

CRANFIELD UNIVERSITY

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DESIGN OF ULTRA-PRECISION PISTON/CYLINDERS FOR DIRECTLY
CALIBRATED HYDRAULIC AMPLIFICATION

School of Aerospace, Transport and Manufacturing

Ultra-Precision Manufacturing

PhD

Academic Year: 2017 - 2022

Supervisor (s): Dr Claudiu Giusca and Prof Saurav Goel

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This thesis is submitted in partial fulfilment of the requirements for the degree of
Doctor of Philosophy

***(NB. This section can be removed if the award of the degree is based solely on
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ABSTRACT

Force calibrations in the Mega Newton range are not currently providing the accuracy required by industry. This work addresses a hydraulic force standards, which perform such calibrations, which work by amplifying the force produced on a small piston/cylinder assembly (PCA) by connecting it hydraulically to a larger PCA. The force standards are currently calibrated using a transducer to the more accurate deadweight machines, but not without a resulting uncertainty of 0.01 % - 0.02 % imparted by the transducer performing the calibration. Because of this, the potential for evaluating uncertainties within the machine from first principles and using it as a primary machine, with traceability maintained to the weights is evaluated. First of all the potential for the resulting reduction in uncertainties is evaluated analytically, then FEA/CFD work is carried out to produce analytical formulae which allow for the design of a directly calibrated system. The analysis is carried out for both the rotational and non-rotational variants of PCA. Finally, the design of a primary hydraulic standard is carried out using FEA/CFD.

Keywords:

Force Metrology, Mega Newton, Pressure Balance, Direct Calibration, Primary Standard, FEA, CFD

ACKNOWLEDGEMENTS

I would like particularly to thank my supervisors Dr Claudiu Giusca based at Cranfield University and Prof Saurav Goel based at London South Bank University as well as my industrial supervisor Dr Andy Knott based at the National Physical Laboratory (NPL), UK. Additionally I want particularly to thank my wife Leila for her continued support, and my children Amin and Ilias for their encouragement, also my parents Anne-Marie and Brian, and my friend Fabian Duarte-Martinez.

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LIST OF ABBREVIATIONS

PCA	Piston/Cylinder Assembly
HFA	Hydraulic Force Amplification
FEA	Finite Element Analysis
CFD	Computational Fluid Dynamics
NPL	National Physical Laboratory
SI	Système Internationale
ppm	Parts per million

1 INTRODUCTION

1.1 Background

Force measurement in the Mega Newton (MN) range is used for numerous applications. For example, the automotive industry (Kleckers and Graef, 2014) and civil engineering (Schäfer, 2010) lead partially the force measurement requirements of up to 0.5 MN. The measurement traceability of the force sensors routinely used in this range demands force standards with associated expanded measurement uncertainties as low as 0.002 % (Kumme et al, 2014). Brake testing of larger locomotives and strength testing of large aircrafts, like the Airbus A380-800 that accommodates up to 544 passengers (Müller et al., 2018) need force measurements greater than 2 MN, with force standards required to provide measurements with an uncertainty of 0.01 %. However, the growing interest in harnessing wind energy led to the development of large wind turbines that call for force measurements of 15 MN when testing their bearings (Schäfer, 2010). The upscaling of wind turbine blades soon will demand 50 MN measurement capability, with associated uncertainties in the range of 0.05 %.

For calibrations of force measurement in the Mega Newton range, the most basic machine used is a deadweight tester (Yaniv and Bartel, 2000) which typically provides measurements with uncertainties as low as 0.002 %, in the range up to about 5 MN (Knott, 2005). Since deadweight testers rely solely on physical constants, they are called ‘primary’ standards, maintaining traceability directly to constants of physics, ‘secondary’ machines rely on being calibrated against primary machines (Salminen et al, 2018). It can be seen From Figure 1-1 how the primary deadweight standards are calibrated against a range of other force standards (hydraulic machines, compression machines), which enable

calibrations to be performed at increased forces (Tegtmeier and Gutsch, 2018). By extending traceability to these ‘secondary’ machines, calibrations are disseminated to industry at increased forces, with quality varying from class 0.5 to class 2.0.

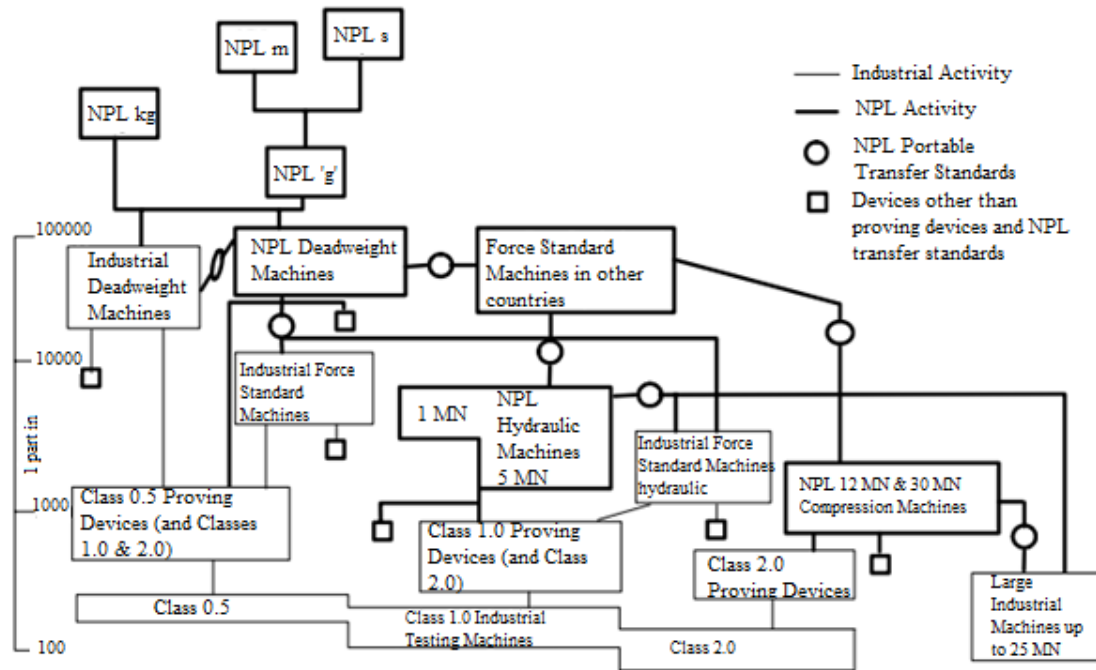


Figure 1-1 Traceability Diagram (Maybank et al, 1999)

In the Mega Newton (MN) force range, force standard machines of four different types (deadweight, hydraulic force amplification, lever and strain gauge) are used. The force standards enable calibrations to be made across the required force range from 0.5 MN to 60 MN or more, but unfortunately there is a trade-off between forces achievable and uncertainty of calibration. As can be seen, deadweight testers are the most accurate machines with uncertainties typically as low as 0.002 % (Zhanget al, 2018), but generally do not provide forces greater than 2 MN; in the median force range of 2 MN to 10 MN hydraulic force amplification and lever machines (Leonov et al, 2018; McIlraith and Clarkson, 2002) provide calibrations with uncertainties of around 0.02 %; at forces

beyond these strain gauge force machines provide uncertainties of around 0.1 % although forces as great as 60 MN or more are achieved (Shen et al, 2018).

At forces beyond 2 MN, amplification is normally needed because of costs associated with such large weight stacks. Amplification is normally done hydraulically, where a small piston/cylinder creates a pressure in the fluid which is amplified due to the larger surface area of a larger piston/cylinder undergoing the created pressure. The pistons within the cylinders can be of two types: either rotational or non-rotational (Durgut, 2018). In rotational PCA's, the piston is rotated within the cylinder which provides lubrication and centrality of the piston. In non-rotational PCA's, centrality is provided for by bearing-like jets which pressurise the sides of the piston. Figure 1-2 (Maybank et al, 1999) shows the smaller and large piston/cylinder sets connected to provide the hydraulic amplification.

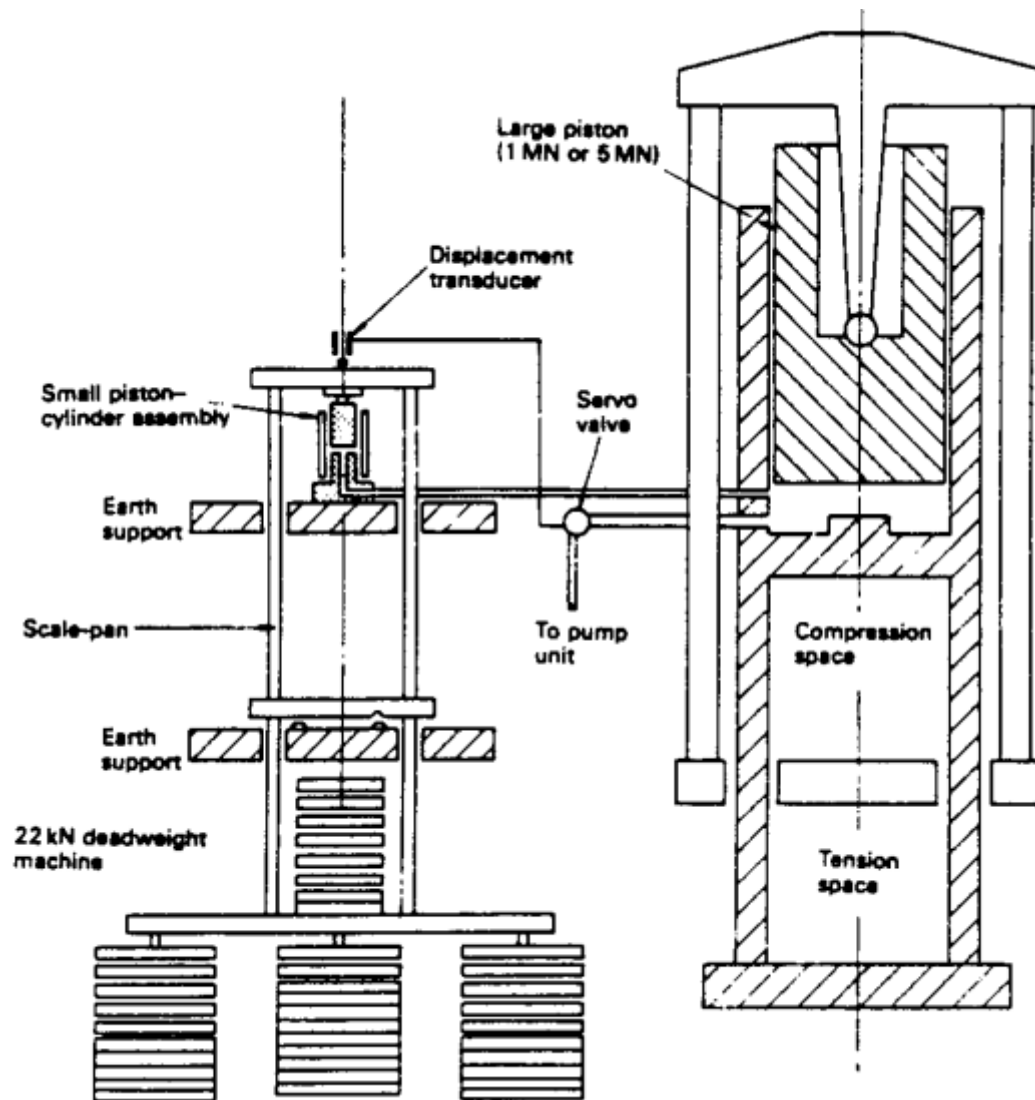


Figure 1-2 HFA Schematic (Maybank et al, 1999)

