



Richard Welsh

Multistatic-Polarimetric SAR for Sparsely Sampled 3D Imaging

School of Defence & Security, Centre for Electronic Warfare,
Information & Cyber
PhD in Multistatic SAR Remote Sensing

Submitted for the award of PhD.

Academic Year: 2021- 2025

Supervisor: Dr. Daniel Andre

Associate Supervisor: Professor Alessio Balleri

March 2025



School of Defence & Security
PhD in Multistatic SAR Remote Sensing

Thesis submitted for the award of a PhD
Academic Year 2021 - 2025

Richard Welsh

Multistatic-Polarimetric SAR for Sparsely Sampled 3D Imaging

Supervisor: Dr. Daniel Andre
Associate Supervisor: Professor Alessio Balleri
March 2025

This thesis is submitted in partial fulfilment of the requirements for
the degree of PhD

© Cranfield University 2025. All rights reserved. No part of this publication may be reproduced without the written permission of the copyright owner.

ABSTRACT

Synthetic Aperture Radar (SAR) provides fine-resolution imagery in all weather conditions during both day and night. SAR can provide 3D imaging capability, enhancing image interpretability and target recognition. Producing sufficient scatterer resolvability and coverage in the vertical imaging direction can be challenging due to the often-impractical data collection requirements of 3D SAR imaging. This thesis presents a novel algorithm for exploiting multistatic sensing opportunities to reduce the sampling requirements of forming high-quality 3D SAR imagery, which is named the SSARVI algorithm. An extension of the SSARVI algorithm to polarimetric SAR analyses is also presented, named the PolSSARVI algorithm. The development of both algorithms addresses limitations in the literature of applying sparse 3D SAR approaches to multistatic SAR geometries. Both simulated and experimental data are used throughout this thesis to validate and optimise the performance of the algorithms developed. Their performance is compared to back projection (BPA)-formed imagery at Nyquist sampling in elevation. The results are presented across three distinct publication chapters.

Keywords: Bistatic, Near-field, Volumetric Processing, Interferometry, Layover, Nyquist Sampling.

ACKNOWLEDGEMENTS

I would like to primarily thank my supervisor, Dr. Daniel Andre, who has been the source of many great technical discussions and was supportive of my scientific development throughout this PhD journey.

...Dstl for their funding of this project.

...the CDS Doctoral Community staff of Mandy Smith, Bea Kingdon and Clare Knock for their unwavering support of research students, especially after the Covid-19 pandemic.

...my family for their support and patience with me during my undertaking of this PhD.

TABLE OF CONTENTS

ABSTRACT	i
ACKNOWLEDGEMENTS.....	iii
LIST OF FIGURES.....	viii
LIST OF TABLES	xiv
LIST OF EQUATIONS.....	xv
LIST OF ABBREVIATIONS	xviii
1 Introduction.....	1
1.1 Motivation	3
1.2 Aims and Objectives	5
1.3 Contribution	6
1.4 Overview of Thesis Structure.....	6
1.4.1 Related Published Works	7
1.4.2 Statement of Originality	7
2 Background	9
2.1 SAR Image Formation	9
2.1.1 SAR Resolution and Coverage	13
2.1.2 Image Formation Algorithms	14
2.1.3 Near-field vs Far-field SAR image formation	15
2.2 Bistatic and Multistatic SAR.....	18
2.2.1 Geometry and Signal Model.....	18
2.2.2 Effects on Imaging Performance	22
2.3 3D SAR.....	26
2.3.1 3D SAR Image Formation	28
2.3.2 SAR Interferometry.....	30
2.3.3 SAR Point Cloud Generation System.....	31
2.4 SAR Polarimetry	32
2.4.1 Overview	32
2.4.2 Development of Bistatic Polarimetric Parameters	35
2.4.3 Example Renderings	39
2.5 Ground Based SAR Laboratory	43
2.6 Summary of Chapter.....	45
3 Volumetric SAR Interferometry for Sparse 3D SAR with Bistatic Geometries	47
3.1 Abstract.....	47
3.2 Introduction	47
3.3 Outline of Approach	48
3.4 Simulations	50
3.5 Measured Data Validation.....	53
3.6 Conclusions	59

4 Laboratory Multistatic 3D SAR with Polarimetry and Sparse Aperture Sampling	61
4.1 Abstract.....	61
4.2 Introduction	61
4.3 Sparse aperture 3D point-cloud algorithm	63
4.4 Laboratory measurements	67
4.5 Simulations	70
4.5.1 Introduction	70
4.5.2 Multistatic Summation Approaches	70
4.5.3 Trajectory Clustering Effects	72
4.6 Full aperture 3D SAR image results.....	75
4.6.1 Multistatic data summation.....	75
4.7 Sparse aperture 3D point-cloud results.....	78
4.7.1 Coherent Multistatic Summation	82
4.7.2 Non-coherent Multistatic Summation.....	85
4.7.3 Non-coherent Combined Bistatic.....	86
4.7.4 Point-Cloud Properties Summary and Polarimetric Results	88
4.8 Conclusion	89
5 Polarimetry for Sparse Multistatic 3D SAR.....	92
5.1 Abstract.....	92
5.2 Introduction	92
5.3 Development of the PoISSARVI Algorithm	95
5.4 Bistatic Polarimetry and Huynen Fork Parameters	99
5.5 Simulations	101
5.6 Cranfield GBSAR Laboratory Overview	105
5.6.1 Experimental Setup	105
5.6.2 Polarimetric Calibration	106
5.7 Methodology	107
5.7.1 Aperture Downsampling.....	107
5.7.2 Sub-Nyquist Imaging Performance Measures.....	108
5.7.3 Huynen Reference Array.....	110
5.8 Experimental Results	112
5.8.1 Sub-Nyquist Point-Clouds	112
5.8.2 Sub-Nyquist Performance of SSARVI and PoISSARVI Algorithms	115
5.8.3 Sparsely Sampled Aperture Polarimetry Performance	118
5.8.4 Discussion.....	125
5.9 Conclusion	125
6 Discussion	128
6.1 Introduction	128
6.2 Summary of Key Findings.....	128
6.2.1 Objective 1: Validate the Accuracy and Performance of the SSARVI Approach.....	128

6.2.2 Objective 2: Optimise the Performance of the SSARVI Approach .	129
6.2.3 Objective 3: Extend the SSARVI Approach to Polarimetric SAR Collections and Analyses	130
6.3 Limitations.....	131
6.3.1 Experimental Uncertainty	131
6.3.2 Thresholding	132
6.3.3 Imaging Performance Metrics	132
6.3.4 Sample Height Randomisation	133
6.3.5 Computation Time Considerations	133
6.4 Place in the Literature	133
7 Closing Remarks	137
7.1 Contribution to Knowledge.....	137
7.2 Expected Impact	138
7.3 Future Research Directions	138
REFERENCES.....	139

LIST OF FIGURES

Figure 2-1 Simulated monostatic SAR collection geometry. SAR trajectory is shown in red, and the target scene shown by the blue asterisk.....	11
Figure 2-2 Region of K-space support. A dynamic range of 50 dB was used. .	12
Figure 2-3: Corresponding SAR image of a point scatterer located at the target scene centre.	12
Figure 2-4 Multistatic collection geometry, with a Transmitter (Tx), multiple receivers (Rx), that are separated through the target scene (T_G) by the bistatic angle (β). The distances d_{RT} and d_{RR} are the same as defined previously.	19
Figure 2-5 Diagram of bistatic SAR wave vectors, K , k_i , and k_s	23
Figure 2-6: Example maximum intensity projections (MIPs) of the backhoe vehicle from the SAR backhoe challenge data set. Formed with full geometry and bandwidth available from the dataset, using the VV linear polarisation channel. The top rendering is a maximum intensity projection in the XZ imaging plane. The bottom rendering is a maximum intensity projection in the YZ imaging plane. A dynamic range of 30 dB was used to present the data in both renderings. Rear-arm labelled 'A', side of cabin labelled 'C'.	27
Figure 2-7: Volumetric plot of SAR backhoe challenge vehicle target. A 30 dB dynamic range was used, in the VV linear polarisation channel. Intensity of the voxels represented by the colourmap. Voxel opacity was determined by the respective brightness of each voxel location.	28
Figure 2-8 Representation of the polarisation ellipse. The measured polarisation channels are E_H and E_V . The orientation the polarisation states is denoted by Θ . The ellipticity of the polarisation states is denoted by τ	35
Figure 2-9: Maximum Intensity Projection of the Backhoe vehicle in the YZ imaging plane, giving a view of the front cross section. Polarimetric information is associated with the colour scheme presented, with red representing an odd number of bounces and green representing an even number of bounces. Multipath is labelled 'M'.	40
Figure 2-10: Maximum intensity projection of the Backhoe vehicle, in the XZ imaging plane. This shows a side view of the tank, with the odd-even bounce decomposition represented by the red-green colourmap.	41
Figure 2-11: MIP of the Backhoe vehicle in the YZ imaging plane, with the skip angle associated with each voxel location. Red represents odd-bounce scattering and green represents even-bounce scattering. Multipath denoted "M".	42

Figure 2-12: MIP of the Backhoe vehicle in XZ imaging plane, giving a side view of the tank structure. The variation of the skip parameter across the vehicle is again represented by the red-green colourmap.	42
Figure 2-13: Experimental set up of Cranfield GBSAR laboratory. The antennas shown here are in the bistatic/multistatic configuration [136].	43
Figure 2-14. Dual-polarimetric horns with a microwave switch on each. Polarimetric switching is here performed automatically by the scanning system [136].	45
Figure 3-1: Flow chart illustrating key principles of new approach.	49
Figure 3-2: Bistatic simulation geometry, consisting of five passes and a single receiver location. Target situated at ($z=0$ m) in height.	50
Figure 3-3: Summed phase factor, R , along a vertical iso-range through the target location ($z=0$ m), with trajectory $\sigma_1=0$, $\sigma_2=0.17$ and $\sigma_3=0.52$	51
Figure 3-4: Simulated summed phase response, R . SNR of simulation was set to 6dB. Transmitter apertures of 5 passes were used, with an increasingly uneven distribution of pass spacings.	52
Figure 3-5: 2D Bistatic SAR geometry, finely sampled in elevation. Transmitter aperture is represented by the blue trajectory diagram (labelled Tx). Stationary receiver position is represented by the red asterisk to the right of the transmitter aperture (labelled Rx).	53
Figure 3-6: Downsampled transmitter height apertures, using a bistatic geometry. Again, blue lines indicate individual SAR heights, fully sampled in azimuth, remaining after each successive downsampling. White spaces consist of fully removed SAR passes.	54
Figure 3-7: Downsampled transmitter height apertures, using a monostatic geometry. Blue lines indicate individual SAR heights, fully sampled in azimuth, remaining after each successive downsampling. White spaces consist of fully removed SAR passes.	55
Figure 3-8: Normalised cross-correlation of sparse point cloud renderings with the corresponding full aperture BPA volumetric SAR geometry images.	56
Figure 3-9 Model tank measurement sparse aperture point cloud renderings, 50% SAR aperture removed for a monostatic SAR geometry.	57
Figure 3-10 Model tank measurement sparse aperture point cloud renderings, 37% SAR aperture removed for a bistatic SAR geometry.	58
Figure 3-11: Example rendering of a T72 point cloud, past the limit of discernibility. Formed using a monostatic collection geometry, with 74% aperture removed.	59

Figure 3-12: Example rendering of a T72 point cloud, past the limit of discernibility. Formed using a bistatic collection geometry, with 71% aperture removed.....	59
Figure 4-1: Flow diagram of the multistatic and polarimetric extension to the SSARVI algorithm.....	65
Figure 4-2: The GBSAR laboratory in a bistatic configuration.	68
Figure 4-3: Multistatic 3D SAR geometry, with 2D SAR aperture transmitter trajectory in blue, and nine fixed receiver positions to the left of the scene.	69
Figure 4-4: Target scene, consisting of a Quarter scale T72 tank model on gravel and reference sphere “S”.....	69
Figure 4-5: Simulation multistatic SAR geometry with five sequential transmitter passes (red) and three simultaneous receivers (blue). The simulated point scatterer location is labelled “T”. The Bistatic Equivalent Monostatic (BEM) trajectories are also shown (yellow, green and magenta for each of the three receivers).....	71
Figure 4-6: Simulation values of summed phase factor, R , along a vertical iso-range arc from multistatic SAR simulations on a point scatterer. The performance is shown for both coherent interferogram summation from multiple receivers, and the non-coherent summed bistatic interferogram approach.....	72
Figure 4-7: Simulation results for ΔR determining the relationship between the sidelobe height and the vertical aperture distribution in the form of the cluster BEM angular extent χ . Results are shown for simulations of 6, 8 and 10 transmitter passes. The vertical dashed line indicates the angular extent corresponding to the Nyquist spacing for the 1 m unambiguous vertical cross-range extent of interest over which ΔR is evaluated.....	74
Figure 4-8: MIPs of an incoherent sum of the nine VV polarisation 3D bistatic SAR images, with a z-projection (a), y-projection (b) and an x-projection (c). The large reference sphere is marked “S”.	76
Figure 4-9: MIPs of a phase corrected coherent sum of the nine VV polarisation 3D bistatic SAR images, with a z-projections (a), y-projection (b) and an x-projection (c). The large reference sphere is marked “S”.....	77
Figure 4-10: Representation of the downsampled transmitter aperture. Blue lines show the transmitter height sampled, for a full aperture width, as a percentage of the initial transmitter aperture removed.	79
Figure 4-11: VV polarisation measurement sub-Nyquist transmitter height sampled multistatic point-cloud renderings. (a) Coherently summed, (b) Non-coherent bistatic summed, (c) Non-coherent bistatic combined.	80

Figure 4-12: Normalised cross-correlation between successive coherent summed bistatic sparse aperture point-cloud renderings and the corresponding initial-sampling point-cloud renderings for each polarisation.....	83
Figure 4-13: Summed phase factor response when coherently summing data from an increasing number of receivers, for the brightest scatterer in each polarisation.	84
Figure 4-14: Normalised cross-correlation between successive non-coherent summed bistatic sparse aperture point-cloud renderings and the corresponding initial-sampling point-cloud renderings for each polarisation.	85
Figure 4-15: Maximum summed phase factor value when non-coherently summing data from an increasing number of receivers.	86
Figure 4-16: Normalised cross-correlation between successive non-coherent combined bistatic sparse aperture point-cloud renderings and the corresponding initial-sampling point-cloud renderings for each polarisation.	87
Figure 4-17: Overlaid point-clouds from each polarisation channel, shown for the three multistatic geometry combination methods: (a) Coherent summation, (b) Non-coherent bistatic summed, (c) Non-coherent bistatic combined. ...	89
Figure 5-1: Flow diagram outlining the PoISSARVI algorithm.	98
Figure 5-2: Sparse bistatic SAR simulation geometry. The transmitter aperture (red) is labelled Tx and receiver position is labelled Rx. The flat plate, dihedral and dipole scatterers are labelled in the scene.....	101
Figure 5-3: Summed interferometric response R_{pol} along vertical iso-range arcs through the location of the flat plate, dihedral and dipole scatterer. These are labelled Plate, Dih and Dip respectively.....	103
Figure 5-4: Summed interferometric responses along the vertical iso-range arc through the location of a horizontal dipole. The PoISSARVI response R_{pol} and the SSARVI response R_{HH} appear overlaid.	104
Figure 5-5: Photographs of model tank target. The wire on top is denoted by "W", the turret of tank denoted by "T", and the reference sphere is denoted by "S".	105
Figure 5-6: Multistatic SAR geometry. The 2D transmitter SAR aperture is shown in red and denoted by Tx. The stationary receiver positions are shown with blue asterisks and denoted Rx1 and Rx2.	106
Figure 5-7: Downsampled transmitter height apertures. Blue lines show heights sampled for a given randomly downsampled aperture. White spaces consist of fully removed SAR passes.....	108
Figure 5-8: Multistatic 3D BPA SAR images of the model vehicle, formed with the full transmitter aperture presented in Figure 5-6: Multistatic SAR geometry.	

The 2D transmitter SAR aperture is shown in red and denoted by Tx. The stationary receiver positions are shown with blue asterisks and denoted Rx1 and Rx2.. Polarimetric Span intensity in dB provides the brightness in these images, and colour is attributed to polarimetric parameter angles. The polarisability angle is shown (top) together with the Skip angle (bottom). “W”, in the top rendering, denotes the position of the wire, which is seen to have a high polarisability value.....	111
Figure 5-9: Sub-Nyquist SAR aperture polarimetric and multistatic T72 rendering showing the Huynen Polarisability angle. Green shows non-polarising scatterers, red shows polarising scatterers. Two different viewing angles of the same rendering are shown.	113
Figure 5-10: Sub-Nyquist SAR aperture polarimetric and multistatic T72 rendering showing the Huynen Skip angle. Green shows Even-bounce scatterers, red shows Odd-bounce scatterers. Two different viewing angles of the same rendering are shown.	114
Figure 5-11: False Alarm Ratio (FAR) of the SSARVI and PolSSARVI approaches for sub-Nyquist SAR aperture samplings below SIS.....	115
Figure 5-12: Normalised cross-correlation of sub-Nyquist formed point cloud renderings with initial SIS aperture. Performed for both SSARVI and PolSSARVI algorithms.....	117
Figure 5-13: The NMSD of all six Bistatic Huynen fork parameters for each aperture sampling.....	118
Figure 5-14 PolSSARVI point-cloud MIPs in the x-z plane, showing a front view of the tank rendering. Colour indicates the Polarisability angle, with brightness determined by the polarimetric span in dB, with a 50dB dynamic range. The renderings were formed at SIS sampling (top) and at 52% ARF-SIS (bottom). The turret labelled ‘T’.....	119
Figure 5-15 PolSSARVI point-cloud MIPs in the x-z plane, showing a front view of the tank rendering. Colour indicates the Skip angle, with brightness determined by the polarimetric span in dB, with a 50dB dynamic range. The renderings were formed at SIS sampling (top) and at 52% ARF-SIS (bottom).	121
Figure 5-16: PolSSARVI point-cloud MIPs in the y-z plane, showing a side view of the tank rendering. Colour indicates the Polarisability angle, with brightness determined by the polarimetric span in dB, with a 50dB dynamic range. The renderings were formed at SIS sampling (top) and at 52% ARF-SIS (bottom).	122
Figure 5-17: PolSSARVI point-cloud MIPs in the y-z plane, showing a side view of the tank rendering. Colour indicates the Skip angle, with brightness determined by the polarimetric span in dB, with a 50dB dynamic range. The renderings were formed at SIS sampling (top) and 52% ARF-SIS (bottom).	123

Figure 5-18: Huynen fork parameters determined at the location of the reference sphere for each aperture sampling. 124

LIST OF TABLES

Table 4-1: Table showing the aperture downsampling limits for point-clouds with minor and major artefacts, for all three receiver combination approaches: coherent multistatic sum (CMS), non-coherent multistatic sum (NCMS), non-coherent bistatic combination (BC)..... 88

LIST OF EQUATIONS

(2-1).....	9
(2-2).....	13
(2-3).....	13
(2-4).....	13
(2-5).....	14
(2-6).....	16
(2-7).....	18
(2-8).....	18
(2-9).....	18
(2-10).....	19
(2-11).....	20
(2-12).....	20
(2-13).....	21
(2-14).....	21
(2-15).....	21
(2-16).....	23
(2-17).....	23
(2-18).....	23
(2-19).....	24
(2-20).....	24
(2-21).....	24
(2-22).....	25
(2-23).....	29
(2-24).....	29
(2-25).....	29
(2-26).....	29
(2-27).....	30
(2-28).....	30

(2-29).....	30
(2-30).....	31
(2-31).....	31
(2-32).....	31
(2-33).....	32
(2-34).....	32
(2-35).....	33
(2-36).....	34
(2-37).....	35
(2-42).....	35
(2-43).....	36
(2-44).....	36
(2-45).....	37
(2-46).....	37
(2-47).....	38
(2-48).....	38
(2-49).....	38
(2-50).....	38
(2-51).....	39
(2-39).....	39
(2-40).....	39
(3-1).....	49
(4-1).....	64
(4-2).....	66
(4-3).....	66
(4-4).....	67
(5-1).....	96
(5-2).....	96
(5-3).....	97

(5-4).....	97
(5-5).....	97
(5-6).....	98
(5-7).....	99
(5-8).....	99
(5-9).....	102
(5-10).....	109
(5-11).....	110
(5-12).....	112

LIST OF ABBREVIATIONS

2D SAR	Two-dimensional Synthetic Aperture Radar
3D SAR	Three-dimensional Synthetic Aperture Radar
ARF-SIS	Aperture Removed From Sub-Initial Sampling
ATR	Automatic Target Recognition
BEM	Bistatic Equivalent Monostatic
BPA	Backprojection Algorithm
CS	Compressive Sensing
CSAR	Circular Synthetic Aperture Radar
DEM	Digital Elevation Map
DFT	Discrete Fourier Transform
DLR	German Aerospace Centre
Dstl	Defence Science and Technology Laboratory
FAR	False Alarm Ratio
FFT	Fast Fourier Transform
FP	False Positive
GBSAR	Ground Based Synthetic Aperture Radar
HPC	High Performance Computing
IFFT	Inverse-Fast Fourier Transform
InSAR	Interferometric Inverse Synthetic Aperture Radar
InSAR	SAR Interferometry
ISAR	Inverse Synthetic Aperture Radar
MF	Matched Filter
MIMO	Multiple-Input Multiple-Output
MIP	Maximum Intensity Projection
NMSD	Normalised-Mean Square Deviation
PFA	Polar Format Algorithm
PolSSARVI	Polarimetric Sparse SAR Volumetric Interferometry
PRF	Pulse Repetition Frequency
PSI	Persistent Scatter Interferometry
Radar	Radio Detection and Ranging
RCS	Radar Cross Section
RF	Radio Frequency
RLS	Regularised Least-Squares
SAR	Synthetic Aperture Radar
SIS	Sub-Initial Sampling
SNR	Signal to Noise Ratio
SPCGS	SAR Point Cloud Generation System
SSARVI	Sparse SAR Volumetric Interferometry
TP	True Positive
UWB	Ultra-Wideband
VNA	Vector Network Analyser
Vol	Volume of Interest

1 Introduction

Synthetic Aperture Radar (SAR) is an imaging technique where radar pulses are combined along the flight path of a radar sensor, forming finely detailed imagery [1], [2]. The coherent combination of signals along a wide aperture from multiple viewing angles produces a finer resolution than would be possible for a conventional stationary antenna system [3]. SAR was initially developed as a military surveillance technique in the 1960s, offering all-weather imaging capabilities during both the day and night [4]. The longer wavelength of radiofrequency (RF) waves can penetrate cloud cover, unlike visible light used in optical imagery [3]. Furthermore, longer wavelength RF can also penetrate building materials, such as wood and concrete, providing the capability of through-wall imaging [5], [6]. Since its original development, SAR has morphed into a highly useful technique for various applications such as urban mapping, terrain mapping, ocean monitoring, and disaster relief support [7], [8].

Recent SAR studies have explored the use of multiple sensors for improving the unambiguous cross-range and spatial resolution of SAR imagery [9], [10]. These are often competing factors when designing SAR systems [3], [11]. Multistatic sensing generally refers to the integration of target-return data from multiple receivers, where at least one receiver has a non-negligible spatial separation from the transmitter [12]. An example of a multistatic SAR system is the TanDEM-X spaceborne SAR mission, originating from the German Aerospace Centre (DLR) [9]. Multistatic SAR geometries have applications to the use of UAV swarm-sensing, as well as satellite constellations [13], [14]. Further satellite constellation studies include the Radarsat mission of the Canadian Space Agency, and COSMO SkyMED of the Italian Space Agency. These constellations use multiple monostatic sensors for providing a wider area of illumination than would be possible with a single sensor [15], [16]. All have been successfully used for various applications, including the formation of fine-resolution imagery of different terrains and improving the accuracy of scatterer height estimates when forming Digital Elevation Maps (DEMs) [17], [18], [19]. There has also been

recent interest in exploiting multiple sensors for passive SAR image formation and the detection of vehicle targets [18], [20].

SAR was originally developed as a two-dimensional imaging technique, in which 3D scatterers are projected onto a 2D imaging plane. The main drawback of conventional 2D SAR is the loss of target height information of elevated scatterers, and the distortion in their position in the image due to an effect known as layover. The development of 3D SAR imaging capability has interested the SAR community since the 1970s, when SAR interferometry, often referred to as InSAR, was first developed for military applications [21]. This method uses the phase difference between two displaced passes to determine target height parameters, which is most commonly used for terrain height mapping [22]. This is now a well-established technique within the SAR community, with the most notable usage being the Space Shuttle radar topography mission [23], [24]. Whilst useful for obtaining the height parameters of certain scatterers, this technique has limited resolvability in the vertical imaging direction.

Fully resolving scatterers in the vertical imaging direction via conventional Fourier-inversion methods requires multiple repeat SAR passes displaced in elevation. This process was first described mathematically by the renowned SAR scientist Jakowatz in 1995 [25]. The collection geometry of such a technique would conventionally require a finely sampled 2D SAR aperture [26]. Fully resolved 3D SAR also has no layover artefacts present in the imagery, allowing for a more accurate reconstruction of targets and greater image interpretability. However, the collection time required for 3D SAR is impractical for most airborne scenarios [27], [28]. Fully resolved 3D SAR imagery has been demonstrated in Ground-Based SAR sensing scenarios, with notable success being achieved in obtaining remote intelligence of building interiors [29].

Radar polarimetry analyses the effect of a target on the direction of oscillation of scattered electromagnetic waves. This technique can be applied to SAR and, by extension, 3D SAR imaging [30], [31], [32]. Polarimetric decompositions can

allow for more information about the target composition and orientation to be deduced [33].

This thesis focuses on 3D SAR algorithm development, specifically utilising multistatic sensing for reducing the sampling requirements of forming finely detailed 3D SAR imagery. The work will focus on the application to vehicle targets, which have finer resolution requirements than large scatterers such as buildings and terrain features. The application of SAR polarimetry to this area will also be investigated.

1.1 Motivation

3D SAR is a highly active discipline within the wider field of SAR, as the greater image interpretability of this technique and the 3D scatterer information obtained are highly useful for target identification purposes [34], [35]. The benefits of 3D SAR have been investigated for use in SAR Automatic Target Recognition (ATR) algorithms, particularly in the field of Inverse SAR (ISAR) [36], [37]. Extending 3D SAR imagery to multi-polarimetric data has also been found to improve the target interpretability for vehicle identification purposes by increasing the number of scattering centres visible to the radar [38], [39]. The collection of SAR data from multiple aspect angles has improved the number of scatterer detections in complex 3D target models and captured more polarimetric information [40], [41]. The use of multiple sensors opens the possibility of exploring multistatic sensing, utilising satellite constellations and swarms of unmanned aerial vehicles (UAVs), for 3D-Polarimetric SAR applications.

There has been increased interest in the use of drones and satellites for remote sensing applications due to their cost-effectiveness and flexibility [42], [43], [44], [45]. This is especially the case when compared to manned aircraft. The use of UAV drone swarms typically allows for an increase in the amount of data that can be collected, with shorter revisit times [46], [47]. For military sensing applications, they allow for more intelligence to be gathered with a far reduced risk of the loss of personnel [48], [49].

Advancements in sensor data-combination methods have allowed for the potential of multiple airborne, manned and unmanned sensors to be exploited for intelligence-gathering scenarios [50], [51]. This allows for the advantages of multistatic SAR sensing, such as finer resolution and larger unambiguous extents, to be more easily exploited. The work presented in this thesis was funded under the wider 'Oberon' project being led by the Defence Science and Technology Laboratory (Dstl) of the United Kingdom [52]. The work presented in this thesis evaluates the potential of the Oberon constellation project and UAV-SAR swarm sensing for reducing the sampling requirements for producing high-quality 3D SAR imagery.

Various techniques have been developed for reducing the collection time of 3D SAR imaging, including the well-established SAR interferometry. This approach typically struggles with complex manmade targets, that typically have significant discontinuities in their height profile [22]. Compressive sensing has also been heavily explored for sparsely formed 3D SAR imagery, producing detailed and accurate 3D mapping of urban and forest scenes [53]. 3D SAR compressive sensing (CS) exploits the physical sparsity of the image scene to form 3D imagery from sparsely sampled data sets [54]. The signal reconstruction techniques involved in CS can also be computationally expensive and the pre-requisite of the sparsity of the target scene may not be valid under certain conditions [55].

The use of polarimetric techniques has improved the accuracy of sparse 3D SAR approaches. This is especially the case for SAR interferometry, where greater interferometric coherence between passes can be achieved with polarimetric diversity, and more accurate scatterer heights can be obtained [56], [57]. Polarimetric collections have also improved the performance of related techniques such as Persistent Scatterer Interferometry (PSI) [58], [59].

For exploiting the advantages of multistatic sensing for forming high-quality 3D SAR imagery with attainable collection geometries, there exists an opportunity for novel research with the formulation and validation of a new approach [60], [61]. The theory developed in this thesis differs from previous studies in this area as it builds upon concepts first presented in the SAR Point Cloud Generation

Systems (SPCGS), which is an algorithm for sparsely sampled 3D SAR imaging [62]. The limitations of this algorithm for multistatic sensing and near-field scenarios are discussed throughout this thesis, in the publication chapters, and in section 2.3.3. The algorithm presented in this thesis accounts for the asymmetrical nature of bistatic, multistatic and near-field SAR geometries, allowing for its implementation using satellite constellations and swarms of UAVs. The SPCGS would be difficult to implement with the SAR sensor platforms discussed here.

1.2 Aims and Objectives

This thesis aims to develop a novel algorithm that can exploit the benefits of satellite constellations and UAV-SAR drone swarms for generating 3D SAR imagery below the Nyquist sampling requirement in elevation. The developed algorithm addresses the challenges of current sparsely sampled 3D SAR approaches applied to multistatic geometries, particularly the SPCGS. The developed algorithm is the Sparse SAR Volumetric Interferometry (SSARVI) algorithm. A polarimetric version of the SSARVI algorithm, known as the Polarimetric SSARVI (PolSSARVI) algorithm, has also been developed.

For the development of both algorithms, the following objectives were formulated:

1. Validate the accuracy of the SSARVI algorithm
 - Produce simulations that show the height of scatterers accurately determined for a series of sparse 2D multistatic SAR apertures.
 - Form experimental imagery 3D SAR imagery, below the Nyquist sampling requirement, of a model vehicle target using multistatic geometries. Conduct initial validation on monostatic and bistatic geometries.
2. Optimise the performance of the SSARVI algorithm
 - Determine the optimal SAR aperture distributions to minimise the presence of false scatterer detections.

- Determine the most effective method of combining data from multiple receivers, focussing on the accuracy of the 3D SAR point clouds produced.
3. Apply the SSARVI algorithm to polarimetric SAR analyses
- Extend the approach to allow for multiple measured polarimetric channels to be combined (the PolSSARVI algorithm).
 - Use the bistatic Huynen fork parameters to add polarimetric information of scatterers to the sub-Nyquist formed 3D point clouds [63].

1.3 Contribution

This thesis contributes a novel algorithm for forming sparsely sampled 3D SAR imagery. The algorithm allows for the exploitation of multistatic SAR sensing, and near-field SAR image formation. The overall contribution of this thesis is distributed across three distinct parts, each presenting new theories and data of value to the SAR community.

The first contribution is a theoretical and experimental validation of the SSARVI approach.

The second contribution is an optimisation of the SSARVI approach, in terms of the distribution in elevation of the SAR apertures and the combination of data from multiple receivers.

The third contribution is an extension and application of the SSARVI approach, in this case for polarimetric analyses. This extension is the PolSSARVI algorithm.

1.4 Overview of Thesis Structure

This thesis is presented in the ‘Thesis by papers’ format, where the novel contributions are presented in a series of journal papers. Together, the journal papers form a coherent body of work under the objectives laid out in section 1.2. The relevant literature is reviewed throughout this thesis.

The work presented in Chapter 3 was published in the IET Journal of Electronics Letters [64]. Some results were added to Chapter 3 that were not included in the published manuscript due to the page limits of the journal. The results were, however, included in the response letter to the reviewers. The link to the publication can be found here: [Volumetric interferometry for sparse 3D synthetic aperture radar with bistatic geometries - Welsh - 2023 - Electronics Letters - Wiley Online Library](#).

The work presented in Chapter 4 was published in the IET Journal of Radar, Sonar and Navigation [65]. Some results were also removed from Chapter 4, to reduce repetition and provide a more logical ordering to the thesis. The link to the publication can be found here: [Laboratory multistatic 3D SAR with polarimetry and sparse aperture sampling - Welsh - 2024 - IET Radar, Sonar & Navigation - Wiley Online Library](#).

The work presented in chapter 5 was published in the IET Journal of Radar, Sonar and Navigation [66]. The link to the publication can be found here: [Polarimetry for Sparse Multistatic 3D SAR - Welsh - 2025 - IET Radar, Sonar & Navigation - Wiley Online Library](#).

Chapter 2 provides an overview of the theory behind the work presented in this thesis. Finally, Chapters 6 and 7 provide an overall discussion and conclusion to this study.

1.4.1 Related Published Works

In addition to the published work mentioned above, work related to this thesis was presented at the following conferences:

- 2022 EUSAR conference in Leipzig, Germany. See [67].
- IET Radar Conference 2022 in Edinburgh, Scotland. See [68].
- SASSAR Conference 2023 in Lerici, Italy. See [69].

1.4.2 Statement of Originality

The work presented in the publication chapters of this thesis formed my intellectual contribution to the field. I was the main author of these publications,

and my co-authors have received the appropriate accreditation in the submitted manuscripts.

Various AI tools, such as ChatGPT and Grammarly, were used to aid with the formatting of the introduction, background and discussion chapters. The tools were mainly used to help with grammar and paragraph structure. The tools were not used in any way to form the intellectual contribution of this thesis. The Scopus AI tool was used to increase the speed of literature searches, but was not used to provide any critical analysis of the source material.

2 Background

The purpose of this background chapter is to provide a theoretical framework for the research presented in the publication chapters of this thesis. Section 2.1 describes the fundamentals of SAR image formation. Section 2.2 describes the geometry and the effects on SAR image formation of using multistatic geometries. Section 2.3 explains extension of SAR imaging principles to the 3D image formation, and gives an overview of different image formation algorithms. Section 2.4 gives an overview of the fundamentals of bistatic SAR polarimetry. Section 2.5 describes the experimental setup.

2.1 SAR Image Formation

The fundamentals of SAR image formation are well understood, and the technique has been continuously developed since its original inception. The goal of SAR image formation is to separate out scatterers in cross-range that would otherwise be overlaid with conventional static radar geometries. SAR image formation constructs a 2D reflectivity function that varies in the x and y imaging directions, often referred to in the literature as $g(x,y)$ [70]. The cross-range direction is typically defined along the x -axis of a target scene, and the range direction along the y -axis. These are both directions in the ground plane of the SAR image. SAR image formation achieves resolution in the cross-range by coherently combining return radar signals, over multiple azimuth angles relative to the target scene, along multiple points of a flightpath. This aperture-synthesis technique also has applications to other areas of signal processing, including sonar [71].

A typical radar would determine the range, R , to a target by measuring the time taken for an individual radar pulse to be transmitted, reflected off a target, and travel back to the receiver. This is shown by the relation:

$$R = \frac{ct}{2} \tag{2-1}$$

where t is the time taken for the pulse to return to the receiver, and c is the speed of light [72]. Radar pulses are often transmitted over a wide range of frequencies, which can be obtained by performing a Fourier transform on the received signal [73]. If a signal has been reflected a moving target, a doppler shift will be visible in the received signal. This thesis assumes a negligible target motion relative to the SAR collection geometry.

SAR measurements collect the range-profile return signals from multiple angles, relative to the target scene, and collate them in the frequency domain. The combined signal represents a 2D area in the spatial frequency domain, the dimensions of which are often referred to as F_x and F_y [74]. The overall 2D spatial frequency area of a SAR signal is often referred to as the K-space support region. The spatial coordinates of a scatterer can be obtained by performing a Fourier transform along each dimension of the K-space support region [75]. This often requires an interpolation step to transform the data from a polar grid to a cartesian grid with coordinates in the x and y dimensions.

An example simulated SAR collection is shown by the geometry presented in (2-1).

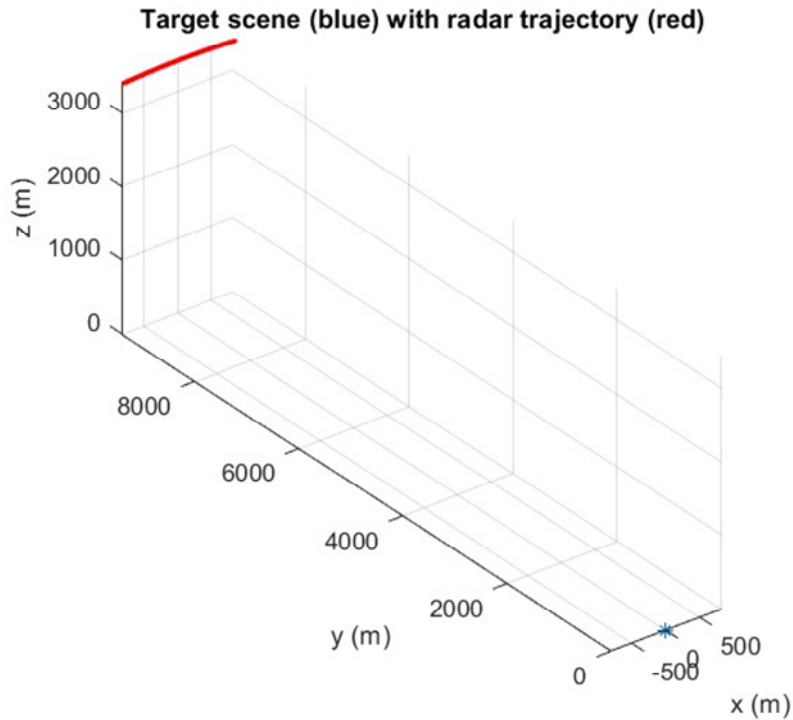


Figure 2-1 Simulated monostatic SAR collection geometry. SAR trajectory is shown in red, and the target scene shown by the blue asterisk.

The total azimuth swept out by the SAR trajectory, relative to the target scene centre, is 10° . The elevation angle between the ground plane and the SAR trajectory is 20° , with the range of the aperture to target scene being 10 km. The corresponding K-space support region is shown in Figure (2-2), with a bandwidth of 10 GHz. The resulting SAR image was formed using a polar-reformatting image formation algorithm and is shown in Figure (2-3). The relevant parameters determining the imaging performance of SAR are detailed in the subsection below.

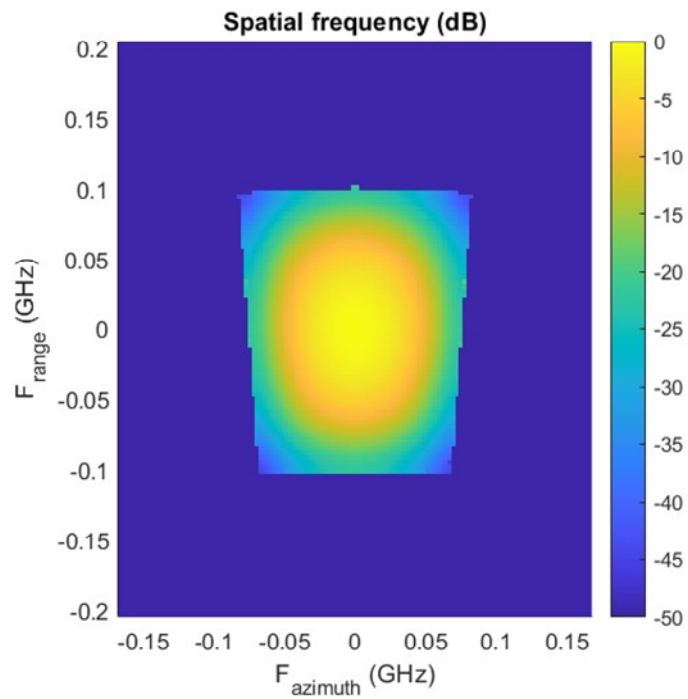


Figure 2-2 Region of K-space support. A dynamic range of 50 dB was used.

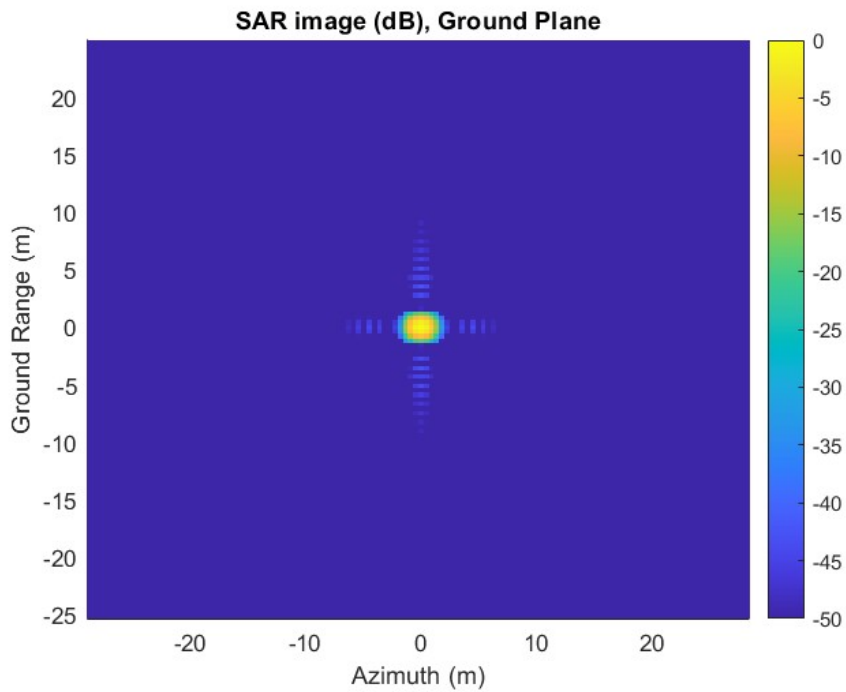


Figure 2-3: Corresponding SAR image of a point scatterer located at the target scene centre.

A critical theorem that enables SAR imaging to produce resolution in the cross-range direction is the projection-slice theorem. This states that performing a Fourier transform at an angle relative to the target scene is equivalent to performing a 2D Fourier transform along the same projection. The importance of this theorem for SAR is demonstrated in the foundational book by Jakowatz [70]. The projection slice theorem enables the variation of the reflectivity of the scene to be obtained in the x and y imaging directions, at each point along a projection through the target scene.

