



Estimation of residual stress in carbon fibre composite laminate using the contour method

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ARTICLE INFO

Keywords:

Carbon-fibre-reinforced polymers
Laminate
Residual stress
Contour method

ABSTRACT

Experimental analysis of residual stress in carbon-fibre-reinforced polymer (CFRP) composites is rare owing to the difficulties of determining residual stress accurately in these materials. This can lead to non-conservatism in design and poor understanding of potential failure mechanisms. In this study we have determined the residual stress in a cross-ply CFRP laminate. For the first time, the contour method of residual stress measurement was applied using wire electric-discharge machining (WEDM), the process that is accepted as the best way to ensure an accurate relaxed surface profile is obtained to back calculate the residual stress. A novel incremental linear smoothing approach was introduced for the measured displacement data which gave better approximation than the conventional spline smoothing method used in the standard contour method for metals. Measurements show tensile residual stress in the direction transverse to the fibres with a maximum value of 40 MPa, and compressive residual stress along the fibre direction with a maximum value of -130 MPa. The accuracy of the calculated residual stress was improved by considering the measured thickness of each ply cluster rather than the nominal thickness. The results of modified contour method are compared with an analytical solution based on the classical laminate theory.

1. Introduction

Carbon-fibre-reinforced polymer (CFRP) composites play a vital role in the aerospace, automotive and rail transport sectors due to their light weight, hence lower fuel consumptions. CFRPs also offer much higher resistance to fatigue load and are corrosion resistant. Modern commercial airliners are composed of up to 50% of composite materials [1]. Composite materials are highly anisotropic in terms of their elastic properties and coefficients of thermal expansion, which are the main sources for generation of residual stress [2]. The reliable and accurate estimation of residual stress is of paramount importance for the integrity of engineering structures. Residual stresses in composites can cause dimensional distortion, warpage, interface debonding, delamination, or cracks within the matrix [3]; all can result in premature failures. Residual stress may also cause micro-cracking in the matrix that can reduce material strength causing premature failure. Such defects may be more deleterious in thin laminates [4]. Non-consideration of the residual stress in structural design is often compensated by a higher safety factor, which can result in weight penalty due to overly conservative design.

In composites, residual stresses are generated due to chemical and

thermal histories, owing to the curing process at elevated temperature and the temperature decrease during the solidification. As a result, the layers shrink differently due to mismatch in thermal and elastic properties. Since all the layers are bonded together, thermal residual stresses are developed [4].

In brief, the residual stress develops within following scales:

- i) Micro-scale (within a ply): this is caused by the mismatch in coefficients of thermal expansion between the matrix and the fibres [5]. During the cooling stage, the volumetric shrinkage of the matrix is higher than the fibres [2], as the fibre has much lower coefficient of thermal expansion than the matrix, by an order of magnitude. Therefore, the resulting thermal residual stresses are compressive in the fibres and tensile in the matrix [5].
- ii) Macro-scale (ply-to-ply): during the cooling stage, the deformation of one ply is restricted by the other plies having different fibre orientations. Therefore, residual stresses are formed in each ply. Various factors affect the magnitude of residual stress generated at the macro level such as [2,4]: temperature difference between the curing and ambient temperatures; fibre volume

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<https://doi.org/10.1016/j.compstruct.2026.120147>

Received 17 June 2025; Received in revised form 22 January 2026; Accepted 8 February 2026

Available online 10 February 2026

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Table 1
Overview of residual stress measurement techniques applied to composites.

Technique (type/scale)	Application and key results	Comments
Contour method (destructive, macro-scale) [9,10]	<ul style="list-style-type: none"> Cross-ply T700S 24 K MTM46, $[90_n, 0_n]_s$ sample, size $145 \times 36 \times 7.4 \text{ mm}^3$ [9] Cut by diamond wire Data smoothing by median filter approach Stress along the fibre: up to -200 MPa Stress transverse to fibre: 10 to 25 MPa Cross-ply layup $[0^\circ, 90^\circ]_{ss}$ [10, Ch.4] Cut by diamond wire Data smoothing by median filter approach Stress along the fibre: -200 to 400 MPa Stress transverse to fibre: 25 to 30 MPa 	<p>The only study [9,10] previously using the contour method, but with a mechanical cut that is likely to have affected the results. Measured stress might be higher than the thermal process induced as the unsymmetrical layup might cause wrapping. A very high magnitude of residual stress (-200 to 400 MPa) along the fibre reported in [10, Ch.4] might pertain to limitations of the data smoothing approach. There is no previous literature available on application of incremental linear smoothing for the contour method. Also, no open literature on wire electric-discharge machining of CFRPs for contour method. Only one paper on metal-CFRP joints [11].</p> <p>IHD combined with 3D finite element model.</p>
Incremental hole drilling (IHD) (semi-destructive, macro-scale) [12]	<ul style="list-style-type: none"> Cross-ply $[0^\circ, 90^\circ]_s$ T300/914 Stress along the fibre: -40 to -60 MPa Stress transverse to fibre: 50 to 100 MPa 	
Deep hole drilling (DHD) (destructive, macro-scale) [13]	<ul style="list-style-type: none"> Tenax HTS 5131 NCF/Hex Ply M36, $200 \times 200 \times 22 \text{ mm}^3$ NCF types 7, 2A, 2B Stress along the fibre (interface with surface ply): 5 to 40 MPa Stress transverse to fibre (surface ply): Less than 10 MPa 	DHD measured residual stress in isotropic materials but can also be applied to orthotropic materials like thick laminated composites.
Layer removal method (destructive, macro-scale) [8,14]	<ul style="list-style-type: none"> Toray T700 UD HS SE84 LV $[0^\circ, 90^\circ]_{4s}$, size $300 \times 30 \times 10 \text{ mm}^3$ [8] Stress along the fibre: -12 to -35 MPa Stress transverse to fibre: 18 to 33 MPa SE 84LV carbon epoxy, $80 \times 10 \text{ mm}^2$ [14] $[0^\circ, 90^\circ]_s$, 2.56 mm thick Stress along the fibre: up to -35 MPa Stress transverse to fibre: up to 35 MPa $[0^\circ, 90^\circ, 0^\circ, 90^\circ]_s$, 5.12 mm thick Stress along the fibre: up to -120 MPa Stress transverse to fibre: up to 115 MPa 	Similar elastic properties through the thickness were used which resulted in complicated stress profile [14].
Slitting method (destructive, macro-scale) [4,8]	<ul style="list-style-type: none"> Toray T700 UD HS SE84 LV $[0^\circ, 90^\circ]_{4s}$, size $300 \times 30 \times 10 \text{ mm}^3$ [8] Drill used for cutting Stress along the fibre direction: -22 to -38 MPa Stress transverse to fibre: 22 to 38 MPa 	The constant stress approximation gave the best result [8].

Table 1 (continued)

Technique (type/scale)	Application and key results	Comments
	<ul style="list-style-type: none"> T300 LY-5052 Aradure-830 $[0^\circ, 90^\circ]_s$, size $60 \times 20 \times 4.8 \text{ mm}^3$ [4, Ch. 5] A mechanical cutter used Stress along the fibre: -12 to -30 MPa Stress transverse to fibre: 5 to 35 MPa 	
Curvature method (destructive, macro-scale) [14]	<ul style="list-style-type: none"> CFRP SE 84LV $[0^\circ, 90^\circ]$ and $[0^\circ, 90^\circ]_s$, size $300 \times 300 \text{ mm}^2$, individual ply thickness 0.32 mm 29 MPa in $[0^\circ, 90^\circ]$ 50 MPa in $[0^\circ, 90^\circ]_s$ 	The greater the curvature of a composite material of specific thickness, the higher the magnitude of residual stress.
Cure referencing method (non-destructive, micro/macro-scale) [15]	<ul style="list-style-type: none"> AS4/3501-6 graphite epoxy Balanced $[0^\circ, 90^\circ, 0^\circ, 90^\circ]_s$ Stress along the fibre: -39 MPa Stress transverse to fibre: 43 MPa Unbalanced $[0^\circ, 90^\circ, 0^\circ, 90^\circ]_s$ Stress along the fibre: -101 MPa Stress transverse to fibre: 40 MPa 	This method is applicable to laminates that are purposely made for measuring residual stress.

fraction; fibre waviness; curing and cooling rate; thickness and fibre orientation of each layer in the laminate; lamination sequence; coefficient of thermal expansion of each layer in the longitudinal and transverse directions; and the mechanical properties of each layer.

- iii) Global-scale (throughout laminate thickness): residual stress at this scale develops throughout the thickness of laminate due to a difference in cooling rate and the temperature or moisture gradient. For example, thicker laminates will experience slower cooling rate at the centre region than the surface [6].

In general, a laminate of orthogonal layup leads to the highest level of residual stress due to mismatch of mechanical and thermal properties. The tensile stress in the resin increases its predisposition to micro-cracking. On the other hand, little or no tensile stress, or having compressive stress in the fibres, reduce their tendency to fracture under tension.

In terms of experiments, CFRP presents a challenge for the experimental determination of residual stress due to its non-crystalline microstructure and highly anisotropic material properties. Non-destructive experimental techniques such as X-ray and neutron diffraction therefore become inapplicable. A few destructive/semi-destructive experimental techniques have been applied, such as incremental hole drilling [7], deep hole drilling, layer removal, and incremental slitting methods [2,8]. A brief overview of various residual stress measurement techniques applied to composites is presented in Table 1. In summary, 10 to 100 MPa in transverse to fibre and -12 to -200 MPa along the fibre are the typical stress ranges reported.