In force, the HFA machines maintain their traceability to deadweight testers, via a calibrated transducer (Tegtmeier and Gutsch, 2018). The problem with this method is that the transducer and calibration themselves have associated uncertainties as large as 0.01 % or more (Maybank et al, 1999; Kleckers and Graef, n.d.). It is therefore desirable to use HFAs as ‘primary’ force standards (Salminen et al, 2018), thereby eliminating the use of the force transducer calibration, and achieving uncertainties as low and comparable to

those of deadweight testers, thereby meeting the needs of industry. Figure 1-3 shows the relevant error sources:

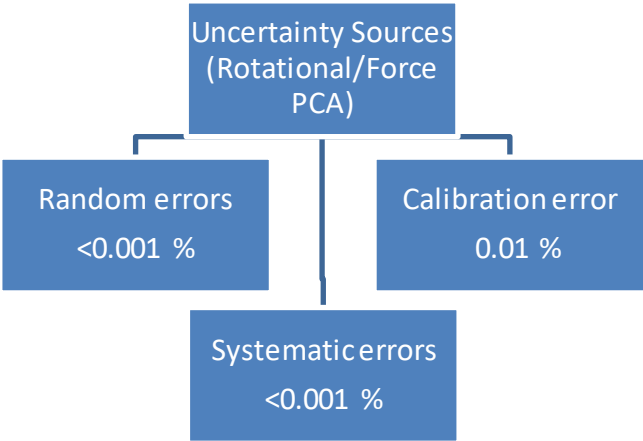


Figure 1-3 Calibration Uncertainty

Figure 1-4 presents a summary of the force measurement uncertainties required to assure the traceability in the Mega Newton range, plotted against the current state of the art (BIPM, 2021), with the contributions of the sources shown in the preceding table. As can be seen, the state-of-the-art has currently not met the requirements of industry, meaning that across industrial measurements, standards are lacking, leading to ambiguity which may have potential legal implications, improving the measurement capability would therefore be beneficial to society on a wide scale.

Table 1-1 Contributions to plot

<u>Publication</u>	<u>Findings</u>
Schäfer (2010)	0.05 % requirements for 50 MN
Müller et al. (2018)	0.01 % requirements for 2 MN
Kumme et al (2014)	0.002 % requirement for 0.5 kN
BIPM (2001)	Capabilities

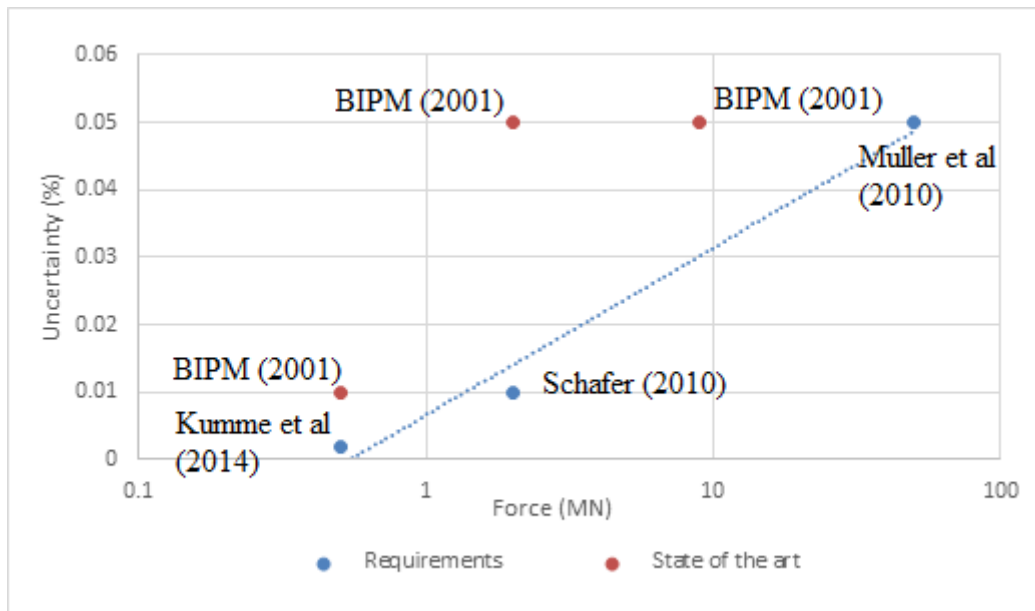


Figure 1-4 Uncertainty Requirements/Capabilities (BIPM, 2001; Schäfer, 2010; Kumme et al, 2014; Müller et al., 2018)

HFA machines currently assure traceability for force measurements up to 20 MN with associated expanded uncertainties of about 0.02 % (200 ppm) (Knott, 2005). The HFAs are secondary standards and are made traceable to the primary standards, deadweight

testers, using transfer standards, which means that their associated measurement uncertainty is theoretically limited to values that are larger than 20 ppm, Figures 1-5 and 1-6 show HFA machines. Enabling use of HFAs without the need for their traceability to deadweight testers, but by controlling the uncertainties in hydraulic amplification and so maintaining traceability directly to the weights would reduce measurement uncertainty significantly, meaning sources of uncertainty in the HFA hydraulics would be the limiting factor, enabling the requirements of industry to potentially be met. The figure below shows the alternative calibration methods, and the following two figures show HFA systems:

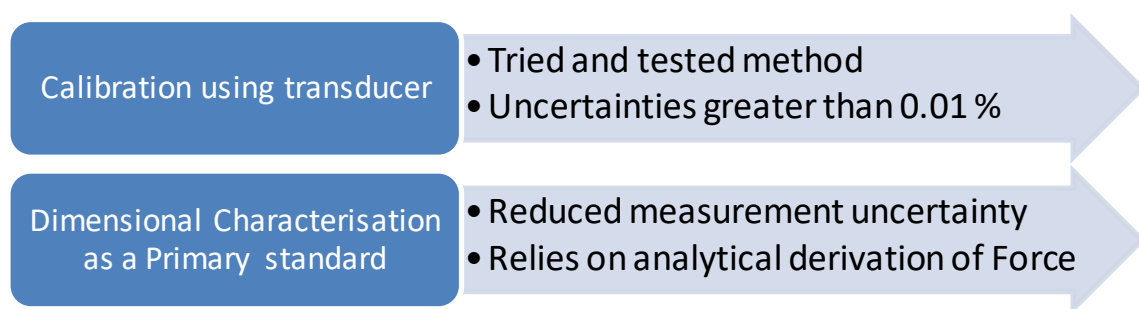


Figure 1-5 Calibration Methods

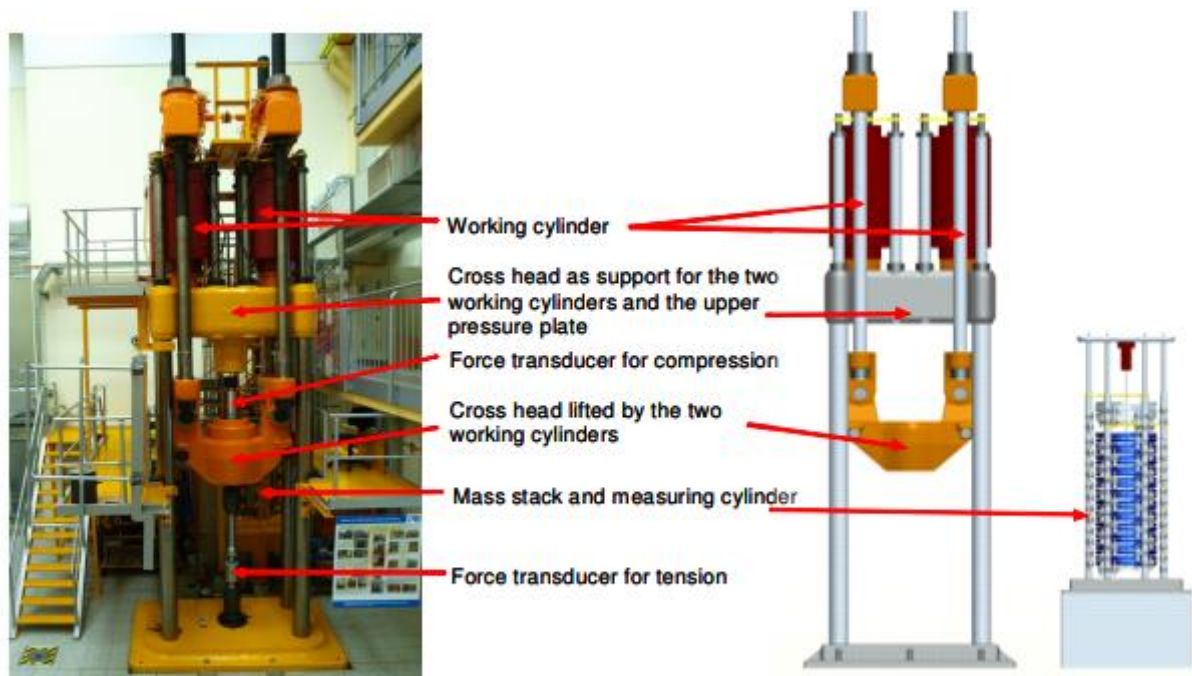


Figure 1-6 Hydraulic Force Standard (Tegtmeier et al, 2009)

Primary pressure balances have been created using oil as well as gas. Peggs and Lewis (1977) discuss dimensional characterisation of piston/cylinders for use in pressure balances, achieving uncertainties as low as 15 ppm (Buonanno and Dell Isola, 2001). Peggs and Lewis rely heavily upon very accurate measurements for piston and cylinder joining faces, which in effect are critical for keeping uncertainties low. Jager et al (1999) likewise discuss characterisation of primary pressure balances using oil, based on very accurate dimensional measurements and the piston/cylinder theory of Dadson, achieving results as low as 5 ppm.

