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HEAT TRANSFERS FROM DISTRICT HEATING PIPES

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Abstract

Experimental and numerical investigations were carried out on air-filled cavities containing heated inner cylinders.

The effect of varying the position of radial spacers on a single cylinder was studied. It was concluded that for central positioning of the cylinder within the cavity, the rate of heat-transfer was minimised at a radial spacer angle of 48° (measured from the vertically downwards radius vector). When the cylinder was positioned at displacement ratio of 0.7, the rate of heat-transfer was minimised at a corresponding spacer angle of 52° . The corresponding reductions in the total rate of heat-transfer were found to be 25% and 31% less than that obtained for the system with no spacers at a cylinder displacement ratio of zero.

Following this research investigation, the behaviour of a two-pipe arrangement, consisting of a hot supply and cooler return pipe within a rectangular sectioned cavity, was studied. Eccentric positioning of both supply and return pipes showed that minimum rates of heat-transfer occur at supply and return pipe displacement ratios of 0.45 and -0.33 respectively. This value of heat-transfer is approximately 20% less than that obtained for a system where supply and return pipe displacement ratios are 0.7 and zero respectively.

As experimental testing has proved to be excessively time consuming (e.g. due to having to wait until a steady-state ensued before measurements were taken) and laborious, a finite-element numerical model was developed and used to predict the heat-transfer between a heated inner cylinder and a cooled outer square duct.

This study investigated eccentricity effects on the rate of heat-transfer for different ratios of duct height to cylinder radius. Solutions were obtained for Rayleigh numbers 1 to 300 and optimal pipe eccentricity for minimum heat-transfer was predicted. These predictions were in good agreement with previous experimental results.

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NOMENCLATURE

A	Area (m^2)
C	Specific heat of a fluid ($Jkg^{-1}K^{-1}$)
D	Cylinder diameter (m)
d	Gap between pipe and cavity wall (m)
d_i	Inner cylinder diameter (m)
d_o	Outer cylinder diameter (m)
$\frac{dT}{dx} \Big _{x=0}$	Temperature gradient in a fluid at the heat transfer surface ($m^{-1}K$)
E	Displacement Ratio $(1 - \frac{2u_r}{Y-D})$ or $(1 - \frac{2u_s}{Y-D})$ for return and supply pipe respectively
G	Gladstone - Dale Constant
g	Acceleration due to gravity (ms^{-2})
Ga	Gap between supply and return pipes (m)
h	Convective heat-transfer coefficient ($Wm^{-2}K^{-1}$)
k	Thermal conductivity of a fluid ($Wm^{-1}K^{-1}$)
k_c	Equivalent thermal conductivity due to combined conduction and convection ($Wm^{-1}K^{-1}$)
L	Length of cavity (m)
M	Constant
N	Number of waves of light
n	Constant
n	Refractive index of gas
P	Pressure of a gas (mm of mercury)

$\dot{Q}_{c,r}$	Steady-state rate of heat loss from the supply pipe due to combined conduction, convection and radiation respectively (W)
$\dot{Q}_t$	Total steady-state rate of heat loss from the supply pipe (W)
r_i	Inner cylinder radius (m)
r_o	Outer cylinder radius (m)
T	Absolute temperature (K)
T_d	Cavity wall temperature (K)
T_m	Arithmetic mean temperature difference across the cavity gap (K)
T_r	Return pipe temperature (K)
T_s	Supply pipe temperature (K)
u_r	Gap above return pipe (m)
u_s	Gap above supply pipe (m)
x	Cavity width (m)
y	Cavity height (m)

Dimensionless Groups

Nu_d	Nusselt number based on 'd'
	$\left(= - \frac{dT}{dx} \Big _{x=0} \cdot \frac{d}{\Delta T} \right)$
$(Nu_d)_m$	Mean Nusselt number integrated around the cylinder's surface

Gr_D	Grashof number based on 'D'
	$\left(= \frac{g \beta \cdot \Delta T \rho D^3}{\mu} \right)$
$Gr_{d,d_i,x}$	Grashof number based on d,d _i ,x
Pr	Prandtl number $\left(\frac{\mu C}{k} \right)$
Ra	Rayleigh number (= Gr Pr)

Greek Symbols

θ	Angular Displacement measured from top of cylinder (See Fig 1.2)
ϕ	Angular Displacement measured from bottom of cylinder (See Fig 1.2)
β	Coefficient of Thermal Expansion (= 1/T _m)
γ	Spacer Position measured from bottom of cylinder (in degrees)
λ_o	Wavelength of Light Source (m)
ρ	Density of fluid (kg m ⁻³)
μ	Dynamic Viscosity (kg m ⁻¹ s ⁻¹)

Subscripts not Included above

1,2,3	Refers to positions of state
R	Refers to reference state
P	Refers to required state

Prefix

Δ	Refers to the difference between two states
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Chapter 1

Introduction and Literature Survey

Introduction and Literature Survey

1.1 Introduction

The long-term growing world scarcity and rising unit costs of conventional fuels are turning attention both towards the achievement of maximum efficiencies in the use of such fuels and towards the exploitation of less convenient but more abundant fuels and energy resources, such as coal, shale, refuse, geothermal and ambient power, and nuclear energy.

Environmental effects generally rule out the location of coal-fired boiler plants, refuse incinerators and nuclear energy complexes in the hearts of city centres or in the immediate vicinity of high-density residential areas.

These factors lead to the siting of heat stations distant from towns and cities. Energy for space heating and domestic water heating is supplied, in the form of hot water (or steam), to customers by transporting it through a network of underground pipelines. This is known as District Heating.

There are four different types of pipeline systems which exist for district heating. These are:

- (a) the single pipe, which is used with most steam systems and no condensate return is provided. It is also used for certain schemes, which distribute highly pressurised and superheated water over long distances.

- (b) the twin pipe system, which is the most common and widely used form of distribution, comprising a flow and separate return line. This system is shown in Figure 1.1.

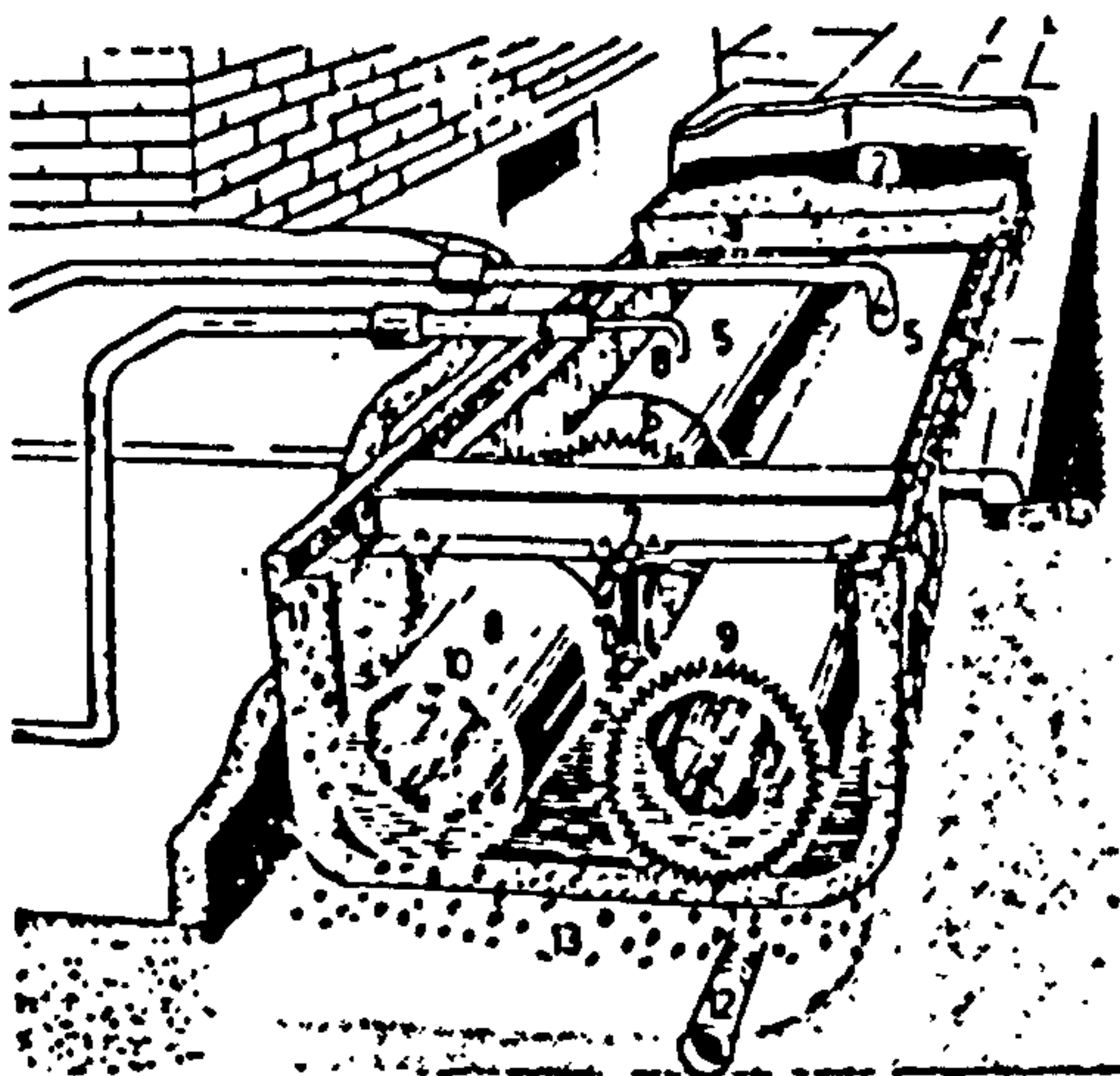


Figure 1.1

- | | |
|--|---|
| 1 Top layer of gravel. | 6 Mineral wool insulation covered with plastic sheet. |
| 2 Asphalt cover. Bitumenmat. | 7 Pipe Support including steel beam, movable pendulum and support ring. |
| 3 Concrete slab. | 8 Steel tube. |
| 4 Feeder line. Plastic pipe culvert with cover of PEH or PVC and insulation of mineral wool, polyurethane foam or similar. | 9 Outgoing flow pipe, 120-80°C. |
| 5 Main or distribution pipeline. | 10 Return flow pipe 70-50°C. |
| | 11 Prefabricated U-section from reinforced concrete. |
| | 12 Drainage pipe of plastic. |
| | 13 Crushed stone. |

- (c) the three-pipe system, which is more expensive than the two-pipe system, but offers considerable advantages over it. Two mains are laid constituting the main supply and return pipes. A much smaller main is added for the supply of hot water and space heating demand in the summer. The common return line serves both supply pipes. With this method two streams of hot water are despatched from the heat station in winter. During the summer months the main heating line is isolated and only the smaller bore pipe functions. In this way heat losses from the network are considerably reduced during the summer.

(d) finally, the four-pipe system, where domestic hot water and space heating hot water are circulated through two separate sets of pipes. This type of system is often discouraged because of problems associated with galvanic corrosion. This happens when two dissimilar metals are in close contact with each other and are exposed to a conductive environment. One metal, perhaps steel, would form the anode and the other metal, say copper, would form the cathode. A potential will exist between them and a current flows. The resulting action causes rapid corrosion of the steel, in the form of rust. A more detailed description of galvanic corrosion is given in (38). Galvanic corrosion occurred on sections of the district heating scheme at Urmston, near Manchester. The domestic hot water was carried through copper pipes and the space heating water through steel pipes. Both sets of pipes were located in the same conduit and thermally insulated with magnesia. When this became damp galvanic action caused rapid corrosion of the steel pipes.

Because in the UK the water table is relatively high, and we live in a damp maritime climate the main problem arises from groundwater corrosion. It is therefore wise to protect the pipe insulation by having an air filled cavity around the buried pipeline to act as a drainage and evaporation channel. With this air layer present, it makes sense to maximise its thermal resistance. This may be achieved by choosing the optimal eccentricity of the pipe, as well as by optimally positioning the radial spacers supporting the pipe.

This investigation deals with these aspects and is thus separated into two parts;

1 Mach-Zehnder interferometric studies of:

- (a) the effect of radial spacers on the heat-transfer from a heated cylinder within a rectangular duct, with the object of determining an optimal design for minimum steady-state rate of heat loss, and
- (b) the heat-transfer from a more conventional two-pipe system, with the aim of minimising the steady-state heat losses from the system by determining optimal relative positions for both the flow and return pipes.

2 A numerical study of the heat-transfer within a square cavity containing a heated cylinder, with the object of determining the optimal eccentricity of the cylinder so that heat losses are minimised.

Steady-state heat-transfer data are usually expressed in the dimensionless form of the Nusselt number, which is a function of Grashof or Rayleigh numbers. The Nusselt number can be defined as the ratio of the actual heat-transfer rate by conduction and convection through the fluid, to the heat-transfer rate that would have occurred by conduction alone.

By means of dimensional analysis, it can be seen (Appendix B) that when convection occurs, the rate of heat-transfer across a rectangular cavity containing a heated cylinder is a function of Grashof number, Gr, the Prandtl number, Pr, and the dimensionless parameters

$$\frac{x}{d}, \frac{y}{d}, \frac{L}{d}, \frac{r_1}{d}$$

and $\frac{r_2}{d}$, although the last term should be

omitted if a single cylinder only is considered (see Appendix B). The notation and geometry of the single and double-pipe systems are shown in Fig 1.2. The dimensionless heat-transfer coefficient (i.e. the Nusselt number) is then a function of these dimensionless quantities, viz

$$Nu = f(Gr, Pr, \frac{x}{d}, \frac{y}{d}, \frac{L}{d}, \frac{r_1}{d}, \frac{r_2}{d})$$

The Nusselt number for both experimental studies is based on the average gap width between the cylinder and the duct wall. The Grashof number for the single pipe study is based on the cylinder diameter, and for the two-pipe study, it is based on the width of the cavity.

All the fluid properties are deduced for the arithmetic mean temperature, T_m .

1.2 Literature Survey

Experimental Studies

Because information concerning natural convection in square or rectangular cavities is limited, much of this literature survey deals with publications of the thermal performance of annular cavities as the flow systems are considered to be similar.

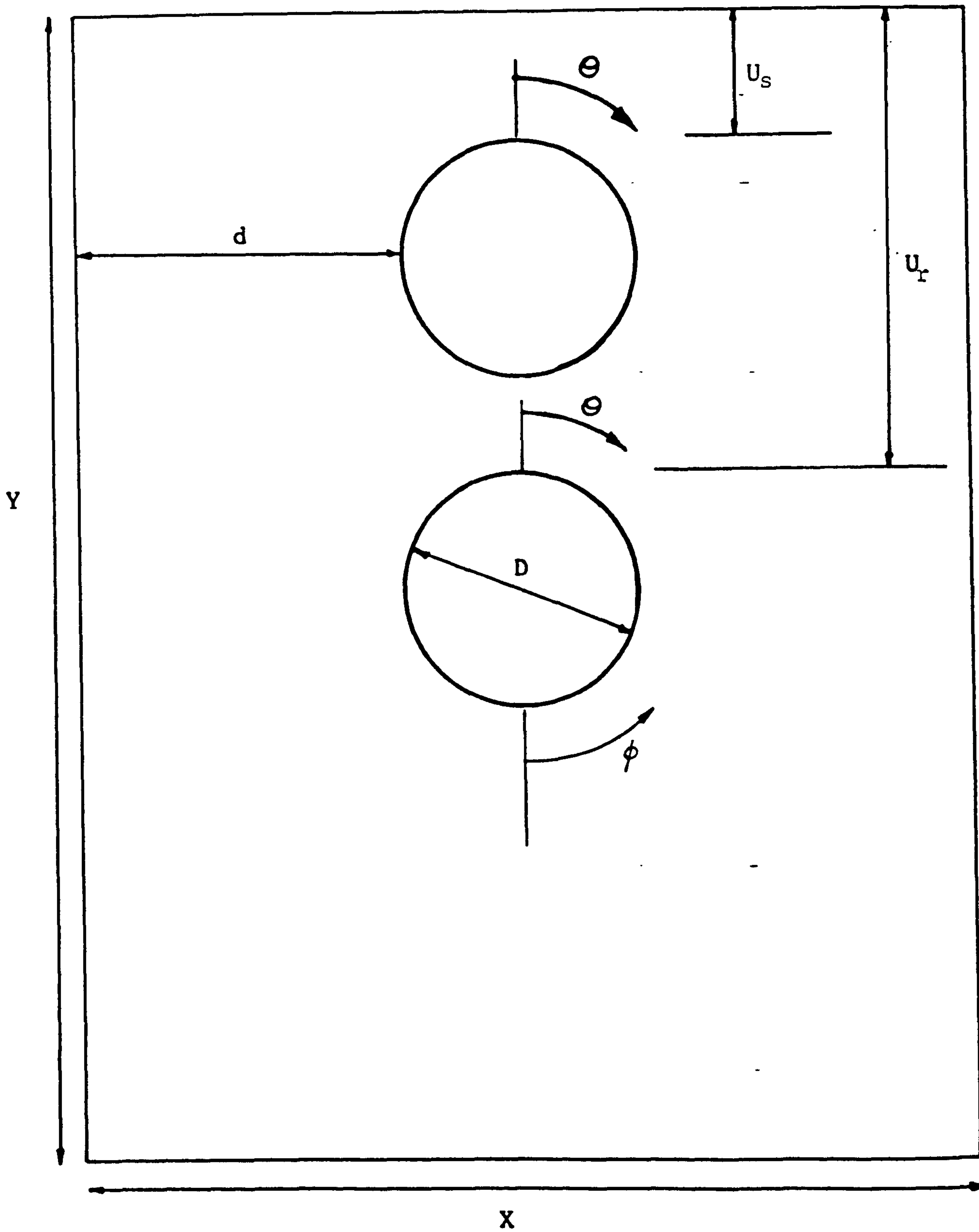


Fig 1.2 Notation and Geometry of the Test Cell
(Identical to single cylinder arrangement)

The earliest recorded work on heat-transfers across annular cavities took place in the 1930's. Beckmann (4) studied thermal insulation effects of thermally floating aluminium foil shields around pipes. The Grashof number, Gr_{d_i} , based on the inner cylinder diameter, the Prandtl number, Pr , and the radius ratio, r_o/r_i , were the parameters used. The following correlation was stated:

$$\frac{k_c}{k} = f(Gr_{d_i}, Pr, r_o/r_i)$$

The gases studied were air, hydrogen and carbon dioxide. Values of k_c , with the radii ratio in the range $1.2 < r_o/r_i < 8.1$. Considerable axial conduction errors along the tube walls and within the fluid could have been avoided if Beckmann had employed end guard heaters. An axial temperature variation of 22.6% of the gap temperature difference was indicated by the nature of the numerical values quoted in the paper. Therefore the two-dimensional assumption of the test set-up must be regarded with suspicion.

Voigt and Krischer (5) investigated the effect of physical size by examining two systems of the same radius ratio, ie $r_o/r_i = 2.72$, but with different inner/outer pipe sizes. They presented $\frac{k_c}{k}$ graphically as a function of

Gr_{d_i} and found that the same correlating equation applied for both cases.

Beckmann's study was extended by Kraussold (6) who used water, transformer oil and machine oil, and was able to account for Prandtl number variations. Suitable end-guards were employed and considered radius ratios of 1.2, 1.5 and 3.0. While Beckmann's experiments, with gas, covered the range of Prandtl number $0.7 < Pr < 0.9$, Kraussold was able to extend this range to $7 < Pr < 4000$. The following empirical correlations were obtained for both his and Beckmann's results with an estimated error of $\pm 20\%$.

$$\frac{k_c}{k} = 1.0 \text{ for } \log(Pr Gr_d) < 3.0$$

$$\frac{k_c}{k} = 0.11 (\text{Pr Gr}_d)^{0.29} \text{ for } 3.8 < \log (\text{Pr Gr}_d) < 6.0$$

$$\frac{k_c}{k} = 0.4 (\text{Pr Gr}_d)^{0.20} \text{ for } \log (\text{Pr Gr}_d) > 6.0$$

Neither author allowed for the separate effects of Grashof or Prandtl numbers as suggested by the dimensional analysis (Appendix B).

About 30 years elapsed until the next significant publication appeared. Liu, Mueller and Landis (7) studied the flow patterns and corresponding temperature profiles of fluid field flows in a horizontal annulus containing air, water and silicone fluid. Temperature profiles and heat-transfer coefficients were obtained for five different radius ratios in the range $1.15 < r_o/r_i < 7.5$ and Prandtl numbers $0.7 < \text{Pr} < 3500$. The average Nusselt number was obtained from an energy balance as:

$$\text{Nu}_d = f (\text{Pr Gr}_d, d/d_i)$$

The test results obtained were more accurate for water and silicone fluid, because the thermocouple measurements of the temperature profiles across the air gap were subject to unknown radiation errors.

Both stable and unstable flows were observed over the temperature difference range $1^\circ\text{C} < \Delta T < 144.6^\circ\text{C}$. Thermocouple readings indicated flow instabilities in several cases and consequently meaningful temperature profiles were obtained only for stable flow conditions. Figure 1.3 shows sketches of flow patterns observed for decreasing values of the radius ratio, r_o/r_i .

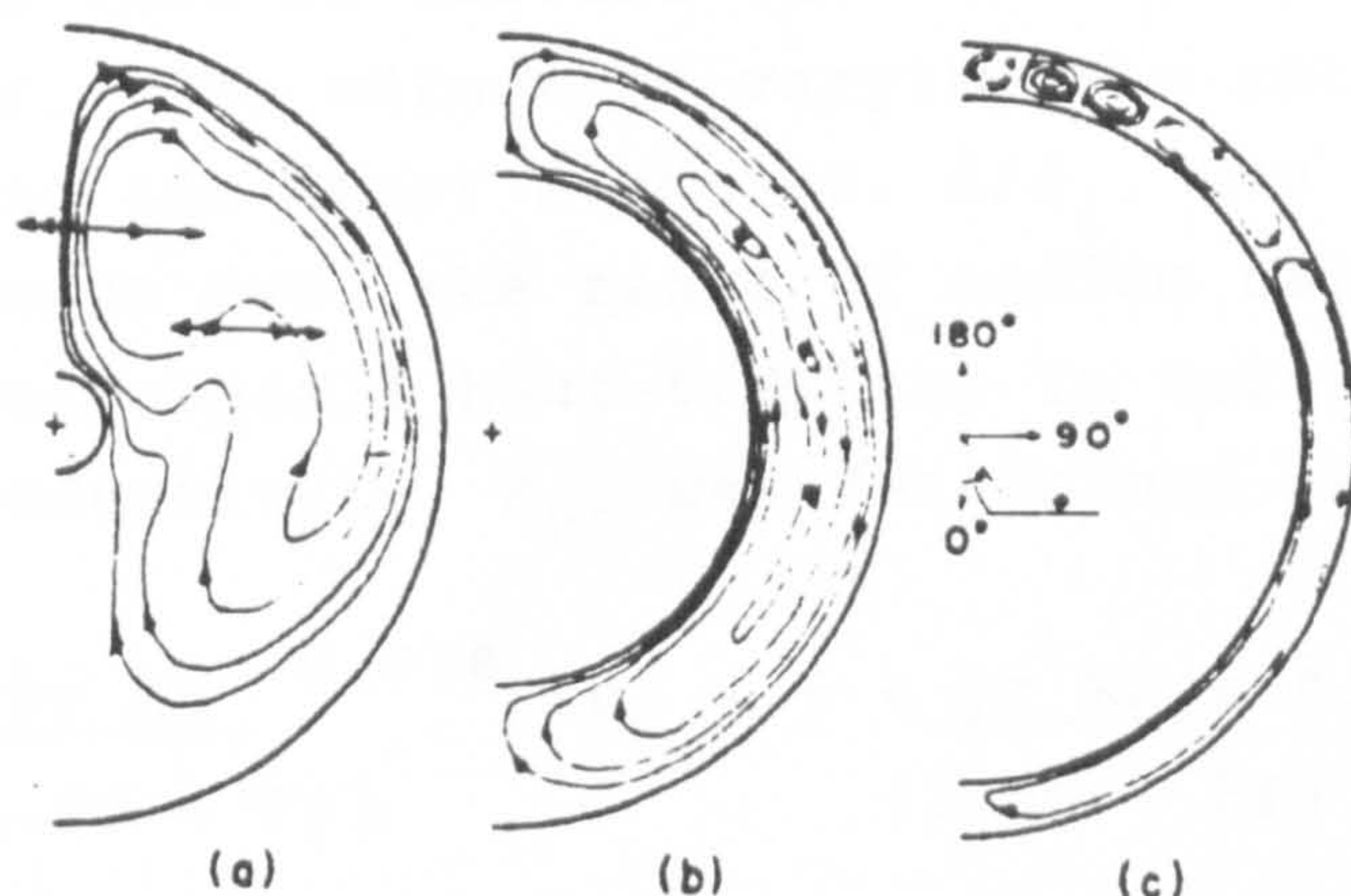


Fig 1.3 Sketches of Flow Patterns

For large values of r_o/r_i the flow remained relatively insensitive to the geometry and was stable for low values of Rayleigh number. Local instabilities near the top and sides were observed as the Rayleigh number increased to ≈ 1600 . These instabilities were governed by the Prandtl number; the more viscous fluids remaining stable at higher Rayleigh number than the less viscous fluids. Stable temperature profiles were found to be dependent primarily on the gap thickness and the fluid properties, and remained relatively insensitive to the temperature difference applied. Pure conduction regimes were observed only for very viscous fluids at low values of Rayleigh number (i.e. $Ra < 6000$). Convection of the air was always detected, even for the lowest Rayleigh number employed ($= 480$). As the Rayleigh number increased, so did the steady-state rate of convection, and at high values ($Ra > 5 \times 10^5$), a temperature gradient inversion (ie. opposite in sign from that which would be expected in view of the temperatures of the immediate boundaries) occurred for angles $90^\circ < \phi < 135^\circ$ measured from the bottom of the cylinder. This is caused by the fast moving thin layer of fluid near the outer cylinder wall.

Liu et al (7) made an initial correlation of the overall heat-transfer based on the silicone fluid data. They then extended this to include the results obtained for air and water. The effect of varying the ratio of the annular gap to the inner diameter, d/d_i , was found to be insignificant over the range of radius ratios tested. The correlated overall heat-transfer in terms of effective conductivity, k_c , was determined as:

$$\frac{k_c}{k} = 0.135 \frac{\text{Pr Ra}}{(1.36 + \text{Pr})}^{0.278} \quad \text{for } 3.5 < \frac{\text{Pr Ra}}{(1.36 + \text{Pr})} < 8.0, \quad 0.25 < d/d_i$$

and

$$\frac{k_c}{k} = 1.0 \quad \text{for } \log \frac{\text{Pr Ra}}{(1.36 + \text{Pr})} < 3.0$$

The data of Beckmann (4) and Kraussold (6) followed the general nature of these correlations. However, a large overestimation of k_c/k occurred in the case of Beckmann (4), and an underestimation of about 10% by Kraussold (6).

McAdams (8) predicted higher values of Nu_{d_i} for steady-state natural convection heat-transfers from a single horizontal cylinder to an infinite plane boundary, than those suggested by Liu et al (7); the difference could be due to the influence of the outer cylinder up to a particular value of r_o/r_i .

The first Mach-Zehnder interferometric study of temperature distributions in, and heat flow rates across, a horizontal annulus was undertaken by Grigull and Hauf (9). Nine different radius ratios were considered in the range $1.3 < r_o/r_i < 6.3$ and the flow medium was air at atmospheric pressure. The results were in good agreement with those of Beckmann (4), and the computer predictions of Crawford and Lemlich (18). Unlike the apparatus used by Beckmann (4), that employed by Grigull and Hauf (9) achieved a constant outer cylinder temperature. The latter measurements did not agree so well with those of Liu et al (7), which were consistently smaller (by ~ 20%).

The temperature difference between the inner and outer cylinder was varied to provide a range of Grashof numbers, based on the gap width d , from 320 to 716000. Three convective flow regimes were identified by the authors. These were:-

- 1 A two-dimensional pseudo-conductive region for $Gr_d < 2400$;
- 2 A region of transition with a three-dimensional convective motion for $2400 < Gr_d < 30\ 000$;
- 3 A two-dimensional region of fully-developed laminar convective motion for $30\ 000 < Gr_d < 716000$.

The results obtained for the fully-convective flow regime were represented by the equation:

$$(Nu_d)_m = (0.2 + 0.145 (d/d_i)^{-(0.02 d/d_i)^{0.25e}}) Gr_d$$

for $2.1 < r_o/r_i < 6.3$

In the pseudo-conductive regime the nature of the flow was found to be independent of Grashof number.

This fact was interpreted by Beckmann (4) to indicate that conduction alone is the prevailing method of heat-transfer. However, flow visualisation and interferograms revealed that convective motions were present even at Grashof numbers as low as 450, see Figure 1.4.

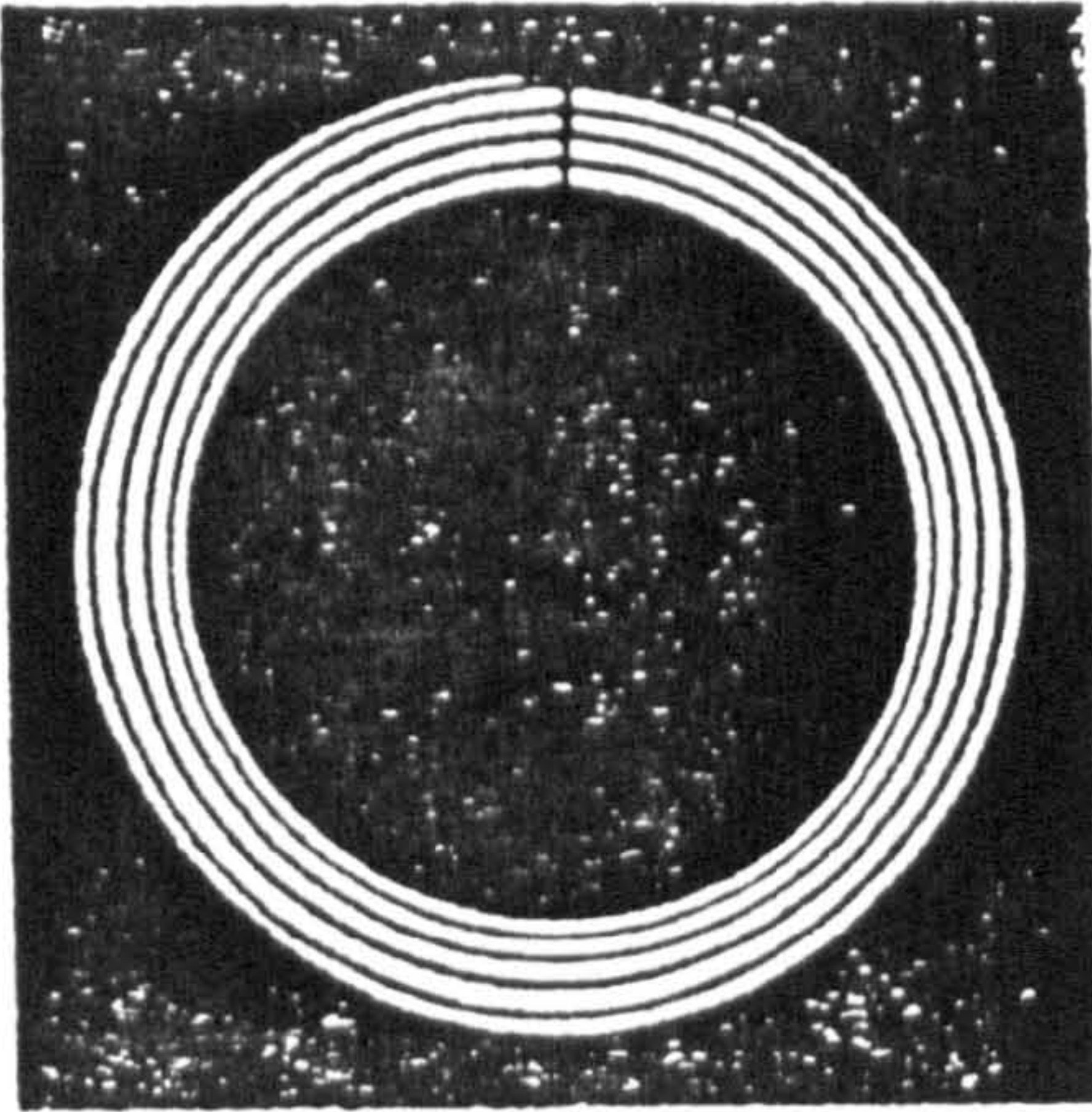
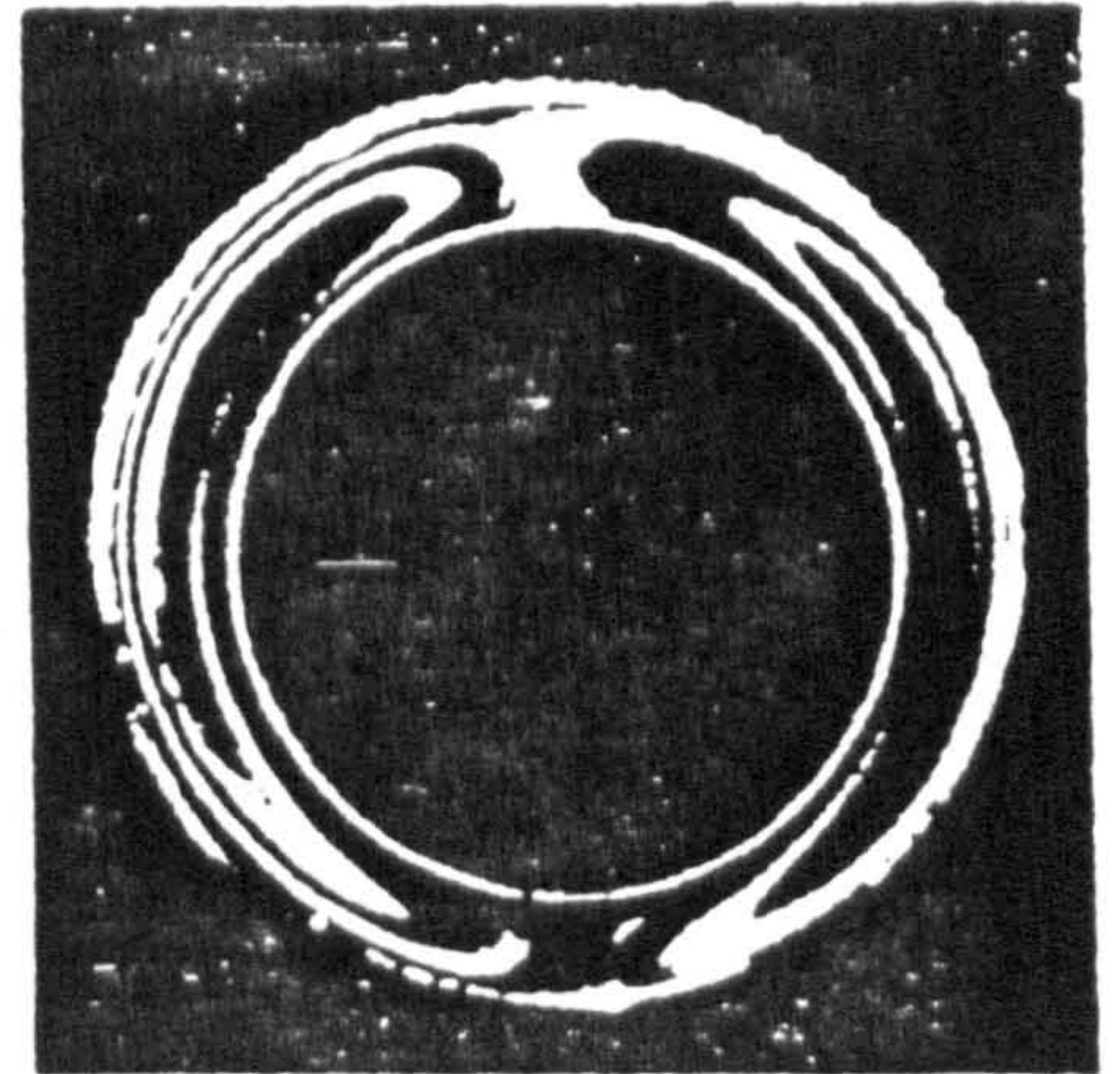


Figure 1.4 Interference photograph and Flow line picture of pseudo-conduction

From the interferogram it can be seen that the isotherms are slightly eccentric, whereas if conduction alone was present, the isotherms would be concentric with the inner cylinder. This effect is further emphasised in the smoke photograph. This evidence suggests that convection diminishes as the Grashof numbers approach zero.

Liu et al (7) also observed that convection was present even at such low Grashof number (~ 680).

The transition regime was characterised by the onset of the change in the flow pattern in the form of three-dimensional vortices in unsteady oscillating-motion. The approximately parallel flow upward along the inner cylinder was separated into zones in the upper region. It then returned in coil-like trajectories or vortices which extended across the gap width in the region $150^\circ < \phi < 180^\circ$. This motion is represented schematically in Figure 1.5.

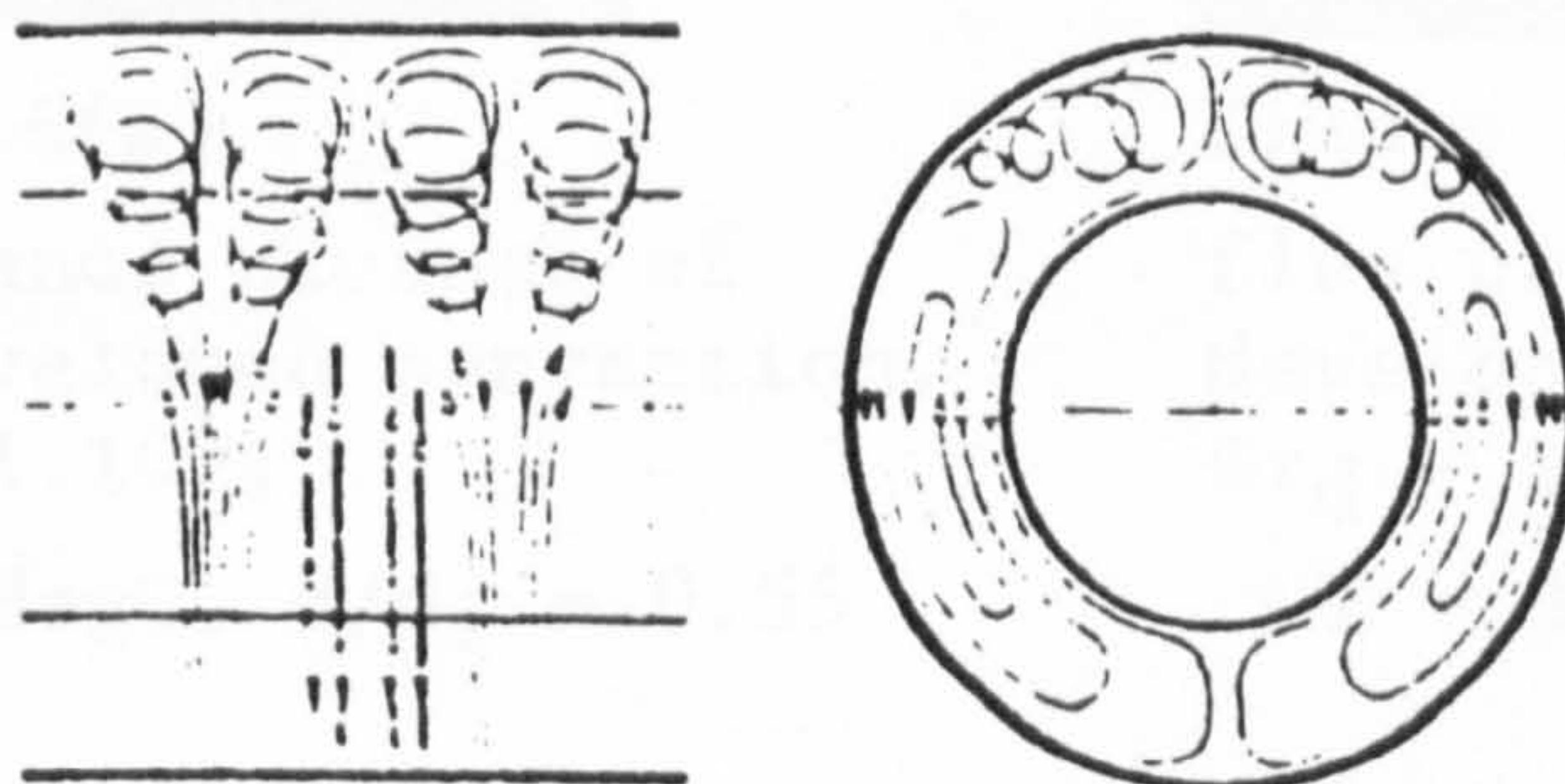


Figure 1.5 Representation of the flow in the transition regime

At higher values of Grashof number, Gr_d , the position of the centre of the vortex oscillated in a circumferential direction. This type of flow was only observed for low values of the radius ratio. The local Nusselt number, Nu_d , reached a minimum at the annular position $\phi = 150^\circ$.

Fully developed two-dimensional laminar convection was observed for Grashof numbers $>30\,000$ and at all radius ratios. As the Grashof number was increased, the centres of the closed flow-lines were located at higher angular positions (measured from the vertically downwards vector) in the annulus. These are shown in the interferograms and flow patterns presented in Figs 1.6 - 1.8. And, as the gap width increased, the flow pattern took on a progressively more kidney-shaped form with increasing Grashof number. The local Nusselt number Nu_d , reached a minimum at $\phi = 180^\circ$ and a maximum at $\phi = 0^\circ$.



Figure 1.6(a)

Interference picture of fully-developed convection.
 $Gr_d = 2.1 \cdot 10^4$,

$\Delta T = 5 \text{ degC}$, $d/d_i = 0.55$

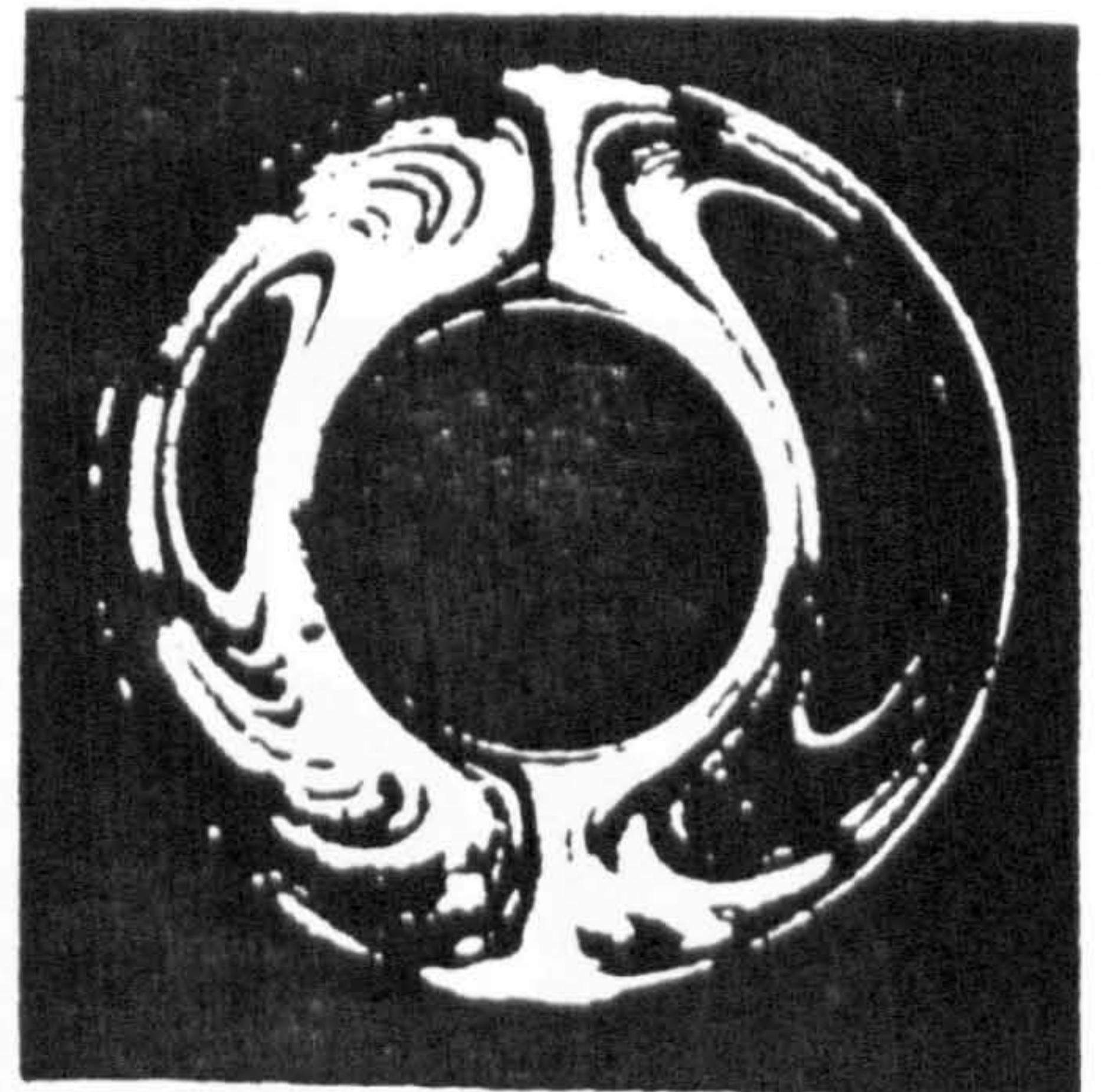


Figure 1.6(b)

Flow picture of fully-developed convection.
 $Gr_d = 1.2 \cdot 10^4$,

$\Delta T = 3 \text{ degC}$, $d/d_i = 0.5$

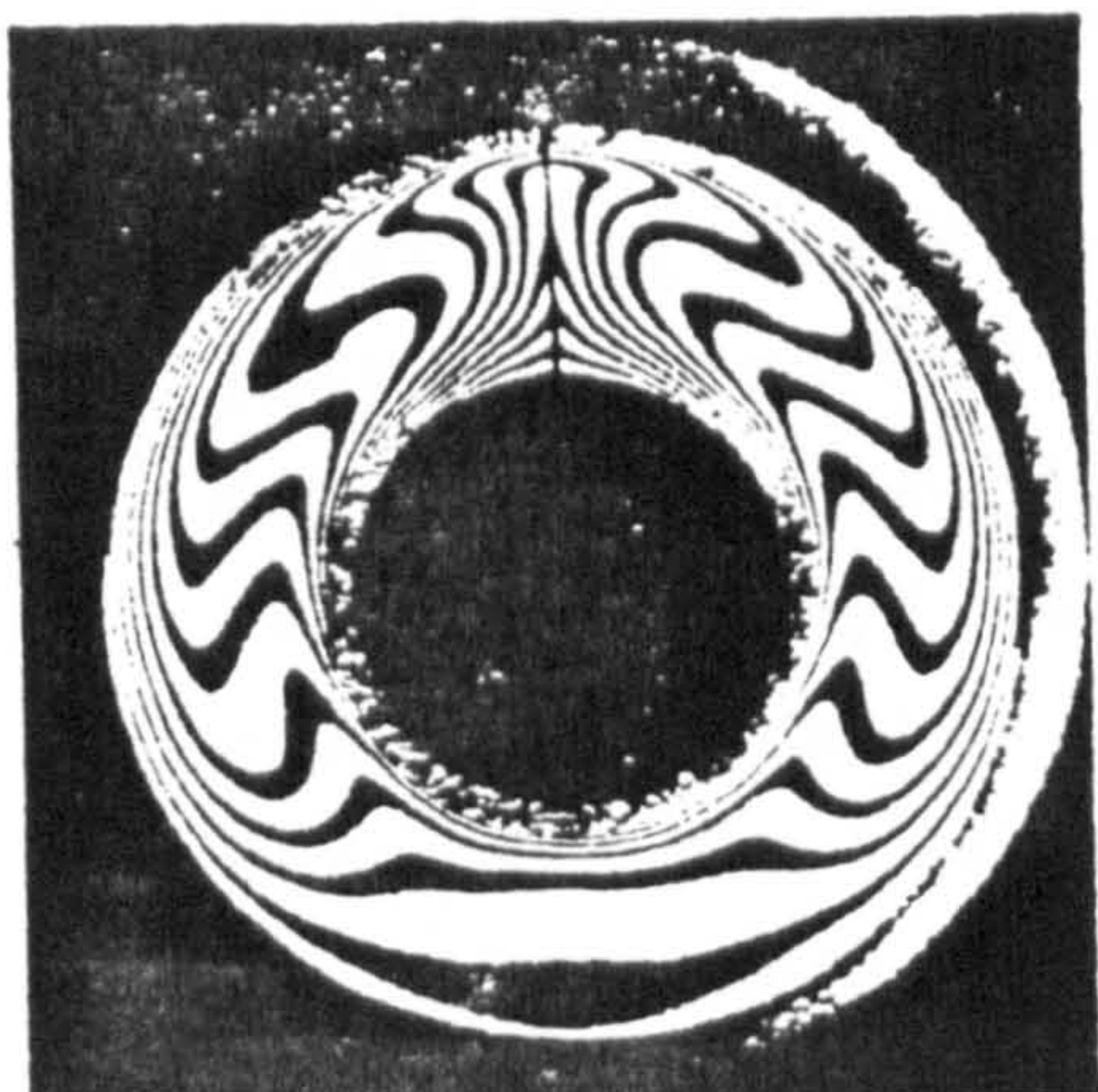


Figure 1.7(a)
Interference picture
of fully-developed
convection.

$$\text{Gr}_d = 7.6 \cdot 10^4,$$

$$\Delta T = 19 \text{ degC}, d/d_i = 0.55$$

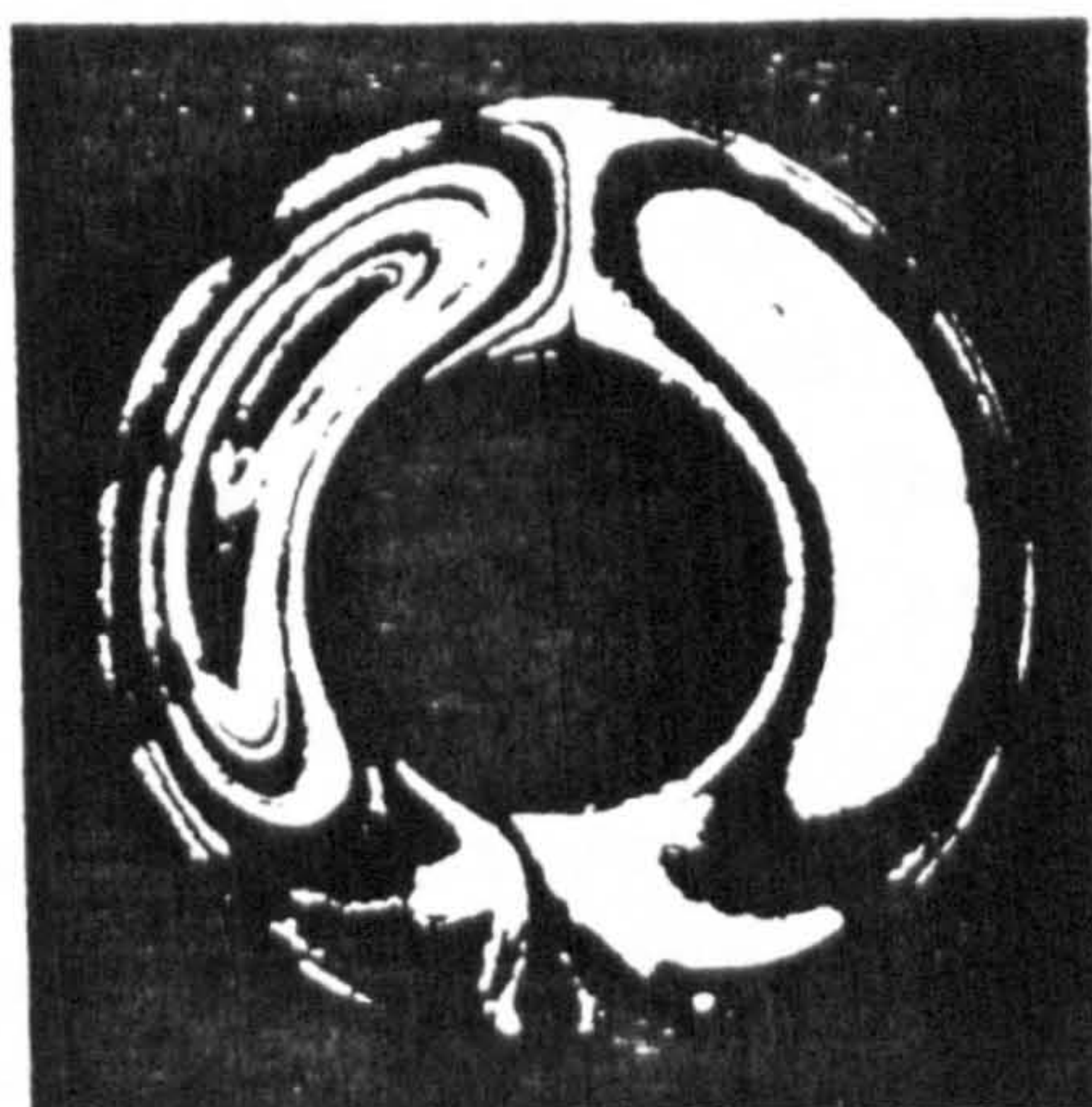


Figure 1.7(b)
Flow picture of fully-
developed convection.

$$\text{Gr}_d = 5 \cdot 10^4,$$

$$\Delta T = 15.5 \text{ degC}, d/d_i = 0.5$$

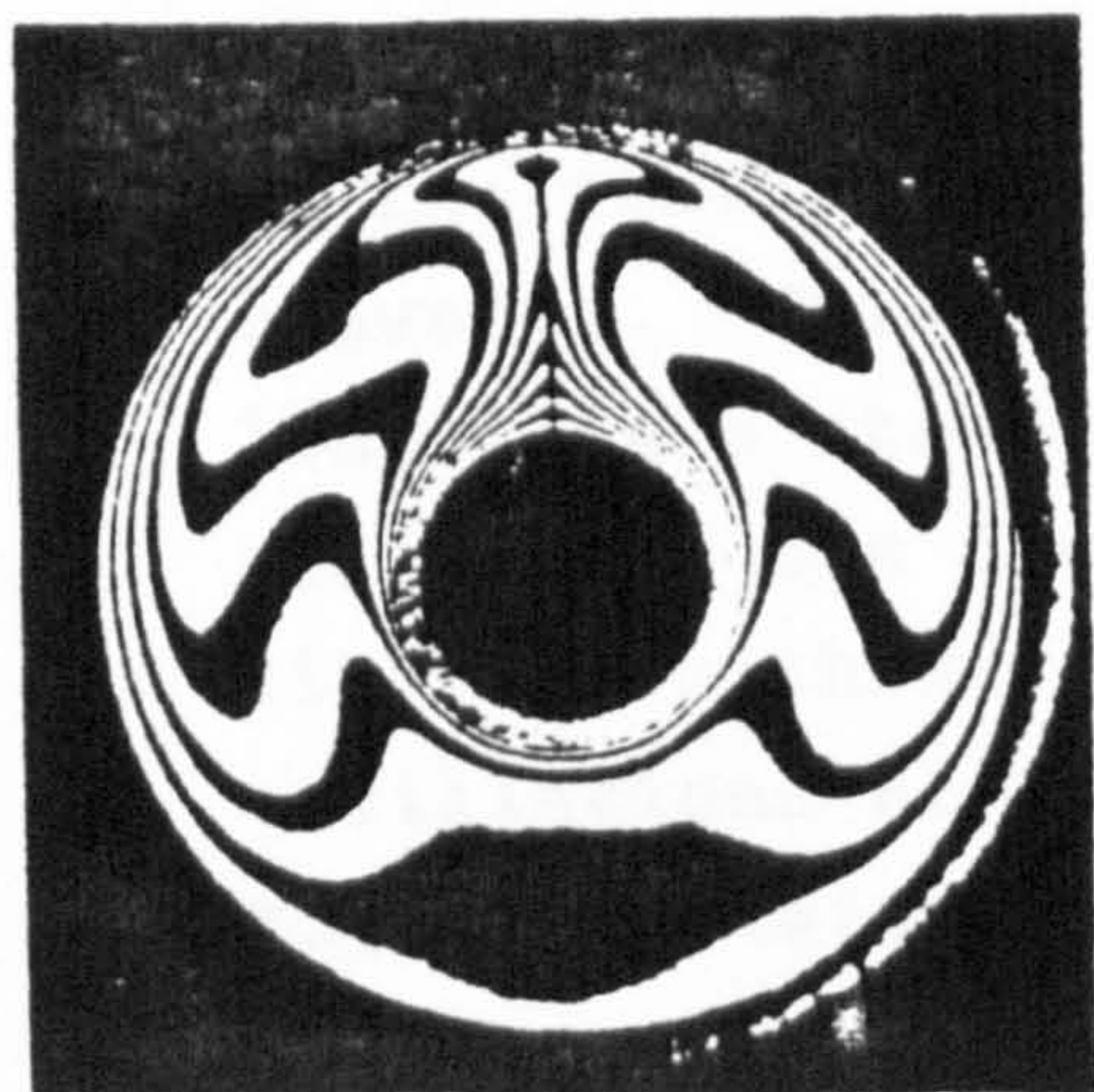


Figure 1.8(a)
Interference picture of
fully-developed convection.

$$\text{Gr}_d = 1.22 \cdot 10^5$$

$$\Delta T = 13 \text{ degC}, d/d_i = 1.08$$

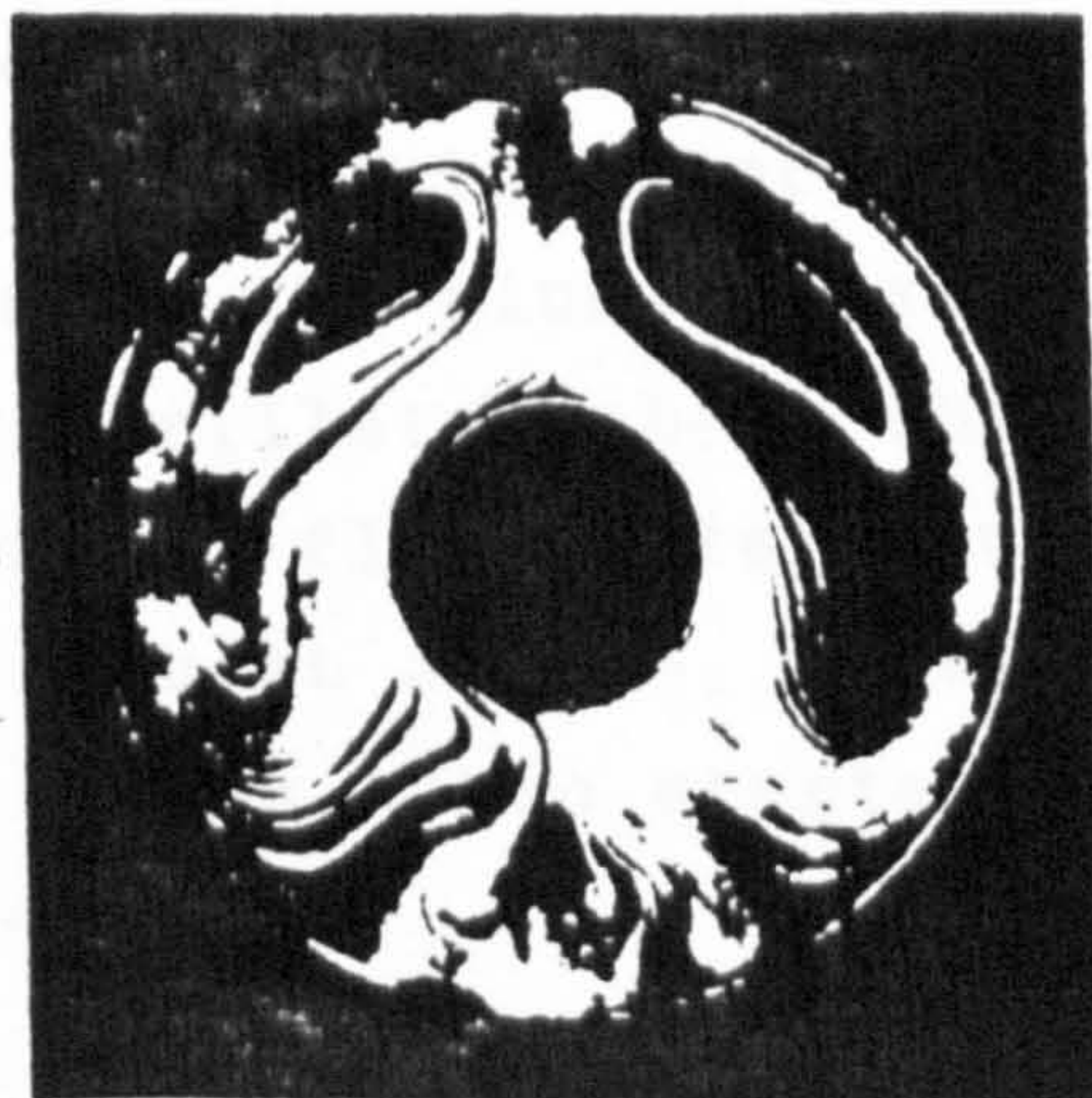


Figure 1.8(b)
Flow picture of fully-
developed convection.

$$\text{Gr}_d = 1.2 \cdot 10^5,$$

$$\Delta T = 14.5 \text{ degC}, d/d_i = 1.0$$

Kidney-shaped, crescent-shaped, and cellular flow patterns were also observed by Bishop and Carley (10) in their investigations of convection between concentric cylindrical annuli. Flow patterns were obtained for radius ratios of 1.23, 1.85, 2.45 and 3.69. Motion pictures of the flow patterns were also made to assist in describing the flows more accurately. For the range of radius ratios and temperature differences studied, two distinct stable-flow patterns were observed. The "crescent-eddy" pattern (Fig 1.3(b)) occurred at all the considered values of ΔT (ie. 3 to 56°C) for $r_o/r_i = 1.23, 1.85$ and 2.45 , while for $r_o/r_i = 3.69$ it only occurred at low values of ΔT ($< 3^\circ\text{C}$). The other flow pattern, the "kidney-shaped eddy" (Fig 1.3 (a)), occurred only at the largest radius ratio ($r_o/r_i = 3.69$) for temperature differences greater than 3°C . This flow pattern was completely stable up to a temperature difference of about 20°C (which led to $Gr_d = 386000$) at which point the flow became unstable. The crescent-eddy type of flow was stable under all operating conditions tested.

The flow instability that was observed for the largest radius ratio was characterised by tangential oscillations of the fluid near the top of the annulus where both flows converge. This region, known as the plume, was then no longer in a steady-state but oscillated about the vertical position. The description of the flow patterns was similar to those reported by Liu et al (7). However, oscillations at the largest radius ratio started at a higher Rayleigh number ($Ra \sim 270000$).

A recent experimental and theoretical-numerical investigation has been carried out by Kuehn and Goldstein (11) to extend the existing knowledge of natural convection in the annulus between horizontal concentric cylinders. Experimental results were obtained from interferometric studies with water or air, at atmospheric pressure, in the annulus for a radius ratio, r_o/r_i , of 2.6. Using a least-squares regression analysis, the correlations for the average equivalent conductivities were found to be:

$$\text{for air} \quad k_c = 0.159 Ra_d^{0.272} \text{ for } 2.1 \times 10^4 < Ra_d < 9.6 \times 10^4 ;$$

$$\text{for water} \quad k_c = 0.234 Ra_d^{0.238} \text{ for } 2.3 \times 10^4 < Ra_d < 9.8 \times 10^5$$

Good agreement was achieved between the experimental measurements and theoretical predictions.

In 1966, Lis (12) carried out an investigation into turbulent flows in horizontal concentric annuli, at radius ratios of 2, 3 and 4.

Three different designs investigated were as follows:

- (a) the plain horizontal annulus
- (b) the annulus containing six, evenly-spaced, longitudinal splitters inclined at an angle of 30° to the diametrical plane through the splitter, Fig 1.9 (a).
- (c) the annulus containing a 'one-start' helical splitter at 300 mm pitch, Fig 1.9 (b).

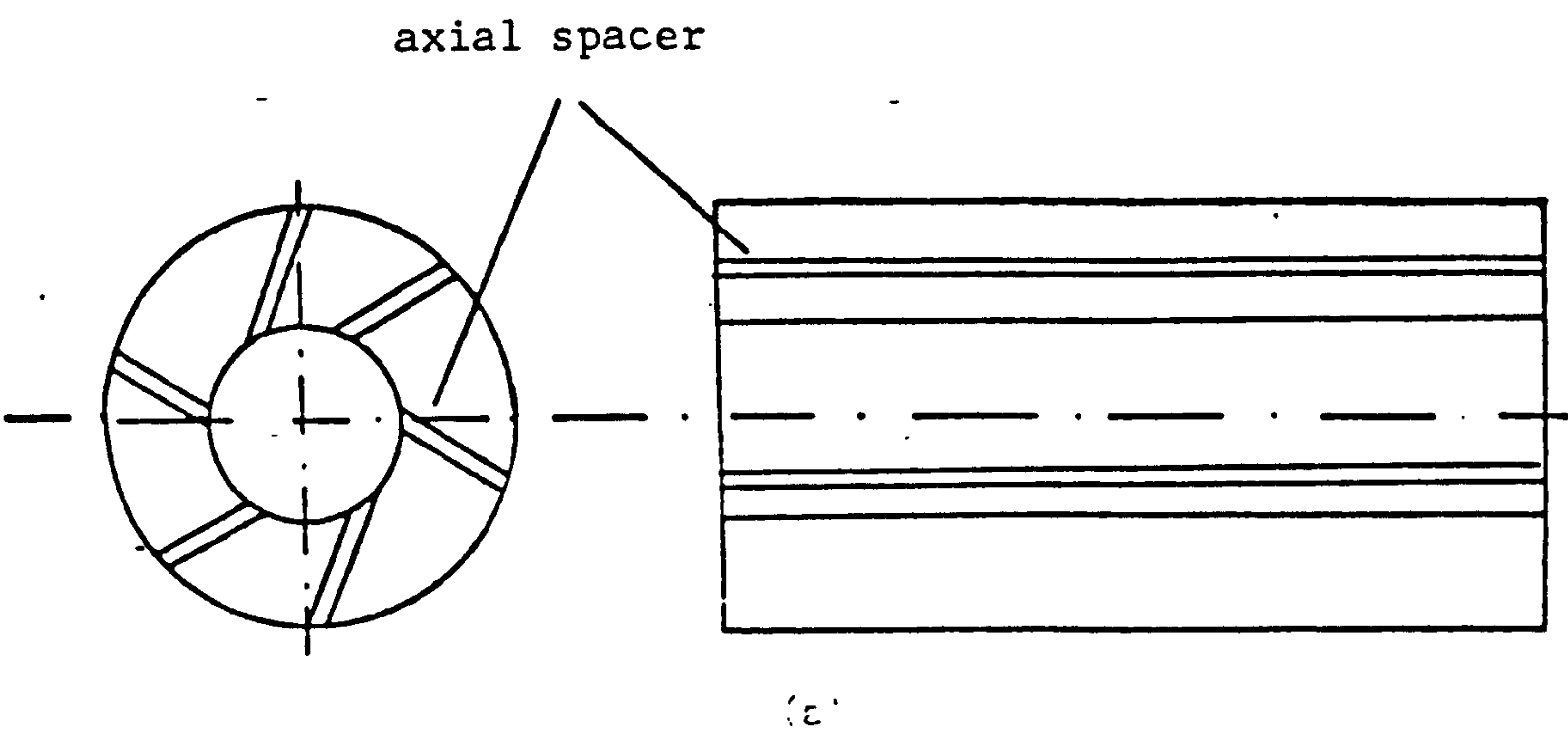
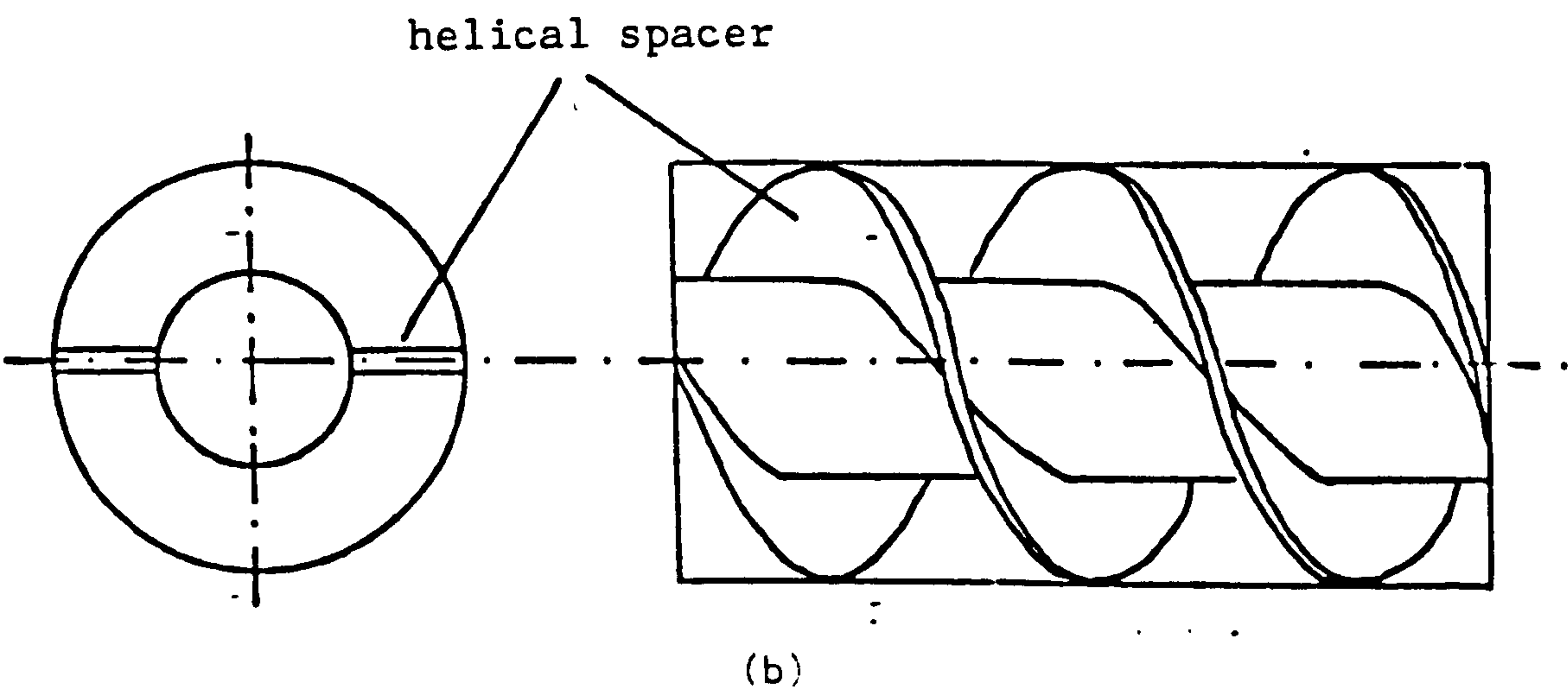


Fig 1.9 Two Types of Axial Spacer design used by Lis (12)

For $Ra_{di} (1 - r_o/r_i)^{6.5} < 5 \times 10^5$, both spacer designs reduced the natural convection heat-transfer rate by less than 20%. It was thought that the effects of the longitudinal spacers would be larger than those of the helical spacer, because of the more extensive blockage involved. Although the convective flows formed in the cells by the spacers were different from those in a single annulus, the flow velocities were relatively unchanged. The correlation for the overall steady-state rate of heat-transfer in the plain annular cavity was:

$$\log_{10} \frac{k_c}{k} = 0.0794 + 0.0625 \log_{10} x + 0.0154 (\log_{10} x)^2$$

6.5

where $x = Ra_{di} (1 - r_i/r_o)$

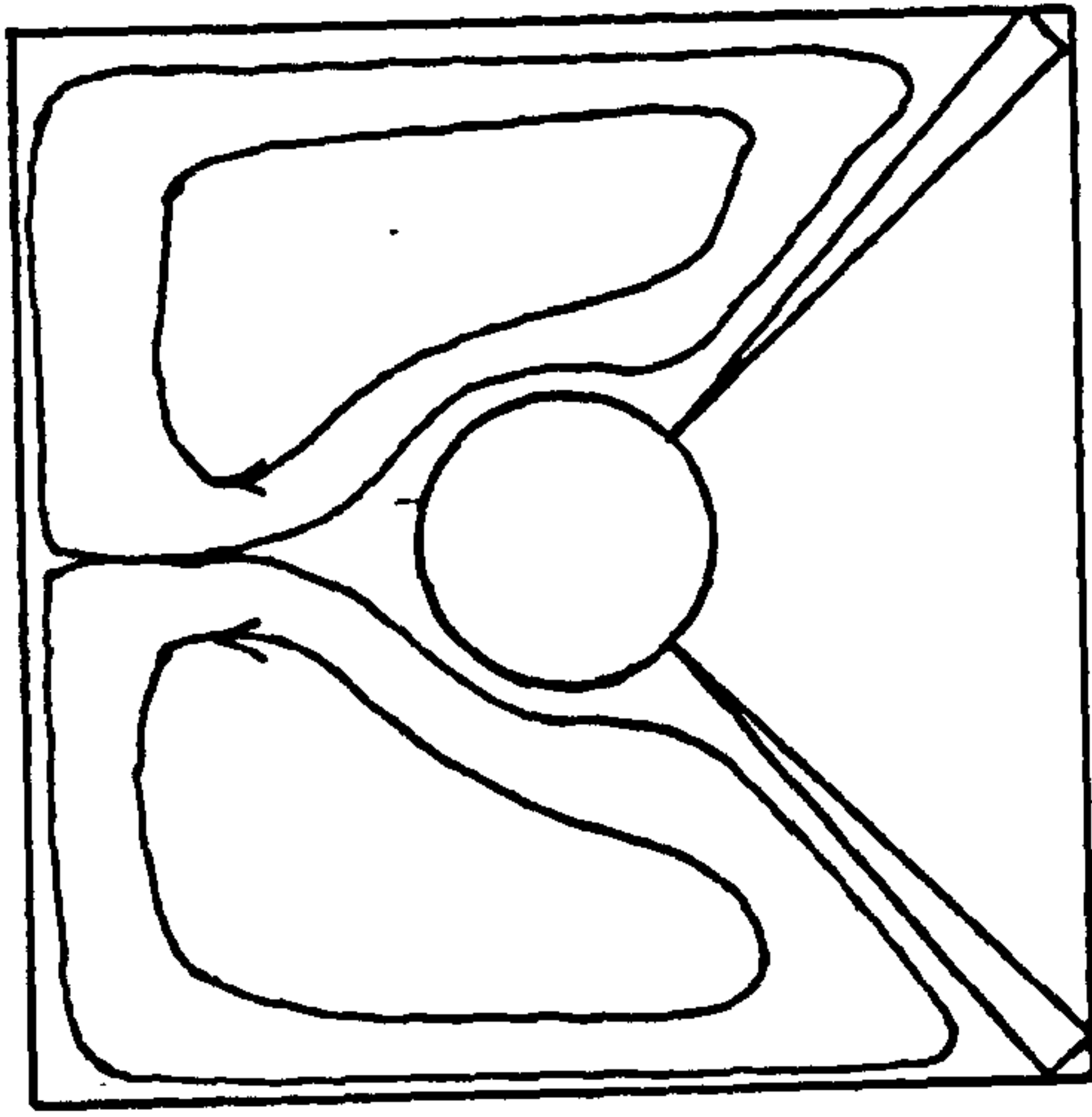
The inclusion of the parameter $(1 - r_i/r_o)$ accounted for the effects of geometry. Examination of this parameter shows that it approaches unity rapidly and asymptotically as the radius ratio, r_o/r_i increases.

This would indicate that the effects exerted by the outer cylinder on the convective flow in the annulus become negligible at large radius ratios.

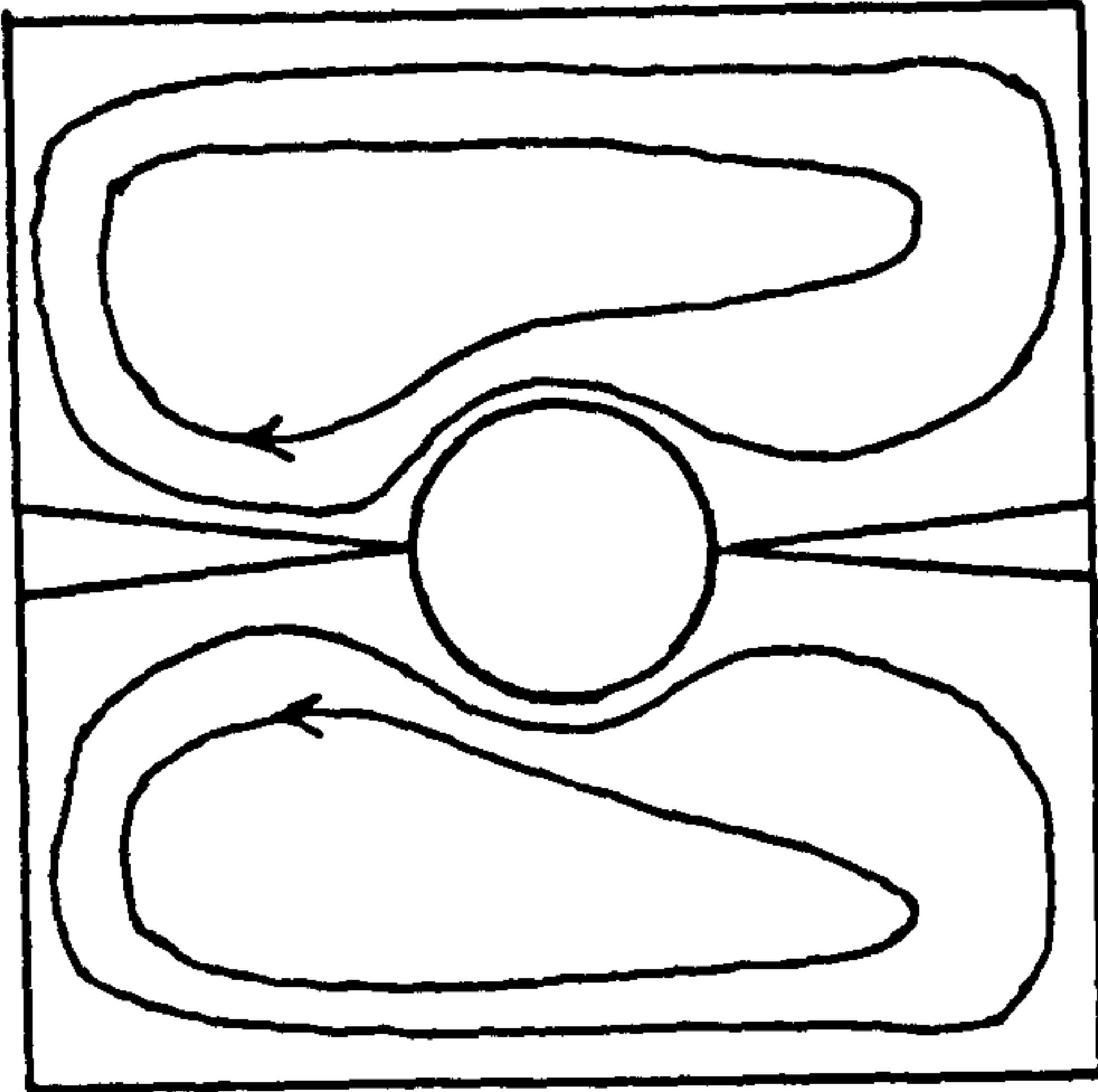
This is in agreement with the practical observations of Senftleben and Gladish (39) who suggested a radius ratio of 500 for the limit of internal flow effects.

Obstruction of the convective flow was also studied by Shilston and Probert (13). They investigated the effects of inserting horizontal and vertical spacers within a horizontal concentric annulus filled with air. Flow patterns and temperature profiles were obtained for a radius ratio of 1.5. The introduction of vertical radial spacers produced a very stable coil-like vortex at the top of the annulus. The extent of the crescent-shaped eddy around the annulus grew with increasing temperature difference. The upper tip of the crescent eddy oscillated circumferentially but did not break away as in the plain cavity. Horizontal radial spacers inserted into the cavity effectively divided the annulus into two sections, both exhibiting very stable crescent-shaped flow patterns. The fluid moved much more slowly around the bottom half of the annulus than it did in both the plain cavity and that containing vertical spacers.

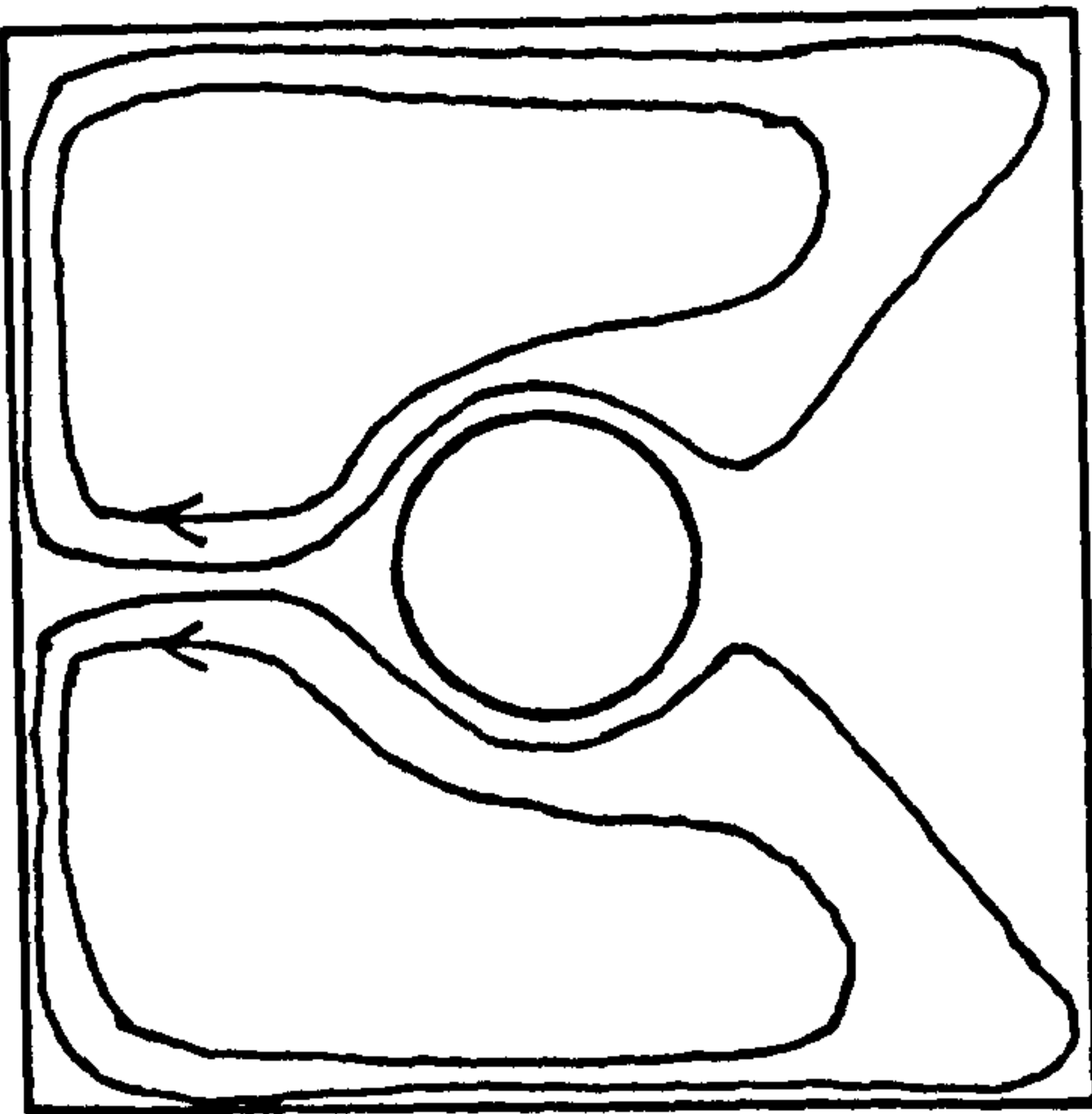
Maximum rates of heat-transfer occurred at the bottom surface of the inner cylinder for both the plain annulus and that containing the horizontal spacers. With the vertical spacer arrangements, maximum heat-transfer occurred at an angle of 30° from the bottom of the inner cylinder. The convective heat-transfer was appreciably reduced by the introduction of horizontal radial spacers, compared with that for the plain cavity arrangement. Norton (14) also studied the effects of positioning radial spacers within the annulus. Radius ratios of 1.307, 1.462 and 1.625 were investigated for four radial spacer positions of 30° , 60° , 90° , 120° and 150° measured from the bottom of the inner cylinder. Interferometric studies revealed that the minimum rate of heat-transfer occurred when the radial spacers were positioned in the lower half of the annulus, at an angle of $60^\circ \pm 10^\circ$. Maximum heat-transfer took place when the spacers were positioned in the top half of the annulus, namely at $\phi = 150^\circ \pm 10^\circ$. Similar results were obtained by Neale (15) when he studied the effects of disturbing the convective flow pattern by obstructing the cavity with spacers but in a square enclosure containing a heated inner cylinder. The spacer positions considered are shown in Fig 1.10, together with their respective flow patterns.



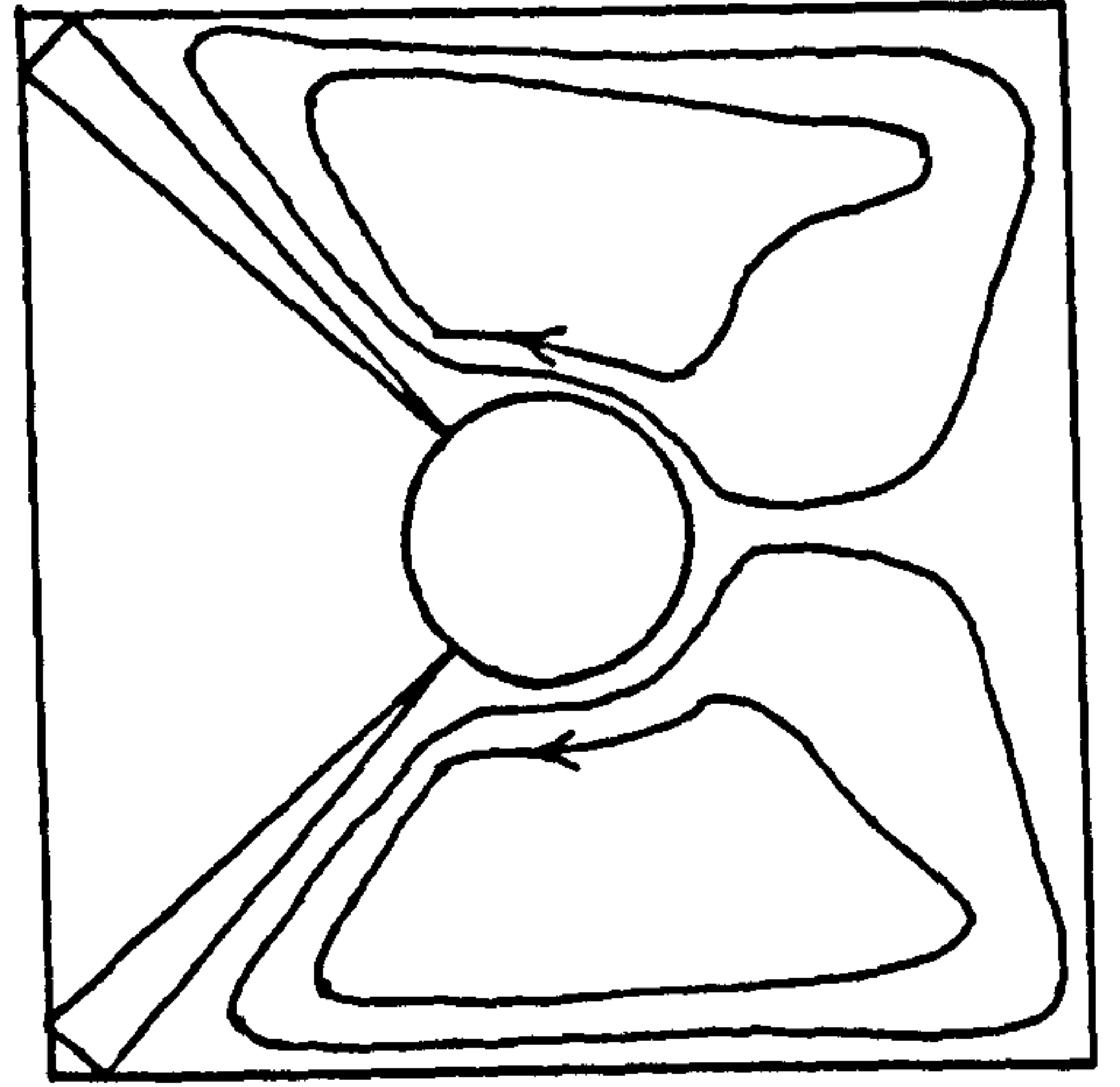
(c)



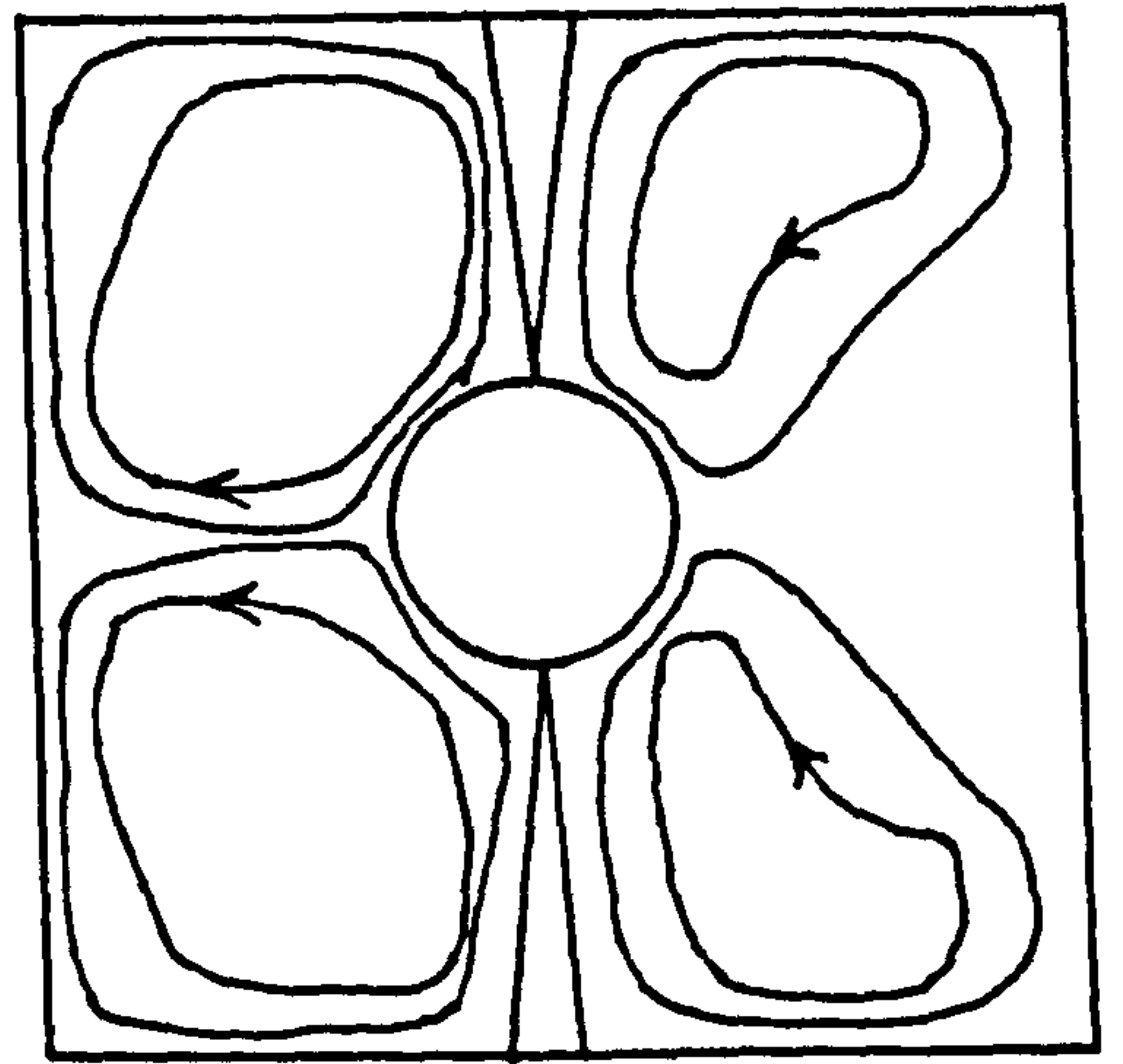
(b)



(a)



(e)



(d)

Fig 1.10 Spacer positions considered by Neale (15)

While the flow patterns were quite different for each position, the convective heat flow appeared to be approximately the same. This was substantiated by results obtained in an interferometric study, which showed that the insertion of spacers had an insignificant effect on the rate of heat-transfer. However, when the spacers were positioned at $\gamma = 45^\circ$ (measured from the bottom of the cylinder), the reduction in heat-transfer amounted to 3.3% of the value obtained for the system with no spacers.

James (16) also carried out a study of the heat-transfer from a heated inner cylinder contained in a square duct. He investigated the effect of vertically displacing the inner cylinder, upwards and downwards. Flow patterns were recorded for displacement ratios of -0.05, 0, 0.3 and 0.7, each at temperature differences of approximately 11 - 12°C and 20 - 24°C. Convective heat flow was observed to be less at higher displacement ratios than at lower cylinder displacements. James also showed, by Mach-Zehnder interferometry that the optimal displacement ratio was approximately 0.7.

Talati (17) was the first investigator to carry out studies on more conventional arrangements. He undertook flow visualisation and Mach Zehnder interferometric studies to determine the least heat loss configuration for a two pipe system ie. flow and return. He investigated several arrangements including "side-by-side" and "one cylinder above the other" for varying relative positions. However, the system which produced the biggest reduction in heat loss proved to be a situation where the flow pipe was placed at a displacement ratio of 0.7 and the return pipe in the centre of the duct. When the temperature difference was maintained at 20°C between the flow pipe and the duct wall the heat loss amounted to approximately 32% lower than that of conventional design ie. "side-by-side" ($E_s = 0$, $E_r = 0$).

Numerical Studies

Just as the literature review of experimental studies deals mostly with natural convection in concentric or eccentric annuli, this also applies to the numerical review, unfortunately for the same reasons.

Due to the complex nature of the governing equations of the phenomena, relatively few theoretical studies have been made that deal successfully with approximate analytical as well as numerical solutions to natural convection in concentric circular cylinders.

The first numerical solution was published by Crawford and Lemlich (18), who used a central-difference method to transform the differential equations into finite-difference equations. They examined the steady two-dimensional flow for a Prandtl number of 0.74 and for radius ratios, r_o/r_i , of 2, 5.7 and 8.

Comparison of the results for the ratio k_c/k was made and the numerical prediction agreed fairly well with the experimental results of Beckmann (4). Abbot (19) solved an almost identical problem and considered radius ratios close to unity.

Mack and Bishop (20) obtained an analytical solution using a power series expansion, and considered radius ratios in the range $1.15 < r_o/r_i < 4.15$ and Prandtl numbers $0.02 < Pr < 6 \times 10^6$, but only for very low Rayleigh numbers. Streamline plots revealed flow patterns very similar to those reported by Bishop and Carley (10).

Powe, Carley and Carruth (21) utilised a finite-difference scheme, similar to that of Crawford and Lemlich (18). They aimed to predict the transition Rayleigh number for which the flow became unstable for a fluid with a Prandtl number of 0.7, ie. air. Stream function and temperature plots were compared with those of Crawford and Lemlich (18), Mack and Bishop (20), and Abbot (19) and a good agreement was obtained.

The procedure for determining the transition Rayleigh number was slow, and to be reliable it required almost prohibitively long computing time. However, the existence of secondary phenomena, in the shape of small rolls turning in the opposite direction to the main cells was reported for $Ra = 6000$, $r_o/r_i = 1.2$ and $Pr = 0.7$.

Kuehn and Goldstein (22) also carried out a numerical study using a finite-difference scheme. Results were in good agreement with those of Crawford and Lemlich (18), but local equivalent conductivities, compared with Mack and Bishop (20), differed by as much as 10%. This could have been due to the slow convergence of the power series used by Mack and Bishop.

Recently, Charrier-Mojtabi et al (23) solved the problem using the implicit alternating scheme on the vorticity-stream function formulation for Rayleigh numbers $10 < Ra < 50\,000$ ($Pr = 0.7$ or 0.02) and radius ratios between 1.2 and 5. For larger radius ratios the method gave identical results to those presented in references 18, 19, 21 and 22.

The introduction of three axial spacers across the concentric annulus was considered by Kwon, Kuehn and Lee (24). A thin-fin approximation was used to model the thermal boundary condition of the spacers for the two-dimensional finite-difference computations. Solutions were obtained for the following range of parameters:
 $9.3 \times 10^3 \leq Ra \leq 2.8 \times 10^4$, $0.5 \leq Pr \leq 10.0$
 and $2 \leq d_o/d_i \leq 3.6$.

Kwon et al showed that total flow recirculation was smaller in a λ -geometry than in the Y geometry. Also, the authors reported that total heat-transfer rates were 3 - 20% less than for simple unobstructed annuli. Streamline and isothermal patterns for λ and Y configurations are shown in Fig 1.11.

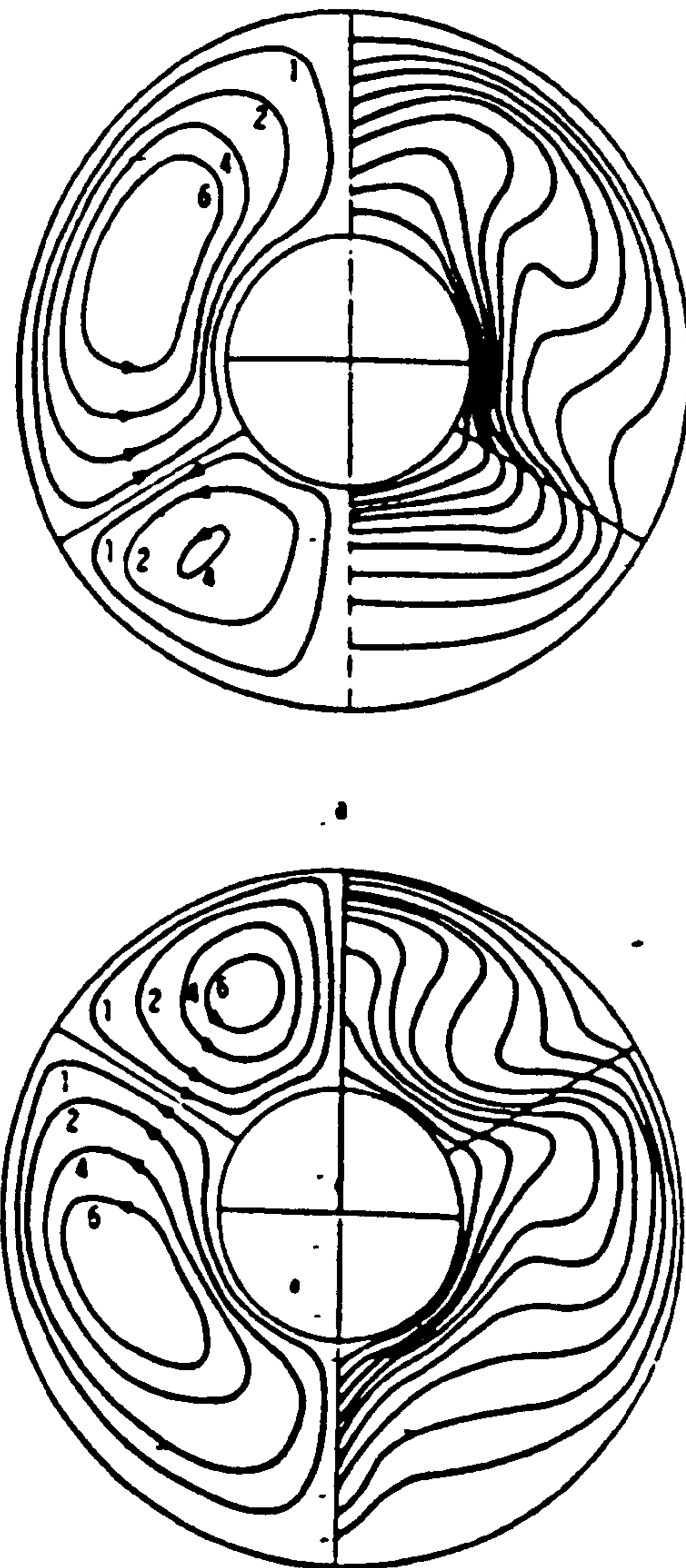


Fig 1.11

Streamlines and Isotherms at $Ra = 10^4$, $Pr = 0.5$, $d_o/d_i = 2.6$
for (a) λ geometry and (b) Y geometry

Several finite-difference schemes have been developed for solving the governing equations describing natural convection between horizontal eccentric cylinders.

