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LPV Systems Analysis Using Pseudospectra

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***Abstract*—Pseudospectra provides a qualitative and quantitative tool for analyzing the stability sensitivity of linear systems. In this paper pseudospectral analysis is extended to Linear Parameter Varying (LPV) systems that are approximated by frozen sets. Two examples of quasi-LPV system models are analyzed, namely an actuated pendulum system and a 2-DOF aeroelastic flutter model. It is shown that use of pseudospectra can overcome some limitations of classical eigenvalue sensitivity as well as providing qualitative information that is straightforward to interpret. Some limitations are discussed and suggestions for further work included.**

***Index Terms*—Perturbation, Eigenvalue sensitivity, Pseudospectra, Aeroelasticity, LPV**

I. Introduction

The eigenvalues and eigenvectors of the system matrix of a dynamic system are extremely important in determining the response of the system. Their properties make them valuable tools for the analysis of the system and the design of feedback controllers. Through the location of the eigenvalues, the system stability can be established, and the dynamic response determined via the matrix exponential of the system matrix through spectral decomposition. The spectrum of a system matrix of finite dimension is simply the set of all eigenvalues of the system matrix.

However, practical systems are generally subject to perturbation and uncertainty. In fact, perturbations in all practical dynamic systems are unavoidable and hence the consideration of the perturbations is an important part in the process of design and analysis of systems and structures. This is, of course, widely recognized and many tools for the analysis of uncertainty have been developed. For feedback systems this is the motivation for nearly the whole of the field of robust control.

Consideration of the behaviour of the spectrum to perturbations in the system matrix is a means to analyze the effect of uncertainty of the system. Because eigenvalues respond differently to each type of perturbation, a variety of perturbation classes had been considered in perturbation theory [1]. Linear perturbation theory has two main approaches depending on the type of perturbation. Classical or qualitative perturbation theory, which is concerned with highly structured perturbations and most of the results deal with operators depending analytically on one complex parameter, and quantitative concerned with unstructured perturbation of matrices [1, p. 398]. Sensitivity analysis lies between those two approaches. Mostly, parameter perturbation does not fit in any category. So, the perturbation structure need to be built in a way that obtain non-conservative results [1, p. 398].

However, in the area of general dynamic systems, there have been developments in the analysis of the spectrum when the system is subjected to perturbations, and one of the main developments is a method often known as 'pseudospectra' [2]. Pseudospectra has been particularly successful in the analysis of dynamic behaviours caused by non-normal operators and matrices. Consider the system represented by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$ and $\mathbf{x} \in \mathbb{R}^n$. This system is considered stable if every eigenvalue of the system matrix $\mathbf{A}$ has a negative real part. If the matrix $\mathbf{A}$ lacks an orthogonal basis of eigenvectors, the system is described as 'non-normal'. A characteristic of some non-normal systems is that, following a small initial perturbation of the system state, $\mathbf{x}(0)$, the subsequent state trajectory given by $\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0)$ deviates very far from the origin even though the system is stable [3] and $\mathbf{x}(0)$ has small magnitude. Such behaviour is undesirable, particularly for some non-linear systems where the trajectory may depart from the domain of attraction resulting in instability, even though linear stability analysis indicates stability. Such behaviour is well-known for fluid systems [3]–[5] and feedback control methods for such systems have been developed, e.g. [6]–[8]. Very often, non-normality occurs in high order and infinite dimension systems. Part of the motivation for spectral analysis (rather than simply considering the eigenvalues) is that it extends to infinite dimension systems [9, p. 80]; fluidic systems being a member of this class. Aeroelastic and aeroservoelastic systems are also infinite dimensional and so one may expect non-normal systems to arise here. Indeed, an aeroelastic model for a Boeing 767 has been posed as a benchmark problem [10], [11]. At this point we should note that dynamic systems perturbations analysis based on pseudospectra often assume no structure in the perturbations of the $\mathbf{A}$ matrix. Much work has been done on assessing the pseudospectra for different perturbation classes, this has been thoroughly explored and detailed in [1]

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using a technique known as spectral level sets. More about the relationships between $\mathcal{H}_\infty$ small gain theorem robust stability analysis and pseudospectra, and between structured singular values/μ-analysis and spectral level sets are detailed in [1].

Note also that classical eigenvalue sensitivity analysis considers the eigenvalue change as a function of changes in the entries of the $\mathbf{A}$ matrix. Although there is an extensive literature on this, many fail to take into account that the eigenvectors are also perturbed [12, p. 131].