While characterisation in the field of force metrology is carried out via a transfer standard as described, in the field of pressure primary characterisation using dimensional measurements of PCA's is common, making low uncertainties in the range of 10 ppm achievable (Dogra et al, 2010; Legras and Molinar, 1991). Piston/cylinder measuring

apparatus used for small PCA's are typically to the accuracy of 20 nm – 50 nm, reflecting the tight tolerances required of the mating faces likewise in pressure as in force. Bermanec et al (2019) for example, carried out dimensional characterisation to determine values of A_0 . They processed their measurements to give A_0 (the zero pressure effective area) in terms of mean values of the measurements obtained, reflecting the median path between surfaces:

$$A_0 = \frac{\pi}{8} (\overline{D_k}^2 + \overline{D_c}^2) \quad (1-1)$$

Where A_0 is zero pressure effective area, D_k is piston diameter and D_c is cylinder diameter. Their results agreed with those obtained by cross floating to within 5 ppm, and their dimensional characterisation had an uncertainty of 18 ppm.

The results achieved previously in pressure are indicative that a primary HFA should be feasible. This however ignores the difference that a large side PCA is required as well as a small side PCA, and also that it is the final force created that is critical, rather than the intermediate pressure. It remains therefore to be determined whether the findings in pressure can be extended to HFAs and that such low uncertainties can be derived in this adjacent field.

While in the field of force metrology traceability is maintained to 'primary' deadweight testers, in the field of pressure piston/cylinders are commonly characterised using dimensional measurement, the rarefied gas flow formula is used which enables prediction of the equilibrium state of the piston/cylinders.

1.2 Aims & Objectives

Based on the background context described, this work aims to evaluate the capability for HFAs to work as primary standards, with traceability maintained directly to the density and volume of the weights. As highlighted earlier, it is shown how apart from the transducer calibration, the uncertainties which would be present in a primary HFA are not excessive, thereby giving validity to the approach.

To achieve the aim, a design methodology for piston/cylinders for an HFA considering both random (time dependent) and pressure dependent error sources is shown. It is first necessary to prove that a primary HFA would improve measurement capability significantly, and that the concept is feasible. Next, it is necessary to create the capability for design of the HFA. To create the capability, analytical formulae are assembled/created and validated using FEA/CFD, which is also used for validation of the analytical formulae used to predict the uncertainties in the HFA. The FEA/CFD is required to ensure that the predictive formulae are representative for the real-world piston/cylinders, where the pressure distribution can be nonlinear, although FEA/CFD uses approximations also. As PCAs in HFAs can be rotational or non-rotational the work is divided accordingly, with an additional section providing confidence in the FEA capability, and one which considers assimilation of the formulae for a directly calibrated HFA. Finally, the designs are carried out within the constraints of a theoretical investigation, making the case for effectiveness of ‘primary’ HFA systems. The objectives are therefore:

1. Evaluate HFA sources of uncertainty including their magnitude and their effect to the generated force, thereby showing that the primary HFA concept would meet the needs of industry and is theoretically feasible

2. Establish analytical formulae which describe uncertainties in rotational PCAs enabling direct traceability, which enables the design of a primary rotational HFA
3. Establish analytical formulae which describe uncertainties in non-rotational PCAs enabling direct traceability, which enables the design of a primary nonrotational HFA
4. Create a computer application which enables design of HFAs, and to show that some designs of directly calibrated HFA are feasible with reduced uncertainties, determining some potential configurations including nominal sizes of PCA's, nominal pressures, forces and amplification ratio; thereby making the case strongly for adoption of 'primary' HFA's

1.3 Chapter plan

The contents of this thesis are arranged to meet the required analysis which will enable analysis and control of the aforementioned uncertainty sources. The contents of the thesis cover four sections: the review of uncertainties in 'secondary' and 'primary' HFA's, the research carried out concerning basic rotational piston/cylinder, the research carried out concerning non-rotational/bearing type piston/cylinders, and additional uncertainty sources involved in the direct calibration of HFA machines to the fundamental constant.

The diagram below lists the sections of each chapter, their presence is then justified:

Chapter 2 Literature Review	Chapter 3 Rotational design	Chapter 4 Nonrotational design	Chapter 5 Application
<ul style="list-style-type: none"> • Review of 'secondary' uncertainty sources • Review of 'primary' uncertainty sources 	<ul style="list-style-type: none"> • Adapted analytical formulae • FEA/CFD • Design and verification 	<ul style="list-style-type: none"> • FEA/CFD • Adapted analytical formulae • Design and verification 	<ul style="list-style-type: none"> • Design considerations

Figure 1-7 Contents

In Chapter 2, the review is carried out of the uncertainty sources in 'primary' and 'secondary' HFA's, this is to prove that by using a 'primary' HFA that the uncertainties which would remain in the system would allow the improved accuracy desirable to meet the needs of industry. In chapter 3 some analytical formulae are first adapted to provide the basis for analytical investigation of rotational PCA's. The FEA/CFD capability is also established, which enables verification of the PCA design to ensure the design is valid, and finally the design is carried out for a rotational HFA, thereby demonstrating that the concept is practical in theory. A repetition is carried out for nonrotational HFA's in Chapter 4, the FEA and analytical capability for carrying out the design first being established, before the design which provides theoretical evidence of the potential improved 'directly calibrated' HFA concept. In the final chapter some design considerations are discussed, and finally the conclusions are drawn from the work.

2 LITERATURE REVIEW

In this literature review an introduction is given to the concepts used to inform the reader about the relevant background material necessary to understand the scope of the work, more specifically concerning metrology vocabulary, work previously carried out in the field of pressure, a description of the experimental PCA's used in the FEA benchmarking, rotational/nonrotational PCA's, bearing theory, manufacturing challenges and metrology capability. Additionally, calculations are presented concerning the uncertainties present in a 'secondary' HFA, as well as those predicted for a 'primary' HFA, the first set of the calculations being carried out to validate that a sufficient gain can be had by converting the system to a 'primary' HFA, the latter also validating the possibility of deriving uncertainty sources directly from analytically predicted results. The diagram below outlines the work contents:

Secondary Standard Model	Primary Standard Model	Experimental Reports	Rotational and Non-rotational Designs	Tolerances
<ul style="list-style-type: none">• Density of weights• Air density• Gravity• Mass of the weights• Fluid Head• Pressure drops• Tilt• Side forces• Aerodynamic forces• Amplification ratio	<ul style="list-style-type: none">• PCA Theory• Large side piston radius• Small side piston radius• Young's Modulus and Poisson's Ratio	<ul style="list-style-type: none">• BNM LNE 200 MPa• PTB 400 MPa	<ul style="list-style-type: none">• Bearing Theory• Bearing	<ul style="list-style-type: none">• Manufacturing Challenges• Dimensional Metrology

Figure 2-1 Literature Review Contents

To give a brief justification of the contents included, the metrology vocabulary and background on the work on pressure are given first to inform the reader about relevant concepts and the context of the work. Then the secondary standard model is evaluated with a section on each uncertainty source, to justify the aim of using a 'primary' HFA

system (that the resulting uncertainties are sufficiently low), this is achieved in the next section on the ‘primary’ standard model. Next the experimental reports which are used for benchmarking of the FEA/CFD in the review are introduced, as is the relevant theory concerning bearings which informs the reader about the workings of nonrotational PCA’s, the experimental bearing report is also introduced. Next, the manufacturing and metrology capabilities are introduced, again to inform the reader about the extant potential for creation of piston/cylinders within the defined manufacturing constraints.

2.1 Metrology Vocabulary

The International Vocabulary of Metrology VIM (VIM, 2004) is the standard document which covers the terms used in metrology, the following definitions are taken from this important document. *Traceability* in measurement concerns the definition of the units of measurement used in practise, and more specifically the way in which they are derived (Gherasimov and Ghita, 2005). It can be said that if the units of measurement are derived relative to a physical standard or fundamental constant of physics, that traceability is maintained to that standard. Traceability of measurement is therefore important because it ensures that measurements are defined in the same way across different industries and in different locations, thereby enabling equivalence of measurements and useful results. In the past, measurements of different type (metre, second, Newton, etc..) were made traceable to physical artefacts which were maintained nationally and internationally to enable standardisation (Pratt et al, 2005); whereas nowadays traceability is maintained to fundamental constants of physics (such as wavelengths of light or vibrations of atoms) which provide constant reference to be used. The SI (système internationale, Robinson and Schlamming, 2016) is the system of units which are commonly used (such as the metre, second, Newton) and these have recently been updated to maintain traceability to

fundamental constants of physics, the table below shows to what traceability is maintained for different units:

Table 2-1 New SI Definitions

<u>Metre</u>	Laser wavelength
<u>Second</u>	Atomic vibrations
<u>Kilogram</u>	Plank constant
<u>Ampere</u>	Elementary charge
<u>Kelvin</u>	Boltzmann constant
<u>Mole</u>	Avogadro constant
<u>Candela</u>	Luminous efficacy of radiation at given wavelength

In the field of force, the newton (N) is a derived unit, being constructed from definitions of length (for calculating weight volume) and mass which are used primarily to define the mass of stacked weights (Kumme et al, 2014). The weights are loaded onto the measuring device used and are thereby used to impart a force. The force is disseminated by calibrations across industry as shown in figure 1-2.

Uncertainty of measurements is defined by Stephanie Bell (1999) as the ‘doubt which exists about the result of any measurement’. In other words the uncertainty is used to represent possible *error* that may exist between the measured value and the theoretical true value. The *error* and the *true value* of the measurand are finite values which can never be known with absolute certainty because of the imperfect nature of all

measurement and are therefore described with the probabilistic definition of *uncertainty*, which represents the band in which the true value is known to lie.