Yao (25) developed a perturbation solution for slightly eccentric cylinders, using a two-parameter expansion in terms of eccentricity and Rayleigh number. The method proved stable for Rayleigh numbers up to 10^4 . However, this range shrinks when the value of r_i/d decreases, agreeing with the criterion found by Mack and Bishop (20).

Prusa and Yao (26) used the SAG (Solution Adaptive Grid), as proposed by Thompson, to extend the results for Rayleigh numbers $100 < Ra < 10\,000$. Only the results in the range $100 < Ra < 300$ were presented in detail. Minimum heat-transfer rates were reported for radius ratio, r_o/r_i , 2.6, Prandtl number 0.7 at an eccentricity of 0.4 ($=$ distance between centres/ r_i) for Grashof numbers up to 5000.

A strongly implicit finite-difference scheme was used by Projahn, Rieger and Beer (27) to obtain a solution for Rayleigh numbers in the range $100 < Ra < 100\,000$ at three different eccentric positions. They reported lower heat-transfer rates for positive vertical eccentricities.

Cho, Chang and Park(28) solved the set of governing equations in bi-polar coordinates using a central-difference scheme for all the derivatives. The method became unstable when the diagonal dominance of the locally linearised coefficient matrix of the finite-difference equations was broken.

The application of the finite element method to this study is still in its infancy. Satake and Reddy (29) used the variational approach, eliminating pressure in the velocity-temperature formulation. Results were obtained for Rayleigh numbers between 1000 and 4.7×10^4 . Streamline and isotherm plots compared very well with those of Charrier-Mojtabi et al (23) and Kuehn and Goldstein (22).

Ratzel, Hickox and Gartling (30) used the NACHOS package program to predict isotherms, flow patterns and overall heat-transfers up to a Rayleigh number of 22 000 and for a negative vertical displacement of the inner cylinder.

Caudiu (31) developed a finite-element model using the Galerkin approach to predict natural convective flow in concentric annuli. Results were obtained for a Rayleigh number, Ra , of 2000 using a relatively coarse mesh (48 elements) and seemed to be in good agreement with experimental observations of previous researchers.

Yeo (32) adapted the same program, developed by Caudiu (31), to obtain results for eccentric cases. For a radius ratio (r_o/r_i) of 1.54 and Prandtl number, Pr , of 0.7 an optimal displacement ratio of 0.3 was determined at a relatively low Rayleigh number of 3000.

The present study will involve the adaptation of the computer program used by Yeo (32) to obtain heat-transfer results for a square cavity containing a heated inner cylinder. Also, attempts will be made to determine the optimal position of the inner cylinder relative to the outer duct so that heat losses are minimised.

Chapter 2

Experimental Apparatus

Experimental Apparatus

Both experimental studies enable use of the same test rig, this consisted of:

2.1 Heated Cylinder Assemblies

The cylinders were manufactured from 650 mm long, 28 mm outside diameter copper tubes, closed at each end, with inlet and outlet flow channels located adjacent to them. The cylinders were heated by passing water through them, which was circulated and heated by Conair Churchill thermocirculators. The inlet and outlet flow channels also provided a means of supporting the cylinders by clamping to vertical steel brackets at each end of the test rig. These brackets allowed horizontal and vertical movement for any position of the cylinders within the duct. The inlet and outlet water temperatures for both pipes were measured differentially by means of thermojunctions set in the water flows in the inlet and outlet flow channels, these being connected to a FLUKE 2200B DATALOGGER.

2.2 The Outer Cavity

The experimental test cell was manufactured from clear Perspex because, had a flow visualisation technique been required, the air flow could have been observed. Also, if metal had been the fabrication material, brazing or welding joints may have distorted the cavity walls, whereas with Perspex cement straight sides and flat faces were relatively easier to achieve.

The size of the model tested in this investigation was chosen to represent the relative size of the underground heating system employed at the Cranfield Institute of Technology. In this system the heating mains are contained in underground rectangular ducts. The outside temperature of the duct will therefore be approximately isothermal and hence the outer wall of the model was cooled with constant temperature water. This was achieved by cooling each face independently by supplying mains-pressure water to a series of flow channels on each face, leading to a constant temperature for each wall. A schematic diagram of the cooling system is shown in Fig 2.1.

Temperature measurement was achieved using thermocouples. Thirty six copper-constantan thermojunctions were sealed in holes flush with the cavity inner surface as in the arrangement shown in Fig 2.2. The leads of these thermocouples passed through the cavity wall to avoid disrupting the air movement within the cavity. They were then connected to a FLUKE 2200B DATALOGGER which provided an instant temperature readout for all thermocouples.

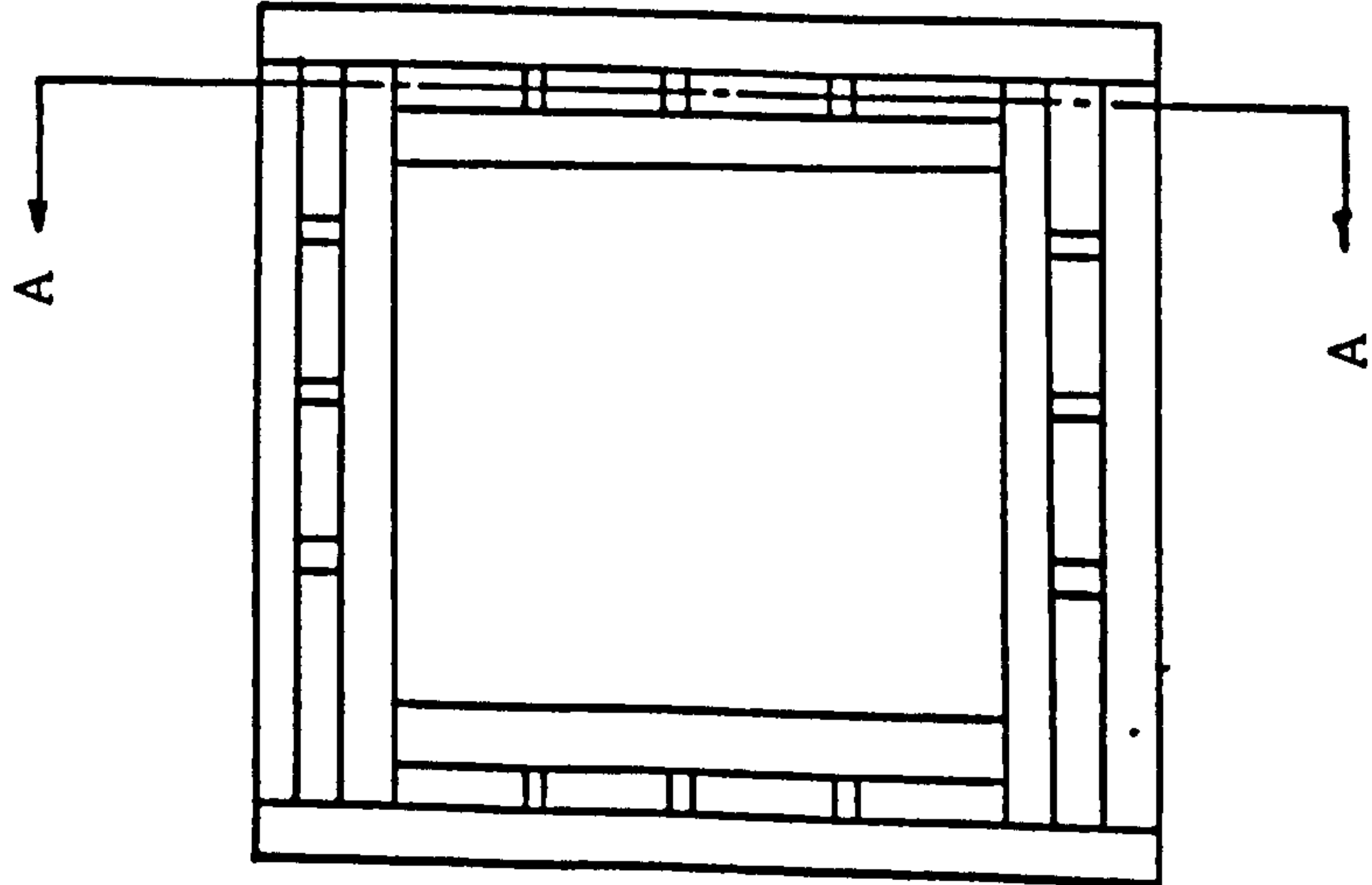
Both ends of the cavity were extended using expanded polystyrene collars to conceal the smaller amount of inner cylinder which protruded at each end. Optically-flat glass plates were fixed flush to the collars to seal the system from the surrounding ambient air.

The spacers used in the investigation were manufactured from plywood, 650 mm long and the width being dictated by the angle at which they were to be positioned in the duct. Both systems are shown in Figs 2.3 (a) and 2.3 (b).

2.3 Mach-Zehnder Interferometer

The 3mW He-Ne stimulated laser Mach-Zehnder interferometer used in this investigation is shown with the test cell in place in Plate 1. It was designed at University College of Swansea, and transferred to Cranfield Insititute of Technology in September 1972. A more detailed description is given in Appendix A.

The interferograms were recorded using a Praktica LTL3 camera. A shutter speed of 1/1000 second was selected and operated by cable release. Photographs were taken on Ilford FP4 pancromatic medium speed film rated at 125 ASA. This film allowed individual processing using Paterson's "Unitol" developer with the recommended times, and fixing with Johnson's "Fiscol".



Section AA

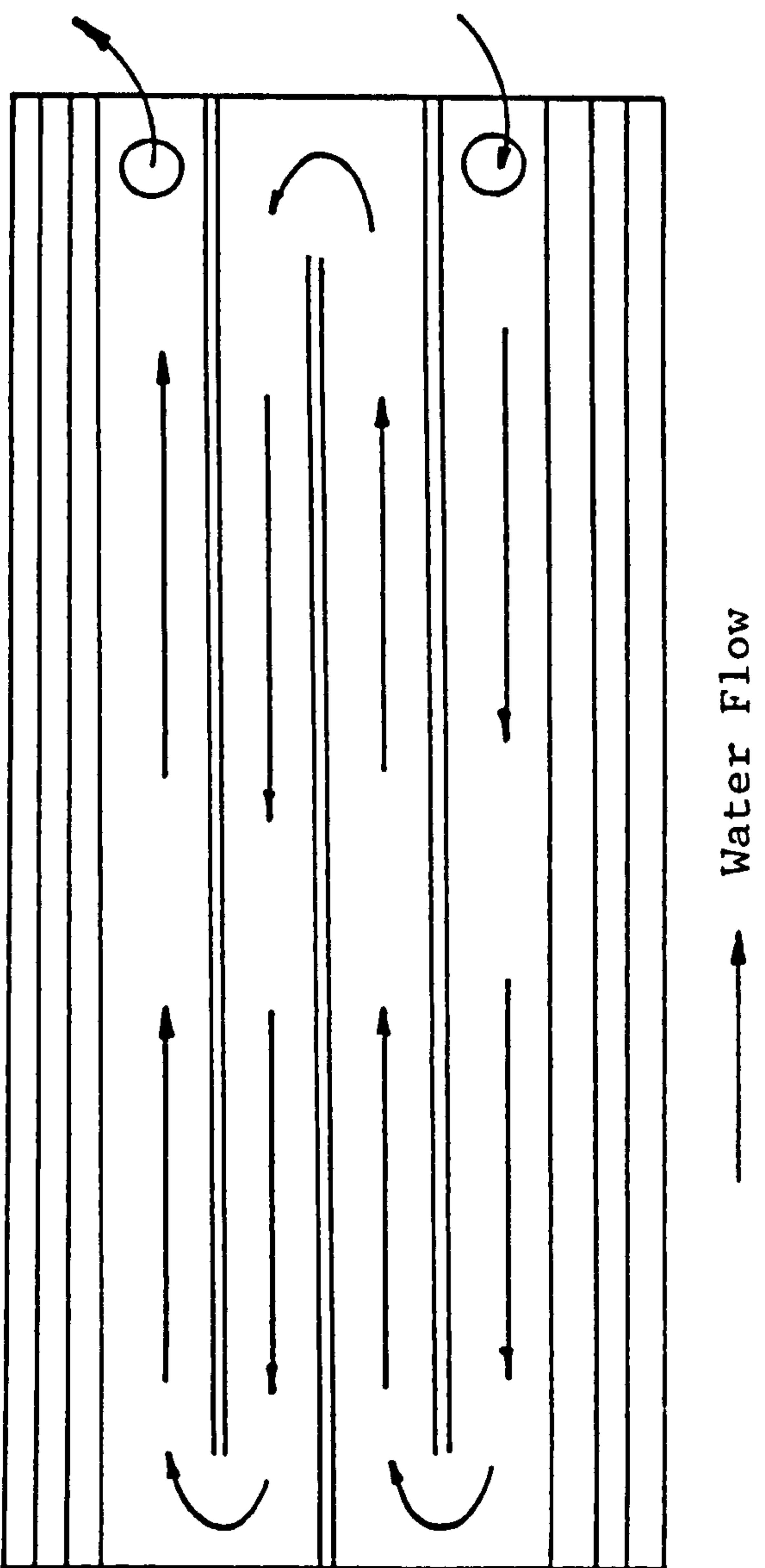


Fig 2.1 Water flow through cavity

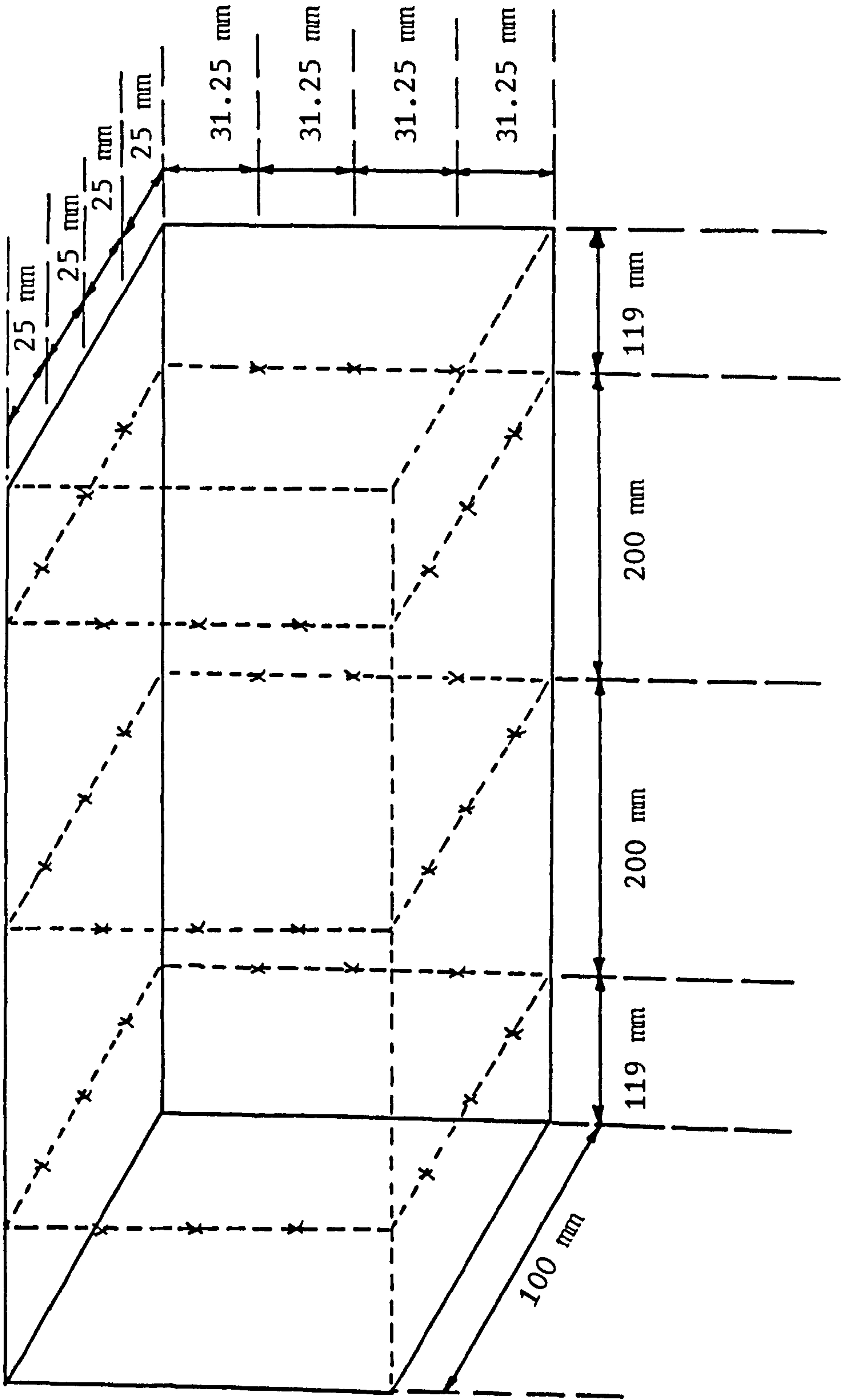


Fig 2.2 Duct and Thermocouple positions

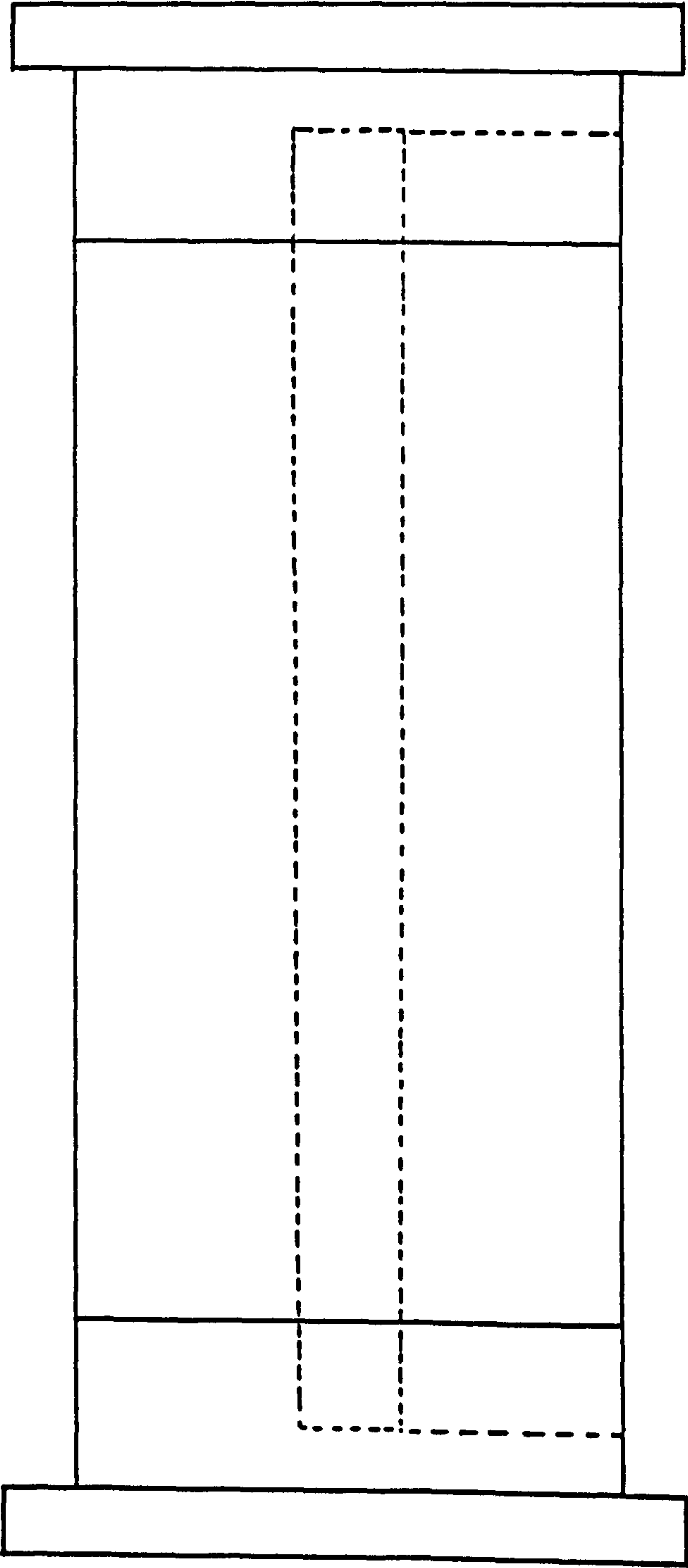


Fig 2.3(a) Arrangement of Test Cell for "Spacer" Set up

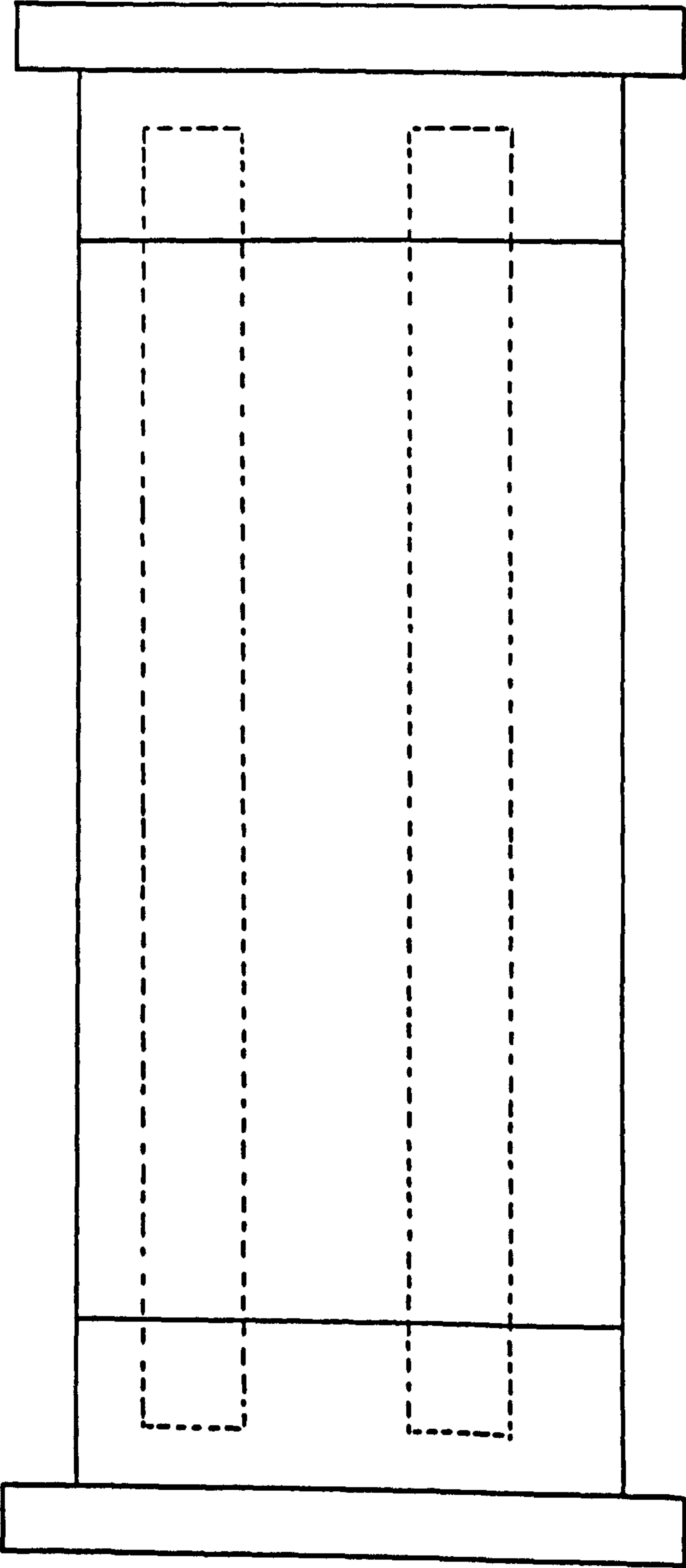


Fig 2.3(b) Arrangement of Test Cell for Twin Pipe Set up

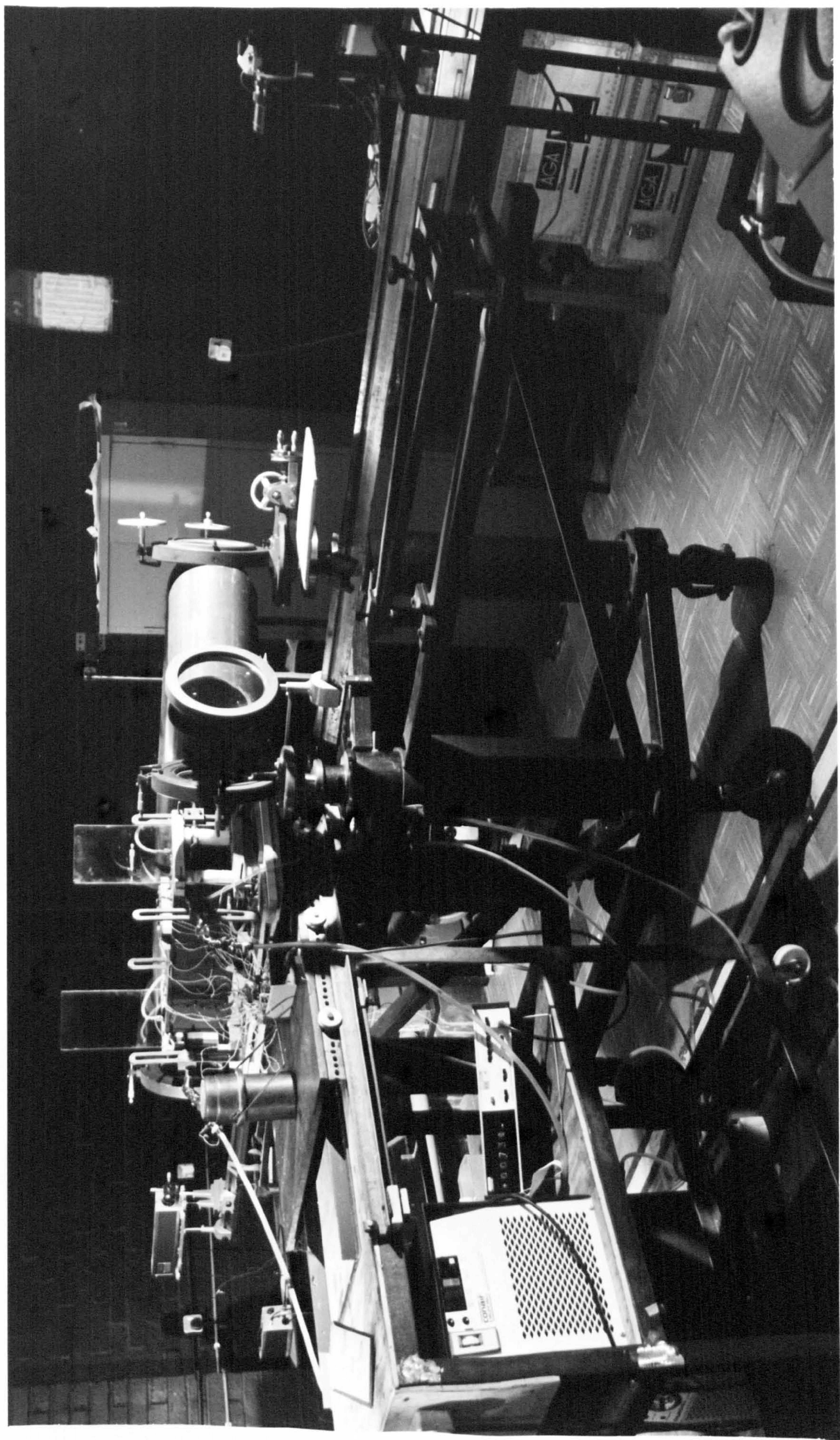


PLATE 1 THE MACH-ZEHNDER INTERFEROMETER

Chapter 3

Mach Zehnder Interferometric Study

Mach Zehnder Interferometric Study

3.1 Introduction

The interferometric technique permits a detailed analysis of the refractive-index distribution in a transparent medium. Therefore it may be used to measure the pressure variations within an isothermal fluid, or alternatively, as in the present study, the temperature variations within an isobaric fluid.

The interferogram provides a photographic record of the interference pattern of the whole field of the parameter (pressure or temperature) being measured, at one instant. Time lags are eliminated, and, as no mechanical probe is inserted into the fluid, the system remains undisturbed by the measurement process.

It is only possible to view a silhouette of the test geometry within the light beam, and therefore the method is only applicable to two-dimensional or axial symmetrical systems. However the interferometric record includes the integrated effect over the full length of the optical path within the test section, which includes end effects in most instances. Details of experimental errors which occur are discussed in Appendix C.

Basic principles and adjustment procedures are given in Appendix A. Details of the instrument design and construction are available elsewhere (33 and 34).

3.2 Experimental Procedure and Apparatus Adjustment

3.2.1 Spacer Study

The test cell was mounted on a wheeled framework which enabled it to be traversed in and out of the test beam. The longitudinal axis of the test cell was adjusted, using a spirit level, so that it was horizontal and parallel to the interferometer test beam. Only one check was required to ensure that the heated cylinder was perfectly horizontal and in correct alignment with the beam. This was achieved by placing a screen between the decollimating lens (L1) and the camera (see Fig A.1) to block the reference beam. The test beam and test cell, and the cylinder appeared as a circular shadow on the screen. The cylinder would appear as an ellipsoid if it was mis-aligned. Also, if a pointer was tapped onto the edge of the cylinder it would appear to sink into the side of the cylinder at the positions out of alignment. If the cylinder was correctly aligned the pointer would just appear to touch the cylinder on all points of its circumference.

This method of aligning the cylinder may appear to be crude but it proved to be perfectly adequate and mis-alignment did not render any interferograms unanalysable.

After a particular set of spacers were inserted into the cavity, the water heater for the cylinder was turned on and the cooling water in the cavity allowed to flow.

The system was left for a period of at least three hours to enable steady-state conditions to be achieved. The infinite fringe (see Appendix A) was checked before the interferogram was photographed. All the thermocouple readings were recorded at the same time as the interferogram was taken. The water heater was then reset so that the cylinder could reach another temperature. A further three hours at least elapsed before steady-state conditions could be reached again and another interferogram taken, together with the corresponding thermocouple readings. Generally, six temperature settings were investigated at each of ten spacer positions and the system with no spacers for cylinder displacement ratio $E = 0$ and $E = 0.7$.

3.2.2 Two Pipe Study

The setting up procedure was identical to that of the previous study, the only difference being the addition of a return pipe positioned vertically below the flow pipe within the rectangular cavity. The return pipe was aligned in the same manner as the flow pipe.

After both pipes were set up at the required eccentricities within the cavity, the water heaters for the pipes were turned on and the cooling water in the cavity allowed to flow.

The system was left, as before, for a period of three hours to enable steady-state conditions to be achieved. The infinite fringe (Appendix A) was checked before the interferogram was photographed. All the thermocouple readings were recorded at the same time as the interferogram was taken. The water heater supplying the flow pipe was then reset so that another temperature could be reached. The return pipe was maintained at a constant temperature. A further three hours elapsed before steady-state conditions could be reached again and another interferogram taken, together with the corresponding thermocouple readings. Generally, four temperature settings were investigated at each of five return pipe positions, for six flow pipe positions ($E_s = 0.8, 0.7, 0.6, 0.5, 0.25$ and 0).

3.3 Analysis of the Interferograms

A4 size prints of the interferograms were obtained for analysis and a travelling microscope was used for accurate measurement. At every 15° radial position around the surface of the cylinder, the fringe distances from the cylinder wall were measured. Only half of each interferogram was analysed because of the symmetrical nature about the vertical of the heat-transfer within the cavity.

At positions of high temperature gradients, the fringe nearest the wall becomes superimposed with the cylinder making it difficult to determine the wall edge precisely and the correct fringes. Two Nusselt number calculations were performed. The first calculation includes the point at the wall, and the second only the fringes that were visible. Comparison between the results showed that the temperature gradient at the cylinder wall was lowered, but by an insignificant amount, in the latter calculation as compared with the former. The Nusselt number calculation was then based on the fringes that were visible.

The temperature gradient at the cylinder wall was obtained by fitting the three points nearest the wall with a straight line using a linear regression analysis. At low temperature differences, three points were not always available and so two points had to suffice.

3.4 Determination of Local Nusselt number, Nu_d

The elementary equation for one-dimensional conduction in the steady-state to a fluid from a surface is given by:

$$q = -kA \left. \frac{dT}{dx} \right|_{x=0} \quad (3.1)$$

the convective heat-transfer from a surface to a fluid is given by:

$$q = hA \Delta T \quad (3.2)$$

By definition,

$$Nu_d = \frac{hd}{k} \quad (3.3)$$

Combining equation (3.1) and (3.2)

$$Nu_d = \frac{- \left. \frac{dT}{dx} \right|_{x=0}}{\Delta T / d} \quad (3.4)$$

From the knowledge of the local Nusselt numbers around the cylinder, the mean Nusselt number $(Nu_d)_m$ was obtained using a numerical-integration technique to determine the area under the graph of local Nusselt number against angular position.

3.5 Results

3.5.1 Spacer Study

The average Nusselt numbers $(Nu_d)_m$ are given in Table 4.1 as a function of Grashof number, Gr_D , and temperature difference, ΔT , for each of the systems studied.

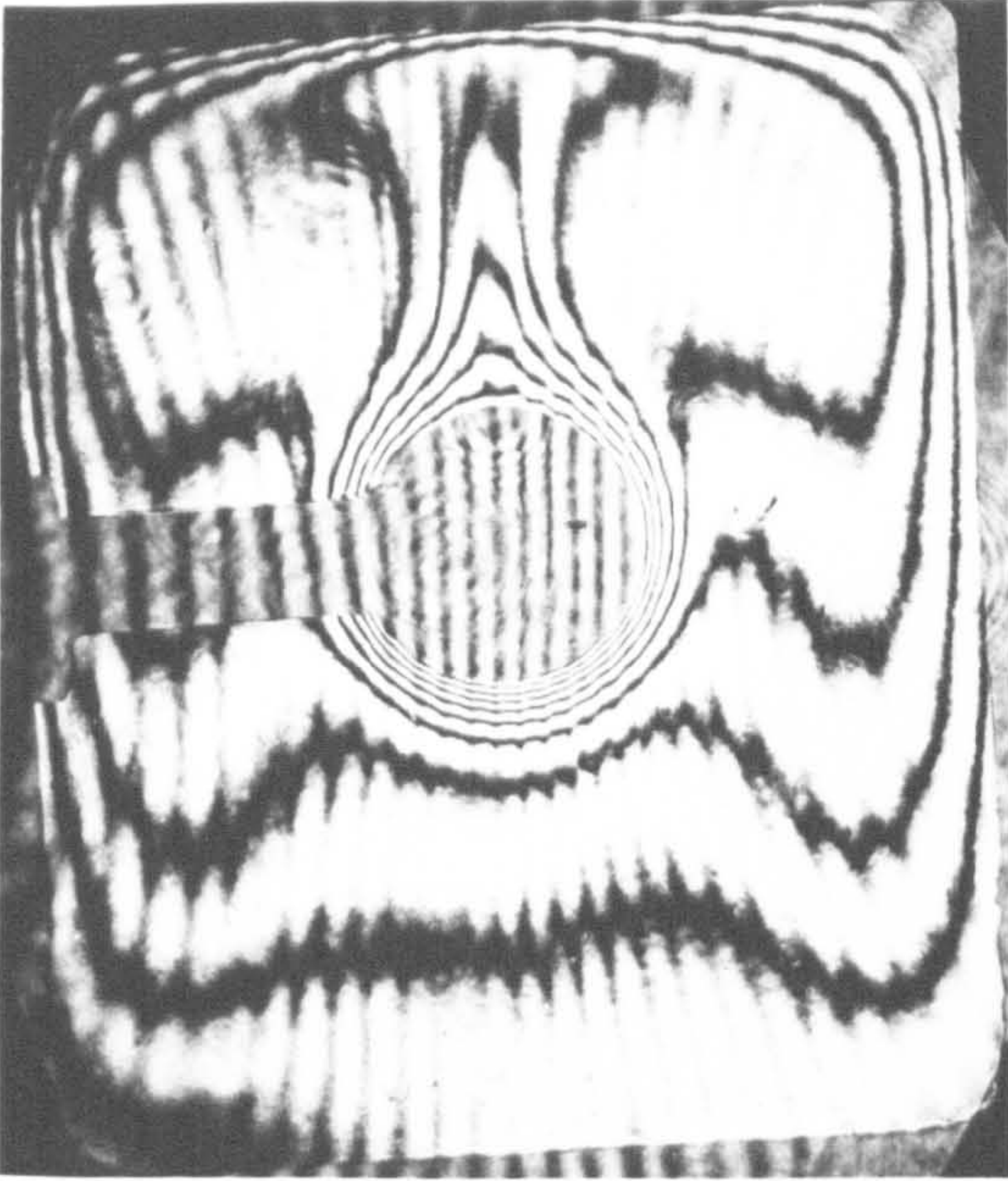
Representative interferograms are shown in Plates 2 - 9.

3.5.2 Two Pipe Study

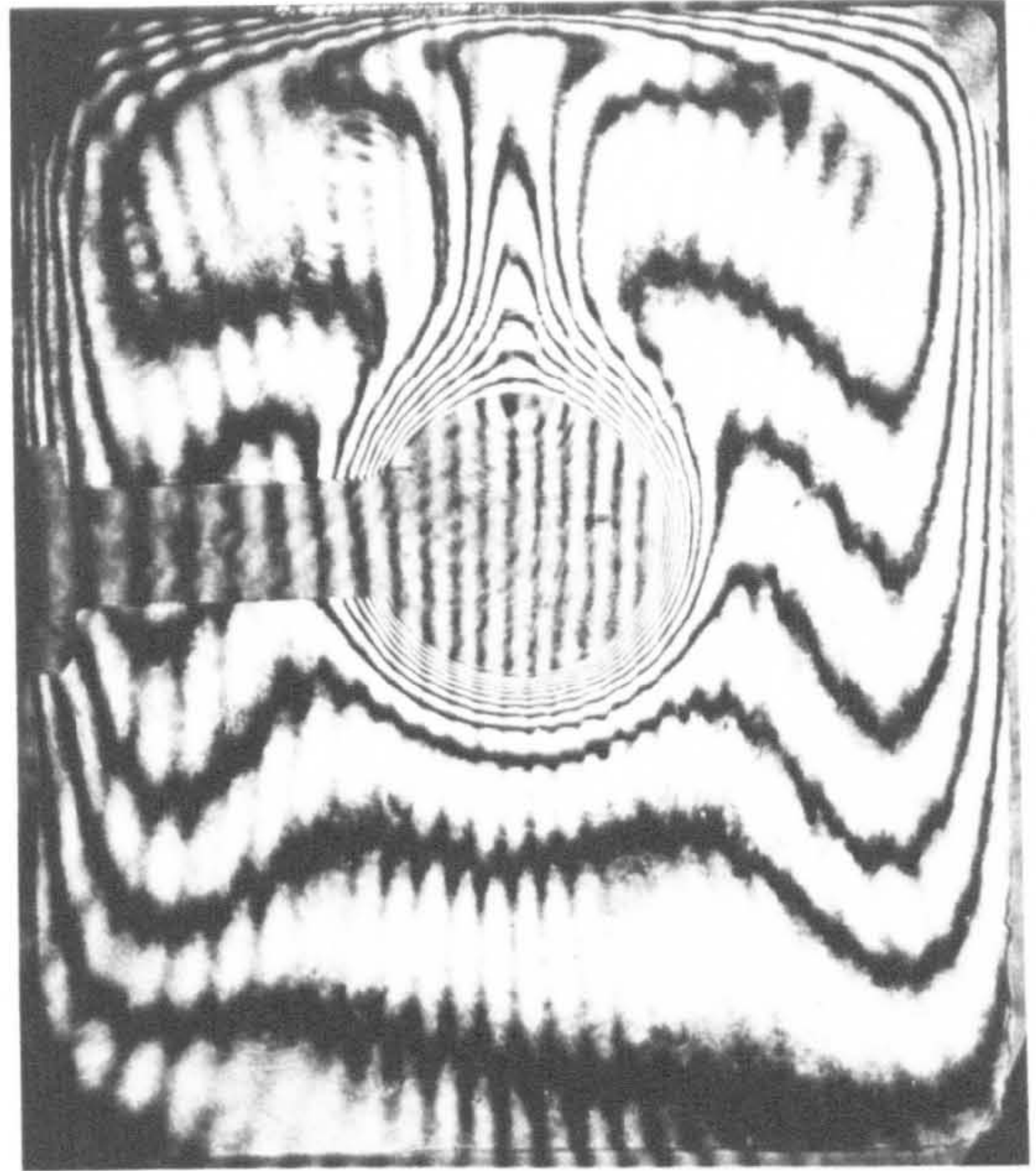
The total heat-transfers, $\dot{Q}_t$, for different supply and return pipe configurations, are given in Table 5.1. The average Nusselt numbers $(Nu_d)_m$ are given in Table 5.2 together with the corresponding Grashof number, Gr_x , and temperature difference, ΔT , for each system studied. Representative interferograms are shown in Plates 10 - 18.

Displacement Ratio = 0, No Spacers

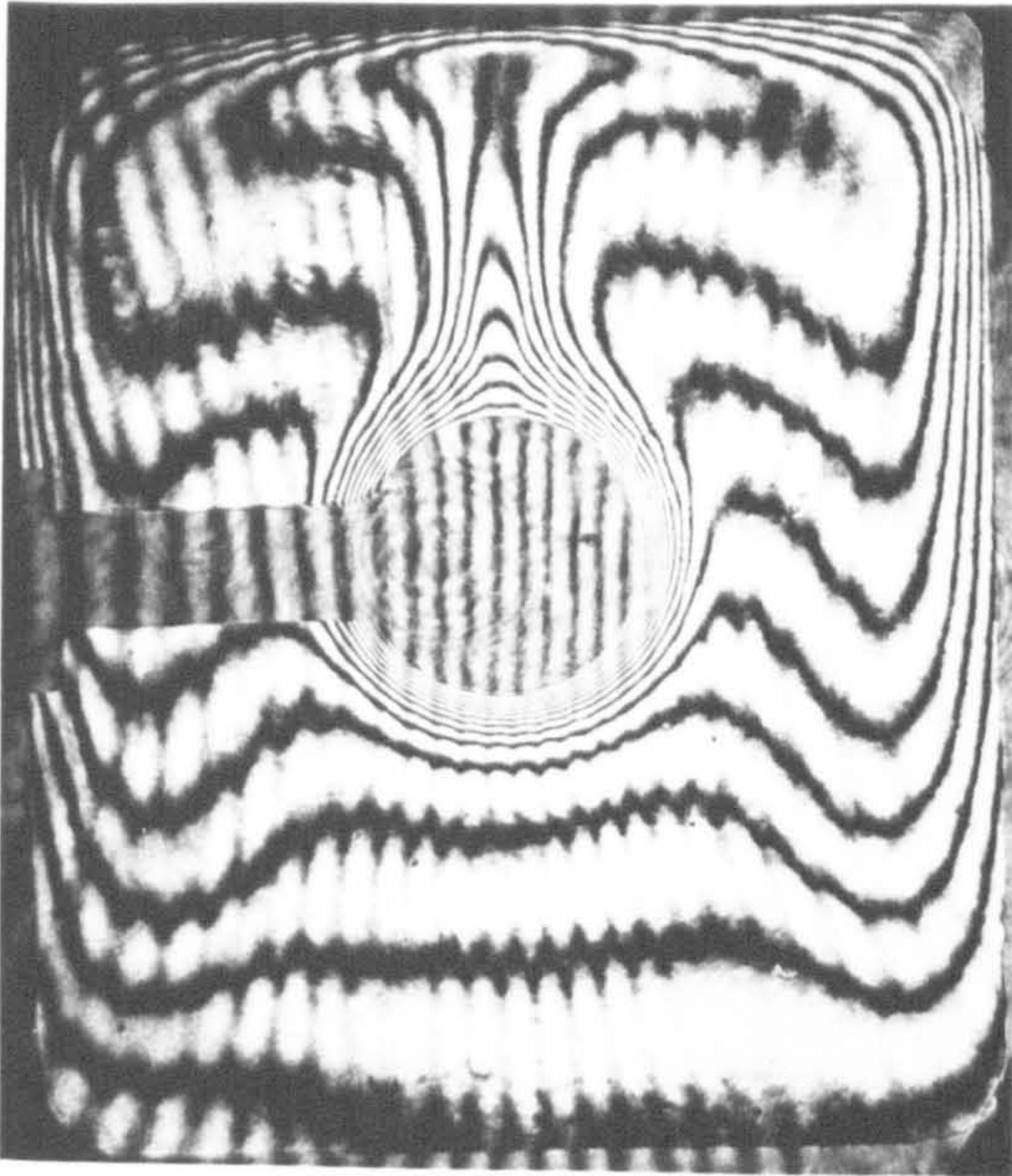
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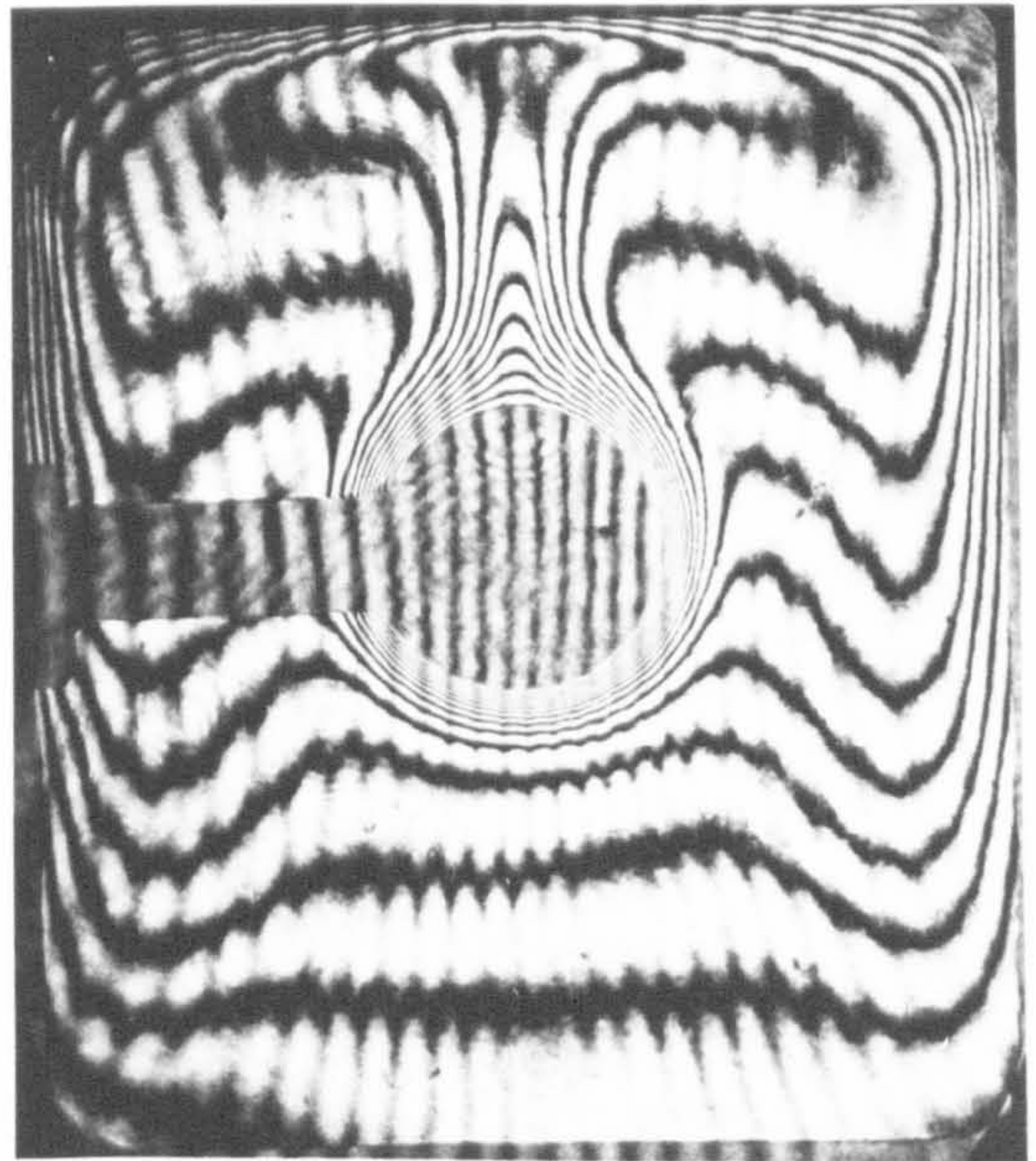
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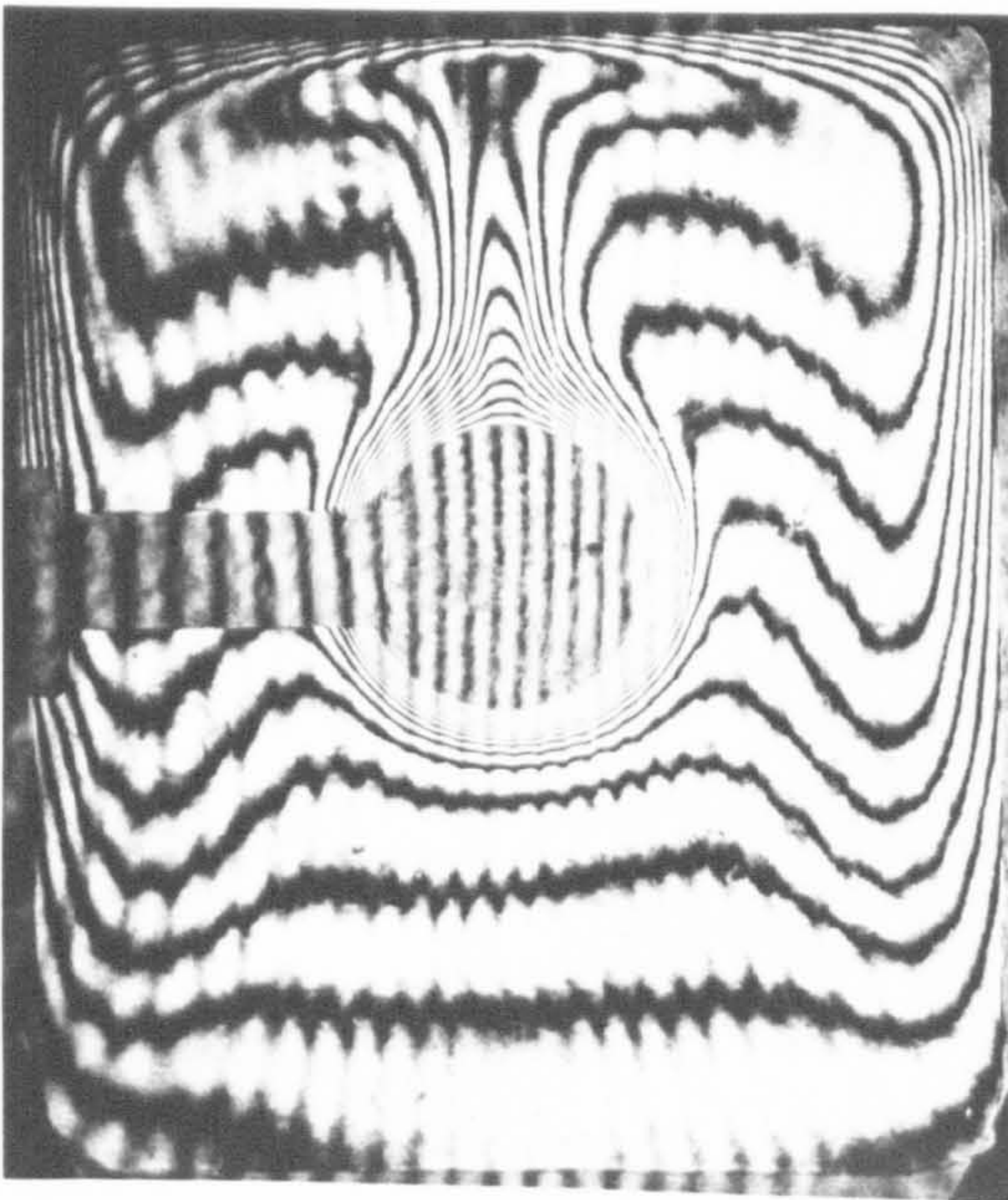
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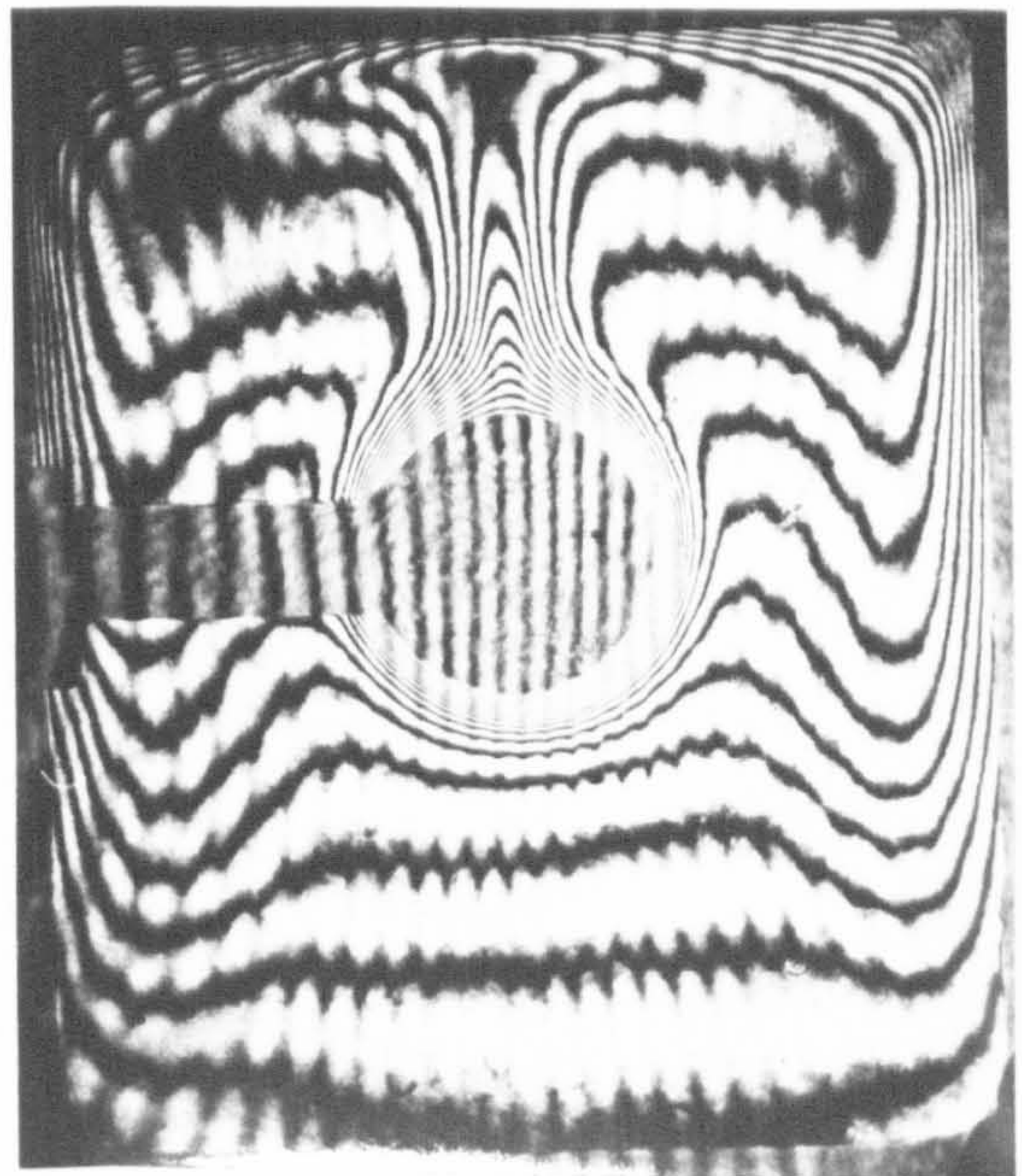
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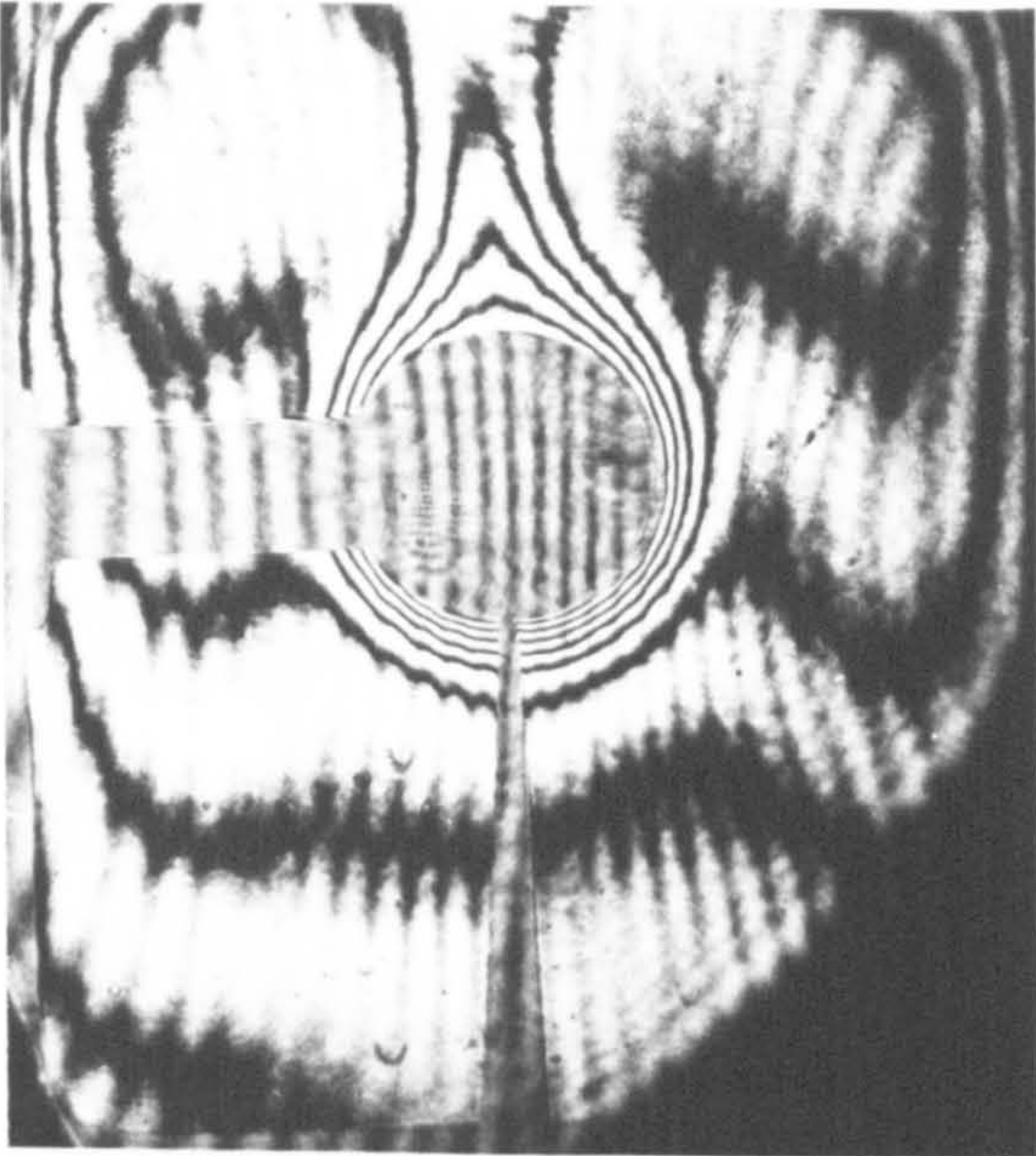


6

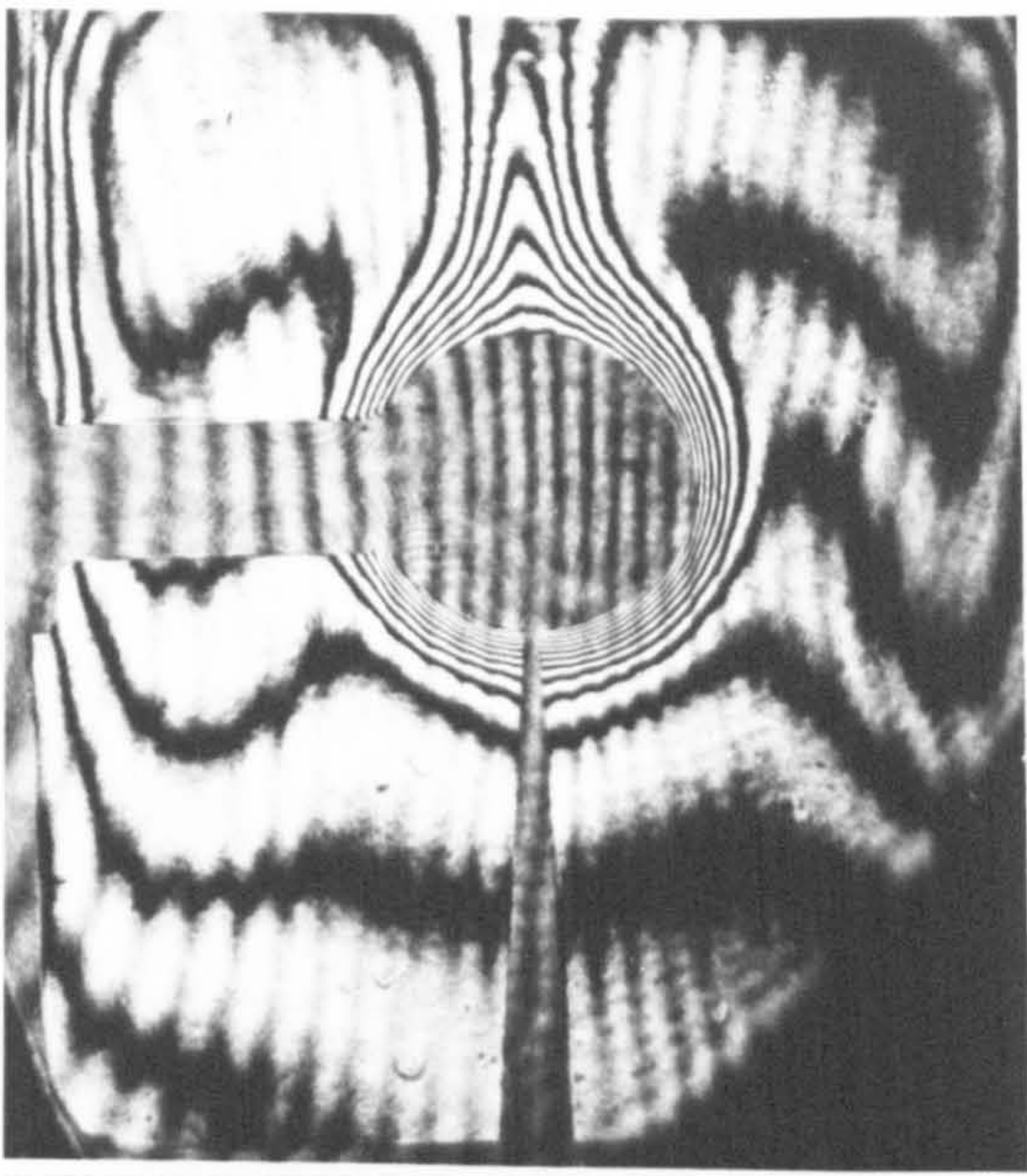


Displacement Ratio = 0, Spacer Angle = 0 Degrees

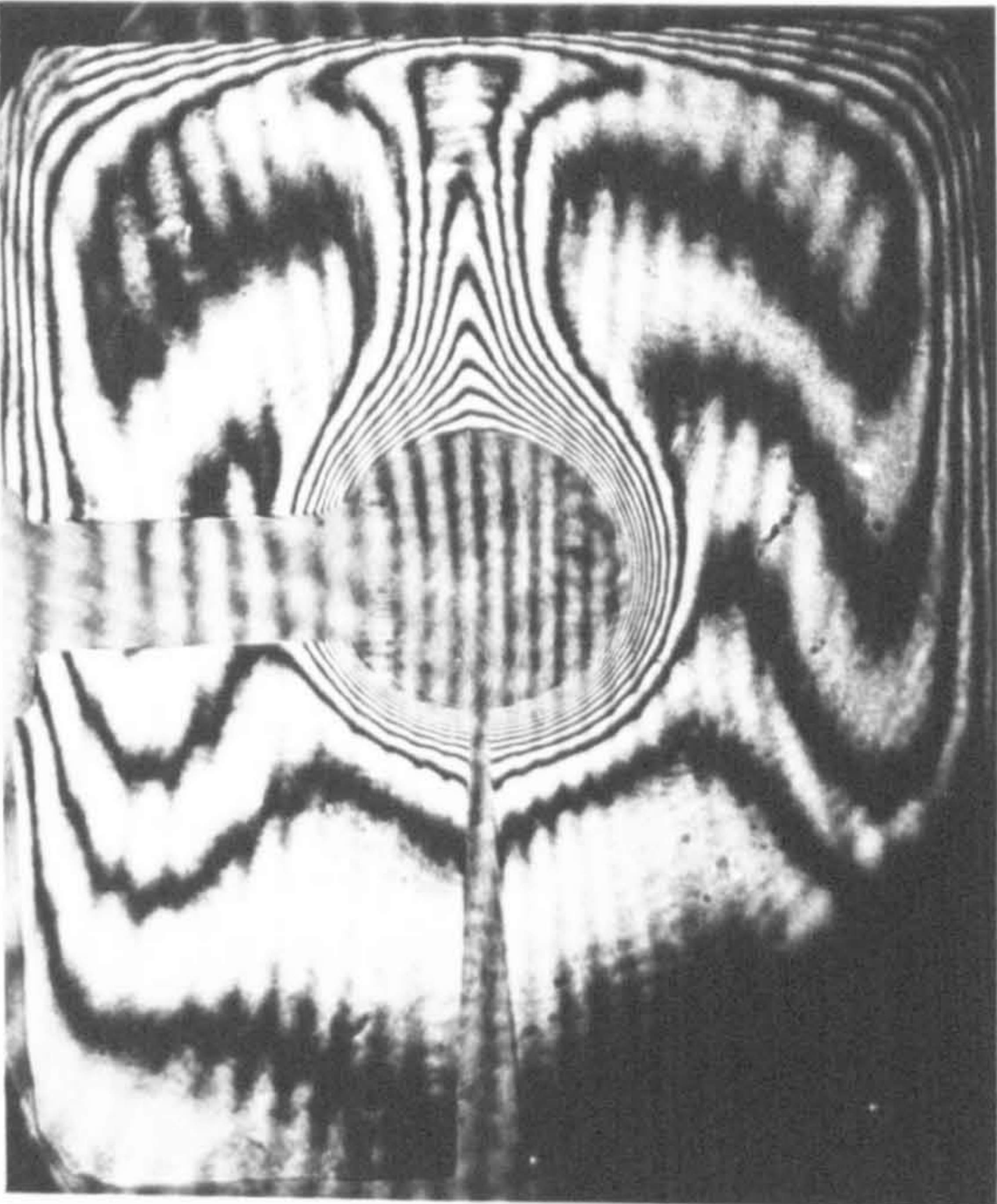
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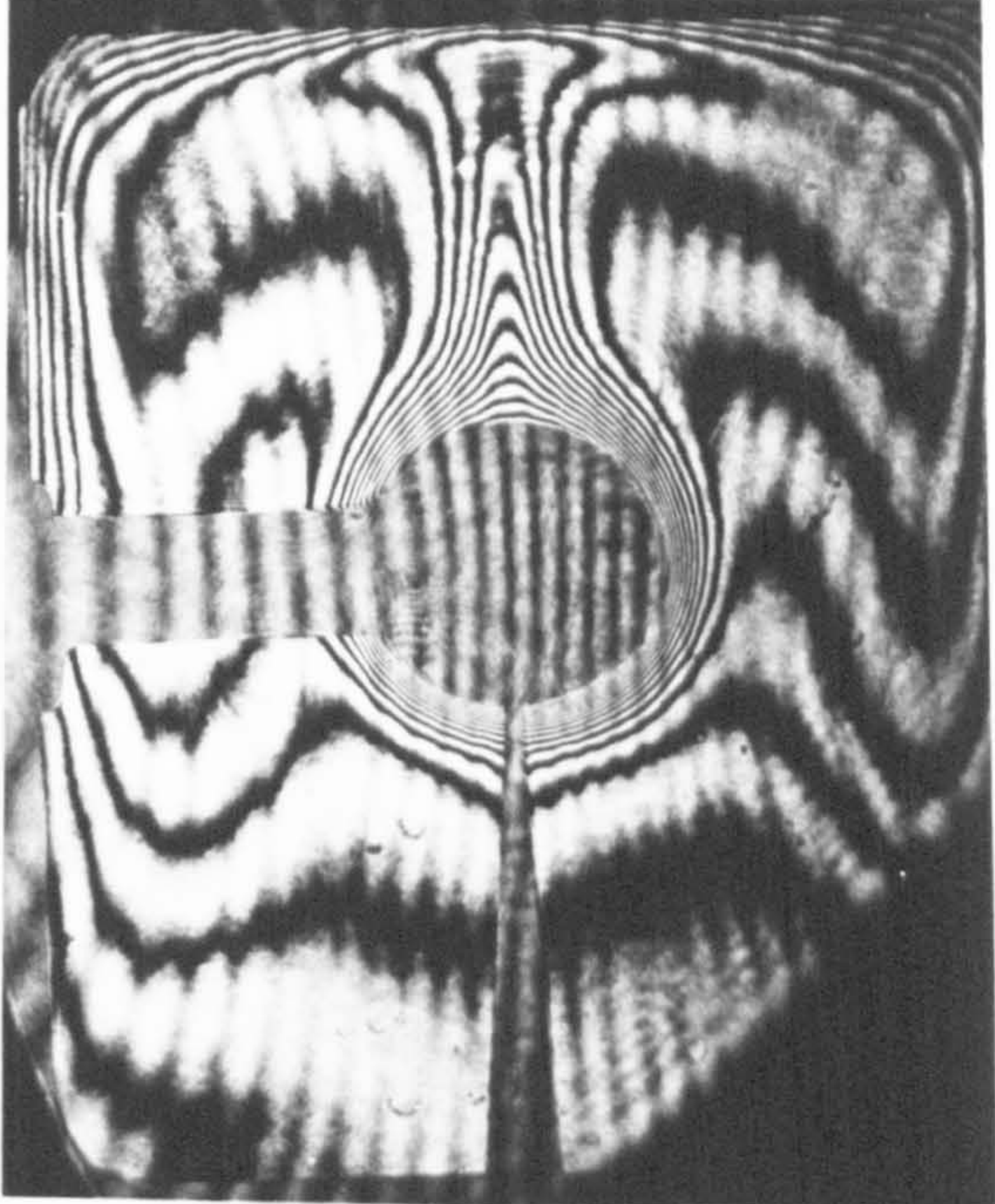
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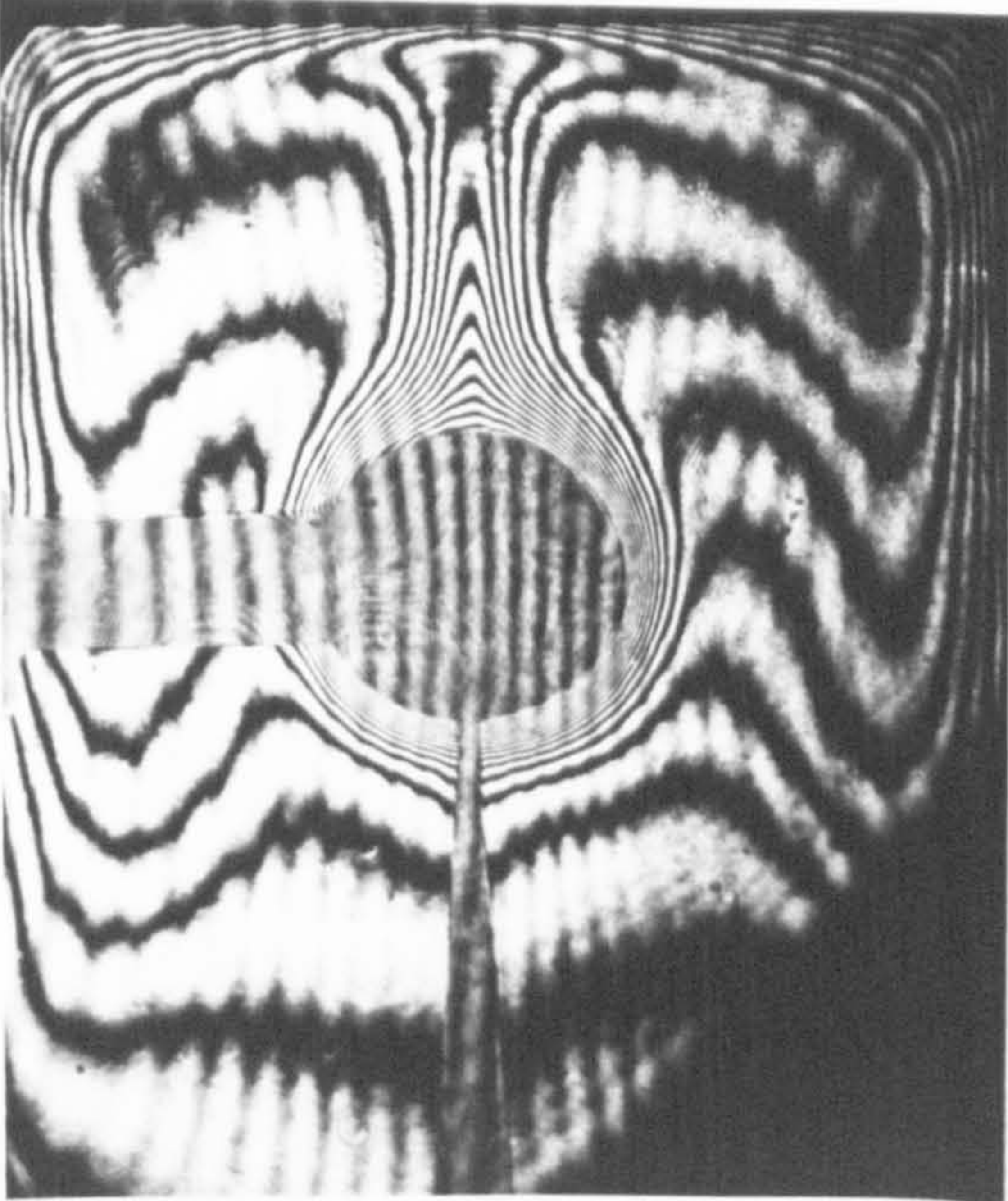
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10



11

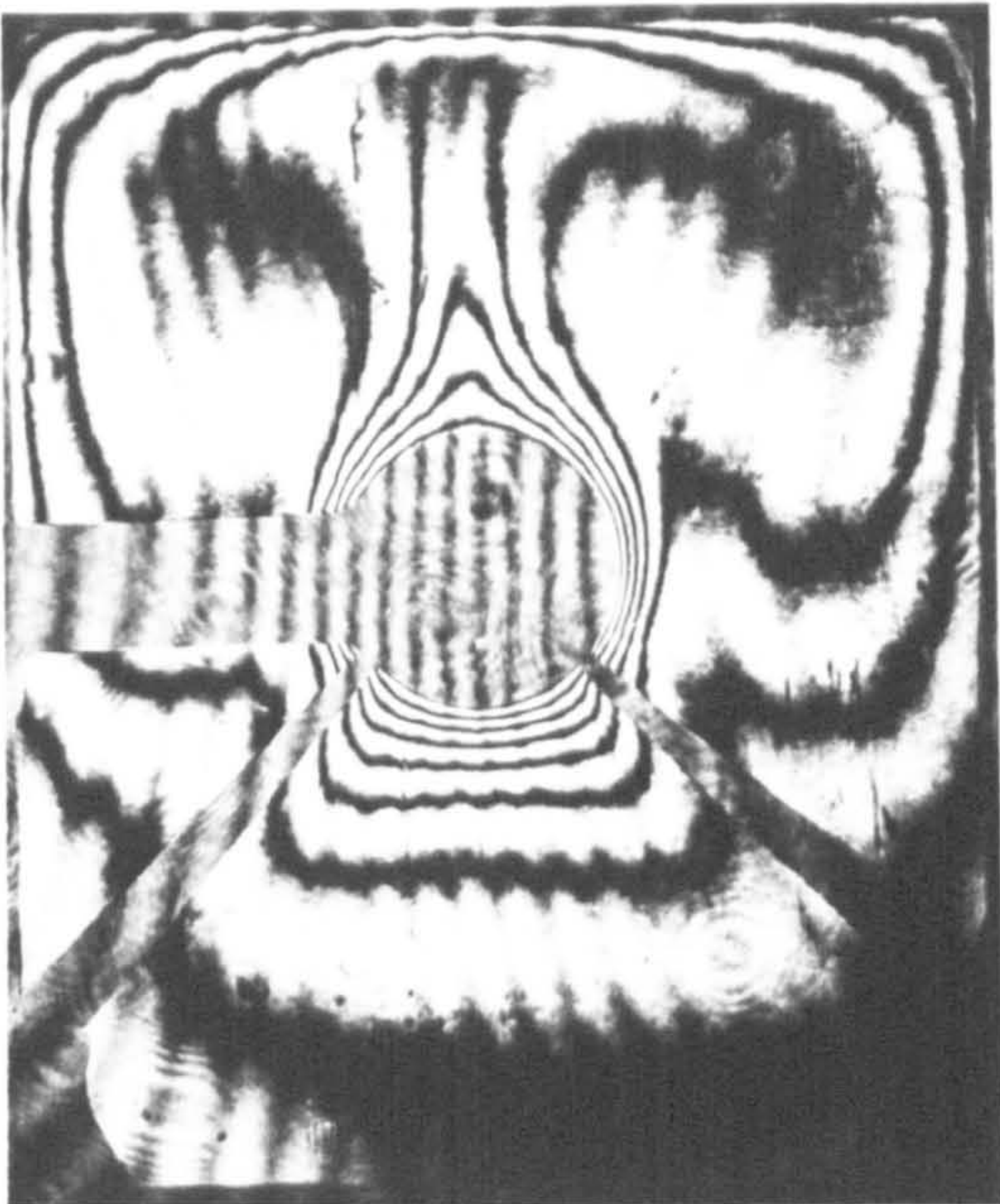


12

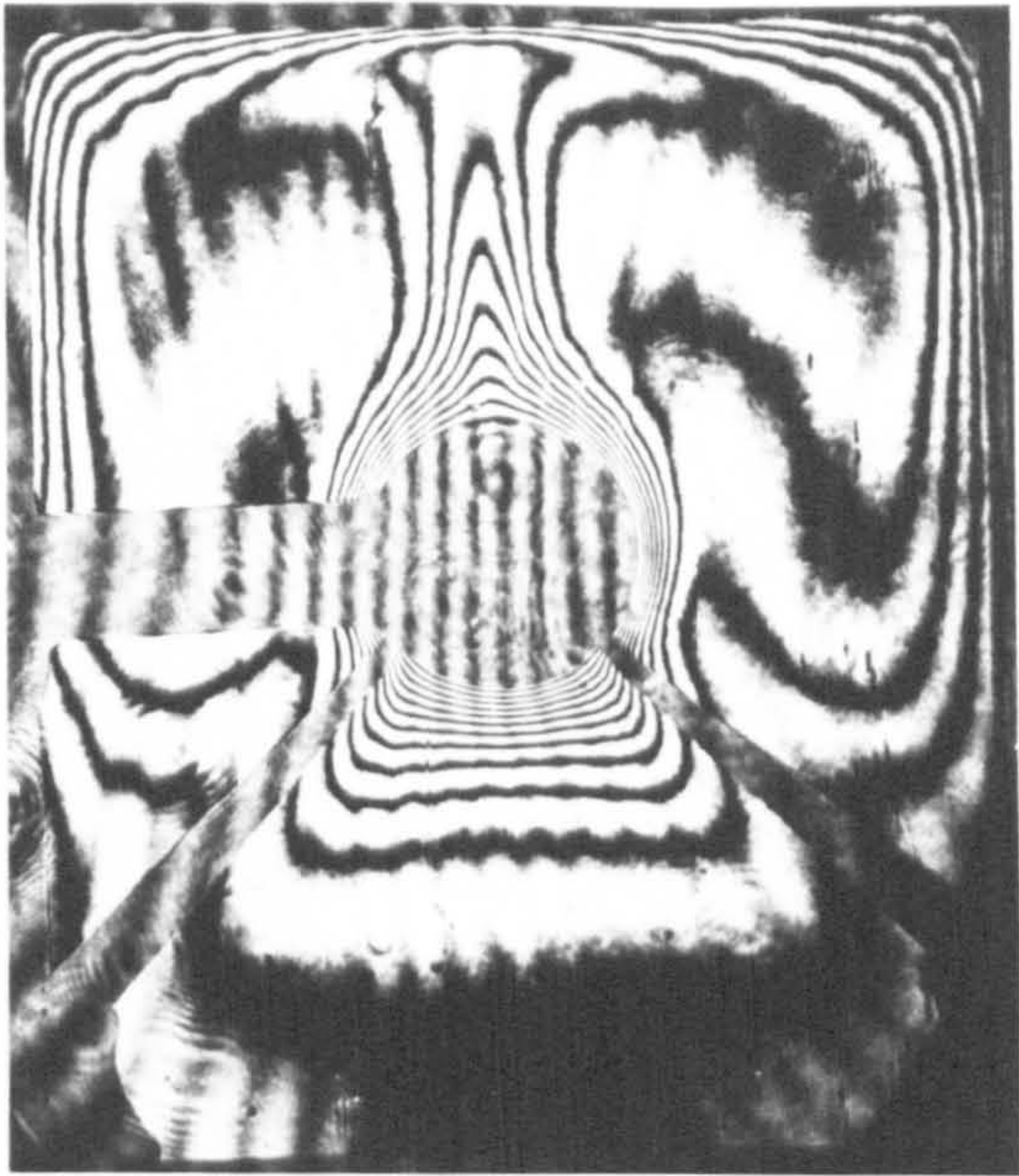


Displacement Ratio = 0, Spacer Angle = 50 Degrees

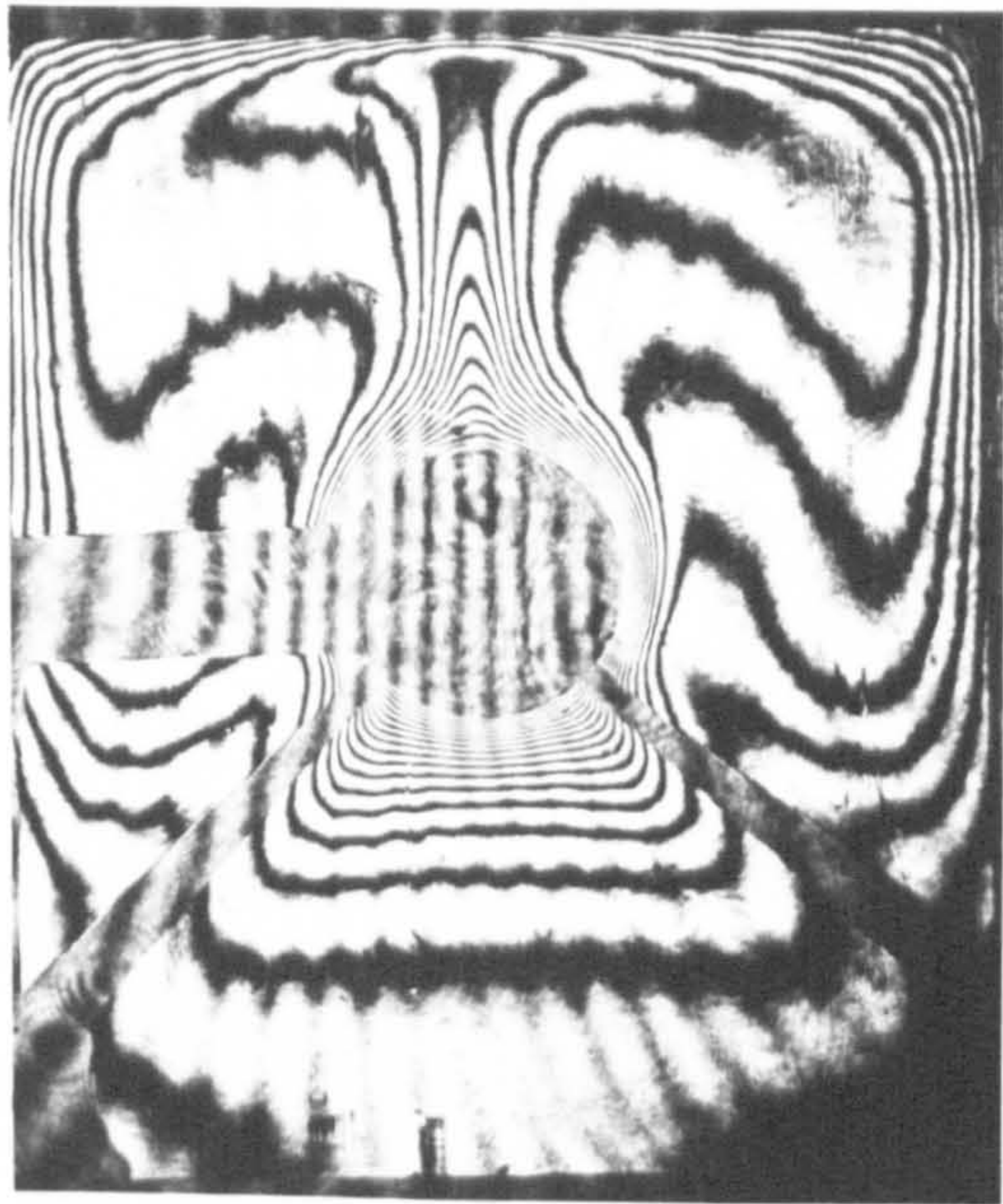
36



37



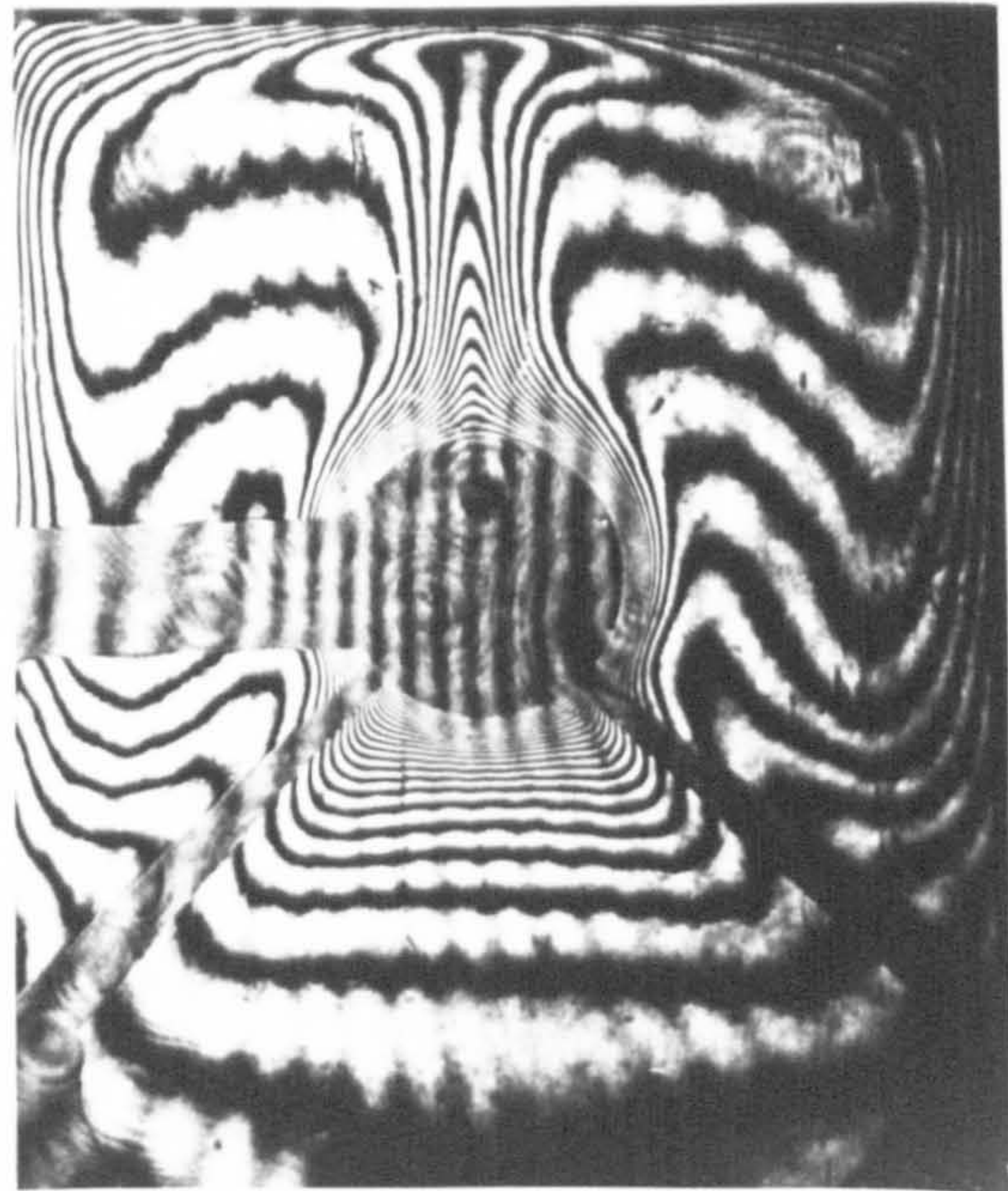
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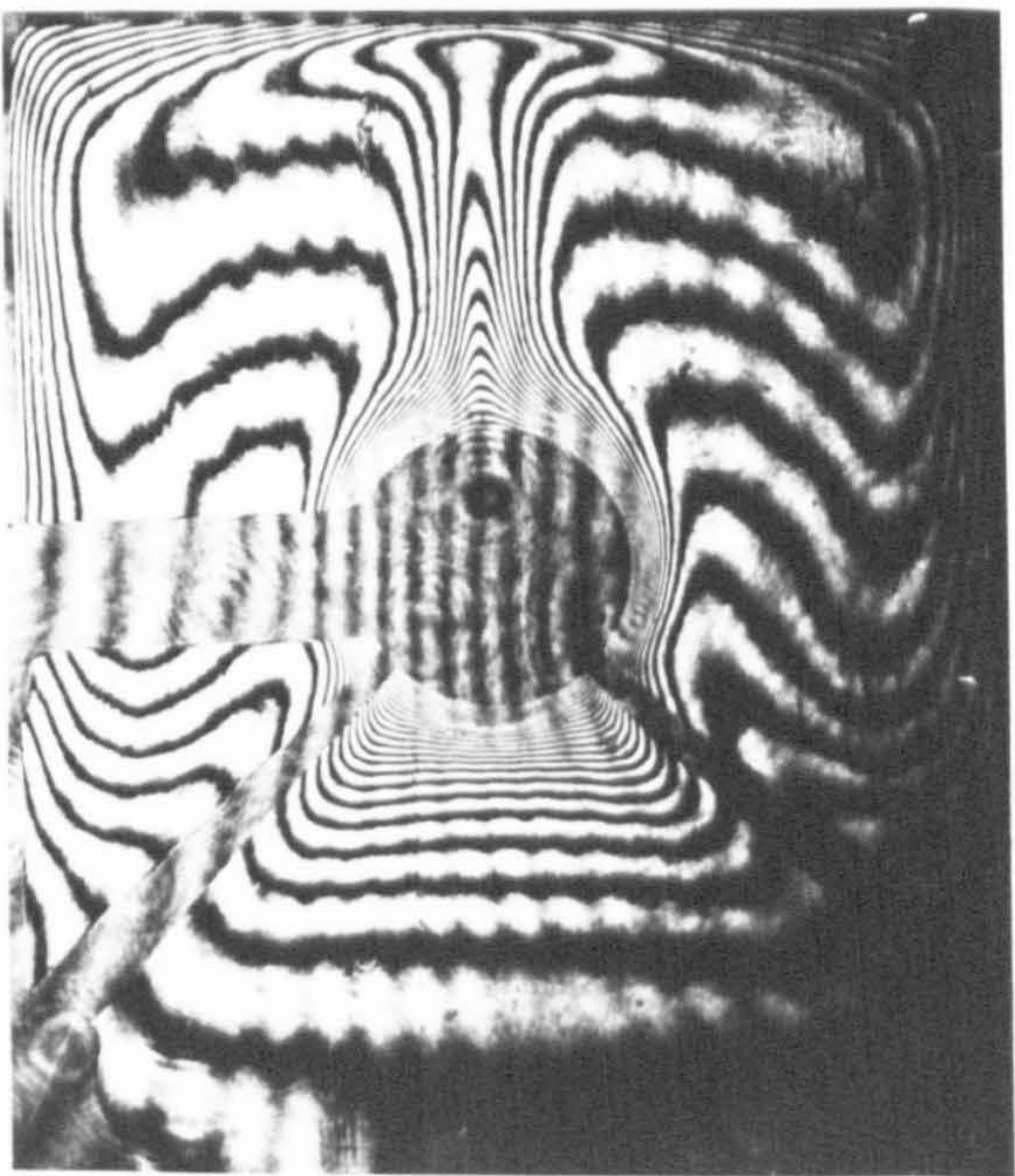
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40

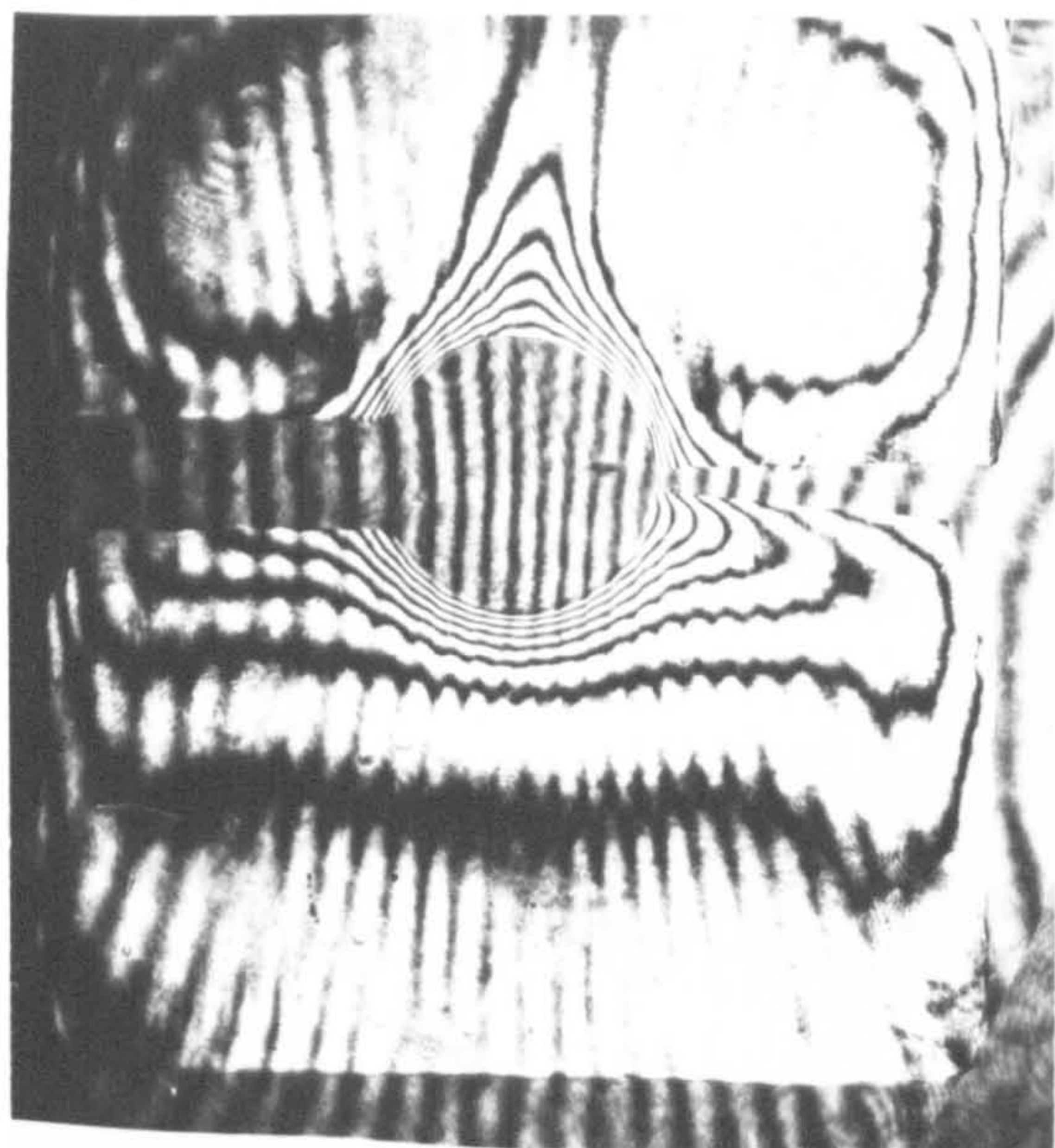


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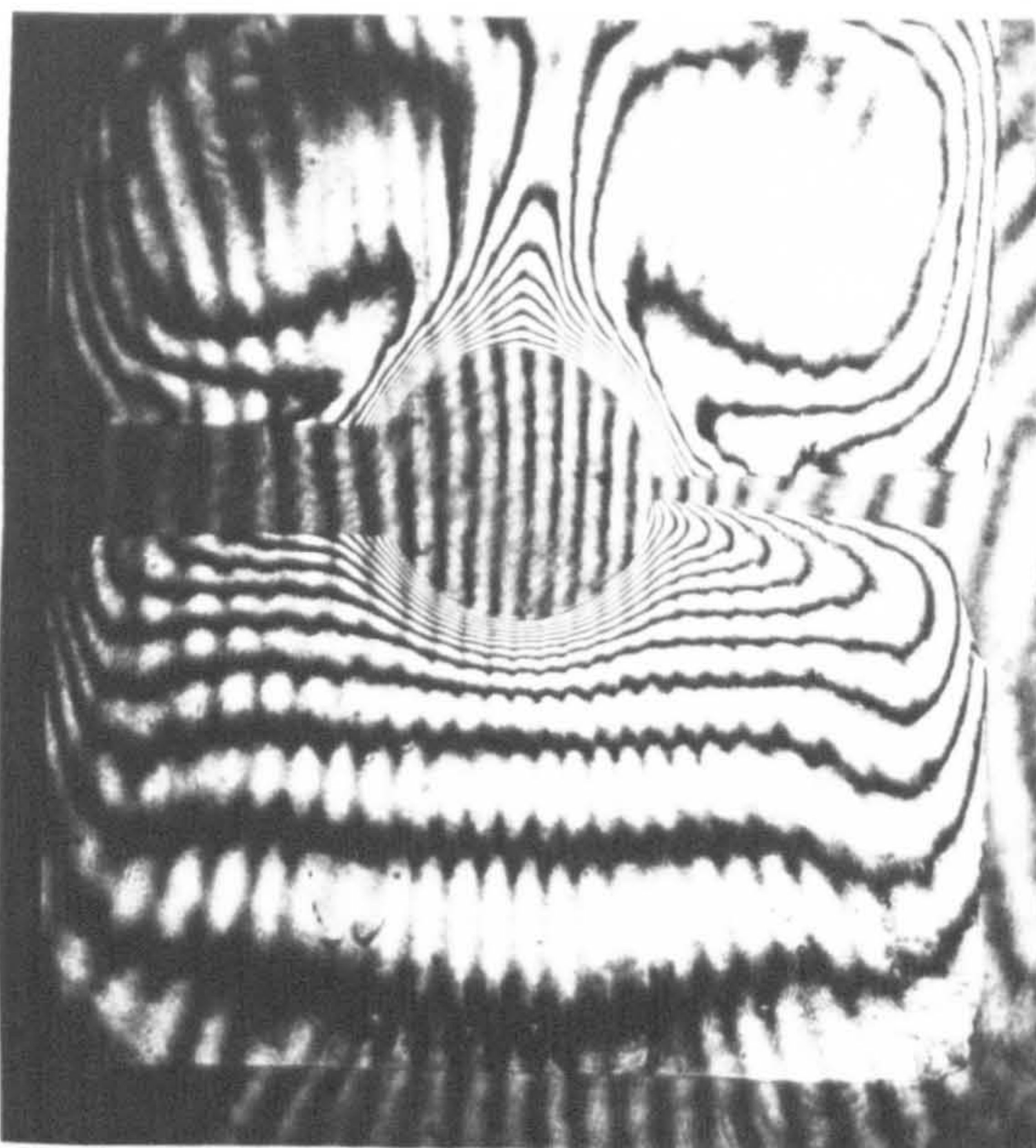


