

A diversity-based strategy for asteroid tour design

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Abstract

The design of multi-target tours among the celestial bodies of the Solar System is one of the key drivers for scientific return in space exploration missions. Such missions could bring valuable insights into the evolution of our Solar System and the available resources by prospecting many targets with a single spacecraft. The generation of a sequence of encountered bodies is a combinatorial optimization problem that can be addressed with heuristic search algorithms. However, solving the optimization problem does not consider the diversity of the body population and heuristic search techniques often produce tours with the same encountered bodies. In this paper, a new search strategy based on the diversity of visited bodies is described. An asteroid tour diversity score is defined to serve as a metric for a new search strategy in the combinatorial optimization problem. The standard Beam Search algorithm is compared to diversity-based optimization for the generation of asteroid tours in the Main Asteroid Belt and Near-Earth Asteroid populations. Unlike Beam Search, the proposed diversity-based strategy does not generate duplicate sequences and facilitates the encounter of more asteroids at higher eccentricities and inclinations, at the cost of higher Δv . A hybrid search strategy is shown to find better balance between asteroid tour diversity and Δv cost. These study cases illustrate the performance of diversity-based search for the preliminary mission design of future asteroid tour missions.

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1. Introduction

The Solar System is populated with many objects besides planets, referred to as small bodies. These objects include asteroids, comets and meteors. They exist with great variations in size, shape, composition and orbit, and can be found in different regions of the Solar System. Over a million bodies exist in the Main Asteroid Belt (MAB), between the orbits of Mars and Jupiter. Other populated regions include the Lagrange points L4 and L5 of Jupiter's orbit (region of Trojan objects) and the Kuiper Belt beyond the orbit of Neptune. Other asteroids have an orbit that passes close to that of Earth. They are known as Near Earth Asteroids (NEAs). The high diversity of the

small body population is of great interest to the scientific community, and many missions have already been launched to these distant objects, for example the Dawn mission to the MAB (Russell et al., 2007), the Lucy mission to Jupiter Trojans (Olkin et al., 2024) and the Hayabusa 2 mission to NEAs (Sarli and Tsuda, 2017).

Space trajectories that can visit multiple bodies with one single spacecraft would enhance the scientific return given by a single space mission. These concepts - commonly referred to as multi-body missions, multi-target missions or tours - were first proposed in 1971 by Bender and Bourke (1971) and Brooks et al. (1972) for prospective missions, discussing asteroid selection, Δv and design requirements. A similar small body tour concept was later studied by Farquhar et al. (1987) for the exploration of both comets and asteroids, inspired by the launch of the two Voyager probes. More recently, NASA's Lucy mission

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(Englander et al., 2019), launched in 2021, aims to explore asteroids in the MAB and to visit multiple bodies in the Trojan population, thus providing insights on small bodies from different regions of the Solar System. Rivkin et al. (2016) proposed a mission concept called MANTIS to explore Main Belt and Near-Earth asteroids. The aim of the mission is to use imaging and spectroscopy to investigate asteroid properties, formation and potential hazard to Earth. Stickle et al. (2024) discusses how asteroid tours can be used as reconnaissance missions among NEAs for planetary defense purposes, providing imaging data and accurate orbit determination. In general, asteroid tours would bring science data from spacecraft operations, providing valuable insights into the small body population.

The complexity of planning multi-target missions to asteroids arises because two problems are involved: a combinatorial optimization problem, relating to the selection of the candidate sequence of asteroids to be visited, and an optimal-control problem, aiming at identifying one or more optimal trajectories for each candidate sequence (Bellome, 2023). Optimality usually refers to minimal Δv consumption and/or overall Time Of Flight (TOF). For this reason, they have been the topic of multiple Global Trajectory Optimization Competitions (GTOC 2, 4, 7, 11 and 12 editions) and China Trajectory Optimization Competitions (CTOC 2, 3, 4 and 5) (Li et al. (2018)).

A two-step approach is generally followed to tackle the trajectory-design problem (Sánchez et al., 2016; Izzo et al., 2007). In the first step, the combinatorial problem is solved using a simplified transfer model, typically Lambert arcs, or Edelbaum-like approaches (Edelbaum, 1961). Then, as a second step, higher-fidelity models are employed to further refine trajectories with multiple deep-space maneuvers or even low-thrust transfers.

To address large-scale combinatorial optimization problems, Beam Search (BS) algorithm has emerged as a popular approach. In such algorithm, the computational burden of exploring different combinations of targets is managed by pruning non-promising routes via the so-called Beam Width (BW). BS has been used in multi-target tour design from the early editions of GTOC competitions to explore massive search spaces. In GTOC5, the Jet Propulsion Laboratory team used a BS algorithm to find an optimal trajectory to perform rendezvous and flybys with multiple asteroids (Petropoulos et al., 2014). A multi-objective version of BS was also employed (Izzo et al., 2016) to optimize for both mass and TOF in the context of a multi-probe exploration of the MAB. More recently, different approaches to multiple rendezvous missions in the asteroid belt have been proposed as part of the 12th edition of GTOC. These strategies were based on BS (Armellin et al., 2025), combined with genetic algorithms (Pang et al., 2025), or tabu lists (Goulet et al., 2025).

A combination of Ant Colony Optimization (ACO) and BS is also proposed in Simões et al. (2017), to find multiple

asteroid rendezvous that are less sensitive to the BW than standard BS. Deterministic and probabilistic BS methods, coupled with ACO or an Evolutionary algorithm, are compared by Li and Baoyin (2021) to find feasible rendezvous sequences, where the first asteroid is chosen via cluster analysis. ACO can also be applied to find optimal Multi-Gravity Assist (MGA) transfers (Ceriotti and Vasile, 2010a; Ceriotti and Vasile, 2010b), showing good performance for finding MGA trajectories applicable to real world scenarios. Optimal rendezvous can also be found using a Particle Swarm Optimization (PSO) algorithm (Pontani et al., 2012), where solutions are found by moving particles representing candidate solutions in the search space. Alternatively, a Tabu search can be applied to tour design (Johnson, 2014), in which the search strategy to find next targets is informed by already visited targets. This strategy, applied to the GTOC 4 Near-Earth Asteroid (NEA) flyby tour problem, has shown to improve existing results. The design of a multiple asteroid rendezvous can also make use of Genetic Algorithms (GA) (Zhang et al., 2015), where a set of candidate solutions is evolved towards better quality solutions. GA can be coupled with a deterministic branch-and-prune strategy (Vasile and De Pascale, 2006), allowing the design of MGA trajectories to asteroids or distant comets to be generated within reasonable computational time. Branch-and-prune appeared in the early 2000s to find feasible solutions in a large combinatorial search space (Vasile and De Pascale, 2006; Izzo et al., 2007). Search algorithms can make use of a Monte Carlo method for generation of target sequences to be allocated to multiple spacecraft (Ye et al., 2025). The optimization of asteroid tours has also been addressed for low-thrust trajectories, using search space pruning heuristics and GA (Alemany, 2009), combining branch-and-prune with a Differential Evolution-based algorithm (Di Carlo et al., 2018), multi-objective GA with artificial neural networks (Viavattene et al., 2022), or a more complex framework based on Hybrid Optimal Control formulations (Englander et al., 2015; Olympio, 2011). Solar-sail trajectory options are also available in Peloni et al. (2018), where a tree-based algorithm is employed to build asteroids tours.

Regarding the optimal-control aspect of such trajectory-design problems, approximate solutions are usually sought to avoid analysing in detail each tour option. Gatto and Casalino (2014) proposes a fast evaluation of low-thrust transfers based on Edelbaum's approximation that can be applied to multiple asteroid rendezvous. More recently, a technique has been presented to rapidly explore a tree of states to find feasible asteroid tours (Long et al., 2022), or for a quick estimation of asteroid-to-asteroid transfer TOF with Deep Neural Networks (DNNs) (Guo et al., 2023). Fast computation of approximated transfer parameters facilitates the estimation of quantities like phasing, initial and final masses with a reduced computational effort compared to the calculation of the true value (Hennes

et al., 2016). Based on the difference in position and velocity of the departure and arrival bodies, a phasing indicator can also be defined (Izzo et al., 2016), which approximates the Δv between different targets by considering simplified linear dynamics.