The *Guide to the Uncertainty of Measurement (GUM)* is the relevant standard used when working with uncertainty, and defines two types of uncertainty: *Type A* (estimation of uncertainty based on statistics) and *Type B* (uncertainty estimates based on other information). GUM defines the way in which uncertainties are converted to *standard uncertainties*, this being done for Type A uncertainties according to the formula:

$$u = \frac{s}{\sqrt{n}}$$

Where u is the standard uncertainty, s is the standard deviation and n is the number of measurements. And for Type B uncertainties (with a rectangular distribution) according to the formula:

$$u = \frac{a}{\sqrt{3}}$$

Where a is the half width between upper and lower limits.

Standard uncertainties can be summed in quadrature to obtain the *combined standard uncertainty*.

Of particular use in the standardisation of measurements is the *measurement model*. The *measurement model* is defined as the analytical formula which is used to predict measured quantity, such as force in force metrology, based upon the input parameters such as mass of the weights, as well as various factors which contribute to the uncertainty of the measurement performed (in force metrology such as variations to air pressure or weight buoyancy). The measurement model is useful because it allows the designer to predict

uncertainty generated in the measured output based upon magnitudes of uncertainty sources present in the design, and thereby derive tolerances which control the uncertainty within requirements. Sensitivity coefficients are derived by differentiation of the measurement model with respect to the parameter of interest, the derived coefficients can then be multiplied by the predicted change in the parameter (using standard values for the other parameters) to gauge the error caused by the predicted deviation.

2.2 Secondary Standard Model

In this section, the theory in relation to HFAs as secondary standards (calibrated *via* a transducer) is discussed to make the case for their use as primary standards. HFAs are pressurised by a pump, which maintains a constant pressure while the transducer is in place, ensuring that the fall rate of the small PCA is kept proportional to that of the large PCA. Pressure regulation is critical in HFA machines (Peters, n.d.), as the pump acts to keep the large PCA at a constant height, thereby maintaining a constant force on the transducer, the figure below shows a typical assembly.

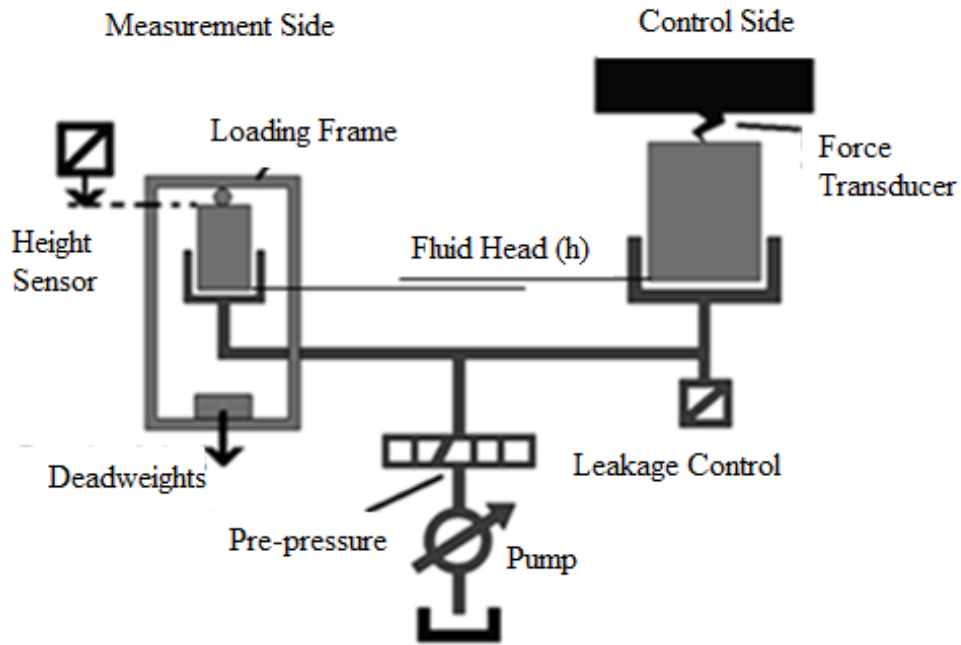


Figure 2-2 HFA Diagram (Adapted Peters, nd)

HFAs are subject to both random and pressure dependent uncertainty sources (Knott, 1999). Sources of random errors in hydraulic force amplification machines are numerous, with eccentric rotation of the piston within the cylinder and its tilting considered as major contributors to uncertainty (Peters, nd). Static influences consist primarily of problems with the loading such as weight eccentricity and defects in coupling weights to the transducer, and effects of variations in temperature, airflow, air pressure, local gravity and piston fall-rate. The pressure dependent influences include mismatched PCA distortion on the small and large sides (Peters, nd).

The overall measurement model for rotational or nonrotational PCA's can be shown to be:

$$F = \cos\phi \cos\theta_w \cos\theta_m \left(1 - \frac{\rho_{atm}}{\rho_m}\right) mgB(P) + F_{\rho_{fluid}gh} + F_{\Delta P} \quad (2-1)$$

The first term refers to the multiplier according to ratio of effective areas, the second to fluid head and the third to pressure drops in the pipework. Thus the measurement model for a secondary HFA, with traceability maintained by calibration, is given. In the following section each individual term in the formula is evaluated to investigate the effects of different sources of uncertainty in secondary HFAs, according to the customarily used method of evaluating sensitivity coefficients by differentiation and multiplying through by predicted uncertainties of the parameter of concern, thereby predicting the uncertainty in force produced. Contrarily, FEA/CFD is used later on to validate the calculations derived by this method, and in determining the exact amplification ratio for the nonrotational PCA's.

2.2.1 Density of Weights (ρ_m)

Maybank *et al* (1999) discuss measurements that are made of the weight density and their accuracy. They discuss taking measurements of the density of a sample of weights and finding a standard deviation of 1 % for steel weights, this result is corroborated by Bartel (2005). They mention also how the density of weights changes with temperature, by about 0.005 % per degree C, meaning that this change is by and large insignificant. The mean density measured for the steel weights was 7907 kg m⁻³.

$$u_F(\rho_m) = \left| \frac{\partial F}{\partial \rho_m} \right| \times u_{\rho_m} = \left| \frac{\rho_a}{\rho_m^2} mgB \right| \times u_{\rho_m} \quad (2-2)$$

$$\frac{u_F(\rho_m)}{F} \cong 0.00015 \times \frac{u_{\rho_m}}{\rho_m} \cong 1 \text{ ppm} \quad (2-3)$$

2.2.2 Density of the Air (ρ_{atm})

As described by Maybank et al (1999), the air density varies according to season, the standard uncertainty being about double in the winter. Air pressure is related to air density by the formula of NPL (2022):

$$\rho_a = \frac{0.348444P - H(0.00252T - 0.020582)}{273.15 + T} \quad (2-4)$$

Where ρ_a is air density, P is air pressure, H is relative humidity, T is temperature

The most appropriate way to determine the uncertainty associated with air pressure is to calculate the way the air pressure varies at the particular site. Maybank et al (1999) describe carrying out this procedure for Heathrow, and found that while considering temperature and humidity effects, standard uncertainty of 0.05 kg m^{-3} for the adopted value of 1.2015 kg m^{-3} . The density uncertainty is corroborated by values given by NPL (2022) of 1 part in 10^3 .

$$u_F(\rho_a) = \left| \frac{\partial F}{\partial \rho_a} \right| \times u_{\rho_a} = \left| \frac{1}{\rho_m} mgB \right| \times u_{\rho_a} \quad (2-5)$$

$$\frac{u_F(\rho_a)}{F} \cong 0.00015 \times \frac{u_{\rho_a}}{\rho_a} \cong 2 \text{ ppm} \quad (2-6)$$

2.2.3 Acceleration due to Gravity (g)

The value of g is not constant at all locations on the earth's surface, this is because of variations in the centripetal acceleration due to height, because the earth is not a perfect sphere and because it consists of different materials around its centre (Moffat-Griffin et

al, 2011). There are also variations in the value of gravity caused by movements of the sun and moon, and in practise the value of gravity changes depending on the height of the mass, meaning that different g values must be used according to the height of the mass in question (Maybank, 1999). Fortunately, gravity can be measured quite accurately using a gravity meter, to accuracies of within 0.1 ppm, and with an expectation to change of less than 0.2 ppm (Hegewald et al, 2011). The standard deviation of each gravity measurement is given as $1.1 \times 10^{-6} \text{ ms}^{-2}$, with a lower uncertainty given by Osman et al (2010) of $0.02 \times 10^{-6} \text{ ms}^{-2}$.

$$u_F(g) = \left| \frac{\partial F}{\partial g} \right| \times u_g = \left| \cos\phi \cos\varphi \cos\theta \left(1 - \frac{\rho_{atm}}{\rho_m}\right) mB \right| \times u_g \quad (2-7)$$

$$\frac{u_F(g)}{F} \cong \frac{u_g}{g} \cong 0.2 \text{ ppm} \quad (2-8)$$

2.2.4 Mass of the Weights (m)

The definition of the masses of the weights is carried out using a mass comparator, to uncertainties as low as 0.1 ppm. The masses are first machined to be slightly overweight, allowing for the final critical mass adjustment. The mass comparator used at NPL uses smaller masses of 300 kg which were calibrated on the NPL equal-arm balance (Maybank et al, 1999). The value is validated by Osman et al (2010) who give a higher value of 28 ppm, which is used here.