Analytical methods such as classical laminate theory (CLT) and numerical simulations such as finite element method (FEM) have been used for residual stress evaluation. However, analytical approaches are less accurate and only applicable to simple geometries or layup sequences. The challenge to the robustness of numerical methods is the need to account for the resin properties and non-linear material behaviour, which are time- and temperature-dependent [4, Ch. 8]. A finite element study based on visco-elastic cure model and temperature independent material properties predicted -70 MPa along the fibre direction and 31 MPa transverse to the fibre for a cross-ply laminate $[0^\circ, 90^\circ]_{4s}$ 330×300

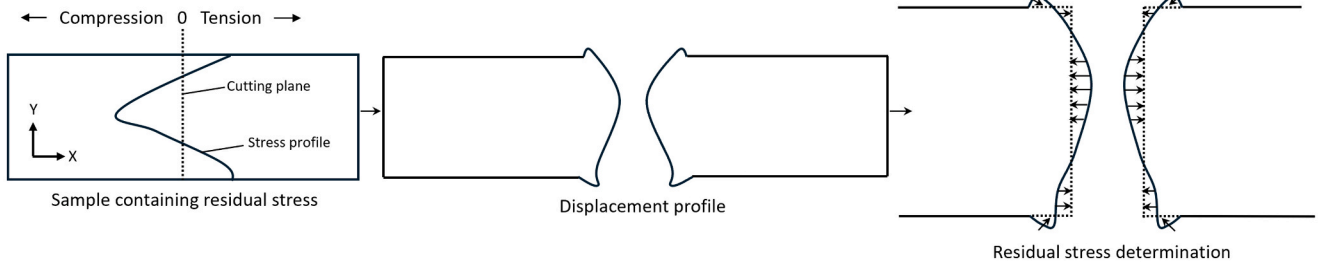


Fig. 1. Illustration of the principle of the contour method adapted from [17].

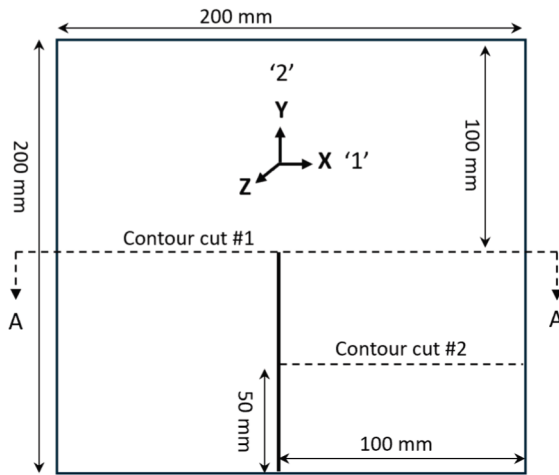
mm² made of T300/5222A carbon fibre/epoxy prepreg with ply thickness 0.125 mm [16].

This paper reports a study of residual stress measurement performed on a CFRP cross-ply laminate $[0_4, 90_4, 0_4, 90_4, 0_4]$ using the modified contour method. The contour method will be introduced in Section 2: it has been used for metallic parts since its invention two decades ago [17]. This paper represents the first application of the contour method to carbon-fibre composites by using wire electric-discharge machining (WEDM). Conventionally, 3-D polynomial/spline smoothing is used for the surface displacement data in contour method. In this study an incremental linear smoothing approach is proposed that works in 1-D i.e., along the laminate thickness, considering the distribution of surface

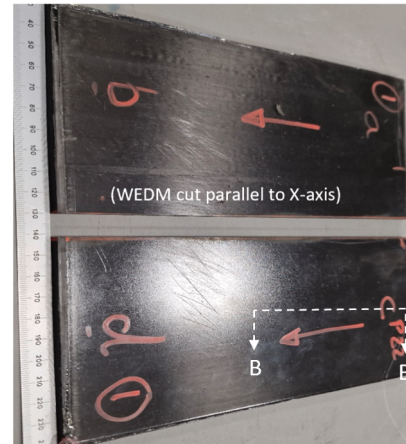
displacements of a cluster of the same ply orientation and the gradients available at interface of cross-ply. Measurement is compared with a solution using the classical laminate theory.

2. Contour method

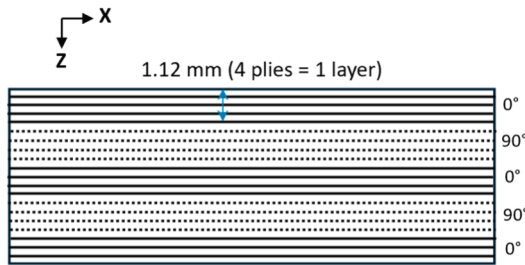
The contour method is a relaxation technique, based on a variant of Bueckner's elastic superposition principle [17]. Bueckner's principle states that "if a cracked body subject to external loading or prescribed displacements at the boundary has forces applied to the crack surface to close the crack together, these forces must be equivalent to the stress distribution in an un-cracked body of same geometry subject to the



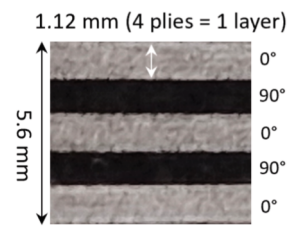
(a)



(b)



(c)



(d)

Fig. 2. (a) sketch showing locations for contour cuts of sample, (b) contour cut #1, (c) & (d) ply orientations through the thickness in relation to the global coordinate X-Z. Note that a "layer" is defined as a cluster of 4 plies of the same orientation.

external loading.” The contour method can determine residual stress normal to the cut-surface in a 2D plane. The principle of the contour method is illustrated in Fig. 1. The main steps involved in the contour method are shown below:

- A single flat cut is performed, to relax the stress component normal to the cut surface. The contour method uses a wire electric-discharge machine (WEDM) to cut the sample of interest into two halves. Re-cutting the material removes the relaxed layer so it is avoided.
- The displacements of the cut surfaces are measured with a co-ordinate measuring machine (CMM). The measured contours/displacements are averaged and smoothed with a curve fitting program.
- The reverse of the smoothed contours is applied as displacement boundary conditions to a 3D finite-element model of one of the cut surfaces. Since the measured displacements are small, therefore the cut part is assumed flat (contrary to Fig. 1) to ease with the build of the FE model. To avoid any rigid body motion, constraints are applied to the corner nodes of the FE model, and a linear-elastic analysis is performed to obtain the original residual stress present in the sample.

The contour method has been applied to a wide variety of metal structures to determine residual stress associated with various processes such as welding [18,19], additive manufacturing [20,21], peening [22], etc. The primary factor in successful application of the contour method remains the cutting technique. Wire electric-discharge machining is used to ensure that a surface is produced that reflects the relaxed strain field, with minimal material removal and no introduction of additional residual stress. Wire EDM remains the most accurate cutting method currently available when compared with other techniques such as mechanical, laser, or water jet cutting, in terms of minimising the cutting errors. Another advantage of using wire EDM for the contour method is that it can perform the cut in a single pass without the need for re-cutting the material, thereby preserving the relaxed contour profile. In contrast, other cutting methods often require multiple passes or re-cut to achieve the desired cut or surface finish. Wire EDM is applicable to electrically conductive materials, for example metals. However, with certain arrangements it has been applied to friction stir welded metal-CFRP joints, to extract the residual stresses using the contour method [11]. Previous studies have also investigated electric-discharge machining (EDM) of polymer-matrix composite [23,24] and wire electric-discharge machining (WEDM) of metal-matrix and ceramic-matrix composites [24], for general cutting purpose. A recent study on wire electric-discharge machining of uni-directional CFRP for general cutting purpose used high-tensile-strength zinc-coated brass wire of 0.25 mm diameter [25,26]. Laser and water jet cutting have also been tried for general cutting of the CFRP in addition to conventional processes like milling, drilling, etc. [27]. A study reported the residual stress in a cross-ply laminate made of T700S 24 K MTM46 using the contour method (also mentioned in Table 1), however the cutting operation was performed with a diamond wire cutter [9].

A single cut requirement for the contour method poses a challenge to select appropriate electrical cutting parameters for wire EDM. For general purpose cutting, the recuts are performed on the same surface to get the desired surface finish. Therefore, the novelty is application of wire electric-discharge machining to CFRP for the contour method. In the contour method, polynomial or spline smoothing in three dimensions is commonly applied to refine the measured displacement data. The cut surface of metals typically exhibits a continuous and smooth profile owing to their homogeneity microstructure and isotropic elastic modulus, therefore smoothing the entire dataset in a single step is both practical and efficient. However, in the case of composites, a new smoothing approach may be required due to the drastic changes in displacement data, at the interfaces between orthogonal plies, and anisotropic elastic modulus. The methodology for wire EDM cutting and displacement data smoothing developed in this study for fibre reinforced

Table 2

Material properties of MTC510-UD300-HS-33%RW [28,29].

Elastic modulus (GPa)		Shear modulus (GPa)	Poisson's ratio	Coefficient of thermal expansion ($10^{-6} \text{ }^{\circ}\text{C}^{-1}$)	
E_{11} (fibre)	E_{22} (resin)	G_{12}	ν_{12}	α_{11} (fibre)	α_{22} (resin)
119.3	8.2	3.6	0.34	-0.14	42

polymer composites is presented in Sections 4.1.1 & 4.1.4.

