity values within specified intervals for quantities of specified kinds.

resolution The smallest change in a quantity being measured that causes a perceptible change in the corresponding indication.

stability of a measuring instrument Stability. The property of a measuring instrument, whereby its metrological properties remain constant in time.

instrumental drift A continuous or incremental change over time in indication, due to changes in metrological properties of a measuring instrument.

maximum permissible measurement error Maximum permissible error. Limit of error. The extreme value of measurement error, with respect to a known reference quantity value, permitted by specifications or regulations for a given measurement, measuring instrument, or measuring system.

Chapter 1

Introduction

Currently, there is not a standard method for measuring a *metre-scale non-specular freeform* surface in the ‘ultra-precise’ measurement regime; specifically, surface features in spatial frequency bandwidths lower than surface roughness. It is critical for manufacturing metrology that measurement systems are produced with the required specifications for ultra-precise measurement of these surfaces.

Of the state-of-the-art metre-scale non-specular surface measurement systems, there is not one method that fulfils the requirements of: high measurement accuracy and resolution, high measurement density, high speed, large measurement volume, and low cost. A system design with these characteristics is essential for the low uncertainty measurement and, subsequently, the production of precise components such as metre-scale optics for the next generation of extremely large telescopes, as well as similarly sized optics for large laser systems.

High measurement accuracy and resolution is a prerequisite for a system employed in the production of metre-scale optics. The precise prescription of optical designs, for telescopes and laser systems, requires low uncertainty measurement of optical surfaces throughout the manufacturing process; with specific challenges early in the manufacturing process, where these surfaces remain non-specular from ‘rough’ machining.

High measurement density is required to measure feature on surfaces: specifically in relatively high spatial frequency bandwidths. If measurement aliasing is to be averted, these small features must be measured multiple times during a single measurement program, which requires a high measurement density relative to the size of features to be measured on a surface.

Measurement speed has implications for both the uncertainty of a measurement and the speed of the manufacturing process. Measurements conducted at high speed reduce the measurement uncertainty due to thermal expansion (drift error) within the machine setup and thermal expansion of sample surfaces. A high measurement speed can also

reduce the total time to manufacture a surface. This is critical to large projects, such as extremely large telescopes and laser systems, that have many optical components to be manufactured.

Speed of measurement also has wider implications for industrial applications. If a manufacturer can reduce the total manufacturing time of a precision surface, through a reduction in surface measurement time, assuming the product is in ‘high demand’, the manufactures potential to generate revenue is increased.

A large measurement volume is essential, given the metre-scale size of components to be manufactured. Any measurement system utilised in the manufacture of these surfaces requires the capability to measure the entirety of a surface without the need to reposition the measurement machine or sample surface. Repositioning during a measurement induces a source of measurement uncertainty: measurement data would be required to be ‘stitched’ together.

Metrologists must also address the cost of these systems, if there is to be industrial relevance. Companies (or governmental organisations) may require several of the same measurement system, operated concurrently, in order to meet production demands; therefore, the measurement system must not be disproportionately expensive to build, maintain, or operate.

1.1 Metre-scale non-specular surfaces

An important example of metre-scale non-specular surfaces, with complex topologies, that require measurement with low measurement uncertainty are optics for large science projects: such as ground-based and space telescopes for astronomy [2–6]; as well as high power laser systems for fundamental physics research [7–12]. At several stages during manufacture, such as grinding, these optics are required to be measured; despite the non-specular nature of surfaces during early stages of manufacture.

The next generation of telescope designs require large diameter mirrors for light collection and focusing; hence, the primary mirrors of these optical systems are constructed with a large number of mirror segments.

1.1.1 Definition of a non-specular surface

As stated, this project endeavours to produce a system enabling the measurement of non-specular surfaces. To that end, it is critical to understand what defines a non-specular surface.

As defined by Leach *et al.* [13], “a surface is optically smooth if the surface height variation within the diffraction limited lateral resolution of the observation is smaller

than $\lambda/4$ ", where λ is the wavelength of light used to illuminate the surface. In this case, optically smooth is the opposite of non-specular and as such a surface is non-specular if the surface height variation is larger than $\lambda/4$. $\lambda/4$ corresponds to a phase change of 90° . A surface which causes a phase change greater than this will produce wholly destructive interference of the light to be observed; as such, information about this surface is lost.

The specularity of a surface is also dependent upon the observation aperture. Observing a ground surface with a very high numerical aperture can result in a speckled surface; however, this type of specular reflection occurs in spatial frequencies much higher than those considered significant to this project (surface roughness), and are therefore classified as non-specular.

1.1.2 Ground-based segmented telescopes

For ground-based telescopes to detect the furthest astronomical phenomenon, these instruments must have the ability to capture light over vast areas. This is due to the relatively low light intensity of these phenomena at the point of the telescope on the surface of Earth. As light traverses space, its intensity decreases proportional to $1/d^2$, where d is the distance travelled from the source of the light. For light to reach the surface of Earth, its intensity is further reduced through scattering in the atmosphere. As a result, alongside requirements for telescopes to operate at high altitudes with low atmospheric humidity; the larger the diameter of the telescope primary mirror, the greater the telescopes ability to collect light for scientific analysis.

Two examples of telescope designs with extremely large primary mirror diameters are the European Extremely Large Telescope (E-ELT) [2], and the Thirty Metre Telescope (TMT) [3]. E-ELT requires 931 off-axis ellipsoidal segments to be manufactured for 6 sections: each with differing prescriptions of 133 segments. A further 133 segments should be produced for reconditioning purposes. These segments form ~ 39 m diameter primary mirror (M1) for the E-ELT. The mirror design is shown in figure 1.1. TMT requires 492 segments based on a ten section design employed by each of the Keck telescopes.

1.1.3 Segmented space telescopes

A solution to light scattering in the atmosphere is to position a telescope in orbit around Earth. This has the added advantage of eliminating imaging error due to other atmospheric seeing effects, such as lensing or scintillation.

The James Webb Space Telescope (JWST), a NASA-ESA project, will be launched to the Sun-Earth gravitationally neutral Lagrange point L2 [14]. The primary mirror will be constructed from 18 lightweighted 1.315 m hexagon beryllium substrate mirror

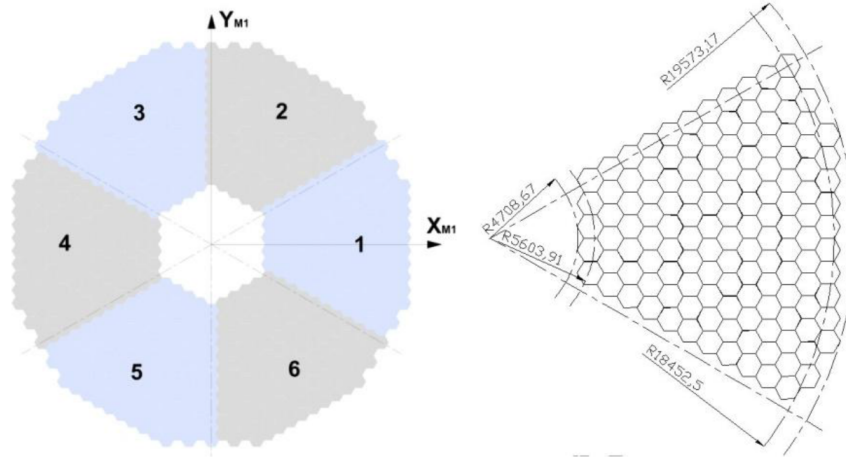


Figure 1.1: Primary mirror segmentation pattern [2]. The segments are grouped into six sectors of 133. Of the 133 segments, each has a unique geometry.

segments [15, 16]. Gaia is an ESA star mapping project. The optical system is designed with lightweighted silicon carbide (SiC) substrate mirror [4].

Building a large telescope for space deployment results in significant design constraints; not least the mass limit for launch into orbit. The mirror substrate for the JWST and Gaia are designed with a lightweighting rib structure to minimise the mass to stiffness ratio. A mirror segment with a honeycomb structure is shown in figure 1.2.

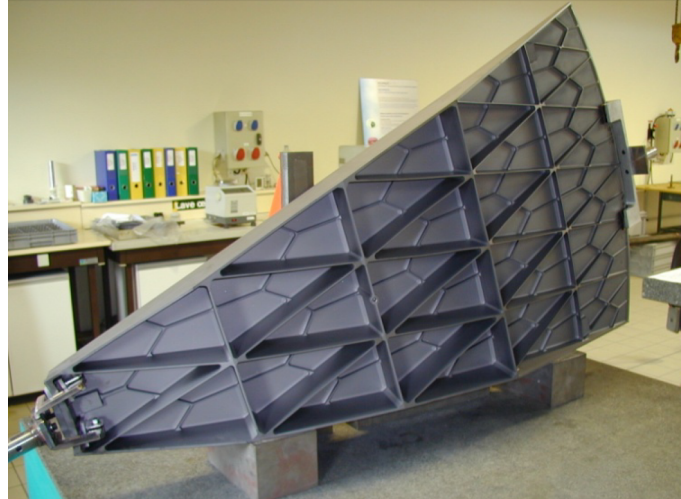


Figure 1.2: The lightweight structure of a SiC optic [4]. The ribs are honeycombed to minimise mass to stiffness ratio.

A consequence of manufacturing an optic with a lightweighting structure is the form error induced by print-through during surface machining, where the machining process requires a force applied to the surface for material removal. This is not the case for chemical machining processes [17]. Quilting is a mid-spatial frequency (MSF)

error caused by the stiffness differential of the optical substrate [18]. This differential is dependent on the position on the optical surface, i.e. the optical surface is more rigid where a rib is directly attached behind it, and less rigid at positions with no direct support from a rib structure.

1.1.4 Laser systems

Extremely large laser systems are required for both fundamental physics research, and the construction of nuclear fusion facilities. These laser systems require vast amounts of precisely manufactured optics. The National Ignition Facility (NIF) uses ~ 7500 metre-sized fused-silica wedge optics to direct a laser beam on to millimetre-scale fusion targets [7, 8]. Fused silica optics are also employed in the laser optical design for the HiPER Laser Fusion Facility [9].

The Orion laser facility consists of 12 laser beams, 10 long pulse and two short pulse. These beams require manipulation via optics in a complex system involving mirrors and lenses [10]. Whilst the Laser MEGAjoule has 240 laser beams; each beam requires dozens of optical components, size $0.4 \text{ m} \times 0.4 \text{ m}$ to $0.4 \text{ m} \times 0.7 \text{ m}$ [11, 12].

1.1.5 Manufacturing process chains

As stated previously, metrology is an integral part of any manufacturing process chain for the production of large optical components. There are several proposed manufacturing process chains for the production of precision optics [19–22]. Figure 1.3 illustrates the manufacturing chain detailed in [19].

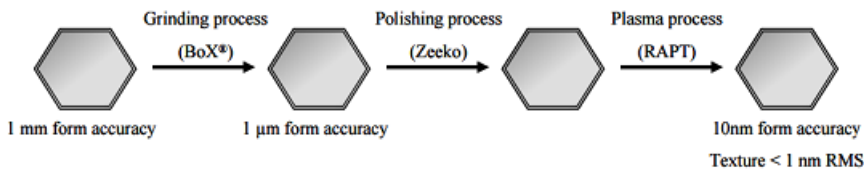


Figure 1.3: A manufacturing process chain proposed for the production of hexagonal segments for M1 of the E-ELT [19].

Figure 1.4 details how a proposed manufacturing process chain is implemented to reduce root-mean-squared (RMS) form error from 5 mm to < 100 nm. It is of note that a significant part of the manufacturing time is spent whilst the surface form error is not being significantly reduced: the neutral polishing step. By removing the requirement for neutral polishing, the overall time to manufacture large optics could be reduced. This could be achieved by reduced uncertainty measurement of surfaces during and after grinding with mechanical contact measurement.

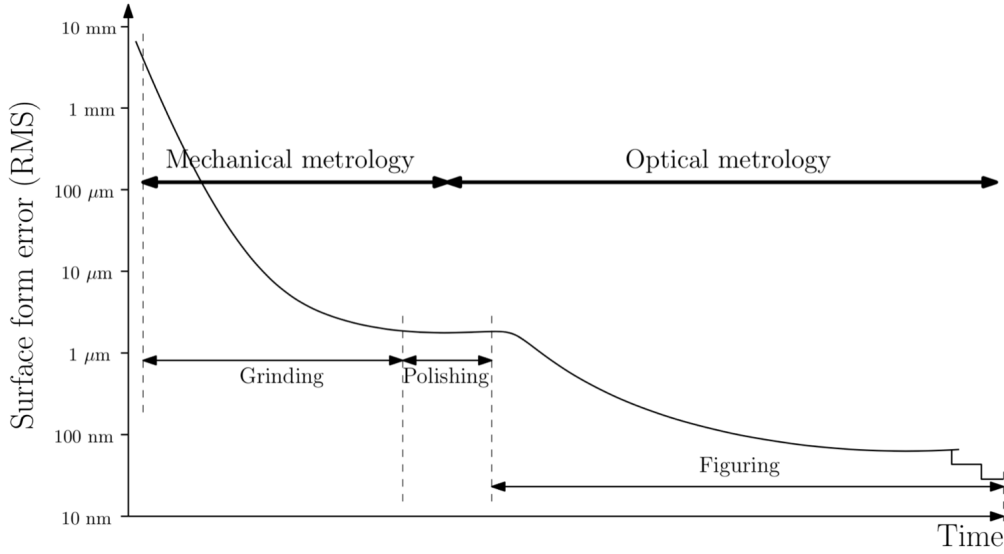


Figure 1.4: A manufacturing process chain proposed for the production of hexagonal segments for M1 of the E-ELT [22].

All propositions for such manufacturing chains must follow the general outline explained by P. C. T. Rees and C. Gray [23]. This explanation has particular importance to metrology within manufacturing chains for the E-ELT, as it details the current standards for measurement at each stage. The process chain described is specific to the requirements of the E-ELT: based on the high number of optical segments; the demanding specifications for RMS form error; time constraints on production; and using mature technologies, for both machining and measurement. The target measurement accuracy for mirrors following all processes prior to polishing (processes that produce non-specular surface finish) is $2\text{ }\mu\text{m}$ [23].

A process common to the manufacturing methods discussed is grinding. Fixed abrasive grinding is a high material removal rate (MRR) process used to machine the required topography into a substrate material. In 1989 a high performance grinding machine, Kodak OAGM 2500, was designed by Cranfield Precision [24]. In a ‘low damage’ mode this machine has a removal rate of $1.6 - 5\text{ mm}^3/\text{s}$, a surface roughness (R_a) of $10 - 50\text{ nm}$, and a form accuracy (P_t) $2 - 5\text{ }\mu\text{m}$ [25].

‘State-of-the-art’ machine-tools such as Cranfield BoX[®] are capable of producing surface form error of $1\text{ }\mu\text{m}$ (P_t) over a metre with MRR of $100 - 200\text{ mm}^3/\text{s}$, R_a of $50 - 150\text{ nm}$, and subsurface damage $< 10\text{ }\mu\text{m}$, material and tool dependent [26].

The surface is produced with a position controlled grinding wheel. The wheel has an abrasive material fixed into the grinding surface. Typically this abrasive is made up of diamond grits embedded in a resin bond. The grinding machine should have high dynamic loop stiffness to maximise MRR but maintain the required surface form accuracy

on the substrate surface [27]. The surface produced by this — high speed process — method results in diffuse (non-specular) reflection of light. As such, polishing is required to produce a specular (reflective) surface.

1.1.6 Current metrology standard

The current expectation for ultra-precise measurement of surfaces, post-grinding, involves a “non-corrective polishing treatment designed to render the optical surface specular and measurable with an optical interferometer at a visible wavelength” [23]. As is clearly shown in figure 1.4, this *neutral* polishing step has no affect upon the surface form error in anything other than high spatial frequency bandwidths. Whilst this process enables the use of a highly accurate measurement method, given that the process has no direct effect on the surface form error, it could be considered as inefficient; the process of neutral polishing and interferometry is expensive in materials, machining time, and measuring time.

1.2 Motivation

The motivation for this project comes from to need to facilitate the production of large optics for scientific research projects in a precise and timely manner. By investigating a measurement setup that can measure the required surfaces early in manufacturing process chains, when the surface is non-specular, with low and traceable measurement uncertainty, both the total manufacturing time and mirror surface form error could be reduced (this will also have positive monetary cost implications).

This PhD project endeavours to determine and implement a method for the measurement metre-scale non-specular freeform surfaces, with uncertainty significantly lower than the current state-of-the-art measurement systems, capable of measuring such surfaces.

1.3 Research methodology

To improve the uncertainty associated with measurement of metre-scale non-specular freeform surfaces, a literature review has been undertaken with the aim of determining the state-of-the-art measurement uncertainty of methods currently capable of measuring this type of surface. Identifying this measurement uncertainty provided a quantity for the criterion that this project aimed to improve upon. The literature reviewed also identified the knowledge gaps that currently limit the surface measurement uncertainty. From the

knowledge gap, the project aim and objectives were constructed. A measurement system was proposed and a series of experiments and simulations were designed and undertaken. Analysis of the results from experiments and simulations enabled the assessment of the measurement system against the project aim.

1.4 Thesis structure

A review of the current literature relevant to this project is presented in Chapter 2. This includes: background theory, systems for the measurement of the defined surfaces, laser tracking systems, and parameters for uncertainty characterisation. Chapter 2 concludes with the detailed the knowledge gap that this PhD project endeavours to fill. Chapter 3 states the hypothesis, aim, and objectives of this research. The method of measurement for the proposed system is detailed in chapter 4, this includes the measurement theory and practical realisation of the setup, as well as an analysis of the measurement setup input error sources. Chapter 5 describes the experimental methods. Chapter 6 details the results of: initial experiments, testing of time independent error sources, short-term measurements of flat samples with the measurement setup, testing of time dependent error sources, long-term measurements of flat samples with the measurement setup, as well as simulations of various real experiments. Chapter 7 is a discussion of experimental and simulated results and chapter 8 states the conclusions of this work and proposes further work.

Appendix A describes surface measurement probes. Appendix B details surface characterisation parameters. Appendix C describes experimentation to test the long-term stability of a non-contact measurement probe. Appendix D described laser tracker error models found in the literature. Appendix E describes standards, definitions, and evaluation techniques relevant to measurement uncertainty. Appendix F is the function for the critical calculation of the proposed measurement system implemented in MATLAB®. Appendix G details the calibration information for equipment for experimentation. Appendix H lists the current publications produced from this research.

Chapter 2

Literature review

Measured positions are defined within a coordinate system. Some coordinate measuring systems measure in Cartesian coordinates directly, whilst others do not. Those that do not required some technique by which the Cartesian coordinates can be calculated, such as multilateration. Laser interferometry enables traceable measurement of displacement. Refractive index change affects the uncertainty of this displacement measurement. This project has concerned itself with the measurement of non-specular surfaces; a surface is non-specular if its surface height variation, on the spatial frequency scale of roughness, is greater than a quarter of the wavelength being used to observe the surface. Coordinate measurement systems that have the capability to measure non-specular surfaces generally work on the principle of contact probe measuring. Some systems utilise metrology frames (low thermal expansion rigid structures) and interferometry to measure the displacement of a measuring probe. These systems are limited by their small volume. It is generally the case that the coordinate measurement uncertainty of a coordinate measuring machine increases with the measurement volume of the machine. Laser trackers are portable coordinate measuring systems. They can measure the displacement of a reflecting target with tracking interferometry. Multilateration can be used to calculate Cartesian coordinates from a network of displacement measurements with multiple (at least four) displacement measuring positions. Systems for the measurement of metre-scale optics include: swing-arm profilometers and optical tower interferometry. These both have significant limitations in terms of measurement system setup.

2.1 Background theory

This section details some of the scientific and engineering theory that form the basis from which the metrology techniques presented in this chapter operate, as well as to help define the types of surface required to be measured.

2.1.1 Coordinate measuring

In order to determine the relative position of two points in three dimensions (3D), it is required that a coordinate system is defined. This is so that the points can be related to each other with a common reference frame. It is typical to define the surface of a sample in terms of Cartesian coordinates: x , y , and z . Other coordinate systems include: cylindrical coordinate systems and spherical coordinate systems. A 3D Cartesian coordinate system must have an origin, $(x_0, y_0, z_0) = (0, 0, 0)$, and defined directions of three orthonormal vectors: $\{\mathbf{e}_1 = (1, 0, 0), \mathbf{e}_2 = (0, 1, 0), \mathbf{e}_3 = (0, 0, 1)\}$, such that any coordinate vector, $(\mathbf{x}, \mathbf{y}, \mathbf{z})$, can be expressed by: $(x, y, z) = x\mathbf{e}_1 + y\mathbf{e}_2 + z\mathbf{e}_3$.

Another aspect to coordinate measuring is establishing a measuring technique. One way to categorise a measuring technique is between those that measure Cartesian coordinates directly, and those that do not — such as interferometry (see section 2.1.2).

If a measurement technique operates in a Cartesian coordinate system, the parameters x , y , and z of the measurement point will be measured directly. If the measurement technique works in cylindrical coordinates — radius, R , angle, θ , and height, z — the coordinates of the measurement point will likely be transformed in to Cartesian coordinates for reporting the measurement; similarly, spherical coordinates — radius, R , horizontal angle, θ , and vertical angle, ϕ — will also be transformed.

Those measuring techniques that do not measure coordinates of a point directly must measure on such a way that enables the calculation of the coordinates of the points measured. An example of a coordinate measuring system of this type utilises range measurements from three unique measuring stations simultaneously measuring the same measurement point, as shown in figure 2.1. Each of the three stations measures the distance to each of the other stations, and the distance to the measured position, p . In this case, the coordinates of the three measurement positions must be known in order to calculate the coordinates of p . As such, the positions of the measuring stations must be verified by some means and their distance measuring technique must be calibrated to the definition of a metre (see section 2.1.2).

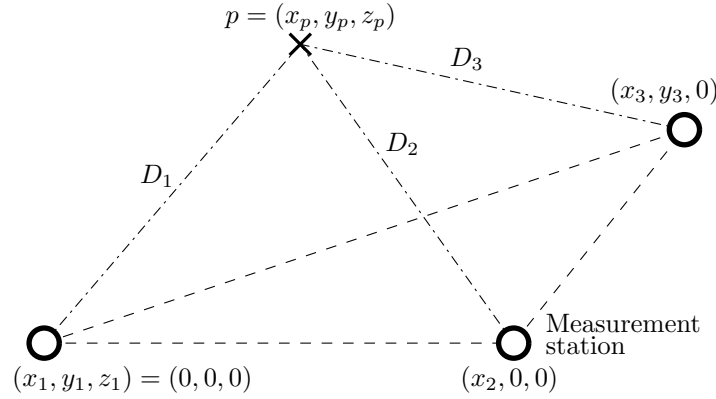


Figure 2.1: The arrangement of a basic range measurement based coordinate measuring system where the positions of the measuring stations within a Cartesian coordinate are known. The measuring stations are measuring the distance between them and the measurement point, p .

From figure 2.1, the measured position p can be calculated by defining a plane through the three measuring stations, the origin of the coordinate system at station one, the x-axis defined by station 2, and solving the equation set

$$\begin{aligned} x_p^2 + y_p^2 + z_p^2 &= D_1^2, \\ (x_p - x_2)^2 + y_p^2 + z_p^2 &= D_2^2, \\ (x_p - x_3)^2 + (y_p - y_3)^2 + z_p^2 &= D_3^2, \end{aligned} \quad (2.1)$$

where x_p , y_p , and z_p are the coordinates of the point being measured; $D_{1,2,3}$ are the measured distances between each station and the measured point; and $x_{2,3}$ and y_3 are the non-zero coordinates of the measuring stations. The equation set shows how this measurement setup has six ‘known’ parameters in comparison to the three unknown coordinates parameters of the measured point. This allows the equation set to be solved and coordinates of the point calculated.

If the positions of the measuring stations were unknown, the equation set in the above form cannot be solved (3 knowns vs 6 unknowns). In this case, one option is to attempt to increase the number of known parameters relative to the number of unknown parameters. Increasing the number of measured points in the setup shown in figure 2.1 will increase the number of measured distance. Therefore, each new measured point increases the number of known parameters by three; however, this also has the effect of increasing the number of unknowns — each measured point has three unknown coordinates. Thus, no matter how many more points are measured, the number of knowns will never exceed the number of unknowns.

If the number of measuring stations is increased from three to four, the number of

knowns per measurement point will increase by four ($D_{1,2,3,4}$) rather than just three in the previous setup with only three measuring stations. If in this setup, one point is measured, there will be an additional three unknowns — the coordinates of the fourth station — relative to the previous setup; however, this setup has the advantage that, for every new point measured, three unknowns are added to the setup, in comparison to the four knowns gained. Therefore, after measuring six points, the number of knowns and unknowns are balanced, and the coordinates of the measured points can be calculated, a so-called self-calibrating measurement system. The first reported application of this technique for coordinate measurement using commercial laser trackers as the measuring stations is described within [28].

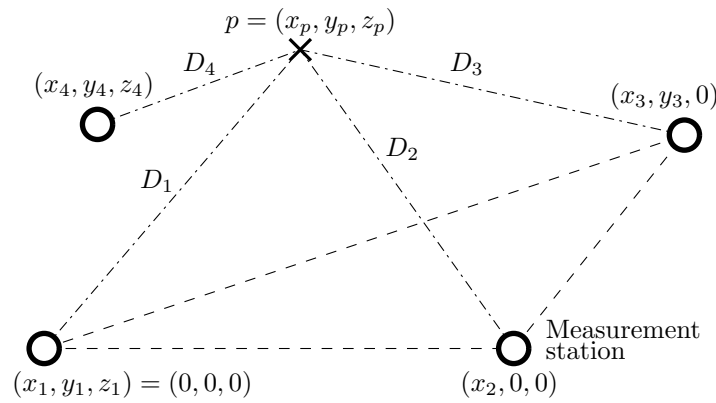


Figure 2.2: The arrangement of a basic range measurement based coordinate measuring system where the positions of the measuring stations within a Cartesian coordinate are unknown. The measuring stations are measuring the distance between them and the measurement point, p .

2.1.2 Laser interferometry

A method for the realisation of the *metre* is to determine the wavelength of an electromagnetic wave in vacuum. The wavelength of a radiation source can be determined from its relationship to the speed of light, c , and frequency, ν : $\lambda = c/\nu$. From this, it is possible to use a technique known as interferometry to provide a *traceable* link between the definition of a metre and a dimensional measurement [29, 30].

Interferometry is based upon the principle of wave superposition. As shown in figure 2.3, a light radiation source is directed at a beam splitter (amplitude splitting). Half the light is reflected onto a reference mirror, reflected back to the beam splitter, where half the light is transmitted onto a light intensity detector. This forms the reference arm of the interferometer. Typically, the reference mirror will remain nominally stationary: this provides a constant phase light source. The measurement arm of the interferometer is formed using the half of the light emitted from the source that is transmitted through

the beam splitter. This light is directed at a measurement mirror and reflected back to the beam splitter. In this way, the measurement and reference parts of the light source superimpose at the detector. At this point, the relationships between light intensity and relative phases, and wavelength and the metre are utilised to measure any displacement of the measurement mirror:

$$I(x, y) \propto \cos(\Delta\varphi) , \quad (2.2)$$

where $I(x, y)$ is spatial light intensity. $\Delta\varphi = \varphi_R - \varphi_M$ where φ_R and φ_M are the phases of the interferometers reference arm and measurement arm, respectively. Intensity is a maximum when the two wavefronts superimpose constructively at $\Delta\varphi = 2n\pi$ where $n = \{0, 1, 2, \dots\}$, and a minimum ($I = 0$) at $\Delta\varphi = (2n + 1)\pi$.

As stated, a displacement of the measurement mirror will cause a change in the measured intensity at the detector. The observed intensity is periodic over a measurement arm path length equal to λ . A displacement of the measurement mirror of one unit length results in a change in path length of 2 unit lengths as the beam is reflected along the path, a so-called ‘double-pass’; therefore, $\Delta\varphi \propto 2D$, where D is the displacement of the measurement mirror. As such, one intensity fringe corresponds to $D = \lambda/2$. By fringe counting at the detector it is possible to equate to measurement mirror displacement:

$$D = \frac{(N + f) \lambda}{2} , \quad (2.3)$$

where N is the fringe order and f is the fringe fraction. With a typical homodyne (single wavelength) displacement interferometry using a HeNe laser radiation source ($\lambda = 632.8 \text{ nm}$) will have a fringe width corresponding to $\lambda/2 = 316 \text{ nm}$. In this way, a method of measuring displacement that is traceable to the definition of a metre, via the ‘known’ wavelength of the HeNe laser, has been realised. It is noteworthy that this method of measurement, homodyne interferometry, has no capability to measure the direction of the displacement of the measurement mirror.

2.1.3 Effect of refractive index and refraction on interferometry

In most practical applications of interferometry, the measurement arm of an interferometer cannot be in a vacuum. For example, measurements required in an industrial environments, such as a manufacturing process chain, would be performed in air. This has a direct impact upon interferometry as the medium the light is passing through changes the light wavelength and speed. These changes are determined by: $\lambda_{\text{med}} = \lambda_{\text{vac}}/n$ and $c_{\text{med}} = c_{\text{vac}}/n$ where n is the refractive index (RI) of the medium (air), λ_{med} and c_{med} and the wavelength and speed of the light in the medium, respectively, and λ_{vac} and c_{vac} are

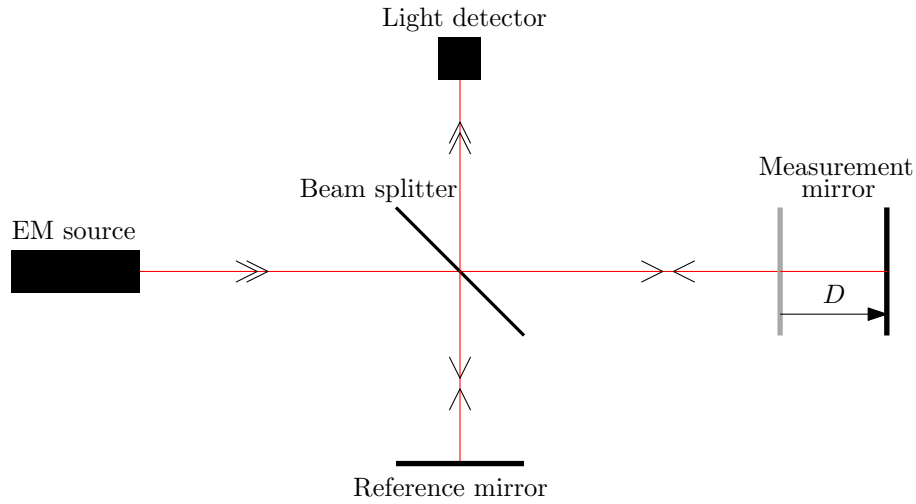


Figure 2.3: The basic design of a Michelson type interferometer for displacement, D , measurement [29]. The detector measures the light intensity as the measurement mirror is displaced. The number of fringes are counted to determine the displacement in terms of light wavelength.

the wavelength and speed of the light in vacuum, respectively.

Refraction of light occurs when the RI of air changes or there is a RI gradient across the light path. With homodyne interferometry, refraction can result in beam bending and therefore increase the length of the light path. The RI of air is dependent on the air pressure, temperature, humidity, and amount of carbon dioxide; therefore, understanding these parameters and quantifying their affect upon a specific interferometric displacement measurement is critical in determining — and compensating — error associated with RI. It is clear that the longer the light path in air the greater the effect of RI: the measurement error has a length dependence [31]. The effect of changes to the RI of air upon the uncertainty of optical length measurements in an industrial environment is discussed by Birch *et al.* within [32].

2.2 Systems for the measurement of non-specular low-slope surfaces

This section provides details of relevant measurement systems with the capability of measuring the form of non-specular surfaces with low-slopes. The systems include small-volume-scale machines that have ‘ultra-low’ measurement uncertainties up to measurement systems with larger volumes. It is the case that larger volume machines generally have large measurement uncertainties than the smaller measurement volume machines detailed below. The designs and measuring techniques of these systems are explained and limitations in measuring the surfaces that this project is aiming to measure

are identified.

2.2.1 Design of coordinate measurement machines

A coordinate measurement machine (CMM) is typically constructed from: a base — where the sample is measured, a measurement probe, and a measurement probe transport system [33]. A critical invention for the detection of contact between a CMM probe stylus tip and a surface contact point is the touch probe [34]: *A probe for determining at what point in space contact is made between an object and a stylus. The stylus is located in an accurately defined rest position relative to a housing and slight deflection of the stylus away from this rest position is detected preferably by electrical switches.*

The highest accuracy CMMs are designed with a bridge to move the probe. This bridge is built on a granite base. The base stands on three or four points, depending upon the need for stability. The bridge design enables any freeform surface to be measured with no change to the configuration or design of the CMM.

The design adheres to the Abbe principle using position encoders on both sides of the bridge that holds the probe. In this way, the Abbe error of each encoder are equal and opposite, and therefore cancel. The thermal stability of these machines is maximised by the choice of large pieces of granite for construction; granite has low thermal diffusivity and the large mass of the design contribute to its stability. The internal crystalline structure of granite makes it a good absorber of kinetic energy (vibrational). Most large CMM stand on 4 points, this is not a kinematic design; however, it provides the stability required for large machines of this size with moving parts. These feet are often equipped with vibration abortion and stabilisation systems, commonly based on active air piston systems.

CMM measurement error

CMM have 21 sources of kinematic error [35]. These are errors inherent in the machine due to imperfect manufacture. Each measurement axis has: one linearity error (positional error), two errors caused by deviation in the orthogonal direction to the axis of motion/measurement, and three angular errors (pitch, roll, and yaw). There are three further errors: squareness errors (perpendicularity errors) between each pair of axis (xy , xz , and yz).

ISO 15530-4 [36] details parameters that affect the task specific measurement uncertainty of a CMM:

- workpiece
 - temperature

- form error
 - clamping
 - contamination;
- probing
 - random errors
 - systematic error sources;
- geometry
 - geometric
 - hysteresis
 - dynamic effects
 - temperature gradients;
- scales
 - temperature
 - scale factor
 - non-linearities;
- measurement strategy
 - number of points
 - distribution
 - probing vector;
- evaluation
 - algorithm
 - precision.

CMM measurement error is minimised by the use of error compensation techniques. Methods for the measurement and compensation of geometric error in CMM measurements are discussed within [37, 38]. Error mapping can be achieved by measuring the systematic errors of a CMM using a more accurate position measurement system. Schwenke *et al.* present a novel method of mapping the geometrical errors a CMM using a single tracking interferometer [39]. The method utilises a displace interferometric

technique between fixed reference points within the measurement volume. The technique enabled the use of compensation techniques to reduce the uncertainty of a CMM measurement. It is of note that this technique is an off-line solution. A similar technique is detailed by Unetsu *et al.* to calibrate a CMM using a laser tracker [40]. This method of tracking is analysed further in section 2.3.5.

2.2.2 Low uncertainty coordinate measurement machines

This section details the capabilities and limitations of machines with measurement uncertainties required to measure the surfaces of the most demanding prescription optical surfaces. A range of low uncertainty CMM are discussed, with error sources and limitations outlined.

Low measurement uncertainty CMM have been developed for micro-systems applications [41–47]. These use measurement systems that are independent of the machine probe transport system, enabling higher accuracy measurement of an artefact.

PTB μ CMM

Brand and Kirchhoff have developed a μ CMM at Physikalisch–Technische Bundesanstalt (PTB) [41]. A standard bridge-type CMM has been adapted with a thermally isolated *metrology frame* to measure the translational displacement of a probe, see figure 2.4. An Invar frame provides the housing where the sample is placed and reference mirrors attached. A Zerodur cuboid with measurement surfaces supports the CMM two-dimensional opto-tactile probe. These surfaces and the reference mirrors are utilised by an interferometer to accurately measure the position of the probing system in three translational dimensions and tilting between probe and artefact. The measurement volume of this set-up is $24\text{ mm} \times 34\text{ mm} \times 35\text{ mm}$. Over a 15 h measurement period the thermal drift of the probe position was measured to be $\pm 100\text{ nm}$. The uncertainty of the measurement of the flat artefact was reported to be $\pm 236\text{ nm}$ ($k = 2.78$, where k is the coverage factor).

This technique has been shown to have low measurement uncertainty. This thermal drift could be further reduced by the selection of materials with very low thermal expansion, such as Zerodur, to replace the Invar frame. This would add significant cost to the design; however, it is reported that without thermal drift the uncertainty of this technique would be reduced to $\pm 61\text{ nm}$ ($k = 2$).

A significant limitation to this technique in the current form is the small measurement volume. Whilst it may be possible to increase the measurement volume, it would not be practical to build a frame capable of measuring metre-scale surfaces due to mass and cost restraints.

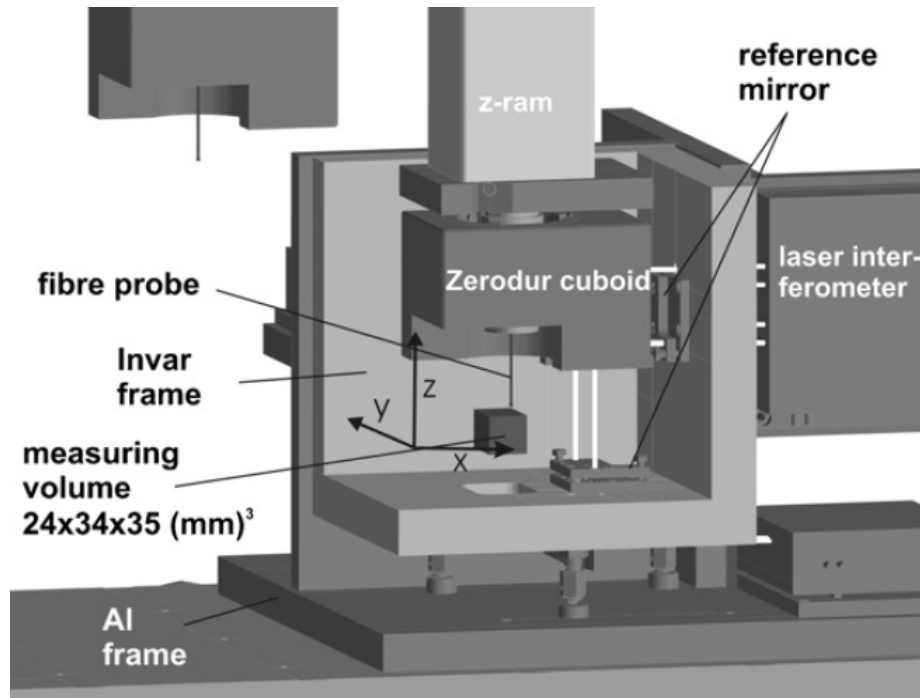


Figure 2.4: The metrology frame design of a small volume high accuracy CMM constructed at the PTB [41].

Isara 400

Donker *et al.* present an ultra-precision CMM, the Isara 400 [44]. The measurement volume of this system is $400 \text{ mm} \times 400 \text{ mm} \times 100 \text{ mm}$. Three interferometers (one per translational axis) are mounted in a SiC metrology frame, along with the probe, and used to measure the position of a mirror table, see figures 2.5 and 2.6. The three-dimensional measurement uncertainty for $k = 2$ is 100 nm. A coverage factor of 2 is equivalent to 95% confidence. A specific measurement probe was designed for this system. This uses three monolithic flexure foils, each with a capacitive sensor. Its absolute measurement errors are $< 10 \text{ nm}$ per axis of the coordinate system and $< 15 \text{ nm}$ in 3D.

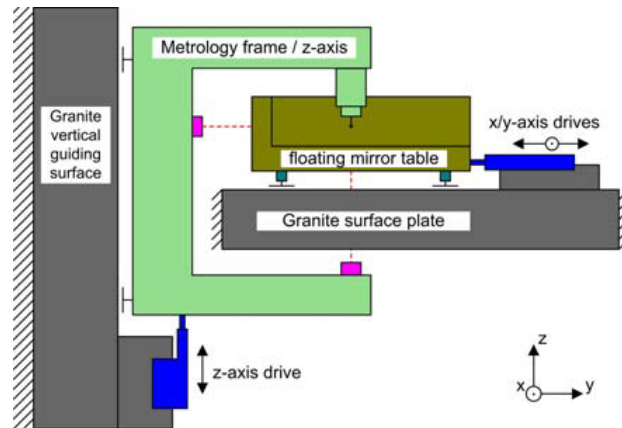


Figure 2.5: The design concept of Isara 400 high accuracy CMM [44].

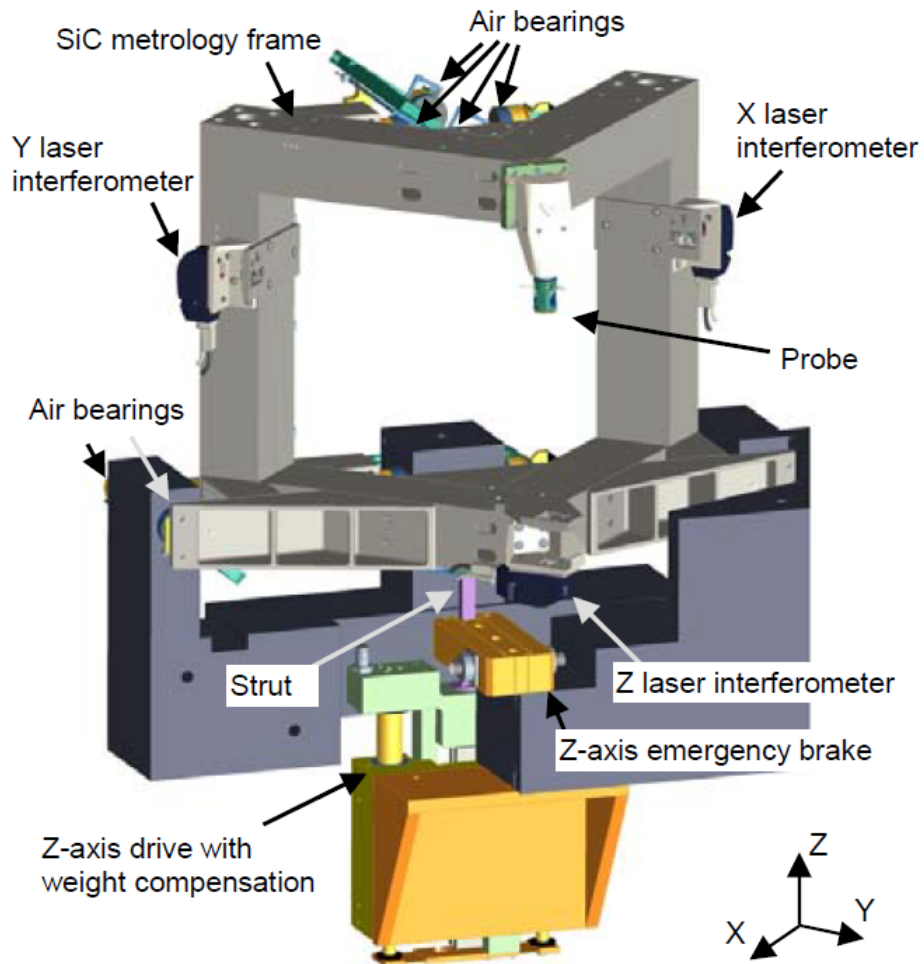


Figure 2.6: The design of Isara 400 high accuracy CMM [44].

NPL metrology frame CMM

Peggs *et al.* describes the design of a compact high-accuracy CMM constructed at the National Physical Laboratory (NPL) [46]. This design also seeks to separate the metrology

system from the load bearing systems. Interferometry is used to measure six degrees of freedom of a probe relative to a metrology frame. Figure 2.7 illustrates the design. The frame is built upon an existing CMM system to reduce the total cost and time to construct.

The metrology frame is constructed from low expansion material, Invar, and kinematically mounted to the CMM base using three balls inserted into three grooves machined into a base plate bolted to the CMM. The frame has three orthogonal surfaces rigidly contested. This provides a cubic measurement volume of 50 mm^3 and a measurement uncertainty of 50 nm .

The use of interferometry within a metrology frame system, such as [41, 44, 46], is not only a high precision displacement measurement technique, but also provides traceability to the definition of a metre from the known stabilised HeNe laser wavelength. This is also the case for an alternative design of a high-accuracy CMM based on multilateration techniques [47]. This technique is reviewed further in section 2.3.5. The CMM has a cubic measurement volume of 600 mm^3 and a measurement uncertainty of 200 nm ($k = 1$). The technique uses a series of interferometers and retro-reflectors as an accurate position measurement technique.

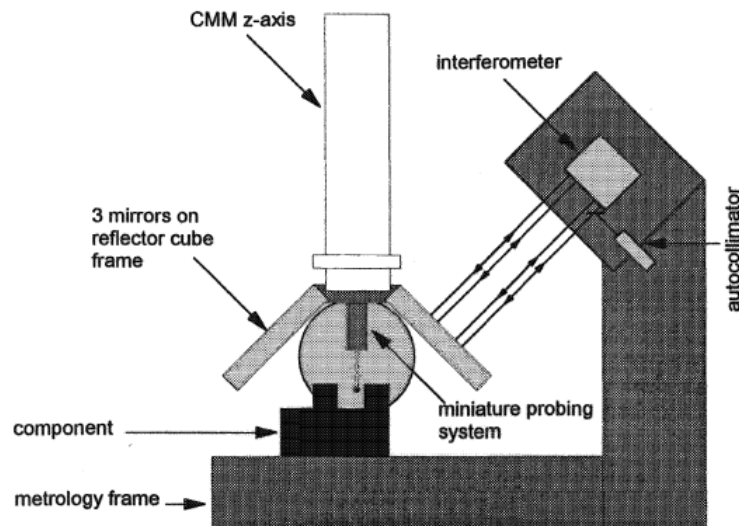


Figure 2.7: The design of a small volume high accuracy CMM constructed at the National Physical Laboratory [46].

Panasonic UA3P 3D profiler

This family of CMM use an atomic force probe in conjunction with interferometry to form a metrology frame [48, 49]. The probe acts with a measuring force of $3 \times 10^{-4} \text{ N}$ and is able to measure surfaces with slope up to 60° . A HeNe frequency stabilised laser is used to measure the displacement of the machines translation stages relative to a reference

mirror. The coordinate axis accuracy is ~ 10 nm in straightness. Table 2.1 details the key parameters of available UA3P 3D profilers.

Table 2.1: The accuracy and measurement volume of Panasonic UA3P 3D profilers.

Model	Measurement vol.(mm)	Measurement accuracy (μm)
UA3P-300	$30 \times 30 \times 20$	0.05 (30 mm)
UA3P-5	$200 \times 200 \times 45$	0.1 (200 mm)
UA3P-700H	$500 \times 500 \times 120$	0.3 (500 mm)

2.2.3 Large co-ordinate measurement machines

In order to measure metre-scale surfaces, the measurement system employed must have a measurement volume capable of measuring the entirety of a surface without the need for moving the sample on its supporting structure. Given the potential size of optics such as the E-ELT primary mirror segments [2] can have diameters of up to ~ 1.5 m, the measurement volume of the system used to measure the optical surface must at least match this diameter. What follows is an evaluation of specifications and limitations of measurement systems capable of measuring surfaces of this size. In general, measurement systems of this size have a relatively high measurement uncertainty, as detailed below.

CMM are commonly used for measurement of manufactured parts for the aerospace sector among many other industries. CMM generally employ a touch-probe to take position measurements of surfaces. These measurements are stored and used by software to determine the form of a surface. For high density coordinate measurements, this type of single-point measurement method is significantly limited by the time required to measure a metre-scale surface. Figure 2.8 shows the Leitz Infinity CMM. It is a bridge-type CMM with a linear stage base.

CMM can be used in a scanning mode. This involves dragging a touch probe over the surface of a substrate. Comley *et al.* suggest that micron level CMM measurement repeatability is achievable for low slope surfaces using scanning routines, within a $\pm 1^\circ\text{C}$ controlled environment. This involves a significant level of post-processing high density scan data to produce topographical maps of a ground surface [51].

A CMM using a touch probe is limited by its measurement speed. It takes many position measurements to produce the form error of large optics. In scanning mode the probe must be moved at low speed resulting in measurements taking several hours. Touch probe based measurements are also limited to measuring early stage machining due to induced surface damage.



Figure 2.8: A medium/large CMM — Leitz Infinity — manufactured by Leitz (Hexagon) [50].

Large CMM products

Table 2.2 details the maximum permissible errors of CMM currently available for purchase. The table consists of the CMM with the lowest $E_{L,MPE}$ from the manufactures that produce CMM with measurement volume capable of measuring E-ELT primary mirror segments. It is clear from the $E_{L,MPE}$ shown in the table, that the most significant contribution to the measurement uncertainty is the variable L , when considering the measurement of a ~ 1.5 m diameter surface. The constant terms of $E_{L,MPE}$ seem to have a dependence upon the size of the machine.

Table 2.2: High accuracy commercial large CMM and associated measurement parameters (18°C to 22°C).

CMM model	Company	Measurement vol. (m)	$E_{L,MPE}$ (μm)
PMM-C [52]	Leitz	$2.4 \times 1.6 \times 0.7$	$\pm 0.9 + L/650$
PMM-C [52]	Leitz	$2.4 \times 1.6 \times 1$	$\pm 1.5 + L/600$
PMM-F [52]	Leitz	$3 \times 2 \times 1$	$\pm 1.7 + L/400$
ACCURA [53]	Zeiss	$1.6 \times 2.4 \times 1$	$\pm 1.2 + L/350$
PRISMO [54]	Zeiss	$1.6 \times 2.4 \times 1$	$\pm 0.5 + L/500$
LK V-HA [55]	Nikon	$2.5 \times 1.5 \times 1.2$	$\pm 1.5 + L/375$

Commercial probes

Table 2.3 details the measurement error of high performance commercial probes that can be used on large CMM. These probes are capable of measuring the smallest deviations in stylus position and are therefore categorised as ‘low-force’. P_{FTU} is the single stylus form error, as detailed in appendix A.

Table 2.3: Form error of high accuracy commercial contact probes.

Probe	Company	P_{FTU} (μm)
LSP-S4 [56]	Leitz	0.4
LSP-S2 [56]	Leitz	0.5
LSP-X5 [56]	Leitz	0.6
RMP600 [57]	RENISHAW	0.25 (2σ)

2.2.4 Summary of CMM

As detailed, there are a number of coordinate measuring techniques currently being utilised within state-of-the-art operating CMM. These systems vary in scale and uncertainty. Figure 2.9 shows how the measurement uncertainty of CMM, with the capability to measure the specified surfaces, vary with measurement volume. The figure shows there are a number of systems capable of measuring with low uncertainty but do not have the required measurement capacity. Conversely, there are several systems with large measurement volumes but relatively high measurement uncertainty.

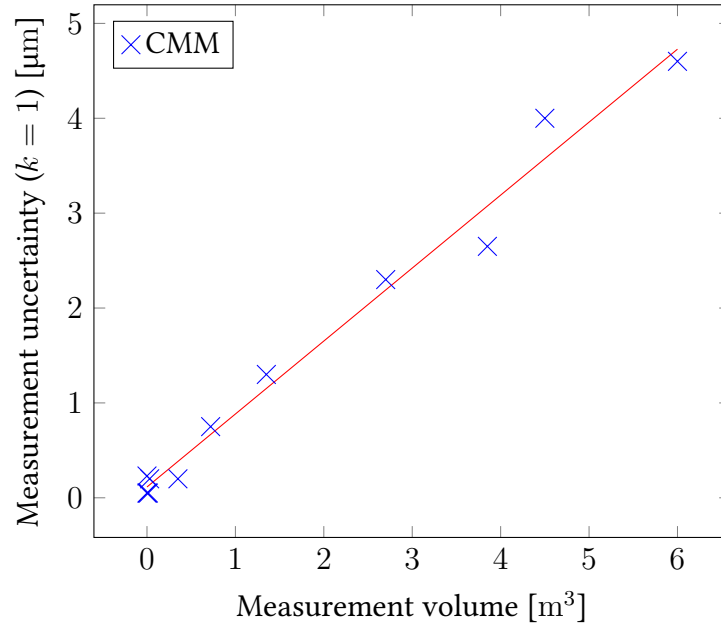


Figure 2.9: The measurement uncertainty of CMM as a function of measurement volume. A straight line best fit is shown.

2.3 Laser tracking systems

The laser tracker was invented at the National Institute of Standards and Technology (NIST) [58]. A comprehensive review of laser trackers for large-scale metrology has been undertaken by Muralikrishnan *et al.* within [59]. A laser tracker records distance to a retro-reflecting target, hereafter referred to as a spherically mounted reflector (SMR), at the same time as two angles in order to measure the SMR's position in spherical coordinates. Figure 2.10 shows the design of a laser tracker with a beam steering mirror. There are two types of laser tracker: with beam steering mirrors, and without beam steering mirrors.

2.3.1 Single laser tracker — ranging techniques

As shown in figure 2.10 a laser source projects laser light through a beam splitter; a portion of this light is split and reflected onto a position sensing device (reference beam); the remaining light passes through the beam splitter onto the steerable mirror towards an SMR. The light is reflected back from the SMR parallel to the incoming beam and back onto the steerable mirror; the beam splitter reflects some of the returning beam onto the position sensing device (measurement beam). This enables any radial movement of the SMR to be measured by the position sensor. The position sensing relies on two techniques: interferometry (IFM) and absolute distance measurement (ADM).

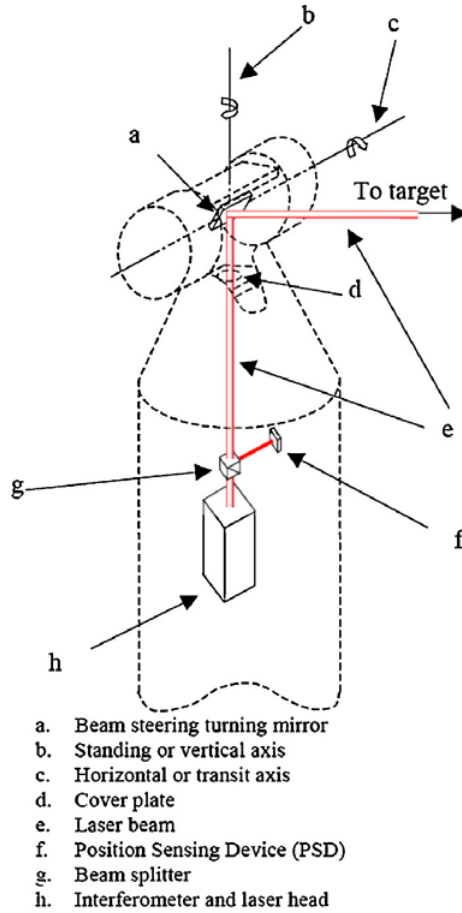


Figure 2.10: The internal design of a laser tracker with a beam steering mirror [59].

Interferometry Part of the laser light is constructed from a HeNe source — wavelength (632.8 nm). This is used for displacement interferometry. In this way the reference beam and the measurement beam are superimposed and used to count the alternating fringes as SMR displacement relative to the tracker home position occurs. If the beam is broken at any time during this measurement the fringe counting is lost and the measurement must be repeated. These setups typically have measurement uncertainty on the order of $0.5 \mu\text{m}/\text{m}$ [60].

Absolute distance meters Absolute distance meters typically use an amplitude, frequency, and polarisation modulating (sinusoidally) laser beam to resolve the distance from the tracker's home position to the SMR by phase measurements. Peggs *et al.* outline that the measurement uncertainty of ADM — $\sim (10 \mu\text{m} + 0.4 \mu\text{m}/\text{m})$ — are now approaching equivalence with IFM systems implemented on laser trackers [60].

The typical measurement range of a laser tracker is on the order of metres; however, the measurement uncertainty increases with the length over which the distance measure-

ment is conducted. This uncertainty is approximately $\pm 10 \mu\text{m}/\text{m}$ for a static target [61]. Error in this angular measurement is a significant source of measurement uncertainty, which increases dramatically with distance to target.

Unetsu *et al.* have analysed the position measurement uncertainty of a laser tracker with a comparison to a high accuracy CMM [40]. An uncertainty of interferometric displacements measurement is reported as

$$u_L = \sqrt{0.3^2 + (0.4 \times 10^{-3}L)^2} \mu\text{m}, \quad (2.4)$$

where L is length (mm). This result is based on a comparison with the CMM measurement of a 300 mm diameter artefact. The most significant sources of uncertainty were reported to be: mechanical error in rotary systems; shape errors in the mirrors; and uncertainty in the interferometry laser wavelength due to environmental factors such as temperature change, air pressure change, and humidity change. A review of laser tracker error models, for both tracker with and without beam steering mirrors, can be found in appendix D.

2.3.2 Commercial trackers

Leica AT960 The Leica AT960 has no beam steering mirror; instead, it utilises a fiber optic arrangement. It has an interferometric displacement measurement MPE of $\pm[0.4 \mu\text{m} + 0.3 \mu\text{m}/\text{m}]$ and an ADM offset (R0 parameter) of $\pm 3 \mu\text{m}$ [62].

Faro Laser Tracker V2 (X & Xi) The Faro Laser Tracker V2 has a beam steering mirror. It has an interferometric displacement measurement MPE of $\pm[2 \mu\text{m} + 0.4 \mu\text{m}/\text{m}]$ and an ADM offset (R0 parameter) of $\pm 10 \mu\text{m}$ [63].

API Radian The API Radian has no beam steering mirror. It has an interferometric displacement measurement MPE of $\pm[1.5 \mu\text{m} + 0.5 \mu\text{m}/\text{m}]$ [64].

2.3.3 Retro-reflectors

ISO 10360-10 [65] defines a retro-reflector/SMR as “a passive device to reflect light back parallel to the incident direction over a range incident of angles”. What follows is a summary of the SMR types, an explanation of the principles on which they work, and their contribution to error in coordinate measuring.

Mirrored corner-cube

A mirrored corner-cube reflector is typically constructed from a solid sphere of material with a corner machined into it. The three planes of the corner intersect (the vertex) at

the centre of the sphere. This SMR type is susceptible to three types of error source: vertex centring, the error between the position of the corner vertex and the sphere centre point; dihedral angle error, the three planes of the corner should ideally be at a normal to each other; and polarization error, as reflections polarize light, this needs to be addressed to avoid polarization mismatch, which can affect the interferometer of a laser tracker. These error sources are detailed within [59, 65, 66]. An example of a mirror corner cube is the Leica Red-Ring Reflector shown in figure 2.11a [67]. This SMR is constructed from steel with a mirrored corner to reflect back the tracker beam. There are three variants available with radii of: 19.05 mm, 11.1125 mm, and $6.35 \text{ mm} \pm 0.0025 \text{ mm}$. Their vertex centring is $< \pm 0.003 \text{ mm}$. They have roundness $< \pm 0.003 \text{ mm}$ and have a viewing angle of $\sim \pm 30^\circ$.

Glass hemispheres

One type of cat's-eye reflector is constructed from two glass hemispheres of different radii. The relationship between the two radii, r_1 and r_2 , is dependent on the glass RI (n) and is governed by: $r_1 = (n - 1) r_2$. The smaller hemisphere is positioned so that the laser beam will enter and is directed to the larger hemisphere [68]. The design is shown in figure 2.11c. An example of this type of reflector is the Leica Cateye reflector, shown in figure 2.11b (left) [69]. It is noteworthy that these glass SMRs also suffer from an equivalent set of errors to the mirrored corner-cube as discussed in section above. The radius of the Leica Cateye reflector is $37.50 \pm 0.025 \text{ mm}$. The vertex centring of the optic is $< 0.01 \text{ mm}$. This design has a much larger viewing angle than the Red Ring Reflector at $\sim \pm 60^\circ$.

Glass spheres

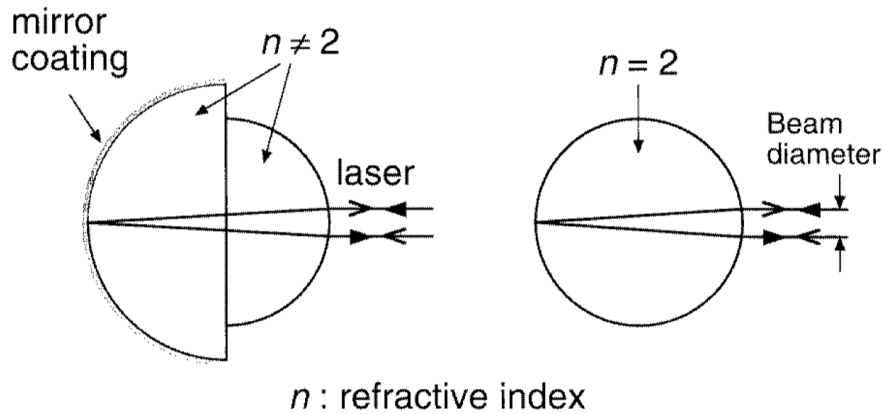
There are a subclass of SMR that provide a significantly expanded incidence angle, so called whole-viewing-angle cat's-eye SMR. The design for this was first presented by Takatsuji *et al.* in 1999 [68]. These are glass spheres of RI, $n = 2$. See figure 2.11c (right). This RI is required when $r_1 = r_2$ in the equation above. When a laser beam is directed at this SMR it is reflected back in parallel. One such material that can be used to manufacture this type of SMR is Ohara glass S-LAH79. This has $n = 1.99613$ at 632.8 nm.



(a) Red-Ring Reflector.



(b) Cat's Eye Reflector.



(c) Half and whole viewing angle reflector designs.

Figure 2.11: (a) A commercial SMR of the mirrored corner cube type. It is constructed from steel with a silvered corner. This SMR is called a Red-Ring Reflector and is manufactured by Leica [67]. (b) A commercial SMR constructed from two glass hemispheres of different radii [69]. (c) The design of an SMR with two glass hemispheres of different radii (left) and the design of a glass sphere SMR that is constructed from glass with RI, $n = 2$ (right) [68].

An example of a commercially available SMR that utilise an $n = 2$ sphere is the Leica Super CatEye Reflector (SCE) [70]. The 0.5 inch glass sphere is embedded in a stainless steel 1.5 inch hemisphere. It has a laser beam acceptance angle of $\pm 75^\circ$ from vertical around a full 360° field of view.



Figure 2.12: A Leica Super CatEye Reflector [70]. An example of a reflector with a glass sphere of refractive index, $n = 2$.

2.3.4 Thermal stability of laser trackers

As discussed in section 2.3.1, a laser tracker measures the displacement of an SMR using a stable light source for ‘single-wavelength’ interferometry. It is common to use a HeNe laser source, $\lambda = 632.8 \text{ nm}$. Typically, this wavelength is emitted stable to $\pm 1 \times 10^{-7} \text{ m}$ [63]. Ideally, a laser tracker would operate in a completely stable environment with no RI changes over the light path between the tracker and SMR; thus, the light wavelength would remain the same as it was emitted from its source and the resulting interferogram will not contain any error due to environmental effects. In reality, the environment in which a tracker operates will not be completely stable. Changes in air temperature, pressure, and humidity over the light path will result in an error in the measured displacement of an SMR, due to the changes in light wavelength and beam bending, caused by RI fluctuations.

The effect of temperature change over time upon tracker stability has been investigated by Ouyang *et al.* within [71]. Here, a CMM is used as a reference. The CMM was moved in a diagonal straight line repeatedly over a 14 hour period whilst a tracker is used to measure a target attached to the CMM motion column. It is shown that, for the first three hours of measurement whilst the tracker is ‘warming up’, the tracker measuring error was $19.6 \mu\text{m}$. After the initial warm-up period the error reduced to $< 10 \mu\text{m}$ until the error starts to increase again after ~ 10 hours stability. The error in distance measurements at a range of temperatures is also reported.

2.3.5 Multilateration

Multilateration is a group term for a number of similar techniques. A famous multilateration coordinate measuring system is the Global Positioning System (GPS) [72]. GPS

uses multiple satellites orbiting Earth measuring the distance (time of flight) to multiple ground stations and moving targets (e.g. mobile phones). The number of measured distances allows for the calibration of the system such that the coordinates of the measuring stations and measured positions can be calculated with uncertainty vastly smaller than the scale of the measurement system.

A similar system has been realised with steerable laser interferometers as the measurement stations and a retro-reflector as the measurement artefact [73]. This system produced coordinate measurement uncertainty on the order of 10^{-7} times smaller than the scale that the system operates over — the range from the interferometer to the target.

Multilateration systems have been used for calibration and verification of coordinate measuring machines using a single LaserTRACER, as reported in [74, 75]. Wendt *et al.* present an ‘online’ metrology frame for a CMM using four LaserTRACER [76]. This setup uses a multilateration routine to measure the error of an SMR mover (a CMM) and compensate measured data. Zhang *et al.* have modelled the uncertainty of the self-calibration of a multilateration system to determine optimised system arrangements based on the number of trackers and their positions relative to the measurement positions [77]. This was tested experimentally using sequential multilateration with one tracker in six measurement station positions. A ball grid was used to measure repeated SMR positions. This showed the coordinates of an SMR could be determined with uncertainty 1.4 times that of the displacement measurement uncertainty of the laser tracker’s interferometric mode. This investigation highlights the potential for laser-based multilateration for measurement of surfaces.

These multilateration implementations are all limited by at least two of the following: a lack of long-term assessment to test the setup measurement uncertainty sensitivity to environmental instability; utilised sequential multilateration with one displacement measuring system in multiple positions; used to calibrate a ‘move robust’ measurement system (a CMM) rather than being used as an ‘online’ measurement system; and not positioning the measurement target at measured position on the surface of a sample (e.g. the target was placed at the top of the CMM stylus inducing an Abbe error between target and stylus tip).

Steerable interferometer based multilateration

The laser-based multilateration technique, used by Greenleaf [73], measures only the displacement — relative to the interferometer’s home position — of the SMR as it is moved to a number of measurement positions. As such, the system has no capability to measure the Cartesian coordinates of the SMR directly. Takatsuji *et al.* have presented the problem to be solved [78]. For a setup that uses four interferometric measuring stations, it is

possible to define the multilateration problem to be solved as:

$$X_i^2 + Y_i^2 + Z_i^2 = (d_{i1} + dp_1)^2, \quad (2.5)$$

$$(X_i - x_2)^2 + Y_i^2 + Z_i^2 = (d_{i2} + dp_2)^2, \quad (2.6)$$

$$(X_i - x_3)^2 + (Y_i - y_3)^2 + Z_i^2 = (d_{i3} + dp_3)^2, \quad (2.7)$$

$$(X_i - x_4)^2 + (Y_i - y_4)^2 + (Z_i - z_4)^2 = (d_{i4} + dp_4)^2. \quad (2.8)$$

where X , Y , and Z are the coordinates of measured positions; $i = 1, \dots, N$ where N is the total number of measured positions; x , y , and z are the coordinates of the measuring stations; and $j = 1, \dots, M$ where M is the number of measuring stations – laser trackers. d_{ij} are the measured displacements of the measurement target relative to the measuring station positions. dp_j are the interferometric deadpaths between each of the measuring stations and the initial measured position. Defining the coordinate system with its origin at measuring station one, $(x_1, y_1, z_1) = 0$; the x-axis from station one to station two, $(x_2, y_2, z_2) = (x_2, 0, 0)$, and the xy-plane defined by the positions of stations one, two, and three, $(x_3, y_3, z_3) = (x_3, y_3, 0)$. Further measuring stations must have non-zero z-coordinates (see section 4.2.6).

Application of a non-linear least squares solver can determine estimates for these unknowns by minimising the residuals between the left-hand and right-hand sides of the equations (see section 4.1.1). This requires an initial guess to the coordinates for the measured positions and measuring stations. This is made possible by aligning the coordinate systems of the laser trackers with the coordinate system of the target motion system; or, using the angular encoders and the ADM of the trackers, if this is an option.

LaserTRACER – Etalon

The Etalon LaserTRACER is a laser tracker optimised for multilateration [39, 47]. Figure 2.13 shows the design of the LaserTRACER. For any measuring station in a multilateration system the stability of the station home position as it tracks the SMR is critical. This is because multilateration utilises only radial displacement measurements for self-calibration; however, if the position of the centre from which this radial displacement is measured has high uncertainty, this will result in high uncertainty in the system coordinate self-calibration. The design of the LaserTRACER is specifically designed to ensure the stability of its home position. This is enabled by a precision sphere, supported by an Invar stem held independent of the LaserTRACER's tracking motion, is used as the home position of the radial measurement. It has a displacement measurement uncertainty of $0.2 \mu\text{m} + 0.3 \mu\text{m}/\text{m}$ for $(k = 2)$.

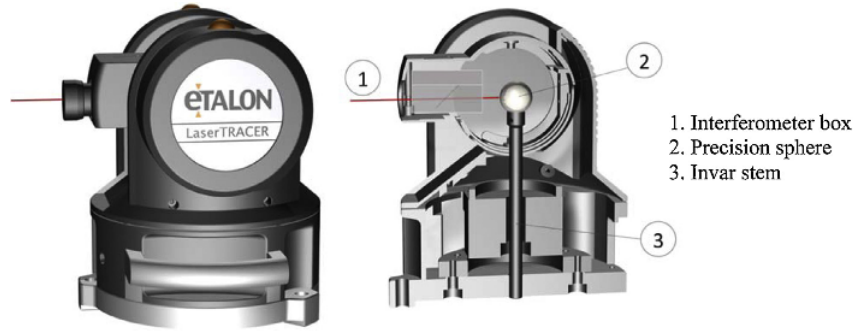


Figure 2.13: Schematic of the Etalon LaserTRACER [59].

2.3.6 Virtual metrology frame

‘Virtual metrology frame’ is a phase first used by Peggs in [79]. The setup has been realised by Greenleaf [73] and Hughes *et al.* have constructed a CMM based upon this concept at the NPL [47]. This uses the initial design for the LaserTRACER to achieve a measurement uncertainty of 200 nm ($k = 1$) over 1 m. There have been numerous reports of this technique being used as an off-line calibration or for verification of metre-scale CMM [39, 40, 74–76].

2.3.7 A multi-target multilateration system using frequency scanning interferometry

The NPL aimed to develop a coordinate measurement system that: has a versatile measurement volume upto $10\text{ m} \times 10\text{ m} \times 5\text{ m}$; is a direct competitor in terms of measurement application and uncertainty to current laser tracking systems; has the ability to self-calibrate its coordinate system whilst the measurement is undertaken and is traceable to the definition of a metre via its measurement principle; and have enough measurement redundancy, such that the uncertainty of each coordinate measurement can be evaluated [80].

The measurement system utilises a multilateration solution to self-calibrate the coordinate system and determine the measured position coordinates. This is made possible via distance measurements from a number of measurement sensors (at least four) that are positioned so that they surround the measurement targets as shown in figure 2.14a. The sensors measure the distance between the sensor and a set of measurement targets at positions of interest upon the sample. The distance measurement technique is based upon infra-red frequency scanning interferometry (FSI). The system has the capability to measure multiple targets simultaneously, therefore, once the measurement sensors and measurement targets have been positioned, the measurement time is short in comparison with a standard laser tracking interferometer.

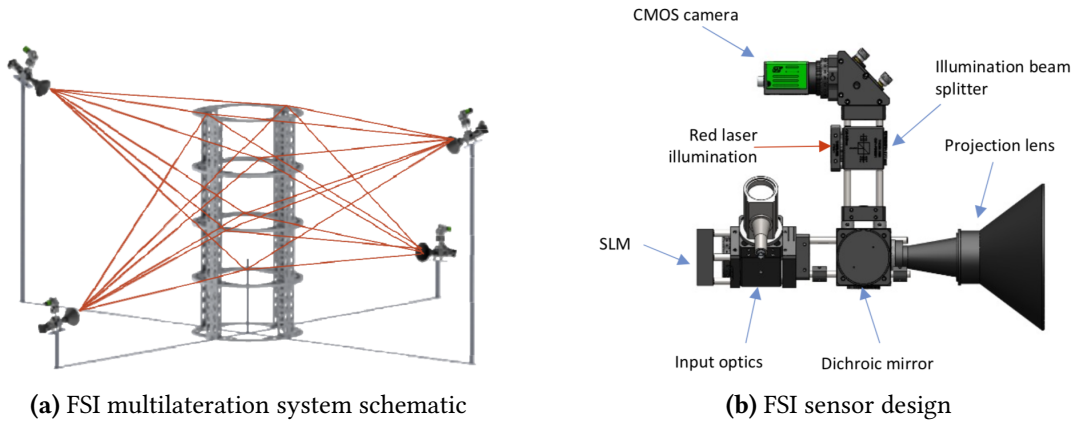


Figure 2.14: (a) A four FSI sensor setup measuring the distance to targets positioned on a measurement rig. (b) The basic design of the FSI sensor [80].

FSI is an absolute distance measuring technique first implemented as a multi-channel measurement system by Dale *et al.* as detailed within [81]. This system uses two frequency scanning lasers along with a gas absorption cell and a reference interferometer. This interferometer is used to determine the optical path difference of the measurement interferometers formed with the measurement targets of this process. The known frequencies of the absorption features of the gas cell are used to provide traceability in length measurements. It is shown that this technique can measure distances over the range of 0.2 m – 20 m with relative uncertainty of 0.41×10^{-6} ($k = 2$).

In the NPL setup, as shown in figure 2.14a, the light not reflected for the reference interferometer is projected onto the measurement targets. The targets are glass sphere SMR, as detailed in section 2.3.3, with RI, $n = 2$. A portion of the laser light is reflected back from the SMR – as the SMR is not coated with any reflective material, most of the light transmits through the glass sphere – into the FSI sensor where it interferes with the reference signal. The intensity of the interference signal is measured by the detector, as detailed in section 2.1.2. The frequency of this signal is directly proportional to the target distance. If there are multiple targets, the signal frequency is a superposition of each of the beat frequencies associated with each target; therefore, resolving these frequencies is critical to determining each target distance. As the measured intensity is dependent on the time-of-flight, taking the Fourier transform of the intensity signal results in a signal in the time-of-flight domain. This signal has a peak for each target measured; therefore, determining the target distance and its uncertainty is dependent on the system's ability to resolve the position of the signal peaks maxima. These distances are used in a set of equations, similar to equations (2.5 – 2.8), in order to solve for the Cartesian coordinates of the measured targets.

An experiment was conducted with the setup shown in figure 2.14a, calculation based on the measured distances to the targets estimated the x- and y-coordinates with an uncertainty $< 30 \mu\text{m}$. The z-coordinates were estimated with uncertainty $< 100 \mu\text{m}$. The relative difference between coordinates in x and y and coordinates in z is due to the positioning of the sensors and measurement targets. The effect of sensor positioning on measurement uncertainty is detailed in section 2.3.5.

2.4 Other methods for metrology of large optics

There are a number of alternative techniques to a metrology frame for the high accuracy measurement of surfaces. Vermeulen *et al.* describe the design of a high-precision 3D-CMM [43]. This design is focused on adherence to the Abbe and Bryan [82] principles such that the measurement system is offset and straightness independent. The measurement volume of this system is 100 mm^3 with an estimated $0.1 \mu\text{m}$ measurement uncertainty.

Peggs has reviewed several techniques for the measurement of rotationally symmetric spherical and aspherical optics [83]. These include, among others: reversal methods, and comparator mode profilometry.

2.4.1 Swing-arm profilometry

A swing arm profilometer (SAP) can be used to measure the topology of large optics with a non-contact optical point probe [84–89]. The probe is positioned at the end of a pivoted ‘swing-arm’. The arm is used to move the across the diameter of a substrate. By rotating the substrate — on a rotary table — a map of its surface topology is measured. Repeatable errors in the motion of the swing-arm can be precisely mapped such that it is removed from the surface measurement data.

The set-up requires careful alignment such that the probe is perpendicular to the optical surface and, as such, measures the surface departure from spherical nominal [89]. It is typical that the probe used to measure the surface is a non-contact interferometer. For a non-specular surface, an infrared light based interferometer has the required wavelength such that ground surfaces produce specular reflections [84].

SAP are capable of measuring most optical surface forms in the ground or polished state to ultra-high accuracy, Valente *et al.* report an accuracy of $0.1 \mu\text{m}$ [84] — precise alignment of measurement path with optic is required. The method also has a high resolution across the probing scan. The limitations are the time and precision required to align the arm and probe is extensive. Furthermore, the radius of curvature (RoC) is not measured directly, and the spatial resolution is low when considering the entire surface as the probe only scans in two dimensions (not in the azimuthal direction). The

measurement time is also a source of measurement uncertainty, for the same reasons as CMM.

Jing *et al.* present a novel method of measuring the surface topology of a polished surface using a SAP system combined with a laser tracker [88]. The laser tracker allows the accurate alignment of the SAP to the sample surface. The measurement uncertainty of this method is analysed. The uncertainty is relatively high: $\pm 8.2 \mu\text{m}$ in the X- and Y-axis, and $\pm 2 \mu\text{m}$ in the Z-axis.

An uncertainty budget for the NPL/UCL SAP [87] is presented by A. Lewis in [90]. Figure 2.15 shows the general assembly design of this SAP. The uncertainty budget shows how the SAP measurement is dependent upon the form error of the master surface used to set the nominal measurement path of the SAP. The measurement uncertainty is highly sensitive to temperature: $+1.73 \mu\text{m}/\text{K}$ in z . The temperature independent uncertainty in z is $\pm 48 \text{ nm}$. The budget shows how as significant part of the radius of curvature measurement uncertainty is derived from the mirror segment sag error and the problem of only measuring partial arc.

The fundamental issue with this design of SAP is the difference between the nominally defined probe measurement path, derived from a spherical master optic, and the real measurement path defined by the measurement surface form, which may be aspheric or freeform. Any deviation of the surface from the nominal path results in a force on the probe resulting in tilt. This tilt is not measured by the SAP, resulting in a significant source of error.

2.4.2 Optical tower interferometry

Optical tower interferometry is employed within many manufacturing process chains for large optics [19–22]. Phase-shifting interferometry (PSI) is a high accuracy method for measuring large optics with specular surface. These systems are specifically based on Fizeau interferometry techniques. The measurement time is low, as the whole surface of the substrate can be measured in one iteration. The measurement of the form error of a surface is produced by a comparison between the measurement surface and a reference surface. This limits the methods ability to measure aspherical and freeform surfaces. If the measurement surface deviates from the form of the reference surface by several microns, the fringes generated by the phase shift become extremely dense and are therefore impossible to resolve. To some extent this limitation can be overcome by the use of computer-generated holograms (CGH) [91–93]. This is a computer-generated reference surface that is designed specifically to the freeform surface being measure; thus, a unique hologram must be programmed for each freeform shape to be measured. In the case of the E-ELT, this would be extremely demanding in terms of time and cost.