Displacement Ratio = 0, Spacer Angle = 90 Degrees

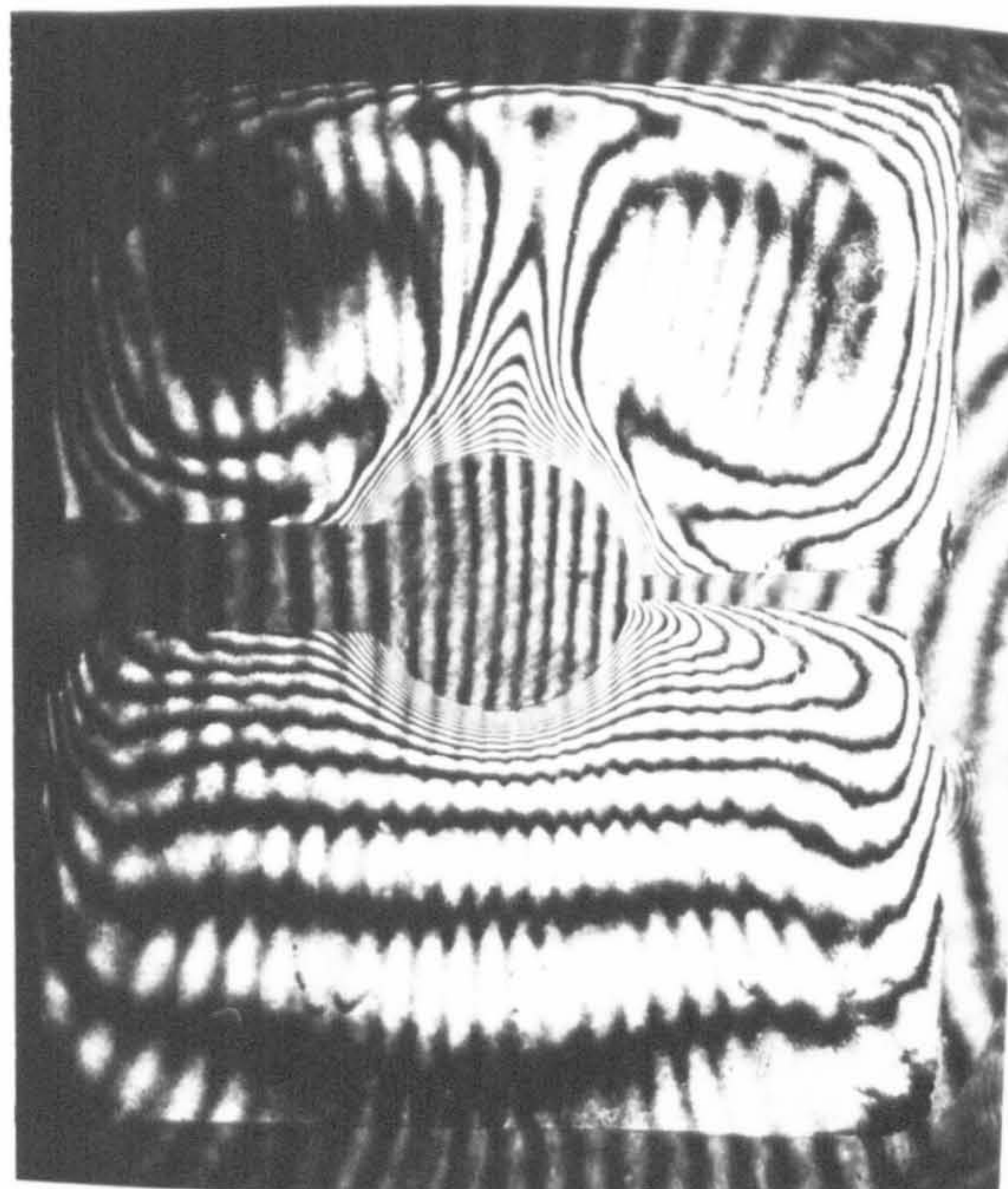
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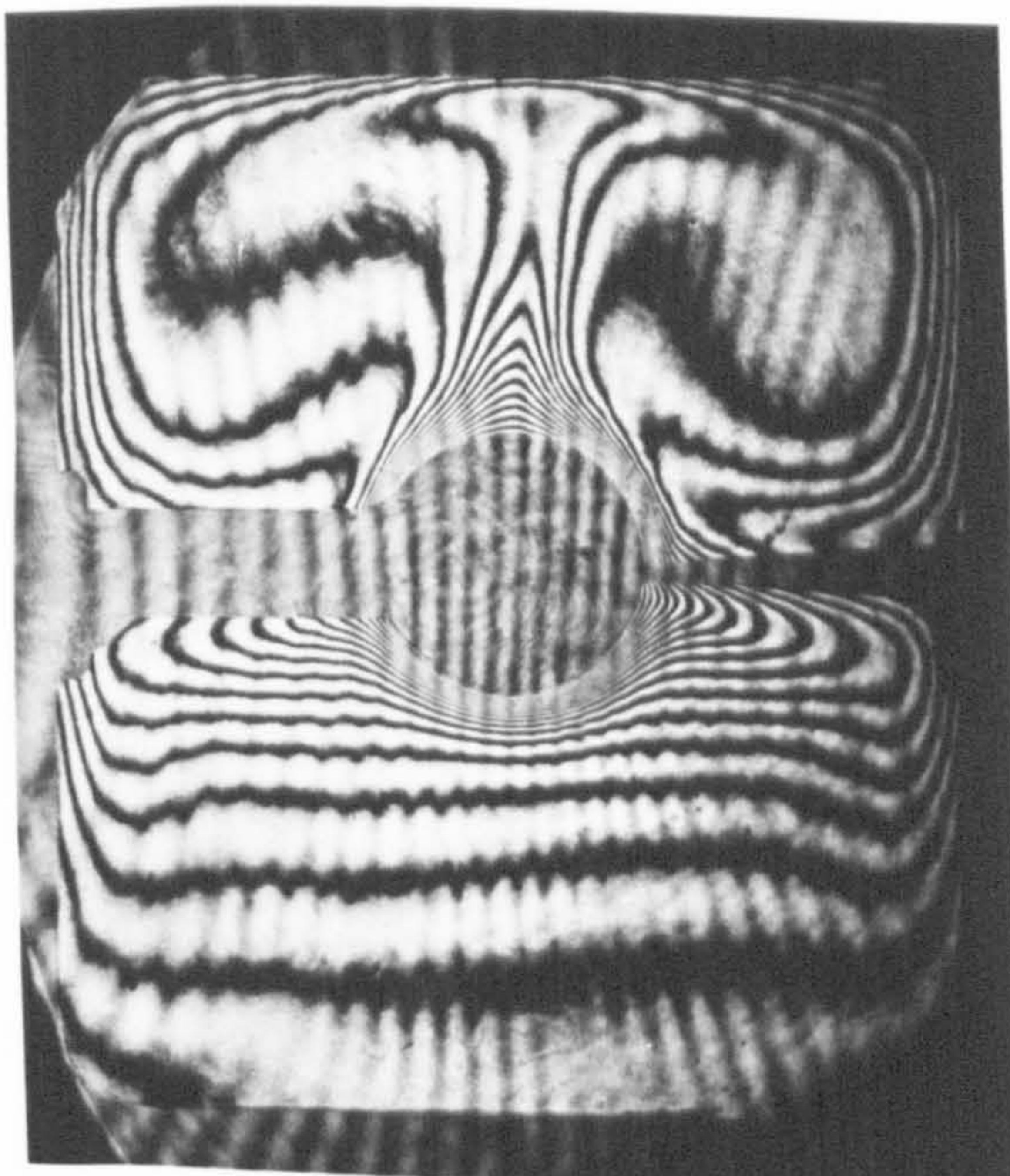
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62



63



64

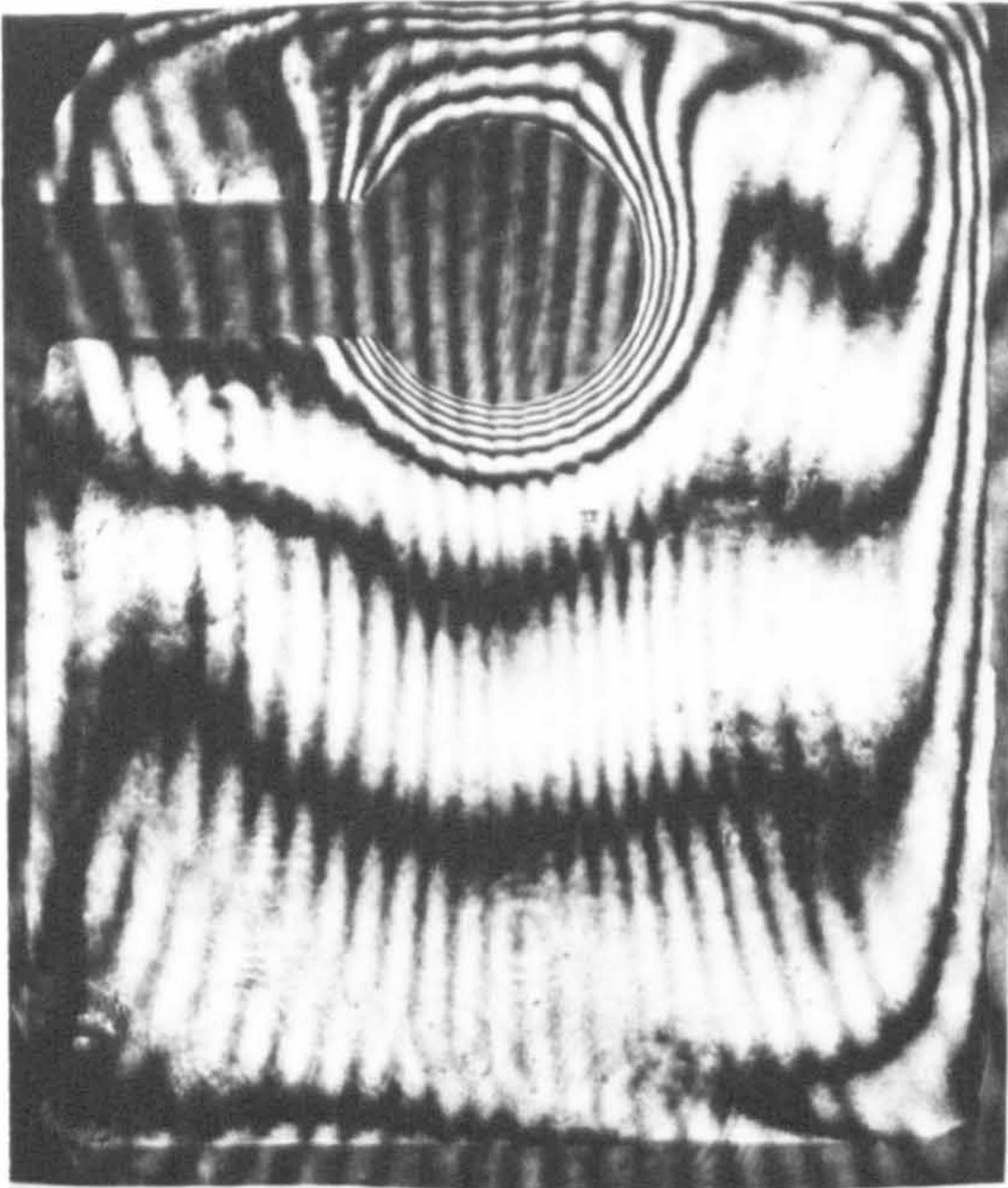


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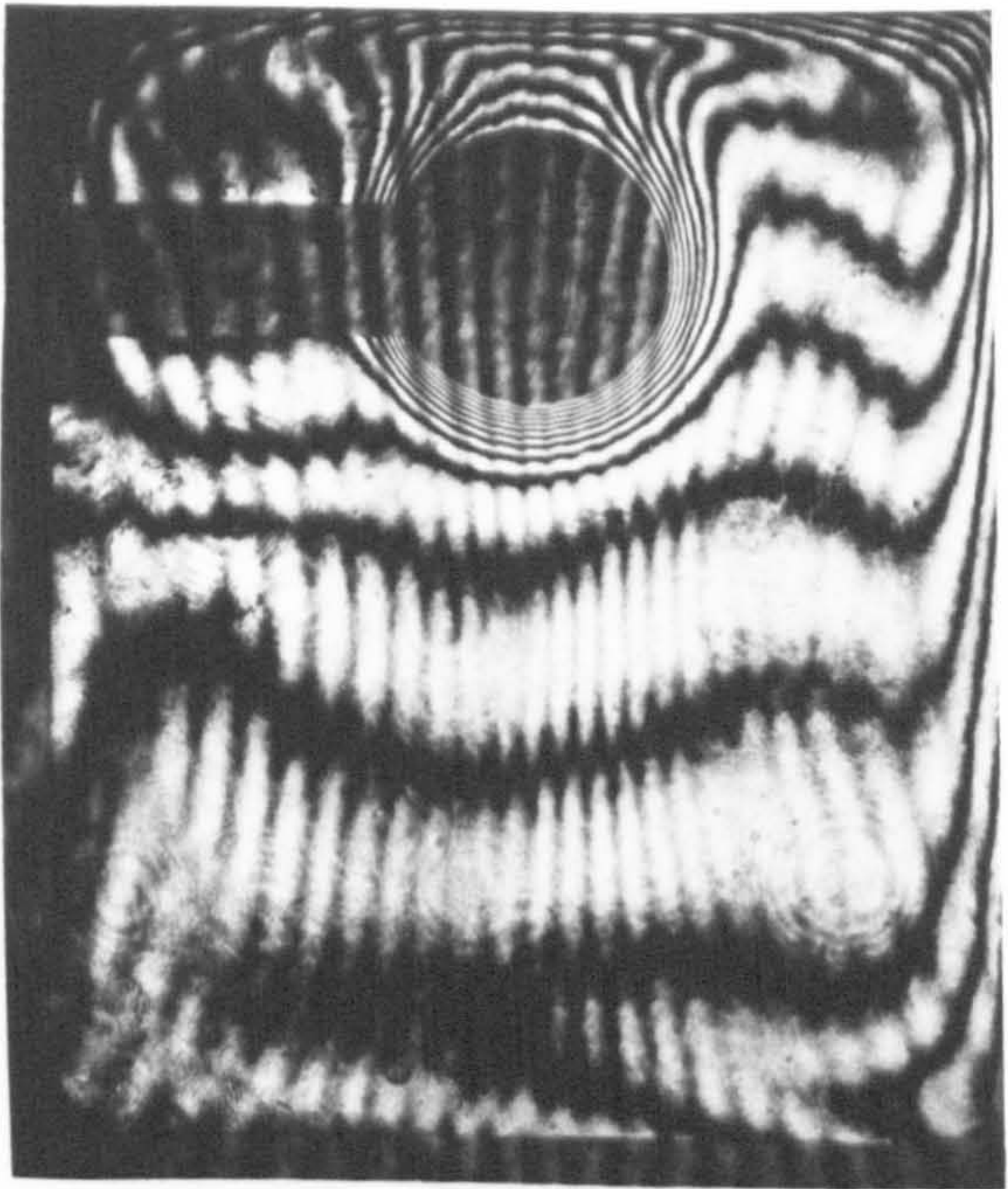


Displacement Ratio = 0.7, No Spacers

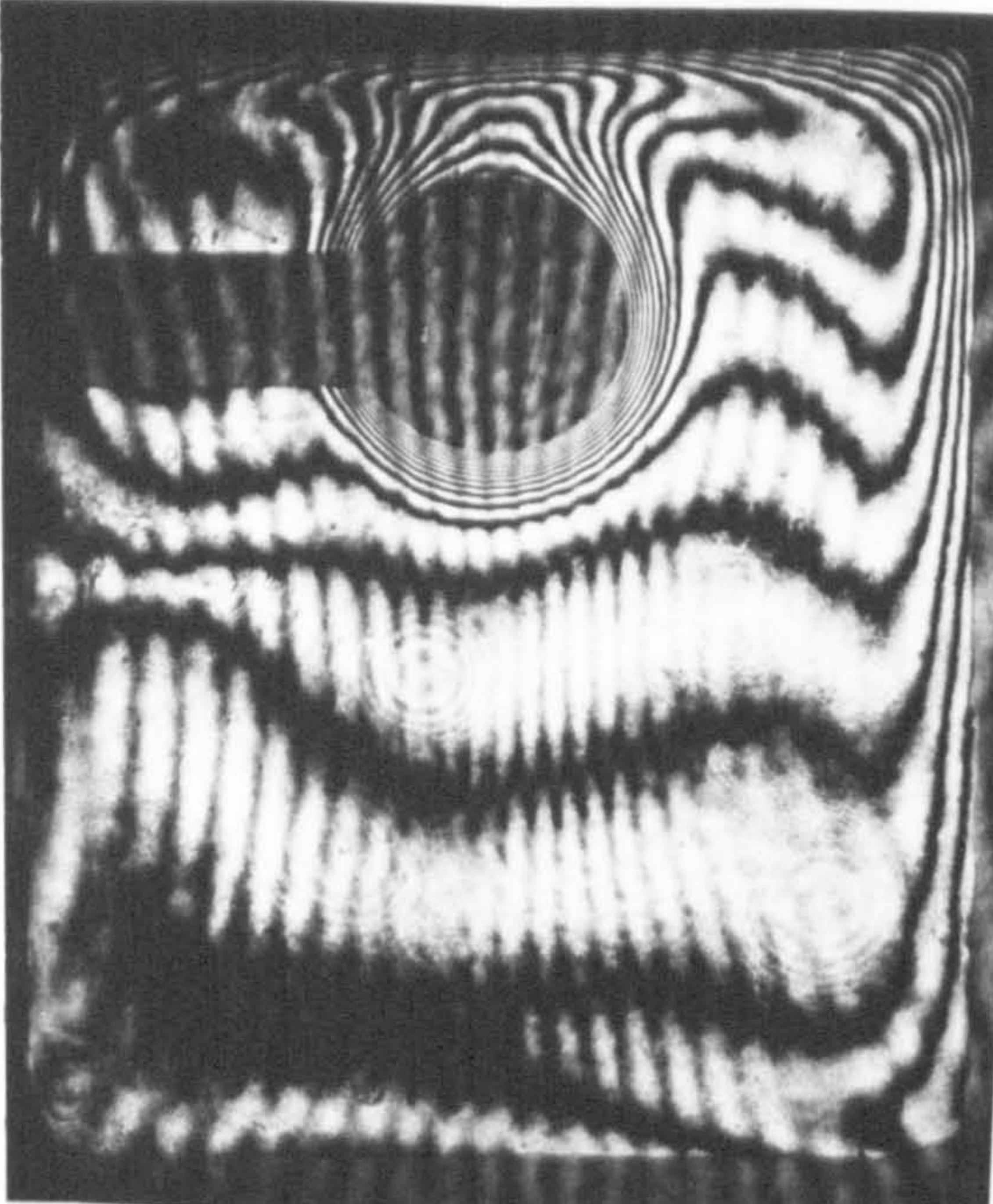
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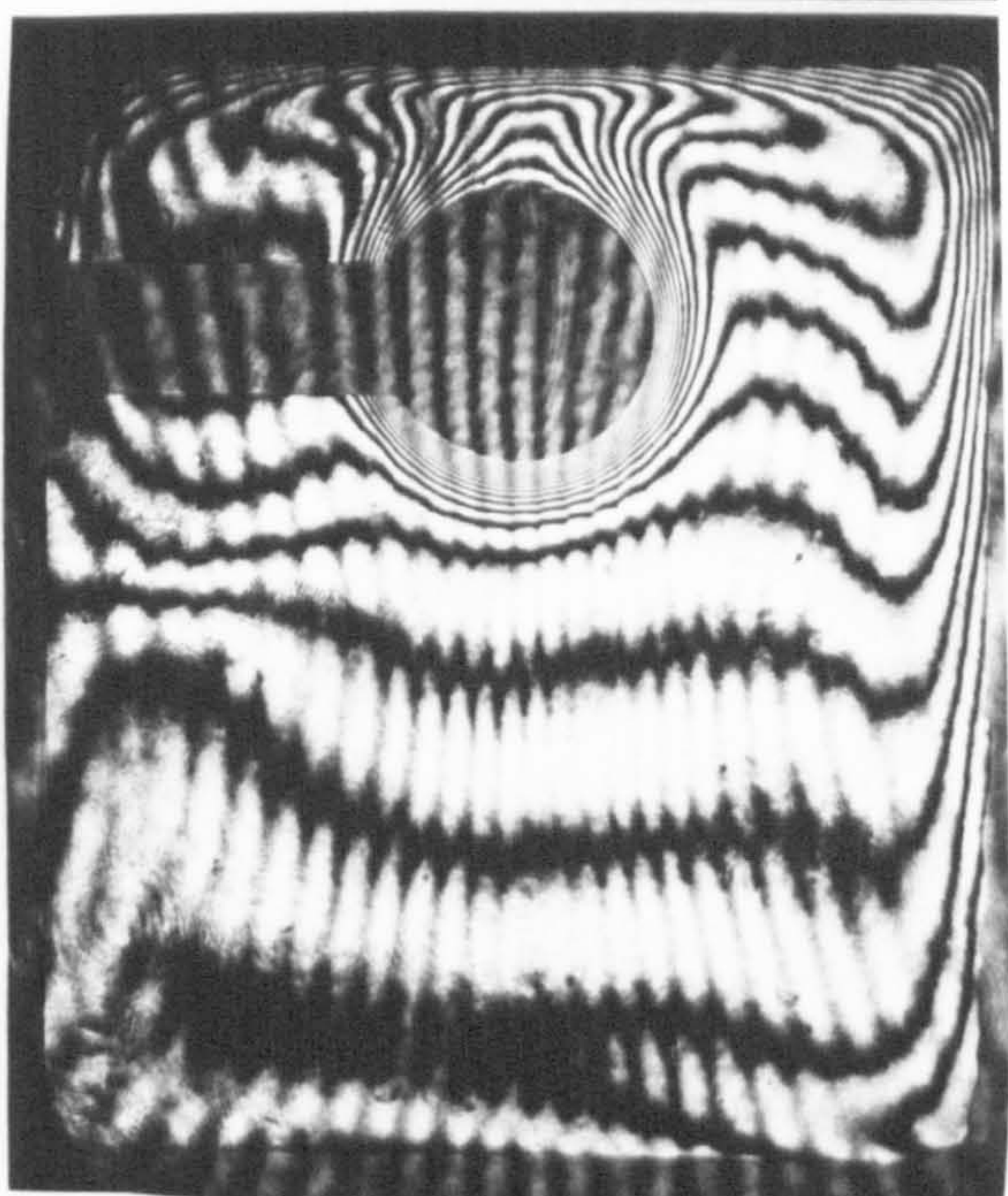
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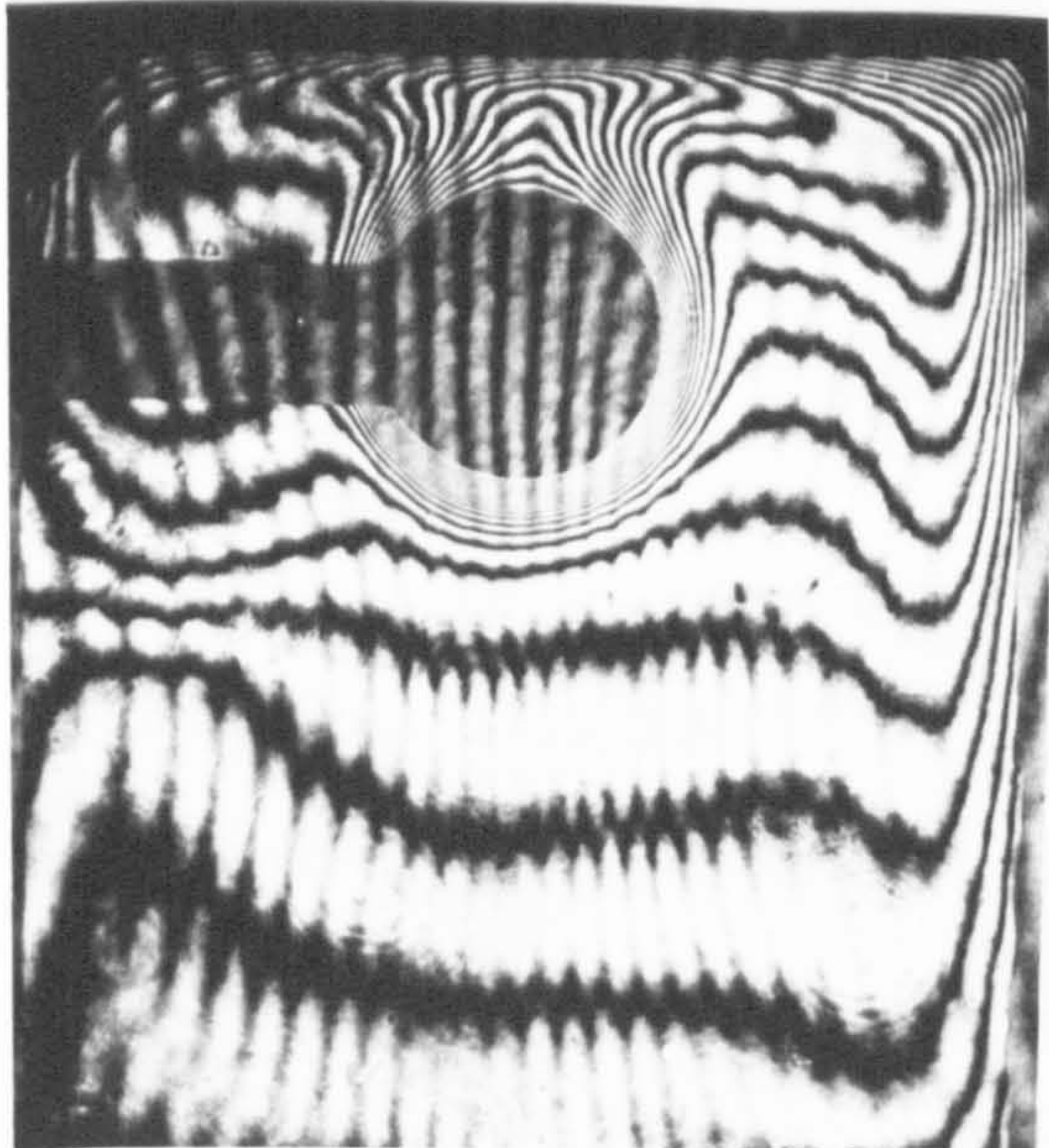
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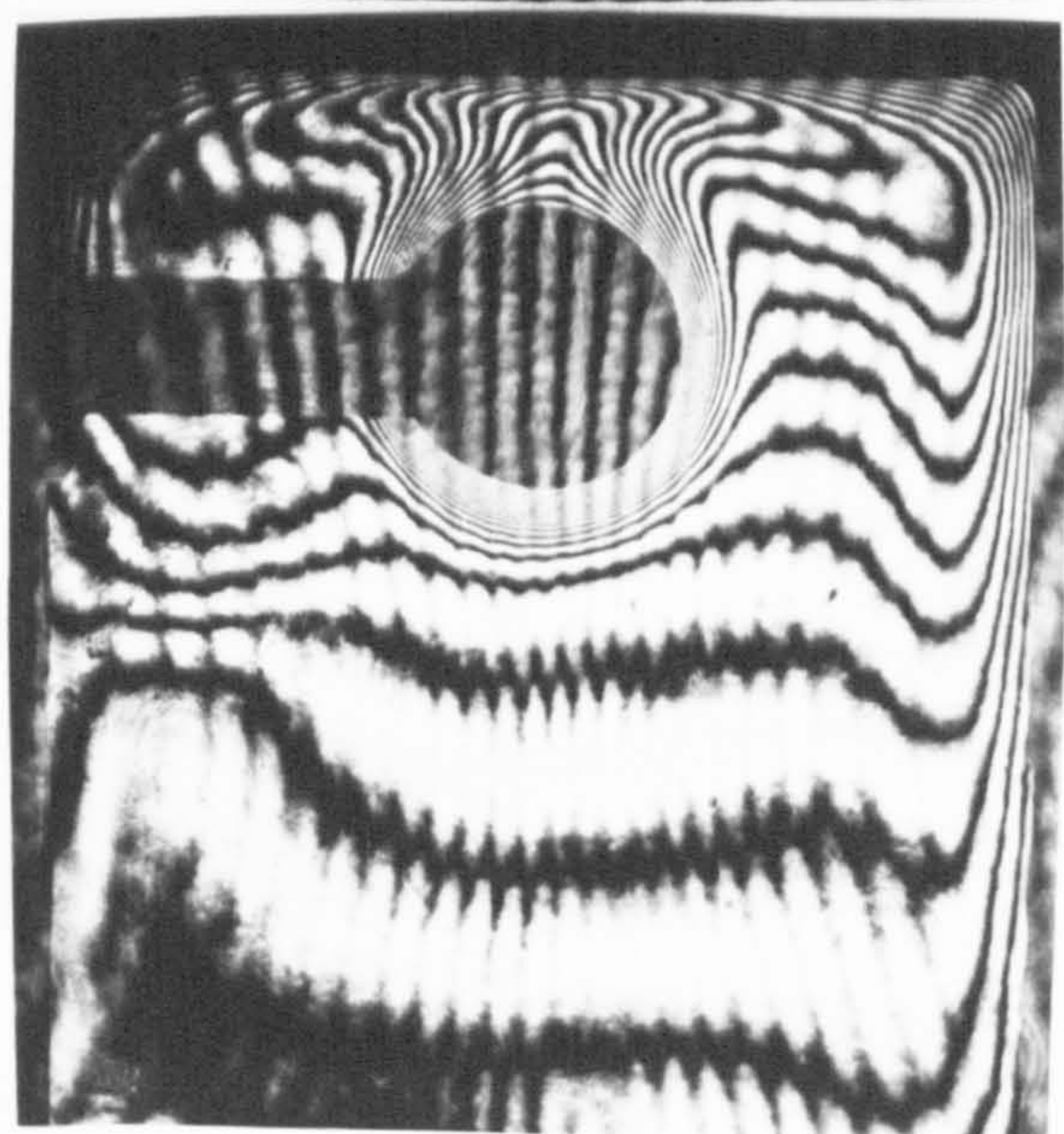
69



70

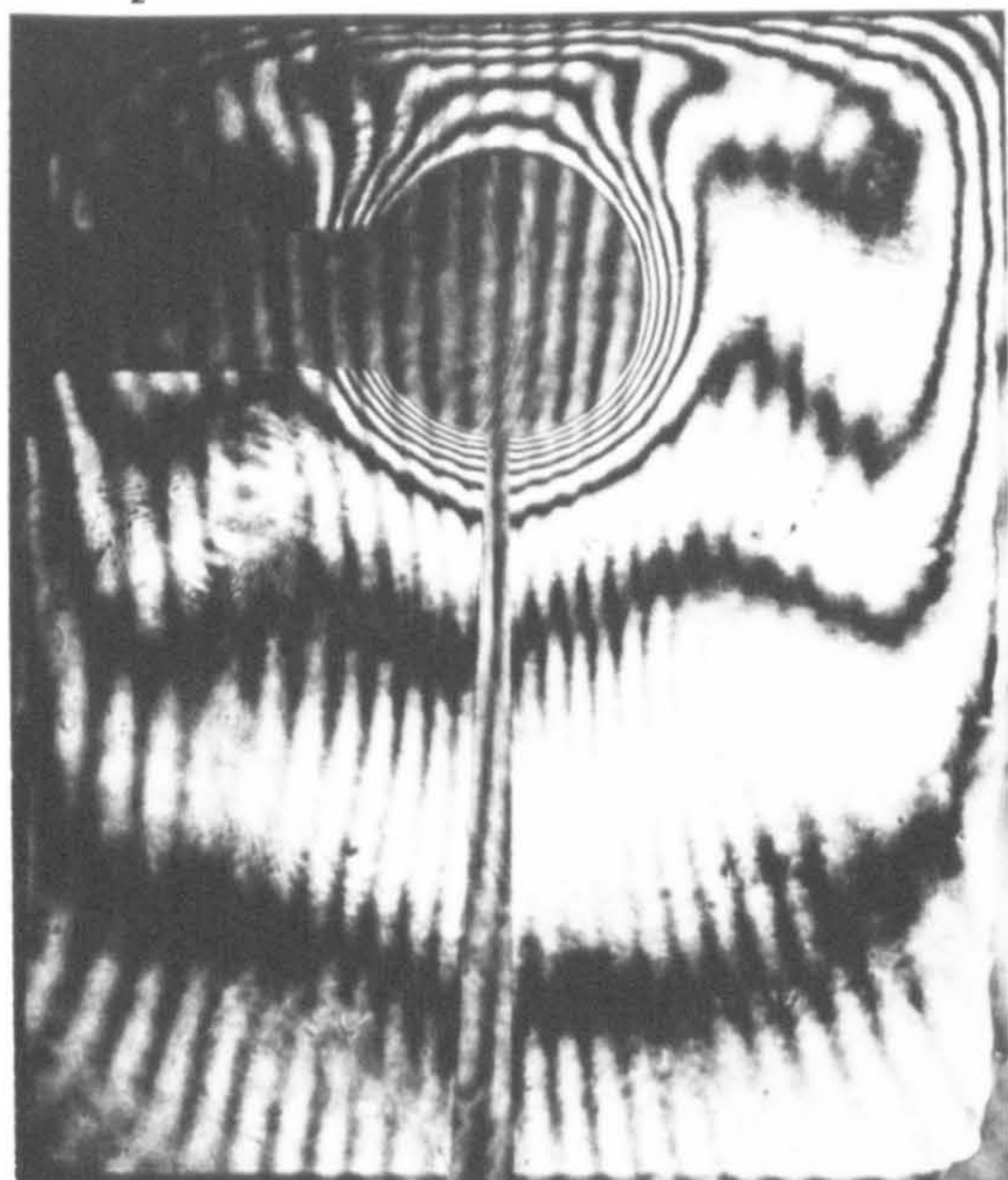


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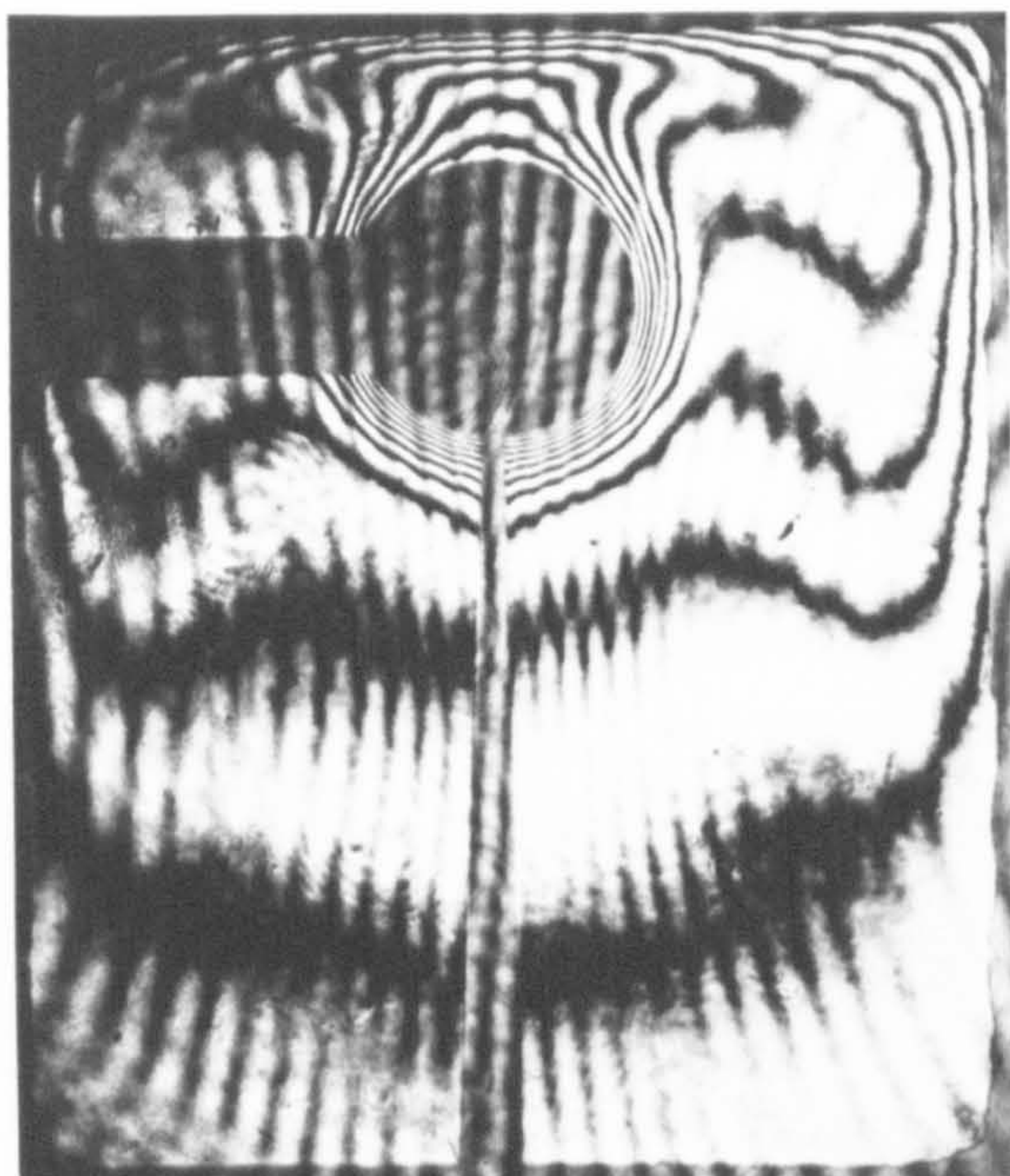


Displacement Ratio = 0.7, Spacer Angle = 0 Degrees

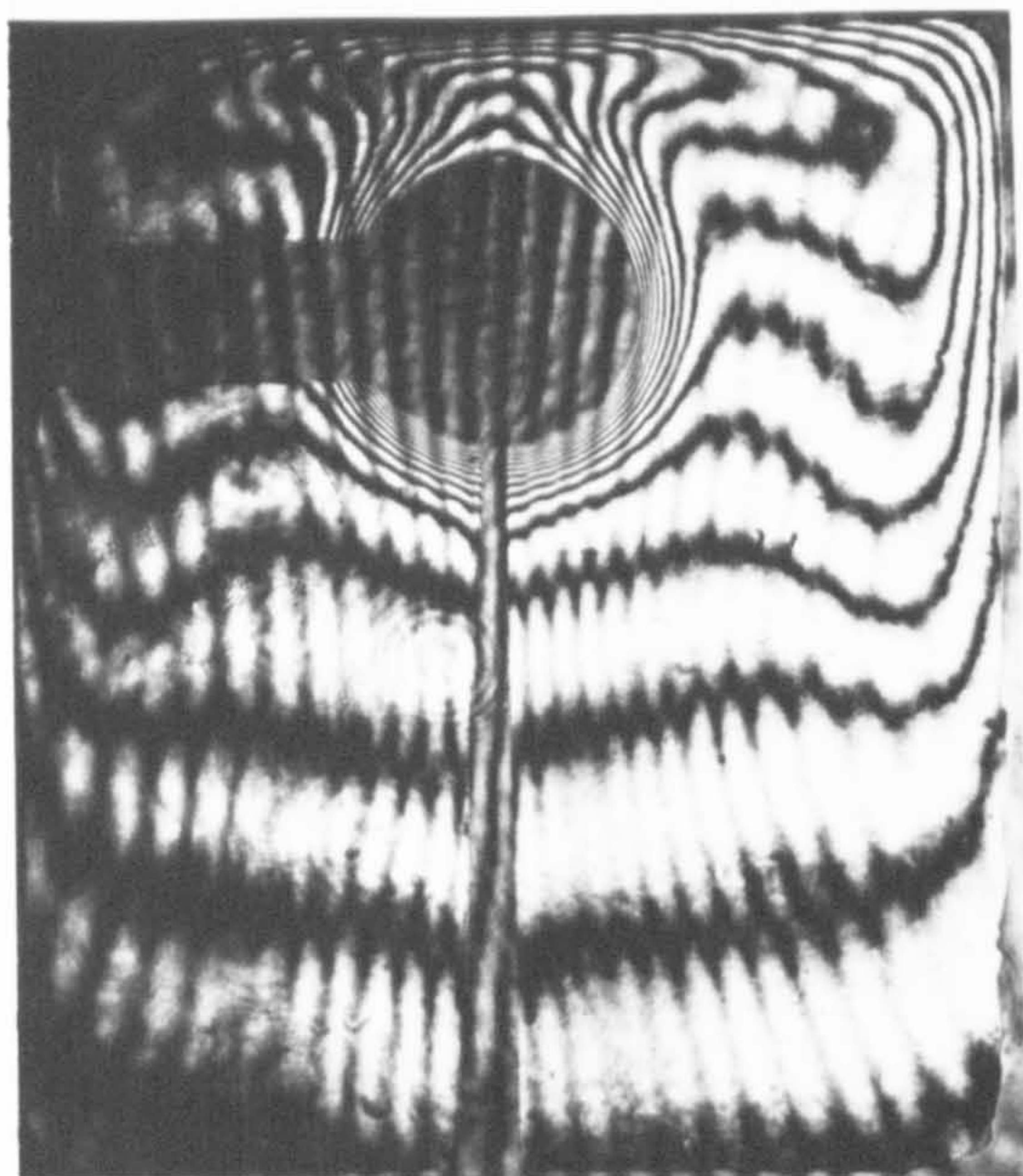
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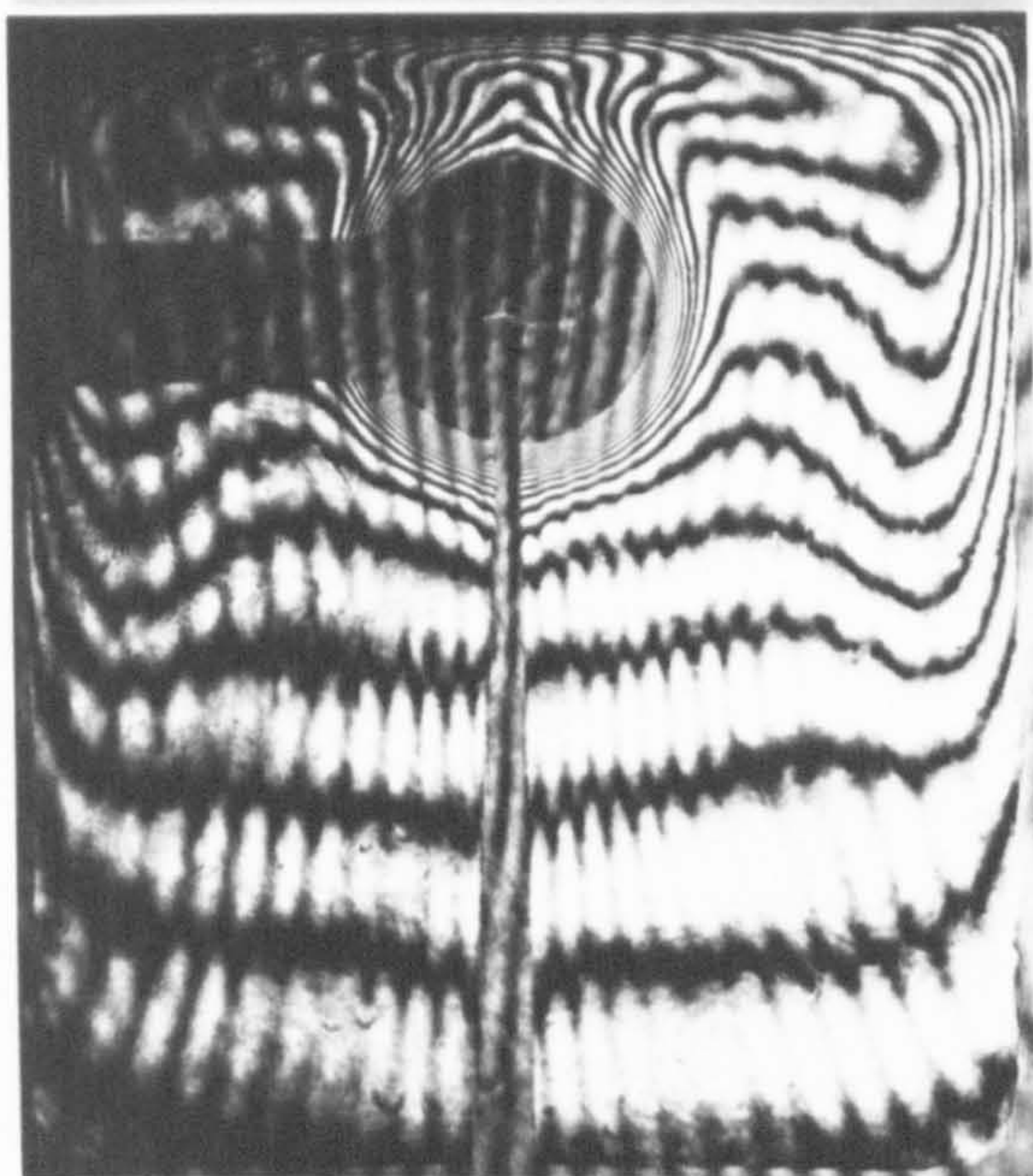
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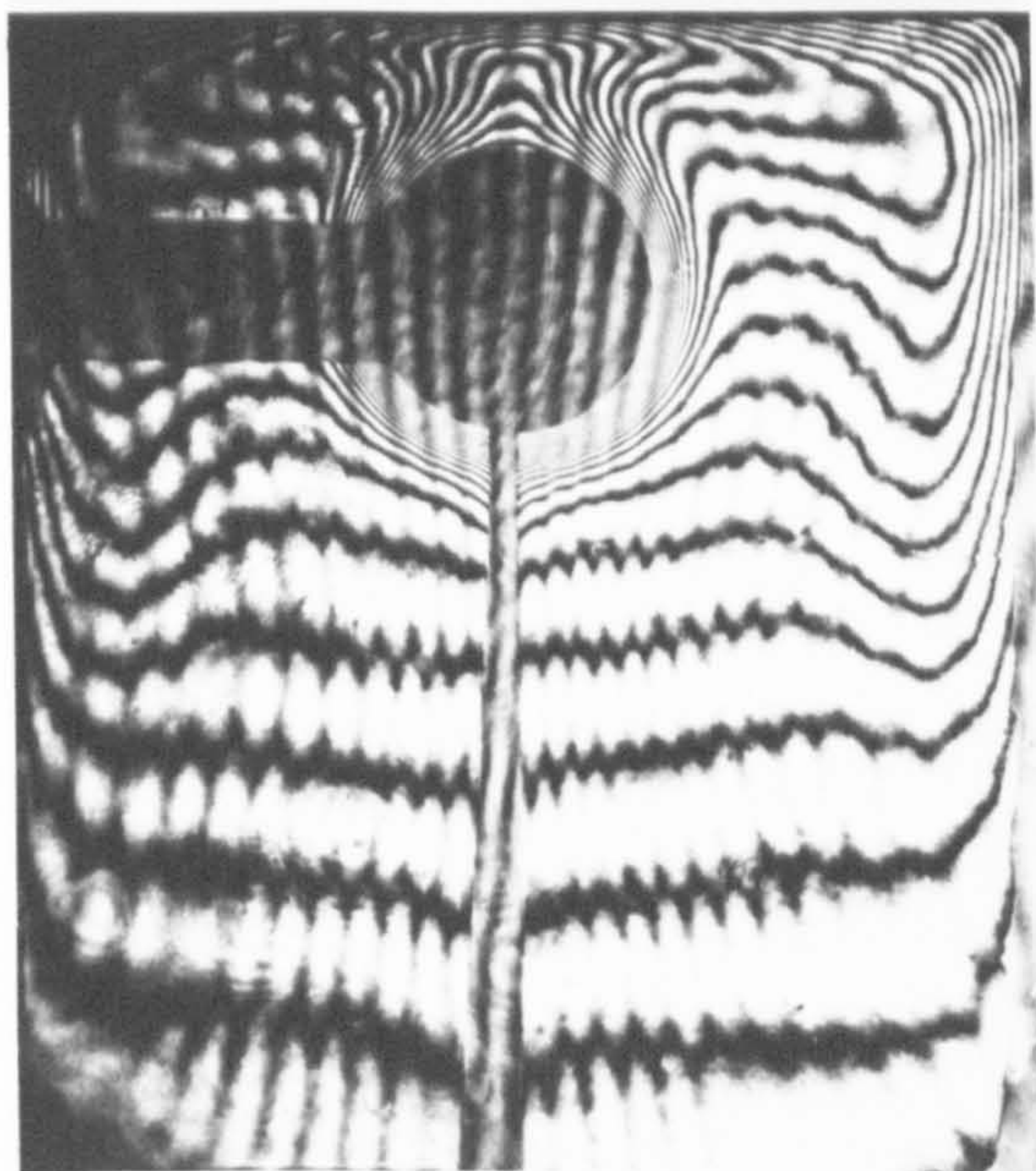
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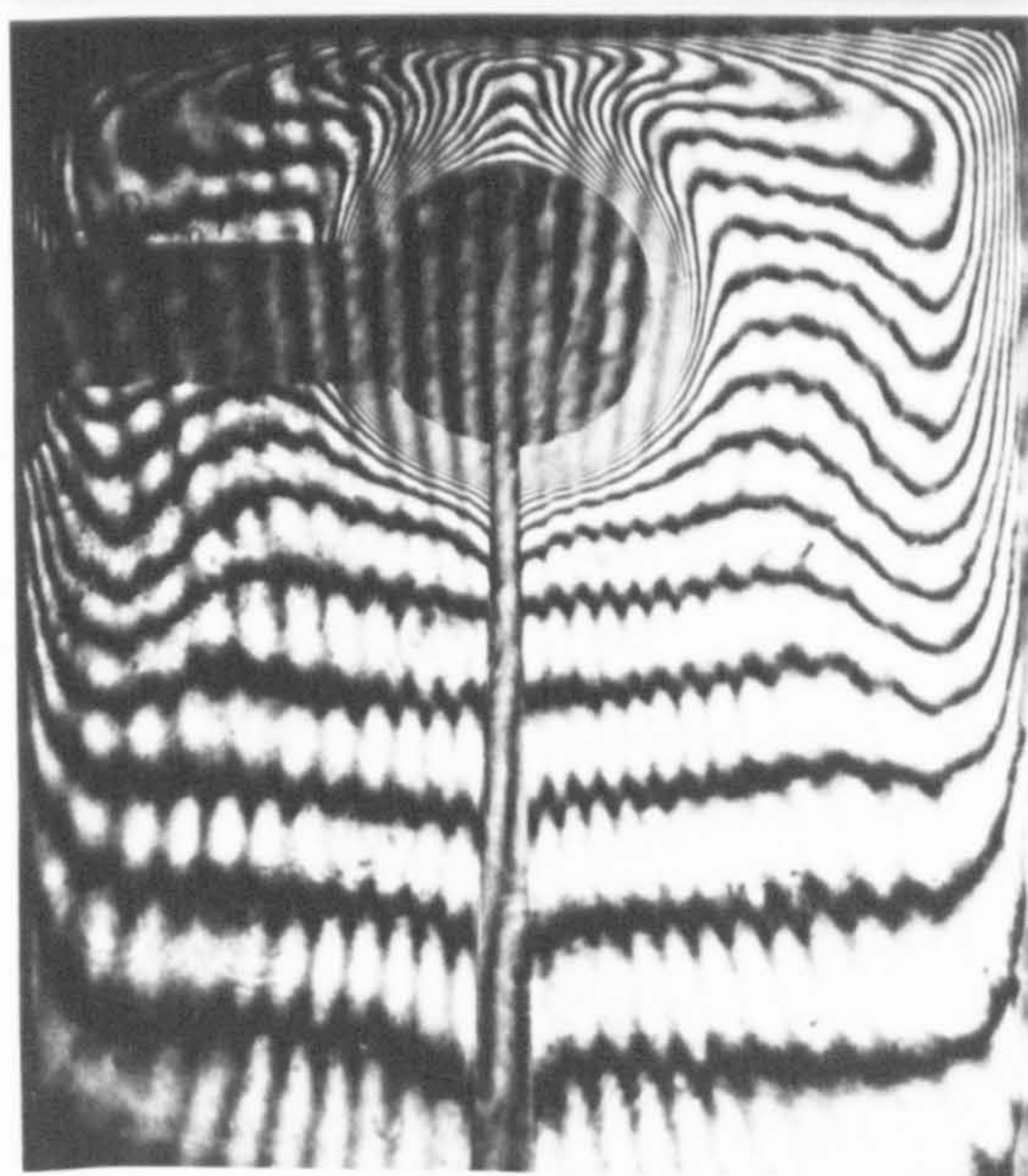
75



76



77



Displacement Ratio = 0.7, Spacer Angle = 50 Degrees

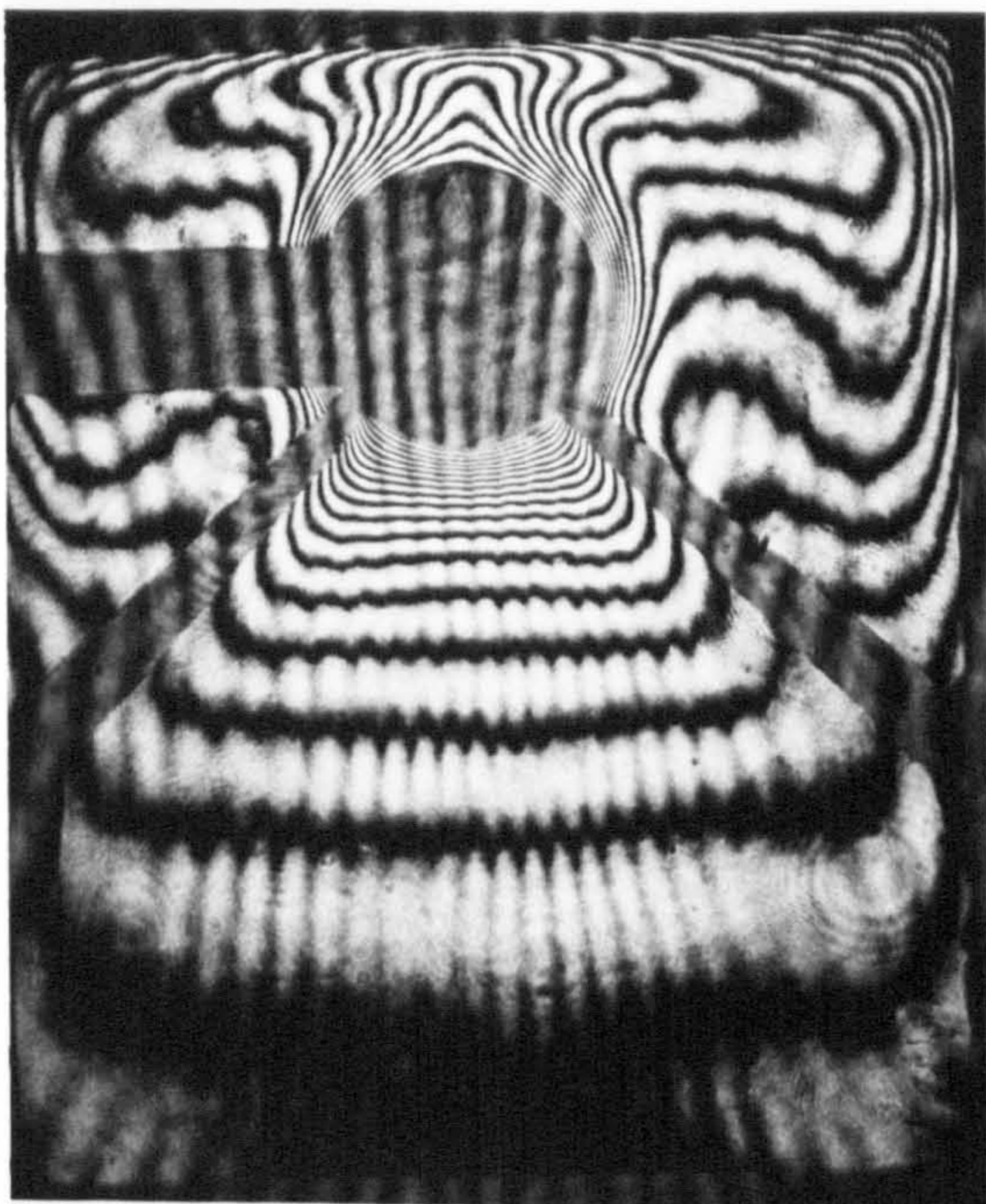
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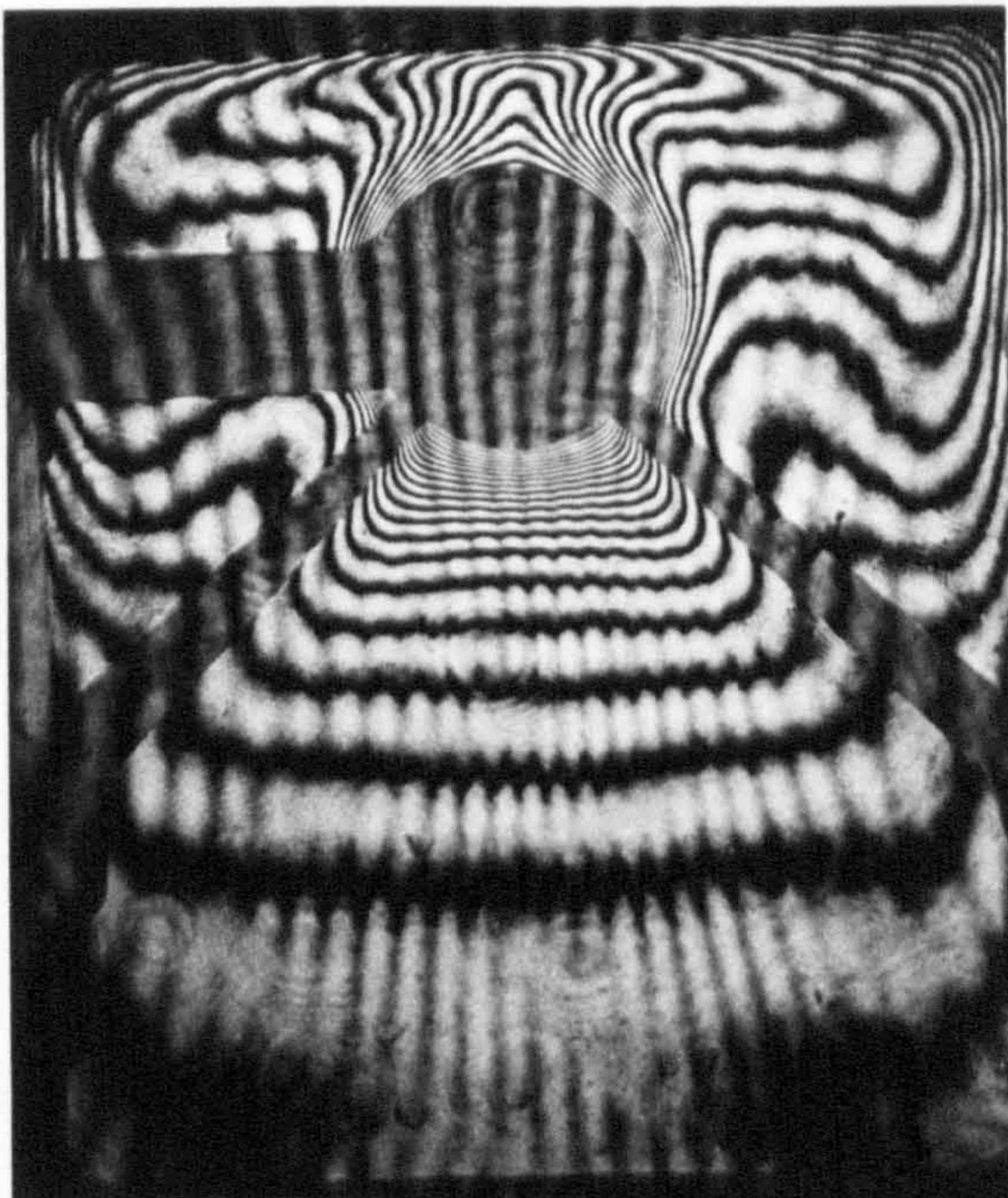
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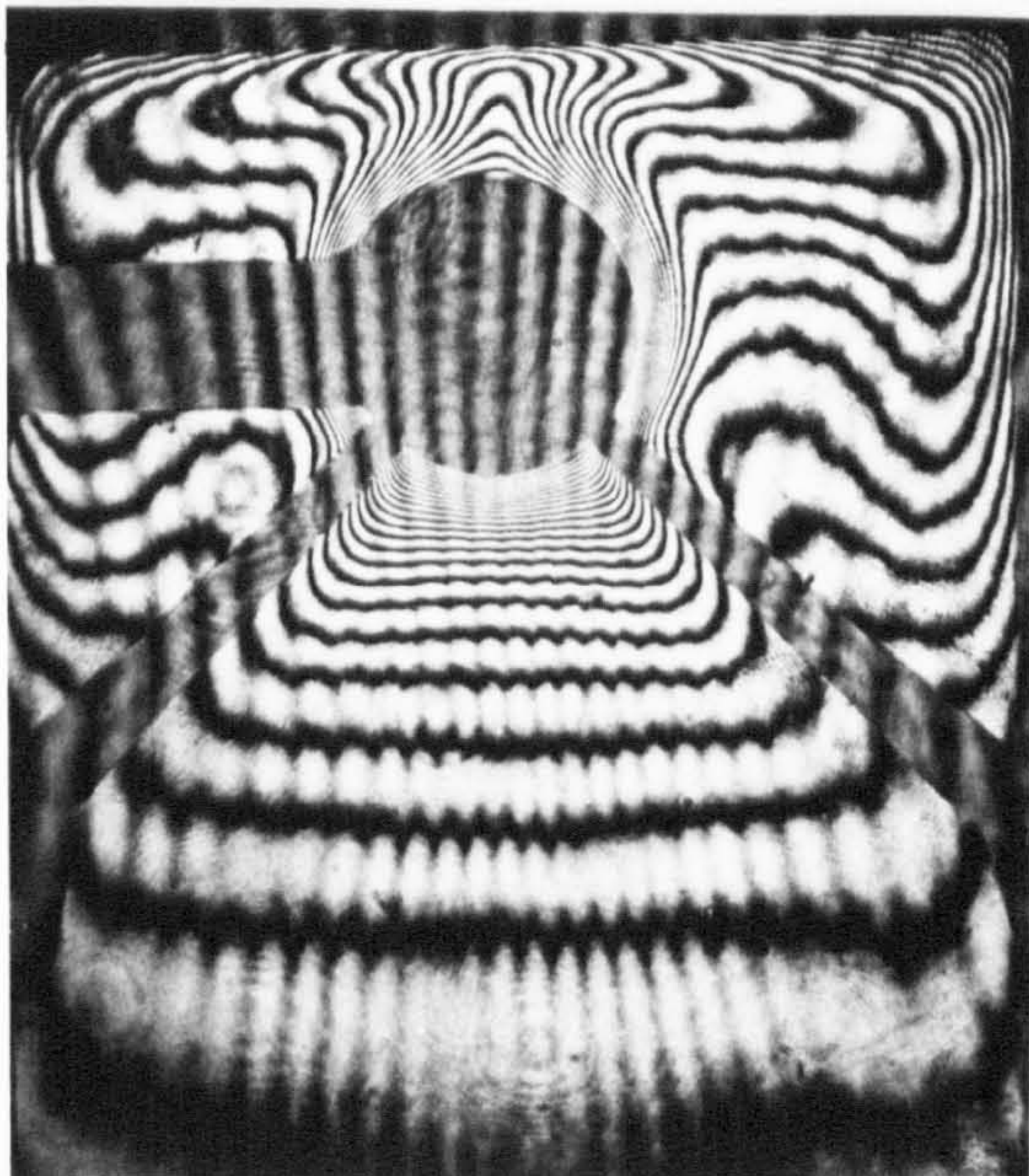
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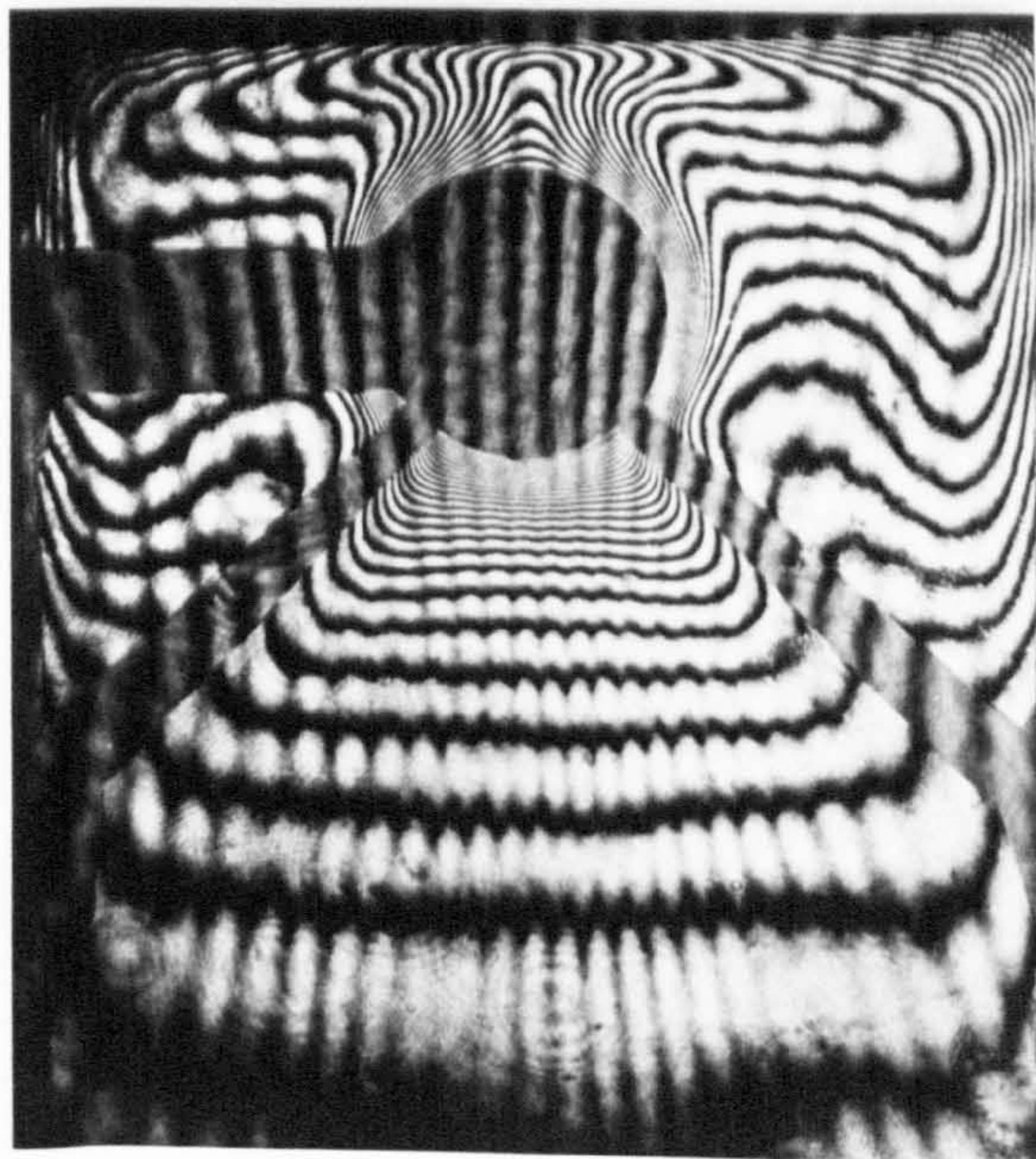
99



100



101



Displacement Ratio = 0.7, Spacer Angle = 90 Degrees

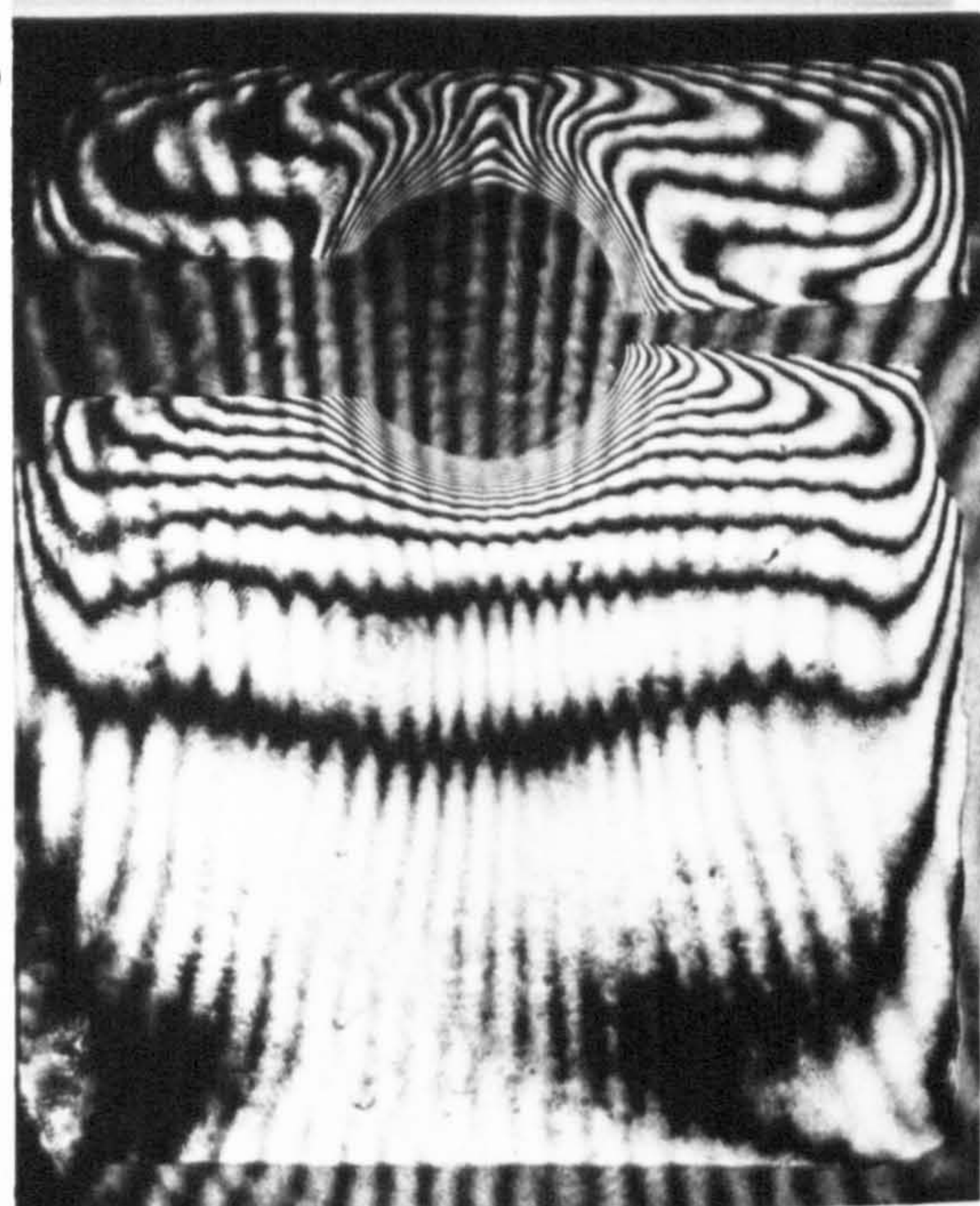
114



115



116



117



118

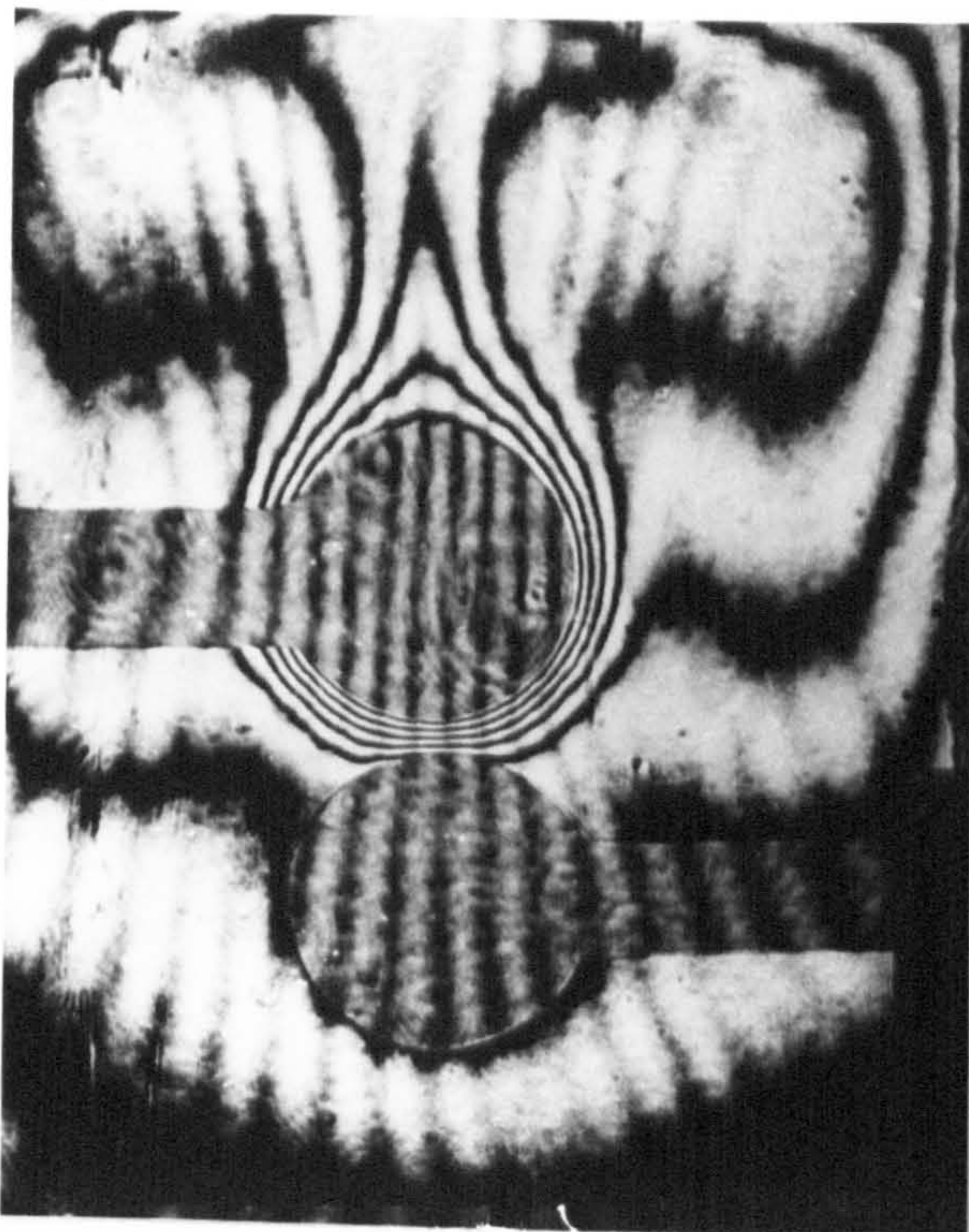


119

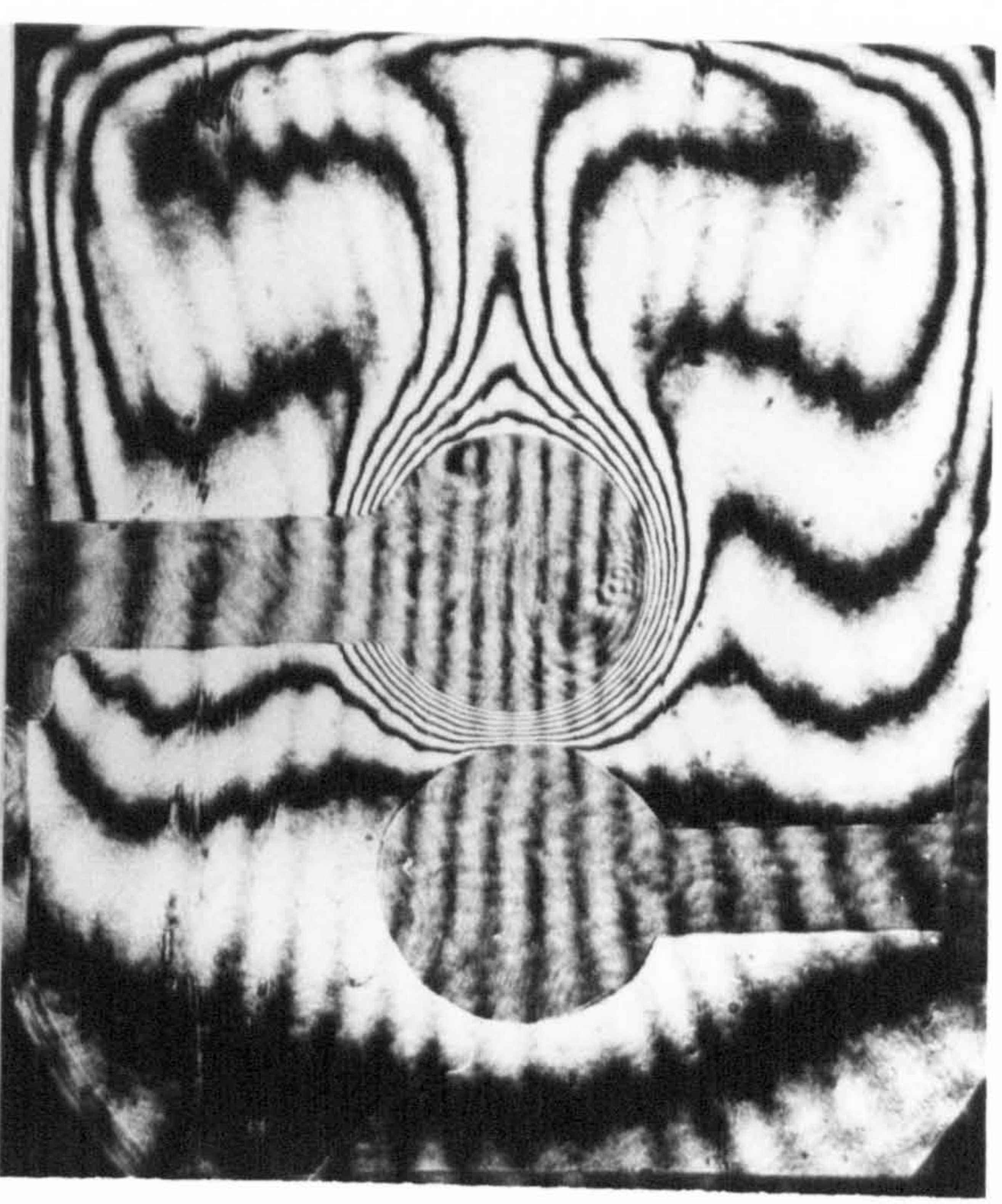


$E_s = 0$
 $E_r = -0.68$

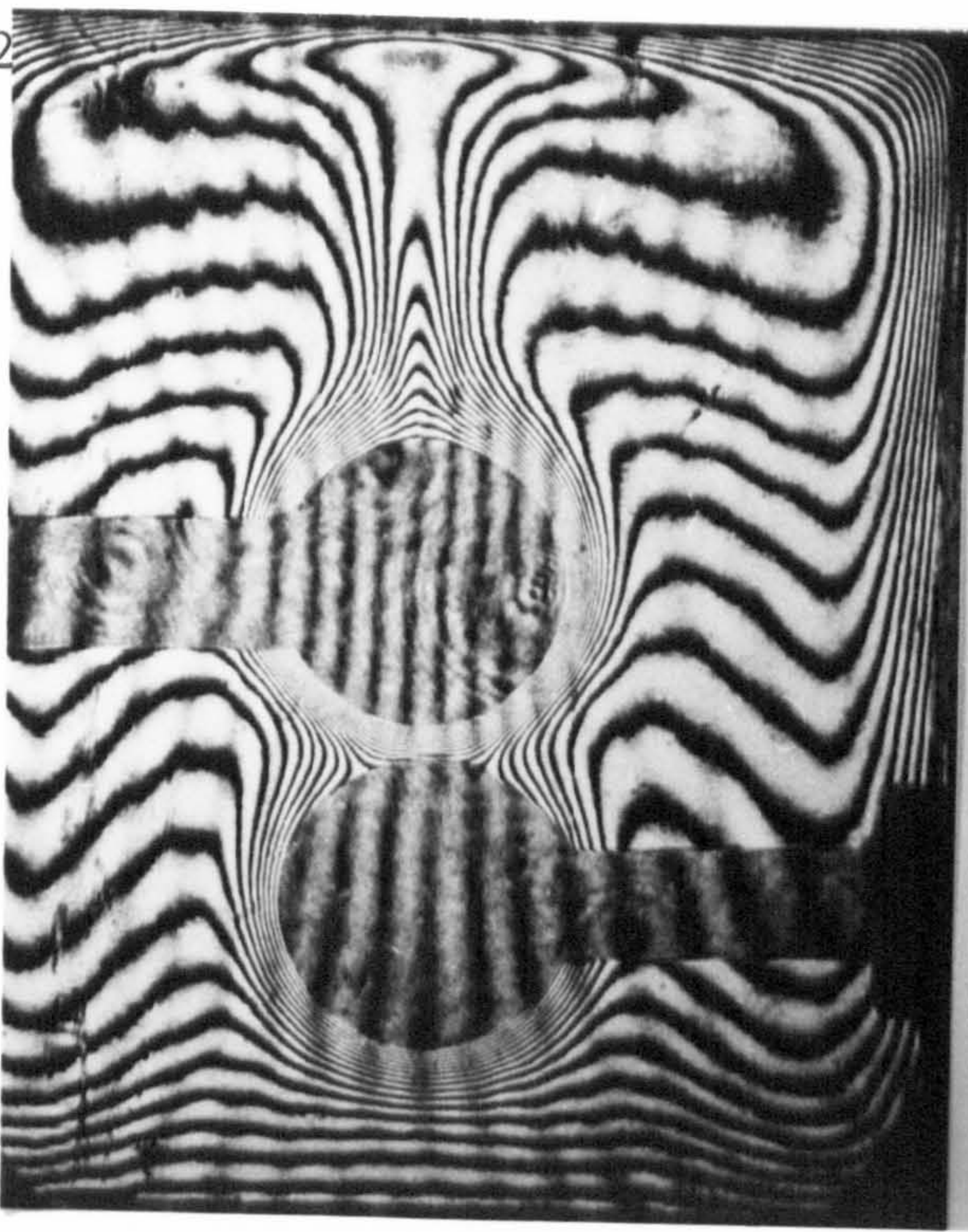
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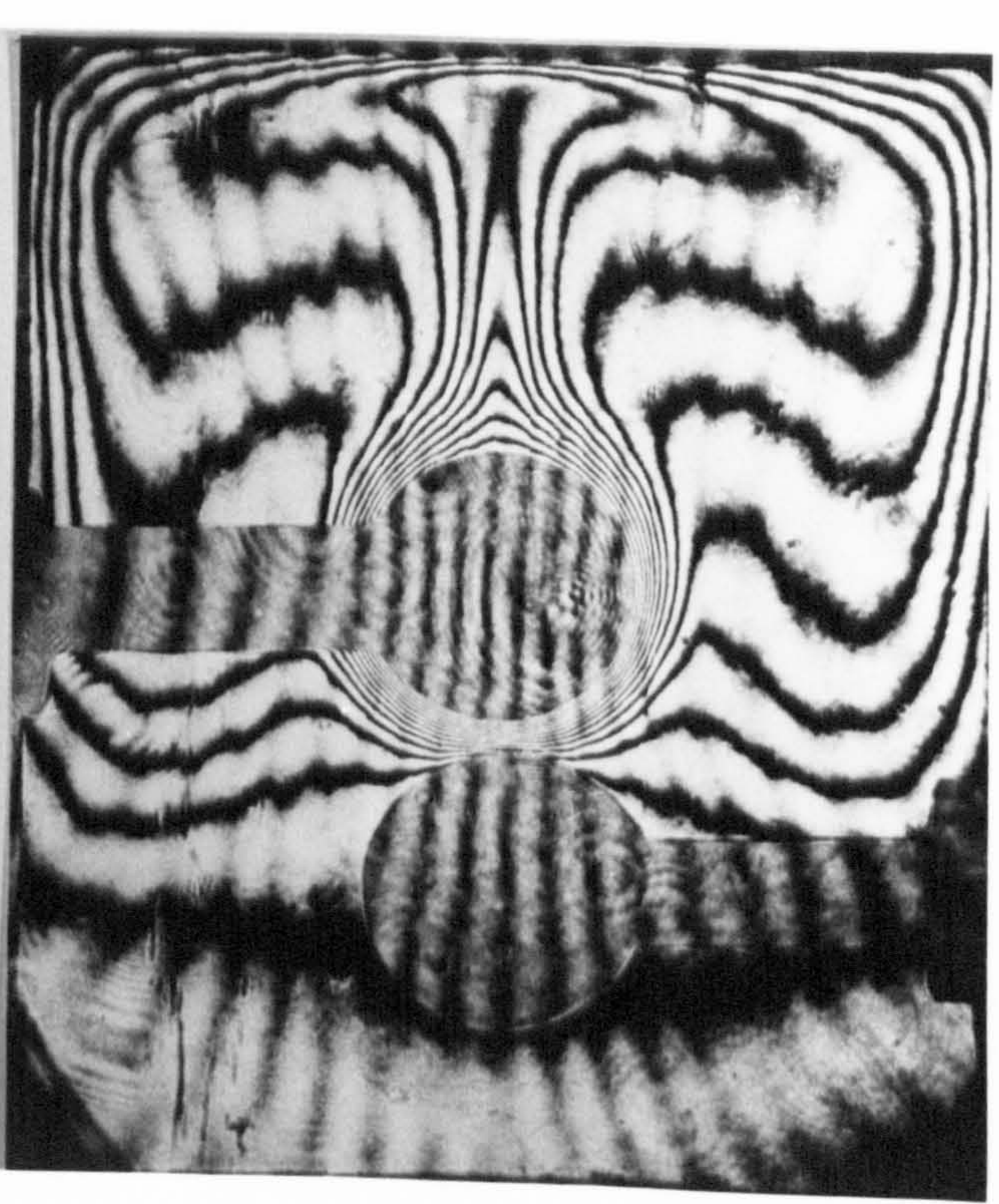
121



122



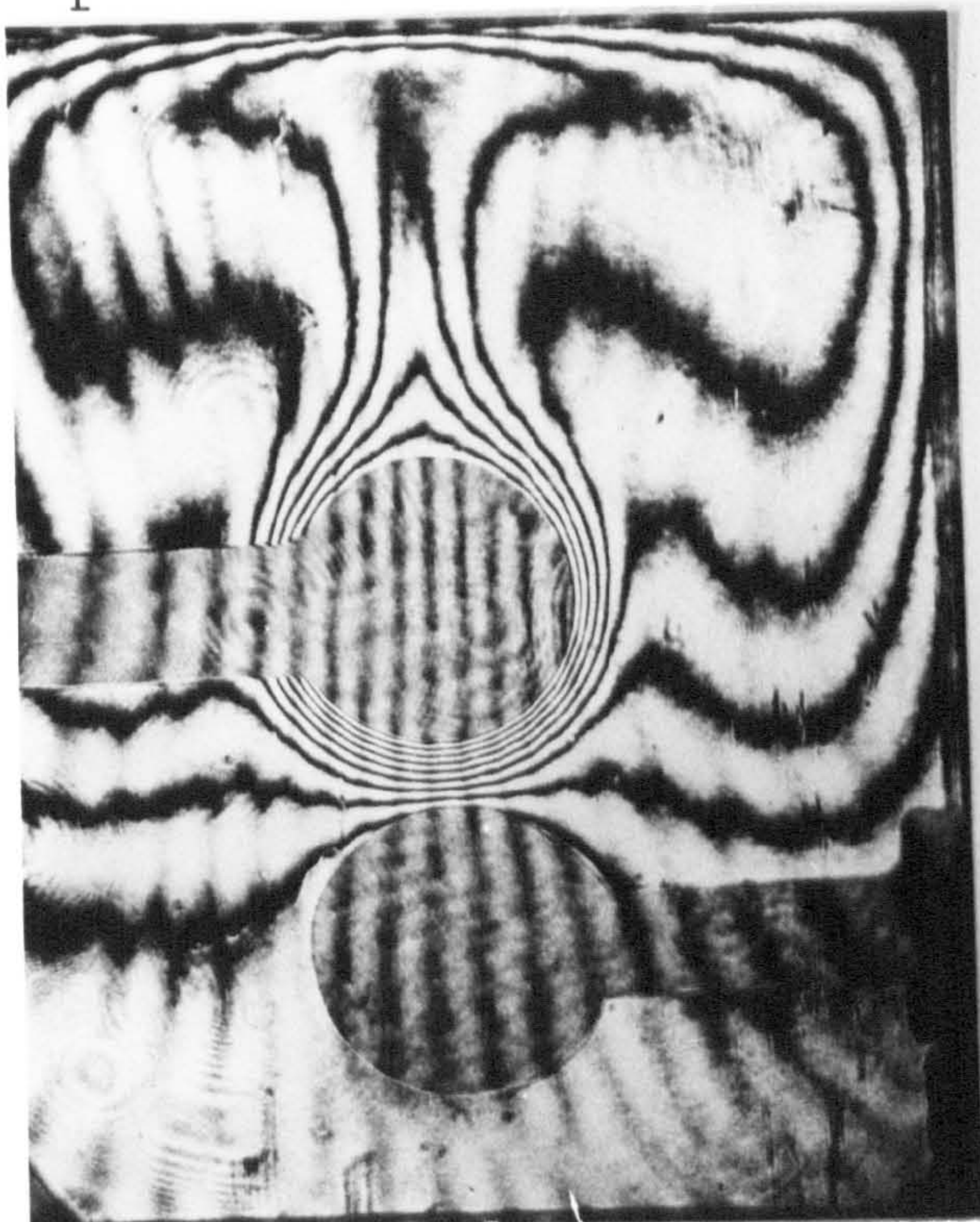
123



$$E_s = 0$$

$$E_r = -0.76$$

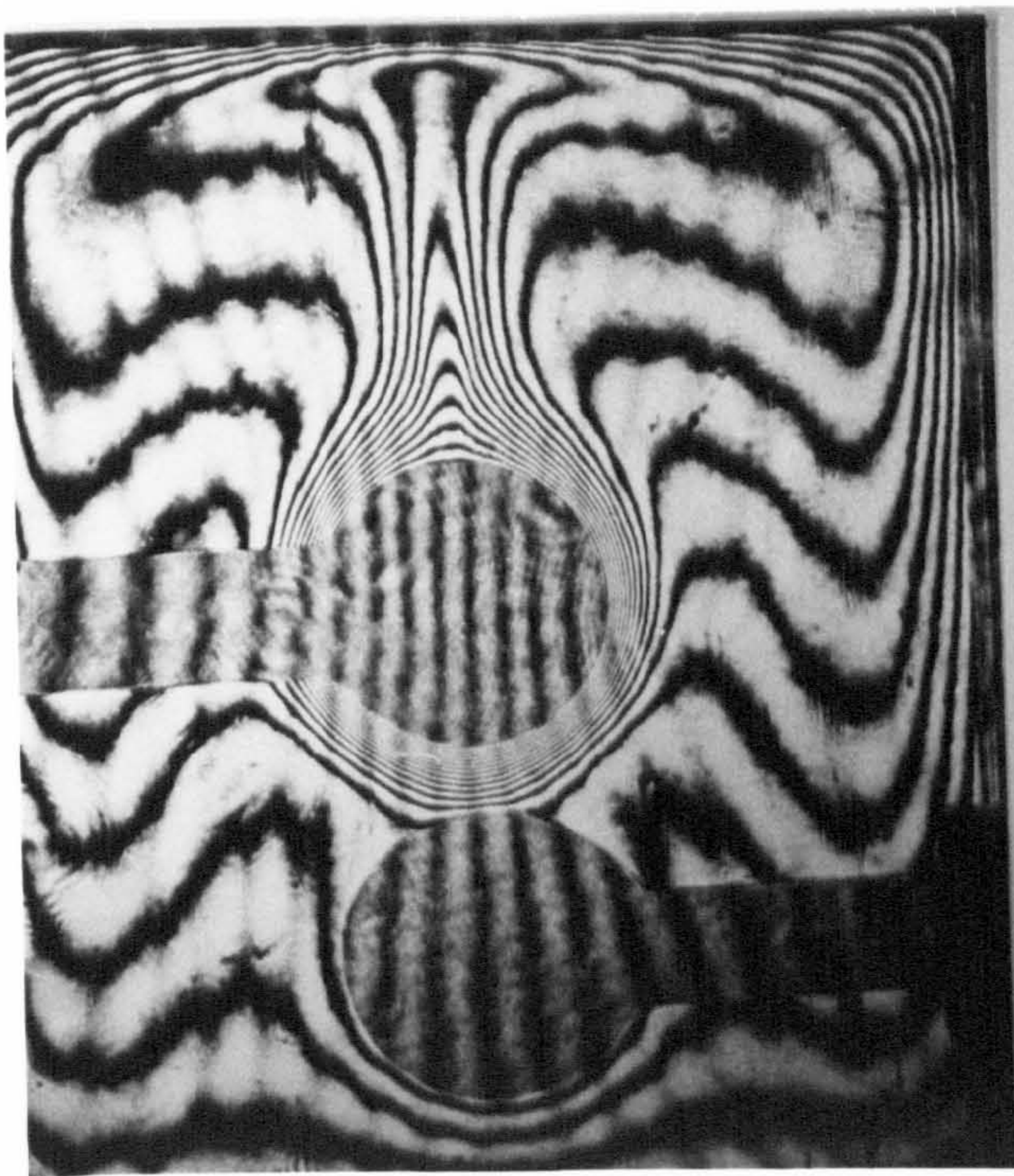
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125