All above approaches scan the search space of the optimization problem, often very large, to find one or multiple sequences of asteroids. Heuristic algorithms can find good solutions efficiently by pruning the search space, thus reducing computational cost. However, they are likely to prune interesting solutions (Alemany, 2009), preventing the search from reaching local optima and restricting diversity of the solution set. The search for asteroid tours beyond optimal solutions is discussed in Long et al., 2024, introducing a stochastic tool for asteroid selection by determining the pool of reachable asteroids in the search space through constraints defined by the user. The tool generates a large number of viable missions with a great diversity in encountered objects but does not directly propose a diversity metric to scan the search space. Moreover, in Bellome et al. (2024), dynamic programming is applied to keep the design space manageable ensuring global optimality in tree-like search spaces.

Beyond the objective of optimizing multi-target missions for Δv and TOF (and combinations thereof), one is interested in also identifying the set of possible tours that are the most diverse possible to provide a catalog of scientifically compelling trajectories in preliminary mission design phases. Typically, arbitrary science merit score based on the object type and the number of encounters is defined, to influence target selection during optimization (Englander et al., 2017). Stuart et al. (2016) proposes the computation of a score after tour generation with ACO, thus ranking tour missions in terms of scientific return. Alternative approaches focus on meeting specific science goals of the mission, like optical navigation time, defined as an objective function (Sarli and Tsuda, 2017), or by imposing constraints on asteroid diameter to reduce the population size to relevant targets for the mission, for example in the case of NEAs for planetary defense (Walker et al., 2021). These approaches generally introduce constraints or a scoring system specific to the mission science goals, which would need to be refined for other mission scenarios. The quality of the solutions can also be assessed post-optimization. For example, Bellome et al. (2024) discusses the diversity of the results by looking at the number of unique asteroids encountered and the regions visited by the tours. Current approaches to diversity in asteroid tour missions in the literature motivate the need of a new search strategy in the optimization process to look for diverse solutions, being applicable to any multi-target missions.

This paper focuses on the combinatorial optimization step of tour missions. The aim is to propose a diversity metric, and use it to explore new search strategies to generate a diverse set of asteroid tours. The concept of diversity in combinatorial problems has been addressed in the mathe-

matical literature, motivated by the search for multiple high-quality solutions instead of a single optimal solution. Diversity can be defined by how different two solutions are, thus introducing the concept of distance between solutions (Ingmar et al., 2020). In a set of solutions, pairwise distances can be combined to assign a diversity measure to each solution. Historically, the search for solutions that maximize diversity by heuristic algorithms is known as the Maximum Diversity Problem (MDP) (Glover et al., 1998). Solutions can be selected such that the minimum distance between them is maximized (Max–Min Diversity Problem), or their sum is maximized (Max–Sum Diversity Problem). To the best of our knowledge, the application of the MDP to spacecraft tour trajectory design has not been addressed in the existing literature.

For the purposes of the present article, diversity refers to the number of unique asteroids that are retained in the final solution set. Given the large population of known asteroids, one cannot manually select a diverse set of bodies to create optimized asteroid tours with feasible trajectories. Therefore, defining and implementing diversity needs to be part of the asteroid tour generation process. This paper introduces a search strategy based on object diversity to generate a set of asteroid sequences that are solutions to the combinatorial optimization problem. The search algorithm that scans the search space relies on a mathematical definition of diversity (Casas et al., 2022; Grabowski et al., 2024) that is hybridized with BS to keep the search space manageable. The aim of this new approach to generate asteroid tours is to improve the diversity of the solution dataset by eliminating duplicate sequences and finding new bodies to visit. A diversity score is introduced in this paper to assess the performance of the diversity-based search and compare it with the standard optimization based on Δv using the BS algorithm. This article extends preliminary results (Grabowski et al., 2024), applying the search strategy to the NEA population and considering variable TOF for asteroid transfers. This approach is used to build asteroid tours within the MAB and the NEA populations, to prove its efficiency and adaptability to different mission scenarios.

The paper is organized as follows: Section 2 provides a model for the combinatorial optimization problem; Section 3 describes the approach to solve the optimization problem based on diversity measurements and introduces the strategy to explore the search space and generate a solution set; Section 4 presents numerical results from the application of two diversity-based algorithms on the MAB and NEA populations; and Section 5 discusses the performance of the diversity-based algorithms.

2. Asteroid tour problem definition

The problem of designing missions that perform successive rendezvous with multiple asteroids involves finding a trajectory to visit a sequence of bodies. Typically, this is addressed as an optimization problem, where the goal is

to find one or several trajectories that minimize one or multiple criteria, defined by an objective function. Generally, the objective function J minimizes the Δv , TOF or final mass of the spacecraft (Shirazi et al., 2018) (or even combination thereof).

Designing asteroids tours thus consists in a global optimization problem that needs to be addressed at two highly-coupled levels: finding the sequence of bodies to be encountered and finding the best trajectory(ies) with respect to the objective function(s) given that sequence. These problems are typically modeled as Mixed Integer Non-Linear Programming (MINLP) problems (Schlueter et al., 2013), formulated as follows:

$$\begin{aligned} \text{Minimize} \quad & J(x, y), \text{ where } x \in \mathbb{R}_{\text{int}}^n \text{ and } y \in \mathbb{Z}_{\text{cont}}^n, n_{\text{int}}, n_{\text{cont}} \in \mathbb{N} \\ \text{Subject to} \quad & g_i(x, y) \leq 0, i = 1, \dots, m \in \mathbb{N} \\ & x_{lb} \leq x \leq x_{ub}, \text{ where } x_{lb}, x_{ub} \in \mathbb{Z}_{\text{int}}^n \\ & y_{lb} \leq y \leq y_{ub}, \text{ where } y_{lb}, y_{ub} \in \mathbb{R}_{\text{cont}}^n \end{aligned} \quad (1)$$

where $J(x, y)$ is the objective function to optimize (note that in case of multi-objective optimization, there is an objective function for each parameter to optimize); x and y are the decision variables in the integer and continuous domain, while n_{int} and n_{cont} refer to the number of continuous and integer variables, respectively; $g_i(x, y)$ are non-linear constraints; (x_{lb}, x_{ub}) and (y_{lb}, y_{ub}) are the lower and upper bounds for x and y variables, respectively.

In this paper, the cost function J will guide the search for the best rendezvous sequences w.r.t. to Δv or a diversity metric. The integer design variables encoded in x represent the sequence of encountered asteroids, while the continuous variables encoded in y include mission events such as launch date, asteroid visiting epochs, deep-space maneuvers and so on. The constraints g_i represent mission parameters such as maximum Δv and mission duration.

For the purposes of the present paper, and similarly to other works (Bellome et al., 2023; Bellome et al., 2024; Armellin et al., 2022), the trajectory of the spacecraft from one object to another is modeled as a Lambert arc. Computing the Lambert arc provides the geometry of the transfer, the Δv and the TOF for a feasible trajectory between the two objects. The spacecraft performs successive rendezvous, matching the position and velocity with those of the visited body. A resulting Δv is sought between asteroids i and j visited at epochs t_0 and $t_0 + \Delta T$, respectively, such that:

$$\Delta v = \|\mathbf{v}_{sc}(t_0) - \mathbf{v}_i(t_0)\| + \|\mathbf{v}_j(t_0 + \Delta T) - \mathbf{v}_{sc}(t_0 + \Delta T)\| \quad (2)$$

where $\mathbf{v}_{sc}(t_0)$ and $\mathbf{v}_{sc}(t_0 + \Delta T)$ are spacecraft velocities at departing and arrival epochs, respectively, solutions of Lambert problem; $\mathbf{v}_i(t_0)$ and $\mathbf{v}_j(t_0 + \Delta T)$ are velocities of asteroids i and j at departing and arrival epochs, respectively.