In this paper we will be using pseudospectral analysis and applying it to an aeroelastic system (albeit a low order model). Pseudospectral analysis has previously been applied to control design for and flutter analysis of a high-order Boeing 767 aircraft model [11]. It is shown that the strengths of the pseudospectral analysis lie in its graphical nature which allow both quantitative and qualitative analysis. Such an approach has an obvious appeal to engineers.

Pseudospectral analysis is a technique for linear models, and it seems that so far its use has been confined to Linear Time Invariant (LTI) systems. In practice, most aeroelastic system models are non-linear, but can be well-approximated by quasi-LPV systems. LPV models can be derived from original non-linear equation models using algebraic manipulation methods. Possibly the first LPV modelling an aeroelastic system was by [13], who presented a Linear Fractional Transformation (LFT) model dependant on Mach number and dynamic pressure for the model based on the NACA Benchmark Active Control Technology (BACT) model [14]. It is a simple model, part of NASA Langley research centre's benchmark models program for studying transonic aeroelastic phenomena. It was developed to collect high quality unsteady aerodynamic data (pressure and loads) near transonic flutter conditions and demonstrate flutter suppression using spoilers. Many studies of this model have been made using different techniques; [15], for example, uses a tensor product modelling approach whilst [16] use gain-scheduling controllers based on LQR control as well as the standard extension of $\mathcal{H}_\infty$ theory to affine and polytopic LPV systems.

In this paper we propose extending pseudospectra and spectral level sets to LPV systems by approximating LPV systems with frozen model sets [17, p.48], the technique used extensively in gain scheduling control.

In the next section, an overview of the pseudospectra theory is provided. Section III considers a simple system, a pendulum with an actuated torque at the pivot. This is modelled as a quasi-LPV system. Then follows a pseudospectral analysis of the system for a set of frozen values of the varying parameter. In Section IV, the analysis is repeated for a more practical example, this being a 2-DOF aeroelastic model, namely the BACT model. The final section provides some conclusions and outlines the further work that is required to establish the usefulness and practicality of applying pseodospectra analysis to LPV systems.

II. Background Theory

A. Classical Eigenvalue Sensitivity

Before explaining pseudospectra, we show the relationship between normality and sensitivity. We also show a limitation of classical sensitivity analysis.

Sensitivity analysis is concerned with changes of the eigenstructure under infinitesimal changes in a single parameter. Consider the dynamic system (1) and let $\{\lambda_k : k = 1 \ldots, n\}$ denote the set of eigenvalues, and $\{y_k : k = 1 \ldots, n\}$ and $\{x_k : k = 1 \ldots, n\}$ denote the set of left and right eigenvectors respectively with the constraint that $y_k^H x_k = 1$ for $k = 1 \ldots, n$. A sensitivity measure of the kth eigenvalue is defined by $\psi_k := \|\partial\lambda_k/\partial\lambda_{\mathbf{A}}\|_F^2$ where $\|\cdot\|_F$ denotes the Frobenius norm and is given by [12, pp 135-137]

$$\psi_k = \|y_k\|_2^2 \|x_k\|_2^2 = \mathrm{tr}\{(y_k x_k^H)(y_k x_k^H)^H\} \tag{2}$$

where $(\cdot)^H$ denotes the complex conjugate transpose of a matrix and $\mathrm{tr}(\cdot)$ the trace.

However if $\mathbf{A}$ is defective (i.e. has repeated eigenvalues with geometric multiplicity less than the algebraic multiplicity), then the left and right eigenvectors for the multiple eigenvalue are orthogonal, and $|x_k^H y_k| = 0$ giving a singularity for the sensitivity measure. This condition arises for the pendulum example of the next section. Hence a singularity-free sensitivity measure is preferred, and this is provided by the pseudospectra. More about the above sensitivity measure can be found in [12, chap.6].

B. Pseudospectra

Pseudospectra has recently found application in analysing the sensitivity of eigenvalues of a system. Quoting [2]: "pseudospectra is a set in the complex plane to which the eigenvalues of a system can be shifted, under a random perturbation of a given size. In this way, one can classify the degree of sensitivity of the system's eigenvalues". Thus for robust stability, the pseudospectra identifies the minimum size of a random perturbation required to shift an eigenvalue such that stability is lost.