2.1.1 SAR Resolution and Coverage

A wider area of K-space support corresponds to a finer resolution in the cross-range [74], [76] . The resolution equations for SAR imaging are given below.

$$\rho_r = \frac{c}{2B} \quad (2-2)$$

This is the resolution in range, ρ_r , which is dependent upon the speed of light and the range of frequencies collected, i.e. bandwidth, given by B [77]. The cross-range resolution is given by:

$$\rho_{cr} = \frac{c}{2f_c \phi_w} \quad (2-3)$$

where f_c refers to the centre frequency of the transmitted signal and ϕ_w refers to the angular width of the SAR trajectory relative to the target [78]. The cross-range resolution is commonly presented in the form shown by equation (2-3), where a large range to the target compared to the scene dimensions is assumed. Note that when using conventional stationary antenna systems, cross-range resolution is determined by the footprint of the antenna beam pattern [79].

The unambiguous range of a SAR collection is given by:

$$L_r = \frac{c}{2\Delta f} \quad (2-4)$$

Where Δf is the spacing of frequencies sampled [80]. The unambiguous cross-range is given by:

$$L_{cr} = \frac{c}{2f_c\phi_{stp}} \quad (2-5)$$

Where ϕ_{stp} is the spacing in angular width spacing between of each individual radar pulse, corresponding to a spatial sampling [81]. ϕ_{stp} is the angular width of the SAR trajectory divided by the number of pulses. The same assumptions are made concerning the relative dimensions of the collection geometry and the target scene size for equation (2-5) as were made for equation (2-3). Note that equation (2-4) is derived from the sampling in Fourier space, and is not the equation typically used for the unambiguous range for static radar, which is inversely proportional to the pulse repetition frequency (PRF).

It is essential that the target scene extents in range and cross-range are smaller than their corresponding unambiguous ranges, and the area of illumination of antenna beam falls within these dimensions, to prevent a phenomenon known as aliasing [82]. This is where features wrap-around and are presented in multiple sections of the image. The minimum sampling size, to prevent aliasing over a specified target area, is referred to as the Nyquist sampling requirement [83]. This is a well-established concept, and is derived from the broader field of signal processing [84], [85]. This form of SAR imaging is often referred as ‘Spotlight-mode’ SAR image formation.

2.1.2 Image Formation Algorithms

The goal of SAR image formation algorithms is to reconstruct $g(x, y)$, from a coherent set of signals, from their representation in the Fourier domain (K-space). The most intuitive mathematical implementation of the SAR image formation process is the matched filter (MF) approach. MF first estimates the impulse response of a scatterer across the whole scene, which is then projected onto the received SAR signal in the Fourier domain. A Fourier transform is then performed to determine the overlap between the impulse response, and each received signal in the range domain [86]. The contribution of each pulse is then summed, which maximises the impulse response at the location of a target and allows the position of scatterers in 2D space to be determined. This process is described by

Gorham and Moore in the well-cited SAR image formation toolbox publication [81]. Whilst this approach is relatively simple and intuitive, it is also computationally expensive.

This paper also describes the backprojection algorithm (BPA), which is highly related to matched filtering. The BPA implements a Fast Fourier Transform (FFT) along the measured frequency domain of each pulse, to separate out the individual contribution of scatterers in the range direction. The FFT of each pulse is then summed and projected onto a target scene, with the response being maximised at the cross-range location of a scatterer within the scene. The FFT is a highly efficient way of implementing the Discrete Fourier Transform (DFT) computationally [87]. The BPA is the most robust method of forming SAR imagery, and is often used as a means of comparison for the development of new SAR algorithms.

The Polar Format Algorithm (PFA) is an image formation algorithm that is designed to offer far greater computation speed than both the MF and BPA approaches [74], [88]. The PFA estimates the 2D reflectivity function of a target scene by the implementation of a 2D FFT on SAR data collated in the frequency domain [89]. Before the 2D FFT is implemented, the collected data is resampled onto a two-dimensional cartesian grid in the spatial frequency domain. Whilst the PFA can produce accurate SAR imagery in an efficient manner, the image formation assumes the wavefronts of the SAR signal are planar [90]. This can introduce artefacts into bistatic and near-field-formed SAR imagery, when using the PFA, unless adapted versions are used [91], [92]. Furthermore, artefacts can also be introduced during the data interpolation step. For research purposes, this thesis will use the BPA as means of comparison for the SSARVI algorithm. This is due to its reliability, as the imagery produced is free from wave-front distortions and interpolation artefacts [11].

2.1.3 Near-field vs Far-field SAR image formation

The near-field and far-field regions refer to the overall distance of the SAR sensor from the target scene, relative to the dimensions of the target scene. Most SAR

collections occur in the far-field regime, where the distance of the SAR sensor to the target is significantly larger than the dimensions of the target. In this case, the wavefronts of the SAR signal are approximated to that of a planar shape. Furthermore, the total angular width swept out by the SAR trajectory remains approximately constant across the entire target scene [93]. This also the case with the elevation angle of the SAR trajectory relative to the desired target.

In the SAR near-field regime, the elevation and total angular width of the SAR trajectory can no longer be approximated as constant. This can lead to distortions in the sampling of the SAR signal in K-space, and makes processing of the layover phenomenon more challenging [94], [95], [96], [97]. Moreover, the wavefronts in the SAR near-field regime are curved, meaning that utilising algorithms such as the PFA require extra processing steps. Near-field scattering effects are noticeable when the SAR sensor operates at relatively small distances to the target scene, whereby the dimensions of the target scene begin to approach a non-negligible size compared to the range to the SAR sensor [11]. This also the case for wide-area SAR imaging, where the curvature of the wavefronts becomes non-negligible at the edges of the target scene [92].

Two common approaches can be used to derive the limits of the near-field to far-field scattering regime. One such approach determines the distance at which the azimuth angle from the left side of the scene to right side of the scene differs by the Nyquist spacing. The Nyquist spacing, of a given unambiguous cross-range, is given by equation (2-6). From this approach, the derived near-field scattering condition is described below.

$$D < \frac{2L_{cr}^2}{\lambda_c} \quad (2-6)$$

D refers to the distance of the SAR platform to the target, and λ_c is the centre wavelength of the radar pulse. This is similar to the well-known Fresnel diffraction limit [98]. Alternatively, the scene size limits can be compared to the range and cross-range resolution [99]. The work presented in the publication chapters of this thesis will use the near-field limit derived from the Nyquist spacing approach.

Conversely, the planar wavefront limit can be used to determine the near-field to far-field boundary distance. This approach determines the range at which the planar-wavefront approximation differs from curved wavefronts at the edges of a target scene by a certain phase difference. The phase difference used to define this boundary region is typically $\frac{\pi}{2}$, although some studies have stricter limits [100].

2.2 Bistatic and Multistatic SAR

Bistatic and multistatic SAR processing are highly related as both involve the use of at least one receiver antenna with a non-negligible spatial separation to the transmitter. As stated previously, multistatic sensing contains more than one receiver antenna in a single SAR aperture. Both collection geometries can have different performance characteristics than conventional monostatic geometries. This due to their different properties in the spatial-frequency/wavenumber (K-space) domain, caused mainly by the change in direction of the resultant scattering vector and the different number of receivers. There can also be a significant difference in the radar cross-section (RCS) from different bistatic angles.

2.2.1 Geometry and Signal Model

The overall distance of a SAR sensor to a target is often referred to as the differential range, denoted by dR . For systems with separated transmitter and receiver antennae, such as bistatic and multistatic geometries, the overall differential range is given by:

$$dR = dR_T + dR_R \quad (2-7)$$

Where dR_T and dR_R are the respective distances from the transmitter and receiver to the target scene. The distance from the transmitter to the target scene is given by:

$$dR_T = \sqrt{(x_T(t) - x_0)^2 + (y_T(t) - y_0)^2 + (z_T(t) - z_0)^2} \quad (2-8)$$

Where the position of the transmitter, at a pulse time t , is given by the location (x_{Tx}, y_{Tx}, z_{Tx}) and the target is at the location (x_0, y_0, z_0) . The distance of the receiver to the scene centre is given by:

$$dR_R = \sqrt{(x_R(t) - x_0)^2 + (y_R(t) - y_0)^2 + (z_R(t) - z_0)^2} \quad (2-9)$$

where the position the receiver, at a pulse time t , is given by (x_R, y_R, z_R) . The collection geometries mainly used in this thesis comprise linear transmitter

passes with stationary receiver positions. A diagram of a multistatic geometry is shown in Figure 2-4.

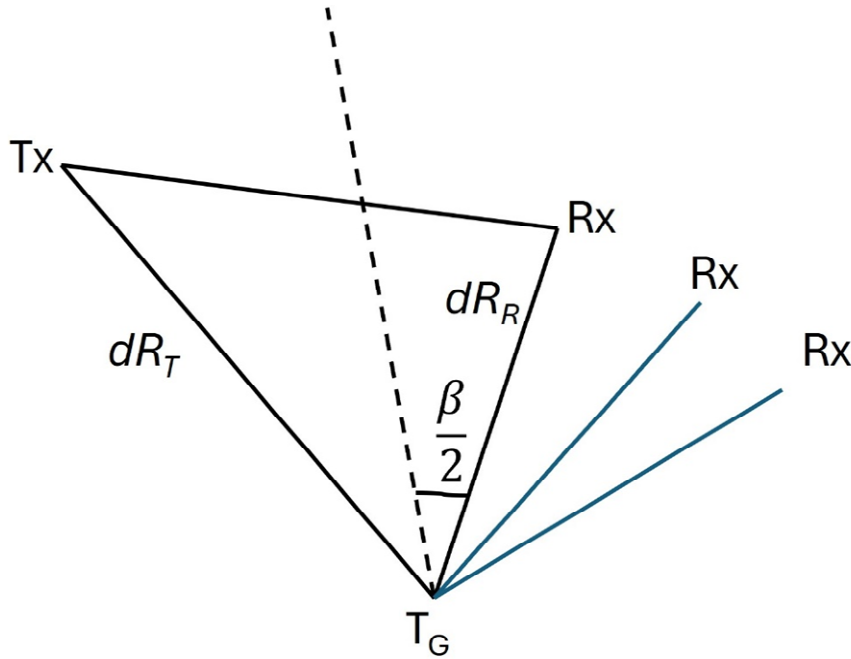


Figure 2-4 Multistatic collection geometry, with a Transmitter (Tx), multiple receivers (Rx), that are separated through the target scene (TG) by the bistatic angle (β). The distances dR_T and dR_R are the same as defined previously.

The bistatic angle, of which a bisector is shown in Figure 2-4, is defined the separation of a transmitter and receiver pair through the location of a target. The bistatic angle is given by the relation:

$$\beta = \arccos(\widehat{R}_T \cdot \widehat{R}_R) \quad (2-10)$$

where $\widehat{R}_T$ is a unit vector along the direction of the distance defined as dR_T , and $\widehat{R}_R$ is a unit vector along the direction of the distance defined as dR_T .

In this thesis, linear SAR transmitter passes will mainly be used for forming imagery. This contrasts with using SAR trajectories that have a change in elevation across the aperture, or are offset at an angle relative to the target scene, which are said to be 'squinted'. The elevation angle of a linear SAR trajectory is given by:

$$\psi = \arccos(\hat{y} \cdot \hat{R}) \quad (2-11)$$

The unit vector $\hat{y}$ is defined in the ground plane along the line of sight of the SAR trajectory to the target scene centre [70]. Note that alterations would be needed for equation (2-11) if the SAR trajectory were to be squinted. $\hat{R}$ is a unit vector from the midpoint of the SAR aperture, directly along the sight of the transmitter to the target scene centre.

The plane intersecting the target scene from the transmitter aperture is known as the slant plane. The slant range is the distance from the midpoint of the SAR aperture to the target scene. For bistatic geometries, the point at which the slant range is defined is more ambiguous than the monostatic case. The mid-aperture point, from which the slant range is defined, lies in the bistatic-equivalent monostatic (BEM) aperture. This is the point of intersection between the bisector of the transmitter and receiver vectors, and the baseline width between them [70], [95], [101]. The bisector of the bistatic angle can be referred to as the bisectrix, this is shown in Figure 2-4.

The signal response, s , of a single pulse, in either a bistatic or multistatic radar system, is given by:

$$s = A \exp \left[\frac{-i2\pi f_k}{c} (dR_T + dR_R) \right] + n \quad (2-12)$$

Where A is the amplitude of the return target signal, f_k is the k^{th} sample of the transmitted frequency range [102], and n is the amplitude of the background noise. A is dependent on several factors, including both the range and RCS of a target. For the results presented in Chapters 3 & 4, A will be treated as constant with to respect range, frequency and orientation. For Chapter 5, which presents polarimetric analyses, where the amplitude of the return signal is expected change according to the polarisation channel measured.

The relative power of the background noise to the return target signal is called the signal-to-noise ratio (SNR), which is given by:

$$SNR = 10 \log_{10} \left(\frac{P_T}{P_N} \right) \quad (2-13)$$

Where P_T is the time-averaged power of the target return signal, and P_N is the time-averaged power of the background noise. This is for a logarithmic power scale, with units of decibels (dB). The effects of background noise on the developed algorithm are briefly considered in Chapter 3. The rest of the results presume a minimal noise floor. The noise term is suppressed in the equations presented in the rest of this chapter.

The combined multistatic signal, S , for a linear SAR transmitter trajectory is given by:

$$S(m, n, f_k) = \sum_{m=1}^{N_R} \sum_{n=1}^{N_P} A \exp \left[\frac{-i2\pi f_k}{c} (dR_T(n) + dR_R(m)) \right] \quad (2-14)$$

Where N_R is the number of receivers, and N_P is the number of pulses within a SAR transmitter flightpath. Although equation (2-14) describes a coherent combination of pulses from each receiver, different combination methods are considered throughout this thesis.

Backprojection was implemented throughout this thesis, both as part of the SSARVI algorithm and as a means of comparison for determining the image quality of the sub-Nyquist formed 3D SAR renderings. For a multistatic SAR system, the relation below shows an implementation of the BPA for determining the location of a target.

$$I(p) = \sum_{m=1}^{N_R} \sum_{n=1}^{N_P} \text{ifft}(S(m, n)) \exp \left(\frac{+i2\pi f_1 dR_{mn}(p)}{c} \right) \quad (2-15)$$

Where $I(p)$ is the brightness of a pixel at the location p , f_1 is the minimum frequency of the transmitted pulse, and ifft is an inverse-Fast Fourier Transform (IFFT) that was implemented in MATLAB. $dR_{mn}(p)$ is the bistatic differential range of each transmitter and receiver pair, described by equation (2-7), to the p^{th} pixel within the target scene. The IFFT was implemented along the spatial

frequency dimension for each pulse at m^{th} and n^{th} receiver and transmitter location. $I(p)$ was interpolated between pixel locations to improve image quality [81]. Various implementations of the BPA were used throughout this thesis, including that shown by equation (2-15). This includes a parallelised version for decreasing the computation time of image formation. The parallelised BPA code was operated on the Cranfield High Performance Computing (HPC) facilities.

An important effect to consider, for bistatic and multistatic collection geometries, is the distortion to the shape of the wavefronts. A separated transmitter and receiver system contains two phase centres, which are the foci of the geometry. The points of equal range, dR , therefore lie along an elliptical curve, as described in the foundational book by Willis, *Bistatic Radar* [103]. This means that points of equal brightness in bistatic and multistatic range profiles, known as Isoranges, are elliptical shaped.

Under certain conditions, the plane wave approximation can be applied to bistatic geometries as well. However, the dimensions are far more restrictive than the monostatic case [104]. The implications of this for certain forms SAR image formation will become clear when the concept of layover is discussed in more detail in section 1.1, as an aspect of 3D SAR image formation.

2.2.2 Effects on Imaging Performance

The imaging performance of multistatic SAR systems is mainly determined by the bistatic angle between a transmitter and receiver pair, and the overall number of receivers. These factors have a significant effect on the shape and the sampling density of the K-space support region.

The impact of the bistatic angle on the imaging performance is best understood by considering the relative directions of the incident and scattered rays, and how they relate to the dimensions of the region of K-space support [105]. The directions of the incident and scattered waves are represented in K-space by their respective wave vectors, denoted by k_i and k_s respectively. The magnitude of the incident and scattered wave vectors given by:

$$|k_i| = |k_s| = k_0 = \frac{2\pi}{\lambda_c} \quad (2-16)$$

Where λ_c refers to the centre wavelength of the transmitted radar signal, and k_0 is the magnitude of an individual scattering component [75]. The resultant wave vector, K , is given by:

$$K = k_i - k_s \quad (2-17)$$

Which is a combination of both the magnitude and direction of the incident and scattered wave vectors. For monostatic SAR, k_i & k_s act in anti-parallel directions, which means that the resultant wave vector, K , is equal to $2k_0$. This is the maximum length of any dimension in K-space. For bistatic SAR, the k_i & k_s vectors are offset by the bistatic angle.

Therefore, the overall length of the bistatic scattering vector in K-space, K_B , is given by:

$$K_B = 2k_0 \cos\left(\frac{\beta}{2}\right) \quad (2-18)$$

Where β is the bistatic angle between the transmitter and receiver, from the target scene centre [106]. For $0 < \beta < \pi$, this results in a reduction in the extent of the K-space dimensions, resulting in a coarser resolution [107]. The bistatic SAR wave vector diagram is represented in Figure 2-5.

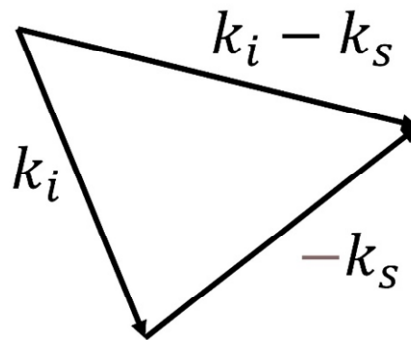


Figure 2-5 Diagram of bistatic SAR wave vectors, K , k_i , and k_s .

The resulting bistatic SAR resolution is given by:

$$\rho_{Bgr} = \frac{c}{2B \cos(\frac{\beta}{2}) \cos(\psi)} \quad (2-19)$$

Where c is the speed of propagation, B is the bandwidth, and ψ is the elevation angle [108]. The range resolution given by equation (2-19) is defined along the line of sight of the BEM mid-aperture point, along the bisector of the bistatic angle. This thesis uses the mean bistatic angle across the whole SAR aperture as an approximation of the resolution equations given in [108] for bistatic collections with a stationary receiver. The $\cos(\psi)$ terms projects the range resolution onto the ground plane.

The offset bistatic geometry has a similar effect with the cross-range resolution, defined from the line of sight of the transmitter axis. The bistatic cross-range resolution is given by:

$$\rho_{Bcr} = \frac{c}{2f_c \phi_{bw} \cos(\frac{\beta}{2}) \cos(\psi)} \quad (2-20)$$

where f_c is the centre frequency of the transmitted signal and ϕ_{bw} refers to the total angular width swept out by the transmitter aperture.

The bistatic angle also affects the unambiguous range of bistatic, and by extension multistatic, SAR collections. Corresponding to the decrease in the overall extent of K-space, there is also a decrease in the spacing between the sample points that is associated with the bistatic angle between the incident and scattered wave vectors. The finer spacing of sample points in spatial frequency causes an increase in the unambiguous range and cross-range, meaning that wider areas can be imaged without showing aliasing artefacts.

The unambiguous range for a bistatic SAR system, projected onto the ground plane, is given by:

$$L_{Bgr} = \frac{c}{2\Delta f \cos(\frac{\beta}{2}) \cos(\psi)} \quad (2-21)$$

The unambiguous cross-range for bistatic SAR is given by:

$$L_{Bcr} = \frac{c}{2f_c \phi_{Bstp} \cos(\frac{\beta}{2}) \cos(\psi)} \quad (2-22)$$

Where ϕ_{Bstp} is the angular spacing between successive pulses, along the transmitter aperture. Equations (2-21) and (2-22) also assume a stationary receiver bistatic geometry, using the mean bistatic angle as in equations (2-19) and (2-20).

The properties of the K-space support region can be improved using multistatic SAR, through the combination of multiple bistatic geometries. Provided the K-space support regions of the individual bistatic geometries are adjacent, the coherent combination of data from multiple angles can increase the overall length and width of the K-space support region [20]. This has been found to improve the spatial resolution of SAR systems employing this data collection approach, especially with low-powered illuminators of opportunity (IoT) [109].

If there is significant overlap between the K-space support regions of the individual bistatic geometries, the data from each receiver can be combined to increase the effective sampling of a SAR collection [110], [111]. Within each K-space support region, there are sample points comprising each frequency collected, and the aspect angle relative to the target scene at which they were sampled. Provided a bistatic geometry can contribute its own unique sample points within the same specified dimensions in K-space, the overall number of K-space samples will increase. This will lead to an improvement in the unambiguous range and cross-range [10]. Various studies have found this approach to suppress aliasing, including for fine resolution imagery where there is often a resolution trade-off associated with improved coverage.

Whilst multistatic SAR geometries have been shown to improve imaging performance, there is still some uncertainty with how best to predict their resulting imaging performance from their representations in the Fourier domain [110], [112]. Furthermore, there are several implementation challenges that result from return radar combining data from multiple aspect angles [113], [114]. This includes signal co-registration error and a shift in RCS of certain targets.

2.3 3D SAR

The aperture synthesis concept can be extended to the vertical imaging plane, allowing for targets to be resolved in height. Conventionally, this would be achieved by conducting many repeat SAR passes at different elevation angles relative to the target scene. This is analogous to the principles from which conventional 2D SAR approaches produce resolution in the cross-range.

Mathematically, this vertical resolution is enabled by an extension of the projection slice theorem to the case of 3-dimensional Fourier transforms. This was first applied to the case of 3D SAR imaging in a publication by Jakowatz in 1995, building on the original tomographic formulation of the 2D case by Munson et. Al [25], [115]. The generalised 3D version of the projection slice theorem states that performing a 2D Fourier transform along a rotated planar slice through a volume is equivalent to performing a 3D Fourier transform along the 3D coordinates of the same planar slice. This applies to both the spatial domain and the Fourier domain [116]. A 3D K-space support region can therefore be constructed from collected return data along multiple vertically displaced SAR trajectories.

Example 3D SAR renderings are shown below, using the 'Backhoe Challenge' public dataset that is available from the US Air Force Research laboratory [117]. This is a simulated data set of a complex model vehicle target, with a 2D polar-monostatic SAR collection geometry. The example vehicle used was a digger, also known as a 'backhoe'.

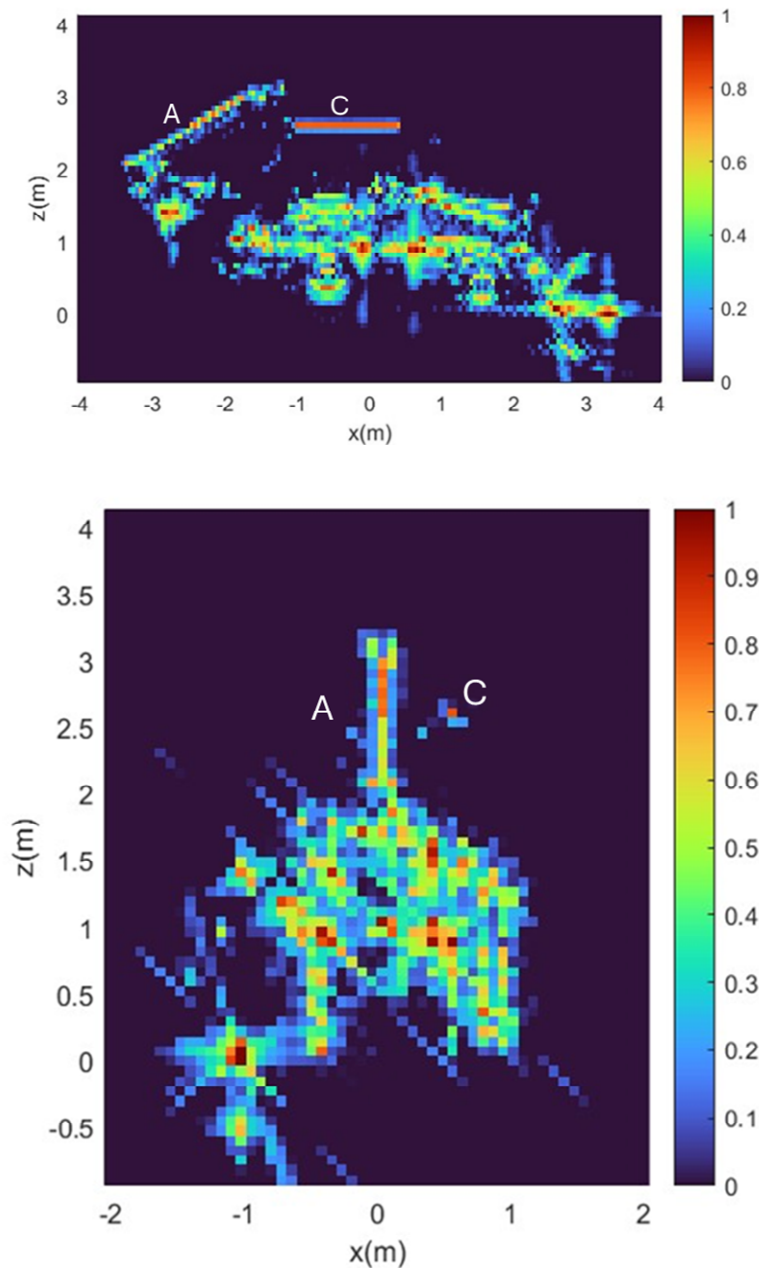


Figure 2-6: Example maximum intensity projections (MIPs) of the backhoe vehicle from the SAR backhoe challenge data set. Formed with full geometry and bandwidth available from the dataset, using the VV linear polarisation channel. The top rendering is a maximum intensity projection in the XZ imaging plane. The bottom rendering is a maximum intensity projection in the YZ imaging plane. A dynamic range of 30 dB was used to present the data in both renderings. Rear-arm labelled 'A', side of cabin labelled 'C'.

The image was formed using the full 2D aperture, with 350 passes displaced in elevation, with the full bandwidth and the VV polarisation channel. Notable features include the rear arm, which is visible towards the left of the YZ MIP and above the main structure in the XZ MIP, and the front shovel which is visible towards the bottom right of the YZ projection and bottom left of the XZ projection. Multipath is visible directly below the shovel, where the scattered ray has experienced multiple reflections before being received by the radar transceiver.

The plot below shows a volumetric 3D plot of the Backhoe vehicle, using the same scene dimensions, collection geometry, bandwidth and polarisation channel as the renderings presented above. This was formed using the VOXView software utilised in Chapter 5 [118].

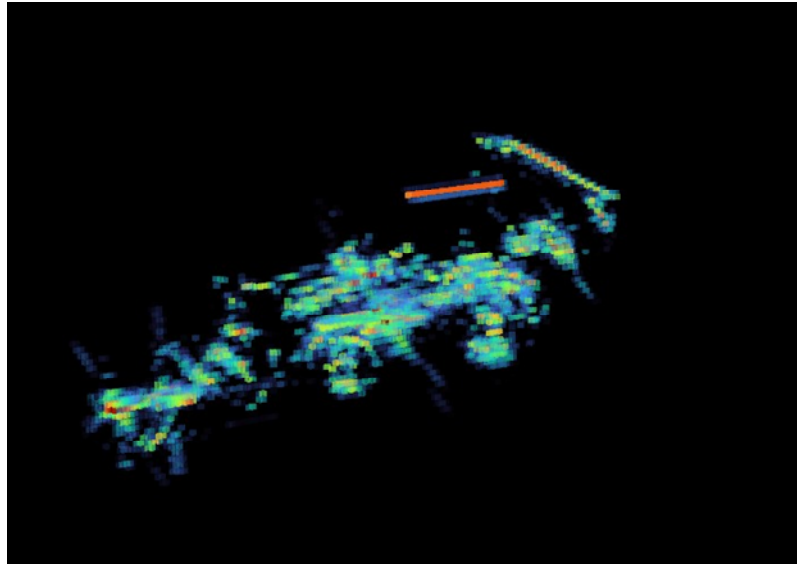


Figure 2-7: Volumetric plot of SAR backhoe challenge vehicle target. A 30 dB dynamic range was used, in the VV linear polarisation channel. Intensity of the voxels represented by the colourmap. Voxel opacity was determined by the respective brightness of each voxel location.

2.3.1 3D SAR Image Formation

For finely sampled 2D SAR apertures, 3D SAR imagery can be formed using the BPA. The signal model of a bistatic 2D SAR aperture is given by:

$$S(\psi, n, f_k) = \sum_{\psi=1}^{N_\psi} \sum_{n=1}^{N_P} A \exp \left[\frac{-i2\pi f_k}{c} (dR_T(\psi, n) + dR_R) \right] \quad (2-23)$$

Where each SAR pass is separated in height, at an elevation angle ψ . N_ψ is the total number of SAR aperture heights sampled. An example of the implementation of the 3D BPA, for a bistatic aperture, is given by:

$$I_{3D}(p) = \sum_{\psi=1}^{N_\psi} \sum_{n=1}^{N_P} \text{ifft}(S(\psi, n)) \exp \left(\frac{+i2\pi f_1 (dR_{T\psi n}(p) + dR_R(p))}{c} \right) \quad (2-24)$$

This describes a coherent summation of signals from each transmitter pass, for the n^{th} pulse of each at pass with an elevation angle of ψ . Multistatic 3D SAR imagery can be formed by combining BPA formed imagery from multiple receiver angles [119]. This can be in the form of both a coherent summation and a non-coherent summation. A finer resolution can be achieved with a coherent summation, but this method is more susceptible to phase errors and a change in RCS of the target between the different bistatic angles.

The resolution in the vertical imaging, ρ_{vcr} , direction is given by:

$$\rho_{vcr} = \frac{c}{2f_c \psi_w} \quad (2-25)$$

Where ψ_w is the total aperture is the vertical angular width of the 2D SAR aperture and is given in radians [34]. The unambiguous cross-range in the vertical imaging direction, L_{vcr} , is given by:

$$L_{vcr} = \frac{c}{2f_c \psi_{stp}} \quad (2-26)$$

Where ψ_{stp} is the angular spacing between each vertically displaced SAR pass. ψ_{stp} is also given in radians [120]. Equations (2-24) and (2-25) are valid for far-field monostatic SAR geometries, where ψ_{stp} and ψ_w can be assumed to be constant across the target scene. Corrections can be made to (2-24) and (2-25), similar to those presented in (2-19) and (2-20), to determine the vertical resolution and unambiguous range for bistatic geometries.

The phenomenon of layover is present when projecting 3D targets onto a 2D SAR imaging plane. Elevated targets appear shifted in the ground range, as they are at a different range to the radar platform than scatterers directly beneath them. The layover point is defined as the intersection of the ground plane and the normal vector to the slant plane [70]. The shifted position in the ground range is given by:

$$y_L = y + h \tan(\psi) \quad (2-27)$$

Where h is the height of a scatterer, y_L is the true position in the y imaging direction of the elevated target, above the ground plane, and y is the position of the scatterer when projected onto the ground plane [26]. This is valid for far-field monostatic geometries, where ψ is approximately constant across the entire scene. For bistatic and multistatic SAR, it is harder to obtain the layover position of scatterers as due to the asymmetrical nature of their geometries.

2.3.2 SAR Interferometry

Due to the impractical sampling requirements of most airborne 3D SAR scenarios, SAR interferometry has proven a reliable technique for requiring 3D target information from a dual-pass aperture [93]. SAR interferometry works by first forming an interferogram from the two SAR images obtained from each pass. The SAR interferogram, denoted here as U , is given by:

$$U(p) = \frac{I_1(p)I_2^*(p)}{\sqrt{|I_1(p)|^2|I_2(p)|^2}} \quad (2-28)$$

Where I_1 is a 2D SAR image formed from the first pass, and I_2 is a 2D SAR image formed of the second pass. The coherent multiplication described in (2-28) is performed at each pixel location, p . The phase of the interferogram formed, typically denoted by φ , is then determined by:

$$\varphi(p) = \arg(U(p)) \quad (2-29)$$

The phase from this calculation is the phase difference between the two SAR images, which is then used to determine the overall height of a scatterer. The

scatterer height above the ground plane and the phase difference between the two SAR images are linked by two relations, the first of which is by:

$$\Delta r \cong \frac{B_i}{r_0 \sin(\xi_i)} \Delta h \quad (2-30)$$

Where Δr is the difference in range to the target between the two SAR flightpaths, B_i is the baseline width between the two passes (which is perpendicular to the line of sight of the platform), r_0 is the range to the target from the first SAR pass, ξ_i is the incidence angle, and Δh is the height above the ground of a scatterer [121]. The second relation relates the phase of the interferogram to the range difference between the two SAR passes, which is given by:

$$\phi = m \frac{2\pi}{\lambda} \Delta r \quad (2-31)$$

Where λ is the carrier wavelength, and m is a propagation factor that considers whether a monostatic or bistatic geometry has been used. ϕ is the overall phase of each pixel in the formed SAR interferogram, which is obtained via a process known as phase unwrapping [122].

2.3.3 SAR Point Cloud Generation System

Conventional phase unwrapping can be sensitive when imaging 3D target structures with significant discontinuities in their height profile, and the presence of layover can add further complications [123]. This was the original motivation behind the development of the SPCGS, the concepts from which this thesis uses to develop a sparse multistatic 3D SAR image formation algorithm [62].

The original literature of the SPCGS describes the summation of a set of 2D-formed interferograms, formed with a series of N sparsely and unevenly distributed passes. This process is given by the equation:

$$MIRF(h) = \frac{1}{K-1} \left| 1 + \sum_{k=1}^{K-1} \exp\{\varphi_{k,meas} - i\beta_k h\} \right| \quad (2-32)$$

Where $MIRF$ is the maximum interferometric response of a set of interferograms formed using the scatterer height estimate, h , which is applied to the measured phase, $\varphi_{k,meas}$. β_k is a term that is derived from the equations presented in (2-29) and (2-30), that relates the baseline width to the overall change in phase between two interferograms [124]. The values of h that maximise the $MIRF$ are taken to be the height of a scatterer.

Performing this process for 2D-formed interferograms requires for the correction of layover in the imagery. In the SPCGS literature, the change in layover, Δl , by:

$$\Delta l = h \frac{\Delta \psi}{\cos(\psi)} \quad (2-33)$$

Whilst the derivation for equation (2-33) is somewhat unclear, it is evident that this requires the grazing angle across the target scene to be constant. This is somewhat challenging for the collection geometries mainly studied in this thesis. The predictability of scatterer layover position is also dependent on the SAR wavefronts being planar. For bistatic and multistatic geometries, the target scene dimensions, under which these assumptions are valid, are for more restricted [104]. This is compared to conventional monostatic geometries.

2.4 SAR Polarimetry

2.4.1 Overview

Radar polarimetry analyses the effect of a scatterer on the electric field strength in a specified direction that is orthogonal to the direction of travel of the reflected radar pulse. The majority of polarimetric SAR analyses are derived from a matrix representation of the transmitted and received polarisation channels, this matrix is known as the Sinclair scattering matrix. The Sinclair matrix describes the transformation effect of a scatterer on the polarisation directions of transmitted radar signals that illuminate a target. The Sinclair matrix is generally presented in the form given below:

$$S = \begin{bmatrix} HH & HV \\ VH & VV \end{bmatrix} \quad (2-34)$$

Where the elements HH , HV , VH and VV describe the relative complex scattering amplitude of the measured polarisation channels. The first letter of each element represents the orientation of the transmitted polarisation, H is horizontal and V is vertical, and the second letter is the orientation of the received polarisation. Conventionally, the HV and VH are treated as being equivalent in what is known as the reciprocity relation. For bistatic geometries, and by extension multistatic geometries, this is not the case. The look angles of the transmitter and receiver can be assumed to differ greatly for bistatic geometries. This means that the defined horizontal and vertical polarisation axes are rotated between the frames of reference of the transmitter and receiver [125].

The horizontal and vertical polarisation directions are defined from the line of sight of the radar platform, from which the target scene is down range [33]. For this thesis, the horizontal polarisation is treated as being parallel to the ground plane and the vertical polarisation is treated as being perpendicular to the ground plane. Rayleigh scattering conditions are assumed in the polarimetric analyses throughout this thesis, where the wavelength of the incident radar signal is significantly smaller than the overall target dimensions [126].

The polarimetric techniques studied in this thesis will focus on coherent approaches, where the analyses are derived from the complex scattering amplitudes of each measured polarisation channel. There are other matrix representations used to formulate polarimetric parameters however, this thesis focuses on methods derived from the Sinclair scattering matrix. Another example of a polarimetric scattering matrix is the Kennaugh matrix, which is larger in dimensions than the Sinclair matrix. The larger dimensions mean that using this matrix is often more convoluted and computationally intensive [127].

The Sinclair matrix is applied to the transmitted polarisation state in the form presented in equation (2-35).

$$\begin{bmatrix} E_H^s \\ E_V^s \end{bmatrix} = \frac{e^{-ikr}}{r} S \begin{bmatrix} E_H^i \\ E_V^i \end{bmatrix} \quad (2-35)$$

Where E refers to the total amplitude of a given polarisation, H or V , and the subscripts “ i ” and “ s ” the incident and scattered polarisations respectively [128]. The range to the target in this case is given by r , and the wavenumber is given by k . Furthermore, the Sinclair matrix can also have a time-dependent factor which is generally suppressed for polarimetric SAR. The propagation factor, present in equation (2-34), is also suppressed for most polarimetric analyses.

A representation of the incident and scattered polarisation states can be formulated, with the time and spatially varying factors suppressed. This is known as the Jones vector, J . A Jones vector representation of the linear polarisation channels, presented in (2-33), is given by:

$$J = \begin{pmatrix} E_H \\ E_V \end{pmatrix} e^{i\delta} \quad (2-36)$$

E_H is the horizontal component of the Jones vector, E_V is the vertical component, and δ is the phase difference between the two channels [126], [128]. This thesis only investigates the use of linear polarisation channels, where there is a negligible phase difference between the complex amplitudes of the transmitted channels.

The Jones vector can be transformed to describe the symmetry of the measured polarisation states, and the orientation about the defined line of sight of the radar. It is noted that there are other polarisation states commonly utilised in polarimetric radar analyses, such as circular polarisations where there is a phase difference between the respective amplitudes of the vertical and horizontal polarisation channels. Although useful, these polarisation states are not studied in this thesis as little is understood of their behaviour when used with bistatic and multistatic SAR geometries.

This is represented by the polarisation ellipse shown in Figure 2-8 where the ellipticity angle is shown by τ and the orientation is shown by Θ . The polarisation ellipse is the effective locus of the measured polarisation, described at a single point in time. The rotation of the scattering matrix, S , is given by:

$$U_{(\theta,\tau)} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\tau & i\sin\tau \\ i\sin\tau & \cos\tau \end{bmatrix} \quad (2-37)$$

The transformation $U_{(\theta,\tau)}$ can be applied to the Jones vector, or the Sinclair scattering matrix [129].

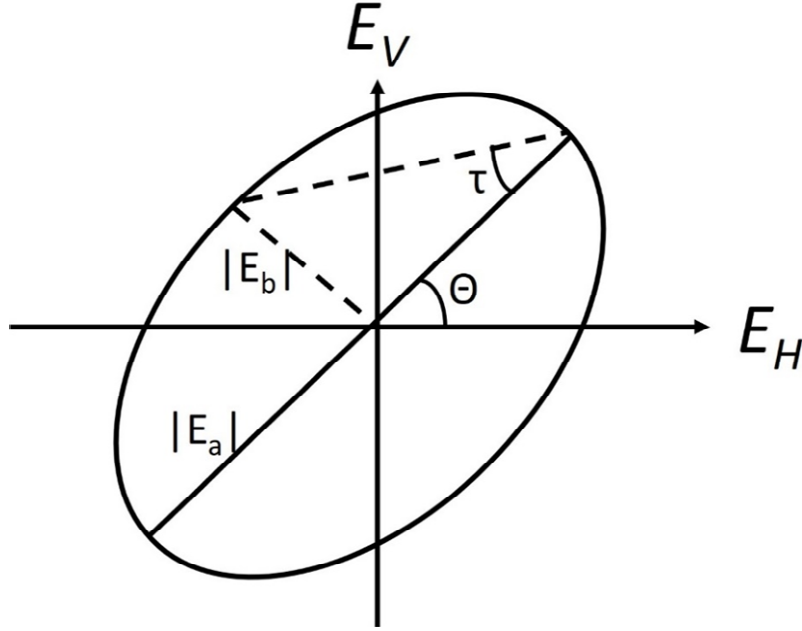


Figure 2-8 Representation of the polarisation ellipse. The measured polarisation channels are E_H and E_V . The orientation the polarisation states is denoted by θ . The ellipticity of the polarisation states is denoted by τ .

2.4.2 Development of Bistatic Polarimetric Parameters

2.4.2.1 Huynen Diagonalisation

The development of bistatic applicable polarimetric analyses has been of significant interest to the radar community since the late 1990s [130]. Bistatic polarimetric analyses are derived from the pioneering work of Huynen, who first developed the roll-invariant scattering matrix representation shown in (2-38). The Huynen parameters, developed in his PhD thesis, were extracted from the diagonalisation of the Sinclair scattering matrix [131]. S is presented in the form:

$$S = U^* S_d U^\dagger \quad (2-38)$$

The matrix U comprises the eigenvectors of the Scattering matrix, S , and the $*$ denotes the complex conjugate. The eigenvectors can be written in the form of the Jones vector, described previously. The ellipticity and orientation parameters, introduced earlier, are contained within the matrix, U . The diagonalised matrix, S_d , contains the eigenvalues of S . The eigenvalues of S contain the polarimetric properties that are independent of the orientation of the scatterer with respect to the radar platform. These properties are the polarisability of a scatterer, and the number of pulse deflections that its surfaces produce when illuminated by a radar. The symbol $\dagger$ refers to the conjugate transpose of the matrix U .

The eigenvalues of the diagonalised scattering matrix, S_d , are written in the form:

$$S_d = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad (2-39)$$

Where both λ_1 and λ_2 correspond to the eigenvectors contained within the matrix, U . The eigenvector corresponding to the largest eigenvalue, usually λ_1 , is used to define the polarisation state. This eigenvector will be referred to as the primary eigenvector in this thesis.

The derivation of equation (2-38) is only valid for symmetric matrices, which in the case of the Sinclair scattering matrix is the reciprocity between the VH and HV polarisation channels. As discussed previously, this is not valid for bistatic and multistatic collection geometries. Lueneburg and Cloude first applied the Singular Value Decomposition (SVD) method for diagonalising the case of non-symmetric scattering matrices, with the main intention being the application to bistatic scattering [132]. Physical meaning was later derived from the diagonalised bistatic scattering matrix, with the pioneering work of Titin-Schnaider [133], [134], [135]. This work is described in section 2.4.2.2 below.