Thus, the only continuous variables needed in y to describe the trajectories are the objects visiting epochs. In this way, the selection of asteroids is transcribed in a discrete combinatorial optimization problem (Bellome et al., 2023; Bellome et al., 2024), where the order in which asteroids

are visited depends on the objective function and constraints. This transcription allows us to define a search space that can be efficiently explored using heuristic search algorithms.

The search space of the problem can be represented with a tree graph, illustrated in Fig. 1. This graph structure is made of nodes connected by branches. All nodes at the same distance from the root node are at the same tree depth. In this problem, a node corresponds to an object visited at a specific epoch. Nodes are assumed to have only discrete values (Armellin et al., 2022), meaning that the visiting epochs are varied over a grid. One path in the tree, made of a succession of nodes, corresponds to a solution of the optimization problem. There exist as many paths as possible solutions. Each path corresponds to an asteroid tour, identifying the sequence of objects and the rendezvous epochs. All branches are also associated with a cost, which corresponds to the value of the objective function between the two nodes.

The dimension of the tree graph can be very large due to all possible object combinations and encounter dates. The tree does not discard previously encountered asteroids (e.g. with a Tabu list), therefore asteroids can appear in a different order in two different paths. In a discrete search space made of n elements, for a sequence of length r , the total number of sequences (where permutations of elements count as different sequences) is defined by the formula:

$$C(n, r) = \frac{n!}{(n-r)!} \quad (3)$$

Thus, efficient strategies are needed to manage the search complexity within reasonable computational time. BS is a popular heuristic algorithm for search space pruning, whose process is illustrated in Fig. 2. The algorithm sorts the sequences based on the objective value and only keeps the best ones at each level of the tree. The number of nodes kept is defined by the BW. The tree is further expanded from these nodes, while the other branches are pruned. The BS algorithm is used as a benchmark in the present paper, as it is often used as the standard approach for addressing multi-target mission design.

3. Diversity-based search strategy

This section introduces the approach to search for diverse asteroid tours. Based on the MDP, a mathematical model of diversity is given in Section 3.1. Section 3.2 provides a definition of a diversity metric for asteroid tours, and Section 3.3 explains the diversity-based tree search strategy used to generate a dataset of diverse asteroid tours.

3.1. Diversity model for asteroid tours

Diversity between two elements can be defined by the differences in their features. These differences are evaluated using a distance measure, which can be non-numerical

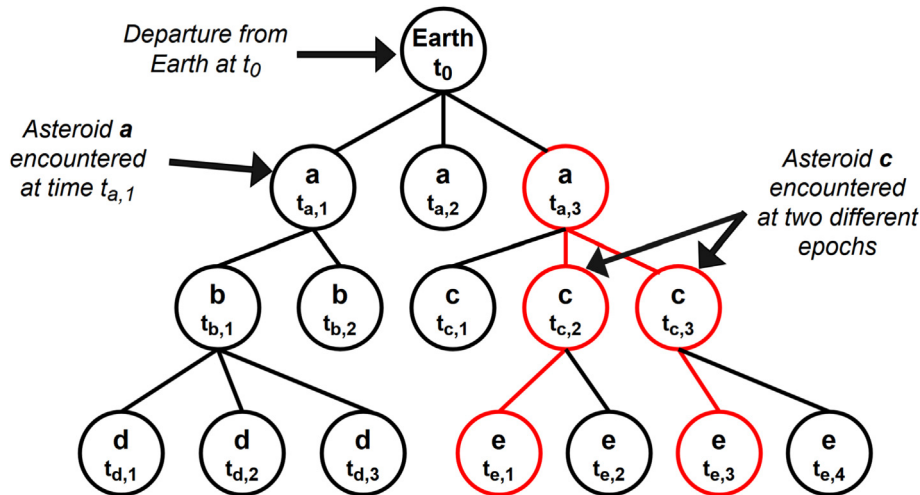


Fig. 1. Tree-graph representation of the problem. The red paths show similar sequences of asteroids but encountered at different epochs. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

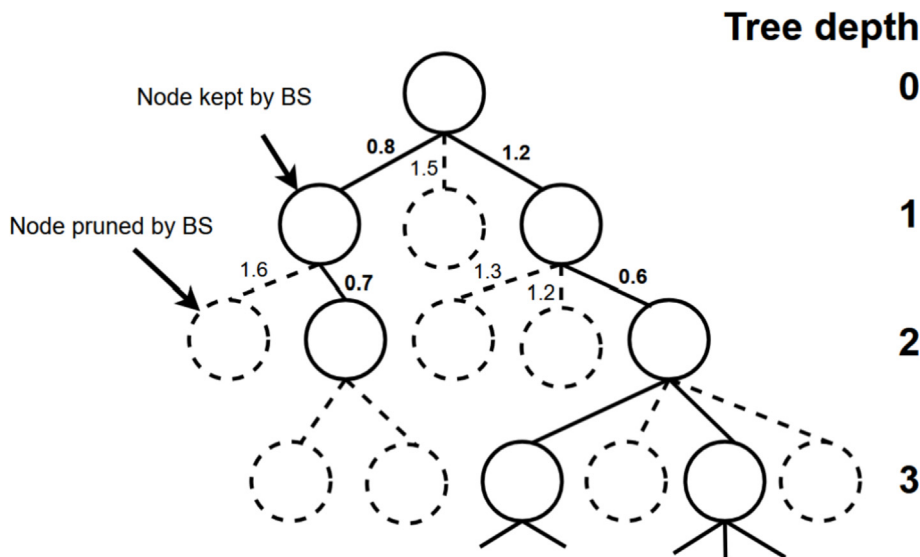


Fig. 2. Exploration of the tree by BS algorithm, with $BW = 2$. An arbitrary cost is assigned to each branch, to be minimized by BS. Pruned nodes are shown in dotted lines.

when dealing with qualitative features (Ingmar et al., 2020; Casas et al., 2022). The greater the differences in the features, the larger the distance. Conversely, if the two elements are identical, the distance is zero.

In order to find the elements that maximize the diversity in a set of size N , it is necessary to calculate all the pairwise distances in the set. Then, M elements can be selected such that the sum of their pairwise distances is maximized. This method of combining diversity distances is known as the Maximum-Sum Diversity Problem. The resolution of the combinatorial optimization problem yields many asteroid tour solutions. The MDP problem can be applied to the selection of asteroid tours that maximize a defined diversity distance.

Let N be a set of n asteroid tours. One wants to select a subset M of size m that maximizes the diversity of the selected sequences. Based on the definition of a diversity

distance $Div(A_i, A_j)$ between two asteroid tours A_i and A_j ($1 \leq (i, j) \leq n$), the MDP formulation is such that the objective function J_{MDP} :

$$J_{MDP}(x) = \sum_{j=1}^n \sum_{i=1}^j Div(A_i, A_j) k_i k_j \text{ is maximized} \quad (4)$$

$$\text{Subject to } \sum_{i=1}^n k_i = m \quad (5)$$

with $k_i = \{0, 1\}$, $i = 1, \dots, n$. k_i is equal to 1 when tour i is selected and 0 otherwise.

m is the number of sequences selected, in this case it corresponds to the BW .

The formulation of the MDP will serve as the base for the definition of a diversity distance between asteroid tours, to compare their contribution to diversity in the set of solu-

tions, as well as the definition of a diversity score assigned to each tour, that will guide asteroid selection in the combinatorial optimization process.