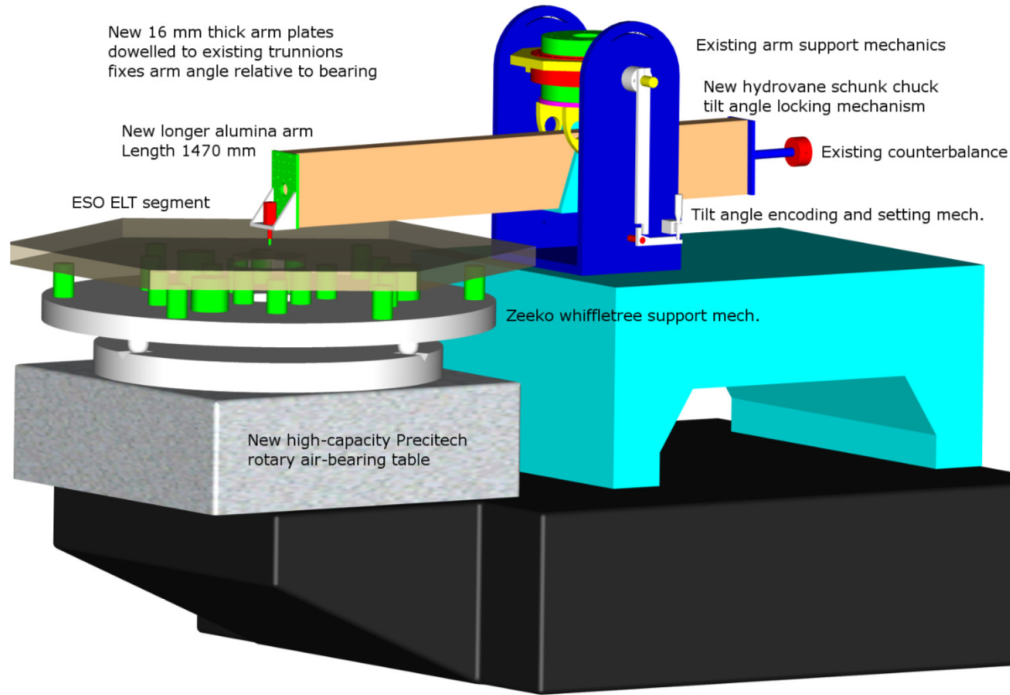


Figure 2.15: The general assembly design of the NPL/UCL swing-arm profilometer [90].

This method is further limited by the need for long length measurement systems dependent on the RoC (or equivalent for aspherics and freeforms); therefore, optical test towers are required such as that detailed in [94]. This is a 4 m test tower with a vibration insensitive interferometric measurement system.

Issues with required length scales are commonly addressed with the implementation of folding mirrors [21] or stitching [95]. Yet these methods induce an additional measurement error. The optical tower detailed in [21] has the capability to measure optics with diameter equivalent to the maximum diameter of E-ELT mirror segments (~ 1.5 m); however, using folding mirrors can induce error in the measurement as the form error of the folding mirror sums with the optic form error. The overall wave front error of this test bench optical tower is 40 nm RMS.

A surface must be specular in order to reflect coherent light for the interferometry to determine the form of the optic. This means a ground surface must be neutral polished in order to be measured for error compensation; this, summed with the long system set-up time, makes this a costly and time-consuming procedure.

2.4.3 Nanomefos

Nanomefos is a non-contact measurement system with design potential for metre-scale optics [96–98]. It is shown in [96] the technique has high measurement speed, high

accuracy (< 100 nm), and low surface damage. A Nanomefos design enabling the measurement of large optics, such as the segments for primary mirror of the E-ELT, is detailed in [98].

The measurement system that the Nanomefos system employs was first proposed by Fainman et al. in [99]. Figure 2.16 is the optical design of this measurement system.

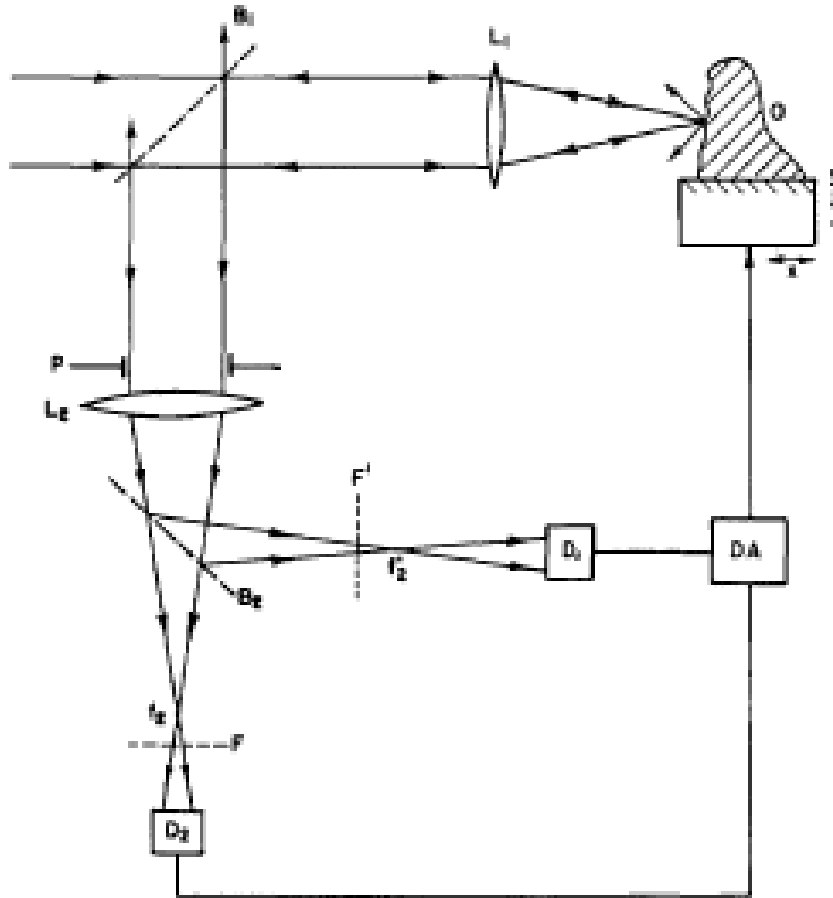


Figure 2.16: The optical set-up of the optical profilometer method employed by the Nanomefos system to measure the form error of optics [99].

As the measurement surface is moved in the x-axis an optical filter arrangement causes an effective shift in the focal positions f_2 and f'_2 . As such, the power measured by the detectors, D_1 and D_2 , changes. In this way the topology of a surface can be measured. The measurement is compared to the idealised form of the surface to calculate form error. The limitation of this method in the current Nanomefos design is its inability to measure a non-specular surface. Height density measurements of a surface to measure MSF may produce high uncertainty in the measurement, given the time to measure with the required surface density. The cost to construct the large scale Nanomefos design may also be a limitation.

2.4.4 Deflectometry

In 2004, Knauer *et al.* present a method of phase measuring deflectometry to measure specular freeform surfaces at high speed [100]. A sinusoidal fringe pattern is projected onto the sample. The reflected pattern is observed and any deviation from the nominal pattern is analysed using well-known phase shift algorithms. The absolute accuracy in measurement of local slope is a few arc minutes, with a repeatability of a few arc seconds. Similar to an optical tower interferometer, this method is limited by a large radius of curvature on an optic, resulting in a long optical path. Furthermore, this method is not able to measure a ground surface directly as the surface is required to be specular.

2.4.5 Stitching interferometry

The basic principle of stitching interferometry is similar to that of an optical tower; however, the measurement of mirrors with diameters of several metres is possible with a relatively small scale system. This is due to the method of performing interferometric measurements of overlapping subaperture regions on a specular surface, and stitching the measurement data together to form a complete topological map [95, 101, 102].

The interferometer is moved laterally to cover the entirety of the surface, and moved in an axis normal to a nominal flat on the surface in order to measure at the varying RoC of a freeform surface. To measure a complex freeform surface to high accuracy, the measurement must be repeated many times at each position to provide redundant data that is used to average out noise. This dramatically increases the time to measure and therefore costs. As with any HeNe Fizeau interferometry, this method requires a specular surface.

2.5 Testing of laser tracker measurement error

The *Guide to the expression of uncertainty in measurement* (GUM) details the general rules for evaluating and expressing measurement uncertainties [103]. A summary of the relevant standards, definitions, and measurement uncertainty evaluation techniques can be found in appendix E. What follows are details of the standards and methods for the evaluation of laser tracker measurement uncertainty.

2.5.1 Testing of a laser tracker as a coordinate measuring system for point-to-point distance

ISO 10360-10 In terms of geometric product specifications, the acceptance and reverification tests for laser trackers specifically measuring point-to-point is set out within ISO 10360-10 [65].

One such test is the two-face test, as described in section D.1. This test is undertaken to determine location errors. For length measurement errors, a significantly more intensive measurement procedure is prescribed. The prescribed test of these errors involves the measurement of 105 lengths. 41 of which are mandatory and the remaining 64 are chosen by the system assessor. The formula for the MPE of the measurement lengths is provided by the system manufacture and thus can be used by the assessor to carry out a system verification. This type of verification requires some specialist equipment, such as: scale bars, target nest mounts — both free standing and on walls, rail-and-carriage systems, gauge blocks, and ball bars. Furthermore, a large area for test measurements is required as the tests must cover at least 66% of the measurement range. It is noteworthy that it would take many hours to carry out these tests on a laser tracking system in full.

2.5.2 Determination of laser tracker measurement error

This section describes methods for assessing measurement error in laser tracking systems. The first method discussed is a network measurement technique for determining quantities for the parameters detailed appendix D [104]. The second assesses range measurement errors a function of azimuth [105].

Network measurement method

Hughes *et al.* present a method that determines all the alignment and angle errors of a laser tracker, along with their associate uncertainties [104]. A laser tracker has errors due to beam misalignments, scale eccentricities, and axis offsets that result in systematic error in the spherical coordinate measurements of a target (see appendix D). This network measurement technique aims to measure and correct these sources of error.

The method utilises an adapted version of the tracker error models presented in appendix D. The method involves the measurement of a number of target positions (R , θ , ϕ) from a number of unique tracker positions — a *network measurement*. By minimising the residuals of a non-linear least squares problem derived from the tracker error model utilising the measurement results, estimates of the various tracker error parameters and their uncertainties are produced. By propagating the uncertainties associated with the network and the target measurements to obtain the uncertainty in Cartesian coordinates

of targets.

Ranging error measurement

A method for assessing laser tracking systems ranging errors as a function of azimuth is presented by Muralikrishnan *et al.* within [105]. Error in the radial motion of a laser tracker occur when the trackers head rotates around the vertical axis and cause error in tracker range measurements. This method utilises a LaserTRACER as a reference measuring machine. As discussed in section 2.3.5, a LaserTRACER has a reference sphere for measuring range. As this sphere is stationary relative to the rotation of the laser head the design of the LaserTRACER aims to eliminate any radial motion error. Instead, the range measurement error is dependent on the form and centring of the stationary reference sphere.

This method aims to move a SMR in a ‘perfect’ circle around a laser tracking system in the plane perpendicular to the trackers nominal vertical axis as a tracker measures the range displacement of the SMR. The deviation from a perfect circle measured by the tracker under test gives the ranging error as a function of azimuth (horizontal) angle.

In practise, the circular path was made using an air-bearing spindle that is equipped with an SMR that moves in a path with 200 mm radius. A secondary SMR was used as a counter balance at the opposite end of the spindle. This SMR is also used to perform the reversal technique to separate the systematic errors of the circular motion of the spindle from the systematic errors of the tracker range measurements. Results of experiment undertaken using this experimental setup show that the LaserTRACER is significantly more accurate at performing short term range measurements of the circular path, in comparison to laser trackers tested. Five laser tracking systems were tested in the same setup. The root-mean-squared of residuals over 10 measurements for the LaserTRACER was 0.11 μm . The equivalent tracker results were on the order of 1.5 to 5 times larger. Critically, the LaserTRACER measurements had a repeatability of 0.02 μm whereas the trackers had repeatability of the same magnitude as their residual measurements.

2.6 Conclusions

Coordinate measurement machines (CMM) with contact probes are well established measurement systems capable of measuring freeform geometries of non-specular surfaces. CMM have 21 kinematic error sources. The measurement uncertainty of CMM is affected by errors associated with: the workpiece under investigation, probing, the CMM geometry, the measurement scales, the measurement strategy, the probing vector, and the evaluation of measured points. Systematic error mapping of CMM is possible utilising single laser tracking interferometry and laser-based multilateration techniques.

There are multiple designs for small volume low uncertainty CMM. These involve the use of costly metrology frames and laser interferometry in multiple axis. In general, the measurement uncertainty of CMM coordinate measurements increase with the measurement volume of the machine.

State-of-the-art measurement of metre-scale non-specular freeform S-filtered – measurement that does not include high spatial frequencies of roughness – surfaces is limited by the uncertainty of measurement probes combined with the uncertainty in probe positioning.

Several existing measurement systems are capable of measuring metre-scale surfaces; some of which have the capability to measure such surfaces with non-specular finish and freeform topology with relatively high uncertainty in measurement: such as the Leitz PMM-C and Zeiss PRISMO. Conversely, there are currently several existing measurement systems capable of measuring with relatively low uncertainty: such as the Isara 400 and PTB μ CMM; however, these systems do not have the required measurement volume. Figure 2.17 shows the relationship between CMM measurement area and CMM measurement uncertainties, u_{CMM} . The measurement specifications shown are for existing operating systems. The trend shows an increase in uncertainty as area increases. The knowledge required to decrease the gradient of this ‘linear trend’ is lacking. There is no measurement system capable of measuring these relatively large surfaces with measurement uncertainty equivalent to the smaller volume techniques. The *knowledge gap* is represented by the red/shaded area.

As identified in the reviewed literature the most significant contributions to the uncertainty of a system measuring metre-scale surfaces are the probe positioning uncertainty (u_{pos}) and the probe measurement uncertainty (u_{probe}). There is no measurement system where these two uncertainties are equivalent, $u_{\text{pos}} \equiv u_{\text{probe}}$: there is no system that utilises one technique to perform the coordinate measurement. Steerable interferometry multilateration could be utilised to create a measurement system that operates with this principle: direct measurement of a CMM stylus-tip as it moves over the surface of a large optic. Such a system has not yet been realised.

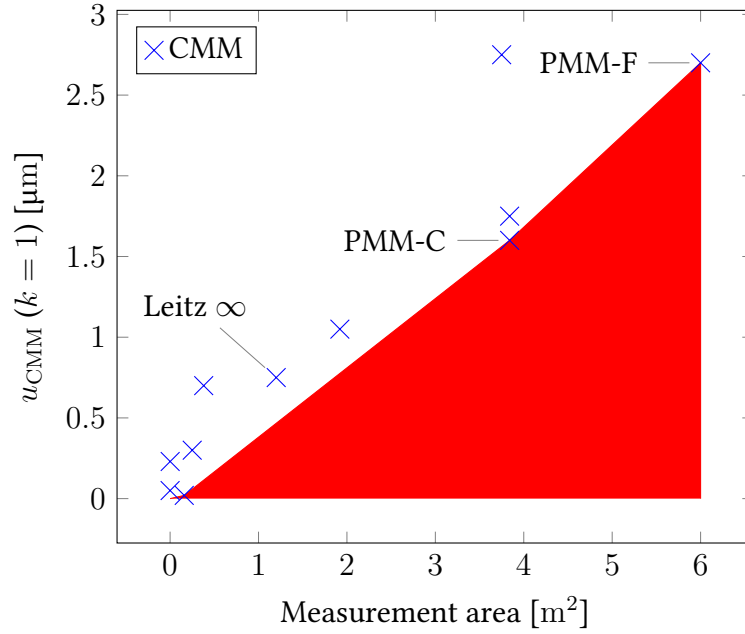


Figure 2.17: The measurement uncertainty of CMM, u_{CMM} , as a function of measurement area ($X \times Y$).

The knowledge required to measure the specified surfaces with an optimised task specific uncertainty does not exist. Some work has been undertaken to optimise the position of laser trackers in a multilateration system [77, 78, 106, 107], these are not exhaustive analyses. More research is required to critically analyse such a system for this measurement task with emphasis on measurement scale and measurement time. The effect of change in the measurement environment on measurement uncertainty over the long times anticipated for the measurement of these surfaces, with the required measurement density, has not been investigated.

Chapter 3

Hypothesis, aim, and objectives

Hypothesis

A laser-based multilateration system can be used to measure a metre-scale low-slope freeform non-specular surface form with z-coordinate measurement uncertainty, $\sigma_z < 1 \mu\text{m}$.

Aim

This project aims to investigate whether a laser based multilateration technique can be used as a method for low uncertainty measurement of metre-scale non-specular freeform S-filtered surfaces to accuracy significantly smaller than current measurement capabilities; this should include measurement of surface RoC. In measurement uncertainty this means

$$\sigma' \leq \frac{\sigma}{2}, \quad (3.1)$$

where σ ($k = 1$) is the state-of-the-art for measurement uncertainty of methods for metre-scale optics. The current state-of-the-art system capable of measuring the specified surface has a maximum permissible error of

$$E_{L,MPE} = \pm \left[0.9 + \frac{L}{650} \right] \mu\text{m}, \quad (3.2)$$

$$\sigma = \frac{E_{L,MPE}}{2}, \quad (3.3)$$

where L is measured in mm. Equation (3.3) describes the relationship between $E_{L,MPE}$ and uncertainty assuming a Gaussian uncertainty distribution.

This system is the PMM-C Leitz CMM using a contact probe head. For a surface of $\sim 1.5 \text{ m}$ the maximum permissible error of the PMM-C is $E_{L,MPE} = \pm 3.2 \mu\text{m}$; therefore

$$\sigma' \leq 0.8 \mu\text{m}.$$

Objectives

- Test the stated measurement system experimentally to determine its uncertainty in z-coordinate measurement of the specified surfaces.
- Build a mathematical model of the measurement system based on analysis of input error sources to predict the system z-coordinate measurement uncertainty via Monte Carlo simulation.
- Determine the effect of long-term high density measurements of the specified surfaces, experimentally and via simulation, on system measurement uncertainty and evaluate measurement error sources.

3.1 Methodology

The methodology for this research is outlined in figure 3.1. In order to investigate the research hypothesis and meet the research aim, three objectives have been defined (see above).

Based on a review of the relevant literature and a theoretical analysis of the proposed laser-based multilateration system, a model of measurement system and the input error sources has been produced, as detailed in chapter 4. This mathematical model had been used to design a series of experiments in order to test both input parameters and the measurement system directly. The input error sources are defined in terms of their time dependency. The measurement system has been tested directly on two flat optics in a number of arrangements.

The mathematical model has been utilised in a Monte Carlo simulation of the measurement system. The theoretical analysis and experimental testing informed the input parameters for this simulation. The simulation aimed to predict the system z-coordinate measurement uncertainty for the real measurements. The simulation has also been used to test the measurement system sensitivity to input parameters.

The effect of long-term measurements upon the system measurement uncertainty has been tested both experimentally and via simulation. The experimental measurement of a surface has been conducted over > 10 h to determine the effect of environmental changes on measurement error. The mathematical model, along with both measured and simulated temperature change, has been used to predict the effect of long-term measurement on the multilateration system coordinate measurement error.

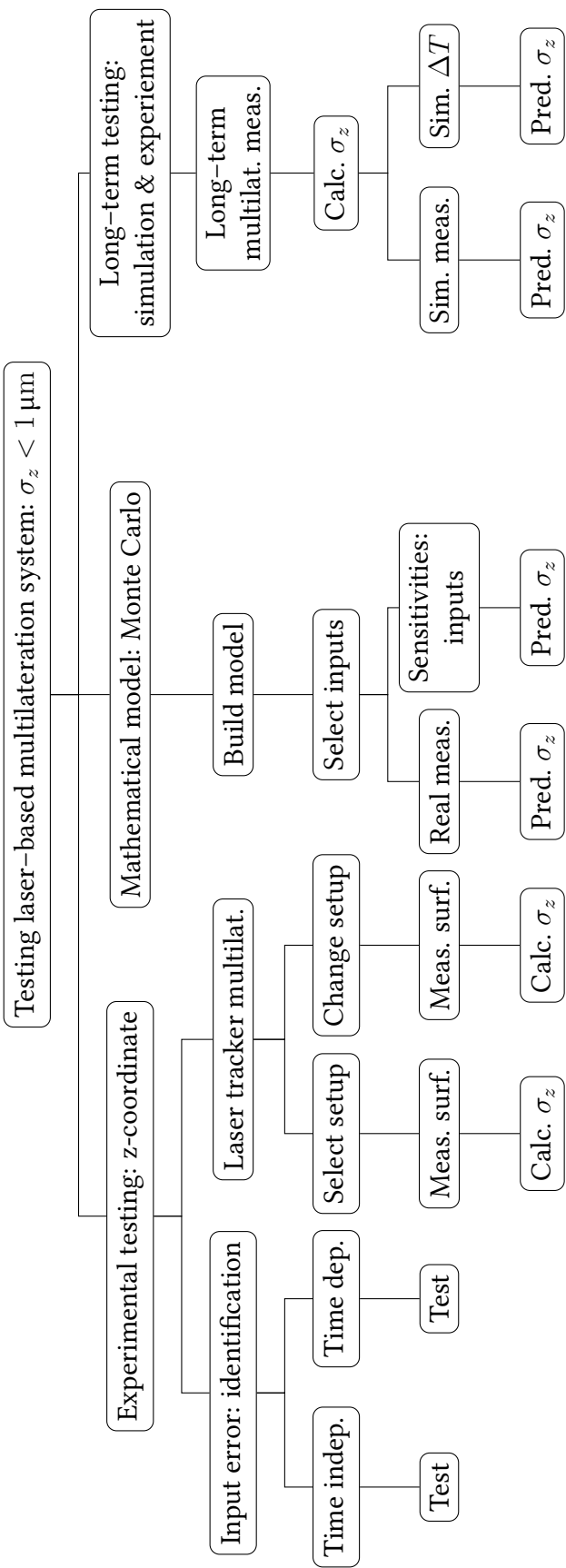


Figure 3.1: A flow chart of the methodology utilised to complete the objectives of this research.

Chapter 4

Method and system theory

The principle for coordinate measurement of a metre-scale low-slope freeform surface is a multilateration algorithm. The system utilises laser-interferometry to measure the range displacement of a target as it probes the sample. The approach aims to minimise the residuals between the measured target ranges and the target coordinates in order to produce an accurate surface form measurement. The minimisation is implemented by a non-linear least squares method. Given the low-slope nature of the sample surfaces, it has been shown that the critical parameter for uncertainty minimisation is the axis perpendicular to the sample surface. The measurement system has been practically realised with a set of AT960LR laser trackers as measuring stations and an SMR-probe that has been integrated into a large CMM. The CMM is utilised as a probe motion system. The input error source parameters have been identified. The parameters can be categorised as time independent and time dependent. An MCA has been described: this aims to enable testing of the sensitivity of the measurement system to change in a specific input parameter. The error sources and the mathematical model of the system enable the design of experiments to empirically test the parameters and their effect on the measurement result.

4.1 Measurement system realisation

The proposed measurement system under investigation, as stated in chapter 3, utilises a laser-interferometry-based multilateration technique. This is implemented using a system of laser trackers to measure the distance to a target as it moves on the surface of a sample. These measurements are used to determine the coordinates of the target. A CMM is used to control the motion of the target.

4.1.1 Principle of the measurement system

The measurement principle, as described by Wendt *et al.* in [76], is a laser-based multilateration technique as used by Greenleaf [73], described in chapter 2. A number of measuring stations are arranged to measure the displacement of a single target simultaneously as the target moves over the surface of a sample. From equations (2.5) – (2.8), the measured displacement of the target can be related to the Cartesian coordinates of the target as it moves in contact with the surface by

$$d_{ij} + d_{0j} + w_{i,j} = \sqrt{(x_i - x_{0j})^2 + (y_i - y_{0j})^2 + (z_i - z_{0j})^2}, \quad (4.1)$$

where $i = 1, \dots, n$ and n is the total number of measured positions, $j = 1, \dots, m$ where m is the number of measuring stations – laser trackers, d_{ij} are the measured displacements of the measurement target relative to the measuring station position j , d_{0j} are the interferometric deadpaths between each of the measuring station j and the initial measured position, x_i, y_i , and z_i are the coordinates of the measured positions, and x_{0j}, y_{0j} , and z_{0j} are the coordinates of measuring station j . w_{ij} are the residuals between the measured range from station to target and the resulting distances determined by a least squares fit.

Figure 4.1 illustrates the system, from which the equations are derived.

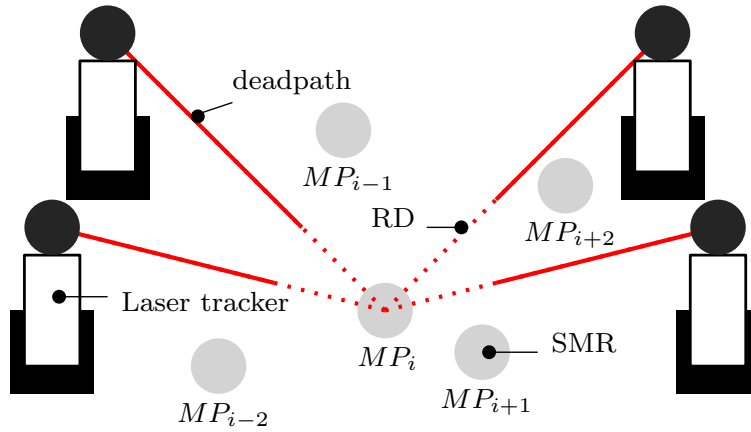


Figure 4.1: Diagrammatic analytical model of measuring principle for calibration of a measurement region. MP are the measured positions of the target (SMR) and RD is the radial displacement measured between each target measurement position.

It is noteworthy that, for each measured position, there are $3 \times n$ (the number of x_i, y_i, z_i) plus $4 \times m$ (the number of $x_{0j}, y_{0j}, z_{0j}, d_{0j}$) unknowns within the set of equations governing the problem to be solved. There are $m \times n$ (the number of d_{ij}) knowns per measured target position. The minimum number of measured positions n is governed by

$$n \geq \frac{4 \times m}{m - 3} ; \quad (4.2)$$

therefore, $m \geq 4$. To measure a useful number of points on a surface it is likely that $n \gg 16$, where 16 is the minimum number of measured points if $m = 4$.

To determine accurate estimates for the coordinates: x_i , y_i , and z_i ; the magnitude of the sum of squared residuals is minimised:

$$F(x, y, z) = \sum w_{ij}^2 \rightarrow \min . \quad (4.3)$$

Treating this as a non-linear least squares problem [108–110], the derivative of the function with respect to x is

$$\frac{\partial F}{\partial x} = 2 \sum_{j=1}^m w_j \frac{\partial w_j}{\partial x} . \quad (4.4)$$

The partial differentials with respect to y and z are similarly formed. Defining the vectors $\vec{w}$ and $\vec{g}$ and the Jacobian matrix, $\mathbf{J}$, such that

$$\vec{g} = 2\mathbf{J}^\top \vec{w} , \quad (4.5)$$

where

$$\mathbf{J} = \begin{pmatrix} \frac{\partial w_1}{\partial x} & \frac{\partial w_1}{\partial y} & \frac{\partial w_1}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial w_j}{\partial x} & \frac{\partial w_j}{\partial y} & \frac{\partial w_j}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial w_m}{\partial x} & \frac{\partial w_m}{\partial y} & \frac{\partial w_m}{\partial z} \end{pmatrix} , \quad \vec{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_j \\ \vdots \\ w_n \end{pmatrix} , \quad \vec{g} = \begin{pmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \\ \frac{\partial F}{\partial z} \end{pmatrix} .$$

Differentiation of the function $w_j(x, y, z)$ yields

$$\frac{\partial w_j}{\partial x} = \frac{x - x_{0j}}{\sqrt{(x - x_{0j})^2 + (y - y_{0j})^2 + (z - z_{0j})^2}} ,$$

$$\frac{\partial w_j}{\partial y} = \frac{y - y_{0j}}{\sqrt{(x - x_{0j})^2 + (y - y_{0j})^2 + (z - z_{0j})^2}} ,$$

$$\frac{\partial w_j}{\partial z} = \frac{z - z_{0j}}{\sqrt{(x - x_{0j})^2 + (y - y_{0j})^2 + (z - z_{0j})^2}} .$$

By expressing the minimisation problem in this way, one group of methods to solve

the minimisation are based on Newton's iterative method. Defining $\vec{X} = (x, y, z)^\top$, the iteration function is expressed by

$$\vec{X}_{\{k+1\}} = \vec{X}_{\{k\}} - (\mathbf{J}_{\{k\}}^\top \mathbf{J}_{\{k\}})^{-1} \mathbf{J}_{\{k\}}^\top \vec{w}_{\{k\}}, \quad (4.6)$$

where $\vec{X}_{\{1\}} = (\tilde{x}, \tilde{y}, \tilde{z})^\top$ for the initial 'guesses' of x , y , and z : $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ respectively. k is the iteration number.

An improved version of Newton's method is known as the Levenberg–Marquardt Algorithm. The changes made to Newton's method to produce this algorithm prevent the iteration from taking excessively large steps [108, 109]. To solve the minimisation problem, for both measured and simulated results, of this laser-based multilateration system the MATLAB® function *lsqnonlin* was used. This function uses the Levenberg–Marquardt algorithm and a trust–region reflective algorithm. The so called 'trust–region' is applied to the steps within the Levenberg–Marquardt algorithm to improve the minimisation.

4.1.2 Solution implementation

This section describes how displacement measurements of a target, both experimental and simulated, are propagated through a multilateration algorithm to determine an estimate of target positions on a sample surface — in Cartesian coordinates. The multilateration algorithm was implemented in MATLAB®. The multilateration function is shown in appendix F.

Measurement inputs

A laser tracker measures an SMR in spherical coordinates (R, θ, ϕ) . When an SMR is moved over the surface of a sample, the trackers measure change in SMR position. From these measurements the range displacement between SMR positions is calculated. Provided that a tracker's beam path — the path that the laser light traverses between the trackers and the SMR — is not broken during the measurement of the SMR position, the range displacements are measured interferometrically. The SMR range displacement between each target measured position is recorded. These measurements are input as the $d_{i,j}$ of $d_{i,j} + d_{0j}$ in equation (4.1). In practice, as the laser tracker is equipped with an ADM as well as an interferometer, the range is input as a single value, $R_{i,j}$, where $R_{i,j} = d_{i,j} + d_{0j}$.

As stated in section 4.1.1, the iterative minimisation method for solving equation (4.1) requires an initial guess for the values of coordinates of measured positions (x_i, y_i, z_i) of the SMR and the coordinates of the laser trackers (x_{0j}, y_{0j}, z_{0j}) . This requires the laser tracker positions to be estimated relative to the origin of a defined coordinate system.

Each tracker position is calculated via tri-lateration using three target measured positions. A tetrahedron is defined with the range to each measured position and the Cartesian coordinates of the three measured positions defining the origin at the first measured point, the x-axis through the second point and the xy-plane through the third point. The relative coordinates of the tracker can therefore be determined by

$$\begin{aligned} x_{0j}^2 + y_{0j}^2 + z_{0j}^2 &= R_{1j}^2, \\ (x_{0j} - x_2)^2 + y_{0j}^2 + z_{0j}^2 &= R_{2j}^2, \\ (x_{0j} - x_3)^2 + (y_{0j} - y_3)^2 + z_{0j}^2 &= R_{3j}^2, \end{aligned}$$

where x_2 is the distance between the first measured point and the second, and x_3 , and y_3 are the x and y coordinates of the third point. The values of these parameters are measured by the respective tracker. These simultaneous equations are solved using a non-linear least square approximation as described for the above.

Minimisation and solution

The input parameters, the measured target displacements and an initial guess for the coordinates of the target, are imported into a function for the equation (4.1). As stated, the first measured target position on the surface of the sample is defined as the coordinate system origin, the xy-plane is defined by the origin and two other target positions — all three positions are chosen to define a nominal flat through the surface. The exact selection of coordinate system is discussed further in section 4.2.

As discussed, an iterative approach to solving the function for the coordinate outputs has been implemented in MATLAB®. The *lsqnonlin* function solver uses the input measured target ranges to determine a set of coordinates for the target by minimising the residuals of the multilateration function. As well as the solver outputting the target coordinates, a value for the minimised sum of squares, as detailed in equation (4.3), and the residual value, $w_{i,j}$, for each target position are also returned.

Critical measurement parameter

As specified, the measurement task, for which the laser-based multilateration system is under investigation, is a coordinate measurement of a low-slope freeform surface. An example of the type of surface required to be measured is a segment of a large telescope mirror: such as the primary mirror (M1) of the E-ELT – this has an ellipsoidal shape. The E-ELT M1 has a diameter of 39 m and an f-number, $f/0.93$ [111]. If it is assumed that the mirror is spherical, the RoC would be ~ 72.5 m.

Figure 4.2 shows how the change in mirror height can be expressed as a function of mirror radius. By defining the centre of the mirror as height equal to zero, with the height increasing to the outer diameter, a change in the radial position of a mirror surface coordinate would result in a height change. The surface height difference between two coordinates along the same mirror radial axis, Δh , is given by

$$\begin{aligned}\Delta h &= \text{RoC} - \delta , \\ \Delta h &= \text{RoC} - \sqrt{\text{RoC}^2 - \Delta R^2} ,\end{aligned}$$

where δ is the distance shown in figure 4.2 and ΔR is the difference in mirror radius of a surface coordinate due to measurement uncertainty. The xy-plane of the coordinate system of this surface is defined by three points coincident with the mirror surface. The maximum surface height difference occurs when ΔR equals x- and y-axes measurement uncertainty; therefore, if the system coordinate measurement uncertainty is relatively large in the x- and y-axes, $100\text{ }\mu\text{m}$ ($k = 2$), $\Delta h \approx 7 \times 10^{-11}$ m. This is not significant in terms of system surface coordinate measurement uncertainty and suggests that uncertainty in coordinate measurement in the xy-plane of the surface is a non-critical parameter: the measurement system should be, where possible, optimised to minimise measurement uncertainty in the z-axis, even in the case where this results in relatively high uncertainties in the x and y directions.

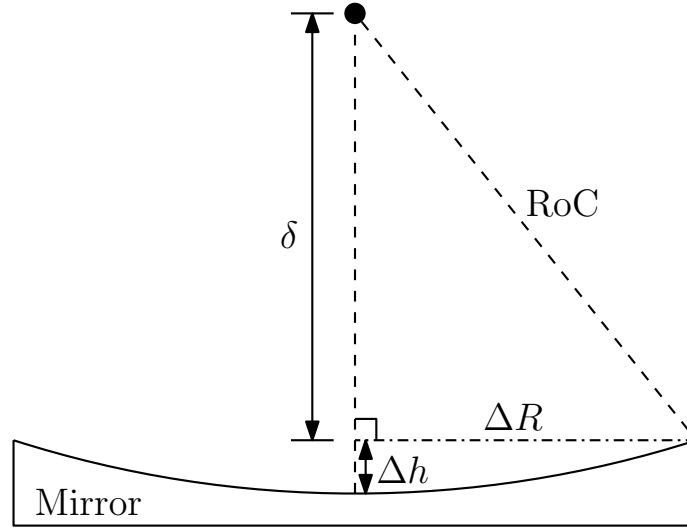


Figure 4.2: The diagrammatic form showing a spherical mirror. The radius of curvature (RoC) is shown as the hypotenuse of a right-angle triangle, from which the change in height, h , can be calculated as a function of mirror radius, ΔR .

4.1.3 Practical realisation

To perform the required measurements for the inputs of the multilateration equation and minimisation solution, the measurement system has been realised in two forms. A sequential multilateration routine, where a single laser tracker is used to measure the displacement of the target as it is moved over the sample surface. The tracker is then moved into five new positions, each time the target range displacement measurement is repeated. The measurement system has also been realised using four laser trackers to simultaneously measure the range displacement of the target. The motion system for the target is a large CMM. The trackers and the sample are positioned on, or connected to, the base of the CMM whilst the measurements are recorded. Figure 4.3 shows the setup for the four tracker measurement system.

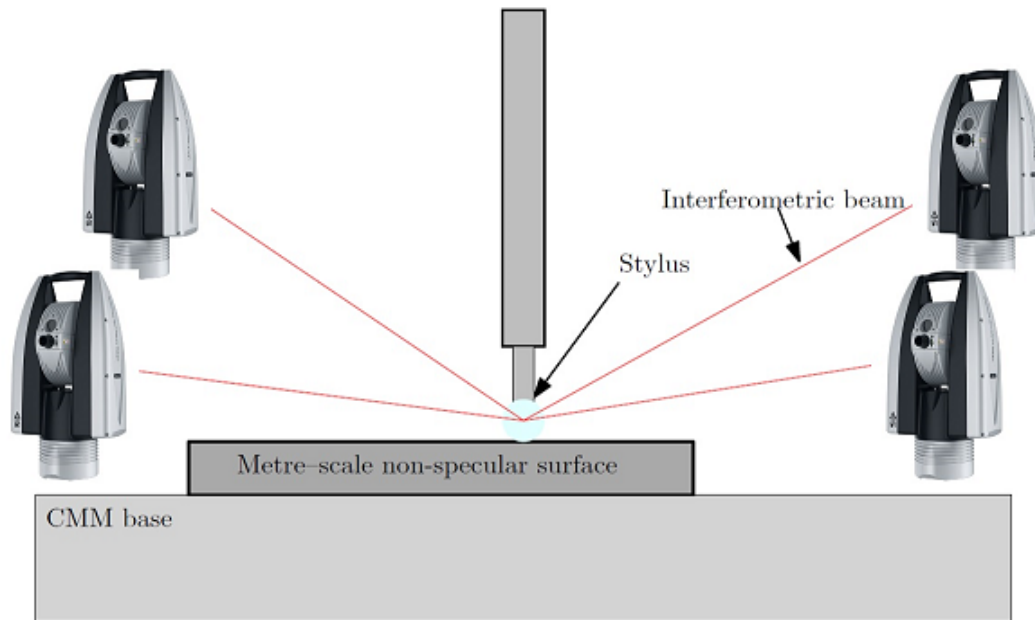


Figure 4.3: Four Leica AT960LRs tracking the centre of a large CMM stylus tip, this tip is probing the surface of large optic.

SMR-probe and position motion system

A critical part of the measurement system is the ability to control the motion of the measurement target. Here, the motion of an SMR is controlled using a large CMM — a Leitz PMM-F 1000. By replacing the ruby-tip of a CMM stylus probe with a Leica 0.5 inch Red-Ring Reflector (RRR), as shown in figure 4.4, the motion of the SMR was controllable within the accuracy to the CMM positioning capability. This has the advantage of utilising the force sensing capability of the CMM probe head to maintain ‘stable’ contact between the SMR and the sample surface. This SMR has a limited viewing angle of $\sim \pm 30^\circ$. An alternative probe was developed (see figure 5.13) that enabled the use of the high viewing angle SCE reflector; enabling the positioning of laser trackers surrounding the sample surface to be measured.

For single point probing — where CMM probing of a surface is performed on a point-by-point basis, as opposed to scanning (or dragging) the probe over a surface for continual coordinate measurement — of the sample surface, a pause was implemented in the CMM point probing controller routine, within the CMM operating software (QUINDOS®), so that the SMR is kept stationary and in contact with the surface whilst the laser trackers measured its position. The pause was implemented prior to retraction of the probe for

CMM point measuring. The probing vector was nominally at a normal to the sample surface. In this way, the trackers are able to follow and measure the CMM stylus tip as it probes the sample surface at the required positions.

The SMR-probe was qualified on the CMM using the CMM reference sphere. The qualification is not required for the multilateration measurement, but enables coincident CMM point measurement. The qualification was implemented by defining a stylus shaft protruding from the mirrored corner cube and specifying an exclusion zone on the SMR surface where the real stylus is fixed (see figure 4.4).

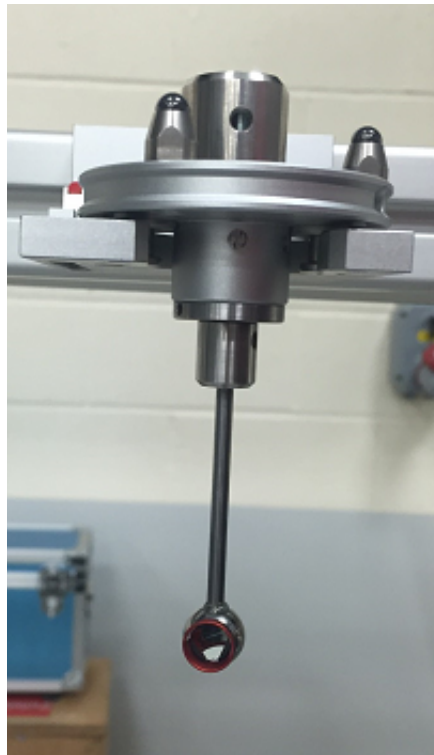


Figure 4.4: A Leica 0.5 inch RRR corner cube retro-reflector fixed to a stylus compatible with a Leitz CMM.

Commercial laser tracker

The measuring stations used for the multilateration system is the commercial Leica Absolute Tracker AT960LR. The trackers are controlled using the software Tracker Pilot®. The trackers use an ADM and angular encoders to determine an initial spherical coordinate measurement of an SMR. Provided the laser beam is not broken, the tracker will continue to measure the SMR with angular encoders and a HeNe wavelength interferometer — for range displacement — continuously as the SMR moves. The beam can be broken either by blocking of the laser path, or by reorientation of the SMR such that the laser beam directed at the SMR is outside of the SMR viewing angle. Both cases result in a loss of

fringe counting from the interferometer; therefore, the ADM must be used to produce a new initial range position and the multilateration routine should be restarted. The restart was preferred for this project due to the relatively high uncertainty in the ADM measurement of SMR range. The time independent uncertainty of this measurement is independent of the order that points are measured. To minimise the effect of time dependent uncertainty sources (drift), the measured positions should be ordered to reduce total measurement time.

Tracker Pilot® enables the programming of a laser tracker measurement of the SMR such that at every 'stable' SMR position the tracker measures the spherical coordinates (R, θ, ϕ) at a rate of 1000 Hz for 2 s–4 s. The specific length of measurement time is determined by a tracker system embedded algorithm that is dependent on the measured range. The required stability of the SMR that results in the triggering of measurement recording is defined by two adjustable parameters within the software: stable time and stable space. These allow user defined stability: the SMR is stable when it remains within a specified geometric space for a specified period of time. In this way, the trackers follow the CMM SMR–probe as it moves over the surface of the sample, and as the CMM probes a point of the surface and the pause begins, each tracker registers a stationary target and records its range displacement based on interferometric fringe counting and its angular position.

An alternative measurement process involves continuously recording the range of the SMR at a defined frequency. The SMR is measured as it scans over the sample surface with the CMM force controlled probing being utilised to maintain contact between the SMR and sample surface. A combination of tracker measurement frequency and CMM scanning speed results in a specific spatial resolution of surface coordinates measured along the scanning direction.

Vibration isolation base

Utilising a Leitz PMM-F as an SMR–probe motion system provided a stable platform for both the sample and the measuring stations. The CMM has a large granite base with active stabilisation control and vibration isolation. In addition, the CMM stands on a vibration isolated floor.

This level of vibration isolation provided a stable motion system for the SMR–probe, such that when it is in contact with the sample surface and probing pause is initiated, the SMR has increased stability. Also, as the laser trackers were mounted either directly onto the CMM base or on a platform resting on the base, the home position of the laser trackers have increased stability for interferometric measurements. Furthermore, as the CMM base has a high thermal mass, changes to its dimensions due to environmental

effects are relatively slow in comparison to the system measurement times. Details of experimental testing of these stability factors are given in chapter 5.

Environmentally controlled workspace

Experimental work was realised within an environment controlled laboratory. Specifically, the temperature was $20 \pm 1^\circ\text{C}$. Testing was undertaken to determine the effect of this level of temperature change on both short and long-term measurements.

4.2 Measurement system error sources

Identification of the error sources that affect the input parameters of the self-calibration measurement routine is critical to analysing the measurement system and simulating its measurement output uncertainties. Understanding these error sources enables estimation and testing of their quantitative effect upon the measurement system. The error sources are defined as either time independent or time dependent. The sensitivity of measurement system output uncertainties to input error magnitude can be tested via Monte Carlo simulation.

4.2.1 Interferometric displacement uncertainty

The critical measurement required to enable the multilateration calculation within the measurement system is the measured displacement of the target. In this case, target displacement is measured using the interferometric mode of a Leica AT960LR laser tracker. This tracker has been assessed under ASME B89.4.19-2006 [66, 112]. The MPE specified for the tracker displacement interferometer is:

$$U = 2 \times \sigma_L , \quad (4.7)$$

where

$$\sigma_L = u_0 + R_{i,j} \cdot u_L , \quad (4.8)$$

U is the expanded uncertainty that covers 95% (two standard deviations, $k = 2$) of the interval described by the distribution, σ_L is standard uncertainty covering one standard deviation ($k = 1$) of the uncertainty distribution, $R_{i,j}$ is the measured range from the j^{th} measuring station to the i^{th} measurement position, and $u_0 = 0.2 \mu\text{m}$ and $u_L = 0.15 \mu\text{m/m}$.

4.2.2 SMR sphericity

The displacement measurement, described above, is a measurement of the range change of the centre of the SMR relative to the tracker; therefore, the error in the sphericity of the SMR affects the coordinate measurement uncertainty of the system. This is due to the uncertainty in the distance and angle between the SMR centre and the point of contact between the SMR and the sample surface.

The Leica 0.5 inch RRR used in the implementation of this measurement system is a mirrored corner cube optic constructed from hardened stainless steel. The centring of the optic is $< \pm 3 \mu\text{m}$ and a radius of $6.35 \text{ mm} \pm 0.0025 \text{ mm}$ ($k = 2$). As shown in figure 4.5, the difference in position of the SMR nominal and optical centres results in a difference in height above the sample surface of these two points. The case that this change will be a maximum occurs if the optical centre moves from the plane parallel to the surface that intersects the nominal centre, as shown by the dashed line, through an angle defined by θ , during the measurement process. This would result in a change in height above the surface, b . If $\theta = 1^\circ, 3^\circ, 5^\circ$, a range of θ considered achievable for a range of SMR-movers from a high accuracy CMM to a robot arm, then $b = 0.05 \mu\text{m}, 0.16 \mu\text{m}, 0.26 \mu\text{m}$ respectively for the RRR. This indicates that the control of the probing motion and contact between SMR and sample surface is a factor to be considered in terms of surface coordinate measurement uncertainty. For this system implementation, the CMM probing motion repeatability and sensitivity will affect the change in θ during the measurement process.

The roundness of the RRR is $\leq 3 \mu\text{m}$ ($k = 2$). Considering the case where (see figure 4.6) $r_1 = 6.35 \text{ mm} + 3 \mu\text{m}$ and $r_2 = 6.35 \text{ mm} - 3 \mu\text{m}$: if it is assumed that, due to the manufacturing process, r decreases uniformly as θ increases, for $\theta = 3^\circ$: $\Delta r = r - r_1 = 6 \mu\text{m} \times 3^\circ/90^\circ = 0.2 \mu\text{m}$. For $\theta = 1^\circ$ and 5° , $\Delta r = 0.07 \mu\text{m}$ and $0.33 \mu\text{m}$ respectively. This shows, as for the error in SMR centring, the ability to control changes in θ during the measurement process affects both the point of contact between surface and SMR, and the SMR radius at the point; therefore, affecting the uncertainty of system surface coordinate measurement.

4.2.3 SMR motion system stability

The SMR motion system is a large CMM (as stated in section 4.1.3). The ability of the CMM to hold the SMR ‘steady’ at the measured position whilst the laser trackers record the displacement of the SMR at a specific measurement position is critical. As the laser tracker measures the SMR position interferometrically at a rate of 1000 Hz for 2 s–4 s, any motion of the SMR during this measurement induces error in the SMR coordinate

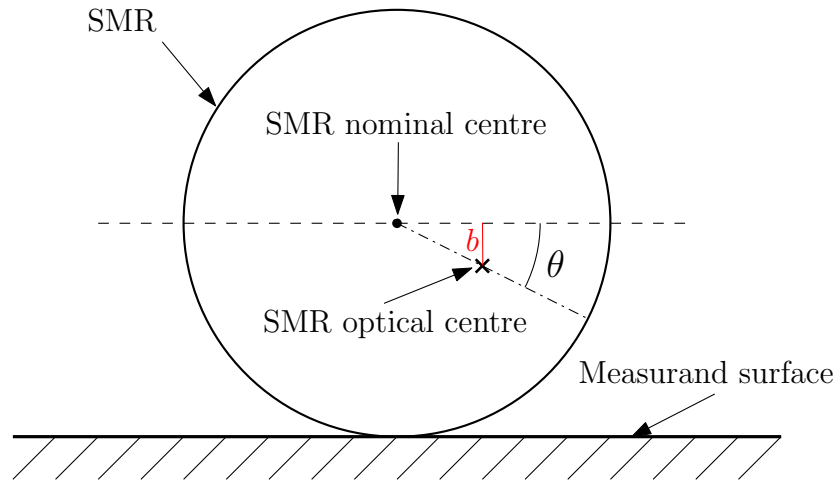


Figure 4.5: The error in coordinate measurement as a result of the difference between the nominal centre of an SMR and the optical centre and a change in θ . Length b is perpendicular to the sample surface. This length is dependent upon the angle between the sample surface and the line between nominal and optical centres, θ . A change in the length of b would result in a change in the distance between the optical centre and the sample surface.

measurement for a single point measured target position. This is not a source of error where the SMR is continuously measured throughout a measurement routine: in the case when the SMR is scanned continuously over the surface of a sample, rather than discrete point measuring. The stability of a motion system is required to be empirically tested to enable input error analysis of a specific measurement system and determination of the effect upon coordinate measurement uncertainty. The experimental testing of this error source is described within section 5.2.4.

4.2.4 CMM–tracker repeatability — sequential multilateration

As detailed in section 2.3.5 and section 4.1.3, sequential multilateration uses one laser tracker in several positions. The displacement of the SMR is measured from each of the tracker positions as the SMR is moved repeatedly into the measured positions by a probe motion system. In this case, the probe motion system is a CMM as shown in figure 4.3; as such, the repeatability of the CMM in positioning the SMR in the same measured positions for each of the tracker positions is a significant limitation to sequential multilateration. This is described in detail within [74]. Any error in the repositioning of the SMR will result in an error in the displacement measured by the tracker between each measured position on a sample surface.

Another source of error that is critical to sequential multilateration is the laser

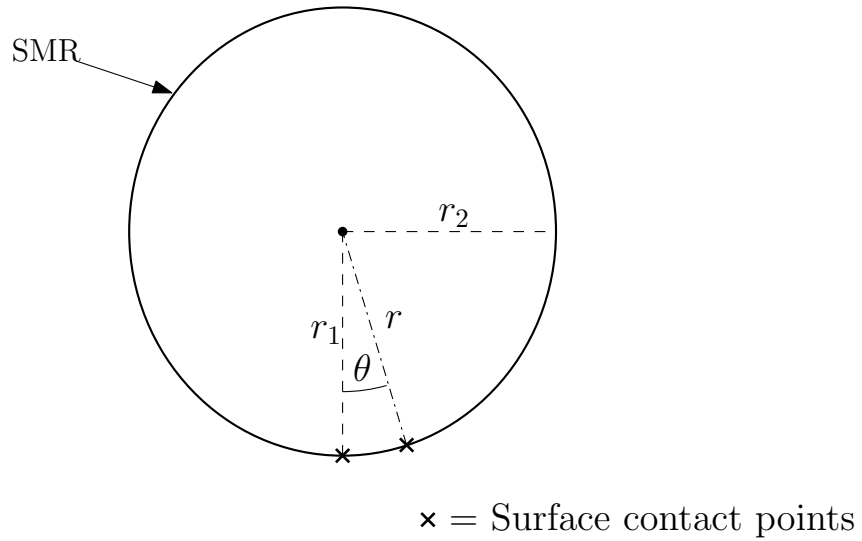


Figure 4.6: A roundness error in an SMR results in a difference between the length of r_1 and r_2 ; therefore, the radius, r , at one contact point is different to the radius, r_1 , at another contact point. As a laser tracking system nominally measures the coordinates of the centre of the SMR, the change in radius of SMR at different contact points could cause uncertainty in the surface measurement.

tracker displacement measurement repeatability; both in the short-term stability of the trackers home position as the tracker rotates to follow SMR displacement, which is contained within the interferometer uncertainty outlined in section 4.2.1, and the long-term environment changes that result in home position drift and interferometric wavelength changes that are discussed in section 4.2.11. The experimental testing of these error sources combined is described within section 5.2.5.

4.2.5 Multilateration solving algorithm

To solve the multilateration equations outlined in section 4.1.1, the nonlinear least squares approach is applied. The difference between the minimised solution of the iterative method to the equations — if the input displacement measurements had no error — and the ‘true’ coordinates of the SMR measured positions is a contribution to the uncertainty of the measurement system [109]; therefore, the choice of method to solve the simultaneous equations affects the system coordinate measurement capability. As stated, here a non-linear least squared approach is applied to solve for the surface coordinates of the measured positions.

4.2.6 Tracker positioning

The tracker positioning criteria described within section 2.3.5 are crucial to minimising the error in the self-calibrating multilateration routine.

Avoiding no solution scenarios

Takatsuji *et al.* show three cases where the self-calibration routine cannot be solved [78]. If parameters x_2 , y_3 , or z_4 — as defined in equations (2.5) – (2.8) — are equal to zero, then there is no unique solution to this equation set. The case of $x_2 = 0$ is not practically possible. For $y_3 = 0$, all of the measuring stations must lie on the x-axis of the coordinate system defined for equations (2.5) – (2.8): all trackers lie along the axis defined by the line between the home position of tracker $j = 1$, (the coordinate system origin) and the home position of tracker $j = 2$. For $z_4 = 0$, all measuring stations must lie in the xy-plane as defined by the plane intersecting the home positions of trackers $j = 1, 2$, and 3 . It follows that, if these parameters are close to zero, the ability of the solving algorithm to determine estimates for the surface coordinates of measured positions is affected.

Arrangement optimisation

As stated in section 2.3.5, Zhang *et al.* have determined an error magnification factor, k , to express the relative ability of a given arrangement of measuring stations to accurately determine estimates of the measured positions [107]:

$$k = \sqrt{\text{Trace}((\mathbf{J}^\top \mathbf{J})^{-1})} . \quad (4.9)$$

The investigation aimed to minimise k . The minimisation produced a most optimal solution, for a system with four trackers, when the trackers are arranged in a tetrahedron. For this specific arrangement, the centre of the tetrahedron circumsphere was outside the body of the tetrahedron. The angle between the vectors from sphere centre to trackers $j = 1, 2$, and 3 , and the xy-plane of the tracker arrangement was, $\theta = 70.53^\circ$. Zhang *et al.* have furthered this work with the addition of real laser tracker measurement data and further tracker arrangements [77]. These studies highlight the effect of tracker arrangement on surface coordinate measurement uncertainty.

Practical considerations

There are further considerations for this specific measurement system and measurement task. Given the nature of the sample, the measurement task requires that all measuring positions enable the tracker to point directly at an SMR whilst it probes at the measured

positions on the surface. This clearly restricts the positioning of the trackers to a hemisphere with its plane coincident with the plane of the sample surface. In reality, the tracker positions are required to be above this surface, in part, due to the non-zero laser beam diameter.

4.2.7 Number of measuring stations and measured positions and their relative arrangement

The minimum number of trackers required for displacement measurement laser-based multilateration has been shown to be four; however, a greater number of measuring positions is an option. For each tracker position that is added, the number of interferometric displacement measurements recorded increases by N – the total number of measured positions. This results in the number of equations definable from equation (4.1) increasing by N . As indicated, this additional constraint information will reduce the relative uncertainty of the self-calibration and therefore reduce the uncertainty in the estimated measured coordinates.

Addition of measured positions also enables the improvement of the arrangement of the positions: the ability to reduce the error magnification factor as detailed in section 4.2.6. Zhang *et al.* have investigated the effect of arrangement of four, five, and six laser trackers on the standard deviation of parameters estimated by self-calibration [77]. The optimal five tracker arrangement was found to be a square-based pyramid with the base facing towards the measured positions. The optimal six tracker arrangement was a triangular prism with a square face directed towards the measured positions. It has been shown that the relative uncertainty for the five tracker arrangement is 22% less than a four tracker arrangement, and the relative uncertainty of the six tracker arrangement is 12% lower than the five tracker arrangement.

The addition of measuring positions is particularly important for sequential multilateration where one tracker can be moved into many measured positions with no need for addition of equipment. Addition of measuring stations not only increases the number of measured displacements but also more data in order to average for the repeatability of the motion system/tracker measurement as outlined in section 4.2.4. There is, clearly, a trade-off between increasing measurement data and increasing total measurement time for sequential multilateration.

Section 2.3.5 shows how a minimum of 16 measured positions are required to solve equations (2.5) – (2.8) when the number of measuring stations is four and the coordinate system origin is on the surface of a sample under measurement. This is the minimum criterion scenario. To measure a metre-scale surface with high surface density, many more measured positions are required. This provides useful measurement data, and also

redundancy in the self-calibration algorithm, reducing the uncertainty of the calculated surface coordinates of the measured positions. Zhang *et al.* have shown that the relative measurement uncertainty of a multilateration can be as much as halved when the number of measured positions is doubled from a grid of 6×6 points to a grid of 12×12 points, depending on the system arrangement [77].

The larger the number of measured positions the more time it takes to complete the measurement. There is, therefore, a trade-off between measurement density and measurement time. Increased measurement times result in monetary costs, and increased measurement uncertainty due to possible instability in the measurement environment.

4.2.8 Distribution of measured positions

The positioning of points measured relative to tracker positions can have a significant effect upon the uncertainty of self-calibration of the measurement system. For the specific task of measuring metre-scale optics, many measured points are required on the optical surface; as discussed, the number of positions measured has an effect on the coordinate measurement uncertainty of the system; however, the position of these measured points is also critical to the uncertainty of the system.

The type of sample for this investigation is a relatively flat surface. There are, in principle, any number of possible orientations that the sample can be placed within the measurement system, provided that the line-of-sight between measured positions and tracker positions is preserved; however, distortion of a sample, in a specific mounting setup, due to gravity should be noted.

There is the option to add non-critical measurements of the target in positions that are not on the surface of the sample, but floating in the space above the surface. This adds a third dimension to the measured target positions. By arranging the measured positions in a 3D grid of points with equal lengths, the measurement points fill a significant part of the measurement volume in each coordinate system axis. This has not been rigorously tested experimentally for its effect upon surface coordinate measurement uncertainty. Wendt *et al.* have shown that length measurement uncertainties of less than $0.4 \mu\text{m}$ within a measuring volume of 1 m^3 are attainable with a system of four LaserTRACERs [76].

These positions affect the self-calibration in a similar way to the positioning of trackers as discussed above. Zhang *et al.* have investigated the effect of dimensions of a 2D grid of measured positions relative to the distance between measuring stations as well as the distance between the grid and the tracker positions [77]. The grid plane was parallel to the plane of the square of a five-tracker pyramid arrangement. This investigation found that when the grid side dimensions were twice the length of the pyramid square, and the distance between grid was 0.4 times the length of the pyramid square, the uncertainty of

the self-calibration was minimised.

Choosing the measured surface position within measured positions

As stated, the sample can be oriented, relative to the measuring station positions, in many ways. For the case of simultaneous multi-tracker multilateration, it is also possible for the sample to be measured at a position within a 3D grid of measurement points. This is possible by recording the displacement of the SMR as it moves to positions on and above the sample surface, followed by the removal of the sample from the measurement system, and recording SMR displacements as it is moved to positions behind where the sample surface was before. In this way the measurement volume can be expanded and the line-of-sight between tracker and target is maintained.

4.2.9 SMR viewing angle

The SMR viewing angle is a key parameter in the system setup: it dictates the size of the zone where trackers can be positioned; it also affects the number and positions of the measured positions. The RRR accepts laser beams within less than one eighth of its spherical surface. As such, the trackers must be positioned within this range if an RRR is used within the measurement system. These position limitations affect the uncertainty of the self-calibration. The SCE has a significantly larger viewing angle and therefore allows greater flexibility in positioning of trackers and measured points. Based on the criteria that affect the uncertainty of self-calibration, it is clearly desirable to implement a measurement with no limited viewing angle.

As detailed in section 2.3.3, an SMR can be constructed from a glass sphere with refractive index, $n \approx 2$, at the wavelength of 632.8 nm. When a laser beam is directed at the SMR, a portion of the light is reflected back, parallel to the incoming beam. This allows the beam to be directed at the SMR from any orientation, thus eliminating the restricted positioning of trackers due to SMR viewing angle size; however, as a result of using a material with high transparency, the intensity of the returning beam is significantly reduced. When a collimated light source is directed at an SMR constructed only from this glass, the resulting returning beam has $\sim 8.8\%$ intensity of the incoming beam. This is typically too low for laser tracking systems.

By coating a portion of the high index of refraction SMR with a ‘reflective’ material, it is possible to increase the intensity of the returning beam. An aluminium coating could result in up to 60% of the laser beam returning to the laser tracker, which should be sufficient for most laser tracking systems to function.

There is clearly some trade-off between viewing angle and light intensity, as coating

of any portion of the glass sphere stops the laser beam from entering the SMR through that portion; thus, reducing the SMR viewing angle. The resulting viewing angle size is dependent on the radius of the glass and the laser beam diameter; however, this is generally significantly larger than the RRR's viewing angle and therefore reduces the limitation of tracker positioning within the measurement system and minimises the uncertainty of the self-calibration.

Figure 4.7 shows the relationship between SMR viewing angle, SMR diameter, and interferometric laser beam diameter. The viewing angle, Θ , is given by

$$\Theta = \pi - 2\theta, \quad (4.10)$$

where

$$\theta = \arcsin\left(\frac{w}{D}\right),$$

w is the laser beam diameter (width), and D is the SMR diameter.

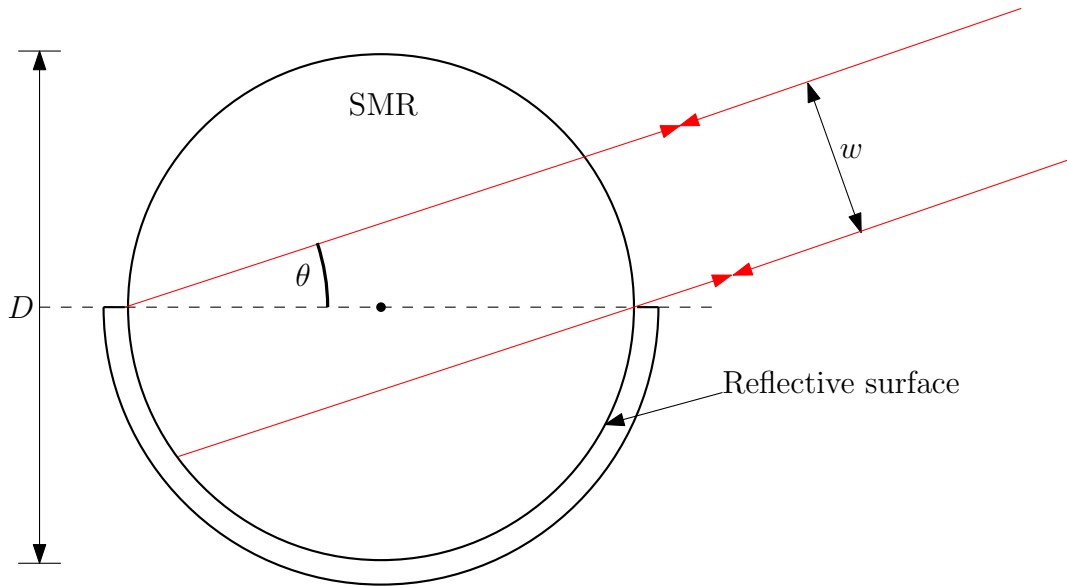


Figure 4.7: An SMR, with diameter D , constructed from $n \approx 2$ glass and one hemisphere adapted with a reflective surface. A laser beam of width, w , is oriented such that θ is minimised whilst the whole laser beam is coincident on the reflective surface.

A Leica AT960LR has a HeNe interferometer laser beam aperture diameter of 5.5 mm and a beam divergence of < 1.5 mrad; therefore, at a range of 1 m, the beam diameter is a maximum of 7.0 mm. The Leica SCE has a diameter of 18 mm; therefore, at this distance, the SCE has a viewing angle, $\Theta \approx 134^\circ$. The viewing angle may be increased if θ is reduced further from the setup shown in figure 4.7; however, the returning laser beam would have a reduced spot size due to beam clipping.

SMR reorientation

One method for quasi-increasing the viewing angle of an SMR is to move it such that the orientation of the viewing section of the SMR is pointed in a new direction. This is particularly relevant to a sequential style of multilateration. With one laser tracker being utilised in multiple measuring station positions, this allows for the SMR to be reorientated on the probe motion system between measurements: whilst the laser tracker is moved from position to position. This type of reorientation is not possible in a four (or more) tracker system as it results in beam break.

The advantage of SMR reorientation allows the positioning of measuring stations to be expanded such that the tracker can be placed in any position relative to the sample surface, provided the line-of-sight between tracker and SMR is preserved. The effect of reorientation on surface coordinate measurement uncertainty should be considered.

As introduced in section 4.2.2, changes in the point of contact between the measurand and SMR during a measurement can result in a change in the distance between SMR centre and surface contact point. Significant reorientation of the SMR result in significant changes to the position of the surface contact point. This may result in significant changes in the distance between and angle to the measured centre of the SMR from the point of surface contact.

4.2.10 Choice of coordinate system position and orientation

The minimum criterion scenario with four measuring station positions — as described by equations (2.5) – (2.8) — and 10 measured positions requires that the origin of the coordinate system be defined at the home position of tracker, $j = 1$, and the xy-plane defined by trackers, $j = 2$ and $j = 3$. In reality, the high-density measurement of a surface requires many more measurement points.

It is possible to define the coordinate system of the measurement by three points on the surface of the sample. This reduces the compounding effect of uncertainty in parameters within the self-calibration as the critical parameter for this measurement, in terms of uncertainty, is the axis normal to the sample surface. By moving the coordinate system onto the sample surface in this way, the z-axis in the coordinate system is close to parallel to the surface normal due to the low-slope surface characteristic.

Moving the coordinate system requires that equations (2.5) – (2.8) are rewritten:

$$(X_i - x_1)^2 + (Y_i - y_1)^2 + (Z_i - z_1)^2 = (d_{i1} + dp_1)^2, \quad (4.11)$$

$$(X_i - x_2)^2 + (Y_i - y_2)^2 + (Z_i - z_2)^2 = (d_{i2} + dp_2)^2, \quad (4.12)$$

$$(X_i - x_3)^2 + (Y_i - y_3)^2 + (Z_i - z_3)^2 = (d_{i3} + dp_3)^2, \quad (4.13)$$

$$(X_i - x_4)^2 + (Y_i - y_4)^2 + (Z_i - z_4)^2 = (d_{i4} + dp_4)^2. \quad (4.14)$$

The addition of unknown parameters within these equations is possible because of the large number of measurement positions and the system redundancy they provide.

4.2.11 Time dependent environmental effect

The above details error sources that are time independent. Here, error sources that occur due to the long time required to perform high-density coordinate measurements of metre-scale surfaces are discussed. Specifically, when measuring over a long period of time, the environment within which the measurement system operates is subject to changes in temperature, pressure, and humidity.

Thermal drift of tracker home position

The distance between the base of a laser tracker and its home position — where range is equal to zero — is approximately $h_S = 0.4$ m. If it is assumed that the base of the tracker remains stationary whilst it is mounted on the CMM base, then changes in h_S as a result of temperature changes in the local environment result in a change in the position of the tracker's home position. If the combined coefficient of thermal expansion for the arrangement of parts within the laser tracker is α_S , the length change of $\Delta h_S = \pm 0.4\alpha_S$.

The length change has a direct effect on the distance between the tracker's home position and the SMR target position: if the temperature of the tracker changes during a surface measurement, the nominal lengths between the tracker position and SMR positions, $R_{i,j}$, also change. This is shown in equation (4.1), a change in h_S results in a change in the coordinates of the trackers (x_{0j}, y_{0j}, z_{0j}) within the measurement system. A method for simulation of this error source is presented in section 4.3. The experimental testing of this error source is described in section 5.3.

Interferometric displacement drift

Section 2.1.3 has detailed the effect of changes in air temperature, pressure, and humidity upon the interferometric wavelength and the distance travelled by a laser beam between

tracker and target. With sufficient measurement of the environment of the beam path, it is possible to apply some level of compensation to an interferometric measurement that has been affected by these changes. The Leica AT960LR has an environment monitoring station with capability to measure refractive index at a specified point — the location of the environment measurement probe. Limitations in the compensation of these effects result in uncertainty in the measurement of target displacement.

The drift in displacement measurements is most significant when the range between tracker and target is large. The longer the beam path the greater the potential for environmental change to affect the measurement. This is also the case when the measurement time is long: the time for environmental changes to affect the displacement measurements is greater.

SMR thermal expansion

As discussed, the self-calibration is sensitive to changes in the distance, r , the SMR radius at the point of contact between SMR and sample surface. Thermal expansion of an SMR changes this distance. The RRR is constructed from stainless-steel ($\alpha_{ss} \approx 15 \times 10^{-6}$) and has a diameter of 12.7 mm: an increase in the RRR temperature of 1°C results in an increase in r of $\sim 0.1 \mu\text{m}$. The SCE is constructed from a glass sphere with a diameter $\sim 18 \text{ mm}$. A hemisphere of the glass is covered with a stainless-steel shell with outer diameter of 25.4 mm. An increase in the SCE temperature of 1°C results in an increase in r of $\sim 0.2 \mu\text{m}$.

4.3 Simulation via Monte Carlo analysis

Estimates for the output parameter uncertainties — target coordinate uncertainties and measuring station coordinate uncertainties — are determined by quantifying the uncertainty of input parameters and propagating them through a mathematical model of the system. The input parameters include: interferometric displacement, CMM positioning, and SMR sphericity.

4.3.1 Measurement system nominal setup

To simulate the measurement system and estimate the task specific measurement uncertainty, the input measurement uncertainties must be propagated through a mathematical model of the system to produce a simulated output result. Based on the mathematical model of the measurement system described in section 4.1, and the input measurement error sources described in section 4.2, the model can be established and probability density

functions assigned. Subsequently, a measurement setup may be selected, including: the position of measuring stations relative to measurement target positions, the number of measuring stations and measurement positions, and the relative positions of measurement positions.

From the selected setup, the nominal distances between each measurement position and measuring station can be calculated. This distance is the range from each tracker to the first measurement position (at $i = 1$, the interferometric deadpath), plus the displacement of the SMR between each measurement position relative to each station position.

4.3.2 Time dependent thermal effects

From the defined nominal setup of a specified measurement task, an estimation of the measurement time, and therefore the temperature change, can be made. This is typically based on *a priori* knowledge of the environment in which the measurement system is operating. Estimates of the effect of this temperature change upon the time dependent input error sources, discussed in section 4.2.11, and therefore the nominal distance measurements, are calculated. This calculation is based on the temperature change, ΔT , the tracker height change, Δh_S , the tracker position and orientation relative to the measured target positions, and the relative time that a specific measured target position is recorded.

Assuming that the north pole of the trackers are perpendicular to the sample surface, the change in range to measured positions as a result of Δh_S is given by

$$\Delta R_{h_S} = (R_{\text{horiz}}^2 + [R_{\text{vert}} + \Delta h_S(\Delta T) - \Delta r_{\text{SMR}}]^2)^{\frac{1}{2}} - R_{\text{nom}} , \quad (4.15)$$

where R_{nom} is the range to measured target position calculated from the nominal experimental setup, R_{horiz} is the length of R_{nom} projected onto the plane of the sample surface, R_{vert} is the length of R_{nom} in the axis perpendicular to the sample surface, and Δr_{SMR} is the change in SMR radius due to temperature change. It is noteworthy that Δr_{SMR} is applied in the opposing direction to R_{vert} as any change to SMR radius acts to raise the measured position in the same direction as tracker height change — based on the assumption that the trackers' north poles are perpendicular to the sample surface.

The change in range measurement as a result of changes in refractive index of the air due to temperature change is modelled by

$$\Delta R_{\text{RI}} = f(\Delta T) , \quad (4.16)$$

where $f(\Delta T)$ is a function that is based on either *a priori* information or experimental test-

ing of the effect of temperature change on a laser tracker's interferometric displacement measuring.

The temperature dependent range measurements are therefore defined by

$$R(\Delta T) = R_{\text{nom}} + \Delta R_{h_s} + \Delta R_{\text{RI}} . \quad (4.17)$$

4.3.3 Uncertainty distributions and Monte Carlo simulation

From equation (4.17), simulated measured displacements are calculated using the input uncertainty distributions of error sources, detailed in section 4.2, and summing:

$$d_{i,j \text{ sim}} = d_{i,j \text{ nom}} + (u_0 + R_{i,j \text{ nom}} \cdot u_L) \cdot P_{\text{int}} + u_{\text{sph}} \cdot P_{\text{sph}} + u_{\text{CMM}} \cdot P_{\text{CMM}} , \quad (4.18)$$

where $d_{i,j \text{ nom}}$ and $R_{i,j \text{ nom}}$ are nominal displacement and range respectively including thermal effects. P_{int} , P_{sph} , and P_{CMM} are pseudo-random numbers generated with a normal distribution. u_{sph} is the standard uncertainty of the SMR sphericity error and u_{CMM} is the standard uncertainty of the CMM-tracker positioning repeatability. The terms with CMM-tracker repeatability are only included in the case where the measurement system operates with a single tracker and sequential multilateration. To produce a large sample (n_{samples}) of simulated displacement measurements for Monte Carlo analysis, equation (4.18) is calculated many times. In this way, a combined pdf of input parameters is estimated. This is used to estimate probability density functions of the sample coordinates.

4.3.4 Error propagation and coordinate measurement uncertainty

The displacements are summed with an initial value of interferometric deadpath and put into a multilateration function defined by equation (4.1). Initial values for measuring station coordinates and measured position coordinates are also input. The *lsqnonlin* solver is applied to reduce the residuals, w_{ij} , whilst keeping the simulated range measurements fixed. In this way, the input parameter distributions are propagated through the model to determine empirical estimates of output pdfs.

Monte Carlo analysis

The multilateration function outputs estimates of measured position coordinate based on the residuals minimisation. There are n estimates of each outputted parameter. The difference, d_i , between each outputted coordinate estimate and the nominal coordinates

of the original system setup are calculated. The means of these differences, $\bar{d}_i$, for each measured position is an estimate of the measurement error at that position. The value of d_i is a result of both systematic and random error sources. Error as a result of long-term thermal drift in the measurement system result in a systematic time dependent error in the coordinate measurement. This error source, along with other systematic errors, can be compensated to some level by the removal of $\bar{d}_i$ for each measured position.

The standard deviations of d_i provides an estimate of the standard uncertainty of each measured position coordinate, $u(d_i)$. The standard uncertainty and the residuals, w_{ij} , of each measured position coordinate provides a prediction of the capabilities of the system given the selected input parameters and measurement setup. The critical axis for measurement uncertainty analysis is the axis normal to the sample surface. The coordinate system of the measurement system is defined such that the normal axis is equivalent to the system z-axis; therefore, the standard uncertainty in this axis, σ_z , provides a specific parameter for surface measurement capability.

4.3.5 Testing sensitivity to input parameter

The sensitivity of the critical output measurement uncertainty to change in the input parameters, outlined in section 4.2, is tested by varying the input parameters over a defined bandwidth. The Monte Carlo (MC) simulation is run for each input parameter value. The comparison between output z-axis uncertainty for each input parameter indicates the measurement's sensitivity to change in a specific correlated or uncorrelated parameter. The sensitivities of the multilateration function to change in the coordinates of measured positions are also expressed by the sensitivity coefficients within the Jacobian matrix of equation (4.5). From the sensitivity analysis, the parameters that cause the most significant uncertainty in the critical z-axis are identified; therefore, where possible, changes to the measurement system can be applied in order to reduce the uncertainty and increase the capability of the system in a specific measurement task.

4.3.6 Simulation of real measurements

The validity of the MC simulation — for the laser-based multilateration system — is tested by comparing data recorded from a real measurement setup to simulated results of the same measurement setup and the same measurement routine.

The measuring station positions and measured target positions are input from the real measurement. The measured positions are subsequently altered to produce a 'nominal' measured position set: for example, if the sample surface is a flat optic, the real measured points are altered to produce a perfect nominal flat coordinate measurement. These

coordinates are used in the same way as the general MC simulation to determine simulated displacement measurements based on measurements of a perfect flat. In this way, the measurement setup is the same as the real experiment, but the measurement is simulated based on a perfect flat sample; therefore, the error in the simulated form measurement is as a result of system measurement error only. The temperature of the environment measured during the real experimental measurement is input to simulate the same temperature changes as the real measurement.

The results of the real measurement in the critical parameter of the z-axis, as well as other parameters, are compared to the equivalent results for the MC simulation. The comparison of both sets of results enable an analysis of the capabilities of the MC simulation for a given experimental measurement system setup; therefore, the degree of confidence in simulations of measurement system setups is informed for the case, where no real experiment has been undertaken. The comparison must take into account the form error of the sample surface used in the real measurement.

4.3.7 Predictive Monte Carlo simulations

Where the degree of simulation confidence is high as a result of an accurate system error model, Monte Carlo analysis (MCA) can be used to predict measurement setups and routines that produce low error in the critical measurement parameters based on a specific sample. Based on a specific measurement task, an initial measuring station arrangement can be tested within the MCA and then iterated to reduce measurement uncertainty in the critical parameter.

4.4 Summary

The measurement system is fundamentally summarised through equation (4.1) and is critically dependent on the ability to solve this equation precisely. By treating this as a non-linear least squares minimisation problem and defining the Jacobian matrix, solutions to equation (4.1) can be determined and the sensitivity of the solution to some parameters can be shown.

From the practical realisation of the laser-tracking-interferometer multilateration system, the input errors have been identified and explained. Some input parameters are dependent upon: the experimental setup, the specific sample, and the choice of SMR. These are fixed throughout the measurement process. Other error sources are time dependent due to changes in the environment where the measurement system is operating. In order to investigate the uncertainty of the measurement system, a thorough analysis of the input error sources should be undertaken and the sensitivity of the measurement

system to these error sources should be determined. This type of analysis enables the optimisation of a measurement system; furthermore, from the identification and analysis of input error sources a set of experiments can be designed to test these parameters. Experimental testing can be conducted on specific input parameters where they are separable. Multiple parameters can be tested together where they are not separable. These correlated error sources can be tested directly as input parameters or by changing the inputs over a specified bandwidth and comparing output coordinate measurements.

A method for predicting the effect of input error sources upon the measurement system coordinate measurement uncertainty is a MCA. By selecting a nominal experimental setup and measurement routine, a set of system input values can be calculated and propagated through the mathematical model of the system to estimate the error in outputted coordinate measurements. Repeating this process many times with randomly varied input ranges produces many estimates of the system error. The distributions of these errors can be used to estimate the standard uncertainty of coordinate measurements.

The MCA requires estimates of input measurement parameter uncertainties. Some of the error sources that affect the input measurements can be determined analytically or with *a priori* knowledge, whilst others require testing experimentally. The validity of an MCA can be tested by comparison to a real measurement result performed with any given measurement setup.