It is well known that the spectrum, $\Lambda(\mathbf{A})$, of a matrix, $\mathbf{A} \in \mathbb{C}^{n\times n}$, is defined as

$$\Lambda(\mathbf{A}) := \{z \in \mathbb{C} : \det(z\mathbf{I} - \mathbf{A}) = 0\}. \tag{3}$$

Thus $\Lambda(\mathbf{A}) = \{z_i\}$ where z_i are the eigenvalues of $\mathbf{A}$. The matrix $(z\mathbf{I} - \mathbf{A})^{-1}$ is known as the resolvent matrix of $\mathbf{A}$. Thus the resolvent of $\mathbf{A}$ is undefined on $\Lambda(\mathbf{A})$, that is $\|(z\mathbf{I} - \mathbf{A})^{-1}\| = \infty$ for $z \in \Lambda(\mathbf{A})$ [18, p. 1].

The ε-pseudospectrum of a matrix $\mathbf{A}$ is defined as the set [18, p. 13]

$$\Lambda_\varepsilon(\mathbf{A}) := \{z \in \mathbb{C} : \varepsilon\|(z\mathbf{I} - \mathbf{A})^{-1}\| > 1\} \tag{4}$$

where $\varepsilon \in \mathbb{R}_+$. Thus it a subset of the complex plane bounded by the ε^{-1} level curve of the norm of the resolvent matrix. An alternative, equivalent definition[1] is given by [18, p. 14] as

[1]Note that here the inequality $\|\mathbf{\Delta}\| < \varepsilon$ is strict. In some works (e.g. [2]) it is semi-strict.

$\Lambda_\varepsilon(\mathbf{A}) = \{z \in \mathbb{C} : z \in \Lambda(\mathbf{A}+\mathbf{\Delta})$ for some $\mathbf{\Delta}$ with $\|\mathbf{\Delta}\| < \varepsilon\}$. Thus the ε-pseudospectrum is the set of eigenvalues for all matrices $\mathbf{A}+\mathbf{\Delta}$ with $\|\mathbf{\Delta}\| < \varepsilon$. It is clear that this can provide a measure of the stability robustness of a system.

Pseudospectra has a graphical representation, and the numerics behind the graphical method for pseudospectra analysis is to compute the singular value decomposition (SVD) at each point on a grid in the complex plane, then send the results to a contour plotter. A MATLAB tool for plotting pseudospctra is available [19]. Pseudospectra is norm dependent, so the matrix norm is formed in a manner that permits them to converge to the appropriate continuous norms as the approximation is refined [2].

The study of the pseudospectrum of a system matrix not only gives information about the eigenvalue sensitivity and stability robustness, it also can give information about the transient performance, particularly for non-normal systems, through the growth or decay of $\left\|e^{t\mathbf{A}}\right\|$ as time advances. Although the information is qualitative, it is provided graphically which is often preferred by practicing engineers. For more details and definition of pseudospectra see [3] and [18].

C. LPV Systems Analysis

Nonlinear system models are often approximated by Linear Time Invariant (LTI) systems, which are more tractable than nonlinear systems. However, nonlinear dynamic systems can be approximated by quasi LPV system more accurately than by LTI systems. An LPV system is a linear system that nonlinearly depends on one or more time-varying parameters. Nonlinear systems are often modelled as LPV systems, in a way that can place the system between LTI and nonlinear and yet can still capture some nonlinear properties of the system.

An LPV system is a linear system, whose state-space descriptions are known functions of a time-varying parameter vector, $\mathbf{p} \in \mathbb{R}^N$. They have the form

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{p})\mathbf{x} + \mathbf{B}(\mathbf{p})\mathbf{u}, \tag{5}$$

$$\mathbf{y} = \mathbf{C}(\mathbf{p})\mathbf{x} + \mathbf{D}(\mathbf{p})\mathbf{u}. \tag{6}$$

If we restrict $\mathbf{p} \in \mathcal{P} \subset \mathbb{R}^N$ where $\mathbf{p}(t)$ is assumed constant, we can approximate the LPV system by a set of LTI systems known as the frozen system set $\mathcal{F}_S$ [17, p. 48]. This approximation is used in this paper.