3. Materials and samples

A plate of carbon-fibre-thermoset resin was used in this study. The prepreg specification was MTC510-UD300-HS-33%RW (SHD0373-300P). The sample consisted of 20 plies with a cross-ply layup $[0_4^{\circ}, 90_4^{\circ}, 0_4^{\circ}, 90_4^{\circ}, 0_4^{\circ}]$. The nominal thickness of a single ply was 0.28 mm. Four plies of the same fibre orientation were clustered together to form a “layer” to get sufficient resolution for each fibre orientation. The size of the plate was $200 \times 200 \times 5.6 \text{ mm}^3$ as shown in Fig. 2. Table 2 shows the material properties according to [28] and the manufacturer data sheet [29]. During the curing process, a cork dam was built around the sample to avoid the spreading of laminate. An autoclave pressure of 2 bar was used. The cure schedule for the sample was at 120°C for 60 min.

The laminate is defined according to an XYZ coordinate system as a global co-ordinate (as shown in Fig. 2). For an individual layer of the same ply orientation, the ‘1’ direction indicates along the fibre (longitudinal to fibre) and the ‘2’ direction is transverse to fibre. In this study, the 0° plies/layers are the fibres orientation that was aligned with the X-axis.

Two contour cuts were used for the modified contour method measurements (as shown in Fig. 2), which are defined as: contour cut # 1 performed on full plate $200 \times 200 \times 5.6 \text{ mm}^3$, and contour cut # 2 conducted after the 1st measurement on plate size $100 \times 100 \times 5.6 \text{ mm}^3$. For both contour cuts, the cut was performed at mid-width location to capture the same stress component, i.e., the stress along the Y-direction as shown in Fig. 2. The measurement for contour cut # 1 was performed at 100 mm from the plate edge, and for contour cut # 2 the cut was conducted at 50 mm distance from the edge after extracting stress results from the contour cut # 1. Both contour cuts were performed along the X-axis.

4. Methodology

4.1. Residual stress measurement by the modified contour method

4.1.1. Contour cut

For the composite material, the contour cut was applied using wire electric-discharge machining (WEDM): the first time that this has been achieved for the contour method. The wire electric-discharge machining offers several advantages including superior surface finish, precise straight cuts, and the ability to avoid re-cutting the relaxed contour in a single pass. The procedure is described briefly as follows. The contour cuts were performed with a Fanuc Robocut α -C600i wire electric discharge machine. A brass wire of 0.25 mm diameter was used. The samples were symmetrically clamped. Thin metal sheets were used at the top and bottom of the specimens during the machine cutting to overcome the lack of electrical conductivity in carbon fibre composites

Table 3

Wire EDM cutting parameters used for CFRP.

No load voltage: VS (V)	Current cnt: CC (A)	On time: ON (μs)	Off time: OFF (μs)	Wire tension: T (N)	Wire feed: WF (m/min)
20	5	50	800	1500	13

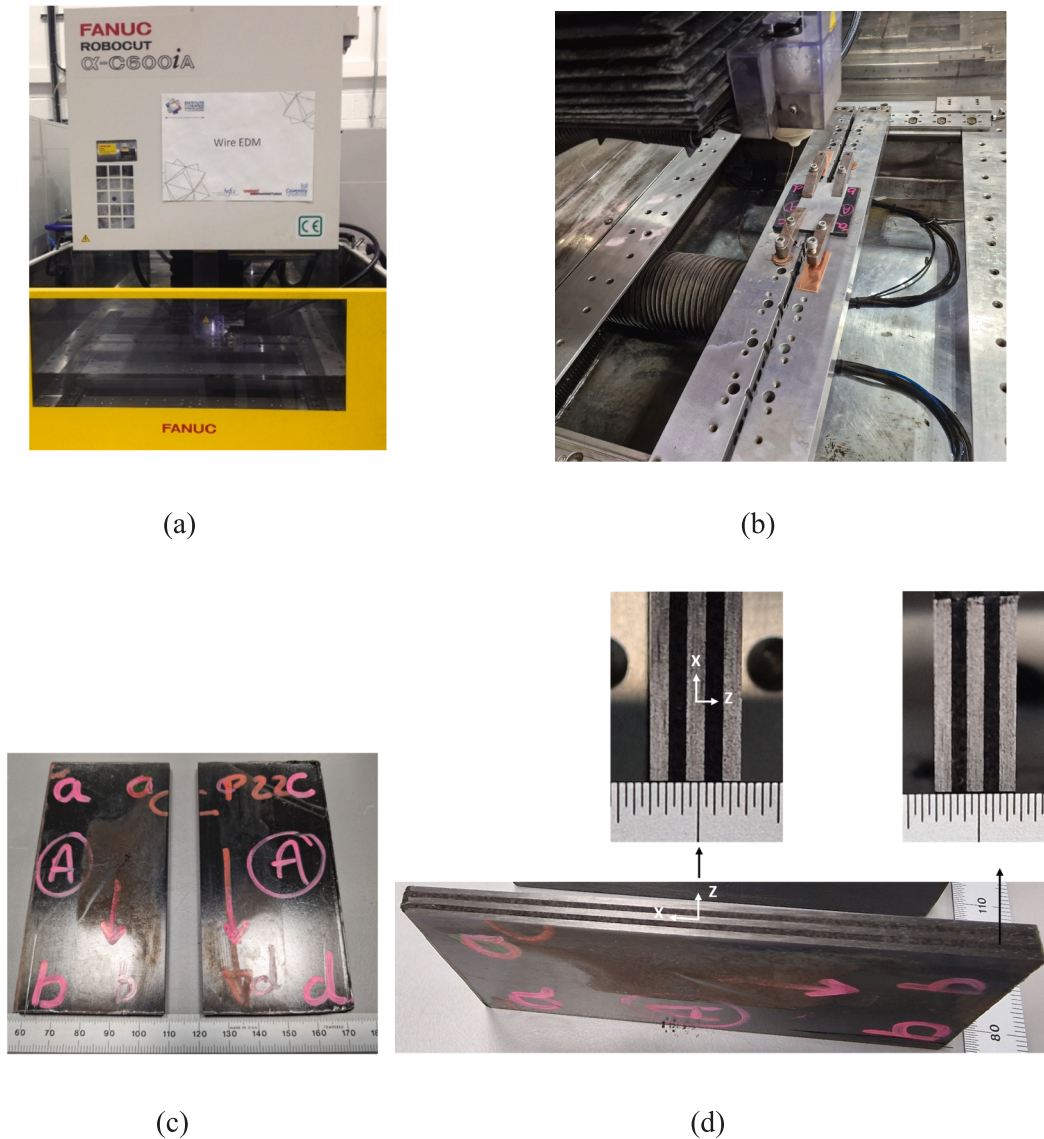


Fig. 3. Contour cut #2 of sample (a) Fanuc wire EDM used for cutting, (b) clamping arrangement for the cut on machine, (c) cut location, (d) cut-surface and its closeup.

[30]. This enabled the machine to generate the required sparks and start cutting. Trial cuts suggested the use of high-energy electrical parameters (such as voltage, current, spark duration/ON time) for the cutting of carbon fibre composite, as it was not possible to cut the carbon fibre composite with low-energy electrical parameters. The chosen wire EDM cutting parameters are shown in Table 3.

The cutting speed was approx. 0.18 mm/min, and the cut was completed without EDM wire breakage. In total, two contour cuts were conducted, as shown in Fig. 2(a), to obtain the stress component normal to cut surface (*i.e.*, along the Y-axis). Fig. 3 shows the Fanuc wire EDM, clamping arrangement, contour cut surface of contour cut # 2 and a zoomed view of the dimensions of the cross-ply cluster (in mm). No cracking or delamination was noticed on the cut-surfaces.

4.1.2. Surface contour measurement

The surface displacement profile of the cut faces was measured with a Zeiss Contura g2 co-ordinate measuring machine (CMM) using a scanning probe of 3 mm diameter. The distance between the measurement points for both in-plane directions of cut surface, as well as from the perimeter, was 0.1 mm. The selected spacing (0.1 mm) was considered sufficient by considering the measured thickness of a cluster

of four plies (*i.e.*, 1.2 mm for the 0° ply cluster & 1 mm for the 90° ply cluster), as the gradient was mainly seen at the interface between the two orthogonal ply clusters (0° and 90°). The accuracy of the employed CMM was 1.5 μm. The averaged displacement values of the relaxed contours were in the range of ± 10 μm *i.e.*, a magnitude greater than the CMM accuracy. Fig. 4 shows Zeiss CMM and an isometric view (for better visibility) of the averaged displacement data for contour cut #1. To examine any viscoelasticity effect on the surface relaxation, the cut-surfaces were remeasured with the CMM after the few weeks of the 1st measurement, however no notable difference was seen.

4.1.3. Displacement data smoothing by conventional approach

The surface displacement data were post-processed with MATLAB analysis routines for data aligning, averaging, cleaning, and smoothing with 3-D cubic splines [31]. A cubic spline smoothing with 1 and 5.5 mm knot spacings was used along thickness (Z) and width (X) directions, respectively.

