

Numerical investigation of wave induced thrust from a submerged hydrofoil

Numerical investigation of wave induced thrust on a submerged hydrofoil

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Submerged flapping hydrofoils have the capability to directly convert wave energy into thrust, offering a sustainable approach to marine propulsion. This research employs Computational Fluid Dynamics (CFD) to analyse the propulsion mechanism of wave-induced flapping hydrofoils. Initially validated through established benchmarks and experimental results with foil in uniform flow, the CFD model was then applied to examine the generation of thrust by flapping hydrofoils in heading regular wave. The study reveals a distinct transition from drag to thrust, characterized by the patterns of vortex flow. For the first time, the influence of pitch stiffness on this propulsion process is extensively explored, identifying optimal wave conditions and pitch stiffness for the application of future eco-friendly marine systems.

I. INTRODUCTION

Addressing the global challenge of climate change, the development of green ocean propulsion systems plays a pivotal role in curtailing CO₂ emissions. Ocean waves represent an underutilized energy source that flapping hydrofoils can convert into thrust, providing both propulsion and stabilisation for marine vessels. This study presents a 'one-fluid' Computational Fluid Dynamics (CFD) framework to understand the dynamics of wave-foil interactions and the resulting thrust, with the goal of improving the environmental and economic efficiency of marine filed.

Flapping hydrofoils can generate thrust from flows, a concept first explained by Knoller¹ and Betz². Wu et al.³ explored energy extraction from wave-foil interactions, while Isshiki⁴ introduced wave devouring propulsion (WDP) using passive flapping hydrofoils. Lighthill⁵ and subsequent researchers like Koochesfahani et al.⁶ confirmed the formation of a jet-like wake, facilitating thrust. Studies have since employed numerical simulations to understand WDP's physics for marine applications, analyzing active flapping hydrofoils under various conditions. Filippas et al.^{7,8} and Gao et al.⁹ investigated the impact of hydrofoil movement and waveform perturbations on thrust generation. Qi et al.¹⁰ compared sinusoidal and non-sinusoidal pitching effects on propulsion, finding that certain pitching profiles enhance propulsive performance. De Silva and Yamaguchi¹¹ studied active oscillation hydrofoils in waves, noting efficient wave energy conversion into propulsion under specific conditions. Wang et al. studied the use of WDP for stabilizing a floating wind platform¹² and for reducing fuel consumption in shipping¹³.

Hydrofoils can transform flow energy into thrust using their flapping motion, often controlled with springs to adjust heave/pitch movements. This enables various flapping modes,

where an infinite stiffness results in pure heave or pitch motion, while reducing the stiffness leads to a coupled flapping motion. Prior research has mainly focused on active flapping, which our study uses as a benchmark to validate our CFD code. Yet, the passive thrust generation by wave-induced flapping remains less explored.

This research presents a novel 'one-fluid' CFD framework to analyse thrust production from passive, wave-induced flapping motions, with a focus on how changes in pitch spring stiffness influence thrust—a topic previously unexplored with CFD simulations. Additionally, the developed methodology can be readily applied to study the impact of heave spring stiffness on thrust generation.

The paper is structured as follows: Section II details the 'one-fluid' framework, hydrofoil geometry, motion modes, flow simulation, and key parameters. Model convergence and validation tests in uniform flow are covered in Section III. Section IV explores the effects of wave and pitch stiffness on thrust in passive hydrofoils under heading waves. Conclusions are summarized in Section V.

II. MATERIAL AND METHOD

A. Numerical solver

The numerical solver used in this study is an 'in-house' developed code that employs the 'one-fluid' formulation, recently proposed by Yang et al.^{14–17}. The 'one-fluid' formulation is a part of the immersed methods family. In contrast to traditional approaches, which typically represent structural dynamics through a set of Newton-Euler equations, this formulation utilises Lagrangian multipliers to impose constraints on motion, thereby ensuring rigidity. Simulations

are performed on a fixed Eulerian Cartesian grid with a uniform mesh size. The ‘one-fluid’ formulation uses the Level Set Method (LSM) to capture the interface between different phases. Compared to other formulations, a key advantage is it only uses fluid solver for the FSI. Additionally, this formulation does not need mesh updating or re-meshing. Both of these features contribute to improved computational efficiency.

Fig.1 shows the calculation logic of ‘one-fluid’ under one single step, which is validated through several benchmarks and experimental tests¹⁸. It is worth mentioning the part related to the solution of the velocity field. The framework considers the elastic force as the external force of the system. The connection between the spring force $\mathbf{F}_{ext}$ and the velocity $\mathbf{u}_r$ of the rigid body can be described as Eq.1, according to the law of conservation of linear momentum.

$$\mathbf{F}_{total} = \mathbf{F}_r + \mathbf{F}_{ext} = m \frac{d\mathbf{u}_r}{dt} + m \frac{d\mathbf{u}_{ext}}{dt} \quad (1)$$

where $\mathbf{F}_{total}$ is the total force on the rigid body at each time step Δt , $\mathbf{u}_{ext}$ is the velocity generated by the external force (i.e. spring force in this case). For more information, please refer to paper¹⁸.

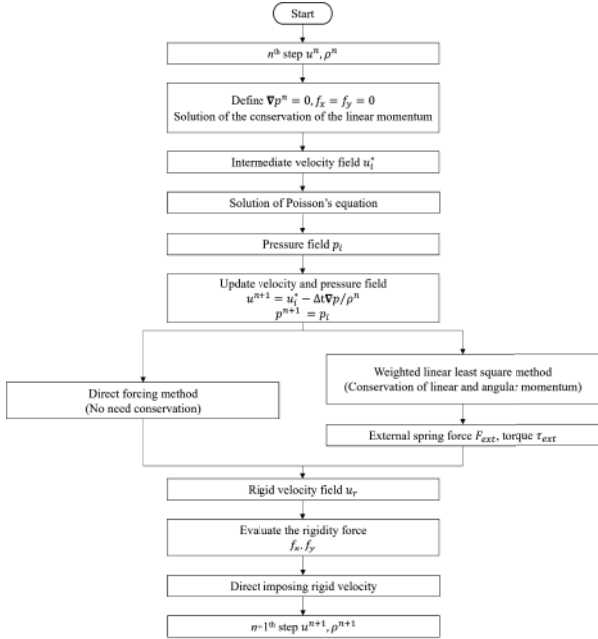


FIG. 1. The flow chart of the ‘one-fluid’ framework.