Chapter 5

Experimental design and procedures

The experimental design and procedures for the testing of the laser-based multilateration system are described herein. The initial experiments focused on testing the setup parameters of the measurement system: testing time for the tracker to reach thermal equilibrium and the tracker measurement of an SMR-probe installed on a CMM-mover. The time independent input error sources were investigated to determine estimates of these parameters for MC simulation of the measurement system to estimate the measurement uncertainty associated with coordinate measurements; these include testing of: the laser tracker interferometer, SMR reorientation, SMR-probing on the CMM, CMM probe pause and triggering of tracker measurements, and CMM-tracker positioning repeatability. Similarly, the time dependent input error sources were also investigated; the effect of temperature change on tracker home position stability and interferometric displacement measurements was tested. A four-tracker multilateration system was tested via measurement of two sample surfaces: a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic and a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. The system was tested in several measurement setups with both an SCE and RRR as measurement targets. The effect of a long-term measurement with this system was tested by measuring the large optic over a 15.5 h time period. CMM measurements were performed for comparison the multilateration coordinate solutions. MC simulation of the real experiments was carried out. The mathematical model of the measurement system was also used to test the sensitivity of the output coordinate solution to various input parameters.

5.1 Initial experiments

A series of tests were required to determine the basic parameters for operation of the measurement system. The time for the tracker to be in thermal equilibrium with the operating environment after turn-on was required to be determined. This was so that

measurements were not affected by tracker temperature changes as a result of heat generated by tracker systems. The ability of the AT960LR to continuously measure the spherical coordinates of an SMR-probe as a CMM moved the probe in a standard probing motion was also tested. A standard probing motion for the CMM uses the default settings within the CMM controller, the probe is stopped by the CMM when a probe deflection is detected, the probe is held on the surface for 20 ms before the probe is retracted and 10 coordinates are recorded – from which a point measurement is determined. The synchronisation of tracker measurements of an SMR and the stable time that the CMM was required to hold the SMR-probe in contact with a sample was also investigated and optimised to reduce the total measurement time. The optimisation involved reducing the pause time to reduce the total measurement time.

5.1.1 Tracker ‘warm-up’ time

Analysing the thermal stability of the measurement system is critical to determining its measurement capabilities. A critical thermal issue for laser tracking systems such as the AT960LR is the time between turning on a tracker and relative interferometric measurement stability. When power is supplied to a laser tracker and it is powered up, heat is generated by both the angular motors and the range measuring systems – ADM and IFM. This heat may cause the dimensions of the tracker to change (internally and/or externally); therefore, it is critical to measure the time for the heat generated to balance with heat loss to the surroundings. This time is dependent upon the temperature of the working environment and the balance between heat generated and heat loss.

Method

An AT960LR was fixed to the large granite base of a Leitz PMM-F CMM. A 0.5 inch RRR was placed at a range of ~ 0.45 m on the CMM base. The RRR was supported by a three-point magnetic holder. Power was supplied to the tracker and the tracker controller was used to turn it on. The tracker was locked onto the RRR as soon as the controller would allow: the tracker locking onto the RRR and measuring its range defined the start of the warm-up period. The range measurement was recorded: measurements were triggered at a rate of 0.5 Hz with range measurement at 1000 Hz for 1 s, the mean of the range measurements was recorded. The measurements were continued for the time period require to reach relative measurement stability. Relative measurement stability is achieved when the range measurement is stable within the MPE of the IFM for 30 minutes – for a range of ~ 0.45 m the MPE is ~ 0.5 μm (see section 2.3.2). The air temperature was measured throughout the range measuring with both the AT960LR weather station and

a type-K thermocouple and Pico Technology TC-08 data logger [113]. This experiment assumes that the laser tracker has been within the local environment for the time require for the laser tracker temperature, prior to power-up, to equal the air temperature. The setup of the experiment is shown in figure 5.1.

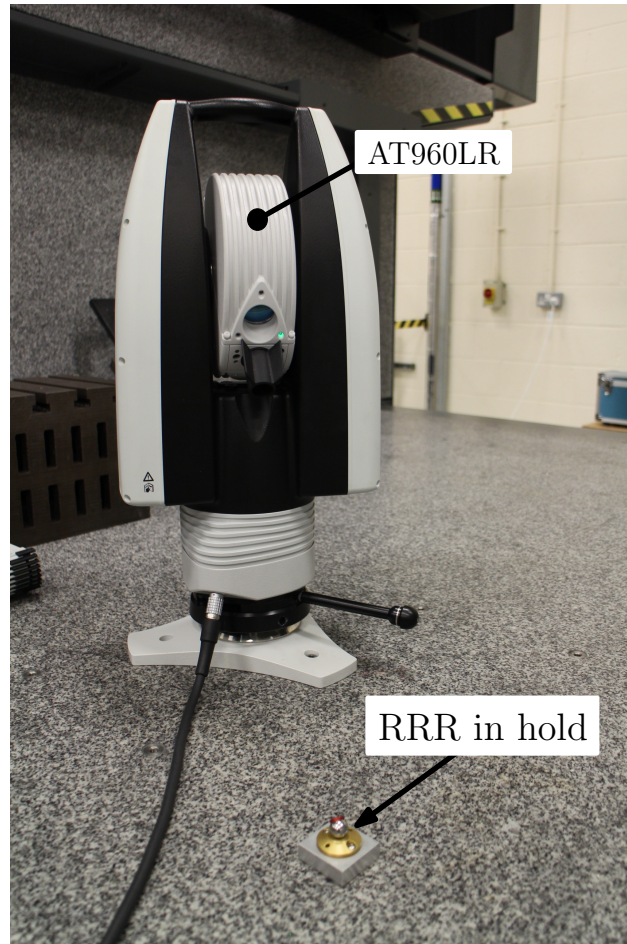


Figure 5.1: An AT960LR laser tracker fixed to the large granite base of a Leitz PMM-F 1000 CMM. The tracker is locked onto an RRR and measuring its spherical coordinates at a rate of 1000 Hz. The RRR has been placed in a three-point magnetic hold that is fixed to the CMM base.

5.1.2 Tracker measurement of CMM probing

Testing of the range measurement of the CMM probing motion enables analysis of the laser tracker's capability to measure the SMR as it is in contact with the sample surface. This measurement indicated whether any changes to the CMM controller probing algorithm was required to enable range measurements of measured positions upon the sample surface.

Method

An SMR-probe (0.5 inch RRR) was installed onto a Leitz PMM-F 1000. An AT960LR was fixed to the base of the CMM. The CMM was programmed, using QUINDOS®, to probe the CMM base with probing parameters set as standard. The tracker was locked onto the RRR and the set to continuously measure the RRR spherical coordinates at 1000 Hz. The CMM probing motion and the tracker measuring where set to begin at the same time. Figure 5.2 shows the tracker setup on the CMM whilst measuring a moving SMR-probe.



Figure 5.2: An AT960LR laser tracker fixed to the large granite base of a Leitz PMM-F 1000 CMM. The tracker is locked onto an SMR-probe that is moving to probe the surface of the CMM base. The tracker is measuring the position of the SMR in spherical coordinates.

5.2 Testing of time independent error sources

The testing of time independent error sources, as described in section 4.2, was undertaken to determine their effect on the multilateration system's coordinate measurement solution uncertainty. The tests included: a buck-in buck-out test of the laser tracking interferometer; testing the effect of SMR reorientation on range displacement measurements; CMM measurement comparison of SMR-probes; testing of the CMM pausing and laser tracker measurement triggering; and CMM-tracker positioning repeatability that affects sequential multilateration only. These tests are used to inform the magnitude of inputs for propagation through the mathematical model of the system in MC simulation. A test of the minimisation algorithms ability to solve the multilateration equations was implemented by utilisation of a nominal measurement setup.

5.2.1 Buck-in buck-out test of interferometer

A buck-in buck-out test of the interferometric displacement measurement is a test of the length measuring capability of the interferometric mode of an AT960LR. The test involved the measurement of two SMR positions that were coincident with the equatorial plane of the laser tracker under investigation. The measured positions were spaced by a distance to be measured. Figure 5.3 illustrates the two sets of measurements undertaken.

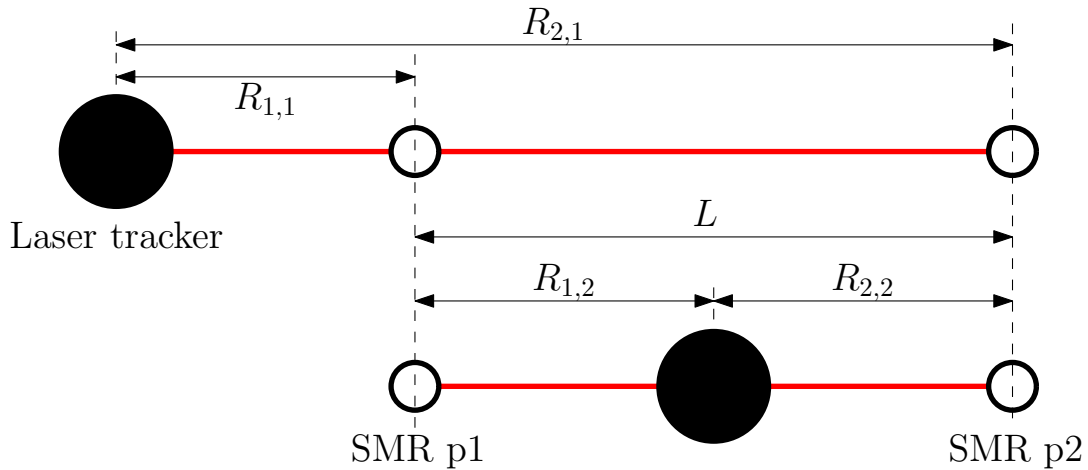


Figure 5.3: The buck-in buck-out test requires two interferometric displacement measurements from a single laser tracker: the measurement of the distance, L , by measuring the ranges, $R_{1,1}$ and $R_{2,1}$; and measuring L by the range measurements, $R_{1,2}$ and $R_{2,2}$.

The comparison between the two displacement measurements for the same distance, L , gives an indication of the error in interferometric displacement measurements that require a laser tracker to rotate over large angles [65, 114]. The R0 error of a laser tracker

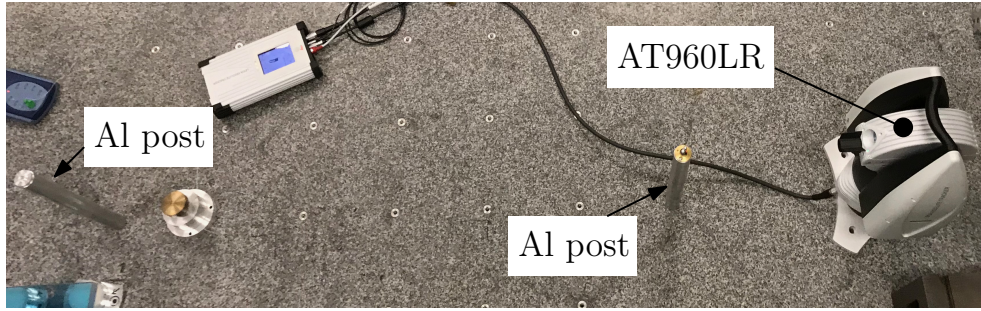
becomes clear from this measurement: the R_0 error is the error in determining the distance from an SMR to a laser trackers geometric origin as stated within ISO 10360 10 [65].

The sources of uncertainty in this displacement measurement are the IFM range displacement measurement uncertainty and the SMR sphericity. The measurement of $R_{2,1}$ has highest uncertainty due to its length. The SMR sphericity may affect the result of L as the SMR is positioned at p1 and p2 for each measurement may result in a different distance between measured SMR centres.

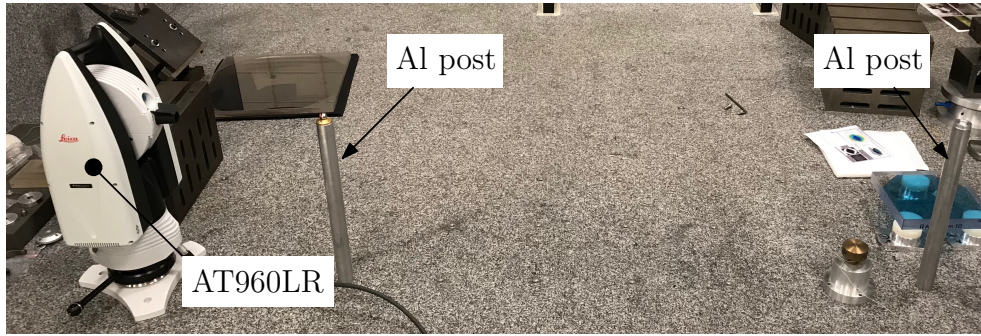
Method

Two aluminium posts with height equal to the height, h_S , of an AT960LR — h_S is the distance from the tracker's base to the home position — were fixed to the base of a Leitz CMM at a distance, L , of ~ 1.2 m. The posts had three-point contact holds installed on the posts that were compatible with a 0.5 inch RRR. The RRR holds define SMR p1 and SMR p2.

To measure $R_{1,1}$ and $R_{2,1}$, the laser tracker was placed in position outside the posts as shown in figures 5.4a and 5.4b. The angle between the vectors defined between the tracker home position and the two SMR positions was minimised [114]. The RRR was placed at p1 and the laser tracker was locked on to measure $R_{1,1}$. This measurement was implemented in Tracker Pilot with stable point measurement: at 1000 Hz for 2 s–4 s. The RRR was then moved, whilst the laser beam path was maintained, to p2 and $R_{2,1}$ was measured as a stable point. This displacement measurement of L was repeated 5 times.



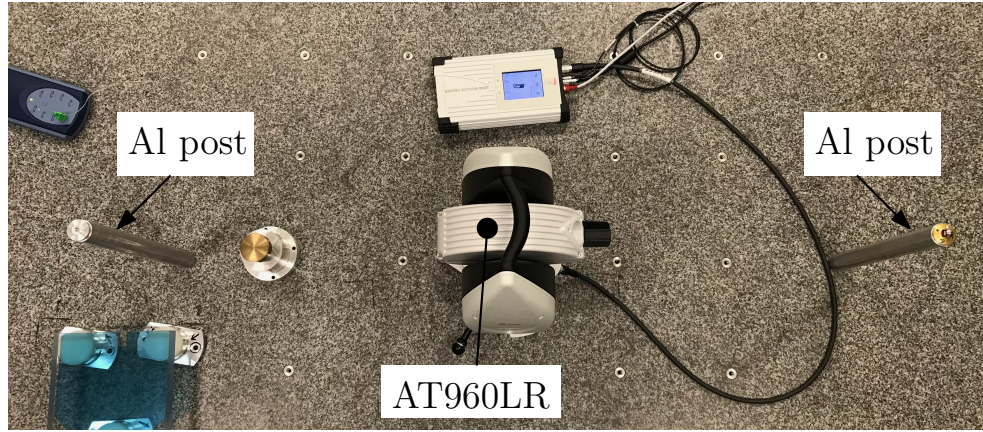
(a) Buck-out testing: top-view



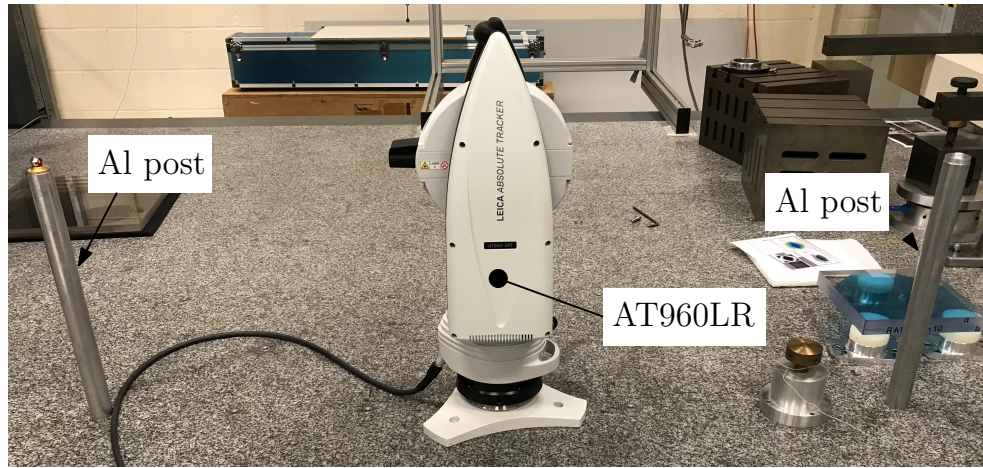
(b) Buck-out testing: side-view

Figure 5.4: (a) A top-view of the buck-out test of a Leica AT960LR interferometer. The laser tracker is placed outside the two posts fixed to the base of a large CMM. (b) A side-view of the buck-out test of a Leica AT960LR interferometer.

To measure $R_{1,2}$ and $R_{2,2}$: the laser tracker was placed in position inside the posts as shown in figures 5.5a and 5.5b. The angle between the vectors defined between the tracker home position and the two SMR positions was set as close to 180° as practical [114]. The RRR was placed at p1 and the laser tracker was locked on to measure $R_{1,1}$. The RRR was then moved, whilst the laser beam path was maintained, to p2 and $R_{2,1}$ was measured as a stable point. This displacement measurement of L was repeated at total of 5 times.



(a) Buck-in testing: top-view



(b) Buck-in testing: side-view

Figure 5.5: (a) A top-view of the buck-in test of a Leica AT960LR interferometer. The laser tracker is placed between two posts fixed to the base of a large CMM. (b) A side-view of the buck-in test of a Leica AT960LR interferometer.

5.2.2 SMR reorientation test

Reorientation of the SMR throughout the measurement of a surface may result in a change in the distance between measured SMR position and the point of contact between SMR and the sample surface (see section 4.2.2). Testing the effect of SMR reorientation on an interferometric displacement measurement of an SMR held in a three-point hold, shown in figure 5.6, indicates the magnitude of possible combined changes to b and r (from figure 4.5 and figure 4.6 respectively). This empirical analysis supplements the theoretical analysis within section 4.2.2.

The test of reorientation of an SMR, specifically within a three-point hold, also gives an indication of the magnitude of this uncertainty source in the testing of interferometric displacement measurements as detailed in section 5.2.1: as the SMR is required to be reorientated — $\sim 180^\circ$ rotation about the axis projected from the top of the posts —

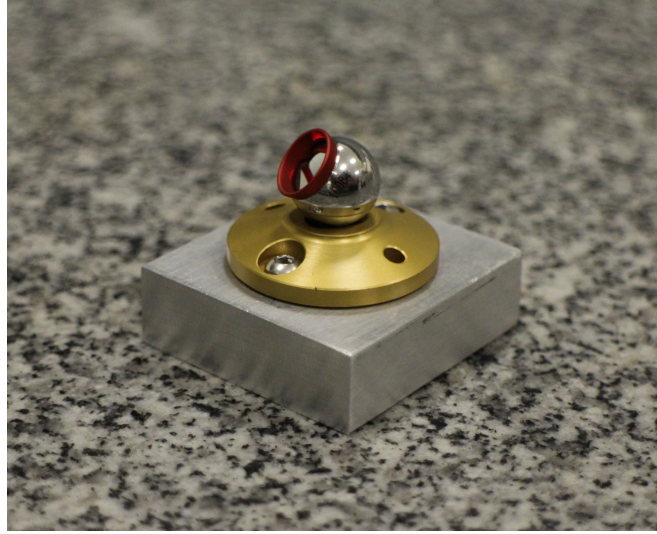


Figure 5.6: An RRR place in a three-point magnetic hold. The hold is fixed to an aluminium plate that enables the hold to be fixed to the granite base of of a CMM.

when moved from one post to the other for the case where the laser tracker is measuring displacement from inside the posts.

Method

The experimental setup for the SMR reorientation testing was the same as shown in figure 5.1: the laser tracker was fixed to the CMM base and an RRR was positioned on the base with a three-point hold at a range of ~ 0.45 m. The tracker was locked onto the RRR and its spherical coordinates were recorded using the stable probing measurement for 10 s. The RRR was then manually reorientated within the three-point hold. The reorientation was through as large an angle as practical, such that it did not cause the laser beam to be coincident with the RRR outside the viewing angle. The reorientation was made up of both vertical and horizontal rotation. The measurement and reorientation were repeated a total of 10 times. The range measurement was repeated a further 10 times, however, the SMR was not reorientated between measurements: this was to provide a comparative measurement of the range measurement repeatability for the measurement setup when the SMR orientation was constant.

5.2.3 SMR-probe CMM testing

As a CMM moves to probe a surface at a measurement point, the stylus tip makes contact with the sample surface and causes a deflection. The probe deflection is detected by the CMM probe head and the motion of the probe is stopped. At this point a pause was implemented in the CMM controller to hold the probe stationary whilst a laser

tracker records the range measurement to the SMR of the probe. After the pause the CMM retracts the probe from the surface. The retraction is based on the deflection of the probe from first contacting the sample surface. As the probe head moves the probe–tip away from the surface, the point of contact between probe–tip and surface changes and the probe position is recorded by the CMM. A comparison between points measured on the surface of a sample by a CMM with a standard stylus probe ruby–tip – typically with roundness $\sim 0.4\text{ }\mu\text{m}$ – and points measured with the SMR–probe indicate the effect of SMR sphericity upon the surface measurement error. The deflection caused by contact between surface and CMM results in a reorientation of the SMR. Determining an estimate for the effect of this magnitude of reorientation upon the surface is similar to the magnitude of reorientation that occurs between laser–based multilateration measurements.

Method

A $0.4\text{ m} \times 0.4\text{ m}$ flat optic was mounted on the base of a CMM with three point–stands, as shown in figure 5.7. The CMM was configured with an SMR–probe (RRR and SCE) and a 10×10 grid of equally spaced points was measured on the flat surface of the optic. The measurement was implemented in QUINDOS® with a standard probing motion and repeated a total of three times per probe type. The CMM was reconfigured with a standard ruby–tipped stylus ($\varnothing 8\text{ mm}$) probe and the measurement of the optic was repeated a further three times.

5.2.4 CMM probe pause and tracker measurement triggering

To measure a single position of a stationary SMR, the AT960LR can be controlled to measure with a stable probing technique. This technique allows the stable space and stable time to be user defined such that the tracker measurement is triggered. It is also possible to implement a pause in the probing motion of the CMM when the force sensitive probe head detects contact between the SMR–probe and a sample surface. The stability of the CMM probe pause affects the coordinate measurement of the SMR. Two aspects of this process, therefore, required testing: the stability of interferometric measurement of the SMR whilst paused in contact with the surface and the pause–time required to trigger the tracker measurement and make a full stable point measurement at 1000 Hz for 2 s–4 s.

To determine the optimal parameters in terms of measuring the SMR surface coordinate with synchronicity and minimising the total measurement time, an iterative approach was undertaken with many combinations of parameters tested.

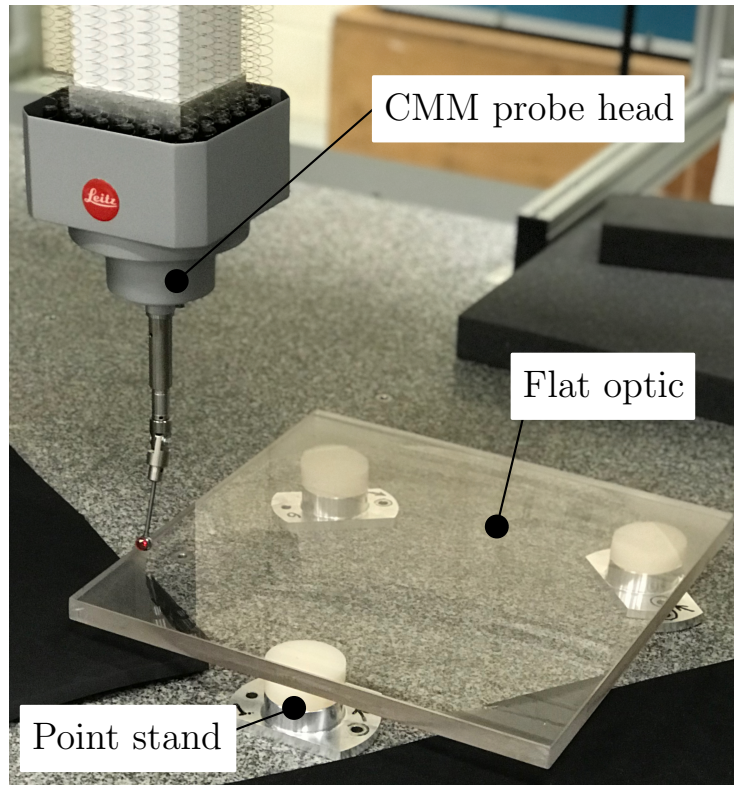


Figure 5.7: A CMM configured with an SMR-probe (RRR) as it moves into measurement positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. The optic is mounted on the CMM base with three point-stands.

Method — pause stability

The same setup as described above for tracker measurement of CMM-probing was used, however, a pause was implemented into the CMM probing routine. The pause was defined within the controller of the CMM. The AT960LR measured interferometrically at a rate of 1000 Hz as the CMM moved the SMR-probe into the surface and the pause in the motion was implemented.

Method — measurement triggering

CMM-pause, stable time and stable space parameters were varied to find a set of values that enabled the SMR-probe to make contact with the sample surface, become stable, and a tracker's interferometric range measurement to be triggered and carried out before the CMM moves the probe away from the surface. This was also varied with the aim of minimising the total measurement time.

5.2.5 CMM–tracker positioning repeatability

As discussed in section 4.2.4, the nature of sequential multilateration induces the error source of motion system positioning repeatability. The repeatability of measured positions for the measurement system was tested by repeatedly measuring the same measured positions and measuring the range displacement from a given laser tracker measuring station position. At the same time, the CMM was used to measure each point repeatedly. The range measurement provides a measure of the combined effect of CMM–tracker positioning repeatability; and, therefore, gives an indication of the magnitude of u_{CMM} from equation (4.18). Using the CMM measurement system to measure its own positioning repeatability gives an indication of the contribution of CMM positioning repeatability to the uncertainty, u_{CMM} .

The two combining sources of error that result in the difference in measurements for repeated positions are: CMM positioning repeatability and laser tracker interferometric displacement measuring repeatability. The difference between u_{CMM} and the standard deviation of CMM positioning gives an estimate of the effect of laser tracker range measuring repeatability; hence, this test supplements the buck-in buck-out test in aiming to determine the interferometric range measuring repeatability. Furthermore, the measurement of surfaces with different orientations relative to the coordinate system of the CMM tests the repeatability of positioning in different motion axes.

Repeating the measurement of a specific sample surface many times, per measuring station position, for a set of measured positions enables the averaging of measurement results and, therefore, the correction of random error. This comes at the expense of measurement time and may result in significant time dependent measurement error.

Method

A $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic tilted in a steel hold at 45° from the plane of the CMM base, as shown in figure 5.8. The CMM was configured with an SMR–probe (RRR). An AT960LR was setup and locked onto the RRR, and after sufficient warm–up time, the CMM moved the probe to measure a 4×4 grid of points was measured. A pause was implemented in the probing routine such that the laser tracker could record a stationary point range measurement before the CMM moved the probe to the next measured position. QUINDOS[®] was used to control the CMM and record the CMM measured positions in the CMM coordinate system. These measurements were repeated a total of 10 times. The grid was measured by the CMM and laser tracker interferometer independently.

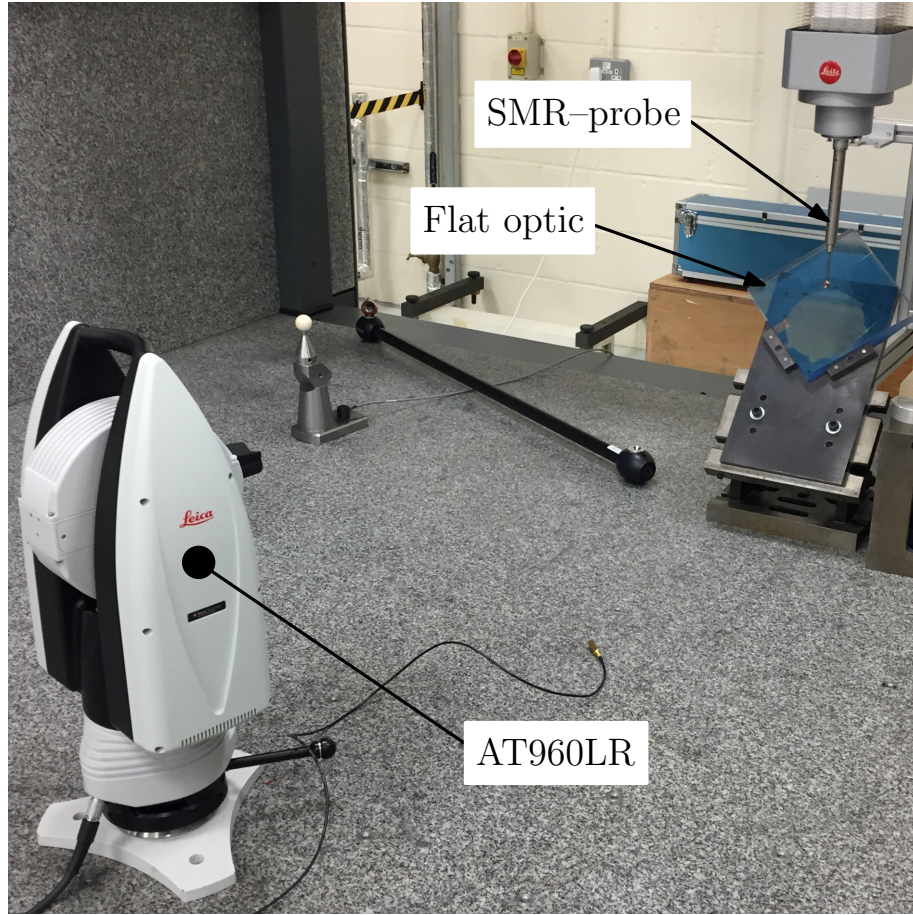


Figure 5.8: An AT960LR laser tracker fixed to the large granite base of a Leitz PMM-F 1000 CMM. The tracker is locked onto an SMR-probe that is moving towards a measured position on the surface of a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic. The optic is tilted in a steel hold at 45° from the plane of the CMM base.

5.2.6 Multilateration solving algorithm validation

To test the ability of the multilateration measurement principle, described in section 4.1.1, to solve equation set (4.1) when implemented within a minimisation utilising the MATLAB[®] function *lsqnonlin*, a nominal measurement setup was characterised in a Cartesian coordinate system: the coordinates of four laser tracker measuring stations and a set of measured positions were defined.

The Cartesian coordinates were used to calculate a set of displacements with effectively no error. These displacements were input into the MATLAB[®] minimisation function to solve the equation set (4.1). The difference between the coordinates determined by the minimisation and the coordinates used to calculate the displacements gives some indication of the ability of the solving algorithm to determine an accurate measurement solution.

5.3 Testing of time dependent error sources

As detailed in section 4.2, the stability of interferometric range measurements is affected by time dependent change in temperature. This change affects the laser trackers home position and the interferometric wavelength; as such, a series of tests to determine the magnitude of these effects were carried out.

5.3.1 Tracker home position stability

It is possible to use the interferometer of a laser tracker to measure the change in tracker home position, due to change in h_S , as a function of time and temperature change. This can be achieved by a comparison between a long-term range measurement of an SMR in two positions: one position on the base of the CMM close to the laser tracker base, and another on a post with height equal to h_S . The range of the two positions should be equal. In the case where the SMR is on the post, any change in h_S as a result of temperature change has a non-significant affect upon the range measurement due to the small-angle change – as the SMR is in the plane of the laser tracker’s equator, expansion in the laser tracker height occurs in an orthogonal direction to the range measurement. For, example, if the distance between the SMR on the top of the post and the laser tracker home position, $D = 0.2$ m, the post height is $h_{\text{post}} = 0.4$ m, and the coefficient of thermal expansion for the post is 20×10^{-6} : for a temperature change, $\Delta T = 1$ K, the change in D is 1.6×10^{-10} m. It is noteworthy that any thermal expansion of the SMR results in the height of the SMR centre changing in the same direction as change in h_S .

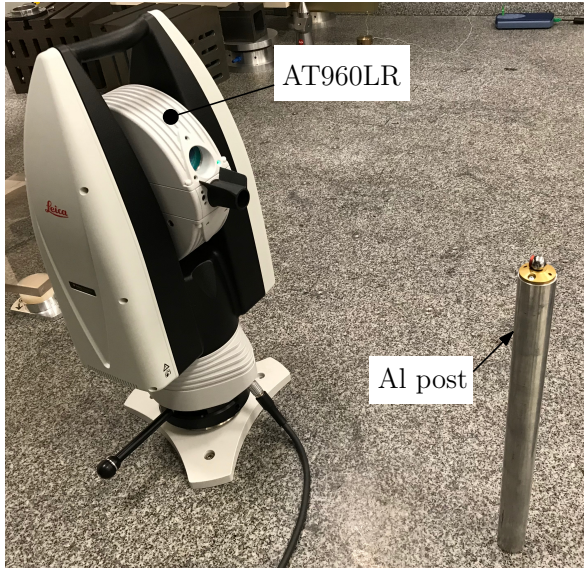
If a change in temperature cause the CMM base to change shape, the measured range may be affected. For the setup with the SMR on a post, if the coefficient of thermal expansion of the granite base is 8×10^{-6} and $D = 2$ m, a long-term temperature change of 1 K would result in a change in D of $16 \mu\text{m}$; however, this type of temperature change was not anticipated during these experiments. It was expected that the temperature changes would have relatively high frequency and overall long-term magnitude to be relatively small. Given the large mass of the CMM base, change in D as a result of thermal expansion in the granite base was expected to be $\ll 16 \mu\text{m}$.

Method

For the range measurement of an SMR at height equal to h_S , a 0.5 inch RRR was installed on a three-point magnetic hold that was fixed to the top of an aluminium post with height such that the SMR height was as required. The post was fixed into the base of a CMM. The laser tracker was fixed directly to the same base. Figure 5.9a shows the experimental setup. After sufficient warm-up time for the laser tracker, a measurement

of the range to the RRR was recorded for ~ 5 h. The measurement was triggered at a rate of 0.5 Hz with the measurement of range at 1000 Hz for 1 s, the mean of the range measurements was recorded.

To measure the change in h_S , an RRR was positioned on the base of the CMM within the three-point hold and moved as close to the base of the laser tracker as practical. The range measurement was undertaken at the same rate and for a similar time period as for the post measurement. The experimental setup is shown in figure 5.9b. The air temperature was measured throughout the range measuring with a type-K thermocouple and Pico Technology TC-08 data logger.



(a) Range measurement at tracker height.



(b) Range measurement of tracker height change.

Figure 5.9: (a) An AT960LR fixed to the granite base of a large CMM. The tracker is locked onto and measuring an RRR in spherical coordinates. The RRR is placed in a three-point hold that is supported on an aluminium post. The post height is such that the RRR height is equal to h_S (the distance between the tracker home position and the tracker base). (b) The laser tracker is locked onto an RRR positioned on the base of the CMM and as close to the tracker base as practical.

5.3.2 Effect of change in refractive index

It is possible to isolate the effect of change in RI on interferometric range displacement measurements by recording the range to a stationary SMR placed in the plane of the laser tracker's equator, as discussed above: change in the recorded range is due to change in the RI of the air that the laser beam traverses. There is a length component to this source of error: the longer the beam path the greater the potential for changes in RI to affect the laser light and, therefore, the measured range. Here, the effect of change in air

temperature was specifically assessed.

Varying the range of stability measurements enables the assessment of distance on this error source. Measuring with ‘zero’ distance provides a reference for non-zero stability measurements: thus, isolating air temperature change.

Method

The experimental setup for testing the effect of temperature change on range displacement measurements was similar to the assessment of tracker home position stability (see figure 5.9a). The RRR was placed on the post at three different range distances — ~ 0 m, 1 m, and 2 m — and each range was recorded for a minimum of 3 h at the same rate as specified above. The air temperature was recorded throughout the range measurements. The experimental setup used for these measurements is shown in figures 5.10 and 5.11.



Figure 5.10: An AT960LR installed on the base of a CMM. The tracker interferometer is locked onto an RRR that is fixed in a three-point hold on an aluminium post. The post height is designed to hold the RRR close to the plane of the laser tracker’s equator. The distance between tracker home position and RRR was minimised.

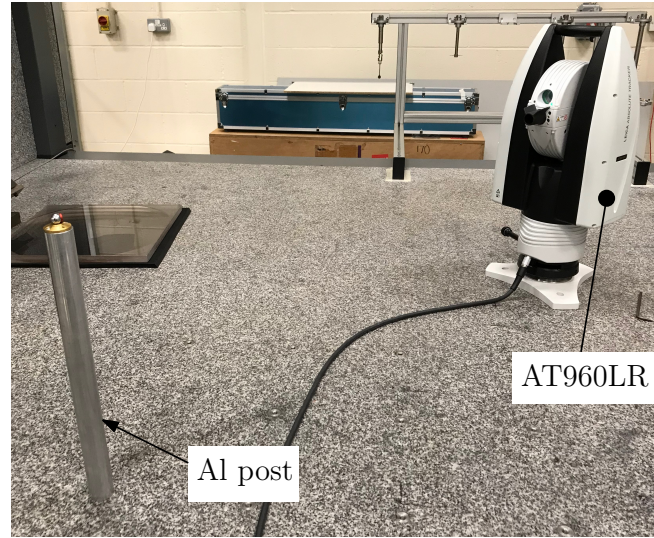


Figure 5.11: An AT960LR installed on the base of a CMM. The tracker interferometer is locked onto an RRR that is fixed in a three-point hold on an aluminium post. The post height is designed to hold the RRR close to the plane of the laser tracker's equator.

5.3.3 Combined long-term stability measurement

A combined stability measurement represents the effect of change in both home position stability and RI. This measurement provides a long-term stability assessment of the laser trackers in a setup similar to a multilateration measurement setup where a sample surface is under investigation.

Method

The method for this assessment is similar to the assessment of changes in RI. The critical difference was that the RRR was positioned on the base of the CMM rather than on the post. The same range and temperature measurements were undertaken. The temperature measuring equipment was mounted above the base of the CMM. The experimental setup is shown in figure 5.12.

5.4 Testing of laser-based multilateration

In this section, a series of measurements with a laser-based multilateration system are described. The aim of these measurements was to provide measurements of surfaces and determine the measurement error. A number of different measurements were performed in order to isolate input parameters to test their effect on the system measurement uncertainty. The effect of tracker arrangement is tested by performing measurements with two measuring station setups. The error induced by a sequential style of multilateration

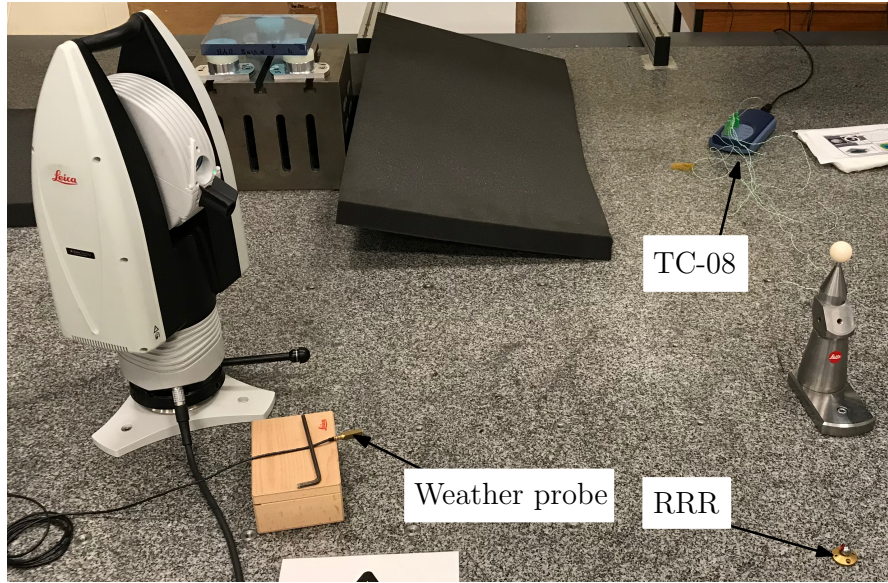


Figure 5.12: An AT960LR fixed to the granite base of a large CMM. The tracker is locked onto and measuring an RRR in spherical coordinates. The RRR is placed in a three-point hold at a specified distance. A type-K thermocouple and Pico Technology TC-08 data logger [113] is measuring the air temperature. A weather station probe is monitoring the temperature, pressure and, humidity of the environment. Both the weather and temperature probes are mounted above the surface of the CMM base.

was analysed by comparison to a non-sequential equivalent measurement. Two sizes of sample surface were used to determine the effect of the arrangement and quantity measured positions upon coordinate measurement solution uncertainty.

For the reasons outlined in section 4.2.6, it is critical to determine the effect of tracker positioning as a source of coordinate measurement error. Using the results of Zhang *et al.* as a benchmark for tracker positions [77], testing of practically achievable positions can be undertaken using MC simulations and an error model of the measurement system, and by undertaking real experimental measurements. The critical parameter that defines the zones, in which measuring stations can be positioned, is the viewing angle of the SMR that is utilised as a probe-tip for measuring surface coordinates.

MC simulation allows for testing of any measuring station position as SMR viewing angle is not a limiting factor. Simulation of a number of tracker positions provides an indication of the effect of positioning upon measurement error. Experimental testing of the measurement system with both the RRR and SCE for a number of real tracker positions also tests this error source. MC simulation of the real experimental setups tests the validity of the simulation in testing these parameters affect on measurement error. The specific dimensions of the experimental setups and measurement routines under investigation were used in the setup of MC simulations of these real experiments. The number of samples propagated through the model, n_{samples} , was set to 50000: this was

to maximise the stability of predicted measurement uncertainties, whilst minimising simulation time.

5.4.1 Four-tracker multilateration – SCE-probe

A laser-based multilateration measurement system, utilising an SCE-probe, was tested. Two samples were used during this investigation: a $0.4\text{ m} \times 0.4\text{ m}$ flat optic and a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic. A grid of CMM floating points was also measured above the surface of the sample.

Method

The $0.4\text{ m} \times 0.4\text{ m}$ flat optic was mounted on three points to the base of the CMM as shown in figure 5.13. The optic was measured in a four-tracker multilateration system utilising an SCE-probe installed on the CMM. Using the SCE as a target enabled the laser trackers to be positioned surrounding the sample surface as shown in figure 5.14. The CMM was used to move the SCE-probe into a 10×10 grid of equally spaced points over the sample surface, with dimensions $0.38\text{ m} \times 0.38\text{ m}$. The stable time was set such that the triggering and measuring of range measurement could be completed.

A grid of 5×5 equally spaced points floating in free space above the sample was also measured. The grid had outer dimensions of $0.5\text{ m} \times 0.5\text{ m}$. This created a virtual flat surface to measure. The CMM was stationary at each point of the grid for 10 s, which enabled the triggering of stable probe measurement of laser tracker range displacement. The stable time was set at 3.5 s and the stable space was set at $100\text{ }\mu\text{m}$. The measurement of the two grids was repeated 10 times to test the repeatability of the measurement solution.

The multilateration measurement of the $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic was similar to the measurement of the $0.4\text{ m} \times 0.4\text{ m}$ flat optic; however, the sample surface was measured with a grid of 6×6 points. The measurement setup is shown in figure 5.15. This sample was measured in two orientations, the second was rotated 180° about the axis normal to the sample surface relative to the first position.

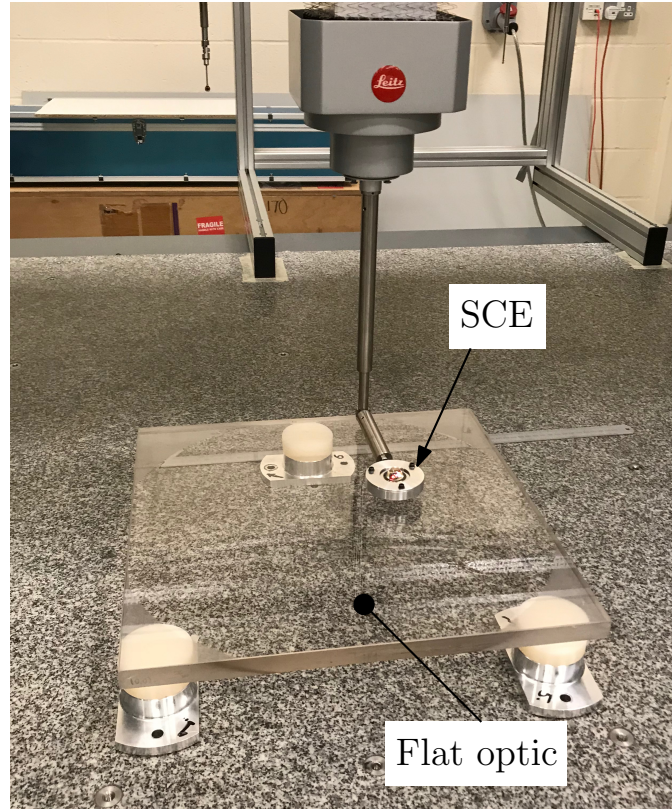


Figure 5.13: A SCE integrated with a Leitz CMM. The CMM is using the SCE to probe the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. The optic is mounted on the granite base of CMM with three point-stands.

5.4.2 Four-tracker multilateration – RRR-probe

The measurement of the $0.4\text{ m} \times 0.4\text{ m}$ flat optic was repeated; in this case, the RRR-probe was installed on the CMM and measured using four laser trackers. As the viewing angle of the RRR is limited, the measuring station positions were changed from the arrangement utilising the SCE-probe. This enabled the testing of a different measuring station arrangement with equivalent measured positions to the SCE-probe measurement.

Method

The measurement setup is shown in figures 5.16 and 5.17. Three of the laser trackers were positioned on steel blocks and one laser tracker was positioned on the base of the CMM. The trackers were arranged in a triangular pyramid. The RRR-probe was tilted at $\sim 12^\circ$ from the normal to the sample surface. A grid of 10×10 points was measured on the surface of the sample. No floating points were measured due to the limited viewing angle of the RRR. The laser tracker and CMM measurement parameters remained the same as described in the above multilateration experiments.

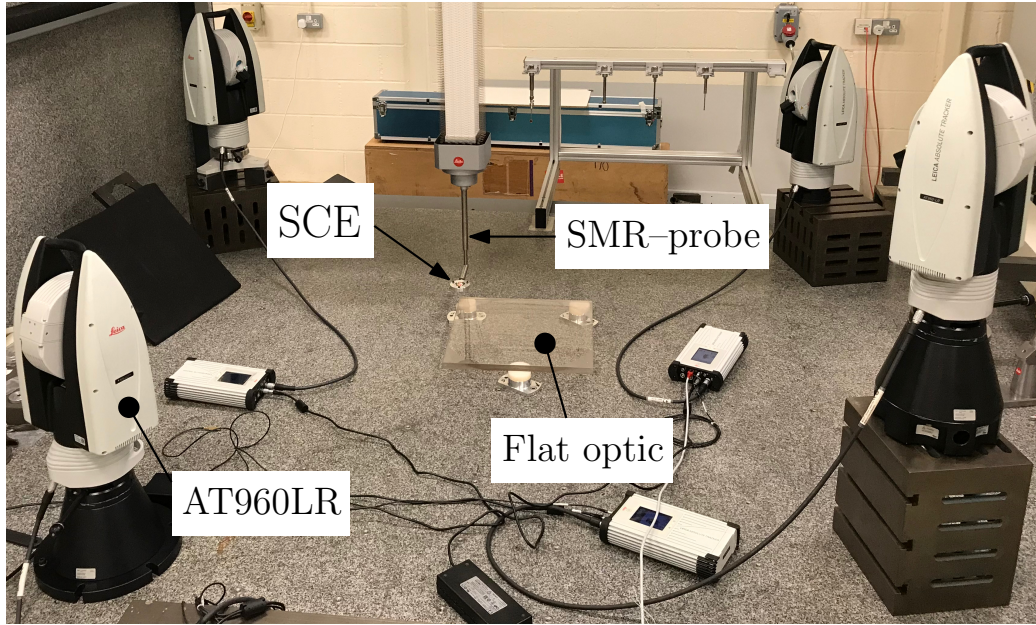


Figure 5.14: Four AT960LRs are installed on the granite base of a large CMM with SCE-probe installed. The CMM is moving the SCE-probe into measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic.

5.4.3 Sequential multilateration — SCE-probe

To test a four laser tracker sequential multilateration system, measurement of the $0.4\text{ m} \times 0.4\text{ m}$ flat optic was repeated with the SCE-probe installed on the CMM.

Method

The measurement setup was the same as shown in figure 5.14. The CMM probing routine was repeated four times with a grid of 10×10 positions on the surface of the sample. For each repeated CMM routine, a different tracker was used to measure the displacement of the SCE target. In this way, a sequential multilateration routine was implemented.

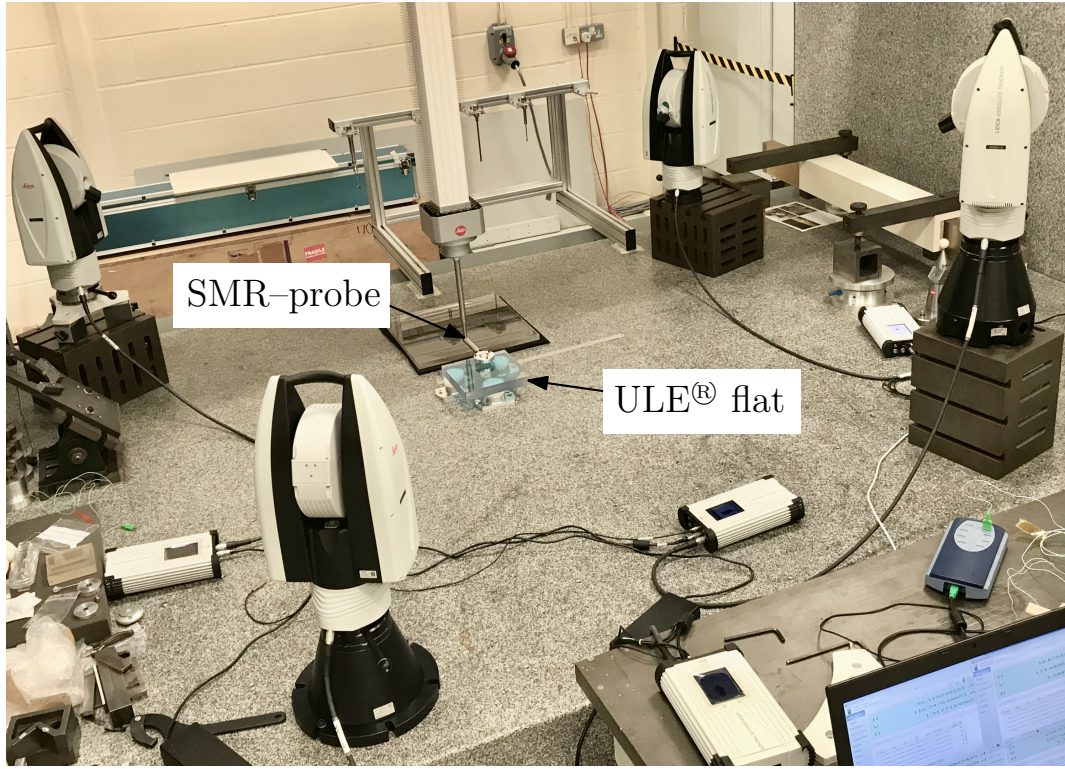


Figure 5.15: Four AT960LRs are installed on the granite base of a large CMM with SCE-probe installed. The CMM is moving the SCE-probe into measured positions on the surface of a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic.

5.4.4 Long-term multilateration measurement

The time dependent measurement error in the multilateration measurement system was tested with a long-term measurement of the $0.4\text{ m} \times 0.4\text{ m}$ flat optic.

Method

The measuring station and sample positions were identical to the non-sequential multilateration measurement with the SCE-probe shown in figure 5.14; however, 34 positions were measured across the surface of the sample. The time between measurement of positions was ~ 28 min. The total measuring time was ~ 15.5 h. The air temperature was measured close to the measurement area at a rate of $1/60$ Hz throughout the measurement period using a type-K thermocouple.

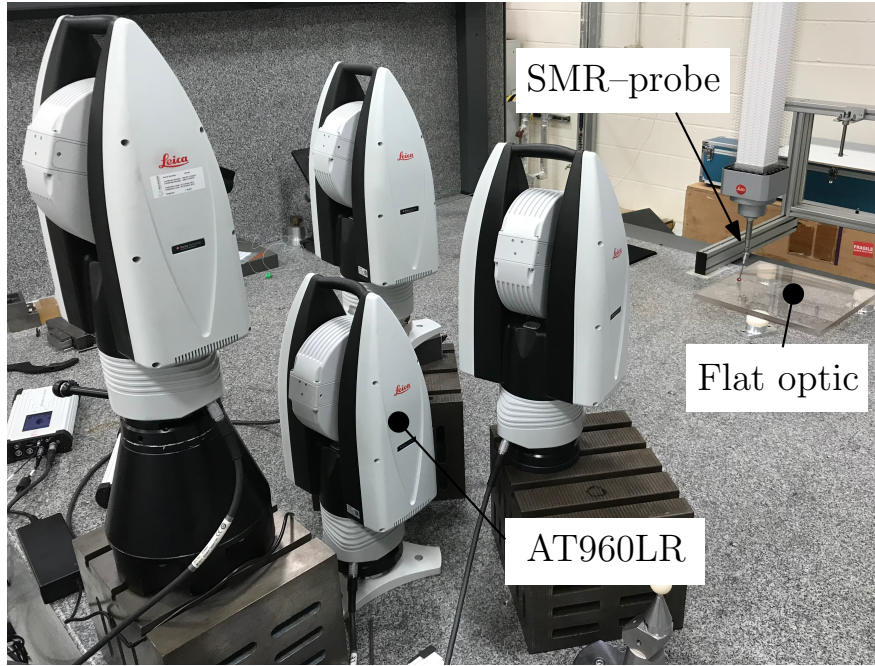


Figure 5.16: Four AT960LRs are positioned on the granite base of a large CMM with RRR-probe installed. The CMM is moving the SCE-probe into measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic.

5.5 Measurements for system verification

The multilateration system under investigation makes use of a number of measuring systems: four laser trackers and a large CMM. As such, in performing measurements of SMR target displacements with laser tracking interferometry, a number of coincident measurements are performed: laser tracker measurements of spherical coordinates, and CMM measurements. These measurements were recorded for direct comparison to the laser-based multilateration system measurement solutions.

5.5.1 CMM comparison

The flat optic samples were measured with a CMM for reference measurement comparison. The measurements included a measurement with a standard ruby-tip probe and direct CMM measurement of multilateration measured positions utilising the probes with SMR targets installed.

Standard probe

A Leitz PMM-F CMM with a standard $\varnothing 8\text{ mm}$ ruby-tip probe installed was used to measure coordinates on the surface of the samples: a $0.4\text{ m} \times 0.4\text{ m}$ flat optic and a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic. The samples were placed on the granite base of the CMM

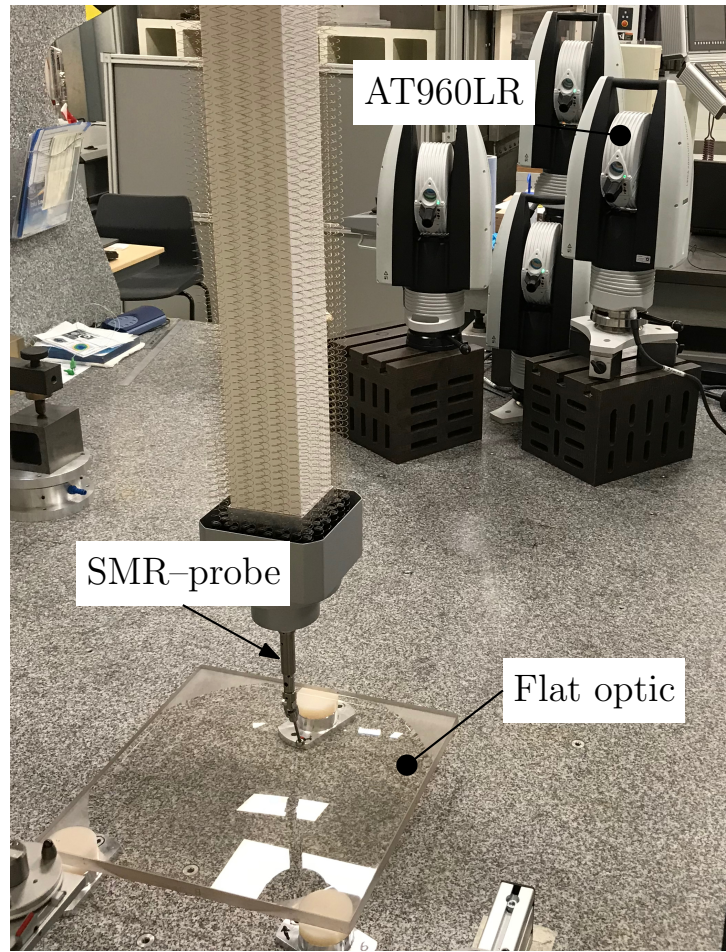


Figure 5.17: A front view of four AT960LRs are positioned on the granite base of a large CMM with RRR-probe installed. The CMM is moving the RRR-probe into measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic.

on three point-stands as for the multilateration measurements. The measured positions formed a grid of 10×10 points on the surface of the sample. A standard probing routine was implemented.

SMR-probes

As the CMM was used as a mover with SMR-probes installed during the multilateration, the CMM was used to record the coordinates of measured positions as a direct comparison to multilateration measurements. In this way, a CMM measurement was recorded on the surface of the samples for each set of measured interferometric displacements.

5.5.2 Single laser tracker measurement

Whilst the multilateration system measured displacement of the SMR measurement target as it was moved into measured positions on the surface of the samples, each laser tracker

was used in its normal operating measurement mode to record the spherical coordinates of the SMR. As the interferometric displacement measurement was triggered, so too was the recording of the SMR angular position relative to each laser tracker. As the ADM was used to measure the initial range to the SMR when the tracker was first locked onto the SMR, this provides a set of spherical coordinates for each measured position for each laser tracker. These measurements were averaged over all four tracker measurement sets.

5.6 MC simulations for sensitivity analysis

In addition the MC simulation of the real multilateration measurements undertaken, input parameters were varied for MC simulations to determine the system measurement uncertainty sensitivity to their changing. With the same setup as described in section 5.4 for the non-sequential measurement of the ULE[®] flat optic, the sensitivity to time independent input errors such as: interferometric displacement measurement uncertainty, SMR sphericity, deadpath error, and measuring station arrangement were investigated. Long-term measurement effects were investigated by simulating long-term temperature change over a long measurement period. The measured surface in the simulations has a perfect flat form – within the limitations of the computation.

5.6.1 Sensitivity to displacement measurement uncertainty

The sensitivity of the four-tracker multilateration system to interferometric displacement measurement uncertainty was tested by varying this as an input. The input u_0 , the constant first-term of the interferometric displacement range measurement uncertainty, was set to $0.1 \mu\text{m}$ ($k = 1$). This is half the magnitude of u_0 for the AT960LR. This input was propagated through the mathematical model of the system along with all other input parameters, which remained the same as the nominal measurement case. The output of the MC simulation was an estimate of the uncertainty in z-coordinate measurement on the sample for the system used in the real measurements, if u_0 was reduced to $0.1 \mu\text{m}$ ($k = 1$).

5.6.2 Sensitivity to SMR sphericity

The sensitivity of the four-tracker multilateration system to the error in range measurement caused by SMR sphericity was tested by increasing the parameter $\theta \rightarrow 5^\circ$. The resulting parameters b and Δr were calculated and input through the mathematical model of the multilateration system. The output of the MC simulation was an estimate of the uncertainty in z-coordinates for the system used in the real measurements with this level of SMR sphericity error.

5.6.3 Sensitivity to deadpath error

The sensitivity of the multilateration system to error in the ADM measurement of interferometer deadpath when the tracker initially locks onto the SMR was tested by reducing by varying u_{dp} as an input parameter. u_{dp} was reduced to $2.5\text{ }\mu\text{m}$ ($k = 1$) for the MC simulation, half the magnitude of u_{dp} for the AT960LR.

5.6.4 MC simulation of optimised setup

The sensitivity of the measurement system to the arrangement of measuring station positions was tested by redefining the positions to a triangular pyramid close to the sample. The measured positions were kept the same as the nominal setup. The simulated interferometric displacement measurements were then calculated based on the redefined setup.

5.6.5 Sensitivity to long-term measurement effects

The effect of time dependent measurement input error on the multilateration measurement system was tested by MC simulation of the real long-term measurement based on measured temperature data, and the measurement setup with a simulated long-term temperature change.

Measured temperature change

The MC simulation of the long-term measurement of the $0.4\text{ m} \times 0.4\text{ m}$ flat optic, with 34 measured positions on the surface of the sample in the setup shown in figure 5.14, was enabled by the measurement of the air temperature close to the laser beam paths. From the interferometric stability measurements described in section 5.3, an estimate of the expected drift in range measured due to air temperature change was determined. The measured temperature profile was input into the mathematical model of the system to determine estimates of the measurement error caused by the measured temperature change.

Sensitivity to temperature change

By defining a function for temperature change over the measurement period, the effect of long-term temperature change was predicted by MC simulation. The function produced a temperature increase of 1.5 K during the measurement of the first 17 positions, and decreased back to $\Delta T = 0\text{ K}$ during the measurement of the second 17 positions. As for the above simulation, an estimate of the expected drift in range measured due to air

temperature change was determined from the interferometric stability measurements. The simulated temperature profile was input into the mathematical model of the system to determine estimates of the measurement error caused by the long-term temperature change.

5.7 Summary

This chapter has described the experimental design and methods in order to test both the measurement system inputs and the measurement system directly. The results of the experiments undertaken are detailed within chapter 6. To carry out measurement tests of the multilateration system under investigation, four Leica AT960LR laser trackers were utilised as measuring stations; as such, the experimental analysis was specifically tailored for the measurement error sources of this system. It is feasible that any measurement system with laser interferometry tracking measurement capability could be used within the measurement system; and therefore, any experimental analysis must be designed to properly evaluate measurement error sources in other systems. This is also the case for the CMM utilised as an SMR-mover and as an independent coordinate measuring system.

Chapter 6

Results

The initial experiment to determine tracker warm-up time show that a time > 5 h is required for the range measurement of an AT960LR to reach relative stability. It has also been shown that the range measurement of a laser tracker cannot resolve the CMM measurement position on a sample surface without alteration the CMM probing routine. The time independent input error sources were investigated. A $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic was measured with a four-tracker multilateration system, utilising an SCE. The standard deviation of z-coordinates measured on the surface of the sample is $\sigma_z = 0.25\text{ }\mu\text{m}$ and $\sigma_z = 0.31\text{ }\mu\text{m}$ for the nominal and reversed sample orientations respectively. Monte Carlo simulation was used to investigate the sensitivity of system measurement uncertainty to a number of input error parameters, including measurement system setup. A $0.4\text{ m} \times 0.4\text{ m}$ flat optic was measured in a number of four-tracker multilateration setups. For the nominal setup, which utilised an SCE, the difference in z-coordinates of the CMM measurement and multilateration solution has a standard deviation of $\sigma_{\Delta z} = 0.73\text{ }\mu\text{m}$. A measurement of this larger optic was also undertaken with a restrictive RRR-probe and a sequential multilateration routine. The time dependent effects of change in RI and tracker home position were investigated. A long-term four-tracker multilateration measurement of the large sample was carried out over a 15.5 h measurement period. The difference between the z-coordinates measured by the CMM and the long-term multilateration have a standard deviation of $\sigma_{\Delta z} = 1.88\text{ }\mu\text{m}$. The measurement error of the long-term multilateration measurement solution was estimated via MC simulation. The sensitivity of measurement error to temperature change was also estimated.

6.1 Initial experiments

This section describes the results of a series of experiments designed to test parameters that affect the viability of the multilateration measurement system under investigation. These experiments, as described in section 5.1, test the time for an AT960LR laser tracker to perform relatively stable range measurements after it is initially powered up. The ability of the tracker range measurement to detect and record a measured position on the surface of a sample, when a Leitz PMM-F is used as a mover of the measurement target, is also tested.

6.1.1 Tracker ‘warm-up’ time

Figure 6.1 shows the AT960LR range measurement of a stationary SMR. The range measurement was started immediately following the turning on of the laser tracker when the tracker was locked onto the target. The AT960LR and SMR were fixed to the base of a Leitz PMM-F CMM. The SMR was positioned such that the range measured was approximately 0.45 m. The measurement was continued at a rate of 0.5 Hz for a period of five hours.

During the first hour of measurement the interferometric range measurement increased relatively rapidly by $\sim 40 \mu\text{m}$. During hours two and three of measurement the rate of change slowed significantly, such that during hours four and five $\Delta R \approx 5 \mu\text{m}$. It is clear that the range measurement has not reached relative measurement stability during the five hour measurement period.

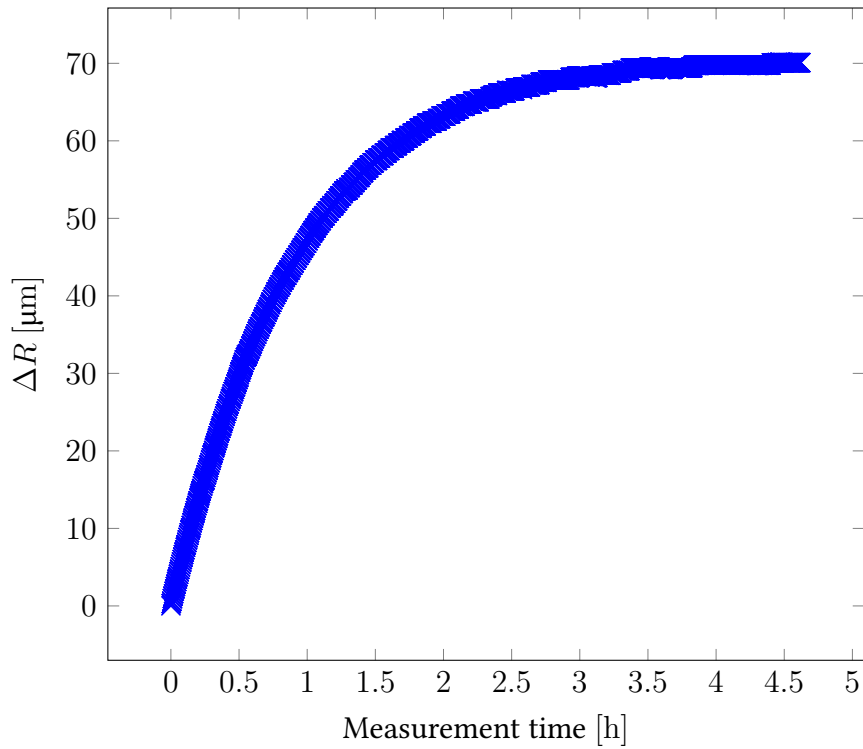


Figure 6.1: The change in range measurement of a stationary SMR by an AT960LR over the period of five hours after the tracker was initially powered-up. The range was measured at approximately 0.45 m. The range measured increased rapidly during hour one. The measurement stabilised significantly between hours two and five.

6.1.2 Tracker measurement of CMM probing

The AT960LR laser tracker measured Cartesian coordinates of an RRR-probe, installed on a Leitz PMM-F, as it moved into and away from a position on the surface of a sample are shown in figure 6.2. This is a laser tracker measurement of the standard controller probing motion of the CMM.

The probe was initially stabilised, close to the origin of the coordinate system shown in the figure, in order to move into a sample surface. The probe was moved into the surface in the direction of $-z$, where the motion was paused, and the CMM measurement was conducted as the probe retracted from the surface; this process occurs in the blue zone of the figure. After the probing routine was completed, the probe was moved away relatively quickly. This results in a scattering of measured points along the y -axis due to the angular tracking of the SMR. The tracking was stabilised and the SMR was measured moving in the direction of $+y$ to the next measurement point.

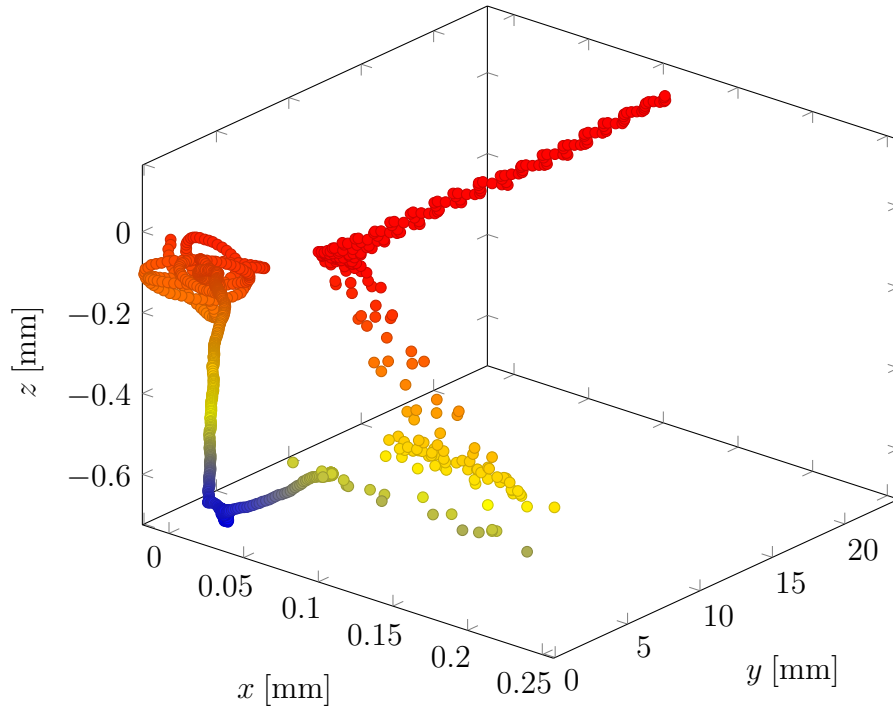


Figure 6.2: AT960LR Cartesian coordinate measurement of the probing motion of a Leitz PMMF with the RRR-probe installed. The origin of the coordinate system is defined by the first measurement. The measurement was conducted at a rate of 100 Hz. The colour change follows change in the z -axis: red is the maximum in z and blue is the minimum.

Figure 6.3 shows the interferometric range measurement of the first 3.5 s of the probing motion shown in figure 6.2. The measurement was conducted at a rate of 1000 Hz.

The period between 0 s and 1 s corresponds with the RRR-probe stabilising and beginning to move into the surface of the sample. The RRR-probe made contact with the sample surface at ~ 1.75 s. Between 2.75 s and 3.5 s the probe is moving away to the to another measurement point. At no time between these two time periods is the probe measured as stable and in contact with the sample surface such that the laser tracker range measurement could record a measured position for any significant length of time — this would be on the order of 2 s.

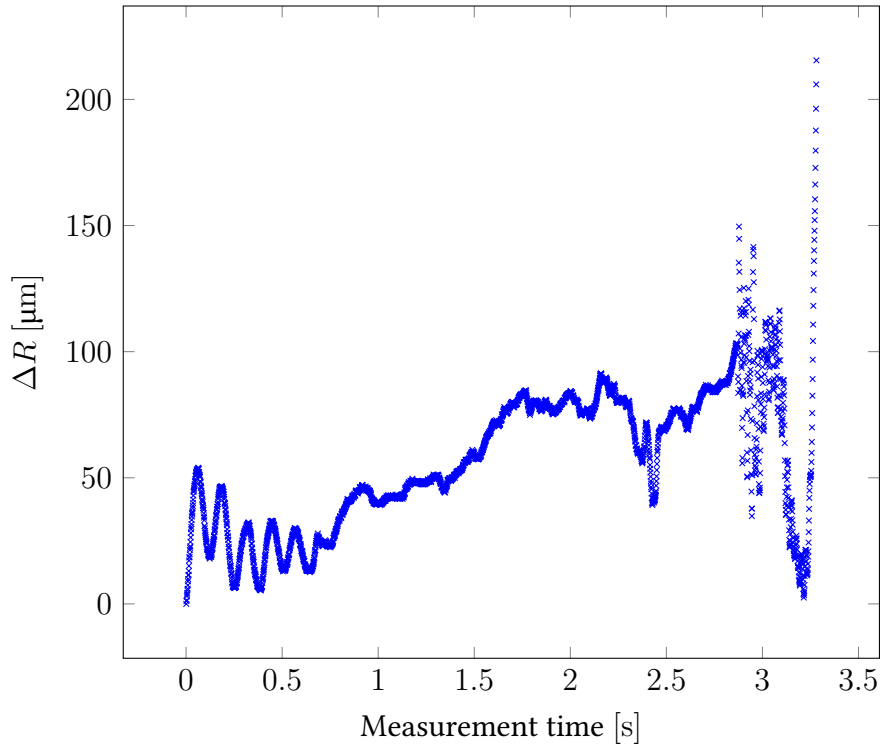


Figure 6.3: AT960LR relative range measurement of the probing motion of a Leitz PMMF with the RRR-probe installed. The probing routine was set as the default for the CMM. The range measurement of the probe does not become stable for any significant time whilst in contact with a sample surface. The measurement was conducted at a rate of 1000 Hz.

6.2 Testing of time independent error sources

This section details a series of experiments to test the time independent error sources identified in the mathematical model of the multilateration measurement system, as described in section 4.2. The experimental design for these tests is detailed in section 5.2. This includes: tests of the interferometric range lateral measurement capabilities; tests to determine range measurement error as a result of SMR reorientation; testing of the repeatability of CMM measurement with a number of unique measurement probes installed; interferometric range measurement stability measurements of a pause CMM probe on the surface of a sample; and the repeatability of CMM-tracker positioning in the case of sequential multilateration.

6.2.1 Buck-in buck-out test of interferometer

Figure 6.4 shows the results of the buck-in buck-out testing of an AT960LR laser tracking interferometer. Both the inside and outside position measurements of L were conducted a total of five times. The deviation from the mean of each measurement set, ΔL , is plotted

in the figure. The SMR positions were selected such that $L \approx 1.2$ m.

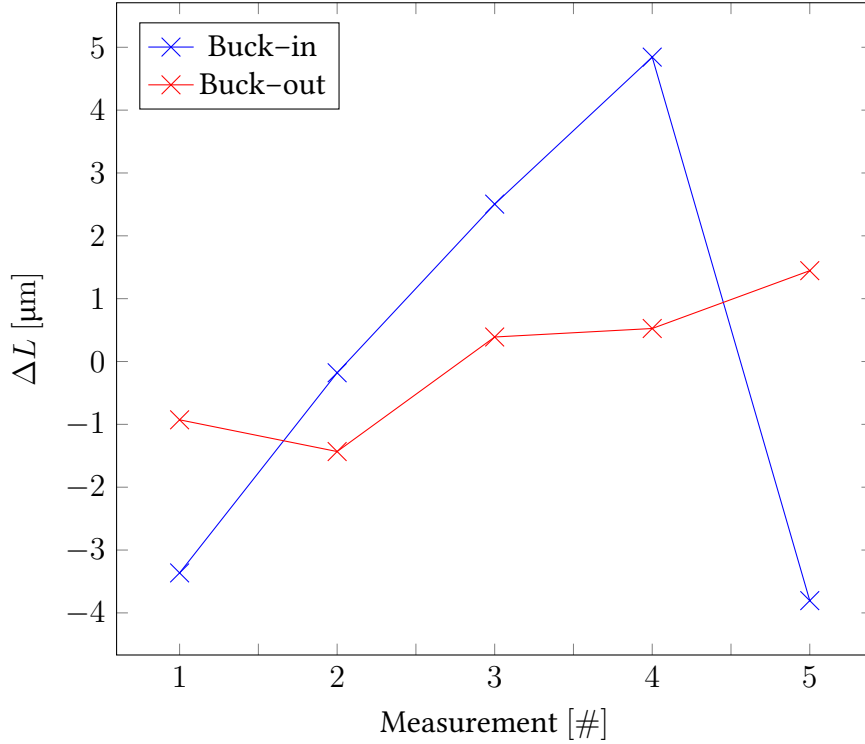


Figure 6.4: Buck-in buck-out testing of an AT960LR. The distance, L , between two SMR positions was measured five times for each tracker position. L was set to approximately 1.2 m. $\Delta L = 0$ was set to the mean of each set of measurement results.

The deviation from the mean is greater for the measurements of L from inside in comparison to outside. The standard deviation of the inside measurements is $\sigma_{L,\text{in}} = 3.7 \mu\text{m}$ and the outside measurements is $\sigma_{L,\text{out}} = 1.2 \mu\text{m}$.

6.2.2 SMR reorientation test

The relative laser tracker range measurements of an SMR, in a three-point magnetic hold on the granite base of a CMM, are shown in figure 6.5. For one set of measurements, the SMR was reorientated between each of 10 measurements; whilst there was no reorientation for the other measurement set. Each measurement is an average of a 10 second measurement with a rate of 1000 Hz. The range was set to $R \approx 0.45$ m.

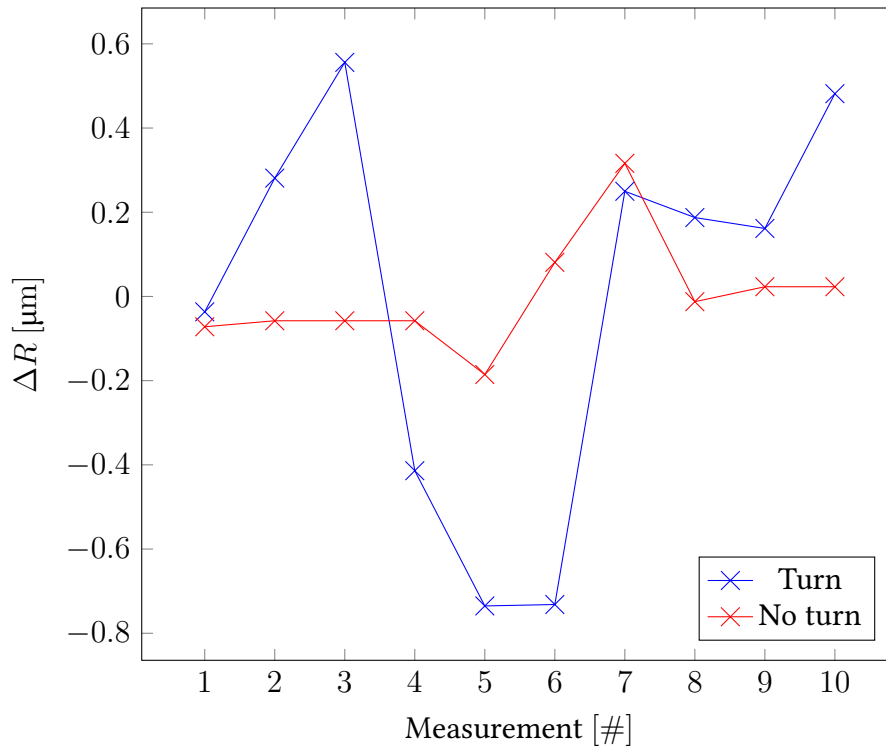


Figure 6.5: The relative range measurement, ΔR , of a 0.5 inch RRR in a three-point hold fixed to the base of a CMM. The measurement was repeated 10 times with an SMR reorientation (turn) between each measurement and 10 times with not reorientations. Each measurement was conducted for 10 s at 1000 Hz. The range was approximately 0.45 m.

