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COLLEGE OF AERONAUTICS

Ph.D. Thesis

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The Aeroelastic Behaviour and Response to
Turbulence of a Class of High Performance
Tailless Sailplanes.

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IN MEMORY OF MY FATHER

SUMMARY

The symmetric flutter and response characteristics of a class of tailless sailplanes are investigated combining rigid-body dynamics with elastic modes of deformation. The methods used are based on the normal mode approach through the use of finite element techniques and unsteady aerodynamic theories in two dimensional flow. The flutter speed and aeroelastic modes are critically examined and the results are compared with those of a tailed sailplane. The flutter speed of the tailless sailplane is found to be low in comparison with those of orthodox layout and a parametric study is undertaken to locate the reason for this; recommendations are made for improvements. A method is proposed for the analysis of the short period oscillation characteristics of flexible aircraft allowing for the effects of unsteady aerodynamics. It is then applied to the type of tailless sailplane under investigation. The response to gusts and turbulence is evaluated using both statistical and discrete gust methods. Comments are made from the calculated results about the aeroelastic behaviour of tailless sailplanes in general. Computer programs developed for detailed aeroelastic calculations are presented.

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NOTATION

A	Aspect ratio, area
$\bar{A}$	Gust response function
A(t)	Indicial admittance (response to unit step disturbances)
A_{ij}	Aerodynamic force coefficients
B	Aircraft pitching moment of inertia about the centre of gravity
C_L	Coefficient of lift ($= \text{Lift} / \frac{1}{2} \rho V^2 S$)
C(k)	Theodorsen function
D	Dissipation function, differential operator ($D \equiv \frac{d}{dt}$)
E	Young's modulus
F	Column of generalised forces
F(t)	Response to a unit step
F(ρ, θ)	Real part of Theodorsen function
G	Shear modulus of rigidity
G(ρ, θ)	Imaginary part of Theodorsen function
H	Gust length, gradient distance
$\bar{H}$	Tuned gust length at which maximum response occurs
$\bar{H}_+$	Tuned gust length at which maximum positive acceleration occurs
$\bar{H}_-$	Tuned gust length at which maximum negative acceleration occurs

H_s	Distance of separation between two single smooth ramp gusts which form a pair
$H_n^{(2)}$	Hankel function of second kind and order n
$\tilde{H}(y)$	Vertical displacement of flutter mode at spanwise station y
$H(i\omega)$	Frequency response function, transfer function
$H(i\omega)_h$	Frequency response function of vertical displacement
$H(i\omega)_\alpha$	Frequency response function of pitching rotation
I	Moment of inertia, second moment of area
I_a	Second moment of area in A - direction (PAFEC)
I_b	Second moment of area in B - direction (PAFEC)
I_f	Mass moment of inertia of fuselage
I_p	Polar moment of inertia about PAFEC axis, mass moment of inertia per unit length
I_{p+f}	Pitching moment of inertia of pilot/fuselage assembly
$\mathcal{I}$	Imaginary part
$\mathcal{I}(4)$	Imaginary part of flutter determinant
J	Bredt-Batho torsional parameter, discrete gust scaling factor
J_n	Bessel function of first kind and order n
K	Stiffness matrix
K_n	Kernel function of order n
L	Unsteady aerodynamic lift, scale length of atmospheric turbulence
M	Mass matrix, unsteady aerodynamic pitching moment

M_{p+f}	Mass of pilot/fuselage assembly
N_0	Characteristic frequency (number of zero crossings per unit time with positive slope)
Q_i	Generalised force corresponding to externally applied force
$Q_{e.f(i)}$	Generalised exciting force corresponding to i th mode
Q_A	Flutter matrix
Q_F	Aerodynamic matrix
Q_{F_m}	Aerodynamic matrix due to forces arising from motion alone
$\mathcal{R}$	Real part
$\mathcal{R}(\Delta)$	Real part of flutter determinant
S	Stiffness matrix, wing area, semi-span
T	Period of short period oscillation, kinetic energy of motion of a system
U	Airspeed, aircraft velocity
U_0	Gust strength parameter, velocity component of aircraft
V	Aircraft speed, potential energy
V_0	Flight speed in steady state
$V_g(x)$	Velocity distribution of discrete gust
V_{g_m}	Maximum velocity of discrete gust
$W(t)$	Step function
$W(x)$	Velocity distribution of a smooth ramp gust
W_0	Gust strength parameter
$\overline{W}_G$	R.M.S. velocity of gust input
Y_n	Bessel function of second kind and order n

z_R	Rigid body vertical displacement of flexural axis intersection of wing root
a_h	Offset of elastic axis from mid chord
b	Semi-chord, span
$\bar{c}$	Standard mean chord
c_r	Wing root chord
c_t	Wing tip chord
g	Structural damping coefficient, acceleration due to gravity
h	Vertical displacement
i	$\sqrt{-1}$
i_B	Moment of inertia coefficient for pitching axis
k	Reduced frequency parameter ($k = \frac{\omega b}{U}$)
k_B	Radius of gyration of aircraft in pitch
l	Length
l_T	Distance of aerodynamic centre of elevon aft of C.G. of aircraft
l_p	Non dimensional rolling moment derivative due to rolling
l_r	Non dimensional rolling moment derivative due to yawing
l_v	Non dimensional rolling moment derivative due to side-slip
l_ξ	Non dimensional rolling moment derivative due to elevons
m	Mass per unit length of wing
m_f	Mass of fuselage

m_q	Non dimensional pitching moment derivative due to rate of pitch
m_u	Non dimensional pitching moment derivative due to velocity increment along Ox
m_w	Non dimensional pitching moment derivative due velocity increment along Oz
$m_{\dot{w}}$	Non dimensional pitching moment derivative due to rate of change of w
n	Number of degrees of freedom
n_p	Non dimensional yawing moment derivative due to rolling
n_r	Non dimensionalyawing moment derivative due to yawing
n_v	Non dimensional yawing moment derivative due to side-slip
n_ξ	Non dimensional elevon yawing derivative
q	Generalised co-ordinate, angular velocity in pitch ($q = \dot{\theta}$)
$\tilde{q}$	Generalised co-ordinate corresponding to flutter, convergent oscillation and harmonic gust
q_r	r th generalised co-ordinate
r	Number of selected modes (number of generalised co-ordinates), non dimensional short period damping
s	Non dimensional short period frequency
t	Time, wing root maximum thickness
$\hat{t}$	Unit of aerodynamic time
t_a	Time to half amplitude
$\bar{t}_+$	Time when maximum positive response due to smooth ramp gust occurs
$\bar{t}_-$	Time when maximum negative response due to smooth ramp gust occurs

u	x - component of aircraft velocity relative to air
w	z - component of aircraft velocity relative to air
x, y, z	Co-ordinates
$\bar{x}$	Distance of aircraft C.G from flexural axis and fuselage centre line intersection
x_q	Non dimensional symmetric force - rotary derivative
x_u	Non dimensional symmetric force - velocity derivative
x_w	Non dimensional symmetric force - velocity derivative
z_q	Non dimensional symmetric force - rotary derivative
z_u	Non dimensional symmetric force - velocity derivative
z_w	Non dimensional symmetric force - velocity derivative
α	Pitching rotation, angle of sweep
$\alpha_r(y)$	Torsional mode shape of the r th mode
γ	Euler constant = 0.5772....
$\gamma(H)$	Maximum response due to a smooth ramp gust of gradient distance H
$\gamma_+(\bar{H})$	Maximum positive response due to tuned gust length $\bar{H}$
$\gamma_-(\bar{H})$	Maximum negative response due to tuned gust length $\bar{H}$
Δ	Flutter determinant
Δn	Acceleration increment
$\delta(\omega)$	Dirac delta function
$\bar{\delta}(y_i, t)$	Displacement of element at point y_i at time t

ϵ	Spectral width parameter
$\eta_{\max}$	Expected maximum response
$\widetilde{H}(y)$	Pitching rotation of flutter mode at spanwise station y
θ	Angle of pitch, parameter in Theodorsen function for generalised motion
θ_0	Rigid body pitching rotation (nose up)
Δ	$\frac{1}{4}$ chord sweep
λ	Complex argument of Theodorsen function ($\lambda = \mu + i\omega$), root of characteristic equation for short period oscillation ($\mu = -r \pm is$), taper ratio
μ	Damping in the short period mode
μ_1	Aircraft longitudinal density parameter
$\bar{\xi}$	R.M.S. value of output acceleration
ρ	Density of air, parameter in Theodorsen function for generalised motion
σ	Standard deviation of specific random variable
$\bar{\sigma}$	Reference gust intensity
σ_i	R.M.S. value of a designated random input
σ_o	R.M.S. value of a designated random output
τ	Aerodynamic time
$\phi(\omega)$	Power spectral density of a designated input (atmospheric turbulence)
$\phi_D(\omega)$	Power spectral density of Dryden spectrum
$\phi_V(\omega)$	Power spectral density of Von Karman spectrum
ϕ_J	J th normal mode
ϕ_m	Modal matrix
$\Psi(\omega)$	Power spectral density of a designated output (normal or angular acceleration)

ω	Frequency
ω_b, ω_h	Flexural frequency
ω_α	Torsional frequency
ω_j	jth natural frequency
ω_{sp}	Frequency of short period oscillation
$[]$	Square or rectangular matrix
$[\]$	Diagonal matrix
$\{ \}$	Column matrix
A.C	Aerodynamic centre
A.R	Aspect ratio
C.G	Centre of gravity
C.L	Centre line (fuselage)
FRP	Fibre reinforced plastic
L.E/C.L	Leading edge and fuselage centre line intersection
M.A.C	Mean aerodynamic chord
S.M.C	Standard mean chord

The Aeroelastic Behaviour and Response
to Turbulence of a Class of High Performance
Tailless Sailplanes.

Volume 1 : Main text and Appendices

CHAPTER 1

1.0 INTRODUCTION

Despite serious attempts made in the past, the history of tailless sailplane design is rather brief. The concept which is appealing on grounds of improved performance, needs authentication to satisfy aeroelastic considerations. In general, sailplanes whether tailed or not with their light wings of high aspect ratio have low structural stiffnesses and are prone to aeroelastic problems even at low speeds. The usually adopted empirical aeroelastic stiffness criteria developed for powered aircraft do not provide useful results when applied to sailplanes and as such there seems to be no adequate method in practice to analyse their aeroelastic problems giving rise to instabilities and structural failures. Thus in the domain of aeroelasticity there is wide scope for both fundamental study of phenomena and for the development of practical techniques for calculations. The complexity of the problem increases when the design of the sailplane is tailless. Because of the absence of a tail such a flying machine would have very little damping in pitch. This makes the longitudinal dynamics of the aeroplane a formidable problem as it involves the dynamic coupling of the rigid body motion with the elastic modes of distortion of the aeroplane.

The aim of this thesis is to study such an interaction and investigate the aeroelastic behaviour and response to gusts and other turbulence of a class of high performance tailless sailplanes whose preliminary design studies show an over-all performance gain of 10% over conventional types. Designers of tailless sailplanes have expressed concern about the aeroelastic phenomena which could prevent realisation of benefits from tailless design. Since it is acknowledged that the longitudinal dynamics of the flexible aeroplane poses a severe problem, an exhaustive treatment in this respect is warranted.

Although the special case of the tailless sailplane is kept in mind, a major portion of the theoretical work applies to all aircraft whether tailed or untailed. However, new methods have been suggested to deal satisfactorily with the problem of uncertainty of the pitching behaviour of the tailless sailplane. Especially for the case with a moderate angle of sweep the wing bending produces changes in incidence along the span which in the presence of aerodynamic forces will introduce twisting in the wing and hence a pitching moment about the centre of gravity of the aircraft. Consequently, the strong likelihood of an aeroelastic instability such as flutter or an unstable gust response or also an unstable coupling between wing bending vibration and short period oscillation demands a thorough investigation. The object of this thesis is therefore, to undertake such an investigation.

- 3 -

CHAPTER 2

2.0 BACKGROUND STUDIES

Although there exists a gap in the knowledge of the aeroelastic behaviour of tailless sailplanes, there is at the same time a wealth of experience in the general field of aeroelasticity. The uniqueness of the problem made it necessary to refer to relevant research papers and specialised text books. Some detail pertinent to the research is gathered and briefly reviewed in this chapter.

2.1 Design Development and Analysis of Tailless Aircraft

The concept of tailless aircraft design is quite old and can be traced back to the period from the early thirties and almost upto the late forties. A significant portion of the earlier attempts was largely confined to powered all-wing aircraft by some aircraft companies whereas efforts to build high aspect ratio tailless sailplanes for better performance were concentrated within a few individuals with personal interest. In an unpublished report (Ref.1) different types of tailless aircraft of the world have been surveyed with details of their features and photographs.

During the late thirties and early forties the Horten brothers in Germany built a series of tailless sailplanes as described by Wilkinson (Ref.2) but their progress did not continue because of the effect of the second world war. Other homebuilders of tailless sailplanes were Feuvél, Marske and Backstorm and details of their work are not known to be readily available. Only a few investigators studied the general design aspect of tailless aircraft. Weyl (Ref.3) published a series of papers and attempted to explain some technical problems of tailless design emphasizing the overall stability and control of such aeroplanes. Lee (Ref.4) made a case for tailless aircraft and discussed to some length the preliminary considerations and also the difficulties of tailless design. His papers point out that

the aeroelastic effects on the swept back wing could be a serious problem.

In this period, very little analytical work was done on the characteristics of tailless aeroplane. Jones (Ref.5) studied the general problems associated with the stability and control of tailless aircraft involving important factors like the location of the aerodynamic centre and its effects on the longitudinal stability, longitudinal trim with high lift devices and the effects of various changes in the shape of the wing on lateral stability. Zbrozek and Mettam (Ref.6) published results on the stability and aeroelastic distortion of a tailless aircraft based on flight test results. The comprehensive tests originally planned could not be completed by them owing to the loss of the aircraft in an accident. Trouncer, Becker and Wright (Ref.7) performed wing tunnel experiments on the stability of tailless sailplanes and obtained interesting results. Other investigators who studied the stability, control and trim of tailless aircraft are Thorpe (Ref.8), Neumark (Ref.9), Johnston (Ref.10), Lippisch (Ref.11), Hill (Ref.12), Lyon et al (Ref.13), Warren et al (Ref.14), Trouncer (Ref. 15) and Wilkinson (Refs.16-17). All these attempts made in the past, show that the then available methods did not account for the contribution of aeroelastic effects to the overall stability of the aeroplane. Some other research on aeronautical aspects was carried out by Dent and Curtis (Ref.18) when they estimated the effect of flaps on pitching moment and lift on tailless aircraft. Lyon and Shepperd (Ref.19) produced results in an interim report on the design of wing sections for tailless aircraft.

Other contributors who investigated this field reported their work in Refs. 20-36.

More recently, Buchanan (Ref.37) produced a design for a tailless sailplane claiming an overall 10% performance gain over conventional types. Serious concern expressed by him like most other designers of tailless sailplanes suggests that the aeroelastic effect with all its complex features has to be thoroughly investigated before undertaking any attempt

to build such an aeroplane. The aeroelastic investigation in this thesis concentrates on the tailless sailplane whose initial design was produced by Buchanan (Ref.37).

2.2

Stability and Control of Aeroplanes

There are quite a number of standard text books on stability and control of aircraft. Some of them are by Babister (Ref.38), Etkin (Ref.39), Duncan (Ref.40), Kolk (Ref.41), Seckel (Ref.42) and Irving (Ref.43). All these texts give sufficient details about the rigid body dynamics of conventional aeroplanes. In particular, the static and dynamic stability neglecting the effects of compressibility and distortion has been discussed in detail, the fundamentals of which involve stick fixed and stick free manoeuvrability, evaluation of longitudinal and lateral stability derivatives to study the oscillatory motion of the aeroplane and its response characteristics. In all these text books, very little account has been taken of the unsteady aerodynamic effects and distortion of the aircraft structure. In the particular context of this investigation, Morelli (Ref.44) seems to be the only known author to write a book on stability and control of sailplanes. He discussed longitudinal equilibrium, static stability and control and also the lateral and directional static stability and control of sailplanes. Throughout his book, the sailplane structure is considered to be rigid.

In the field of stability and control there has been in the past some confusion between the notational usage of British and American systems. Hopkin (Ref.45) cleared the confusion and proposed a scheme of notation which brought consistency and uniformity to many research workers in this field. Although there exists a sizeable number of research papers and reports in the field of stability and control, they are omitted for brevity and also because they do not cater for the direct need of the present research. The analysis in the rigid body domain is rather less important in the context of this work.

2.3

Classical Aeroelasticity

As in the case of the stability and control of aircraft, there are many text books on classical aeroelasticity of which the leading contributors are Freberg and Kemler (Ref.46), Scanlan and Rosenbaum (Ref.47), Fung (Ref.48), Bisplinghoff, Ashley and Halfman (Ref.49) and Bisplinghoff and Ashley (Ref. 50). These text books are quite comprehensive and present details about classical aeroelastic phenomena such as flutter, divergence and control reversal. However, there are some limitations in these texts about certain structural and aerodynamic aspects of aeroelasticity. In particular the finite element method (which was not so much in use when these texts were written) and the unsteady aerodynamic theory with three dimensional effects have rarely been advocated. However, many of the existing problems of aeroelasticity could be very well tackled using these texts without much loss of generality. Ref.49 which is quite frequently used by many aeroelasticians, gives an exhaustive bibliography at its end. The Manual of Aeroelasticity (Ref.51) provides background material for some of the practical aspects of aeroelasticity. Recently Stacey (Ref.52) and Nahas (Ref.53) have used the finite element method for the structural part of the flutter analysis of the Cranfield Aerobat A1 and a project aeroplane - respectively. The number of research papers in the classical field of aeroelasticity are too many and are not reported here again for brevity. However, the classical papers of Collar (Ref.54) and Duncan (Ref.55) are worth mentioning. Refs. 56-57 review the state of the art for the past decade whereas Refs. 58-60 are quite fundamental and original research work.

2.4

Dynamics of Deformable Aircraft

The flexibility of the aeroplane is considered to be of basic importance to the consideration of the present research. The literature survey in this section therefore, provides the most relevant information pertaining to the subject matter of this thesis.

On most conventional aeroplanes, there is very little coupling between the rigid body motion and the elastic deformation. To deal with aeroplanes of considerable flexibility, it is important to combine rigid-body motion with elastic distortion, which means combining two major disciplines namely, stability and control on one side and flutter and response on the other. In recent years Taylor and Woodcock (Ref.61) made a noteworthy effort to establish an integrated approach and argued that having a common objective the stability and control analyst and the flutter and response analyst should not work in watertight compartments. Taylor's attempt was to satisfy the need by an extension to the deformable aircraft of concepts developed for the rigid aircraft in classical stability and control theory whereas Woodcock, for his part, relied on an extension of ideas from the domain of flutter and gust response. However, what was mainly an academic problem for Taylor and Woodcock proved vital for the requirement of the present research. The amount of published material on deformable aeroplanes with an integrated approach is small. Milne studied respectively the dynamics of deformable bodies (Ref.62), analysis of coupled dynamical systems (Ref.63) and dynamics of deformable aeroplanes (Ref.64) and came to many useful conclusions. The method of modified derivatives or the quasi-static method for predicting the effect of flexibility of the aeroplane structure on the rigid body motion has been used for many years. There are some references dealing with specific cases. Taylor (Ref.65) discussed the method in the context of general motion of a flexible aeroplane. Apart from the report by Duncan (Ref.66) the method seemed to be dealt with on a heuristic basis and has often been applied without adequate understanding of its application. Some of the criticisms came from Hancock and Milne (Ref.67) and Milne (Ref.64). The method when applied, reorganizes the modified rigid aeroplane equations of motion by a process of back substitution and those terms which would lead to an increase in the order of the rigid aeroplane characteristic polynomial are ignored. Such a process will often be straight forward but may run into difficulties with the modified equations as

explained in Ref. 63. Moreover, the method of modified derivatives concentrates on the modified system for the roots of small modulus. This may be acceptable from the point of view of application, the method's neglect of the system as a whole and in particular, of the complex approximate solutions to the initial value and response problem means that the status of the approximation is not clear.

Some of the earlier attempts in predicting the dynamics of deformable aeroplanes are due to Lyon and Ripley (Ref.68), Lyon (Ref.69), Fingado and Taylor (Ref.70), Paraghamian and Lohman (Ref.71), Roskam (Refs.72-73) and Huss and Donegan (Ref.74). More recently, serious attempts have been made by Wells (Ref.75), Kemp (Ref.76) and Swaim and Fullman (Ref.77). Ref.78 by the Boeing Company contains very useful information and is a novel step towards the solution of the stability problem of elastic aeroplanes. Undoubtedly, one of the main reasons for the relative scarcity of such studies is that more complex mathematical techniques are required to solve such problems.

2.5 Response of Aeroplanes to Gusts and Turbulence

The response of aeroplanes to gusts and turbulence has been investigated by a great many research workers. However, there has always been a clear distinction between two approaches to the problem - namely, the Power Spectral Density (PSD) method used by American research workers and the Discrete Gust Approach used by the British. Leading investigators who used the PSD approach are Pr  ss and Mazelsky (Ref. 79), Diederich (Refs.80-81), Liepmann (Refs. 82-83), Etkin (Ref.84) and Payne and Cox (Ref.85). Those using discrete gust approach include Mitchell (Refs 86-87), Jones (Refs. 88-89) and Kaynes (Ref.90). Mitchell (Ref.91) Houbolt and Kordes (Ref.92) and Houbolt (Ref. 93) used both approaches and obtained useful results. More recently, Jones (Ref.94) made a significant contribution using both discrete gust and PSD methods and drew the conclusion that ultimately both methods give similar results. Ref.95 is a recent report which gives revised requirements for the flying

qualities of service aeroplanes and has upgraded many of the requirements stipulated in Av.P.970 (Ref.135) particularly with regard to gusts and atmospheric turbulence. Based on Jones' research (Ref.94) Purcell (Ref.96) recently produced a computer program which evaluates response of an aircraft to a single and a pair of smooth ramp gusts. In almost all cases solutions have been arrived at for conventional aeroplanes where deformability did not play a significant role.

Those who used transfer function representation of aeroplanes to solve response problems as early investigators are Coleman et al (Ref.97), Lin (Ref.98) and Filotas (Ref.99).

Later Morito Ii et al (Ref.100) presented a technique with an analytical evaluation of unsteady aerodynamic forces and dynamic response of a flexible aeroplane to continuous turbulence in supersonic flight by applying the PSD method whereas Schmitz (Ref.101) proposed a normal mode solution to the equations of motion of flexible aeroplanes. Filotas (Ref.102) studied the response of finite wings to sinusoidal gusts on vertical oscillations and obtained expressions containing both the vertical oscillation (Theodorsen) and sinusoidal gust (Sears) cases and explicitly presented the aspect ratio dependence. In a similar attempt Kobayakawa et al (Ref.103) investigated gust response of an aeroplane both theoretically and experimentally assuming that the gust varied sinusoidally and also taking into consideration the aerodynamic interference between the wing panel and the tail panel. Other informative research papers are by Mitchell (Ref.104), Dempster and Roger (Ref.105), Bisplinghoff et al (Ref. 106), Houbolt (Ref. 107), Pratt (Ref. 108), and Traill-Nash and Chandivert (Ref.109). The number of zero crossings by the response quantity per unit time with a positive slope and the probability of exceeding a certain value have been dealt by Davis and Swaim (Ref.110) and Pritchard (Ref.111). Cartwright and Longuet-Higgins (Ref.112) studied the statistical distribution of the maxima of a random function and applied the results to sea waves and pitching and rolling motion of a ship.

2.6 Unsteady Aerodynamics and Arbitrary Motion of Aerofoils

Although the standard texts in aeroelasticity mentioned in section 2.3 deal with some aspect of non-stationary aerofoil theory, they do not completely cover the case when the aerofoil motion is arbitrary. Earlier research relating to aerofoil theory for non-uniform motion has been adequately dealt with by Von-Karman and Sears (Ref.113) and Sears (Ref.114). In a later period Jones (Ref.115) determined the aerodynamic forces acting on wings of finite span in non-uniform motion in an incompressible, inviscid fluid. Following Jones' work Luke and Dengler (Ref.116) produced tables for the Theodorsen function for generalised motion of an aerofoil. Based on Luke and Dengler's work tables for the Theodorsen function are drawn in this thesis for ranges which have not been covered by them. In recent years, Davies (Ref.117) presented a method on the basis of further development of the lifting surface of Multhopp type to calculate the linearised values of generalised aerodynamic force coefficients on a harmonically oscillating flat plate wing. He programmed his refined procedure in ICL 1900 FORTRAN (Ref.118).

Zwaan (Ref.119) gives an excellent review of unsteady aerodynamics and its integration in aeroelastic analysis.

2.7 Finite Element Method

The finite element method is applied in this thesis to obtain natural frequencies and mode shapes of the aeroplane. The package developed in Nottingham University, U.K., known as PAFEC (Program for Automatic Finite Element Calculations) is used. Details of PAFEC can be found in Ref.120-121. The aspects of finite element structural idealisation can be found in Refs. 122-124.

2.8 Other References

Quite a few other publications have been consulted of which the most relevant ones are

discussed here . Refs. 125-126 supplied valuable information towards deciding the scale length of the gust while performing the response analysis of the aeroplane. Refs. 127-128 provided useful material for idealising the aircraft structure and also for deducing such structural parameters as torsional stiffness, location of shear centre etc.

A truncated modal analysis has been proposed in this thesis arguing on a semi-intuitive basis that a sufficiently accurate solution can be obtained by using a system of equations in which the effects of modes with frequencies outside the range of interest are ignored. Literature concerning the residual flexibility approach making a partial allowance for the effects of the neglected higher modes in truncated modal analysis can be found in Refs. 129-130. Furthermore Refs. 131-133 suggest useful mathematical tools whereas Ref. 134 covers a wide range of ideas for realistic calculation of aircraft loadings. Refs. 135-137 serve the purpose of providing guidelines of airworthiness requirement for service aeroplanes.

Interestingly enough, the uncertainties of one of the tailless sailplanes were reported in a private communication (Ref.138) between the author and a pilot who is also a sailplane researcher. While flying the tailless sailplane - BK81, the pilot experienced violent flapping motion of the fuselage indicating an overall instability. He vividly recalled that an upward movement of the fuselage was accompanied by a downward movement of the wing tip. The speed of the flight was reported to be in the higher ranges.

CHAPTER 3

3.0 SCOPE OF THE INVESTIGATION

3.1 Description of the Aircraft and Identification of the Problem

The general layout of the tailless sailplane named as the Ricochet whose preliminary design studies have been made by Buchanan (Ref.37) is shown in Fig.1. The task herein is to investigate the symmetric flutter and response characteristics of the aeroplane. The elevons may play an important role in the symmetric flutter and response characteristics but have in this work been assumed to be rigidly fixed to the wing with their angular settings at zero because of the non-availability of stiffness data for the control circuit. The complexity of the problem demanded a rigorous analysis involving the coupling of the rigid body freedoms with those of the elastic deformations. The longitudinal dynamics of the flexible aeroplane being considered to be critically important, is given a detailed treatment. In particular, the specific areas which have been investigated are:- (1) the evaluation of flutter speed and aeroelastic modes at flutter speed, (2) a parametric study on possible improvements of flutter characteristics, (3) the pitching behaviour involving the short period oscillation, (4) the response to random atmospheric turbulence and also to discrete gusts.

The results of the flutter calculations for the Ricochet* are compared with those of a standard sailplane, the Kestrel-22m⁺. Except in the case of the results of the flutter calculations for which the Kestrel is used as a reference to relate the behaviour of the aeroplane the remaining investigations are centred only on the Ricochet as an attempt to predict its aeroelastic character. Fig.2 shows the wing layout of the Ricochet whereas Figs. 3 and 4 show the inner and outer wing spars respectively. Fig.5 shows the root rib which illustrates the aerofoil section. Other relevant details of the Ricochet can be found in Appendix A and Ref.37.

* The name Ricochet is consistently used for the tailless sailplane under investigation.

⁺ In what follows the Kestrel-22m is consistently referred to as the Kestrel although there are other versions of the sailplane

3.2 Assumptions

In view of the magnitude of the problem, it is customary to make simplifying assumptions, the nature of which may vary according to the objective of the analysis. Basically the problem is identified as a mathematical model where the aeroplane is idealised structurally and aerodynamically. Apart from the linearity assumption which has been maintained throughout the following points need special mention.

3.2.1 Structural Idealisation

Sailplanes whether tailless or tailed mostly have large slender wings which can be idealised as beam elements in the finite element idealisation without any loss of generality. In the case of the Ricochet, the fuselage is compact so that the pilot/fuselage assembly is idealised as a single lumped mass. The finite element idealisation of the Ricochet is shown in Fig. 6. In the case of the Kestrel, the rear fuselage, fin and rudder assembly and the tailplane are all idealised as beam elements whereas the front fuselage and the pilot are idealised as lumped mass elements. Fig. 37 shows the idealised Kestrel. The relevant details needed for structural idealisation of the Ricochet and Kestrel are reported in Appendices A and B respectively. Only the bending and torsional displacements associated with the structure are considered and no allowance is made for the spanwise direct flexibility of the aeroplane. Since the finite element package PAFEC (Programme for Automatic Finite Element Calculations) is used, some of the assumptions with regard to the selection of element type are inherent in it. However, the procedure followed in solving the eigen value problem to obtain natural frequencies and mode shapes is described in Appendix I.

3.2.2 Aerodynamic Idealisation

The unsteady aerodynamic forces acting on the aircraft may be obtained theoretically by

methods such as strip theory and vortex-lattice theory. Because of the large aspect ratio of sailplane wings, strip theory based on Theodorsen's flat plate unsteady aerodynamic theory is assumed to be adequate for the problem. However, alternatively provision has been made to use a program developed by Davies (Ref. 118) 118 which accounts for the three dimensional effects of the airflow. It is assumed that the lift is developed entirely by the wing and the contribution from fuselage and other units is neglected. The thickness effect of the aerofoil is not considered as is obvious from the flat plate theory.

3.2.3 Further Assumptions in Flutter, Short Period Oscillation and Response Analysis

In all stages of the analysis, only the symmetric motion is considered. Thus in evaluating flutter speed, aeroelastic modes and other response quantities, only the symmetric mode shapes are used which by implication mean that the ailerons (or elevons) are not moved. The two rigid body modes in the pitching plane, namely heave and pitch, are incorporated as well as the selected elastic modes in symmetric motion.

In investigating the short period oscillation characteristics, the unsteady aerodynamic effects have been evaluated for the elastic aeroplane and to study the same behaviour of the rigid aircraft, the standard method employing the stability derivatives which ignores the unsteady effects, is used for the sake of comparison. No attempt has been made to study the short period oscillation characteristics using the 'Quasi-static method' which makes partial allowance for the flexibility (see Ref. 65).

In the case of the response analysis turbulence is considered to be isotropic when the Power Spectral Density (PSD) method is used. No spanwise variation of the atmospheric turbulence is taken into account. The exciting turbulence is represented by the Dryden and the Vonkarman spectrum in the PSD analysis of the output response.

The assumptions made in finding the response

to discrete gusts are mainly two fold. The first is that the gust velocity which has only vertical component is uniformly spread along the span of the aircraft and secondly the superposition theory is considered to be valid which again implies the linearity assumption so that the response to a discrete gust of arbitrary profile can be calculated from the response to a unit step with the use of Duhamel's integral.

Unlike powered aircraft, sailplanes are slow flying machines and the horizontal component of gusts and turbulence will not be small compared with the aircraft velocity and thus may have significant effects on the response characteristics. However, such an investigation is beyond the scope of this thesis and only the vertical component is considered. Furthermore, any variation of the intensity of gusts and turbulence and their time dependency at various spanwise stations while passing over the swept wing is neglected at all stages of the response calculations.

All calculations made are within the limits of the above assumptions.

3.3

Summary of the Work

The work done in this thesis can be summarized as follows:-

- (i) Formulation, evaluation and interpretation of results from the analysis of flutter speed and aeroelastic modes at flutter speed of the tailless sailplane - the Ricochet. The comparison of these results with the tailed sailplane - the Kestrel-22m.
- (ii) A parametric study on the effect of varying significant design parameters of the Ricochet in order to locate the cause of flutter and suggest ways for possible improvement.
- (iii) Development of a method and its computer use to predict the short period oscillation characteristics of the Ricochet accounting for its flexibility and also the unsteady aerodynamic effects.

- (iv) Further development of the transfer function approach to evaluate the acceleration response of deformable aeroplanes to continuous random atmospheric turbulence and the calculation of results for the Ricochet at various spanwise stations.
- (v) Determination of the acceleration response of the flexible Ricochet to an isolated smooth ramp gust and evaluation of the worst response cases. Investigation of the worst response cases due to a pair of smooth ramp gusts.

3.4 Development of Computer Programs

Throughout the course of this research most of the time has been devoted to the writing, testing and developing of computer programs, the background details of which are given in Appendix L. However, the following programs were supplied by external agencies:

- (i) PAFEC - developed by Nottingham University U.K.

This is a finite element package (Program for Automatic Finite Element Calculations) to solve static and dynamic structural problems. The version of PAFEC used in the calculations reported here is PAFEC 70+. A later version of the package, namely PAFEC 75 is now replacing PAFEC 70+ but the eigen value routines are unaltered.

- (ii) Program developed by D. E. Davies, Structures Department, Royal Aircraft Establishment, Farnborough, U.K.

This program calculates the linearised values of generalised airforce coefficients on a harmonically oscillating flat plate wing in subsonic flow by further developing the lifting surface theory of Multhopp type.

- (iii) Program developed by A. G. Purcell, Mathematics and Computation Department Royal Aircraft Establishment, Farnborough U.K.

This program evaluates the critical cases of the response of an aircraft to a single and pair of smooth ramp gusts (one-minus-cosine) when the response to a unit step gust is known.

- (iv) Program initially developed by C.G.B. Mitchell, Royal Aircraft Establishment, Farnborough, U.K. and later modified by and available in the College of Aeronautics, Cranfield Institute of Technology, U.K.

This program evaluates both longitudinal and lateral stability of a rigid aircraft by inputting its geometric and inertial details.

3.5 Units and System of Axes

The S.I. unit has been consistently used in this thesis. The system of axes used in the right handed set of cartesian co-ordinates the same in general adopted in the stability and control analysis. The x-axis coincides with the fuselage axis and is positive forward, the y-axis is in the spanwise direction and is positive starboard and the z-axis is positive downwards.

CHAPTER 4

4.0

ANALYSIS OF FLUTTER SPEED AND AEROELASTIC MODES AT FLUTTER SPEED OF THE RICOCHET AND COMPARISON OF RESULTS WITH THOSE FOR THE KESTREL

The flutter speed and aeroelastic modes of the Ricochet are investigated using normal modes to define the generalised co-ordinates. The finite element package PAFEC (Ref.120) is used to obtain the natural frequencies and modes of vibration. The structure is idealised as a set of beam and lumped mass elements. As has been previously mentioned, the longitudinal dynamics of the Ricochet is the main subject of the research and as a result the flutter analysis has been confined to the case of motion symmetric about the plane of symmetry of the aircraft. Strip theory based on the Theodorsen expressions for lift and moment is used to give the unsteady aerodynamic forces in the flutter analysis. However, provision has been made to use a program developed by Davies (Ref.119) which obtains the aerodynamic forces in a more sophisticated procedure using Multhopp's lifting surface theory in unsteady motion (Ref.118). Strip theory is justified because of the high aspect ratio of the sailplane wing and the program developed by Davies has not in fact so far been used for flutter calculations mainly because of its excessive requirement of computer time. The program, however, was used in an early stage to obtain aerodynamic forces at one value of the frequency parameter and it did not give significantly different results from the simpler theory.* In order to provide a comparison of the results of flutter calculation, similar analysis was performed on the conventional sailplane Kestrel. The details of the analytical method used to formulate flutter determinant are given in Appendix D. However, the outline of the complete procedure is briefly summarized as follows:-

- (i) Structural idealisation of the sailplane as an assembly of beam and lumped mass elements and evaluation of the finite element properties as required for input to the PAFEC analysis.

* For a value of the reduced frequency parameter (k) to be 0.1, the difference between the results obtained using Davies' program and the one using strip theory was less than 2 percent.

- (ii) Use of PAFEC routines for dynamic solution to determine natural frequencies and normal modes of vibration of the sailplane structure.
- (iii) Selection of the normal modes to be used in the flutter analysis and reduction of the generalised mass and stiffness matrices in diagonal form.
- (iv) Calculation of the aerodynamic matrix in terms of the generalised co-ordinates from unsteady aerodynamic forces.
- (v) Formulation of flutter determinant and its solution.
- (vi) Evaluation of flutter modes (Aeroelastic modes at flutter speed).

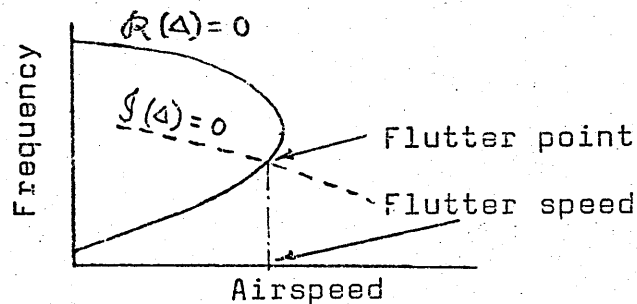
4.1

Method of Solution of Flutter Matrix

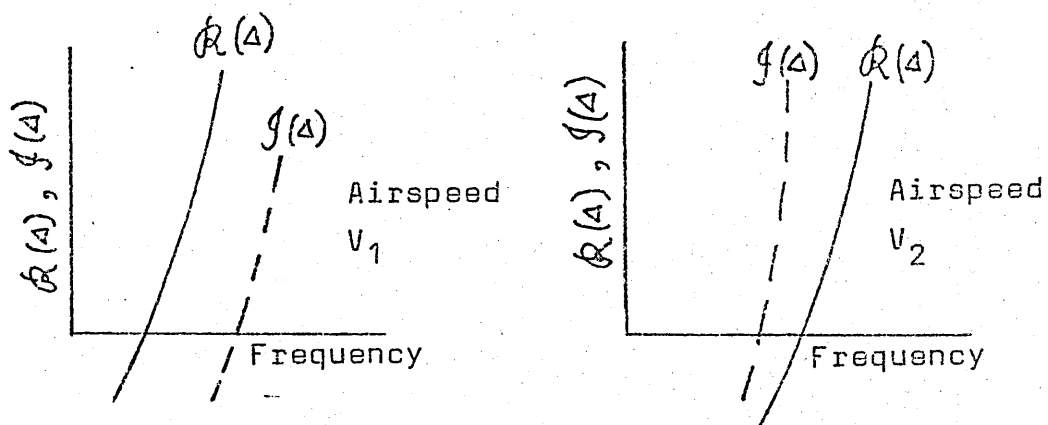
The solution of the flutter stability determinantal equation (see equations D.6.1. and D.6.1.8; Appendix D) is a complex eigenvalue problem because the determinant is a complex function of two variables, the airspeed and the frequency. The equations which make up the aerodynamic terms are such that no explicit solution for the vanishing of the determinant can be derived. Methods of solution include those due to Theodorsen (Ref.58), Smilg and Wasserman (Ref.47 Chapter 13) and Duncan, Collar and Lyon (Ref.55). The method used here to solve the flutter determinant is essentially Theodorsen's method with some minor modification. The Theodorsen method (see Ref. 48 Section 6.11) transforms the flutter determinant to the $\sqrt{X} \left(= \frac{\omega_x}{\omega} \right), \frac{1}{k} \left(= \frac{U}{\omega_b} \right)$

plane whereas in this thesis an air-speed is selected and then the flutter determinant is evaluated for a range of frequencies. The zeros of the real part of the determinant are determined for single values of air-speed by noting the frequencies at which the sign of $\mathcal{R}(\Delta)$ changes with variation of frequency. Linear interpolation is used to determine the zero from the two values of the $\mathcal{R}(\Delta)$ on either side of the root. The zeros of the imaginary part

of the determinant $f(\Delta)$ are found similarly. By repeating this process for a range of air-speeds real and imaginary root loci are drawn and the intersection point is determined. The adjoining sketch shows a schematic view representing the nature of solution of the flutter determinant in a practical situation.



Moreover, if the real and imaginary part of the determinant are plotted against frequency for different air speeds then just before and after the flutter speed, the plots will be as shown below:



In the above sketches, the cross over of the curves for the real and imaginary parts indicates the existence of a flutter speed between V_1 and V_2 . The method described in this section is followed in the Computer program to locate the flutter speed and frequency.

4.2. Aeroelastic Modes at Flutter Speed

In the previous section a method has been outlined

to determine the speed by identifying the frequency and airspeed at which the flutter determinant vanishes completely. Once the flutter speed and frequency are found out it is of significant interest to study the flutter modes.

As in any eigen value problem the eigenvectors are found by deleting one row of say, the r th order matrix QA (see equations D.6.1. and D.6.1.8 Appendix D) and solving for $(r-1)$ of the co-ordinates in terms of the r th. In the Computer program developed, the latter variable has been taken to be that corresponding to the elastic mode of highest frequency used in the flutter calculation. Matrix QA being complex, this leads to the other variables (co-ordinates) being complex numbers and so defining the relative contributions of the modes to the flutter motion both in magnitude and phase. The results of this calculation leads to the modal column of generalised co-ordinates $\tilde{q}$. The flutter modes in terms of the vertical displacement and pitching rotation can be found by multiplying the modal matrix by $\tilde{q}$. At a spanwise station y the aeroelastic modes in bending (vertical displacement) and torsion (pitching rotation) can be expressed as

$$\begin{Bmatrix} \tilde{H}(y) \\ \tilde{\Theta}(y) \end{Bmatrix} = \begin{bmatrix} h_1(y) & h_2(y) & \dots & h_r(y) \\ \alpha_1(y) & \alpha_2(y) & \dots & \alpha_r(y) \end{bmatrix} \begin{Bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \\ \dots \\ \tilde{q}_r \end{Bmatrix}$$

.....4.2.1

where $h_1(y)$, $h_2(y)$ --- $h_r(y)$ correspond to the bending mode shapes of the r -modes at spanwise station y and $\alpha_1(y)$, $\alpha_2(y)$ --- $\alpha_r(y)$ are the corresponding torsional mode shapes. $\tilde{H}(y)$ and $\tilde{\Theta}(y)$ are complex quantities and their magnitude and phase give the relative measure of the vertical displacement (bending) and pitching rotation (torsion) vector at the spanwise station y . It follows that the column matrix $\{\tilde{q}\}$ with elements $\tilde{q}_1, \tilde{q}_2$ --- $\tilde{q}_r$ gives the contribution of the normal modes of vibration to the flutter motion:

$$\tilde{H}(y) = \sum_{i=1}^r h_i \tilde{q}_i$$

$$= h_1 \tilde{q}_1 + h_2 \tilde{q}_2 + \dots + h_r \tilde{q}_r$$

.....(4.2.2)

and

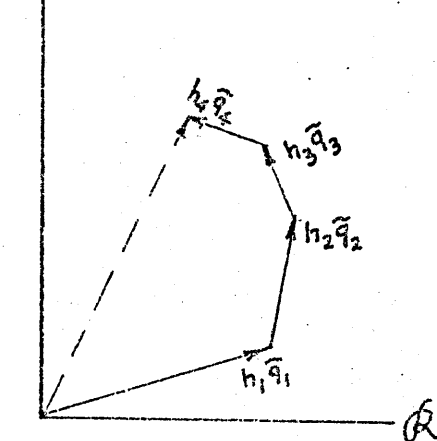
$$\tilde{H}(y) = \sum_{i=1}^n \alpha_i \tilde{q}_i$$

$$= \alpha_1 \tilde{q}_1 + \alpha_2 \tilde{q}_2 + \dots + \alpha_r \tilde{q}_r$$

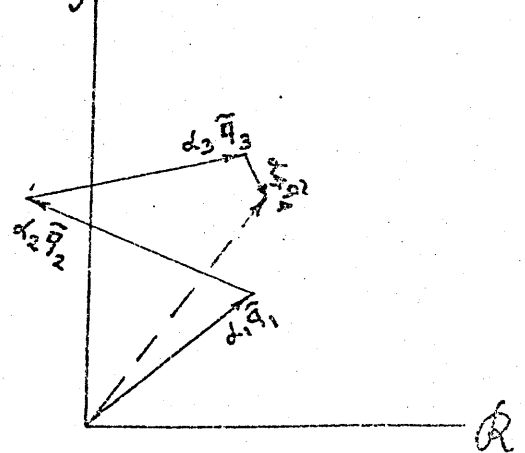
.....(4.2.3)

In the description of the results later the contributions of the various modes to the flutter motion at different stations is shown by vector diagrams of the type of those given below for the case $r = 4$. In some of the practical figures the contribution of several modes is so small as to be impossible to show.

Vertical displacement vector



Pitching rotation vector



—————→ Component vector

- - - - -→ Resultant vector

The real and imaginary parts of $\tilde{H}(y)$ and $\tilde{H}(y)$ are also plotted along the span. For many cases the variation of their absolute values and phase angles along the span gives meaningful description of the flutter modes and these plots are also given.

4.3

Results of Calculations (Ricochet)

The layout of the Ricochet and some general description of the aircraft have already been referred to in Chapter 3. The structure of that part of the Ricochet on one side of the plane of symmetry as idealised into beam and lumped mass elements is shown in Fig.6. The two rigid body and four elastic modes obtained from the PAFEC analysis for the case of unballasted and ballasted wings are respectively shown in Figs 7-11 and Figs. 12-16. These normal modes are used in the flutter analysis of the two independent cases of the unballasted and ballasted wings of the Ricochet and a close inspection of the mode shapes is of value. Figs. 7 and 12 show the rigid bodymodes for the two cases produced by the eigenvalue algorithm used by PAFEC. For convenience in later description of the results these have been replaced by heave and pitch. The aeroplane has not been restrained and these rigid body modes have been included in the flutter analysis. The first elastic mode of the Ricochet ($\omega_1 = 18.28$ rad/sec) with unballasted wings is shown in Fig.8*. It will be noted that this mode is primarily bending of the swept wing accompanied by a nearly constant pitching rotation serving to bring the pitching angular momentum to zero. Fig. 9 shows the second elastic mode ($\omega_2 = 68.6$ Rad/sec) which again is dominated by the vertical displacement with a small mean value of pitching rotation but with proportionately greater twist. Fig.10 shows the third elastic mode ($\omega_3 = 158.96$ rad/sec) which is fairly coupled between the vertical displacement and pitching rotation.

*The Ricochet has a sweep of 13° in its leading edge and in all its mode shapes the terms vertical displacements and pitching rotations are justifiably more appropriate than the terms bending and torsional motions.

The fourth elastic mode ($\omega_4 = 285.726$ rad/sec) shown in Fig. 11 is mainly a torsional mode indicated by a large amount of pitching rotation and with practically no amount of vertical displacement. The mode shapes for the ballasted case are shown in Figs. 12-16. (The Ricochet wings have provision for water ballast of 50 Kg in each and the ballast tanks run from 0.94 m to 4.38 m from the plane of symmetry). The first two elastic modes shown by Figs. 13 and 14 ($\omega_1 = 17.9$ rad/sec and $\omega_2 = 53.4$ rad/sec) of the ballasted Ricochet do not seem to be very much different from those of the unballasted case (see Figs. 8 and 9) although there is a greater deviation in the frequency values of the second elastic mode. The third elastic mode shown in Fig. 15 ($\omega_3 = 133.9$ rad/sec) closely resembles the mode shape of the corresponding unballasted case ($\omega_3 = 168.96$ rad/sec) in Fig. 10 although the frequencies are quite well separated. The effect of the water ballast has, however, caused a marked difference in the shape of the fourth elastic mode ($\omega_4 = 238.09$ rad/sec) shown in Fig. 16. The unballasted fourth elastic mode (see Fig. 11) is largely torsional whereas in the ballasted case the bending inertia of the water ballast seems to have brought down the frequency of a higher order largely bending mode.

