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Studies in Hypersonic Viscous Interactions

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SUMMARY

The literature on the leading edge viscous interaction phenomena has been reviewed. Theoretical studies of the the leading edge bluntness and the boundary-layer displacement effects have been made by using a combination of 'tangent wedge + centrifugal force' pressure law relation and also using the 2nd order tangent wedge formula suggested by Lees. These analyses, when applied to concave surfaces show a mild oscillatory behaviour compared to the highly oscillatory results of Cheng's theory. Further, an integral approach due to Chernyi has been applied to concave surface flows and the damping of the oscillations present in Cheng's theory is demonstrated.

Experiments have been conducted at a Mach number Of 12.2 in a gun tunnel, on flat plates and on a concave surface with a sharp leading edge. The results are compared with the predictions of various interaction theories.

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LIST OF SYMBOLS

$$A : 0.664 \frac{\gamma-1}{2} (1 + 2.6 T_w/T_0)$$

$$a : \gamma + 1 \quad \text{unless otherwise defined}$$

$$b : \sigma \quad - , , -$$

$$C : \text{Proportionality constant in the linear viscosity temperature relationship } \frac{\mu_w}{\mu_\infty} = c \frac{T}{T_\infty}, \quad c = \frac{\mu_w T_0}{\mu_\infty T_w}$$

$$C_p : \text{Pressure coefficient } 2(p_w - p_\infty) / \gamma p_\infty M_\infty^2$$

$$C_1, C_2 : \text{Constants (see equations 3.3.2 and 3.3.3)}$$

$$C_{d_N} : \text{Leading edge drag coefficient}$$

$$C_f : \text{Skin friction coefficient defined by equation 3.1.28}$$

$$D_N : \text{Nose drag}$$

$$f : \int \psi u_c dx$$

$$g : H/He$$

$$H : \text{Total specific enthalpy}$$

$$I : \text{Integral defined by } \int_0^\infty (g - f'^2) d\eta \quad \& \quad I_0 = \frac{I}{0.4696 (1 + 2.6 g_w)}$$

$$k : \text{proportionality constant in the leading edge solution for the pressure } p \propto x^m, \text{ unless defined otherwise.}$$

$$K_s : \frac{\gamma(\gamma-1)}{4} m_\infty^3 \frac{C_p t}{L_s} ; \quad K_e : M_\infty^3 C_p t / x$$

$$L_c : (A^2 \bar{x}^2 x) / (4 \gamma M_\infty^4 \alpha^4)$$

$$L_s : A^2 \bar{x}^2 x$$

$$l_1, l_2, l_3 : \text{as defined by equations 7.1.48 a, b and c}$$

$$M : \text{Mach number}$$

$$m : \text{Exponent in the strong interaction solution } P \propto X^m$$

$$N : \rho / \rho_w \mu_w$$

$$n : \text{Exponent in } y_w \propto x^n$$

$$n_1, n_2, n_3 : \text{as defined by equations 7.1.49}$$

$$P : p_w / p_\infty$$

$$p : \text{Static pressure unless defined otherwise.}$$

P_r : Prandtl number
 q : Heat transfer rate
 R : Gas Constant
 R : $\int P \, dx$
 R_{ex} : Reynolds number based on x
 R_{le} : Leading edge Reynolds number
 St : Stanton number
 T : Temperature
 T^* : Characteristic temperature for calculation of C
 t : leading edge thickness
 U, U_∞ : Free stream velocity
 $\bar{V}$: Rarefaction parameter. = $M/R_e^{1/2}$
 u, v : Velocity component parallel to x and y axis
 x, y : Coordinates along and normal to the wall
 y_w : Wall shape
 y_e : edge of the boundary layer or effective body shape
 Y_e : $M_\infty y_e / L_c$
 z : $y_e / \alpha_c L_c$

d_c, d_s, d_N : body incidence parameter defined corresponding to the length scales L_c , L_s and L_N

β : Pressure gradient parameter defined by equation 3.1.14

γ : Ratio of specific heats

δ^* : Boundary layer displacement thickness (eqn 3.1.17)

Δ^* : $M \delta^* / L_s$

μ : Viscosity

ξ, η : Transformed variables defined in Section 3.1

ℓ : x / L_c

ρ : Density ; $\sigma = 1 - \frac{\gamma^2 - 1}{\gamma}$

$\bar{\gamma}$: Viscous interaction parameter $M_\infty^3 c^{1/2} Re_x^{-1/2}$

$\bar{\chi}_e$: $\frac{2}{\gamma+1} A \bar{\chi}$

λ : defined by equation 5.3.14

Subscripts: & Superscripts

b, B : Body surface

e : Refers to edge of the entropy layer or effective body surface as defined in the particular context.

w : Wall shape

∞ : Free stream conditions

' : Denotes differentiation once.

Abbreviations:

TW : Tangent wedge pressure law.

TW+CFGL : Tangent wedge + Centrifugal pressure law.

Note: Other symbols not defined in this list will be defined appropriately in the particular context.

CHAPTER 1 : INTRODUCTION.

The subject of hypersonic viscous interaction is important for an understanding of the mutual coupling between the viscous and inviscid flow fields prevailing under high Mach number and low Reynolds number flight conditions. Under these conditions, the laminar boundary-layer on a slender body can be much thicker because of high temperatures and hence low densities, and can cause external flow deflections of magnitude comparable or even higher than those produced by the body itself. These flow deflections caused by the boundary-layer modify the flow characteristics and the resulting surface pressure distribution can be much different from the inviscid prediction. When the boundary-layer is much thinner as in high Reynolds number flows, the problem can be solved at least to a first approximation by regarding the outer inviscid flow to be unaffected by the presence of the boundary-layer. However, this procedure becomes inapplicable or too inaccurate in certain regions where the flow is considerably modified by the boundary-layer growth, e.g. strong interaction region of a sharp flat plate in hypersonic flow. In such regions, the determination of the surface pressure, skin friction and the heat transfer rates requires a simultaneous consideration of the viscous and inviscid flow fields. In addition, other factors such as nose bluntness, curvature of the leading edge shock wave and the body curvature can further affect the boundary-layer growth and hence, modify the interaction phenomena. In the present study, the flow situations where both the displacement effect and the nose bluntness are important have been given primary attention.

Considerable amount of theoretical and experimental studies have been made in the past two decades regarding the hypersonic viscous interaction effects on two-dimensional and axisymmetric bodies. The procedure adopted in many of the theoretical studies is to determine the inviscid flow past a hypothetical body defined by either the edge of the displacement thickness of the boundary-layer when the leading edge is sharp or the edge of the entropy layer for the case of a blunt leading edge. This scheme, depending on the particular theory employed for determining the inviscid flow in conjunction with the boundary-layer analysis gives results showing varying degree of agreement with the experimental observations. The most significant work in this area is due to Cheng et al (Ref. 32). They showed for the limiting case of $M \rightarrow \infty$ and $\gamma \rightarrow 1$, the pressure gradient effect on the boundary-layer growth can be very small and calculated the pressure by using the Newton-Busemann formula for the inviscid flow. Though the analysis, as they mention it, was applicable to a general curved after-body shape, the results were obtained only for the case of a flat plate and the agreement demonstrated qualitatively with flat plate heat transfer measurements at $M = 12$. Following this work, there have been several studies (discussed in Chapter II) to understand the combined effects of displacement, bluntness and incidence at hypersonic speeds and modifications to Cheng's theory have been suggested to give better predictions.

The extension of Cheng's method to surfaces other than a flat plate afterbody has been made only recently. Stollery (Ref. 22) showed that the strong viscous interaction theory of Cheng predicts highly unrealistic oscillatory solutions for some concave surfaces. This oscillatory behaviour was attributed to the centrifugal term in the Newton-Busemann formula used in Cheng's analysis and an alternative analysis using tangent-wedge approximation in conjunction with Lees' (Ref. 31) boundary-layer theory was suggested. The use of tangent-wedge approximation was found to suppress the oscillatory behaviour and predict a smooth approach to the inviscid values downstream. The analysis was later extended to the case of hypersonic flow past a blunted wedge (Ref. 44).

Confining our attention to the case of viscous interaction on a concave surface, the work of Stollery highlights the importance of a proper choice of the pressure law to describe the interaction phenomena. However, the choice of a pressure law such as the tangent wedge approximation or modified Newtonian which is often considered to be in error (Refs. 36, 74) for not accounting for the centrifugal pressure variation, casts a doubt on the role of inviscid turning effects in governing the viscous interaction phenomena on concave surfaces. The Newton-Busemann formula used in Cheng's analysis is obtained as the leading approximation of the shock-layer analysis for the limiting case of $M \rightarrow \infty$ and $V \rightarrow 1$, and when higher order approximations are considered, one can expect the unrealistic oscillations predicted by the zeroth order theory to be significantly modified. For the problem of hypersonic wedge flow, Cheng and Kirsch (Ref. 42) have shown that the higher approximations almost completely fade away the oscillations present in the zeroth order theory. Similarly, for the case of viscous interaction on a concave surface, one may speculate to obtain satisfactory results without discarding the inviscid turning effects if higher approximations are considered. Perhaps, another interesting question which may be worth investigating is - whether the Cheng's zeroth order theory correct strictly for $V = 1$, at least in some sense suggest for $V \neq 1$, the possible formation of a neck where the leading edge shock wave and the compression waves from the effective body surface coalesce to form the concave shock-wave. The physical reality of the subsequent peaks suggested by Cheng's theory is highly doubtful and the oscillations might get damped quickly because of other higher order effects not considered in the theory.

The objective of the present study is to investigate further the viscous interaction phenomena without ignoring the inviscid turning effects. Alternative analyses which give satisfactory agreement for the case of a flat plate are presented. These analyses when extended to concave surface flows exhibit for large downstream distances a mild oscillatory behaviour in contrast to the quantitatively unrealistic oscillations predicted by Cheng's theory. Experimentally, pressure and heat transfer rate measurements on sharp and blunt flat plates and on a concave surface with sharp leading edge made in a gun tunnel are presented and the results compared with theoretical predictions. The experiments were carried out at a Mach number of 12.2 in the

No. 1 gun tunnel of the Imperial College*.

The application of the various methods suggested for the viscous interaction analysis is also considered to the case of bluntness dominated flows.

In the next Chapter, a review of the literature on the viscous interaction phenomena followed by a description of the scope of the present work is given. This is mainly a detailed version of the points discussed in the present chapter along with other relevant information. Part II deals with the theoretical studies and their comparison with earlier works and some experimental results. Finally, the experimental work, the results obtained and their comparison with the viscous interaction theories is presented in part III.

*

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CHAPTER II

REVIEW OF PREVIOUS LITERATURE AND SCOPE OF THE PRESENT WORK

2.0 Viscous interaction on a sharp flat plate:

The hypersonic flow past a sharp flat plate represents a spectrum of flow regimes from thin boundary-layer limit far downstream to kinetic regime near the leading edge where the continuum concepts breakdown. It has been observed that the interaction of the leading edge with the external flow produces a shock-wave (Fig 1), which can be quite strong to start with and will degenerate into an acoustic wave far downstream. Very near the leading edge for some distance downstream, the entire region between the shock-wave and the body-surface is filled by the viscous boundary-layer growing from the leading edge. This region is known as the merged layer. In this layer, the rarified gas effects become increasingly important as the leading edge is approached. It has been suggested by Talbot (Ref 1) that the merged layer effects begin to appear when the rarefaction parameter $\bar{V} \approx 0.2 - 0.3$. Downstream of the merged layer, the shock wave and the boundary-layer are distinctly separated by an inviscid region, - the separation distance increasing downstream. Near the leading edge in this region, the growth of the boundary-layer strongly influences the shock-wave growth and the resulting pressures on the plate will be much higher than the freestream value. Far downstream, because of relatively small displacement effect of the boundary-layer, the interaction effect tends to become weaker and the pressure on the plate reaches the free stream value asymptotically. These regions are called strong and weak interaction regions respectively. The growth of the boundary-layer and the interaction effects can be much different in merged, strong and weak interaction regions. In the present study, only the regions downstream of the merged layer are considered.

In the treatment of viscous interaction problems, there have been mainly two approaches employing different flow models shown in figures 2a,b. The flow model (Fig 2a) in which a distinct inviscid region separating the boundary-layer and the shock-wave is considered, was first used by Becker (Ref 2). Becker observed experimentally at $M = 6.9$, that the pressures near the leading edge of a sharp flat plate were much higher than the free stream value, and presented some theoretical estimates accounting for the rate of boundary-layer growth. This method was later improved by Bertram (Ref 3) who proposed an approximate method for evaluating the displacement effects on two-dimensional bodies. A more extensive treatment of the weak interaction problem was given later by Lees and Probstein (Ref 4).

The second model shown in Fig 2b adopted by Shen (Refs 5 and 6) assumes that the edge of the boundary-layer coincides with the shock-wave. Shen considered the viscous effects on insulated wedges and flat plates, and showed that the compressible boundary-layer equations are still valid in the presence of viscous interaction provided $(\delta^*/x)^2 \ll 1$. The boundary-layer was calculated using Karman

momentum integral method and a linear velocity profile, and the pressure was determined according to the strong shock relations. He obtained the now familiar strong interaction limit,

$$\frac{p_w}{p_\infty} \propto \chi^{-1/2}, \quad \delta \propto \chi^{3/4}$$

Li and Nagamatsu (Ref 7) later extended Shen's approach by using a fourth degree polynomial for the velocity profile. Though Shen's analysis gave the correct dependence for the pressure and the boundary-layer growth, Lees (Ref 8) pointed out that the flow model assumed by Shen was incompatible with the equation of continuity. In order to obviate this inconsistency, Lees used the flow model shown in Fig 2a in which the boundary-layer and the shock-wave are separated by an inviscid region. The pressure distribution was calculated using the tangent-wedge approximation on the effective body described by the displacement surface. With this refined approach, it was shown that Shen's results are numerically different by a factor of $(\gamma+1)/2$. It appears that in most of the later works, the revised model used by Lees has been adopted. Lees' method is more general since the tangent-wedge approximation used in the analysis is valid in both the strong and weak interaction regions.

In particular, the problem of strong interaction has received much interest and matching of the viscous and inviscid flowfields has been considered by Stewartson (Ref 11), and Li and Nagamatsu (Ref 9, 10 and 12). Stewartson showed the existence of self-similar solutions for the strong interaction problem and the possibility of matching with the similar solutions for hypersonic flow past a $\frac{3}{4}$ power law body. Li and Nagamatsu (Ref 12) treated the case of non-insulated wall using the method of similar solutions of the compressible boundary-layer equations. In all these studies, only the displacement effect was considered. Since the leading edge shock-wave is highly curved, the flow behind has an entropy gradient normal to the streamlines and external vorticity is created. Lees (Ref 13) and Oguchi (Ref 14) corrected the strong interaction results to account for the effect of external vorticity. Bush (Ref 15), and Lee and Cheng (Ref 16) have treated this problem in detail using asymptotic expansions and have shown that in order to get a uniformly valid temperature distribution across the entire flow field, it is necessary to introduce an intermediate layer between the inviscid region and the boundary-layer to account for the external vorticity. Bush showed that the direct matching of the boundary-layer with the inviscid region is possible only for the case of non-linear viscosity-temperature relationship. Lee and Cheng later carried out a similar three-layer analysis assuming a linear viscosity-temperature law and showed that a separate analysis of the intermediate layer in this case is not strictly required and the conventional two-layer approach is sufficient.

In most of the studies mentioned so far, only the limiting theories of either strong or weak interaction have been considered. These theories have been described in detail by Hayes and Probstein (Ref 17), and Moore (Ref 18). Recent investigations in the theory of strong interaction have been summarised by Mikhailov, Neiland and Sychev

(Ref 19). In cases of practical interest, the intermediate region between the strong and weak interaction limit is important. In this region either of the theories may not be accurate enough. To obtain solutions in this region, it is necessary to integrate the full equations of the viscous and inviscid flow fields from a point close to the leading edge where the initial conditions are given by the strong interaction theory, and proceeding downstream. References 20 to 30 deal with this problem, in general. In these studies, the boundary-layer growth has been calculated either by ignoring the pressure gradient effects or using a local similarity method, or by integrating the complete boundary-layer equations. For the inviscid flow calculation, the tangent-wedge approximation has been widely used because of its applicability in both the strong and weak interaction regions.

It was suggested by Lees (Ref 31) that the effect of streamwise pressure gradient on the boundary-layer flow can be small when the wall is highly cooled relative to the freestream stagnation temperature. This approximation considerably simplifies the momentum equation since the pressure gradient terms are ignored, and the energy and momentum equations can be solved independently. This leads to the well known result relating skin friction and heat transfer (for unit Prandtl number),

$$f''(0) = \frac{g'(0)}{1 - g_w} = 0.4696 \quad 2.0.1$$

Moore (Ref 18) points out that for certain ranges of the pressure gradient parameter β , Lees' cold wall similarity approach is rather inaccurate. However, the simplicity of the method has resulted in its extensive application. Cheng et al (Ref 32) also arrive at the same result as Eq 2.0.1 and have shown that ignoring the pressure gradient term can be justified in the limit of $M_e \gg 1$ and $\gamma \rightarrow 1$. In the limit of this hypersonic approximation, the pressure gradient term β will be at most of the order $\epsilon (= (\gamma-1)/(\gamma+1))$, and appears only as a higher order effect. Though the resulting equation for the boundary-layer is the Blasius equation in Cheng's as well as Lees' analysis, it should be realised that the neglect of pressure gradient terms is because of conceptually different reasons. Sullivan (Refs 20 and 21) combining the tangent-wedge rule and Lees' boundary-layer theory presented a simple formulation which can cover both the strong and weak interaction regions. The method gave good agreement with some experimental results. Stollery (Ref 22) extended this method to a variety of two-dimensional shapes and demonstrated the applicability of the method to problems involving no upstream influence. This will be discussed in a subsequent Section.

Though the assumption of negligible pressure gradient can be justified in the mathematical limit of $\gamma \rightarrow 1$, or for a highly cooled wall, it is found that it can lead to considerable numerical errors in the evaluation of skin friction and heat transfer rates for values of γ of practical interest. In order to overcome some of these limitations of the zero pressure gradient theory, Dewey (Ref 23) used a local similarity approach for the boundary-layer along with

the tangent-wedge approximation for the inviscid flow field. Within the scope of the local similarity approach for the boundary-layer it was possible to vary the pressure gradient parameter β in the streamwise direction. The agreement of the method with the experimental results was found to be good.

Numerical solutions of the boundary-layer equations including the displacement effect have been obtained by Flugge-Lotz and Blottner (Ref 25), Garvine (Ref 26), and Werle and Wornom (Ref 27). A finite difference scheme was employed for the boundary-layer analysis and the tangent-wedge formula was used to calculate inviscid flow by Flugge-Lotz and Blottner, and Werle and Wornom. Garvine used a linearised interaction model relating the pressure at the outer edge of the boundary-layer with the vertical velocity. The more accurate results of Werle and Wornom agree very closely with Sullivan's method in the range of the viscous interaction parameter $\bar{\lambda}$ ($= M_\infty^3 C^{1/2} \mu$) from 1 to 40. The agreement suggests that in some cases, the approximate method of Sullivan may be quite adequate. Chan (Ref 30) used a method of combining integral relations and the similarity solutions for the boundary-layer and used tangent-wedge formula for the inviscid flow. Yalamanchili and Jeng (Ref 29) similarly used the tangent-wedge formula but the non-similar boundary-layer equations were integrated by the method of steepest descent (Meksyn, Ref 82). All these studies, whilst refining the boundary-layer analysis, have treated the inviscid flow to be adequately described by the tangent-wedge formula. A refinement in the analysis of the inviscid flow field was adopted by Mann and Bradley (Ref 28), who used the method of characteristics for the inviscid flow calculation and a numerical procedure for the boundary-layer. Richey (Ref 24) used the tangent-wedge approximation for the two-dimensional case and the method of characteristics for the axisymmetric case.

With most of the methods discussed above, it is not necessary to distinguish between strong and weak interaction regions and the solution can be obtained over the entire range of interaction, once the initial data corresponding to the strong interaction limit is specified.

2.1 Effect of the leading edge bluntness:

The first theoretical study involving both the effects of the leading edge bluntness and the boundary-layer displacement effect was made by Cheng et al (Ref 32). Earlier studies dealt with either bluntness dominated flows (Refs 33 to 35) or viscous dominated flows discussed in Sec 2.0. In treating the inviscid hypersonic flow problem past slender bodies, it is usual to invoke the 'equivalence principle' and solve the corresponding unsteady flow problem in one less space dimension. The leading edge blunting in the steady flow problem is effectively equivalent to the release of a finite amount of energy instantaneously at a point in the unsteady problem. This similarity between the two fields is known as the 'Blast Wave Analogy'. For the analogy to hold good, it is necessary that the body must be slender and also $(M_\infty)^{-1} \rightarrow 0$. Under these approximations, the small

disturbance equations of the hypersonic flow past a slender body, and the exact equations for the unsteady flow with the time variable replaced by x/U_∞ , are identical. A detailed account of the blast wave analogy is given by Hayes and Probstein (Ref 36), and Guiraud et al (Ref 37). In this Section, we shall confine our interest to other approximate methods.

For the hypersonic flow past a blunted flat plate, the flow field consists mainly of three different regions (Fig 3). The region adjacent to the boundary-layer known as the entropy-layer contains fluid particles which have passed through much stronger portions of the leading edge shock-wave and hence the temperature and the specific entropy in this region will be much higher. The flow field between the shock-wave and the edge of the entropy-layer is called shock-layer. Cheng et al proposed an elegant method to this combined problem of bluntness and displacement on slender bodies with arbitrary after-body shape. Assuming the shock-layer to be very thin, and considering the limit $M \gg 1$ and $V \rightarrow 1$, they showed that the pressure at the outer edge of the entropy-layer can be obtained by using the Newton-Busemann formula,

$$p_e = \rho_\infty U^2 (y_e'^2 + y_e y_e'') \quad 2.1.1$$

By considering the momentum balance in the streamwise direction, another equation relating the pressure p_e , thickness of the entropy-layer y_e and the leading edge drag D_N was obtained. The effect of the pressure gradient on the boundary-layer growth and the variation of pressure across the entropy-layer were shown to have only higher order effects. With these approximations, the basic parameters governing the problem were identified. Considering the simplicity of the theory, the analysis predicted the trends correctly and agreed reasonably with the experimental heat transfer measurements on flat plates at $M = 12$. Kemp (Ref 38) found that Cheng's analysis did not give satisfactory agreement with measurements in helium flows and in order to improve the predictions, he proposed certain modifications. The Newton-Busemann formula used by Cheng was modified as

$$p_e = \frac{2}{\gamma+1} \rho_\infty U^2 (y_e'^2 + y_e y_e'') \quad 2.1.2$$

Kemp's modifications gave better agreement in the strong interaction region. When the centrifugal force reduces to zero, as in the case of flow past a blunted wedge for large downstream distances, the Eq 2.1.2 represents the correct shock pressure rise according to the oblique shock theory.

Another interesting feature of Cheng's analysis is the oscillatory approach of the pressure and shock-wave shape to the downstream limits when applied to blunted wedges. The physical reality of this oscillatory behaviour has been doubted by the authors and others (Ref 37). The authors suggest that for realistic values of $\gamma \neq 1$, the oscillatory decay may not exist since the amplitude of oscillation

was observed to be quite small even for $\gamma = 1$. It was supposed for a long time that a similar method due to Chernyi (Refs 39 and 40) did not exhibit any such oscillatory approach to downstream conditions. The non-existence of oscillations in Chernyi's approach was suspected to be due to the inclusion of the kinetic energy term in the analysis (Ref 36, p 364). It has been demonstrated recently by Schnieder (Ref 41) that Chernyi's approach also predicts an oscillatory approach in pressure and shock shape but the wave length and amplitude of oscillations are different. Cheng and Kirsch (Ref 42) considered an analogous unsteady problem of an intense explosion with an expanding contact surface to treat the problem of inviscid hypersonic flow past blunted slender bodies of arbitrary after-body shape. The case of the contact surface expanding linearly with time is equivalent to the steady hypersonic flow past a blunted wedge. For this case, their results show that when higher order solutions are considered, the oscillations present in the zeroeth order theory almost vanish. Good agreement for surface pressure on blunted wedge was obtained with the characteristics solution of Cleary and Axelson (Ref 43).

Another approach to improve Cheng's zeroeth order theory to give the correct downstream limit for the pressure was suggested by Stollery (Ref 22) for the pure viscous interaction problem. This method was later applied to the case of blunted wedges by Stollery, Pimputkar and Bates (Ref 44). They used the tangent-wedge approximation in place of the Newton-Busemann rule in Cheng's analysis and integrated the resulting equations numerically. This modified analysis suppressed the oscillatory behaviour and the downstream conditions were reached monotonically. Near the leading edge, the method appears to overestimate the pressure. It is felt that the estimates of the/can be improved for the case of a blunted wedge, by /method incorporating modifications similar to those in Kemp's analysis. This point will be discussed subsequently. It would also be interesting to see whether a similar analysis using the tangent-cone approximation for blunted cones, predicts the undershoot in pressure.

2.2 Experimental studies on flat plates:

Numerous experimental studies (Refs 32, 38, 45 - 61) have been made to investigate the effect of the boundary-layer displacement and the leading edge thickness on flat plates in hypersonic flow. These studies have helped to evaluate the usefulness of the various theoretical methods discussed in Sections 2.0 and 2.1.

References 45- 53 deal with tests on flat plates at zero incidence. In general, these tests indicate that viscous interaction theories using tangent-wedge approximation for determining the inviscid flow field, reasonably predict the induced pressures for sharp leading edges from weak interaction region to moderately strong interaction region ($\tilde{x} = 30-40$). However, the heat transfer rates appear to be slightly underestimated by most of the methods. For blunt leading edges, good correlation for the shock-wave shape and the pressure has been obtained using the blast-wave analogy (Refs 54 - 55). The analogy fails very near the leading edge where the perturbations in the flow field are not small, and also far downstream where the

requirement of a strong shock is not met. Marvin (Ref 52) reports good predictions for shock wave shape between 2-60 times the nose thickness. The effect of blunting the leading edge was observed to result in an increase in heat transfer rates downstream corresponding to the sharpening leading edge values (Ref 52 and 53). However, Neal (Ref 54) and Stone (Ref 55) found that small blunting of the leading edge reduced the heat transfer rates corresponding to the sharp leading edge values, while the pressure distribution was not affected much. Stone made impact pressure surveys and found that the boundary-layer thickened due to nose blunting and reduced the heat transfer coefficients.

The effect of incidence on heat transfer rates for sharp and blunt flat plates was studied by Cheng et al (Ref 32). They compared the heat transfer measurements at $M = 12$ in air, with their theoretical analysis for $M \gg 1$ and $V \rightarrow 1$ taking into account both the displacement and the bluntness effects. The measurements were generally found to be 15 - 20 % higher than the theoretical predictions. Allegre and Bisch (Ref 57) made pressure measurements on sharp and blunt flat plates at incidence in the strong viscous interaction region at $M = 18$ in nitrogen. For the sharp leading edge, good agreement with Cheng's theory (Ref 32) was found, but for the blunt leading edge the agreement was rather poor. Stollery (Ref 44, 59) has made pressure measurements on a sharp flat plate at incidence at $M = 14.8$ and has compared his measurements and also Holden's measurements (Ref 58) on a sharp flat plate at zero incidence with Sullivan's method. The comparison shows that the pressures are generally overestimated by Sullivan's method. More detailed experiments involving both heat transfer and pressure measurements over sharp and blunt flat plates at incidence in the strong region ($\bar{\chi} = 60 - 300$) at $M = 42$, in helium have been made by Kemp (Ref 38). The measurements agreed with Cheng's theory after some modifications discussed in Sec 2.2. Vidal and Bartz (Ref 60) conducted tests at $M = 18 - 24$, spanning flow regimes from free molecule flow to thin boundary-layer flow. Their data for pressure and heat transfer on sharp flat plates showed good agreement with Cheng's theory in the strong interaction region. Similar heat transfer measurements on a sharp flat plate at Mach numbers 22.8 - 86.8 in helium have been made by Nagamatsu and Sheer (Ref 61) covering a wide range of the viscous interaction parameter, $\bar{\chi} = 11 - 16000$, and find good agreement with theoretical predictions of Li and Nagamatsu (Ref 12) for ranges of $\bar{\chi}$ not affected by rarified gas effects.

In general, it appears that for sharp flat plates in the strong interaction region, Cheng's theory with Kemp's corrections gives good predictions. Most of the studies mentioned in this Section have generally established an understanding of the displacement effect and the bluntness effect in the vicinity of the leading edge and for some distance downstream. The quantitative agreement of results with theoretical predictions does not appear to be very satisfactory for the case of blunt leading edges. Another interesting point is the far downstream effect of a blunt leading edge on the heat transfer rates. Perhaps, this would require some more theoretical and experimental study in the light of Neal's and Stone's (Ref 54 and 55)

work who find the heat transfer rates to be decreased for small leading edge bluntness.

2.3 Viscous interaction on curved surfaces:

The study of viscous interaction effects on surfaces other than flat plate afterbody, in the presence of strong leading edge interaction has received attention only recently. The nature of the viscous interaction mechanism depends on the shape of the after-body. In particular, when the after-body is a concave surface, the flow phenomena is quite complicated because of the interaction between the leading edge shock-wave and the compression waves generated from the body surface. Other after-body shapes of practical interest are convex surfaces, and compression and expansion corners. The study of compression and expansion corner flows is beyond the scope of the present thesis, though some of the analyses of the present work are still applicable when the upstream influence is not significant. The concave surface flows are considered in detail with occasional references to convex surfaces.

From a practical point of view, concave surface flows are important in the design of hypersonic inlets where the boundary-layer is subjected to adverse pressure gradients in addition to the viscous-inviscid interaction. Even the treatment of the inviscid hypersonic flow past such surfaces appears to be rather limited, considering the abundant literature existing on the corresponding convex surfaces. Sullivan (Ref 62) obtained similar solutions for the inviscid hypersonic flow on power-law concave surfaces and showed that they are the correct first approximation to the exact inviscid equations for $M_\infty \rightarrow \infty$. Thus, he was able to calculate the pressures on concave surfaces, and compared with measurements on a cubic surface. The agreement was found to be good over the limited region downstream of the leading edge where the similar solutions are expected to be a good approximation. A detailed comparison of the similar solutions with other approximate inviscid theories was given in Ref 63.

For the viscous hypersonic flow on concave surfaces, the boundary-layer growth can significantly modify the effective body-shape and the inviscid calculations ignoring the displacement effects may be incorrect in certain regions of the flow. The region near the leading edge will be much affected by the viscous interaction effects. Because of the dominant incidence effect, the interaction effects become negligible far downstream and the inviscid predictions are quite accurate in this region. Richey (Ref 24) presented an analysis for compression surfaces taking the displacement effect of the boundary-layer into consideration. The inviscid flow was determined by the tangent-wedge rule near the leading edge region and in regions of adverse pressure gradient reverse Prandtl-Meyer turning (Simple wave flow theory) was employed. The boundary-layer growth was calculated using a local similarity approach. The analysis covered both the strong and weak interaction regions and the method predicted a steady decrease in the boundary-layer thickness (super-critical behaviour $\frac{d\delta^*}{dp} < 0$) in the adverse pressure gradient region as observed experi-

mentally. Thus, the physical features of the flow on a compression surface (Fig 4) were described correctly by the analysis. Since, on compression surfaces, the viscous effects are important only near the leading edge, Stollery (Ref 46) suggested that the the Newton-Busemann rule corrected for the strong viscous interaction effects may be a good approximation for the pressure, i.e.,

$$\frac{p_w}{p_\infty} = \frac{3}{4} \sqrt{\gamma(\gamma+1)} \bar{\chi}_e + \gamma M_\infty^2 (\gamma_w^2 + \gamma_w \gamma_w'') \quad 2.3.1$$

The first term in Eq 2.3.1 represents Cheng's strong interaction solution and the other terms represent the inviscid pressure calculated ignoring the viscous effects. The Eq 2.3.1 is simple and can be applied to complicated slender shapes in a straightforward manner. However, comparing with the pressure distribution due to viscous interaction effects predicted by Richey (curve BB of Fig 4), *LGA and* the trend towards approaching the inviscid limit according to Eq 2.3.1 seems to be different. Eq 2.3.1 suggests that the pressure is everywhere higher than the inviscid value, *Whereas,* the pressure distribution in Fig 4 represents a lag in the pressure rise due to viscous interaction effects. *w*

The boundary-layer on a concave surface is essentially growing in an adverse pressure gradient and there are available a number of standard methods for treating this problem (e.g., Ref 65 and 66), provided the inviscid pressure distribution is known. Fitzhugh (Ref 67 and 68) has made a detailed study of the various approximate methods in the light of exact numerical studies of the boundary-layer equations, for boundary-layers in adverse pressure gradient. His results represent only the first approximation and cannot be expected to hold good when the displacement and other effects normally considered to be of higher order assume first order importance.

A more rigorous extension of Cheng's analysis, rather than the empirical modification suggested by Eq 2.3.1. was made later by Stollery (Ref 22). The original Cheng's viscous interaction was integrated for a variety of different after-body shapes such as concave and convex surfaces, and compression and expansion corners. Except for concave surfaces, the analysis was found to predict correctly the main flow features in the strong interaction region. However the analysis predicted highly oscillatory approach in pressure, heat transfer and boundary-layer thickness to the downstream inviscid limit. The amplitude of the oscillation was quantitatively unrealistic, so as to provide any useful estimates of interaction effects. To overcome this limitation, Stollery modified the analysis by substituting the extensively used tangent-wedge rule in place of the Newton-Busemann rule (Eq 2.1.1) used in Cheng's analysis. The modified analysis, which only accounts for the shock pressure rise removed the oscillatory behaviour completely and a smooth transition to downstream conditions was obtained. To demonstrate the applicability of the method, the pressure and heat transfer rates were calculated for a number of cases and compared with experimental measurements. In particular, the method gave good agreement for

for pressure on a cubic surface, for which Cheng's theory yields quantitatively unrealistic results. A more detailed investigation of Cheng's method and its modified version and their comparison with experimental pressure and heat transfer measurements at $M = 12.25$, was made by Mohammadian (Ref 69). The sub-critical ($ds^*/dp > 0$) and super-critical ($ds^*/dp < 0$) nature of the boundary-layer on concave surfaces was also pointed out by Mohammadian. The modified Cheng's analysis agreed well with the measurements, except for heat transfer rates on the concave surface for which the predictions were considerably lower*. Further comparisons for the concave surface were presented by Stollery (Ref 70), by using the well known Lees-Reeves integral technique (Ref 71) with subsequent modifications by Holden (Ref 72), and Klineberg and Lees (Ref 73). With this approach, better agreement was obtained for the heat transfer rate, but pressures were considerably overestimated.

The effect of the longitudinal curvature can be quite important on the boundary-layer growth over curved surfaces. Wornom and Werle (Ref 86) solved the boundary-layer equations taking into account the displacement and the longitudinal curvature effects. They employed a finite difference method for the boundary-layer and used the tangent-wedge relation for the pressure on the displacement body. The results agreed well with the pressure measurements of Mohammadian on the cubic surface.

2.4 Scope of the present work:

In the present work, the interaction effect has been studied by varying the viscid-inviscid coupling. This is in contrast with the earlier works as we have seen in Sections 2.0 - 2.3, where refinement in the viscous flow field analysis has been adopted extensively, whilst retaining the tangent wedge approximation to describe the inviscid flow field. The present investigation was motivated by the fact that the widely used tangent wedge formula has been criticised on many occasions (Ref 36 and 74) for not accounting for centrifugal effects, and at the same time the Newton-Busemann rule which is free of this criticism is found to give oscillatory results of unrealistic magnitude for concave surfaces (Stollery, Ref 22). In order to further investigate the effect of retaining the centrifugal pressure variation in the inviscid flow field analysis, the viscous interaction problem on flat plates and concave surfaces has been studied by modifying the tangent-wedge approximation. Basically, two modifications have been considered:

- (i) a straight forward correction for the centrifugal effects to the tangent-wedge rule.
- and
- (ii) modification to account for the centrifugal effects and for variation in the flow inclination angle between the body and the shock wave as suggested by Lees (Ref 74).

* than the measurements

For the boundary-layer, Lees' local flat plate similarity has been used for simplicity. The application of the suggested modifications to bluntness dominated flows is also considered. The results are compared with Cheng's theory and its modified version due to Stollery (Ref 22). In order to facilitate comparison and to present some generalizations, the treatment of the present analysis is carried out along with Cheng's and modified Cheng's methods, wherever necessary.

Finally, the application of Chernyi's integral method has been considered for the bluntness as well as viscous dominated flows and it is demonstrated that the oscillations present in Cheng's zeroeth order theory are heavily damped because of the inclusion of the after-body drag and kinetic energy terms in the analysis.

On the experimental side, measurement of heat transfer and pressure have been made on blunt flat plates and on a concave surface with a sharp leading edge at $M = 12.25$. The measurements on blunt flat plates was undertaken to study the downstream effect of the blunt leading edge. The concave surface was designed so that measurements can be made in a region as near to the leading edge as possible, where the growth of the leading edge shock wave is strongly influenced by the body curvature. The results are compared with the predictions of viscous interaction theories.

PART II THEORETICAL STUDIES

CHAPTER III

PRELIMINARY CONSIDERATIONS.

In this Chapter, the governing equations for the viscous and the inviscid flow fields will be presented. For the boundary-layer analysis, Lees' flat plate theory is used and its limitations discussed in the light of local similarity approach. In the treatment of the inviscid flow field, only the pressure laws extensively used in the viscous interaction analysis and their modifications for the present analysis are considered. Within the scope of these pressure laws, the sub-critical and super-critical behaviour of the boundary-layer on concave surfaces is illustrated. The analysis of the inviscid flow field using Chernyi's integral approach is deferred to Chapter VII.

3.1 Local flat plate similarity theory:

The compressible laminar boundary-layer equations for the steady flow of a perfect gas past a two-dimensional body can be written as

$$\text{Continuity:} \quad \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0 \quad 3.1.1$$

$$x \text{ Momentum :} \quad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \quad 3.1.2$$

$$y\text{-Momentum:} \quad \frac{\partial p}{\partial y} = 0 \quad 3.1.3$$

$$\text{Energy:} \quad \rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y} \left\{ \frac{\mu}{Pr} \frac{\partial H}{\partial y} + \mu \left(1 - \frac{1}{Pr} \right) u \frac{\partial u}{\partial y} \right\} \quad 3.1.4$$

$$\text{Eqn of State:} \quad p = \rho R T \quad 3.1.5$$

In the derivation of the above equations, it is assumed that the thickness of the boundary-layer is small in comparison with the longitudinal radius of curvature of the body. The boundary conditions are

$$y = 0; \quad u = 0, v = 0$$

$$H = H_w \quad \text{For the heat transfer case}$$

$$\frac{\partial H_w}{\partial y} = 0 \quad \text{For the insulated wall} \quad 3.1.6$$

$$y \rightarrow \infty ;$$

$$u = u_e$$

$$H = H_e$$

3.1.7

Applying the well known Howarth-Dorodnitsyn transformation

$$\xi = \int_{\eta}^{\infty} \rho_w \mu_w u_e dx$$

$$\eta = \frac{u_e}{\sqrt{2\xi}} \int \rho dy$$

and with

$$f' = u/u_e \quad \text{and} \quad g = H/H_e$$

the equations 3.1.1 - 3.1.7 can be reduced to

$$(N f'')' + f f'' + 2 \frac{d \ln u_e}{d \ln \xi} \left(\frac{\rho_e}{\rho} - f'^2 \right) = 0 \quad 3.1.8$$

$$\left(\frac{N}{Pr} g' \right)' + f g' + \frac{u_e^2}{H_e} \left\{ N \left(1 - \frac{1}{Pr} \right) f' f'' \right\}' = 0 \quad 3.1.9$$

$$\text{where} \quad N = \rho \mu / \rho_w \mu_w$$

with boundary conditions

$$\begin{aligned} \eta = 0; \quad & f = f' = 0 \\ & g = g_w \quad (\text{Heat transferring wall}) \\ & g'_w = 0 \quad (\text{Insulated wall}) \quad 3.1.10 \\ \eta \rightarrow \infty; \quad & f' = g = 1 \quad 3.1.11 \end{aligned}$$

For the case of hypersonic flow ($M \gg 1$, $u_e^2/H_e \approx 2$), the above equations 3.1.8 and 3.1.9 can be further simplified assuming g_w to be constant along the wall and a linear viscosity-temperature relation, to

$$f''' + f f'' + \beta (g - f'^2) = 0 \quad 3.1.12$$

$$g'' + Pr f g' + 2 (Pr - 1) (f' f'')' = 0 \quad 3.1.13$$

$$\text{where} \quad \beta = - \frac{\gamma - 1}{\gamma} \frac{\int \rho_e dx}{p_e^2} \frac{dp_e}{dx} \quad 3.1.14$$

In the flat plate similarity rule, the effect of the pressure gradient term β in Eqn 3.1.12 is considered to be small for the case of highly cooled walls since the term $\beta(q-f'^2) \ll 1$, over the range of integration (Ref 31). It has also been argued by Sullivan (Ref 20) that the contribution of the pressure gradient terms can be small when the pressure gradient is favourable even if the wall is not cold. To simplify the problem, the Prandtl number is assumed unity and the equations 3.1.12 and 3.1.13 are integrated subject to the boundary-conditions 3.1.10 and 3.1.11, to give

$$f''(0) = 0.4696 \quad 3.1.15$$

$$q = 1 - (1-g_w)(1-f') \quad 3.1.16$$

From equation 3.1.14, it can be seen that β can be small when $V \rightarrow 1$, even if the cold wall assumption is not met. This is the basis for neglecting the pressure gradient effects in Cheng's analysis (Ref 32). The resulting equations for the boundary-layer are same in Lees' flat plate similarity approach and in Cheng's analysis.

Displacement thickness: In order to calculate the viscoid-inviscid interaction, it is necessary to know the variation of displacement thickness along the wall. By definition, the displacement thickness δ^* is written as

$$\delta^* = \int_0^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy \quad 3.1.17$$

or in the transformed coordinates

$$\delta^* = \frac{(2\xi)^{1/2}}{\rho_e u_e} \int_0^\infty \left(\frac{\rho_e}{\rho} - \frac{u}{u_e}\right) d\eta \quad 3.1.18$$

The integrand in equation 3.1.18 can be simplified further by making use of the hypersonic approximation $\frac{h_e}{He} \ll 1$

$$\frac{\rho_e}{\rho} - \frac{u}{u_e} = \frac{q - f'^2 \frac{u_e^2}{2He}}{h_e/He} - f'$$

and

$$\frac{u_e^2}{2He} = 1 - \frac{h_e}{He}$$

$$\left(\frac{\rho_e}{\rho} - \frac{u}{u_e}\right) \approx \frac{He}{h_e} (q - f'^2) \quad 3.1.19$$

Substituting 3.1.19 in 3.1.18, we get

$$\delta^* = \frac{(2\xi)^{1/2}}{\rho_e u_e} \frac{H_e}{h_e} I \quad 3.1.20$$

$$\text{where } I = \int_0^\infty (g - f'^2) d\eta \quad 3.1.21$$

For unit Prandtl number and zero pressure gradient, the integral I can be evaluated using equation 3.1.16 and we obtain

$$I = 0.4696 (1 + 2.6 g_w) \quad 3.1.22$$

The displacement thickness cannot be calculated directly from equation 3.1.20 since the pressure distribution is unknown and the determination of which is a part of the problem. For further analysis, assuming the flow to be everywhere hypersonic, we write

$$\xi = \int_0^x \rho_w M_w u_e dx \approx \frac{U_\infty M_w}{R T_w} \int_0^x p_w dx$$

$$\rho_e u_e \approx \frac{p_w}{R T_e} U_\infty$$

$$H_e/h_e = 1 + \frac{u_e^2}{2h_e} \approx \frac{U_\infty^2}{2C T_e}$$

Substituting for ξ , $\rho_e u_e$ and H_e/h_e in equation 3.1.20, we get

$$\delta^* = \frac{1}{\sqrt{2}} \frac{\gamma-1}{\gamma} \left(\frac{M_w}{R T_w} \right)^{1/2} \frac{\left(\int_0^x p_w d\tilde{x} \right)^{1/2}}{p_w} I$$

$$\text{or } \frac{M_\infty \delta^*}{\chi} \approx \frac{1}{\sqrt{2}} \frac{\gamma-1}{\gamma} M_\infty^3 \left[\frac{U_\infty^{3/2}}{M_\infty^2} \left(\frac{M_w}{R T_w} \right)^{1/2} \frac{1}{\chi^{1/2}} \right] \frac{\left(\int_0^x p_w d\tilde{x} \right)^{1/2}}{p_w} I \quad 3.1.23$$

The expression within the square bracket can be written as

$$\frac{U_\infty^{3/2}}{M_\infty^2} \left(\frac{M_w}{R T_w} \right)^{1/2} \frac{1}{\chi^{1/2}} = \gamma^{1/2} p_\infty^{-1/2} R_{ex} C^{1/2}$$

$$\text{where } C = \frac{M_w T_\infty}{M_\infty T_w}$$

So, the equation 3.1.23 reduces to

$$\frac{M_\infty \delta^*}{\chi} = \frac{\gamma-1}{\sqrt{2}} \frac{\gamma}{P} \frac{I}{P} \left(\int_0^x P \frac{d\tilde{x}}{\chi} \right)^{1/2} \quad 3.1.24$$

where $\bar{\chi} = M_\infty^3 C_{ex}^{-1/2}$, Viscous interaction parameter
 $P = p_w / p_\infty$

writing $I_0 = \frac{I}{0.4696(1 + 2.6 g_w)}$

and $A = 0.4696 \cdot \frac{\gamma-1}{\sqrt{2}} (1 + 2.6 g_w)$

we get

$$\frac{M_\infty \delta^*}{\chi} = I_0 \frac{A \bar{\chi}}{P} \left(\int_0^x P \frac{d\bar{x}}{\chi} \right)^{1/2} \quad 3.1.25$$

Choosing a length scale $L_s = A \bar{\chi} \chi$ and with $X = \chi / L_s$, equation 3.1.25 can be written as

$$\frac{M_\infty \delta^*}{L_s} = I_0 \frac{\left(\int_0^X P d\bar{x} \right)^{1/2}}{P} \quad 3.1.26$$

It may be noted that, in general, the value of I_0 depends on the value of the pressure gradient parameter β , Prandtl number and the viscosity-temperature relation used in the analysis. In the present flat plate similarity approach for zero pressure gradient, unit Prandtl number and linear viscosity-temperature relation, I_0 will be unity, and equation 3.1.26 reduces to

$$\frac{M_\infty \delta^*}{L_s} = \frac{\left(\int_0^X P d\bar{x} \right)^{1/2}}{P} \quad 3.1.27$$

Skin friction and Heat transfer: The local skin friction coefficient c_f and the Stanton number St are defined as

$$c_f = \frac{\mu (\partial u / \partial y)_w}{\frac{1}{2} \rho_\infty U_\infty^2} \quad 3.1.28.$$

and $St = \frac{q_w}{\rho_\infty U_\infty c_p (T_0 - T_w)} \quad 3.1.29$

Following a similar analysis as earlier, it can be shown

$$A M_\infty^3 St = \frac{g_w'}{\sqrt{2} \cdot P_\infty (1 - g_w)} \cdot \frac{P}{\left(\int_0^X P d\bar{x} \right)^{1/2}} \quad 3.1.30$$

$$\text{and } AM_{\infty}^3 C_f = \sqrt{2} f''(0) \cdot \frac{P}{\left(\int_0^x P d\tilde{x}\right)^{1/2}} \quad 3.1.31$$

For the local flat plate similarity approach, substituting from equation 3.1.16, we get (for $P_T = 1$)

$$2 St = C_f = \frac{1}{AM_{\infty}^3} - \frac{P}{\left(\int_0^x P d\tilde{x}\right)^{1/2}} \quad 3.1.32$$

The equations 3.1.27 and 3.1.32 along with a suitable pressure law to describe the inviscid flow field are the basic equations used in the viscous interaction analysis.

The local flat plate boundary-layer theory affords great simplicity in the analysis by ignoring the pressure gradient effects. Though the neglect of the pressure gradient effects can be justified either for a cold wall or in the mathematical limit of $V \rightarrow 1$, it is found that for certain ranges of the pressure gradient parameter of practical interest, this can lead to serious numerical errors in the evaluation of the integral I_0 , heat transfer and skin friction (Ref 18, 74). The assumption of unit Prandtl number can be relaxed easily and this can give more accurate values of the boundary-layer parameters even if the pressure gradient effect is not considered. Dewey (Ref 23), Sullivan (Ref 21), and Cohen and Reshotko (Ref 75) have given the values of I_0 , f_w'' and g_w' for different values of Prandtl number and the pressure gradient parameter β . The values of the integral I_0 , f_w'' and $g_w'/Pr(1-g_w)$ have been plotted for a Prandtl number of 0.723 and for various β in figures 5 - 7, using the results of Rogers (Refs 76 and 77). It may be observed from figures that zero pressure gradient and unit Prandtl number results are applicable over a restricted range of the parameter β . Discrepancies in the skin friction derivative f_w'' can be as much as 100% for an adiabatic wall. Despite these inaccuracies, the local flat plate boundary-layer theory has been extensively used because of its simplicity. The success of the method may be partly due to other uncertainties involved in correctly describing the inviscid flow field which might compensate for the inaccuracies and give acceptable agreement with measurements.

3.2 Governing equations for the inviscid flow field.

There are available a number of methods for determining the inviscid flow field past slender bodies at hypersonic speeds. Although exact solutions can be obtained by resorting to numerical methods, when the interest is mainly confined to the determination of the properties on the body surface, recourse is made to approximate methods. Particularly, in dealing with the problem of viscous-inviscid interaction, these approximate methods relating the pressure at a point on the body to the local geometric properties of the body shape offer

great simplicity. Some such approximate methods employed in the viscous interaction problem are discussed below.

Newtonian approximations: These are applicable to blunt as well as slender bodies. For slender bodies (with the approximation $\sin \theta \approx \theta$ and $\cos \theta \approx 1$), we have

a. Simple Newtonian formula:
$$\frac{p_w}{p_\infty} = 1 + \gamma M_\infty^2 \left(\frac{dy_e}{dx} \right)^2 \quad 3.2.1$$

b. Newton-Busemann formula:
$$\frac{p_w}{p_\infty} = \gamma M_\infty^2 \left(y_e'^2 + y_e y_e'' \right) \quad 3.2.2$$

The Newton-Busemann formula also corresponds to the leading approximation in the theory of thin shock layers for $V \rightarrow 1$. It incorporates a correction to the shock pressure rise for the centrifugal pressure variation across the shock layer.

Tangent wedge approximation:

In this extensively used approximation, the local pressure at a point on the body surface is assumed to be equal to that on an equivalent wedge having the same surface inclination. The tangent wedge approximation assumes that the flow properties are constant between the body-surface and the shock wave at a given streamwise location. The two-dimensional oblique shock relations simplified for the limiting case of infinite Mach number are used for calculating the surface pressure and can be written as

$$\frac{p_w}{p_\infty} = 1 + \gamma M_\infty^2 \left(\frac{dy_e}{dx} \right)^2 \left[\frac{\gamma+1}{4} + \left\{ \left(\frac{\gamma+1}{4} \right)^2 + \frac{1}{M_\infty^2 \left(\frac{dy_e}{dx} \right)^2} \right\}^{1/2} \right] \quad 3.2.3$$

For $M_\infty \frac{dy_e}{dx} \ll 1$, the equation 3.2.3 can be written as

$$\frac{p_w}{p_\infty} = 1 + \gamma M_\infty^2 \frac{dy_e}{dx} + \gamma \frac{(\gamma+1)}{4} M_\infty^2 \left(\frac{dy_e}{dx} \right)^2 + \frac{\gamma(\gamma+1)}{32} M_\infty^2 \left(\frac{dy_e}{dx} \right)^2 + \dots \quad 3.2.4$$

Similarly for $M_\infty \frac{dy_e}{dx} \gg 1$,

$$\frac{p_w}{p_\infty} = 1 + \frac{\gamma(\gamma+1)}{2} M_\infty^2 \left(\frac{dy_e}{dx} \right)^2 + \frac{2\gamma}{\gamma+1} + O \left[M_\infty^{-2} \left(\frac{dy_e}{dx} \right)^{-2} \right] \quad 3.2.5$$

The equation 3.2.4 has been used by many investigators to calculate the weak viscous interaction effects. The first three terms of 3.2.4 are identical with the Busemann series for supersonic airfoils. The tangent wedge approximation has the advantage that it can cover both the strong and weak interaction regions, whereas

the Newton-Busemann relation is valid only in the strong interaction region since far downstream the pressure does not asymptote to the free stream value.

Proposed modifications:

In spite of their extensive use in the viscous interaction problem, the Newton-Busemann formula and the tangent wedge approximation have been subjected to criticisms and modifications to overcome some of the objections have been attempted. In the Newton-Busemann relation the shock pressure rise is correctly represented only for $M_\infty dy_e/dx \gg 1$. When dealing with flow past general two-dimensional surfaces such as concave or convex surfaces, the condition $M_\infty dy_e/dx \gg 1$, may not hold good everywhere in the flow field and the Newton-Busemann relation may not be a good approximation for such flows. In particular, for the case of a wedge in hypersonic flow, the Newton-Busemann relation does not give the correct value for the pressure far downstream where the centrifugal force becomes zero. As discussed in Sec 2.1. Kemp obviated this difficulty by empirically modifying the equation 3.2.2 by substituting $2\gamma/(\gamma+1)$ in place of γ . Even with this modification

it should be realised that the resulting equation is correct only in the limit of $M_\infty dy_e/dx \gg 1$, and cannot cover the weak interaction region on a flat plate. Further, the modification introduces an error in the centrifugal correction term which may not be serious considering the uncertainties involved in assessing the extent of centrifugal correction necessary for realistic values of $\gamma \neq 1$. Perhaps, a more refined modification, which at least removes some of the objections levelled against Newton-Busemann rule can be obtained by substituting the tangent-wedge rule 3.2.3 for the shock pressure rise term $\gamma M_\infty^2 (dy_e/dx)$ in equation 3.2.2. Alternatively, such a modification can be looked upon as a correction to the basic tangent wedge rule to account for the centrifugal effects. The complete formula can be written as

$$\frac{p_w}{p_\infty} = 1 + \gamma M_\infty^2 \left(\frac{dy_e}{dx} \right)^2 \left[\frac{\gamma+1}{4} + \left\{ \left(\frac{\gamma+1}{4} \right)^2 + \frac{1}{M_\infty^2 \left(\frac{dy_e}{dx} \right)^2} \right\}^{1/2} \right] + \gamma y_e \frac{d^2 y_e}{dx^2} \quad 3.2.6$$

The equation 3.2.6 can cover both the strong and weak interaction regions and incorporates the correct coefficient for the centrifugal term. In addition, it represents the correct limit for the pressure on a wedge. It is important to note that the modifications in equation 3.2.6 is not just an outcome of heuristic reasoning, but has the justification, as we shall see later that it represents a logical improvement to the basic tangent wedge approximation to account for the centrifugal effects. While the application of equation 3.2.6. to the viscous interaction problem was in progress, the author came across a recent report by Sullivan (Ref 63) suggesting a combination of 'tangent wedge plus centrifugal' formula based on similar reasonings. However, the analysis of Sullivan is limited to inviscid flows past cusped concave surfaces, whereas in the present work the main interest is to study the viscous-inviscid interaction and also the leading

edge bluntness effects. The form of the equation used by Sullivan is

$$C_p = \frac{2(1-\epsilon)}{M_\infty^2} \left[k_s^2 - 1 \right] + 2B(x) \cdot \frac{d^2 B(x)}{dx^2} \quad 3.2.7$$

where

$$C_p = 2(p_w - p_\infty) / \rho_\infty U_\infty^2$$

$$k_s = \frac{M_\infty dB/dx}{2(1-\epsilon)} + \left[\left\{ \frac{M_\infty dB/dx}{2(1-\epsilon)} + 1 \right\} \right]^{1/2}$$

and

$$\epsilon = (\gamma-1)/(\gamma+1) ; B(x) : \text{Body shape}$$

It can be shown that equations 3.2.6 and 3.2.7 are identical. The tangent wedge approximation assumes the curvature of the leading edge shock wave to be small and is considered to be in error for two reasons (Refs 36 and 74):

- (i) It neglects the centrifugal effects, and
- (ii) It does not account for the variation in flow inclination angle between the shock wave and the body.

Lees (Ref 74) has shown that these two effects are of opposite sign and tend to cancel each other. The degree of cancellation depends on the value of γ and for $\gamma \rightarrow 1$, the correction for the flow inclination will be much smaller than that for the centrifugal effects. Lees modified the tangent wedge approximation for these two effects and suggested the following formula.

$$\frac{p_w}{p_\infty} = \gamma \frac{(\gamma+1)}{2} M_\infty^2 \left(\frac{dy_e}{dx} \right)^2 + \gamma \left(1 - \frac{\gamma^2-1}{\gamma} \right) y_e \frac{d^2 y_e}{dx^2} M_\infty^2 \quad 3.2.8$$

The equation 3.2.8 is called 2nd order tangent wedge in Ref 74.

Hayes and Probstein argue (Ref 36) that the errors involved in estimating the correction for the flow inclination angle are of the same order as the uncertainties involved in the centrifugal pressure correction. Hence they account only for the centrifugal effects and suggest the following modification.

$$\frac{p_w}{p_\infty} = 1 + \frac{\gamma M_\infty^2 (dy_e/dx)^2}{1-\epsilon} + \gamma M_\infty^2 \frac{y_e}{R_b} \quad 3.2.9$$

where

$$\epsilon = (\gamma-1)/(\gamma+1)$$

R_b = Radius of curvature of the body
(+, Concave)

$$\frac{1}{R_b} = \frac{d^2 y_e}{dx^2} > 0$$

For slender bodies, equation 3.2.9 can be written as

$$\frac{p_w}{p_\infty} = 1 + \frac{\gamma M_\infty^2 (dy_e/dx)^2}{1-\epsilon} + \gamma M_\infty^2 y_e \frac{d^2 y_e}{dx^2} \quad 3.2.10$$

For $M_\infty dy_e/dx \gg 1$, the equation 3.2.6 and 3.2.10 are identical and this suggests that the correction incorporated in equation 3.2.6 is justified as long as the flow inclination effects are small. The equation 3.2.6 may be more useful since it represents the shock pressure rise correctly even for moderate values of $M_\infty dy_e/dx$. However, in the weak interaction region, the effect of the centrifugal terms is of higher order compared to the pressure increment associated with small flow deflections, and may not introduce serious errors.

Now, considering the modifications represented by equations 3.2.6. and 3.2.8, for γ greater than unity the corrections suggested can be significantly different in two cases. Lees (Ref 74) has compared equation 3.2.8 with the exact value for the initial pressure gradient on plane ogive and the agreement seems to be better than 5% for $1 < \gamma < 1.5$. Lees' modification suggests that for $\gamma = 1.4$, the effective correction can be much smaller than that given by purely centrifugal effects. In the present work, the application of both the methods will be considered to the study of viscous interaction and bluntness effects. For convenience, the equation 3.2.6 will be referred as 'Tangent/plus centrifugal' or for brevity 'TW + CFGL'. Similarly, the equation 3.2.8 will be called '2nd order Tangent Wedge'.

The surface pressures calculated from various approximate formulae discussed in this Section are compared with the results of hypersonic self-similar solutions (Sullivan, Ref 63) in figures 8 and 9. For power-law surfaces ($y_e \propto x^n$), the equations 3.2.1 to 3.2.3, 3.2.6 and 3.2.8 can be written as ($M_\infty \frac{dy_e}{dx} \gg 1$)

Newtonian	:	$\frac{p_w}{\gamma p_\infty M_\infty^2 (dy_e/dx)^2} = 1.0$
Newton-Busemann	:	$\text{---} = 1 + \frac{n-1}{n}$
Tangent Wedge(TW)	:	$\text{---} = (\gamma+1)/2$
TW + CFGL	:	$\text{---} = \frac{\gamma+1}{2} + \frac{n-1}{n}$
2nd order TW	:	$\text{---} = \frac{\gamma+1}{2} + \frac{n-1}{n} \left(1 - \frac{\gamma-1}{\gamma}\right)$

For $V^* = 1.4$, the pressures predicted by the TW + CFGL formula are slightly higher than the similar solutions. Sullivan suggests that the error in using the TW + CFGL formula as compared to the exact solution for finite M_∞ should be less than that of hypersonic self-similar solutions. The Newton-Busemann rule estimates are slightly lower than the similar solutions and the difference keeps decreasing as η increases. The results of tangent wedge approximation, 2nd order tangent wedge and Newtonian rule are much lower.

3.3 Behaviour of the hypersonic boundary-layers on power law surfaces:

The asymptotic behaviour of the boundary-layer on a power-law surface can be determined without solving the complete viscous interaction problem. From equation 3.1.27, the non-dimensional displacement thickness Δ^* is

$$\Delta^* = \frac{M_\infty S^*}{L_s} = \frac{\left(\int_0^x P dx \right)^{1/2}}{P}$$

Assuming the inviscid pressure distribution follows a power-law $P = k X^m$, Δ^* can be written as

$$\Delta^* = \frac{1}{[k(m+1)]^{1/2}} X^{1/2 - m/2} \quad 3.3.1$$

For $m = 0$, the boundary-layer will be growing under constant pressure and we have the well known flat plate boundary-layer result $\Delta^* \propto X^{1/2}$. For $m > 0$, the boundary-layer is subjected to an adverse pressure gradient and the behaviour depends on the value of m .

- (i) For $m < 1$, the boundary-layer thickness Δ^* increases with X as pressure is increasing downstream, i.e. $\frac{d\Delta^*}{dP} > 0$ (known as sub-critical behaviour)
- (ii) For $m = 1$, the boundary-layer thickness Δ^* will be constant for all X even though the pressure is increasing
- (iii) For $m > 1$, the boundary-layer thickness Δ^* decreases with increasing X , though the pressure keeps increasing i.e. $\frac{d\Delta^*}{dP} < 0$ (known as super-critical behaviour)

The sub- and super-critical behaviour discussed above is entirely dependent on the pressure distribution and is independent of the mechanism causing the pressure distribution. For example, the pressure distribution can be imposed on a flat plate by an external body. In particular, when we are dealing with concave surface flows, the pressure distribution far downstream is mainly determined by the body incidence. If the body-shape is given by $Y_w = X^n$, it may be easily verified that all the pressure laws for large X and the self-similar solutions behave like

$$P = C_1 X^m \quad 3.3.2$$

where $m = 2n - 2$

$$C_1 = \text{Constant}$$

The value of C_1 will be different for different pressure laws. Depending on the value of n , the boundary-layer will be of either subcritical or super-critical type as shown below.

Sub-critical	:	$m < 1$	corresponds	$n < 3/2$
Constant thickness	:	$m = 1$	corresponds	$n = 3/2$
Super-critical	:	$m > 1$	corresponds	$n > 3/2$

The behaviour of the boundary-layer thickness for large X on concave surfaces is obtained by substituting for k and m in equation 3.3.1 and we get

$$\gamma^{1/2} \Delta M_{\infty} \alpha_s = C_2 X^{3/2 - n} \quad 3.3.3$$

where C_2 is a constant whose value depends on n and γ only, and will be different for different pressure laws. The actual expressions for C_1 and C_2 have been tabulated in Table 3.1.

The sub- and super-critical behaviour of the boundary-layer on concave surfaces was pointed out by Mohammadian (Ref 69) by considering the viscous interaction analysis of Cheng (using Newton-Busemann law) and also its modified version using the tangent wedge approximation. From the present analysis, it may be seen that the sub- and super-critical behaviour is not restricted to Cheng's analysis or to its modified version but is true for all pressure laws. It is also important to note that this behaviour of the boundary-layer is not because of the viscous interaction effects but is entirely determined by the inviscid pressure distribution far downstream. The character of decay or growth of the boundary-layer may be modified by the viscous interaction effects to a certain extent.

In figure 10, the asymptotic behaviour of boundary-layer as given by equation 3.3.3 is shown plotted for $\gamma = 1.4$, and for three pressure laws (Newton-Busemann, TW and TW + CFGL). The sub-critical and super-critical behaviour can be explained as follows. The flat plate boundary-layer theory used does not necessarily imply that the body is a flat plate but is related to an equivalent flat plate by a coordinate transformation involving the local pressure (Ref 18). When the pressure is constant, the boundary-layer

TABLE 3.1

Method	C_1	C_2
Newtonian	$\gamma M_\infty^2 \alpha_s^2 n^2$	$[n^2 (2n-1)]^{-1/2}$
Newton-Busemann	$\gamma M_\infty^2 \alpha_s^2 \cdot n (2n-1)$	$[n (2n-1)^2]^{-1/2}$
Tangent Wedge	$\gamma \frac{(\gamma+1)}{2} M_\infty^2 \alpha_s^2 n^2$	$\left[\frac{\gamma+1}{2} n^2 (2n-1) \right]^{-1/2}$
TW + CFGL	$\gamma M_\infty^2 \alpha_s^2 \left[\frac{\gamma+1}{2} n^2 + n(n-1) \right]$	$\left[\left\{ \frac{\gamma+1}{2} n^2 + n(n-1) \right\} (2n-1) \right]^{-1/2}$
2nd order TW	$\gamma M_\infty^2 \alpha_s^2 \left[\frac{\gamma+1}{2} n^2 + \sigma \cdot n(n-1) \right]$	$\left[\left\{ \frac{\gamma+1}{2} n^2 + \sigma \cdot n(n-1) \right\} (2n-1) \right]^{-1/2}$

Self-similar
solutions

obtained by numerical integration
(See Reference 63)

Note: $Y_w = X^{n_s}$, $Y_w = \frac{y_{w0}}{d_{sh} s}$, $X = x/L_s$
 $\sigma = 1 - \frac{\gamma^2 - 1}{\gamma}$

grows parabolically downstream. Since the pressure is also increasing downstream, the local unit Reynolds number also increases downstream which causes the boundary-layer thickness to decrease. For $n = 3/2$, the two effects tend to cancel each other giving almost a constant boundary-layer thickness. When $n > 3/2$, the growth of the boundary-layer is much slower compared to the thinning effect due to increase in the local unit Reynolds number and the boundary-layer thickness decreases downstream. Similarly, from a reverse argument it follows that the boundary-layer thickness increases downstream for $n < 3/2$.

CHAPTER IV

PURE VISCOUS INTERACTION PROBLEM

The complete solution of the viscid-inviscid interaction problem requires a simultaneous solution of the equations derived in Chapter III. In the following analysis, the displacement surface is assumed to represent a stream line and the pressures are calculated using the pressure laws described in Section 4.2. The solution for the case of the tangent wedge approximation and the Newton-Busemann rule when applied to general two-dimensional shapes have been given by Stollery (Ref 22). The original work of Cheng (Ref 32) using the Newton-Busemann rule/with only flat plates. /dealt

4.1 Cheng's analysis:

The governing equations for this can be written as

$$\frac{M_\infty \delta^*}{\chi} = \frac{A \bar{\chi}}{P_w} \left[\int_0^{\chi} P_w \frac{dx}{\chi} \right]^{1/2} \quad 4.1.1$$

$$P_w = \gamma M_\infty^2 \left(y_e'^2 + y_e y_e'' \right) \quad 4.1.2$$

where $y_e = y_w + \delta^*$: Effective body shape

y_w ; Geometric body shape

$$A = 0.664 \frac{\gamma-1}{2} \left(1 + 2.6 \frac{T_w}{T_0} \right)$$

for local similarity

$$= 0.664 \frac{\gamma-1}{\gamma+1} \left(1 + 2.6 \frac{T_w}{T_0} \right)$$

for Cheng's analysis.

The difference in definition for A arises from the fact that Cheng's analysis uses a consistent approximation $[(\gamma-1)/(\gamma+1)] \ll 1$ for both the boundary-layer analysis and the inviscid flow field. The use of the Newton-Busemann rule which is the leading approximation of the thin shock layer theory, implies that the shock-wave coincides with the edge of the boundary-layer and an inviscid region separating the boundary-layer and the shock wave cannot be defined. The equations 4.1.1 and 4.1.2 are combined to give

$$(y_e - y_w) \left[(y_e y_e')^{1/2} \right]' = \left(\frac{A^2 \bar{\chi}^2 \chi}{4 \gamma M_\infty^4} \right)^{1/2} \quad 4.1.3$$

The right hand side of equation 4.1.3 is a constant. For a

flat plate at zero incidence ($y_w = 0$), equation 4.1.3 admits a solution

$$\frac{4\gamma M_\infty^4 y_e}{A^2 \bar{x}^2 x} = \frac{2^{3/2}}{3^{1/4}} \left(\frac{4\gamma M_\infty^4}{A^2 \bar{x}^2} \right) \quad 4.1.4$$

Corresponding expressions for pressure, displacement thickness and Stanton number are given by

$$P = (3/4 \gamma)^{1/2} A \bar{x} \quad 4.1.5a$$

$$\frac{M_\infty S^*}{x} = \frac{2}{(3\gamma)^{1/4}} (A \bar{x})^{1/2} \quad 4.1.5b$$

$$\frac{St}{St_0} = \frac{1}{2} (3\gamma)^{1/4} (A \bar{x})^{1/2} \quad 4.1.5c$$

where $St_0 = 0.332 (C/Re_x)^{1/2}$

For the case of a general curved surface ($y_w \propto x^n$), the equation 4.1.3 can be written as (Ref 22)

$$(z \mp \zeta^n) \left\{ (z z')^{1/2} \right\}' = 1 \quad 4.1.6$$

where $z_w = \mp \zeta^n$ Equation of the surface

$$z_w = \frac{y_w}{\alpha_c h_c} ; \zeta = x/h_c$$

$$h_c = (A^2 \bar{x}^2 x) / (4\gamma M_\infty^4 \alpha_c^4) ; z = \frac{y_e}{\alpha_c h_c}$$

and

$$S^* / \alpha_c h_c = z \mp \zeta^n \quad 4.1.7a$$

$$\frac{P}{\gamma M_\infty^2 \alpha_c^2} = (z z')' \quad 4.1.7b$$

$$\frac{A \cdot St}{0.332 \times 4\gamma \alpha_c^3} = \frac{1}{(z \mp \zeta^n)} = \left(\frac{S^*}{\alpha_c h_c} \right)^{-1} \quad 4.1.7c$$

It may be noted that for the case of a general curved surface, the equation 4.1.6 cannot be integrated in closed form and requires to be integrated numerically for the given wall shape. The initial conditions for integration are obtained from the strong interaction solution given by equation 4.1.4. Stollery (Ref 22) showed that for concave surfaces the equation 4.1.6 yields quantitatively

unrealistic oscillatory solutions. The severity of the oscillations predicted by the theory was attributed to the over-correction of the Busemann centrifugal term in the Newton-Busemann pressure law. The oscillations in pressure and heat transfer were found to be about the value given by

$$\text{for } n = 2; \frac{P}{\gamma M_{\infty}^2 \alpha_c^2} = 96 \gamma^2 \left[\frac{M_{\infty}^2 \alpha_c^2}{A \bar{\chi}} \right]^4 \text{--- Inviscid Value } 4.1.8a$$

$$\frac{A St}{0.332 \times 4 \gamma \alpha_c^3} = (18 \gamma)^{1/2} \frac{M_{\infty}^2 \alpha_c^2}{A \bar{\chi}} \quad 4.1.8b$$

$$n = 3; \frac{P}{\gamma M_{\infty}^2 \alpha_c^2} = 3840 \gamma^4 \left[\frac{M_{\infty}^2 \alpha_c^2}{A \bar{\chi}} \right]^8 \text{--- Inviscid Value } 4.1.8c$$

Cheng's analysis is based on the approximation that $\frac{\gamma-1}{\gamma+1} \ll 1$ or $\gamma \rightarrow 1$. It is quite possible in many practical cases, in particular for Helium, γ can be quite different from unity and can lead to unsatisfactory predictions even for flat plates. Kemp (Ref 38) found that Cheng's theory underestimated the pressure and heat transfer rates in the strong interaction in helium at Mach 42. To improve the predictions, the Cheng's theory was modified using the pressure formula 3.2.2 and following relations were obtained for a flat plate at zero incidence in the strong interaction region.

$$\frac{M_{\infty} S^*}{\chi} = 1.525 \left[A_1^2 \left(\frac{A_1}{B} \right)^{1/2} \bar{\chi}_e \right]^{1/2} \quad 4.1.9a$$

$$P = \frac{3^{1/2}}{2} \left[\frac{A_1^{3/2}}{B^{1/2}} \gamma \bar{\chi}_e \right] \quad 4.1.9b$$

$$\epsilon \left[0.664 + 1.73 \frac{T_w}{T_0} \right] M_{\infty}^3 St = 0.219 \left[\frac{A_1^{3/4}}{B^{1/4}} \gamma^{1/2} \bar{\chi}_e^{3/2} \right] \quad 4.1.9c$$

$$\text{where } A_1 = \frac{\gamma+1}{2}; \quad B = \gamma$$

The equations 4.1.9a, b and c reduce to Cheng's results when $A_1 = B = 1$. The bracketed variables on right hand side can correlate experimental results in gases having different γ . The modification introduced by Kemp gave good agreement with experimental results in the strong interaction region.