The simulation platform in this study employs a high-performance computing system featuring an Intel Core i7-10700 CPU 2.90GHz, 32GB of RAM, a 954GB SSD, and a 1.8TB HDD, running on Windows 10 Enterprise (version 21H2). This setup provides robust computational power and ample memory and storage capacity, essential for conducting CFD simulations efficiently.

1. ‘One-fluid formulation - Governing equations

Vortices are key to thrust generation from the hydrofoil¹⁹. Our aim is to solve the incompressible Navier-Stokes equations to capture these phenomena.

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\nabla \mathbf{u}) \mathbf{u} \right] = -\nabla p + \nabla \cdot 2\mu \mathbf{D}(\mathbf{u}) + \rho \mathbf{g} \quad \text{in } \Omega \quad (2a)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega \quad (2b)$$

$$\mathbf{D}[\mathbf{u}] = 0 \quad \text{in } \Omega_r \quad (2c)$$

where $\rho = \rho_a H_a + \rho_w H_w + \rho_r H_r$ is the density field, $\mathbf{u}$ is the velocity vector field, $\mathbf{g}$ is the gravitational acceleration, μ is the dynamic viscosity and $\mathbf{D}(\mathbf{u}) = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$ is the strain rate tensor. The subscripts indicate the different phases, i.e., a for air, w for water, and r for rigid body.

The LSM is used for capturing the interface between the fluid phases in the numerical domain, while the characteristic function χ of any region Ω_α can be expressed as

$$\chi = H_\alpha(\phi) \quad \alpha = a, w, r \quad (3)$$

where

$$H_\alpha(x) = \begin{cases} 1 & \text{if } x \in \Omega_\alpha \\ 0 & \text{if } x \notin \Omega_\alpha \end{cases} \quad (4)$$

is a Heaviside function for a phase α . $x = (x, y)$ is the coordinate of any point in the domain. H_α is introduced to identify different phase regions. Once the Heaviside function has been constructed, the corresponding density and viscosity field can be assigned. For more information refer to Yang’s study¹⁵.

2. Numerical and discretisation scheme

All the simulations are performed with a fixed Eulerian Cartesian grid and are numerically discretised using a grid-based immersed method. The Cartesian staggered Finite Volume scheme (Marker-and-Cell) is adopted for the spatial discretisation of the study domain. The convective terms of the Navier-Stokes equations are discretised with a second-order Total Variation Diminishing (TVD) Runge-Kutta scheme in the conservative framework. The second-order accurate Runge-Kutta scheme processes the time treatment of the computational momentum and level set equations. Adaptive time steps are applied to the calculation of the Courant-Friedrichs-Lewy (CFL) number¹⁵, which for all the simulations presented in this paper is kept equal to 0.5.

3. Boundary conditions

This study impose Dirichlet boundary condition for the wave generation and absorption. The initial flow profiles are

derived from experimental inlet settings. Specifically, the uniform flow and regular heading waves are referenced from the studies by Read et al.²⁰ and Isshiki et al.^{21,22}, respectively.

Convergence tests and initial validation of the computations, see in Section III, are performed with one hydrofoil subject to the uniform flow. The flow velocity U_{flow} is the same throughout the domain and is described by Eq.5 and Fig.6.

$$u(x, y, t) = U_{flow} \quad (5a)$$

$$v(x, y, t) = 0 \quad (5b)$$

The current flows steadily along the x positive direction with zero velocity in the y direction. The flow enters the domain from the left boundary and exits through the right boundary. To address concerns regarding the trailing wake, it is important to note that both the left and right boundaries are positioned sufficiently distant. It ensures uniform flow at both boundaries, maintaining the conservation of mass while minimising the influence of wake dynamics on the boundary conditions.

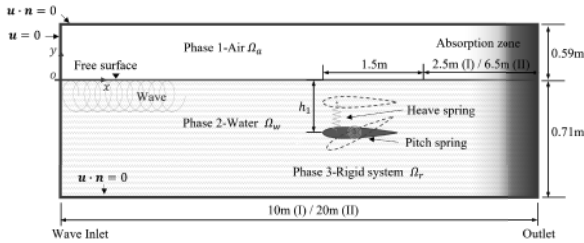


FIG. 2. The schematic diagram of the passive hydrofoil simulation.

For the study of wave-induced performance, see in Section IV, the hydrofoil is exposed to regular waves. The schematic diagram for passive foil simulations is shown in Fig.2. For these simulations, wave generation and absorption are set on the left and right-hand side of the numerical domain, see also Fig.2. The flow conditions, in the form of horizontal and vertical velocity, at the inlet are calculated using linear wave theory as:

$$u(x, y, t) = \frac{\pi H}{T} \frac{\cosh[k(y+d)]}{\sinh(kd)} \cos(kx - \omega_w t) \quad (6a)$$

$$v(x, y, t) = \frac{\pi H}{T} \frac{\sinh[k(y+d)]}{\sinh(kd)} \sin(kx - \omega_w t) \quad (6b)$$

where u and v are the instantaneous horizontal and vertical particle velocities at (x, y) location, H is the wave height, T is the wave period, d is the water depth, $k = \frac{2\pi}{\lambda}$ is the wave number, $\omega_w = \frac{2\pi}{T}$ is the angular wave frequency, and λ the wavelength calculated with the linear dispersion equation, Eq.7. Using linear wave theory, the wave conditions in the numerical domain were tuned to match the wave measured in the experiments of Isshiki et al.²². It is noted, that the use of linear wave theory for generating non-linear waves is a limitation of this study, which can lead to the generation of spurious long wave components. The application of a more precise wave generation procedure goes beyond the scope of the present

work and the spurious effects partly mitigated through the use of an absorption zone that the end of the numerical fume.