$$u_F(m) = \left| \frac{\partial F}{\partial m} \right| \times u_m = \left| \cos\phi \cos\varphi \cos\theta \left(1 - \frac{\rho_{atm}}{\rho_m}\right) gB \right| \times u_m \quad (2-9)$$

$$\frac{u_F(m)}{F} \cong \frac{u_m}{m} \cong 0.1 \text{ ppm} \quad (2-10)$$

2.2.5 Fluid Head ($\rho_{fluid}gh$)

The fluid head represents a source of uncertainty in the amplification of force, so the nominal value can be determined to assess whether significant uncertainty is introduced in the system. Therefore, for a height of 1 mm, the resulting pressure is of approximately 10 Pa, which at the lower force range with a line pressure of 0.5 MPa results in an error of 0.002 %, a fluid density of 1000 kgm⁻³ is assumed. It is noteworthy that the fluid head can also be corrected for, but that the low-level uncertainties involved indicate that the fluid head is not of particular significance. The uncertainty associated with the fluid head is in terms of the measured height difference, which can be assumed to be measurable to the millimetre at a minimum, therefore a standard uncertainty of 1 mm is assumed. Here the pressure is assumed to be 0.5 MPa and the effective area is given for a 4 mm radius piston:

$$u_F(h) = \left| \frac{\partial F}{\partial h} \right| \times u_h = |\rho_{fluid}gA| \times u_h \quad (2-11)$$

$$\frac{u_F(h)}{F} = \frac{|\rho_{fluid}gA| \times u_h}{F} \cong \frac{gA}{F} \cong 20 \text{ ppm} \quad (2-12)$$

2.2.6 Pressure Drops in Pipework (ΔP)

Pressure drops in the pipework are generally nonsignificant, because of the minimal flowrates ($Q < 10^{-5}$), therefore the pressure drop in the pipework, as well as the changes caused by changes to flowrate are generally non-significant. What has been reported is that the flow can accelerate uncontrollably at the excessively high pressure range where fluid starts to flow at increased rates, thereby causing significant pressure drops (Peters, nd). The value of Q alters with piston eccentricity according to the formula:

$$Q = \frac{\pi d \delta^3 P}{12 \eta L} \left(1 + 1.5 \frac{e^3}{\delta^3} \right) \quad (2-13)$$

Where d is piston diameter, δ is clearance gap, P is line pressure, η is fluid viscosity, L is engagement length, e is piston eccentricity

Which indicates that the flowrate can normally be amplified by a factor of 2.5 due to piston eccentricity, thereby leading to a range for uncertainty in Q (u_Q) of 10^{-5} to 10^{-8} . This therefore indicates that the maximal variation in Q is expected when the maximum pressure is applied, and the piston is experiencing eccentricity. It is noteworthy, however, that the system designer has freedom to design the pipework with sufficiently large radius

$$u_F(Q) = \left| \frac{\partial F}{\partial Q} \right| \times u_Q = \left| \frac{8 \eta l A}{\pi r^4} \right| \times u_Q \quad (2-14)$$

$$\frac{u_F(Q)}{F} = \frac{\left| \frac{8 \eta l}{\pi r^4} \right| \times u_Q}{F} \cong \frac{100 A \times u_Q}{F} \cong 0.1 ppm \quad (2-15)$$

The above calculation assumes that the force is in the Mega Newton range and that the pipework is of radius 40 mm, but as the uncertainty in the value of Q is likely to be in the range of $10^{-5} - 10^{-8}$, it can be seen how the resulting value is easily within acceptable uncertainty limits.

It is noteworthy that the literature (Dadson, 1965) has always assumed that piston eccentricity does not significantly affect distortion coefficient (λ), but this has not been thoroughly investigated so is highlighted here as a gap in the research and is investigated later on in this work in the Rotational PCA section.

2.2.7 Tilt (θ)

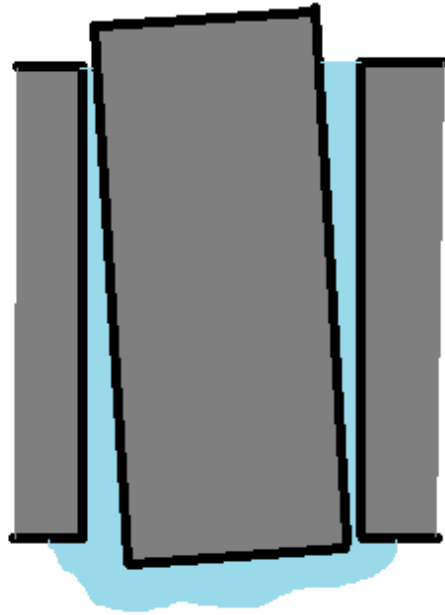


Figure 2-3 Piston Tilt

The non-concentric tilt of the piston within the cylinder, as shown in the figure above, is known to enhance the force imparted on the piston by the fluid as a direct ratio (Peggs and Lewis 1977). Tilt is controlled by maintaining a surplus ratio of piston/cylinder engagement length (L) to the gap (h), such that the tilt angle (θ) precludes excessive changes to the effective area of the piston. Elementary calculations show that angles as low as 0.05° should be maintained to keep uncertainties as low as 0.001 %, corresponding to a ratio of $2h/L$ of 0.05 (or 20:1), which is readily achievable in most standard PCAs. Formally, the uncertainty in the $\cos\theta$ is given by the maximum possible value of θ , given by:

$$u_F(\theta) = \left| \frac{\partial F}{\partial \theta} \right| \times u_\theta = \left| -\sin\theta_W \cos\phi \cos\theta_m \left(1 - \frac{\rho_{atm}}{\rho_m} \right) mgB \right| \times u_Q \quad (2-16)$$

$$\begin{aligned} \frac{u_F(\theta)}{F} &= \frac{\left| -\sin\theta_W \cos\phi \cos\theta_m \left(1 - \frac{\rho_{atm}}{\rho_m} \right) mgB \right| \times u_{\cos\theta}}{F} \cong \frac{|mgB| \times u_{\cos\theta}}{F} \quad (2-17) \\ &\cong \frac{25 \times 10^6 \times 100 \times u_\theta}{25 \times 10^8} < 0.1ppm \end{aligned}$$

2.2.8 Side Forces

In loading the transducer, it is possible for sideways forces to be introduced because of angular loading of the transducer (Ferrero, 1990). The sideways force is equivalent to (Maybank et al, 1999):

$$error \text{ (\%)} = (1 - \cos\phi) \times 100\% \quad (2-18)$$

It can be seen that to achieve errors as low as required, the machine surfaces need to be parallel to within 0.1° to achieve uncertainties as low as 0.001 %, which is readily achievable. The magnitude of the uncertainty relevant in the consideration of side forces is the angle of the contact plates itself (ϕ), which is controllable such that $\cos\phi$ has less of an impact than 1.5 ppm, so the relevant uncertainty is given below. A coverage factor of 2 applies because of the normal distribution of the tilting.

$$\begin{aligned} u_F(\cos\phi) &= \left| \frac{\partial F}{\partial \cos\phi} \right| \times u_{\cos\phi} \quad (2-19) \\ &= \left| -\sin\phi \cos\phi \cos\theta \left(1 - \frac{\rho_{atm}}{\rho_m} \right) mgB \right| \times u_{(\cos\phi)} \\ &\cong |mgB| \times u_{(\cos\phi)} \end{aligned}$$

$$\frac{u_F(\cos\phi)}{F} \cong u_{(\cos\phi)} < 0.1 \text{ ppm} \quad (2-20)$$

2.2.9 Aerodynamic Forces

Wind flow over the weight stack has been shown to cause significant uncertainties in the force from the masses, Maybank et al (1999) predicted uncertainties as high as 0.022 % from this error source. The wind travels over the individual weight and creates a pressure differential which elevates or suppresses the weight. The air pressure exerted is calculated by the long Bernoulli equation (which is omitted here), suffice it to say that efforts are made in HFA design to protect against airflow, and variations in the produced force are checked for to ensure adequate guarding against airflow isn't required.

2.2.10 Amplification Ratio ($B(P)$)

The overall uncertainties for the amplification ratio are quite high (0.01 % - 0.02 %), reflecting the high measurement uncertainties in secondary HFAs due to errors in the calibration using the transfer standard (Singh et al, 2018). The Table 2-2 shows uncertainties in transducers found in a comparison in the MN range (Bartel, 2012). In secondary standards the amplification ratio is effectively calibrated at several different pressures to ensure that the mass applied results in the desired force, whereas in a direct pressure dependence is critical and the amplification must be known at the design stage with given uncertainties. Transducers are known to have uncertainties of approximately 0.01 %, and uncertainties in calibration methods can increase the uncertainty. Therefore, it can be seen how the majority of the uncertainty in the secondary HFA is due to calibration uncertainty, and results in uncertainty of $B(P)$ in the range of 0.01 % - 0.02 %, as shown by the relative expanded uncertainty in the table below, which is given to show

the relative error, alongside the absolute values shown in the expanded uncertainty column.

With regards to thermal effects, these are almost negligible in the PCA because of the abundance of metallic parts which act as a heat sink to dissipate thermal effects, and the lack of a significant heat source. The table below shows uncertainties associated with some transducers used in calibrations, and the following figure shows HFA uncertainty sources:

Table 2-2 Transducer Uncertainties (Bartel, 2012)

<u>Force (MN)</u>	<u>Transducer</u>	<u>K=2 Relative Expanded Uncertainty (%)</u>
2	1	0.0093
4	1	0.0072
2	2	0.0110
4	2	0.0042