126



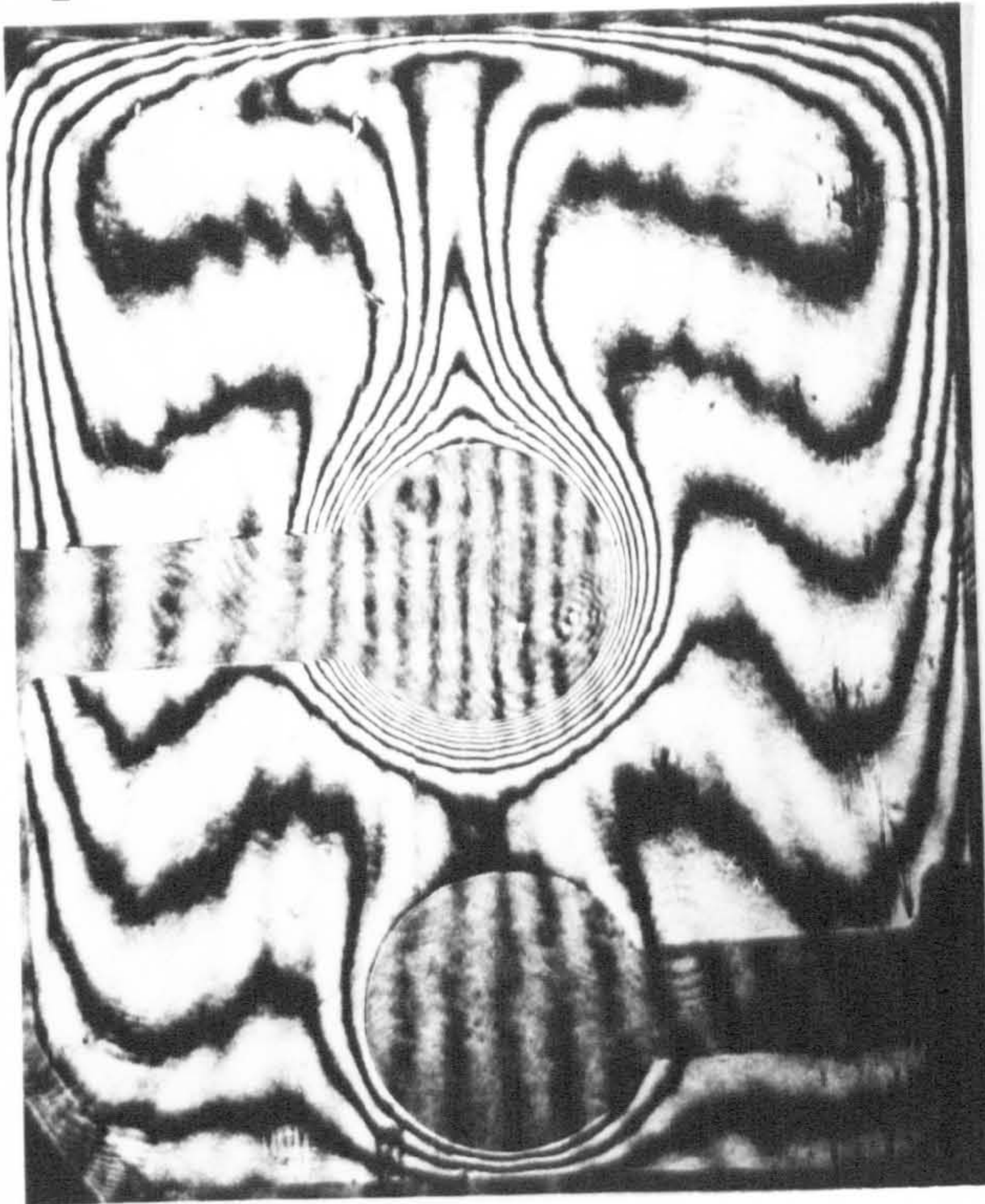
127



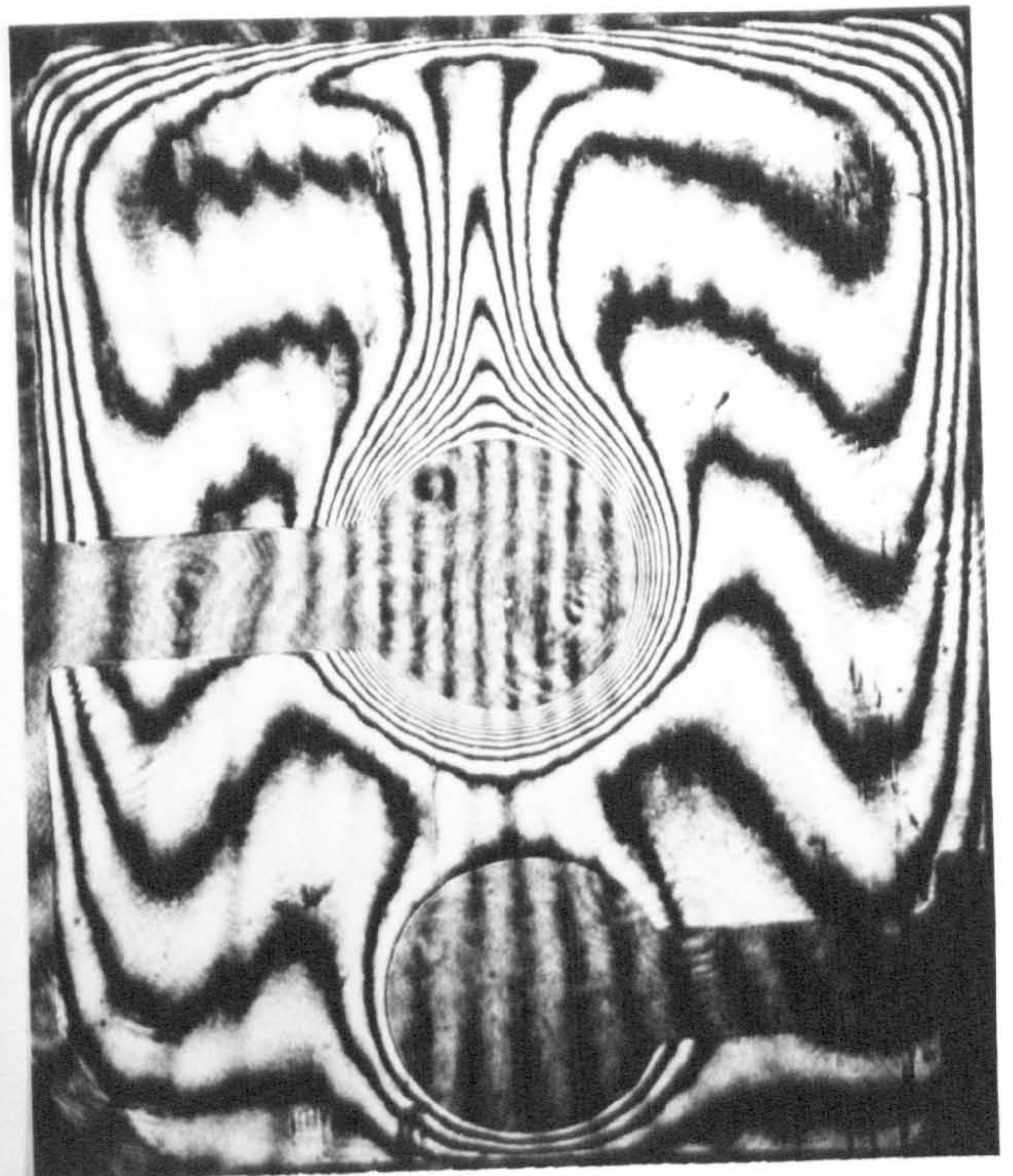
$$E_s = 0$$

$$E_r = -0.98$$

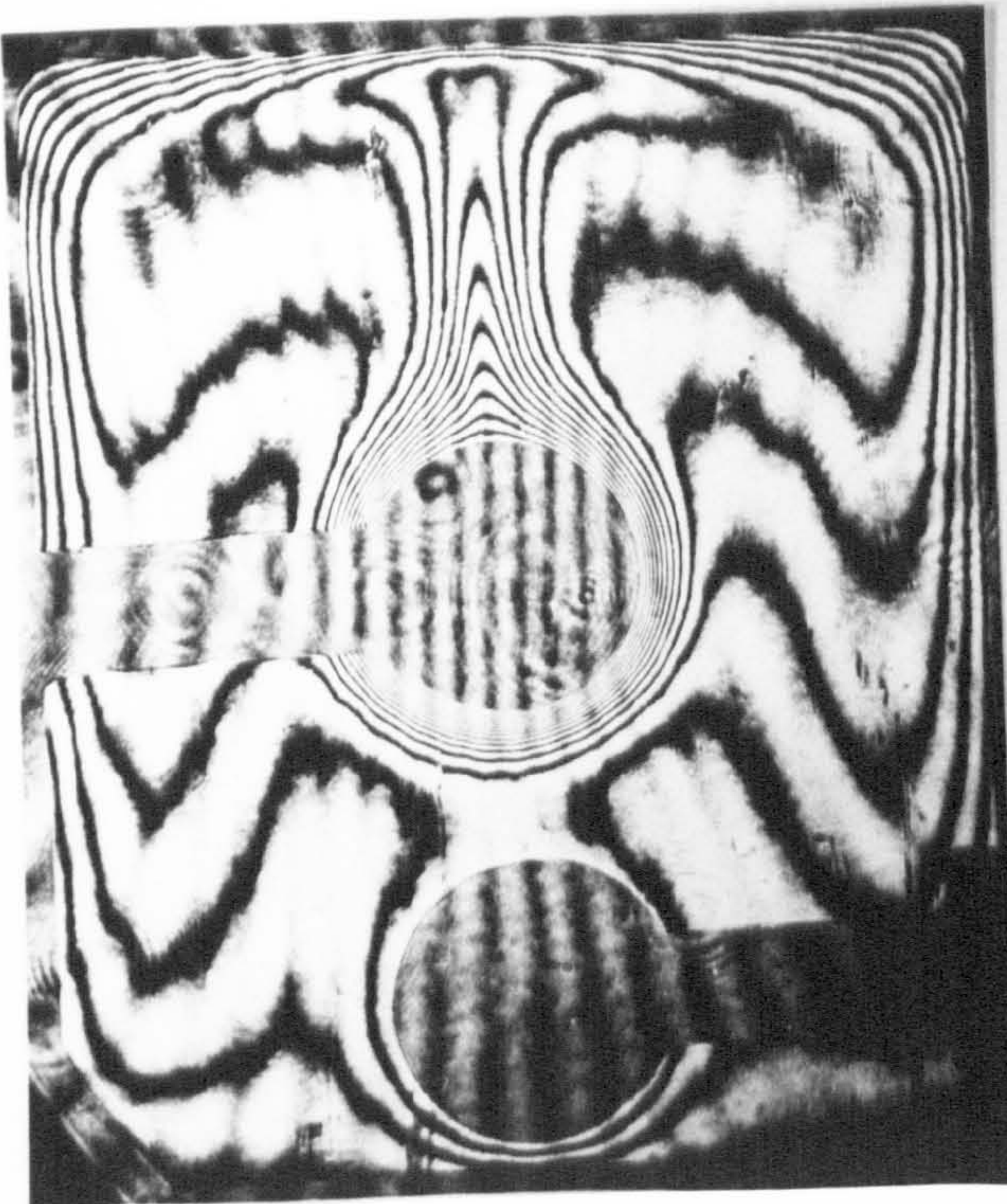
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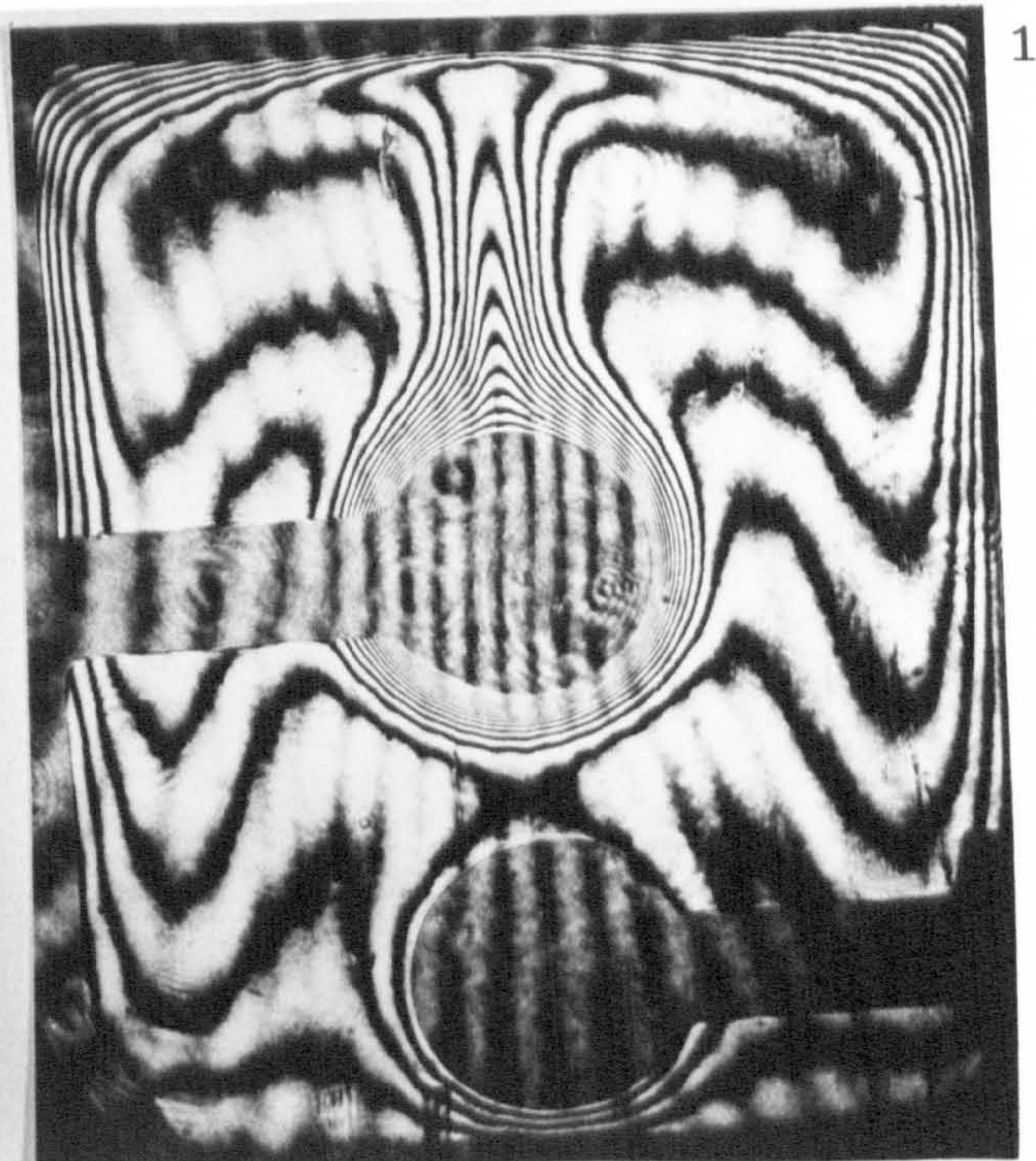
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138



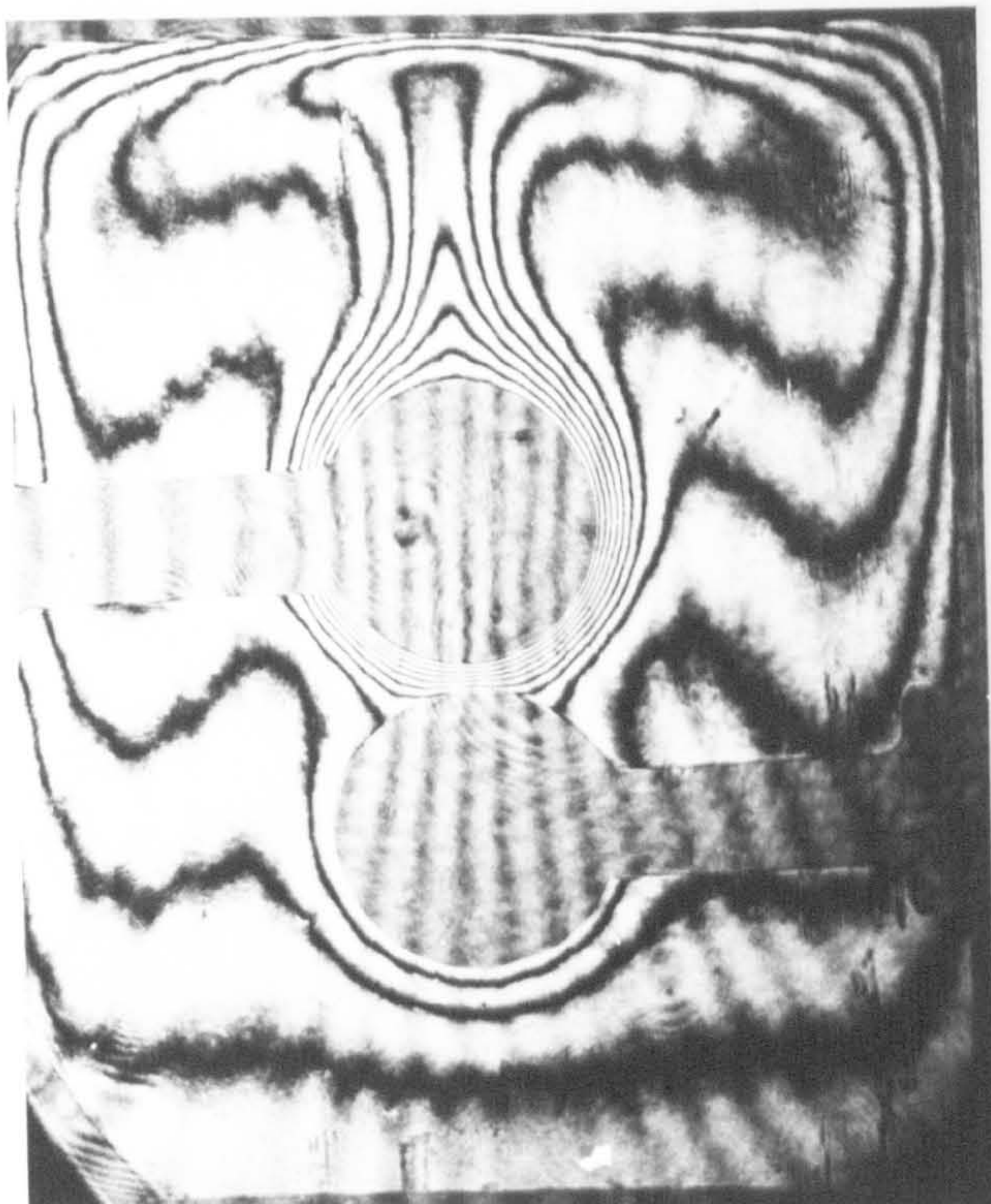
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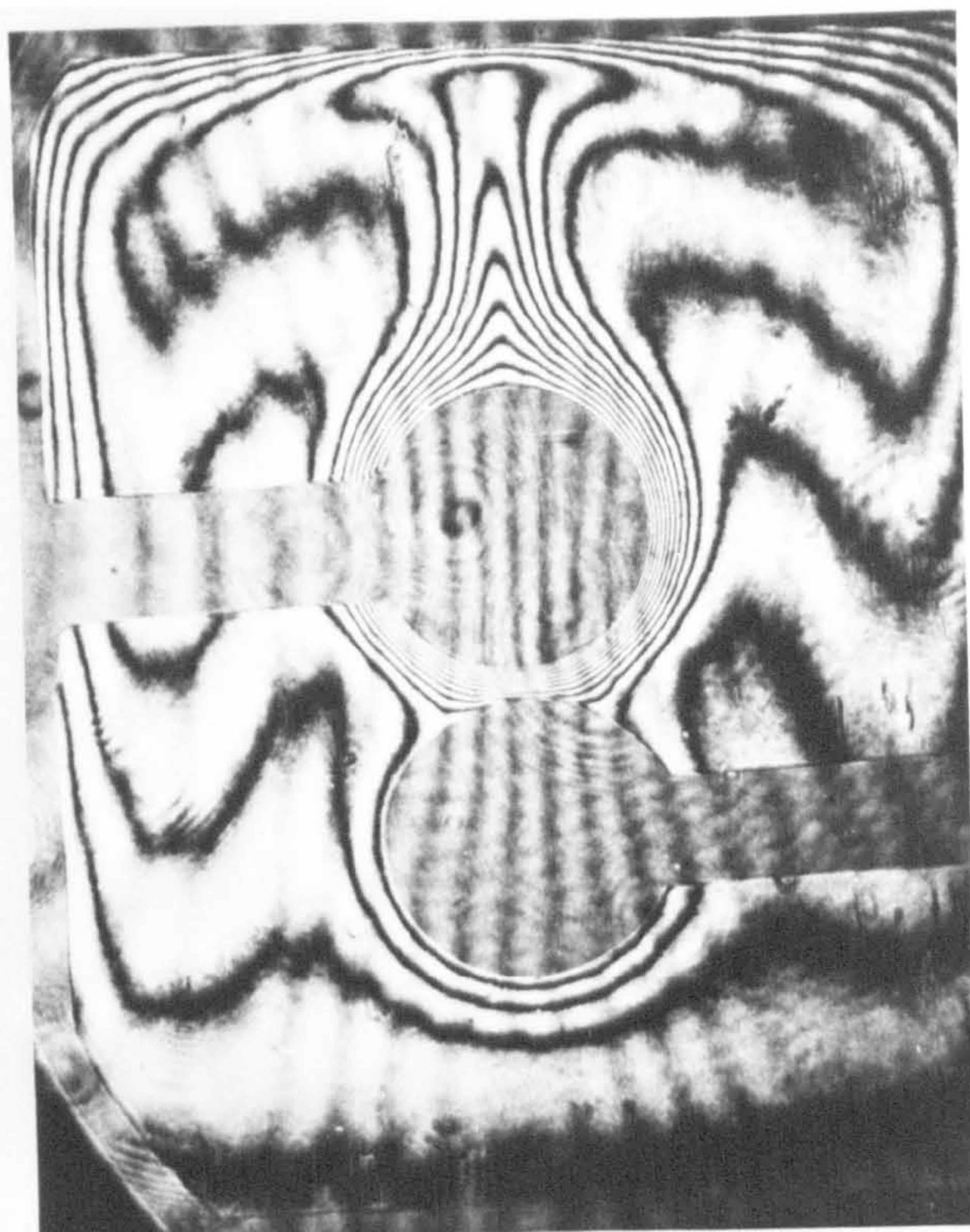
$$E_s = +0.25$$

$$E_r = -0.50$$

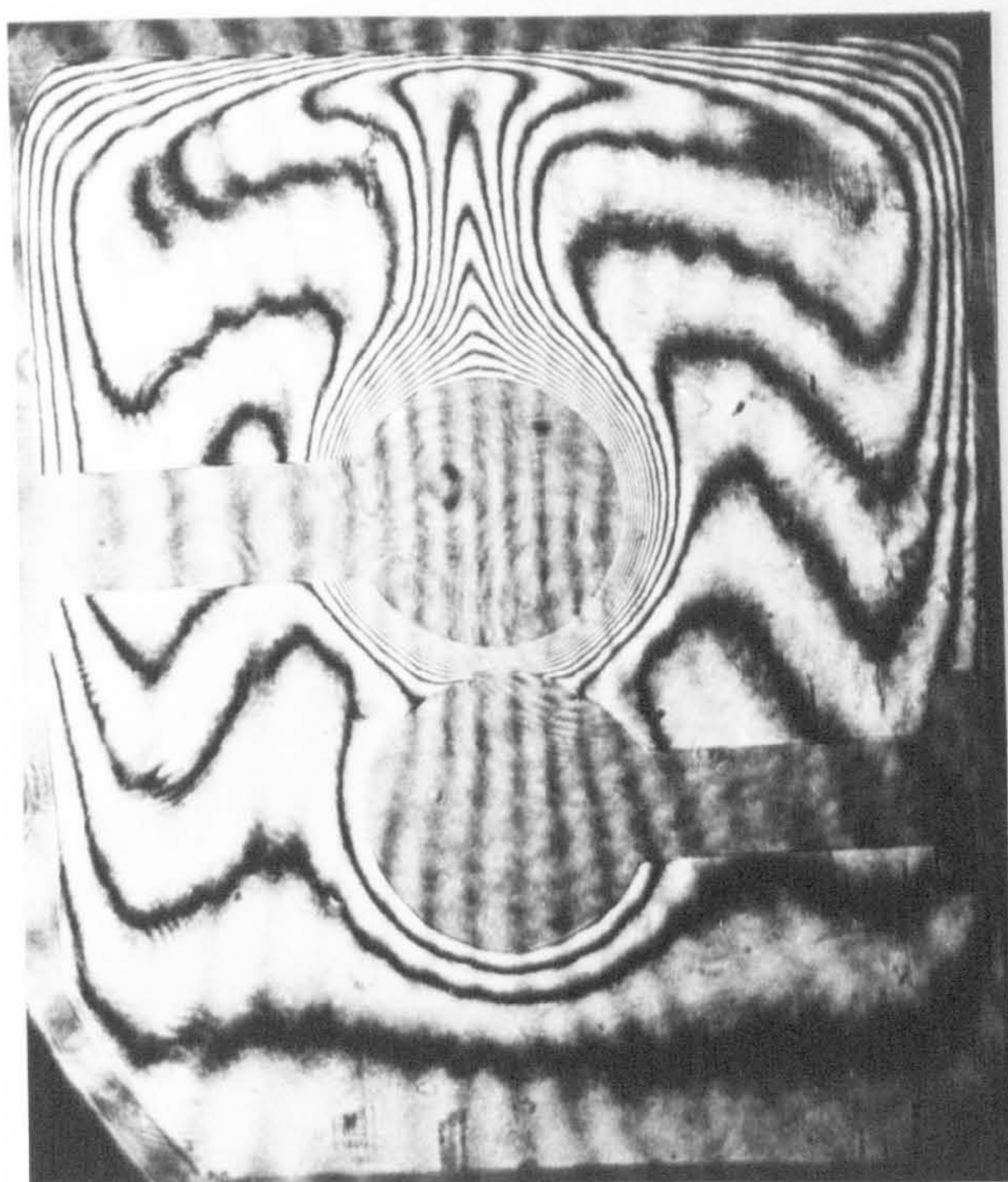
140



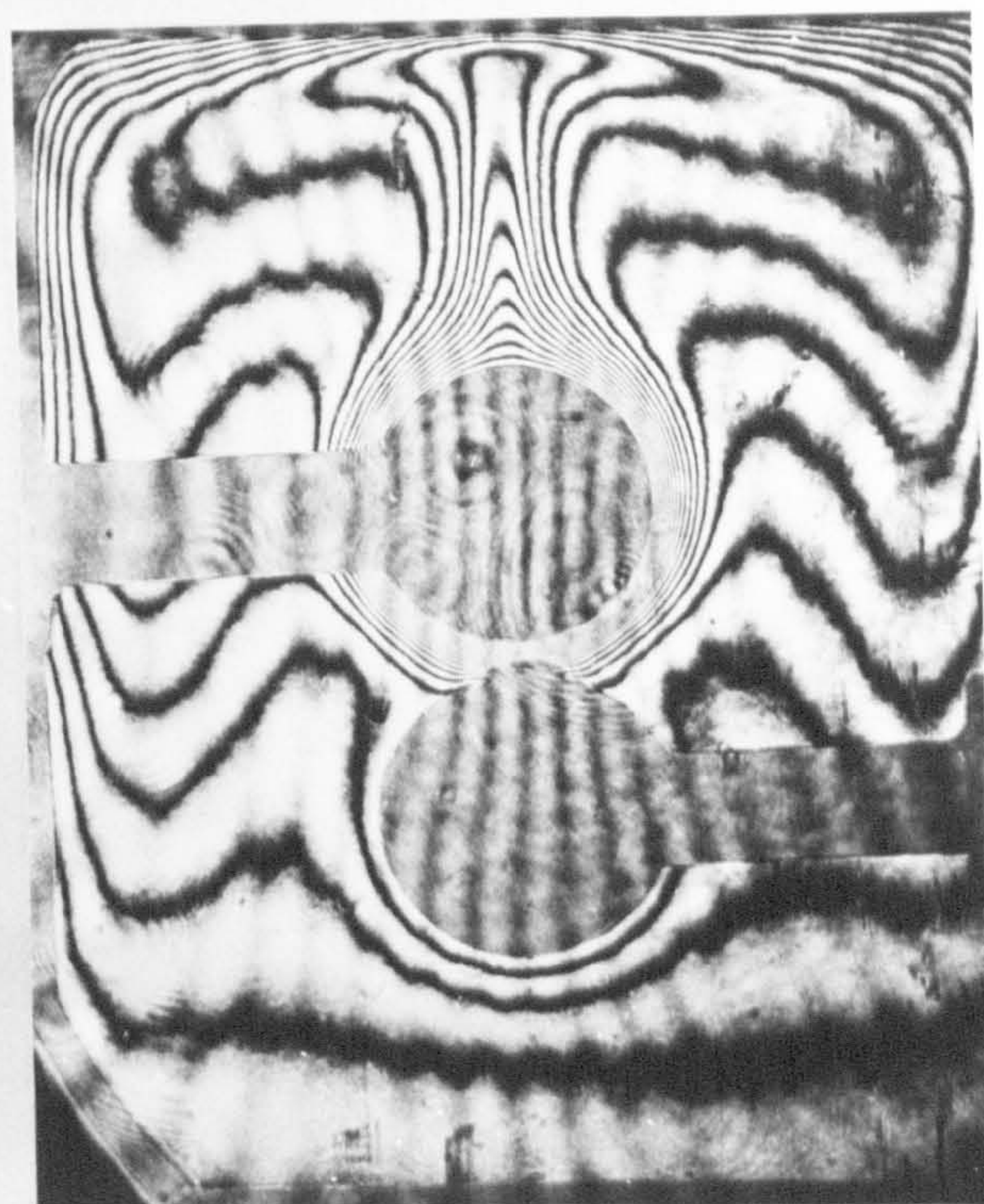
141



142



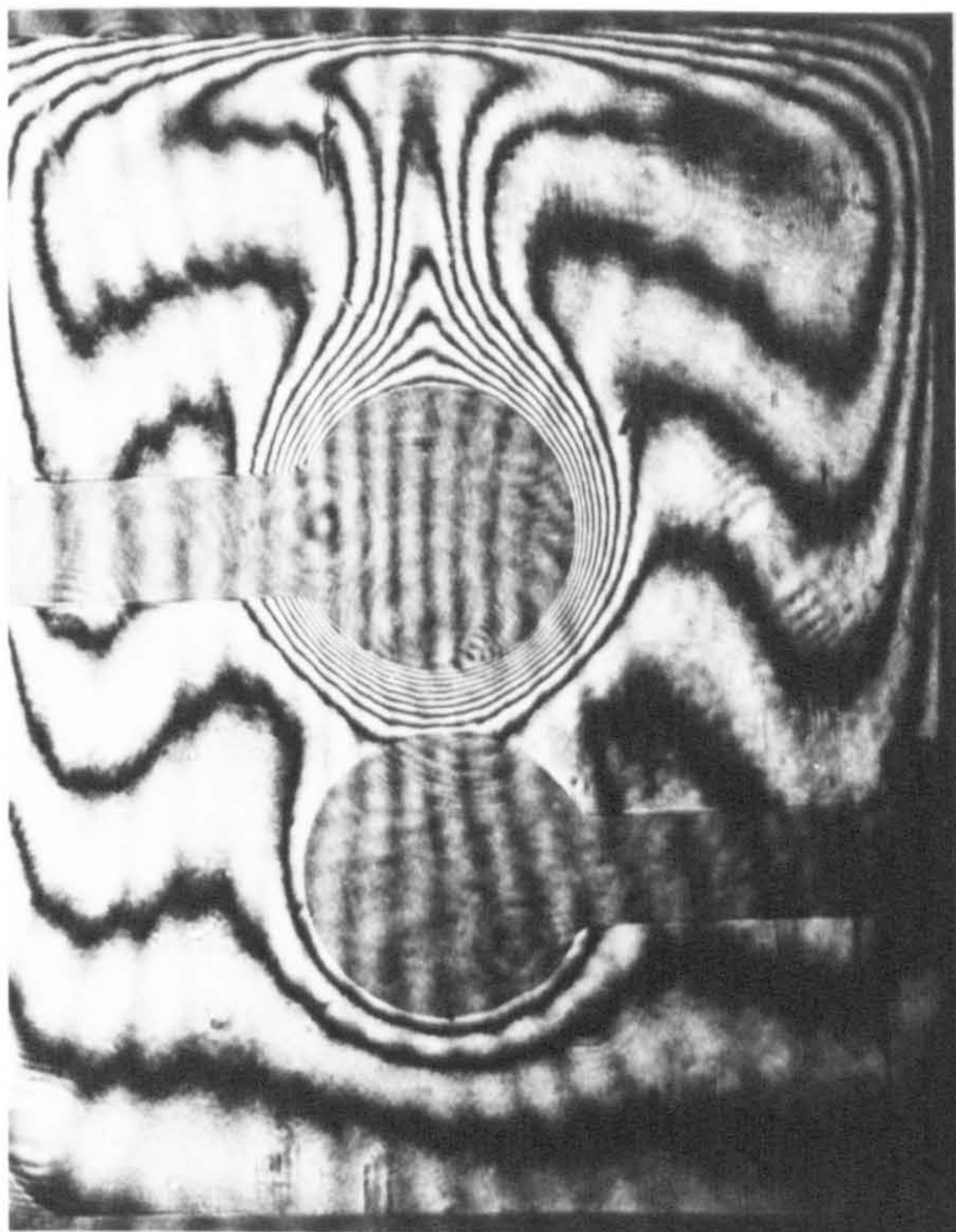
143



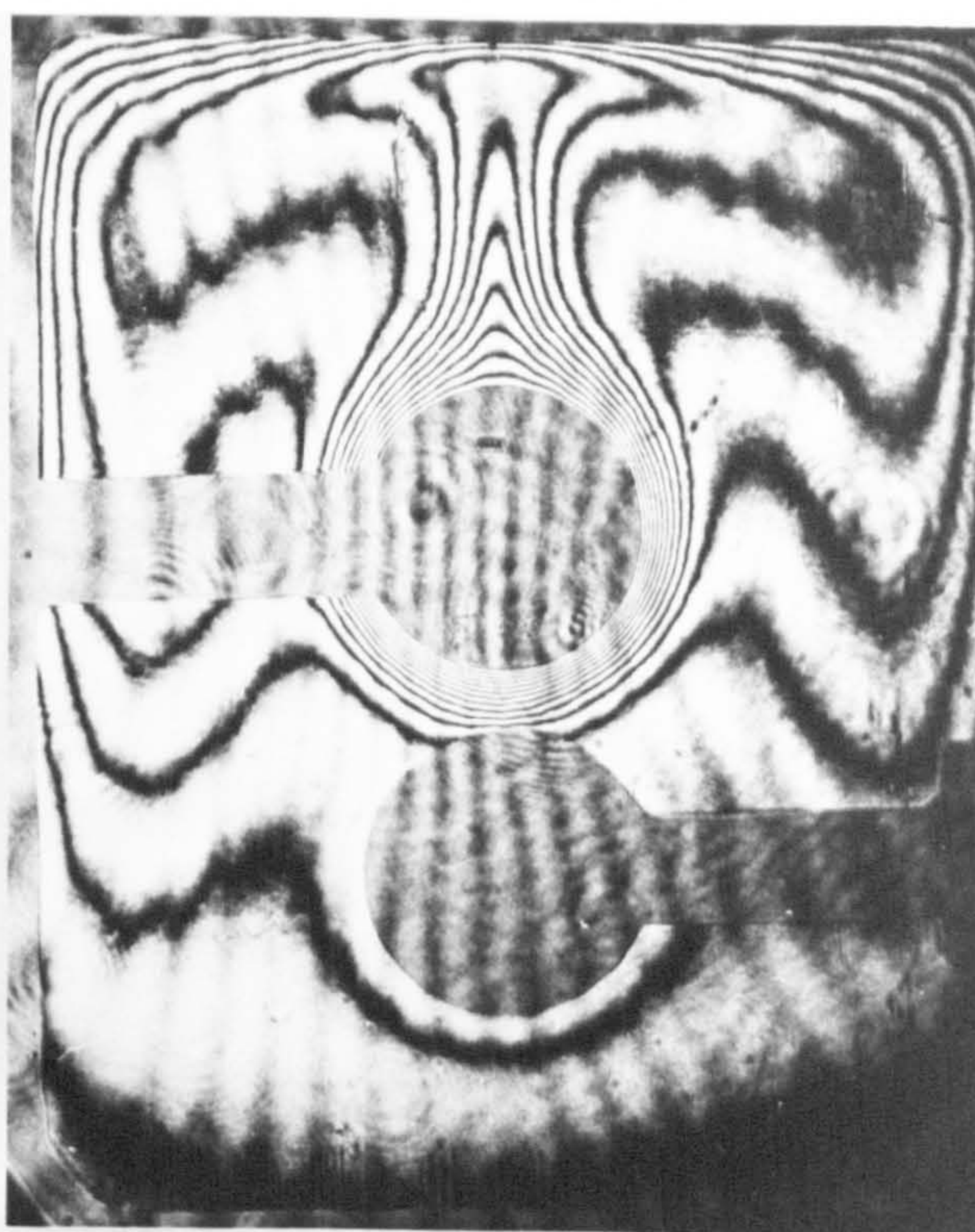
$$E_s = +0.25$$

$$E_r = -0.59$$

144



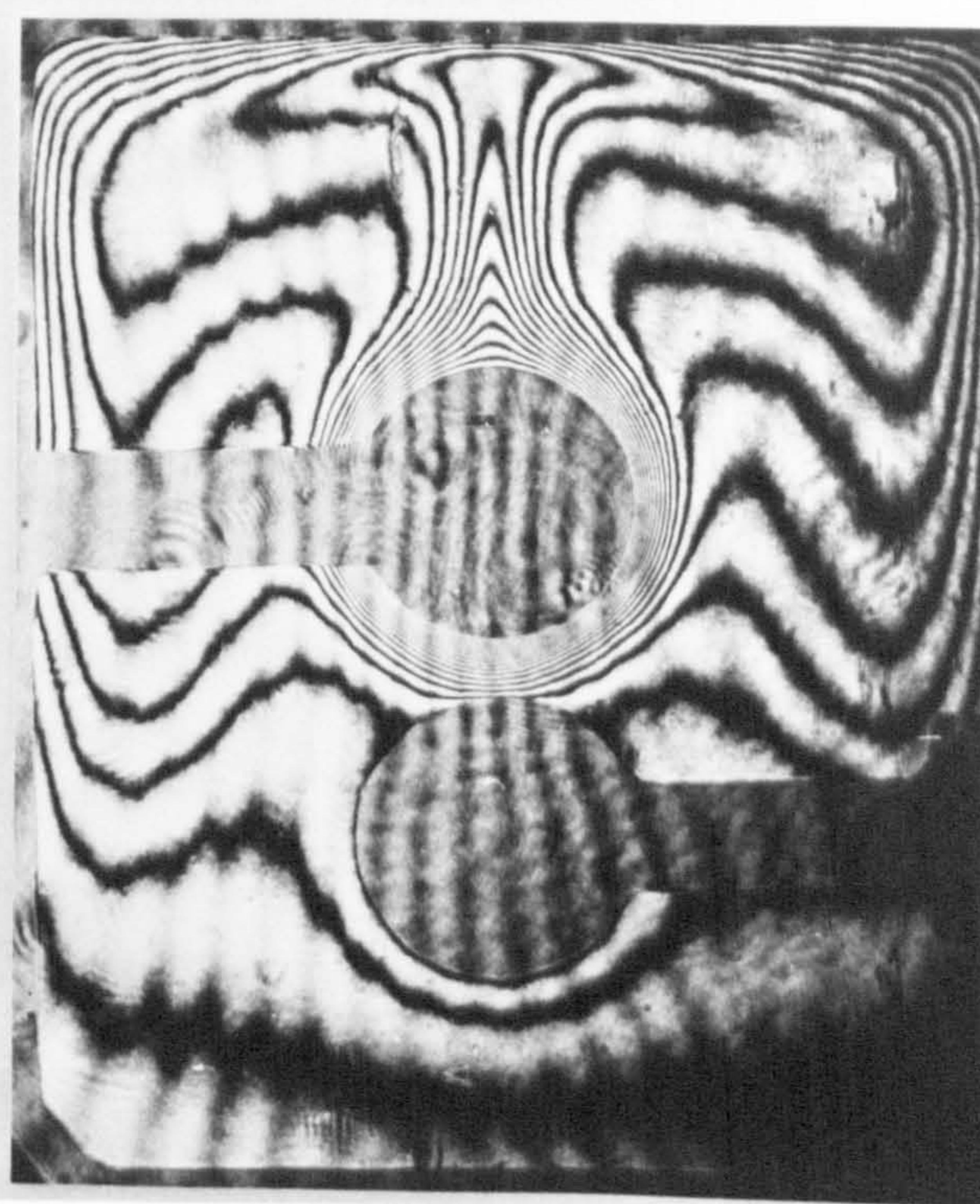
145



146



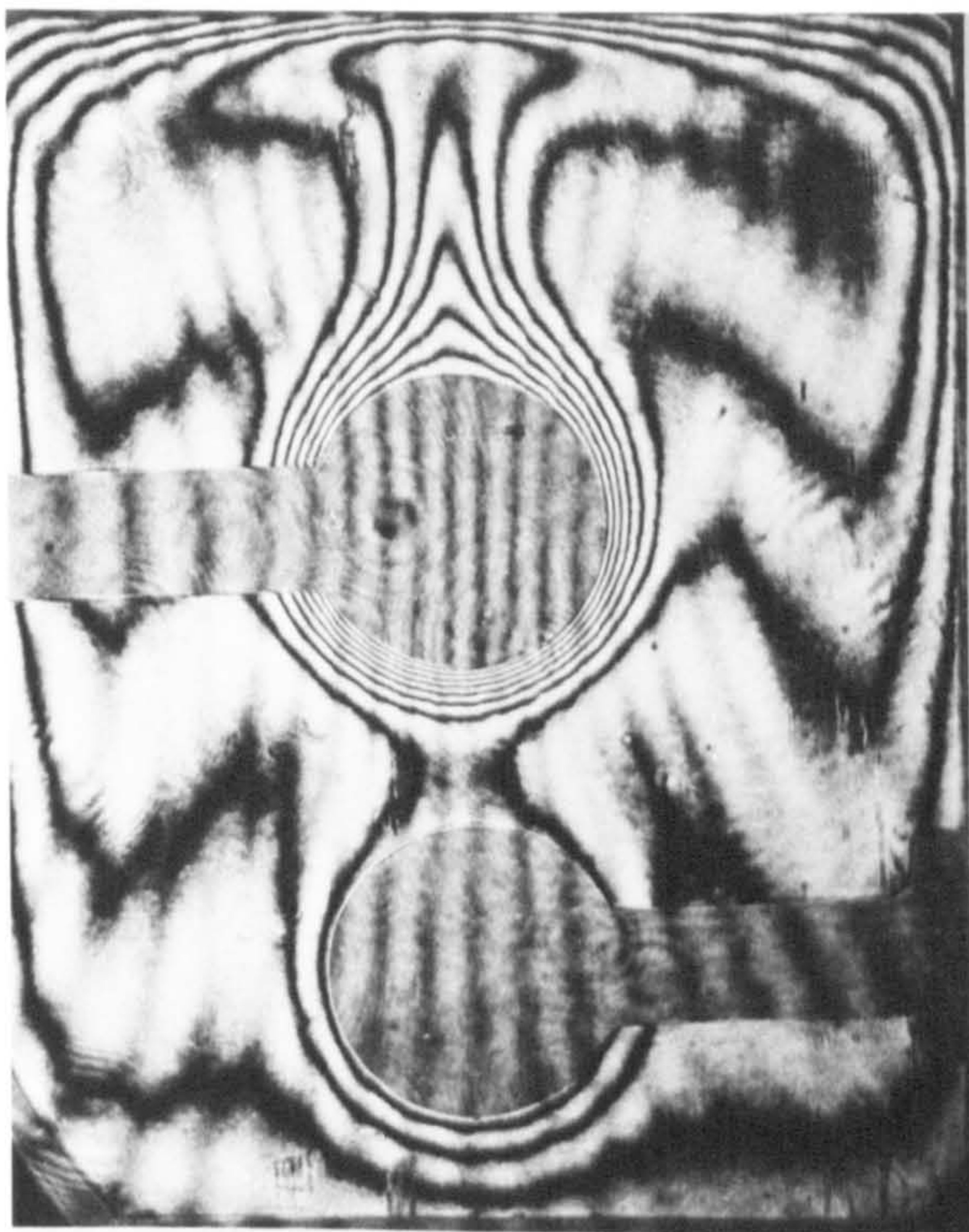
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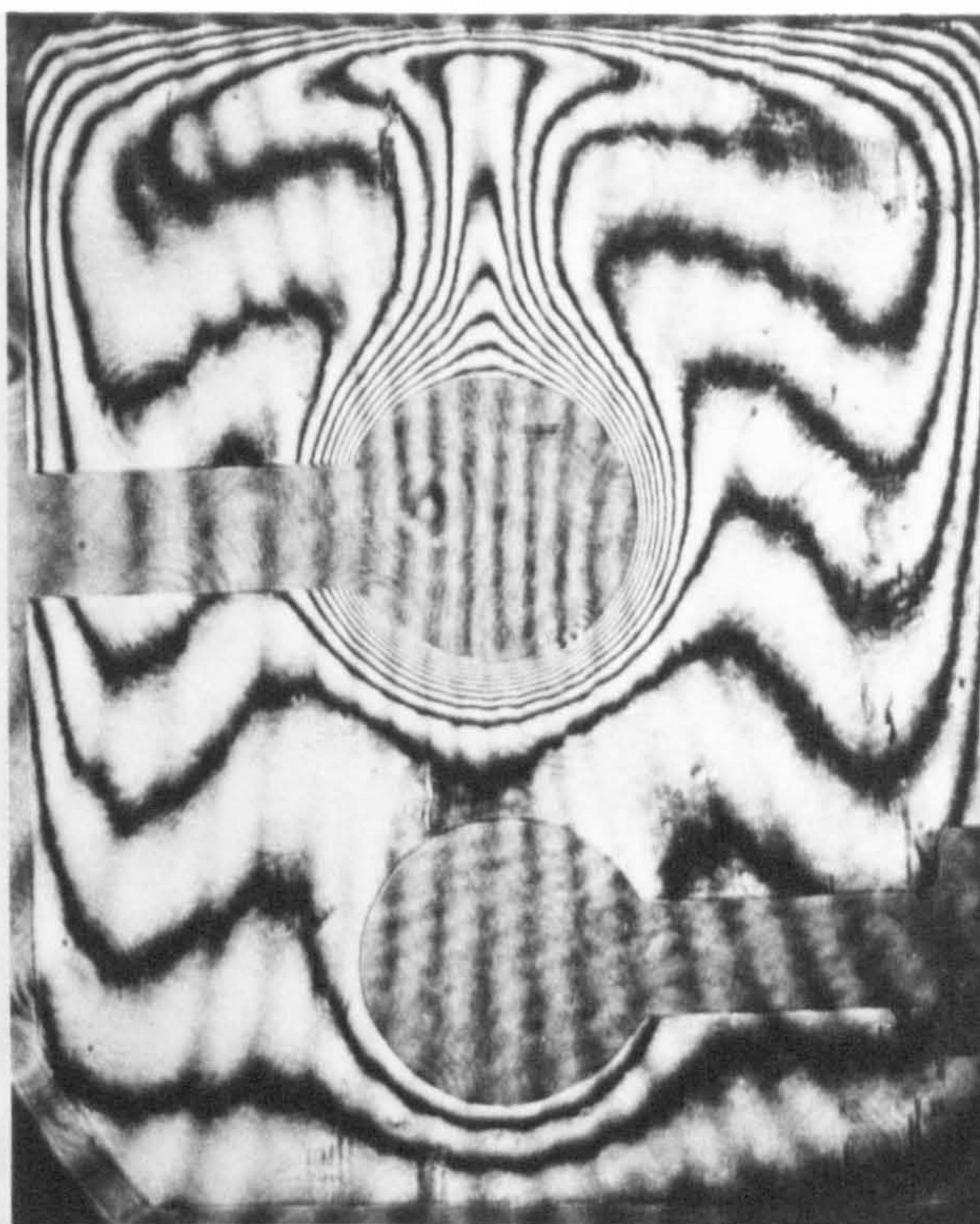
$$E_s = +0.25$$

$$E_r = -0.81$$

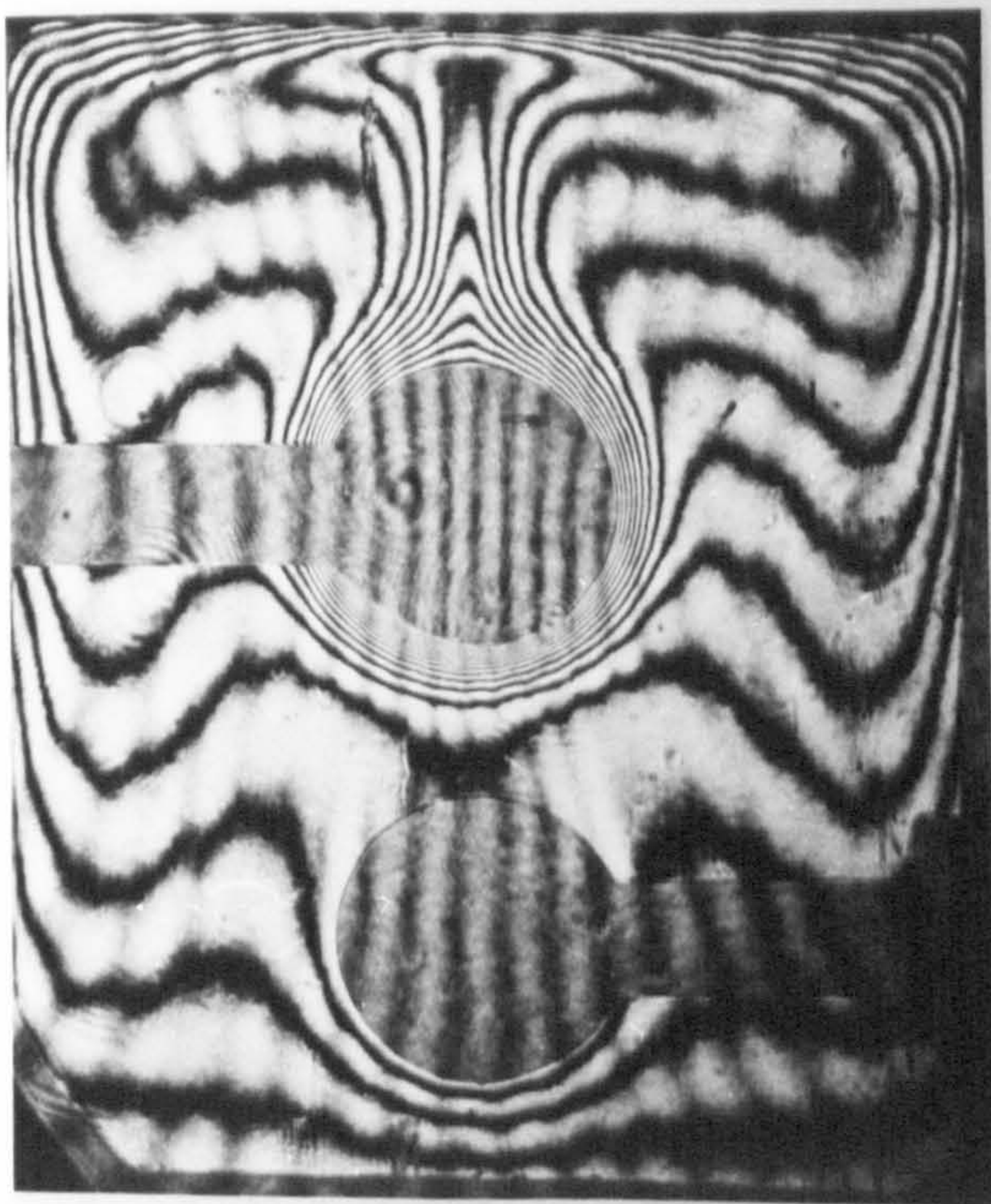
156



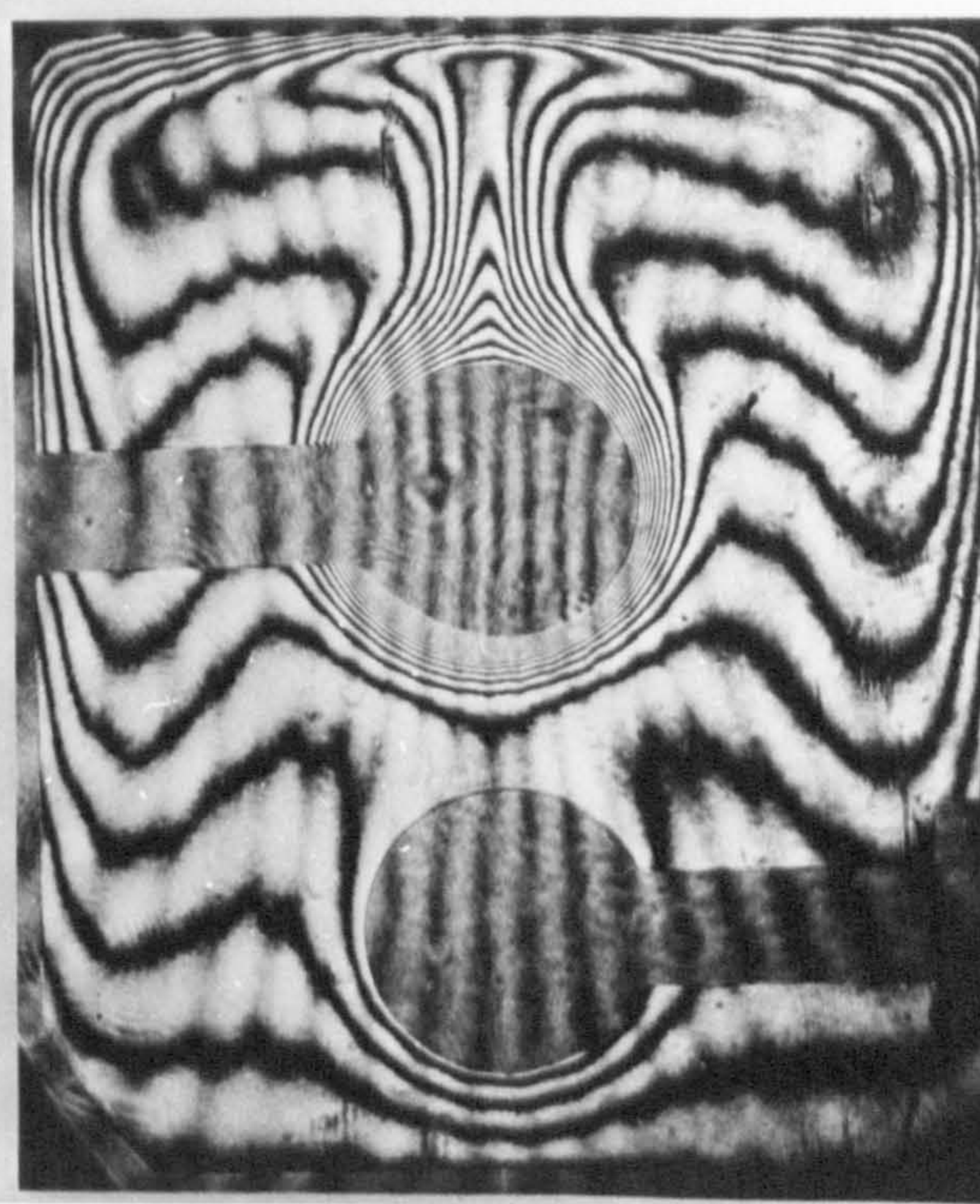
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158



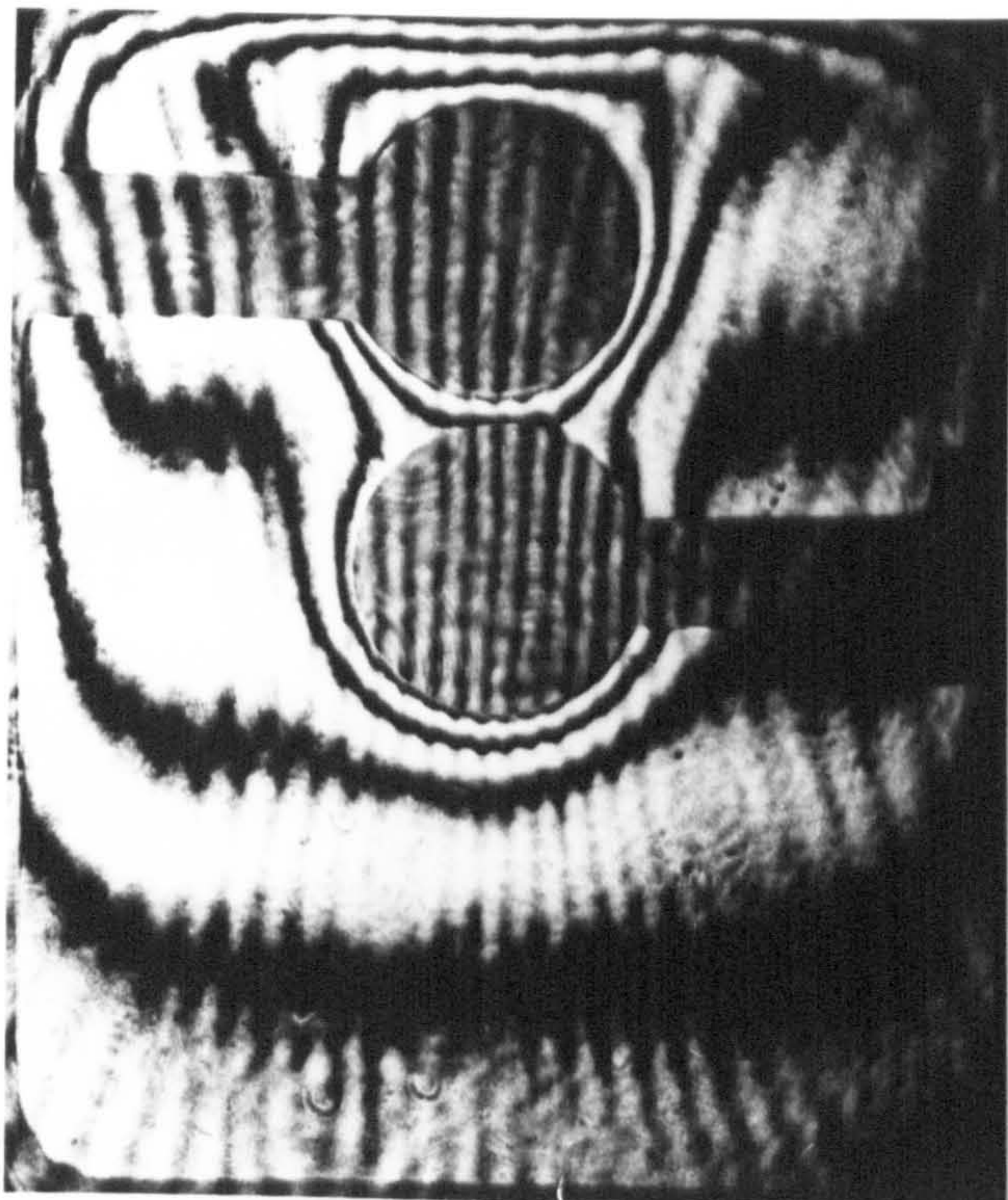
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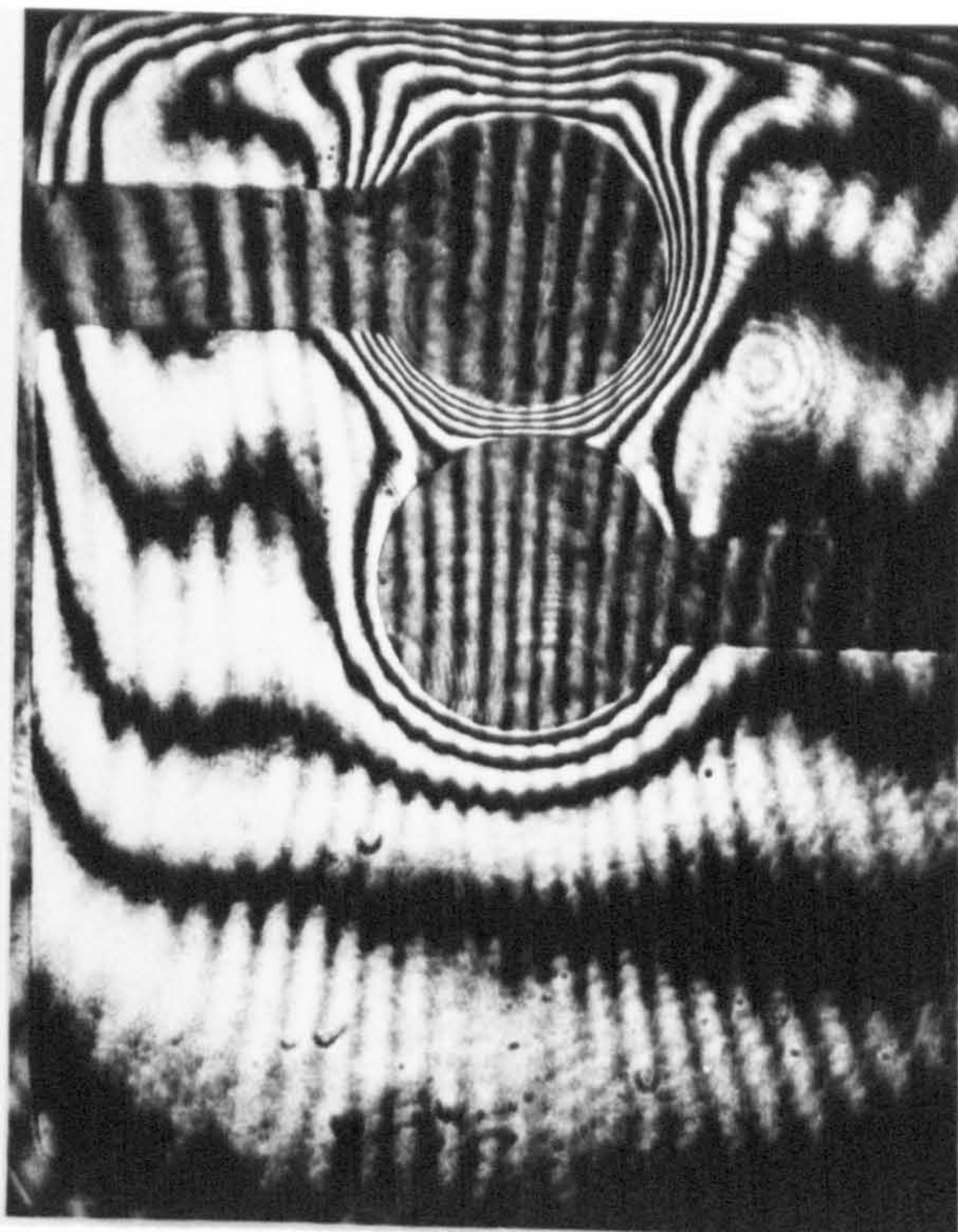
$$E_s = +0.80$$

$$E_r = +0.10$$

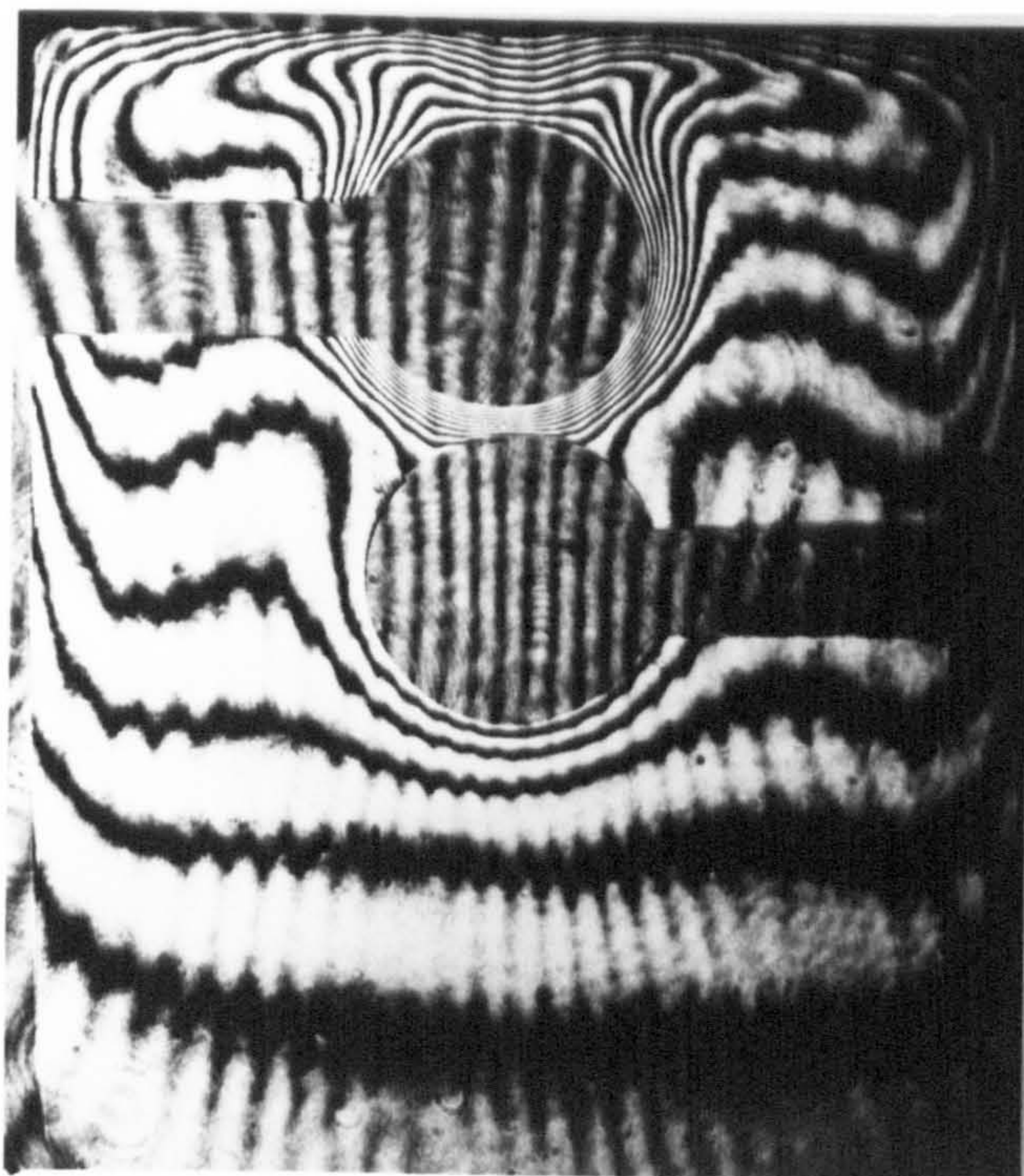
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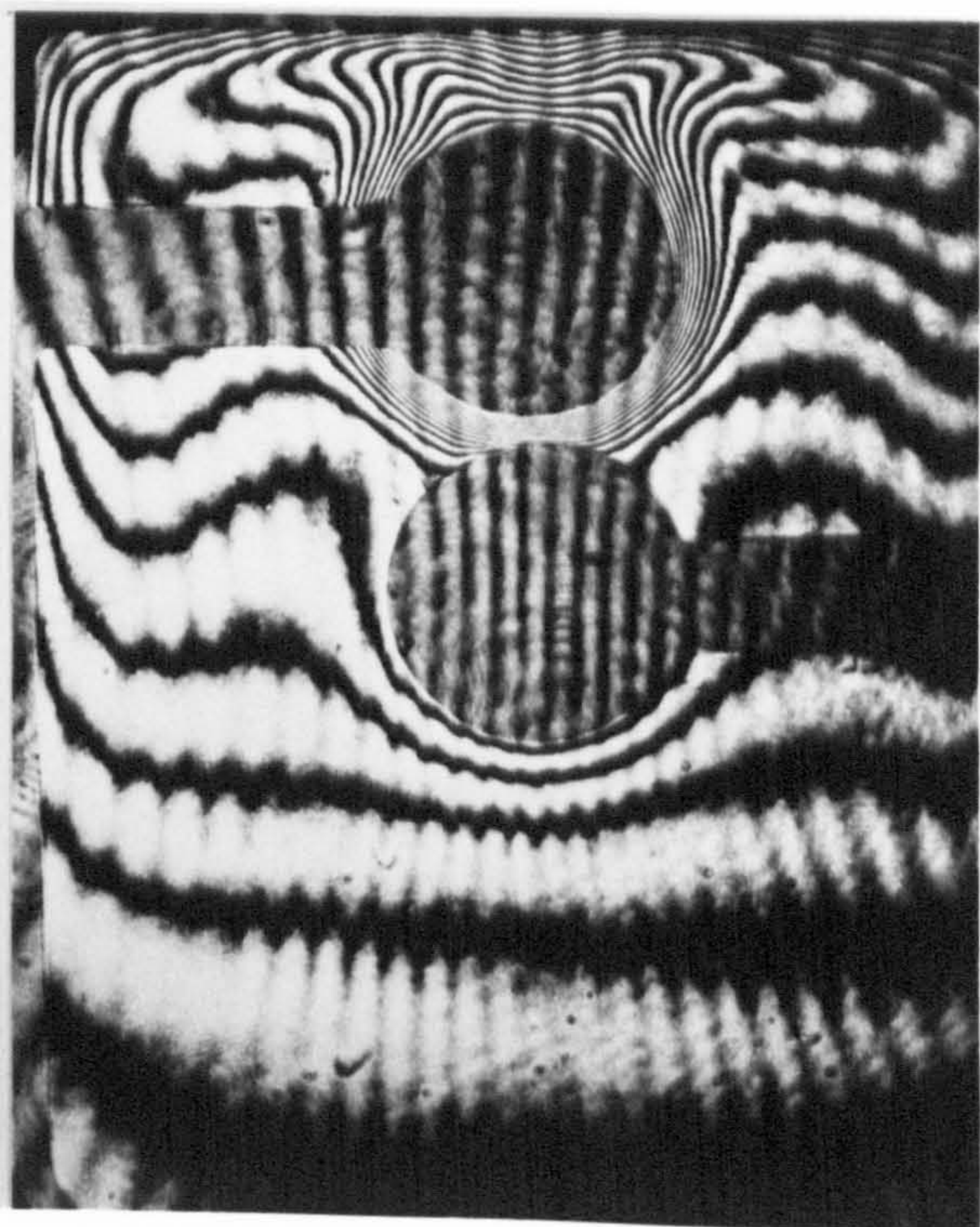
206



207



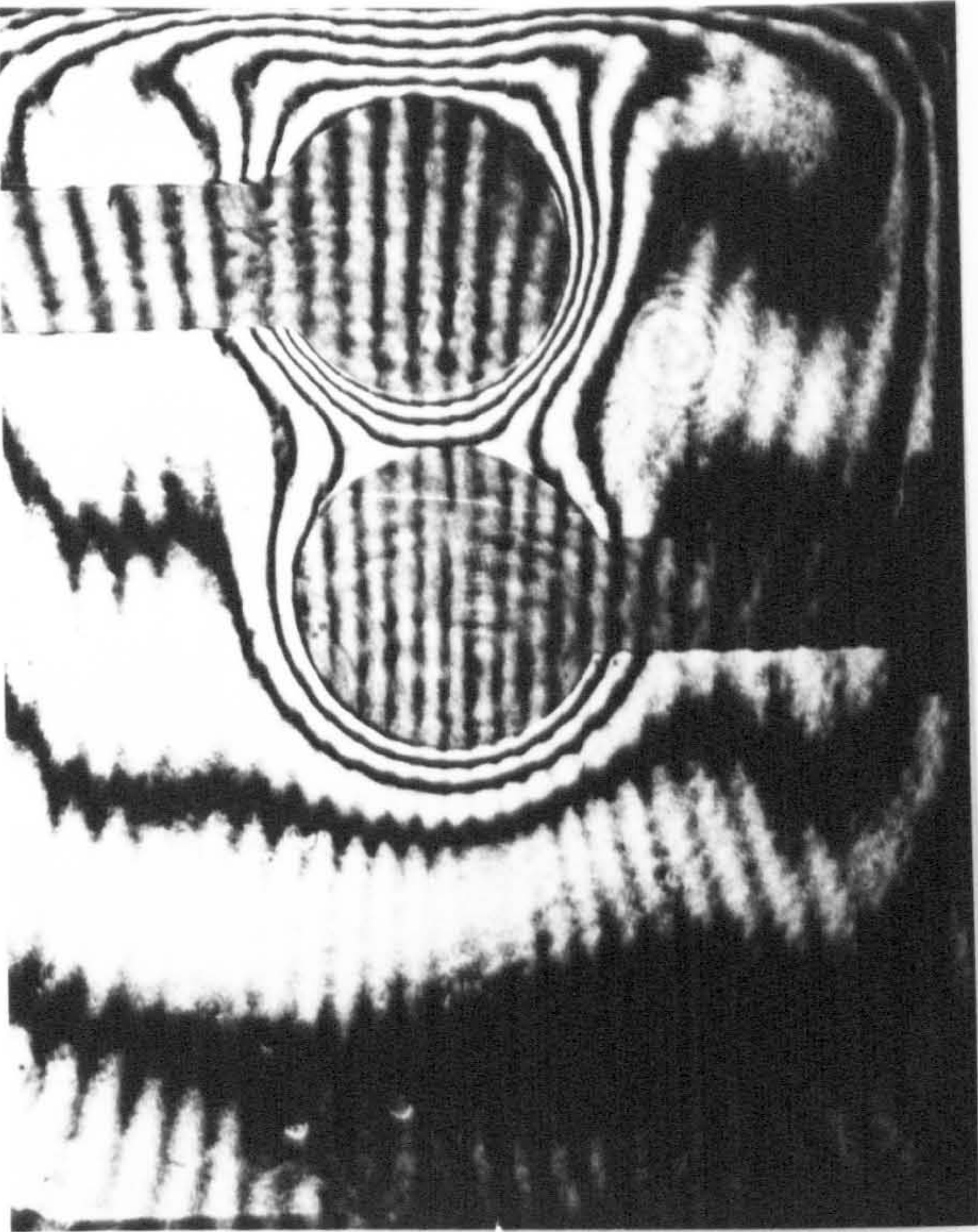
N.A.



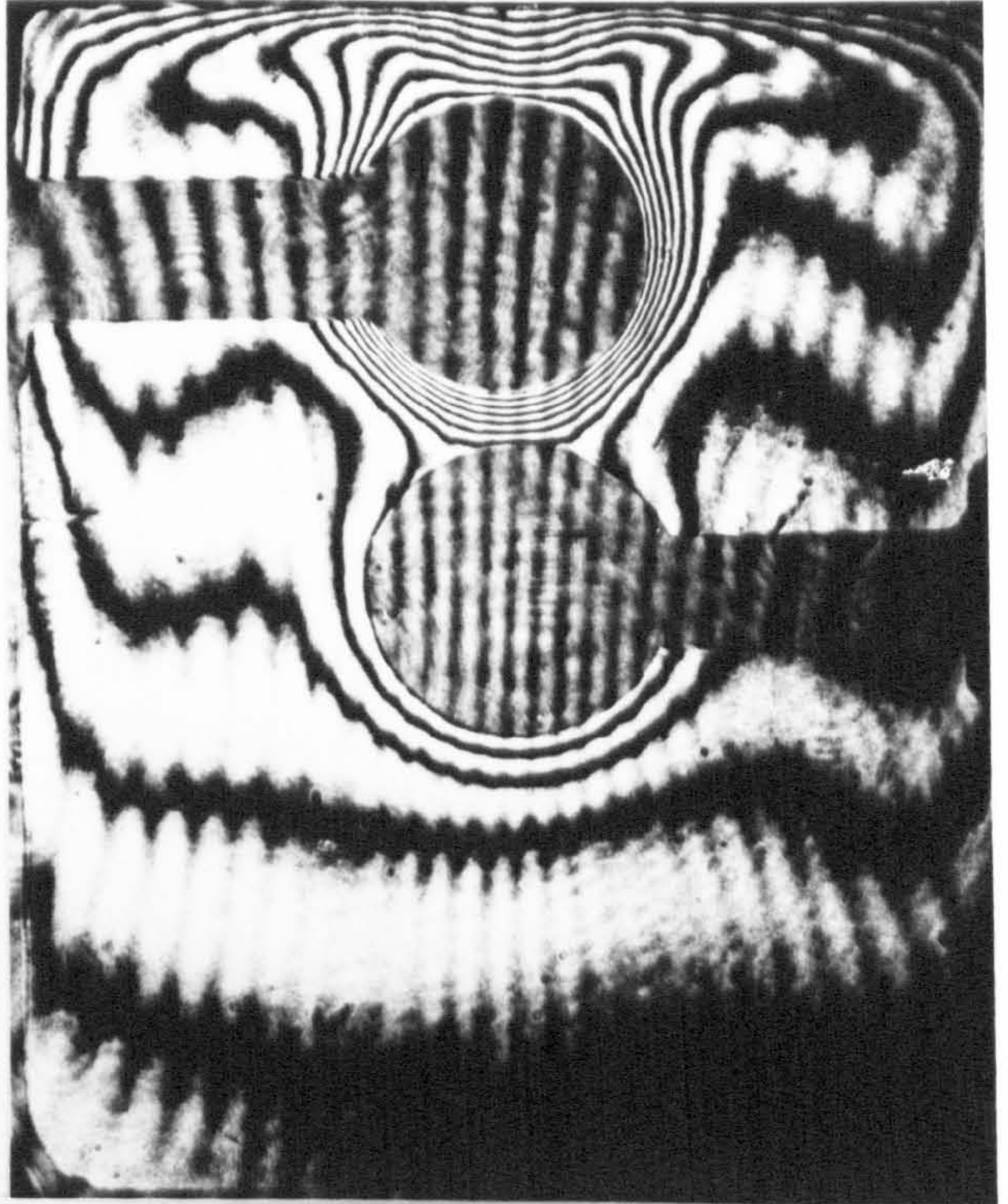
$$E_s = +0.80$$

$$E_r = +0.03$$

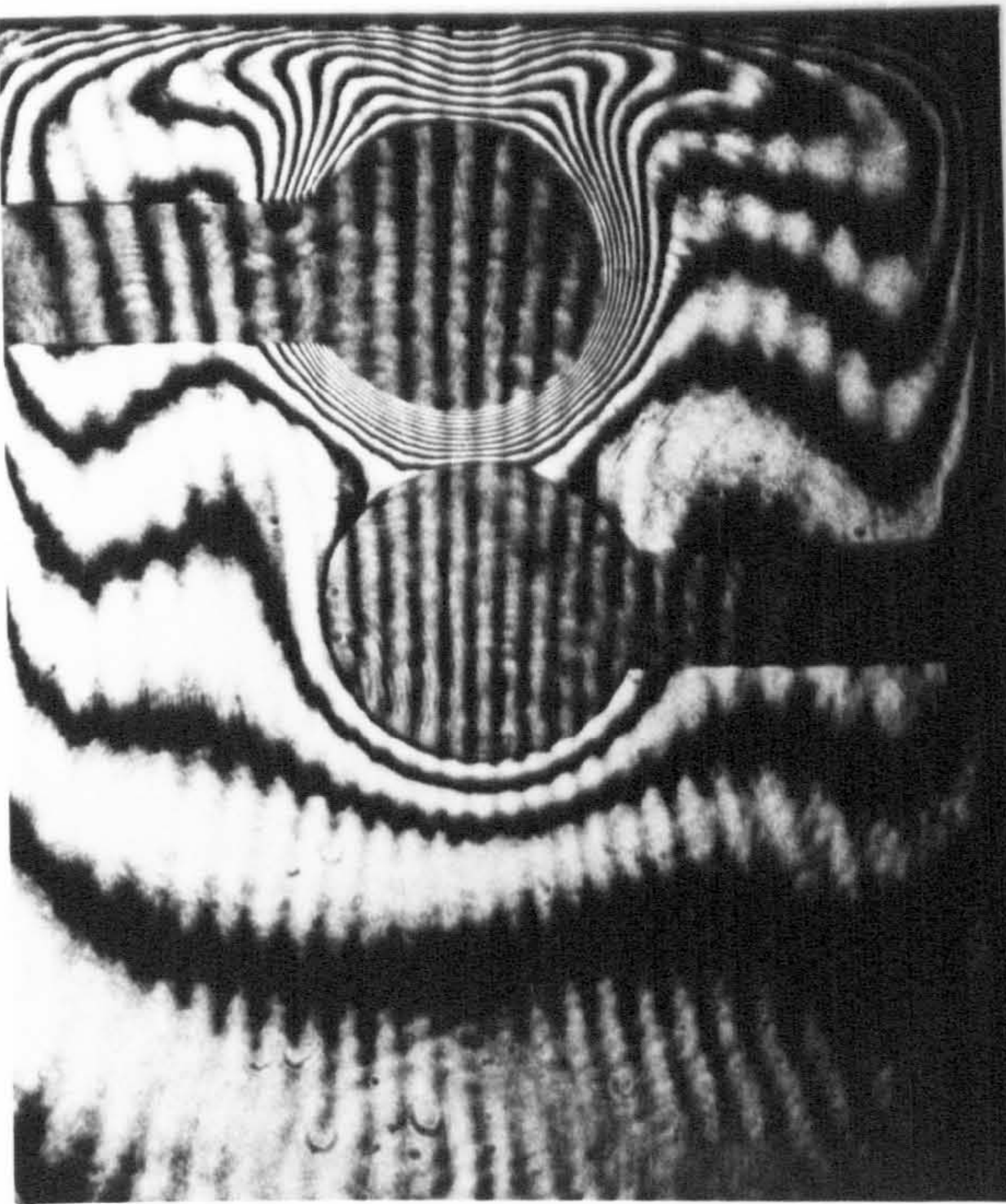
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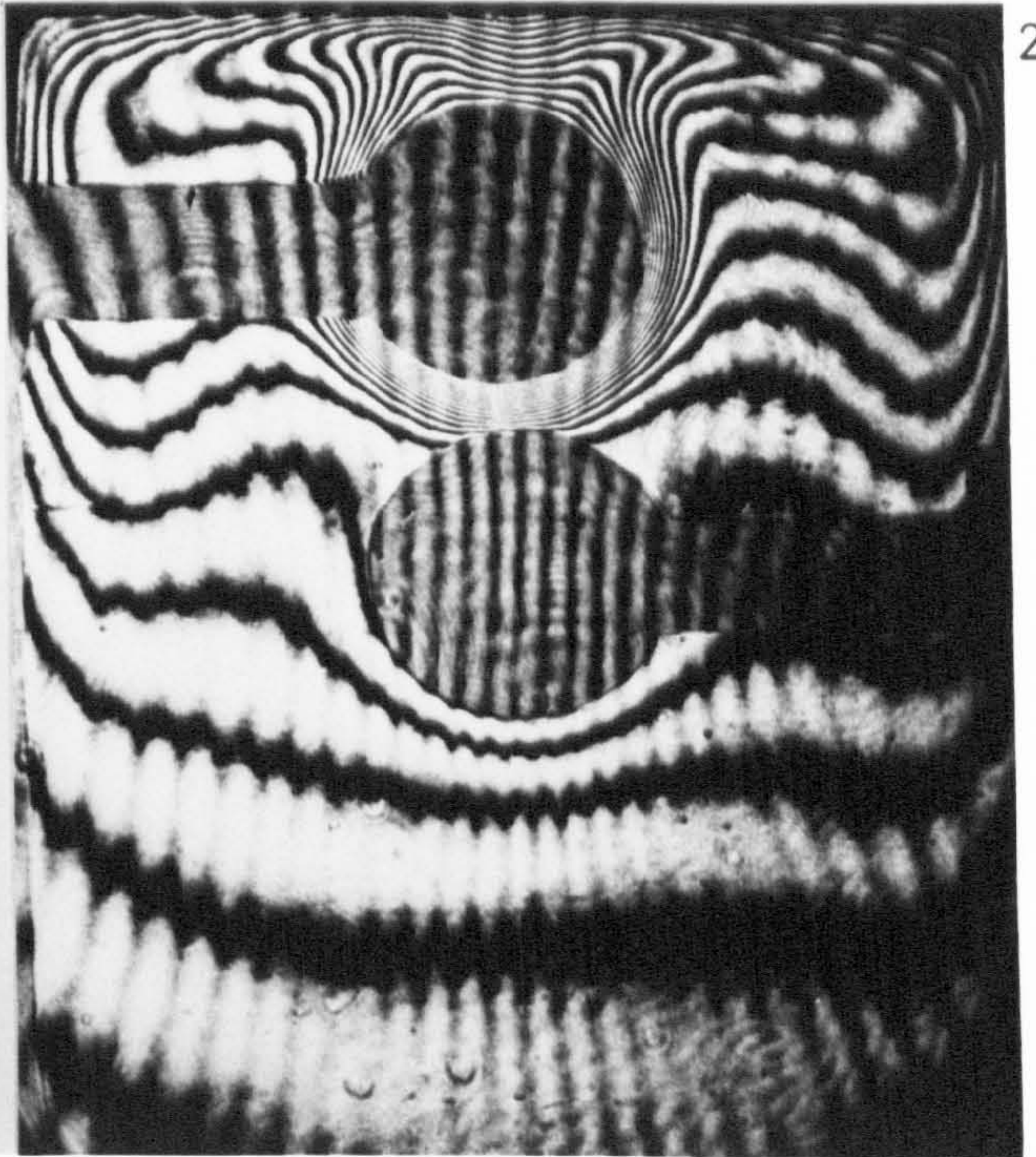
209



210



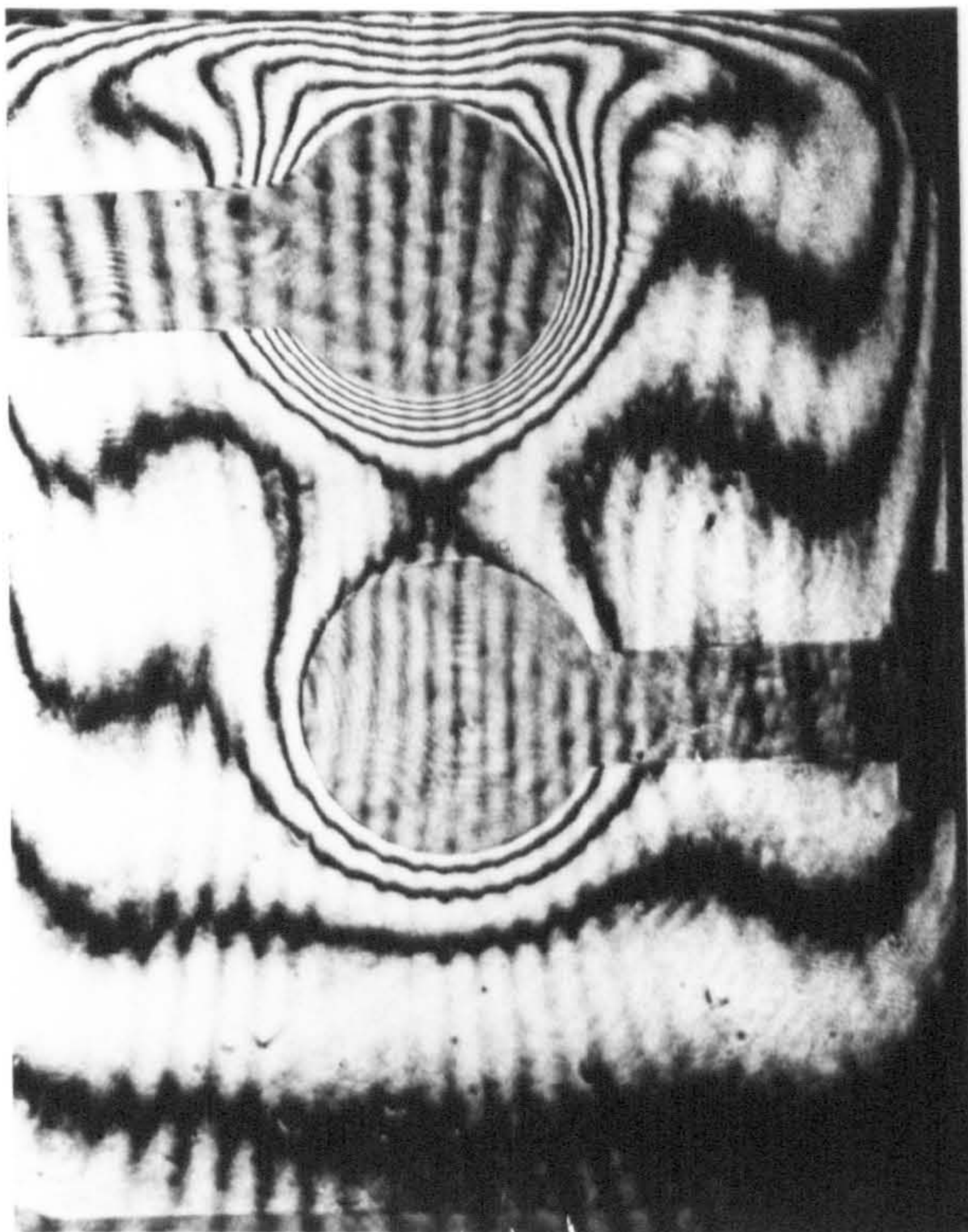
211



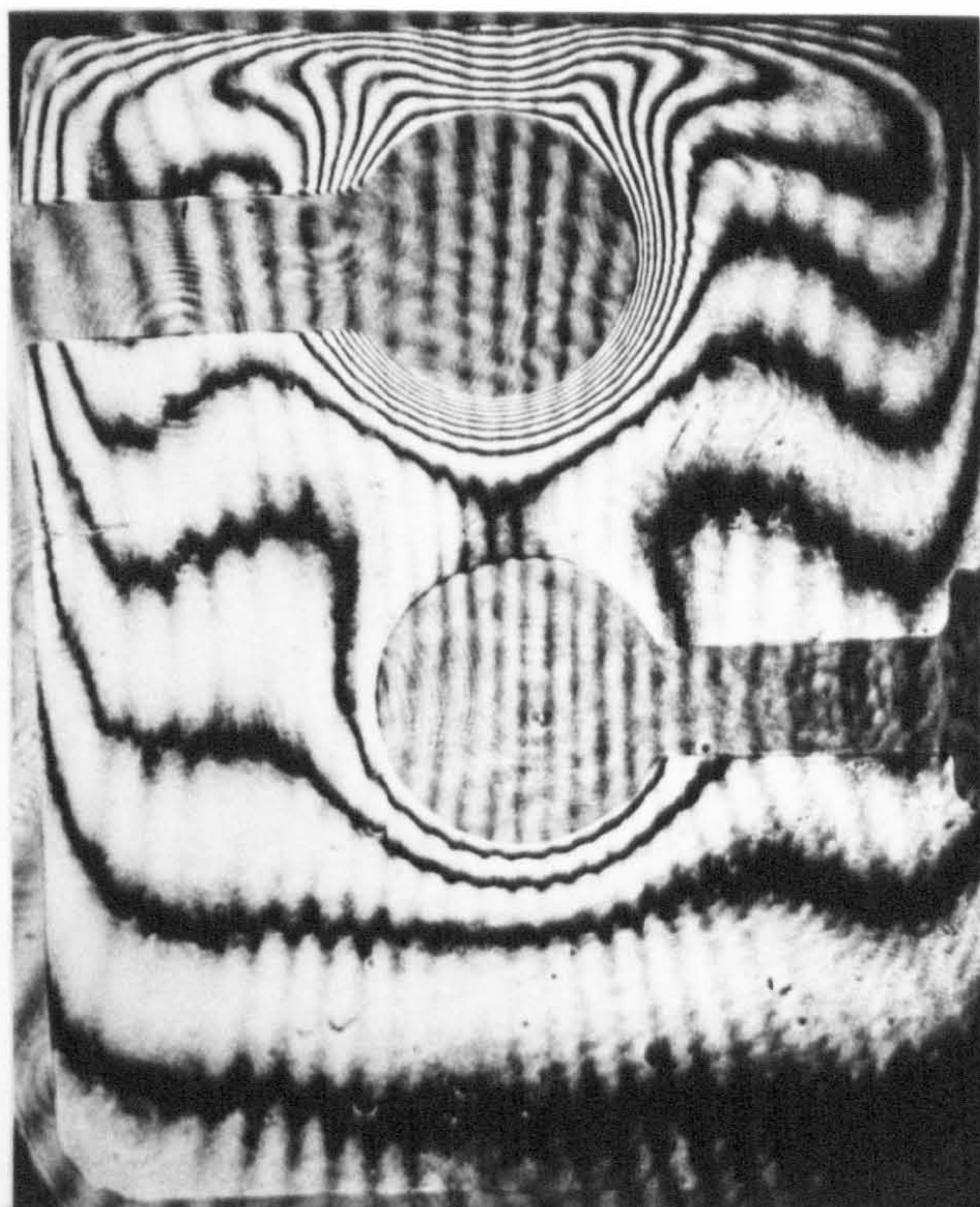
$$E_s = +0.80$$

$$E_r = -0.20$$

219



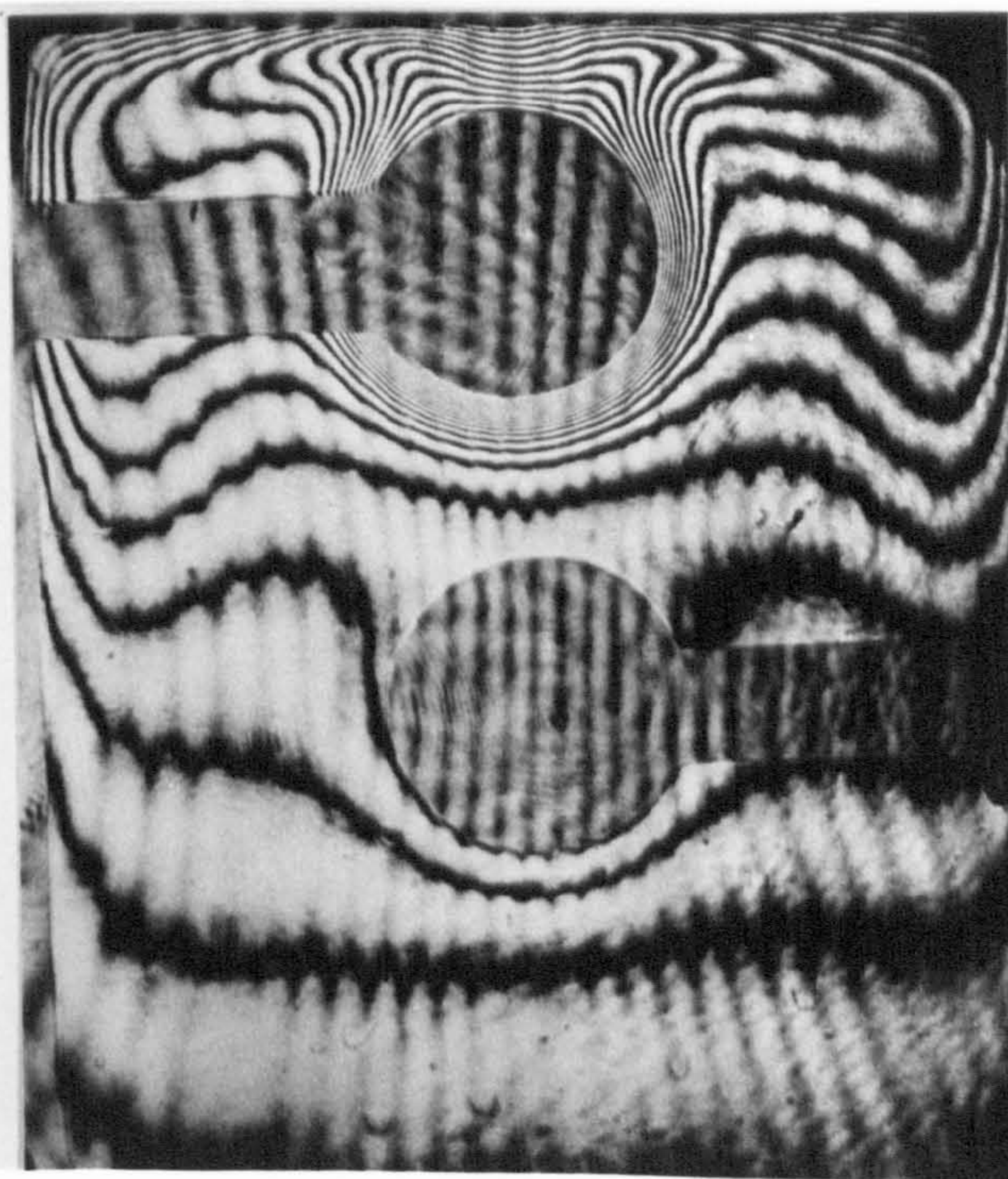
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Chapter 4

Discussion of Results obtained from the Spacer Study

Discussion of Results obtained from the Spacer Study

4.1 Problems encountered

The investigations undertaken experienced no problems with equipment malfunctions. However, difficulty was encountered in analysing some interferograms. At high temperature-differences there were a large number of fringes close to the cylinder wall which were also very close together, consequently in some cases they were poorly defined, see Plate 19. The fringes must be clear around all of half of the cylinder for complete analysis of the interferogram to be achieved. In Plate 19 the fringes are not defined sufficiently well in the areas just above the spacer to enable the interferogram to be analysed. Therefore, interferograms could not be analysed at temperature differences greater than $\approx 25^{\circ}\text{C}$.

Also, for large temperature differences, slight distortion and hence misalignment of the cylinder was evident but did not prevent the interferograms from being analysed. Plate 19 exhibits misalignment. This can be observed by the existence of another fringe which seems superimposed on the bottom surface of the cylinder. However, the change in Nusselt number was found to be negligible if the extra fringe was included or not when the results were computer analysed. Misalignment occurred because the test cell was "knocked" at some stage during the experiment. The only solution is for the operator to take greater care during experimentation.

4.2 Variation of Local Nusselt number around the cylinder

The local Nusselt number, Nu_d , was obtained at 15° intervals around one half of the cylinder for each system configuration investigated: symmetry applying about the vertical plane through the cylinder axis. Graphs of Nu_d against angular position are shown in Figures 4.1 to 4.20.

For the system at zero displacement ratio with no spacers, the value of Nu_d is at a minimum at the top of the cylinder. It rises rapidly at first but flattens out to reach a maximum at the bottom of the cylinder. This curve is characteristic for all values of ΔT .

For the system with a vertical radial spacer ($\gamma = 0^\circ$) a similar curve is observed, with Nu_d reaching a maximum at the spacer.

Positioning of spacers at $\gamma = 10^\circ$ effectively divides the flow system into two parts. The section of the curve $0^\circ < \theta < 170^\circ$ is similar to the system with no spacers and for $\gamma = 0^\circ$. The local Nusselt number reaches a maximum at approximately $\theta = 135^\circ$. Below the spacer (i.e. $170^\circ < \theta < 180^\circ$), the curve falls off slightly. Similarly-shaped curves are obtained for $\gamma = 20^\circ$.

As the positions of the spacers are moved around to between $\gamma = 30^\circ$ and $\gamma = 60^\circ$, the curve changes shape. The initial section, from $\theta = 0^\circ$ to the spacer, remains similar to the two previous systems, except that the maximum value of Nu_d occurs at approximately $\theta = 120^\circ$. Below the spacer, the curve falls off slightly before rising again steadily to $\theta = 180^\circ$, the value of Nu_d at this angle being slightly less than the maximum value.

As γ is increased to 70° the first section of the curve ($0 < \theta < 110^\circ$) rises rapidly to a maximum value of Nu_d at approximately $\theta = 100^\circ$. At $\gamma = 80^\circ$ and $\gamma = 90^\circ$, the curves rise rapidly to a maximum value of Nu_d at approximately $\theta = 60^\circ$ before falling slightly, close to the spacer. Below the spacer, the value of the Nu_d rises in a smooth manner similar to the curve described for the system with no spacers.

For the systems with $E = 0.7$ similar sets of curves are observed, except that the rise in local Nusselt number, from $\theta = 0^\circ$, is not as rapid as that for the corresponding curve at $E = 0$. Overall, the maximum Nusselt numbers are not as large as those obtained for the systems of zero displacement ratio.

4.3 Correlation and Interpretation of Data

From the obtained values of the overall mean Nusselt numbers $(Nu_d)_m$ and corresponding Grashof numbers (Table 3.1), a series of correlations were determined. Dimensional analysis indicates that:

$$Nu = f \left(Gr, Pr, \frac{x}{d}, \frac{y}{d}, \frac{r_1}{d}, \frac{L}{d} \right) \quad (\text{Appendix B})$$

x , y , r_1 and L were constant and the Prandtl number remained at 0.707 for the range of mean air temperatures involved. Therefore, for this study,

$$(Nu_d)_m = f (Gr_D)$$

which can be written

$$(Nu_d)_m = M Gr_D^n$$

where M and n are constants. Theory and experiment show a power law dependence of this type, with $n = 0.25$ for laminar flow (37). As this study involved laminar flow, the correlations were assumed to follow the equation.

$$0.25$$

$$(\text{Nu}_d)_m = M \text{Gr}_D$$

For each spacer position the value of " M " was determined by taking the average of the values of " M " obtained when the overall mean Nusselt number and the corresponding Grashof numbers were fitted to the above equation. Table 4.2 gives the values of " M " obtained for six-point curves. Figures 4.21 to 4.40 show the six-point correlations (where possible).

The value of " M " is an indicator of the rate of heat-transfer. For the system at zero displacement ratio, the rate of heat-transfer is at a minimum at $\gamma = 48^\circ$, and for the system at displacement ratio 0.7 it is at a minimum at $\gamma = 52^\circ$. Figure 4.41 shows a gradual decrease in " M " for the spacer positions up to approximately $\gamma = 50^\circ$, where it reaches a minimum, and then rapidly increases up to $\gamma = 90^\circ$. The shape of the curve indicates that " M " would increase further as γ is increased.

From the data, the value of " M " at a spacer position

$\gamma = 48^\circ$ (for zero displacement ratio) is 25% lower than that obtained for the system with no spacers.

Furthermore, when the cylinder is placed at displacement ratio 0.7, the value of " M " at spacer position $\gamma = 52^\circ$ is 31% lower than that obtained for the system with no spacers (and zero displacement ratio).

4.4 Implications of the Results

A substantial reduction in heat loss is possible by the insertion of radial spacers in the rectangular cavity. Perhaps by considering other values of the displacement ratio E , of the hot cylinder, more reductions in heat loss may be possible.

Table 4.1

Average Nusselt Number with variation in
Grashof Number and Temperature difference

$(Nu_d)_m$	Gr_D	ΔT $(T_s - T_d), (^\circ C)$	Interferogram
<u>E=0, no spacers</u>			
5.32	24587	8.16	1
5.90	36116	12.54	2
6.12	42724	15.22	3
6.25	47896	17.56	4
6.53	54273	20.31	5
6.60	58734	22.56	6
<u>E=0, $\gamma = 0^\circ$</u>			
4.73	21818	7.35	7
5.49	39096	13.60	8
5.63	43722	16.00	9
6.08	51983	19.41	10
6.19	58939	22.40	11
6.19	61920	23.80	12
<u>E=0, $\gamma = 10^\circ$</u>			
4.51	19130	6.51	13
5.13	32044	11.30	14
5.35	38066	13.80	15
5.53	45159	16.74	16
5.89	55727	21.00	17
5.93	56130	21.70	18
<u>E=0, $\gamma = 20^\circ$</u>			
5.12	35241	12.20	19
5.49	42241	14.97	20
5.59	48677	17.70	21
5.83	55117	20.61	22
5.97	66545	25.00	23
<u>E=0, $\gamma = 30^\circ$</u>			
3.84	15473	5.21	24
4.65	30798	10.3	25
5.03	43885	15.5	26
5.14	48503	17.62	27
NA			28
5.81	73686	27.10	29

E=0, γ=40°

4.10	27756	9.01	30
4.66	45140	15.20	31
4.92	56220	19.21	32
5.06	61805	21.81	33
5.29	72291	25.90	34
5.48	76847	28.10	35

E=0, γ=50°

4.15	30482	9.95	36
4.59	45102	15.10	37
4.98	62160	21.00	38
5.05	67302	23.41	39
5.29	79056	27.70	40
5.31	83223	30.10	41

E=0, γ=60°

4.12	25090	8.20	42
4.65	40386	13.70	43
4.85	48420	16.91	44
5.05	57325	20.31	45
5.14	61435	22.41	46
5.32	69740	26.00	47

E=0, γ=70°

5.04	36043	11.71	48
5.46	49799	16.70	49
5.68	55051	19.14	50
NA			51
NA			52
NA			53

E=0, γ=80°

5.26	32685	10.61	54
5.82	46463	15.80	55
6.10	57019	19.61	56
6.23	62439	22.21	57
NA			58
NA			59

E=0, γ=90°

5.27	26645	8.81	60
5.89	41961	14.45	61
6.09	46428	16.41	62
6.39	55281	19.90	63
NA			64
NA			65

E=0.7 no spacers

4.34	24434	8.15	66
4.87	39284	13.56	67
5.08	44897	16.20	68
5.28	53980	19.60	69
5.59	66709	24.41	70
5.57	67104	25.50	71

E=0.7, $\gamma = 0^\circ$

4.47	27545	9.27	72
4.78	35997	12.31	73
5.00	43846	15.40	74
5.27	54072	19.31	75
5.51	66498	23.90	76
NA			77

E=0.7, $\gamma = 10^\circ$

4.65	33984	11.20	78
4.94	43178	14.61	79
5.23	54702	18.76	80
5.32	58443	20.65	81
5.51	68201	24.51	82
5.51	68137	25.50	83

E=0.7, $\gamma = 20^\circ$

4.56	33927	11.20	84
4.77	41928	14.21	85
4.98	48811	16.91	86
5.28	62973	22.01	87
5.28	62372	22.61	88
5.47	73210	27.00	89

E=0.7, $\gamma = 30^\circ$

4.60	40761	13.01	90
4.93	53987	18.00	91
5.15	63069	21.41	92
5.38	72134	25.01	93
5.45	76154	27.50	94
5.50	79176	29.20	95

E=0.7, $\gamma = 50^\circ$

4.02	34500	11.20	96
4.39	50264	16.91	97
4.54	57044	19.70	98
4.68	62970	22.31	99
4.88	76319	27.30	100
4.94	78843	29.10	101

E=0.7, $\gamma=60^\circ$

4.02	33524	10.91	102
4.39	45908	15.61	103
4.66	55949	19.40	104
4.85	67012	23.61	105
4.95	70728	25.30	106
5.24	79176	29.20	107

E=0.7, $\gamma=70^\circ$

4.13	29262	9.70	108
4.57	43085	14.71	109
4.79	51851	18.21	110
5.08	66003	23.31	111
5.29	76319	27.30	112
NA			113

E=0.7, $\gamma=90^\circ$

5.22	31436	10.30	114
5.65	43600	14.91	115
5.80	49785	17.50	116
6.12	60363	21.55	117
6.15	63031	23.30	118
6.39	70408	26.51	119

Table 4.2

0.25

Correlations of results assuming $(Nu_d)_m = M Gr_D$
for Six-Point Curves.