As mentioned earlier the solution of the flutter determinant formed by equations D.6.1 and D.6.1.8 of Appendix D is a double eigen value problem involving the airspeed and frequency in the flutter analysis. A typical example showing the results followed from the solution of flutter determinant to locate the flutter point of the Ricochet with unballasted wings is shown in Fig. 17 for the case when the structural damping is zero. Fig. 17 demonstrates the root locus plots of the real and imaginary parts of the flutter determinant against the airspeed. Their intersection gives the flutter point. The first set of calculations is made for the unballasted case with no structural damping. The flutter speed is found to be 38.8 m/sec (see Fig. 17).

Once the flutter point is determined the aero-elastic modes at flutter speed are evaluated

- 10 -

based on the method described in section 4.2. In Fig. 18 the vertical displacement and pitching rotation vectors along three points on the wing of the unballasted Ricochet with no structural damping are shown in an Argand Diagram. The three spanwise stations chosen are the tip, midspan and root of the wing. The vertical displacement vectors in Fig. 18 leading the pitching rotation vectors assume larger values as the wing root is approached from the wing tip whereas the pitching rotation vectors lagging the vertical displacement vectors are very nearly of constant magnitude along the span. The vectors represented by Fig. 18 are the resultant vectors of the vertical displacements and pitching rotations at the three spanwise stations obtained by the use of equations (4.2.2) and (4.2.3). Figs 19-21 show the contributions of various normal modes which constitute the flutter mode for each of the aforementioned points on the wing. The adjoining tables are given with these figures for clarity and also to show the small contributions from the higher modes which are impossible to draw. Fig. 22 illustrates the plots of the real and imaginary parts of the vertical displacement and pitching rotation vectors respectively along the half span of the wing whereas Fig. 23 shows the plot of their absolute values and phase angles respectively in its top and bottom halves.

The same analysis is carried out for the unballasted case with a non-zero value of structural damping coefficient (g). With a structural damping coefficient (g) of 0.05, the analysis shows the expectedly small improvement in the flutter speed from 38.8 m/sec to 39.35 m/sec. Figs. 24-29 illustrate the aeroelastic modes for this case with structural damping coefficient 0.05. Using the same procedure, the ballasted wings case of the Ricochet is investigated and the analysis shows the flutter speed with zero structural damping to be at 38 m/sec implying that the water ballast does not affect the flutter speed to any appreciable extent. Figs. 30-35 explain the aeroelastic modes for this ballasted case.

The results of the flutter analysis for the

Ricochet are summarized in Table 1. In all cases of flutter analysis mentioned in this section and elsewhere the density of the air is taken to be 1.225 Kg/m^3 (sea level value) and the speeds obtained are all equivalent airspeeds.

4.4 Results of Calculation (Kestrel)

The general arrangement of the Kestrel sailplane is shown in Fig. 36. The wings, rear fuselage, tailplane and the fin and rudder assembly are idealised as beam elements whereas the front fuselage and the pilot are idealised as lumped mass elements. The idealised structure of one side of the plane of symmetry is shown in a schematic view in Fig. 37. The Kestrel is a glass fibre sailplane and its other distinguished feature apart from geometric dimensions when compared with the Ricochet is the fact that the wings are straight. There is provision for water ballast in the Kestrel* but only the unballasted case is investigated for the comparison. The aircraft is given an increased number of six elastic degrees of freedom to include the fundamental torsion mode, together with the two rigid body modes in the symmetric motion. These modes are used as input data for flutter calculations and an inspection of them is worthwhile. The two rigid body modes of the Kestrel shown in Fig. 38 are, unlike the Ricochet case, pure heave and pure pitch because of the straight wings. The first elastic mode ($\omega_1 = 11.8 \text{ rad/sec}$) of the Kestrel is shown in Fig. 39 which by observation is the first bending mode of the aeroplane without having any substantial amount of torsion at all. Fig 40 again is an uncoupled one demonstrating the second bending mode ($\omega_2 = 44.4 \text{ rad/sec}$) of the Kestrel. Figs. 41 and 42 respectively show the third ($\omega_3 = 61.95 \text{ rad/sec}$) and fourth ($\omega_4 = 100.1 \text{ rad/sec}$) elastic modes which are both coupled

*The manufacturer because of its company policy did not disclose many of the technical details of the Kestrel. The description of the ballast tank is one of those.

with reasonable amounts of bending and torsion. The fifth elastic mode shown in Fig. 43 ($\omega_5 = 112.1$ rad/sec) seems to be the fundamental torsion which is justifiably included in the flutter analysis. Fig. 44 shows the sixth elastic mode ($\omega_6 = 115.8$ rad/sec) of the Kestrel which is again a coupled mode with frequency very nearly equal to the previous one and is included in the analysis.

The first set of calculations which has been made on the unballasted Kestrel is with no structural damping. The flutter speed is found to be 70.6 m/sec. The aeroelastic modes are determined in a similar way like in the case of the Ricochet. Fig. 45 shows the vertical displacement and pitching rotation vectors* along four spanwise stations namely, the tip, midspan, 1.1 m from the wing root and the wing root respectively at the flutter speed of the Kestrel when the structural damping coefficient is set to zero. As it is seen in Fig. 45 the vertical displacement and pitching rotation vectors get reduced as the root is approached from the wing tip. How the resultant vectors shown in Fig. 45 for the four spanwise stations are formed by individual contribution of normal modes through their respective generalised co-ordinates is explained in Figs. 46-49 respectively for each of the spanwise stations mentioned.

The adjoining tables with Figs 46-49 facilitate the reading of the vector diagrams along the real and imaginary axes. Fig. 50 gives the real and imaginary parts of the vertical displacements and pitching rotations respectively along the semi-span of the Kestrel whereas Fig. 51 represents their absolute values and phase angles respectively. Similar flutter calculations for the Kestrel are carried out for a structural damping coefficient of 0.08 and as expected the flutter speed rose to 73.7 m/sec from its earlier value of 70.6 m/sec at zero structural damping.

*Although the terms bending and torsional vectors can be quite appropriately used for the Kestrel, the terms vertical displacement and pitching rotation vectors are sometimes retained in this thesis for uniform consistency.

The aeroelastic modes for this non-zero structural damping case are illustrated through Figs. 52-58. The results of the flutter calculations for the Kestrel are summarised in Table 2.

4.5 Discussion of Results

4.5.1 Discussion of Results (Ricochet)

The flutter speed of the Ricochet with its wings unballasted and also in the absence of any structural damping is 38.8 m/sec (see Fig. 17 and Table 1). This is very low and far below the estimate of 150 m/sec by Buchanan (Ref.37). The range of flying speeds for sailplanes is roughly between 30-70 m/sec in normal flight and therefore, this flutter speed is unacceptable. A structural damping of 2.5% ($\eta = 0.05$) increases the flutter speed slightly to 39.35 m/sec (see Table 1). The wing ballast does not seem to have any significant effect in changing the flutter speed to any appreciable extent. The ballasted case of the Ricochet with no structural damping results to a flutter speed of 38.0 m/sec.

The aeroelastic modes of the Ricochet at flutter speed for various cases have been shown in Figs. 18-35 and generally in all cases it is found that the flutter modes involving the vertical displacements and pitching rotations are mainly influenced by the rigid body motions. A further inspection of flutter modes in Figs. 18-23, Figs. 24-29 and Figs. 30-35 clearly reveal that the wing root of the Ricochet undergoes larger deflection than the wing tip. A study of the results given in the adjoining tables with Figs. 19-21, Figs. 25-27 and Figs. 31-33 (representing contribution of various normal modes to flutter modes) makes it clear why the tip deflection is relatively smaller than that of the wing root. Referring to the tables shown along with the Figs. 19, 25 and 31, it is indicated that at the wing tip, the elastic deformation as vertical displacement is counter balanced by the rigid body vertical motion of opposite sign.

However, for a much clearer understanding the aeroelastic modes need further investigations. Considering the real part of the vertical displacement and pitching rotation along the semi-span of the Ricochet for the case shown in Fig. 22 (as a typical example from one of the several cases investigated) and analysing further how the flutter modes are comprised, the explanation becomes evident in Fig. 59. The top half of Fig. 59 shows that the chief contributors to the real part of the resulting vertical displacement shown in Fig. 22 are the two rigid body and first elastic modes. The algebraic sum of the displacements arising from these individual modes (① + ② + ③ in Fig 59) leads to the resulting real part of the vertical displacement in Fig. 22. Similarly, the bottom half of Fig. 59 explains how the real part of the resulting pitching rotation along the span in Fig. 22 is made up of individual normal modes and their algebraic sum. The large values indicated in the scales of the aeroelastic modes in all cases of the Ricochet are due to the fact that the generalised co-ordinates are evaluated in terms of the co-ordinate of mode 6 (4th elastic) which happens to make a small contribution to the flutter of the Ricochet. The bottom halves of Figs. 23, 29 and 35 reveal the well known fact that in a flutter situation the torsional motion (pitching rotation) lags the bending motion (vertical displacement).

4.5.2 Discussion of Results (Kestrel)

It is reported by the manufacturer that the Kestrel has suffered a wing flutter problem. One pilot report states that the Kestrel wings flutter at a speed of about 140 Kts (i.e. 72 m/sec) and this is corroborated by the results of the investigation cited in Table 2. The two flutter speeds obtained for the Kestrel with structural damping coefficient $g = 0.0$ and $g = 0.08$ are respectively 70.6 m/sec and 73.7 m/sec which are not well above the safety margin. The aeroelastic modes along the semi-span showing the vertical displacement and pitching rotation vectors are illustrated in Figs. 45-51 and Figs. 52-53 for the two respective cases with zero and

nonzero structural damping. The flutter modes of the Kestrel suggest that unlike the Ricochet the wing root of the Kestrel does not undergo any appreciably large displacement (neither the vertical displacement nor the pitching rotation) as compared to its wing tip. Figs. 46-49 and Figs. 53-56 together with the adjoining tables illustrate the contribution of various normal modes to the flutter mode and it is well indicated that the rigid body modes are not significant. An inspection of Figs. 50 and 51 for the case with no structural damping and Figs. 57 and 58 for the case with structural damping ($g = 0.08$) suggests that the relative magnitude of the torsional displacement as compared with the bending is appreciably high in the resulting motion. The phase angle distributions along the semi-span for the two cases ($g = 0.0$ and $g = 0.08$) shown by the bottom half of Fig. 51 and 58 respectively illustrate the expected lag of the torsional motion behind the corresponding bending. However, the change in phase angle distribution near the wing root (see Figs. 51 and 58) is certainly not so obvious. The only possible explanation is probably the influence of the fuselage inertia near the root. As in the case of the Ricochet, a similar investigation is carried out to split one of the flutter modes and see what are the predominant normal modes composing it. The real part of the vertical displacement and pitching rotation for the zero structural damping case shown in Fig. 50 has been examined. Figs. 60 and 61 respectively show how the real parts of the flutter mode in Fig. 50 have resulted. The algebraic sum of ① to ⑧ in Fig. 60 corresponds to the real part of the vertical displacement shown in the top half of Fig. 50 whereas the algebraic sum of ① to ⑧ in Fig. 61 corresponds to the real part of the pitching rotation shown in the bottom half of Fig. 50. It is clear from Figs. 60 and 61 that the chief contributors to the flutter mode are the first and fifth elastic mode ③ and ⑦ respectively which are the fundamental bending and torsional modes of the Kestrel. Since the investigation demonstrates that the flutter of Kestrel wings is due to the coupling between the fundamental bending and torsional mode an attempt has been made to more accurately establish these two modes through a ground resonance test (see Appendix H) of the Kestrel. The results of

the ground resonance test as shown in Table 45 are in good agreement with the PAFEC output.

4.6

Comparison of Results and Conclusions

It is of interest to compare the results obtained for the Ricochet with those of the Kestrel. First of all the flutter speed of the Ricochet at 38.8 m/sec (unballasted wings, no structural damping) is far below that expected for a standard sailplane. The Kestrel analysis has indicated a flutter speed of 70.6 m/sec (unballasted wings, no structural damping) which agrees very well with flight test results. The type of flutter which is shown by the analysis has different characteristics in the case of each sailplane. In the case of the Ricochet, as is seen in previous sections, the flutter occurs mainly as a result of the coupling between the rigid body motions and the first elastic mode of distortion. The instability is dictated by the overall motion of the aeroplane and the fuselage movement is considerably larger than that of the wing tip. On the other hand the flutter behaviour of the Kestrel is quite normal and does not cause any appreciable movement of the fuselage. The rigid body modes have little significance in contrast to the case of the Ricochet. The analysis of the Kestrel has given the straight forward implication that its wings flutter in the orthodox flexure-torsion manner combining fundamental bending and torsional modes (demonstrated by ground resonance test (see Appendix H) and also by theoretical calculations using PAFEC). In the case of the Ricochet, no experimental evidence is available to support the phenomena. However, a private communication (Ref.138) provides some authenticity to justify the theoretical results obtained on the Ricochet. Ref.138 is the description of a flutter-like instability given by an experienced pilot who had the opportunity to fly the tailless sailplane BKBl. Flapping motion of the fuselage is said to have occurred with the BKBl as a practical experience but not at a low speed as the result shows in the case of the Ricochet.

Such a low flutter speed of the Ricochet has

raised questions and doubts about the suitability of this sailplane. The expected uncertainties which give rise a low flutter speed were anticipated by the original designer of the Ricochet (Ref.37). However, the marked discrepancy in the flutter speeds between the Ricochet and the Kestrel has led to further investigation on the possible improvement of flutter characteristics of the Ricochet. The effect on flutter speed of the variation of basic design parameters of the Ricochet is explored in the following chapter.

4.7 Note about Presentation of Flutter Modes in Figs. 18-35 and Figs. 45-58

As explained in section 4.2 the flutter modes have been normalised with respect to the generalised co-ordinates of highest natural frequency. This has meant that in the calculations $\bar{q}_6$ in the case of the Ricochet and $\bar{q}_8$ in the case of the Kestrel have been taken to be -1.0. The values of real and imaginary parts of deflections in the figures are quoted relative to this base which, of course, has no absolute significance. Thus relative magnitudes and phases are only of significance especially in Figs. 18, 24, 30 and Figs. 45,52.

4.8 Note about Selection of Normal Modes and Convergence Criteria in the Flutter Analysis of the Ricochet

As mentioned in earlier sections, in the flutter analysis of the Ricochet, four elastic modes together with the two rigid body modes in the pitching plane have been chosen to describe the generalised co-ordinates. This has been initially attempted to include the fundamental torsional mode (4th elastic) of the Ricochet. In order to have a check on the convergence of the results, the number of elastic modes in the analysis was increased from four to six. The flutter analysis carried out with the increased number of modes did not produce any significantly different results*

*In most of the cases, using a total number of six modes flutter speed, frequency and aeroelastic modes have been obtained. The results computed with eight modes in the unballasted case of the Ricochet were only 0.1 percent different from those obtained with six modes.

However, in a later stage as it has been found that only the first elastic mode combined with the two rigid body modes, (heave and pitch) is primarily responsible for the flutter of the Ricochet, a subsequent analysis with only two elastic modes has given sufficiently accurate results.

CHAPTER 5

5.0 FURTHER INVESTIGATION INTO THE AEROELASTIC INSTABILITY OF THE RICOCHET - A PARAMETRIC STUDY FOR POSSIBLE IMPROVEMENT OF ITS FLUTTER CHARACTERISTICS

5.1 Purpose of the Study

The existence of flutter of the Ricochet at 38 m/sec poses two fundamental questions -

(i) Why has this sailplane such a low flutter speed? and (ii) Are there any ways for possible improvement of its flutter characteristics? In the light of these two questions attempts have been made to discover whether or not any simple modification of the present design of the Ricochet can overcome its flutter difficulty.

5.2 Distinguishing Features of the Ricochet Compared with Sailplanes of Similar Class

Sailplanes having a span of 15 m are generally said to be of 15 m class. The Ricochet is, therefore, a member of this class. The standard Cirrus sailplane which is also of 15 m class and whose particulars are listed in Appendix C, can be chosen a basis for comparison when exploring variation of the structural and inertial properties of the Ricochet. The Standard Cirrus has almost identical aspect ratio and wing area to the Ricochet (see Appendices A and C). There are two marked differences between these two aeroplanes; one inertial and the other geometrical both of which can be used as variables in the parametric study. The first characteristic is in the pitching moment of inertia value. The absence of a tail has made the Ricochet have a pitching moment of inertia (about its C.G.) of only 95 Kgm^2 for the unballasted case and 100 Kgm^2 for the ballasted case whereas the pitching moment of inertia of the Standard Cirrus (about its C.G.) is about 5 times this value. The second characteristic is the angle

of sweep of 13° in the leading edge of the Ricochet. Like any other conventional sail-plane the Standard Cirrus has no sweep. Before taking up the parametric study other worth mentioning points are:

- (i) The Ricochet has fuselage and wing weights comparable with the Standard Cirrus,
- (ii) The fuselage length of the Ricochet is half of that of the Standard Cirrus, and
- (iii) Equipment and pilot weight etc. are substantially the same for the two aeroplanes.

5.3 Scope of the Parametric Study

The areas in which the parametric study can be made effective are considered in the context of a realistic redesign of the Ricochet.

These areas are as follows:

- (i) The effect of the C.G. position on the flutter speed.
- (ii) The effect of the pitching moment of inertia of the pilot fuselage assembly on the flutter speed.
- (iii) The effect of the angle of sweep on the flutter speed.
- (iv) The effect of variable sweep with variable pitching moment of inertia of the pilot fuselage assembly on the flutter speed.

The parametric study has been attempted only for the unballasted case of the Ricochet. The water ballast does not seem to make a great deal of difference in the flutter characteristics.

5.4 Effect of the C.G. Position on the Flutter Speed of the Ricochet

The C.G. position of the wing is fixed and so is the angle of sweep. The only way the C.G. of the complete aircraft can be changed is by varying the C.G. position of the pilot fuselage

assembly. The C.G. position of the pilot fuselage assembly is moved forward and backward by an amount 0.1 m in the present analysis which means an overall C.G. shift of 0.064 m as indicated by Table 3. In the bottom half of Table 3, the natural frequencies are listed for these two cases of C.G. positions along with the original case for analysis (case A) for the Ricochet. The natural frequencies have not been greatly affected by the change in C.G. position as can be seen in Table 3. The mode shapes are also not altered to any large extent. However, the first elastic mode which was found to couple with the rigid body modes in the flutter of the original Ricochet is shown in Fig. 62 for the two C.G. positions. The results of the flutter analysis for these two cases are shown in Table 4 as case B and case C respectively, whereas case A corresponds to the original analysis. As is clear from Table 4, the forward shift of the C.G. reduces the flutter speed by a small amount to 36.5 m/sec with an increased frequency of oscillation to 10.8 rad/sec whereas the aft C.G. increases the flutter speed to 44.0 m/sec with a reduced frequency (reduction in frequency) of oscillation of 4.3 rad/sec.

5.5

Effect of the Pitching Moment of Inertia on the Flutter Speed of the Ricochet

Because of the absence of a tail, the Ricochet has a very low pitching moment of inertia of around 100 Kg m^2 (about its C.G.) when compared with the pitching moment of inertia of the order of 500 Kg m^2 of similar sailplanes of orthodox layout. So the question arises as to whether it is the low pitching moment of inertia which causes the Ricochet to flutter at such a low speed. However, it is worth remembering that before the analysis was carried out the idealisation procedure followed for the Ricochet and the Kestrel is of similar nature as far as the wing idealisation as beam elements is concerned. But when the fuselage and pilot idealisation of these two aeroplanes is compared, there is a significant difference in the structural elements involved (see Figs. 5 and 37). In the case of the Ricochet, the pilot fuselage assembly is just a lumped mass having some inertia properties (mass and moment of

inertias) and is located at a point on the fuselage axis with respect to the reference set of co-ordinate axes (see Fig. 6). However, in the case of the Kestrel, the fuselage is a series of beam elements as well as two lumped mass elements for the front fuselage and pilot respectively (see Fig. 37). Thus in the parametric study of the Ricochet the changes that can be brought about will have to be the employment of various values of the pitching moment of inertia of the pilot fuselage assembly (I_{p+f}^*). The mass of the pilot fuselage assembly (M_{p+f}) which is around 180 Kg for the Ricochet has not been changed for practical reasons (It may be pointed out here that as an academic exercise the flutter problem of the Ricochet was solved with an increased mass of the fuselage. An increment of 25 Kg did not produce any appreciably different results). Since symmetric motions are always being dealt with, only one half of the values of I_{p+f} and M_{p+f} (respectively the pitching moment of inertia and mass of the pilot fuselage assembly) are used in the analysis. The value of $\frac{1}{2} I_{p+f}$ in the original analysis of the Ricochet was 31.2 Kgm^2 and this has been increased to a maximum value of 250 Kgm^2 and the intermediate values of $\frac{1}{2} I_{p+f}$ are chosen by linearly interpolating the radius of gyration. The natural frequencies of the five cases with different pitching moment of inertias (including the original case of the analysis) are reported in Table 5. An inspection of Table 5 suggests that except the small variation in the first natural frequency, the remaining higher frequencies of vibration are almost invariant. As has been emphasised before, the first elastic mode of the Ricochet is important as it couples with the rigid body motions to give rise to a flutter condition of the basic aircraft. It is worth mentioning how the first elastic mode changes with increased pitching moment of inertia of the pilot fuselage assembly. Fig. 8 shows this mode for the original Ricochet (case A of Tables 5 and 6) where there exists a certain amount of pitching rotation along with the primary constituent of vertical displacement.

*The pitching moment of inertia of the pilot fuselage assembly is taken about the C.G. of the assembly.

As the pitching moment of inertia of the pilot fuselage assembly is gradually increased according to the steps shown in Tables 5 and 6, the amount of pitching rotations associated with vertical displacements in the first elastic mode gets further and further reduced and the two motions become more and more uncoupled. Out of four additional cases (B to E) reported in Tables 5 and 6, two have been illustrated (cases B and C) for demonstration purpose to show how the first elastic mode behaves with the increase in the pitching moment of inertia of the pilot fuselage assembly. The mode shapes for the two cases are shown in Fig. 63. Figs. 8 and 63 describe the comparison of the first elastic mode for the three cases. As regards the flutter calculations, the results of all cases cited in Table 5 are shown in Table 6. An observation of the results in Table 6 suggests that flutter speed improves as expected with the increase in pitching moment of inertia and for a respectable flutter speed of 86 m/sec, the half value of the pitching moment of inertia of the pilot fuselage assembly ($\frac{1}{2} I_{P+F}$) needs to be 175.58 Kgm² which, of course, is without changing any other inertial or structural parameters of the Ricochet. A plot of the results given in Table 6 is shown in Fig. 64 suggesting that flutter speed varies almost linearly with the pitching moment of inertia of the pilot fuselage assembly which is a phenomenon particularly pertinent to the Ricochet analysis. The nature of the flutter modes for the cases with increased pitching moment of inertia listed in Tables 5 and 6 (cases B to E) has remained the same. However, a typical example of one of the cases (case B, $\frac{1}{2} I_{P+F} = 66.08 \text{ Kgm}^2$) with increased pitching moment of inertia is shown in Fig. 65 which illustrates the absolute values of the vertical displacement and pitching rotation and their phase difference along the span. The other interesting observation from the results shown in Table 6 is about the flutter frequency. It has already been noticed that the Ricochet flutters in an unusual way and the analysis of the basic aircraft showed a low flutter frequency (8.74 rad/sec) which is below the first natural frequency (18.28 rad/sec) of the

aircraft. The results in Table 6 reveal that increase in pitching moment of inertia brings the flutter frequency further down which again means that the instability is an overall instability rather than a conventional wing flutter problem. The private communication (Ref.138) about the behaviour of a real tailless sailplane gives some evidence to this effect.

5.6 Effect of the Sweep on the Flutter Speed of the Ricochet

The sweep in the leading edge of the Ricochet is perhaps the special feature which distinguishes it from other type of conventional sailplanes. The bending of a swept wing produces changes in incidence along the span which make the aerodynamic interaction with the elastic and inertia forces much more conducive to flutter than in the case of a straight wing. The design study of the Ricochet (Ref.37) explains why the aeroplane has to have a sweep of 13° . Some of the reasons attributed are the outcome of computer results on the basis of wing plan form optimization, a reasonable C.G. margin and also the desired wing loading. The C.G. margin for the Ricochet is between 16.5 and 21% of the standard mean chord (SMC) for the forward and aft C.G. configuration (measured from the leading edge of the SMC) as mentioned in Appendix A. The same margin is all through maintained for the analysis presented here. A change in sweep changes the aerodynamic centre (A.C.) and the C.G. position of the aeroplane which are recalculated in each case. The C.G. position of the pilot fuselage assembly is so adjusted that the margin between the A.C. and the C.G. of the complete aircraft remains constant in each case for investigation due to a change in sweep. Increase in the angle of sweep is out of the question. Sweep is not only a structural penalty, it reduces the aerodynamic efficiency considerably at subsonic speed and the basic purpose of sailplane design to achieve a high lift/drag ratio will be seriously hampered. The Ricochet had to be provided with a sweep of 13° because there was no way out to reach a reasonable design compromise. However, without altering any other parameters,

two values of sweep angles 5° and 0° have been tried in the parametric study (the second case is obviously the example of straight wing). Table 7 shows the natural frequencies of vibration of the three cases with variable sweep including the original case with 13° . As the Table 7 shows, the frequencies are unaltered except the minor change in the frequency value of the first mode. The mode shapes corresponding to the first natural frequency of the 13° , 5° and 0° sweep cases are respectively shown in Fig. 8 and by (1) in Figs. 66 and 67. As expected, the sweep couples the two modes (vertical displacement and pitching rotation) and the reduction in sweep decouples the mode (A theoretical method on how sweep couples the modes of a uniform swept wing is presented in Appendix J). Table 8 shows the results of the flutter analysis with the three different sweep angles. It is evident from results shown in Table 8 that reduction in sweep increases the flutter speed. Five degrees sweep of the Ricochet wings slightly increases the flutter speed to 40 m/sec. However, the straightening up of the wings (sweep = 0°) has a more dramatic effect and brings up the flutter speed to 54 m/sec. It is also interesting to note (see Table 8) that the flutter frequency goes up as the wings are straightened. The aeroelastic modes showing the vertical displacements and pitching rotations along the semi-span of the original Ricochet with 13° sweep are given by Figs. 22 and 23 whereas Figs. 68 and 69 correspond to the cases of 5° and 0° sweep respectively. Further observation of the aeroelastic modes for the 5° and 0° sweep cases (see Figs. 68 and 69) and a comparison of those with that of the original Ricochet case (see Figs. 22 and 23) with 13° sweep support the fact that the contribution from the first elastic mode is relatively higher in the reduced sweep angle cases of 5° and 0° than in the case of the original Ricochet with 13° sweep. The investigations made here are all for cases with change in sweep alone and the pitching moment of inertia of the pilot fuselage assembly which is one of the critical elements to affect the flutter speed is maintained to retain its original value ($\frac{1}{2} I_{P+F} = 31.2 \text{ Kgm}^2$).

In the section follows the pitching moment of inertia is changed together with the variable sweep cases to investigate whether or not a

compromise between the two is possible for an acceptable flutter speed of the Ricochet.

5.7 Effect of Both Variable Pitching Moment of Inertia and Sweep on the Flutter Speed of the Ricochet

The pitching moment of inertia of the pilot fuselage assembly of the Ricochet is varied when its sweep is already changed. The cases for 5° and 0° sweep are treated separately. For the first case when the sweep of the Ricochet wing is made to be 5° , the natural frequencies of vibration are reported in Table 9 for three different values of pitching moment of inertia of one half of the pilot fuselage assembly. In Table 9, case A corresponds to the pitching moment of inertia value of the original Ricochet ($\frac{1}{2} I_{p+F} = 31.2 \text{ Kgm}^2$). No significant variation in the natural frequencies for variable moment of inertias is found to occur as seen in Table 9. The first elastic mode for the two cases (cases A and B) of Table 9 is respectively shown in Fig. 66. As in previous cases, it is found (see Fig. 66) that increase in pitching moment of inertia results in decoupling of the modes with reduction in pitching rotation. The results of the flutter analysis for cases cited in Table 9 are shown in Table 10. The selection of an appropriate pitching moment of inertia value in Table 10 lies between case B and case C to bring the flutter speed within acceptable limits. For the values of $\frac{1}{2} I_{p+F}$ as 75 Kgm^2 (case B) and 175 Kgm^2 (case C) the respective flutter speeds are 60 m/sec and 117 m/sec . About the flutter frequencies reported in Table 10, the behaviour is quite similar to the 13° sweep case (see Table 6) where the reduction in flutter frequency resulted due to the increase in the pitching moment of inertia values. The aeroelastic modes at flutter speed for this 5° sweep case are shown in Figs. 68 and 70 for two cases (cases A and B) taken from Tables 9-10.

In a similar way the pitching moment of inertia of the pilot fuselage assembly is changed for the straight wing case and the corresponding

natural frequencies are not found to be significantly different (see Table 11). Case A in Table 11 has an equal amount of the pitching moment of inertia of the original Ricochet whereas in case B and case C the figures have been increased to some arbitrarily higher values. The first elastic mode corresponding to case A and case C of Table 11 is shown in Fig. 67 which again indicates less pitching rotation in the higher pitching moment of inertia case. The flutter calculation results of the straight-wing case presented in Table 11 are reported in Table 12 which are particularly interesting. As seen in Table 12, a value of $\frac{1}{2} I_{p+F}$ as 50 Kgm² is sufficient

to keep the flutter speed of the Ricochet at 85 m/sec which in comparison with the Kestrel flutter speed (70.6 m/sec) is a good result. However, it is not out of place to mention that the structural details of the Ricochet and the Kestrel presented respectively in Appendices A and B reveal an important fact that the torsional stiffness of the Ricochet wings is quite high in comparison (see Figs. 118 and 120). Therefore, it is likely that a conventional sailplane having wings of the Ricochet type will flutter at a relatively high speed. It is also interesting to note from the results in Table 12 that unlike 13° and 50° sweep cases, the flutter frequency of the straight-wing Ricochet goes up with the increase in the pitching moment of inertia values. The flutter modes of two cases from Table 12 (cases A and B) are shown in Figs. 69 and 71 which reinforce an earlier mentioned point that in the case of straight-wing, the contribution of the first elastic mode to the flutter mode becomes higher in comparison with the swept wing cases.

5.8 Salient Features of Results from the Parametric Study

Although the results of the parametric study with the variation of geometric and inertial parameters of the Ricochet have already been discussed to some length in sections 5.4-5.7, some salient features of the calculated results are well worth consideration.

The effects of the C.G position of the Ricochet on its flutter characteristics have been discussed in section 5.4 and the corresponding results have been reported in Tables 3 and 4. The aft C.G improves the flutter characteristics (see Table 4) and an increment of flutter speed by about 6 m/sec from the original value of 38 m/sec is reported to have occurred. The reason for this improvement as has been explained earlier, is the fact that in the case of the aft C.G configuration, there is less amount of pitching rotation associated with the first elastic mode (see Fig.62) which is primarily responsible for the flutter of the Ricochet. Moreover, the aft C.G position increases the pitching moment of inertia of the complete aircraft which is again a contributory factor to this change. However, for the fwd C.G configuration the converse of what has been said so far, is true. In the analysis, the C.G has been shifted forward and backward by an amount of 6 cm from its original position. A much wider C.G margin is not feasible because the slight change in the C.G position is only the direct result of the effect of the water ballast as well as the pilot's sitting position and the inclination of his back. However it is important to examine the effect of the C.G position on the ratio of the vertical displacement and pitching rotation at flutter speed and also the variation of their phase difference. The behaviour has been illustrated in Fig. 137 where the relative magnitude of the vertical displacement is higher for the aft C.G configuration as compared to the case of the original Ricochet. The associated lower values of the pitching rotation so indicated in Fig. 137 for the aft C.G have resulted in the larger values of the phase difference between the vertical displacement and pitching rotation to satisfy the requirements for the flutter condition. In the case of the fwd C.G configuration, the opposite is true (see Fig. 137) where comparatively lower values of the ratio of the vertical displacement to pitching rotation have resulted into lower values of the lag of the latter motion to the former at flutter speed.

In section 5.5 the effects of the pitching moment of inertia of the pilot/fuselage assembly

on the flutter characteristics of the Ricochet have been discussed in sufficient details with illustrations given by Figs. 23, 65 and 64 and Tables 5 and 6. It has been observed that an increase in the pitching moment of inertia value not only increases the flutter speed but also reduces the amount of pitching rotation effected in the resulting flutter mode (see Figs. 23 and 65). In Fig. 138 the ratio of the vertical displacement and pitching rotation and the phase difference of the two are shown for increasing values of the pitching moment of inertia as included in the analysis. It is quite clear from Fig. 138 that for higher values of the pitching moment of inertia the associated pitching rotation in the flutter mode is lesser but the corresponding lag of the pitching rotation has eventually become higher in the higher pitching moment of inertia cases.

The effect of sweep on the flutter behaviour of the Ricochet has been dealt in section 5.6 with detailed discussion of results. Much reduction in sweep is not possible mainly because the achievable control efficiency may become far less than that required. However, like the previous cases of the parametric study, the relative magnitudes of the vertical displacement and pitching rotation and their corresponding phase difference for the swept and unswept cases are illustrated in Fig. 139. It has already been noticed that in the case of the unswept wing, there had been a comparatively greater contribution from the first elastic mode (see Figs. 23 and 69) to the causation of flutter of the Ricochet and in this regard the results shown in Fig. 139 are particularly interesting.

An overall general comment about the flutter modes of the Ricochet is the fact that although there is a substantial amount of elastic deformation associated with the vertical motion at flutter speed, the Ricochet wings having high torsional stiffness do not seem to have any amount of twist in the resulting flutter mode and everywhere there exists an almost constant pitching rotation of the complete aircraft.

5.9

Conclusions from the Parametric Study

It is clear from the results of the parametric study (see Tables 5, 8, 10 and 12) that the low value of the pitching moment of inertia as well as the angle of sweep are responsible for the low flutter speed of the Ricochet. The real compromise involves in changing both these parameters to obtain an acceptable flutter speed. It must be noted that appropriate C.G margin has to be achieved whenever the sweep is changed. Straightening the wings improves the flutter characteristics of the Ricochet and brings the limit within acceptability only when the pitching moment of inertia is increased which is difficult, if not impossible, with a tailless layout. In the existing form of design of the Ricochet out of 100 Kgm² of its total pitching moment of inertia about 38 Kgm² comes as a contribution from its pair of swept wings. Straightening up the wings will result into limited supply of the pitching moment of inertia values from the wings and the required amount cannot be achieved without providing a tail unit. Thus to make the Ricochet an operational flying machine, the idea is confronted and the concept of tailed design comes back again. However, as long as the aeroplane is not flown within the flutter affected zone, the low frequency instability giving rise to the flutter can be of no principal concern. It is much of practicality left to the designer to decide design parameters from the results of the parametric study and make a tangible approach to any further design of tailless sailplanes.

CHAPTER 6

6.0 ANALYSIS OF SHORT PERIOD LONGITUDINAL OSCILLATIONS OF THE RICOCHET

It is clear from the work described in Chapters 4 and 5 that the flutter problem of the Ricochet is a serious one. Although the flutter speed achievable from the existing design is unacceptably low, no attempt has been made (apart from the recommendations in Chapter 5 as a result of the parametric study) to redesign the aeroplane. The subsequent analyses in this thesis emphasize the treatment of the aircraft as originally projected to establish if there are other important aspects which could jeopardize the existing design.

The Ricochet without having a tail is expected to suffer loss of rotary damping. The effect can be very important and in some cases may be so strong as to make damping of the short period oscillation nearly zero or even negative, thus making the aircraft uncontrollable. It is considered that the phugoid motion is less critical than that of the short period oscillation, although in the case of the Ricochet, the damping in the phugoid mode is expected to be low because of its low parasite drag. However, the period of the phugoid mode is higher (in comparison to that of the short period oscillation) and thus allows a longer time for the pilot to control the aircraft. The analysis of the phugoid motion of the Ricochet is, therefore, excluded.

An analytical method is developed to study the short period oscillation characteristics of the Ricochet when the effects of its flexibility together with the unsteady aerodynamics in the oscillatory mode are taken into account. The matter is investigated to ascertain whether the damping related to the motion of the aircraft in the pitching plane is adequate or not. The effect of flexibility and that of the unsteady aerodynamics on the loss of short period damping are considered to be important features of the theory developed. An alternative analysis for the rigid aeroplane using

a conventional method employing stability derivatives ($z_w, m_w, m_q, m_{\dot{w}}$) on which the properties of the short period oscillation depend, is carried out to compare the results. The calculations are based on determination of short period frequency, damping factor and time to half-amplitude of various oscillatory modes. In the analysis, the forward speed of the aircraft is assumed constant implying the non-intrusion of phugoid motion. It is argued in general terms that only the short period motion can couple with the elastic modes of distortion and scarcely any phugoid motion can result from longitudinal manoeuvres of the Ricochet. The control surfaces are assumed to be held rigid with their angular settings at zero.

6.1 Outlines of the Proposed Method

The development of the method to deal satisfactorily with the problem by taking both flexibility and unsteady aerodynamics into consideration is primarily dependent on the extension of the Theodorsen function $C(k)$ for non harmonic arbitrary motion of an aerofoil. In the flutter problem formulated in Appendix D, sinusoidal motion is assumed to exist between the transition of stable and unstable oscillatory motion, so that above the flutter speed the motion is divergent and below the flutter speed the motion is convergent. The Theodorsen expressions used for lift and moment in the flutter analysis are valid for aerofoil oscillation of form $e^{i\omega t}$ ($=e^{ik\tau}$ where $k = \frac{\omega b}{U}$ and $\tau = \frac{Ut}{b}$). The Theodorsen function $C(k)$ used in the flutter analysis (see Appendix D) is expressed in terms of Hankel functions which are again combinations of Bessel functions of first and second order and also of first and second kind. Now for non-harmonic oscillation (convergent or otherwise) of an aerofoil, the expressions for lift and moment can be obtained if the Theodorsen function $C(k)$ is properly calculated for complex values of k . This useful extension of the Theodorsen function $C(k)$ for motion of form $e^{\lambda t}$ (where $\lambda = \mu + i\omega$) is found in the

work of Luke and Dengler (Ref. 116). However, for an application of this extension, the vertical displacement (h) and the pitching rotation (α) at various points on the wing are given appropriate time dependence. The term k in $C(k)$ is regarded as a complex argument so that, $k = \frac{\omega b}{U} - i \frac{\mu b}{U}$. In the flutter

problem the airspeed (U) and the frequency parameter (k) are both unknown and the value of μ which is a measure of damping is zero, thus implying that the motion is simple harmonic. Now, if an airspeed (U) is chosen (preferably below the flutter speed), the stability determinant involving λ and hence, μ and ω (see equation E.12.6, Appendix E) can be solved for the zeros of both real and imaginary part. The real part μ gives the short period damping and the imaginary part ω gives the short period frequency of oscillation at that particular speed. Based on this process, the suggested method has been programmed. The details of the method involving the formulation of the stability determinant are reported in Appendix E. The classical method of analysing the short period oscillation of rigid aeroplanes employing aerodynamic derivatives is also appended therein.

6.2. Calculation of Results and Discussions

The aircraft is given two rigid and four elastic degrees of freedom as in the case of flutter analysis described in Chapters 4 and 5.* The investigation is carried out for a forward speed of the aircraft less than or equal to its flutter speed thus ensuring a convergent oscillation in the short period mode. The flutter speed of the Ricochet limited the range of speeds of the investigation between 20-38 m/sec of which the lower values tend to be unrealistic. The damping, frequency, period and time to half amplitude of the short period mode of the elastic Ricochet for unballasted wings are given in Table 13. The corresponding results for the rigid aircraft when stability derivatives are used are provided in Table 14. Similar analysis was carried out for the ballasted wings case and the results so obtained for the elastic and rigid Ricochet are respectively contained in Tables 15 and 16. With

*The effect of structural damping is neglected in the analysis.

data taken from Tables 13-16, the short period modes of the rigid and elastic Ricochet for both unballasted and ballasted wings related to a forward speed of 30 m/sec are illustrated in Fig.72 to serve as a typical example. The solid lines as indicated in Fig.72 describe the rate of pitch for the elastic aircraft whereas the dotted curves correspond to the case of the rigid aircraft when the conventional method employing stability derivatives is used. The significant difference in the results between the two cases shown in Fig. 72 explains the effect of flexibility. A comparative study of the rigid and elastic Ricochet may first be made for the unballasted case from the results provided in Tables 13 and 14. In the lower range of speeds (20-30 m/sec) the short period damping μ (unit:sec⁻¹) for the elastic case is comparable with those for the rigid case. The other interesting point is that the short period damping for the elastic aeroplane is constant in that range. As the speed of the aircraft is increased the damping in the resulting motion of the elastic aircraft case reduces and finally becomes zero at the flutter speed (see Table 13). The same behaviour of reduction in damping with increase in speed is not observed in the case of the rigid aircraft (see Table 14). On the other hand, the results are of the opposite nature. This is because the unit of aerodynamic time $\hat{t}$ is dependent on the forward speed of the aircraft V_0 and is inversely proportional to it when expressed with other constant parameters ($\hat{t} = \frac{m}{\rho s V_0}$). Therefore, an increase in forward speed results in reduction in the value of $\hat{t}$ which in turn increases the short period frequency and damping in the real time domain (see equations E.2.8., E.2.9 and E.2.25, Appendix E) Further details to this effect can be found in Ref.38. The short period frequencies shown in Tables 13-14 for the rigid and elastic aircraft indicate that they increase with increase in speed for both cases. The values are, however, higher in the elastic aircraft case. The period (T) and the time to half amplitude (t_a) are accordingly smaller. The results shown in Tables 15-16 for the ballasted Ricochet depict similar trends as in the unballasted case.

Results of the elastic aeroplane for both unballasted and ballasted cases given in Tables 13 and 15 respectively show that the ballast leads to lower damping but higher frequency in the short period mode. An inspection of Tables 14 and 16 indicates similar observations from calculated results for the rigid aircraft.

An effective way of showing the variation of roots of the short period oscillation as one parameter (speed in this case) varies, is to use a root-locus plot, in which the real and imaginary parts of the roots are plotted in an Argand diagram. The results displayed in Tables 13-16 are respectively transferred to the root-locus diagrams in Figs. 73-76. The elastic Ricochet results for unballasted and ballasted wings are shown in Figs. 73 and 75 respectively. The plots indicate that the frequency of the short period oscillation in the speed range 20-30 m/sec increases with speed, the damping being practically unchanged. Above the speed value of 30 m/sec and upto the flutter speed the reduction in damping and increase in frequency may be observed in Figs. 73 and 75. The plots for the cases of the rigid aircraft with unballasted and ballasted wings are respectively shown in Figs. 74 and 76. Since the stability derivative approach has been used to obtain the plots, an increase in speed is accompanied by a rise in both short period frequency and damping for reasons already explained. One of the advantages of the root-locus plot is that the number of cycles to half the amplitude is easily obtained from the plot (The number of cycles to half amplitude

$$= \frac{t_a}{T} = \frac{\log_e 2}{2\pi} \cdot \left(\frac{\omega}{-\mu} \right).$$

6.3

Remarks and Conclusions

In the short period oscillation characteristics of the Ricochet, there have been significant differences between the results of the rigid and flexible aeroplane. In the analysis of flexible aeroplane, unsteady aerodynamic effects have been taken into account. It would be difficult to make any decisive comments on whether the flexibility or the unsteady

aerodynamics played a greater role for the wide differences in the results between the rigid and flexible aircraft. The modified derivative approach (also designated as the quasistatic method which is referred to later on as an area for future work), may provide the answer. It is expected that the results from such an approach would lie between the results of the two extreme cases investigated. The distortion effects should not be neglected in any case in dealing with an aeroplane of the Ricochet type while evaluating its short period characteristics. The analysis for speeds greater than 38 m/sec has not been carried out because of the exceedingly low value of the flutter speed taking cognizance of the fact that the aircraft behaviour with inaccurate initial data may lead to quite unreliable results. Because of the non-inclusion of flexibility effects, the classical approach of the analysis for stability characteristics using aerodynamic derivatives can be misleading even from the point of uncritical interpretation. Again, since the results obtained are confined to a very low range of speeds, the aircraft's ride quality at higher speed cannot be predicted. However, the short period damping and frequency obtained for the low range of speeds lie within the bounds of the requirements of service aeroplanes (Refs. 95 and 135). It must be pointed out that the flutter speed of a conventional aircraft is much higher than the speed of investigation of its stability characteristics. In this regard, the norm has not been followed to the letter because of the unexpectedly low flutter speed of the Ricochet. Although it has not been attempted in this thesis, the method outlined in section 6.1 could have been applied for the two rigid body modes only by simply suppressing the elastic modes of vibration and the results so obtained would be interesting to examine.

CHAPTER 7

7.0 RESPONSE OF THE RICOCHET TO CONTINUOUS RANDOM ATMOSPHERIC TURBULENCE

The purpose of this chapter is to present a detailed analysis of the acceleration response* at various wing stations of the Ricochet due to random atmospheric turbulence allowing for the dynamic flexibility of the aircraft. Power Spectral Density (PSD) method is applied to obtain frequency decompositions of the acceleration response through the use of transfer function representation of the aircraft. The aerodynamic forces which arise due to the motion and external excitation are both evaluated using unsteady aerodynamic theories. The atmospheric turbulence is represented by the well known Dryden and Von Karman spectra. The full details of the method used are described in Appendix F. The analysis is again concerned with longitudinal motion only and therefore the disturbed motion is symmetrical with respect to the longitudinal plane of symmetry. While passing through the turbulence, the aircraft is assumed to maintain a constant forward speed (U). The response parameters evaluated as unknown quantities are explained in the following sections.

7.1 The Gust Response Function

In the PSD method the root mean square (r.m.s.) value of the output response is determined from the knowledge of the input PSD. The output response in the present analysis is the acceleration (normal and angular) at various spanwise stations whereas the input is described by the Dryden and Von Karman spectrum. The PSD distributions of these two input spectra for various scale lengths (L) are shown in Figs. 77-78 for the case when the r.m.s. gust velocity $\bar{w}_G$ (also indicated as σ_i) is unity. Now if σ_o is the r.m.s.

*The normal and angular (or pitching) accelerations are considered to be the meaningful quantities to calculate. Only the vertical component of the turbulence is considered in the analysis although the horizontal component in some cases may have significant effects as well.

value of the output acceleration corresponding to the input gust velocity of r.m.s. value σ_i (or $\bar{w}_G$), the ratio of the output

to input r.m.s. values, $\bar{A} = \frac{\sigma_o}{\sigma_i}$ is defined

as the gust response function. The gust response function is calculated by the use of the transfer function of the aircraft (see Appendix F) and is dependent only on the normalised spectrum of the turbulence and not on its r.m.s. intensity. To find the actual acceleration at a point on the aircraft, the gust response function $\bar{A}$, has to be multiplied by the

r.m.s. value of the gust velocity embedded in the turbulence. Figs. 77-78 showing the normalised Dryden and Von Karman spectra can be appropriately scaled for any particular value of the r.m.s. gust velocity $\bar{w}_G$ (or σ_i).

Although the transfer function is a property of the aircraft, because of the dependency of normalised PSD on the scale length of the turbulence, the gust response function ($\bar{A}$) is expected to vary along with the scale length.

7.2

Response Due to Various Grades of Turbulence

In the previous section, it has already been explained that the r.m.s. value of the actual response (normal and angular acceleration) can be obtained from the gust response function by multiplying it with the r.m.s. gust velocity (also known as gust intensity) of the turbulence. The gust response function varies with scale length (L) which in turn varies almost linearly with the height at which the turbulence occurs. In a recent report Bennett (Ref. 95) has revised the design requirements that are given in Av.P 970 (Ref.135) for critical gust cases applicable to service aeroplanes. He has defined a scalar reference intensity $\bar{w}$ for the atmospheric disturbance which has the dimensions of an r.m.s. turbulence velocity and is in some cases identical with r.m.s. values of actual turbulence velocity components. Once the gust response function $\bar{A}$ is evaluated, the reference intensity for the turbulence $\bar{w}$ is to be

decided to use it as a scaling factor to obtain the actual or absolute response. The qualitative description of the turbulence intensity as light, moderate, severe and extreme depending upon the value of the reference intensity $\bar{\sigma}$, has earlier been proposed by Bennett (Ref. 95) and is reproduced in Table 17 from his work. For a given reference intensity $\bar{\sigma}$, the following relations between the scale length L and the r.m.s. gust velocity $\bar{w}_G$ hold (see page 58, Ref. 95).

$$\frac{\bar{w}_G}{(L)^{\frac{1}{3}}} = \frac{\bar{\sigma}}{(750)^{\frac{1}{3}}} \quad \dots(7.2.1)$$

for height ≤ 750 m

and

$$\bar{w}_G = \bar{\sigma} \quad \dots(7.2.2)$$

for height above 750 m

where the scale length L is in metre.