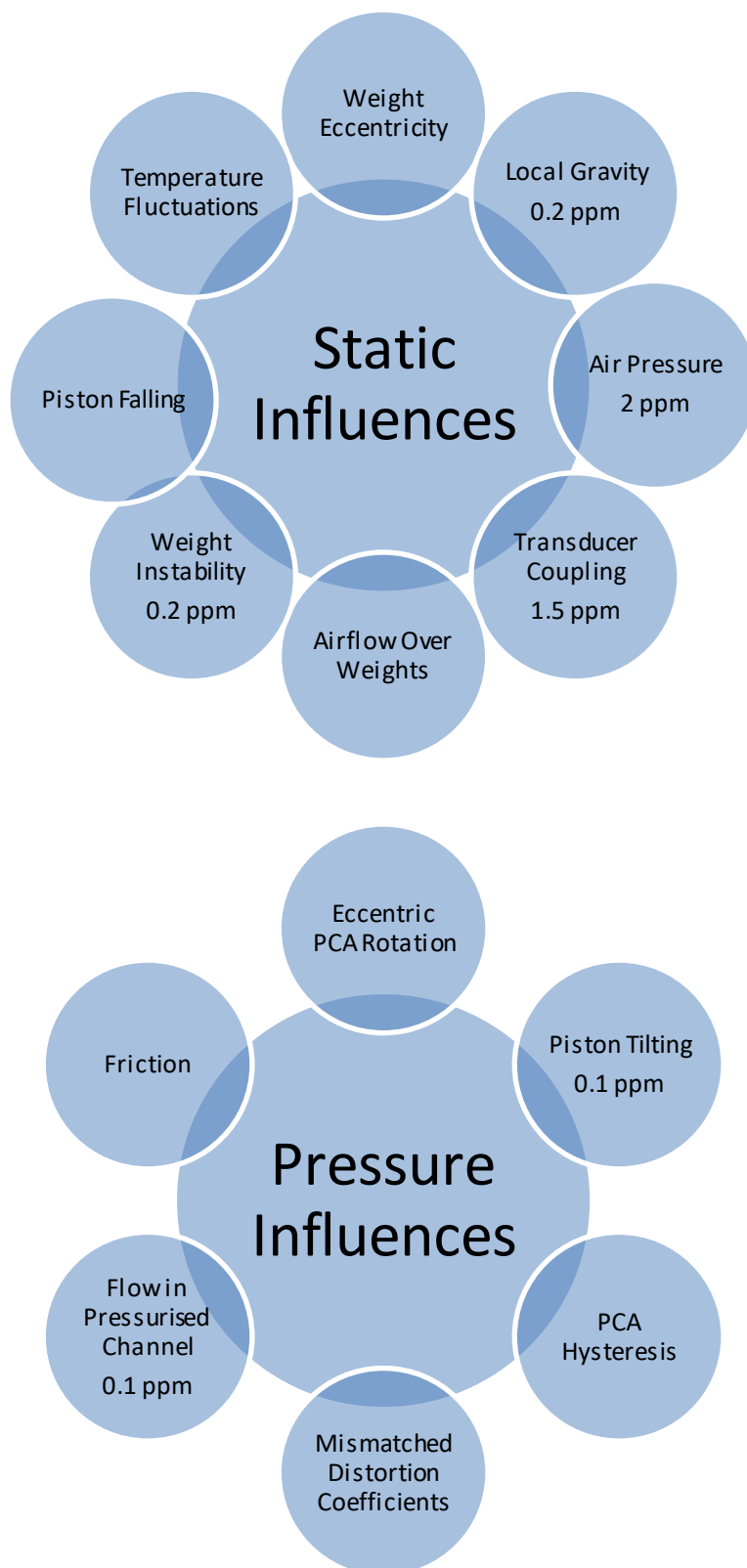


Figure 2-4 HFA Uncertainties

2.2.11 Conclusion

As can be seen from the foregoing analysis, uncertainties in the basic components (weight density, air density, gravity etc..) for a secondary HFA do not contribute significantly to uncertainty in the system, contributing towards uncertainties of less than 0.01 %. It can be seen how the uncertainties caused by these factors are all in the range of tens of ppm or less, making them non-significant when compared with the calibration uncertainty. It is the uncertainties in the calibration of the amplification ratio ($R(P)$) which give rise to significant uncertainties in the system (of 0.01 %). It can be seen however that the potential exists for direct traceability of the force to the weights within the system because of the low uncertainty contributions of the basic components.

The terms for tilt, head and pressure drop in the pipework also seem not to add significant uncertainties in the system, seeming to contribute negligible uncertainties only. The significant uncertainty which remains in the system is that of the amplification ratio, which in reality constitutes of the ratios of A_{eff} . It is noteworthy that A_{eff} is proportional to the force produced and the pressure, so uncertainties in A_{eff} can be taken as representative of uncertainties in the HFA machine. To achieve the desired uncertainties it is necessary to control uncertainties in A_{eff1} and A_{eff2} to within 0.001 %, thereby enabling improvement in force measurement of an order of magnitude. Therefore, by analysing this term for amplification ratio in more detail, the feasibility of maintaining traceability directly to the weights can be evaluated in the next section. The table below summarises the different uncertainty sources that were derived previously:

Table 2-3 Summary of Uncertainty Sources

<u>Uncertainty Source</u>	<u>Relative Expanded Uncertainty</u>
Density of Weights (ρ_m)	1 ppm
Density of the Air (ρ_{atm})	2 ppm
Acceleration due to Gravity (g)	0.2 ppm
Mass of the Weights (m)	1 ppm
Fluid Head ($\rho_{fluid}gh$)	0.1 ppm - 20 ppm
Pressure Drops in Pipework ($\frac{Q8\eta l}{\pi r^4}$)	< 0.1 ppm
Tilt (θ)	< 0.1 ppm
Side Forces	1.5 ppm
Aerodynamic Forces	< 2ppm
Amplification Ratio (R)	100 ppm – 200 ppm

In conclusion, the table above demonstrates that by using the HFA without the calibration for amplification ratio has the potential to profoundly reduce uncertainties of the machine approaching uncertainties as low as 20 ppm. This provides confidence that the use of a ‘primary’ HFA standard could be used to meet the demands of industry for more accurate measurements in the 2 MN – 20 MN range, provided that the amplification ratio can be controlled without excessive uncertainty being caused, which is the topic of the next chapter. The figure below again shows the potential calibration methods:

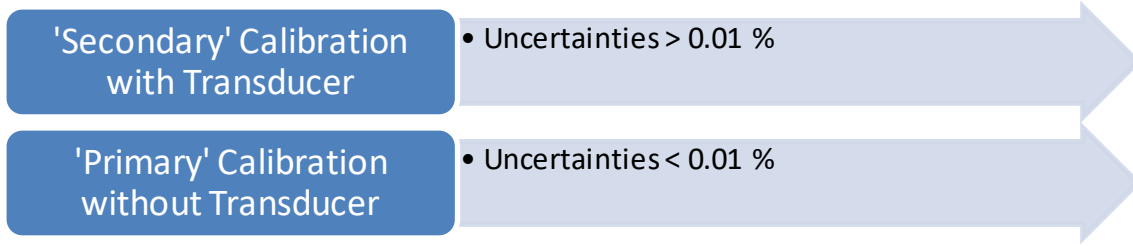


Figure 2-5 Uncertainties in Primary/Secondary HFA

It is noteworthy that there exist some additional uncertainty sources which have been discussed in the literature (Peters, n.d.), including hysteresis of the piston/cylinder and discrimination of the reading. It is decided that these functional sources of uncertainty will be beyond the scope of this investigation, as has been the case in the literature of these uncertainty sources not having had significant effect upon the achievement of low uncertainties in hydraulic PCA's. The designer of a direct HFA may wish to consider these for completeness.

2.3 Primary Standard Model

Using the formulae described above, in this section the terms concerning the amplification ratio (B) are introduced, along with terms for temperature dependence and the basic formulae describing B , which includes the distortion coefficient (λ), the coefficient for thermal expansion (α) and temperature change (ΔT).

$$B = \frac{A_{eff2}}{A_{eff1}} \quad (2-21)$$

$$B = \frac{A_{02}(1 + \lambda_2 P)(1 + \alpha \Delta T)}{A_{01}(1 + \lambda_1 P)(1 + \alpha \Delta T)} \quad (2-22)$$

At this stage the need emerges of introducing some of the theory concerning PCA's, this is done in the next section, and the analysis is picked up again afterwards:

2.3.1 PCA Theory

As preparation for the investigation into the feasibility of a secondary HFA with traceability maintained directly to weights, the relevant PCA theory is introduced. It is necessary to consider ‘systematic’ errors in PCAs in order to allow for ‘direct’ traceability to the weights of the HFA system, and avoid use of a transducer. In this section, the predominant systematic uncertainty sources are described to give an overview. The dimensions of concern are primarily the diameter of the piston and cylinder, along with the inner diameter of the cylinder, as well as their length. Analytical formulae have long been used in the study of PCA’s, so their use is deemed appropriate here. A PCA is shown diagrammatically below with the relevant parameters highlighted:

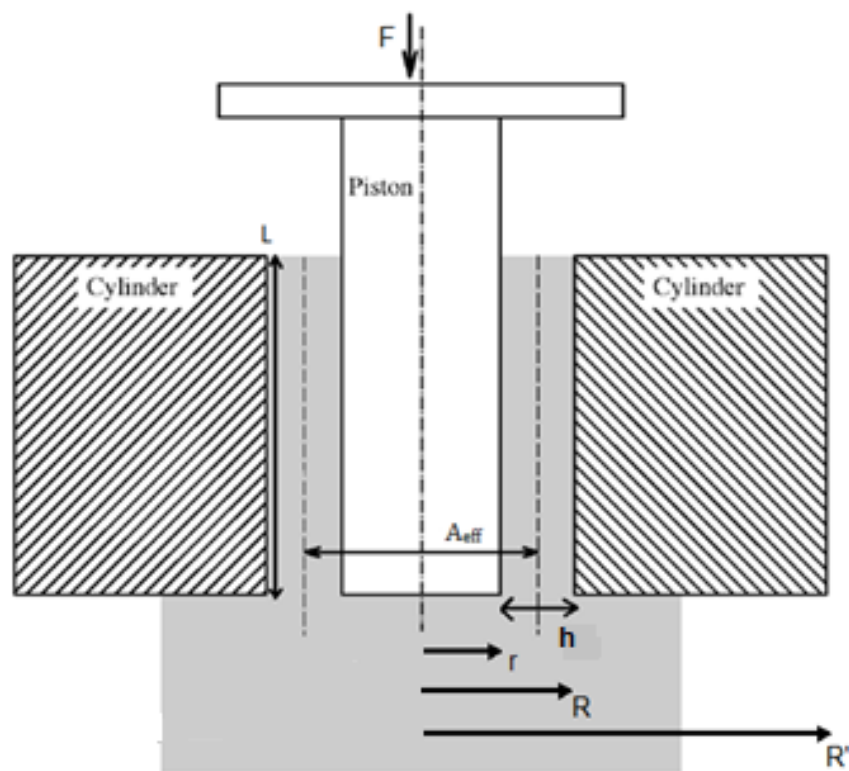


Figure 2-6 PCA Schematic (Adapted from Esward, 2002)

It is noteworthy that it is also necessary to determine the zero-distortion effective area, used for determining the amplification ratio. This has been described by the formula below by Dadson (1985):

$$A_0 = \pi r^2 \left(1 + \frac{h}{r}\right) \quad (2-23)$$

where r is piston radius and h is gap width.