2.4.2.2 Bistatic Huynen Parameter Development

The diagonalised bistatic scattering matrix, S_d , is given by the relation:

$$S_d = U^* S V^\dagger \quad (2-40)$$

Where S is again the Sinclair scattering matrix, and U and V are separate matrices that contain an independent set of coneigenvectors. U contains the eigenvectors of the matrix $S^\dagger S$, and V contains the eigenvectors of the matrix $SS^\dagger$. The eigenvalues obtained from the two combined matrices are the same.

The basis vectors contained within U and V can be both be viewed as a Jones vector representation of an ‘incident’ and ‘scattered’ polarisation state. Both U and V contain separate parameters for the orientation and ellipticity of their respective polarisation states. These are denoted as: Θ_i , Θ_s , τ_i and τ_s where the subscript ‘i’ denotes incident and the subscript ‘s’ denotes scattered, respectively, for the orientation angle Θ and ellipticity angle τ .

A computationally efficient approach, for extracting the bistatic Huynen fork parameters from the Sinclair matrix, was given by Xu et al. in 2022 [63]. From their approach, the incident orientation angle is given by:

$$\Theta_i = \frac{1}{2} \arctan \left[\frac{2|E_{vx}||E_{vy}|\cos(\Delta\phi_v)}{|E_{vx}|^2 - |E_{vy}|^2} \right] \quad (2-41)$$

Where E_{vx} is the horizontal component of the primary eigenvector contained within the matrix V . E_{vy} is the vertical component of the same eigenvector within V . $\Delta\phi_v$ is the phase difference between the polarisation states of the vertical and horizontal components within the same eigenvector. The parameter Θ_i was determined over the range $-45^\circ \leq \Theta_i < 0$. The scattered orientation angle is given by:

$$\Theta_s = \frac{1}{2} \arctan \left[\frac{2|E_{ux}||E_{uy}|\cos(\Delta\phi_u)}{|E_{ux}|^2 - |E_{uy}|^2} \right] \quad (2-42)$$

Similar to equation (2-41), $|E_{ux}|$ and $|E_{uy}|$ are the magnitudes of the primary eigenvector within the matrix, U . $\Delta\phi_u$ also refers to the phase difference between the horizontal and vertical components within the same eigenvector. The parameter Θ_s was extracted over the range $0 \leq \Theta_s < 45^\circ$.

The symmetry parameters were also presented in the same publication. The incident symmetry parameter, τ_i , is given by:

$$\tau_i = \frac{1}{2} \arcsin \left[\frac{2|E_{vx}||E_{vy}|\sin(\Delta\varphi_v)}{|E_{vx}|^2 + |E_{vy}|^2} \right] \quad (2-43)$$

Which uses the same magnitudes and phase differences as those used in (2-41). Similarly, the scattered symmetry, τ_s , parameter is given by:

$$\tau_s = \frac{1}{2} \arcsin \left[\frac{2|E_{ux}||E_{vy}|\sin(\Delta\varphi_u)}{|E_{ux}|^2 + |E_{uy}|^2} \right] \quad (2-44)$$

Where the magnitudes and phase differences correspond to those use in (2-42). An example of a scatterer that is considered highly symmetrical is a sphere, where the absolute value of the sum of the τ_i & τ_s is approximately zero. Conversely, a dipole is a highly asymmetrical scatterer, where both the τ_i & τ_s are approximately zero. In some publications, the symmetry angle is referred to as the ellipticity angle; this terminology is not used in this thesis.

Huynen, and later Titin-Schnaider, wrote the diagonalised scattering matrix, now denoted as S_A , in the form:

$$S_A = \mu e^{i\kappa} \begin{bmatrix} e^{2i\nu} & 0 \\ 0 & \tan^2\gamma e^{-2i\nu} \end{bmatrix} \quad (2-45)$$

Where ν is known as the skip parameter, and γ is the polarisability parameter. μ and κ are separate components that do not play any role in characterising the polarimetric properties of a scatterer, thus they are ignored in the literature.

The polarisability angle, γ , is given by:

$$\gamma = \arctan \left(\sqrt{\frac{\lambda_2}{\lambda_1}} \right) \quad (2-46)$$

The polarisability angle is defined between 0° and 45° , where 0° indicates a high level of polarisability and 45° a low level of polarisability. The skip angle, ν , is given by:

$$\nu = -\frac{1}{4} \arg\left(\frac{S_A(2,2)}{S_A(1,1)}\right) \quad (2-47)$$

The skip angle determines the number of bounces and is also valid between 0° and 45°. 0° is typical of a scatterer that produces an odd number of bounces, and 45° is expected of a scatterer that produces an even number of bounces.

2.4.3 Example Renderings

This subsection compares renderings presenting the Huynen-derived skip parameter with the more conventional odd-even bounce polarimetric decomposition, using MIPs formed of the ‘Backhoe Challenge’ dataset described in section 1.1.

2.4.3.1 Odd-Even Bounce Decomposition Plots

The total power radiated by the odd-bounce component, P_{odd} , is given by:

$$P_{odd} = \frac{1}{2} |HH + VV|^2 \quad (2-48)$$

This relation is derived from the scattering matrix of a flat plate scatterer, which is described in the publication presented in Chapter 5. The total power radiated by the even-bounce component is given by:

$$P_{even} = \frac{1}{2} |HH - VV|^2 + 2|HV|^2 \quad (2-49)$$

P_{odd} and P_{even} are often combined in a single image by assigning a red-green colourmap to each component. The odd and even-bounce power components shown in the polarimetric MIPs presented below.

Figure 2-9 shows a MIP in the YZ imaging plane, where the projected voxel corresponds to the maximum polarimetric-span brightness of each voxel in the defined X-axis.

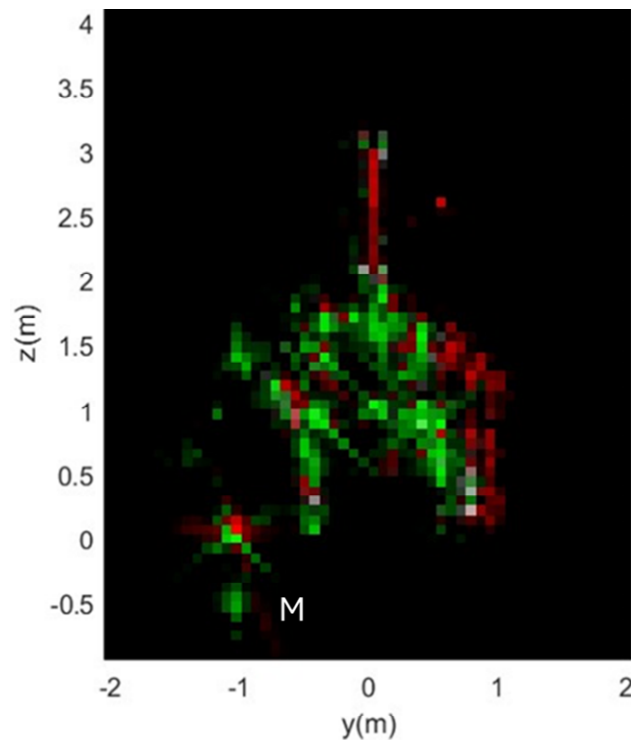


Figure 2-9: Maximum Intensity Projection of the Backhoe vehicle in the YZ imaging plane, giving a view of the front cross section. Polarimetric information is associated with the colour scheme presented, with red representing an odd number of bounces and green representing an even number of bounces. Multipath is labelled 'M'.

The multipath visible below the shovel, towards the bottom left of Figure 2-9, shows an even number of bounces. This is an expected feature of multipath propagation. Figure 2-10 shows the odd-even bounce decomposition, projected in the XZ imaging plane. Here, the projected voxel corresponds to that of the largest polarimetric-span brightness in the Y-axis. The top of the cabin side, and the rear arm, visibly show a strong single bounce scattering response. The front of the shovel shows an even bounce scattering response, as would be typical of a dihedral scatterer.

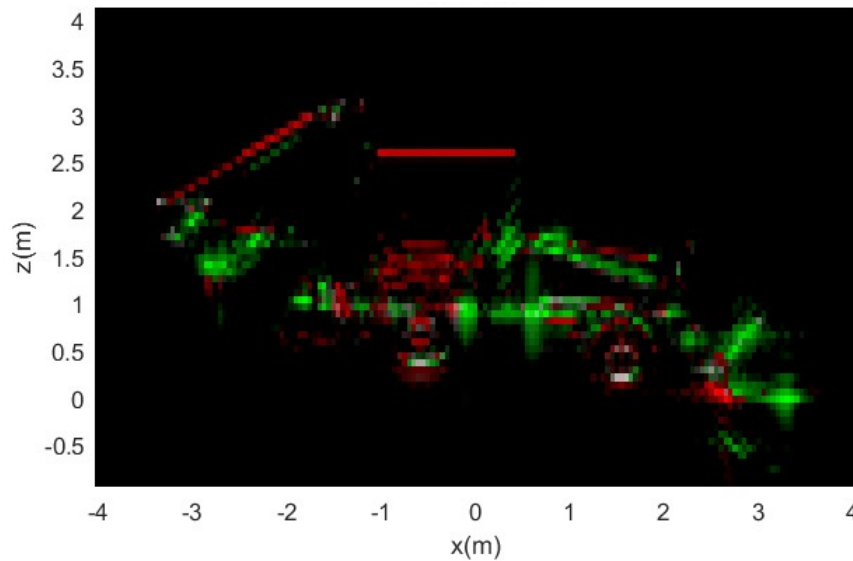


Figure 2-10: Maximum intensity projection of the Backhoe vehicle, in the XZ imaging plane. This shows a side view of the tank, with the odd-even bounce decomposition represented by the red-green colourmap.

2.4.3.2 Skip Parameter Plots

This sub-section shows the plot of the skip parameter that corresponds to the MIPs presented in section 2.4.3.1. Figure 2-11 shows the MIP in the YZ-imaging plane, with the skip parameter associated with the brightest voxel, again determined by the respective polarimetric-span intensity, in the X-axis. The corresponding skip parameter plot in the XZ-imaging plane is shown in Figure 2-12.

Comparing the plots of the skip parameter to those showing the odd-even bounce decomposition, there is little difference between the two methods. This is most evident in the strong odd-bounce scattering response in Figure 2-11 and Figure 2-12, showing the upper right hand side of the backhoe cabin. The multipath below the shovel also shows an even bounce response when determined by the skip parameter.

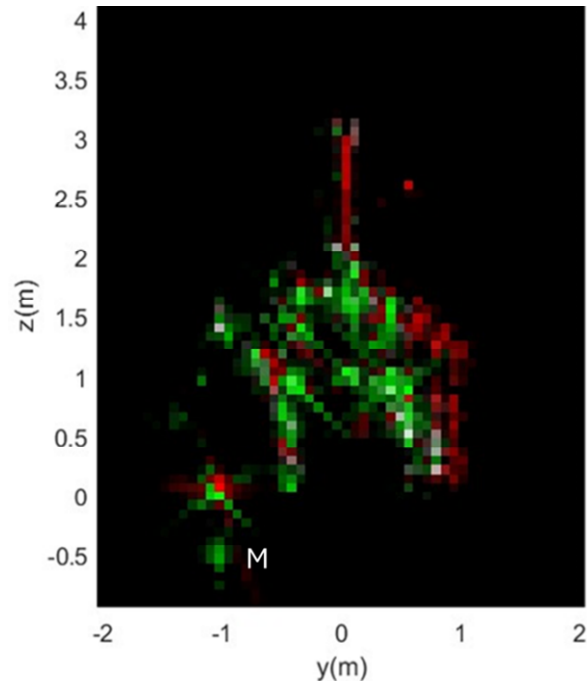


Figure 2-11: MIP of the Backhoe vehicle in the YZ imaging plane, with the skip angle associated with each voxel location. Red represents odd-bounce scattering and green represents even-bounce scattering. Multipath denoted “M”.

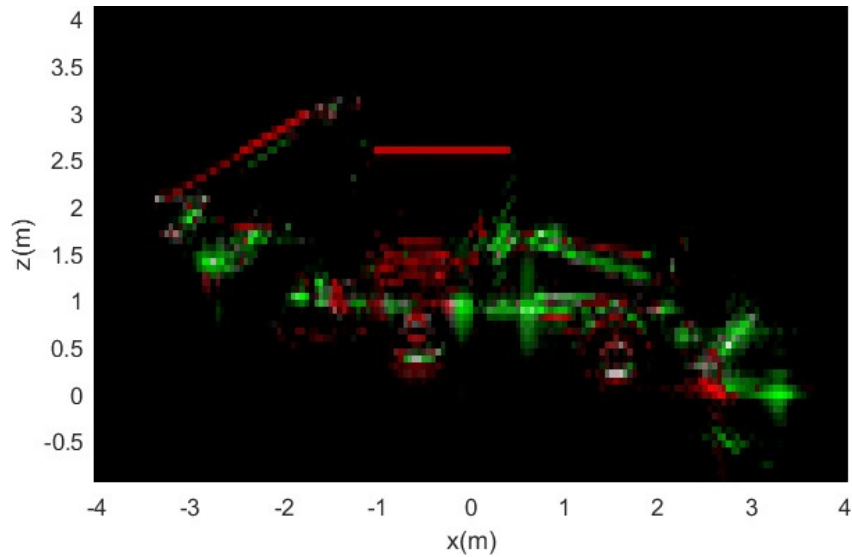


Figure 2-12: MIP of the Backhoe vehicle in XZ imaging plane, giving a side view of the tank structure. The variation of the skip parameter across the vehicle is again represented by the red-green colourmap.

2.5 Ground Based SAR Laboratory

The measurements conducted at the Cranfield GBSAR laboratory are described throughout the publication chapters of this thesis. This section gives an overview of the experimental set up at this facility.

The SAR scanning laboratory and system is shown in Figure 2-13, with each antenna mounted on a separated frame. Each frame contains an independent moveable arm that can sweep out a finely sampled 2D SAR aperture. The antennas can also be mounted on the same axis for a monostatic collection.



Figure 2-13: Experimental set up of Cranfield GBSAR laboratory. The antennas shown here are in the bistatic/multistatic configuration [136].

The GBSAR scanner system is a custom-built apparatus that controls and changes the position of antennae, providing an adjustable SAR aperture with finer than 1mm accuracy for positioning. To reduce doppler ambiguities, and aid with algorithm development, each antenna was stationary for the duration of each pulse. This is in addition to the target being stationary.

A detailed summary of the hardware deployed in the GBSAR laboratory can be found here [136]. The main hardware included:

- Two antenna horns:

- these were A-Info UWB dual polarisation Antenna horns, 1–10 GHz, LB-SJ-10100,
- An Agilent (now Keysight) Vector Network Analyser (VNA) model N5230A,
- Microwave cables with sufficient length for bistatic positioning,
- Hewlett Packard wide band microwave amplifier connected to the receive VNA port,
- Two, Two-dimensional vertical plane positional scanner in system,
 1. The first two-axis scanner, generally with just the transmitter, but also taking the receiver for monostatic scans, allowing positioning in a vertical rectangle 3500 mm in the horizontal and 1450 in the vertical;
 2. The second two-axis scanner, generally with just the receiver, allowing positioning in a vertical rectangle 1285 mm in the horizontal and 1450 in the vertical.
- Two scanner controller boxes, each controlling one of the two-axis scanners.
- Control and data recording computer connected to the VNA and two scanner controller boxes.

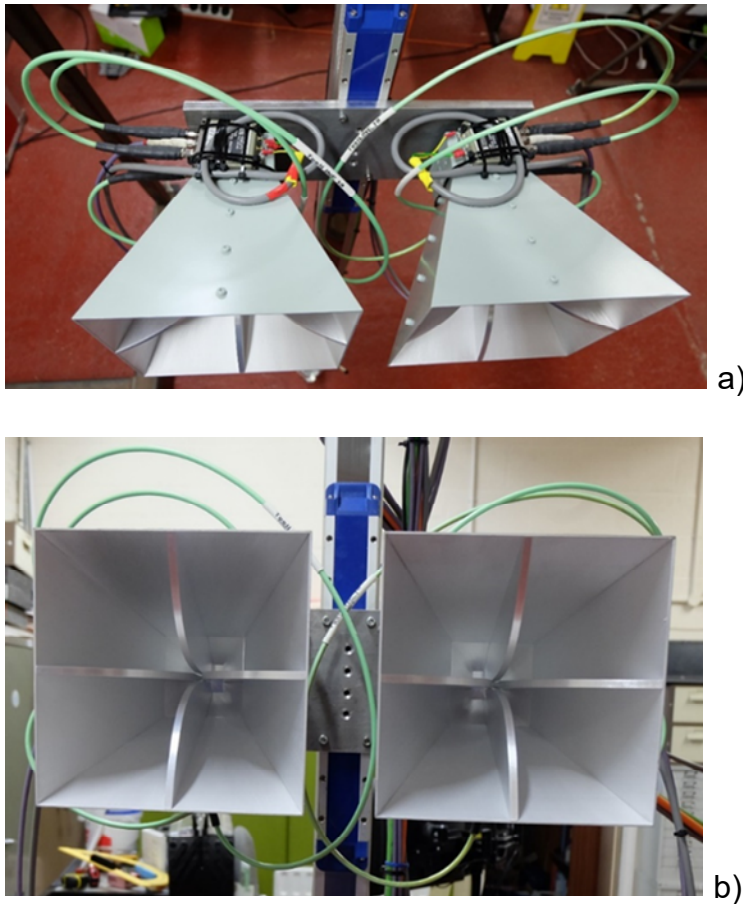


Figure 2-14. Dual-polarimetric horns with a microwave switch on each. Polarimetric switching is here performed automatically by the scanning system [136].

2.6 Summary of Chapter

This chapter gives an overview of the main theoretical concepts and publications that are relevant to the work carried out in this thesis. The fundamental mathematical principles behind the operation of SAR were introduced, which is mainly the Fourier slice theorem. This underpins all of SAR, and therefore this thesis. The effect of bistatic and multistatic geometries on SAR image formation was then discussed, and then the challenges that arise from this. The extension of SAR principles to 3D SAR was discussed, and various sparse 3D SAR image formation techniques were presented. The one that is most relevant to the SSARVI algorithm developed in this thesis, the SPCGS, was discussed in detail. An overview was given of tangential 3D SAR image formation algorithms. The bistatic Huynen fork parameters were introduced, which were used to extend the

SSARVI algorithm to polarimetric imaging in the final publication chapter of this thesis. Finally, an overview of the Cranfield GBSAR laboratory was given.

3 Volumetric SAR Interferometry for Sparse 3D SAR with Bistatic Geometries

3.1 Abstract

Synthetic Aperture Radar (SAR) renderings in 3D provide additional target information when compared to 2D by separating out features overlaid in height. However, the required 2D SAR aperture, when Nyquist sampled, necessitates large scanning times that would be impractical for most realistic collections. This research has developed a novel volumetric approach to sparse aperture 3D SAR imaging, that is applicable to bistatic SAR near-field geometries, a generalization of far-field cases. This approach is first demonstrated in simulation and then applied to a measured scene containing a model vehicle target, producing sub-Nyquist sampled 3D SAR renderings.

3.2 Introduction

Conventional finely sampled volumetric images can be impractical to produce from realistic SAR collections, as achieving suitable resolution and coverage in the vertical direction requires a finely sampled 2D SAR aperture, necessitating many radar platform horizontal flight passes with varying height over the scene [27], [137]. Producing volumetric renderings with few passes would typically be insufficient to satisfy the Nyquist sampling criterion in the vertical direction, leading to image wrap-around aliasing artefacts [3].

Several methods have been developed for forming 3D renderings from sparsely sampled SAR datasets, allowing for fewer collection passes over a scene, including SAR interferometry, the SAR Point Cloud Generation System, and 3D SAR compressive sensing [55], [62], [138].

Here, a novel interferometry-based approach for sparsely sampled aperture 3D SAR imaging has been developed, that can be used for bistatic and SAR near-field geometries, which are generalisations of both monostatic and SAR far-field

geometries. The SAR near-field is defined as the regime where the range $d < 2L_{cr}^2/\lambda_c$ for centre wavelength λ_c and for a scene with cross range extent L_{cr} . No phase unwrapping or intensive signal reconstruction methods are required for the developed approach, as is the case for interferometry and compressive sensing respectively. Conventional SAR interferometry is sensitive to structures with significant discontinuities in their height profile, which is problematic for imaging complex targets [139]. The measured data used to demonstrate the algorithm was collected at the Cranfield University Ground-Based SAR (GBSAR) laboratory [67], [68].

3.3 Outline of Approach

Sparse 3D point cloud renderings have previously been formed from simulated far-field interferometric SAR data using monostatic collection linear trajectories [119]. The approach presented in [62], [140] utilises the change in layover of scatterers when processing the interferogram phase factor of the returned signals, assuming a constant radar trajectory height for each of the separate passes. However, the assumptions made, for example that the grazing angle is constant across the scene, are not generally valid for SAR near-field geometries. Bistatic geometries, where the transceivers have a non-negligible separation, add further complexities [95], [141], [142].

The new approach described here exploits volumetric processing of interferograms to account for the effects of layover in the processing of the data in a natural way. Volumetric interferograms are formed from volumetric SAR images generated with the Backprojection Algorithm (BPA) [143], which accounts for complications associated with bistatic and SAR near-field geometry imaging [144].

Linear horizontal radar platform passes are employed to form the individual volumetric SAR images, and these linear passes should be distributed in height in an irregular manner to avoid strong aliasing artefacts in the final 3D scene rendering. Although the initial individual volumetric images have no vertical resolution, when they are combined, 3D information is obtained due to the variation of collection grazing angle across the set.

As outlined in Figure 3-1, N sparsely and non-evenly distributed in height horizontal linear SAR apertures are collected and formed into volumetric SAR images. Having chosen one of these to be the common reference image, N-1 volumetric interferograms are then formed.

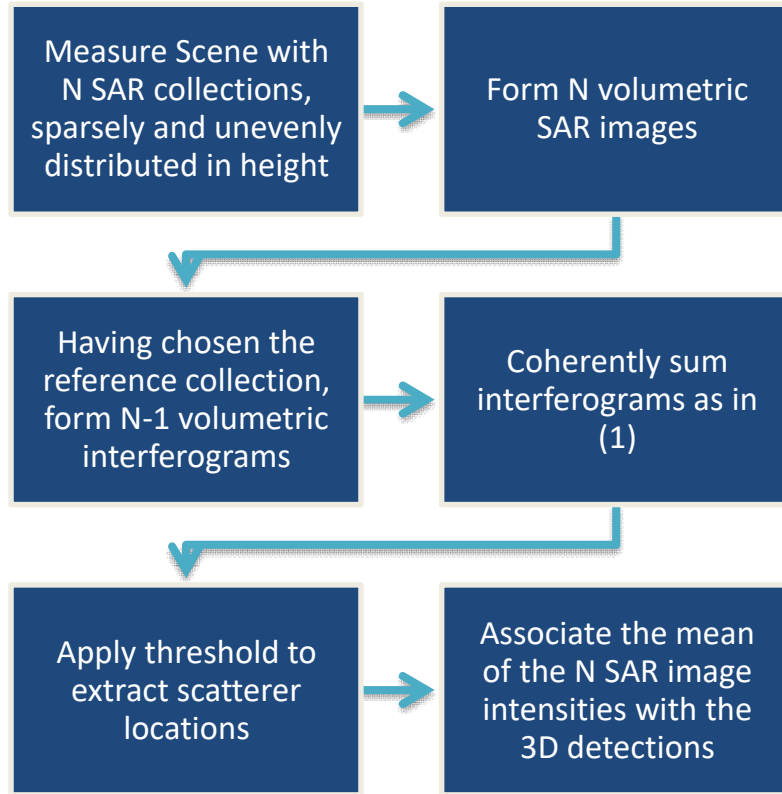


Figure 3-1: Flow chart illustrating key principles of new approach.

The phase factors of the interferograms are coherently summed for each voxel, x , giving the volumetric summed phase factor response at the voxel,

$$R(x) = \frac{1}{N} \left| 1 + \sum_{n=1}^{N-1} e^{i\varphi_n(x)} \right| \quad (3-1)$$

Where $\varphi_n(x)$ is the phase at voxel x of the n^{th} interferogram. Point scatterer detections are determined by applying a threshold to $R(x)$. Scatterer detections surpassing the threshold form part of an output 3D point cloud rendering. To each

of these detections, it is useful to associate the corresponding mean sparse-volumetric SAR image intensity value at the voxel.

3.4 Simulations

Bistatic near-field SAR simulations were conducted with sparse 2D apertures, consisting of five linear horizontal transmitter SAR apertures distributed in height with a stationary receiver positioned to the side. An example rendering of the simulation geometry is shown in Figure 3-2 below.

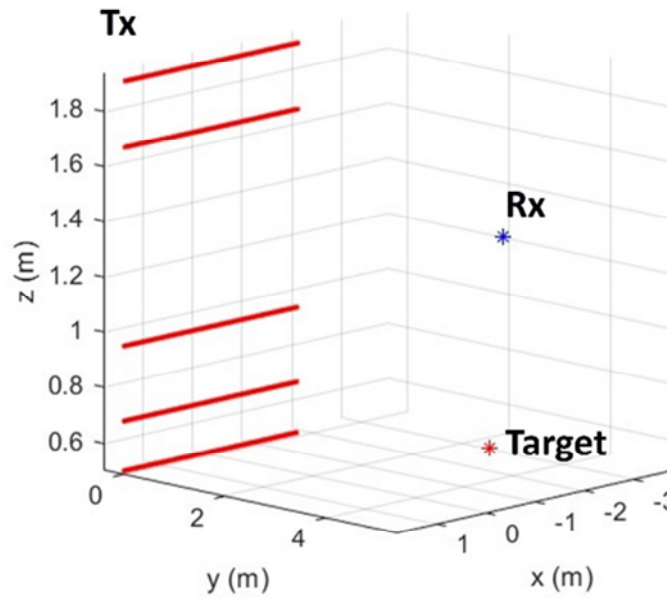


Figure 3-2: Bistatic simulation geometry, consisting of five passes and a single receiver location. Target situated at ($z=0$ m) in height.

Three height distributions, labelled ($\hat{\sigma}_1 - \hat{\sigma}_3$), were implemented, with fixed overall vertical extent of 1.4 m. The standard deviation of normalized transmitter pass height spacing (pass height spacing divided by its mean) ($\hat{\sigma}$) provides a measure of unevenness of the height sampling. The frequency range measured was 6-10 GHz and the distance to the target from the transmitter aperture was 5.0 m.

Figure 3-3 shows R along a vertical iso-range line through a point scatterer situated at 0 m in height, for the three SAR geometries ($\hat{\sigma}_1 - \hat{\sigma}_3$), as follows:

- a) an even distribution of passes is employed with height spacings of 35 cm, so that $\hat{\sigma} = 0$. This results in large scatterer height ambiguities 52 cm from the true scatter position, with $R = 1$, so that unique scatterer localization is not possible,
- b) two of the inner passes are offset by 5 cm so that $\hat{\sigma} = 0.17$ resulting in a reduction in ambiguity sidelobe height to just over $R = 0.8$, and thus an accurate height estimation of the scatterer position can be made with appropriate threshold in R ,
- c) the transmitter pass heights are further randomized so that $\hat{\sigma} = 0.52$, and sidelobes have been further suppressed to under $R = 0.7$.

The examples show that with unevenly spaced trajectories, the application of a suitable threshold to R can allow the detection of point scatterers at their correct locations.

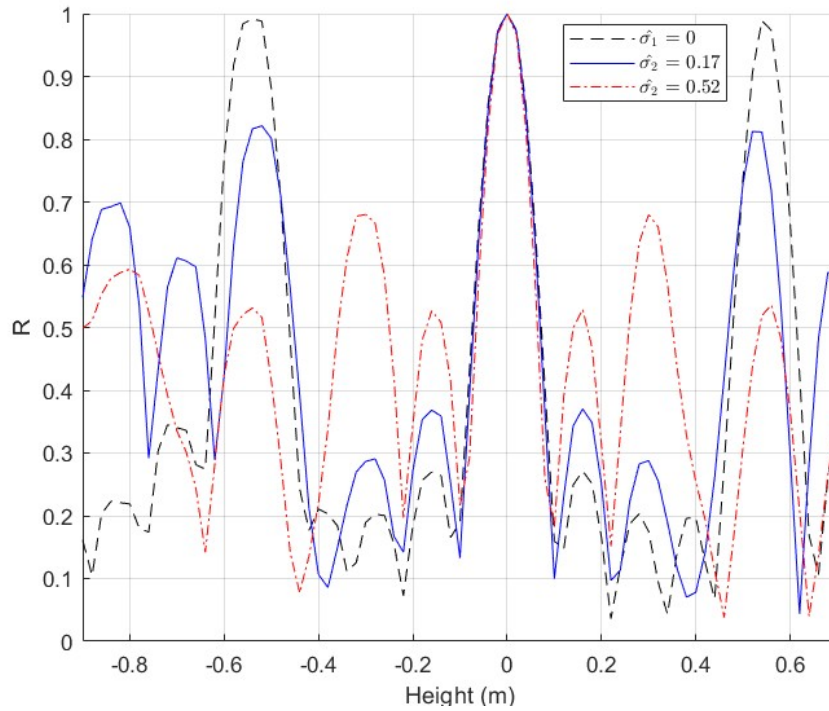


Figure 3-3: Summed phase factor, R , along a vertical iso-range through the target location ($z=0$ m), with trajectory $\hat{\sigma}_1=0$, $\hat{\sigma}_2=0.17$ and $\hat{\sigma}_3=0.52$.

The simulations presented in Figure 3-4 show the summed phase factor response, R , in the presence of background noise. The simulation used a signal-to-noise ratio (SNR) of 6dB, and comparable collection geometries to those used for Figure 3-3.

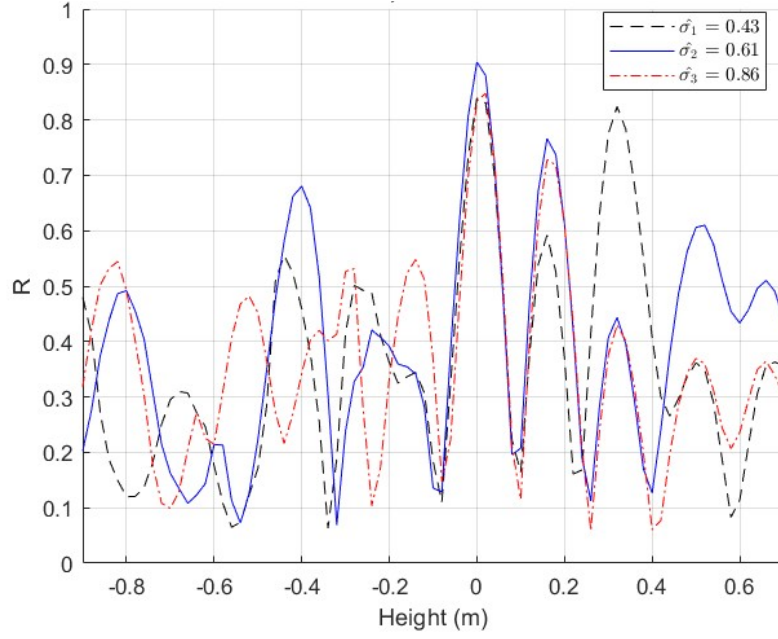


Figure 3-4: Simulated summed phase response, R . SNR of simulation was set to 6dB. Transmitter apertures of 5 passes were used, with an increasingly uneven distribution of pass spacings.

For all three pass distributions, in Figure 3-4, the value of R at the scatterer height was below 1. The sidelobe height increases in the presence of background noise, with $\hat{\sigma}_1$ and $\hat{\sigma}_2$ being of a comparable pass distribution to that used in Figure 3-3. The sidelobe heights for these apertures were $R = 0.82$ and 0.75 respectively, which is considerably higher than those presented in Figure 3-3. The sidelobe height $\hat{\sigma}_3$ was $R = 0.72$. The difference between the peak of R and the largest sidelobe, denoted as was 0.03 , 0.13 and 0.11 for $\hat{\sigma}_1$, $\hat{\sigma}_2$ and $\hat{\sigma}_3$ respectively.

It was generally found that if trajectory distributions with $\hat{\sigma}$ greater than approximately 0.6 were employed, ΔR began to decrease. This behaviour can be attributed to trajectory clustering. For scenarios of low SNR, lower sidelobes are required for correct target estimation, such that a more uneven distribution of pass spacings are required for greater sidelobe reduction. A more typical summed phase factor response, with similar sidelobe heights to simulations without background noise, was observed with SNR values in the range of 12-14 dB. A distinct peak close to the maximum value of 1 was also observed within this range of SNR values. These observations were made using comparable collection geometries to those used in Figure 3-3 and Figure 3-4.

3.5 Measured Data Validation

The measured scene consisted of a quarter scale tank model situated on gravel. A finely sampled vertical 2D SAR transmitter aperture was initially measured, for monostatic and bistatic geometries, where in the bistatic geometry the receiver was fixed and offset to the side. The SAR transmit apertures had vertical and horizontal extents of 1.4 m and 3.5 m, respectively. A frequency range of 6.62-10 GHz was measured in the VV linear polarisation channel. A diagram of the fully collected bistatic aperture is shown in Figure 3-5.

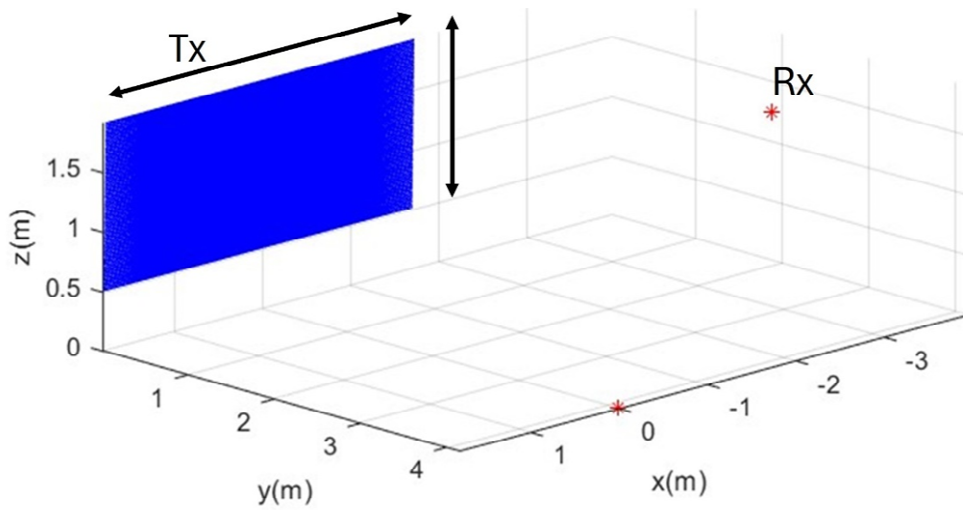


Figure 3-5: 2D Bistatic SAR geometry, finely sampled in elevation. Transmitter aperture is represented by the blue trajectory diagram (labelled Tx). Stationary receiver position is represented by the red asterisk to the right of the transmitter aperture (labelled Rx).

The finely sampled 2D SAR geometry presented in Figure 3-5, along with the corresponding monostatic geometry, was repeatedly downsampled for determining the minimum sampling requirements of the developed algorithm. The apertures were downsampled by removing random passes from the middle of the aperture, whilst keeping the overall extent constant.

The imaging performance of the developed algorithm was determined over a subset of the downsampled geometries. The most well-sampled aperture, within the downsampled subset, had a mean pass spacing that was equivalent to the Nyquist spacing for a 1.8 m high Volume of Interest (Vol) when using

conventional image formation algorithms. This aperture sampling is referred to as the sub-initial sampling (SIS), of which the mean pass spacing was 7.8 cm and 5.3 cm for bistatic and monostatic geometries, respectively. The apertures downsampled to below SIS can be considered sub-Nyquist sampled in elevation. The individual transmitter heights sampled of the SIS apertures and below are presented in Figure 3-6 and Figure 3-7.

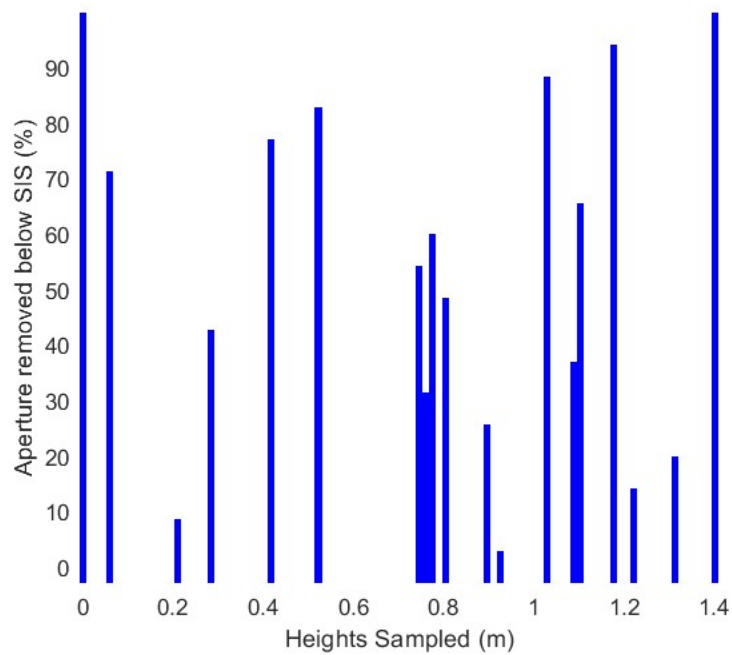


Figure 3-6: Downsampled transmitter height apertures, using a bistatic geometry. Again, blue lines indicate individual SAR heights, fully sampled in azimuth, remaining after each successive downsampling. White spaces consist of fully removed SAR passes

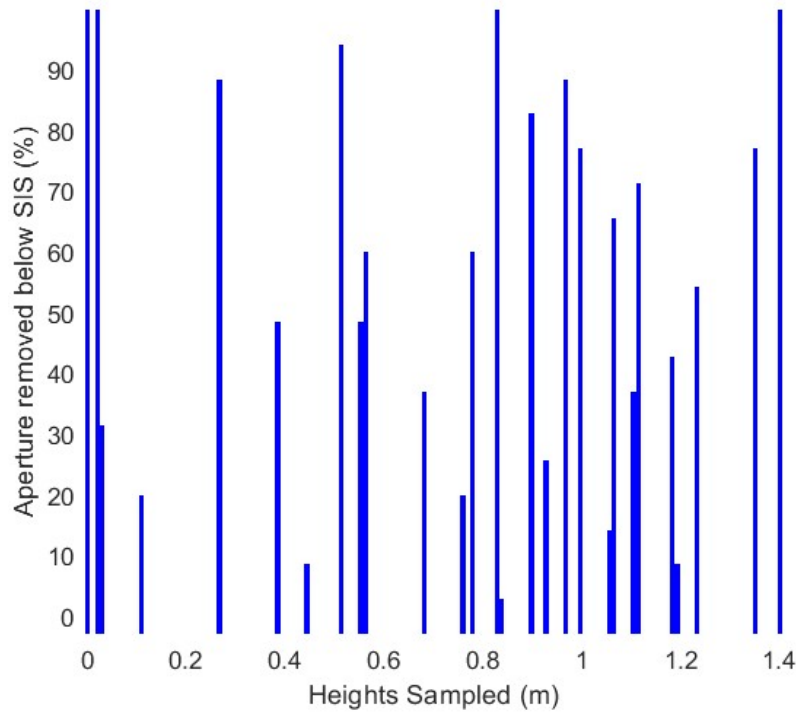


Figure 3-7: Downsampled transmitter height apertures, using a monostatic geometry. Blue lines indicate individual SAR heights, fully sampled in azimuth, remaining after each successive downsampling. White spaces consist of fully removed SAR passes.

It is noted that all of the transmitter apertures presented in Figure 3-6 and Figure 3-7 contain unevenly distributed SAR pass heights, as is required for the developed algorithm. Whilst the mean spacing of the first bistatic and monostatic SIS aperture corresponded to the Nyquist spacing for the Vol, parts of the aperture were sub-Nyquist sampled due to the uneven distribution of passes. The apertures downsampled to below SIS had a larger mean pass spacing, and thus lower sampling rate, than the Nyquist sampling requirement.

The developed algorithm was assessed by comparing the sparsely sampled 3D point cloud images produced to finely sampled BPA-formed volumetric renderings. Figure 3-8 shows normalised cross-correlation results between full-Nyquist BPA images (bistatic and monostatic) and the corresponding downsampled aperture point cloud results. For generating the sub-Nyquist point clouds, a histogram-based thresholding was implemented in the developed algorithm. The threshold was set by the difference between the maximum value of R and its modal value across the scene, and then selecting voxels in the upper

25% of this range. Similar approaches are taken in other image processing applications [145].

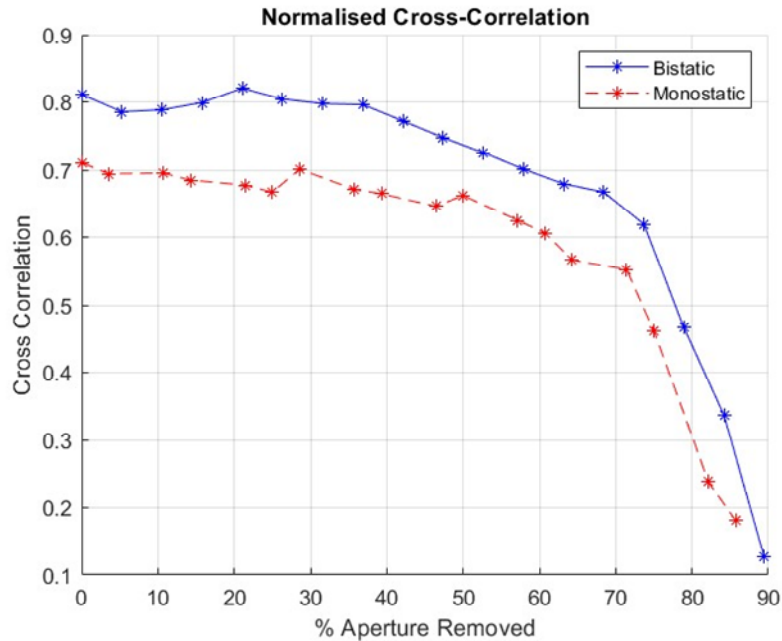


Figure 3-8: Normalised cross-correlation of sparse point cloud renderings with the corresponding full aperture BPA volumetric SAR geometry images.

