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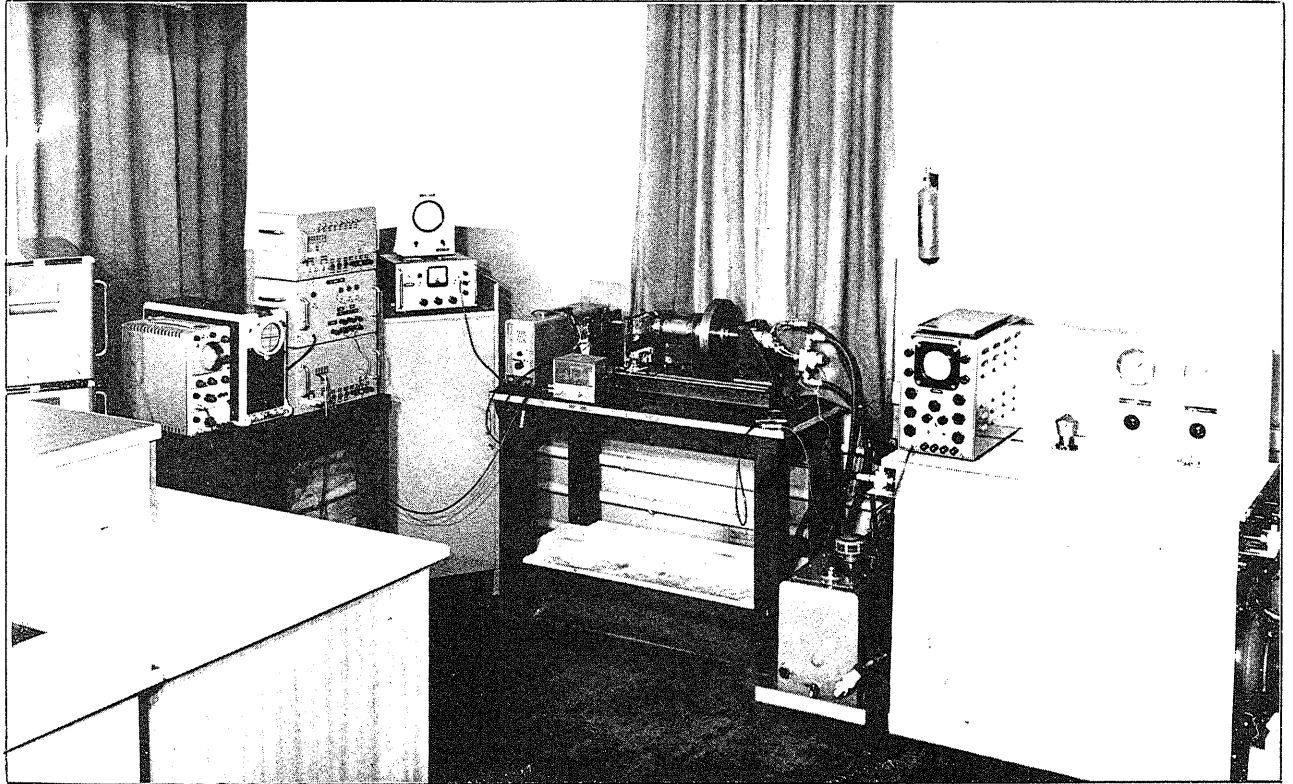
THE COLLEGE OF AERONAUTICS

DEPARTMENT OF PRODUCTION AND INDUSTRIAL ADMINISTRATION

An investigation into high precision control
of the relative angular position of two
shafts over a range of speeds and ratios

- by -

M. Healey



GENERAL VIEW OF LABORATORY TEST RIG

FRONTISPIECE

He who never changed his mind never corrected his mistakes.

Anon.

SUMMARY

The paper sets out to describe how a servo-mechanism may be applied to generative gear grinding machines and an appropriate specification is drawn up.

The system involves separate motor drives to the two shafts, the angular positions of which are monitored with radial diffraction gratings. One of the signals is frequency divided to achieve the speed ratio between the two shafts, the resultant similar frequency signals being phase compared, any difference producing an error signal thus completing the servo loop. Hydraulic motors are used as prime movers. The transfer functions of the loop are developed and compared with measured open loop results. It is clearly shown that with the components at present available the loop cannot be made stable. The specification of more appropriate hardware is discussed and future work outlined, but the conclusion is reached that this system cannot be used for its primary purpose on a generative gear grinding machine.

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A SERVO CONTROLLED GENERATIVE GEAR GRINDING MACHINE

1. Factors Affecting the Standards of Gear Boxes

The requirements of the highly competitive motor car and aircraft industries have led to constant demands for better gear boxes as defined by strength and size, wear, noise and accuracy of motion. The improved design of gear box housings, shafts and bearings and in some cases new tooth profiles, have all contributed to better gear boxes. This thesis, however, will be based on the problems associated with the manufacture of the actual gears.

The factors to be considered are wear, noise, accuracy of motion and fatigue strength.

Surface hardening, correct profiling and good surface finishes have improved the wear and noise factors.

Reduction of pitch errors and correct profiling have improved the accuracy of motion.

Fatigue strength presents more problems. It is dependant upon a) the tensile stresses in the tooth surface, particularly at the root, due to machining, hardening and grinding, and b) excessive loading of specific teeth due to profile and pitch errors.

In general precision gears are rough cut, hardened and ground. Similar gears can then be compared by pitch and profile errors and by the fatigue strength of individual teeth.

Finally different production methods should be compared for quality of the finished work piece and also it's cost. The cost should include depreciation, running, insurance etc., costs of the machines and, indirectly, the cycle, setting and handling times.

The topic of gear grinding and gear grinding machines is dealt with further in Appendix A, from which we can extract the following information about the two available types of machine, i.e. the generative and the form grinding machines. The generative grinder uses a basic rack and generates the involute profile, while the form grinder has a pre-shaped grinding wheel. Surface heating during cutting on the form grinder is higher than on the generative grinder, so that the fatigue strength of the generated gear is likely to be higher. For the continuous generative grinding machine, as described in section 2, the pitch error is of the order of 0.0005" on a 10" diameter gear compared to 0.0001" for a form ground gear. In addition any profile can be incorporated with negligible errors on a form grinder whereas pitch errors produced in the generative grinder will result also in profile errors.

For these reasons the form grinder, typified by the 'ORCUTT' machine is in common use. However, the time taken to grind a gear is much shorter using a generative grinder.

Thus a generative gear grinding machine with the pitch accuracy of a form grinding machine would be desirable. Other factors affecting the ultimate accuracy of a gear such as centring will be mainly independent of the method of machining.

2. Generative Gear Grinding Machines

In the system under consideration the grinding wheel, which is a single start spiral of a basic rack form, is rotated at a constant speed suitable for grinding conditions. In all other respects the system is the same as used on gear hobbing machines (Fig. 1.1).

The workpiece is attached to a shaft which is at right angles to the grinding wheel shaft and rotated at exactly $\frac{1}{N}$ times the speed of the grinding wheel in order to cut a gear having N teeth. Existing machines typified by the Coventry Gauge and Tool Company's Matrix 40 and 61 machines achieve the speed ratio by using a gear train, a spare set of gears providing a range for N. This drive may be further complicated by cutting helical gears, since the angle between the two drives will now be (90° - helix angle) thus involving differential gearing.

The limit of accuracy is set by back lash and wind-up of the drive train which unavoidably has a long developed length. This is further accentuated by the desirability, for profile reasons, of only using the machine to finish out rough cut gears, as the back lash and wind-up tend to follow the errors in the rough cut gears. The ultimate accuracy of the production machines is achieved by taking the finishing cut with the workpiece 'floating' i.e. with the drive disconnected, the workpiece being in mesh with the grinding wheel.

Gears are manufactured with cumulative pitch errors of about .0005" on a 10" diameter gear.

3. Proposed System for a Servo Controlled Generative Gear Grinding Machine

3.1 Degree of Accuracy

It is aimed to grind gears of up to 10" diameter within a tolerance of $\pm .0001$ " on the pitch, i.e. ± 4 seconds of arc.

3.2 Review of Previous Work

A years work has previously been done at the College of Aeronautics under the guidance of Professor J. Loxham by Mr. W.B. Harley and Mr. Arthur. Mr. Arthur looked into the problems of producing a reference signal from

diffraction gratings and Mr. Harley, with assistance from his parent firm of Coventry Gauge and Tool Ltd., investigated the best methods of developing a servo-controlled machine. This has resulted in the assembly of a suitable experimental rig including a hydraulic power supply and a centred grating. The phase discriminator and frequency divider were designed and made by Elliot Bros. Ltd. It is partially as a result of this work that the following specification has been written.

N.B. Error correction of a gear hobbing machine using diffraction gratings and frequency dividers has been carried out by the N.E.L. on a David Brown Gear Hobbing machine. (Fig. 1.1).

3.3 Specification

The speed of the grinding wheel is governed by cutting speed and therefore will depend upon the diameter of the wheel and type of grit etc. For the initial design it is proposed to assume a 12" dia. grinding wheel running, therefore, at about 1800 to 2500 r.p.m.

It is aimed to maintain the range of present existing machines so that gears with 10 to 250 teeth can be cut. Thus the workpiece must rotate over a range of speeds from 8 to 250 r.p.m.

The drives to grinding wheel and workpiece will be by separate motors synchronised by an electronic servo. Hydraulic motors controlled by electro-hydraulic spool valves have been selected as the most suitable drive. While it is undesirable, it may be necessary to incorporate a reduction gear between the motor and workpiece because of the low speeds.

The reference signals for the servo-mechanism will be derived from radial diffraction gratings attached to the rotating shafts. A fixed reference grating with similar radial lines is placed close to the moving

grating and a lamp shines colimated light through both gratings onto a photo cell. The relative movement of the ruled lines causes a shuttering effect, thus giving an a.c. output from the photo-cell. Movement of one line pitch will cause the photo cell to give an output which is roughly one cycle of a sine wave (see Appendix B).

A grating with 1000 lines rotating at 2000 r.p.m. will give an output of frequency 33 Kc/s. The spacing of adjacent lines is 21.6 minutes of angle. With the required accuracy of ± 4 seconds of angle this required a resolution of 1.1° of an electrical cycle.

Similarly with a grating with 10,800 lines rotating at 2000 r.p.m. the output frequency will be 360 kc/s, the spacing of adjacent lines 2 minutes of angle and the required resolution 12° of an electrical cycle.

The first system has the advantages of working at a lower frequency but requires higher resolution. The second requires a lower resolution but by working at the higher frequency introduces more complicated optical and electronic components.

To obtain the required ratio between the speeds of the two shafts, which is equal to the number of teeth to be cut, N , the output signal from the reference grating on the grinding wheel spindle will be frequency divided N times and then compared to the output from the workpiece reference. A phase comparator will be used which produces a d.c. voltage proportional to the difference in phase of the two signals. This error signal will control the speed of the workpiece motor (see Fig. 1.2).

To obtain the ultimate accuracy of the cut gear, it is standard practice to machine the gear nearly to size, remove it from the machine and measure it, and replace it for a final cut. This entails re-synchronising the

workpiece in a particular phase - it may synchronise onto any one of the lines on the radial grating. A possible answer to this problem is suggested in Appendix E.

Finally to make the machine universal it must be able to cut helical gears. Since the workpiece and grinding wheel shafts are driven separately the helix angle can be set up as desired. However, as the grinding wheel is traversed along the axis of the blank, relative change in the shaft positions dependant upon the helix angle must ensue. Since the 'twist' and traverse must be synchronous it seems likely that the best method is to move the position of the reference grating, probably using a sine-bar principle worked from the traverse.

4. Proposed Investigations

4.1 In the system described the closed loop must provide accurate control over a wide speed range (8 - 250 r.p.m.) and with a wide range of gear blank sizes involving different inertias inside the control loop.

A labrig has been manufactured comprising a shaft driven by an hydraulic motor, carrying the ruled glass disc and a tacho-generator (Fig. 1.3). The inertia of the shaft can be varied by two detachable steel discs. The motor is controlled by an electro-hydraulic spool valve, the hydraulic power being derived from an induction motor driven gear pump in series with a pressure regulator giving 0 to 2000 psi.

The ruled glass disc at present employed has 1000 ruled lines and two reading heads fixed diametrically opposite can be employed.

4.2 The labrig is to be synchronised to an oscillator to run at about 100 r.p.m., thus representing the work spindle carrying a typical gear blank, the oscillator representing the frequency divided output from the grinding

spindle (Fig. 1.4). The following tests must be performed.

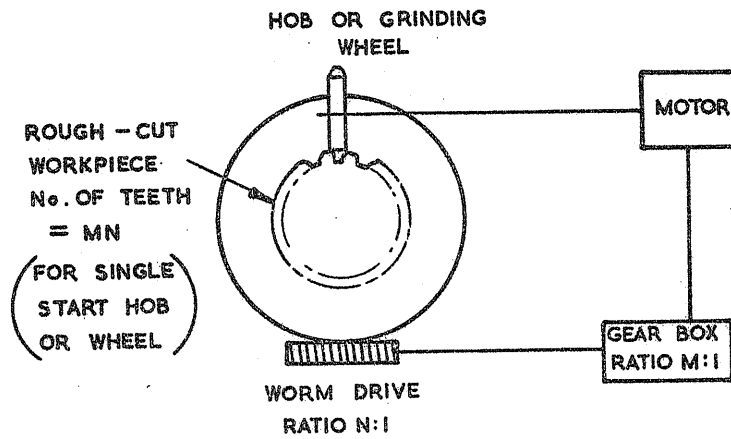
- a) Production of suitable reference signals.
- b) Measurement of transfer functions of the mechanical parts including the spool valve.
- c) Design of a suitable amplifier and stability network to obtain closed loop control of the system.
- d) Measurement of the resolution obtained from this system.

4.3 Repeat 4.2 including the frequency divider between the oscillator and the input to the servo. The oscillator frequency is to be 40 Kc/s constant. A range of speed and inertia of the labrig shaft must be tried up to 200 and possibly 250 r.p.m., and as low as possible. The low speed range is primarily limited by the motor and will warrant special investigation, including the possibility of a gear box, which will however be inside the servo loop.

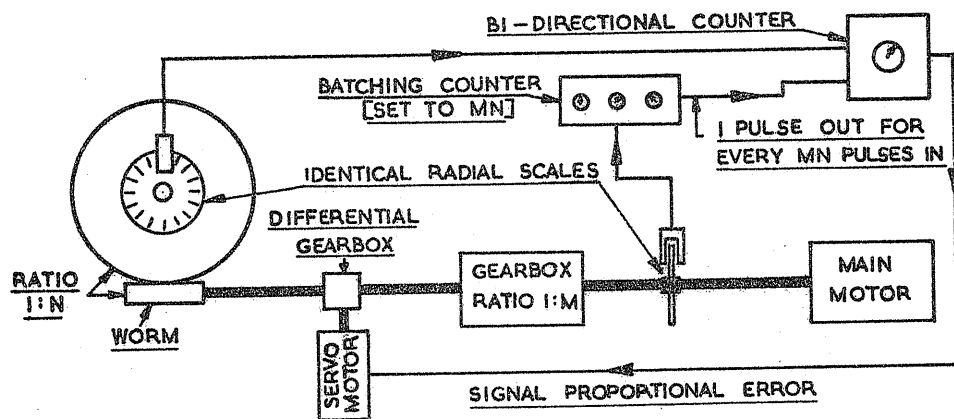
4.4 Some means of locking the loop must be employed. Possibly this will be done by means of a secondary loop providing coarser speed control during the locking-in period but this point will require special investigation.

4.5 The second system utilising 10,800 line discs must be investigated and a critical appraisal of the type of disc to be used must be made. This will involve repeating the foregoing tests with special reference to 4.2(a). Due to the higher frequency of up to 400 Kc/s some trouble in the optical system and in the selection of the photocell is anticipated.

4.6 A final test must be the actual cutting of gears. It is therefore possible that an existing machine may be modified to incorporate the new system.

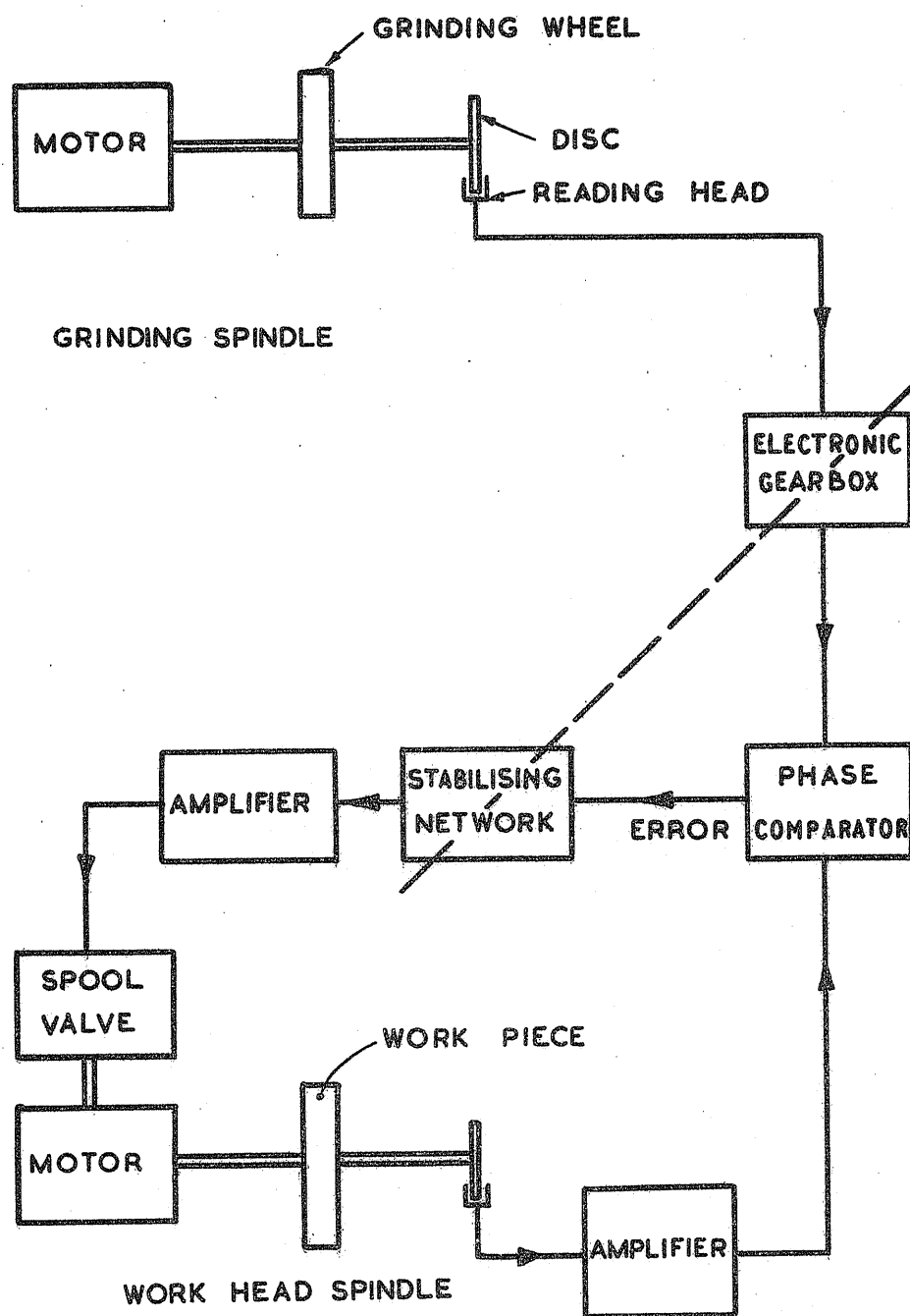


PRINCIPLE OF GEAR HOBGING OR GRINDING MACHINE



ERROR-CORRECTION OF A GEAR - HOBGING MACHINE

FIG. 1.1



PROPOSED LAYOUT OF SERVO CONTROLLED GENERATIVE GRINDING MACHINE.

FIG. 1·2

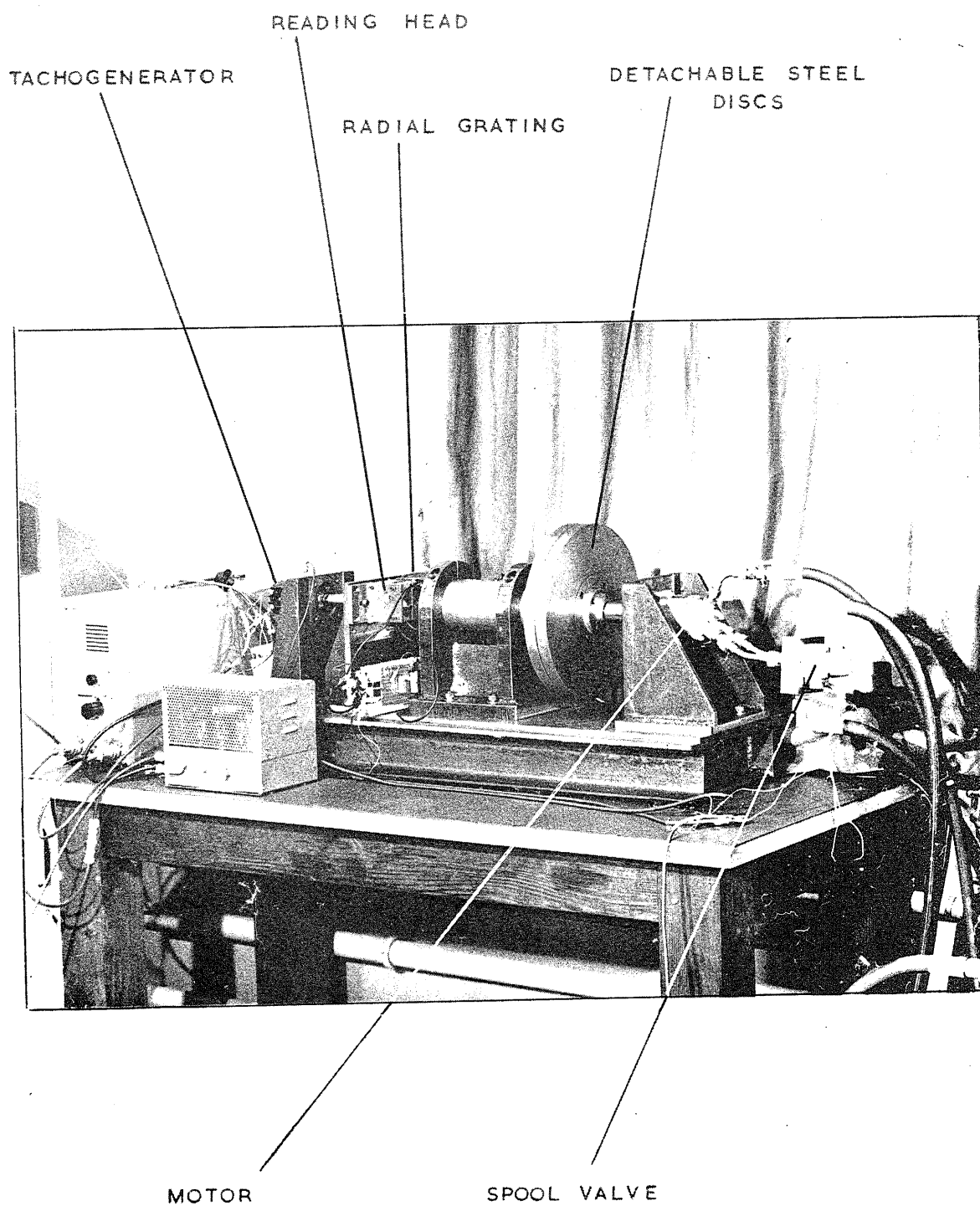
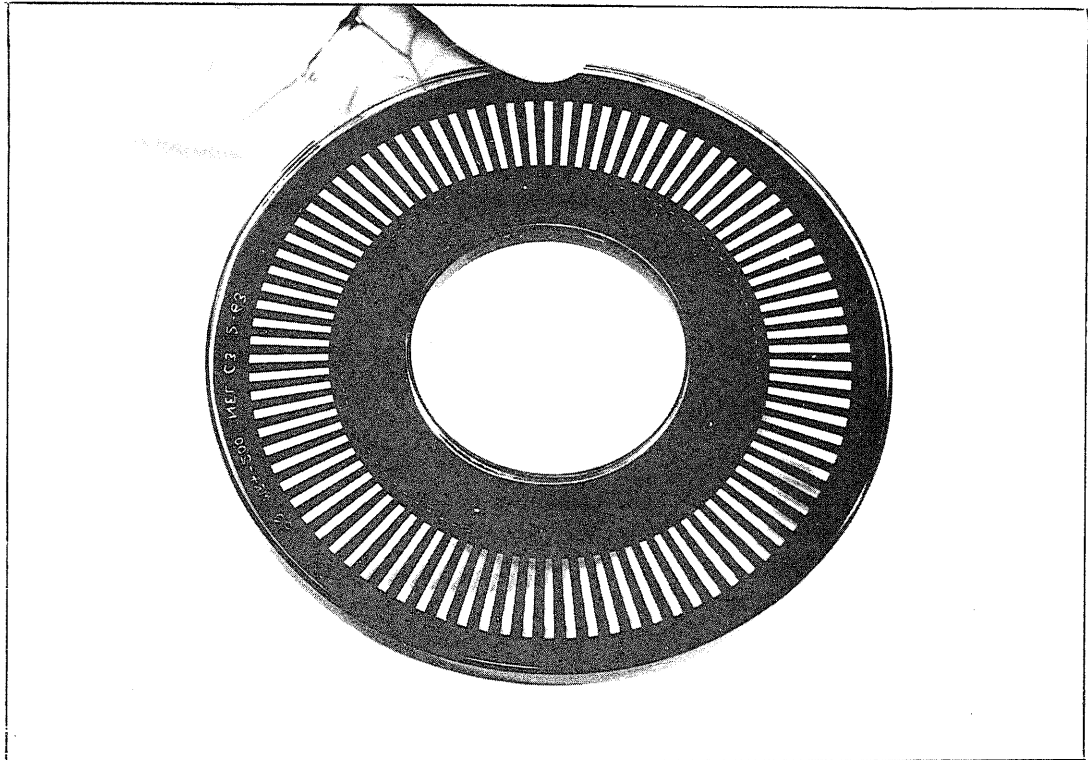


FIG. 1-3 (a)