For LTI systems, the pseudospectra in (4), as a function of ε, can be rewritten as the set

$$\Lambda_\varepsilon(\mathbf{A}) = \{z \in \mathbb{C} : \|(z\mathbf{I}-\mathbf{A})^{-1}\| > \varepsilon^{-1}\}. \tag{7}$$

that is[2]

$$\Lambda_\varepsilon(\mathbf{A}) = \{z \in \mathbb{C} : \underline{\sigma}(z\mathbf{I}-\mathbf{A}) < \varepsilon\}. \tag{8}$$

where $\underline{\sigma}(\cdot)$ denotes the minimum singular value of a matrix.

From the definition of pseudospectra, for the LPV model, it is a set of complex numbers such that when we perturb $\mathbf{A}$, the graph gives a measure of how sensitive eigenvalues are. Mathematically, we measure the resolvent and compare it to ε. For the LPV model analysis we should consider the set $\mathcal{P}$, and analyse by plotting the set $\{z : \|z\mathbf{I}-\mathbf{A}(\mathbf{p})\|_{\mathbf{p}\in\mathcal{P}} < \varepsilon\}$. Clearly this is intractable.

From (8), we can approximate the pseudospectra for an LPV system by plotting the pseudospectra for the frozen system set. For an LPV system with $A(\mathbf{p})$ where $\mathbf{p}(t) \in \mathcal{P}$, we freeze $\mathbf{p}$ so that $\dot{\mathbf{p}} = 0$, and define a set of frozen values

$$\mathcal{P}_f := \{\mathbf{p}(t) \in \mathcal{P} : \dot{\mathbf{p}} = 0\}. \tag{9}$$

where $\mathcal{P}_f \subset \mathcal{P}$. Then we can define a set of spectra for the frozen system set, $\mathcal{F}_S$, to be

$$\Lambda_S(\mathcal{F}_S) := \{z \in \mathbb{C} : \det(z\mathbf{I}-\mathbf{A}(\mathbf{p})) = 0, \mathbf{p} \in \mathcal{P}\}. \tag{10}$$

Now we can extend (8) to define the pseudospectra for a frozen system set, $\mathcal{F}_S$, to be

$$\Lambda_{S\varepsilon}(\mathcal{F}_S) := \{z \in \mathbb{C} : \min_{\mathbf{p}\in\mathcal{P}_f} \underline{\sigma}(z\mathbf{I}-\mathbf{A}(\mathbf{p})) < \varepsilon\}. \tag{11}$$

or equivalently

$$\Lambda_{S\varepsilon}(\mathcal{F}_S) := \{z \in \mathbb{C} : \min_{\mathbf{p}\in\mathcal{P}_f} \left\|(z\mathbf{I}-\mathbf{A}(\mathbf{p}))^{-1}\right\| > 1/\varepsilon\}. \tag{12}$$

III. Actuated Pendulum System

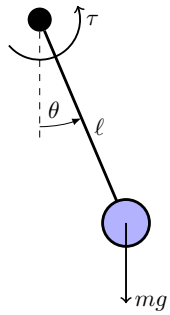


Fig. 1. Simple pendulum actuated at the pivot.

Perhaps one of the most studied and analysed nonlinear systems is the simple pendulum. Here we add an actuator to the pivot to provide control. The system is shown in Figure 1 and consists of a mass, m, attached to a light, stiff rod of length, ℓ, which is attached at the other end to a motor which supplies a torque, τ at the pivot which is assumed friction free. The angle with the vertical is denoted by θ and g is the gravitational acceleration constant. The nonlinear model is given by

$$\dot{\theta} = \omega, \qquad m\ell^2\dot{\omega} = -mg\ell\sin\theta + \tau. \tag{13}$$

The equilibrium (trim) condition triplet, $(\hat{\omega}, \hat{\theta}, \hat{\tau})$, is obtained when $\dot{\theta} = 0$ and $\dot{\omega} = 0$ giving $\hat{\omega} = 0$ and $\hat{\tau} = mg\ell\sin\hat{\theta}$.