$$\lambda = \frac{gT^2}{2\pi} \tanh(kd) \quad (7)$$

The relaxation zone is applied to gradually decrease the velocities u and v in the absorption zone to zero. This technique is known to mitigate spurious wave reflections and improve the quality of the simulation results. The length of the wave absorption zone at the outlet (right hand side boundary) varies in accordance with the incoming wavelength and it is set to 2.5 m for $\lambda < 3.75$ m and 6.5 m for $\lambda < 7.70$ m. At the absorption zone wave reflection is prevented through the application of the relaxation method proposed by Bihs et al.²³, Eq.8.

$$\Gamma(\tilde{x}) = 1 - \frac{e^{(\tilde{x}^{3.5})} - 1}{e - 1} \quad \text{for } \tilde{x} \in [0, 1] \quad (8a)$$

$$u_r = \Gamma(\tilde{x})u_c \quad (8b)$$

$$v_r = \Gamma(\tilde{x})v_c \quad (8c)$$

Here, $\Gamma(\tilde{x})$ represents the relaxation function, with $\tilde{x}$ being a scale factor ranging from 0 to 1, denoting the normalized distance from the wave inlet boundary to the absorption zone. In the absorption zone, u_r and v_r are the relaxed velocity components, contrasting with u_c and v_c , the original computational velocities.

B. Hydrofoil characteristics

1. Geometry and motions

The study validates simulation results against experimental findings by Read et al.²⁰ and Isshiki et al.^{21,22}, focusing on two NACA hydrofoil types: NACA0012 and NACA0015. Each hydrofoil is described by its chord length c and maximum thickness D , with D being a percentage of c . While 3D applications consider the hydrofoil's span s , this 2D analysis normalizes s to 1.

Hydrofoil motions in simulations are either active oscillations (enforced around a fixed position) or passive flapping (flow-induced combining heave and pitch), as depicted in Fig.3. Active motions include vertical heave or rotational pitch around a pivot point P . In accordance with the experiments of Read et al.²⁰, P is set at the chord and a distance of $c/3$ from the leading edge of the hydrofoil. Pure heave and pitch motions are dictated in the numerical domain through Eq.9a and 9b. The flapping hydrofoil motion emerges from the combination of heave and pitch.

$$h(t) = h_0 \sin(\omega t) \quad (9a)$$

$$\theta(t) = \theta_0 \sin(\omega t + \psi) \quad (9b)$$

where $h(t)$ is the displacement of the hydrofoil in the y direction or in other words the instantaneous vertical position of the hydrofoil axis. $\theta(t)$ is the instantaneous angle formed between the in flow velocity (U_∞) and the chord, also referred to

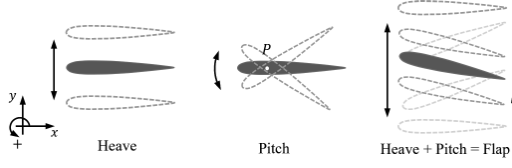


FIG. 3. The motions of the hydrofoil.

as the angle of rotation. h_0 and θ_0 is the heave and pitch amplitude. ω is the angular frequency (rad/sec), $2\pi f$, where f is the oscillating frequency, and ψ is the phase difference between the pitch and heave motion. When it comes to propulsion efficiency, Platzer et al.²⁴ suggested $\psi = 90^\circ$ as the optimal phase difference between pitch and heave.

A hydrofoil can generate either a drag force or a thrust force, depending on the wake patterns. When a von Kármán vortex street (vK)²⁵ develops, the velocity of the flow within the wake is diminished relative to the incoming or ambient flow velocity. This leads to the hydrofoil entering a regime dominated by drag. However, when the vortices in the wake align at the hydrofoil's trailing edge, a unique condition arises. The velocity of the flow within the wake now matches the incoming or ambient flow velocity, and the hydrofoil experiences a regime of neutrality. Further evolution of these conditions can lead to thrust generation. When a reversed von Kármán vortex street (RvK) develops, it accelerates the flow behind the hydrofoil, leading to thrust production. The emergence of a reversed RvK wake signals the onset of the drag-to-thrust wake transition. However, there's a delay in the hydrofoil's transition towards thrust generation. This delay occurs because the minor velocity surplus cannot overcome the profile drag, velocity fluctuations, or pressure differences^{26–29}. The three fundamental wake patterns are illustrated in Fig.4.

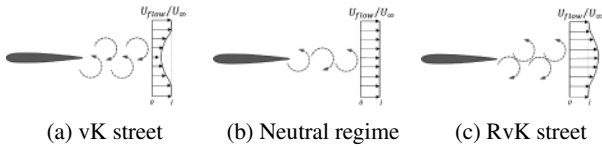


FIG. 4. The illustration for the drag-to-thrust wake transition of the flapping hydrofoil.

For passive hydrofoils controlled by springs, the switch between drag and thrust depends on spring stiffness, affecting pitch angle change and amplitude. Simulations represent this effect by applying an instantaneous force and moment on the hydrofoil, as outlined by Eq.10 and depicted in Fig.2.

$$F_h(t) = k_h \Delta y \quad (10a)$$

$$M_p(t) = k_p \Delta \theta \quad (10b)$$

where, $F_h(t)$ is the heave force and $M_p(t)$ is the pitch moment. Δy and $\Delta \theta$ are the vertical and rotational deviation of the hydrofoil from its initial position at time t , respectively, while k_h and k_p are the heave and pitch stiffness. This definition enables the direct comparison with the experiments of Isshiki

et al.^{21,22} for which k_h is set to 10388 N/m and k_p to 129.24 N·m/rad. The pitch stiffness effect is then explored by keeping k_h constant and varying k_p for different incoming flow conditions. This topic will be revisited in Section IV B. To remove the effect of the buoyancy force, the hydrofoil's density has been matched to that of the fluid $\rho = \rho_{hydrofoil} = \rho_{fluid}$. We have included the gravity force for the wave dynamics.