ABOVE: A 90 LINE RADIAL DIFFRACTION GRATING

BELOW: A 1000 LINE RADIAL DIFFRACTION GRATING

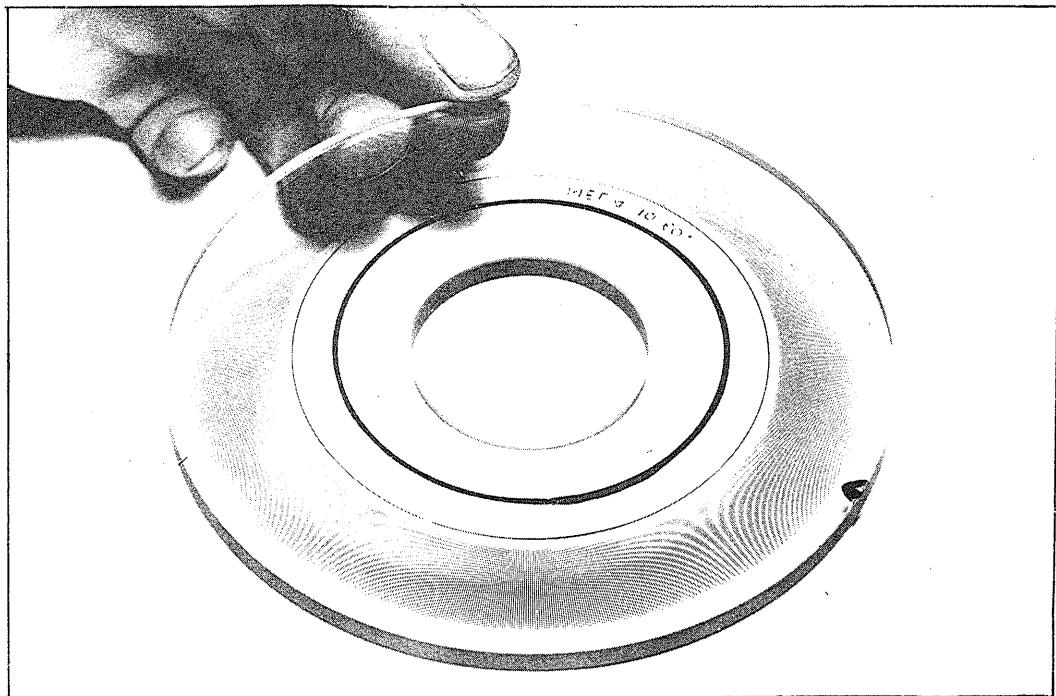
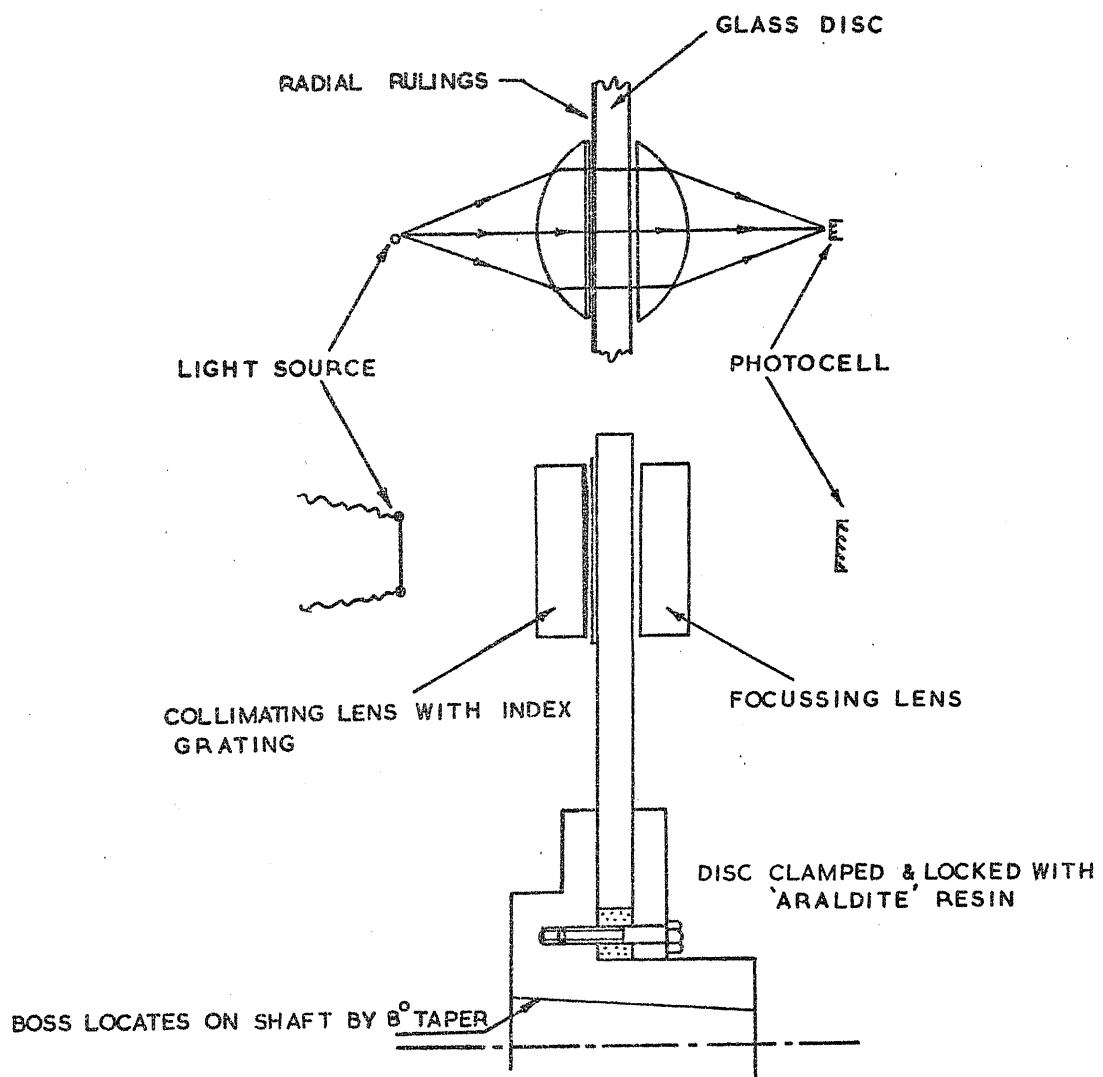
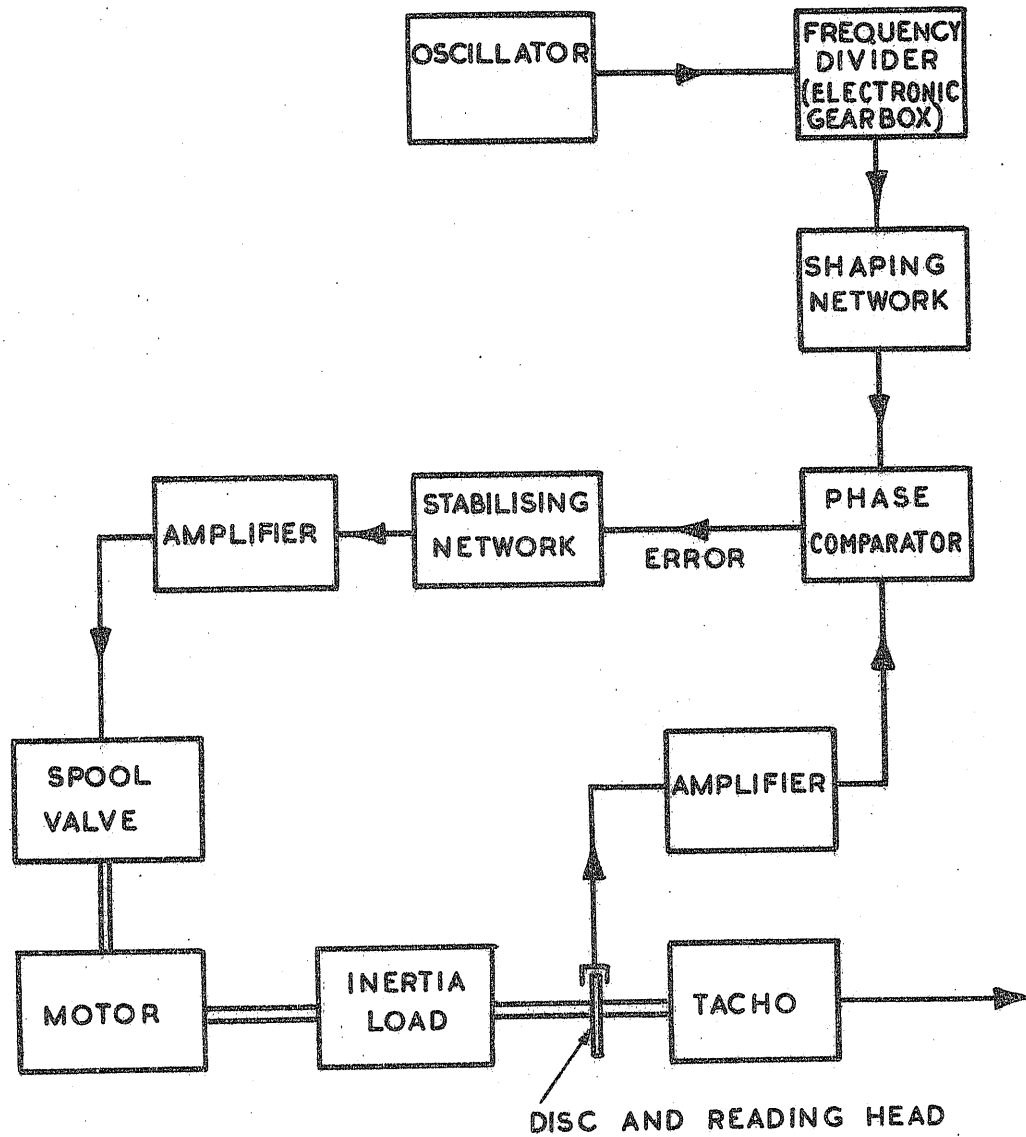


FIG. 1-3 (b)



ARRANGEMENT OF OPTICAL SYSTEM

FIG. 1-3 (c)



LABRIG LAYOUT

FIG. 1-4

THEORETICAL STUDY OF THE SYSTEM

1. Open loop system

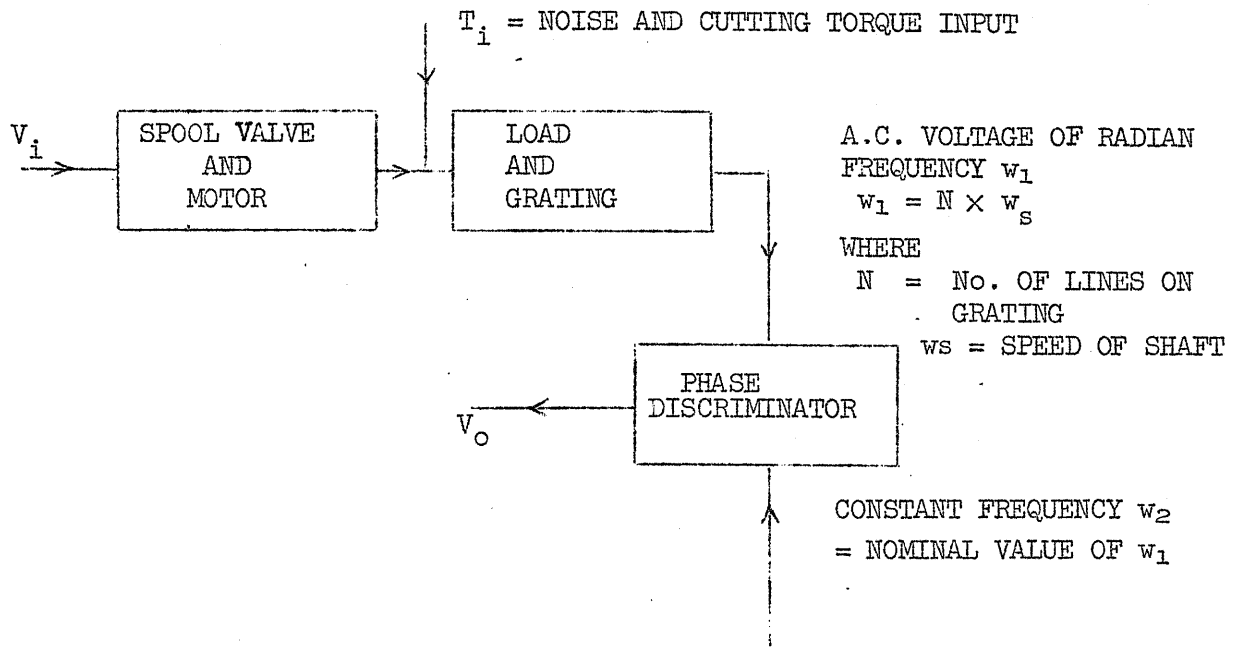


FIG. 2.1

NOTES:- V_i = d.c. value = normal steady state value to make $w_1 \doteq w_2$

V_o = output from phase discriminator which is design controlled variable.

2. Input Variables

- i) V_i = reference voltage. Initial design will assume constant V_i so that sensitivity function to V_i can be ignored - must be checked later.
- ii) T_i - Sensitivity function of V_o to T_i i.e. $S_{T_i}^{V_o}(s)$ is the main design limit - Closed loop system $\therefore$ will have T_i as input variable.

- iii) w_2 - Sensitivity function of w_2 is of high importance and should be allowed for in design, i.e. $S_{w_2}^V(s)$
- iv) Plant - in particular the motor. Motor time constants tend to depend on rotating friction which may vary.

However as a high loop gain is essential the following design will assume that the sensitivity function $S_{Plant}^V(s)$ will be adequate without special compensation.

Thus a minimum of two design degrees of freedom would be desirable to allow adjustment of $S_{T_i}^V(s)$ and $S_{w_2}^V(s)$

NOTE:- in iv) above the following assumptions have been made

- a) that inertia of load is fixed
- and b) that speed of motor is fixed.

Both these are not practically applicable but will be considered as part of a later design.

3) Desired Operation of System.

- 3.1. If w_1 and w_2 outputs are externally controlled and brought approximately in phase before closing the loop, then the system can be designed to lock allowing V_o as maximum transient swing not exceeding the equivalent of $\pm 135^\circ$ of phase shift, over which range the phase discriminator can be considered linear.
- 3.2. In a practical system the above limitations would be undesirable and a new design allowing for the non-linearity of the phase discrimination when $w_1 \neq w_2$ must be considered.

4. Desired Performance of System

4.1 From the previous specification the maximum allowable phase difference is $1.1 \times 10^{-3}N$ degrees electrical where N = number of lines on grating.

Changes in both load Torque, T_i , and reference frequency, w_2 , will contribute to this error in the practical system. However, if the block diagrams of Figs. 2.2, 2.3 and 2.4, p.23 are studied it is clear that only one design degree of freedom is available. To design for both $S_{T_i}^{V_o}$ and $S_{w_2}^{V_o}$ would require a compensation network to be added between the input points of T_i and w_2 , which is impossible. Note that three design variables, K_1 , K_2 and H_2 are available but all contribute to the same design degree of freedom.

The design will therefore consider $S_{T_i}^{V_o}$ only, the value for $S_{w_2}^{V_o}$ thus resulting being used to set the limit to variations in w_2 .

The maximum anticipated input torque is 32 lb-ins. (see 7, p.29) and will most likely be equivalent to a ramp function of say 30 lb-in/sec. slope. The actual form and magnitude of the load torque could be the basis for a separate study. Thus the steady state error = K_4 (see part 3, 2.4, p.34)

$$= \frac{4}{32} = .125 \text{ secs. of arc. mech./lb-in.}$$

4.2 Transient and Frequency Responses

This system is a regulator, the output, V_o , being required to remain constant when the inputs, primarily T_i and w_2 , vary. Since some error is required to actuate the servo, the steady state gain and errors can be calculated. Transient errors should desirably be kept less than the steady state error so that the frequency response of the closed loop,

$\frac{V_o}{T_i}$ as a function of jw , should be as low as possible with no peaks.

Thus a step-function change in load torque will produce an over damped response in the position error.

However, the frequency response for $V_o/\Delta w_s$ should also be low, which is not compatible with the similar requirement for Torque input. Briefly to counteract changes in load torque the system should have a high inertia and to be able to follow change in reference speed the system should have a low inertia, 'inertia' here being the effective closed loop inertia and not simply the load inertia. As previously mentioned, the design will focus around the torque input, the resulting design setting limits to the change in w_2 .

5. Transfer Functions of the System

5.1 Summary of Symbols

$G_m(s)$ = Transfer function of motor, spool valve and load in
rads/sec/volt.

$G_p(s)$ = T.F. of phase discriminator in volts/rad/sec.

$G_T(s)$ = T.F. relating load torque in rad/sec/lb-in.

$K_1(s)$ = T.F. of tachometer in volts/rad/sec.

K_2 and K_3 = system gains (frequency independant)

$K_2.H_2(s)$ = T.F. of compensating network

Note:- $H_2(jw) = 1$ when $w = 0$.

In parts 2 and 3 $H_2(s)$ has been considered as 1. e.g. Fig. 2.4, p. 24

K_4 = maximum allowable error (secs. of arc mech.)/lb-in. load torque.

5.2 The Phase Discriminator

$$G_p(s) = \frac{V_o(s)}{\Delta w_s(s)}$$

Let frequency of signal from gratings = w_G

then $w_G = N \cdot w_s$ where N = no. of lines on grating

and w_s = shaft speed (rad/sec).

Let the two inputs to phase discriminator be

$$a_1 = A \sin w_G t$$

$$\text{and } a_2 = A \sin (w_G - \Delta w_G) t$$

where Δw_G is difference in input frequencies.

Now $V_o = K_p \times \text{phase difference}$

K_p is constant in Volts/radian

$$= K_p \left[\int w_G dt - \int (w_G - \Delta w_G) dt \right]$$

$$= K_p \int \Delta w_G dt$$

$$= K_p N \int \Delta w_s dt$$

$$\therefore V_o(s) = K_p N \frac{w_s(s)}{s}$$

$$\therefore G_p(s) = \frac{V_o(s)}{w_s(s)} = K_p N \cdot \frac{1}{s}$$

$$\therefore G_p(jw) = \frac{K_p N}{jw}$$

5.3 The Motor, Spool Valve and Load

The Motor

Total flow to motor

$$Q = Q_m + Q_L + Q_c \text{ in}^3/\text{sec}$$

Q_m = useful flow to motor

Q_L = leakage flow to motor

Q_c = equivalent compressibility flow

Also let T_L = load torque lb-ins.

dm = motor displacement in^3/rad .

P = differential line pressure applied to motor lb/in^2

P_s = supply pressure lb/in^2

L = leakage coefficient ($\text{in}^3/\text{sec}/\text{lb}/\text{in}^2$)

J = total inertia lb-in-sec^2

F = friction torque $\text{lb-in}/\text{rad}/\text{sec}$.

V = volume of oil in motor and pipes in^3

B = bulk modulus of oil lb/in^2

Then

a) $Q_m = d_m \cdot \dot{w}_s$

b) $Q_L = P.L.$

Now P provides the motor torque neglecting drop in lines so that

$$\text{Total torque } T = dm \times P = J \dot{w}_s + Fw_s + T_L$$

$$\text{Hence } Q_L = \frac{L}{dm} [J \dot{w}_s + F w_s + T_L]$$

c) Q_c

Let ΔV = change in V due to compression

$$\therefore \frac{\Delta V}{V} = \frac{P}{B}$$

$$Q_c = \frac{dV}{dt} = \frac{V}{B} \frac{dP}{dt}$$

$$= \frac{V}{B} \cdot \frac{1}{dm} [J \ddot{w}_s + F \dot{w}_s + \dot{T}_L]$$

$$\therefore Q = dm w_s + \frac{L}{dm} [J \dot{w}_s + F w_s + T_L] + \frac{V}{B} \cdot \frac{1}{dm} [J \ddot{w}_s + F \dot{w}_s + \dot{T}_L] \quad (1)$$

The Spool Valve

A constant supply pressure, electrohydraulic flow control valve was used. Type DOWTY-MOOG Series 21.

Let

P = load pressure - lbs/in²

Q = output flow - in³/sec.

i = input current to spool valve torque motor - amps.

v = input voltage to spool valve - volts

R = resistance of spool valve coil - ohms

T_c = time constant of spool valve coil - secs.

T_v = low frequency approximation to spool valve time constant - secs.

K_v = rated flow - in³/sec/amp.

K_q = slope of flow/pressure characteristic (fig. 3.6) in³/sec/lb/in²

ζ_v = damping coefficient of valve

w_{nv} = undamped natural frequency of valve - rads/sec.

From DOWTY characteristics, with $P = 0$

$$\frac{Q(s)}{i(s)} = K_v \cdot \frac{1}{1 + \frac{2\ell_v}{w_{nv}} s + \frac{1}{w_{nv}^2} s^2}$$

For frequencies up to 50 c/s the following approximation is adequate, viz:-

$$\frac{Q(s)}{i(s)} = K_v \cdot \frac{1}{1 + sT_v}$$

Allowing for change in load

$$Q(s) = K_v \frac{1}{1 + sT_v} i(s) - K_q P(s)$$

Also $dm.P(s) = T(s) = \text{total torque}$

Finally

$$\begin{aligned} Q(s) &= K_v \cdot \frac{1}{1 + sT_v} i(s) - \frac{K_q}{dm} \cdot T(s) \\ &= \frac{K_v}{R} \cdot \frac{1}{1 + sT_c} \cdot \frac{1}{1 + sT_v} v(s) - \frac{K_q}{dm} T(s) \end{aligned}$$

substitute in (1)

$$\begin{aligned} \frac{K_v}{R} \cdot \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} V(s) &= dm w_s(s) + \frac{(L+Kq)}{dm} [F+sJ] w_s(s) \\ &\quad + \frac{(L+Kq)}{dm} T_L(s) \\ &\quad + \frac{V}{Bdm} [Js^2 w_s(s) + F s w_s(s) + s T_L(s)] \end{aligned}$$

(See Fig. 2.2)

put $L + Kq = L_1$

$$\begin{aligned} &= \left\{ \left[dm + \frac{L_1 F}{dm} \right] + s \left[\frac{L_1 J}{dm} + \frac{VF}{Bdm} \right] + s^2 \frac{VJ}{Bdm} \right\} w_s(s) \\ &\quad + \left\{ \frac{L_1}{dm} + s \cdot \frac{V}{Bdm} \right\} T_L(s) \end{aligned}$$

$$\text{put } dm + \frac{L_1 F}{dm} = d'$$

Then

$$\frac{K_v}{R} \cdot \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} v(s) = d'[1 + bs + as^2]w_s(s) + \frac{L_1}{dm}[1 + s \frac{V}{BL_1}]T_L(s)$$

$$\text{where } b = \frac{1}{d'dm} \left[L_1 J + \frac{VF}{B} \right]$$

$$\text{and } a = \frac{VJ}{d'dmB}$$

$$\text{Also put } \frac{V}{BL_1} = T_T$$

Hence

$$w_s(s) = \frac{K_v}{R \cdot d'} \cdot \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} \cdot \frac{1}{1+bs+as^2} v(s) \\ - \frac{L_1}{dm \cdot d'} \cdot \frac{1+sT_T}{1+bs+as^2} T_L(s) \quad (\text{See Fig. 2.3})$$

$$\therefore w_s(s) = G_m(s) \cdot v(s) - G_T(s) \cdot T_L(s)$$

$$\text{where } G_m(s) = \frac{K_v}{Rd'} \cdot \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} \cdot \frac{1}{1+bs+as^2}$$

$$\text{and } G_T(s) = \frac{L_1}{dm d'} \cdot \frac{1+sT_T}{1+bs+as^2}$$

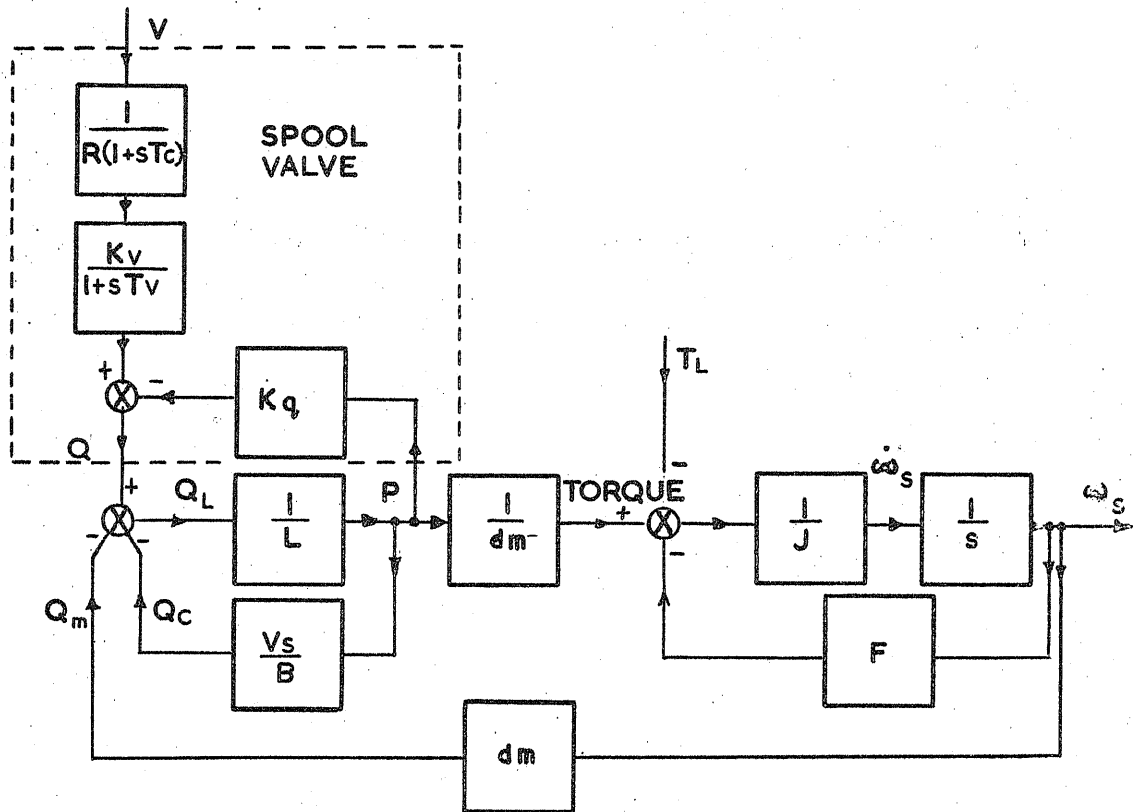


FIG 2.2

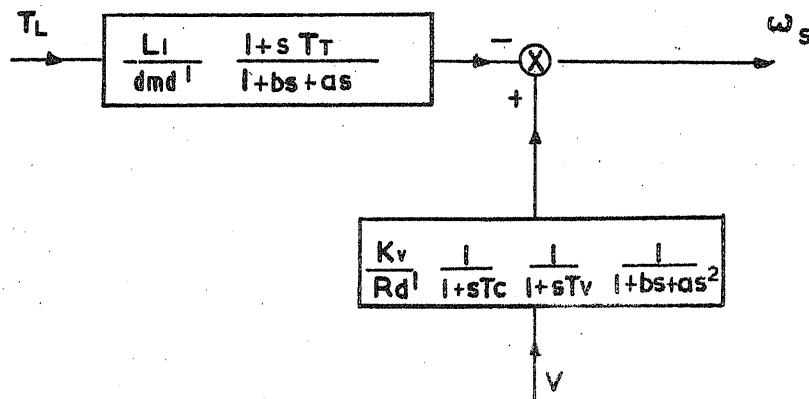


FIG 2.3

6. Investigation of Velocity Feedback Stabilisation

Tacho - generator mounted on load shaft.

Tach output = $K_1 \omega_s(s)$ volts.

NOTE:- K_1 must be investigated for noise output over working frequency range and may be a function of s .