E = 0.7		E = 0	
Spacer Position		Spacer Position	
(degrees)		(degrees)	
Y	M	Y	M
NO SPACERS	0.348	NO SPACERS	0.425
0	0.346	0	0.390
10	0.343	10	0.383
20	0.337	20	0.373
30	0.325	30	0.347
40	-	40	0.320
50	0.294	50	0.314
60	0.298	60	0.327
70	0.317	70	0.367
80	-	80	0.394
90	0.390	90	0.412

FIGURE 4. 1

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
PIPE DISPLACEMENT RATIO = 0
NO SPACERS, PLAIN CAVITY

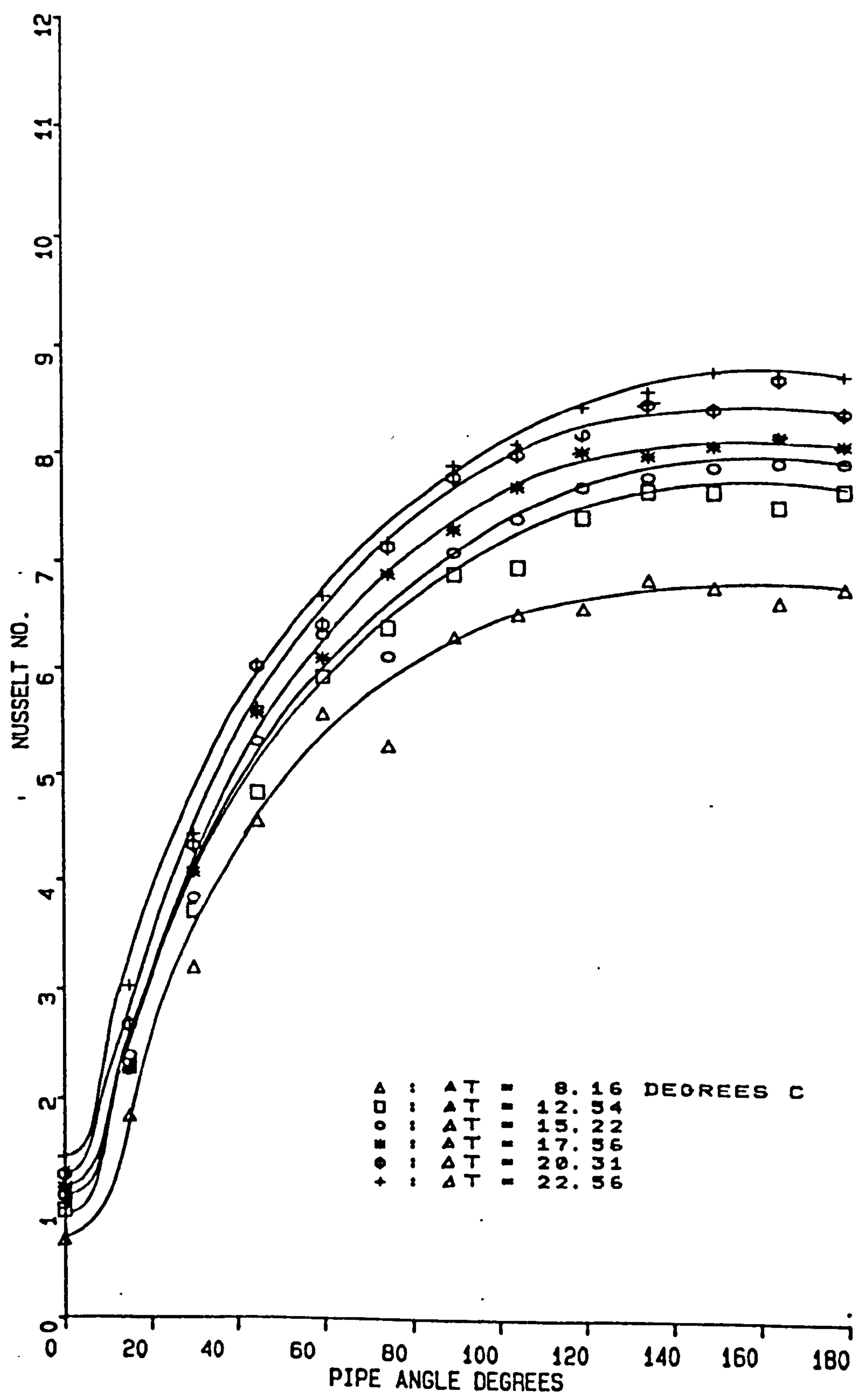


FIGURE 4. 2

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 0 DEGREES

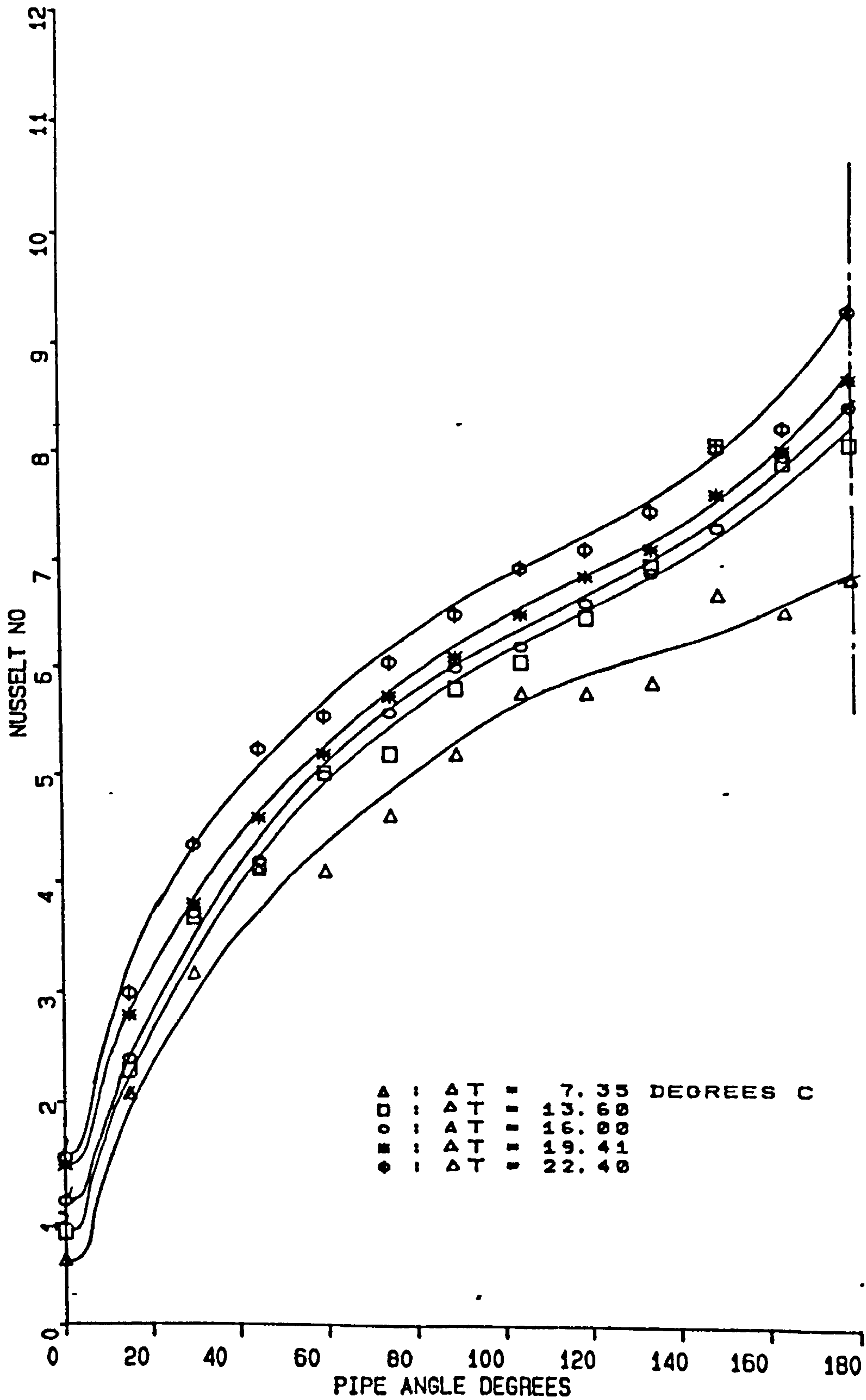


FIGURE 4.3

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 10 DEGREES

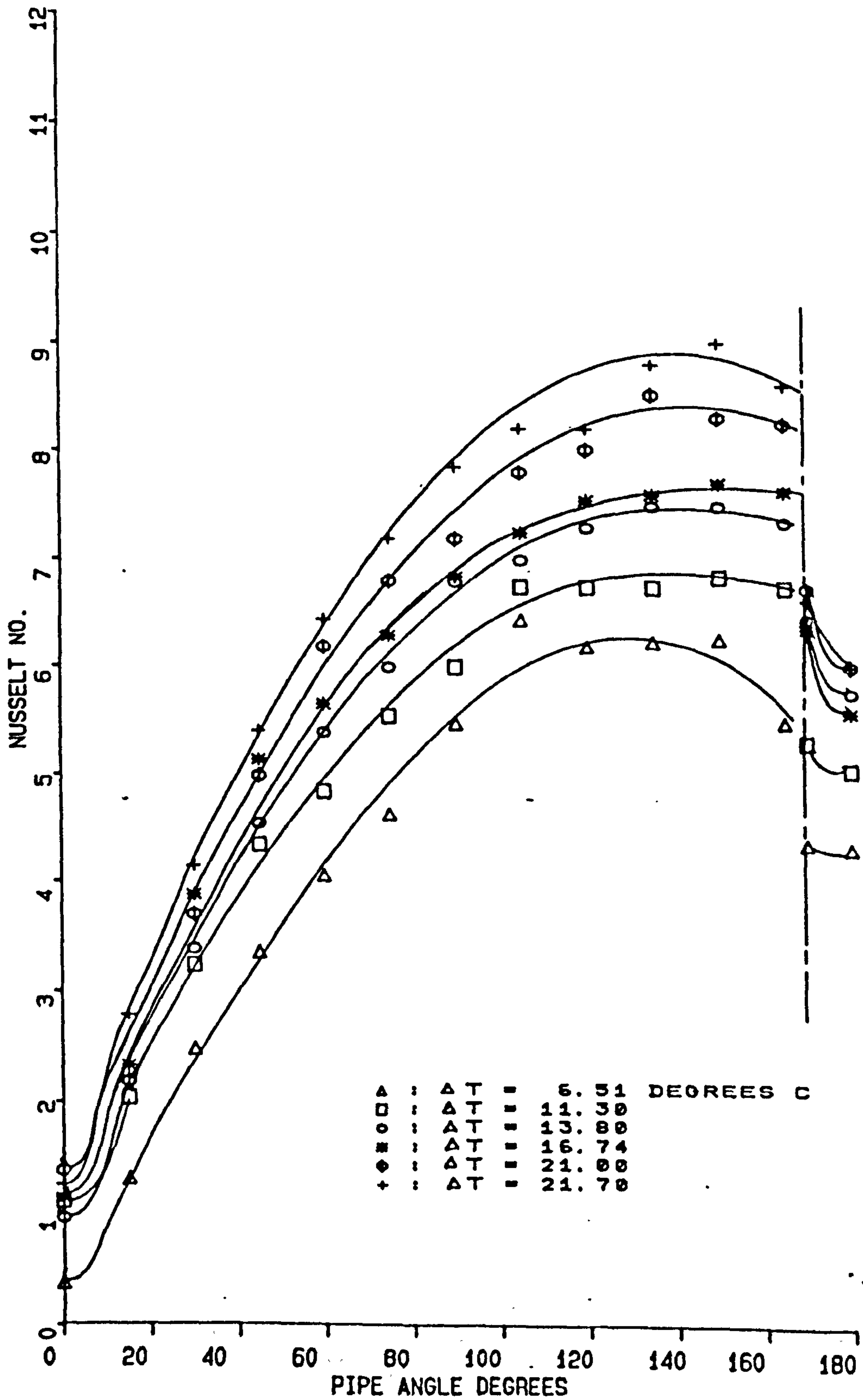


FIGURE 4. 4

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 20 DEGREES

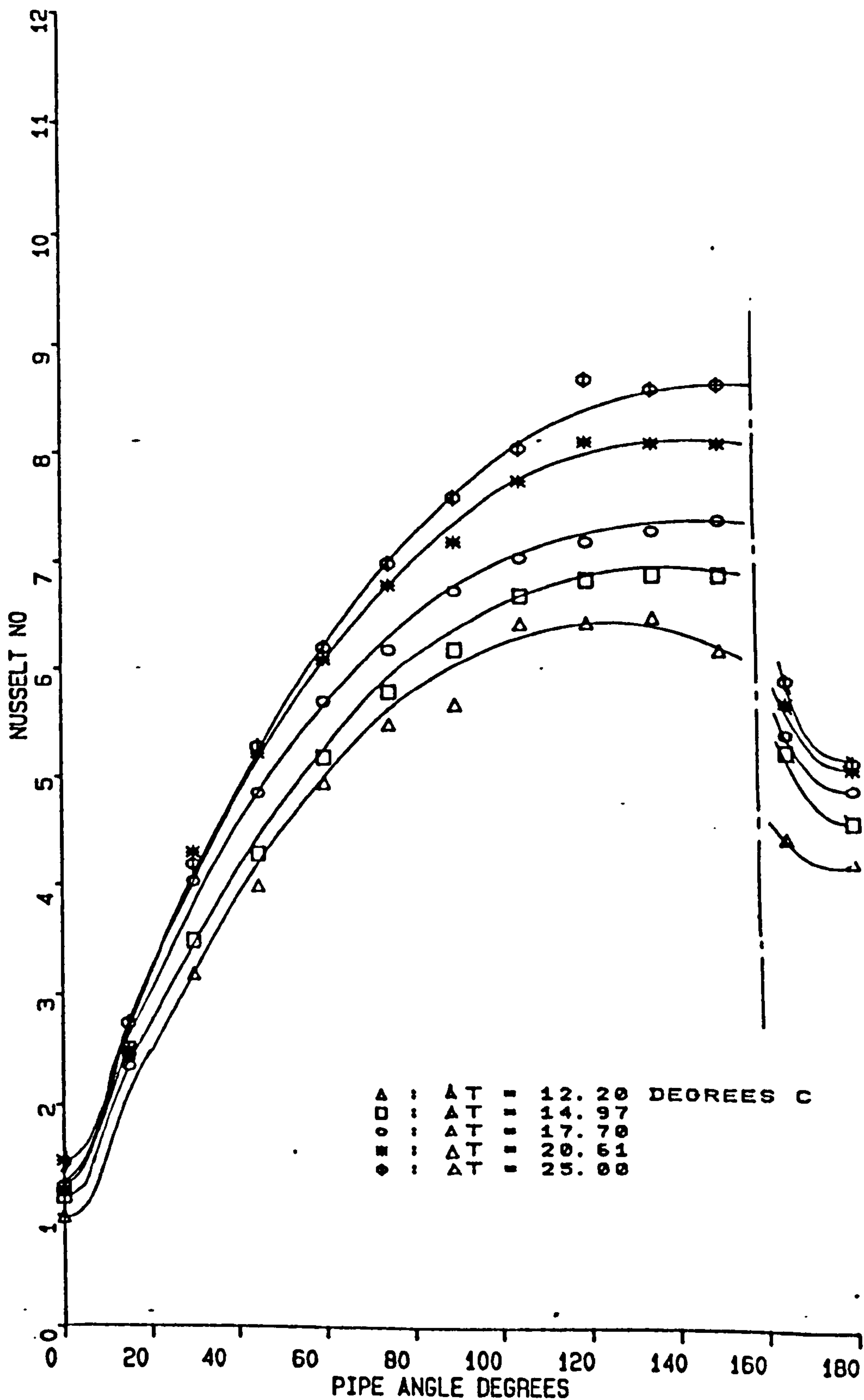


FIGURE 4.5

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 30 DEGREES

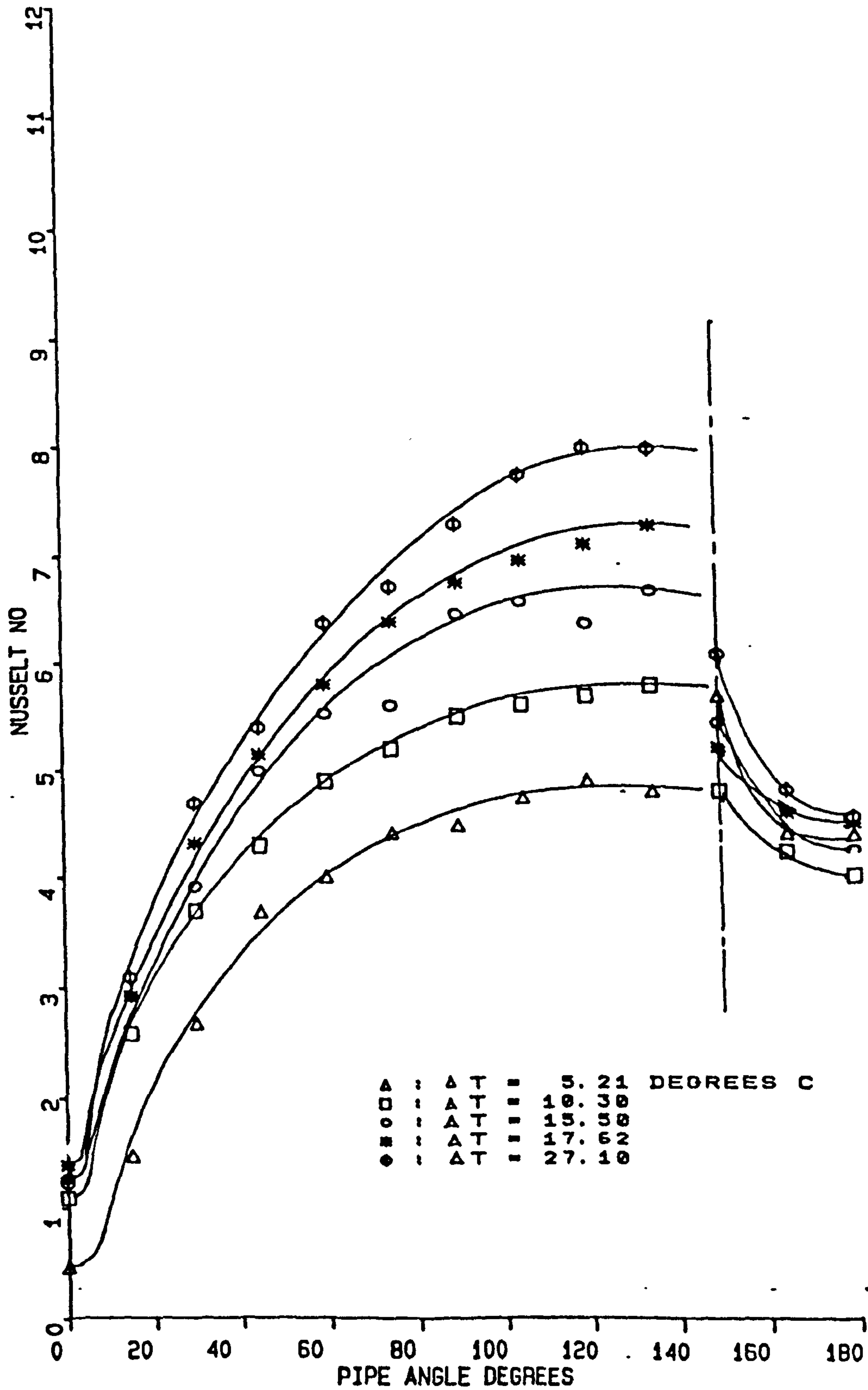


FIGURE 4. 6

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 40 DEGREES

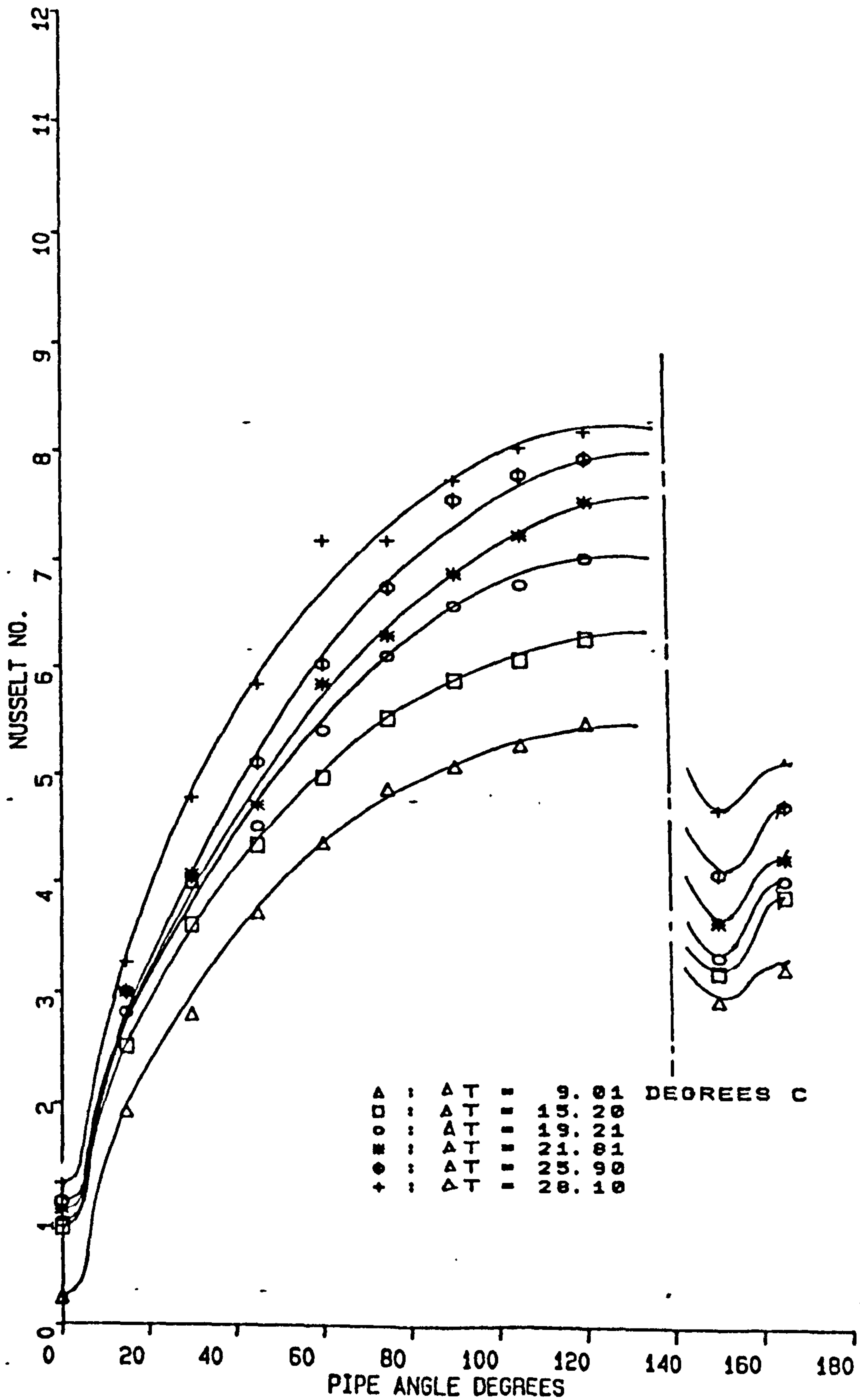


FIGURE 4.7

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 50 DEGREES

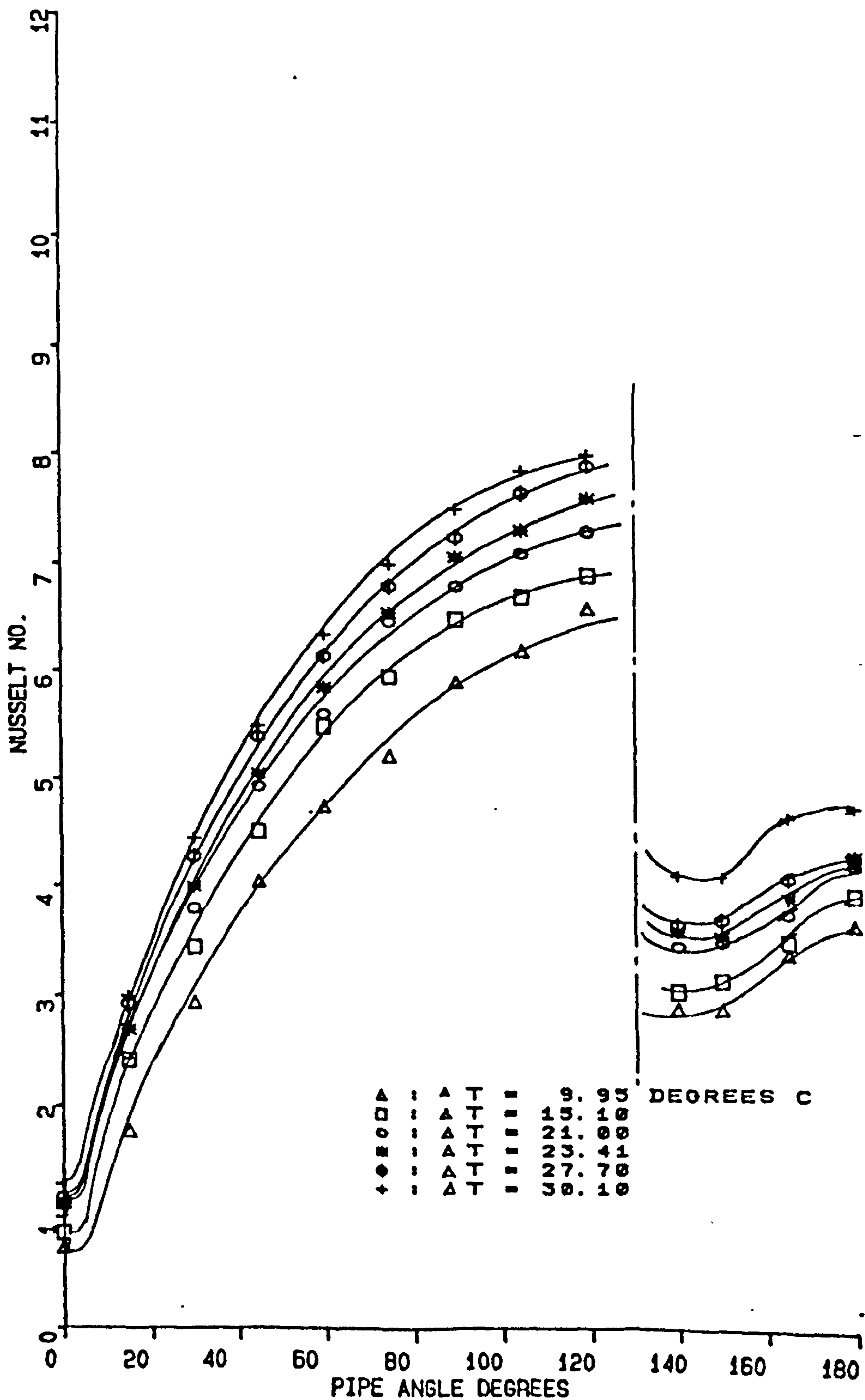


FIGURE 4.8

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 60 DEGREES

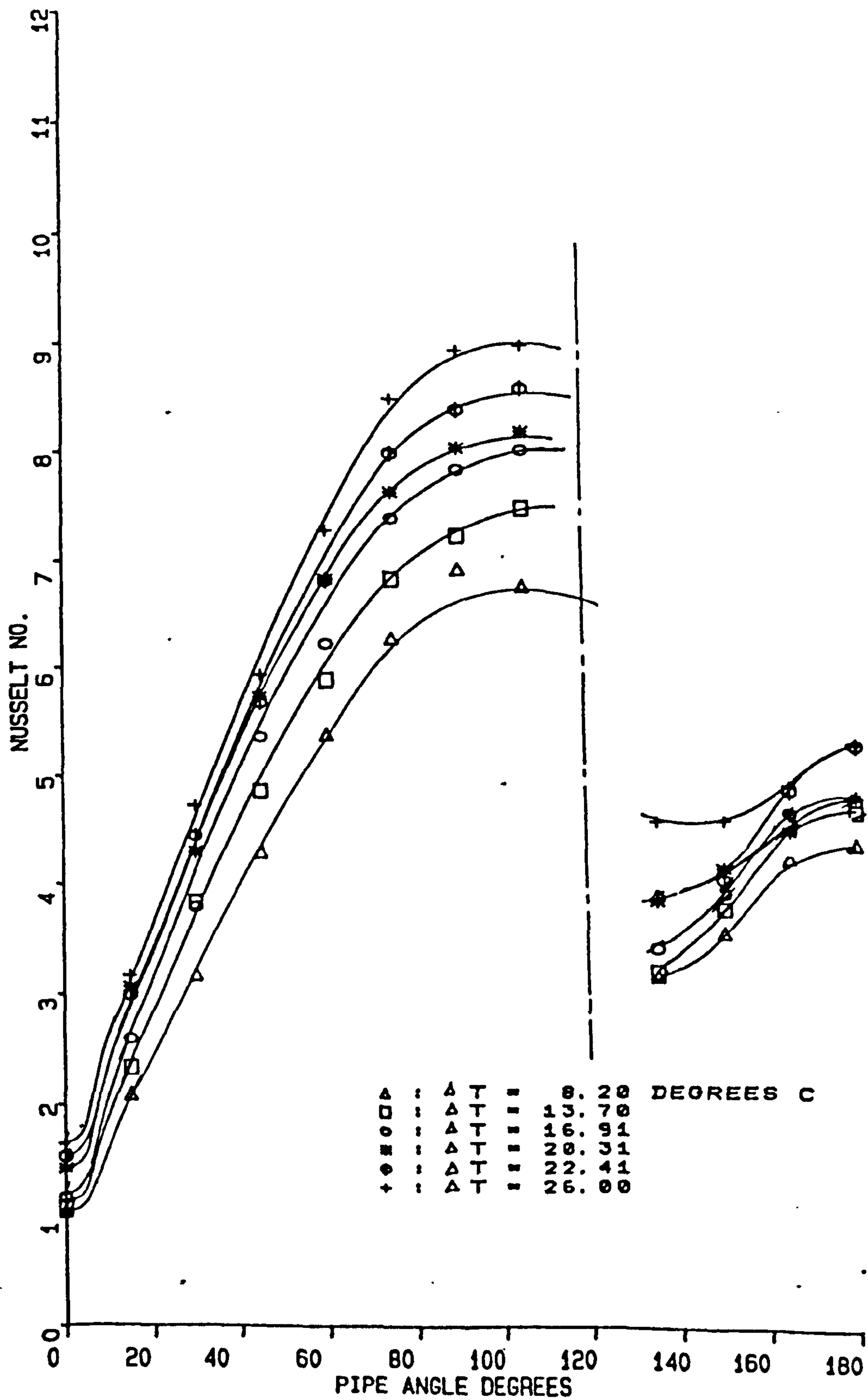


FIGURE 4.9

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 70 DEGREES

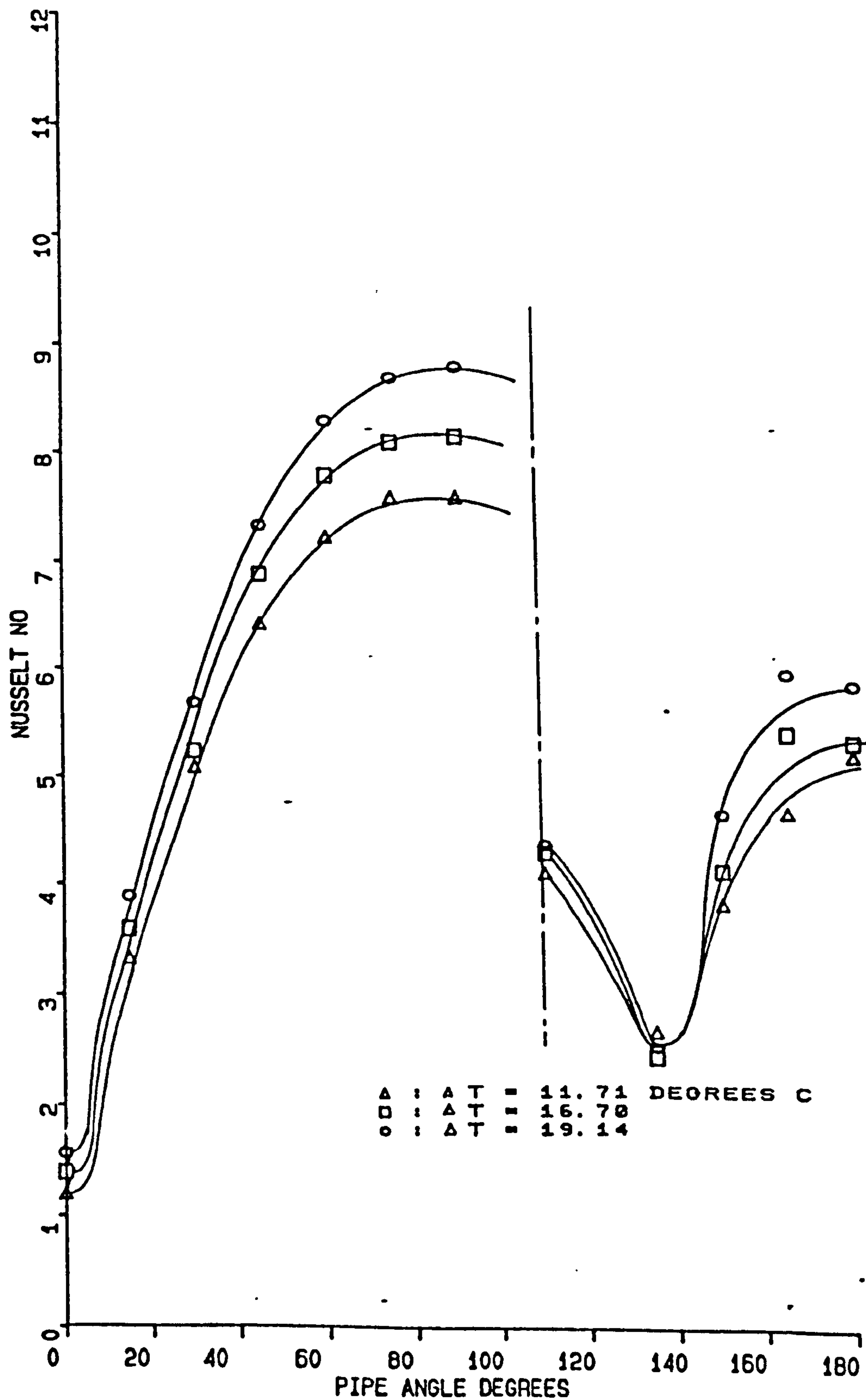


FIGURE 4. 10

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 80 DEGREES

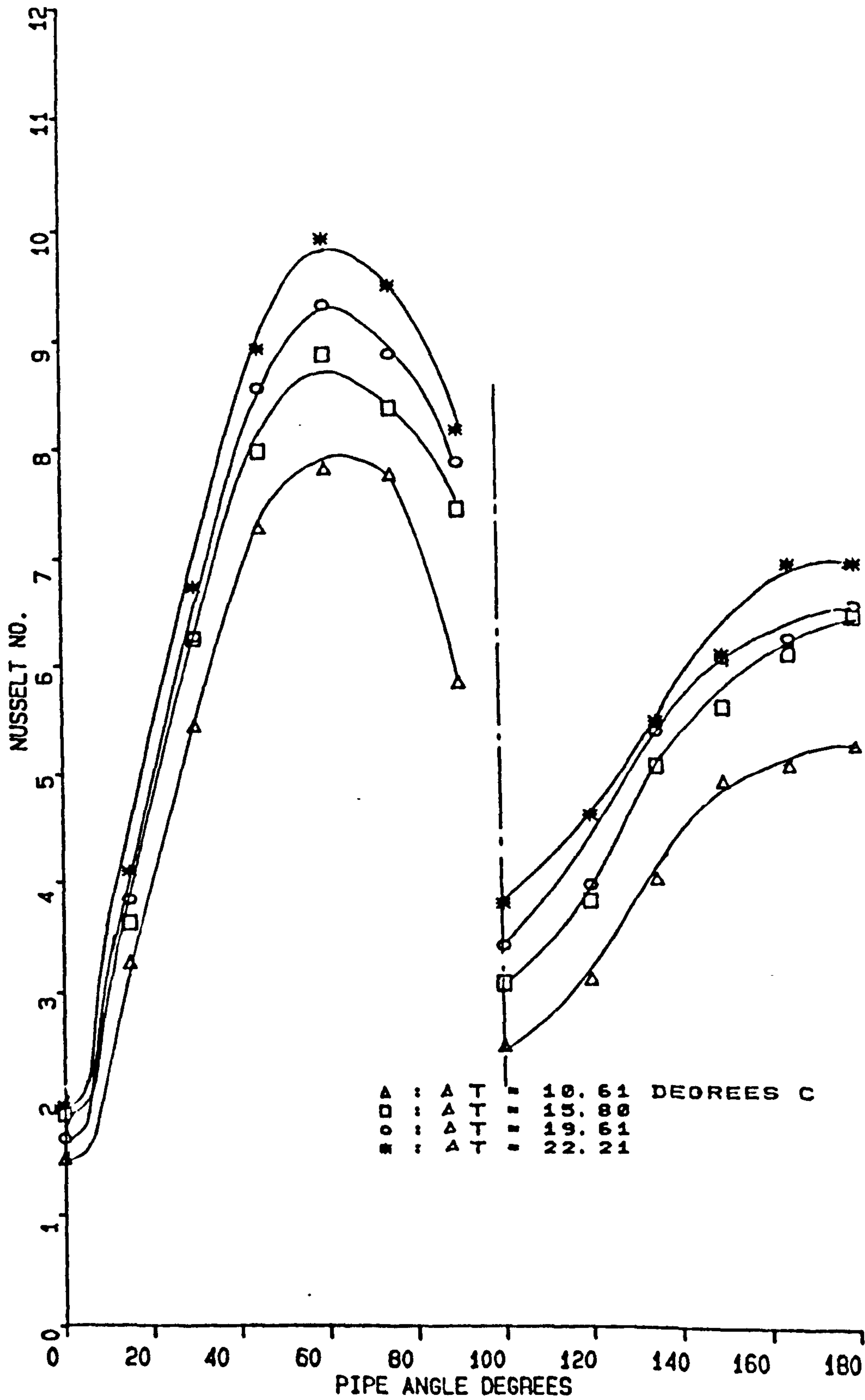


FIGURE 4.11

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0
 SPACER ANGLE = 90 DEGREES

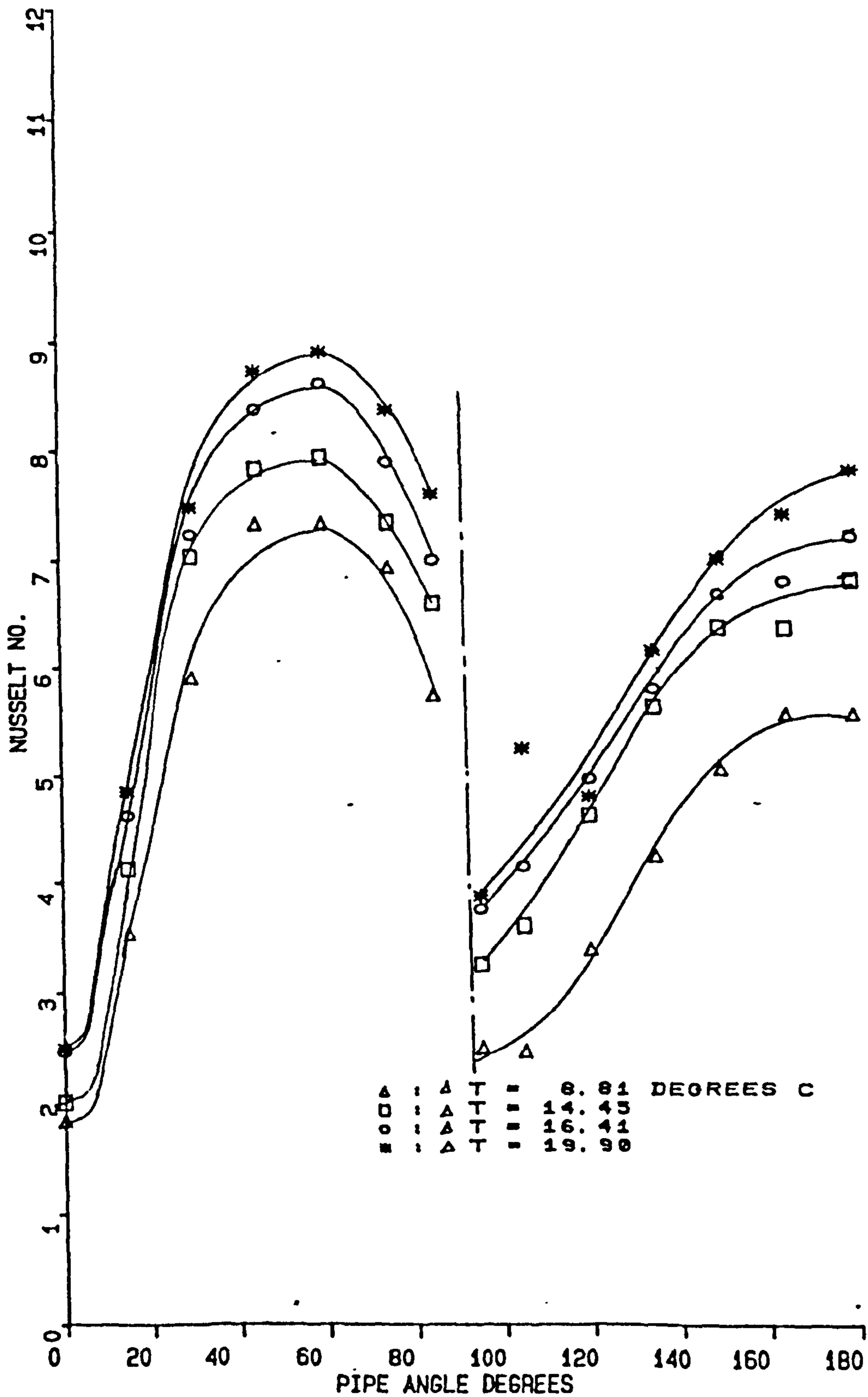


FIGURE 4. 12

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0.70
 NO SPACERS, PLAIN CAVITY

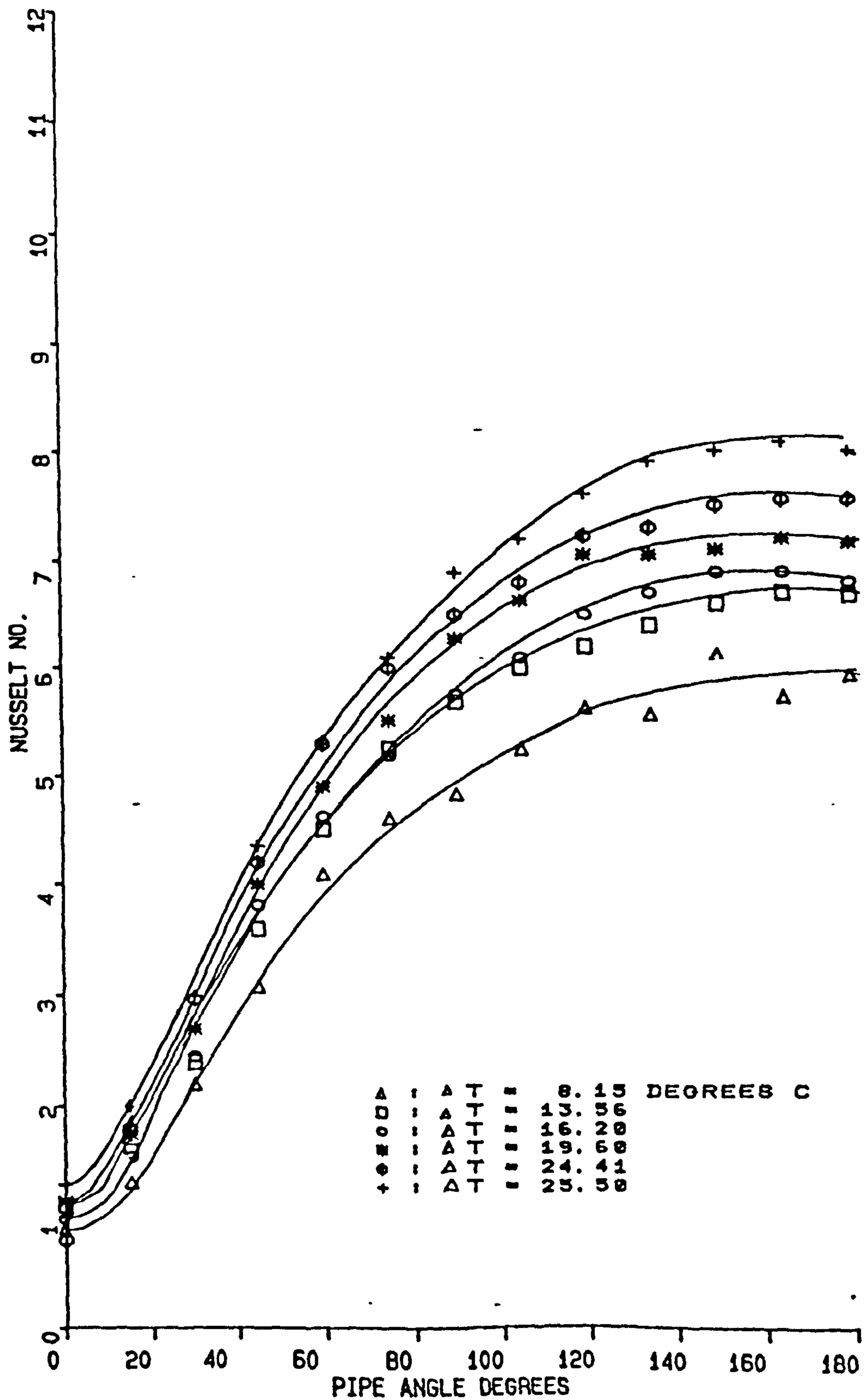


FIGURE 4. 13

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0.70
 SPACER ANGLE = 0 DEGREES

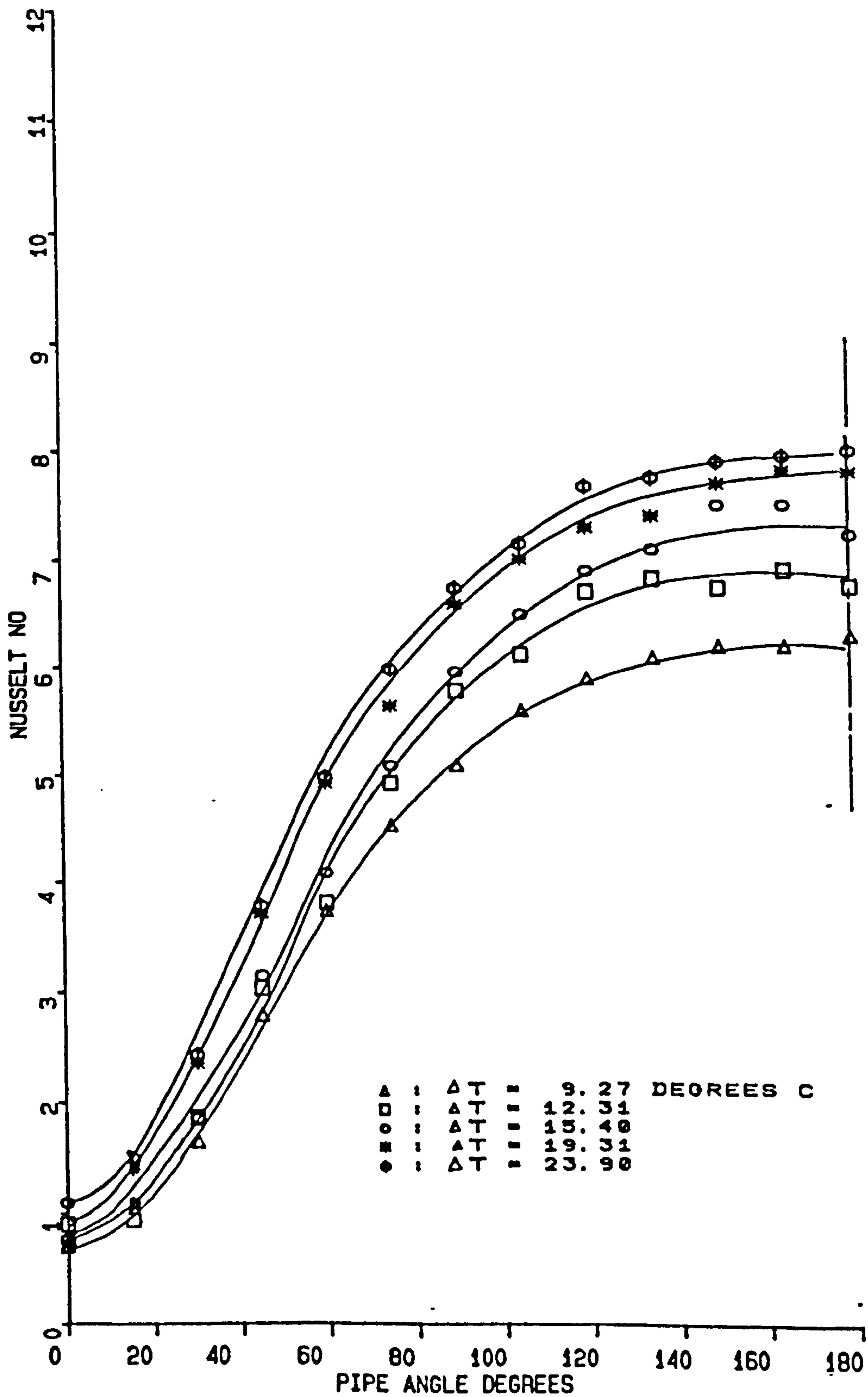


FIGURE 4. 14

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0.70
 SPACER ANGLE = 10 DEGREES

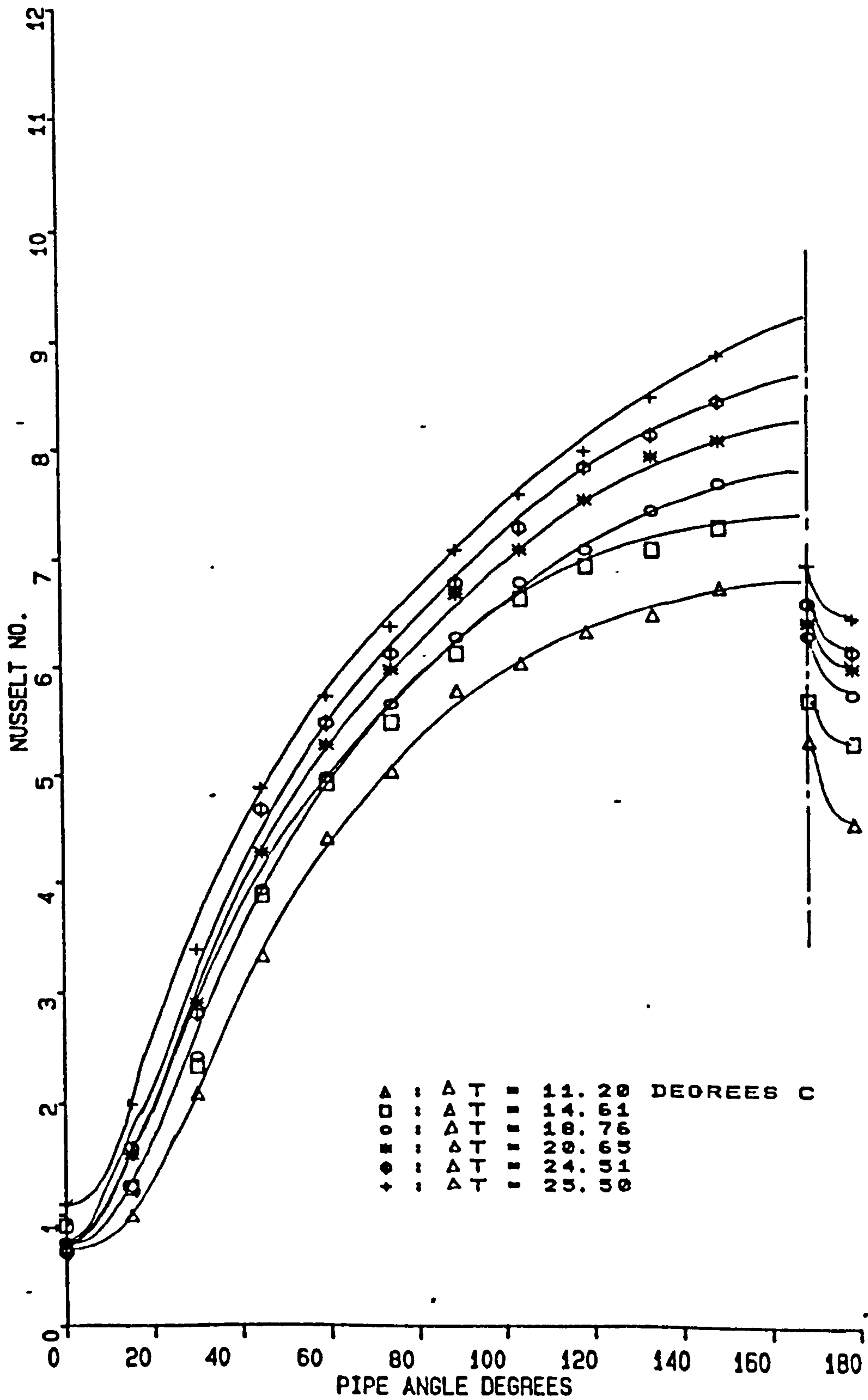


FIGURE 4.15

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0.70
 SPACER ANGLE = 20 DEGREES

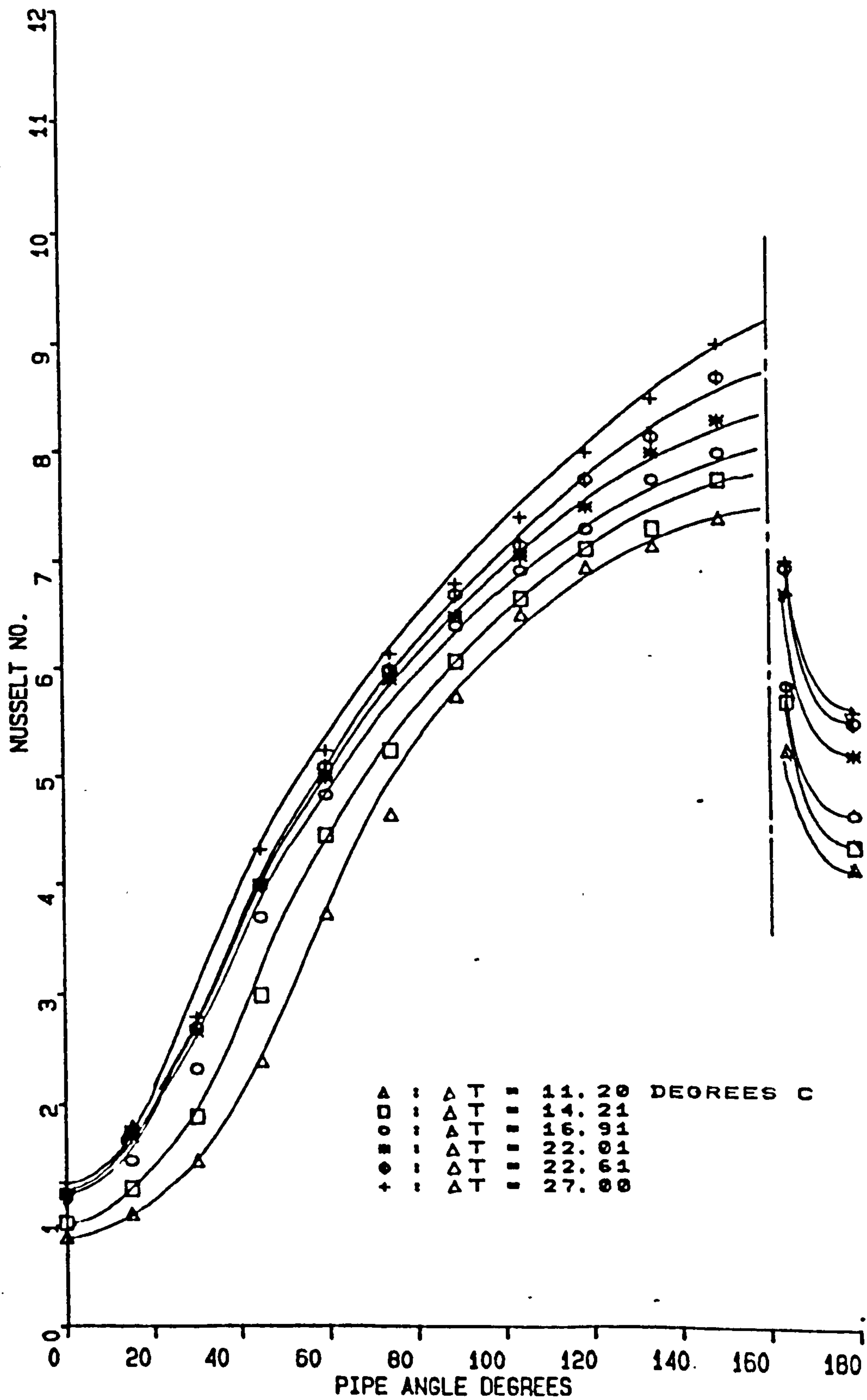


FIGURE 4. 16

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0.70
 SPACER ANGLE = 30 DEGREES

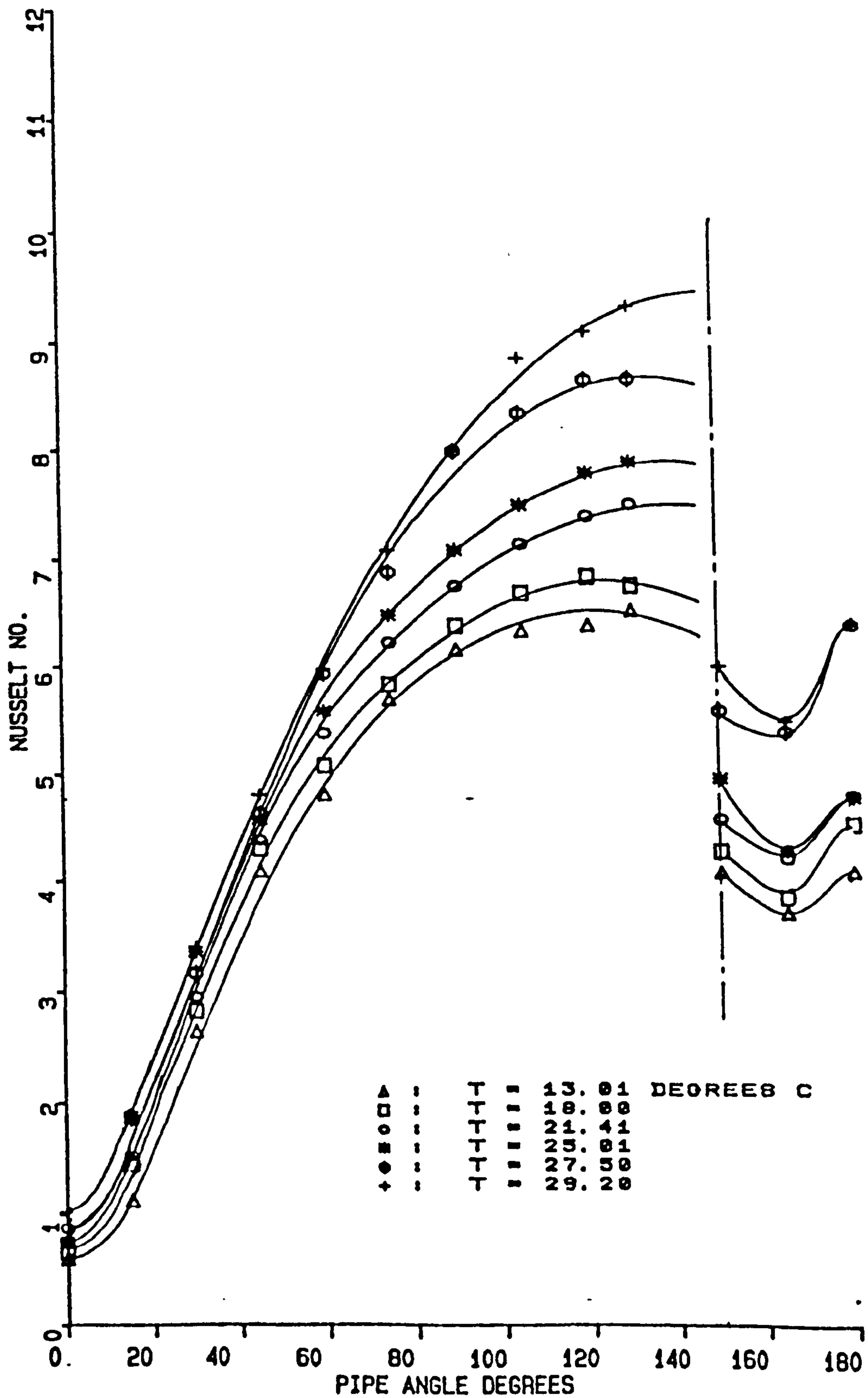


FIGURE 4.17

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0.70
 SPACER ANGLE = 50 DEGREES

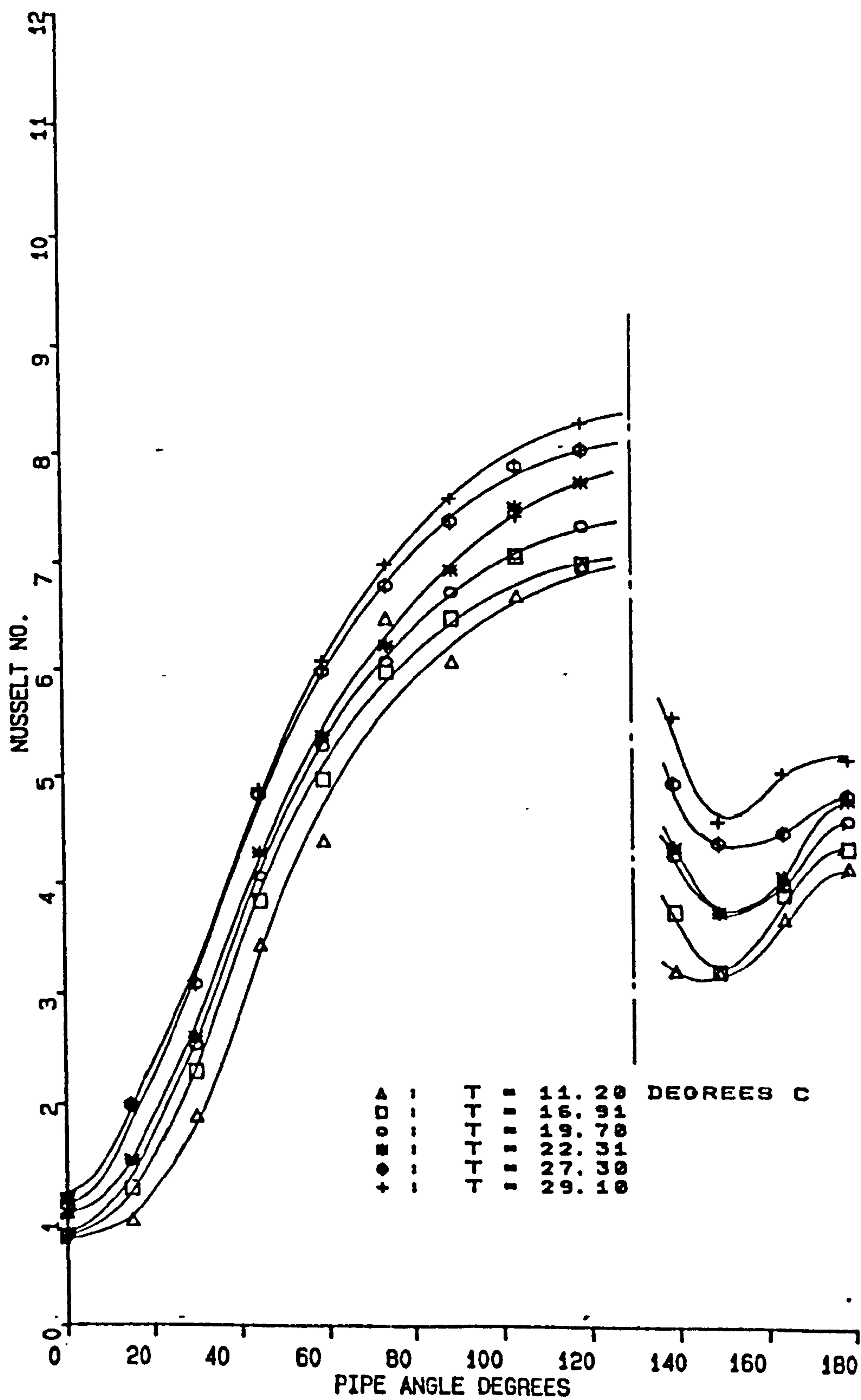


FIGURE 4. 18

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0.70
 SPACER ANGLE = 60 DEGREES

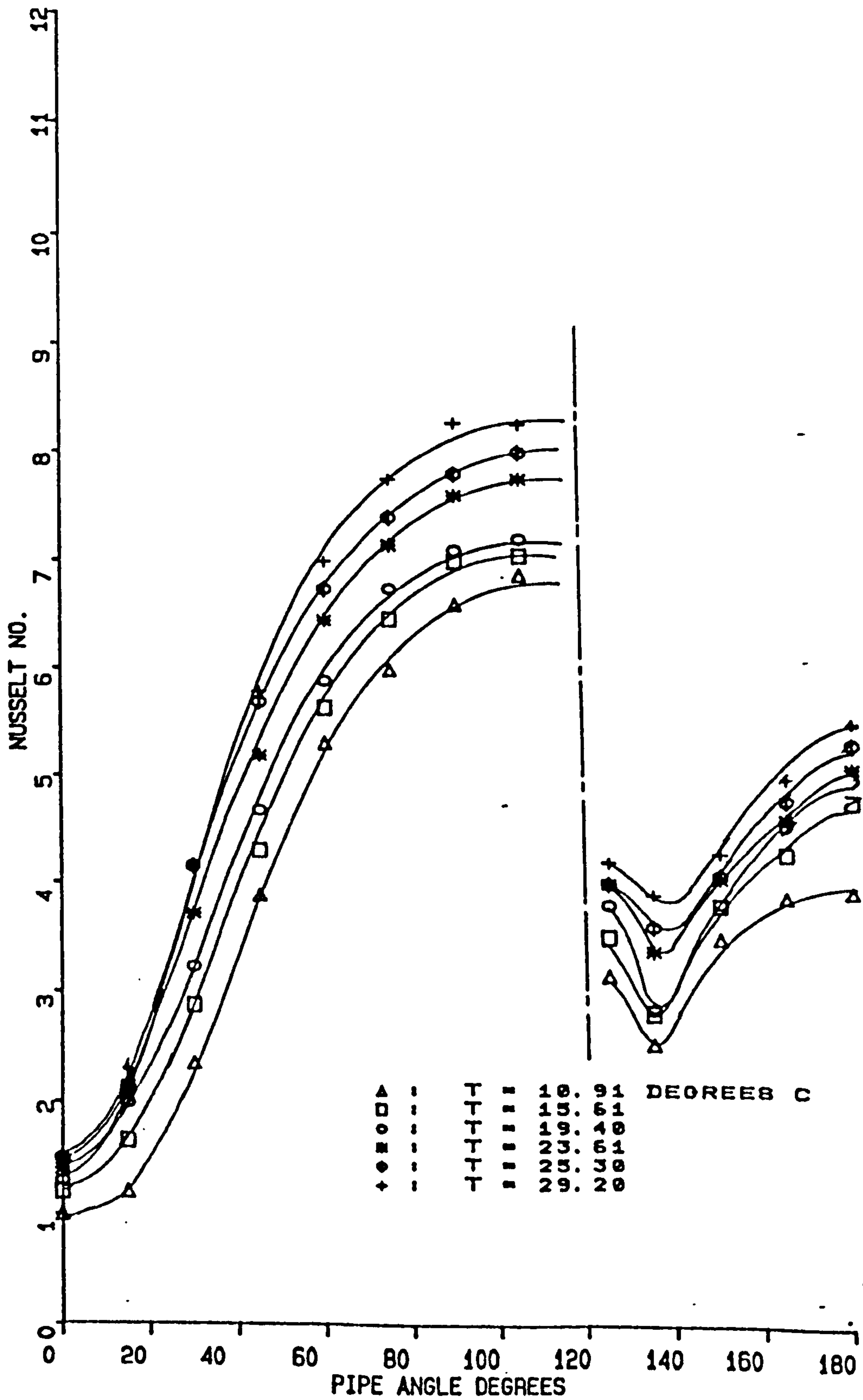


FIGURE 4.19

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0.70
 SPACER ANGLE = 70 DEGREES

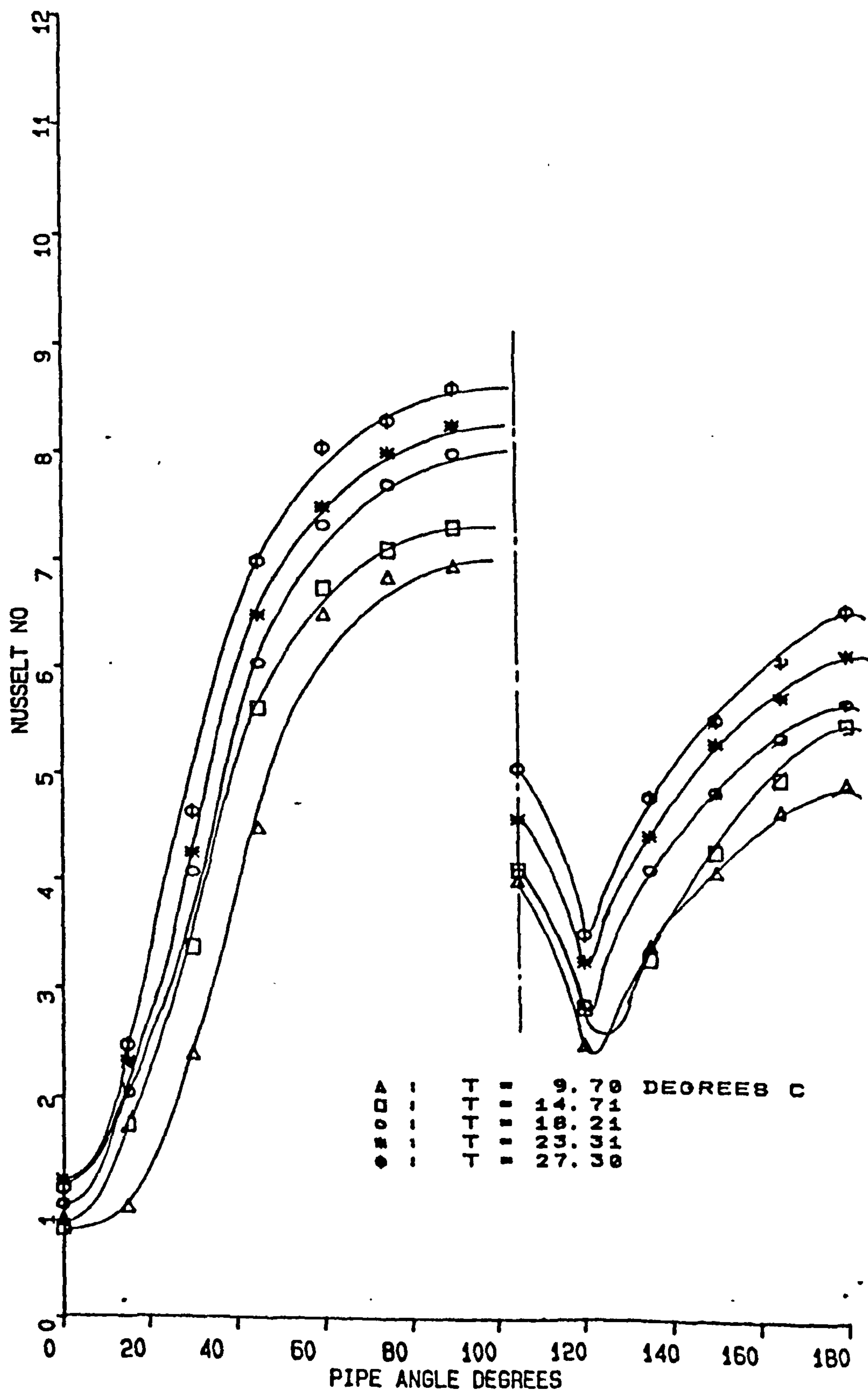


FIGURE 4.20

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 PIPE DISPLACEMENT RATIO = 0.70
 SPACER ANGLE = 90 DEGREES

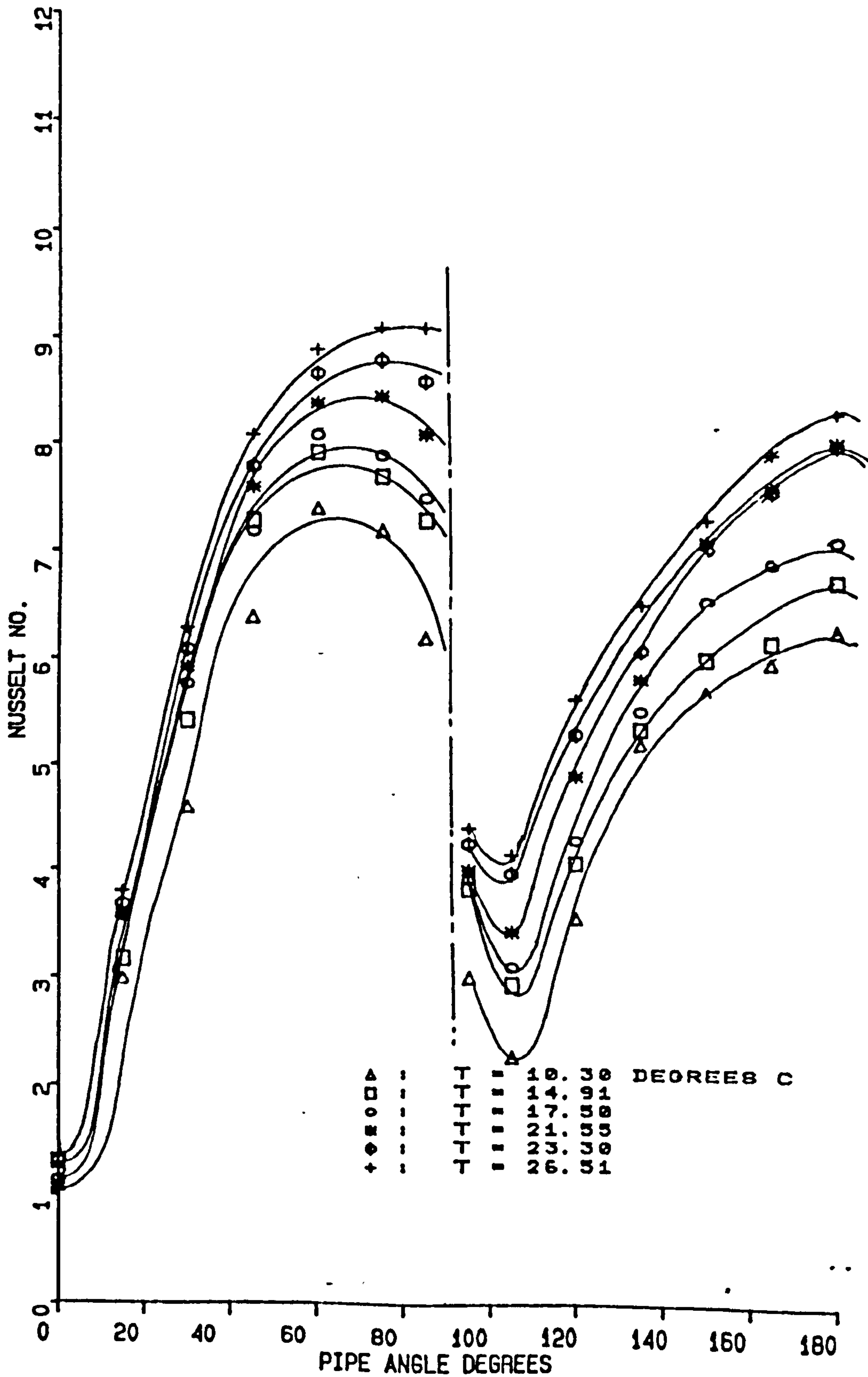


Figure : 4.21

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
No Spacers

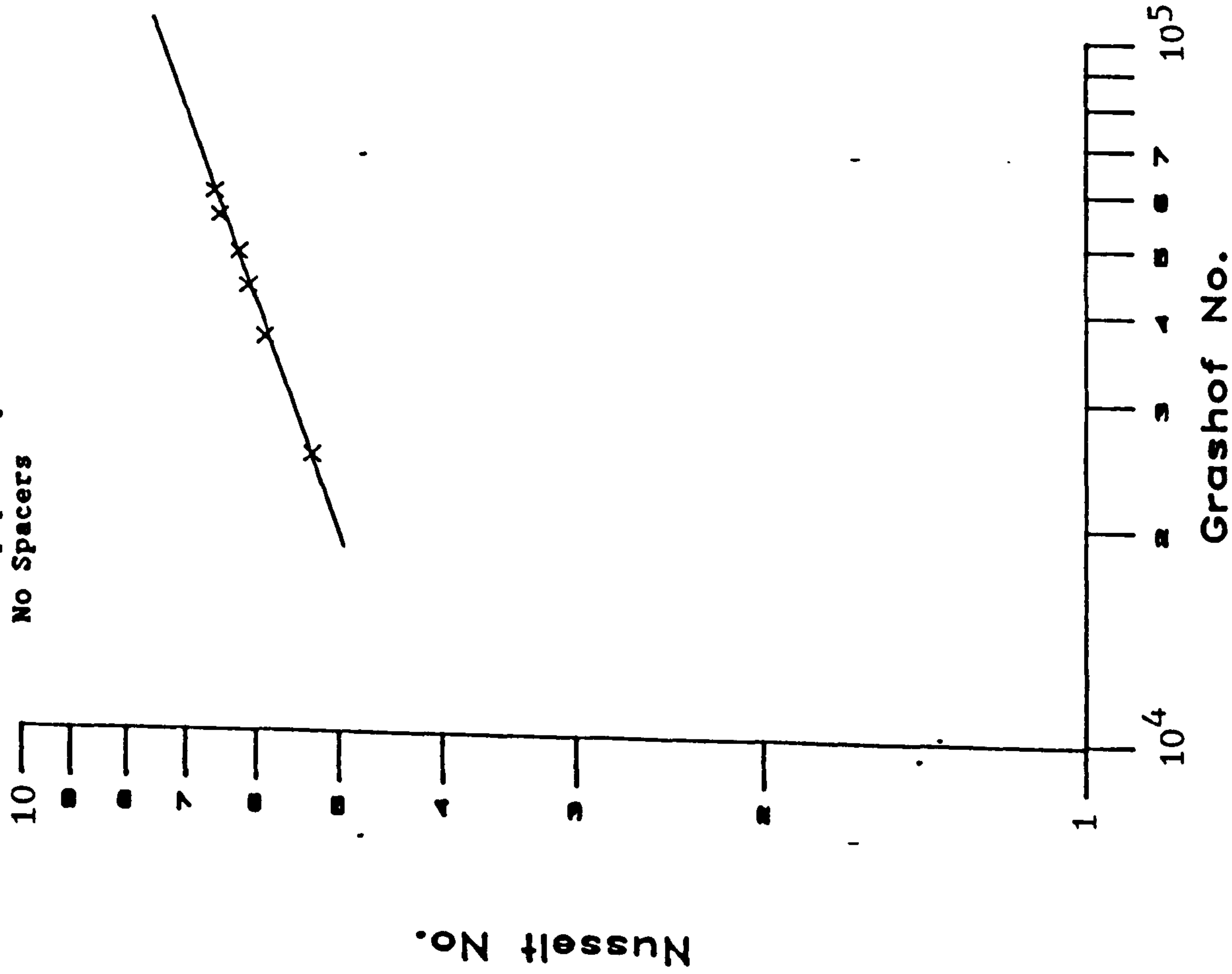


Figure : 4.22

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
Spacer Position = 0 (degrees)

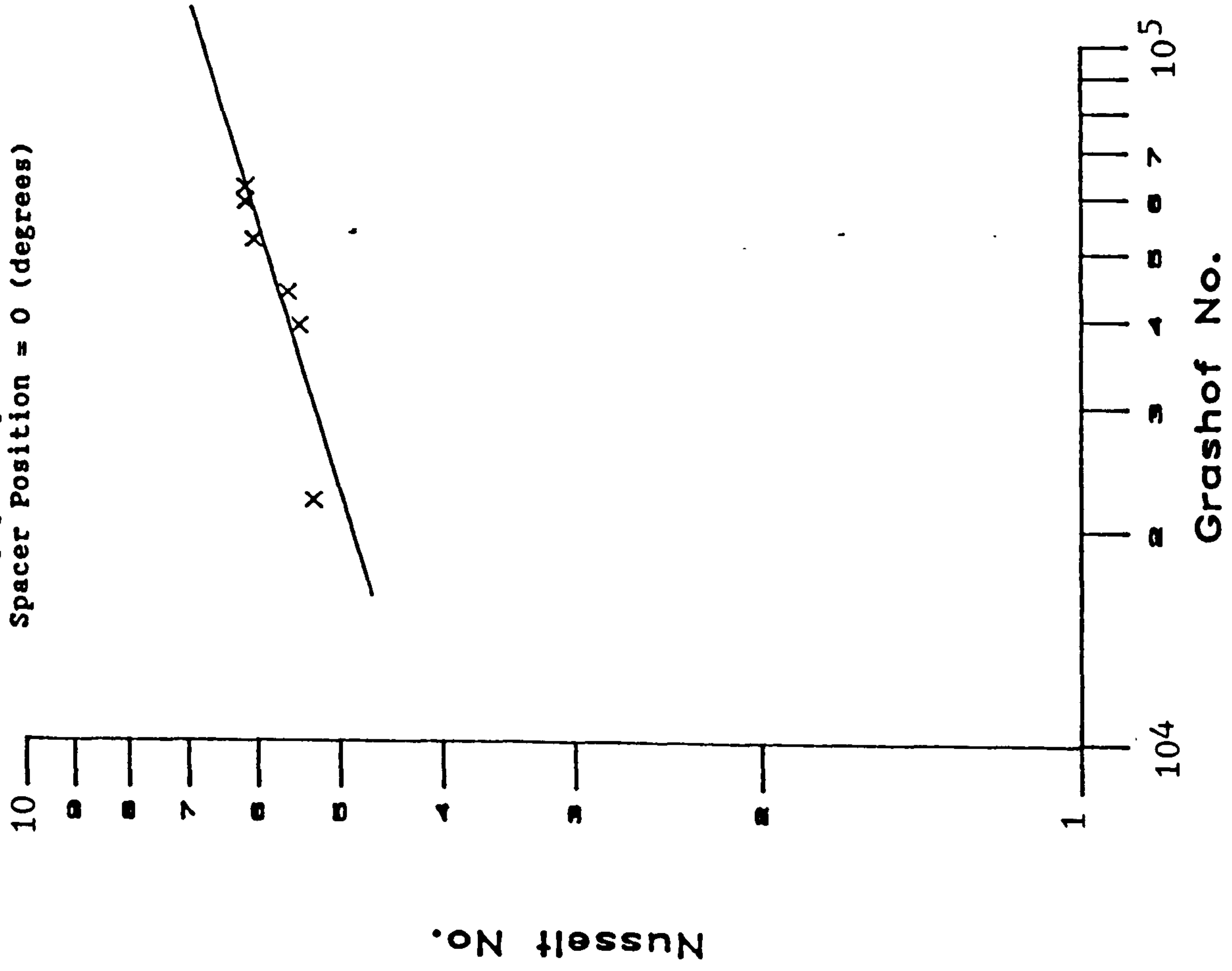


Figure : 4.23

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
Spacer Position = 10 (degrees)

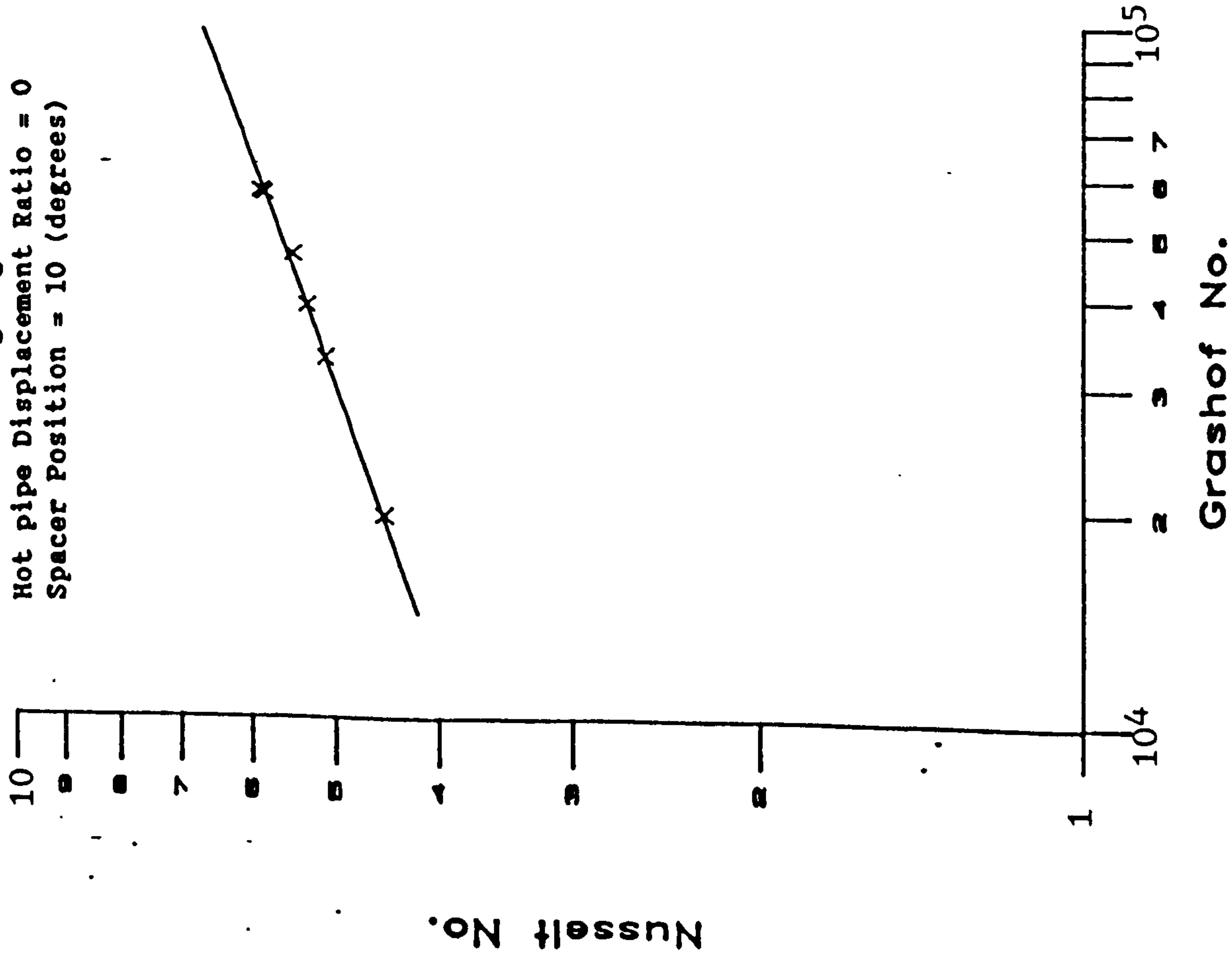


Figure : 4.24

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
Spacer Position = 20 (degrees)

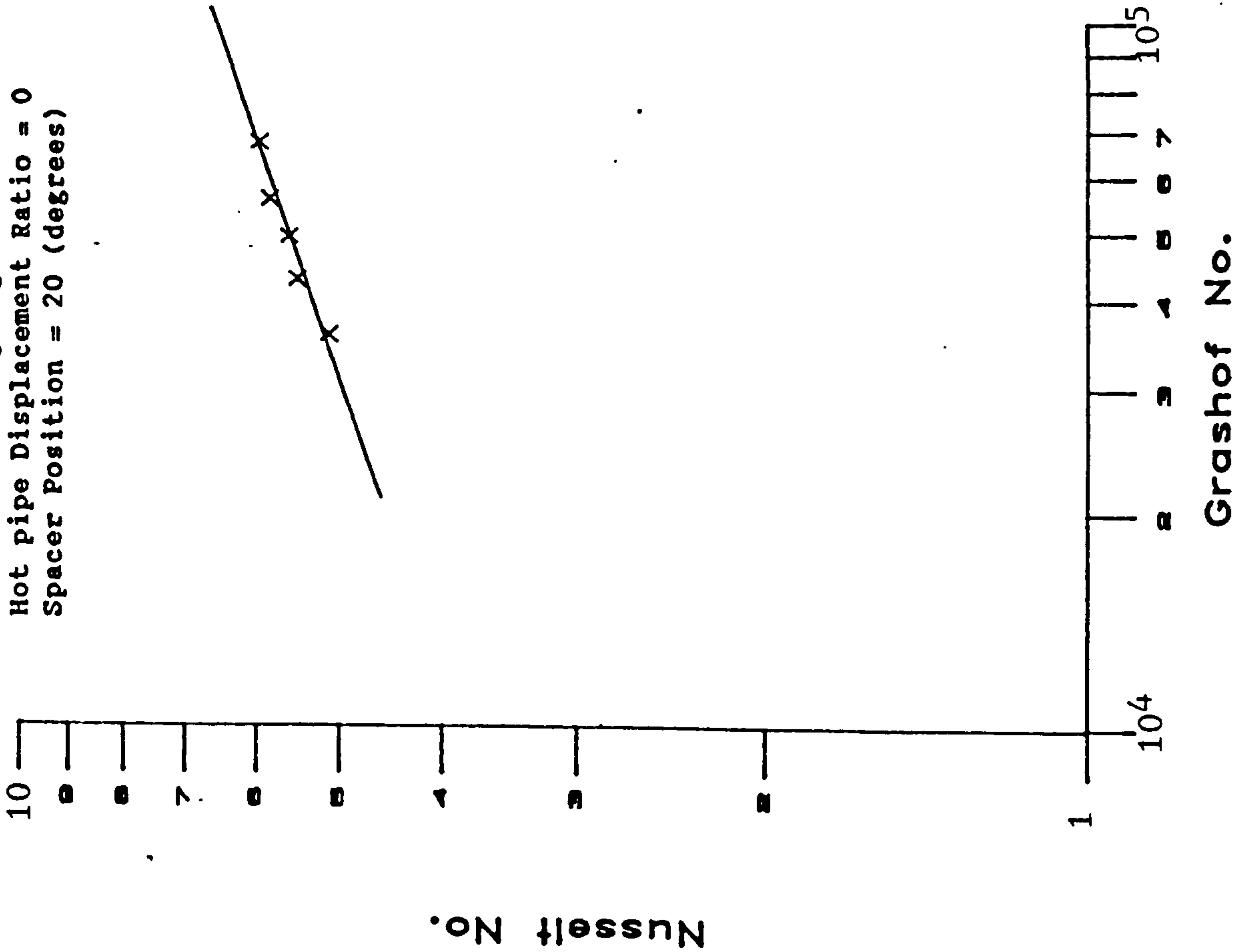


Figure : 4.25

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
Spacer Position = 30 (degrees)

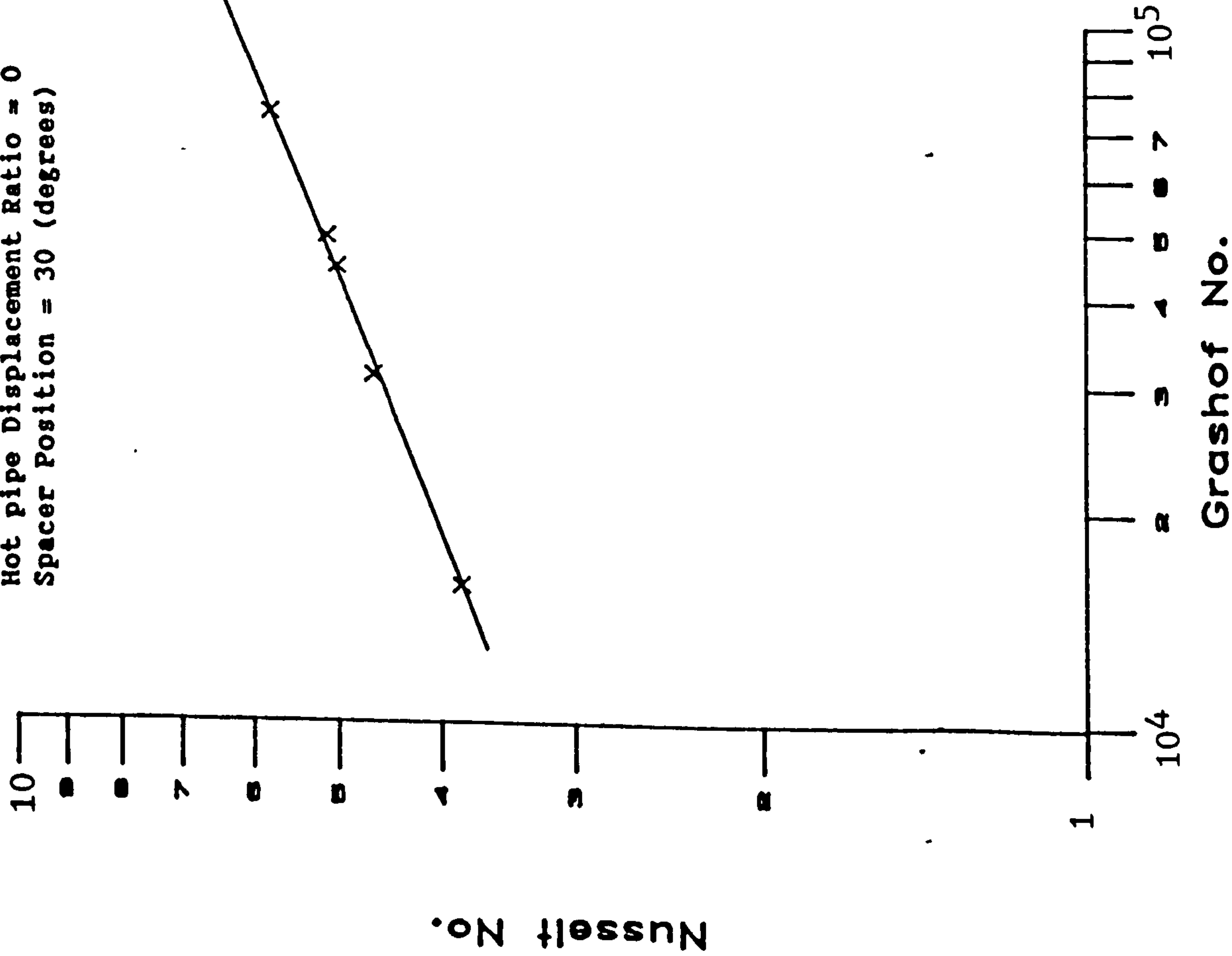


Figure : 4.26

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
Spacer Position = 40 (degrees)

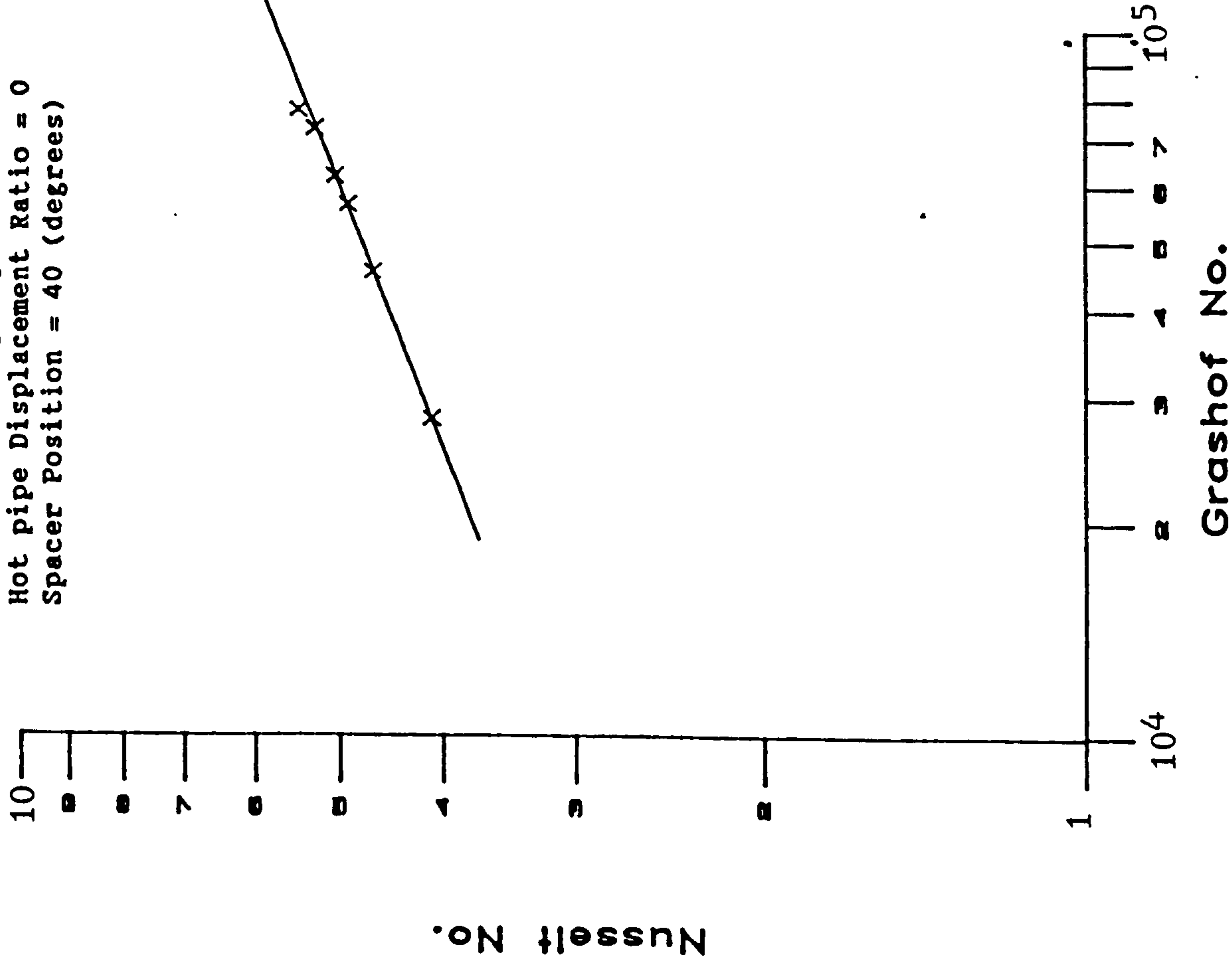


Figure : 4.27

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
Spacer Position = 50 (degrees)