A small perturbation model is obtained by linearising about the trim condition and defining the state variables to be $x_1 = \theta - \hat{\theta} = \delta\theta$, $x_2 = \delta\omega$, the control to be $u = \tau - \hat{\tau} = \delta\tau$, giving

$$\begin{bmatrix}\dot{\delta\theta}\\ \dot{\delta\omega}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -(g/\ell)\cos\hat{\theta} & 0\end{bmatrix}\begin{bmatrix}\delta\theta\\ \delta\omega\end{bmatrix} + \begin{bmatrix}0\\ 1/(m\ell^2)\end{bmatrix}\delta\tau. \tag{14}$$

[2]This is computed, for example, in Higham's MATLAB routine `pscont.m` by a grid map over a subset of the complex plane, $\mathbb{C}$, where at each point on the grid the resolvent is calculated by $\min(\text{svd}(z(i,j) \star I - \mathbf{A}))$ [2]

To represent the system as a quasi-LPV model, the time varying parameter is taken to be $\mathbf{p} = \cos\hat{\theta}$ and we get the quasi-LPV model in the form of (5) with

$$\mathbf{A}(\mathbf{p}) = \begin{bmatrix} 0 & 1 \\ -(g/\ell)\mathbf{p} & 0 \end{bmatrix}. \tag{15}$$

and $\mathbf{p} \in [-1, 1]$.

Assuming a frozen system set $\mathcal{P}_f$, the spectrum for a $\mathbf{p} \in \mathcal{P}_f$ is

$$\Lambda(\mathbf{p}) = \left\{ \pm\sqrt{-g\mathbf{p}/\ell} \right\}. \tag{16}$$

The variation of the real and imaginary parts with $\mathbf{p} \in \mathcal{P}_f$ and $\ell = 1$ is shown in Figure 2, showing that when $\mathbf{p} = 0$, the stability of the system switches from marginally stable to unstable. Furthermore, at this point the eigenvalues are equal and the matrix, $\mathbf{A}$, is defective.

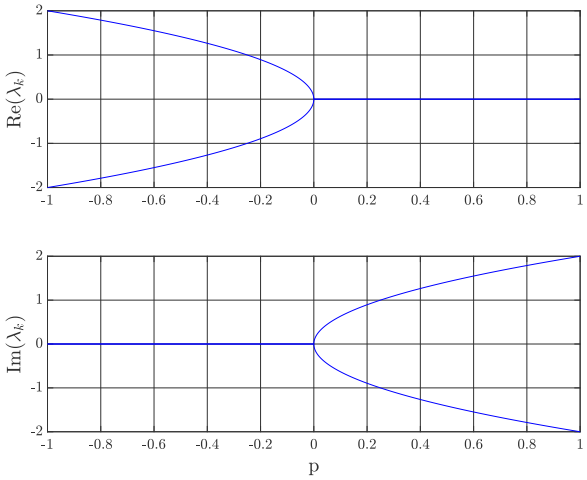


Fig. 2. Eigenvalue variation with $\mathbf{p}$ for the actuated pendulum model – real and imaginary parts.

Figure 3 shows the sensitivity measure, ψ_k, (note $\psi_1 = \psi_2$) for $\mathbf{p} \in \mathcal{P}$. At point $\mathbf{p} = 0$, where the $\mathbf{A}$- matrix is defective, the eigenvalues sensitivity measure defined by (2) is infinite. For the case shown, the eigenvalues become less sensitive as we move from $\mathbf{p} = \pm 1$ towards the defective point, until a minimum point with $\psi_k = 1$ after which their sensitivity increases rapidly as the defective point is approached. In fact, the location of the minimum sensitivity depends on the ratio g/ℓ and is minimum at $\mathbf{p} = \pm 1$ for $g/\ell \leq 1$. For the case shown, $g/\ell = 9.81$.

Because of the singularity at the defective point, eigenvalues sensitivity analysis has some limitations. However, the pseudospectra is singularity free. Figure 4 shows the pseudospectra for a subset of frozen points, $\{\mathbf{p}_i\} \subset \mathcal{P}_f$ where $\{\mathbf{p}_i\} = \{-1, -0.5, 0, 0.5, 1\}$. The pseudospectra actually show the norm of the resolvent contours, $\|(z\mathbf{I}-\mathbf{A})^{-1}\| = 1/\varepsilon$. The locii of the eigenvalues of $\mathbf{A}(\mathbf{p})$ are also shown for $\mathbf{p} \in \mathcal{P}_f$. As for the sensitivity plot in Figure 3, eigenvalues are most sensitive near the defective point since they have larger pseudospectra sets. The pseudospectra sets for $\mathbf{p} = \pm 0.5$ are the smallest shown. The analysis was performed using MATRIXCOMP toolbox with MATLAB [2].