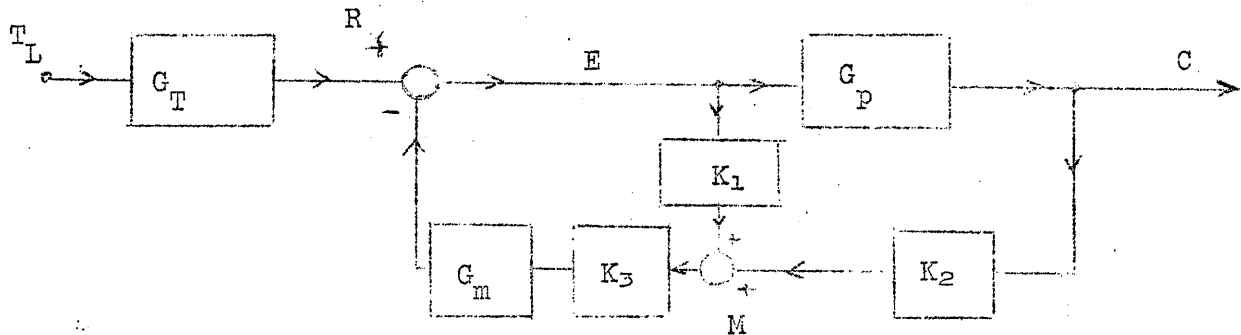


Fig. 2.4

Hence

$$C = G_p E \quad (\text{all functions of } s)$$

$$E = R - K_3 G_m M.$$

$$M = K_1 E + K_2 C = \left(\frac{K_1}{G_p} + K_2 \right) C$$

$$\therefore C = G_p \left[R - K_3 G_m C \left(\frac{K_1}{G_p} + K_2 \right) \right]$$

$$\therefore C \left[1 + K_3 G_m (K_1 + K_2 G_p) \right] = R G_p.$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_p(s)}{1 + K_3 G_m(s) (K_1 + K_2 G_p(s))}$$

$$\therefore \frac{V_o(s)}{R(s)} = \frac{G_p(s)}{1 + K_3 G_m(s) (K_1 + K_2 G_p(s))}$$

$$\text{where } R(s) = G_T(s) T_L(s)$$

$$= \frac{K_{pN}}{s} \cdot \frac{1}{1 + \frac{K_3 K_V}{R d'} \cdot \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} \cdot \frac{1}{1+bs+as^2} (K_1 + \frac{K_2 K_{pN}}{s})}$$

We can now neglect T_c and T_v . If they are not small enough in practice we can make

$$K_3 = \frac{K_3(1 + sT_c)(1 + sT_v)}{(1 + sT_1)(1 + sT_2)}$$

where T_1 and T_2 are negligible

Hence

$$\begin{aligned} \frac{V_o(s)}{R(s)} &= \frac{K_p N}{s} \cdot \frac{sR \cdot d'(1 + bs + as^2)}{sRd'(1 + bs + as^2) + K_3K_v(K_2K_pN + sK_1)} \\ &= K_p N R d' \cdot \frac{(1 + bs + as^2)}{K_2K_3K_vK_pN + (Rd' + K_1K_3K_v)s + Rd'b s^2 + Rd'as^3} \\ &= \frac{Rd'}{K_2K_3K_v} \cdot \frac{(1 + bs + as^2)}{1 + \beta s + \alpha s^2 + \gamma s^3} \end{aligned}$$

where $\beta = \frac{Rd' + K_1K_3K_v}{K_2K_3K_vK_pN}$

$$\alpha = \frac{Rd'b}{K_2K_3K_vK_pN}$$

$$\gamma = \frac{a}{b} \cdot \alpha.$$

The overall response

$$\begin{aligned} \frac{V_o(s)}{T_L(s)} &= \frac{Rd'}{K_2K_3K_v} \cdot \frac{(1 + bs + as^2)}{1 + \beta s + \alpha s^2 + \gamma s^3} \cdot \frac{L_1}{dm d'} \cdot \frac{1 + sT_T}{1 + bs + as^2} \\ &= \frac{L_1 R}{K_2K_3K_v dm} \cdot \frac{1 + sT_T}{1 + \beta s + \alpha s^2 + \gamma s^3} \end{aligned}$$

K_3 only occurs as a product of K_2 and could $\therefore$ be unity. However, it may be desirable to limit the value of K_1 thereby varying K_3 so K_2 will possibly need modifying. There are though only 2 design variables K_1K_3 and K_2K_3 .

K_2K_3 will be set by the required closed loop overall sensitivity.

Thus:-

$$\frac{V_o \max}{T_L \max} = \frac{R L_1}{K_2 K_3 K_V \cdot d m.}$$

7. Estimation of maximum input torque

Range of gears to be cut 10 to 250 teeth say from 2 inch to 10 inch diameter.

The input torque will occur when the cutting wheel makes contact, due to error in rough cutting, with only one flank of one tooth. Thus the estimated cutting force is the force perpendicular to the tooth face due to removing the error. Confusion between this force and the main cutting force which will be axial must not arise.

For a given error the size of the metal removed will depend upon the radius of the tooth profile, viz.

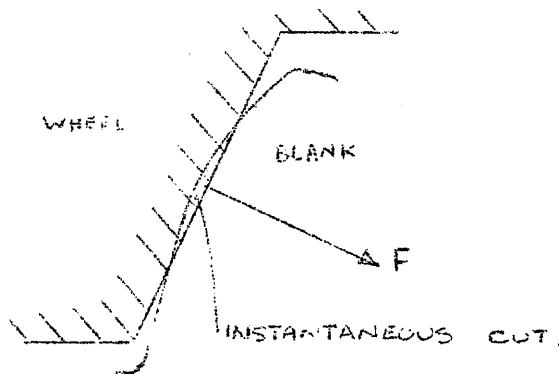


Fig. 2.5

Thus the maximum value of F will occur for maximum error and maximum radius of profile. A safety factor will be included by assuming that F is tangential to the gear blank.

Grants Odontograph

Ref. Machinery Handbook pp 676-7.

Grants odontograph is a table compiled to enable approximate outlines of cycloidal or involute teeth to be drawn as though comprised of two radii, viz.

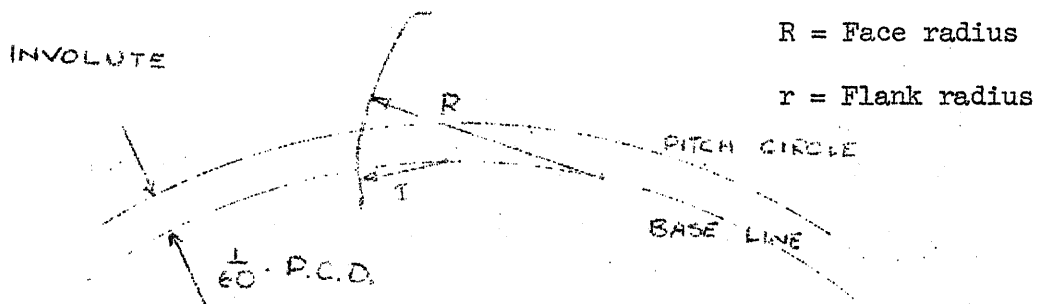


Fig.2.6

An equivalent constructor for cycloidal gears is also shown.

Quoting from the table for involute teeth (which give larger radii than cycloidal).

$$10 \text{ teeth} \quad R = \frac{2.28''}{DP} \quad r = \frac{0.69''}{DP}$$

$$20 \text{ teeth} \quad R = \frac{3.32''}{DP} \quad r = \frac{1.89}{DP}$$

$$52-60 \text{ teeth} \quad R = r = \frac{5.74''}{DP}$$

$$181-360 \text{ teeth} \quad R = r = \frac{21.62}{DP}$$

$$\text{Diametrial pitch} \quad DP = \frac{\text{No. of teeth}}{\text{Pitch circle dia.}}$$

$$\text{If } R = \frac{x}{D.P.} = \frac{x \times D}{N}$$

D = pitch circle dia.

N = No. of teeth

then R will be a maximum when D is maximum and $\frac{x}{N}$ is maximum. By inspection $\frac{x}{N}$ is a maximum for a 10 tooth gear.

Hence we can assume a maximum possible tooth form radius

$$R = \frac{2.28 \times 10}{10} = 2.28 \text{ inches, say } 2\frac{1}{4} \text{ inches.}$$

Size of cut

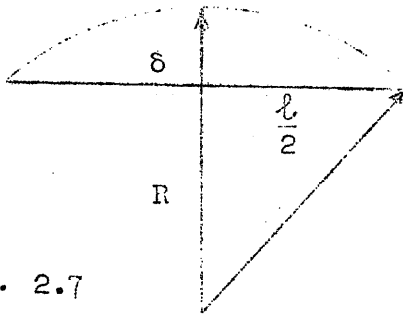


Fig. 2.7

$$R^2 = \frac{l^2}{4} + (R - \delta)^2$$

$$\therefore 2R\delta = \frac{l^2}{4} \text{ if } \delta^2 \ll R$$

$$\therefore l = \sqrt{8R\delta}$$

Hence

δ	l
.0005"	.095"
.001"	.134"
.002"	.19"
.003"	.23"
.004"	.27"

Reference to tests of vertical force applied to surface grinding performed at College of Aeronautics by Mr. J. Purcell.

$$\text{Width of wheel} = \frac{1}{4}''$$

Depth of cut

Vertical force

.0005"	12 lb
.001"	21 lb
.002"	37 lb

The absolute reliability of these results is doubtful as they were a side issue of a test on actual cutting forces.

Force is proportional to width of wheel.

We will assume that an error in the gear blank δ of 0.001" corresponding to a width of cut = .134" will require the same force as a rectangular cut 0.0005" deep.

i.e.

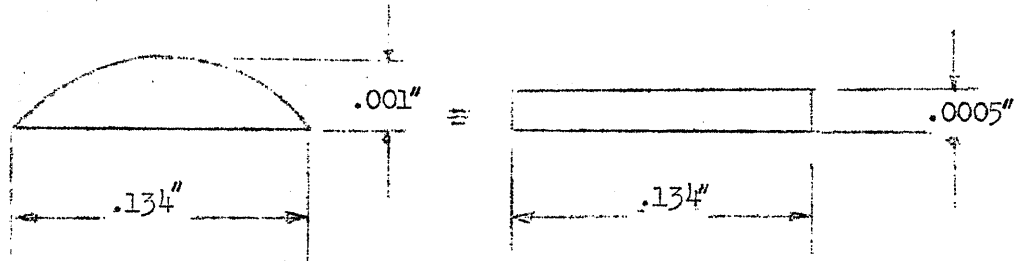


Fig. 2.8

$$\begin{aligned}\text{Then maximum cutting force} &= \frac{.134}{.25} \times 12 \\ &= 6.4 \text{ lbs.}\end{aligned}$$

$$\therefore \text{Max. Torque} = 5 \times 6.4 = 32 \text{ lbs-ins.}$$

.... *

THE MEASURED OPEN LOOP PERFORMANCE

1. Speed - Voltage Characteristics

Tests have been performed to measure the speed against spool current for the system at 1000 and 2000 psi supply pressures and with none, one or two inertia discs attached. Typical results are shown plotted in Fig. 3.1.

Deductions made from this are clearly that using the Vickers 3906-30 motor speeds of less than 100 RPM should be avoided. N.B. Due to initial design specifications the minimum working frequency of the phase discriminator is 4 kc/s. Using a 1000 line disc this is equivalent to a speed of 240 RPM. Hence a basic speed of 250 RPM is being used.

From part 2, page 22, it was shown that for steady state value with no load torque:-

$$\frac{\text{speed}}{\text{input voltage to coil}} = \frac{K_v}{R d'}$$

where K_v is spool valve constant in³/sec/amp. which is proportional to $\frac{\text{supply pressure} - \text{load pressure}}{\text{load pressure}}$

R is coil resistance

$$\text{and } d' = d_m + \frac{LF}{d_m}$$

where d_m = motor displacement

L = motor leakage

F = viscous friction constant

The speed/voltage graphs show clearly that inertia J has no effect on this term, as was theoretically predicted.

Thus slope of speed/voltage graph

$$= \frac{K_v}{R d'}$$

At 1000 psi supply pressure for Vickers 3906-30 motor and Dowty MOOG.
series 21 spool value

$$K_v = 1000 \text{ in}^3/\text{sec}/\text{amp.}$$

$$R = 1000 \text{ ohms.}$$

$$d_m = .015 \text{ in}^3/\text{rad.}$$

$$F \text{ is unknown}$$

$$L = 4.4 \times 10^{-6} \text{ in}^3/\text{sec}/\text{lb}/\text{in}^2$$

Assume $d' = d_m$

$$\text{then } \frac{K_v}{R d_m} = \frac{1000}{1000 \cdot .015} = 67 \text{ rad/sec/volt.}$$

Measured slope of speed/voltage curve

$$= 75 \text{ rad/sec/volt.}$$

Q.E.D.

2. Open Loop Frequency Response

These tests were performed using a SOLATRON TRANSFER FUNCTION ANALYSER measuring the speed of the shaft with a tachometer, with the output filtered by an R-C network.

2.1 Important Problems Encountered

2.1.1. Input signals of amplitude greater than 0.1 volts r.m.s.

caused a fall in the mean speed of rotation. This was located to reversal of the spool valve although motor speed variation was small.

Study of the speed/voltage curves will show that with a mean speed of 250 RPM peak voltage swing to stay on the linear part of the curve must be about 0.2 volts. Thus the input voltage was limited to 0.1 volts r.m.s.

2.1.2. The piece of chart included - Fig.3.2(b), shows a non sinusoidal speed response to a sinusoidal input voltage. However, by applying a high frequency (80 c/s) a.c. signal to the coil of about 0.02 volts r.m.s., this non-linearity was eliminated - Fig. 3.2(a).

2.2 Measured Results

The curves plotted - Fig. 3.3, show the amplitude and phase of the speed output for voltage input to coil. Fig. 3.4, is included to show the accuracy of the theoretical approach specified in part 2. The estimated effect of the spool valve is taken from Dowty published information, and for the coil by measuring the time constant. (see 2.3).

Steady state response tends to 36 db

$$= 63 \text{ rads/sec/volt.}$$

The small discrepancy between this and the slope of the speed/voltage curves is due to the use of different measuring devices.

2.2.1 No Inertia Discs 250 RPM 1000 PSI.

When phase shift = 90° $f = 8 \text{ c/s.}$

$$\text{Assuming } G_m(s) = \frac{K}{1 + bs + as^2}$$

$$\text{then } G_m(jw) = \frac{K}{1 + b \cdot jw + a(jw)^2} = \frac{K}{(1-aw^2) + j bw}$$

$$\therefore \phi = 90^\circ \text{ when } w^2 = \frac{1}{a}$$

$$\text{i.e. } a = \frac{1}{w^2} = \frac{1}{w_n^2}$$

∴ natural undamped resonant frequency $w_n = 8 \text{ c/s.}$

$$= 50 \text{ rad/sec.}$$

$$\therefore a = \frac{1}{w_n^2} = \frac{1}{50^2} = 0.4 \times 10^{-3}$$

Similarly when $bw = 1 - aw^2$ $\phi = 45^\circ$

From Fig. 3.3 $\phi = 45^\circ$ at 3.5 c/s.

i.e. $w = 22 \text{ rads/sec.}$

$$\therefore b = \frac{1 - aw^2}{w} = \frac{1 - .4 \times 10^{-3} \times 22^2}{22}$$

$$= .037$$

$$\therefore G_m(s) = \frac{63}{1 + .037s + 0.4 \times 10^{-3} s^2}$$

2.2.2 Two Inertia Discs 250 RPM 1000 PSI

From the curve shape it can be seen that the expression has real roots given by $wT = 1$ at 0.15 c/s and about 8 c/s.

$$\text{i.e. } G_m(s) = \frac{63}{(1 + \frac{1}{2\pi \times .15} s)(1 + \frac{1}{2\pi \times 8} s)}$$

$$= \frac{63}{(1 + 1.06s)(1 + .02s)} = \frac{63}{1 + 1.08s + .0212s^2}$$

2.3 Spool valve Dowty Series 21.

A current was fed into the spool valve coil under working conditions and the frequency adjusted until current and supply voltage were 45° out of phase. This occurred at 65 c/s. The resistance was nominally 1000 ohms.

$$\text{Hence } \frac{i(s)}{v(s)} = \frac{1}{R} \frac{1}{1 + sT_c}$$

$$\text{where } T_c = \frac{1}{2\pi 65} = .0025 \text{ secs.}$$

From measurements made by the supplier

$$\frac{Q(s)}{i(s)} = \frac{1000}{1 + 1.2 \cdot 10^{-3}s + .75 \times 10^{-6}s^2} \quad \text{in}^3/\text{sec}/\text{amp.}$$

at 1000 psi

Thus, for example, a more accurate representation of $G_m(s)$ for the system with two discs is:-

$$G_m(s) = \frac{.63}{(1 + 1.06s)(1 + .02s)(1 + .0025s)(1 + 1.2 \times 10^{-3}s + .75 \cdot 10^{-6}s^2)}$$

See Fig. 3.4

2.4 Phase Discriminator

$$\begin{aligned} K_p &= .036 \text{ volts/degree electrical} \\ &= 2 \text{ volts/radian electrical} \end{aligned}$$

$$\begin{aligned} \text{Maximum allowable error} &= \pm 4 \text{ seconds mechanical} = \pm 2.10^{-5} \text{ radians mechanical} \\ &= \pm 2.10^{-5} \text{N radians electrical} \end{aligned}$$

$$\therefore V_{o \max} = K_p \cdot 2 \times 10^5 \text{N volts.}$$

$$\text{Maximum applied load torque} = 32 \text{ lb-ins}$$

(part 2, p. 29)

$$\therefore \frac{V_{o \max}}{T_L \max} = \frac{2 \times 10^{-5}}{32} K_p \text{ N} = K_4 K_p \text{ volts/lb.in.}$$

where K_4 = Allowable mech. error/lb.in. load torque

$$\text{Steady state response } \frac{V_o}{T_L} = \frac{L_1 R}{K_2 K_3 K_v d m} \quad - \text{ from p. 25.}$$

$$\begin{aligned} \therefore \frac{L_1 R}{K_2 K_3 K_v d m} &= K_4 K_p N \\ &= \frac{2 \times 10^{-5}}{32} \times 2N \\ &= 1.25 \times 10^{-6} N \text{ volts/lb-in.} \end{aligned}$$

Also

$$\begin{aligned} G_p(j\omega) &= \frac{K_p \cdot N}{j\omega} = \frac{2 \cdot 10^3}{j\omega} \\ &= 66 \text{ db at } \omega = 1 \text{ i.e. } 0.16 \text{ c/s.} \end{aligned}$$

2.5 Desired Value of $K_2 K_3$

With a torque applied of 8 lb-ins voltage across coil was increased by 0.4 volts to return speed to 250 RPM.

From part 2, page 22, we can see that for constant w_s

$$T_L \cdot \frac{L_1}{d m d'} = V \cdot \frac{K_v}{R d'} \text{ in steady state.}$$

But the slope of the speed/voltage curve (see p.30)

$$= \frac{K_v}{R d'} = 63 \text{ (see page 32)}$$

Hence

$$8 \frac{L_1}{d m d'} = .4 \cdot 63$$

$$\therefore \frac{L_1}{d m d'} = 3.2 \text{ rads/sec/lb-in.}$$

From 2.4

$$\begin{aligned} \frac{L_1 R}{K_2 K_3 K_v d m} &= K_4 K_p N \\ &= 1.25 \times 10^{-6} N \end{aligned}$$

Hence

$$K_2 K_3 = \frac{L_1 R}{K_v d m \times 1.25 \times 10^{-6} N}$$

$$= \frac{L_1}{dmd'} \cdot \frac{R d'}{K_v} \cdot \frac{10^6}{1.24 N}$$

$$\therefore K_2 K_3 = 3.2 \times \frac{1}{63} \times \frac{10^6}{1.24 N} = \frac{41}{N \cdot 10^{-3}}$$

$$= 41 \text{ for a 1000 line grating.}$$

Put $K_3 = 1$

then $K_2 = 41$.

..... *

$$\therefore 20 \log_{10} 41 = 32.3 \text{ db.}$$

Hence the open loop frequency response function with no feed back loops is

$G_m(j\omega) \cdot G_p(j\omega) \cdot K_2$ which is plotted in Fig. 3.5.

It can be seen that, uncorrected, with 180° phase shift the loop gain is approximately 100 db

100 db being a large margin to correct it seems that a more suitable motor could be used.

3. Optimum choice of Motor and Valve

Open loop frequency response function is

$$G_2(j\omega) \cdot G_p(j\omega) \cdot K_2 = \frac{K_2 K_v}{R d'} \cdot \frac{K_p N}{j\omega} \cdot \frac{1}{[1 + c_1(j\omega) + c_2(j\omega)^2 + \dots]}$$

For a given closed loop sensitivity we have shown that (2.4):-

$$\frac{L_1 R}{K_2 K_v d m} = K_4 K_p N \quad (K_3 = 1)$$

$$\therefore G_m \cdot G_p \cdot K_2 = \frac{L_1}{d m d'} \cdot \frac{1}{K_4} \cdot \frac{1}{j\omega(1 + c_1(j\omega) + \dots)}$$

Thus to reduce the open loop gain in order to simplify stability requirements either L_1 must be reduced or dm increased.

It can be shown from measured results that

$$d' = dm + \frac{L_1 F}{dm} \approx dm$$

$$\text{and } L_1 = L + Kq \approx Kq$$

Thus dm is dependant upon the motor and L_1 upon the spool valve.

Note that the leakage of the motor is negligible.

Fig. 3.6 shows the effect of change in load pressure on flow. The slope of this curve is Kq and it can be seen that it is not a constant.

We can see therefore that:-

$$\begin{aligned} Kq &\approx \frac{\text{steady flow}}{2 \times \text{supply pressure}} \\ &= \frac{K_v \times I}{2 \times P_s} \\ &= \frac{dm \cdot w_s}{2 \times P_s} \quad \text{in steady state.} \end{aligned}$$

Thus the open loop gain at constant speed (w_s) and supply pressure (P_s) is proportional to

$$\frac{L_1}{dm d'} \approx \frac{Kq}{dm^2} \propto \frac{1}{dm}$$

The practical value will be proportional to between $\frac{1}{dm}$ and $\frac{1}{dm^2}$

4. COMPARISON OF VICKERS 3906-30 AND ROL VANE HT-10 MOTORS

	3906-30 and DOWTY MOOG VALVE	HT-10 and TELEHOIST VALVE
Pressure to develop 30 lb-in torque - PSI	2000	190
Speed Range - RPM	100-6000	5-1000
Displacement d_m - $\text{in}^3/\text{rev.}$	.095	1.0
Volume under compression - in^3	.57	.77
Leakage	Negligible compared to K_q	

In conclusion the H.T. 10 motor is more suitable because

- i) d_m is 10 times greater and the required open loop gain therefore 10 times (20 db) smaller.
- ii) Torque output is higher
- iii) Speed range is suitable for direct drive.

The disadvantage is that the larger volume under compression will cause more phase shift at higher frequencies, the effect of which will be small.

N.B. increasing the supply pressure from 1000 to 2000 psi will approximately halve K_q thereby reducing the required open loop gain a further 6 db.

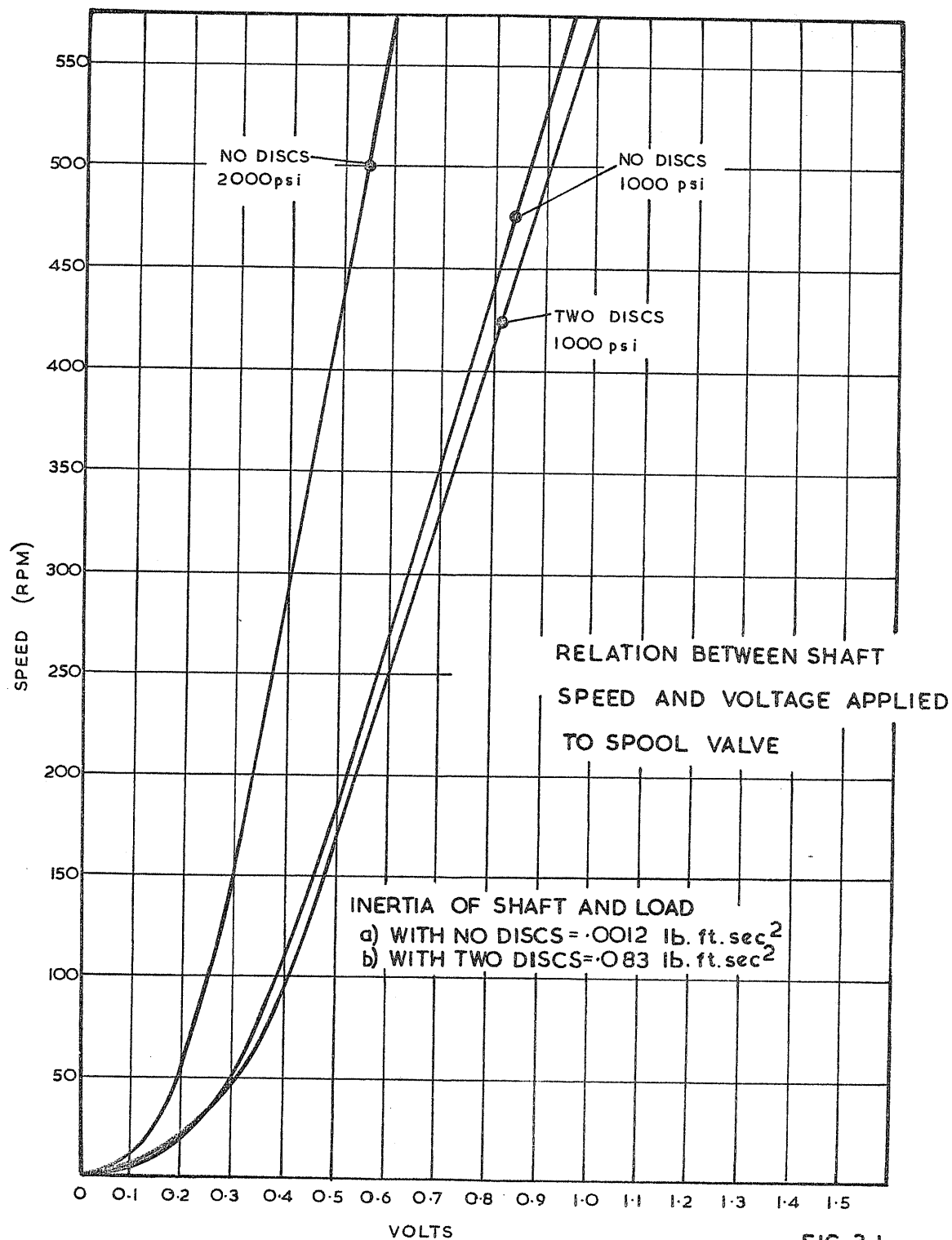
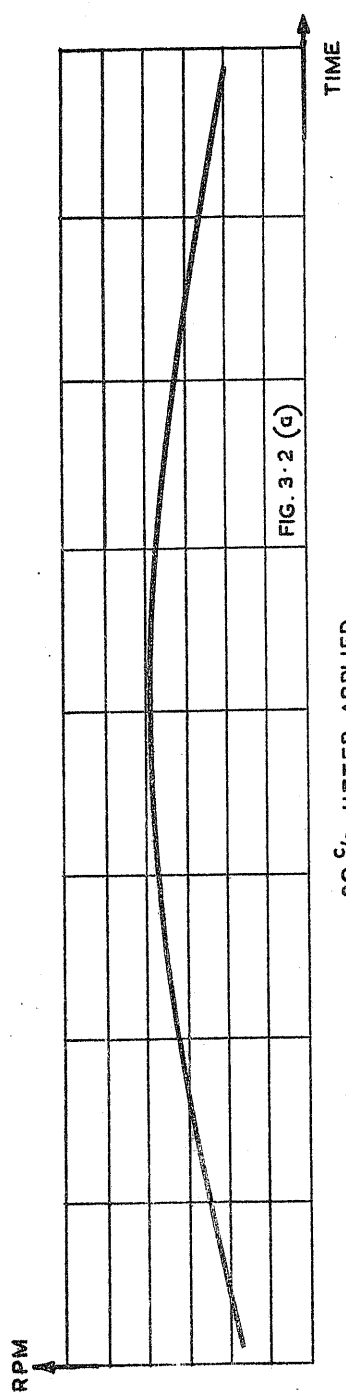
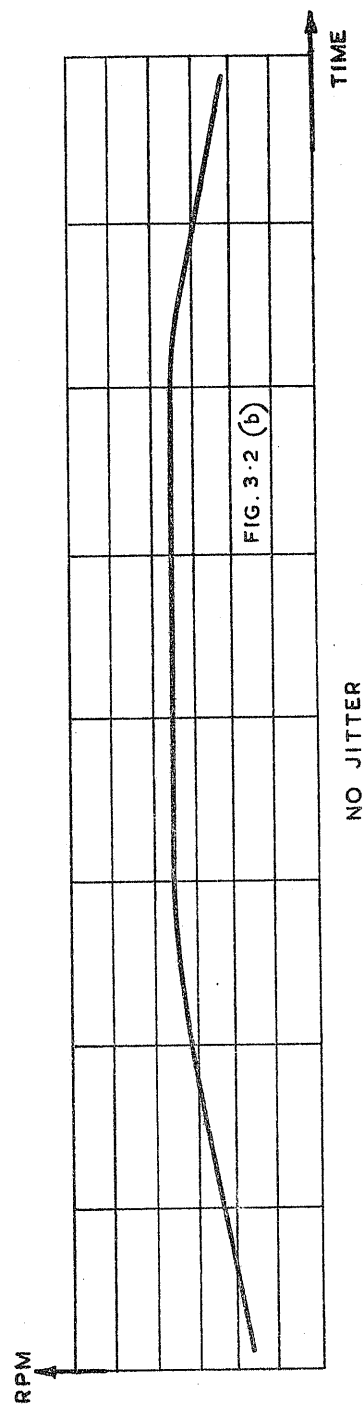