In the present analysis equation (7.2.1) is used because the aircraft is assumed to fly at a height below or equal to 750 m. Now, the value of $\bar{w}_G$ in the input spectra of turbulence

shown in Figs. 77-78 is decided by the equation (7.2.1) for a specific length (L), only when the reference intensity ($\bar{\sigma}$) of the turbulence is

set to a fixed quantity. However, this ultimately leads to the fact that a corresponding change in scale length (L) is so compensated by the effective change in the r.m.s. gust velocity ($\bar{w}_G$) such that for a fixed reference intensity

$\bar{\sigma}$ of the turbulence (see Table 17) the response

becomes independent of the scale length (see Ref.95). In the calculation of the r.m.s. values of the normal and angular accelerations at various spanwise stations of the

Ricochet, the first three values of the reference intensity cited in Table 17 are used.

7.3

Probability Distribution of the Output Acceleration

When the r.m.s. value of the acceleration response is known from the procedure explained in sections 7.1 and 7.2, the probability density function of the acceleration can be found out using a normal (Gaussian) distribution. Therefore, if the distribution of aeroplane acceleration increments in the turbulent air is considered to be normal with a zero mean, then the probability density distribution of the acceleration increments Δn , is completely described by the standard deviation σ (the r.m.s. value in this case) of the acceleration as

$$f(\Delta n) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\Delta n}{\sigma}\right)^2} \dots (7.3.1)$$

The above distribution can be normalised with a zero mean and a standard deviation equal to unity so that an advantage from the tabulated error functions can be taken of in determining the probability of exceeding a certain value of acceleration.

It is a reasonable assumption that the response follows a normal (Gaussian) distribution and in such a case the probability that the variable x (or Δn) will exceed k times its root mean square value $\sqrt{\bar{x}^2}$ (or $\sqrt{\sigma^2}$) for large k is (see Ref.82)

$$P \left\{ x > k \sqrt{\bar{x}^2} \right\} \cong \frac{e^{-\frac{1}{2} k^2}}{k \sqrt{2\pi}} \dots (7.3.2)$$

Some general and broad limits of the probability of exceeding a certain value can be fixed, at least approximately even without knowing the probability distribution of the response by using the well known Chebycheff's inequality

$$P \left\{ |x - \bar{x}| > k \sqrt{\bar{x}^2} \right\} \leq \frac{1}{k^2} \quad \dots (7.3.3)$$

i.e. the probability that x differs from its mean value by k times its root mean square value $\sqrt{\bar{x}^2}$ is less than or equal to $\frac{1}{k^2}$.

However, in the calculation of results in this thesis, the approximate equation (7.3.3) is not used and the probability of exceeding a certain limit is evaluated from the distribution of the probability density function.

7.4

Exceedances

It is important in many cases to find the number of exceedances with positive slope of a certain level of the acceleration per unit time. If σ_0 is the r.m.s. value of the output acceleration and $\phi_0(\omega)$ is its PSD distribution then the number of zero crossings per second with positive slope (called the characteristic frequency) N_0^* is given by (see Ref. 111)

$$N_0 = \frac{1}{2\pi\sigma_0} \left[\int_{-\infty}^{\infty} \omega^2 \phi_0(\omega) d\omega \right]^{\frac{1}{2}} \quad \dots (7.4.1)$$

where,

$$\sigma_0 = \left[\int_{-\infty}^{\infty} \phi_0(\omega) d\omega \right]^{\frac{1}{2}} = \bar{A} \sigma_i \quad \dots (7.4.2)$$

*The parameter N_0 denoting the number of times per second the response time history crosses the zero mean value with a positive slope is sometimes called the radius of gyration of the spectrum about the vertical axis.

An inspection of equations (7.4.1) and (7.4.2) indicates that the number of zero crossings does not depend on the r.m.s. value of the gust velocity ($\bar{w}_G$ or σ_i) of the input turbulence but depends on its normalised PSD distribution (see Figs. 77-78) which relates the structural parameter and the gust response function through the transfer function of the aircraft. The number of exceedances per second and with positive slope ($\frac{dx}{dt} > 0$) of the level x for a single normal process (with zero mean) is (see Ref.111)

$$N_x = N_0 e^{-\frac{x^2}{2 \bar{A} \sigma_i^2}} \quad \dots(7.4.3)$$

$$= N_0 e^{-\frac{x^2}{2 \sigma_0^2}} \quad \dots(7.4.4)$$

where N_0 , the number of zero crossing per second with positive slope is given by equation (7.4.1).

7.5. The Distribution of Maxima from the Statistical Analysis

Cartwright and Longuet-Higgins (Ref.112) studied the statistical distribution of the maximum values of a random function by considering it to be the sum of an infinite number of sine waves in random phase. They applied their results to sea waves and to the pitching and rolling motion of a ship. The ratio of the maximum value to the r.m.s. value of the response can be expressed as (see equation 6.1 of Ref.112)

$$\frac{\eta_{\max}}{\sigma} = \sqrt{\ln N_0} + \frac{\gamma}{2 \sqrt{\ln N_0}} \quad \dots(7.5.1)$$

where

$\eta_{\max}$ = the expected maximum response

σ = the r.m.s. value of the response

N_0 = the no. of zero crossings per sec with positive slope given by equation (7.4.1)

and γ = Euler constant = 0.5772....

Equation (7.5.1) is accurate to within 3% only and applies only to narrow band processes. A more accurate expression for the ratio for random processes which are not narrow banded can be written as (see Ref.112 again)

$$\frac{\eta_{\max}}{\sigma} = \sqrt{\ln \sqrt{1-\epsilon^2} N_0} + \frac{\gamma}{2 \sqrt{\ln \sqrt{1-\epsilon^2} N_0}} \quad \dots(7.5.2)$$

where ϵ is the spectral width parameter given by

$$\epsilon^2 = \frac{m_0 m_4 - m_2^2}{m_0 m_4} \quad \dots(7.5.3)$$

where

$$m_n = \int_0^{\infty} \omega^n \phi_0(\omega) d\omega \quad \dots(7.5.4)$$

Although the response PSD obtained in the present analysis is not narrow banded, the approximate formula given by equation (7.5.1) is used to determine the maximum response to

economize the computer time needed for evaluating the spectral width parameter (ϵ) expressed by equation (7.5.3)

7.6 Scope of the Calculations

As in the case of previous analyses, the aircraft is given two rigid and four elastic degrees of freedom in the symmetric motion. Strip theory has again been justified while calculating the two different types of oscillatory unsteady aerodynamic forces which act on the aircraft due to its motion and the external excitation respectively (see Appendix F). The response quantities calculated are the normal and angular accelerations* at various points on the structure with a particular emphasis on the former. Although both Dryden and Von Karman spectra are used to obtain the preliminary results, the detailed analysis is confined to the latter representation of the atmospheric turbulence because of its universal acceptance and close agreement with experimental data. The scale lengths of the atmosphere turbulence represented by the two spectra are respectively chosen as 75m, 150m and 300m at an aircraft speed of 30m/sec (see Figs.77-78) to evaluate the gust response function $\bar{A}$ in

each case. While determining the actual value of the absolute response the reference intensities are decided for the first three grades of the turbulence cited in Table 17 and thus omitting the fourth case designated as 'extreme' turbulence. The reasons are two fold. Firstly, the determination of the transfer function of a flexible aircraft from a sinusoidal gust input is based on the assumption $W_G \ll U$ (see Appendix F and Ref.49) where W_G is the maximum velocity of the sinusoidal gust and U is the forward speed of the aircraft ($U = 30$ m/sec in the present analysis); the assumption is likely to be violated in the 'extreme' turbulence case given in Table 17. The second reason is that sailplanes are not normally flown in extreme weather conditions.

The r.m.s. value of the normal acceleration at the pilot C.G. is evaluated from the normal and angular accelerations of the wing root

*The symmetric motion being considered everywhere the term angular acceleration is synonymous with pitching acceleration. The effect of structural damping is not considered in the analysis.

bearing in mind that the pilot-fuselage mass is a compact lumped mass rigidly connected to the wing root. Although remarks about the ride quality of the aircraft have been made from the calculated results, no attempt has been made to determine the stress distributions in the airframe from the calculated response to check whether the design satisfies the strength requirements or not.

In determining the mean square value of the response, numerical integration is carried out to evaluate the integrals of the PSD curves whose frequency limits should ideally be from zero to infinity. In the preliminary set of calculations, the upper limit of the frequency is truncated at a suitable point where the convergence criterion is adequately satisfied. However, to account for the tail end of the PSD curves which extend upto infinity, a log-log plot (of the PSD versus frequency) is made which eventually became a straight line in the high frequency range and thus indicated an exponential decay of the response PSD curves. The rate of the decay is determined from the slope of the log-log plot and this enabled the computation of the rest of the integration upto infinity. However, in the present calculation, a finite upper limit of integration of about $\omega = 150$ rad/sec gave sufficiently accurate results.

The maximum normal acceleration at the pilot C.G. is calculated from its r.m.s. value using a statistical distribution. The method assumes the random process to be narrow banded which is not quite true in the present analysis. Therefore, the maximum response so obtained cannot be fully trusted and the results concerning only the r.m.s. values can be judged with all reliability. The other limitations on the use of PSD approach and transfer function representation of flexible aircraft are explained in Appendix F.

7.7

Appraisal and Discussion of Calculated Results

The Dryden and Von Karman spectra to which the Ricochet with a forward speed of 30 m/sec is subjected to are respectively shown in

Figs. 77 and 78 for various scale lengths of the turbulence. The difference between the two spectra in the frequency range between 5 to 20 rad/sec has been shown in Fig. 79 for a typical scale length of 75 m. It is clear from Fig. 79 that the Dryden spectrum having less energy content than the Von Karman will underestimate the response PSD particularly at higher frequencies. In addition to this fact, the Von Karman spectrum being based on statistical data is manipulated with more rigour at various stages of the calculations. The transfer functions* which relate the input PSD to the output PSD of the normal accelerations for the three spanwise station of the unballasted Ricochet are shown in Fig. 80. It is observed in Fig. 80 that the first peak of the transfer function corresponds to the short period frequency whereas the two subsequent peaks correspond to the first and second structural modes of the free vibrational characteristics. Quite clearly the flexibility which has been taken into account has important effects particularly near the tip region (see Fig. 80) unlike the case normally found with the rigid aircraft where the peak in the transfer function corresponds to the short period frequency alone. The present analysis did not show any more significant contribution of structural modes to the transfer function apart from the first two.+

The PSD distributions of the normal acceleration per unit r.m.s. gust input (i.e. $\frac{\Phi_0(\omega)}{\bar{W}_G^2}$)

at three spanwise stations namely tip, midspan and the root of the wing are respectively shown in Figs. 81-83, Figs 84-86 and Figs. 87-89 for various scale lengths of both Dryden and Von Karman spectra. Since the plotted quantities are the PSDs of normal acceleration divided by the mean square gust intensity

$\bar{W}_G^2$, the area under each curve shown in

* The terms transfer function, frequency response function, mechanical admittance and admittance function are quite often used synonymously, the reciprocal of each is the mechanical impedance.

+This is in contrast to the flutter mode where only the first elastic mode was significant

Figs. 81-89 gives the gust response function (described in section 7.1) in the respective case. As is evident from the input spectra shown in Figs. 77-79, the results in Figs. 81-89 indicate that the response due to the Von Karman spectrum is higher than that in the case with the Dryden. For a specific point on the wing the variation of gust length (e.g. see Figs. 81-83 for the wing tip) has an effect that the increase in gust length results in the decrease in the gust response function $\bar{A} = \frac{\sigma_o}{\sigma_i}$ as the area

under the output PSD curves per unit r.m.s. gust velocity reduces with the increase in scale lengths. It is also important to note that the effects of the structural modes are important for the calculated response PSD at the tip (see Figs. 81-83) and midspan (see Figs. 84-86) of the wing. However, at the root the contribution from the structural modes is practically nil (see Figs. 87-89) and all that matters is the short period mode. Analysis is carried out in a similar way on the ballasted Ricochet and the transfer functions obtained for the three spanwise stations (tip, midspan and the root) are shown in Fig. 90. The PSD's of the normal accelerations per unit r.m.s. gust input for a particular scale length of 75m for both the Dryden and Von Karman spectra are shown in Figs. 91-93 for the respective three points on the ballasted wing. Here again the effect of flexibility is not significant at the root (see Fig. 93) whereas it is important at the midspan and tip of the wing (see Figs. 91 and 92).

The gust response functions of the normal and angular accelerations at various spanwise stations of the unballasted Ricochet are respectively given in Tables 18 and 19 for different scales of turbulence. Similar results for the ballasted case are provided in Tables 20 and 21. The results quoted in Tables 18-21 have to be multiplied by the respective r.m.s. gust intensity ($\bar{w}_G$) to obtain

the actual r.m.s. accelerations. It is observed from the results in Tables 18 and 20 that for both unballasted and ballasted cases, the gust response function of the normal acceleration gradually reduces from the tip towards the

root for any particular scale length of the turbulence. This is a phenomenon expected with an aircraft having considerable flexibility. However, the results in Tables 19 and 21 showing the gust response functions of angular accelerations for the unballasted and ballasted cases respectively show a different situation. The gust response function of the angular acceleration for a particular scale of turbulence is found to increase slightly from the tip towards the root. A further observation of results in Tables 18-21 indicates that the gust response function of the normal acceleration for a particular scale length assumes a higher value in the unballasted case whereas the angular acceleration response function is smaller.

The gust response functions listed in Tables 18-21 will enable calculations of r.m.s. values of the actual response (normal or angular acceleration) from a known reference gust intensity ($\bar{G}$). The value of the r.m.s. gust velocity ($\bar{W}_G$) embedded in the turbulence (see Figs. 77-78) is related to the reference gust intensity ($\bar{G}$) by equation (7.2.1).

However, it has already been explained that both the gust response function ($\bar{A}$) and the r.m.s. gust velocity ($\bar{W}_G$) vary with the scale length of the turbulence in such a way that for a fixed reference intensity $\bar{G}$ their product which gives the r.m.s. value of the absolute response remains the same for all scale lengths. This implies that for a fixed reference intensity ,

the r.m.s. value of the response is independent of the scale length of the turbulence. From the categorized grades of turbulence in Table 17, the light, moderate and severe turbulence cases are investigated in this thesis. The three aforementioned cases have reference intensities of 0.9, 1.8 and 3.7 m/sec respectively and the selection of their values is primarily based upon the statistical distributions presented in Fig. 94 (Ref.97) which gives the probability of equalling or exceeding a given reference intensity. The gust response

functions only corresponding to the Von Karman spectrum as input are now considered in evaluating the response due to various grades of turbulence. Tables 22 and 23 for the respective unballasted and ballasted case show the r.m.s. normal accelerations at three span-wise stations due to the Von Karman spectrum under various grades of turbulence. The corresponding values of the r.m.s. angular accelerations are contained in Tables 24 and 25. A comparison of results given in Tables 22 and 23 indicates that for a particular grade of turbulence, the r.m.s. normal acceleration at a point is higher in the unballasted case whereas the results shown in Tables 24 and 25 suggest that the corresponding r.m.s. angular acceleration is slightly lower in that case. Table 26 provides the r.m.s. normal accelerations at the pilot C.G. for both unballasted and ballasted cases under various grades of turbulence. The results presented in Table 26 are well within the acceptable limit. The order of magnitude of the r.m.s. normal acceleration being less than 0.5g is in agreement with pilots' normal experience on meeting turbulence. The unballasted case gives a higher r.m.s. normal acceleration at the pilot C.G. as shown in Table 26. The maximum normal acceleration at the pilot C.G. of the unballasted Ricochet due to the three selected grades of turbulence is stipulated in Table 27. These results are obtained using equation (7.5.1) which is a simplified expression applicable only to a narrow banded random process. It is therefore subject to an unfair relevancy when it is already known that the PSD distributions of the output response of the Ricochet are not narrow banded. Although the amount of inaccuracies cannot be pinpointed, the results in Table 27 do not show any uncharacteristic feature and are all well within the tolerable limit.

The probability of exceeding a particular normal acceleration by various points on the unballasted wing of the Ricochet under various grades of turbulence is evaluated following the method described in section 7.3, Table 28 shows the probability of exceeding normal acceleration values of 1, 2 and 3 m/sec² by the tip, midspan and root of the unballasted wing under different grades of turbulence. As expected the probability of exceedance increases

with the severity of the turbulence and also when the point considered is further away from the root. The results in Table 28 may also provide information related to the calculation of limiting values of the estimated stress levels. The probability of equalling or exceeding a particular value has been determined from the probability density function with the use of equation (7.3.1). Typical distributions of the probability density function of normal accelerations of the wing tip under light and moderate turbulence are shown in Fig. 95.

Based on the procedure described in section 7.4, the numbers of occasions of crossing zero or any particular normal acceleration per unit time with positive slope (i.e. the no. of crossings in one direction) by various points of the unballasted Ricochet are calculated under various grades of turbulence. It is worth noting that the no. of zero crossings is independent of the reference intensity of the turbulence whereas the number of crossings of a particular value is directly related to it. Table 29 gives the number of times in a second the normal accelerations of the three points on the wing will cross the zero level with positive slope. The numbers of occasions of crossing normal acceleration values of 1, 2 and 3 m/sec² per second with positive slope (i.e. in one direction) by the three points on the wing are provided in Table 30.

7.8

Remarks about the Calculated Results

The Ricochet did not show any uncharacteristic behaviour from the ride quality point of view. The normal accelerations at the pilot C.G. are found to be what is generally expected. However, the speed investigated being low at 30 m/sec, the response characteristics at higher speeds cannot be easily predicted. The r.m.s. normal and angular accelerations obtained will prove useful to calculate design loads in dynamic conditions. The flexibility played a significant role, particularly near the tip region and it may well be necessary to check whether the dynamic distribution of stresses can cause any overloading in the structure at higher speeds. This demonstrates the need to

account for the aircraft response and in particular the effects of aircraft pitching and wing bending in the loading calculations.

CHAPTER 8

8.0 RESPONSE OF THE RICOCHET TO DISCRETE GUSTS

Although the PSD method used in Chapter 7 has wide applications, some of the response problems are more adequately treated by the discrete gust approach (see Refs. 89 and 94). An analysis of the acceleration response of the flexible Ricochet due to discrete gusts* is carried out here in detail. The method used is based on Refs. 94 and 96 with a particular application to flexible aircraft. It is assumed that the aircraft encounters a discrete gust of a specific profile - "One-minus-cosine" (smooth ramp gust) whose velocity distribution $V_g(x)$ is shown in Fig. 96 where H is the gradient distance (sometimes called the gust length) and V_{g_m} is its maximum velocity (intensity of the gust). The distribution of the gust velocity is given by (see Fig. 96)

$$V_g(x) = \begin{cases} 0 ; & x < 0 \\ \frac{1}{2} V_{g_m} \left(1 - \cos \frac{\pi x}{H} \right) ; & 0 \leq x \leq H \\ V_{g_m} ; & x > H \end{cases}$$

...(8.1)

and

$$\begin{aligned} V_{g_m} &= H^{\frac{1}{3}} U_0 ; & H \leq L \\ &= L^{\frac{1}{3}} U_0 ; & H \gg L \end{aligned}$$

...(8.2)

where L is the scale length in equations (8.2)

*Only the vertical component of the gust is considered in the analysis. The horizontal component in some cases may be important because sailplanes have comparatively low forward speeds. Structural damping is not considered in the analysis.

U_0^* in equations (8.2) which decides the maximum velocity of the gust is considered to be an overall 'gust strength parameter' which specifies a particular family of equi-probable design gusts (see Ref. 94 and Fig.99). Bennett in his recent report (Ref.95) on revised flying requirements of service aeroplanes has expressed U_0 in terms of reference intensity ($\bar{\sigma}$) of the gust as (in SI units)

$$U_0 = 1.45 J \left(\frac{1}{750} \right)^{\frac{1}{3}} \bar{\sigma} \quad ; \quad H \leq L$$

...(8.3)

$$= 1.45 J \left(\frac{L}{750} \right)^{\frac{1}{3}} \bar{\sigma} \quad ; \quad H \geq L$$

The first of equations (8.3) is applicable for the present analysis because the gradient distance (H) appropriately chosen for calculation is much smaller than the scale length (L). The factor J in equations (8.3) is a discrete gust scaling factor which is still a subject of considerable research. Strictly speaking J is a function of altitude and also of the type of turbulence. Until further information is available, the recommended value of $J = 4$ (see Ref. 95) is used in the analysis. Therefore, equations (8.1), (8.2) and (8.3) together with Fig. 96 completely define the profile of the discrete gust (single smooth ramp) to which the flexible aircraft is subjected. The aircraft behaviour is assumed to be linear and calculations have been made for $U_0 = 1$. The results for other values of U_0 given by equations (8.3) are treated by linear scaling. A pair of ramp gusts which is a combination of two single smooth ramp gusts of opposite sign is shown in Fig. 97. In the calculation procedure, the length of the gust (or the gradient distance) H is varied to give the most critical

* U_0 has the dimensions of $\frac{[Vel]}{[Length]^{\frac{1}{3}}}$

aircraft response either to a single smooth ramp gust or to a combination of two. The forward speed of the aircraft is set to a constant value. The program developed by Purcell (Ref.96) based on the theory presented by Jones (Ref.94) has been fitted to the main program. The response quantities of interest are the normal and angular accelerations of the wing root of the Ricochet. The full details of the method and a significant portion from Refs. 94 and 96 are presented in the Appendix G.

8.1

Response to a Single Smooth Ramp Gust

Mitchell (Ref. 86) has introduced a Fourier transform method to find the response of a flexible aircraft to a unit step and thus avoided the direct evaluation of the Küssner functions. If the transfer function (or the frequency response) of an aeroplane is known Bromwich's integral formula (Refs. 49 and 132) provides an explicit formula for calculating the response to a unit step (see Appendix G). The response to a unit step is known as the indicial admittance of the aeroplane. Once the response to a unit step is known, the response to a gust of any arbitrary profile which can be approximated as n-step gusts can be found out through the linearity assumption of simple addition. However, for a smooth ramp gust (one-minus-cosine) shown in Fig. 96, the direct evaluation of the Duhamel's integral is possible and the question of approximating the gust profile by a finite number of steps does not arise (see Appendix G and Ref.96). Firstly from the response of the flexible aeroplane to a unit step gust velocity, the response to a single smooth ramp gust is found for a series of trial values of the gust length (gradient distance) H . Then from the maximum response $\gamma(H)$ due to a single smooth ramp gust for each of the trial set of gradient distances (H), a safeguarded quadratic scheme outlined by Purcell (Ref. 96) is used to find the particular gradient distance $\bar{H}$ which causes the maximum positive or negative response ($\gamma_+(\bar{H})$ or $\gamma_-(\bar{H})$).

Fig. 98 gives an idea how the tuned gust

length $\bar{H}$ is found out when the gust strength parameter U_0 is considered to be unity.

Equation (G.2.1.7) in Appendix G gives the measure of sharpness of the peak in $\gamma(H)$ at the point $\bar{H}$ and is defined as gust length sensitivity by Jones (Ref.94). So the calculation procedure involves the evaluation of the extreme positive and negative response $\gamma_+(\bar{H})$ and $\gamma_-(\bar{H})$ respectively and the corresponding time $\bar{t}_+$ and $\bar{t}_-$ when they occur from a supplied set of trial values of gradient distances H .

8.2

Response to a Pair of Smooth Ramp Gusts

The possible resonance effect associated with a pair of smooth ramp gusts of opposite sign (see Fig.97) is also assessed. The distance H_s in Fig. 97 is the distance of separation of the two gusts. The most adverse pair of smooth ramp gusts is deduced as follows.

After having calculated the particular gradient distances $\bar{H}_+$ and $\bar{H}_-$ for which the maximum positive and negative response respectively $\bar{\gamma}_+$ and $\bar{\gamma}_-$ occur and the corresponding time $\bar{t}_+$ and $\bar{t}_-$ of their occurrence, the primary peak with the larger of $\bar{\gamma}_+$ and $\bar{\gamma}_-$ is identified, the smaller becoming the worst overswing (see Ref.96 and Appendix G for details). Then the maximum possible response is $\bar{\gamma}_+ + |\bar{\gamma}_-|$ occurring at $\bar{t}_+$ or $\bar{t}_-$ whichever is later.

If it is assumed for clarity that $\bar{t}_- > \bar{t}_+$ then as explained in Appendix G (and also in Ref. 96) the worst gust pair is a ramp gust of length $\bar{H}_-$ followed by one of opposite sign of length $\bar{H}_+$ ($H_1 = \bar{H}_-$ and $H_2 = \bar{H}_+$ in Fig. 97) and separated by a distance H_s (see Fig. 97) given by

$$H_s = V (\bar{t}_- - \bar{t}_+) - \bar{H}_- \quad \dots(8.2.1)$$

where V is the forward speed of the aircraft.

8.3 Brief Outlines of the Computer Program

Although the development of the computer program to find the response to a discrete gust has been discussed to some length in section L.3.2 of Appendix L, it is necessary to mention here the salient features of the program based on that of Purcell (Ref. 96). The program developed by Ref. 96 derives the response to various types of ramp gusts when the response to a unit step is already known. The main input data needed for the program are the response to a unit step in the time domain, the forward speed of the aircraft and the trial gust lengths within the range of which the maximum response is expected to occur. Purcell's program is connected to the main program which evaluates the indicial admittance of the aircraft (i.e. the response to a unit step gust) from its transfer function using a fourier transform. In the computation of results, the gust strength parameter U_0 (see equations 8.2 and Figs. 98-99) is considered to be unity so that the magnitude of the largest response peak $U_0 \gamma(\bar{H})$ associated with the tuned gust length $\bar{H}$ becomes $\gamma(\bar{H})$, the tuned response (see Fig. 98). The normal and angular accelerations at the wing root of both unballasted and ballasted Ricochet due to a single and pair of smooth ramp gusts are investigated and the worst cases of the extreme response are spotted.

8.4 Numerical Calculation of Results

The transfer functions of the flexible Ricochet (see Figs. 80 and 90) which have earlier been calculated in Chapter 7 to investigate the response to continuous atmospheric turbulence are again used through equations (G.1.3) - (G.1.13) of Appendix G in evaluating the response due to a unit step gust velocity. The calculations of response quantities are confined

to the normal and angular accelerations of the wing root of the Ricochet at its forward speed of 30 m/sec. The normal accelerations of the wing root for both unballasted and ballasted Ricochet due to a unit step gust velocity input are respectively shown in Figs. 100 and 106. The corresponding angular accelerations are shown in Figs. 103 and 109. The results displayed in Figs. 100, 106, 103 and 109 are employed to investigate the response to a single and pair of smooth ramp gusts whose profiles have earlier been defined in Figs. 96 and 97 respectively. First of all, the two independent isolated single smooth ramp gusts are determined which give rise to the two extreme responses namely, the maximum positive and the maximum negative response at the wing root of the Ricochet. These two isolated gusts are evaluated for a unit gust strength parameter and are identified by their respective gust lengths. Finally they are added with the appropriate distance of separation and one acting in the opposite direction of the other to form the most adverse pair (see Fig. 97) causing the worst response.

To find the particular single smooth ramp gusts which give rise to the maximum positive and maximum negative normal accelerations at the wing root of the unballasted Ricochet, trial gust lengths of value 5, 10, 20 and 30 metres are chosen as shown in Tables 31 and 32 respectively to determine the most adverse case. The interpolated gust lengths corresponding to the maximum positive normal acceleration with the adequate convergence criterion (0.001m in the present analysis) satisfied are shown in Table 31. The calculations settle down to the required tuned gust length $\bar{H}$ which causes the maximum positive normal acceleration at the wing root of the unballasted Ricochet (see bottom half of Table 31). In this particular case, the tuned gust length $\bar{H}$ as shown in Table 31 is 9.33m. The corresponding maximum normal acceleration (γ_+) and the time of its occurrence (t^+) are respectively 5.47 m/sec² and 0.285 sec as reported. The sensitivity factor indicating the sharpness of the peak of $\gamma(H)$ versus H curve at the tuned gust length $\bar{H}$ is found to be 0.326. The time variation of the normal acceleration of the wing root for this

particular case is illustrated in Fig. 101. Similar investigations are made to obtain the maximum negative normal acceleration at the wing root of the unballasted Ricochet by inputting the trial gust lengths of 5, 10, 20 and 30m into Purcell's program. The results obtained are stipulated in Table 32. The time plot of the normal acceleration due to the tuned gust which causes maximum negative acceleration at the wing root is shown in Fig. 102. In a very similar way the ballasted wing cases for maximum positive and negative normal accelerations are investigated and the results so obtained are respectively reported in Tables 33 and 34. The corresponding plots of the normal accelerations against time for this ballasted case due to the single smooth ramp gusts are illustrated in Figs. 107-108. Analysis has also been carried out to find out the maximum positive and negative angular accelerations at the wing root of both the unballasted and ballasted Ricochet due to isolated single smooth ramp gusts. The computed results for the two cases are respectively stipulated in Tables 35-36 and Tables 37-38. The illustrations are accordingly described in Figs. 104-105 and Figs. 110-111. The most adverse pair of smooth ramp gusts is then constructed by using the two tuned gusts which cause respectively the maximum positive and negative responses (see section 8.2). The worst response cases of the wing root normal accelerations of the Ricochet due to single and pair of smooth ramp gusts for both unballasted and ballasted cases are contained in Table 39. The corresponding angular acceleration cases are reported in Table 40. The gust strength parameter (U_0) is considered to be unity in all the calculations reported above. Now with the use of equations (8.3) and Table 17, the gust strength parameter U_0 for various grades of turbulence can be calculated and finally be used as scaling factor to obtain the actual response. The maximum normal accelerations at the wing root of the Ricochet due to single smooth ramp gusts with varied grade of gust intensity are given in Table 41. The corresponding results for a pair of smooth ramp gusts are reported in Table 42. It has been found that the particular gust lengths which cause the maximum normal acceleration at a particular time do not give rise to an

appreciably large amount of angular acceleration. The maximum normal accelerations at the pilot C.G. of the unballasted Ricochet due to a single and pair of smooth ramp gusts with varied grade of intensities are given in Table 43. A comparison between the results obtained from the PSD approach in Chapter 7 and the discrete gust method in this Chapter is made in Table 44.

8.5

Discussion of Results and Remarks

The wing root of the Ricochet at a forward speed of 30 m/sec is analysed under varied conditions of discrete gusts. The maximum normal accelerations at the wing root due to both single and pair of smooth ramp gusts have occurred in the unballasted case. A similar observation has been made while using the PSD method. The worst response (normal acceleration) due to a single smooth ramp gust occurs when the tuned gust length is 9.33m in the unballasted case (see Table 31). The pair of smooth ramp gusts which causes the extreme normal acceleration at the wing root of the unballasted Ricochet is made up of the two single smooth ramp gusts - one acting opposite to the other. The first gust in this extreme case has a length of 10.3m and second 9.3m and their distance of separation is 1.7m (see Table 39). The corresponding plots against the time axis suggest that the response is of practical importance only upto about two seconds after the start of excitation. The worst response cases of the angular accelerations at the wing root are summarized in Table 40. Unlike the maximum normal acceleration case, it is found in Table 40 that the ballasted Ricochet wing root experiences a slightly higher angular acceleration. This has also been observed while following the PSD approach in Chapter 7. However, the plots in the time domain shown in Figs. 104-105 and Figs. 110-111 respectively for the unballasted and ballasted cases suggest that angular accelerations upto a time of about 1.5 sec after the start of the excitation are of physical significance. An inspection of Table 41 which gives the maximum normal acceleration at the wing root of the Ricochet due to a single smooth ramp gust under varied grades of turbulence, indicates that the gust with severe intensity ($\bar{q} = 3.7\text{m/sec}$)

can cause a maximum normal acceleration of 12.92 m/sec^2 at the wing root of the unballasted Ricochet. The maximum normal acceleration due to a pair of gusts also occurs in the unballasted case and is of magnitude 20.124 m/sec^2 in the severe grade of gust intensity ($\bar{u} = 3.7 \text{ m/sec}$) as reported in

Table 42. Normally, sailplane designers become concerned when any part of the structure exceeds an approximate limit of the normal acceleration of $3g$. It would be desirable to check the distribution of stresses in the airframe due to severe gust conditions. The maximum normal acceleration at the pilot C.G. which expectedly occurs in the severe grade of gust intensity ($\bar{u} = 3.7 \text{ m/sec}$) is 20.3 m/sec^2 . Although this normal acceleration is tolerable to an average pilot, it can be unpleasant at times. In Table 44 a comparison has been made between the results obtained from the discrete gust method and the PSD approach used earlier in Chapter 7. The ratio of the maximum response from the discrete gust method to that of the r.m.s. response obtained from the PSD method is considered to be the standard basis of comparison. The ratio which has turned out to be 3.94 and 3.75 respectively for the unballasted and ballasted cases agrees very well with the figures obtained by the gust analysts in the U.K. (The gust analysts in the Royal Aircraft Establishment, U.K. have reported the ratio to be within 3 to 4). Since the complete analysis has been carried out for an aircraft speed of 30 m/sec , it would be of necessity to examine the aircraft behaviour to discrete gusts at higher speeds.

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CHAPTER 9

9.0

SUMMARY OF DISCUSSION AND CONCLUSIONS

As has been described earlier, this work stems from the promising performance gains indicated in the design study of a tailless sailplane (the Ricochet design - Ref.37) and the uncertainty as to whether this achievement would be made impossible by aeroelastic limitations. The present research does indeed indicate severe and unacceptable limitations of this nature on the design as put forward. The parametric study undertaken has moreover indicated that it is unlikely that any tailless sailplane of the Ricochet type will be free from similar limitations as the main geometric variations which can improve the flutter characteristics are those which would lead to the loss of the performance advantages claimed for the design. This will be particularly true for other classes of tailless sailplane with aspect ratios and material of construction similar to the Ricochet. The parametric study mentioned above does not take into account any variation of aspect ratio and material of construction of the basic aircraft. However, specific conclusions drawn from the Ricochet analysis are extended to the general class of tailless sailplanes.

The research has revealed that the Ricochet in its present form with 13° sweep and 100 Kg m^2 pitching moment of inertia is unacceptable because of its low symmetric flutter speed at 38 m/sec. The type of flutter found with the Ricochet consists of an overall flapping motion of the complete aircraft dominated by the up and down movement of the fuselage and centre wing. An upward movement of the fuselage at flutter speed is accompanied by a downward movement of the wing tip and vice versa. Unlike conventional aeroplanes, the fuselage of the Ricochet undergoes maximum displacement at the flutter speed and the resulting motion is not due to the coupling between bending and torsional modes usually found in classical flexure-torsion flutter but is the direct result of the coupling between the short period oscillation and the first bending mode in the symmetric motion.

Significant improvements in the flutter characteristics of the Ricochet result from increasing its pitching moment of inertia and/or reducing its angle of sweep as attempted in the parametric study in Chapter 5. However, to remain within the scope of tailless design with particular reference to the Ricochet, it may not be realistic to reduce the sweep mainly from the point of stability and control efficiency; and so without any alteration of sweep, a pitching moment of inertia value in the order of 250 Kg m^2 (of which a small fraction of about 40 Kg m^2 may be supplied by the swept wings) would be needed for an acceptable flutter speed. It is difficult, if not impossible, to achieve this pitching moment of inertia from a tailless layout particularly in the class of tailless sailplanes to which the Ricochet belongs.

To comment on possible flutter characteristics of the general class of tailless sailplanes, it must be stressed that the aircraft behaviour is predominantly dependent on both the structural properties involving inertia and stiffness distributions as well as the aerodynamic properties associated with the geometry of the aircraft. Changes in these two properties will expectedly produce changes in the flutter characteristics to a certain extent. Structural modifications to the Ricochet whose metallic wings have as much torsional stiffness as is necessary, might include carbon fibre spar reinforcement to improve the flexural stiffness. This would be helpful because of the large part which flexural modes play in the Ricochet flutter mode. The FRP in general has higher bending stiffness than the aluminium alloy and is thus a novel alternative. The relative positions of the mass and shear centre of the wing sections are unlikely to be able to be altered sufficiently to affect the flutter speed because of the small chord of the high aspect ratio wing. The scope for modifications of aerodynamic parameters such as the aspect ratio, wing chord, taper ratio etc. is rather limited if the sailplane has to remain within the 15m class span. However, the situation can be improved by incorporating design modifications on structural and aerodynamic parameters. But for the class of tailless sailplanes like the Ricochet, there is no replacement for the inadequacy of the pitching moment of inertia value of the aeroplane. Even if the torsional rigidity provided to the wings is sufficiently

high, there still exists a possibility that the short period mode can couple with the first bending mode to give rise to an overall instability of the flutter type without involving elastic distortion of any torsional mode. Of course, if such an interaction of the rigid body motion with the elastic modes in bending occurs at a sufficiently high speed, the instability can be of no serious concern. It may be emphasized again that an important way to bring about a change in the design of the Ricochet is to use composite materials like the FRP instead of the aluminium alloy. However, before taking such a step, it would be useful to study the effect of the relative magnitude of the bending and torsional stiffnesses of the wing on the flutter behaviour of the aeroplane.

The reduction in both aspect ratio and sweep may improve the flutter characteristics of a tailless sailplane. But this can have adverse effect on the higher performance and overall controllability of the aircraft and therefore, presents a difficult compromise. On the basis of the information available to date, it is quite apparent that none of the tailless sailplanes which existed in the past resembles to any appreciable extent the Ricochet with the high aspect ratio of its metallic wings ($A.R = 22$). Evidence (Ref.1) indicates that the historic tailless sailplanes like the BKB, Horten gliders and Fauvel* had wings (mainly of wooden construction with larger areas and thicker chords) of aspect ratios ranging between 10 to 12. The BKB and the Horten gliders had a reasonable amount of sweep in the leading edge of approximately the same magnitude of 13° as in the case of the Ricochet whereas the Fauvel construction - interestingly enough, did not rely on sweeping the wings but had thicker chords with larger wing areas. It is again a matter of achievable objectives to benefit from the earlier design in the overall perspective of high performance gain.

The flutter analysis of the Kestrel has shown excellent agreement with experimental and flight test results and has provided an important

* see Jane's World Sailplanes and Motor Gliders, First published in 1978 by Macdonald and Jane's Publishers Ltd., London.

basis of reference for comparison. Apart from the marked difference in the flutter speed (the Kestrel wings have a flutter speed of 70 m/sec), the distinguishing feature of the calculated results is that the rigid body modes do not play any significant role in the flutter characteristics of Kestrel wings whereas they are the chief contributors to the flutter motion of the Ricochet. It has been established that the Kestrel wings flutter because of the interaction between the fundamental bending and torsional mode. This is a normal phenomenon with wings of conventional aeroplanes. The other interesting observation in the comparative study is about the frequency of oscillation at flutter speed. The flutter frequency for the Ricochet is 8.7 rad/sec whereas it is 53.8 rad/sec for the Kestrel wings. The low frequency instability found with the Ricochet, being an overall instability of the complete aircraft, indicates a kind of uncertainty in its pitching behaviour whereas the analysis in the case of the Kestrel presents itself as a distinct case of a wing flutter problem.

The short period oscillation characteristics of the Ricochet suggest that the flexibility of the aircraft is an important factor which decides the short period frequency and the associated damping in the mode. The investigation shows that the flexibility has caused increase in the frequency and reduction in the damping of the short period mode. However, the speeds chosen for the analysis are not appreciably below the flutter speed. When the flutter speed of 38 m/sec of the Ricochet is approached from a lower speed of 25 m/sec, it is found that the short period frequency gradually increases with the reduction in damping. Finally at the flutter speed, the short period frequency coincides with the flutter frequency giving rise to a self sustained motion with no damping. As expected the rigid body treatment of the sailplane leads to erroneous results. Since the calculation of the short period oscillation characteristics of the Ricochet is limited to the lower part of its speed range, the analysis of its modified design at higher speeds would be of interest. It is important in any case to include the effect of flexibility while evaluating the short period mode.

The response of the Ricochet to random atmospheric

turbulence indicates that the r.m.s normal acceleration at points further away from the root is dependent on the flexible modes of distortion of the wing. However, at the prescribed aircraft speed of 30 m/sec, the flexibility has not influenced the response at the wing root. The r.m.s. normal acceleration at the tip of the Ricochet is, in general, found to be about 2.5 times than that at the wing root. This result is in contrast to the flutter modes of the Ricochet which indicated a larger deflection at the root rather than at the tip. The presence of the external exciting force in the forced vibration case is the contributory factor to this change. It would be worth investigating whether large normal acceleration at the wing tip due to turbulence could cause overloading in the structure. In the severe grade of turbulence (gust intensity : 3.7 m/sec) the r.m.s value of the normal acceleration at the wing tip of the Ricochet is 8.3 m/sec². The r.m.s. normal acceleration at the pilot C.G corresponding to this condition has been calculated to be 4.8 m/sec². This is well within the acceptable range. The calculated number of times a point on the deformable aircraft crosses a particular valued normal acceleration with positive slope per unit time would provide the basis for the study of stress cycles at critical points on the wing. Provided the aircraft is redesigned, the response characteristics at speeds appreciably higher than 30 m/sec can be explored with profit.

The analysis of the response to discrete gusts has been restricted to the wing root of the Ricochet apart from its constant forward speed of 30 m/sec. The maximum normal acceleration at the wing root and also at the pilot C.G is found to be approximately of the value 2g in a condition when the aircraft is subjected to a pair of smooth ramp gusts(one-minus-cosine profile) with the severe grade of intensity (gust intensity : 3.7 m/sec). Even though this normal acceleration can be tolerated by a pilot, it can be unpleasant at times. The acceleration in the flexible part of the aeroplane due to various types of discrete gusts has to be investigated to fulfil the strength criteria. Of course, the normal or angular accelerations at various spanwise stations do not bear an explicit implication of the loading on the

structure unless the net bending moments and shear forces are calculated with the effects of flexibility at various points on the wing. The time dependency of the root response obtained for varied gust conditions may further enable an assessment of the ride quality of the aeroplane. The ratio of the maximum response obtained from the discrete gust analysis to the r.m.s response obtained from the PSD method for the Ricochet is found to be within the range of 3 to 4. This agrees very well with the experiences of gust analysts in the U.K. It is desirable to repeat the calculations at higher speeds when the aeroplane is redesigned for a much higher flutter speed.

The computer programs developed during the course of this research are fairly general purpose and can be used for aeroelastic calculations under conditions stipulated in this thesis.

In the final comment, it can be said as a contradiction to the unhappy conclusion registered in this thesis that there is still room for exploration of sensitive design variables to arrive at a reasonable compromise and make the tailless sailplane design a possible possibility.

CHAPTER 10

10.0 RECOMMENDATION FOR FURTHER WORK

Although the longitudinal dynamics of the tailless sailplane (The Ricochet) under investigation has been given extensive treatment, there are quite a few important areas where further research can be carried out. First of all, the information laid down about the aeroelastic characteristics of the Ricochet would provide critical data for designing a tailless sailplane, particularly to achieve an acceptable flutter speed. The response calculations will then have to be repeated for the modified aircraft at speeds which are more realistic than those necessitated by the existing design. In the analysis of symmetric flutter and response characteristics of the Ricochet, elevon motions have not been considered in this thesis. A complete investigation will need to include those freedoms.

Using composite materials like the FRP instead of aluminium alloy, could be an alternative way to bring about a change in the design of the Ricochet. However, this would first require the investigation of the effect of the relative magnitude of the bending and torsional stiffnesses of the wing on the aeroelastic behaviour and response characteristics of the basic aircraft. The effect of other aircraft parameters like the sweep, pitching moment of inertia and also the influence of the aspect ratio on the results should also be included.

The antisymmetric flutter of tailless sailplanes (in particular - the Ricochet) involving movements of the elevons has not been studied in this thesis and is therefore an important area for future investigation. The analysis of aileron (elevon) reversal speed is also an important area for future research. Of course, the phenomenon of aileron reversal is important from the point of control effectiveness rather than failure or damage of any structural part.

It is not expected that a tailless sailplane may suffer from classical divergence phenomenon in the same fashion as of a conventional tailed aircraft. The only divergence-like phenomenon

likely to occur is that of the central portion of the wing twisting relative to the tips. This needs investigation before any decisive remark about the associated uncertainties can be made.

With particular reference to the Ricochet, there are two extreme methods used in this thesis to analyse the short period oscillation characteristics. In one case, the net effects of the flexibility as well as unsteady aerodynamics have been accounted for whereas in the other case the aircraft has been considered rigid and to this effect, neither the flexibility nor the unsteady aerodynamics has been taken into account. It would be of considerable interest to apply a quasi-static method using modified aerodynamic derivatives to examine whether the results obtained with this method lie within the bounds of those using the above two methods.

A tailless aircraft in general is expected to suffer some loss of damping in its phugoid mode because of its low parasite drag. In this regard, the analysis of the phugoid motion of a tailless aircraft is an interesting area of exploration.

Sailplanes are slow flying machines and the effect of horizontal gusts and turbulence on them may be significantly important for their response characteristics as it affects the dynamic pressure distribution over the aircraft. An investigation in this regard should be included in any further work.

During the complete analysis in this thesis, the control surfaces of the aircraft have been considered to be rigidly connected to the main body with their angular settings at zero. The behaviour of the flexible aircraft subjected to various types of control inputs in the presence of gusty atmospheric condition with its unsteady aerodynamic effects would be a useful study. This is a challenging area, both in the context of the Ricochet analysis and also of academic interest involving additional efforts which will combine the discipline of the stability and control with that of the flutter and response.

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APPENDIX A

A.0. PARTICULARS OF THE RICOCHET

A.1. Structural Details of the Ricochet

The general arrangement of the Ricochet and some relevant details have already been shown in Figs. 1-5. Most of the structural and aerodynamic parameters of the Ricochet have been deduced from the drawings of the design study described in Ref.37. The details of the calculations made are not reported here but the gist of the calculations is produced in the form of graphs for final presentation. All important parameters needed for aeroelastic calculations are deduced and in some cases estimated whenno relevant data were available. Fig. 112 shows the distribution of the metallic area along the semi-span of the Ricochet Wing.* The second moment of area (I_a) distribution of the principal bending carrying members along the semi-span is shown in Fig. 113. The polar moment of inertia (sectional) $I_a + I_b$ along the semi-span is shown in Fig. 114 whereas the distribution of the shear centre and mass centre of the wing along the semi-span of the Ricochet are respectively shown in Figs. 115 and 116. The bending and torsional stiffnesses represented by EI and GJ are respectively shown in Figs. 117 and 118. The torsional stiffness (called structural GJ) is evaluated applying Bredt-Batho theory to torsional cell boxes of the wing structure. PAFEC (Program for Automatic Finite Element Calculation) uses the polar second moment of area (I_p in PAFEC notation) for beam elements for calculating polar mass moment of inertia ($I_p \times \rho$), and the torsional stiffness ($I_p \times G$). It is obvious that the true material density (ρ) and shear modulus (G) which are both material properties can only be used for torsional inertia

*The Ricochet wings are made of 6061-T6 Aluminium alloy whose density is 2720 Kg/m³.

properties and not for torsional stiffnesses because the section used for the aircraft wing is neither circular nor a solid one. The restriction is overcome by using some effective values of G defined as $G_{\text{eff}} = \frac{G_{\text{true}} \times J}{I_p}$

in the PAFEC analysis. G_{true} is the true shear modulus of the material; J is the Bredt-Batho torsional parameter and I_p is the second polar moment of area. All the details presented in Figs. 112-118 are used in the structural idealisation of the Ricochet wings as a series of beam elements.