The standard deviation of the measurement set with reorientations is $\sigma_{\text{turn}} = 0.47 \mu\text{m}$; and $\sigma_{\text{no turn}} = 0.13 \mu\text{m}$ for the set with no reorientations.

6.2.3 SMR-probe CMM testing

Figure 6.6 shows the results of the relative z-coordinate measurements of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic as measured by a Leitz CMM with three unique contact measurement probes installed: an $\varnothing 8 \text{ mm}$ ruby-tip probe, an RRR-probe, and an SCE-probe. The large flat optic was measured with a 10×10 grid of points.

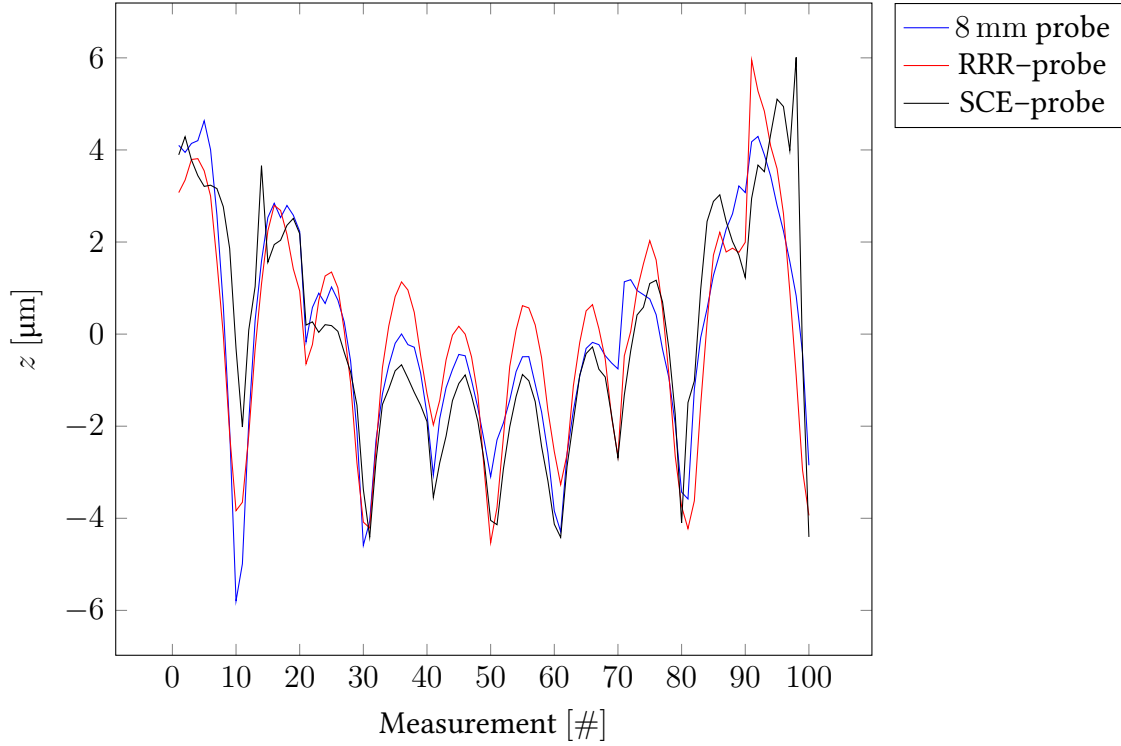


Figure 6.6: Leitz PMM-F relative z -coordinate measurement of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic repeated with three probe types: $\varnothing 8 \text{ mm}$ ruby-tip probe, RRR-probe, and SCE-probe. The measurement was conducted with 100 points in a 10×10 grid. Each line represents the mean of three measurements with each probe.

Taking the results of z for the $\varnothing 8 \text{ mm}$ ruby probe as a reference, the mean average difference between the RRR-probe results and ruby probe is, $\Delta z_{\text{RRR}} = 0.79 \mu\text{m}$. Equivalently for the SCE-probe, the difference is, $\Delta z_{\text{SCE}} = 0.95 \mu\text{m}$.

6.2.4 CMM probe pause and tracker measurement triggering

Testing of the AT960LR laser tracker displacement measurement of a Leitz CMM probe head with an RRR-probe installed was conducted. This enables the determination of the ability of the laser tracking controller to determine the point at which to record a point measurement on the surface of a sample.

Pause stability

The interferometric range measurement of an RRR-probe as it moves into the surface of a sample is shown in figure 6.7. $\Delta R = 0$ was set to the measured range at the end of the two second measurement.

The range measurement of the probe was initiated before the probe began to move towards the measured position on the surface. The velocity of the probe stabilises as it

approaches the surface. The CMM probing head detects a deflection of the probe had and the probe becomes relatively stable, as measured by the laser trackers interferometer.

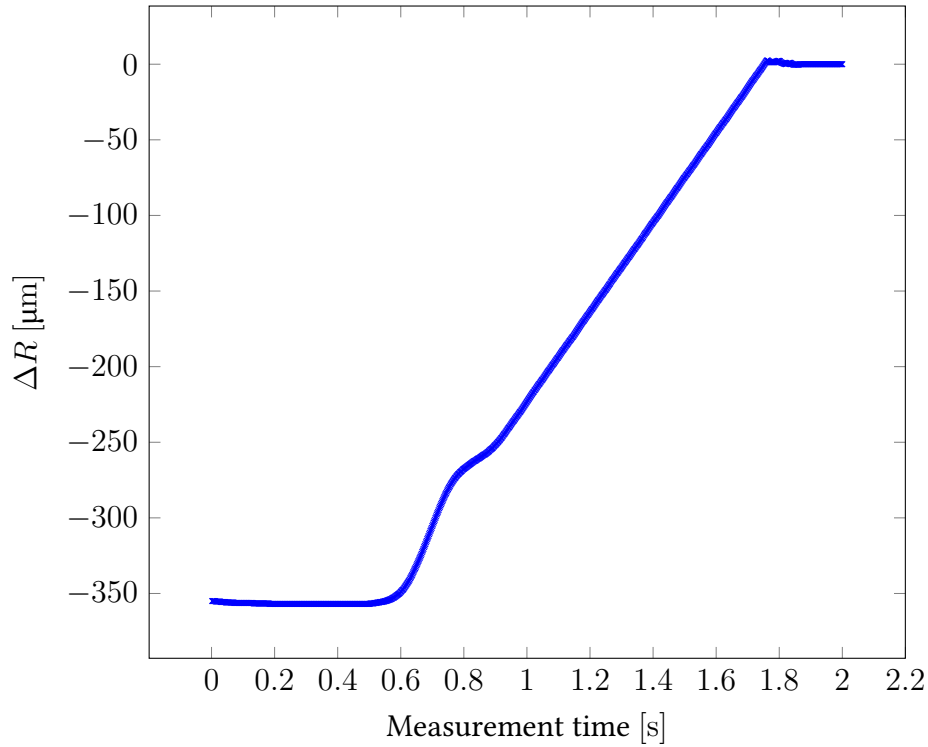


Figure 6.7: AT960LR relative range measurement, ΔR , of an RRR-probe installed on a Leitz PMM-F as it probes the surface of a sample with a ‘pause’ implemented into the probing routine. $t = 0$ s is the start of the probing motion. At $t = 2$ s the probe is held with relatively stability on the surface of a sample. The range measurement was conducted at a rate of 1000 Hz.

Figure 6.8 shows the final 0.25 s of the range measurement. At $t = 1.75$ s the CMM is driving the probe into the surface of a sample. Between $t = 1.75$ s and 1.9 s the probe contacts the surface and reaches relatively stability with a CMM probe pause. The RRR is held stationary within the resolution of the AT960LR interferometer in the axis of the range measurement.

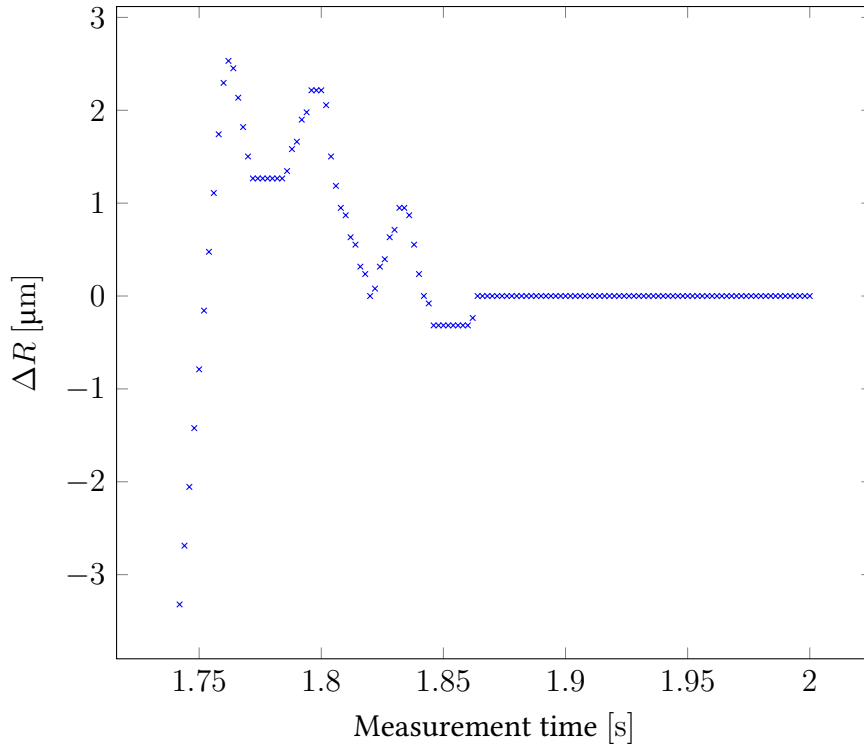


Figure 6.8: AT960LR relative range measurement, ΔR , of an RRR-probe installed on a Leitz PMM-F as it probes the surface of a sample with a ‘pause’ implemented into the probing routine. After an initial stabilisation period of ~ 0.1 s the range measurement becomes stable as measured by laser tracker.

Measurement triggering

It was determined that with a CMM probing routine pause of 5 s, the laser tracker controller was able to detect the stability of the measurement probe and measure the probes position in spherical coordinates at a rate of 1000 Hz for 2 – 4 s.

6.2.5 CMM-tracker positioning repeatability

The repeatability of the AT960LR laser tracker range measurement of the Leitz CMM probing positioning on the surface of a flat optic was tested as described in section 5.2.5. The standard deviation of range measurements of an RRR-probe repeatedly moved into positions on the surface of a sample are shown in figure 6.9. The 16 measured positions were organised in a 4×4 grid. Each position was repeated 10 times.

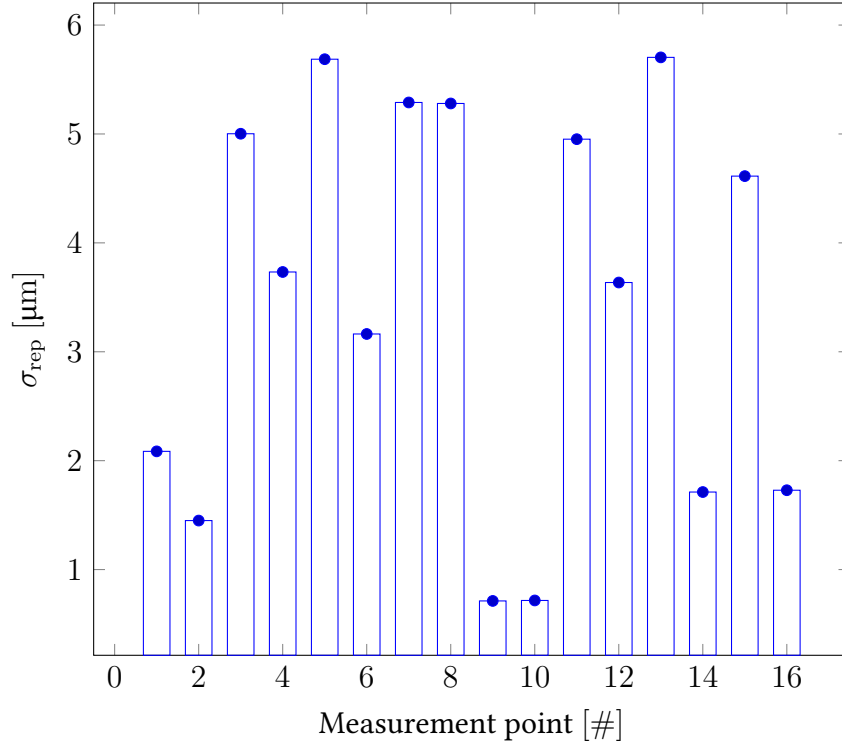


Figure 6.9: The variation in repeated AT960LR range measurements of a Leitz PMM-F with an RRR-probe installed. The measurement routine was 4×4 grid of points on a flat sample. The grid measurement was repeated 10 times. The standard deviation, σ_{rep} , of range measurement for each grid point is plotted.

From the figure, it is clear that the variation in repeatability between points is significant. The square root of the mean variance of range measurements per position is $\sigma_{\text{rep,rmv}} = 3.9 \mu\text{m}$. The minimum standard deviation is at point 9: $\sigma_{\text{rep,min}} = 0.7 \mu\text{m}$ and the maximum is at point 13: $\sigma_{\text{rep,max}} = 5.7 \mu\text{m}$.

6.2.6 Multilateration solving algorithm validation

The multilateration solving algorithm was validated by inputting a set of nominal displacements for the experimental setup utilised in the four-tracker multilateration measurements, as described in section 5.2.6. Figure 6.10 shows the z-coordinates of the multilateration solution based on the nominally calculated displacements, $d_{i,j \text{ nom}}$.

The z-coordinates have a standard deviation of $\sigma_z = 4 \times 10^{-5} \mu\text{m}$. The peak-to-valley of the measured z-coordinates is approximately $2.5 \times 10^{-10} \text{ m}$.

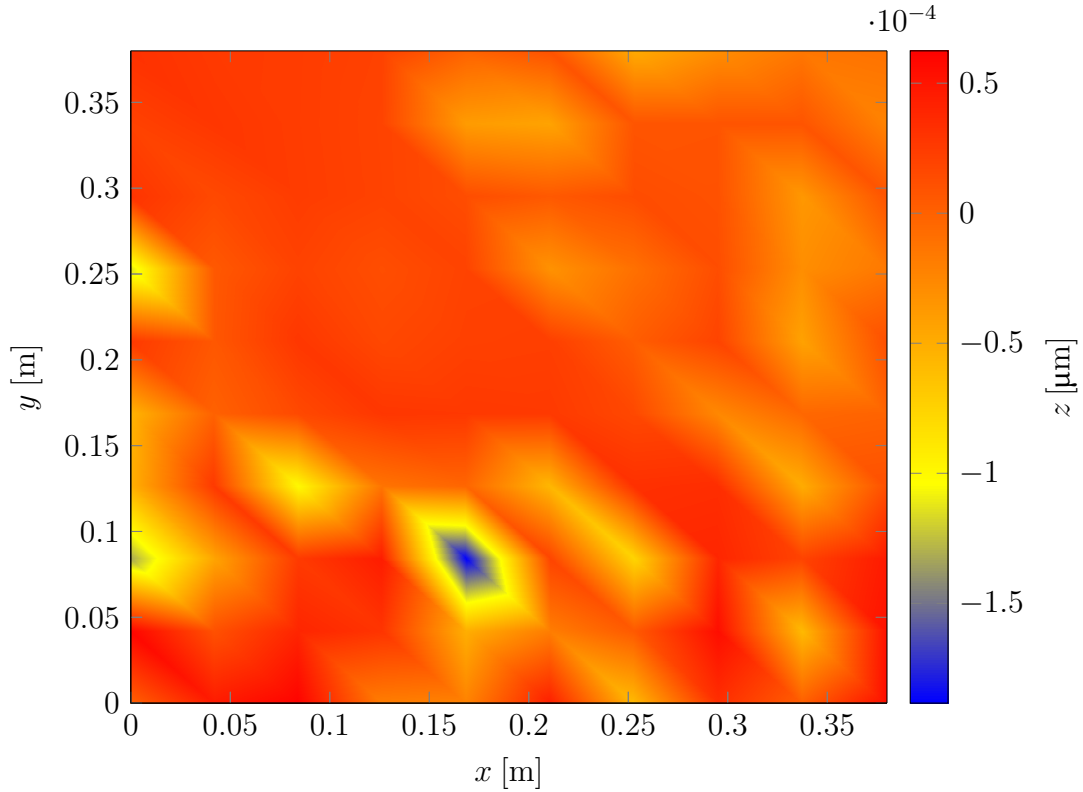


Figure 6.10: A linear interpolation of a multilateration solution for z -coordinates of a $0.4 \text{ m} \times 0.4 \text{ m}$ nominal flat based on a nominal experimental setup. The measured positions on the surface of the flat were in a 10×10 grid of points.

6.3 CMM measurement of flat optics

The results of CMM reference measurements of the flat optics used for this investigation are detailed in this section. The measurement procedure is detailed in section 5.5.1. A Leitz PMM-F, with a standard ruby-tip probe with an $\varnothing 8 \text{ mm}$ and nominal probing parameters selected, was used to measure the form error of the sample surfaces used to investigate the measurement uncertainty of the proposed multilateration system: a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic and a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic.

6.3.1 ULE[®] flat optic

The z -coordinates of a Leitz PMM-F measurement of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic plotted against the xy -coordinates in figure 6.11. A linear interpolation of results is shown. An $\varnothing 8 \text{ mm}$ ruby-tip probe was used to measure a 10×10 grid of points on the surface of the sample. A standard probing routine was implemented.

The measured z -coordinates have a standard deviation of $\sigma_z = 0.19 \mu\text{m}$. The peak-to-valley of the measured z -coordinates is approximately $0.8 \mu\text{m}$. The measurement of this

flat appears to be relatively high in noise; however, it is clear that the points at the more central part of the measurement are lower than the outer parts of the measurement.

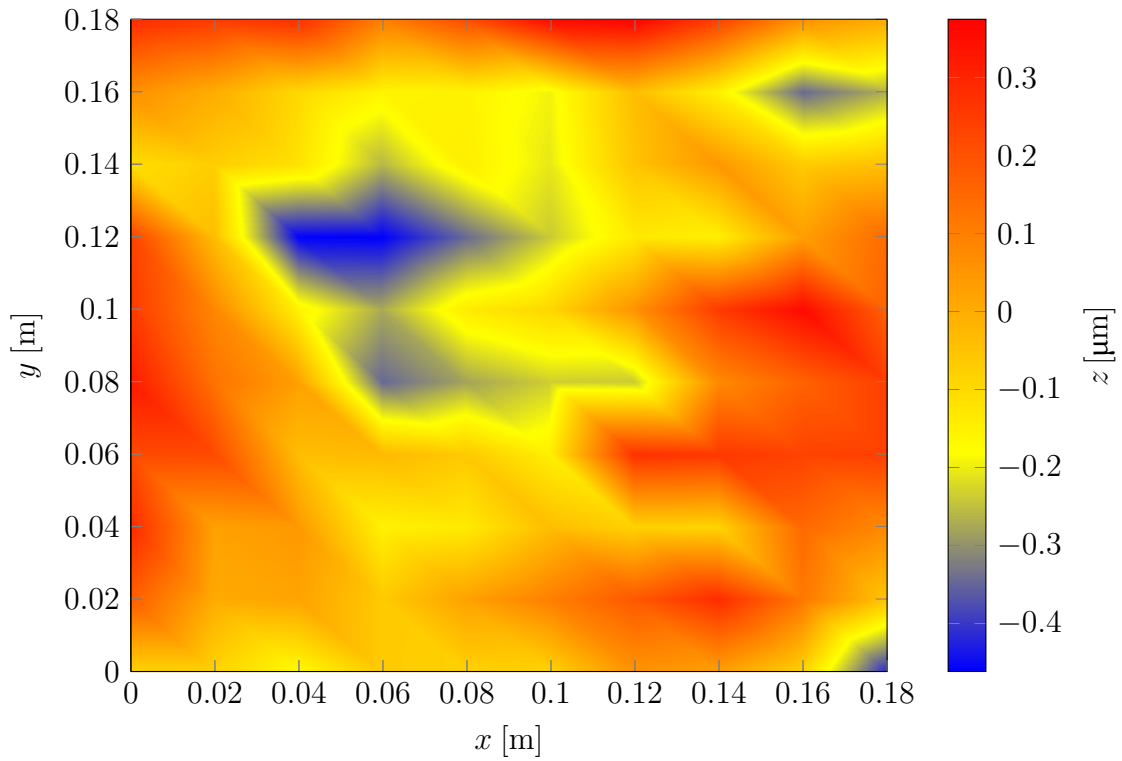


Figure 6.11: A linear interpolation of a reference measurement of $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic with a Leitz PMM-F. A standard ruby-tip probe with an $\varnothing 8\text{ mm}$ was installed on the CMM. The measured positions on the surface of the flat were in a 10×10 grid of points.

6.3.2 $0.4\text{ m} \times 0.4\text{ m}$ flat optic

The z -coordinates of a Leitz PMM-F measurement of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic plotted against the xy -coordinates in figure 6.12. A linear interpolation of results is shown. As for the above measurement of the smaller part, $\varnothing 8\text{ mm}$ ruby-tip probe was used to measure a 10×10 grid of points on the surface of the sample and standard probing routine was implemented.

The measured z -coordinates have a standard deviation of $\sigma_z = 2.3\text{ }\mu\text{m}$. The peak-to-valley of the measured z -coordinates is approximately $10\text{ }\mu\text{m}$. There is a clear form to the measured z -coordinates: a saddle shape.

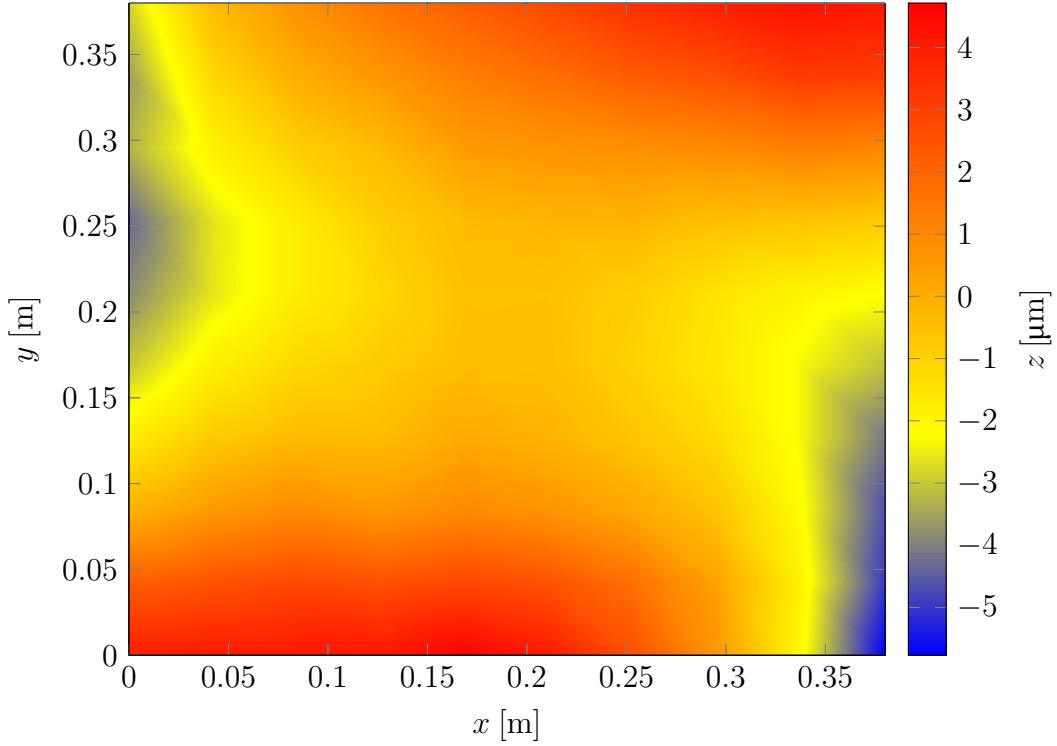


Figure 6.12: A linear interpolation of a reference measurement of $0.4\text{ m} \times 0.4\text{ m}$ flat optic with a Leitz PMM-F. A standard ruby-tip probe with an $\varnothing 8\text{ mm}$ was installed on the CMM. The measured positions on the surface of the flat were in a 10×10 grid of points. A standard probing routine was implemented.

6.4 Four-tracker multilateration — ULE[®] flat optic

This section describes a four AT960LR laser tracker multilateration measurement solution and compares the results to a CMM reference measurement. The sample under investigation was a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic. The measurement procedure was described in section 5.4.

6.4.1 System setup

The measurement setup of a four laser tracker multilateration measurement is shown in figures 6.13, 6.14, and 6.15. This estimate of the measured positions coordinates was determined by one of the laser trackers measuring in spherical coordinates using ADM, IFM, and angular encoders. The coordinates of the remaining three laser trackers were calculated using a trilateration routine, as detailed in section 4.1.2, by matching coordinates of the measured positions as measured by each tracker. The origin of the coordinate system is defined by a measured position: a corner point of the floating grid.

The measured positions form a 6×6 grid of points on the surface of the $0.2\text{ m} \times 0.2\text{ m}$

ULE[®] flat optic, and a 5×5 grid of floating points above the sample surface.

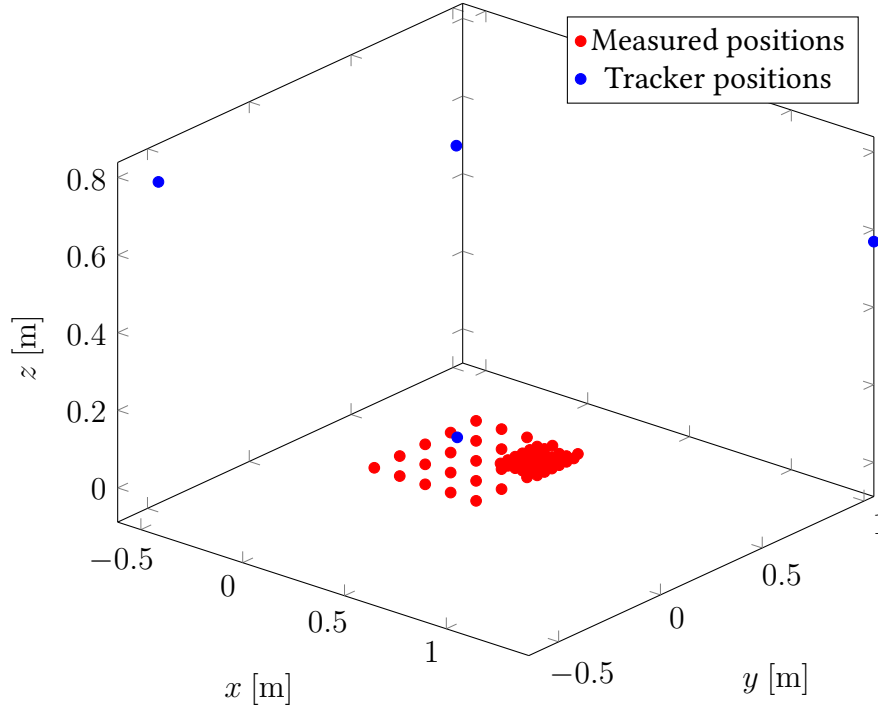


Figure 6.13: The measurement setup for a four AT960LR multilateration measurement of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic. The measuring station (laser tracker) positions surround the measured positions at differing heights above the surface. The measured positions include: a 6×6 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface. The floating-point grid size is $0.5 \text{ m} \times 0.5 \text{ m}$.

Figure 6.14 shows how the measuring stations (laser trackers) surround the measured positions.

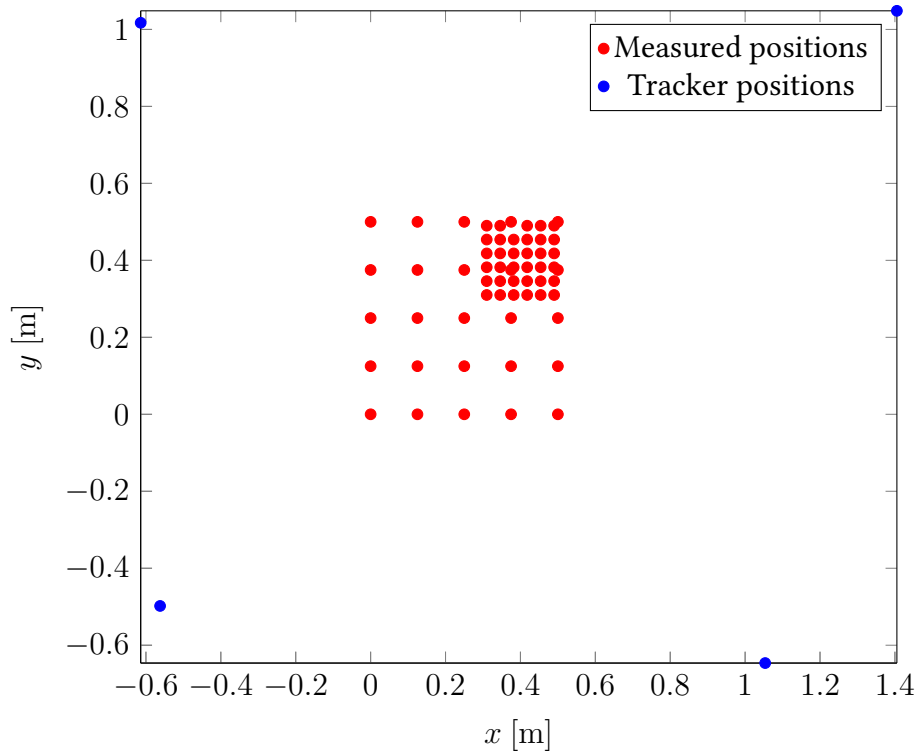


Figure 6.14: A top-view of the measurement setup for a four AT960LR multilateration measurement of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic. The measuring station (laser tracker) positions surround the measured positions at differing heights above the surface. The measured positions include: a 6×6 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface. The floating-point grid size is $0.5 \text{ m} \times 0.5 \text{ m}$.

Figure 6.15 shows how the height of the laser trackers vary in the z -axis of the coordinate system.

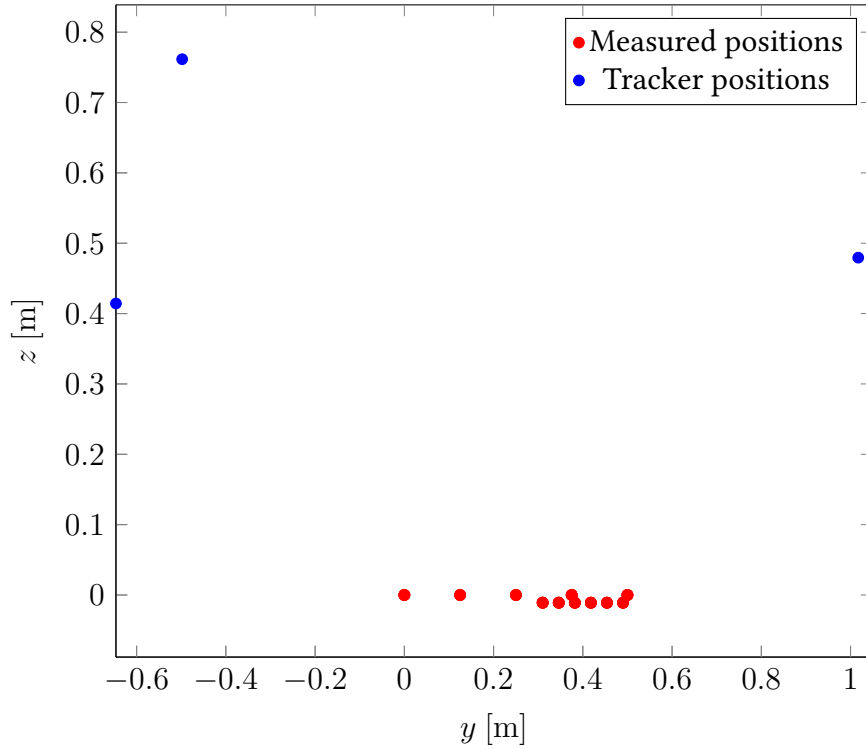


Figure 6.15: A side-view of the measurement setup for a four AT960LR multilateration measurement of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic. The measuring station (laser tracker) positions surround the measured positions at differing heights above the surface. The measured positions include: a 6×6 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface. The floating-point grid size is $0.5 \text{ m} \times 0.5 \text{ m}$.

6.4.2 Measurement results

A linear interpolation of the z -coordinates of a four-tracker multilateration measurement solution of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic plotted against the xy -coordinates in figure 6.16. An SCE-probe was used to measure a 6×6 grid of points on the surface of the sample. A paused probing routine was implemented. The coordinate system is defined by a first-order polynomial surface through the determined coordinates were the origin of the coordinate system is set at the first measured position in x and y .

The measured z -coordinates have a standard deviation of $\sigma_z = 0.25 \mu\text{m}$. The peak-to-valley of the measured z -coordinates is approximately $1 \mu\text{m}$. The measurement of this flat appears to be relatively high in noise; however, the z -coordinates at $(x, y) = (0, 0)$ and $(x, y) = (0.18, 0.18)$ fall away from the nominal plane.

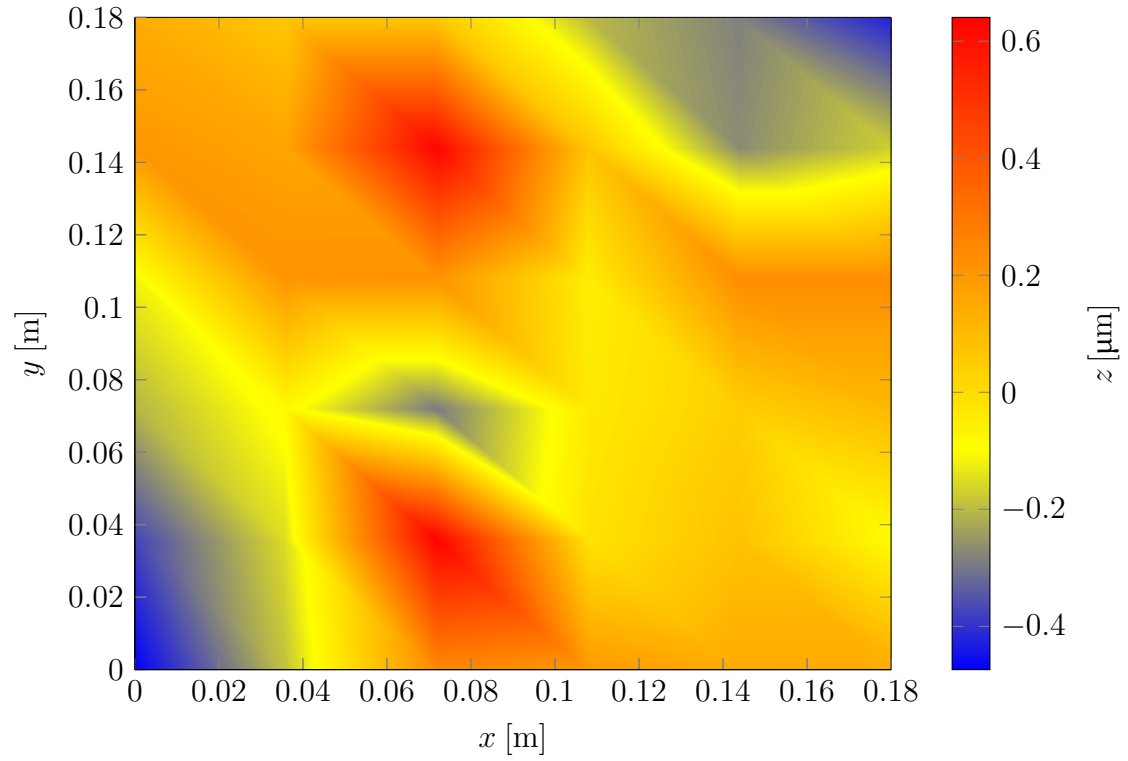


Figure 6.16: A linear interpolation of multilateration determined z -coordinates of a 6×6 grid of measured positions of an SCE-probe on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic using four AT960LR laser trackers. The z -coordinates of measured positions are shown as a function of x - and y -coordinates.

Figure 6.17 shows a second-order polynomial fit of the z-coordinates shown in figure 6.16. This emphasises the saddle nature of the results.

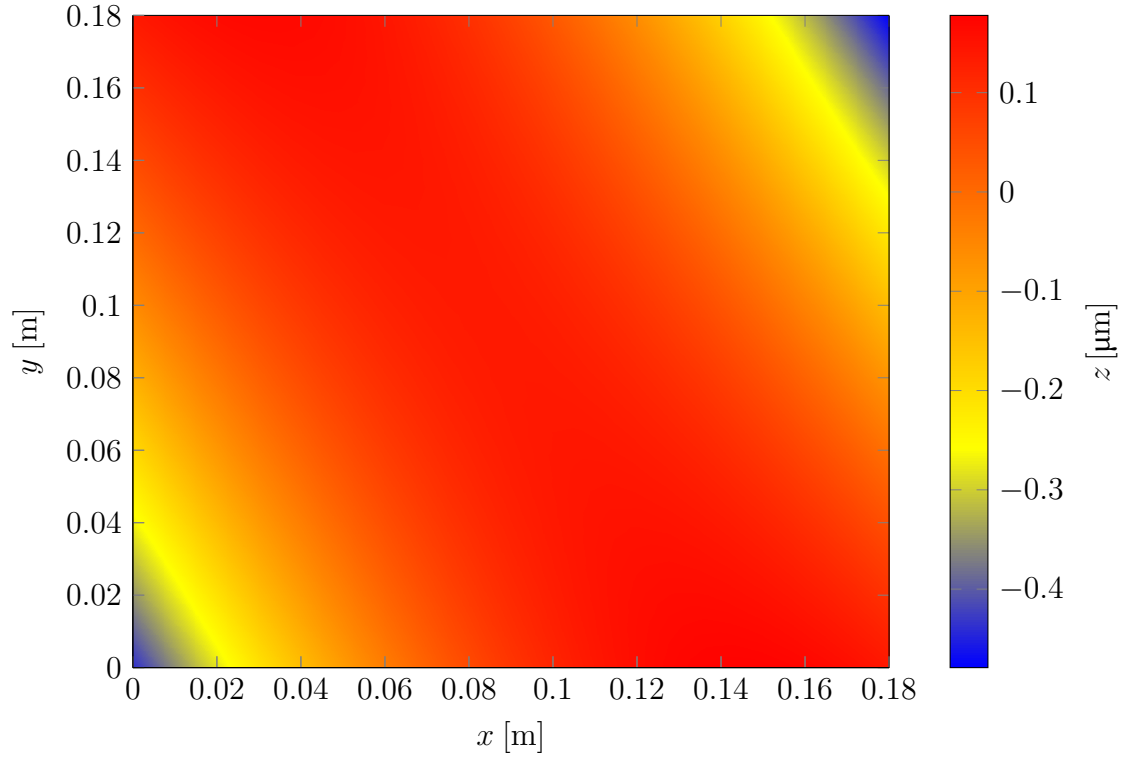


Figure 6.17: A second-order polynomial fit of multilateration determined z-coordinates of a 6×6 grid of measured positions of an SCE-probe on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic using four AT960LR laser trackers.

Figure 6.18 shows a linear interpolation of the z-coordinates of the ULE[®] flat optic shown in figure 6.16 with the second-order fit, shown in figure 6.17, removed. With this fit removed, $\sigma_z = 0.19 \mu\text{m}$.

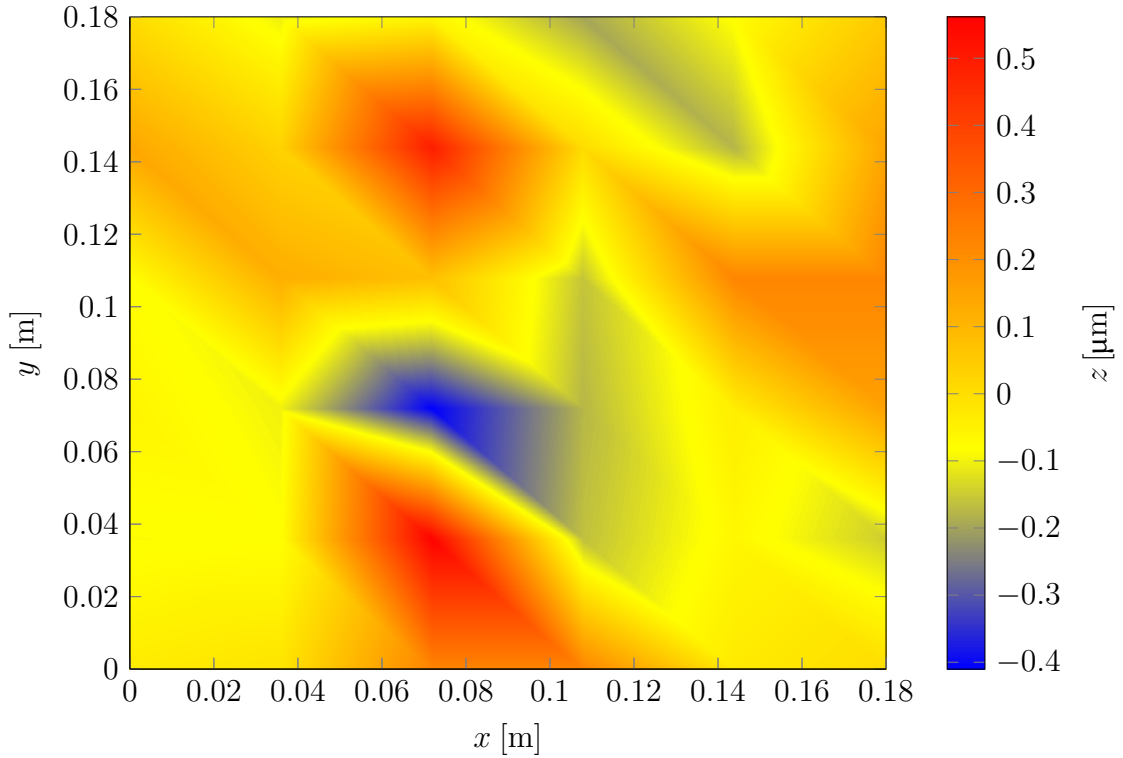


Figure 6.18: A linear interpolation of multilateration determined z-coordinates, with a second-order polynomial fit removed, of a 6×6 grid of measured positions of an SCE-probe on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic using four AT960LR laser trackers. The z-coordinates of measured positions are shown as a function of x- and y-coordinates.

A linear interpolation of the z-coordinates of a Leitz PMM-F measurement of the $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic is plotted against the xy-coordinates in figure 6.19. This measurement corresponds to the above multilateration measurement solution as each point was measured pseudo-simultaneously by the CMM with the SCE-probe installed.

The measured z-coordinates have a standard deviation of $\sigma_z = 0.19 \mu\text{m}$. The peak-to-valley of the measured z-coordinates is approximately $0.6 \mu\text{m}$. It is noteworthy the standard deviation of z-coordinates in this measurement is equal to the standard deviation of the equivalent multilateration solution with a second-order fit removed.

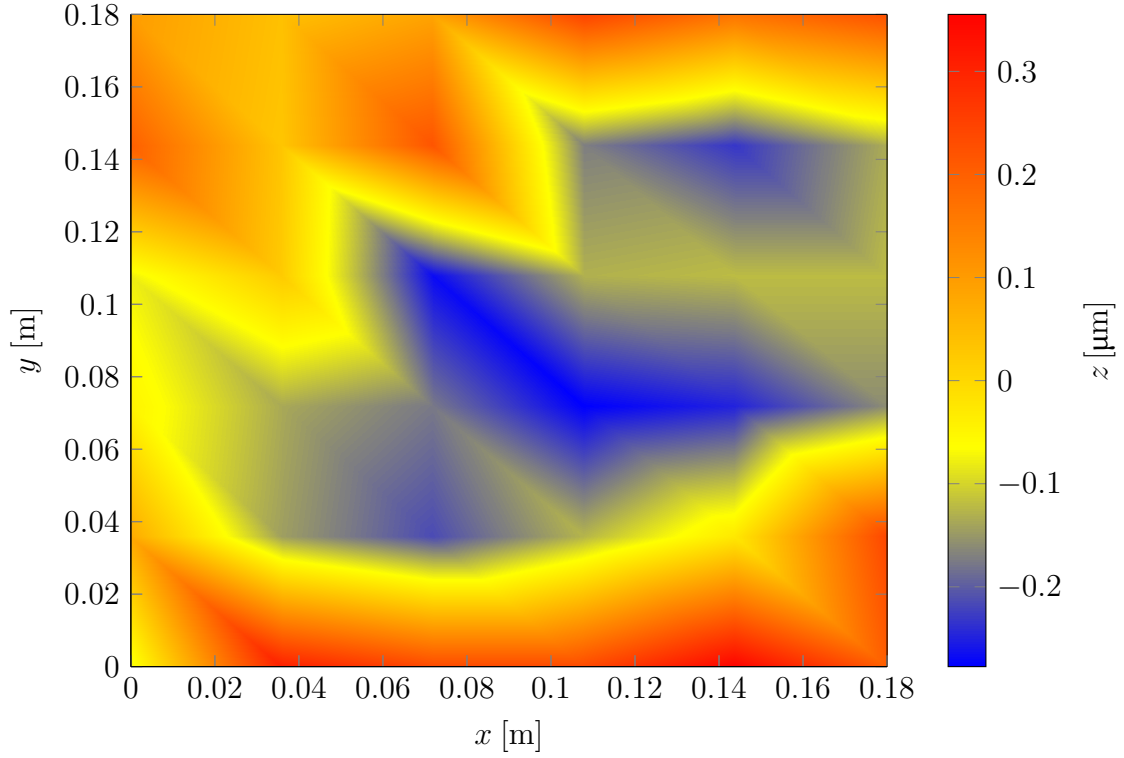


Figure 6.19: A linear interpolation of Leitz PMM-F measured z -coordinates of a 6×6 grid of points, with SCE-probe installed, on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE® flat optic. The z -coordinates of measured positions are shown as a function of x - and y -coordinates and are defined by a first-order polynomial surface through the measured coordinate positions. The origin of the coordinate system is defined at the first measured position in x and y .

6.4.3 Minimisation residuals

The l^2 -norm of the residuals, $w_{i,j}$, of the minimisation for the multilateration equations, based on the laser tracker interferometric displacement measurements of the measured positions on the surface of the $0.2 \text{ m} \times 0.2 \text{ m}$ ULE® flat optic, is:

$$|\mathbf{w}|_2 = \sqrt{\sum_{k=1}^n |w_k|^2} = 1.5 \times 10^{-12} \text{ m.}$$

Figure 6.20 shows the residuals of the 240 equations that make up the equation set (4.1) for the measurement of the 60 measured positions in the setup.

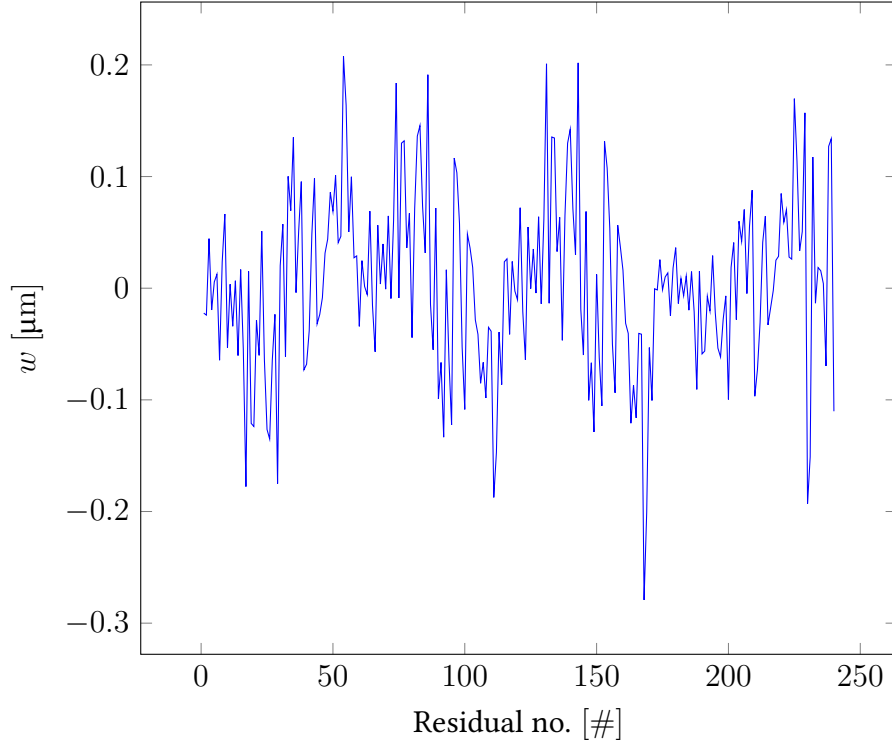


Figure 6.20: The residuals of the least squares minimisation for a multilateration measurement with four AT960LR laser trackers. The sample was a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic. Each laser tracker displacement measurement has a corresponding residual, $w_{i,j}$.

The residuals per laser tracker, j , are plotted in figure 6.21. There are four residuals per measured position corresponding to the four displacement measurements for each position. The measurement of 36 points on the surface of the sample and 25 floating points results in a nominal total number of residuals of 244. The figure shows how the solving algorithm has approached the minimisation by balancing the residuals for tracker $j = 1$ with tracker $j = 3$, and balancing the residuals for tracker $j = 2$ with tracker $j = 4$ to some extent.

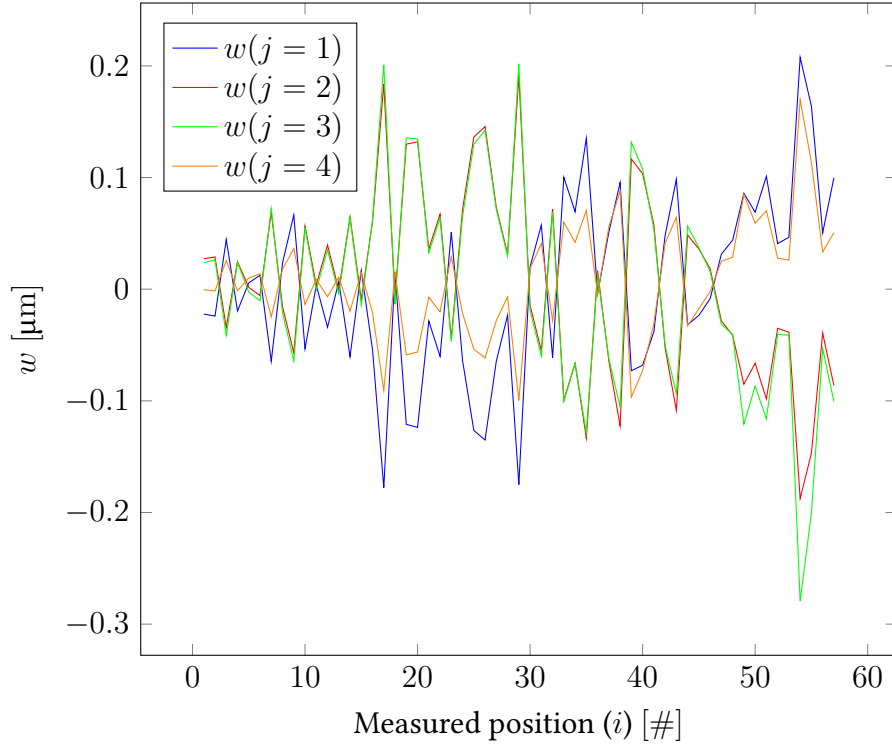


Figure 6.21: The residuals of the least squares minimisation for a multilateration measurement per AT960LR laser tracker (j) for each measured position (i): nominally, 36 measured positions on the surface of a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic, plus 25 floating positions above the optic.

6.4.4 Z-coordinate comparison

The z-coordinates of the measured positions on the surface of the ULE[®] flat optic, as measured by each of the four laser trackers and the multilateration solution, are shown in figure 6.22 as a function of the 36 nominal measurement positions. It is noteworthy that there are 35 measured positions shown of the nominal 36 measured positions on the sample surface as a measurement position was ‘missed’ by a number of the laser trackers. The missed measurement occurred due to a lack of full synchronization between CMM pause and tracker measurement triggering.

Each set of z-coordinates was determined from a first-order polynomial fit of coordinates measured. The standard deviations of z-coordinates measured by the laser trackers (utilising ADM, IFM, and angular encoders) are $\sigma_z = 5.4\text{ }\mu\text{m}$, $7.8\text{ }\mu\text{m}$, $6.3\text{ }\mu\text{m}$, and $7.1\text{ }\mu\text{m}$.

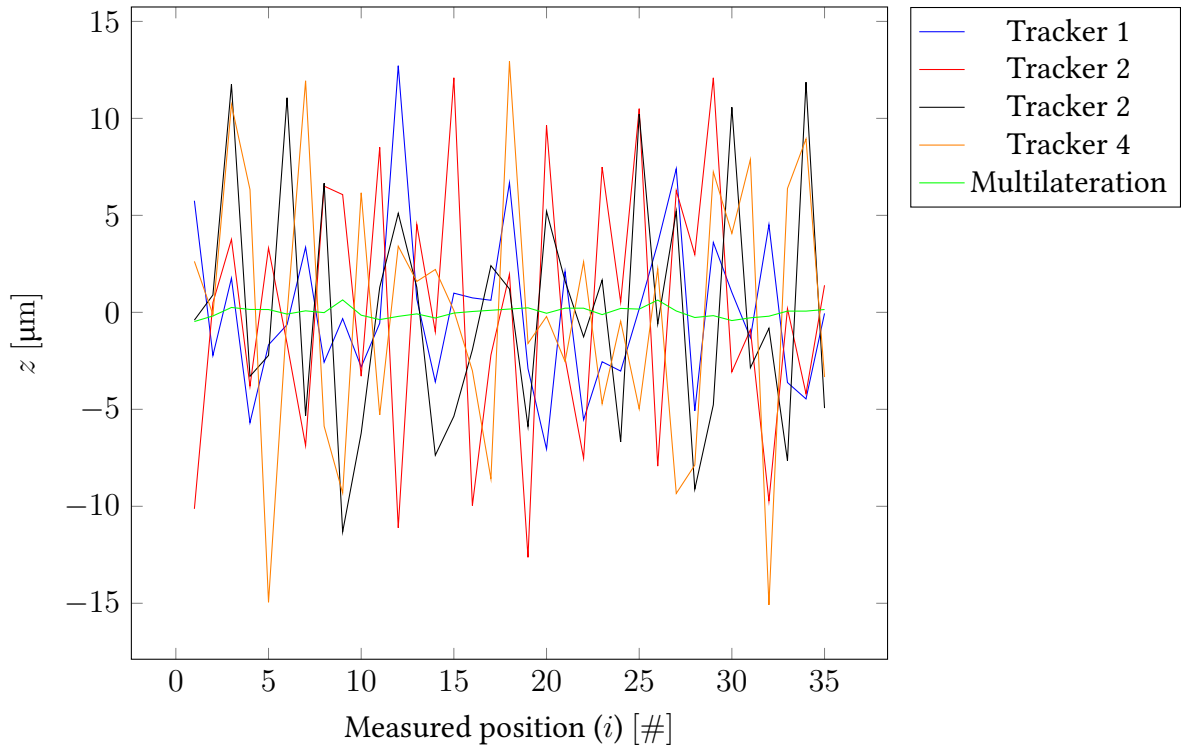


Figure 6.22: The z -coordinate measurements of a 6×6 grid of measured positions on the surface of a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic; as measured by each of the four AT960LR laser trackers individually, and as determined via four-tracker multilateration. The z -coordinates are determined from a first-order polynomial fit of points for each measurement set: four laser tracker grid measurements and the multilateration solution.

Figure 6.23 shows the mean of tracker measurements of z -coordinates, as well as the z -coordinates of the multilateration solution and the equivalent CMM measurement. The standard deviation of the mean of tracker measurements is $\sigma_z = 4.0 \mu\text{m}$.

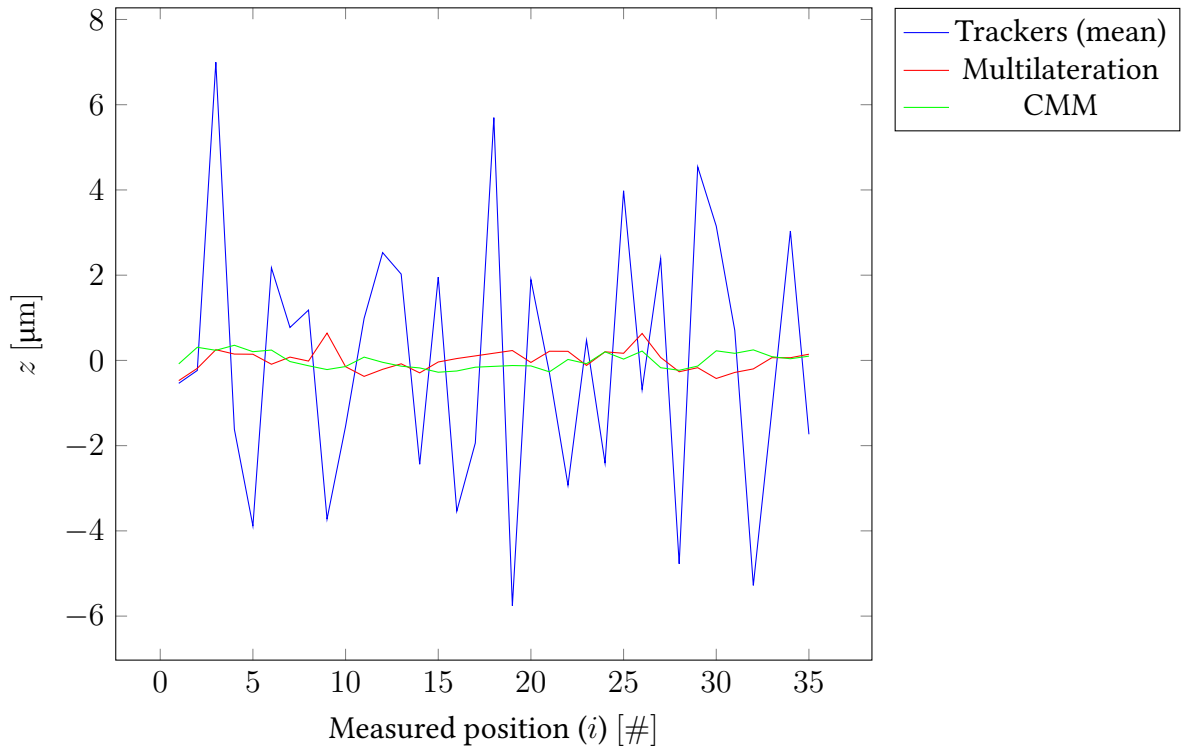


Figure 6.23: The z -coordinate measurements of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic, as determined via: the mean of four laser tracker measurements, four-tracker multilateration, and a CMM measurement. The z -coordinates are determined from a first-order polynomial fit of points for each measurement set: the mean of four laser tracker grid measurements, CMM measurement, and the multilateration solution.

Figure 6.24 shows the z -coordinates of the multilateration solution and the equivalent CMM measurement, as well as the multilateration solution with the second-order fit removed. The z -coordinates are determined from a first-order polynomial fit of points for each measurement set: CMM measurement and the multilateration solution. The most significant difference the multilateration z -coordinates, with and without the fit removed, occurs at the first and last points of the data sets where the saddle shape of the z -coordinates is most apparent.

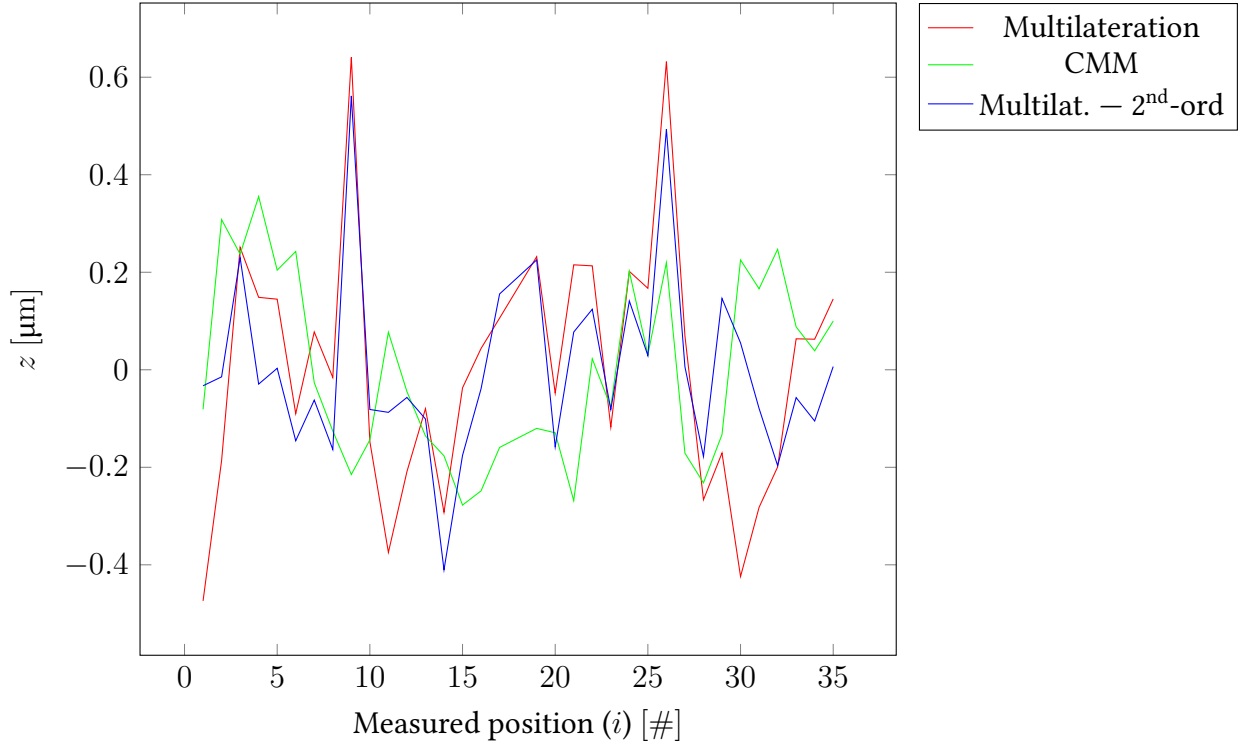


Figure 6.24: The z -coordinate measurements of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic, as determined via: four-tracker multilateration, a CMM measurement, and the multilateration solution with a second-order polynomial fit removed.

6.5 Four-tracker multilateration —reversed ULE[®] flat

This section describes a four AT960LR laser tracker multilateration measurement solution and compares the results to a CMM reference measurement. The sample under investigation was a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic in a reversed orientation — rotated 180° about the axis normal to the measured surface. The measurement procedure was described in section 5.4.

6.5.1 System setup

As for section 6.4, the measurement setup of a four laser tracker multilateration measurement is shown in figures 6.25, 6.26, and 6.27. In this setup, the ULE[®] flat optic was placed in a reversed orientation.

As stated previously, this estimate of the measured positions coordinates was determined by one of the laser trackers measuring in spherical coordinates using ADM, IFM, and angular encoders; and the coordinates of the remaining three laser trackers were calculated using a trilateration routine. The origin of the coordinate system is defined by a measured position: a corner point of the floating grid.

Similar to the first setup, the measured positions form a 6×6 grid of points on the surface of the $0.2 \text{ m} \times 0.2 \text{ m}$ ULE® flat optic, and a 5×5 grid of floating points above the sample surface. Additionally, four floating measured positions were added in order to expand the measured area of the system.

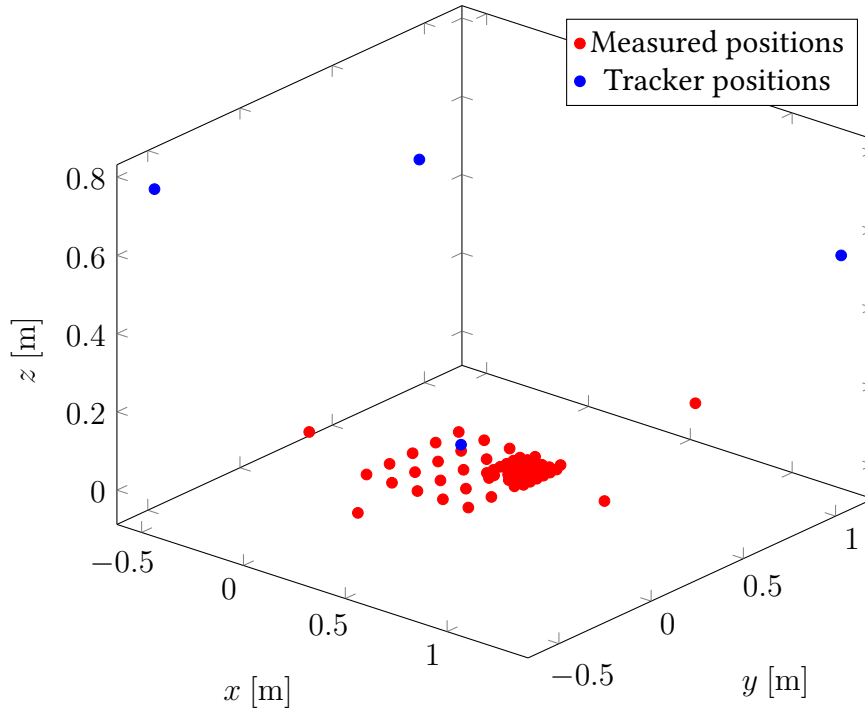


Figure 6.25: The measurement setup for a four AT960LR multilateration measurement of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE® flat optic in a reversed orientation. The measuring station (laser tracker) positions surround the measured positions at differing heights above the surface. The measured positions include: a 6×6 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface, plus a further four floating at the extremes of the system measurement area. The floating-point grid size is $0.5 \text{ m} \times 0.5 \text{ m}$.

Figures 6.26 and 6.27 show how the tracker positions for the setup, detailed in section 6.4, are equivalent to this setup — the only changes were the orientation of the sample and the addition of expanded measured positions.

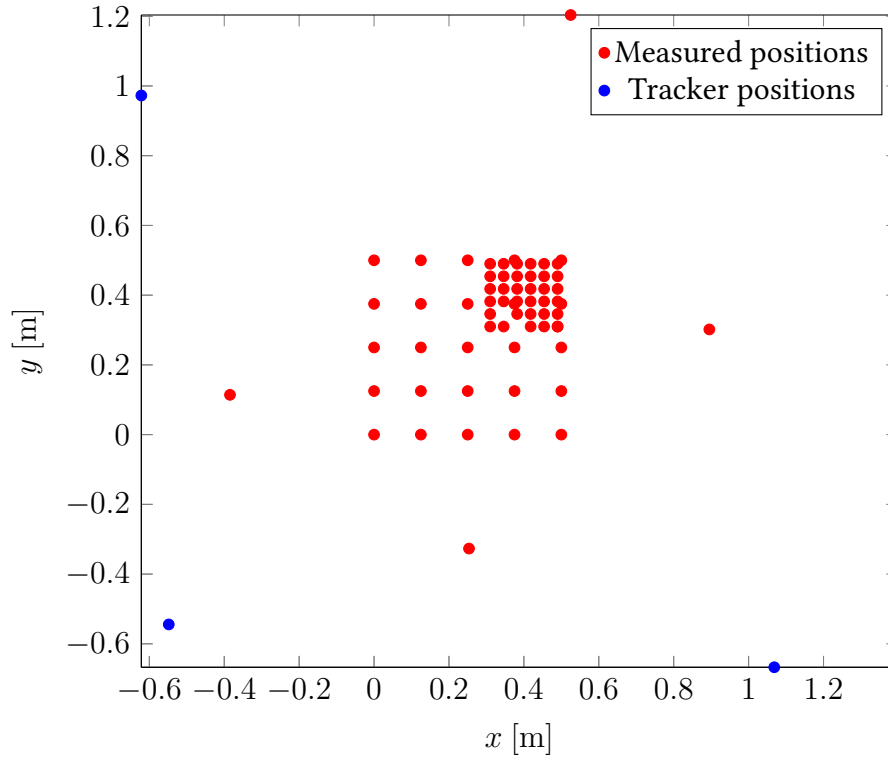


Figure 6.26: A top-view of the measurement setup for a four AT960LR multilateration measurement of a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic in a reversed orientation. The measuring station (laser tracker) positions surround the measured positions at differing heights above the surface. The measured positions include: a 6×6 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface, plus a further four floating at the extremes of the system measurement area. The floating-point grid size is $0.5\text{ m} \times 0.5\text{ m}$.

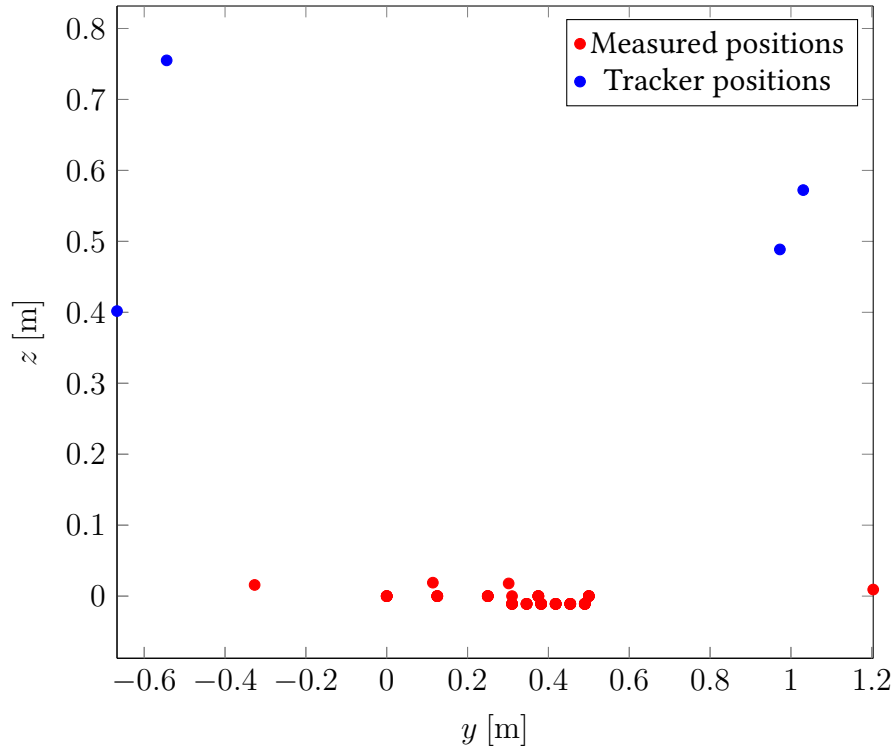


Figure 6.27: A side-view of the measurement setup for a four AT960LR multilateration measurement of a $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic in a reversed orientation. The measuring station (laser tracker) positions surround the measured positions at differing heights above the surface. The measured positions include: a 6×6 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface, plus a further four floating at the extremes of the system measurement area. The floating-point grid size is $0.5\text{ m} \times 0.5\text{ m}$.

6.5.2 Measurement results

A linear interpolation of the z-coordinates of a four-tracker multilateration measurement solution of a $0.2\text{ m} \times 0.2\text{ m}$ ULE® flat optic, in a reversed orientation, plotted against the xy-coordinates in figure 6.28. As for the measurement setup detailed in section 6.4, an SCE-probe was used to measure a 6×6 grid of points on the surface of the sample.

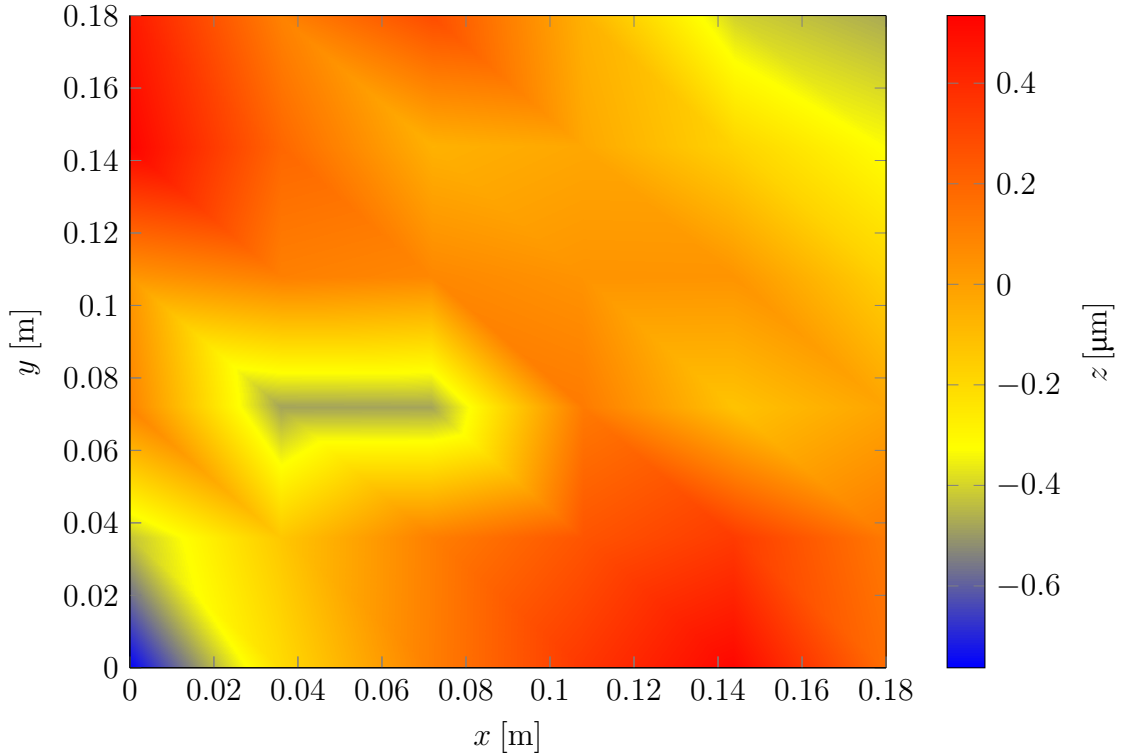


Figure 6.28: A linear interpolation of multilateration determined z-coordinates of a 6×6 grid of measured positions of an SCE-probe on the surface of a $0.2\text{ m} \times 0.2\text{ m}$ ULE® flat optic, in a reversed orientation, using four AT960LR laser trackers. The z-coordinates of measured positions are shown as a function of x- and y-coordinates.

The measured z-coordinates have a standard deviation of $\sigma_z = 0.31\text{ }\mu\text{m}$. The peak-to-valley of the measured z-coordinates is approximately $1.2\text{ }\mu\text{m}$. The measurement of this flat has less noise than the measurement solution shown in figure 6.16; and, similar to the previously described measurement solution, the z-coordinates at $(x, y) = (0, 0)$ and $(x, y) = (0.18, 0.18)$ fall away from the nominal plane.

Figure 6.29 shows a second-order polynomial fit of the z -coordinates shown in figure 6.16. This emphasises the saddle nature of the results.

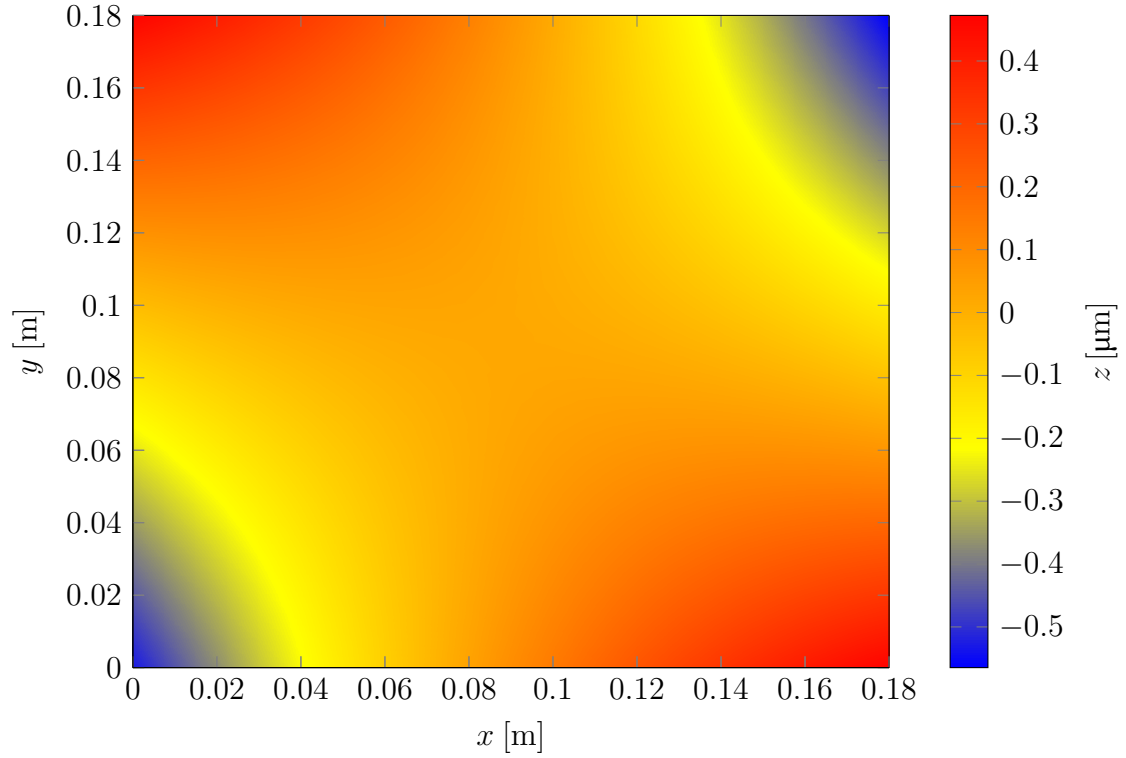


Figure 6.29: A second-order polynomial fit of multilateration determined z -coordinates of a 6×6 grid of measured positions of an SCE-probe on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE® flat optic, in a reversed orientation, using four AT960LR laser trackers. The origin of the coordinate system is defined at the first measured position in x and y .

Figure 6.30 shows a linear interpolation of the z-coordinates of the ULE[®] flat optic shown in figure 6.28 with the second-order fit, shown in figure 6.29, removed. With this fit removed, $\sigma_z = 0.18 \mu\text{m}$.