Modified Cheng theory: Stollery proposed an alternative analysis by substituting the tangent wedge approximation (equation 3.2.3) in place of the Newton-Busemann formula used in Cheng's analysis. The governing equations for the analysis are obtained by combining equations 3.1.27 and 3.2.3 as

$$\frac{dP}{dx} = \frac{P^2}{2R} \left\{ 1 + 2R^{1/2} \left[M_\infty \alpha_s \frac{dy_w}{dx} - Q \right] \right\} \quad 4.1.10a$$

$$\frac{dR}{dx} = P \quad 4.1.10b$$

where

$$Q = \frac{P-1}{r \left[1 + \frac{r+1}{2r} (P-1) \right]^{1/2}}$$

$$\chi = x/L_s; \quad L_s = A^2 \bar{\chi}^2 x$$

When $M_\infty \alpha_s = 0$, the above equations reduce to the case of a flat plate at zero incidence and the pressure distribution depends on the viscous interaction parameter $\bar{\chi}$ through L_s . For other surfaces, for each $M_\infty \alpha_s$ there is a different solution. The pair of equations 4.1.10a and b are solved numerically by forward integration starting near the leading edge. Near the leading edge, the equations admit a solution of the form $P = k_{TW} X^m$ and by substitution it may be found

$$m = -\frac{1}{2}, \quad k_{TW} = \frac{3}{4} \left[r(r+1) \right]^{1/2} \quad 4.1.10c$$

The results of the calculations obtained by numerical integration by Stollery indicate for a flat plate at zero incidence, a smooth variation in pressure and heat transfer rates from strong to weak interaction region. A similar variation was found for the case of a concave surface instead of the quantitatively unrealistic oscillatory behaviour predicted by Cheng's analysis.

4.2 Present analysis:

In the present analysis, the 'TW + CFGL' and '2nd order Tangent Wedge' pressure laws discussed in Section 3.2. will be used to investigate the viscous interaction effects. First, the application of the TW + CFGL rule will be considered since it can cover both the strong and weak interaction regions. As in the previous analyses, equation 3.1.27 will be used for the boundary-layer growth but for the inviscid flow field equation 3.2.6 will be used.

A. Flat plate at zero incidence: For this case, y_e happens to be zero and the effective body shape y_e is given by the edge of the displacement thickness δ^* of the boundary-layer. The equations 3.1.27 and 3.2.6 can be written as

$$P y_e = \left(\int_0^x P d\bar{x} \right)^{1/2}$$

or differentiating

$$\frac{dP}{dx} = \frac{1}{y_e} \left(\frac{1}{2y_e} - P \frac{dy_e}{dx} \right) \quad 4.2.1$$

and

$$P = 1 + r \left(\frac{dy_e}{dx} \right)^2 \left[\frac{r+1}{4} + \left\{ \left(\frac{r+1}{4} \right)^2 + \frac{1}{(dy_e/dx)^2} \right\}^{1/2} \right] + r y_e \frac{d^2 y_e}{dx^2} \quad 4.2.2$$

$$\text{where } y_e = \frac{M_\infty \delta^*}{L_s} \\ X = x/L_s ; \quad L_s = A \bar{\alpha} \gamma$$

The equations 4.2.1 and 4.2.2 form the basic equations for further analysis and are required to be solved for P and Y_e . The complete solution can only be obtained by numerical integration. The inviscid equation is of second order compared to the first order tangent wedge equation. To facilitate numerical integration, the equations 4.2.1 and 4.2.2 are written as

$$\frac{dP}{dx} = \frac{1}{y_e} \left(\frac{1}{2y_e} - P \frac{dy_e}{dx} \right) \\ \frac{d^2 y_e}{dx^2} = \left(\frac{1}{r y_e} \right) \left[P - 1 - r \left(\frac{dy_e}{dx} \right)^2 \left\{ \frac{r+1}{4} + \left[\left(\frac{r+1}{4} \right)^2 + \frac{1}{(dy_e/dx)^2} \right]^{1/2} \right\} \right] \quad 4.2.3$$

In order to start the integration, the behaviour of P near the leading edge is required. This is obtained by writing $P = k X^m$, where k and m are to be determined. Using equation 4.2.1, the corresponding expressions for Y_e , dy_e/dx and $d^2 y_e/dx^2$ can be written as

$$y_e = k_1 X^{m_1} \quad 4.2.4a$$

$$\frac{dy_e}{dx} = k_2 X^{m_2} \quad 4.2.4b$$

$$\frac{d^2 y_e}{dx^2} = k_3 X^{m_3} \quad 4.2.4c$$

where

$$k_1 = [k(m+1)]^{-1/2} ; \quad m_1 = \frac{1-m}{2} \\ k_2 = k_1 m_1 ; \quad m_2 = -(1+m)/2 \\ k_3 = k_2 m_2 ; \quad m_3 = -(3+m)/2$$

Substituting for Y_e , Y_e' and Y_e'' in equation 4.2.3 and noting that near the leading edge $P \gg 1$ and $Y_e' \gg 1$, k and m can be determined by equating the coefficient and power of X , as

$$m = -\frac{1}{2} ; \quad k_{TW+CFGL} = \left[r(r+1) \frac{9}{16} - \frac{3}{8} r \right]^{1/2} \quad 4.2.5a,b$$

If the centrifugal forces are neglected and only the tangent wedge rule is considered, the term $\frac{3}{8} \gamma$ in equation 4.2.5b will be absent and the expression for k will be same as equation 4.1.10c. By comparing the expression for k, equations 4.1.10c and 4.2.5b, the direct effect of centrifugal force in alleviating the pressure can be seen. Knowing the behaviour near the leading edge, the integration can now be started at a point where the values* of dP/dx and $d^2\gamma_e/dx^2$ and the leading edge solution 4.2.4 agree within the desired accuracy. The complete solution was obtained by this method and the results are discussed in Chapter VI. Knowing the values of P by numerical integration, the heat transfer rates can be calculated from

$$\frac{0.2}{\gamma+1} AM_{\infty}^3 St = \frac{0.0664}{\gamma+1} PR^{-1/2} \quad 4.2.6$$

$$\text{where } R = \int_0^x P d\bar{x} \quad 4.2.7$$

To facilitate comparison with other theories and experimental results, the pressure and heat transfer rates have been plotted against $\bar{\chi}_e = \frac{2}{\gamma+1} A \bar{\chi} = \frac{2}{\gamma+1} \bar{\chi}^{1/2}$

Substituting the value of γ in equation 4.2.5b, the strong interaction solution for pressure can be written as

$$\gamma = 5/3 \quad P = 1.3693 A \bar{\chi} \quad 4.2.8a$$

$$\gamma = 1.4 \quad P = 1.1683 A \bar{\chi} \quad 4.2.8b$$

Now, we shall express the pressure and heat transfer rates in terms of the parameters defined by Kemp in equation 4.1.9, so that a direct comparison of the present theory with strong interaction results can be made. The expression for P can be written as

$$P = k \left[\frac{2}{\gamma(\gamma+1)} \right]^{1/2} \left[\left(\frac{\gamma+1}{2} \right)^{3/2} \gamma^{1/2} \bar{\chi}_e \right] \quad 4.2.9$$

The quantity within square brackets is the same as Kemp's variable and k is the constant in the leading edge solution $P = k X^m$. Substituting appropriate values of k, we get

$$\text{TW Rule: } k = \frac{3}{4} [\gamma(\gamma+1)]^{1/2} ; P = \frac{1}{2} \cdot \frac{3}{4} \left[\left(\frac{\gamma+1}{2} \right)^{3/2} \gamma^{1/2} \bar{\chi}_e \right]$$

$$\text{or } P = 1.061 \left[\left(\frac{\gamma+1}{2} \right)^{3/2} \gamma^{1/2} \bar{\chi}_e \right] \quad 4.2.10$$

* as calculated by using equations 4.2.1 and 4.2.3

$$\text{TW + CFGL Rule: } k = \left[\gamma (\gamma + 1) \left(\frac{3}{4} \right)^2 - \frac{3}{8} \gamma \right]^{1/2}$$

$$P = \frac{3^{1/2}}{2} \left(\frac{3}{2} - \frac{1}{\gamma + 1} \right)^{1/2} \left[\left(\frac{\gamma + 1}{2} \right)^{3/2} \gamma^{1/2} \bar{\chi}_e \right]$$

$$\text{For } \gamma = 1.4 \quad P = 0.901 \left[\left(\frac{\gamma + 1}{2} \right)^{3/2} \gamma^{1/2} \bar{\chi}_e \right] \quad 4.2.11$$

$$\gamma = 5/3 \quad P = 0.919 \left[\left(\frac{\gamma + 1}{2} \right)^{3/2} \gamma^{1/2} \bar{\chi}_e \right] \quad 4.2.12$$

$$\text{Kemp's Solution: } P = 0.866 \left[\left(\frac{\gamma + 1}{2} \right)^{3/2} \gamma^{1/2} \bar{\chi}_e \right] \quad 4.2.13$$

From the above equations, it may be noticed that for the case of the tangent wedge approximation, when the pressure is expressed as a function of Kemp's variable, the result does not depend on the value of γ as in Kemp's theory. If the TW + CFGL relation is employed the result depends on the value of γ , but the dependence is rather weak. Similarly, the heat transfer rate can be expressed as

$$E \left(0.664 + 1.73 \frac{T_w}{T_o} \right) M_\infty^3 St = \text{Constant} \left[\left(\frac{\gamma + 1}{2} \right)^{3/4} \gamma^{1/4} \bar{\chi}_e^{3/2} \right] \quad 4.2.14$$

The value of constant in equation 4.2.14 depends on γ only for the TW + CFGL relation and is given by

$$= \frac{0.664}{2^{5/4}} \left[\left(\frac{3}{4} \right)^2 - \frac{3}{8(\gamma + 1)} \right]^{1/4}$$

The numerical values of the constant are tabulated below

Kemp's theory (for all γ) :	= 0.219
Tangent Wedge (for all γ) :	= 0.242
TW + CFGL ($\gamma = 1.4$) :	= 0.223
($\gamma = 5/3$) :	= 0.225
($\gamma = 1.0$) :	= 0.219

As in the case of pressure, the dependence of heat transfer on γ when expressed in terms of Kemp's variable is very weak. The TW + CFGL rule and Kemp's theory agree very well and the tangent wedge estimates are higher in the strong interaction region.

The solution valid far downstream in the weak interaction

region can be obtained from equation 4.2.3, by expanding in powers of Y_e' (i.e. $Y_e' < 1$) as

$$P = 1 + \gamma Y_e' + \left[\gamma Y_e' Y_e'' + \frac{\gamma(\gamma+1)}{4} Y_e'^2 \right] + \dots \quad 4.2.15$$

Since, in the weak interaction region $Y_e \propto \bar{x}^{1/2}$, we get

$$P = 1 + \frac{\gamma}{2} A \bar{x} + O(A^2 \bar{x}^2) \quad 4.2.16$$

It may be noted from 4.2.15 that the effect of centrifugal force in the weak interaction region is of higher order and the first two terms correspond to the tangent wedge solution. (i.e. $O(\bar{x})$ compared with $O(\bar{x}^{1/2})$, $\bar{x} \gg 1$)

Analysis for power-law bodies: The scope of the analysis using the TW + CFGL rule will now be extended to a general power-law surface and the simplification of the equations when the incidence parameter is large will be considered.

The effective body shape y_e will now be given by the geometric body shape y_w plus the displacement thickness. Thus, we will now have three equations for determining the pressure, boundary-layer thickness and the effective body shape

$$\begin{aligned} y_e &= y_w + \delta^* ; \quad y_w = k x^n \\ \frac{M_\infty \delta^*}{L_s} &= \frac{\left(\int_0^x P dx \right)^{1/2}}{P} \\ P &= 1 + \gamma M_\infty^2 \left(\frac{dy_e}{dx} \right)^2 \left[\frac{\gamma+1}{4} + \left\{ \left(\frac{\gamma+1}{4} \right)^2 + \frac{1}{M_\infty^2 \left(\frac{dy_e}{dx} \right)^2} \right\}^{1/2} \right] + \gamma y_e y_e'' M_\infty^2 \end{aligned}$$

Choosing a length scale $L = A \bar{x}^2$, the above equations can be written in the non-dimensional form as

$$\frac{dP}{d\bar{x}} = \frac{1}{\Delta^*} \left(\frac{1}{2\Delta^*} - P \frac{d\Delta^*}{d\bar{x}} \right) \quad 4.2.17$$

$$\frac{d^2 y_e}{d\bar{x}^2} = \frac{P - 1 - \gamma \left(\frac{dy_e}{d\bar{x}} \right)^2 \left[\frac{\gamma+1}{4} + \left\{ \left(\frac{\gamma+1}{4} \right)^2 + \frac{1}{\left(\frac{dy_e}{d\bar{x}} \right)^2} \right\}^{1/2} \right]}{\gamma y_e} \quad 4.2.18$$

and

$$\Delta^* = \frac{M_\infty \delta^*}{L_s} = y_e - M_\infty \alpha_s y_w \quad 4.2.19$$

where

$$Y_w = X^n \quad : \quad \text{Body shape}$$

$$Y_w = y_w / \alpha_s l_s$$

$$X = x / L_s ; \quad Y_e = M_\infty y_e / L_s$$

The equations 4.2.17-19 are of the same type as for the case of a flat plate at zero incidence and when $M_\infty \alpha_s = 0$, they reduce to equations 4.2.1 and 4.2.3. Near the leading edge, the dominating effect is that due to displacement and the boundary-layer growth is similar to that in the strong interaction region of a flat plate at zero incidence and the effect of incidence becomes important downstream. In the limit, the pressure reaches the inviscid value asymptotically. As in the earlier analyses, the equation is singular at the origin and the solution is obtained by starting the integration at a point close to the leading edge and making use of the strong interaction solution given by equations 4.2.5a and b. For each $M_\infty \alpha_s$ and n , there is a separate solution. As in the case of tangent wedge analysis, the important parameters governing the flow are $M_\infty \alpha_s$ and $A \bar{x}$. For $M_\infty \alpha_s \rightarrow 0$, the flow is everywhere similar to that on a flat plate at zero incidence and higher approximations can be obtained by expanding P and Y_e in powers $M_\infty \alpha_s$. For $M_\infty \alpha_s \gg 1$, the equations 4.2.17-19 can be expanded in inverse powers of $M_\infty \alpha_s$ using the transformed variables (Appendix A)

$$p_i = \frac{P}{(M_\infty \alpha_s)^{2/(4n-3)}} \quad 4.2.20a$$

$$y_i = y_e (M_\infty \alpha_s)^{3/(4n-3)} \quad 4.2.20b$$

$$x_i = x (M_\infty \alpha_s)^{4/(4n-3)} \quad 4.2.20c$$

Substituting for P , Y_e and X in 4.2.17 and 4.2.18, we get

$$p_i(y_i \cdot x_i^n) = \left[\int_0^{x_i} p_i d\chi_i \right]^{1/2} \quad 4.2.21$$

$$p_i = \gamma y_i^2 \left[\frac{\gamma+1}{4} + \left\{ \left(\frac{\gamma+1}{4} \right)^2 + \frac{1}{(M_\infty \alpha_s)^{2/(4n-3)} y_i^{1/2}} \right\}^{1/2} \right] + \gamma y_i y_i'' + (M_\infty \alpha_s)^{-2/(4n-3)} \quad 4.2.22$$

It may be noted that for an inclined flat plate $n = 1$, and the expansion is in powers of $(M_\infty \alpha_s)^{1/2}$ and even the zeroeth order terms may be quite accurate and as n increases, it may become necessary to keep higher order terms. The zeroeth order approximation can be written as

$$p_{i0} (y_{i0} - \kappa_i^n) = \left[\int_0^{\kappa_i} p_{i0} d\kappa_i \right]^{1/2}$$

4.2.23

$$p_{i0} = \frac{\gamma(\gamma+1)}{2} y_{i0}'^2 + \gamma y_{i0} y_{i0}''$$

4.2.24

where p_{i0} and y_{i0} are the zeroeth order terms in the expression for p_i and y_i for $(M_\infty \alpha_c)^{-1} \rightarrow 0$. If only the tangent wedge rule is employed, the analysis is still applicable and only the term $\gamma y_{i0} y_{i0}''$ will be absent in equation 4.2.24. For $M_\infty \alpha_c \gg 1$, it is possible to relate the pressure distribution and heat transfer on different bodies belonging to a particular family of shapes by using the variables p_i, y_i and κ_i . It may be verified easily by using the relation between $M_\infty \alpha_c$ and $M_\infty \alpha_c^*$ that the variable differs from the non-dimensional variable ξ in Cheng's analysis (equation 4.1.6) only by a constant. So, by using Cheng's variables in equation 4.1.6, it is possible to treat the case of $M_\infty \alpha_c \gg 1$ for all the pressure laws in general. This will be attempted in the following Section.

4.3 Generalization for $M_\infty \frac{dy_e}{dx} \gg 1$:

With $I_0 = 1$ in equation 3.1.25, the general equation for $M_\infty \frac{dy_e}{dx} \gg 1$ can be written as

$$\frac{M_\infty \delta^*}{\chi} = \frac{A \bar{\chi}}{P} \left(\int_0^\chi P \frac{d\xi}{\chi} \right)^{1/2} \quad 4.3.1$$

$$y_e = y_w + \delta^*$$

$$\text{and} \quad P = \gamma M_\infty^2 \left(a y_e'^2 + b y_e y_e'' \right) \quad 4.3.2$$

The values of a and b for various pressure laws are

i)	Cheng's analysis :	$a = 1$	$b = 1$
ii)	TW + CFGL :	$a = \frac{\gamma+1}{2}$	$b = 1$
iii)	2nd order TW :	$a = \frac{\gamma+1}{2}$	$b = 0 = 1 - \frac{\gamma-1}{\gamma}$
iv)	Tangent Wedge :	$a = \frac{\gamma+1}{2}$	$b = 0$

As in Section 4.1, writing

$$\ell = \frac{\chi}{L_c} ; \quad L_c = \frac{A^2 \bar{\chi}^2 \chi}{4 \gamma M_\infty^4 \alpha_c^4}$$

$$\bar{z} = \frac{ye}{\alpha_c l_c} ; \quad \bar{z}_N = \zeta^n : \quad \text{wall shape}$$

the equations 4.3.1 and 4.3.2 can be written as

$$\frac{P}{\gamma M_\infty^2 \alpha_c^2} (\bar{z} - \bar{z}_w) = 2 \left[\int_0^{\bar{\ell}} \frac{P}{\gamma M_\infty^2 \alpha_c^2} d\bar{\ell} \right]^{1/2} \quad 4.3.3$$

$$\text{and} \quad \frac{P}{\gamma M_\infty^2 \alpha_c^2} = a \bar{z}'^2 + b \bar{z} \bar{z}'' \quad 4.3.4$$

Combining equations 4.3.3 and 4.3.4, the generalized form of Cheng's equation can be written as

$$(\bar{z} - \bar{z}_w) \left[\left\{ \int_0^{\bar{\ell}} (a \bar{z}'^2 + b \bar{z} \bar{z}'') d\bar{\ell} \right\}^{1/2} \right]' = 1 \quad 4.3.5$$

For Cheng's theory, $a = b = 1$ and equation 4.3.5 reduces to equation 4.1.6. The expression for the boundary-layer thickness, pressure and heat transfer will be

$$\frac{\delta^*}{\alpha_c l_c} = \bar{z} - \bar{z}_w \quad 4.3.6a$$

$$\frac{P}{\gamma M_\infty^2 \alpha_c^2} = a \bar{z}'^2 + b \bar{z} \bar{z}'' \quad 4.3.6b$$

$$\frac{A \cdot St}{0.332 \times 4 \gamma \alpha_c^3} = \frac{1}{\bar{z} - \bar{z}_w} = \left(\frac{\delta^*}{\alpha_c l_c} \right)^{-1} \quad 4.3.6c$$

From the above analysis, it may be seen that the parameter governing the combined effect of incidence and viscous interaction is $(M_\infty^2 \alpha_c^2 / A \bar{x})$. It should be noted that if the length scale used is $A \bar{x}^2 \bar{x}$, the results still depend on $M_\infty \alpha_c$ separately even for $M_\infty \frac{dy_c}{dx} \gg 1$.

The analysis for $M_\infty \frac{dy_c}{dx} \gg 1$ will be useful when dealing with concave surface flows. Near the leading edge, the displacement effect will be strong and hence $M_\infty \frac{dy_c}{dx} \gg 1$, and far downstream also this condition is satisfied because the incidence is large.

The strong interaction solution of equation 4.3.5 for pressure can be obtained by putting $\bar{z} \propto \bar{\ell}^m$ and this gives for the pressure

$$\frac{P}{\gamma M_{\infty}^2 \alpha_c^2} = \left[\frac{3(3a-b)}{2} \right]^{\frac{1}{2}} \zeta^{-\frac{1}{2}} \quad 4.3.7$$

Substituting for a and b in 4.3.7, the strong interaction solution for any of the pressure laws can be written.

For $\gamma = 1$, a and b will be equal to unity for all pressure laws except tangent wedge. Knowing the behaviour near the leading edge, equation 4.3.5 can be integrated for any given body shape z_w . The results of the calculations obtained by numerical integration^w are discussed in Chapter VI.

CHAPTER V

LEADING EDGE BLUNTNESSE EFFECTS

In the present Chapter, the analysis of viscous interaction theories for sharp nosed slender bodies discussed in the previous Chapter will be modified to the case where the leading edge is slightly blunted. The effect of the blunting of leading edge is to produce a very strong bow shock wave which induces on the after body pressures which are much higher than the free stream value. The analysis of the problem requires a consideration of the combined effects of bluntness, displacement and incidence. This is treated in Sec 1. For certain after-body shapes, the displacement effect can be much smaller compared to the bluntness and incidence effects. Therefore, the inviscid problem with nose bluntness and incidence only, is considered in Section 2 and some closed form solutions obtainable when the after-body is a wedge are pointed out.

5.1. Bluntness and Displacement Effects:

The flow pattern for the hypersonic flow past a blunt nosed slender body is shown in figure 3. The blunt leading edge introduces a strong entropy gradient in the downstream flow which can significantly modify the pressure and heat transfer on the after-body. The effective shape of the body is given by the edge of the entropy layer. For $\gamma \rightarrow 1$, Cheng (Ref 32) shows that the pressure variation across the entropy layer is small and the pressure on the after-body can be assumed to be equal to that at the edge of the entropy layer. On the basis of the thin shock layer theory, the pressure p at the outer edge of the entropy layer y_e is given by the Newton-Busemann formula (equation 3.2.2). However, in order to determine the pressure and shock shape, one more relation between p_e and y_e is required. By an analysis of the flow within the entropy layer Cheng obtains the integral relation between the pressure and the cross-sectional area as

$$(y_e - y_b)^\gamma \phi_e = \text{constant} \quad 5.1.1$$

Consistent with the thin shock-layer assumption, the equation 5.1.1 was further simplified by putting $\gamma = 1$. As $x \rightarrow 0$, y_b tends to zero, y_e also tends to zero in such a way that the product $y_e p_e$ is constant. This constant is related to the drag of the blunt nose and is determined by considering the balance of momentum in the streamwise direction. This leads to the following simple pressure-area relationship.

$$\frac{\phi_e (y_e - y_b)}{\gamma - 1} = \frac{D_N}{2} \quad 5.1.2$$

where D_N is the drag of the blunt nose. The equation 5.1.2

along with a suitable pressure law determines the problem completely provided y_b is known. When the boundary-layer displacement effect is small, y_b is the actual shape of the body. In the presence of the displacement effect, one more equation relating the displacement thickness with the pressure, derived in Section 3.1 will be required. Expressing equation 5.1.1 in non-dimensional variables, the governing equation can now be written as

$$P(Y_e - Y_b) = K_s \quad 5.1.3$$

$$Y_B = M_\infty \alpha_s Y_W + \frac{\left(\int P dx\right)^{1/2}}{P} \quad 5.1.4$$

$$P = f(\gamma, Y_e) \quad 5.1.5$$

where

$$X = x/L_s ; \quad Y_e = \frac{M_\infty y_e}{L_s}$$

$$Y_w = X^n ; \quad Y_W = \frac{y_w}{\alpha_s L_s}$$

$$L_s = A^2 \bar{\chi}^2 \chi$$

$$K_s = \frac{\gamma(\gamma-1)}{4} \frac{M_\infty^3 C_{DNT}}{L_s} ; \text{ Constant.}$$

The second term on right hand side of equation 5.1.4 represents the modification of the actual body-shape by the displacement. The function $f(\gamma, Y_e)$ represents a suitable pressure law such as

- (i) Newton-Busemann
(used by Cheng et al(Ref 32)) $P = \gamma (Y_e'^2 + Y_e Y_e'')$
- (ii) Tangent Wedge
(Stollery et al, Ref 44.) $P = 1 + \gamma Y_e'^2 \left[\frac{\gamma+1}{4} + \left\{ \left(\frac{\gamma+1}{4} \right)^2 + \frac{1}{Y_e'^2} \right\}^{1/2} \right]$
- (iii) TW + CFGL (Present) $P = 1 + \gamma Y_e'^2 \left[\frac{\gamma+1}{4} + \left\{ \left(\frac{\gamma+1}{4} \right)^2 + \frac{1}{Y_e'^2} \right\}^{1/2} \right] + \gamma Y_e Y_e''$
- (iv) 2nd Order TW (Present) $P = \gamma \frac{(\gamma+1)}{2} Y_e'^2 + \gamma Y_e Y_e'' \left(1 - \frac{\gamma^2-1}{\gamma} \right)$

5.1.6a,b,c,d

For the case of a sharp leading edge, K_s will be zero and the analysis of the previous chapter will hold good. For given

values of Mach number and leading edge drag coefficient, K , can be interpreted as the ratio of two lengths, namely - the tip thickness t and the viscous interaction length scale L . The equations 5.1.3 to 5.1.5 can be integrated numerically for the given body shape Y_w , incidence parameter $M_\infty \alpha_s$ and constant K . However, the equations are singular at the origin and the integration can only be started at a point close to the leading edge. The solution valid near the leading edge can be obtained by assuming $P = k X^m$ and by substitution in 5.1.3 and 5.1.5, we get the values of k and m for various pressure laws. The result is

$$\begin{aligned}
 \text{(i) Newton-Busemann Rule:} \quad P &= \left(\frac{2}{9} \gamma k_s^2 \right)^{1/3} X^{-2/3} & 5.1.7a \\
 \text{(ii) Tangent Wedge} \quad :: \quad P &= \left[\frac{2}{9} \gamma (\gamma+1) k_s^2 \right]^{1/3} X^{-2/3} & 5.1.7b \\
 \text{(iii) TW + CFGL} \quad : \quad P &= \left(\frac{2}{9} \gamma^2 k_s^2 \right)^{1/3} X^{-2/3} & 5.1.7c \\
 \text{(iv) 2nd order TW} \quad : \quad P &= k_s^{2/3} \left[\frac{2}{9} \gamma (\gamma+1) - \frac{2}{9} \gamma \sigma \right]^{1/3} X^{-2/3} & 5.1.7d
 \end{aligned}$$

where $\sigma = 1 - \frac{\gamma^2 - 1}{\gamma}$

The behaviour of P near the leading edge being known, the full equations 5.1.3 - 5.1.5 can be integrated and other related quantities like heat transfer and skin friction can be calculated. From equation 3.1.32, we have

$$C_f = 2S_t = \frac{0.664}{A M_\infty^3} \frac{P}{\left(\int_0^x P dx \right)^{1/2}} \quad 5.1.8$$

Near the leading edge $P = a_0 X^{-2/3}$ where a_0 is the coefficient of $X^{-2/3}$ in equations 5.1.7a - d, and Equation 5.1.8 can be written as

$$C_f = 2S_t = \frac{0.664}{A M_\infty^3} a_0^{1/2} \frac{1}{\sqrt{3}} X^{-5/6} \quad 5.1.9$$

To facilitate integration, equations 5.1.3 and 5.1.4 are combined to give

$$\frac{dP}{dx} = \frac{1}{S_e} \left[\frac{P}{2(P\delta_e - K_s)} - P \frac{d\delta_e}{dx} \right] \quad 5.1.10$$

$$\text{where} \quad \delta_e = Y_e - M_\infty \alpha_s Y_w \quad 5.1.11$$

The pressure laws can be written as

$$\begin{aligned}
 \text{(i) Newton-Busemann : } \quad \frac{d^2 y_e}{dx^2} &= \frac{P - r y_e'^2}{r y_e} & 5.1.12 \\
 \text{(ii) TW + CFGL} \quad \frac{d^2 y_e}{dx^2} &= \frac{P - 1 - r y_e'^2 \left[\frac{r+1}{4} + \left\{ \left(\frac{r+1}{4} \right)^2 + \frac{1}{y_e'^2} \right\}^{1/2} \right]}{r y_e} & 5.1.13 \\
 \text{(iii) 2nd order TW : } \quad \frac{d^2 y_e}{dx^2} &= \frac{P - \frac{r(r+1)}{2} y_e'^2}{r y_e} & 5.1.14 \\
 \text{(iv) Tangent Wedge : } \quad \frac{dy_e}{dx} &= \frac{P-1}{r \left\{ 1 + \frac{r+1}{2r} (P-1) \right\}^{1/2}} & 5.1.15
 \end{aligned}$$

The equation 5.1.10 can be integrated numerically along with any one of the pressure laws. The results of numerical integration for the case of a blunted wedge are discussed along with experimental results in Part III. Cheng et al (Ref 32) obtained a closed form solution of equations 5.1.3, 5.1.4 and 5.1.12 for a blunt plate with displacement only ($M_\infty \alpha_s = 0$). Their results indicate $10 \leq \tilde{x}_e / k_e^{2/3} < 2$, for both the bluntness and the displacement effect will be important.

5.2 Bluntness dominated flows:

The analysis of Section 5.1 will now be simplified to the case where the displacement effect is negligible. Particularly for curved after body shapes and for blunt flat plates at large angle of attack, the dominating effect will be due to bluntness near the leading edge and body incidence far downstream. For such surfaces, ignoring the displacement effect simplifies the problem and in some cases leads to closed form solutions.

The governing equations in this will be only equation 5.1.2 with a suitable pressure law. First, the case of a blunt flat plate at zero incidence will be treated using the Newtonian formula (equation 3.2.1) for the pressure. The equations can be written as

$$p_e y_e = (r-1) D_N / 2 \quad 5.2.1$$

$$\text{and } p_e = 1 + r M_\infty^2 (dy_e/dx)^2 \quad 5.2.2$$

or choosing a length scale L , the equations 5.2.1 and 5.2.2 can be written as

$$p_e y_e = \frac{r(r-1)}{4} \frac{M_\infty^3 G_{D,t}}{L} = K : \text{Constant} \quad 5.2.3$$

$$P = 1 + \gamma \left(\frac{dy_e}{dx} \right)^2 \quad 5.2.4$$

$$\text{where } X = x/L, \quad Y = (M_\infty y_e)/L$$

Combining equations 5.2.3 and 5.2.4, we get

$$K = y_e \left[1 + \left\{ \frac{dy_e}{d(X/\gamma^{1/2})} \right\}^2 \right] \quad 5.2.5$$

It is interesting to note an analogy between equation 5.2.5 which describes the hypersonic flow past a blunt flat plate at zero incidence and the classical equation for the Brachystochrone*. By analogy, the solution of equation 5.2.5 can be easily written as

$$\frac{X}{\gamma^{1/2}} = K \sin^{-1} \left(\frac{y_e}{K} \right)^{1/2} - (K y_e - y_e^2)^{1/2} + K_2 \quad 5.2.6$$

The constant K_2 will be zero since at $X=0$, $y_e = 0$ and the solution can be written as

$$\frac{X}{\gamma^{1/2}} = K \left[\sin^{-1} \left(\frac{y_e}{K} \right)^{1/2} - \frac{y_e}{K} \left(\frac{K}{y_e} - 1 \right)^{1/2} \right] \quad 5.2.7$$

The equation 5.2.7 represents a cycloid in the transformed coordinates $X/\gamma^{1/2}$ and y_e , where K is the diameter of the generating circle. The choice of the length scale is still arbitrary and can be chosen in such a way to make $K=1$. For $X \rightarrow 0$, it may be verified by expanding 5.2.7, the pressure behaves as

$$P = \left(\frac{4}{9} \gamma K^2 \right)^{1/3} X^{-2/3} \quad 5.2.8$$

which is the correct behaviour when the blast wave effect of the leading edge is strong (Ref 32). It must be noted that in using equation 5.2.7 that it is applicable only in the region where $P \gg 1$ since the Newtonian formula is valid only in the limit of infinite Mach number. This example is considered, only to exhibit the analogy with the Brachystochrone equation.

Blunt flat plate at incidence: For blunt flat plates at incidence the governing equations can be written as

$$P (y_e - M_\infty \theta x) = \frac{\gamma(r-1)}{4} M_\infty^3 \frac{G t}{L} = K \quad 5.2.9$$

The Brachystochrone equation determines the shape of the curve between two given points A and B for which a particle released with zero initial velocity at A and acted upon by gravity alone would descend to point B in the shortest time possible. The equation for such a curve is same as equation 5.2.5. (Ref 84 and 85).

where θ : angle of attack

$$X = x/L, \quad Y_e = (M_\infty y_e)/L$$

The equation 5.2.9 can be integrated in conjunction with any one of the pressure laws (equations 5.1.11 to 5.1.15). In general, the equations need to be integrated numerically. Cheng (Ref 32) found that when the Newton-Busemann relation is used, the results exhibit an oscillatory approach to downstream inviscid conditions. Stollery et al (Ref 44) obviated this difficulty by using the tangent wedge approximation (equation 5.1.15) and showed by numerical integration that the use of the tangent wedge approximation suppresses the oscillatory decay. However, the resulting equations when the tangent wedge approximation is used can be integrated in closed form as shown below. The governing equations are

$$P(Y_e - M_\infty \theta X) = K \quad 5.2.10$$

$$\text{and} \quad \frac{dY_e}{dX} = \frac{P-1}{\gamma \left[1 + \frac{\gamma+1}{2\gamma} (P-1) \right]^{1/2}} \quad 5.2.11$$

By substituting $t = Y_e - M_\infty \theta X$ and

$$\frac{dY_e}{dX} = M_\infty \theta + dt/dX \quad 5.2.12$$

and eliminating P from 5.2.10, the equation 5.2.11 can be written as

$$\frac{dt}{dX} = \frac{\frac{K}{t} - 1}{\gamma \left[1 + \frac{\gamma+1}{2\gamma} \left(\frac{K}{t} - 1 \right) \right]^{1/2}} - M_\infty \theta \quad 5.2.13$$

$$\text{or} \quad \frac{dt}{dX} = \frac{dt}{F(t)} \quad 5.2.14$$

The function $F(t)$ represents the terms on the right hand side of equation 5.2.13. The equation 5.2.14 can be integrated between given limits X_1 to X , and t_1 to t

$$\int_{X_1}^X dX = \int_{t_1}^t \frac{dt}{F(t)}$$

or

$$X - X_1 = I_1(t) - I_1(t_1) \quad 5.2.15$$

$$\text{where } I_1 = \int \frac{dt}{F(t)} \quad 5.2.16$$

The integral I_1 can be evaluated in closed form (Appx C) and the result is

$$I_1 = \frac{K(r+1)^2}{4r(b_1-b_0)} \left[\frac{b_1}{(p-b_1)(p^2-a^2)} - \frac{b_0}{(p-b_0)(p^2-a^2)} + \frac{b_1}{2a} \left\{ E(p, -a, b_1) - E(p, a, b_1) \right\} - \frac{b_0}{2a} \left\{ E(p, -a, b_0) - E(p, a, b_0) \right\} \right] \quad 5.2.17$$

$$\text{where } E(p, a, b) = \frac{1}{(a+b)^2} \log \frac{p+a}{p-b} - \frac{1}{(a+b)(p-b)}$$

$$a^2 = (r-1)/2r$$

$$b_{0,1} = \frac{1}{2} \left[\frac{r+1}{2} M_\infty \theta \pm \left\{ \left(M_\infty \theta \cdot \frac{r+1}{2} \right)^2 + 4 \right\}^{1/2} \right]$$

$$p = \left[1 + \frac{r+1}{2r} \left(\frac{K}{t} - 1 \right) \right]^{1/2}$$

$$K/t = p = p_\infty/p_\infty$$

5.2.18 a -e

In the particular case, when the integration is from the leading edge, the lower limits X_1, t_1 in equation 5.2.16 happen to be zero. As $t \rightarrow 0$, from equation 5.2.18d it follows that p tends to infinity. It may be verified by expanding equation 5.2.17 that for $p \gg 1$, I_1 is proportional to p^{-3} so that I_1 tends to zero as p tends to infinity. Therefore, the solution 5.2.15 reduces to

$$X = I_1 \quad 5.2.19$$

where I_1 is given by equation 5.2.17. The leading edge solution for $X \rightarrow 0$ from equation 5.2.17 is obtained by expanding for $p \gg 1$ and the result for the pressure is

$$P = \left[\frac{2}{9} r(r+1) K^2 \right]^{1/3} X^{-2/3} \quad 5.2.20$$

The equation 5.2.20 is the same as that obtained in equation 5.1.7b. If we substitute $P = (K/t)$, the equation 5.2.17 is a function of the pressure P and the incidence parameter $M_\infty \theta \frac{r+1}{2}$, for given values K and r . By giving values of P and $M_\infty \theta$, X can be calculated from equation 5.2.19.

For calculating heat transfer rates, we need to know $\int P dx$, and it is possible to obtain a closed form solution for $\int P dx$ as (see Appendix C)

$$\int_0^X P dx = \frac{K(r+1)}{b_1 - b_0} \left[\frac{b_0}{2} \left\{ \frac{1}{a+b_0} \log \frac{p-b_0}{p+a} + \frac{1}{-a+b_0} \log \frac{p-b_0}{p-a} \right\} - \frac{b_1}{2} \left\{ \frac{1}{a+b_1} \log \frac{p-b_1}{p+a} + \frac{1}{-a+b_1} \log \frac{p-b_1}{p-a} \right\} \right] \quad 5.2.21$$

The definitions for b_0 , b_1 , a and p in equation 5.2.21 are the same as in equations 5.2.18b - d. The constant of integration in equation has been put equal to zero since for

$$X \rightarrow 0, \quad \int P dx \sim \int X^{-2/3} dx \rightarrow 0$$

and right hand side of equation 5.2.21 also tends to zero for $p \rightarrow \infty$. With equations 5.2.19 and 5.2.21 it is possible to calculate P and $\int P dx$ for any $M_\infty \theta$. The results for particular values of $M_\infty \theta$ are presented in Chapter VI.

Similar closed form solutions can be obtained if the Newtonian formula (equation 3.2.1) is used instead of the tangent-wedge approximation. By combining equations 5.2.4 and 5.2.10, we get

$$(Y_e - M_\infty \theta X) \left[1 + r \left(\frac{dY_e}{dX} \right)^2 \right] = K \quad 5.2.22$$

Writing $Y_e - M_\infty \theta X = t$, equation 5.2.22 simplifies to

$$r \left(\frac{dt}{dX} + M_\infty \theta \right)^2 = \frac{K}{t} - 1$$

Or

$$\frac{dX}{r^{1/2}} = \frac{dt}{\left(\frac{K}{t} - 1 \right)^{1/2} - M_\infty \theta r^{1/2}} \quad 5.2.23$$

Integrating equation 5.2.23 from 0 to X_0 and 0 to t , we get

$$\frac{x}{r^{1/2}} = I_3 \Big|_0^t \quad 5.2.24$$

$$\text{where } I_3 = \int \frac{dt}{\left(\frac{k}{t} - 1\right)^{1/2} - M_\infty \theta r^{1/2}}$$

The integral I_3 has been evaluated in Appendix C and the solution of 5.2.24 can be written as

$$\begin{aligned} \frac{x}{r^{1/2}} = & \frac{t}{F_1(t)} + \frac{ka_1}{(1+a_1^2)} \log \frac{k/t}{F_1^2(t)} + \frac{k(a_1^2-1)}{(1+a_1^2)^2} \left[\frac{\pi}{2} - \tan^{-1} \left(\frac{k}{t} - a_1 \right)^{1/2} \right] \\ & - \frac{k}{F_1(t)} \cdot \frac{1}{(1+a_1^2)} \end{aligned} \quad 5.2.25$$

$$\text{where } F_1(t) = \left(\frac{k}{t} - 1 \right)^{1/2} - a_1$$

$$a_1 = M_\infty \theta r^{1/2}$$

In a similar manner, the $\int P dx$ can also be evaluated and the result is (Appendix C)

$$\int_0^x P dx = \frac{2kr^{1/2}}{(1+a_1^2)} \left[\frac{a_1}{2} \log \frac{k/t}{F_1^2(t)} + \frac{\pi}{2} - \tan^{-1} \left(\frac{k}{t} - 1 \right)^{1/2} \right] \quad 5.2.26$$

It appears that for the present TW + CFGL and 2nd order tangent wedge pressure laws, solutions can only be obtained by numerical integration and closed form solutions are not possible. The equations to be integrated are given in the following Section on curved after bodies.

Governing equations for power-law after body shapes:

The power law shapes are given by

$$\frac{y_w}{\theta_c \ell_c} = \left(\frac{x}{\ell_c} \right)^n \quad \text{or} \quad \gamma_w = \ell_c^n \quad 5.3.1$$

where ℓ_c : length scale

Substituting equation 5.3.1 in 5.1.2,

$$\frac{P}{\gamma M_\infty^2 \theta_c^2} (Z - Z_w) = \frac{\gamma-1}{4} \frac{C_D t}{\theta_c^3 l_c} \quad 5.3.2$$

The length scale l_c can be chosen such that right hand side of equation is unity, i.e.

$$l_c = \frac{(\gamma-1) C_D t}{4 \theta_c^3} \quad 5.3.3$$

For Cheng's theory $\gamma \approx 1 + 2\epsilon$, and the length scale l_c will be $(\epsilon C_D t / 2 \theta_c^3)$.

The governing equations can now be written as

$$(Z - \varphi_c^n) \left(\frac{P}{\gamma M_\infty^2 \theta_c^2} \right) = 1 \quad 5.3.4$$

and the corresponding pressure laws will be

$$(i) \text{ Newton-Rusemann } \frac{P}{\gamma M_\infty^2 \theta_c^2} = Z'^2 + Z Z'' \quad 5.3.5$$

$$(ii) \text{ TW + CFGL } \frac{P}{\gamma M_\infty^2 \theta_c^2} = \frac{1}{\gamma M_\infty^2 \theta_c^2} + Z'^2 \left[\frac{\gamma+1}{4} + \left\{ \left(\frac{\gamma+1}{4} \right)^2 + \frac{1}{M_\infty^2 \theta_c^2 Z'^2} \right\}^{1/2} \right] + Z Z'' \quad 5.3.6$$

$$(iii) \text{ 2nd order TW } \frac{P}{\gamma M_\infty^2 \theta_c^2} = \frac{\gamma+1}{2} Z'^2 + 5 Z Z'' \quad 5.3.7$$

$$(iv) \text{ Tangent wedge } Z' = \frac{(P-1) / \gamma M_\infty^2 \theta_c^2}{\left[\frac{\gamma-1}{2 \gamma M_\infty^2 \theta_c^2} + \frac{(\gamma+1)P}{2 \gamma M_\infty^2 \theta_c^2} \right]^{1/2}} \quad 5.3.8$$

Equations 5.3.5 to 5.3.8 can be integrated after substituting for $\frac{P}{\gamma M_\infty^2 \theta_c^2}$ from equation 5.3.4. For tangent wedge and TW + CFGL

rules, the result depends on $M_\infty \theta_c$. However for $M_\infty \theta_c \gg 1$, equations 5.3.5 to 5.3.8 can be expressed in generalized form as

$$(Z - \varphi_c^n) (a Z'^2 + b Z Z'') = 1 \quad 5.3.9$$

where a and b will have the following values for various pressure laws.

Newton-Busemann

$$a = 1, \quad b = 1$$

TW + CFGL

$$a = \frac{\gamma+1}{2}, \quad b = 1$$

2nd order TW

$$a = \frac{\gamma+1}{2}, \quad b = \delta$$

Tangent Wedge

$$a = \frac{\gamma+1}{2}, \quad b = 0$$

In order to start the numerical integration, we need the behaviour near the leading edge. By substituting $Z = \frac{1}{2} \zeta_c^m$ in equation 5.3.9, we get

$$Z \sim \left[\frac{2}{9} (2a-b) \right]^{-1/3} \zeta_c^{2/3} \quad 5.3.10$$

and from equation 5.3.4, the pressure is given by

$$\frac{P}{\gamma M_\infty^2 \theta_c^2} = \left[\frac{2}{9} (2a-b) \right]^{1/3} \zeta_c^{-2/3} \quad 5.3.11$$

The particular case when the after body is a flat plate at incidence $n = 1$ in equations 5.3.4 and 5.3.9, and closed form solutions can be obtained when $b = 0$ (i.e. Tangent wedge rule). With $n = 1$ and $b = 0$, equation 5.3.9 becomes

$$(Z - \zeta_c) a Z'^2 = 1 \quad 5.3.12$$

The solution of equation 5.3.12 is given by

$$-\frac{\gamma+1}{4} \zeta_c = \lambda + \frac{\lambda^2}{2} + \text{Log}(1-\lambda) \quad 5.3.13$$

$$\text{where } \lambda = \left(\frac{\gamma+1}{2} \cdot \frac{\gamma M_\infty^2 \theta_c^2}{P} \right)^{1/2} \quad 5.3.14$$

The analytical solution given by equation 5.3.13 is the approximate form of the complete solution obtained in equation 5.2.17 for $M_\infty \theta_c \gg 1$. However, this can be quite accurate even for moderate values of $M_\infty \theta_c$. For $M_\infty \theta_c = 1$ and $P = 28$, the values of $2\theta_c^3 x / \zeta_c^4$ calculated by two equations differ only by 4%.

Of particular interest is the application of equation 5.3.9 to concave surface flows for which the viscous interaction equation 4.3.5 was found to give highly oscillatory solution when $a = b = 1$ (Ref 22). The results of numerical integration are discussed in the next Chapter.

CHAPTER VI

RESULTS AND DISCUSSION

In this Chapter, the results obtained by integrating the equations derived in Chapters IV and V will be presented. The integration of the governing equations was carried out by splitting into a number of first order equations and using the Runge-Kutta method. The Runge-Kutta subroutine was checked by integrating the Blasius equation (equation 3.1.12 with $\beta=0$) and comparing the results with the tabulated results (Ref 80). The computations were performed on an ICL 1905 computer using double precision arithmetic and with variable stepsize. The stepsize was checked after a number of specified operations to ensure the desired accuracy is maintained. The stepsize checking was carried out by integrating with the given stepsize as well as with half its value and ascertaining that the two results differ less than the specified accuracy. If the difference was more the process was repeated till the desired accuracy was achieved. The integration was started at a point where the difference between the leading edge solutions and the values calculated directly from the governing equations agreed within the specified accuracy. Using this method, the governing equations for Cheng's and other modified theories were integrated for flat plates and concave surfaces.

6.1 Pure viscous interaction:

In figure 11, results obtained by numerical integration for a flat plate at zero incidence (equations 4.2.1 and 4.2.3, TW+CFGL) are compared with the series expansion for the strong and weak interaction limits and it can be seen that they are in good agreement over a considerable range of $A\bar{x}$. For $0.3 < A\bar{x} < 10$, it may be necessary to use the complete solution. As in the case of tangent wedge rule (Sullivan, Ref 20; Stollery, Ref 22), the present TW + CFGL rule predicts a smooth transition from the strong interaction region to weak interaction region. The pressure distribution in the strong interaction region is shown plotted in figure 12 in terms of Kemp's variables (Equations 4.2.10 - 13). The TW + CFGL rule and Kemp's theory agree closely and the tangent wedge predictions are slightly higher. The dependence of the pressure and heat transfer on γ when expressed in Kemp's variables is rather very weak. In the strong interaction region, the difference between the TW + CFGL rule and TW approximation for the pressure is about 15% for $\gamma = 1.4$ and 13.5% for $\gamma = 5/3$. Corresponding differences in the heat transfer coefficient is about 7.5% for $\gamma = 1.4$ and 6.3% for $\gamma = 5/3$. As the weak interaction region is approached, the difference between the two methods is reduced because of the higher order effect of the centrifugal forces (equation 4.2.15). Far downstream, the centrifugal forces become zero and the TW + CFGL and TW rules become identical. Similarly, in the other extreme limit of merged (not considered in the present work) where the boundary-layer growth is almost linear, the centrifugal terms vanish and the TW+ CFGL rule reduces to TW approximation. The effect of γ on the pressure distribution and heat transfer is shown in figures 13a,b. The

interaction effects appear to be generally higher for $\gamma = 5/3$ than for $\gamma = 1.4$. Comparison of the TW + CFGL results with measurements is made in figures 14, 15 and 16. From figure 14 and 15, it may be seen that the present rule gives good agreement with Holden's measurements for the pressure distribution, and with Kemp's and other correlated results in the strong interaction region. Though, the induced pressures are predicted favourably, heat transfer rates are generally underestimated by the present as well as other methods compared to most of the experimental data. This underestimation in heat transfer rates may be due to the simplifying assumption of unit Prandtl number in the analysis. It appears that the present TW + CFGL rule with local flat plate similarity may serve as a good prediction method for pressures on flat plates since the tangent wedge estimates are generally higher.

For the case of general power-law surface, equations 4.2.17 to 4.2.19 were integrated for the present TW + CFGL rule. In figure 17 the pressure distribution on a sharp flat plate at incidence is shown for two values of the incidence parameter $M_\infty \alpha_s$. For $M_\infty \alpha_s = 0.1$, the flow is everywhere similar to that on a flat plate at zero incidence. As $M_\infty \alpha_s$ increases, the incidence effect dominates downstream and the pressure reaches the wedge value asymptotically.

The most severe test for any pressure law to be used in the interaction problem is its application to concave surfaces. The inclusion of centrifugal terms in a pressure law can give rise to oscillatory behaviour when applied to concave surfaces. As mentioned in Section 1, Stollery (Ref 22) showed that the Newton-Busemann rule which accounts for the centrifugal pressure rise predicts highly oscillatory results on concave surfaces. So, in order to study the effect of the centrifugal term in the present TW + CFGL rule, computations were made for the case of a concave surface $y_w \propto x^2$ and $M_\infty \alpha_s = 1.0$. The pressure distribution and heat transfer rates predicted by the present rule are compared with tangent wedge results and with Cheng's theory in figures 18a,b. It may be noted that the present method does exhibit an oscillatory behaviour similar to that predicted by Cheng's theory but the severity of oscillations is very much less in magnitude. However, in comparing Cheng's theory with the TW + CFGL analysis and also with the tangent wedge approximation (i.e., Modified Cheng), figures 18a,b need to be interpreted with some caution because of the different length scales involved in two analyses. The results for Cheng's theory plotted in figures 18a,b have been taken from Reference 22. The TW + CFGL results are close to the tangent wedge results and present the correct comparison between two theories because of the same length scale $L_s (= A^2 \bar{x}^2 \chi)$ used in the analyses. However, the length scale used in Cheng's theory is different ($L_c = \frac{A^2 \bar{x}^2 \chi}{4 \gamma M_\infty^4 \alpha_c^4}$), and hence the value of the

incidence parameter $M_\infty \alpha$ will be different in two cases. This is the reason for using subscripts 's' and 'c' for $M_\infty \alpha$ in the present work. The relation between $M_\infty \alpha_c$ and $M_\infty \alpha_s$ is worked out in Appendix B, and is given by

$$M_\infty \alpha_s = (4\gamma)^{n-1} (M_\infty \alpha_c)^{4n-3} \quad 6.1.1$$

Only for $n = 1$, will the definition of the incidence parameter $M_\infty \alpha$ be the same. For other values of n , each $M_\infty \alpha_c$ corresponds to a different value of $M_\infty \alpha_s$. The difference between two parameters $M_\infty \alpha_s$ and $M_\infty \alpha_c$ is not just one of magnitude. If the length scale L is used in the analysis using the Newton-Busemann rule as in Cheng's theory, for each $M_\infty \alpha_s$ there is a separate solution for a given power-law surface. This can be seen easily by writing the governing equations 4.1.1. and 4.1.2 as

$$\gamma_e - M_\infty \alpha_s \gamma_w = \frac{\left(\int_0^x P dx \right)^{1/2}}{P} \quad 6.1.2$$

$$\text{and} \quad P = \gamma \left[\left(\frac{d\gamma_e}{dx} \right)^2 + \gamma_e \frac{d^2 \gamma_e}{dx^2} \right] \quad 6.1.3$$

If the length scale L_c is used instead of L_s , equations 6.1.2 and 6.1.3 reduce to

$$(Z - Z_w) \frac{P}{\gamma M_\infty^2 \alpha_c^2} = 2 \left(\int_0^\xi \frac{P}{\gamma M_\infty^2 \alpha_c^2} d\eta \right)^{1/2} \quad 6.1.4$$

$$\text{and} \quad \frac{P}{\gamma M_\infty^2 \alpha_c^2} = Z'^2 + Z Z'' \quad 6.1.5$$

The equations 6.1.4 and 6.1.5, when combined give Cheng's viscous interaction equation 4.1.6. So, the results do not depend on $M_\infty \alpha_c$ if L is used as the reference length scale. Apparently, it appears to the present author that this difference between $M_\infty \alpha_s$ and $M_\infty \alpha_c$ has not been noticed in References 22 and 44, and the results of Cheng's theory and the tangent wedge approximation are compared as $\frac{P}{\gamma M_\infty^2 \alpha_s^2}$ versus $\frac{M_\infty^2 \alpha_c^2}{A \bar{x}}$. The figures 18a,b* (or figures

5,6* of Ref 22, and 9a,b* of Ref 44) can still be considered to represent the right comparison only if the abscissa and the ordinate are interpreted as $\frac{M_\infty^2 \alpha_c^2}{A \bar{x}}$ and $\frac{M_\infty^2 \alpha_s^2}{A \bar{x}}$, $\frac{P}{\gamma M_\infty^2 \alpha_c^2}$ and $\frac{P}{\gamma M_\infty^2 \alpha_s^2}$

for the respective theories. In order to provide a correct comparison of different theories, equations 6.1.2 and 6.1.3 were integrated for $M_\infty \alpha_s = 1.0$ and the results are shown in figures 19a,b along with TW + CFGL and TW predictions. However, this difference in length scales does not ^{negate} the conclusion of Reference 22 that Cheng's theory results are quantitatively unrealistic. For the particular case considered, the oscillatory behaviour of the TW + CFGL rule appears to be very much less. The heat transfer results also

* However, owing to an error in plotting, the first peak in pressure is omitted. Otherwise the pressure and heat transfer peaks occur almost simultaneously (confirmed during private discussion).

also indicate a similar oscillatory behaviour. The boundary-layer growing under the influence of the leading edge shock-wave, first increases in thickness reaching a maximum value at the point where the pressure is minimum and then starts decreasing in thickness since the boundary-layer is super-critical for $Y = X^2$ (Section 3.3). Correspondingly, the pressure starts increasing and eventually reaches the inviscid value far downstream. The approach to the inviscid value is monotonically increasing when the tangent wedge rule is used. However, when the centrifugal effects are considered as in the TW + CFGL and in Newton-Busemann rules, the downstream conditions are approached in an oscillatory manner. The boundary-layer starts decreasing in thickness, reaches a minimum and then increases again. This pulsating behaviour of the boundary-layer repeats again and again until it is damped far downstream. Under each neck where the boundary-layer reaches a minimum thickness, the outer inviscid flow has to negotiate small radii of curvature which gives rise to very high centrifugal pressures and, correspondingly the pressure and heat transfer rate reach a maximum value beneath each neck. Cheng's theory is valid only in the limit of $M_\infty \frac{dy}{dx} \gg 1$ and $\gamma \rightarrow 1$. As it is pointed by the present theory using TW + CFGL rule, the downstream oscillatory behaviour is much less severe in magnitude for $M_\infty \alpha_s = 1$. Since the TW + CFGL rule accounts correctly for the shock pressure rise, it appears that the oscillatory behaviour is dependent strongly on the relative pressure rise due to shock and centrifugal effects. In figure 20, the viscous interaction pressure distribution is compared with the inviscid prediction. The viscous interaction solution fluctuates about the inviscid value and for Cheng's theory the rise and drop in pressure appears to take place over a narrow region about the maximum value and the distribution exhibits a spiky behaviour. In the case of TW + CFGL rule, the oscillations are spread out and the distribution appears almost like a continuous rise with intermediate plateau regions. However, the oscillations might become more severe as $M_\infty \alpha_s$ increases. Another factor which might affect the amplitude of oscillation is the variation in γ . Figure 21 shows the pressure and heat transfer rate for $\gamma = 1.1$ and $5/3$, and $M_\infty \alpha_s = 1.0$. It seems that for moderate values of $M_\infty \alpha_s (\approx 1)$, the change in γ from 1.1 to $5/3$ does not change the oscillatory behaviour very much. In figure 22a,b, the results are plotted for a family of shapes $Y = X^n$ with $n = 1.0, 1.5$ and 2.0 . For $n = 1.0$, there is no oscillatory behaviour and as n increases the amplitude and frequency of oscillations also increase. This behaviour is similar to that found by Mohammadian for the Newton-Busemann rule but the amplitudes are very much less. The position of the first peak shifts towards the leading edge as n increases.

The effect on the oscillatory behaviour of increasing $M_\infty \alpha_s$ for the TW + CFGL rule is studied for the worst case of $M_\infty \alpha_s \rightarrow \infty$. In this limit, the governing equations can be considered to be a particular case of the generalized Cheng's equation 4.3.5, with $a = (r+b)/2$ and $b = 1$. The results for the pressure distribution and heat transfer rate are shown in figures 23a, 23b, for a family of shapes $Z = \omega^{-1/n}$. From these results, it may be seen that the oscillatory behaviour is more severe compared to the case of $M_\infty \alpha_s = 1$. So, it appears that

the extent of shock pressure rise controls the amplitude of the oscillatory behaviour. By varying $M_\infty \alpha_s$ in the TW + CFGL rule, we are varying the shock pressure rise to realistic values, keeping the centrifugal corrections constant. However, the 2nd order tangent wedge theory (equation 3.2.8) suggests that the extent of the centrifugal pressure rise for γ greater than unity could be very much smaller than the limiting value of $\gamma h_\infty M_\infty^2 \gamma''$ assumed in the TW + CFGL analysis. So, in order to study the effect of the reduction in the effective centrifugal force for realistic values of γ , the equation 4.3.5 was integrated for

$$a = \frac{\gamma+1}{2} \quad \text{and} \quad b = \delta = 1 - \frac{\gamma^2-1}{\gamma}$$

The above values of a and b correspond to the case of 2nd order tangent wedge theory and the results of numerical integration are shown in figures 24 - 26 for $\gamma = 1.4$. From the pressure and heat transfer rate results in figures 24 and 25, the oscillatory behaviour almost vanishes compared to the highly oscillatory results of Cheng's theory (figures 19a and 19b). The pressure and heat transfer initially start decreasing away from the leading edge, reach a minimum value and start increasing again. At some point downstream, a slight over shoot corresponding to the inviscid prediction and a subsequent mild oscillatory decay to the asymptotic limit is observed. The over-shoot in pressure and heat transfer appear to be negligible for $n < 2$. The downstream asymptotic values are given by

$$\begin{aligned} \frac{P}{\gamma M_\infty^2 \alpha_c^2} &= a z_w'^2 + b z_w z_w'' \\ &= [a n^2 + b n(n-1)] \ell^{2n-2} \end{aligned} \quad 6.1.6$$

$$\frac{A \cdot St}{0.332 \times 4 \gamma \alpha_c^3} = \left[\{a n^2 + b n(n-1)\} (2n-1) \right] \frac{\ell^{1/2} n^{-3/2}}{2} \quad 6.1.7$$

In the second order tangent wedge theory, the centrifugal force is a function of γ and for $\gamma = 1$, it reduces to the Newton-Busemann rule. This suggests that the downstream oscillatory behaviour may be strongly dependent on γ . In figure 26, the effect of γ on pressure and heat transfer is shown. The amplitude of oscillations is strongly attenuated as γ is increased. This is due to the reduction in centrifugal effects. For zero centrifugal effect, as in simple tangent-wedge approximation or Newtonian formula, Stollery (Ref 22) has shown the oscillatory behaviour to vanish completely.

In figure 27, the boundary-layer thickness is plotted for the surface $Y_w = X^2$, according to TW + CFGL THEORY. This clearly exhibits the supercritical behaviour discussed in detail in Section 3.3.

6.2 Bluntness dominated flows:

The closed form solution given by equation 5.2.19 for the case of a blunt flat plate at incidence using the tangent wedge approximation, was solved for different values of $M_\infty \theta$ and the results are shown in figure 28 along with Cheng's zeroeth order results. The values of X were obtained by calculating the integral I_1 for various values of the pressure. When the tangent wedge approximation is used, for each value of incidence θ , there is a different downstream limit corresponding to the wedge value. Near the leading edge tangent wedge estimates are high compared to Cheng's zeroeth order theory. Far downstream, Cheng's theory predicts an undershoot in pressure compared to the inviscid value, and a subsequent decaying oscillatory approach to the downstream limit. A higher order theory by Cheng and Kirsch (Reference 42) considering the corresponding problem of an intense explosion with a moving contact surface, almost removes this undershoot in pressure and predicts close agreement with the wedge value downstream.

Figure 29 shows comparison of the present TW + CFGL results with tangent wedge and Cheng's theory. The governing equations for this case are 5.3.4 and 5.3.6. As in the tangent wedge case, for each $M_\infty \theta$, there is a different solution. The presence of centrifugal terms in the present rule give rise to oscillatory approach downstream. Since, the viscous interaction results indicated that the oscillations become more predominant for large values of $M_\infty \theta$, comparison has been made in the present case for $M_\infty \theta = 10^4$. The TW + CFGL results are similar to that of Cheng's theory. However, the oscillations are about the wedge value and amplitudes appear to be slightly less. Near the leading edge, the present estimates are in between the tangent wedge and Cheng's theory values.

In figure 30, the results when the 2nd order tangent wedge theory is used for the pressure law relation, are given for various values of γ . This particular case corresponds to $a = (\gamma+1)/2$, $b = \gamma$ and $n = 1$ in equation 5.3.9. As earlier in the viscous interaction problem (figure 26), the downstream oscillatory behaviour is strongly dependent on γ . For $\gamma = 1$, the results are identical to Cheng's theory. As γ increases, the amplitude of oscillations diminish fast and for $\gamma = 1.4$, they are not even noticeable within the accuracy of plotting. The downstream oscillatory approach occurring whenever the centrifugal force is considered can be shown analytically, by considering the asymptotic solution of equation 5.3.9 for $n = 1$. For the case of 2nd order tangent wedge theory, the asymptotic solution of equation 5.3.9 for $\xi_c \rightarrow \infty$ can be written as (Appendix D)

$$\frac{P}{\gamma M_\infty^2 \theta_c^2} = \frac{\gamma+1}{2} \left[1 - \frac{\gamma+1}{2} \xi_c^{\frac{1}{4} - \frac{\gamma+1}{2\sigma}} A \cos \left(\frac{\gamma+1}{\sqrt{\sigma}} \xi_c^{1/2} + B \right) + \dots \right] \quad 6.2.1$$

where A and B are constants.

The equation 6.2.1 clearly exhibits the oscillatory behaviour for $\zeta_c \rightarrow \infty$. When $\gamma = 1$ and $\sigma = 1$, equation 6.2.1 reduces to the case of Cheng's theory

$$\frac{P}{\gamma M_\infty^2 \theta_c^2} \approx 1 - \zeta_c^{-3/4} A \cos(2 \zeta_c^{1/2} + B) + \dots \quad 6.2.2$$

The equation 6.2.2 is the same as that obtained by Cheng et al (equation 2.26 of Reference 32).

For the case of TW + CFGL rule, the behaviour is dependent on γ and is obtained by putting $\sigma = 1$

$$\frac{P}{\gamma M_\infty^2 \theta_c^2} \approx \frac{\gamma+1}{2} \left[1 - \zeta_c^{\frac{1}{4} - \frac{\gamma+1}{2}} A \cos\left\{(\gamma+1)\zeta_c^{1/2} + B\right\} + \dots \right] \quad 6.2.3$$

From equation 6.2.3, it may be seen that for $\gamma > 1$, the amplitude of oscillation decreases much faster as $\zeta_c \rightarrow \infty$.

By substituting $\sigma = 1 - \frac{\gamma^2 - 1}{\gamma}$ in equation 6.2.1, the corresponding behaviour for the 2nd order tangent wedge theory is obtained. For $\gamma = 1.4$, the result is

$$\frac{P}{\gamma M_\infty^2 \theta_c^2} \approx 1.2 \left[1 - 1.2 \zeta_c^{-3.57} A \cos\left\{1.78 \times 2.4 \zeta_c^{1/2} + B\right\} + \dots \right] \quad 6.2.4$$

The equation 6.2.4 reveals the strong damping behaviour for $\gamma = 1.4$, thus providing further evidence to the results of numerical integration (figure 30). As regards the application of the 2nd order tangent wedge theory for $\gamma = 5/3$, it warrants some discussion. As γ increases, σ decreases and becomes zero when $\gamma = 1.618$. For this value of γ , equation 6.2.1 suggests infinite damping and the governing differential equation 5.3.9 reduces to the case of pure tangent wedge since the centrifugal term is zero. Perhaps, this infinite damping provides additional support for the non-oscillatory solution obtained for the tangent wedge rule (figure 28). For values of $\gamma > 1.618$, the centrifugal correction becomes negative since $\sigma < 0$. The change in the direction of centrifugal forces for $\gamma > 1.618$ does not appear to be satisfactory on physical grounds, since the direction is determined by the geometry of the body. It appears that the correction for the variation in flow inclination between the body and the shock wave in the 2nd order tangent wedge formula, turns out to be too inaccurate for values of γ away from unity. As mentioned in Section 3.2, Hayes and Probstein (Reference 36) argue that the errors involved in estimating the correction for the flow inclination variation is of the same order as the inaccuracies in centrifugal corrections and hence suggest only a centrifugal correction. For $\gamma = 5/3$, σ is negative and the solution started diverging when integrating numerically. This is the reason why the results have been presented for $\gamma = 1.4$ only, when considering the 2nd order tangent wedge theory. It may be seen from equation 6.2.1

that when δ is negative, the amplitude starts increasing for increasing ℓ_c . In spite of this deficiency, it is felt that the 2nd order tangent wedge might serve as a lower bound for the extent of corrections involved in the simple tangent wedge rule for $\gamma = 1.4$.

In the analysis discussed so far and in reference 44, different pressure laws have been used in place of the Newton-Busemann rule to give improved estimates for pressures on blunt flat plates, for realistic values of γ . At the same time, the pressure-area relationship given by equation 5.1.2 which is valid only in the limit of $\gamma \rightarrow 1$, has been retained. This introduces an inconsistent approximation in the analysis. This objection can be obviated by incorporating corrections to equation 5.1.2 as suggested by Kemp (Ref 38). Fortunately, these corrections do not alter the analysis and introduce only a minor change in the length scale. Assuming the shock wave to collapse on the edge of the entropy layer ($y_s = y_e$), Kemp approximates Cheng's pressure area relation 5.1.1 by

$$\left(\frac{y_s}{\ell}\right)^\gamma (1-f) p = \frac{\bar{M}}{\ell^\gamma} \quad 6.2.5$$

The function f was linearly approximated by $\frac{\gamma+1}{2} \frac{y_b}{y_s}$ and in order to make equation 6.2.5 dimensionally correct, $\bar{M}$ was assumed to be $\frac{\gamma-1}{2} \frac{D_N}{\ell^{\gamma-1}}$ instead of $\frac{\gamma-1}{2} D_N$ as in Cheng's theory. Assuming $\left(\frac{y_s}{\ell}\right)^{1-\gamma} \approx 1$, he obtains the improved pressure-area relation

$$p_e \left(y_s - \frac{\gamma+1}{2} y_b\right) = \frac{\gamma-1}{2} D_N \quad 6.2.6$$

In order to use equation 6.2.6 in our case along with the pressure laws 5.1.6b,c,d we need one more relation connecting the shock shape y_s with the effective body shape y_e . A reasonable approximation for y_s correct for $M_\infty \gg 1$, would be

$$y_s \approx \frac{\gamma+1}{2} y_e$$

This simplifies equation 6.2.6 to

$$p_e (y_e - y_b) = \frac{\gamma-1}{\gamma+1} D_N$$

or from equation 5.3.2

$$\frac{p}{\gamma M_\infty^2 \theta_c^2} (Z - Z_w) = \frac{\gamma-1}{\gamma+1} \frac{C_D t}{2 \theta_c^3 \ell_c}$$

If the length scale ℓ_c is interpreted as $\frac{C \rho t}{2 \theta_c^2}$, the governing equation will be the same as equation 5.3.4. In figure 29, the results with these corrections are shown in dotted lines. In later analysis, the results are presented with these corrections by interpreting ℓ_c as $\frac{C \rho t}{2 \theta_c^3}$ rather than $\frac{\gamma-1}{4} \frac{\rho t}{\theta_c^3}$.