2. Main and dimensionless parameters

In fluid dynamics, hydrofoils are subject to both drag and lift forces, illustrated in Fig.5. As elaborated earlier, the formation of a RvK vortex street indicates a shift from drag to thrust, occurring when the wake's velocity surpasses the surrounding flow's speed, thus generating thrust. In this study, the thrust force opposes drag and is denoted as F_x , while the lift force, acting vertically upward, is labeled F_y . These forces are quantified using dimensionless coefficients for lift (C_l) and drag (C_d), calculated by normalizing the force on the hydrofoil against the dynamic pressure of the incoming flow and the hydrofoil's platform area, as shown in Equation 11.

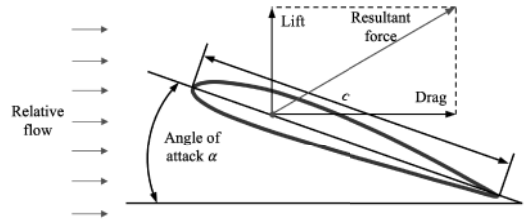


FIG. 5. The forces on the hydrofoil facing flows.

$$C_l = \frac{F_L}{\frac{1}{2} \rho U_\infty^2 c} \quad (11a)$$

$$C_d = \frac{F_D}{\frac{1}{2} \rho U_\infty^2 c} \quad (11b)$$

where, F_L and F_D is the lift and drag force and c is the hydrofoil chord. $F_T = -F_D$ and $C_t = -C_d$ are the thrust force and coefficient. In addition, Eq.11 refers to instantaneous coefficient values and the time-averaged, e.g. over one wave period T , forces and coefficients ($\overline{F_D}$, $\overline{F_L}$ and $\overline{C}$) are calculated as:

$$\overline{C} = \frac{1}{T} \int_0^T C(t) dt \quad (12)$$

$$\overline{F_x} = \frac{1}{T} \int_0^T D(t) dt \quad (13)$$

$$\overline{F_y} = \frac{1}{T} \int_0^T L(t) dt \quad (14)$$

where, $\overline{C}$ can be any of $\overline{C_l}$, $\overline{C_d}$ and $\overline{C_t}$.

The interaction between a moving hydrofoil and the incoming flow is described through the non-dimensional Strouhal

and Reynolds numbers. The chord based Re is calculated from Eq.15a, while several expressions for the Strouhal number have been proposed. Typically, the peak-to-peak oscillating amplitude, A , of the hydrofoil's trailing edge and its oscillating frequency, f , are normalised with the characteristic flow velocity, U_∞ , for estimating St_A , Eq.15b^{19,30}. Read et al.²⁰ reported optimum thrust generation for $0.2 < St_A < 0.4$.

$$Re = \frac{\rho c U_\infty}{\mu} \quad (15a)$$

$$St_A = \frac{fA}{U_\infty} \quad (15b)$$

where ρ is the density of the surrounding fluid, U_∞ is the characteristic flow speed, μ is the dynamic viscosity of the fluid, f is the oscillating frequency and A is the oscillating amplitude, which is often taken as the width of the wake. For forced harmonic oscillating motions, $2h_0$ is an estimate for the width of the hydrofoil wake, thus A can be approximated with $2h_0$, and since $f = \frac{\omega}{2\pi}$ Eq.15b is also written as Eq.16.

$$St_A = \frac{\omega h_0}{\pi U_\infty} \quad (16)$$

III. HYDROFOIL IN UNIFORM FLOW

A. Fixed hydrofoil

This section assesses the convergence of drag and lift coefficients for a NACA0012 hydrofoil in uniform flow, depicted in Fig.6. The simulation uses a numerical tank sized 7 m by 7 m, positioning the hydrofoil vertical centrally, one meter from the inlet on the left boundary. Fig.6 shows the coordinate system starting at the hydrofoil's leading edge, extending positively towards the trailing edge and upward from the front. The hydrofoil's density matches the fluid's at $\rho=1$ kg/m³, with a dynamic viscosity of $\mu=10^{-3}$ kg/m·sec. The incoming flow velocity is set at $U_\infty=1$ m/s, with an angle of attack (AoA) α of -15° and the chord length c is 1 m. Consequently, the Reynolds number is calculated as $Re = 1000$, fitting within the typical range for flapping hydrofoils of $100 < Re < 10000$ ³¹. Direct Numerical Simulation is utilized for these low Reynolds number flows in the fixed hydrofoil scenario.

Convergence tests were conducted using three different mesh sizes, as detailed in Table.I. The results for both lift coefficient (C_l) and drag coefficient (C_d) are presented in Fig.7, demonstrating satisfactory convergence. The period between 20 and 30 seconds was specifically chosen for analysis because, during this interval, the wake reaches full development. In the fixed case, the numerical scheme becomes a direct forcing method, and achieved expected results. The consistently positive C_d indicates a constant, though periodically fluctuating, drag force acting on the hydrofoil, with no thrust generation observed.

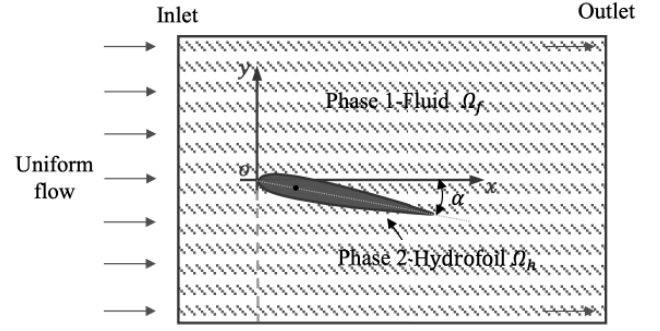


FIG. 6. A schematic diagram of the simulation for the fixed hydrofoil in uniform flow.

TABLE I. Grid resolution for numerical comparison of fixed hydrofoil model.