A.2 Aerodynamic and Other Geometric Details of the Ricochet

A.2.1 Wings

Type	: Metallic
Span (b)	: 15m
Area (S)	: 10.26 m ²
Aspect Ratio (A)	: 21.93
Wing Root Chord (c_r)	: 0.73 m
Wing Tip Chord (c_t)	: 0.50 m
Wing Crank Chord (c_m)	: 0.73 m
Standard Mean Chord ($\bar{c}$)	: 0.683 m
Mean Aerodynamic Chord ($\bar{c}$)	: 0.689 m
Taper Ratio (λ)	: 0.685
$\frac{1}{4}$ Chord Sweep (Λ)	: 13°
Dihedral (Γ)	: 0.0°
Aerofoil Section	: Eppler

A.2.2 Elevons

Type	:	Plain
Span	:	2.625 m
Area (each)	:	0.283 m ²
Percentage Chord	:	17.5
Maximum Deflection Up	:	26.67 degrees
Maximum Deflection Down	:	13.33 degrees

The elevons extend from 60 to 95 percent of the semi-span.

A.2.3. Flaps

Inboard Segment:

Span	:	2.625 m
Area (each)	:	0.335 m ²
Percentage Chord	:	17.5
Maximum Deflection Up	:	10 degrees
Maximum Deflection Down	:	80 degrees
Lift Curve Slope (a_2)	:	0.70
Centre of Pressure (h_c)	:	-0.134

Outboard Segment:

Span	:	1.875 m
Area (each)	:	0.24 m ²
Percentage Chord	:	17.5

Maximum Deflection
Up : 6.7 degrees

Maximum Deflection
Down : 56.7 degrees

The inboard flap extends from 5 to 35 percent semi-span, the outboard flap from 35 to 60 percent semi-span.

A.2.4 Fuselage

Overall length : 3.4 m

Maximum Width : 0.585 m

Maximum Height : 0.82 m

A.3 Inertia Properties of the Ricochet

A.3.1 Masses

Wings (both) : 99.8 Kg

Fuselage : 55.33 Kg

Empty Mass : 155.1 Kg

Instruments : 6.8 Kg

Batteries : 2.27 Kg

Equipped Mass : 164 Kg

Maximum Pilot Mass
with above equipment : 113.4 Kg

Maximum Water Ballast : 90.7 Kg

Maximum All Up Mass : 368.3 Kg

Nominal mass used for
Stability and Control Calculations

(i) Without Ballast : 254.9 Kg

(ii) With Ballast : 345.6 Kg

A.3.2 Moments of Inertia

Taken about aircraft centre of gravity at all up masses of 254.9 Kg without ballast and 345.6 Kg with ballast.

Forward Centre of Gravity

Rolling Moment of Inertia (without Ballast)	: 1751 Kg m ²
Rolling Moment of Inertia (with Ballast)	: 2421 Kg m ²
Pitching Moment of Inertia (without Ballast)	: 95 Kg m ²
Pitching Moment of Inertia (with Ballast)	: 100 Kg m ²
Yawing Moment of Inertia (without Ballast)	: 1832 Kg m ²
Yawing Moment of Inertia (with Ballast)	: 2616 Kg m ²

Aft Centre of Gravity

Rolling Moment of Inertia (without Ballast)	: 1751 Kg m ²
Rolling Moment of Inertia (with Ballast)	: 2421 Kg m ²
Pitching Moment of Inertia (without Ballast)	: 93 Kg m ²
Pitching Moment of Inertia (with Ballast)	: 99 Kg m ²
Yawing Moment of Inertia (without Ballast)	: 1832 Kg m ²
Yawing Moment of Inertia (with Ballast)	: 2592 Kg m ²

A.4 Centre of Gravity Margins

Forward (percentage standard
mean chord) : 16.5% SMC

Aft (percentage standard
mean chord) : 21% SMC

A.5 Aerodynamic Stability and Control Derivatives

Two typical sets of the derivatives for the rigid aircraft are given below:

	C.G.forward	C.G.aft
Weight	345.6 Kg	254.9 Kg
Height	0.0 km	4.57 km
Speed (TAS)	118 Kts	55 Kts
Centre of Gravity	16.5% SMC	21% SMC
Aircraft Lift curve Slope	5.68	5.68
Static Margin	6.28% SMC	1.79% SMC
Manoeuvre Margin	9.7% SMC	4.8% SMC

Longitudinal, non dimensional, Stability Derivatives

x_u	-0.0093	-0.0191
x_w	0.0595	0.3264
x_q	0.0	0.0
z_u	-0.1452	-0.7972
z_w	-2.845	-2.85
z_q	-0.8883	-0.761

m_u	0.0	0.0
m_w	-0.1718	-0.051
m_q	-1.254	-1.311
$m_{\dot{w}}$	0.0	0.0

Longitudinal Control Derivative

Elevon Lift Curve Slope (a_2)	0.82	0.82
Elevon Centre of Pressure Location (h_e)	1.04	1.04
Inner Flap Lift Curve Slope (a_2)	0.7	0.7
Inner Flap Centre of Pressure Location	-0.134	-0.134

Lateral, non dimensional, Stability Derivatives

l_v	-0.0171	-0.0196
n_v	0.0079	0.0116
y_v	-0.0829	-0.0829
l_r	0.0422	0.2316
n_r	-0.028	-0.0269
l_p	-0.7206	-0.7212
n_p	-0.0199	-0.1094

Lateral Control Derivatives

Elevon Rolling Derivatives	-0.2598	-0.2173
Elevon Yawing Derivatives	0.0057	0.0263

Spoiler Rolling
Derivative (for full
deflection)

-0.0394

-0.0423

Spoiler Yawing
Derivative (for full
deflection)

-0.0056

-0.0056

APPENDIX B

B.0 PARTICULARS OF THE KESTREL

Slingsby's Kestrel-22m sailplane which reported to have suffered a wing flutter problem in the region of 140 Kts came to Cranfield at a late stage during the course of the main investigation. Structural and other details of the Kestrel have been evaluated and the results are declared in the form of graphs. The particulars of the Kestrel are not disclosed in sufficient details as a matter of the manufacturer's company policy. The FRP construction of the aeroplane offered certain difficulties in idealisation but they are overcome with suitable approximations.

B.1 Structural Details of the Kestrel

The general arrangement of the Kestrel has already been shown in Fig. 36. The bending and torsional stiffnesses of the wings, fuselage, tailplane and fin and rudder assembly are found out by theoretical calculations and then checked with experimental results. There was a fair degree of agreement between the two set of results, the experimental values are given more importance in the data preparation and in some cases a compromise between the theoretical and experimental values has been reached. Fig. 119 shows the EI distribution of the Kestrel wing along the semi-span whereas Fig. 120 shows its GJ distribution. The mass distribution along the semi-span of the Kestrel is shown in Fig. 121. The mass moment of inertia (I_p) per unit length along the semi-span is shown in Fig. 122. The EI and GJ distributions along the fuselage of the Kestrel (starting from the rear rigging point and up to the tail wheel axle) are respectively shown in Figs. 123 and 124. The rear rigging point from which the measurements have been taken is at a distance 2.57m aft of the nose of the aeroplane. Fig. 125 shows the mass distribution per unit length along the fuselage starting from and beyond the rear rigging point. Figs. 126 and 127 respectively show the EI and GJ distributions of the fin

along its height, whereas Fig. 128 gives its mass distribution per unit length. Similarly Figs. 129 and 130 show the EI and GJ distributions of the tailplane along its semi-span whereas Fig. 131 gives the corresponding mass distribution per unit length.

B.2 Geometric and Inertial Properties of the Kestrel

Material of Construction	: FRP
Span (b)	: 22m
Wing Area (S)	: 15.44 m ²
Aspect Ratio (A)	: 31.34
Wing Root Chord (c _r)	: 0.900 m
Wing Tip Chord (c _t)	: 0.360 m
Sweep Angle	: 0.0°
Mass of the Inboard Wing (each)	: 90 Kg
Mass of the Outboard Wing (")	: 25 Kg
Fuselage Mass with Equipments	: 180 Kg
Mass of Tailplane including Elevators	: 11 Kg
Mass of the Rudder Extension	: 1 Kg

APPENDIX C

C.O. PARTICULARS OF THE 15M CLASS STANDARD CIRRUS

The Standard Cirrus sailplane is constructed from glass-fibre reinforced plastic and being in the 15m class falls within the category of the Ricochet so that a comparison of the structural, aerodynamic and inertial parameters of the two aeroplanes can effectively be made.

C.1 Aerodynamic and Other Geometric Details of the Standard Cirrus

C.1.1 Wings

Type	:	FRP
Span (b)	:	15m
Area (S)	:	10 m ²
Aspect Ratio (A)	:	22.5
Wing Root Chord (c_r)	:	0.915 m
Wing Tip Chord (c_t)	:	0.35 m
Wing Crank Chord (c_m)	:	0.64 m
Standard Mean Chord ($\bar{c}$)	:	0.664 m
Mean Aerodynamic Chord ($\bar{\bar{c}}$)	:	0.707 m
Taper Ratio (λ)	:	0.3825
$\frac{1}{4}$ Chord Sweep (Λ)	:	0.4 Degrees
Dihedral (Γ)	:	4.0 Degrees
Aero Twist Root/Tip (θ_a)	:	1.5 Degrees
Aerofoil Section	:	Wortmann FX.66-S-196

C.1.2 Ailerons

Type	:	Plain
Span	:	2.6 m
Area (Each)	:	0.314 m ²
Percentage Chord	:	25.0
Maximum Deflection Up	:	26.67 Degrees
Maximum Deflection Down	:	13.33 Degrees

C.1.3 Horizontal Tail

Type	:	All Moving
Span	:	2.4 m
Area (S_T)	:	1.022 m ²
Aspect Ratio	:	5.66
Tip Chord	:	0.30
Root Chord	:	0.66
Taper Ratio	:	0.455
$\frac{1}{4}$ Chord Sweep	:	13.2 Degrees
Tail Arm (L_T from $\frac{1}{4}$ Chord M.A.C. Wing to $\frac{1}{4}$ Chord M.A.C. Tail)	:	3.724

C.1.4 Vertical Tail

Area of Fin and rudder (S_F)	:	1.282 m ²
Area of Rudder	:	0.348 m ²
Aspect Ratio	:	1.33
Tail Arm L_F	:	3.6
Maximum Deflection	:	25 Degrees

C.1.5 Fuselage

Overall Length	:	6.3 m
Maximum Width	:	0.61 m
Maximum Height	:	0.84 m

C.2 Inertia Properties of the Standard Cirrus

C.2.1. Masses

Wings (both)	:	118 Kg
Fuselage (including rudder)	:	93 Kg
Horizontal Tailplane	:	6.8 Kg
Empty Weight	:	217.7 Kg
Instruments	:	6.8 Kg
Batteries	:	2.27 Kg
Equipped Weight (W_E)	:	227.0 Kg
Maximum Pilot Weight (with above Equipment) (W_p)	:	113.4 Kg
Maximum Water Ballast (W_b)	:	59 Kg
Maximum All Up Weight (W)	:	390 Kg

C.2.2 Moment of Inertia

Taken about aircraft centre of gravity, at all up weights of 317.5 Kg without ballast and 376.4 Kg with ballast.

Forward Centre of Gravity

Rolling Moment of Inertia (without Ballast)	: 1655 Kg m ²
Rolling Moment of Inertia (with Ballast)	: 1994 Kg m ²
Pitching Moment of Inertia (without Ballast)	: 583 Kg m ²
Pitching Moment of Inertia (with Ballast)	: 583 Kg m ²
Yawing Moment of Inertia (without Ballast)	: 2238 Kg m ²
Yawing Moment of Inertia (with Ballast)	: 2577 Kg m ²

Aft Centre of Gravity

Rolling Moment of Inertia (without Ballast)	: 1655 Kg m ²
Rolling Moment of Inertia (with Ballast)	: 1994 Kg m ²
Pitching Moment of Inertia (without Ballast)	: 570 Kg m ²
Pitching Moment of Inertia (with Ballast)	: 586 Kg m ²
Yawing Moment of Inertia (without Ballast)	: 2225 Kg m ²
Yawing Moment of Inertia (with Ballast)	: 2580 Kg m ²

C.3 Centre of Gravity Margins

Forward (percentage standard mean chord)	: 47.39% SMC
Aft (percentage standard mean chord)	: 24.87% SMC

APPENDIX D

D.O. USE OF NORMAL MODES IN FLUTTER ANALYSIS

Among all aeroelastic phenomena, flutter of aeroplane wings, in general, is a serious problem. This is particularly true for sailplanes whose wings are the most deformable part of the body. Taking advantage of the symmetry of the aeroplane structure, the wing flutter problem is generally divided into symmetric and antisymmetric analysis*. In this appendix, the formulation of the flutter determinant in the symmetric case only using normal modes of vibration is discussed in detail. The modal method enables the use of lifting line or lifting surface theory to evaluate the generalised airforces in flutter calculation. The finite element method being a powerful technique can be used to obtain the natural frequencies and mode shapes.

D.1 General Approach to the Flutter Problem

Flutter is a phenomenon of self-excited or unstable oscillation arising out of the simultaneous action of elastic, inertia and aerodynamic forces upon a mass or a system of masses. The purpose of this section is to outline a general approach to the flutter problem through the use of some well known principles namely, the finite element method and unsteady aerodynamic theory. The mass and stiffness matrices generated from the finite element idealisation are transformed into modal co-ordinates to give generalised mass and stiffness and similar treatment is made to the aerodynamic forces to give a generalised aerodynamic matrix. Finally, these three matrices are assembled to form the flutter matrix. The flutter equations are solved as usual for the sinusoidal oscillation which is the transition

* Symmetry reduces the magnitude of the problem reducing the size of the matrices.

between stable and unstable oscillatory motion. Use of normal modes in flutter analysis is quite common now a days and the specific merits of this method have been discussed in Ref.49.

D.2 Choice of Generalised Co-ordinates

Since flutter of an aeroplane wing may result from the coupling of bending and torsional motions, an inspection of the natural frequencies normally suggests that at least the first and second bending modes and the first torsional mode of the wing will have to be considered in order to describe the flutter. Bisplinghoff et al (Ref. 49) suggested some guidelines for the selection of mode shapes for a flutter analysis. In earlier days it was felt by many investigators that it was unnecessary to introduce the body freedoms of the aeroplane as whole into the flutter problems. This outlook has changed particularly since the introduction of swept wings and in present day flutter calculations, the effect of rigid-body freedoms is often considered to be important. The normal modes for an unrestrained aircraft contain the rigid-body motions in the standard eigen value solution. The generalised mass associated with the rigid-body motion is calculated in the same manner as in the case of an elastic motion. The stiffness associated with a rigid-body motion is, of course, zero.

D.3 Formation of Inertia and Elastic Matrices

If the finite element method is used to find the normal (or orthogonal) mode shapes and natural frequencies of the aeroplane, (as it has been done in this thesis) the assembled mass and stiffness matrices of the structure in the finite element idealisation are diagonalised and thus transformed into generalised mass and stiffness matrix form. This is achieved by post multiplying the mass and stiffness matrices by the modal matrix (that is the matrix formed by the columns of the selected normal modes) and premultiplying the resultant matrices by the transpose of the modal matrix.

In matrix form, the generalised mass and stiffness matrices are

$$[M] = [\phi_m]^T [M] [\phi_m] \quad \dots(D.3.1)$$

$$[S] = [\phi_m]^T [S] [\phi_m] \quad \dots(D.3.2)$$

where,

$[M]$ is the diagonal mass matrix

$[S]$ is the diagonal stiffness matrix

$[\phi_m]$ is the modal matrix formed by the columns of the selected modes so that each column of $[\phi_m]$ represents a normal mode.

D.4 Formation of Aerodynamic Matrix Using Strip Theory

In the formulation of the flutter matrix the matrix needed other than the mass and stiffness is the aerodynamic matrix expressed in terms of generalised co-ordinates. In this appendix, the oscillatory aerodynamics developed by Theodorsen (Refs. 58, 49) is used to formulate the aerodynamic matrix.

The aerodynamic forces entering into an ordinary flutter calculation are the increments of forces caused by the deformed motion of the surface of the aircraft in a uniform airstream. The 'static' forces which act on the undeformed system, which are ordinarily used in aeroplane performance and strength calculations, do not enter into the flutter phenomena; and are balanced by the forces of gravity. Only the air force increments or 'dynamic' air forces resulting from the unsteady flow caused by changing deformation of the surface occur in the equations

of flutter. The determination of the dynamic air forces is complicated by the unsteady flow condition and only a few solutions for particular surfaces are available. Theodorsen (Ref. 58) has derived expressions for the forces for two dimensional flow over a rigid airfoil

executing a general sinusoidal oscillation in heave and pitch. The Theodorsen expressions are quite standard and are extensively used for flutter analysis. In recent years Davies (Ref. 117) has developed a method to determine the subsonic oscillatory airforce coefficients by extending the lifting surface theory of Multhopp type leading to a computer program (Ref. 118) which requires too extensive computer facilities to be useful for this work. In any case, the aerodynamic forces are introduced into the equations of motion by the generalised force terms and since a general expression for the air forces is not available, the method of expressing them in the form of generalised forces is discussed in the subsequent sections.

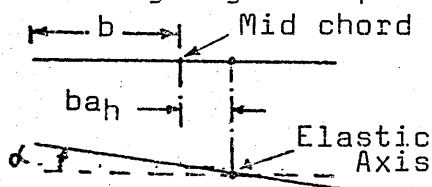
D.4.1 Torsion-Bending Aerodynamic Forces

The torsion-bending oscillatory aerodynamic force and moment on an aerofoil based on two dimensional flat plate theory of Theodorsen are given in Refs. 48-49 and 58 as

$$\begin{aligned} \frac{L}{\rho \pi b U^2} = & -k^2 \left(\frac{h}{b} - a_h \alpha \right) + ik \alpha + 2C(k) \left[\alpha \right. \\ & \left. + i \frac{h}{b} k + \left(\frac{1}{2} - a_h \right) ik \alpha \right] \\ & \dots (D.4.1.1) \end{aligned}$$

$$\begin{aligned} \frac{M}{\rho \pi b^2 U^2} = & \left(\frac{1}{2} + a_h \right) \cdot 2 \cdot C(k) \left[\alpha + \frac{ikh}{b} + \left(\frac{1}{2} - a_h \right) ik \alpha \right] \\ & - k^2 a_h \left(\frac{h}{b} - a_h \alpha \right) - \left(\frac{1}{2} - a_h \right) ik \alpha + \frac{k^2}{8} \alpha \\ & \dots (D.4.1.2) \end{aligned}$$

Where L is lift force defined positive up and h , the vertical displacement defined positive down, M is the pitching moment and α is the angular rotation - both defined positive when leading edge is up. The adjoining sketch of



the airfoil as a representative flat plate explains the nomenclature of the various terms used in equations (D.4.1.1) and (D.4.1.2).

Bisplinghoff et al (Ref. 49) modified these expressions for a wing whose leading edge has an angle of sweep Λ . The derived expressions in Ref (49) for an oscillating aerofoil in subsonic flow with the effect of sweep are same as Theodorsen's except the fact that the right hand side of equations (D.4.1.1) and (D.4.1.2) are to be multiplied by $\cos \Lambda$ where Λ is the sweep angle. In matrix form these expressions for lift and moment can be expressed as

$$\begin{Bmatrix} L \\ M \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} \quad \dots(D.4.1.3)$$

where

$$A_{11} = -\pi \rho U^2 \left[-k^2 + 2C(k) \right] \cos \Lambda \quad \dots(D.4.1.4)$$

$$A_{12} = -\pi \rho U^2 b \left[(a_h k^2 + ik) + 2C(k) \left\{ 1 + ik \left(\frac{1}{2} - a_h \right) \right\} \right] \cos \Lambda \quad \dots(D.4.1.5)$$

$$A_{21} = \pi \rho U^2 b \left[2C(k) \left(\frac{1}{2} + a_h \right) - k^2 a_h \right] \cos \Lambda \quad \dots(D.4.1.6)$$

$$A_{22} = \pi \rho U^2 b^2 \left[2C(k) \left(\frac{1}{2} + a_h \right) \left\{ 1 + ik \left(\frac{1}{2} - a_h \right) \right\} \right. \\ \left. + \frac{k^2}{8} + k^2 a_h^2 + \left(a_h - \frac{1}{2} \right) ki \right] \cos \Lambda$$

...(D.4.1.7)

The signs of A_{11} and A_{12} are reversed to make L positive downwards corresponding to the sign of the deflection, h .

The Theodorsen function $C(k)$ used in the above expressions is mathematically defined by Bessel or Hankel functions as:

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + iH_0^{(2)}(k)}$$

...(D.4.1.8)

where

$$H_n^{(2)}(k) = J_n(k) - i Y_n(k)$$

...(D.4.1.9)

where J_n are Bessel functions of the first kind of order n , and Y_n are Bessel functions of the second kind of order n . J_n and Y_n can be expressed as power series.

In reference to the present analysis the Bessel functions used in the computer programme are J_0 , J_1 , Y_1 and Y_2 which follow from the solution of the Bessel's differential equation. These functions are given by

$$J_0(x) = 1 - \frac{x^2}{2^2(1!)^2} + \frac{x^4}{2^4(2!)^2} - \frac{x^6}{2^6(3!)^2} + \dots$$

...(D.4.1.10)

$$J_1(x) = \frac{x}{2} - \frac{x^3}{2^3 1! 2!} + \frac{x^5}{2^5 2! 3!} - \dots$$

...(D.4.1.11)

$$Y_0(x) = \frac{2}{\pi} \left(\gamma + \log_e \frac{x}{2} \right) J_0(x) - \frac{2}{\pi} \sum_{r=1}^{\infty} \frac{(-1)^r (x/2)^{2r}}{(r!)^2} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r} \right)$$

...(D.4.1.12)

$$Y_1(x) = \frac{2}{\pi} \left(\gamma + \log_e \frac{x}{2} \right) J_1(x) - \frac{2}{\pi x} - \frac{1}{\pi} \sum_{r=0}^{\infty} \frac{(-1)^r (x/2)^{2r+1}}{r! (r+1)!} \left\{ 2 \left(1 + \frac{1}{2} + \dots + \frac{1}{r} \right) + \frac{1}{r+1} \right\}$$

...(D.4.1.13)

Where $\gamma = 0.57721566 \dots$ is Euler's constant

There are approximate expressions for $C(k)$ given in Ref. 48. The approximation leads to less than 10 percent error from the actual $C(k)$ values evaluated from Bessel's function.

D.4.2 Derivation of Aerodynamic Matrix in Generalised Co-ordinates

A general method of setting up the airforce matrix in terms of generalised co-ordinates is described in this section. Through the use of a finite number of generalised displacements and co-ordinates, the problem of a continuous structural system, which theoretically has an infinite number of degrees of freedom can be reduced to one which has only the few degrees of freedom

which are of practical interest. Another advantage of the use of generalised co-ordinates is that, if the generalised displacements 'q' of a system with whatever constraints or boundary conditions exist can be considered as independent, the equations of motion of the system can be expressed in the very simple and general Lagrangian form as

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) + \frac{\partial V}{\partial q_i} - Q_i = 0, \quad (i=1, 2 \dots r)$$

...(D.4.2.1)

where q_i = independent generalised displacements which specify the displacements of the system about an equilibrium position,

$$\dot{q}_i = \frac{\partial q_i}{\partial t}$$

T = kinetic energy of motion of system, which is a quadratic in $\dot{q}_i$ with constant co-efficients as a result of the assumption of small displacements,

V = potential (elastic strain) energy of system which is a quadratic in q_i with constant co-efficient as a result of the assumption of small displacements about an equilibrium position,

Q_i = generalised forces corresponding to externally applied forces.

There is one Lagrangian equation (D.4.2.1) for each displacement q_i ; therefore, the motion of the system is completely determined by the full set of equations. Each equation expresses the equilibrium of all the forces on the system with respect to a generalised displacement q_i , and each term in the equation may be identified with a particular generalised force. The term $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right)$ is the generalised inertia force

corresponding to q_i , the term $\frac{\partial V}{\partial q_i}$ is the gener-

alised elastic force, and in a flutter problem Q_i is the generalised air-force. In section D.3 the generalised mass and stiffness matrices have already been derived so that $M_i \ddot{q}_i$ where

M_i is the generalised mass in the i th mode

corresponds to $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right)$ and $K_i q_i$ where K_i is

the generalised stiffness in the i th mode

corresponds to $\frac{\partial V}{\partial q_i}$ of equation (D.4.2.1). We are

only left with the derivation of generalised forces.

The generalised forces $Q_1, Q_2 \dots Q_r$ corresponding to any set of forces acting on a system are defined as the coefficients of the differentials $\delta q_1, \delta q_2, \dots \delta q_r$, which specify an arbitrary displacement of the system, in the expression for the work done by the forces during the arbitrary displacement. Thus, if δW denotes the work,

$$\delta W = Q_1 \delta q_1 + Q_2 \delta q_2 + \dots + Q_r \delta q_r$$

...(D.4.2.2)

is the defining equation for Q_i . In matrix notation

$$\delta W = \begin{bmatrix} \delta q_1 & \delta q_2 & \dots & \delta q_r \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_r \end{Bmatrix} \dots (D.4.2.3)$$

Now the aerodynamic matrix in terms of generalised co-ordinates $q_1, q_2 \dots q_r$ is derived as follows:

The vertical deflection and rotation of a station y along the span at time t is given by

$$\begin{Bmatrix} h(y,t) \\ \alpha(y,t) \end{Bmatrix} = \begin{bmatrix} h_1(y) & h_2(y) & \dots & h_r(y) \\ \alpha_1(y) & \alpha_2(y) & \dots & \alpha_r(y) \end{bmatrix} \begin{Bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_r(t) \end{Bmatrix}$$

...(D.4.2.4)

where, $h_i(y)$ is the vertical deflection at station y along the span for the i th mode

$\alpha_i(y)$ is the rotation about the reference axis at station y along the span for the i th mode.

$h_i(y)$ and $\alpha_i(y)$ are produced as the orthogonal normal modes of vibration by the eigen value solution.

Now the generalised force Q_i corresponding to the generalised co-ordinate q_i are found by calculating the work done δW by the air forces given by equations (D.4.1.1) and (D.4.1.2) in varying the generalised co-ordinate from q_i to $q_i + \delta q_i$ and integrating the air forces over the semi-span S , as follows:

$$\delta W_i = \delta q_i \int_0^S \{ L(y) h_i(y) + M(y) \alpha_i(y) \} dy$$

...(D.4.2.5)

In matrix form

$$\begin{Bmatrix} \frac{\delta W_1}{\delta q_1} \\ \frac{\delta W_2}{\delta q_2} \\ \vdots \\ \frac{\delta W_r}{\delta q_r} \end{Bmatrix} = \int_0^S \begin{bmatrix} h_1(y) & \alpha_1(y) \\ h_2(y) & \alpha_2(y) \\ \vdots & \vdots \\ h_r(y) & \alpha_r(y) \end{bmatrix} \begin{Bmatrix} L(y) \\ \alpha(y) \end{Bmatrix} dy \quad \dots (D.4.2.6)$$

Substituting for $\begin{Bmatrix} L(y) \\ M(y) \end{Bmatrix}$ from equation (D.4.1.3) of section D.4.1

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_r \end{Bmatrix} = \begin{Bmatrix} \frac{\delta W_1}{\delta q_1} \\ \frac{\delta W_2}{\delta q_2} \\ \vdots \\ \frac{\delta W_r}{\delta q_r} \end{Bmatrix} = \int_0^S \begin{bmatrix} h_1(y) & \alpha_1(y) \\ h_2(y) & \alpha_2(y) \\ \vdots & \vdots \\ h_r(y) & \alpha_r(y) \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} h(y,t) \\ \alpha(y,t) \end{Bmatrix} dy \times$$

Substituting $\begin{Bmatrix} h(y,t) \\ \alpha(y,t) \end{Bmatrix}$ from equation D.4.2.4)

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_r \end{Bmatrix} = \int_0^S \begin{bmatrix} h_1(y) & \alpha_1(y) \\ h_2(y) & \alpha_2(y) \\ \vdots & \vdots \\ h_r(y) & \alpha_r(y) \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} h_1(y) & h_2(y) & \dots \\ \alpha_1(y) & \alpha_2(y) & \dots \\ \dots & h_r(y) & \dots \\ \dots & \alpha_r(y) & \dots \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_r \end{Bmatrix} dy$$

$$= \begin{bmatrix} QF_{11} & QF_{12} & \dots & QF_{1r} \\ QF_{21} & QF_{22} & \dots & QF_{2r} \\ \vdots & \vdots & & \vdots \\ QF_{r1} & QF_{r2} & \dots & QF_{rr} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_r \end{Bmatrix}$$

$$= [QF] \cdot \{q\} \quad \dots (D.4.2.7)$$

where $QF(i,j) = \int_0^S (A_{11} h_i h_j + A_{21} h_j \alpha_i + A_{12} h_i \alpha_j + A_{22} \alpha_i \alpha_j) dy$

$$\dots (D.4.2.8)$$

Equation (D.4.2.8) gives the elements of aerodynamic matrix in terms of the generalised co-ordinates.

The theory presented is adequate for the type of problems dealt in this thesis. Sailplanes having high aspect ratios fly at subsonic speed and the limitation imposed on strip theory does not affect the analysis. However the assumptions made in accepting the Theodorsen expression for lift and moment are:

- (i) lift and moment are linear functions of h and α
- (ii) the airflow is two dimensional, i.e. the aerofoil is assumed to be of infinite aspect ratio
- (iii) the aerodynamic coefficients are calculated for a flat plate aerofoil at zero mean angle of attack.

D.5 Formation of Air Force Matrix Using Lifting Surface Theory

Davies (Ref.117) presented a method for the

calculation of the linearised values of generalised air force coefficients on a harmonically oscillating flat plate wing in subsonic flow by further developing the lifting surface theory of Multhopp type. His method involving the integral equation of Kernel function is reasonably complex and is not reproduced here for brevity. However, he programmed his procedure in ICL 1900 FORTRAN (Ref.118) to evaluate the aerodynamic matrix $[QF]$ which has been formulated in section (D.4) using strip theory.

D.6 Formation of Flutter Stability Determinant

The flutter matrix is formed by algebraically summing the generalised mass and stiffness matrices along with the aerodynamic matrix derived in sections (D.4) and (D.5). For a system without structural damping the equations which form the flutter matrix are given by

$$[QA] \{q\} = \left[-\omega^2 [M] + [S] - [QF] \right] \{q\}$$

...(D.6.1)

where $[QA]$ is the flutter matrix
 $[M]$ is the generalised mass matrix
 $[S]$ is the generalised stiffness matrix
 $[QF]$ is the aerodynamic matrix - a complex matrix of functions of ω and U
 ω is the frequency of vibration
 U is the aircraft speed
 $\{q\}$ is the generalised co-ordinate vector

By setting the equation (D.6.1) to zero, the flutter stability determinant is formed.

D.6.1 Inclusion of Structural Damping

There is always some inherent amount of structural damping present in the structure which under normal circumstances is advantageous from the flutter point of view. Structural damping is expressed as a dissipation function of the form (see Ref.47)

$$D = \frac{1}{2} \sum_i M_i g_i \frac{\bar{\omega}_i^2}{\omega} \dot{q}_i^2 \quad \dots(D.6.1.1)$$

where M_i is the generalised mass corresponding to the i th mode of vibration

q_i is the i th generalised co-ordinate

$\bar{\omega}_i$ is the i th natural frequency

g_i is the structural damping coefficient of the i th mode

ω is the frequency of oscillation

In matrix form D can be written as

$$D = \frac{1}{2\omega} \left[\bar{\omega}^2 g M \right] \left\{ \dot{q}^2 \right\} \quad \dots(D.6.1.2)$$

Lagrange's equations including the dissipation function give the generalised force due to the structural damping as

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) + \frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i \quad \dots(D.6.1.3)$$

From equation (D.6.1.2)

$$\frac{\partial D}{\partial \dot{q}_i} = \frac{1}{\omega} \left[\bar{\omega}^2 g M \right] \left\{ \dot{q} \right\} \quad \dots(D.6.1.4)$$

For simple harmonic motion

$$q = q_0 e^{i\omega t} \quad \dots(D.6.1.5)$$

$$\therefore \dot{q} = i\omega q \quad \dots(D.6.1.6)$$

Substituting in equation (D.6.1.4)

$$\frac{\partial D}{\partial \dot{q}} = i \left[\bar{\omega}^2 g M \right] \left\{ q \right\} \quad \dots(D.6.1.7)$$

Including the structural damping in equation (D.6.1.) of section D.6, the flutter-stability determinant becomes

$$\begin{aligned} [QA] \left\{ q \right\} &= \left[M \right] \left[-\omega^2 + i\bar{\omega}^2 g \right] + \left[S \right] \\ &+ \left[QF \right] \left\{ q \right\} \quad \dots(D.6.1.8) \end{aligned}$$

Structural damping generally has only small effect on the oscillatory motion because the structural damping coefficient lies in the range 0.0 to 0.08 for built up aircraft construction.

APPENDIX E

E.0 SHORT PERIOD OSCILLATION CHARACTERISTICS OF RIGID AND FLEXIBLE AEROPLANES

A longitudinal disturbed motion of an aircraft following an arbitrary initial disturbance, consists normally of two oscillatory modes namely, the short period and the phugoid. The short period oscillation for a conventional aircraft with a period no more than a few seconds (sometimes below 1 sec) is usually so strongly damped as to become not recordable after two or three periods. The phugoid oscillation of a conventional aircraft has period of the order of one minute (up to 2 minutes or more at high speed) and its natural damping is always low, the practical importance of this damping, being, however, very much less than that of the more rapid (short period) oscillation.

The short period oscillation of a rigid aeroplane is investigated by standard techniques involving longitudinal aerodynamic derivatives ($\dot{z}_w$, m_w , m_q and $m_{\dot{w}}$) and normally without taking any account of the flexibility and/or unsteady aerodynamic effects. A method has been outlined in this appendix on how to satisfactorily deal with the short period oscillation characteristics of a flexible aeroplane with proper account of the unsteady aerodynamics. Conventional method based on the use of stability derivatives for investigating the short period oscillation characteristics is also described for a ready reference.

E.1 Development of the Proposed Method

In Appendix D, sinusoidal motion has been assumed in the formulation of flutter determinant on the basis of the argument that the flutter speed marks the transition between stable and unstable oscillatory motion so that above the flutter speed the motion is divergent and below that speed it is convergent. The

Theodorsen expressions for lift and moment used in Appendix D in the formulation of the flutter determinant are essentially for aerofoil oscillation of form $e^{i\omega t}$. The expressions for lift and moment for non-harmonic oscillation (convergent or divergent) of an aerofoil can also be obtained if the Theodorsen function $C(k)$ where $k = \frac{\omega b}{U}$ are properly evaluated

for k where k is a complex quantity. In flutter problem ω (or k) has been treated as a real variable and now this ω (or k) can be replaced by a complex quantity λ which implies the non-harmonic aerofoil motion of form $e^{\lambda t}$. Thus

since the generalisation of the Theodorsen function $C(\lambda)$ is possible, the stability determinant involving λ (where $\lambda = \mu + i\omega$) can be solved for real and imaginary parts of λ at a particular chosen air speed. If the air speed chosen is above the flutter speed the real part μ of λ will turn out to be positive indicating divergent oscillation. On the other hand if air speed chosen is below the flutter speed, μ which is a measure of damping will turn out to be negative indicating convergent oscillation. The imaginary part ω of λ in both cases gives the respective short period frequency.

E.1.1 The Theodorsen Circulation Function for Generalised Motion

The Theodorsen circulation function $C(k)$ for generalised motion has been developed by Luke and Dengler (Ref. 116). An extract from Luke and Dengler's work is reproduced here.

In the treatment of aerodynamic forces on an oscillating aerofoil, it is assumed that the motion follows the law e^{ikt} where t is the time variable and

$$ik = \mu + i\omega \quad \dots(E.1.1.1)$$

Thus ω gives the frequency of motion, while μ defines the rate of decay. If $\mu < 0$ the motion is convergent or stable; if $\mu > 0$, the motion is divergent or unstable. When $\mu = 0$, the motion

is of constant amplitude and is said to be neutrally stable.

To accomplish the derivation of the Theodorsen function $C(k)$ for general motion effectively, let us consider the definition by Theodorsen (Ref.58)

$$C(k) = \frac{\int_1^{\infty} \frac{x}{x^2-1} U dx}{\int_1^{\infty} \frac{x+1}{x^2-1} U dx} \quad \dots(E.1.1.2)$$

where it is supposed that

$$U = U_0 e^{i \left[k \left(\frac{s}{b} - x \right) + \phi \right]} \quad \dots(E.1.1.3)$$

and that $\omega > 0$. It is well known in the theory of Bessel functions (Ref.132) that

$$K_n(z) = \int_0^{\infty} e^{-z \cosh y} \cosh ny dy \quad \dots(E.1.1.4)$$

which is valid if $|\arg z| < \frac{\pi}{2}$. This requires that $\text{Re}(z) > 0$

$$\text{Let, } z = ik \quad \dots(E.1.1.5)$$

Thus if $\text{Re}(z) > 0$, it follows that $\mu > 0$, so that the motion is assumed to be divergent.

Combining equations (E.1.1.2) to (E.1.1.5) with the well known relation

$$K_n(z) = \frac{\pi}{2} i e^{\left(\frac{1}{2}\right) n \pi i} H_n^{(1)}(iz) \quad \dots(E.1.1.6)$$

It is readily deduced that

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + i H_0^{(2)}(k)} \quad \dots(E.1.1.7)$$

which is the generalised Theodorsen function $C(k)$. Although it is true that, in the development of the equation (E.1.1.7) it was necessary to require $\mu > 0$, equation (E.1.1.7) has no such a restriction, and therefore, by the method of analytic continuation, one can argue that equation (E.1.1.7) is valid for all μ 's.

The Theodorsen function defined in equation (E.1.1.7) can be expressed as

$$C(z) = \frac{H_1^{(2)}(z)}{H_1^{(2)}(z) + i H_0^{(2)}(z)} = F(\rho, \theta) + iG(\rho, \theta) \quad \dots(E.1.1.8)$$

where

$$z = \rho e^{i\theta} = \omega - i\mu = k, \quad \rho \gg 0, \quad |\theta| \leq \frac{\pi}{2} \quad \dots(E.1.1.9)$$

Thus if the motion is convergent, $0 < \theta \leq \frac{\pi}{2}$
if the motion is divergent $-\frac{\pi}{2} \leq \theta < 0$.

However, if the motion is harmonic, $\theta = 0$ and, therefore, $\rho = k$. In this case, of course, values of $C(k)$ are well known. Since the intention of this section is to study the aerodynamic system for non-harmonic motion, it is important and necessary to know the values of

$C(z)$ for general z . Luke and Dengler (Ref. 116) tabulated $C(z)$ for the following ranges of ρ and θ

$\rho : 0(0.1)0.3(0.02)0.5(0.05)1.0(0.5)10.0$

$\theta : -5^\circ(5^\circ)30^\circ$

E.1.2 The Basic Equations of Motion

The undisturbed aircraft motion is considered to be level flight, at speed V , the x-axis being directed forward, the y-axis in the star-board spanwise direction and the z-axis vertically downward. The origin is the intersection of the flexural axis with the fuselage centre line. In the mathematical model, the two rigid body modes heave and pitch are required to define the aircraft short period motion associated with elastic modes in the symmetric motion.

If finite element representation of the structure is adopted, then the general motion of the aircraft can be assumed to be compounded of displacements in the two rigid body modes and in $(n-2)$ independent natural vibrational modes of the elastic structure whose shapes are defined relative to the axis system. Thus for small perturbations, the displacements $\bar{\delta}_i$ ($i = 1, 2 \dots n$) of the element at point y_i on the spanwise station is given by

$$\left\{ \bar{\delta} (y_i, t) \right\} = [\phi] \left\{ q (t) \right\} \quad \dots(E.1.2.1)$$

The last $(n-2)$ columns of the square matrix $[\phi]$ describe the natural mode shapes of the flexible structure relative to the axis system while the first two columns, correspond to the rigid-body displacements.

The equation

$$\{\phi_j\}^T [M] \{\phi_j\} = M_j \dots (E.1.2.2)$$

defines the generalised masses, M_j ($j = 1, 2 \dots n$) of the structural modes for which corresponding generalised stiffnesses K_j ($j = 1, 2 \dots n$) are defined by

$$K_j = M_j \omega_j^2 \dots (E.1.2.3)$$

where ω_j are the natural frequencies of the aircraft. The rigid-body modes are zero-frequency structural modes (i.e. $\omega_1 = \omega_2 = 0$) with zero generalised stiffness ($K_1 = K_2 = 0$). The equations of motion may now be written in terms of $n \times n$ matrices in a similar way like the flutter determinant in sections (D.6) and (D.6.1) of Appendix D, using generalised co-ordinates $\{q\}$

$$[MD^2 + K] \{q\} = \{F\} \dots (E.1.2.4)$$

with $D \equiv \frac{d}{dt}$. The generalised forces $\{F\}$ can be derived for aerofoil motion of $e^{\lambda t}$ type where $\lambda = \mu + i\omega$ using the work done (by the generalised force) concept described in section D.4.2 of Appendix D, with appropriate modification of the Theodorsen function having the complex reduced frequency parameter λ . Finally, for a chosen airspeed the stability equation for oscillation of form $q = \tilde{q}e^{\lambda t}$ can be written in the following matrix form

$$[\lambda^2 [M] + [K]] \tilde{q}e^{\lambda t} = [QF] \tilde{q}e^{\lambda t} \dots (E.1.2.5)$$

The generalised air force matrix $[QF]$ is primarily

a function of λ (and hence of μ and ω .) when an airspeed is already chosen. The stability determinant based on equation (E.1.2.5) is

$$\left| \lambda^2 [M] + [K] - [QF] \right| = 0$$

...(E.1.2.6)

The solution of the determinantal equation (E.1.2.6) is a general algebraic eigen value problem for which the QR Algorithm has proved to be one of the most effective methods of solution (see Ref.133). The matrix is first transformed into an upper Heisenberg form; i.e. a matrix of an upper triangular form with one extra sloping line below the leading diagonal. Then the calculation of the eigen values can be effected by the use of standard scientific subroutines developed by Numerical Algorithms Groups. However, the real part of λ gives the short period damping μ (unit: sec^{-1}) and the imaginary part gives the short period frequency $\omega_{s.p}$ (unit: rad/sec).

E.2 The Short Period Oscillation of the Rigid Aircraft Using Stability Derivatives

The conventional approach of a stability analyst to study the short period oscillation of an aircraft is by using its stability derivatives. In this method the aircraft is generally considered to be a rigid structure and therefore, the distortion effects are totally neglected. Also, in the evaluation of aerodynamic derivatives, the unsteady aerodynamic effects are not usually considered.

Suppose an aircraft, initially in equilibrium, is given a symmetric disturbance, e.g. due to a sudden up-gust. The incidence of the aircraft will change and the aircraft will start to pitch. If the aircraft is stable, the angle of incidence will soon return to its original value; this movement is sometimes called the rapid incidence adjustment of the whole aircraft. There will also be a change in the aircraft forward speed, but this change will be small compared

with the changes in the angles of incidence and pitch. In the theory outlined in this section, it is assumed that the aircraft speed

remains constant. The matter presented here is mainly taken from Ref.38 although the longitudinal motion of an aircraft following a small perturbation in steady flight has been covered in many other texts (Refs. 39-44). The notation and the axes system used by Babister (Ref.38) are consistently followed. The axes are fixed in the aircraft and move with the aircraft. Ox and Oz are in the plane of symmetry of the aircraft, with Oz downwards and Oy is starboard (see Ref.38).

Small disturbances from the steady trimmed rectilinear motion in the symmetric plane with no bank, sideslip or yaw are considered. In steady flight, the aircraft is moving forward with a uniform velocity and with no angular rotation. Let U_0 and W_0 be the velocity components of the centre of gravity in steady flight in which there is no sideslip and no velocity components parallel to Oy . Let $U_0 + u$, v , $W_0 + w$ be the velocity components of the centre of gravity in the disturbed motion, with angular velocities p , q , r about the axes Ox , Oy , Oz respectively. If u , v , w , p , q and r are considered to be small quantities of the first order compared with U_0 and also if terms of the second degree and higher are neglected, the equations of the longitudinal free motion in the symmetric plane as derived by Babister (Ref.38) for the case $W_0 = 0$, $U_0 = V_0$ where V_0 is the flight speed in the steady state can be written as (non-dimensionalised derivatives involved are x_u , x_w , x_q , z_u , z_w , z_q , m_u , m_w , $m_{\dot{w}}$, m_q) :

$$\frac{d\hat{u}}{d\tau} - x_u \hat{u} - x_w \hat{w} - \frac{x_q}{\mu_1} \frac{d\theta}{d\tau} + \frac{C_L}{2} \theta = 0$$

...(E.2.1)

$$- z_u \hat{u} + \frac{d\hat{w}}{d\tau} - z_w \hat{w} - \left(1 + \frac{z_q}{\mu_1}\right) \frac{d\theta}{d\tau} = 0$$

...(E.2.2)

$$-\mu_1 m_u \hat{u} - \mu_1 m_w \frac{d\hat{w}}{d\tau} - \mu_1 m_w \hat{w} + i_B \frac{d^2\theta}{d\tau^2}$$

$$-m_q \frac{d\theta}{d\tau} = 0 \quad \dots(E.2.3)$$

where

$$\hat{u} = \frac{u}{V_o} \quad \dots(E.2.4)$$

$$\hat{w} = \frac{w}{V_o} \quad \dots(E.2.5)$$

$$\frac{d\theta}{d\tau} = \hat{q} = \hat{t}q \quad \dots(E.2.6)$$

$$\mu_1 = \frac{m}{\rho S l_T} = \frac{V_o \hat{t}}{l_T} \quad \dots(E.2.7)$$

The non-dimensional aerodynamic time τ is given by

$$\tau = \frac{t}{\hat{t}} \quad \dots(E.2.8)$$

$$\text{where } \hat{t} = \frac{m}{\rho S V_o} = \frac{V_o C_L}{2g} \quad \dots(E.2.9)$$

In equation (E.2.9), m is the mass of the aircraft, ρ is the density of air at sea level and S is the wing area of the aircraft. Other parameters involved are same as described in Ref.38.

The pitching moment equation (E.2.3) is sometimes written in an alternative form.

Let,

$$m'_u = \frac{m_u}{i_B}, \quad m'_w = \frac{m_w}{i_B}, \quad m'_{\dot{w}} = \frac{m_{\dot{w}}}{i_B}$$

$$m'_q = \frac{m_q}{i_B} \quad \dots(E.2.10)$$

where

$$i_B = \frac{B}{m l_T^2} = \left(\frac{k_B}{l_T} \right)^2 \quad \dots(E.2.11)$$

In equation (E.2.11), B is the pitching moment of inertia about the aircraft centre of gravity, m is the mass of the aircraft, l_T is the distance of the aerodynamic centre of the elevon (tail-plane in case of a tailed aeroplane) and k_B is the radius of gyration of the aircraft in pitch. The equation (E.2.3) becomes on dividing by i_B

$$- \mu_1 m'_u \hat{u} - \mu_1 m'_{\dot{w}} \frac{d\hat{w}}{d\tau} - \mu_1 m'_w \hat{w} + \frac{d^2 \theta}{d\tau^2}$$

$$- m'_q \frac{d\theta}{d\tau} = 0 \quad \dots(E.2.12)$$

The method of derivations of the non-dimensionalised aerodynamic derivatives $x_u, x_w, x_q, z_u, z_w, z_q, m_u, m_w$, and $m_{\dot{w}}$ which occurred in the equations of longitudinal symmetric motion are given in Ref.38.

As mentioned before, for the short period oscillation, the motion involves mainly pitching motion with no change in forward speed. On the basis of this assumption, it is quite legitimate to assume that the change in forward speed of

the aircraft can be neglected for the short period mode as a result of which the terms in $\hat{u}$ in equations (E.2.1) - (E.2.3) and (E.2.12) can be omitted so that the new set of equations becomes

$$\frac{d\hat{w}}{d\tau} - z_w \hat{w} - \left(1 + \frac{z_q}{\mu_1}\right) \hat{q} = 0 \quad \dots(E.2.13)$$

$$-\mu_1 m'_w \frac{d\hat{w}}{d\tau} - \mu_1 m'_w \hat{w} + \frac{d\hat{q}}{d\tau} - m'_q \hat{q} = 0 \quad \dots(E.2.14)$$

where $\hat{q} = \frac{d\theta}{d\tau}$

In the usual way of solving such a system of linear differential equations with constant coefficient, it is assumed that there is a solution of the form

$$\hat{w} = \rho_1 e^{\lambda t}, \quad \hat{q} = \rho_2 e^{\lambda t} \quad \dots(E.2.15)$$

where ρ_1 , ρ_2 and λ are constants.