Concave surface flows:

It is often considered that leading edge bluntness effects in concave surface flows will be much less important (Reference 62). On physical grounds, this is reasonable to expect since the body incidence controls the flow everywhere on such surfaces except in the vicinity of leading edge. However, theoretical investigation of such flows instead of tacitly assuming the leading edge effect to be unimportant, has not received much attention. Particularly, in the light of Stollery's work (Reference 22) on viscous interaction over concave surfaces exhibiting unrealistic results predicted by Cheng's theory, it would be worth investigating to see whether, methods normally employed for convex surface do give satisfactory results for concave surfaces. In order to study the applicability of various methods the generalized Cheng's equation 5.3.4 for bluntness-dominated flows, was solved for different values of n , and the results for the pressure distribution are presented in figures 31-36.

The downstream oscillatory approach predicted by Cheng's theory for the case of a blunt flat plate at incidence appears to become very severe for concave surfaces, as can be seen from figure 31. The oscillatory feature is similar to that found for the case of viscous interaction but the amplitude of oscillation appears to be much higher. For $Z_w = \ell_c^2$, oscillations are poorly damped even after about 7 cycles. Near the leading edge where the distribution is similar to that on a flat plate, the pressure is decreasing as the entropy layer produced by the blunt leading edge is thickening. At a point downstream, the pressure reaches a minimum and starts increasing. Correspondingly, the entropy layer of the blunt leading edge thins down to a minimum and again increases. Following this, the entropy layer undergoes successive thinning and thickening until eventually, the oscillations are damped far downstream. This phenomenon is similar to the oscillatory feature exhibited in the case of viscous interaction problem. At each point, where the entropy layer reaches a minimum thickness, the pressure attains a maximum value. In figures 32 and 33, the results for the TW + CPGL rule (equation 5.3.6) are shown for $M_\infty \theta_c \rightarrow \infty$ and $M_\infty \theta_c = 1$, respectively, for various concave surfaces. The results are similar to that of Cheng's theory but the amplitude of oscillation is much less. As found for the case of viscous interaction, oscillations tend to become comparatively less severe for $M_\infty \theta_c = 1$.

The corresponding results for the 2nd order tangent wedge theory are shown in figure 34, for $\gamma = 1.4$. Comparing with Cheng's theory

results (figure 31), we see the significant attenuation in the amplitude and even for $Z_w = \ell_c^2$, the oscillations are almost unnoticeable after two cycles. For $Z_w = \ell_c^{1/5}$, the inviscid conditions are approached almost without any observable overshoot.

When the tangent wedge approximation is used, results (figure 35) are completely non-oscillatory and the inviscid conditions are reached monotonically. As n increases, the downstream conditions are reached much faster. In figure 36, the effect of $M_\infty \theta_c$ for the surface $Z_w = \ell_c^{1/5}$ is shown. For $M_\infty \theta_c = 1$, it may be seen that the minimum value of the non-dimensional pressure $p/\gamma M_\infty^2 \theta_c^2$ is lower than that for $M_\infty \theta_c \rightarrow \infty$. The tangent wedge results suggest that the effect of a blunt leading edge is to cause a lag in the pressure rise compared to that for a sharp leading edge.

The thinning of the leading edge entropy layer far downstream on concave surfaces can be understood by studying the behaviour of equation 5.3.4 for $\ell_c \rightarrow \infty$. Writing $\Delta_e = Z - Z_w$, the entropy layer thickness is given by

$$\Delta_e \sim \frac{1}{(\gamma M_\infty^2 \theta_c^2)} \ell_c \rightarrow \infty$$

From equation 3.3.2, for $\ell_c \rightarrow \infty$,

$$\frac{p}{\gamma M_\infty^2 \theta_c^2} = \frac{C_1}{\gamma M_\infty^2 \theta_c^2} \ell_c^{2n-2}$$

therefore,

$$\Delta_e \sim \frac{\gamma M_\infty^2 \theta_c^2}{C_1} \ell_c^{2-2n}$$

For $n = 1$, corresponding to a blunt flat plate at incidence the non-dimensional entropy layer thickness reaches a constant value for $\ell_c \rightarrow \infty$. For $n > 1$, as in concave surface flows, Δ_e decreases with increasing ℓ_c . This behaviour is similar to the super-critical behaviour of the boundary-layer discussed in Section 3.3.

In the present Section, we have discussed only the inviscid problem. The displacement effect of the boundary-layer can further modify the flow field. In order to study the combined effect of bluntness and displacement, equation 5.1.10 requires to be integrated along with the particular pressure law under consideration. This was carried out for the case of blunt flat plates and the results show that the leading edge thickness generally increases the heat transfer over a significant region near the leading edge, but far downstream, it causes a decrease compared to the sharp leading edge value. Since the results were obtained for some particular cases corresponding to experimental conditions, they will be discussed in Part III.

The results presented in this Chapter show the effect of the incidence parameter $M_\infty \theta_c$ or $M_\infty \alpha_c$ and the extent of centrifugal force in controlling the displacement and bluntness induced interactions. However, in practice the reality of such poorly damped oscillations (e.g., figure 20) is highly doubtful. Far downstream, other effects normally considered to be of higher order, such as afterbody drag etc., might strongly damp the oscillations.

In the next Chapter, a treatment accounting for the higher order effects ignored in Cheng's zeroth order theory is presented. The approach is based on using an integral method (similar to Chernyi's method for blunted wedges and cones). The results obtained demonstrate a heavily damped oscillatory approach to the downstream inviscid conditions on concave surface flows for the case of a sharp leading edge as well as a blunt leading edge.

Before concluding, it is worthwhile to mention the possibility of application of the methods discussed in this Chapter to the axisymmetric case. In general, provided the appropriate pressure-area relation is used in conjunction with the correct form of the pressure law (Reference 81), the analysis of Section 6.2 can be extended to the axisymmetric case. Perhaps, the most interesting case would be to study whether the tangent cone formula predicts the undershoot in pressure on blunted cones. It may be recalled that the corresponding tangent wedge formula for the blunted wedge approached the wedge value asymptotically from the high pressure near the leading edge. In Appendix E, the governing equations for the tangent cone method to determine pressure on blunted cones are presented. For $M_\infty \theta_c \rightarrow \infty$, the equations are simpler to solve and the results of the numerical integration are given in figure 37 along with other theories and some experimental data. The tangent cone method shows the undershoot in pressure with subsequent asymptotic approach to cone value. Cheng's zeroth order theory results show an oscillatory approach to cone value. Since the main purpose of the present work is restricted to two-dimensional flows, detailed studies have not been made for the axisymmetric case. However, the favourable comparison of the tangent cone method warrants a more detailed study of the various methods to the axisymmetric case. / order

CHAPTER VII

AN INTEGRAL APPROACH TO THE INTERACTION PROBLEM

In the previous Chapters, we have shown the inadequacy of Cheng's zeroeth order theory for certain concave surface flows and suggested alternative avenues of approach which greatly modify the downstream oscillatory behaviour. The use of the tangent wedge approximation as pointed out by Hayes and Probestein (Reference 36) must be considered as empirically based in the range of its application. The improved forms of the tangent wedge approximation to account for the centrifugal effect and also for variation in flow inclination angle highlight important parameters controlling the oscillatory behaviour. Whilst these improvements have at least helped to overcome some of the objections (as discussed in Section 3.2) levelled against the basic tangent wedge approximation, they are still subject to some limitations. The validity of such improvements cannot be strictly defended beyond a certain degree of empiricism and there is no means of effecting rigorous further modifications to improve the predictions. In spite of its highly oscillatory results, Cheng's theory is free from this weakness since it is based on the theory of thin shock layers and affords to seek further improvements by expanding in terms of ϵ . Though such a method is not impracticable, considering the severity of the oscillatory behaviour, it is rather difficult to assess the exact extent to which the higher order analysis should be carried out to obtain satisfactory results. An alternative approach which provides a means to account for the higher order effects ignored in Cheng's zeroeth order theory is afforded by using an integral approach for the analysis of the inviscid flowfield. This approach has been adopted in the present work using Chernyi's integral method. The application of this method is considered to study the effect of displacement and leading edge bluntness effects. For $\gamma \rightarrow 1$ and Mach number tending to infinity, Chernyi's method is identical with Cheng's theory. However, the two methods differ when higher order approximations in Cheng's theory are considered. The results obtained in this Chapter demonstrate the damping of the oscillatory behaviour for practical values of γ and show a significant improvement over Cheng's results.

7.1 Analysis:

In this Section, the governing equations for Chernyi's integral approach will be described. The analysis is on the lines of Hayes and Probestein (Reference 36) who have dealt with integral methods in detail. Here, only the equations required for further analysis will be given. For the inviscid hypersonic flow of a perfect gas with constant specific heats past a plane or axisymmetric body (figure 7.1.1), the conservation equations in the integral form can be written as

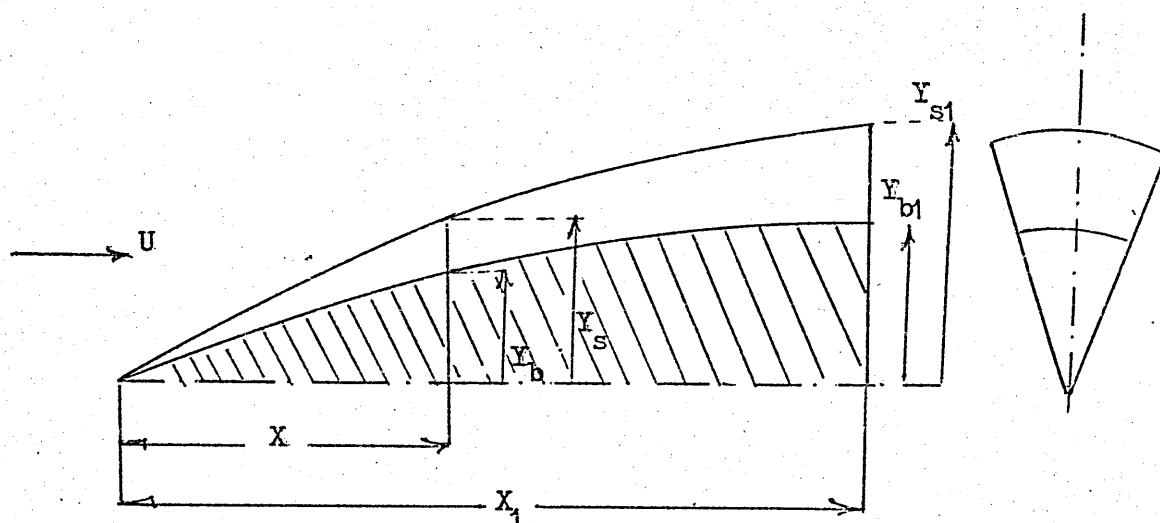


Fig . 7.1.1 : Contour for Integral conservation equations. (Ref 36).

Mass:

$$\frac{\rho_{\infty} U y_{s1}}{(1+j)} = \int_{y_{b1}}^{y_{s1}} \rho u y^j dy \quad 7.1.1$$

X - Momentum:

$$(\rho_{\infty} + \rho_{\infty} U^2) \frac{y_{s1}}{1+j} = \int_{y_{b1}}^{y_{s1}} (\rho + \rho u^2) y^j dy + \int_0^{y_{b1}} \rho_b y_b^j dy_b \quad 7.1.2$$

Y - Momentum:

$$\rho_{\infty} \int_0^{x_1} y_s^j dx = - \int_{y_{b1}}^{y_{s1}} \rho u v y^j dy + \int_0^{x_1} \rho_b y_b^j dy + j \int_0^{x_1} \int_{y_b}^{y_s} \rho y^{j-1} dy dx \quad 7.1.3$$

Energy:

$$\rho_{\infty} U \left(e_{\infty} + \frac{\rho_{\infty}}{\rho_{\infty}} + \frac{U^2}{2} \right) \frac{y_{s1}}{1+j} = \int_{y_{b1}}^{y_{s1}} \rho u \left(e + \frac{p}{\rho} + \frac{u^2}{2} + \frac{v^2}{2} \right) y^j dy \quad 7.1.4$$

$j = 0$: Plane flow,

$j = 1$: Axisymmetric

The conservation equations are simplified further by assuming the shock layer to be very thin and the pressure to be constant across the layer. Following Hayes and Probstein, the governing equations can be written as

$$\frac{\rho_{\infty} U v_{s1} y_{s1}^{1+j}}{1+j} = I_N + \int_0^{x_1} (p_b - p_{\infty}) y_s^j dx \quad 7.1.5$$

and

$$\begin{aligned} \frac{D_N}{2\pi^j} + \int_0^{y_{b1}} p_b y_b^j dy_b &= \frac{p_{b1}}{\gamma-1} \frac{y_{s1}^{1+j} - y_{b1}^{1+j}}{1+j} + \frac{\rho_{\infty}}{2} \frac{v_{s1}^2 y_{s1}^{1+j}}{1+j} \\ &\quad - \frac{p_{\infty} y_{s1}^{1+j}}{(\gamma-1)(1+j)} \end{aligned} \quad 7.1.6$$

where I_N corresponds to a concentrated impulse at the nose and D_H is the drag of the blunt nose.

In arriving at equations 7.1.5 and 7.1.6, it is assumed that the velocity U is constant across the shock layer. The equation 7.1.5 is further simplified by making use of the shock continuity equation,

$$v_s = (1-\epsilon) U y_s' \quad 7.1.7$$

or, using oblique shock relations for ϵ (the density ratio across the shock),

$$v_s = \frac{2 U y_s'}{\gamma+1} \left(1 - \frac{1}{M_{\infty}^2 y_s'^2} \right) \quad 7.1.8$$

Substituting for v_s , equations 7.1.5 and 7.1.6 can be solved for the pressure p_b and shock shape y_s . By combining equations 7.1.5 and 7.1.8, we get

$$p_b - p_{\infty} = \frac{2 \rho_{\infty} U^2}{\gamma+1} \left[y_s'^2 - M_{\infty}^{-2} + \frac{y_s y_s''}{1+j} \left(1 + M_{\infty}^{-2} y_s'^2 \right) \right] \quad 7.1.9$$

In arriving at equation 5.1.12 in Cheng's theory, it is assumed that the second term on the left hand side and the third and fourth terms on the right hand side of equation 7.1.6 are negligible. Similarly in the limit of $\gamma \rightarrow 1$ and $M \rightarrow \infty$, the equation 7.1.9 reduces to the Newton-Busemann formula used in Cheng's theory. In the following we put $j = 0$, since we will be dealing with only two-dimensional flows.

Leading edge bluntness effect on power law surfaces:

For the case of general power law surface $y_b \propto x^n$, we will choose the following non-dimensional variables

$$Z_s = \frac{y_s}{\alpha_N L_N}$$

$$Z_b = \frac{y_b}{\alpha_N L_N} ; \quad x_N = \frac{x}{L_N}$$

$$\text{where} \quad L_N = \frac{C_D t}{2 \alpha_N^3} \quad (\text{length scale}) \quad 7.1.10$$

From equations 7.1.8 and 7.1.9,

$$u_s = \frac{2U\alpha_N}{\gamma+1} \frac{dZ_s}{dx_N} \left(1 - \frac{1}{M_\infty^2 \alpha_N^2 Z_s'^2} \right) \quad 7.1.11$$

$$\frac{p_b}{p_\infty U^2 \alpha_N^2} = \frac{p_\infty}{p_\infty U^2 \alpha_N^2} + \frac{2}{\gamma+1} \left[\left(\frac{dZ_s}{dx_N} \right)^2 - \frac{1}{M_\infty^2 \alpha_N^2} + Z_s^2 \frac{d^2 Z_s}{dx_N^2} \left(1 + \frac{1}{M_\infty^2 \alpha_N^2 Z_s'^2} \right) \right] \quad 7.1.12$$

Substituting 7.1.11 and 7.1.12 in equation 7.1.6 and dropping subscripts '1', we get

$$\begin{aligned} \frac{\gamma+1}{2} \frac{D_N}{\rho_\infty U^2 \alpha_N^2 L_N} + \frac{\gamma+1}{2} \int_0^{x_N} \frac{p_b}{\rho_\infty U^2 \alpha_N^2} \frac{dz_b}{dx_N} dx_N &= \frac{z_s - z_b}{\gamma-1} \frac{p_b}{\rho_\infty U^2 \alpha_N^2} \frac{\gamma+1}{2} \\ &+ \frac{z_s}{\gamma+1} \left(\frac{dz_s}{dx_N} \right)^2 \left[1 - \frac{1}{M_\infty^2 \alpha_N^2 z_s^2} \right] - \frac{p_\infty}{\rho_\infty U^2 \alpha_N^2} \frac{z_s}{\gamma-1} \frac{\gamma+1}{2} \end{aligned} \quad 7.1.13$$

For $M_\infty \alpha_N \gg 1$, equation 7.1.12 shows that

$$\frac{p_b}{\rho_\infty U^2 \alpha_N^2} = \frac{2}{\gamma+1} \frac{d}{dx_N} \left(z_s \frac{dz_s}{dx_N} \right)$$

and equation 7.1.13 reduces to

$$\begin{aligned} \frac{\gamma+1}{4} + \int_0^{x_N} \frac{dz_b}{dx_N} \frac{d}{dx_N} \left(z_s \frac{dz_s}{dx_N} \right) dx_N &= \frac{z_s - z_b}{\gamma-1} \frac{d}{dx_N} \left(z_s \frac{dz_s}{dx_N} \right) \\ &+ \frac{z_s}{\gamma+1} \left(\frac{dz_s}{dx_N} \right)^2 \end{aligned} \quad 7.1.14$$

For the case when the after body is a wedge $z_b = x_N$, and the equation 7.1.14 reduces to

$$\frac{\gamma+1}{4} + z_s \frac{dz_s}{dx_N} = \frac{z_s - z_b}{\gamma-1} \frac{d}{dx_N} \left(z_s \frac{dz_s}{dx_N} \right) + \frac{z_s}{\gamma+1} \left(\frac{dz_s}{dx_N} \right)^2 \quad 7.1.15$$

The equation 7.1.15 when expressed in dimensional variables y_s , y_b and x is the same as equation 5.8.12 of Reference 36, and equation 1 of Reference 41. For the case of general power-law surface, the above simplification is not possible and the solution of equation 7.1.14 can only be obtained by computing the integral

numerically or by differentiating the complete equations once and then integrating the resulting differential equation. Employing the latter approach, the equation 7.1.14 can be differentiated with respect to X_N to give

$$\begin{aligned} \frac{dz_b}{dX_N} \frac{d}{dX_N} \left(z_s \frac{dz_s}{dX} \right) &= \frac{z_s - z_b}{r-1} \frac{d^2}{dX_N^2} \left(z_s \frac{dz_s}{dX_N} \right) + \frac{1}{r-1} \left(\frac{dz_s}{dX_N} - \frac{dz_b}{dX_N} \right) \frac{d}{dX_N} \left(z_s \frac{dz_s}{dX_N} \right) \\ &+ \frac{2z_s}{r+1} \frac{dz_s}{dX_N} \frac{d^2 z_s}{dX_N^2} + \frac{1}{r+1} \left(\frac{dz_s}{dX_N} \right)^3 \end{aligned} \quad 7.1.16$$

or writing $z_s \frac{dz_s}{dX} = Y_1$

$$\frac{dY_1}{dX_N} \cdot \frac{dz_b}{dX_N} = \frac{z_s - z_b}{r-1} \frac{d^2 Y_1}{dX_N^2} + \frac{1}{r-1} \left(\frac{Y_1}{z_s} - \frac{dz_b}{dX_N} \right) \frac{dY_1}{dX_N} + \frac{1}{r+1} \left[\frac{2Y_1}{z_s} \frac{dY_1}{dX_N} - \left(\frac{Y_1}{z_s} \right)^3 \right] \quad 7.1.17$$

The equation 7.1.17 is a general form of Chernyi's differential equation for the shock shape z_s for the given body shape $z_b = X_N^n$. To facilitate numerical integration, equation 7.1.17 can be written as

$$\frac{dz_s}{dX_N} = \frac{Y_1}{z_s} \quad 7.1.18a$$

$$\frac{dY_1}{dX_N} = Y_2 \quad 7.1.18b$$

$$\begin{aligned} \frac{dY_2}{dX_N} &= \frac{r-1}{z_s - z_b} \left[\frac{dY_1}{dX_N} \cdot \frac{dz_b}{dX_N} - \frac{1}{r-1} \frac{dY_1}{dX_N} \left(\frac{Y_1}{z_s} - \frac{dz_b}{dX_N} \right) - \frac{2}{r+1} \frac{Y_1}{z_s} \frac{dY_1}{dX_N} \right. \\ &\quad \left. + \frac{1}{r+1} \left(\frac{Y_1}{z_s} \right)^3 \right] \end{aligned} \quad 7.1.18c$$

The equations 7.1.18a - c are singular at the origin and in order to start the integration, we need the behaviour for small X_N . This is obtained by carrying out a series expansion

$$Z_S \sim k_3 X_N^{m_3} + \dots$$

By substituting for Z_S and $Z_b = X_N^n$ in equation 7.1.14 and taking the limit $X_N \rightarrow 0$, we get

$$\frac{\gamma+1}{4} + \dots \approx \frac{k_3^3 m_3 (2m_3-1)}{\gamma-1} X_N^{3m_3-2} + \frac{1}{\gamma+1} k_3^3 m_3^2 X_N^{3m_3-2} \quad 7.1.19$$

By equating the power and coefficient of X_N , we get for k_3 and m_3

$$m_3 = 2/3 \quad 7.1.20a$$

$$k_3 = \left[\frac{9(\gamma+1)^2(\gamma-1)}{8(3\gamma-1)} \right]^{1/3} \quad 7.1.20b$$

So, for $X_N \rightarrow 0$

$$Y_1 \sim k_3^2 m_3 X_N^{2m_3-1} \quad 7.1.21a$$

$$Z_S \sim k_3 X_N^{m_3} \quad 7.1.21b$$

$$Y_2 \sim k_3^2 m_3 (2m_3-1) X_N^{2(m_3-1)} \quad 7.1.21c$$

Using equations 7.1.21a-c as initial conditions, the system of differential equations 7.1.18a-c can be integrated. The non-dimensional pressure is given by

$$\frac{p_b}{\gamma p_{b0} M_{b0}^2 \alpha_N^2} = \frac{2}{\gamma+1} Y_2 \quad 7.1.22$$

It may be noted from equation 7.1.14 that when $\gamma = 1$, it reduces to

$$(Z_S - Z_b) \frac{d}{dX_N} \left(Z_S \frac{dZ_S}{dX_N} \right) = 1 \quad 7.1.23$$

The equation 7.1.23 is the same as equation 5.3.9 (with $a = b = 1$) obtained in Cheng's theory.

The numerical results obtained by integrating equations 7.1.18a - c are presented in Section 7.2 for concave surface flows. It must be noted that the advantage of using the integral method is that it does not stipulate that the major drag contribution should be at the leading edge (Mirels, Reference 87). Particularly for concave surface flows for which the afterbody drag can be very significant, the integral method may be very useful.

Application to Viscous Interaction:

In the presence of viscous interaction, the body shape will be modified by the boundary-layer growth and y_b is unknown in equation 7.1.6. In order to solve for the shock shape y_s and the effective body shape y_b , we need one more equation relating y_b and the boundary-layer displacement thickness δ^* . This is furnished by equation 3.1.26. To facilitate comparison with the results of Chapter IV, we will choose the following variables

$$\begin{aligned} Y_S &= \frac{M_\infty y_s}{L_S} & Y_B &= \frac{M_\infty y_b}{L_S} \\ X &= \frac{x}{L_S} & P_b &= \frac{p_b}{p_\infty} = \frac{p_w}{p_\infty} \end{aligned}$$

With the new variables, equations 7.1.6 to 7.1.9 can be written as (with $j = 0$ for two-dimensional flows)

$$\frac{DN M_\infty}{2 p_\infty L_S} + \int_0^X P_b \frac{dY_B}{dX} dX = \frac{P_b (Y_S - Y_B)}{\gamma - 1} + \frac{1}{2} \frac{p_\infty v_s^2 Y_S}{p_\infty} - \frac{Y_S}{\gamma - 1} \quad 7.1.24$$

$$v_s = \frac{2U Y_S'}{M_\infty (\gamma + 1)} \left(1 - \frac{1}{Y_S'^2} \right)$$

$$\text{or } \frac{1}{2} \frac{p_\infty v_s^2 Y_S}{p_\infty} = \frac{2\gamma Y_S'^2}{(\gamma + 1)^2} \left(1 - \frac{1}{Y_S'^2} \right)^2 Y_S$$

$$\text{and } P_b = 1 + \frac{2\gamma}{\gamma + 1} \left[Y_S'^2 - 1 + Y_S Y_S'' \left(1 - \frac{1}{Y_S'^2} \right) \right] \quad 7.1.25$$

From equation 3.1.26

$$\frac{M_\infty \delta^*}{L_s} = I_0 \frac{\left(\int_0^x P_b d\xi \right)^{1/2}}{P_b} \quad 7.1.26$$

If the geometric shape of the body is $y_w \propto x^n$, then δ^* is related to y_b by

$$\begin{aligned} \frac{M_\infty \delta^*}{L_s} &= \frac{M_\infty}{L_s} (y_b - y_w) \\ &= Y_B - M_\infty \alpha_s Y_w \end{aligned} \quad 7.1.27$$

where $Y_w = \frac{y_w}{\alpha_s L_s} = X^n$

Combining 7.1.26 and 7.1.27,

$$(Y_B - M_\infty \alpha_s Y_w) = I_0 \frac{\left(\int_0^x P_b d\xi \right)^{1/2}}{P_b} \quad 7.1.28$$

With equations 7.1.24, 7.1.25 and 7.1.28, we can solve for the unknowns P_b , Y_b and Y for the given body shape Y_w . For a sharp leading edge, the first term representing the nose drag in equation 7.1.24 will be zero and in the limit of $\gamma \rightarrow 1$, we get $Y_s = Y_b$ and the shock wave collapses on to the effective body Y_b defined by the boundary-layer edge. However, for γ different from unity, under the present treatment, it is possible to treat the boundary-layer and the shock-wave to be distinct.

Now, the retention of the term I in equation 7.1.28 needs some discussion. In the earlier analyses (Chapters IV and V), I_0 was assumed to be unity on the basis of cold wall similarity or in the limit $\epsilon \rightarrow 0$, as in Cheng's theory. In Cheng's analysis, consistent with the approximation $\epsilon \rightarrow 0$, the kinetic energy and after body drag terms were considered to be of higher order in the inviscid flow field analysis. However, in the present approach we are not justified in putting $I_0 = 1$, and its variation with X needs to be considered since we have retained the kinetic energy and afterbody drag terms in the description of the inviscid flow field. A most straight forward approach to consider the variation in I_0 is to use local similarity method (Dewey, Reference 23). This method is easily applicable for flat plates at angle of attack or at zero incidence. However, in regions of adverse pressure gradient as on concave surfaces, this approach may not lead to more correct results. This point will be discussed later. For the present, we will assume I_0 to be a function of ξ .

As in the earlier cases, equations 7.1.24, 7.1.25 and 7.1.28 are singular at the origin and in order to facilitate numerical integration, we need a series solution for small X . This is obtained by writing

$$P_b \approx k X^m + \dots \quad 7.1.29a$$

$$Y_b \approx k_2 X^{m_2} + \dots \quad 7.1.29b$$

$$Y_s \approx k_3 X^{m_3} + \dots \quad 7.1.29c$$

where the constants k, k_2, k_3, m, m_2, m_3 are to be determined.

Substituting for P_b, Y_b and Y_s in equations 7.1.25 and 7.1.28, we get for $X \rightarrow 0$

$$k_2 = \frac{I_{0L}}{[k(1+m)]^{1/2}} \quad ; \quad m_2 = \frac{1-m}{2} \quad 7.1.30a, b$$

$$k = \frac{2r}{r+1} \cdot k_3^2 m_3 (2m_3-1) \quad ; \quad m = 2(m_3-1) \quad 7.1.31a, b$$

where I_{0L} = value of I_0 for $X \rightarrow 0$.

From equation 7.1.24, we have for the case of a sharp leading edge

$$k k_2 \frac{m_2}{m_1+m_2} X^{m_1+m_2} + \dots \approx k X^m \frac{(k_3 X^{m_3} - k_2 X^{m_2})}{r-1} + \frac{2r}{r+1} k_3^2 m_3 X^{3m_3-2} - \frac{k_3}{r-1} X^{m_3} \quad 7.1.32$$

Comparing the order of magnitude of various terms for $X \rightarrow 0$, it may be seen that $m_2 = m_3$ (i.e. the shock wave shape and the boundary-layer shape are similar) and therefore from equations 7.1.30b and 7.1.31b, we get

$$m_2 = m_3 = 3/4 \quad ; \quad m = -1/2 \quad 7.1.33$$

Substituting for m, m_2 and m_3 in equation 7.1.33,

$$3 k k_2 X^{1/4} + \dots = \frac{k(k_3 - k_2)}{r-1} X^{1/4} + \frac{2r}{(r+1)^2} k_3^2 \left(\frac{3}{4}\right)^2 X^{1/4} - \frac{k_3}{r-1} X^{3/4} + \dots$$

For $X \rightarrow 0$, the last term is of higher order than the remaining terms and equating the coefficients of $X^{1/4}$,

$$3 k k_2 = \frac{k(k_3 - k_2)}{\gamma - 1} + \frac{2\gamma}{(\gamma + 1)^2} \left(\frac{3}{4}\right)^2 k_3^3 \quad 7.1.34$$

From equations 7.1.30a and 7.1.31a

$$k_2 = I_{OL} \left(\frac{2}{k}\right)^{1/2} \quad 7.1.35$$

$$\text{and } k = \frac{3\gamma}{4(\gamma + 1)} k_3^2 \quad 7.1.36$$

Now, the values of k , k_2 and k_3 can be determined by using equations 7.1.33, 7.1.34 and 7.1.35, as

$$k = \frac{2^{1/2} \cdot \frac{3\gamma - 1}{\gamma - 1}}{\left\{\frac{3}{\gamma(\gamma + 1)}\right\}^{1/2} + \left\{\frac{4(\gamma + 1)}{3\gamma}\right\}^{1/2} \frac{1}{\gamma - 1}} I_{OL} \quad 7.1.37$$

$$k_2 = \left(\frac{2}{k}\right)^{1/2} I_{OL} \quad 7.1.38$$

$$\text{and } k_3 = \left[\frac{4(\gamma + 1)}{3\gamma} k \right]^{1/2} \quad 7.1.39$$

The values k , k_2 and k_3 depend on γ

$$\gamma = 1.4 \quad P_b = 1.6463 I_{OL} X^{-1/2}$$

$$Y_B = 1.1022 I_{OL}^{1/2} X^{3/4}$$

$$Y_s = 1.9399 I_{OL}^{1/2} X^{3/4}$$

$$\gamma = 5/3 \quad P_b = 2.1125 I_{OL} X^{-1/2}$$

$$Y_B = 0.9730 I_{OL}^{\frac{1}{2}} X^{\frac{3}{4}}$$

$$Y_S = 2.1229 I_{OL}^{\frac{1}{2}} X^{\frac{3}{4}}$$

From the above leading edge solutions, it may be noticed that for $\gamma = 5/3$, the induced pressures are much higher in the strong interaction region than for $\gamma = 1.4$. Further, the shock shape is about twice the distance from the wall compared to the boundary-layer edge. The shock shape and the boundary-layer thickness are proportional to the square root of I_{OL} , whereas the induced pressure is directly proportional to I_{OL} . In order to make a correct comparison of results for various γ , we need to know the value of I_{OL} which depends on β (the pressure gradient parameter). From equation 3.1.4, β is

$$\beta = - \frac{\gamma-1}{\gamma} \frac{\int p_e dx}{p_e^2} \frac{dp_e}{dx}$$

$$\text{or} \quad \beta = - \frac{\gamma-1}{\gamma} \frac{\int P_b dx}{P_b^2} \frac{dP_b}{dx} \quad 7.1.40a$$

Substituting $P_b = k X^m$

$$\beta = - \frac{\gamma-1}{\gamma} \frac{m}{m+1} \quad 7.1.40b$$

With $m = -\frac{1}{2}$, in the strong interaction region

$$\beta = \frac{\gamma-1}{\gamma} \quad 7.1.41$$

Thus, the value of β depends on γ and is equal to 0.2857 and 0.4, for γ equal to 1.4 and 5/3 respectively. In general, I_{OL} depends on β , wall enthalpy g_w and Prandtl number Pr . As discussed in Section 3.1, the value of I_{OL} can vary by a factor of 1 to 1.6 from cold wall conditions to adiabatic wall. Corresponding variation in I_{OL} ($\int g_w^{1/2} dx$) could be as much as by a factor of 3 (Dewey, Reference 76). The values of I_{OL} are given in Table 7.1.1.

For $g_w = 0$, using the values of Rogers (Table 7.1.1), we have

$\gamma = 1.4$	$P_b = 1.1066 X^{-\frac{1}{2}} ;$	$\gamma = 5/3$	$P_b = 1.3146 X^{-\frac{1}{2}}$
	$Y_S = 1.5905 X^{\frac{3}{4}} ;$		$Y_S = 1.6747 X^{\frac{3}{4}}$
	$Y_B = 0.9037 X^{\frac{3}{4}} .$		$Y_B = 0.7676 X^{\frac{3}{4}}$

TABLE 7.1.1

Ref	β	ω	Pr	g_w	I_1	I_{oL}
Dewey (Ref 23)	0.2857	0.7	0.7	0.15	0.4660	0.7139
				0.40	0.7751	0.8091
				0.60	1.0080	0.8385
				0.8025	1.2380	0.8541
Rogers (Ref 78 * 79)		1.0	0.723	0.0		0.6722
				0.2		0.7507
				0.6		0.7984
				g_{aw}		0.8054
Dewey (Ref 23)	0.4	0.7	0.7	0.15	0.4361	0.6681
				0.40	0.7256	0.7574
				0.60	0.9432	0.7846
				0.7977	1.1530	0.7987
Rogers (Ref 78, * 79)		1.0	0.723	0.0		0.6223
				0.2		0.7000
				0.6		0.7458
				g_{aw}		0.7520

* I_{oL} obtained from a quadratic fit for the data $0 \leq \beta \leq 0.5$

The values of pressure calculated by the present method are very close to the results of TW + CFGL rule obtained in equations 4.2.8a, b. Knowing the solution near the leading edge, the system of differential equations 7.1.24, 7.1.25 and 7.1.28 can be integrated starting from a point close to the leading edge and marching downstream. Since I_o

in equation 7.1.28 is a function of X , the equation 7.1.28 can be differentiated to give

$$\frac{dP}{dx} = P \left[\frac{I_0^2}{2 P \Delta^{*2}} - \frac{1}{\Delta^*} \frac{d\Delta^*}{dx} + \frac{1}{I_0} \frac{dI_0}{dx} \right] \quad 7.1.42$$

where $\Delta^* = Y_B - M_\infty \alpha_S Y_W$

In the local similarity method, the value of β is assumed to be a constant determined by the local values of the flow parameters. This implies a power law variation for the pressure and hence $d\beta/dx$ can be put equal zero. Even otherwise, the term $\frac{1}{I_0} \frac{dI_0}{dx}$ can be small if the pressure is varying slowly with respect to X , and I_0 could be treated as a constant over the small stepsize range of integration. The full implications of the local similarity approach have been discussed by Dewey (Ref 23). So, the equation 7.1.42 can be written as

$$\frac{dP}{dx} = P \left[\frac{I_0^2(\beta)}{2 P \Delta^{*2}} - \frac{1}{\Delta^*} \frac{d\Delta^*}{dx} \right] \quad 7.1.43$$

If we put $I_0 = 1$, equation 7.1.43 reduces to equation 4.2.17 for the cold/similarity approach. By differentiating equation 7.1.24 (wall

$$\begin{aligned} \frac{\gamma}{\gamma-1} P_b \frac{dY_B}{dx} &= \frac{dP_b}{dx} \cdot \frac{Y_S - Y_B}{\gamma-1} + \frac{P_b}{\gamma-1} \frac{dY_S}{dx} + \frac{4\gamma}{(\gamma+1)^2} Y_S \frac{dY_S}{dx} \frac{d^2 Y_S}{dx^2} \left(1 - \frac{1}{Y_S'^2}\right) \\ &\quad + \left(\frac{2\gamma}{\gamma+1}\right)^2 \left(\frac{dY_S}{dx}\right)^3 \left(1 - \frac{1}{Y_S'^2}\right) - \frac{1}{\gamma-1} \frac{dY_S}{dx} \end{aligned} \quad 7.1.44$$

$$\frac{d^2 Y_S}{dx^2} = \frac{P_b - 1 - \frac{2\gamma}{\gamma+1} (Y_S'^2 - 1)}{\frac{2\gamma}{\gamma+1} Y_S \left(1 + \frac{1}{Y_S'^2}\right)} \quad 7.1.45$$

With equations 7.1.43, 7.1.44 and 7.1.45, we can evaluate the interaction effects for given body shape and given wall conditions.

When the leading edge is blunt, the boundary-layer growth near the leading edge is dictated by the inviscid pressure distribution caused by the blunt leading edge. In order to obtain initial

conditions for this case, the series expansion 7.1.29a,b,c are substituted in equation 7.1.24 and noting that the drag term is balanced by the first two terms on the right hand side, we get

$$P_b = k X^m, \quad k = \frac{4}{9} \frac{\gamma}{\gamma+1} k_3^2, \quad m = -2/3 \quad 7.1.46a$$

$$Y_B = k_2 X^{m_2}, \quad k_2 = \left(\frac{2}{k}\right)^{1/2} \bar{I}_0, \quad m_2 = 5/6 \quad 7.1.46b$$

$$Y_s = k_3 X^{m_3}, \quad k_3 = \left[\frac{9}{4} \frac{(\gamma+1)^2 K_s}{\gamma(3\gamma-1)} \right]^{1/3}, \quad m_3 = 2/3 \quad 7.1.46c$$

$$\text{where} \quad K_s = \frac{\gamma(\gamma-1)}{4} M_\infty^3 \frac{C_D t}{L_s}$$

The values of k_2 and m_2 were determined using 7.1.28. The relations 7.1.46b suggests that the boundary-layer thickness $Y_B \rightarrow 0$ much faster than the shock shape Y_s for $X \rightarrow 0$. The relations 7.1.46a,c are the same as obtained for the inviscid problem in equation 7.1.20, when expressed in the corresponding non-dimensional variables for P_b and Y_s . The pressure gradient parameter is given by

$$\beta = -\frac{\gamma-1}{\gamma} \frac{m}{m+1} = 2 \frac{\gamma-1}{\gamma} \quad 7.1.47$$

$$\text{For } \gamma = 1.4, \quad \beta = 0.5714$$

$$\gamma = 5/3 \quad \beta = 0.8$$

Knowing the pressure by numerical integration, the values of Stanton number can be calculated by using equations 3.1.30 and 3.1.31.

Inviscid limit for $X \rightarrow \infty$:

Far downstream, the disturbance induced by the leading edge (either blunt or sharp) would have become very small and the incidence effect will be dominating. Particularly, on concave surfaces the incidence effect will be increasing rapidly downstream and correspondingly one would expect the leading edge bluntness and the boundary-layer displacement effect to become less important, and the inviscid pressure distribution will be a good approximation. To determine the inviscid limit, we can look for a series expansion of the type (for $X \rightarrow \infty$)

$$P_b \approx l_1 X^{n_1} + \dots \quad 7.1.48a$$

$$Y_B \approx l_2 X^{n_2} + \dots \quad 7.1.48b$$

$$Y_s \approx l_3 X^{n_3} + \dots \quad 7.1.48c$$

From equation 7.1.28, we get for $X \rightarrow \infty$,

$$Y_B = M_\infty \alpha_s Y_W = M_\infty \alpha_s X^n$$

therefore $l_2 = M_\infty \alpha_s$, $n_2 = n$

Substituting for P_b , Y_B and Y_s in equations 7.1.24 and 7.1.25

$$\begin{aligned} \text{Constant} + \frac{l_1 l_2}{l_1 + l_2} n_2 X^{n_1 + n_2} &\approx \frac{l_1 X^{n_1} (l_3 X^{n_3} - l_2 X^{n_2})}{\gamma - 1} + \\ &+ \frac{2\gamma}{(\gamma + 1)^2} l_3^3 n_3^2 X^{3n_3 - 2} - \frac{l_3 X^{n_3}}{\gamma - 1} \end{aligned} \quad 7.1.49$$

and $l_1 X^{n_1} + \dots \approx \frac{2\gamma}{\gamma + 1} l_3^2 X^{2n_3 - 2} \cdot n_3 (2n_3 - 1) + \dots$ 7.1.50

From equation 7.1.50 for $X \rightarrow \infty$,

$$n_1 = 2n_3 - 2 \quad 7.1.51a$$

$$l_1 = \frac{2\gamma}{\gamma + 1} l_3^2 n_3 (2n_3 - 1) \quad 7.1.51b$$

From equation 7.1.49, comparing the order of magnitude of various terms, we get $n_2 = n_3 = n$. This result also follows from the similar solutions. Substituting for n_1 and n_2 in equation 7.1.49,

$$\begin{aligned} \text{Constant} + \frac{l_1 l_2}{n_1 + n_2} X^{3n - 2} &\approx \frac{l_1 (l_3 - l_2)}{\gamma - 1} X^{3n - 2} + \frac{2\gamma}{(\gamma + 1)^2} l_3^3 n^2 X^{3n - 2} \\ &- \frac{l_3}{n - 1} X^n \end{aligned} \quad 7.1.51c$$

For the case of a wedge $n = 1$, and for $X \rightarrow \infty$, we get by equating the coefficients of X

$$\frac{l_1 l_2}{n_1 + n_2} = \frac{l_1 (l_3 - l_2)}{\gamma - 1} + \frac{2\gamma}{(\gamma + 1)^2} l_3^3 n^2 - \frac{l_3}{n - 1} \quad 7.1.52a$$

and $n_2 = n_3 = 1$; $n_1 = 2n_3 - 2 = 0$

$$l_1 = \frac{2\gamma}{\gamma + 1} l_3^2 \quad 7.1.52b$$

From equations 7.1.52a and b, we can solve for l_3

$$l_3 = \frac{1}{2} \left[\frac{\gamma+1}{2} M_\infty \alpha_s + \frac{\gamma+1}{2} \left\{ M_\infty^2 \alpha_s^2 + \frac{4}{\gamma^2} \right\}^{1/2} \right]$$

For $M_\infty \alpha_s \gg 1$,

$$l_3 = \frac{\gamma+1}{2} M_\infty \alpha_s \quad 7.1.53a$$

$$l_1 = \frac{\gamma(\gamma+1)}{2} M_\infty^2 \alpha_s^2 \quad 7.1.53b$$

Equations 7.1.53a and b are the familiar strong shock results.

For the case of a general concave surface $n > 1$, the last term on the right hand side of equation 7.1.51c is of higher order and by equating the coefficient of X^{3n-2} , we have

$$\frac{l_1 l_2}{n+n_2} = \frac{l_1 (l_3 - l_2)}{\gamma-1} + \frac{2\gamma}{(\gamma+1)^2} l_3^3 n_3^2 \quad 7.1.54$$

From equations 7.1.51a and b,

$$n_1 = 2n - 2 \quad ; \quad n_1 + n_2 = 3n - 2$$

$$l_1 = \frac{2\gamma}{\gamma+1} n(2n-1) l_3^2$$

Substituting for l_1 , n_1 and n_2 simplifying the resulting expression, we get

$$l_3 = \frac{2n-1}{3n-2} \frac{n\gamma + 2n-2}{[\gamma(3n-1) + (n-1)]} (\gamma+1) l_2 \quad 7.1.55$$

$$\text{and} \quad l_1 = \frac{2\gamma}{\gamma+1} n(2n-1) l_3^2 \quad 7.1.56$$

Since l_2 is known from the geometry of the body shape, l_3 and hence the inviscid pressure and shock shapes can be evaluated. The values of l_3 and l_2 are given Table 7.1.2 for various n and γ . The inviscid limits calculated in the present section helps to serve as a check against the results of numerical integration for large X .

TABLE 7.1.2

n	l_3		l_1	
	$\gamma = 1.4$	$\gamma = 5/3$	$\gamma = 1.4$	$\gamma = 5/3$
1	1.2 l_2	1.3333 l_2	0.96 l_2^2	2.2222 l_2^2
1.5	1.1022 l_2	1.1789 l_2	4.2521 l_2^2	5.2121 l_2^2
2.0	1.08 l_2	1.1429 l_2	8.1648 l_2^2	9.7959 l_2^2

7.2 Numerical results and discussion:

First we will consider the inviscid flow past concave surfaces with blunt leading edge. In general, the results depend on the value of the incidence parameter $M_\infty \alpha_N$ and γ as can be seen from equations 7.1.12 and 7.1.13. For fixed γ , results presented in Chapter VI using the TW + CFGL theory show that the oscillatory behaviour becomes very severe as $M_\infty \theta_c \rightarrow \infty$. In view of this, computations were carried out for the present case for $M_\infty \alpha_N \rightarrow \infty$. The governing equations 7.1.18a - c, were integrated numerically as described in Chapter VI, using the initial conditions 7.1.21a - c. In order to have a check on the numerical integration, the pressure distribution was calculated for a blunted wedge ($Z_b = X_N$) for which Schnieder's results (Reference 41) obtained by integrating equation 7.1.15 are available. The agreement was found to be satisfactory. The results for the surface pressure on a family of shapes $Z_b = X_N^n$ are presented in figure 38 for $\gamma = 1.4$. Comparing this with Cheng's zeroeth order theory results (figure 31), we find a significant improvement in the downstream behaviour. To facilitate a more realistic comparison, the results of two theories are plotted in figure 39 using the same length scale L_N . It may be noted that we have denoted the incidence parameter in this thesis by $M_\infty \alpha_N$, $M_\infty \theta_c$ or $M_\infty \alpha_s$ corresponding to different length scales employed in the analysis. For the case of a wedge, all these have the same value. The results of figures 31 and 32 are related by (Appendix F) for $n = 2$ and $\gamma = 1.4$

$$\frac{2\alpha_N^3 \chi}{C_{\theta t}} = 0.64 \frac{2\theta_c^3 \chi}{C_{\theta t}} \quad 7.2.1$$

$$\text{and} \quad \frac{P}{\gamma M_\infty^2 \alpha_N^2} = 0.41 \frac{P}{\gamma M_\infty^2 \theta_c^2} \quad 7.2.2$$

Near the leading edge, the flow is mainly governed by the blast wave effect of the blunt leading edge and the pressure distribution is similar to that on a flat plate. In this region, there is not significant difference in results obtained by using Cheng's zeroeth order theory and Chernyi's method. At a point downstream, the pressure reaches a minimum and starts increasing. In this region as mentioned earlier, Cheng's theory predicts violent oscillations in pressure distribution about the inviscid value, similar to that found by Stollery (Reference 22) for viscous interaction on a concave surface. These oscillations represent a repeated thinning and thickening of the entropy layer as it approaches the body surface. This oscillatory behaviour is present even for a blunted wedge, but is normally ignored since the amplitude of oscillation is small. However, as we have seen now, the oscillatory behaviour assumes values of unrealistic magnitude as the surface becomes more and more concave. However, when Chernyi's method is used, we observe from figure 39, that the pressure at the first instant slightly overshoots above the inviscid value and then approaches the inviscid value with an undershoot and subsequent heavily damped oscillations. The amplitude of the initial overshoot appears to be a strong function of γ for a given concave surface. In figure 40, results for a family of shapes for $\gamma = 5/3$ are presented. The extent of overshoot is greatly reduced for $\gamma = 5/3$ and the pressure appears to approach the downstream asymptotic value much faster. The reason for the strong damping present in Chernyi's method, as can be seen from the present analysis appears to be the inclusion of the afterbody drag and /due to kinetic energy terms in the analysis. The neglect of these terms as in Cheng's theory is only correct in a small region near the leading edge where the flow is dominated by bluntness. However, far downstream, it is reasonable to expect the after body drag to dominate over the leading edge drag. Guiraud et al (Ref 37) point out that even for the case of zero bluntness, it is necessary to retain the after-body drag term in order to describe how the asymptotic value of pressure is reached. A more rigorous comparison of the present results can be made if higher order approximations are considered in Cheng's theory. This has not been attempted in the present work. However, for the case of a wedge various theories are compared in figure 41 with exact results obtained by the method of characteristics. (Reference 43). As it has been pointed by Mikhailov et al (Reference 19), the difference between Cheng's and Chernyi's method is not significant for a blunted wedge. Far downstream, Chernyi's method approaches the wedge value. For concave surfaces, only the limiting case of $M_\infty \rightarrow \infty$ has been considered and for finite values of M_∞ the observed overshoot may be still very much less.

Now, we will consider the application of the present method to the viscous interaction problem on a flat plate. Using equations 7.1.24, 7.1.25 and 7.1.28 and with the initial conditions given by equations 7.1.29a, b and c, we can determine the pressure, boundary-layer thickness and the shock wave shape by a forward integration, for the given body shape. The solution is dependent on the wall conditions through the integral I_0 , ratio of specific heats γ ,

and the incidence parameter $M_\infty \alpha_s$ for inclined plates or curved surfaces. The variation of the integral I (Reference 78,79) over a range of the pressure gradient parameter β is shown in figure 5. Within the scope of the local similarity approach, we can account for the variation in the value of I from strong to weak interaction region. In figures 42 and 43, the results obtained for a flat plate at zero incidence are shown for $g_w = 0$ and 0.2. These results were obtained by fitting a second degree curve for the boundary-layer parameters I_0 and $\bar{g}_w'/P_w(1-g_w)$ in the range $0 \leq \beta \leq 0.5$. Some computations were made with a 7th degree curve fit over a much wider range of β from -0.3 to 2.0 covering both the favourable and adverse pressure gradient regions, and these results did not differ much from the 2nd degree curve fit results in the range of β variation on a flat plate. However, the use of the higher degree curve was abandoned since it was not effective in extremes of its range. From figure 42, we note a smooth variation in pressure from strong to weak interaction region. The induced pressure is more sensitive to variations in wall temperature since it is directly proportional to I_0 , and increases with increase in wall enthalpy. The present approach affords the shock shape and the boundary-layer thickness to be calculated directly as a part of the problem and the results are shown in figure 43. The shock wave tends to move away from the boundary-layer with increasing downstream distance. Comparison with pressure and heat transfer measurements (Reference 58) of the present approach is shown in figure 14 in dotted lines. The agreement seems to be satisfactory. The improved prediction in heat transfer rate may be due to the inclusion of Prandtl number effects in the analysis. In order to fully evaluate the advantage of the method, more detailed comparison over a range of wall temperatures would be necessary. However, the favourable agreement obtained for the flat plate is encouraging enough to investigate further its application to concave surface flows which happens to be a severe test for any method used in the viscous interaction analysis.

Although, the application of equations 7.1.43 to 7.1.45, to concave surface flows appears to be straight forward, the use of the local similarity method for the boundary-layer analysis presents some difficulties. The boundary-layer on a concave surface is essentially growing in an adverse pressure gradient and is bound to separate whenever the gradients are severe. The actual values of the pressure gradient parameter β , when separation occurs have been studied in detail for various wall conditions by Rogers (Reference 78,79). However, in the present viscous interaction analysis, we know from the results of Chapter VI, that whenever the centrifugal effects are retained in the analysis of the inviscid flow field, the highly oscillatory behaviour of the pressure introduces very much larger pressure gradients than the boundary-layer can withstand. These severe pressure gradients would predict spurious premature separation of the boundary-layer, thus making the further analysis complicated. This difficulty would offset the advantage afforded by the local similarity approach and the resulting solution may be subject to a lot of errors. In order to obviate this difficulty, in studying concave surface flows, we shall use local flat plate similarity approach and assume $I_0 = 1$. This assumption can be

justified only on the basis of cold wall similarity approach. The variation in I from unity for cold walls is about $\pm 30\%$ in the range $-0.3 \leq \beta \leq 0.3$ and the amplitude of oscillation in pressure predicted by Cheng's theory can be an order of magnitude larger than this value. For example, for the concave surface $y \propto x^2$, the maximum value of the first overshoot in pressure is about 3 times the local inviscid value when the leading edge is sharp, and is about 20 times (figure 38) when the leading edge is blunt. This suggests that major difficulties arise from the inviscid flow model used in the analysis and it is doubtful whether effecting further improvements in the boundary-layer analysis would be of any practical use unless the inviscid flow field analysis is significantly improved. It should be borne in mind that even assuming a local similarity approach is subject to errors since the boundary-layer profiles may not be similar in regions of adverse pressure gradient.

The results of computations carried out for a family of shapes $Y_w = X^n$ for $M_\infty \alpha_s = 1$ are shown in figures 44 - 46. Comparing figure 44 with Cheng's theory results (figure 19a), we note that near the leading edge region where the flow is mainly governed by the displacement effect, prediction of both methods are similar. However far downstream where the dominating effect is that due to incidence, the present method using Chernyi's integral method for the inviscid flow field approaches the inviscid conditions very rapidly while Cheng's theory results are highly oscillatory. The pressure rises rapidly after reaching a minimum value, slightly overshoots and again reaches a minimum and then decays rapidly to the inviscid value. For X greater than 5 times the characteristic viscous interaction length scale $A^2 \bar{\chi}^2 \chi$, the difference between the viscous interaction result and the inviscid value for pressure is almost indistinguishable. This behaviour is similar to that observed for the case of nose bluntness (figure 38). From figure 45, it may be observed that for $\gamma = 5/3$, the initial overshoot present for $\gamma = 1.4$ almost vanishes. The pressure after increasing rapidly is sensibly constant for a short distance and starts increasing again. These results show a marked improvement over Cheng's zeroth order theory results. A more direct comparison of various methods is given in figure 46.

In figure 47, the effect of increasing the incidence parameter $M_\infty \alpha_s$ on the pressure distribution is shown. For $M_\infty \alpha_s = 5$, the extent of overshoot appears to increase slightly and the incidence effect sets in much earlier than for $M_\infty \alpha_s = 1$.

The shock-wave shape and the boundary-layer growth on concave surface $Y = X^2$ are shown in figures 48 a and b. The boundary-layer growing from the leading edge first starts increasing in thickness as though developing on a flat plate. At some distance downstream it reaches a maximum value and then thins down to a neck, and starts increasing again. It may be noted from figure 48b that subsequent oscillations after the first neck are highly damped and the boundary-layer thins down very rapidly. The shock wave shape also shows a similar behaviour in the initial portion near the leading edge, but starts increasing rapidly downstream after the formation of

the first neck.

The analysis of the present Chapter shows a significant improvement over the highly oscillatory zeroeth order Cheng's theory results. The analysis can be applied to other shapes such as flat plates at incidence, compression and expansion corners provided the upstream influence is not significant.

PART III EXPERIMENTAL STUDIES, RESULTS,
CONCLUSIONS AND RECOMMENDATIONS

CHAPTER 8

APPARATUS AND TESTS

8.0 Tunnel and test conditions:

The present investigation was conducted in the Imperial college No. 1 gun tunnel. A full description of the facility and its operating ranges is given by Stollery et al (Ref 88) and Needham (Ref 89).

The tunnel basically consists of a long barrel at the end of which a convergent-divergent nozzle is fitted. The other end of the barrel is connected to a drive chamber through a diaphragm section. The drive vessel is pressurised to the required pressure depending on the running conditions by compressed air. When the diaphragm separating the drive vessel and the barrel is burst, a light aluminium alloy piston is accelerated inside the barrel and the working gas (air) between the piston face and the nozzle end is compressed. Because of the multiple reflections of the incident shock between the barrel end and the piston face, the working gas is heated non-isentropically and hence, high stagnation enthalpies are obtained. The compressed gas inside the barrel expands through the nozzle to the designed Mach number. The working section is fitted at the end of the nozzle and the dump chamber is evacuated (< 0.5 mm Hg) prior to a run. A microphone pick-up mounted on the barrel provides the necessary signal for triggering the oscilloscope beams and the Schlieren system.

The tunnel is equipped with a conventional single pass Schlieren system. The spark source stabilised by an argon jet, is triggered through a time delay unit from the instant the tunnel primary diaphragm is burst.

The present tests were conducted in the Mach 12 contoured nozzle, with an exit diameter of 9". The nozzle has been calibrated by Mohammadian (Reference 90). Allowing for the nozzle wall boundary-layer, the useful test core is about 5" diameter at the nozzle exit and 4.5" diameter at a station 8" downstream. The exit flow from the nozzle has a Mach number gradient of approximately 0.015/inch, and the average test section Mach number was found to be 12.25. All the tests were conducted at atmospheric pressure in the barrel. The flow conditions for the tests were

$$A. \quad \text{Flat plate model:} \quad p_o = 1600 \text{ psi}$$

$$T_o = 1300^\circ \text{ K}$$

$$Re_{\text{unit}} = 0.86 \times 10^5 / \text{inch}$$

B. Concave surface model: i) As for the plate model

and ii) $p_o = 800 \text{ psi}$

$T_o = 1037^\circ \text{ K}$

$Re_{\text{unit}} = 0.644 \times 10^5 / \text{inch}$

All the tests were conducted under cold wall conditions with $g_w = 0.22$

8.1 Pressure measurements:

For model static pressure measurements, two types of pressure transducers were employed. For low pressures, Pitran pressure sensitive transistor type transducer was used. This had a linear calibration for $\pm 0.25 \text{ psi}$ with sufficient overload capability permitting use up to 1.5 psi differential using non-linear calibration. Full details of the Pitran transducer and associated instrumentation are described in Reference 91. The transducer was calibrated using a solartron digital volt meter, against a water manometer at frequent intervals during the test programme. For higher pressures, a CEC strain gage type transducer (0-10 psia) was used. This was calibrated against a mercury manometer. The output of the transducers was recorded directly on Tektronix 502A dual beam oscilloscopes equipped with Polaroid land cameras. Allowing for various errors, the overall accuracy of pressure measurements is estimated to be $\pm 5\%$

8.2 Heat transfer measurement:

Heat transfer measurements were made using thin film (platinum) resistance thermometers deposited chemically on the surface of pyrex glass. Because of very low heat capacity, these films respond within a few microseconds to the surface temperature variations. The platinum film heat transfer gage forms one of the arms of a wheatstone bridge and the change in resistance due to rise in surface temperature during a run produces an out-of balance voltage proportional to the change of temperature. This voltage is amplified and the temperature-time history is directly converted to heat transfer rates by an electric analogue circuit. The latter makes use of the analogy between a one-dimensional network, having distributed resistance and capacitance, and the one-dimensional heat flow through a solid material. The heat transfer equipment (comprising the amplifier, analogue network and their associated instrumentation) used in the present experiments is described by Hunter (Reference 92). The overall accuracy of heat transfer measurements is estimated to be about $\pm 10\%$

8.3 Models:

Flat plate model: A sketch of the flat plate model is given in figure 49. The model was 6.5" long and 4" wide with interchangeable leading edge pieces of different tip thickness. The 0.0005" (Sharp leading edge) and 0.0312" thick leading edges had bevels of 34 and 30° respectively. For the largest tip thickness of 0.23", the plate was used without any leading edge piece. The model was instrumented with heat transfer measurement and also had static pressure holes for pressure measurements. The heat transfer insert consisted of a 6" long and 0.75" wide, 1/16" pyrex glass with chemically deposited platinum films (Reference 93) at the locations shown in figure 49. The gauges were constructed by drawing a thin line of liquid bright platinum paint (Hanovia - 05X) using a draughting pen, and allowed to air dry for a short interval in a dust free atmosphere. Following this, the glass was heated, starting from room temperature in an electric oven, to a temperature of 500°C. During this process, the oven door was kept slightly open to allow the volatile gases to escape. The door was then closed and the temperature taken to the softening point of the pyrex glass (680°C) which was allowed to soak at this temperature for 5 minutes. The glass was then removed from the furnace and cooled in open air. Silver paste was painted to the ends of the film to form the current leads, and the glass was then reheated to about 600°C. By this process 24 films were deposited on the pyrex glass at 1/4" interval. The resistance of the films varied from 55 to 80 ohms. Copper leads were then soldered to the ends of the silver paint and the glass insert was flush mounted in the steel plate using peisin wax. The films were calibrated for temperature coefficient of resistance before installing in the model.

Since earlier tests of Mohammadian on concave and convex surfaces (Reference 90) had shown good two-dimensionality, a single model was employed for both heat transfer and pressure tests. The static pressure holes were 0.063 ϕ , to allow pressures to be recorded during the tunnel running time. The pressure holes were blanked with plasticin and also the tubing (projected from the model) covered with one end sealed plastic sleeves. Plate 1 shows the flat plate model mounted on the variable incidence support designed for these tests. The pressure transducers (Pitran) were mounted inside the model support.

To avoid flow around the sides of the model, side plates were used.

Concave surface model: For the concave surface, two separate models for each pressure and heat transfer measurements were fabricated. Both the models had sharp leading edges (0.001" leading edge thickness for the pressure model and 0.0002" for the heat transfer model). The equation of the curve for the concave surface was

$$y_w = 0.082x^2, \quad y_w \text{ and } x \text{ in inches.}$$

* platinum thin film gauges

The shape of the concave surface was determined so that the measurements can be made in a region where the viscous interaction effects and the incidence effects are both significant. This requirement introduced very large turning angles (40°) and hence the dominating effect was due to surface incidence over a considerable portion of the model, except for a short distance downstream of the leading edge. It is quite possible that the large turning angles of the model do not meet with the requirement of slender body theory. However, it was hoped that some reliable measurements can be made in a region near the leading edge, where the body can be considered to be slender. The surface had continuous curvature from the leading edge to the trailing edge.

The location of pressure tapings on the model is shown in figure 50a. As in the case of blunt flat plate model, the pressure holes were sealed when not in use. The same model support fabricated for the flat plate model was used for the concave surface pressure model. The length of the tubing between the transducer (housed inside the model support) and the static pressure holes was kept about 3 - 4". The model mounted on the surface is shown plate 2.

The concave surface heat transfer model is shown in figure 50b and Plate 3. The platinum thin films were deposited on pyrex glass substrate by the technique described earlier in this Section. Pyrex* glass sheets cut to the required size of 4"x4.5"x1/16" were purchased. A steel mould was used for bending the glass sheet. The steel mould was later machined for the pressure model. The glass sheet was bent by allowing it to sag on the mould by heating it to the softening temperature in an electric oven. The holes were then drilled in the glass using an ultrasonic drill at the required locations, for taking the leads. An aluminium plate bent to the same shape with the holes drilled at the desired locations served as a guide for drilling holes in the glass sheet. The platinum films were then deposited by painting with Hanovia 05X and silver connecting leads made by the described earlier. The subsequent heating of the glass sheet after drilling the holes helped to anneal, and remove any residual stresses in the glass near the holes. Copper leads were soldered to the silver paint and the glass sheet was then mounted flush on an aluminium support with a steel leading edge machined to the desired contour. The holes were then blanked with peisin wax. The glass sheet was fixed to the support by peisin wax and the slots in the support were filled with Bees wax so that the leads were held strongly in position. Finally the leads were connected to a Canon 100 pin plug. A photograph of the completed model is shown in Plate 3. */ method*

The accuracy of the contour of the final model was checked by using an optical projector with a magnification of 20. The contour was found to be smooth with no abrupt discontinuities and was found to be within 0.005" of the desired contour. No direct check on the slope of the contour was made.

* Jencons & Co., Scientific Instruments Ltd., Hemel Hempstead.

** Optical Measuring Tools Ltd. (Newall).

8.4 Gauge Calibration:

The heat transfer gauges were calibrated individually for temperature coefficient of resistance and a few sample calibrations were carried out for the ρk for the pyrex glass substrate.

The temperature coefficient of resistance of the films was found by heating the glass insert in a silicone oil bath from room temperature to 60°C , and measuring the change in resistance at suitable intervals. The silicone oil bath was heated in a Gallenkamp water bath. The temperature was maintained uniform by constant stirring. The resistance of the films was measured by passing a current of about 1mA through the gauge and noting the voltage across the gauge on a digital voltmeter with a resolution of $10\text{ }\mu\text{V}$. The temperature coefficient of resistance was determined by this procedure for each film independently and was found to be $\pm 4\%$ of the value $2.3 \times 10^{-3}/^{\circ}\text{C}$. Individual calibrations were found to be very linear, and calibrations are expected to be better than 2% .

For the determination of ρk for the pyrex glass substrate the joule heating method used by Skinner (Ref 94) was adopted. No detailed calibration for all the films was made. Only a couple of films were pulsed in both air and glycerol by applying a step current to the gauge. The value of $(\rho k)^{1/2}$ obtained by this method was found to agree closely with the value of $0.0745\text{ Btu/ft}^2\text{ }^{\circ}\text{F}$ (Ref 92,94). This value has been used in the reduction of the experimental heat transfer data.

CHAPTER 9

EXPERIMENTAL RESULTS AND DISCUSSION

In this Chapter, the experimental results obtained on flat plates and on concave surfaces will be discussed. Comparison of the measurements with the viscous interaction theories has been made. For flat plates, the following theories have been used for comparison.

- (i) Newton Busemann Rule with Lees' boundary-layer theory
- (ii) Tangent Wedge (Mod. Cheng) - , -
- (iii) TW + CFGL with - , -

No comparison with the 2nd order tangent wedge theory has been made, since the difference between modified Cheng using TW rule and the 2nd order TW rule is small because of relatively low centrifugal effects.

To provide a better comparison of various theories, the results are shown plotted on the actual scale rather than log-log. The use of log-log correlation often makes the comparison impressive in spite of bad agreement when a direct comparison is made.

9.1 Flat plate studies:

9.1.1 Sharp flat plate heat transfer rates:

Figure 51 shows the heat transfer rates measured on a sharp flat plate ($Re_x \approx 45$) at zero incidence along with theoretical predictions. In the absence of viscous interaction, the heat transfer rate on a flat plate

$$\dot{q} = \rho U C_p (T_0 - T_\infty) St$$

$$St = 0.332 \left(\frac{C}{Re_x} \right)^{1/2}$$

As can be seen from figure 51, the estimates of the flat plate results are lower near the leading edge, whereas the agreement is better far downstream where the interaction is weak. In the calculation of C , Sutherland's viscosity formula has been used throughout this work, since the same formula has been used for Reynolds number calculation. C is given by

$$C = \frac{\mu(T^*)}{\mu(T_\infty)} \cdot \frac{T_\infty}{T^*}$$

Following Cheng et al (Ref 32), the characteristic temperature T^* was calculated using

$$\gamma^* = \frac{T_0}{6} \left(1 + 3 \frac{T_w}{T_0} \right)$$

The agreement of experimental data with the viscous interaction theory of Reference 22 (Modified Cheng) seems to be good. The difference between modified Cheng and TW + CFGL is very small for the present experimental conditions and hence a separate comparison for the TW + CFGL rule has not been made. This measurement on the sharp flat plate at zero incidence was undertaken to serve as a check on instrumentation (heat transfer) employed for the present tests.

Figures 52a and b show the effect of incidence on heat transfer measurements for two angle of attack 5° and 10° . The good agreement of the modified Cheng and the present TW + CFGL method with experimental data is expected because of the correct wedge limit for the pressure given by two theories. The estimates using Newton-Busemann rule appear to be generally lower. For $\alpha = 10^\circ$ the Newton-Busemann rule seems to agree better near the leading edge.

It appears from the present heat transfer measurement on a sharp flat plate that the estimates of the tangent wedge and TW + CFGL rules are quite satisfactory. The TW + CFGL prediction may give slightly improved predictions in the strong interaction region. However considering the experimental variations, there is little difference between the two methods. *to choose*

9.1.2 Blunt flat plate heat transfer studies:

In figure 53, the heat transfer rates and Stanton number for blunt flat plates at zero incidence are shown, for leading edge thickness of $t = 0.0312''$ and $t = 0.23''$ along with sharp flat plate data. It appears from the results that the heat transfer rates are generally increased by leading edge thickness in the vicinity of the leading edge but far downstream, for $t = 0.0312''$, the values are slightly lower than that for a sharp flat plate. For $t = 0.23''$, the values appear to be higher everywhere compared to the sharp flat plate results. A similar bluntness effect has been found by Stone (Reference 54) and Neal (Reference 55). Their heat transfer data (using calorimetric technique) measured far downstream from the leading edge indicated that progressively blunting the leading edge caused first a decrease and then an increase relative to sharp leading edge values. The present theoretical predictions using modified Cheng theory appears to agree qualitatively with this behaviour, but the extent of decrease in heat transfer rate and the point downstream of which the values start decreasing do not appear to be predicted satisfactorily. The present results suggest that a slight blunting of the leading edge decreases the heat transfer rates downstream and large blunting increases. In order to gain a better understanding of this leading edge bluntness effect, integration of the governing equations for modified Cheng results was carried out over a wide range of the viscous interaction parameter and the results are shown plotted

in figure 54. From the results, it is evident that as the leading edge is progressively blunted, at a point downstream, the heat transfer rates start decreasing relative to the sharp leading edge value. The point at which the values start decreasing appears to move farther away downstream as the leading edge thickness increases. A similar behaviour is also found for the case of the flat plate at incidence (figure 55). So, it appears from the present computations that the decrease in heat transfer rate due to blunting is not restricted to slight blunting, but is true for all leading edge thickness provided measurements can be made sufficiently far downstream.

Regarding the agreement of the present experimental results, it is found that modified Cheng theory estimates are higher and as we approach the leading edge, the difference keeps increasing. The estimates of the Newton-Busemann rule appear to agree well with the experimental results. Far downstream, all the methods appear to estimate the heat transfer satisfactorily for the bluntest leading edge.

The decrease in heat transfer rate for small bluntness was found to be due to an increase in boundary-layer thickness (Stone, Reference 54). In the present tests, no boundary-layer survey has been attempted.

The measured heat transfer rates are shown in figure 56 a-c for angles of attack of 5° , 10° and 15° respectively. In general, as the incidence increases, the bluntness effect gradually diminishes. Considering the experimental accuracy, the results for all the leading edge thickness for $\theta = 15^\circ$, are close over most of the region. At angle of attack of 5° and 10° , for leading edge thickness of $0.0312''$, the results appear to be slightly lower than the corresponding sharp leading edge values. Away from the leading edge, the incidence effect dominates and the heat transfer rates approach wedge values. In figure 56d, the heat transfer rates for the bluntest leading edge are shown for various angles of attack. As the angle of attack increases, the heat transfer rates also increase because of the increased pressure.