FIG 3-1



VELOCITY OF SHAFT WITH SINUSOIDAL INPUT OF FREQUENCY .04 %/s

FIG. 3.2

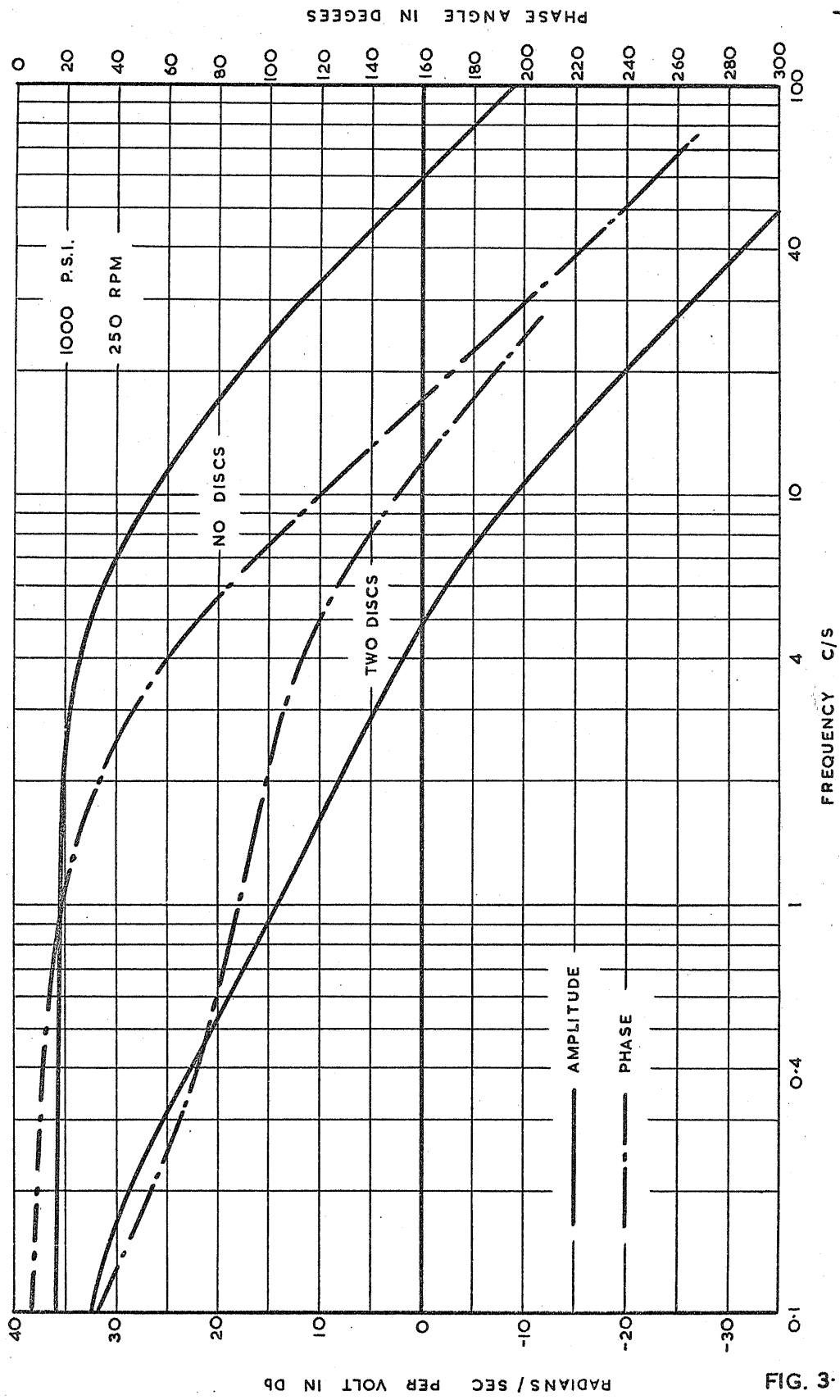


FIG. 3-3

FREQUENCY RESPONSE OF MOTOR AND SPOOL VALVE. G_m
 VICKERS 3906 -30 MOTOR DOWTY MOOG 21 VALVE

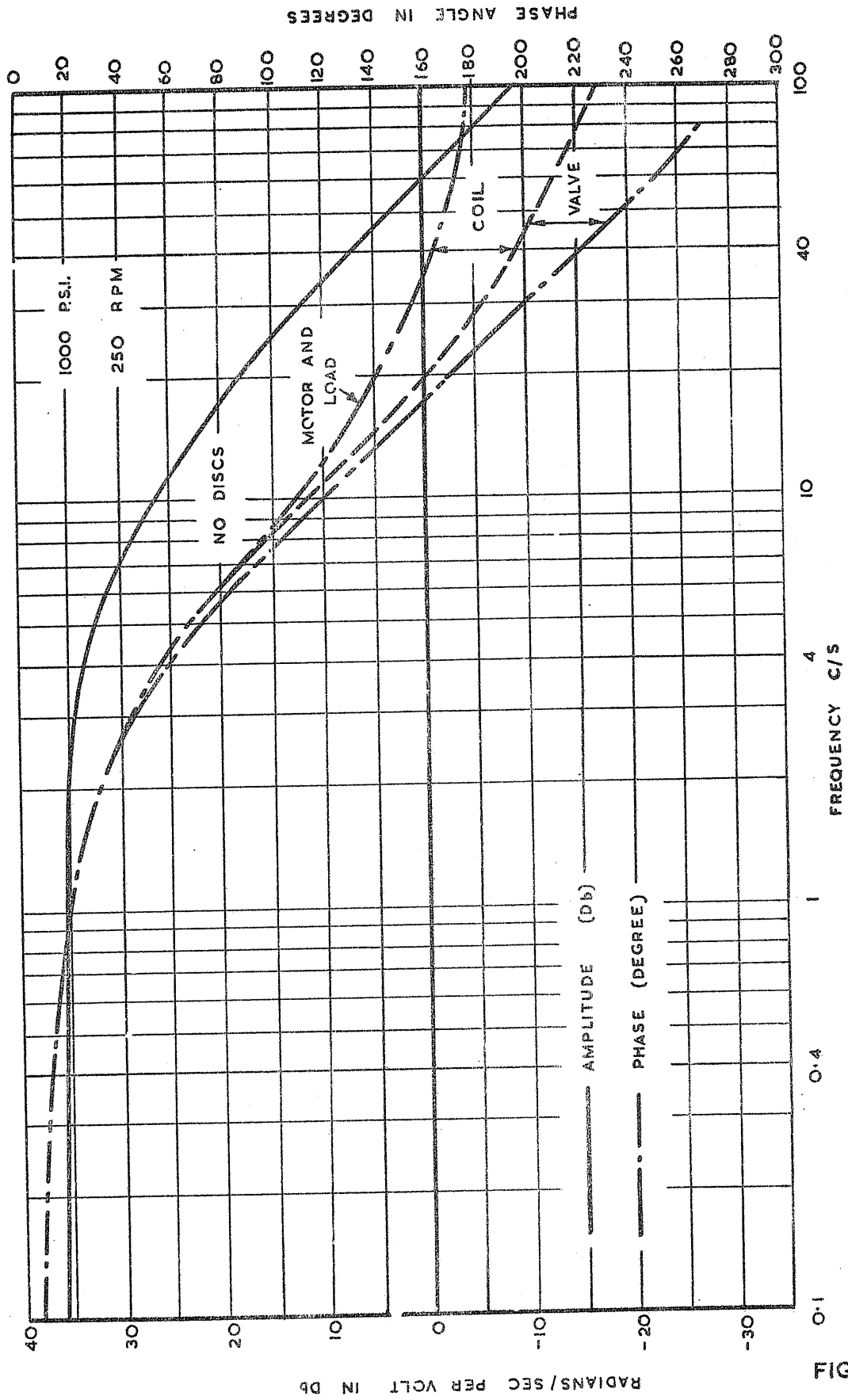
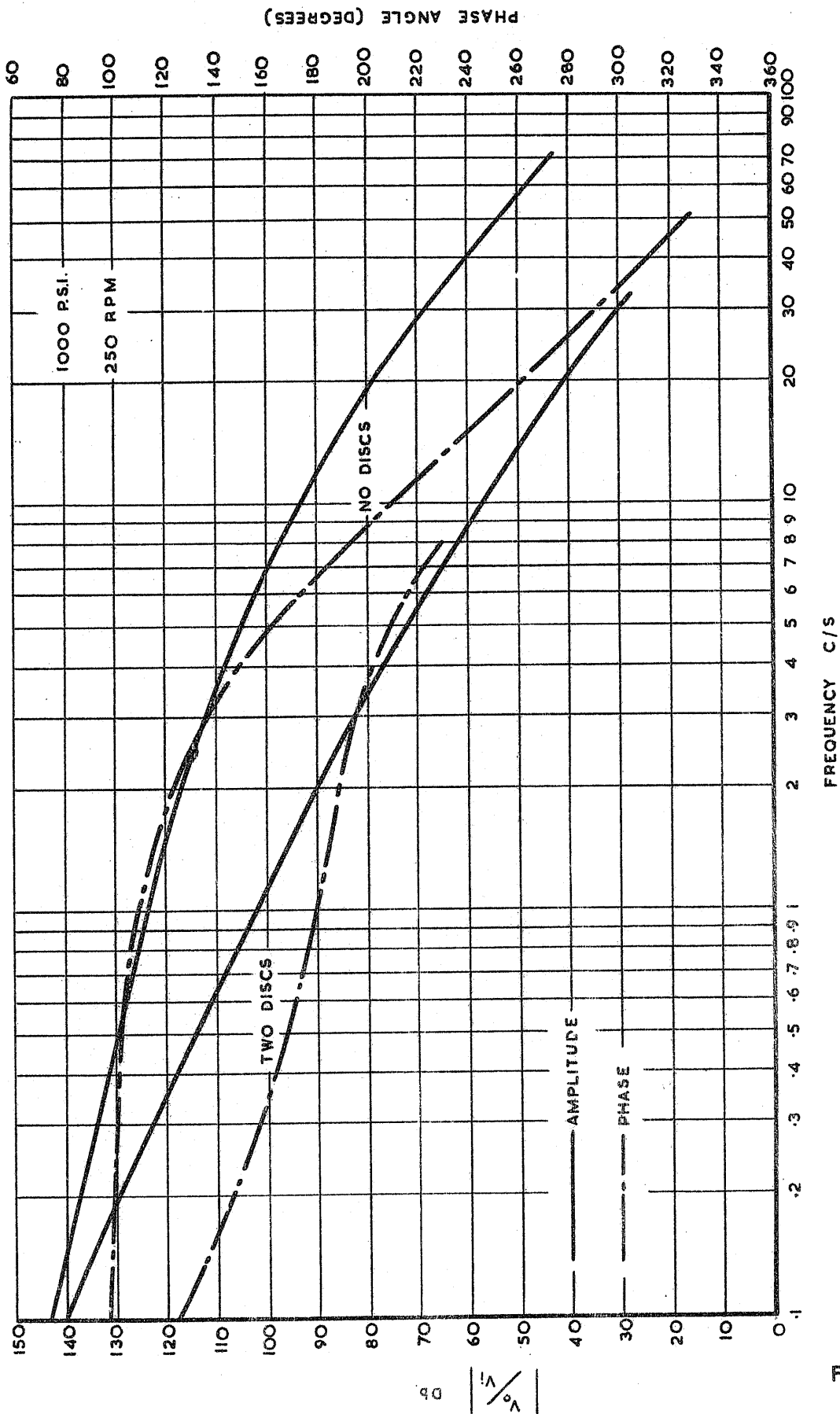
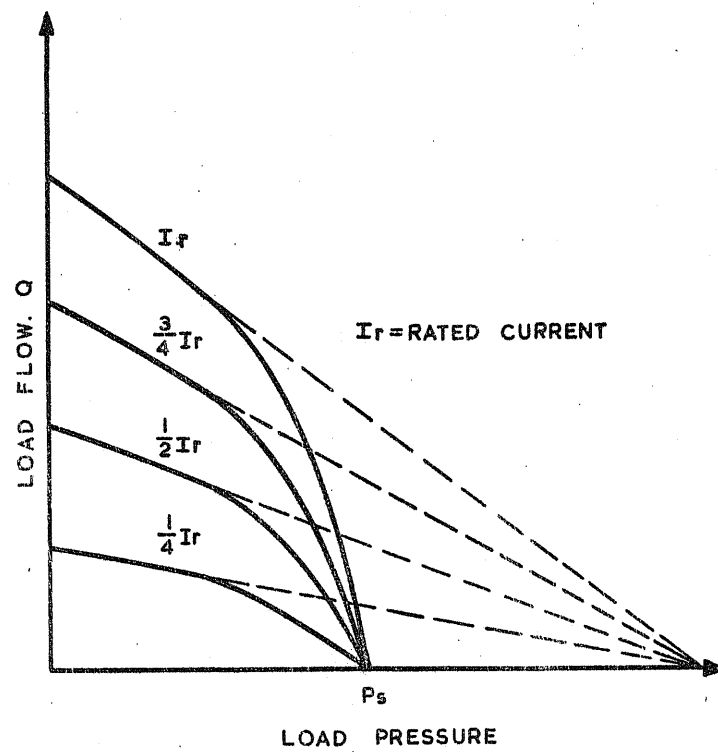


FIG. 3.4





EFFECT OF LOAD PRESSURE
ON FLOW THROUGH SPOOL
VALVE WITH VARIOUS COIL
CURRENTS.

FIG. 3-6

STABILITY CONSIDERATIONS AND SOURCES OF ERROR

1. The Spool Valve Coil

This time constant can be eliminated by driving the coil from a current source.

2. The Effect of Load Inertia

It is probably worthwhile initially to look at the effects of various load inertias.

If we consider a stable closed loop system subjected to an input torque then it has already been shown in part 2 (section 6, page 24) that the steady state error is independent of inertia and so only the transient term will differ. Further only the s^2 and s^3 terms of the denominator will be affected, both increasing with increase in J .

If we now consider the Open Loop frequency response (part 3, Fig. 3.5), it is immediately clear that the system with higher inertia will be easier to stabilise, although the resulting transient response may not be as good. It is not, however, felt to be possible from the above preamble to make any conclusive statements about the system.

We must also remember that the system will be required to follow changes in the reference frequency corresponding to speed changes in the Grinding Wheel shaft. A step function change in the reference frequency will require the shaft to change speed sharply. An assessment of the response of the closed loop system to a change in w will be included later but it seems probable that the lower the load inertia the better will be this response.

With the aforementioned points in mind it was decided to concentrate

on the system with the higher inertia (two steel discs on the load shaft) mainly on the grounds that this is the most applicable in respect to the ultimate grinding machine, a fact which outweighs the probable advantage of lower inertia.

3. Direct Velocity Feedback

The use of direct velocity feedback has already been considered as an opening gambit in part 2 section 6. However, a brief glance at the open loop frequency response, which has been repeated here as Fig. 4.1, considering the high inertia system only, will show that much more than direct velocity feedback is required to produce a stable system, e.g. -100 db gain margin!

All the same some useful information may be gleaned from mathematically interpreting the stability of this system.

From part 2 section 6, , p.24.

$$\frac{V_o}{T_L} = \frac{L_1 R}{K_2 K_V d_m} \cdot \frac{1 + sT_T}{1 + \beta s + \alpha s^2 + \gamma s^3}$$

having put $K_3 = 1$ where the important terms are

$$\beta = \frac{R d' + K_1 K_V}{K_2 K_V K_p N}$$

$$\alpha = \frac{R d' b}{K_2 K_V K_p N}$$

$$\text{and } \gamma = \frac{R d' a}{K_2 K_V K_p N}$$

From the results obtained in parts 2 and 3 the following approximate values can be quoted.

$$R = 1000 \text{ ohms}$$

$$d' = 0.015 \text{ in}^3/\text{radian}$$

$$K_v = 1000 \text{ in}^3/\text{sec}/\text{Amp}$$

$$K_2 = 41$$

$$K_p = 2 \text{ volts/radian}$$

$$N = 1000 \text{ lines}$$

$$\left. \begin{array}{l} a = 0.0212 \\ b = 1.08 \end{array} \right\} \text{ see p. 22 and p. 33.}$$

K_1 is the magnitude of the velocity feedback. in volts/rad/sec.

Thus for examination by Rouths Criterion the characteristic equation

$$1 + \beta s + \alpha s^2 + \gamma s^3$$

becomes

$$1 + \frac{K_1 + .015}{82,000} \cdot s + 2.10^{-7} s^2 + .04.10^{-7} s^3.$$

Hence

$$s^3 + 50 s^2 + 3.1 \times 10^3 (K_1 + .015) s + 2.5 \cdot 10^8 = 0$$

Rouths Array

$$\begin{array}{l|l} s^0 & P_0 \\ s & P_1 \\ s^2 & 50 \quad 2.5 \cdot 10^8 \\ s^3 & 1 \quad 3.1 \times 10^3 (K_1 + .015) \end{array}$$

$$P_1 = \frac{1}{50} \left| \begin{array}{cc} 50 & 2.5 \cdot 10^8 \\ 1 & 3.1 \cdot 10^3 (K_1 + .015) \end{array} \right|$$

$$= \frac{1}{50} (50 \times 3.1 \cdot 10^3 (K_1 + .015) - 2.5 \cdot 10^8)$$

For p_1 to be positive i.e. no change of sign in L.H. column which indicates stability, then

$$K_1 > \frac{2.5 \cdot 10^8}{1.55 \cdot 10^5} - .015$$

$$> 1600$$

If p_1 is positive p_0 must also be positive. In coming to this result we started with an approximation for the closed loop transfer function which neglected the time constants of the spool valve. Even neglecting these time constants, however, the value of K_1 required for indicated stability is far too large.

Thus this system is unsuitable.

4. Transfer Functions of the System

From the previous section it is obvious that a more accurate transfer function is required. For this we can quote from Part 3, 2.3 p.34. where the transfer function of the motor, valve and load was shown to be:-

$$G_m(s) = \frac{63}{(1 + 1.06s)(1 + .02s)(1 + 1.2 \cdot 10^{-3}s + .75 \cdot 10^{-6}s^2)}$$

The units of G_m are radians per sec/volt.

The time constant of the coil has been eliminated.

To check this function the amplitude and phase were plotted at various frequencies. The phase points were all correct, but, the amplitude points were all 5 db more than those plotted in part 3, Fig.

3.3.

Hence, using a pen recorder instead of the Transfer Function Analyser the amplitude was checked at various frequencies. These results showed the previous curve to be correct in form, but, shifted down by approximately 7 db. It was also shown earlier from part 3, Fig. 3.1, that a value of 75 radians per sec/volt should be expected at $\omega = 0$ a figure which the 7 db

shift upholds. Hence a correction must be made so that

$$G_m(s) = \frac{75}{(1 + 1.06s)(1 + .02s)(1 + 1.2 \cdot 10^{-3}s + .75 \cdot 10^{-6}s^2)}$$

$$= \frac{75}{1 + 1.08s + .022s^2 + 2.6 \cdot 10^{-5}s^3 + 1.6 \cdot 10^{-8}s^4}$$

With this new figure the value of K_2 suggested in part 3, 2.5, p. 35 must be checked.

It was shown that

$$\frac{L_1}{dmd'} = \frac{.4 \times 75}{8} = 3.75 \text{ rads/sec/lb.in.}$$

This figure can be checked in two ways

- (a) With coil current constant, an applied torque of 3.6 lb.in. changed the speed from 250 to 200 RPM.

This gives

$$\frac{L_1}{dmd'} = \frac{50 \times 2\pi}{60 \times 3.6}$$

$$= 1.45 \text{ rads/sec/lb.in.}$$

- (b) From part 2, 5.3, p. 22 the first order term of the motor and load response is

$$b = \frac{1}{dmd'} \left[L_1 J + \frac{VF}{B} \right]$$

with no inertia discs $J = .0012$ and $b = .037$

with two inertia discs $J = .083$ and $b = 1.08$.

rewriting

$$b = \frac{L_1}{dmd'} \cdot J + \frac{VF}{Bdmd'}$$

$$\therefore 1.08 = \frac{L_1}{dmd'} \cdot .083 + K$$

$$\text{and } .037 = \frac{L_1}{dmd'} \cdot .0012 + K$$

$$\text{where } K = \frac{VF}{Bdmd'}$$

Hence

$$\begin{aligned} \frac{L_1}{dmd'} &= 12.75 \text{ rads/sec/lb.ft. and } K = .022. \\ &= 1.06 \text{ rads/sec/lb.in.} \end{aligned}$$

Thus three different values of $\frac{L_1}{dmd'}$ have been derived with no obvious answer to the reason for such a discrepancy. With a safety factor in mind therefore the larger value of 3.75 was used giving the value of K_2 as before, e.g. $\frac{41}{N \times 10^{-3}} = 32 \text{ db for a 1000 line grating.}$

5. Theoretical Closed Loop Response and Limits of Error

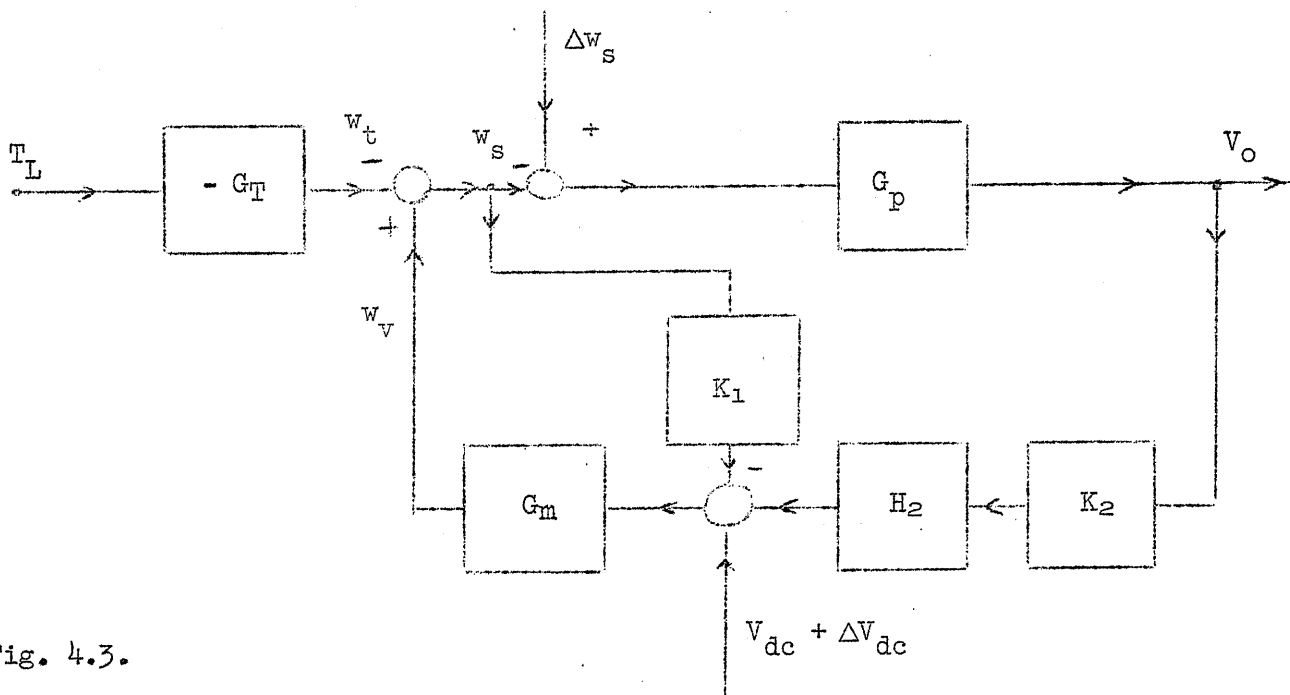


Fig. 4.3.

The input quantities are

T_L the input torque

Δw_s the change in reference frequency

and ΔV_{dc} the change in reference voltage

K_1 is a function of s .

H_2 is a function of s introduced to shape K_2 and having a value of unity at d.c.

The input to the phase discriminator will also contain errors due to the diffraction grating output.

5.1 The Effect of Change in Reference Frequency

Put T_L and $\Delta V_{dc} = 0$.

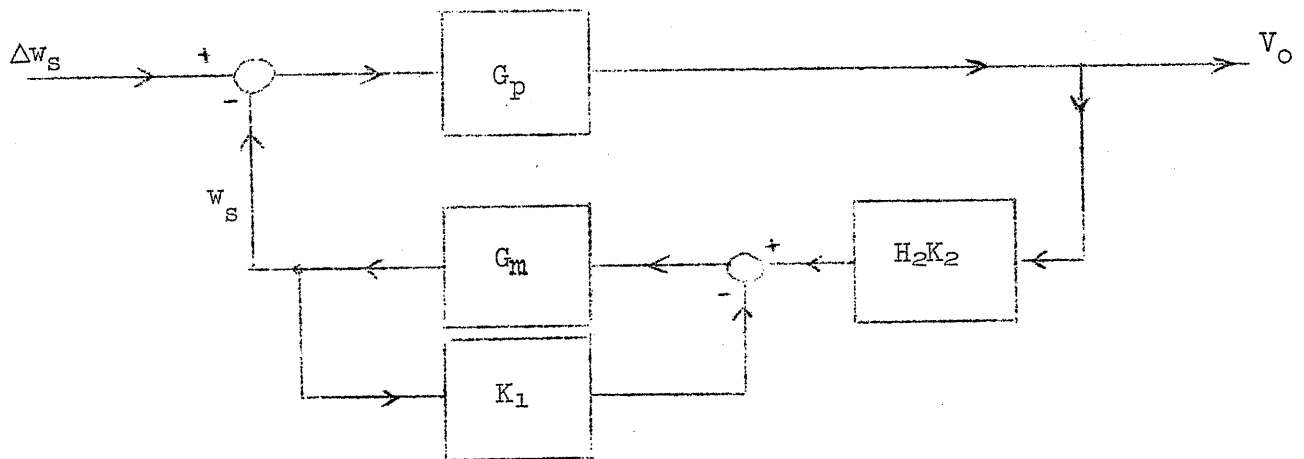


Fig. 4.4

Hence

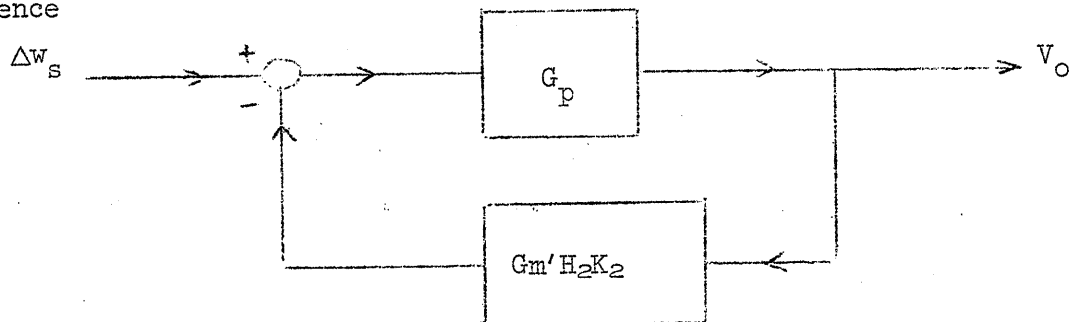


Fig. 4.5

where

$$G'_m = \frac{G_m}{1 + G_2 K_1}$$

$$\begin{aligned} \frac{V_o}{\Delta w_s} &= \frac{G_p}{1 + G_p \cdot G'_m H_2 K_2} \\ &= \frac{G_p(1 + G_m K_1)}{1 + G_m(K_1 + H_2 K_2 G_p)} \end{aligned}$$

∴ when $w = 0$ assuming $G_m K_1 \gg 1$

and $H_2 K_2 G_p \gg K_1$

$$\text{then } \frac{V_o}{\Delta w_s} = \frac{K_1}{K_2 H_2} \text{ volts/rad/sec.}$$

This is a straight forward velocity error.