	Mesh size Δx (m)	Numbers of cells
1	0.0156	448×448
2	0.0078	896×896
3	0.0039	1792×1792

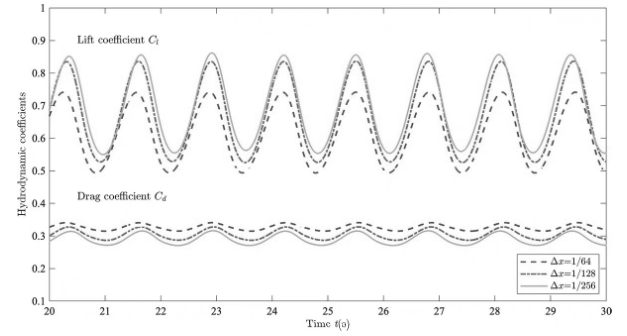


FIG. 7. Validation of convergence of fixed hydrofoil facing uniform flow.

B. Active flapping hydrofoil

This subsection examines the active flapping hydrofoils in uniform flows, highlighting the transition from drag to thrust as demonstrated in previous studies^{20,32–34}. The experimental findings of Read et al.²⁰ serve as benchmarks for this analysis. They conducted experiments with a towed NACA0012 hydrofoil undergoing harmonic (sinusoidal) oscillation in heave and pitch, measuring lift, thrust forces, and pitching torque across various angles of attack and ψ .

The towing speed was 0.4 m/s, and the hydrofoil chord length was 0.1 m, resulting in a Reynolds number of $Re=4 \times 10^4$. This setup is replicated in a 2D numerical tank sized $0.5 \text{ m} \times 0.6 \text{ m}$, similar to the configuration shown in Figure 6(a). The numerical simulation adopts a uniform flow at the towing speed of 0.4 m/s, with the hydrofoil's chord length also at 0.1 m and matching the fluid density. Heave and pitch displacements are induced using Eq.9 with $h_0=0.075$ m and $\psi=-90^\circ$. This configuration produced a trajectory that allows for a symmetric evolution of $\alpha_{eff}(t)$, closely following the

experimental design. The pivot point is set at a third of the chord length from the leading edge, with an initial angle of attack of -15° . The simulations and experiments span Strouhal numbers St_A from 0.08 to 0.44 and frequency f ranges from 0.21 to 1.17 Hz, utilizing Direct Numerical Simulation to fully resolve the fluid dynamics.

Numerical results, compared with experimental data in Fig.8, show the thrust coefficients C_t plotted against the amplitude-based Strouhal number St_A . The findings indicate that C_t rises with St_A . However, across all St_A values, the calculated C_t consistently falls below the experimental C_t , except at $St_A=0.44$. The difference may be attributed to the limitations of using 2D simulations, as discussed in Mittal and Balachandar's study³⁵.

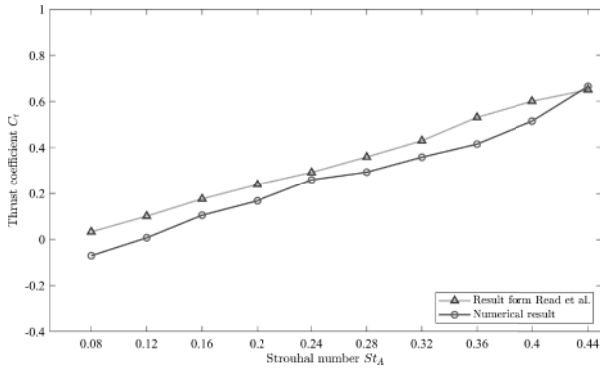


FIG. 8. The comparison of the thrust coefficient C_t under different St_A between the experiment (the upper line) and active hydrofoil simulation ($\Delta x = \frac{1}{1280}$ m).

The Strouhal number St_A , as defined, reflects the ratio of the hydrofoil's trailing edge speed to the flow velocity. Increasing the hydrofoil's amplitude enhances motion-induced thrust, as shown by higher C_t values for larger St_A , as shown in Fig.8. This relationship is further illustrated by analyzing the wake and thrust development for varying St_A values, with visual examples in Fig.9. The figures capture the fluctuating forces on the hydrofoil across a motion cycle, oscillating between thrust and drag. Significantly, the forces on the hydrofoil oscillate between thrust and drag within a motion cycle. At specific times (t_0 , $t_0 + T$, and $t_0 + 1/2T$) (Fig.9 columns (a), (c) and (e)), when the hydrofoil returns to its initial position, implying a zero heave velocity and maximum AoA, thrust is produced, except at $St_A = 0.08$ time $t_0 + T$. Conversely, at times (t_0 , $t_0 + 1/4T$, and $t_0 + 3/4T$) (Fig.9 columns (b) and (d)), the heave acceleration is zero, and the AoA in the pitch direction reaches its minimum ($=0^\circ$), consistently generating drag.

IV. HYDROFOIL IN WAVES: PASSIVE FLAPPING HYDROFOIL

A. The effect of heading wave

This section explores the interaction between waves and hydrofoils under regular wave conditions with varying spring

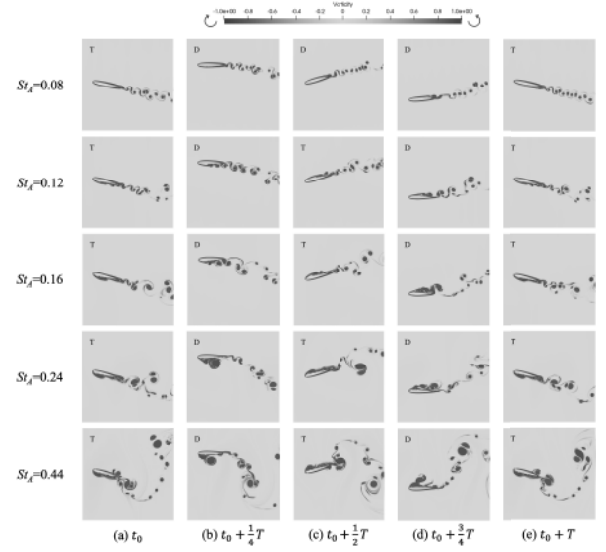


FIG. 9. Snapshot series of the motion of active hydrofoil and its wake in one cycle ($St_A=0.08$ (1st row), 0.12(2nd row), 0.16(3rd row), 0.24(4th row), 0.44(5th row), $\Delta x = \frac{1}{1280}$ m. The force status is marked in the top left corner, where D is for drag and T is for thrust.).

stiffness, comparing the results with Isshiki et al.'s physical model tests^{21,22}. The setup for numerical simulations is illustrated in Fig.2, highlighting that computational domain sizes adjust according to the wavelengths discussed in Table II. Shorter wavelengths in group I use a 10 m by 1.3 m domain, while longer waves in group II require a 20 m by 1.3 m domain, optimizing calculation efficiency. The simulation space includes air, water, and hydrofoil phases. A NACA0015 hydrofoil, with a chord length of 0.4 m, is positioned 0.077 m below the air-water interface, mirroring the experimental setup. The simulations employ restoring forces and moments to simulate the experimental springs (Eq.10), controlling heave and pitch movements with stiffness values of $k_h=10388$ N/m and $k_p=129.24$ N·m/rad, matching the physical tests.