Individual comparisons of Stanton number with different theories is shown in figures 57 and 58. The degree of agreement of various theories with experimental results depends on both leading edge thickness and angle of attack. In general, it appears that the tangent wedge approximation estimates give improved comparisons far downstream and remove the oscillatory behaviour. Near the leading edge, where the bluntness effect is important, the estimates appear to be slightly higher. In this region, the Newton-Busemann rule and the present TW + CFGL estimates appear to be better.

Flat plate pressure measurements: The pressure distribution was measured for the bluntest leading edge only, at zero incidence and at an angle of attack of 10° and the results are shown in figures 59 and 60. The pressures induced by the blunt leading edge are much higher compared to the sharp leading edge values (Figure 59). For

this leading edge thickness, the tip Reynolds number is 19940, and it is reasonable to expect the blunt ness effect to dominate over the viscous effects. The predictions of all the theories appear to be higher than the measurements. It appears from the trend of the distribution that further downstream Cheng's theory predictions may be better. The tangent wedge estimates are higher as much as by about 100% and the TW + CFGL estimates are in between Cheng's and tangent wedge estimates. Very near the leading edge, all the theories, fail because the small disturbance requirement is not met.

The prediction of the Newton Busemann rule appears to be good for the angle of attack case.

From the limited experimental data on pressure measurements, it appears that the tangent wedge and TW + CFGL estimates may be very inaccurate for bluntness dominated flows. Perhaps the predictions of the tangent wedge method may be better when both the bluntness and displacement effects are present.

Schlieren photos of the shock wave shape was taken at zero angle of attack and at incidence. Plate 4 shows shock shapes for the bluntest leading edge, and it may be seen that as the incidence increases, the shock is much nearer to the body. A summary of shock wave shape at zero angle of attack is given in figure 61.

9.2 Concave Surface Flow Studies.

The tests on the concave surface were conducted at a Mach number of 12.25 and representative Schlieren photographs are shown in Plates 5, 6 and 7. From Plate 5, we note that the leading edge shock wave first starts moving away from the body, reaches a maximum distance and then comes very close to the body. In this region, there is a significant interaction between the leading edge shock wave and the compression waves generated by the effective body surface. Downstream of this region, a well defined concave shock wave is formed where the compression waves coalesce. Because of the dominating incidence effect, the boundary-layer thins down very rapidly and it is rather difficult from the photographs to clearly define the edge of it. From the photograph it appears that a neck might form in this region before the boundary-layer thins down far downstream. In order to fully appreciate the interaction in this region, it may be necessary to conduct tests at a higher Mach number (or at higher values of the viscous interaction parameter) so that the boundary-layer is quite thick. In the present tests, the value of the viscous interaction length scale ($L_s = A' \bar{\chi}^2 \chi$) is 2.75" for $p_o = 800$ psi. Hence the value of the effective interaction parameter $A' \bar{\chi}$ at a distance of 1" downstream of the leading edge will be 0.6, and $M_o^2 \sqrt{\gamma} / A' \bar{\chi}$ is about 4.5. This indicates that the incidence effects set in very rapidly. In order that the combined effect of displacement and incidence is important, it is necessary that $(M_o^2 \sqrt{\gamma} / A' \bar{\chi}) \gg 1$, and at the same time $A' \bar{\chi} \gg 1$. This condition is not strictly met on the present tests. However, in the design of present tests,

the main objective was to cover, at least, a flow region within available experimental conditions, where both the concave shock wave and the leading edge shock wave are present. The thinning of the boundary-layer to a minimum appears to be rather more clear in Plate 7 taken at an angle of attack of -5° .

Since, the boundary-layer thickness is rather thin and is difficult to measure accurately from the photograph, comparison with theoretical predictions has not been made. However, the shock shape can be taken as an indication of the boundary-layer as shown by figure 48b, where the formation of the neck in the boundary-layer is reflected in the shock-wave shape also. A comparison of figure 48b, with plate 5, indicates at least qualitatively the formation of the first neck in the shock-wave shape. It should be noted that it may be possible that the transformation of the convex leading edge shock wave to the downstream concave shockwave may be much more smooth than that suggested by figure 48b, for finite Mach number. Hayes and Probestein (Reference 36) mention that an alternative choice due to Lunay (Not referred to by the present author) of the basic assumptions in the integral methods gives an equation similar to Chernyi's method, but with the kinetic energy term doubled. Using this method for the inviscid analysis, it may be possible that the oscillations are damped much faster than those suggested by the results of Chapter 7.

It has not been possible to account for the normal density gradient shown by a black line in the region of the neck formation. The term 'neck' has been used with the restricted definition to denote the region where the curvature of the leading edge shock wave changes from convex to concave, if only one shock is assumed to be present in the flow field. A detailed boundary-layer and shock-layer traverse in the neck region at higher values of the viscous interaction parameter might throw some light on this aspect.

From Plate 5, it may be observed that near the trailing edge, the flow seems to be rather disturbed for some unknown reason. To see whether the nozzle outlet flow had any effect on this disturbance, a Schlieren photograph was taken with the front half of the model well inside the nozzle. The flow feature was still the same. However, since the primary interest in the present tests was to investigate the formation of the neck region, attention has been paid to the flow upstream of the disturbance. In this region the flow appears to be satisfactory as can be judged from Schlieren photographs.

The slight differences between Plate 5 and 6 is perhaps due to different leading edge conditions of the respective heat transfer and pressure models, and variations in Schlieren setting.

In figure 62, the heat transfer measurements, at a free stream Reynolds number of 0.644×10^5 and $M = 12.25$ are shown for zero incidence and for a negative angle of attack of 5° . No measurement was taken at positive angles of attack, since the model blocked the tunnel flow even the diffuser was moved well forward. For $\alpha = 0^\circ$, heat transfer rate is sensibly constant for a short distance downstream of the leading edge and then rises very rapidly. After reaching

a maximum, at a distance of about $\frac{3}{4}$ chord length, there is a drop in the heat transfer rates. This drop seems to be closely associated with the disturbed region discussed earlier. The tests were conducted with and without side plates. The side plates were used to avoid spillage from sides. The side plates extended above the shock wave over almost the entire length of the model. Of course, this obstructed the Schlieren flow visualization. However from the heat transfer measurements with and without sideplates we see that over most of the model length, the two measurements agree very well and further, the drop in heat transfer rate near the trailing edge seems to be less with sideplates than without. This suggests that the flow quality may be better with sideplates and there is no reason to suspect extraneous interference from the sideplates.

Hence, all the tests were conducted with side plates except when taking Schlieren pictures.

For negative angle of attack of 5° , the trend of the heat transfer distribution is similar to that at zero incidence. However, the region of nearly constant heat transfer rate is extended further downstream of the leading edge, and the position of peak heat transfer rate is also shifted downstream. This is to be expected since the incidence effects set in more slowly for negative angle of attack and the flow is like on a flat plate over a longer length of the model from the leading edge. *that*

In comparing the measurements, with theoretical predictions, only the viscous interaction theories discussed in Part I, have been used. Other standard methods (such as Lees Reeves) for compressible boundary layers with known pressure distribution have not been used. This is not because the latter methods are not applicable, but the main interest was to study the viscous-inviscid coupling without ignoring the centrifugal effects.

In figure 63, the measured heat transfer distributions on the concave surface at zero incidence are compared with theoretical results. The modified Cheng theory agrees very well with the measurements for about 40% of the chord from the leading edge. Downstream of this region where the heat transfer is rising rapidly, the agreement appears to be rather poor. The reason may be partly due to simplifying assumption in the boundary-layer analysis. The TW + CFGL results are oscillatory. Though, the TW + CFGL results are less oscillatory compared to Cheng's as seen from Chapter 6, the rise in heat transfer rate appears to be predicted too early. The subsequent oscillations in Cheng's or in TW + CFGL theories stem mainly from the limitations of theory, in not accounting for other higher order effects. This is demonstrated by the analysis of Chapter 7 using Chernyi's Integral approach

The present method of Chapter 7 using Chernyi's approach for the viscous interaction analysis appears to predict the rapid rise in heat transfer rate fairly well. The subsequent drop in heat transfer rate after reaching the maximum value appears to be rather large compared to the experimental observations. However, a rigorous comparison cannot be made in this region since the measured distribution may be subject to trailing edge effects and also the Schlieren flow visualization (Plate 5) indicated a rather disturbed flow in this region. Provided, the inviscid flow field analysis is improved further

the drop predicted by the theory may be modified.

In general it appears that the experimental results are generally higher on concave surface. The reason for this is perhaps due to effects:

(i) The strong concave shock wave introduces in the flow behind, high entropy gradients and hence vorticity. This effect is also present in the strong interaction of a flat plate at zero incidence, and can give rise to increased heating (discussed in Chapter 2). Since the downstream shockwave can be much stronger than the leading edge shock wave, the vorticity heating effect may be more severe.

(ii) Another effect which may be worth investigating is the centrifugal instability of flow over concave surfaces. At very high speeds of the external flow, the flow may be centrifugally unstable. This instability is found to generate vortices parallel to the main stream which are called Gortler vortices (Ref 80). It is quite possible that these vortices may increase the heat transfer rates. Thomman (Ref 95) when studying turbulent boundary-layers on concave surfaces found a similar increase in heat transfer due to curvature effects.

In figure 64 and 65, the measured pressure distribution on the concave surface at a free stream Reynolds number of 0.64×10^5 and 0.86×10^5 respectively, are shown and are compared with viscous interaction theories.

The measured distribution (pressure) is sensibly constant for a short distance downstream from the leading edge, then starts increasing rapidly. However, after some distance downstream, the rate of increase is lowered and the pressure distribution has a kink in this region before increasing again. The region where the kink appears in the pressure distribution appears to be very close to the region of 'neck' formation in the shock wave (figure 65). The pressure distribution shown in figure 65 at higher Reynolds number was conducted with the view that the higher pressures on the model can be measured accurately.

In contrast to heat transfer measurements where the measured heat transfer rates were higher than the theoretical predictions, the pressures appear to be lower considerably in the region downstream of the kink. The modified Cheng agrees well with the measured distribution over a significant portion of the model. The trend of the pressure distribution is predicted well in the region near the leading edge by the present method using Chernyi's integral approach. Though this method gives improved results compared with Cheng's theory, still the quantitative agreement is not very satisfactory, with the experimental results. Of the various theories considered, it appears that the modified Cheng estimates for pressure appear to be superior.

CHAPTER 10: CONCLUSION

The problem of leading edge bluntness and boundary-layer displacement effect has been studied by modifying the tangent wedge rule for centrifugal effects. The analysis is applied to flat plates and concave surfaces. On concave surfaces, these methods predict a mild oscillatory behaviour compared to the highly oscillatory results of Cheng's theory. Further, by adopting an integral approach (Chernyi's), it is shown that consideration of afterbody drag and other higher effects heavily damp the oscillations present in Cheng's theory.

Experimental studies on flat plates at $M=12.25$ indicated

- (i) On sharp plates, the heat transfer rates appear to be predicted satisfactorily by both the modified Cheng and TW + CFGL methods.
- (ii) On blunt plates, the heat transfer rates agreed well with the modified Cheng theory far downstream. In regions where bluntness is important the modified Cheng appears to overestimate. In this region either Newton Busemann or TW+ CFGL happens to be a good approximation.
- (III) From the limited pressure measurements made, it appears that the Newton- Busemann is a good approximation when considering cases of large bluntness.
- (iv) At zero angle of incidence, *with a blunt leading edge ($t=0.0312$)* the heat transfer rate appeared to decrease far downstream relative to the sharp leading edge values. This behaviour was qualitatively confirmed by theoretical calculations using modified Cheng theory.

Experiments conducted on the concave surface indicated

- (i) The Schlieren photograph of the flow suggested the possibility of the formation of a neck in shock shape in the region, where the leading edge shock wave approaching the wall, merges with the compression waves generated by the effective body surface. However, further confirmation of this behaviour from experiments conducted at higher values of the viscous interaction parameter is required.
- (ii) Of the various theories, the modified Cheng appears to predict satisfactorily the pressures on the concave surface. The heat transfer predictions of the present method using Chernyi's integral approach, agree well in the region of rapid rise in heat transfer.

CHAPTER 11: RECOMMENDATIONS

To evaluate the correctness of the various methods used in viscous interaction analysis of the concave surface flows, exact calculations using the method of characteristics for the inviscid flow field analysis would be very helpful. Further detailed study of the inviscid flows on concave surfaces with nose bluntness is required.

In order to study the rather vaguely defined neck region observed in the present investigation, experiments at a higher Mach number or at higher values of the viscous interaction parameter need to be conducted.

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APPENDIX AGOVERNING EQUATIONS FOR $M_{\infty} \alpha_s \rightarrow \infty$

The equations 4.2.17 - 19, can be written as

$$P (y_e - \delta^{-1} x^n) = \left[\int P dx \right]^{1/2} \quad A-1$$

$$P = 1 + r y_e'^2 \left[\frac{r+1}{4} + \left\{ \left(\frac{r+1}{4} \right)^2 + \frac{1}{y_e'^2} \right\}^{1/2} \right] + r y_e y_e'' \quad A-2$$

where $\delta = (M_{\infty} \alpha_s)^{-1}$

In order to determine the stretching factors, we shall write the variables as

$$\begin{aligned} P &= p_i \delta^{n_1} \\ y &= y_i \delta^{n_2} \\ x &= x_i \delta^{n_3} \end{aligned}$$

where p_i, y_i are the new dependent variables and x_i the independent variable and n_1, n_2 and n_3 are to be determined. Substituting in equations 3-1 and 3-2, we get

$$p_i \delta^{n_1} (y_i \delta^{n_2} - \delta^{n_3-1} x_i^n) = \delta^{\frac{n_1+n_3}{2}} \left[\int p_i dx_i \right]^{1/2} \quad A-3$$

$$\begin{aligned} p_i \delta^{n_1} &= 1 + r \delta^{2(n_2-n_3)} y_i' \left[\frac{r+1}{4} + \left\{ \left(\frac{r+1}{4} \right)^2 + \frac{\delta^{-2n_2 n_3}}{y_i'^2} \right\}^{1/2} \right] \\ &+ r y_i y_i'' \delta^{2(n_2-n_3)} \end{aligned} \quad A-4$$

In order to retain all the significant terms, we get by equating the powers of δ

$$n_2 = n n_3 - 1 \quad A-5a$$

$$n_1 + n_2 = \frac{n_1 + n_3}{2} \quad A-5b$$

$$n_1 = 2(n_2 - n_3) \quad A-5c$$

Solving for n_1 , n_2 and n_3 , we get

$$n_1 = -\frac{2}{4n-3}; \quad n_2 = \frac{3}{4n-3}; \quad n_3 = \frac{4}{4n-3}$$

Using these values of n_1 , n_2 and n_3 , the equations B_1 and B_2 can be written as

$$p_i (y_i - \kappa_i^n) = \left[\int p_i d\kappa_i \right]^{1/2} \quad A-6$$

$$p_i = \left(\frac{1}{M_{ss}} \right)^{\frac{2}{4n-3}} + r y_i'^2 \left[\frac{r+1}{4} + \left\{ \left(\frac{r+1}{4} \right)^2 + \frac{1}{(M_{ss})^{2/4n-3}} \cdot \frac{1}{y_i'^2} \right\}^{1/2} \right] \quad A-7$$

$$+ r y_i y_i''$$

The values of n_1 , n_2 and n_3 for different values of

n are

n	n_1	n_2	n_3
1	-2	3	4
2	-2/5	3/5	4/5
3	-2/9	3/9	4/9

APPENDIX B:RELATION BETWEEN LENGTH SCALES L_c AND L_s :

The surface is given in the non-dimensional form as

$$Y_w = X^n \quad \text{or} \quad Z_w = \zeta^n \quad \text{B-1}$$

where

$$Y_w = \frac{y_w}{\alpha_s h_s} \quad X = x/h_s$$

$$Z_w = y_w/\alpha_c h_c \quad \zeta = x/h_c$$

In the physical plane, the two equations B-1 represent the same surface

$$y_w = \text{Constant} \cdot x^n \quad \text{B-2}$$

It must be noted that the value of α is dependent on the length scale L . The subscripts c and s denote the length scales used in Cheng's analysis and in the modified Cheng method (Stollery, Reference 22). From B-1 and B-2, substituting for Y_w , Z_w , X and ζ , we obtain

$$\alpha_c h_c^{1-n} = \alpha_s h_s^{1-n} = \text{Constant}$$

$$\text{or} \quad \frac{M_\infty \alpha_c}{M_\infty \alpha_s} = \left(\frac{h_c}{h_s} \right)^{n-1} \quad \text{B-3}$$

Only for the case of a flat plate at incidence ($n=1$), the definition for $M_\infty \alpha$ will be same in both cases.

From Section 4.1, we have

$$L_c = \frac{A^2 \bar{\chi}^2 \kappa}{4 \gamma M_\infty^4 \alpha_c^4}$$

$$\text{and} \quad L_s = A^2 \bar{\chi}^2 \kappa$$

$$\text{or} \quad \frac{h_c}{h_s} = \frac{1}{4 \gamma M_\infty^4 \alpha_c^4} \quad \text{B-4}$$

Substituting for L_c/L_s in equation B-3, we get

$$\frac{M_{\omega d_c}}{M_{\omega d_s}} = \left(\frac{1}{4 \gamma M_{\omega d_c}^4} \right)^{n-1}$$

$$\text{or } M_{\omega d_c} = \left(\frac{1}{4 \gamma} \right)^{\frac{n-1}{4n-3}} \cdot (M_{\omega d_s})^{\frac{1}{4n-3}} \quad \text{B-5}$$

$$\text{or } M_{\omega d_s} = (4 \gamma)^{n-1} \cdot (M_{\omega d_c})^{4n-3} \quad \text{B-6}$$

$$\text{For } n = 1, \quad \text{for all } \gamma, \quad M_{\omega d_c} = M_{\omega d_s}$$

$$n = 2, \quad \text{for } \gamma = 1.4, \quad M_{\omega d_c} = 0.708 M_{\omega d_s}$$

$$n = 3, \quad \text{for } \gamma = 1.4, \quad M_{\omega d_c} = 0.682 M_{\omega d_s}$$

APPENDIX C1) Evaluation of Integral I_1

$$I_1 = \int \frac{dt}{F(t)}$$

$$F(t) = \frac{\frac{k}{t} - 1}{r \left[1 + \frac{r+1}{2r} \left(\frac{k}{t} - 1 \right) \right]^{1/2}} - M_\infty \theta \quad \text{C-1}$$

Putting $p^2 = 1 + \frac{r+1}{2r} \left(\frac{k}{t} - 1 \right)$

We have $\frac{k}{t} - 1 = \frac{2r}{r+1} (p^2 - 1)$

and $F(t) = \frac{2}{r+1} \frac{p^2 - 1}{p} - M_\infty \theta$

Substituting for $F(t)$, I_1 can be written as

$$\frac{2}{r+1} I_1 = \int \frac{p dt}{p^2 - k_2 p - 1} \quad \text{C-2}$$

where $k_2 = M_\infty \theta \cdot \frac{r+1}{2}$

By partial fractions, the integrand can be simplified and C - 2 can be written as

$$\frac{2I_1}{r+1} = \int \frac{dt}{b_0 - b_1} \left[\frac{b_0}{p - b_0} - \frac{b_1}{p - b_1} \right] \quad \text{C-3}$$

where $b_1 + b_0 = k_2$

$$b_0 b_1 = -1$$

or
$$\frac{2(b_1 - b_0) I_1}{r+1} = I_{11}(p, b_1) - I_{11}(p, b_0) \quad \text{C-4}$$

where
$$I_{11} = \int \frac{b}{p-b} dt$$

Now, $I_{11}(p, b)$ can be integrated. To facilitate further integration I_{11} is written as

$$I_{11}(p, b) = \int \frac{b dt}{p-b} = \int \frac{dt}{dp} \frac{b db}{p-b} \quad \text{C-5}$$

Integrating C-5 by parts, and substituting for t , we get

$$\frac{1}{b} I_{11}(p, b) = \frac{t}{p-b} + \int \frac{t db}{(p-b)^2}$$

$$\frac{2r}{r+1} \frac{I_{11}(p, b)}{b k} = \frac{1}{(p-b)(p^2 - a^2)} + E_1(p, a, b) \quad \text{C-6}$$

where

$$E_1(p, a, b) = \frac{1}{2a} \left[-E(p, a, b) + E(p, -a, b) \right] \quad \text{C-7}$$

$$E(p, a, b) = \int \frac{dp}{(p+a)(p-b)^2} \quad \text{C-8}$$

$$a^2 = \frac{r-1}{2r}$$

The integral $E(p, a, b)$ can be evaluated easily in closed form as

$$\begin{aligned}
 E(p, a, b) &= \frac{1}{(a+b)^2} \int \left\{ \frac{1}{p+a} - \frac{1}{p-b} + \frac{a+b}{(p-b)^2} \right\} dp \\
 &= \frac{1}{(a+b)^2} \log \frac{p+a}{p-b} - \frac{1}{(a+b)(p-b)} \quad \text{C-9}
 \end{aligned}$$

Therefore, I_{11} can be written as

$$\frac{\gamma+1}{2\gamma} \frac{I_{11}(p, b)}{bK} = \frac{1}{(p-b)(p^2-a^2)} + \frac{1}{2a} \left[-E(p, a, b) + E(p, -a, b) \right] \quad \text{C-10}$$

Substituting for $I_{11}(p, b)$, in C-4, we get

$$\begin{aligned}
 I_1 &= \frac{K(\gamma+1)^2}{4\gamma(b_1-b_0)} \left[\frac{b_1}{(p-b_1)(p^2-a^2)} - \frac{b_0}{(p-b_0)(p^2-a^2)} \right. \\
 &\quad + \frac{b_1}{2a} \left\{ E(p, -a, b_1) - E(p, a, b_1) \right\} \\
 &\quad \left. - \frac{b_0}{2a} \left\{ E(p, -a, b_0) - E(p, a, b_0) \right\} \right] \quad \text{C-11}
 \end{aligned}$$

where

$$a^2 = \frac{\gamma-1}{2\gamma}$$

$$k_2 = \frac{r+1}{2} M_\infty \theta$$

$$b_{0,1} = \frac{1}{2} \left[k_2 \pm (k_2^2 + 4)^{1/2} \right]$$

$$p = \left[1 + \frac{r+1}{2r} \left(\frac{k}{t} - 1 \right) \right]^{1/2}$$

$$\frac{k}{t} = P = \frac{p_w}{p_\infty}$$

and $E(p, a, b)$ is given by equation C-9.

2) Evaluation of $\int P \, dx$ for Tangent wedge approximation

$$\text{Let } I_2 = \int P \, dx$$

From equation 5.2.14, I_2 can be written as

$$I_2 = \int \frac{k}{t} \frac{dx}{dt} dt = \int \frac{k}{t F(t)} dt \quad \text{C-12}$$

$$\text{where } F(t) = \frac{k/t - 1}{r \left[1 + \frac{r+1}{2r} \left(\frac{k}{t} - 1 \right) \right]^{1/2}} - M_\infty \theta$$

$$\text{with } p^2 = 1 + \frac{r+1}{2r} \left(\frac{k}{t} - 1 \right)$$

we have,
$$\frac{k}{t} = \frac{2r}{r+1} \left(p^2 - \frac{r-1}{2r} \right) \quad \text{C-13}$$

and
$$F(t) = \frac{2}{r+1} \frac{p^2-1}{p} - M_\infty \theta \quad \text{C-14}$$

$$= \frac{2}{(r+1)p} (p-b_0)(p-b_1) \quad \text{C-15}$$

where

$$k_2 = \frac{r+1}{2} M_\infty \theta; \quad b_{0,1} = \frac{1}{2} \left[k_2 \pm (k_2^2 + 4)^{1/2} \right] \quad \text{C-16}$$

From equation C-13, by differentiation and substituting for t , we get

$$\frac{k}{t} dt = - \frac{2pk}{p^2-a^2} dp; \quad a^2 = \frac{r-1}{2r} \quad \text{C-17}$$

Substituting C-15 and C-17, in C-12

$$I_2 = -k(r+1) \int \frac{p^2}{p^2-a^2} \frac{dp}{(p-b_0)(p-b_1)} \quad \text{C-18}$$

By partial fractions of the integrand, C-18 can be written as

$$- \frac{(b_1-b_0) I_2}{(r+1)k} = b_1 I_{21}(p, a, b_1) - b_0 I_{21}(p, a, b_0) \quad \text{C-19}$$

where
$$I_{21}(p, a, b) = \int \frac{p dp}{(p^2-a^2)(p-b)} \quad \text{C-20}$$

$$\begin{aligned}
 \text{or } I_{21} &= \int \left\{ \frac{1}{p-a} + \frac{1}{p+a} \right\} \frac{dp}{2(p-b)} \\
 &= \frac{1}{2} \left[E_2(p, -a, b) + E_2(p, a, b) \right] \quad \text{C-21}
 \end{aligned}$$

$$\begin{aligned}
 \text{where } E_2(p, a, b) &= \int \frac{dp}{(p+a)(p-b)} \\
 &= \frac{1}{a+b} \log \frac{p-b}{p+a} \quad \text{C-22}
 \end{aligned}$$

therefore

$$I_{21} = \frac{1}{2} \left[\frac{1}{a+b} \log \frac{p-b}{p+a} + \frac{1}{-a+b} \log \frac{p-b}{p-a} \right] \quad \text{C-23}$$

Substituting C-23 in C-19, we get

$$\begin{aligned}
 I_2 &= \frac{k(r+1)}{b_1-b_0} \left[\frac{b_0}{2} \left\{ \frac{1}{a+b_0} \log \frac{p-b_0}{p+a} + \frac{1}{-a+b_0} \log \frac{p-b_0}{p-a} \right\} \right. \\
 &\quad \left. - \frac{b_1}{2} \left\{ \frac{1}{a+b_1} \log \frac{p-b_1}{p+a} + \frac{1}{-a+b_1} \log \frac{p-b_1}{p-a} \right\} \right]
 \end{aligned}$$

where

$$\begin{aligned}
 a &= \frac{r-1}{2r} ; \quad b_{0,1} = \frac{1}{2} \left[k_2 \pm (k_2^2 + 4)^{1/2} \right] \\
 \phi &= \left[1 + \frac{r+1}{2r} \left(\frac{k}{t} - 1 \right) \right]^{1/2} ; \quad \frac{k}{t} = P = \frac{\dot{p}_w}{\dot{p}_\infty} \quad \text{C-24}
 \end{aligned}$$

3) Evaluation of Integral I_3

$$I_3 = \int \frac{dt}{F_1(t)}$$

$$F_1(t) = \left(\frac{k}{t} - 1 \right)^{1/2} - M_\infty \theta r^{1/2} \quad \text{C-25}$$

Substituting $\chi = \left(\frac{k}{t} - 1 \right)^{1/2} \quad \text{C-26}$

and $a_1 = M_\infty \theta r^{1/2}$

$$I_3 = \int \frac{dt}{d\chi} \cdot \frac{d\chi}{\chi - a_1}$$

Integrating by parts

$$I_3 = \frac{t}{\chi - a_1} + \int \frac{t d\chi}{(\chi - a_1)^2}$$

from C-26 $t = \frac{k}{1 + \chi^2} \quad \text{C-27}$

therefore

$$I_3 = \frac{t}{F_1(t)} + k I_{31} \quad \text{C-28}$$

where $I_{31} = \int \frac{d\chi}{(1 + \chi^2)(\chi - a_1)^2} \quad \text{C-29}$

I_{31} can be integrated in closed form by splitting the integrand into partial fractions

$$I_{31} = \int \left[\frac{2a_1}{(1+a_1^2)^2} \frac{x}{(1+x^2)} + \frac{a_1^2-1}{(a_1^2+1)^2} \frac{1}{1+x^2} - \frac{2a_1}{(1+a_1^2)^2} \frac{1}{x-a_1} + \frac{1}{(1+a_1^2)} \frac{1}{(x-a_1)^2} \right] dx$$

or by integration

$$I_{31} = \frac{a_1}{(1+a_1^2)^2} \log \frac{1+x^2}{(x-a_1)^2} + \frac{1}{(1+a_1^2)} \left[-\frac{1}{x-a_1} + \frac{a_1^2-1}{a_1^2+1} \tan^{-1} x \right]$$

C-30

Substituting C-30 in C-28, and putting $1+x^2 = \frac{k}{t}$ and $x-a_1 = F_1(t)$

$$I_3 = \frac{t}{F_1(t)} + \frac{ka_1}{(1+a_1^2)^2} \log \frac{k/t}{F_1^2(t)} + \frac{k}{1+a_1^2} \left[-\frac{1}{F_1(t)} + \frac{a_1^2-1}{a_1^2+1} \tan^{-1} \left(\frac{k}{t} - 1 \right)^{1/2} \right] \quad \text{C-31}$$

$$F_1(t) = \left(\frac{k}{t} - 1 \right)^{1/2} - M_\infty \theta \gamma^{1/2} ; \quad a_1 = M_\infty \theta \gamma^{1/2}$$

$$\text{For } t \rightarrow 0, \quad \frac{t}{F_1(t)} \sim k^{-1/2} t^{3/2} \rightarrow 0$$

$$\frac{k/t}{F_1^2(t)} \sim 1$$

$$\frac{1}{F_1(t)} \rightarrow 0$$

$$\therefore \lim_{t \rightarrow 0} I_3 = \frac{k(a_1^2-1)}{(1+a_1^2)^2} \cdot \frac{\pi}{2}$$

4) Evaluation of $\int P dx$ for Newtonian rule : From equation 5.2.23

$$\int P dx = r^{1/2} I_4 = r^{1/2} \int \frac{k/t}{F_1(t)} dt \quad \text{C-32}$$

where $F_1(t) = \left(\frac{k}{t} - 1\right)^{1/2} - M_\infty \theta r^{1/2}$

writing $\chi = \left(\frac{k}{t} - 1\right)^{1/2}$ and $a_1 = M_\infty \theta r^{1/2}$, equation C-32 becomes

$$I_4 = - \int \frac{2 K \chi}{(1+\chi^2)(\chi-a_1)} d\chi$$

By splitting into partial fractions and integrating, we get

$$I_4 = \frac{2K}{1+a_1^2} \left[\frac{a_1}{2} \log \frac{1+\chi^2}{(\chi-a_1)^2} - \tan^{-1} \chi \right] \quad \text{C-33}$$

or

$$I_4 = \frac{2K}{(1+a_1^2)} \left[\frac{a_1}{2} \log \frac{k/t}{F_1^2(t)} - \tan^{-1} \left(\frac{k}{t} - 1\right)^{1/2} \right] \quad \text{C-34}$$

$$\lim_{t \rightarrow 0} I_4 = - \frac{2K}{(1+a_1^2)} \frac{\pi}{2} \quad \text{C-35}$$

APPENDIX DASYMPTOTIC BEHAVIOUR OF THE SOLUTION FOR A BLUNTED WEDGE.

The governing equations can be written as

$$(Z - \ell_c) \frac{P}{r M_\infty^2 \theta^2} = 1 \quad D-1$$

$$\frac{P}{r M_\infty^2 \theta^2} = \frac{r+1}{2} Z'^2 + \sigma Z' Z'' \quad D-2$$

where

$$Z = y_e / \ell_c ; \quad \ell_c = x \theta / \ell_c$$

$$\ell_c = \frac{r-1}{4} \frac{C_D t}{\theta^2} ; \quad \sigma = 1 - \frac{r^2-1}{r}$$

For $\ell_c \rightarrow \infty$, the shock slope tends towards the value for a sharp wedge and in view of this, we can seek a solution of the form

$$Z = A \ell_c + B + y_1(\ell_c) + \dots \quad D-3$$

The function y_1 in D-3 represents the higher correction and in the following analysis terms of order y_1^2 are ignored. By substituting D-3, in D-1 and D-2 the value of constants A and B, and the governing equation for the function y_1 are obtained as follows.

$$Z' = A + y_1'$$

$$Z'^2 = A^2 + 2A y_1' + O(y_1'^2)$$

$$Z Z'' = (A \ell_c + B) y_1'' + O(y_1'^2)$$

Substituting for $\frac{P}{r M_\infty^2 \theta^2}$, Z'^2 and $Z Z''$ in D-2, we get

$$[\ell_c(A-1) + B + y_1 + \dots] \left[\frac{r+1}{2} (A^2 + 2A y_1') + \sigma (A \ell_c + B) y_1'' + O(y_1'^2) \right] = 1 \quad D-4$$

Equating terms of order ℓ_c , 1 and y_1 on both sides, we get

$$A = 1, \quad B = \frac{2}{r+1}$$

$$\sigma \ell_c \frac{d^2 y_1}{d \ell_c^2} + 2 \left(\frac{r+1}{2} \right) \frac{dy_1}{d \ell_c} + \left(\frac{r+1}{2} \right)^2 y_1 = 0$$

$$\text{or } \ell_c \frac{d^2 y_1}{d \ell_c^2} + \frac{2k}{\sigma} \frac{dy_1}{d \ell_c} + \frac{k^2}{\sigma} y_1 = 0 \quad \text{D-5}$$

$$\text{where } k = \frac{r+1}{2}$$

The equation D-5 is of the form

$$\frac{d^2 y_1}{d \kappa^2} + \left(a + \frac{b}{\kappa} \right) \frac{dy_1}{d \kappa} + \left(\alpha + \frac{\beta}{\kappa} + \frac{\gamma}{\kappa^2} \right) y_1 = 0 \quad \text{D-6}$$

with $a = 0$, $\alpha = 0$, $b = 2k/\sigma$ and

$$\beta = k^2/\sigma ; \gamma = 0$$

The solution of equation D-6 can be written as (Ref 83)

$$y_1 = \kappa^{1/2 - b/2} e^{-a\kappa/2} C_\gamma \left(\xi \right) \quad \text{D-7}$$

where $C_\gamma \left(\xi \right)$ is any solution of

$$\xi^2 \frac{d^2 y}{d \xi^2} + \xi \frac{dy}{d \xi} + (\xi^2 - \gamma^2) y = 0 \quad \text{D-8}$$

$$\xi = 2 \left[\left(\beta - \frac{ab}{2} \right) \kappa \right]^{1/2} \quad \text{D-9}$$

$$\gamma = \left[(b-1)^2 - 4\gamma \right]^{1/2} \quad \text{D-10}$$

Substituting for a , b , α , β and γ

$$\xi = \frac{2k}{\sqrt{\sigma}} \kappa^{1/2}$$

$$\gamma = \left(\frac{2k}{\sigma} - 1 \right)^{1/2}$$

Therefore

$$y_1 = \ell_c^{1/2 - \frac{k}{\sigma}} C_\gamma \left(\frac{2k}{\sqrt{\sigma}} \ell_c^{1/2} \right) \quad \text{D-11}$$

The general solution of Bessel's differential equation D-8 can be written as

$$y = C_1 J_2(\xi) + C_2 Y_2(\xi)$$

D-12

where C_1 and C_2 are arbitrary constants.

For $\xi \rightarrow \infty$, J_2 and Y_2 behave as (Ref 84)

$$\lim_{\xi \rightarrow \infty} J_2(\xi) = \frac{\cos(\xi - \pi/4 - 3\pi/2)}{\sqrt{\pi\xi/2}} \quad \text{D-13}$$

$$\lim_{\xi \rightarrow \infty} Y_2(\xi) = \frac{\sin(\xi - \pi/4 - 3\pi/2)}{\sqrt{\pi\xi/2}} \quad \text{D-14}$$

Substituting for $J_2(\xi)$ and $Y_2(\xi)$ in D-12, we get

$$y \approx \left(\frac{2}{\pi\xi}\right)^{1/2} \left[C_1 \cos(\xi - \pi/4 - 3\pi/2) + C_2 \sin(\xi - \pi/4 - 3\pi/2) \right]$$

$$\text{or } y \approx \left(\frac{2}{\pi\xi}\right)^{1/2} A_1 \cos(\xi + B_1) \quad \text{D-15}$$

The constants A_1 and B_1 cannot be determined since they depend on the initial conditions. Substituting $\xi = \frac{2k}{\sqrt{\sigma}} \zeta_c^{1/2}$ in D-15

$$y_1 = \zeta_c^{-1/4} A \cos\left(\frac{2k}{\sqrt{\sigma}} \zeta_c^{1/2} + B\right) + \dots \quad \text{D-16}$$

where A and B are constants. Using equations D-11 and D-16, the asymptotic solution for y_e can be written as

$$Z = \zeta_c + \frac{2}{\gamma+1} + \zeta_c^{1/4 - \frac{k}{\sigma}} A \cos\left(\frac{2k}{\sqrt{\sigma}} \zeta_c^{1/2} + B\right) + \dots \quad \text{D-17a}$$

The equation D-17a clearly illustrates the oscillatory behaviour in the shock shape and hence in the pressure from equation D-1.

$$\frac{p}{\gamma M_\infty^2 \theta^2} = \frac{1}{Z - \zeta_c} \approx \frac{\gamma+1}{2} \left[1 - \frac{\gamma+1}{2} \zeta_c^{1/4 - \frac{k}{\sigma}} A \cos\left(\frac{2k}{\sqrt{\sigma}} \zeta_c^{1/2} + B\right) + \dots \right]$$

Particular Cases:

D-17b

(i) Newton-Busemann Rule: This corresponds the Cheng's analysis and can be obtained by putting $\gamma = 1$ in equations D-2 and D-17b

$$\text{For } \gamma = 1; \quad k = 1, \quad \sigma = 1$$

Therefore
$$\frac{P}{\gamma M_{\infty}^2 \theta^2} \approx 1 - \frac{1}{\zeta_c^{3/4}} A \cos(2 \zeta_c^{1/2} + B) + \dots \quad D-18$$

The equation D-18 is the same as obtained by Cheng et al (Ref 32).

(ii) TW + CFGL Formula: By putting $\sigma = 1$, in equations D-2 and D-17 the resulting equation will hold good for TW + CFGL formula.

$$\frac{P}{\gamma M_{\infty}^2 \theta^2} \approx \frac{\gamma+1}{2} \left[1 - \frac{\gamma+1}{2} \zeta_c^{1/4 - \frac{\gamma+1}{2}} A \cos\left\{(\gamma+1)\zeta_c^{1/2} + B\right\} + \dots \right] \quad D-19$$

Comparing D-18 and D-19, it may be noted for $\gamma > 1$, the amplitude of the oscillation is damped much faster with the TW + CFGL approximation than in Cheng's analysis.

(iii) 2nd order Tangent wedge: In this case the oscillatory behaviour depends on the value of σ given by

$$\sigma = 1 - \frac{\gamma^2 - 1}{\gamma}$$

For $\gamma = 1.4$, D-17 can be written as

$$\frac{P}{\gamma M_{\infty}^2 \theta^2} \approx \frac{\gamma+1}{2} \left[1 - \frac{\gamma+1}{2} \zeta_c^{-3.57} A \cos\left\{1.78(\gamma+1)\zeta_c^{1/2} + B\right\} + \dots \right] \quad D-20$$

From equation D-20, it follows that the oscillations are heavily damped than in the earlier cases (i) and (ii)

APPENDIX E

APPLICATION OF THE TANGENT-CONE APPROXIMATION FOR BLUNTED CONES:

From Cheng's zeroeth order theory (Ref 32), we have the pressure area relation for the axisymmetric case

$$\frac{D_N}{2\pi} = \frac{p_b}{r-1} \frac{y_e^2 - y_b^2}{2} \quad \text{E-1}$$

In arriving at equation ^{E-1}, it is assumed that the shock layer is thin and the pressure is constant across the entropy layer. For the pressure on the body p_b , Cheng used the Newton-Busemann relation for the axisymmetric case

$$p_e = p_\infty U^2 \left(y_e'^2 + y_e y_e''/2 \right) \quad \text{E-2}$$

To simplify the problem, we shall consider the limiting case of $M_\infty \theta_c \gg 1$, where θ_c is the semi-cone angle (figure E-1). From Cox and Crabtree (Ref 81) for $M_\infty \theta_c \gg 1$, the tangent cone approximation can be written as

$$\frac{p_b}{r p_\infty M_\infty^2} = \frac{(r+1)(r+7)}{(r+3)^2} \left(\frac{dy_e}{dx} \right)^2 \quad \text{E-3}$$

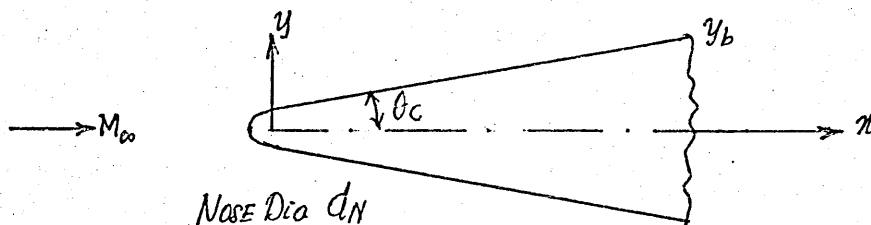


Figure E-1

Substituting $D_N = C_{DN} \frac{\pi d_N^2}{4} \frac{1}{2} p_\infty M_\infty^2$, equation E-1 can be written as

$$\frac{p_b}{r p_\infty M_\infty^2} (y_e^2 - y_b^2) = \frac{C_{DN} d_N^2}{8} (r-1) \quad \text{E-4}$$

$$\text{or} \quad \frac{p_b}{r p_\infty M_\infty^2} (y_e^2 - y_b^2) = e \frac{r+1}{8} C_{DN} d_N^2 \quad \text{E-5}$$

$$\text{where} \quad e = \frac{r-1}{r+1} \quad \text{E-6}$$

Choosing a length scale $L_t = \sqrt{C_{DN} e} d_N$, equations E-3 and E-5 can be written as

$$P_T (Y_T^2 - X_T^2) = \frac{\gamma+1}{8}$$

E-7

and

$$P_T = \frac{(\gamma+1)(\gamma+7)}{(\gamma+3)^2} \left(\frac{dY_T}{dX_T} \right)^2$$

E-8

where

$$P_T = \frac{p_b}{\gamma p_\infty M_\infty^2 \theta_c^2}$$

$$Y_T = \theta_c y_e / L_T$$

and

$$X_T = \gamma \theta_c^2 / L_T$$

The equations E-7 and E-8 are singular at the origin and the solution valid near the leading edge can be obtained by substituting $P_T \propto X_T^m$ in E-7 and E-8. This leads to the leading edge solution

$$P_T = \left[\frac{(\gamma+1)^2 (\gamma+7)}{32 (\gamma+3)^2} \right]^{1/2} X_T^{-1}$$

E-9

The equations E-7 and E-8 can now be integrated with the initial condition given by equation E-9. For $\gamma = 1.4$, equation E-9 reduces to

$$P_T = \frac{0.2795}{X_T}$$

E-10

APPENDIX F

RELATION BETWEEN LENGTH SCALES L_N AND l_c

We have the length scales L_N and l_c given by

$$l_c = \frac{\epsilon C_0 t}{2 \theta_c^3} \quad F-1$$

$$L_N = \frac{C_0 t}{2 \alpha_N^3} \quad F-2$$

Combining equations F-1 and F-2.

$$L_N = l_c \left(\frac{\theta_c}{\alpha_N} \right)^3 \cdot \frac{1}{\epsilon}$$

For a power law body $y_b \propto x^n$, we have from Appendix B

$$\frac{\theta_c}{l_c^{n-1}} = \frac{\alpha_N}{L_N^{n-1}}$$

Combining equations F-3 and F-4

$$\frac{M_\infty \theta_c}{M_\infty \alpha_N} = \left[\epsilon \left(\frac{M_\infty \alpha_N}{M_\infty \theta_c} \right)^3 \right]^{n-1}$$

$$\text{or } \frac{M_\infty \theta_c}{M_\infty \alpha_N} = \epsilon^{\frac{n-1}{3n-2}} \quad F-5$$

$$\text{and } \frac{l_c}{L_N} = \epsilon^{\frac{1}{3n-2}} \quad F-6$$

For $n=1$, for all γ

$$= 1.5$$

$$= 2.0$$

$$M_\infty \theta_c = M_\infty \alpha_N$$

$$M_\infty \theta_c = \epsilon^{1/5} M_\infty \alpha_N$$

$$M_\infty \theta_c = \epsilon^{1/4} M_\infty \alpha_N$$

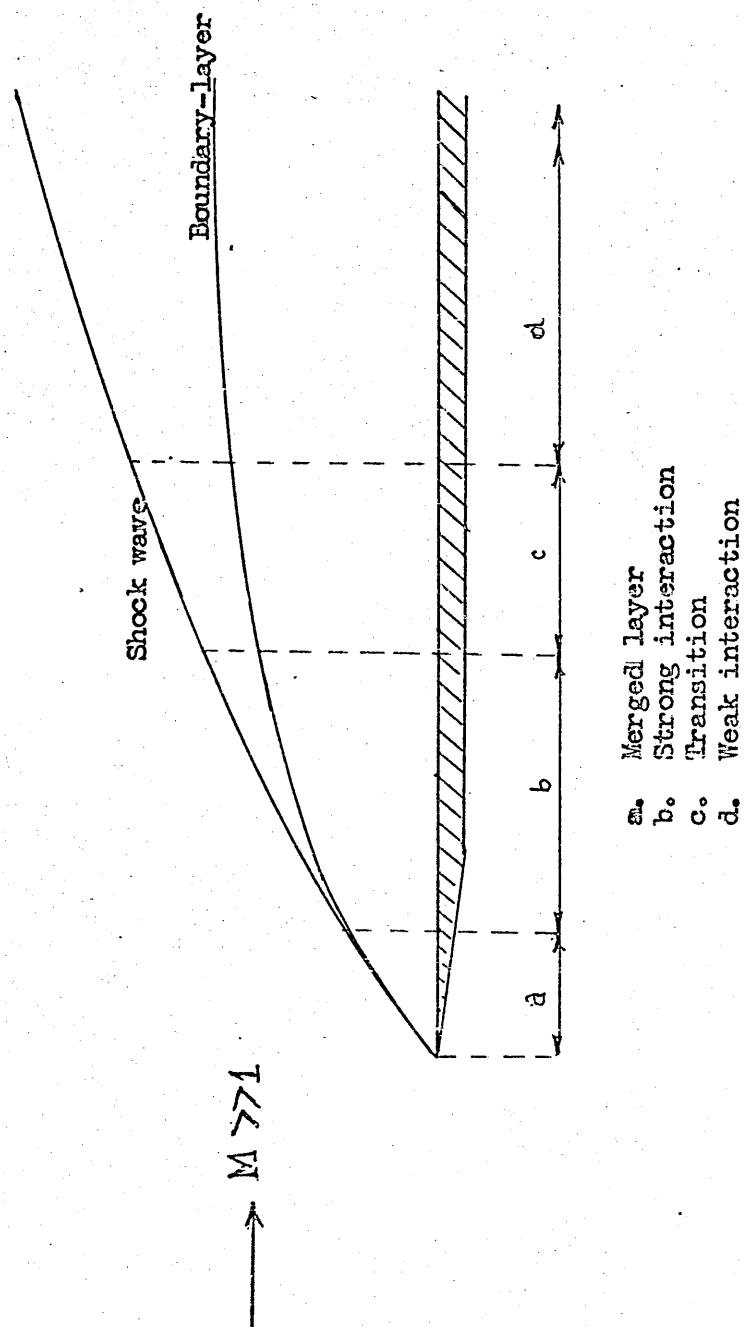
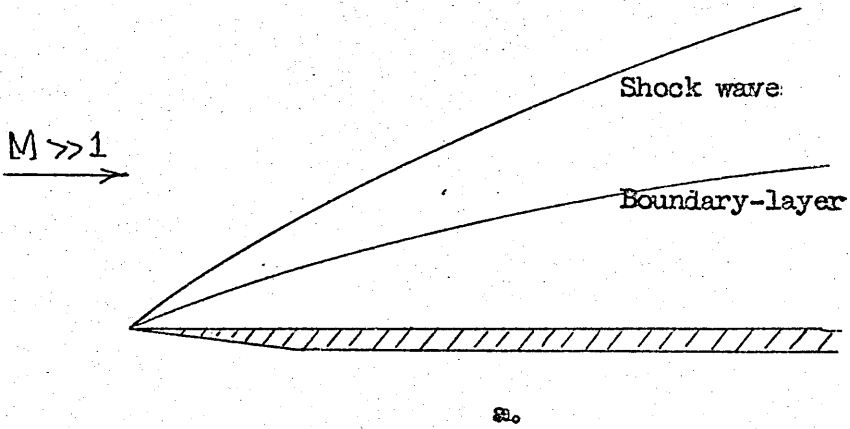
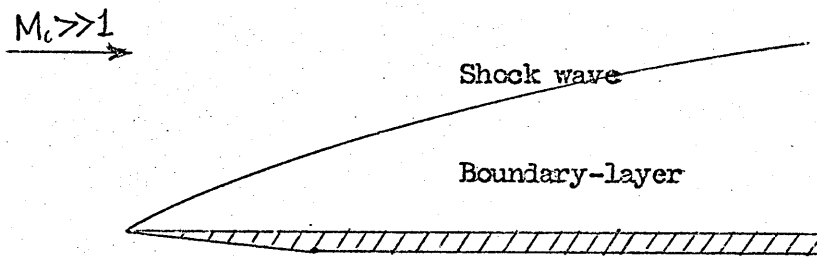


Fig 1. Viscous interaction on a sharp flat plate



a.



b.

Fig 2. Flow models used in viscous interaction analysis.

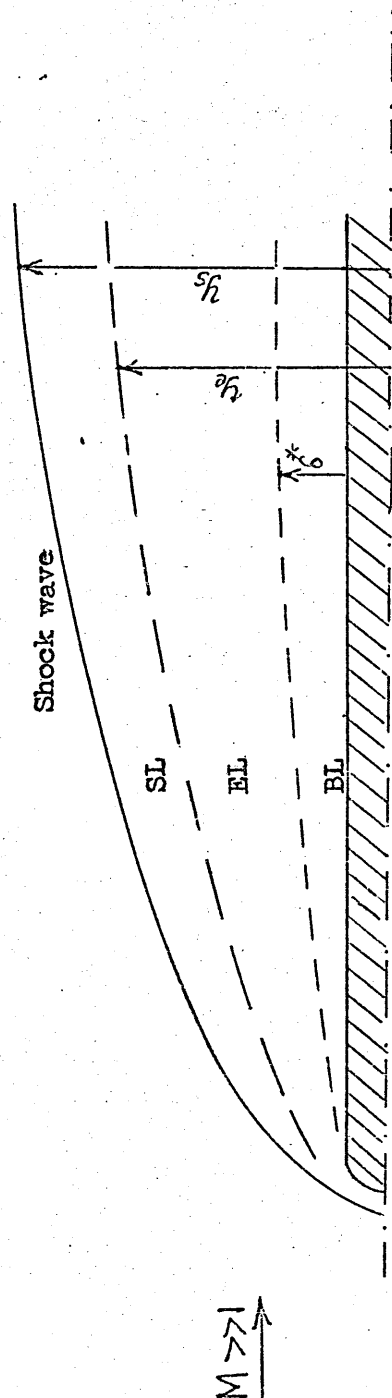
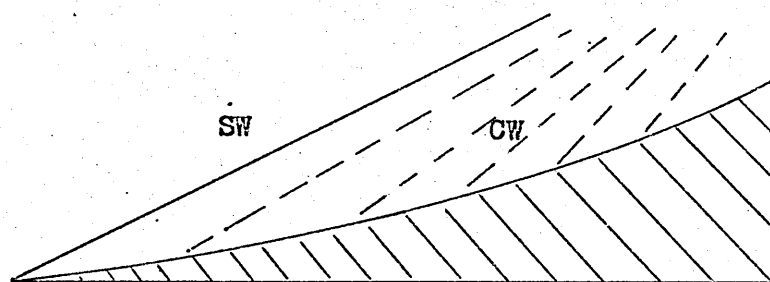
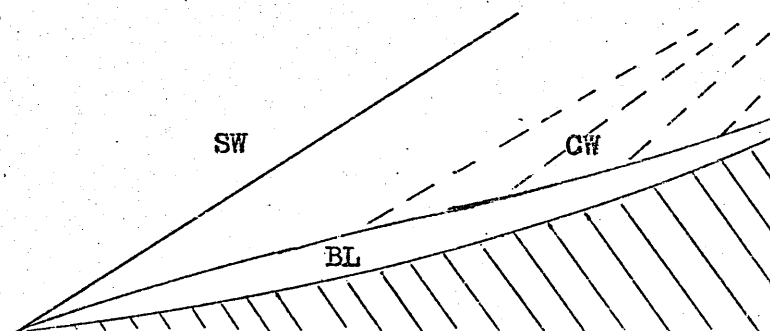


Fig 3. Hypersonic flow past a blunt flat plate.

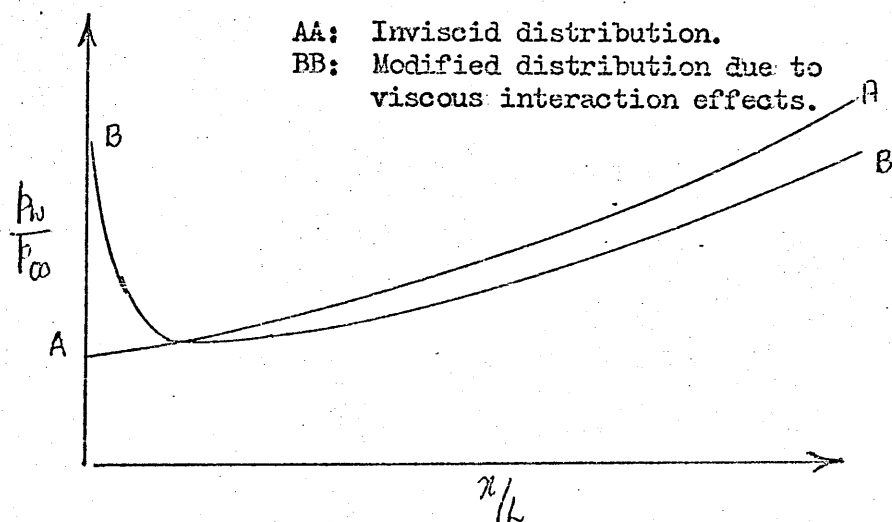
SL: Shock layer
 EL: Entropy layer
 BL: Boundary-layer



a. Inviscid flow.



b. Viscous flow



c. Typical pressure distribution.

Fig 4. Viscid-inviscid interaction on a compression surface. (Ref. 24).

SW: Shock wave
CW: Compression waves
BL: Boundary-layer

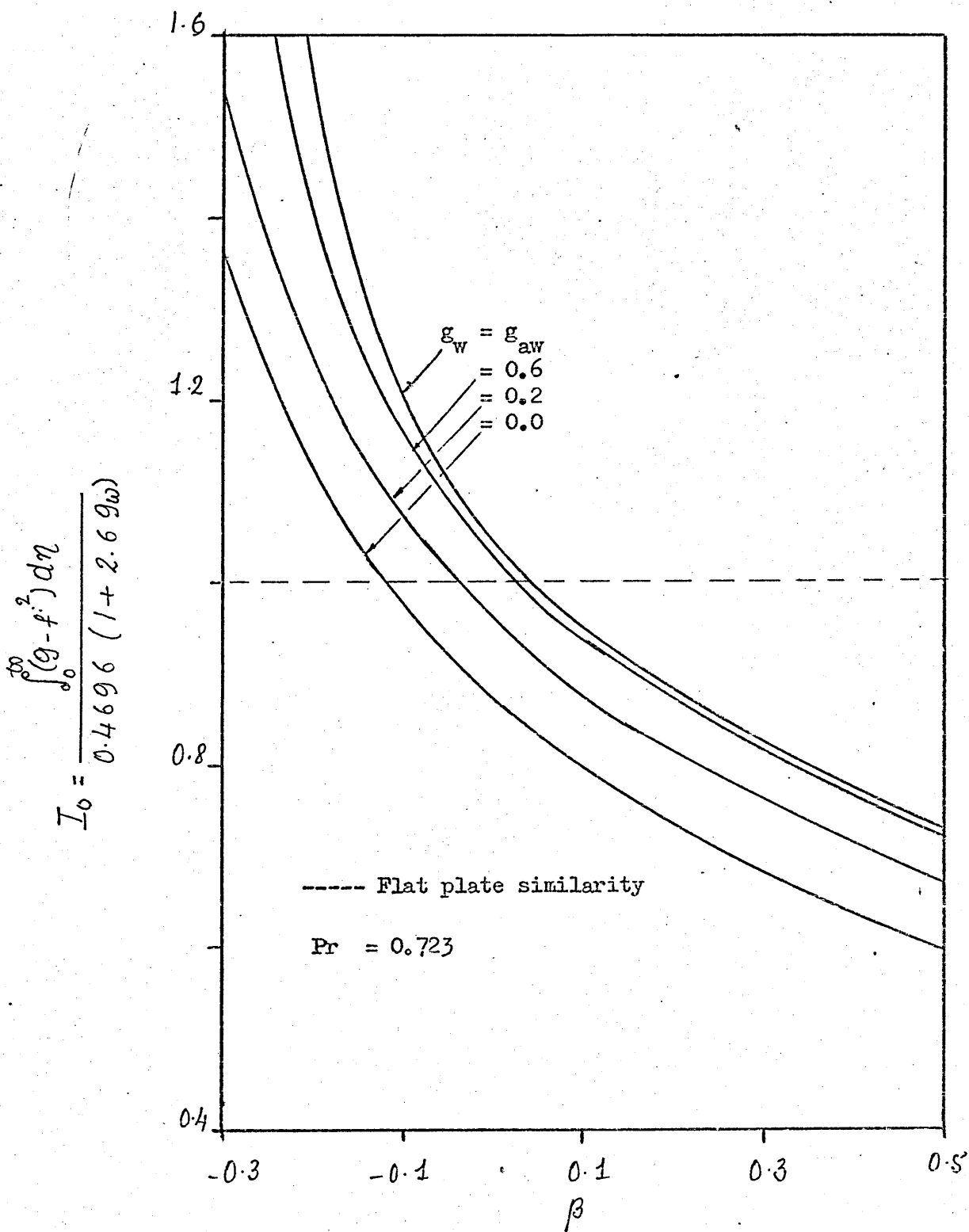


Fig 5: Variation I_0 with pressure gradient parameter β .
(Data from Refs 78,79)

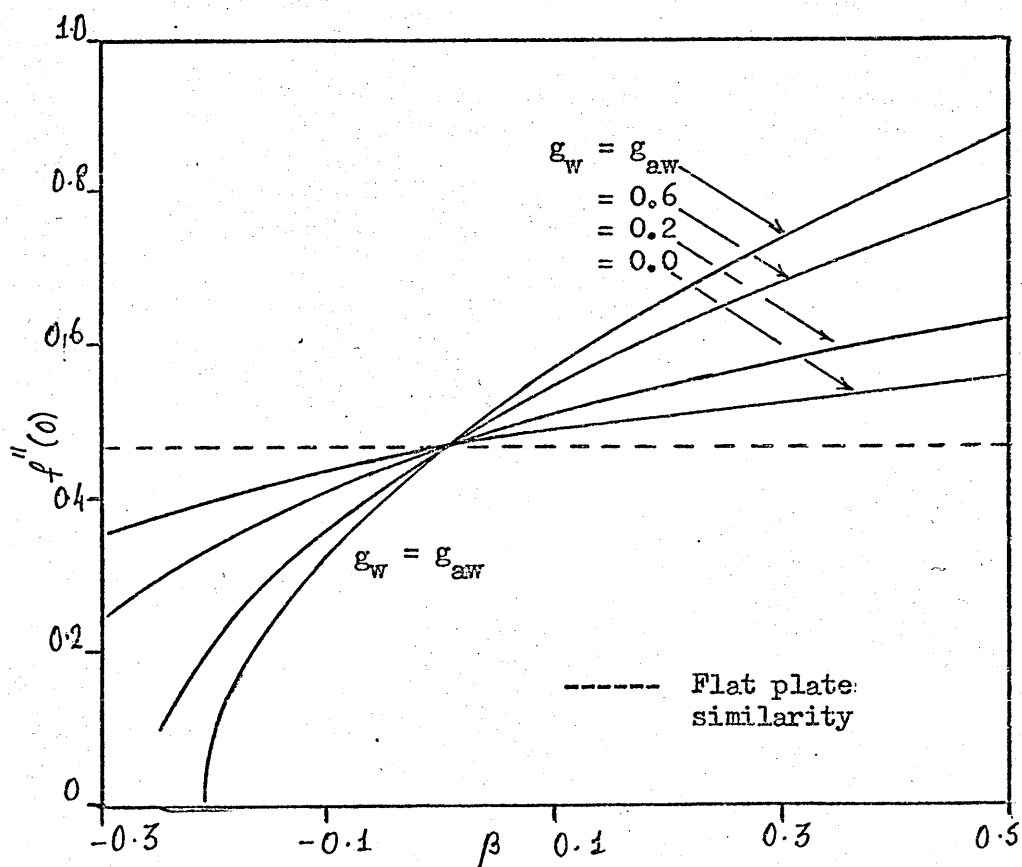


Fig 6: Variation of $f''(0)$ with β (Data from Ref 78, 79)

$$\text{Pr} = 0.723$$

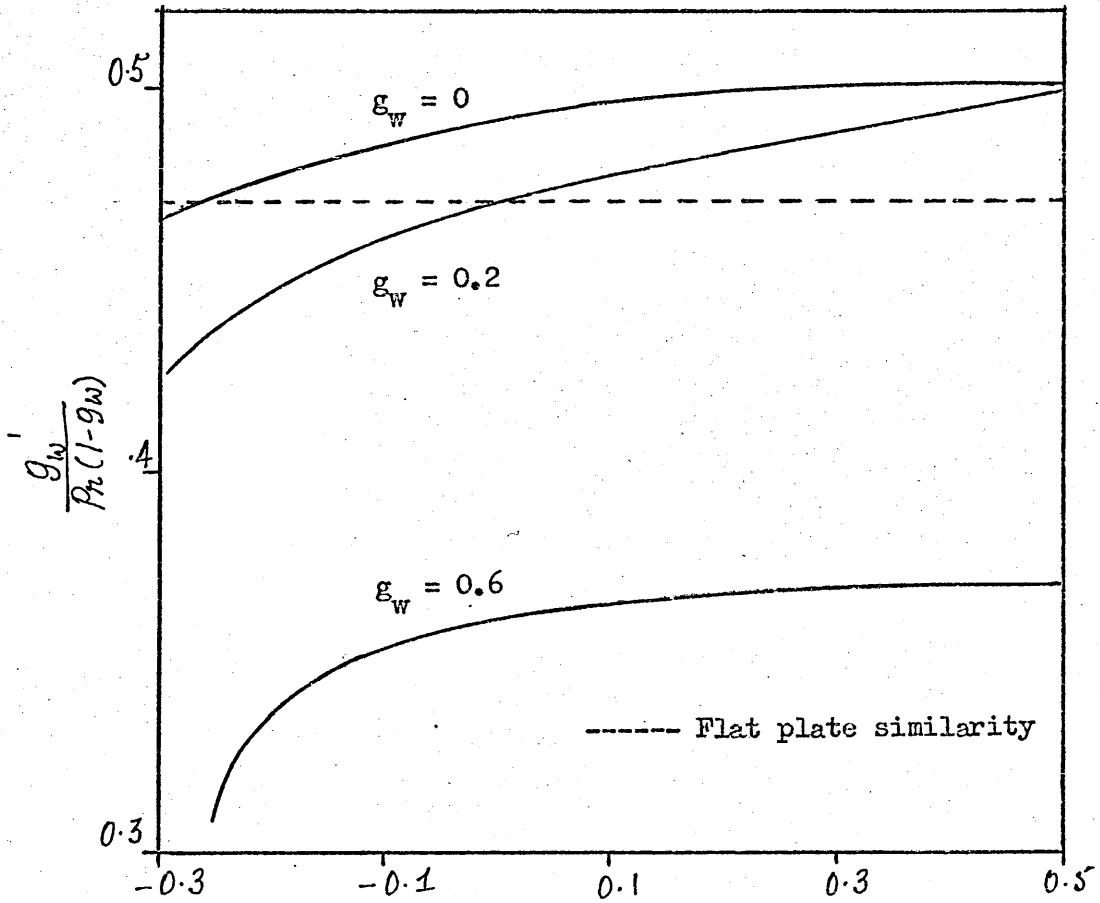


Fig 7. Variation of $\frac{g'_w}{\rho_\infty (1 - g_w)}$ with β (data from Ref 78, 79)

Pr = 0.723

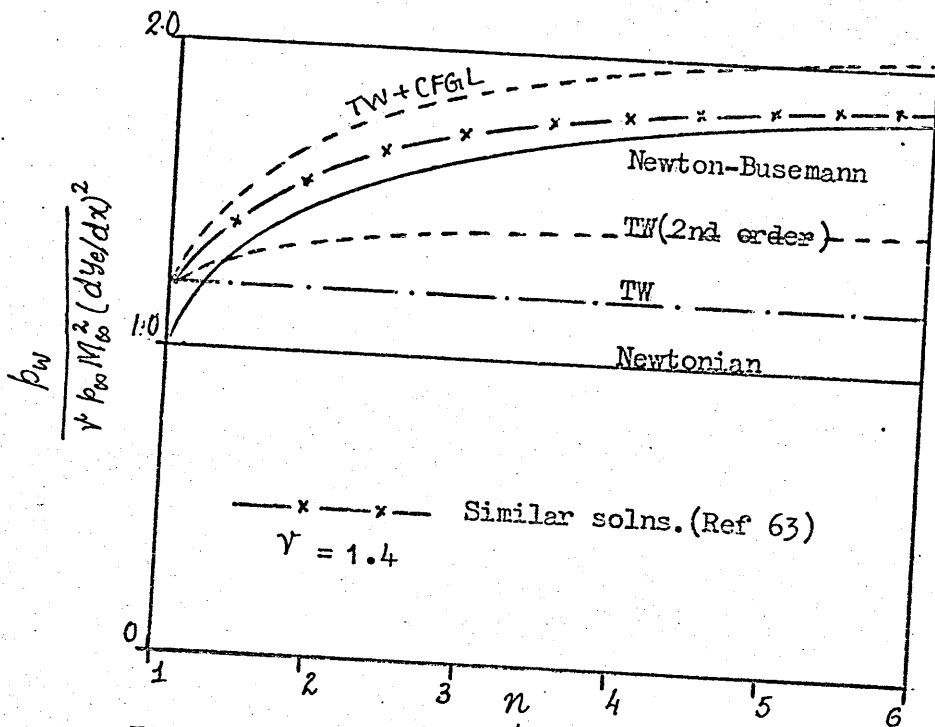


Fig 8: Variation of $\frac{p_w}{\gamma p_\infty M_\infty^2 (dy_e/dx)^2}$ with n

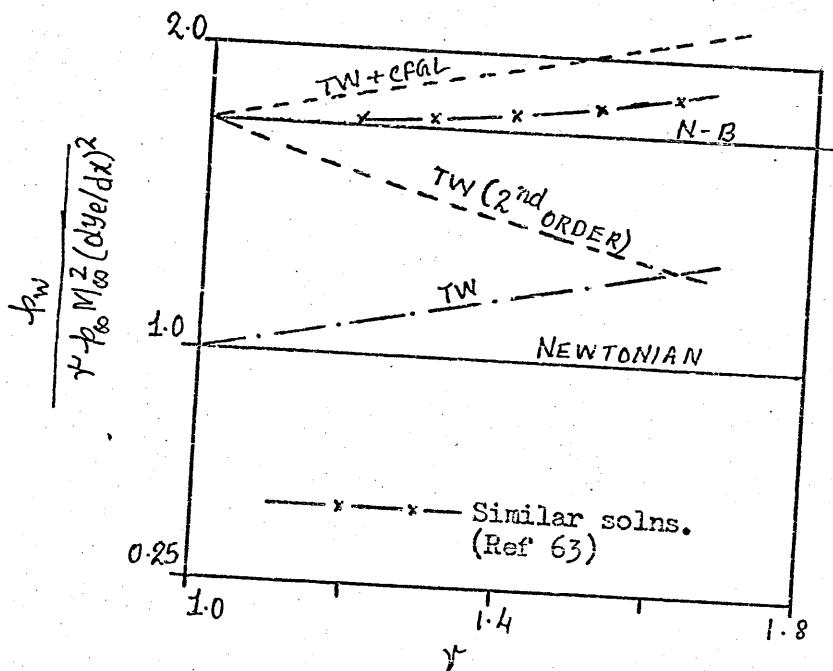


Fig 9: Variation of $\frac{p_w}{\gamma p_\infty M_\infty^2 (dy_e/dx)^2}$ with γ
 $n = 4$.

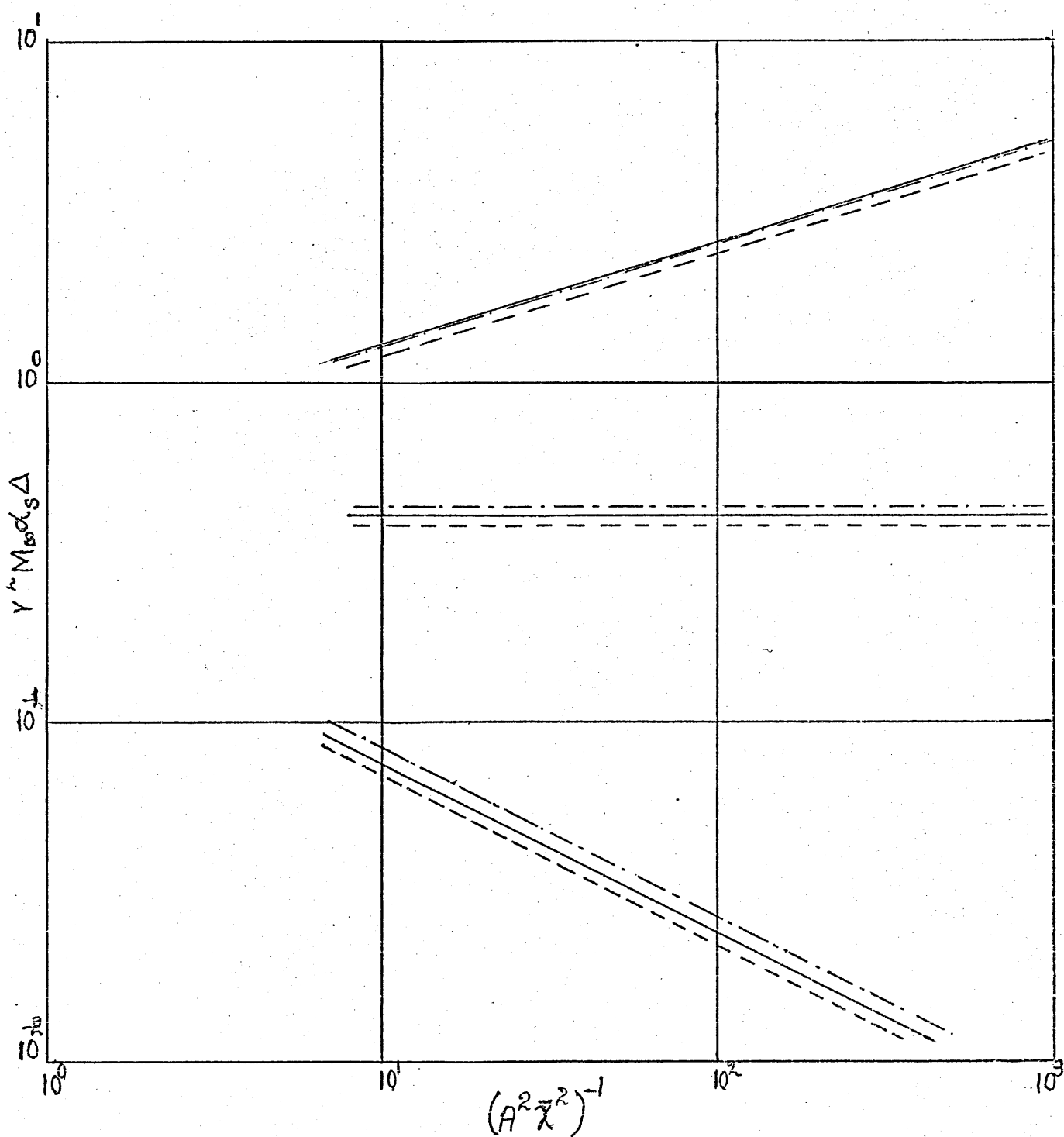


Fig 10. - Asymptotic behaviour of boundary layer thickness.