Now for 1 second of arc error

i.e. $\frac{N}{3600}$ degrees electrical ($N = \text{No. of lines}$)

$$\text{then output from discriminator} = \frac{.036 \times N}{3600}$$

$$= N \times 10^{-5} \text{ volts/second of arc.}$$

$$\therefore \text{Change in speed } \Delta w_s = \frac{K_2 H_2}{K_1} \cdot N \times 10^{-5}$$

$$= \frac{.41}{N \cdot 10^{-3}} \cdot \frac{N \cdot 10^{-5}}{K_1} \text{ in steady state } (H_2 = 1)$$

$$= \frac{.41}{K_1} \text{ rads/sec.}$$

$$= \frac{.41 \times 60 \times n}{2\pi K_1} \text{ RPM of grinding wheel when cutting } n \text{ teeth}$$

$$= \frac{.41 \times 60 \times 10}{2\pi K_1} = \frac{40}{K_1} \text{ RPM minimum.}$$

Now with 2% speed control of the grinding wheel running at 2000 RPM

$$\text{then } \frac{40}{K_1} = .02 \times 2000$$

$$\therefore K_1 = 1.$$

In conclusion:

with $K_1 = 1$, 2% change in speed of grinding wheel will cause 1 second of arc error on 10 tooth gear and $\frac{1}{25}$ second of arc error on 250 tooth gear.

with $K_1 = 10$, 0.2% change in speed will cause the same errors.

We must therefore limit K_1 so that at $w = 0$ $K_1 = 1$ volt/rad/sec.

The transient response to change in Δw_s should preferably be such that the Bode plot of $\frac{V_o}{\Delta w_s}$ rolls away rapidly. However, as long as there is no increase in response as w increases this will be satisfactory. Thus K_1 should 'roll off' before H_2 .

Nothing more can be said about the transient response at this stage.

5.2 The Effect of Change in Load Torque

Put Δw_s and $\Delta V_{dc} = 0$.

From part 2.6 p. 24 with $K_3 = 1$ and K_2 replaced by $K_2 H_2$

$$\frac{V_o}{R} = \frac{G_p}{1 + G_m(K_1 + K_2 H_2 G_p)}$$

$$\therefore \frac{V_o}{T_L} = \frac{G_T G_p}{1 + G_m(K_1 + K_2 H_2 G_p)}$$

If $K_2 H_2 G_p \gg K_1$ and $G_m K_2 H_2 G_p \gg 1$.

then

$$\frac{V_o}{T_L} = \frac{G_T}{G_m K_2 H_2}$$

the value of K_2 was calculated from this equation so it is only necessary to consider the transient response, which will be covered later.

5.3 The Effect of Change in Reference Voltage

Put Δw_s and $T_L = 0$.

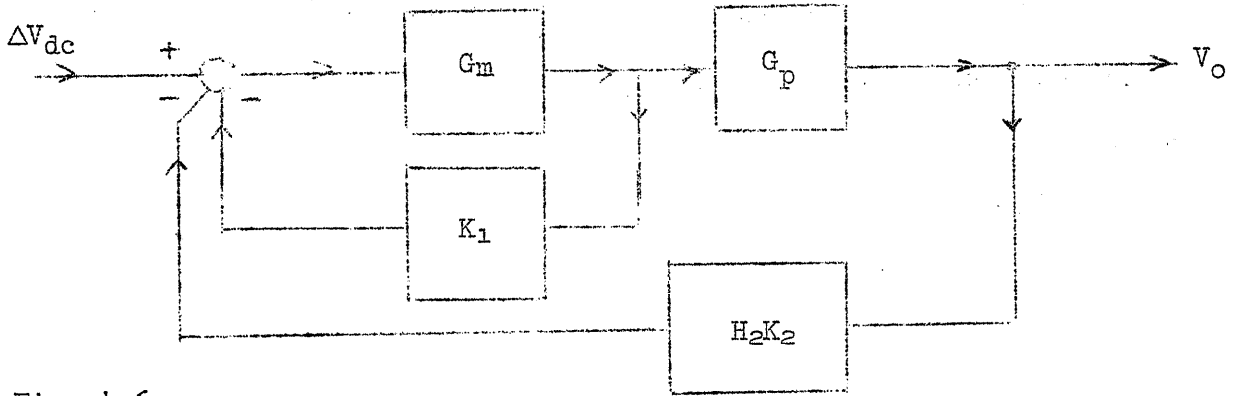


Fig 4.6

$$\begin{aligned} \therefore \frac{V_o}{\Delta V_{dc}} &= \frac{G_m G_p}{\frac{1 + G_m K_1}{1 + \frac{G_m G_p H_2 K_2}{1 + G_m K_1}}} \\ &= \frac{G_m G_p}{1 + G_m (K_1 + G_p H_2 K_2)} \end{aligned}$$

if $G_p H_2 K_2 \gg K_1$ and $G_m G_p H_2 K_2 \gg 1$

$$\frac{V_o}{\Delta V_{dc}} = \frac{1}{H_2 K_2}$$

$$\therefore \text{at d.c. } \frac{V_o}{\Delta V_{dc}} = \frac{1}{K_2} = \frac{N \times 10^{-3}}{41}$$

V_o for 1 sec of arc error = $N \times 10^{-5}$ volts

$$\begin{aligned} \therefore \Delta V_{dc} &= \frac{41}{N \times 10^{-3}} N \cdot 10^{-5} \\ &= 0.41 \text{ volts/sec error.} \end{aligned}$$

This figure is the total change in voltage applied to the coil and should include drift in the output of any d.c. amplifiers used to provide K_1 and K_2 . Obviously the reference voltage itself will not cause any problems unless for practical reasons it is injected before one of the

d.c. amplifiers.

This figure, however, is of most importance in assessing the allowable value of ripple and noise of specific frequencies due to the phase discriminator and the tachometer. e.g. a Tachometer with 1% ripple and producing 1 volt/rad/sec. ($K_1 = 1$) gives

$$\Delta V_{dc} = \frac{1 \times 250 \times 2\pi \times .01}{60} = .26 \text{ volts at 250 RPM.}$$

This however corresponds to a ripple frequency of 120 c/s with a 30 segment commutator at which frequency the approximation $\frac{1}{H_2 K_2}$ no longer holds and the numerator $G_1 G_p$ will cause a fall in response. The final value of H_2 therefore is the dependant factor and the problem of commutator ripple is quite acute.

It is possible that the a.c. signal from the diffraction grating may be frequency detected and utilised to provide K_1 since the base ripple frequency is high.

N.B. It must be remembered that while ripple in the range 100 c/s to 250 c/s will probably cause no error, since the response of the system is negligible at these frequencies, the spool valve itself will respond possibly causing saturation and obviously reducing the life of the valve considerably. On the other hand, however, a little ripple in this range will possibly provide the desirable 'jitter' of the spool.

5.4 The Effect of Errors in the Grating Signal

Refer to Appendix B for the theory of Diffraction Gratings.

Errors in the reference signals derived from the radial gratings are due to the following causes.

- 5.4.1 Errors in the manufacture of the actual grating. The averaging effect of the index piece which covers about $\frac{1}{20}$ th of the moving grating will improve the accuracy. Since the quality of radial gratings is being improved by N.P.L. and N.E.L. (see appendix B) errors due to this source are being neglected.
- 5.4.2 Jitter in the phase discriminator. The phase discriminator (Appendix D) samples the grating signals at one point in there cycle, e.g. at the instant when the serrasoidal output voltage is zero. Variation in the triggering voltage of the bi-stable elements will therefore correspond to an apparent change in phase. This effect is random and will have a mean value of zero. It also occurs at the high repetition frequency of the number of lines on the grating, N , times the shaft speed. i.e. a minimum of 130 c/s with $N = 1000$ lines and $w_s = 8$ RPM. This error has already been considered under the heading of change in reference voltage (5.3).
- 5.4.3 Error due to sampling rate. Since the relative phase is compared at one point on the reference voltage cycle only, this is essentially a sampling technique. The effect of this is that the transfer function of the phase discriminator is not $G_p(s) = \frac{K_p N}{s}$ (part 2, 5.2. p.18) at frequencies near to the sampling frequency. Time will not allow an immediate investigation of this point and as initial synchronisation is being attempted at speeds of 250 RPM i.e. about 4 Kc/s reference frequency, the approximation for $G_p(s)$ will suffice. (see however, 5.4.5, p.60).

5.4.4 Eccentricity error.

Let the eccentricity = h

mean spacing of lines = d

mean radius of lines = R

No. of lines = N

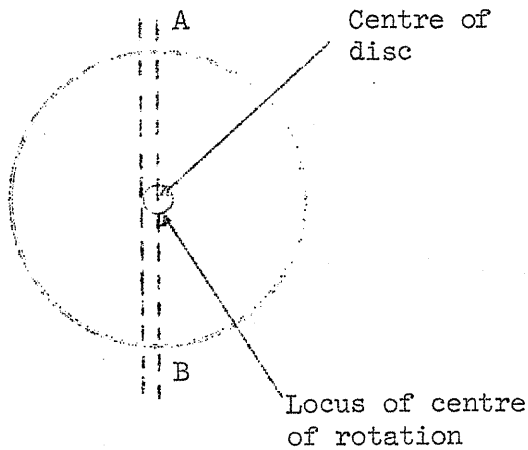


Fig. 4.7

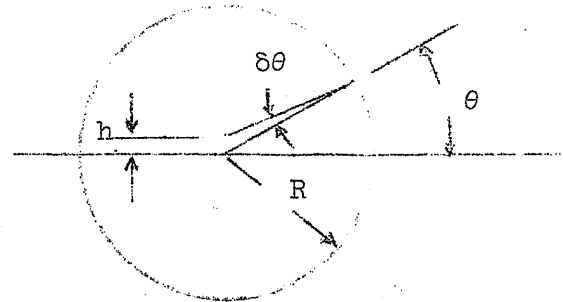


Fig. 4.8

At points A and B, the rulings are effectively displaced by h .
If we say that the effect is a phase lag at A then there will be a phase lead at B.

At any angle θ the effective displacement is $x = h \cos \theta$.

$$\begin{aligned} \text{Thus the phase error } \phi &= \frac{2\pi \cdot h \cos \theta}{d} \\ &= m \cos \theta \end{aligned}$$

If the disc is now rotated at a velocity w w.r.t. the fixed index piece, the photo cell output

$$a = A(1 + \cos Nwt)$$

assuming no eccentricity and perfect 'shuttering' of the light, where $\theta = wt$.

With the modulation reduced due to fringe blurring etc. $a = A(1 + \alpha \cos Nwt)$

With eccentricity h this signal will be phase modulated, thus:-

$$a = A(1 + \alpha \cos [Nwt + m \cos wt])$$

This phase modulation represents an error of $\frac{m}{N}$ radians mechanically.

$$\therefore \text{Error} = \frac{m}{N} = \frac{2\pi h}{dN}$$

$$= \frac{h}{R} \text{ radians} = \delta\theta.$$

Now if two photo cells are used, placed diametrically opposite and there exact position within 1 ruled line space adjusted, then two output signals will be derived thus:-

$$a_1 = A_1 (1 + \alpha_1 \cos [Nwt + m \cos wt])$$

$$\text{and } a_2 = A_2 (1 + \alpha_2 \cos [Nwt - m \cos wt])$$

thus adding these two signals

$$\begin{aligned} a = a_1 + a_2 &= A_1 + A_2 + A_1 \alpha_1 \cos (Nwt + m \cos wt) \\ &\quad + A_2 \alpha_2 \cos (Nwt - m \cos wt) \end{aligned}$$

Filtering with a H.P. filter and adjusting the photo cell sensitivities so that $A_1 \alpha_1 = A_2 \alpha_2 = A$

$$\begin{aligned} \text{then } a &= A [\cos Nwt \cos (m \cos wt) - \sin Nwt \sin (m \cos wt) \\ &\quad + \cos Nwt \cos (m \cos wt) + \sin Nwt \sin (m \cos wt)] \\ &= A \cos (m \cos wt) \cos Nwt. \end{aligned}$$

This signal is now Amplitude modulated but not phase modulated and therefore will introduce no phase errors. An ambiguous zero, however, could occur if the amplitude ever fell to zero.

$$\therefore A \cos (m \cos wt) > 0$$

$$\text{e.g. } m \cos wt < \frac{\pi}{2}$$

since $\cos wt$ will be cycling between $+1$ and -1 once per revolution of

the disc

$$m < \frac{\pi}{2}$$

$$\text{Now } m = 2\pi \frac{h}{d} < \frac{\pi}{2}$$

$$\therefore h < \frac{d}{4} < \frac{\pi R}{2N}$$

This indicates that all phase error due to eccentricity can be removed but second order errors will creep in due to the fact that the lines at A and B, Fig. 4.7 are not only shifted sideways but are also 'skewed' by $\delta\theta$ thus causing amplitude modulation of the two signals a_1 and a_2 . Thus it is essential to get the centring as good as possible.

In conclusion, a 1000 line commercial disc has a mean diameter of approx. 4" and a 10,800 line disc 9.25" diameter. Thus, for 1000 line disc $h < .003"$ and for 10,800 line disc $h < .00067"$.

The ultimate degree of accuracy of the reference signal must therefore be assumed adequate until the N.E.L. have completed work to produce more accurate radial gratings.

5.4.5 The Number of Lines on the Gratings

So far it has been clearly shown that the number of lines on the grating has no effect on the system theoretically. It must of course be remembered that if N is changed corresponding changes in K_2 must be made.

The choice of N therefore is for purely practical reasons.

The following points therefore must be considered.

- a) Output frequency. The lower frequency limit is set by the rate of sampling and the transfer function of the phase discriminator. The upper limit is set by maximum working frequency of the photo

cells and frequency dividers. Roughly then a range of 150 c/s to 1 Mc/s is allowable.

- b) Physical size of the grating. 1000 line discs are about $4\frac{1}{2}$ " diameter and the 10,800 line discs about 10" diameter. One of the discs will need to run at up to possibly 2500 RPM and it is felt that a 10" diameter glass disc running at this speed may be dangerous.
- c) Centreing. This is dependent upon pitch of lines which of course depends upon N and the diameter. However, we have previously shown that with a 1000 line disc eccentricity must not exceed .003" and with a 10,800 line discs .00067".

In this case, with range of shaft speeds of say 8 to 2400 RPM possible, frequencies of 130 c/s to 40 kc/s will be developed with a 1000 line disc and correspondingly 1.4 Kc/s to 430 Kc/s with a 10,800 line disc.

The choice of the number of lines available is strictly limited, once again pending further development from the N.E.L.

It is possible that the grating signal may be frequency discriminated and used for velocity feedback should a conventional tachogenerator prove unsuitable for any reason. This would necessitate a higher minimum frequency than 150 c/s.

The choice of N must finally be decided after studying the sampling rate of the phase discriminator.

5.4.6 The Effect of the Rate of Sampling the Grating Signal

Since the phase is sampled at one point of the photo cell output only, the phase discriminator is a form of non-linearity. Using a 1000 line grating the minimum frequency is 130 c/s so that the system would act as an effective low-pass filter allowing a describing function to be utilized to represent the transfer function of the phase discriminator. Forewarning of the difficulties encountered in a similar problem attempted at the N.E.L. lead to the following approach being applied. First, however, it must be observed that the obvious approach of measuring the transfer function was not expedient due to lack of apparatus capable of phase modulating a signal in the range 150 c/s to 4 Kc/s at response frequencies.

Let time between samples = τ secs.

Let samples occur at $t = 0; \tau, 2\tau$ etc.

Assume system to be in balance for $t < 0$. Let the load torque change from T to $T + \Delta T$ lb.in. For $t > 0$.

The system is now open loop until $t = \tau$, when another sample will occur, assuming that V_{in} is constant from $t = 0$ to $t = \tau$. Thus ΔV_o when $t = \tau$ will give an indication of the error due to sampling.

Thus:-

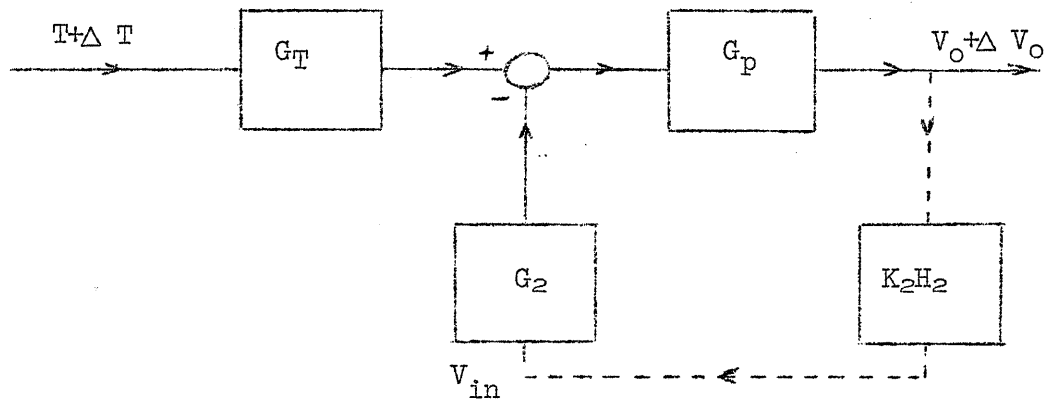


Fig. 4.9

Hence

$$\begin{aligned}\Delta V_o &= G_T \cdot G_p \cdot \Delta T \\ \therefore \Delta V_o(s) &= \frac{K_p N}{s} \cdot \frac{L_1}{dmd'} \cdot \frac{1+sT_T}{1+bs+as^2} \cdot \Delta T(s) \\ &= \frac{K_p N}{s} \cdot \frac{L_1}{dmd'} \cdot \frac{1+sT_T}{(1+sT_1)(1+sT_2)} \cdot \Delta T(s)\end{aligned}$$

Now $K_p = 2$ and $\frac{L_1}{dmd'} = 3.75$

$$T_T = 3.5 \times 10^{-4} \quad T_1 = 1.06 \quad \text{and} \quad T_2 = .02.$$

It has also been shown that $V_o = N \cdot 10^{-5}$ volts/sec. of arc., so that the mechanical angular error, ϕ is:-

$$\phi(s) = \frac{7.5 \cdot N}{N \cdot 10^{-5}} \cdot \frac{1+sT_T}{s(1+sT_1)(1+sT_2)} \Delta T(s)$$

Now assume that the torque changes at a constant rate of 1 lb.in./sec. then

$$\phi(s) = 7.5 \times 10^5 \cdot \frac{1+sT_T}{s(1+sT_1)(1+sT_2)} \cdot \frac{1}{s}.$$

The solution to this for small values of t is:

$$\phi(t) = \frac{7.5 \cdot 10^5}{T_1 T_2} \left[\frac{t^4}{4!} + \frac{T_T t^3}{3!} \right] \text{ secs. of arc mechanical.}$$

The maximum error will occur for the longest sampling period τ which coincides with minimum shaft speed.

$\therefore$ with a 1000 line disc at the minimum shaft speed of 8 RPM

$$\tau = \frac{60}{8 \times 1000} = 7.5 \cdot 10^{-3} \text{ secs.}$$

Hence

$$\phi = \frac{7.5 \cdot 10^5}{1.06 \times .02} \left[\frac{(7.5 \times 10^{-3})^4}{24} + \frac{3.5 \times 10^{-4}(7.5 \times 10^{-3})^3}{6} \right]$$
$$= \underline{\underline{5.4 \times 10^{-3}}} \text{ secs. of arc error.}$$

Thus, we can safely use the 1000 line grating, although it may still be necessary to evaluate the full transfer function of the discriminator for stability considerations.

5.4.7. Preferable Gratings

From the preceding sections 5.4.5 and 5.4.6. it would seem that a grating of about 2000 lines would be the most suitable.

5.5 The Effect of Changes in System Parameters

Of the parameters involved, G_m , K_1 , K_2 , H_2 , G_p and G_T , changes in G_m will be significant enough to make the remaining terms appear constant.

Thus, if we apply the principle of superposition,

$$V_o = \frac{K_1}{K_2 H_2} \Delta w_s + \frac{G_T}{G_m K_2 H_2} T_L + \frac{1}{H_2 K_2} \Delta V_{d.c.} + \text{Error due to Gratings}$$

Thus a fall in G_m of 6 db would cause a somewhat less than a doubling of the error.

With the motor running at low speeds the pulsating torque produced would correspond to a fluctuation in G_m which is a further factor in favour of the ROL-VANE motor if the system is to be run without reduction gearing.

6. The Effect of Reduction Gearing Between Motor and Load

6.1 Transfer functions

Compare with part 2; 5.3, p. 19.

Let gear ratio = $n : 1$ [a worm and wheel gearbox]

Let motor speed = w_m rads/sec.

Let shaft speed = w_s rads/sec.

Let Inertia of shaft and load = J_L lb-ft-sec²

Let Inertia of motor and worm = J_m lb-ft-sec²

Let Friction force due to load = F_L lb-ft/rad/sec.

Let Friction force due to motor and worm = F_m lb-ft/rad/sec.

then $w_m = n w_s$

Motor output torque = $\dot{d}m.P$

$$= J_m \dot{w}_m + F_m w_m + \frac{1}{n} [J_L \dot{w}_s + F_L w_s + T_L]$$

$$\therefore Q_L = \frac{L}{\dot{d}m} \left[J_m \dot{w}_m + \frac{1}{n} J_L \dot{w}_s + F_m w_m + \frac{1}{n} F_L w_s + \frac{1}{n} T_L \right]$$

$$Q_m = \dot{d}m w_m$$

$$Q_c = \frac{V}{B} \frac{dP}{dt}$$

$$= \frac{V}{B} \cdot \frac{1}{\dot{d}m} \left[J_m \ddot{w}_m + \frac{J_L}{n} \ddot{w}_s + F_m \dot{w}_m + \frac{F_L}{n} \dot{w}_s + \frac{1}{n} \dot{T}_L \right]$$

$$\begin{aligned} \therefore Q &= \dot{d}_m . n . w_s + \frac{L}{n . \dot{d}m} \left[(J_L + n^2 J_m) \dot{w}_s + (F_L + n^2 F_m) w_s + T_L \right] \\ &+ \frac{V}{B} \cdot \frac{1}{n . \dot{d}m} \left[(J_L + n^2 J_m) \ddot{w}_s + (F_L + n^2 F_m) \dot{w}_s + \dot{T}_L \right] \end{aligned}$$

$$\begin{aligned} \text{Also } Q &= \frac{K_v}{R} \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} v(s) \cdot \frac{K_q'}{\dot{d}_m} T_{TOT} \\ &= \frac{K_v}{R} \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} v(s) - \frac{K_q'}{\dot{d}_m n} \cdot [(J_L + n^2 J_m) \dot{w}_s + (F_L + n^2 F_m) w_s + T_L] \end{aligned}$$

Thus

$$\begin{aligned} \frac{K_v}{R} \cdot \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} v(s) &= n . \dot{d}m w_s \\ &+ \frac{L + K_q'}{n \dot{d}m} \left[(J_L + n^2 J_m) s w_s(s) + (F_L + n^2 F_m) w_s(s) + T_L \right] \\ &+ \frac{V}{n B \dot{d}m} \left[(J_L + n^2 J_m) s^2 w_s(s) + (F_L + n^2 F_m) s w_s(s) + s T_L \right] \end{aligned}$$

$$L + K_q' = L_1' \quad \text{and} \quad \dot{d}m + \frac{L_1' F'}{n^2 \dot{d}m} = d''$$

$$\text{where } F' = F_L + n^2 F_m$$

$$\text{also } J' = J_L + n^2 J_m$$

Hence

$$\begin{aligned} \frac{Kv}{R} \cdot \frac{1}{1+sT_c} + \frac{1}{1+sT_v} v(s) &= \left\{ n \cdot d_m + \frac{L_1' F'}{n d_m} \right. \\ &\quad + s \left(\frac{L_1' J'}{n d_m} + \frac{V F'}{B n d_m} \right) + s^2 \frac{V J}{B n d_m} \left. \right\} w_s(s) \\ &\quad + \left\{ \frac{L_1'}{n d_m} + \frac{s V}{B n d_m} \right\} T_L(s) \\ &= n d'' (1+b' s+a' s^2) w_s(s) + \frac{L_1'}{n d_m} (1+s \frac{V}{B L_1'}) T_L(s) \end{aligned}$$

where $b' = \frac{1}{n^2 d'' d_m} (L_1' J' + \frac{V F'}{B})$

and $a' = \frac{V J'}{n^2 d'' d_m B}$

We have previously noted $\frac{V}{B L_1'} = T_T^n$.

Hence

$$\begin{aligned} w_s(s) &= \frac{Kv}{n d'' R} \cdot \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} \cdot \frac{1}{1+b' s+a' s^2} v(s) \\ &\quad - \frac{L_1'}{n^2 d'' d_m} \cdot \frac{1+sT_T}{1+b' s+a' s^2} T_L(s) \end{aligned}$$

Thus

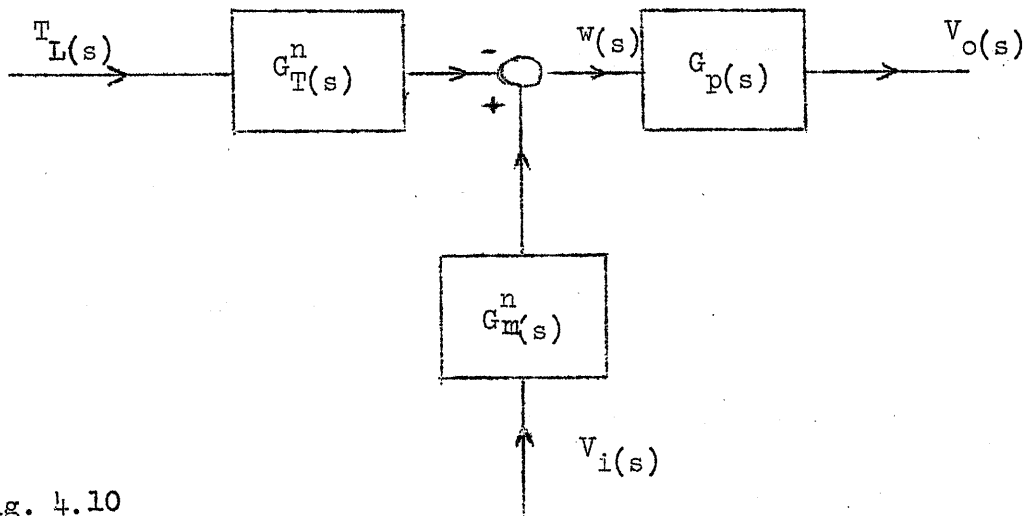


Fig. 4.10

$$\text{where } G_T^n(s) = \frac{L_1'}{n^2 d'' dm} \cdot \frac{1+sT_T}{1+b's+a's^2}$$

$$\text{and } G_m^n(s) = \frac{K_v}{nd''R} \cdot \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} \cdot \frac{1}{1+b's+a's^2}$$

From part 2, p.25

Overall closed loop response with $K_3 = 1$

$$\begin{aligned} \frac{V_o(s)}{R(s)} &= \frac{G_p(s)}{1+G_m^n(s)(K_1+K_2G_p(s))} \\ &= \frac{K_p N}{s} \cdot \frac{1}{1 + \frac{K_v}{nRd''} \cdot \frac{1}{1+sT_c} \cdot \frac{1}{1+sT_v} \cdot \frac{1}{1+b's+a's^2} \cdot \frac{(K_1+K_2K_p N)}{s}} \end{aligned}$$

neglecting T_c and T_v

$$\begin{aligned} \frac{V_o(s)}{R(s)} &= \frac{K_p N}{s} \cdot \frac{s n R d'' (1+b's+a's^2)}{s n R d'' (1+b's+a's^2) + s K_1 K_v + K_2 K_p N K_v} \\ &= K_p N R d'' n \cdot \frac{1+b's+a's^2}{K_2 K_v K_p N + (nRd'' + K_1 K_v)s + nRd'' b's^2 + nRd'' a's^3} \\ &= \frac{nRd''}{K_2 K_v} \cdot \frac{1+b's+a's^2}{1 + \beta's + \alpha's^2 + \gamma's^3} \end{aligned}$$

$$\text{where } \beta' = \frac{nRd'' + K_1 K_v}{K_2 K_v K_p N}$$

$$\alpha' = \frac{nRd'' b'}{K_2 K_v K_p N}$$

$$\gamma' = \frac{nRd'' a'}{K_2 K_v K_p N}$$

Hence

$$\begin{aligned}\frac{V_o(s)}{T_L(s)} &= \frac{V_o(s)}{R(s)} \cdot G_T^n(s) \\ &= \frac{n R d''}{K_2 K_V} \cdot \frac{1+b's+a's^2}{1+\beta's+\alpha's^2+\gamma's^3} \cdot \frac{L_1'}{n^2 d'' dm} \cdot \frac{1+sT_T}{1+b's+a's^2} \\ &= \frac{R L_1'}{n K_2 K_V dm} \cdot \frac{1+sT_T}{1+\beta's+\alpha's^2+\gamma's^3}\end{aligned}$$

∴ In steady state

$$\begin{aligned}\frac{V_o}{T_L} &= \frac{R L_1'}{n K_2 K_V dm} \\ &= K_4 K_p N = 1.25 \times 10^{-6} \text{ N volts/lb-in (p. 34)}\end{aligned}$$

Also $L_1 \approx K_q$ and $K_q \propto dm$ for w_s constant (p. 37)

Thus $L_1' \approx K_q'$ is proportional to $n dm$

$$\therefore L_1' = n L_1$$

Thus

$$\begin{aligned}K_2^n &= \frac{R L_1'}{n K_V dm 1.24 \cdot 10^{-6}} \\ &= \frac{1}{n} \cdot \frac{n L_1}{dm d'} \cdot \frac{R d'}{K_V} \cdot \frac{10^6}{1.25 N} \\ &= K_2 \\ &= 41\end{aligned}$$

Allowing 80% efficiency for the gear reduction this figure becomes

$$\begin{aligned}K_2^n &= \frac{41}{.8} \approx 50 \\ &= 34 \text{ db}\end{aligned}$$

Further

$$\begin{aligned} G_m^n &= \frac{d'}{d''} \cdot \frac{1}{n} \cdot G_m \\ &= \frac{1}{1.1 \times 25} \cdot G_m = \frac{1}{27.5} \cdot G_m \quad (\text{see 6.2(c)}) \end{aligned}$$

The open loop gain for a given closed loop gain is reduced in the ratio

$$\frac{G_m^n \cdot K_2^n}{G_m \cdot K_2} \approx 27 \text{ db (see below).}$$

6.2 Comparison of System With and Without Gearing

The following table of figures of a system with gear ratio $n:1$ of 25:1 are approximated values, as yet uncorrelated by measurement. These results, however, should show where any marked differences can be expected.