Accurate measurement of the PCA without distortion is critical in a directly calibrated system because it informs the value of A_0 which is used, and therefore the knowledge of the value of amplification ratio and final force output at the transducer. Simplified formulae have also been derived for distortion coefficient of the PCA, which describes how the pressurised area is derived from A_0 :

$$A_{eff} = A_0(1 + \lambda P) \quad (2-24)$$

Where λ represents the distortion coefficient. The simplified formula assumes the pressure gradient and resulting distortion of piston/cylinder walls to be of constant gradient:

$$\lambda = \frac{1}{E} \left(2\sigma + \frac{R^2}{R'^2 - R^2} \right) \quad (2-25)$$

Where E is Young's modulus, σ is Poisson's ratio, R is cylinder inner radius and R' is cylinder outer radius

Having introduced the formulae relevant to the formulation, the derivations are picked up again here:

It can be seen also how the amplification ratio is almost linear except for variations between λ_1 and λ_2 . In fact, in the case where $\lambda_1 = \lambda_2$ within the uncertainty, the amplification ratio is easily derived as the ratio of A_{02} to A_{01} . To further consider the equation, formulae for λ are added from the formula of Dadson (1985):

$$B = \frac{A_{02} \left[1 + \frac{1}{E} \left(2\sigma + \frac{R_2^2}{R_2'^2 - R_2^2} \right) P \right]}{A_{01} \left[1 + \frac{1}{E} \left(2\sigma + \frac{R_1^2}{R_1'^2 - R_1^2} \right) P \right]} \quad (2-26)$$

It can be seen from the above how, depending upon the assumption that the material properties match, that it is only necessary to match the ratio of outer cylinder (R') to inner cylinder (R) to obtain a matching distortion coefficient. Next, terms for A_0 are expanded using formula of Dadson (1965), and the gap is shown in terms of piston/cylinder radii:

$$B = \frac{\pi r_2^2 \left(1 + \frac{(R_2 - r_2)}{r_2} \right) \left[1 + \frac{1}{E} \left(2\sigma + \frac{R_2^2}{R_2'^2 - R_2^2} \right) P \right]}{\pi r_1^2 \left(1 + \frac{(R_1 - r_1)}{r_1} \right) \left[1 + \frac{1}{E} \left(2\sigma + \frac{R_1^2}{R_1'^2 - R_1^2} \right) P \right]} \quad (2-27)$$

Where B is amplification ratio, r is piston radius, R is cylinder inner radius, E is Young's modulus, R' is cylinder outer radius and P is line pressure

At this stage it is worth reflecting that the measurement model above can be said to be representative of a 'primary' HFA machine, in that the exact terms which describe the amplification ratio have been described according to the known theory, which allows the uncertainty in these terms to be controlled for, providing an alternative pathway from the mass 'm' and the produced force 'F'. In this case the experimental calibration is no longer needed, and the characterisation relies rather on dimensional measurements of the PCA along with the controlled uncertainties in those measurements, which enables the

prediction of the value of F with a controlled uncertainty. It is noteworthy that there will be a given uncertainty in the model, particularly concerning the simplification of the linearly varying pressure distribution.

In the sections which follow, the different components of the formula are considered in terms of their uncertainty, thereby evaluating how feasible dimensional control of amplification ratio is in a primary standard:

2.3.2 Large side radius of the piston (r_2)

It can be seen how, broadly speaking the ratio of effective areas is directly proportional to the force produced, such that uncertainties in one value of A_{eff} will lead to equivalent uncertainty values in the force produced. It is assumed for the purposes of the analysis that both the piston and cylinder are perfectly straight and cylindrical. Therefore, it can be seen how values for r can be taken as critical, with uncertainties associated with its measurement being multiplied by double the nominal value of r . It is assumed that distortion in the piston/cylinder varies linearly. Because of this, very accurate machines have been devised for the accurate measurement of pistons, such as by Miles et al (2005) who devised equipment with measurement uncertainty as low as 20 nm.

$$u_B(r_2) = \left| \frac{\partial B}{\partial r_2} \right| \times u_{r_2} \quad (2-28)$$

$$= \left| \frac{2\pi r_2 \left(1 + \frac{(R_2 - r_2)}{r_2} \right) \left[1 + \frac{1}{E_2} \left(2\sigma_2 + \frac{R_2^2}{R_2'^2 - R_2^2} \right) P \right]}{\pi r_1^2 \left(1 + \frac{(R_1 - r_1)}{r_1} \right) \left[1 + \frac{1}{E_1} \left(2\sigma_1 + \frac{R_1^2}{R_1'^2 - R_1^2} \right) P \right]} \right|$$

$$\times u_r$$

$$\frac{u_R(m)}{B} \cong \frac{r_2}{r_2'^2} \times u_r \cong \frac{1}{r_2} \times u_r < 10 \text{ ppm} \quad (2-29)$$

2.3.3 Small side radius of the piston (r_1)

The effect of the small side piston radius is likewise given by differentiation of the amplification ratio and the use of the important sensitivity coefficients. The standard uncertainty in the radius remains confined according to the stringent metrology capability of 20 nm and the similar manufacturing capabilities. A larger nominal radius of the piston/cylinder of 100 mm is opted for:

$$\frac{u_B(r_2)}{B} \cong \frac{2r_2}{r_2'^2} \times u_r \cong 1 \text{ ppm} \quad (2-30)$$

$$u_B(r_1) = \left| \frac{\partial B}{\partial r_1} \right| \times u_{r_1} \quad (2-31)$$

$$\begin{aligned} &= \pi r_2'^2 \left(1 + \frac{2\delta_2}{r_2} \right) [1 \\ &+ \frac{1}{E} \left(2\sigma + \frac{R_2'^2}{R_2'^2 - R_2^2} \right) P] \ln \left| \pi r_1'^2 \left(1 + \frac{2(R_1 - r_1)}{r_1} \right) [1 \right. \\ &+ \frac{1}{E} \left(2\sigma + \frac{R_1'^2}{R_1'^2 - R_1^2} \right) P] \left| 2\pi r_1 \left(1 + \frac{2(R_1 - r_1)}{r_1} \right) [1 \right. \\ &+ \frac{1}{E} \left(2\sigma + \frac{R_1'^2}{R_1'^2 - R_1^2} \right) P] \times u_r \end{aligned}$$

2.3.4 Young's Modulus (E) and Poisson's Ratio (σ)

The other parameters being also of diameters and being less stringent than the r value, the Young's Modulus (E) is considered next. Young's modulus is commonly measured experimentally, Sabuga (2001) describes the use of strain gauges to determine Young's

modulus experimentally for use in piston/cylinder assemblies. He used strain gauges on Tungsten Carbide PCAs, and obtained tolerances of measurement as high as 7 GPa (13 %). However, Salama et al (2012) describe characterising Young's modulus and Poisson's ratio for pressure balances using resonant ultrasonic spectroscopy (RUS) with uncertainties as low as 0.03 % and 0.05 % respectively. Therefore, it can be seen how the Young's modulus term influences the percentage increase in the A_{eff} , therefore indicating a resulting uncertainty as low as approximately 0.0005 % on effective area. The Poissons ratio likewise affects a term which describes the increase in A_{eff} , so again the resulting uncertainty is likely to be of the order of 0.0005 %.

It appears therefore, upon a preliminary inspection, that a primary HFA is feasible, in that uncertainties can be maintained sufficiently low to make the concept possible.

$$u_R(E) = \left| \frac{\partial B}{\partial E_2} \right| \times u_{r_2} \quad (2-32)$$

$$= \left| \frac{\pi r_2^2 \left(1 + \frac{2(R_2 - r_2)}{r_2} \right) \left[-\frac{1}{E^2} \left(2\sigma + \frac{R_2^2}{R_2'^2 - R_2^2} \right) P \right]}{\pi r_1^2 \left(1 + \frac{2(R_1 - r_1)}{r_1} \right) \left[1 + \frac{1}{E} \left(2\sigma + \frac{R_1^2}{R_1'^2 - R_1^2} \right) P \right]} \right| \times u_E$$

$$\frac{u_R(E)}{B} \cong \frac{-\frac{1}{E^2} \left(2\sigma + \frac{R_2^2}{R_2'^2 - R_2^2} \right) P}{1 + \frac{1}{E} \left(2\sigma + \frac{R_1^2}{R_1'^2 - R_1^2} \right) P} \times u_E \cong \frac{-\frac{1}{E^2} P}{1 + \frac{1}{E} P} \times u_E \quad (2-33)$$

$$\cong -\frac{1}{E^2} P \times u_E < 0.1 \text{ppm}$$

$$u_B(\sigma) = \left| \frac{\partial B}{\partial \sigma} \right| \times u_\sigma \quad (2-34)$$

$$= \left| \frac{\pi r_2^2 \left(1 + \frac{2(R_2 - r_2)}{r_2} \right) \left[\frac{2}{E_2} P \right]}{\pi r_1^2 \left(1 + \frac{2(R_1 - r_1)}{r_1} \right) \left[1 + \frac{1}{E_1} \left(2\sigma + \frac{R_1^2}{R_1'^2 - R_1^2} \right) P \right]} \right| \times u_\sigma$$

$$\frac{u_R(\sigma)}{B} \cong \frac{\left[\frac{2}{E} P \right]}{1 + \frac{1}{E} \left(2\sigma + \frac{R_2^2}{R_2'^2 - R_2^2} \right) P} \times u_\sigma \cong \frac{2}{E} P \times u_\sigma \cong 1ppm \quad (2-35)$$

2.3.5 Conclusion

It can be seen from the foregoing conclusions that uncertainties expected in a directly calibrated HFA are not excessive in terms of the uncertainty caused in the amplification ratio, indicating that a primary HFA may be a feasible option. This proposition is supported by evidence from the field of pressure, where PCA's are characterised to within 15 ppm (Buonanno and Dell Isola, 2001). The largest uncertainty contributor identified was the gap of the small PCA at 0.002 %, which still sits comfortably beneath the uncertainty limit desired. It's noteworthy also that in the system design the parameters can be altered to reduce this further. It seems that with the advancements in measurement capability, and the non-criticality of the material properties, that it should be possible to define the amplification ratio with low enough uncertainty to enable traceability to be maintained within the hydraulic system instead of resorting to the use of calibrations.