3.2. Diversity score definition

The definition of the diversity distance between two asteroid tours relies on the objects encountered. To adhere with the concept of diversity distance introduced in Section 3.1, this distance will be low for two sequences that are almost identical (e.g. when just one asteroid differs between the two sequences, as illustrated in Fig. 3a), and will be high when the two sequences are completely different (when the two sequences visit completely different bodies, as shown in Fig. 3b).

In this paper, a simple diversity distance is defined based on the total transfer Δv of the two sequences and the number of visited objects in common. For two sequences A and B of the same length, the diversity distance $Div(A, B)$ is formulated as follows:

$$Div(A, B) = \frac{\Delta v(A) + \Delta v(B)}{2} \frac{L - nb_{common}}{L} \quad (6)$$

where L is the length of the sequences, i.e., the number of objects it contains (distance is compared between sequences of same length); nb_{common} is the number of bodies in common between the two sequences; $\Delta v(A)$ and $\Delta v(B)$ are the accumulated cost of sequences A and B , respectively.

The first factor in Eq. 6 is the average total Δv between the two sequences. The second one corresponds to the ratio of different objects to the total number of object encounters. Sequences that differ significantly in the objects encountered will result in higher diversity distances. One can also verify that for two identical sequences, the diversity distance is zero, regardless of the encounter epochs. One can note that two sequences visiting the same asteroids in a different order also yields a diversity distance of zero, as they can be considered of equal value in terms of diversity.

The diversity distance relies on the assumption that sequences with a high Δv encompass more diverse objects. In other words, high Δv tours are more likely to improve the diversity of the solution set by enabling visits to a wider

range of orbits that contain objects with greater variations in size and physical properties.

For a given set of N asteroid sequences, all being the same length, the pairwise diversity distances can be calculated. For each sequence, a diversity score J_{div} is then defined, based on the average diversity distance to all other sequences of the set. An example of the diversity score calculation is given in Fig. 4, for a set of three sequences.

The diversity score is formulated as follows:

$$J_{div}(A_i) = \frac{\sum_{k=1, k \neq i}^N Div(A_i, A_k)}{m - 1} \quad (7)$$

This score represents how well a sequence contributes to the diversity of the set compared to all other solutions. It estimates the relative diversity of a sequence, providing a new way of finding and comparing solutions in a set of any size. It can be used directly in the optimization process, thus providing a novel way of exploring the tree for interesting solutions.

3.3. Tree exploration algorithm for diverse tour generation

The tree of possible asteroid tours is explored using Δv -based BS or diversity-based strategies. The algorithm takes as input a population of objects, here asteroids, with a known orbit. The output of the tree exploration algorithm is a set of asteroid sequences that are optimized with respect to some criterion (here Δv or diversity score).

To guide the search for heuristics, a search strategy based on object diversity is presented. The aim of such a diversity-based algorithm is to select only the nodes in the tree that improve the diversity of the solution set. For example, the selection of the node will favor objects that have not already been visited in any other sequence in the set.

The first Diversity Search (DS) strategy is based on the diversity score in Eq. 7. Like BS, the algorithm is characterized by a BW. At each step of the tree search, DS keeps only unique sequences in terms of objects visited, by eliminating duplicates with a higher Δv . It then calculates the diversity score, ranks the sequences and fills the BW with the “most diverse” solutions.

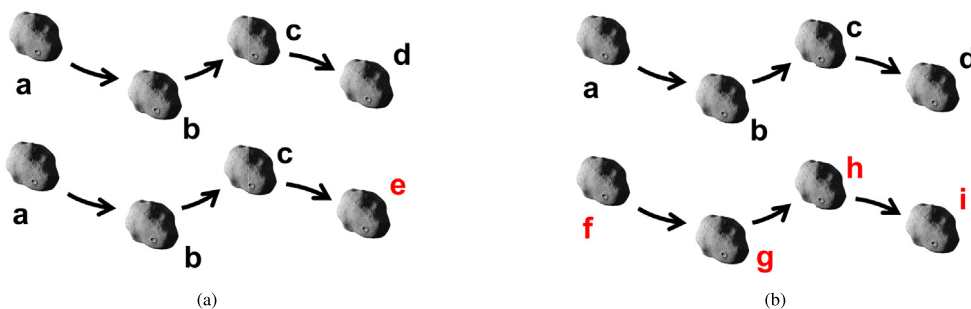


Fig. 3. Diversity score comparison between two almost identical (a) and two distinct asteroid tours (b).

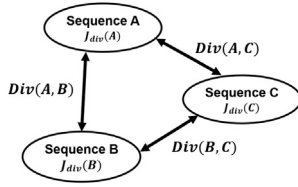


Fig. 4. Pairwise diversity distances in a set of three asteroid tours.

One can note from Eq. 6 that DS maximizes a function that includes total Δv . The BW is filled purely with sequences with the highest diversity score J_{div} , which will lead to the generation of high- Δv sequences in the solution set. This remark leads to the consideration of another approach, compared to DS in Section 4, which consists in identifying unique sequences, filling half of the BW with the most diverse sequences, and then filling the other half with the lowest Δv sequences. This new search procedure, referred to as Hybrid Search (HS), allows the algorithm to explore the tree in a different way than BS and DS, and may not find the best solutions from these two search algorithms. HS does not search for solutions that are Pareto optimal, but it fills a subset of the BW with solutions based solely on their diversity score ranking, and the rest of the BW based on Δv . This means that there might be best Δv solutions with a better diversity score, or best diversity solutions with a better Δv , than the solutions retained by HS. Therefore, it is somehow different from a multi-objective optimization. However, it is worth noting that the hybridization enables a more sounding exploration of the search space even providing lower Δv compared to a pure BS in some cases (please, refer to Section 4). The aim of HS is to keep a low average Δv for the solution set while still searching for diverse solutions.

Lambert problems are solved for the object-to-object transfers, providing information on Δv and TOF. When using large datasets of objects, computing all the Lambert arcs to the next reachable asteroids can be a heavy task. The pool of asteroids' candidates can be reduced by calculating the Improved Orbital Indicator (IOI) as in Hennes et al. (2016), Izzo et al. (2016). This provides a fast approximation of the Δv requiring less computational effort when solving optimal transfers between asteroids. It is expressed as follows:

$$d_{IOI} = |\mathbf{v}_{2,ind} - \mathbf{v}_{1,ind}| \quad (8)$$

where $\mathbf{v}_{1,ind}$ and $\mathbf{v}_{2,ind}$ are vectors related to departing and arrival objects, respectively, defined as follows:

$$\mathbf{v}_{i,ind} = \left[\frac{1}{\Delta T} \mathbf{r}_i(t_0) + \mathbf{v}_i(t_0), \frac{1}{\Delta T} \mathbf{r}_i(t_0), \frac{1}{\Delta T} \mathbf{r}_i(t_0 + \Delta T) - \mathbf{v}_i(t_0 + \Delta T), \frac{1}{\Delta T} \mathbf{r}_i(t_0 + \Delta T) \right] \quad (9)$$

where t_0 is the departure epoch, $t_0 + \Delta T$ is the arrival epoch and $\mathbf{r}_i$ and $\mathbf{v}_i$ are position and velocity vectors of object i , and $i = \{1, 2\}$.

The evaluation of the IOI for a set of asteroid candidates for the next transfer allows mission designers to

reduce the computational effort when defining a pool of reachable asteroids for which Lambert arcs will be computed.