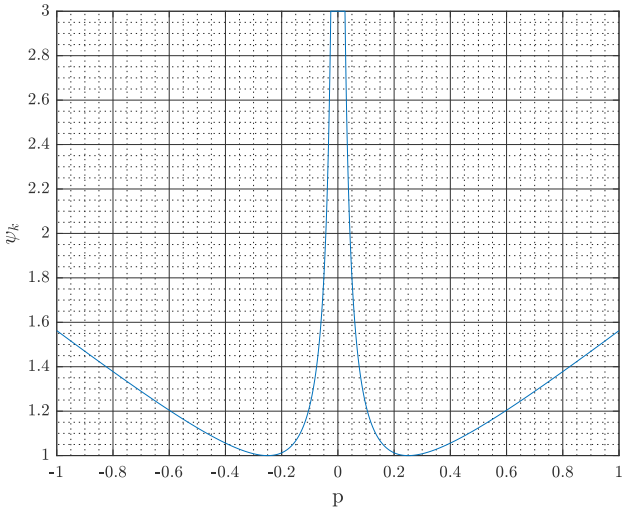


Fig. 3. Sensitivity variation with $\mathbf{p}$ for the actuated pendulum model.

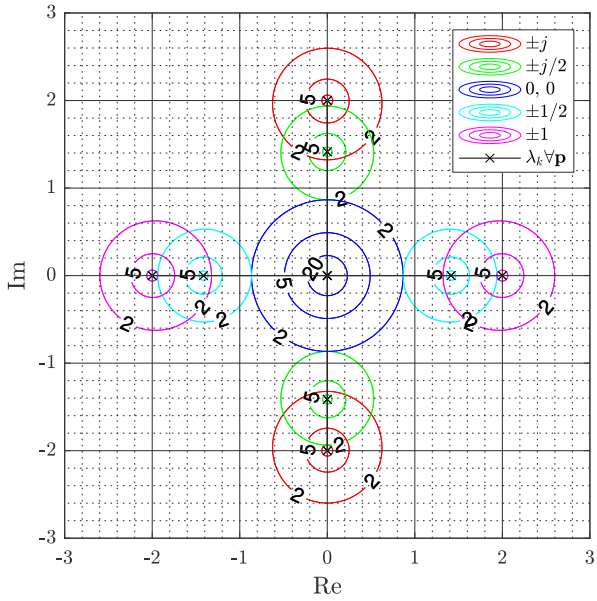


Fig. 4. Pseudospectra contours for $\|(z\mathbf{I} - \mathbf{A})^{-1}\| = (2, 5, 20)$ for actuated pendulum model for $\{\mathbf{p}_i\} = \{-1, -0.5, 0, 0.5, 1\}$.

Figure 5 shows the full frozen system set pseudospectra, $\Lambda_{S\varepsilon}(\mathcal{F}_S)$ for the quasi-LPV model of the open-loop actuated pendulum. From the classical eigenvalues analysis of Section III that the eigenvalues are most sensitive around the origin. But what we see from this plot is that as the perturbations increase, the sensitivity around the origin actually falls away more rapidly so for larger perturbations, the sensitivity is even less for $\mathbf{p}$ near zero and the unperturbed eigenvalues near the origin. Changing the perturbation level can give more information about the system behaviour.

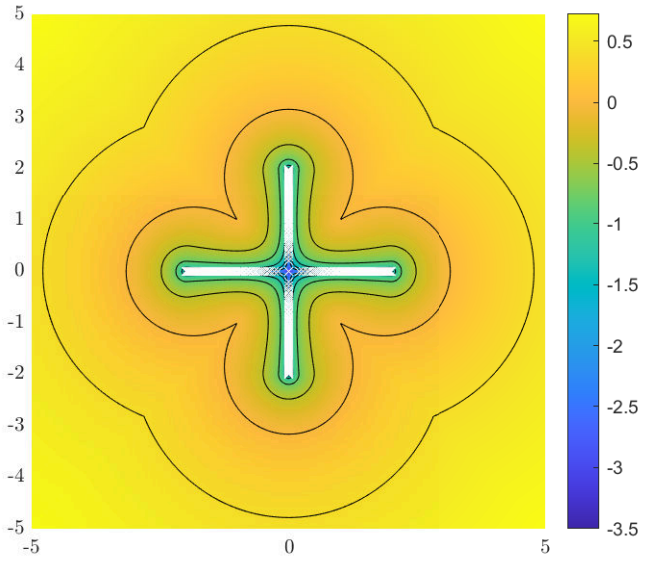


Fig. 5. Pseudospectra sets $\Lambda_{S\varepsilon}(\mathcal{F}_S)$ for $\varepsilon = [10^{-2} : 0.4 : 10^{2}]$ for open-loop actuated pendulum quasi-LPV model (colour bar shown in log 10).