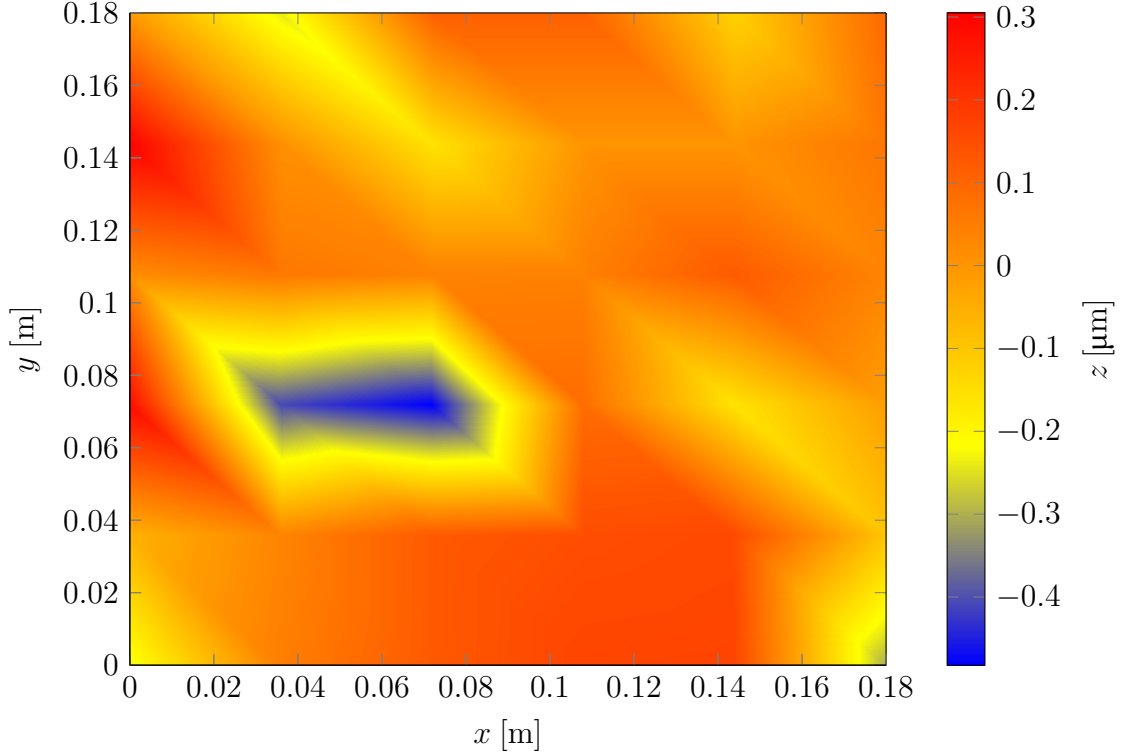


Figure 6.30: A linear interpolation of multilateration determined z-coordinates, with a second-order polynomial fit removed, of a 6×6 grid of measured positions of an SCE-probe on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic, in a reversed orientation, using four AT960LR laser trackers. The z-coordinates of measured positions are shown as a function of x- and y-coordinates.

A linear interpolation of the z-coordinates of a Leitz PMM-F measurement of the $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic, in a reversed orientation, is plotted against the xy-coordinates in figure 6.31. This measurement corresponds to the above multilateration measurement solution.

The measured z-coordinates have a standard deviation of $\sigma_z = 0.17 \mu\text{m}$. The peak-to-valley of the measured z-coordinates is approximately $0.8 \mu\text{m}$. Similar to the case described in section 6.4, the standard deviation of z-coordinates in this measurement is approximately equal to the standard deviation of the equivalent multilateration solution with a second-order fit removed.

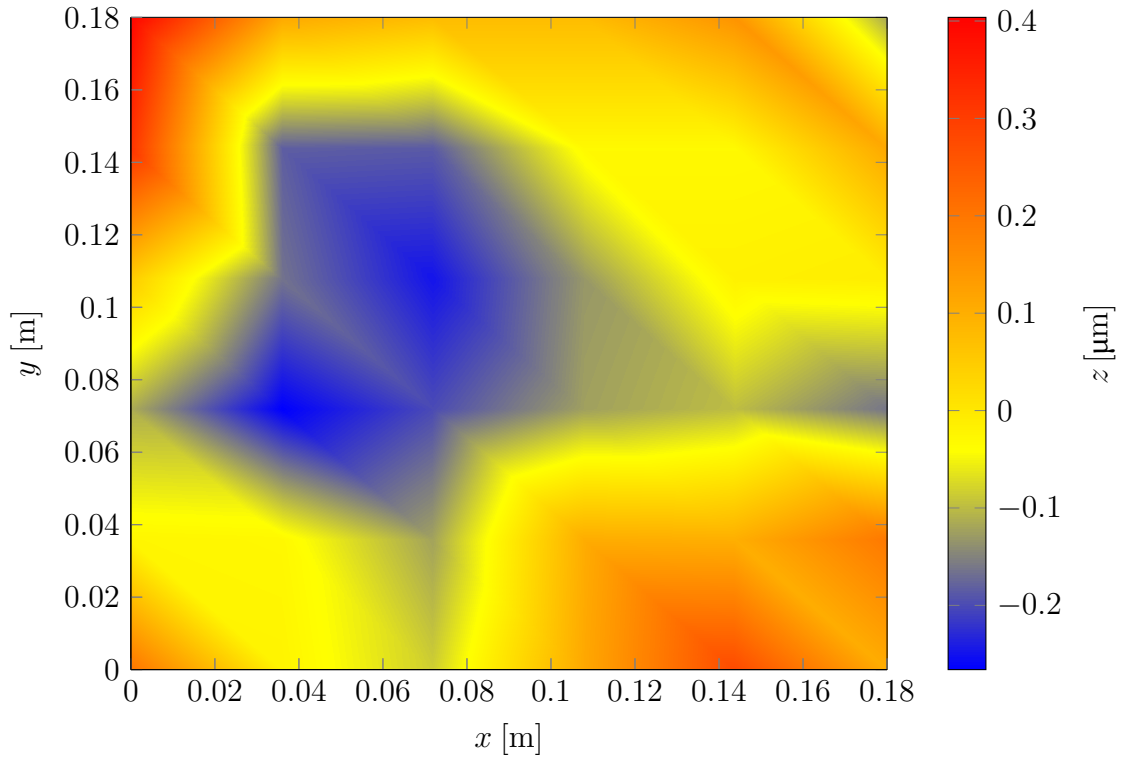


Figure 6.31: A linear interpolation of Leitz PMM-F measured z -coordinates of a 6×6 grid of points, with SCE-probe installed, on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE® flat optic in a reversed orientation. The z -coordinates of measured positions are shown as a function of x - and y -coordinates.

6.5.3 Minimisation residuals

The l^2 -norm of the residuals, $w_{i,j}$, of the minimisation for the multilateration equations, based on the laser tracker interferometric displacement measurements of the measured positions on the surface of the $0.2 \text{ m} \times 0.2 \text{ m}$ ULE® flat optic, in a reversed orientation, is: $|\mathbf{w}|_2 = 7.5 \times 10^{-12} \text{ m}$.

Figure 6.32 shows the residuals of the 256 equations that make up the equation set (4.1) for the measurement of the 63 measured positions in the setup.

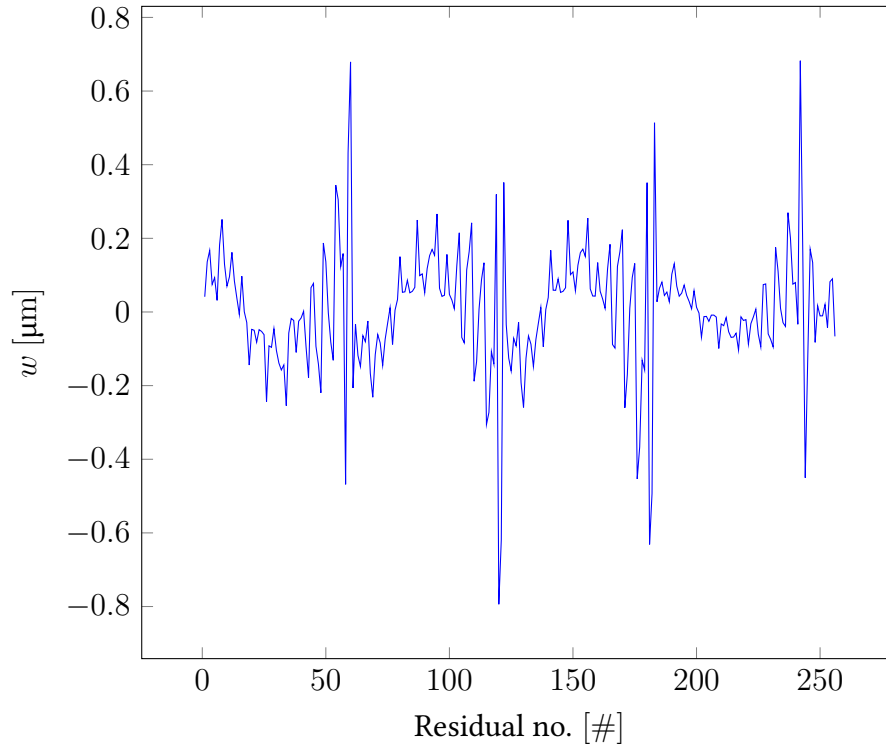


Figure 6.32: The residuals of the least squares minimisation for a multilateration measurement with four AT960LR laser trackers. The sample was a $0.2\text{ m} \times 0.2\text{ m}$ ULE® flat optic in a reversed orientation. Each laser tracker displacement measurement has a corresponding residual, $w_{i,j}$.

The residuals per laser tracker, j , are plotted in figure 6.33. The figure shows how the solving algorithm has approached the minimisation by balancing the residuals for each tracker's measurement set in order to find a solution. It is clear that the measured positions, $i = 253\text{--}256$, have significantly greater residuals, w , than other measured positions. These positions are the expanded area floating points.

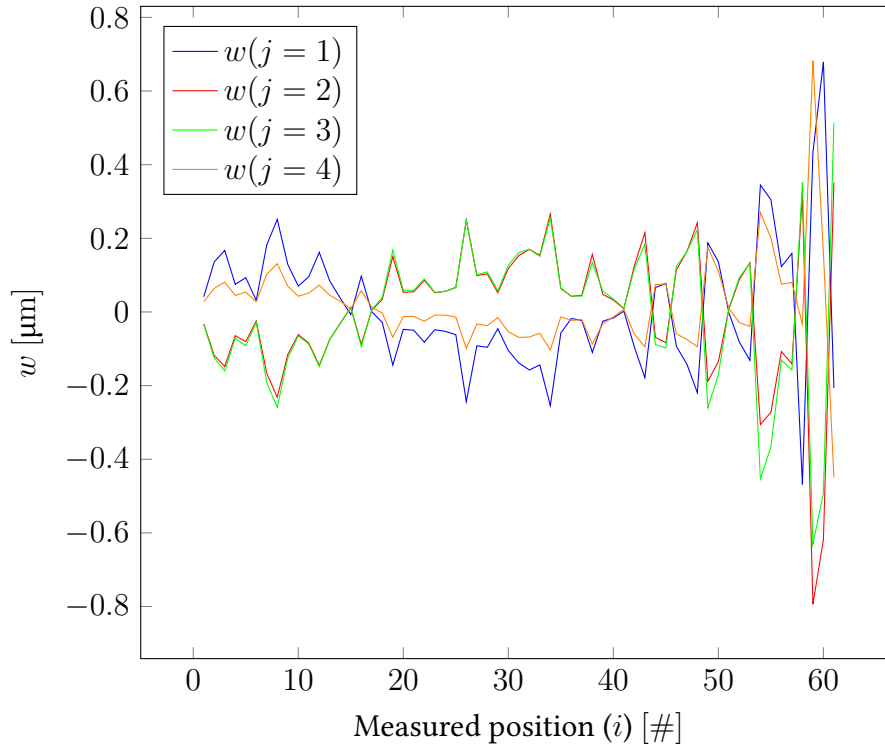


Figure 6.33: The residuals of the least squares minimisation for a multilateration measurement per AT960LR laser tracker (j) for each measured position (i): nominally, 36 measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE® flat optic in a reversed orientation, plus 25 floating positions above the optic and four expanded floating positions.

6.5.4 Z-coordinate comparison

The z-coordinates of the measured positions on the surface of the ULE® flat optic, in a reversed orientation, as measured by each of the four laser trackers and the multilateration solution, are shown in figure 6.34 as a function of the 36 nominal measurement positions. Similar to the measurement detailed in section 6.4, there are 35 measured positions shown of the nominal 36 measured positions on the sample surface as a measurement position was ‘missed’ by a number of the laser trackers due to a lack of measurement synchronization.

Each set of z-coordinates was determined from a first-order polynomial fit of coordinates measured. The standard deviations of z-coordinates measured by the laser trackers (utilising ADM, IFM, and angular encoders) are $\sigma_z = 7.3 \text{ μm}$, 7.4 μm , 6.3 μm , and 9.8 μm .

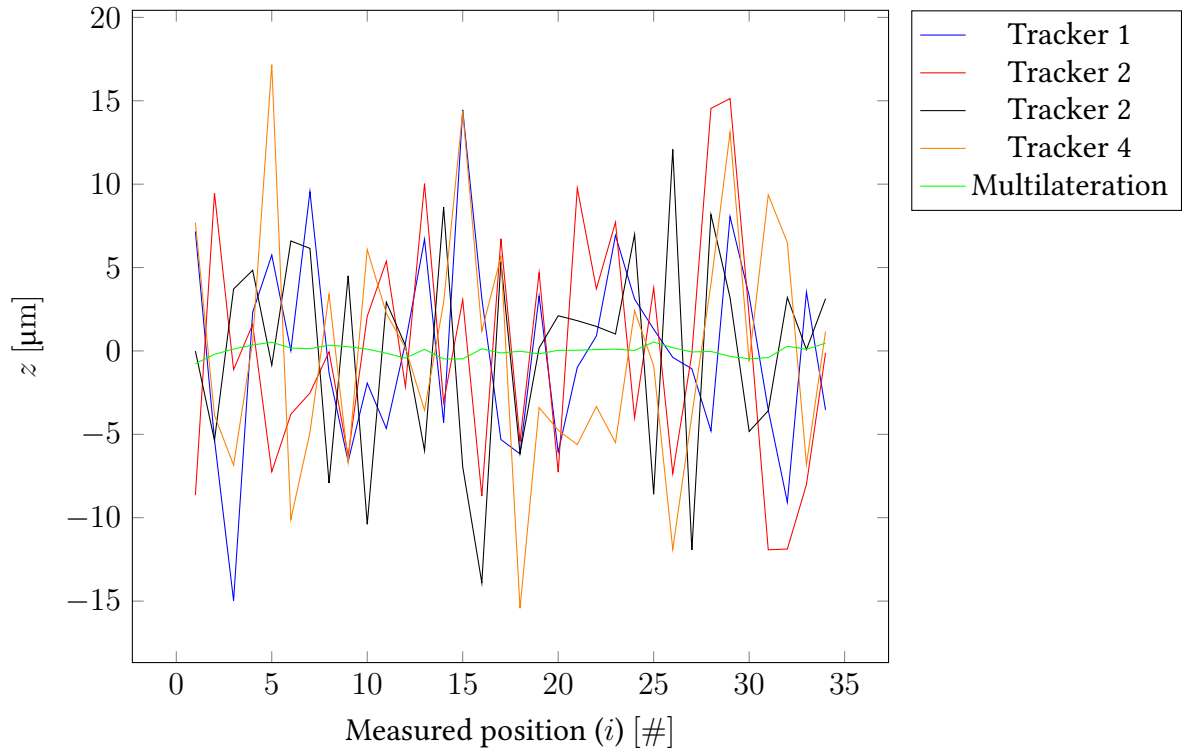


Figure 6.34: The z -coordinate measurements of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic in a reversed orientation; as measured by each of the four AT960LR laser trackers individually, and as determined via four-tracker multilateration.

Figure 6.35 shows the mean of tracker measurements of z -coordinates, as well as the z -coordinates of the multilateration solution and the equivalent CMM measurement. The standard deviation of the mean of tracker measurements is $\sigma_z = 3.9 \mu\text{m}$. This is similar to the measurement of the flat optic in its origin orientation.

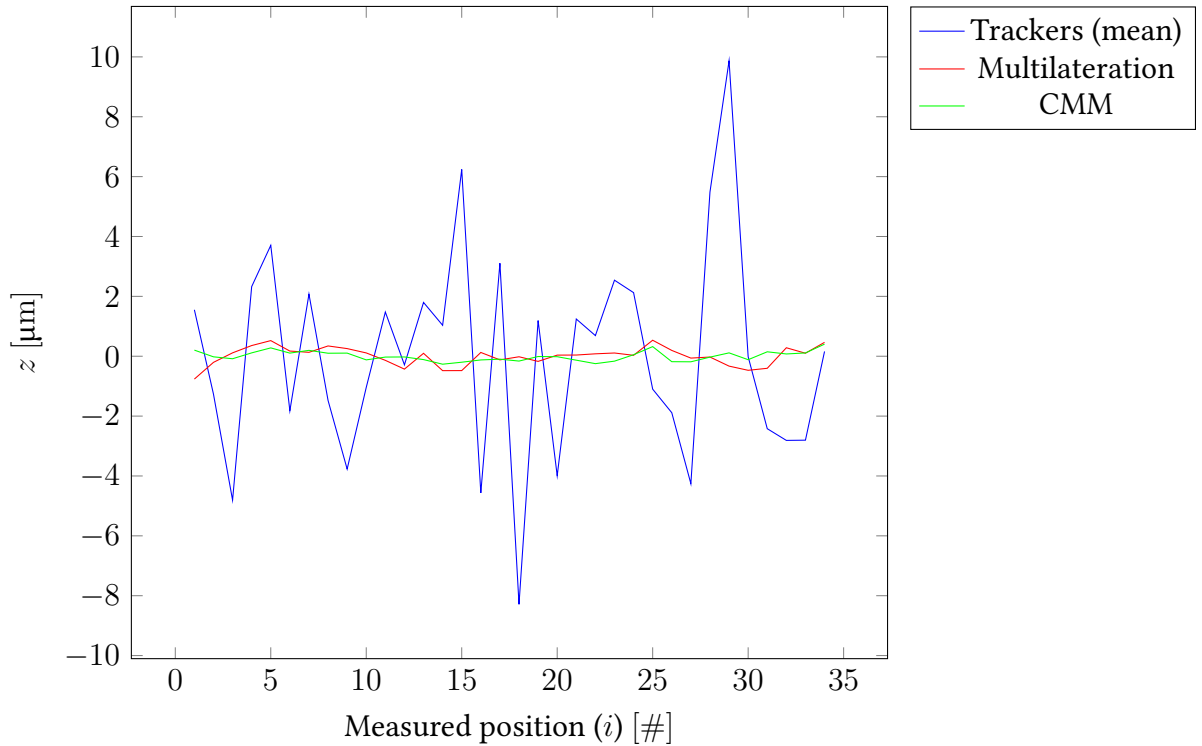


Figure 6.35: The z -coordinate measurements of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE® flat optic in a reversed orientation, as determined via: the mean of four laser tracker measurements, four-tracker multilateration, and a CMM measurement. The z -coordinates are determined from a first-order polynomial fit of points for each measurement set: the mean of four laser tracker grid measurements, CMM measurement, and the multilateration solution.

Figure 6.36 shows the z -coordinates of the multilateration solution and the equivalent CMM measurement, as well as the multilateration solution with the second-order fit removed. As for the measurement of the ULE® flat optic in its original orientation, the most significant difference the multilateration z -coordinates, with and without the fit removed, occurs at the first and last points of the data sets where the saddle shape of the z -coordinates is most apparent.

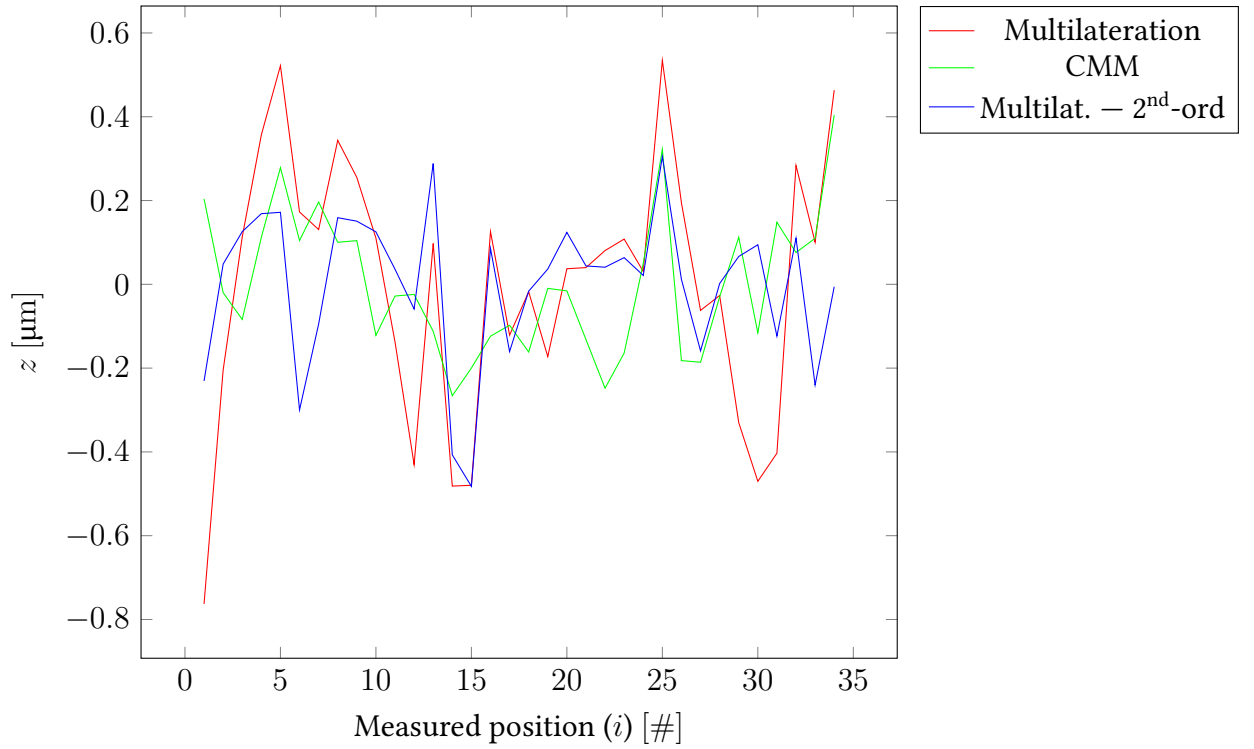


Figure 6.36: The z -coordinate measurements of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ ULE[®] flat optic in a reversed orientation, as determined via: four-tracker multilateration, a CMM measurement, and the multilateration solution with a second-order polynomial fit removed.

6.6 Monte Carlo simulation of ULE[®] flat measurement

A series of Monte Carlo simulations for the measurement of a $0.2\text{ m} \times 0.2\text{ m}$ flat optic to determine estimates of the measurement uncertainty of the system under investigation. The predicted sensitivity of measurement uncertainty to a number of parameters is detailed; these include: interferometric measurement uncertainty, SMR sphericity, deadpath error, and measurement system setup arrangement. It is of note that the simulation time was approximately 0.2 s per n_{samples} .

6.6.1 Nominal parameters

To determine an estimate of uncertainty of the measurement described in section 6.4, a set of nominal parameter values for equation (4.18) were selected. These parameter values are shown in table 6.1. Other system setup parameters are defined as equivalent to the setup of the measurement under analysis. The value of SMR sphericity were defined based on $\theta = 1^\circ$. The interferometric displacement uncertainty and ADM deadpath uncertainty are the manufacturer determined values for the AT960LR. As this measurement used four laser trackers simultaneously, u_{CMM} . No time dependent parameters were considered due to the short time of the real measurement.

The square root mean variance of $\sigma_{z,i}$, the standard deviation 50000 predicted measurement per measured position, of the measured positions on the surface of the sample is $\sigma_z = 1.07\text{ }\mu\text{m}$. This prediction is a factor of 4.3 greater than the real measured value of $\sigma_z = 0.25\text{ }\mu\text{m}$ for this measurement setup.

Table 6.1: Monte Carlo simulation parameters and results for the measurement of the small flat.

Parameter	Setting/result
n_{samples}	50000
m	4
n	$35 + 25$
u_0	$0.2\text{ }\mu\text{m}$
u_{L}	$0.15\text{ }\mu\text{m}$
u_{dp}	$5\text{ }\mu\text{m}$
b	$0.05\text{ }\mu\text{m}$ ($\theta = 1^\circ$)
Δr	$0.07\text{ }\mu\text{m}$ ($\theta = 1^\circ$)
σ_z	$1.07\text{ }\mu\text{m}$ ($k = 1$)

A linear interpolation of the standard deviations, $\sigma_{z,i}$, of z-coordinates of measured positions are shown as a function of x- and y-coordinates in figure 6.37.

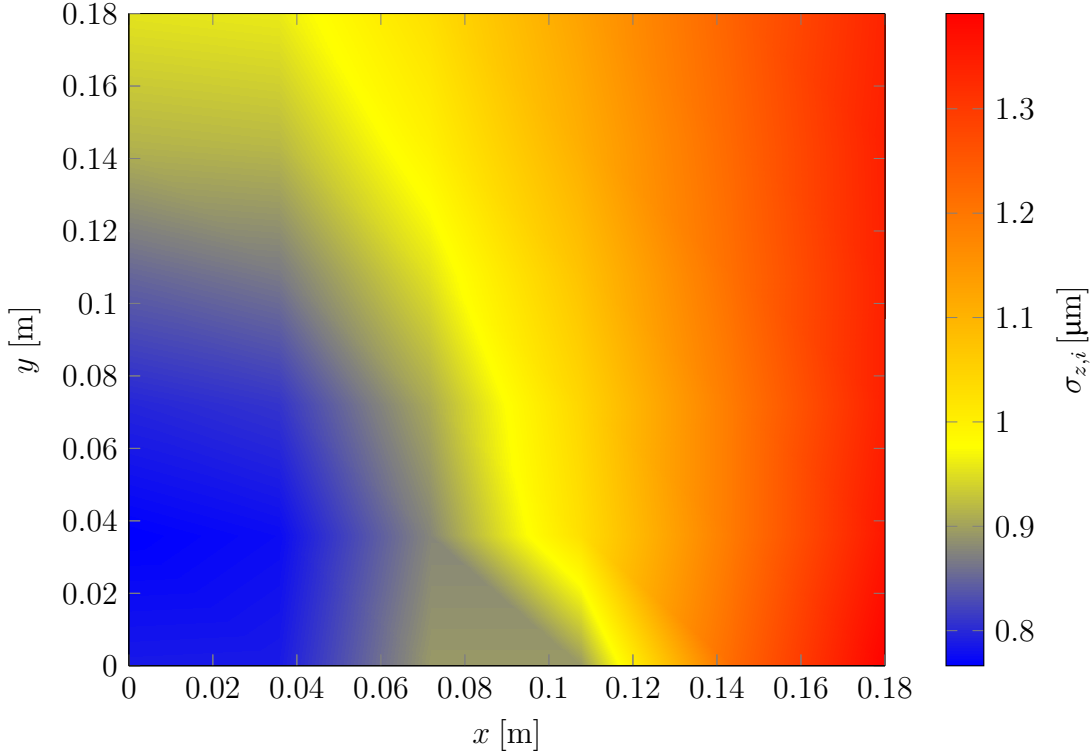


Figure 6.37: A linear interpolation of MC simulated ($n_{\text{samples}} = 50000$) multilateration determined standard deviations, $\sigma_{z,i}$, of z-coordinates of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ flat sample using four AT960LR laser trackers.

The predicted measurement uncertainty is lowest, $\sigma_{z,i} \approx 0.75 \mu\text{m}$, close to the corner of the sample at $(x, y) = (0, 0)$. These coordinates correspond to the corner of the sample that is coincident with the corner of the 25 floating positions as shown in figure 6.14. The uncertainty is highest, $\sigma_{z,i} \approx 1.6 \mu\text{m}$, along the line at $x = 0.18$.

6.6.2 Sensitivity to interferometric displacement measurement uncertainty

To determine an estimate of uncertainty of the measurement described in section 6.4, with a reduced first-term interferometric displacement measurement uncertainty was selected as an input, as shown in table 6.2. All other parameters are equal to those selected for the nominal simulation above.

The square root mean variance of $\sigma_{z,i}$ of the measured positions on the surface of the sample is $\sigma_z = 0.85 \mu\text{m}$. This predicts a reduction of 21% on the equivalent simulated result for the nominal AT960LR interferometric uncertainty.

Table 6.2: Monte Carlo simulation parameters, to test error sensitivity to interferometric displacement measurement uncertainty, and the corresponding results for the measurement of the small flat.

Parameter	Setting/result
n_{samples}	50000
m	4
n	$35 + 25$
u_0	$0.1 \mu\text{m}$
u_L	$0.15 \mu\text{m}$
u_{dp}	$5 \mu\text{m}$
b	$0.05 \mu\text{m} (\theta = 1^\circ)$
Δr	$0.07 \mu\text{m} (\theta = 1^\circ)$
σ_z	$0.85 \mu\text{m} (k = 1)$

A linear interpolation of the standard deviations, $\sigma_{z,i}$, of z-coordinates of measured positions are shown as a function of x- and y-coordinates in figure 6.38.

The predicted measurement uncertainty is lowest, $\sigma_{z,i} \approx 0.6 \mu\text{m}$, close to the corner of the sample at $(x, y) = (0, 0)$. The uncertainty is highest, $\sigma_{z,i} \approx 1.1 \mu\text{m}$, along the line at $x = 0.18$. The form of the predicted uncertainties remains the same as the nominal analysis.

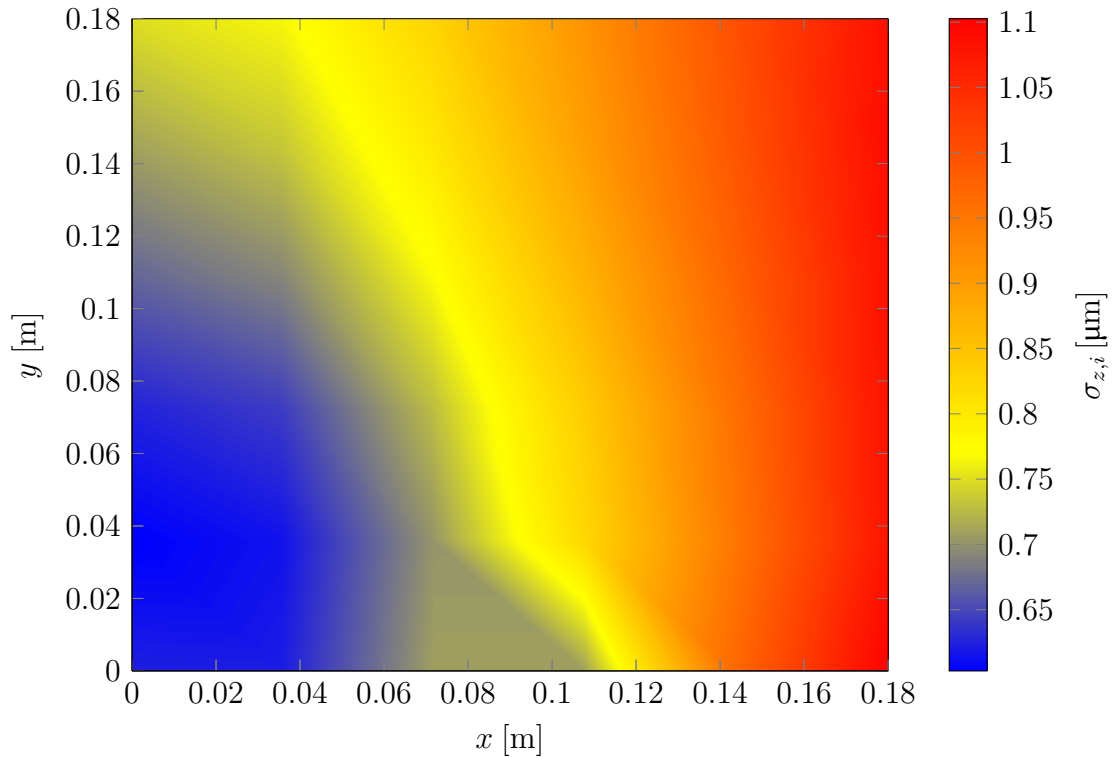


Figure 6.38: A linear interpolation of MC simulated ($n_{\text{samples}} = 50000$) multilateration determined standard deviations, $\sigma_{z,i}$, of z-coordinates of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ flat sample using four laser trackers with reduced first-term displacement measurement uncertainty.

6.6.3 Sensitivity to SMR sphericity

To determine an estimate of uncertainty of the measurement described in section 6.4 with SMR sphericity errors $b = 0.26 \mu\text{m}$ and $0.33 \mu\text{m}$ that correspond to $\theta = 5^\circ$ as shown in table 6.3. All other parameters are equal to those selected for the nominal simulation above.

The square root mean variance of $\sigma_{z,i}$ of the measured positions on the surface of the sample is $\sigma_z = 1.62 \mu\text{m}$. This prediction is an increase of approximately 51% on the measurement uncertainty of the equivalent simulated result for the nominal SMR sphericity error for $\theta = 1^\circ$.

Table 6.3: Monte Carlo simulation parameters, to test error sensitivity to SMR sphericity error, and the corresponding results for the measurement of the small flat.

Parameter	Setting/result
n_{samples}	50000
m	4
n	$35 + 25$
u_0	$0.2 \mu\text{m}$
u_L	$0.15 \mu\text{m}$
u_{dp}	$5 \mu\text{m}$
b	$0.26 \mu\text{m} (\theta = 5^\circ)$
Δr	$0.33 \mu\text{m} (\theta = 5^\circ)$
$\sigma_z,$	$1.62 \mu\text{m} (k = 1)$

A linear interpolation of the standard deviations, $\sigma_{z,i}$, of z-coordinates of measured positions are shown as a function of x- and y-coordinates in figure 6.39.

The predicted measurement uncertainty is lowest, $\sigma_{z,i} \approx 1.5 \mu\text{m}$, along the line on the sample at $x = 0$. The uncertainty is highest, $\sigma_{z,i} \approx 1.9 \mu\text{m}$, along the line at $x = 0.18$. The form of the predicted uncertainties remains the similar to the nominal case.

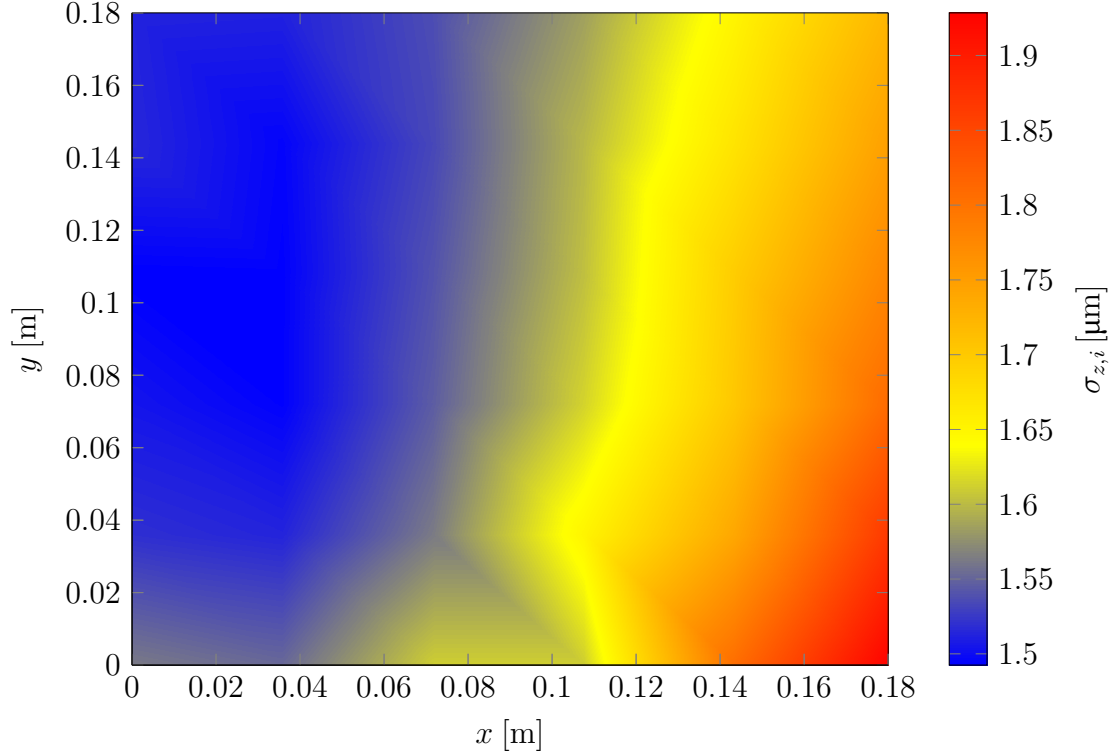


Figure 6.39: A linear interpolation of MC simulated ($n_{\text{samples}} = 50000$) multilateration determined standard deviations, $\sigma_{z,i}$, of z-coordinates of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ flat sample using four laser trackers and SMR sphericity errors that correspond to $\theta = 5^\circ$.

6.6.4 Sensitivity to deadpath error

To determine an estimate of uncertainty of the measurement described in section 6.4 with deadpath measurement uncertainty reduced to $u_{dp} = 2.5 \mu\text{m}$. All other parameters are equal to those selected for the nominal simulation above, as shown in table 6.4.

The square root mean variance of $\sigma_{z,i}$ of the measured positions on the surface of the sample is $\sigma_z = 1.03 \mu\text{m}$. This predicts a reduction of approximately 4% on the measurement uncertainty of the equivalent simulated result for the nominal deadpath error, $u_{dp} = 5 \mu\text{m}$.

Table 6.4: Monte Carlo simulation parameters, to test error sensitivity to deadpath error, and the corresponding results for the measurement of the small flat.

Parameter	Setting/result
n_{samples}	50000
m	4
n	$35 + 25$
u_0	$0.2 \mu\text{m}$
u_L	$0.15 \mu\text{m}$
u_{dp}	$2.5 \mu\text{m}$
b	$0.05 \mu\text{m} (\theta = 1^\circ)$
Δr	$0.07 \mu\text{m} (\theta = 1^\circ)$
σ_z	$1.03 \mu\text{m} (k = 1)$

A linear interpolation of the standard deviations, $\sigma_{z,i}$, of z-coordinates of measured positions are shown as a function of x- and y-coordinates in figure 6.40.

The predicted measurement uncertainty is lowest, $\sigma_{z,i} \approx 0.75 \mu\text{m}$, close to the corner of the sample at $(x, y) = (0, 0)$. The uncertainty is highest, $\sigma_{z,i} \approx 1.35 \mu\text{m}$, along the line at $x = 0.18$. The form of the predicted uncertainties remains the same as the nominal analysis.

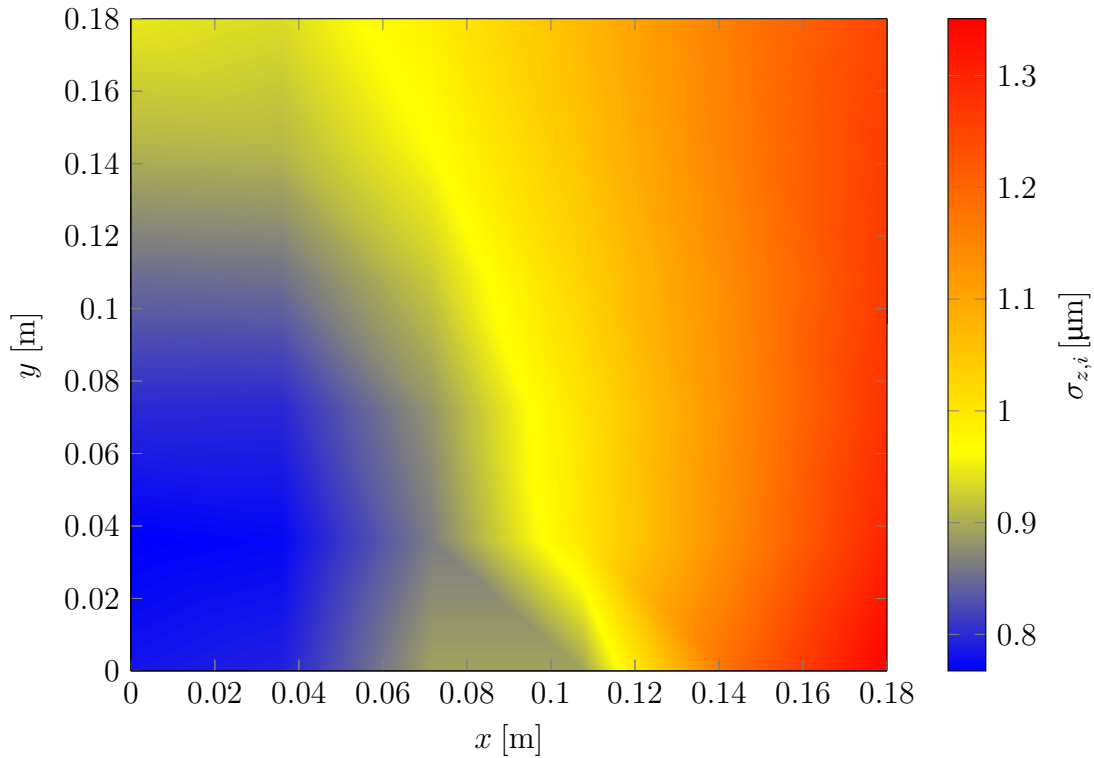


Figure 6.40: A linear interpolation of MC simulated ($n_{\text{samples}} = 50000$) multilateration determined standard deviations, $\sigma_{z,i}$, of z-coordinates of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ flat sample using four laser trackers with a reduced deadpath error, $u_{dp} = 2.5 \mu\text{m}$.

6.6.5 MC simulation of optimised setup

The measurement setup of a Monte Carlo simulated four laser tracker multilateration measurement is shown in figures 6.41, 6.42, and 6.43. The measured positions remain the same as the setup described in section 6.4; however, the measuring station positions have been redefined to reduce the contribution of measurement setup on measurement solution uncertainty. The measuring station are positioned in a triangular pyramid above the measured positions.

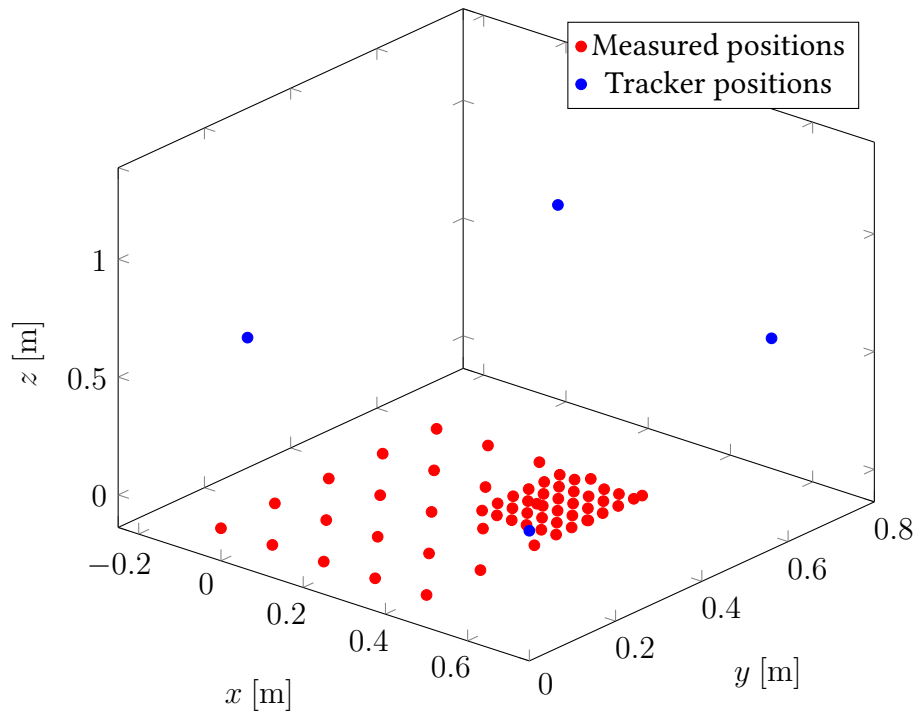


Figure 6.41: The measurement setup for a Monte Carlo simulated four AT960LR multilateration measurement of a $0.2 \text{ m} \times 0.2 \text{ m}$ flat optic. The measuring station (laser tracker) positions have been optimised to reduce the measurement uncertainty contribution of system setup. The measured positions include: a 6×6 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface. The floating-point grid size is $0.5 \text{ m} \times 0.5 \text{ m}$.

Figure 6.42 shows how three of the laser tracker positions surround the sample; whilst the fourth tracker is positioned directly over the sample.

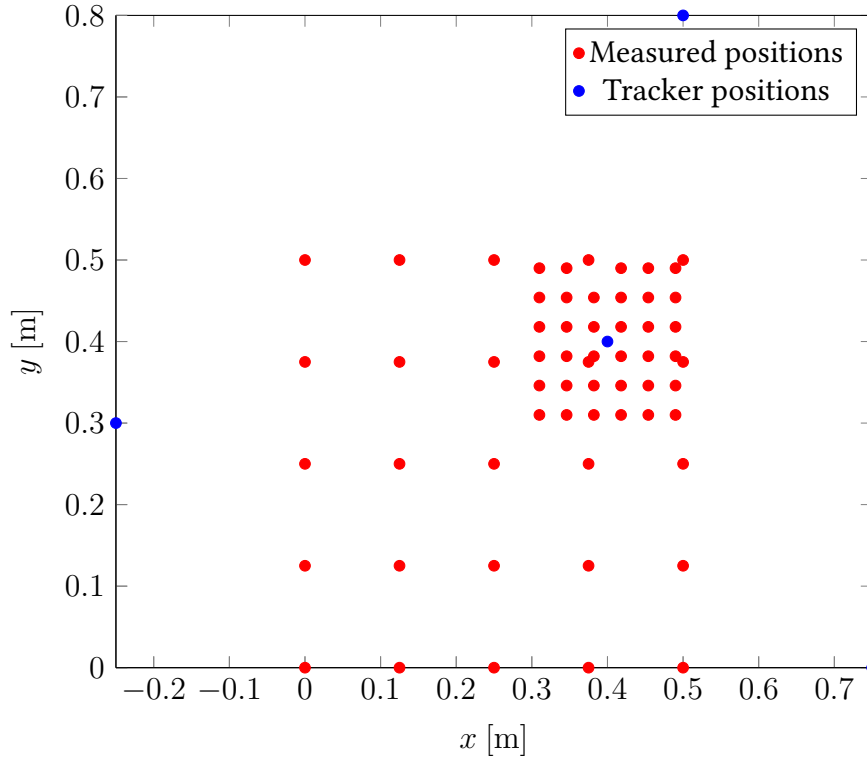


Figure 6.42: A top-view of the measurement setup for a Monte Carlo simulated four AT960LR multilateration measurement of a $0.2\text{ m} \times 0.2\text{ m}$ flat optic. The measuring station (laser tracker) positions have been optimised to reduce the measurement uncertainty contribution of system setup. The measured positions include: a 6×6 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface. The floating-point grid size is $0.5\text{ m} \times 0.5\text{ m}$.

Figure 6.43 shows how three of the tracker's height above the sample are equal and relatively close to the sample in the z-axis; by comparison, the fourth laser tracker height is relatively large.

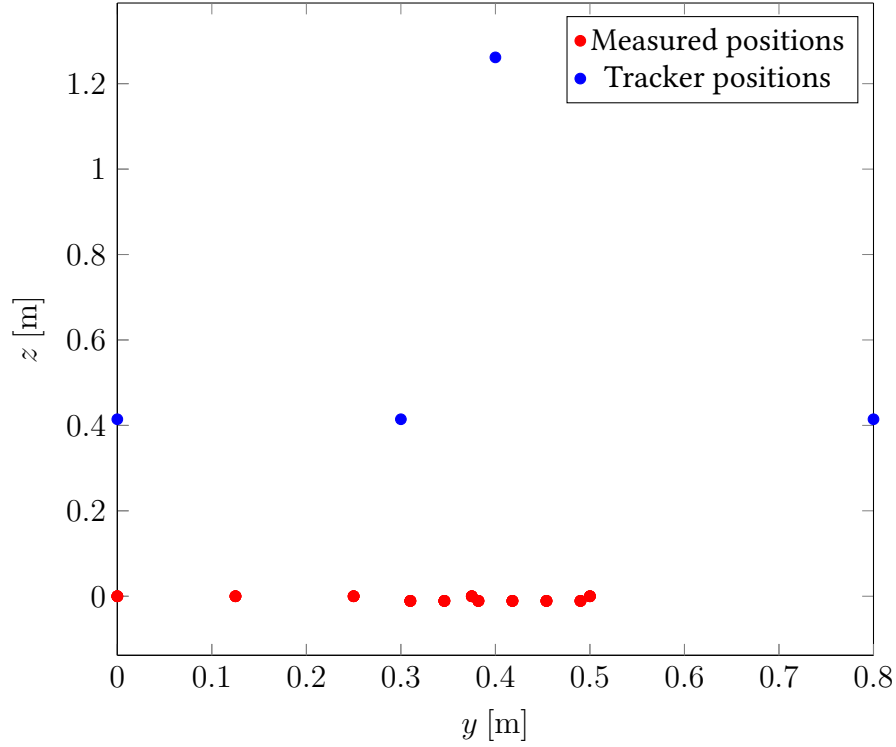


Figure 6.43: A side-view of the measurement setup for a Monte Carlo simulated four AT960LR multilateration measurement of a $0.2 \text{ m} \times 0.2 \text{ m}$ flat optic. The measuring station (laser tracker) positions have been optimised to reduce the measurement uncertainty contribution of system setup. The measured positions include: a 6×6 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface. The floating-point grid size is $0.5 \text{ m} \times 0.5 \text{ m}$.

To test the sensitivity of measurement uncertainty to the arrangement of measuring station positions relative to measured positions, the system setup as described in section 6.4 was redefined with different measuring station positions as shown above. As shown in table 6.5, all other parameters remained the same as the nominal measurement simulation.

The square root mean variance of $\sigma_{z,i}$ of the measured positions on the surface of the sample is $\sigma_z = 0.47 \mu\text{m}$. This predicts a reduction of approximately 56% on the measurement uncertainty of the equivalent simulated result by redefining the positions of measuring stations into this triangular pyramid above the measured positions.

Table 6.5: Monte Carlo simulation parameters and results for the measurement of a $0.2 \text{ m} \times 0.2 \text{ m}$ flat optic with an optimised measuring positions setup.

Parameter	Setting/result
n_{samples}	50000
m	4
n	$35 + 25$
u_0	$0.2 \mu\text{m}$
u_L	$0.15 \mu\text{m}$
u_{dp}	$5 \mu\text{m}$
b	$0.05 \mu\text{m} (\theta = 1^\circ)$
Δr	$0.07 \mu\text{m} (\theta = 1^\circ)$
σ_z	$0.47 \mu\text{m} (k = 1)$

A linear interpolation of the standard deviations, $\sigma_{z,i}$, of z-coordinates of measured positions are shown as a function of x- and y-coordinates in figure 6.44.

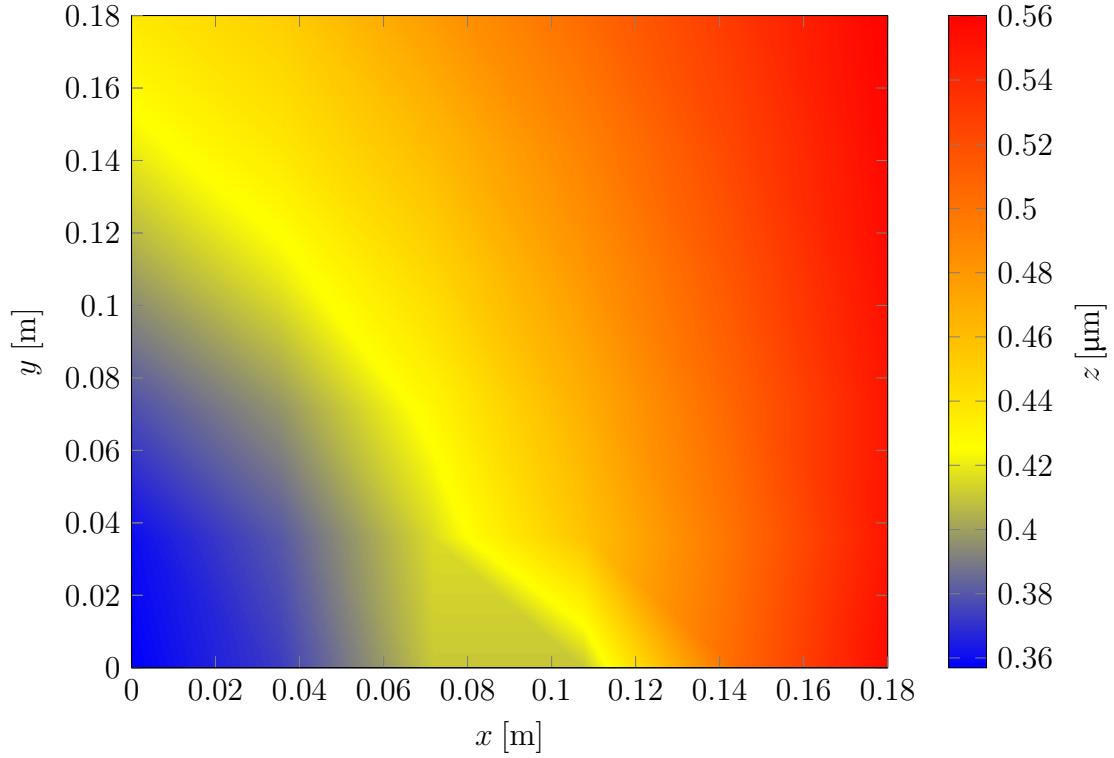


Figure 6.44: A linear interpolation of MC simulated ($n_{\text{samples}} = 50000$) multilateration determined z-coordinates of a 6×6 grid of measured positions on the surface of a $0.2 \text{ m} \times 0.2 \text{ m}$ flat sample using four AT960LR laser trackers. The measuring positions were chosen to minimise the uncertainty contribution of system setup. The z-coordinates of measured positions are shown as a function of x- and y-coordinates.

The predicted measurement uncertainty is lowest, $\sigma_{z,i} \approx 0.36 \mu\text{m}$, close to the corner of the sample at $(x, y) = (0, 0)$. The uncertainty is highest, $\sigma_{z,i} \approx 0.56 \mu\text{m}$, along the line at $x = 0.18$. The form of the predicted uncertainties is similar to the nominal case.

6.7 Four-tracker multilateration — large flat with SCE

This section describes a four AT960LR laser tracker multilateration measurement solution for a large optic and compares the results to a CMM reference measurement. The sample under investigation was a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. An SCE-probe was used as a measurement target.

6.7.1 System setup

The measurement setup of a four laser tracker multilateration measurement is shown in figure 6.45. Again, the estimate of the measured positions coordinates was determined by one of the laser trackers measuring in spherical coordinates. The coordinates of the remaining three laser trackers were calculated using a trilateration routine. The system setup is equivalent to the setup detailed in section 6.4; in this case, there were 100 positions measured on the surface of the large sample in a 10×10 grid.

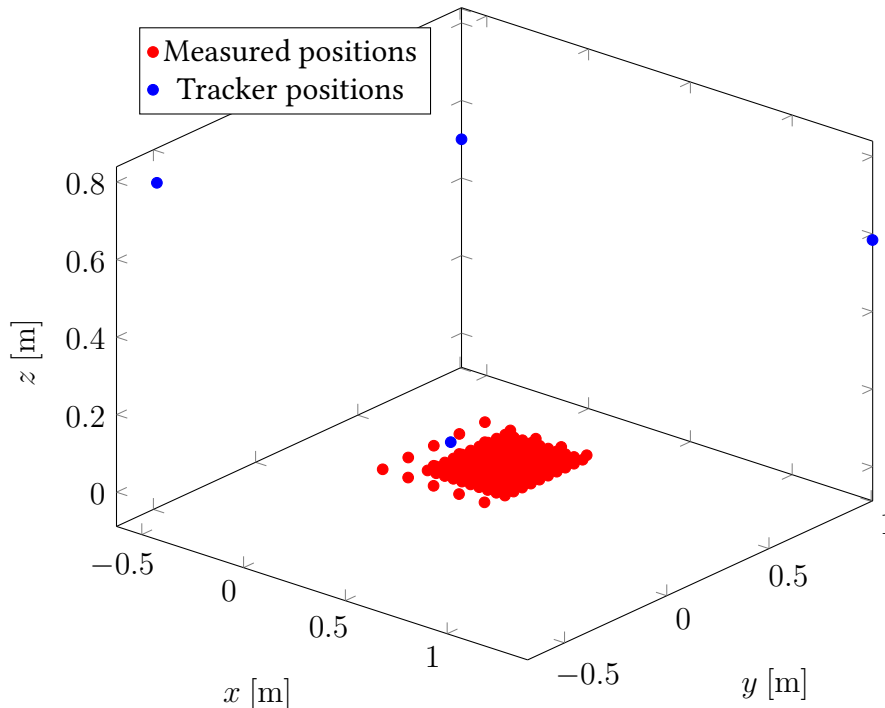


Figure 6.45: The measurement setup for a four AT960LR multilateration measurement of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. An SCE-probe was installed on the CMM used to move the probe into the measurement positions. The measuring station (laser tracker) positions surround the measured positions at differing heights above the surface. The measured positions include: a 10×10 grid of points on the surface of the sample and a 5×5 grid of floating points above the sample surface. The floating-point grid size is $0.5\text{ m} \times 0.5\text{ m}$.

6.7.2 Measurement results

A linear interpolation of the z-coordinates of a four-tracker multilateration measurement solution of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic plotted against the xy-coordinates in figure 6.46. An SCE-probe was used to measure a 10×10 grid of points on the surface of the sample.

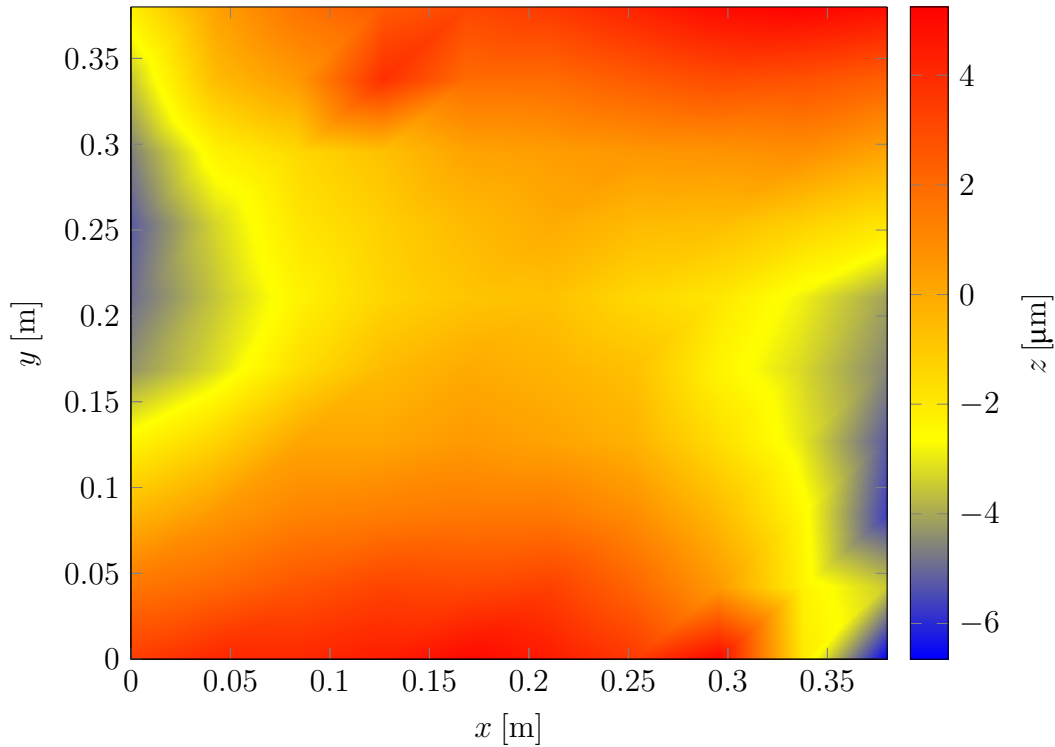


Figure 6.46: A linear interpolation of multilateration determined z-coordinates of a 10×10 grid of measured positions of an SCE-probe on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic using four AT960LR laser trackers. The z-coordinates of measured positions are shown as a function of x- and y-coordinates.

The measured z-coordinates have a standard deviation of $\sigma_z = 2.81\text{ }\mu\text{m}$. The peak-to-valley of the measured z-coordinates is approximately $11\text{ }\mu\text{m}$. The multilateration solution for the measurement of this flat appears to have a clear form of an inverted saddle.

A linear interpolation of the z-coordinates of a Leitz PMM-F measurement of the $0.4\text{ m} \times 0.4\text{ m}$ flat optic is plotted against the xy-coordinates in figure 6.47. This measurement corresponds to the above multilateration measurement solution.

The measured z-coordinates have a standard deviation of $\sigma_z = 2.48\text{ }\mu\text{m}$. The peak-to-valley of the measured z-coordinates is approximately $10\text{ }\mu\text{m}$. The form of this measurement is similar to the form of the equivalent multilateration solution, however, it has a more exaggerated saddle shape.

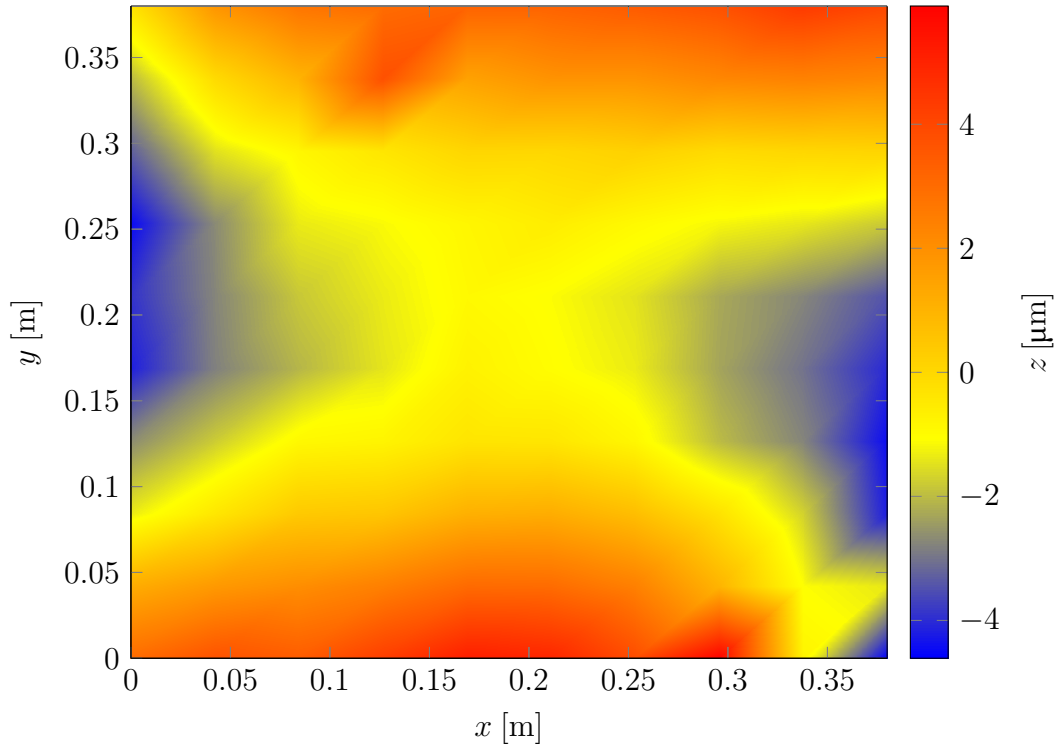


Figure 6.47: A linear interpolation of Leitz PMM-F measured z -coordinates of a 10×10 grid of points, with SCE-probe installed, on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. The z -coordinates of measured positions are shown as a function of x - and y -coordinates and are defined by a first-order polynomial surface through the measured coordinate positions. The origin of the coordinate system is defined at the first measured position in x and y .

6.7.3 Minimisation residuals

The l^2 -norm of the residuals, $w_{i,j}$, of the minimisation for the multilateration equations, based on the laser tracker interferometric displacement measurements of the measured positions on the surface of the $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic is: $|\mathbf{w}|_2 = 4.8 \times 10^{-12} \text{ m}$.

Figure 6.48 shows the residuals of the 500 equations that make up the equation set (4.1) for the measurement of the 125 measured positions in the setup.

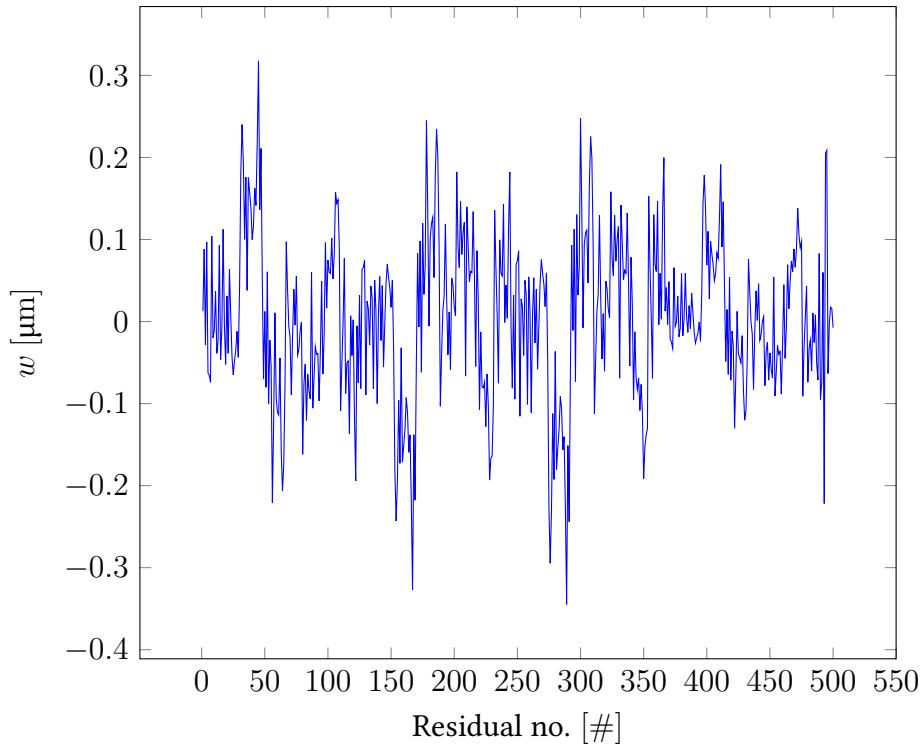


Figure 6.48: The residuals of the least squares minimisation for a multilateration measurement with four AT960LR laser trackers. The sample was a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. Each laser tracker displacement measurement has a corresponding residual, $w_{i,j}$.

The residuals per laser tracker, j , are plotted in figure 6.49. There are four residuals per measured position corresponding to the four displacement measurements for each position. The measurement of 100 points on the surface of the sample and 25 floating points results in a total of 500 residuals. The figure shows how the solving algorithm has approached the minimisation by balancing the residuals for each tracker to find a minimised solution to the equation set.

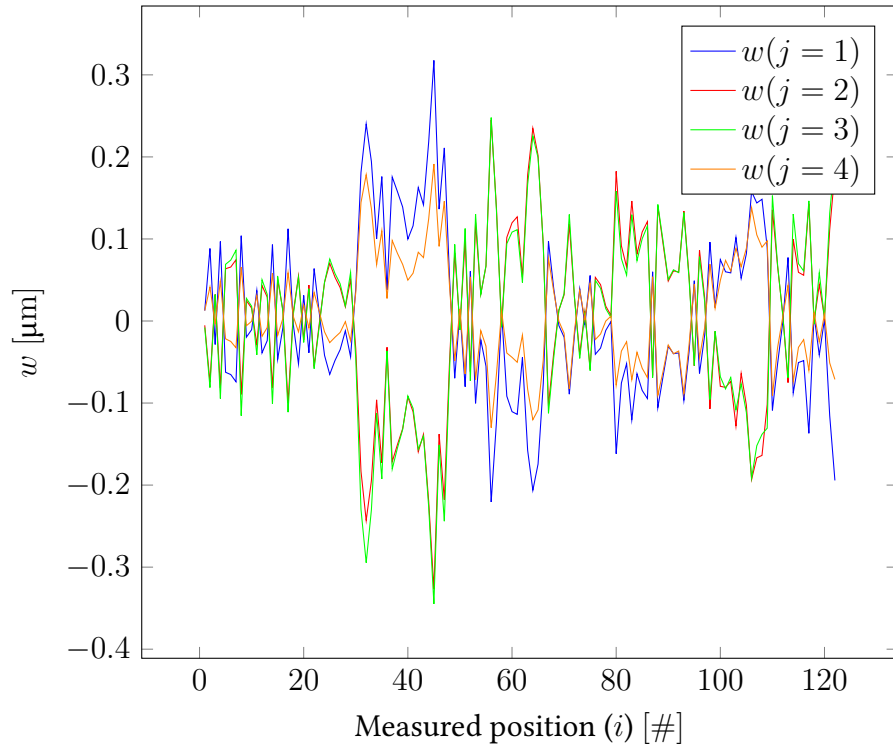


Figure 6.49: The residuals of the least squares minimisation for a multilateration measurement per AT960LR laser tracker (j) for each measured position (i): 100 measured positions on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic, plus 25 floating positions above the optic.

6.7.4 Z-coordinate comparison

Figure 6.50 shows the z -coordinates of the multilateration solution and the equivalent CMM measurement, as well as the difference between the multilateration solution and the CMM measurement.

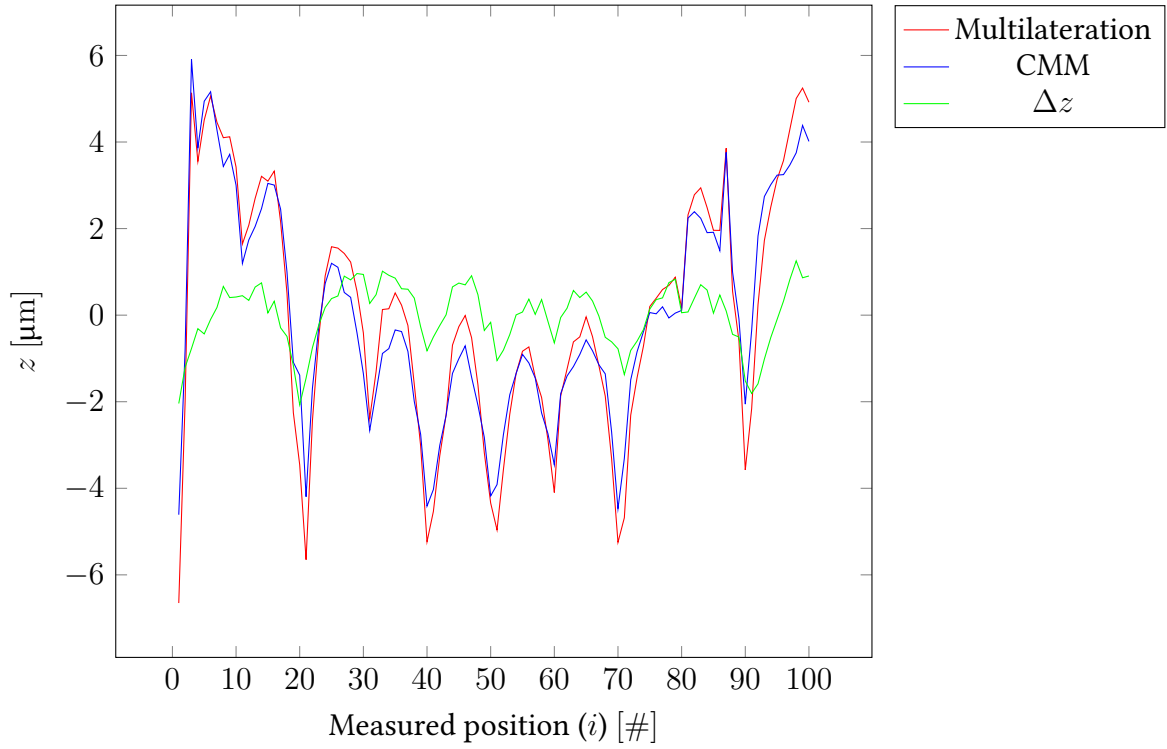


Figure 6.50: The z -coordinate measurements of a 10×10 grid of measured positions on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic, as determined via: four-tracker multilateration and a CMM measurement. The difference, Δz , between the z -coordinates of the multilateration and CMM measurements is plotted.

Figure 6.51 shows a linear interpolation of the difference between the multilateration solution and the CMM measurement, Δz , of the large flat optic. The standard deviation of Δz is $\sigma_{\Delta z} = 0.73 \mu\text{m}$. The peak-to-valley of $\Delta z \approx 3.3 \mu\text{m}$. It is noteworthy that the form of Δz is similar to the form of the multilateration solutions for the measurements of the ULE[®] flat optic, detailed in sections 6.4 and 6.5.

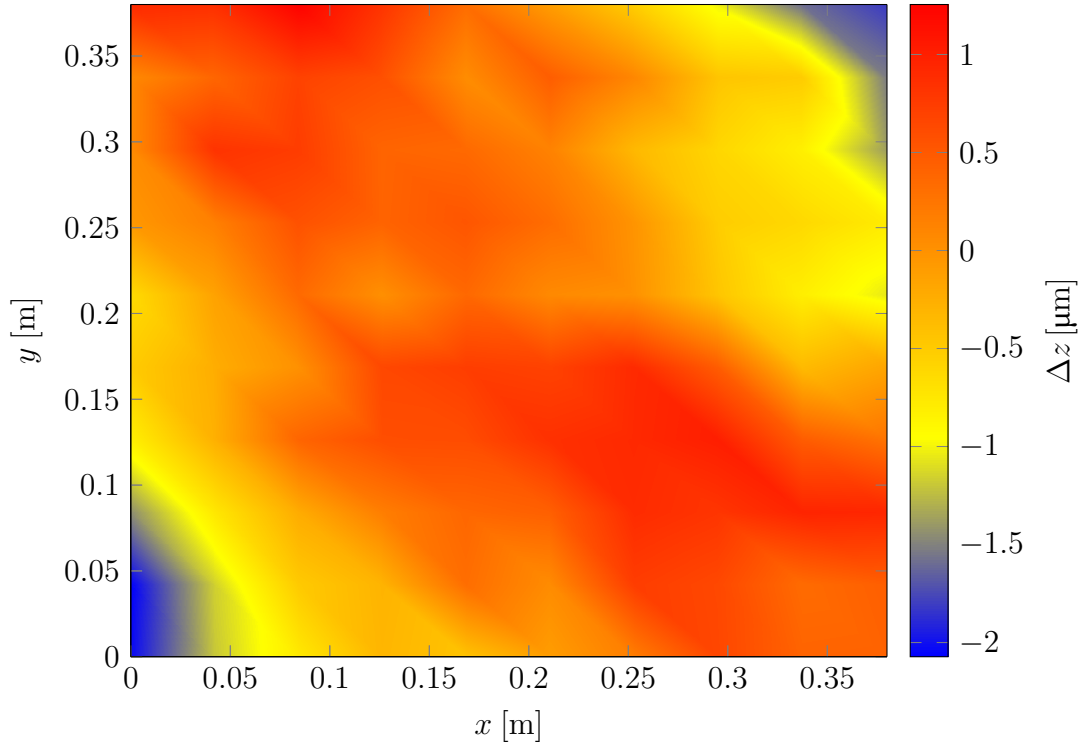


Figure 6.51: A linear interpolation of the difference, in z , between a four-tracker multilateration solution and a CMM measurement, with an SCE-probe, of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. The flat surface was measured with a 10×10 grid of points.

Figure 6.52 shows a linear interpolation of the difference between the multilateration solution and the CMM measurement, Δz , of the large flat optic with a second-order polynomial fit removed. The standard deviation of Δz is $\sigma_{\Delta z} = 0.28 \mu\text{m}$. The peak-to-valley of $\Delta z \approx 1.25 \mu\text{m}$.

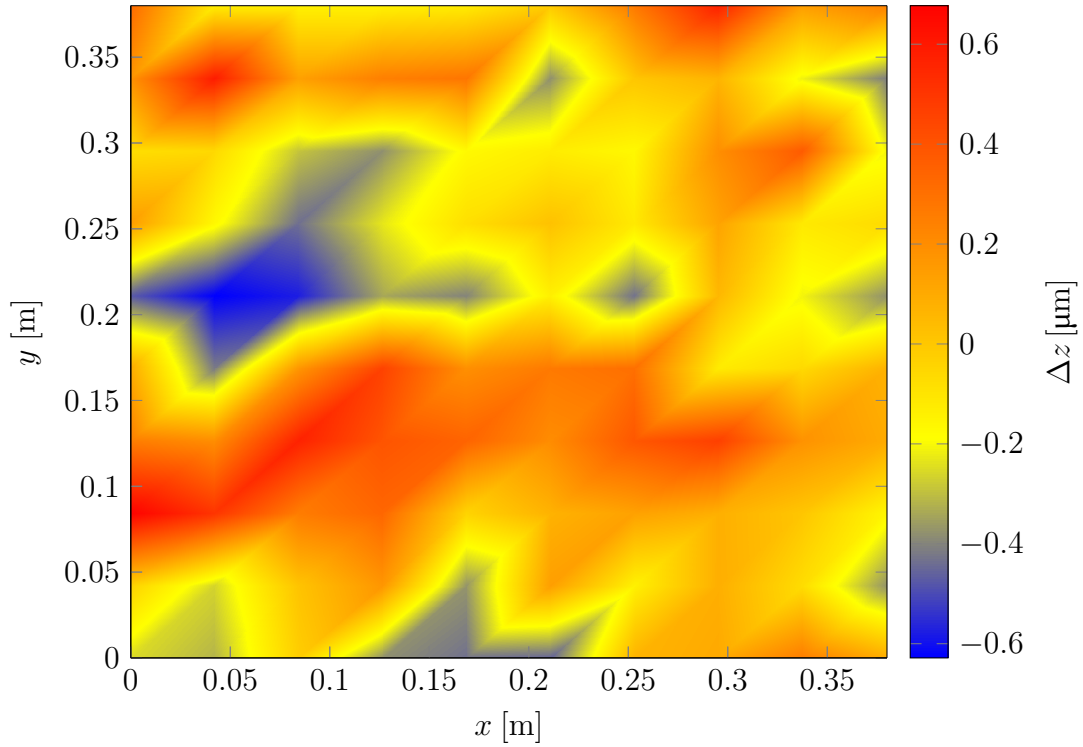


Figure 6.52: A linear interpolation of the difference, in z , between a four-tracker multilateration solution and a CMM measurement, with a second-order polynomial fit removed, of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. The flat surface was measured with a 10×10 grid of points.