Figure 3-8 initially shows a high normalised cross-correlation with the full-Nyquist BPA rendering, for both monostatic and bistatic geometries, with a low rate of decline in correlation until 70% sub-initial sampling (SIS) as the sampling density decreases (by “percentage SIS” it is meant the percentage of aperture removed from the initial uneven sampling). Only minor erroneous detections are present in the SIS ranges from 58-74% and 39-71% for the monostatic and bistatic renderings respectively.

Example point cloud renderings without errors are presented in Figure 3-9 and Figure 3-10 with SIS values of 50% and 37% for monostatic and bistatic, the tank structure is discernible with cannon and main body as distinct features. The gravel below the tank is also visible. Here, the monostatic and bistatic apertures have 14 and 12 passes respectively, with mean height spacings of 10.9 cm and 12.8 cm respectively.

The clustering of points detected for the bistatic rendering is visibly larger than the monostatic rendering due to the coarser underlying SAR resolution of this antenna geometry as derived from the overall SAR aperture extents. There are no aliasing artefacts present in either of the renderings, which was not the case for the corresponding conventionally formed BPA sub-Nyquist SAR aperture volumetric images.

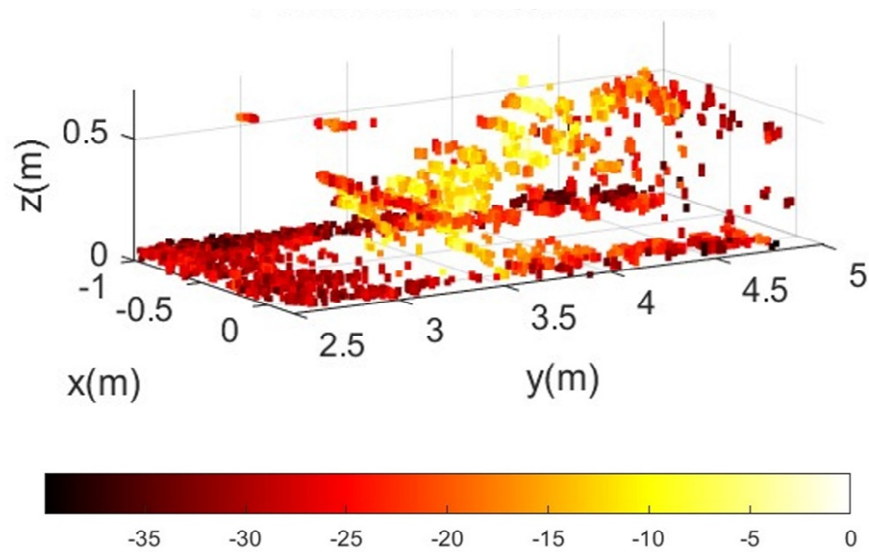


Figure 3-9 Model tank measurement sparse aperture point cloud renderings, 50% SAR aperture removed for a monostatic SAR geometry.

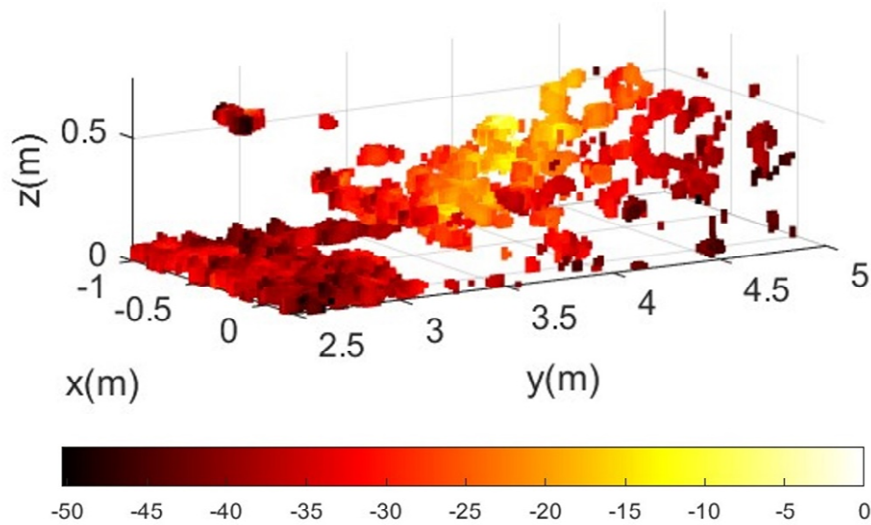


Figure 3-10 Model tank measurement sparse aperture point cloud renderings, 37% SAR aperture removed for a bistatic SAR geometry.

Erroneous detections increase dramatically after the 74% and 71% sub-Nyquist limits for the monostatic and bistatic case respectively. After this limit, the tank structure is not discernible from erroneous background detections. This limit corresponds with the sharp decrease in correlation with the BPA rendering (Figure 3-6). The mean spacing between passes at this limit was 47.0 cm and 23.6 cm for the bistatic and monostatic geometries respectively, which for evenly sampled heights would have given vertical unambiguous cross-ranges smaller than the height of the tank structure when using the BPA algorithm on evenly sampled data. The model tank point cloud renderings at these respective apertures are shown in Figure 3-11 and Figure 3-12

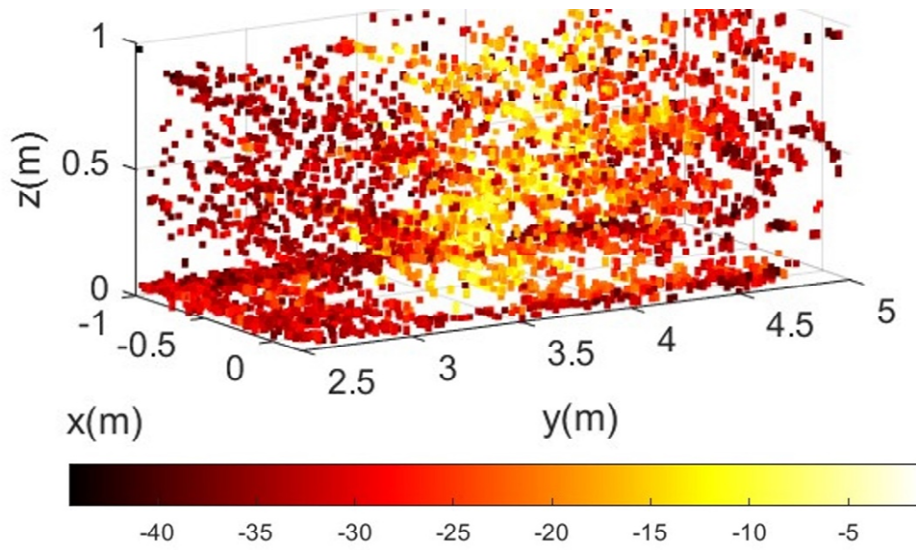


Figure 3-11: Example rendering of a T72 point cloud, past the limit of discernibility. Formed using a monostatic collection geometry, with 74% aperture removed.

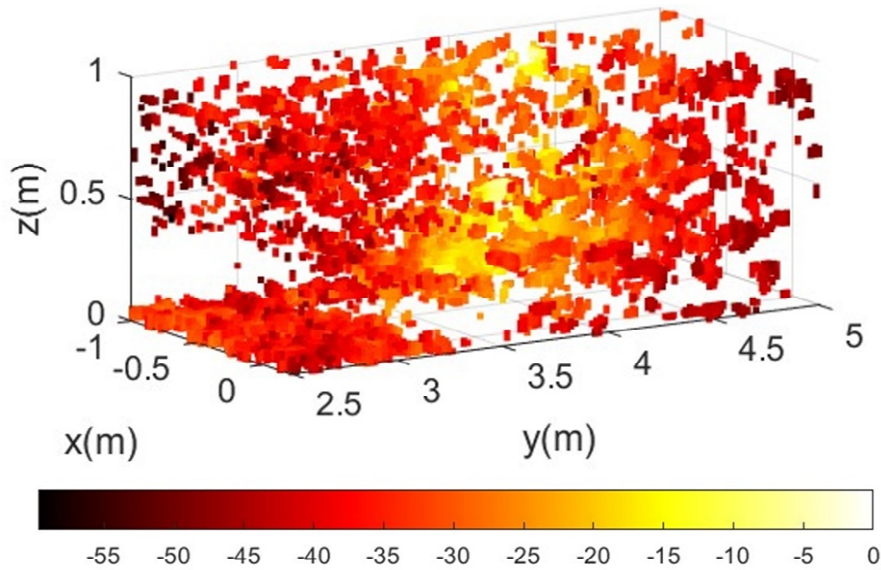


Figure 3-12: Example rendering of a T72 point cloud, past the limit of discernibility. Formed using a bistatic collection geometry, with 71% aperture removed.

3.6 Conclusions

The developed 3D point cloud imaging approach produces accurate height estimates of scatterers using bistatic SAR near-field geometries, which is a

generalisation of the monostatic far-field case. The simulations showed the effect of pass distribution on sidelobe height, and also the effect of background noise. Simulations were validated with complex model target laboratory measurements, forming sub-Nyquist SAR aperture point cloud renderings. Good quality results were obtained for SIS up to 50% and 37% for the monostatic and bistatic cases respectively. The nature of the SAR near-field geometries tested show the versatility of the approach.

The determined downsampling limits indicate the minimum sampling needed to obtain useful 3D target information with no aliasing artefacts. The approach is applicable to non-linear sparse aperture SAR collections employing formation flying between pairs of transceivers, including UAV's and satellite constellations as examples.

Future research will go into more detail on the effect of pass distribution on sidelobe height and will demonstrate the new approach on multistatic geometries. The incorporating of polarimetric information will also be explored.

4 Laboratory Multistatic 3D SAR with Polarimetry and Sparse Aperture Sampling

4.1 Abstract

With the advent of constellations of SAR satellites, and the possibility of swarms of SAR UAV's, there is increased interest in multistatic SAR image formation. This may provide advantages including allowing three-dimensional image formation free of clutter overlay; the coherent combination of bistatic SAR geometries for improved image resolution; the collection of additional scattering information, including polarimetric. The polarimetric collection may provide useful target information, such as its orientation, polarisability, or number of interactions with the radar signal; distributed receivers would be more likely to capture any bright specular responses from targets in the scene, making target outlines distinct. Highlight results from multistatic polarimetric SAR experiments at the Cranfield University GBSAR laboratory are presented, illustrating the utility of the approach for fully sampled 3D SAR image formation, and for sparse aperture SAR 3D point-cloud generation with a newly developed volumetric multistatic interferometry algorithm.

4.2 Introduction

With the advent of constellations of Synthetic Aperture Radar (SAR) satellites, and the possibility of swarms of SAR Unmanned Aerial Vehicles (UAV)'s [146], [147], there is increased interest in multistatic SAR image formation.

These types of collections may provide advantages such as finer spatial resolution improvement, and allowing three-dimensional (3D) image formation free of clutter overlay [35], [109], [148], [149].

The coherent combination of bistatic SAR geometry collections allow for improved image resolution; the collection of additional scattering information, including polarimetric [150], [151], [152].

Polarimetric collections allow for more scene data to be collected and may provide additional target information, such as scatterer orientation, polarisability,

or number of inter-actions with the radar signal [131], [133], [134], [135]. Furthermore, distributed receivers would be more likely to capture any bright specular responses from targets in the scene, making target outlines distinct.

Obtaining 3D SAR imagery is desirable for object recognition, though providing sufficient SAR aperture sampling in the vertical dimension can be challenging. Hence there is a need for the development of new image formation algorithms that only require sparsely sampled apertures [27], [54], [153].

Several approaches to forming 3D SAR scene renderings from sparsely sampled collections have been developed. These include dual-pass SAR interferometry, 3D SAR compressive sensing and the SAR Point-Cloud Generation System (SPCGS) [3], [55], [62]. Dual-pass SAR interferometry is used to construct Digital Elevation Models (DEMs) of target scenes and can involve phase unwrapping methods. This method often struggles with manmade targets due to scene height discontinuities which can give rise to several scatterers of different height being overlaid. The approach developed for the SPCGS estimates 3D locations of scatterers by summing the phase factors from a series of 2D SAR interferograms collected with a sparse set of elevation angles. This method utilises the change in layover within a stack of 2D images, with the assumption that the SAR collections are monostatic, are in a far-field scenario, and have constant grazing angle across the scene. The approach has been successful in forming target renderings from simulated monostatic datasets [140]. However, the assumptions made for this approach would not generally hold true for data collected with SAR satellite constellations and UAV-SAR drone swarms. This is due to their use of multiple receiver locations being significantly separated from that of the transmitter. Processing these collection geometries can introduce distortions to the SAR scatterer layover due to their asymmetrical nature [95], [141], [154].

Sparse reconstruction methods, such as compressive sensing, have been the focus of recent tomographic SAR research. This approach has mainly achieved success in areas such as urban mapping. Such scenarios can be relied upon for having largely empty scenes, which is exploited in the sparse reconstruction approach [22]. The application to vehicle targets has been limited thus far, with

simple 3D renderings being produced of model tanks [155]. Applying this technique to sparse 3D swarm sensing may be complicated by the need to estimate scatterer reflectivity for multiple bistatic angles, although simulated 3D renderings have been formed [156].

Results from multistatic polarimetric SAR experiments at the Cranfield University GBSAR laboratory [157] are presented, illustrating the utility of the approach for fully sampled 3D SAR image formation, and for sparse aperture sampled SAR 3D point-cloud generation.

Section 4.3 describes the extension of a newly developed 3D point-cloud algorithm which operates with multistatic geometries apertures and is valid in the most general SAR near-field multistatic geometries. The use of all linear polarisation channels is also considered.

Section 4.4 describes the laboratory SAR measurements undertaken. The simulations in section 4.5 presents the workings of the sparse 3D aperture approach, defining parameters that would enable multistatic trajectories to be designed for an optimal performance of the new approach. Section 4.6 presents fully sampled 2D SAR aperture results whilst section 4.7 presents sparsely sampled SAR aperture 3D point-cloud results, for each measured polarisation channel. Concluding remarks are presented in section 4.8.

4.3 Sparse aperture 3D point-cloud algorithm

A new sparse SAR aperture imaging approach, Sparse SAR Volumetric Interferometry (SSARVI) is described here. It is a generalisation of the SPCGS algorithm, in this case applicable to multistatic and SAR near-field imaged scenes. Previously the performance of the algorithm was demonstrated for monostatic and bistatic scenarios for a single polarisation channel [64], whereas here it is demonstrated on multistatic collections. It exploits the benefits of volumetric processing, allowing laid over scatterers to be processed and distinguished in a natural manner [144]. A wide variety of multistatic SAR geometry scenarios can be accommodated by the algorithm, however in the results presented here, multiple simultaneous fixed receiver positions are used,

together with multiple horizontal passes of the transmitter at different heights. It is noted that a randomised trajectory spacing in height suppresses vertical aliasing artefacts [62], [140]. In the multistatic scenario, this randomised spacing should be imparted to Bistatic Equivalent Monostatic (BEM) trajectories which follow the bistatic bisectors of the corresponding transceiver locations. This new approach does not require the intensive signal reconstruction processes, or phase unwrapping, used for other sparse 3D SAR techniques [95].

Within a multistatic collection, having chosen the reference bistatic collection, the other bistatic collections can then be combined with the reference to form volumetric interferograms. Each volumetric interferogram contributes vertical SAR iso-range arcs at the scatterer locations (with no vertical resolution themselves), but when combining different interferograms, the arcs cross at the scatterer 3D location, allowing the scatterer heights to be determined by the locations of maximum interferogram phase factor sum, to be introduced next.

Considering N bistatic SAR geometry collections (comprising the multistatic SAR collection), the sum of the interferogram phase factors is evaluated at each voxel location:

$$R(x) = \frac{1}{N} \left| \mathbf{1} + \sum_{n=1}^{N-1} e^{i\varphi_n(x)} \right| \quad (4-1)$$

Where $\varphi_n(x)$ is the phase at voxel x of the n th interferogram. Voxel locations where the summed phase R has surpassed a designated threshold, form part of a point-cloud rendering of the target being imaged. Detections are determined by the use of a threshold, which can be implemented through histogram-based approaches that are common in image processing [144]. The point-clouds can then be augmented by associating the mean voxel intensity of the contributing SAR data, at the position of each individual point detection. The multistatic extension of the SSARVI approach is summarised by the flow diagram in Figure 4-1.

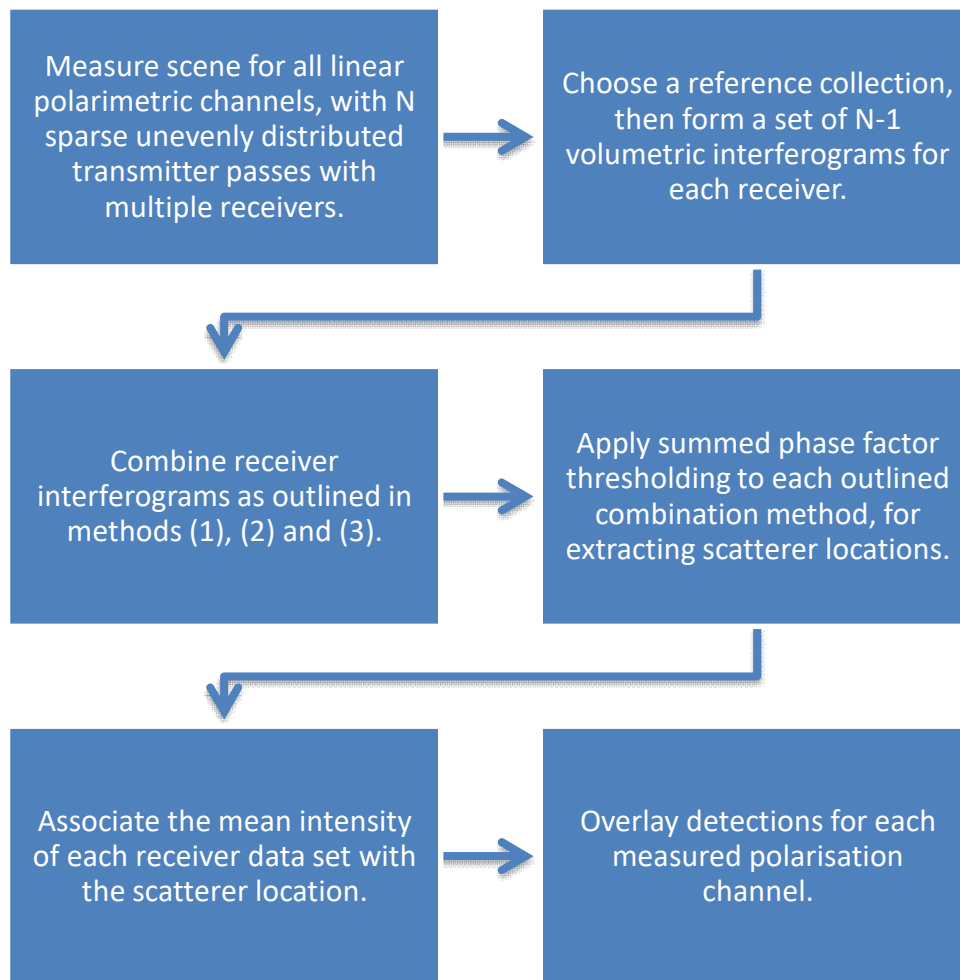


Figure 4-1: Flow diagram of the multistatic and polarimetric extension to the SSARVI algorithm.

Many approaches to combining SAR data from contributing bistatic geometries are possible, however three approaches are here employed, attuned to the particular SAR measurements undertaken in the laboratory. The three approaches may offer benefits in the case of complex targets, to be commented on below. The effect on performance of each method of combination is considered both theoretically and experimentally. This is due to the improvement in resolution associated with multistatic geometries, but also the challenges of combining multiple received signals. The main challenges arise from varying signal returns from complex volumetric targets over a wide baseline and signal co-registration [12], [158], [159], [160].

Let there be N transmitter passes undertaken, with M receivers. For the m^{th} receiver, the sum of interferometric phase factors corresponding to the N transmitter passes, $R_m(x)$, is obtained via equation (4-1). The three approaches to multistatic point-cloud generation used are:

1. **Coherent multistatic sum:** a coherent combination of all $N-1$ interferograms for all transmitter (Tx) and receiver (Rx) combinations, with the threshold being applied to the absolute value of the phase factor sum:

$$Det = \frac{1}{M} \left| \sum_{m=1}^M R_m(x) \right| > Th \quad (4-2)$$

Where Det is the set of point-cloud detections and Th is the threshold derived from the histogram-based approach. For omnidirectional point scatterers, this could offer finer resolution than other approaches, however it could prove overly sensitive when applied to variable bistatic RCS scatterers.

2. **Non-coherent bistatic sum:** a coherent sum of all the bistatic geometry interferograms corresponding to each receiver trajectory, and then an absolute value sum of these values:

$$Det = \frac{1}{M} \sum_{m=1}^M |R_m(x)| > Th \quad (4-3)$$

Where the threshold is applied to the final sum. This approach could offer greater stability to complex target RCS variation than the fully coherent summation approach, when applied to experimental data.

3. **Non-coherent bistatic combination:** a combination of the detections obtained from each receiver - a threshold is applied separately to the coherent summed interferograms corresponding to each receiver, and these detections are combined:

$$Det = \bigcup_{m=1}^M Det_m , \quad (4-4)$$

where $Det_m = R_m(x) > Th_m$

This approach may be most robust to variation in scatterer RCS from complex targets, however, of the three methods this approach would offer the least multistatic based spatial resolution improvement.

The performance of each of these methods is demonstrated on a simulated isotropic point scatterer and on multi-polarisation experimental data from a complex target measurement.

4.4 Laboratory measurements

The GBSAR laboratory conducts microwave measurements with a Vector Network Analyser (VNA), which generates a stepped frequency waveform. The system was set up for indoor use, with two Ultra-Wideband horn antennas allowing measurements within the range 1-10 GHz. The Antennae can be mounted in a bistatic or pseudo monostatic configuration.

When in a bistatic configuration, seen in Figure 4-2, each antenna is mounted on a different two-dimensional vertical SAR aperture scanner. One SAR aperture was 3.50 m wide by 1.46 m high and was used for the transmitter, and the other was 1.29 m wide by 1.46 m high and was used for the receiver, as seen in Figure 4-3.

When a SAR scene is left undisturbed, and transmitter trajectories are precisely repeated, the combination of repeat scans with the receiver in different locations (or trajectories) are equivalent to a multistatic scan with multiple receivers operating simultaneously. Using this approach, multistatic scans have been collected in the GBSAR lab, even though the VNA has only two ports.



Figure 4-2: The GBSAR laboratory in a bistatic configuration.

The multistatic SAR geometry chosen for this collection is shown in Figure 4-3. The transmitter traverses the vertical rectangular aperture in blue, with Nyquist positional sampling, and nine fixed receiver positions to the left of the scene, numbered 1-9 in the figure, provide nine bistatic 3D SAR collections. For sparse aperture image analysis, the collected dataset is downsampled.

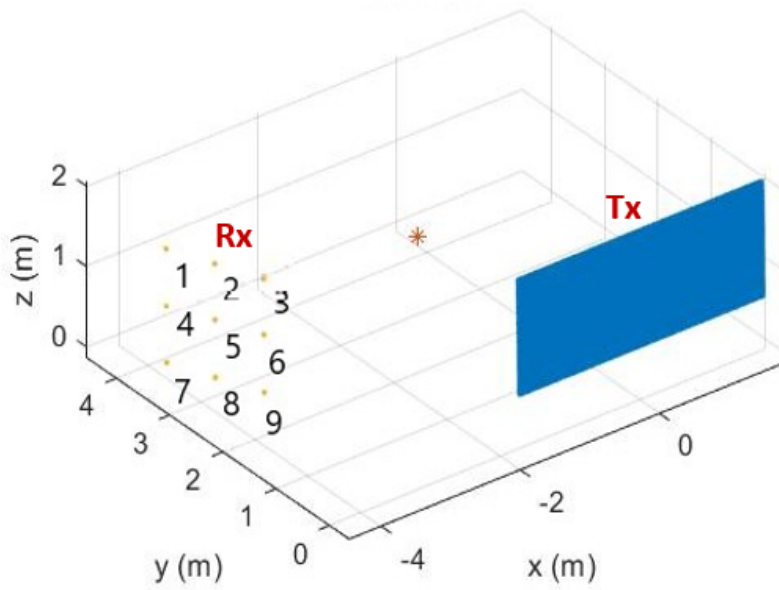


Figure 4-3: Multistatic 3D SAR geometry, with 2D SAR aperture transmitter trajectory in blue, and nine fixed receiver positions to the left of the scene.

The measured target scene for the results presented, consisted of a quarter scale T72 tank model placed on gravel and reference spheres, seen in Figure 4-4. The gravel provides a speckle background, with target shadowing visible depending on SAR geometry. The use of a stationary target allows for optimal algorithm development.



Figure 4-4: Target scene, consisting of a Quarter scale T72 tank model on gravel and reference sphere “S”.

4.5 Simulations

4.5.1 Introduction

To explore the performance of the SSARVI algorithm for multistatic SAR collections, simulations of a single point scatterer were conducted and are described in this section.

Previously it was found that randomized spacing trajectory distributions in height with a normalized trajectory spacing standard deviation (NSTD) of approximately 0.6 gave the best peak to sidelobe performance [64]. However, for higher NSTD values the peak to sidelobe performance was seen to drop again. It was hypothesised that this drop was due to the clustering of trajectories that occurs for higher NSTD values. In this section we therefore investigate how trajectory clustering affects peak to sidelobe levels. In these simulations, for N transmitter trajectories, the clustering is performed only on the 2nd to $(N-1)^{\text{th}}$ passes, which we call the mid-aperture trajectories (of which there are $N-2$), leaving the top-most and end-most passes unchanged as these define the overall aperture extent.

4.5.2 Multistatic Summation Approaches

For an initial simulation, the SAR geometry consisted of five transmitter height horizontal pass trajectories traversed sequentially, with three fixed position receivers at different heights operating simultaneously, seen in Figure 4-5. The geometry is comparable to a subset of the measurement geometry seen in Figure 4-3. The Bistatic-Equivalent Monostatic (BEM) passes contributed by each transmitter and receiver pair are shown in the figure – for each receiver they are coloured yellow, green, and magenta respectively. The frequency range was 7 – 8 GHz. Note that BEM antenna positions are in the bistatic direction (bisection of antenna directions from the scene centre), and at a range which is the same as the bistatic range.

The vertical extent of the transmitter aperture was 1.28 m, with heights {0.63, 0.19, -0.01, -0.21, -0.65} m, with the three fixed receiver positions offset to the right at mean bistatic angle of 5.7° . This combination gave a mean slant range of 6.8 m to the target. For an image unambiguous vertical cross-range of 1 m, the transmitter pass vertical spacing required is 2.46° in elevation. However, starting from even spacing, the second and fourth transmitter passes were here offset by 0.38° in elevation towards the mid-aperture, which had the effect of suppressing aliasing artefacts over the vertical cross-range extent.

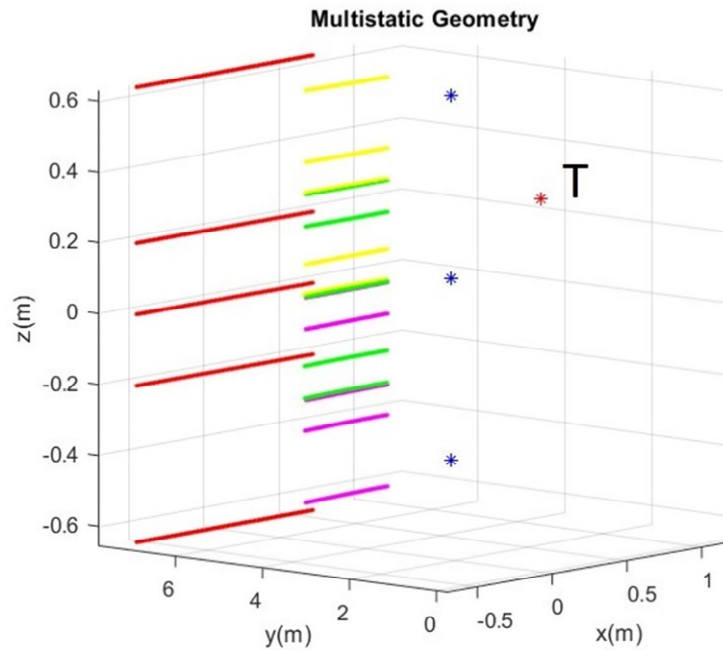


Figure 4-5: Simulation multistatic SAR geometry with five sequential transmitter passes (red) and three simultaneous receivers (blue). The simulated point scatterer location is labelled “T”. The Bistatic Equivalent Monostatic (BEM) trajectories are also shown (yellow, green and magenta for each of the three receivers).

The simulation results presented in Figure 4-6 show height estimates for the scatterer in the form of the interferometric phase factor sum value, R . The R values plotted are extracted along a vertical iso-range arc through the point scatterer, thus showing vertical sidelobe structures resulting from different transceiver aperture distributions in height.

Figure 4-6 shows R values for coherent and non-coherent bistatic interferogram sums corresponding to the three receiver geometries shown in Figure 4-5. The

peak of R represents the estimate of the scatterer height in the SSARVI calculation process, and is correctly collocated with the scatterer height, at 0.4 m. Both summation methods show the suppression of ambiguities, with the sidelobe heights being significantly lower than the main peak at the scatterer position. For the given geometry, the coherent summation of interferograms across receivers shows the greatest suppression of sidelobes, when compared to the non-coherent sum.

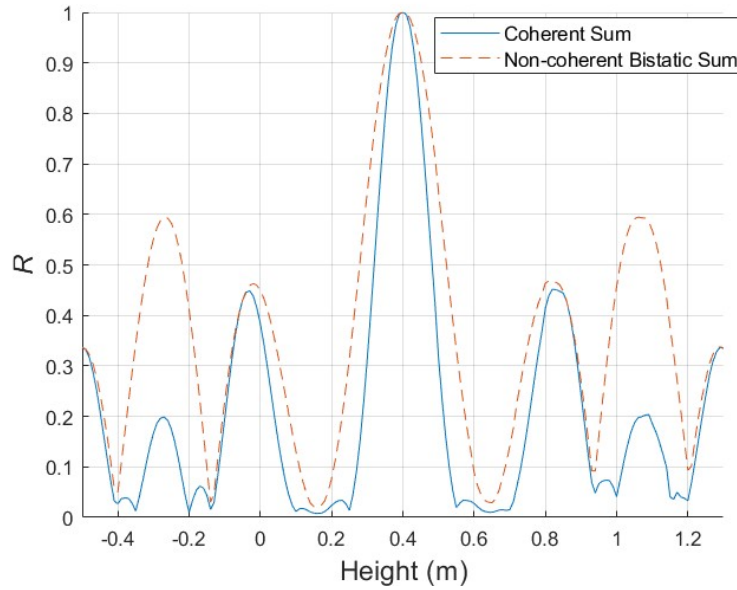


Figure 4-6: Simulation values of summed phase factor, R , along a vertical iso-range arc from multistatic SAR simulations on a point scatterer. The performance is shown for both coherent interferogram summation from multiple receivers, and the non-coherent summed bistatic interferogram approach.

4.5.3 Trajectory Clustering Effects

The R sidelobe height gives rise to false detections in the SSARVI algorithm, which can be influenced by the number of passes and the unevenness of the spacing between them. It has been found that generally, but only to an extent, greater unevenness in trajectory height sampling leads to greater sidelobe suppression [64]. However, it is thought that when the unevenness is taken to the extreme within a fixed extent, passes can cluster together, effectively reducing the trajectory sampling, which is prejudicial to R sidelobe suppression. The clustering effect is investigated here.

Multiple simulations were conducted, using a generalised bistatic aperture using the mid receiver position shown in Figure 4-5, and with the same isotropic point-target. The transmitter passes were varied from evenly distributed, to a centrally clustered distribution as described in section 4.5.1, with top and bottom passes defining the constant extent of the transmitter aperture, and the mid-aperture cluster between them.

For multiple simulations as described above, Figure 4-7 presents the peak to sidelobe level, ΔR , as a function of the cluster width, for N transmitter passes. The degree of clustering is here given by the BEM (mid-aperture) cluster extent in degrees, which we denote this by χ . Smaller values of χ therefore represent a higher degree of clustering. The curves in the figure correspond to 6, 8 and 10 transmitter passes, and show the evolution of ΔR with χ , starting from an evenly sampled in height transmitter aperture. The transmitter mid-aperture passes were sequentially displaced towards the middle of the aperture, keeping the very top and very bottom passes unchanged. For the 6-pass case, the initial sampling rate matches the Nyquist sampling rate for a 1 m unambiguous vertical cross-range extent. Considering the separation of BEM passes, this corresponded to a spacing angular extent of 0.98° .

It is noted that the 1 m unambiguous vertical cross-range extent is especially significant, as this gives the extent over which sidelobe levels are considered in the evaluations of ΔR , hence the 0.98° value is marked in the figure with a vertical dashed line and is denoted χ_1 .

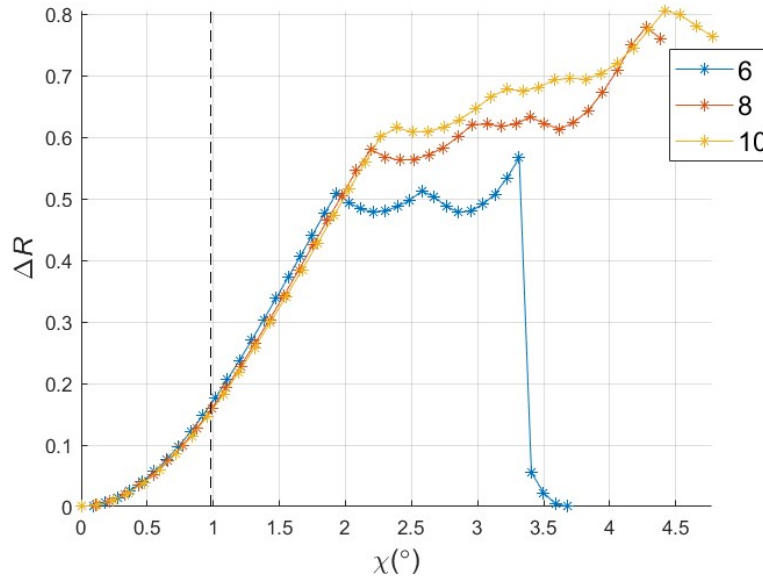


Figure 4-7: Simulation results for ΔR determining the relationship between the sidelobe height and the vertical aperture distribution in the form of the cluster BEM angular extent χ . Results are shown for simulations of 6, 8 and 10 transmitter passes. The vertical dashed line indicates the angular extent corresponding to the Nyquist spacing for the 1 m unambiguous vertical cross-range extent of interest over which ΔR is evaluated.

The three curves in Figure 4-7 initially behave in the same way with increasing χ : when χ is small, ΔR starts at zero and progressively increases until χ is approximately double χ_1 . This behaviour is associated with the cluster initially behaving as a single pass, so that there are effectively three passes in total, giving rise to a small unambiguous cross-range extent and poor ΔR performance.

For χ greater than about $2\chi_1$, ΔR performance increase is reduced and attains a maximum, where the greater the number of passes, the greater the maximum. For 10 passes, ΔR attains 0.88.

Hence, it has been shown with an example simulation that excessive trajectory clustering can reduce the sidelobe performance substantially, due to multiple trajectories behaving as a single trajectory. In the example it is shown that for a given desired unambiguous vertical cross-range extent, the extent of clustered trajectories should not be finer than approximately two times the corresponding Nyquist sample spacing, otherwise the trajectory distribution becomes sub-optimal.

4.6 Full aperture 3D SAR image results

4.6.1 Multistatic data summation

A multistatic polarimetric laboratory collection was performed, providing thirty-six bistatic 2D SAR aperture scans: nine bistatic scans, for each of four linear polarisation channels. With this data, thirty-six 3D volumetric bistatic SAR image results were generated with the bistatic Backprojection SAR image formation algorithm (BPA).

An incoherent sum of the nine bistatic 3D SAR image VV polarisation results, provided initial volumetric images for inspection, corresponding maximum-intensity projections (MIPs) are presented in Figure 4-8, with the down-range direction in the positive y-axis direction. The 3D structure of the target and scene is evident. The shadowing to the rear of the model is not discernible due to the wide transmitter SAR aperture, however the shadow to the right of the vehicle, associated with the static receiver directions, is evident. The large sphere to the rear-right of the scene is marked “S”, and several associated multipath features are visible, one directly at ground level below the sphere, and the other symmetrically below ground level.

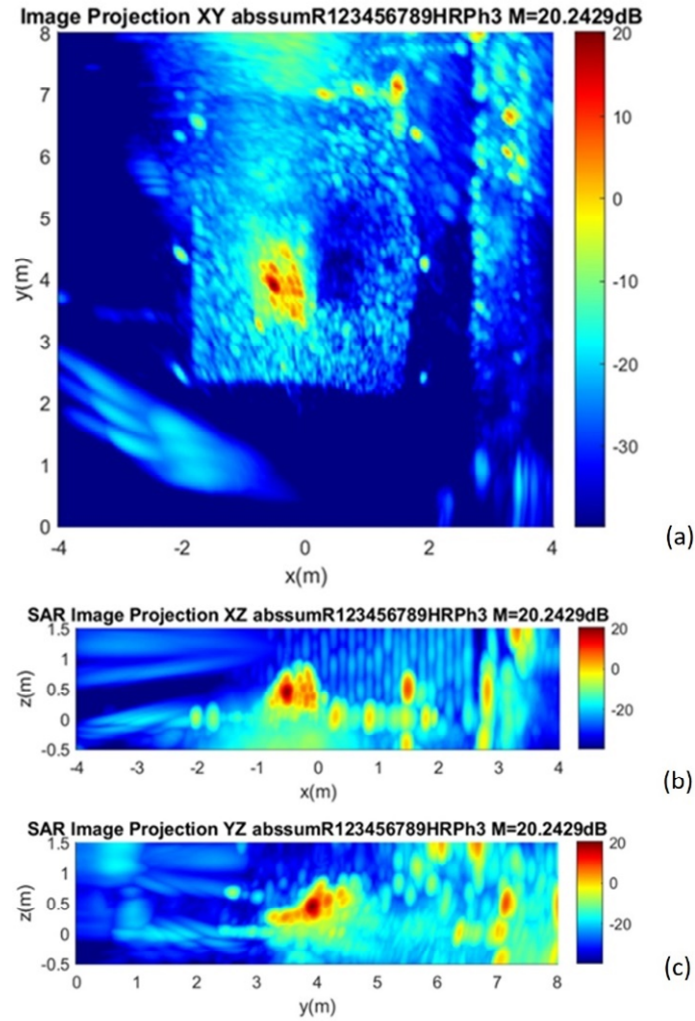


Figure 4-8: MIPs of an incoherent sum of the nine VV polarisation 3D bistatic SAR images, with a z-projection (a), y-projection (b) and an x-projection (c). The large reference sphere is marked “S”.

Having evaluated the incoherent sum result, a coherent sum of the nine bistatic 3D SAR image VV polarisation results was performed. Coherent sums are however very sensitive to positional errors in SAR geometry information. It is estimated that a 4 mm range error in antenna ground truth, could give rise to as much as 90° of phase error at 10 GHz. Such a recorded error size would be possible in the laboratory antenna position ground-truth taking process in the directions perpendicular to the linear scanner actuators. (However, in the directions of the linear scanner actuators the accumulated positional errors after many thousands of moves were confirmed to be lower than 0.1 mm). Indeed, after forming an initial coherent sum the result was seen to be defocussed.

A data-driven phase correction algorithm was developed, maximizing the intensity of the brightest scene scatterer in any given VV image pair coherent sum, providing an overall phase correction value for each of the nine bistatic SAR geometries. The result of the phase corrected nine VV polarisation image coherent sum is presented in Figure 4-9, in the form of MIPs.

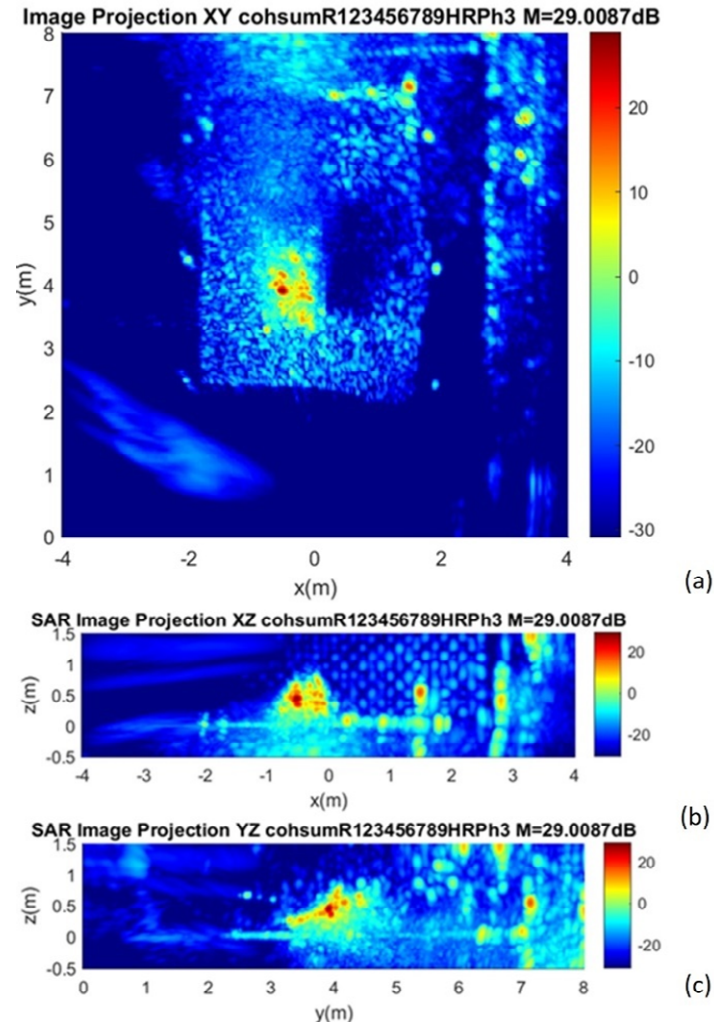


Figure 4-9: MIPs of a phase corrected coherent sum of the nine VV polarisation 3D bistatic SAR images, with a z-projections (a), y-projection (b) and an x-projection (c). The large reference sphere is marked “S”.

An improvement in SAR resolution in all three spatial dimensions has resulted from the phase corrected coherent summation, over the incoherent summation, providing finer detail across the main target and scene.

4.7 Sparse aperture 3D point-cloud results

The SSARVI technique is applied to downsampled versions of the experimental dataset described in section 4.4, with the aim of determining a sub-Nyquist sampling limit for good quality results and determining effective ways of combining multistatic data from the different receivers when the scene consists of a complex target, as opposed to the idealised point scatterer investigated in section 4.5. The results are shown for the four measured polarisation channels, to determine their utility.