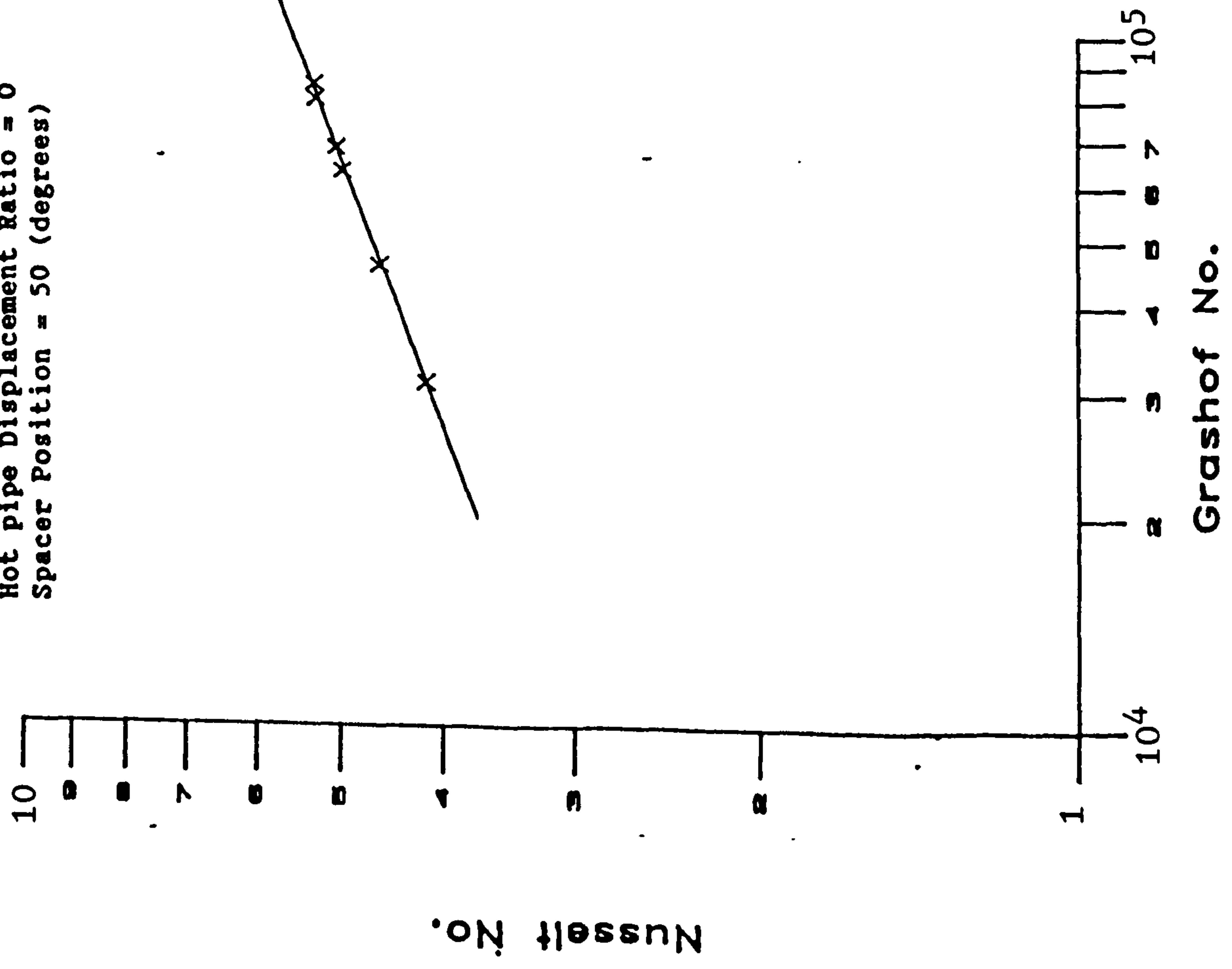


Figure : 4.28

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
Spacer Position = 60 (degrees)

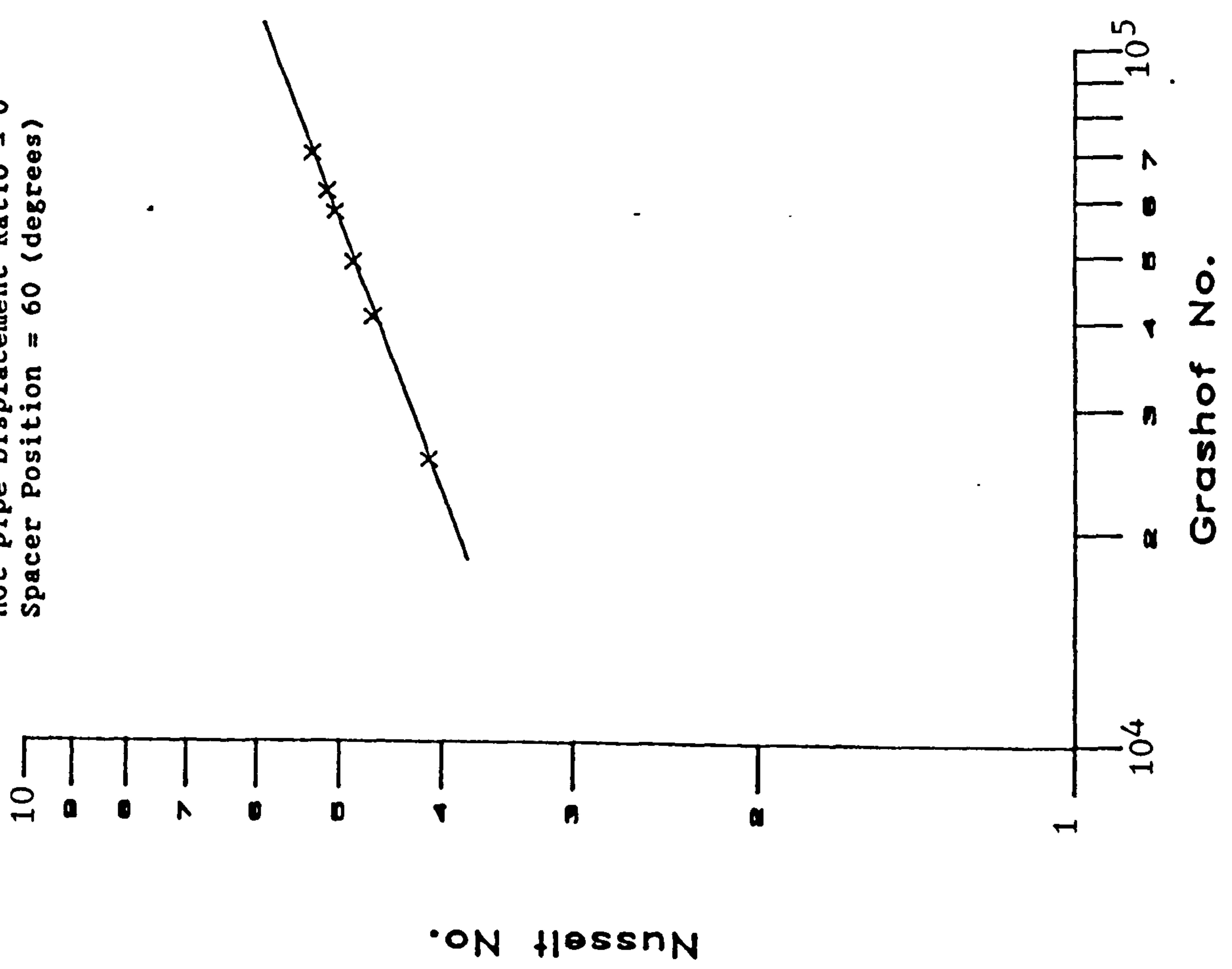


Figure : 4.29

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
Spacer Position = 70 (degrees)

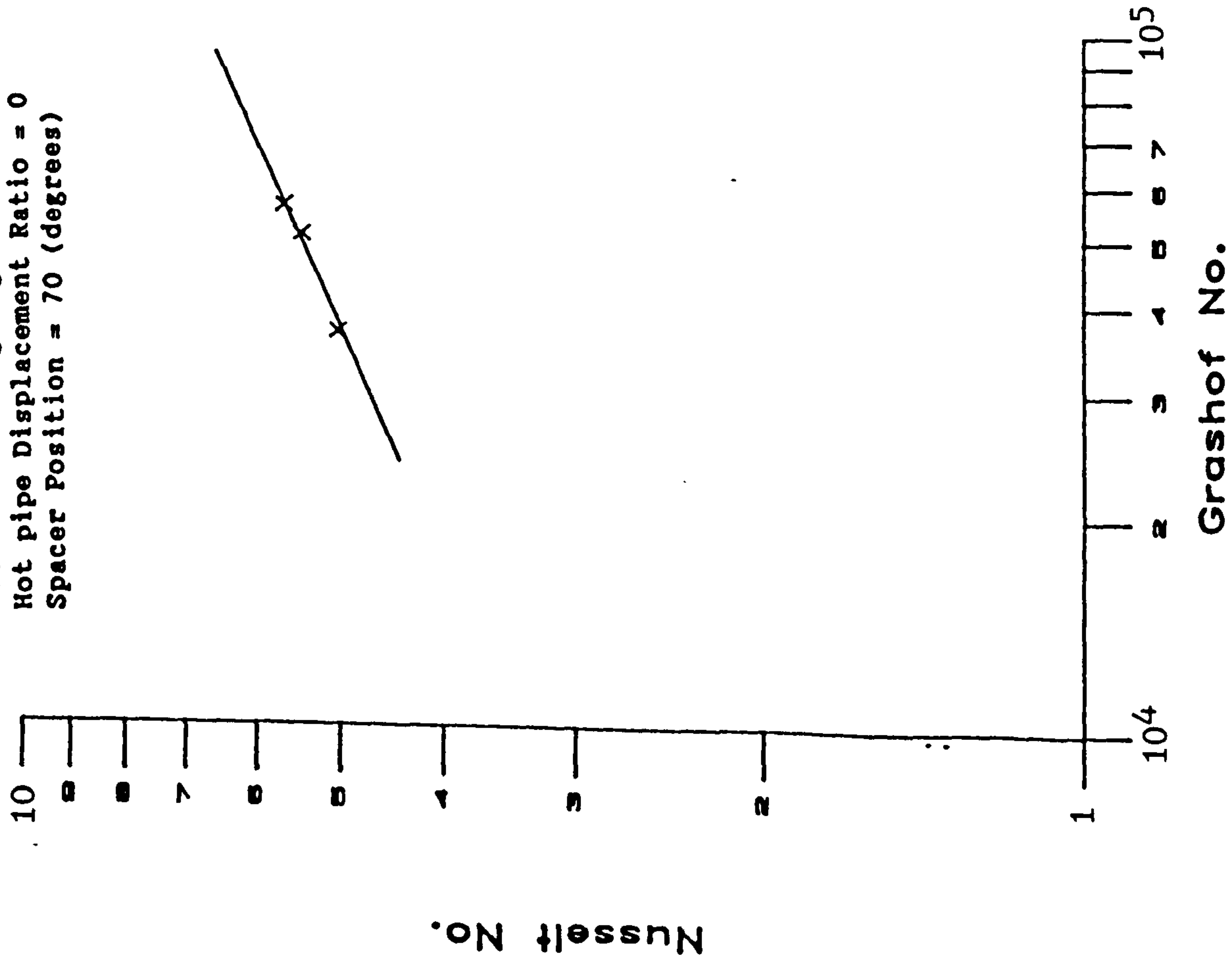


Figure : 4.30

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0
Spacer Position = 80 (degrees)

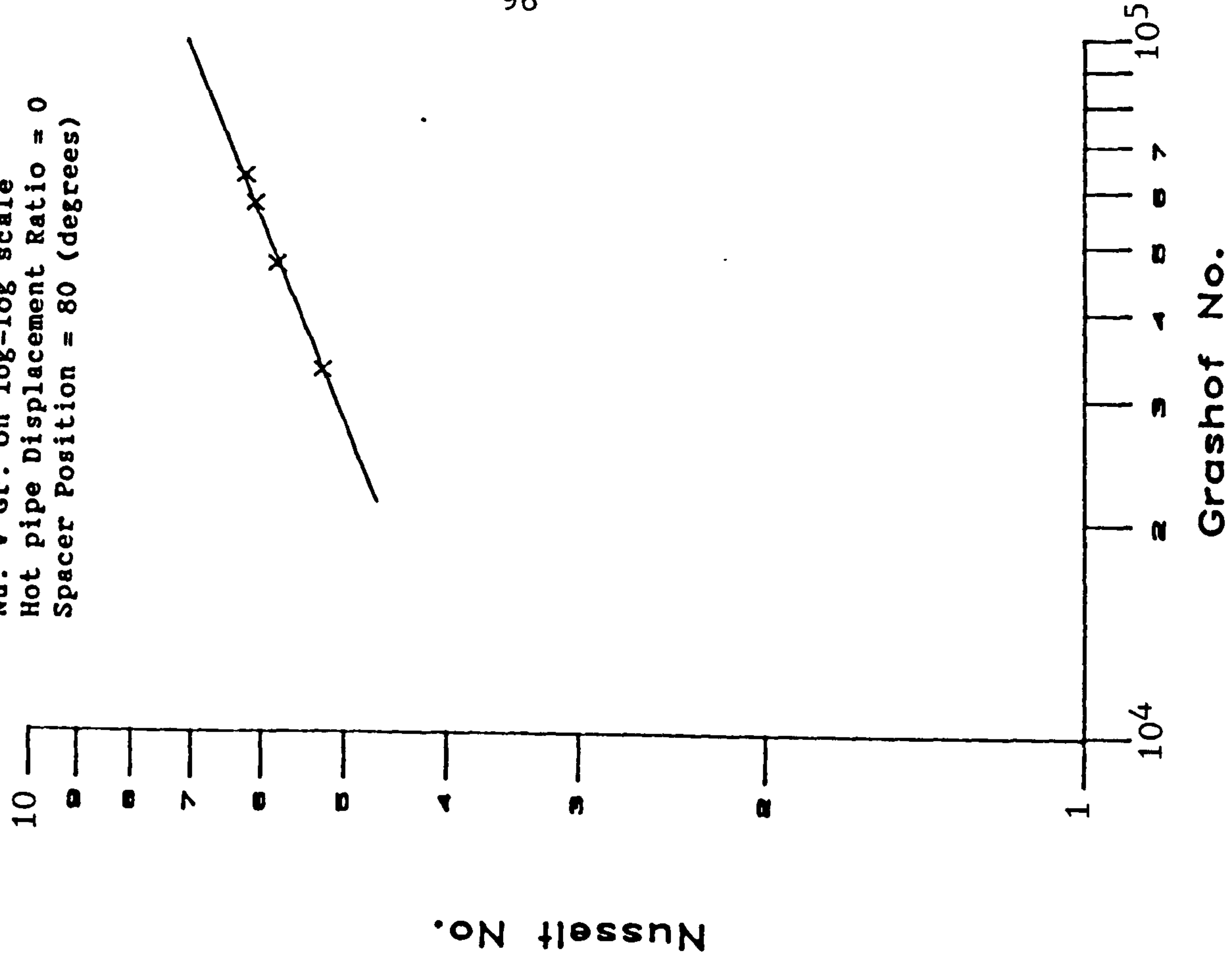


Figure : 4.31

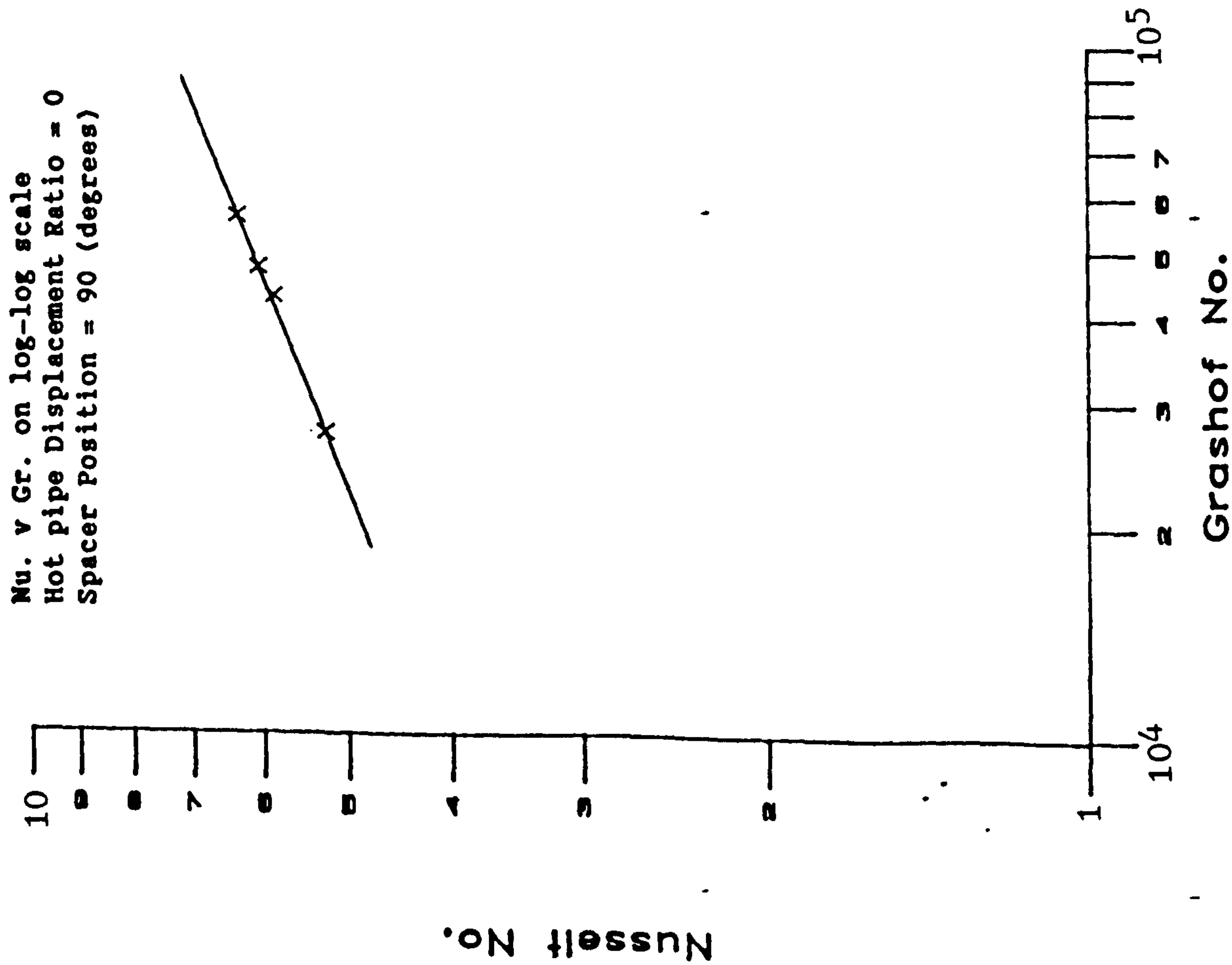


Figure : 4.32

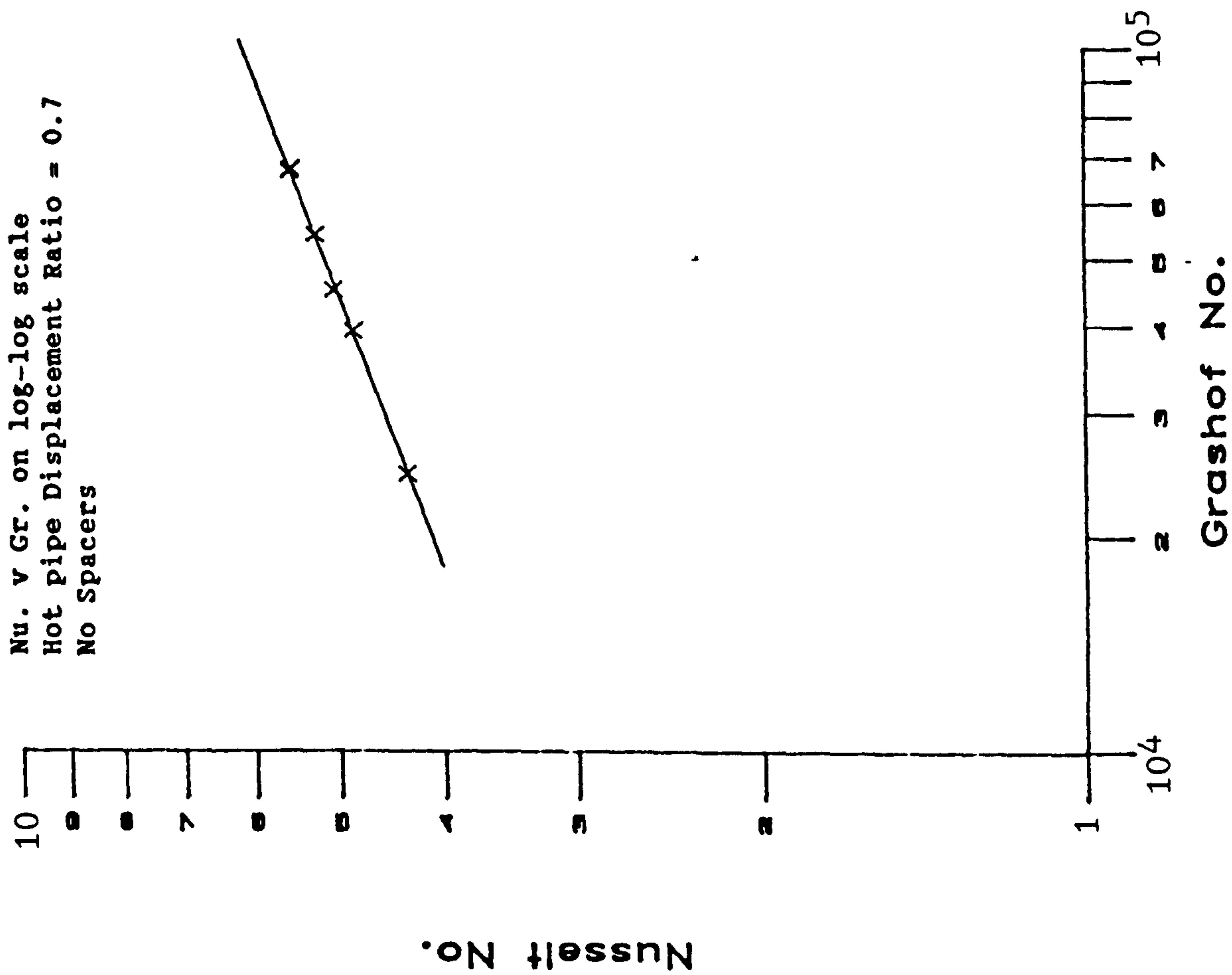


Figure : 4.33

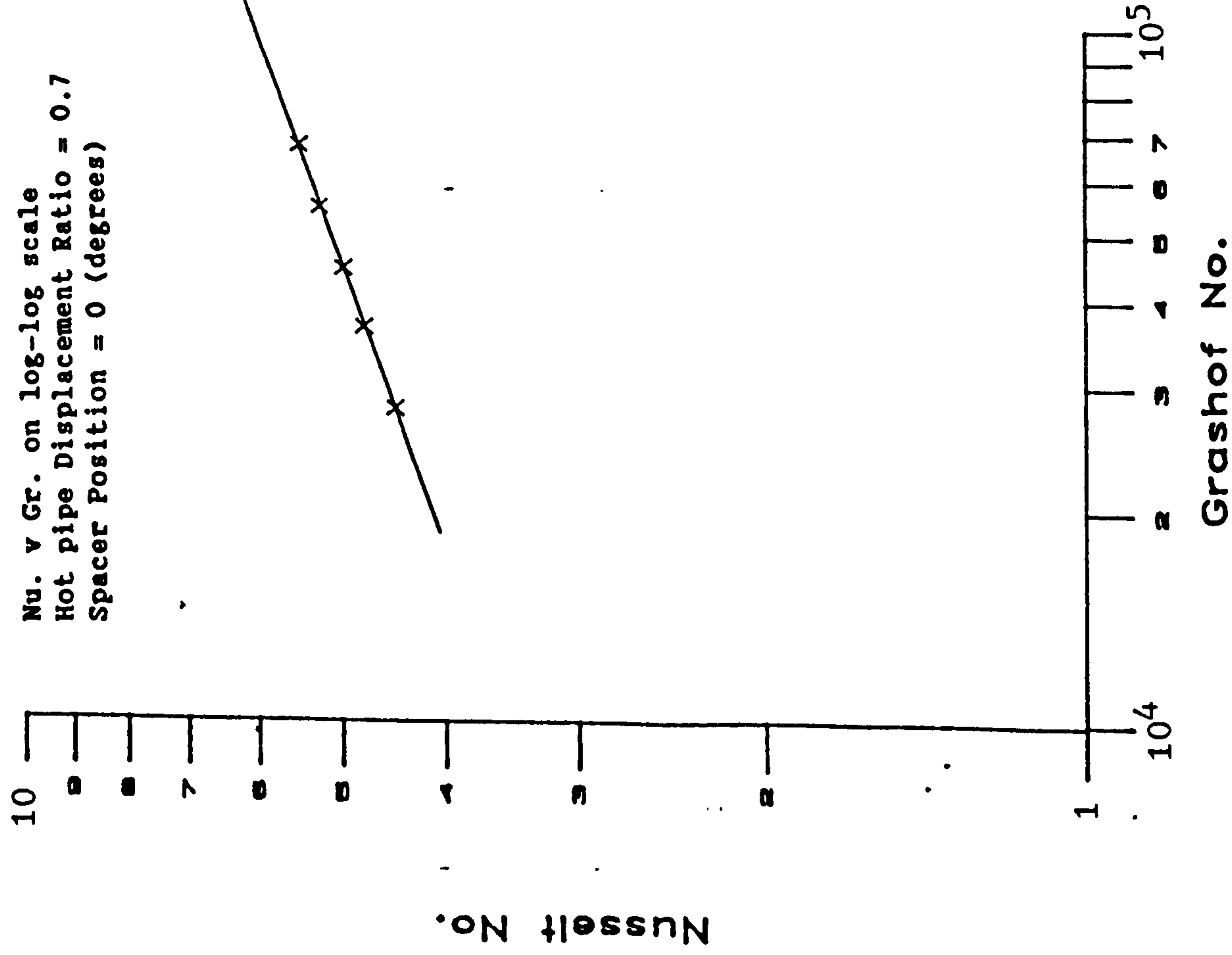


Figure : 4.34

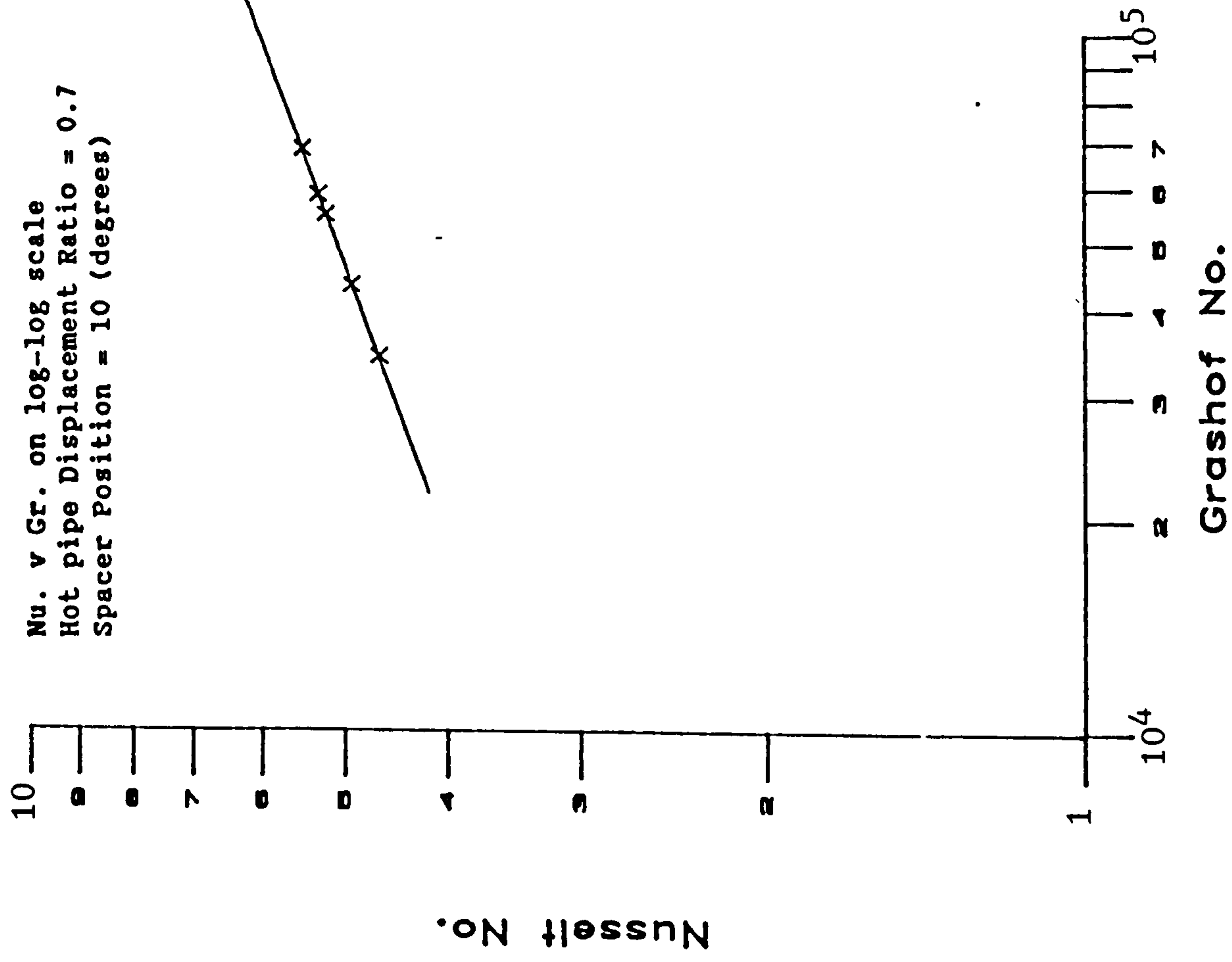


Figure : 4.35

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0.7
Spacer Position = 20 (degrees)

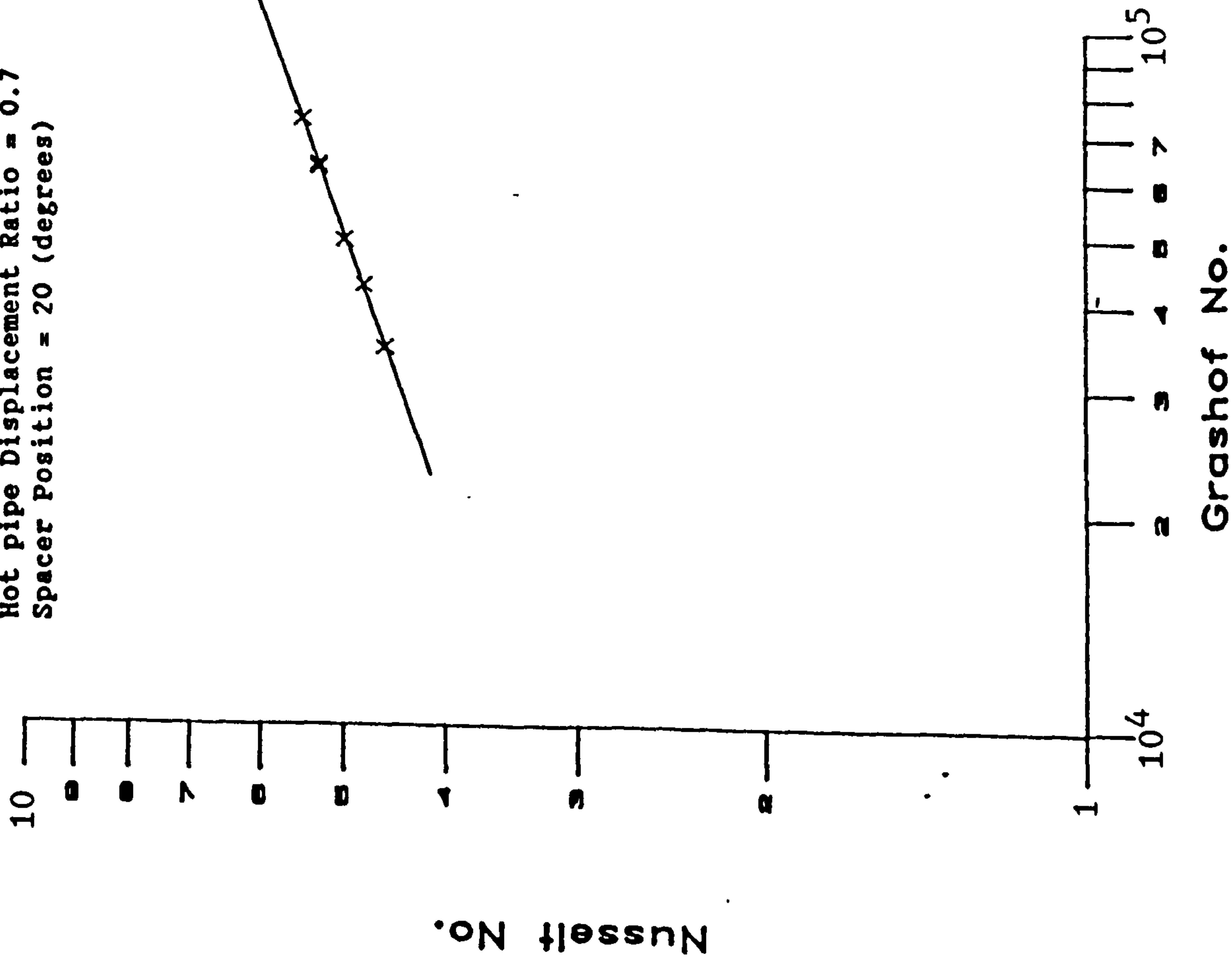


Figure : 4.36

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0.7
Spacer Position = 30 (degrees)

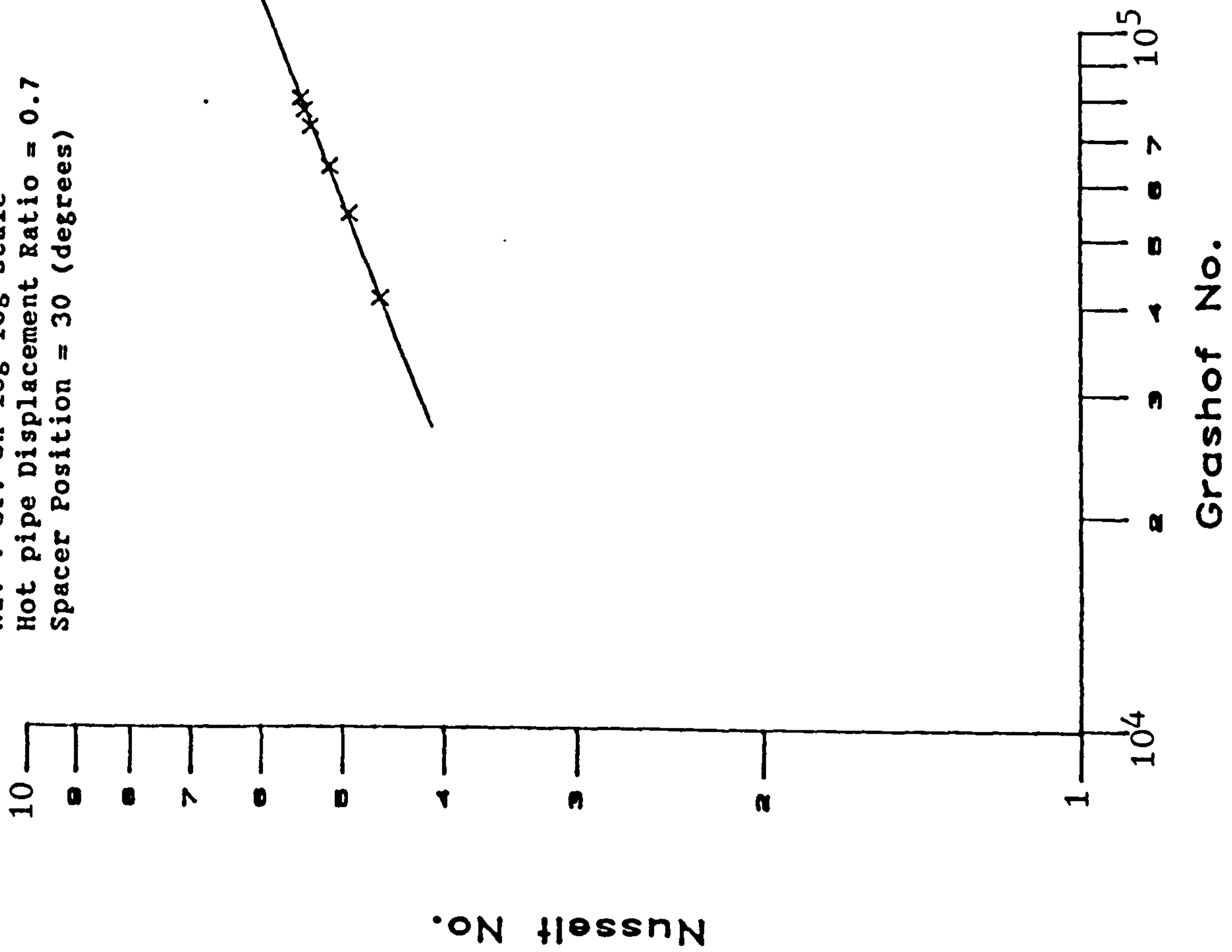


Figure : 4.37

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0.7
Spacer Position = 50 (degrees)

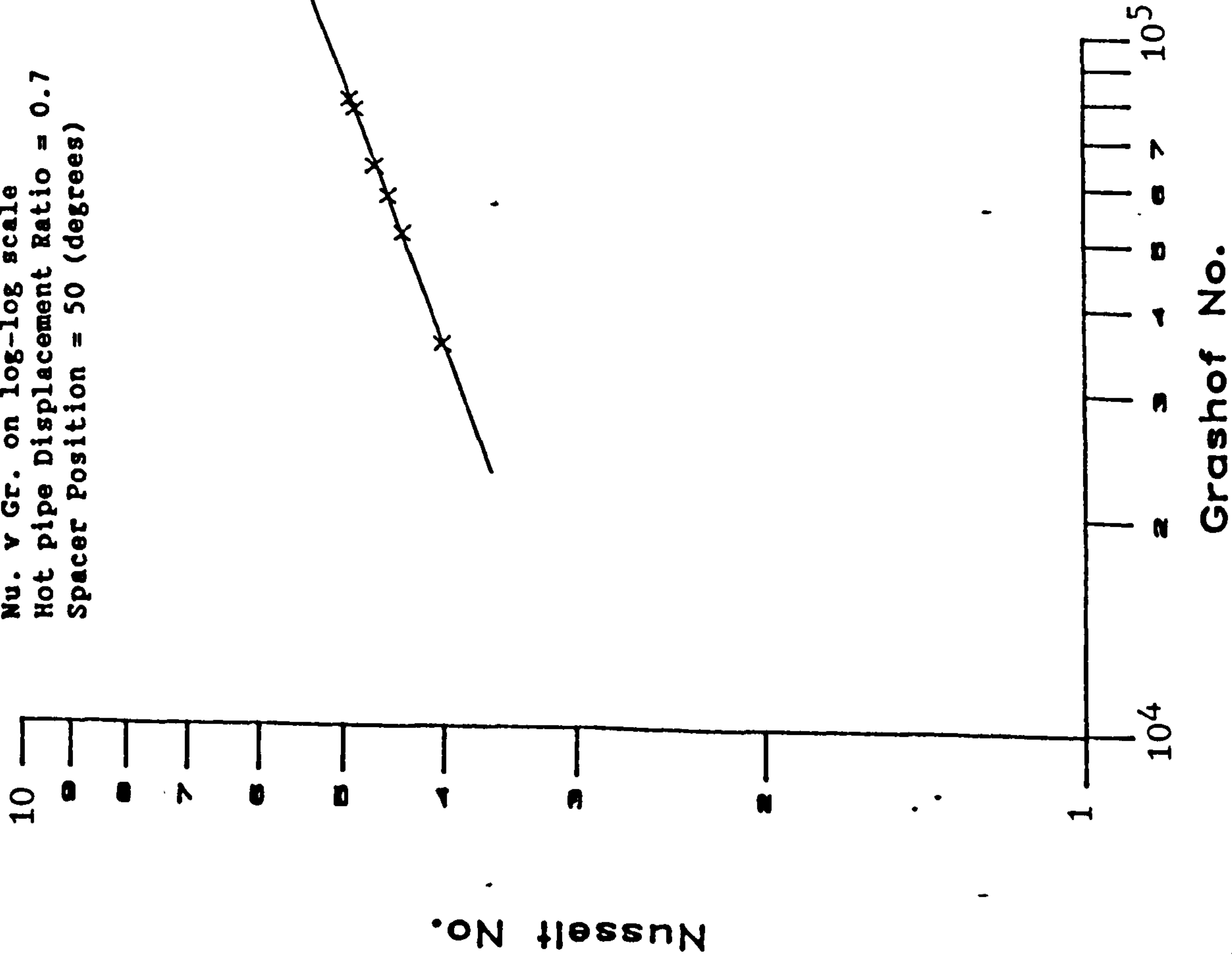


Figure : 4.38

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0.7
Spacer Position = 60 (degrees)

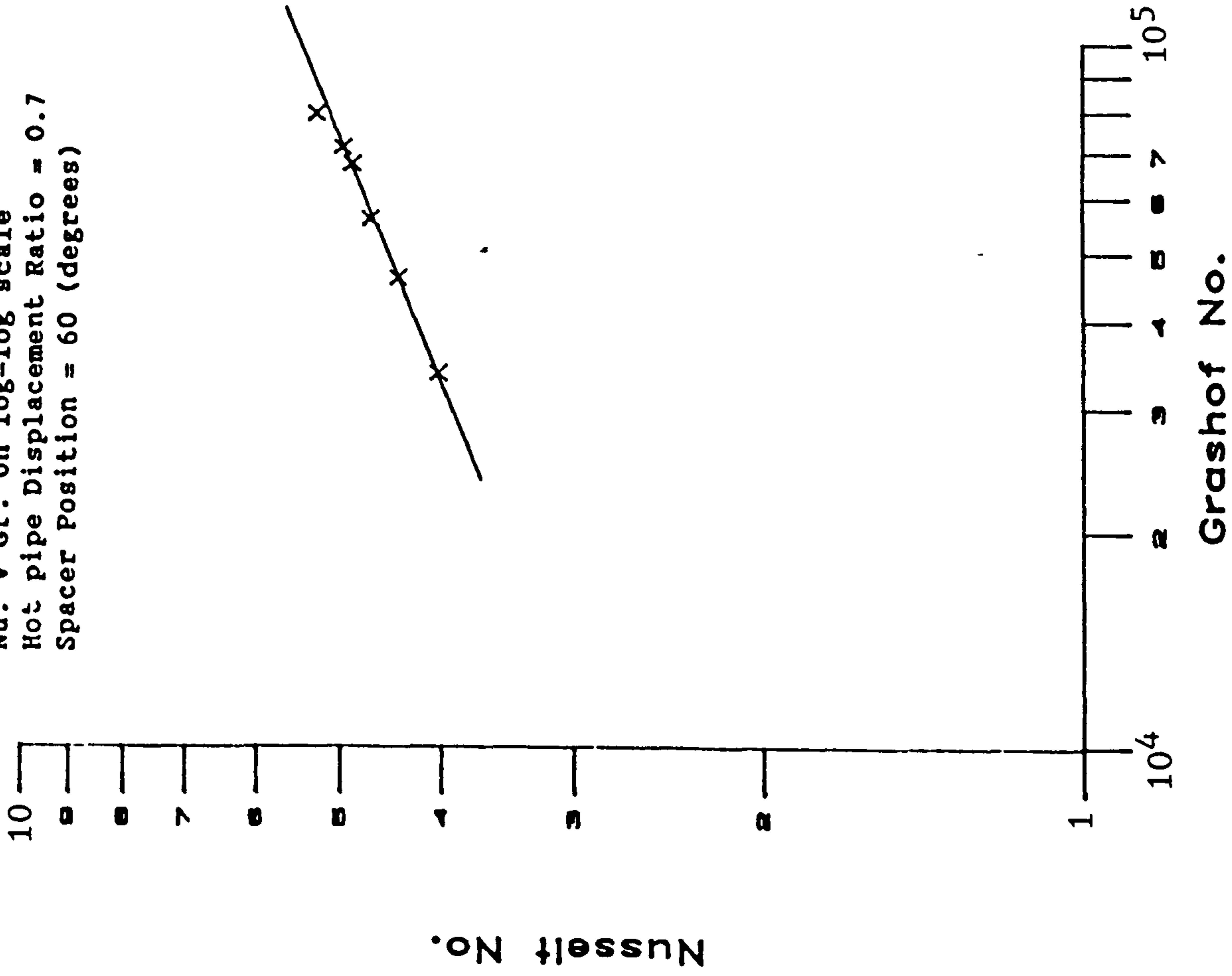


Figure : 4.39

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0.7
Spacer Position = 70 (degrees)

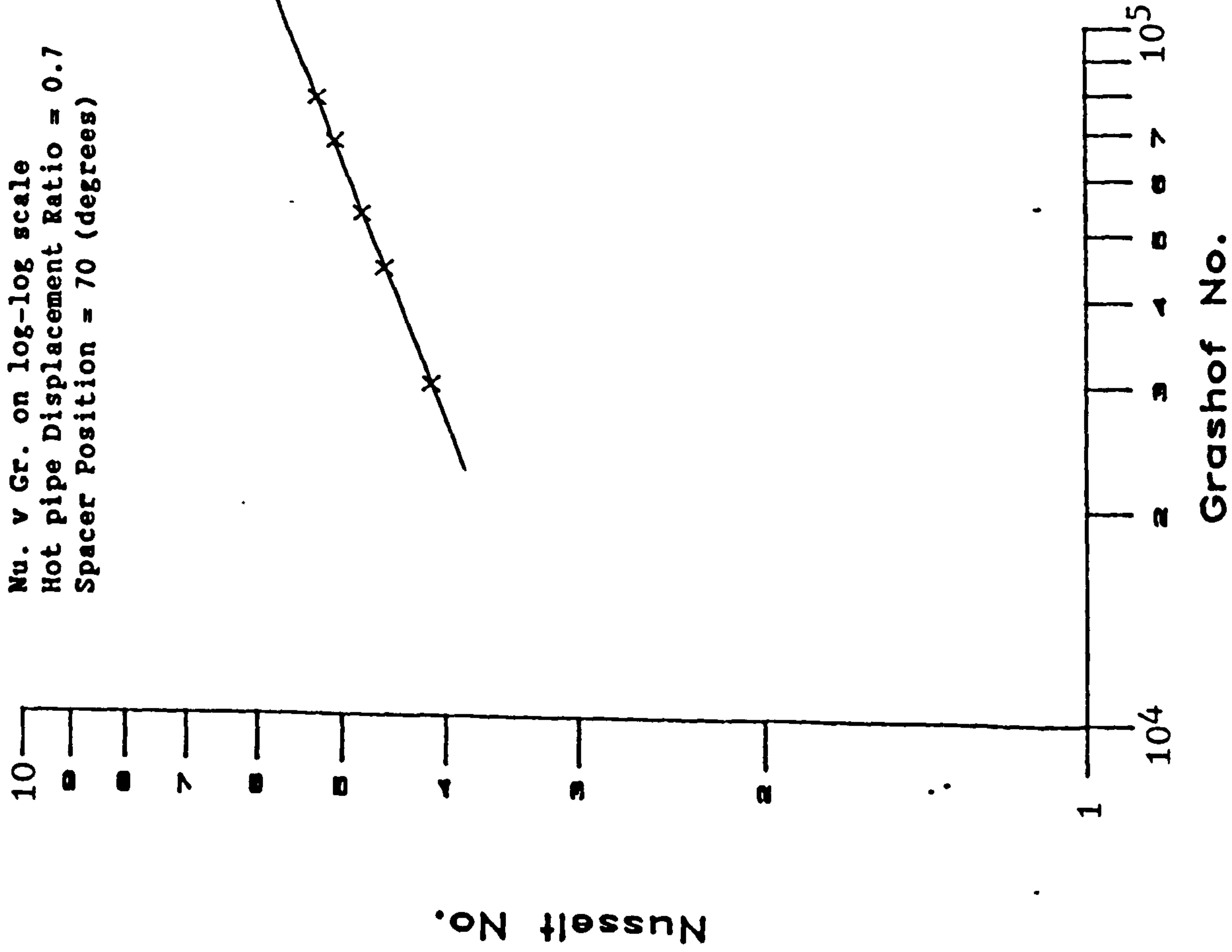


Figure : 4.40

Nu. v Gr. on log-log scale
Hot pipe Displacement Ratio = 0.7
Spacer Position = 90 (degrees)

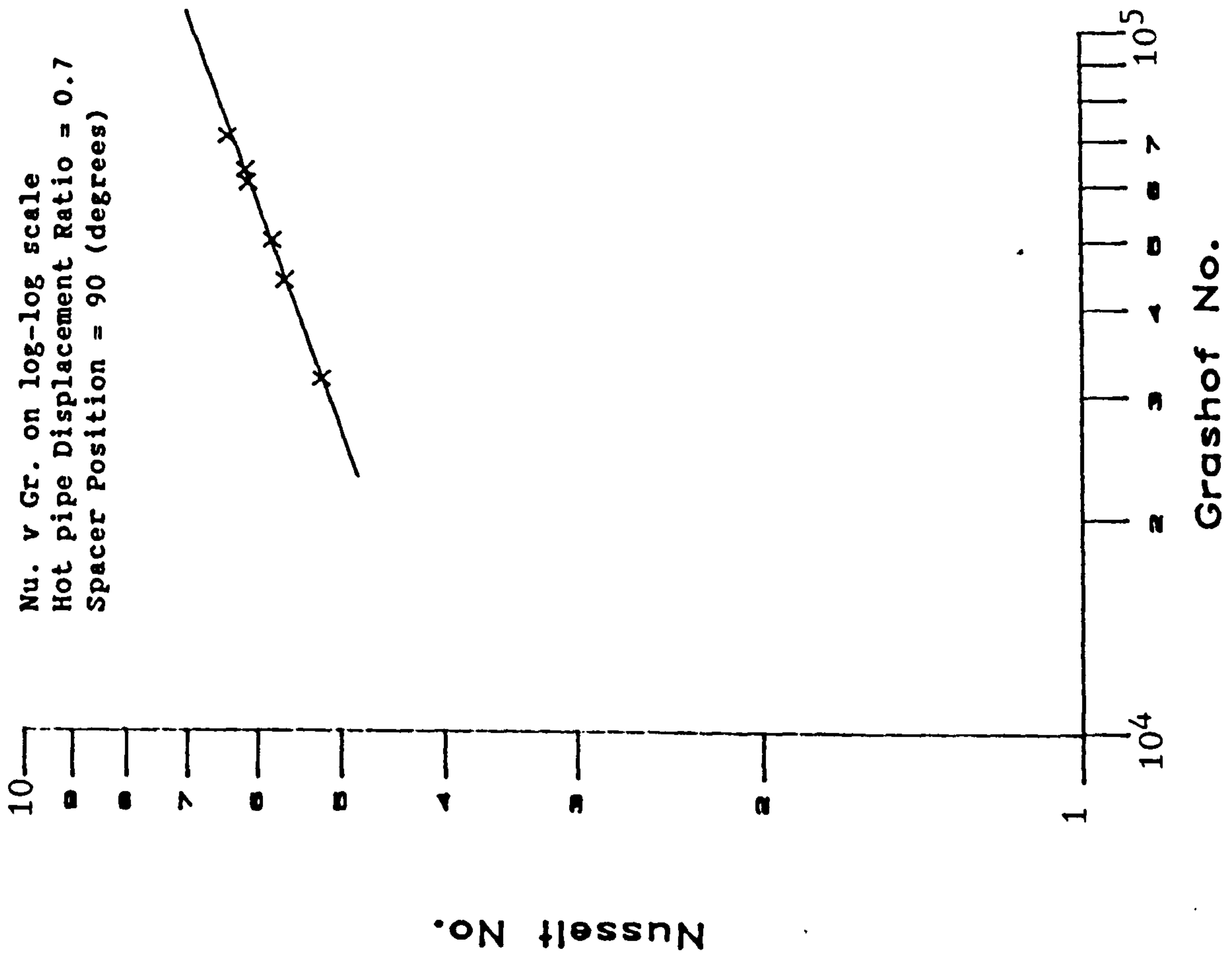
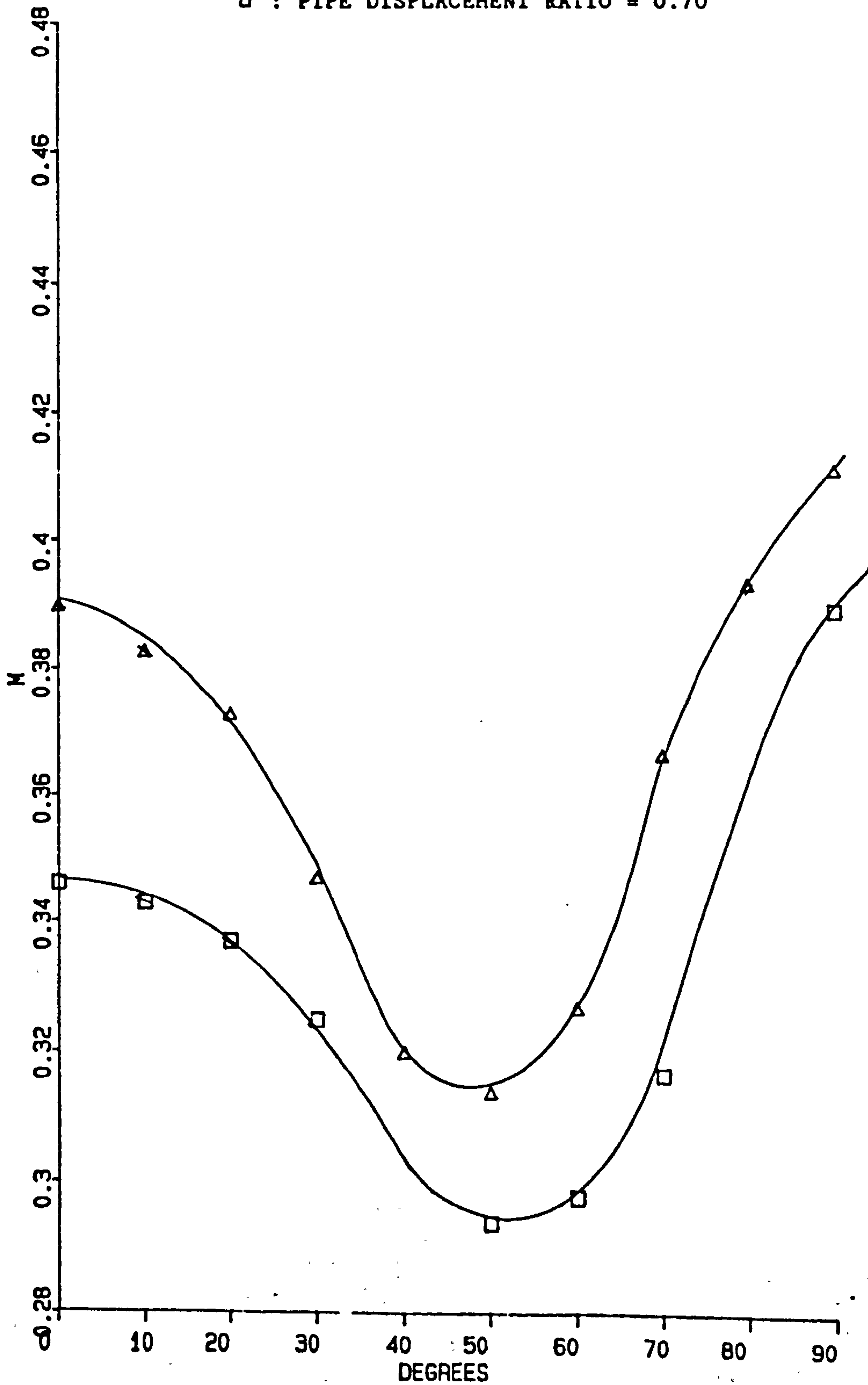


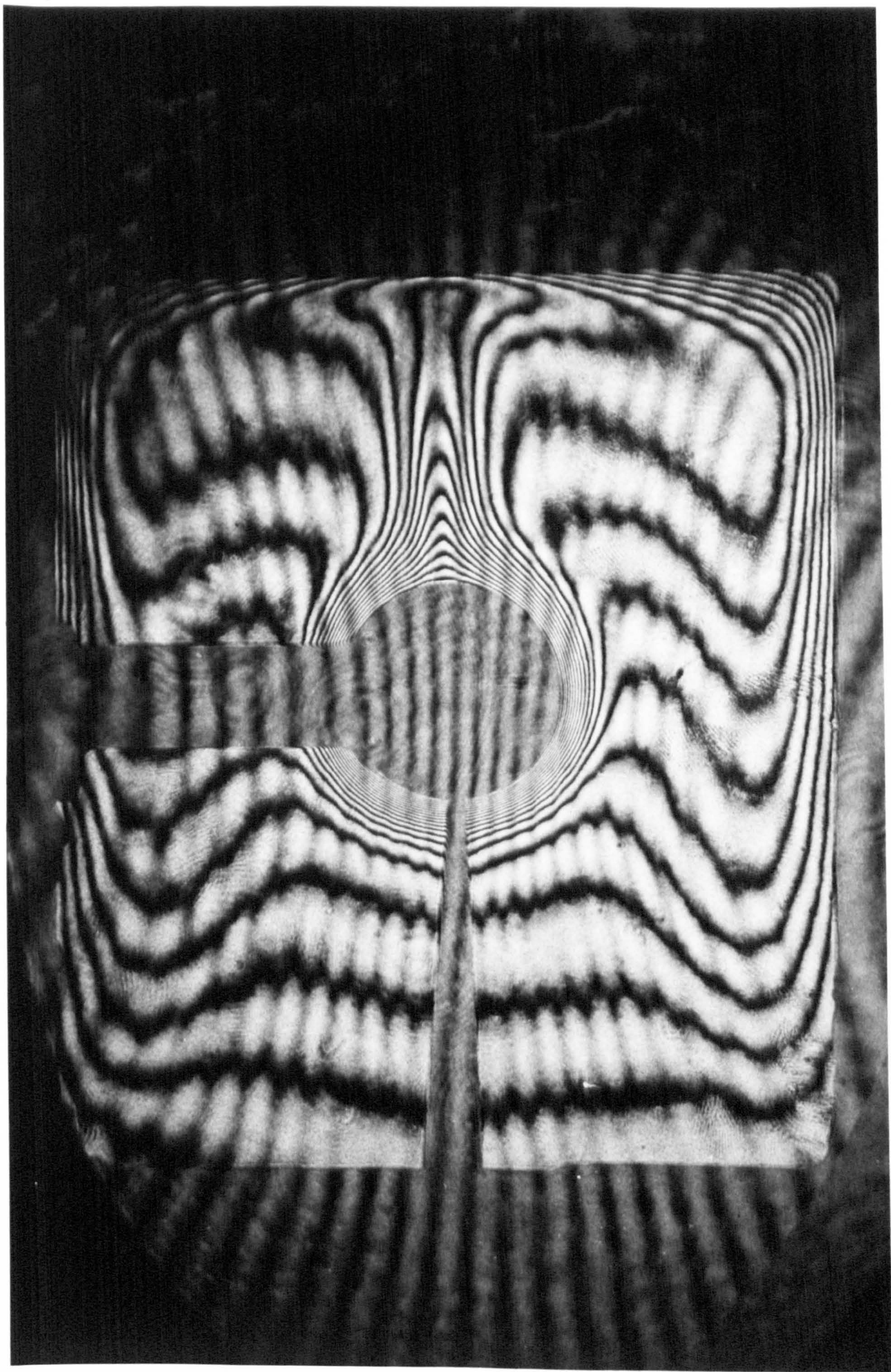
FIGURE 4. 41

GRAPH OF ' M ' VERSUS SPACER ANGLE

▲ : PIPE DISPLACEMENT RATIO = 0
□ : PIPE DISPLACEMENT RATIO = 0.70



High Temperature Interferogram



Chapter 5

Discussion of Results obtained from the Two-pipe study

Discussion of Results obtained from the Two-pipe study

The problems encountered during this investigation were similar to those encountered during the spacer study and have already been discussed in Chapter 4.

5.1 Correlation and Interpretation of Data

Work in the past has shown that for one pipe in a rectangular duct, the following correlation applies for fully developed natural convective flow:-

$$(\text{Nu}_d)_m = M \text{Gr}^{0.25}$$

where "M" is a constant dependent only on the system geometry. However, due to the large number of variables involved in the case tested (ie two cylinders in a rectangular enclosure), the value of "M" is expected to be a complex function of the system geometry and the three temperature differences available (ie T_0 , T_2 and T_3) to transfer heat through the air. At present insufficient data are available to enable a reasonable determination of this relationship. From Figs. 5.1 to 5.6 it can be seen that the power index of the Grashof number increases as the supply pipe is placed at positions of higher displacement ratio.

To obtain comparative results, a relation between the steady-state rate of heat loss, $\dot{Q}_t$, and temperature difference, ΔT , is shown in Table 5.1. At an arbitrary temperature-difference ($\Delta T = 20^\circ\text{C}$), values of total heat-transfer, $\dot{Q}_t$, were extracted and then plotted against the distance (in mm) between the supply and return pipe, Fig. 5.36. From this graph, the total rate of heat-transfer, $\dot{Q}_t$, was then plotted against the supply pipe displacement ratio, E_s , for the optimal distance between pipes, Fig. 5.37.

5.2 Variation of Local Nusselt Number around the Supply Pipe

Heat-transfers from only the supply pipe were considered in this investigation, because the suppliers of heat in district-heating schemes are mainly interested in distributing hot water to their customers with minimum heat loss.

For all configurations the isotherm patterns formed similar shapes but the fringes became more closely packed as the temperature difference, ΔT , was increased.

For the supply pipe position such that $E_s = 0$, the isotherms were more closely packed towards the bottom of the cylinder ($\theta = 180^\circ$), Fig. 5.7. Hence the maximum value of local Nusselt number, Nu_d occurred there. However, as the return pipe was moved vertically downwards, Fig. 5.9, the position of the maximum value of Nu_d occurred at $135^\circ < \theta < 150^\circ$. When the return pipe is moved further downwards, the maximum value of Nu_d occurred at $\theta = 180^\circ$, Fig. 5.11.

For the supply pipe position such that $E_s = 0.8$, similar trends are observed for the position of the maximum value of Nusselt number, Nu_d . The major difference is that a minimum value of Nu_d is observed at $30^\circ < \theta < 45^\circ$ due to the plume from the supply pipe being reduced because of the short distance between the pipe and the cavity roof, Figs. 5.31 to 5.35.14

5.3 Implications of the results

It can easily be seen from Figures 5.36 and 5.37 that minimum heat-transfer from the supply pipe in a 'two-pipe' system in a rectangular cavity occurs at a supply pipe displacement ratio of 0.45 with a gap of about 10 mm between the two pipes.

Table 5.1

Steady-State Heat Losses from the hot supply pipe
across the rectangular cavity
($T_r = 30^\circ\text{C}$)

Geometrical Description	T_s ($^\circ\text{C}$)	ΔT ($T_s - T_d$), ($^\circ\text{C}$)	$\dot{Q}$ Conv+Cond (W)	$\dot{Q}$ Rad (W)	$\dot{Q}$ Total (W)
$E_s = 0$	29.5	6.6	0.83	1.77	2.60
$E_r = -0.68$	33	9.7	1.40	2.69	4.09
($G_a = 4.31$ mm)	36	23.3	-	-	-
	38.7	14.8	2.28	4.27	6.55
$E_s = 0$	32.2	10	1.11	2.72	3.83
$E_r = -0.76$	35.2	15.4	2.22	4.17	6.39
($G_a = 8.17$ mm)	38.3	18.3	2.73	5.07	7.80
	41.4	21.2	3.21	6	9.21
$E_s = 0$	32.2	12.5	1.48	3.32	4.8
$E_r = -0.81$	35.0	15.5	2.08	4.19	6.27
($G_a = 10.58$ mm)	38.4	18.5	2.65	5.14	7.79
	41.0	21.2	3.14	5.98	9.12
$E_s = 0$	32.8	12.8	1.64	3.43	5.07
$E_r = -0.86$	35.3	15.3	2.17	4.17	6.34
($G_a = 13$ mm)	37.9	17.7	2.67	4.92	7.59
	41.2	20.5	3.02	5.84	8.86
$E_s = 0$	32.7	11.5	1.53	3.13	4.66
$E_r = -0.96$	35.4	14.2	2.03	3.93	5.96
($G_a = 17.82$ mm)	38.8	17.2	2.59	4.88	7.47
	41.3	19.6	2.99	5.64	8.63
$E_s = +0.25$	32.3	14.1	1.58	3.68	5.26
$E_r = -0.50$	35.4	17.6	2.20	4.69	6.89
($G_a = 7.69$ mm)	38.5	20.5	2.86	5.59	8.45
	41.4	23.2	3.23	6.46	9.69
$E_s = +0.25$	32.3	15.3	1.73	3.97	5.70
$E_r = -0.59$	35.4	17.6	2.17	4.71	6.88
($G_a = 12.03$ mm)	38.9	20.6	2.61	5.67	8.28
	41.1	22.8	3.07	6.36	9.43

$E_S = +0.25$	32.4	14.3	1.63	3.77	5.40
$E_r = -0.67$	35.4	16.2	1.92	4.40	6.32
(Ga = 15.9 mm)	38.3	19.4	2.47	5.36	7.83
	41.7	22.4	2.93	6.34	9.27
$E_S = +0.25$	32.4	12.7	1.38	3.40	4.78
$E_r = -0.73$	35.0	15.5	1.98	4.22	6.20
(Ga = 18.78 mm)	38.2	18.6	2.50	5.17	7.67
	40.8	21.0	2.90	5.94	8.84
$E_S = +0.25$	32.4	11.1	1.25	3.02	4.27
$E_r = -0.81$	35.3	14.4	1.83	3.98	5.81
(Ga = 22.64 mm)	38.0	16.5	2.13	4.66	6.79
	41.9	19.7	2.61	5.72	8.33
$E_S = +0.50$	30.4	14.4	1.47	3.62	5.09
$E_r = -0.24$	33.3	17.3	1.98	4.46	6.44
(Ga = 7.205 mm)	36.0	19.6	2.34	5.18	7.52
$E_S = +0.50$	30.8	13.0	0.86	3.35	4.21
$E_r = -0.32$	33.2	14.6	1.22	3.87	5.09
(Ga = 11.065 mm)	36.8	17.9	1.65	4.88	6.53
	40.3	21.3	2.29	5.94	8.23
$E_S = +0.50$	30.7	12.5	1.02	3.25	4.27
$E_r = -0.40$	33.4	14.7	1.40	3.92	5.32
(Ga = 14.925 mm)	36.6	18.5	1.93	5.02	6.95
	40.4	21.9	2.45	6.11	8.56
$E_S = +0.50$	30.5	10.7	0.76	2.83	3.60
$E_r = -0.54$	33.5	16.8	1.94	4.44	6.38
(Ga = 21.68 mm)	36.3	17.5	1.95	4.78	6.73
	39.0	19.7	2.28	5.50	7.78
$E_S = +0.50$	31.6	12.0	1.09	3.20	4.29
$E_r = -0.62$	34.4	14.4	1.42	3.93	5.35
(Ga = 25.54 mm)	38.1	17.4	1.90	4.89	6.79
	40.0	19.5	2.38	5.53	7.91
$E_S = +0.60$	30.0	13.1	1.71	3.72	4.98
$E_r = -0.07$	34.7	17.2	2.68	4.52	7.20
(Ga = 3.872 mm)					

$E_S = +0.60$	30.1	12.5	1.46	3.19	4.65
$E_r = -0.17$	35.0	18.6	2.47	4.89	7.36
(Ga = 8.652 mm)	39.9	22.8	3.42	6.21	9.63
$E_S = +0.60$	30.1	13.6	1.48	3.45	4.93
$E_r = -0.26$	35.0	17.4	2.31	4.64	6.95
(Ga = 12.995 mm)	39.9	22.1	3.42	6.10	9.52
$E_S = +0.60$	30.2	13.2	1.45	3.40	4.85
$E_r = -0.32$	35.0	16.8	2.35	4.51	6.86
(Ga = 15.89 mm)	40.1	21.9	3.41	6.08	9.49
$E_S = +0.60$	30.4	12.6	1.5	3.29	4.79
$E_r = -0.41$	35.0	16.8	2.49	4.58	7.07
(Ga = 20.232 mm)	39.7	20.9	3.41	5.84	9.25
$E_S = +0.70$	33.0	11.1	1.51	3.02	4.53
$E_r = -0.01$	37.8	14.7	2.42	4.18	6.60
(Ga = 5.757 mm)	41.6	18.3	3.34	5.34	8.68
$E_S = +0.70$	34.0	12.3	1.62	3.37	5.00
$E_r = -0.05$	38.0	15.3	2.29	4.35	6.64
(Ga = 7.687 mm)	42.1	18.5	3.06	5.42	8.48
$E_S = +0.70$	33.7	11.8	1.22	3.24	4.46
$E_r = -0.10$	38.1	15.2	2.10	4.33	6.43
(Ga = 10.1 mm)	42.4	18.8	2.98	5.52	8.50
$E_S = +0.70$	33.8	11.7	1.34	3.23	4.57
$E_r = -0.20$	38.7	15.2	2.14	4.37	6.51
(Ga = 14.925 mm)	41.6	18.1	2.91	5.29	8.20
$E_S = +0.80$	29.5	7.5	0.94	2.00	2.94
$E_r = +0.10$	33.8	11.1	1.91	3.07	5.00
(Ga = 5.275 mm)	37.4	13.7	2.56	3.92	6.48

$E_s = +0.80$	27.6	7.7	0.62	1.95	2.57
$E_r = +0.03$	32.0	10.3	1.24	2.78	4.02
(Ga = 8.652 mm)	35.6	12.8	1.90	3.49	5.49
	39.5	15.8	2.65	4.57	7.22
$E_s = +0.80$	29.8	9.0	0.76	2.37	3.13
$E_r = -0.03$	34.0	11.5	1.37	3.19	4.56
(Ga = 11.547 mm)	38.1	14.3	2.05	4.12	6.17
	41.8	16.6	2.53	4.94	7.47
$E_s = +0.80$	34.0	11.6	1.49	3.22	4.71
$E_r = -0.10$	38.0	14.6	2.23	4.19	6.42
(Ga = 14.925 mm)	41.5	16.0	2.47	4.77	7.24
$E_s = +0.80$	30.1	9.1	1.03	2.42	3.45
$E_r = -0.20$	33.9	11.1	1.57	3.09	4.66
(Ga = 19.75 mm)	37.8	13.6	2.19	3.93	6.12
	42.1	16.9	2.98	5.03	8.01

Table 5.2

Steady-State Natural Convection through the air from
the Supply Pipe across the rectangular cavity
($T_r = 30^{\circ}\text{C}$)

Geometrical Description	T_s ($^{\circ}\text{C}$)	ΔT ($T_s - T_d$), ($^{\circ}\text{C}$)	-6 $10 Gr_x$	$(Nu_d)_m$	Interferogram
$E_s = 0$ $E_r = -0.68$	29.5	6.6	0.75	5.36	120
	33	9.7	1.08	6.13	121
	36	23.3	-	-	122
	38.7	14.8	1.58	6.40	123
$E_s = 0$ $E_r = -0.76$	32.2	10	1.13	4.73	124
	35.2	15.4	1.73	6.05	125
	38.3	18.3	2.01	6.19	126
	41.4	21.2	2.28	6.24	127
$E_s = 0$ $E_r = -0.81$	32.2	12.5	1.43	5.01	128
	35	15.5	1.74	5.64	129
	38.4	18.5	2.03	5.96	130
	41.0	21.2	2.29	6.11	131
$E_s = 0$ $E_r = -0.86$	32.8	12.8	1.46	5.41	132
	35.3	15.3	1.71	5.95	133
	37.9	17.7	1.95	6.27	134
	41.2	20.5	2.20	6.07	135
$E_s = 0$ $E_r = -0.96$	32.7	11.5	1.30	5.62	136
	35.4	14.2	1.58	6.00	137
	38.8	17.2	1.86	6.25	138
	41.3	19.6	2.09	6.30	139
$E_s = +0.25$ $E_r = -0.50$	32.3	14.1	1.64	4.74	140
	35.4	17.6	2.00	5.23	141
	38.5	20.5	2.28	5.80	142
	41.4	23.2	2.53	5.73	143
$E_s = +0.25$ $E_r = -0.59$	32.3	15.3	1.79	4.78	144
	35.4	17.6	2.00	5.18	145
	38.9	20.6	2.28	5.26	146
	41.1	22.8	2.49	5.55	147

$E_S = +0.25$	32.4	14.3	1.66	4.82	148
$E_T = -0.67$	35.4	16.2	1.82	4.97	149
	38.3	19.4	2.15	5.29	150
	41.7	22.4	2.42	5.39	151
$E_S = +0.25$	32.4	12.7	1.45	4.61	152
$E_T = -0.73$	35.0	15.5	1.74	5.37	153
	38.2	18.6	2.05	5.59	154
	40.8	21.0	2.27	5.70	155
$E_S = +0.25$	32.4	11.1	1.26	4.79	156
$E_T = -0.81$	35.3	14.4	1.60	5.35	157
	38.0	16.5	1.80	5.37	158
	41.9	19.7	2.08	5.46	159
$E_S = +0.50$	30.4	14.4	1.72	4.34	160
$E_T = -0.24$	33.3	17.3	2.03	4.83	161
	36.0	19.6	2.24	5.00	162
$E_S = +0.50$	30.8	13.0	1.53	2.83	163
$E_T = -0.32$	33.2	14.6	1.68	3.54	164
	36.8	17.9	2.00	3.86	165
	40.3	21.3	2.32	4.45	166
$E_S = +0.50$	30.7	12.5	1.47	3.48	167
$E_T = -0.40$	33.4	14.7	1.68	4.03	168
	36.6	18.5	2.08	4.37	169
	40.4	21.9	2.40	4.62	170
$E_S = +0.50$	30.5	10.7	1.24	3.04	171
$E_T = -0.54$	33.5	16.8	1.95	4.88	172
	36.4	17.5	1.96	4.65	173
	39.0	19.7	2.16	4.80	174
$E_S = +0.50$	31.6	12.0	1.38	3.87	175
$E_T = -0.62$	34.4	14.4	1.62	4.15	176
	38.1	17.4	1.90	4.54	177
	40.0	19.5	2.11	5.04	178
$E_S = +0.60$	30.0	13.1	1.56	5.59	179
$E_T = -0.07$	34.7	17.2	1.97	6.54	180

$E_S = +0.60$	30.1	12.5	1.74	4.97	181
$E_r = -0.17$	35.0	18.6	2.14	5.59	182
	39.9	22.8	2.53	6.22	183
$E_S = +0.60$	30.1	13.6	1.62	4.65	184
$E_r = -0.26$	35.0	17.4	1.99	5.57	185
	39.9	22.1	2.43	6.40	186
$E_S = +0.60$	30.2	13.2	1.58	4.66	187
$E_r = -0.32$	35.0	16.8	1.91	5.87	188
	40.1	21.9	2.40	6.46	189
$E_S = +0.60$	30.4	12.6	1.50	5.04	190
$E_r = -0.41$	35.0	16.8	1.91	5.87	191
	39.7	20.9	2.29	6.46	192
$E_S = +0.70$	33.0	11.1	1.25	5.76	193
$E_r = -0.01$	37.8	14.7	1.59	6.87	194
	41.6	18.3	1.93	7.49	195
$E_S = +0.70$	34.0	12.3	1.37	5.55	196
$E_r = -0.05$	38.0	15.3	1.65	6.22	197
	42.1	18.5	1.94	6.81	198
$E_S = +0.70$	33.7	11.8	1.17	4.36	199
$E_r = -0.10$	38.1	15.2	1.64	5.75	200
	42.4	18.8	1.96	6.50	201
$E_S = +0.70$	33.8	11.7	1.30	4.83	202
$E_r = -0.20$	38.7	15.2	1.62	5.86	203
	41.6	18.1	1.90	6.63	204
$E_S = +0.80$	29.5	7.5	0.86	5.36	205
$E_r = +0.10$	33.8	11.1	1.23	7.27	206
	37.4	13.7	1.43	7.80	207
$E_S = +0.80$	27.6	7.7	0.91	3.46	208
$E_r = +0.03$	32.0	10.3	1.17	5.10	209
	35.6	12.8	1.40	6.23	210
	39.5	15.8	1.68	6.94	211

$E_S = +0.80$	29.8	9.0	1.04	3.61	212
$E_r = -0.03$	34.0	11.5	1.28	5.03	213
	38.1	14.3	1.53	5.97	214
	41.8	16.6	1.72	6.26	215
$E_S = +0.80$	34.0	11.6	1.29	5.42	216
$E_r = -0.10$	38.0	14.6	1.57	6.37	217
	41.5	16.0	1.66	6.36	218
$E_S = +0.80$	30.1	9.1	1.05	4.84	219
$E_r = -0.20$	33.9	11.1	1.23	5.96	220
	37.8	13.6	1.46	6.70	221
	42.1	16.9	1.75	7.24	222

FIGURE 5. 1

LOG NUSSELT NO. V LOG GRASHOF NO.
SUPPLY PIPE DISPLACEMENT RATIO = 0

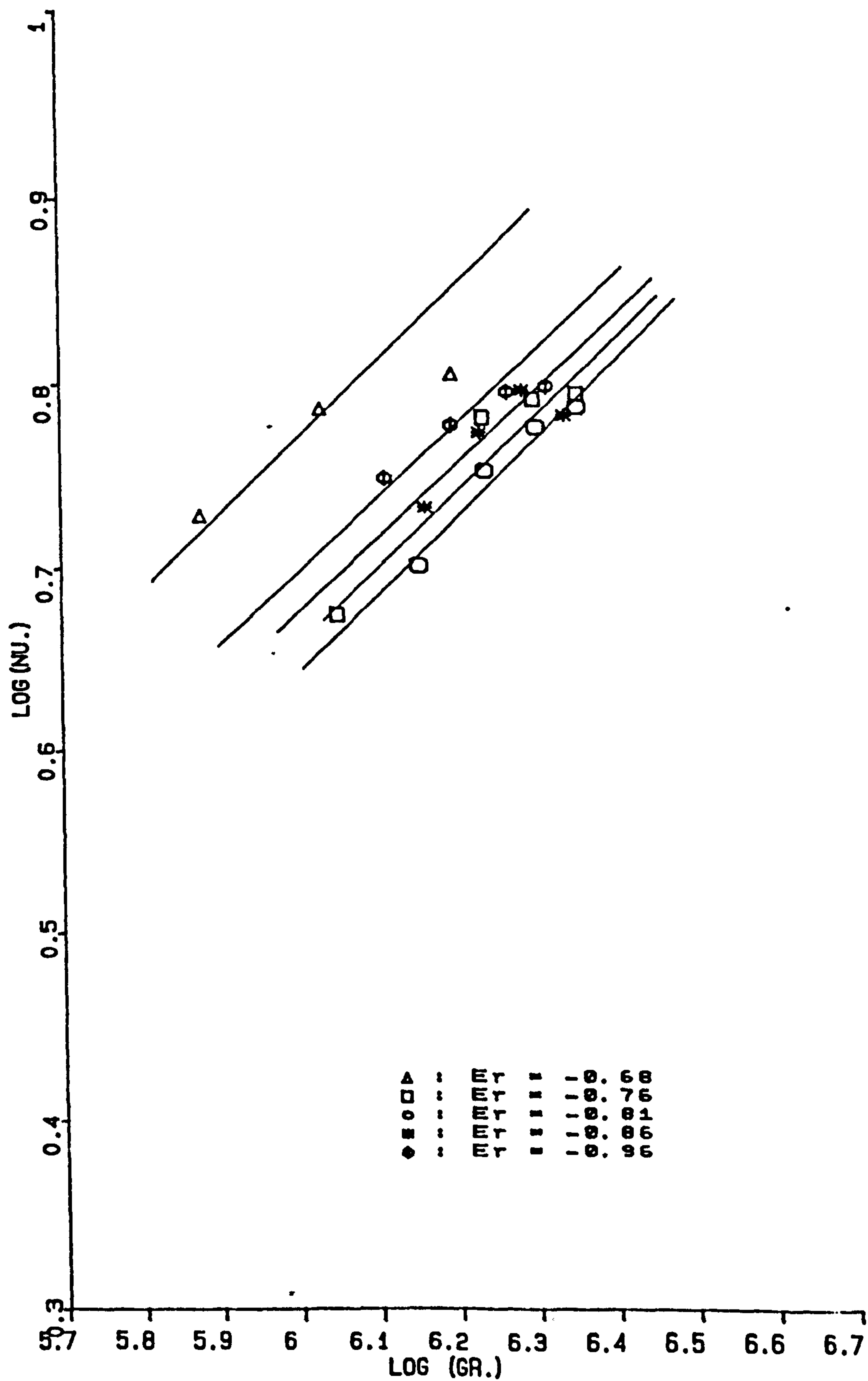


FIGURE 5. 2

LOG NUSSELT NO. V LOG GRASHOF NO.
 SUPPLY PIPE DISPLACEMENT RATIO = 0.25

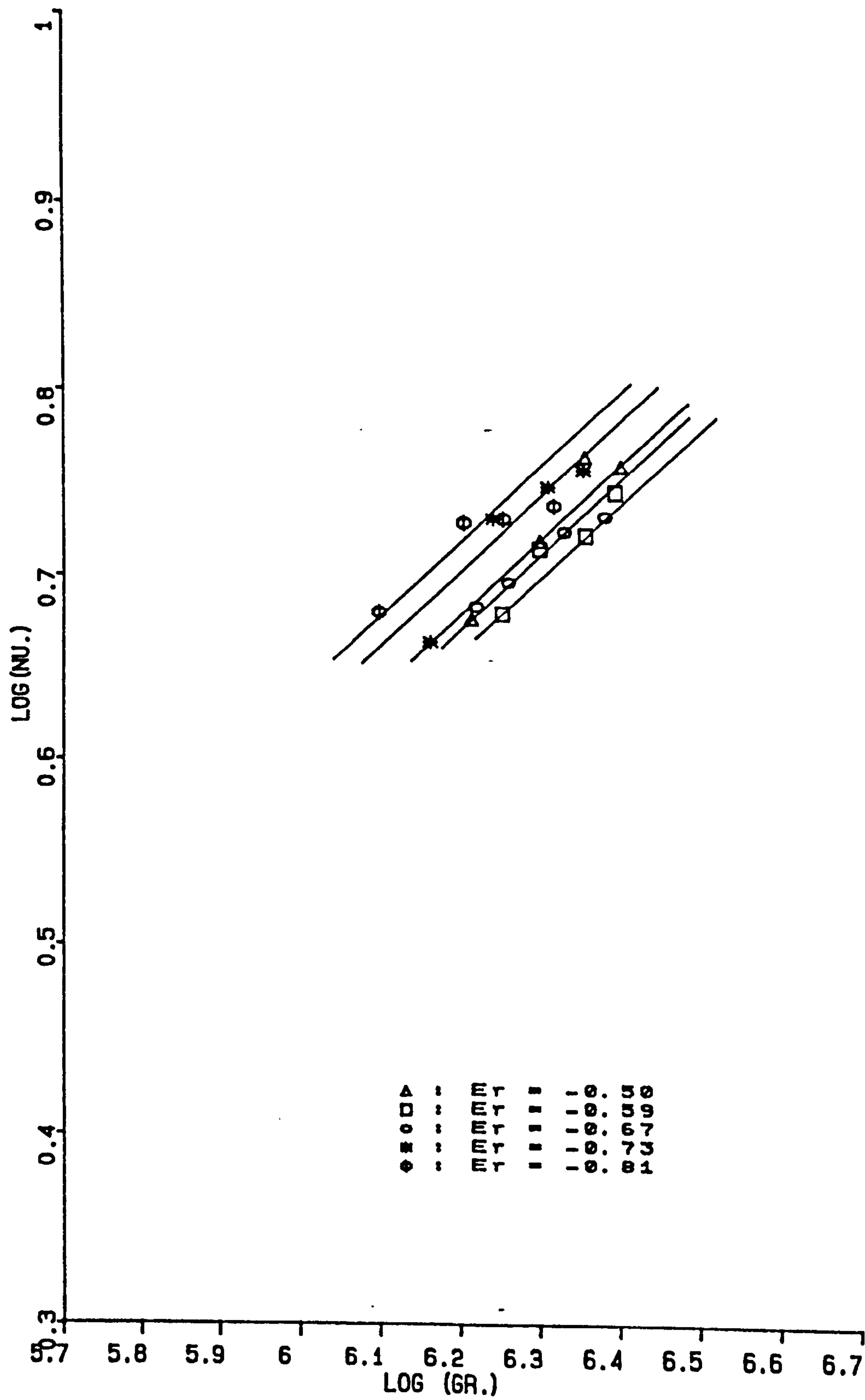


FIGURE 5.3

LOG NUSSELT NO. V LOG GRASHOF NO.
SUPPLY PIPE DISPLACEMENT RATIO = 0.50

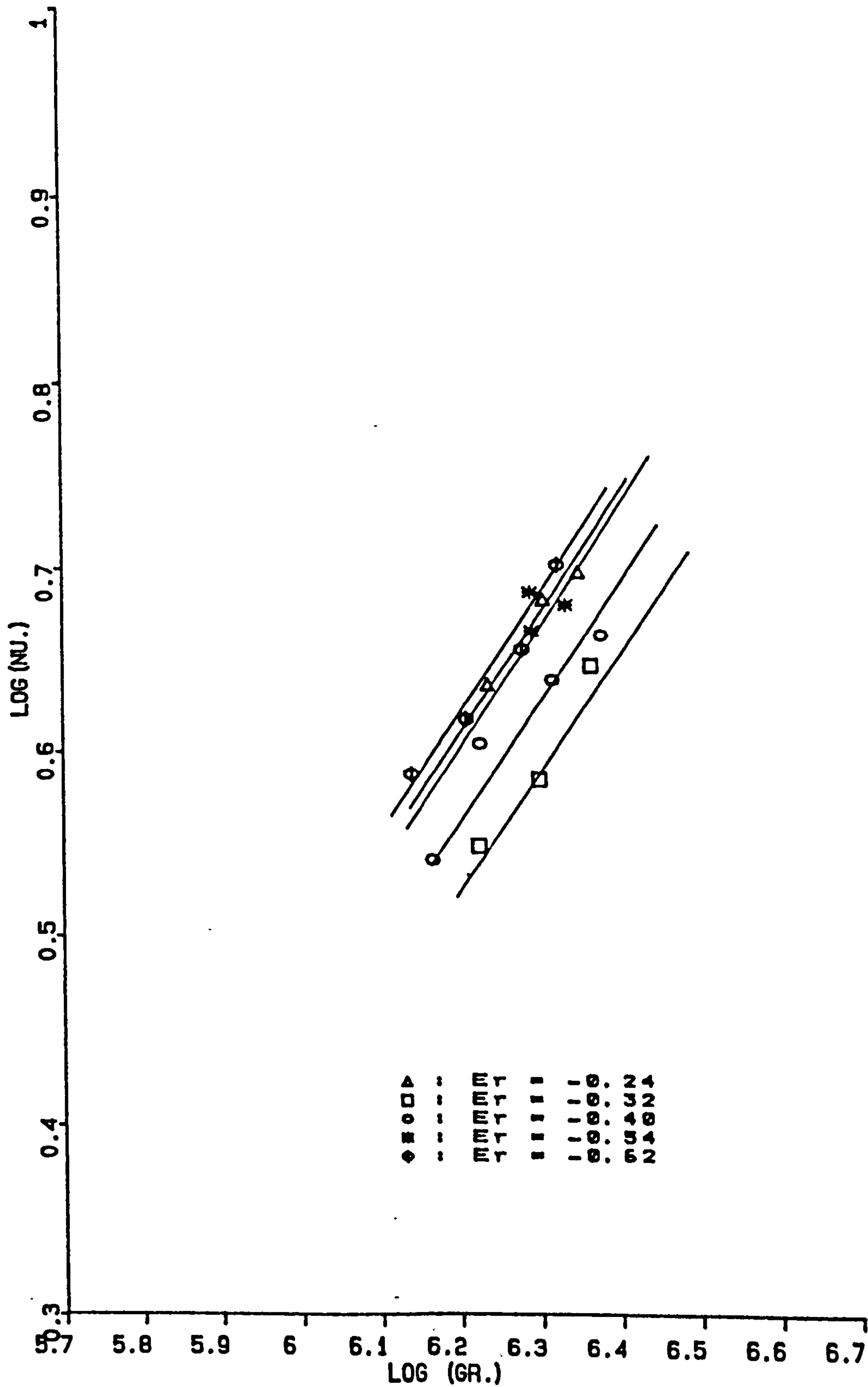


FIGURE 5. 4

LOG NUSSELT NO. V LOG GRASHOF NO.
 SUPPLY PIPE DISPLACEMENT RATIO = 0.60

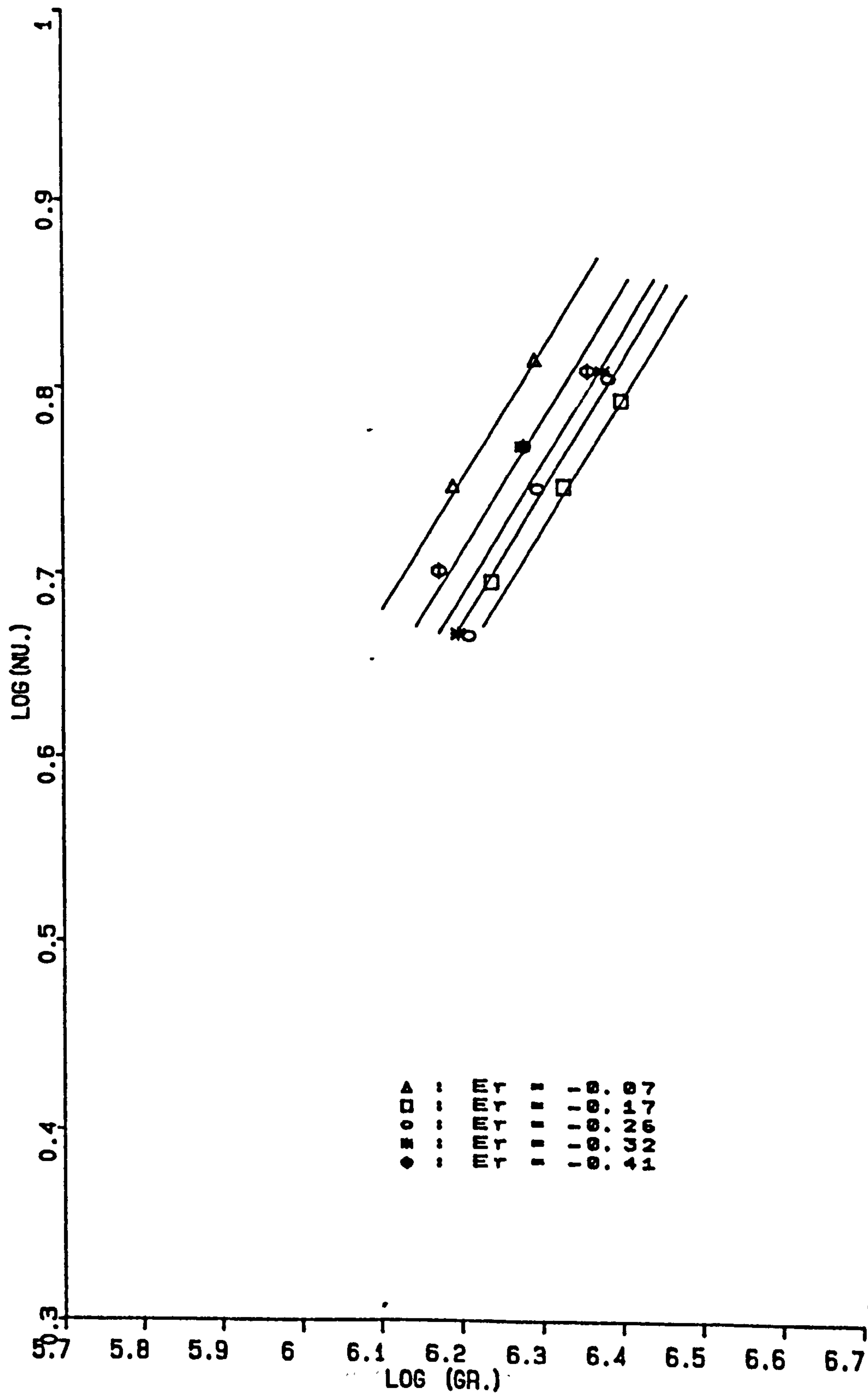


FIGURE 5. 5

LOG NUSSLETT NO. V LOG GRASHOF NO.
 SUPPLY PIPE DISPLACEMENT RATIO = 0.70

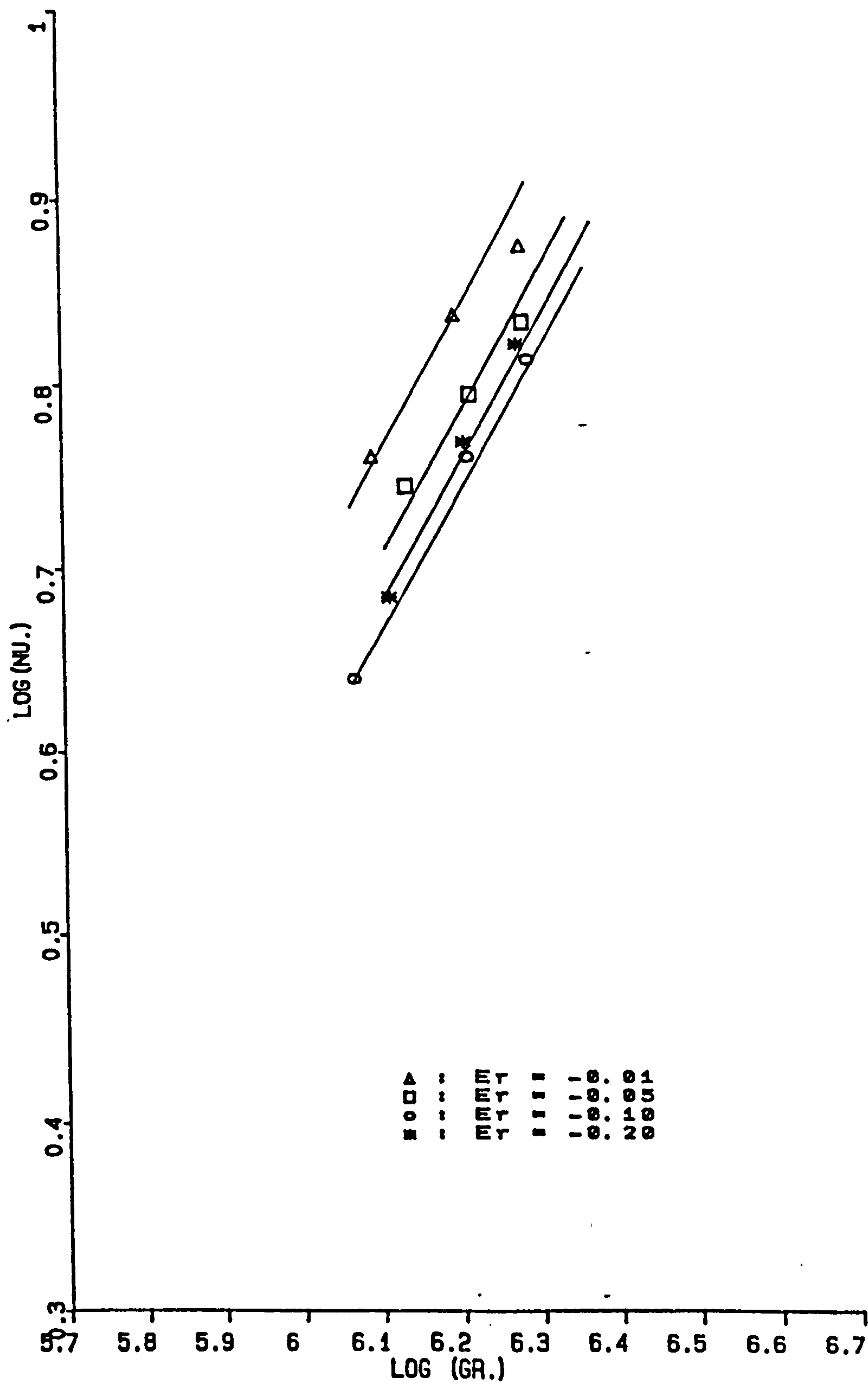


FIGURE 5. 6

LOG NUSSELT NO. V LOG GRASHOF NO.
 SUPPLY PIPE DISPLACEMENT RATIO = 0.80

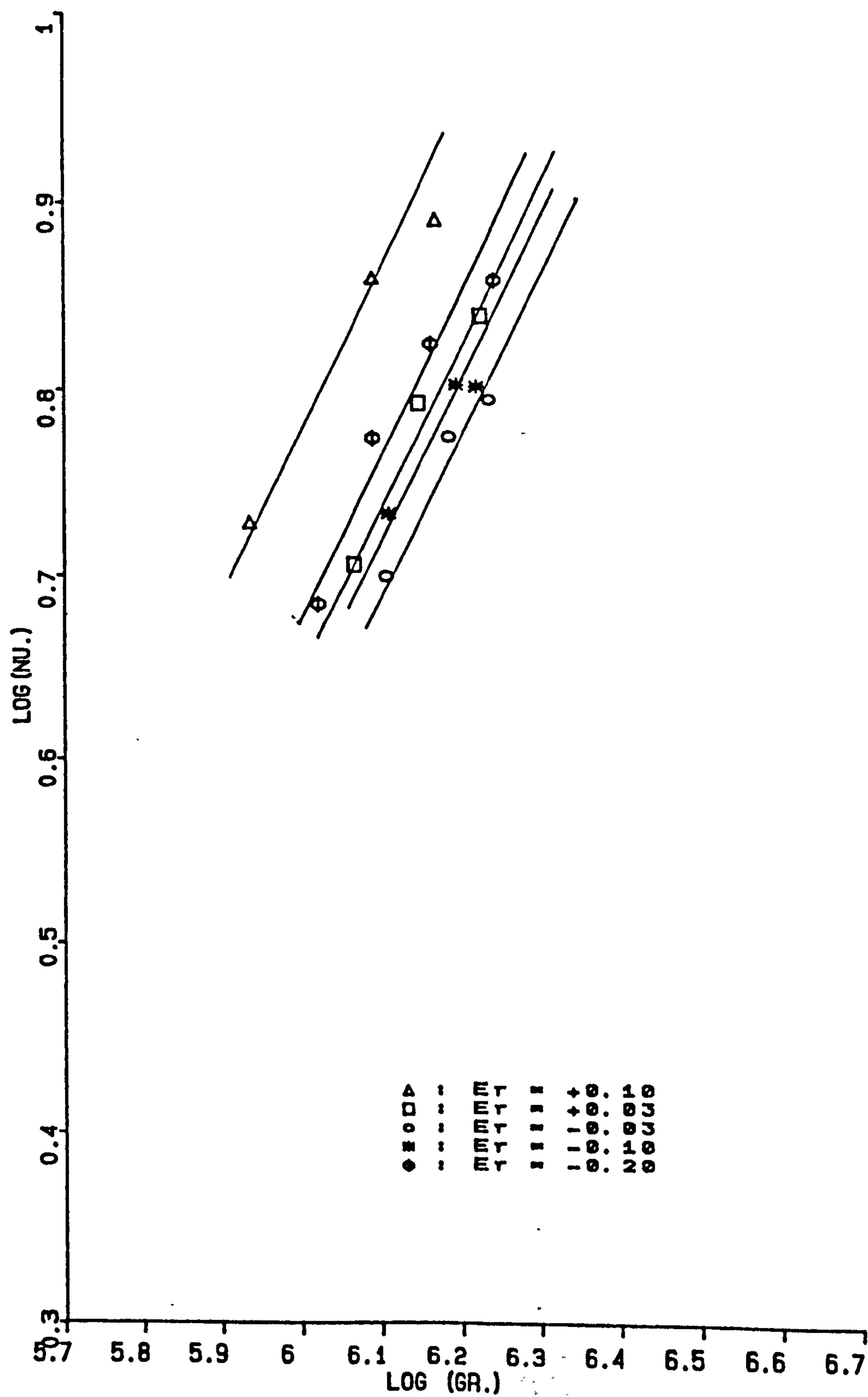


FIGURE 5.7

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0
 RETURN PIPE DISPLACEMENT RATIO = -0.68