IV. A 2-DOF Aeroelastic System

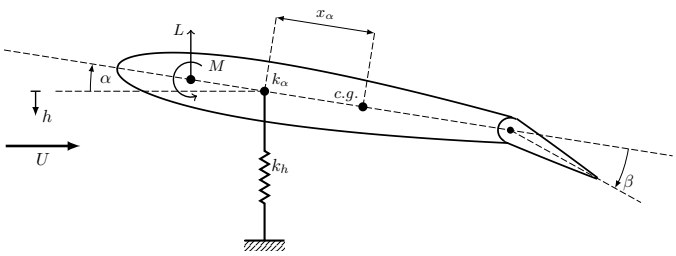


Fig. 6. 2-DOF aeroelastic model.

In this section, some of the analysis is repeated for the BACT aeroelastic system model [14].

The system is shown in Figure 6. It consists of a rigid rectangular wing equipped with a trailing edge control surface. The system has two degrees of freedom; plunge $h(t)$ and pitch α. The use of this model allows direct comparison with previous studies about the aeroelastic wing section [14].The trailing edge deflection is denoted by β, but is not used in this paper.

The equations of motion for the model in Figure 6 are given by [20]

$$\begin{bmatrix} m & mx_\alpha b \\ mx_\alpha b & I_\alpha \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} c_h \dot{h} \\ c_\alpha \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h h \\ k_\alpha \alpha \end{bmatrix} = \begin{Bmatrix} -L \\ M \end{Bmatrix}, \quad (17)$$

where m is the mass of the wing, I_α is the mass moment of inertia about the elastic axis, c_h and c_α are the aerodynamic damping coefficients, α is the pitch angle, x_α is the non-dimensional distance (with respect to the semi-chord, b) between the elastic axis and center of mass, k_α is the rotational spring constant, k_h the vertical spring constant, U is the free stream velocity, L is the aerodynamic lift force and M is the aerodynamic moment which are both dependent upon α, β and U. Details about the experimental data and results for this model can be found in [20] and [21].

LPVTools[3] is used here to generate the quasi-LPV model of the BACT system with the free stream velocity, U, and pitch angle, α, as the varying parameters, that is $\mathbf{p} = \begin{bmatrix} U, & \alpha \end{bmatrix}^T$.

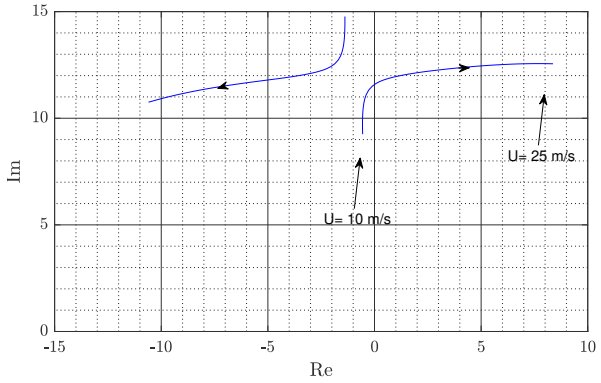


Fig. 7. Spectra for $\mathbf{p} \in \mathcal{P}_f$ for 2-DOF aeroelastic model.

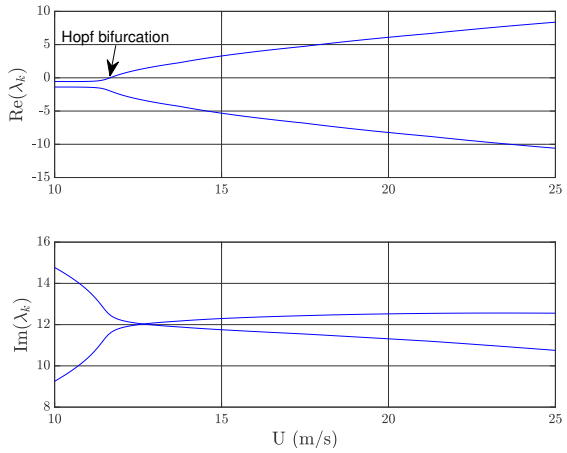


Fig. 8. Variation of eigenvalues with free stream velocity U for 2-DOF aeroelastic model with $\alpha = 0$ – real and imaginary parts.