Figure 4-10 shows the repeated random downsampling implemented for the transmitter aperture in the vertical direction, with the original aperture being presented in Figure 4-3. The most well sampled aperture size within the subset geometries presented in Figure 4-10 is referred to as the sub-initial sampling (SIS) in the figure. To provide an unambiguous vertical image extent across a 1.8 m high Volume of Interest (Vol), a conventional bistatic BPA based imaging approach would require a transmitter spacing of 7.8 cm between transmitter passes for the measured frequencies. For each method of receiver-data combination, passes were sequentially removed until the tank structure became indistinguishable from erroneous detection artefacts.

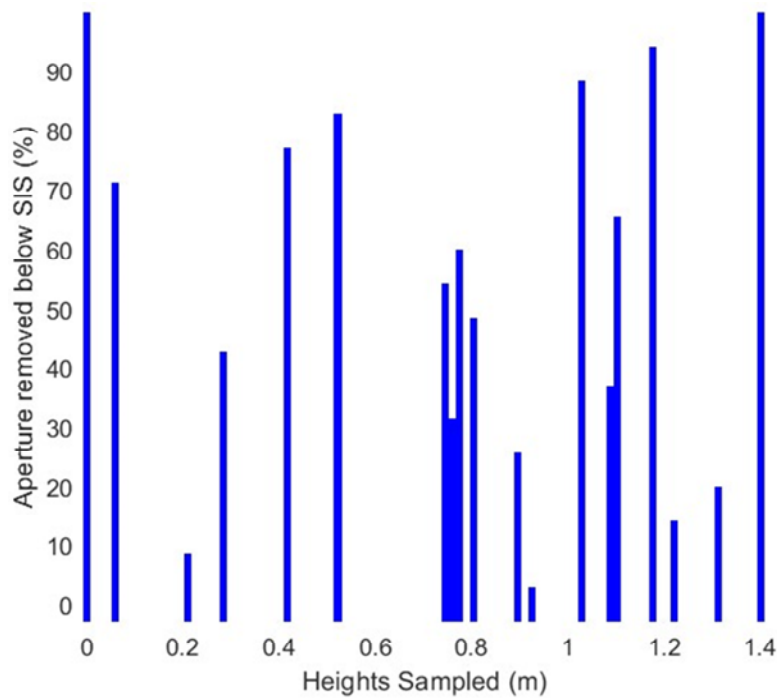


Figure 4-10: Representation of the downsampled transmitter aperture. Blue lines show the transmitter height sampled, for a full aperture width, as a percentage of the initial transmitter aperture removed.

A threshold was determined for the tank structure using a subset of the geometry shown in Figure 4-3. Using a single receiver position, interferogram phase factors from all transmitter passes were summed forming a combined volumetric interferogram (the phase factor sum is always divided by the number of elements in sum, hence giving a mean). Background artefacts were then reduced by systematically increasing the summed phase factor lower threshold. The threshold at which they were removed was determined to be optimal at the point which is 75% from the modal R value to the maximum R value across the whole Vol. This thresholding approach was maintained in the formation of the sub-Nyquist aperture point-cloud renderings, which enabled the appearance of artefacts to be associated with the performance of the SSARVI algorithm, providing the downsampling limits presented.

Figure 4-11 compares example sub-Nyquist point-cloud renderings formed from each of the three combination approaches described in section 3, employing the VV linear polarisation data.

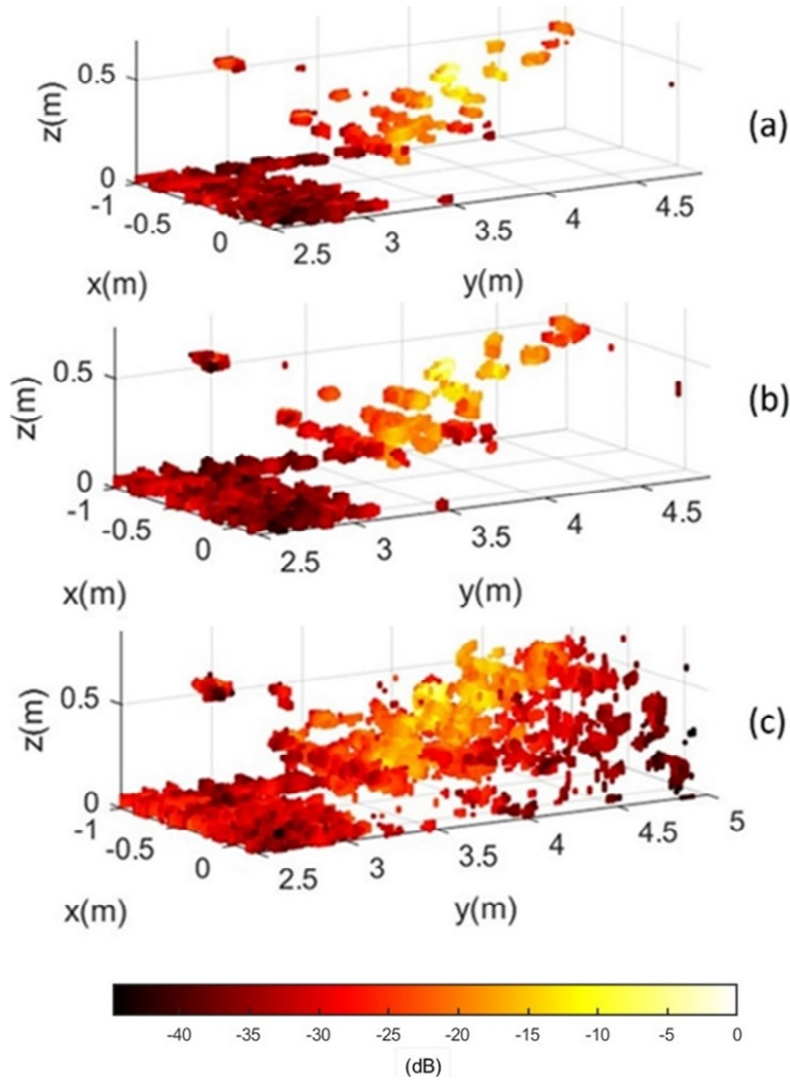


Figure 4-11: VV polarisation measurement sub-Nyquist transmitter height sampled multistatic point-cloud renderings. (a) Coherently summed, (b) Non-coherent bistatic summed, (c) Non-coherent bistatic combined.

The renderings presented were formed using 10 randomly spaced transmitter passes, extracted from the radar geometry shown in Figure 4-3, employing receivers 3, 6 and 9, which are in a single vertical column. The resulting transmitter aperture had a mean pass spacing of 14.1 cm, which would give a maximum unambiguous vertical cross-range extent of 1.2 m when using conventional image formation. This sampling corresponds to 47% removal of data from the initial aperture seen in Figure 4-9.

From the point-cloud detections, the tank structure is clearly visible for all three methods of receiver combination, with the main body appearing the brightest. The cannon barrel is visible, and the gravel is visible. The point-cloud rendering shown in Figure 4-11c ("combined") shows the most points detected out of the three methods, and the most discernible tank structure in the point-cloud formation. With the "summed" combination approaches, both coherent and non-coherent, there is a section of the rear body missing from the point-cloud rendering with the cannon also less well defined.

In determining the effect of removing passes from the transmitter aperture, a 3D cross correlation was performed between consecutive sub-Nyquist formed point-cloud renderings and the initial sampling point-cloud rendering. The cross-correlation measures similarity between two 3D point-clouds, represented as 3D images. The point-cloud renderings formed with the initial sampling rate were free of erroneous detections, therefore significant decreases in correlation with aperture downsampling can be reasonably attributed to the appearance of erroneous detection artefacts. Sections 4.7.1 to 4.7.3 present the results with this measure, for each receiver combination method, to highlight trends in image quality.

The correlation results are compared to the point-clouds themselves by a process of direct visual inspection, for each sparse aperture employed. The direct inspection process was used to determine the aperture downsampling limits for the cases:

- 1) where only minor artefacts are present, and
- 2) where major but not overwhelming artefacts are present.

The case of "major but not overwhelming artefacts" is included as these renderings are deemed to still provide useful results, as the target can still be discerned. Beyond this point however, the target is no longer discernible. The two sparse aperture downsampling limits for point-cloud formation are summarised in Table 4-1 for the different polarisations and receiver data combination methods.

To determine the effect of using increasing amounts of receiver aperture data (increasing fixed receiver positions), the maximum summed phase factor value, R , for the brightest scatterer in the scene is presented, for the different polarisations and multistatic data combination approaches. The results for each method of combining receiver data are also presented in section 4.7.1 to section 4.7.3. The receiver positions used are those shown in Figure 4-3, with the full width of receivers spanning a total width of 18.6° in azimuth, and 17.0° in elevation. A summary of the properties of the point clouds from each receiver is presented in section 4.7.4.

4.7.1 Coherent Multistatic Summation

Figure 4-12 shows the downsampling aperture point-cloud correlation performance for the *coherent* multistatic summed approach, for the four polarisations against their corresponding initial sampling point-cloud rendering. The receiver column employed in generating the Figure 4-11 results (receivers 3, 6, 9) were used in generating subsequent multistatic point-cloud correlation results.

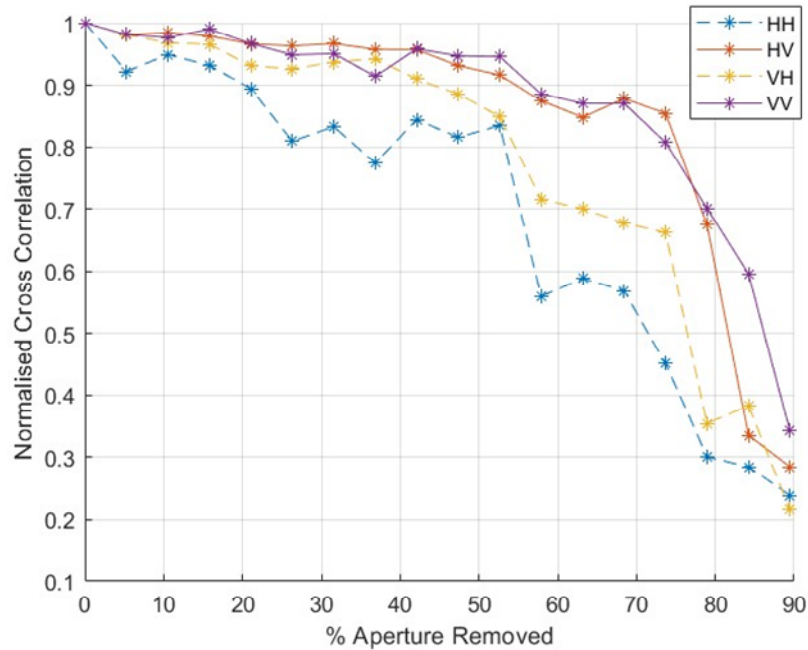


Figure 4-12: Normalised cross-correlation between successive coherent summed bistatic sparse aperture point-cloud renderings and the corresponding initial-sampling point-cloud renderings for each polarisation.

The correlation values necessarily start at 1 for all polarisations, and then show a decline as passes were removed from the aperture. For the VV and HV channels, the correlation remained above 0.8 until 74% of the original aperture had been removed. Inspection of the point-cloud renderings indicated that this corresponds to sparse apertures for which only a small number of erroneous detections were present. Although the correlation values for the remaining polarisations were lower, inspection of point-clouds showed consistency across all polarisations for given aperture sparsity. Hence the correlation values should be taken to represent trends rather than exact point-cloud detection performance.

The correlation then showed a rapid decline, so that up until 78% aperture removal there were major but not overwhelming erroneous detections. Beyond the 78% aperture removal, the target is overwhelmed by image artefacts so that no useful target information is obtained. The aperture downsampling limit values are provided in Table 4-1.

To illustrate the effect of adding measured data from an increasing number of receivers, Figure 4-13 shows the summed phase factor, R , at the location of the brightest scatterer in the scene for each polarisation. This scatterer was located on the vehicle turret, and is visible in Figure 4-4. The values of R , for the three multistatic combination methods, were generated using an increasing number of receivers, with a fixed transmitter aperture. The transmitter aperture consisted of 14 passes, corresponding to 26% initial aperture removal. The first three receiver positions plotted, correspond to those used to form the renderings and correlation results shown in Figure 4-11 and Figure 4-12.

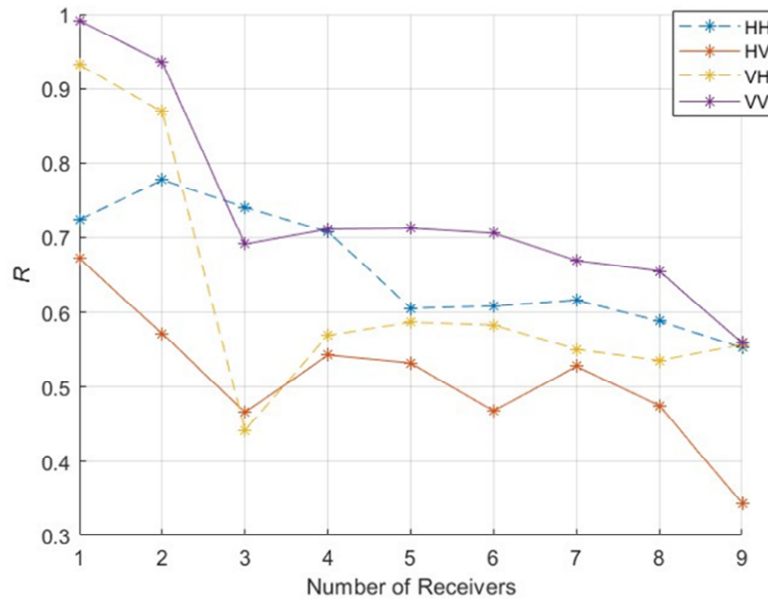


Figure 4-13: Summed phase factor response when coherently summing data from an increasing number of receivers, for the brightest scatterer in each polarisation.

Most of the polarisations initially show a pronounced decrease in R , which then became less pronounced. Direct inspection of point-clouds revealed that the decrease was associated with a decrease in the number of detections. This decrease in complex phase factor sum is likely associated with the scatterer variability in bistatic complex RCS. The isotropic point scatterer simulation results in Figure 4-6 showed no such decrease in R .

4.7.2 Non-coherent Multistatic Summation

Figure 4-14 shows the downsampling aperture point-cloud correlation performance for the non-coherent multistatic summed approach, for the four polarisations against their corresponding initial sampling point-cloud rendering, and the same geometries as in section 4.7.1.

The correlation values mostly remain above 0.8 until 47% of the initial aperture is removed, after which they show a faster decline. Point-cloud inspection revealed that only minor artefacts were present until 68% of the initial aperture was removed. This aperture downsampling point occurred earlier than in the coherently summed approach. The last usefully interpretable point-cloud renderings, where ‘major but not overwhelming’ artefacts are present, occurred when 73% of the initial sampled aperture was removed. The downsampling values are summarised in Table 4-1.

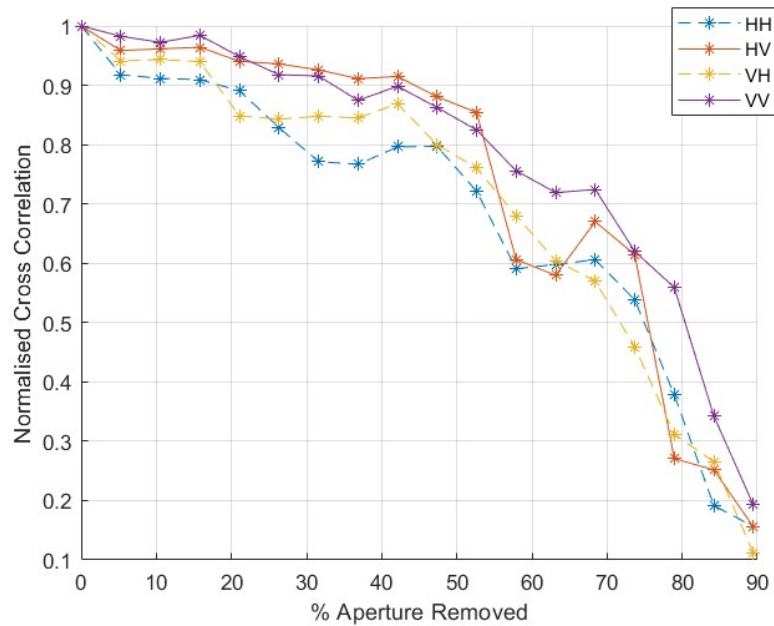


Figure 4-14: Normalised cross-correlation between successive non-coherent summed bistatic sparse aperture point-cloud renderings and the corresponding initial-sampling point-cloud renderings for each polarisation.

For the non-coherent multistatic summation, to illustrate the effect of adding measured data from an increasing number of receivers, Figure 4-15 shows the summed phase factor, R , at the location of the brightest scatterer in the scene for

each polarisation. This scatterer was located on the vehicle turret and is visible in Figure 4-4, and the conditions for the generation of the results were as in the corresponding ones in section 4.7.1.

Most of the polarisations initially show a decrease in R , after which they then mostly stabilize and, in some cases, increase slightly. Likely due to this trend, when comparing the corresponding point-clouds with the coherent summed approach point-clouds, more detections were visible in the non-coherent summation case. The additional detections are distributed across the main body of the vehicle. It is likely that this property comes about due to the reduced sensitivity of the non-coherent multistatic addition approach to the RCS variation between bistatic geometries. The reduced sensitivity, however, has led to lower aperture removal limits when compared to the coherent summation case, as seen in Table 4-1.

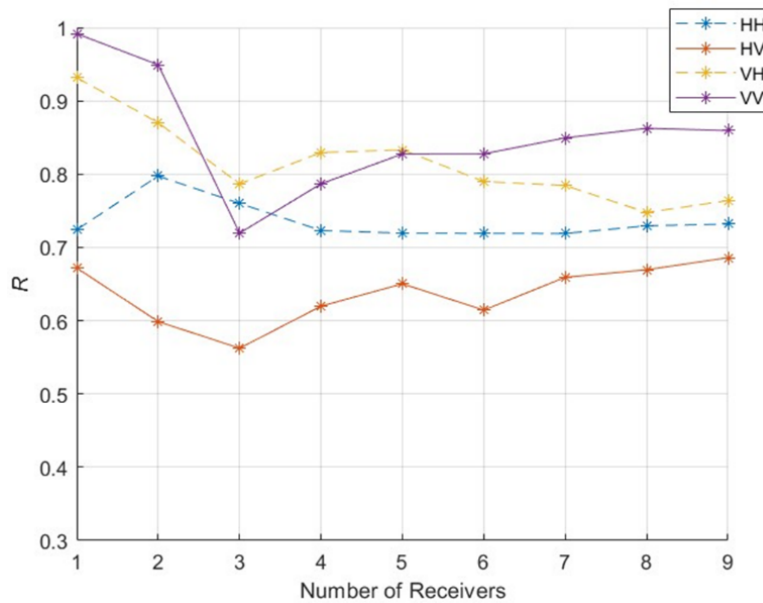


Figure 4-15: Maximum summed phase factor value when non-coherently summing data from an increasing number of receivers.

4.7.3 Non-coherent Combined Bistatic

Figure 4-16 shows the downsampling aperture point-cloud correlation performance for the *non-coherent combined* bistatic approach, for the four

polarisations against their corresponding initial sampling point-cloud rendering, and the same geometries as in Figure 4-10.

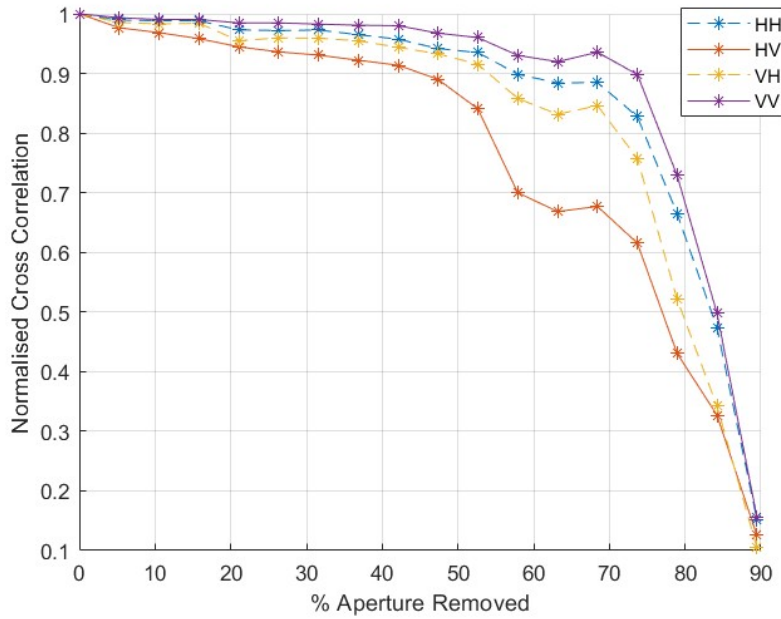


Figure 4-16: Normalised cross-correlation between successive non-coherent combined bistatic sparse aperture point-cloud renderings and the corresponding initial-sampling point-cloud renderings for each polarisation.

The downsampled aperture point-cloud correlation curves are smooth and relatively high valued in comparison to the summed approaches, however more artefacts were present. Only minor artefacts were present up until between 32% and 42% depending on the polarisation, and major artefacts were introduced at 57%, both earlier limits than in the summation approaches (sections 4.7.1, 4.7.2). These limits are shown in Table 4-1. Generally, the combination approach showed significantly more initial detections and a more complete initial vehicle structure. Example point-clouds were presented in Figure 4-11(c), illustrating this.

It is seen that with the combination approach, for a given transmitter aperture, incorporating an increasing number of receivers adds further detections. This is in contrast with the previous two approaches, which both showed a decrease in the number of detections as receivers were added. This approach is the least sensitive to variations in bistatic RCS however, it generally requires more passes than the summation approaches due to the faster onset of erroneous detections.

4.7.4 Point-Cloud Properties Summary and Polarimetric Results

The point-cloud sparse aperture artefact limits for minor and major artefacts, are presented in Table 4-1. Regarding false scatterer artefacts, for a given aperture sparsity, the coherent summation gives the least, followed by the non-coherent summation and then the non-coherent combination approach. Similarly, it was observed that for a given aperture sparsity where little or no false scatterers were present, the coherent summation approach gave the fewest scatterer detections over the vehicle, followed by the non-coherent summation and then by the non-coherent combination, as seen in the example results in Figure 4-11.

Table 4-1: Table showing the aperture downsampling limits for point-clouds with minor and major artefacts, for all three receiver combination approaches: coherent multistatic sum (CMS), non-coherent multistatic sum (NCMS), non-coherent bistatic combination (BC).

Pol.	Downsampling limit for only minor artefacts (%)			Downsampling limit for only major artefacts (%)		
	CMS	NCMS	BC	CMS	NCMS	BC
HH	73	68	32	78	73	57
HV	73	68	42	78	73	57
VH	73	68	36	78	78	57
VV	73	68	42	78	73	57

Regarding the point-clouds formed in the different polarimetric channels for a given multistatic SAR geometry and combination approach, it was found that the actual detections differed across the channels. Figure 4-17 shows this for the three-receiver scenario with 26% of passes removed from the transmit aperture, by overlaying the four point-clouds in a single plot with different colours for different polarisations. It is noted that the SSARVI algorithm and thresholding process was applied to the channels separately.

The result shows that each polarisation contributes a significant amount of different scatterer detections, so that when they are combined, a more complete outline of the vehicle is provided. Of the three multistatic combination approaches, the coherent summation one benefits most from the overlay of the polarimetric channels as for any given aperture it has the least detections.

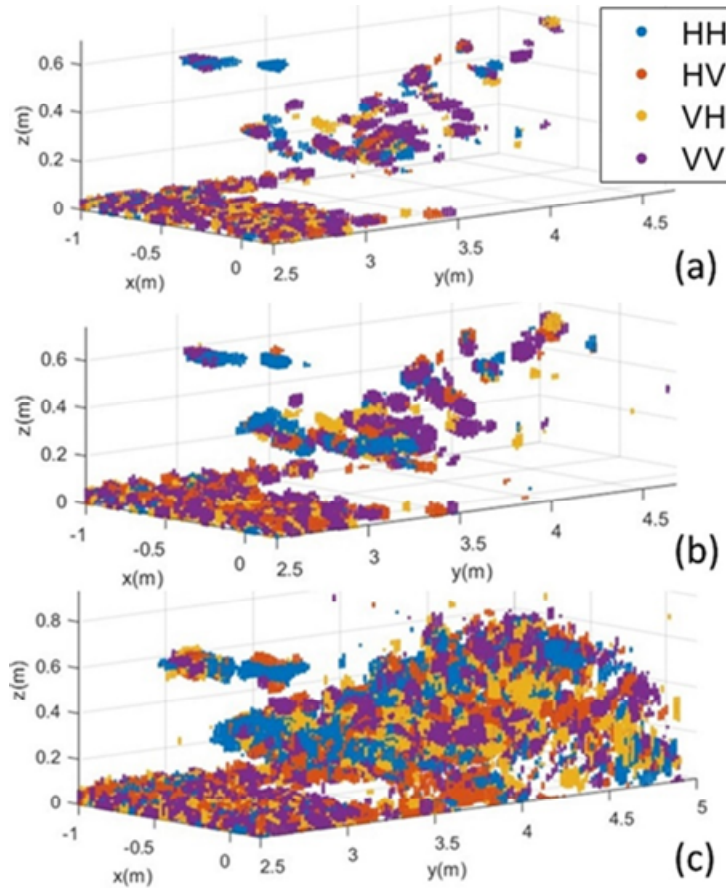


Figure 4-17: Overlaid point-clouds from each polarisation channel, shown for the three multistatic geometry combination methods: (a) Coherent summation, (b) Non-coherent bistatic summed, (c) Non-coherent bistatic combined.

4.8 Conclusion

Multistatic polarimetric SAR measurements of a complex target and scene were conducted at the Cranfield University GBSAR laboratory, to determine benefits of these type of collection geometries.

Results have been presented which combine multiple bistatic SAR collections to provide 3D SAR (volumetric) images of the scene. Full Nyquist sampled measured data were collected and image processed with the back-projection image formation algorithm (BPA), and downsampled versions of the collected data were processed with the novel, and here extended to multistatic SAR geometries, sparse aperture point-cloud formation algorithm, SSARVI.

For the Nyquist sampled datasets, incoherent summation of the individual bistatic collections provides useful BPA derived results, allowing a clear determination of the target, however with the SAR geometries utilised, a phase corrected coherent summation of the nine bistatic SAR collections was shown to provide BPA results with a much-improved SAR resolution in all three spatial dimensions.

A multistatic SAR geometries extension of the SSARVI approach for generating sub-Nyquist SAR aperture 3D point-cloud renderings was presented. With simulations, it was shown that excessive clustering of trajectories can reduce the sidelobe performance substantially, essentially due to multiple trajectories behaving as a single trajectory. When designing non-even trajectories for collections, trajectory cluster extents below the Nyquist spacing are suboptimal, as the advantages of having the clustered multiple trajectories are then lost. With a generalised bistatic example, it was shown that for a given desired unambiguous vertical cross-range extent, the extent of clustered trajectories should not be finer than approximately two times the corresponding Nyquist sample spacing, otherwise the trajectory distribution becomes sub-optimal.

A comparison of three different methods for combining the multistatic data from different receivers was made. Out of the three approaches implemented, it was found that coherent summation of the data was the most effective approach to suppressing artefact detections but gave rise to fewest point-cloud detections overall. The non-coherent summing approach showed similar results but with more detections and more artefacts for any given sparse SAR aperture. Overlaying the detections in the 'Non-coherent Bistatic Combined' approach gave rise to the most detections but required more passes for suppressing higher levels of detection artefacts.

For the coherent and non-coherent summed methods, it was found that increasing the number of receiver datasets used for point-cloud formation resulted in a decrease in the number of points detected. The “non-coherent bistatic combined” approach led to the most complete outline of the target but contained the most artefact detections. These different approach behaviours are likely due to their varying sensitivity to the variation in bistatic RCS across the different receiver datasets.

It was found that each polarisation channel contributes a significant amount of different scatterer detections, so that when they are combined, a more complete outline of the vehicle is provided. Of the three multistatic combination approaches, the coherent summation one benefits most from the overlay of the polarimetric channels as for any given aperture it has the least detections.

Further research will include developing the SSARVI approach for polarimetric decompositions other than the linear set, to provide a polarimetric characterisation of detections. Further effects expected in realistic conditions should also be considered, such as SAR geometry errors, noise, and target motion. It is possible that different summed phase factor thresholds might be appropriate for different scenes. Potentially electromagnetic scattering simulation could allow the generation of target specific thresholds [140].

5 Polarimetry for Sparse Multistatic 3D SAR

5.1 Abstract

There is significant interest in multistatic SAR image formation, due to the increased development of satellite constellations and UAV swarms for remote sensing applications. The exploitation of the finer resolution and wider coverage of these geometries has been shown to reduce the often-impractical data collection requirements of 3D SAR imagery; this offers advantages such as improved target identification and the removal layover artefacts. This paper presents a novel polarimetric generalisation of the SSARVI algorithm, which was previously developed to exploit these sparse aperture multistatic collections for 3D SAR image formation. The new algorithm presented here, denoted the PolSSARVI algorithm, combines polarimetrically weighted interferograms for determining the 3D scatterer locations from sparse aperture polarimetric collections. The bistatic generalised Huynen fork polarimetric parameters are then determined for the multistatic PolSSARVI 3D SAR renderings. This new approach was tested on both simulated and experimental data. Experimental imagery was formed using measurements from the Cranfield GBSAR laboratory.

5.2 Introduction

When compared to conventional monostatic Synthetic Aperture Radar (SAR), multistatic SAR image formation can be used to improve the resolution and coverage of SAR systems and capture additional target information from multiple aspect angles [10], [149], [152]. This has led to increased interest in SAR missions employing satellite constellations and UAV-SAR drone swarms [15], [111], [147], [161]. The advantages of multistatic SAR could be exploited for 3D imaging scenarios, which often have impractically high sampling requirements [162]. Moreover, multiple aspect angle collections make multistatic geometries beneficial for capturing additional target scattering information, including polarimetric [163].

3D SAR has the benefit of separating target features overlaid in height, and thus removing layover distortions from imagery. However, producing sufficient

resolution and coverage in the vertical direction conventionally requires many repeat passes over multiple aperture heights with Nyquist sampled spacing [26], [27], [28]. With conventional 3D SAR imaging algorithms, a reduction in the number of passes can make the collection more realisable, however would reduce image quality with aliasing ambiguities.

Various techniques have been developed for reducing the sampling requirements of 3D SAR whilst maintaining image quality, these include differential SAR interferometry, 3D SAR Compressive Sensing, and the SAR Point-Cloud Generation System (SPCGS) [22], [62], [164]. SAR interferometry relies on a phase unwrapping process to generate scatterer height estimates [3], and has primarily seen success with Digital Elevation Mapping. However, the phase unwrapping step can introduce ambiguities for scenes with large discontinuities in their height profile [139]. 3D SAR compressive sensing has seen significant interest, with renderings of complex vehicle targets being successfully produced [155]. This approach requires the target scene to be sparsely populated, which is then exploited in the signal reconstruction. The approach can be computationally expensive [55]. However, recently introduced methods based on sparse-prior optimisation are attempting to improve this [165].

The SPCGS algorithm processes the change in layover within a stack of 2D SAR images obtained from different radar platform trajectory heights, and sums the interferometric contribution to generate scatterer height estimates [62], [140]. This can be challenging for bistatic, multistatic SAR geometries, and in particular in SAR near-field geometries where the elevation angle across the scene is spatially variant. Under these conditions, the layover position of scatterers is difficult to calculate due to significant distortions of the contributing SAR iso-range arcs (constant range contours) [95]. The Sparse SAR Volumetric Interferometry (SSARVI) approach was originally developed to overcome this problem [64], [65].

This paper contributes a new polarimetric generalisation of the multistatic SSARVI algorithm, which we denote PolSSARVI. The PolSSARVI algorithm utilises the novel volumetric interferometry-based approach, developed in the previous SSARVI literature, to account for the layover distortions attributed to

using multistatic and near-field SAR collections. This paper extends the capability of this approach further, by combining multiple volumetric interferograms for each measured polarisation channel, for a sparsely sampled 2D multistatic aperture. The polarimetric interferograms are each weighted by their combined polarimetric span intensity. This combined polarimetric approach was devised to better account for the variability of scatterer brightness in different polarisation channels, and enable the association of polarimetric information with 3D scatterer detections. Examples of such scatterers, including a discussion on polarimetric scattering variability, can be found here [33]. The new methodology introduced by the PolSSARVI algorithm is explained in greater detail in section 5.3.

Previous studies of the SSARVI algorithm were based on the overlay of individual polarisation channel detections, which although contributing more detections, did not provide polarimetric scatterer properties [65]. The polarimetric information determined for the 3D targets presented in this study is obtained from bistatic Huynen fork parameters, which were developed from the pioneering work of Huynen and later Titin-Schnaider [131], [133], [134], [135]. The development of these parameters is explained in more detail in section 5.4.

Recent studies highlight the utility of extending SAR imaging techniques to include polarimetric analyses, particularly for improving target identification [166]. Prior to this work, polarimetry has been combined with sparsely sampled 3D SAR approaches, with meaningful polarimetric scatterer information being obtained using 3D SAR Compressive Sensing (CS) [167]. Whilst useful, this approach also uses the expensive signal reconstruction processes associated with single-channel compressive sensing. Recent publications have improved the scatterer identification of the polarimetric 3D CS-based approach, employing mixed-regularisation reconstruction methods and novel polarimetric decompositions [168], [169]. The polarimetric decompositions used in these approaches, however, are not applicable to the multistatic geometries used in this study.

The most established sparse polarimetric 3D SAR technique is polarimetric SAR interferometry [170]. This technique uses the phase unwrapping process utilised by conventional SAR interferometry. The polarimetric diversity is used to optimise

the coherence between SAR passes, which leads to smaller phase ambiguities and more accurate scatterer height estimates [56]. Recent publications have exploited this for 3D Interferometric Inverse-SAR (InISAR) image formation, forming complex point cloud models of a full-size model tank [39], [59], [171]. Like the PolSSARVI algorithm, these publications do not use phase unwrapping process due to its sensitivity with complex targets. These studies found that the use of multiple polarisations was able to produce more accurate and stable point cloud models of a model tank target, leading to improved target identification. Whilst the algorithms in these publications are different to the one presented in this paper, they show that performance gains are possible when using multi-polarimetric data.

The detection performance of PolSSARVI, for a range of sub-Nyquist sampled apertures, will be compared to the non-polarimetric SSARVI approach previously presented, using both simulated and experimental data. It will also be shown that PolSSARVI provides accurate polarimetric information, associated with sub-Nyquist formed 3D renderings of targets. The extraction of bistatic polarimetric parameters used in this study has been previously validated at the Cranfield GBSAR laboratory, using finely sampled SAR collection geometries [163].

Section 5.5 presents simulation results. Section 5.6 describes the Cranfield GBSAR laboratory. Section 5.7 outlines the experimental procedure and polarimetric features of the model target. Section 5.8 compares the PolSSARVI algorithm to the SSARVI algorithm and assesses the accuracy of polarimetric data obtained with the PolSSARVI algorithm, followed by a discussion. Concluding remarks are provided in Section 5.9.

5.3 Development of the PolSSARVI Algorithm

The SSARVI algorithm described in previous work, [64], [65] uses a set of volumetric interferograms that are formed for a single polarisation, using N sparsely and unevenly distributed in height SAR passes. To be more specific, for the case of multistatic collections, the passes as described by their Bistatic Equivalent Monostatic (BEM) trajectories, should be sparsely and unevenly distributed in height. For a given scene centre, a BEM antenna position is defined

along the bisecting line (bisectrix) of the transmitter and receiver positions relative to the scene centre, and at the bistatic range from the scene centre. $N-1$ volumetric interferograms are formed using a common reference image, and the phase factors of each interferogram are summed as follows:

$$R(\mathbf{x}) = \frac{1}{N} \left| 1 + \sum_{n=1}^{N-1} e^{i\varphi_n(\mathbf{x})} \right| \quad (5-1)$$

where $\varphi_n(\mathbf{x})$ is the phase at image voxel $\mathbf{x}$ of the n^{th} interferogram. Voxels, with a value of R , surpassing a threshold are classed as a 3D scatterer detection location. This approach utilises volumetric processing when forming a set of sparsely sampled interferograms, which removes the need to account for the layover distortions when summing the interferograms [144]. A consistent elevation angle across the imaged scene is not required for this approach, which also allows for bistatic, multistatic and SAR near-field geometries to be processed more easily.

However, a weakness of this SSARVI approach is that it acts only on single polarimetric channels at a time – it may for example be the case that some objects spread their scattering response over several polarimetric channels. In this case it would be advantageous to detect these responses in a joint manner, helping to raise a joint polarimetric phase factor sum R above the scatterer detection threshold.

For the generalised polarimetric algorithm, termed PolSSARVI, data is again collected from N sparsely sampled and unevenly distributed in height SAR passes, though now for all four polarisation channels (in the linear basis: HH, HV, VH, VV). Firstly, for a given pass n , across each set of four polarisations labelled p , the four complex values at a given SAR image voxel, $a_{np}(\mathbf{x})$, can be regarded as a two-by-two polarimetric scattering matrix (with four elements labelled p), whose polarimetric Span is given by:

$$S_n(\mathbf{x}) = \sum_{p=1}^4 |a_{np}(\mathbf{x})|^2 \quad (5-2)$$

The n^{th} Span-normalised polarimetric interferogram for polarisation p is here defined as follows:

$$\Gamma_{np}(\mathbf{x}) = \frac{a_{np}(\mathbf{x})a_{Np}^*(\mathbf{x})}{\sqrt{S_n(\mathbf{x})S_N(\mathbf{x})}} \quad (5-3)$$

where without loss of generality, pass N is chosen to be the reference pass. Note that with this definition, the magnitude of each polarimetric interferogram is generally not equal to one.

Now we define the sum over all of the collections of interferograms for a given polarisation as follows:

$$\beta_p(\mathbf{x}) = \frac{1}{N} \sum_{n=1}^N \Gamma_{np}(\mathbf{x}) \quad (5-4)$$

Finally, we may now define the generalised polarimetric phase factor sum, to be used for scatterer detections, as follows:

$$R_{\text{pol}}(\mathbf{x}) = S_N(\mathbf{x}) \sum_{p=1}^4 \frac{|\beta_p(\mathbf{x})|^2}{|a_{Np}(\mathbf{x})|^2} \quad (5-5)$$

It is the case that $R_{\text{pol}} \leq 1$, thus a threshold value less than one, to be determined, is applied for scatterer detections.

For each detection and for each of the four polarisations, the mean SAR image complex pixel value may be obtained $1/N \sum_{n=1}^N a_{np}(\mathbf{x})$, providing what can be regarded as a two-by-two polarimetric scattering matrix (with four elements, labelled p). Thus, for each of these detections, the bistatic Huynen fork parameters, or any other polarimetric decomposition, can be determined. The whole PolSSARVI process is described in the flow diagram in Figure 5-1.

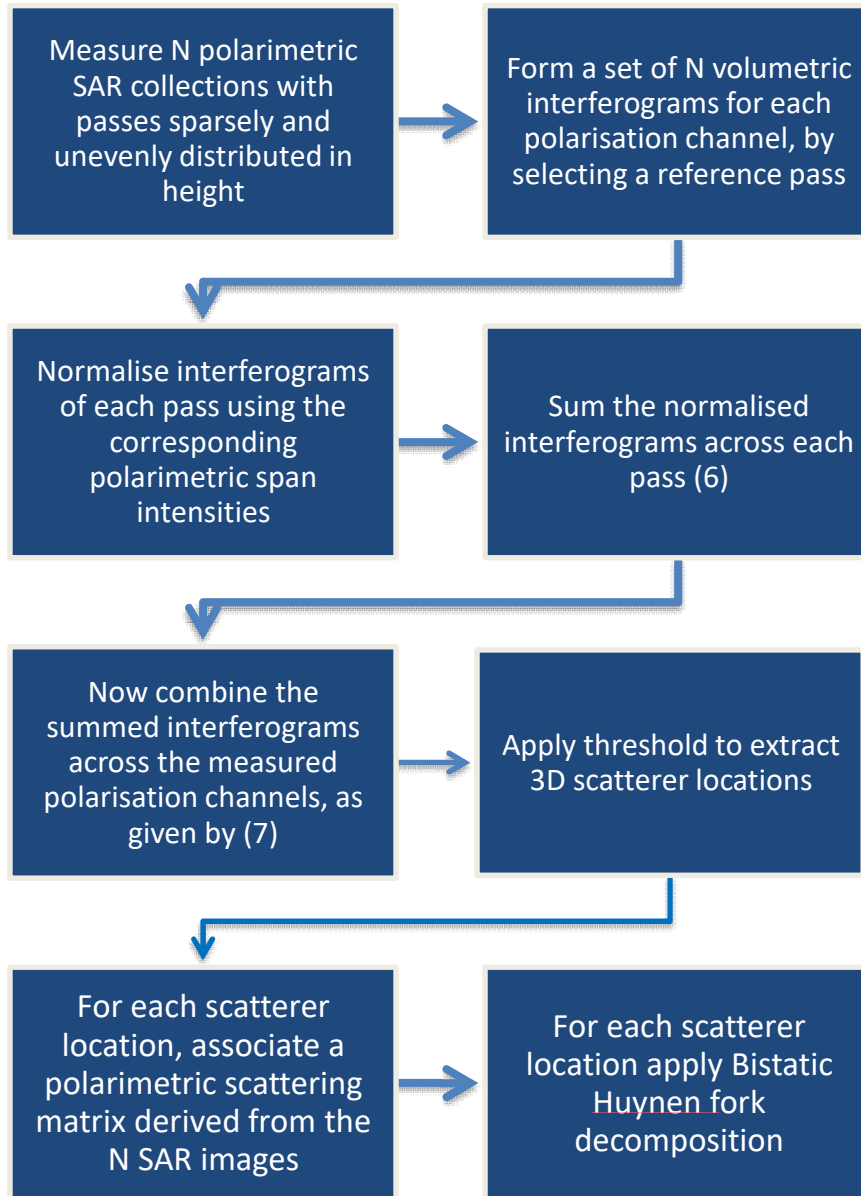


Figure 5-1: Flow diagram outlining the PolSSARVI algorithm.