Figure 6.53 shows a linear interpolation of the standard deviations of 10 repeated four-tracker multilateration z-coordinate measurement solutions of the large flat optic. The square root mean variance of z is $\sigma_z = 0.46 \mu\text{m}$. The peak-to-valley of $\sigma_{z,i} \approx 0.8 \mu\text{m}$.

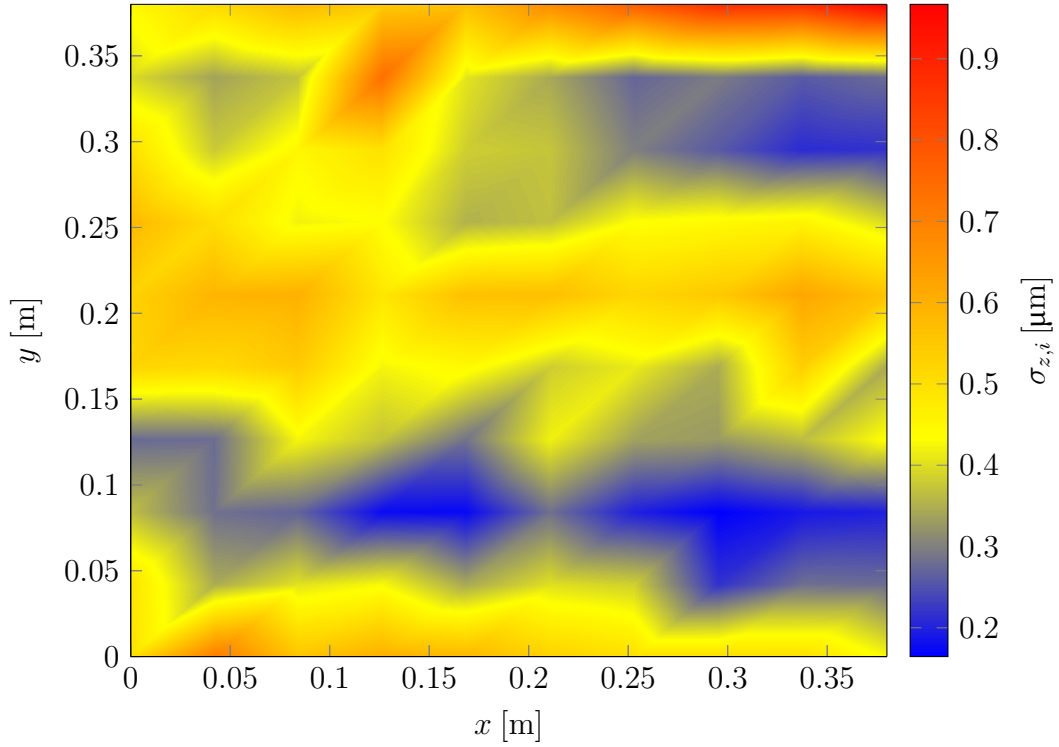


Figure 6.53: A linear interpolation of the standard deviations of 10 repeated four-tracker multilateration z-coordinate measurement solutions, using an SCE-probe, of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. The flat surface was measured with a 10×10 grid of points 10 times.

6.8 Four-tracker multilateration — large flat with RRR

This section describes a four AT960LR laser tracker multilateration measurement solution for a large optic and compares the results to a CMM reference measurement. The sample under investigation was a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. An RRR-probe was used as a measurement target. As an RRR has a limited viewing angle, the system setup arrangement was restricted.

6.8.1 System setup

The measurement setup of a four laser tracker multilateration measurement is shown in figures 6.54, 6.55, and 6.56. The measured positions form a 10×10 grid on the surface of the large sample. As for the previous system setups, the estimate of the measured positions coordinates was determined by one of the laser trackers measuring in spherical coordinates. The coordinates of the remaining three laser trackers were calculated using a trilateration routine.

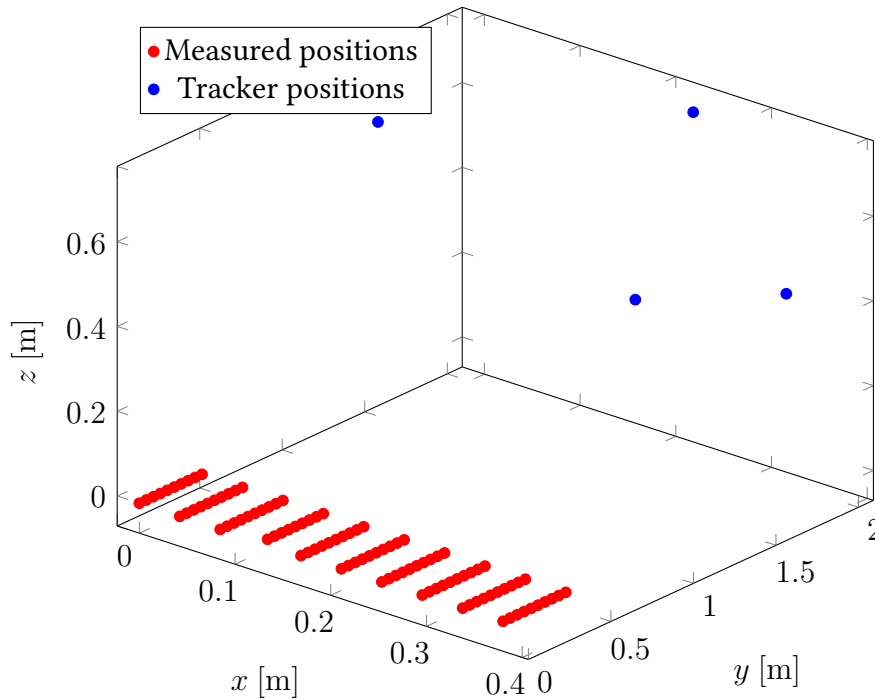


Figure 6.54: The measurement setup for a four AT960LR multilateration measurement of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. An RRR-probe was installed on the CMM used to move the probe into the measurement positions. The measuring station (laser tracker) positions are offset to one side of the measured positions at differing heights above the surface.

Figure 6.55 shows how the measuring station (laser trackers) positions are offset to one side of the measured positions.

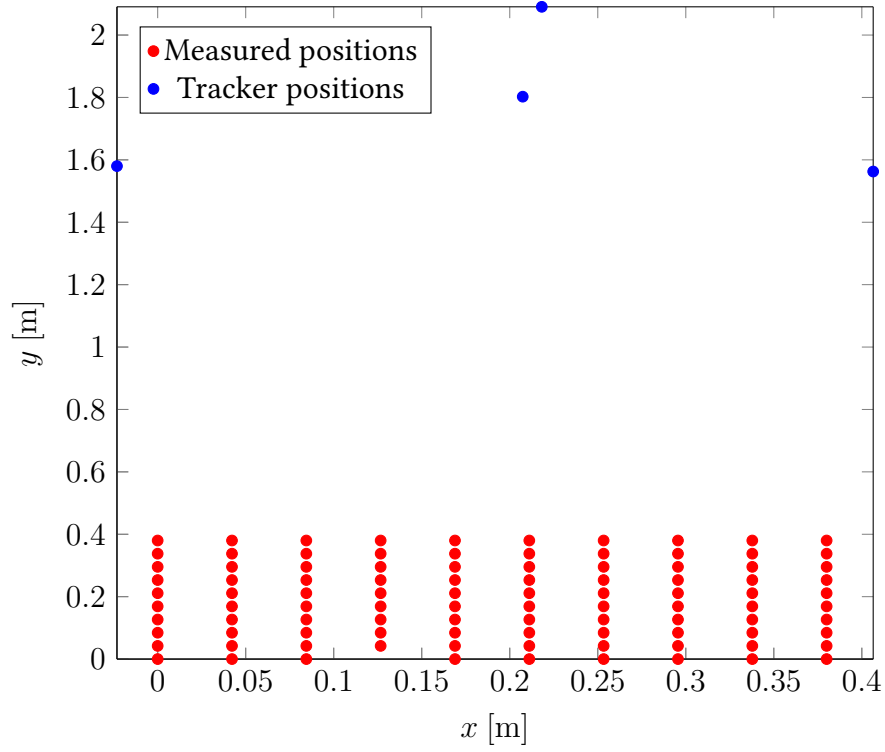


Figure 6.55: A top-view of the measurement setup for a four AT960LR multilateration measurement of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. An RRR-probe was installed on the CMM used to move the probe into the measurement positions.

Figure 6.56 shows how the measuring station positions are at differing heights above the sample surface positions.

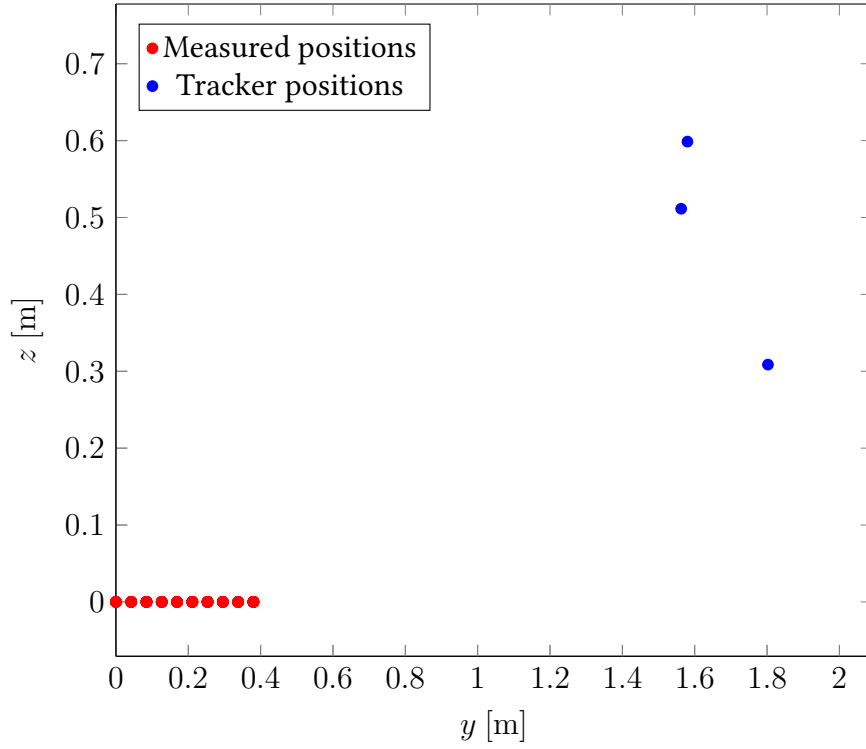


Figure 6.56: A side-view of the measurement setup for a four AT960LR multilateration measurement of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. An RRR-probe was installed on the CMM used to move the probe into the measurement positions.

6.8.2 Measurement results

A linear interpolation of the z-coordinates of a four-tracker multilateration measurement solution of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic plotted against the xy-coordinates in figure 6.57. An RRR-probe was used to measure a 10×10 grid of points on the surface of the sample.

The measured z-coordinates have a standard deviation of $\sigma_z = 1.93\text{ }\mu\text{m}$. The peak-to-valley of the measured z-coordinates is approximately $10\text{ }\mu\text{m}$. The multilateration solution for the measurement of this flat appears to have relatively high noise.

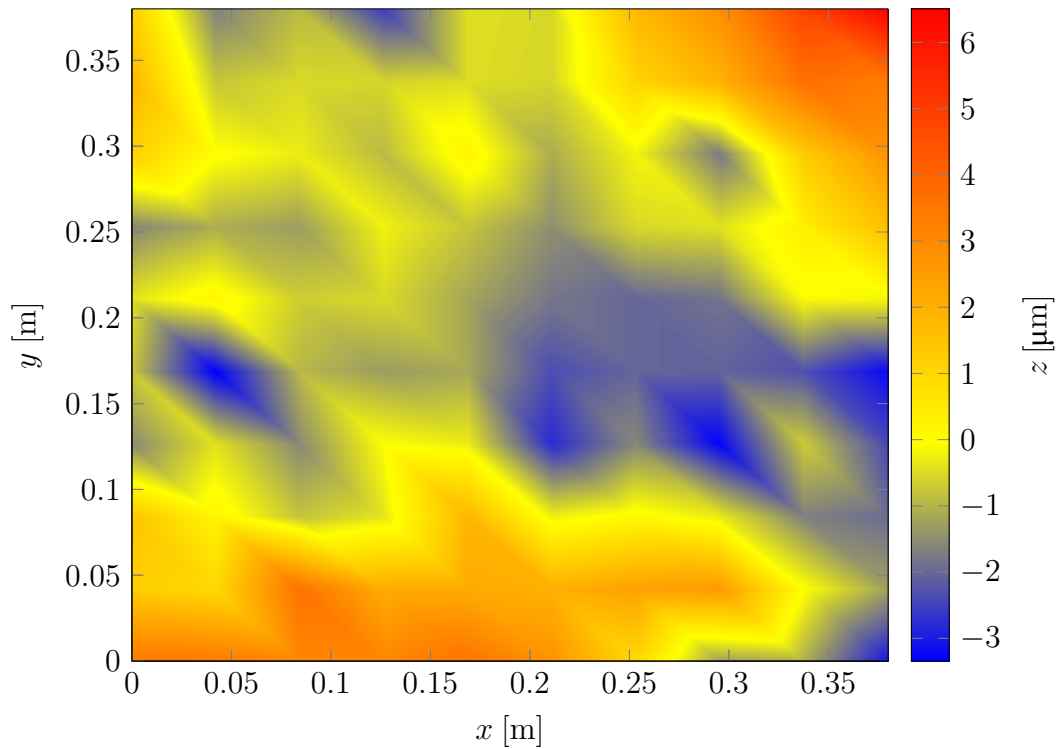


Figure 6.57: A linear interpolation of multilateration determined z-coordinates of a 10×10 grid of measured positions of an RRR-probe on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic using four AT960LR laser trackers. The z-coordinates of measured positions are shown as a function of x- and y-coordinates.

A linear interpolation of the z -coordinates of a Leitz PMM-F, with an RRR-probe installed, measurement of the $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic is plotted against the xy -coordinates in figure 6.58. This measurement corresponds to the above multilateration measurement solution.

The measured z -coordinates have a standard deviation of $\sigma_z = 1.61 \text{ } \mu\text{m}$. The peak-to-valley of the measured z -coordinates is approximately $10 \text{ } \mu\text{m}$. The form of this measurement is similar to the form of the equivalent multilateration solution, however, with significantly lower noise.

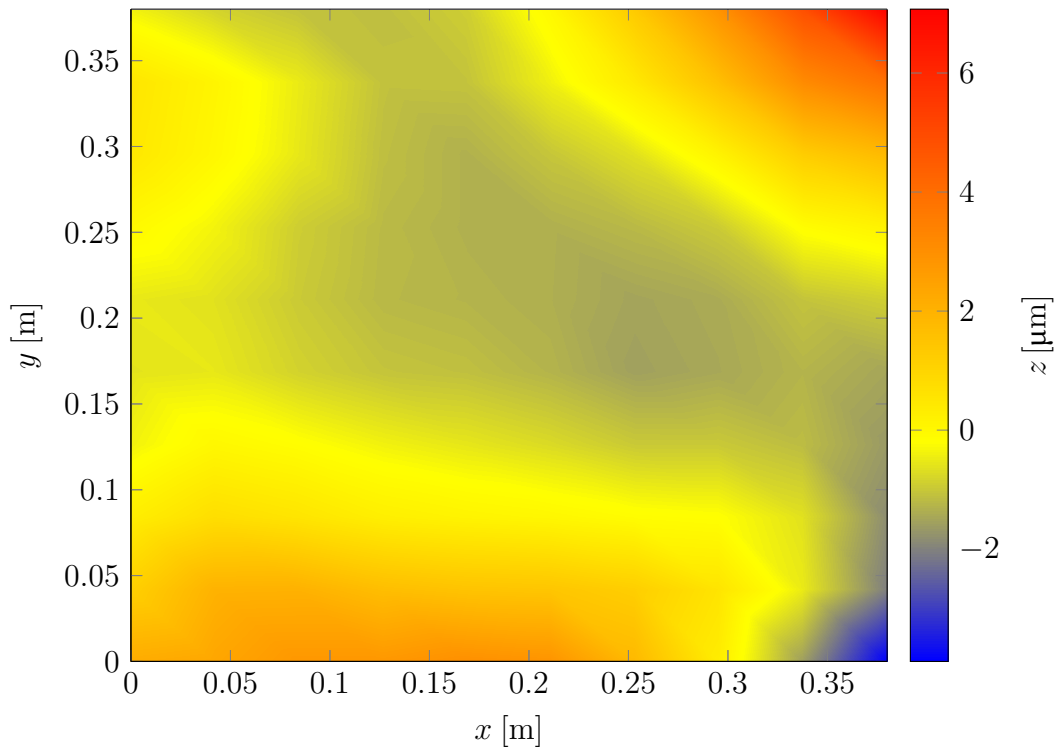


Figure 6.58: A linear interpolation of Leitz PMM-F measured z -coordinates of a 10×10 grid of points, with RRR-probe installed, on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. The z -coordinates of measured positions are shown as a function of x - and y -coordinates and are defined by a first-order polynomial surface through the measured coordinate positions. The origin of the coordinate system is defined at the first measured position in x and y .

6.8.3 Minimisation residuals

The l^2 -norm of the residuals, $w_{i,j}$, of the minimisation for the multilateration equations, based on the laser tracker interferometric displacement measurements of the measured positions on the surface of the $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic is: $|\mathbf{w}|_2 = 2.1 \times 10^{-12} \text{ m}$.

Figure 6.59 shows the residuals of the 400 equations that make up the equation set (4.1) for the measurement of the 100 measured positions in the setup. The magnitude of residuals follows an increasing trend the higher the residuals number.

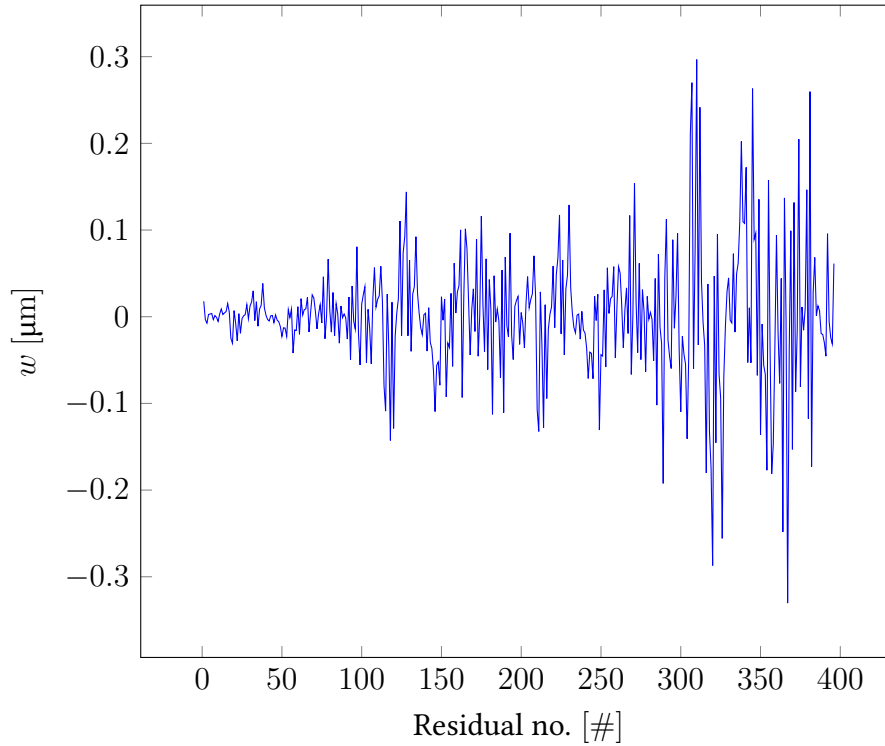


Figure 6.59: The residuals of the least squares minimisation for a multilateration measurement with four AT960LR laser trackers and CMM with RRR-probe installed. The sample was a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. Each laser tracker displacement measurement has a corresponding residual, $w_{i,j}$.

The residuals per laser tracker, j , are plotted in figure 6.60. There are four residuals per measured position corresponding to the four displacement measurements for each position. The figure shows how the solving algorithm has approached the minimisation by balancing the residuals for each tracker to find a minimised solution to the equation set. Here, it appears the residuals of trackers, $j = 1, 2, 3$ have been balanced against tracker, $j = 4$. This suggests the error of displacement measurements in tracker, $j = 4$, are relatively high.

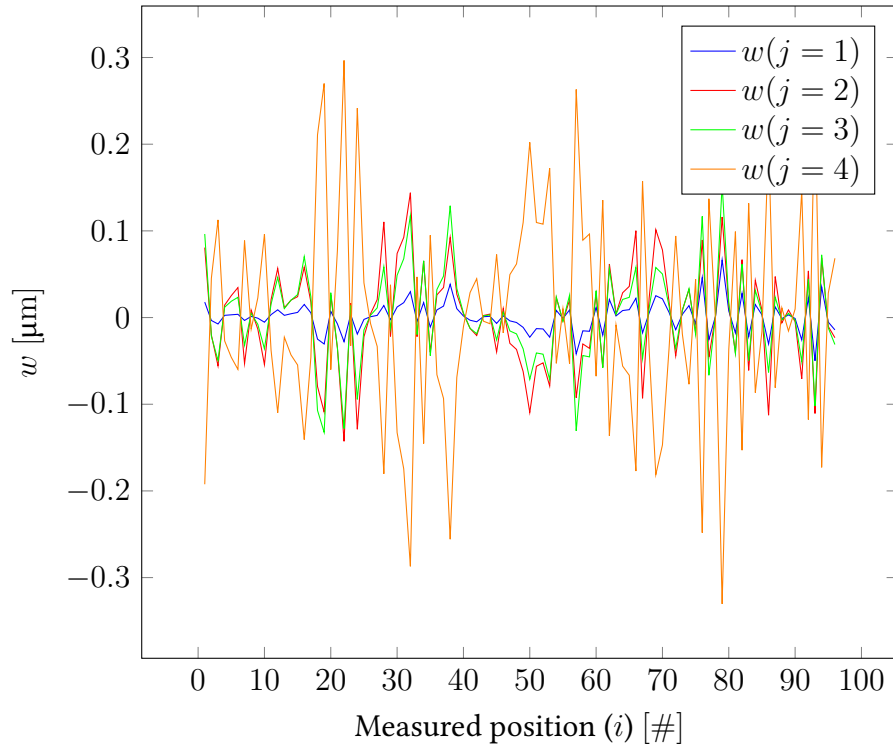


Figure 6.60: The residuals of the least squares minimisation for a multilateration measurement per AT960LR laser tracker (j) for each measured position (i): 100 measured positions on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. An RRR-probe was installed on a CMM and used as a measurement target.

6.8.4 Z-coordinate comparison

Figure 6.61 shows a linear interpolation of the difference between the multilateration solution and the CMM measurement, Δz , of the large flat optic. The standard deviation of Δz is $\sigma_{\Delta z} = 0.93 \mu\text{m}$. This is an increase of approximately 27% on the equivalent $\sigma_{\Delta z}$ for the measurements described in section 6.7. The peak-to-valley of this RRR measurement is $\Delta z \approx 4.8 \mu\text{m}$.

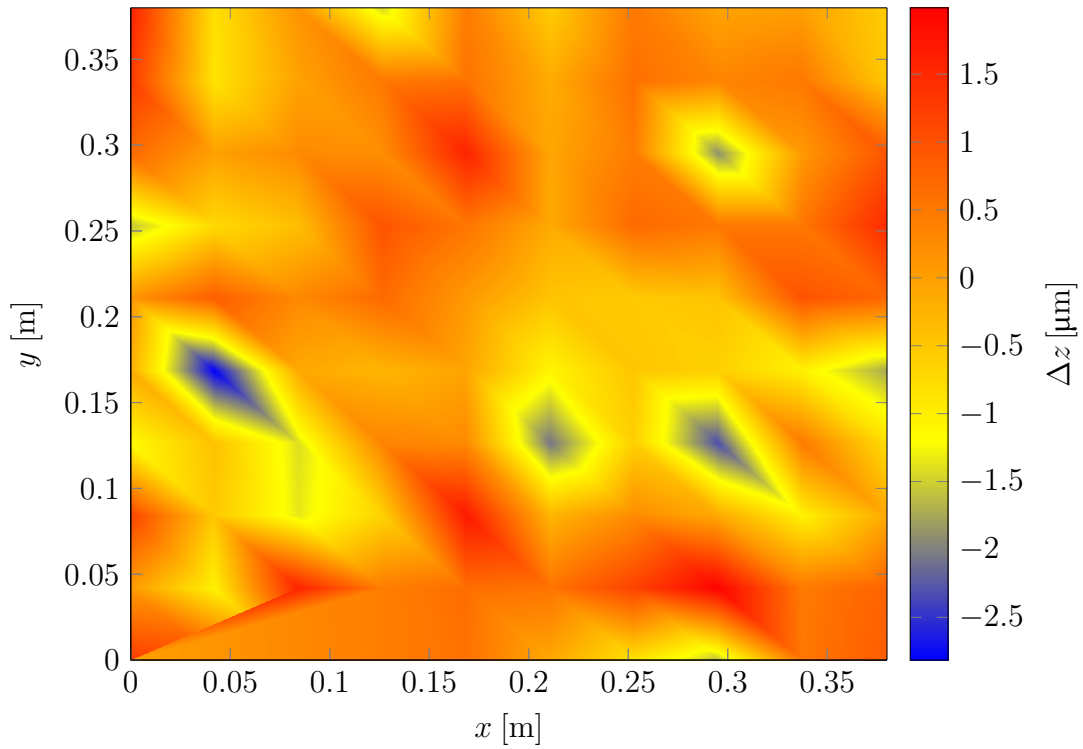


Figure 6.61: A linear interpolation of the difference, in z , between a four-tracker multilateration solution and a CMM measurement, with an RRR-probe, of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic.

6.9 Four-tracker sequential multilateration — large flat with SCE

A sequential multilateration measurement of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. This was implemented with a SCE-probe installed on a Leitz PMM-F CMM. The SCE measurement target was moved into the measured positions on the surface four times. Each laser tracker measured the target in sequence, one per set of grid points to which the SCE was moved.

6.9.1 System setup

The measurement setup of a four laser tracker multilateration measurement is shown in figure 6.62. The system setup is equivalent to the setup detailed in section 6.4; in this case, there were 100 positions measured on the surface of the large sample in a 10×10 grid.

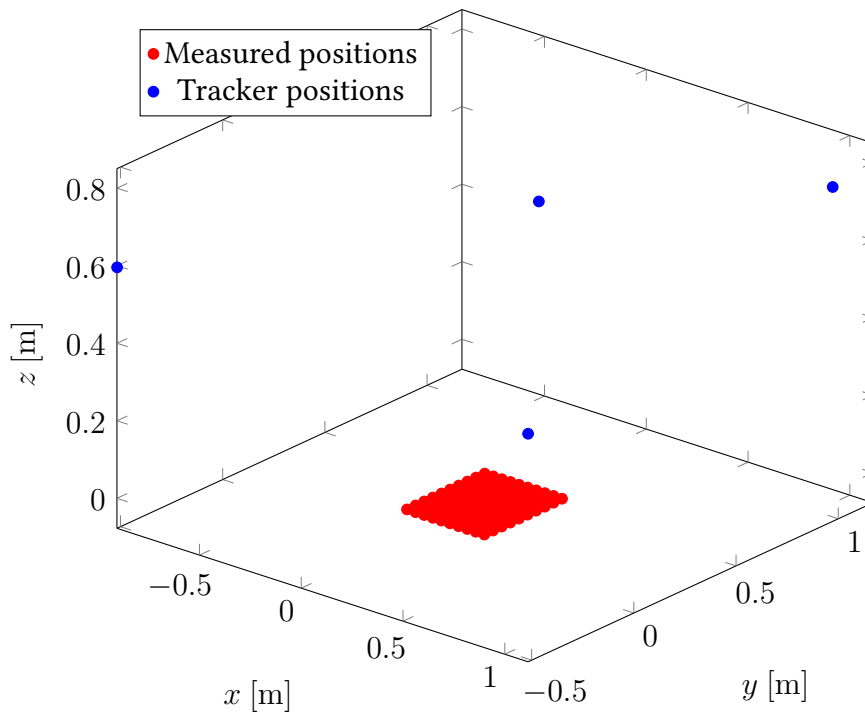


Figure 6.62: The measurement setup for a four AT960LR multilateration measurement of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. An SCE-probe was installed on the CMM used to move the probe into the measurement positions. A sequential multilateration routine was implemented. The measuring station (laser tracker) positions surround the measured positions at differing heights above the surface. The measured positions are a 10×10 grid of points on the surface of the sample.

6.9.2 Measurement results

A linear interpolation of the z -coordinates of a four-tracker sequential multilateration measurement solution for a $0.4\text{ m} \times 0.4\text{ m}$ flat optic plotted against the xy -coordinates in

figure 6.63.

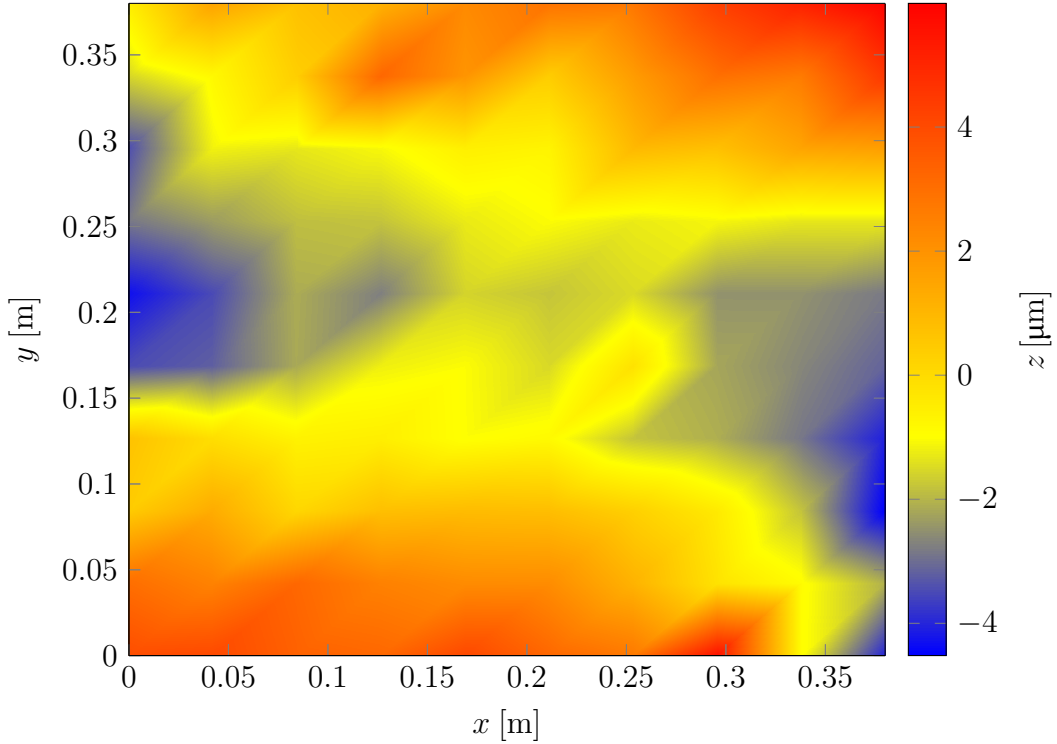


Figure 6.63: A linear interpolation of multilateration determined z -coordinates of a 10×10 grid of measured positions of an SCE-probe on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic using four AT960LR laser trackers. A sequential multilateration routine was implemented. The z -coordinates of measured positions are shown as a function of x - and y -coordinates and are defined by a first-order polynomial surface through the determined coordinates. The origin of the coordinate system is defined at the first measured position in x and y .

The measured z -coordinates have a standard deviation of $\sigma_z = 2.44 \mu\text{m}$. The peak-to-valley of the measured z -coordinates is approximately $10 \mu\text{m}$. The multilateration solution for the measurement of this flat appears to have a clear form of an inverted saddle similar to the CMM measurement shown in figure 6.47; however, the level of noise appears to be higher.

6.9.3 Minimisation residuals

The l^2 -norm of the residuals, $w_{i,j}$, of the minimisation for the multilateration equations, based on the laser tracker interferometric displacement measurements of the measured positions on the surface of the $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic is: $|\mathbf{w}|_2 = 3.9 \times 10^{-11} \text{ m}$. This is approximately a factor of eight times the equivalent l^2 -norm for the non-sequential multilateration measurement detailed in section 6.7.

Figure 6.64 shows the residuals of the 400 equations that make up the equation set (4.1) for the measurement of the 100 measured positions in the setup.

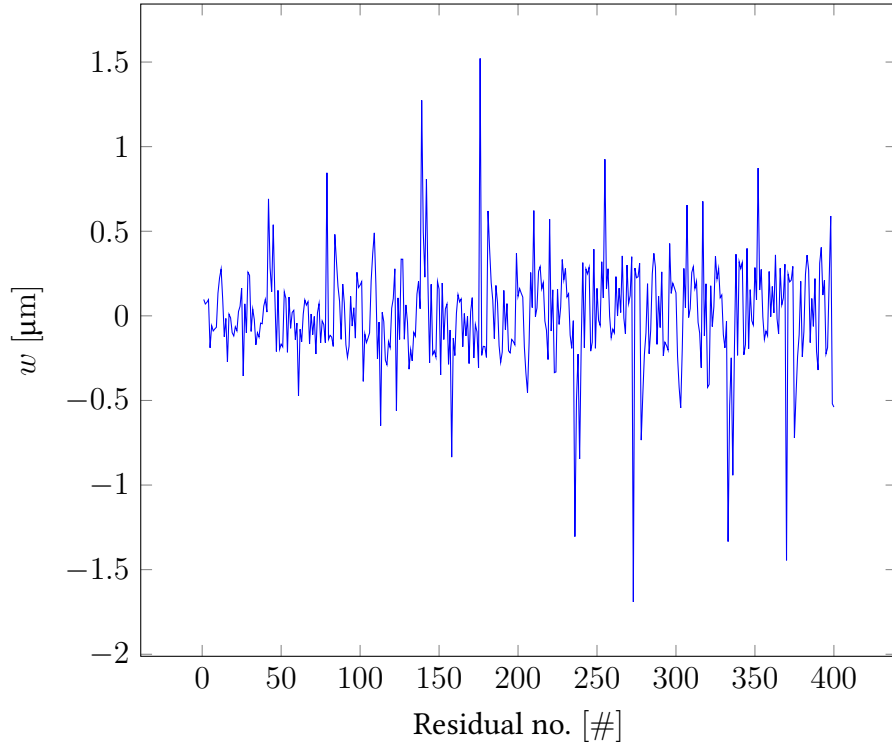


Figure 6.64: The residuals of the least squares minimisation for a sequential multilateration measurement with four AT960LR laser trackers. The sample was a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. Each laser tracker displacement measurement has a corresponding residual, $w_{i,j}$.

The residuals per laser tracker, j , are plotted in figure 6.65. The figure shows how the solving algorithm has approached the minimisation by balancing the residuals for each tracker to find a minimised solution to the equation set. It is of note that that the magnitude of the residuals for this sequential multilateration measurement are significantly higher than those for the non-sequential equivalent measurement.

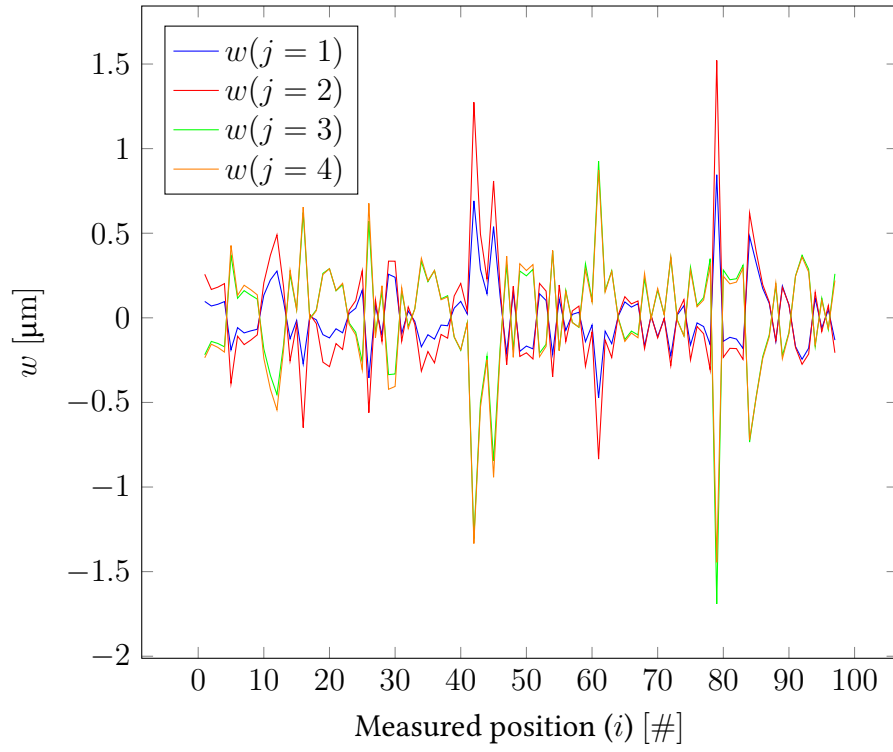


Figure 6.65: The residuals of the least squares minimisation for a sequential multilateration measurement per AT960LR laser tracker (j) for each measured position (i): 100 measured positions on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic, plus 25 floating positions above the optic.

6.9.4 Z-coordinate comparison

Figure 6.66 shows a linear interpolation of the difference between the multilateration solution and the CMM measurement, Δz , of the large flat optic. The standard deviation of Δz is $\sigma_{\Delta z} = 0.92 \mu\text{m}$. This is an increase of approximately 26% on the equivalent $\sigma_{\Delta z}$ for the measurements described in section 6.7. The peak-to-valley measurement is $\Delta z \approx 5 \mu\text{m}$. The form of Δz is a saddle shape.

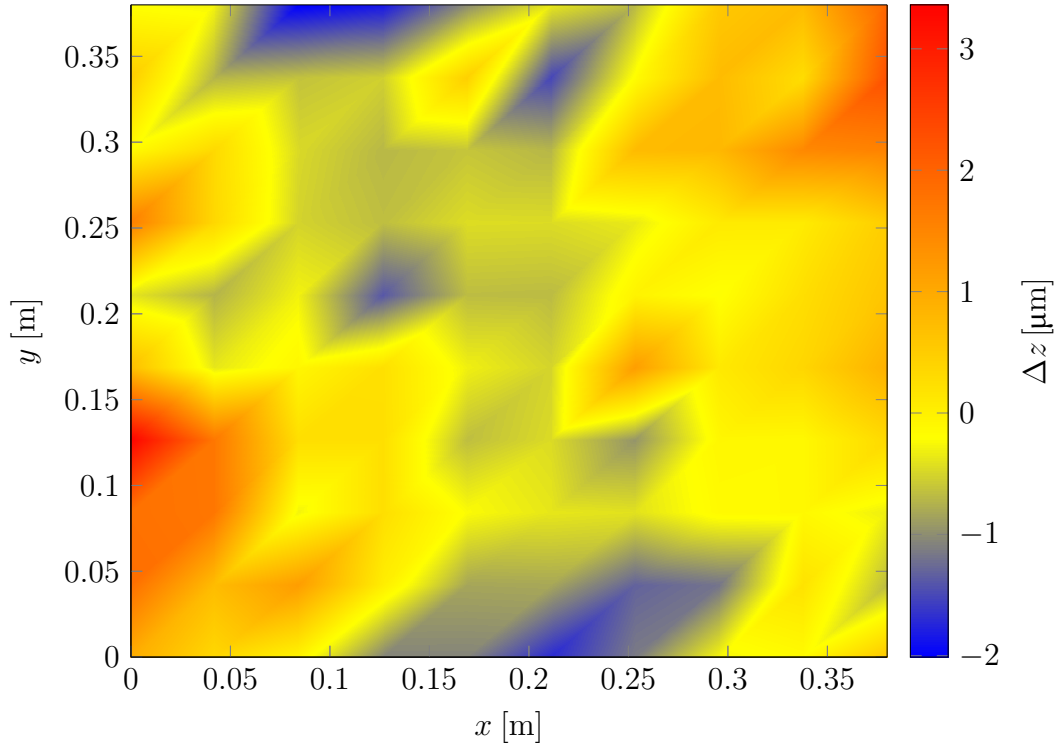


Figure 6.66: A linear interpolation of the difference, in z , between a four-tracker sequential multilateration solution and a CMM measurement, with an SCE-probe, of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. The flat surface was measured with a 10×10 grid of points.

6.10 Monte Carlo simulation of a 0.4 m $\times$ 0.4 m flat measurement

Monte Carlo simulations for the measurement of a 0.4 m $\times$ 0.4 m flat optic to determine estimates of the measurement uncertainty of the system under investigation. This is a time independent evaluation of the system z-coordinate measurement uncertainty.

To determine an estimate of uncertainty of the measurement described in section 6.7, a set of nominal parameter values for equation (4.18) were selected. These parameter values are shown in table 6.6. Other system setup parameters are defined as equivalent to the setup of the measurement under analysis.

The square root mean variance of $\sigma_{z,i}$, the standard deviation 50000 predicted measurement per measured position, of the measured positions on the surface of the sample is $\sigma_z = 1.13 \mu\text{m}$. This prediction is similar to the square root mean variance for the measurement of the smaller ULE[®] flat optic, $\sigma_z = 1.07 \mu\text{m}$.

Table 6.6: Monte Carlo simulation parameters and results for the measurement of the large flat.

Parameter	Setting/result
n_{samples}	50000
m	4
n	100 + 25
u_0	0.2 μm
u_L	0.15 μm
u_{dp}	5 μm
b	0.05 μm ($\theta = 1^\circ$)
Δr	0.07 μm ($\theta = 1^\circ$)
σ_z	1.13 μm ($k = 1$)

A linear interpolation of the standard deviations, $\sigma_{z,i}$, of z-coordinates of measured positions are shown as a function of x- and y-coordinates in figure 6.67.

The predicted measurement uncertainty is lowest, $\sigma_{z,i} \approx 0.75\text{ }\mu\text{m}$, close to the corners of the sample at $(x, y) = (0, 0)$ and $(x, y) = (0, 0.38)$. The uncertainty is highest, $\sigma_{z,i} \approx 1.6\text{ }\mu\text{m}$, close to the corner of the sample at $(x, y) = (0.38, 0)$. $\sigma_{z,i} \approx 0.59\text{ }\mu\text{m}$ close to the corner of the sample at $(x, y) = (0.38, 0.38)$.

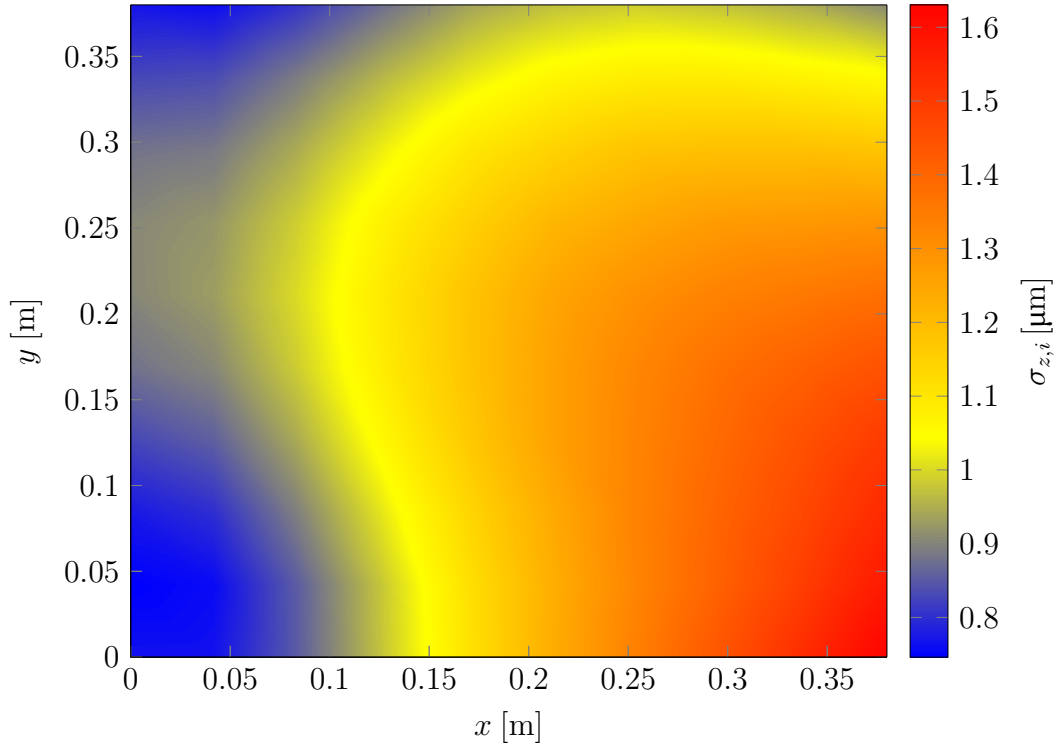


Figure 6.67: A linear interpolation of MC simulated ($n_{\text{samples}} = 50000$) multilateration determined standard deviations, $\sigma_{z,i}$, of z-coordinates of a 10×10 grid of measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat sample using four AT960LR laser trackers.

6.11 Time dependent error sources

This section details the results of a series of tests, described in section 5.3, to determine the effect of time dependent error sources on the measurement system input parameters. The tracker home position stability, the effect of changes to RI, and the combined effect of these to parameters were investigated.

6.11.1 Tracker home position stability

Figure 6.68 shows the relative range measurement of an RRR in a three-point magnetic hold on an aluminium post with height equal to h_S , as a function of measurement time. Both the tracker and post were fixed to the base of a large CMM, as show in figure 5.9a. A second-order exponential fit of ΔR has been removed from the range data set. This was in order to compensate for drift in the range measurement cause by setup instability.

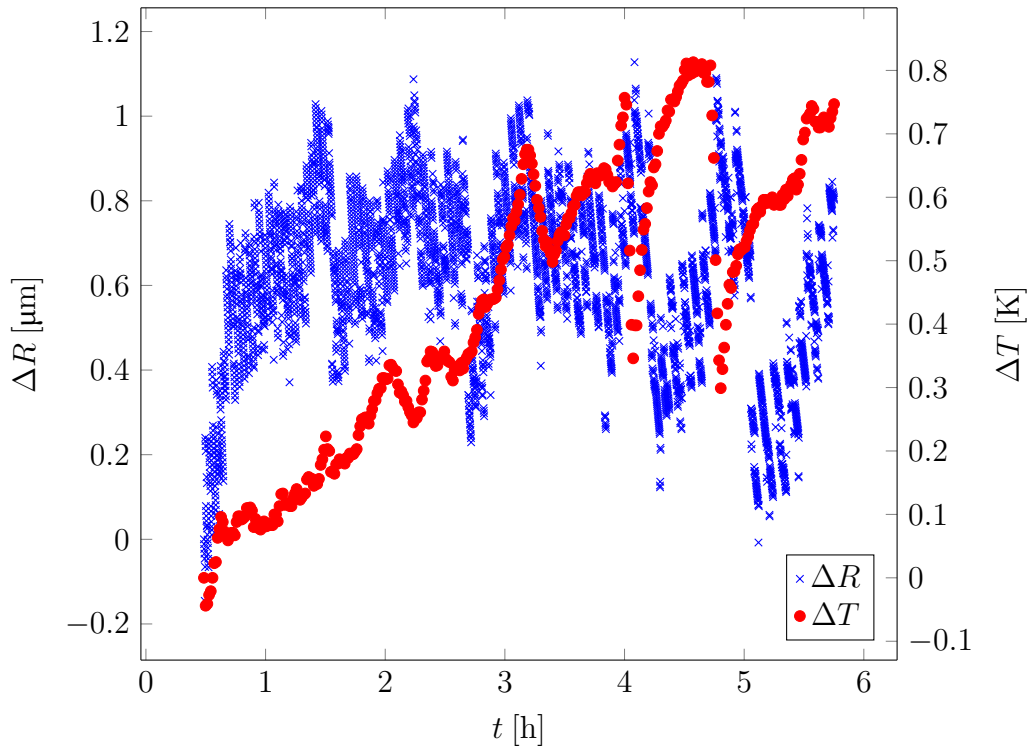


Figure 6.68: The relative AT960LR range measurement of an RRR in a three-point hold, fixed to the top of an aluminium post with height equal to h_S , as a function of time. The simultaneously measured air temperature of the tracker's laser beam path is also plotted. A second-order exponential fit of ΔR has been removed from the range data set.

It is clear from the figure that there is some level of correlation between the change in range measured and the temperature measured over the 6 h measurement. The change in temperature is followed by a change in range measurement. During hour six, the

air temperature increased by ~ 0.45 K; over the same time period, the measured range increased by ~ 0.8 μm .

Figure 6.69 shows results of the equivalent measurement setup as above, however, in this case the RRR in hold was positioned on the base of the CMM, as shown in figure 5.9b. The nominal range was the same as above.

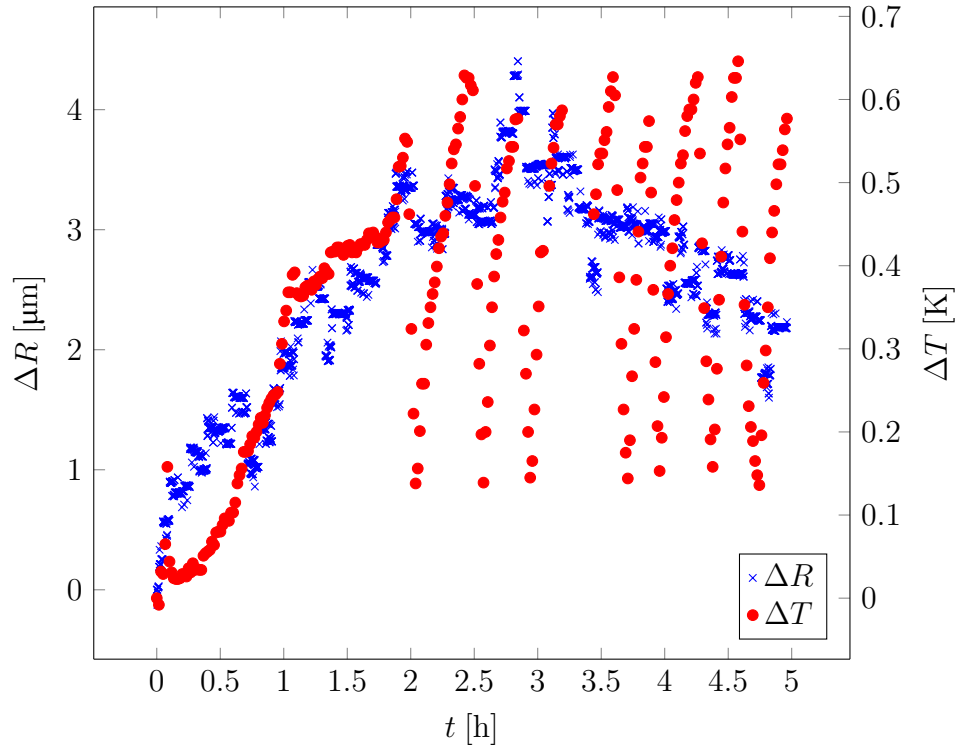


Figure 6.69: The relative AT960LR range measurement of an RRR in a three-point hold, fixed to the base of a large CMM, as a function of time. The simultaneously measured air temperature of the tracker's laser beam path is also plotted.

There is a clear correlation between the change in range measured and the change in temperature measured during the first two hours of measurement. During this period the air temperature increased by ~ 0.5 K and the measured range increased by ~ 3.5 μm . Between $t = 2$ h and $t = 5$ h, the frequency of the temperature change appears to have been higher than the change in range measured; however, the range does follow the longer-term trend of the change in temperature during this time period.

6.11.2 Effect of change in refractive index

The effect of change in refractive index to range measurements was tested at a minimum range, $R = 0.2$ m, a medium range of $R = 1$ m, and a relatively long-range, $R = 2$ m.

Short-range

Figure 6.70 shows the measured range of a stationary RRR fixed to an aluminium post, such that it is at the minimum possible range, over a period of five hours. Both the tracker and post were fixed to the base of a large CMM, as shown in figure 5.10. The nominal range from the tracker's home position to the RRR was $R = 0.2$ m.

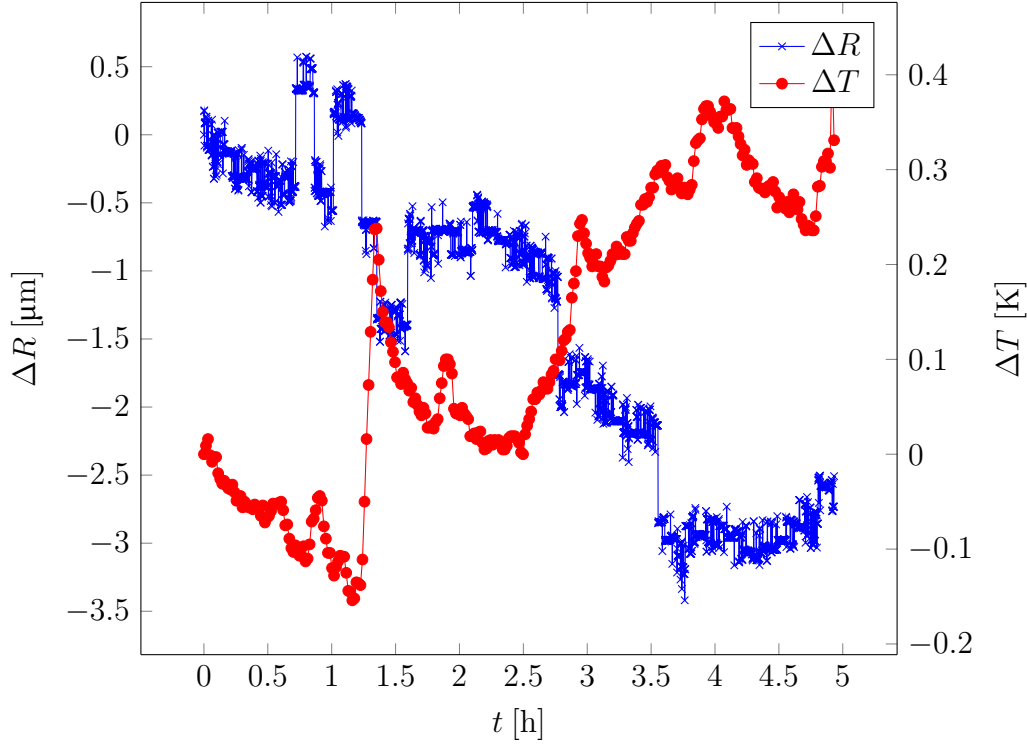


Figure 6.70: The relative AT960LR range measurement of an RRR in a three-point hold, fixed to the top of an aluminium post with height equal to h_S , as a function of time. The nominal range from the tracker's home position to the RRR was $R = 0.2$ m. The simultaneously measured air temperature of the tracker's laser beam path is also plotted.

The short-range stability measurement appears to be inversely correlated with the change in temperature measured. Over the five hour measurement, the temperature measured increased by ~ 0.5 K and the measured range decreased by ~ 3 μm ; however, during the first hour of measurement, the temperature and range both decrease. At $t \approx 1.25$ h the temperature rapidly increased by ~ 0.35 K. At this point the correlation inverts.

1 m range

Figure 6.71 shows the measured range of a stationary RRR fixed to an aluminium post over a period of three hours. Both the tracker and post were fixed to the base of a large CMM, as shown in figure 5.11. The nominal range from the tracker's home position to

the RRR was $R = 1$ m.

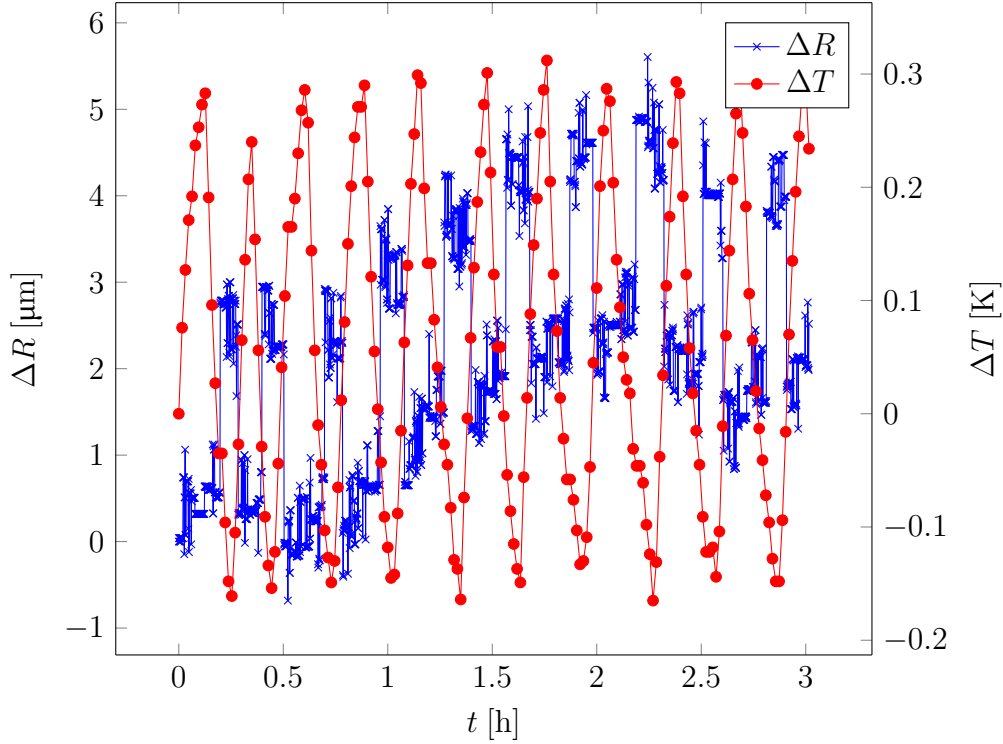


Figure 6.71: The relative AT960LR range measurement of an RRR in a three-point hold, fixed to the top of an aluminium post with height equal to h_S , as a function of time. Both the tracker and post were fixed to the base of a large CMM. The nominal range from the tracker's home position to the RRR was $R = 1$ m. The simultaneously measured air temperature of the tracker's laser beam path is also plotted.

The frequency of change in air temperature and measured range, nominally 1 m, are equivalent. The temperature oscillated between -0.15 K and 0.3 K, relative to the $\Delta T = 0$ at $t = 0$, at a frequency of 3.5 per hour. The measured range also changed at a similar rate, with amplitude ~ 2.5 μm ; however, the running average of ΔR increased by ~ 2 μm over the three hour measurement period. There is also delay between change in temperature and change in range measured.

Figure 6.72 shows the measured range of a stationary RRR fixed to an aluminium post over a period of three hours. Both the tracker and post were fixed to the base of a large CMM, as shown in figure 5.11. The nominal range from the tracker's home position to the RRR was $R = 2$ m. A second-order exponential fit of ΔR has been removed from the range data set. This was in order to compensate for drift in the range measurement cause by setup instability.

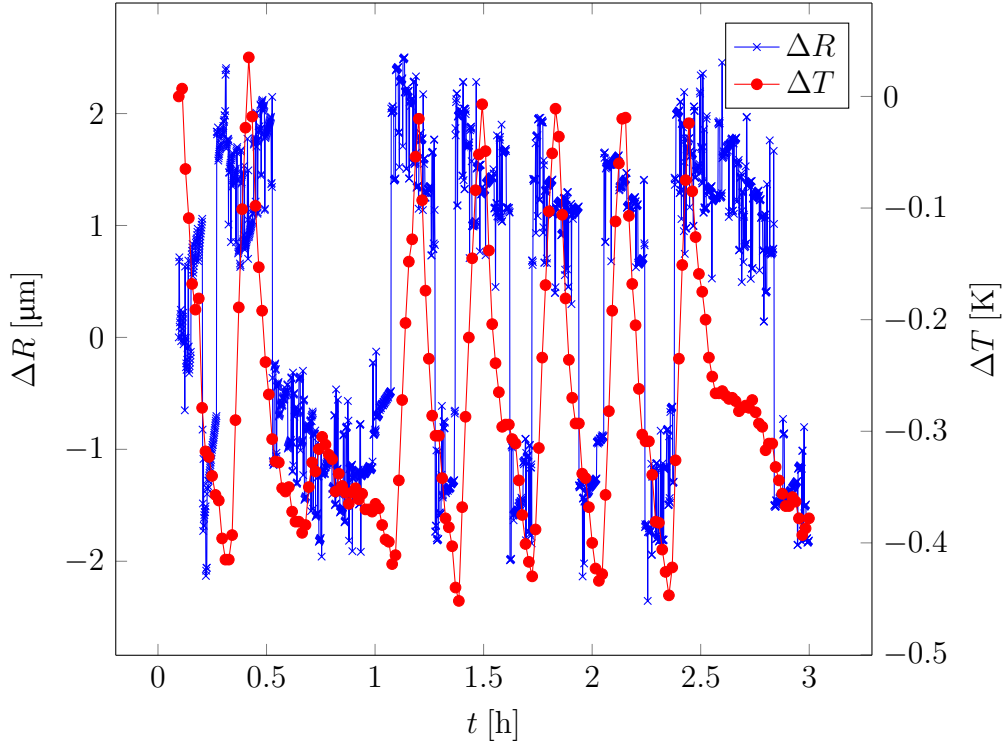


Figure 6.72: The relative AT960LR range measurement of an RRR in a three-point hold, fixed to the top of an aluminium post with height equal to h_S , as a function of time. The nominal range from the tracker's home position to the RRR was $R = 2$ m. The simultaneously measured air temperature of the tracker's laser beam path is also plotted. A second-order exponential fit of ΔR has been removed from the range data set.

The frequency of change in air temperature and measured range, nominally 2 m, are equivalent. The temperature oscillated between 0 K and -0.45 K, relative to the $\Delta T = 0$ at $t = 0$. The measured range also changed at a similar rate, with amplitude $\sim 4 \mu\text{m}$; however, between $t = 0.5$ h and $t = 1$ h the measured range remained relatively stable. During the same time period the temperature remained similarly stable.

6.11.3 Combined long-term stability measurement

The combined effect of RI change and tracker home position change, as a result of a change in temperature, were investigated with nominal ranges $R = 1$ m and $R = 2$ m. This effect was further investigated with a 1.5 inch RRR on the granite base of a CMM.

1 m range

Figure 6.73 shows the measured range of a stationary RRR in a three-point magnetic hold, on the base of a CMM, over a period of four hours, as shown in figure 5.12. The nominal range from the tracker's home position to the RRR was $R = 1$ m.

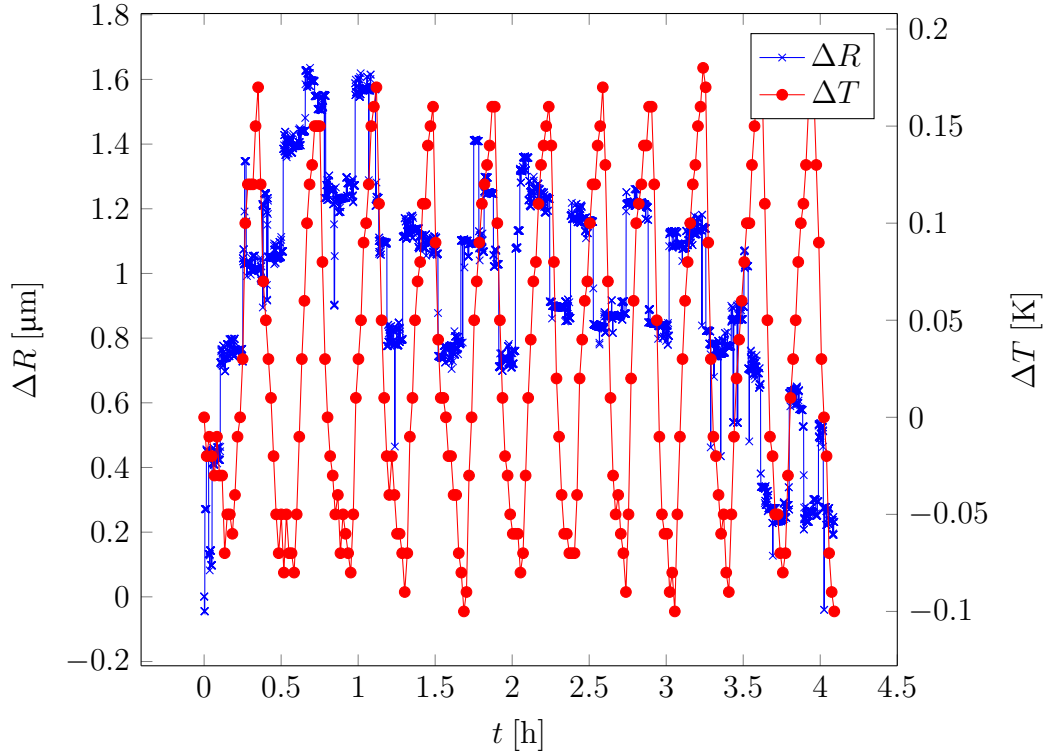


Figure 6.73: The relative AT960LR range measurement of an RRR in a three-point hold, fixed to the base of a large CMM, as a function of time. The tracker was also fixed to the base the CMM. The nominal range from the tracker's home position to the RRR was $R = 1$ m. The simultaneously measured air temperature of the tracker's laser beam path is also plotted.

Similar to the correlation shown in figure 6.71, this frequency changes of the measured range and temperature are equivalent with a delay between temperature and measured range change. In this case, the temperature oscillated between -0.1 K and 0.18 K, relative to the $\Delta T = 0$ at $t = 0$. At frequency equivalent to the change in temperature, the measured range changed with an amplitude of $\sim 0.35 \mu\text{m}$; however, between $t = 0$ h and $t = 1$ h the measured range increased by $\sim 1.4 \mu\text{m}$. Between $t = 1$ h and $t = 3$ h the measured range remained relatively stable; followed by an decrease of $\sim 0.5 \mu\text{m}$ between $t = 3$ h and $t = 4$ h.

2 m range

Figure 6.74 shows the measured range of a stationary RRR in a three-point magnetic hold, on the base of a CMM, over a period of 12 hours, as shown in figure 5.12. The nominal range from the tracker's home position to the RRR was $R = 2$ m.

Over the 12 hour measurement, the measured range increases by $\sim 2\mu\text{m}$; whilst the average temperature increases by 0.1 K. Figure 6.75 shows the first three hours of these measurement sets.

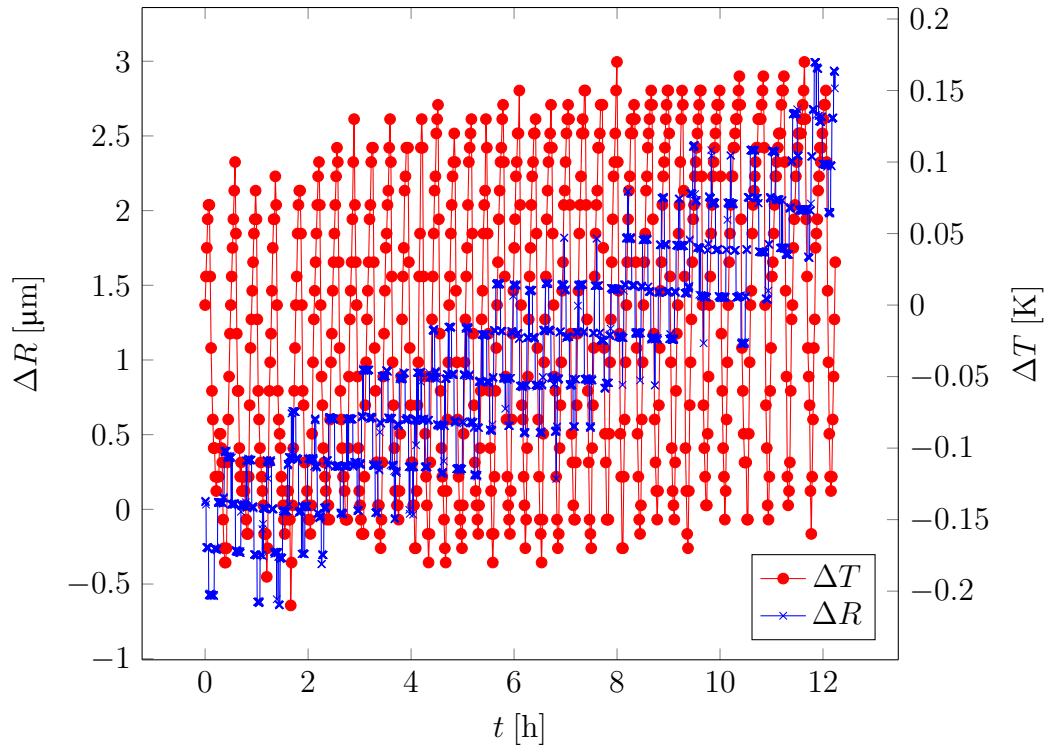


Figure 6.74: The relative AT960LR range measurement of an RRR in a three-point hold, fixed to the base of a large CMM, as a function of time. The tracker was also fixed to the base the CMM. The nominal range from the tracker's home position to the RRR was $R = 2$ m. The simultaneously measured air temperature of the tracker's laser beam path is also plotted.

The frequency of change in air temperature and measured range, nominally 2 m, are equivalent. The temperature oscillated at a frequency of 2.7 per hour, with an amplitude of 0.3 K. The measured range also changed at a similar rate, with amplitude $\sim 1 \mu\text{m}$. The running average of ΔR increased by $\sim 0.5 \mu\text{m}$ over the three hour measurement period. There is also a between change in temperature and change in range measured.

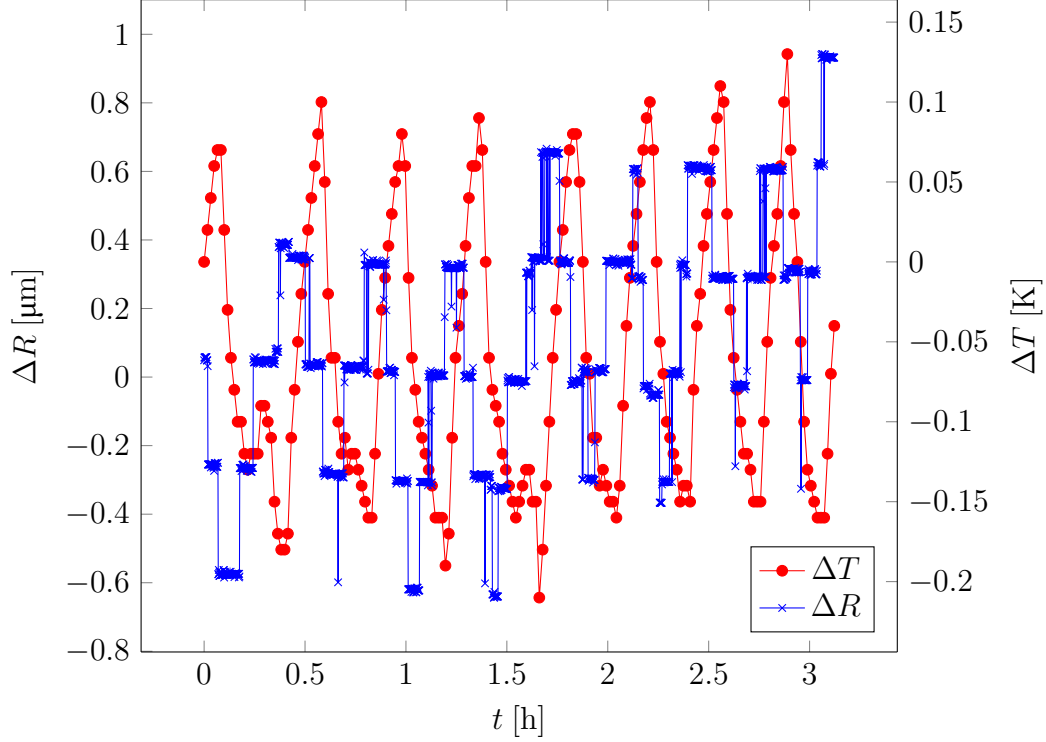


Figure 6.75: The relative AT960LR range measurement of an RRR in a three-point hold, fixed to the base of a large CMM, as a function of time. The tracker was also fixed to the base the CMM. The nominal range from the tracker's home position to the RRR was $R = 2 \text{ m}$. The simultaneously measured air temperature of the tracker's laser beam path is also plotted.

1.5 inch RRR

Figure 6.76 shows the measured range of a stationary 1.5 inch RRR in a three-point magnetic hold, on the base of a CMM, over a period of 33 hours. The nominal range from the tracker's home position to the RRR was $R = 0.9$ m.

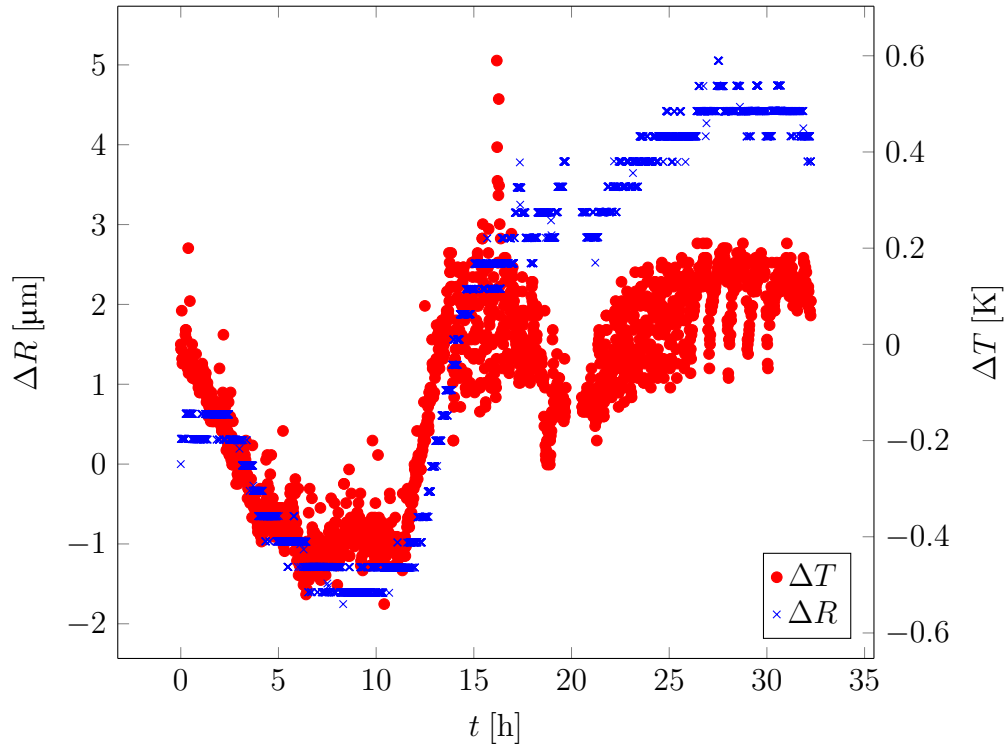


Figure 6.76: The relative AT960LR range measurement of a 1.5 inch RRR in a three-point hold, fixed to the base of a large CMM, as a function of time. The nominal range from the tracker's home position to the RRR was $R = 0.9$ m. The simultaneously measured air temperature of the tracker's laser beam path is also plotted.

Up to $t = 17$ h, the temperature and measured range follow a similar trend. From this point, the trend of change diverges; however, the shape of the trends remains similar throughout the remainder of the measurement period. Between $t = 10$ h and $t = 15$ h the temperature increases by 0.6 K. During the same time period, the measured range increases by ~ 4 μm .

6.12 Long-term four-tracker multilateration measurement

This section describes a long-term four AT960LR laser tracker multilateration measurement solution for a large optic and compares the results to a CMM reference measurement. The sample under investigation was a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. An SCE-probe was used as a measurement target.

The long-term measurement of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic was conducted with an equivalent measurement setup to that shown in section 6.7; however, 34 positions were measured across the surface of the sample. The total measurement time was 15.5 h. The time between the measurement of each position was 28 minutes.

The air temperature was measured over the 15.5 h measurement period. It oscillated with at a rate of 2.5 per hour and had an amplitude of $\sim 0.3\text{ K}$. The average temperature increased by $\sim 0.1\text{ K}$ over the measurement period.

6.12.1 Measurement result

A linear interpolation of the z-coordinates of a four-tracker multilateration measurement solution of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic plotted against the xy-coordinates in figure 6.77.

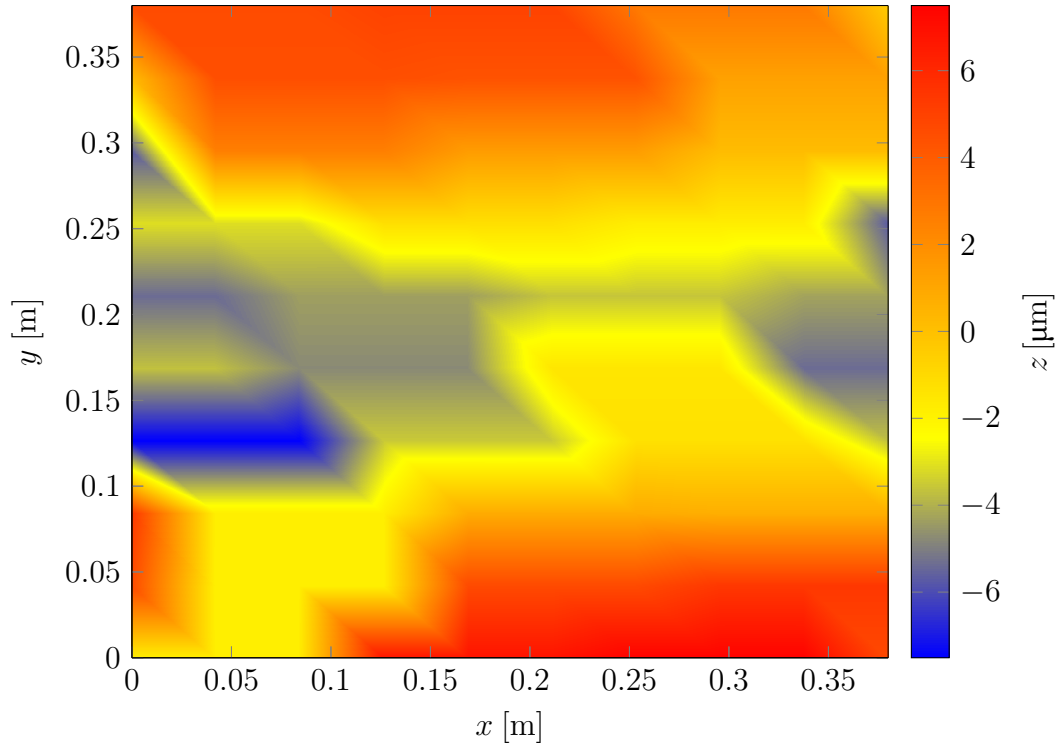


Figure 6.77: A linear interpolation of multilateration determined z-coordinates of 34 measured positions of an SCE-probe on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic using four AT960LR laser trackers. The z-coordinates of measured positions are shown as a function of x- and y-coordinates.