A set of frozen values $\mathcal{P}_f$ with a single frozen $\alpha = 0$ and a set of $U \in [10, 25]$ m/s is defined. The locii of the spectra $\Lambda(A(\mathbf{p}))$ for all $\mathbf{p} \in \mathcal{P}_f$ is shown in Figure 7. It can be seen that there are no defective points, but that there exists a Hopf bifurcation at a frequency $\omega \simeq 11.5$ rad/s. It has been shown previously [15], [20], [21] that the system enters a limit-cycle oscillation at airspeeds greater than the flutter speed. Figure 8 showing the variation of the real and imaginary parts of the spectrum of the system matrix $\mathbf{A}$ with free stream velocity U. The real part of the complex conjugate pair of eigenvalues switches from negative to positive (the Hopf bifurcation) at $U \simeq 11.7$ m/s – this predicts the flutter speed.

[3]LPVTools is a MATLAB toolbox for modelling, analysis, and synthesis of parameter varying control systems [22], [23].

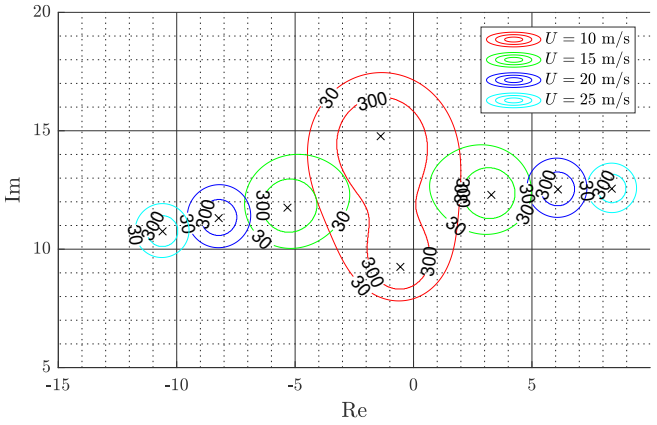


Fig. 9. Pseudospectra contours for $\|(z\mathbf{I} - \mathbf{A})^{-1}\| = (30, 300)$ for 2-DOF aeroelastic model for $U \in \{10, 15, 20, 25\}$ and $\alpha = 0$.

The ε-pseudospectra contours are shown in Figure 9 for a single frozen $\alpha = 0$ and a set of $U \in \{10, 15, 20, 25\}$. They show that the eigenvalues are more sensitive at lower speeds and near the Hopf bifurcation point. This means that the flutter speed is sensitive to the system uncertainty. The sets are no longer circular, which indicates some non-normality.

V. Conclusion

The paper considers two nonlinear systems that can be modelled as quasi-LPV models. By approximating the LPV model by a frozen set model, pseudospectral analysis can be performed. For the actuated pendulum system, we see that for systems where there exists some values of frozen $\mathbf{p}$ such that $\mathbf{A}$ is defective, then classical spectral sensitivity analysis becomes difficult because the sensitivity becomes infinite at the defective point. However, the pseudospectral contours are finite and thus can be used more easily for computational design. Furthermore, plotting the pseudospectral contours for frozen subsets of $\mathcal{P}$ enables graphical, qualitative analysis of the sensitivity and gives more information than the sensitivity function. We envisage that this has potential appeal to practising engineers.

For the 2-DOF aeroelastic model, the pseudospectra near the bifurcation point is of much interest, space restrictions have limited the details that are included here. Note that it is fairly straightforward to extend pseudospectra to analysis where there is some structure in the system perturbations $\mathbf{\Delta}$; see [1] and [18] for details. Note in particular for the 'unstructured' pseudospectral analysis presented here, $\mathbf{A}$ and $\mathbf{\Delta}$ can be complex whereas in practical applications they are usual real matrices.

Although neither of the two systems analyzed here displayed any non-normality, it is known via pseudospectral analysis that some aeroelastic systems are non-normal, in particular a high order aeroelastic model of the Boeing 767 [11]. Analysis of the Boeing model and the Pendulum and BACT systems with feedback controllers that reduce the sensitivity is presented in [24], along with work on the relationship between pseudospectra, non-normality and transient growth. Work on reliably evaluating $\Lambda_{S\varepsilon}$ from (11), particularly for multivariable $\mathbf{p}$, remains.

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