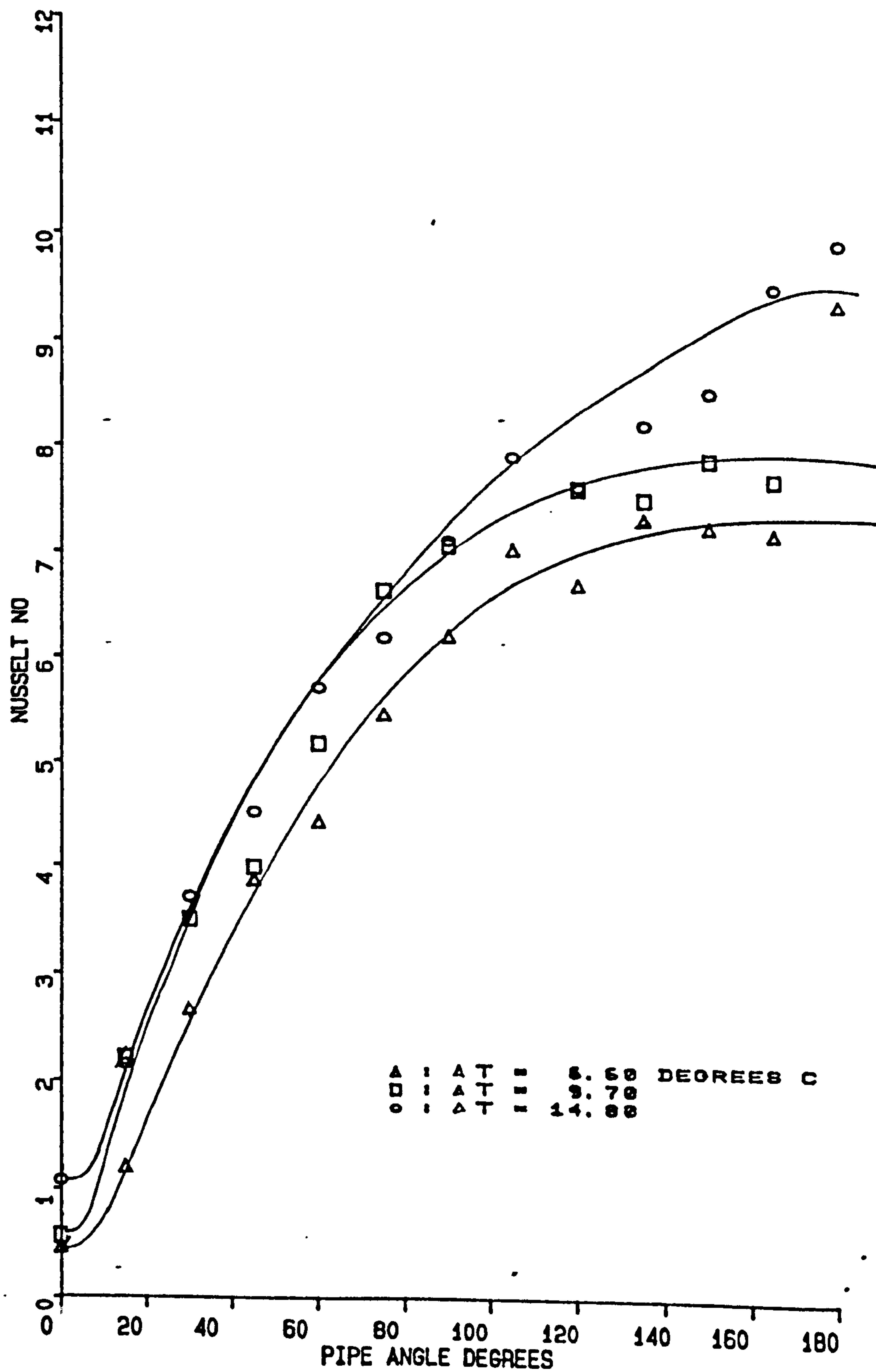


FIGURE 5.8

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0
 RETURN PIPE DISPLACEMENT RATIO = -0.76

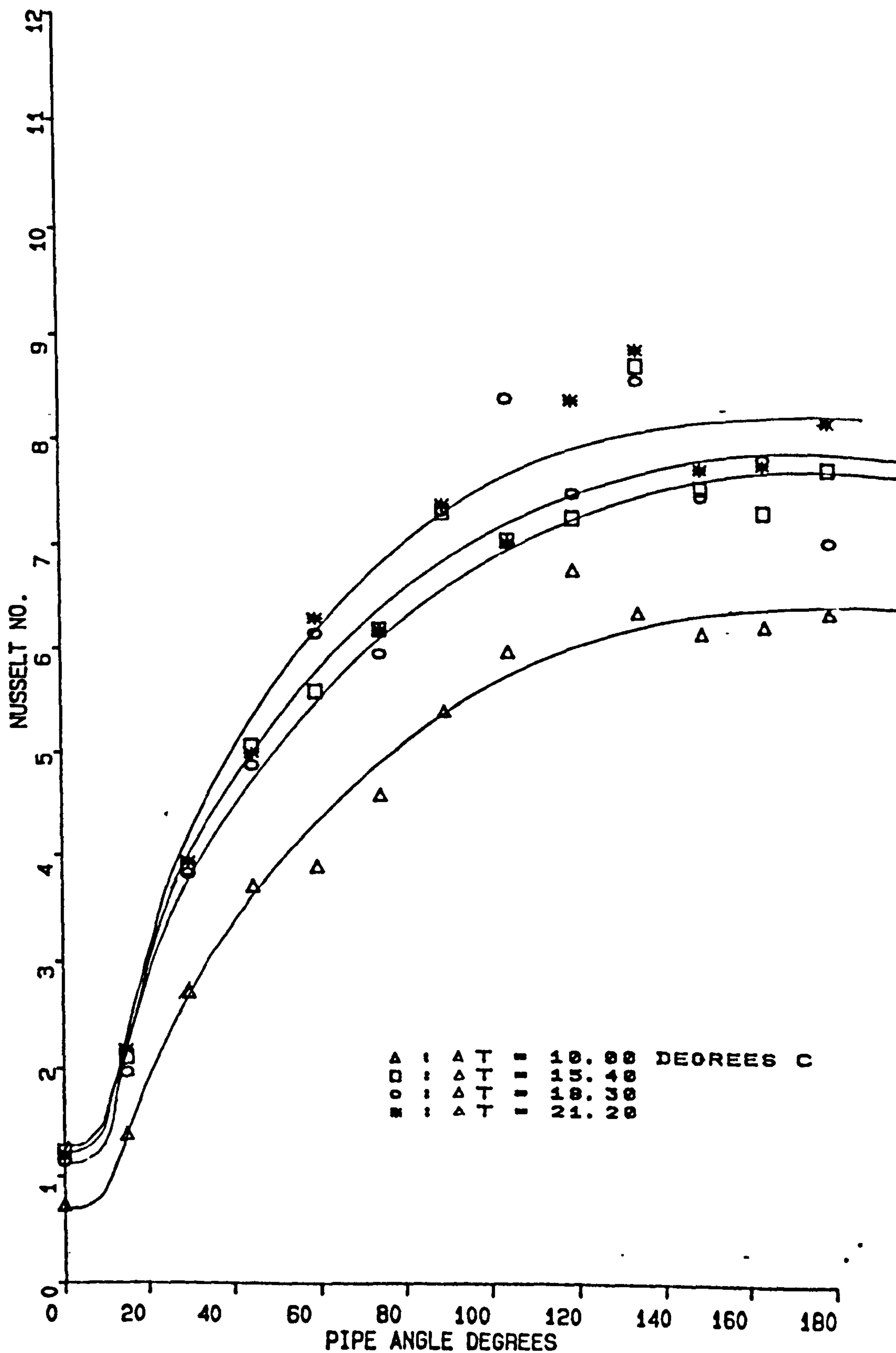


FIGURE 5.9

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0
 RETURN PIPE DISPLACEMENT RATIO = -0.81

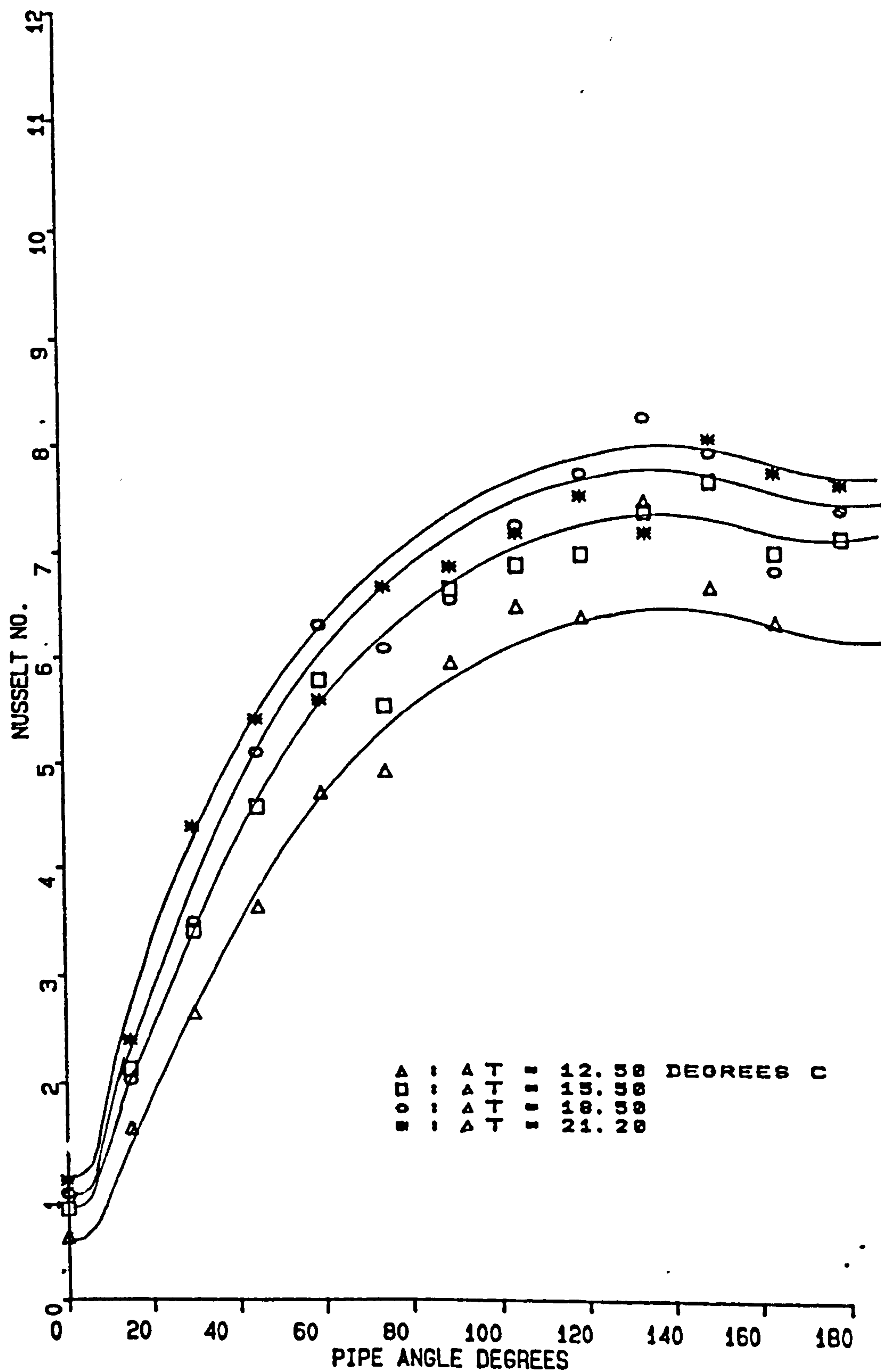


FIGURE 5. 10

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0
 RETURN PIPE DISPLACEMENT RATIO = -0.86

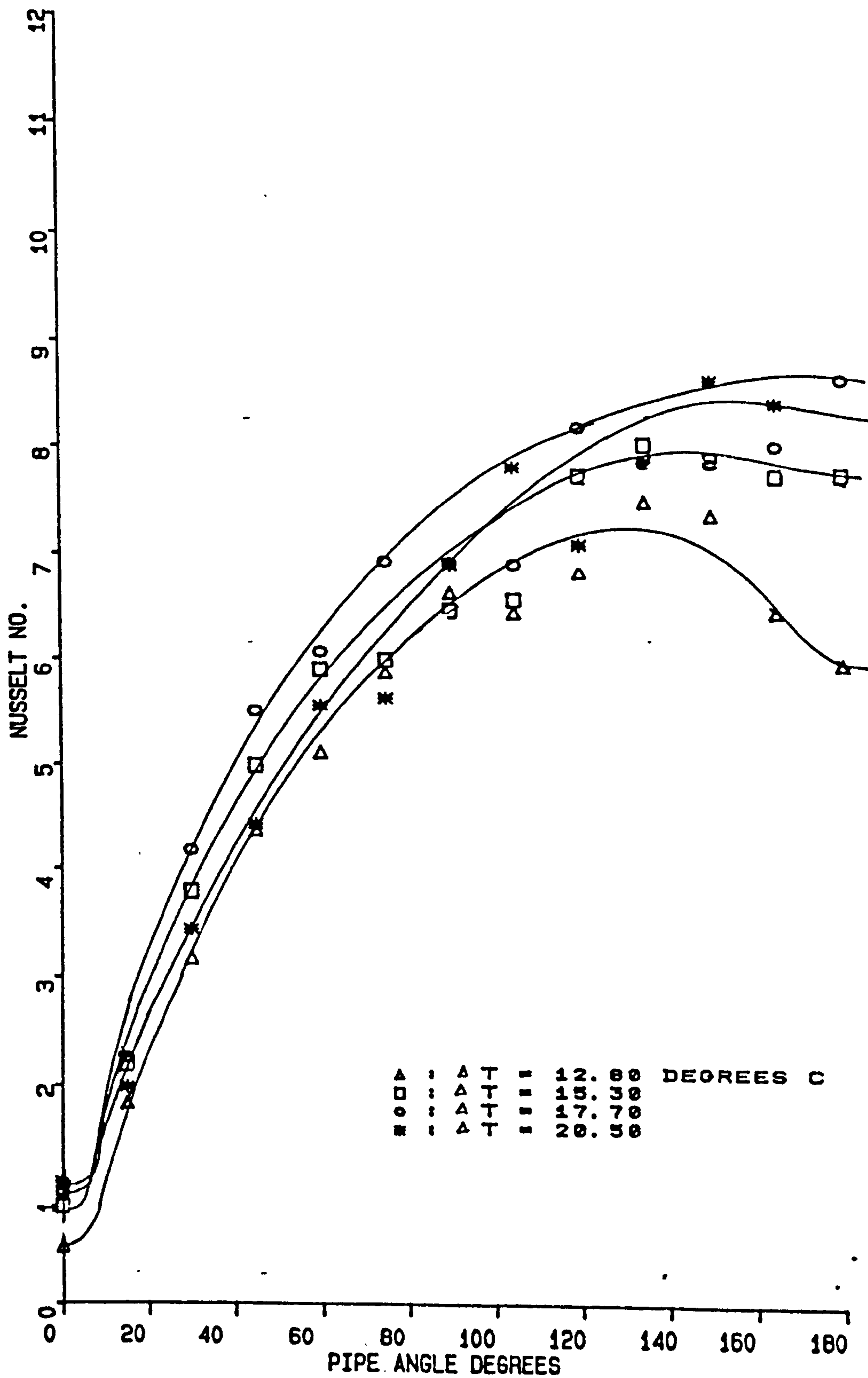


FIGURE 5. 11

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0
 RETURN PIPE DISPLACEMENT RATIO = -0.96

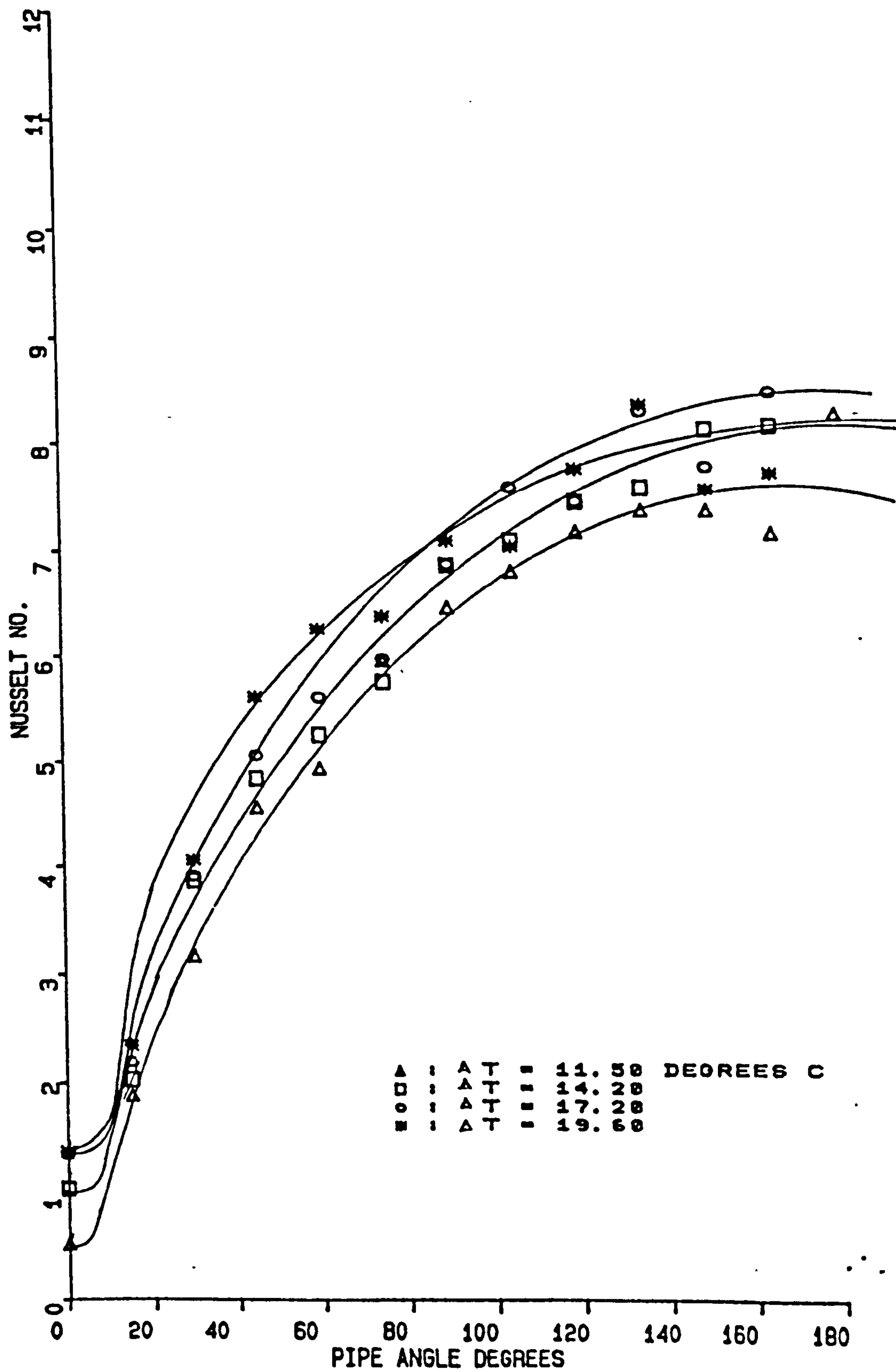


FIGURE 5.12

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.25
 RETURN PIPE DISPLACEMENT RATIO = -0.50

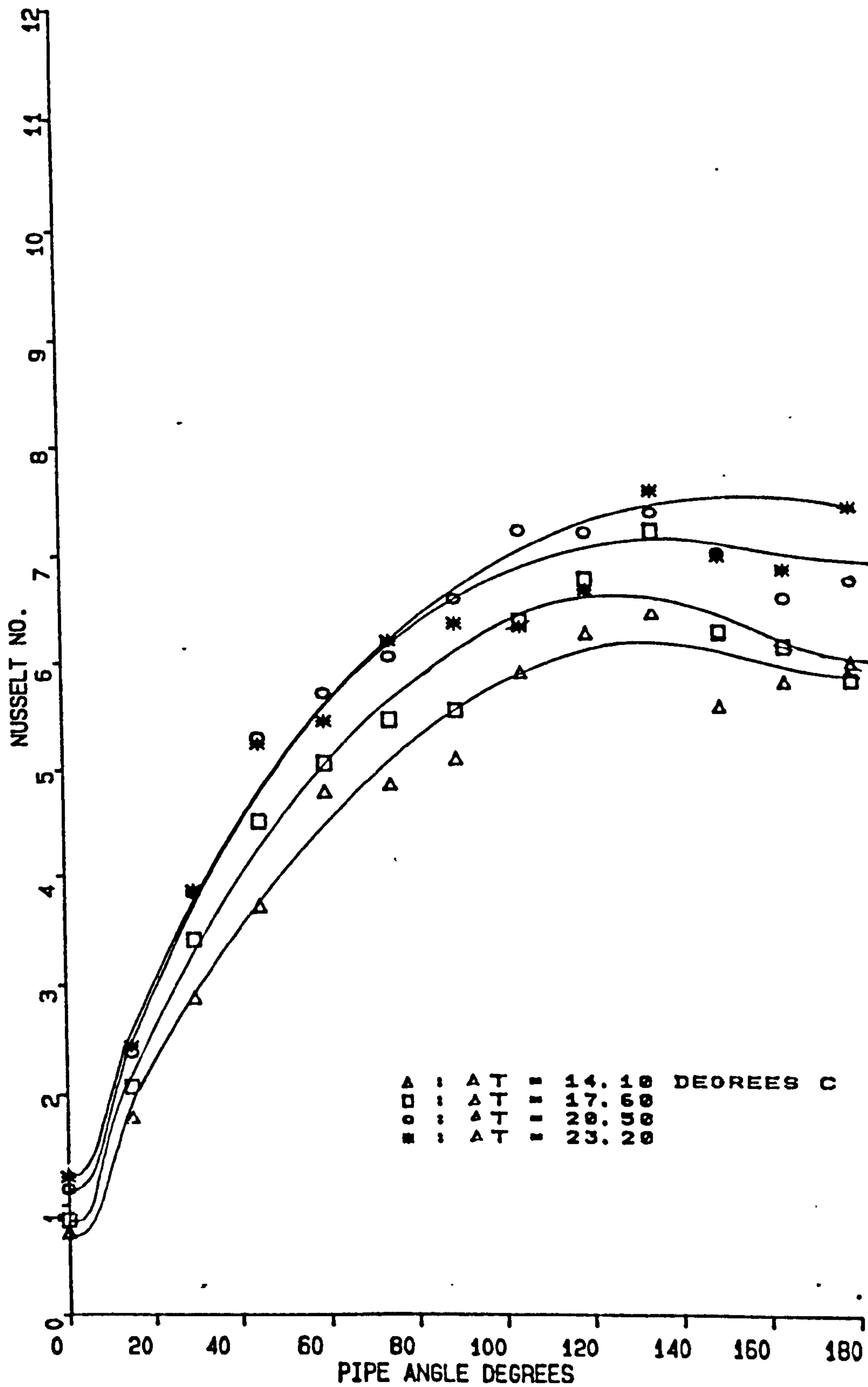


FIGURE 5.13

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.25
 RETURN PIPE DISPLACEMENT RATIO = -0.59

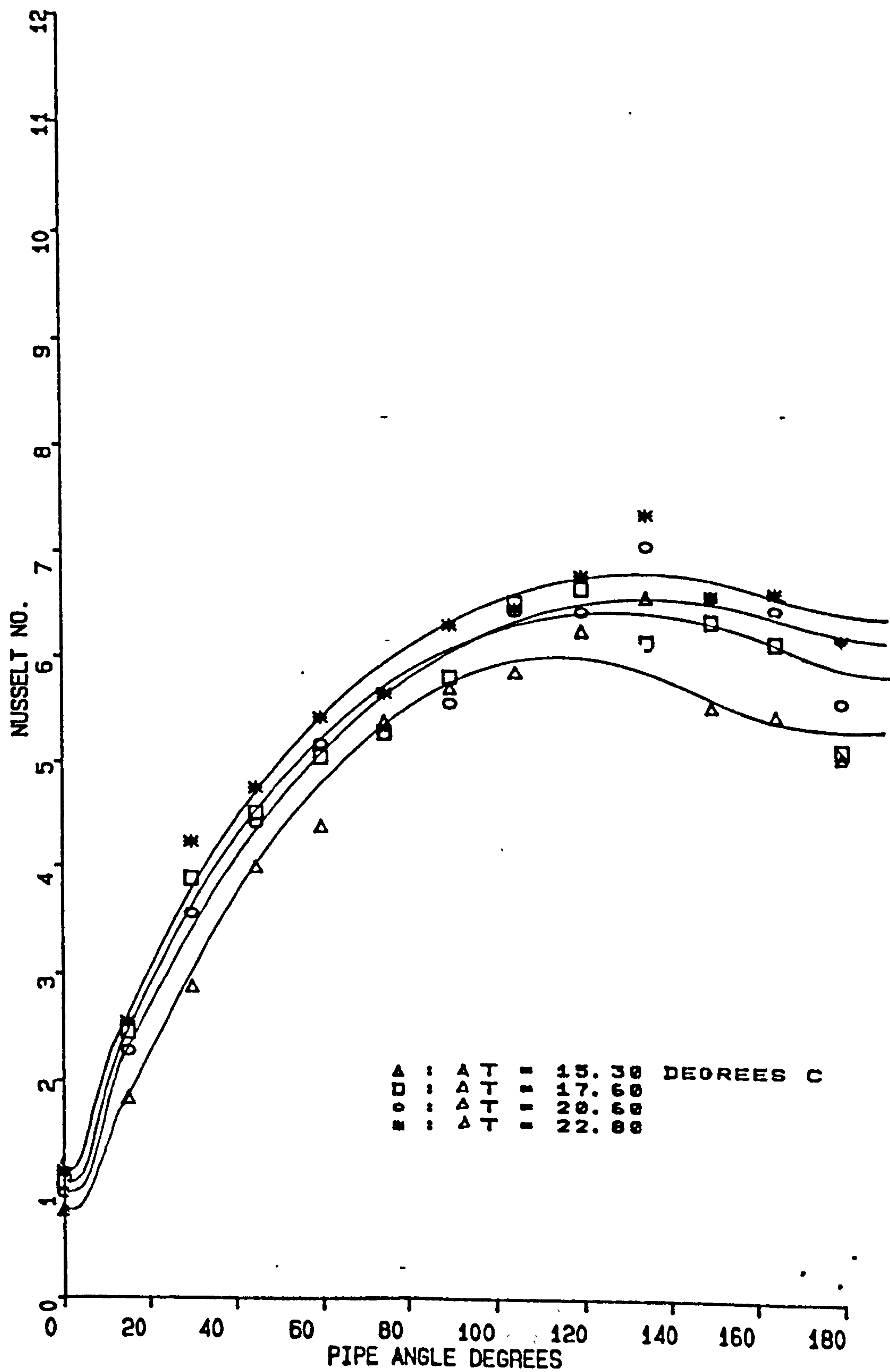


FIGURE 5.14

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.25
 RETURN PIPE DISPLACEMENT RATIO = -0.67

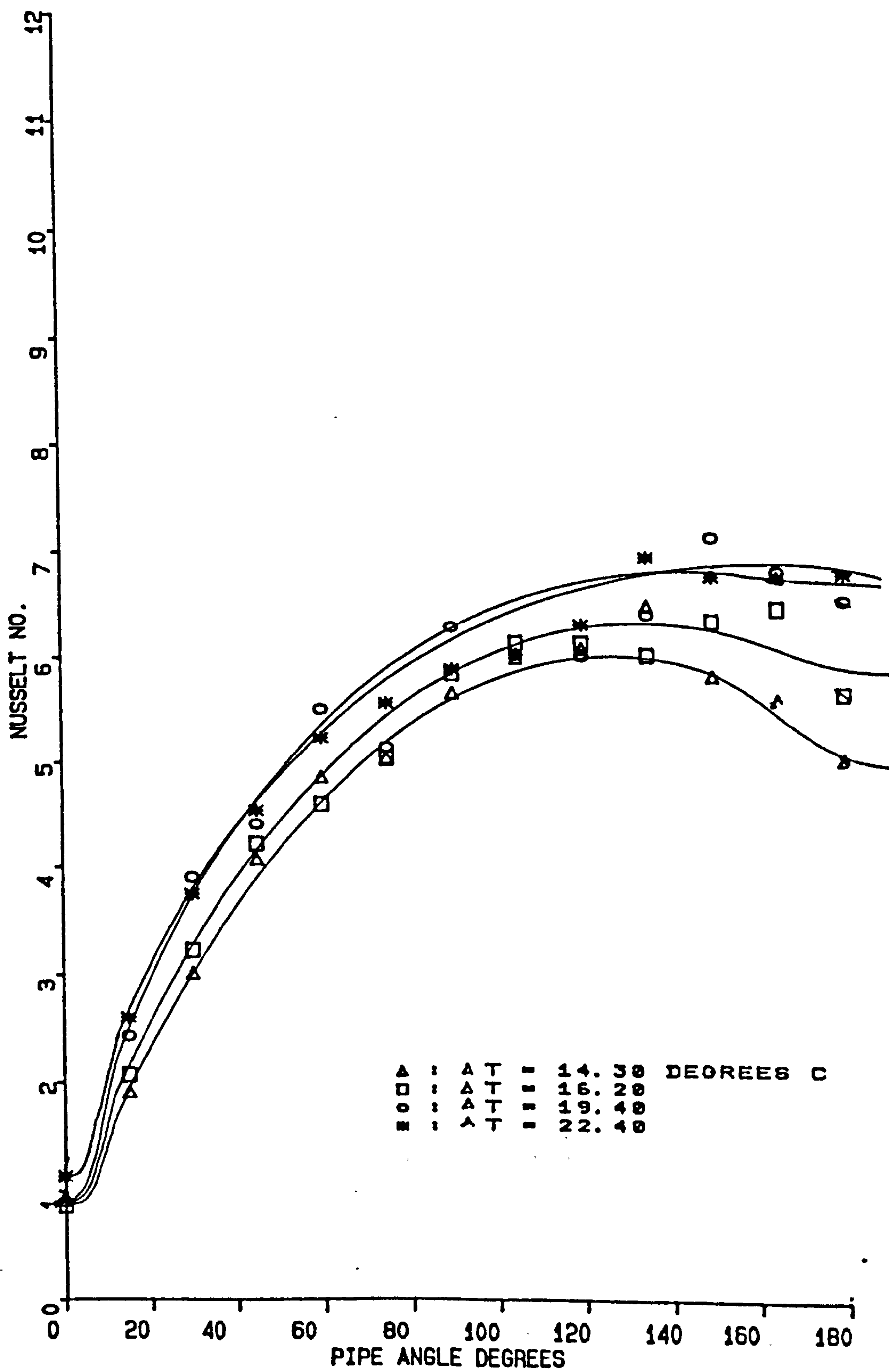


FIGURE 5. 15

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.25
 RETURN PIPE DISPLACEMENT RATIO = -0.73

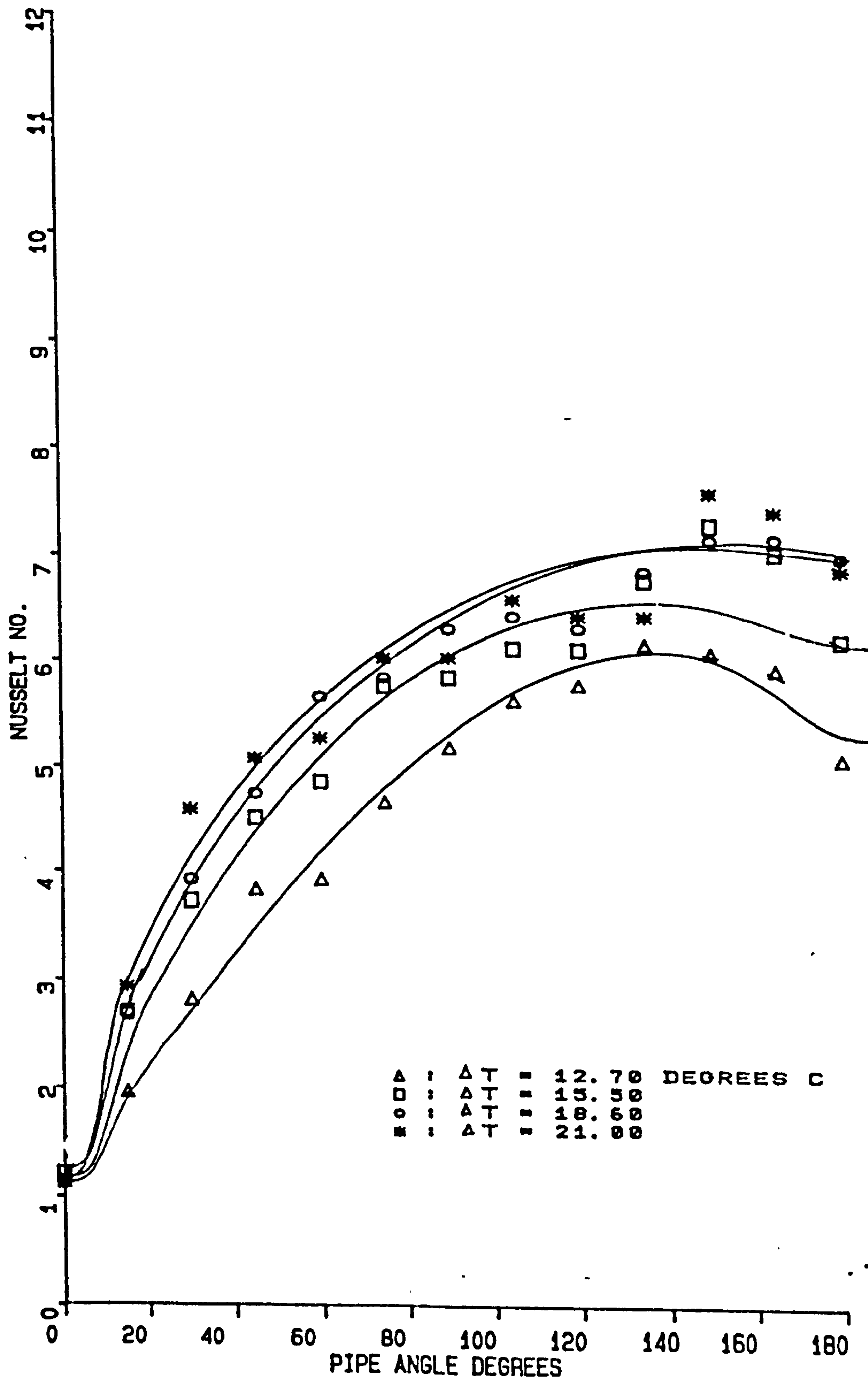


FIGURE 5.16

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.25
 RETURN PIPE DISPLACEMENT RATIO = -0.81

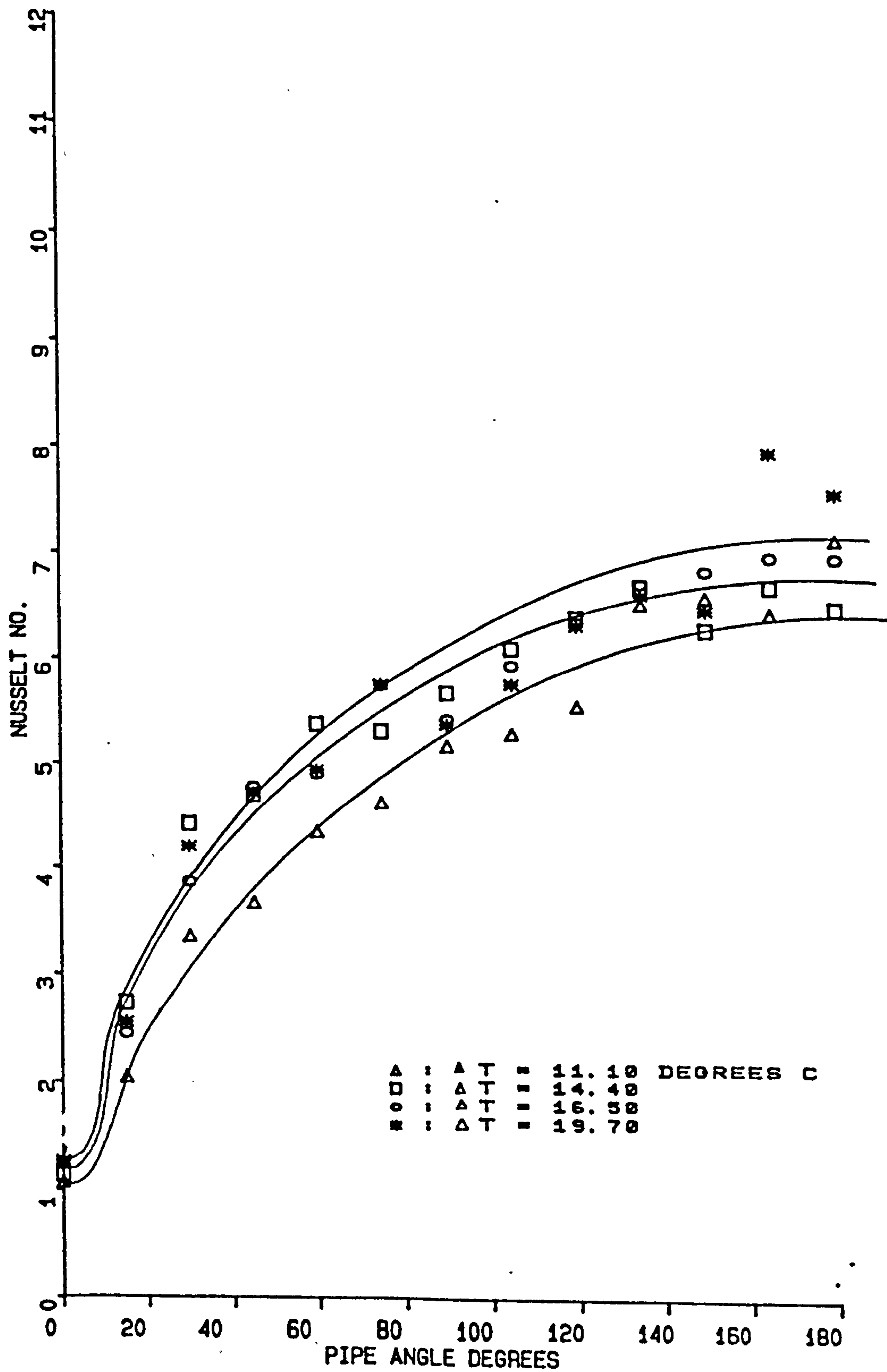


FIGURE 5.17

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.50
 RETURN PIPE DISPLACEMENT RATIO = -0.24

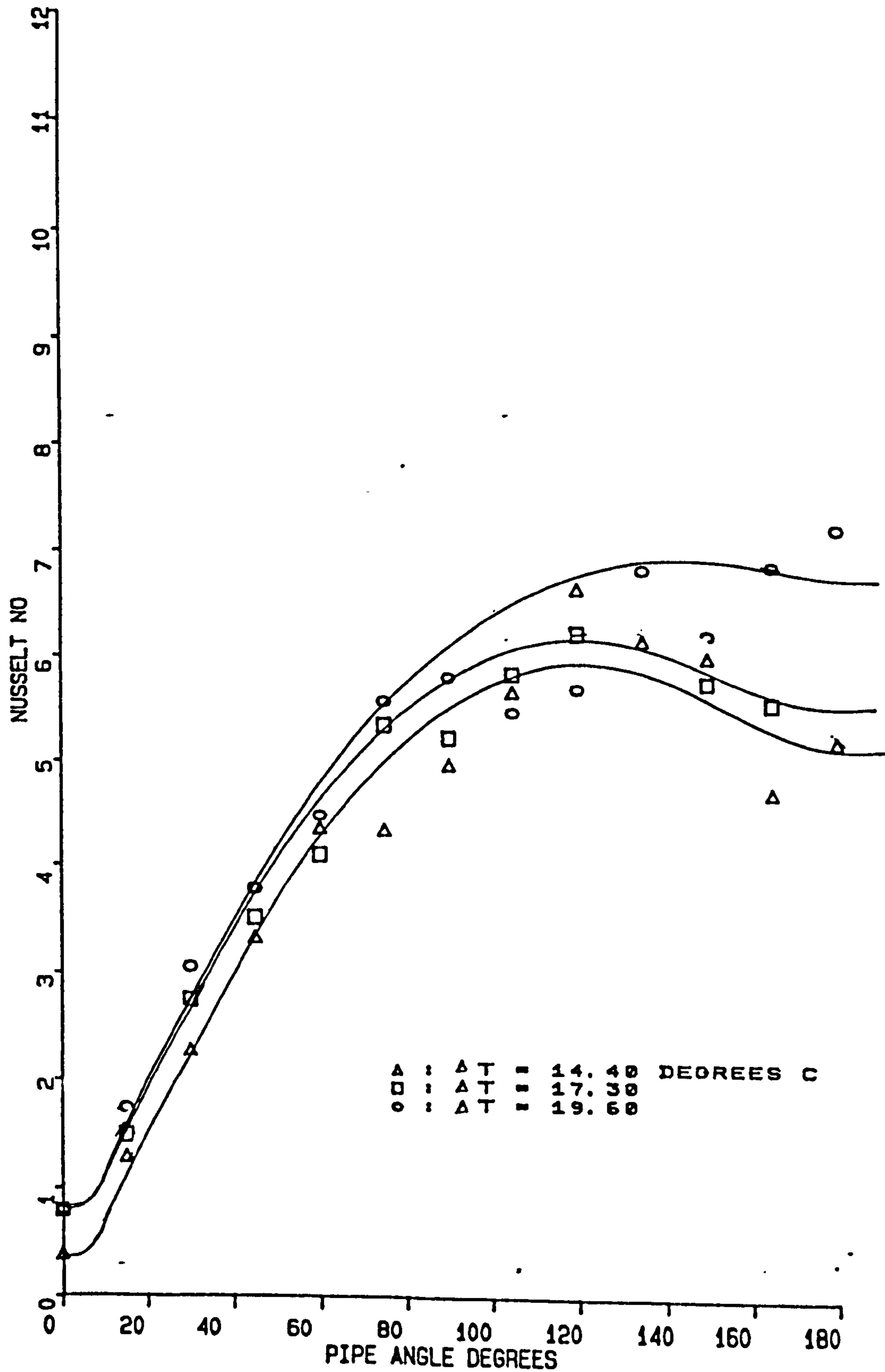


FIGURE 5.18

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.50
 RETURN PIPE DISPLACEMENT RATIO = -0.32

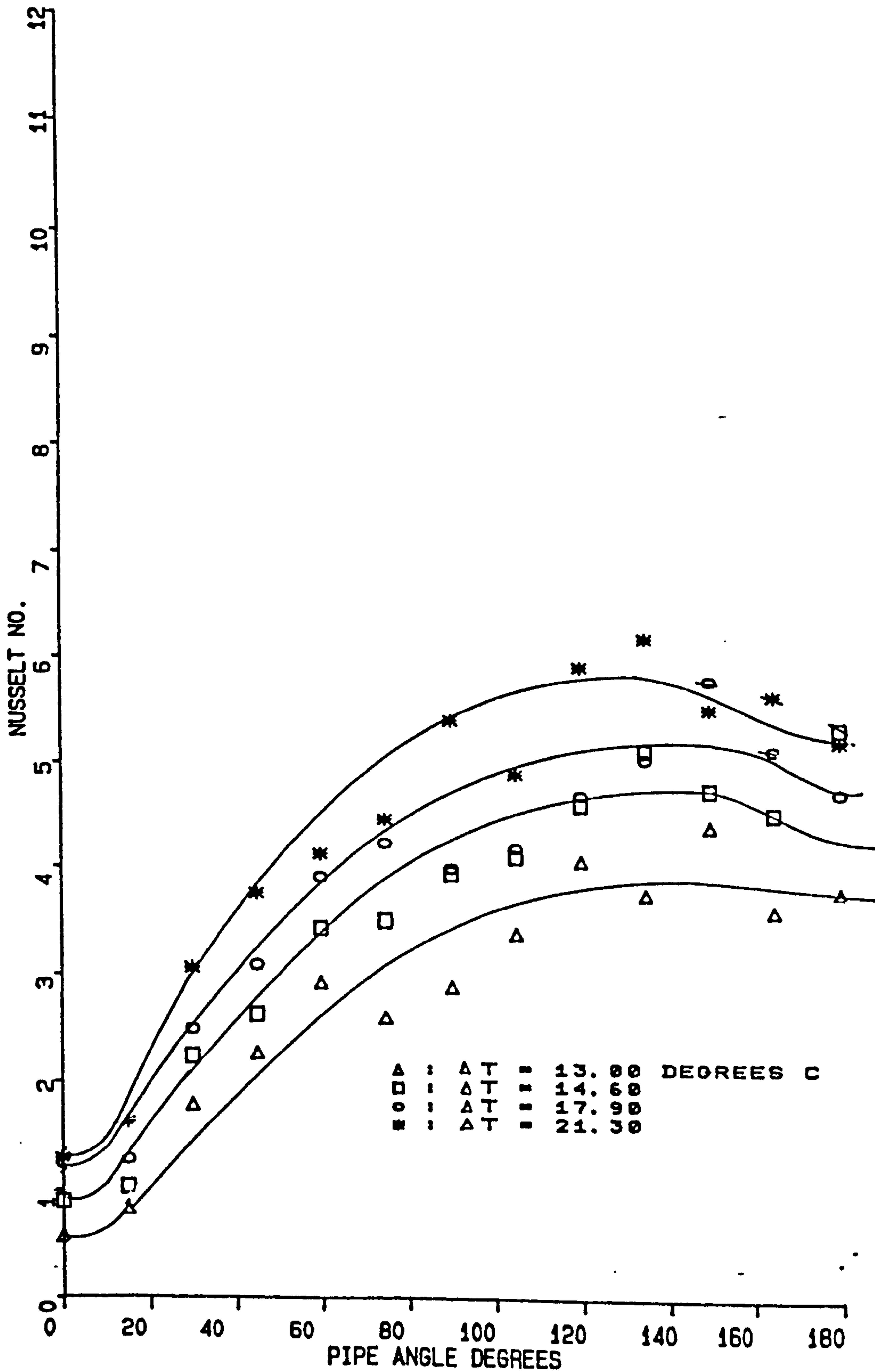


FIGURE 5.19

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.50
 RETURN PIPE DISPLACEMENT RATIO = -0.40

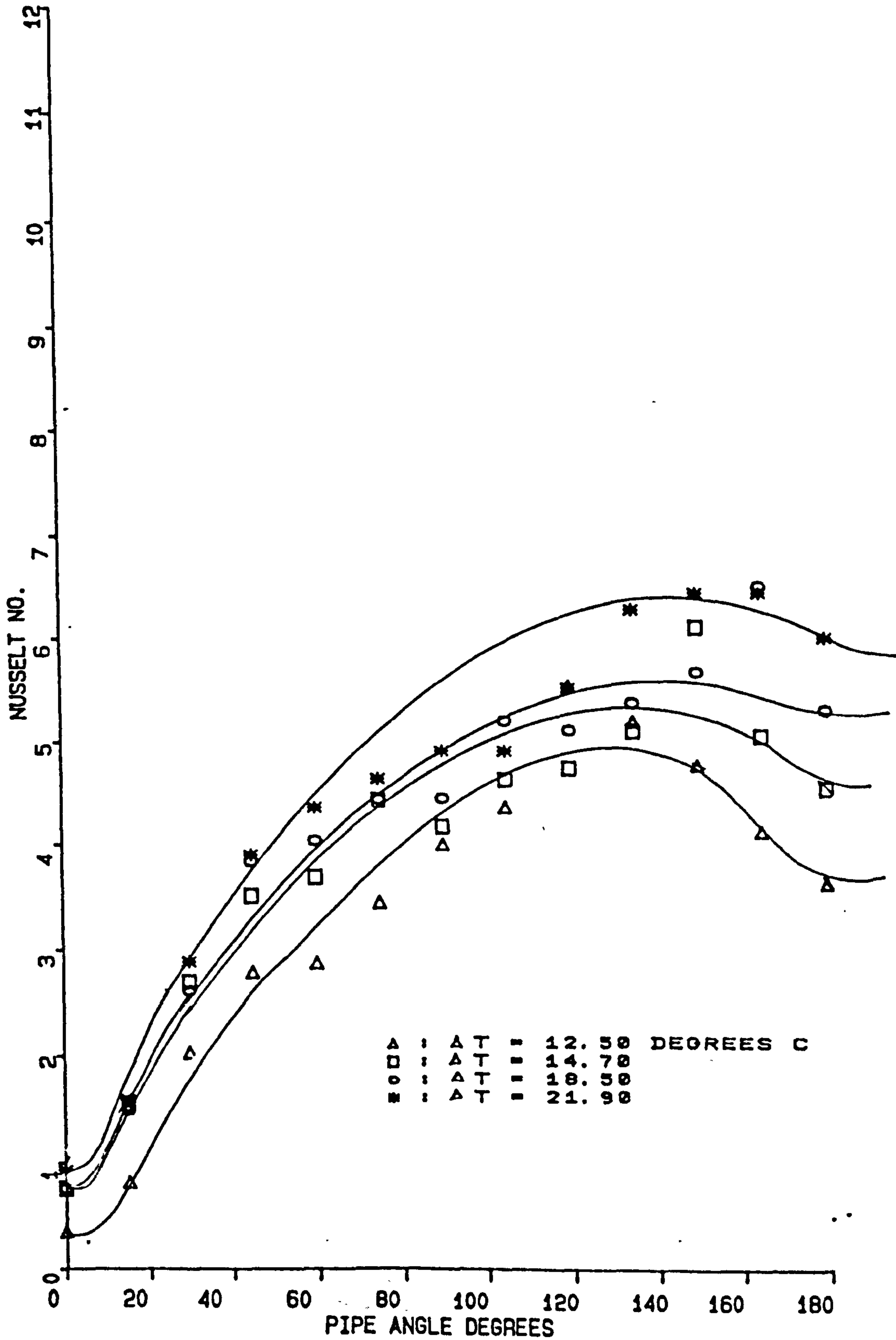


FIGURE 5.20

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.50
 RETURN PIPE DISPLACEMENT RATIO = -0.54

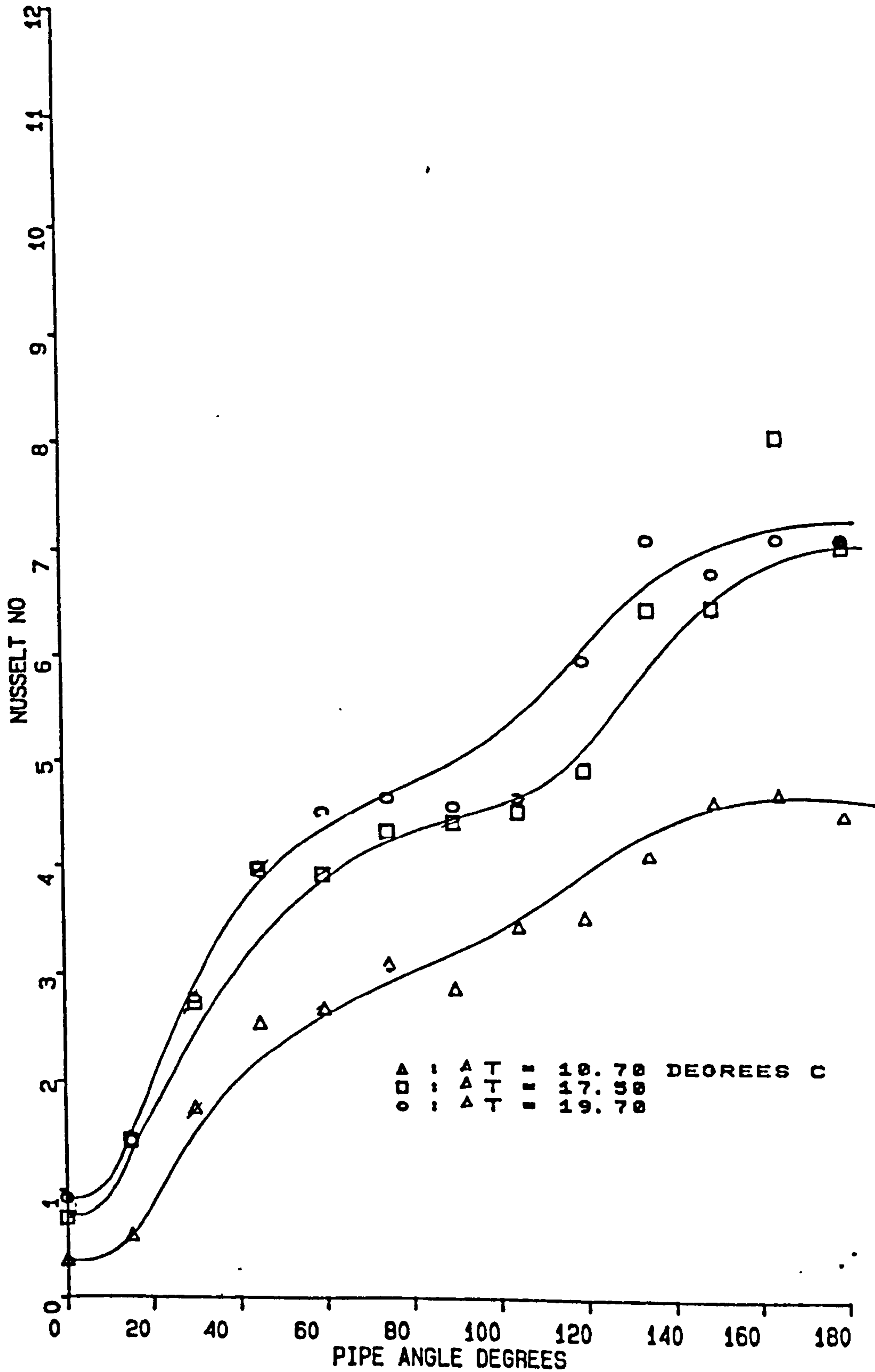


FIGURE 5.21

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.50
 RETURN PIPE DISPLACEMENT RATIO = -0.62

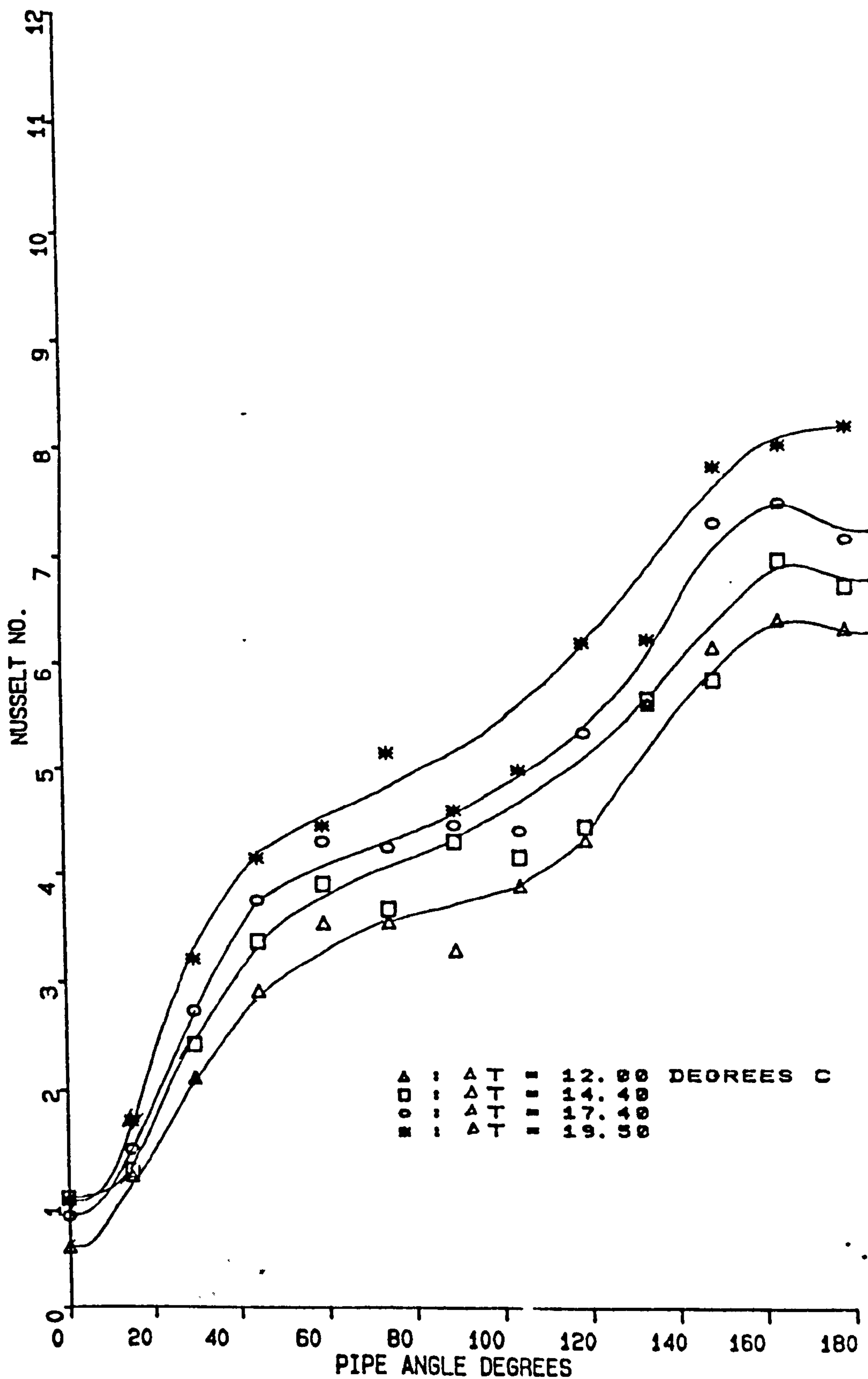


FIGURE 5. 22

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
SUPPLY PIPE DISPLACEMENT RATIO = 0.60
RETURN PIPE DISPLACEMENT RATIO = -0.07

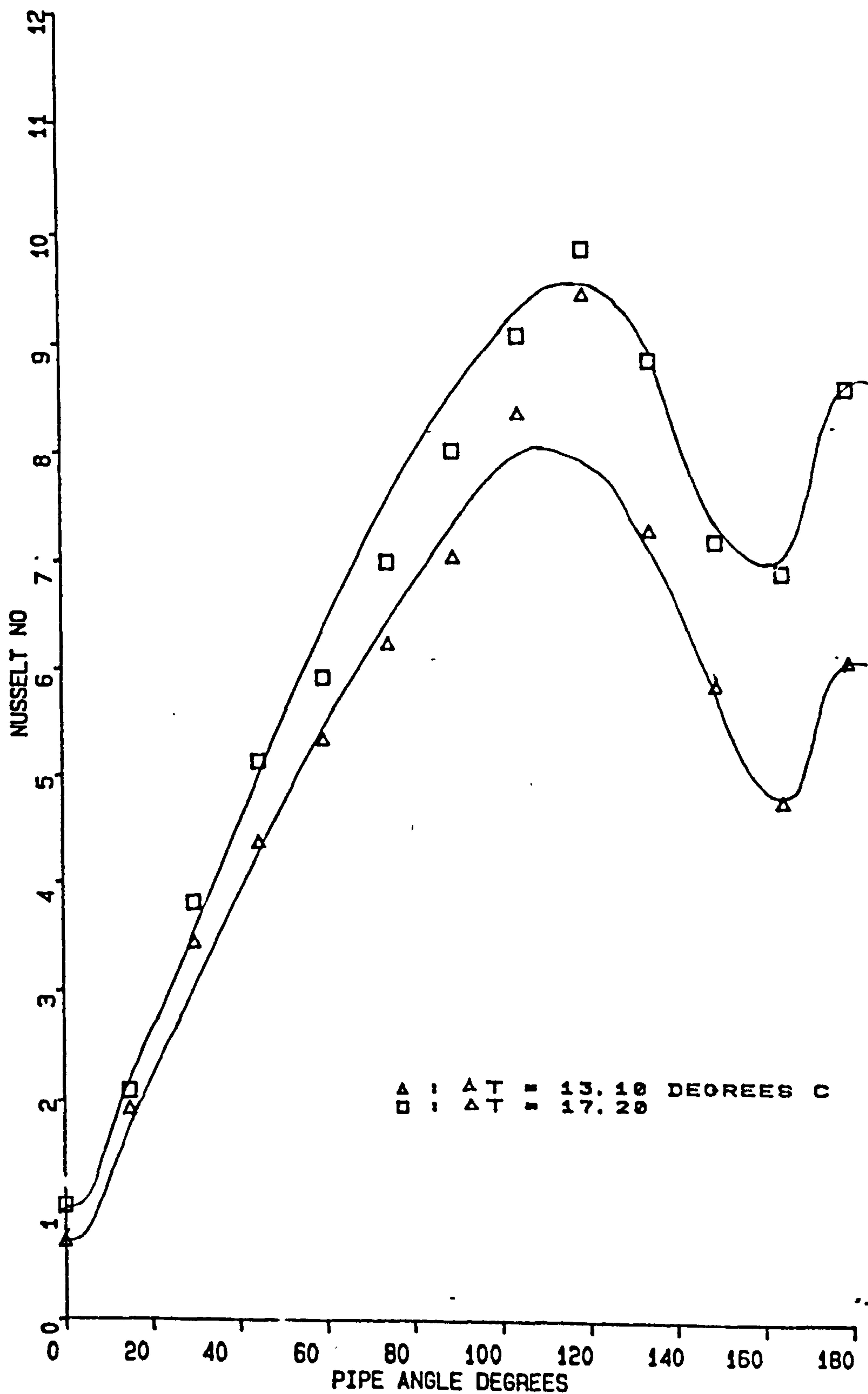


FIGURE 5.23

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
SUPPLY PIPE DISPLACEMENT RATIO = 0.60
RETURN PIPE DISPLACEMENT RATIO = -0.17

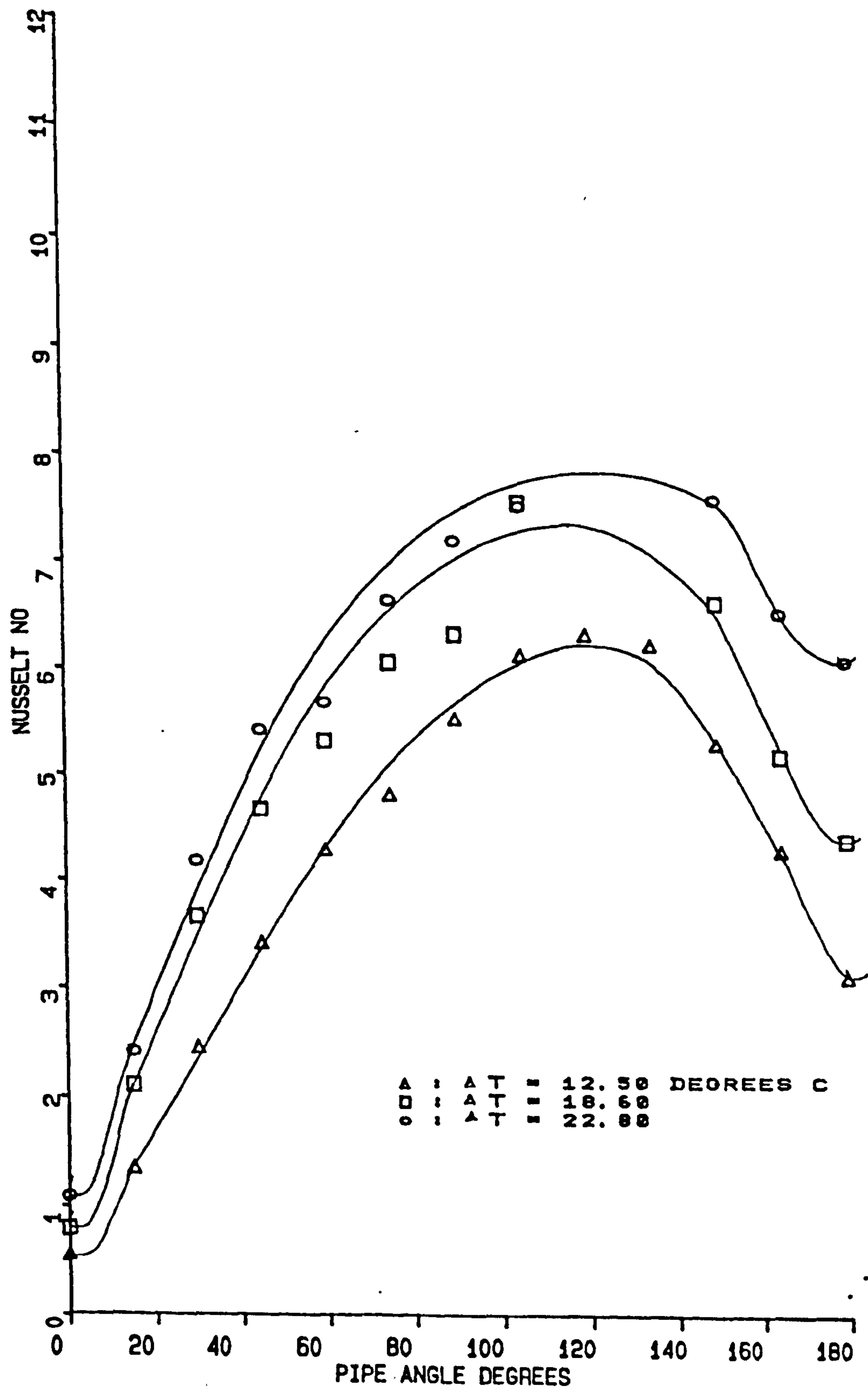


FIGURE 5.24

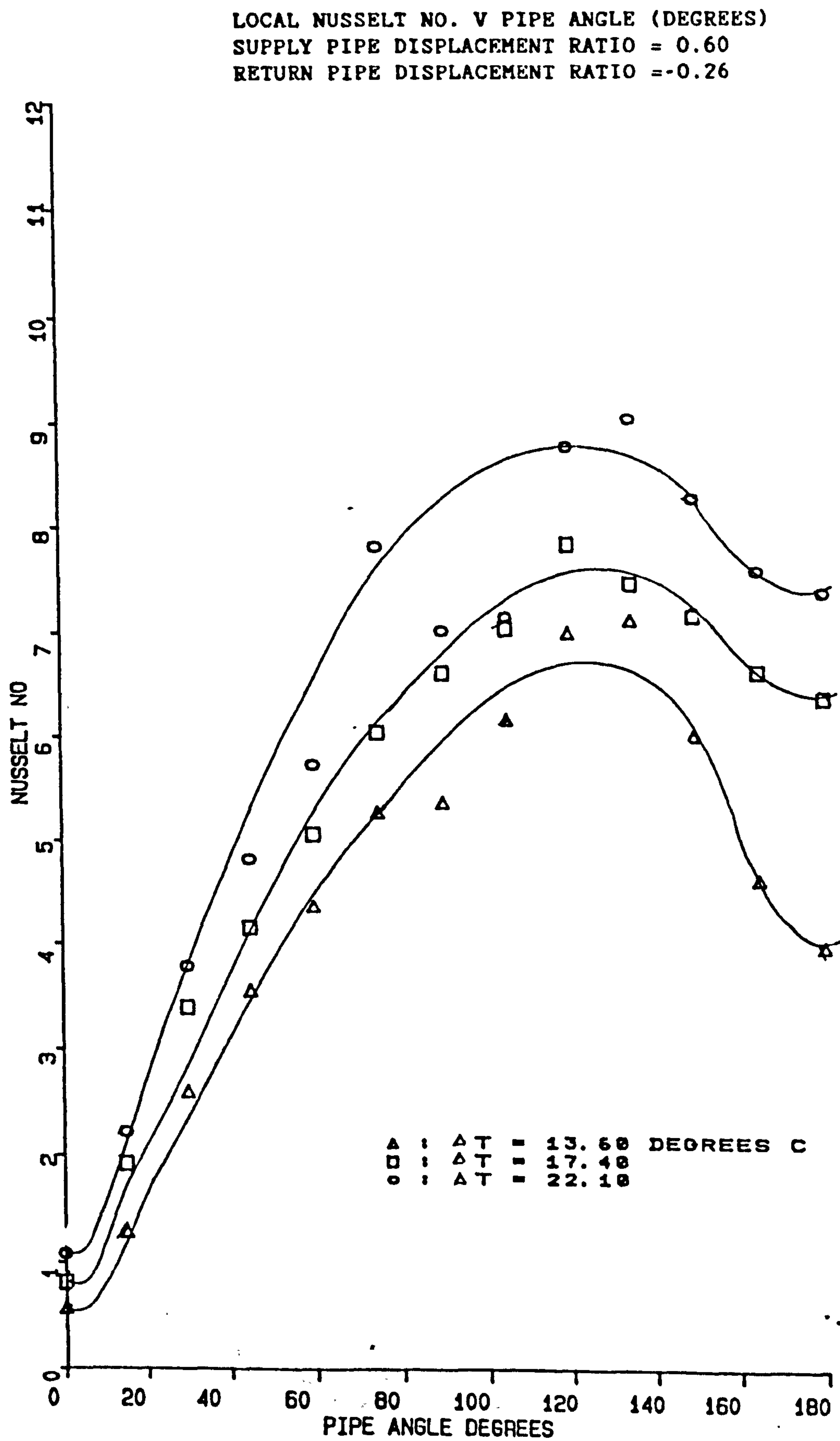


FIGURE 5.25

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
SUPPLY PIPE DISPLACEMENT RATIO = 0.60
RETURN PIPE DISPLACEMENT RATIO = -0.32

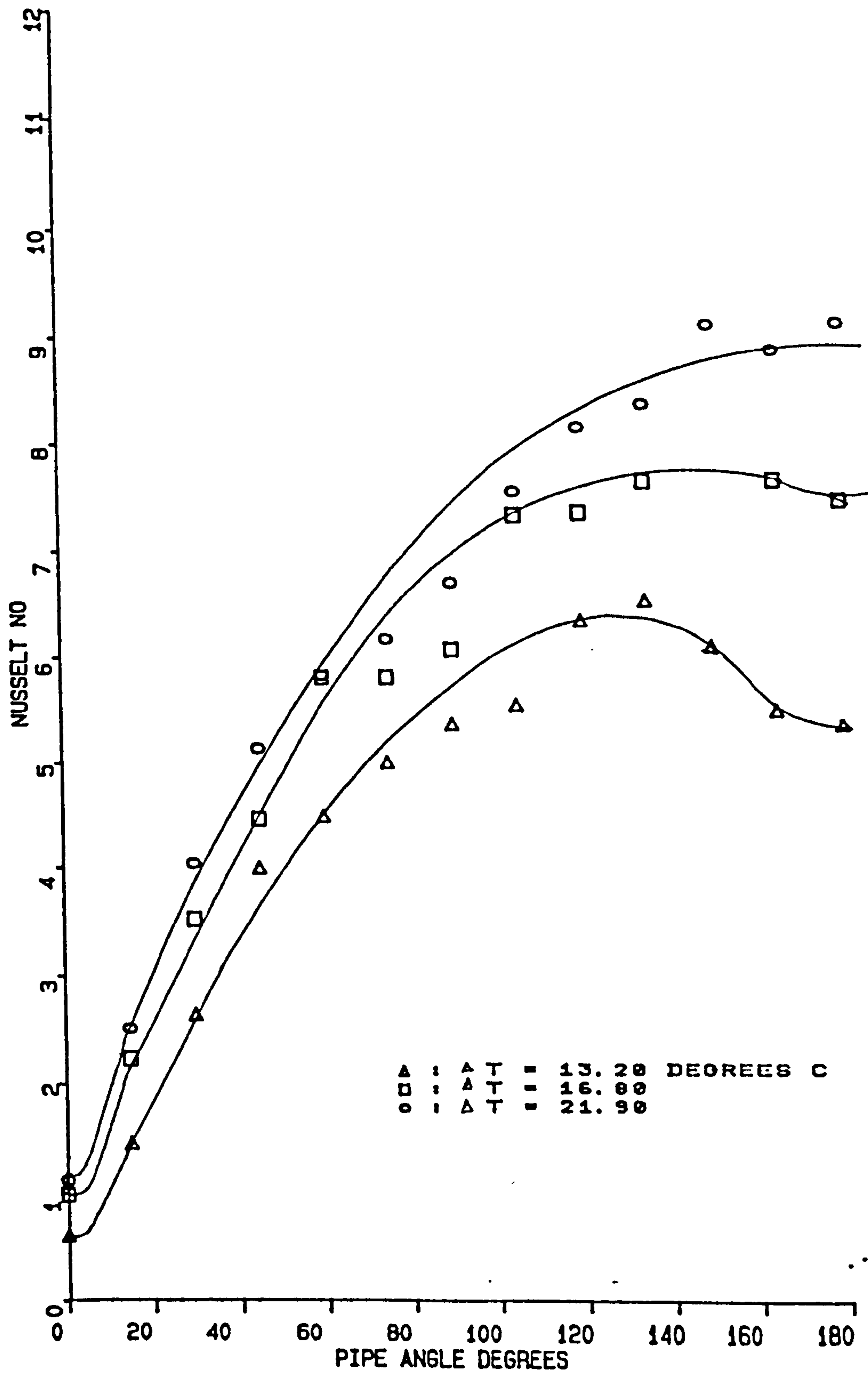


FIGURE 5.26

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.60
 RETURN PIPE DISPLACEMENT RATIO = -0.41

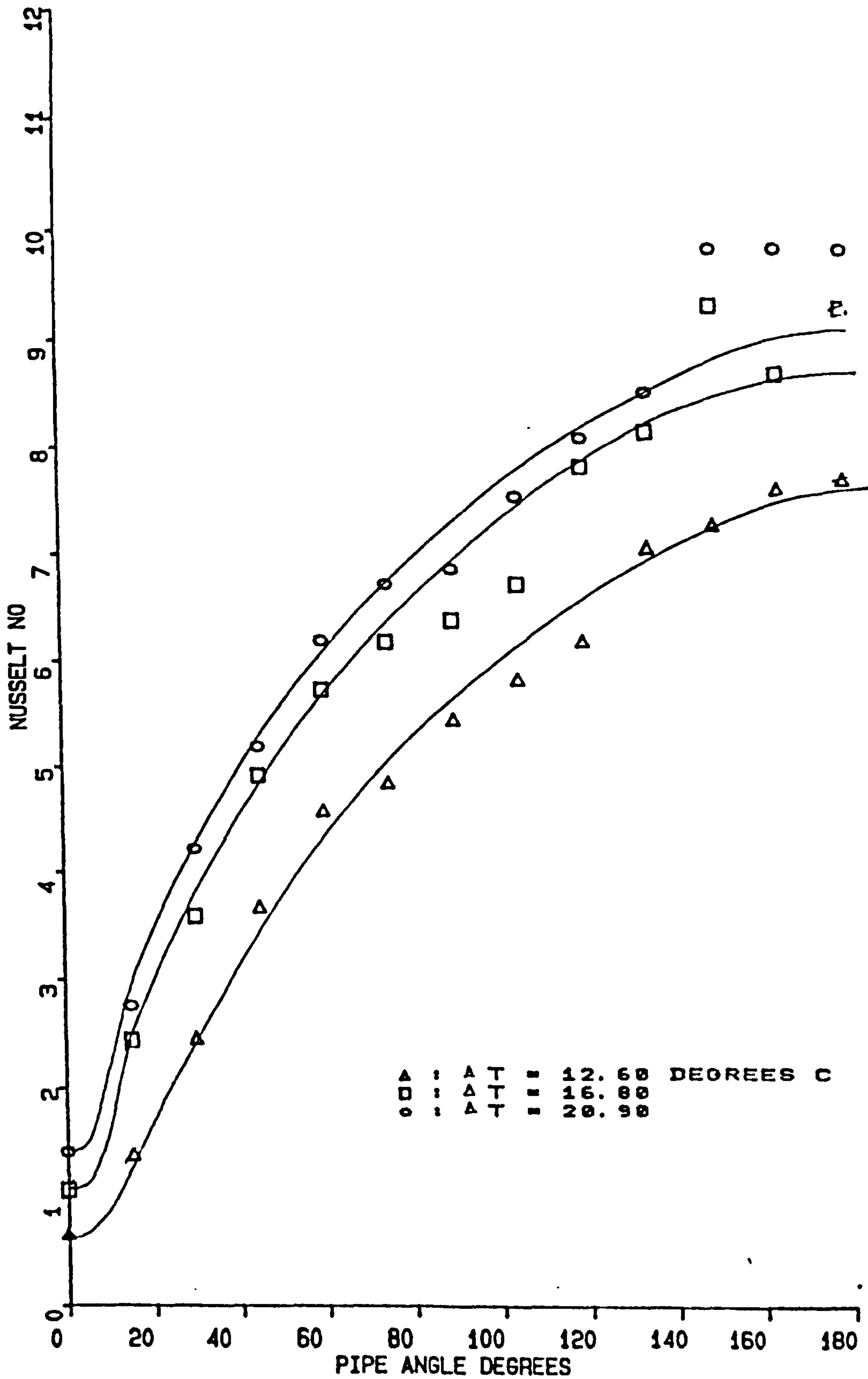


FIGURE 5. 27

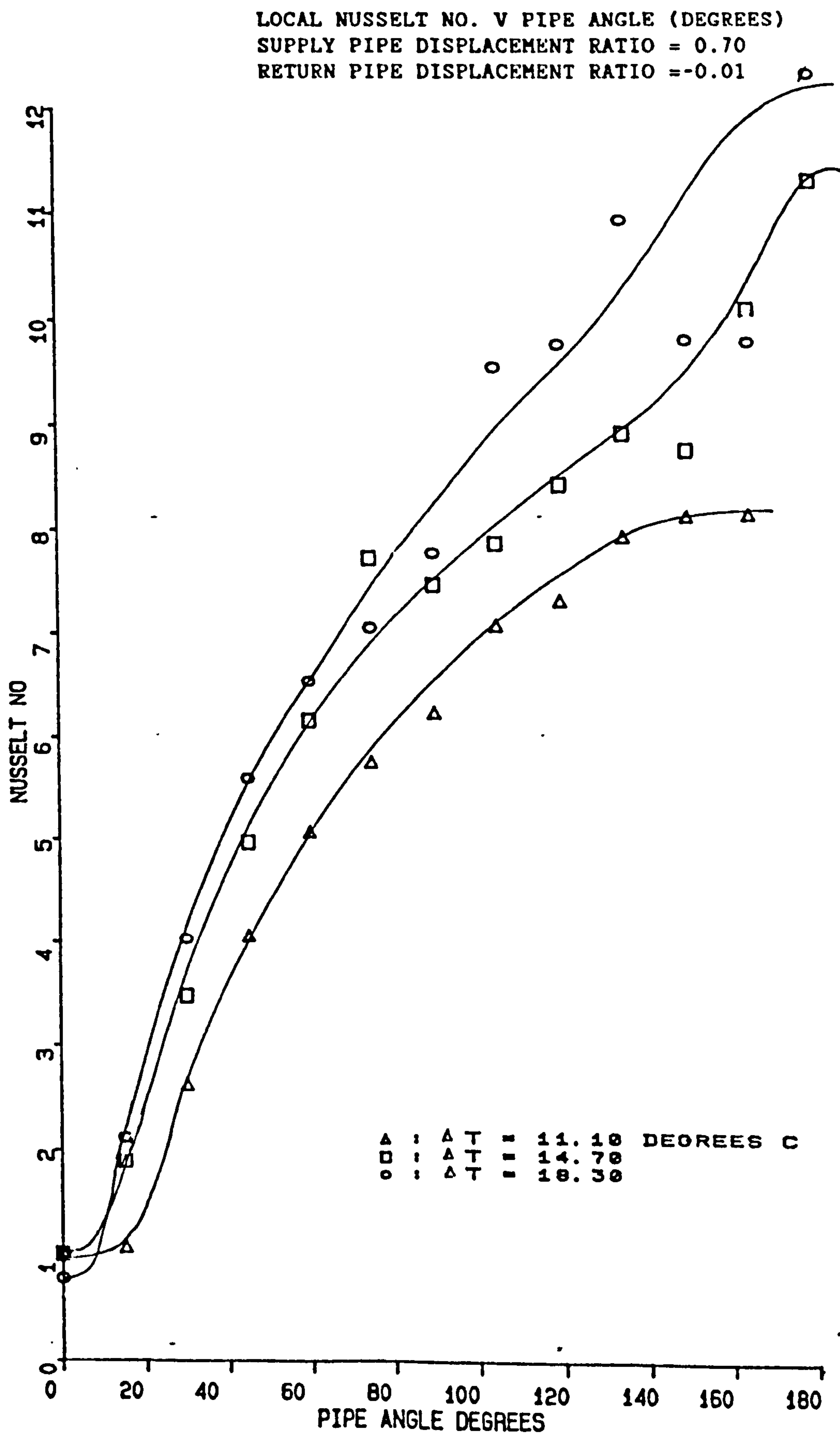


FIGURE 5. 28

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.70
 RETURN PIPE DISPLACEMENT RATIO = -0.05

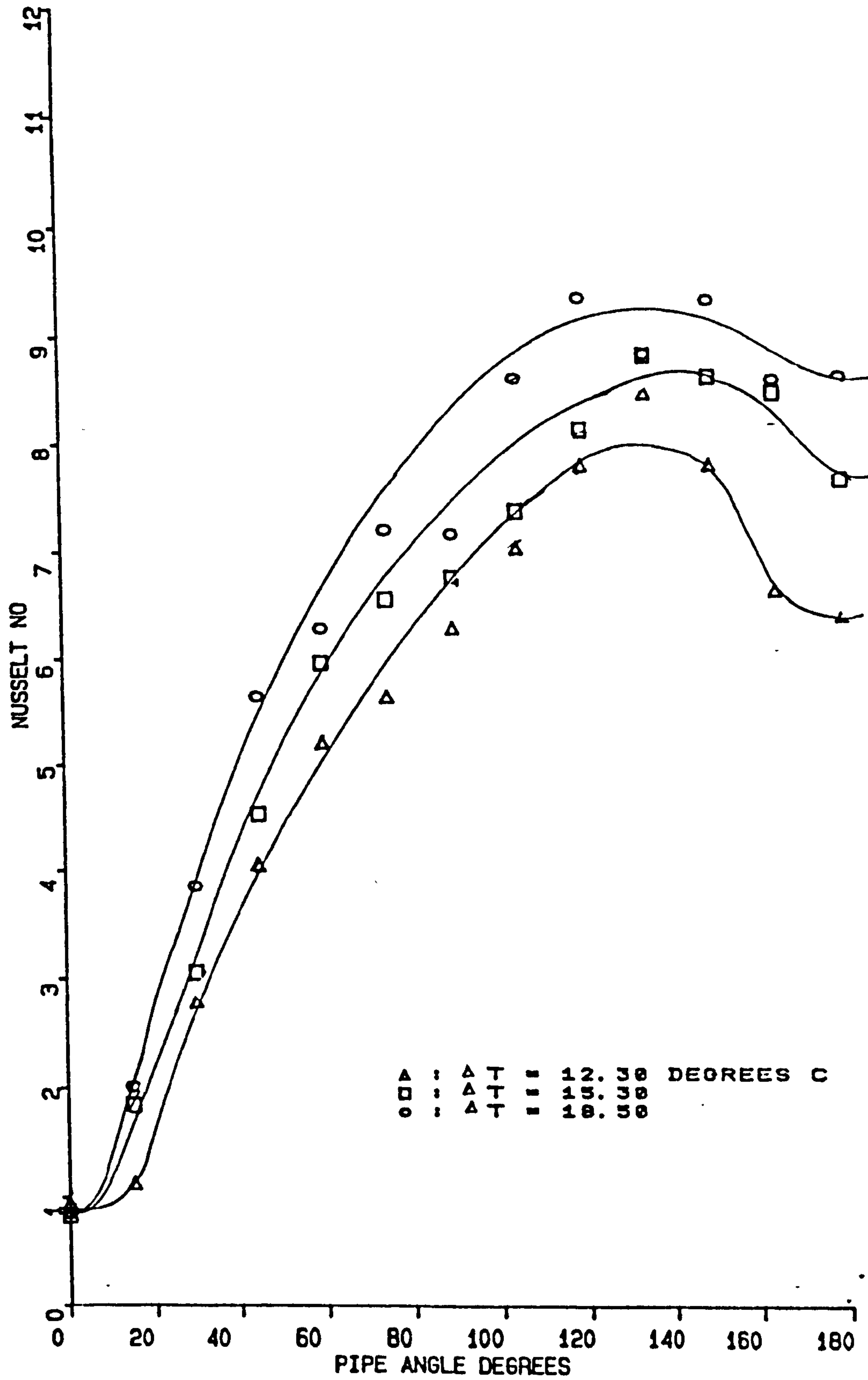


FIGURE 5. 29

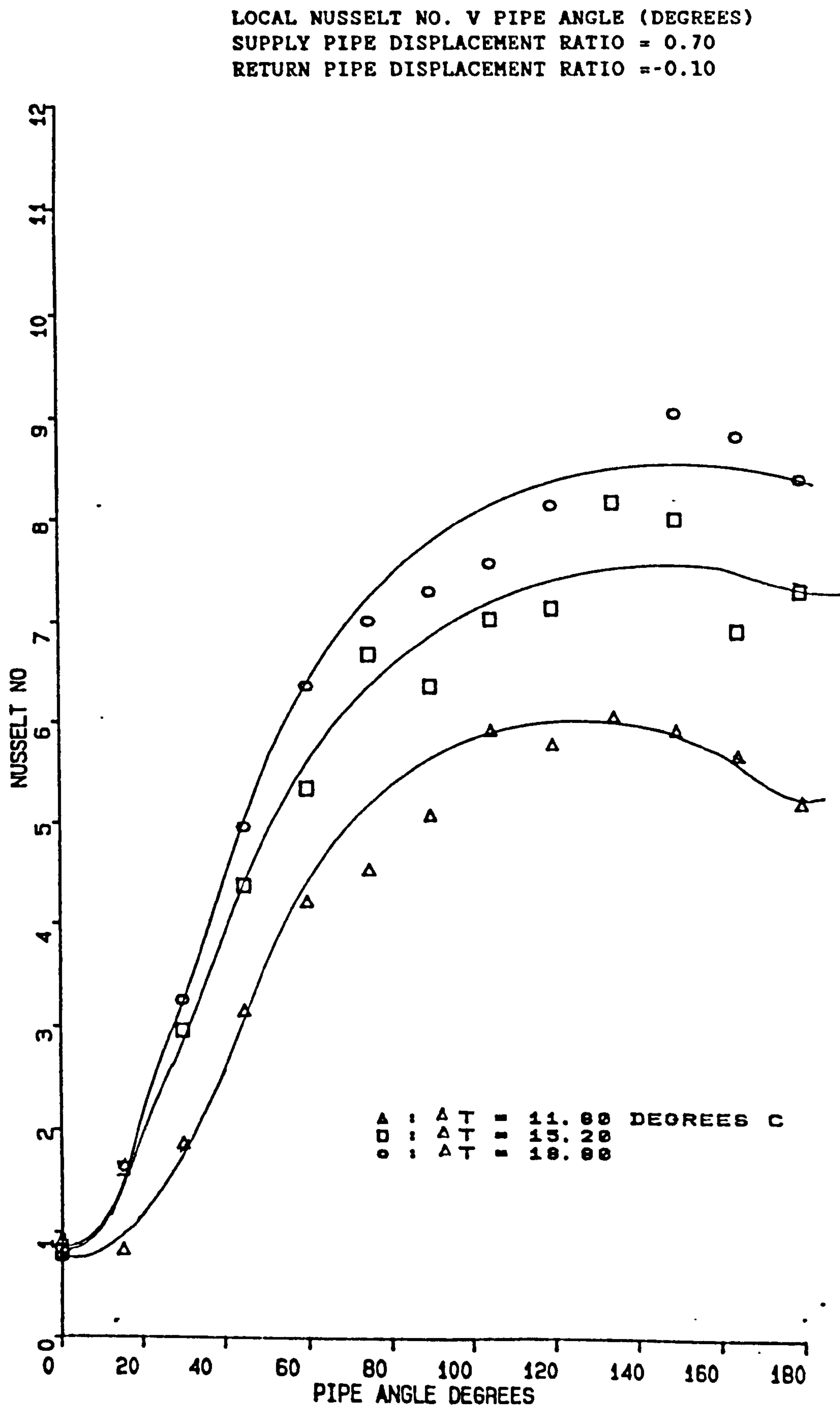


FIGURE 5.30

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
SUPPLY PIPE DISPLACEMENT RATIO = 0.70
RETURN PIPE DISPLACEMENT RATIO = -0.20

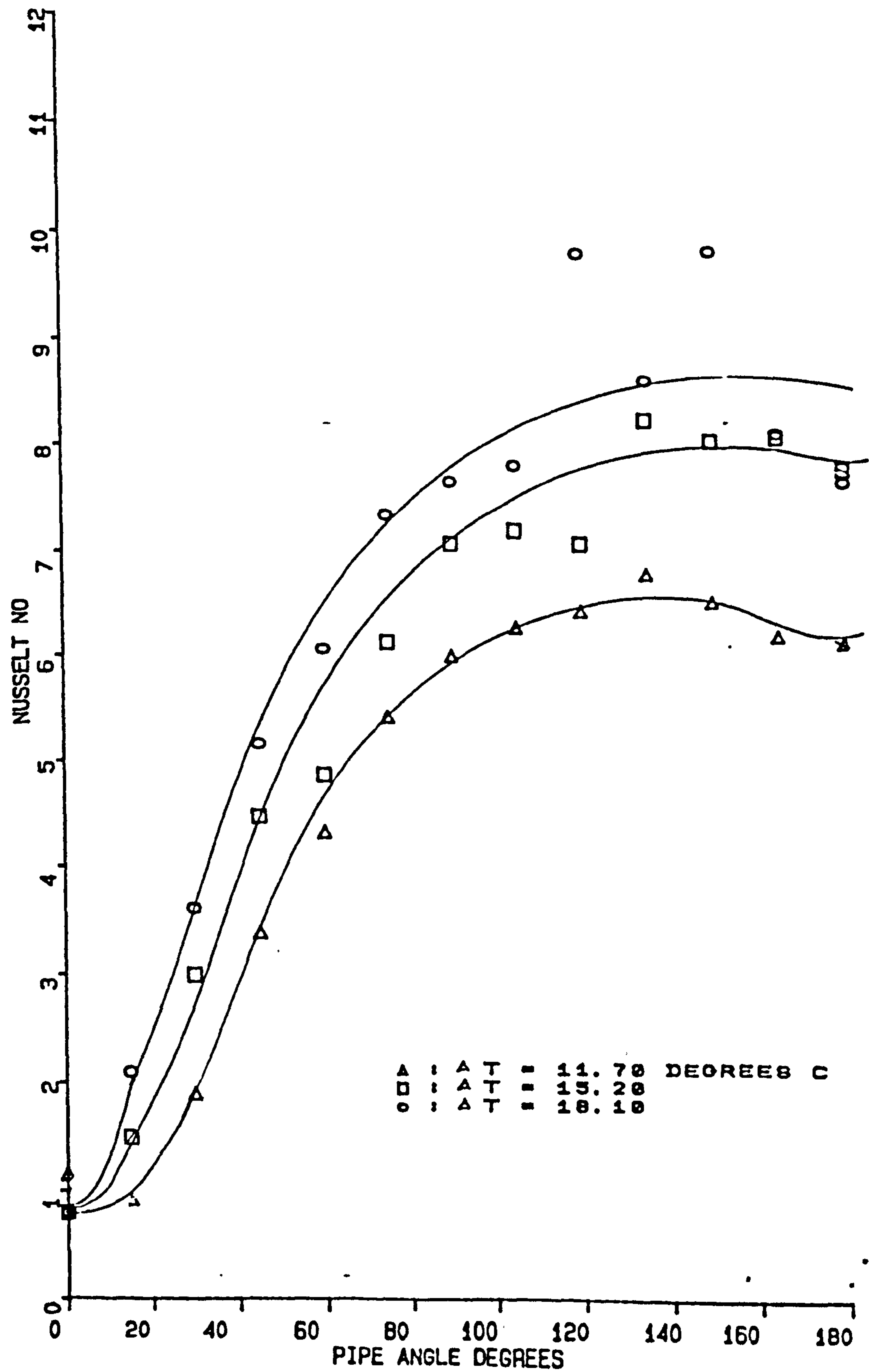


FIGURE 5.31

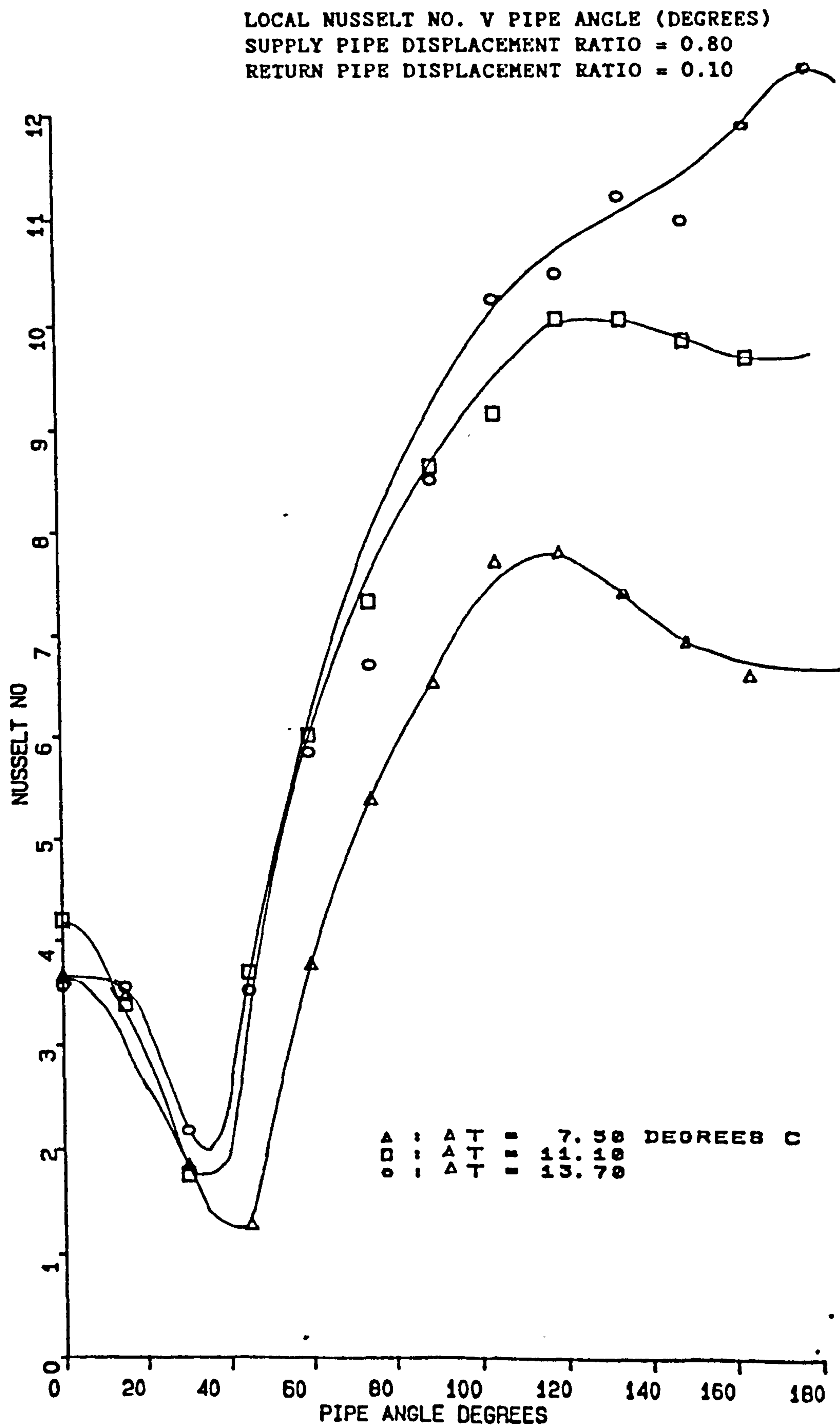


FIGURE 5.32

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
SUPPLY PIPE DISPLACEMENT RATIO = 0.80
RETURN PIPE DISPLACEMENT RATIO = 0.03

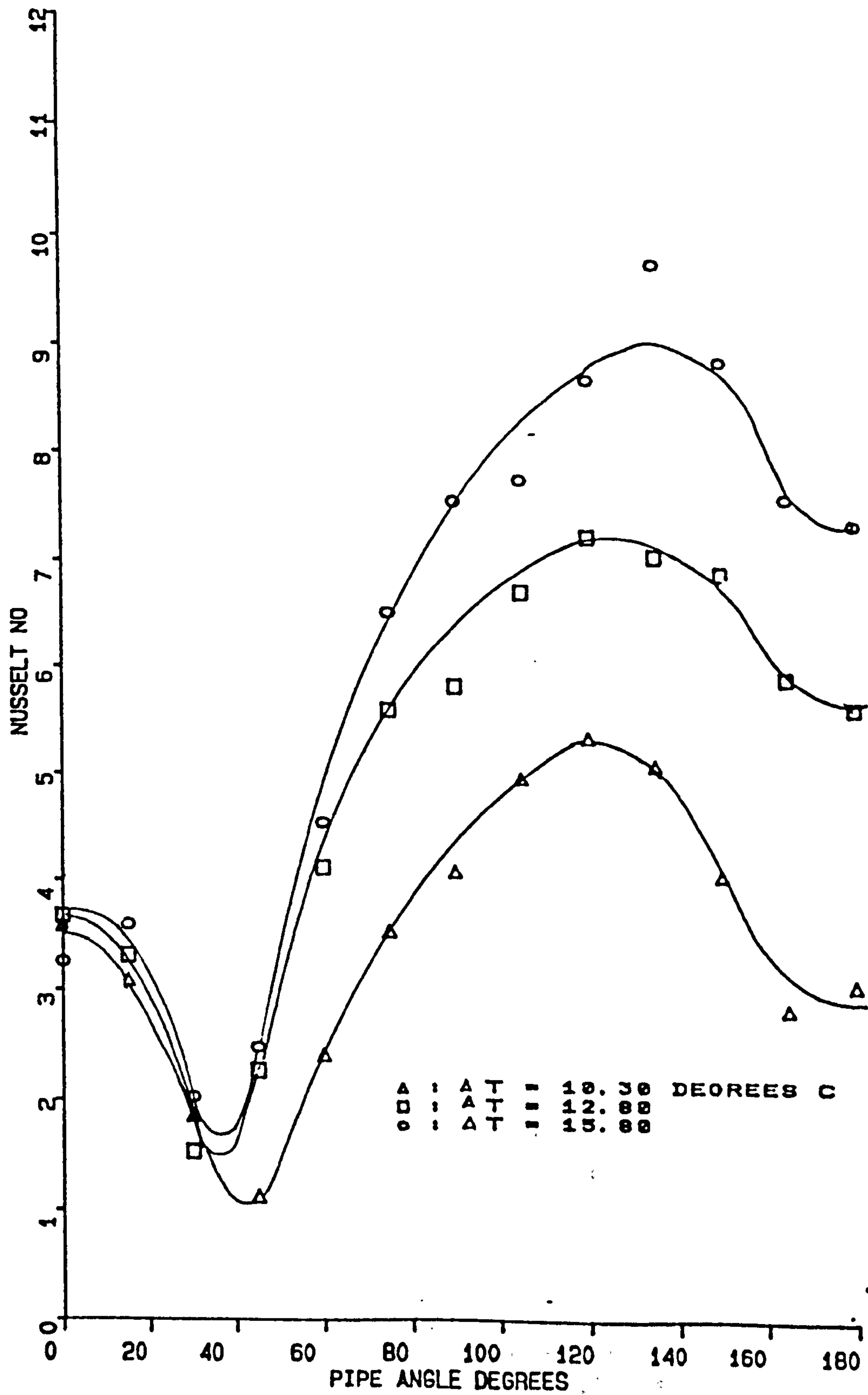


FIGURE 5.33

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.80
 RETURN PIPE DISPLACEMENT RATIO = -0.03