The long-term multilateration measured z-coordinates have a standard deviation of $\sigma_z = 3.99\text{ }\mu\text{m}$. The peak-to-valley of the measured z-coordinates is approximately $15\text{ }\mu\text{m}$. This is in comparison to $\sigma_z = 2.81\text{ }\mu\text{m}$ and peak-to-valley of $11\text{ }\mu\text{m}$, for the measurement described in section 6.7. The equivalent multilateration solution for the measurement of this large flat with 34 measured positions, but with the time between points on the order of seconds, have a standard deviation of $\sigma_z = 2.93\text{ }\mu\text{m}$ and peak-to-valley of $11\text{ }\mu\text{m}$.

6.12.2 Minimisation residuals

The l^2 -norm of the residuals, $w_{i,j}$, of the minimisation for the multilateration equations, based on the laser tracker interferometric displacement measurements of the measured positions on the surface of the $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic is: $|\mathbf{w}|_2 = 1.3 \times 10^{-11} \text{ m}$. This is approximately a factor of 2.4 times the equivalent l^2 -norm for the non-sequential multilateration measurement detailed in section 6.7.

Figure 6.78 shows the residuals of the 236 equations that make up the equation set (4.1) for the measurement of the 59 measured positions in the setup. The mean of the residuals is $\bar{w} = 0.14 \mu\text{m}$.

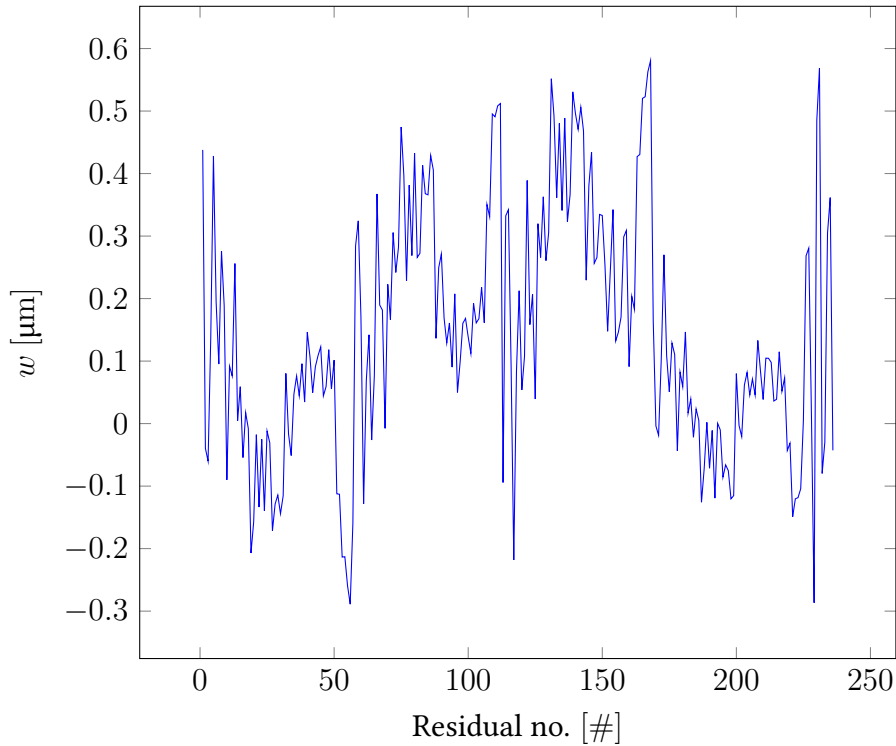


Figure 6.78: The residuals of the least squares minimisation for a multilateration measurement with four AT960LR laser trackers. The measurement was conducted over a 15.5 h time period. The sample was a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. Each laser tracker displacement measurement has a corresponding residual, $w_{i,j}$.

The residuals per laser tracker, j , are plotted in figure 6.79. The figure shows how the solving algorithm has approached the minimisation by balancing the residuals for each tracker to find a minimised solution to the equation set.

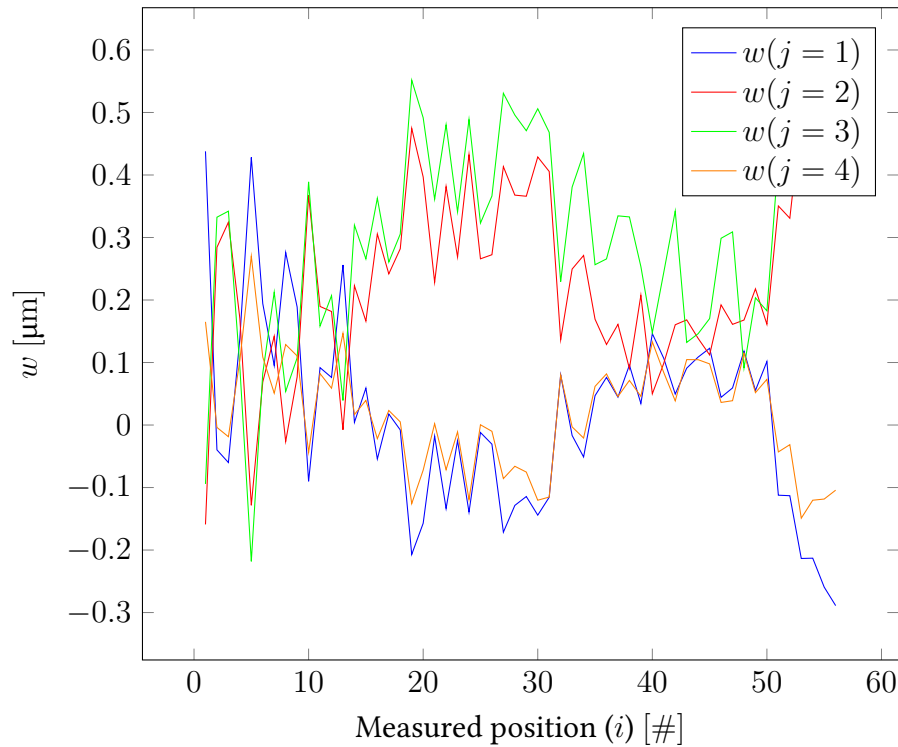


Figure 6.79: The residuals of the least squares minimisation for a long-term multilateration measurement per AT960LR laser tracker (j) for each measured position (i): 34 measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic, plus 25 floating positions above the optic.

6.12.3 Z-coordinate comparison – long- and short-term

Figure 6.80 shows a comparison in the z -coordinates measured by the long-term measurement of the large flat optic and both the equivalent long-term CMM and short-term multilateration measurements. This has been achieved by using the equivalent short-term CMM measurement as a reference measurement. The difference, in z , between the three measurements – long-term multilateration and CMM measurements, and short-term multilateration measurement – and the short-term CMM measurement are shown in the figure.

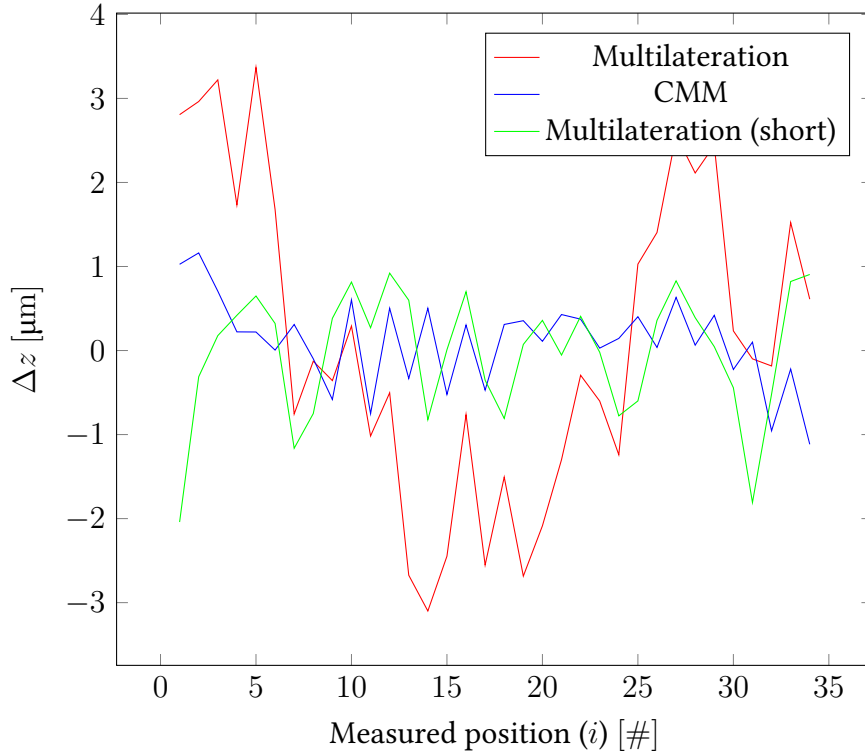


Figure 6.80: Measurement of 34 measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic was conducted with both short- and long-term four-tracker multilateration and CMM measurements. The difference in z-coordinates of the long-term measurements and short-term CMM measurement, and the difference in z-coordinates the short-term multilateration solution and short-term CMM measurement are plotted. The long-term measurements were conducted over 15.5 h.

The difference between long-term multilateration and short-term CMM measurements is significantly larger than either of the other measurements — long-term CMM or short-term multilateration. The standard deviation of the difference between long-term multilateration and short-term CMM measurements is $\sigma_{\Delta z} = 1.88\text{ }\mu\text{m}$, with a peak-to-valley of approximately $6\text{ }\mu\text{m}$.

Figure 6.81 shows the difference between the long-term multilateration measurement solution and the equivalent short-term multilateration solution; and the difference between the long-term multilateration solution and the equivalent short-term CMM measurement, as shown in figure 6.80.

The standard deviation of the difference between long-term multilateration and short-term multilateration solutions is $\sigma_{\Delta z} = 1.94\text{ }\mu\text{m}$, with a peak-to-valley of approximately $8\text{ }\mu\text{m}$.

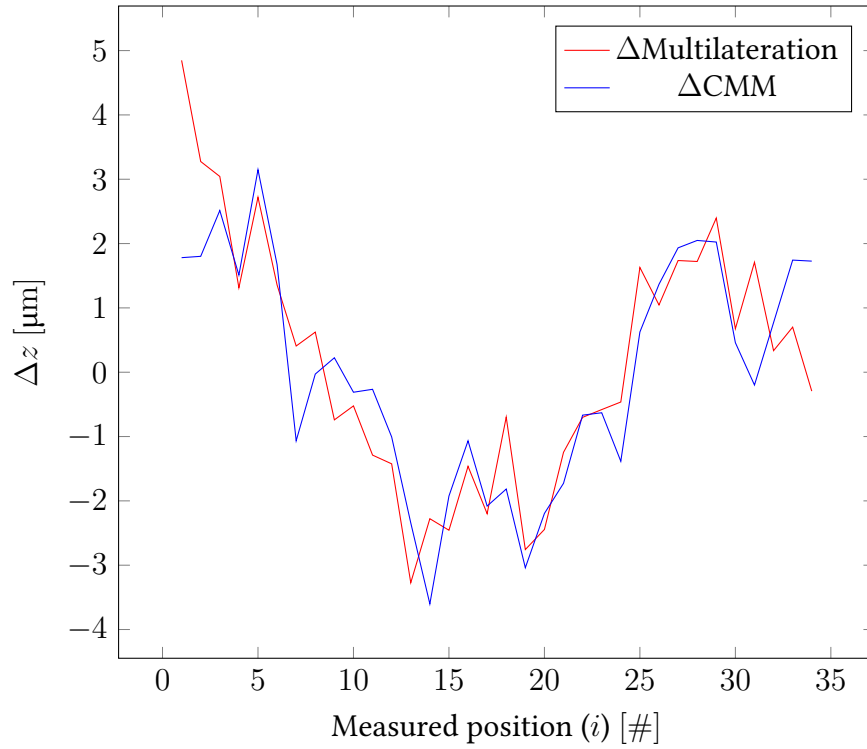


Figure 6.81: Measurement of 34 measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic was conducted with both short- and long-term four-tracker multilateration and CMM measurements. The difference in z-coordinates of the long-term multilateration solution and short-term multilateration solution, and the difference in z-coordinates the long-term multilateration solution and short-term CMM measurement are plotted. The long-term measurements were conducted over 15.5 h.

Figure 6.82 shows a linear interpolation of the difference between the long-term multilateration solution and the short-term CMM measurement, Δz , of the large flat optic. The form of Δz follows a half-cylinder shape.

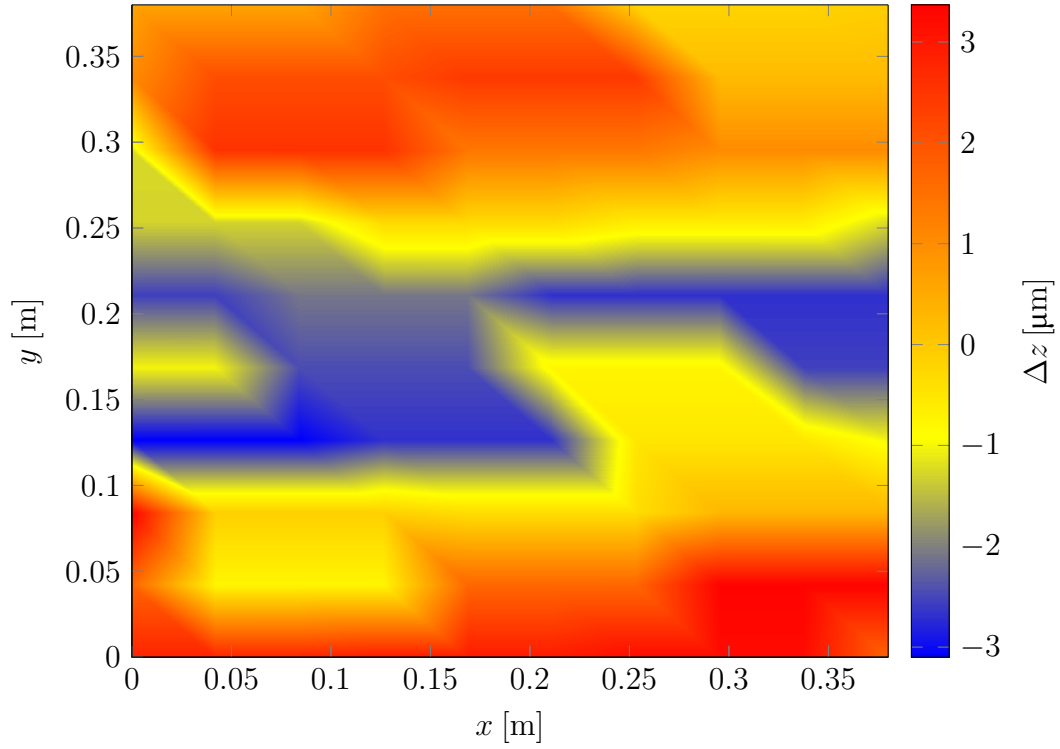


Figure 6.82: A linear interpolation of the difference, in z , between a long-term four-tracker multilateration solution and a short-term CMM measurement, with an SCE-probe, of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic. The flat surface was measured with 34 points.

6.13 MC simulation of long-term measurement

Monte Carlo simulations for the measurement of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic to determine estimates of the measurement uncertainty of the system under investigation. This is a time dependent evaluation of the system z-coordinate measurement uncertainty. The nominal setup was simulated in which the temperature function used was based on the measured temperature during the measurement period of the experiment described in section 6.12. A simulation of the measurement, where the temperature function described a steady increase and then decrease in temperature, was also carried out.

6.13.1 Nominal setup

To determine an estimate of uncertainty of the measurement described in section 6.12, a set of nominal parameter values for equation (4.18) were selected. These parameter values are shown in table 6.7. Other system setup parameters are defined as equivalent to the setup of the measurement under analysis: specifically, the local air temperature change as defined by, f_T , that describes the measured air temperature as a function of time, t .

Table 6.7: Monte Carlo simulation parameters and results for the long-term measurement of $0.4 \text{ m} \times 0.4 \text{ m}$ flat optic. f_T describes the measured air temperature as a function of time.

Parameter	Setting/result
n_{samples}	50000
m	4
n	$34 + 25$
u_0	$0.2 \mu\text{m}$
u_L	$0.15 \mu\text{m}$
u_{dp}	$5 \mu\text{m}$
b	$0.05 \mu\text{m} (\theta = 1^\circ)$
Δr	$0.07 \mu\text{m} (\theta = 1^\circ)$
ΔT	f_T
α_S	$10 \times 10^{-6} \text{ K}^{-1}$
ΔR_T	$7.5 \times 10^{-6} \text{ K}^{-1}$
σ_z	$1.92 \mu\text{m} (k = 1)$

Two additional parameters were included in this simulation to take into account time dependent effects of temperature change: Δh_S , change in tracker height, and

interferometric displacement drift, taking the form of ΔR . These parameters are detailed within section 4.2.11.

To determine the values of Δh_S , an estimate of the combined coefficient of thermal expansion for a laser tracker was set at $\alpha_S = 10 \times 10^{-6} \text{ K}^{-1}$, as stated in table 6.7.

The drift of interferometric range displacement measurements, as a result of air temperature change, was set to $\Delta R_T = 7.5 \times 10^{-6} \text{ K}^{-1}$. This is based on the range measurements shown in figure 6.71.

The square root mean variance of $\sigma_{z,i}$ of the measured positions on the surface of the sample is $\sigma_z = 1.92 \mu\text{m}$. A linear interpolation of the standard deviations, $\sigma_{z,i}$, of z-coordinates of measured positions are shown as a function of x- and y-coordinates in figure 6.83.

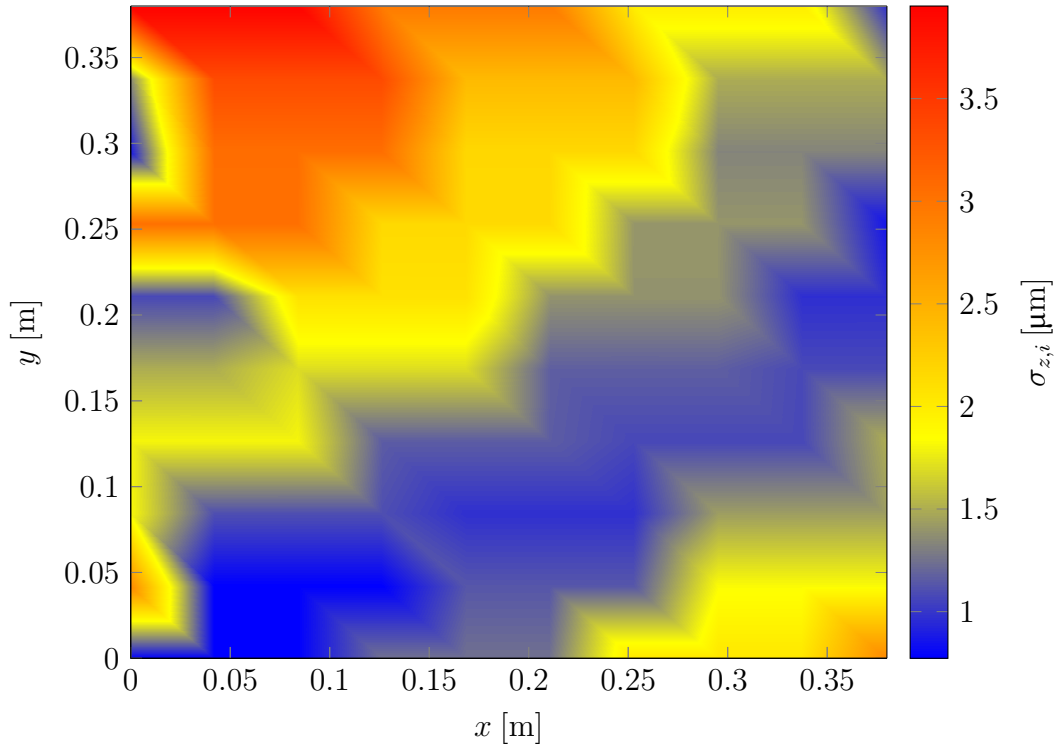


Figure 6.83: A linear interpolation of MC simulated ($n_{\text{samples}} = 50000$) multilateration determined standard deviations, $\sigma_{z,i}$, of z-coordinates of 34 measured positions on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat sample using four AT960LR laser trackers over a 15.5 h measurement.

The predicted measurement uncertainty is lowest, $\sigma_{z,i} \approx 0.8 \mu\text{m}$, close to the corner of the sample at $(x, y) = (0, 0)$. The predicted uncertainty is relatively low along the line between points $(x, y) = (0, 0)$ and $(x, y) = (0.38, 0.25)$. The uncertainty is highest, $\sigma_{z,i} \approx 3.95 \mu\text{m}$, close to the corner of the sample at $(x, y) = (0, 0.38)$.

It is of note that this prediction is effectively independent of temperature change: the prediction predicts the measurement uncertainty due to random measurement error and as well as the uncertainty contribution of measurement setup.

Systematic error analysis

Figure 6.84 shows the predicted systematic measurement error due to the change in temperature measured during the measurement described in section 6.12. A linear interpolation of the mean z -coordinates of MC simulated for 500000 sample multilateration solutions determined from 34 measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat sample using four AT960LR laser trackers over a 15.5 h measurement.

The standard deviation of the mean predicted z -coordinates is $\sigma_{\bar{z}} = 1.89\text{ }\mu\text{m}$. These z -coordinates appear to have no form. The peak-to-valley of this systematic error due to measured temperature change is $\sim 7\text{ }\mu\text{m}$.

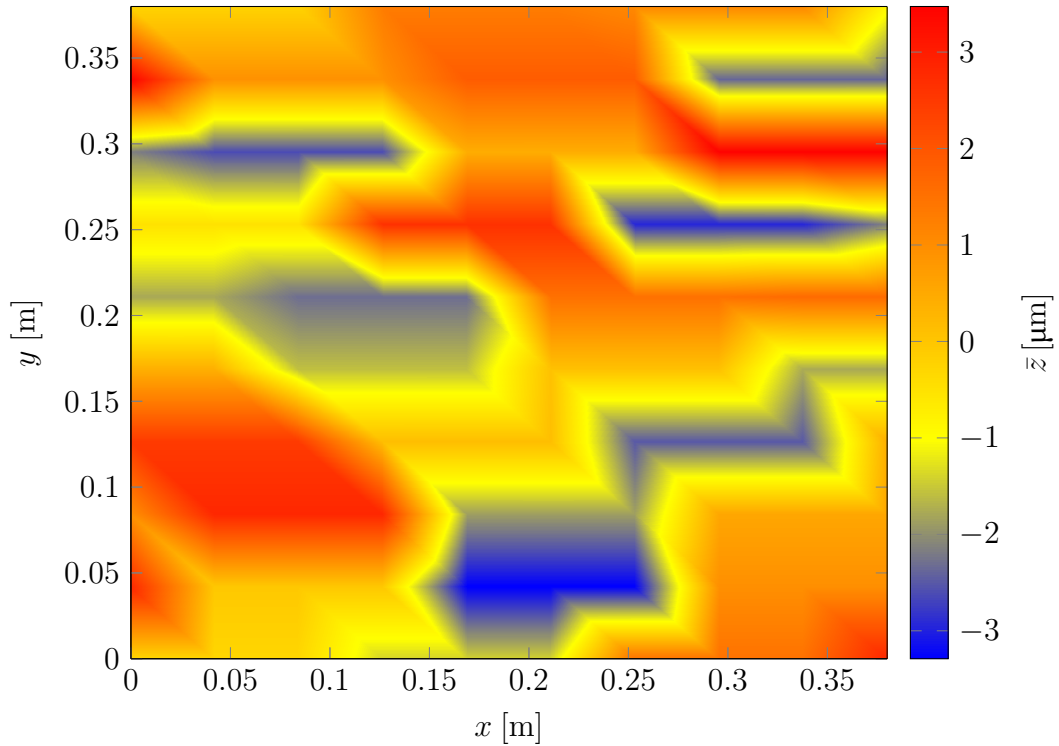


Figure 6.84: A linear interpolation of the mean z -coordinates of MC simulated ($n_{\text{samples}} = 50000$) multilateration determined from 34 measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat sample using four AT960LR laser trackers over a 15.5 h measurement.

Figure 6.85 shows the mean z-coordinates as a function of measurement number.

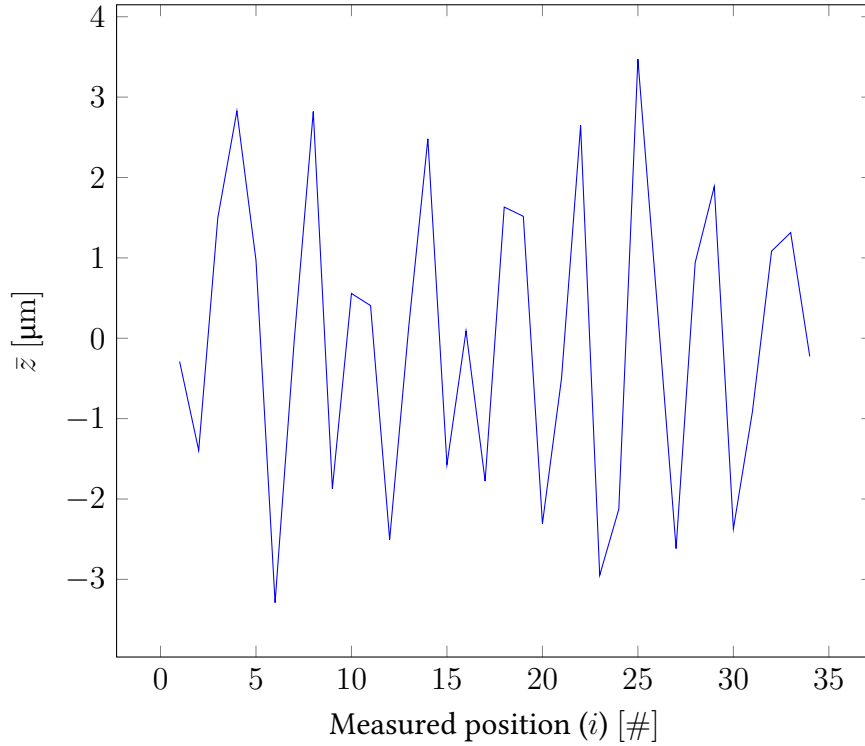


Figure 6.85: The mean z-coordinates of MC simulated ($n_{\text{samples}} = 50000$) multilateration determined from 34 measured positions on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat sample using four AT960LR laser trackers over a 15.5 h measurement.

6.13.2 Sensitivity to temperature change

To determine an estimate of uncertainty of the measurement described in section 6.12, a set of nominal parameter values for equation (4.18) were selected. These parameter values are shown in table 6.8. Other system setup parameters are defined as equivalent to the setup of the measurement under analysis: with exception to the local temperature change. This is defined by the function, f_T , which describes the temperature as a function of time, t .

Table 6.8: Monte Carlo simulation parameters and results for the long-term measurement of $0.4\text{ m} \times 0.4\text{ m}$ flat optic. f_T describes the simulated temperature change as a function of time.

Parameter	Setting/result
n_{samples}	50000
m	4
n	$34 + 25$
u_0	$0.2\text{ }\mu\text{m}$
u_L	$0.15\text{ }\mu\text{m}$
u_{dp}	$5\text{ }\mu\text{m}$
b	$0.05\text{ }\mu\text{m}$ ($\theta = 1^\circ$)
Δr	$0.07\text{ }\mu\text{m}$ ($\theta = 1^\circ$)
ΔT	f_T
α_S	$10 \times 10^{-6}\text{ K}^{-1}$
ΔR_T	$7.5 \times 10^{-6}\text{ K}^{-1}$
σ_z	$1.93\text{ }\mu\text{m}$ ($k = 1$)

For this temperature sensitivity simulation, all time independent parameters were set equal to the nominal setup. The function, f_T , was defined such that the simulated temperature increased by 1.5 K over the first half of the measurement period; subsequently, the temperature decreased by 1.5 K over the second half of the measurement period.

As for the case described above, the square root mean variance of $\sigma_{z,i}$ of the measured positions on the surface of the sample is $\sigma_z = 1.93\text{ }\mu\text{m}$.

Systematic error analysis

Figure 6.86 shows the predicted systematic measurement error due to the change in temperature defined by the function, f_T . A linear interpolation of the mean z-coordinates of MC simulated for 500000 sample multilateration solutions determined from 34 measured positions on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat sample using four AT960LR laser trackers.

The standard deviation of the mean predicted z-coordinates is $\sigma_{\bar{z}} = 6.47\text{ }\mu\text{m}$. The peak-to-valley of this systematic error due to measured temperature change is $\sim 22\text{ }\mu\text{m}$. It is noteworthy that the form of this predicted systematic measurement error, a half-cylinder shape, is similar the that of the z-coordinates of the real measurement detailed in section 6.12.

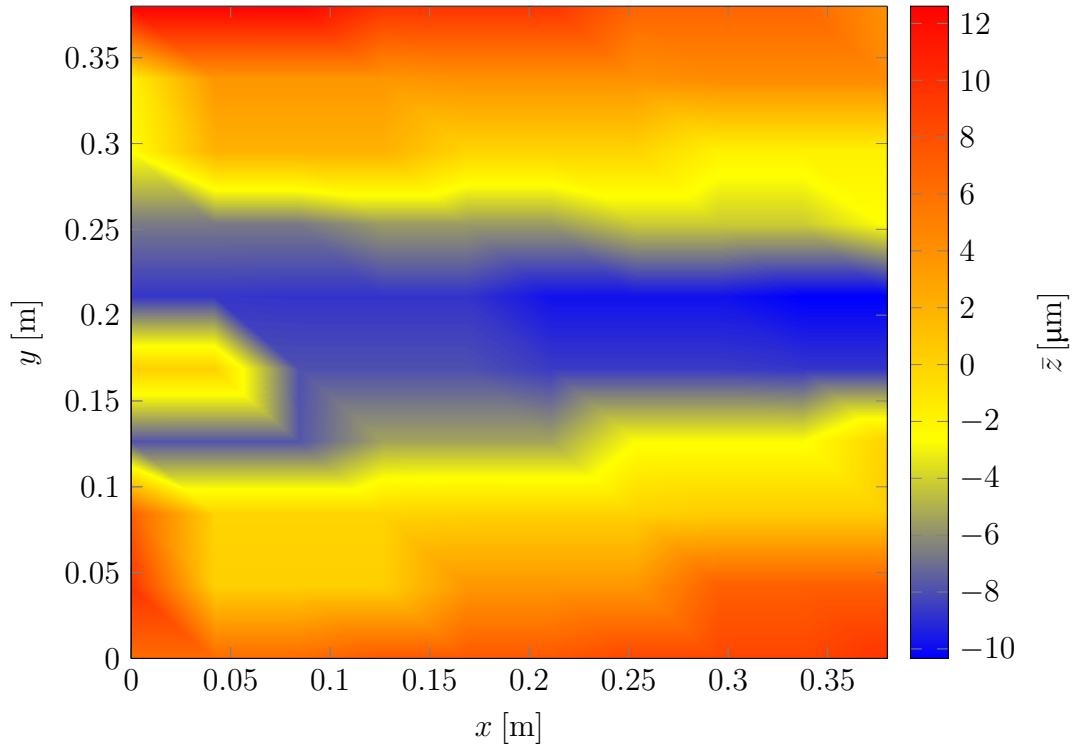


Figure 6.86: A linear interpolation of the mean z -coordinates of an MC simulated ($n_{\text{samples}} = 50000$) multilateration determined from 34 measured positions on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat sample using four AT960LR laser trackers over a 15.5 h measurement. The simulated temperature was varied with a magnitude of 1.5 K..

Figure 6.87 shows the mean z-coordinates as a function of measurement number.

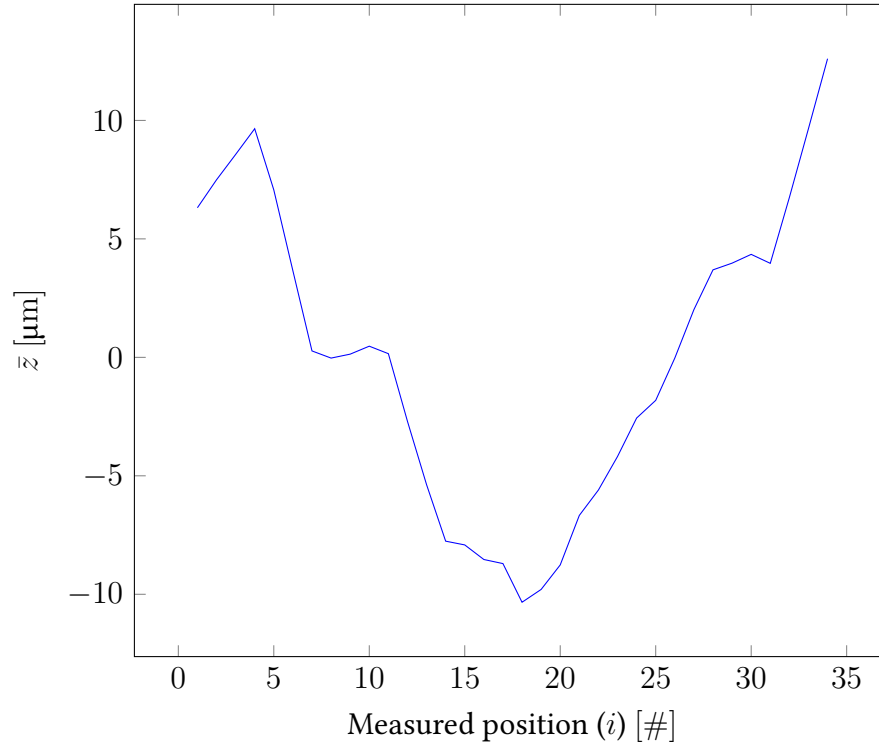


Figure 6.87: The mean z-coordinates of an MC simulated ($n_{\text{samples}} = 50000$) multilateration determined from 34 measured positions on the surface of a $0.4 \text{ m} \times 0.4 \text{ m}$ flat sample using four AT960LR laser trackers over a 15.5 h measurement.

6.14 Summary

The initial experiments aimed to test parameters that affect the viability of the measurement system. It was shown that after five hours of the laser trackers being powered up, the range measurement had not reached relative stability. It was further shown that the range measurement of a laser tracker could not resolve a measurement point when the measurement target is installed on a CMM with standard probing parameters enabled.

A series of time independent input error sources were investigated. The interferometric range measurement capability of the AT960LR was tested using a buck-in buck-out style experiment. There was significant difference in the measurement of distance, L , between the two measurement setups. The effect of SMR reorientation showed an increase in standard deviation of $0.34 \mu\text{m}$ between the measurements with no reorientation and the measurements with reorientations. CMM measurement of a large flat was conducted with three unique measurement probes for comparison. The laser tracker range measurement stability of a CMM probe, whilst paused, was investigated; and the repeatability of CMM-tracker positioning on the surface of a flat optic was also tested for its effect on

sequential multilateration.

A Leitz PMM-F CMM was used, with a ruby-tip probe, to measure the z-coordinates of two sample surfaces: a small ULE[®] flat optic and a larger 0.4 m × 0.4 m flat optic. These measurements were used as references.

A four-tracker multilateration measurement system was used to measure the optics. The ULE[®] flat optic was measured in both a nominal and reversed orientation was a nominal 6 × 6 grid of points. The CMM was used as a mover with the SCE-probe installed. A simultaneous measurement was taken with the CMM for comparison. The larger flat was measured with a number of measurement setups. The measured positions were in a 6 × 6 grid of points. It was measured in the same measurement setup as for the smaller optic. It was also measured with a setup using a restrictive RRR-probe and a setup utilising sequential multilateration. The measurements minimisation solution residuals were detailed. The z-coordinates of each multilateration solution was compared with its equivalent CMM measurement.

Monte Carlo simulation was used to estimate the measurement uncertainty of the measurements carried out and to test the sensitivity of measurement uncertainty to error in input parameters. The sensitivity to interferometric error, SMR sphericity, deadpath error, and laser tracker arrangement were all tested through simulation.

A number of time dependent errors were tested; specifically, the effect of change to RI and tracker home position due to change in the local temperature. These parameters were also tested in their combined form.

A long-term four-tracker multilateration measurement was conducted. The sample was the larger flat optic. 34 points were measured across the surface with half an hour between each measurement. This measurement was simulated using Monte Carlo analysis inputting the measured air temperature at the time of the real measurement. The sensitivity of this measurement system was tested by simulating a variable temperature throughout the measurement. This resulted in a systematic error in the measurement.

Table 6.9 summarises the results.

Table 6.9: A summary of the results presented within this chapter.

Parameter name	Parameter symbol	Result
Tracker ‘warm-up’ time		> 5 h
Buck-in	$\sigma_{L,\text{in}}$	3.7 μm
Buck-out	$\sigma_{L,\text{out}}$	1.2 μm
SMR reorientation	σ_{turn}	0.47 μm
SMR reorientation	$\sigma_{\text{no turn}}$	0.13 μm
Probe comparison (RRR)	Δz_{RRR}	0.79 μm
Probe comparison (SCE)	Δz_{SCE}	0.95 μm
CMM-tracker repeatability	$\sigma_{\text{rep,srmv}}$	3.9 μm
Multilateration validation	σ_z	4×10^{-11} m
CMM measurement (small flat)	σ_z	0.19 μm
CMM measurement (large flat)	σ_z	2.3 μm
Multilateration (small flat)	σ_z	0.25 μm
Multilat. 2 nd -order fit removed (small flat)	σ_z	0.19 μm
Multilateration — reversed (small flat)	σ_z	0.31 μm
MC nominal (small flat)	σ_z	1.07 μm
MC interferometer (small flat)	σ_z	0.85 μm
MC SMR sphericity (small flat)	σ_z	1.62 μm
MC deadpath (small flat)	σ_z	1.03 μm
MC optimised (small flat)	σ_z	0.47 μm
Multilat. SCE (large flat)	$\sigma_{\Delta z}$	0.73 μm
Multilat. SCE 2 nd -order fit removed (large flat)	$\sigma_{\Delta z}$	0.28 μm
Multilat. repeatability SCE (large flat)	σ_z	0.46 μm
Multilat. RRR (large flat)	$\sigma_{\Delta z}$	0.93 μm
Multilat. sequential SCE (large flat)	$\sigma_{\Delta z}$	0.92 μm
MC nominal (large flat)	σ_z	1.13 μm
Multilat. long-term (large flat)	$\sigma_{\Delta z}$	1.88 μm

Chapter 7

Discussion

Abstract

The measurement results have enabled a significant analysis of uncertainty in the multilateration measurement system. The initial experiments informed the required measurement setup time and CMM/mover parameters. Testing of time independent parameters has enabled analysis of their system measurement uncertainty contributions and informed the mathematical model used in Monte Carlo (MC) simulations. The four-tracker multilateration measurements of the small ULE[®] flat optic provide a task-specific measurement error for analysis and comparison to MC simulation. The MC simulations of the setup in measuring the small flat predict the measurement uncertainty of the task and the sensitivity of the system to time independent input error sources. The four-tracker multilateration measurements of the large flat optic provide a task-specific measurement error, over a larger measurement area, for analysis and comparison to MC simulation; this included testing: sensitivity to measuring station positions and a sequential multilateration system. Testing of time dependent parameters has enabled analysis of their system measurement uncertainty contributions and informed the mathematical model used in MC simulations for long-term measurements. The measurement of the large flat optic over a long time period has provided a measurement error of the system that can be analysed for long-term influences. The MC simulation of the system sensitivity to temperature change has informed the measurement uncertainty analysis. With further investigation and optimisation of the measurement system, its measurement uncertainty can be reduced significantly.

7.1 Initial experiments

The initial experiments (see section 6.1) have shown that, in order for the multilateration experimental setup to function with relative stability, the time between tracker ‘power-up’ and undertaking a measurement should be > 5 h; and a pause in the probing routine

of the CMM/mover is required to measure a probing position as a measured position by the multilateration system. In an ideal scenario, power-up would occur during the day before any planned measurement.

Figure 6.1 shows that the stability of the laser tracker would reach a level within the resolution of the range measuring interferometer within a day of power-up. In reality, the local temperature is subject to change; it is therefore difficult to determine long-term range measurement stability $< 2 \mu\text{m}$.

Figures 6.2 and 6.3 show that a laser tracker range measurement cannot resolve a measured position of a CMM with a standard probing routine as the probe-tip was not in contact with the sample surface for sufficient time to trigger the recording of range measurement by an AT960LR laser tracker. Even if continuous measurement of the probe-tip was practical (long-term), figure 6.3 shows that a single measured position cannot be resolved when a standard probing motion is used. This led to the implementation of a pause (minimum 5 s) in the CMM probing routine.

7.2 Testing of time independent error sources

The testing of time independent input error sources — as defined in section 4.2 — was undertaken to determine estimates of the magnitude of these effects for utilisation in an Monte Carlo (MC) analysis of the multilateration measurement setup under investigation (see section 6.2).

The buck-in buck-out testing of the laser tracking interferometer on an AT960LR showed how error in the measurement of a length, L , may be induced in the case where the laser tracker must rotate through a large lateral axis to measure the displacement of an SMR. As shown in figure 6.4, this error has random and systematic elements.

The buck-out measurement was relatively stable when compared to the buck-in measurement (see table 6.9). This is expected due to the angle change required to perform the measurement of L was small from the outside position: with a good setup ΔL should have been relatively close to zero. A significant part of the ΔL for the outside measurement may have been caused by possible SMR reorientation during the placement of the SMR in the three-point magnetic hold: this placement was performed by hand.

The buck-in measurement has a relatively large variance. This indicates some random measurement error as a result of the large angular change required to perform the measurement. It is noteworthy that the first four measurements of L_{in} follow a linearly increasing trend, and the fifth measurement returns to a similar value to the first

measurement; however, the relatively small number of measurement repeats (five) makes it difficult to make conclusions about this pattern.

The SMR reorientation test (see figure 6.5) found that, over 10 repeated range measurements of an SMR, the standard deviation of ΔR with reorientation between measurements is 3.6 times higher than the standard deviation of ΔR without reorientations (see table 6.9). This error is on the order of the magnitude of the error sources associated with SMR sphericity (described in section 4.2.2); however, it was not possible to measure the magnitude of SMR reorientation as the turn was done by hand.

The results of the SMR reorientation testing show the effect of sphericity error upon the measurement of range; and therefore, the effect of repeatability and stability of the SMR–mover’s probing motion and surface contact point, specifically for single point measuring of the kind conducted in this implementation of the multilateration system under investigation.

The CMM measurement of the large flat optic with three unique measurement probes installed (as shown in figure 6.6) tested the probes effect on z-coordinates measured on the surface of the optic (see table 6.9). The aim of this test was to determine the extent that the different probe forms would affect the positioning of the centre of the SMR measurement target on the surface of a sample. The $\varnothing 8$ mm ruby standard probe measurement was used as a reference — Leitz manufactured. Given the relatively high peak-to-valley of the form of the flat optic, $\sim 10\text{ }\mu\text{m}$, and the high frequency of change in z for this probing routine (the arrangement of measurements on the surface), the difference between z-coordinate measurement could be as a result of inaccurate repositioning of the probes for each measurement set: due to misalignment of the sample and the coordinate system of the CMM between coordinate systems setup for each probe. It is therefore not possible, from this set of measurements, to determine whether the difference in z-coordinates is as a result of the probe shapes. To some extent, this is due to the relatively small difference in z-coordinates measured, indicating that the effect under investigation has only a small effect on z .

Following the implementation of a pause in the CMM probing routine, the positional stability of the paused probe was measured interferometrically by a laser tracker. As shown in figure 6.8, the paused probe in contact with a sample surface was stable relative to the resolution of the range measuring interferometer. This enabled the selection of a pause of 5 s to trigger and measure the range of a point on a surface.

The repeatability of CMM–tracker positions was tested to determine its effect on

measured positions for sequential multilateration. The standard deviation of laser tracker interferometric range measurements of repeated points on the surface of a flat optic show how the range measurements vary with repeats. This is a direct measurement of an input error source for sequential multilateration measurements with the setup of the measurement system under investigation. The variation in repeatability between measured positions was significant (see table 6.9). This test should be repeated with a greater number of measured positions spread over a larger measurement area to analyse the effect of measurement area on CMM–tracker positioning.

The multilateration solving algorithm was validated by simulating a ‘true’ set of ranges, $R_{i,j}$, for the measurement setup utilised within this investigation. These ranges were propagated through the multilateration algorithm to determine the coordinates of the measurement coordinates. The predicted z-coordinates, as shown in figure 6.10 and detailed in table 6.9, indicate that, for the measurement setup utilised in this investigation, the multilateration solving algorithm is not a negligible source of coordinate measurement uncertainty.

7.3 CMM measurement of flat optics

The CMM measurement of the optics (see section 6.3) used for sample surfaces in the investigation of the multilateration system are shown in figures 6.11 and 6.12. These measurements were used as a reference for comparison to the multilateration measurement solutions that followed. The noise level of the measurement of the ULE[®] flat optic shown in figure 6.11 suggests that the form error of the surface is close to the measurement uncertainty of the CMM. This is in contrast to the measurement of the large optic, which shows a clear saddle form error (see figure 6.12).

7.4 Four–tracker multilateration — ULE[®] flat optic

A small ULE[®] flat optic was measured (see sections 6.4 and 6.5) with a nominal 6×6 grid of points by the four–tracker multilateration arrangement (as shown in figures 6.13 and 6.25). The optic was measured in two orientations, nominal and reversed, to assess the difference between systematic errors in the measurement setup and form error of the sample surface.

Figures 6.14 and 6.26 show how the measuring station positions were evenly distributed in order to surround the measurement area; however, the distance between the measuring station positions and measured positions is large relative to the size of the measurement area. This distance was caused by the inability of the laser trackers

to measure the SCE when in close proximity to each other. When the SCE is moved within ~ 1 m of the AT960LR the ADM is not able to measure the range to the SCE. As a result, the tracker will not record the range displacement via the interferometer, despite the interferometric system still operating. This tracker controller limitation could be overcome with further development.

Figures 6.15 and 6.27 show the z-coordinate distribution of the measuring station positions. The z-coordinates of the laser trackers were compressed relative to the z-coordinates of the measured positions. The trackers' height above the sample surface is large relative to their z-coordinate distribution.

Despite the relatively high noise of the multilateration solution for the measurement of the optic in the nominal position, shown in figure 6.16, there is a clear saddle shape to the z-coordinates. With this form removed, the standard deviation of z-coordinates is equal to the equivalent z-coordinates measured by the CMM (see table 6.9).

Figure 6.21 shows the residuals, per laser tracker (j). Points 1–35 are measured positions on the surface of the sample. The first 16 measured positions have corresponding residuals that fall within a bandwidth of $\pm 0.07 \mu\text{m}$. The remaining points vary significantly within a bandwidth of $\pm 0.2 \mu\text{m}$.

Figures 6.22 and 6.23 show the significance of the difference in z-coordinates measured by the trackers in normal operating mode and the multilateration solution that utilises the range measurements for multilateration. Figure 6.24 shows the z-coordinates of the equivalent CMM measurement, multilateration solution, and multilateration solution with a second order polynomial fit removed. With the saddle shape removed from the multilateration solution, the significant difference in z-coordinates at point 9 remains. This difference does not correlate with the residuals for this point, which fall within the relatively low bandwidth of $\pm 0.07 \mu\text{m}$. There is some correlation between the difference and size of residuals between points 15 and 25; however, further experimental data would be required to confirm a relationship.

The form of the multilateration solution for the measurement of the optic, in a reversed orientation, has a similar form to the equivalent multilateration solution with the optic in the nominal orientation, as shown in figure 6.28. The form follows the same shape and direction for both measurement solutions. As for the previous setup, when this form is removed, the standard deviation in z-coordinates is similar to the equivalent CMM measurement (see table 6.9).

The residuals for the minimisation solution generally fall in a similar bandwidth to the residuals of the nominal setup, $\pm 0.25 \mu\text{m}$. The exemptions to this are some of the floating points, specifically in the expanded measurement area. Figure 6.26 shows the

expanded area measured positions. The relatively high magnitude of residuals for these positions may be as a result of several factors: the low density in measured positions close to these points, the large distance between these points and some of the laser trackers — resulting in higher interferometric displacement measurement error, and the stability of CMM floating positions. The stability of floating positions applies an error source to all such positions in a measurement routine, and thus requires further investigation. More generally, given the relatively low number of measured positions on the surface of the optic, it is not possible to confirm any correlation between the difference in z-coordinates, as measured by the CMM and the multilateration solution, and the magnitude of the corresponding residuals.

The saddle form of the two measurements of the optic remains the same in shape and direction for both the nominal and reversed orientations. This indicates the presence of a systematic error source. This is supported by the lack of a similar form in the equivalent CMM measurements. The source of this error may be the measurement setup arrangement. This breaks down into a number of parameters, including: arrangement of measuring stations relative to measured positions, distance between measuring stations and measured positions, the arrangement of measured positions within the measurement volume, the density of measured positions — critically at positions close to the sample surface, and the density of measured positions on the surface of the sample.

As stated, the ability to reduce the distances in this system was limited by the SCE-tracker interaction. The effect of arrangement and positioning of laser trackers has been investigated via MC simulation. An increase in the density of points at the sample surface would both increase the accuracy of the measurement and provide greater insight into the shape of the measurement form error. The relatively low density of points was selected to minimise measurement time; therefore, minimising the effect of environmental changes that could cause a form error if a rapid air temperature change occurred, resulting in a change in RI of the laser beam-path. The effect of temperature change on the measurement was investigated in the long-term multilateration measurements and MC simulation.

The source of variation in residuals for the measured positions include: CMM floating point stability, air flow across the beam-path during a tracker measurement causing RI changes, SMR beam clipping, and synchronisation between the four laser trackers and CMM measurements. The synchronisation of measurements is a factor caused by the independent triggering of each tracker measurement. This results in a difference in the time of measurement triggering; therefore, if the SMR moves during range measurement, this results in a relative increase in the residual at that point.

7.5 Monte Carlo simulation of ULE[®] flat measurement

MC simulation was used to determine an estimate for the measurement uncertainty in the multilateration solutions for the measurement of the ULE[®] flat optic, and to determine the sensitivity of measurement uncertainty of the system to a number of input error sources (see section 6.6). Table 6.9 outlines the results of these simulations.

The predicted uncertainty of points measured on the sample for the nominal experimental setup used in the above discussed measurement are shown in figure 6.37. The form of $\sigma_{z,i}$, as plotted against x and y , shows how the estimated uncertainty for measured positions increases with x and y . The increase in estimated uncertainty occurs closer to the centre of the 5×5 grid of floating points, as shown in figure 6.14, in the x -direction. It is noteworthy that the direction of the form increase is similar to the direction of the saddle shape in the real measurement solutions.

An MC simulation was used to estimate the measurement uncertainty sensitivity to interferometric displacement measurement error by simulation of the measurement discussed above, but with a reduction in the first-term of the measurement uncertainty, as detailed in table 6.2. As previously stated, this reduction resulted by a predicted 21% decrease in the square root mean variance, σ_z . This indicates a significant sensitivity to interferometric measurement error and shows the potential for reduced measurement uncertainty in real measurements.

A similar sensitivity analysis was carried out to determine the effect of an increased error in range measurement as a result of SMR sphericity. This was achieved by simulating a greater variation in surface contact points on the SMR, equivalent to $\theta = 5^\circ$. This resulted in an increase of 51% in σ_z relative to the nominal case. The form of $\sigma_{z,i}$ as a function of x and y : $\sigma_{z,i}$ increases with x only. This result indicates the significance of either maintaining a stable probing point on an SMR, or ensuring the sphericity error of an SMR and its optical centring error are minimised.

Figure 6.40 shows predicted uncertainty when deadpath error is halved, in comparison to the nominal case. This suggests a relative insensitivity to ADM range measurement error (a reduction of 4% in σ_z was predicted).

The sensitivity of measurement uncertainty in the multilateration measurement setup due to measuring station positioning was analysed via MC simulation of the arrangement shown in figure 6.41. A reduction of 56% in σ_z , by relocation of measuring stations, shows the significance of this parameter on measurement uncertainty. It is of note that the form of $\sigma_{z,i}$, as a function of x and y , remains broadly similar to the nominal case. This suggests the cause of this spatial distribution of measurement uncertainty is related to measured position arrangement.

7.6 Four-tracker multilateration — large flat with SCE

A multilateration measurement, using a similar setup to the measurements of the ULE[®] flat, was undertaken using a larger flat optic as a sample (see section 6.7). This was done to test the effect of measured area size on the measurement uncertainty of the multilateration system under investigation.

The residuals for measured positions 1–30 are within a bandwidth of $\pm 0.12 \mu\text{m}$; whilst the residuals of the remaining points lie within a significantly greater magnitude bandwidth. This pattern is similar to the residuals of the measured position with the ULE[®] flat optic in a nominal position.

As shown in figure 6.51, the difference in z-coordinates measured by the CMM and the multilateration solution has no apparent correlation to the residuals of the corresponding measured positions. This comparison has been applied as the raw z-coordinates have a significant form error — the change in z-coordinates (Δz) across the surface is large compared to the target measurement uncertainty — due to the shape of the optic used as a sample.

The form of difference in z-coordinates, Δz , has a saddle with a slope along the line between $(x, y) = (0, 0)$ and $(x, y) = (0.38, 0.38)$. This form is similar to the multilateration solutions to the measurement of the small ULE[®] flat. Despite an increase in measured area in the xy-plane of the sample surface, the form of the measurement error has continued. A $0.2 \text{ m} \times 0.2 \text{ m}$ central section of Δz as a function of x and y has a peak-to-valley similar to the measurement of the small ULE[®] flat.

As both the optics were measured in a similar position within the measurement area, close to a corner of the floating grid of points, it is clear that this form error is not position dependent within the measurement area. It is, however, dependent on the position of the sample measured positions within the overall measured positions — including floating points — and the distribution of measured positions on the xy-plane of the sample surface. The presence of the saddle shape, with constant orientation for the multilateration solution of both the small sample and the large sample — when the SCE-probe was utilised — suggests that this was caused by a systematic error source.

Figure 6.52 shows the noise in Δz ; this includes both the measurement noise of the multilateration measurement solution and the CMM measurement. As the standard deviation of Δz is correlated between the measurement uncertainty of the multilateration solution and the CMM result, it can only be used as an indication of the multilateration setup measurement uncertainty. Further work to measure a sample surface of this size, verified by traceable means, should be conducted to determine the multilateration setup measurement uncertainty for this size of surface. It is noteworthy, however, that the magnitude of $\sigma_{\Delta z} = 0.28 \mu\text{m}$ is similar to the combined square root variance of

the multilateration solution with a second-order polynomial fit removed and CMM measurement of the smaller sample: $\sqrt{\sigma_{z,\text{multilat}}^2 + \sigma_{z,\text{CMM}}^2} \approx 0.28 \mu\text{m}$.

The standard deviations, $\sigma_{z,i}$ of 10 repeated 10×10 grid measurements of the large optic are shown in figure 6.53. The magnitudes of $\sigma_{z,i}$ are greatest along the lines defined by $y = 0 \text{ m}$, 0.2 m , and 0.38 m . $\sigma_{z,i}$ are lowest along the line defined by $y = 0.08 \text{ m}$ and relatively low along the line defined by $y = 0.3 \text{ m}$. This repeated pattern in the y direction is perpendicular to the probing raster direction of the measured positions. This suggests that the magnitude or the $\sigma_{z,i}$ for repeated points measured is dependent on measurement position rather than measurement time (i.e. this measurement uncertainty is time independent). It is noteworthy that, because of the relatively large form of this sample surface (see figure 6.12), the standard deviations, $\sigma_{z,i}$, include a combination of the random error in the multilateration solution and the CMM positional repeatability (the precision of the CMM in returning to the same measured positions for each measurement set).

7.7 Four-tracker multilateration — large flat with RRR

As detailed in section 6.8, the multilateration measurement solution for the larger optic with an RRR as the measurement target has relatively high noise in comparison to the measurement with the SCE-probe (see figure 6.57). The system arrangement (see arrangement in figure 6.54) may be the partial cause of the noise level. This was restricted by the viewing angle of the RRR; however, SMR beam clipping is likely to be a significant source of the noise.

Beam clipping occurs when an SMR is orientated in such a way that only a fraction of an incoming laser beam enters the reflecting part of the SMR; therefore, only a fraction of the beam is returned to the laser tracker for range measurement. This may have the effect of inducing error into a displacement measurement.

Figure 6.60 shows how the residuals of the minimisation utilising measurements for laser tracker $j = 4$ have significantly greater magnitude than those for the other three trackers. As stated, it seems that the minimisation used the first three trackers measurements to balance with the fourth trackers measurements. This suggests that the measurements of displacement by tracker $j = 4$ have relatively high measurement error. The relatively small viewing angle of the RRR could have resulted in beam clipping that may be the cause of this measurement error.

In comparison to the multilateration measurement of the large optic with an SCE-probe, the distribution of laser trackers in the measurement utilising an RRR-probe is

limited to one side of the optic and at a relatively large distance from the optic; although, the shape of the arrangement of trackers is more similar to the triangular pyramid used in the MC simulation to investigate the effect of tracker arrangement on estimated measurement uncertainty. The limited distribution of the trackers in x , y , and z may have contributed to both the high level of noise and the 27% increase in $\sigma_{\Delta z}$ from the equivalent measurement with an SCE. The increased distance — relative to the nominal setup — between trackers and sample contributed to this error as the interferometric displacement measurement uncertainty is length dependent (u_L).

7.8 Four-tracker sequential multilateration — large flat with SCE

Figure 6.63 shows the clear form of the sequential multilateration measurement solution that, as stated, is similar to the equivalent non-sequential measurement (see section 6.9). A source of the measurement noise was CMM-tracker positioning repeatability, as investigated in section 6.2. The measurement did not use any floating measured positions in an attempt to isolate the measurement from possible CMM floating position instability.

As detailed, the l^2 -norm for this measurement solution is approximately a factor of eight times the equivalent l^2 -norm for the non-sequential multilateration measurement. This indicates the relatively high measurement uncertainty in this measurement solution.

The comparison between z -coordinates of sequential and non-sequential measurements shows an increase in $\sigma_{\Delta z}$ of approximately 26% on the equivalent non-sequential measurement; but the form of the sequential measurement is clearly similar to the non-sequential measurement, suggesting that the error of this measurement is smaller in magnitude than the form of the sample in z .

The form of the $\sigma_{\Delta z}$ for the sequential measurement solution is also a saddle shape; however, the direction of the saddle is shifted through 90° about the z -axis relative to the non-sequential equivalent. More investigation is required to determine the source of this form shift.

The magnitude of $\sigma_{\Delta z}$ and the Δz peak-to-valley for the sequential measurement are similar to those of the measurement with the RRR-probe; however, given the dependence of the sequential multilateration measurement to CMM-tracker positioning repeatability, this is not a comparison of equivalent systems. It is reasonable to expect that the measurement error of the sequential multilateration solution would be greater if an SMR-mover with less precision than a Leitz PMM-F was used.

7.9 MC simulation of large flat measurement

An MC simulation of the multilateration measurement of the large flat optic with the nominal measurement setup (utilising the SCE-probe) predicted a square root mean variance, $\sigma_z = 1.13 \mu\text{m}$ (see section 6.10). Notably, this is not significantly greater than the equivalent simulated σ_z for the measurement of the small flat ($\sigma_z = 1.07 \mu\text{m}$). This result indicates that the measurement uncertainty of the multilateration setup does not increase significantly with the measured area within the total measurement volume.

Figure 6.67 shows the standard deviations of predicted measurements per measured position, $\sigma_{z,i}$, plotted against x and y . The distribution of predicted z-coordinate measurement uncertainty is different to the equivalent prediction for the measurement of the small flat. The general trend shows an increase in $\sigma_{z,i}$ with x . The relationship between z-coordinate measurement uncertainty and the position of measured points within the measurement area requires further investigation; specifically, the effect of positioning and size of surface of the sample relative to floating measured positions and relative to the measuring station positions.

7.10 Time dependent error sources

The time dependent input error sources were investigated in section 6.11. The experiments aimed to test the effect of change in RI and tracker home position on range displacement measurements. This involved tests that separate these error sources and combine them.

The tests of tracker home position stability indicate a relationship between change in range measured of a ‘stationary’ SMR and change in measured air temperature. The measurement with the RRR on a post aimed to isolate the measurement from changes in h_S ; thus, providing a reference from which to assess the equivalent test with the RRR on the base of the CMM in the same plane as the tracker base. Due to issues with the setup of the RRR on the post test, a fit of the data set was removed such that the relationship between air temperature change (ΔT) and ΔR could be established; hence, the results have not provided a reliable reference measurement. This measurement should be repeated.

For the case where the RRR was on the CMM base, the relationship between ΔT and ΔR is clear. For the first two hours of measurement, the temperature increased at a relatively steady rate. During this period, the measured range, ΔR also increases at the steady rate. During the period where the temperature changed rapidly (after two hours), as shown in figure 5.9b, it is not clear that the direct relationship between ΔT and ΔR holds; however, the decrease in measured range during this time may be due

to some latent effect on the h_S , a decrease in range indicates a decrease in h_S . The stability of home position due to temperature change requires a more detailed analysis. This measurement suggests that when the air temperature change occurs relatively slowly, there was sufficient time for the temperature change to significantly affect the range measurement; however, when the temperature change is more rapid, ΔR does not maintain correlation. This is possibly due to the large thermal mass of the tracker that required relatively constant heating in order to affect the range measurement.

A short-range measurement of an RRR on an aluminium post aimed to provide a reference measurement to compare to equivalent measurements with longer range. Due to reasons of experimental setup, the minimum range achievable was $R \approx 0.2$ m. The inverse trend relationship between ΔT and ΔR requires further investigation; however, it is clear that the magnitude of change in the measured range suggests that a shorter range is required to further isolate the effect of RI change on range measured. Without further information, this measurement cannot be used as a reference.

Figure 6.71 illustrates the effect of relatively high frequency temperature change on the range measurements of an RRR on a post. For $R = 1$ m, $\Delta R \approx 2.5 \mu\text{m}$ when $\Delta T \approx 0.45$ K. The high frequency of this range measurement change indicates its source is a change in RI; given the mass of a laser tracker, change in h_S at this frequency with this level of temperature change is not expected. To identify the source of the long-term drift in this measurement, more investigation is required.

Figure 6.72 shows the effect of high frequency temperature change on the range measurements. The increased range, $R = 2$ m, has resulted in a larger magnitude of $\Delta R \approx 4 \mu\text{m}$ for the same magnitude of temperature change as the equivalent $R = 1$ m test. The removal of a fit from the range data set, due to experimental setup instability, reduces the confidence in this trend analysis; however, the matching of trends between measured range — with fit removed — and temperature increases the level of confidence.

During the combined measurement with $R = 1$ m, the high frequency temperature change has ~ 0.66 times the magnitude of ΔT during the equivalent measurement when the RRR was on a post; the high frequency change in measured range is ~ 0.14 times the magnitude of ΔR during the equivalent measurement; however, the change in range measured during the first hour of measurement has ~ 0.64 times the magnitude of ΔR during the equivalent measurement. This indicates that, during the initial increase in temperature, the measured range followed ΔT as expected; however, some other factor caused the magnitude of this trend to reduce in the short-term but result in a long-term drift.

Relative to the equivalent RRR on post measurement, the combined measurement

with $R = 2$ m has a small magnitude change in ΔR at high frequency. Over the entire 12 h, the measured temperature increased by 0.1 K and the measured range increased by ~ 2 μm . This is almost double the proportional high frequency measured range change that occurred during the $R = 2$ m measurement where the RRR was placed on a post: the ratio between ΔR and ΔT for the equivalent post measurement was 10:1; the ratio for the combined measurement was 20:1. This indicates that a long-term temperature change results in a greater increase in measured range, implying that the combined measurement is affected by factors other than change in RI. Given the thermal mass of a laser tracker, it is expected that change in h_S occurs over relatively low frequency ΔT . This suggests that change in h_S is, at least partially, the cause of the high ratio between ΔR and ΔT for the long-term combined measurement.

For the combined $R = 2$ m measurements, the effect of ΔT on ΔR can be modelled by a right-angle triangle with hypotenuse, R , and vertical side, $h_S \approx 0.4$ m; therefore, assuming $\alpha_S = 10$ $\mu\text{m}/\text{m}$, if $\Delta T = 0.1$ K then $\Delta R \approx 0.1$ μm . Clearly this does not account for any significant part of the drift in measured range; therefore, more investigation into the stability of range measurements is required, including an increases sensitivity analysis of change in h_S as this analysis is hindered by RI effects.

The range stability test using a 1.5 inch RRR provides further evidence of the relationship between ΔR and ΔT . There is a clear trend between the two parameters; however, the long-term relationship does drift apart. This also provided another magnitude ratio between ΔR and ΔT that is inconsistent with previous measurements, re-emphasising the need for further analysis to determine measured range sensitivity to these and possible unanticipated parameters.

The long-term stability testing of the laser tracker to assess the effect of time dependent error sources was limited by measurement of air temperature only. Any further investigation of these error sources should include measurement of air humidity and pressure to determine the effect of refractive index change upon range measurements.

7.11 Long-term four-tracker multilateration measurement

The long-term measurement of the large flat optic aimed to determine the effect of environmental temperature change, ΔT , on the measurement uncertainty of the four-tracker multilateration system (see section 6.12). Figures 6.80 and 6.81 shows how Δz varies as a function of measured position; and therefore, as the time between measuring each point was ~ 0.5 h, this shows how Δz varies as a function of time.

The differences in z for the long-term measurement of the large flat, relative to the

short-term measurements, follows a U-shape as a function of time; with the greatest magnitude of difference at the beginning, middle, and end of the measurement period. The magnitude of this temporal distribution is caused by defining the coordinates by a first-order polynomial fit of the data. Given the significance of change in RI on the rate of ΔR , it is expected that a significant fraction of this z-coordinate measurement error is caused by ΔT and the corresponding change in RI across the laser beam-path.

As stated, the l^2 -norm of the residuals, $w_{i,j}$, of the minimisation for the long-term measurement is 2.4 times the equivalent short-term measurement solution with 100 points measured on the surface. This indicates a higher measurement uncertainty in the measurement. Furthermore, the mean of $\bar{w} = 0.14 \mu\text{m}$ is 38 times the magnitude of the stated equivalent measurement solution. This indicates that a systematic error source, in the form of measurement drift, had a significant effect on the long-term measurement solution.

It is noteworthy that σ_z for the z-coordinates of the short-term multilateration solution with 34 points is not significantly greater in magnitude, approximately 4%, than the equivalent short-term measurement with 100 points. This suggests that the effect of the number of measured positions, on the order of tens, does not have a significant effect on measurement uncertainty when greater than 30 points are measured; however, the greater the number of points taken over a sample provide information about the form of the sample; and the distribution of measurement uncertainty throughout a measurement area.

7.12 MC simulation of long-term measurement

The MC simulation of the long-term measurement of the large flat utilised the real temperature measurement data for the nominal case. The MC simulation parameters were selected to match the previous analysis with the addition of α_S and ΔR_T to simulate the effect of time dependent parameters on the measurement solution. As previously stated, the investigation of these time dependent parameters requires further analysis to determine accurate estimates for parameter values; however, these values enable the simulations to indicate the overall effect these parameters have on measurement uncertainty, rather than predicting an accurate value for the measurement uncertainty.

Figure 6.83 shows the distribution of predicted measurement uncertainty as a result of random error sources. It is noteworthy that this result remains the same for simulations that do not consider temperature change. Figure 6.84 shows the distribution of systematic error (measurement drift) in the measurement solution. For simulations that do not

consider temperature change, this result is several orders of magnitude smaller. The noise in the predicted drift is caused by the high frequency change in temperature measured during the real measurement. It is noteworthy that the peak-to-valley of the predicted measurement uncertainties is similar to the peak-to-valley of Δz for the real long-term measurement. A relatively small lag in the reaction of ΔR to ΔT would result in a significant change to the systematic error predicted by the simulation. Further investigation into the thermal inertia of the measurement system would improve the accuracy of the MC simulated predicted measurement uncertainties.

The sensitivity of long-term measurements to long-term temperature change was analysed by MC simulation. The random error predicted is equal to the real temperature simulation as these are long-term temperature independent. As stated, the form of the predicted systematic measurement error (measurement drift), as shown in figure 6.86, is also a half-cylinder shape. This suggests that an increase in temperature results in a decrease in z-coordinate measured, and a decrease in temperature results in an increase in z-coordinate measured. The results for an equivalent simulation utilising the inverse f_T maintains this predicted trend.

The similarity in the forms of this predicted systematic measurement error and the real measurement indicates that the model is capable of predicting some aspects of the system sensitivity to temperature change; however, as previously stated, further investigation is required. If the accuracy of the model is improved, temperature measurements close to the beam-path of the laser trackers could be used to remove some level of systematic error from measurement solutions. This would require precise temporal synchronisation in measurement of temperature and tracker displacement measuring.

7.13 Measurement system and MC simulations

The measurement solutions for the small flat optic and short-term measurement of the large flat optic show the capabilities of the measurement system for these specific measurement setups. Using the CMM measurement of the large flat as a reference measurement indicates that the measurement uncertainty of the multilateration measurement system, in this setup, is $< 1 \mu\text{m}$ ($k = 1$). A significant part of the error in this measurement is long spatial wavelength form error; therefore, the removal of this systematic error would result in a significant reduction in measurement error.

The MC simulation to determine system sensitivity to the measurement input parameter errors indicate that, with the implementation of achievable changes to the measurement system, the measurement uncertainty of the system could be significantly

improved; for example: measuring station positioning changes, and a reduction to the first-term of the measurement uncertainty of the interferometric displacement measurement. The simulation also indicates that, if all other simulated parameters were to remain the same, implementation of the measurement setup with a less complex SMR-mover, such as a robot arm, would result in a relatively small increase in σ_z provided $\theta \leq 5^\circ$.

The MC simulation of measured position optimisation suggests that at least a 52% reduction in measurement uncertainty is possible, relative to the measurement setup utilised in real measurements undertaken. If this improvement was applied to the measurement solutions for the small flat measurement, σ_z would be approximately the same as for the corresponding CMM measurements. It is noteworthy that this is the case when the second-order polynomial fits of the z-coordinates of the solutions are removed.

As discussed, there are further input measurement parameters for the multilateration measurement setup that can be altered to affect the uncertainty in the system measurement solution, such as: increasing the number of measured positions, changing the distribution of measured positions, increasing the velocity of the mover to reduce the effect of time dependent error sources, and increasing the sphericity and optical centring of the SMR.

Increasing the number of measured positions provides more information that can be used by the multilateration solving algorithm to reduce measurement residuals. If the measurement setup was utilised for a high spatial density measurement of a metre-scale optic, this would necessarily require many more measured positions than was utilised for the multilateration solution of z-coordinates on the samples in this analysis. Whilst increasing the number of CMM measurements potentially provides more information about the surface, this has no inherent effect on the coordinate measurement uncertainty of the CMM.

Altering the arrangement of measuring station positions may enable an increase in the z-distribution of measured positions. This would be possible if the measuring stations were moved to enable viewing of an SMR with a greater volume in z ; therefore, the testing of this parameter would be further enabled. By inputting more information along the z-axis to the solver, the uncertainty in measurement of z-coordinates should be reduced.

Increasing the velocity of an SMR-mover would reduce the measurement time and therefore the effect of time dependent errors. Other than the physical velocity limit of the mover, the limiting factor would be the trade-off between SMR velocity and spatial density of points measured: an AT960LR measures range at a temporal frequency of 1000 Hz; therefore, an alternative solution could be to reduce the magnitude of ΔT across the beam-path of the laser interferometer throughout the measurement period. In any case, the dynamic range measurement stability of a high velocity SMR should be investigated.

From the multilateration solutions for the measurement of the small and large flat optic, the form of the measurement solutions seems to be a systematic error present in the three z -coordinate sets: σ_z for the small optic in both orientations and $\sigma_{\Delta z}$ for the large optic. The magnitude of the random error in the solution for the large optic measurement is similar to the random error in the measurement solutions for the small optic.

Given that the two samples were placed within the measurement volume at similar coordinates, and the measuring stations remained in the same positions for all measurements, it is reasonable to assume that the systematic and random error would remain the same for a larger optic measured within the same measurement setup. It is possible that the random error would increase towards the extremes of the measurement area, where R is potentially large, and therefore interferometric measurement uncertainty is relatively high; however, this may be offset by the fact that when R is large for one or more of the measuring stations, it is relatively small for other measuring stations. To determine the measurement uncertainty for a larger part of the measurement area of the system, a larger flat optic should be measured within the same measurement setup.

The results obtained show that, with further investigation and optimisation, the multilateration measurement system can measure the z -coordinates of a metre-scale low-slope sample with uncertainty at least as low as a CMM measurement.

Chapter 8

Conclusions

Conclusions

The hypothesis tested was:

A laser-based multilateration system can be used to measure a metre-scale low-slope freeform non-specular surface form with z-coordinate measurement uncertainty, $\sigma_z < 1\ \mu\text{m}$.

This research has made significant progress in proving this hypothesis by actualising the research objectives.

The results of this research show that a laser-based multilateration system can measure large optics with uncertainty, at least, equivalent to a state-of-the-art CMM. This measuring technique could be utilised to reduce the time to manufacture a large optic by reducing the requirement for neutral polishing. Implementing a system of four laser trackers on a vibration isolated base, with a motion arm (robot) to move an SMR over the surface of an optic, could be relatively cost effective, in comparison to state-of-the-art CMM;. This type of measurement setup could be optimised to measure optics of different size scales by moving the laser trackers to fit an optic within the measuring volume: minimising measurement uncertainty. This would not be possible with a CMM, which has a fixed measurement volume.

Laser-based multilateration could also be utilised on larger scales, not just for measuring individual optics, but also for measurement of optic positioning within large segmented mirrors. With the construction of smaller scale tracking interferometers, this technique could be utilised within machining systems for *in situ* metrology. This could improve the accuracy of machining processes and reduce manufacturing times.

A four-tracker multilateration system was tested experimentally to determine its uncertainty in z-coordinate measurement of two sample surfaces. A $0.2\text{ m} \times 0.2\text{ m}$ ULE[®] flat optic was measured with a nominal grid of 6×6 points in two orientations; the measured z-coordinates have a standard deviation of $\sigma_z = 0.25\text{ }\mu\text{m}$ in the nominal orientation and $\sigma_z = 0.31\text{ }\mu\text{m}$ in the reversed orientation. Both sets of coordinates have a saddle shape, which when removed, results in $\sigma_z = 0.19\text{ }\mu\text{m}$ and $\sigma_z = 0.17\text{ }\mu\text{m}$ respectively.

The standard deviations, for this specific measurement setup and routine, are indicative of an uncertainty in the measurement of this, relatively small, sample surface of $\sigma_z < 1\text{ }\mu\text{m}$. This is the case for both the coordinates with and without the systematic error (saddle form) removed. The small optic was measured with relatively low point density, further work should include a significantly higher density measurement of the optic to increase confidence in σ_z . The measurement of a sample of this size limited the assessment of the measurement setup uncertainty in measuring metre-scale surfaces for reasons that are self-evident; however, the measurement area, that the sample was measured within, was metre-scale. As such, the laser tracker range measurements were greater than a metre; hence, this measurement does give some indication of the measurement capabilities of the multilateration setup for the required sample surface areas. Further repeated measurements of the sample both in the same position and in different positions within the measurement area would assist this investigation.

A 10×10 grid of points was measured on the surface of a $0.4\text{ m} \times 0.4\text{ m}$ flat optic with the four-tracker multilateration setup. The standard deviation of the difference between the multilateration measurement solution and the equivalent CMM measurement is $\sigma_{\Delta z} = 0.73\text{ }\mu\text{m}$. As with the measurement solutions for the small ULE[®] flat optic, a saddle shape is apparent. In this case, the standard deviation includes both the CMM z-coordinate measurement error and the error in multilateration solution for z-coordinates.

For a set of multilateration solutions of 10 repeated measurements of 10×10 grid of points on the surface, the square root mean variance of the z-coordinates of measured positions is $\sigma_z = 0.46\text{ }\mu\text{m}$. Here, the standard deviation includes both the z-coordinate multilateration solution error and the CMM positioning repeatability error.

As both $\sigma_{\Delta z}$ and σ_z , for the large optic, include a combination of errors in the multilateration solution for z-coordinates measured and errors due either CMM measurement error or CMM positioning repeatability, it is not possible to claim that these results show $\sigma_z < 1\text{ }\mu\text{m}$; however, the form of Δz is similar to the form of z for the small optic and both $\sigma_{\Delta z}$ and σ_z are significantly less than $1\text{ }\mu\text{m}$. These measurement results give a strong indication that, for this measurement setup and routine, the uncertainty in measured z-coordinates does meet requirements. With further work, that includes either (or both):

the measurement of a similar large optic, but with form error significantly less than the sample surface used within this investigation – thereby minimising the effect of CMM positioning repeatability on the measurement uncertainty; or undertaking a more rigorous investigation of CMM positioning repeatability. From these, CMM positioning repeatability error could either be removed as an influence factor or isolated from the measurement result.

The effect of measuring station positioning on z-coordinate measurement uncertainty was tested by changing the arrangement of laser trackers and repeating the measurement of the larger optic with an RRR-probe. The difference in z-coordinates for the multilateration solution and CMM measurements have a standard deviation of $\sigma_{\Delta z} = 0.93 \mu\text{m}$: a 27% increase on the measurement solution for the nominal setup. This shows the sensitivity of the uncertainty in z-coordinates to measuring station arrangement; and indicates how further work could be undertaken to reduce the contribution of the setup to the measurement uncertainty.

A sequential style of multilateration was tested, using the nominal measurement setup, and shown to have a standard deviation, $\sigma_{\Delta z} = 0.92 \mu\text{m}$: a 26% increase on the measurement solution for the nominal setup. This shows how the multilateration technique can be utilised to measure surfaces in a setup with a single measuring station. It is of note that, in addition to this result including the CMM z-coordinate measurement error (used to calculate Δz), the result also includes error due to the CMM positioning repeatability; therefore, investigation of any sequential multilateration setup should determine the SMR-mover positioning repeatability, due to the sensitivity of measurement uncertainty to this parameter.

A mathematical model of the multilateration system was built and tested; the input error sources were analysed; and MC simulations were used to predict the uncertainty in z-coordinate measurement of real experimental measurements undertaken on sample surfaces. The model was also utilised to test the sensitivity of uncertainty in z-coordinate measurement to change in the magnitude of input errors.

The model was validated by determining ‘true’ displacement measurements for the nominal experimental setup and propagating them through the multilateration algorithm. The z-coordinates of the solution, based on ‘true’ displacement measurements, have a standard deviation on the order of 10^{-11} m ; therefore, it is concluded that the multilateration algorithm had a negligible effect upon the measurement uncertainty for the multilateration setups used.

Testing of time independent input error sources was used to inform the inputs implemented in the MC simulations. These tests showed the significance of SMR sphericity and probing repeatability upon the laser tracker measurement of range. It is concluded

that: either the SMR sphericity error should be minimised, or the repeatability error of probing motion of the SMR–mover should be minimised. This is so that the distance between probe surface contact point and SMR optical centre remains stable, such that this has a negligible effect on the z-coordinate measurement uncertainty of a multilateration system of the kind investigated here.