In Fig.10, the time-averaged advance speeds of a passive hydrofoil are plotted across a range of wavelengths, comparing numerical simulation results with experimental and calculated data from Isshiki et al.^{21,22}. The advancing speed, U , is defined as the average speed of the foil after it has reached a steady motion following several wave cycles. This speed is determined by measuring the distance the foil travels in the x-direction over a specific period and then dividing that distance by the elapsed time. The graph demonstrates a consistent trend across all three data sets, with the numerical results slightly lower than the experimental findings but closely following the calculated results. This close alignment suggests that the numerical simulations are capturing the key physical phenomena with a high degree of accuracy, despite the inherent simplifications of 2D modeling. The convergence between the numerical and experimental data, especially at longer wavelengths, indicates that the simulations are particularly effective at capturing the hydrofoil's interactions with such wave conditions. The persistent lower thrust predictions

TABLE II. Wave conditions in the semi-active flapping hydrofoil simulations.

Group	No.	Wavelength λ (m)	Wave height $2a$ (m)	Wave steepness
I	1	2.00	0.096	0.048
	2	2.37	0.108	0.045
	3	2.92	0.123	0.042
	4	3.75	0.143	0.038
II	5	4.75	0.159	0.034
	6	5.92	0.170	0.029
	7	6.88	0.174	0.025
	8	7.70	0.175	0.023

in the simulations are likely due to the two-dimensional approach's limitations in fully representing the intricate three-dimensional flows and interactions observed in actual environments.

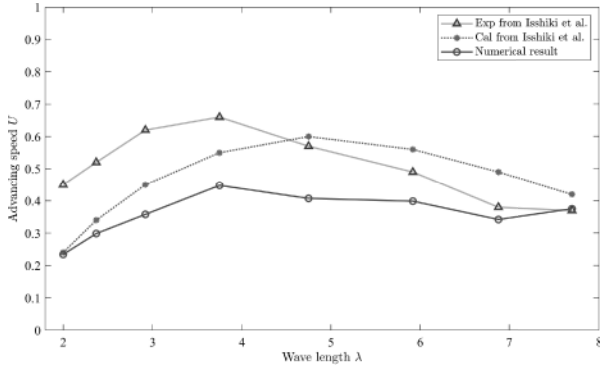


FIG. 10. The comparison between the experiment, calculation and simulation for passive hydrofoil ($\Delta x = \frac{1}{160}$ m.)

Fig.11 captures the hydrofoil's interaction with regular waves, specifically with a wavelength of 3.75 m where it achieves its highest advance speed within the studied range. The snapshots cover the moment the hydrofoil encounters a head-on wave and its subsequent motion over six wave periods. Fig.12 illustrates the thrust variation corresponding to Fig.11, with the horizontal axis intuitively representing the states from t_0 to t_5 as snapshots. The red dots in the figure indicate the thrust at each moment. Here, the thrust in the direction of the foil's motion is considered positive. Comparing the snapshots, it can be observed that during the progression from t_0 to t_5 , the force is predominantly positive but fluctuates, demonstrating the transition and generation of thrust and drag. At t_4 , a neutral vortex street forms, leading to mild propulsion. After t_5 , an RvK vortex street develops, visibly boosting the hydrofoil's self-propelled motion against the incoming waves. Overall, thrust dominates over drag, resulting in noticeable forward motion.

Fig.13 shows the hydrofoil's motion and wake over one wave period, where the chord line orientation and vortex street evolution are presented. As the hydrofoil tilts under the wave crest (Fig.13(a)), a negative AoA generates an RvK vortex street, propelling it against the wave. This force fluctuates with the wave's motion, transitioning between thrust and drag, ultimately resulting in net positive thrust. By the end of one

cycle, the pattern repeats with the next wave crest (Fig.13(e)), indicating a consistent thrust-generation mechanism, as confirmed by studies like those of Ishihiki et al.^{21,22}.

Fig.14 illustrates how the hydrofoil's non-dimensional displacement varies with wavelength over time, with the advance velocities indicated next to each line and also shown in Fig.10. The displacement initially rises mildly due to the hydrofoil's interaction with first waves, then peaks and falls sharply within each period, reflecting the dynamic motion between wave crests and troughs. This cyclic motion results in net positive displacement.

B. The effect of the pitch stiffness

This section examines how pitch stiffness influences thrust generation, affecting the hydrofoil's pitch amplitude and its propulsion performance. Originally, the pitch spring stiffness (k_p) was 129.24 N·m/rad in Section IV. To deepen the investigation, simulations with k_p values between 0 and 12900 N·m/rad were conducted. When the foil encounters waves, it begins to flap with the wave motion. The presence of pitch stiffness causes the foil's pitch angle to deviate from zero, and the spring attempts to restore it to its equilibrium position. As a result, the flapping frequency of the foil does not synchronize exactly with the wave frequency. The resulting thrust coefficients (C_t) are plotted against k_p in Figure 15.