It is noted that alternative phase factor sum expressions are possible, giving varying levels of detection performance. In particular, one may define a generalised polarimetric phase factor sum term which has its form in correspondence with (5-1):

$$R_{\text{pol2}}(\mathbf{x}) = \left| \sum_{p=1}^4 \beta_p(\mathbf{x}) \right| \quad (5-6)$$

$$= \frac{1}{N} \left| 1 + \sum_{p=1}^4 \sum_{n=1}^{N-1} \Gamma_{np}(x) \right|$$

However, the detection performance for $R_{\text{pol}2}$ has been found to be no better, and sometimes worse, than that for R_{pol} in (5-5), likely due sensitivity due to the complex summation over polarisation terms.

5.4 Bistatic Polarimetry and Huynen Fork Parameters

Bistatic radar polarimetry has seen a significant degree of interest from the 1990s onwards [130], [132]. Bistatic polarimetry is generally more complicated than monostatic, as cross-polarisation terms are no longer equivalent. In this paper, we employ a robust bistatic polarimetric decomposition developed by Titin-Schnaider which builds on work by Huynen [133], [134], [135]. Huynen showed that the scattering matrix of a target illuminated monostatically, S , can be written in the diagonalised form:

$$S = U(\theta, \tau)^* S_d(\gamma, \nu) U(\theta, \tau)^\dagger \quad (5-7)$$

Where the superscript ‘*’ represents the complex conjugate, and ‘†’ represents the complex conjugate transposed. θ, τ, γ and ν are the four real-independent Huynen fork parameters describing polarimetric properties of the scatterer; U is a unitary matrix constructed for the diagonalisation of S ; S_d is a diagonalised matrix that contains information about the polarisability of the scatterer, and nominally about whether the number of reflections associated with the scattered ray is odd or even [131].

Titin-Schnaider showed that Huynen diagonalisation approach could be extended to the bistatic case as follows:

$$S = U(\theta_i, \tau_i)^* S_d(\gamma, \nu) V(\theta_s, \tau_s)^\dagger \quad (5-8)$$

Where $\theta_i, \tau_i, \theta_s, \tau_s$ are redefined to include the orientation and symmetry of the scatterer for incident “ i ” and “ s ” scattered wave directions; U and V are unitary matrices which diagonalise the asymmetric scattering matrix for bistatic radar

geometries. The approach developed by Titin-Schnaider was further developed by Xu et. Al [63], presenting a computationally efficient algorithm.

The Huynen fork parameters are summarised as follows:

- Orientation / Tilt angle, θ : the angle of the dipole type scattering orientation to the horizontal plane about the line from the transceiver to the scatterer, in the anticlockwise direction, when looking down-range. The range of definition is $-90^\circ \leq \theta \leq 90^\circ$. In the monostatic case there is one orientation parameter, however in the bistatic case this is replaced by two: θ_i denoting the incident direction orientation relating to the transmitter, and θ_s denoting the scattered direction orientation relating to the receiver;
- Symmetry angle, τ : a parameter indicating how symmetric a scatterer is about the orientation direction. The range of definition is $0^\circ \leq |\tau| \leq 45^\circ$ where 0° indicates high symmetry and $|\tau|=45^\circ$ indicates low symmetry. In the monostatic case there is only one symmetry parameter, however in the bistatic case this is replaced by two: τ_i denoting the incident direction symmetry relating to the transmitter, and τ_s denoting the scattered direction symmetry relating to the receiver;
- Skip angle, v : a parameter indicating whether the scattered ray has had an odd or even number of interactions (bounces) with the target. The range of definition is $0^\circ \leq |v| \leq 45^\circ$ where 0° indicates an Odd-bounce interaction and $|v|=45^\circ$ indicates an Even-bounce interaction. Odd-bounce type scatterers include spheres, flat plates and trihedral multi-bounce. Even-bounce scatterers include dihedral multi-bounce;
- Polarisability angle, γ : a parameter indicating how polarising a scatterer is – a measure of how much the incident polarisation is projected on to any particular orientation. The range of definition is $0^\circ \leq \gamma \leq 45^\circ$ where 0° indicates high polarisability and 45° low. Scatterers with medium to high polarisability include rods and the straight edges of extended objects. Scatterers with low polarisability

include spheres, flat plates, and corner reflectors with multi-bounce, including both dihedrals and trihedrals.

The six Huynen parameters extracted for the experimental work of this paper were obtained via singular value decomposition [63], with the main experimental point-cloud renderings being presented for the rotationally independent parameters v and γ .

5.5 Simulations

Simulations were conducted to verify the effectiveness of the PoISSARVI approach, and compare its combined interferometric response to the response of the more basic SSARVI approach across multiple polarisation channels. Simulations were performed on isotropic point scatterers and used a sparsely sampled 2D bistatic SAR aperture, seen in Figure 5-2. For simplicity, scatterers were given the *monostatic* polarimetric response of a flat plate, a dihedral and of dipole scatterers. The antennae geometry and scatterer locations of the first simulation are shown in Figure 5-2.

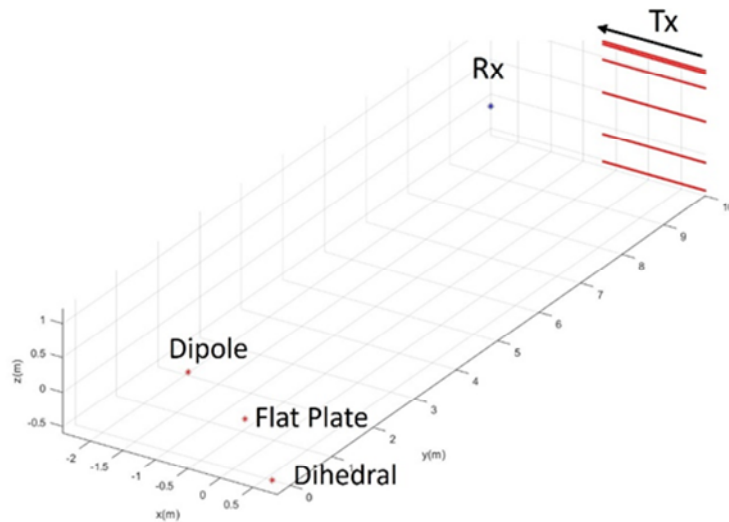


Figure 5-2: Sparse bistatic SAR simulation geometry. The transmitter aperture (red) is labelled Tx and receiver position is labelled Rx. The flat plate, dihedral and dipole scatterers are labelled in the scene.

The vertical extent of the transmitter aperture is 1.74 m, with a mean pass separation of 2° in elevation to the scene centre at the origin. The mean bistatic angle was 14.4°. This is the angle between the mid-transmitter aperture point and the receiver location, through the scene centre. Using a frequency range of 7-8 GHz, this would give a theoretical unambiguous vertical cross-range extent of 1.11 m. The unambiguous extent is approximately equal to half of the 2 m vertical target extent. The point scatterers were situated between 9.5 m and 10.5 m in ground range from the transmitter aperture. The dihedral and dipole scatterers were orientated respectively at 22.5° and 45° anticlockwise to the down-range direction, so that in the form of a two-by-two scattering matrix their polarimetric responses in the linear polarimetric basis were set as follows:

$$S_{\text{plate}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5-9)$$

$$S_{\text{dih}}(22.5^\circ) = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$S_{\text{dip}}(45^\circ) = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\text{where, } S = \begin{bmatrix} HH & HV \\ VH & VV \end{bmatrix}$$

Figure 5-3 presents the summed interferometric response R_{pol} along vertical iso-ranges (constant range) arcs through the three scatterers.

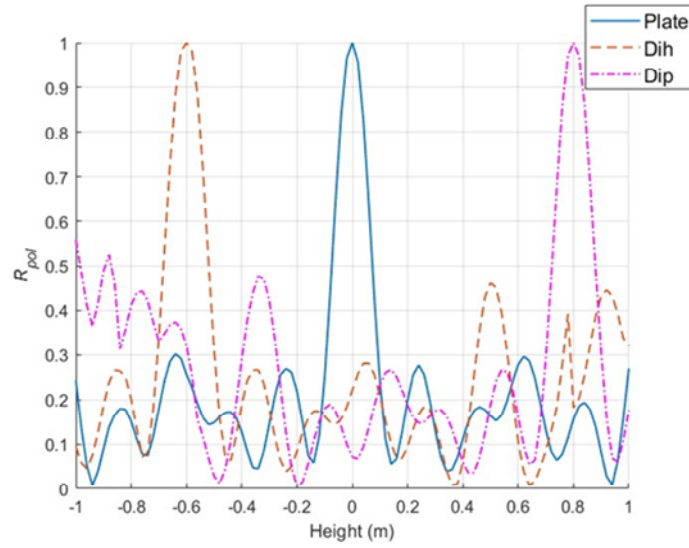


Figure 5-3: Summed interferometric response R_{pol} along vertical iso-range arcs through the location of the flat plate, dihedral and dipole scatterer. These are labelled Plate, Dih and Dip respectively.

The R_{pol} curves attain maximum values of 1 when passing through their respective scatterers at the correct heights of -0.6 m, 0 m and 0.8 m. The R_{pol} curves have low sidelobes. Using conventional image formation algorithms, with a transmitter aperture sampled evenly in elevation, would result in aliased peaks which would significantly increase scatterer height ambiguity and result in false detections.

To contrast the performance of the new PolSSARVI algorithm in comparison to the SARVI algorithm, an additional simulation is run, using the previous bistatic SAR geometry of Figure 5-2, though with only a single scatterer, consisting of a horizontal dipole at 0 m height. A low level of noise is added to each polarimetric channel to illustrate algorithm behaviour where scatterer polarimetric responses are very low. In the channels with signal present, the signal to noise ratio (SNR) was 60 dB.

In Figure 5-4, vertical iso-range plots for R_{pol} (PolSSARVI) and the SSARVI derived R for the single HH and HV polarimetric channels are presented.

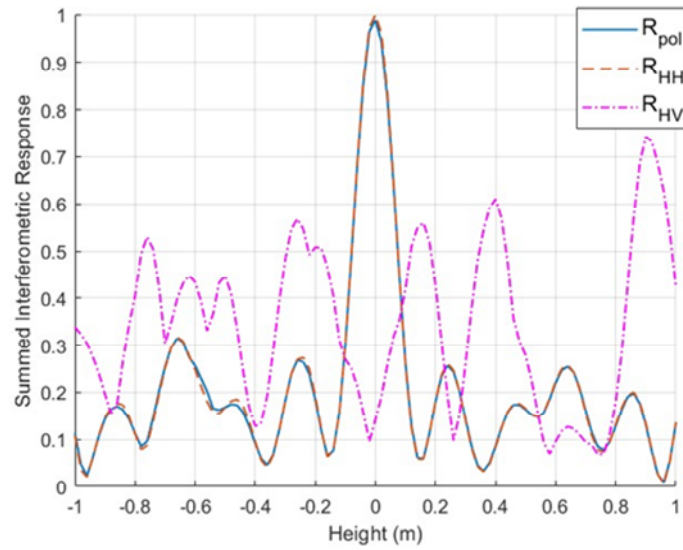


Figure 5-4: Summed interferometric responses along the vertical iso-range arc through the location of a horizontal dipole. The PolISSARVI response R_{pol} and the SSARVI response R_{HH} appear overlaid.

Dipoles are polarising scatterers, showing a strong response in a single polarisation orientation, and in this case the strong response is in the HH channel only. The combined polarisation PolISSARVI approach provides a clear and correctly located peak. The SSARVI approach also provides a correct detection in the HH channel (R_{pol} and R_{HH} appear overlaid), however in the HV channel, as expected, the scatterer is not correctly detected. The highest peak in R_{HV} is in the wrong location, so that the SSARVI threshold application step would lead to an erroneous detection.

The performance of the PolISSARVI approach has been found to be invariant to polarimetric scatterer type and orientation, whereas the application of the SSARVI approach on individual polarisations followed by a combination of the detections, leads to false detections when scatterers show a dim response in any given polarimetric channel.

5.6 Cranfield GBSAR Laboratory Overview

5.6.1 Experimental Setup

The SSARVI and PolSSARVI algorithms were applied to an experimental multistatic SAR data set, collected at the Cranfield GBSAR laboratory. The laboratory contains a representative target scene, with the quarter-scale model tank situated on a gravel ground surface, shown in Figure 5-5. The model tank target is approximately 80 cm in height, and 2.5 m in length. The gravel scene is approximately 3.5 m in width and 4.5 m in length, and is marked by the sphere reflectors shown around the edges. A large reference sphere towards the back of the target scene is visible in Figure 5-5, and is elevated slightly off the ground. The phase-history data was recorded using a Keysight Vector Network Analyser (VNA), which also linked the two antennas by high fidelity microwave cables.



Figure 5-5: Photographs of model tank target. The wire on top is denoted by "W", the turret of tank denoted by "T", and the reference sphere is denoted by "S".

The transmitter and receiver are situated on independent antenna rails, each capable of sweeping out 3.5 m in width and 1.9 m in height. Using this setup, a finely sampled 2D multistatic aperture was used as the initial collection geometry, consisting of 95 vertically spaced horizontal transmitter passes with a 1.5 cm

spacing in height, together with two fixed receiver positions to the left of the aperture, depicted in Figure 5-6. The collection was fully polarimetric with a frequency range of 6.62 – 10 GHz.

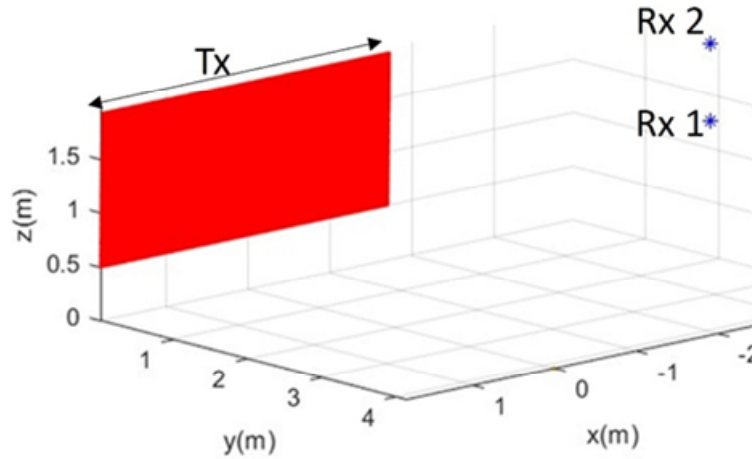


Figure 5-6: Multistatic SAR geometry. The 2D transmitter SAR aperture is shown in red and denoted by Tx. The stationary receiver positions are shown with blue asterisks and denoted Rx1 and Rx2.

The transmitter aperture extent was 3.5 m wide and 1.5 m high, providing azimuth and elevation transmitter apertures of 44° and 17.7° . These values were determined relative to the scene centre, which was 4.15 m from the transmitter aperture. The mean bistatic angles to the receiver positions Rx1 and Rx2 were 61.4° and 62.8° respectively, giving ground-range resolutions of 5.7 cm and 5.4 cm, and cross-range resolutions of 5.5 cm and 5.4 cm respectively. The theoretical vertical resolutions for Rx1 and Rx2 were 13.6 and 13.5 cm respectively. The resolution values were determined from the BEM aperture relative to the scene centre.

5.6.2 Polarimetric Calibration

Polarimetric calibration is achieved by measurement of calibration scattering targets with known scattering response, following Sarabandi et al. [172]. Metal spheres were employed as calibration scatterers, since their scattering matrix can be computed accurately. Additionally, a strong cross-polarisation scatterer was also employed, consisting of four parallel metal rods. The rods were attached to a polystyrene backing board and acted as a grid of dipole scatterers. The rods

were 60cm long with 20cm separation. The polystyrene backing was cut into an octagon, allowing the grid to be easily rotated in 45° steps for tests with different configurations.

5.7 Methodology

5.7.1 Aperture Downsampling

For conventional image formation with the laboratory scenario, a minimum transmitter pass spacing height of 7.8 cm in height would be required to suppress aliasing artefacts over a 1.8 m high volume of interest (Vol). This aperture spacing is a subset of the very finely sampled SAR geometry presented in Figure 5-6, and shall be referred to here as the Sub-Initial Sampling (SIS). However, in accordance with the SSARVI and PolSSARVI requirements, for the downsampling procedure, it was convenient that whilst maintaining the required average spacing, the SIS had randomized heights. For algorithm evaluation, the transmitter aperture was repeatedly downsampled from the SIS to form sparse 2D apertures, with a mean pass vertical sampling below the minimum Nyquist sampling requirement. This was achieved by removing passes randomly from heights within the transmitter vertical aperture extent. The subsampled transmitter apertures are presented in Figure 5-7.

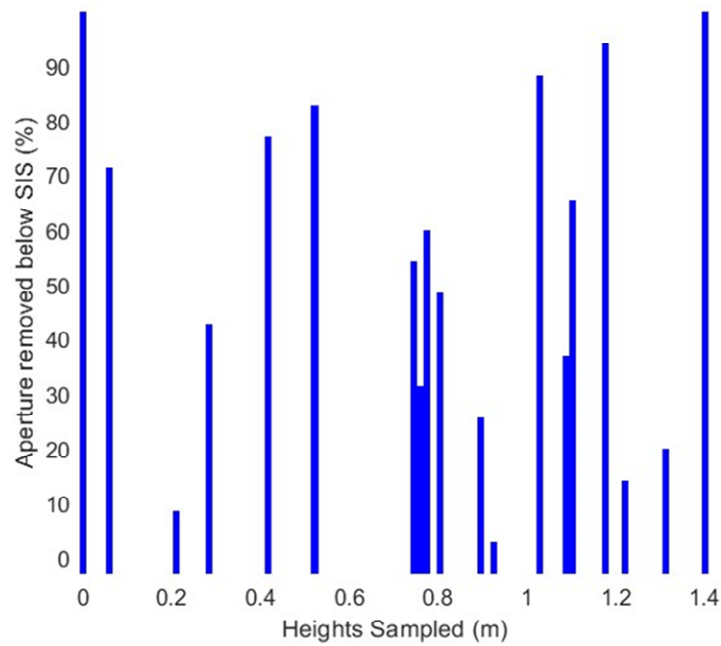


Figure 5-7: Downsampled transmitter height apertures. Blue lines show heights sampled for a given randomly downsampled aperture. White spaces consist of fully removed SAR passes.

The first subsampled geometry, presented in Figure 5-7, consisted of a mean transmitter pass spacing of 7.8 cm. This is the SIS aperture, at the minimum mean spacing required for suppressing artefacts over the Vol around the target. For the range of sparse apertures presented, the performance of the PoISSARVI and SSARVI approaches were compared. The performance was determined by the sub-Nyquist performance measures presented below.

5.7.2 Sub-Nyquist Imaging Performance Measures

Two main measures were used to determine the performance of the PoISSARVI and SSARVI algorithms with sub-Nyquist sampled apertures. These included a measure of the level of false detections present in the point-clouds, and of the accuracy of the bistatic Huynen fork parameters.

The level of false detections was determined by measuring the False Alarm Ratio (FAR) for a range of aperture samplings below Nyquist sampling. The FAR is given by:

$$FAR = \frac{FP}{TP + FP} \quad (5-10)$$

where FP denotes the number of False Positive erroneous detection of artefacts, and TP denotes the number of True Positive correct detections [173].

The TP and FP scatterer detections were determined from two 3D binary mask images for the model tank target. To form the masks, two separate span polarisation volumetric images were formed from fully sampled SAR aperture data with the backprojection algorithm (BPA), one for each receiver. The binary mask of each image was formed by applying a span-intensity threshold at 50 dB below the maximum of each image. The TP and FP scatterer locations of each mask image was thus determined, and combined into a single data set.

In addition to the FAR measurements, further quantitative analysis of the sub-Nyquist aperture performance of the SSARVI and PolSSARVI algorithms was provided by normalised cross-correlation results. The normalised cross-correlation was performed between the 3D point clouds formed with the SIS aperture, and the successively downsampled apertures. The polarimetric span intensity was associated for each detection location in the point clouds used in the cross-correlation. This approach, applied to both algorithms, gave a numerical indication of image quality.

The polarimetric properties of the model tank structure were determined by the Huynen parameters. The accuracy of the Huynen parameters obtained for the sub-Nyquist formed PolSSARVI detections was determined by making a comparison with Huynen parameter arrays obtained from the full SAR aperture BPA images, which will be referred to as Huynen Reference Arrays. The BPA-formed Huynen parameters obtained for the model tank target were validated in previous work [65].

The accuracy of the Huynen parameters obtained for the PolSSARVI detections was determined by making a comparison with Huynen parameter arrays obtained from the full SAR aperture BPA images, which will be referred to as Huynen

Reference Arrays. For a quantitative comparison, the normalised-mean squared deviation (NMSD) was employed for any given Huynen parameter:

$$\bar{\sigma}^2 = \frac{\sum_{i=1}^{D_j} |F(x_i) - F_{RA}(x_i)|^2}{D_i G^2} \quad (5-11)$$

where i enumerates detections of which there are D_j in total for the j^{th} sparse aperture; $F(x_i)$ is the value of a particular Huynen parameter for detection i at location x_i ; $F_{RA}(x_i)$ is the corresponding value of the same Huynen parameter reference array; G is the range of the specific Huynen parameter.

It is noted that there were slight variations in the location of scatterer detections for the different aperture samplings, however the NMSD results in section 0 were restricted to evaluation over ‘TP’ detections only, which reduced the variations.

5.7.3 Huynen Reference Array

The Huynen parameters were determined for full SAR aperture BPA images from each receiver separately. For each Huynen parameter and image position, the mean value over the two receiver antennas were used to form the Huynen parameter reference arrays used in (5-11). The Huynen reference arrays were used to colour the Span polarisation BPA SAR images presented in Figure 5-8 (formed with full SAR apertures). VoxView software [118] was used to form the 3D renderings, where both the brightness and the opacity of each voxel is determined by the polarimetric span-intensity. The polarimetric span-intensity is presented with a dB scale, with a dynamic range of 50 dB. The figure shows renderings for both the polarisability angle and the Skip angle.

The top rendering shows the polarisability angle across the tank scene. Non-polarising scatterers appear as green and polarising scatterers as red. The tank structure is seen as mainly non-polarising, except for the wire labelled as ‘W’ on the top of the tank. The bottom rendering shows the Skip angle across the tank scene, with red indicating an Odd-bounce response and green indicating an Even-bounce response. A mixture between Odd and Even bounce scattering mechanisms is seen to be present across the tank structure, which may be expected due to the complex nature of the target. The reference sphere at the

back of the scene allowed a validation check on the polarimetry: it was seen to show the expected strong single/Odd-bounce and non-polarising scattering.

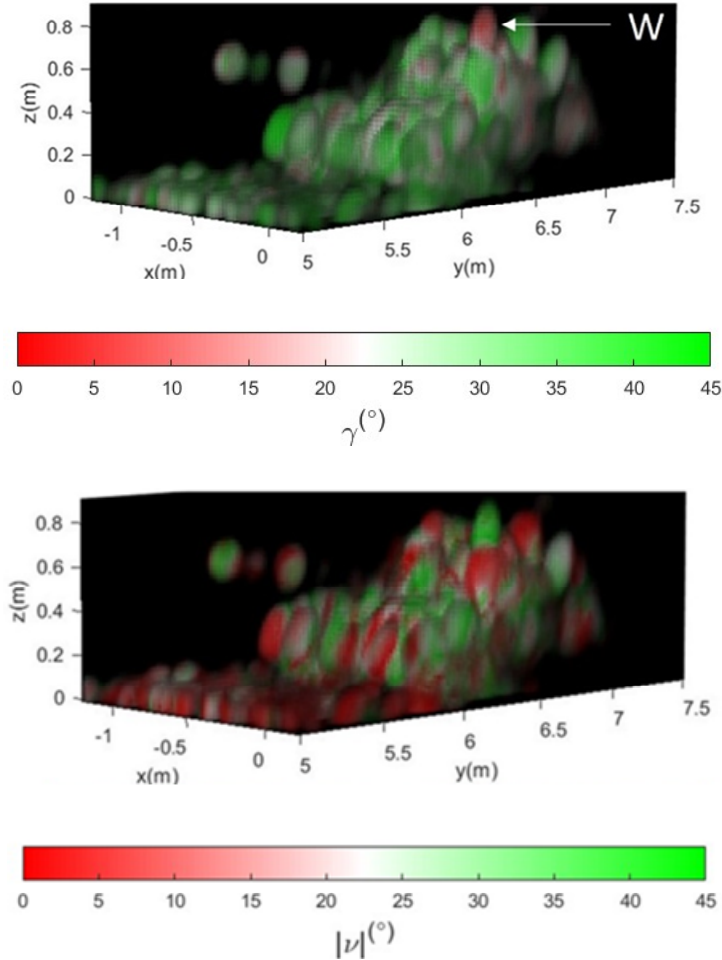


Figure 5-8: Multistatic 3D BPA SAR images of the model vehicle, formed with the full transmitter aperture presented in Figure 5-6: Multistatic SAR geometry. The 2D transmitter SAR aperture is shown in red and denoted by Tx. The stationary receiver positions are shown with blue asterisks and denoted Rx1 and Rx2.. Polarimetric Span intensity in dB provides the brightness in these images, and colour is attributed to polarimetric parameter angles. The polarisability angle is shown (top) together with the Skip angle (bottom). “W”, in the top rendering, denotes the position of the wire, which is seen to have a high polarisability value.

5.8 Experimental Results

5.8.1 Sub-Nyquist Point-Clouds

The SSARVI and PolSSARVI thresholding value used (see Figure 5-1) is described numerically as:

$$T = \alpha(\max(R) - \text{mode}(R)) + \text{mode}(R) \quad (5-12)$$

where α , referred to here as the threshold factor where $0 \leq \alpha \leq 1$, and was varied to determine an optimal threshold T . The modal values of the interferometric phase factor sum R , were determined for both SSARVI and PolSSARVI approaches [145].

The threshold factor was determined by sequentially increasing its value on finely sampled experimental imagery until the background noise and sidelobe artefacts were no longer visible in the point-clouds. For the PolSSARVI algorithm, there were no visible background erroneous detections above $\alpha = 0.5$. For the SSARVI algorithm, there were no visible background artefacts above $\alpha = 0.75$. These values of α were adopted for subsequent point cloud generation.

Example renderings of the PolSSARVI algorithm being used to form sub-Nyquist sampled multistatic polarimetric 3D point-clouds are presented in Figure 5-9. The polarisability angle is represented through colour. Two different orientations are presented, the top one with the tank barrel pointing left, and the bottom one pointing right. The renderings were produced using VoxView, as was the case for Figure 5-8.

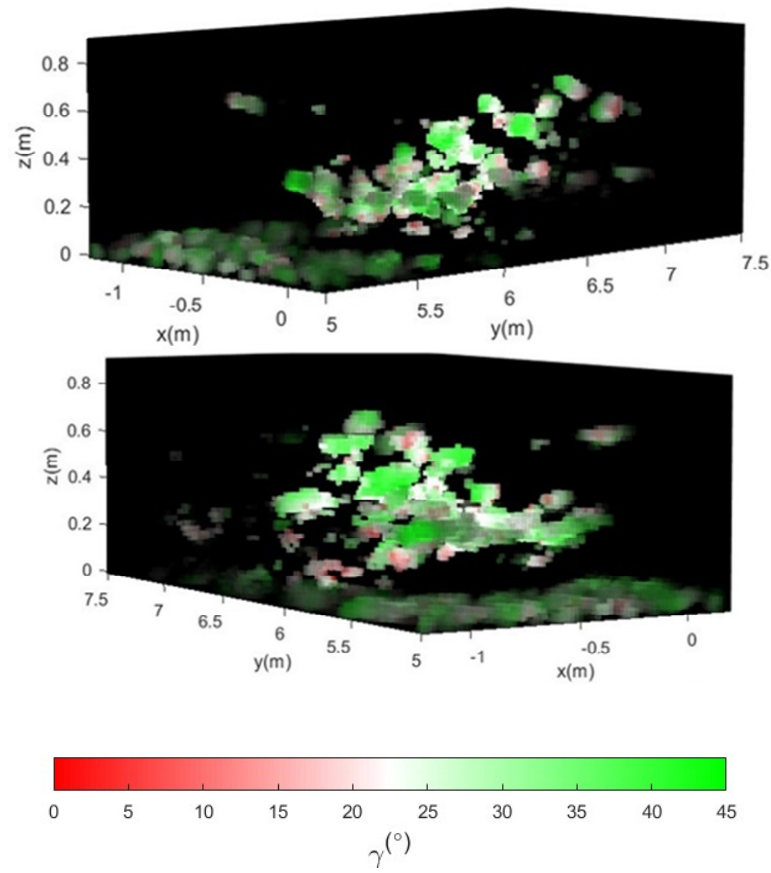


Figure 5-9: Sub-Nyquist SAR aperture polarimetric and multistatic T72 rendering showing the Huynen Polarizability angle. Green shows non-polarising scatterers, red shows polarising scatterers. Two different viewing angles of the same rendering are shown.

Figure 5-9 was formed using a sparse multistatic geometry, using 57% Aperture Removed From SIS (ARF-SIS) with a mean spacing 18.1 cm. The two stationary receiver locations, that were shown in Figure 5-6, were used to form the imagery. The thresholding was applied to the data from each receiver separately, with the detection points from each then overlaid. No false scatterer detections are present.

As was the case in the BPA rendering, the result shows a predominantly non-polarising scattering response for the PolSSARVI point-cloud, though a small number of polarising scatterers can be seen. The wire, visible in Figure 5-8a, was not visible in the PolSSARVI rendering.

In correspondence with Figure 5-9, Figure 5-10 shows the Huynen Skip angle determined at each point in the point-cloud rendering, formed from a 57% ARF-SIS aperture.

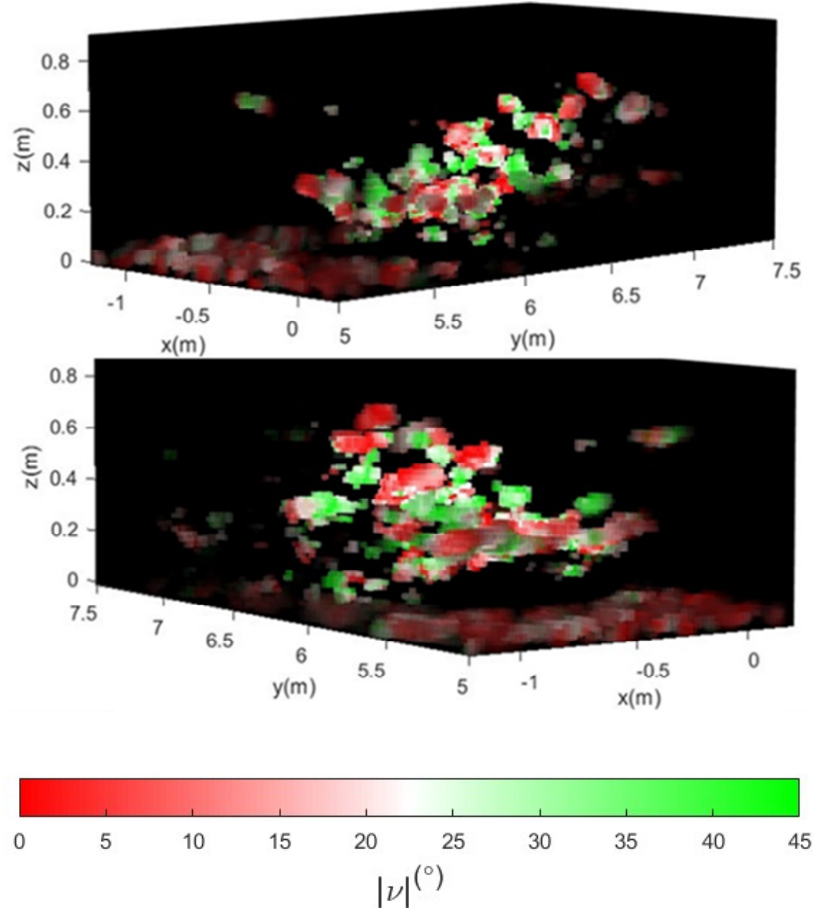


Figure 5-10: Sub-Nyquist SAR aperture polarimetric and multistatic T72 rendering showing the Huynen Skip angle. Green shows Even-bounce scatterers, red shows Odd-bounce scatterers. Two different viewing angles of the same rendering are shown.

The main tank structure shows a mixture of Odd and Even-bounce scattering, as was the case for the full aperture BPA rendering in Figure 5-8. The cannon shows a similar response to the corresponding locations in the full aperture BPA image. The reference sphere at the back of scene correctly showed a distinct Odd-bounce scattering response for all aperture samplings.

5.8.2 Sub-Nyquist Performance of SSARVI and PolSSARVI Algorithms

The False Alarm Ratio (FAR) was determined for PolSSARVI and SSARVI algorithms, using a multistatic data set that was sub-Nyquist sampled in elevation. The apertures used to measure the FAR were presented in Figure 5-7, and were formed by sequentially downsampling the SIS aperture, as described in section 5.2.

Figure 5-11 compares the FAR of the SSARVI and PolSSARVI approaches. The FAR measured at each aperture sampling is given by equation Figure 5-10. The 'TP' and 'FP' scatterer locations were determined from the corresponding mask images as described in 5.7.2.

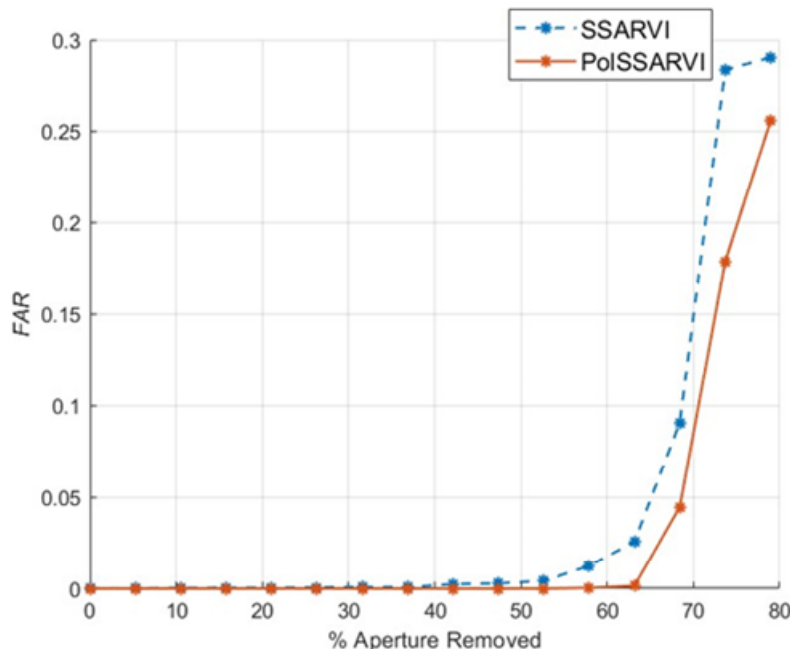


Figure 5-11: False Alarm Ratio (FAR) of the SSARVI and PolSSARVI approaches for sub-Nyquist SAR aperture samplings below SIS.

As the transmitter aperture was downsampled from 0% to 42% ARF-SIS, the SSARVI approach showed a negligible FAR. After 42% aperture removal below SIS, the erroneous artefact detections became more visible around the target and the measured FAR started to increase. The level of artefact detections started to have a significant presence on the rendering at 57% ARF-SIS, at which point the

FAR started to increase at a sharper rate. The tank structure was difficult to discern at 63% aperture removal below SIS, at which point the measured FAR was approximately 0.02. The tank structure became indiscernible from artefact detections at 68% ARF-SIS, at which point the measured FAR was approximately 0.1.

For the PoISSARVI approach, there were no visible artefact detections up to 57% ARF-SIS. At 63% ARF-SIS, there was a minor level of erroneous detections, with a negligible effect on the FAR. The FAR started to increase significantly at 68% ARF-SIS. The tank structure became indiscernible from erroneous detections at 73% ARF-SIS, with a FAR of approximately 0.17.

Comparing the two approaches, the PoISSARVI approach showed fewer erroneous detections over a larger range of aperture samplings below Nyquist sampling. The tank structure also became indiscernible at a lower aperture sampling for PoISSARVI, which implies that it provides an improved performance compared to SSARVI, with lower aperture sampling requirements.

The FAR was found useful for indicating the presence of artefacts, with values below about 0.1 providing scatterer point-clouds with some utility, and below about 0.01 providing point-clouds of a good quality.

The FAR measurements are supplemented by the normalised cross-correlation results between the sub-Nyquist aperture point clouds and the finely sampled aperture point clouds for both algorithms, shown in Figure 5-12.

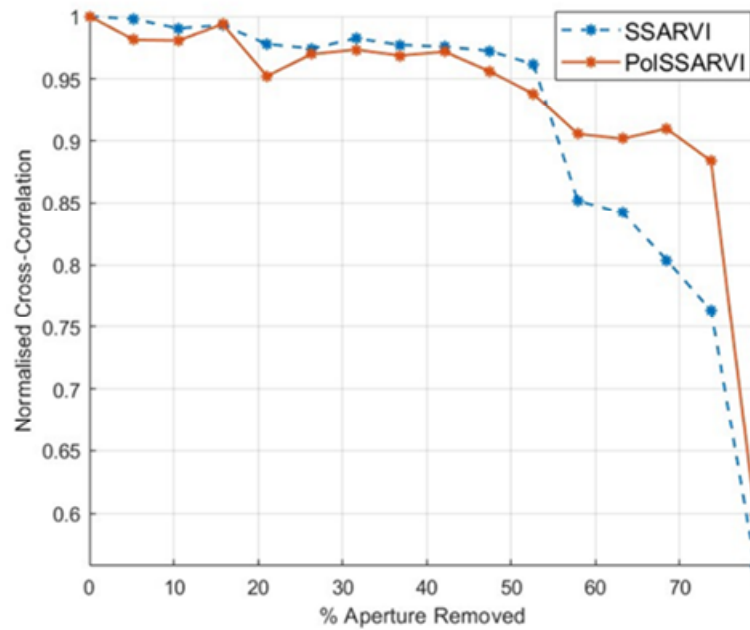


Figure 5-12: Normalised cross-correlation of sub-Nyquist formed point cloud renderings with initial SIS aperture. Performed for both SSARVI and PoISSARVI algorithms.

The sub-Nyquist sampled aperture point clouds formed with the SSARVI algorithm show an initial high cross-correlation with the corresponding SIS aperture point cloud. This continues until 47% ARF-SIS, after which the measured cross-correlation starts to noticeably decline. The cross-correlation decreases to 0.85 at 57% ARF-SIS which is noted as being the aperture sampling at which the discernibility of the tank becomes limited due to the presence of artefact detections. The measured cross-correlation continues to decrease after 63% ARF-SIS, the point at which the tank structure becomes indiscernible.

The measured cross-correlation for the sub-Nyquist aperture point clouds formed of the PoISSARVI algorithm, also shows a high correlation over a similar downsampling range when compared to the SSARVI algorithm. The values obtained for the PoISSARVI algorithm are marginally lower compared to those obtained from the SSARVI algorithm until 52% ARF-SIS. At 57% ARF-SIS, the PoISSARVI algorithm has a higher degree of correlation with the finely sampled point clouds, which is the point at which distinguishability of the tank structure is limited for the SSARVI algorithm. For the PoISSARVI algorithm, only minor artefacts are present at this aperture sampling. The normalised cross-correlation then only shows a significant decrease at 73% ARF-SIS, which is the point at which the tank structure becomes indiscernible from artefact detections.

There was reasonable agreement between the determined FAR at each downsampled aperture, and the corresponding measured cross-correlation. The decrease in cross-correlation for the SSARVI algorithm at 57% ARF-SIS corresponded to a sharp increase in the FAR for the same aperture. This was the point at which there was limited discernibility for the tank structure in the SSARVI-formed point clouds, and only minor artefacts for the PolSSARVI-formed point clouds. As the measured cross-correlation for the PolSSARVI algorithm stayed above 0.9 at 57% ARF-SIS, this suggests that 0.9 is an optimal value for classifying good quality 3D point cloud imagery.

5.8.3 Sparsely Sampled Aperture Polarimetry Performance

The NMSD of the Huynen parameters, defined in equation (5-11), obtained for the PolSSARVI approach, are shown in Figure 5-13. They are presented for the sub-Nyquist apertures shown in Figure 5-7. The NMSD of the Huynen parameters was determined for the sub-Nyquist point-cloud detections across the structure of the target for different aperture samplings.

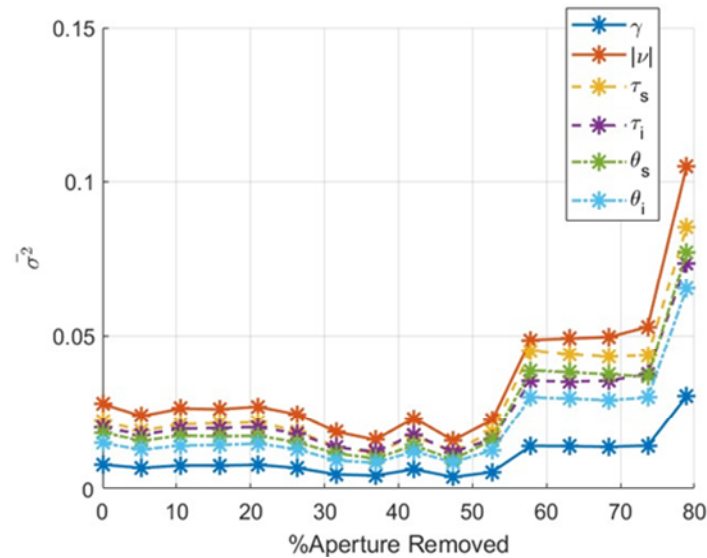


Figure 5-13: The NMSD of all six Bistatic Huynen fork parameters for each aperture sampling.

All the Huynen parameters show the same general trend of maintaining a stable NMSD until about 57% ARF-SIS, after which they go through increases. The

curves do have different NMSD levels however, due to a constant difference between the point cloud detection polarisations and the corresponding BPA derived Huynen reference arrays (see section 5.7.3).