Substituting equation (E.2.15) in equations (E.2.13) and (E.2.14) and eliminating ρ_1 and ρ_2 , the characteristic equation for λ is given by

$$\begin{vmatrix} \lambda - z_w & -\left(1 + \frac{z_q}{\mu_1}\right) \\ -\mu_1 m'_w \lambda - \mu_1 m'_w & \lambda - m'_q \end{vmatrix} = 0 \quad \dots(E.2.16)$$

The characteristic equation given by the above determinantal equation (E.2.16) can be reduced to the quadratic form

$$\lambda^2 + B_1 \lambda + C_1 = 0 \quad \dots(E.2.17)$$

where

$$B_1 = - (z_w + m'_q + \mu_1 m'_w + z_q m'_w) \quad \dots(E.2.18)$$

$$C_1 = - (\mu_1 m'_w + z_q m'_w - z_w m'_q) \quad \dots(E.2.19)$$

From equation (E.2.17) the solution of the quadratic is given by

$$\lambda = - \frac{B_1}{2} \pm i \sqrt{C_1 - \frac{B_1^2}{4}} \quad \dots(E.2.20)$$

Thus if $\lambda = - r \pm i s$ is $\dots(E.2.21)$

$$r = \frac{B_1}{2} \quad \dots(E.2.22)$$

$$s = \sqrt{C_1 - \frac{B_1^2}{4}} \quad \dots(E.2.23)$$

r gives the measure of damping and s the frequency of short period oscillation. The reference time is the aerodynamic time τ in the analysis.

The period T in seconds is given by

$$T = \frac{2\pi}{s} \hat{t} \quad \dots(E.2.24)$$

and the frequency of oscillation in rad/sec is

$$\omega_{s.p} = \frac{s}{\hat{t}} \quad \dots(E.2.25)$$

The time t_a (in secs) to half amplitude is given by

$$t_a = \frac{\log_e 2}{r} \hat{t} \quad \dots(E.2.26)$$

where $\hat{t}$ in equations (E.2.24 - E.2.26) which relates the aerodynamic time to the actual time is given by equation (E.2.9).

APPENDIX F

F.0

APPLICATION OF THE POWER SPECTRAL DENSITY
METHOD FOR DYNAMIC RESPONSE OF FLEXIBLE
AEROPLANES TO CONTINUOUS RANDOM ATMOSPHERIC
TURBULENCE

The structural response of aeroplanes in continuous rough air has always been a matter of concern. Power spectral methods of analysis are often applied in order to obtain a frequency decomposition of the response. The excitation and response can be described statistically in this method by their spectra. The spectrum of the response is related to that of the excitation through the appropriate transfer function which is a property of the aircraft. The spectrum of a random variable is simply a statement of the distribution of the square of the variable with frequency. A transfer function is the output/input relationship between the continuous harmonic response of some chosen parameter, such as acceleration and some form of continuous harmonic excitation of which gust is an example. For an aircraft responding to turbulence a typical transfer function is a complex function of frequency, indicating that the response is not in phase with the excitation. The transfer function is related to the frequency response function by the fact that the former is the square of the modulus of the latter.

Many of the papers published in this field have treated the aeroplane as rigid-body and in so doing have dealt either only with the degree of freedom of vertical motion or with vertical motion and pitch. On the basis of this rigid-body assumption, externally applied forces, such as air loads are put into equilibrium with aerodynamic and inertial forces which arise as a result of translational and rotational motion of the aeroplane as a rigid-body and these forces are considered to comprise the total force system acting on the aeroplane. Dealing with flexible aeroplanes this procedure leads to errors in two ways (Ref.49). Deformation of the structure may induce additional aerodynamic forces which affect the overall response of the aeroplane. Furthermore, if the dynamic

response is the direct result of rapidly applied external forces such as atmospheric turbulence, the aeroplane is not only caused to translate and rotate, but structural vibrations are also induced. The latter have a significant effect upon the internal stress distribution in the structure and a somewhat lesser influence on the overall response of the aeroplane. The main interest in this Appendix is to study the effect of aeroelastic phenomena on the response of the flexible aeroplane when it is subjected to continuous random atmospheric turbulence. Thus the complete analysis of the motion requires the application of rigid-body and elastic dynamics, aerodynamics and the added excitation of atmospheric turbulence. If the longitudinal motion only is of interest the disturbed motion will be symmetrical with respect to the longitudinal plane of symmetry. While passing through the turbulence, the aircraft is assumed to maintain a constant forward speed.

F.1

Basic Concepts and Relationship in Power Spectral Analysis

The input-output relation of the power spectral densities (PSDs) of an aircraft flying through continuous random turbulence is governed by the following well known formula (Ref.49)

$$\psi(\omega) = |H(i\omega)|^2 \phi(\omega) \quad \dots(F.1.1)$$

where $\phi(\omega)$ is the power spectral density (PSD) of the input turbulence, $\psi(\omega)$ is the power spectral density (PSD) of the output response and $H(i\omega)$ is the transfer function (or mechanical admittance or frequency response function) of the aircraft. As regards the analysis in this thesis, $\psi(\omega)$ is the PSD function of the normal or angular acceleration at various points on the aeroplane, $\phi(\omega)$ is the PSD function of the gust velocity experienced by the aeroplane and $H(i\omega)$ is the aeroplane normal or angular acceleration response function for a sinusoidal gust velocity input. $H(i\omega)$ is defined such that $H(i\omega)e^{i\omega t}$ is the system response to the sinusoidal input $e^{i\omega t}$ and is sometimes called the mechanical admittance of the linear

system. Thus from equation (F.1.1) it is seen that the calculation of the response PSD function to a random disturbance entails a knowledge of the PSD of the forcing function and the mechanical admittance of the system. The relationship applies to stationary and ergodic random functions as described in Ref.49. From the property of the PSD function $\phi(\omega)$ and $\psi(\omega)$ respectively satisfy the following equations

$$\overline{W}_G^2 = \int_0^{\infty} \phi(\omega) d\omega \quad \dots(F.1.2)$$

$$\overline{\ddot{x}}^2 = \int_0^{\infty} \psi(\omega) d\omega \quad \dots(F.1.3)$$

where $\overline{W}_G^2$ is the mean square velocity of the gust which constitute the turbulence and $\overline{\ddot{x}}^2$ is the

mean square acceleration (normal or angular) of points on the aircraft structure. Although in a random response analysis following a power spectral density approach, it is not possible to calculate an explicit time history of response of the aircraft, analysts are content with the mean square values which are evaluated from the area of the PSD curves. The present study in this appendix as mentioned earlier is primarily concerned with vertical gust velocity inputs and acceleration outputs at various spanwise points on the wing.

The power spectral method is based on the following assumptions:

- (i) the turbulence is isotropic and the power spectrum of the turbulence is invariant for all types and intensities of turbulence and there is no spanwise variation
- (ii) the atmospheric turbulence is divided into patches, each with a particular value, $\overline{W}_G$ of the r.m.s. of the vertical gust velocity
- (iii) the turbulence in each patch is considered

to be of stationary Gaussian random form.

- (iv) the patches of the turbulence are long enough for the aircraft's response to settle to a steady state, in the sense of the frequency response
- (v) the equations of motion of the aircraft in disturbed air are linear which permit the definition of a transfer function.

F.2 Calculation of the Frequency Response Function From the Response to a Harmonic Gust

F.2.1 The Aerodynamic Forces

The aerodynamic forces which arise on an aircraft following a disturbance are of two types. The first type of forces results from the motion of the aircraft which do not depend directly on the excitation but only on the response to it. The other type of aerodynamic forces which arise on the aircraft structure is due to the excitation itself (gust or turbulence*) and which are not in anyway related to the response of the aircraft. The latter set of forces (excitation forces) can be considered to be the forces on the aircraft if it were exposed to the excitation while being restrained against responding. The two set of forces mentioned above can be separated under the linearity assumption which has already been made in this thesis. The generalised oscillatory aerodynamic forces of the aforementioned two types are separately dealt with in the subsequent two sections.

F.2.1.1 Generalised Forces Due to the Aircraft Motion

Forces resulting from the motion of the aircraft have already been considered in the Appendix D where the flutter-stability determinant has been

*Gust and turbulence are often used synonymously

formulated in the absence of any external exciting forces. The generalised air force coefficients have been derived in Sections D.4.1 and D.4.2 of Appendix D for free oscillatory motion of the wing using Theodorsen expressions for lift and moment. The air force matrix $[QF]$ so derived in generalised co-ordinates in Section D.4.2. of Appendix D will hereafter be referred to as $[QF_m]$ where the suffix 'm' denotes that the forces arise solely from the motion of the aircraft. The use of normal modes and the techniques employed to obtain $[QF_m]$ are all explained in the appropriate sections of Appendix D.

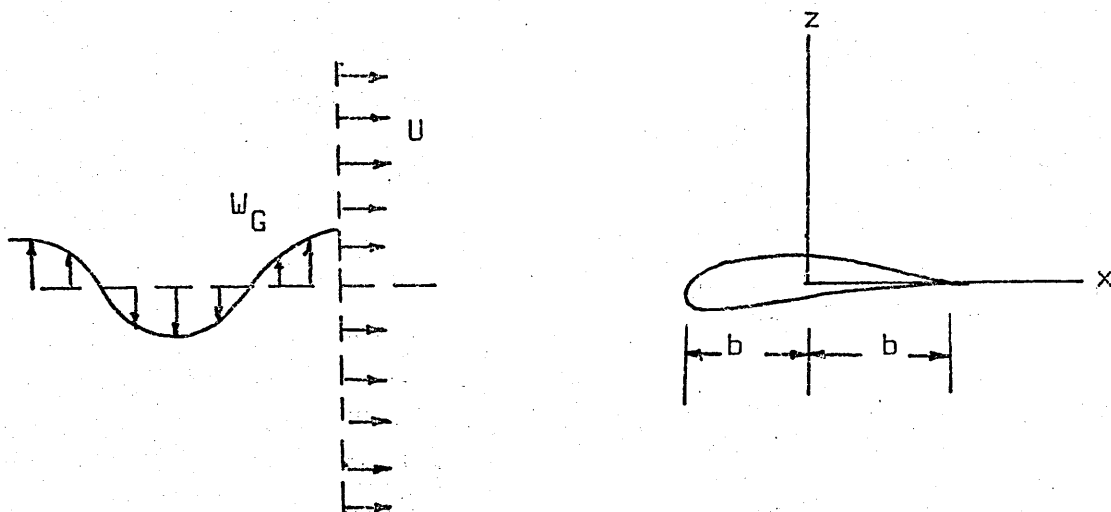
F.2.1.2 Generalised Forces Due to a Harmonic Excitation

The aerodynamic forces arising on an aerofoil while encountering a harmonic excitation - a sinusoidal gust in particular have been derived by quite a number of research workers of which Ref. 49 and Ref. 103 are typical. A sinusoidal gust can be described physically by the real part of (see Ref.49)

$$w_G(x - Ut) = \bar{w}_G e^{i\omega\left(t - \frac{x}{U}\right)} = \bar{w}_G e^{ik(s - x)}$$

...(F.2.1.2.1)

when the aerofoil as shown in the sketch below faces the air velocity U with an embedded gust velocity distribution of w_G ($w_G \ll U$). It is assumed that the turbulence (gust) to be embedded in the atmosphere in such a way that it moves past the aerofoil at velocity U without any appreciable change in w_G for any particular particle during its encounter. This is expressed in a way that w_G is a function of x and t in the combination $(x - Ut)$



An aerofoil encountering a vertical gust $w_G(x-Ut)$

In equation (F.2.1.2.1)

$$k = \frac{\omega b}{U}$$

$$s = \frac{Ut}{b}$$

$$\tilde{x} = \frac{x}{b}$$

...(F.2.1.2.2)

Bisplinghoff et al (Ref.49) have used Schwarz' solution to obtain lift and moment per unit span due to the above typed gust which are respectively

$$L = 2\pi \rho U b \bar{w}_G \left\{ C(k) \left[J_0(k) - i J_1(k) \right] + i J_1(k) \right\} e^{i\omega t}$$

...(F.2.1.2.3)

$$M = b \left(\frac{1}{2} + a \right) L$$

...(F.2.1.2.4)

where J_0 , J_1 are Bessel functions of first kind with order zero and one respectively; $C(k)$ is the Theodorsen function (discussed in Appendix D); b is the semi-chord; ab is the distance (aft) of the flexural axis from the mid-chord point. The lift due to the sinusoidal gust acts at the quarter-chord point (by superposition, the same must be true for any other gust which is a function of $(x - Ut)$ only). In equations (F.2.1.2.3) and (F.2.1.2.4) the upward lift and nose-up pitching moment are positive.

The generalised force $Q_{e.f}^{(i)}$ due to this gust corresponding to the generalised co-ordinate q_i is found in a similar way like the generalised forces due to the motion of the wing as described in Section D.4.2 of Appendix D by calculating the work done δW_i by the air forces in varying q_i to $q_i + \delta q_i$ (the suffix 'e.f' in Q indicates excitation of external force)

$$\delta W_i = \delta q_i \int_0^S \left\{ L(y) h_i(y) + M(y) \alpha_i(y) \right\} dy \quad \dots (F.2.1.2.5)$$

where $h_i(y)$ and $\alpha_i(y)$ are orthogonal mode shapes produced by the eigen value solution and S is the semi-span

Equation (F.2.1.2.5) can be written as

$$\frac{\delta W_i}{\delta q_i} = \int_0^S \left\{ L(y) h_i(y) + M(y) \alpha_i(y) \right\} dy \quad \dots (F.2.1.2.6)$$

In matrix notation

$$\frac{\delta W_i}{\delta q_i} = \int_0^S \left[h_i(y) \quad \alpha_i(y) \right] \begin{Bmatrix} L(y) \\ M(y) \end{Bmatrix} dy \quad \dots (F.2.1.2.7)$$

$$\text{or } \begin{Bmatrix} \frac{\delta w_1}{\delta q_1} \\ \frac{\delta w_2}{\delta q_2} \\ \vdots \\ \frac{\delta w_r}{\delta q_r} \end{Bmatrix} = \int_0^S \begin{bmatrix} h_1 & \alpha_1 \\ h_2 & \alpha_2 \\ \vdots & \vdots \\ h_r & \alpha_r \end{bmatrix} \begin{Bmatrix} L(y) \\ M(y) \end{Bmatrix} dy = \begin{Bmatrix} Q_{e.f}^{(1)} \\ Q_{e.f}^{(2)} \\ \vdots \\ Q_{e.f}^{(r)} \end{Bmatrix}$$

...(F.2.1.2.8)

$$\text{where, } Q_{e.f}^{(i)} = \int_0^S \{ L(y) h_i(y) + M(y) \alpha_i(y) \} dy$$

...(F.2.1.2.9)

The lift $L(y)$ and moment $M(y)$ are given by equations (F.2.1.2.3) and (F.2.1.2.4) wherein the semichord (b) and the reduced frequency parameter vary along the span of the wing. In programming equation (F.2.1.2.9) the sign of lift $L(y)$ is to be reversed to account for the fact that the downward lift is positive for the axis system chosen. This leads to changing the sign for the pitching moment $M(y)$ as well because in the axis system chosen the nose up pitching moment is positive and in equation (F.2.1.2.4) M has been expressed as a multiple of L . Thus equation (F.2.1.2.9) gives the generalised force $Q_{e.f}^{(i)}$ due to the external force corresponding to the generalised co-ordinate q_i .

F.2.2 Equations of Motion and Solution of the Frequency Response Function

Following a similar approach to the formulation

of the flutter problem, the n Lagrangian equations can be expressed as

$$\left[-\omega^2 \begin{bmatrix} M \end{bmatrix} + \begin{bmatrix} S \end{bmatrix} - \begin{bmatrix} Q_{F_m} \end{bmatrix} \right] \{q\} = \{Q_{e.f}\}$$

...(F.2.2.1)

where $\begin{bmatrix} M \end{bmatrix}$ and $\begin{bmatrix} S \end{bmatrix}$ are respectively, the generalised mass and stiffness matrices, $\begin{bmatrix} Q_{F_m} \end{bmatrix}$ is the square matrix with complex coefficient of the generalised aerodynamic forces arising from the motion of the aircraft and $\{Q_{e.f}\}$ is the column matrix of the generalised aerodynamic forces with complex coefficients arising from the sinusoidal gust as the external exciting force.

Equation (F.2.2.1) can be written as

$$\left[-\omega^2 \begin{bmatrix} M \end{bmatrix} + \begin{bmatrix} S \end{bmatrix} - \begin{bmatrix} Q_{F_m} \end{bmatrix} \right] \{\tilde{q}\} e^{i\omega t} = \{\tilde{Q}_{e.f}\} e^{i\omega t}$$

...(F.2.2.2)

$$\therefore \{\tilde{q}\} = \left[-\omega^2 \begin{bmatrix} M \end{bmatrix} + \begin{bmatrix} S \end{bmatrix} - \begin{bmatrix} Q_{F_m} \end{bmatrix} \right]^{-1} \{\tilde{Q}_{e.f}\}$$

...(F.2.2.3)

where

$$\{\tilde{q}\} = \begin{Bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \\ \vdots \\ \tilde{q}_r \end{Bmatrix}$$

...(F.2.2.4)

r = No. of modes considered

Therefore, the frequency response functions of the vertical displacement and pitching rotation at a spanwise station y are respectively given by

$$H(i\omega)_h = \begin{Bmatrix} \hat{q}_1 \\ \hat{q}_2 \\ \vdots \\ \hat{q}_r \end{Bmatrix} \begin{bmatrix} h_1(y) & h_2(y) & \dots & h_r(y) \end{bmatrix} \quad \dots(F.2.2.5)$$

$$= \{ \hat{q} \} [h(y)] \quad \dots(F.2.2.6)$$

$$H(i\omega)_\alpha = \begin{Bmatrix} \hat{q}_1 \\ \hat{q}_2 \\ \vdots \\ \hat{q}_r \end{Bmatrix} \begin{bmatrix} \alpha_1(y) & \alpha_2(y) & \dots & \alpha_r(y) \end{bmatrix} \quad \dots(F.2.2.7)$$

$$= \{ \hat{q} \} [\alpha(y)] \quad \dots(F.2.2.8)$$

F.3 The Power Spectral Density (PSD) Representation of Atmospheric Turbulence

It is extremely difficult to construct an accurate realistic model of the atmosphere because the physical processes which generate atmospheric turbulence are not completely understood. However, the problem of air turbulence has been studied theoretically by many research workers and different approximate analytical expressions for the power spectral density function have been proposed. The agreement between measured and calculated PSD functions within the approximation

of the theory is usually good. Out of many suggested analytical expressions for the PSD functions of one dimensional turbulence in an isotropic gust field, there are two, namely the Dryden Spectrum and the Von Karman Spectrum which have been universally accepted as the statistical knowledge of atmospheric turbulence. Although the former is a simple algebraic expression, the latter gives better agreement with experimental data.

The PSD of the atmospheric turbulence given by Dryden is

$$\phi_D(\omega) = \bar{W}_G^2 \frac{L}{\pi U} \frac{1 + 3 \left(\frac{\omega L}{U}\right)^2}{\left[1 + \left(\frac{\omega L}{U}\right)^2\right]^2} \quad \dots(F.3.1)$$

whereas the exposition of the theory of turbulence by its PSD given by von Karman is

$$\phi_V(\omega) = \bar{W}_G^2 \frac{L}{\pi U} \frac{1 + 2.6667 \left(1.339 \frac{\omega L}{U}\right)^2}{\left[1 + \left(1.339 \frac{\omega L}{U}\right)^2\right]^{11/6}} \quad \dots(F.3.2)$$

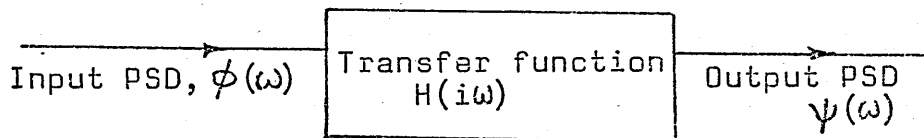
In equations (F.3.1) and (F.3.2), $\bar{W}_G^2$ is the mean square value of the gust velocity, L is the scale of turbulence, U is the forward speed of the aircraft and ω is the frequency in rad/sec. The distributions of these two spectra for various scale lengths of turbulence have been shown in Figs. 77-78.

Details about the Dryden and von Karman spectra and other relevant information like the selection of scale length of turbulence can all be found in Ref. 125 where spectra of various scale lengths are also discussed.

The form of PSD functions of atmospheric turbulence represented by equations (F.3.1) and (F.3.2) defines statistical quantities of interest. The PSD's defined are for the gust velocities and their integration over the entire range of frequencies gives the mean square value of the respective input gust velocity.

F.4 Summary of the PSD Method

The PSD method of finding the response of an aircraft to atmospheric turbulence involves the evaluation of the transfer function of the aircraft and the knowledge of the input PSD so that the output PSD can be found out in a straight forward way as has been explained in earlier sections of this Appendix. The fundamental equation (F.1.1) which governs the input/output relation can be solved when the transfer functions are known from equations (F.2.2.5-F.2.2.8) and the input PSDs are given by (F.3.1) and (F.3.2). The following block diagram summarizes the procedure



$$= |H(i\omega)|^2 \phi(\omega)$$

APPENDIX G

G.0 THE RESPONSE OF FLEXIBLE AEROPLANES TO DISCRETE GUSTS

Although the PSD method described in Appendix F is widely used, gust analysts in British industry have reservations about its suitability under certain circumstances. As a possible alternative to this method in the domain of response analysis, research workers in Britain use the 'Discrete Gust Method' emphasizing on all its merits. When compared with the statistical approach some of the response problems are more adequately treated by the discrete gust approach which accounts for non-Gaussian distributions* and at the same time incorporates relevant spectral information (Refs. 88, 89, 94 and 95). In general, the discrete gust method assumes a gust of a specific profile, usually "one-minus-cosine", (smooth ramp gust) which the aircraft is assumed to encounter. The length of the gust of a particular intensity is varied to give the most critical aircraft response either to a single gust or to a combination of many. It is considered that discrete gust is a rational representation of the gust configuration and this appendix is devoted to those aspects of the response problem which depend on the important parameters like gust intensity, gradient and its profile. The spanwise distribution of non uniform gust velocity may have important effects upon the aeroplane response but is omitted and made beyond the scope of this appendix. Analytical method has been directed here towards the calculation of acceleration response (normal and angular) of flexible aircraft subject to step gusts and to single and pair of smooth ramp gusts with identification of critical response cases. The response to a smooth ramp gust can be evaluated from the response to a unit step gust with the use of some mathematical technicality. However

*W.S.Pi and C. Hwang have recently studied PSD of non-Gaussian model for aircraft response analysis. See AIAA Journal, Vol.16, No. 7 July 1978, pp.641-642.

in all cases, the aircraft is represented by a linear system.

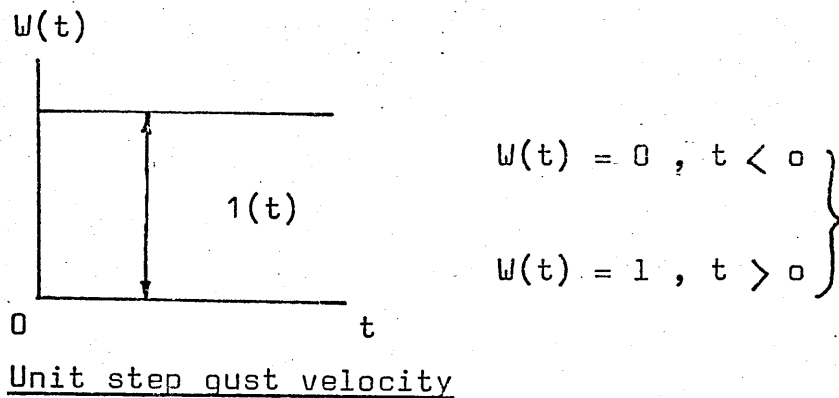
It is worth mentioning a few points about the two methods of gust response analysis, namely, the PSD and Discrete Gust methods which are in common use throughout aircraft industry and research institutions. The author thinks that although gusts do occur that are so different from the surrounding continuous turbulence that they can be considered to be isolated discrete events, there can be little doubt that a model of atmospheric turbulence as a continuous random process is very close to reality. However, the author does think that the discrete gust method, by incorporating the variation of intensity with gradient distance, can equally include the correct scaling with respect to wavelength, and the choice between the two approaches probably rests on more fundamental issues (Ref.94).

Although the discrete approach can only handle gusts of a particular shape, clearly it is possible to calculate the full dynamic response of a flexible aircraft to the discrete gusts which usually consider a number of gust shapes (ramp, sin-cosine etc) of various lengths and seek a "worst" gust for the particular flight condition. From a practical point of view in some instances, it is possible to find cases where isolated discrete gusts stand out in a clearly identifiable manner and there should not persist a reluctance to dispense with a discrete gust model as a means of representing an atmospheric disturbance.

G.1 Response to a Unit Step

Mitchell (Ref.86) has introduced a Fourier transform method to find the response of a flexible aircraft to a unit step and thus avoided the direct evaluation of the Küssner functions. If the transfer function (or the frequency response) of the aeroplane is known Bromwich's integral formula (Refs.49 and 132) provides an explicit formula for calculating the response to unit step which is also known as indicial admittance of the aeroplane. The theory presented here is by and large taken from Ref. 49.

Let us consider a gust with a velocity history



$$\left. \begin{aligned} W(t) &= 0, \quad t < 0 \\ W(t) &= 1, \quad t > 0 \end{aligned} \right\}$$

...(G.1.1)

The Fourier transform of this is

$$\bar{W}(\omega) = \pi \delta(\omega) + \frac{1}{i\omega} \quad \dots(\text{G.1.2})$$

where $\delta(\omega)$ is the Dirac delta function which does not pose a problem in the case of the transfer function for acceleration which tends to zero as ω tends to zero and the transform response does not become infinite at $\omega = 0$.

Now if $H_r(i\omega)$ is the transfer function (admittance function) of the aeroplane then the indicial admittance (response to a unit step) is given by

$$A_r(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\omega=+\infty} \frac{H_r(i\omega)}{i\omega} e^{i\omega t} d\omega \quad \dots(\text{G.1.3})$$

where the path of integration makes an infinitesimal loop below the origin. As it is seen from the above equation that the integrand has a pole at the origin i.e. when $\omega = 0$, the integrand is infinite and unless $H_r(0) = 0$,

the integral will not converge. In dealing with the transfer function of acceleration which tends to zero as ω tends to zero, the integral

$$\int_{\omega = -\infty}^{\omega = +\infty} H_r(i\omega) \delta(\omega) d\omega$$

becomes zero and so the response to unit step given by equation (G.1.3) can be evaluated without any difficulty in integration. However equation (G.1.3) can be written in the following form

$$A_r(t) = \frac{H_r(0)}{2\pi} \int_{\omega = -\infty}^{\omega = +\infty} \frac{e^{i\omega t}}{i\omega} d\omega + \frac{1}{2\pi} \int_{\omega = -\infty}^{\omega = +\infty} \left[\frac{H_r(i\omega) - H_r(0)}{i\omega} \right] e^{i\omega t} d\omega$$

...(G.1.4)

It can be shown that by applying residue theorem (Ref.49) that

$$\frac{1}{2\pi} \int_{\omega = -\infty}^{\omega = +\infty} \frac{e^{i\omega t}}{i\omega} d\omega = 1(t) \quad \dots(G.1.5)$$

Equation (G.1.4) with the help of equation (G.1.5) reduces to

$$A_R(t) = H_R(0) \cdot 1(t) + \frac{1}{2\pi} \int_{\omega = -\infty}^{\omega = +\infty} \frac{H_R(i\omega) - H_R(0)}{i\omega} e^{i\omega t} d\omega$$

...(G.1.6)

We make the following substitution in equation (G.1.6)

$$e^{i\omega t} = \cos \omega t + i \sin \omega t \quad \dots(G.1.7)$$

$$H_R(i\omega) = R_R(\omega) + i I_R(\omega) \quad \dots(G.1.8)$$

so that equation (G.1.6) becomes

$$A_R(t) = R_R(0) \cdot 1(t) + \frac{1}{2\pi} \int_{\omega = -\infty}^{\omega = +\infty} \left[\frac{R_R(\omega) - R_R(0)}{\omega} \sin \omega t \right.$$

$$\left. + \frac{I_R(\omega)}{\omega} \cos \omega t \right] d\omega - \frac{i}{2\pi} \int_{\omega = -\infty}^{\omega = +\infty} \left[\frac{R_R(\omega) - R_R(0)}{\omega} \times \right.$$

$$\left. \cos \omega t - \frac{I_R(\omega)}{\omega} \sin \omega t \right] d\omega$$

...(G.1.9)

where the assumption $I_R(0) = 0$ requires merely that when the forcing function is statically applied the lag is zero which is a valid assumption in aeroelastic systems.

Because $A_r(t)$ is real, the last term in equation (G.1.9) must vanish. Moreover, we know (see Ref.49)

$$\left. \begin{aligned} \int_{\omega = -\infty}^{\omega = +\infty} \frac{\sin \omega t}{\omega} d\omega &= \pi \quad \text{for } t > 0 \\ &= -\pi \quad \text{for } t < 0 \end{aligned} \right\} \quad \dots(G.1.10)$$

The equation (G.1.9) finally becomes

$$A_r(t) = \frac{1}{2} R_r(0) + \frac{1}{2\pi} \int_{\omega = -\infty}^{\omega = +\infty} \left[\frac{R_r(\omega)}{\omega} \sin \omega t + \frac{I_r(\omega)}{\omega} \cos \omega t \right] d\omega \quad \dots(G.1.11)$$

The equation (G.1.11) provides an explicit formula for computing the indicial admittance $A_r(t)$ if the real and imaginary parts of the mechanical admittance functions are known. The equation (G.1.11) can be reduced to a simpler form by putting the conditions that $R_r(\omega)$ is an even function so that $R_r(-\omega) = R_r(\omega)$, and $I_r(\omega)$ is an odd function so that $I_r(-\omega) = -I_r(\omega)$. These conditions are generally satisfied in aeroelastic systems so that we obtain from equation (G.1.11).

$$A_r(t) = \frac{1}{2} R_r(0) + \frac{1}{\pi} \int_{\omega = 0}^{\omega = \infty} \left[\frac{R_r(\omega)}{\omega} \sin \omega t + \frac{I_r(\omega)}{\omega} \cos \omega t \right] d\omega \quad \dots(G.1.12)$$

Since $H_r(\omega)$ and $I_r(\omega)$ are most of the time not available in explicit function form but are available as plotted curves, the summation form of equation (G.1.12) is more useful for practical application and computational procedure:

$$A_r(t) = \frac{1}{2} R_r(0) + \frac{2}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} \left[R_r(n\omega_0) \sin n\omega_0 t + I_r(n\omega_0) \cos n\omega_0 t \right] \quad \dots(G.1.13)$$

The frequency ω_0 is arbitrarily selected so as to obtain satisfactory convergence. The criterion for convergence is that $R_r(n\omega_0)$ and $I_r(n\omega_0)$ approach zero for large values of n . When the indicial admittance has been obtained the response to an arbitrary disturbance can be computed by application of Duhamel's integral.

G.2 Response to Smooth Ramp Gusts

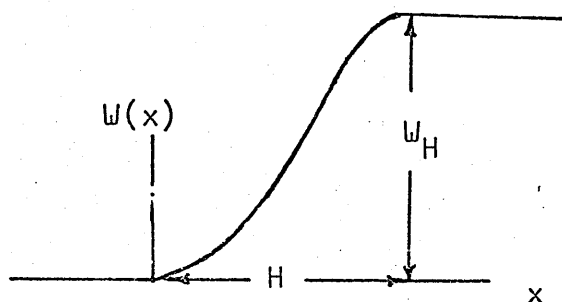
If the response $F(t)$ of an aircraft (may be displacement or acceleration) flying at a certain speed to a unit step is known, the response to a family of ramp gusts and the critical gust length at which the extreme response occurs can be found out. The problem has been treated by Purcell (Ref.96) fairly recently as a mathematical and computational exercise. The computer program developed by Purcell (Ref.96) has been extensively used in this thesis to calculate the worst response cases due to a single and pair of smooth ramp (one-minus-cosine) gusts. A portion of the theory (Ref.96) based on which Purcell wrote his program is presented here to give an insight to what has really been done.

A single smooth ramp (one-minus-cosine) gust

velocity can be mathematically represented by (Refs.94-96)

$$W(x) = \frac{1}{2} W_H \left(1 - \cos \frac{\pi x}{H} \right) \dots (G.2.1)$$

where $W_H = W_0 H^{\frac{1}{3}}$, W_0 being the gust strength parameter (Ref.94). The sketch below shows the above gust



Single Smooth Ramp Gust

Firstly, the problem is to determine a gust length $\bar{H}$ which causes the worst response, $\bar{\gamma}$ and then to extend it to deduce the most adverse gust pair. The problem with some complicating feature is basically one of numerical integration and interpolation.

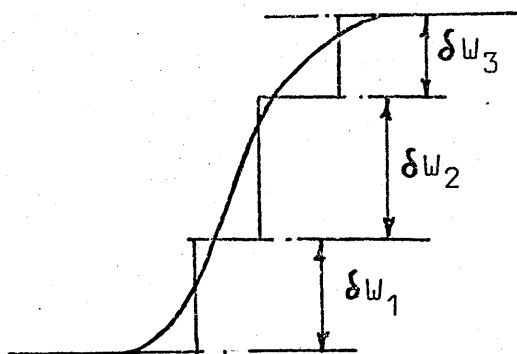
G.2.1 Consideration of the Theory

The theory developed by Purcell (Ref.96) is briefly recapitulated to show how the response to a smooth ramp gust can be evaluated from the knowledge of the response to a step input.

Let the step response be $F(t)$ and the ramp response be $\phi(H,t)$. The calculation of ϕ essentially involves the numerical integration of $F(t)$. Strictly speaking $F(t)$ should be $F(V,t)$ where V is the aircraft velocity. However, it is customary to regard the aircraft velocity as constant during an encounter with turbulence. (It is difficult to determine ϕ and hence $\gamma(H)^*$)

* $\gamma(H)$ is the maximum response corresponding to the gust length H .

at different speeds and no attempt appears to have been made by anyone to date).



As shown in the adjoining sketch, a ramp gust can be thought of as a series of n step gusts. Thus it is necessary to combine n step responses, each scaled and appropriately time shifted. The linearity assumption is essential if the

means of combination is simple addition. The response at time t' will be given by (Ref.96).

$$\phi(H, t') = \sum_{i=1}^J \delta w_i F(t' - \tau_i) \quad \dots(G.2.1.1)$$

where $n \delta t = \frac{H}{V}$; $\tau_i = i \delta t$; $J = \min(n, t'/\delta t)$

letting $\delta t \rightarrow 0$

$$\phi(H, t') = \int_0^{\min(t', \frac{H}{V})} \left. \frac{dw}{dt} \right|_{\tau} F(t' - \tau) d\tau \quad \dots(G.2.1.2)$$

Making the transformation $t = t' - \tau$ in equation (G.2.1.2)

$$\phi(H, t') = \int_{\max(0, t' - \frac{H}{V})}^{t'} \left. \frac{dw}{dt} \right|_{t' - t} F(t) dt \quad \dots(G.2.1.3)$$

The limits reflect the assumption that $\frac{dw}{dt} = 0$
when $t < 0$ or $t > H/V$

Expressing equation (G.2.1) as a function of
time and setting the gust strength parameter,
 $W_0 = 1$ we get,

$$\ddot{w}(t) = \frac{1}{2} H^{\frac{1}{3}} \left(1 - \cos \frac{\pi V t}{H} \right) \quad \dots (G.2.1.4)$$

Using equation (G.2.1.4) in equation (G.2.1.3)
we obtain

$$\phi(H, t') = \frac{\pi}{2} \frac{V}{H^{\frac{2}{3}}} \int_{\max(0, t' - \frac{H}{V})}^{t'} \sin \frac{\pi V}{H} (t' - t) F(t) dt \quad \dots (G.2.1.5)$$

There is no difficulty in interpolation to find
 t_γ at which the maximum response $\gamma(H)$ occurs
because $\frac{\partial \phi}{\partial t}$ for the smooth ramp (one-minus-
cosine) gust as seen from the above equation
is everywhere smooth

$$\frac{\partial \phi}{\partial t'} = \frac{\pi^2 V^2}{2H^{\frac{5}{3}}} \int_{\max(0, t' - \frac{H}{V})}^{t'} \cos \frac{\pi V}{H} (t' - t) F(t) dt \quad \dots (G.2.1.6)$$

Instead of evaluating the above integral Purcell
(Ref.96) elected to fit a cubic spline through
 $\phi(t)$ and to interpolate the required extrema
from it having located the maximum and minimum
tabulated points by inspection.

What Purcell's program (Ref.96) calculates can be summarised as follows.

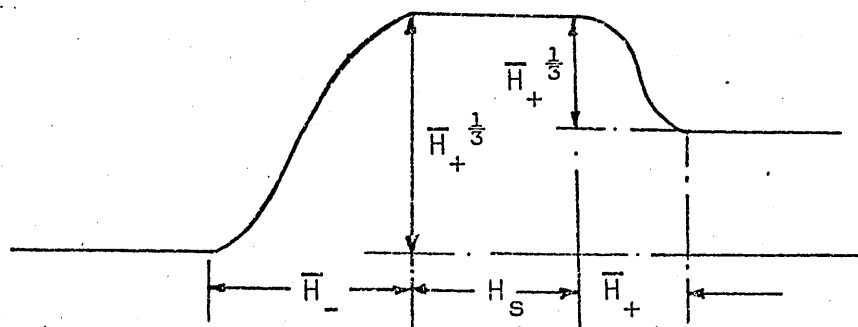
(i) the peak responses corresponding to the maximum positive and negative values of the response and the trial gust lengths at which these peaks occur: the primary peak of $\phi(t)$ and the largest overswing are determined for each trial value of H supplied by the user. The basic procedure in the program is to search the set of $\phi(t)$ for the largest positive and negative values and to then interpolate $\gamma(H)$ and t_γ using $\frac{d\phi}{dt}$. Then it seeks the extreme positive and negative responses $\gamma_+(H)$ and $\gamma_-(H)$ respectively.

(ii) the program then evaluates the most adverse gust pair situation one is acting in the opposite direction of the other and the corresponding time and maximum response and their distance of separation: having calculated $\bar{\gamma}_+$, $\bar{t}_+$, $\bar{H}_+$, $\bar{\gamma}_-$, $\bar{t}_-$, and $\bar{H}_-$, the primary peak with the larger of $\bar{\gamma}_+$ and $\bar{\gamma}_-$ is identified. The maximum possible response is then

$\bar{\gamma}_+ + |\bar{\gamma}_-|$ occurring at $\bar{t}_+$ or $\bar{t}_-$ whichever is the later. If for clarity it is assumed $\bar{t}_- > \bar{t}_+$ then as explained in Ref.96 the worst gust pair is a ramp gust of length $\bar{H}_-$ followed by one of opposite sign of length $\bar{H}_+$ separated by H_s where

$$H_s = V (\bar{t}_- - \bar{t}_+) - \bar{H}_-$$

The situation is explained in the following sketch



Critical Gust Pair: the extreme response is

$$\bar{\gamma}_+ + |\bar{\gamma}_-| ; \text{ Gust Strength Parameter } W_0 = 1$$

(iii) the program calculates the gust length sensitivity (λ) defined by Jones (Ref.94) which is a measure of sharpness of the peak in $\gamma(H)$: the gust sensitivity λ at the tuned gust length $\bar{H}$ defined by Jones is given by

$$\lambda = \frac{1}{\log_e 2} \left[\frac{2 \gamma(\bar{H}) - \gamma(2\bar{H}) - \gamma(\bar{H}/2)}{2 \pi \gamma(\bar{H})} \right]^{\frac{1}{2}}$$

...(G.2.1.7)

In all the above cases the program calculates the response characteristics for a unit value of the gust strength parameter W_0 and because of the linearity assumption, the response to various gust intensities (and hence for various gust strength parameters) can be evaluated by simply multiplying the response obtained from Purcell's program by the appropriate value of the gust strength parameter W_0 .

APPENDIX H

H.0

GROUND RESONANCE TEST ON THE KESTREL

The author combined his efforts with those of the British Aerospace personnel who were asked by the CAA (Airworthiness Division) to carry out a ground resonance test on the Kestrel for a successful solution of its flutter problem. The test is necessary to substantiate theoretical calculations of the static effectiveness and the flutter characteristics of the wing. The main objective is thus to measure wing bending and torsional frequencies and mode shapes for the purpose of verifying the structural data used in the theoretical calculations.

The instability of the Kestrel wing has been found to be due to the coupling between the fundamental wing bending and fundamental torsion modes (see Chapter 4). Therefore, particular efforts have been made to establish these two modes. The details of the ground resonance test can be found in BAC Report No. BAe/D/SLY/2. The fundamental bending and torsional frequencies are reported in Table 45 together with the theoretical results using PAFEC.

APPENDIX I

I.0 USE OF PAFEC IN EVALUATING NATURAL FREQUENCIES AND MODE SHAPES

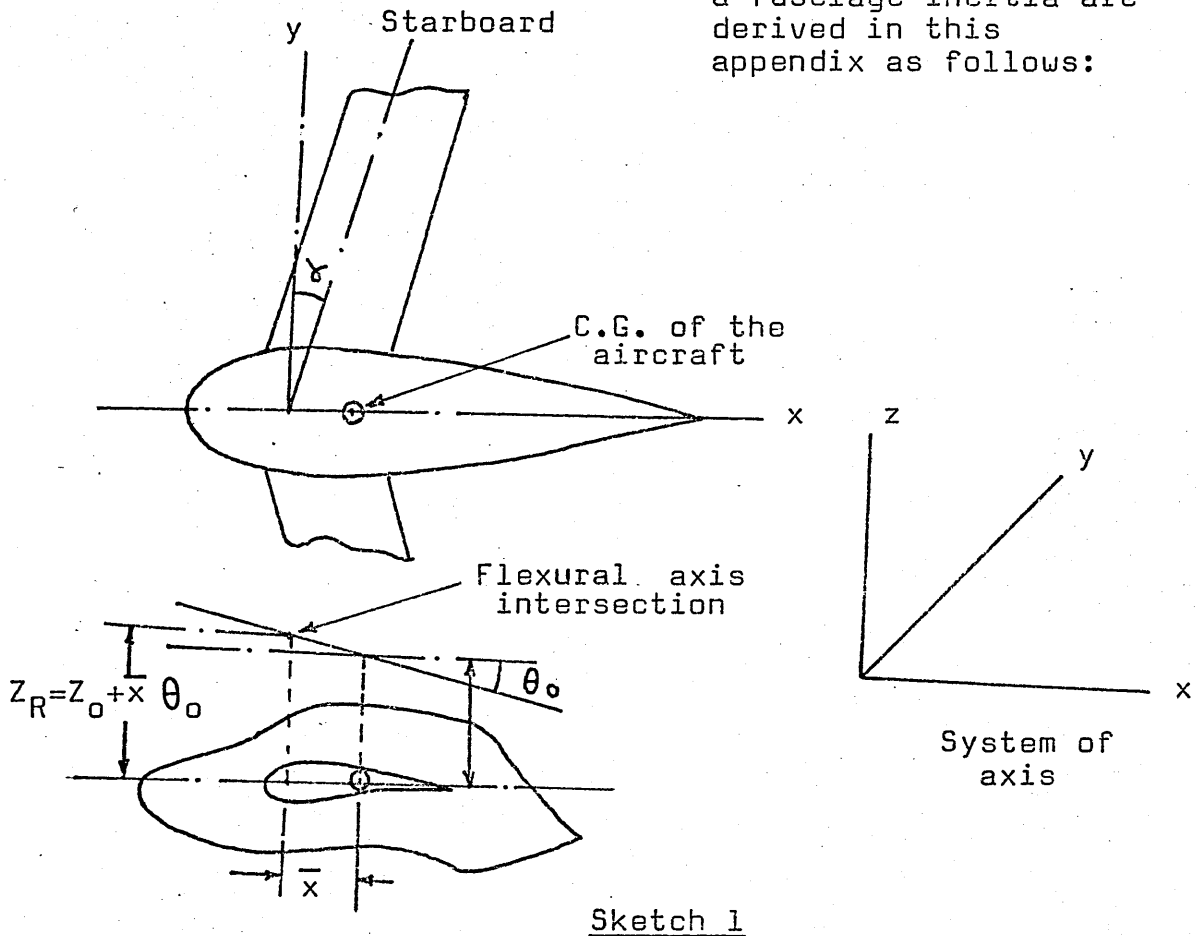
The program for automatic finite element calculation (PAFEC) developed by Nottingham University has been used in this thesis for evaluation of natural frequencies and mode shapes of the Ricochet and the Kestrel sailplanes. Details of the package can be found in Ref.120. Only a brief description of the calculation procedure of the eigen value problem is outlined here. Beam and lumped mass elements have been used to idealise the sailplane structures. After the master mass and stiffness matrices $[M]$ and $[K]$ are respectively assembled from the elemental matrices, one of the subroutines used in PAFEC, factorizes $[M]$ using Choleski decomposition to give $[L]$ where $[L]^T [L] = [M]$. Then the matrix $[L]^{-1} [K] [L]^T$ is formed and eigen values are found using the Householder tridiagonalisation. After the eigen values and eigen vectors are determined, one of the PAFEC routines carries out a transformation and normalisation so that $\{x_i\} [M] \{x_j\} = 1$ for $i = j$ and 0 for $i \neq j$.

The process is dependent upon the mass matrix $[M]$ being positive definite which is always a case for physical systems. Full details of the process and its limitations are given in Ref.133.

APPENDIX J

J.0 COUPLED BENDING AND TORSIONAL VIBRATION OF A UNIFORM SWEEP WING IN SYMMETRIC MOTION WITH A FUSELAGE INERTIA

The free vibrational equations of motion of a uniform swept wing with a fuselage inertia are derived in this appendix as follows:



In the above sketch

α = Angle of sweep

$\bar{x}$ = Distance of the aircraft C.G. from the flexural axis and fuselage centre line intersection (i.e. the origin)

z_R = Rigid-body vertical displacement of the flexural axis intersection of the wing root

θ_o = Rigid-body rotation (nose-up)

Let,

EI = Bending rigidity of the uniform wing

GJ = Torsional rigidity of the uniform wing

l = Length of the uniform wing along its flexural axis

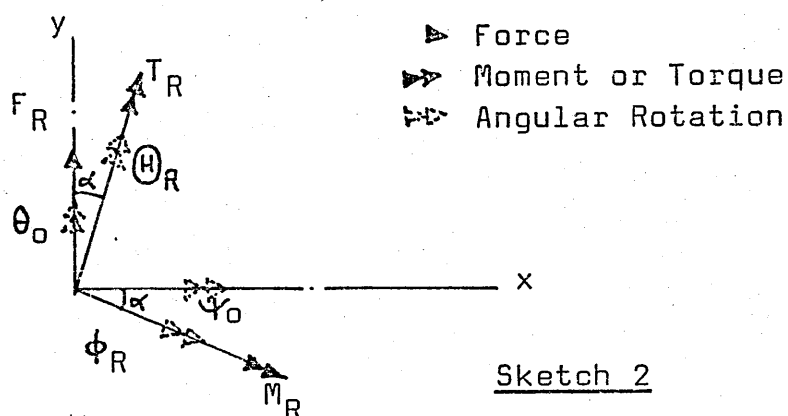
m_f = One half mass of the fuselage

I_f = One half mass moment of inertia of fuselage

m = Mass per unit length of the wing

I_p = Mass moment of inertia per unit length of the wing

The force and moment balance at the flexural axis intersection of the wing root is shown below in Sketch 2.