The MC simulations of the measurement of the small ULE® sample, with nominal measurement setup parameters selected, predicts that the z-coordinate measurement solution uncertainty increases with x and y . This effect occurs in a similar way to the other MC simulations carried out for this sample with altered parameters. It is concluded that further investigation is required to confirm if this uncertainty distribution occurs with other measurement setups, specifically when the sample is positioned differently within the measurement area (different in x , y , and z as well as different in orientation).

The MC simulation results for a reduced interferometric displacement measurement input uncertainty indicate that the z-coordinate multilateration measurement solution uncertainty is highly sensitive to this parameter. It is concluded that optimisation of this parameter, by selection of measuring stations with u_0 (the constant first term of the interferometric displacement measurement uncertainty) less than that of the measuring stations utilised in this experimental investigation, a significant reduction in the system measurement uncertainty could be achieved. The MC simulations further illustrate the requirement that the effects of SMR sphericity on the measurement uncertainty should be mitigated or reduced as indicated by the time independent input error analyses. The uncertainty in absolute distance measurement (ADM), utilised by the laser trackers to measure an initial distance to the SMR, has a relatively small effect upon measurement uncertainty, as predicted by the MC simulation to test sensitivity to this parameter.

Significantly, the MC simulation of the measurement for an optimised arrangement of measuring stations showed an estimated reduction in z-coordinate multilateration measurement solution uncertainty of 56%. This illustrates the sensitivity of measurement uncertainty to this parameter and shows how, without any change to measurement equipment or other measurement parameters, the uncertainty can be reduced.

The MC simulation of the nominal multilateration measurement of the large flat in this measurement setup predicted a square root mean variance with magnitude 5.5% greater than the equivalent simulation for the small flat. This indicates that, despite the measurement area on the sample surface increasing by a factor of (approximately) four, the z-coordinate measurement uncertainty does not significantly increase. This provides further evidence for the argument that the measurement uncertainty is more sensitive to change in the size of the measurement volume than to change in the size of the area of points measured on the surface of a sample; i.e. the sensitivity of measurement

uncertainty to measuring position arrangement is greater than the sensitivity to positions measured area. This is in agreement with the MC simulations for an optimised measuring station position setup.

The results of experimental measurements undertaken suggest that the nominal measurement setup has the capability to measure a surface, with dimension $0.4\text{ m} \times 0.4\text{ m}$, with $\sigma_z < 1\text{ }\mu\text{m}$ for short-term measurements (measurements with a sufficiently stable environment). This conclusion is supported by the measurement results for: the multilateration measurement solution for the small flat, $\sigma_z \approx 0.3\text{ }\mu\text{m}$; the difference between the measurement solution for the large flat and the equivalent CMM measurement, $\sigma_z \approx 0.7\text{ }\mu\text{m}$; the square root mean variance of 10 repeated multilateration measurement solutions for the large flat, $\sigma_z \approx 0.4\text{ }\mu\text{m}$.

The results of the MC simulations indicate that a multilateration measurement of a metre-scale ($\sim 1\text{ m} \times \sim 1\text{ m}$) sample, with a setup with minimal alteration from the setup utilised within this investigation, can measure the surface z-coordinates with the required uncertainty. This is indicated by MC simulation of: the small and large flats with the nominal measurement setup, and the MC simulation of the small flat with MC simulation of the optimised measuring station arrangement. As stated, the increase in predicted σ_z for the nominal measurement of the large flat in comparison to the small flat is approximately 5.6%. The simulation of the optimised measuring station setup indicated a reduction in z-coordinate measurement solution uncertainty of approximately 50%.

The effect of long-term high-density measurements of a sample surface was investigated by performing a long time period measurement of a surface. The measurement was simulated by inputting measured temperature, as a function of time, into the model to predict the measurement uncertainty. A simulated long-term temperature change was used as an input within an MC simulation.

Change in the local environment, specifically temperature change (ΔT), of the measurement setup was shown to affect the range measurement stability of the measuring stations (laser trackers). Change in range measured (ΔR) to a stationary SMR, as a function of time, was determined to be caused primarily by the effect of refractive index (RI) change — which was both time and length dependent — and drift in the home position of a measuring station. A series of long-term stability assessments were carried out to isolate and measure these effects. This investigation did not determine a clear relationship between ΔR and ΔT , or indeed ΔR and R itself. The results show that for some time periods a clear trend was observed; however, these trends did not occur over the full time period of the measurements. Further investigation into these effects should be undertaken. Air pressure and humidity changes (RI) should be monitored, and potential thermal inertia of the laser tracker body (tracker home position drift), to

determine accurate multilateration input error source magnitudes. It is clear that ΔT , and therefore change in RI, has a significant effect on ΔR .

The comparison of short-term CMM measurement and a short-term multilateration solution with a long-term (15.5 h measurement) multilateration measurement solution for the large flat measurement shows how time dependent error sources affect the surface z -coordinate measurement. The standard deviation of the difference, in z , of positions measured by the CMM and the long-term multilateration measurement was $\sigma_{\Delta z} = 1.88 \mu\text{m}$: a factor of approximately 2.4 times the equivalent short-term multilateration measurement solution.

The MC simulation of this long-term multilateration measurement solution has shown the significant effect of thermal drift on the measurement. These simulations were based on both real temperature measurements of the time period of the real measurement and simulated long-term temperature change.

The simulations that utilised the real temperature measurements did not predict a measurement with systematic form error (due to drift) similar to the real measurements; this is concluded to be due to the poor temporal synchronisation of temperature measurements with position measurements. The simulation does illustrate the potential effect of high frequency temperature change that can result in drift that appears as measurement noise.

The simulations based on a long-term temperature trend produce a drift error that has form, in z , similar to the measurement solution for the real experimental measurement. This indicates that the model is capable of simulating the sensitivity of the measurement to temperature change to some extent. With improvements to the accuracy of the model (further work), and well synchronised measurement of temperature near the laser beam-paths, this drift could be significantly compensated.

8.1 Further work

Details of further work, based on chapter 7 and the conclusions above, are outlined within this section. The reasons for this further work are explained and experimental methods are briefly defined.

8.1.1 Time independent error sources

This investigation would benefit from: a re-characterised buck-in buck-out test; a more accurate assessment of CMM-tracker positioning repeatability; and further measurement to determine the effect of SMR CMM probing on the measurement of z-coordinates.

The buck-in buck-out test should enable a more accurate assessment of the range measurement error as a result of a laser trackers lateral motion as it tracks a moving SMR. This would provide further detail in the mathematical model of the system and improve the accuracy of MC simulation uncertainty estimates. The investigation should use SMR posts with increased thickness to provide more rigid holding of the SMR to reduce the effect of vibrations in the posts. The measurements should be repeated a minimum of 20 times per tracker position with the tracker being moved inside and outside several times. This would increase the accuracy of the error assessment and highlight any outliers.

The CMM-tracker positioning repeatability should be tested on a nominally flat metre-scale sample surface. If the form error of the surface is small enough that it has a negligible effect on the z-coordinate measurement, this surface can be used to isolate the repeatability of the CMM-tracker measurement. This measurement will include a combination of CMM positioning repeatability error and laser tracker range measurement repeatability error. If the CMM measured coordinates are used to assess the CMM repeatability, then the two effects can be isolated. If the measurement has relatively high spatial density — 100×100 points — the relationship between the position of the measurement and measurement repeatability can be investigated. These accurate parameter results can enable an MC simulation of the sequential multilateration routine.

8.1.2 Reference measurement

A high accuracy traceable measurement of a large flat optic would enable calibration of the sample used within the investigation of the multilateration setup. This would provide a reference measurement of the sample such that, specifically for the large flat, the multilateration result could be isolated from the CMM measurement. This could be achieved by expanding the interferometric laser beam of systems detailed within section 2.4.2; enabling the use of the CMM purely as an SMR-mover and not a comparison measurement system.

8.1.3 Four-tracker multilateration

Investigation of the measurement setup could be furthered/improved by: synchronisation of measurement systems, increased density of measured positions on sample surfaces, measurement of the sample in positions throughout the measurement volume, use of a calibrated (with traceability) surface, further repeated measurements, increasing the number of measuring station positions for sequential multilateration, and automating the simulation of the measurement setup to determine optimised measuring station positions.

Synchronisation of measurements of the measuring stations is critical to measurement uncertainty as time between measurements of SMR range is time where the SMR may have moved. This would be a significant source of error if the SMR-mover had poor probing stability (this was not the case for the CMM used within this investigation). For this investigation, a CMM was used as a reference measuring system; hence, synchronisation between the CMM measurement and laser tracker measurements was critical.

The first step of measurement synchronisation would be to trigger the laser tracker range measurements from an external source: the CMM. When the CMM controller implements the pause in the probing routine, a signal could be sent to the laser tracking controller to trigger the measurement. This would be an improvement upon the procedure used here: each tracker measuring individually to detect a stationary target.

High-density measurement of the small flat optic would increase the accuracy of the measurement of σ_z by way of increasing the number of measured positions on the surface. Further measurements should have a minimum of 10×10 points on the surface. This would also provide further insight into the spatial distribution of measurement uncertainties, especially if the measurement is repeated many times.

If a larger — metre-scale — flat was able to be utilised, these high-density measurements could be applied over a larger surface area; thus, enabling investigation into the spatial dependence of measurement uncertainty over a larger area. Again, this investigation would be enhanced with many repeated measurements; however, the time for measurement of a sample should be noted to minimise the effect of time dependent parameters at this stage of the investigation.

Measurement of the sample surface with different positions and with different orientations (rotations about all three axis) throughout the measurement volume — including relative to any floating positions measured — would enable a rigorous investigation of the dependency of measurement uncertainty upon measured positions spatial distribution throughout the measurement volume. Where possible, all samples should be measured with at least 10 repeats with high spatial density.

Measurements of samples should be repeated with different arrangements of measuring stations to determine the sensitivity of z-coordinate multilateration measurement solution uncertainty to change in this parameter. Following these tests, it should be possible to run a ‘trial and error’ style iterative approach to optimise the positions of laser trackers, along with use of literature recommendations, in order to minimise measurement solution error. Further to this, the simulation of the measurement setup could be advanced by including automation of determining optimised positions for the measuring stations. This could be implemented by repeating the MC simulations with new measuring station positions moved relative to the previous positions along randomly generated vectors; if the measurement uncertainty estimation was lower, the station would be moved along this vector again and the MC simulation repeated; if the measurement uncertainty estimation was higher, a new random vector would be generated and the simulation repeated.

This optimisation should include minimisation of the range to reduce the effect of refractive index on range measurement error. If the investigation is to continue using the Leica AT960LR and SCE, work must be done to enable the measurement of the SCE range less than 1 m.

8.1.4 Time dependent error sources and long-term multilateration

The evaluation of measurement input error sources that are time dependent would be improved by: undertaking a long-term multilateration measurement within a stable environment (temperature, humidity, and ambient pressure) to provide a reference; undertaking a long-term multilateration measurement within a controlled environment (temperature, humidity, and ambient pressure) where these sources of error can be varied to test the sensitivity of z-coordinate measurement uncertainty; repeating the tests of stationary SMR range measurements in a stable environment; and a more rigorous analysis of the laser tracker home position drift — including a more accurate estimation of the effective coefficient of thermal expansion of the laser tracker (from base to home position).

By repeating the long-term multilateration measurement on a metre-scale surface within a nominally stable environment — ± 0.05 K over the measurement period is achievable temperature control — and also in a controlled environment where, for example, the temperature could be steadily changed over the measurement period, and undertaking a comparison of z-coordinate measurements, would make it possible to determine the magnitude of measurement solution sensitivity to temperature change. These experiments

would benefit from an increased spatial density on the sample surface: thus, increasing the temporal resolution of points measured on the surface.

Determination of accurate sensitivities of laser tracker range measurements to change in RI and home positions are critical to the accuracy of the mathematical model of a multilateration measurement setup and of any subsequent MC simulations. By repeating the stationary SMR (on post) measurements in an environmentally controlled space where, for example, temperature can be varied steadily over the measurement period to determine a more accurate magnitude of change in range measured per unit temperature change per unit length, $\Delta R/R/T$, as a result of change in RI only. Future experiments should increase the thickness of the SMR post and a wait, of a determined time, should be applied to ensure setup stability before measurements are recorded. These results can then be utilised to decouple the effect of RI from interferometric measurements of home position drift: provided sufficiently accurate temperature, pressure, and humidity measurements close to the laser beam-path.

Thank you for taking the time to read this thesis!

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Appendix A

Surface measurement probes

Section 2.3.5 describes systems and techniques for precision measurement of a probes position, which could be retrofit onto existing machines to improve total measurement accuracy. The measurement uncertainty of a system is also dependent on the probe used by a system. This section provides detail of precision probes; these fit into two categories, contact probes and non-contact probes.

A.1 Contact probes

Contact probes use a hard material as a tip on a stylus. ISO 25178-601:2010 defines a stylus as a mechanical device consisting of a tip and an arm. The tip is moved into a surface. When tactile contact between a surface and the stylus is made this generates a force, which cause the stylus to move relative to the probe transport system. A probe should be designed to detect even extremely small forces generated on a stylus tip in order to measure a surface precisely. These probes can be calibrated using the procedures outlined in ISO 25178-701:2010.

In 1992, McMurtry *et al.* produced a patented design for a contact probe head [115]. This probe is designed for co-ordinate measuring machines. The the system enables the calculation of the orientation of a stylus when it is in contact with a surface. The force provided by the interaction between surface and style tip. This for causes a tilt in the stylus relative to the probe head. This tilt causes electrical contact at a specific position within the probe head design. This position enables the calculation of the new orientation of the stylus. This knowledge is critical to calculating the point on the stylus tip which makes contact with the measurand surface.

ISO 10360-5:2010 defines the single stylus form error, P_{FTU} , as the error within which the range of radii can be calculated using a least-squares approach of measured points on a test sphere. This is the ISO parameter used to access the performance of a contact

probe.

The size of a stylus tip can affect the measurement ability of a probe. As discussed, spatial aliasing can occur when the measurement density of a system is low relative to the spatial wavelengths of features on a surface. Similarly, spatial aliasing can occur if a stylus tip is large relative to these feature. A large stylus tip cannot reach the smallest feature and therefore cannot measure them.

If a stylus tip is small relative to the spatial wavelength of interest, or the measurement aims to record a nominal surface such as a flat, the tip could ‘fall’ into a surface defect. This would result in a measurement error, the size of which would be dependent on the algorithm used to process the measurement data.

Contact measurement probes of this type are limited to surfaces with robust or unfinished surfaces. This is due to the potential for surface indentation or damage at the point of interaction between the surface of the measurand and the stylus tip.

A.2 Non-contact probes

ISO series ISO 25178 provides the classifications for and detail of non-contact probes. These probes use light to measure the distance to a surface. What will follow is a summary of each of these systems as defined by the ISO and detail of commercially available probes.

A.2.1 Phase-shifting interferometric microscopy

PSI microscopy is defined in ISO 25178-603:2010 as a method for surface topology measurement whereby an optical microscope with illumination of a known effective wavelength is integrated with an interferometric attachment and produces multiple successive optical images with interferometric fringes from which the profile or area surface topology image is calculated. The maximum spatial frequency that can be resolved by such a system is given by the Rayleigh criterion.

As detailed in section 2.4.2, PSI is used in the measurement of specularly reflecting surfaces. A phase detection system is employed to measure the interference of a single wavelength light beam in the case of homodyne techniques, and two variable wavelength light beams in heterodyne techniques.

Perhaps the simplest method of PSI is the Michelson interferometer which makes use of a homodyne technique. By analysing the interference pattern generated between a reference beam and the reflected light, it is possible to determine the shape of the surface relative to a know reference [116].

A.2.2 Coherence scanning interferometry

Coherence scanning interferometry (CSI) is defined in ISO 25178-604:2010 as a term that describes a group of methods for surface topology measurement. These methods involve the localisation of interference fringes during a scan over a surface to determine a surface topology map. A number of terms are used to describe this technique; these include, among others: Mirau correlation microscope, white light interferometry, vertical scanning interferometry, and rough surface profiler.

CSI makes use of the short coherence length of the light source which results in a modulation envelope of intensity variation. A coherence function decays in a predictable manner, allowing the measurement of absolute position as opposed to simply displacement [116].

A.2.3 Contact probes — radius correction methods

Geometric error correction in contact stylus tip probe measurement systems is a mature research area. Wozniak *et al.* have developed an envelop method for the determination of the corrected measurement point on a stylus tip [117]. A review of methods developed over the past 25 years is presented. These include normal vector methods such as: non-uniform rational basis spline (NURBS) best fit curve techniques; surface fitting techniques; recalculation of the measurement point by estimation of the measurement profile; least squares approaches; multi-cross product approaches; use of computer aided designs or nominal surface identification; force sensitive tips enabling position calculation; and electrically conductive tip position calculation. These methods as based on calculation of the position on the stylus tip that interacts with the surface can be calculated relative to the known centre of the tip itself. By measuring in many positions over a surface profile, these techniques are able to accurately predict the true position of the tip–surface interaction relative to the tip centre.

Appendix B

Surface characterisation

Whitehouse describes surface characterisation “to mean the breakdown of the surface geometry into basic components based usually on some functional requirement” [118]. Typically, these components consist of *roughness*, *waviness*, and *form*. Roughness describes a surface at the highest spatial frequencies; whilst waviness is a MSF phenomenon; and form describes the lowest spatial frequencies.

Roughness

As stated, surface roughness parameters describe a surface at high spatial frequencies. Its relevance to this project is due the relationship between roughness and surface specularity, as detailed in section 1.1.1.

ISO 4287:1998+A1:2009 defines the roughness profile as being “derived from the primary profile of a surface by suppressing the longwave component using the profile filter λ_c ”, where λ_c defines the boundary between roughness and waviness in terms of spatial frequency. It is noteworthy that this interpretation is only relevant for two-dimensional profile analysis of a surface.

ISO 25178-2 defines a set of parameters that describe areal surface texture. This is of specific relevance to applications where the three-dimensional areal surface topology is of importance. These parameters characterise the frequency, shape, and lay of surface features in both lateral directions on the surface.

Assessment of the roughness of a surface is not always critical, dependent on the stage of manufacture. As discussed in section 1.1.6, during the manufacturing process, polishing is used to reduce the roughness of a surface in order to produce specular reflections; therefore, during the grinding step of manufacture, the roughness is not a parameter that is required to be controlled.

Waviness

ISO 4287:1998+A1:2009 defines the waviness profile as derived by applying the filters λ_c and λ_f to the primary surface profile. λ_c suppresses the shortwave spatial frequencies, whilst λ_f suppresses the longwave spatial frequencies. This is done after removal of the nominal form of the surface profile using a best-fit least-squares method.

W_q is defined as the RMS deviation from the idealised form of a surface. It is calculated using

$$W_q = \sqrt{\frac{1}{N} \sum_{i=1}^N W_i^2}, \quad (\text{B.1})$$

where i is a data point and N is the total number of data points measured. W_i is the profile height at the data point i .

ISO 10110-8:2010 defines RMS waviness to be within the spatial wavelength bandwidth of 0.08 mm to 2.5 mm for W_q , if no spatial bandwidth is specified.

MSF surface errors can cause flaring of an image in an optical system. It is critical to understand and quantify this effect. Methods for the assessment of MSF image errors are developed in [119].

Form

ISO 4287:1998+A1:2009 defines P_t as the sum of the height of the largest peak on the substrate surface and the depth of the deepest valley measured on the substrate surface. Hence it is called Peak-to-Valley.

PVr is a parameter defined by C. J. Evans [120]. It combines the PV of a 36-term Zernike fit with the root mean squared (RMS) of the residual. It is proposed that this characterisation of a surface provides a more robust assessment of surface form accuracy than simply defining in terms of PV.

parameter that combines the PV of a 36-term Zernike fit and the root mean square of the residual. This provides automatic filtering, is insensitive to system resolution, and relates directly to imaging performance via the Marechal criterion.

P_q is defined as the RMS deviation from the idealised form of a surface. It is calculated using

$$P_q = \sqrt{\frac{1}{N} \sum_{i=1}^N P_i^2}, \quad (\text{B.2})$$

where i is a data point and N is the total number of data points measured. P_i is the profile height at the data point i .

It is important to be able to measure these parameters on a non-specular surface as form error can result in image degradation within the optical system. For a grinding system to correct this type of error it must first be measurable.

Angle and slopes

Local slope defines the change in height over a surface. The slope can be defined over a given spacial wavelength, resulting in the parameters $P\Delta_q$, $R\Delta_q$, and $W\Delta_q$. These parameters are defined in ISO 4287:1998+A1:2009.

B.0.1 Surface filtering

As previously stated, filtering a measurement data set allows a user to analyse specific spatial frequency bandwidths. Filtering can be divided into two categories: instrument aliasing and post processing.

Instrument aliasing is a consequence of an instruments measurement density on a surface. The Nyquist criterion states that, for a two-dimensional feature to be analyse, it must be measured at least twice along its profile; therefore, selection of the measurement density can be used to filter features which occur above a certain frequency threshold [121]. Conversely, the measurement density of some instruments is limited by characteristics such as stylus tip size. This is more commonly know as instrument aliasing and can even induce low frequencies by the folding of frequencies around the sampling frequency [121].

Primary profile/surface

ISO 3274:1996 defines the primary profile as the total profile after application of the short wavelength filter, λ_s . Similarly, a primary surface is defined in ISO 25178-2:2012 as the surface portion obtained when a surface portion is represented by a specific primary mathematical model with an associated nesting index. This model is generally obtained via the application of an S-filter.

S-filter

Figure B.1 shows the boundary that the S-filter acts from. This figure is taken directly from ISO 25178-2:2012. The S-filter removes the smallest lateral spatial wavelengths and as discussed results in the primary surface.

L-filter

Used for analysing form and roughness Application of the L-filter results in the removal of the large scale lateral components of the measurement data set. Figure B.1 shows the

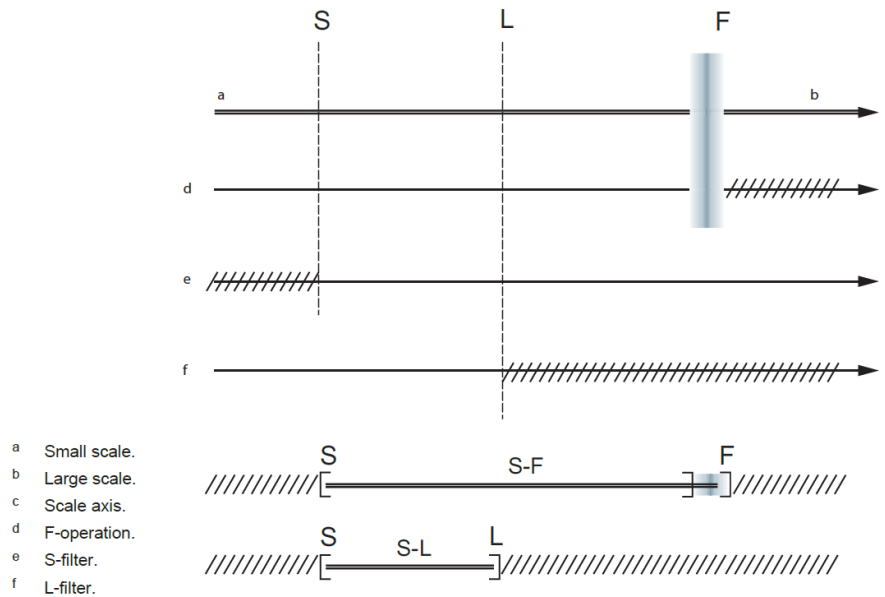


Figure B.1: Relationships between the S-filter, L-filter, F-operation and S-F and S-L surfaces as defined by ISO 25178-2:2012.

boundary that the L-filter acts from.

F-operation (ISO)

The F-operation results in the removal of form from the primary surface, as defined in ISO 25178-2:2012. This is not a filtration technique; however, it is detailed here as it removes a set of spatial frequencies from a data set. The operation is represented by a fuzzy line in figure B.1 as it does not produce a clear frequency cut-off as with filtration techniques.

S-F and S-L surfaces

The application of an F-operation to a primary surface results in a S-F surface where the analysis of MSF and roughness is critical. Similarity, by applying an L-filter to an S-F surface an S-L surface results. This enables the analysis of roughness alone. The relationship between these surfaces and the relevant filters and operations is shown in figure B.1.

It is noteworthy that there is no defined standard surface, within ISO 25178-2:2012, that isolates MSF for analysis.

B.0.2 Surface categories

There are several classifications of shape that optics can be categorised within. The shape of an optic is selected based upon the requirements of an optical system. The most simple optics have spherical shape and are therefore easily machined and measured. Some

systems require aspheric optics with a degree of symmetry, such as ellipsoidal — such as the E-ELT described in section 1.1.2 — or paraboloidal. Other systems require off-axis spaces. The most complex surfaces are freeform and can also be off-axis.

Spherical optics

A spherical optic has a constant radius of curvature (RoC). This shape of optic was common in early telescopes. Given a constant RoC, this type of optic can be machined without the use of computational numerical control (CNC) systems, which were unavailable during most of the 20th century.

Furthermore, this geometry is easily measured using interferometry after the optic has been polished to a specular surface. Metrology systems, such as that described in [94], use a reference surface to measure surface form error; spherical optics are measured with a spherical reference; therefore, one spherical reference can measure all spherical surfaces, provided the metrology system can accommodate the RoC.

Whilst spherical optics are the most easy to produce and measured, they do not enable the most precise telescope systems. Spherical aberrations occur as a spherical surface is not the ideal shape to focus light onto specific focal point. This phenomena is fully characterised in [122]. Furthermore, due to the nature of a spherical shape, this type of optic cannot be combined to create a large segmented mirror. This is not to say that freeform segments cannot be used to create a large spherical optic.

Asperics & Freeform

Aspheric optics, such as the elliptical primary mirror (M1) of the E-ELT [2], are constructed to reduce the optical error due to aberrations that affect spherical optics; however, aspheric geometries are complex to manufacture, adding time and cost to the production of the optic. Specifically, the metrology associated with aspheric surfaces is more complex relative to a more simple spherical surface geometry.

Interferometric surface metrology, which uses a reference surface to measure form error, cannot be universally applied to any aspheric surface. An elliptical reference can be used to measure any elliptical surface; however, an parabolic reference cannot be used to measure a elliptical surface, as clearly there is a form difference; therefore, in order to use an interferometric method, a different reference is required for each surface form. This is high in cost and increases the time to measure.

Alternatively, the form of of aspheric surfaces can be measured using a co-ordinate measurement machine (CMM). Whilst this method of measurement provides a high degree of versatility in terms of the surface shape that can be measured, there is a significant loss in measurement accuracy and resolution relative to interferometric methods. CMMs, and

several other alternative measurement methods are analysed and compared in chapter 2.

Freeform optics need not have any rotational axis of symmetry. Similar to aspheric optics, freeform optics require a CNC machine tool to machine this type of surface into a substrate material. It is conceivable that some freeform surfaces are more demanding, in terms of machine tool performance and capabilities, than the shape of an aspheric surface.

Interferometric measurement of freeform surfaces, similar to aspherics, requires different reference surfaces dependent on the form of the surface being measure; however, this is made more demanding by the fact that there are no sub-categories of freeform equivalent to those such as elliptical or parabolic aspherics. A new reference surface may be required for every freeform shape required to be measured. As stated, this is discussed further in chapter 2.

The advantages of using freeforms to construct telescope structures are numerous. As for aspherics, freeform geometries can be optimised to reduce aberrations; therefore fewer components are required in the optical system to compensate for this error; thus, reducing the overall mass and volume of the system. This type of optimisation is of particular importance to space telescopes, where mass and volume are critical to the design.

Freeforms can be manufactured to specification, such that they are segments of a large mirror. In this way, significantly larger mirror can be constructed than from a single monolithic substrate.

Surface analysis

Optics, such as those required for the construction of the E-ELT or JWST, have extremely demanding surface topology specifications across a range of spatial frequencies [2, 123]. In order for a manufacturer to produce optics to requirements, it is necessary to produce specifications for optical surfaces within clearly defined spatial frequency bandwidths.

For example L-3 SSG–Tinsley has fabricated JWST mirror segments to a set of RMS specifications for spatial frequency bandwidths. The results of the machining process are shown in table B.1.

Table B.1: The surface measurement results of JWST mirror fabrication program at L-3 SSG–Tinsley [123].

Spatial bandwidth (mm)	Specification (nm RMS)	Average performance (nm RMS)
> 222	20	12.3
222 to 0.08	7	6.1
< 0.08	4	3.2

Optic manufactures and metrologists can use parametrisation to produce the results shown in this table, enabling the construction a telescope such as JWST to the required parameters to meet performance requirements.

Appendix C

Non-contact probe experiment

A method for assessing measurement precision and stability of optical probes

C.1 Introduction

Non-contact optical probes, such as chromatic probes, have potential to replace contact probes for some applications [124]. The use of non-contact probes in existing contact probe based systems could decrease measurement time and uncertainty [124]. Some optical probing techniques are capable of measuring non-specular metre-scale surfaces – for instance segmented freeform optics post-grinding – in the mid- to low-spatial frequency bandwidths (S-filter). Current measurement techniques use contact probe based systems where measurement precision is a limiting factor. State-of-the-art contact probes have a measurement uncertainty of $0.25\text{ }\mu\text{m}$ (2σ) [57] due to errors including: the probe-surface interface position, flexibility in the probe, and thermal expansion of the probe. Equivalent non-contact optical probes claim accuracies up to an order of magnitude higher [125]; employing these probes could therefore improve current measurement systems.

In this chapter, two methods are presented: the first assesses the long term stability of a probe; whilst the second investigates the accuracy of a probe. These techniques contribute to the assessment of the viability of using chromatic probes in the measurement of large non-specular surfaces.

C.2 Measurement principle

ISO 25178-602:2010 defines a chromatic probe as a device that converts the height of a point on the surface of a measurand into a signal during measurement, using the confocal chromatic dispersion of a white light source. Confocal chromatic microscopy is a surface topology measurement method of a confocal microscope with a chromatic objective integrated with a detection device whereby the surface height at a single point is sensed by the wavelength of light reflected from the surface.

A chromatic confocal probe measures the distance to a surface using the principle of axial chromatic dispersion within a single point optical sensor [124, 126].

Figure C.1 shows how an axial chromatic dispersion lens can be used to split the constituent wavelengths of white light, thus realising spectral encoding. Each wavelength of the light source is focused at a different point along the axis of the lens; therefore, each focal position corresponds to a point within the measuring range. The measuring range is given by

$$\text{measuring range} = \text{focal distance}(\lambda_{\max}) - \text{focal distance}(\lambda_{\min}), \quad (\text{C.1})$$

where $\lambda_{\max}$ is the longest wavelength in the range, $\bar{\lambda}$, and $\lambda_{\min}$ is the shortest.

When an object is positioned within the measuring range, light is reflected back to the probe. The light that is focused at the specific position of the objects surface is reflected with the highest intensity. An optical fibre is utilised as a pin hole, such that only light focused at it's core is transmitted to the detector. In this way, the probe–fibre pair behaves as a confocal microscope [124, 126].

The transmitted light directed onto a detector for spectral decoding. The detector is a spectrometer type CCD. Given that the light reflected back up the fibre by the object surface consists of only a single wavelength, the spectrometer can be used to analyse and identify the wavelength; thus, the position of the measured surfaces within the measuring range is decoded [124, 126].

The characteristics of the probe described above, lead to their implementation as a non-contact optical alternative to a stylus probe, in surface topology metrology systems. Similar to stylus systems, the dimensional capability of such systems is merely dependent upon the precision stages the probe and measurand object are mounted on. Several case studies of chromatic probes implemented in surface topology systems can be found in [124].

To integrate a chromatic confocal probe into a (CMM), consideration must be taken to analyse how the characteristics of the probe affect the CMM's measurement uncertainty for surface topology. Specifically the spot diameter of the light focuses by the

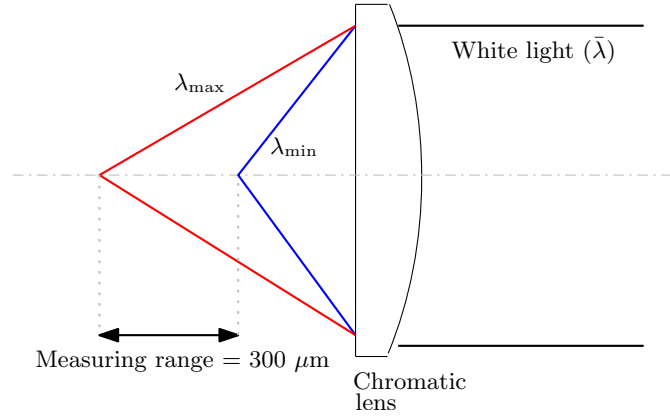


Figure C.1: A white light source with wavelength range, $\bar{\lambda}$, is directed into a chromatic lens. The light is dispersed axially to generate a unique focal point for each wavelength [124, 126].

chromatic lens, which determines the lateral resolution of the probe and thus contributes to the resolution of the CMM. Whilst the spot diameter differs for over the range of wavelength, a mean value can be derived from: the probe lens numerical aperture; the probe head magnification; the optical fibre core diameter; and the mean focal distance of the light [124].

Changes to the environment that the probe operates have an effect on measurement precision. Environmental factors such as temperature, humidity, and vibrations can have an effect upon the mechanical, electronic, and optical systems of a non-contact probe of this type.

Table C.1: High accuracy commercial non-contact chromatic probes and associated measurement parameters [125].

Probe (Measuring range)	Company	Accuracy (μm)	Resolution (μm)
CHRocodile SE (100 μm)	Precitec	$\sim 40 \times 10^{-3}$	1×10^{-5}
CHRocodile SE (300 μm)	Precitec	$\sim 100 \times 10^{-3}$	3×10^{-5}
CHRocodile SE (350 μm)	Precitec	$\sim 125 \times 10^{-3}$	3.5×10^{-5}

C.2.1 Error classification

All probes have an associated measurement uncertainty caused by errors in the measurement systems. A probe's purpose is to allow the recoding of a position on a surface relative to a given reference frame. Error caused by the probe-surface interaction can be categorised into four types [127] geometric, kinematic, stiffness, and thermal.

Geometric error occurs due to the error in the identification of the position of a specific point where the probe interacts with the measurand surface. Given that all probes, contact and non-contact, have a finite tip or spot size: there is a finite interaction

area between the probe and the surface; and an uncertainty in the position of the probe surface interaction over the probe area. This affects the system accuracy and repeatability.

The presence of kinematic error is an indication that the total measurement accuracy of a system is limited by the accuracy of the probe positioning system, independent of the probe accuracy. This error is an assessment of a systems ability to position a probe precisely within the measurement volume.

More, but not exclusively, relevant to contact probe systems; stiffness error occurs due to the finite rigidity of a measurement system. For instance, flexibility in a stylus results in a error in the position of the stylus tip when a measurement is recorded.

Changes in temperature cause non-linear expansion within a complex multi-material-type equipment such as a position measurement probe. Changes in the dimensions of a probe affect the repeatability of a probe measurement. Modelling such a non-linear phenomenon is not trivial, and is therefore a significant cause of error in a precision metrology probe.

There is currently a lack of standardisation for the calibration, testing, and analysis of the performance of non-contact distance measurement probes. The types of error that affect the measurement uncertainty of these probes can be divided into two categories: lateral and axial. Lateral errors occur in the measurement of position in the plane normal to the measurement axis of the probe. Axial position measurement error occurs in the axis of measurement.

In the case of a chromatic probe, axial error could be affected by the spatial amplitude and frequency of the measurand surface. Furthermore, the non-uniform amplitude of light frequencies generated by the source could also result in measurement uncertainty. Lateral error would be dependent of the diameter of the spot formed on the surface by the optics and light source used in the probe. The factors require investigating to quantify these affects. Nourira *et al.* present a method for the characterisation of error sources of chromatic probes [128]. This method uses a metrology frame CMM and a laser interferometer to analyse the effect of measurand: colour, material, roughness, axial and radial motion, tilt angle, and changing measurement angle.

C.3 Stability evaluation test

The time to measure a metre-scale freeform surface is significant. If an optical probe based system is to be used to measure such a surface, the probe's measurement stability should be analysed.

C.3.1 Method

A CHRcodile SE 300 μm probe was held in a fixed position by an aluminium probe holder with a measurand surface placed within the probe's measurement range. Figure C.2 shows the experimental set-up. The artefact was constructed from a machinable ceramic and its surface was non-specular. The measurement of position was recorded over eight hours.

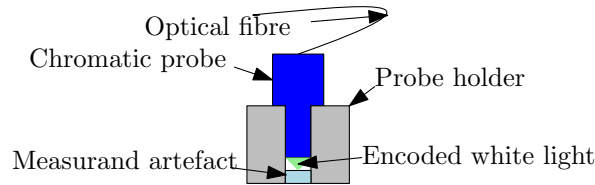


Figure C.2: A chromatic probe held by a probe holder to measure a constant distance to the artefact surface.

C.3.2 Results

Figure C.3 shows the position measurement of a CHRcodile SE 300 μm probe. This is shown with the temperature of the probe holder and the air and figure C.4 shows a fast Fourier transform (FFT) of the position measurement.

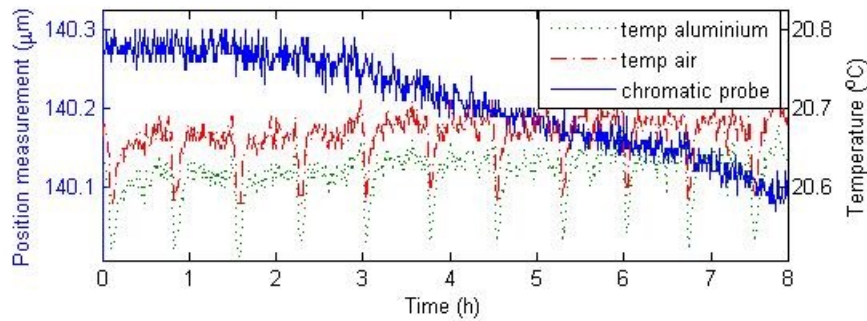


Figure C.3: The position measurement of a chromatic probe over an eight hour period, the temperature of an aluminium probe holder, and air temperature.

The probe was stable to $\pm 20 \text{ nm}$ during hours one and two; then changed linearly throughout the remaining measurement period by $200 \pm 20 \text{ nm}$. The FFT shows the presence of low frequency changes in the position measured. Investigation is required to determine the cause of these changes. The temperature increased over this time period. A low thermal expansion holder has been designed with multiple clamping positions to determine a thermally decoupled probe mounting position and insulate the holder from thermal effects.

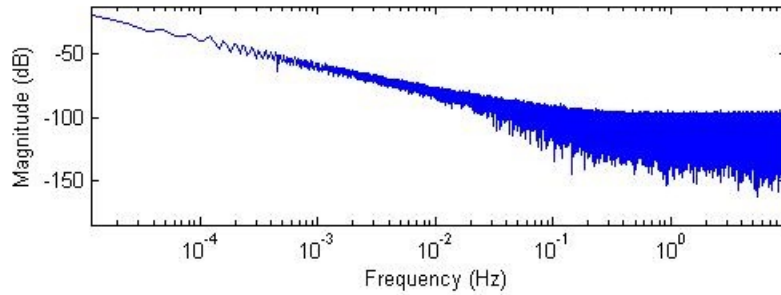


Figure C.4: A fast Fourier transform of the position measurement of a chromatic probe over an eight hour period.

C.4 Accuracy testing

In order to validate whether a probe has the capability to measure a metre-scale freeform surface, assessment of the probe's measurement accuracy is required.

C.4.1 Method

A CHRcodile SE 300 μm probe was held with orientation control and aligned normal to the measurand surface. Figure C.5 shows the experimental set-up. The probe was incrementally displaced along its measurement axis. Its position measurement was recorded and the deviation from the 'true' displacement calculated.

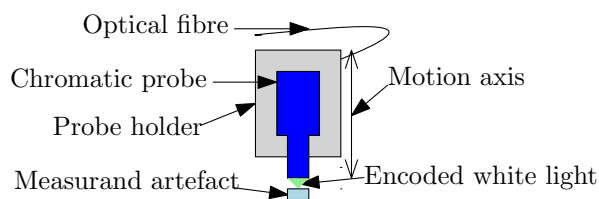


Figure C.5: A chromatic probe held by a probe holder with orientation control. The holder was set-up on a precision linear axis.

C.4.2 Results

Figure C.6 shows the difference between displacement measured by the probe and the 'true' displacement measured by a precision motion axis. As the probe was displaced through its measurement range from 10 μm to 15 μm the maximum deviation from true displacement was 36.4 ± 4.5 nm. From 290 μm to 285 μm , the maximum deviation was 84.6 ± 3.4 nm.

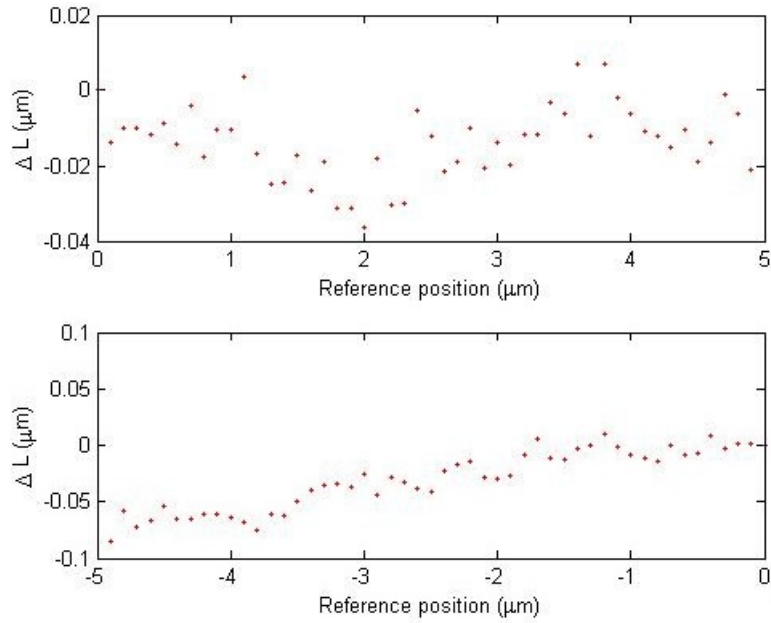


Figure C.6: . The difference between the displacement measured by the CHRcodile SE 300 μm and the true displacement as measured by a precision motion system (ΔL), moving from 10 μm to 15 μm (top) and 290 μm to 285 μm (bottom) as measured by the chromatic probe.

C.5 Conclusion

The stability of a chromatic probe was 200 ± 20 nm over an eight hour measurement period. The stability test should be thermally insulated from its environment for further investigation. Maximum deviation over a 5 μm displacement was 84.6 ± 3.4 nm. The entire measuring range should be assessed for further accuracy investigation.

Appendix D

Laser tracker error models

Error models for laser trackers with and without beam steering mirrors

Geometric error models for laser trackers are derived from an understanding of the systematic error sources of this measurement system. The models can be used to map and compensate these errors. There are two model types: error models for trackers with a beam steering mirror, and models for trackers without a beam steering mirror.

D.1 Laser trackers with a beam steering mirror

The geometric error model for trackers with a beam steering mirror is set out by Loser and Kyle [114]. The relevant error sources are summarised clearly within [59], these include among others: alignment offset parameters, alignment tilt parameters, encoder eccentricities and vertical index offset, birdbath distance, and ADM offset. These parameters coalesce to cause an error in the measurement of horizontal angle, vertical angle, and range. Equations (D.1) – (D.3) show how knowledge of these parameters can be used to correct measurements:

$$\begin{aligned} H_c = H_m + \frac{1}{\sin(V_m)} & \left[I_x \cos(H_m) - I_y \sin(H_m) \right. \\ & \left. + \frac{O1_x \cos(H_m) - O1_y \sin(H_m) + O2_x + H_{\text{off}}}{R_m} \right] - \frac{i \sin(V_m/2) + c}{\cos(V_m/2)} \\ & + E_y \sin(H_m) - E_x \cos(H_m), \end{aligned} \quad (\text{D.1})$$

$$\begin{aligned}
V_c = V_m - & \left[I_x \sin(H_m) + I_y \cos(H_m) \right] \\
& - \frac{O1_x \sin(H_m) + O1_y \cos(H_m) + O2_y + V_{\text{off}}}{R_m} \\
& - \cos\left(\frac{V_m}{2}\right) \left[\frac{2}{R_m} \left(e \cos\left(\frac{V_m}{2}\right) + f \right) + K_x \right] + K_y \sin\left(\frac{V_m}{2}\right), \quad (D.2)
\end{aligned}$$

and

$$R_c = R_m - 2 \sin\left(\frac{V_m}{2}\right) \left[e \cos\left(\frac{V_m}{2}\right) + f \right], \quad (D.3)$$

where H_m , V_m , and R_m are the measured horizontal angle, vertical angle, and range, respectively. H_c , V_c , and R_c are the corrected horizontal angle, vertical angle, and range, respectively. H_{off} and V_{off} arise from corrections applied to the point sensing device, which detects the laser light reflected back from the SMR, internally and thus provides and correction when the tracker is measuring a moving SMR. The error source parameters are detailed in Table D.1. This included whether the parameter's sensitivity to a two-face check. A two-face check is two measurements of a stationary target (SMR): one initial measurement of the target (H_m , V_m , and R_m) from the 'front face' of the tracker, and a second measurement of the target after the tracker is rotated 180 degrees horizontally and 180 degrees vertically to measure from the trackers 'back face'. The difference between the measurements from the two faces of the tracker is recorded. Repeating this process with the target in a number of positions within the tracker coordinate system, the magnitude of the error sources, detailed in the table as sensitive to this test, can be estimated. The same holds for the error model of tracker's without beam steering mirrors discussed in section D.2. It is noteworthy that, j , the vertical index offset, is applied directly to the measurement of the vertical angle, V_m , and therefore does not appear in equations (D.1) – (D.3). The sources of these errors are explained fully within [114].

D.2 Laser trackers without a beam steering mirror

The geometric error model for trackers without a beam steering mirror is set out by Muralikrishnan *et al.* within [129]. The error sources include the geometric misalignment parameters that arise from these laser tracker designs. As is the case for trackers with beam steering mirrors, the error model is used to correct coordinate measurement of targets. The model is described by

Table D.1: Details of parameters within the error model for trackers with a beam steering mirror [59, 114].

Parameter	Description	Two-face sensitive
<u>Alignment offset parameters</u>		
e	Transit axis offset	Yes
f	Mirror offset	No
$O1_x, O1_y$	Beam offset	No
$O2_x, O2_y$	Cover plate offset	Yes
<u>Alignment tilt parameters</u>		
c	Mirror tilt	Yes
i	Transit axis tilt	Yes
I_x, I_y	Beam axis tilt	No
<u>Encoder eccentricity and vertical index offset</u>		
E_x, E_y	Horizontal encoder eccentricity	Yes
K_x, K_y	Vertical encoder eccentricity	K_x is not, K_y is
j	Vertical index offset	Yes

$$\begin{aligned}
H_c = H_m + \frac{x_{1t}}{R_m \sin(V_m)} + \frac{x_{4t}}{\sin(V_m)} + \frac{x_5}{\tan(V_m)} + x_{6x} \cos(H_m) \\
- x_{6y} \sin(H_m) + x_{9a} \sin(2H_m) + x_{9b} \cos(2H_m) ,
\end{aligned} \tag{D.4}$$

$$\begin{aligned}
V_c = V_m - \frac{x_{1m}}{R_m} + \frac{x_2 \cos(V_m)}{R_m} + x_3 + x_{7n} \cos(V_m) - x_{7z} \sin(V_m) \\
+ x_{10a} \sin(2V_m) + x_{10b} \cos(2V_m) ,
\end{aligned} \tag{D.5}$$

and

$$R_c = R_m + x_2 \sin(V_m) + x_8 , \tag{D.6}$$

where the parameters denoted by x are defined in table D.2. The sources of these errors are explained fully within [129].

Table D.2: Details of parameters within the error model for trackers without a beam steering mirror [59, 129].

Parameter	Description	Two-face sensitive
x_{1t} and x_{1m}	Beam offset	Yes
x_2	Transit offset	Yes
x_3	Vertical index offset	Yes
x_{4t}	Beam tilt	Yes
x_5	Transit tilt	Yes
x_{6x} and x_{6y}	Horizontal angle encoder offset	Yes
x_{7n} and x_{7z}	Vertical angle encoder offset	x_{7n} is, x_{7z} is not
x_8	Bird-bath error	No
x_9 and x_{10}	Second order scale errors in the encoders	No

Appendix E

Standards, definitions, and evaluation

Measurements of the dimensions of an object, as for any measurement, is simply an estimate of the true dimensions of said object. It is therefore obligatory that the measured estimate has a quantitative indication of its quality; in other words, the measurement must include an uncertainty in the measured value. This allows measurement results to be compared to the nominal or prescribed dimensions of the sample. The general rules for evaluating and expressing measurement uncertainties are detailed within [103], titled *Uncertainty of measurement. Guide to the expression of uncertainty in measurement* (GUM).

E.1 Uncertainty in measurement

As stated, the GUM details the universal definitions [103], concepts, and specification for expressing uncertainty. What follows is a summary of relevant terms used in the assessment of uncertainty in coordinate measurement.

E.1.1 Definitions and concepts

Uncertainty (of measurement) A parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the sample.

Standard uncertainty Uncertainty in the form of the standard deviation of a set of measurement results.

Type A evaluation A method of evaluating uncertainty by the statistically analysing a series of measurements. Type A standard uncertainty is obtained from a probability density function derived from an *observed* frequency distribution.

Type B evaluation A method of evaluating uncertainty by any other means than statistically analysing a series of measurements. Type B standard uncertainty is obtained from an *assumed* probability density function based on the degree of belief that an event will occur.

Expanded uncertainty A quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.

Coverage factor A numerical factor used to relate the standard uncertainty to the expanded uncertainty.

Random error An error due to an unpredictable or stochastic temporal and spatial variations of effects that influence the measurement. It is not possible to compensate for the random error but it is possible for them to be reduced by increasing the number of observations.

Systematic error An error caused by a systematic effect. If this effect can be quantified and has significant size relative to the required accuracy of the measurement then a correction can be applied in order to error compensate to some level.

E.1.2 Evaluating standard uncertainty

Practically, a measurement is always subject to some uncertainty. This uncertainty can result from many different input parameters. In some cases these parameters are mutually independent, and in others they are not [103].

The GUM recommends that a model is formulated that accounts for the input parameters that influence a measurement. A measurement's dependence on a number of inputs, X_i ($i = 1, \dots, N$), can be expressed as a function:

$$Y = f(X_1, X_2, \dots, X_N), \quad (\text{E.1})$$

where Y denotes the measurand and X_i the input parameter quantities.

A probability density function (pdf) is used to describe what is known about a parameter that influences the measured result; the best estimate of the value of the parameter is given by the expectation value of the pdf; this knowledge can be inferred by repeated measurement of a parameter in a Type A assessment or using the available information to predict the possible variability in the quantity of the parameter in a Type B assessment.

Defining η and ξ as possible values of Y and X_i respectively, the pdf of Y is denoted by $g_Y(\eta)$; therefore the best estimate of Y is given by

$$y = \int_{-\infty}^{\infty} g_Y(\eta) \eta \, d\eta . \quad (\text{E.2})$$

The standard deviation of this pdf is the standard uncertainty of the estimate y ,

$$u(y) = \sqrt{\int_{-\infty}^{\infty} g_Y(\eta) (\eta - y)^2 \, d\eta} . \quad (\text{E.3})$$

A pdf can take the form of a variety of different distributions, these include: Gaussian, rectangular, triangular, trapezoidal, U-shaped, and t-distributions. For a Type B evaluation, if the only available information indicates that the value of a parameter lies in a known interval, it follows from the principle of maximum entropy that the parameter is distributed uniformly over the interval, a rectangular distribution. This is the most common distribution for an evaluation of this type; however, for a system with multiple inputs, each input parameter may have a unique distribution.

E.1.3 Combined standard uncertainty

The combined standard uncertainty, $u_c(y)$, is defined by the GUM as an “estimated standard deviation associated with the output estimate or measurement result y .” For the case where the input quantities, X_i , are uncorrelated, the combined variance is given by

$$u_c^2(y) = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) , \quad (\text{E.4})$$

where x_i is an estimate of the input, X_i , f is the function defined in equation (E.1), and $u(x_i)$ is the standard uncertainty of the input. The partial derivative terms, $\partial f / \partial x_i$, are known as the *sensitivity coefficients*. This describes the sensitivity of output, y , to changes in the input estimates, x_i . For a small change on the input, Δx_i , the change in y is given by $(\Delta y)_i = (\partial f / \partial x_i) \Delta x_i$. If $\Delta x_i = u(x_i)$ then $u_i(y) \equiv (\partial f / \partial x_i) u(x_i)$ and the combined uncertainty can be written as

$$u_c^2(y) = \sum_{i=1}^N u_i^2(y) . \quad (\text{E.5})$$

If some of the input quantities, X_i , are correlated then the combined uncertainty must take this into account. In this case, the standard combined variance associated with a measurement result is expressed as

$$u_c^2(y) = \sum_{i=1}^N \sum_{j=1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) , \quad (\text{E.6})$$

where $u(x_i, x_j)$ is the estimated covariance associated with x_i and x_j . The degree to which these two inputs are correlated is characterised by the *correlation coefficient* given by

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i)u(x_j)} , \quad (\text{E.7})$$

where $-1 \leq r(x_i, x_j) \leq 1$. If x_i and x_j are uncorrelated then $r(x_i, x_j) = 0$. It is therefore more simple to interpret equation (E.6) by replacing the covariance term with

$$2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i)u(x_j)r(x_i, x_j) .$$

E.1.4 Expanded uncertainty

As defined in section E.1.1, the expanded uncertainty (U) covers a large fraction of the interval described by the distribution of measurements. It is given by

$$U = k u_c(y) , \quad (\text{E.8})$$

where k is a coverage factor. From this the result of a measurement can be reported as

$$Y = y \pm U = y \pm k u_c(y) . \quad (\text{E.9})$$

E.2 Estimation and expression of system uncertainty

ISO 14253 In terms of geometric product specification (i.e. coordinate measurement) the expanded uncertainty is

$$U = k \times u_c \rightarrow k = 2 , \quad (\text{E.10})$$

unless otherwise stated.

The *maximum permissible error* (MPE) of a measurement result determined by a

measurement system is given by

$$MPE = \frac{u_{xx}}{b}, \quad (\text{E.11})$$

where u_{xx} is the standard deviation of uncertainty component xx and b is a parameter dependent on the pdf of u_{xx} . The value of b is determined by the rules set out in ISO 14253: if y follows a Gaussian distribution, $b = 0.5$; if y follows a uniform distribution, $b = 0.6$; and if y follows a U-shaped distribution, $b = 0.7$. Figure E.1 shows illustrates the shape of these distributions [130].

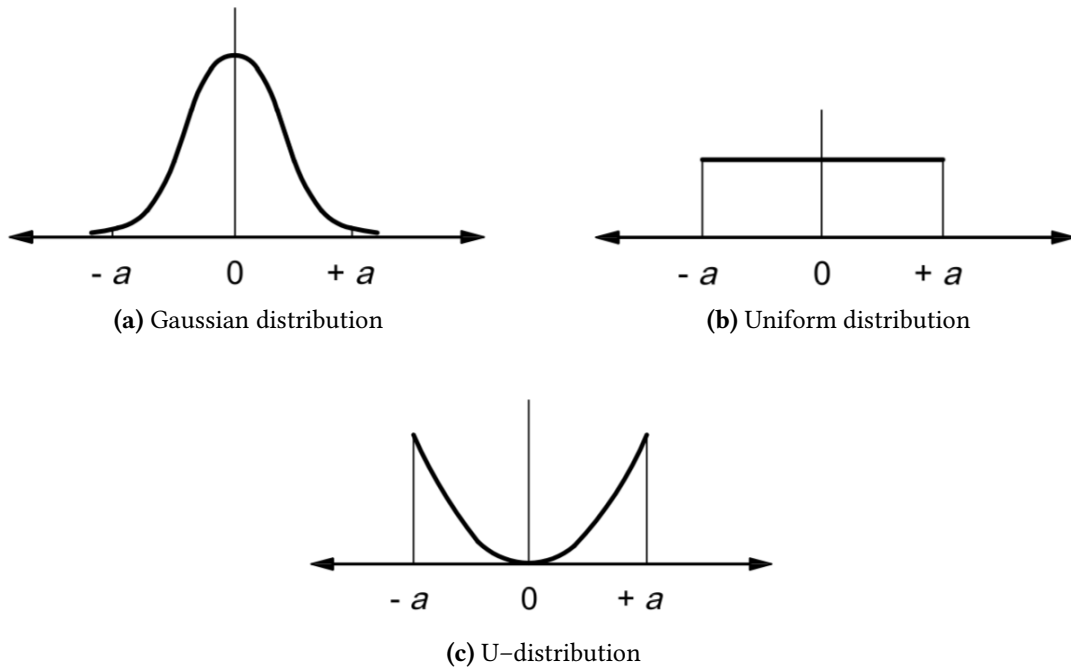


Figure E.1: Three example probability distributions. These are used for transforming limits of variation, a , into uncertainty components, u_{xx} [130].

E.3 Estimation of task specific uncertainty

ISO 15530 ISO 15530 [131] details techniques for determining task specific measurement uncertainty for CMM. These enable the evaluation of the measurement uncertainty for a specific measurement task for a specific measurement system based on error sources, such as: sampling strategy, environmental effects, and the influence of the measuring system. Assessment can be undertaken using: sensitivity analysis, calibrated workpieces (ISO 15530-3) or standards, or using computer simulation by employing a mathematical model of a measurement process (ISO 15530-4).

Task-specific measurement uncertainty (TSMU) is the expanded uncertainty, using

$k = 2$, of a specific measurement result. It is noteworthy that this uncertainty takes into account all uncertainty sources associated with the details of the measurement process including: CMM, probing system, sampling strategy, workpiece location and orientation, fixturing, contamination, and thermal environment. Furthermore, it is expected that the different parameters of a feature have different uncertainties associated with them. Influences on task specific uncertainty can be categorised as: instrumentation factors, measurement plan factors, or extrinsic factors. In order to evaluate these uncertainty sources, several different techniques may be required. The sources are combined using the law of propagation of uncertainty, which yields the combined standard uncertainty, and then multiplied by k gives the expanded uncertainty. The uncertainty budget lists: uncertainty sources, source combination, and an expression of expanded uncertainty.

Experimental validation of the uncertainties predicted by two different commercial software packages that aim to predict the TSMU of CMM measurement results is reported in [132]. Further use of the ISO 15530 in uncertainty estimation of CMM can be found in [133].

E.3.1 Type A evaluation

ISO 15530-3 [134] describes the principle, procedure, measuring equipment, and execution of CMM uncertainty analysis using calibrated workpieces. A master calibrated workpiece can be used to evaluate the instrumentation, measurement plan, and many extrinsic uncertainty sources. These master workpieces (artefacts) can be expensive and only allow for limited uncertainty assessment due to their specific shapes; however, where an artefact of a similar shape to the sample is available, this uncertainty assessment method allows for a precise uncertainty estimation.

E.3.2 Type B evaluation

ISO 15530-4 [36] describes how a CMM uncertainty assessment can be undertaken by identifying measurement uncertainty source quantities and their distributions quantified by properties, such as standard deviation. These parameters can be propagated through a mathematical model of the measurement system to simulate a measurement result. A Monte Carlo style generation of many input quantities can be used to determine an estimate for the uncertainty and distribution of many simulated system measurement results. This type of analysis is only as valid as the mathematical model used in the system simulation. Reducing the input uncertainties to a single output measurement uncertainty is a useful tool.

The Principle of the Propagation of Distributions The pdf for a measurand is obtained by “combining all possible values of the input quantities through the model according to their distributions” [135]. This process constitutes the principle and is defined by

$$g_Y(\eta) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} g_{X_1, \dots, X_N}(\xi_1, \dots, \xi_N) \delta(\eta - f(\xi_1, \dots, \xi_N)) d\xi_1 \dots d\xi_N, \quad (\text{E.12})$$

where $g_{X_1, \dots, X_N}(\xi_1, \dots, \xi_N)$ is the combined pdf of the inputs parameters.

Monte Carlo method Monte Carlo methods (MCM) can be applied to propagate the distributions discussed above in order to evaluate the uncertainty of a measurement, thus solving equation (E.12) [103, 131, 135–137]. MCM use a random combination of possible input values to allow the propagation of input distributions. Using multiple combinations it is possible to determine an empirical estimate of the pdf for the measurand, Y .

The MCM generates values for $\xi_1, \dots, \xi_N$ at random from the input pdf's. These are generated by

$$\xi_i = G_{X_i}^{-1}(\rho_i), \quad (i = 1, \dots, N), \quad (\text{E.13})$$

where ρ_i is a random value from a uniform distribution within the range $[0, 1]$ and G_{X_i} is the distribution function of X_i . The generated values can then be input into the model to output values of Y :

$$\eta = f(\xi_1, \dots, \xi_N). \quad (\text{E.14})$$

Repeating this random selection multiple, n , times produces multiple estimates of the measurand, Y . These estimates can be formed into a histogram that is an estimate of $g_Y(\eta)$.

E.4 Sources of error that affect a CMM measurement

ISO 23165 [138] explains the sources of error that affect a CMM measurement, these include: the calibration procedure, thermal expansion of the CMM or sample, temperature measurement error for thermal compensation, uncertainty in the alignment of the sample with the CMM co-ordinate system, and fixturing.

Errors that are specific to detailed measurement of metre-scale non-specular optics with a scanning technique include: thermal expansion over a multi-hour measurement, therefore increasing the uncertainty further; the friction between the stylus and the

optical surfaces the stylus is scanned across the surface may cause vibrations in the stylus head, which can cause error in position measurement; if a CMM measures in touch probing mode, whilst time dependent errors may be reduced by measurement speed, the low density of measurement data can result in aliasing.

E.5 Assessment of CMM performance and the standard for performance expression

ISO 10360 The standard practise for the assessment of CMM performance is detailed within ISO 10360 [139]. The measurement accuracy of a CMM is quantified with the parameter know as maximum permissible error. MPE should be provided by the machine manufacture and take one of the following three forms for size measurement:

$$E_{L,MPE} = \pm \text{minimum of } \left(A + \frac{L}{K} \right) \text{ and } B, \quad (\text{E.15})$$

$$E_{L,MPE} = \pm \left(A + \frac{L}{K} \right), \quad (\text{E.16})$$

$$E_{L,MPE} = \pm B, \quad (\text{E.17})$$

where A is a positive constant provided by the manufacture given in microns, K is a dimensionless positive constant provided by the manufacture, L is the measured size given in millimetres, and B is the maximum permissible error in microns as provided by the manufacture. The ISO details a test that can be implemented by the CMM user to check the performance against the manufacture provided $E_{L,MPE}$.

It is noteworthy that a different parameter is required to express the measurement error of a CMM when used in scanning mode. ISO 10360-4:2000 [140] details the error of indication in terms of the maximum permissible scanning probing error, MPE_{Tij} . This is valid for a specified maximum permissible time for scanning test, MPT_{τ} . These values are provided by the manufacture and testable as defined in the ISO. Validation of CMM to ISO 10360 has been undertaken in [141, 142].

Appendix F

Multilateration function — implemented in MATLAB[®]

A MATLAB[®] function for the multilateration function as described in equation (4.1), specifically for a four measuring station ($m = 4$) setup.

```
1 function zero = pos4lt_vect_realdatal(XYZ)
2
3 global d1_abs
4 global d2_abs
5 global d3_abs
6 global d4_abs
7
8 global d1_abs_px
9 global d2_abs_px
10 global d3_abs_px
11 global d4_abs_px
12
13
14 global d1_abs_pxy
15 global d2_abs_pxy
16 global d3_abs_pxy
17 global d4_abs_pxy
18
19 global d1_abs_zero
20 global d2_abs_zero
21 global d3_abs_zero
22 global d4_abs_zero
23
24
25 M = 4;
```



```

26 % 200rev plus pexp
27 % MP = 60-3+4;
28
29 % 200
30 % MP = 60-3;
31
32 % 400
33 MP = 125-3;
34
35 % 400 long
36 % MP = 34+25 - 3;
37
38 % 400rrr
39 % MP = 99-3;
40
41 % 400 sce seq
42 % MP = 100 - 3;
43
44
45 x = XYZ(1:MP);
46 y = XYZ(MP+1:2*MP);
47 z = XYZ(2*MP+1:3*MP);
48
49 x01 = XYZ(3*MP+1);
50 x02 = XYZ(3*MP+2);
51 x03 = XYZ(3*MP+3);
52 x04 = XYZ(3*MP+4);
53
54 y01 = XYZ(3*MP+M+1);
55 y02 = XYZ(3*MP+M+2);
56 y03 = XYZ(3*MP+M+3);
57 y04 = XYZ(3*MP+M+4);
58
59 z01 = XYZ(3*MP+2*M+1);
60 z02 = XYZ(3*MP+2*M+2);
61 z03 = XYZ(3*MP+2*M+3);
62 z04 = XYZ(3*MP+2*M+4);
63
64 x_px = XYZ(3*MP+2*M+5);
65 x_pxy = XYZ(3*MP+2*M+6);
66 y_pxy = XYZ(3*MP+2*M+7);
67
68
69
70 zero1 = ((x - x01).^2 + (y - y01).^2 + (z - z01).^2).^0.5 - ...

```

```
(d1_abs);
71
72 zero2 = ((x - x02).^2 + (y - y02).^2 + (z - z02).^2).^0.5 - ...
    (d2_abs);
73
74 zero3 = ((x - x03).^2 + (y - y03).^2 + (z - z03).^2).^0.5 - ...
    (d3_abs);
75
76 zero4 = ((x - x04).^2 + (y - y04).^2 + (z - z04).^2).^0.5 - ...
    (d4_abs);
77
78 zero5 = ((x_px - x01).^2 + (y01).^2 + (z01).^2).^0.5 - (d1_abs_px);
79
80 zero6 = ((x_px - x02).^2 + (y02).^2 + (z02).^2).^0.5 - (d2_abs_px);
81
82 zero7 = ((x_px - x03).^2 + (y03).^2 + (z03).^2).^0.5 - (d3_abs_px);
83
84 zero8 = ((x_px - x04).^2 + (y04).^2 + (z04).^2).^0.5 - (d4_abs_px);
85
86 zero9 = ((x_pxy - x01).^2 + (y_pxy - y01).^2 + (z01).^2).^0.5 - ...
    (d1_abs_pxy);
87
88 zero10 = ((x_pxy - x02).^2 + (y_pxy - y02).^2 + (z02).^2).^0.5 - ...
    (d2_abs_pxy);
89
90 zero11 = ((x_pxy - x03).^2 + (y_pxy - y03).^2 + (z03).^2).^0.5 - ...
    (d3_abs_pxy);
91
92 zero12 = ((x_pxy - x04).^2 + (y_pxy - y04).^2 + (z04).^2).^0.5 - ...
    (d4_abs_pxy);
93
94 zero13 = ((x01).^2 + (y01).^2 + (z01).^2).^0.5 - (d1_abs_zero);
95
96 zero14 = ((x02).^2 + (y02).^2 + (z02).^2).^0.5 - (d2_abs_zero);
97
98 zero15 = ((x03).^2 + (y03).^2 + (z03).^2).^0.5 - (d3_abs_zero);
99
100 zero16 = ((x04).^2 + (y04).^2 + (z04).^2).^0.5 - (d4_abs_zero);
101
102
103
104 zero = [zero1, zero2, zero3, zero4, zero5, zero6, zero7, zero8, ...
    zero9, zero10, zero11, zero12, zero13, zero14, zero15, zero16];
```

Appendix G

Calibration information

Below is the calibration certificate for the laser tracker Leica AT960LR that was utilised in the testing of long-term measurement uncertainty sources. The certificate certifies that, at the time the experimentation was carried out, the laser tracker was compliant with manufacturer specifications.

No up-to-date calibration information is available for the trackers and CMM utilised in the multilateration experiments; however, as the typical calibration testing carried out by the manufacturer does not include direct testing of the displacement measuring interferometer, this information is not of significant interest to this research. Whilst the CMM was used as a comparison measurement technique and up-to-date verification of its measurement capability would be of interest to this research, confidence in the measurements performed by the CMM come in the for of the results of surface form measurements of the optics used in this research. Similarly, the results of the form measurements using multilateration provide confidence in the interferometric laser tracker measurements.



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-  WHITE LIGHT SCANNERS
-  METROLOGY SOFTWARE SOLUTIONS
-  CAD / CAM
-  STATISTICAL PROCESS CONTROL
-  AUTOMATED APPLICATIONS
-  MICROMETERS, CALIPERS AND GAUGES



HEXAGON
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Calibration Certificate Metrology

Calibration Certificate Metrology with measurement values issued by Manufacturer

Product : Leica Absolute Tracker AT960-LR
Article No.: 576863
Serial No. : 750796

Certificate No. : 750796-19092017
Inspection Date: September 19, 2017
P.O. Number :

Customer : Hexagon Manufacturing Intelligence

Certification Location:
Milton Keynes

Status : After Inspection

Compliance

The Calibration Certificate Metrology with measurement values issued by Manufacturer corresponds to the Customer Certificate Metrology in accordance with DIN 55 350 Part 18-4.2.2.

Certificate :

We hereby certify that the product described has been tested with the following result:

☒ **Compliance** The test results are within the specification of the product.

The test equipment used is traceable to national standards or to recognized procedures.
This is established by our Quality Management System, audited and certified to ISO 9001 by an independent national accreditation authority.

Hexagon Manufacturing Intelligence,
September 19, 2017

Service Engineer

Certificate No. 750796-19092017

Art. No. 5003909

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United Kingdom
www.hexagonmi.com



Specifications

a) Measurement Uncertainty of a spatial length (MPE) of 2500 mm observed at a distance of:	2.0 m :	≤	± 0.033 mm
	20.0 m :	≤	± 0.190 mm
b) Measurement Uncertainty (MPE) of an ADM measurement:		≤	± 0.010 mm
c) Repeatability of an ADM measurement throughout the entire working range (Expanded Repeatability (k=2)):		≤	± 0.005 mm
d) Embedded Meteostation Temperature:		≤	± 0.3 °C
e) Embedded Meteostation Pressure:		≤	± 1.0 hPa
f) Embedded Meteostation Relative Humidity:		≤	± 5.0 %

Test Results

a) Maximum observed deviation of measurements at the spatial distance of:	2.0 m :	-0.022 mm
	20.0m :	-0.069 mm
b) Maximum observed deviation of all performed ADM measurements compared to built-in interferometer:		-0.009 mm
c) Maximum observed Repeatability of ADM:		0.003 mm
d) Maximum observed deviation of Temperature:		0.0°C
e) Maximum observed deviation of Pressure:		0.4 hPa
f) Maximum observed deviation of Relative Humidity:		-2.0 %

Calibration Certificate Metrology with measurement values issued by Manufacturer

Measurement Report



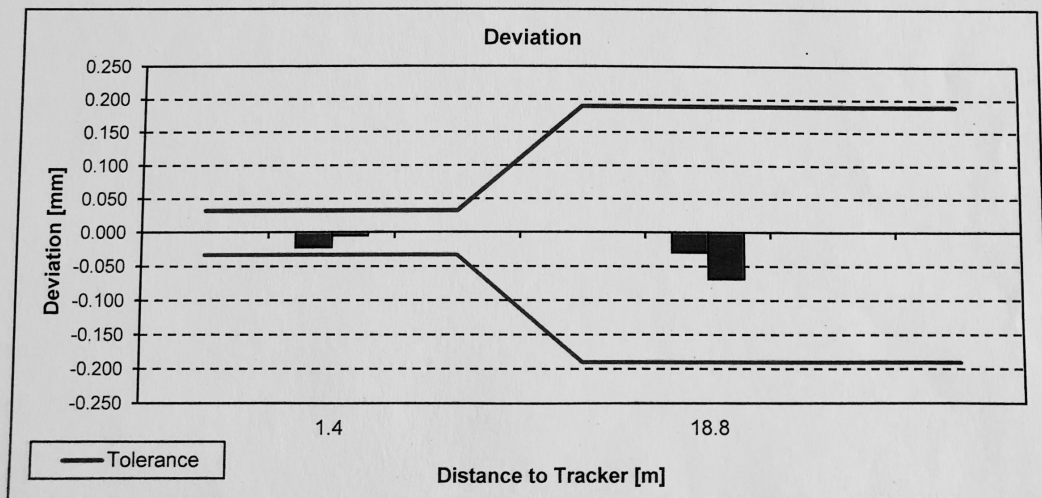
a) Spatial Length Measurement (Scale Bar)

Inspection Date: September 19, 2017
Inspected by: J. Harris

Product: Leica Absolute Tracker AT960-LR
Serial No.: 750796

Temperature: 20.7 hPa
Pressure: 1014.6 hPa
Humidity: 52.9 %
Reflector Serial No.: 16634

Reference distance [mm]:					2549.991 mm			
Temperatures of scale bar:					@ Reference measurements:	19.9°C		
					@ Scale Bar measurements:	20.5°C		
Distance to System	Ref.-Dist corrected	Measured Distances		Repeatability	Measured - Reference		Max. Dev.	Tolerance
		Run 1	Run 2	$\Delta R1 - R2$	Run 1	Run 2	Run 1/2	
[m]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]
1.4	2549.993	2549.971	2549.988	0.017	-0.022	-0.005	-0.022	± 0.033
18.8	2549.993	2549.963	2549.924	0.040	-0.030	-0.069	-0.069	± 0.190



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Measurement Report

c) Distance Measurement (ADM)

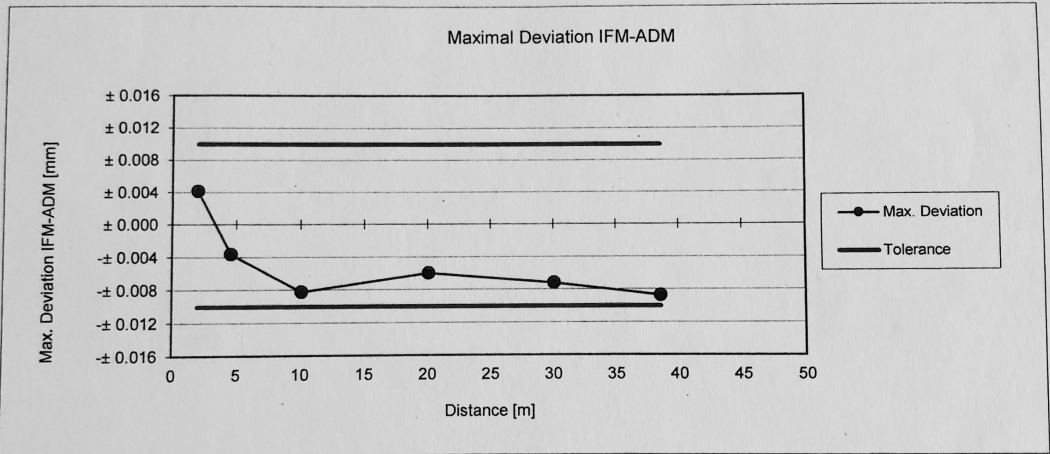
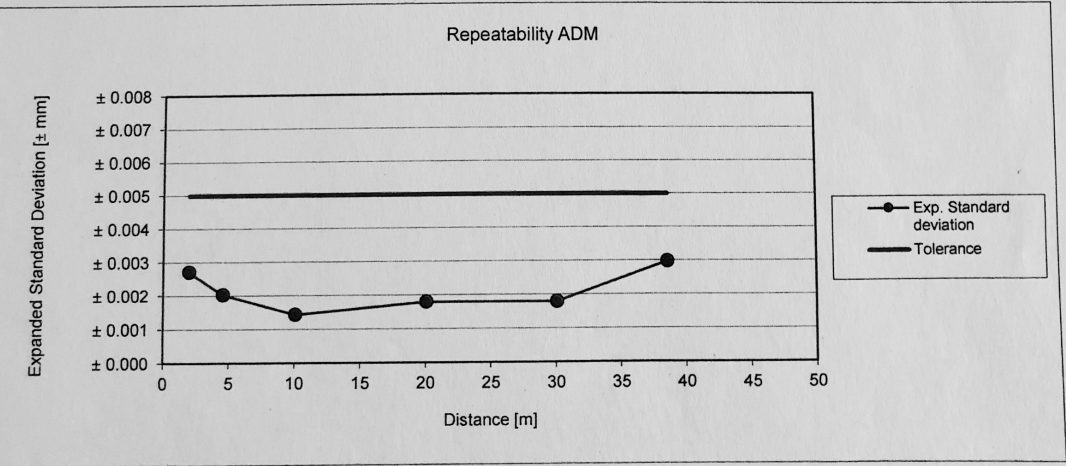
Inspection Date: September 19, 2017
Inspected by: J. Harris

Temperature: 20.7 °C
Pressure: 1014.6 hPa
Humidity: 52.9 %
Reflector Serial No.: 16634

Product: Leica Absolute Tracker AT960-LR
Serial No: 750796



Distance approximate	Max. Deviation IFM-ADM	Standard Deviation Repeatability	Expanded Standard Deviation (k=2)
[m]	[mm]	[mm]	[mm]
2.0	0.0042	± 0.0014	± 0.0027
4.6	-0.0036	± 0.0010	± 0.0020
10.1	-0.0082	± 0.0007	± 0.0014
20.1	-0.0060	± 0.0009	± 0.0018
30.1	-0.0072	± 0.0009	± 0.0018
38.6	-0.0087	± 0.0015	± 0.0030



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Measurement Report

d) Embedded Meteostation (Temperature / Pressure / Humidity)



Inspection Date: September 19, 2017
Inspected by: J. Harris

Product: AT Controller
Serial No: 750796
Ext. Temp. Sensor Serial No: 2837FA5A0600007F

Results

	Reference value	Actual value	Deviation	Verdict
Temperature Air	20.7 °C	20.7 °C	0.0 °C	passed
Temperature Object	20.7 °C	20.9 °C	0.2 °C	passed
Pressure	1014.6 hPa	1015.0 hPa	0.4 hPa	passed
Relative Humidity	52.9 %	50.9 %	-2.0 %	passed

Note: Accuracy of Air Temperature and Relative Humidity is valid with connected external Air temperature sensor only.

Appendix H

Publications

J. Norman, X. Tonnellier, P. Morantz. A method for assessing measurement precision and stability of optical probes. *euspen's 15th International Conference & Exhibition, Nottingham, UK*, 2015.

J. Norman, X. Tonnellier, P. Morantz. A laser based multilateration system for measurement of low-slope surfaces. *euspen's 17th International Conference & Exhibition, Hannover, DE*, 2017.

J. Norman, C. Giusca, X. Tonnellier. Assessment of a laser-based multilateration system for measurement of low-slope metre-scale surfaces. *Laser Metrology and Machine Performance XIII, Sheffield, UK*, 2019.