At 73% ARF-SIS it is noted that the target structure became indiscernible from erroneous detections, and for more sparse sampling, all the NMSD curves increase further. The Polarisability parameter seems to be the least sensitive to

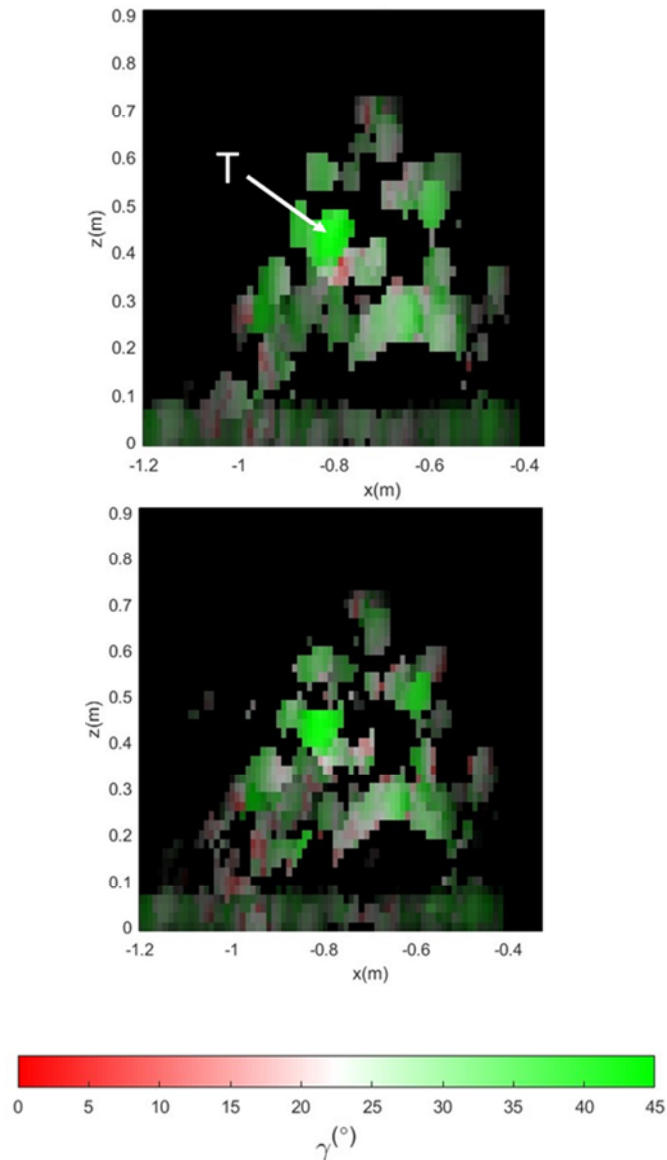


Figure 5-14 PoISSARVI point-cloud MIPs in the x-z plane, showing a front view of the tank rendering. Colour indicates the Polarisability angle, with brightness determined by the polarimetric span in dB, with a 50dB dynamic range. The renderings were formed at SIS sampling (top) and at 52% ARF-SIS (bottom). The turret labelled ‘T’.

aperture downsampling and experienced the least variation in NMSD. This is likely because the scene showed an overwhelmingly non-polarising scattering response.

Polarimetric point-cloud MIPs are shown in Figure 5-14 through Figure 5-17. The renderings were formed at aperture samplings of 52% ARF-SIS, showing a negligible presence of erroneous detections and a Huynen parameter NMSD below 0.1. The MIPs in Figure 5-14 provide a front-view of the target, showing the maximum polarimetric span in the y-direction in a dB scale with 50 dB dynamic range, with its associated Polarisability angle represented by colour.

The top rendering shows the polarisability angle of the tank structure, formed at the SIS aperture. The scattering centre on the turret of the tank, labelled “T”, is the brightest scatterer. Overall, little difference can be seen in the polarisability angle when comparing with the 52% ARF-SIS in the lower figure, confirming the Huynen parameter NMSD results in Figure 5-13.

Figure 5-15 presents MIPs formed with the same aperture sampling and polarimetric-span intensity projection as those presented in Figure 5-14, though the colour here represents the Skip angle. Again, there is a good deal of similarity between the images with the different aperture samplings.

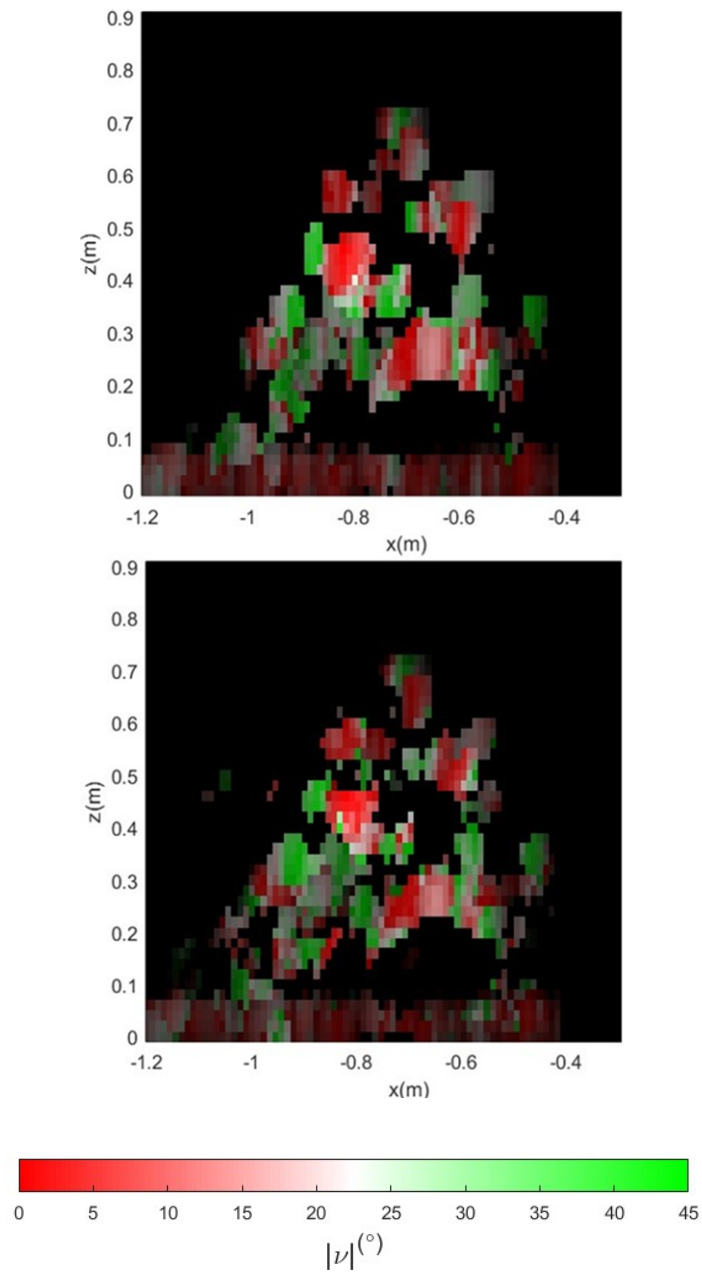


Figure 5-15 PolSSARVI point-cloud MIPs in the x-z plane, showing a front view of the tank rendering. Colour indicates the Skip angle, with brightness determined by the polarimetric span in dB, with a 50dB dynamic range. The renderings were formed at SIS sampling (top) and at 52% ARF-SIS (bottom).

Figure 5-16 and Figure 5-17 show MIPs corresponding to those in Figure 5-14 and Figure 5-15, but now with an x-direction projection, providing a side view of the target with the cannon visible, extending beyond the front of the target. Only minor changes in the location of the detections and their polarisation are

noticeable between the differently sampled aperture results. The target can be seen to be mostly non-polarising, and there is some variability in the Skip parameter.

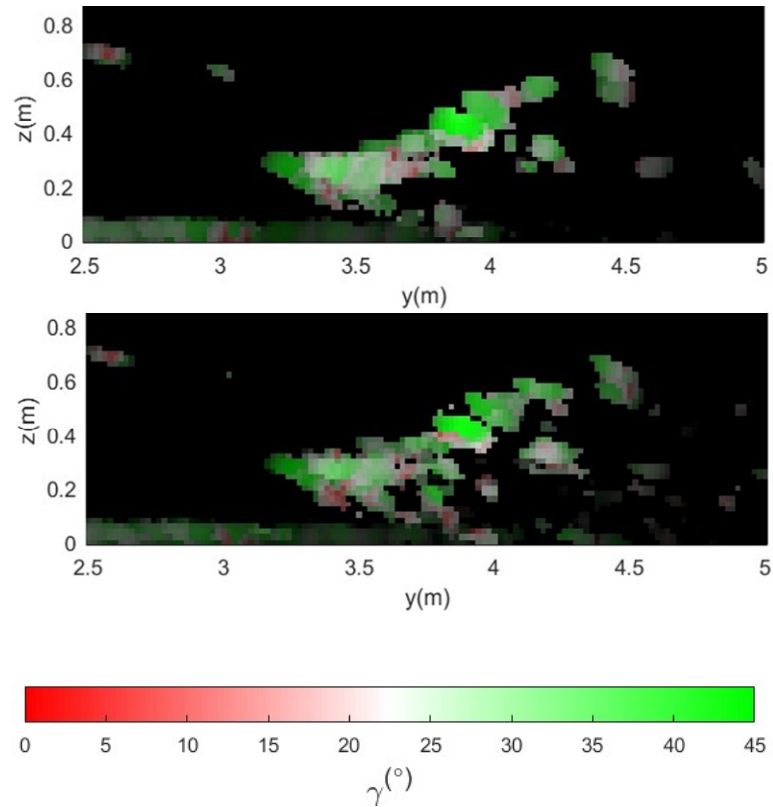


Figure 5-16: PoISSARVI point-cloud MIPs in the y-z plane, showing a side view of the tank rendering. Colour indicates the Polarisability angle, with brightness determined by the polarimetric span in dB, with a 50dB dynamic range. The renderings were formed at SIS sampling (top) and at 52% ARF-SIS (bottom).

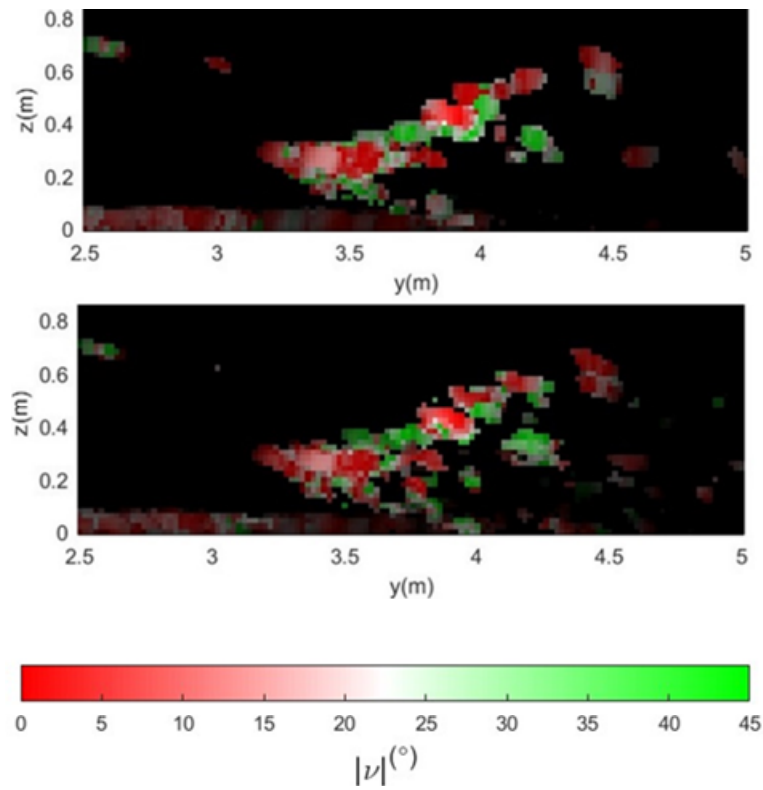


Figure 5-17: PolSSARVI point-cloud MIPs in the y-z plane, showing a side view of the tank rendering. Colour indicates the Skip angle, with brightness determined by the polarimetric span in dB, with a 50dB dynamic range. The renderings were formed at SIS sampling (top) and 52% ARF-SIS (bottom).

Figure 5-18 shows the Huynen parameters determined at the location of the reference sphere, located to the rear of the target scene, labelled “S” in Figure 5-5. These values were extracted for the sparse apertures presented in Figure 5-7. For all the apertures, the Huynen parameters were measured at a common point on the sphere.

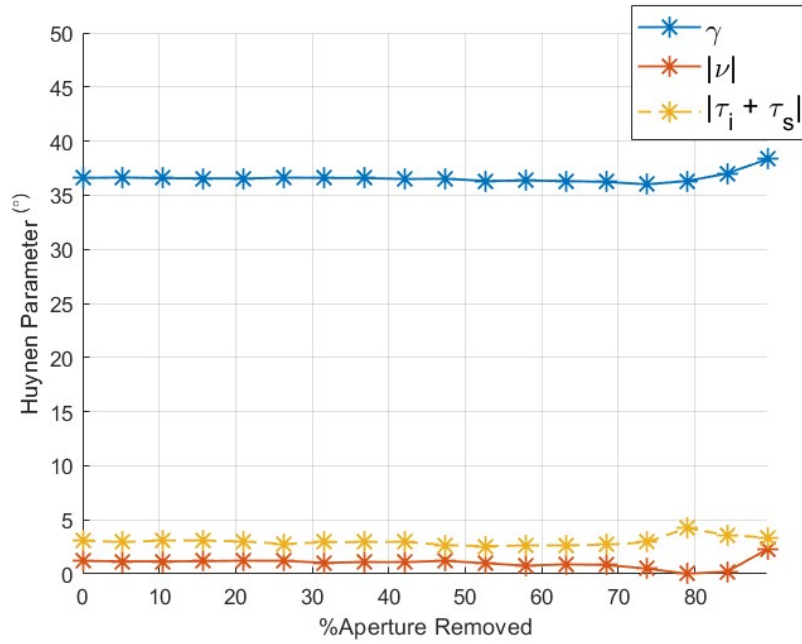


Figure 5-18: Huynen fork parameters determined at the location of the reference sphere for each aperture sampling.

The expected Huynen parameter values for a sphere are: 45° for the Polarisability angle γ , 0° for the Skip angle ν , and 0° for the symmetry angles τ_i and τ_s . The orientation parameters Θ_i and Θ_s offer no physical meaning for non-polarising scatterers [133], [134], [135].

The sphere results are in agreement with the expected values, and are stable over the sparse apertures implemented, with the Polarisability angle at about 37° , the Skip angle at about 1.0° , and the sum of the symmetry angles at about 2.7° .

It is noted that a more substantial validation of Huynen parameter extraction in the GBSAR laboratory was presented in [163] where canonical scatterers including a metal dihedral in different orientations were measured in a monostatic geometry. The use of dihedrals for validation in the current multiple bistatic scenario would have been more difficult however, given the range of different bistatic geometries used.

5.8.4 Discussion

Taking into account the NMSD results in Figure 5-13, the stability of the measured Huynen parameters up to 52% ARF-SIS suggests that this is a good minimum aperture sampling for obtaining accurate polarimetric information. This aperture is significantly below the Nyquist sampling requirement and is therefore a large reduction on what would conventionally be required. There was also little observed difference in the point cloud detection locations from 0% to 52% ARF-SIS, which was evidenced in the MIPs presented in Figure 5-14 through Figure 5-17. A negligible FAR was also seen over the same range of sub-Nyquist sampled apertures, shown in Figure 5-11.

The tank structure was seen to be largely non-polarising in both the Huynen reference array and in the various sampled aperture point clouds, so consequently the polarisability angle showed the smallest NMSD, with the smallest variation. The other Huynen parameters showed a period of stability up to 52% ARF-SIS followed by substantial increases, most strongly seen for the Skip parameter. This may be because these parameters, and more-so for the skip parameter, had the greatest variability across the target structure, as can be seen in the point-cloud MIPs (e.g. see Figure 5-17).

There was little variation in the Huynen fork parameters determined on the sphere for all downsampled apertures (see Figure 5-18). This suggests that the Huynen parameters are most stable for bright and isolated scatterers.

5.9 Conclusion

This paper presented and validated the PolSSARVI algorithm, which is a polarimetric extension of the multistatic SSARVI algorithm [64], [65]. Simulations showed that accurate scatterer height locations were determined for polarimetrically diverse scatterers at sub-Nyquist aperture sampling, demonstrated using a near-field bistatic collection geometry.

Simulations showed how the polarimetric combination approach in PolSSARVI reduces false target detections when compared to SSARVI, arising from when scatterers have a relatively dim response in a particular polarisation channel.

With experimental data, it was shown that the PolSSARVI approach had a lower measured FAR compared to the SSARVI approach where detections from each measured channel were overlaid. For aperture sampling lower than 52% ARF-SIS the PolSSARVI FAR became significantly lower than that for SSARVI. Furthermore, it was observed that the minimum aperture sampling limit for discerning the target structure was 5% less for the PolSSARVI algorithm than for the SSARVI.

Both algorithms exhibited a significant initial similarity with the finely sampled point cloud renderings, as evidenced by the normalised cross-correlation measurements presented in Figure 5-12. The SSARVI algorithm demonstrated a relatively rapid decrease in cross-correlation beginning at 52% ARF-SIS, which coincided with an increase in FAR at the same aperture sampling. In contrast, the PolSSARVI algorithm exhibited a more gradual decline in cross-correlation, indicating that the imagery produced maintains a higher degree of accuracy at a smaller aperture sampling.

The accuracy of the Huynen parameters determined on the sub-Nyquist PolSSARVI-formed point clouds was determined by their NMSD relative to the corresponding values obtained for the Huynen reference arrays formed from BPA full aperture SAR images. A minimal NMSD (lower than 0.03), and low variation in NMSD, was shown across the tank structure between SIS and 52% ARF-SIS apertures. There was little variation in the Huynen fork parameters determined on the sphere for all downsampled apertures which suggests that the Huynen parameters are most stable for bright and isolated scatterers.

Overall, for the current experimental setup and target measured, it was found that most sparse aperture sampling, which provided negligible FAR, high correlation with finely sampled aperture imagery, and accurate Huynen parameter estimations, was 52% ARF-SIS.

For this work, non-coherently overlaying of detections from different bistatic geometries was the method chosen for combining the data from each receiver. The future of this research should therefore focus on assessing different kinds of multistatic receiver data combinations for the PolSSARVI approach, including coherent combination. As with the SSARVI approach, the threshold for the PolSSARVI approach was determined experimentally for the target. It is possible that the threshold for scatterers displaying different polarimetric properties may differ from that observed in this study. In addition to conducting further measurements, thresholds could potentially be derived from electromagnetic simulations of other complex polarimetric targets, and statistical-based methods could also be implemented [140], [174]. The use of CS related methods, discussed earlier, could also be incorporated [165], [168]. Alternative polarimetric phase factor summation methods, such as that given by equation (5-6), could also be investigated further.

6 Discussion

6.1 Introduction

The purpose of this chapter is to summarise the main findings of this thesis, the limitations of the study, and discuss its wider place within the literature. The links between the publication chapters are discussed, relating to the specific objectives they answer, and how they form a unified body of work. Some of the discussion points raised in the publication chapters are expanded upon in this section.

6.2 Summary of Key Findings

The main aim of this thesis was to develop a novel algorithm for exploiting satellite constellations and UAV-SAR swarm sensing to form high-quality sub-Nyquist sampled 3D SAR imagery. The aim was divided into three objectives answered in the publication chapters presented in this thesis. The results for each objective are summarised below.

6.2.1 Objective 1: Validate the Accuracy and Performance of the SSARVI Approach

The accuracy of the SSARVI approach was first validated in section 3.4, using simulations of an isotropic point scatterer in 3D space. The simulations used a bistatic geometry for a near-field target scene, with a sub-Nyquist sampled aperture in elevation. The algorithm produced accurate scatterer height estimates with a geometry that would have caused significant distortions to the scatterer layover properties and K-space support regions. The sub-Nyquist spacing in elevation would have produced aliasing in the vertical imaging direction using conventional image formation algorithms.

Experimental validation was presented in section 3.5 by forming 3D point clouds of a model tank target using the GBSAR laboratory apparatus. The point clouds were formed with sub-Nyquist sampling in elevation, using monostatic and bistatic collection geometries for a near-field target scene. The 3D point cloud renderings formed were highly interpretable until the downsampling limits

mentioned, showing a high measured cross-correlation with finely sampled BPA imagery of the same target. These showed that the SSARVI algorithm could determine accurate 3D locations of complex target structures, suppressing false scatterer detections well below the Nyquist sampling requirement. As with the simulations, the experimental SAR geometry would have also produced layover distortions and K-space irregularity.

6.2.2 Objective 2: Optimise the Performance of the SSARVI Approach

Chapter 4 presented an optimisation of the SSARVI algorithm. This first involved the extension of the algorithm to multistatic geometries, in line with the broader thesis aims of exploiting satellite constellations and swarms.

A coherent summation of receiver data decreased the simulated sidelobe height and produced fewer false detections than the other combination methods investigated. This approach had the smallest aperture sampling requirements, producing interpretable 3D point clouds with the fewest number of passes, but also produced the fewest detections across the tank structure. The non-coherent combining of receiver data produced the most detections, as it was the least sensitive to the change in RCS across the tank structure. However, this showed the most erroneous detections over the same downsampled aperture sizes compared to other methods of combining multistatic data. Table 4-1 summarises the sub-Nyquist performance of each method.

The results suggest that the smaller minimum aperture sizes of a coherent combination of multistatic data can exploit target scenes with highly isotropic scatterers. More complex targets with a more varied structure may be better suited to non-coherent approaches.

In both cases, the presence of false detections can be minimised by the non-even displacement of SAR passes to within the limits found in this thesis. This was also investigated as part of the optimisation objective of this thesis. A highly uneven distribution of SAR heights was found to improve the peak-to-sidelobe difference, reducing the ambiguity on the height of scatterer detections. However, the over-

displacement of passes towards the centre of the 2D SAR aperture eventually resulted in larger sidelobes. The deterioration in peak-to-sidelobe difference started when the extent of the mid-aperture cluster approached a size that was twice the Nyquist spacing. This is summarised in section 4.5.

6.2.3 Objective 3: Extend the SSARVI Approach to Polarimetric SAR Collections and Analyses

The use of multiple measured polarisation channels contributed more detection points of the tank structure, as shown in the point cloud renderings presented in section 4.7. The downsampling limits were also found to be similar between the measured polarisation channels, indicating that extending the polarimetric capability of the SSARVI algorithm would be plausible further. The PolSSARVI algorithm was presented in section 5.3, which extended the SSARVI algorithm by combining polarimetrically weighted interferograms across multiple linear polarisation channels.

The PolSSARVI algorithm accurately determined the heights of different types of polarimetric scatterers, using near-field bistatic geometries that were sub-Nyquist sampled in elevation. Simulations also showed that the PolSSARVI algorithm could suppress ambiguities that would be present in the SSARVI approach when scatterers have a dim response in a particular polarisation channel.

Experimental imagery, presented in section 5.8, validated the PolSSARVI algorithm for producing multistatic sub-Nyquist sampled polarimetric 3D SAR imagery. The bistatic Huynen fork parameters were used to add polarimetric information to the sub-Nyquist-formed PolSSARVI point clouds. The FAR of the PolSSARVI algorithm was determined to be lower over a range of sub-Nyquist aperture sizes compared to the non-combined polarimetric SSARVI algorithm. The bistatic Huynen fork parameters determined on the tank structure showed a small deviation as the aperture was downsampled to significantly below the Nyquist spacing. A similar stability was shown when the Huynen fork parameters were determined on the reference sphere for the same set of downsampled SIS apertures. The experimental results show that the PolSSARVI algorithm produces sufficiently accurate polarimetric 3D scatterer information for sub-

Nyquist sampled apertures and suppresses artefact detections more effectively than the SSARVI approach.

6.3 Limitations

This section summarises the limitations considered throughout this thesis. Some of the limitations mentioned in the paper chapters are discussed in greater detail here, and further discussion points are also introduced in this section.

6.3.1 Experimental Uncertainty

Some overlap in the sub-Nyquist performance limits of the multistatic combination methods discussed in section 4.7.4 may be possible due to the downsampling interval employed in the determination of these limits. Notably, the difference in the sub-Nyquist performance limits between the two methods corresponded to the selected downsampling interval. Utilising the same data collection with consistent downsampling geometries mitigated this uncertainty to a large degree. The sub-Nyquist performance limit of the PolSSARVI algorithm was also marginally lower than that of the SSARVI algorithm. The lower FAR of the PolSSARVI approach, compared to the SSARVI approach, over a wide range of apertures means there is little uncertainty of its performance advantage.

Experimental uncertainty was considered for the sphere measurements presented in section 5.8. An ideal sphere scatterer would be expected to have a polarisability angle of 45° , compared to the 37° that was shown in the results. When performed for finely sampled BPA imagery, the polarisability angle was also approximately 37° . Thus, the difference was not attributed to the PolSSARVI algorithm. This result still shows the sphere to show a predominantly non-polarising scattering mechanism and is close to the 5° phase angle margin of error found within the calibration literature [172]. The other Huynen parameters determined for the sphere were significantly closer to their expected theoretical value. A more substantial validation of these parameters was also given in previous publications [163].

6.3.2 Thresholding

The thresholding process, outlined in Chapters 3, 4 and 5, involved systematically increasing the threshold until background noise artefacts were removed. Whilst this approach is valid for the analysis of the SSARVI and PolSSARVI algorithms presented in this thesis, the threshold may be different for other targets. Threshold estimates are given for different scatterers in the SPCGS literature. However, the application of the developed algorithms to real data collections may require a generalised thresholding approach.

The SSARVI and PolSSARVI algorithms could make use of adaptive thresholding algorithms that have been developed for use in SAR, where targets are separated from clutter and background noise via the use of statistical models [174]. Moreover, as stated in Chapters 4 and 5, electromagnetic simulations could be used to estimate thresholds for different target structures.

6.3.3 Imaging Performance Metrics

Throughout this thesis, various metrics were utilised to evaluate the imaging performance of the SSARVI and PolSSARVI algorithms. The impact of pass clustering was analysed by measuring the peak-to-sidelobe difference (ΔR) as detailed in section 4.5.3. It is anticipated that smaller ΔR values could lead to an increased FAR in the experimental results; however, additional data collection with a larger aperture extent would be necessary to confirm this hypothesis. This is attributed to the overlay of displaced passes in height when applying this approach to the dataset discussed in section 4.7.

Normalised cross-correlation calculations between sub-Nyquist and full Nyquist SSARVI point clouds were employed to evaluate image interpretability, as outlined in Chapters 3 and 4. This metric, along with visual inspections of artefact detections, proved valuable for classifying trends in image accuracy as the transmitter aperture was downsampled. However, it was less effective for quantifying the presence of erroneous detections from different receiver combination methods. This led to a greater reliance on visual inspections in Chapter 4, and the inclusion of the FAR measure in Chapter 5.

6.3.4 Sample Height Randomisation

The simulations presented in section 4.5.3 show how the performance of the SSARVI algorithm could be optimised by a non-even distribution of SAR pass heights. The uneven distribution of each simulated aperture was quantified by the pass cluster ratio, denoted as χ , which was the width of the mid-aperture cluster as a proportion of the Nyquist elevation spacing for a given aperture. This measure does not consider pseudo-randomised SAR pass distributions, which could be implemented to further optimise the performance of the SSARVI algorithm and, by extension, the PolSSARVI algorithm. Various apertures of pseudo-randomly distributed pass heights could be generated by utilising algorithms designed for phase encoding, such as the Barker code. Passes could also be displaced within the SAR aperture according to a distribution of co-prime numbers.

6.3.5 Computation Time Considerations

This thesis primarily considers the data collection requirements of sparse 3D SAR imaging; as such, the relative computing time was not investigated. Whilst avoiding the expensive reconstruction processes of 3D SAR compressive sensing, the SSARVI and PolSSARVI algorithms use volumetric processing, which requires more processing time than would be the case for conventional 2D SAR processing. Further algorithm development could research optimal volumetric scene sizes to reduce the computational time required for SSARVI and PolSSARVI algorithms. The image formation time of each volumetric interferogram could be reduced by utilising the PFA image formation discussed in section 2.1.2. The processing requirements of the PolSSARVI algorithm could potentially be reduced in future work by investigating its usage with hybrid polarimetric techniques [39].

6.4 Place in the Literature

Several sparse 3D SAR approaches were discussed at various points throughout this thesis, each contributing a different method for forming high-quality 3D SAR point clouds with smaller aperture sampling requirements. This section will

compare the SSARVI and PolSSARVI algorithms to newer methods employing signal reconstruction techniques and novel SAR transmission modes. SAR interferometry and Circular SAR (CSAR) have been explored in several comprehensive studies and have been heavily researched since the 1980s onwards [137], [175]. The application of CSAR collection techniques to the algorithms presented in this thesis would likely be feasible. The comparison of the SSARVI and PolSSARVI algorithms to the SPCGS has been suitably well-established throughout this thesis.

The compressive sensing (CS) based 3D SAR approach, as mentioned at various points throughout this thesis, provides scatterer resolvability in the vertical imaging direction, with a smaller vertical aperture extent than would be conventionally required. The required aperture extents for conventional 3D SAR image formation are described by the Rayleigh resolution requirement in the vertical imaging direction, which is given in section 2.3.1 by equation (2-25).

CS has been mainly applied to urban imaging scenarios. Xu and Bamler have presented a comprehensive investigation of this technique across multiple publications [53], [176], [177]. Their study achieved a near-100% increase in the number of scatterers resolved, compared to conventional approaches, with a vertical aperture extent of approximately 270 m. With a consistent spacing between passes, this corresponds to a 50% reduction in the number of SAR passes required. The imaging results presented were of the Las Vegas cityscape collected from TerraSAR-X mission data [178].

The algorithms developed in this thesis do not exploit the physical sparsity of a target scene as CS does, nor do they utilise the associated signal reconstruction techniques. However, like with CS-based approaches, a reduction in the number of SAR passes for forming interpretable 3D imagery was shown. In compressive sensing-based 3D SAR studies, the sparse apertures investigated are often compared against the Rayleigh resolution requirements, whereas in this thesis, the sparse aperture sizes were compared against the minimum Nyquist spacing in elevation. Both are valid when discussing sparsely formed 3D SAR imagery.

Comparable CS studies to this thesis have investigated a range of complex manmade targets. Results were presented using a downsampled 2D SAR aperture for a model tank target by Tian et al. [155]. This publication reduced the sampling requirements of 3D SAR by downsampling a 2D SAR aperture in azimuth. This was implemented on an aperture 43% below the minimum 2D SAR aperture sampling for forming fully volumetric renderings. Interpretable 3D renderings were shown, with minimal cross-range sidelobes present in the imagery.

Compressive sensing was also applied to a UAV-SAR swarm sensing collection for 3D SAR imagery, with simulated model ship targets being one of the renderings presented [156]. Sparsely sampled imagery was formed by optimising the distribution of UAVs for exploiting the CS-based signal reconstruction approaches, with the target structures shown being largely free of artefacts. Whilst the collection was stated to be sparsely distributed, a relative comparison in relation to the Rayleigh and Nyquist criteria was not given. A comparable monostatic CS-based study, investigating the algorithms use for the 3D imaging of aircraft carriers, showed a 90% reduction in the scene sampling required for producing an interpretable rendering of the vehicle's 3D structure [179]. CS has also been extended to incorporate SAR polarimetric techniques, where accurate 3D polarimetric information has been retrieved for vehicle targets using a sub-Nyquist aperture size [168].

A similar type of sparse 3D SAR method to the CS-based approaches is the regularised least-squares (RLS) method. Rather than exploit the physical sparsity of a target scene, these approaches impose the constraint of a uniform scattering response on the RCS over a small range of elevation angles. This method was notably applied to the GOTCHA data set from the US Air Force Research laboratory, which consists of multiple vertically displaced circular SAR trajectories [153]. 3D scatterer information was obtained using a sub-set geometry, with 6° width in elevation, for vehicle targets present in the target scene. This is a large reduction on the 20° elevation width of the full collection geometry in this example.

It is also noted that CSAR collections were employed in this data set, which also provide a certain degree of resolvability in the vertical imaging direction.

RLS and CS-based approaches are similar in that significant prior knowledge of the reflectivity of the target scene is assumed when forming the imagery. Comparatively, the SSARVI and PolSSARVI algorithms only require a minimum estimate of the summed interferometric response of a target for setting a threshold. In [176], [177], [180], Xu and Bamler found the LS approach to be less effective than the CS approach at resolving scatterers in the vertical imaging direction, with aperture extents below the minimum Rayleigh-derived requirement.

Recent studies have combined the RLS and CS-based approaches and extended their analyses to polarimetric SAR [168]. Like with the PolSSARVI approach, this was found to produce fewer false detections compared to the single-polarisation approach, improving the interpretability of the formed 3D imagery. The collection geometry used for this study, when using conventional 3D SAR image formation, would have had a resolution of 0.69 m in the vertical direction. Detailed 3D imagery of targets that were around 1.5 – 2.0 m in height were presented for wide-area target scenes.

Multiple Input-Multiple Output (MIMO) SAR can be considered an extension of the multistatic SAR sensing investigated in this thesis. MIMO collections exploit the angular diversity of both multiple transmit and receive geometries to improve the properties of the k-space support region. Provided the apertures are distributed in an optimal manner, the width and spatial sampling of k-space support can be improved, meaning finer resolution and wider coverage are achievable [181], [182]. Tebaldini showed that an optimised distribution of 4-5 SAR passes can achieve the vertical imaging resolution and ambiguity suppression of a conventional 2D SAR system comprising 17-26 displaced SAR passes. It is, therefore, likely that a further improvement in the SSARVI and PolSSARVI algorithms could be obtained by extending their capability to incorporate MIMO-SAR data collections.

7 Closing Remarks

This section summarises the intellectual contribution of this thesis to the SAR community, the impact of the findings, and the future directions of this research.

7.1 Contribution to Knowledge

This thesis has contributed a comprehensive new approach for exploiting multistatic SAR for producing high-quality sub-Nyquist formed 3D SAR imagery, eventually incorporating polarimetric information. The SSARVI algorithm utilises volumetric processing to account for the distortion in the layover position of scatterers in the formed interferograms, allowing for their summed interferometric response to be maximised at the 3D location of scatterers. This is a novel implementation of the theory first presented by the SAR Point Cloud Generation System, optimised for multistatic geometries.

The development of the SSARVI algorithm was spread across three distinct publication chapters, with Chapter 3 contributing a simulated and experimental validation of the SSARVI algorithm, primarily focusing on bistatic geometries.

Chapter 4 contributed an optimisation of the SSARVI algorithm, whereby the sub-Nyquist sampling requirements of the SSARVI algorithm were reduced by extending the approach to multistatic geometries. The effect of the number of receivers on the detectability of scatterers was also investigated. An optimised non-even pass distribution was also presented in Chapter 4, for minimising false detections. The distribution of passes corresponded to a mid-pass cluster width of twice the Nyquist spacing.

Chapter 5 presented the PoISSARVI algorithm, which extended the capability of the SSARVI algorithm by incorporating polarimetric diversity into the formation of sparsely sampled volumetric interferograms. This new approach was shown to provide greater suppression of false detections in both simulated and experimental results. Furthermore, a novel application of the Bistatic Huynen Target Parameters was contributed in Chapter 5, where they were used to associate polarimetric information with the sub-Nyquist formed 3D point clouds.

7.2 Expected Impact

The novel algorithms introduced in this thesis enable the utilisation of a broader range of unconventional collection geometries in the generation of sub-Nyquist sampled 3D SAR imagery. The results demonstrate a significant reduction in the sampling requirements for 3D SAR when compared to conventional image formation algorithms, such as the BPA.

This decrease in sampling requirements enhances the feasibility of 3D SAR imaging for real-world applications involving airborne sensors, particularly those focused on complex vehicle targets. This is particularly relevant for SAR missions that utilise satellite constellations and swarms of UAVs. The improved attainability of 3D polarimetric SAR imagery, facilitated by the development of the PolSSARVI algorithm, would be of considerable interest to research in SAR ATR.

7.3 Future Research Directions

The work presented in this thesis regarding the SSARVI and PolSSARVI algorithms establishes a theoretical foundation for future research into sparse 3D multistatic SAR imaging. As previously discussed, the thresholding approach could be further generalised using statistical-based methods. These methods could complement both the experimental framework outlined in this thesis and the electromagnetic simulations mentioned earlier.

The aperture size requirements for the SSARVI and PolSSARVI algorithms could be reduced by using MIMO-SAR geometries and optimisation techniques like compressive sensing and least-squares regularisation, as discussed in the previous chapter. Additionally, exploring pseudo-randomisation algorithms could also be a potential future research direction.

The PolSSARVI algorithm could benefit from the integration of coherence optimisation techniques that are commonly employed in conventional polarimetric SAR interferometry. The simplified version of the PolSSARVI algorithm presented in section 5.3 also warrants further investigation.

REFERENCES

- [1] W. Brown, 'Synthetic Aperture Radar', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 3, 1967.
- [2] K. Tomiyasu, 'Tutorial Review of Synthetic-Aperture Radar (SAR) with Applications to Imaging of the Ocean Surface', *Proc. IEEE*, vol. 66, no. 5, pp. 563–583, 1978, doi: 10.1109/PROC.1978.10961.
- [3] A. Moreira, P. Prats-Iraola, M. Younis, G. Krieger, I. Hajnsek, and K. P. Papathanassiou, 'A tutorial on synthetic aperture radar', *IEEE Geosci. Remote Sens. Mag.*, vol. 1, no. 1, pp. 6–43, 2013, doi: 10.1109/MGRS.2013.2248301.
- [4] S. W. Lasswell, 'History of SAR at Lockheed Martin (previously Goodyear Aerospace)', in *Radar Sensor Technology IX*, SPIE, May 2005, pp. 1–12. doi: 10.1117/12.603927.
- [5] B. Corbett, D. Andre, and M. Finnis, 'Through-wall detection and imaging of a vibrating target using synthetic aperture radar', in *Electronics Letters*, Institution of Engineering and Technology, Jul. 2017, pp. 991–995. doi: 10.1049/el.2017.1570.
- [6] D. Andre, R. Sabiers, and M. Finnis, 'Through-Wall Multistatic Polarimetric 3D SAR', in *EUSAR*, 2022.
- [7] M. Inggs, 'Synthetic Aperture Radar during the 50 Years of the Aerospace and Electronic Systems Society', *IEEE Aerosp. Electron. Syst. Mag.*, vol. 38, no. 1, pp. 22–31, Jan. 2023, doi: 10.1109/MAES.2022.3216263.
- [8] D. A. Yocky, D. E. Wahl, and C. V. Jakowatz, 'Terrain Elevation Mapping Results From Airborne Spotlight-Mode Coherent Cross-Track SAR Stereo', *IEEE Trans. Geosci. Remote Sens.*, vol. 42, no. 2, pp. 301–308, Feb. 2004, doi: 10.1109/TGRS.2003.817683.
- [9] G. Krieger *et al.*, 'TanDEM-X: A satellite formation for high-resolution SAR interferometry', in *IEEE Transactions on Geoscience and Remote Sensing*, Nov. 2007, pp. 3317–3340. doi: 10.1109/TGRS.2007.900693.
- [10] G. Krieger, N. Gebert, and A. Moreira, 'Unambiguous SAR signal reconstruction from nonuniform displaced phase center sampling', *IEEE Geosci. Remote Sens. Lett.*, vol. 1, no. 4, pp. 260–264, Oct. 2004, doi: 10.1109/LGRS.2004.832700.
- [11] B. Rigling, W. Garber, and L. Gorham, 'Wide-Area, Persistent SAR Imaging: Algorithm Tradeoffs', *IEEE A&E Magazine*, vol. 29, no. 1, pp. 14–20, Jan. 2014.
- [12] G. Krieger, I. Hajnsek, K. P. Papathanassiou, M. Younis, and A. Moreira, 'Interferometric synthetic aperture radar (SAR) missions employing formation

- flying', *Proc. IEEE*, vol. 98, no. 5, pp. 816–843, 2010, doi: 10.1109/JPROC.2009.2038948.
- [13] Y. Zhou, B. Rao, and W. Wang, 'UAV swarm intelligence: Recent advances and future trends', *IEEE Access*, vol. 8, pp. 183856–183878, 2020, doi: 10.1109/ACCESS.2020.3028865.
- [14] Z. Wang, Z. Zeng, G. Xu, Z. Xiong, and S. Zhou, 'UAV Swarm SAR New Concept and Fast Time Domain Imaging Algorithm', in *2021 CIE International Conference on Radar (Radar)*, IEEE, Dec. 2021, pp. 333–336. doi: 10.1109/Radar53847.2021.10028651.
- [15] G. Seguin, S. Srivastava, and D. Auger, 'Evolution of the RADARSAT Program', *IEEE Geosci. Remote Sens. Mag.*, vol. 2, no. 2, pp. 56–58, Jun. 2014, doi: 10.1109/mgrs.2014.2321381.
- [16] M. Virelli, A. Coletta, and M. L. Battagliere, 'ASI COSMO-SkyMed: Mission Overview and Data Exploitation', *IEEE Geosci. Remote Sens. Mag.*, vol. 2, no. 2, pp. 64–66, Jun. 2014, doi: 10.1109/mgrs.2014.2317837.
- [17] M. Zink *et al.*, 'TanDEM-X: 10 Years of Formation Flying Bistatic SAR Interferometry', *IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens.*, vol. 14, pp. 3546–3565, 2021, doi: 10.1109/JSTARS.2021.3062286.
- [18] A. A. Thompson, 'Overview of the RADARSAT Constellation Mission', *Can. J. Remote Sens.*, vol. 41, no. 5, pp. 401–407, Sep. 2015, doi: 10.1080/07038992.2015.1104633.
- [19] F. Nunziata *et al.*, 'Sea Surface Wind Estimation Using COSMO-SkyMed SAR Imagery', in *2022 IEEE International Workshop on Metrology for the Sea; Learning to Measure Sea Health Parameters, MetroSea 2022 - Proceedings*, Institute of Electrical and Electronics Engineers Inc., 2022, pp. 378–382. doi: 10.1109/MetroSea55331.2022.9950880.
- [20] U. Nithirochananont, M. Antoniou, and M. Cherniakov, 'Passive coherent multistatic sar using spaceborne illuminators', *IET Radar Sonar Navig.*, vol. 14, no. 4, pp. 628–636, 2020, doi: 10.1049/iet-rsn.2019.0425.
- [21] M. Richards, 'A Beginners Guide to Interferometric SAR Concepts and Signal Processing AESS Tutorial IV', *IEEE Aerosp. Electron. Syst. Mag.*, vol. 22, no. 9, pp. 5–29, 2007.
- [22] C. Rambour, A. Budillon, A. C. Johnsy, L. Denis, F. Tupin, and G. Schirinzi, 'From Interferometric to Tomographic SAR: A Review of Synthetic Aperture Radar Tomography-Processing Techniques for Scatterer Unmixing in Urban Areas',

- IEEE Geosci. Remote Sens. Mag.*, vol. 8, no. 2, pp. 6–29, Jun. 2020, doi: 10.1109/MGRS.2019.2957215.
- [23] B. Rabus, M. Eineder, A. Roth, and R. Bamler, 'The shuttle radar topography mission - A new class of digital elevation models acquired by spaceborne radar', *ISPRS J. Photogramm. Remote Sens.*, vol. 57, no. 4, pp. 241–262, 2003, doi: 10.1016/S0924-2716(02)00124-7.
- [24] L. C. Graham, 'Synthetic Interferometer Radar for Topographic Mapping', *Proc. IEEE*, vol. 62, no. 6, 1974.
- [25] C. Jakowatz and P. Thompson, 'A New Look at Spotlight Mode Synthetic Aperture Radar as Tomography: Imaging 3-D Targets', *IEEE Trans. IMAGE Process.*, vol. 4, no. 5, pp. 699–703, 1995.
- [26] C. D. Austin, E. Ertin, and R. L. Moses, 'Sparse signal methods for 3-D radar imaging', *IEEE J. Sel. Top. Signal Process.*, vol. 5, no. 3, pp. 408–423, Jun. 2011, doi: 10.1109/JSTSP.2010.2090128.
- [27] J. Elgy, D. Andre, and M. Finnis, 'Volumetric SAR near-field upsampling and basebanding', *Electron. Lett.*, vol. 56, no. 12, pp. 622–624, Jun. 2020, doi: 10.1049/el.2019.4123.
- [28] D. André, 'An analysis of 3D SAR from single pass nonlinear radar platform trajectories', in *Algorithms for Synthetic Aperture Radar Imagery XVII*, SPIE, Apr. 2010, p. 769908. doi: 10.1117/12.850093.
- [29] D. Andre, B. Faulkner, and M. Finnis, 'Low-frequency 3D synthetic aperture radar for the remote intelligence of building interiors', in *Electronics Letters*, Institution of Engineering and Technology, Jul. 2017, pp. 984–987. doi: 10.1049/el.2017.1584.
- [30] A. Budillon, A. C. Johnsy, and G. Schirinzi, 'Urban tomographic imaging using polarimetric SAR data', *Remote Sensing*, vol. 11, no. 2. MDPI AG, Jan. 2019. doi: 10.3390/rs11020132.
- [31] Z. Jiao *et al.*, 'Urban 3D imaging using airborne TomoSAR: Contextual information-based approach in the statistical way', *ISPRS J. Photogramm. Remote Sens.*, vol. 170, pp. 127–141, Dec. 2020, doi: 10.1016/j.isprsjprs.2020.10.013.
- [32] O. Ponce *et al.*, 'Fully polarimetric high-resolution 3-D imaging with circular SAR at L-band', *IEEE Trans. Geosci. Remote Sens.*, vol. 52, no. 6, pp. 3074–3090, 2014, doi: 10.1109/TGRS.2013.2269194.
- [33] S. W. Chen, M. D. Li, X. C. Cui, and H. L. Li, 'Polarimetric Roll-Invariant Features and Applications for Polarimetric Synthetic Aperture Radar Ship Detection: A

- comprehensive summary and investigation', *IEEE Geosci. Remote Sens. Mag.*, vol. 12, no. 1, pp. 36–66, Mar. 2024, doi: 10.1109/MGRS.2023.3328472.
- [34] M. Martorella, *Multidimensional radar imaging*. Institution of Engineering and Technology (IET), 2019.
- [35] J. A. Jackson, B. D. Rigling, R. L. Moses, S. Member, J. Jackson, and R. Moses, 'Canonical Scattering Feature Models for 3D and Bistatic SAR', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 46, no. 2, pp. 525–541, 2010.
- [36] J. Cai, M. Martorella, Q. Liu, Z. Ding, E. Giusti, and T. Long, 'Automatic Target Recognition Based on Alignments of Three-Dimensional Interferometric ISAR Images and CAD Models', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 56, no. 6, pp. 4872–4888, Dec. 2020, doi: 10.1109/TAES.2020.3005654.
- [37] A. Lupidi, E. Giusti, S. Tomei, S. Ghio, and M. Martorella, 'Target Recognition by Means of 3D ISAR Images', in *Proceedings of the IEEE Radar Conference*, Institute of Electrical and Electronics Engineers Inc., 2024. doi: 10.1109/RadarConf2458775.2024.10548952.
- [38] M. A. Saville, J. A. Jackson, and D. F. Fuller, 'Rethinking vehicle classification with wide-angle polarimetric SAR', *IEEE Aerosp. Electron. Syst. Mag.*, vol. 29, no. 1, pp. 41–49, 2014, doi: 10.1109/MAES.2014.130057.
- [39] A. Kumar, E. Giusti, and M. Martorella, 'Hybrid polarimetry inverse synthetic aperture radar', *IET Radar Sonar Navig.*, vol. 19, no. 1, Jan. 2025, doi: 10.1049/rsn2.70004.
- [40] F. Salvetti, M. Martorella, E. Giusti, and D. Stagliano, 'Multiview Three-Dimensional Interferometric Inverse Synthetic Aperture Radar', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 55, no. 2, pp. 718–733, Apr. 2019, doi: 10.1109/TAES.2018.2864469.
- [41] E. Giusti, A. Kumar, S. Ghio, and M. Martorella, 'Fully polarimetric multi-aspect 3D InISAR', in *23rd International Radar Symposium (IRS)*, IEEE, 2022, pp. 184–189.
- [42] S.-Y. Tan, 'Remote Sensing Applications and Innovations via Small Satellite Constellations', in *Handbook of Small Satellites: Technology, Design, Manufacture, Applications, Economics and Regulation*, S. P. J. N and Madry, Eds., Springer International Publishing, 2020, pp. 1–31. doi: 10.1007/978-3-030-20707-6_44-1.
- [43] A. C. Watts, V. G. Ambrosia, and E. A. Hinkley, 'Unmanned aircraft systems in remote sensing and scientific research: Classification and considerations of use', *Remote Sens.*, vol. 4, no. 6, pp. 1671–1692, Jun. 2012, doi: 10.3390/rs4061671.