$(H)_R$ is the angular rotation at the wing root corresponding to the torque T_R acting along the direction of the flexural axis. ϕ_R is the angular rotation and M_R is the moment both

acting in a direction perpendicular to the flexural axis. F_R is the vertical force at the root and θ_0 is the nose up pitching rotation whereas ψ_0 is the rolling rotation. It can be shown that the following relation holds

$$T_R = - \left(\frac{GJ}{I} k_2 l \tan k_2 l \right) \Theta_R \quad \dots (J.1)$$

where

$$k_2 l = \omega l \sqrt{\frac{I_D}{GJ}} \quad \dots (J.2)$$

where, ω is the natural frequency of vibration

Also,

$$\begin{Bmatrix} F_R \\ \frac{M_R}{I} \end{Bmatrix} = \frac{EI}{l^3} \frac{(k_1 l)^3}{(1 + \cosh k_1 l \cos k_1 l)} x$$

$$\begin{bmatrix} -(\cosh k_1 l \sin k_1 l + \cos k_1 l + l \sinh k_1 l) & - \frac{(\sinh k_1 l \sin k_1 l)}{k_1 l} \\ - \frac{(\sinh k_1 l \sin k_1 l)}{k_1 l} & \frac{(\sinh k_1 l \cos k_1 l - \cosh k_1 l \sin k_1 l)}{(k_1 l)^2} \end{bmatrix} x$$

$$\begin{Bmatrix} x_R \\ l\phi_R \end{Bmatrix} \quad \dots (J.3)$$

where

$$k_1 l = l \omega^{\frac{1}{2}} \left(\frac{m}{EI} \right)^{\frac{1}{4}} \quad \dots (J.4)$$

Equation (J.3) can be written in a simplified form

$$\begin{Bmatrix} F_R \\ M_R \end{Bmatrix} = \frac{EI}{l^3} \begin{bmatrix} f_{11} & l f_{12} \\ l f_{12} & l^2 f_{22} \end{bmatrix} \begin{Bmatrix} z_R \\ \phi_R \end{Bmatrix} \quad \dots (J.5)$$

where f_{11} , f_{12} and f_{22} are non dimensional functions of $k_1 l$ given by

$$f_{11} = - (k_1 l)^3 (\cosh k_1 l \sinh k_1 l + \cosh k_1 l \sinh k_1 l) / (1 + \cosh k_1 l \cosh k_1 l) \quad \dots (J.6)$$

$$f_{12} = - (k_1 l)^2 \sinh k_1 l \sinh k_1 l / (1 + \cosh k_1 l \cosh k_1 l) \quad \dots (j.7)$$

$$f_{22} = (k_1 l) (\sinh k_1 l \cosh k_1 l - \cosh k_1 l \sinh k_1 l) / (1 + \cosh k_1 l \cosh k_1 l) \quad \dots (j.8)$$

Now if θ_0 is the nose up pitching rotation and ψ_0 is the rolling rotation at the root, then the following relationships hold (see the force diagram shown in Sketch 2)

$$\Theta_R = \theta_o \cos\alpha \quad \dots(J.9)$$

$$\phi_R = -\theta_o \sin\alpha \quad \dots(J.10)$$

$$\Theta_R = \psi_o \sin\alpha \quad \dots(J.11)$$

$$\phi_R = \psi_o \cos\alpha \quad \dots(J.12)$$

Referring to the same force diagram in Sketch 2.

The pitching moment applied to the wing

$$= T_R \cos\alpha - M_R \sin\alpha \quad \dots(J.13)$$

The rolling moment applied to the wing

$$= T_R \sin\alpha + M_R \cos\alpha \quad \dots(J.14)$$

Now from the Sketch 1 shown previously

$$Z_R = Z_o + \bar{x} \theta_o \quad \dots(J.15)$$

The upward force applied to the fuselage is $-m_f \omega^2 Z_o$ and the nose-up moment applied to the fuselage about the C.G. is $-I_f \omega^2 \theta_o$ so that the moment about the wing-flexural axis intersection with fuselage centre line is $-\omega^2 (I_f \theta_o - \bar{x} m_f Z_o)$. Therefore, the total force applied vertically and the total nose-up pitching moment applied at the wing-flexural axis intersection are respectively given by

$$F_T = -m_f \omega^2 Z_o + F_R \quad \dots(J.16)$$

$$M_T = -\omega^2 (I_f \theta_o - \bar{x} m_f Z_o) + T_R \cos \alpha - M_R \sin \alpha$$

...(J.17)

With the use of equation (J.1), (J.5) and (J.9), equations (J.16) and (J.17) become

$$F_T = (-m_f \omega^2 + \frac{EI}{l^3} f_{11}) Z_R + \left[(m_f \bar{x} \omega^2 - \frac{1EI}{l^3} f_{12}) \sin \alpha \right] \theta_o$$

...(J.18)

and

$$M_T = (\omega^2 \bar{x} m_f - \frac{EI}{l^2} f_{12} \sin \alpha) Z_R$$

$$+ (-I_f \omega^2 - m_f \bar{x} \omega^2 - GJk_2 \cos^2 \alpha \tan k_2 l$$

$$+ l^2 f_{22} \sin^2 \alpha) \theta_o \quad \dots(J.19)$$

In the free vibrational condition $F_T = 0$ and $M_T = 0$ so that the following ratios R_1 and R_2 can be derived from equations (J.18) and J.19).

$$R_1 = \left(\frac{Z_R}{\bar{x} \theta_o} \right) = \frac{\sinh \lambda_1 \sin \lambda_1 + \frac{\bar{x}}{X} \frac{m_f}{m} (1 + \cosh \lambda_1 \cos \lambda_1) \lambda_1^2}{(\cosh \lambda_1 \sin \lambda_1 + \sinh \lambda_1 \cos \lambda_1) \lambda_1 + \lambda_1^2 \frac{m_f}{m_w} (1 + \cosh \lambda_1 \cos \lambda_1)}$$

...(J.20)

and

$$\begin{aligned}
 & (\cosh \lambda_1 \sin \lambda_1 - \sinh \lambda_1 \cos \lambda_1) + \frac{\lambda_2}{\lambda_1} \times \\
 & (1 + \cosh \lambda_1 \cos \lambda_1) \left(\frac{GJ}{EI} \right) \cot^2 \alpha \tan \lambda_2 \\
 & + \left[\frac{m_f}{m} \left(\frac{\bar{x}}{X} \right)^2 + \frac{I_f}{m_w} X^2 \right] \times \\
 R_2 = \frac{Z_R}{X \theta_0} = & \frac{\lambda_1^3 (1 + \cosh \lambda_1 \cos \lambda_1)}{\lambda_1 (\sinh \lambda_1 \sin \lambda_1) + \left(\frac{\bar{x}}{X} \right) \left(\frac{m_f}{m_w} \right) \lambda_1^3 (1 + \cosh \lambda_1 \times} \\
 & \cos \lambda_1) \\
 & \dots (J.21)
 \end{aligned}$$

where

$$\lambda_1 = k_1 l$$

$$\lambda_2 = k_2 l$$

$$X = l \sin \alpha$$

$$m_w = m l = \text{mass of the wing}$$

$$\dots (J.22)$$

The values of the ratios R_1 and R_2 in equations (J.20) and (J.21) are plotted against the frequency ω and their intersections yield the natural frequencies of coupled vibration. Equations (J.20) and (J.21) are both coupled by $X (= l \sin \alpha)$ so that the substitution of the sweep angle $\alpha = 0$ in equations (J.18-J.21) leads to the uncoupled motions of the bending and torsional vibration of the uniform wing.

The frequencies and mode shapes were determined for a uniform wing with a fuselage mass using the analytical equations (J.20) and (J.21). The finite element package PAFEC was then used to

to compare the results. An excellent agreement was found between the two sets of results.

APPENDIX K

K.0 INERTIA PROPERTIES OF THE PILOT

In the case of sailplane structures, the pilot inertia plays an important role in aeroelastic calculations because the weight and moment of inertia of the pilot are significant fractions of those of the complete aeroplane. Moreover, it has been observed in Chapter 5 that the flutter speed is dependent on the pitching moment of inertia and is quite sensitive to it (see Fig. 64). The mass moment of inertia values of a pilot could not be traced in the existing literature. In this appendix, the calculated values of the mass moment of inertia of a pilot about the three perpendicular set of axes passing through his C.G. are provided. Fig.132 shows the standard dimensions of an average airman* referred to and taken from the Av.P 970 (Ref.135) whereas Fig 1 describes his sitting position. The individual parts of the pilot body have been idealised to be of cylindrical sections.

Contributions of various parts to the moment of inertias about the three axes passing through the pilot's C.G. are reported in Table 46. (The aircraft axes being parallel to this set). Table 47 summarises the final results of the calculation. The data for the heavy and light pilot shown in Table 47 have been derived by scaling from those of the medium pilot.

* The author wishes to draw attention to the fact that dimensions in Fig.132 are in Imperial units.

APPENDIX L

L.0

BACKGROUND TO COMPUTER PROGRAM DEVELOPMENT

Four major programs have been developed during the course of this thesis. However, there are a number of subroutines which are common to all the programs. Use has been made of existing programs available from other sources and this is noted where appropriate. The four computer programs mentioned above accomplish the following tasks:

- (i) evaluates flutter speed and aeroelastic modes at flutter speed of an aircraft wing using normal modes of vibration
- (ii) investigates the short period oscillation characteristics of a flexible aeroplane
- (iii) calculates the acceleration response of a flexible aeroplane to continuous random atmospheric turbulence using statistical method and transfer function approach.
- (iv) determines the acceleration response of a flexible aeroplane to a unit step gust velocity and feeds the output into Purcell's program (Ref.96) to derive the worst response cases for smooth ramp (one-minus-cosine) gusts.

However, before running any of the programs mentioned above, PAFEC is first used to obtain the natural frequencies, mode shapes and the generalised mass and stiffness matrices. There are two minor programs used with subfile names JRB1 and JRB2 each requiring PAFEC to prepare data for any of the four aeroelastic calculations. The data are first written on magnetic tapes and are finally transferred onto a MAXIMOP file (in the subfile name DAT2) so that each of the aeroelastic programs can operate independently of PAFEC. The first of the two minor programs JRB1 requires the PAFEC data (printed in the subfile name DAT1) for preliminary organisation and set up of the system whereas the second one prepares the relevant information for the subsequent aeroelastic calculations. The listing of the program and data subfiles JRB1, DAT1 and JRB2 follows.

L.O.1 Listing of the Program Subfile JRB1

```
LIBRARY(PAFEC36234)
LIBRARY(COMP)
PROGRAM(D50E)
USE 4=MT0/UNFORMATTED(INPUTMEDTAPE)/
USE 3=MT1/UNFORMATTED(BANERJEE )/
INPUT 5=CRO
OUTPUT 6=LPO
COMPRESS INTEGER AND LOGICAL
END
MASTER INPUT DATA
DIMENSION DLIST(500),CPDDC(13,8),S(1),PLO(1),B(13),AH(13),C(13)
CALL R18000(DLIST,S,CPDDC,PLO,IP,0,0,0)
WRITE(4) DLIST,CPDDC,S
ENDFILE 4
REWIND 4
10 READ(5,10) II,JJ,KK,IJO
   FORMAT(4I5)
   DO 20 I=1,II
20 READ(5,15) B(I),AH(I),C(I)
15 FORMAT(3F0,0)
   DO 25 I=1,II
25 AH(I)=AH(I)/B(I)
   C(I)=C(I)/B(I)
   WRITE(3) II,JJ,KK,IJO
   WRITE(3) (CPDDC(I,2),I=1,II), B,AH,C
ENDFILE 3
REWIND 3
STOP
END
SUBROUTINE EXIT
STOP
END
FINISH
```

VIBRATION OF TAILLESS AIRCRAFT-SYMMETRIC CASE

[illegible]

```

LIBRARY(pAFEC36234)
LIBRARY(COMP)
PROGRAM(D50F)
INPUT 5=CP0
OUTPUT 6=LP0
USE 3=MT1/UNFORMATTED(BANERJEE  )/
USE 4=MT0/UNFORMATTED(INPUTHETAPE)/
COMPRESS INTEGER AND LOGICAL
END
MASTER TAIL-LESS AIRCRAFT
DIMENSION DLIST(500),CPDDC(13,8),S(38,39),PLO(1,1),Z(38,38),DM(38,
138),S1(38,39),W1(38,38),OMEGA(38),AMODE(38,38),W2(38,38),S2(38,38)
2,CHECK(13,13)
EQUIVALENCE (AMODE(1,1),Z(1,1)),(S(1,1),W1(1,1)),(DM(1,1),W2(1,1))
READ(3) II,JJ,KK,IJ0
READ(3)
IJ3=JJ
READ(4) DLIST,CPDDC,S(1,1)
REWIND 4
IP = DLIST(22)+0.1
CALL R34000(DLIST,S,CPDDC,PLO,IP)
CALL R50200(DLIST,S,CPDDC,PLO,IP)
CALL R18720(DLIST,IP,IDF,ILO,IW,IM,ID,IX)
IDF1=IDF+1
CALL MATCOP(S1,S,IDF,IDF1)
K=3
CALL R11120(S1,S2,ID,IDF,IW,IM)
WRITE(3) ((S1(3*I,3*J)),I=1,11),J=1,11)
BACKSPACE 3
READ (3) CHECK
WRITE(6,10) ((CHECK(1,J),J=1,11),I=1,11)
10  FORMAT (7F15.3)
CALL R62000(DLIST,S,CPDDC,PLO,IP,DM,Z)
IFREQ=DLIST(36)-0.9
DO 1 ICI=1 ,IJ3
IC4=IFREQ+ICI
OMEGA(ICI)=DLIST(IC4)
I=ICI
DO 1 J=1,IDF
1  AMODE(J,ICI)=S(ICI,J)
CALL EMATPR(AMODE,IDF,IJ3,K)
CALL EMATPR(OMEGA,IJ3,0,K)
CALL MATMUL(W1,S1,AMODE,IDF,IDF,IJ3)
CALL MATMUL(W2,S2,AMODE,IDF,IDF,IJ3)
CALL TRANS(S2,AMODE,IJ3,IDF)
CALL MATMUL(S1,S2,W1,IJ3,IDF,IJ3)
CALL MATMUL(W1,S2,W2,IJ3,IDF,IJ3)
100 WRITE(6,100)
FORMAT(27H DIAGONAL STIFFNESS MATRIX,///)
CALL EMATPR(S1,IJ3,IJ3,K)
101 WRITE (6,101)
FORMAT(21H DIAGONAL MASS MATRIX,///)
CALL EMATPR(W1,IJ3,IJ3,K)
WRITE (3) (OMEGA(I),I=IJ0,IJ3),((S1(I+IJ3*(J-1),1),I=IJ0,IJ3),
3J=IJ0,IJ3),((W1(I+IJ3*(J-1),1),I=IJ0,IJ3),J=IJ0,IJ3),
3((AMODE(3*I-2,J)),I=1,11),J=IJ0,IJ3),((AMODE(3*I,J),
4I=1,11=1),AMODE(38,J)),J=IJ0,IJ3)
WRITE(3)((AMODE(I,J),I=1,38),J=1,6)
ENDFILE 3
REWIND 3
STOP
END

```

```

SUBROUTINE EXIT
STOP
END
SUBROUTINE R30200(DLIST,S,CPDDC,PLO,IP)
DIMENSION DLIST(1000),S(1),CPDDC(IP,1),PLO(1),SE(6,7),IEPA(1),IDG(
16),DCA(3,3)
CALL R18200(DLIST,IL,ISN,IB,MW,NW,NA,N,NEI,ISSN,INE,IPROP)
CALL R18710(DLIST,IM,IK,ID,ID1,IT,IDT,ILO)
DO 1 IC=1,6
1 IDG(IC)=IC
DO 100 IC=1,IL
IDE=6
CALL R18231(DLIST,IEPA,XYZM,XX,YY,ZZ,XY,XZ,YZ,EX,EY,EZ)
WRITE(6,99) IEPA,XYZM,XX,YY,ZZ,XY,XZ,YZ,EX,EY,EZ
99 FORMAT(15,1E10,5,9E10,3)
DLIST(87)=DLIST(87)+XYZM
4 CALL UNIT(DCA,3)
INE=1
5 CALL NULL(SE,IDE,7)
SE(1,2),SE(2,3),SE(3,4) = XYZM
SE(4,5)=XX+XYZM*(EY*EY+EZ*EZ)
SE(5,6)=YY+XYZM*(EZ*EZ+EX*EX)
SE(6,7)=ZZ+XYZM*(EY*EY+EX*EX)
SE(4,6)=-XY
SE(4,7)=-XZ
SE(5,7)=-YZ
SE(1,7)=-EY*XYZM
SE(2,7)=EX*XYZM
SE(1,6)=EZ*XYZM
SE(3,6)=-EX*XYZM
SE(2,5)=-EZ*XYZM
SE(3,5)=EY*XYZM
100 CALL R14000(DLIST,S,PLO,CPDDC,SE,IEPA,IDG,DCA,IP,ID,ID1,IDE,INE,
1IT,IDT)
WRITE(6,999) DLIST(87)
999 FORMAT(28H MASS OF ELEMENTS SO FAR IS ,E15,5)
RETURN
END
FINISH

```

L.1

Program for Analysis of Flutter Speed and
Aeroelastic Modes

The program was originally written by Stacey (Ref.52) and has undergone some major modifications. The program is stored in the subfile name JRB3 on the MAXIMOP file and requires data subfiles DAT2 (already produced by JRB1, DAT1 and JRB2) and DAT3 for its run. The program JRB3 reads the natural frequencies, mode shapes and the generalised mass and stiffness matrices together with the geometric parameters of the wing from the data subfile DAT 2. Then the program determines the aerodynamic matrix in terms of the generalised coordinates using strip theory (alternatively, using Davies' program - Ref.118). The flutter determinant is formed by assembling the generalised mass, stiffness and aerodynamic matrices. The complex determinant so formed is solved for air-speed and frequency using one of the scientific subroutines developed by the Numerical Algorithm Group (NAG). Once this complex eigen value problem is solved, the corresponding flutter modes are evaluated by setting one of the generalised coordinates (which is a complex quantity) to a known value. The complete procedure is explained in Fig. 133 in a block diagram. The flutter program JRB3 is a developed version of the program in Ref.52. The subroutine AIRMAT has been modified for the angle of sweep and the subroutine FMODE which evaluates the aeroelastic modes at flutter speed is a new addition. The listings of the flutter program JRB3 and the required data subfiles DAT2 and DAT3 are given in the subsequent sections. The listing of the subfile RES1 which follows DAT3 is a typical example of output results of the flutter program JRB3.

L.1.1 Listing of the Program Subfile JRB3

```

LIBRARY(SUBGROUPNAGF)
PROGRAM(D50D)
INPUT 1=CR0
OUTPUT 2=LPO
COMPRESS INTEGER AND LOGICAL
TRACE 2
END
MASTER SYMMETRIC CALCULATIONS
COMPLEX PP,RR,YY,YYZ1,YYZ2
DIMENSION PP(5,5),RR(5,1),YY(6,1),WKSPC(5),YYZ1(11,1),YYZ2(11,1)
DIMENSION S(2000)
C  XK = VALUES OF X AT INTEGRATION POINTS
C  II = NO NODES    JJ = NO NODES    KK = NO INTEGRATION POINTS
200 READ(1,200)II,JJ,KK,IJ0
    FORMAT(4I0)
    JJ=JJ-IJ0+1
    IJ0=IJ0-1
    LLB=JJ-1
    WRITE(2,9) II,JJ,KK,IJ0
9    FORMAT(3X,15HNO WING NODES =,I2,5X,10HNO NODES =,I2,5X,23HNO INTEG
    2RATION POINTS =,I2,5X,33HNO RIGID BODY MODES DISREGARDED =,I2)
    I1=II+1
    N=I1-1
    N2=N*(N+1)/2
    M1=1
    M2=M1+I1
    M3=M2+II
    M4=M3+II
    M5=M4+II*JJ
    M6=M5+II*JJ
    M7=M6+KK
    M8=M7+KK
    M9=M8+KK
    M10=M9+KK*JJ
    M11=M10+KK*JJ
    M12=M11+II
    M13=M12+II
    M14=M13+II
    M15=M14+II
    M16=M15+KK
    M17=M16+JJ
    M18=M17+N2
    M19=M18+II
    M20=M19+JJ*JJ
    M21=M20+JJ*JJ
    M22=M21+KK*2
    M23=M22+KK*2
    M24=M23+KK*2
    M25=M24+KK*2
    M26=M25+KK*2
    M27=M26+JJ*JJ*2
    M28=M27+JJ*JJ*2
    M29=M28+II
    M30=M29+II
    M31=M30+II*II
    M32=M31+II*II
    M33=M32+II*II
    M34=M33+II
    M35=M34+II
    M36=M35+II

```

```

M37=M36+II
M38=M37+II
M39=M38+II
M40=M39+II
M41=M40+II
M42=M41+II
M43=M42+II
M44=M43+II*II
WRITE(2,5) M44
5  FORMAT(5X,15HCOEFF REQUIRED =,I4)
   IF(M44.GT,2000) STOP
   CALL FLUTTR ( S(M1),S(M2),S(M3),S(M4),S(M5),S(M6),S(M7),S(M8),
2S(M9),S(M10),S(M11),S(M12),S(M13),S(M14),S(M15),S(M16),S(M17),
3S(M18),S(M19),S(M20),S(M21),S(M22),S(M23),S(M24),S(M25),S(M26),
4S(M27),S(M28),S(M29),S(M30),S(M31),S(M32),S(M33),S(M34),S(M35),S(M
536),S(M37),S(M38),S(M39),S(M40),S(M41),S(M42),S(M43),      II,JJ
6,KK,N2,II,LLB,PP,RR,YY,WKSPC,YYZ1,YYZ2)
  STOP
  END
  SUBROUTINE FLUTTR ( Y,B,AH, H,A,XK,BB,AHH,HH,AA,HHH,AAA,BBB,
2AHHH,ZK,W,HK,WKSP,XM,XS,A11,A12,A21,A22,QQ,QA,QF,DY,C,S,S1,EVV,EV
3,WKSP2,T10,T4,WN,T1,T2,TT1,TT2,WWN,DIV,II,JJ,KK,N2,II,LLB,
4PP,RR,YY,WKSPC,YYZ1,YYZ2)
  COMPLEX A11,A12,A21,A22,QQ,QA,QF,Q1,Q2
  COMPLEX PP,RP,YY,YYZ1,YYZ2
  DIMENSION Y(II),R(II),AH(II),H(II,JJ),A(II,JJ),XK(KK),BB(KK),
2AHH(KK),HH(KK,JJ),AA(KK,JJ),HHH(II),AAA(II),BBB(II),AHHH(II),
3ZK(KK),W(JJ),HK(N2),WKSP(II),XM(JJ,JJ),XS(JJ,JJ),A11(KK),A12(KK),
4A21(KK),A22(KK),QQ(KK),QA(JJ,JJ),QF(JJ,JJ),DY(II),C(II),S(II,II),
5S1(II,II),EVV(II,II),EV(II),WKSP2(II),T10(II),T4(II),WN(II),T1(II)
6,T2(II),TT1(II),TT2(II),WWN(II),DIV(II,II)
  DIMENSION Q(6,6)
  DIMENSION YYZ1(KK,1),YYZ2(KK,1),PP(LLB,1),RR(LLB,1),WKSPC(LLB),
1YY(JJ,1)
1  FORMAT(7X,15HSPANWISE CO-ORD,3X,10HSEMI-CHORD,5X,19HELASTIC AXIS 0
2FFSET,2X,20HAILERON HINGE OFFSET,15X,3H(Y),10X,3H(B),15X,4H(A),
320X,3H(C))
2  FORMAT(5X,37HCOUPLED NATURAL FREQUENCIES (RAD/SEC))
3  FORMAT(5X,33HGENERALIZED STIFFNESS MATRIX (XS))
4  FORMAT(5X,28HGENERALIZED MASS MATRIX (XM))
6  FORMAT(5X,23HBENDING MODE SHAPES (H))
7  FORMAT(5X,24HPITCHING MODE SHAPES (A))
8  FORMAT(5X,25HINTERPOLATED Y,B,AH,C,H,A)
14  FORMAT(5X,12HAIRDENSITY =,E10.4,5X,26HSTRUCTURAL DAMPING COEFF =,F
15.3)
  READ(1,201)(Y(I),I=1,II)
  READ(1,201)(B(I),I=1,II)
  READ(1,201)(AH(I),I=1,II)
  READ(1,201)(C(I),I=1,II)
201  FORMAT(20F0.0)
  READ(1,202)((S(I,J),I=1,II),J=1,II)
  READ(1,202)(W(I),I=1,JJ)
  READ(1,202)((XS(I,J),I=1,JJ),J=1,JJ)
  READ(1,202)((XM(I,J),I=1,JJ),J=1,JJ)
  READ(1,202)((H(I,J),I=1,II),J=1,JJ)
  READ(1,202)((A(I,J),I=1,II),J=1,JJ)
202  FORMAT(30F0.0)
  READ(1,17) LL,R0,G
17  FORMAT(10,E0.0,F0.0)
  WRITE(2,14) R0,G
  WRITE(2,1)
  DO 11 I=1,II
11  WRITE(2,12) Y(I),B(I),AH(I),C(I)
12  FORMAT(5X,4F18.5)
  WRITE(2,82)
  DO 22 I=1,JJ

```



```

22      W(I)=W(I)*6.283185307
C      CLEAR ALL OFF-DIAGONAL TERMS IN XS AND XM
      IF(IJ0.EQ.0) XS(1,1),XS(2,2)=0.0
      IF(IJ0.EQ.1) XS(1,1)=0.0
      DO 200 III=1,JJ-1
      DO 200 KKK=III+1,JJ
      XS(III,KKK)=0.0
      XS(KKK,III)=0.0
      XM(III,KKK)=0.0
200    XM(KKK,III)=0.0
C      CLEARING OF OFFDIAGONAL TERMS IS OVER
      WRITE(2,2)
      WRITE(2,10) (W(I),I=1,JJ)
      WRITE(2,82)
10     FORMAT (5X,F10.3)
      WRITE(2,3)
      CALL XPRINT(XS,JJ,JJ)
      WRITE(2,4)
      CALL XPRINT(XM,JJ,JJ)
      WRITE(2,6)
      CALL XPRINT(H,II,JJ)
      WRITE(2,7)
      CALL XPRINT(A,II,JJ)
      AKK=KK-1
      DX=(Y(1)-Y(1))/AKK
      DO 5 K=1,KK
      AK=K-1
5      XK(K)=Y(1)+DX*AK
      CALL EXTRAP(II,JJ,KK,N2,DX,Y,XK,H,A,B,AH,HK,HH,AA,BB,AHH,
1      HHH,AAA,BBB,AHHH)
      WRITE(2,8)
      DO 15 K=1,KK
      CC=0.9999
15     WRITE(2,12) XK(K),BB(K),AHH(K),CC
      WRITE(2,82)
      CALL XPRINT(HH,KK,JJ)
      CALL XPRINT(AA,KK,JJ)
C      U = AIRSPEED      WF = FLUTTER FREQUENCY (RAD/SEC)
      READ(1,115) OM,XI,R,KL
115    FORMAT(3F0.0,10)
      IF(OM.EQ.0.0) GOTO 118
      CALL OFFMAS(JJ,KK,KL,OM,XI,R,HH,AA,0)
      DO 116 I=1,JJ
      DO 116 J=1,JJ
116    XM(I,J)=XM(I,J)+0(I,J)
118    READ(1,119) WFI,DWF,WEMAX
119    FORMAT(3F0.0)
      WRITE(2,16)
16     FORMAT(5X,9H VELOCITY,11X,10H FREQUENCY,10X,9H REAL DET,11X,
19H IMAG DET)
      READ(1,117) U,DU,UMAX,UC
117    FORMAT(4F0.0)
81     QARP,QAIP=0.0
      IF(U.GT.UMAX) GOTO 160
      IF(UC.NE.0.0.AND.U.GE.UMAX) READ(1,117) U,DU,UMAX,UC
      IF(U.LT.0.000001) U=0.000001
      WRITE(2,82)
82     FORMAT(//)
      WF=WFI
      LRO,LJO,LR,LI=0
90     CALL AIRMAT(II,JJ,KK,RO,DX,U,WF,7K,HH,AA,BB,AHH,A11,A12,A21,A22,
100,QF)
      LR,LI=0
C      FORM FLUTTER DETERMINANT--QA
      DO 50 I=1,JJ
      DO 50 J=1,JJ

```

```

50  QA(I,J)=XM(I,J)*(-WF*WF+W(I)*W(I)*CMPLX(0.0,G))+XS(I,J)-QF(I,J)
C  WRITE(2,12) ((QA(I,J),J=1,JJ),I=1,II)
C  SOLVE FLUTTER DETERMINANT
    IFAIL = 0
    CALL F03ADF(QA,JJ,JJ,QAR,QAI,WKSP,IFAIL)
    IF(IFAIL.EQ.0) GOTO 120
    WRITE(2,110) IFAIL
110  FORMAT('FAILURE I = ',15)
    STOP
120  CONTINUE
    WRITE(2,125) U,WF,QAR,QAI
125  FORMAT(5X,2(F10.3,10X),2(F10.4,10X))
C  CHECK IF SIGN OF QAR OR QAI HAS CHANGED
    SIGNR=QAR*QARP
    IF(SIGNR.LT.0.0) CALL XINT(QAR,QARP,WF,DWF,WFO,1,LR)
    JJK=JJ-1
    SIGNI=QAI*QAIP
    IF(SIGNI.LT.0.0) CALL XINT(QAI,QAIP,WF,DWF,WFO,-1,LI)
    IF(SIGNR.LT.0.0.AND.SIGNI.LT.0.0) CALL FMODE(PP,RR,YY,
3WKSPC,QI,II,JJ,KK,RO,DX,U,WFO,ZK,HH,AA,BB,AHH,A11,A12,A21,
4A22,QO,QF,JJK,XS,XM,H,W,YYZ1,YYZ2,AMODE,G)
    QARP=QAR
    QAIP=QAI
    WF=WF+DWF
    LI0=LI+LI0
    LR0=LR+LR
    IF((LR0+LI0).GE.LL) GOTO 80
    IF(WF.GT.WFMAX) GOTO 80
    GOTO 90
80  U=U+DU
    GOTO 81
160  STOP
    RETURN
    END
    SUBROUTINE EXTRAP(II,JJ,KK,N2,DX,X,XK,H,A,B,AH,HK,HH,AA,BB,AHH,
1HHH,AAA,BBB,AHHH)
    DIMENSION X(II),XK(KK),H(II,JJ),A(II,JJ),B(II),AH(II),HK(N2),
1HH(KK,JJ),AA(KK,JJ),BB(KK),AHH(KK),HHH(II),AAA(II),BBB(II),AHHH(I
2I)
    N=II-1
    DO 150 K=1,KK
    XD=XK(1)
    IF (K.GT.1) XD=XK(K)-XK(K-1)
    DO 100 J=1,JJ
    DO 105 I=1,II
105  HHH(I)=H(I,J)
    CALL E01AAF(X,HHH, HK,II,N2,N,XD)
    XD=0.0
    HH(K,J)=HK(N2)
    DO 125 I=1,II
125  AAA(I)=A(I,J)
    CALL E01AAF(X,AAA, HK,II,N2,N,G.0)
    AA(K,J)=HK(N2)
100  CONTINUE
    DO 130 I=1,II
130  BBB(I)=B(I)
    CALL E01AAF(X,BBB,HK,II,N2,N,0.0)
    BB(K)=HK(N2)
    DO 140 I=1,II
140  AHHH(I)=AH(I)
    CALL E01AAF(X,AHHH,HK,II,N2,N,0.0)
    AHH(K)=HK(N2)
150  CONTINUE
    RETURN
    END

```

```

SUBROUTINE AIRMAT(II,JJ,KK,RO,DX,U,WF,ZK,HH,AA,BB,AHH,A11,A12,A21,
1A22,QQ,QF)
COMPLEX A11,A12,A21,A22,QQ,QF
DIMENSION HH(KK,JJ),AA(KK,JJ),BB(KK),AHH(KK),A11(KK),A12(KK),
1A21(KK),A22(KK),QQ(KK),QF(JJ,JJ),ZK(KK)
DO 200 K=1,KK
ZK(K)=BB(K)*WF/U
CALL AIRCOF(U,RO,ZK(K),AHH(K),BB(K),A11(K),A12(K),A21(K),A22(K))
C
23 WRITE(2,23) A11(K),A12(K),A21(K),A22(K)
200 FORMAT(8F12.3)
CONTINUE
DO 300 L=1,JJ
DO 300 M=1,JJ
DO 250 K=1,KK
QQ(K)= A11(K)*HH(K,L)*HH(K,M)+A21(K)* HH(K,M)*AA(K,L)+A12(K)*
1HH(K,L)*AA(K,M) + A22(K)*AA(K,L)*AA(K,M)
C
26 WRITE(2,26) QQ(K),K,L,M
250 FORMAT(2F10.3,3I10)
CONTINUE
CALL SIMPSN (KK,DX,QQ,QF(L,M))
C
27 WRITE(2,27) QF(L,M),L,M
300 FORMAT(2F10.3,2I10)
CONTINUE
RETURN
END
SUBROUTINE SIMPSN(KK,HX,Y,AREA)
COMPLEX Y,AREA,ODD,EVEN
DIMENSION Y(KK)
HX=ABS(HX)
ODD=(0.0,0.0)
EVEN=(0.0,0.0)
DO 10 I=1,KK-2,2
ODD=ODD+Y(I)
10 EVEN=EVEN+Y(I+1)
AREA=HX/3.0*(4.0+EVEN+2.0*ODD+Y(KK)-Y(1))
RETURN
END
SUBROUTINE AIRCOF(U,RO,XK,AH,B,A11,A12,A21,A22)
COMPLEX A11,A12,A21,A22,C11,C12,CK
C
CALCULATION OF THEODORSENS FUNCTIONS
IF(XK.NE.0.0) GOTO5
CK=CMPLX(1.0,0.0)
GOTO 50
5 IF (XK=0.45) 10,20,20
10 CA=0.0455/XK
CB=0.3/XK
GOTO 40
20 IF(XK=0.55) 30,30,35
30 CA=(0.04575-0.045*XK)/XK
CB=(0.21+0.2*XK)/XK
GOTO 40
35 CA=0.041/XK
CB=0.32/XK
40 C11=CMPLX(1.,-CA)
C12=CMPLX(1.,-CB)
CK=1.-0.165/C11-0.335/C12
50 CONTINUE
C
WRITE(2,60) CK,XK
60 FORMAT (3F20.5)
CO=3.14159265*RO*U*U
A11=CO*(-XK*XK+2.*CK*CMPLX(0.0,XK))
A11=-A11
A12=CO*(CMPLX(AH*XK*XK,XK)+2.*CK*CMPLX(1.,XK*(0.5-AH)))*B
A12=-A12
A21=CO*B *(2.*CK*CMPLX(0.0,XK*(0.5-AH))-XK*XK*AH)
CX=CO*B*B
A22=CX*(2.*(0.5-AH)*CK*CMPLX(1.0,XK*(0.5-AH))+CMPLX(XK*XK/8.,XK

```

```

1 *XK*AH*AH, (AH-0.5)*XK))
C EFFECT OF SWEEP IS TAKEN INTO CONSIDERATION
SWEEP=13.0*4.0*ATAN(1.0)/180.
ECONST=COS(SWEEP)
A11=A11+ECONST
A12=A12+ECONST
A21=A21+ECONST
A22=A22+ECONST
RETURN
END
SUBROUTINE XINT(Y1,Y2,WF,DWF,WFO,IP,L)
L=1
WFO=ABS(Y1)*DWF/(ABS(Y1)+ABS(Y2))
WFO=WF-WFO
IF(IP) 5,5,10
5 WRITE(2,20) WFO
RETURN
20 FORMAT(10X,37H FREQUENCY WHEN IMAGINARY DET ZERO = ,F10.5,/)
10 WRITE(2,30) WFO
30 FORMAT(10X,32H FREQUENCY WHEN REAL DET ZERO = ,F10.5,/)
RETURN
END
SUBROUTINE XPRINT(A,II,JJ)
DIMENSION A(II,JJ)
IF(JJ.EQ. 3) WRITE(2, 3) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ. 4) WRITE(2, 4) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ. 5) WRITE(2, 5) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ. 6) WRITE(2, 6) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ. 7) WRITE(2, 7) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ. 8) WRITE(2, 8) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ. 9) WRITE(2, 9) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ. 10) WRITE(2, 10) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ. 11) WRITE(2, 11) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ. 12) WRITE(2, 12) ((A(I,J),J=1,JJ),I=1,II)
3 FORMAT(5X, 3F10.4)
4 FORMAT(5X, 4F10.4)
5 FORMAT(5X, 5F10.4)
6 FORMAT(5X, 6F10.4)
7 FORMAT(5X, 7F10.4)
8 FORMAT(5X, 8F10.4)
9 FORMAT(5X, 9F10.4)
10 FORMAT(5X, 10F10.4)
11 FORMAT(5X, 11F10.4)
12 FORMAT(5X, 12F10.4)
WRITE(2,100)
100 FORMAT(//)
RETURN
END
SUBROUTINE OFFMAS(JJ,KK,KL,OM,XI,R,HH,AA,Q)
DIMENSION AA(KK,JJ),HH(KK,JJ),Q(JJ,JJ)
DO 5 I=1,JJ
DO 5 J=1,JJ
5 Q(I,J)=OM*(HH(KL,I)+HH(KL,J)+R*AA(KI,I)+HH(KL,J)+R*AA(KL,J)
2*HH(KL,I))+(OM*R+R*XI)*AA(KL,I)*AA(KL,J)
CALL XPRINT(Q,JJ,JJ)
RETURN
END
SUBROUTINE EMODE(PPP,RRR,YY,WKSPC,QA,II,JJ,KK,RO,DX,U,WFO,ZK,HH,AA
1,BB,AHH,A11,A12,A21,A22,QQ,QF,JJK,XS,XM,H,W,YYZ1,YYZ2,AMODE,G)
COMPLEX A11,A12,A21,A22,QQ,QF,QA,PPP,RRR,YY,YYZ1,YYZ2
DIMENSION H(KK,JJ),AA(KK,JJ),BB(KK),AAH(KK),A11(KK),A12(KK),A21(KK)
2,A22(KK),QQ(KK),QF(JJ,JJ),ZK(KK),XS(JJ,JJ),XM(JJ,JJ),PPP(JJK,JJK)
3,RRR(JJK,1),YY(JJ,1),WKSPC(JJK),QA(JJ,JJ),W(JJ),HH(KK,JJ)

DIMENSION YYZ1(KK,1),YYZ2(KK,1)
CALL A1RMAT(II,JJ,KK,RO,DX,U,WFO,ZK,HH,AA,BB,AHH,A11,A12,A21,A22,Q

```

```

40,0F)
DO 50 I=1,JJ
DO 50 J=1,JJ
50 QA(I,J)=XM(I,J)*(-WFO*WFO+W(I)*W(I)*CMPLX(0.0,G))+XS(I,J)-QF(I,J)
DO 51 I=1,JJ=1
DO 51 J=1,JJ=1
51 PPP(I,J)=QA(I,J)
DO 52 I=1,JJ=1
52 RRR(I,1)=QA(I,JJ)
IFAIL=1
LL=JJ-1
CALL F04ADF(PPP,LL,RRR,IL,LL,1,YY,JJ,WKSPC,IFAIL)
IF(IFAIL.EQ.0) GO TO 20
WRITE(2,53)IFAIL
20 WRITE(2,54)(YY(I,1),I=1,LL)
YY(JJ,1)=CMPLX(-1.0,0.0)
WRITE(2,60)
WRITE(2,61)
WRITE(2,54)((YY(I,1),I=1,JJ))
DO 56 I=1,KK
YYZ1(I,1)=CMPLX(0.0,0.0)
YYZ2(I,1)=CMPLX(0.0,0.0)

DO 56 K=1,JJ
56 YYZ1(I,1)=YYZ1(I,1)+HH(I,K)*YY(K,1)
YYZ2(I,1)=YYZ2(I,1)+AA(I,K)*YY(K,1)
WRITE(2,55)
WRITE(2,61)
WRITE(2,54)(YYZ1(I,1),I=1,KK)
WRITE(2,62)
DO 63 I=1,KK
RYYZ1=REAL(YYZ1(I,1))
AIYYZ1=AIMAG(YYZ1(I,1))
AMYYZ1=SQRT(RYYZ1**2+AIYYZ1**2)
ATHET1=ATAN(AIYYZ1/RYYZ1)
ADTHET1=ATHET1*180.0/3.1415926
WRITE(2,64)RYYZ1,AIYYZ1,AMYYZ1,ATHET1,ADTHET1
63 CONTINUE
WRITE(2,65)
WRITE(2,61)
WRITE(2,54)(YYZ2(I,1),I=1,KK)
WRITE(2,62)
DO 66 I=1,KK
RYYZ2=REAL(YYZ2(I,1))
AIYYZ2=AIMAG(YYZ2(I,1))
AMYYZ2=SQRT(RYYZ2**2+AIYYZ2**2)
ATHET2=ATAN(AIYYZ2/RYYZ2)
ADTHET2=ATHET2*180.0/3.14159265
WRITE(2,64)RYYZ2,AIYYZ2,AMYYZ2,ATHET2,ADTHET2
66 CONTINUE
53 FOPMAT(1H ,23HERROR IN F04ADF IFAIL =,I2)
54 FOPMAT(1H ,2(2X,F16.9,8X))
55 FOPMAT(1H ,5X,28HAEROELASTIC MODES IN BENDING)
60 FOPMAT(1H ,5X,53HAEROELASTIC MODES INTERMS OF GENERALISED CO-ORDIN
1ATES)
61 FOPMAT(1H ,5X,9HREAL-PART,10X,14HIMAGINARY-PART)
62 FOPMAT(1H ,2X,9HREAL-PART,5X,9HIMAG-PART,5X,7HMODULUS,5X,10HPHASE(
2RAD),5X,10HPHASE(DEG))
64 FOPMAT(1H ,2X,F12.5,2X,F12.5,2X,F12.5,5X,F11.6,5X,F11.6)
65 FOPMAT(1H ,28HAEROELASTIC MODES IN TORSION)
RETURN
END
FINISH

```

13	6	11	1	5	10
4.80200	7.50300	4.20100	6.60200	3.60100	5.40200
2.40100	1.80100	1.20000		0.60000	
0.00000	0.25000	0.36500	0.30800	0.36500	0.33100
0.35500	0.36500	0.36500	0.36500	0.36500	0.36500
0.36500	0.16000	0.01754	-0.05844	-0.20548	-0.12387
-0.13310	-0.20548	-0.20548	-0.20548	-0.20548	-0.20548
-0.20548	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.56588E 05	-0.45760E 05	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
-0.50055E 06	-0.45760E 05	0.17760E 06	-0.94262E 05	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	-0.94262E 05	0.00000E 00	
0.27441E 06	-0.11774E 06	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	-0.11774E 06	0.34486E 06	-0.14498E 06	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	-0.14498E 06	0.00000E 00	
0.43568E 06	-0.18153E 06	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	-0.18153E 06	0.61153E 06	-0.27536E 06	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	-0.27536E 06	0.00000E 00	
0.75633E 06	-0.28457E 06	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	
0.00000E 00	-0.28457E 06	0.81066E 06	-0.29374E 06	0.00000E 00	

0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	-0.29376E 06
0.85241E 06	-0.29822E 06	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	-0.29822E 06	0.00000E 00	0.90133E 06	-0.29999E 06
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.94996E 06	-0.30279E 06	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	-0.30279E 06	0.00000E 00	0.10036E 07	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
0.16238E 01	0.00000E 00	0.00000E 00	0.00000E 00	0.14096E-01
-0.10360E-01	-0.15912E-02	0.29088E 01	0.10919E 02	
0.25891E 02	0.45475E 02	0.75252E-06	0.45574E-03	-0.32944E-03
-0.75051E-07	0.55974E-04	0.97284E-06	-0.51107E-06	-0.51107E-06
0.30747E-03	0.10039E-04	-0.82138E-05	-0.14020E-04	-0.14020E-04
0.90816E-05	0.97351E-05	0.75975E 02	-0.21195E-03	-0.21195E-03
0.45738E-03	0.25400E-04	-0.32895E-03	0.10960E-04	0.10960E-04
0.20619E-03	0.10705E 04	-0.19707E-03	-0.19263E-04	-0.19263E-04
-0.21455E-03	0.73910E-05	0.20496E-03	-0.19293E-03	-0.19293E-03
0.30986E-03	-0.73910E-05	0.56043E-04	-0.13649E-04	-0.13649E-04
0.64928E 04	-0.28001E-04	-0.28716E-04	0.18568E 05	0.18568E 05
0.25153E-04	-0.18847E-04	-0.12733E-10	0.17053E-11	0.17053E-11
0.22744E 00	-0.72760E-11	-0.79581E-11	0.22744E 00	0.22744E 00
-0.30695E-11	0.65212E-12	-0.35243E-11	-0.75033E-11	-0.75033E-11
0.39108E-10	0.10758E-11	0.22744E 00	0.17053E-11	0.17053E-11
-0.12506E-10	0.34561E-10	0.27285E-11	0.18190E-11	0.18190E-11
-0.48885E-11	0.11369E-11	0.56843E-11	-0.68212E-12	-0.68212E-12
0.90949E-12	0.22744E 00	-0.53433E-11	0.40146E-11	0.40146E-11
-0.35538E-11	-0.25011E-11	0.71054E-12	-0.81855E-11	-0.81855E-11
0.22744E 00	-0.20843E-11	-0.31264E-11	0.22744E 00	0.22744E 00
0.68212E-12	-0.61107E-12	0.33306E-01	0.34547E-01	0.34547E-01
0.30217E-01	0.32074E-01	0.38260E-01	0.30492E-01	0.30492E-01
0.35779E-01	0.37020E-01	0.43206E-01	0.44438E-01	0.44438E-01
0.40733E-01	0.41065E-01	0.66347E-01	0.56747E-01	0.56747E-01
0.45679E-01	0.80817E-01			

0.47077E-01	0.37477E-01	0.27807E-01	0.18137E-01
0.85366E-02	-0.11333E-02	-0.10734E-01	-0.20404E-01
-0.30004E-01	-0.39674E-01	-0.13753E 00	-0.10549E 00
-0.84953E-01	-0.65833E-01	-0.48463E-01	-0.33870E-01
-0.21554E-01	-0.11698E-01	-0.45420E-02	-0.11133E-03
0.13961E-02	0.16403E-04	-0.42573E-02	-0.18702E 00
-0.63258E-01	0.61191E-02	0.56868E-01	0.85679E-01
0.94133E-01	0.87458E-01	0.71019E-01	0.49093E-01
0.26035E-01	0.57774E-02	-0.82437E-02	-0.13560E-01
-0.18326E 00	0.14617E-01	0.85996E-01	0.92018E-01
0.48563E-01	-0.12694E-01	-0.64339E-01	-0.91754E-01
-0.90783E-01	-0.66989E-01	-0.31023E-01	-0.52047E-03
0.13373E-01	-0.12338E-01	0.43858E-02	0.68832E-02
0.20606E-02	-0.33546E-02	-0.59384E-02	-0.46661E-02
-0.58641E-03	0.32821E-02	0.58742E-02	0.61427E-02
0.53451E-02	0.54598E-02		
-0.89264E-02	-0.89264E-02	-0.89264E-02	-0.89264E-02
-0.89264E-02	-0.89264E-02	-0.89264E-02	-0.89264E-02
-0.89264E-02	-0.89264E-02	-0.89264E-02	-0.89264E-02
0.69568E-01	0.69568E-01	0.69568E-01	0.69568E-01
0.69568E-01	0.69568E-01	0.69568E-01	0.69568E-01
0.69568E-01	0.69568E-01	0.69568E-01	0.69568E-01
0.69568E-01	0.69568E-01	0.69568E-01	0.69568E-01
0.31996E-01	0.32613E-01	0.33410E-01	0.31621E-01
0.35149E-01	0.36091E-01	0.37051E-01	0.34306E-01
0.30115E-01	0.40136E-01	0.41138E-01	0.38063E-01
-0.26452E-01	-0.20796E-01	-0.13046E-01	-0.29554E-01
0.15662E-02	0.58013E-02	0.86012E-02	-0.50843E-02
0.93871E-02	0.76436E-02	0.48619E-02	0.97433E-02
-0.63306E-01	-0.47414E-01	-0.24028E-01	0.12006E-02
0.11349E-01	0.12161E-01	0.57590E-02	-0.12830E-02
-0.13538E-01	-0.19562E-01	-0.20443E-01	-0.42730E-02
-0.66668E-02	0.80334E 00	0.77216E 00	-0.15853E-01
0.66848E 00	0.59532E 00	0.51466E 00	0.72861E 00
0.38258E 00	0.30913E 00	0.23492E 00	0.45270E 00
0.70691E-01	-0.11327E-01		0.15184E 00

L.1.3 Listing of the Data Subfile DAT3

10	0.1225E 01	0.05
0.0	0.0 0.0 0	
8.4	0.1 8.6	
39.35	0.05 30.45	0.0

L.1.4 Listing of the Result Subfile RES1

VELOCITY	FREQUENCY	REAL DET	IMAG DET	
39.300	8.400	-.2264E 14	-.3501E 14	
39.300	8.500	0.4150E 13	-.4820E 13	
FREQUENCY WHEN REAL DET ZERO =		8.48451		
39.300	8.600	0.3176E 14	0.2769E 14	
FREQUENCY WHEN IMAGINARY DET ZERO =		8.51482		
39.350	8.400	-.2772E 14	-.3266E 14	
39.350	8.500	-.1064E 13	-.2349E 13	
39.350	8.600	0.2640E 14	0.3029E 14	
FREQUENCY WHEN REAL DET ZERO =		8.50387		
FREQUENCY WHEN IMAGINARY DET ZERO =		8.50720		
AEROELASTIC MODES INTERMS OF GENERALISED CO-ORDINATES				
REAL-PART		IMAGINARY-PART		
-845.748521157	-208.891635053			
-190.703783135	0.507565978			
-238.106926087	-11.807218247			
8.903143796	3.004864749			
-0.956921307	-0.317203339			
AEROELASTIC MODES IN BENDING				
REAL-PART		IMAGINARY-PART		
-9.698559635	-5.151050016			
-14.370346729	-5.534670247			
-18.794863802	-5.934156682			
-22.885003332	-6.362199648			
-26.450500429	-6.831700166			
-29.341901897	-7.340670360			
-31.521736626	-7.864467827			
-32.813657327	-8.380402169			
-33.122495938	-8.852638679			
-32.219642310	-9.256801084			
-30.192253398	-9.556819268			
REAL-PART	IMAG-PART	MODULUS	PHASE(RAD)	PHASE(DEG)
-9.69856	-5.15105	10.98150	0.488229	27.973440
-14.37033	-5.53467	15.39933	0.367635	21.063952
-18.79486	-5.93416	19.70942	0.305827	17.522621
-22.88501	-6.36220	23.75292	0.271160	15.536320
-26.45050	-6.83170	27.31851	0.252759	14.482001
-29.34190	-7.34067	30.24620	0.245145	14.045791
-31.52174	-7.86445	32.48790	0.244501	14.008892
-32.81364	-8.38040	33.86690	0.250049	14.326732
-33.12250	-8.85264	34.28511	0.261165	14.963660
-32.21964	-9.25680	33.52303	0.279768	16.029510
-30.19226	-9.55682	31.66868	0.306554	17.564254

AEROELASTIC MODES IN TORSION

REAL-PART		IMAGINARY-PART			
-14.210309672		1.459968337			
-14.238453355		1.464027477			
-14.226590777		1.467303795			
-14.277978198		1.478584741			
-14.360712765		1.491637983			
-14.467372752		1.499757525			
-14.612772692		1.501027841			
-14.785915824		1.493841050			
-15.002742808		1.477134881			
-15.230946324		1.450947516			
-15.483437252		1.420228115			
REAL-PART	IMAG-PART	MODULUS	PHASE(RAD)	PHASE(DEG)	
-14.21031	1.45997	14.28511	-0.102381	-5.865991	
-14.23845	1.46403	14.31352	-0.102462	-5.870641	
-14.22660	1.46730	14.30207	-0.102775	-5.888554	
-14.27798	1.47858	14.35433	-0.103189	-5.912305	
-14.36071	1.49164	14.43797	-0.103498	-5.930011	
-14.46737	1.49976	14.54490	-0.103296	-5.918416	
-14.61273	1.50103	14.68967	-0.102361	-5.864865	
-14.78592	1.49384	14.86110	-0.100690	-5.769094	
-15.00275	1.47713	15.07529	-0.098141	-5.623082	
-15.23095	1.45095	15.29990	-0.094977	-5.441753	
-15.48344	1.42023	15.54844	-0.091470	-5.240827	

39.400 8.400 -.3280E 14 .3030E 14
 39.400 8.500 -.6285E 13 0.1352E 12
 FREQUENCY WHEN IMAGINARY DET ZERO = 8.49956

39.400 8.600 0.2104E 14 0.3289E 14
 FREQUENCY WHEN REAL DET ZERO = 8.52300

39.450 8.400 -.3789E 14 -.2792E 14
 39.450 8.500 -.1151E 14 0.2632E 13
 FREQUENCY WHEN IMAGINARY DET ZERO = 8.49138

39.450 8.600 0.1568E 14 0.3552E 14
 FREQUENCY WHEN REAL DET ZERO = 8.54234

L.2

Program for Analysis of Short Period Oscillation Characteristics of a Flexible Aeroplane

A program very similar to the flutter program described in section L.1 is developed to analyse the short period oscillation characteristics of a flexible aircraft. The main difference from the flutter program is that in this program the Theodorsen function and the corresponding lift and moment are evaluated for motion of the form $e^{\lambda t}$ where $\lambda = \mu + i\omega$. A particular airspeed is chosen (preferably below the flutter speed) and the corresponding values of μ and ω are found. The μ value gives the amount of damping (sec^{-1}) and the latter provides the frequency (rad/sec) of the short period mode. The computation method is described in the block diagram shown in Fig.134. The program JRB4 (subfile name) which evaluates the short period frequency and damping requires the data subfiles DAT2 (given in section L.1.2), DAT4 and DAT5. Many of the subroutines in the program JRB4 are common with those in the flutter program JRB3. However, the subroutines AIRMAT and AIRCOF are developed as an extension of the earlier ones to account for the Theodorsen functions of complex arguments. The data subfile DAT4 contains the constant airspeed value and range of short period frequency and damping values within which the stability determinant is expected to be zero. DAT5 contains the Theodorsen functions for non-harmonic motion which are evaluated from the computation of Bessel and Hankel functions for complex arguments.