4.1.4. Displacement data smoothing by novel approach

In the case of CFRP, there is a significant difference in the modulus of elasticity for along the fibre and transverse to fibre directions. For a

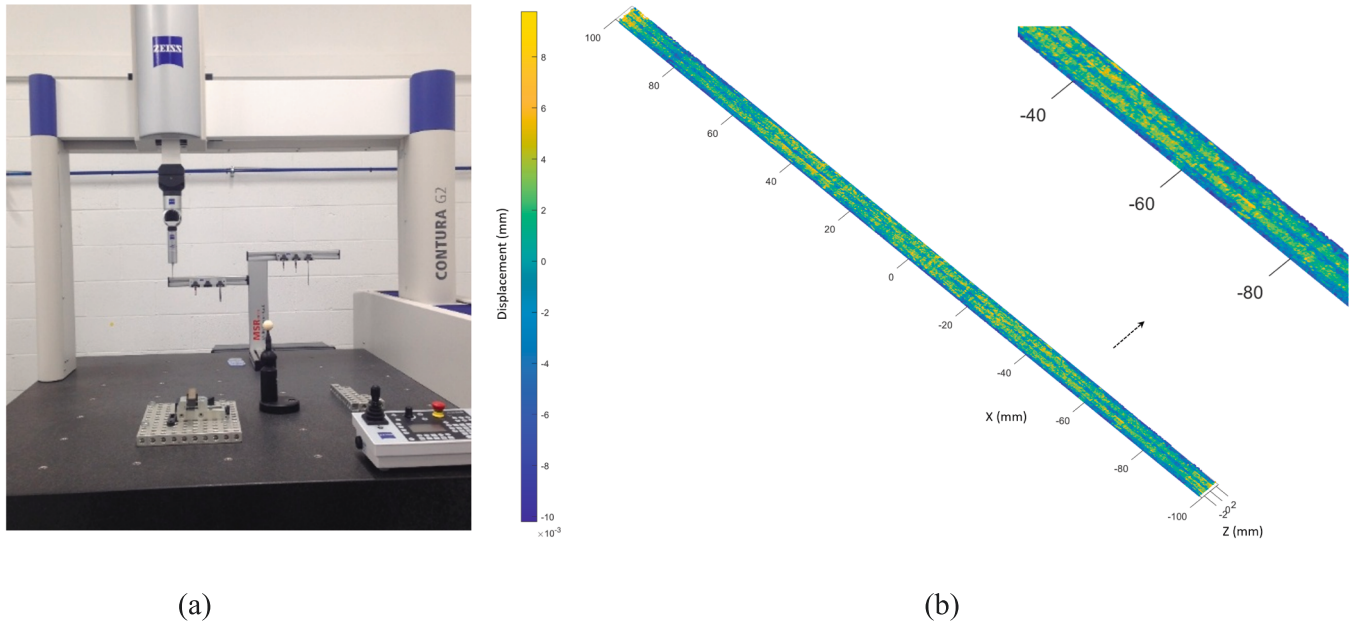


Fig. 4. (a) Zeiss CMM used for surface displacement measurement, (b) averaged displacement data of contour cut # 1 and its closeup.

cross-ply layup, the surface contour from the stress release will be different for along the fibre and transverse to fibre directions. Also, a significant shift in the displacement data can be present at the boundary/interface of cross-ply. Thus, a conventional smoothing routine may not work efficiently for the CFRP. A close look of the displacement data suggested that the change within each layer can be approximated by a 1-D linear smoothing. Therefore, an incremental linear smoothing approach was selected: *i.e.*, linear smoothing of each layer separately, in five steps/increments for cross-ply layup $[0^\circ_4, 90^\circ_4, 0^\circ_4, 90^\circ_4, 0^\circ_4]$. The measured displacements were in the Y-direction perpendicular to the cut-plane. The evaluated stress is in the Y-direction, as in Fig. 2.

All displacement data along the thickness direction (Z) were divided into five groups considering the trend and shift in the data. Fig. 5(a) & (b) shows two increments of linear smoothing along the thickness (Z) for the displacement data of contour cut # 2. A similar procedure was conducted for three remaining increments along the thickness (Z). A single linear fit was performed on displacement data along the thickness (Z) and over 10 mm along the sample width (X). The displacement data over a 10 mm step along the width (X) were selected to obtain a sufficient amount of data which showed a similar trend through the thickness (Z axis).

In this way, linear smoothing was approximated with increments of 10 mm along the width (X) for the whole cut surface. Finally, all linear fittings were averaged for each group. The range of scatter of linear fits along the width direction (X) is approx. 0.001 mm. A slight deviation in the slope of linear fits can also be noticed. Fig. 5(c) shows the displacement data pertaining to average of linear smoothing of each group for contour cut # 2. This data with reverse sign was then input to FE model of the cut surface as a displacement boundary condition. The displacement data belonging to contour cut # 1 was processed in a comparable way.

4.1.5. Finite element analysis (FEA)

Finite element analysis (FEA) is an integral part of the contour method. A 3D finite-element model of one cut-half was built using hexahedral continuum elements with reduced integration (C3D8R) in the Abaqus/CAE 6.14-1 software package. Fig. 6 shows the FE model used. For the contour method, the mesh size on the cut-surface of the FE model matters the most. Away from the cut-surface, the mesh size does not significantly affect the stress estimation as stresses on the cut-surface

only are reported. A mesh size of 0.1 mm (along thickness, Z direction) $\times$ 0.5 mm (along the sample width, X) was selected on the cut surface, and a biased mesh size in the third direction (Y). Any further smaller mesh size on the cut-surface was found to be computationally expensive, and it did not affect the results. The smoothed displacements were then applied to the FE model as the boundary conditions. Additional displacement constraints were applied at two corner nodes of the cut surface to avoid rigid body motion, and finally linear-elastic analysis was performed to estimate residual stresses present in sample before the cutting. Material properties such as E_{11} and ν_{12} for along the fibre direction and E_{22} and ν_{21} for transverse to fibre direction were used in FE analysis. The FE model and procedure for stress analysis were the same for both cases, *i.e.*, 3-D spline smoothing and 1-D incremental linear smoothing. The FE model developed for contour cut # 2 was also used for contour cut # 1. Although the width of contour cut # 2 is half of contour cut # 1, however as we can see from Fig. 5(c), the displacement profile input to the FE model is same along the width (X) (*i.e.*, there is no change in the displacement data along the width). Thus, to save the computational time and keep similar mesh resolution, a single FE model was used for both contour cuts.

4.2. Stress analysis using the classical laminate theory

For the case of residual stress verification by classical laminate theory (CLT), only stress arising from the temperature difference (ΔT) between room temperature (T_{room}) and the glass transition temperature (T_g) was considered. In our study, T_g was evaluated by a dynamic mechanical analysis (DMA) test. DMA was performed at room temperature (20°C) to determine the glass transition temperature (T_g) of the same laminate using a small sample size of $40 \times 10 \times 2 \text{ mm}^3$. The measured glass transition temperature was found to be 118°C. The main plate axis (global) are represented by x, y & z, and material axis (local) are represented by 1 & 2 (see Fig. 2). '1' direction is along the fibre and '2' direction is transverse to fibre. The fibre direction "1" is aligned to the main plate x-axis. Material properties used in this calculation were the elastic moduli (E_{11} , E_{22} , G_{12}) and Poisson's ratio (ν_{12}) and the coefficients of thermal expansion (α_{11} , α_{22}) as listed in Table 2. Only the σ_{yy} result from the classical laminate theory analysis was used for comparison with the modified contour method. The analysis procedure has followed the established theory and procedures as in these references