- [44] A. S. Milas, A. P. Cracknell, and T. A. Warner, 'Drones—the third generation source of remote sensing data', *Int. J. Remote Sens.*, vol. 39, no. 21, pp. 7125–7137, Nov. 2018, doi: 10.1080/01431161.2018.1523832.
- [45] D. I. of Ukraine (GUR), 'Results of Using "People's Satellite" ICEYE'. Jun. 2024. [Online]. Available: <https://gur.gov.ua/en/content/prosto-kosmos-rezultaty-vykorystannia-narodnoho-suputnyka-iceye.html>
- [46] M. Dunbabin and L. Marques, 'Robots for environmental monitoring: Significant advancements and applications', *IEEE Robot. Autom. Mag.*, vol. 19, no. 1, pp. 24–39, Mar. 2012, doi: 10.1109/MRA.2011.2181683.
- [47] Y. Li *et al.*, 'Unmanned Aerial Vehicle Remote Sensing for Antarctic Research: A review of progress, current applications, and future use cases', *IEEE Geosci. Remote Sens. Mag.*, vol. 11, no. 1, pp. 73–93, Mar. 2023, doi: 10.1109/MGRS.2022.3227056.
- [48] A. Colmer and R. Ellem, 'Remote Undersea Surveillance Insights Paper', Australian Department of Defence, 2021.
- [49] K. Lewis, 'Challenges in military remote sensing', in *Electro-Optical Remote Sensing, Detection, and Photonic Technologies and Their Applications*, SPIE, Oct. 2007, p. 67390J. doi: 10.1117/12.753508.
- [50] M. D. Mura, S. Prasad, F. Pacifici, P. Gamba, J. Chanussot, and J. A. Benediktsson, 'Challenges and Opportunities of Multimodality and Data Fusion in Remote Sensing', in *Proceedings of the IEEE*, Institute of Electrical and Electronics Engineers Inc., Sep. 2015, pp. 1585–1601. doi: 10.1109/JPROC.2015.2462751.
- [51] M. J. Guitton, 'Fighting the Locusts: Implementing Military Countermeasures Against Drones and Drone Swarms', *Scand. J. Mil. Stud.*, vol. 4, no. 1, pp. 26–36, 2021, doi: 10.31374/sjms.53.
- [52] M. Chapman, 'Brexit Britain is creating its own network of surveillance satellites', *Wired UK*. Dec. 2018.
- [53] X. X. Zhu and R. Bamler, 'Demonstration of super-resolution for tomographic SAR imaging in urban environment', *IEEE Trans. Geosci. Remote Sens.*, vol. 50, no. 8, pp. 3150–3157, 2012, doi: 10.1109/TGRS.2011.2177843.
- [54] A. Budillon, A. Evangelista, and G. Schirinzi, 'Three-dimensional SAR focusing from multipass signals using compressive sampling', *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 1 PART 2, pp. 488–499, Jan. 2011, doi: 10.1109/TGRS.2010.2054099.

- [55] G. Xu, B. Zhang, H. Yu, J. Chen, M. Xing, and W. Hong, 'Sparse Synthetic Aperture Radar Imaging From Compressed Sensing and Machine Learning: Theories, applications, and trends', *IEEE Geosci. Remote Sens. Mag.*, vol. 10, no. 4, pp. 32–69, Dec. 2022, doi: 10.1109/MGRS.2022.3218801.
- [56] S. R. Cloude, 'Polarimetric SAR interferometry', *IEEE Trans. Geosci. Remote Sens.*, vol. 36, no. 5 PART 1, pp. 1551–1565, 1998, doi: 10.1109/36.718859.
- [57] R. Iglesias, D. Monells, X. Fabregas, J. J. Mallorqui, A. Aguasca, and C. Lopez-Martinez, 'Phase quality optimization in Polarimetric differential SAR Interferometry', *IEEE Trans. Geosci. Remote Sens.*, vol. 52, no. 5, pp. 2875–2888, 2014, doi: 10.1109/TGRS.2013.2267095.
- [58] V. D. Navarro-Sanchez, J. M. Lopez-Sanchez, and L. Ferro-Famil, 'Polarimetric approaches for persistent scatterers interferometry', *IEEE Trans. Geosci. Remote Sens.*, vol. 52, no. 3, pp. 1667–1676, 2014, doi: 10.1109/TGRS.2013.2253111.
- [59] A. Kumar, E. Giusti, F. Mancuso, S. Ghio, A. Lupidi, and M. Martorella, 'Three-Dimensional Polarimetric InSAR Imaging of Non-Cooperative Targets', *IEEE Trans. Comput. Imaging*, vol. 9, pp. 210–223, 2023, doi: 10.1109/TCI.2023.3248942.
- [60] G. Fasano, A. Renga, and M. D'Errico, 'Formation geometries for multistatic SAR tomography', *Acta Astronaut.*, vol. 96, no. 1, pp. 11–22, Mar. 2014, doi: 10.1016/j.actaastro.2013.11.024.
- [61] S. DeGraaf, C. Twigg, and L. Phillips, 'Mono- and multistatic polarimetric sparse aperture 3D SAR imaging', in *Algorithms for Synthetic Aperture Radar Imagery XV*, SPIE, Apr. 2008, p. 697004. doi: 10.1117/12.786287.
- [62] R. Carande and D. Cohen, 'SAR Point Cloud Generation System'. U. S. Patent Office, 2014.
- [63] Z. Xu, X. Ai, F. Zhao, and S. Xiao, 'A New Bistatic Huynen Target Parameters Extraction Algorithm: Experimental Validation and Data Analysis', *IEEE Trans. Antennas Propag.*, vol. 70, no. 10, pp. 9694–9705, Oct. 2022, doi: 10.1109/TAP.2022.3184548.
- [64] R. Welsh, D. Andre, and M. Finnis, 'Volumetric interferometry for sparse 3D synthetic aperture radar with bistatic geometries', *Electronics Letters*, vol. 59, no. 12. John Wiley and Sons Inc, Jun. 2023. doi: 10.1049/ell2.12851.
- [65] R. Welsh, D. Andre, and M. Finnis, 'Laboratory multistatic 3D SAR with polarimetry and sparse aperture sampling', *IET Radar Sonar Navig.*, vol. 18, no. 1, pp. 184–197, Jan. 2024, doi: 10.1049/rsn2.12528.

- [66] R. Welsh, D. Andre, and M. Finnis, 'Polarimetry for Sparse Multistatic 3D SAR', *IET Radar Sonar Navig.*, 2025.
- [67] R. Welsh, D. Andre, and M. Finnis, 'Multistatic 3D SAR Imaging with Coarse Elevation and Azimuth Sampling', in *EUSAR 14th European Conference on Synthetic Aperture Radar, Leipzig, Germany: electronic proceedings*, 2022.
- [68] D. Andre, R. Welsh, and M. Finnis, 'Laboratory Multistatic Polarimetric 3D SAR', in *Radar Conference*, 2022.
- [69] R. Welsh, D. Andre, and M. Finnis, 'LABORATORY MULTISTATIC POLARIMETRIC SPARSE APERTURE 3D SAR INVESTIGATION', in *Proceedings of the Institute of Acoustics*, Sep. 2023.
- [70] C. Jakowatz, D. Wahl, P. Eichel, D. Ghiglia, and P. Thompson, *Spotlight-Mode Synthetic Aperture Radar: A Signal Processing Approach*, 1st ed. Springer, 1996.
- [71] R. E. Hansen, H. J. Callow, T. O. Sabo, and S. A. V. Synnes, 'Challenges in seafloor imaging and mapping with synthetic aperture sonar', *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 10 PART 1, pp. 3677–3687, Oct. 2011, doi: 10.1109/TGRS.2011.2155071.
- [72] S. Liaquat, N. M. Mahyuddin, and I. H. Naqvi, 'An End-to-End Modular Framework for Radar Signal Processing: A simulation-based tutorial', *IEEE Aerosp. Electron. Syst. Mag.*, vol. 39, no. 9, pp. 98–118, Sep. 2024, doi: 10.1109/MAES.2023.3334689.
- [73] M. Desai and K. Jenkins, 'Convolution backprojection image reconstruction for spotlight mode synthetic aperture radar', *IEEE Trans. Image Process.*, vol. 1, no. 4, pp. 505–517, 1992.
- [74] A. W. Doerry, 'Basics of Polar-Format Algorithm for Processing Synthetic Aperture Radar Images'. Sandia National Laboratories, 2012. [Online]. Available: <http://www.ntis.gov/help/ordermethods.asp?loc=7-4-0#online>
- [75] J. F. Shaeffer, B. A. Cooper, R. C. Bauckd, and N. A. ' Talcott, 'A Review of Bistatic k-Space Imaging for Electromagnetic Prediction Codes for Scattering and Antennas', *IEEE Antennas Propag. Mag.*, vol. 39, no. 5, 1997.
- [76] D. Andre, D. Blacknell, C. Finch, M. Nottingham, and D. Muff, 'Life in the Frequency Domain: SAR CCD over Mountains and the Physics of Vibrating Targets', in *Proceedings of the Defence Applications of Signal Processing*, Aug. 2011.
- [77] S. J. A. M. W. L, *Principles of Modern Radar, Volume 2 - Advanced Techniques*. Institution of Engineering and Technology (The IET), 2013. [Online]. Available:

- <https://app.knovel.com/hotlink/toc/id:kpPMRVAT01/principles-modern-radar/principles-modern-radar>
- [78] C. Ozdemir, *Inverse synthetic aperture radar imaging with MATLAB algorithms*. John Wiley & Sons, 2021.
 - [79] H.-J. Li and Y.-W. Kiang, '10 - Radar and Inverse Scattering', in *The Electrical Engineering Handbook*, W.-K. CHEN, Ed., Academic Press, 2005, pp. 671–690. doi: <https://doi.org/10.1016/B978-012170960-0/50047-5>.
 - [80] W. G. Carrara, R. S. Goodman, and R. M. Majewski, *Spotlight synthetic aperture radar : signal processing algorithms*. 1995. [Online]. Available: <https://api.semanticscholar.org/CorpusID:108599478>
 - [81] L. A. Gorham and L. J. Moore, 'SAR image formation toolbox for MATLAB', in *Algorithms for Synthetic Aperture Radar Imagery XVII*, SPIE, Apr. 2010, p. 769906. doi: 10.1117/12.855375.
 - [82] S. Member and H. Schmidt, 'Azimuthal Ambiguities in Synthetic Aperture Sonar and Synthetic Aperture Radar Imagery', *IEEE J. Ocean. Eng.*, vol. 17, no. 1, 1992.
 - [83] X. Mao, 'Spherical Geometry Algorithm for Space-borne Synthetic Aperture Radar Imaging', *IEEE Trans. Geosci. Remote Sens.*, 2024, doi: 10.1109/TGRS.2024.3427014.
 - [84] C. E. Shannon, 'Communication in the Presence of Noise', *Proc. IRE*, vol. 37, no. 1, pp. 10–21, 1949, doi: 10.1109/JRPROC.1949.232969.
 - [85] H. Ogawa, 'Sampling Theory and Isao Someya: A Historical Note', *Sampl. Theory Signal Image Process.*, vol. 5, no. 3, pp. 247–256, Sep. 2006, doi: 10.1007/BF03549454.
 - [86] D. Sundararajan, *Fourier Analysis-A Signal Processing Approach*, vol. 42. Springer, 2018.
 - [87] K. R. Rao, D. N. Kim, and J. J. Hwang, *Fast Fourier Transform: Algorithms and Applications*, vol. 32. Springer, 2010. [Online]. Available: <http://www.springer.com/series/4748>
 - [88] Jr. Charles V. Jakowatz, D. E. Wahl, D. A. Yocky, B. K. Bray, Jr. Wallace J. Bow, and J. A. Richards, 'Comparison of algorithms for use in real-time spotlight-mode SAR image formation', in *Algorithms for Synthetic Aperture Radar Imagery XI*, SPIE, Sep. 2004, pp. 108–116. doi: 10.1117/12.548203.
 - [89] R. Deming, M. Best, and S. Farrell, 'Polar format algorithm for SAR imaging with Matlab', in *Algorithms for Synthetic Aperture Radar Imagery XXI*, SPIE, Jun. 2014, p. 909306. doi: 10.1117/12.2050681.

- [90] R. W. Deming, 'Tutorial on Fourier space coverage for scattering experiments, with application to SAR', in *Algorithms for Synthetic Aperture Radar Imagery XVII*, SPIE, Apr. 2010, p. 769904. doi: 10.1117/12.849541.
- [91] B. D. Rigling and R. L. Moses, 'Polar format algorithm for bistatic SAR', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 40, no. 4, pp. 1147–1159, Oct. 2004, doi: 10.1109/TAES.2004.1386870.
- [92] L. A. Gorham and B. D. Rigling, 'Scene size limits for polar format algorithm', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 52, no. 1, pp. 73–84, Feb. 2016, doi: 10.1109/TAES.2015.140853.
- [93] D. Andre, D. Blacknell, and M. Finnis, 'An Improved SAR-Near-Field Spatially Variant Basebanding Technique', in *Proceedings of the Institute of Acoustics*, 2018.
- [94] D. B. André, D. Blacknell, and K. Morrison, 'Spatially variant incoherence trimming for improved SAR CCD', in *Algorithms for Synthetic Aperture Radar Imagery XX*, SPIE, May 2013, p. 87460K. doi: 10.1117/12.2015377.
- [95] Z. Manzoor, M. T. A. Qaseer, and K. M. Donnell, 'Image Distortion Characterization due to Equivalent Monostatic Approximation in Near-Field Bistatic SAR Imaging', *IEEE Trans. Instrum. Meas.*, vol. 69, no. 7, pp. 4898–4907, Jul. 2020, doi: 10.1109/TIM.2019.2957868.
- [96] B. Fan, J. K. Gao, H. J. Li, Z. J. Jiang, and Y. He, 'Near-field 3D SAR imaging using a scanning linear MIMO array with arbitrary topologies', *IEEE Access*, vol. 8, pp. 6782–6791, 2020, doi: 10.1109/ACCESS.2019.2961247.
- [97] D. S. Plotnick, P. L. Marston, and T. M. Marston, 'Fast nearfield to farfield conversion algorithm for circular synthetic aperture sonar', *J. Acoust. Soc. Am.*, vol. 136, no. 2, pp. EL61–EL66, Aug. 2014, doi: 10.1121/1.4885486.
- [98] J. W. Goodman, *Introduction to Fourier optics*. Roberts and Company publishers, 1969. [Online]. Available: <https://api.semanticscholar.org/CorpusID:118908270>
- [99] J. Walker, 'Range-Doppler Imaging of Rotating Objects', *Proc. IEEE*, vol. 16, no. 1, pp. 23–52, 1980.
- [100] B. D. Rigling, I. L. R. Moses, and S. Member, 'Taylor Expansion of the Differential Range for Monostatic SAR', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, no. 1, pp. 60–64, 2004.
- [101] T. Tsao, M. Slamani, P. Varshney, D. Weiner, H. Schwarzlander, and S. Borek, 'Ambiguity Function for a Bistatic Radar', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 33, no. 3, pp. 1041–1051, 1997.

- [102]R. Wang and Y. Deng, *Bistatic SAR System and Signal Processing Technology*. Springer, 2018.
- [103]N. J. Willis, *Bistatic Radar*. SciTech Publishing, 2005. [Online]. Available: <https://app.knovel.com/hotlink/toc/id:kpBR000005/bistatic-radar/bistatic-radar>
- [104]D. Mao and B. D. Rigling, 'Scene Size Limits for Bistatic Polar Format Algorithm', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 54, no. 5, pp. 2554–2567, Oct. 2018, doi: 10.1109/TAES.2018.2822138.
- [105]A. Theodosiou and P. Lopez-Dekker, 'On the Sensitivity to Height and Motion of Bistatic SAR Interferometry: A Spectral View', *IEEE Trans. Geosci. Remote Sens.*, vol. 62, pp. 1–12, 2024, doi: 10.1109/TGRS.2024.3399598.
- [106]H. Xie, D. An, X. Huang, and Z. Zhou, 'Research on spatial resolution of one-stationary bistatic ultrahigh frequency ultrawidebeam-ultrawideband SAR based on scattering target wavenumber domain support', *IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens.*, vol. 8, no. 4, pp. 1782–1798, Apr. 2015, doi: 10.1109/JSTARS.2015.2414412.
- [107]A. J. Devaney, 'Geophysical Diffraction Tomography', *IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING*, no. 1. 1984.
- [108]W. Dower and M. Yeary, 'Bistatic SAR: Forecasting Spatial Resolution', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 55, no. 4, pp. 1584–1595, Aug. 2019, doi: 10.1109/TAES.2018.2873076.
- [109]F. Santi, M. Antoniou, and D. Pastina, 'Point spread function analysis for GNSS-based multistatic SAR', *IEEE Geosci. Remote Sens. Lett.*, vol. 12, no. 2, pp. 304–308, 2015, doi: 10.1109/LGRS.2014.2337054.
- [110]M. Kazemi, Z. Kavehvas, and M. Shabany, 'K-Space Analysis of Aliasing in Millimeter-Wave Imaging Systems', *IEEE Trans. Microw. Theory Tech.*, vol. 69, no. 3, pp. 1965–1973, Mar. 2021, doi: 10.1109/TMTT.2021.3053238.
- [111]S. Tebaldini and F. Rocca, 'Multistatic Wavenumber Tessellation: Ideas for High Resolution P-Band SAR Missions', in *International Geoscience and Remote Sensing Symposium*, 2017, pp. 2412–2415.
- [112]D. M. Sheen and T. E. Hall, 'Reconstruction techniques for sparse multistatic linear array microwave imaging', in *Passive and Active Millimeter-Wave Imaging XVII*, SPIE, Jun. 2014, p. 90780I. doi: 10.1117/12.2053814.
- [113]E. Rodrigues-Silva, M. Rodriguez-Cassola, G. Krieger, and A. Moreira, 'GNSS-Based Phase Synchronization for Bistatic and Multistatic Synthetic Aperture

- Radar', *IEEE Trans. Geosci. Remote Sens.*, vol. 62, pp. 1–14, 2024, doi: 10.1109/TGRS.2024.3406797.
- [114]P. J. Beasley, N. Peters, C. Horne, and M. A. Ritchie, 'Global Navigation Satellite Systems disciplined oscillator synchronisation of multistatic radar', *IET Radar Sonar Navig.*, vol. 18, no. 1, pp. 23–40, Jan. 2024, doi: 10.1049/rsn2.12475.
- [115]D. J. Munson, J. O'Brien, and W. K. Jenkins, 'A Tomographic Formulation of Spotlight-Mode Synthetic Aperture Radar', *Proc. IEEE*, vol. 71, no. 8, 1983.
- [116]R. M. Mersereau and A. V. Oppenheim, 'Digital Reconstruction of Multidimensional Signals from Their Projections', *Proc. IEEE*, vol. 62, no. Issue, pp. 1319–1338, 1974.
- [117]'Backhoe Sample Public Release and Visual-D Challenge Problem'. U.S. Air Force Research Laboratory, Jan. 2010. [Online]. Available: <https://www.sdms.afrl.af.mil/request/data request.php#Visual-D>
- [118]T. Marston, 'VOXview', *MATLAB Central File Exchange*. Jun. 2024. [Online]. Available: <https://uk.mathworks.com/matlabcentral/fileexchange/78745-voxview>
- [119]B. D. Rigling and R. L. Moses, 'Three-dimensional surface reconstruction from multistatic SAR images', *IEEE Trans. Image Process.*, vol. 14, no. 8, pp. 1159–1171, Aug. 2005, doi: 10.1109/TIP.2005.851690.
- [120]T. M. Marston and J. L. Kennedy, 'Volumetric acoustic imaging via circular multipass aperture synthesis', *IEEE J. Ocean. Eng.*, vol. 41, no. 4, pp. 852–867, Oct. 2016, doi: 10.1109/JOE.2015.2502664.
- [121]R. Bamler and P. Hart, 'Synthetic Aperture Radar Interferometry', *Inverse Probl.*, vol. 14, pp. 1–54, 1998.
- [122]H. Yu, Y. Lan, Z. Yuan, J. Xu, and H. Lee, 'Phase unwrapping in InSAR : A review', *IEEE Geosci. Remote Sens. Mag.*, vol. 7, no. 1, pp. 40–58, Mar. 2019, doi: 10.1109/MGRS.2018.2873644.
- [123]F. Mancuso, E. Giusti, B. Ng, and M. Martorella, 'Dual-Frequency Phase Unwrapping for 3D InSAR Imaging of Non-Cooperative Targets', *IEEE Trans. Radar Syst.*, vol. 2, pp. 434–445, Apr. 2024, doi: 10.1109/trs.2024.3392073.
- [124]D. C. Ghiglia and D. E. Wahl, 'Interferometric Synthetic Aperture Radar Terrain Elevation Mapping from Multiple Observations', in *Proceedings of the IEEE 6th Digital Signal Processing Workshop*, 1994, pp. 33–36.
- [125]N. Trouvé, E. Colin-Koeniguer, P. Fargette, and A. D. Martino, 'Influence of Geometrical configurations and polarization basis definitions on the analysis of

- bistatic polarimetric measurements', *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 6 PART 2, pp. 2238–2250, Jun. 2011, doi: 10.1109/TGRS.2010.2093533.
- [126] A. B. Kostinski and W.-M. Boerner, 'On Foundations of Radar Polarimetry', *IEEE Trans. ANTENNAS Propag.*, 1986.
- [127] R. Cloude and E. Pottier, 'A Review of Target Decomposition Theorems in Radar Polarimetry', *IEEE Trans. Geosci. REMOTE Sens.*, vol. 34, no. 2, 1996.
- [128] I. Hansjek and Y.-L. Desnos, *Polarimetric Synthetic Aperture Radar: principles and applications*, vol. 25. Springer, 2021. [Online]. Available: <http://www.springer.com/series/6477>
- [129] S. W. Chen, X. S. Wang, and M. Sato, 'Uniform polarimetric matrix rotation theory and its applications', *IEEE Trans. Geosci. Remote Sens.*, vol. 52, no. 8, pp. 4756–4770, 2014, doi: 10.1109/TGRS.2013.2284359.
- [130] A.-L. Germond, E. Pottier, and J. Saillard, 'Foundations of Bistatic Radar Polarimetry Theory', in *IET Radar Conference*, 1997, pp. 833–837.
- [131] J. R. Huynen, 'PHENOMENOLOGICAL THEORY OF RADAR TARGETS', 1970.
- [132] E. Lueneburg and S. R. Cloude, 'Bistatic Scattering', in *Wideband Interferometric Sensing and Imaging Polarimetry*, SPIE, Dec. 1997, pp. 56–68. doi: 10.1117/12.283852.
- [133] C. Titin-Schnaider, 'Characterization and recognition of bistatic polarimetric mechanisms', *IEEE Trans. Geosci. Remote Sens.*, vol. 51, no. 3, pp. 1755–1774, 2013, doi: 10.1109/TGRS.2012.2208118.
- [134] C. Titin-Schnaider, 'Physical meaning of bistatic polarimetric parameters', *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 5, pp. 2349–2356, May 2010, doi: 10.1109/TGRS.2009.2038063.
- [135] C. Titin-Schnaider, 'Polarimetric characterization of bistatic coherent mechanisms', *IEEE Trans. Geosci. Remote Sens.*, vol. 46, no. 5, pp. 1535–1546, May 2008, doi: 10.1109/TGRS.2008.916078.
- [136] D. Andre, R. Welsh, A. Hagelberg, and A. Rattan, 'LabSAR Phase 1 Technical Report', CUCDSGBSAR211201/1, Dec. 2021.
- [137] L. Moore, L. Potter, and J. Ash, 'Three-dimensional position accuracy in circular synthetic aperture radar', *IEEE Aerosp. Electron. Syst. Mag.*, vol. 29, no. 1, pp. 29–40, 2014, doi: 10.1109/MAES.2014.130076.
- [138] T. Tanaka, I. N. S. Parwata, and P. E. Yastika, 'Synthetic Aperture Radar Interferometry: Utilizing Radar Principles', *IEEE Geosci. Remote Sens. Mag.*, vol. 8, no. 4, pp. 111–116, Dec. 2020, doi: 10.1109/MGRS.2019.2942952.

- [139]F. Baselice, G. Ferraioli, and V. Pascazio, 'Three dimensional reconstruction using COSMO-SkyMed high-resolution data', in *2011 Microwaves, Radar and Remote Sensing Symposium, MRRS-2011 - Proceedings*, 2011, pp. 161–164. doi: 10.1109/MRRS.2011.6053625.
- [140]M. Woollard, D. Blacknell, H. Griffiths, and M. A. Ritchie, 'SARCASTIC v2.0—High-Performance SAR Simulation for Next-Generation ATR Systems', *Remote Sens.*, vol. 14, no. 11, p. 2561, May 2022, doi: 10.3390/rs14112561.
- [141]Z. Manzoor, M. T. Ghasr, and K. M. Donnell, 'Image distortion characterization due to equivalent monostatic approximation in near field bistatic SAR imaging', in *I2MTC 2017 - 2017 IEEE International Instrumentation and Measurement Technology Conference, Proceedings*, Institute of Electrical and Electronics Engineers Inc., Jul. 2017. doi: 10.1109/I2MTC.2017.7969810.
- [142]G. Li *et al.*, 'A new structure-based coregistration method for near-field ground-based MIMO tomographic SAR', in *IEEE International Geoscience & Remote Sensing Symposium*, 2019.
- [143]J. L. Bauck, 'A Rationale for Backprojection in Spotlight Synthetic Aperture Radar Image Formation', *IEEE Aerosp. Electron. Syst. Mag.*, vol. 37, no. 5, pp. 44–66, May 2022, doi: 10.1109/MAES.2021.3100886.
- [144]D. B. Andre and K. Morrison, 'A volumetric approach to SAR interferometry', in *Proceedings of the Institute of Acoustics*, 2014.
- [145]J. C. Russ and F. B. Neal, *The Image Processing Handbook: Seventh edition*. CRC Press, 2016. doi: <https://doi.org/10.1201/b18983>.
- [146]A. Horne *et al.*, 'Exploration of Multidimensional Radio Frequency Imaging to Derive Remote Intelligence of Building Interiors'. 2018.
- [147]H. Kuang, J. Chen, W. Yang, W. Liu, and X. Zhu, 'Fully three-dimensional UAV SAR imaging with multi-azimuth angle observation', in *IEEE International Geoscience and Remote Sensing Symposium*, 2017.
- [148]F. Liu, M. Antoniou, Z. Zeng, and M. Cherniakov, 'Point spread function analysis for BSAR with GNSS transmitters and long dwell times: Theory and experimental confirmation', *IEEE Geosci. Remote Sens. Lett.*, vol. 10, no. 4, pp. 781–785, Jul. 2013, doi: 10.1109/LGRS.2012.2223655.
- [149]F. Santi, M. Bucciarelli, D. Pastina, M. Antoniou, and M. Cherniakov, 'Spatial Resolution Improvement in GNSS-Based SAR Using Multistatic Acquisitions and Feature Extraction', *IEEE Trans. Geosci. Remote Sens.*, vol. 54, no. 10, pp. 6217–6231, Oct. 2016, doi: 10.1109/TGRS.2016.2583784.

- [150]A. K. Mishra, M. Bernard, and M. A. Kumar, 'Automatic Target Recognition using Multipolar Bistatic Synthetic Aperture Radar Images', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 46, no. 4, pp. 1906–1920, 2010.
- [151]T. Zeng *et al.*, 'Multiangle BSAR Imaging Based on BeiDou-2 Navigation Satellite System: Experiments and Preliminary Results', *IEEE Trans. Geosci. Remote Sens.*, vol. 53, no. 10, pp. 5760–5773, Oct. 2015, doi: 10.1109/TGRS.2015.2430312.
- [152]C. Hu, Z. Chen, X. Dong, and C. Cui, 'Multistatic geosynchronous sar resolution analysis and grating lobe suppression based on array spatial ambiguity function', *IEEE Trans. Geosci. Remote Sens.*, vol. 58, no. 9, pp. 6020–6038, Sep. 2020, doi: 10.1109/TGRS.2020.2969573.
- [153]C. D. Austin, E. Ertin, and R. L. Moses, 'Sparse multipass 3D SAR imaging: applications to the GOTCHA data set', in *Algorithms for Synthetic Aperture Radar Imagery XVI*, SPIE, May 2009, p. 733703. doi: 10.1117/12.820323.
- [154]T. Zeng, M. Cherniakov, and T. Long, 'Generalized approach to resolution analysis in BSAR', *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, no. 2, pp. 461–474, Apr. 2005, doi: 10.1109/TAES.2005.1468741.
- [155]H. Tian, C. Dong, H. Yin, and L. Yuan, 'Airborne sparse flight array SAR 3D imaging based on compressed sensing in frequency domain', *J. Syst. Eng. Electron.*, vol. 34, no. 1, pp. 56–67, Feb. 2023, doi: 10.23919/JSEE.2022.000125.
- [156]H. Ren *et al.*, 'Swarm UAV SAR for 3-D Imaging: System Analysis and Sensing Matrix Design', *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, doi: 10.1109/TGRS.2022.3221775.
- [157]B. Corbett, D. Andre, and M. Finnis, 'Localising vibrating scatterer phenomena in synthetic aperture radar imagery', *Electron. Lett.*, vol. 56, no. 8, pp. 395–398, Apr. 2020, doi: 10.1049/el.2019.4114.
- [158]H. A. Zebker and J. Villasenor, 'Decorrelation in interferometric radar echoes', *IEEE Trans. Geosci. Remote Sens.*, vol. 30, no. 5, pp. 950–959, 1992, doi: 10.1109/36.175330.
- [159]G. Gao, L. Yao, W. Li, L. Zhang, and M. Zhang, 'Onboard Information Fusion for Multisatellite Collaborative Observation: Summary, challenges, and perspectives', *IEEE Geosci. Remote Sens. Mag.*, vol. 11, no. 2, pp. 40–59, Jun. 2023, doi: 10.1109/MGRS.2023.3274301.

- [160]D. H. T. Minh and S. Tebaldini, 'Interferometric Phase Linking: Algorithm, application, and perspective', *IEEE Geosci. Remote Sens. Mag.*, vol. 11, no. 3, pp. 46–62, Sep. 2023, doi: 10.1109/MGRS.2023.3300974.
- [161]M. Zink *et al.*, 'TanDEM-X: The new global DEM takes shape', *IEEE Geosci. Remote Sens. Mag.*, vol. 2, no. 2, pp. 8–23, 2014, doi: 10.1109/MGRS.2014.2318895.
- [162]B. Hawkins, M. Anderson, S. Prager, S. J. Chung, and M. Lavalley, 'Experiments with Small UAS to Support SAR Tomographic Mission Formulation', in *International Geoscience and Remote Sensing Symposium (IGARSS)*, Institute of Electrical and Electronics Engineers Inc., 2021, pp. 643–646. doi: 10.1109/IGARSS47720.2021.9554770.
- [163]D. Andre, R. Sabiers, and M. Finnis, 'Through-Wall Multistatic Polarimetric 3D SAR', *NATO STO Rev.*, no. STO-MP-SET 273, pp. 1–12, 2022, doi: 10.14339/STO-SET-273-05.
- [164]X. X. Zhu and R. Bamler, 'Superresolving SAR tomography for multidimensional imaging of Urban areas: Compressive sensing-based TomoSAR inversion', *IEEE Signal Process. Mag.*, vol. 31, no. 4, pp. 51–58, 2014, doi: 10.1109/MSP.2014.2312098.
- [165]M. Wang, S. Wei, R. Shen, Z. Zhou, J. Shi, and X. Zhang, '3D SAR Imaging Method Based on Learned Sparse Prior', *J. Radars*, vol. 12, no. 1, pp. 36–52, Feb. 2023, doi: 10.12000/JR22101.
- [166]J. Wang, S. Quan, S. Xing, Y. Li, H. Wu, and W. Meng, 'PSO-based fine polarimetric decomposition for ship scattering characterization', *ISPRS J. Photogramm. Remote Sens.*, vol. 220, pp. 18–31, Feb. 2025, doi: 10.1016/j.isprsjprs.2024.11.015.
- [167]E. Aguilera, M. Nannini, and A. Reigber, 'Multisignal compressed sensing for polarimetric SAR tomography', *IEEE Geosci. Remote Sens. Lett.*, vol. 9, no. 5, pp. 871–875, 2012, doi: 10.1109/LGRS.2012.2185482.
- [168]H. Bi, J. Feng, S. Jin, W. Yang, and W. Xu, 'Mixed-Norm Regularization-Based Polarimetric Holographic SAR 3-D Imaging', *IEEE Geosci. Remote Sens. Lett.*, vol. 21, pp. 1–5, 2024, doi: 10.1109/LGRS.2024.3353801.
- [169]S. Demirci and Y. Izumi, 'Application of H/ α Decomposition to Limited and Dual-Polarimetric 3D SAR Data of Civilian Vehicles', *IEEE Access*, vol. 11, pp. 83589–83602, 2023, doi: 10.1109/ACCESS.2023.3301619.

- [170]F. Xue, X. Wang, F. Xu, and Y. Wang, 'Polarimetric SAR Interferometry: A Tutorial for Analyzing System Parameters', *IEEE Geosci. Remote Sens. Mag.*, vol. 8, no. 2, pp. 83–107, Jun. 2020, doi: 10.1109/MGRS.2019.2954756.
- [171]K. Wu and X. Xu, 'A Fast ISAR Tomography Technique for Fully Polarimetric 3-D Imaging of Man-Made Targets', *IEEE Trans. Geosci. Remote Sens.*, vol. 60, 2022, doi: 10.1109/TGRS.2021.3097725.
- [172]K. Sarabandi, F. T. Ulaby, and M. A. Tassoudji, 'Calibration of Polarimetric Radar Systems With Good Polarization Isolation', *IEEE Trans. Geosci. Remote Sens.*, vol. 28, no. 1, pp. 70–75, 1990, doi: 10.1109/36.45747.
- [173]P. J. Roebber, 'Visualizing multiple measures of forecast quality', *Weather Forecast.*, vol. 24, no. 2, pp. 601–608, Apr. 2009, doi: 10.1175/2008WAF2222159.1.
- [174]C. Zhang, G. Gao, J. Liu, and D. Duan, 'Oriented Ship Detection Based on Soft Thresholding and Context Information in SAR Images of Complex Scenes', *IEEE Trans. Geosci. Remote Sens.*, vol. 62, pp. 1–15, 2024, doi: 10.1109/TGRS.2023.3340891.
- [175]M. Xing, V. Pascazio, and H. Yu, 'A Special Issue on Synthetic Aperture Radar Interferometry [From the Guest Editors]', *IEEE Geosci. Remote Sens. Mag.*, vol. 8, no. 1, pp. 6–7, Mar. 2020, doi: 10.1109/mgrs.2020.2965784.
- [176]X. X. Zhu and R. Bamler, 'Very high resolution spaceborne SAR tomography in urban environment', *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 12, pp. 4296–4308, Dec. 2010, doi: 10.1109/TGRS.2010.2050487.
- [177]X. X. Zhu and R. Bamler, 'Tomographic SAR inversion by L1-norm regularization-the compressive sensing approach', *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 10, pp. 3839–3846, Oct. 2010, doi: 10.1109/TGRS.2010.2048117.
- [178]X. X. Zhu, Y. Wang, S. Montazeri, and N. Ge, 'A review of ten-year advances of multi-baseline SAR interferometry using terraSAR-X data', *Remote Sens.*, vol. 10, no. 9, Sep. 2018, doi: 10.3390/rs10091374.
- [179]P. Wang, M. Liu, and Z. Wang, 'Three-dimensional SAR imaging of sea targets with low PRF sampling', *IET Radar Sonar Navig.*, vol. 12, no. 3, pp. 294–300, Mar. 2018, doi: 10.1049/iet-rsn.2017.0363.
- [180]X. X. Zhu and R. Bamler, 'Demonstration of super-resolution for tomographic SAR imaging in urban environment', *IEEE Trans. Geosci. Remote Sens.*, vol. 50, no. 8, pp. 3150–3157, 2012, doi: 10.1109/TGRS.2011.2177843.

- [181]M. Younis, F. Q. D. Almeida, M. Villano, S. Huber, G. Krieger, and A. Moreira, 'Digital Beamforming for Spaceborne Reflector-Based Synthetic Aperture Radar, Part 1: Basic imaging modes', *IEEE Geosci. Remote Sens. Mag.*, vol. 9, no. 3, pp. 8–25, Sep. 2021, doi: 10.1109/MGRS.2021.3060543.
- [182]M. Younis, F. Q. D. Almeida, M. Villano, S. Huber, G. Krieger, and A. Moreira, 'Digital Beamforming for Spaceborne Reflector-Based Synthetic Aperture Radar, Part 2: Ultrawide-swath imaging mode', *IEEE Geosci. Remote Sens. Mag.*, vol. 10, no. 4, pp. 10–31, Dec. 2022, doi: 10.1109/MGRS.2022.3200871.