For each case the closed loop sensitivity is 4 secs of arc error for 32 lb-in load torque.

a) F = Friction due to motor alone

$$F' = F_L + n^2 F_m$$

$F_L \ll F_m$ and F_m will be bigger than F due to the worm wheel friction

$\therefore$ say $F' = 10 n^2 F$.

b) $J = J_m + J_L = J_L$

$$J' = J_L + n^2 J_m$$

Typically $J_L = .1 \text{ lb-ft-sec}^2$ down to $10^{-3} \text{ lb-ft-sec}^2$

and $J_m = 2 \cdot 10^{-4} \text{ lb-ft-sec}^2$.

$\therefore$ if $n = 25$

$$J' = .225 \text{ to } .125 \text{ lb-ft-sec}^2.$$

Note the normal advantage of gearing is that changes in load inertia are less significant.

N.B. Comparisons will be made on the two systems with two inertia discs in place.

Thus $J' = 2 J$

$$\begin{aligned} \text{c) } d' &= d_m + \frac{L_1 F}{d_m} \\ &= d_m \text{ from measured results.} \end{aligned}$$

$$d'' = d_m + \frac{L_1' F'}{n^2 d_m}$$

We have assumed that $\frac{F'}{n^2} = 10 F$ so that we can estimate that

$$d'' = 1.1 d'$$

$$\text{d) } b = \frac{1}{d' d_m} \left[L_1 J + \frac{V F}{B} \right] = 1.08$$

$$\begin{aligned} b' &= \frac{1}{n^2 d'' d_m} \left[L_1' J' + \frac{V F'}{B} \right] \\ &= \frac{1}{n^2 \cdot 1.1 d' d_m} \left[L_1' \cdot 2J + \frac{10 n^2 \cdot V F}{B} \right] \end{aligned}$$

It was shown in part 4, 4.0 that with two inertia discs $\frac{V F}{B}$ is about 2% of $L_1 J$ (.022 in 1.08).

However, multiplied by $10 n^2 = 6250$ it now becomes about 120 times larger than $L_1 J$

$$\begin{aligned} \text{thus } b' &= \frac{120}{1.1 n^2} \cdot b = 0.18 b \\ &= 0.19. \end{aligned}$$

$$e) \quad a = \frac{VJ}{d' dmB} = .021$$

$$\begin{aligned} a' &= \frac{VJ'}{n^2 d'' dmB} \\ &= \frac{V.2J}{n^2 1.1 d' dmB} \\ &= 3.5 \cdot 10^{-3} a \\ &= 7.4 \cdot 10^{-5} \end{aligned}$$

$$f) \quad G_T \Big|_{W=0} = \frac{L_1'}{d' dm}$$

$$\begin{aligned} G_T^n \Big|_{W=0} &= \frac{L_1'}{n^2 d'' dm} = \frac{nL_1}{n^2 d'' dm} \\ &= \frac{1}{27.5} \cdot G_T \Big|_{W=0} \end{aligned}$$

g) We have already shown that

$$G_m^n \Big|_{W=0} = \frac{1}{27.5} G_m \Big|_{W=0}$$

N.B. obviously since a' and b' are considerably lower it is important that T_T should not become appreciable i.e. T_T should be $\ll b'$.

Thus, using manufacturers figures

$$T_T = \frac{V}{BL_1} = \frac{.5}{2 \cdot 10^5 \cdot 7.2 \cdot 10^{-3}} = 3.5 \cdot 10^{-4} \text{ secs. Q.E.D.}$$

Summary

Parameter	n = 1:1	n = 25:1
G_m rads/sec/volt	75 (37.5 db)	2.73 (8.7 db)
K_2	41 (32.3 db)	50 (34 db)
$G_m G_p K_2$ (at $w=1$ when $K_p = 66$ db)	135.8 db	108.7 db
b } with maximum J_L	1.08	0.19
a }	0.021	$7.4 \cdot 10^{-5}$
b } with minimum J_L	.037	0.19
a }	4×10^{-4}	$3.7 \cdot 10^{-5}$
		Note that change in parameters with change in load is negligible

6.3 Effect of Change in Reference Frequency - With Gearing

With T_L and $\Delta V_{dc} = 0$, from 5.1 p. 52.

$$\frac{V_o}{\Delta w_s} = \frac{G_p(1 + \frac{n}{G_m K_1})}{1 + \frac{n}{G_m}(K_1 + H_2 K_2 G_p)}$$

Now $H_2 K_2 G_p \gg K_1$ and $G_m H_2 K_2 G_p \gg 1$.

$$\therefore \frac{V_o}{\Delta w_s} = \frac{G_p(1 + \frac{n}{G_m K_1})}{\frac{n}{G_m} H_2 K_2 G_p} = \frac{1 + \frac{n}{G_m K_1}}{\frac{n}{G_m} K_2 H_2}$$

$\therefore$ as $w \rightarrow 0$

$$\frac{V_o}{\Delta w_s} = \frac{1 + \frac{n}{G_m K_1}}{\frac{n}{G_m} K_2}$$

$$\therefore \Delta w_s = \frac{\frac{n}{G_m} K_2}{1 + \frac{n}{G_m K_1}} V_o \text{ in steady state.}$$

Now $V_o = 10^{-2}$ volts for 1 sec. of arc error with 1000 line disc

$$\text{Also } \left. \frac{n}{G_m} \right]_{w=0} = 2.73 \text{ rads/sec/volt and } \left. K_2 \right]_{w=0} = 2$$

So that

$$\Delta w_s = \frac{2.73 \times 2 \cdot 10^{-2}}{1 + 2.73 K_1}$$

Put $K_1 = 0$ at $w = 0$

then

$$\Delta w_s = 5.46 \cdot 10^{-2} \text{ rads/sec.}$$

$\therefore$ Change in Grinding wheel speed = $5.46 \cdot 10^{-2} n$ rads/sec.

where n = no. of teeth cut.

$$= 5.46 \times 10^{-2} \times \frac{60}{2\pi} \times 10 \text{ or } 5.46 \times 10^{-2} \times \frac{60}{2\pi} \times 250 \text{ RPM}$$

$$= 5.2 \text{ RPM or } 130 \text{ RPM}$$

With a grinding wheel speed of say 2000 RPM this represents 0.26% or 6.5% speed control. This lower figure of 0.26% is unattainable by normal commercial speed controllers.

If K_1 is > 0 this situation becomes worse

∴ K_1 at $w = 0$ must be zero.

Increase in K_2 will help as it will also increase the accuracy due to load torque but stability problems, particularly with $K_1 \Big|_{w=0} = 0$, will become more acute.

Since all the above figures are calculated and not correlated since a gear box is not yet available, attempts to interpret transients are pointless.

6.4 The Effect of Change in Load Torque - With Gearing

This has already been considered in the steady state in calculating K_2^n and as previously mentioned transient responses cannot yet be considered.

6.5 The Effect of Change in Reference Voltage - With Gearing

From 5.3, p. 55, but with $K_1 = 0$

$$\frac{V_o}{\Delta V_{dc}} = \frac{G_m^n G_p}{1 + G_m^n G_p H_2 K_2^n}$$

∴ as $w \rightarrow 0$

$$\frac{V_o}{\Delta V_{dc}} = \frac{G_m G_p}{G_m G_p K_2^n} = \frac{1}{K_2^n}$$

$$\therefore \Delta V_{dc} = K_2^n V_o$$

$$= 2.10^{-2} \text{ volts/sec of arc error.}$$

This is best considered as an equivalent input drift to the amplifier providing $K_2 H_2$ of 10^{-2} i.e. 10 millivolts which is quite acceptable.

6.6 The Effect of Errors in Grating Signal

This will be unaffected by gearing.

6.7 The Effect of Changes in System Parameters - With Gearing

As in 5.5, p.65 the only parameter liable to serious variation is G_m^n , which due to the gearbox now will include further errors.

In this case

$$V_o = \frac{1}{G_m^n K_2} \Delta w_s + \frac{G_m^n T_L}{G_m^n K_2 H_2} T_L + \frac{1}{H_2 K_2} \Delta V_{dc} + \text{Error due to gratings}$$

Changes in G_m^n would have more effect on the error in this system

than the system with direct drive motor, but the gearing will eliminate pulsating torque due to the higher speed of the motor.

6.8 Choice of motor

With $n = 25:1$ and shaft speeds in the range 8 to 250 RPM, motor speeds will be in the range 200 to 6250 RPM which eliminates the ROL-VANE motor and makes the VICKERS motor, as tested, ideal. Whether this motor in conjunction with the existing MOOG spool valve will produce enough torque, with an allowance in hand for transient requirements, depends upon the gear box and cannot yet be assessed properly, however:-

$$\text{max. flow} = d_m \times w_s \times n$$

$$= .095 \times \frac{6250}{60} = 1.0 \text{ in}^3/\text{sec.}$$

The series 21 MOOG servo-valve in service is rated for $10 \text{ in}^3/\text{sec}$ flow with the valve fully open at 1000 psi drop across the valve (1500 psi

supply pressure). With a maximum supply pressure readily available of 2000 psi all but the extreme speeds should be achievable. It should be noted, however, that both motor and spool valve are rated to work up to 3000 psi. The series 21 valves can be obtained for up to 20 in³/sec. maximum flow.

7. Factors Affecting the Choice of Whether or Not to Use Gearing

Obviously, as has been considered throughout the earlier work in this report, it is very desirable to avoid gearing on the following counts:-

- a) In a system of this accuracy all possible sources of error such as backlash and transmission errors in a gearbox should be avoided.
- b) The design of a gearbox to give a ratio of say 25:1, capable of running at up to 250 RPM output speed with a maximum accuracy must involve worm and wheel gearing. It involves extra expense in the resulting machine tool and more important provides a source for deterioration of performance with age which will swamp all other ageing problems.
- c) Inclusion of the gearbox will increase the complexity of the mechanical design.

In favour of including gearing we can list:-

- d) The reduction in open loop gain and of the magnitude of the constants a and b make stability far easier. In fact from the proceeding discussions, Part 4 sections 1 - 5, we can deduce that with no gearing a stable system will be impossible to

produce, at least with suitable transient responses.

- e) The choice of a suitable motor is easier.
- f) Variation in Load Inertia causes only small changes in system.

IT WOULD THEREFORE APPEAR THAT THIS SPECIFICATION CANNOT BE FULFILLED USING DIRECT DRIVE.

8.0 The effect of Errors in the Tachometer

As suggested in 5.1 p.53 the system with no gearing was connected up with velocity feedback using a permanent magnet tacho-generator to give $K_1 = 1 \text{ volts/rad/sec.}$ At this stage the main loop involving the phase discriminator was left open the shaft being simply speed controlled.

We will now consider this minor loop only. To check the degree of speed consistency more accurately than metering the tachometer output voltage, a double beam oscilloscope was employed on one beam of which the grating output was displayed. The output of a precision oscillator, at the nominal frequency of the grating signal, was fed to the second beam.

With no velocity feedback it was found impossible to keep the two signals displayed still. Closing the loop, with $K_1 = 1$, the improvement in consistency of shaft speed was very marked, so that only small infrequent adjustment of the oscillator frequency was necessary. All this is of course obvious and expected and is simply a lead to the following.

The important observation made was that while the frequency of the two signals could be held constant there was a phase variation of about 90° , when running at 250 RPM, at shaft frequency.. This was only apparent in the closed loop condition. Thus closing the minor loop has caused a once a revolution phase modulation, which will act as an undesirable input in

the main loop.

Possible causes are:-

- a) Periodic error in the grating
- b) Unbalanced load
- c) Uneven friction
- d) Uneven motor torque
- e) Tachometer errors

a) can be eliminated since the two signals taken diametrically opposite show only small phase modulations which will be due to small centring errors.

In fact a), b), c) and d) can be eliminated since they would show on open loop and closed loop.

Thus the error is due to the tachometer.

Let grating signal = $A \sin (N \omega_s t + \phi)$

where ϕ = phase modulation due to variation in ω_s .

Assume that ϕ is sinusoidal at frequency ω_s . The amplitude of $\phi = \pm 90^\circ$ electrical when $\omega_s = 4$ c/s (240 RPM).

Thus $\phi = \frac{\pi}{2} \sin 2\pi \cdot 4t$

Then the total phase of the grating signal

$$\phi_{\text{elec.}} = N\omega_s t + \frac{\pi}{2} \sin 8\pi t$$

$$\text{and } \phi_{\text{mech.}} = \omega_s t + \frac{\pi}{2N} \sin 8\pi t$$

$\therefore$ actual shaft speed

$$\omega_s' = \frac{d\phi}{dt} = \omega_s + \frac{8\pi^2}{2N} \cos 8\pi t$$

$\therefore$ if $N = 1000$

$$\omega_s' = \omega_s + \frac{1}{25} \cos 8\pi t.$$

Let the reference voltage $V_{DC} = R$

Nominal tachometer constant = K_1 volts/rad/sec.

Let actual tachometer constant $K_1' = K_1 + \Delta K_1$

where $\Delta K_1 = \hat{\Delta K_1} \sin w_s t$

Then

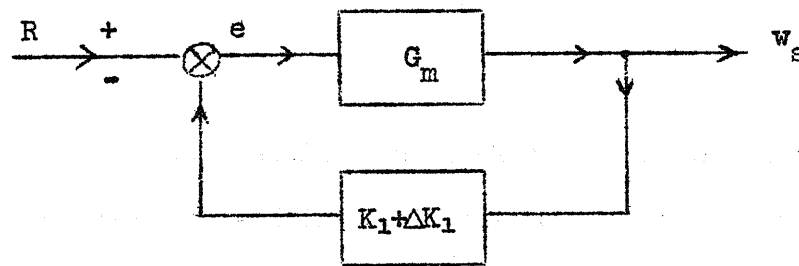


Fig. 4.11

$$w_s = G_m e$$

$$= G_m [R - (K_1 + \Delta K_1)w_s]$$

$$\therefore R = \frac{1 + G_m K_1}{G_m} w_s + \Delta K_1 w_s$$

$$= K_1 w_s + \Delta K_1 w_s$$

$$\therefore w = \frac{R}{K_1 + \Delta K_1} = \frac{R}{K_1} \cdot \frac{1}{1 + \frac{\Delta K_1}{K_1}}$$

$$= \frac{R}{K_1} \left(1 - \frac{\Delta K_1}{K_1} + \dots \right)$$

$$= \frac{R}{K_1} \left(1 - \frac{\Delta K_1}{K_1} \right) \quad \text{if } \Delta K_1 \ll K_1$$

But $w = w_s' = w_s + \frac{1}{25} \cos 8\pi t$

and $w_s = \frac{R}{K_1}$ rads/sec.

Thus $\frac{R}{K_1} \cdot \frac{\Delta K_1}{K_1} = - \frac{1}{25} \cos 8\pi t \text{ rads/sec.}$

$$\therefore \frac{\hat{\Delta K_1}}{K_1} \cdot \frac{R}{K_1} = \frac{1}{25}$$

$$\therefore \frac{\hat{\Delta K_1}}{K_1} = \frac{K_1}{R} \cdot \frac{1}{25} = \frac{1}{25w_s}$$

$$= \frac{1}{25 \cdot 8\pi}$$

$$= \underline{\underline{0.16\%}}$$

This figure should not be confused with linearity. It is the variation in the tachometer constant during a cycle. It has not been possible to check this figure due to the lack of a drive with speed consistent to a degree better than .16%.

8.1 Comparison of System with and without Minor Loop

Comparison will be made for the same value of w_s .

a) No minor loop

Variation in w for constant R will be due to variation in G_m .

$\therefore$ Let $G_m' = G_m + \Delta G_m$

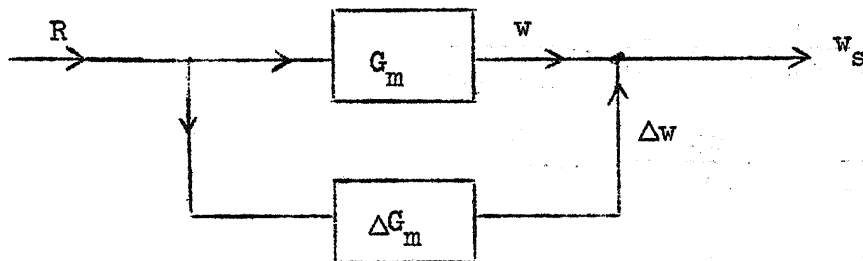


FIG. 4.12

b) Tachometer feedback

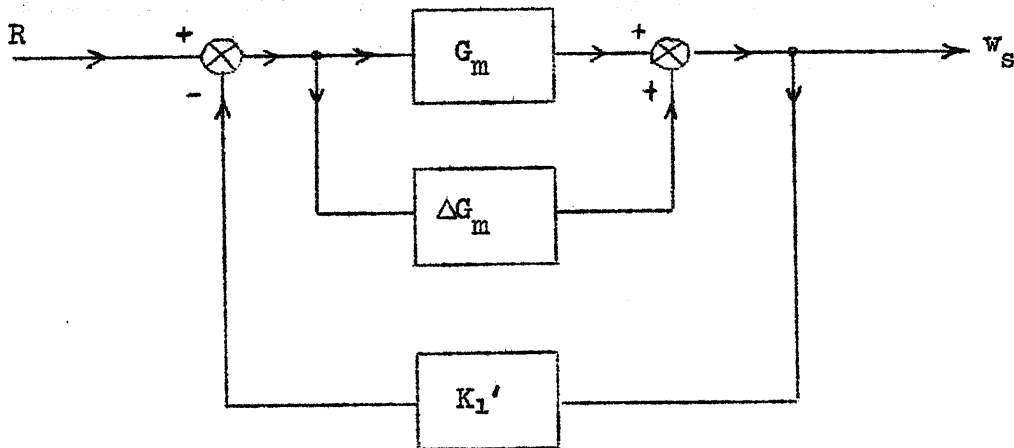


Fig. 4.13

If $G_m K_1 \gg 1$ then: -

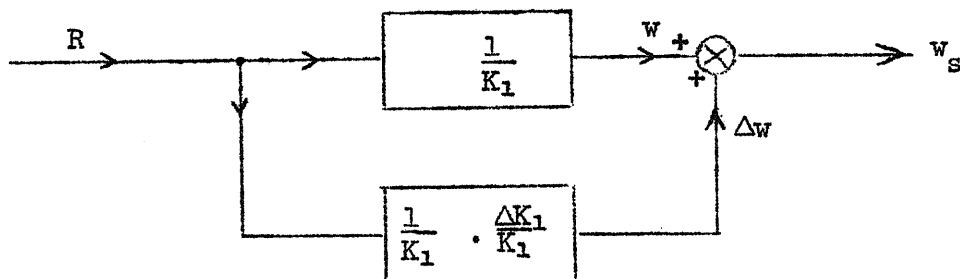


FIG. 4.14

In case a) Δw is due to ΔG_m . It is likely to be large and random.

In case b) Δw is due to ΔK_1 . It will be much smaller than the open loop case but is periodic at a relatively high frequency (4 c/s).

Thus steady state errors in the overall system will be reduced by using tachometer feedback but it is possible that the transients due to the periodic error will result in greater overall system errors. This was borne out

in practice when an early attempt to close the major loop showed better results with no tachometer feedback, despite a theoretically indicated improvement in stability.

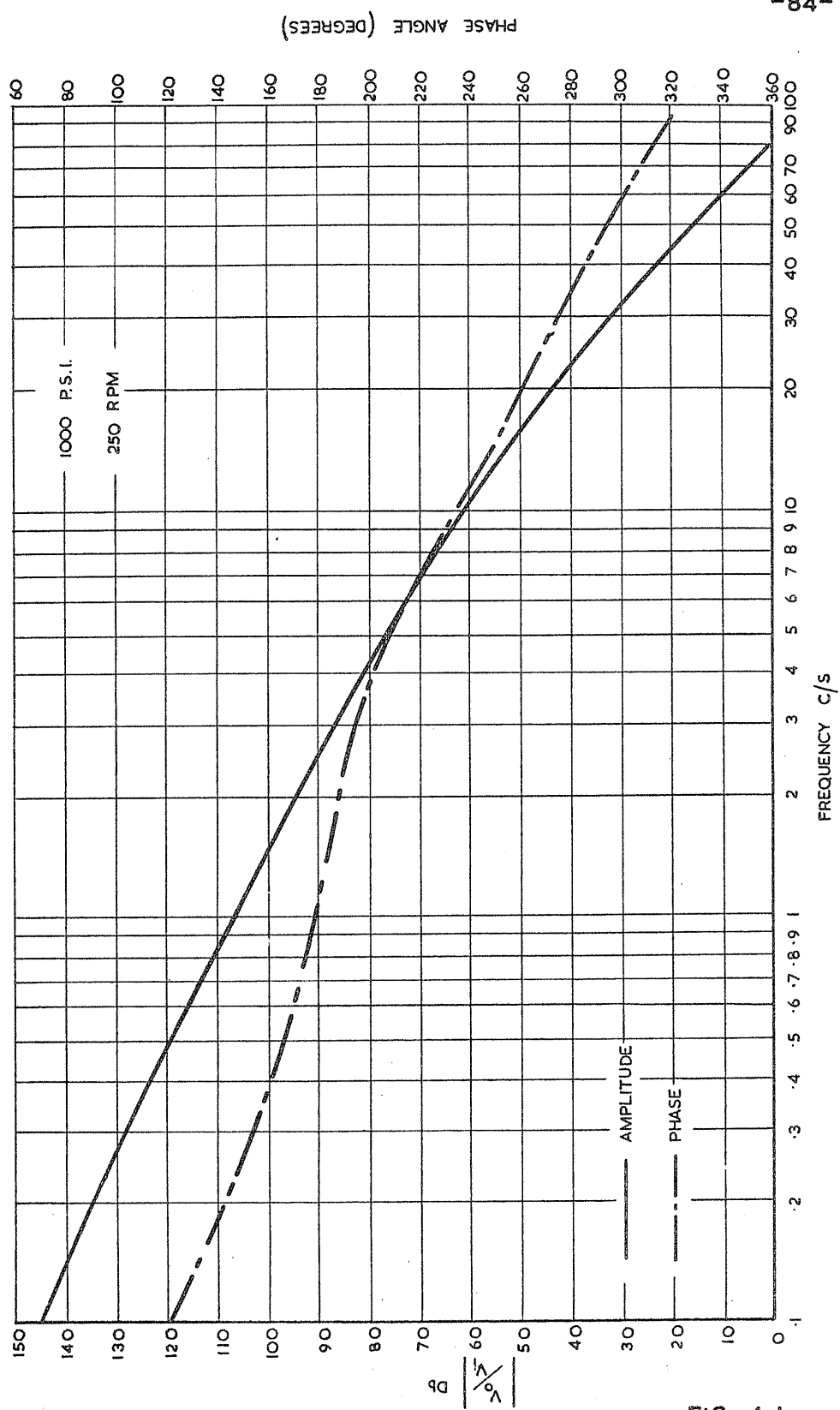
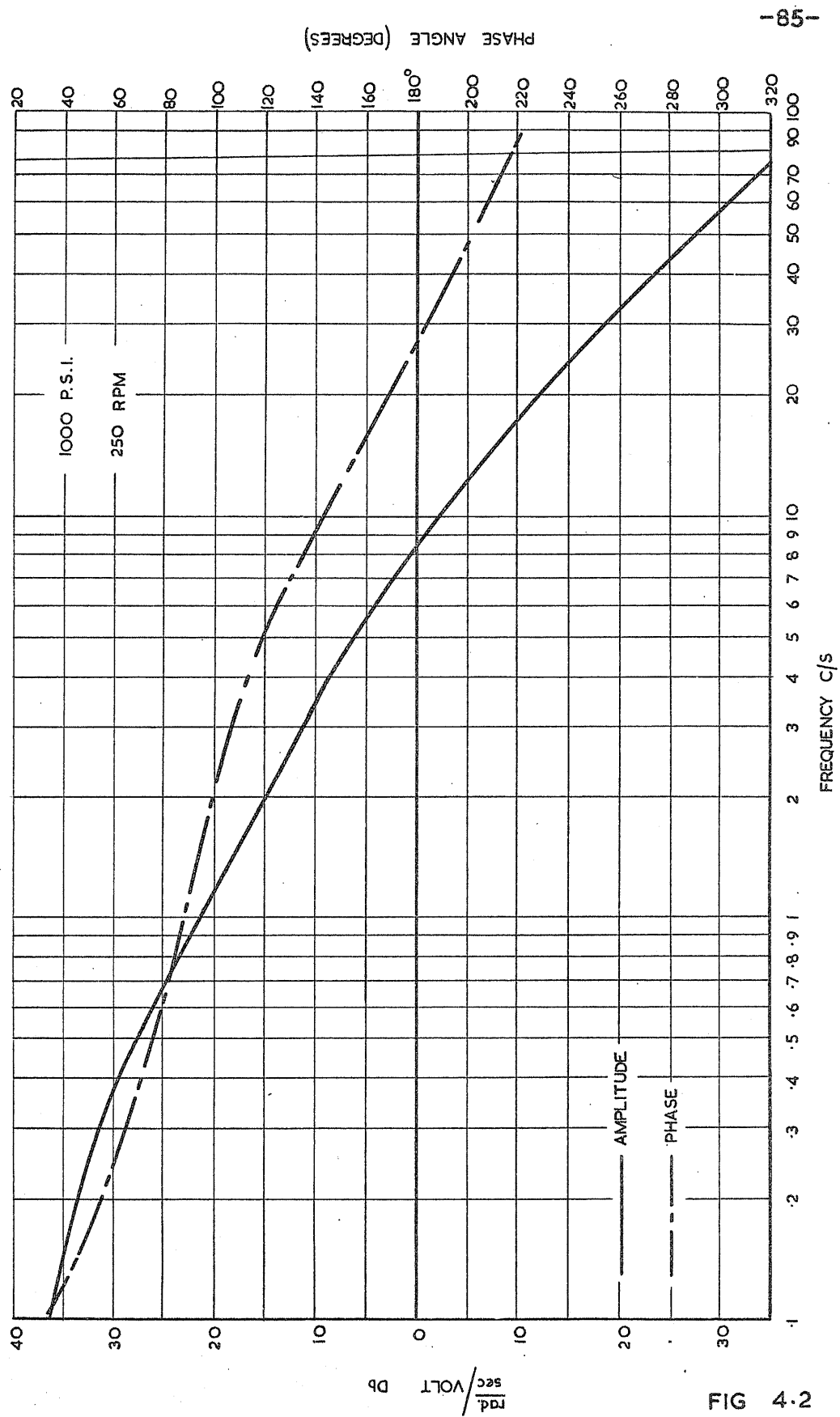


FIG 4-1

OVERALL OPEN LOOP FREQUENCY RESPONSE, $G_m G_p K^2$



CONCLUSIONS

1. Introduction

The preceeding work has shown clearly that the desired degree of accuracy cannot be achieved using the small Vickers motor with no reduction gearing. Further it has been suggested by the calculations in Part 4, p.74 as yet unverified, that the velocity lag error caused by change in speed of the reference shaft will render the desired accuracy unattainable even with perfect reduction gearing. The specification of an ideal motor was outlined in part 3, 3 p.36 and is enlarged upon below. It has been decided then to continue the investigation in order to determine just what accuracy can be obtained.