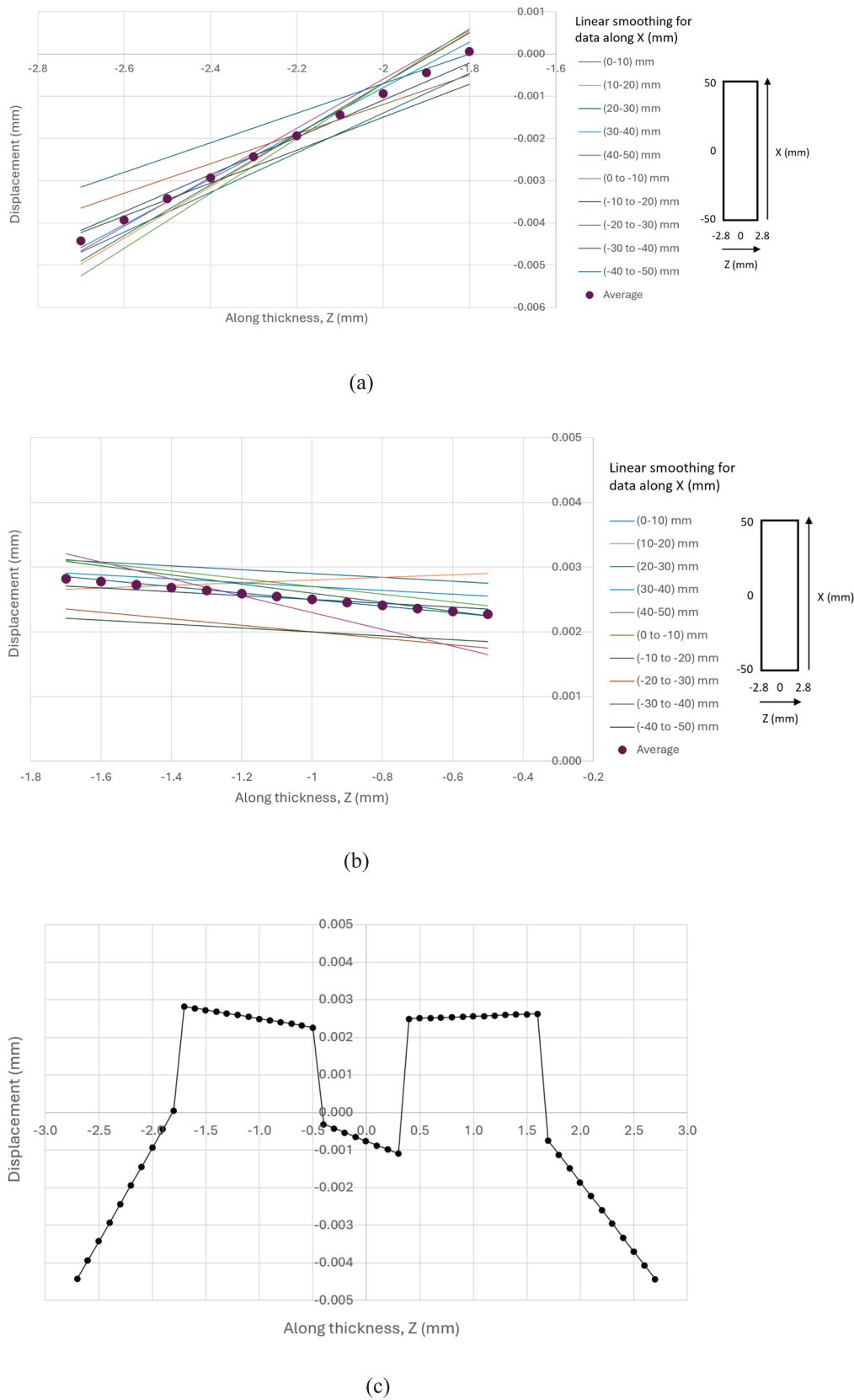


Fig. 5. Incremental linear smoothing in 1-D of averaged displacement data of contour cut # 2 along thickness (Z) (a) -2.7 to -1.8 mm, (b) -1.7 to -0.5 mm, and shown for data over 10 mm distance along the width (X), (c) average of incremental linear fittings showing sharp shifts in displacement profile around interfaces.

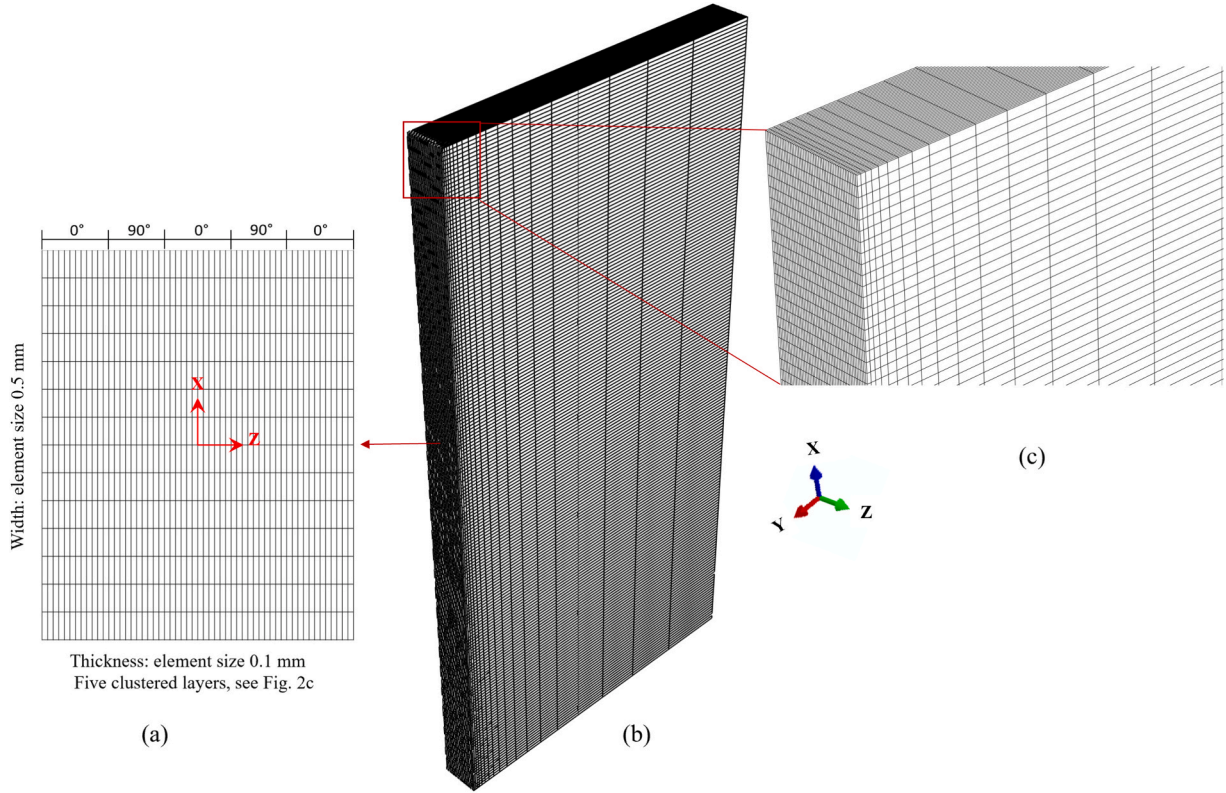


Fig. 6. Finite element model of the composite plate: (a) the cut plane, element size 0.1 x 0.5 mm, (b) FE model of one-half of the plate, (c) zoomed view of the plate length-wise direction; element size: 0.2, 0.2, 0.3, 0.4, 0.5, 0.6, 0.9, 1.1, 1.4, 1.7, 2.2, 2.7, 3.4, 4.2, 5.2, ..., 9.9 mm.

[32–36].

Step 1: Calculate the reduced stiffness matrix $[Q]$ for each ply (lamina) using the material properties (E_{11} , E_{22} , G_{12} , ν_{12}) as shown in [Appendix A](#).

Step 2: Calculate the transformed reduced stiffness matrix $[\bar{Q}]$ based on $[Q]$ and fibre angle (θ) (relative to the x-axis) for each ply (lamina) as shown in [Appendix A](#).

Step 3: Determine the $[A]$, $[B]$ & $[D]$ matrices based on $[\bar{Q}]$ and the distances from the mid plane to the top and bottom of a ply, combine them into one ABD matrix and the inverse of ABD matrix is calculated for the laminate as shown in [Appendix A](#).

Step 4: Calculate the coefficients of thermal expansion (CTEs) with regard to the global axes (x , y) from CTEs with regard to the material axes (1, 2) for each ply (lamina).

$$\text{i.e., } \begin{pmatrix} \alpha_{xx} \\ \alpha_{yy} \\ \alpha_{xy} \end{pmatrix} \text{ from } \begin{pmatrix} \alpha_{11} \\ \alpha_{22} \\ 0 \end{pmatrix}$$

Step 5: Calculate the equivalent force resultants and the equivalent moment resultants as a result of the temperature difference (ΔT). N represents force and M is bending moment.

$$\text{i.e., } \begin{pmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{pmatrix} \text{ and } \begin{pmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{pmatrix}$$

Step-6: Calculate the mid-plane thermal strains and curvatures from the following equation to represent the deflections of the laminate. ϵ^0 represents normal strain, γ^0 is shear strain and κ is bending curvature at mid plane.

$$\begin{pmatrix} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \gamma_{xy}^0 \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{pmatrix} = \begin{pmatrix} AB \\ BD \end{pmatrix}^{-1} \begin{pmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{pmatrix} \quad (1)$$

Step 7: Determine the residual strains in the x-y coordinate system for each ply (lamina). ϵ represents normal strain, γ is shear strain and z is the distance from mid-plane of the laminate. The superscript R stands for residual.

$$\text{Actual strain caused by } \Delta T = \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \gamma_{xy}^0 \end{pmatrix} + z \begin{pmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{pmatrix} \quad (2)$$

$$\text{Free thermal strain} = \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{pmatrix}^{\text{Free}} = \Delta T \begin{pmatrix} \alpha_{xx} \\ \alpha_{yy} \\ \alpha_{xy} \end{pmatrix} \quad (3)$$

$$\begin{aligned} \text{Residual strain (actual strain} - \text{free thermal strain)} &= \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{pmatrix}^R \\ &= \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{pmatrix} - \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{pmatrix}^{\text{Free}} \end{aligned} \quad (4)$$