The listings of the program subfile JRB4 and the data subfiles DAT4 and DAT5 are provided in the subsequent sections. Then an example of typical sample output RES2 follows.

```

LIBRARY(SUBGROUPNAGF)
LIBRARY(SUBGROUPPCRA)
PROGRAM(D50D)
COMPACT DATA
INPUT 1=CRO
OUTPUT 2=LPO
COMPRESS INTEGER AND LOGICAL
TRACE 2
END
MASTER SYMMETRIC CALCULATIONS

```

THIS PROGRAM EVALUATES THE SHORT PERIOD FREQUENCY AND DAMPING
OF A FLEXIBLE AIRCRAFT

```

COMPLEX PP7RR,YY,YYZ1,YYZ2
DIMENSION S(2000)
DIMENSION XRO(63),AFX(63),AGX(63)
DIMENSION PP(5,5),RR(5,1),YY(6,1),
7WKSPC(5),YYZ1(11,1),YYZ2(11,1)
C XK = VALUES OF X AT INTEGRATION POINTS
C II = NO NODES JJ = NO MODES KK = NO INTEGRATION POINTS
READ(1,590)II,JJ,KK,IJO
590 FORMAT(4I0)
JJ=JJ-IJO+1
LLB=JJ-1
IJO=IJO-1
WRITE(2,9) II,JJ,KK,IJO
9 FORMAT(5X,15HNO WING NODES =,12,5X,10HNO MODES =,12,5X,23HNO INTEG
2RATION POINTS =,12,5X,33HNO RIGID BODY MODES DISREGARDED =,12)
II=II+1
N=II-1
N2=N*(N+1)/2
M1=1
M2=M1+II
M3=M2+II
M4=M3+II
M5=M4+II*JJ
M6=M5+II*JJ
M7=M6+KK
M8=M7+KK
M9=M8+KK
M10=M9+KK*JJ
M11=M10+KK*JJ
M12=M11+II
M13=M12+II
M14=M13+II
M15=M14+II
M16=M15+KK
M17=M16+JJ
M18=M17+N2
M19=M18+II
M20=M19+JJ*JJ
M21=M20+JJ*JJ
M22=M21+KK*2
M23=M22+KK*2
M24=M23+KK*2
M25=M24+KK*2
M26=M25+KK*2
M27=M26+JJ*JJ*2
M28=M27+JJ*JJ*2
M29=M28+II
M30=M29+II

```

```

M31=M30+I1*I1
M32=M31+I1*I1
M33=M32+I1*I1
M34=M33+I1
M35=M34+I1
M36=M35+I1
M37=M36+I1
M38=M37+I1
M39=M38+I1
M40=M39+I1
M41=M40+I1
M42=M41+I1
M43=M42+I1
M44=M43+I1*I1
WRITE(2,5) M44

```

```

5  FORMAT(5X,15HCORE REQUIRED =,I4)
   IF(M44.GT.2000) STOP
   CALL SPFRQC ( S(M1),S(M2),S(M3),S(M4),S(M5),S(M6),S(M7),S(M8),
2S(M9),S(M10),S(M11),S(M12),S(M13),S(M14),S(M15),S(M16),S(M17),
3S(M18),S(M19),S(M20),S(M21),S(M22),S(M23),S(M24),S(M25),S(M26),
4S(M27),S(M28),S(M29),S(M30),S(M31),S(M32),S(M33),S(M34),S(M35),S(M
536),S(M37),S(M38),S(M39),S(M40),S(M41),S(M42),S(M43),      I1,JJ
6, KK,N2,I1,LLB,PP,RR,YY,YYZ1,YYZ2,WKSPC)
   STOP
   END
   SUBROUTINE SPFRQC ( Y,B,AH, H,A,XK,BB,AHH,HH,AA,HHH,AAA,BBB,
2AHHH,ZK,W,HK,WKSP,XH,XS,A11,A12,A21,A22,QQ,QA,QF,DY,C,S,S1,EVV,EV
3,WKSP2,T10,T4,WN,T1,T2,TT1,TT2,WWN,DIV,I1,JJ,KK,N2,I1,LLB,PP,
4RR,YY,YYZ1,YYZ2,WKSPC)
   COMPLEX A11,A12,A21,A22,QQ,QA,QF,Q1,Q2,ALDA,ZK
   COMPLEX PP,RR,YY,YYZ1,YYZ2
   DIMENSION Y(I1),B(I1),AH(I1),H(I1,JJ),A(I1,JJ),XK(KK),BB(KK),
2AHH(KK),HH(KK,JJ),AA(KK,JJ),HHH(I1),AAA(I1),BBB(I1),AHHH(I1),
3ZK(KK),W(JJ),HK(N2),WKSP(I1),XM(JJ,JJ),XS(JJ,JJ),A11(KK),A12(KK),
4A21(KK),A22(KK),QQ(KK),QA(JJ,JJ),QF(JJ,JJ),DY(I1),C(I1),S(I1,I1),
5S1(I1,I1),EVV(I1,I1),EV(I1),WKSP2(I1),T10(I1),T4(I1),WN(I1),T1(I1)
6,T2(I1),TT1(I1),TT2(I1),WWN(I1),DIV(I1,I1)
   DIMENSION Q(6,6)
   DIMENSION XRO(63),AFX(63),AGX(63)
   DIMENSION PP(LLB,LLB),RR(LLB,1),WKSPC(LLB),YY(JJ,1),YYZ1(KK,1),
7YYZ2(KK,1)
1  FORMAT(7X,15HSPANWISE CO-ORD,3X,10HSEMI-CHORD,5X,19HELASTIC AXIS O
2FFSET,2X,20HAILERON HINGE OFFSET,15X,3H(Y),10X,3H(B),15X,4H(A'),
320X,3H(C))
2  FORMAT(5X,37HCOUPLED NATURAL FREQUENCIES (RAD/SEC))
3  FORMAT(5X,33HGENERALIZED STIFFNESS MATRIX (XS))
4  FORMAT(5X,28HGENERALIZED MASS MATRIX (XM))
6  FORMAT(5X,23HBENDING MODE SHAPES (H))
7  FORMAT(5X,24HPITCHING MODE SHAPES (A))
8  FORMAT(5X,25HINTERPOLATED Y,B,AH,C,H,A)
14  FORMAT(5X,12HAIRDENSITY =,E10.4,5X,26HSTRUCTURAL DAMPING COEFF =,F
15,3)

```

```

LL=10
RO=1.225
G=0.0
WRITE(2,14) ROTG
READ(1,201)(Y(I),I=1,II)
READ(1,201)(B(I),I=1,II)
READ(1,201)(AH(I),I=1,II)
READ(1,201)(C(I),I=1,II)
201  FORMAT(20F0.0)
      READ(1,202)((S(I,J),I=1,II),J=1,II)
      READ(1,202)(W(I),I=1,JJ)
      READ(1,202)((XS(I,J),I=1,JJ),J=1,JJ)
      READ(1,202)((XM(I,J),I=1,JJ),J=1,JJ)
      READ(1,202)((H(I,J),I=1,II),J=1,JJ)
      READ(1,202)((A(I,J),I=1,II),J=1,JJ)
202  FORMAT(200F0.0)
      WRITE(2,1)
      DO 11 I=1,II
11    WRITE(2,12) Y(I),B(I),AH(I),C(I)
12    FORMAT(5X,4F18.5)
      WRITE(2,82)
      DO 22 I=1,JJ
22    W(I)=W(I)*6.283185307
C      CLEAR ALL OFF-DIAGONAL TERMS IN XS AND XM
      IF(IJO.EQ.0) XS(1,1),XS(2,2)=0.0
      IF(IJO.EQ.1) XS(1,1)=0.0
      DO 200 III=1,JJ-1
      DO 200 KKK=III+1,JJ
      XS(III,KKK)=0.0
      XS(KKK,III)=0.0
      XM(III,KKK)=0.0
200  XM(KKK,III)=0.0
C      CLEARING OF OFFDIAGONAL TERMS IS OVER
      WRITE(2,2)
      WRITE(2,10) (W(I),I=1,JJ)
      WRITE(2,82)
10    FORMAT (5X,F10.3)
      WRITE(2,3)
      CALL XPRINT(XS,JJ,JJ)
      WRITE(2,4)
      CALL XPRINT(XM,JJ,JJ)
      WRITE(2,6)
      CALL XPRINT(H,II,JJ)
      WRITE(2,7)
      CALL XPRINT(A,II,JJ)
      AKK=KK=1
      DX=(Y(II)-Y(1))/AKK
      DO 5 K=1,KK
      AK=K-1
5      XK(K)=Y(1)+DX*AK
      CALL EXTRAP(II,JJ,KK,N2,DX,Y,XK,H,A,B,AH,HK,HH,AA,BB,AHH,
1 HHH,AAA,BBB,AHHH)
      WRITE(2,8)
      DO 15 K=1,KK
      CC=99999
15    WRITE(2,12) XK(K),BB(K),AHH(K),CC
      WRITE(2,82)
      CALL XPRINT(HH,KK,JJ)
      CALL XPRINT(AA,KK,JJ)
C      U = AIRSPEED      WF = FLUTTER FREQUENCY (RAD/SEC)

```

```

66 DO 66 I=1,51
   XRO(I)=0.01*FLOAT(I-1)
   DO 67 I=32,41
     J=I-32
67   XRO(I)=0.32+0.02*FLOAT(J)
   DO 68 I=42,51
     J=I-42
68   XRO(I)=0.55+0.05*FLOAT(J)
   DO 69 I=52,57
     J=I-52
69   XRO(I)=1.5*FLOAT(J)*0.5
   DO 70 I=58,63
     J=I-58
70   XRO(I)=5.0*FLOAT(J)
   READ(1,850)U
   READ(1,850)AROWI,DAROW,AROWM
   READ(1,850)ATHI,DATH,ATHM
850  FORMAT(10F0.0)
   AROW=AROWI
   ATH=ATHI
   WRITE(2,16)
16   FORMAT(3X,4HAROW,10X,1HU,13X,2HMU,6X,5HOMEGA,2X,8HREAL DET,
25X,8HIMAG DET)
81   QARP,QAIP=0.0
   IF(ATH.GT.ATHM) GOTO 160
   WRITE(2,82)
82   FORMAT(/)
   AROW=AROWI
   LRO,LIO,LR7LI=0
   DO 63 I=1,63
     READ(1,65)AFX(I),AGX(I)
63   FORMAT(2F0.0)
65   WF=AROW*COS(ATH*3.14159/180.0)
90   AMU=-AROW*SIN(ATH*3.14159/180.0)
   ALDA=CMPLX(AMU,WF)
   CALL AIRMAT(II,JJ,KK,RO,DX,U,WF,ZK,HH,AA,BB,AHH,A11,A12,A21,A22,
1QQ,QF,AMU,AROW,XRO,AFX,AGX)
   LR,LI=0
   FORM FLUTTER DETERMINANT==QA
C   DO 50 I=1,JJ
     DO 50 J=1,JJ
50   QA(I,J)=XM(I,J)*(ALDA**2+W(I)*W(I)*CMPLX(0.0,G))+XS(I,J)-QF(I,J)
C   WRITE(2,12) ((QA(I,J),J=1,JJ),I=1,JJ)
C   SOLVE FLUTTER DETERMINANT
   IFAIL = 0
   CALL F03ADF(QA7JJ,JJ,QAR,QAI,WKSP,IFAIL)
   IF(IFAIL.EQ.0) GOTO 120
   WRITE(2,110) IFAIL
110  FORMAT('FAILURE I = ',15)
   STOP
120  CONTINUE
   WRITE(2,125) AROW,U,AMU,WF,QAR,QAI
125  FORMAT(3X,F5.2,2X,3(F12.5,5X),2(E10.4,5X))
C   CHECK IF SIGN OF QAR OR QAI HAS CHANGED
   SIGNR=QAR*QARP
   IF(SIGNR.LT.0.0) CALL XINT(QAR,QARP,AROW,DAROW,WFO,1,LR)
   SIGNI=QAI*QAIP
   IF(SIGNI.LT.0.0) CALL XINT(QAI,QAIP,AROW,DAROW,WFO,-1,LI)
   JJK=JJ-1
   IF(SIGNR.LT.0.0.AND.SIGNI.LT.0.0) CALL FMODE(PP,RR,YY,
3WKSPC,QA,II,JJ,KK,RO,DX,U,WFO,ZK,HH,AA,BB,AHH,A11,A12,A21,
4A22,QQ,QF,JJK,XS,XM,H,W,YYZ1,YYZ2,ATH,XRO,AFX,AGX,AROW,G)
   QARP=QAR

```

```

QAIP=QAI
AROW=AROW+PAROW
LI0=LI+LI0
LR0=LR0+LR
IF((LR0+LI0).GE.LL) GOTO 80
IF(AROW.GT.AROWH) GOTO 80
GOTO 90

80   ATH=ATH+DATH
    GOTO 81
160  STOP
    RETURN
    END
    SUBROUTINE EXTRAP(II,JJ,KK,N2,DX,X,XK,H,A,B,AH,HK,HH,AA,BB,AHH,
1   HHH,AAA,BBB,AHHH)
    DIMENSION X(II),XK(KK),H(II,JJ),A(II,JJ),B(II),AH(II),HK(N2),
1   HH(KK,JJ),AA(KK,JJ),BB(KK),AHH(KK),HHH(II),AAA(II),BBB(II),AHHH(I
2   I)
    N=II-1
    DO 150 K=1,KK
    XD=XK(K)
    IF (K.GT.1) XD=XK(K)-XK(K-1)
    DO 100 J=1,JJ
    DO 105 I=1,II
    HHH(I)=H(I,J)
105  CALL E01AAF(X,HHH, HK,II,N2,N,XD)
    XD=0.0
    HH(K,J)=HK(N2)
    DO 125 I=1,II
125  AAA(I)=A(I,J)
    CALL E01AAF(X,AAA, HK,II,N2,N,0.0)
    AA(K,J)=HK(N2)
100  CONTINUE
    DO 130 I=1,II
130  BBB(I)=B(I)
    CALL E01AAF(X,BBB,HK,II,N2,N,0.0)
    BB(K)=HK(N2)
    DO 140 I=1,II
140  AHHH(I)=AH(I)
    CALL E01AAF(X,AHHH,HK,II,N2,N,0.0)
    AHH(K)=HK(N2)
150  CONTINUE
    RETURN
    END
    SUBROUTINE AIRMAT(II,JJ,KK,RO,DX,U,WF,ZK,HH,AA,BB,AHH,A11,A12,A21,
1   A22,QQ,QF,AMU,AROW,XRO,AFX,AGX)
    COMPLEX A11,A12,A21,A22,QQ,QF,ZK
    DIMENSION HH(KK,JJ),AA(KK,JJ),BB(KK),AHH(KK),A11(KK),A12(KK),
1   A21(KK),A22(KK),QQ(KK),QF(JJ,JJ),ZK(KK)
    DIMENSION XRO(63),AFX(63),AGX(63)
    DO 200 K=1,KK
    ZK(K)=CMPLX(BB(K)*WF/U,-AMU*BB(K)/U)
    AAB=BB(K)*AROW/U
    AFF=UTSQ(AAB,XRO(1),AFX(1),63)
    AGG=UTSQ(AAB,XRO(1),AGX(1),63)
    CALL AIRCUF(U,RO,ZK(K),AHH(K),BB(K),A11(K),A12(K),A21(K),A22(K),
3   AFF,AGG)
    WRITE(2,23) A11(K),A12(K),A21(K),A22(K)
23  FORMAT(8F12,3)
200  CONTINUE

```



```

DO 300 L=1,JJ
DO 300 M=1,JJ
DO 250 K=1,KK
  QQ(K)= A11(K)*HH(K,L)*HH(K,M)+A21(K)* HH(K,M)*AA(K,L)+A12(K)*
  1 HH(K,L)*AA(K,M) + A22(K)*AA(K,L)*AA(K,M)
C 26 WRITE(2,26) QQ(K),K,L,M
  250 FORMAT(2F10.3,3I10)
  CONTINUE
  CALL SIMPSN (KK,DX,QQ,QF(L,M))
C 27 WRITE(2,27) QF(L,M),L,M
  300 FORMAT(2F10.3,2I10)
  CONTINUE
  RETURN
END
SUBROUTINE SIMPSN(KK,HX,Y,AREA)
  COMPLEX Y,AREA,ODD,EVEN
  DIMENSION Y(KK)
  HX=ABS(HX)
  ODD=(0.0,0.0)
  EVEN=(0.0,0.0)
  DO 10 I=1,KK-2
    ODD=ODD+Y(I)
    10 EVEN=EVEN+Y(I+1)
    AREA=HX/3.0*(4.0*EVEN+2.0*ODD+Y(KK)-Y(1))
  RETURN
END
SUBROUTINE AIRCOF(U,RO,XK,AH,B,A11,A12,A21,A22,AFF,AGG)
  COMPLEX A11,A12,A21,A22,C11,C12,CK,XK
C  CALCULATION OF THE ODDORSENS FUNCTIONS
  CK=CMPLX(AFF,-AGG)
  CO=3.14159265*RO*U*U
  A11=CO*(-XK*XK+2.*CK*XK*CMPLX(0.0,1.0))
  A11=-A11
  A12=CO*(AH*XK*XK+XK*CMPLX(0.0,1.0)+2.*CK*(1.0+
  2XK*CMPLX(0.0,(0.5-AH))))*B
  A12=-A12
  A21=CO*B*(2.*CK*XK*CMPLX(0.0,(0.5-AH))-XK*XK*AH)
  CX=CO*B*B
  A22=CX*(2.*(0.5-AH)*CK*(1.0+XK*CMPLX(0.0,(0.5-AH)))+XK*XK/8.
  3+XK*XK*AH*AH
  1+XK*CMPLX(0.0,(AH=0.5)))
C  EFFECT OF SWEEP IS TAKEN INTO CONSIDERATION
  SWEEP=13.0*4.0*ATAN(1.0)/180.
  ECONST=COS(SWEEP)
  A11=A11+ECONST
  A12=A12+ECONST
  A21=A21+ECONST
  A22=A22+ECONST
  RETURN
END

```

```

SUBROUTINE XINT(Y1,Y2,AROW,DWF,WFO,IP,L)
L=1
WFO=ABS(Y1)*DWF/(ABS(Y1)*ABS(Y2))
WFO=AROW-WFO
IF(IP) 5,5,10
5 WRITE(2,20) WFO
RETURN
20 FORMAT(10X,37H RO=VALUE WHEN IMAGINARY DET ZERO = ,F10.5,/)
10 WRITE(2,30) WFO
30 FORMAT(10X,32H RO=VALUE WHEN REAL DET ZERO = ,F10.5,/)
RETURN
END
SUBROUTINE XPRINT(A,II,JJ)
DIMENSION A(II,JJ)
IF(JJ.EQ.3) WRITE(2,3) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ.4) WRITE(2,4) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ.5) WRITE(2,5) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ.6) WRITE(2,6) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ.7) WRITE(2,7) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ.8) WRITE(2,8) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ.9) WRITE(2,9) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ.10) WRITE(2,10) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ.11) WRITE(2,11) ((A(I,J),J=1,JJ),I=1,II)
IF(JJ.EQ.12) WRITE(2,12) ((A(I,J),J=1,JJ),I=1,II)
3 FORMAT(5X,3F10.4)
4 FORMAT(5X,4F15.4)
5 FORMAT(5X,5F10.4)
6 FORMAT(5X,6F15.4)
7 FORMAT(5X,7F15.4)
8 FORMAT(5X,8F10.4)
9 FORMAT(5X,9F10.4)
10 FORMAT(5X,10F10.4)
11 FORMAT(5X,11F10.4)
12 FORMAT(5X,12F10.4)
WRITE(2,100)
100 FORMAT(/)
RETURN
END
SUBROUTINE OFFMAS(JJ,KK,KL,OM,XI,R,HH,AA,Q)
DIMENSION AA(KK,JJ),HH(KK,JJ),Q(JJ,JJ)
DO 5 I=1,JJ
DO 5 J=1,JJ
5 Q(I,J)=OM*(HH(KL,I)*HH(KL,J)+R*AA(KL,I)*HH(KL,J)+R*AA(KL,J)
2*HH(KL,I))* (OM*R*R*XI)*AA(KL,I)*AA(KL,J)
CALL XPRINT(Q,JJ,JJ)
RETURN
END
SUBROUTINE FMODE(PPP,RRR,YY,WKSPC,QA,II,JJ,KK,RO,DX,U,WFO,ZK,HH,AA
1,BB,AHH,A11,A12,A21,A22,QQ,QF,JJK,XS,XM,H,W,YYZ1,YYZ2,ATH,XRO,
4AFX,AGX,AROW,G)
COMPLEX A11,A12,A21,A22,QQ,QF,QA,PPP,RRR,YY,YYZ1,YYZ2
COMPLEX BLAM,ZK
DIMENSION H(KK,JJ),AA(KK,JJ),BB(KK),AAH(KK),A11(KK),A12(KK),A21(KK)
2,A22(KK),QQ(KK),QF(JJ,JJ),ZK(KK),XS(JJ,JJ),XM(JJ,JJ),PPP(JJK,JJK)
3,RRR(JJK,1),YY(JJ,1),WKSPC(JJK),QA(JJ,JJ),W(JJ),HH(KK,JJ)
DIMENSION YYZ1(KK,1),YYZ2(KK,1),XRO(63),AFX(63),AGX(63)
AROW=WFO
CALL AIRMAT(II,JJ,KK,RO,DX,U,WFO,ZK,HH,AA,BB,AHH,A11,A12,A21,A22,Q
4Q,QF,AMU,AROW,XRO,AFX,AGX)
ZZI=WFO*COS(ATH*3.14159/180.0)
ZZR=-WFO*SIN(ATH*3.14150/180.0)
BLAM=CMPLX(ZZR,ZZI)
DO 50 I=1,JJ
DO 50 J=1,JJ
50 QA(I,J)=XM(I,J)*(BLAM**2+W(I)*W(I)*CMPLX(0.0,G))+XS(I,J)-QF(I,J)

```

```

DO 51 I=1,JJ=1
DO 51 J=1,JJ=1
51 PPP(I,J)=QA(I,J)
DO 52 I=1,JJ=1
52 RRR(I,1)=QA(I,JJ)
IFAIL=1
LL=JJ-1
CALL F04ADF(PPP,LL,RRR,LL,LL,1,VY,JJ,WKSPC,IFAIL)
IF(IFAIL.EQ.0) GO TO 20
WRITE(2,53)IFAIL
20 WRITE(2,54)(VY(I,1),I=1,LL)
VY(JJ,1)=CMPLX(-1,0,0,0)
WRITE(2,60)
WRITE(2,61)
WRITE(2,54)((VY(I,1),I=1,JJ))
DO 56 I=1,KK
YYZ1(I,1)=CMPLX(0,0,0,0)
YYZ2(I,1)=CMPLX(0,0,0,0)

DO 56 K=1,JJ
YYZ1(I,1)=YYZ1(I,1)+HH(I,K)*VY(K,1)
56 YYZ2(I,1)=YYZ2(I,1)+AA(I,K)*VY(K,1)
WRITE(2,55)
WRITE(2,61)
WRITE(2,54)(YYZ1(I,1),I=1,KK)
WRITE(2,62)
DO 63 I=1,KK
RYYZ1=REAL(YYZ1(I,1))
AIYYZ1=AIMAG(YYZ1(I,1))
AMYYZ1=SQRT(RYYZ1**2+AIYYZ1**2)
ATHET1=ATAN(AIYYZ1/RYYZ1)
ADTHET1=ATHET1*180,0/3,1415926
63 WRITE(2,64)RYYZ1,AIYYZ1,AMYYZ1,ATHET1,ADTHET1
CONTINUE
WRITE(2,65)
WRITE(2,61)
WRITE(2,54)(YYZ2(I,1),I=1,KK)
WRITE(2,62)
DO 66 I=1,KK
RYYZ2=REAL(YYZ2(I,1))
AIYYZ2=AIMAG(YYZ2(I,1))
AMYYZ2=SQRT(RYYZ2**2+AIYYZ2**2)
ATHET2=ATAN(AIYYZ2/RYYZ2)
ADTHET2=ATHET2*180,0/3,14159265
WRITE(2,64)RYYZ2,AIYYZ2,AMYYZ2,ATHET2,ADTHET2
66 CONTINUE
53 FORMAT(1H ,23HERROR IN F04ADF IFAIL =,I2)
54 FORMAT(1H ,2(2X,F16,9,8X))
55 FORMAT(1H ,5X,28HAEROELASTIC MODES IN BENDING)
60 FORMAT(1H ,5X,53HAEROELASTIC MODES INTERMS OF GENERALISED CO-ORDIN
1ATES)
61 FORMAT(1H ,5X,9HREAL=PART,10X,14HIMAGINARY=PART)
62 FORMAT(1H ,2X,9HREAL=PART,5X,9HIMAG=PART,5X,7HMODULUS,5X,10HPHASE(
2RAD),5X,10HPHASE(DEG))
64 FORMAT(1H ,2X,F12,5,2X,F12,5,2X,F12,5/5X,F11,6,5X,F11,6)
65 FORMAT(1H ,28HAEROELASTIC MODES IN TORSION)
RETURN
END
FINISH

```

L.2.2 Listing of the Data Subfile DAT4

```

25,0
3,0 1,0 7,0
20,0 2,5 25,0

```

L.2.3 Listing of the Data Subfile DAT5

1.00000	0.00000
0.99553	0.05059
0.98390	0.08657
0.96943	0.11585
0.95343	0.14026
0.93666	0.16075
0.91956	0.17799
0.90248	0.19248
0.88562	0.20462
0.86914	0.21474
0.85315	0.22312
0.83770	0.23000
0.82285	0.23559
0.80860	0.24005
0.79497	0.24355
0.78196	0.24620
0.76955	0.24813
0.75773	0.24942
0.74648	0.25018
0.73577	0.25046
0.72559	0.25034
0.71591	0.24987
0.70671	0.24910
0.69795	0.24807
0.68963	0.24682
0.68172	0.24538
0.67419	0.24377
0.66702	0.24203
0.66020	0.24018
0.65371	0.23823
0.64752	0.23620
0.63601	0.23195
0.62554	0.22754
0.61599	0.22303
0.60729	0.21848
0.59933	0.21394
0.59204	0.20942
0.58535	0.20497
0.57921	0.20058
0.57356	0.19629
0.56834	0.19209
0.55699	0.18206
0.54762	0.17270
0.53981	0.16403
0.53327	0.15600
0.52775	0.14857
0.52307	0.14170
0.51907	0.13534
0.51564	0.12945
0.51269	0.12398
0.51014	0.11890
0.49705	0.08316
0.49331	0.06310
0.49229	0.05051
0.49222	0.04197
0.49248	0.03582
0.49287	0.03121
0.49367	0.02477
0.49439	0.02051
0.49499	0.01749
0.49548	0.01523
0.49590	0.01349
0.49624	0.01211

	1.000000	0.000000
	0.997325	0.050945
	0.986780	0.087655
	0.973030	0.117795
	0.957480	0.143080
	0.940935	0.164430
	0.923895	0.182480
02	0.906240	0.197710
	0.889705	0.210520
04	0.872970	0.221230
	0.856670	0.230125
03	0.840870	0.237450
	0.825640	0.243410
03	0.811000	0.248180
	0.796970	0.251925
10	0.783560	0.254775
	0.770755	0.256850
	0.758545	0.258255
	0.746920	0.259085
	0.735850	0.259405
	0.725325	0.259295
	0.715310	0.258810
	0.705795	0.258005
	0.696740	0.256920
	0.688135	0.255595
	0.679960	0.254075
	0.672180	0.252370
05	0.664775	0.250530
	0.657735	0.248565
0	0.651035	0.246495
	0.644655	0.244340
10	0.632795	0.239835
	0.622025	0.235155
12	0.612220	0.230370
	0.603290	0.225550
14	0.595140	0.220735
	0.587690	0.215950
16	0.580865	0.211235
	0.574610	0.206595
18	0.568865	0.202060
	0.563570	0.197625
20	0.552090	0.187040
	0.542660	0.177185
22	0.534840	0.168065
	0.528330	0.159635
24	0.522860	0.151855
	0.518250	0.144665
26	0.514335	0.138020
	0.510995	0.131875
28	0.508145	0.126175
	0.505695	0.120890
30	0.493570	0.083900
	0.490545	0.063330
32	0.490030	0.050505
	0.490325	0.041850
34	0.490860	0.035645
	0.491455	0.031005
36	0.492550	0.026550
	0.493465	0.020295
38	0.494205	0.017285
	0.494800	0.015045
40	0.495295	0.013315
	0.495705	0.011945

	1.00000	0.00000
	0.99912	0.05130
41	0.98966	0.08874
	0.97663	0.11974
43	0.96153	0.14590
	0.94521	0.16811
	0.92823	0.18697
	0.91000	0.20294
50	0.89379	0.21642
	0.87680	0.22772
52	0.86019	0.23713
	0.84404	0.24490
54	0.82843	0.25123
	0.81340	0.25631
56	0.79897	0.26030
	0.78516	0.26335
58	0.77196	0.26557
	0.75936	0.26709
60	0.74736	0.26799
	0.73593	0.26835
62	0.72506	0.26825
	0.71471	0.26775
64	0.70488	0.26691
	0.69553	0.26577
	0.68664	0.26437
	0.67820	0.26277
70	0.67017	0.26097
	0.66253	0.25903
72	0.65527	0.25695
	0.64836	0.25476
74	0.64179	0.25248
	0.62958	0.24772
76	0.61851	0.24277
	0.60845	0.23771
80	0.59929	0.23262
	0.59095	0.22753
82	0.58334	0.22248
	0.57638	0.21750
84	0.57001	0.21261
	0.56417	0.20783
86	0.55880	0.20316
	0.54719	0.19202
88	0.53770	0.18167
	0.52987	0.17210
90	0.52339	0.16327
	0.51797	0.15514
92	0.51343	0.14763
	0.50960	0.14070
94	0.50635	0.13430
	0.50360	0.12837
96	0.50125	0.12288
	0.49009	0.08464
98	0.48778	0.06356
	0.48777	0.05050
30	0.48843	0.04173
	0.48924	0.03547
32	0.49004	0.03080
	0.49143	0.02433
34	0.49254	0.02008
	0.49342	0.01708
36	0.49412	0.01486
	0.49469	0.01314
38	0.49517	0.01178

L.2.4 Listing of the Result Subfile RES2

AROW	U	MU	OMEGA	REAL DET	IMAG DET
3.00	25.00000	-1.02606		2.81908	-3022E 14
4.00	25.00000	-1.36808		3.75877	-2350E 14
5.00	25.00000	-1.74010		4.69866	0.340E 13
	RO-VALUE	WHEN REAL DET ZERO =	4.78750		-5928E 14
					-7530E 14
					-5666E 14
6.00	25.00000	-2.05212		5.63816	0.6530E 14
	RO-VALUE	WHEN IMAGINARY DET ZERO =	5.63206		0.3169E 14
7.00	25.00000	-2.30414		6.57785	0.1592E 15
					0.2115E 15
3.00	25.00000	-1.14805		2.77164	-2697E 14
4.00	25.00000	-1.53073		3.69552	-2350E 14
5.00	25.00000	-1.91342		4.61940	-4036E 13
6.00	25.00000	-2.29610		5.54328	0.3028E 14
	RO-VALUE	WHEN REAL DET ZERO =	5.14260		-15966E 14
					-7287E 14
					-4677E 14
					0.6760E 14
	RO-VALUE	WHEN IMAGINARY DET ZERO =	5.49661		

L.3

Programs for Response Calculations

The response (normal or angular acceleration) of the flexible aeroplane due to random atmospheric turbulence and discrete gusts of smooth ramp profile (one-minus-cosine) is computed by applying a transfer function approach. Fig.135 shows the block diagram of the analytical procedure for both statistical and discrete gust methods.

L.3.1

Computation of Response Due to Random Atmospheric Turbulence

The Power Spectral Density (PSD) of the output response (normal or angular acceleration) is calculated from the knowledge of the input PSD of the atmospheric turbulence. A transfer function representation has been adopted to evaluate the output PSD. By calculating the response to a harmonic gust, the transfer function of the flexible aeroplane is determined. The force due to the motion and that due to the excitation are calculated taking unsteady aerodynamic effects into account. The former type of forces are those which enter into flutter calculations and are evaluated in the same way as in section L.1 using Theodorsen expressions for lift and moment whereas the latter types arising from the harmonic gust are calculated by using Schwarz' solution (see section F.2.1.2, Appendix F). The generalised co-ordinates are obtained as response quantities and are finally multiplied by the modal functions of a specific point on the aircraft to determine the transfer function of that particular point. The frequency of the harmonic gust is varied and the transfer function of points on the deformable aeroplane is obtained for wide range of frequencies. The square of the transfer function is ultimately multiplied by the input PSD of the turbulence to calculate the output PSD of the normal or angular acceleration. The method and the sequential operations are described in the block diagram in Fig.136.

The response program is stored in the subfile JRB5 and the required data in the subfile DAT6 both of which have been listed in sections

L.3.1.1 and L.3.1.2. The subroutines which are developed for the response program JRB5 are as follows.

SUBROUTINE EFQEF

This subroutine evaluates external forces on the wing due to a sinusoidal gust and expresses them in terms of the generalised co-ordinates.

SUBROUTINE TURBULENCE

This subroutine evaluates the transfer function of normal and angular accelerations at various points on the wing and finally calls the subroutine TURB1 to calculate the PSD of the normal and angular acceleration due to the Dryden and von Karman atmospheric spectrum respectively.

SUBROUTINE UNSTP

This subroutine evaluates the normal and angular acceleration in the time domain due to a unit step gust velocity - using a fourier transform. The program calls the subroutine UNST and outputs the results on cards allocated to Channel 3 so that they can be directly fed into the program developed by Purcell (Ref.96).

Following the main program JRB5 and the data subfile DAT6 respectively in sections L.3.1.1 and L.3.1.2, a sample output RES3 of the response program has been listed in section L.3.1.3.

```

LIBRARY(SUBGROUPPAGE)
PROGRAM(D50G)
INPUT 1=CRQ
OUTPUT 2=LPO
OUTPUT 3=CPQ
COMPRESS INTEGER AND LOGICAL
TRACE 2
END

```

```

C
C
MASTER RESPONSE CALCULATIONS

```

```

C
C
COMPLEX QQQ, OEF, CCT, ALFT, AMT, RESQ

```

```

C
C
DIMENSION S(24000)

```

```

C
C
XK = VALUES OF X AT INTEGRATION POINTS
C
C
II = NO NODES JJ = NO NODES KK = NO INTEGRATION POINTS
C
C

```

```

200 READ(1,200) II, JJ, KK, IJO, NN, NT
FORMAT(610)

```

```

C
C
JJ=JJ-IJO+1
IJO=IJO-1
WRITE(2,0) II, JJ, KK, IJO
9 FORMAT(5X, 15HNO WING NODES =, 12, 5X, 10HNO MODES =, 12, 5X, 23HNO INTEG
2RATION POINTS =, 12, 5X, 33HNO RIGID BODY MODES DISREGARDED =, 12)

```

```

I1=II+1
N=II-1
N2=N*(N+1)/2
M1=1
M2=M1+I1
M3=M2+I1
M4=M3+I1
M5=M4+I1*JJ
M6=M5+I1*JJ
M7=M6+KK
M8=M7+KK
M9=M8+KK
M10=M9+KK*JJ
M11=M10+KK*JJ
M12=M11+I1
M13=M12+I1
M14=M13+I1
M15=M14+I1
M16=M15+KK
M17=M16+JJ
M18=M17+I2
M19=M18+I1
M20=M19+JJ*JJ
M21=M20+JJ*JJ
M22=M21+KK*2
M23=M22+KK*2
M24=M23+KK*2
M25=M24+KK*2
M26=M25+KK*2
M27=M26+JJ*JJ*2
M28=M27+JJ*JJ*2

```

```

M29=M28+I I
M30=M29+I I
M31=M30+I I*I I
M32=M31+I I*I I
M33=M32+I I*I I
M34=M33+I I
M35=M34+I I
M36=M35+I I
M37=M36+I I
M38=M37+I I
M39=M38+I I
M40=M39+I I
M41=M40+I I
M42=M41+I I
M43=M42+I I
M44=M43+I I*I I
M45=M44+KK*2
M46=M45+JJ*1*2
M47=M46+JJ
M48=M47+JJ*1*2
M49=M48+NN
M50=M49+NN*JJ
M51=M50+NN*JJ
M52=M51+KK*NN
M53=M52+KK*NN
M54=M53+KK*NN
M55=M54+KK*NN
M56=M55+NN
M57=M56+NN
M58=M57+NN*KK
M59=M58+NN*KK
M60=M59+NN*KK
M61=M60+NN*KK
M62=M61+NN
M63=M62+NN
M64=M63+NT*KK
M65=M64+NT*KK
ICORE=M65*2+2000

```

5

```

WRITE(2,5) ICORE
FORMAT(5X,15HCORE REQUIRED =,15)
IF(CORE.GT.50000) STOP
CALL RSPNSE ( S(M1),S(M2),S(M3),S(M4),S(M5),S(M6),S(M7),S(M8),
2S(M9),S(M10),S(M11),S(M12),S(M13),S(M14),S(M15),S(M16),S(M17),
3S(M18),S(M19),S(M20),S(M21),S(M22),S(M23),S(M24),S(M25),S(M26),
4S(M27),S(M28),S(M29),S(M30),S(M31),S(M32),S(M33),S(M34),S(M35),S(M
536),S(M37),S(M38),S(M39),S(M40),S(M41),S(M42),S(M43),
6S(M44),S(M45),S(M46),S(M47),S(M48),S(M49),S(M50),S(M51),S(M52),
7S(M53),S(M54),S(M55),S(M56),S(M57),S(M58),S(M59),S(M60),S(M61),
8S(M62),S(M63),S(M64),I I,JJ,KK,NT,I I,NN,NT)
STOP
END

```

C

C****

C

```

SUBROUTINE RSPNSE ( Y,B,AH, Y,A,XK,RR,AHH,HH,AA,HHH,AAA,BBB,
2AHHH,ZK,W,WK,WKSP,XM,XS,A11,A12,A21,A22,QQ,QA,QF,DY,C,S,S1,EVV,EV
3,WKSP2,T10,T4,WK,T1,T2,TT1,TT2,WWN,DIV,QQQ,QEF,WKSPCE,RESQ,
4WW,TER,TFI,THR,THI,TAR,TAI,PHID,PHIV,RDH,RDA,RVH,RVA,AAT,AHT,
5ANX,AAV,I I,JJ,KK,NT,I I,NT)
COMPLEX A11,A12,A21,A22,QQ,QA,QF,Q1,Q2
COMPLEX QQQ,QEF,CCT,ALFT,ANT,RESQ
DIMENSION Y(I I),B(I I),AH(I I),H(I I,JJ),A(I I,JJ),XK(KK),BB(KK),
2AHH(KK),HH(KK,JJ),AA(KK,JJ),HHH(I I),AAA(I I),BBB(I I),AHHH(I I),
3ZK(KK),W(JJ),WK(N2),WKSP(I I),XM(JJ,JJ),XS(JJ,JJ),A11(KK),A12(KK),
4A21(KK),A22(KK),QQ(KK),QA(JJ,JJ),QF(JJ,JJ),DY(I I),C(I I),S(I I,I I),
5S1(I I,I I),EVV(I I,I I),EV(I I),WKSP2(I I),T10(I I),T4(I I),WK(I I),T1(I I)