2. Results Achieved with Vickers Motor

It has proved impossible to produce a stable closed loop with sufficient open loop gain and low enough closed loop bandwidth to cause the servo to lock consistently. Note that network shaping using a tachometer for velocity feedback was not possible as indicated in part 4,8 p. 78. This motor with reduction gearing, however, may still be suitable as indicated in 3.2, p. 89.

The following is typical of the designs attempted.

The specification was lowered drastically to 45° electrical (2.7 minutes of arc mechanical) phase shift for an input torque of 1 lb-in. This was considered adequate to allow for noise inputs so that the system should 'lock' provided that no load torque is applied and that the reference frequency is kept constant.

$$\text{Thus } K_4 = \frac{\text{Allowable mech error}}{\text{max. load torque}} \quad (\text{see pp. 34-36})$$

$$= \frac{\pi}{4} \times \frac{1}{1000}$$

$$\approx 8 \times 10^{-4} \text{ radians/lb-in.}$$

$$\text{and thus } K_4 K_p = 16 \times 10^{-4}$$

$$\text{so that } K_2 = \frac{3.2}{63} \cdot \frac{10^4}{16 \cdot 10^3} = .032$$

$$= -30 \text{ db.}$$

This reduced specification corresponds to a reduction in gain of
 $32.3 + 30 = 62.3 \text{ db.}$

The resultant open loop gain, $G_m G_p K_2$ is plotted in Figure 5.1 from which it can be seen that the system still has a negative gain margin of 43 db and a negative phase margin of 55° .

Addition of the compensation network, H_2 , comprising 1 lag and 2 lead networks produces an open loop frequency response which indicates a stable closed loop.

The closed loop response, volts out per torque in, can then be derived, thus:-

$$\frac{V_o}{T} = G_T \cdot \frac{G_p}{1 + G_p \cdot G_m K_2 H_2} \quad \begin{array}{l} \text{see 5.2 p. 54.} \\ [K_1 = 0] \end{array}$$

Thus at low frequencies, when $1 < G_p G_m K_2 H_2$

$$\frac{V_o}{T} \approx \frac{G_T}{G_m K_2 H_2}$$

and at high frequencies when $1 > G_p G_m K_2 H_2$

$$\frac{V_o}{T} \approx G_T G_p$$

The approximate closed loop response is thus plotted in Fig. 5.2. It is immediately obvious that a response of this shape will produce exponential errors in response to step inputs of a magnitude far in excess of the steady state error.

In conclusion the rise in the high frequency response of the stable closed loop thus obtained can only be reduced at the expense of reduction in steady state gain. A statistical optimum response could obviously be achieved, providing a statistical knowledge of the input torque could be found, but it is obvious that even the optimum will fall sadly short. The loop described was tried practically and produced the expected result of a locked system which varied in phase in a random manner over nearly 360° and jumped a cycle of the phase discriminator about once every 10 seconds.

3. The Ideal Specification For the Motor

3.1 The basic requirement for the motor can be determined by considering a given closed loop gain, i.e. K_4 max. error/lb-in load torque, and thus finding the minimum open loop gain, i.e. $G_m G_p K_2$ to give this closed loop gain. This was discussed initially in part 3, 3 p. 36, where it was shown that the open loop gain required is inversely proportional to something between d_m and d_m^2 . It was also shown in part 4, 6.1, pp. 65 that the open loop gain for a given K_4 is inversely proportional to the gear ratio n .

Ideally then the product $n \cdot d_m$ should be as high as possible. The practical limit will in fact be set by the maximum speeds of particular motors providing n does not exceed say 100 which would otherwise probably

introduce backlash problems. Other factors listed below must also be considered:-

a) Increase in dm will also increase the compressed volume of oil V , the motor inertia J_m , the motor leakage L and the viscous friction constant F_m . All these factors effect the frequency response of the open loop system. If L were to become comparable with the spool valve 'droop' K_q , it could counteract the increase in dm .

b) The larger the value of $n.dm$, the smaller will become the spool valve displacement necessary to produce the error correcting torque. Now this system is greatly simplified from many hydraulic systems in that all servo positioning action takes place about a constant shaft speed and therefore a mean spool valve opening. In other words the spool valve never works near the more critical closed position. In fact all frequency response measurements are taken as a perturbation about a mean valve opening, the result being a particularly linear servo, as shown by the high approximation to the calculated figures as indicated in Fig. 3.4. This linearity is not likely to be maintained if the changes in valve opening during servo action are infinitely small. It is obvious that mechanical stick-slip is also eliminated in any case.

3.2 Comparison of Motors Available

We will consider the two variables as n and dm .

We have shown that approximately:-

$$G_m \propto \frac{1}{n \, dm} \quad - \text{part 4, 6.1 p. 68}$$

and $K_2 \propto \frac{L_1}{n \, dm} \quad - \text{part 4, 6.1 p. 69.}$

It was indicated on p. 37 that L_1 would increase with $n \cdot dm$ making K_2 roughly constant. Thus the open loop gain required for a given value of K_4 is inversely proportional to between $n \cdot dm$ and $n \cdot dm^2$. Finally it was shown in part 4, 6.3, p. 74 that with no tacho feedback, $K_1 = 0$, then velocity lag error due to change in reference frequency w_2 is also proportional to $G_m K_2$. Thus the advantage gained in improving the stability problem, by reducing the open loop gain, increases the velocity lag errors.

The open loop frequency response should also be increased with increase in n and dm as the a and b constants in the transfer function of $G_m(s)$ i.e. p. 71 and 72 should both decrease. It is therefore proposed to compare steady state values only.

Three motors are considered, for a maximum shaft speed of 250 RPM

- the VICKERS 3906-30 as used in the preceeding tests
- a HARTMAN ROL-VANE H.T, 10
- an experimental BOULTON PAUL radial piston motor.

	VICKERS		ROL-VANE	BOULTON PAUL
$dm \text{ in}^3/\text{rev.}$	.095		1.0	2.2
maximum speed RPM	6000		1000	1500
$n = \frac{\text{max. speed}}{250}$	1	24	4	6
$\left\{ \begin{array}{l} n \cdot dm \\ n \cdot dm^2 \end{array} \right.$	.095	2.3	4	13.2
	.009	.216	4	29
$n \cdot dm$ } rationalized to VICKERS	1	24	42	140
$n \cdot dm^2$ } MOTOR with direct drive, n = 1.	1	24	445	3,200

The indicated reduction in required open loop gain of between 140 and 3,200 (i.e. 43 to 70 db) available using the Boulton-Paul motor should make it possible to design a theoretically stable closed loop.

The disadvantages are:-

- a) heating due to the excessive flow of oil, i.e. $55 \text{ in}^3/\text{sec.}$ at 1500 RPM.
- b) Non linearity being introduced due to small spool valve displacements. The value of supply pressure, P_s , should be chosen to ensure the valve being about 90% open with the motor running at maximum speed. It may be necessary at lower shaft speeds to reduce P_s and increase K_2 accordingly.
- c) Errors due to change in reference frequency, w_2 , or reference voltage, $V_{d.c.}$, will be large

4. Final Considerations of the Servo Controlled Gear Grinding Machine

In the light of the preceeding work the following points can be made.

- a) with further work using a more suitable motor and gearbox there is a small chance of making a stable loop with a steady state error less than 4 secs. of arc for 32 lbs-in. input torque, although transient errors will almost certainly be far greater.
- b) If such a system is achieved the speed control of the grinding wheel shaft will have to be better than 1% when cutting a 100 tooth gear to keep velocity lag errors below 4 secs. of arc.
- c) The problem of synchronising the system in a particular phase is going to introduce complex electronic circuits and possibly extra

stability problems.

- d) To cut helical gears the reading heads must be moved in synchronism with the cutting traverse. It was shown in part 4, 5.4, p.60 that eccentricity larger than .003" can cause a lost signal. The reading heads must therefore be moved concentric to the workpiece shaft within say .001".

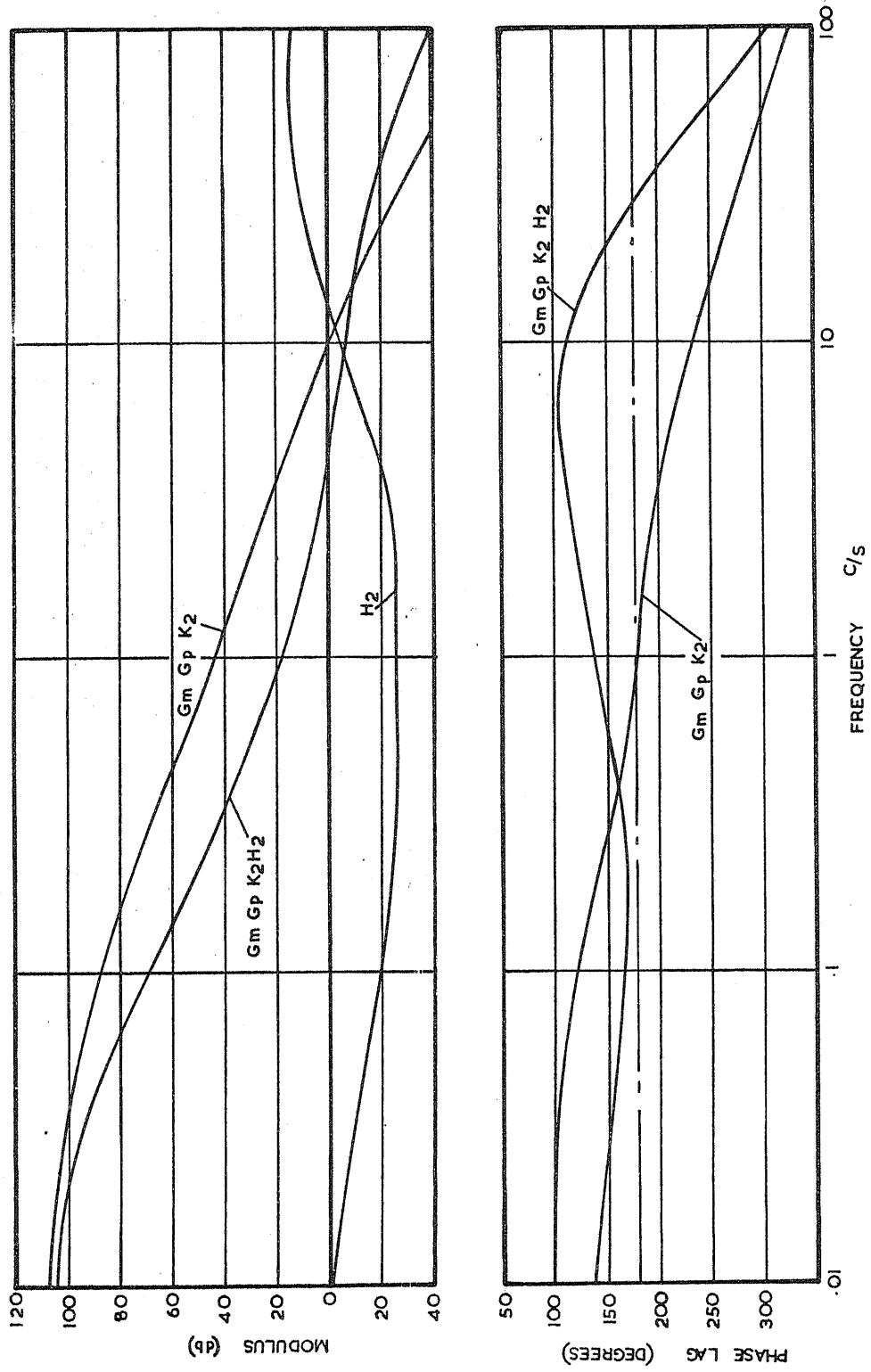
It is therefore concluded that such a form of gear grinding machine is impractical.

It is recommended that in consideration of generative gear grinding machines any further work should be directed towards an error corrected device such as the NEL - David Brown gear hobbing machine as shown in Fig. 1.1. This unfortunately throws away the advantage of changing the gear ratio by simple electronic switching only, since both frequency division and mechanical change gears are required.

5. Proposed Future Investigation

Work is at present being instigated to check different motors and gearbox ratios as previously suggested.

Final conclusions can only be drawn when it is known just how accurately the system can be locked, but it does seem possible that this system may lend itself to statistically derived optimum frequency responses.



OPEN LOOP RESPONSE USING VICKERS MOTOR

FIG 5.1

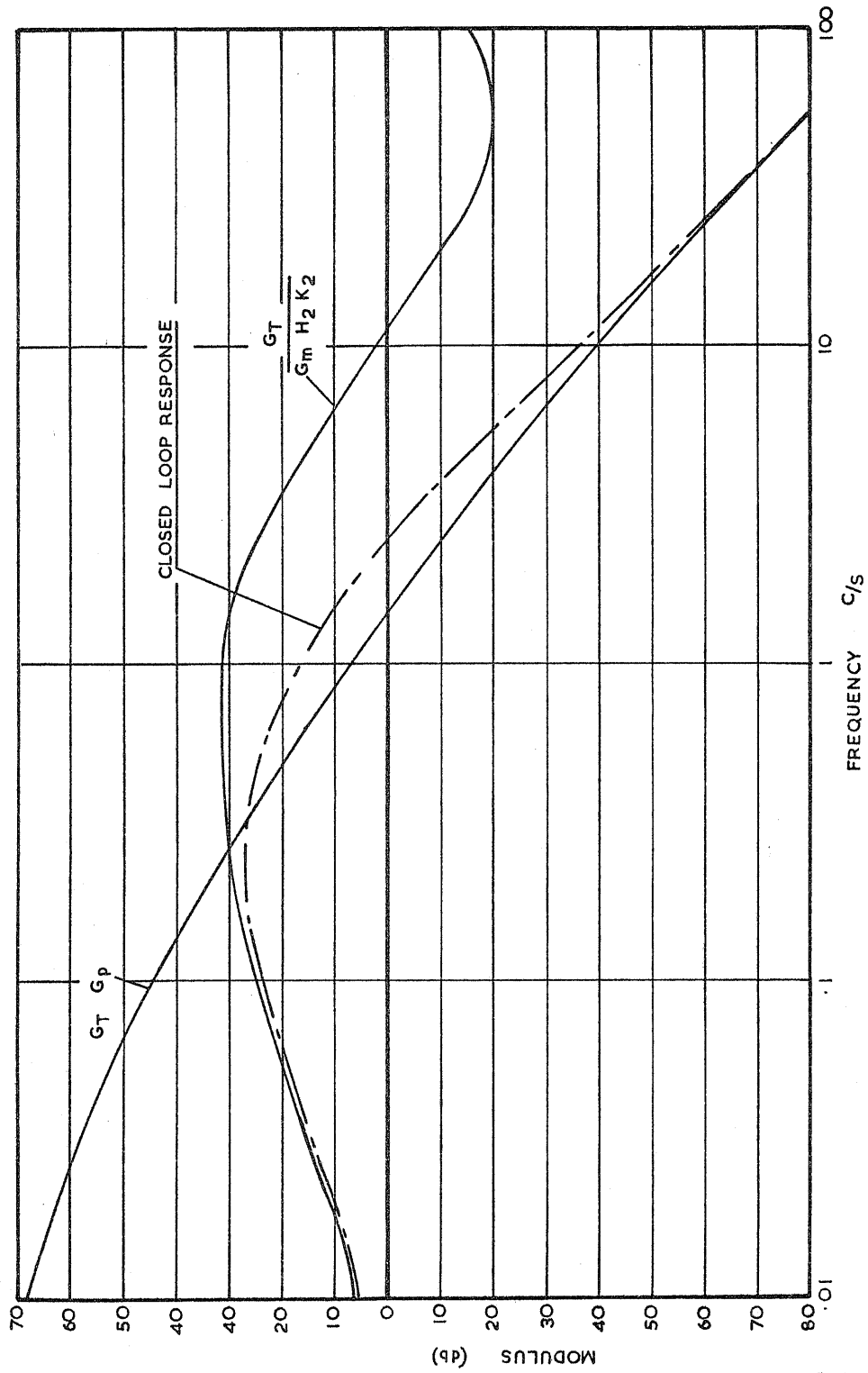


FIG 5-2

CLOSED LOOP RESPONSE USING VICKERS MOTOR

GEAR GRINDING AND GEAR GRINDING MACHINES

1. Gear Terminology

It is suggested that for details of gear design Machinery Handbook or some specialised publication should be consulted. However, certain terms are used throughout this text, the definitions of which many readers will need reminding of.

Pitch - the uniform spacing along the pitch circle of adjacent teeth.

Pitch Circle - the basic datum relative to which the gear teeth are projected.

Diametral Pitch and Module

D.P. is the number of teeth per inch of pitch circle diameter.

$$\therefore \text{D.P.} = \frac{N}{\text{P.C.D. (ins)}} = \frac{\pi}{\text{pitch}} \quad [N = \text{number of teeth}]$$

Module is the reciprocal of D.P. but with pitch circle diameter in millimetres.

$$\therefore M = \frac{\text{P.C.D. (m.m.)}}{N}$$

Cumulative Pitch Error - the difference between design length and actual length between corresponding points on teeth not adjacent.

Adjacent Pitch Error - the difference between design and actual pitch.

Helix Angle - the angle which a gear tooth makes with the generator of the cylinder on which the tooth lies.

British Standard Specifications on Gears

Machine cut gears	BS 436-40
Instrument gears	BS 9867 Pt 1 1952
Turbine gears	BS 1807 Pt 1 1952

These standards must be criticised however, since many gears, unsuitable in accuracy for aircraft use, would fulfill the specification.

2. General Considerations

Gearing was one of the earliest forms of transmitting power, preceding even the simple belt drive. The introduction of marine engines and automobiles followed by the aeroplane greatly increased the demands for better gearing, particularly with respect to reliability. The size of gearboxes also had to be reduced, particularly in aircraft and so we arrive at the modern gear which is required to be of minimum size and maximum accuracy. The size of a gear is related to the power to be transmitted and the peak tensile stresses set up in the tooth surface. The accuracy of transmission depends upon the pitch and profile errors of the gear, which are in fact not unrelated to the stresses. The effects of noise and wear are related mainly to surface finish and lubrication, although various materials such as fibre, bronze and, most usual, steel can effect these factors.

Before considering the actual gears it must be remembered that good, rigidly mounted bearings and concentric shafts are essential.

The profile of the tooth can be considered the basic variable in gearing. A correct profile should ensure line contact between two teeth in mesh at all instances, with a rolling action and thus no sliding. This is commonly arrived at by using an involute form which is generated by a basic straight sided rack moving in mesh with the gear (this factor is the basis of generative machining). Other forms such as cyclic and the more recent Novikov profiles are in use.

Here we will only consider the common spur gears, as used for straight drives, with teeth parallel to, or helical with respect to, the axis of rotation, although much of what is said is applicable to bevel, hypoid

bevel, worm and wheel gearing etc. We must also make reservations about the size of the gears since we are considering machines capable of grinding gears up to about 15" in diameter thus eliminating larger marine gears from the further discussion.

It has now been established that the following factors must be considered:

- a) Wear and noise
- b) Accuracy of transmission
- c) Failure of any tooth.

Wear and noise are dependent upon correct profiling, good surface finishes and surface hardness as well as the material used. Since we are primarily concerned with gear grinding it is necessary to consider only the common steel gear, remembering that there are different types of steel. A hardened and ground gear will best fulfill all these requirements although there are some suggestions that too good a surface finish can cause problems with inability to retain lubricants.

Accuracy of transmission depends upon pitch and profile errors as well as concentricity, bearings, etc. Again, due to the greater precision of the grinding process, ground gears are preferable.

Failure of a tooth depends upon the peak loading of the tooth and the fatigue strength of the individual tooth. It has been shown that a typical tooth bends say .001" under load so that an error of .001" could cause .002" tooth deflection. Thus the accuracy of pitch and profile are important to tooth failure. It has also been shown that due to this effect a tooth of a normal hobbed gear is likely to be subjected to as much as 10 times the peak load of a tooth on a precision ground gear when doing the same work, once

again favouring the ground gear. This however, is not so when considering the fatigue strength of gear teeth. This is mainly dependant upon the tensile stresses in the tooth surface, particularly at the tooth root. The ground gear has commonly a residual tensile stress in the tooth which reduces the allowable stress due to loading while the cut gear is normally unstressed. This point however, demands special attention since the hardening process usually employed prior to grinding sets up residual compressive stresses in the tooth which would be advantageous. These stresses are removed and replaced by tensile stresses due to the severe local heating, while grinding, which varies with the type of grinding process as described later. A College of Aeronautics report by Mr. J. Purcell has indicated that by keeping the cut of the grinding wheel to one tenth of its nominal cut at all instances i.e. no rough cuts, then some of this compressive stress can be retained. This work is yet to be verified with respect to gear grinding but promises well, despite the increase in the grinding cycle time. The possibility of grinding first and hardening second is at present unsuitable, however desirable, due to changes in the material size caused by the heating. This point is obviously under consideration, particularly with local heating by induction. It should be borne in mind that changes in size after hardening and then grinding are not impossible under certain faulty conditions.

Tests made by MIRA indicated that a tooth of a form ground gear varies from 40 to 60% of the fatigue strength of a tooth of a hobbed gear and a tooth of a generative ground gear from 50 to 80%. Thus we can conclude that a precision hardened and ground gear is about 5 times less likely to

failure under the same loading as a hobbed gear. It must be stressed that the preceding statement is very general and is based on static considerations. Some work has been done at Oxford on the dynamic behaviour of gears in mesh, but is as yet not conclusive.

The disadvantage of ground gears is obvious - COST!

3. Gear Grinding Machines

As previously stated, this article is limited to considering gears less than 15" diameter.

There are two basic types of gear grinding machine, the form grinder and the generative grinder.

The form grinder grinds one tooth at a time using a wheel pre-shaped to the tooth profile (Fig. A.1). Pitching is achieved with a dividing head so that pitch errors less than $\pm 0.0001''$ on a 10" diameter gear can be achieved. The profile accuracy is determined by the dressing of the wheel. Profile modifications are easily incorporated providing no more problem than a pure involute. Variations in this type of gear grinder, which is typified by the ORCUTT machines, include single flank grinding for greater control of pitch and rotary dividing heads, geared to the table traverse for cutting helical gears.

Generative grinders generate the profile from a basic rack; - a straight sided rack generating an involute profile. Three types exist either single tooth, reciprocating or continuous. The flank or single tooth grinder as typified by the MAAG machine sets the pitch with a dividing head and uses two saucer shaped grinding wheels which are traversed axially along the gear and reciprocated to form the gear flanks (Fig. A.3). The reciprocating motion is generated from a basic rack. Since at any instant

the saucer shaped wheel is in contact with the flank at two small areas only this process is often called point contact grinding. The Maag machine uses this point contact to eliminate coolant. Reciprocating machines use a grinding wheel having four or five Vee grooves (as a Vee belt pulley) which is reciprocated axially. The gear is also rotated in synchronism with grinding wheel axial movement thereby completely grinding one tooth and parts of adjacent teeth. The gear is then indexed and the process completed (Fig. A.4). Continuous generative gear grinders work on the same principle as a hobbing machine. The grinding wheel is also of basic rack form i.e. V-grooved, but in the form of a single start spiral. The gear is driven through a change gear box with a ratio equal to the number of teeth to be cut. (Fig. A.5). Such a machine is marketed as the MATRIX 40 and 61. The REISHAUER machine is similar except that two synchronous motors are used instead of a linked gear box. This reduces the problems of cutting helical gears which otherwise needs differential gearing.

These machines can now be compared on the following grounds:-

- a) Initial cost
- b) Accuracy of pitch
- c) Accuracy of profile including modified profiles
- d) Time to grind complete gear
- e) Problems of wheel dressing particularly for modified profiles
- f) Stressing of tooth root due to grinding 'burns'.

Any errors in pitch in reciprocating or continuous generating processes will also result in profile errors.

Obviously no one system is clearly superior on all counts, so that the following summary is a generalisation.

The cheapest and most accurate machine is probably the form grinder. This machine has the disadvantages of long grinding cycle time and high stressing of tooth root, due to the large contact area of the formed grinding wheel. The generative grinders are quicker but slightly less accurate, the Reishauer approaching the accuracy of the Orcutt but at much higher initial cost. The generated gear is usually less stressed due to lower heating since the grinding contact is a line. This is not quite true of the Maag machine since it is as accurate in pitch as the Orcutt with as long a grinding cycle time. Also the point rather than line contact, used with no coolant, in general, results in stresses higher than a Matrix ground gear but less than an Orcutt ground gear. Modified tooth profiles require complex dressing equipment for use with generative grinders but can be relatively easily incorporated in the form grinder.

In conclusion the type of gear grinding machine most suited to a particular job will depend mainly on economics which in turn are governed by batch sizes, type of gear, depreciation of machines, etc.