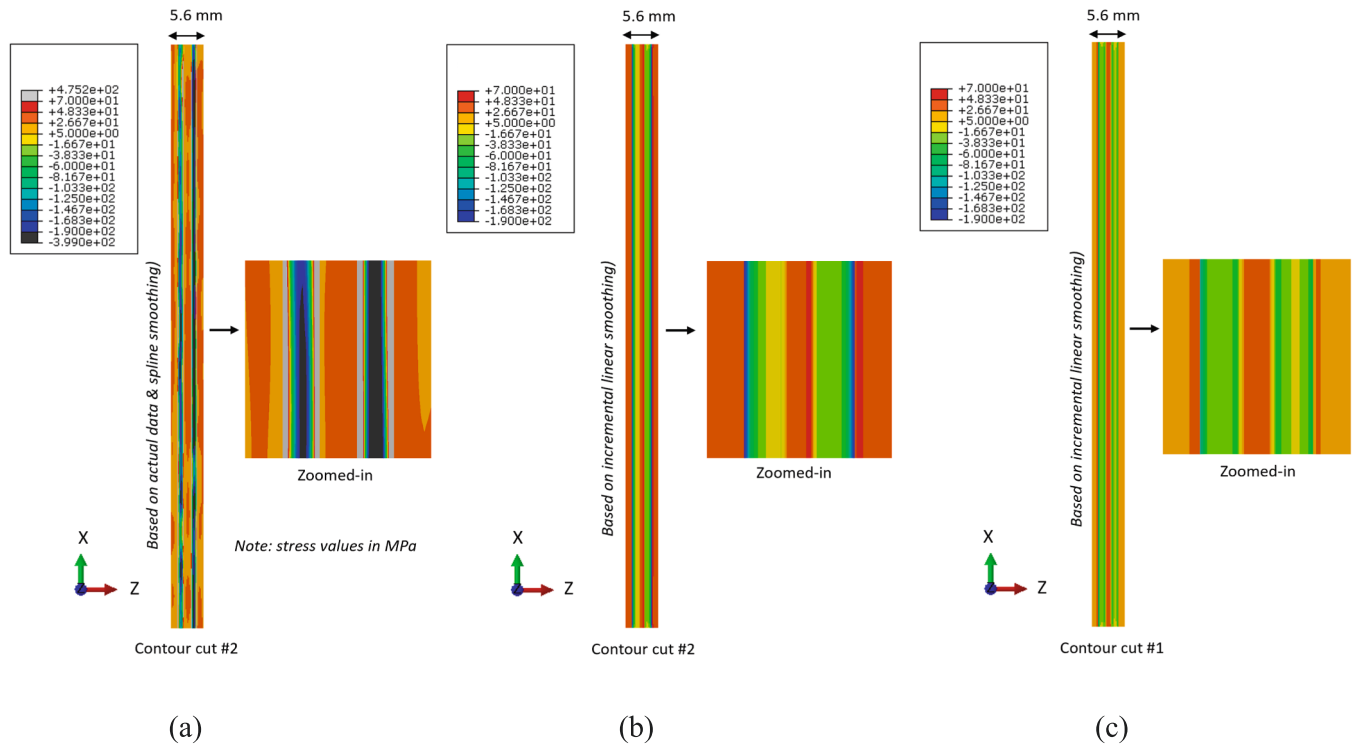


Fig. 7. Residual stress based on nominal thickness of layers: (a) stress map of contour cut # 2 obtained with 3-D spline smoothing, (b) stress map of contour cut # 2 obtained with 1-D incremental linear smoothing, (c) stress map of contour cut # 1 obtained with 1-D incremental linear smoothing. Note: residual stress is along Y-direction as shown in Fig. 2 (i.e., for the 0° ply, the plotted stress is transverse to fibre; for the 90° ply, the plotted stress is along the fibre).

Step 8: Finally determine the thermal residual stresses in the x-y coordinate system for each ply (lamina). σ represents normal stress and τ is shear stress.

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{pmatrix}^R = [\bar{Q}] \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{pmatrix}^R \quad (5)$$

5. Results and discussion

Two cases were considered for residual stress analysis. For one case, the nominal thickness of layers was considered, i.e., 1.12 mm for each layer which is a cluster of four plies. For the other case, the measured thickness of layers was incorporated, i.e., 1.2 mm for 0° ply cluster and 1 mm for 90° ply cluster. Correspondingly, 0.28 mm thickness was used for a single ply for the case of nominal layer thickness, and for the case of measured layer thickness, 0.3 mm was used for a 0° ply and 0.25 mm was used for a 90° ply. For contour method, the measured stress component was in the Y-direction (i.e. σ_{yy}), perpendicular to the cut-plane, as per Fig. 2.

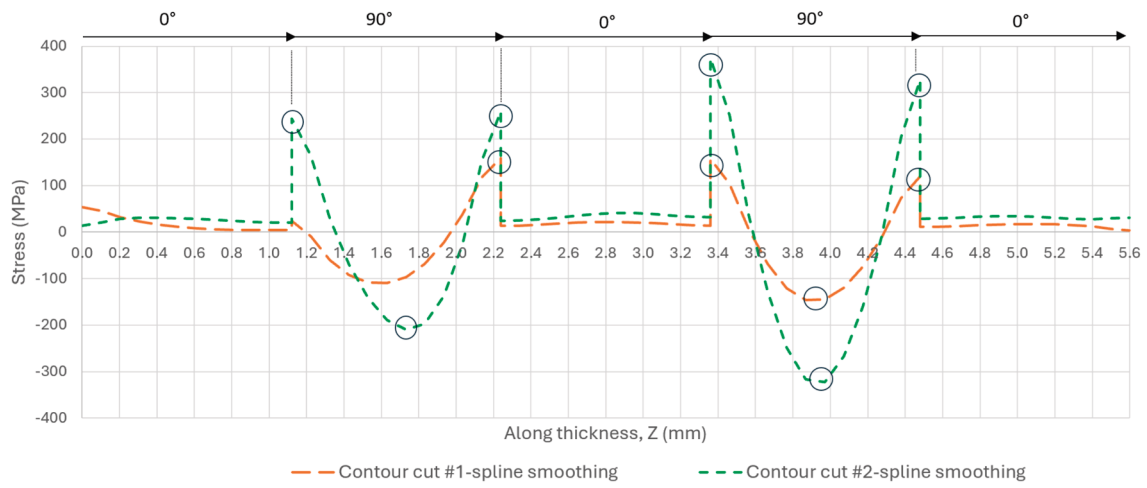
5.1. Residual stress based on nominal thickness of a clustered layer

Fig. 7(a) shows the stress map for contour cut # 2 using the spline smoothing method. Fig. 7(b) & 7(c) shows the stress maps based on 1-D incremental linear smoothing method for contour cut # 2 and 1, respectively. The result based on 3-D spline smoothing showed tensile stress in 0° layers (transverse to fibre), and the 90° layers (along the fibre) showed compressive stress in the centre and tensile stress at the interface with 0° layers. The results based on 1-D incremental linear approximation of the displacement data showed tensile stress in the 0° layers and the compressive stress was seen in the 90° layers. Tensile residual stress of lower magnitude, 10 to 35 MPa in 0° ply (transverse to fibre), has been

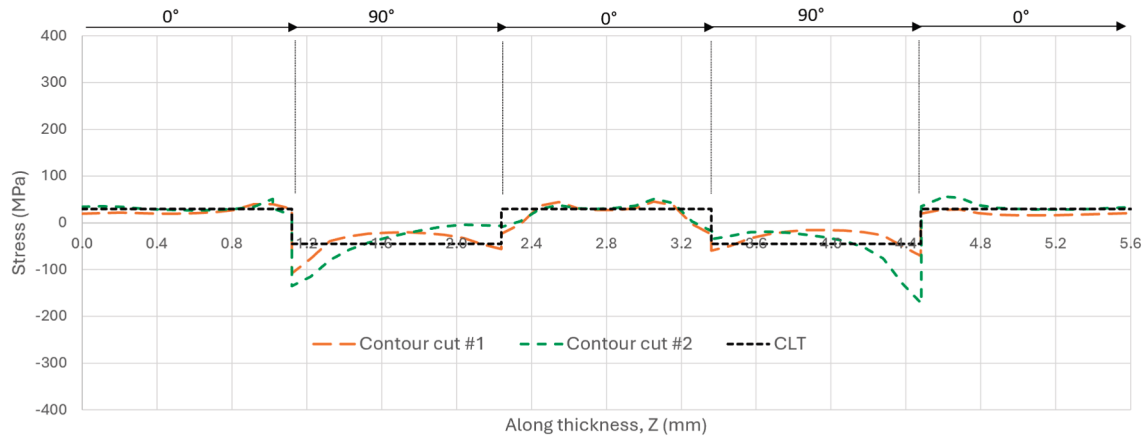
reported [8,9]. Previous studies mostly have shown less magnitude of tensile and compressive stresses in 0° and 90° plies respectively (see Table 1).

Fig. 8(a) shows stress profiles based on 3-D spline smoothing for contour cut # 1 and 2 extracted from similar locations along the width (X) of the contour cuts. Lower stresses in 0° layers (transverse to fibre) and 90° layers (along the fibre) are seen in contour cut # 1 than in contour cut # 2. The stress peaks in 90° layers (along the fibre) show unusually high magnitude of tensile and compressive stress (as highlighted on Fig. 8a). However, these high stresses arise from the poor data smoothing and approximation approach. The 3-D spline smoothing did not work well for the cross-ply layup. Various spline orders and knot spacings were trialled, however no significant improvement in the stress profile was achieved. Less effect in terms of stress magnitude seen for 0° layers pertains to the small values for elastic modulus and Poisson's ratio for transverse to fibre direction. Another study (as mentioned in Table 1) also noted a similar stress profile and very high magnitude of residual stress in 90° plies (along the fibre) i.e., -200 to 400 MPa for cross-ply layup [0°, 90°]_{SS} using the contour method, however the displacement data smoothing was performed using a median filter approach and the sample cutting was performed with a diamond wire cutter [10]. Thus, 1-D incremental linear smoothing/approximation offered a better fit for the displacement data of CFRP than other smoothing approaches, as it linearly smoothed the data of each ply cluster separately and therefore takes in to account a large gradient/shift present in data at the interface of orthogonal ply cluster, whereas the spline smoothing tends to fit the data set altogether and is unable to account for the linearity and sudden shift present. Fig. 8(b) shows stress profiles based on 1-D incremental linear smoothing for both contour cuts. The magnitudes of maximum residual stress obtained with contour method for contour cut #1 & 2 are shown in Table 4. The classical laminate theory results are also shown in Table 4.