```

```
6, T2(I1), TT1(I1), TT2(I1), WVN(I1), DIV(I1, I1)
  DIMENSION QQQ(KK), QEF(JJ, 1), WKSPACE(JJ), PESQ(JJ, 1)
```

```
  DIMENSION WU(N), TFR(N, JJ), TFI(N, JJ), THR(KK, N), THI(KK, N), TAR(KK, N),
  7TAI(KK, N), PHID(N), PHIV(N), RDH(N, KK), RDA(N, KK), RVH(N, KK), RVA(N, KK),
  8AAT(N), AHT(N), AHX(NT, KK), AAY(NT, KK)
```

```
14  FORMAT(5X, 12H AIRDENSITY =, E10.4, 5X, 26H STRUCTURAL DAMPING COEFF =, F
  15, 3)
```

```
  READ(1, 201) U, DWF, WG, SL, T0, DT, TMX, RO, G, SPA
```

```
  READ(1, 201) (Y(I), I=1, I1)
```

```
  READ(1, 201) (B(I), I=1, I1)
```

```
  READ(1, 201) (AH(I), I=1, I1)
```

```
  READ(1, 201) (C(I), I=1, I1)
```

```
201  FORMAT(20F0.0)
```

```
  READ(1, 202) ((S(I, J), I=1, I1), J=1, I1)
```

```
  READ(1, 202) (W(I), I=1, JJ)
```

```
  READ(1, 202) ((XS(I, J), I=1, JJ), J=1, JJ)
```

```
  READ(1, 202) ((XM(I, J), I=1, JJ), J=1, JJ)
```

```
  READ(1, 202) ((H(I, J), I=1, I1), J=1, JJ)
```

```
  READ(1, 202) ((A(I, J), I=1, I1), J=1, JJ)
```

```
202  FORMAT(175F0.0)
```

```
  SWP=COS(SPA*3.1415926/180.0)
```

```
  WRITE(2, 14) RO, G
```

```
  WRITE(2, 82)
```

```
  DO 22 I=1, JJ
```

```
22  W(I)=U(I)*6.283185307
```

```
  CLEAR ALL OFF-DIAGONAL TERMS IN XS AND XM
```

```
  IF(IJ0.EQ.0) XS(1, 1), XS(2, 2)=0.0
```

```
  IF(IJ0.EQ.1) XS(1, 1)=0.0
```

```
  DO 200 III=1, JJ-1
```

```
  DO 200 KKK=III+1, JJ
```

```
  XS(III, KKK)=0.0
```

```
  XS(KKK, III)=0.0
```

```
  XM(III, KKK)=0.0
```

```
200  XM(KKK, III)=0.0
```

```
  CLEARING OF OFFDIAGONAL TERMS IS OVER
```

```
  WRITE(2, 82)
```

```
  AKK=KK-1
```

```
  DX=(Y(II)-Y(1))/AKK
```

```
  DO 5 K=1, KK
```

```
  AK=K-1
```

```
5  XK(K)=Y(1)+DX*AK
```

```
  CALL EXTRAP(II, JJ, KK, N2, DX, Y, XK, H, A, B, AH, HK, HH, AA, BB, AHH,
  1HHH, AAA, BBB, AHHH)
```

```
  WRITE(2, 82)
```

```
82  FORMAT(/)
```

```
  DO 90 LI=1, N
```

```
  WF=(LI-1)*DWF
```

```
  WW(LI)=WF
```

```
  CALL AIRMAT(II, JJ, KK, RO, DX, U, WF, ZK, HH, AA, BB, AHH, A11, A12, A21, A22,
  1QQ, QF, SUP)
```

```
  FORM FLUTTER DETERMINANT=QA
```

```
  DO 50 I=1, JJ
```

```
  DO 50 J=1, JJ
```

```
50  QA(I, J)=XM(I, J)*(WF*WF+U(I)*W(I)*CMPLX(0.0, G))+XS(I, J)-QF(I, J)
```

```
  CALL EFOFF(II, JJ, KK, RO, DX, U, WF, HH, AA, BB, AHH, QQQ, QEF, WG)
```

```

IFAIL = 0
CALL F04ADF(OAT,JJ,QEF,JJ,JJ,1,RESQ,JJ,WKSPCE,IFAIL)
IF(IFAIL.EQ.0) GOTO 120
WRITE(2,110) IFAIL
110  FORMAT('FAILURE I = ',15)
STOP
120  CONTINUE
DO 752 J=1,JJ
TFR(LI,J)=REAL(RESQ(J,1))
TFI(LI,J)=AIMAG(RESQ(J,1))
752  CONTINUE
90   CONTINUE
WRITE(2,859)
859  FORMAT(1H,39H ADMITTANCE FUNCTION OF THE DISPLACEMENT)
WRITE(2,858)
858  FORMAT(1H,4H MODE,1X,9H FREQUENCY,5X,9H REAL-PART,5X,9H IMAG-PART,5X,
19H ABS-VALUE,5X,6H SQUARE,6X,8H PHASE(R),4X,8H PHASE(D))
861  FORMAT(1H,13,1X,F7.2,2X,4(F14.5,1X),2X,2(F8.3,2X))
WRITE(2,857)
857  FORMAT(1H,39H ADMITTANCE FUNCTION OF THE ACCELERATION)
WRITE(2,858)
DO 862 J=1,JJ
DO 862 I=1,N,2
O9=WW(I)
A9=TFR(I,J)*WW(I)**2
B9=TFI(I,J)*WW(I)**2
C9=A9**2+B9**2
D9=SQRT(C9)
IF(ABS(A9).LT.0.0000001) A9=0.0000001
E9=ATAN(B9/A9)
F9=E9*180.0/3.1415926
WRITE(2,851)J,O9,A9,B9,D9,C9,E9,F9
862  CONTINUE
DO 851 I=1,3
IF(I.EQ.1)SL=75.0
IF(I.EQ.2)SL=150.0
IF(I.EQ.3)SL=300.0
CALL TURBULENCE(WW,TFR,TFI,HH,AA,THR,THI,TAR,TAI,PHID,PHIV,RDH,
1RDA,RVH,RVA,N,JJ,KK,UG,SL,U,DWF)
851  CONTINUE
CALL UNSTP(WW,TFR,TFI,HH,AA,THR,THI,TAR,TAI,AAT,AHT,AHX,AAV,N,NT,
2JJ,KK,TG,DT,THX,DWF)
STOP
RETURN
END

```

C
C****
C

```

SUBROUTINE EXTRAP(I1,JJ,KK,N2,DX,X,XK,H,A,B,AH,HK,HH,AA,BB,AHH,
1HHH,AAA,BBB,AHHH)
DIMENSION X(I1),XK(KK),H(I1,JJ),A(I1,JJ),B(I1),AH(I1),HK(N2),
1HH(KK,JJ),AA(KK,JJ),BB(KK),AHH(KK),HHH(I1),AAA(I1),BBB(I1),AHHH(I
21)
N=I1-1
DO 150 K=1,KK
XD=XK(K)
IF (K.GT.1) XD=XK(K)-XK(K-1)
DO 100 J=1,JJ
DO 105 I=1,I1
105  HHH(I)=H(I,J)
CALL F01AAF(X,HHH,1HK,I1,N2,N7XD)
XD=0.0
HH(K,J)=HK(N2)
DO 125 I=1,I1
125  AAA(I)=A(I,J)
CALL F01AAF(X,AAA,1HK,I1,N2,N70,0)

```

```

100  AA(K,J)=HK(N2)
      CONTINUE
      DO 130 I=1,II
130   BBB(I)=B(I)
      CALL E01AAF(X,BBB,HK,II,N2,N,0,0)
      BB(K)=HK(N2)
      DO 140 I=1,II
140   AHH(I)=AH(I)
      CALL E01AAF(X,AHH,HK,II,N2,N,0,0)
      AHH(K)=HK(N2)
150   CONTINUE
      RETURN
      END

```

C
C****
C

```

      SUBROUTINE AIRMAT(II,JJ,KK,RO,DX,U,UF,ZK,HH,AA,BB,AHH,A11,A12,A21,
1A22,QQ,QF,SWP)
      COMPLEX A11,A12,A21,A22,QQ,QF
      DIMENSION HH(KK,JJ),AA(KK,JJ),BB(KK),AHH(KK),A11(KK),A12(KK),
1A21(KK),A22(KK),QQ(KK),QF(JJ,JJ),ZK(KK)
      DO 200 K=1,KK
      ZK(K)=BB(K)*UF/U
      CALL AIRCOF(U,RO,ZK(K),AHH(K),BB(K),A11(K),A12(K),A21(K),A22(K),
3SWP)
200   CONTINUE
      DO 300 L=1,JJ
      DO 300 M=1,JJ
      DO 250 N=1,KK
      QQ(K)= A11(K)*HH(K,L)*HH(K,M)+A21(K)* HH(K,M)*AA(K,L)+A12(K)*
1HH(K,L)*AA(K,M) + A22(K)*AA(K,L)*AA(K,M)
250   CONTINUE
      CALL SIMPSN (KK,DX,QQ,QF(L,M))
300   CONTINUE
      RETURN
      END

```

C
C****
C

```

      SUBROUTINE SIMPSN(KK,HX,Y,AREA)
      COMPLEX Y,AREA,ODD,EVEN
      DIMENSION Y(KK)
      HX=ABS(HX)
      ODD=(0,0,0,0)
      EVEN=(0,0,0,0)
      DO 10 I=1,KK-2,2
      ODD=ODD+Y(I)
10   EVEN=EVEN+Y(I+1)
      AREA=HX/3.0*(4.0*EVEN+2.0*ODD+Y(KK)-Y(1))
      RETURN
      END

```

C
C****
C

```

      SUBROUTINE AIRCOF(U,PD,XK,AH,B,A11,A12,A21,A22,SWP)
      COMPLEX A11,A12,A21,A22,C11,C12,CK
      CALCULATION OF THEODORSE'S FUNCTIONS
      IF(XK.NE.0.0) GOT05
      CK=CMPLX(1,0,0,0)
      GOT0 50
5   IF (XK=0.45) 10,20,20
10  CA=0.0455/XK
   CB=0.5/XK
   GOT0 40
20  IF(XK=0.55) 30,30,35
30  CA=(0.06575-0.045*XK)/XK

```

```

      CB=(0.21+0.2*XX)/XX
      GOTO 40
35    CA=0.044/XX
      CB=0.32/XX
40    C11=CMPLX(1.,-CA)
      C12=CMPLX(1.,-CB)
      CK=1.-0.165/C11-0.335/C12
50    CONTINUE
      CO=3.14159265*RO*U*U
      A11=CO*(-XX*XX+2.*CK*CMPLX(0.0,XX))
      A11=-A11
      A12=CO*(CMPLX(AH*XX*XX,XX)+2.*CK*CMPLX(1.,XX*(0.5-AH)))*B
      A12=-A12
      A21=CO*B*(2.*CK*CMPLX(0.0,XX*(0.5+AH))-XX*XX*AH)
      CX=CO*B*B
      A22=CX*(2.*(0.5+AH)*CK*CMPLX(1.0,XX*(0.5-AH))+CMPLX(XX*XX/8.,XX
1    *XX*AH*AH,(AH-0.5)*XX))
C    EFFECT OF SWEEP IS TAKEN INTO CONSIDERATION
      A11=A11*SWP
      A12=A12*SWP
      A21=A21*SWP
      A22=A22*SWP
      RETURN
      END

C
C****
C
      SUBROUTINE EFREF(II,JJ,KK,RO,DX,U,WF,HH,AA,DB,AHH,
1    QQQ,QEF,UG)
      COMPLEX H1,H0,CCT,ALFT,AMT,QQQ,QEF,IIC
      REAL J0,J1
      DIMENSION QQQ(KK),QEF(JJ,1),HH(KK,JJ),AA(KK,JJ),BS(KK),AHH(KK)
      CONS=2.*4.*0*ATAN(1.0)*U*UG*RO*GOS(13.0*3.14159/180.)
      IIC=CMPLX(0.0,1.0)
      DO 52 J=1,JJ
      DO 53 K=1,KK
      ZK=BS(K)*WF/U
      J0=S17ADF(ZK)
      J1=S17ADF(ZK)
      CCT=CMPLX(1.0,0.0)
      IF(ZK.NE.0.0)GO TO 56
      GO TO 57
56    IF=0
      Y0=S17ACF(ZK,IF)
      IF=0
      Y1=S17ADF(ZK,IF)
      H1=J1-IIC*Y1
      H0=J0-IIC*Y0
      CCT=H1/(H1+IIC*H0)
57    ALFT=-CONS*BS(K)*(CCT*CMPLX(J0,-J1)+CMPLX(0.0,J1))
      AMT=-ALFT*BS(K)*(0.5+AH(K))
53    QQQ(K)=HH(K,J)*ALFT+AA(K,J)*AMT
      CALL SIMPSN(KK,DX,QQQ,QEF(J,1))
52    CONTINUE
      RETURN
      END

C
C****
C
      SUBROUTINE TURBULENCE(W,TER,TFI,HH,AA,THR,THI,TAR,TAI,PHID,PHIV,
1    RDH,RDA,RVH,RVA,N,JJ,KK,UG,SL,U,OW)
C    RESPONSE TO TURBULENCE
      DIMENSION W(N),TER(N,JJ),TFI(N,JJ),HH(KK,JJ),AA(KK,JJ),THR(KK,
3    3N)
      1,THI(KK,N),TAR(KK,N),TAI(KK,N),PHID(N),PHIV(N),RDH(N,
7    KK),

```

```

C      ZRDA(N, KK), RVH(N, KK), RVA(N, KK)
C      N=NO OF INTG. POINT
C      N=NO OF FREQUENCIES AS WELL
C      JJ=NO OF MODES
C      KK=NO OF POINTS ON WING(NO OF INTG. POINTS OF STAGE)
C      MULTIPLY THE REAL AND IMAGINARY PARTS OF THE T.F. BY MODAL MATRICES
C      THEN MULTIPLY BY -(W(I)**2) FOR ACCELERATION T.F
C
C      DO 4 I=1, KK, 5
C
C      DO 4 J=1, N
C      THR(I, J), THI(I, J), TAR(I, J), TAI(I, J)=0.0
C
C      DO 5 I=1, KK, 5
C
C      DO 5 J=1, N
C      DO 5 K=1, JJ
C      THR(I, J)=THR(I, J)+HH(I, K)*TFR(J, K)*(-W(J)**2)
C      THI(I, J)=THI(I, J)+H4(I, K)*TFI(J, K)*(-W(J)**2)
C      TAR(I, J)=TAR(I, J)+AA(I, K)*TFR(J, K)*(-W(J)**2)
C      TAI(I, J)=TAI(I, J)+AA(I, K)*TFI(J, K)*(-W(J)**2)
C      CALL TURB1(WG, SL, U, DW, PHID, PHIV, RDH, RDA, RVH, RVA, KK, N, W,
C      2THR, THI, TAR, TAI)
C      RETURN
C      END
C
C****
C
C      SUBROUTINE TURB1(WG, SL, U, DW, PHID, PHIV, RDH, RDA, RVH, RVA, KK, N, W,
C      9THR, THI, TAR, TAI)
C      DIMENSION PHID(N), PHIV(N), RDH(N, KK), RDA(N, KK), RVH(N, KK), RVA(N, KK)
C      DIMENSION W(N), THR(KK, N), THI(KK, N), TAR(KK, N), TAI(KK, N)
C      PI=3.1415926
C      CONST=WG**2*SL/(PI*U)
C      WRITE(2, 99) SL
C      99  FORMAT(1H, 10X, 13HSCALE=LENGTH=, F8.2)
C      WRITE(2, 7)
C      7   FORMAT(1H, 5X, 9HFREQUENCY, 5X, 14H DRYDEN=SPECTRA, 5X, 14H KARMAN=SPECTR
C      3A)
C      DO 5 I=1, N
C      X=W(I)*SL/U
C      PHID(I)=CONST*(1.+3.*X**2)/((1.+X**2)**2)
C      PHIV(I)=CONST*(1.+8./3.*(1.339*X)**2)/((1.+(1.393*X)**2)**(11./6.
C      5))
C      WRITE(2, 8) W(I), PHID(I), PHIV(I)
C      8   FORMAT(1H, 5X, F9.2, 5X, E14.7, 5X, E14.7)
C      CONTINUE
C
C      DO 6 I=1, KK, 5
C
C      WRITE(2, 9)
C      9   FORMAT(1H, 13HPOINT ON WING, 5X, 9HFREQUENCY, 10X, 15H DRYDEN-RESPONSE,
C      410X, 15H KARMAN-RESPONSE)
C      DO 6 J=1, N
C      RDH(J, I)=(THR(I, J)**2+THI(I, J)**2)*PHID(J)
C      RDA(J, I)=(TAR(I, J)**2+TAI(I, J)**2)*PHID(J)
C      RVH(J, I)=(THR(I, J)**2+THI(I, J)**2)*PHIV(J)
C      RVA(J, I)=(TAR(I, J)**2+TAI(I, J)**2)*PHIV(J)
C      WRITE(2, 10) I, W(J), RDH(J, I), RDA(J, I), RVH(J, I), RVA(J, I)
C      10  FORMAT(5X, I4, 10X, F9.2, 5X, E14.4, 2X, E14.4, 5X, F11.4, 2X, E11.4)

```



```

6 CONTINUE
C
C
C DO 11 J=1, KK, 5
C
C
15 WRITE(2,15) J
   FORMAT(1H, '3HKK=', I4)
   WRITE(2,16)
16   FORMAT(1H, '52HMEAN SQUARE LINEAR ACCELERATION USING DRYDEN SPECTRA
5)
   CALL SIMPSON1(N, DW, RDH, AREA, J)
   WRITE(2,17)
17   FORMAT(1H, '53HMEAN SQUARE ANGULAR ACCELERATION USING DRYDEN SPECTR
6A)
   CALL SIMPSON1(N, DW, RDA, AREA, J)
   WRITE(2,18)
18   FORMAT(1H, '52HMEAN SQUARE LINEAR ACCELERATION USING KARMAN SPECTRA
7)
   CALL SIMPSON1(N, DW, RVH, AREA, J)
   WRITE(2,19)
19   FORMAT(1H, '53HMEAN SQUARE ANGULAR ACCELERATION USING KARMAN SPECTR
8A)
   CALL SIMPSON1(N, DW, RVA, AREA, J)
11 CONTINUE
   RETURN
   END

```

C
C****
C

```

SUBROUTINE SIMPSON1(N, DX, Y, AREA, J)
  DIMENSION Y(N, J)
  ODD=0.0
  EVEN=0.0
  DO 3 I=1, N=2, 2
    ODD=ODD+Y(I, J)
3  EVEN=EVEN+Y(I+1, J)
  AREA=DX/3.*(4.*EVEN+2.*ODD+Y(N, J)-Y(1, J))
  BREA=SQRT(AREA)
  WRITE(2,4) AREA, BREA
4  FORMAT(1H, '16HMEAN SQUARE ACCLN=, E16.8, 5X, 12HR. M.S VALUE=, E16.8)
  RETURN
  END

```

C
C****
C

```

SUBROUTINE INSTP(W, TFR, TFI, HH, AA, THR, THI, TAR, TAI, AAT, AHT, AHX, AAY,
1 N, NT, JJ, KK, TO, DT, THX, DW)

```

```

C
C
C RESPONSE TO UNIT STEP
  DIMENSION W(N), TFR(N, JJ), TFI(N, JJ), HH(KK, JJ), AA(KK, JJ), THR(KK, N),
1 THI(KK, N), TAR(KK, N), TAI(KK, N), AAT(N)
  DIMENSION AHT(N), AHX(NT, KK), AAY(NT, KK)

```

C
C

```

DO 4 I=1, KK, 5

```

C
C

```

DO 4 J=1, N
  THR(I, J), THI(I, J), TAR(I, J), TAI(I, J)=0.0

```

4
C
C

```

DO 5 I=1, KK, 5

```

C
C

```

DO 5 J=1,N
DO 5 K=1,JJ
THR(I,J)=THR(I,J)+HH(I,K)*TFR(J,K)*(-W(J))
THI(I,J)=THI(I,J)+HH(I,K)*TFI(J,K)*(-W(J))
TAR(I,J)=TAR(I,J)+AA(I,K)*TFR(J,K)*(-W(J))
5 TAI(I,J)=TAI(I,J)+AA(I,K)*TFI(J,K)*(-W(J))
CALL UNST(THR,THI,TAR,TAI,AHT,AAT,TO,DT,N,JJ,W,KK,TMX,DW,NT,
3AHX,AAY)
STOP
END

```

C
C****
C

```

SUBROUTINE UNST(THR,THI,TAR,TAI,AHT,AAT,TO,DT,N,JJ,W,KK,TMX,DW,
2NT,AHX,AAY)
DIMENSION THR(KK,N),THI(KK,N),AHT(N),AAT(N),W(N)
DIMENSION TAR(KK,N),TAI(KK,N),AHX(NT,KK),AAY(NT,KK)
PI=3.1415926

```

C
C
C
C

```

DO 16 I=1,KK,5

```

```

WRITE(2,16)I
16 FORMAT(1H,15)
WRITE(2,5)
5 FORMAT(1H,5X,4HTIME,5X,21HVERTICAL ACCELERATION,5X,20HANGULAR ACC
1ELERATION)
DO 18 L=1,NT
T=(L-1)*DT
DO 4 J=1,N
4 AHT(J)=THR(I,J)*SIN(W(J)*T)+THI(I,J)*COS(W(J)*T)
AAT(J)=TAR(I,J)*SIN(W(J)*T)+TAI(I,J)*COS(W(J)*T)
CALL SIMPSON(N,DW,AHT,AREA)
AREA=AREA/PI
AHX(L,1)=-AREA
CALL SIMPSON(N,DW,AAT,BREA)
BREA=BREA/PI
AAY(L,1)=BREA
WRITE(2,6)T,AREA,BREA
6 FORMAT(1H,2X,F6.3,3X,E16.8,10X,E16.8)
18 CONTINUE

```

C
C
C
C

```

DO 21 I=1,KK,5

```

```

WRITE(3,22)TMX
DO 21 L=1,NT
WRITE(3,22)AHX(L,1)
22 FORMAT(F20.9)
21 CONTINUE

```

C
C

```

DO 23 I=1,KK,5

```

C
C

```

WRITE(3,22)TMX
DO 23 L=1,NT

```

```

WRITE(3,22)AAY(L,1)
23 CONTINUE
RETURN
END

```

C

C

```
SUBROUTINE SIMPSON(N,DX,Y,AREA)
  DIMENSION Y(N)
  ODD=0.0
  EVEN=0.0
  DO 3 I=1,N/2,2
    ODD=ODD+Y(I)
    EVEN=EVEN+Y(I+1)
    AREA=DX/3.*(4.*EVEN+2.*ODD+Y(N)+Y(1))
  RETURN
  END
  FINISH
```

**** END OF STREAM CA504S

615

0.85366E-02	-0.11333E-02	-0.10734E-01	-0.20404E-01
-0.30004E-01	-0.39674E-01	-0.13753E-00	-0.10549E-00
-0.84953E-01	-0.65833E-01	-0.48663E-01	-0.33870E-01
-0.21554E-01	-0.11698E-01	-0.45420E-02	-0.11133E-03
0.13961E-02	0.16603E-04	-0.42573E-02	-0.18702E-00
-0.63258E-01	0.61191E-02	0.56868E-01	0.85679E-01
0.94133E-01	0.87458E-01	0.71019E-01	0.49093E-01
0.26035E-01	0.57774E-02	-0.82437E-02	-0.13560E-01
-0.18326E-00	0.14617E-01	0.85996E-01	0.92018E-01
0.48563E-01	-0.12694E-01	-0.64339E-01	-0.91754E-01
-0.90783E-01	-0.66989E-01	-0.31923E-01	-0.52947E-03
0.13373E-01	-0.12338E-01	0.43858E-02	0.68832E-02
0.20606E-02	-0.33546E-02	-0.59684E-02	-0.46661E-02
-0.58641E-03	0.32821E-02	0.58742E-02	-0.61427E-02
0.53451E-02	0.54598E-02		
-0.89264E-02	-0.89264E-02	-0.89264E-02	-0.89264E-02
-0.89264E-02	-0.89264E-02	-0.89264E-02	-0.89264E-02
-0.89264E-02	-0.89264E-02	-0.89264E-02	-0.89264E-02
0.69568E-01	0.69568E-01	0.69568E-01	0.69568E-01
0.69568E-01	0.69568E-01	0.69568E-01	0.69568E-01
0.69568E-01	0.69568E-01	0.69568E-01	0.69568E-01
0.31996E-01	0.52613E-01	0.33410E-01	0.34306E-01
0.35149E-01	0.36091E-01	0.37051E-01	0.38063E-01
0.39115E-01	0.40136E-01	0.41138E-01	-0.29554E-01
-0.26452E-01	-0.20796E-01	-0.13046E-01	-0.50843E-02
0.15662E-02	0.58013E-02	0.86012E-02	0.97433E-02
0.93871E-02	0.76436E-02	0.48619E-02	0.12906E-02
-0.63306E-01	-0.47414E-01	-0.24028E-01	-0.12830E-02
0.11349E-01	0.12161E-01	0.57590E-02	-0.42730E-02
-0.13538E-01	-0.19562E-01	-0.20443E-01	-0.15853E-01
-0.66668E-02	0.80354E-00	0.77216E-00	0.72861E-00
0.66848E-00	0.59552E-00	0.51666E-00	0.45270E-00
0.38258E-00	0.30913E-00	0.23192E-00	0.15184E-00
0.70691E-01	-0.11327E-01		

L.3.1.3 Listing of the Result Subfile RES3

KK = 1 corresponds to tip
KK = 6 corresponds to midspan
KK = 11 corresponds to root

1									
KK=	MEAN	SQUARE	LINEAR	ACCELERATION	USING	DRYDEN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.63621480E 01	R.M.S	VALUE=	0.25223301E 01		
	MEAN	SQUARE	ANGULAR	ACCELERATION	USING	DRYDEN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.51859598E 00	R.M.S	VALUE=	0.72013608E 00		
	MEAN	SQUARE	LINEAR	ACCELERATION	USING	KARMAN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.14687587E 02	R.M.S	VALUE=	0.38324388E 01		
	MEAN	SQUARE	ANGULAR	ACCELERATION	USING	KARMAN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.99717177E 00	R.M.S	VALUE=	0.99858489E 00		
6									
KK=	MEAN	SQUARE	LINEAR	ACCELERATION	USING	DRYDEN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.17289857E 01	R.M.S	VALUE=	0.13149090E 01		
	MEAN	SQUARE	ANGULAR	ACCELERATION	USING	DRYDEN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.56029136E 00	R.M.S	VALUE=	0.74852613E 00		
	MEAN	SQUARE	LINEAR	ACCELERATION	USING	KARMAN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.30572856E 01	R.M.S	VALUE=	0.17485095E 01		
	MEAN	SQUARE	ANGULAR	ACCELERATION	USING	KARMAN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.10582603E 01	R.M.S	VALUE=	0.10287178E 01		
11									
KK=	MEAN	SQUARE	LINEAR	ACCELERATION	USING	DRYDEN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.14690546E 01	R.M.S	VALUE=	0.12120456E 01		
	MEAN	SQUARE	ANGULAR	ACCELERATION	USING	DRYDEN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.71822378E 00	R.M.S	VALUE=	0.84748084E 00		
	MEAN	SQUARE	LINEAR	ACCELERATION	USING	KARMAN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.22956780E 01	R.M.S	VALUE=	0.15151495E 01		
	MEAN	SQUARE	ANGULAR	ACCELERATION	USING	KARMAN	SPECTRA		
	MEAN	SQUARE	ACCLN=	0.13856291E 01	R.M.S	VALUE=	0.11771275E 01		

L.3.2 Computation of Response due to Discrete Gusts

The computation of response of the flexible aircraft to discrete gusts is mainly effected by the use of a program recently developed by Purcell (see Ref.96). The program evaluates the response to a single and pair of smooth ramp (one-minus-cosine) gusts when the response to a unit step is used as its input data. The theory of determining the response to a unit step from the transfer function of the aero-plane using Bromwich's integral has already been explained in Appendix G. The response to a unit step gust velocity is calculated within the subroutine UNST of the program JRB5 (see section L.3.1.1) and the results are printed on cards via output Channel 3. These cards are fed as input to Purcell's program which investigates the critical cases due to a single and pair of smooth ramp gusts respectively. The left hand side of Fig.135 outlines the computation procedure in the block diagram. A typical sample output from Purcell's program is listed in the following section. It is to be noted that in the output listing the unit of length feet (FT) is to be read as meter (M).

L.3.2.1 A typical Example of Output Results from Purcell's Program

AIRCRAFT RESPONSE TO SMOOTH RAMP GUSTS

AIRCRAFT VELOCITY = 30. FT/SEC

RAMP RESPONSE CALCULATED AT 0.05 SEC INTERVALS UP TO T = 5.00 SECS

GUSTS OF LENGTH H < 1.5 FT ARE EFFECTIVELY STEPS
SUCH H(I) IN THE DATA ARE IGNORED

CUBIC SPLINE FIT TO STEP RESPONSE

T	F(T)	DF/DT	D2F/DT2	D3F/DT3
0.0000E 00	1.1515E 00	8.3413E 01	-2.3352E 03	2.9800E 04
5.0000E-02	3.0241E 00	3.9051E 00	-8.4516E 02	2.9800E 04
1.0000E-01	2.7837E 00	-1.1031E 00	6.4483E 02	-2.7189E 04
1.5000E-01	2.9682E 00	-2.8471E 00	-7.1460E 02	1.9116E 04
2.0000E-01	2.3308E 00	-1.4682E 01	2.4119E 02	-1.1107E 04
2.5000E-01	1.6668E 00	-1.6506E 01	-3.1414E 02	7.6265E 03
3.0000E-01	6.0772E-01	-2.2680E 01	6.7186E 01	-2.2590E 03
3.5000E-01	-3.9522E-01	-1.6497E 01	1.8014E 02	-4.4727E 03
4.0000E-01	-1.0881E 00	-1.3081E 01	-4.3498E 01	8.3657E 03
4.5000E-01	-1.6222E 00	-4.7985E 00	3.7479E 02	-1.0386E 04
5.0000E-01	-1.6100E 00	9.5795E-01	-1.4453E 02	8.7246E 03
5.5000E-01	-1.5610E 00	4.6372E 00	2.9170E 02	-1.0073E 04
6.0000E-01	-1.1744E 00	6.6309E 00	-2.1195E 02	8.4137E 03
6.5000E-01	-9.3250E-01	6.5504E 00	2.0873E 02	-8.1817E 03
7.0000E-01	-5.1451E-01	6.7600E 00	-2.0035E 02	6.6900E 03
7.5000E-01	-2.8758E-01	5.1050E 00	1.3415E 02	-5.0773E 03
8.0000E-01	2.9580E-02	5.4659E 00	-1.1971E 02	3.3399E 03
8.5000E-01	2.2281E-01	3.6551E 00	4.7281E 01	-2.3656E 03
9.0000E-01	4.1538E-01	3.0620E 00	-7.1002E 01	1.3902E 03
9.5000E-01	5.0869E-01	1.2497E 00	-1.4923E 00	-7.6106E 02
1.0000E 00	5.5346E-01	2.2375E-01	-3.9545E 01	1.5905E 02
1.0500E 00	5.1853E-01	-1.5547E 00	-3.1593E 01	8.6118E 02
1.1000E 00	4.1924E-01	-2.0579E 00	1.1466E 01	-9.4032E 02
1.1500E 00	3.1109E-01	-2.6599E 00	-3.5550E 01	1.3557E 03
1.2000E 00	1.6190E-01	-2.7427E 00	3.2237E 01	-9.4882E 02
1.2500E 00	4.5296E-02	-2.3169E 00	-1.5204E 01	9.8322E 02
1.3000E 00	-6.9070E-02	-1.8481E 00	3.3957E 01	-9.5486E 02
1.3500E 00	-1.3892E-01	-1.3438E 00	-1.3786E 01	9.9769E 02
1.4000E 00	-2.0256E-01	-7.8594E-01	3.6099E 01	-9.2609E 02
1.4500E 00	-2.1602E-01	-1.3861E-01	-1.0205E 01	5.7326E 02
1.5000E 00	-2.2377E-01	6.7493E-02	1.8457E 01	-3.3600E 02
1.5500E 00	-2.0431E-01	5.7056E-01	1.6572E 00	1.0128E 02
1.6000E 00	-1.7160E-01	7.4002E-01	6.7212E 00	-2.0840E 02
1.6500E 00	-1.2854E-01	8.5559E-01	-3.6986E 00	1.4688E 02
1.7000E 00	-8.7324E-02	8.5425E-01	3.6453E 00	-1.2961E 02
1.7500E 00	-4.2755E-02	8.7450E-01	-2.8353E 00	-1.1986E 02
1.8000E 00	-5.0700E-03	5.8291E-01	-8.8232E 00	2.2597E 02
1.8500E 00	1.7747E-02	4.2397E-01	2.4705E 00	-2.0992E 02
1.9000E 00	3.7661E-02	2.8510E-01	-8.0253E 00	9.8373E 01
1.9500E 00	4.3934E-02	6.8009E-03	-3.1047E 00	8.8061E 00
2.0000E 00	4.0574E-02	-1.3753E-01	-2.6664E 00	7.5394E 01

2.0500E 00	3.1935E-02	-1.7660E-01	1.1033E 00	-1.4809E 02
2.1000E 00	2.1399E-02	-3.0654E-01	-0.3010E 00	2.1454E 02
2.1500E 00	2.6651E-03	-3.5342E-01	4.4259E 00	-9.2393E 01
2.2000E 00	-1.1398E-02	-2.4762E-01	-1.9379E-01	-1.6715E 01
2.2500E 00	-2.4370E-02	-2.7820E-01	-1.0295E 00	7.6193E 01
2.3000E 00	-3.7930E-02	-2.3444E-01	2.7801E 00	2.4572E 00
2.3500E 00	-4.6175E-02	-9.2359E-02	2.9030E 00	-9.0922E 01
2.4000E 00	-4.9059E-02	-6.0862E-02	-1.6431E 00	8.3486E 01
2.4500E 00	-5.2416E-02	-3.8660E-02	2.5312E 00	4.1797E 00
2.5000E 00	-5.1098E-02	9.3124E-02	2.7402E 00	-1.0687E 02
2.5500E 00	-4.5243E-02	9.6548E-02	-2.6032E 00	8.3659E 01
2.6000E 00	-4.1927E-02	7.0962E-02	1.5798E 00	-2.6363E 00
2.6500E 00	-3.6459E-02	1.4666E-01	1.4480E 00	-7.8503E 01
2.7000E 00	-2.8952E-02	1.2092E-01	-2.4772E 00	4.3261E 01
2.7500E 00	-2.5101E-02	5.1140E-02	-3.1417E-01	4.3994E 01
2.8000E 00	-2.2020E-02	9.0424E-02	1.8855E 00	-9.7880E 01
2.8500E 00	-1.7181E-02	6.2351E-02	-3.0085E 00	5.2993E 01
2.9000E 00	-1.6720E-02	-2.1832E-02	-3.5884E-01	4.2147E 01
2.9500E 00	-1.7382E-02	1.2909E-02	1.7485E 00	-7.7915E 01
3.0000E 00	-1.6174E-02	2.9399E-03	-2.1473E 00	2.8950E 01
3.0500E 00	-1.8108E-02	-6.8236E-02	-6.9978E-01	5.9295E 01
3.1000E 00	-2.1139E-02	-2.9107E-02	2.2650E 00	-7.5410E 01
3.1500E 00	-2.1355E-02	-1.0120E-02	-1.5055E 00	1.3891E 01
3.2000E 00	-2.3453E-02	-6.8031E-02	-8.1095E-01	6.6387E 01
3.2500E 00	-2.6485E-02	-2.5595E-02	2.5084E 00	-7.0746E 01
3.3000E 00	-2.6103E-02	1.1391E-02	-1.0289E 00	4.6027E-01
3.3500E 00	-2.6810E-02	-3.9480E-02	-1.0059E 00	6.7048E 01
3.4000E 00	-2.8645E-02	-5.9648E-03	2.3465E 00	-5.8793E 01
3.4500E 00	-2.7235E-02	3.7869E-02	-5.9314E-01	-1.7352E 01
3.5000E 00	-2.6444E-02	-1.3478E-02	-1.4607E 00	7.2452E 01
3.5500E 00	-2.7435E-02	4.0506E-03	2.1619E 00	-5.0520E 01
3.6000E 00	-2.5582E-02	4.8994E-02	-3.6415E-01	-2.7747E 01
3.6500E 00	-2.4166E-02	-3.8978E-03	-1.7515E 00	7.1770E 01
3.7000E 00	-2.5055E-02	-1.7613E-03	1.8370E 00	-3.6946E 01
3.7500E 00	-2.3617E-02	4.3905E-02	-1.0331E-02	-3.9350E 01
3.8000E 00	-2.2254E-02	-5.7998E-03	-1.9779E 00	7.1242E 01
3.8500E 00	-2.3532E-02	-1.5640E-02	1.5843E 00	-2.4034E 01
3.9000E 00	-2.2837E-02	3.3407E-02	3.7758E-01	-4.8862E 01
3.9500E 00	-2.1712E-02	-8.7916E-03	-2.0655E 00	6.8156E 01
4.0000E 00	-2.3314E-02	-2.6872E-02	1.3423E 00	-1.1337E 01
4.0500E 00	-2.3216E-02	2.6070E-02	7.7540E-01	-5.6799E 01
4.1000E 00	-2.2126E-02	-6.1591E-03	-2.0646E 00	6.2384E 01
4.1500E 00	-2.3715E-02	-3.1407E-02	1.0547E 00	1.9940E 00
4.2000E 00	-2.3926E-02	2.3818E-02	1.1543E 00	-6.3943E 01
4.2500E 00	-2.2624E-02	1.6066E-03	-2.0428E 00	5.5413E 01
4.3000E 00	-2.3943E-02	-3.1267E-02	7.2784E-01	1.4733E 01
4.3500E 00	-2.4280E-02	2.3541E-02	1.4645E 00	-6.8912E 01
4.4000E 00	-2.2717E-02	1.0626E-02	-1.9811E 00	4.6521E 01
4.4500E 00	-2.3693E-02	-3.0277E-02	3.4495E-01	2.7769E 01
4.5000E 00	-2.4197E-02	2.1682E-02	1.7334E 00	-7.2283E 01
4.5500E 00	-2.2452E-02	1.7999E-02	-1.8807E 00	3.6346E 01
4.6000E 00	-2.3146E-02	-3.0604E-02	-6.3409E-02	4.0667E 01
4.6500E 00	-2.3908E-02	1.7059E-02	1.9699E 00	-7.3810E 01
4.7000E 00	-2.2131E-02	2.3293E-02	-1.7206E 00	2.4587E 01
4.7500E 00	-2.2605E-02	-3.2003E-02	-4.9124E-01	5.3911E 01
4.8000E 00	-2.3696E-02	1.0824E-02	2.2043E 00	-7.5626E 01
4.8500E 00	-2.1975E-02	2.6506E-02	-1.5770E 00	1.9143E 01
4.9000E 00	-2.2222E-02	-2.3416E-02	-6.1928E-01	3.6937E 01
4.9500E 00	-2.3648E-02	-1.3239E-02	1.2270E 00	3.6937E 01
5.0000E 00	-2.2007E-02	9.4282E-02	3.0739E 00	3.6937E 01

ESTIMATED EXTREMES OF RESPONSE TO RAMP GUSTS

INDIC	GUST LENGTH	GAMMA(+)	T(+)	GAMMA(-)	T(-)
0	5.00	4.9503E 00	1.8368E-01	-2.7352E 00	5.7954E-01
0	10.00	5.4471E 00	2.9720E-01	-3.0500E 00	6.7919E-01
0	20.00	4.1067E 00	4.3910E-01	-2.4600E 00	8.9958E-01
0	30.00	2.9088E 00	5.2794E-01	-1.3743E 00	1.1525E 00
0	8.53	5.4571E 00	2.6578E-01		
0	9.04	5.4602E 00	2.7858E-01		
0	9.13	5.4682E 00	2.8049E-01		
0	9.44	5.4696E 00	2.8714E-01		
0	9.33	5.4703E 00	2.8507E-01		
0	9.24	5.4699E 00	2.8317E-01		
0	9.39	5.4701E 00	2.8612E-01		

CONVERGENCE CRITERION SATISFIED 1 DRYHBR < 0.0099
REQUIRED ACCURACY = 0.010

IT= 7 9.33 5.4703E 00 2.8507E-01

RESPONSE TO SMOOTH RAMP GUST OF LENGTH 9.3 FT (MULTIPLY BY 10.630034 TO GET PHI(T))

0.0000E 00	2.6244E-02	1.1626E-01	2.4793E-01	3.8857E-01	4.9032E-01	5.0976E-01	4.3207E-01	3.0312E-01	1.4287E-01
-2.0267E-02	-1.5656E-01	-2.4688E-01	-2.8406E-01	-2.7695E-01	-2.3732E-01	-1.8209E-01	-1.2132E-01	-6.3253E-02	-1.0906E-02
3.2573E-02	6.6125E-02	8.7726E-02	9.6616E-02	9.3525E-02	8.0329E-02	6.0291E-02	3.7033E-02	1.4103E-02	-4.2777E-03
-2.2188E-02	-3.3150E-02	-3.9140E-02	-4.0624E-02	-3.8339E-02	-3.3082E-02	-2.6040E-02	-1.8130E-02	-1.0426E-02	-3.6417E-03
1.6974E-03	5.2238E-03	6.9279E-03	7.0739E-03	5.8566E-03	3.7111E-03	1.0903E-03	-1.7041E-03	-4.3665E-03	-6.5708E-03
-8.2429E-03	-9.3178E-03	-9.7253E-03	-9.5969E-03	-9.0662E-03	-8.1980E-03	-7.1795E-03	-6.1784E-03	-5.2262E-03	-4.4360E-03
-3.8943E-03	-3.5486E-03	-3.4069E-03	-3.4899E-03	-3.6899E-03	-3.9647E-03	-4.3201E-03	-4.6553E-03	-4.9299E-03	-5.1778E-03
-5.3381E-03	-5.3923E-03	-5.4065E-03	-5.3552E-03	-5.2294E-03	-5.1067E-03	-4.9769E-03	-4.8204E-03	-4.7069E-03	-4.6266E-03
-4.5416E-03	-4.5061E-03	-4.5106E-03	-4.5023E-03	-4.5245E-03	-4.5743E-03	-4.5973E-03	-4.6198E-03	-4.6436E-03	-4.6669E-03
-4.6646E-03	-4.6805E-03	-4.6587E-03	-4.6286E-03	-4.6255E-03	-4.5939E-03	-4.5544E-03	-4.5504E-03	-4.5272E-03	-4.4923E-03
-4.5001E-03									

RESPONSE TO SMOOTH RAMP GUST OF LENGTH 4.7 FT (MULTIPLY BY 16.874177 TO GET PHI(T))

0.0000E 00	4.9688E-02	1.8442E-01	2.7919E-01	2.8386E-01	2.5968E-01	2.0044E-01	1.1559E-01	1.8184E-02	-6.7186E-02
-1.2786E-01	-1.5640E-01	-1.5531E-01	-1.3591E-01	-1.0605E-01	-7.3458E-02	-4.2019E-02	-1.4262E-02	1.0487E-02	3.0234E-02
4.4329E-02	5.1771E-02	5.2438E-02	4.6574E-02	3.6529E-02	2.4105E-02	1.1148E-02	-3.6124E-04	-9.5939E-03	-1.6325E-02
-2.0283E-02	-2.1618E-02	-2.1054E-02	-1.8674E-02	-1.5041E-02	-1.0912E-02	-6.6828E-03	-2.6944E-03	3.9701E-04	2.5605E-03
3.8694E-03	4.1221E-03	3.6021E-03	2.6494E-03	1.2581E-03	3.1025E-04	-1.7022E-03	-2.9951E-03	-4.0591E-03	-4.6707E-03
-4.9944E-03	-5.0771E-03	-4.7870E-03	-4.3469E-03	-3.8830E-03	-3.2840E-03	-2.7293E-03	-2.3426E-03	-1.9743E-03	-1.7197E-03
-1.6794E-03	-1.6683E-03	-1.7171E-03	-1.9191E-03	-2.0915E-03	-2.2289E-03	-2.4450E-03	-2.5854E-03	-2.6334E-03	-2.7277E-03
-2.7530E-03	-2.6789E-03	-2.6612E-03	-2.6149E-03	-2.4482E-03	-2.4375E-03	-2.3988E-03	-2.2944E-03	-2.2699E-03	-2.2806E-03
-2.2240E-03	-2.2330E-03	-2.2846E-03	-2.2586E-03	-2.2753E-03	-2.3379E-03	-2.3151E-03	-2.3148E-03	-2.3674E-03	-2.3553E-03
-2.3172E-03	-2.3518E-03	-2.3152E-03	-2.2747E-03	-2.3116E-03	-2.2816E-03	-2.2347E-03	-2.2753E-03	-2.2593E-03	-2.2091E-03
-2.2588E-03									

0 4.67 4.8805E 00 1.7469E-01
0 18.67 4.3023E 00 4.2463E-01

LAMBDA (u) = 0.267 LAMBDA = 0.326

0 9.00 -3.0337E 00 6.5855E-01
 0 11.05 -3.0466E 00 7.0041E-01
 0 10.32 -3.0511E 00 6.8552E-01
 0 10.42 -3.0510E 00 6.8759E-01
 0 10.21 -3.0510E 00 6.8348E-01

CONVERGENCE CRITERION SATISFIED : DH/HBAR < 0.0090
 REQUIRED ACCURACY = 0.010

I = 5 10.32 -3.0511E 00 6.8552E-01

RESPONSE TO SMOOTH RAMP GUST OF LENGTH 10.3 FT (MULTIPLY BY 9.944554 TO GET PHI(T))

0.000E 00	2.3825E-02	1.0665E-01	2.3179E-01	3.7373E-01	4.9177E-01	5.4533E-01	5.0110E-01	3.7644E-01	2.1520E-01
3.8855E-02	-1.1808E-01	-2.3411E-01	-2.9507E-01	-3.0500E-01	-2.7496E-01	-2.2068E-01	-1.5652E-01	-9.1432E-02	-3.1942E-02
1.9401E-02	5.9623E-02	8.7859E-02	1.0214E-01	1.0335E-01	9.2708E-02	7.3509E-02	4.9333E-02	2.4168E-02	9.2127E-04
-1.8241E-02	-3.2118E-02	-4.0611E-02	-4.3931E-02	-4.2819E-02	-3.8196E-02	-3.1233E-02	-2.2896E-02	-1.4404E-02	-6.6334E-03
-2.5246E-04	4.3025E-03	6.8613E-03	7.6150E-03	6.8127E-03	4.8162E-03	2.1326E-03	8.5960E-04	-3.8192E-03	-6.4093E-03
-8.4351E-03	-9.8403E-03	-1.0541E-02	-1.0604E-02	-1.0176E-02	-9.3580E-03	-8.2993E-03	-7.1969E-03	-6.1394E-03	-5.2108E-03
-4.5227E-03	-4.0716E-03	-3.8376E-03	-3.8408E-03	-4.0173E-03	-4.2880E-03	-4.6443E-03	-5.0204E-03	-5.3434E-03	-5.6227E-03
-5.8333E-03	-5.9329E-03	-5.9616E-03	-5.9320E-03	-5.8230E-03	-5.6856E-03	-5.5475E-03	-5.3870E-03	-5.2445E-03	-5.1449E-03
-5.0544E-03	-4.9948E-03	-4.9850E-03	-4.9817E-03	-4.9925E-03	-5.0368E-03	-5.0705E-03	-5.0950E-03	-5.1363E-03	-5.1565E-03
-5.1547E-03	-5.1650E-03	-5.1568E-03	-5.1258E-03	-5.1117E-03	-5.0893E-03	-5.0431E-03	-5.0297E-03	-5.0129E-03	-4.9781E-03
-4.9702E-03									

RESPONSE TO SMOOTH RAMP GUST OF LENGTH 5.2 FT (MULTIPLY BY 15.785906 TO GET PHI(T))

0.000E 00	4.5561E-02	1.7664E-01	2.9212E-01	3.1485E-01	2.9114E-01	2.3353E-01	1.4320E-01	3.8271E-02	-5.9197E-02
-1.3084E-01	-1.6824E-01	-1.7307E-01	-1.5399E-01	-1.2299E-01	-8.4927E-02	-5.2281E-02	-2.0531E-02	7.0274E-03	3.0048E-02
4.6466E-02	5.6050E-02	5.7933E-02	5.2687E-02	4.2268E-02	2.8919E-02	1.6698E-02	1.6273E-03	-8.9595E-03	-1.6893E-02
-2.1735E-02	-2.3725E-02	-2.3387E-02	-2.1118E-02	-1.7301E-02	-1.2825E-02	-8.1493E-03	-3.6998E-03	-9.9440E-03	-2.4655E-03
4.0517E-03	4.5355E-03	4.0896E-03	3.1077E-03	1.6495E-03	-5.1134E-03	-1.6384E-03	-3.0762E-03	-4.2964E-03	-5.0626E-03
-5.4672E-03	-5.5978E-03	-5.3528E-03	-4.8873E-03	-4.3724E-03	-3.7426E-03	-3.1165E-03	-2.6529E-03	-2.2494E-03	-1.9460E-03
-1.8560E-03	-1.8463E-03	-1.8994E-03	-2.0748E-03	-2.2813E-03	-2.4412E-03	-2.6591E-03	-2.8338E-03	-2.9059E-03	-2.9945E-03
-3.0369E-03	-2.9783E-03	-2.9415E-03	-2.8979E-03	-2.7768E-03	-2.7047E-03	-2.6566E-03	-2.5582E-03	-2.5106E-03	-2.5162E-03
-2.4717E-03	-2.4645E-03	-2.5178E-03	-2.5044E-03	-2.5107E-03	-2.5694E-03	-2.5662E-03	-2.5582E-03	-2.6036E-03	-2.5901E-03
-2.5563E-03	-2.5882E-03	-2.5681E-03	-2.5275E-03	-2.5442E-03	-2.5291E-03	-2.4801E-03	-2.5032E-03	-2.5014E-03	-2.4533E-03
-2.4833E-03									

0 5.16 -2.7571E 00 5.8281E-01
 0 20.63 -2.3946E 00 9.1439E-01

LAMBDA (M) = 0.253 LAMBDA = 0.321

ESTIMATED WORST RESPONSE TO AN ISOLATED GUST: 5.4703E 00
 OCCURS WHEN H = 9.3

ESTIMATED WORST RESPONSE TO A PAIR OF GUSTS: 8.5214E 00
 WHICH OCCURS WHEN THE FIRST GUST IS OF LENGTH, H1 = 10.3

AND THE SECOND GUST IS OF LENGTH, H2 = 9.3
 THE TWO BEING SEPARATED BY A DISTANCE, HS = 1.7