Algorithm 1. Pseudo-code for tree exploration

```

Initialize Earth, starting epoch, pool of asteroids and
the pruning algorithm (either BS, DS or HS)
Compute first legs using Lambert arc from Earth to all
asteroids in the pool
for next transfer do
    Generate reduced pool using IOI criterion
    Compute next legs using Lambert arcs from the
    current object to all objects in the reduced pool
    Apply either BS, DS or HS to select legs for further
    expansion
end for
  
```

The exploration of the tree is characterized by pruning with either a BS, DS or HS strategy, to keep the search space of a reasonable size with sequences of interest, and expanding to search for the best next nodes, at each level of the tree. The overall pipeline is provided in the pseudo-code in Algorithm 1. In particular, all sequences depart from Earth at a given epoch, corresponding to the root node of the tree. To determine the first transfer leg, a Lambert arc is computed to all asteroids in the pool, applying constraints on Δv and TOF. Because one wants to match the velocity of the asteroid to rendezvous with, a constraint on the relative velocity is also applied. For the next legs, the pool of reachable asteroids is first reduced by calculating the IOI, and the Lambert arc is computed for each asteroid in the reduced pool. The set of sequences is updated with the corresponding transfer and Δv . At each level of the tree, when Lambert arcs have been computed, pruning is then applied using either BS, DS or HS. If BS is selected, sequences with the best Δv are kept. If DS or HS is selected, the diversity score is computed for each unique solution, which are compared one to the other and are ranked by their diversity score. The nodes that are kept for further expansion fill the BW and correspond to either best Δv branches (BS), best diversity score branches (DS) or a combination of both (HS). The process is repeated for the next leg of the tour, until the sequences reach the desired length. Such a procedure can then be replicated for different Earth departing epochs.

4. Numerical results

This section presents an application of the pipeline defined in Section 3.3 on two study cases: the exploration of the MAB and NEA populations. For both cases, the pipeline generates asteroid tours departing from Earth that rendezvous with successive asteroids. Multiple rendezvous trajectories can serve as a baseline for the preliminary design of future missions to visit asteroids. The results pre-

sented here consider a launch window of 5 years, starting Jan 1st, 2025. The duration of the launch window is chosen to increase chances to find various asteroid tours with all search algorithms while keeping reasonable computational time. Inner Belt asteroids, the closest Main Belt objects to Earth, have an orbital period around 3 years, and NEAs have periods up to 3 years, ensuring multiple opportunities for rendezvous to occur in the 5-year window. The optimization scenario for the generation of rendezvous sequences are defined with the following parameters:

- The range of departing and arrival Δv at the body.
- The launch window (t_0, t_f) and search step Δt .
- The transfer TOF between each body, fixed or defined by a range.

A comparison is given between asteroid tours generated by Δv -based BS and the DS and HS algorithms defined in Section 3.2. The results are compared on solution sets of the same size containing asteroid sequences of a fixed length. The comparison is based on the following metrics:

- Number of unique solutions generated
- Number of unique asteroids encountered
- Average total Δv of a sequence in the set
- Minimum Δv in the set
- Average diversity score
- Maximum diversity score

The comparison also examines the sets of the asteroids' populations by studying reachable orbital elements and asteroid groups of the visited populations. The following key points summarize the performance of the Diversity Search algorithm:

1. The diversity-based algorithm eliminates all duplicate sequences at each step of the search, therefore all sequences in the solution set are unique.
2. The constraint of keeping only unique solutions facilitates the discovery of more bodies to be reached, participating in the diversity of the solutions.
3. The improvement in the quality of the solutions leads to an increased average Δv of the solution set, balanced by more variation in Δv in the HS results.

4.1. Main Asteroid Belt exploration

The pipeline described in Section 3.3 is first applied to the Main Asteroid Belt. A population of 16,256 asteroids between Mars and Jupiter (semi-major axis between 2 and 4.5 AU) is considered. The input data is the Keplerian elements of all asteroids, retrieved from the GTOC7 problem data ¹.

¹ https://sophia.estec.esa.int/gtoc_portal/wp-content/uploads/2014/09/gtoc7_asteroids_data.txt, last accessed November 2025

The performance of BS, DS and HS is assessed on the scenario described in Table 1. Due to the size of the population, each search algorithm generates 500 asteroid tours that visit 10 bodies, departing from Earth. The launch window starts on Jan 1st, 2025 and extends to 5 years, looking for launch opportunities every 10 days. In this study case, the transfer TOF between each asteroid varies from 100 to 400 days.

The comparison of the results from the search algorithms is given in Table 2. BS finds only two unique asteroid tour with many duplicates, while DS and HS guarantee to fill the BW with unique solutions. Tours with BS visit 10 unique asteroids in total. This means that the two unique sequences contain the same asteroids visited in a different order, which yields a diversity score of zero for each tour. Unlike BS, DS and HS approaches find rendezvous opportunities with many more unique asteroids, which is a direct consequence of keeping only unique solutions during the search. As a result, average Δv is higher with both strategies. However, the minimum Δv tour is actually found by HS. This is due to HS exploring the tree differently than BS and DS. At every search iteration, HS selects nodes (or objects) in the tree with the highest diversity score, and nodes with the lowest Δv . The nodes selected for their high diversity score provides more opportunities to find new low Δv solutions at the next steps of the search, while still contributing to the diversity of the generated set of solutions. As a consequence, nodes that would be kept by BS are discarded by HS, because the diversity metric allows the algorithm to keep nodes leading to more optimal Δv solutions.

Fig. 5a illustrates the Δv distribution in all three sets. All BS tours are found in the 15–20 km/s Δv range, results with DS and HS are more spread. HS generates both lowest and highest Δv solutions, all sets considered, while all tours found with DS have a Δv above 40 km/s. This confirms that HS found new low Δv tours that were not found by BS. The distribution of the diversity score in Fig. 5b shows that HS tours maintain some level of diversity, with tours in the 15–20 km/s range and tours with a diversity score comparable to DS. Because the two unique solutions obtained with BS contain the same asteroids, the diversity score is zero.

Table 1
Scenario parameters of the tree exploration for the MAB population.

Earth departure	
Maximum v_∞ (km/s)	5
Maximum Δv arriving at first asteroid (km/s)	5
Start date of launch window	2025–01-01
Size of launch window (years)	5
Asteroid to asteroid transfer	
Maximum Δv leaving an asteroid (km/s)	2
Maximum Δv arriving at an asteroid (km/s)	2
Range of TOF between targets (days)	100–400
Number of asteroids in sequence	10

Table 2
Comparison of BS and DS when searching for asteroid tours in the MAB.

BW = 500	BS	DS	HS
Number of solutions	500	500	500
Number of unique solutions	2	500	500
Number of unique asteroids	10	717	603
Average Δv (km/s)	18.87	42.54	30.88
Minimum Δv (km/s)	18.72	41.28	17.23
Average diversity score (km/s)	0	31.42	24.93
Maximum diversity score (km/s)	0	38.20	33.84

For each algorithm, the most diverse trajectory from each solution set is given in Fig. 6. Each dot along the trajectory represents a rendezvous between the spacecraft and an asteroid from the MAB. From the 3D plot, it can be noted that the asteroid tours generated with DS reaches greater inclinations and heliocentric distances, enabling rendezvous with asteroids that are not visited in the BS dataset.

The BS algorithm generates only two distinct solutions in terms of encountered objects. This can be explained by the variable TOF, which favors optimal solutions that follow the same sequence of bodies but vary in encounter epochs. This search strategy limits the object diversity to only 10 asteroids visited among the 500 solutions. On the other hand, all solutions generated by DS and HS are unique. The number of unique asteroids visited increases at each search step with DS and HS, reaching 717 objects as shown in Fig. 7. This shows that keeping unique nodes during tree search results in a greater exploration of the search space from the second search iteration. This improved performance in the diversity of the solutions is confirmed by a higher diversity score (see Table 2), meaning a great variety of solutions is found.