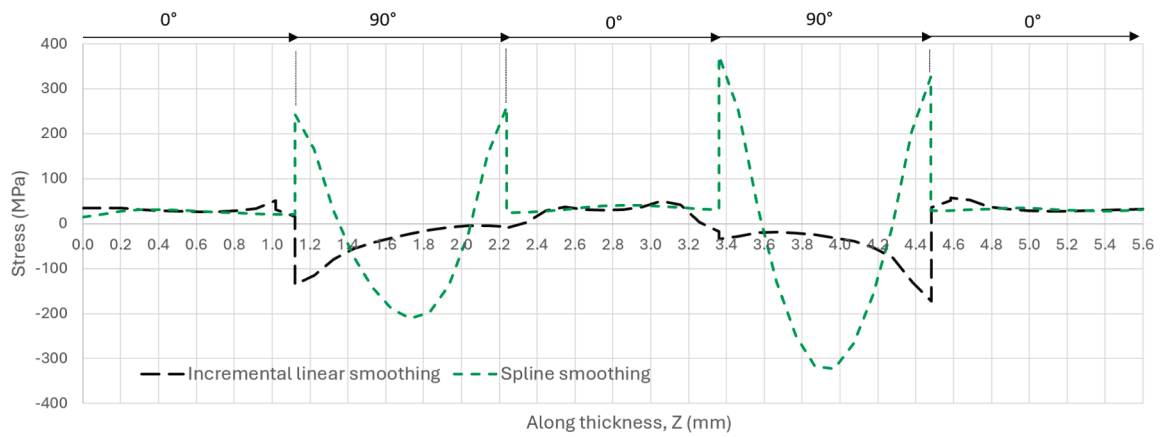
Contour cut # 2 showed comparatively higher compressive stress particularly near the interface with 0° layers. High compressive residual



(a)



(b)



(c)

Fig. 8. Stress profiles for both contour cuts (a) using the 3-D spline smoothing method, (b) the 1-D incremental linear smoothing method and comparison with classical laminate theory, (c) incremental linear smoothing versus spline smoothing for contour cut # 2. *Note: residual stress is along Y-direction as shown in Fig. 2 (i.e., for the 0 ply, the plotted stress is transverse to fibre; for the 90 ply, the plotted stress is along the fibre).*

Table 4

Maximum values of residual stress estimated by contour method and classical laminate theory using nominal ply thickness (unit: MPa).

Method	Stress transverse to fibres	Stress along the fibre direction
Contour method (cut #1)	45	-110
Contour method (cut #2)	55	-170
Classical laminate theory (CLT)	30	-44

stress in 90° ply (along the fibre) has been reported particularly near the interface of cross-ply laminate [9]. At the interface higher stresses were measured by IHD [12] and slitting [4, Ch. 5] on both sides of 0° and 90° plies. The higher stresses estimated in our study could be due to four plies being clustered together, and the mismatch of stiffness is bigger than the single ply variations. The difference in stress distribution seen between the two cuts might be due to variations in material shrinkage and cooling rates at separate locations of the same plate during the curing process [2,11].

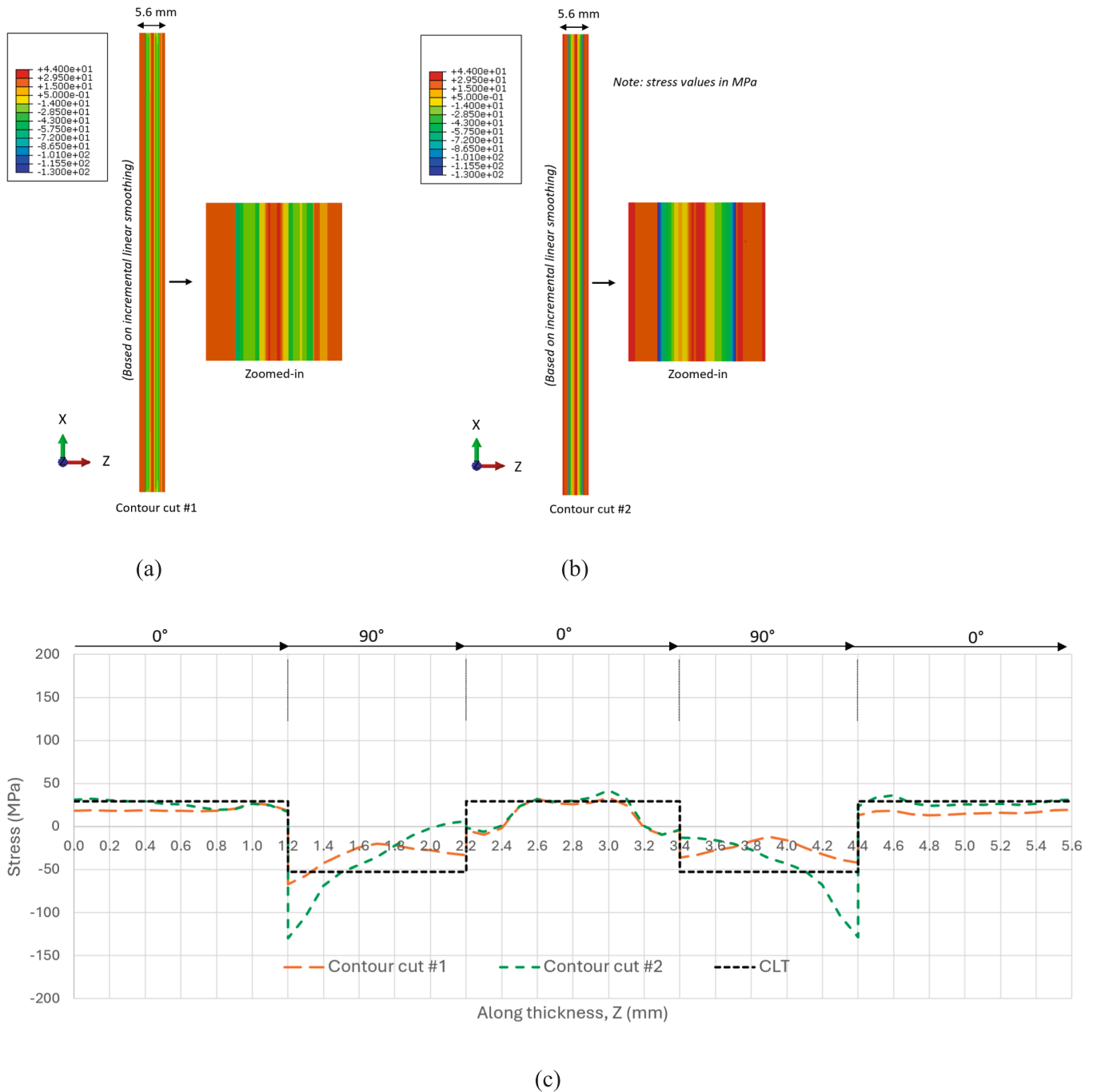


Fig. 9. Residual stress based on 1-D incremental linear smoothing for measured thickness of layers: (a) stress map of contour cut # 1, (b) stress map of contour cut # 2, (c) comparison of stress profiles obtained with modified contour method and classical laminate theory. Note: residual stress is along Y-direction as shown in Fig. 2 (i. e., for the 0° ply, the plotted stress is transverse to fibre; for the 90° ply, the plotted stress is along the fibre).

Table 5

Maximum values of residual stress estimated by contour method and classical laminate theory using measured ply thickness (unit: MPa).

Method	Stress transverse to fibres	Stress along the fibre direction
Contour method (cut #1)	30	−70
Contour method (cut #2)	40	−130
Classical laminate theory (CLT)	29	−52

The modified contour method results are compared with the classical laminate theory stress values (also shown in Fig. 8b). The difference between the modified contour method and classical laminate theory stress values is less in the 0° layers at the surface of contour cuts. Near the interface of 0° layers and 90° layers, the difference between both approaches is greater. The stress profile at interface obtained with the modified contour method is unique as it is based on the relaxed contour pertaining to mismatch of the material properties. Classical laminate theory assumes uniform distribution of stress throughout the ply thickness and does not account for the interface stresses [37]. Fig. 8(c) shows a comparison of stress profile between 1-D incremental linear smoothing and 3-D spline smoothing for contour cut # 2 and indicates the improvement achieved in stress distribution with incremental linear smoothing.

5.2. Residual stress based on measured thickness of layers

The residual stress results incorporating correction for the actual measured thickness of the layers are shown in Fig. 9. This is based on measurement of the cut surface along the thickness direction (Z). During the heating process, the fibres contract and the resin expands [6], thus resulting in different ply thicknesses. Fig. 9(a) & 9(b) shows stress maps based on 1-D incremental linear smoothing for contour cuts # 1 & 2, respectively. These results showed almost similar stress profile in 0° & 90° layers compared to the case of nominal thickness of layers and incremental linear smoothing. However, some variation in the magnitude of residual stresses can be noticed. The magnitudes of maximum residual stress obtained with contour method for contour cut #1 & 2 are shown in Table 5. The classical laminate theory results are also shown in Table 5.