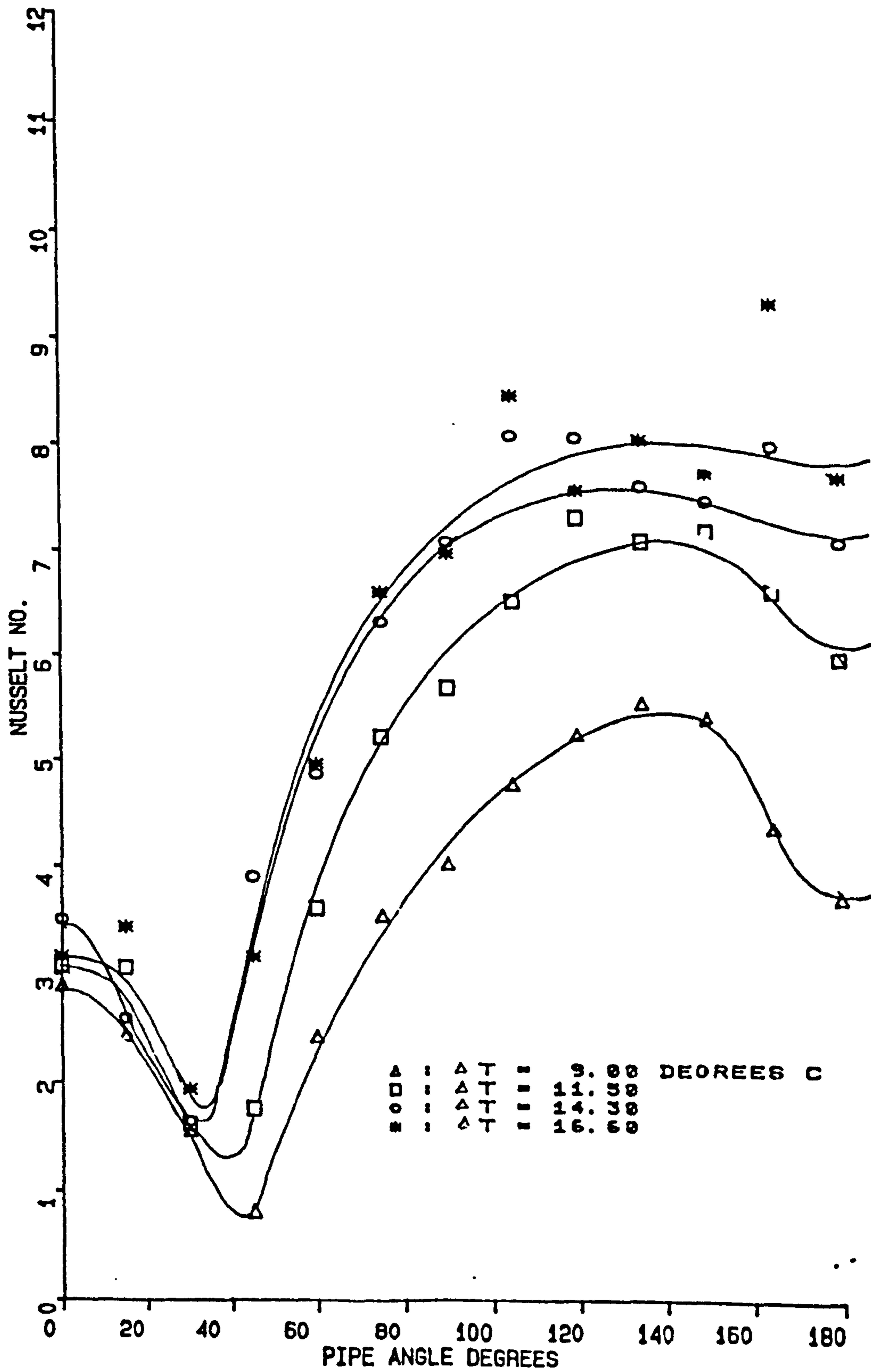


FIGURE 5.34

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.80
 RETURN PIPE DISPLACEMENT RATIO = -0.10

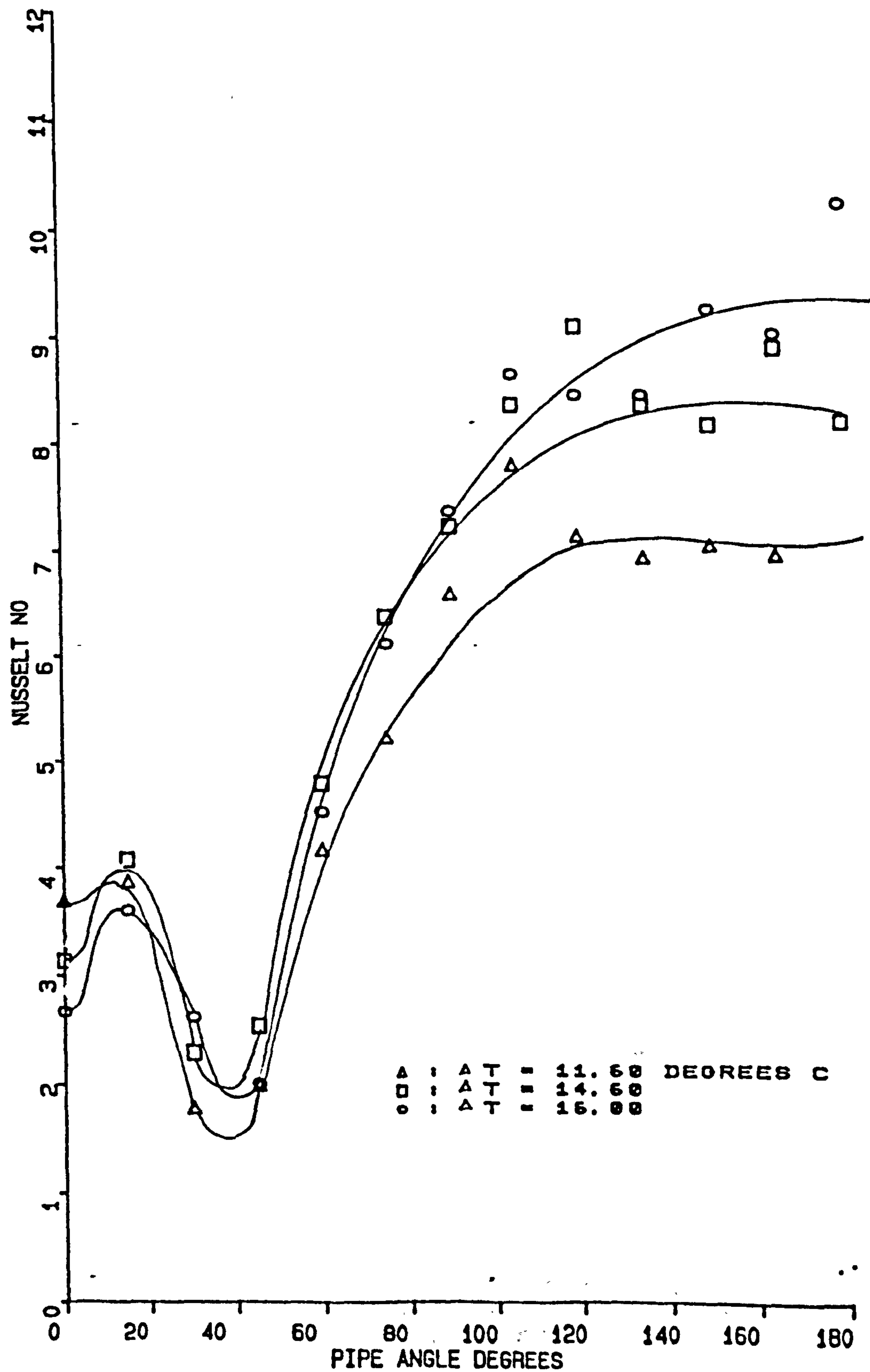


FIGURE 3.35

LOCAL NUSSELT NO. V PIPE ANGLE (DEGREES)
 SUPPLY PIPE DISPLACEMENT RATIO = 0.80
 RETURN PIPE DISPLACEMENT RATIO = -0.20

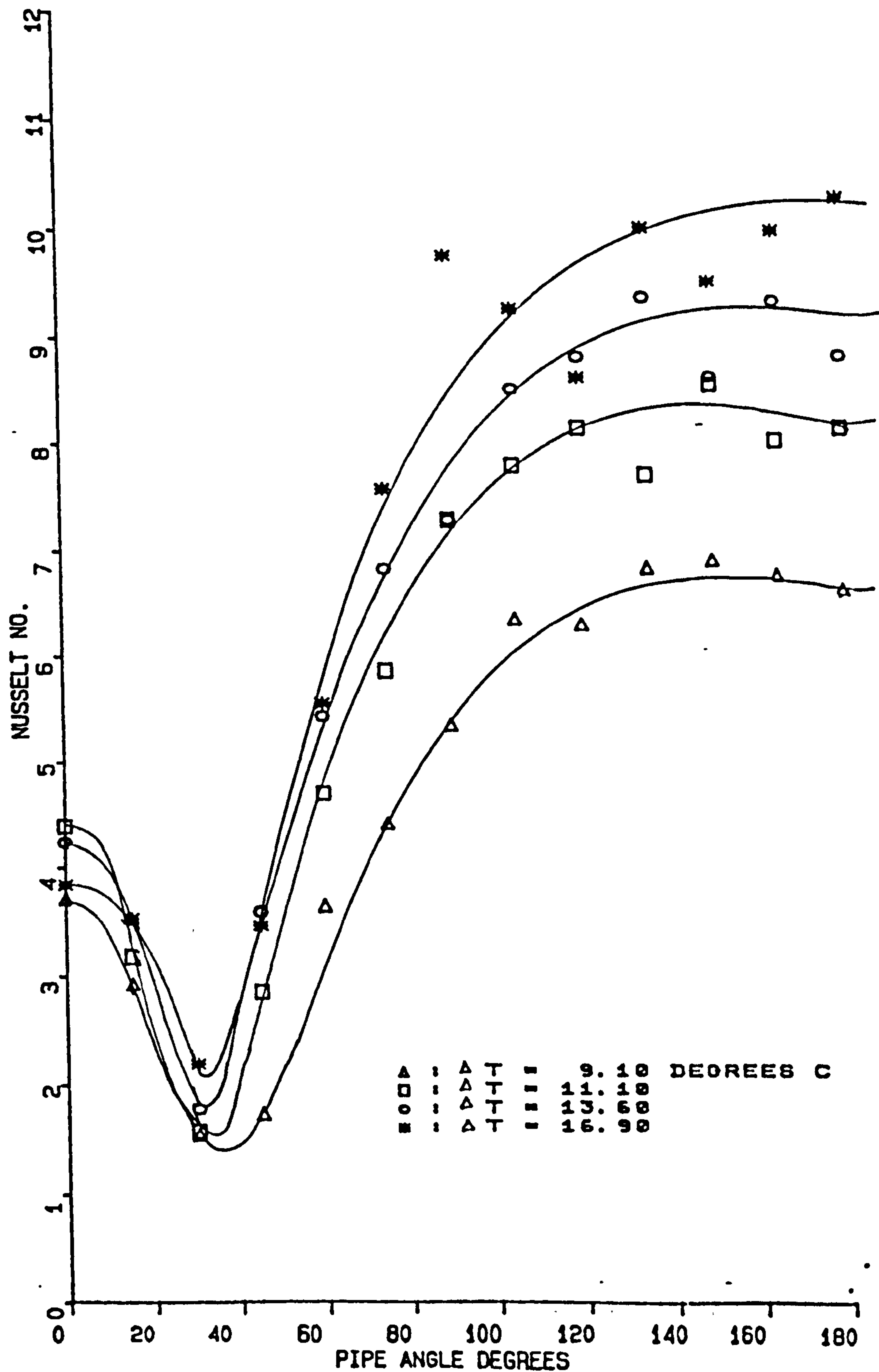


FIGURE 5.36

TOTAL HEAT TRANSFER Q_t (W) FROM SUPPLY PIPE ✓
 DISTANCE BETWEEN SUPPLY AND RETURN PIPES (mm)

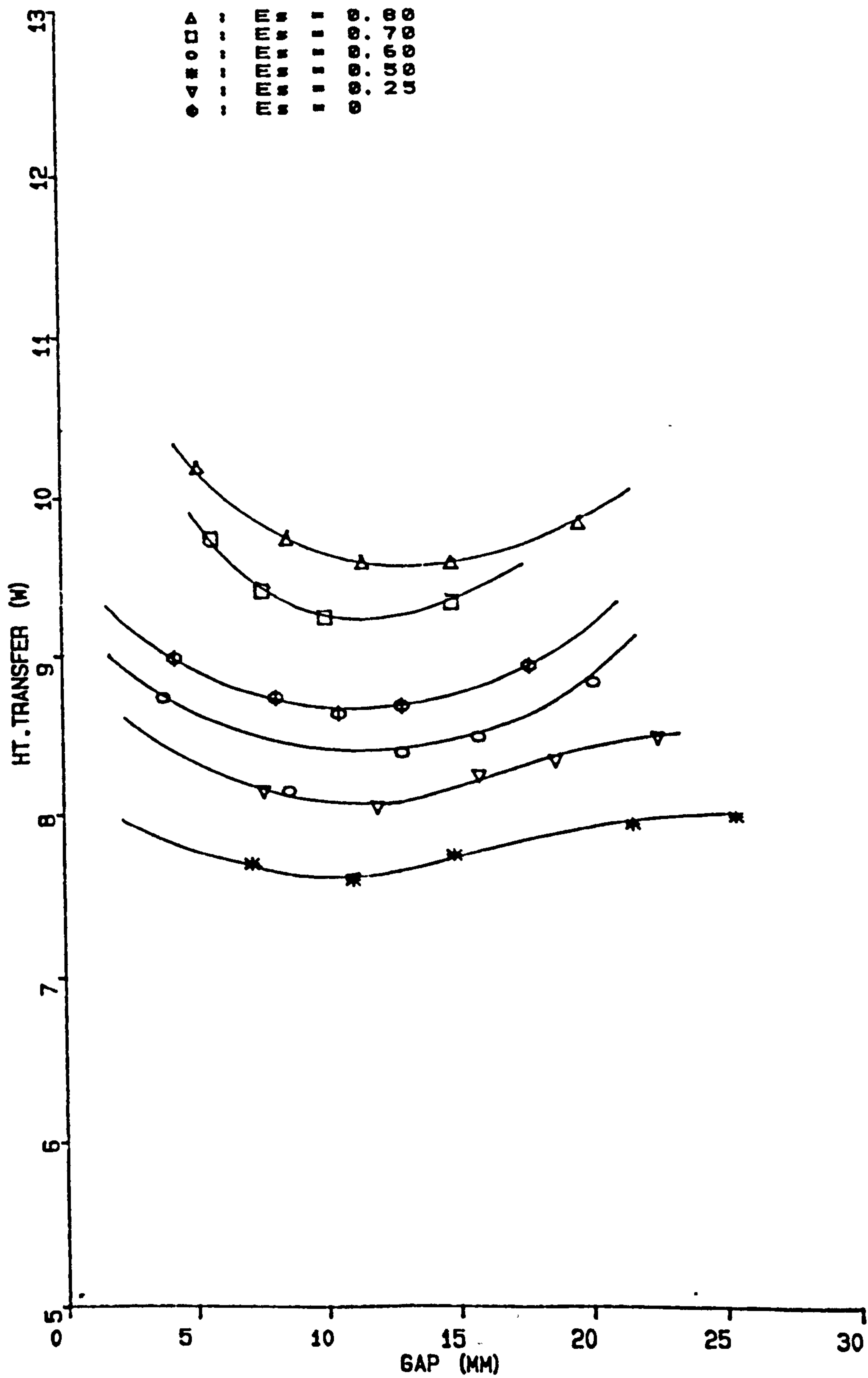
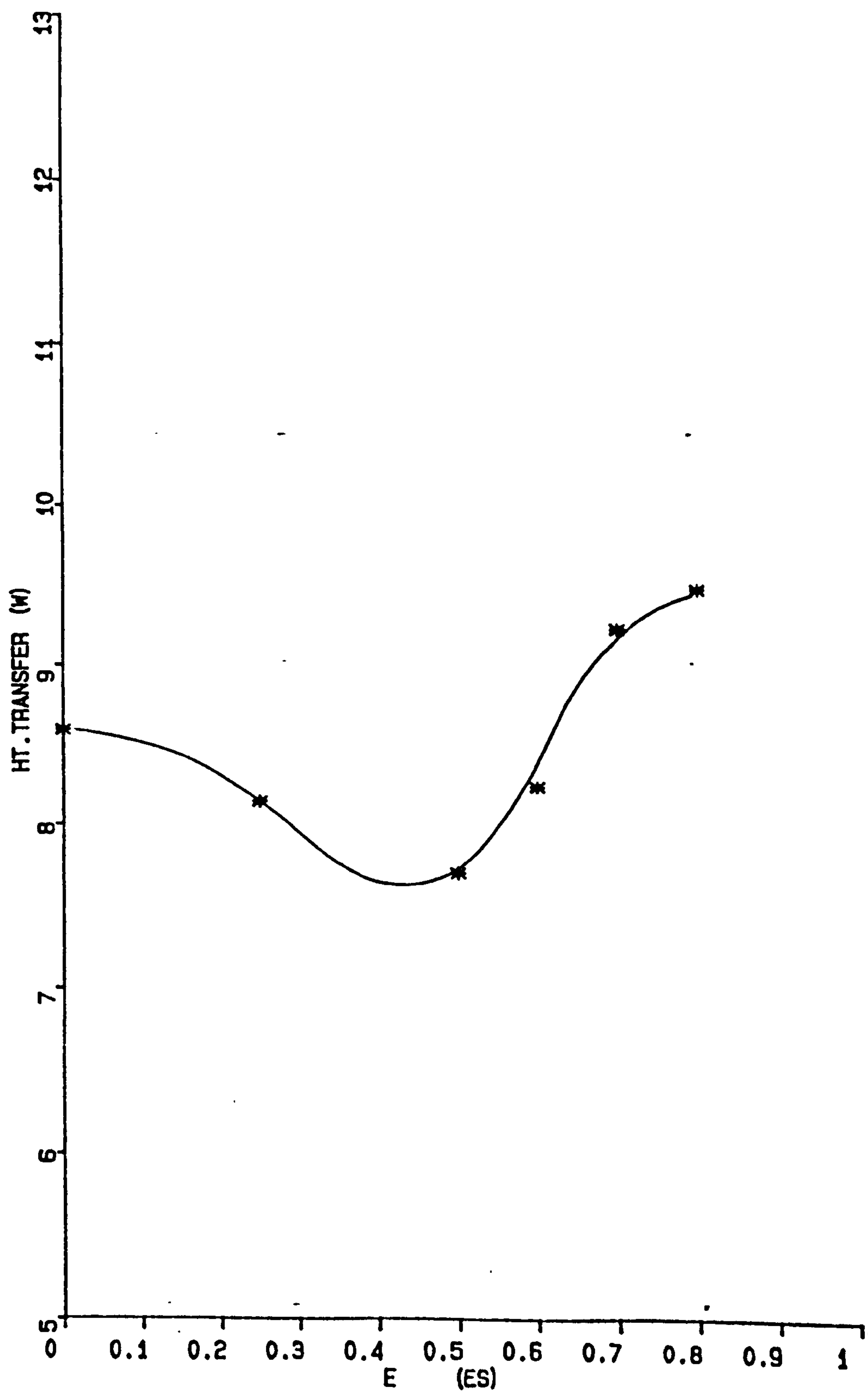


FIGURE 5.37

TOTAL HEAT TRANSFER Q_t (W) FROM SUPPLY PIPE V
SUPPLY PIPE DISPLACEMENT RATIO AT OPTIMUM DISTANCE
BETWEEN PIPES



Chapter 6

Numerical Study of the heat-transfers within a
rectanqular cavity containing a heated cylinder

Numerical Study of the heat-transfers within a
rectangular cavity containing a heated cylinder

6.1 Introduction

As experimental investigation has proved excessively time consuming and laborious, a computer program was developed to model the cavity flow problem, using the finite-element method.

The first such numerical study at CIT was undertaken by Caudiu (31) who wrote a program to investigate the configuration of one cylinder concentrically placed within another. This work was extended by Yeo (32) to include eccentric positioning of the inner cylinder. Forien (41) adapted the program to study eccentric placement in a rectangular duct. It is intended to run this program with the aim of predicting the optimal position of a hot cylinder within the cavity, so that heat losses are minimised.

6.2 Main Program

The solution is assumed to be symmetrical about the vertical plane, consequently the mesh is generated by program MAILLAGE for only half of the domain. The program NEWPROG uses this mesh data and solves equations of mass, momentum and energy using a Gaussian elimination method.

The program structure, both external and internal, is shown in Figure 6.1.

A more comprehensive explanation of solution and program subroutines is given in references 32 and 41.

6.3 Range of Tests performed

Test runs were performed with a mesh of 64 elements at a radius ratio of 1.33 for Rayleigh number 1 to 10,000 in logarithmic increments of 0.5 and at values of displacement ratio from -0.4 to 0.8 in steps of 0.1. An example of a mesh used is shown in Figure 6.2.

Tests were also performed at radius ratios of 1.5 with 80 elements and 2.0 with 64 elements, at a Rayleigh number of 100.

6.4 Eccentricity Effects on Heat-Transfer

The variation of mean Nusselt number with hot pipe position is shown in Figures 6.3 to 6.5 for different radius ratios. Cylindrical annulus data obtained experimentally by Shilston (42) are included for similar values of radius ratio but higher Rayleigh numbers.

It should be noted that the experimental 'Nusselt numbers' are relative to the value for the concentric case (ie 'Nu' = 1 at $E = 0$) so they do not bear direct comparison with the numerically-derived data for the square duct configuration.

The results may be summarised as follows:

Radius Ratio Square / Cylinder	Rayleigh No Square / Cylinder	Optimal E Square / Cylinder
1.33 / 1.31	All	0.18 / 0.24
1.50 / 1.54	100	0.25 / 0.52
2.00 / 2.43	100	0.48 / 0.58

The radius ratio for a square duct configuration is defined as the ratio of half duct height to inner cylinder radius.

These results are presented in Figure 6.6 and show a trend towards increasing optimal displacement ratio with increasing radius ratio for both square and cylindrical ducts. The experimental result of James (16) for a square duct, radius ratio of 3.57, is included. Good agreement is shown with the result obtained by extrapolating the numerical results to higher radius ratios.

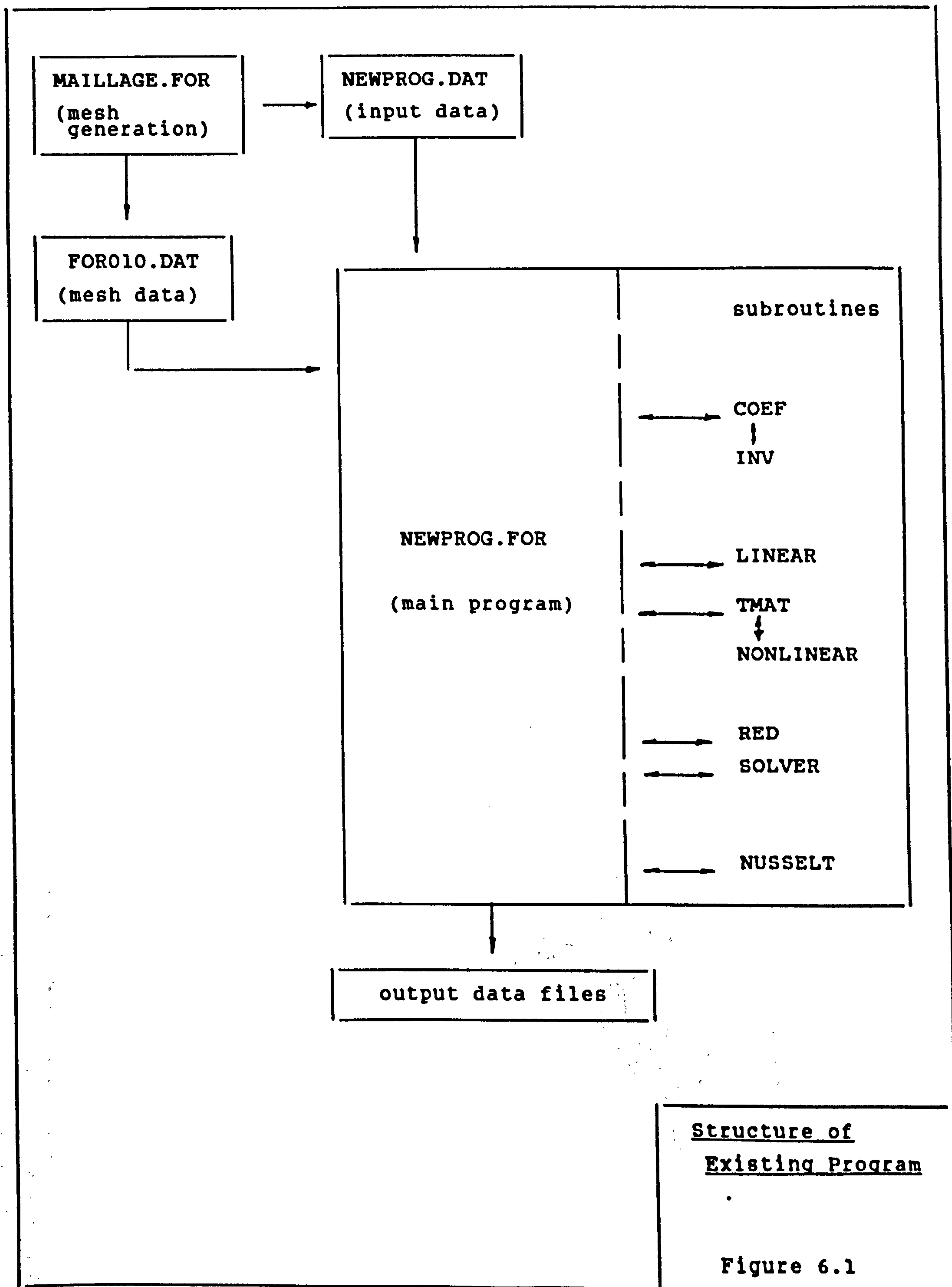
For all Rayleigh numbers the optimal cylinder position is about the same for a radius ratio of 1.33. However, at $E = 0.3$, the values of Nusselt number do not follow the same trend as at other values of displacement ratio. Also, at $E = -0.3$ nearly all solutions diverged and in the only convergent case, for $Ra = 300$, the results differed greatly from those at displacement ratios -0.2 and -0.4 , (compare contour plots in Figures 6.7 to 6.9). It would appear that the mesh is not fine enough or the distribution of elements are not suitable to model the flow adequately.

6.5 Limitations of the Numerical Solution

In several cases the solution would not converge because the flow could not be modelled adequately. To a certain extent this was overcome by increasing the number of elements in the mesh employed. However the CPU time required for convergence then increased dramatically. At present, a 64-element mesh requires a 35 minute initial calculation followed by 20 minutes per iteration until convergence is reached. This is excessive.

Because of the number of elements involved solutions could neither be reached at high radius ratios nor at high Rayleigh numbers.

Solution times could also be reduced by choosing a suitable convergence factor. As the operator has to choose this factor, it would be desirable to have a more advanced procedure for sensing convergence or divergence. This may permit less stable situations mathematically to be modelled.



**FINITE ELEMENT ANALYSIS OF
NATURAL CONVECTION IN
AIR-FILLED CAVITIES**

Number of elements = 64

Number of nodes = 109

Number of equations to be solved = 398

**Height = 2.000
Inner radius = 1.000
Yshift = 0.000
Displacement ratio = 0.000
Radius ratio = 2.000
Number of angular divisions = 8
Number of radial divisions = 4**

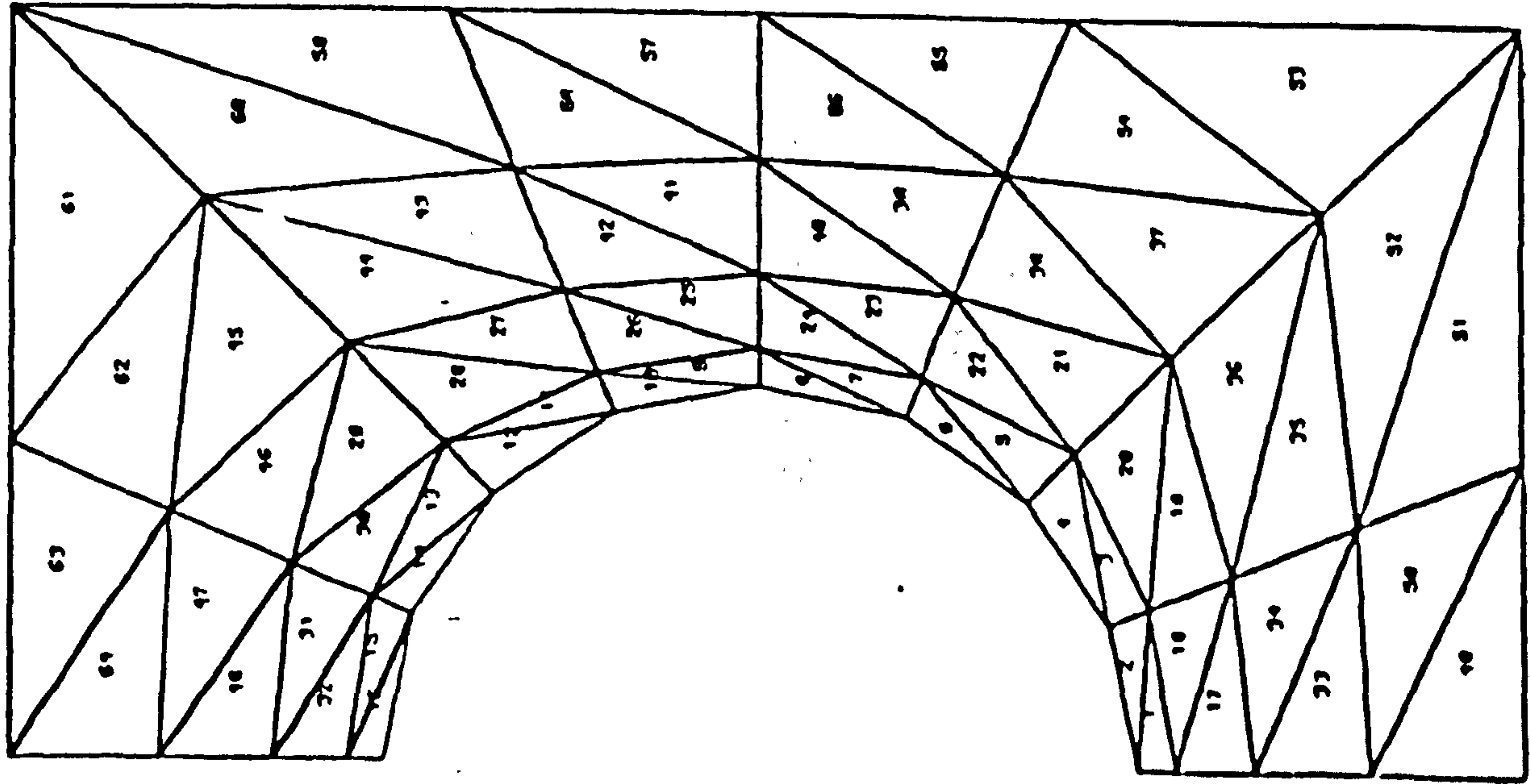


Figure 6.2

FIGURE 6.3

VARIATION OF MEAN NUSSELT NUMBER WITH DISPLACEMENT RATIO
FOR A HOT INNER CYLINDER CONTAINED WITHIN SQUARE
AND CYLINDRICAL CAVITIES

RADIUS RATIO : SQUARE CAVITY , 1.33
CYLINDRICAL ANNULUS , 1.31

Δ : SQUARE CAVITY , RA. = 100
 $\square$: CYLINDRICAL ANNULUS , HIGH GRASHOF NO.
 $*$: CYLINDRICAL ANNULUS , LOW GRASHOF NO.

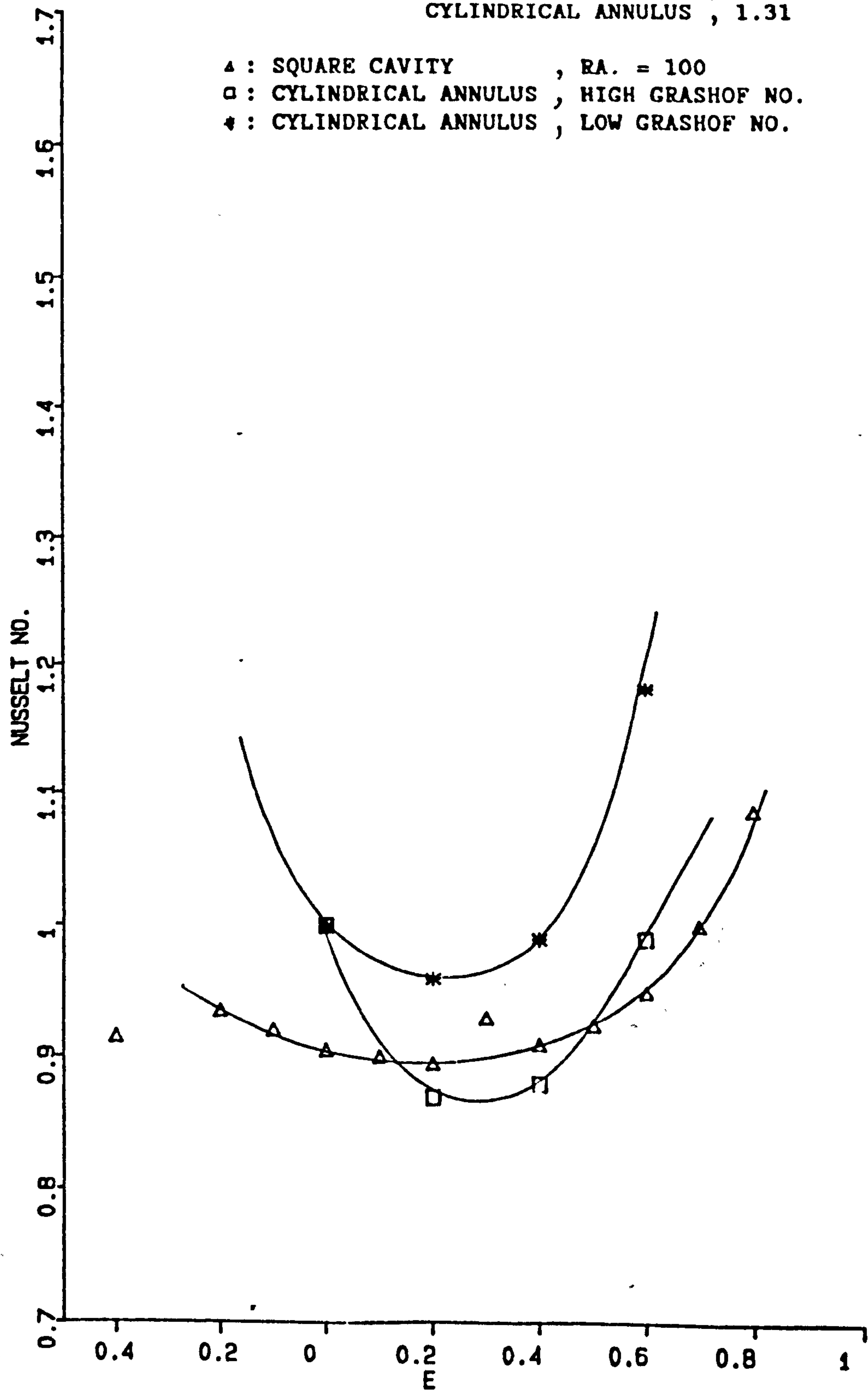


FIGURE 6. 4

VARIATION OF MEAN NUSSELT NUMBER WITH DISPLACEMENT RATIO
FOR A HOT INNER CYLINDER CONTAINED WITHIN SQUARE
AND CYLINDRICAL CAVITIES

RADIUS RATIO : SQUARE CAVITY , 1.50
CYLINDRICAL ANNULUS , 1.54

Δ : SQUARE CAVITY , RA. = 100
 $\square$: CYLINDRICAL ANNULUS

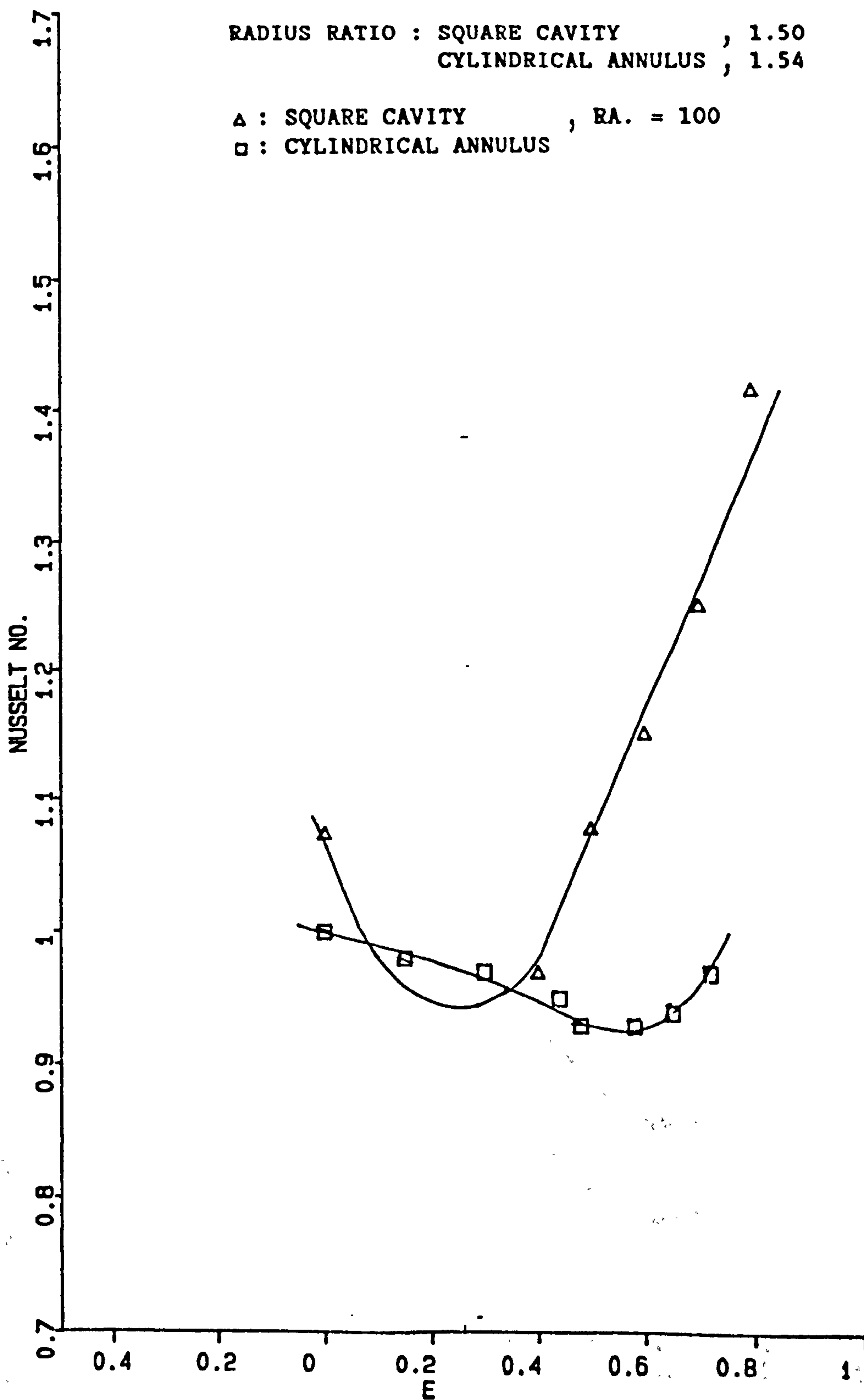


FIGURE 6. 5

VARIATION OF MEAN NUSSELT NUMBER WITH DISPLACEMENT RATIO
FOR A HOT INNER CYLINDER CONTAINED WITHIN SQUARE
AND CYLINDRICAL CAVITIES

RADIUS RATIO : SQUARE CAVITY , 2.00
CYLINDRICAL ANNULUS , 2.43

Δ : SQUARE CAVITY , RA. = 100
 $\square$: CYLINDRICAL ANNULUS , HIGH GRASHOF NO.
* : CYLINDRICAL ANNULUS , LOW GRASHOF NO.

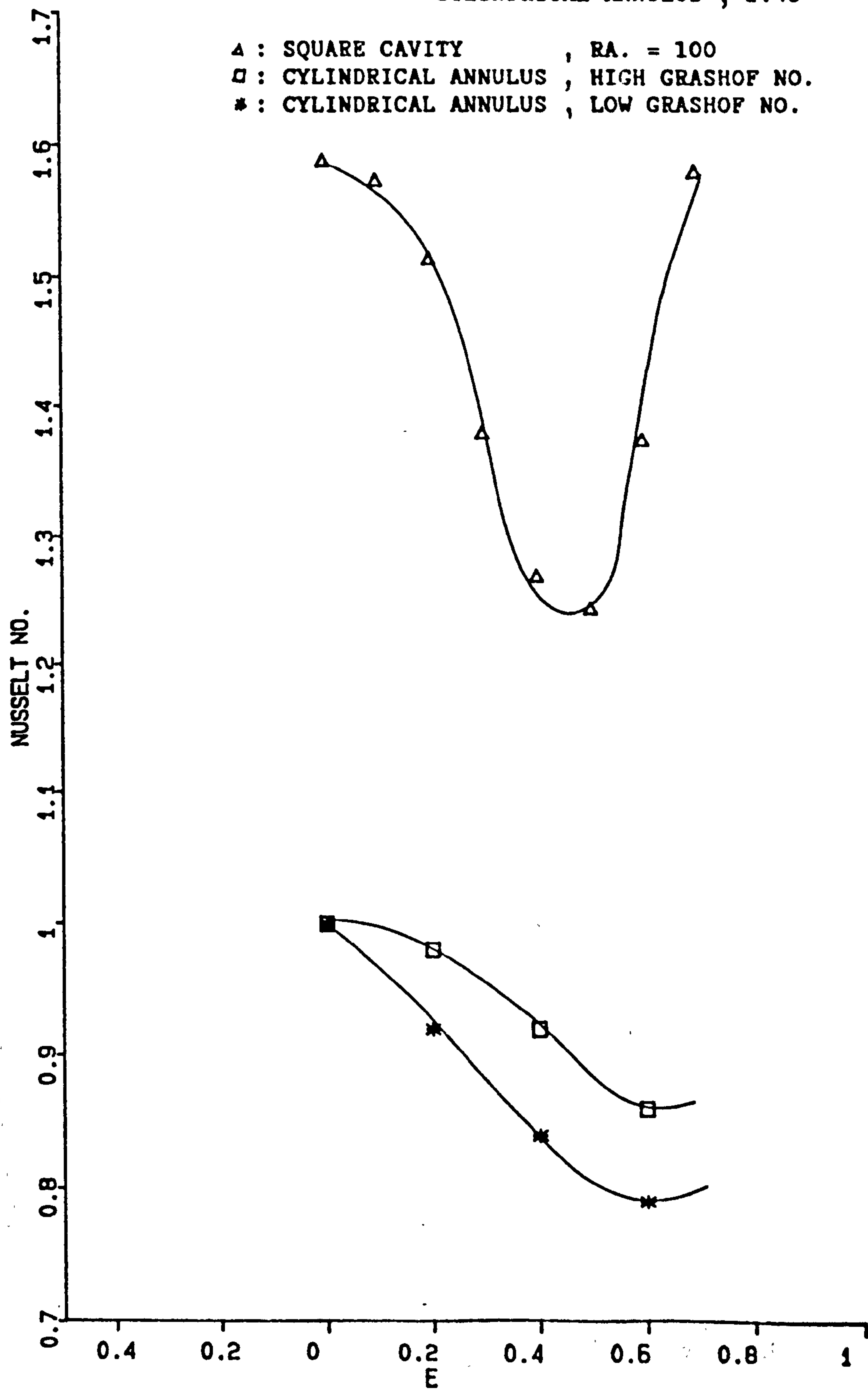
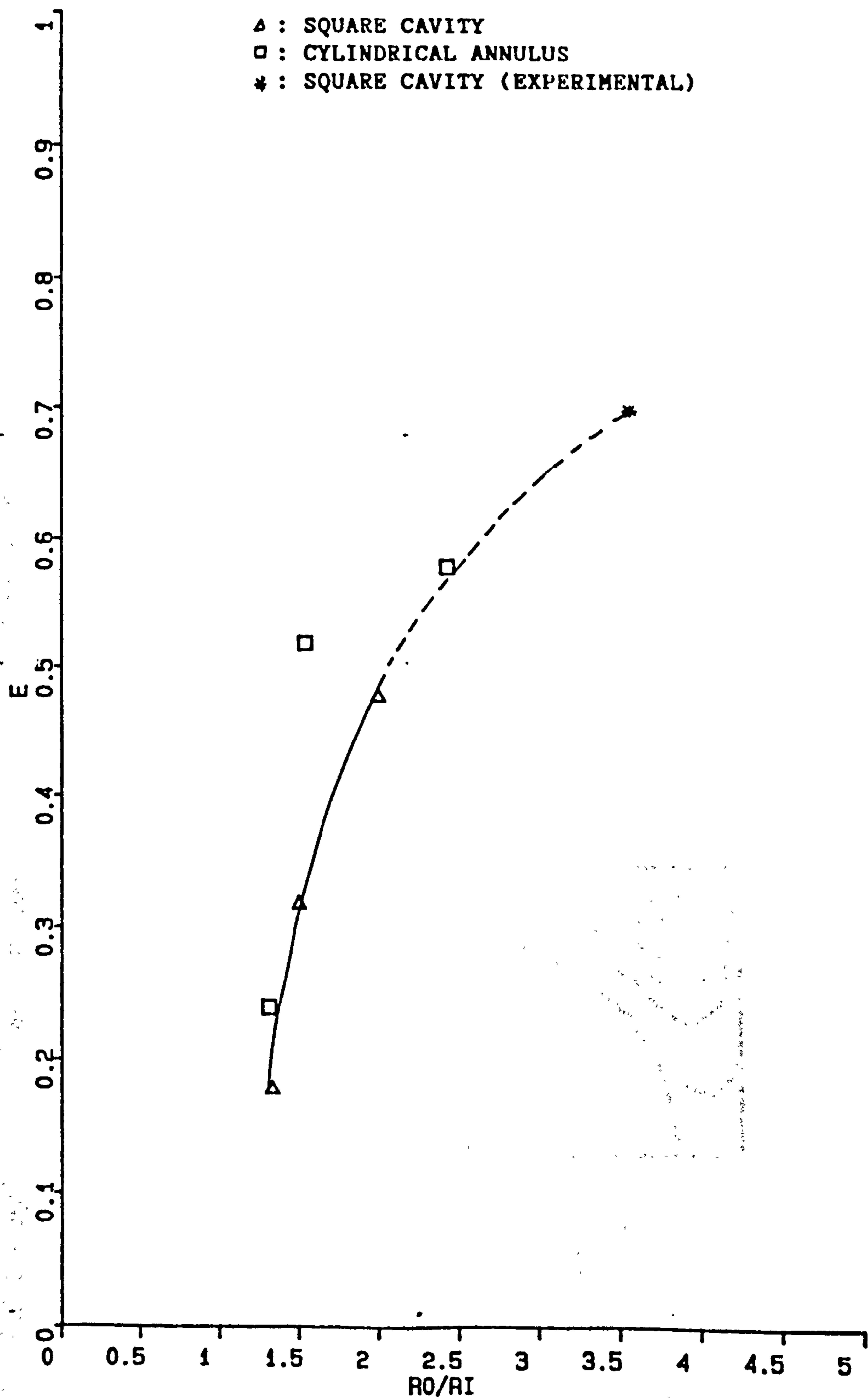


FIGURE 6.6

OPTIMAL DISPLACEMENT RATIO V RADIUS RATIO (R_0/R_I) FOR A
HOT INNER CYLINDER CONTAINED WITHIN SQUARE AND
CYLINDRICAL CAVITIES

- Δ : SQUARE CAVITY
 $\square$: CYLINDRICAL ANNULUS
 $*$: SQUARE CAVITY (EXPERIMENTAL)



Radius Ratio = 1.33

Displacement Ratio = -0.20

Ra = 300.00

Pr = 0.70

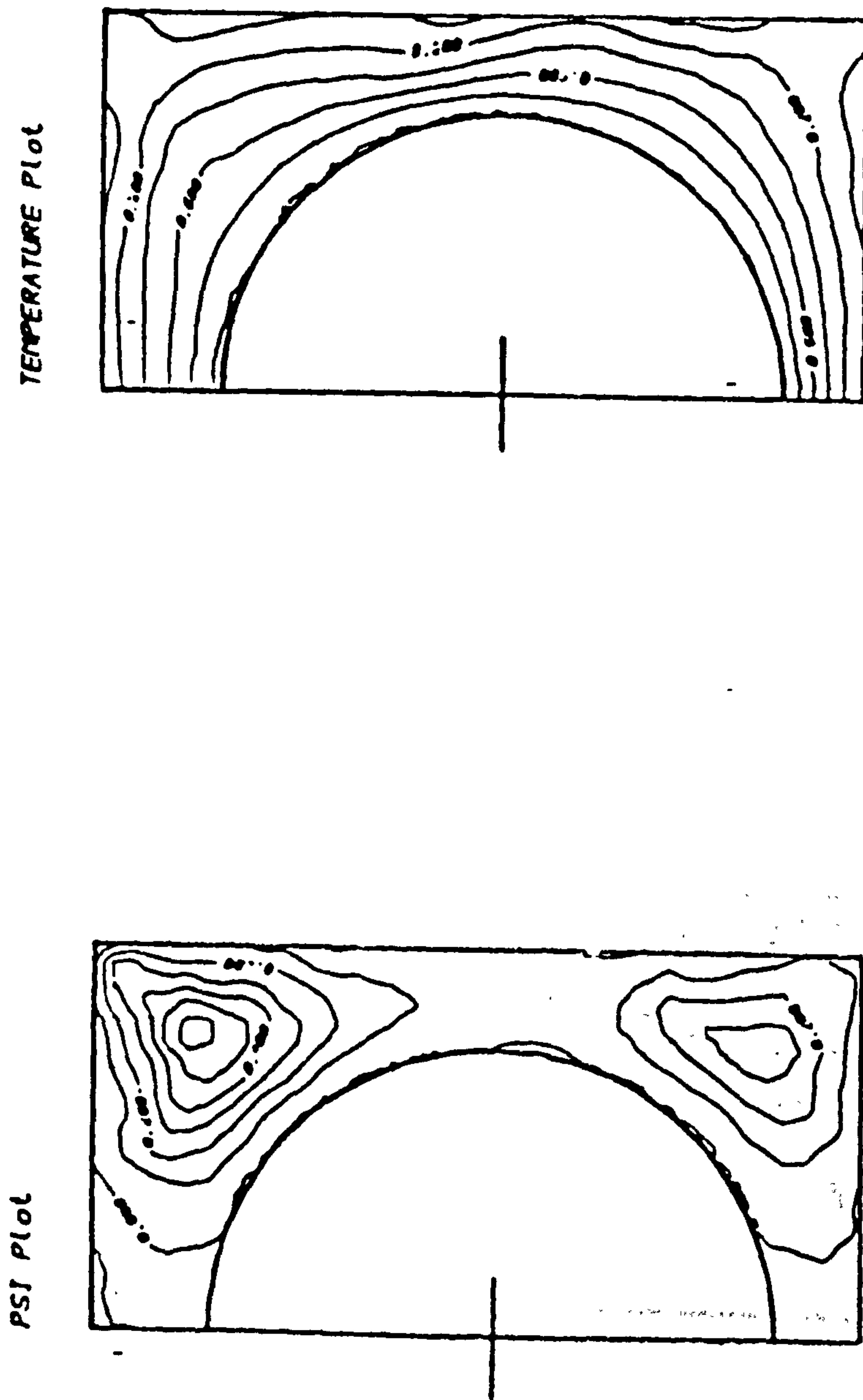


Figure 6.7

Analysis of natural convection
between a hot cylinder and a
cool, enclosing, rectangular duct.

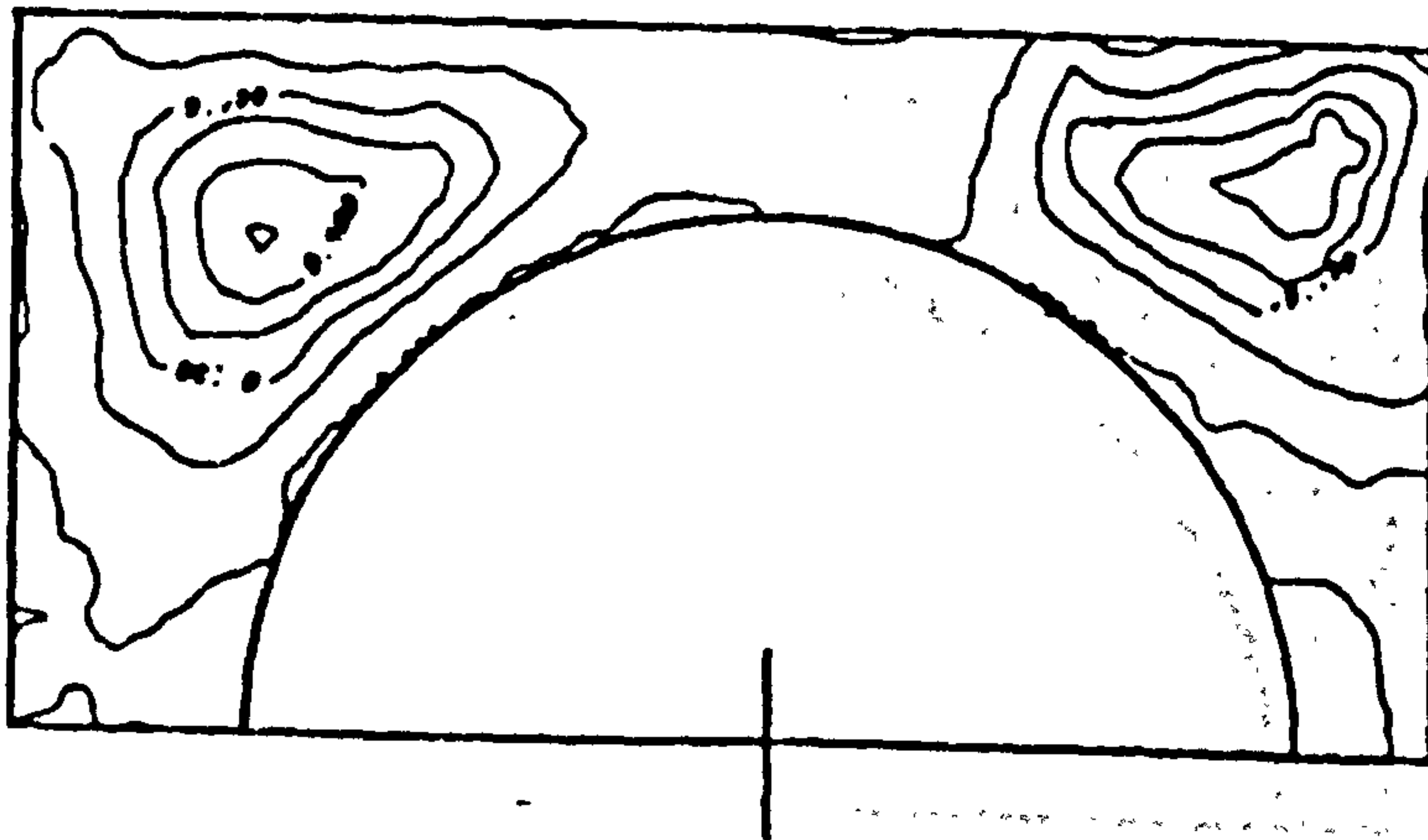
$Pr = 0.70$

$Ra = 300.00$

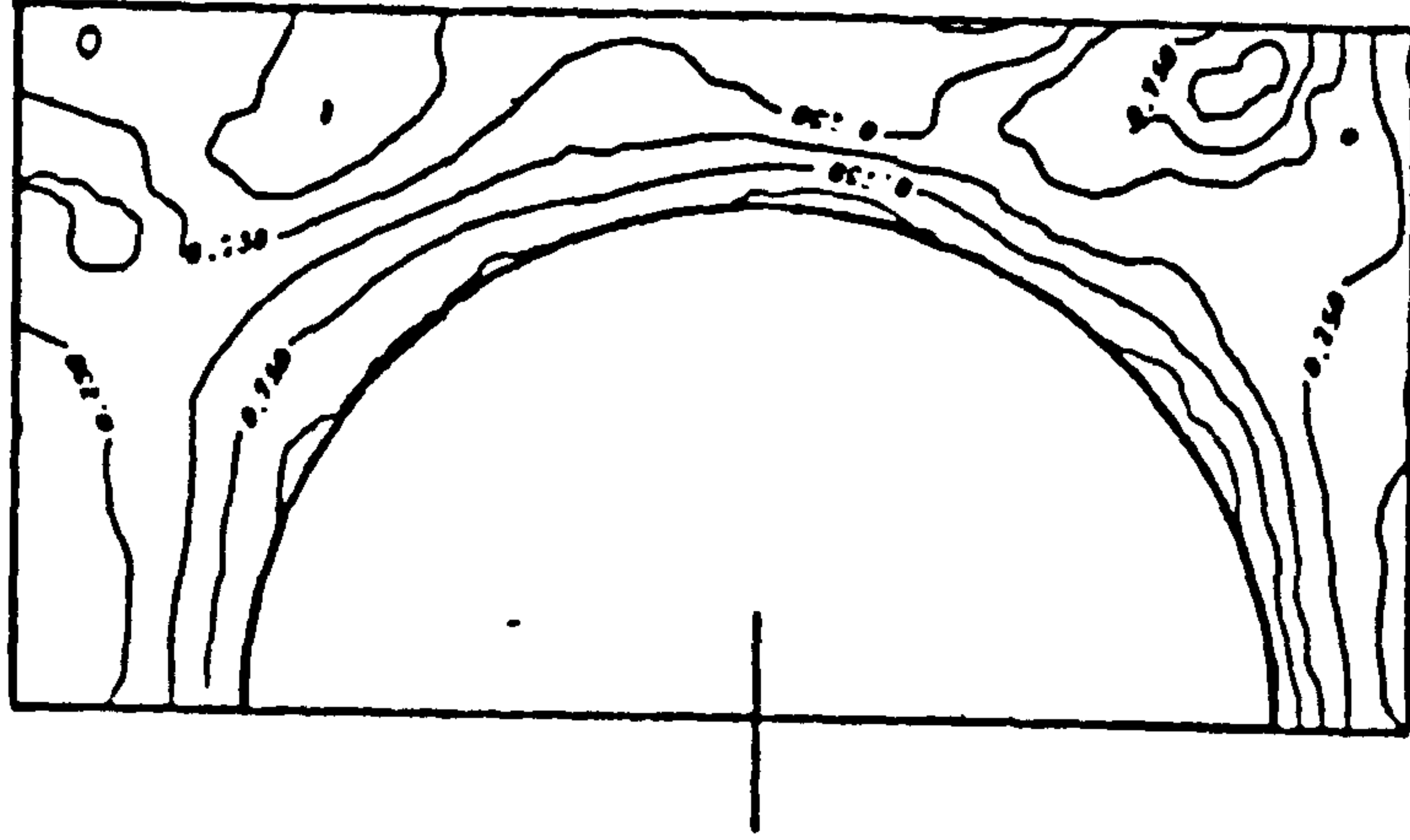
Displacement Ratio = 0.30

Radius Ratio = 1.33

PSI Plot



TEMPERATURE Plot



Analysis of natural convection
between a hot cylinder and a
cool, enclosing, rectangular duct.

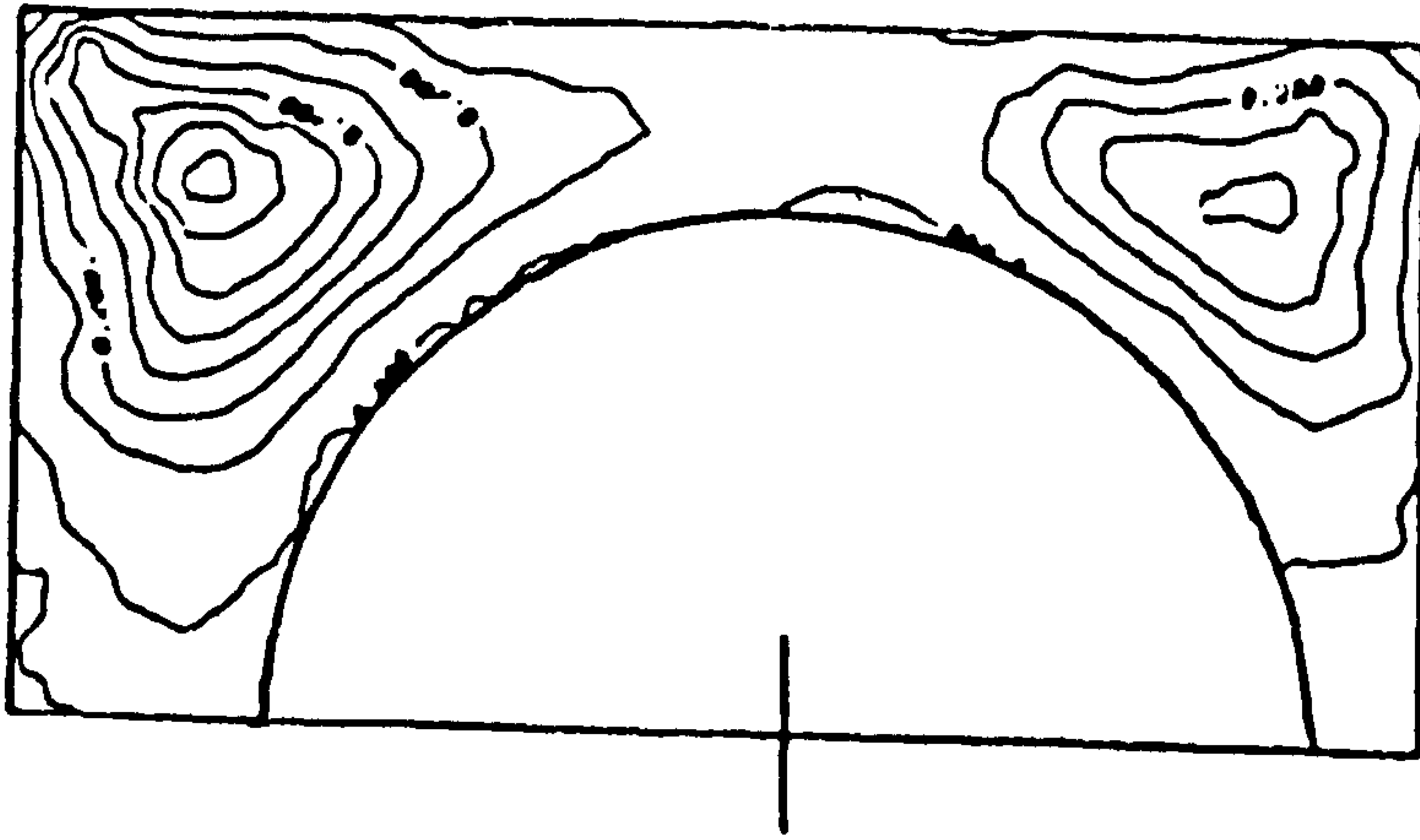
Figure 6.8

$Pr = 0.70$ $Ra = 300.00$

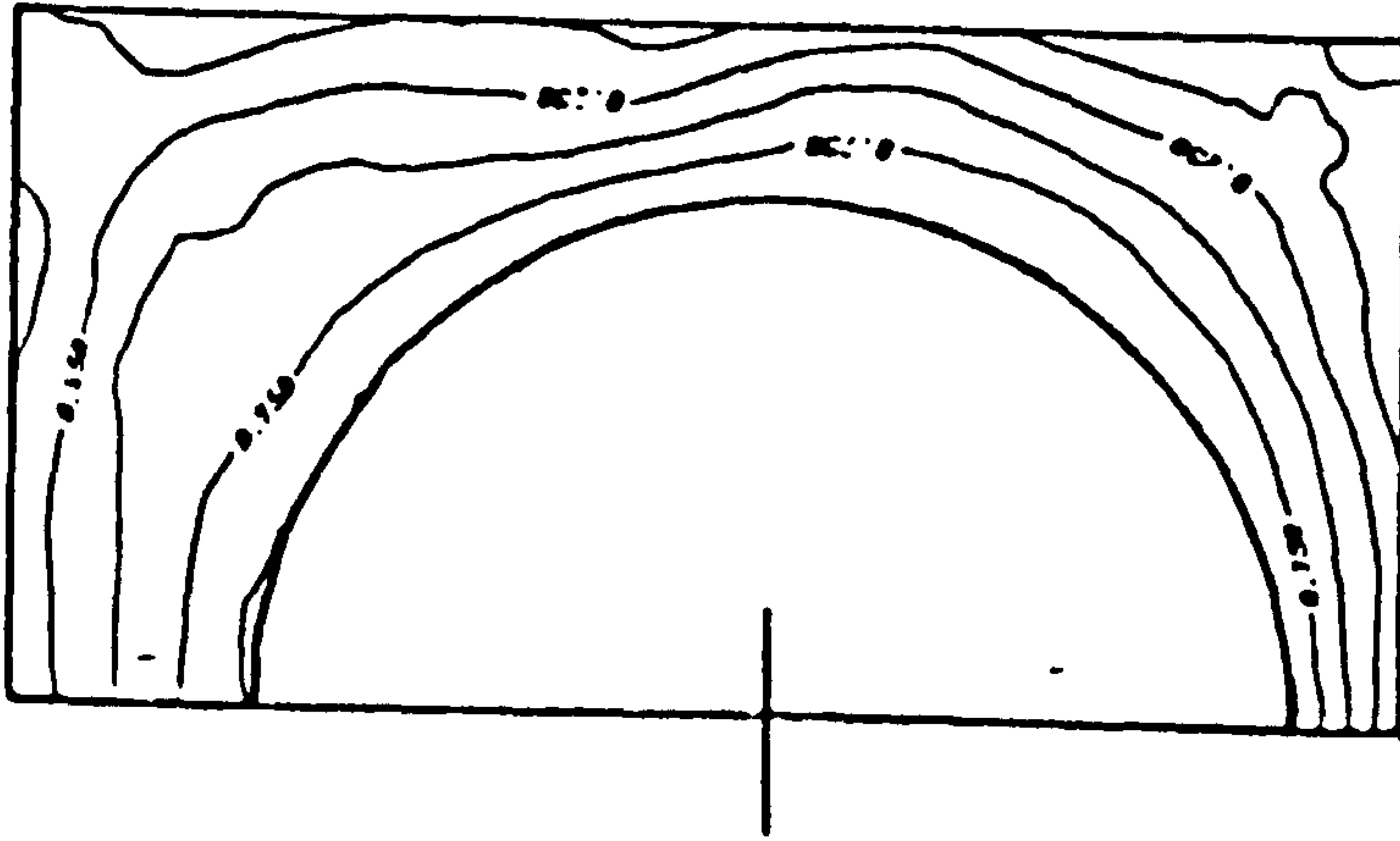
Displacement Ratio = 0.40

Radius Ratio = 1.33

PSI Plot



TEMPERATURE Plot



Analysis of natural convection
between a hot cylinder and a
cool, enclosing, rectangular duct.

Figure 6.9

Chapter 7

Conclusions and Recommendations

Conclusions and Recommendations

Both experimental studies show that heat-transfers across rectangular cavities can be minimised by implementing simple modifications.

7.1 Spacer Study

It has been shown that by inserting radial spacers and considering several angular positions there is minimum steady-state rate of heat-transfer across the cavity at $\gamma = 48^\circ$ (measured from the bottom of the cylinder). This optimal rate of heat-transfer being 25% less than that obtained for the system which has no spacers.

Furthermore, when the cylinder is placed at a displacement ratio of 0.7 the optimal spacer position was found to be $\gamma = 52^\circ$. The value of heat-transfer in this instance is 31% less than that obtained for the system with no spacers (and central positioning).

The scope for further studies appears limited, however, the introduction of radial spacers could influence the optimal eccentric displacement of the cylinder. Therefore, several system configurations could be investigated with a view to determining an optimal position for both the cylinder and the radial spacers.

7.2 Two Pipe Study

Talati (17) showed that a reduction in the rate of heat-transfer across a rectangular cavity of 32% is available by choosing a "hot-above-cold" configuration ($E_s = 0.7$, $E_r = 0$), as opposed to a conventional "side-by-side" configuration ($E_s = 0$, $E_r = 0$).

The present study showed that heat-transfers across a rectangular cavity can be optimised by eccentric positioning of both supply and return pipes in a central vertical plane. This optimum occurred at a supply pipe displacement ratio of 0.45 and a return pipe displacement ratio of -0.33, yielding a gap of about 10 mm between the pipes. The value of the total rate of heat-transfer was found to be 20% less than that found by Talati (17) for a system in which the supply and return pipes were placed at displacement ratios of 0.7 and 0 respectively.

With such large reductions in the rate of heat-transfer available, the use of optimal configurations would be economically worthwhile over the lifetime of the pipeline.

Further research could investigate the effects of inserting radial spacers within a cavity for the two-pipe system, with the aim of reducing heat losses even further.

7.3 Numerical Investigation

A numerical study was made of the steady-state natural convective heat-transfers between a hot horizontal inner cylinder contained within a relatively cold outer duct using a finite-element method.

The program successfully predicted the optimal position of the pipe for minimum heat-transfer. For radius ratios of 1.33, 1.50 and 2.0, these displacement ratios are 0.18, 0.25 and 0.48 respectively. Although the heat-transfer studied was at low Rayleigh numbers, results show there is a trend towards increasing optimal displacement ratio with increasing radius ratio for both square and cylindrical cavities. Extrapolating these results up to higher radius ratios showed good agreement with the experimental result obtained by James (16) at a radius ratio of 3.57.

- 8 McADAMS, W.H. "Heat Transmission"
McGraw-Hill, New York, 3rd Ed.,
1954.
- 9 GRIGULL, U.
and
HAUF, W. "Natural Convection in Horizontal
Cylindrical Annuli."
Proc. 3rd Int. Heat-Transfer
Conf., paper 60, Vol. 2, pp
182-195, 1966.
- 10 BISHOP, E.H.
and
CARLEY, C.T. "Photographic Studies of Natural
Convection between Concentric
Cylinders."
Proc. Heat-Transfer and Fluid
Mechanics Inst.
Stanford University Press, U.S.A.
pp 63-78, 1966.
- 11 KUEHN, T.H.
and
GOLDSTEIN, R.J. "An Experimental Study of Natural
Convection Heat-Transfer in
Concentric Annuli."
J. Heat-Transfer, Vol. 100 pp
635-640, 1978.
- 12 LIS, J. "Experimental Investigation of
Natural Convective Heat-Transfer
in Simple Obstructed Horizontal
Annuli."
Proc. 3rd Int. Heat-Transfer
Conf., Vol. 2, pp 196-204, 1966.
- 13 SHILSTON, M.J.
and
PROBERT, S.D. "Effects of Horizontal and
Vertical Spacers on the Heat-
Transfer across a Horizontal
Annular, Air-Filled Cavity."
Applied Energy Vol. 4, pp 21-37,
January 1978.
- 14 NORTON, B. "Mach-Zehnder Study of
Horizontal Annular Cavities."
MSc. Thesis, Cranfield Institute
of Technology. pp 84-95, 1978.

- 15 NEALE, A.J. "Optimising the Design of a Square Duct enclosing a Higher Temperature Cylinder: A Case Study for District Heating Pipe Design." MSc. Thesis, Cranfield Institute of Technology, 1983.

- 16 JAMES, S.P. "Pipe Position Optimisation for District Heating Schemes: A Mach-Zehnder Interferometer Study." MSc. Thesis, Cranfield Institute of Technology, 1981.

- 17 TALATI, A. "District Heating Pipe Work Design." MSc. Thesis, Cranfield Institute of Technology, 1983.

- 18 CRAWFORD, L.
and
LEMLICH, R. "Natural Convection in Concentric Horizontal Annuli." I.E.C. Fund., Vol. 1, pp 260-264, 1962.

- 19 ABBOTT, M.R. "A Numerical Method for Solving the Equations of Natural Convection in a Narrow Concentric Cylindrical Annulus with a Horizontal Axis", Quart. J. Mech. Appl. Math., Vol. 17, pp 471-481, 1964.

- 20 MACK, L.R.
and
BISHOP, E.H. "Natural Convection between Horizontal Concentric Cylinders for Low Rayleigh Numbers", Quart. J. Mech. Appl. Math. Vol. 21, pp 223-241, 1968.

- 21 POWE, R.E.
CARLEY, C.T.
and
CARRUTH, S.L. "A Numerical Solution for Natural Convection in Cylindrical Annuli", Trans. ASME, J Heat-Transfer, Vol. 93, pp 210-220, 1971.

- 22 KUEHN, T.H.,
and
GOLDSTEIN, R.J. "An Experimental and Theoretical Study of Natural Convection in the Annulus between Horizontal Concentric Cylinders", J Fluid Mech., Vol. 74, pp 695-719, 1976.

- 23 CHARRIER-MOJTABI, M.C.,
MOJTABI, A.
CALTAGIRONE, J.P. "Numerical Solution of a Flow due to Natural Convection in Horizontal Cylindrical Annulus", Trans. ASME, J Heat-Transfer, Vol. 101, pp 171-235, 1979.

- 24 KWON, S.S.,
KUEHN, T.H.
and
LEE, T.S. "Natural Convection in the Annulus between Horizontal Circular Cylinders, with Three Axial Spacers." J. Heat-Transfer, Vol. 104, pp 118-124, January 1981.

- 25 YAO, L.S. "Analysis of Heat-Transfer in Slightly Eccentric Annuli", Trans, ASME, J Heat-Transfer, Vol. 102, pp 279-284, 1980.

- 26 PRUSA, J.
and
YAO, L.S. "Natural Convection Heat-Transfer between Eccentric Horizontal Cylinders", Trans. ASME, J Heat-Transfer, Vol. 105, pp 108-116, 1983.

- 27 PROJAHN, U.,
RIEGER, H.
and
BEER, H. "Numerical Analysis of Laminar Natural Convection between Concentric and Eccentric Cylinders." Numerical Heat-Transfer, Vol. 4, pp 131-146, 1981.

- 28 CHO, C.H.,
CHANG, K.S.,
and
PARK, K.H. "Numerical Simulation of Natural Convection in Concentric and Eccentric Horizontal Cylindrical Annuli." ASME, Vol. 104, pp 624-630, 1982.

- 29 SATAKE, A.,
 and
 REDDY, J.N. "Natural Convection between Concentric (Horizontal) Circular Cylinders by a Penalty Finite Element Method." 2nd Int. Conf. on Numerical. Meth. in Laminar and Turbulent Flow, Venice, 1981.

- 30 RATZEL, A.C.,
 HICKOX, C.E.
 and
 GARTLING, D.K. "Techniques for Reducing Thermal Conduction and Natural Convection Heat Losses in Annular Receiver Geometries", Trans ASME, J Heat-Transfer, Vol. 101, pp 108-113, 1979.

- 31 CAUDIU, E.J.N. "Finite Element Analysis of the Natural Convection in Concentric Horizontal Annuli". MSc Thesis 1972, Cranfield Institute of Technology.

- 32 YEO, A.K. "Finite Element Analysis of Natural Convection in Eccentric Annuli." MSc Thesis 1983, Cranfield Institute of Technology.

- 33 BROOKS, R.G.,
 PROBERT, S.D.
 and
 MAXWELL, J. "18 cm Field of View Mach-Zehnder Interferometer for Heat-Transfer Studies." Trans. Inst. Measurement Control, 1, pp T9-T116.

- 34 GEBHART, B.,
 and
 KNOWLES, C.P. "Design and Adjustment of a 20 cm Mach-Zehnder Interferometer." Review of Scientific Instruments, Vol. 37, pp 12-15, 1966.

- 35 A.I.P. "Index of Refraction of Gases and Vapours." American Institute of Physics Handbook.

- 36 WILKIE, D.
and
FISHER, S.A. "Measurement of Temperature by
Mach-Zehnder Interferometry."
Proc, Inst. Mech. Eng. Vol. 178,
Part 1, 17, 1963-64.
- 37 TRITTON, D.J. "Physical Fluid Dynamics."
Van Nostrand Reinhold P. 147,
1979.
- 38 COBURN, S.K. "Corrosion Source Book".
- 39 SENFTLEBEN, H.
and
GLADISCH, H. "Wärmeübergang in gasen zwischen
koaxialen zylindern, Zeitschrift
für Physik, Vol. 125, pp
629-652, 1949.
- 40 LANGHAAR, H.L. "Dimensional Analysis and Theory
of Models", Krieger, 1979.
- 41 FORIEN, T.B. "Analysis of Natural Convection
in air-filled cavities".
MSc Thesis, Cranfield Institute
of Technology, 1985.
- 42 SHILSTON, M.J. "Heat-Transfers across air
filled cavities".
Phd Thesis, Cranfield Institute
of Technology, 1982.

Appendix A

The Measurement of Temperature using the

Mach-Zehnder Interferometer

A.1 The Basic System and its adjustment

The interferometer is based on a 60° angle horizontal parallelogram, with four mirrors, one mounted at each corner. A schematic representation of the interferometer is shown in Figure A.1. The mirrors are circular, of 19 cm effective diameter, and because of the angle of incidence of the light, they produce an elliptical field with a horizontal minor axis of 15 cm, and a vertical major axis of 19 cm.

The light source is provided by a 3mW laser which passes through a microscope objective lens, producing a diverging beam. It then passes through a pin hole filter which "cleans up" the edges of the beam which is then made parallel by a collimating lens. The objective lens being placed at the focal point of the collimating lens. This beam is sufficient to cover the semi-silvered mirror S1. This mirror splits the input beam into two parts. The reference beam which passes through S1 then passes through undisturbed air to mirror M1, which is fully reflecting and then onto the second semi-silvered mirror S2. The other part of the input beam is reflected by S1 to the fully silvered mirror M2 from where it passes through the test cell to S2. At S2 both beams recombine to form the output. The output beam is then either viewed on a screen or photographed to provide a permanent record.

The optical path length of a beam is defined as:

$$n.dL \quad (A.1)$$

Where L is the geometric length and n the refractive index along the path.

Interference fringes are produced at the output when the optical path length of the working beam differs from that of the reference beam. This can be achieved by altering the actual length or changing the refractive index of one of the beams.

All four mirrors are mounted on gimbals so that they can be rotated about their horizontal and vertical axes, the movement being controlled by micrometer screws. Mirror M1 is also provided with a traverse mechanism, so that it may be moved either in or out in a direction perpendicular to the plane of the mirror surface. Initially, fringes of the required orientation and spacing are localised in the plane of the test section. The fringe contrast is maximised by careful adjustment of the above controls, ie. by adjusting the physical path lengths of the system.

The system uses the basic principle that temperature changes, and therefore density changes, in the field of the working beam, affect the optical path length. Thus the original interference pattern becomes distorted with temperature change.

The interferometer can be used in three different modes to yield the positions of the isotherms in the field of view:

- (a) Initially the interferometer is adjusted so that one fringe covers the whole field of view (nominally infinite spacing). The introduction of temperature gradients in the test cell consequently produce fringes which are lines of constant fringe shift, and therefore isotherms.

- (b) The instrument can be adjusted to produce parallel fringes having a small finite fringe spacing. This wedge interferogram is then distorted by the test object, and the isotherms can be identified by locating points of equal fringe shift.
- (c) A double exposure of the distorted and undistorted wedge interferogram yields Moire fringes which are lines of equal fringe displacement, ie. isotherms. This mode of operation has the advantage of eliminating the effects of imperfections in the interferometer plates as both the distorted and undistorted fringe patterns suffer equally as a result of these defects.

Due to the difficulty in operating procedure and the problems of analysing the results, only the infinite fringe spacing method was used in this investigation.

A.2 Relationship between the Interference Fringes and Temperature

At infinite fringe spacing one fringe covers the whole field of view. The test section and the reference section temperatures are equal. As temperature gradients are introduced into the path of the working beam, there is a change in air density across the cavity causing a corresponding change in refractive index. The infinite fringe is disturbed, due to the change in optical path length, and the fringes produced are isotherms.

The refractive index (n) of a fluid is related to its density (ρ) by the Lorentz-Lorentz formula:

$$\frac{n^2 - 1}{n^2 + 2} \cdot \frac{1}{\rho} = \text{a constant} \quad (\text{A.2})$$

Assuming air to behave as an ideal gas at the moderate pressures and temperatures encountered in natural convection situations, the corresponding equation of state can be applied,

$$P = \rho RT \quad \text{A.3)}$$

factorising equation (A.2), gives:

$$\frac{(n-1)(n+1)}{n+2} = \frac{1}{\rho} \quad \text{(A.4)}$$

This reduces to,

$$\frac{n-1}{\rho} = G \quad \text{(A.5)}$$

based on the fact that the refractive index of air does not differ very much from unity. Equation (A.5) expresses the Gladstone-Dale law, G , being the Gladstone-Dale constant.

For a change in refractive index from n_1 to n_2 ,

$$n_1 - n_2 = G (\rho_1 - \rho_2) \quad \text{(A.6)}$$

The number of waves of light (N) along the path of length (L) through the test section is given by:

$$N = \frac{nL}{\lambda_0} \quad \text{(A.7)}$$

where λ_0 is the wavelength of light.

Combining equations (A.6) and (A.7)

$$\Delta N = \frac{GL}{\lambda_0} (\rho_1 - \rho_2) \quad \text{(A.8)}$$

Substituting for G from equation (3.5) gives:

$$\Delta N = \frac{n_1 - 1}{\rho_1} \cdot \frac{L}{\lambda_0} \cdot (\rho_1 - \rho_2) \quad (A.9)$$

For a gas at constant pressure, however, this equation becomes,

$$\Delta N = (n_1 - 1) \frac{L}{\lambda_0} \cdot \frac{\Delta T}{T_2} \quad (A.10)$$

Where $\Delta T = (T_2 - T_1)$ and, therefore, by substituting T_2 equal to $\Delta T + T_1$ in equation (A.10) and rearranging the equation, the relationship between fringe shift and temperature change is obtained in terms of a reference temperature and the corresponding refractive index of the gas in the test section.

$$\Delta T = \frac{T_1 \cdot \Delta N}{L/\lambda_0 \cdot (n_1 - 1) - \Delta N} \quad (A.11)$$

Therefore if $T_2 = T_P$ (the absolute temperature at the required point), and $T_1 = T_R$ (the absolute reference temperature), then,

$$T_P = T_R \frac{\Delta N}{L/\lambda_0 \cdot (n_R - 1) - \Delta N} = 1 \quad (A.12)$$

where (n_R) is the refractive index at the reference temperature. To determine the value of n_R , the following formula (35) is used.

$$n_R - 1 = \frac{T_0}{T_1} (n_0 - 1) \quad (A.13)$$

Where (n_R) is the refractive index for temperature (T_R) and (n_0) the refractive index at 0°C . The value of (n_0) nominally used in equation (A.13) is,

$$n_0 = 1.0002936$$

Hence, given the above information, the temperature at any fringe position can be calculated.

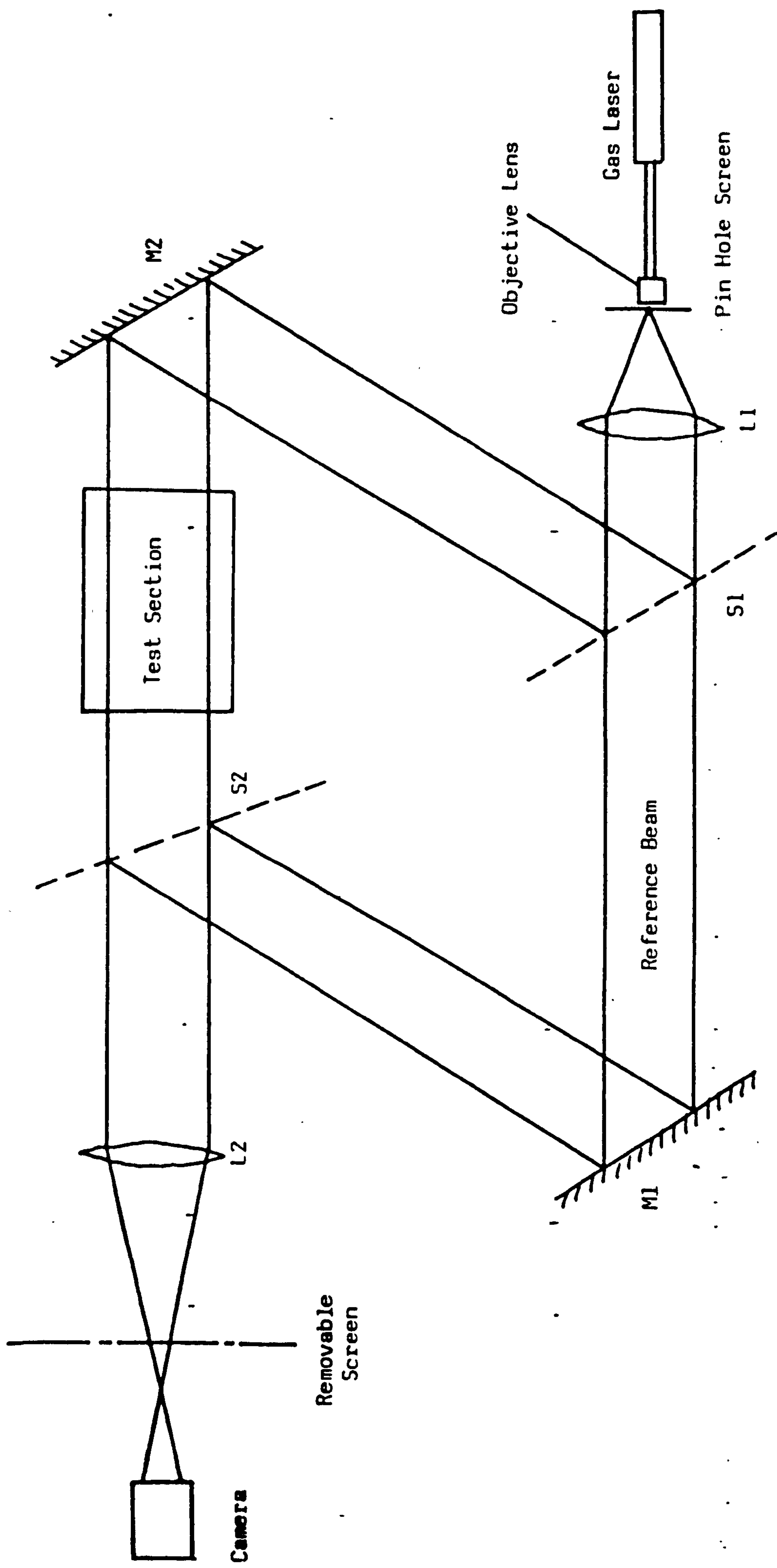


FIG. A.1 Schematic View of the Mach Zehnder Interferometer

Appendix BDimensional AnalysisDimensional Analysis of Natural Convection

The physical quantities pertinent to this investigation, together with their symbols and dimensions are listed below:-

<u>Physical Quantity</u>	<u>Symbol</u>	<u>Dimensions</u>
Average gap width	d	L
Buoyancy acceleration	$g\beta$	$Lt^{-2} T^{-1}$
Density	ρ	ML^{-3}
Dynamic viscosity	μ	$ML^{-1} t^{-1}$
Thermal conductivity	K	$QL^{-1} T^{-1} t^{-1}$
Temperature difference	T	T
Specific heat	C	$QM^{-1} T^{-1}$
Heat-transfer coefficient	h	$Qt^{-1} L^{-2} T^{-1}$
Horizontal dimension of cavity	x	L
Vertical dimension of cavity	y	L
radius of supply pipe	r_1	L
radius of return pipe	r_2	L
Effective length of cavity	L	L

Applying Buckingham's rule (40), the solution to the problem should contain 8 dimensionless groups. To determine these groups we write down the product of the variables, each raised to an unknown power:

$$= (d)^{a_1} (g\beta)^{a_2} (\rho)^{a_3} (\mu)^{a_4} (K)^{a_5} \\ (T)^{a_6} (C)^{a_7} (h)^{a_8} (x)^{a_9} (y)^{a_{10}} \\ (r_1)^{a_{11}} (r_2)^{a_{12}} (L)^{a_{13}}$$

where $a_1, a_2, \dots$ etc are the power exponents.

Substituting the dimensional formulae,

$$\begin{aligned}
 &= (L)^{a_1} (Lt^{-2}T^{-1})^{a_2} (ML^{-3})^{a_3} (ML^{-1}t^{-1})^{a_4} \\
 &\quad (QL^{-1}T^{-1}t^{-1})^{a_5} (\Delta T)^{a_6} (QM^{-1}T^{-1})^{a_7} \\
 &\quad (Qt^{-1}L^{-2}T^{-1})^{a_8} (L)^{a_9} (L)^{a_{10}} (L)^{a_{11}} \\
 &\quad (L)^{a_{12}} (L)^{a_{13}}
 \end{aligned}$$

By equating the sum of the exponents of each primary dimension to zero, we obtain the following set of equations:

$$a_1 + a_2 - 3a_3 - a_4 - a_5 - 2a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} = 0 \quad \text{for L .. B.1}$$

$$-2a_2 - a_4 - a_5 - a_8 = 0 \quad \text{for t .. B.2}$$

$$-a_2 - a_5 + a_6 - a_7 - a_8 = 0 \quad \text{for T .. B.3}$$

$$a_5 + a_7 + a_8 = 0 \quad \text{for Q .. B.4}$$

$$a_3 + a_4 - a_7 = 0 \quad \text{for M .. B.5}$$

subtracting equation B.2 from 2 x (B.3)
we get:-

$$-a_4 + a_5 - 2a_6 + 2a_7 + a_8 = 0$$

subtracting equation B.4 from equation B.6
we have:

$$-a_4 - 2a_6 + a_7 = 0$$

or

$$a_4 = a_7 - 2a_6 \quad \text{B.7}$$

Similarly the values of the other exponents are found in terms of

$a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}$ and a_{13}

Thus

$$a_1 = 3a_6 + a_8 - a_9 - a_{10} - a_{11} - a_{12} - a_{13}$$

$$a_2 = a_6$$

$$a_3 = 2a_6$$

$$a_4 = a_7 - 2a_6$$

$$a_5 = -a_7 - a_8$$

Hence:-

$$= (d)^{3a_6+a_8-a_9-a_{10}-a_{11}-a_{12}-a_{13}} (g\beta)^{a_6} (\mu)^{a_7-2a_6} (k)^{-a_7-a_8} (\Delta T)^{a_6} (C)^{a_7} (h)^{a_8} (x)^{a_9} (y)^{a_{10}} (r_1)^{a_{11}} (r_2)^{a_{12}} (L)^{a_{13}}$$

We now dispose of the six arbitrary exponents by setting each in turn equal to unity, and simultaneously setting those remaining to zero.

$$\begin{aligned} \text{For } a_6=1, \quad a_7=a_8=a_9=a_{10}=a_{11}=a_{12}=a_{13} &= 0 \\ &= \frac{d^3 \cdot g\beta \cdot \rho^2 \cdot \Delta T}{\mu^2} = \text{Grashof number, Gr} \end{aligned}$$

$$\begin{aligned} \text{For } a_7=1, \quad a_6=a_8=a_9=a_{10}=a_{11}=a_{12}=a_{13} &= 0 \\ &= \frac{C}{k} = \frac{\mu/\rho}{k/c} = \text{Prandtl number, Pr} \end{aligned}$$

$$\begin{aligned} \text{For } a_8=1, \quad a_6=a_7=a_9=a_{10}=a_{11}=a_{12}=a_{13} &= 0 \\ &= \frac{hd}{k} = \text{Nusselt number, Nu} \end{aligned}$$

$$\begin{aligned} \text{For } a_9=1, \quad a_6=a_7=a_8=a_{10}=a_{11}=a_{12}=a_{13} &= 0 \\ &= \frac{x}{d} \end{aligned}$$

$$\begin{aligned} \text{For } a_{10}=1, \quad a_6=a_7=a_8=a_9=a_{11}=a_{12}=a_{13} &= 0 \\ &= \frac{y}{d} \end{aligned}$$

$$\begin{aligned} \text{For } a_{11}=1, \quad a_6=a_7=a_8=a_9=a_{10}=a_{12}=a_{13} &= 0 \\ &= \frac{r_1}{d} \end{aligned}$$

$$\begin{aligned} \text{For } a_{12}=1, \quad a_6=a_7=a_8=a_9=a_{10}=a_{11}=a_{13} &= 0 \\ &= \frac{r_2}{d} \end{aligned}$$

$$\begin{aligned} \text{For } a_{13}=1, \quad a_6=a_7=a_8=a_9=a_{10}=a_{11}=a_{12} &= 0 \\ &= \frac{L}{d} \end{aligned}$$

The required function containing 8 dimensionless arguments takes the form:-

$$f \left(\text{Nu}, \text{Gr}, \text{Pr}, \frac{x}{d}, \frac{y}{d}, \frac{r_1}{d}, \frac{r_2}{d}, \frac{L}{d} \right) = 0$$

or

$$\text{Nu} = f \left(\text{Gr}, \text{Pr}, \frac{x}{d}, \frac{y}{d}, \frac{r_1}{d}, \frac{r_2}{d}, \frac{L}{d} \right)$$

If the system contained just one cylinder the second to last term may be omitted, hence:

$$\text{Nu} = f \left(\text{Gr}, \text{Pr}, \frac{x}{d}, \frac{y}{d}, \frac{r_1}{d}, \frac{L}{d} \right)$$

Similarly, if the system was square and not rectangular

$$x = y,$$

$$\text{Nu} = f \left(\text{Gr}, \text{Pr}, \frac{x}{d}, \frac{r_1}{d}, \frac{L}{d} \right)$$

Appendix C

Interferometer Errors

The use of the interferometer for heat-transfer investigations is subject to a number of sources of error.

- (1) End effects
- (2) Refraction errors
- (3) Imperfections in the optical plates
- (4) Vibration
- (5) Evaluation errors

C.1 End Effects

Although the camera is focused on a plane crossing the test section, the fringe pattern formed by the interferometer is the result of integrating the whole temperature field within the total optical path. This includes the end effects of the test chamber, and any other disturbance along the beam path.

It is assumed in the relationship between fringe shift and temperature difference (equation A.11) that the density remains constant along any one light ray, and that the density at the ends of the cavity is constant and equal to the ambient air density. The end plates help to satisfy this condition to a certain extent, but end effect errors are inherent within the system and consequently the absolute values of Nusselt number obtained will be affected, although the system used is relatively long which reduces the significance of end effects and they will not affect the comparison between the different systems studied.

C.2 Refraction Errors

The relationship between fringe shift and temperature assumes that the light beam travels in a straight line through the test section. Due to the refraction of the light rays by the density gradients being measured, the light ray actually traces a curved path. This results in a modification of the fringe shift, and a loss of detail near the hot cylinder wall. Unfortunately there is not sufficient information in the measured temperature profile to enable corrections to be calculated. This source of error is important when large temperature differences are being measured as this leads to larger density gradients, but a degree of correction is possible by the correct focusing of the camera.

Referring to Figure C.1, a light beam T, entering the test section assumes a parabolic path curvature under the influence of the constant gradient of refractive index of the working fluid for the immediate vicinity of the wall in a thermal boundary layer. The light beam travels in a straight line outside the test section, and traverses the splitter mirror and the decollimating lens. The beam will appear to have emanated from a point P within the test section if it is projected backwards in a straight line before it enters the lens. This point is $0.33L$ from the front end of the test cell, where L is the length of the test cell. The corresponding reference beam R, is reflected from the splitter mirror along a path in exact alignment with the point P. It then traverses the lens and meets the test beam T, at I where the image of the combined effect of T and R is formed in a plane F-F. This is the focal plain in which the camera is to be placed. It should be focused on the point P, this being the point from which both the test beam T, and the reference beam R, appear to have emanated.

For this investigation a screen was placed at the correct distance, with the test section traversed out of the test beam, to enable the camera to be accurately focused.

C.3 Optical Plate Imperfections

Imperfections in the optical plates can lead to large errors. The optical plates used for this study have been in use for a number of years, and have probably suffered from the effects of fatigue. This error is only significant when a small number of fringes are present; as the temperature difference is increased the error due to optical plate imperfections is reduced.

C.4 Vibration

The Mach-Zehnder interferometer is very sensitive to vibration, which can cause perturbation of the fringe pattern. This may result in blurred interferograms unless photographs are taken with a very fast shutter speed, or fringe patterns which do not represent the actual temperature distribution.

In order to isolate the interferometer from vibration, the location of the instrument was chosen so that it would suffer least from sources of vibration. It was also mounted on four partially inflated tyred wheels. However, although these precautions were insufficient to eliminate totally, local heavy machine vibration, no photograph was found to suffer significantly from vibration blurring effects.

C.5 Evaluation Errors

Generally, the largest source of error in the evaluation of the interferograms arises from the judgement of the individual performing the analysis. The hot and cold surfaces are often not clearly defined around their whole perimeter, due to refraction, thus giving poor definition of the surface. Under good conditions it is possible to locate the centre of the fringe to within ± 0.025 of the fringe spacing (36). With normal experimental conditions, accuracies of ± 0.1 of the fringe spacing may be obtained. The maximum possible evaluation error would then result in a $\pm 10\%$ error in the local Nusselt number, although $\pm 2\%$ would be a more typical error in most cases.

C.6 Summary

Although the interferometer has several sources of error, the most important is that involved in the evaluation of the interferograms. Those errors which only vary with temperature difference, do not affect the comparisons between the different systems being investigated.

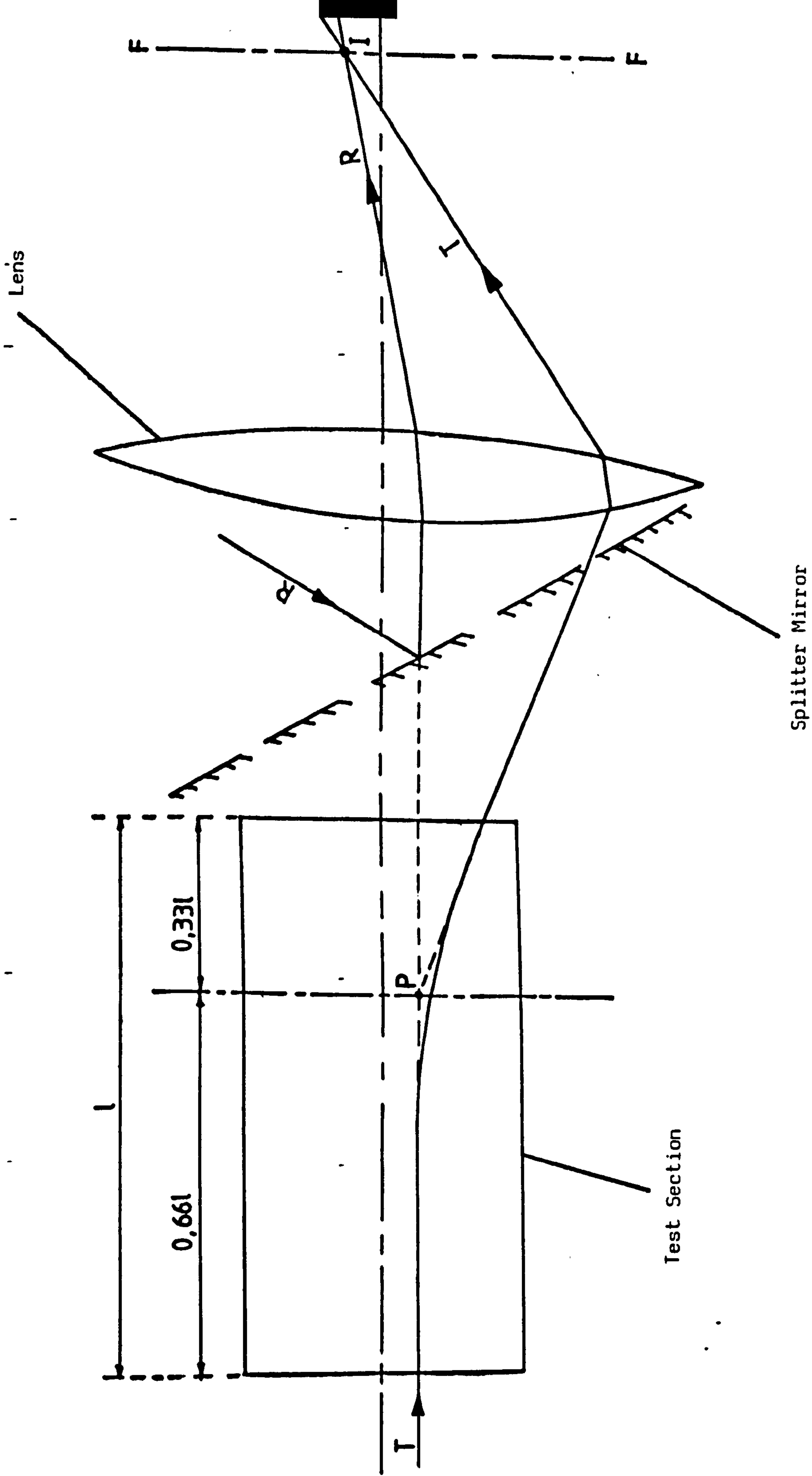


FIG. C.1 Geometry Associated with Focusing of the Camera