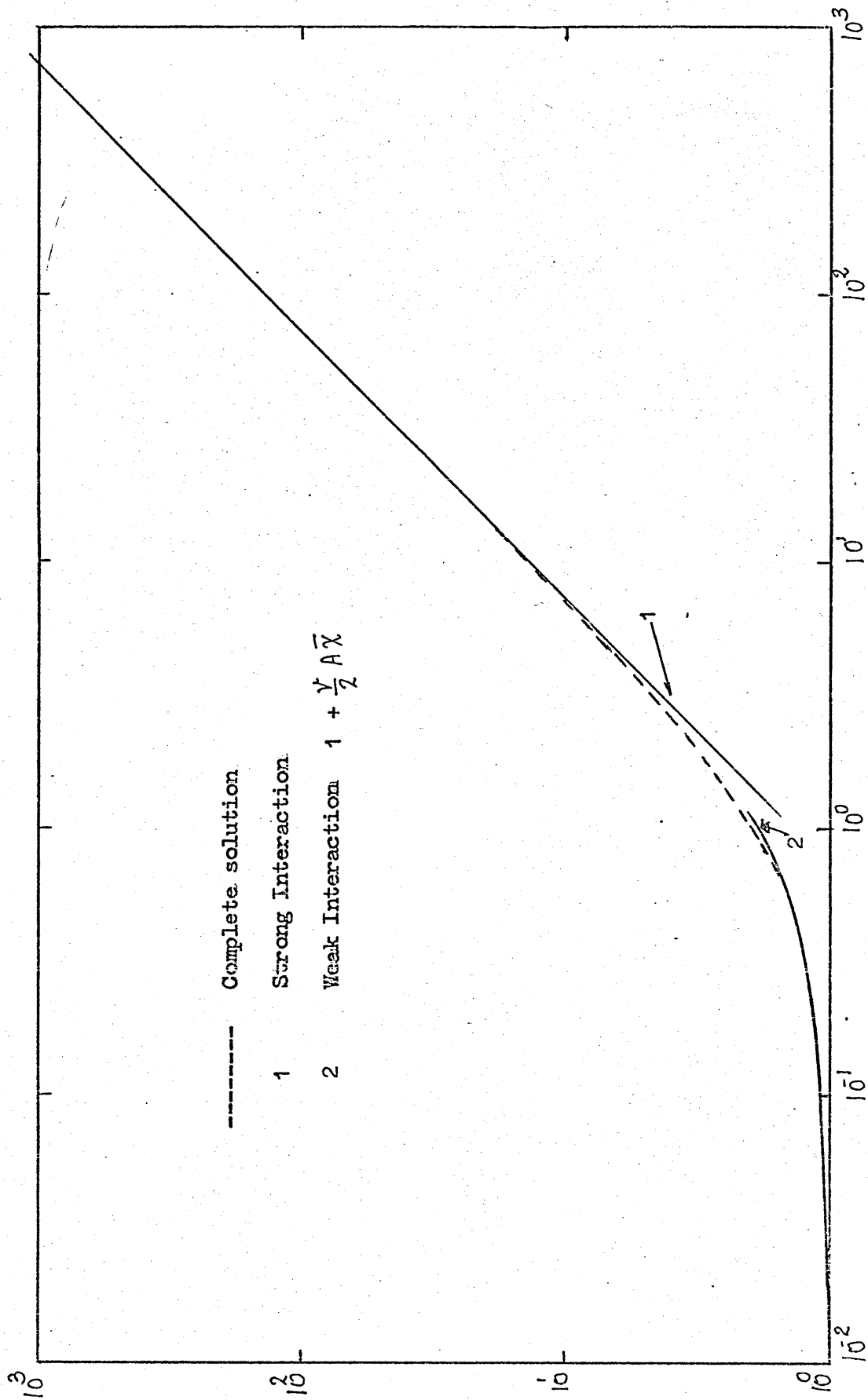


Fig 11; Pressure distribution on a flat plate using TW + CFGL Rule. ($\gamma = 5/3$).

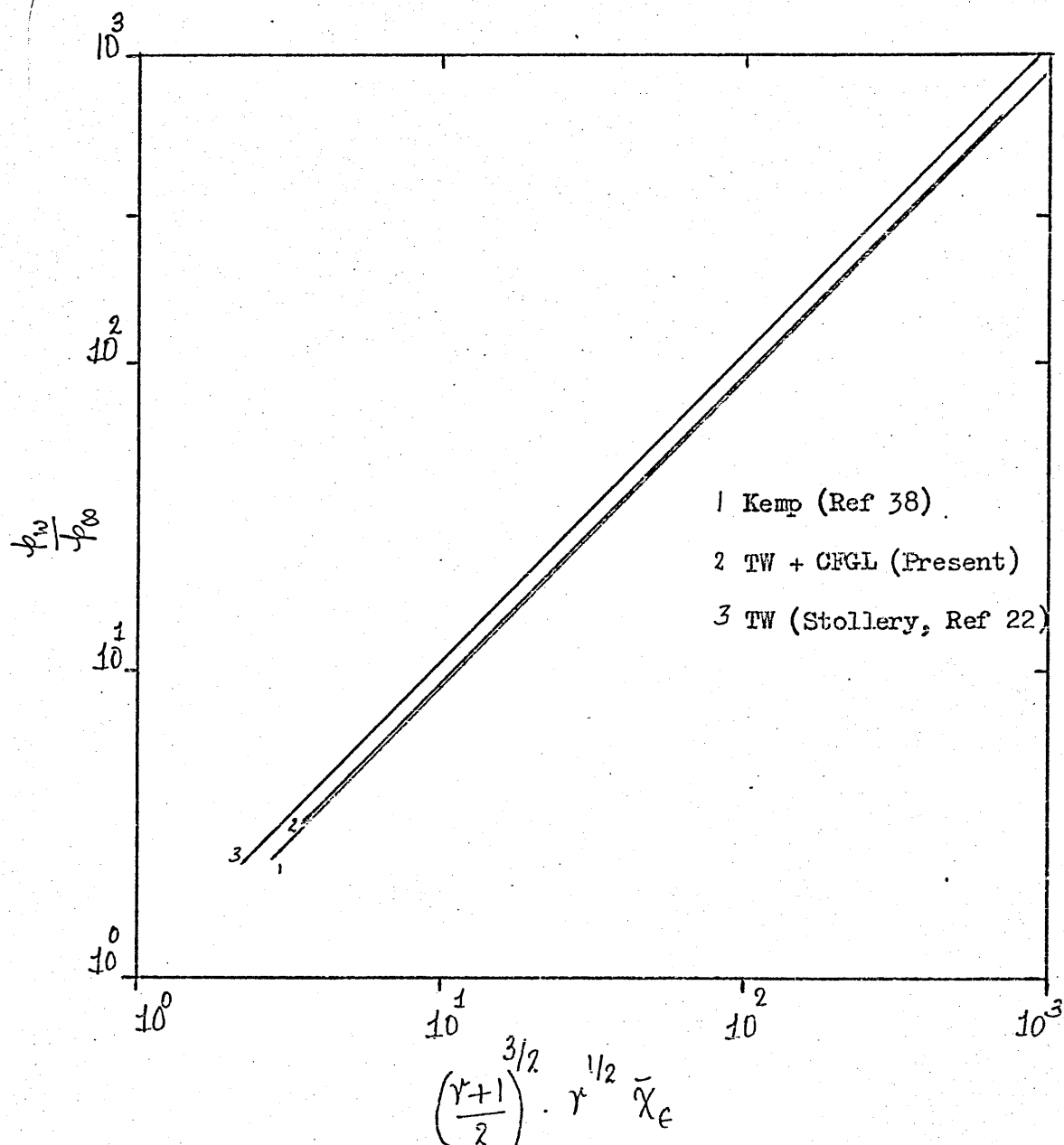


Fig 12: Comparison of pressure distribution in the strong interaction region.

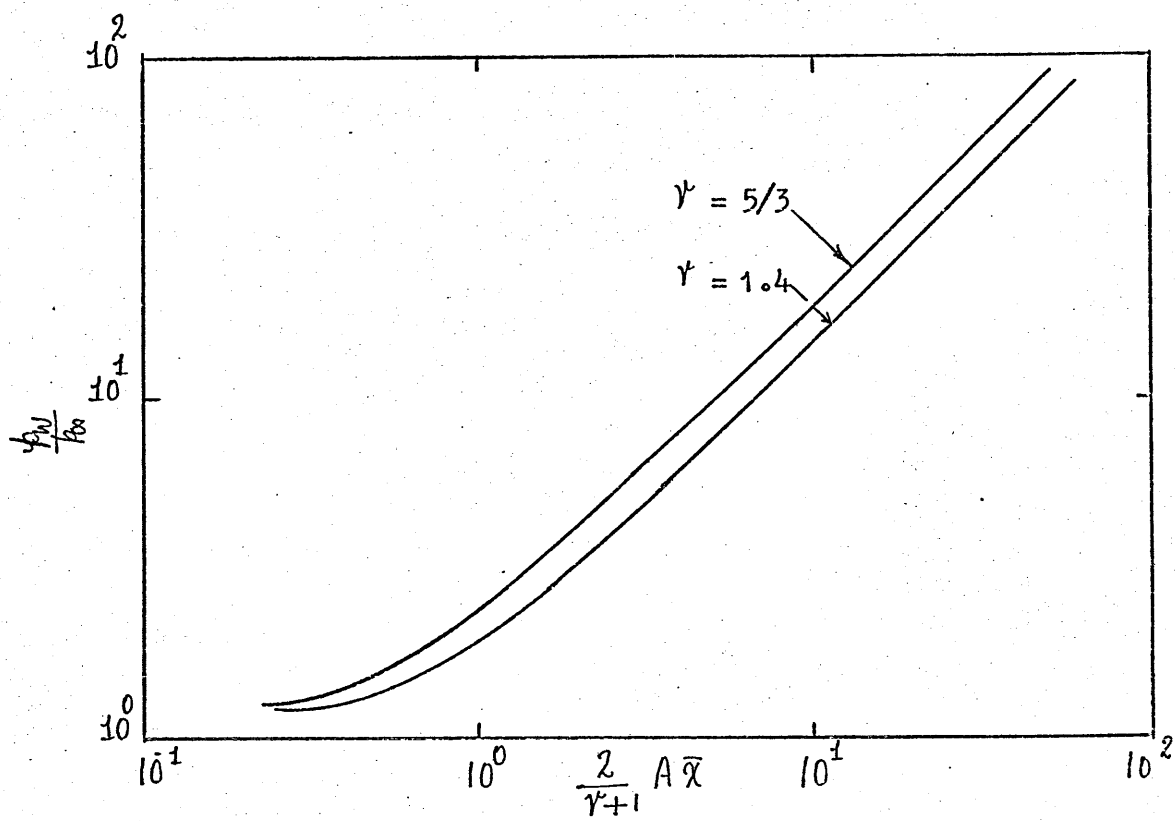


Fig 13a: Pressure distribution on a flat plate for different γ ,
using TW + CFGL Rule

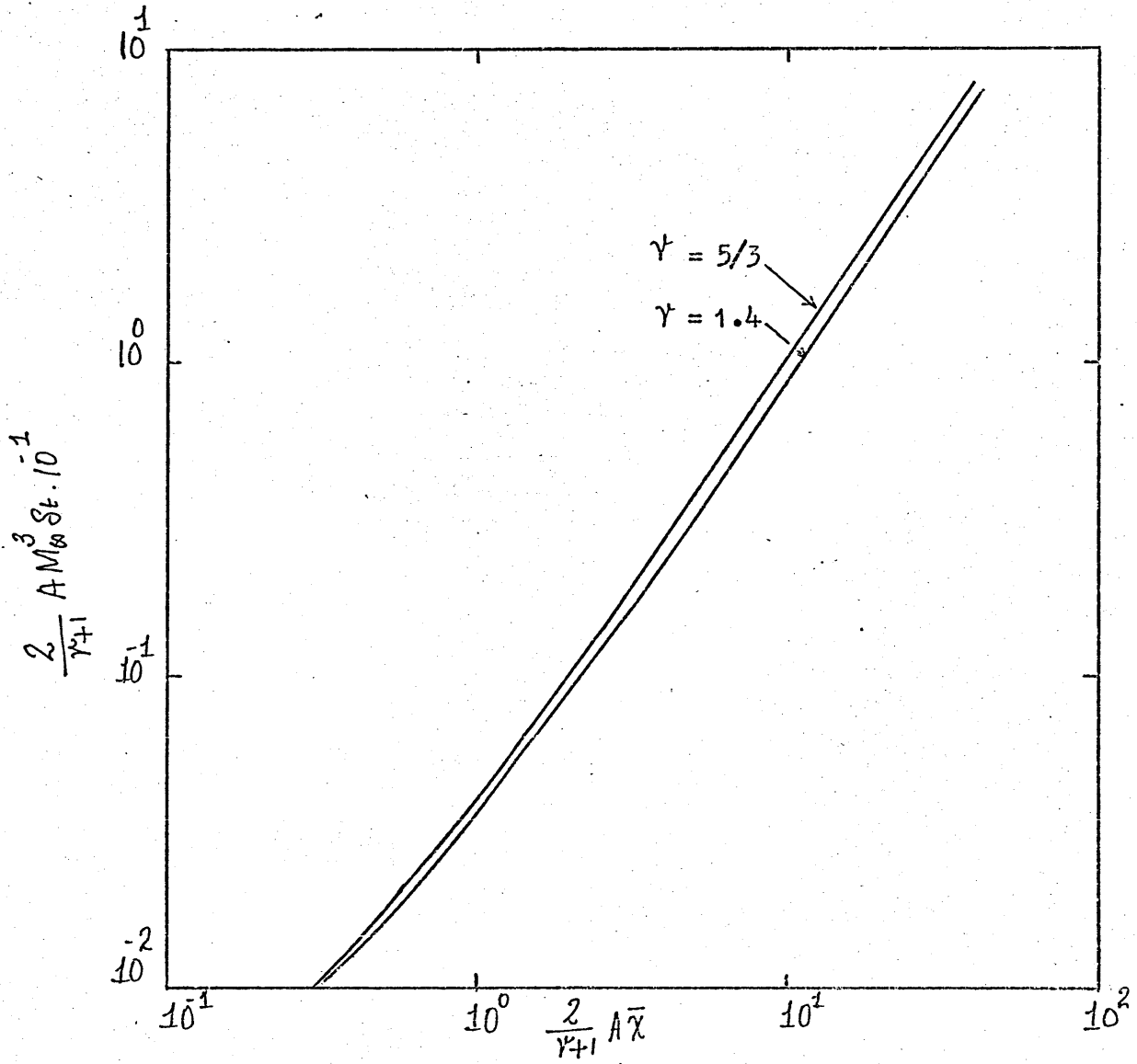


Fig 13b: Heat transfer on a flat plate for different γ using TW + CFGL rule.

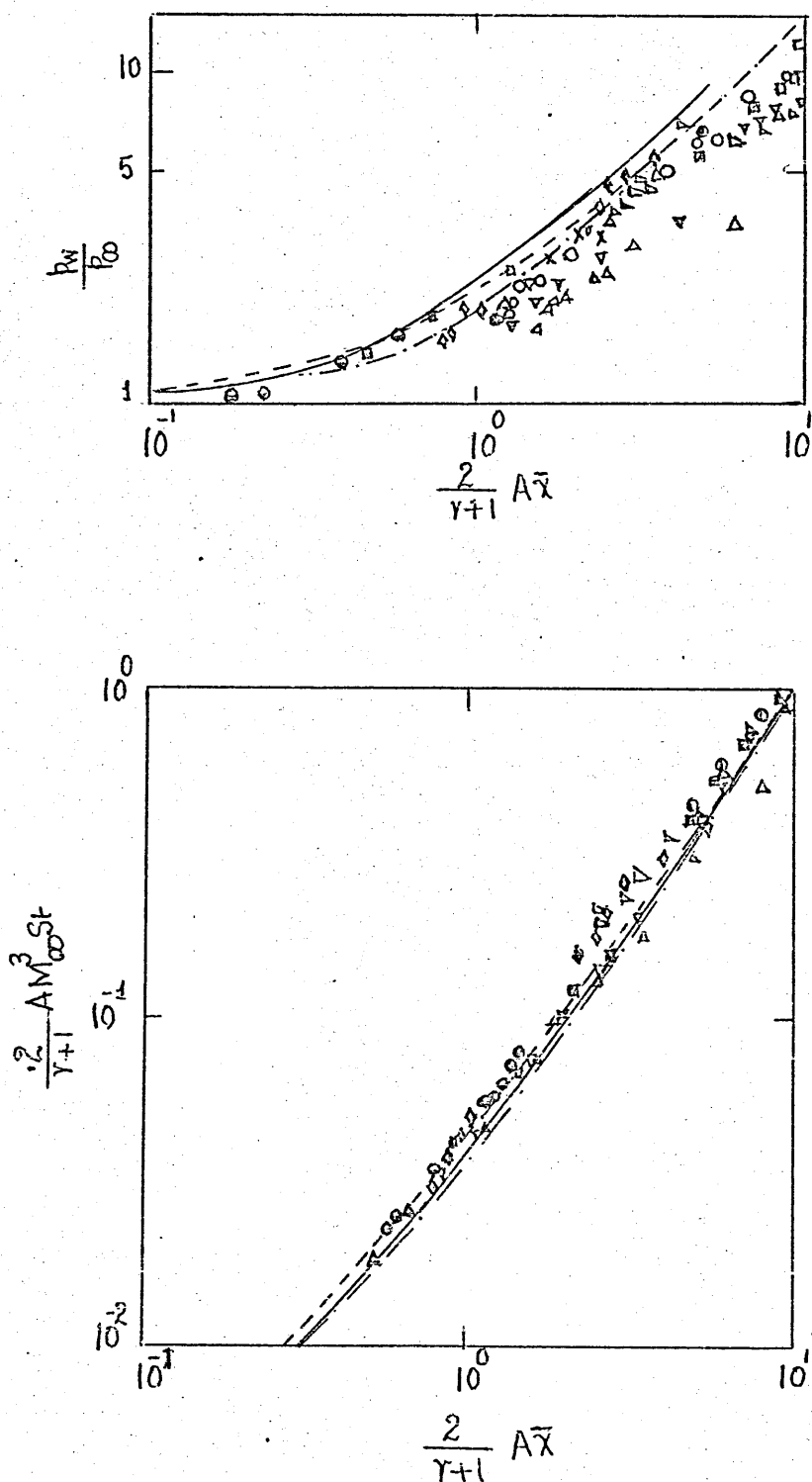


Fig 14: Comparison of theoretical predictions of pressure distribution and heat transfer with experimental results of Holden (Ref 58).
 — Stollery (TW); ---- Present TW+CEGL;
 - · - Present (using Chernyi's integral method, $g_w=0$, $Pr=0.723$)

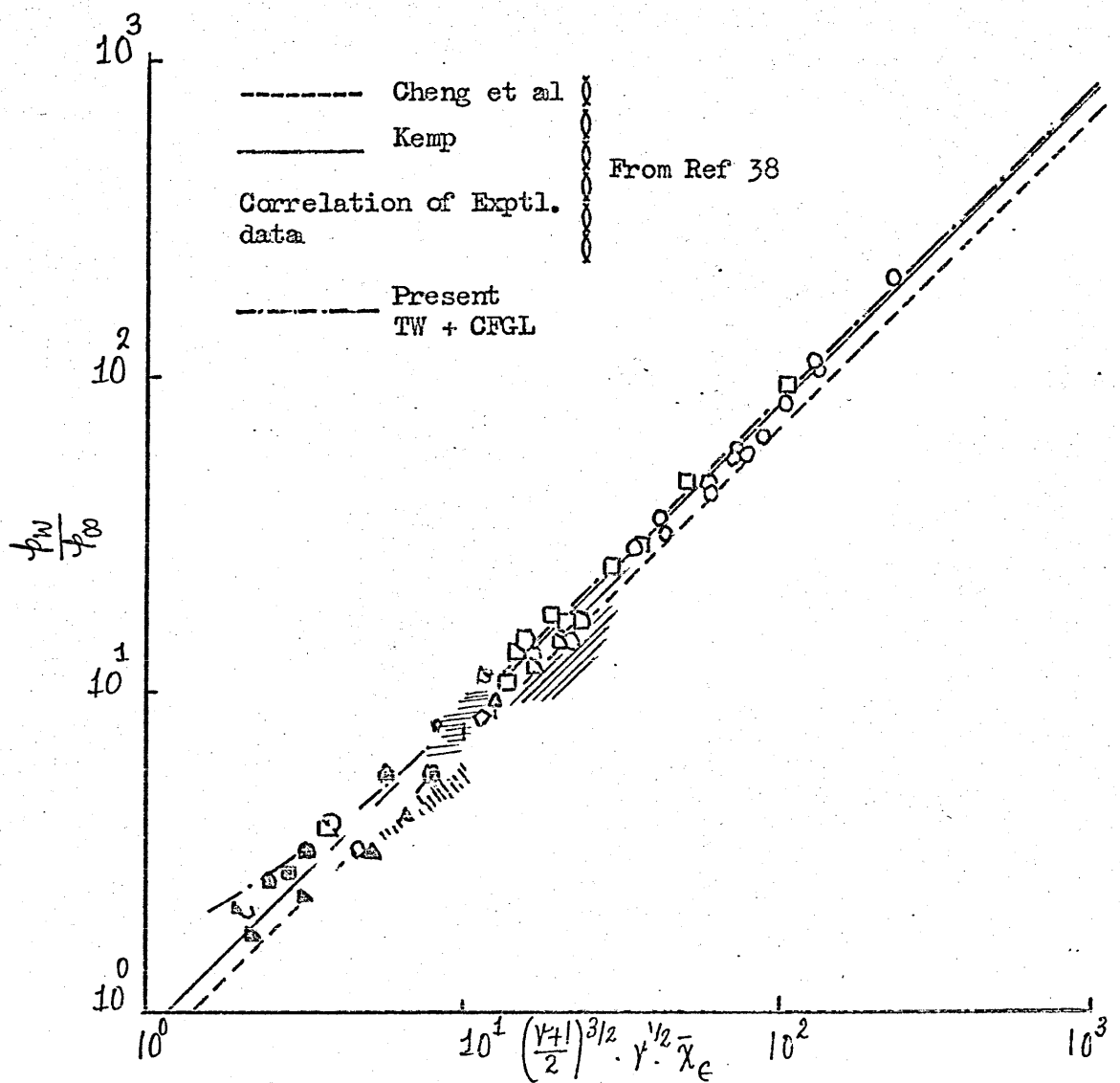


Fig 15: Pressure distribution on a flat plate at zero incidence.

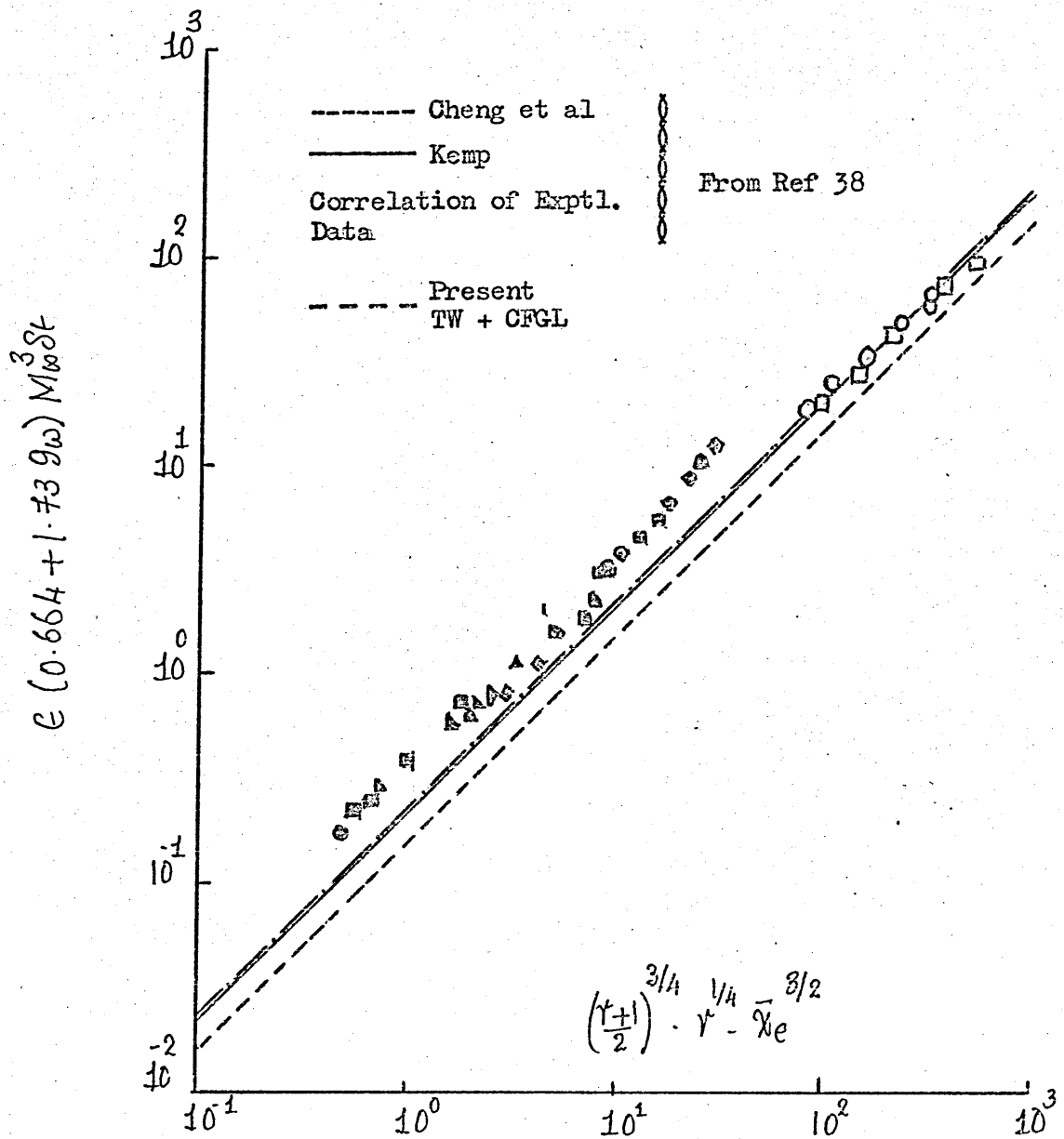


Fig 16: Heat transfer on a sharp flat plate at zero incidence.

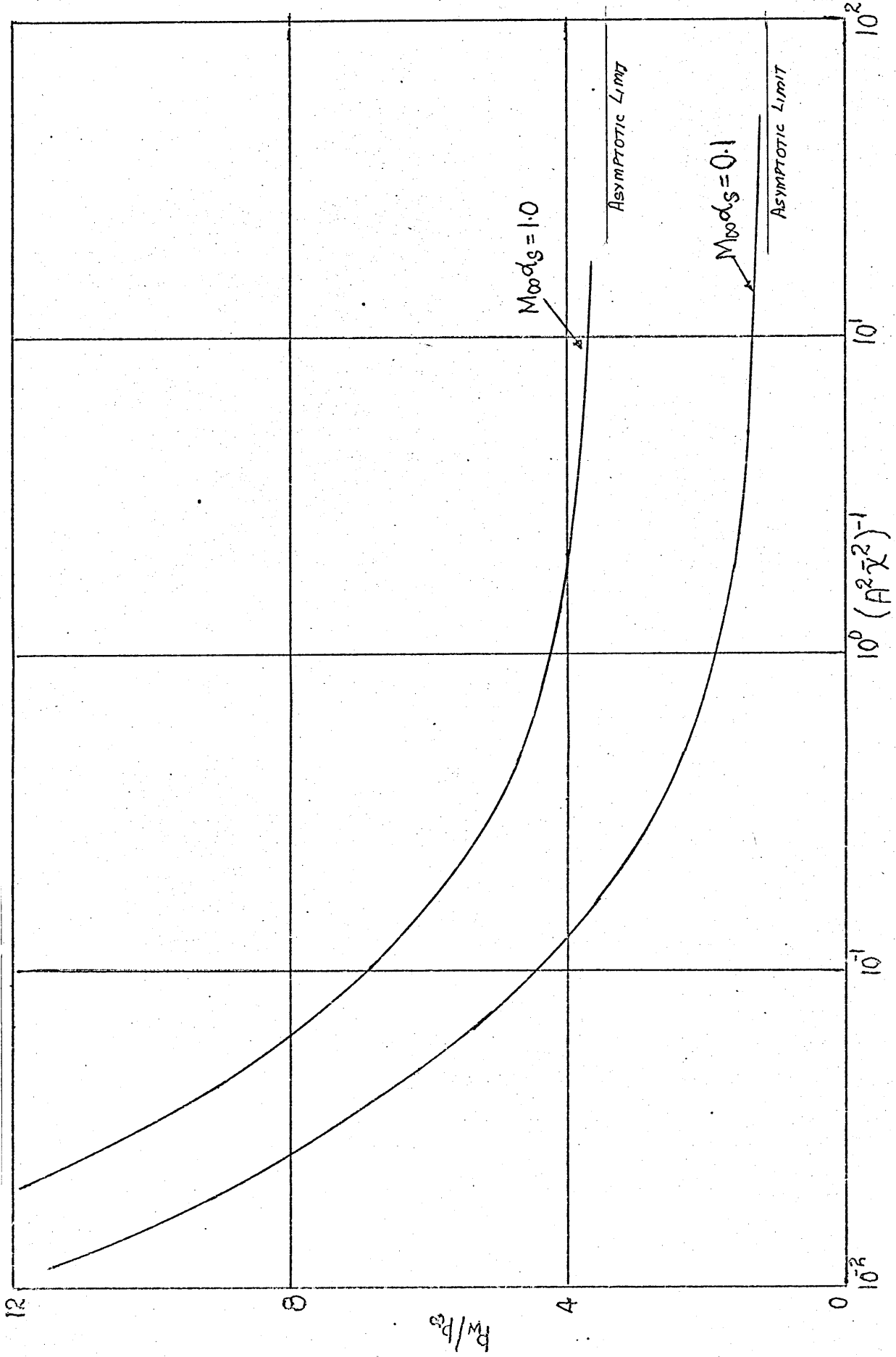


Fig 17. Pressure distribution on a sharp flat plate at incidence according to TW + CFGL theory (Viscous interaction).

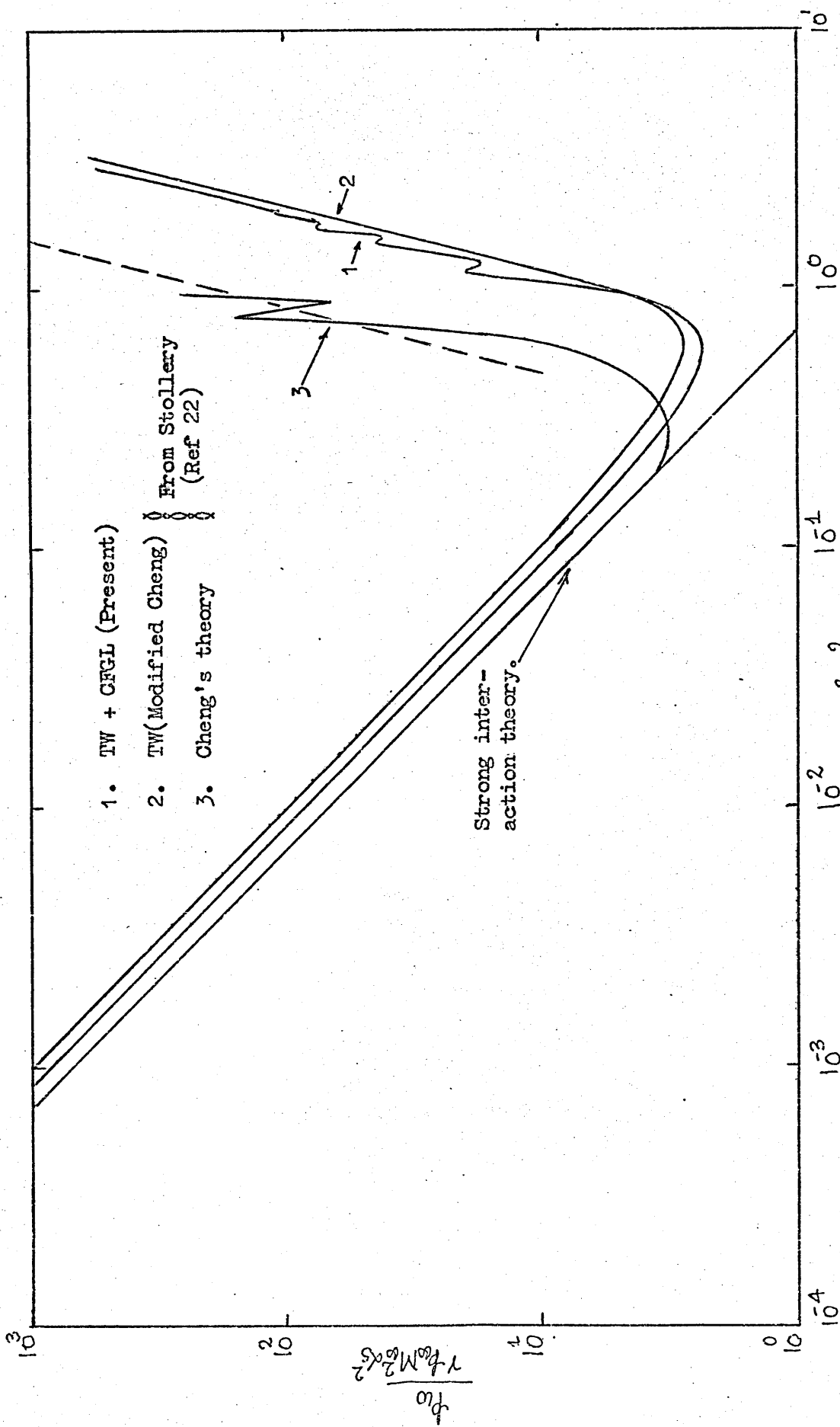


Fig 18a: Pressure distribution on a concave surface $\gamma_w \neq \gamma$, $M_0 \alpha_3 = 1$, $\gamma = 1.4$

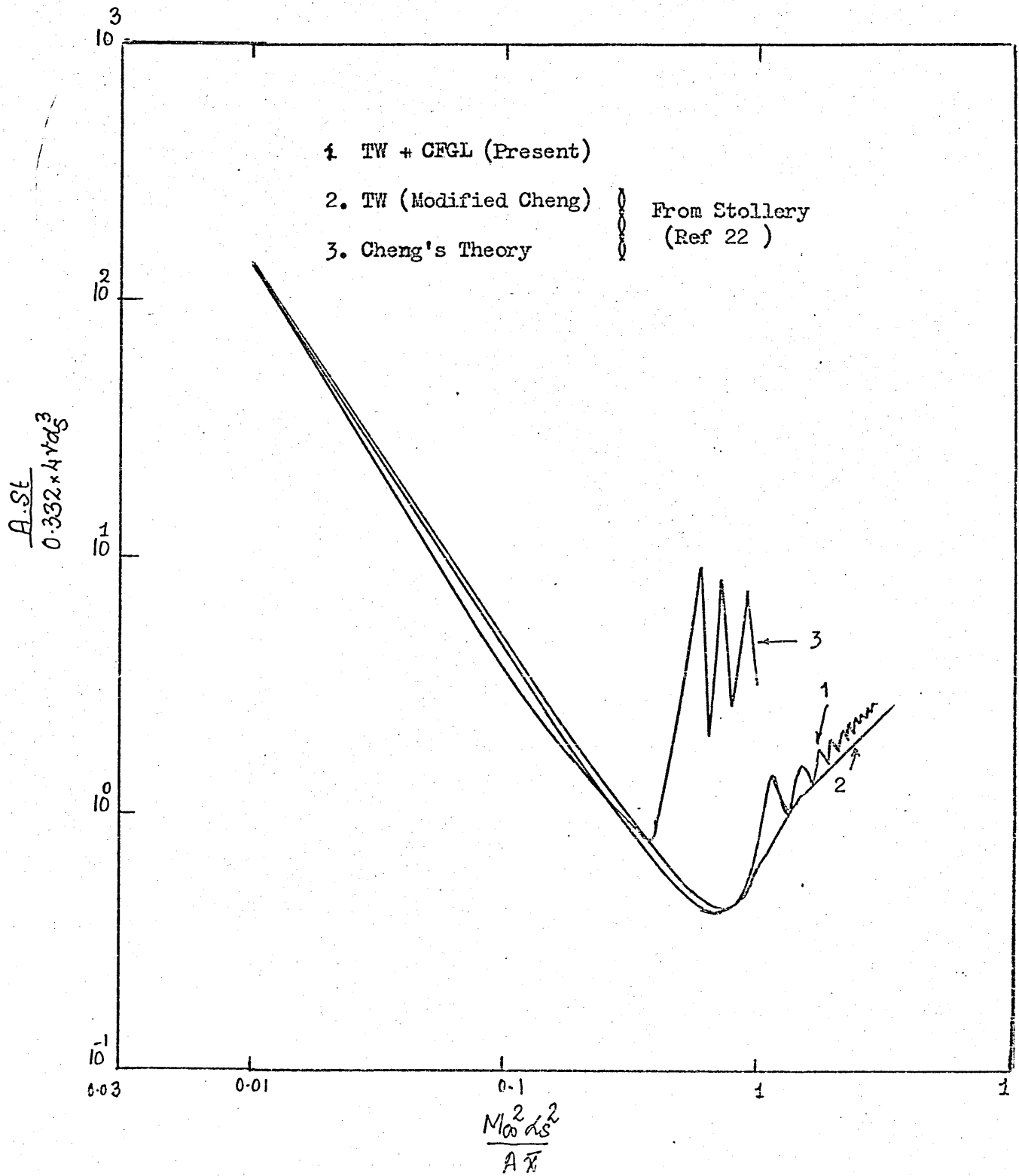


Fig 1.8b: Heat transfer on a concave surface, $\gamma_w = X^2$, $M_\infty \alpha_s = 1$
 $\gamma = 1.4$

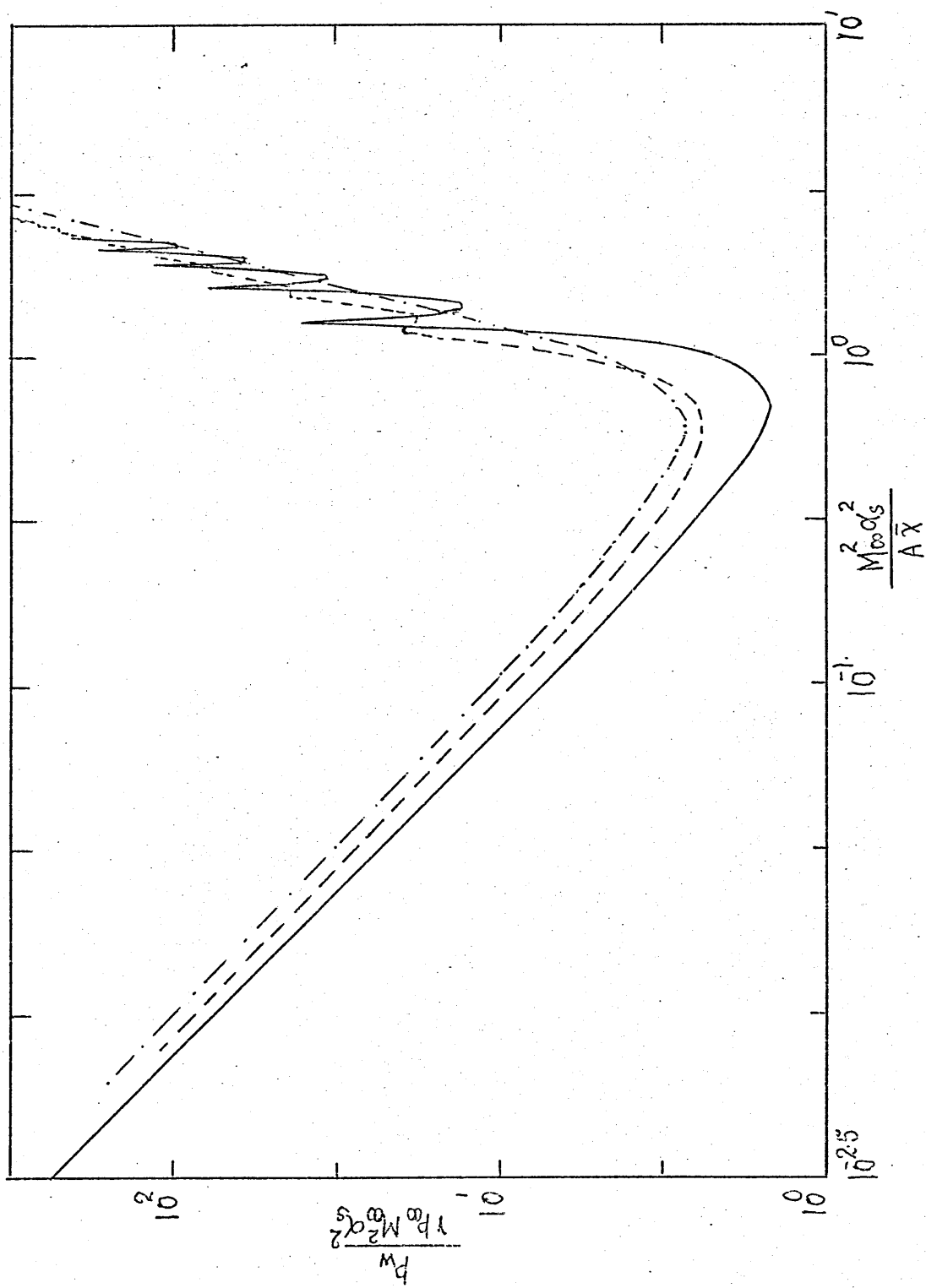


Fig 19a. Pressure distribution on concave surface $\gamma = 1.4$, $M_\infty \alpha_s = 1$; — Cheng's theory; ---- TW+CFGL; -.-.- TW (Modified Cheng).

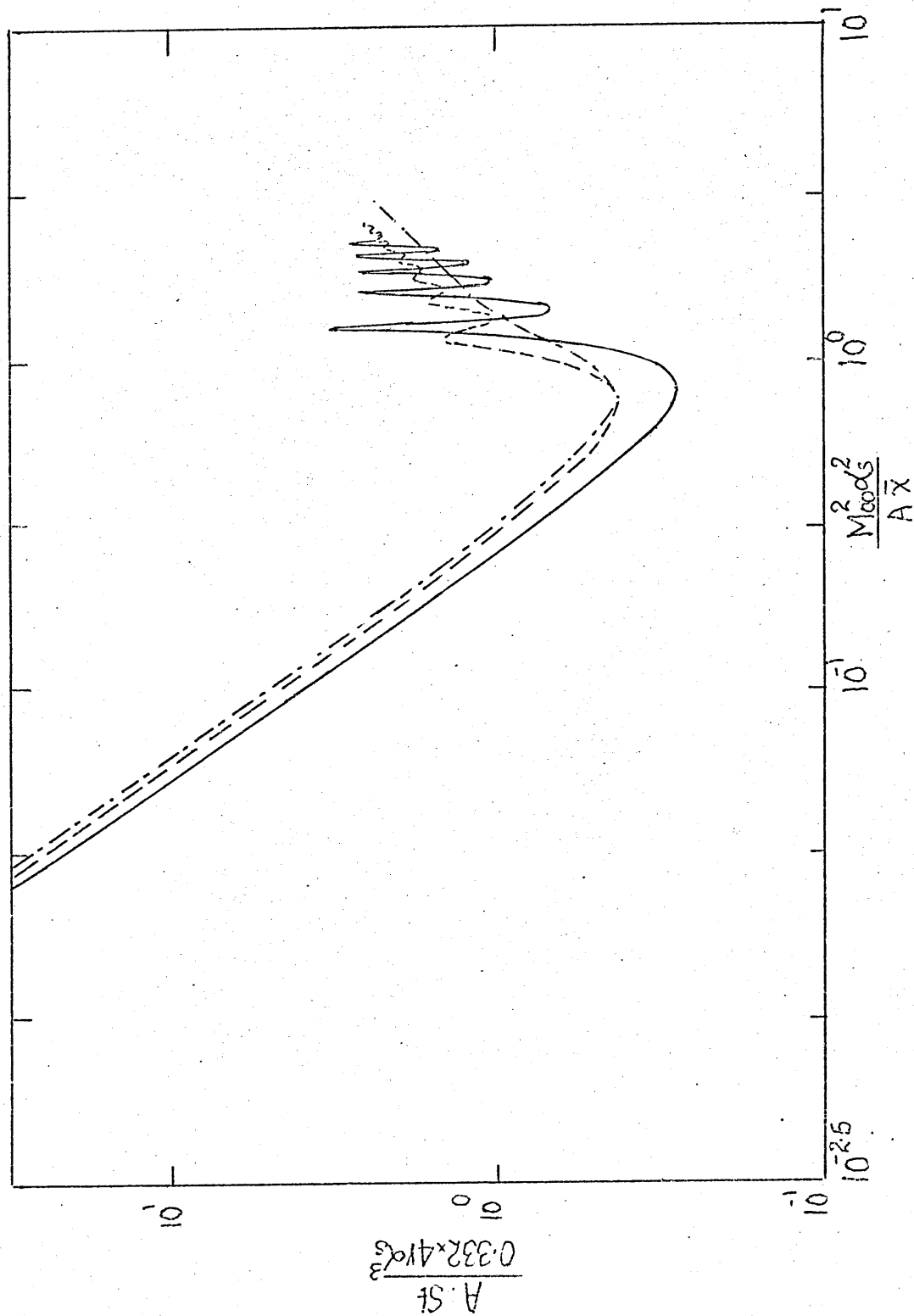


Fig 19b. Heat transfer on concave surface $Y_w = X^2$, $M_{\infty} \alpha_s = 1$, $\gamma = 1.4$;
—— Cheng's theory; ----- TW+OFG; -.-.- TW
(Modified Cheng).

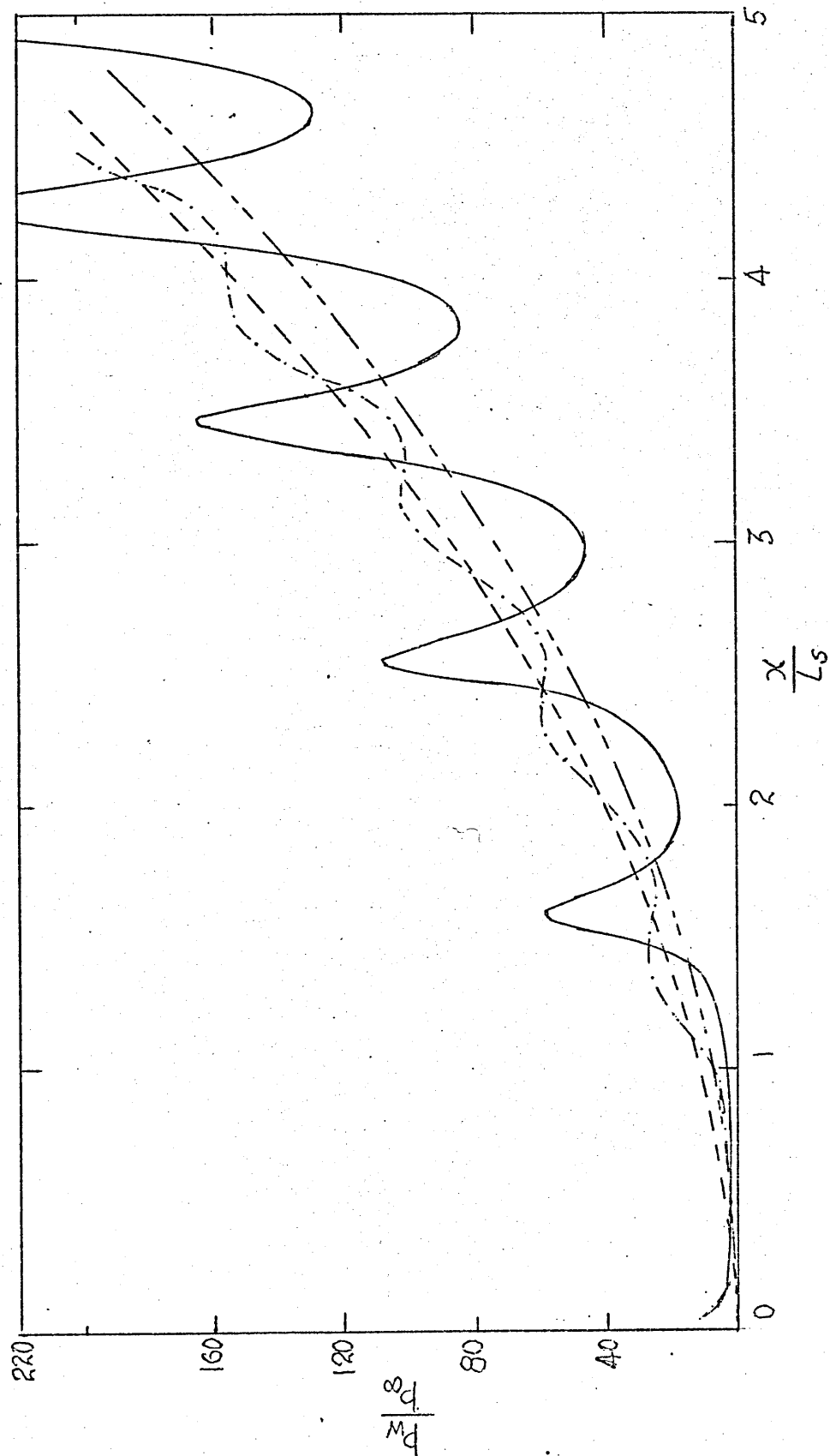


Fig 20. Comparison of viscous interaction solution with inviscid theories on concave surface $Y_w = X^2$, $M_\infty = 1$, $\gamma = 1.4$.
 — Viscous, --- Inviscid, -.-.- Cheng's theory.
 -.-.- Viscous, ----- Inviscid; TW + CRGL theory.

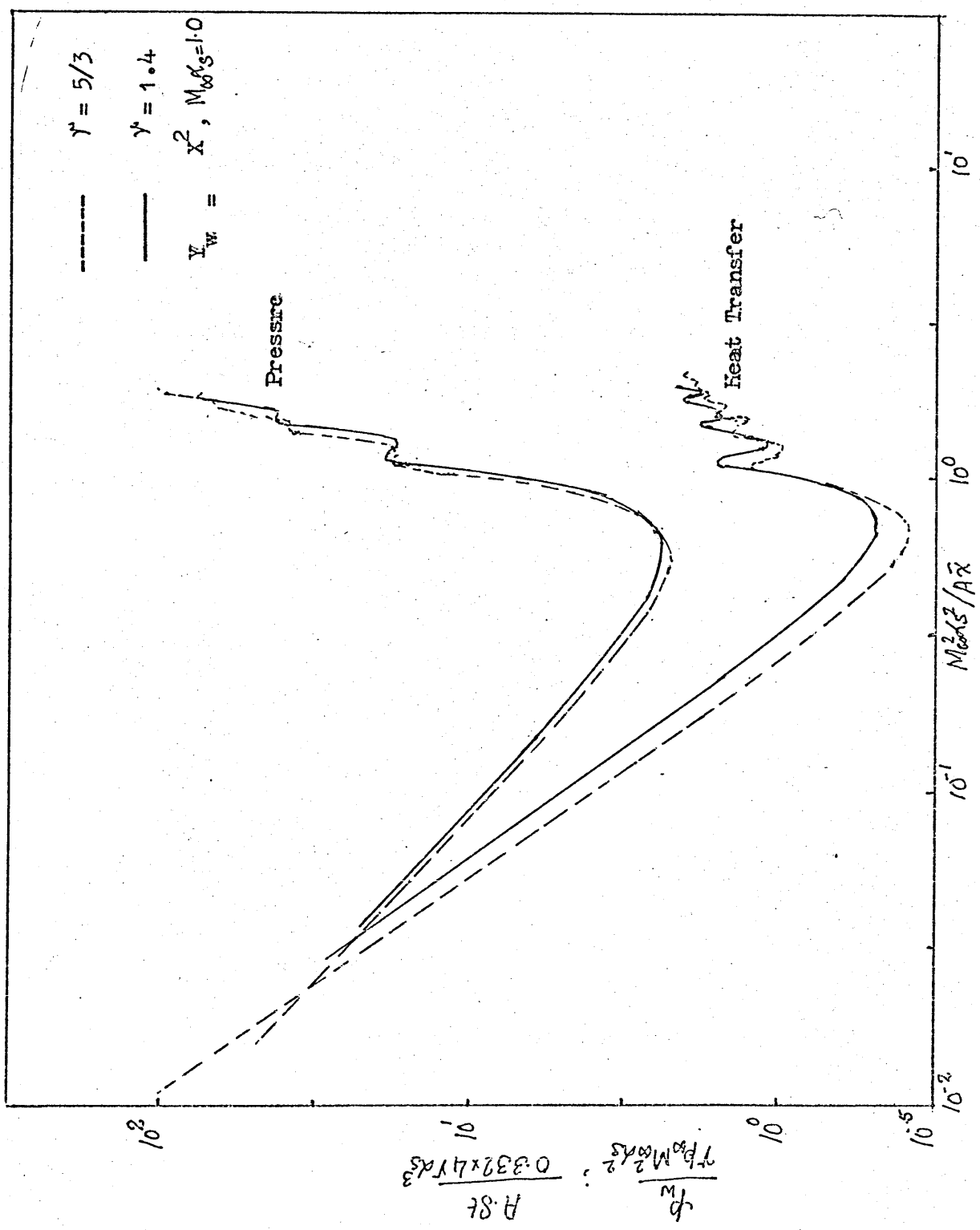


Fig 21: Effect of γ on pressure and heat transfer rate (TW+CFGL)

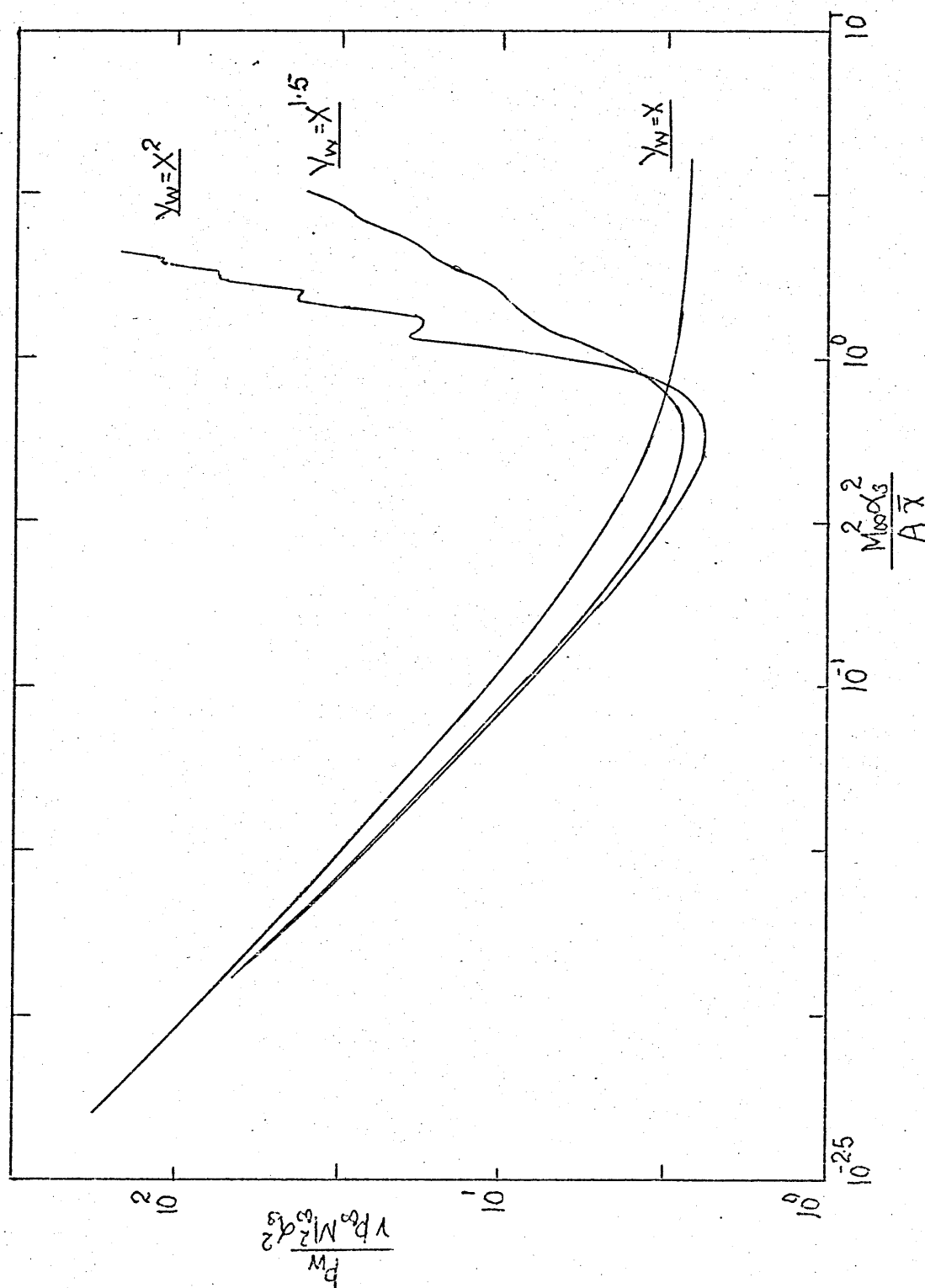


Fig 22a. Pressure distribution on a family of shapes $Y = X^n$, according to TW + CFGL theory, $M_\infty \alpha_3 = 1$, $\gamma = 1.4$.

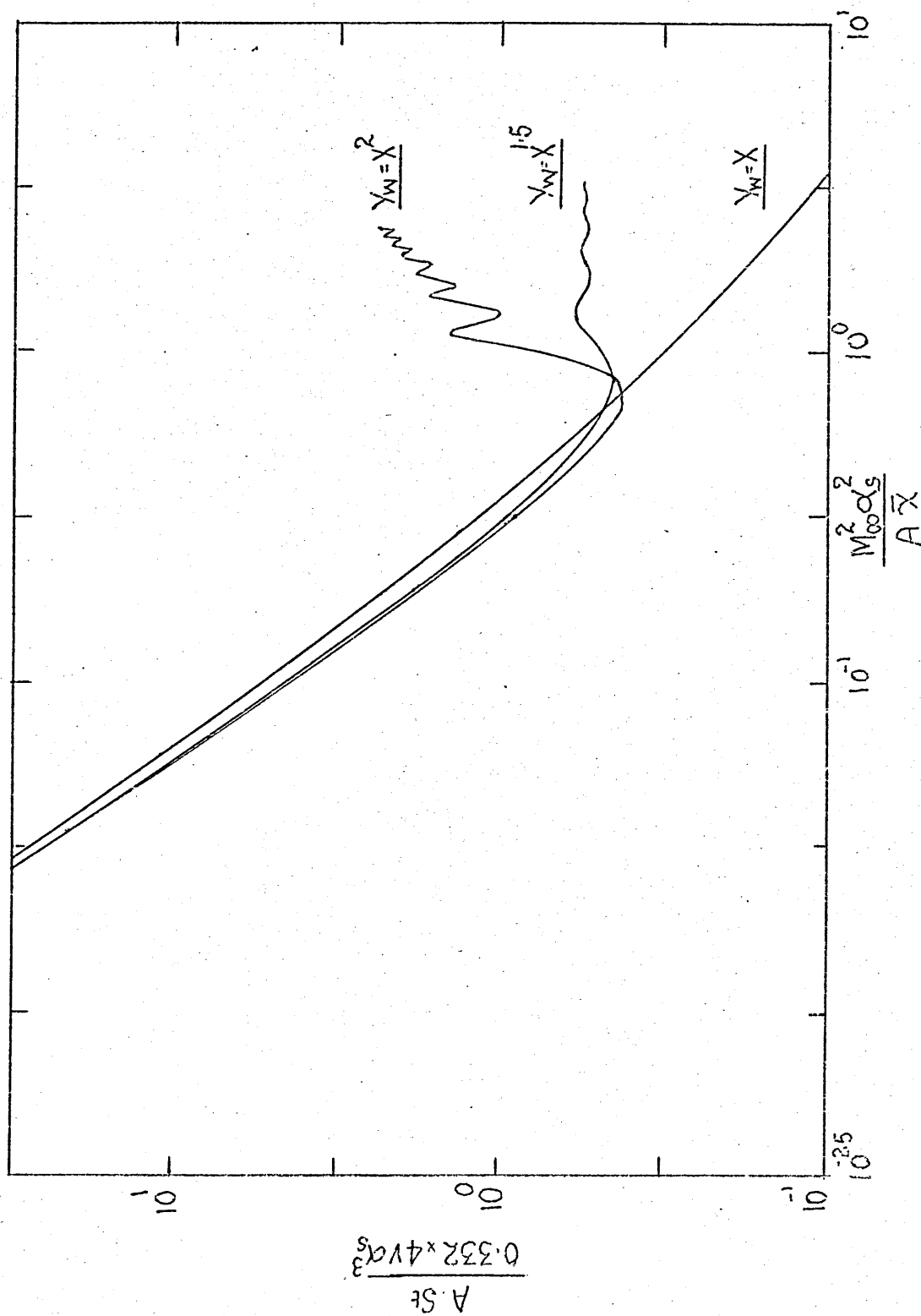


Fig 22b. Heat transfer to a family of shapes $Y_W = X^n$, according to TW + CFGL theory, $M_\infty \alpha_s = 1$, $\gamma = 1.4$.

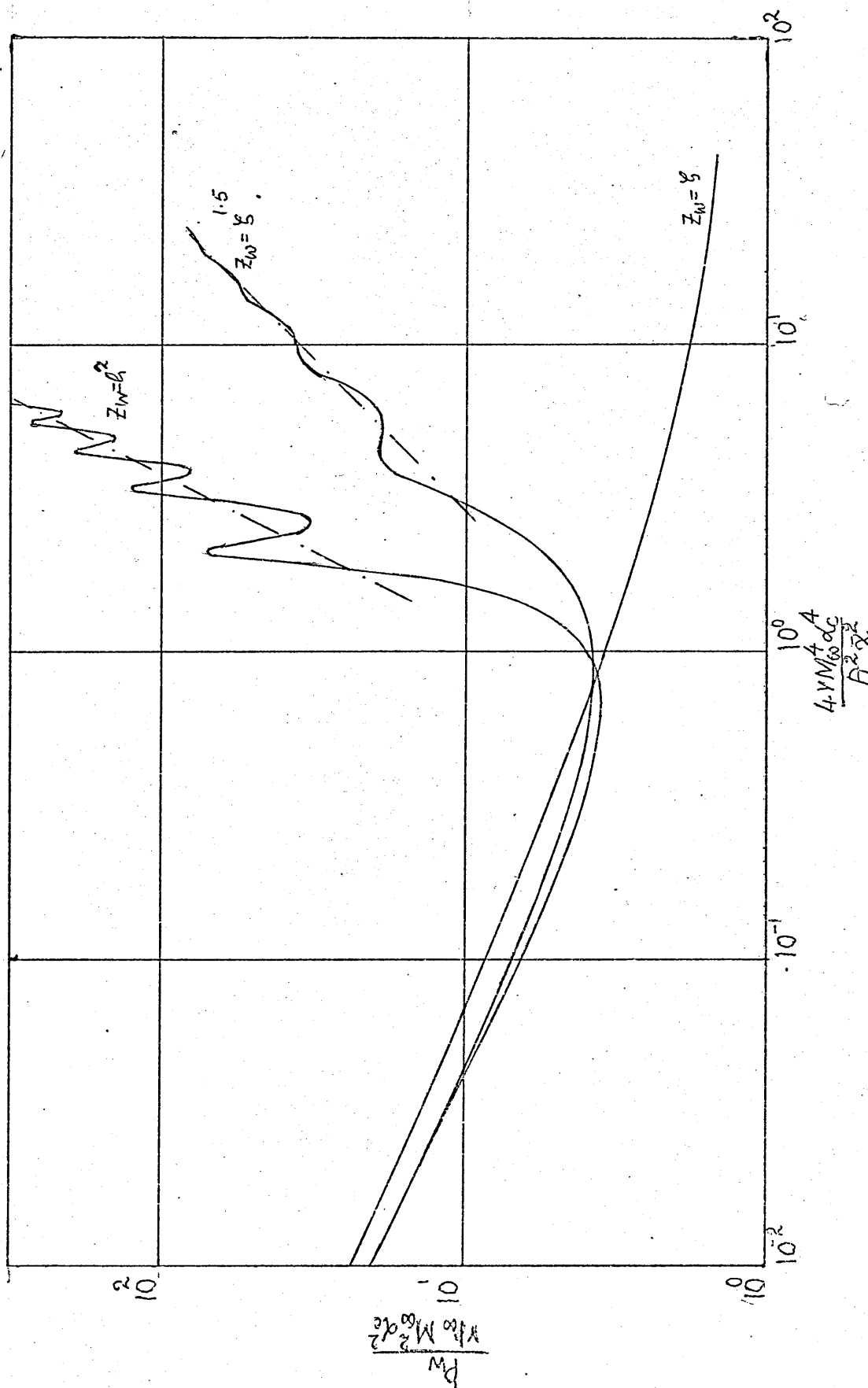


Fig 23a. Pressure distribution on family of shapes $z = z_w^n$, according to TW + CFGL theory ($M_\infty = 1.4$) and $r = 1.4$. --- Inviscid prediction.

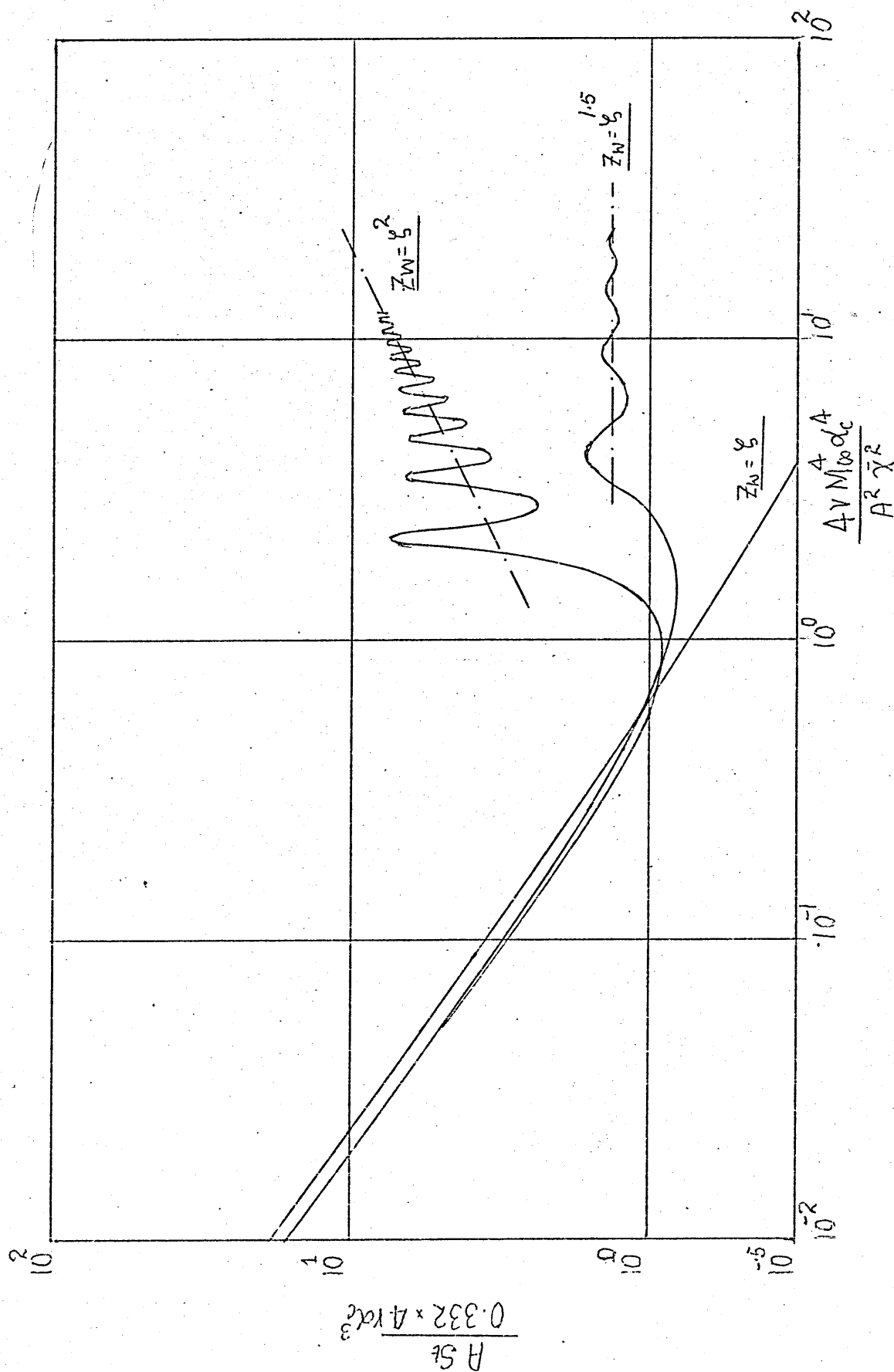


Fig 23b. Heat transfer to a family of shapes $z = z_w^n$, according to TW + CFGL theory ($M_{\infty} \rightarrow \infty$) and $\gamma = 1.4$. --- Asymptotic limit.