The modified contour method results are also compared with the classical laminate theory stress values as shown in Fig. 9(c). The

difference in terms of stress magnitudes between the experimental and analytical method is reduced for this case in comparison to the case of nominal thickness of plies. However, at interface the difference between modified contour method and classical laminate theory results still exist as CLT does not account for the interface stresses resulting from the change in material properties.

5.3. Comparison of residual stresses obtained with nominal and measured thickness of layers

Comparing the modified contour method results obtained from the cases of nominal and measured thickness of layers as shown in Fig. 10 for contour cut # 1, it can be noticed that the stress profile is almost identical, however in case of measured thickness of plies there is a reduction in the magnitude of peak tensile and compressive stress. Although the nominal thickness of the single ply is 0.28 mm, during the heating process the carbon fibres contract and the resin expands (as also clear from the CTEs for fibre and resin given in Table 2). As a result, the thickness of resin dominant layers increases, and the thickness of layers dominated by carbon fibres decreases, after the curing. This can also be estimated from the closeup of Fig. 3. The residual stress analysis based on the measured thickness of layers seems to be practical.

The advantage with proposed approach is achievement of an experimentally-determined through thickness stress profile, and particularly the stress state at interface of orthogonal plies. The modified contour method can be applied to relatively thick layups, in contrast to most other techniques. The wire electric-discharge machining imparts fewer cutting errors in comparison to other cutting techniques i.e., mechanical, laser, water jet, etc. In principle, the incremental linear smoothing approach should work for the layups where there are sudden changes in the residual stress between layers. The actual thickness values of the plies may need to be considered. For future studies, a detailed analysis of the effect of wire EDM on resins may need to be considered along with the impact on the release of residual stress. Also, there may be a limitation on the thickness of the composite material that can be successfully cut with the wire electric discharge machining.

6. Conclusions

The modified contour method enabled to determine the residual stress through the thickness of laminate. Residual stress was estimated along the fibre and transverse to fibre in a cross-ply CFRP laminate. The

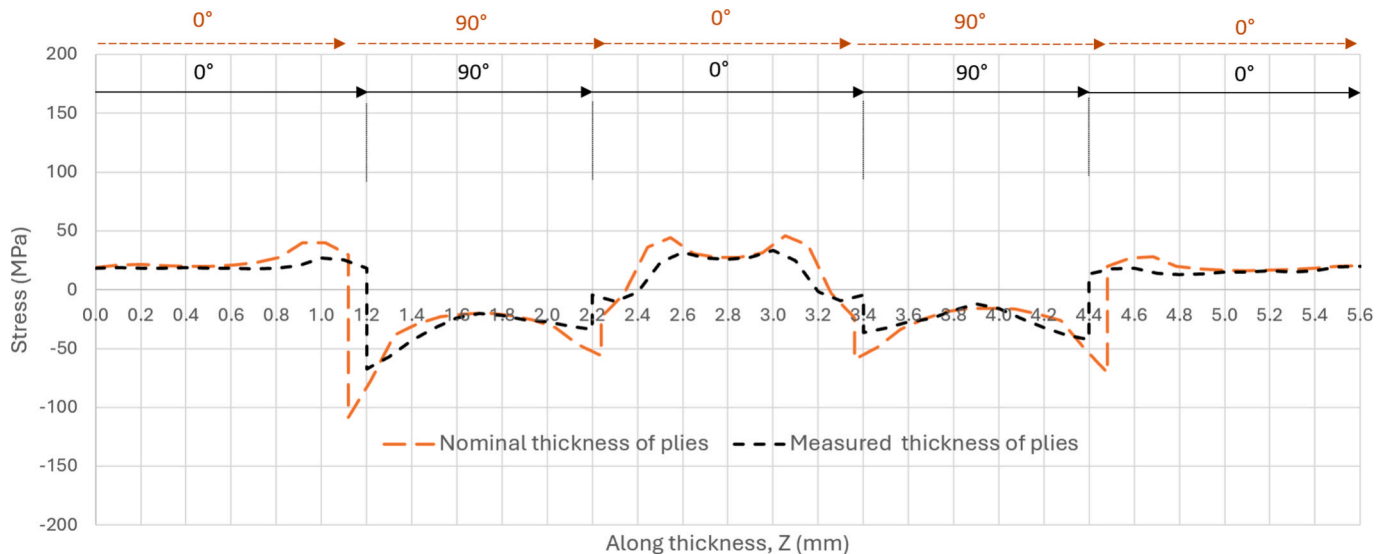


Fig. 10. Comparison of residual stress profiles obtained with modified contour method and based on nominal & measured thickness of plies for contour cut # 1. Note: residual stress is along Y-direction as shown in Fig. 2 (i.e., for the 0° ply, the plotted stress is transverse to fibre; for the 90° ply, the plotted stress is along the fibre).

following conclusions were drawn from this study:

- As a first step, wire electric-discharge machining was used to cut the CFRP for the contour method. Wire EDM cutting was possible by employing carefully selected electrical cutting parameters and using sacrificial thin metal sheets at the top and bottom of the sample. The cutting successfully revealed the displacements profile associated with the relaxation of the residual stresses.
- An incremental linear smoothing approach in 1-D was introduced for the displacement data to extract reliable residual stresses. The 1-D incremental linear smoothing approach gave better representation of the measured displacement data of the CFRP cut surfaces than the 3-D spline smoothing method.
- Results showed tensile stress ranging from 30-40 MPa in 0° layers (transverse to fibre), and compressive stress ranging from -70 to -130 MPa in 90° layers (along the fibre).
- For the contour method, measured ply or layer thickness should be used due to the high differences in the elastic properties and the coefficients of thermal expansion between the fibre and resin materials.
- At interfaces of 0° layers and 90° layers, the results of modified contour method and the classical laminate theory did not agree well as classical laminate theory does not account for the interface stresses.

Funding sources

This research did not receive any specific grant from funding

Appendix A

The reduced stiffness matrix $[Q]$ and transformed stiffness matrix $[\bar{Q}]$ is calculated as follows [32–34]:

$$Q = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \quad (A.1)$$

$$Q_{11} = \frac{E_{11}^2}{E_{11} - \nu_{12}^2 E_{22}}, \quad Q_{12} = \frac{\nu_{12} E_{11} E_{22}}{E_{11} - \nu_{12}^2 E_{22}}, \quad Q_{22} = \frac{E_{11} E_{22}}{E_{11} - \nu_{12}^2 E_{22}}, \quad Q_{66} = G_{12} \text{ and} \\ \bar{Q} = \begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{pmatrix} \quad (A.2)$$

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\ \bar{Q}_{12} &= \bar{Q}_{21} = Q_{12} (\sin^4 \theta + \cos^4 \theta) + (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta \\ \bar{Q}_{16} &= \bar{Q}_{61} = (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta - (Q_{22} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\ \bar{Q}_{26} &= \bar{Q}_{62} = (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta - (Q_{22} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta) \end{aligned}$$

Where, θ is angle between fibre and X-axis.

The A_{ij} , B_{ij} , D_{ij} matrices are calculated, and the ABD matrix is assembled as below.

$$A_{ij} = \sum_{k=1}^n \{ \bar{Q}_{ij} \}_k (z_k - z_{k-1}) \quad (A.3)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n \{ \bar{Q}_{ij} \}_k (z_k^2 - z_{k-1}^2) \quad (A.4)$$

agencies in the public, commercial, or not-for-profit sectors.

CRediT authorship contribution statement

Bilal Ahmad: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Xiang Zhang:** Writing – review & editing, Visualization, Project administration, Conceptualization. **Hua Guo:** Writing – review & editing, Resources, Methodology, Data curation. **Michael E. Fitzpatrick:** Writing – review & editing, Visualization, Methodology. **David Ayre:** Writing – review & editing, Resources, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

Authors would like to thank Jim Hurley from Cranfield University for building the sample and Steve Damms at The Institute for Advanced Manufacturing and Engineering, Coventry University for his help with the Wire EDM. M.E. Fitzpatrick is grateful for funding from the Lloyd's Register Foundation, a charitable foundation helping to protect life and property by supporting engineering-related education, public engagement, and the application of research.

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n \{Q_{ij}\}_k (z_k^3 - z_{k-1}^3) \quad (\text{A.5})$$

Where, z is the distance from mid-plane of the laminate (positive above and negative below the mid plane). And z_k and z_{k-1} are distances from the mid plane to the top and bottom of a ply (lamina) respectively.

$$ABD = \begin{pmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{pmatrix} \quad (\text{A.6})$$

And the inverse of ABD matrix is calculated.

$$(a, b, d) = (A, B, D)^{-1} \quad (\text{A.7})$$

Data availability

Data will be made available on request.

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Estimation of residual stress in carbon fibre composite laminate using the contour method

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2026-05-01

Elsevier

<https://doi.org/10.1016/j.compstruct.2026.120147>

Ahmad B, Zhang X, Guo H, et al., (2026) Estimation of residual stress in carbon fibre composite laminate using the contour method. Composite Structures, Volume 383, May 2026, Article number 120147