Fig.15 illustrates the influence of pitch stiffness k_p on thrust generation C_t , revealing two distinct response regions. Initially, at low stiffness, thrust is negligible. As stiffness increases, the thrust coefficient rises sharply (region (I)), peaking at a stiffness of 129.24 N·m/rad. Beyond this peak, further increases in stiffness result in a decrease in thrust (region (II)). The relationship between thrust generation and stiffness is linked to the Strouhal number's optimal range for effective propulsion. As highlighted earlier, optimal C_t is achieved when $0.2 < St_A < 0.4$ ²⁰. The system's flow dynamics are influenced by varying wavelengths (from 2.00 m to 5.92 m), which affect the flow velocity and consequently the Strouhal number. At higher stiffness levels, the system becomes more rigid, leading to reduced flapping amplitude, which impacts the Strouhal number. The changes in thrust due to variations in stiffness and wavelength suggest that there is an optimal stiffness level that maximizes thrust across different wave scenarios. This behavior is also supported by Wang et al.'s experiments^{36,37}.

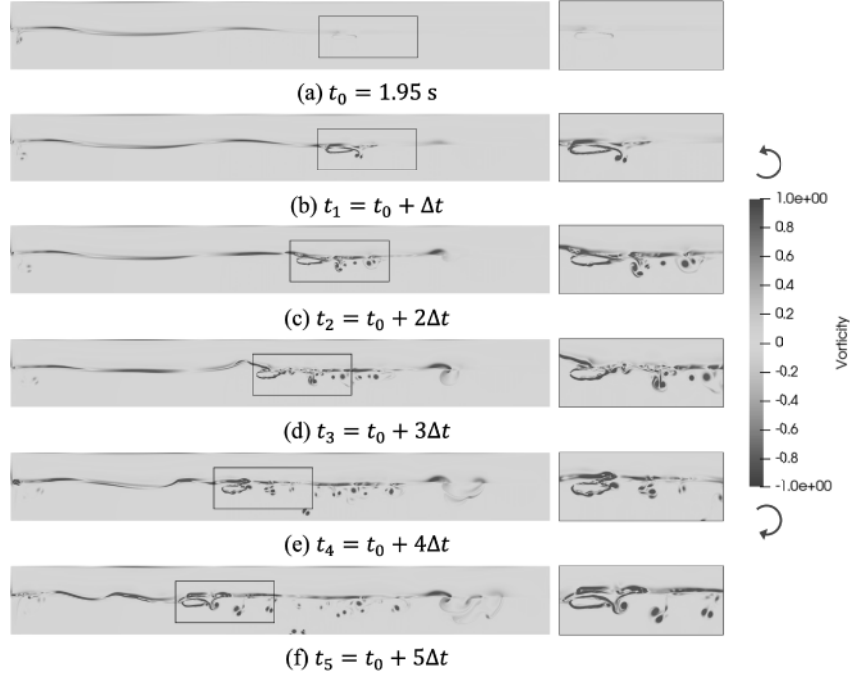


FIG. 11. The position of the passive hydrofoil in wave (wavelength=3.75 m, time interval $\Delta t = 1.70$ sec is the wave period). The series of snapshots on the right are the zoomed-in parts.

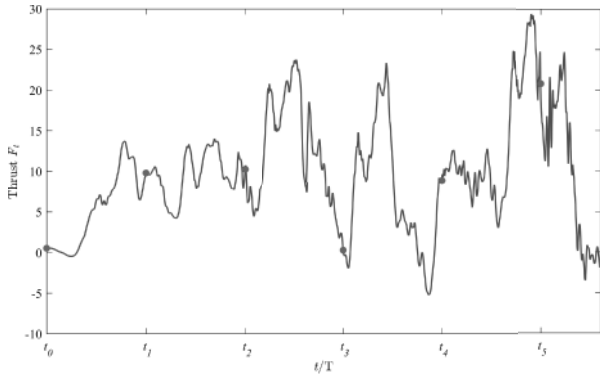


FIG. 12. Thrust variation of the passive hydrofoil in wave corresponding to foil snapshots.

forward. As pitch stiffness rises, the pitch motion narrows, generating smaller vortices at higher frequencies, as observed in parts (c) and (d) of the Fig.16. The optimal stiffness for maximum thrust suggests starting with 129.24 N·m/rad for a NACA0015 hydrofoil with a 0.4 m chord. While pitch stiffness optimization can enhance thrust, the maximum C_l remains wavelength-dependent. For those investigating the ideal pitch stiffness for maximum thrust, starting with a pitch stiffness of 129.24 N·m/rad for a 0.4 m NACA0015 hydrofoil is advisable. It is also noteworthy that although optimizing the pitch stiffness will lead to a higher thrust coefficient, the maximum threshold for C_l is mostly defined by the wavelength, see Fig.15.

V. CONCLUSIONS

Fig.16 illustrates how the hydrofoil's position and the vorticity field vary with pitch stiffness k_p , all under wave conditions with a 3.75 m wavelength. The images captured at $t_0=8.99$ sec show the fully-developed vortex streets behind the hydrofoil. The rotational angle deviation of the hydrofoil, $\Delta\theta$, diminishes with increased pitch stiffness, leading to different hydrofoil responses to identical wave conditions. At zero pitch stiffness (Fig.16(a)), there's no restoring force, resulting in a hydrofoil that moves passively with the wave and only a weak vorticity field occurs at the leading and trailing edge. However, even a small stiffness value creates a regulating moment (Fig.16(b)), enabling a controlled pitch motion and a transition from drag to thrust, propelling the hydrofoil

This study applied CFD to examine wave-induced thrust on hydrofoils, with a special focus on the effect of pitch stiffness. The numerical approach was verified with established benchmarks, including the performance of a foil in uniform flow, to confirm its accuracy for analyzing thrust generation mechanisms in varied hydrodynamic conditions.

Investigations into the foil in uniform flow highlighted how alternating vortex streets—RvK and vK—correspond with shifts in thrust and drag. These dynamics are essential for propelling the hydrofoil against waves and are crucial for performance optimization. The study also assessed the hydrofoil's response to heading regular waves, examining a range of pitch stiffness up to 12900 N·m/rad and wavelengths up to 7.70 m.