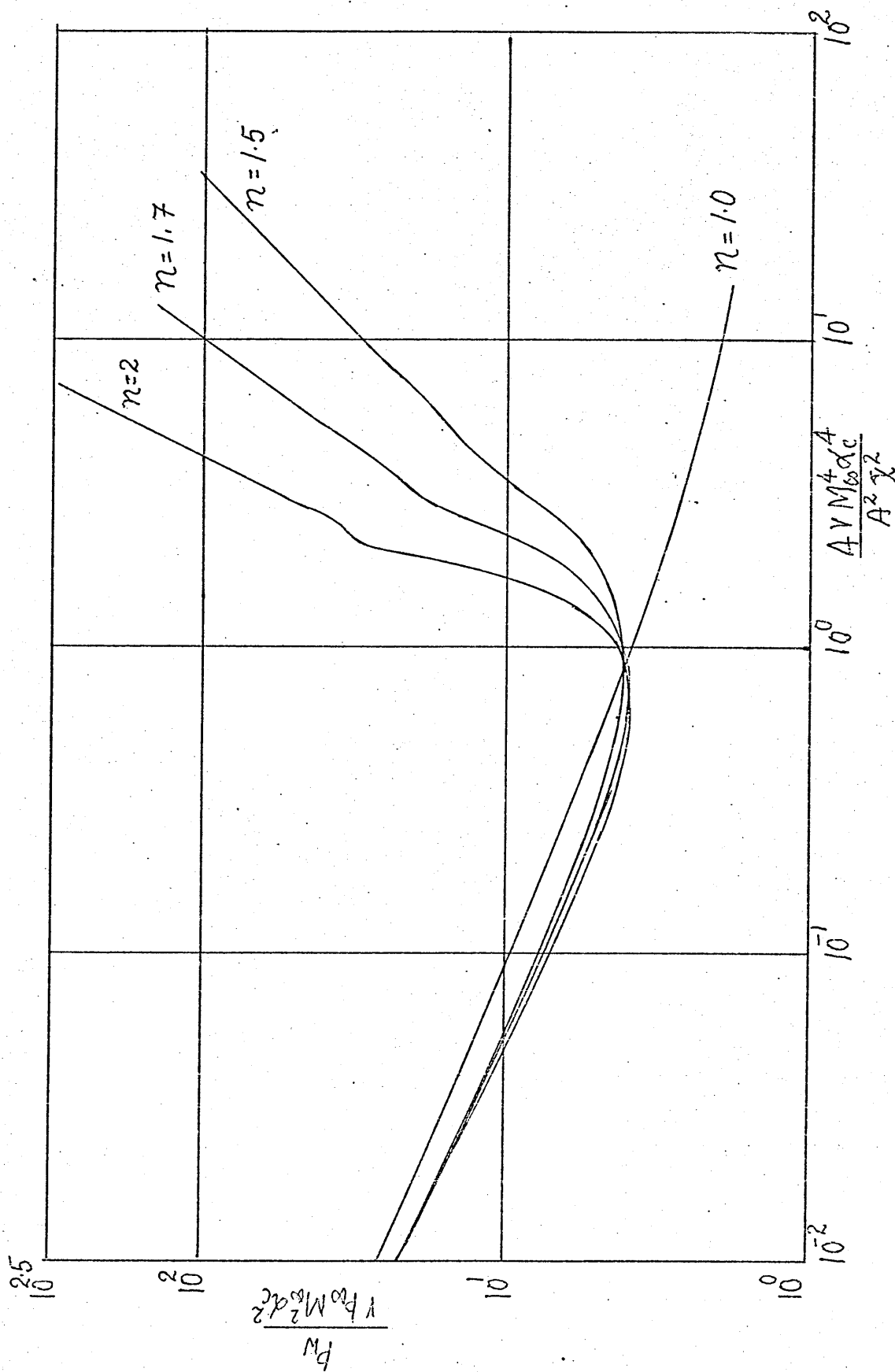


Fig 24. Pressure distribution on a family of shapes $z_w = z^n$, according to 2nd order tangent wedge theory; $\gamma = 1.4$.

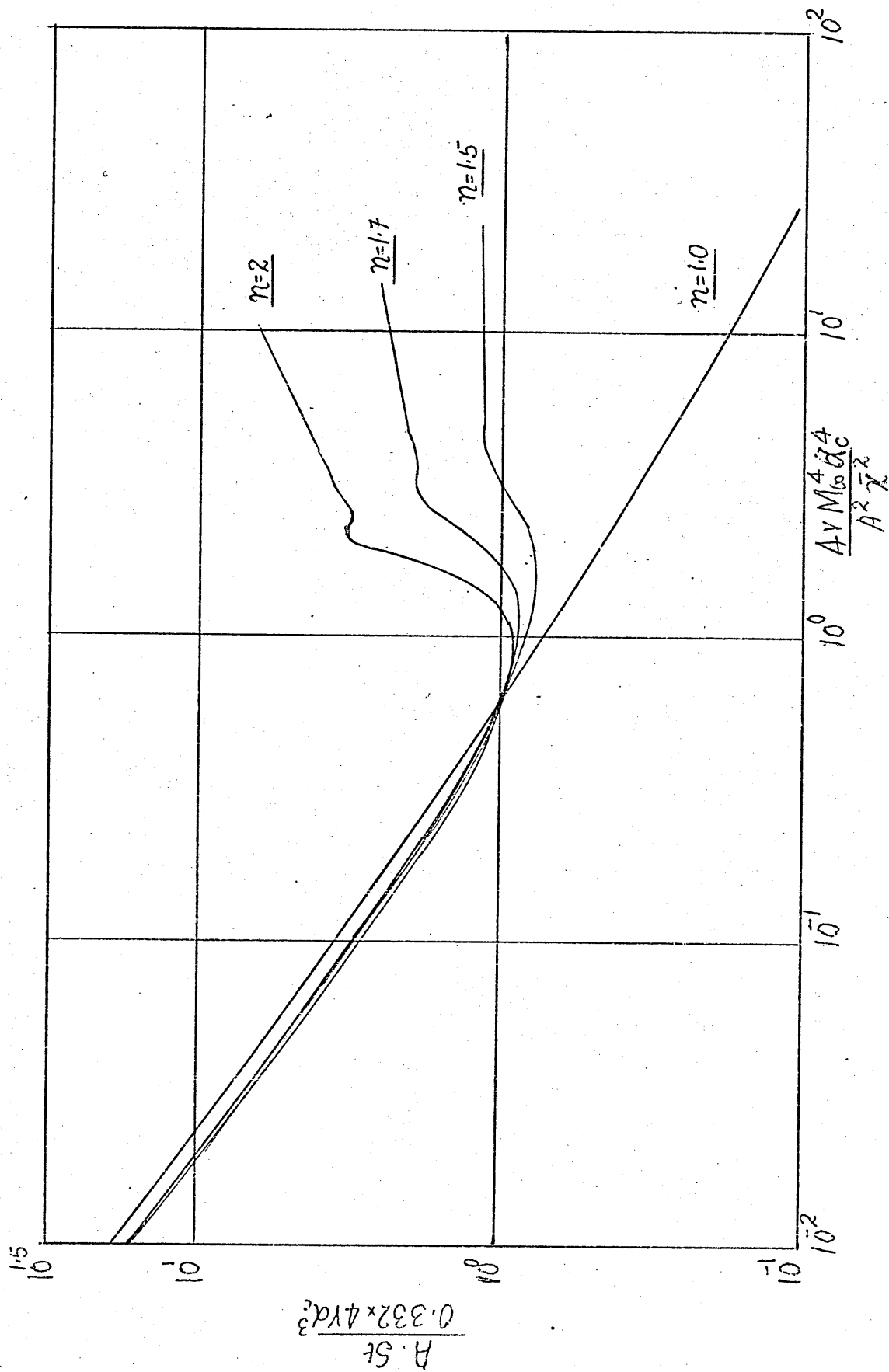


Fig 25. Heat transfer to a family of shapes $z_\gamma = z^n$, according to 2nd order tangent wedge theory; $\gamma = 1.4$.

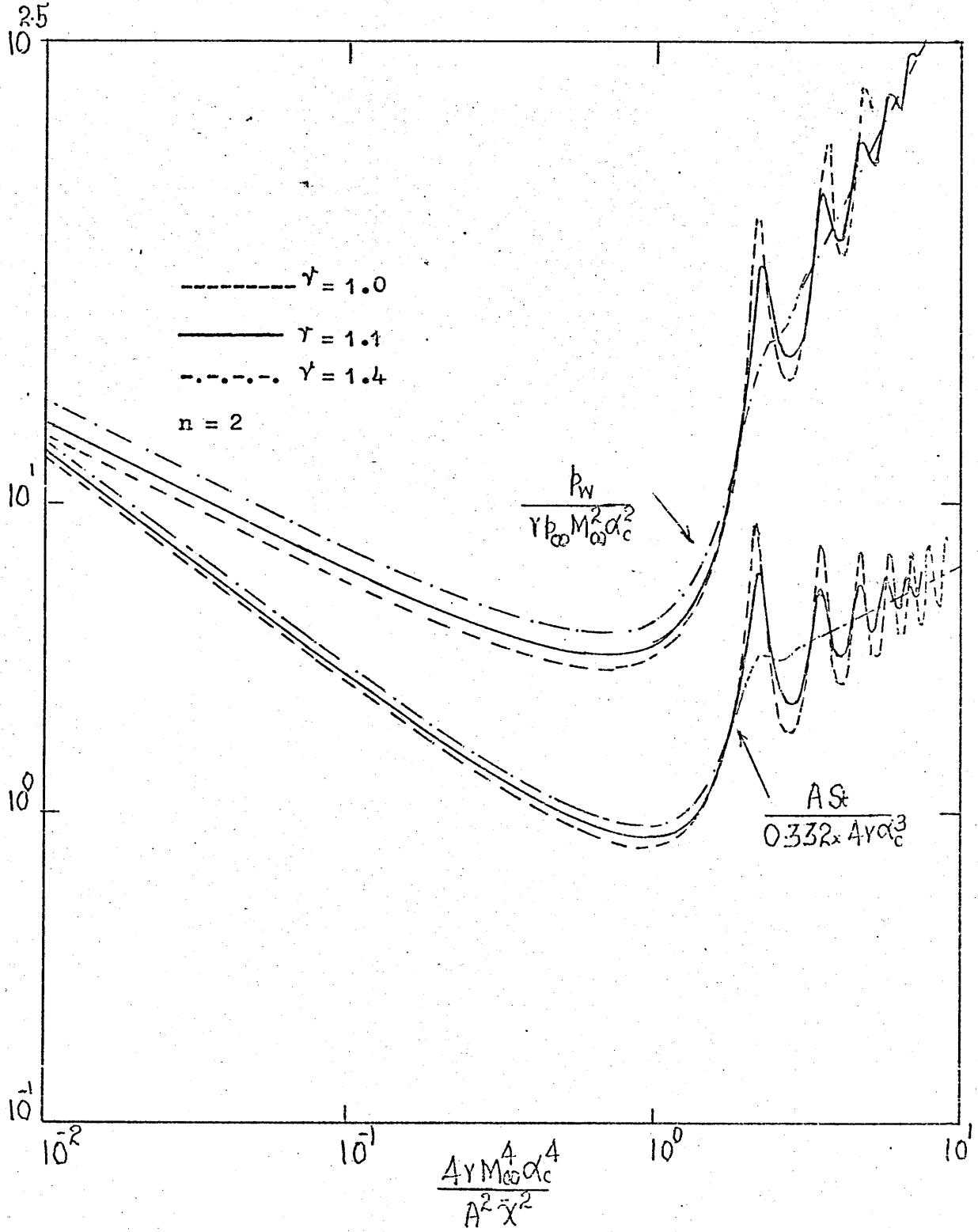


Fig 26. Effect of γ on pressure distribution and heat transfer rate on a concave surface $z_w = \xi^n$, according to 2nd order TW theory. ($a=1$, $b=5$ in Eq. 4.3.5)

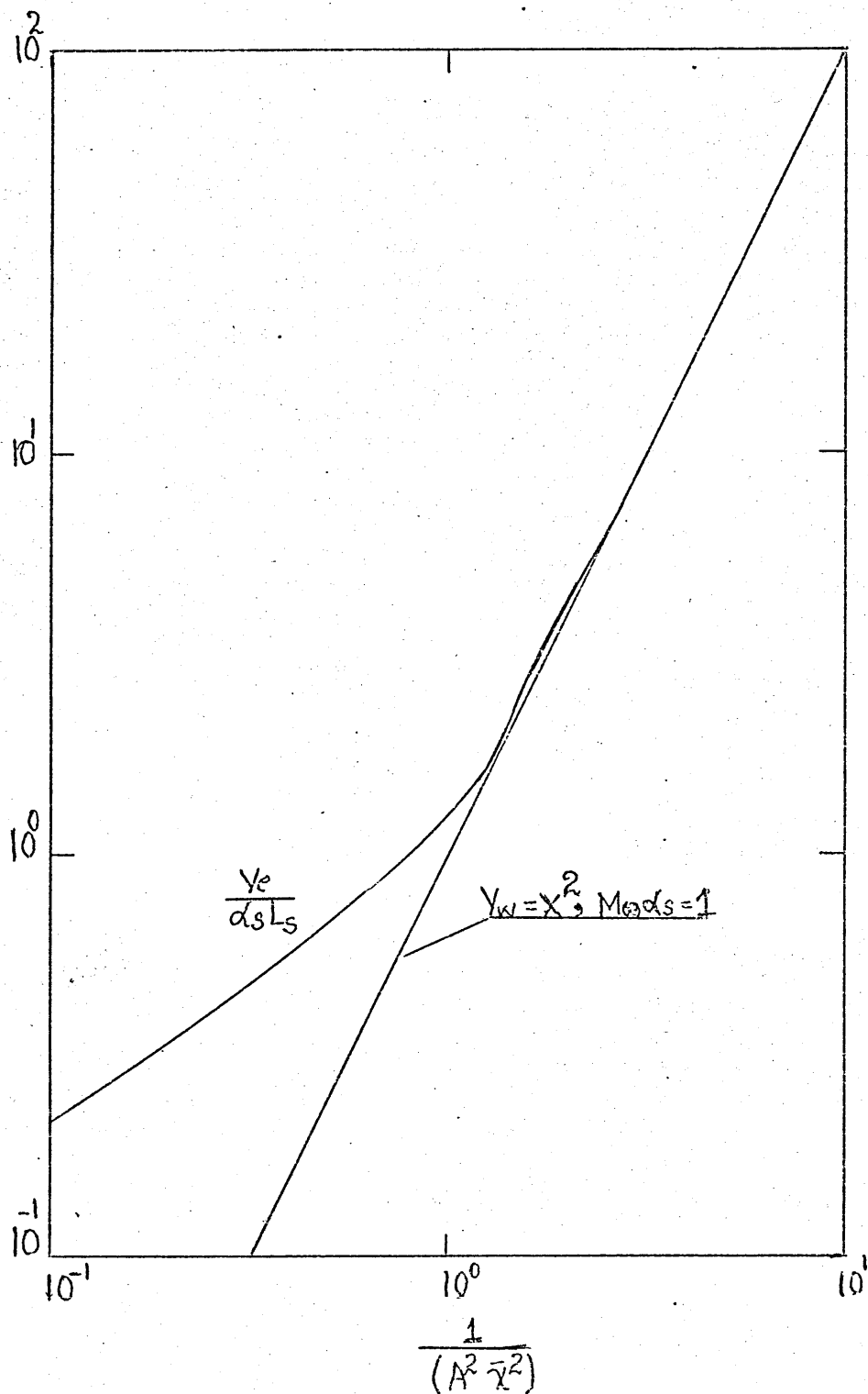


Fig 27. Variation of boundary thickness on surface $Y_w = X^2$ according to TW + CFGL theory, $\gamma = 1.4$.

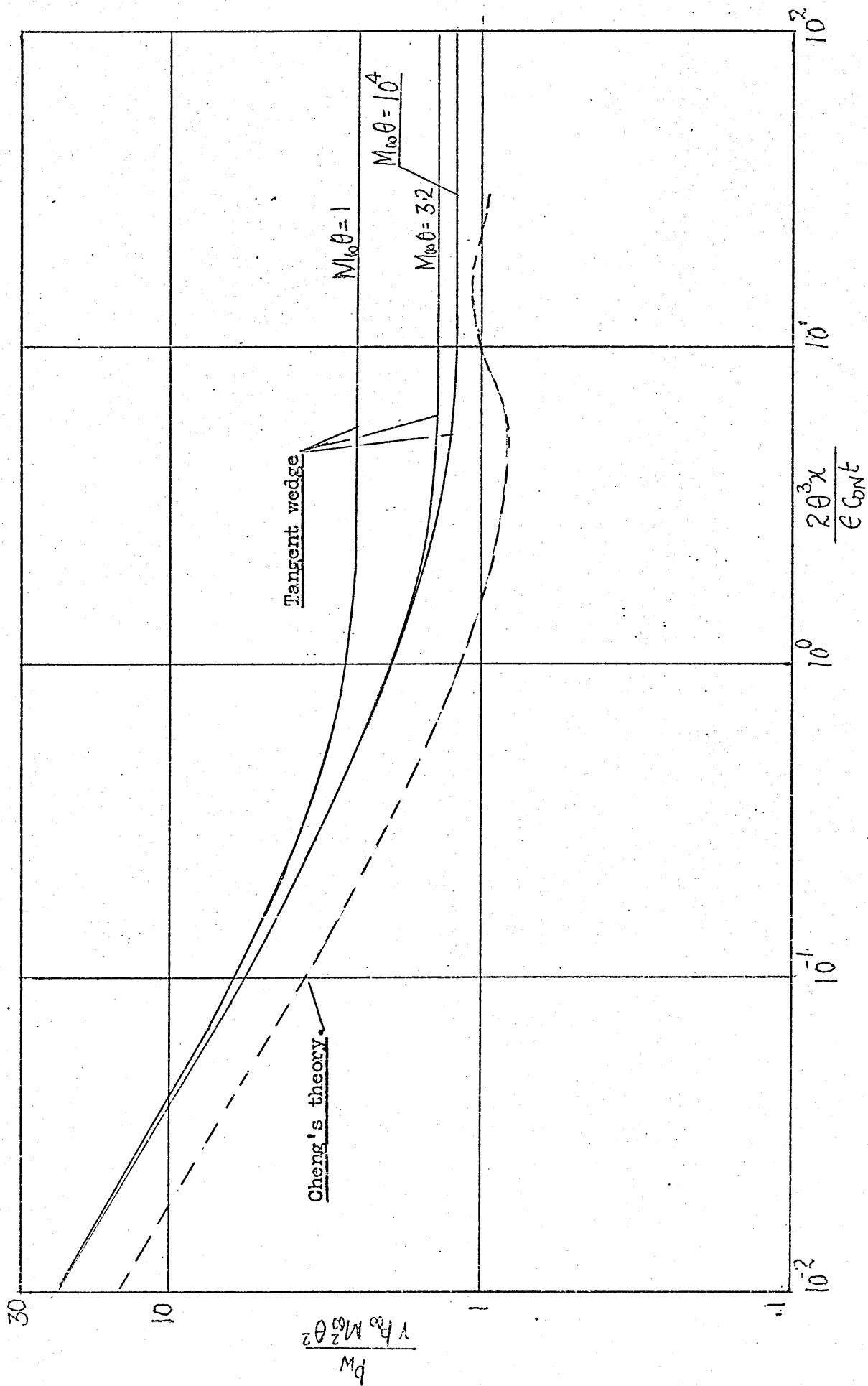


Fig. 28. Inviscid pressure distribution on a blunt flat plate at incidence. Comparison of TW (analytical solution) and Cheng's theory; $\gamma = 1.4$.

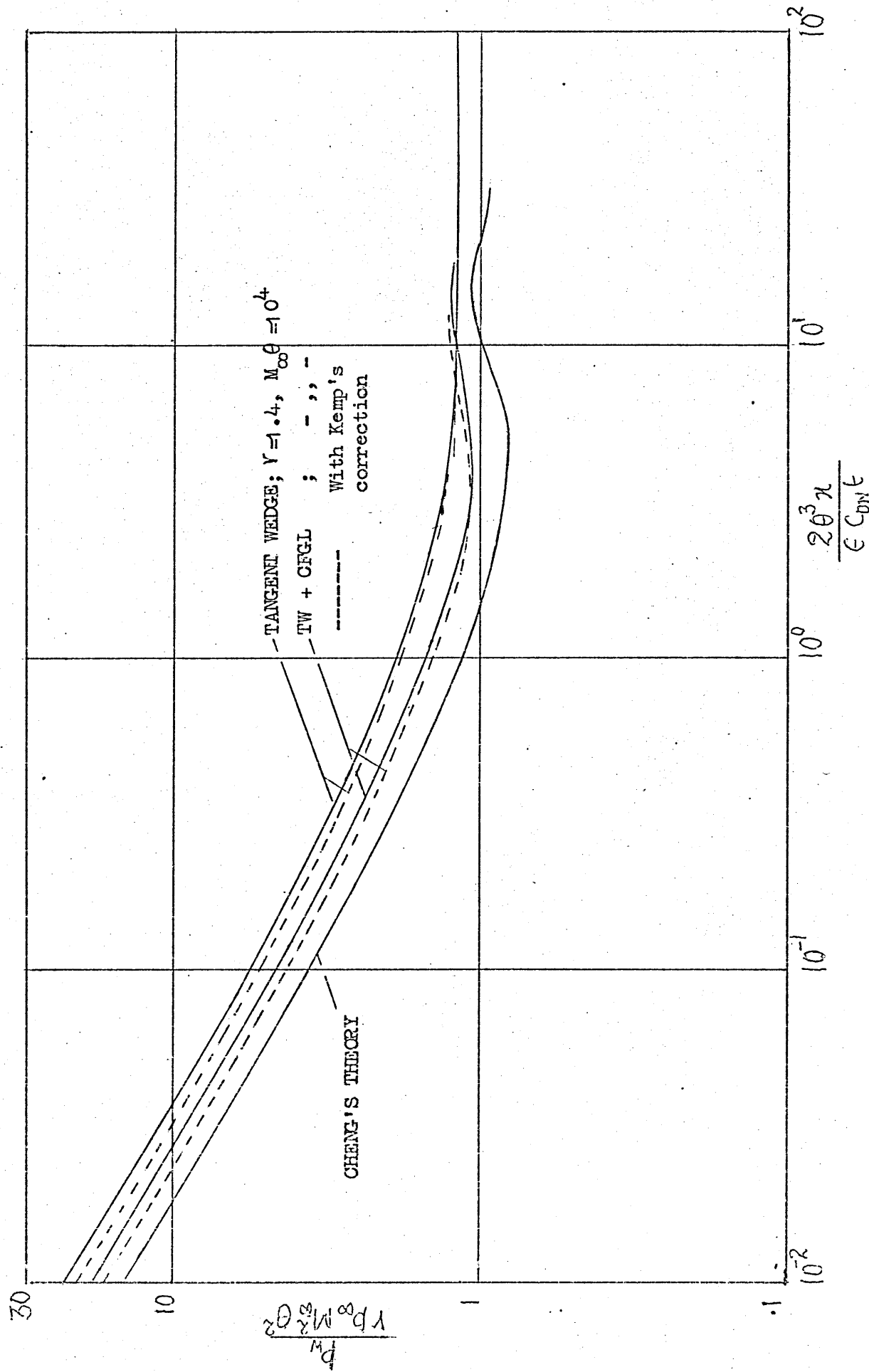


Fig 29. Inviscid pressure distribution on a blunt flat plate at incidence.

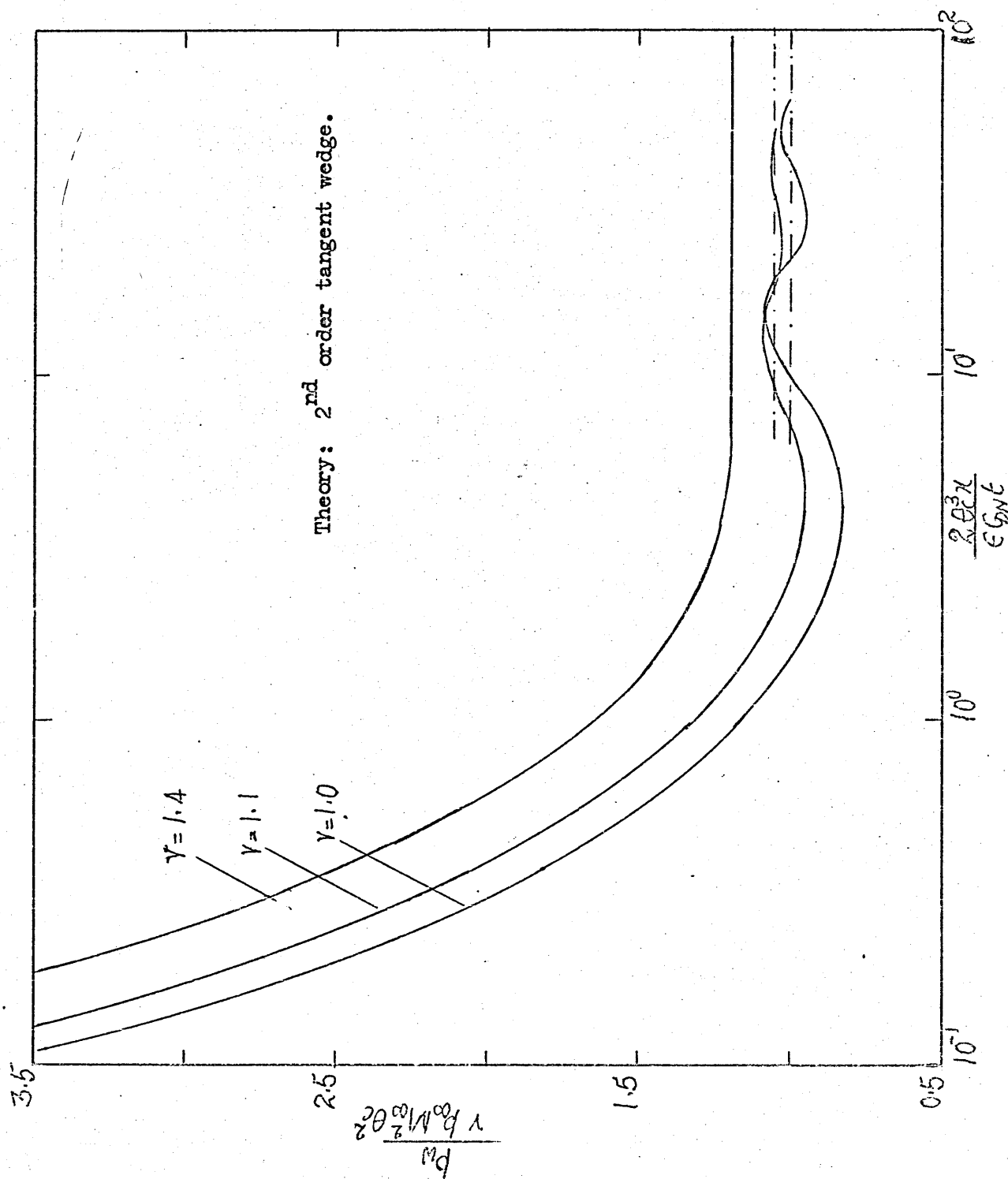


Fig 30. Pressure distribution on a bluted wedge .

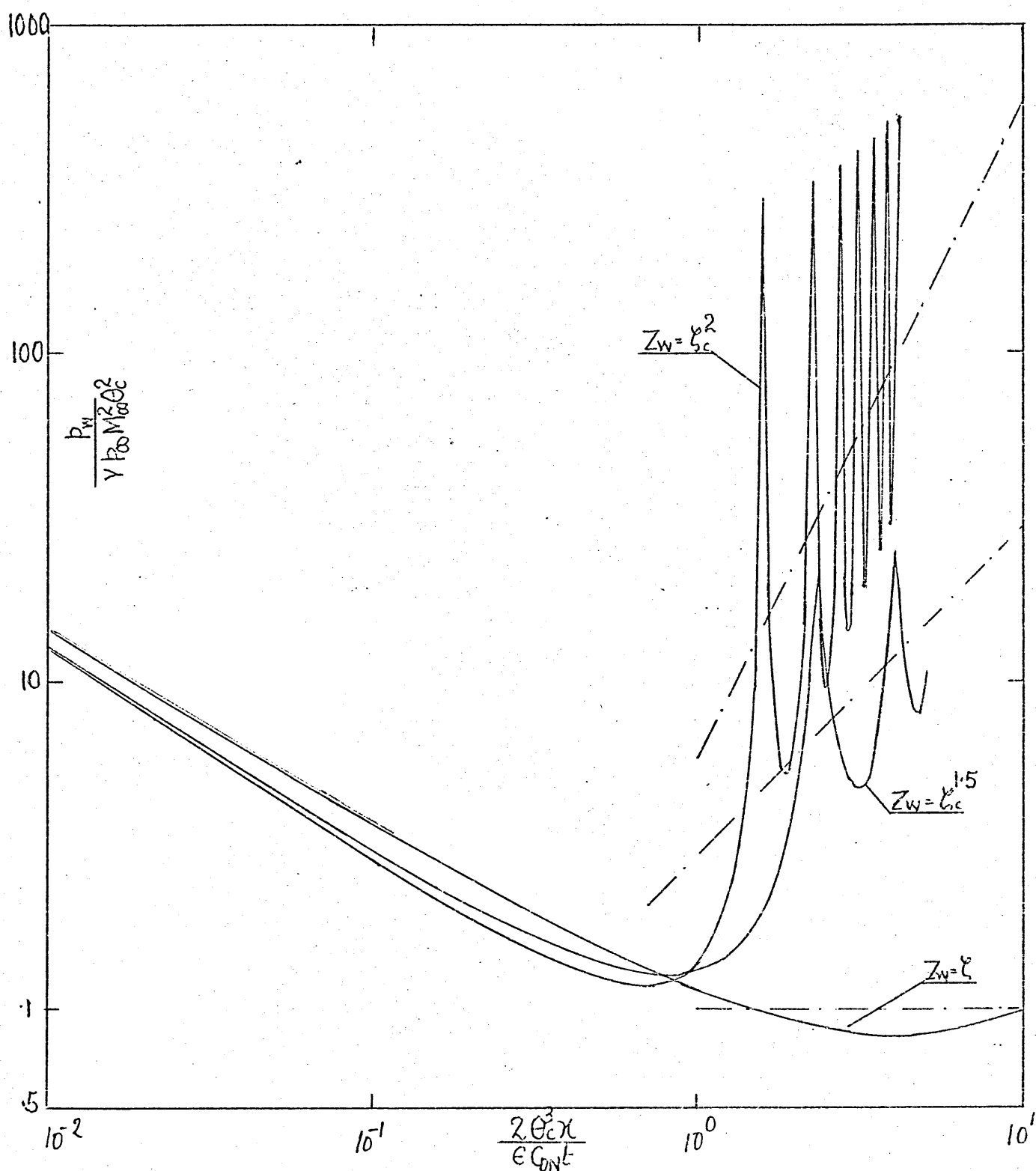


Fig 31. Effect of leading edge bluntness on inviscid pressure distribution over concave surfaces (Cheng's theory).

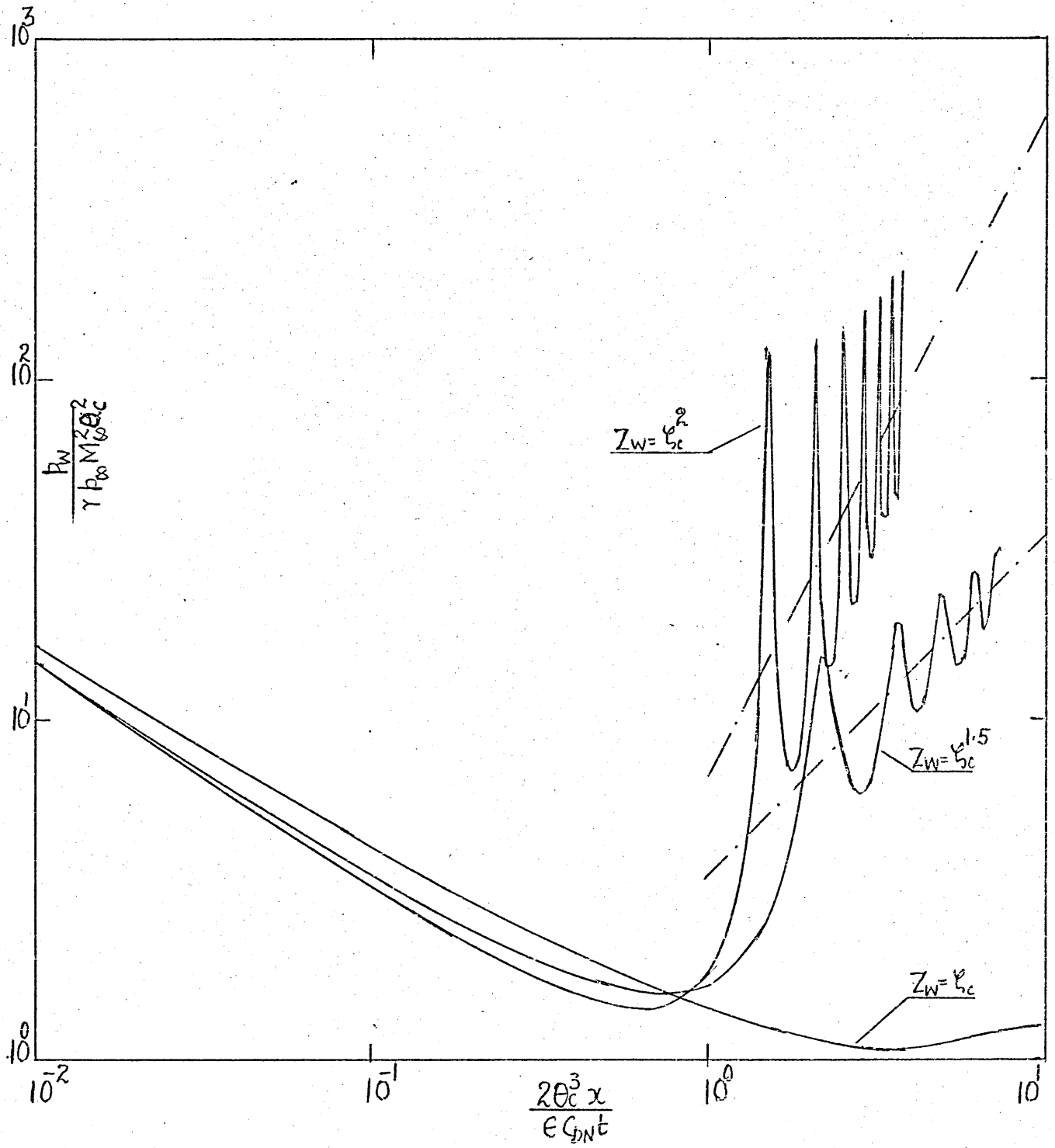


Fig 32. Inviscid pressure distribution on concave surfaces with blunt leading edge according to TW + CFGL theory for $M_\infty \theta_c \rightarrow \infty$, $\gamma = 1.4$.

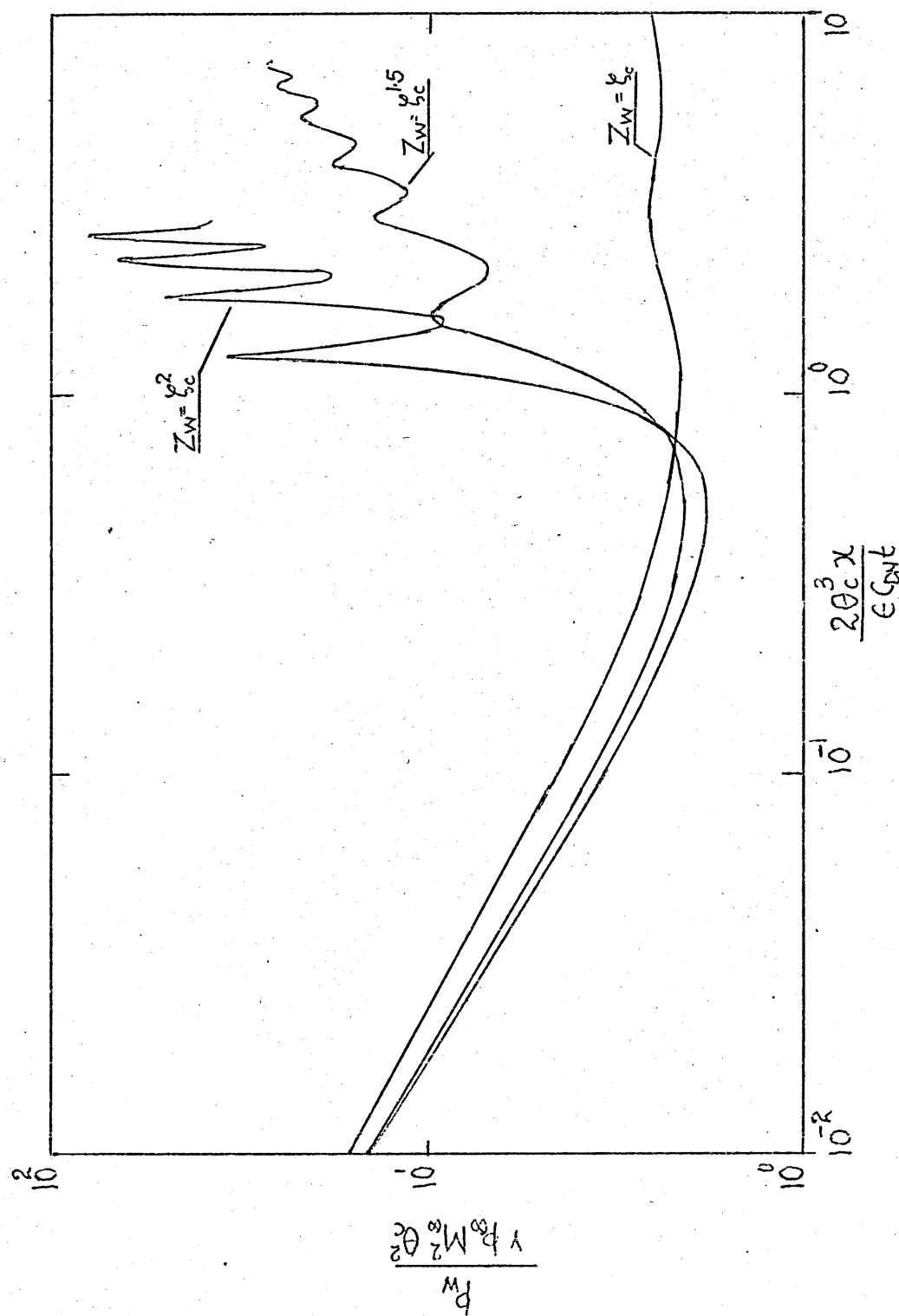


Fig 33. Inviscid pressure distribution on concave surfaces $Z_n = \zeta_c^n$ with blunt leading edge according to $TW + QFGL$ theory; $\gamma = 1.4$, $M_\infty \theta_c = 1$.

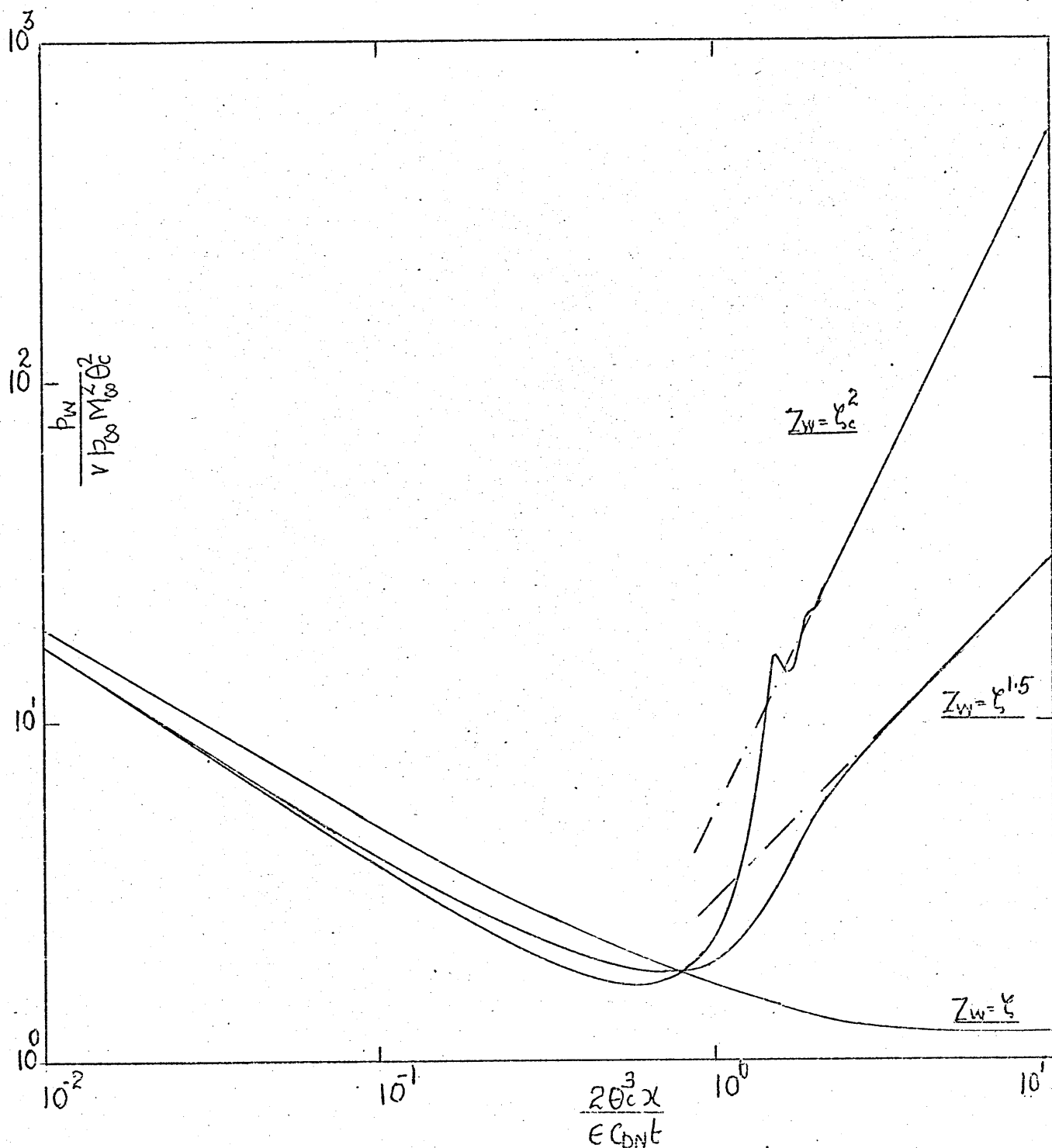


Fig 34. Inviscid pressure distribution on concave surfaces with blunt leading edge according to 2nd order tangent wedge theory, $\Gamma = 1.4$.

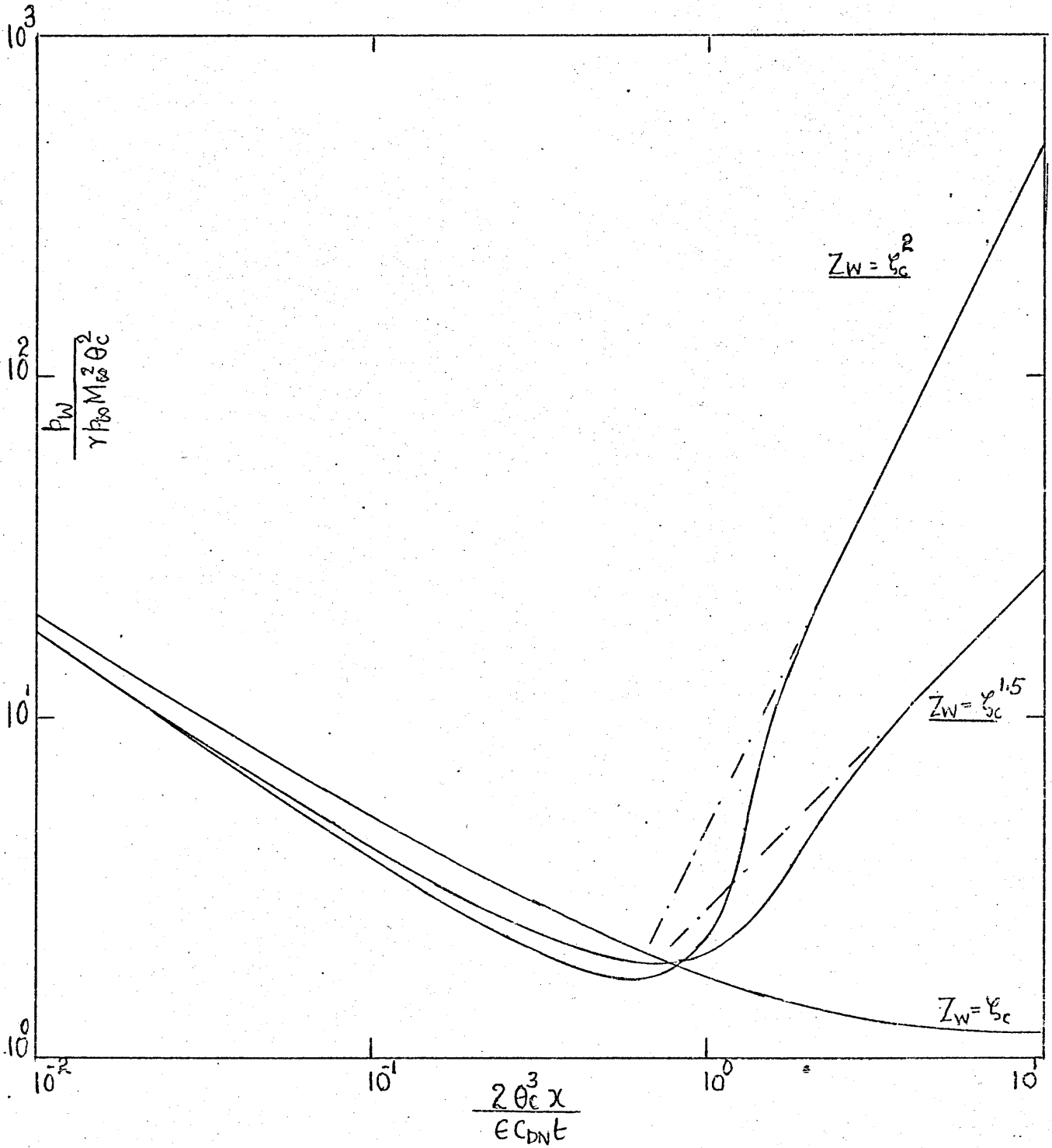


Fig 35. Inviscid pressure distribution on concave surfaces with blunt leading edge using tangent wedge approximation.
 $\gamma = 1.4$, $M_\infty \theta_c = 10^4$.

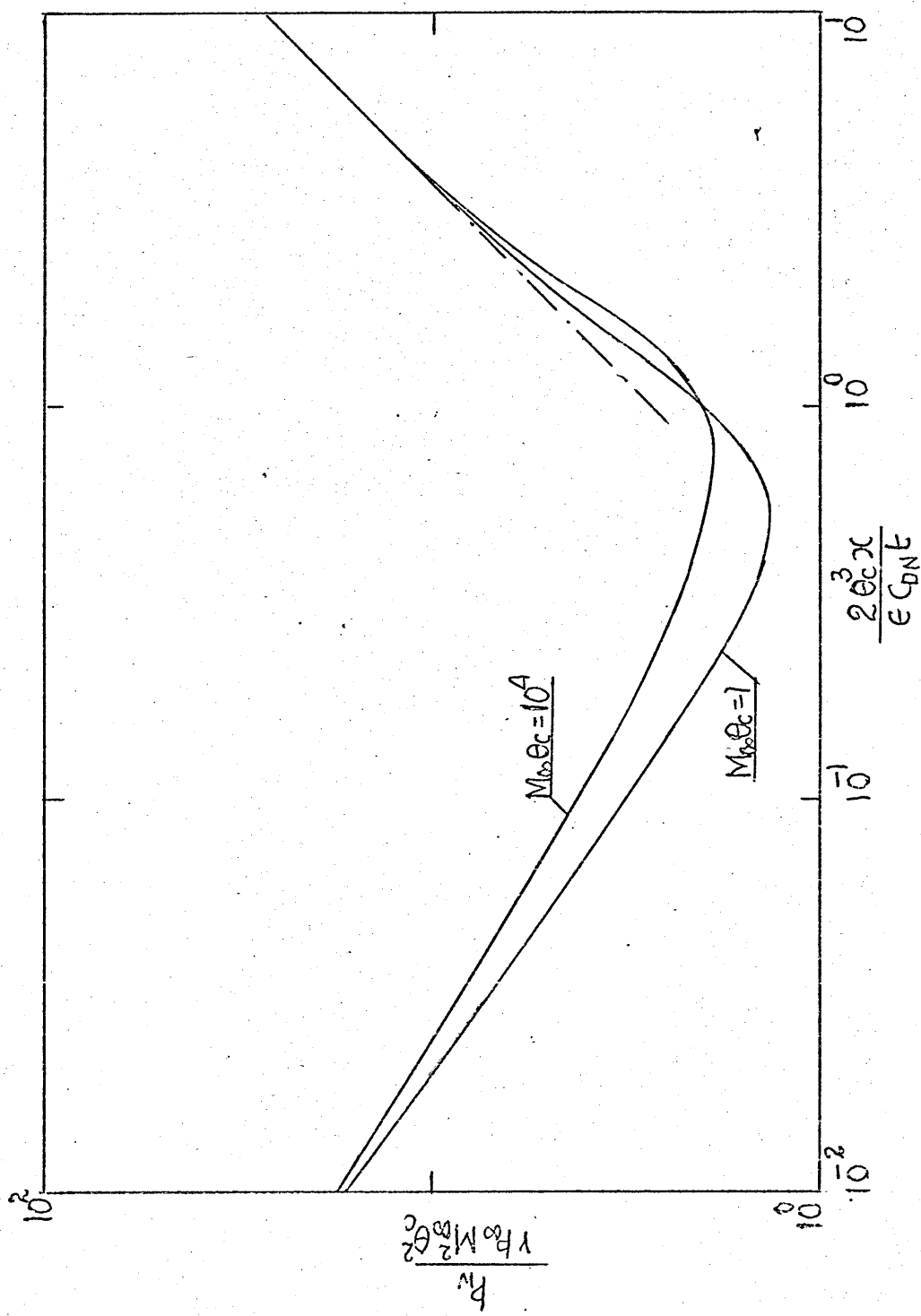


Fig 36. Effect of $M_\infty \theta_c$ on concave surface $Z = \psi_c^{1.5}$ with blunt leading edge using tangent wedge approximation. $\gamma = 1.4$.

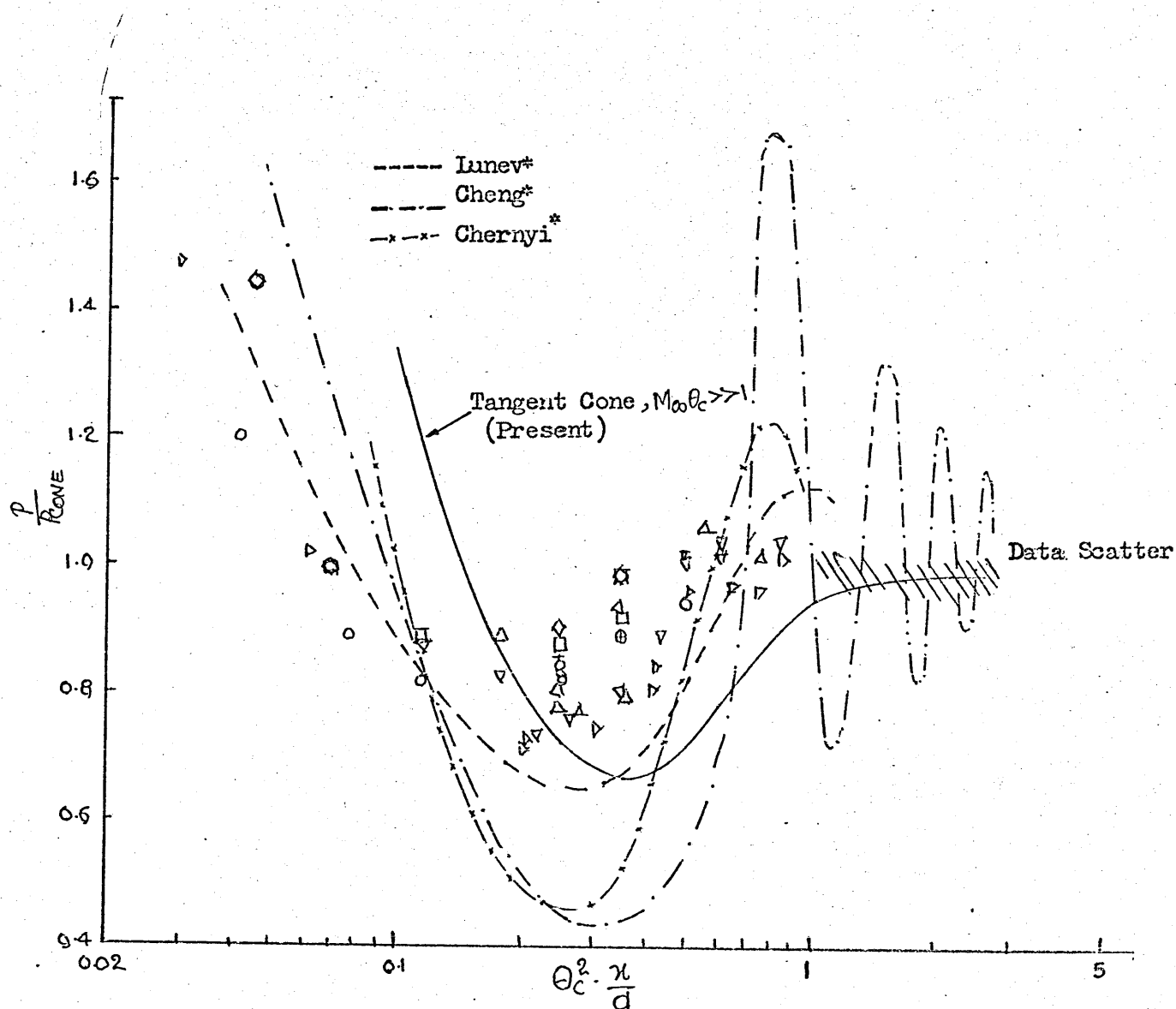


Fig 37: Inviscid flow about a blunt cone, $\gamma = 1.4$, $C_{DN} \approx 1$

(* & other experimental data
from Dewey, Reference 76)

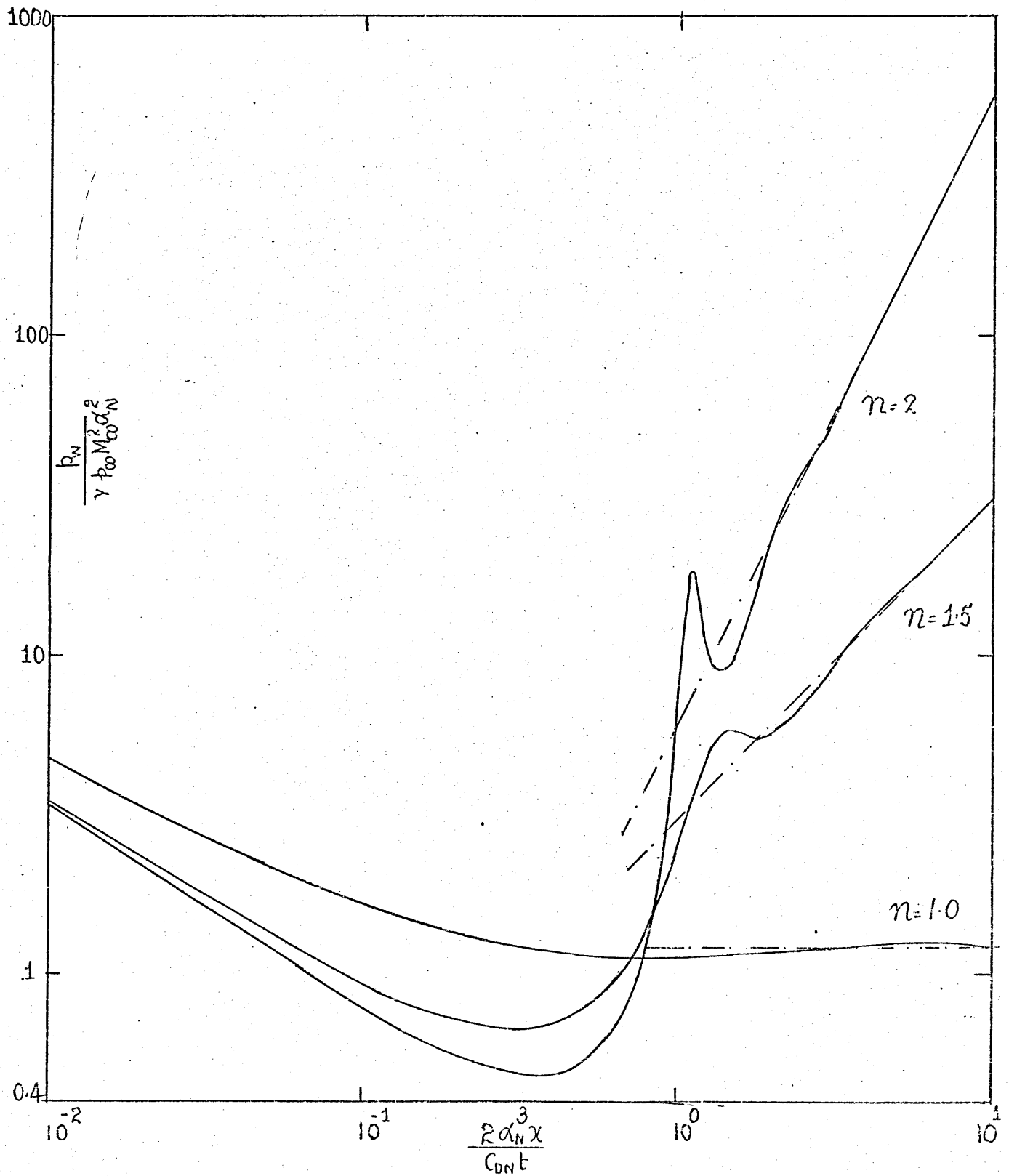


Fig 38. Inviscid surface pressure distribution on a family of shapes $z_b = X_H^n$, with blunt leading edge, using Chernyi's method.
 ($M_{\infty} \alpha_N \rightarrow \infty$, $\gamma \approx 1.4$)

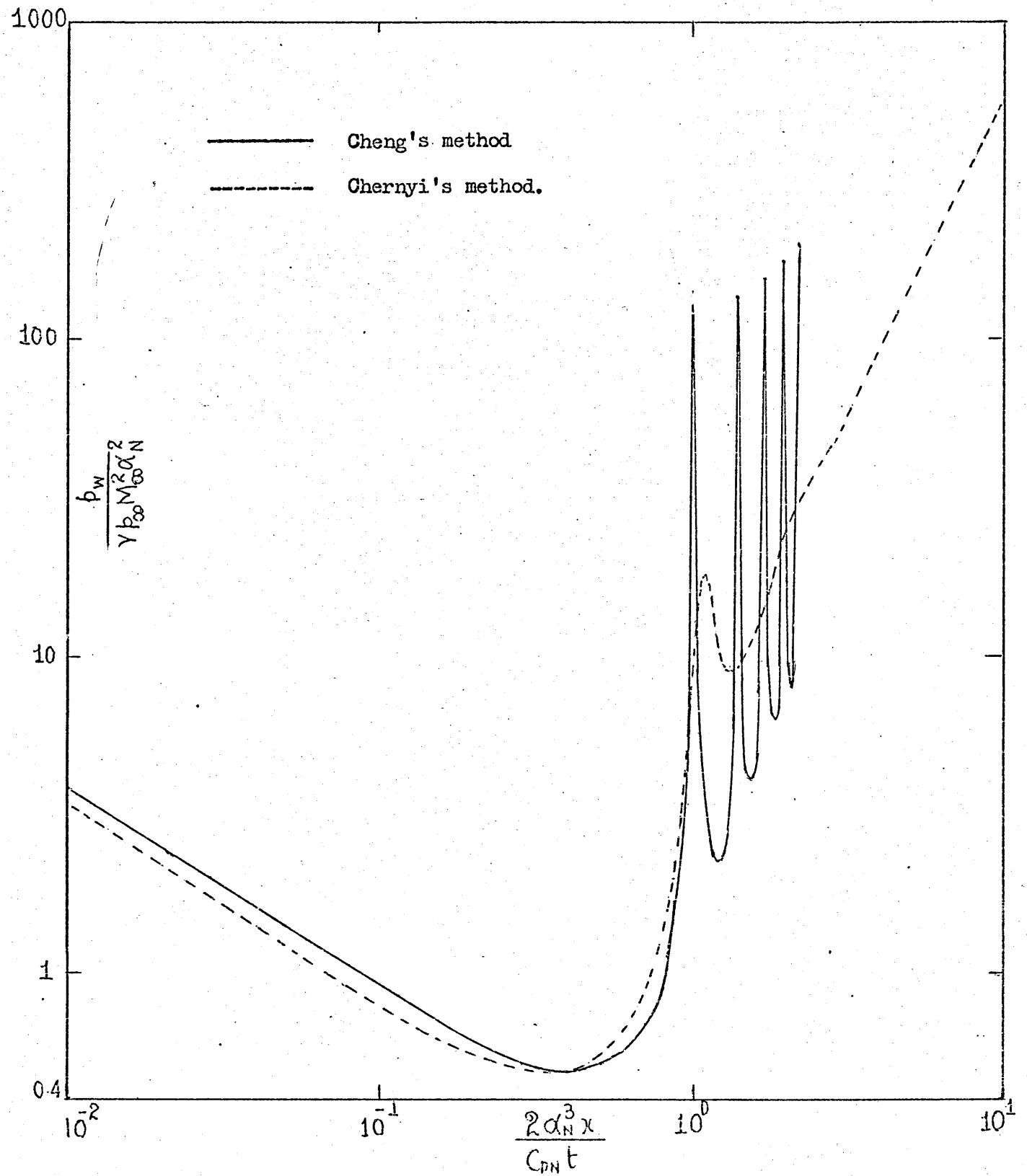


Fig 39. Comparison of Cheng's and Chernyi's method for pressure distribution on concave surface $z_b = X_N^n$, with blunt leading edge.
 ($M_\infty \alpha_N \rightarrow \infty$, $\gamma = 1.4$)

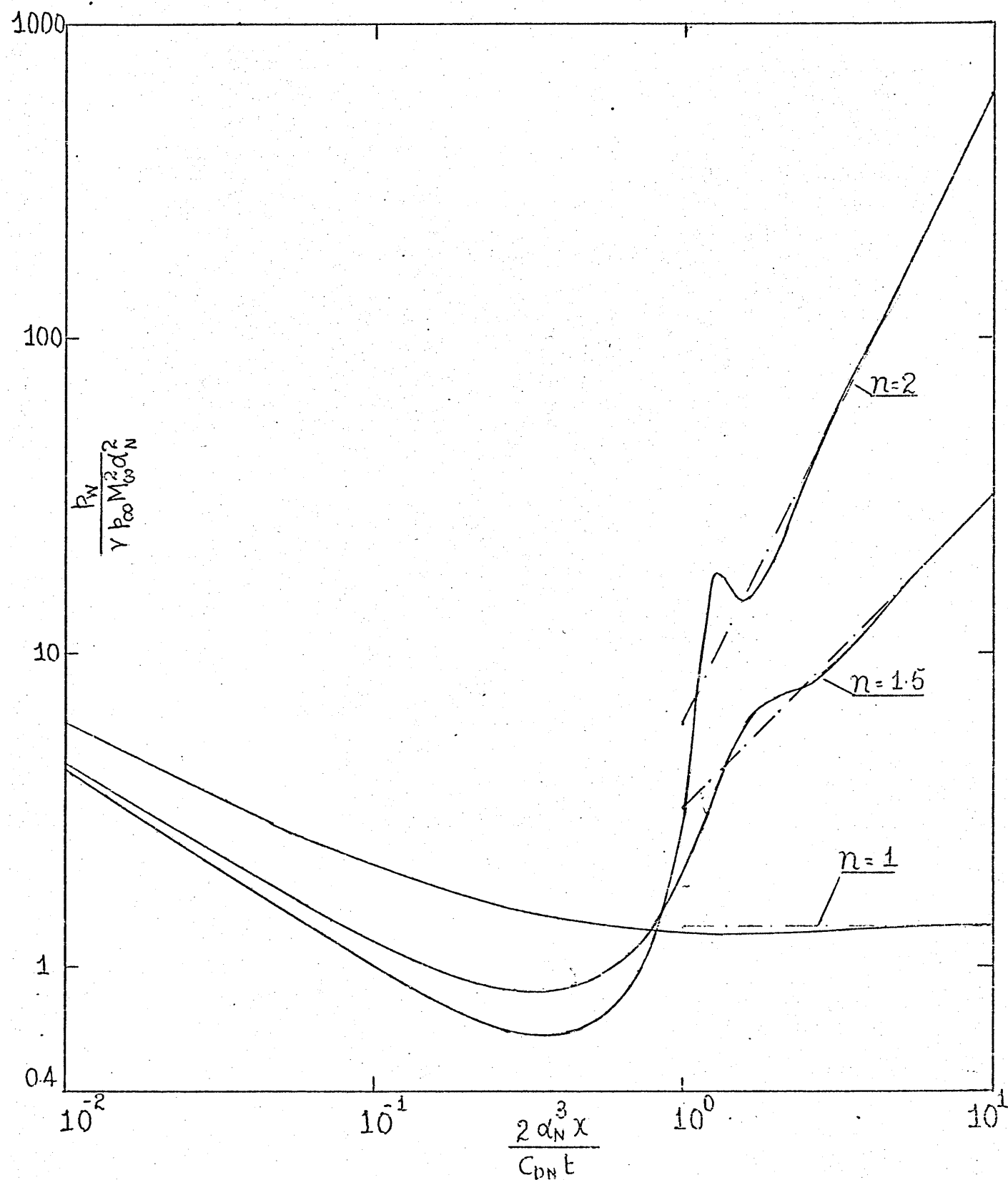


Fig 40. Inviscid surface pressure distribution on a family of shapes $z_b = X_N^n$, with blunt leading edge, using Chernyi's method. ($M_\infty \alpha_N \rightarrow 0$, $\gamma = 5/3$.)