However, the Δv of the solutions increased, as shown in Table 2, because more distant and varied targets are reached. DS does not find good Δv solutions, but the best solutions in terms of diversity score. Therefore, the improvement in the diversity of the solution set induces

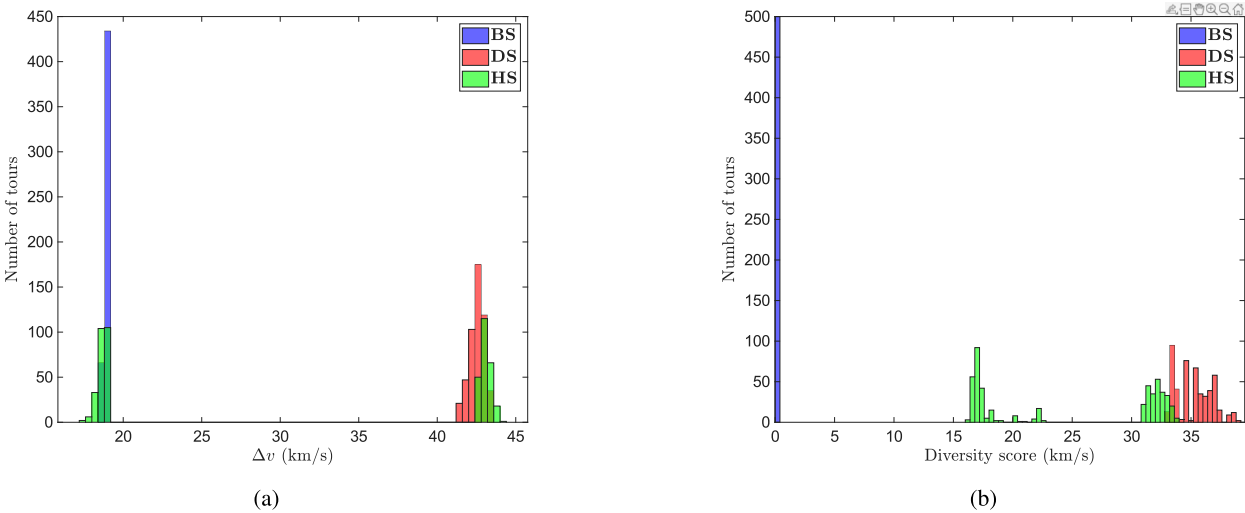


Fig. 5. Distribution of Δv (a) and diversity score (b) in generated MAB sequences.

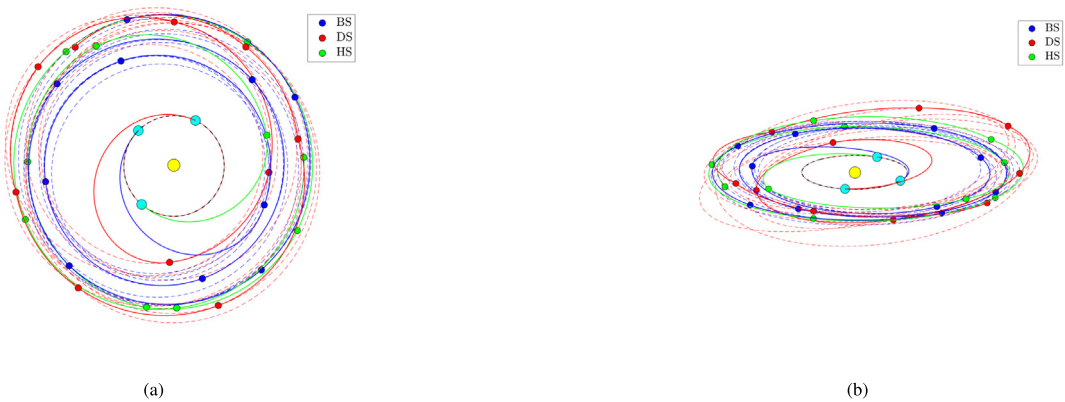


Fig. 6. Ecliptic projection (a) and 3D trajectory plot (b) of most diverse asteroid tours in the MAB for each search algorithm.

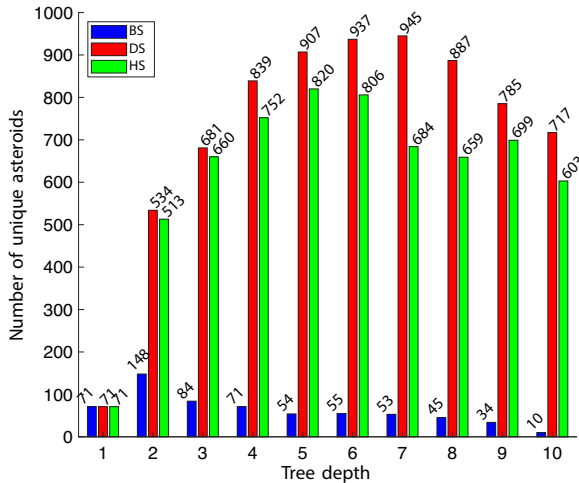


Fig. 7. Number of unique asteroids encountered in the MAB at each step of the tree exploration.

an additional cost in the average Δv for an asteroid tour. DS and HS search the tree differently with a bias towards object originality, leading to new paths in the tree graph that are usually pruned by BS.

Encountered bodies are also represented in a semi-major axis/eccentricity map in Fig. 8. Asteroids found by DS and HS are more spread in terms of semi-major axis and can have higher eccentricity. On the other hand, most bodies from the BS population have similar semi-major axis and eccentricity and are more likely to belong to the same asteroid family.

Asteroids in the MAB can be grouped into three different categories based on their semi-major axis a and eccentricity e , represented in Fig. 8. They are defined as follows:

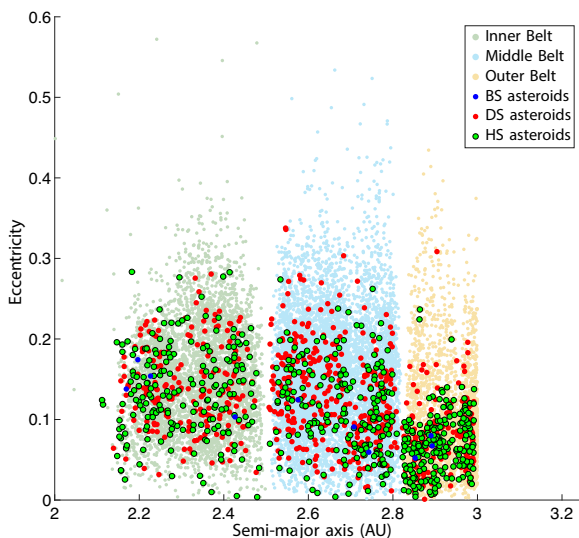


Fig. 8. Semi-major axis and eccentricity map of encountered bodies for BS (blue), DS (red) and HS (green) for MAB asteroids. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 3

Distribution of encountered objects in MAB regions.

MAB family	Inner	Middle	Outer	Total
BS	4	3	3	10
DS	214	326	177	717
HS	192	194	217	603

- Inner Belt ($a < 2.5$ AU)
- Middle Belt ($2.5 < a < 2.82$ AU)
- Outer Belt ($a > 2.82$ AU)

These groups are representative of the diversity of asteroids in the MAB and can have different sizes, shapes and compositions. Each asteroid group is represented by a different color in Fig. 8. The number of objects encountered from each group in the BS and DS solutions is summarized in Table 3.

The DS and HS algorithms generate solutions with more asteroids encountered across all groups. Besides finding more asteroids, DS and HS find solutions that are more spread throughout the entire Belt, thereby achieving encounters in the Inner, Middle, and Outer Belts in similar proportions.

DS is thus advantageous compared to standard approaches for exploring asteroid tours in the Main Asteroid Belt. Duplicate solutions are efficiently eliminated by DS, and many more asteroids are encountered in the generated asteroid tours. However, this improvement in diversity negatively impacts the Δv of the solution set, and optimal Δv solutions found by BS are not kept by DS. HS provides a better balance between Δv and diversity score, proposing a relatively high number of new asteroids encountered compared to BS while keeping some low- Δv solutions. This balance between finding optimized asteroid tours and diverse ones can be adjusted by the user by tuning the proportion of the BW filled by Δv -optimal and diversity-optimal solutions.

4.2. Near-Earth asteroids exploration

The DS algorithm is also applied to the exploration of the NEA population. The dataset is taken from the GTOC 4 problem data ² and it is made of 1,436 asteroids.