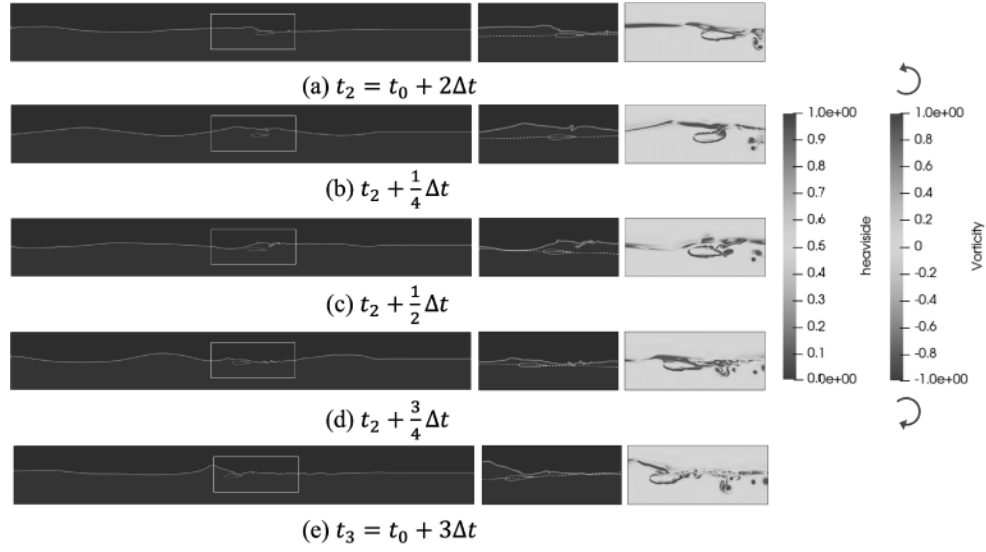


FIG. 13. The position of the passive hydrofoil in one wave period (wavelength=3.75 m, $t_0 = 1.95$ sec, time interval $\Delta t = 1.70$ sec is the wave period). The snapshots on the left show the Heaviside function, while those on the right show the zoomed-in vortex fields. The dash lines are the chord lines.

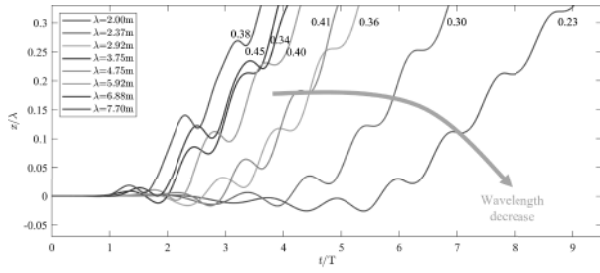


FIG. 14. The non-dimensional displacement x/λ of the passive hydrofoil with time over wave period t/T facing different waves ($\Delta x = \frac{1}{160}$ m).

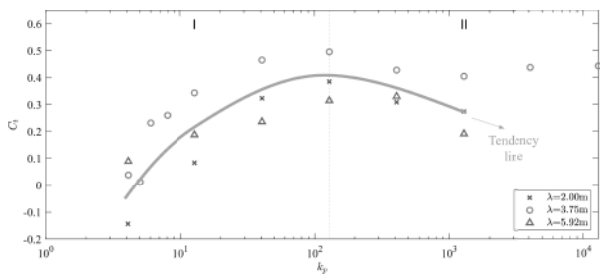


FIG. 15. The thrust coefficient C_t of the passive hydrofoil with different pitch stiffness k_p ($\Delta x = \frac{1}{160}$ m).

- There are two distinct responses to pitch stiffness: an initial increase in thrust coefficient with increasing stiffness, followed by a decrease as the hydrofoil becomes overly stiffer.
- A pitch stiffness of 129.24 N·m/rad marks the transition between the two responses, recommended as an optimal starting point for maximizing thrust in regular waves.

These conclusions provide a pathway for further refinement of hydrofoil applications in wave-dominated environments.

AUTHOR STATEMENTS

J. Xing: Numerical simulations and methodology, formal analysis, writing, editing; D. Stagonas: Original concept development, methodology, formal analysis, writing, supervision, review and editing; C. Zhang: Review and Editing; J. Yang: Review and editing; P. Hart: Review and Editing; L. Yang: Numerical simulations and methodology, original concept development, formal analysis, reviewing, editing, supervision and project administration.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests that could have appeared to influence the work reported in this paper.

The findings indicate that:

- Hydrofoil advance speeds vary with wavelength, reaching peak efficiency at 3.75 m wavelength.
- A nonzero pitch stiffness is necessary for the hydrofoil to counteract incoming waves.

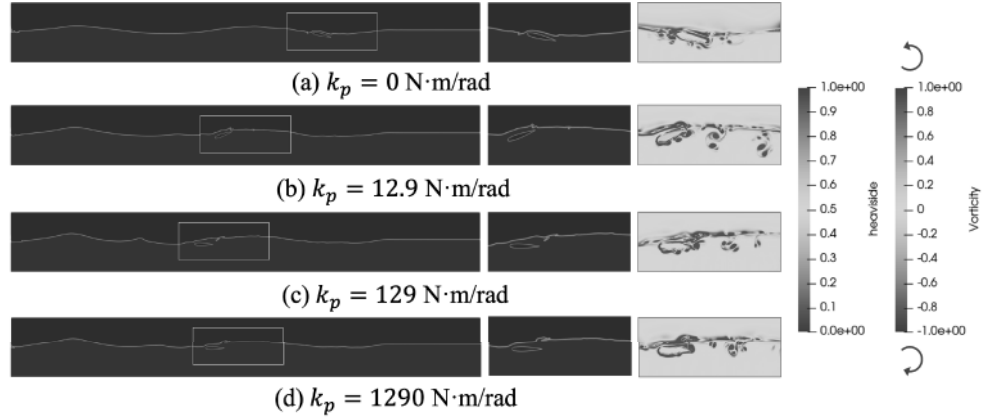


FIG. 16. The snapshots of the passive hydrofoil under different pitching spring stiffness k_p (wavelength=3.75 m, $t_0 = 8.99$ sec). The series of snapshots on the left shows the Heaviside fields, while those on the right show the zoomed-in Heaviside and vortex fields.

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Numerical investigation of wave induced thrust on a submerged hydrofoil

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