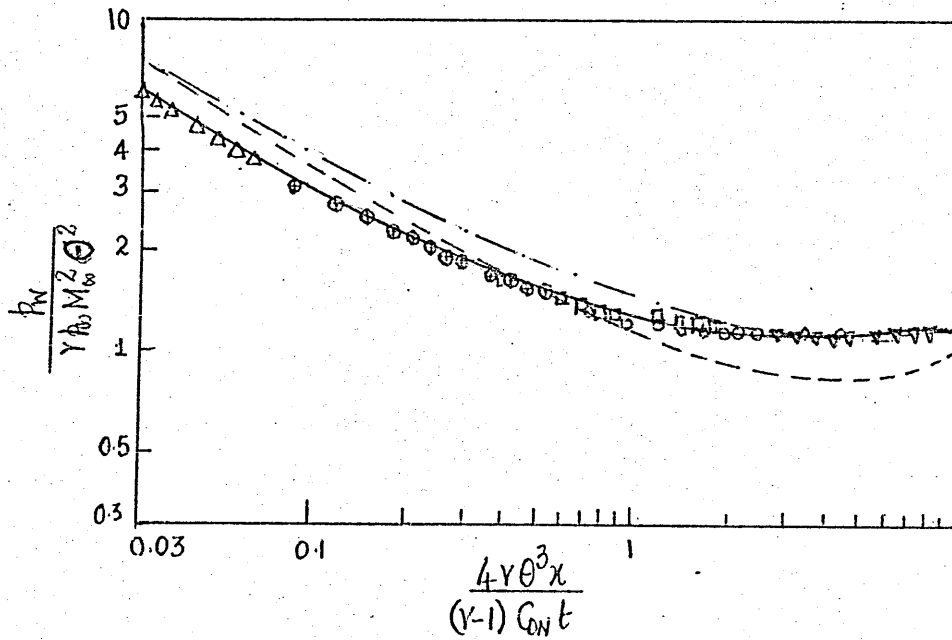


Fig 41: Pressure distribution on a blunted wedge. $\gamma = 1.4$

- Cheng's zeroth theory *
- Cheng & Kirsch* (Ref 42)
- · - · - Chernyi's Method
(from present calculations)

Δ , 5° ; $\oplus$, 10° ; $\square$, 15° ; $\circ$, 20° ; ∇ , 25° *
(Characteristics Solution: Cleary & Axelson
Reference 43)

(* taken from figure 4 of Ref 42)

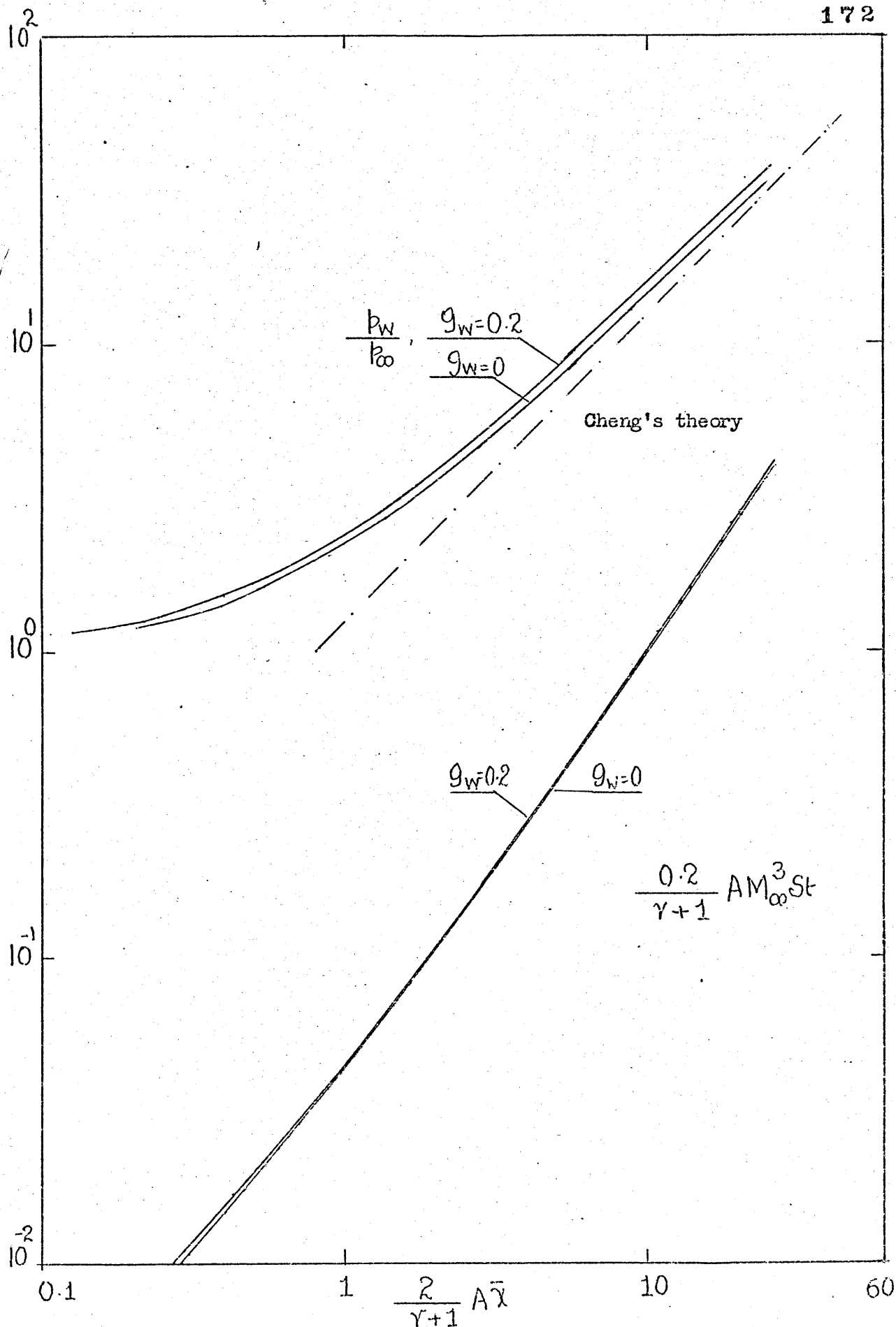


Fig 42. Pressure and heat transfer on a sharp flat plate at zero incidence; $\gamma = 1.4$ (Using Chernyi's method)

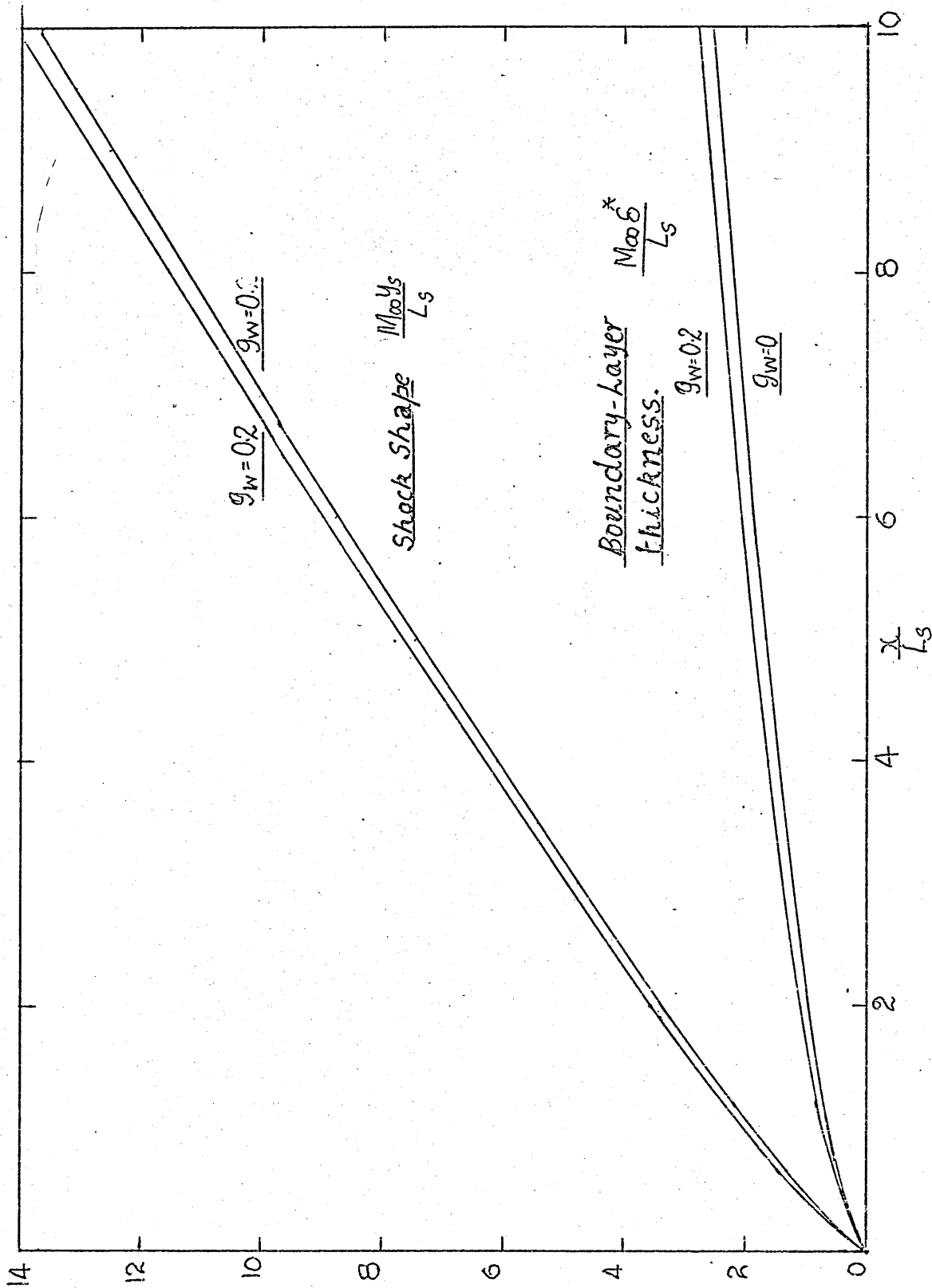


Fig 43. Shock shape and boundary-layer growth on a sharp flat plate at zero incidence (Using Chernyi's method), $\gamma = 1.4$.

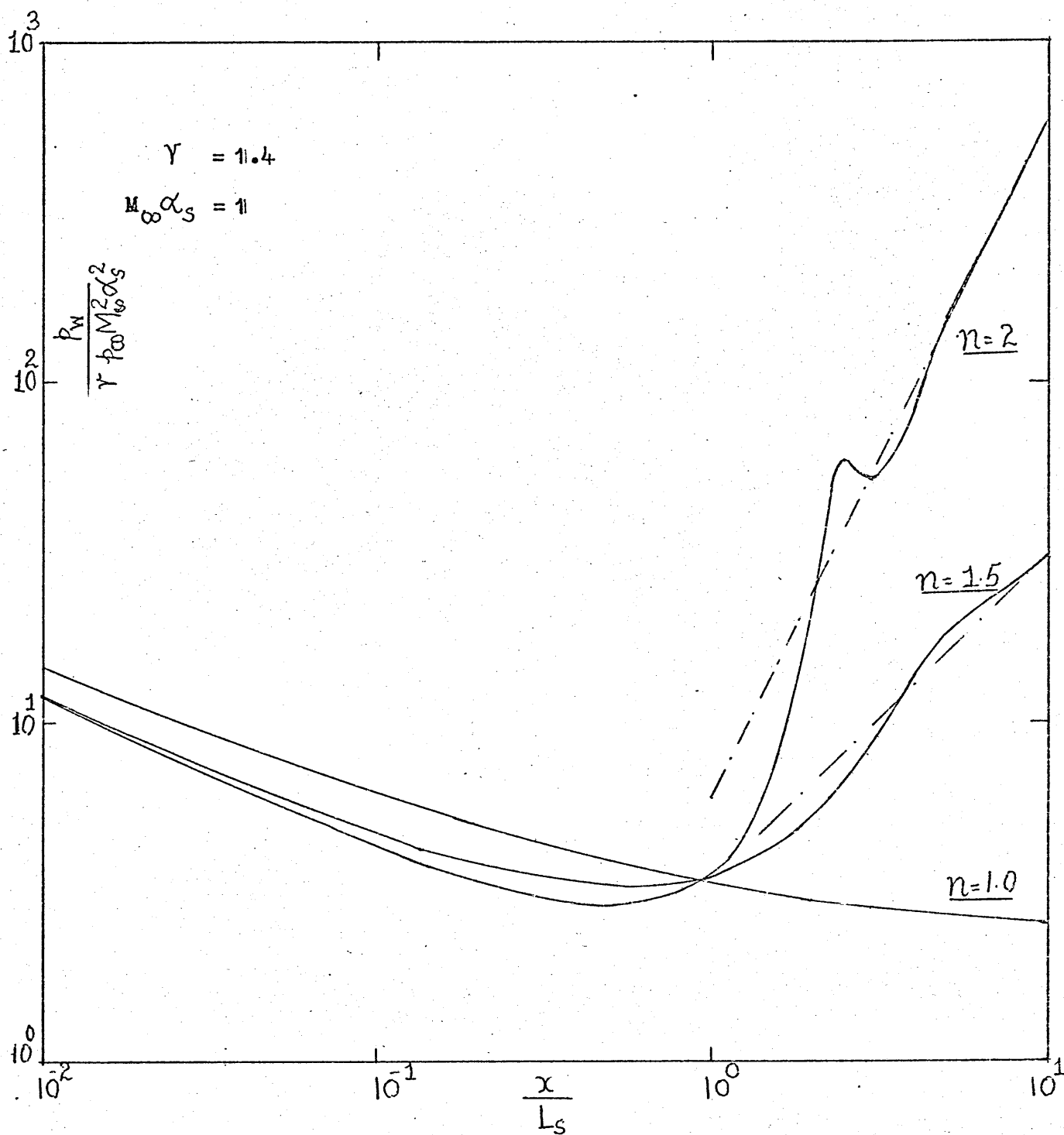


Fig 44. Pressure distribution on a family of shapes $Y_w = x^n$
 (Viscous interaction theory using Chernyi's method
 with Lees' boundary-layer theory)

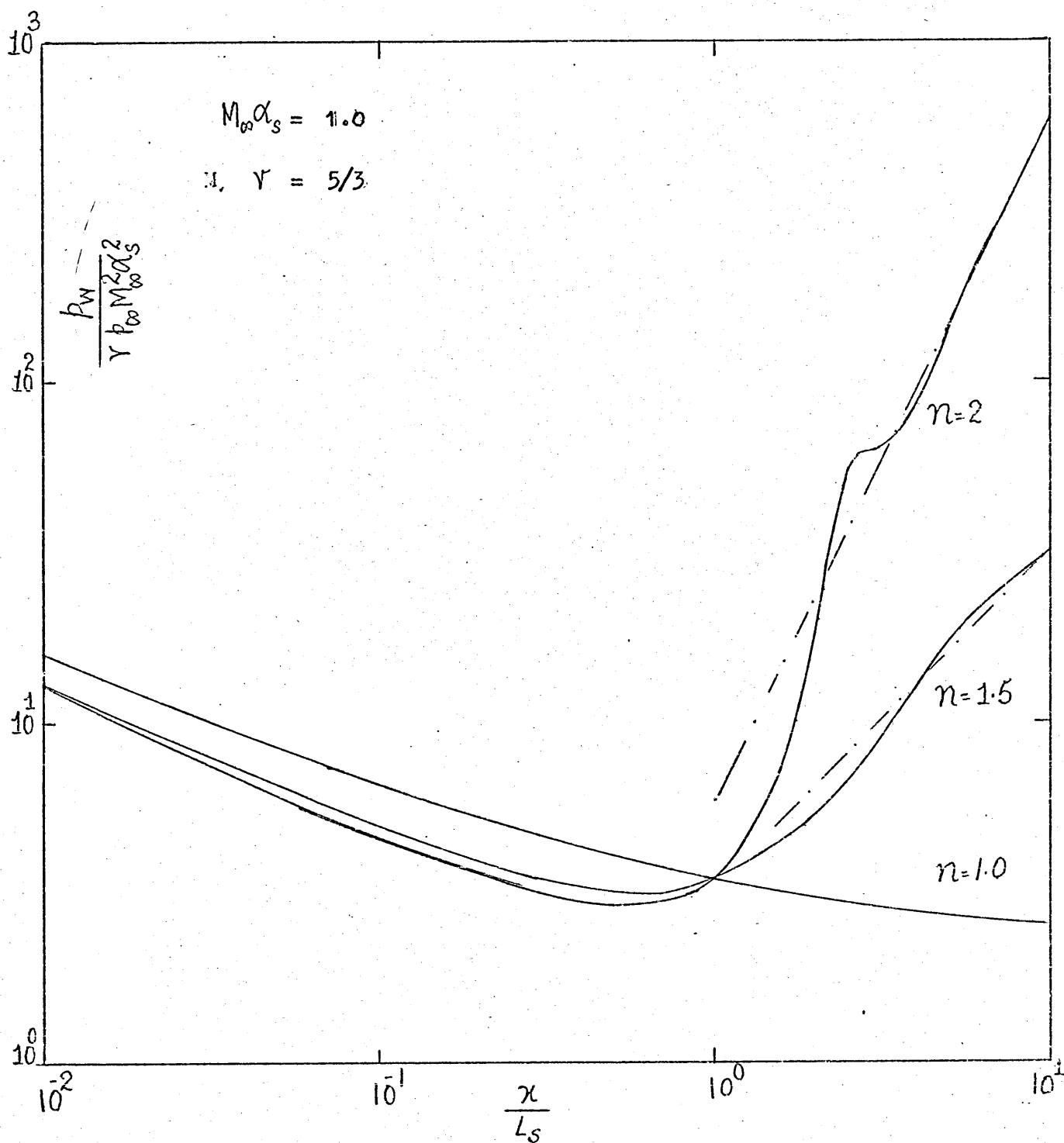


Fig 45. Pressure distribution on a family of shapes $Y_w = X^n$
 (Viscous interaction theory using Chernyi's method
 with Lees' boundary-layer theory)

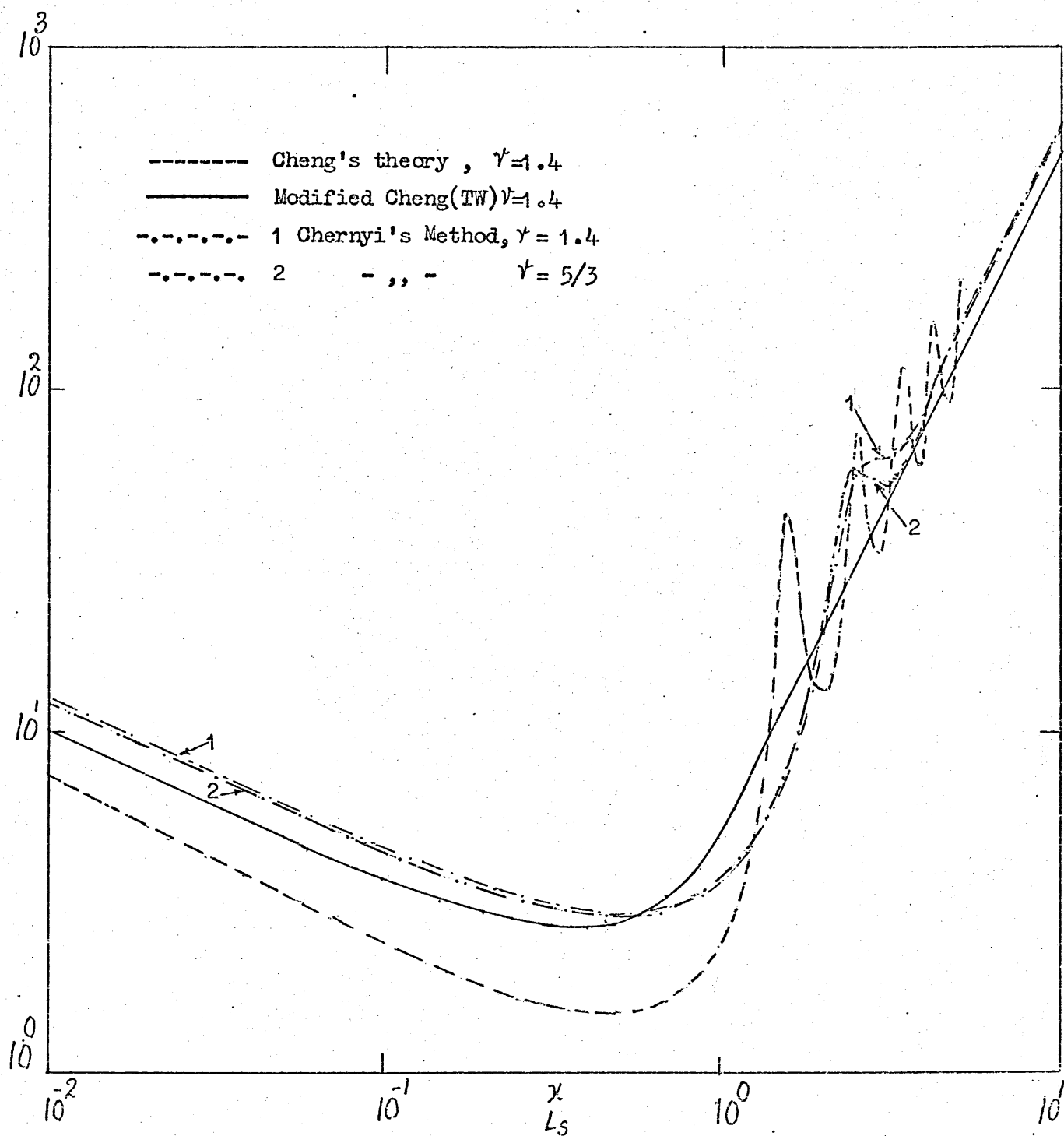


Fig 46. Pressure distribution on concave surface $Y_w = X^2$,
 $M_\infty \alpha_s = 1$. Comparison of various theories.

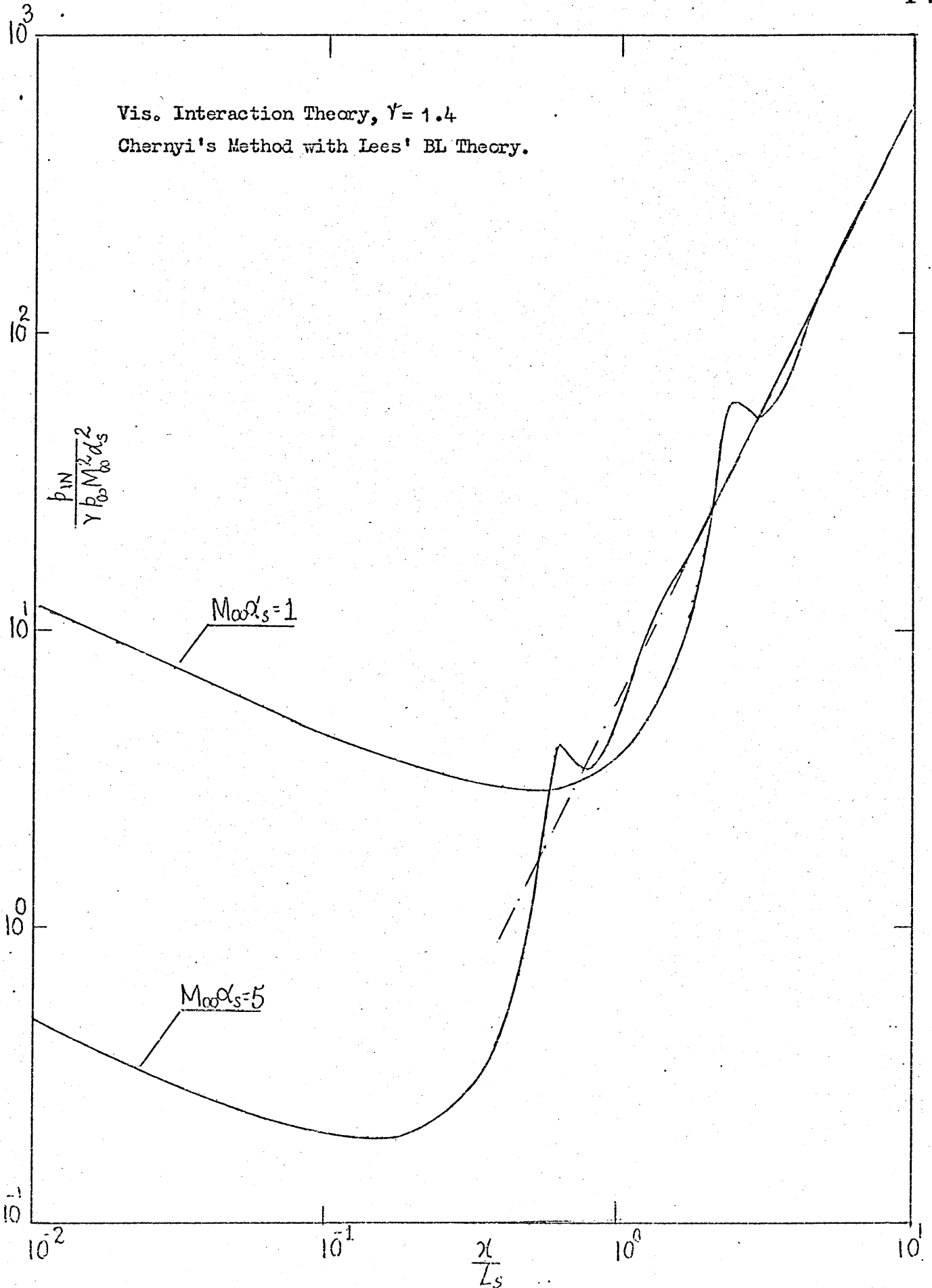


Fig 47. Effect of incidence parameter $M_\infty \alpha_s$ on pressure distribution on concave surface $Y_w = \chi^2$

Viscous interaction theory, $\gamma = 1.4$

Chernyi's method with Lees' BL theory.

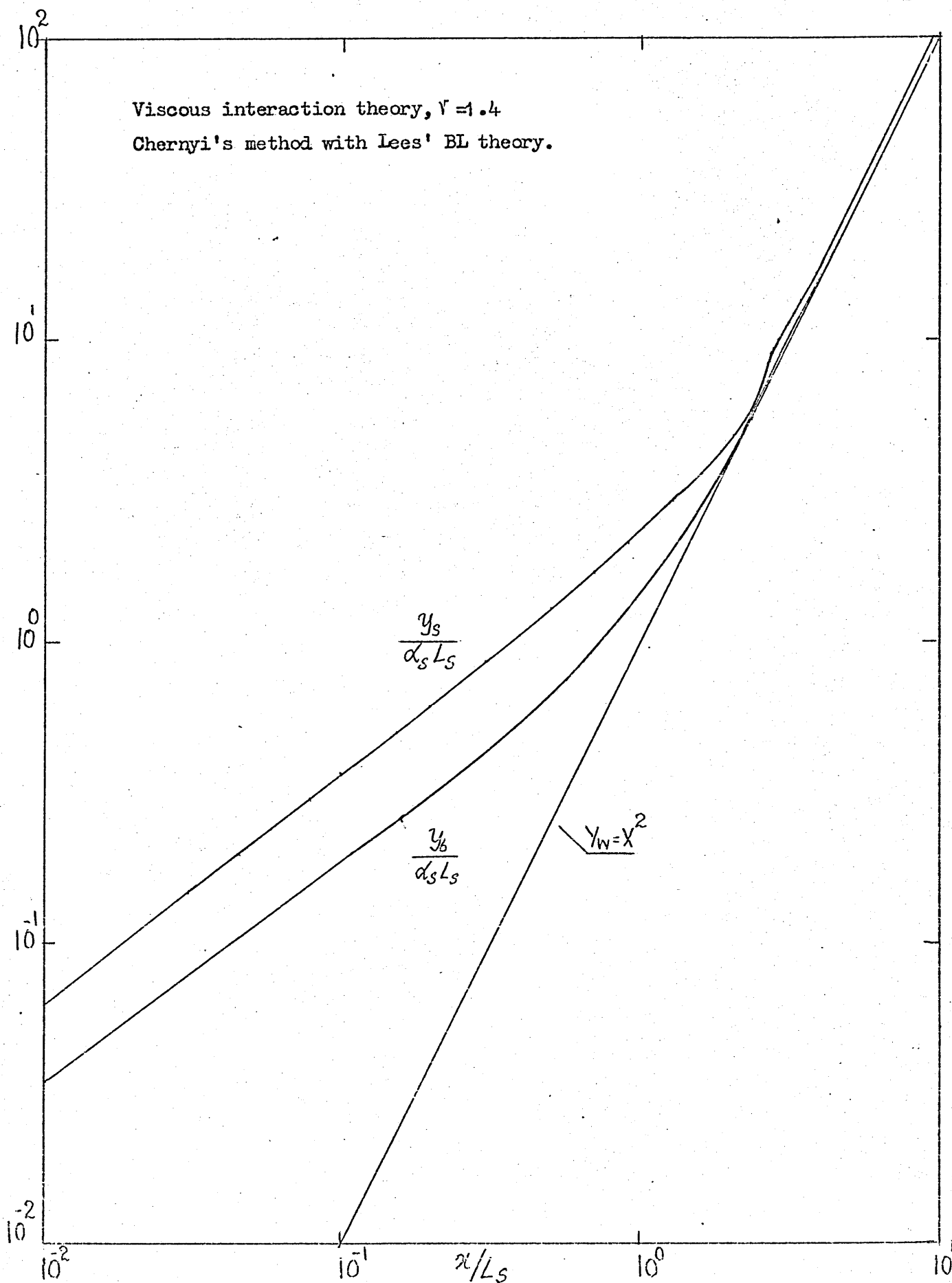


Fig 48a. Shock wave and boundary-layer growth on concave surface surface $y_w = x^2$, $M_\infty \alpha_s = 1$.

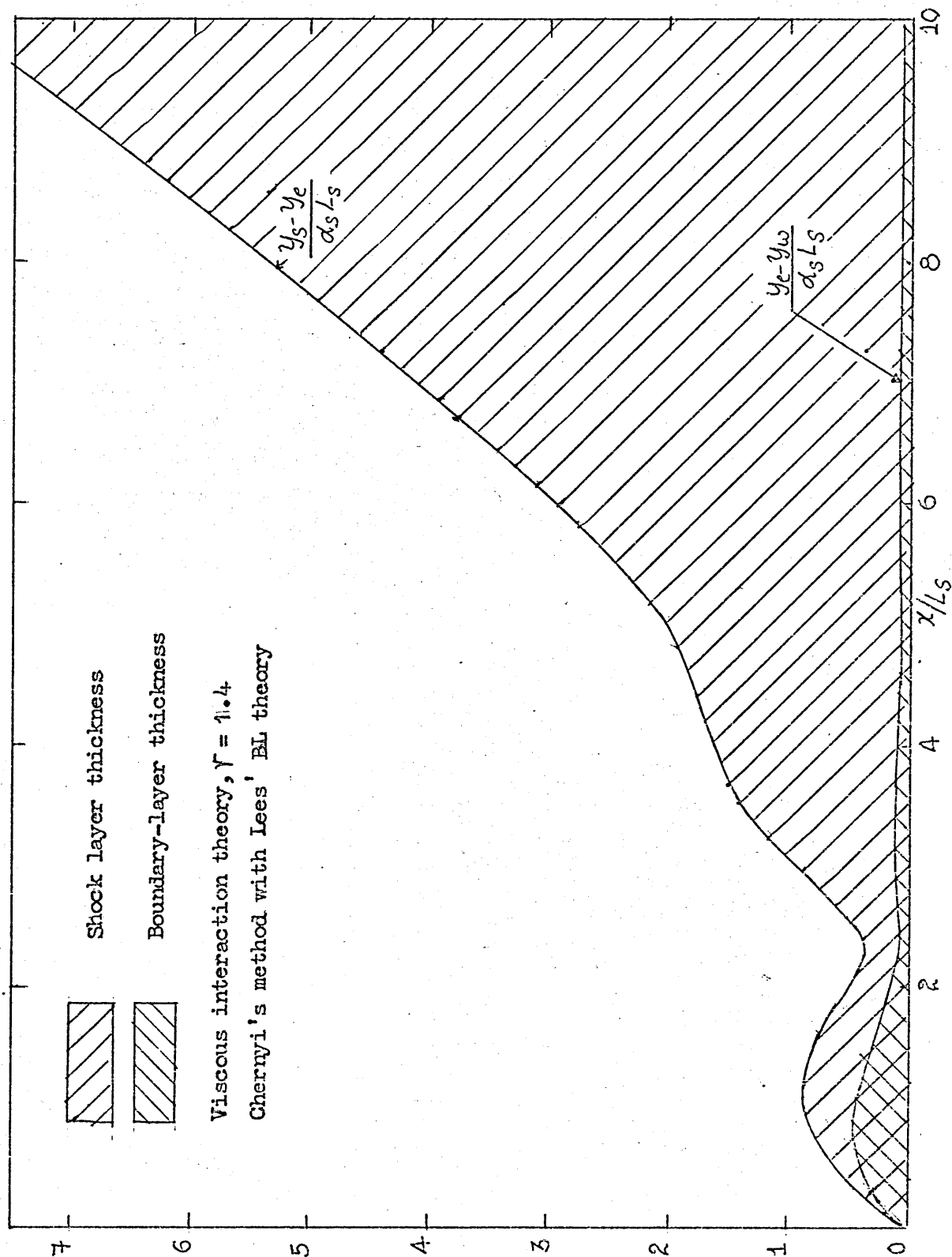


Fig 48b. Shock wave and boundary-layer growth on concave surface
 $\gamma = 1.4$, $M_\infty = 1$, $\omega_s = 1$.

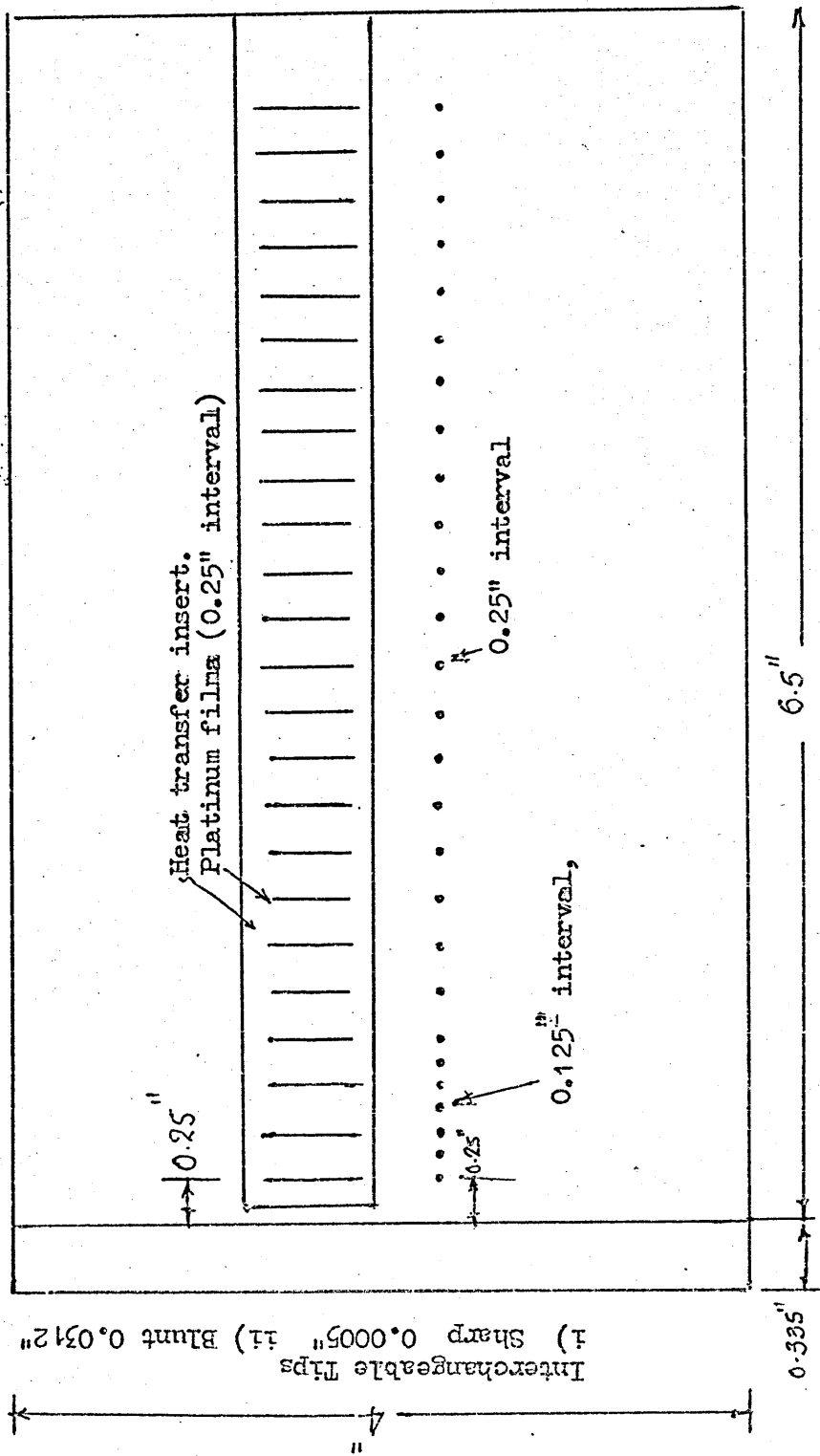


Fig 49: Layout of the flat plate model.
Thickness of the Plate = 0.23"

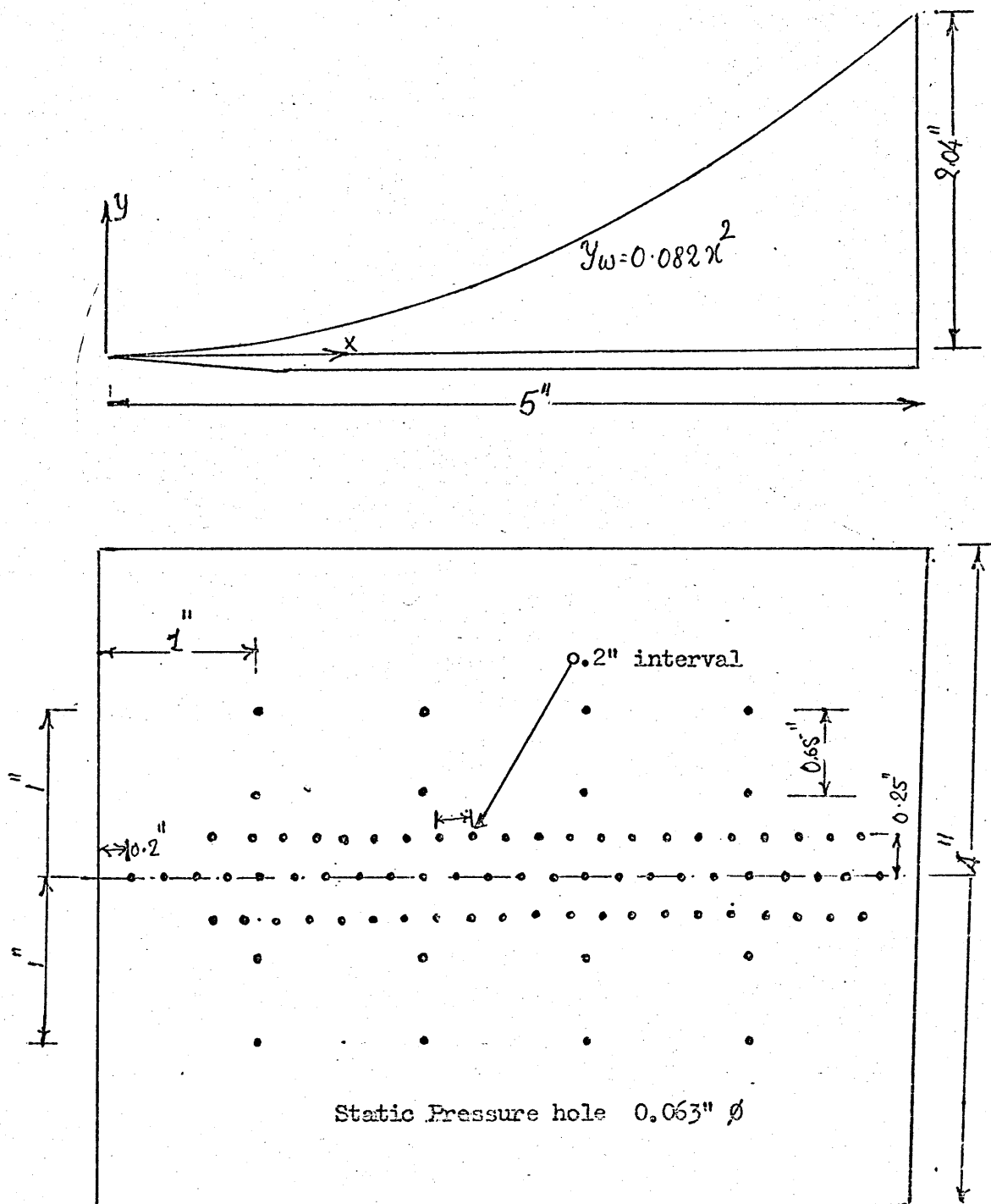


Fig 50a: Layout of Concave surface pressure model

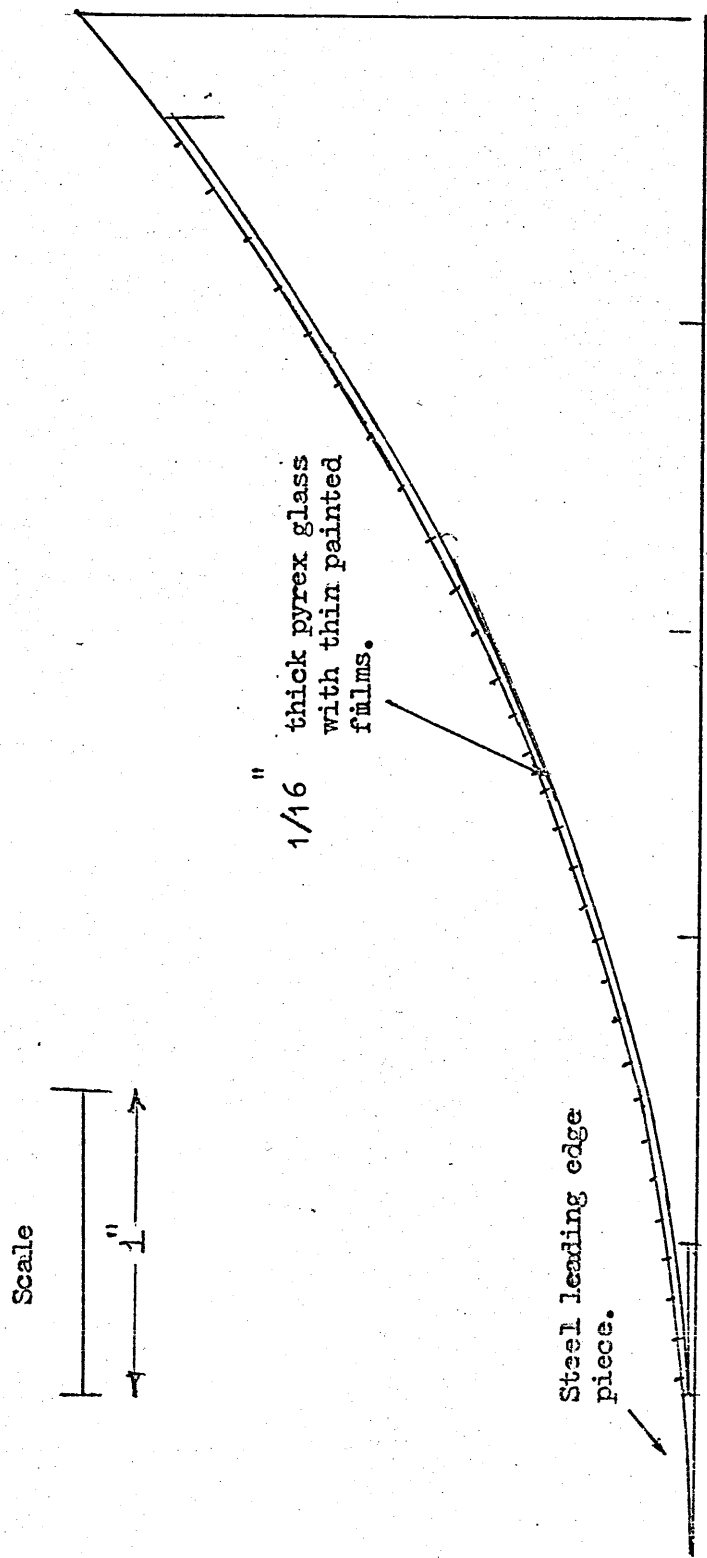


Fig 50b: Centerline layout of thin film heat transfer gages.
(Location of gauges shown by a dash)

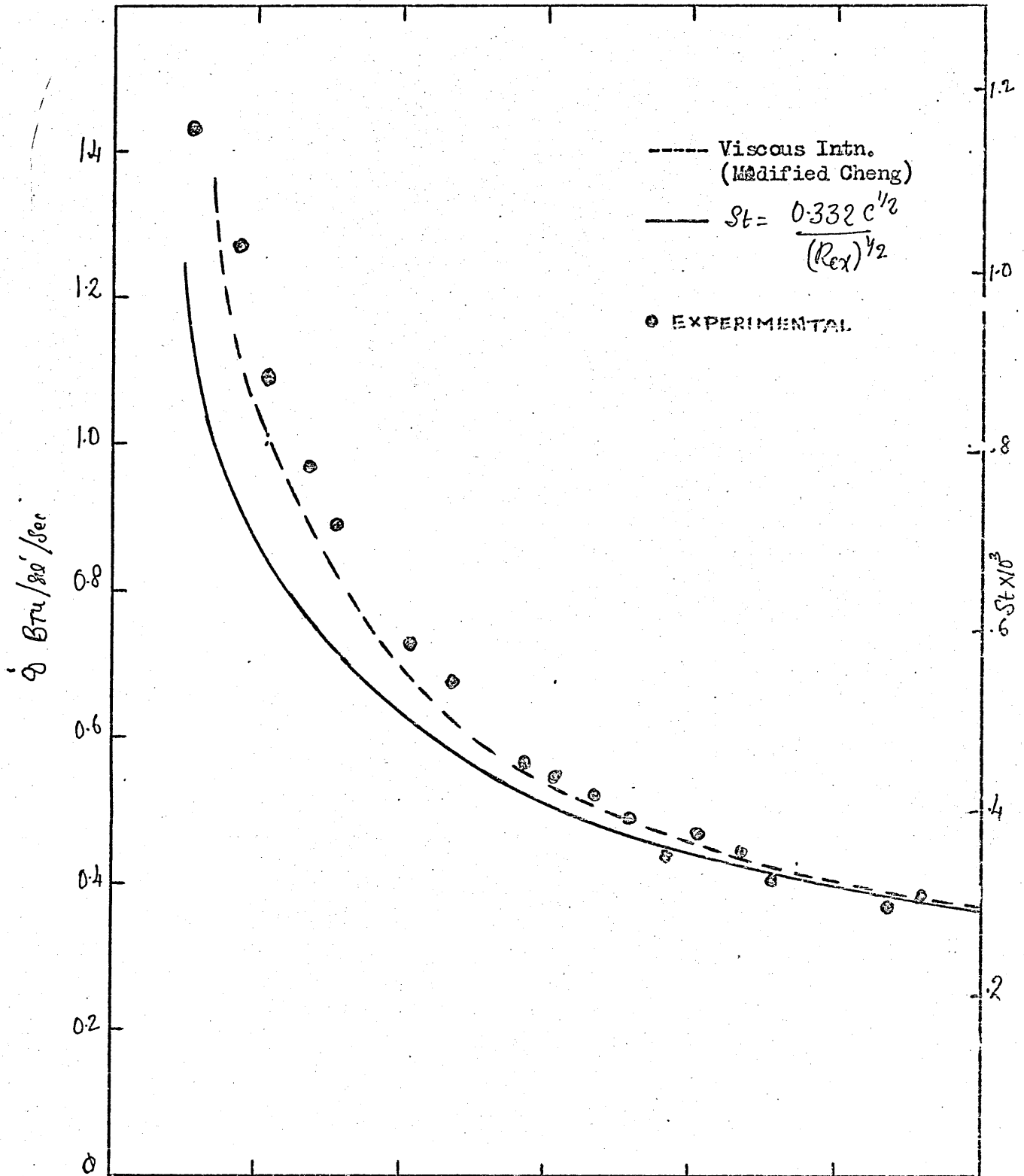


Fig 51: Heat transfer to a sharp flat plate at zero incidence.
 $M = 12.25$, $Re_{unit} = 0.86 \times 10^5$.

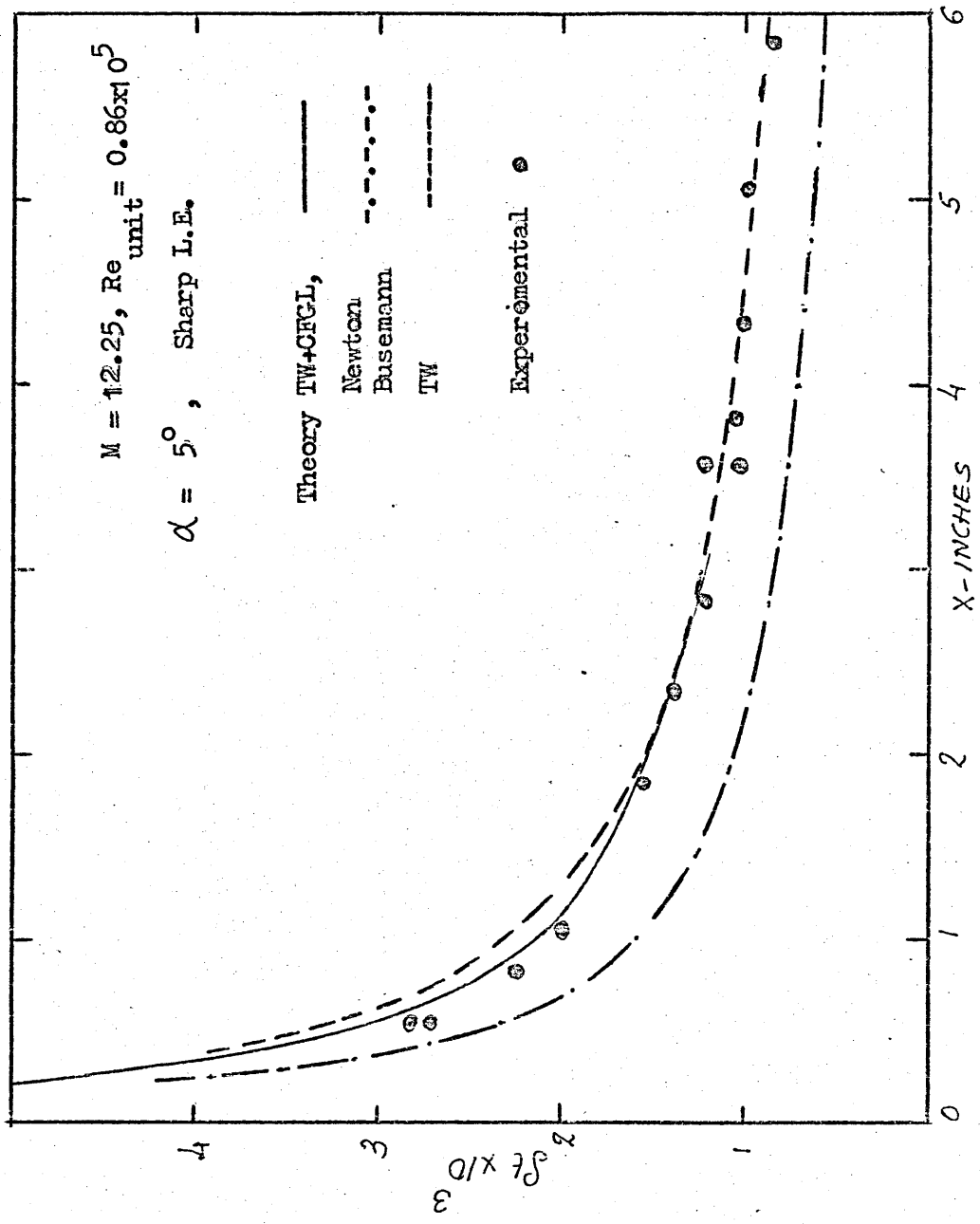


Fig 52a: Experimental heat transfer on a sharp flat plate

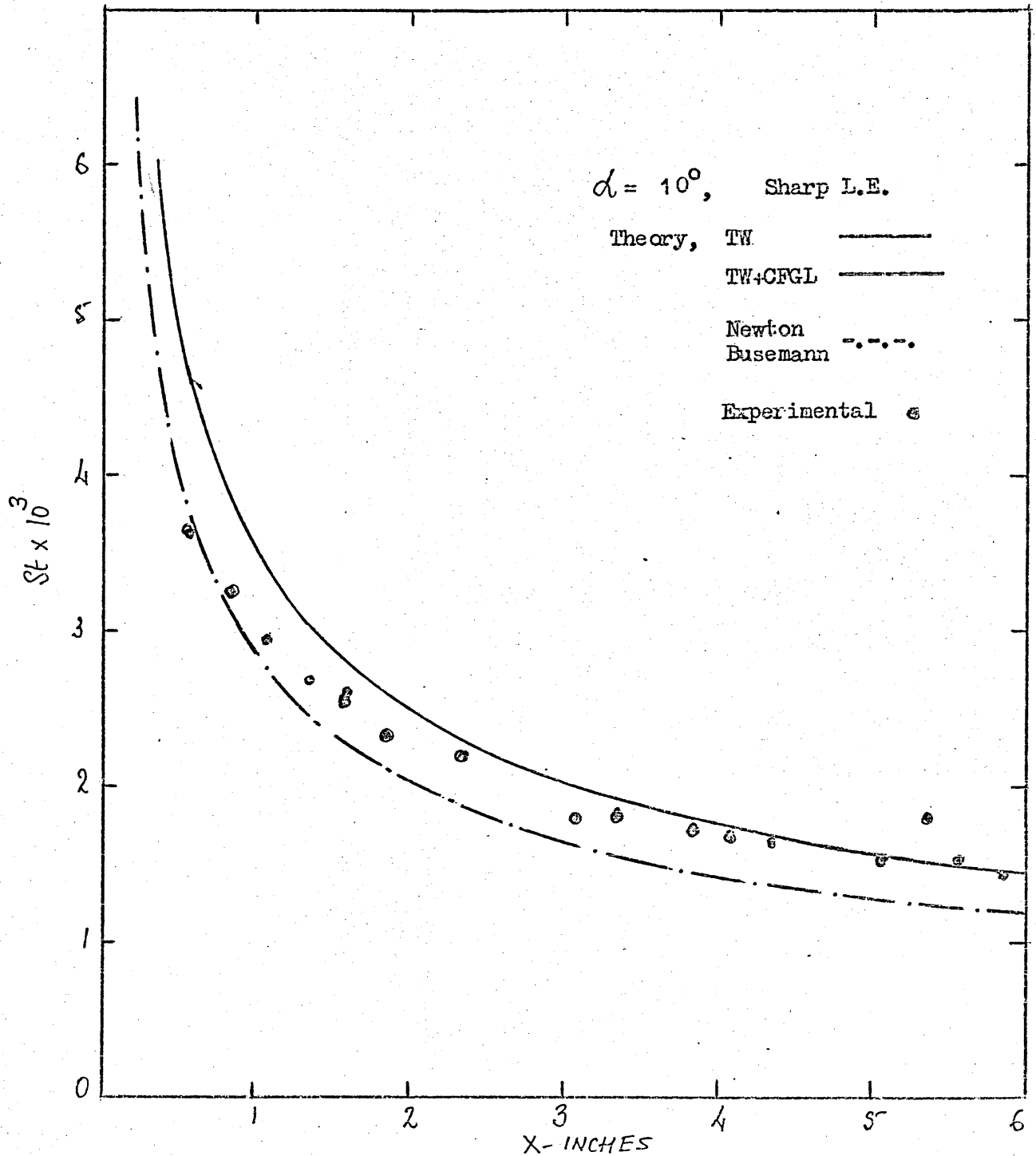


Fig 52b: Experimental Heat transfer on a sharp flat plate at incidence.

$$M = 12.25, Re_{unit} = 0.86 \times 10^5$$

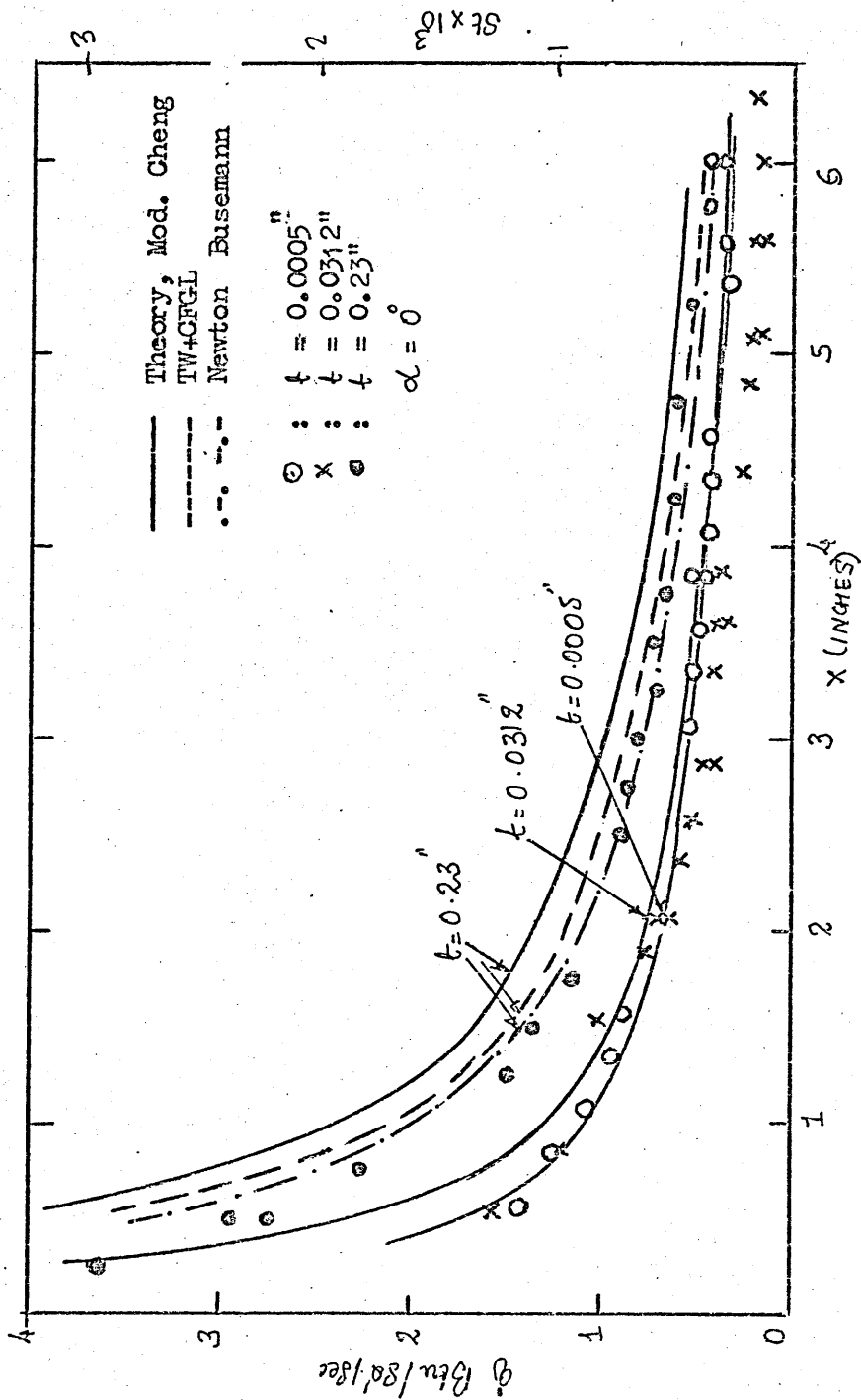


Fig 53: Heat transfer to a blunt flat plate at zero incidence

$M = 12.25, Re_{unit} = 0.86 \times 10^5$

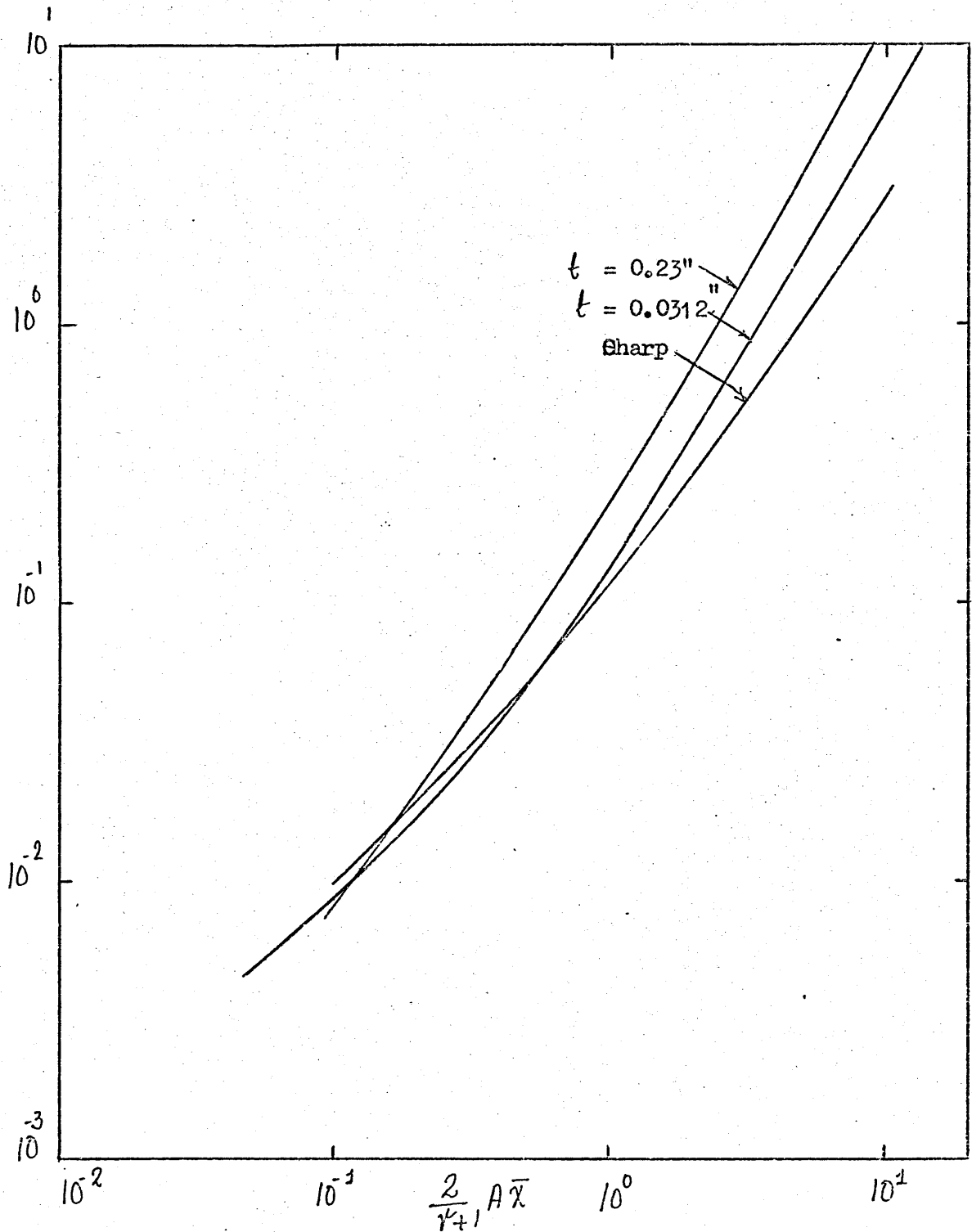


Fig 54: Effect of leading edge bluntness on flat plate heat transfer (Modified Cheng Theory) at zero incidence.
 $M = 12.25$, $Re_{unit} = 0.86 \times 10^5$.

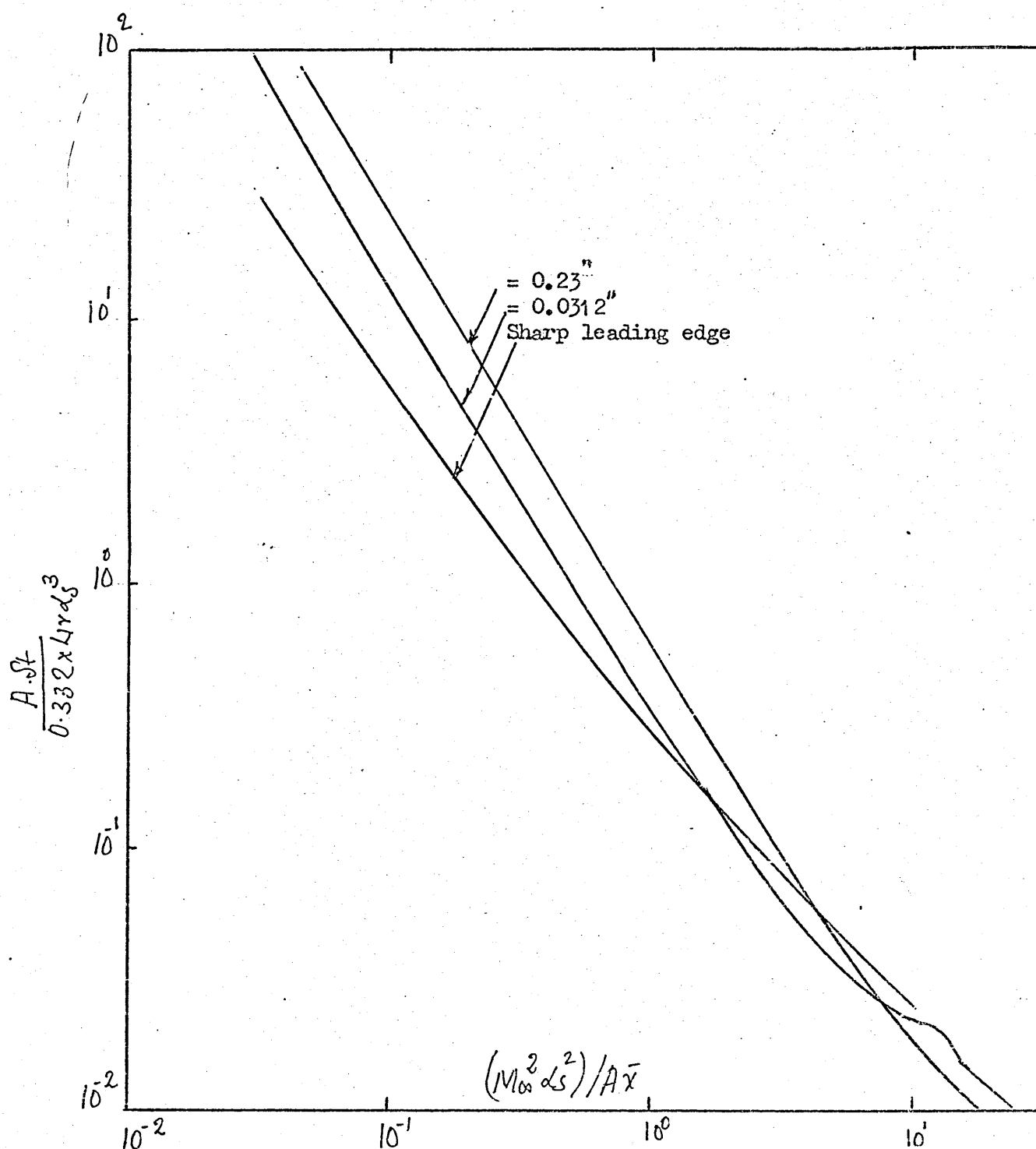


Fig 55: Effect of leading edge thickness on the heat transfer for a flat plate at zero incidence. TW+ CFGL (Theory).

$$M = 12.25, Re_{unit} = 0.86 \times 10^5$$

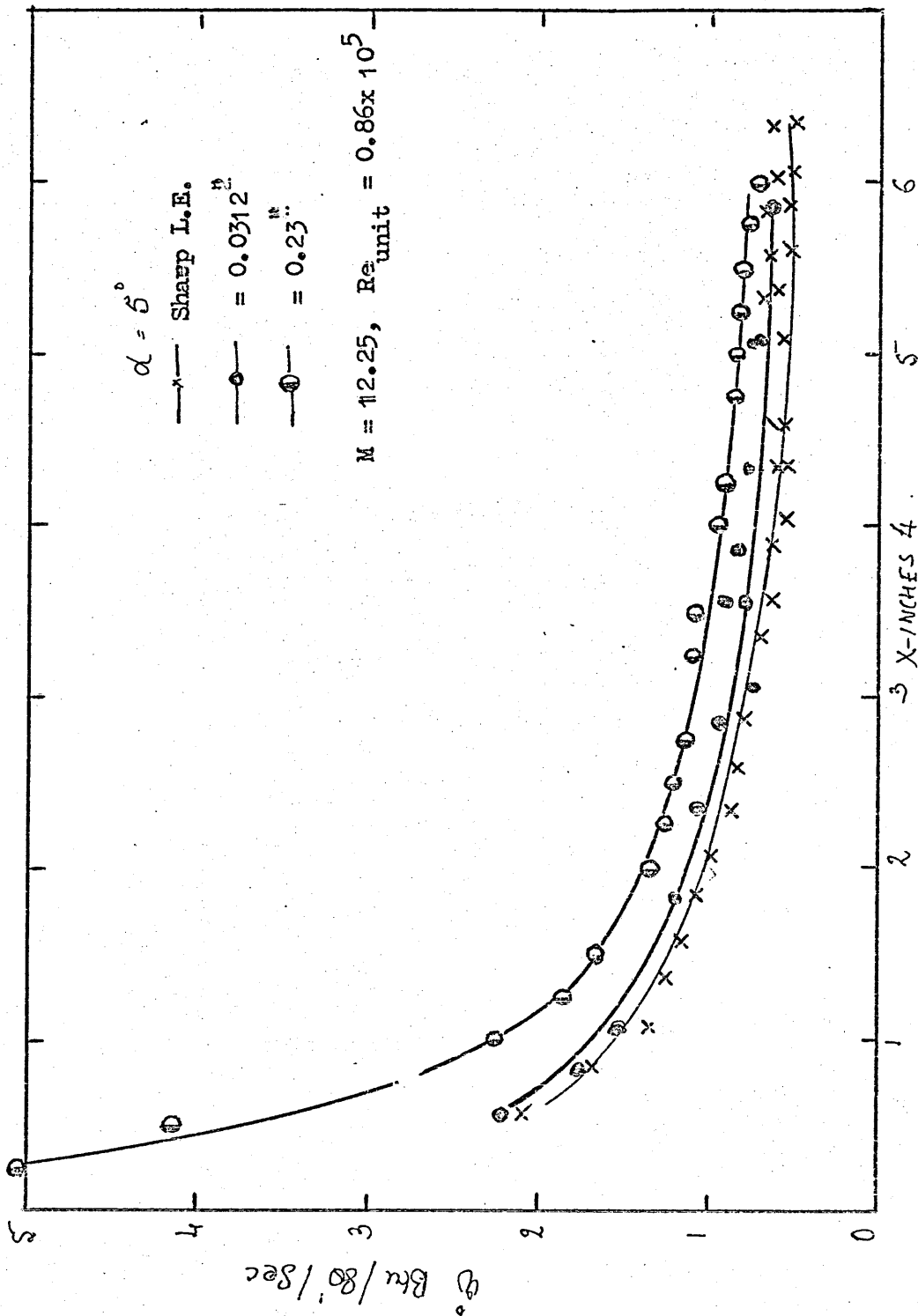


Fig 56a: Experimental heat transfer to a flat plate at incidence.