By definition, NEAs have a perihelion lower than 1.3 AU and have orbits that pass close to the Earth with some relatively high eccentric orbits. NEA trajectories are closely studied for planetary defense purposes, and it is suggested that there are still objects to be discovered in that region. Asteroids tours of NEAs are thus particularly interesting for reconnaissance or deflection missions, as part of Planetary Defense programs. These types of missions usually require top-level mission design for a large range of scenarios to characterize the object, refine its orbit or plan deflec-

² https://sophia.estec.esa.int/gtoc_portal/wp-content/uploads/2012/11/gtoc4_problem_data.txt, last accessed December 2024

Table 4
Scenario parameters of the tree exploration for the NEA population.

Earth departure	
Maximum v_{∞} (km/s)	5
Maximum Δv arriving at first asteroid (km/s)	5
Start date of launch window	2025-01-01
Launch window (years)	5
Asteroid to asteroid transfer	
Maximum Δv leaving an asteroid (km/s)	3.5
Maximum Δv arriving at an asteroid (km/s)	3.5
TOF range between targets (days)	100 to 400
Number of asteroids in sequence	10

Table 5
Comparison of BS and DS when searching for asteroid tours in the NEA population.

BW = 500	BS	DS	HS
Number of solutions	500	500	500
Number of unique solutions	30	500	500
Number of unique asteroids	43	376	339
Average Δv (km/s)	37.24	50.67	46.52
Minimum Δv (km/s)	33.20	40.35	37.48
Average diversity score (km/s)	12.36	41.82	38.10
Maximum diversity score (km/s)	15.30	47.57	45.68

tion strategies. The application of the diversity-based search strategies are relevant as it enables the generation of a high diversity of solutions for more efficient exploration of the NEA population.

The scenario used to generate sequences with BS, DS and HS is described in Table 4. The same launch window of five years is chosen, starting Jan 1st, 2025. In this case, a TOF ranging between 100 and 400 days is selected. The maximum Δv constraint for departure and arrival at aster-

oids is set to 3.5 km/s to facilitate the generation of more tours in a smaller population compared to the MAB.

BS, DS and HS are compared on the same performance parameters. The results of the comparison between search algorithms are summarized in Table 5. In this study case again, DS generates encounters with 376 unique objects, and all asteroid sequences are unique. Sequences generated by DS, which feature higher Δv values, result in a high diversity distance between each sequence. Example tours of NEAs obtained with BS, DS and HS are given in Fig. 9. These figures show that the spacecraft trajectory obtained from DS and HS expands much further from Earth, encountering more distant objects at higher eccentricities and inclinations. DS and HS tours increase the population of encountered asteroids in the dataset compared to BS results, which only proposes 30 unique tours out of 500 solutions, with most targets visited many times.

The Δv distribution, given in Fig. 10a, reveals that BS has all its asteroid tours concentrated in the 33–38 km/s, while DS and HS find tours in a wider range, most of them in the 40–45 km/s interval for HS. HS does not find lower Δv solutions than BS, yet Δv is improved compared to DS. Fig. 10b shows distribution of the tours in terms of diversity score. While all BS tours have a relatively low diversity score because there are only 30 unique tours, HS tours have a slightly lower or comparable score to DS tours, indicating that diversity in the set of solutions is preserved.

Similarly to the MAB case, DS and HS finds more unique asteroids from the first step of the search, as illustrated in Fig. 11. The number of encountered asteroids is the same between DS and HS until the 5th step of the search, showing that HS is still able to find the same high-diversity tours as DS to some extent.

The population of NEAs is classified into four groups, based on their distance from the Sun. The orbital classification is defined as follows (a being the semi-major axis, q the perihelion distance and Q the aphelion distance):

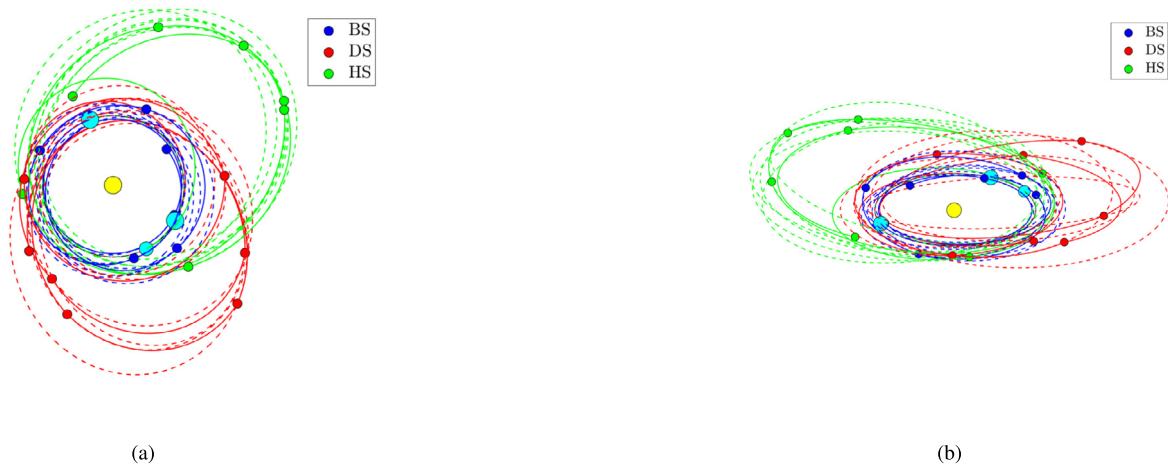


Fig. 9. Ecliptic projection (a) and 3D trajectory plot (b) of most diverse asteroid tours in the NEAs for each search algorithm.

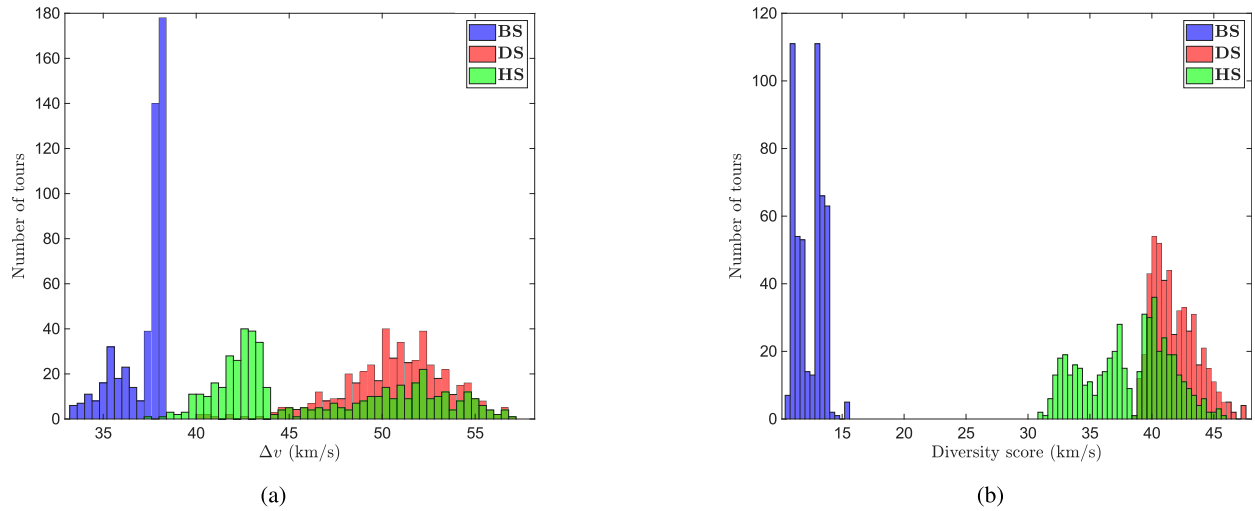


Fig. 10. Distribution of Δv (a) and diversity score (b) in generated MAB sequences.