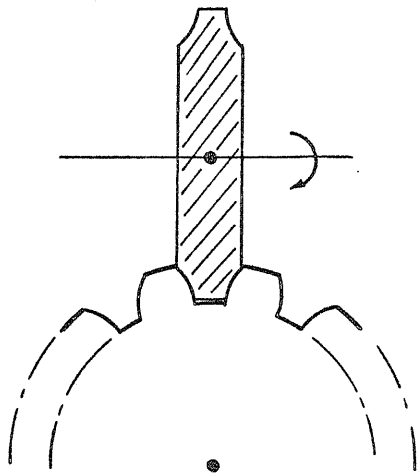
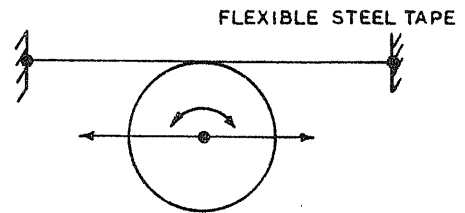


FIG A.1 FORM GRINDING



LINKAGE FOR PRODUCING
ROTARY / LINEAR MOTION

FIG A.2

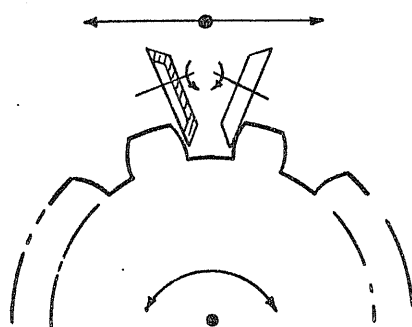
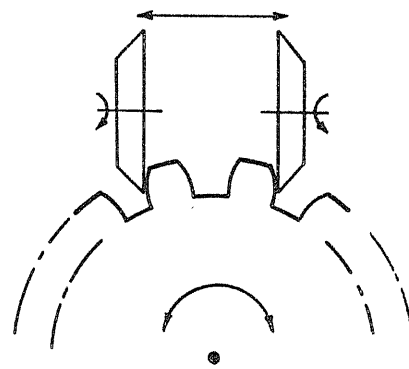


FIG A.3

OR



POINT CONTACT, FLANK GRINDING

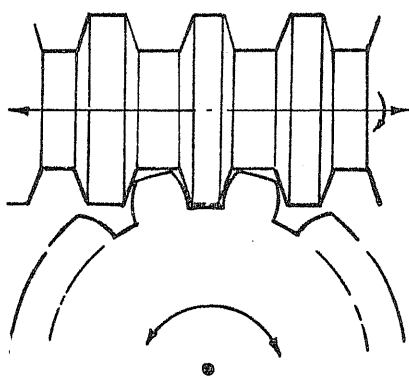


FIG A.4 RECIPROCATING GRINDING

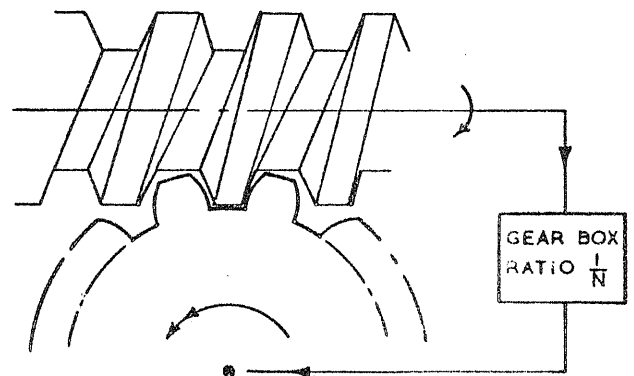


FIG A.5 CONTINUOUS GRINDING

TYPES OF GEAR GRINDING MACHINES

THE PRINCIPLES OF DIFFRACTION GRATINGS

Please refer to College of Aeronautics Note M and P 2 entitled
'Diffraction Gratings - Their Principles and Applications to Machine
Tools'.

THE FREQUENCY DIVIDER

The frequency dividing circuit employed in this system is a gating device which accepts a continuous train of pulses and passes one out for every N fed in.

The specification is:-

N is to cover the range of gears to be cut i.e. 10 to 250 teeth, the maximum value of N being in fact $2^8 = 256$.

The input frequency is to be about 40 Kc/s for use with 1,000 line gratings or 430 Kc/s for use with 10,800 line gratings.

The circuit block diagram is shown in Fig. C.1.

The roughly sinusoidal signal derived from the diffraction grating attached to the grinding wheel head is fed into a pulse shaping network to produce a continuous train of pulses at the same frequency as the input. The frequency divider comprises 8 binary elements connected in cascade, each via an 'and' gate. The first gate connecting Binaries 1 and 2 is a '2¹' gate; the second gate connecting Binaries 2 and 3 is a '3¹' gate; the third gate is a '4¹' gate; and so on.

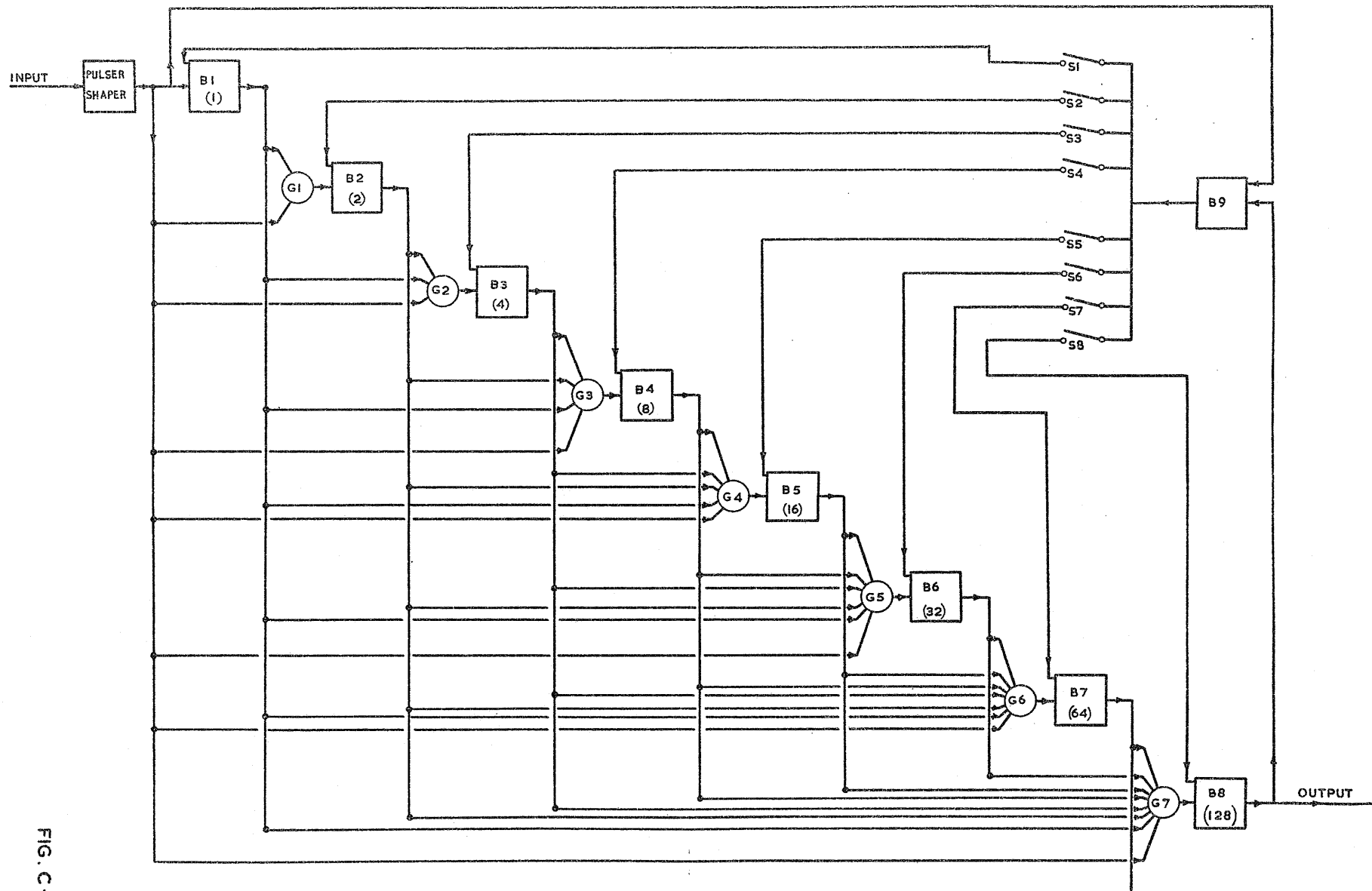
Consider then that all binaries are in the '0' state initially. The first pulse will set B1 to the '1' state so that the second pulse fed to G1, will complete the inputs to that gate, thus triggering B2 to the '1' state. This same pulse will also put B1 back to the '0' state. 127 pulses will set B1 to B7 in the '1' state. The next pulse will complete all 8 inputs to G7, triggering B8 to the '1' state and B1 to B7 back to the '0' state. B8 changing from the '0' to the '1' state produces an output pulse. A further 128 pulses will put B8 back to the '0' state and another 128 to the '1' state again. Note then that an output pulse

occurs on the 128th pulse but for every 256 pulses thereafter.

To reduce the division ratio from 256 it is simply necessary to set the appropriate binaries to the '1' state instead of starting with all binaries set at the '0' state. This is done each cycle by feeding the output pulse via the switches S1 to S8 to the appropriate binary so that with all switches closed, an output pulse will set all binaries to the '1' state ensuring that they are always in this state. Thus every input pulse produces an output pulse. To divide therefore by a number N, the binary number corresponding to 256-N must be set up after, in fact by, each output pulse. This is equivalent to making the number represented by the switches left open equal to N-1.

The feedback pulse is derived from B9 which is triggered to the '1' state by the output pulse and switched back to the '0' state by the next input pulse.

This unit is basically constructed with VENNERS ELECTRONICS plug in sub-assemblies and was designed and constructed by ELLIOT BROS. of Boreham Wood.



FREQUENCY DIVIDER CIRCUIT

CODE
B BINARY UNIT
G 'AND' GATE

THE PHASE DISCRIMINATOR

The discriminator used in this system is a pulse phase comparator. The circuit layout is shown in Fig. D.1.

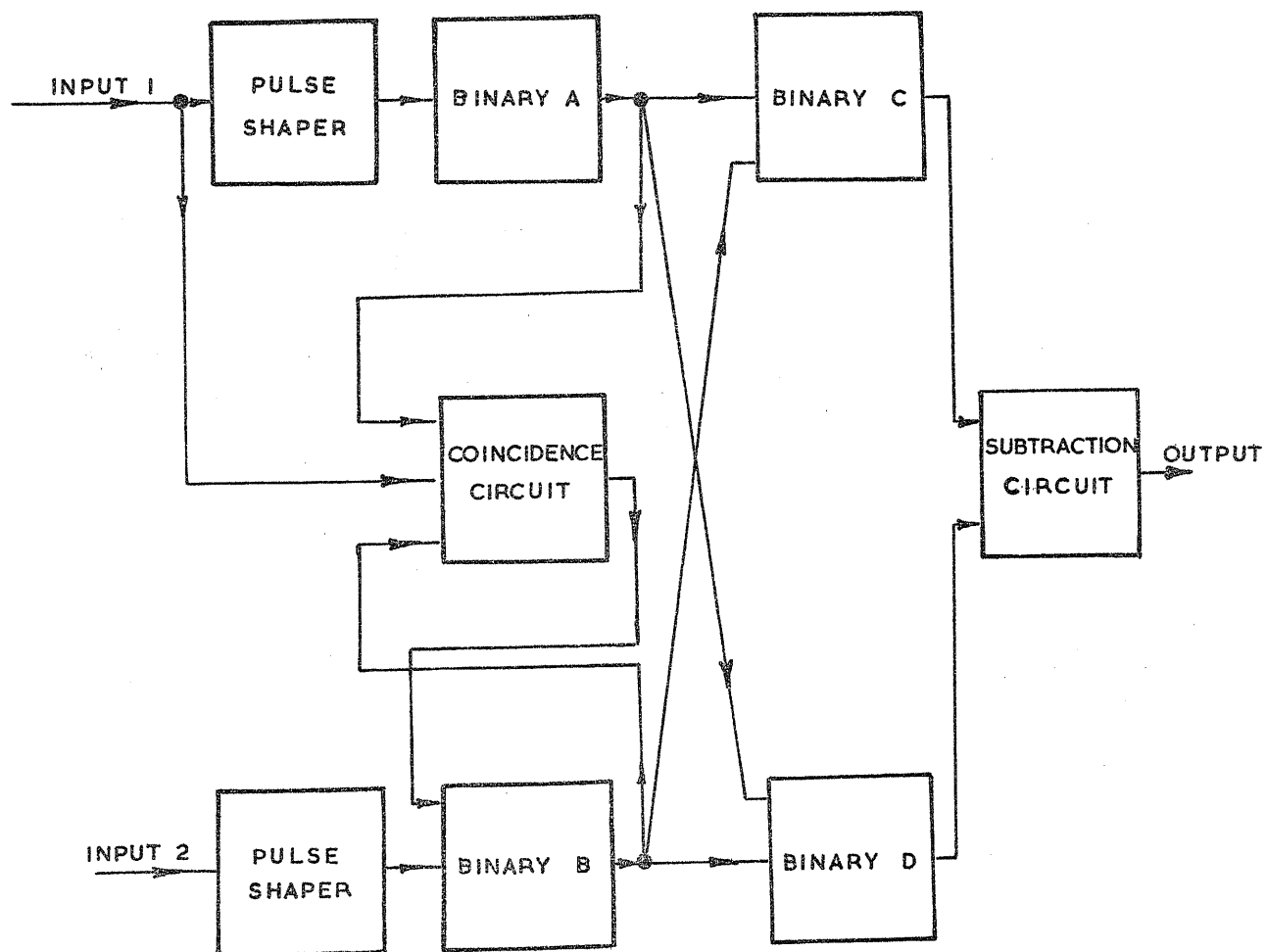
The two input waveforms to be compared, which must be of similar frequency and which will be roughly sinusoidal, are fed into pulse shaping networks to convert the signals into trains of positive going rectangular pulses. The pulses are fed into bistable units labelled binaries A and B which convert the pulse trains into equal mark-space ratio square waves at half the input frequency. The square waves are fed to binaries C and D such that C is triggered on by the rising edge of the output from A and triggered off by the trailing edge of B. Similarly D is triggered on by the rising edge of B and off by the trailing edge of A. The output is derived by subtracting D from C.

The waveforms produced are shown in Figs. D.2; D.3; and D.4.

It can be noted from these diagrams that V_o is proportional to the phase between waveforms A and B over a range of $\pm 180^\circ$ as shown in fig. D.6(a). However, since A and B are at half the input frequency, this represents a phase range of $\pm 360^\circ$ between the input waveforms thus giving 2 distinct values for the output for a phase difference of ϕ between the input waves, as shown on fig. D.6(a). Thus we must limit the range of the comparator to $\pm 90^\circ$ between A and B. This is achieved by the coincidence circuit, which introduces an extra pulse into B whenever the phase difference exceeds $\pm 90^\circ$. The state of B is sampled midway through the 'on' state of A at which time B should also be in the 'on' state, as in figs. D.2 and D.3. Should B be in the 'off' state as in fig. D.4 an extra pulse is developed and fed into B, putting it into the 'on' state.

Thus the switching state of D.4 will be changed to that of D.3 as shown in D.5.

This unit is basically constructed with Venner Electronics plug in sub-assemblies and was designed and constructed by ELLIOT BROS. of Boreham Wood.



THE PHASE DISCRIMINATOR

FIG. D · I

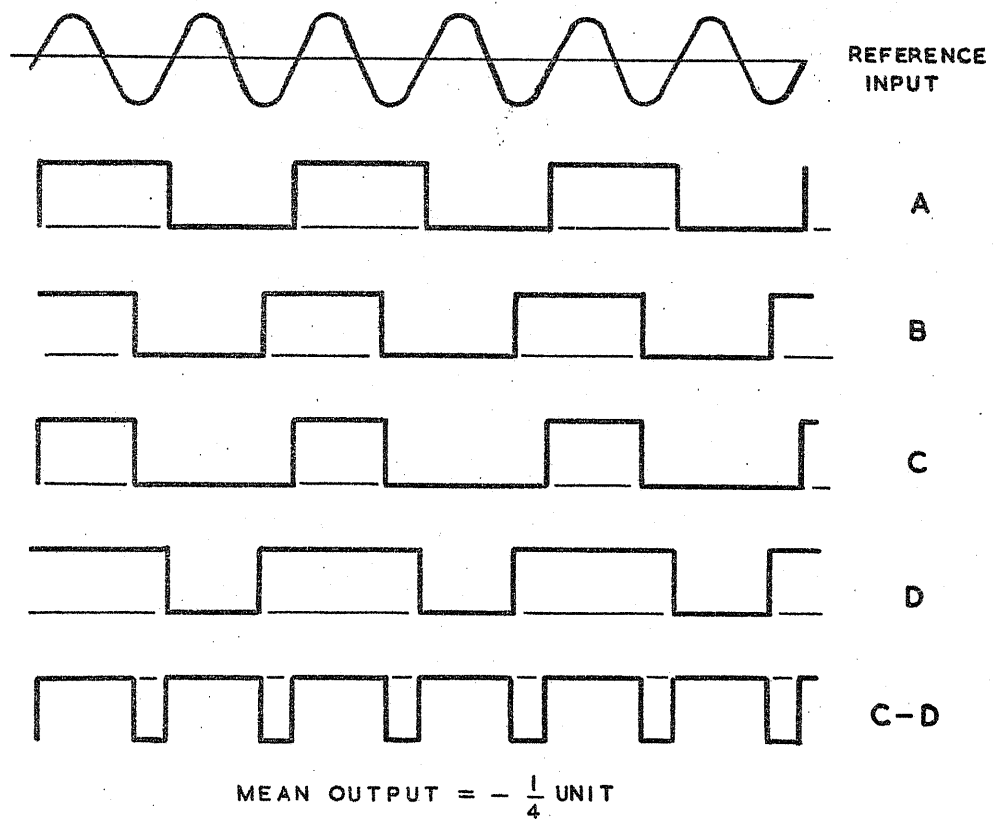
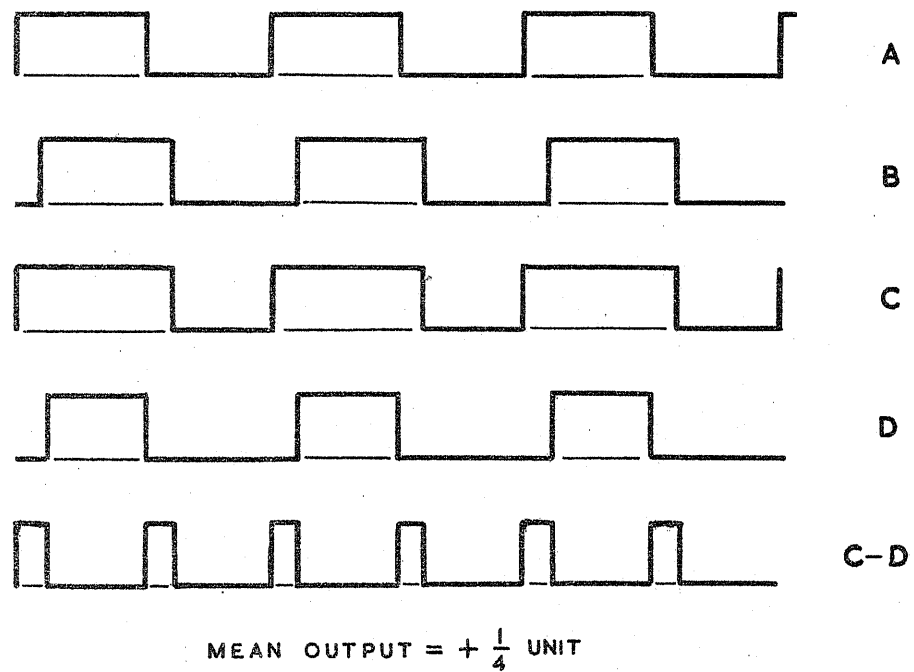


FIG D-2



SWITCHING WAVEFORMS

FIG D-3

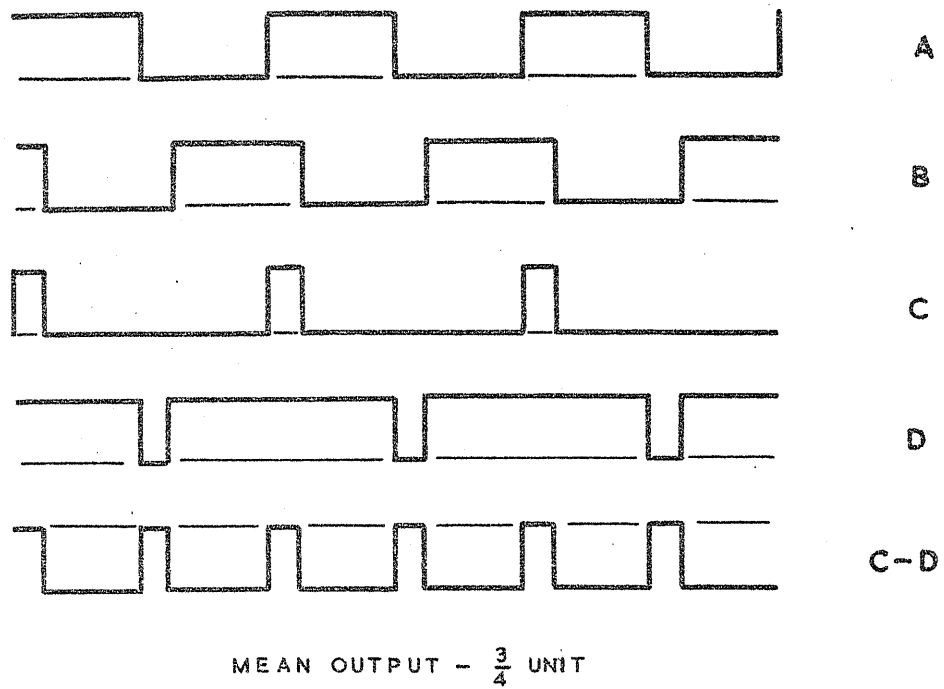


FIG D-4

WITH COINCIDENCE CIRCUIT

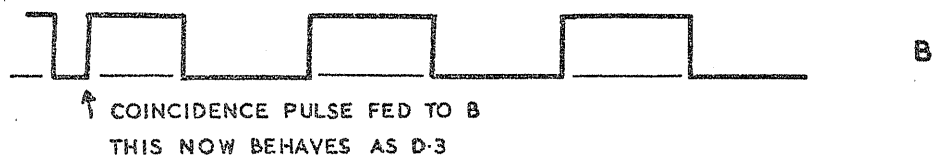
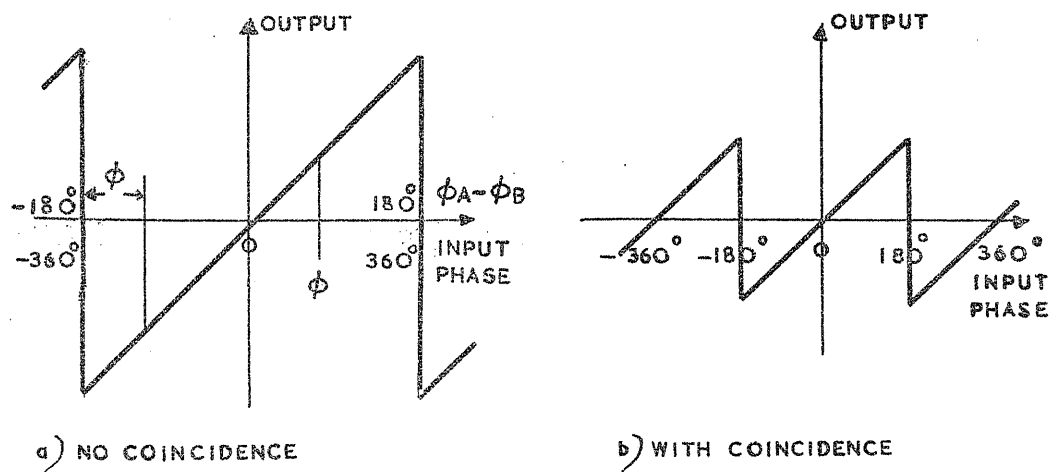


FIG D-5



PHASE DISCRIMINATOR OUTPUT

FIG D-6

APPENDIX E

A SUGGESTED SCHEME FOR SYNCHRONISING THE SYSTEM IN A PARTICULAR PHASE

Using a grating with N lines it would be possible to synchronise the system in any one of N different phases with no ability to detect any difference between any of these states. It is likely that finish machining of rough cut gears will be undertaken and also that a gear may be removed for measurement and replaced for finishing. Both conditions will require synchronisation in a one specific phase.

A possible scheme for achieving this is outlined below.

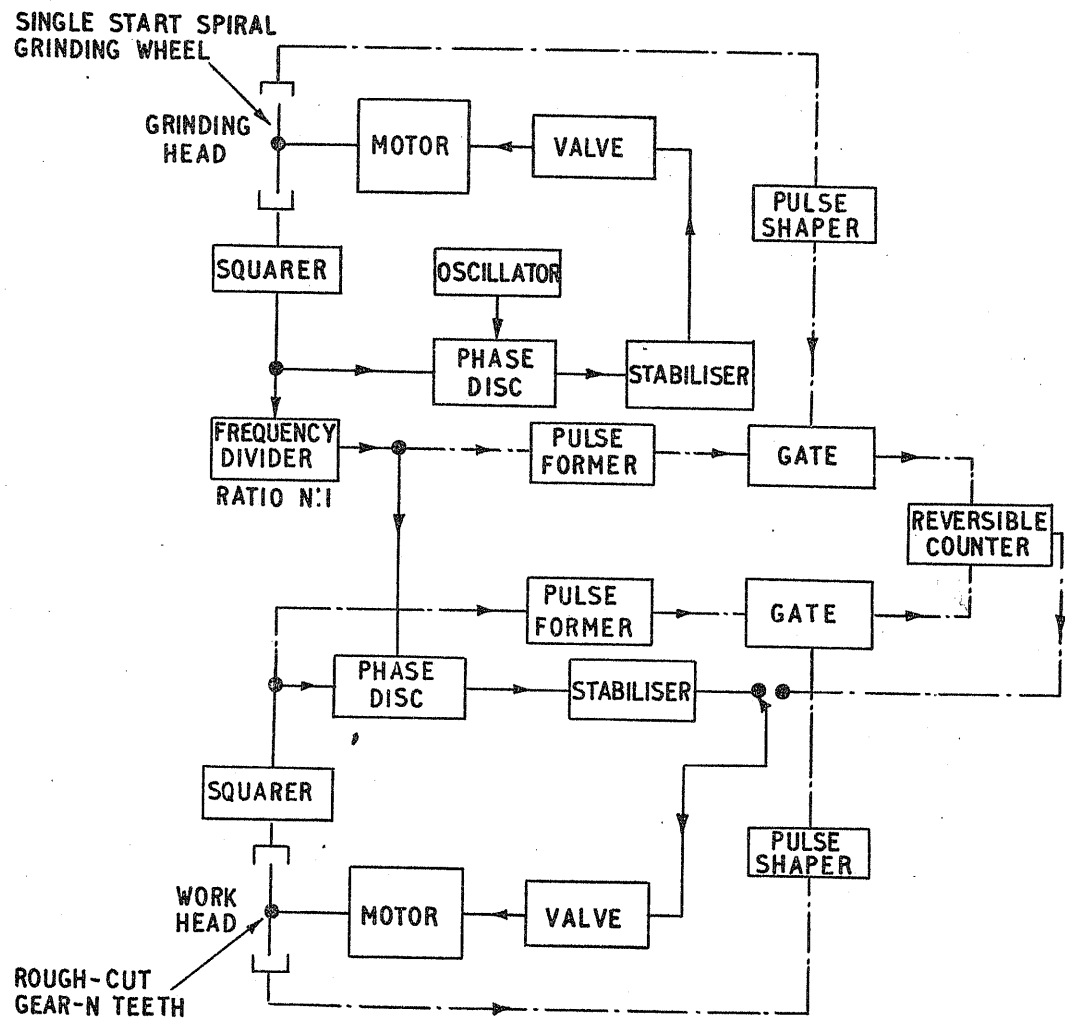
Each grating must carry one extra mark, indicating a datum point in the cycle. It would be desirable that the grinding wheel spindle be kept rotating at constant speed at all times, so that the work head spindle must be brought up to roughly the required speed. When this is achieved the 'datum' mark on the grinding head grating will produce a pulse which opens a 'gate' thus passing pulses from the grating proper through a frequency divider, into a store. The 'datum' mark on the work head grating also opens a 'gate' feeding its own grating signals direct into the same store in such a way as to subtract from the frequency divided pulses from the grinding head. The resultant number of pulses in the store actuates the servo valve on work head motor in such a way as to reduce the number of pulses in the store. When the number of pulses in the store is reduced to zero a relay would change the servo-valve over to the output from the phase discriminator to function as previously discussed.

It is very likely however, that the datum points as set by the

extra marks on the grating are not always convenient so that datum shift can be achieved by feeding in more add or subtract pulses from an external source into the store. This of course is limited to increments of $\frac{360}{N}$ degrees of the workpiece.

In making the store, which would be a reversible binary counter, it would be desirable to make the number of lines on the grating, N , equal to 2^r where r is a whole number. A suitable value would be $r = 10$ giving a grating with 1024 lines. Such a grating would be suitable for the rest of the system, availability being the deciding factor.

In the included diagram a further refinement has been indicated in the inclusion of a speed control of the grinding wheel head by locking the grating signal to a base oscillator frequency. Such a system is more refined and far more accurate than a normal speed controller, but is probably difficult to stabilise.



BLOCK DIAGRAM OF PROPOSED CONTROL SYSTEM

FIG. E.1.