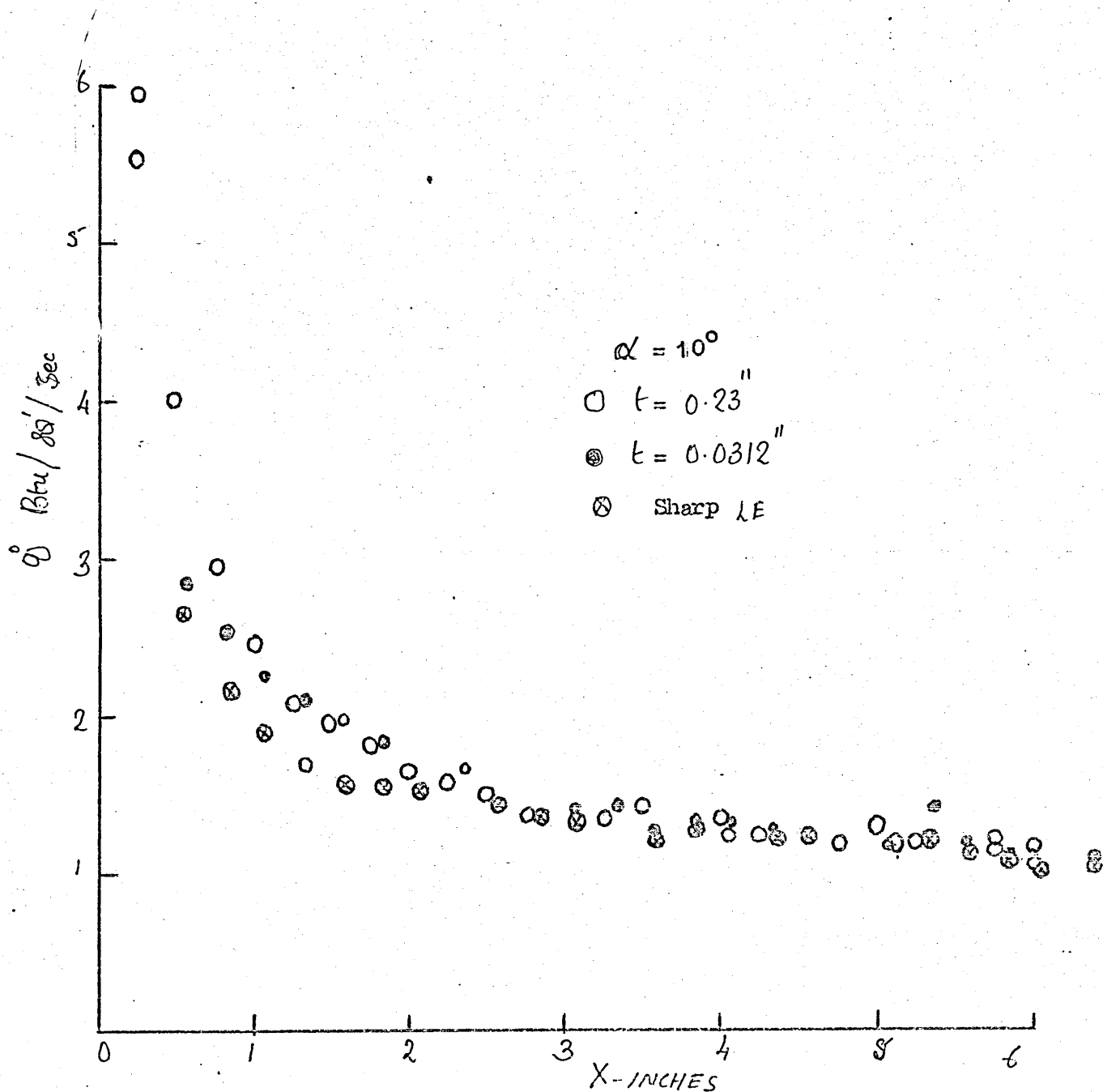


Fig 56b: Experimental heat transfer to the flat plate at incidence.

$$M = 12.25, \quad Re_{unit} = 0.86 \times 10^5.$$

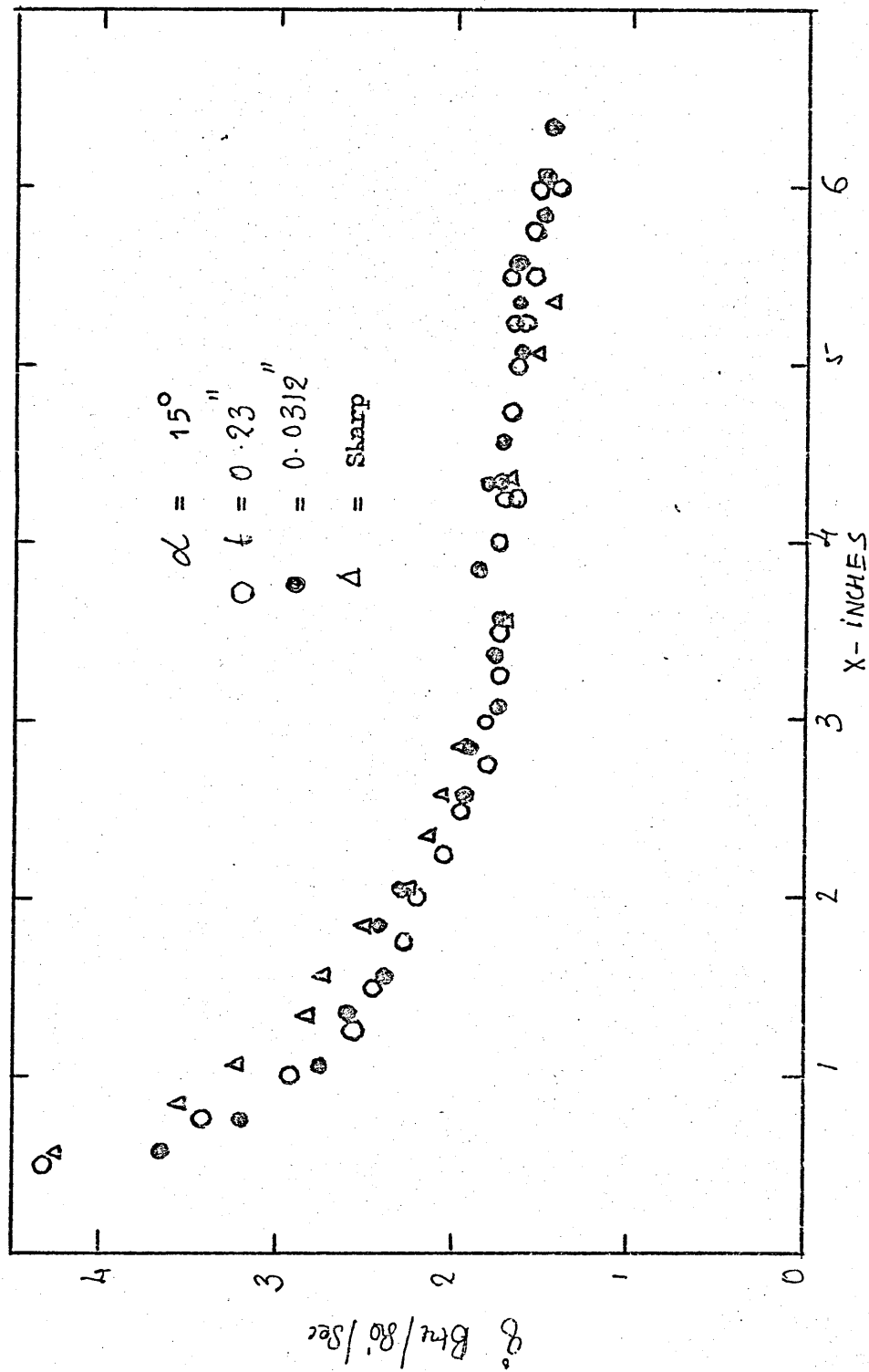


Fig 56c: Experimental heat transfer to the flat plate at incidence

$M = 12.25, Re_{unit} = 0.86 \times 10^5.$

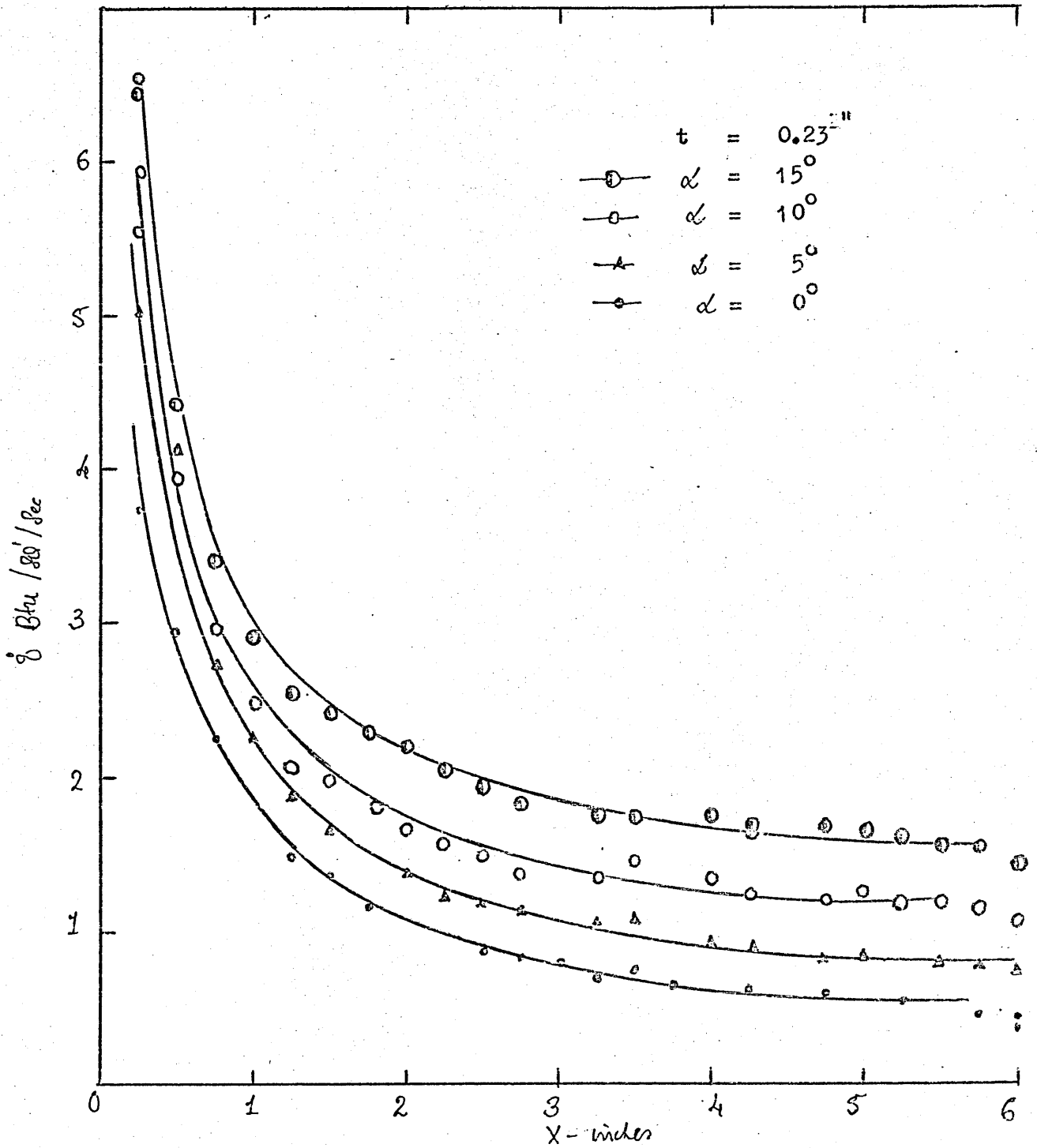


Fig 56d: Experimental Heat Transfer rate to a blunt flat plate at different angles of attack.

$$M = 12.25, \quad Re_{unit} = 0.86 \times 10^5$$

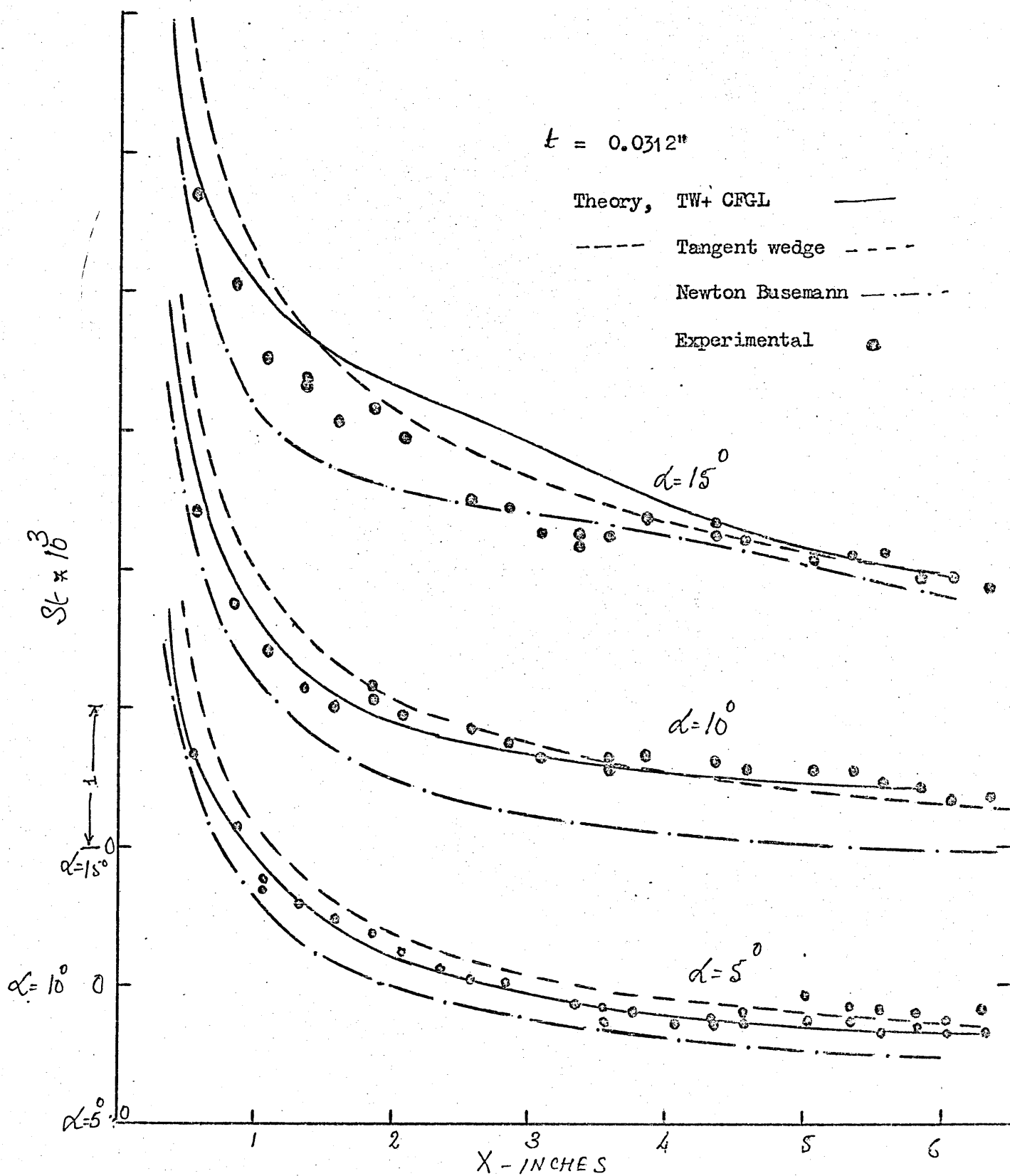


Fig 57 : Comparison of measured heat transfer rates with theoretical predictions. (Blunt flat plate)

$$M = 12.25, \text{Re}_{\text{unit}} = 0.86 \times 10^5$$

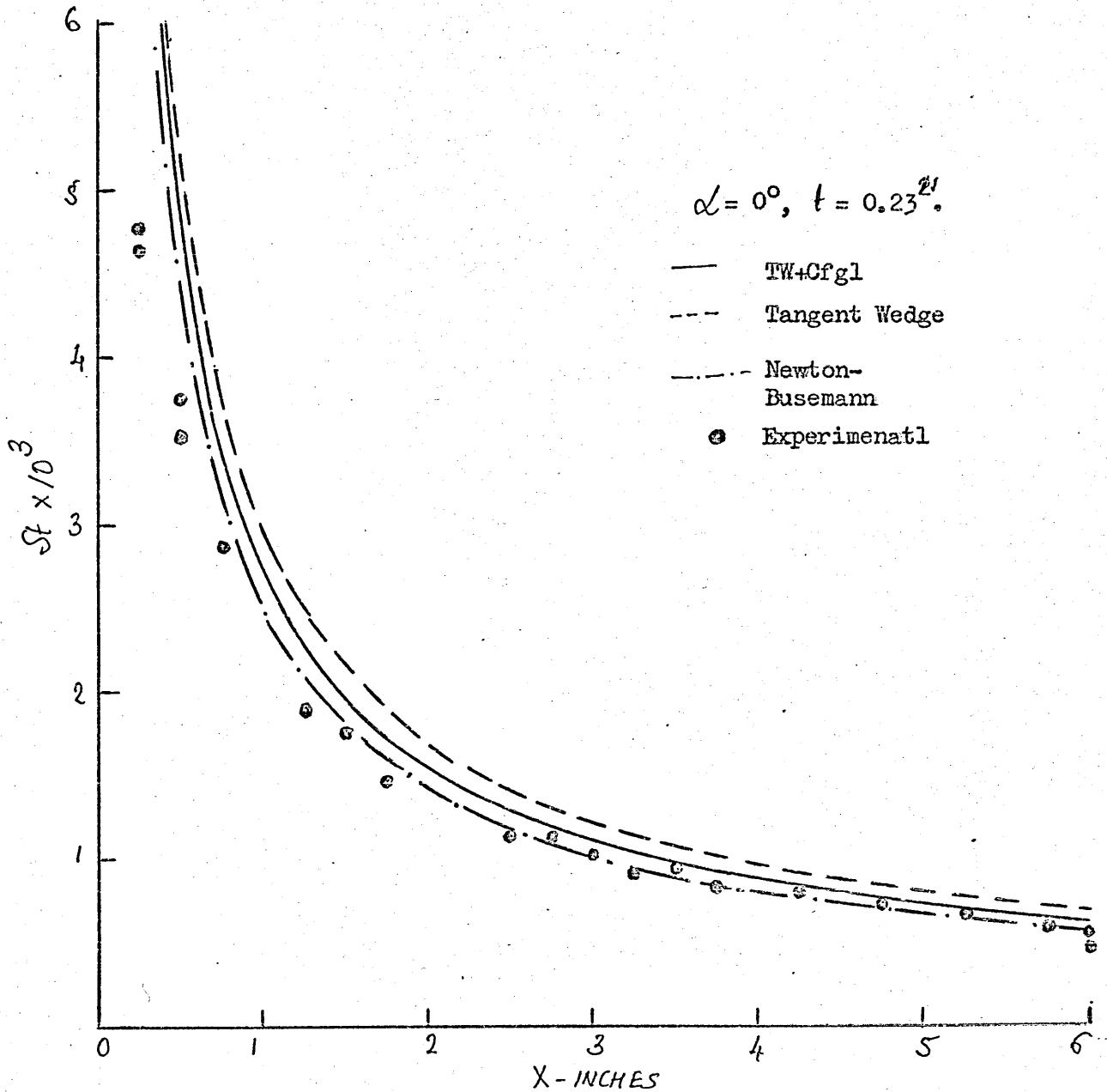


Fig 58A: Comparison of heat transfer measurements with theoretical predictions. (Blunt flat plate)
 $M = 12.25, Re_{unit} = 0.86 \times 10^5$.

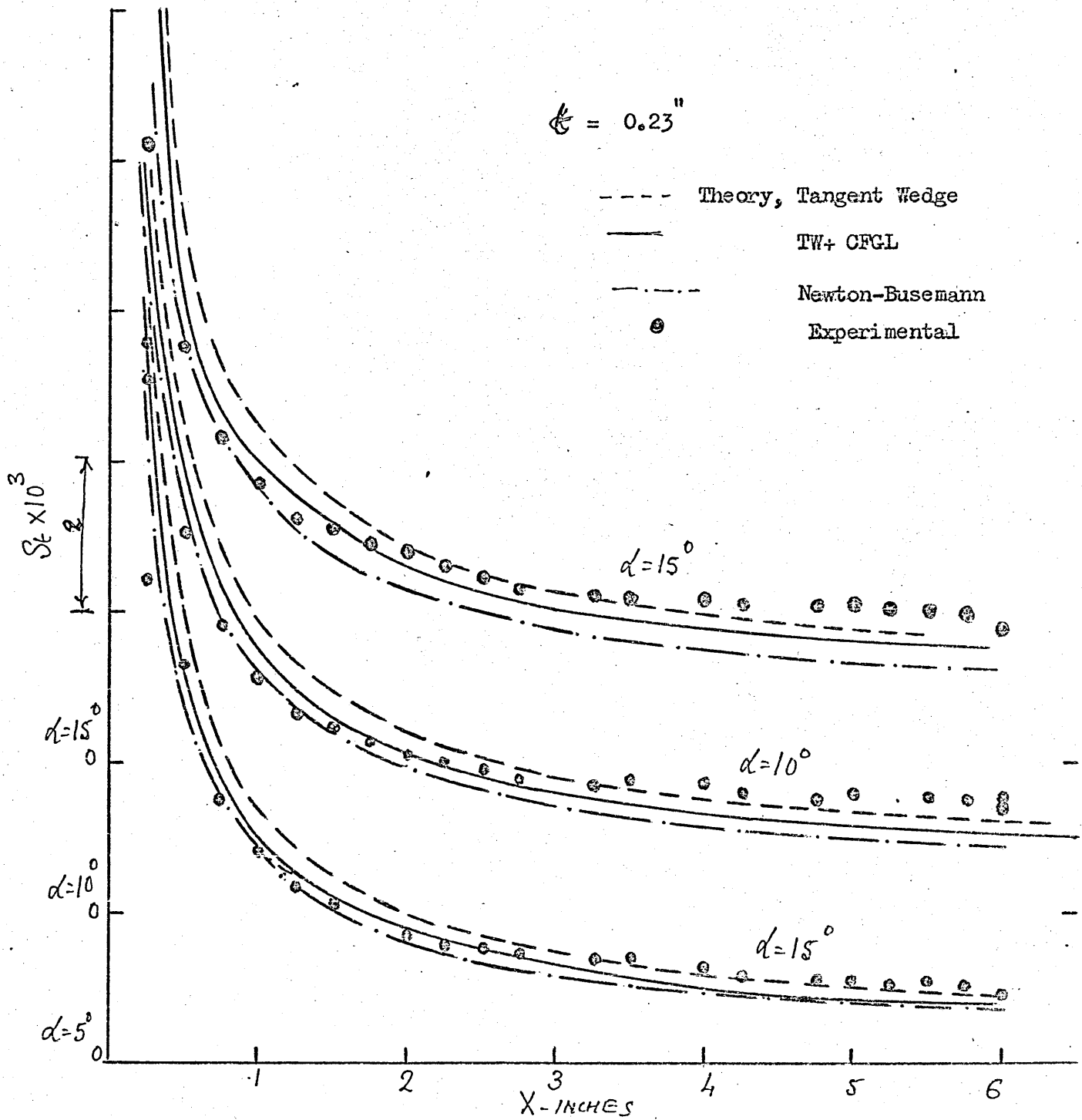


Fig 58b: Comparison of heat transfer measurements with theoretical predictions (Blunt flat plate)

$M = 12.25$, $Re_{unit} = 0.86 \times 10^5$.

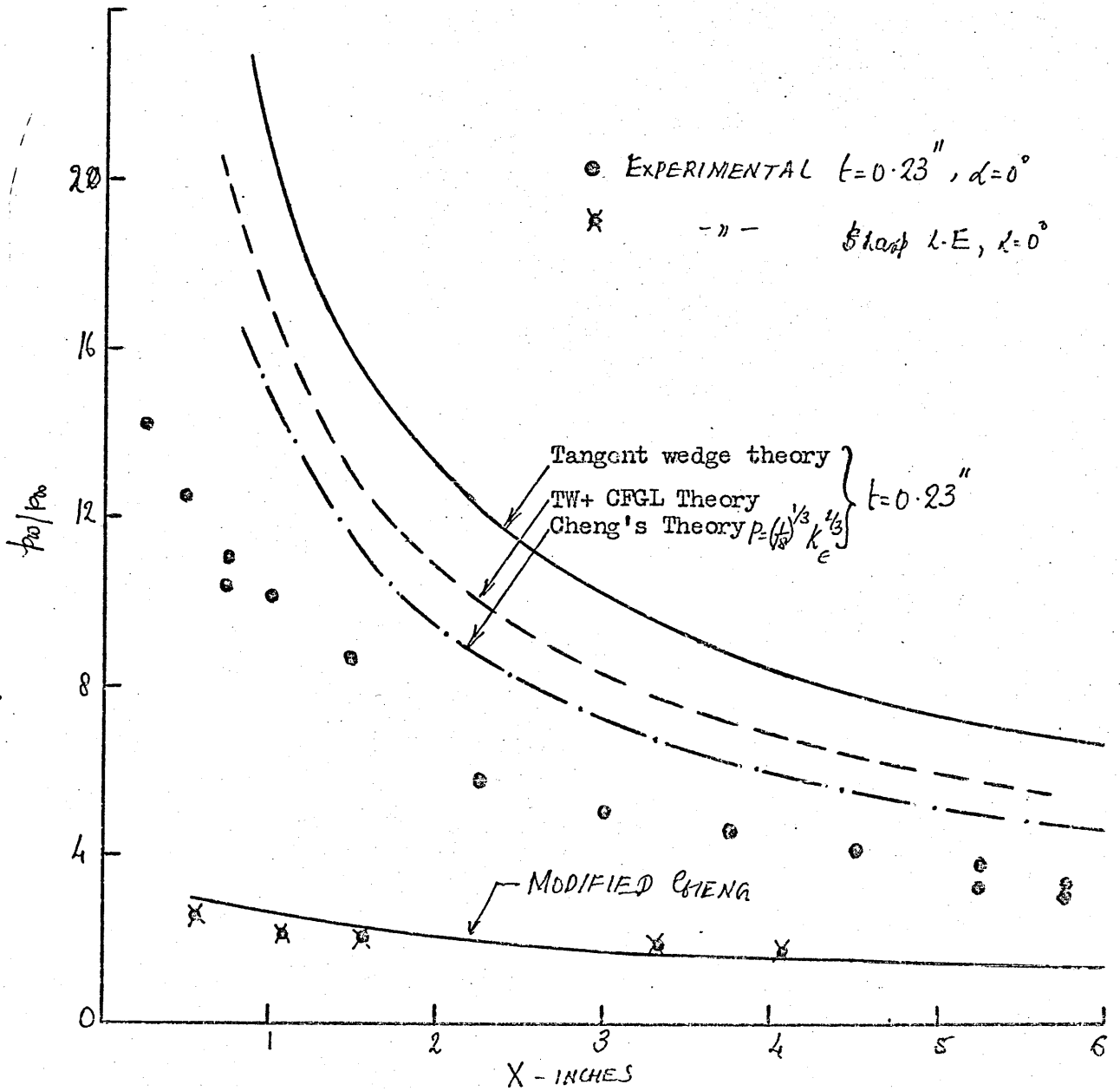


Fig 59: Pressure on sharp and blunt flat plates at zero incidence

$$M = 12.25, Re_{unit} = 0.86 \times 10^5.$$

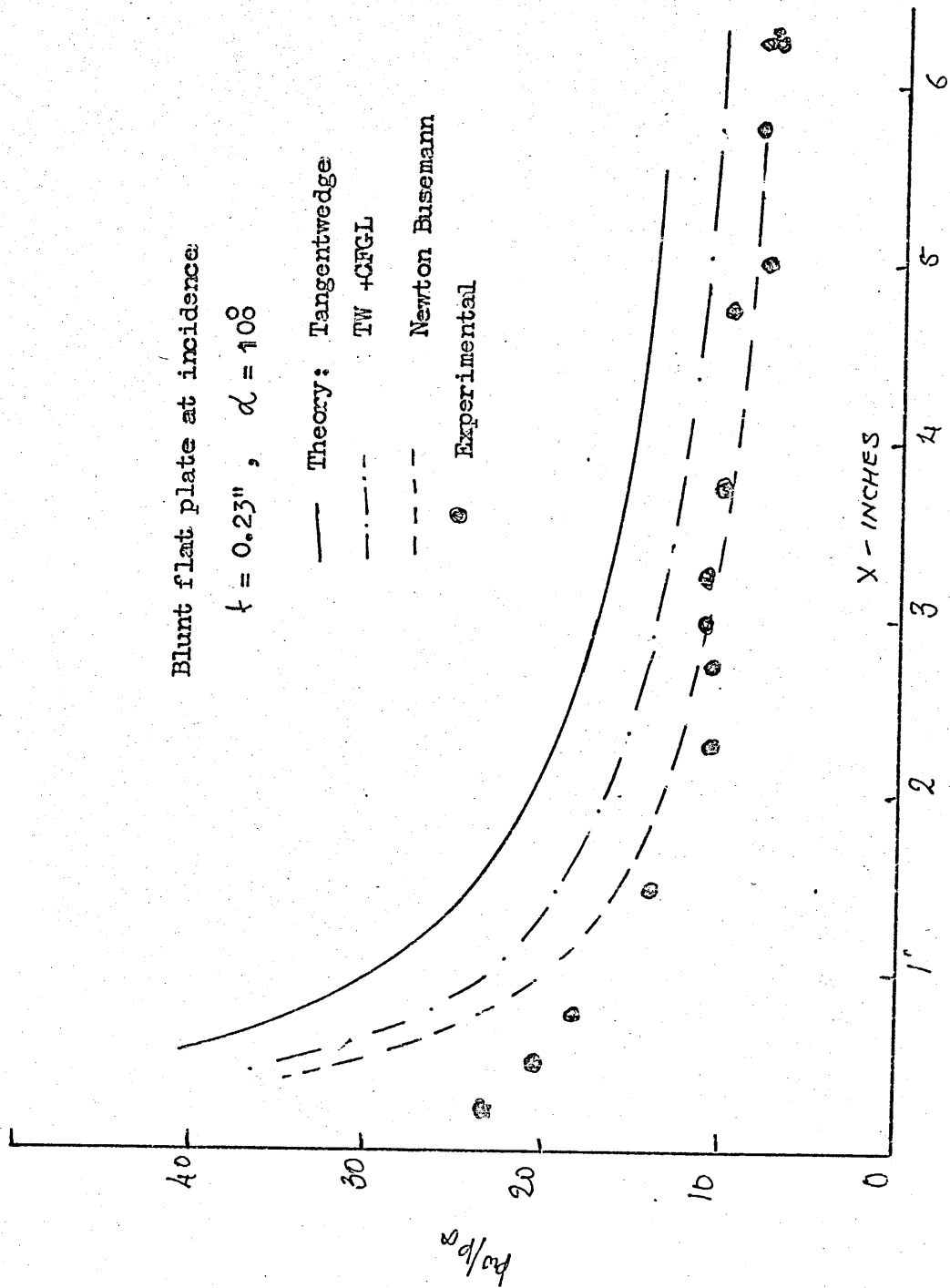


Fig 60: Comparison of theoretical and experimental pressuredistribution

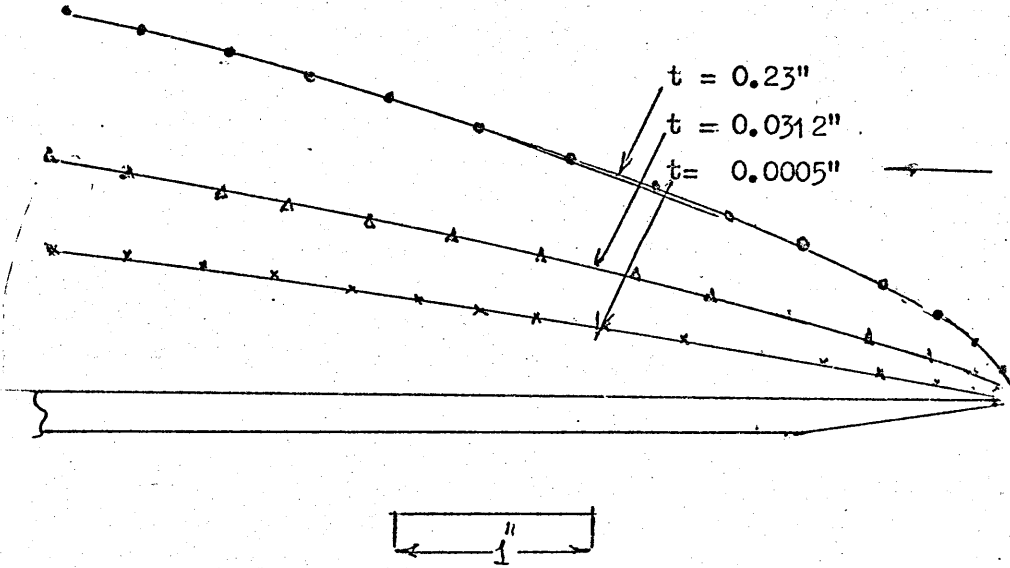


Fig 61: Measured Shock wave shapes for flat plate at zero angle of attack.

$$M = 12.25, Re_{unit} = 0.86 \times 10^5.$$

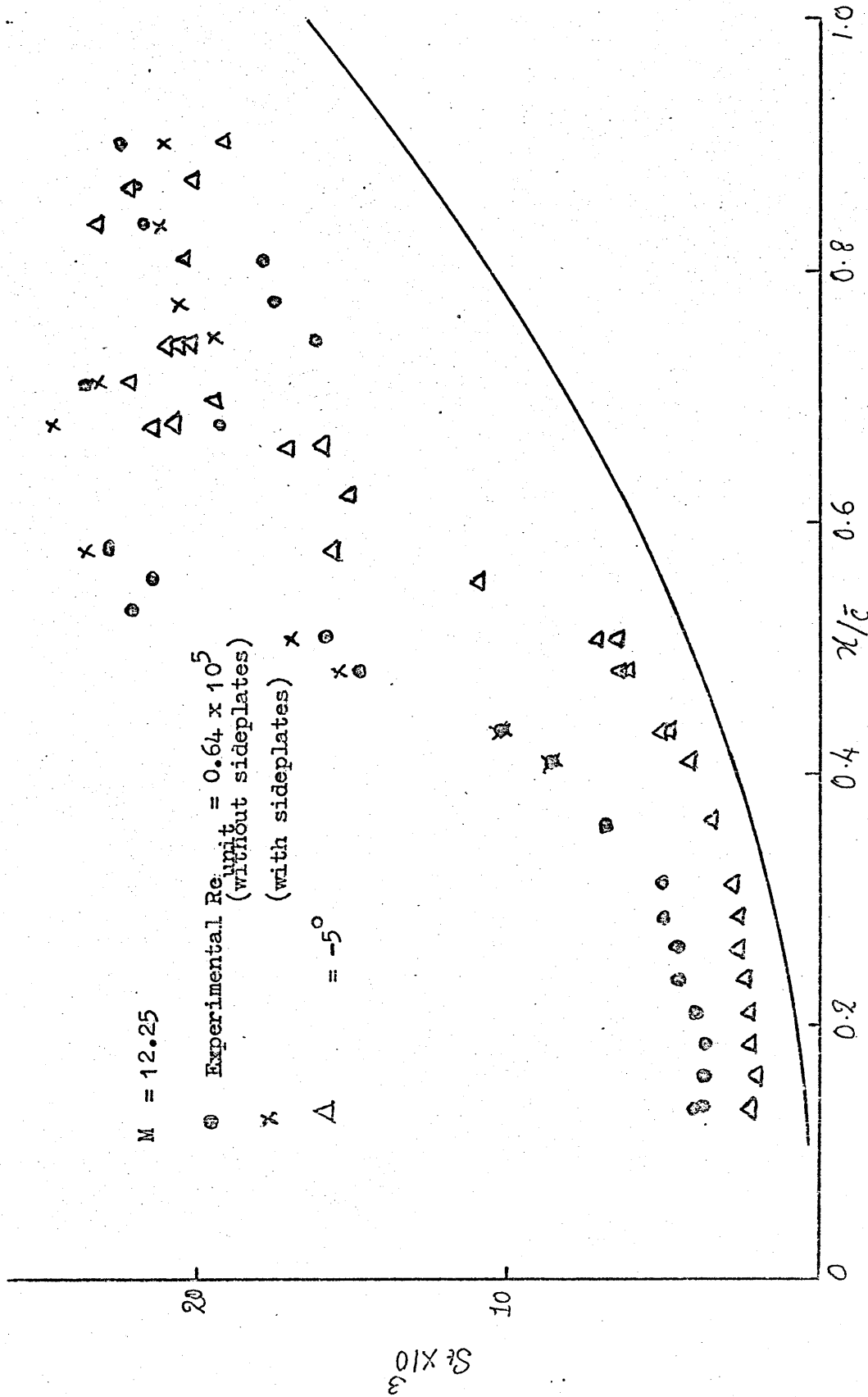


Fig 62: Measured heat transfer rates on the Concave surface $\gamma_w = 0.082 \chi^2$

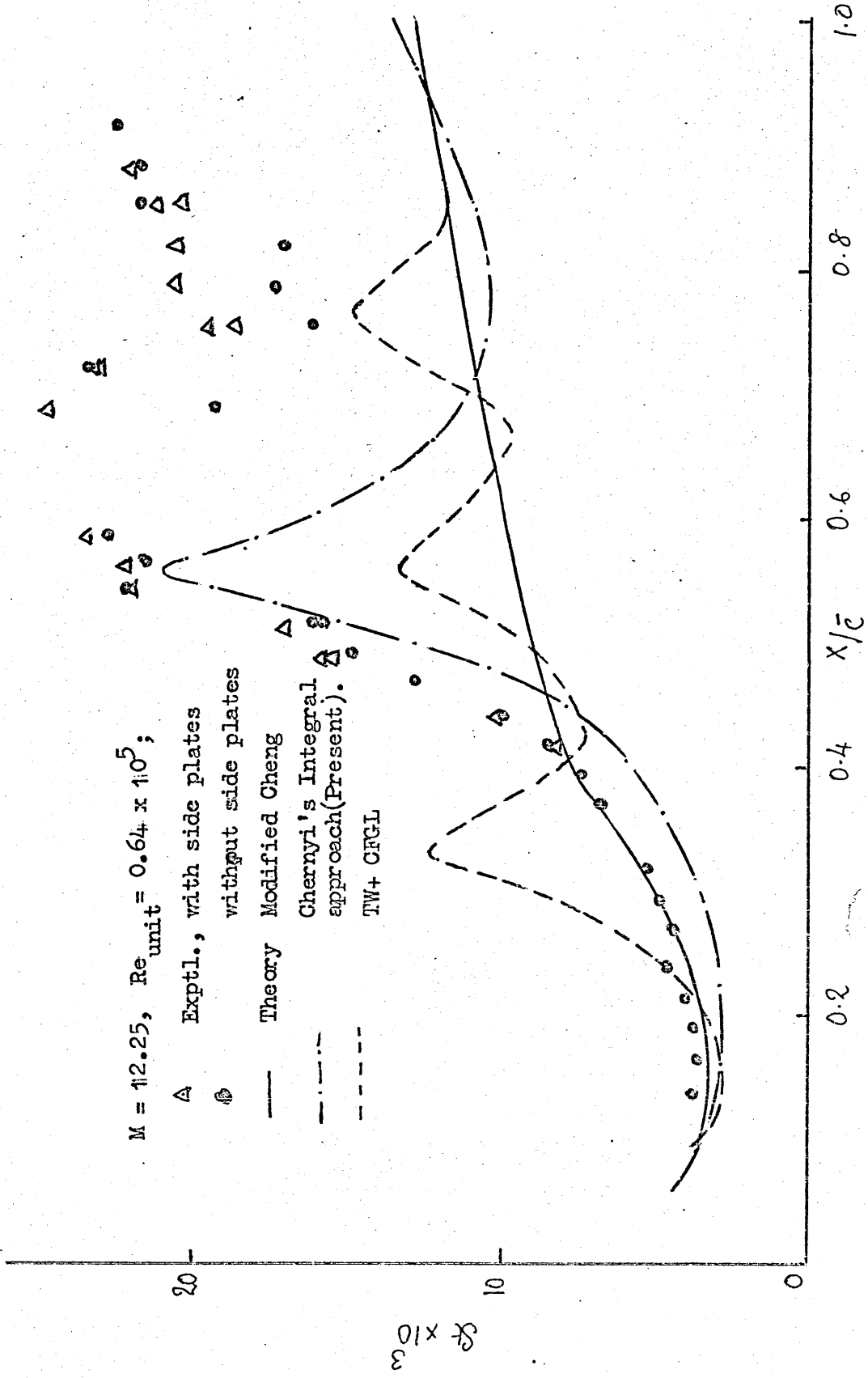


Fig 63: Comparison of experimental heat transfer measurements on the concave surface $y_w = 0.022x$ with viscous interaction theories.

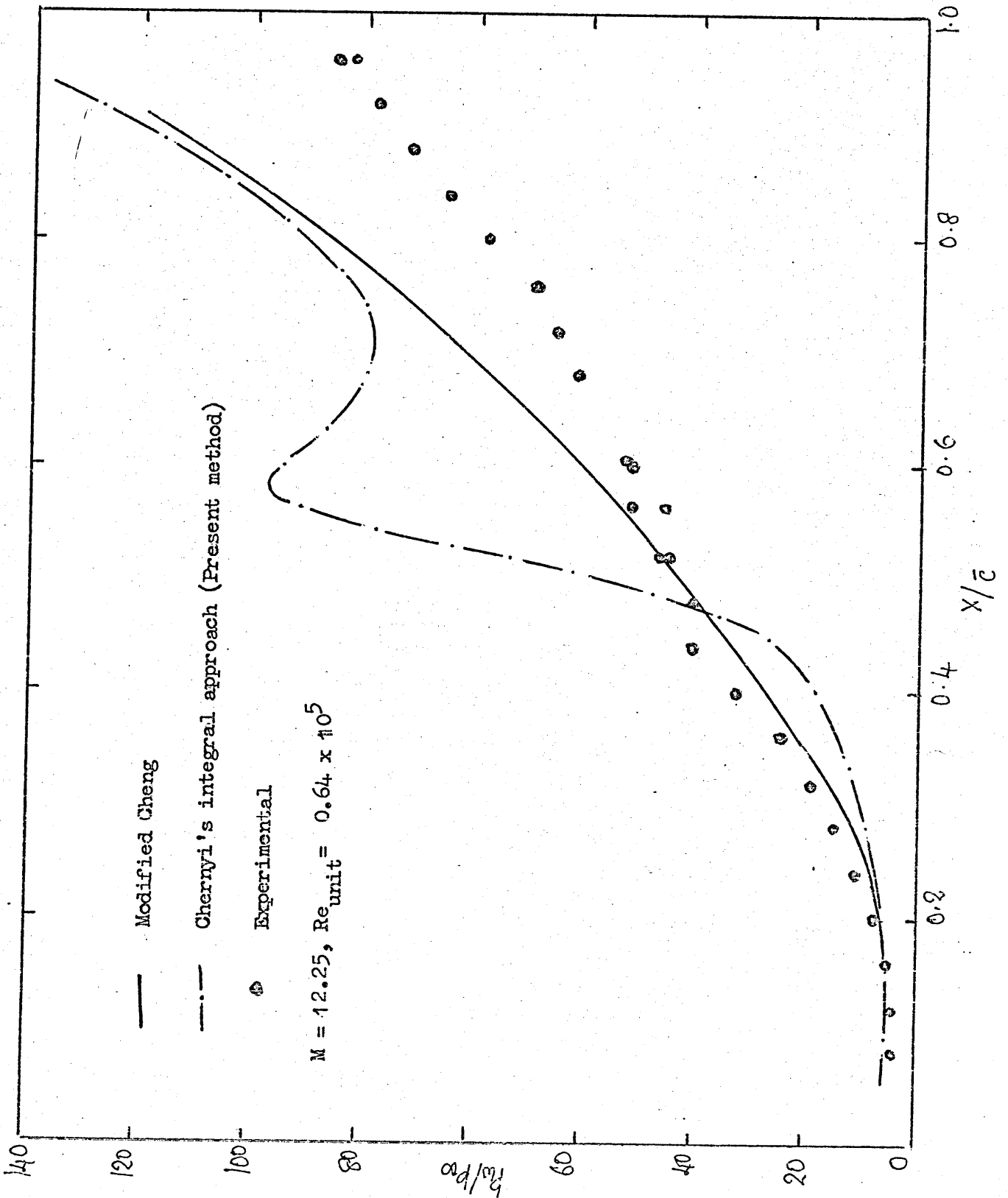


Fig 64: Comparison of pressure distribution on concave surface $\gamma_0.082 \chi^2$ with viscous interaction theories. ($\bar{c} = 5''$)

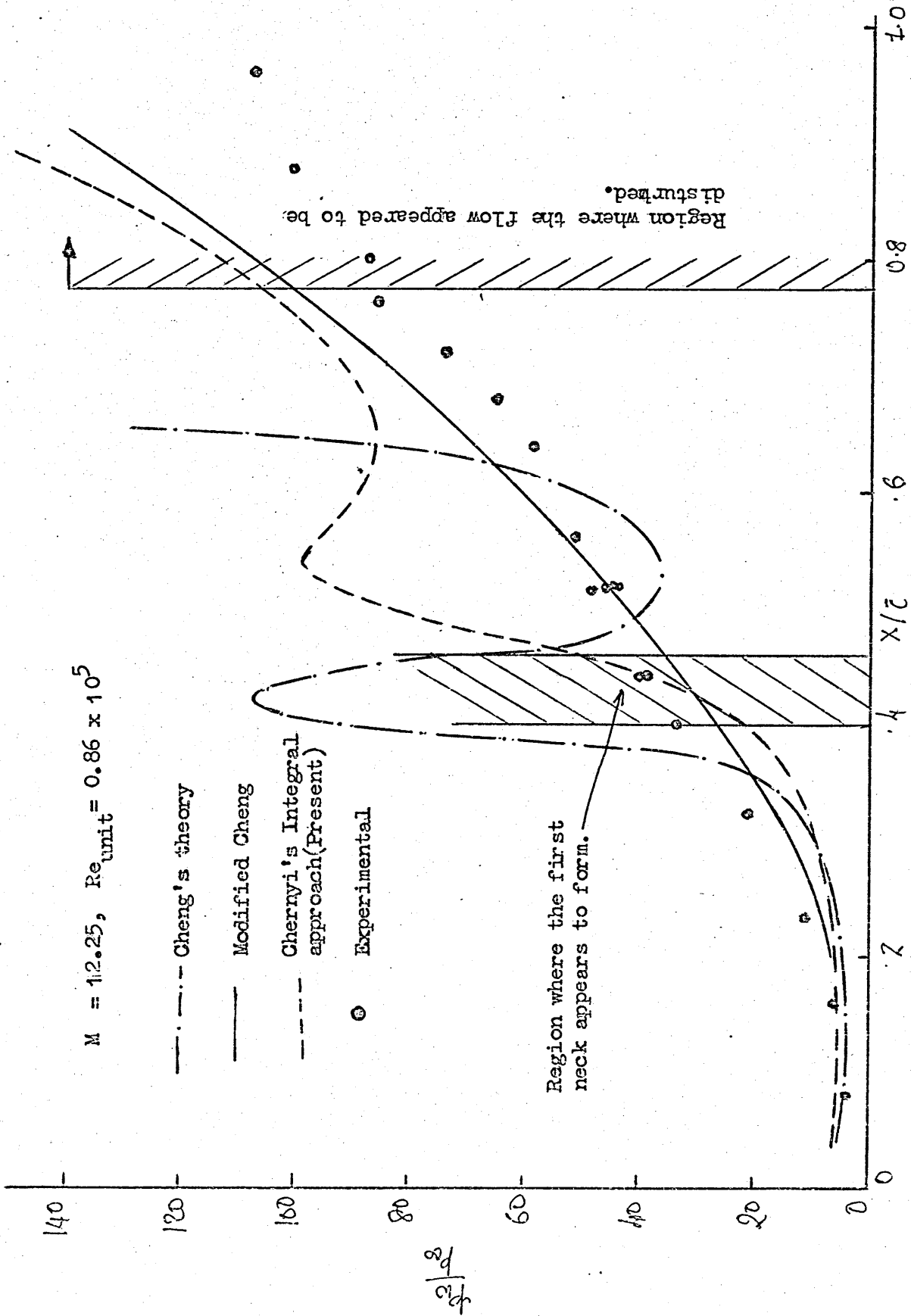
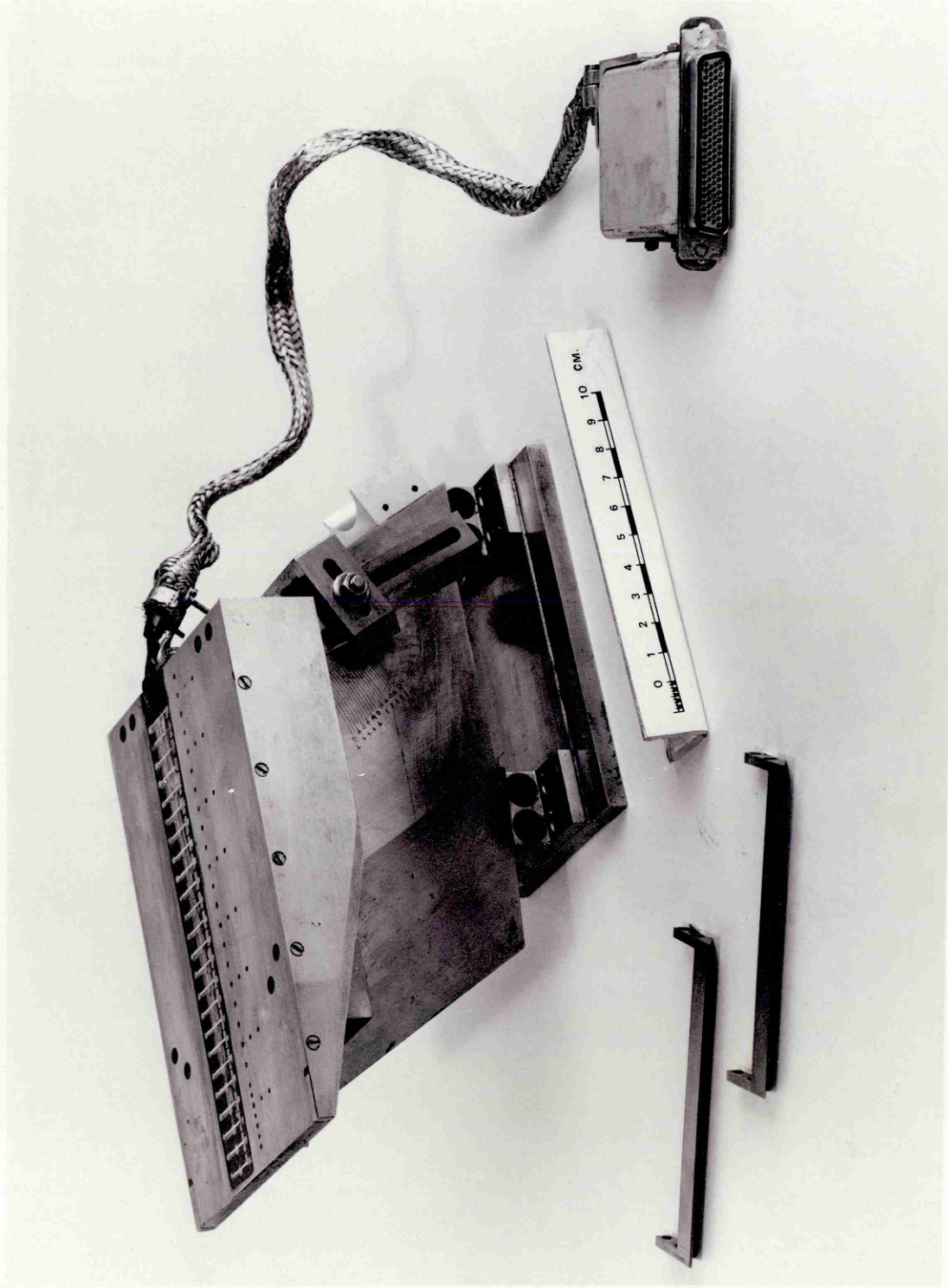


Fig 65: Comparison of pressure distribution on concave surface with viscous interaction theories. ($\bar{z} = 5$)



Plata 1. Flat plate model and support
with different leading edge pieces

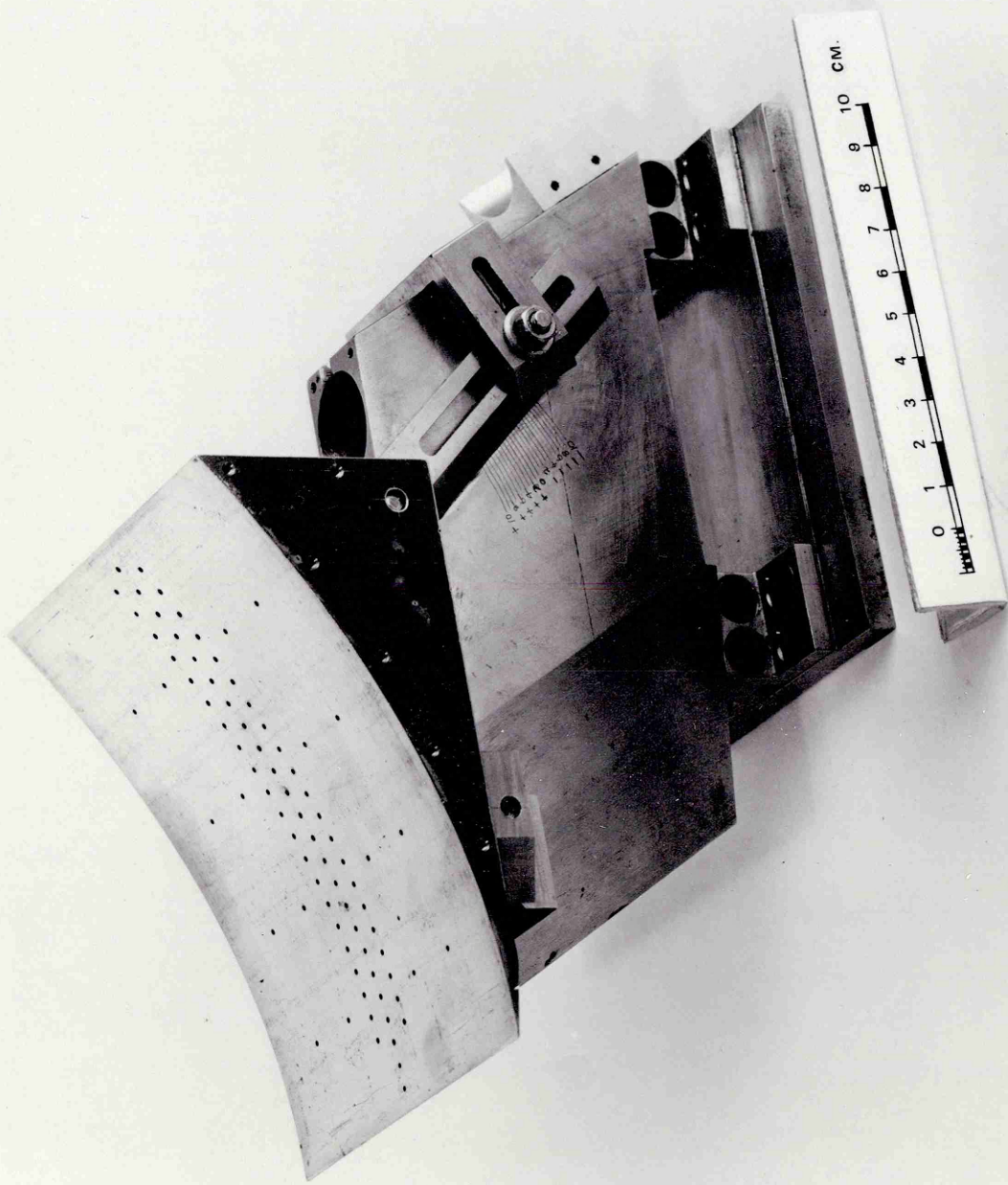


Plate 2: Concave surface orography model
(side plates removed)

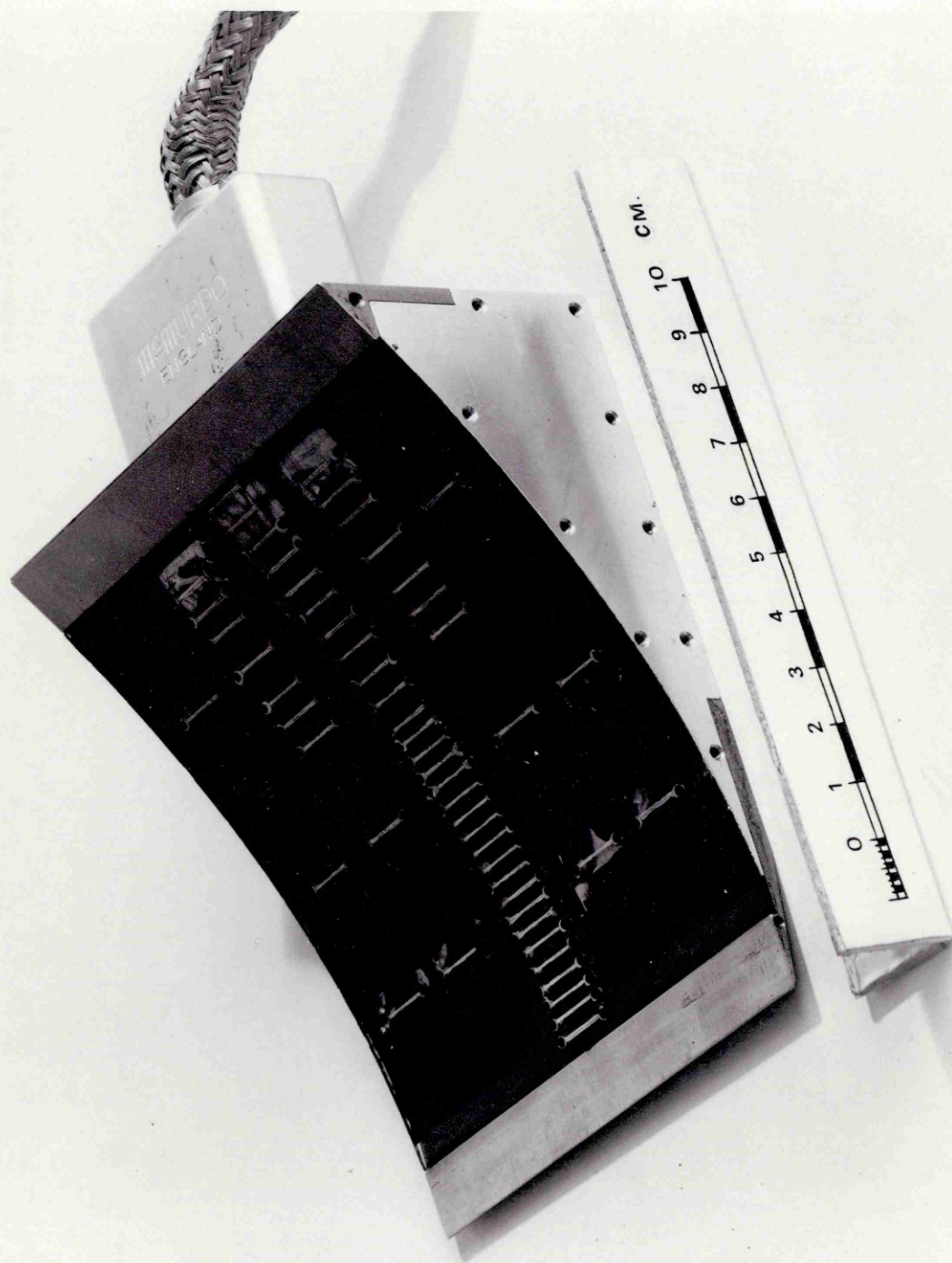
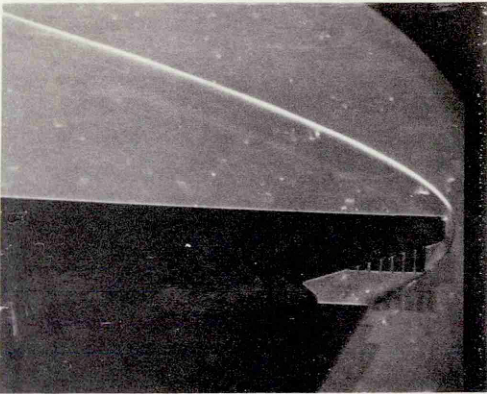
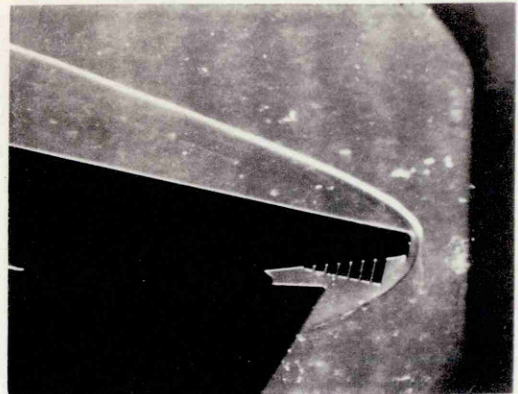


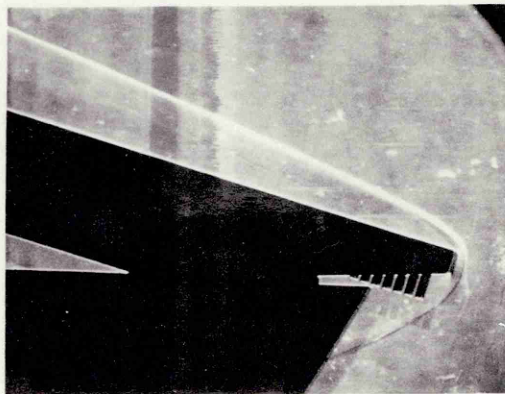
Plate 3: Concave surface heat transfer model



$\alpha = 0^\circ$



$\alpha = 10^\circ$



$\alpha = 15^\circ$

Plate 4: Schlieren picture for the blunttest leading edge $t = 0.23$,
 $M = 12.25$, $Re_{unit} = 0.86 \times 10^5$.

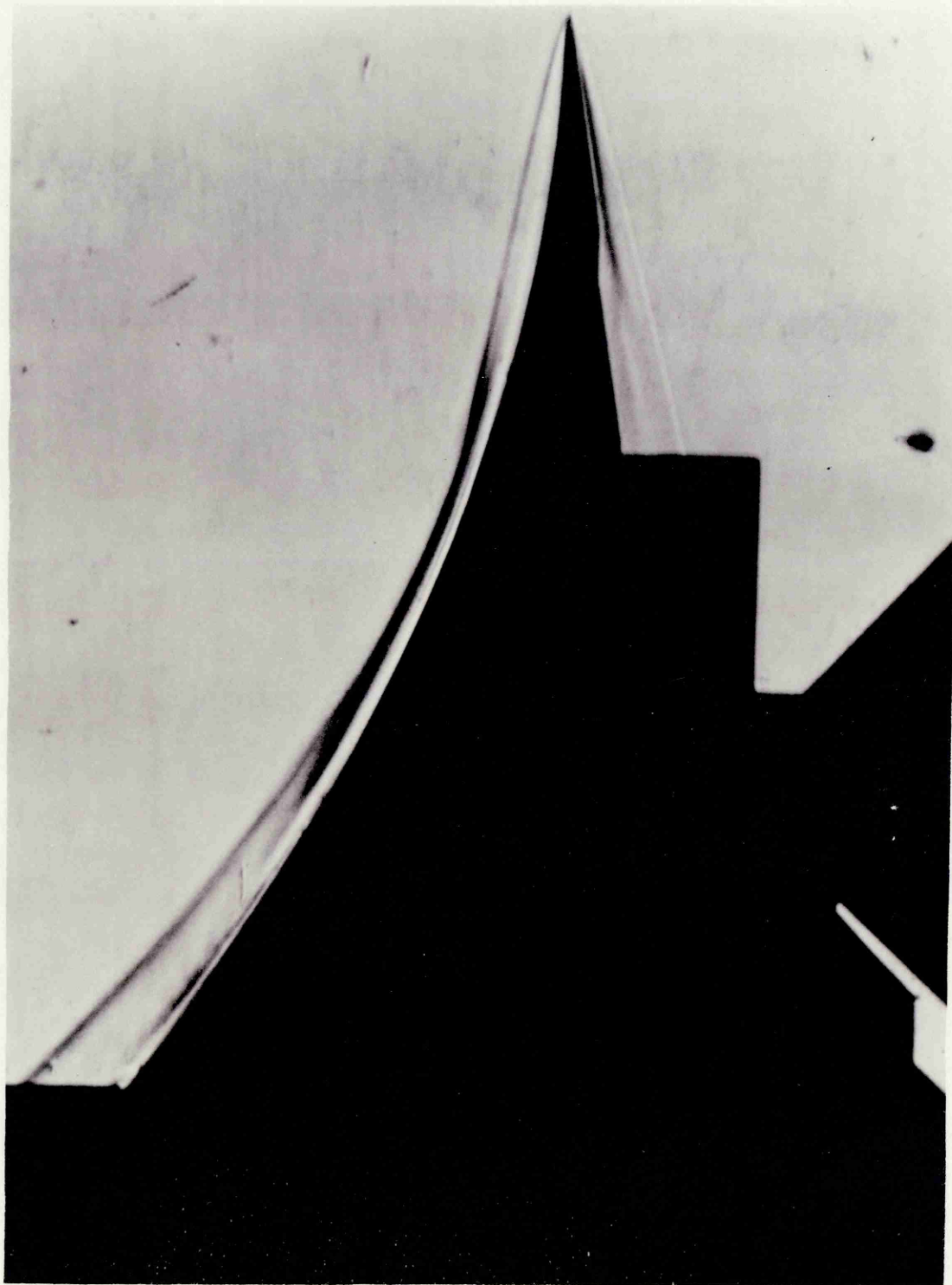


Plate 5: Schlieren photograph of the heat transfer (Concave)
model at zero incidence, $M = 12.25$, $Re \text{ unit} = 0.64 \times 10^5 / \text{in}$

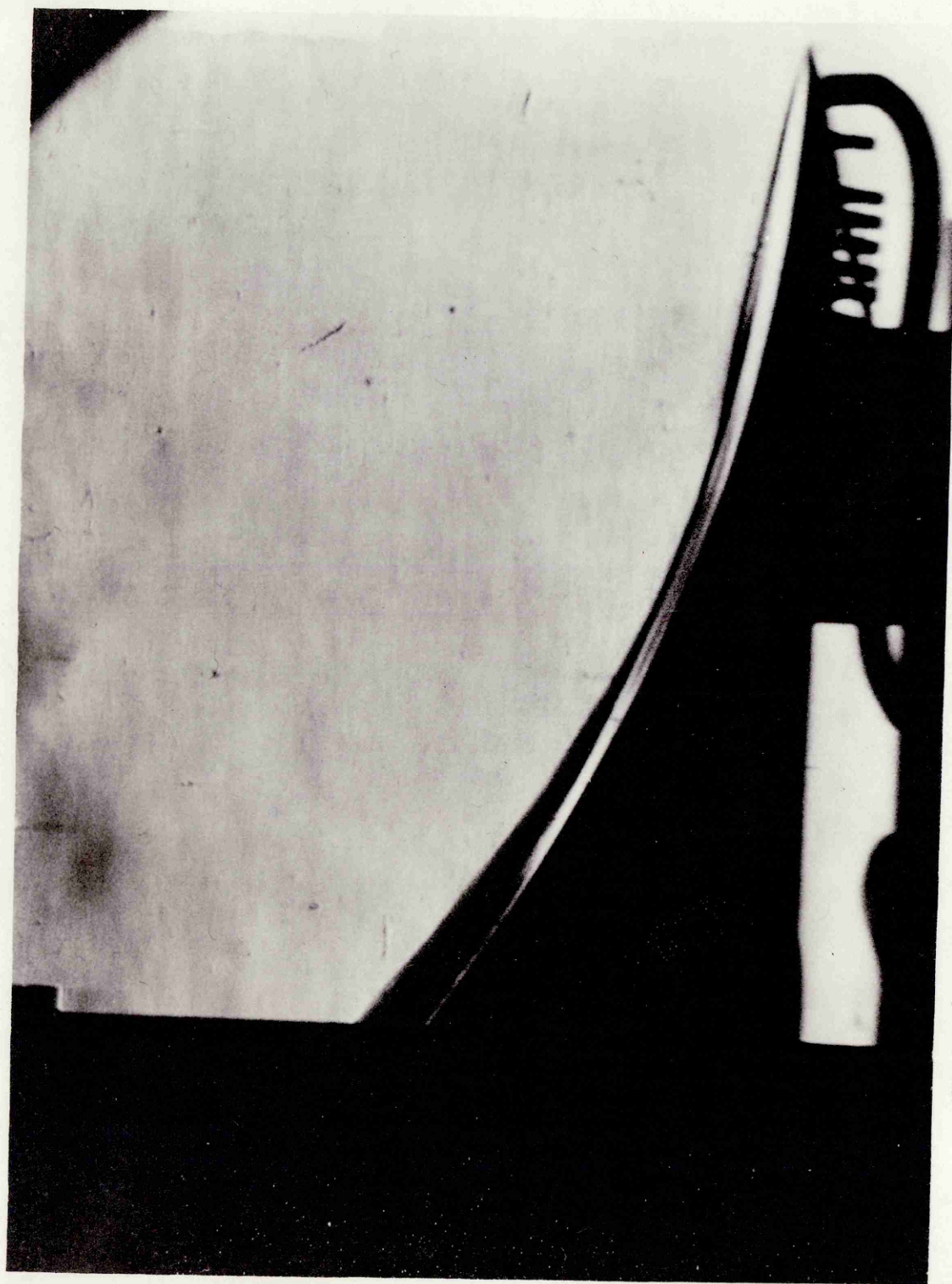


Plate 6: Schlieren photograph of the concave surface pressure
model at zero incidence, $M = 12.25$, $Re_{unit} = 0.64 \times 10^5 / \text{inch}$

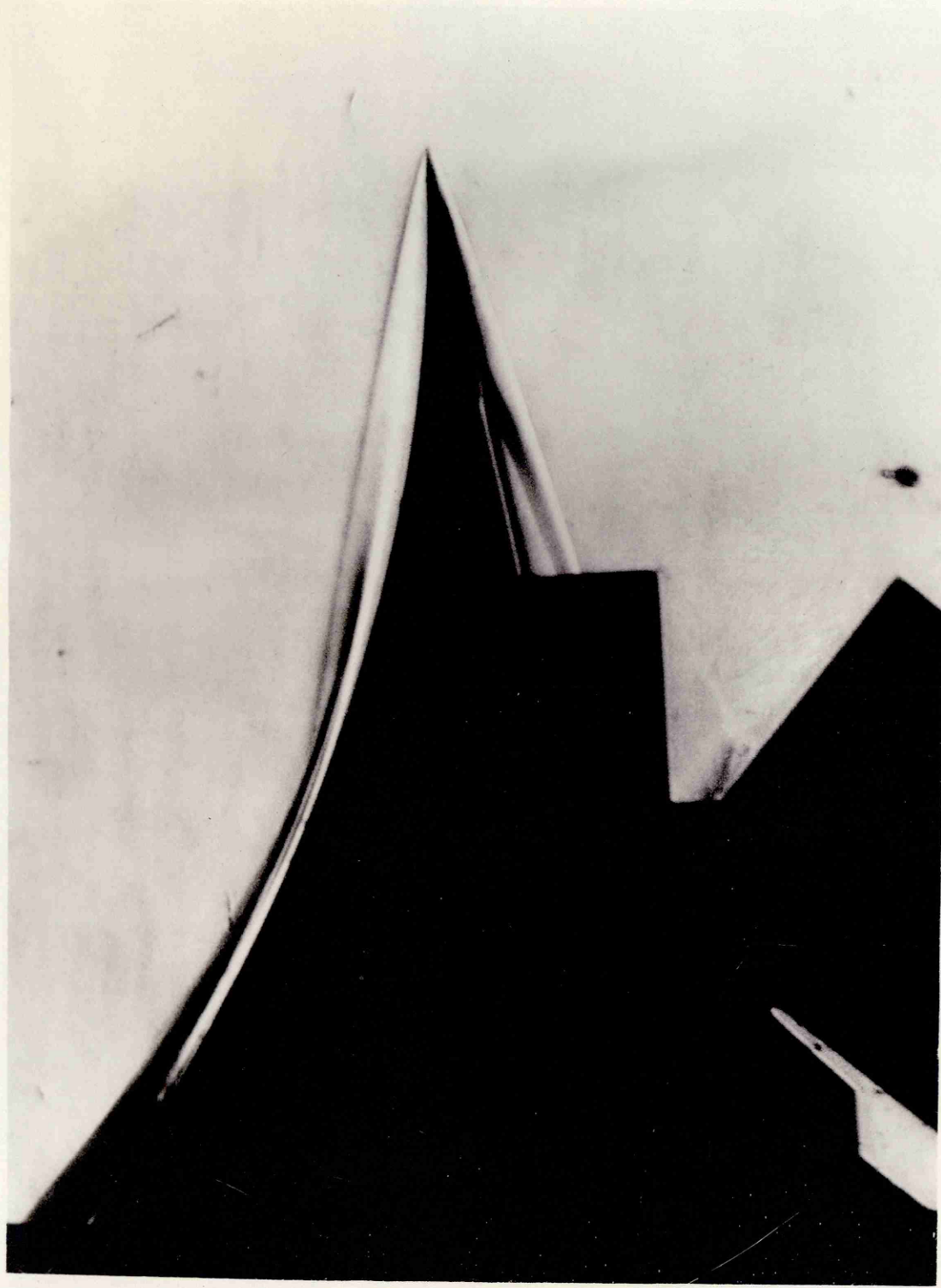


Plate 7 : Schlieren photograph of the concave surface heat transfer model at -5° angle of attack
 $M = 12.25$, $Re_{unit} = 0.64 \times 10^5 / \text{inch}$