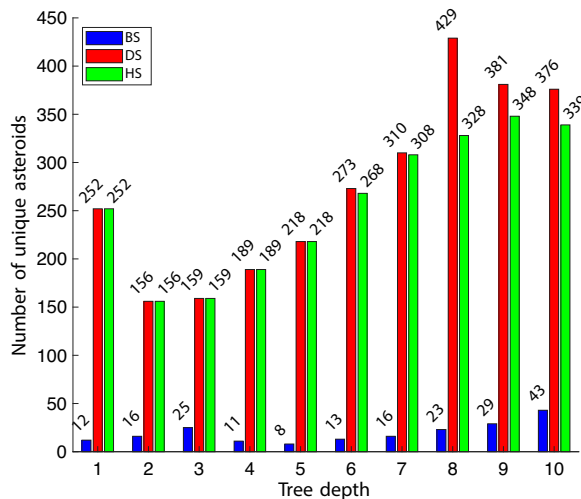


Fig. 11. Number of unique asteroids encountered in the NEA population at each step of the tree exploration.

- Amors ($a > 1.0$ AU and $1.017 < q < 1.3$ AU), with an orbit included between Earth's and Mars'.
- Apollos ($a > 1.0$ AU and $q < 1.017$ AU), crossing Earth orbit with a semi-major axis greater than Earth's.
- Atens ($a < 1.0$ AU and $Q > 0.983$ AU), crossing the Earth orbit with a semi-major axis smaller than Earth's.
- Atiras ($a < 1.0$ AU and $Q < 0.983$ AU) that are interior to Earth orbit.

Fig. 12 confirms that greater semi-major axes and eccentricities can be reached in diversity-based approaches, offering more opportunities for Amors and Apollos encounters in generated NEA tours. In particular, the number of Amors and Apollos objects visited has increased for both DS and HS, shown in Table 6 but no Atens and Atiras objects are encountered, as these are fewer and most of their orbit is within Earth's.

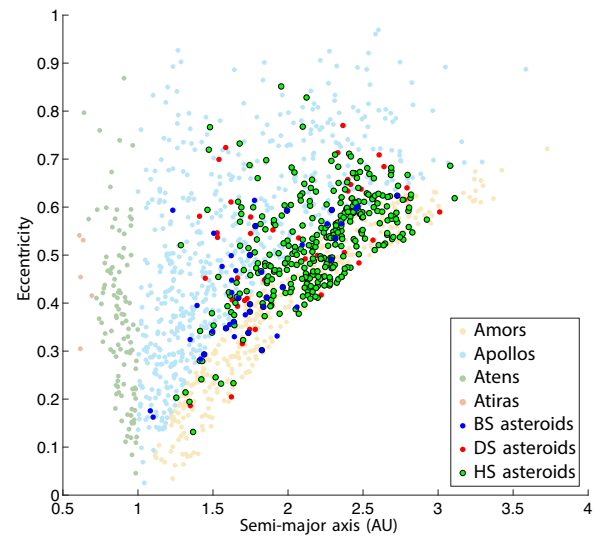


Fig. 12. Semi-major axis and eccentricity map of encountered bodies for BS (blue), DS (red) and HS (green) for NEAs. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The DS and HS algorithms also demonstrate more efficiency in the generation of asteroid tours in the NEA population, in a scenario where BS only finds a limited number of sequences. More distant objects are encountered in the DS and HS solutions, providing more opportunities to visit important NEA families such as Amors and Apollos. This improvement in the quality of the solutions is made at the cost of a higher total Δv as in the previous case. The optimal Δv solution obtained with BS is not found by DS nor HS.

5. Discussion of diversity-based search strategy

The diversity-based search algorithms demonstrated an improvement in the diversity of encountered objects.

Table 6
Distribution of encountered objects in NEA families.

NEA family	Amors	Apollos	Atens	Atiras	Total
BS	19	24	0	0	43
DS	196	180	0	0	376
HS	185	154	0	0	339

For both asteroid populations considered in this paper, the DS and HS algorithms generate unique sequences of asteroids and finds trajectories that visit more asteroids than BS results. However, the enhanced diversity of the solutions results in an increased average Δv . This is due to the definition of the diversity distance, which is based on the assumption that maximizing the Δv will increase the possibility of finding more diverse asteroid tours. Therefore, the defined diversity-based approach is conflictual with Δv -based optimization, pruning the optimum Δv solution that is found with BS. The hybrid search algorithm mitigates these results by retaining both low- Δv and high diversity solutions, with the aim of generating a richer set of feasible asteroid tours that would not be found with BS. The diversity-based search strategy presented in this paper does not guarantee to find local optima or good Δv solutions. Instead, it enables greater exploration of the search space by providing a measure of how much potential scientific value different asteroid tours could bring. With HS, this can lead to potentially new low Δv solutions as illustrated with the MAB exploration results. As a future work, a multi-objective optimization based on Δv and diversity can be explored, with the goal being to find in the solution set both Δv -optimal solution obtained with BS and diversity-optimal solution from DS. The inclusion of the total Δv in the diversity metric facilitates the discovery of new asteroid tours pruned by Δv -based BS. The high Δv of asteroid tours obtained with a diversity-based search could be contained by using a Δv bound, at the cost of limiting diversity in the solutions. This would define a maximum admissible value of the diversity score for a sequence, and limit the search for new sequences. Further work on using a bound could show how it can be used to generate asteroid tours with reasonable Δv and satisfactory diversity.

The limitations of the diversity score lie in its definition. Unlike Δv , which is independent of the solution set, the diversity score of a sequence is measured against all other solutions. Therefore, the diversity scores of the same sequence in two different set of solutions cannot be compared. In addition, the computational effort does not scale well for larger populations, as the increase in the number of operations (calculation of all pairwise diversity distances) is defined by $n!$, as given in Eq. 3. However, the overall performance of the code could be improved using parallelization.

The current dynamical model considers impulsive rendezvous maneuvers with asteroids where trajectories are computed with Lambert arcs, so further refinement of sequences should extend the model to include finite multiple deep-space maneuvers or even low-thrust trajectories. In any case, the DS and HS algorithms are versatile and can be applied to various multi-target missions, including on-orbit servicing and multiple flyby missions, also including gravity assist with Solar System planets as in [Bellome et al. \(2024\)](#), as well as adaptable to any model, from multiple deep-space maneuvers to low-thrust trajectories.

6. Conclusion

This paper addressed the combinatorial optimization problem of selecting destinations for multi-body space missions, with a particular focus on enhancing the diversity of encountered bodies. It introduces a search strategy that uses a diversity score metric to improve search space exploration and keep diverse solutions, enhancing mission variety and scientific value compared to traditional methods. Two diversity-based search algorithm, DS and HS, are compared to a standard Δv -based BS. Both generate asteroid tours that encounters a wider range of objects across the MAB and NEA regions. While this results in a higher total Δv in DS results, HS balances diversity score and Δv to find possibly feasible solutions that may have not been found by BS, although such solutions are not guaranteed to belong to the Pareto front, as in a true multi-objective setting. The development of a dedicated multi-objective optimization framework is therefore deferred to future work.

The performance of this diversity-based search algorithm is tested in exploring the Main Asteroid Belt and Near-Earth Asteroid populations. By avoiding the generation of duplicate sequences, the results demonstrate that the proposed approach increases the number of unique asteroids encountered. Additionally, it shows a marked improvement in identifying asteroids with higher eccentricities and inclinations, which are often of greater scientific interest, and provides more opportunities for tours in different asteroid families.

Overall, the diversity-based search strategy developed in this paper provides a robust tool for the preliminary mission design of future asteroid tour missions. It can be applied on a variety of scenarios with asteroid datasets of variable size. The diversity-based algorithms ensure a richer set of targets in generated rendezvous tours, thereby enhancing the potential scientific returns and resource prospects of these missions.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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