2.3.5.1 FEA/CFD

FEA/CFD analysis of PCAs relies on the same formula described above in the PCA Theory section, hence, this section is introduced here to discuss the formulation further. In this work FEA/CFD was used to validate the designs that have been based on the analytical formula in a valid way, therefore it is also necessary to ensure that the accuracy of the FEA/CFD is robust. The Dadson formula shown in equation assumes linearly varying pressure distribution in the clearance, and therefore linearly varying piston/cylinder distortions. However, the formula is derived from the more complicated formula for distortion coefficient which considers the non-linearity of pressure distribution:

$$\lambda = \frac{1}{1 + h/r} \left[\frac{U(0) + u(0)}{rp} + \frac{1}{rp^2} \int_0^L p(z) \left(\frac{dU}{dz} + \frac{du}{dz} \right) dz \right] \quad (2-36)$$

Where λ is distortion coefficient, h is clearance gap, r is piston radius, $U(0)$ and $u(0)$ are deflections at base of cylinder and piston respectively, p is line pressure and U and u are deflections of the cylinder and piston respectively

It can be seen how both formulae are derived from the same first principles, with the main difference that the pressure distribution ($p(z)$), and gradients of distortion ($\frac{dU}{dz}, \frac{du}{dz}$), are assumed to be of constant value in the simplified version. The values for the distortion at the base ($U(0)$ and $u(0)$) have also been replaced using the Lamé equations (of Beukers et al, 2004).

The above formula is used in the FEA/CFD to process the distortions calculated in the procedure. To carry out the analyses, researchers have in the past analytically calculated the pressure distribution in the gap based upon the pressure dependent viscosity and

density and geometry of the piston/cylinders (Buonanno et al, 1999); the normal course is for the calculated pressure distribution to be imported into the FEA to determine the PCA distortions given the peculiarities of its geometry such as mounting points and external diameters. In either case, whether the pressure distribution is calculated analytically or using software, the outputs from the FEA/CFD are in most cases the distortions of the piston and cylinder as well as the resulting pressure distributions.

These parameters, however, are not immediately useful and must be subjected to further processing before they can be made use of, with formulae for distortion coefficient or effective area (which correspond to the previously introduced formulae for distortion coefficient but without simplifying assumptions).

While the simplifications are certainly useful for predicting distortion coefficient, unfortunately the pressure distribution in the piston/cylinder clearance and the resulting distortions are not uniformly distributed (Fitzgerald and Sutton, 2005). Particularly in piston/cylinders where the gap is very narrow, or the pressure very great (Sabuga, et al, 2006), the pressure adopts a non-linear distribution meaning that the simplified version is inaccurate to the extent that the pressure distribution is nonlinear. The figure below of Yagmur and Bagli (2009) for example shows the curvature in the pressure distribution:

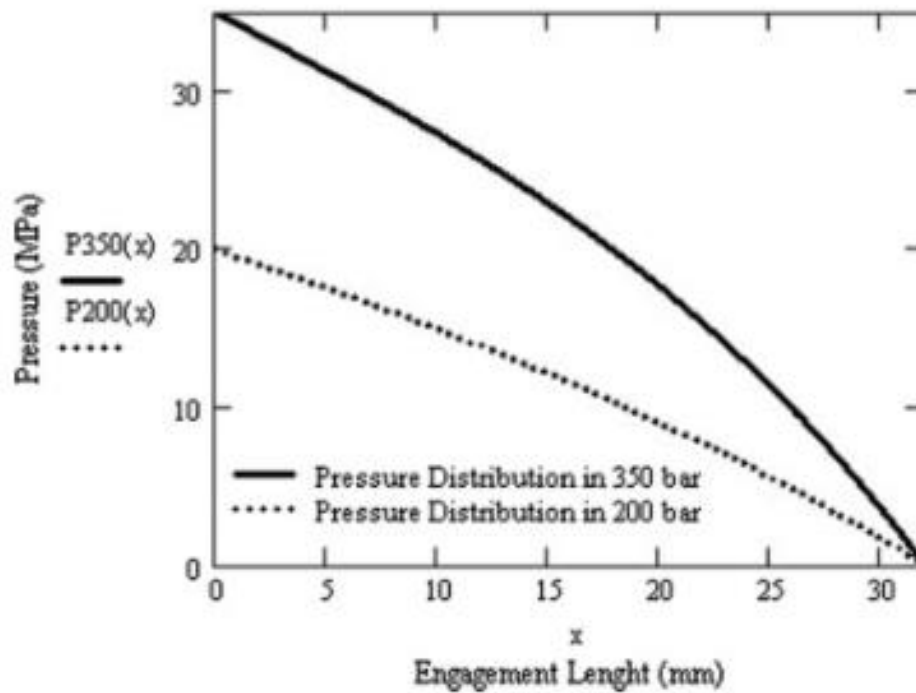


Figure 2-7 Pressure Distribution (Yagmur and Bagli, 2009)

It is for this reason that preliminary calculations using Dadson's simplified model must be verified using FEA/CFD if reliable information about distortion coefficient is to be obtained. With regards to the accuracy of results, the distortion coefficient is normally computed to within 3 % accuracy (Yagmur and Bağli, 2009), which corresponds to a standard uncertainty approximately of the order of 10^{-14} , while there is more uncertainty in the computation of fluid flow, which can vary from what the computation predicts.

Knowledge gaps in the literature are centred principally around non-rotational PCA's, which have yet to be investigated using FEA/CFD in the literature. A gap exists concerning the analytical formulae which are used to derive the distortion coefficient in non-rotational PCA's, and likewise in the method of processing FEA/CFD results for non-rotational PCA's, which have previously used distortions along the engagement length (which is not appropriate for nonrotational PCA's).

2.4 Experimental Reports

2.4.1 BNM LNE 200 MPa

Because FEA/CFD must be benchmarked to experimental results to be meaningful, two reports of experimental use of piston/cylinders are covered here which are used in the benchmarking. The BNM LNE 200 MPa pressure balance has been developed at the Laboratoire Nationale de Essais (LNE) and at the organisation Degranges et Huot from 1975 to 1985 (Molinar et al, 1998). The BNMLNE 200 MPa is used as a primary standard for pressure, and consists of 5 PCAs used in conjunction, the PCAs can operate in both controlled-clearance or free-deformation mode. They are made of Tungsten Carbide with 6 % Cobalt, and have nominal effective areas of 50.02 mm². The table below shows the nominal dimensions of the 5 assemblies, the PCA is also shown pictorially:

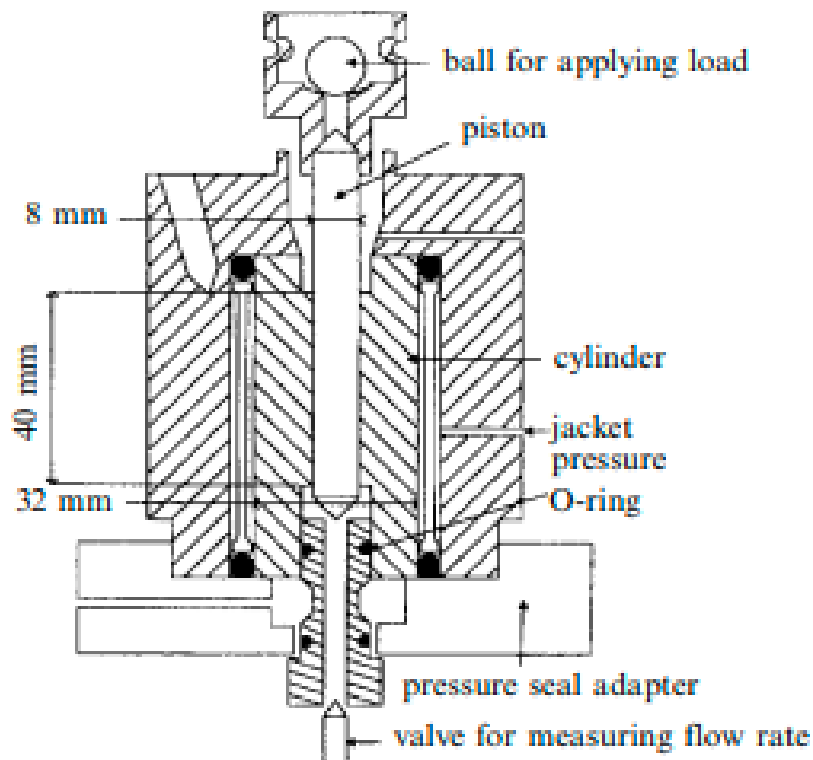


Figure 2-8 BNM LNE 200 MPa PCA (Molinar et al, 1998)

Table 2-4 PCA Characteristics

<u>Parameter</u>	<u>Value</u>
Piston Diameter (mm)	8.000266
Clearance (μm)	0.576
λ (10^{-12} Pa ⁻¹)	0.786

The PCAs are known to have relative uncertainties of 8 ppm at 120 MPa and 10 ppm at 200 MPa in the controlled clearance mode (Molinar, 1998).

The material properties of the PCAs given by the manufacturer are as shown in the table below:

Table 2-5 Tungsten Carbide Material Properties (Molinar et al, 1998)

Young's Modulus (E)	628 GPa
Young's Modulus Expanded Uncertainty	5 %
Poisson's Ratio (μ)	0.218
Poisson's Ratio Expanded Uncertainty	2 %

Diethyl-hexyl-sebacate (DHS) is the used as the oil, with pressure dependent density and dynamic viscosity as described by the equations:

$$\rho = 9126657 + 0.752097p - 1.64485e^{-3}p^2 + 1.45625e^{-6}p^3 \quad (2-37)$$

$$\eta = 0.0215541 + 1.90036e^{-3}p^{8.8101} \quad (2-38)$$

Measurements carried out on PCA units #4 and #5 showed that the undistorted clearance gap was $0.540 \mu\text{m} \pm 0.044 \mu\text{m}$ and $0.272 \mu\text{m} \pm 0.028 \mu\text{m}$ respectively. For each PCA, the mean piston radius (r_0) and the mean cylinder radius (R_0) were calculated from A_0 that was obtained from the cross-float experiments at a low pressure and h_0 . The mean radii and R_0 of the 200 MPa PCU N°4 were calculated to be $r_0 = (4.000\,143 \pm 0.000\,030)$ mm $R_0 = (4.000\,683 \pm 0.000\,030)$ mm. The mean radii r_0 and R_0 of the 200 MPa PCU N°5 were calculated to be $r_0 = (4.000\,116 \pm 0.000\,025)$ mm, $R_0 = (4.000\,388 \pm 0.000\,025)$ mm. The uncertainties mentioned for h_0 , r_0 and R_0 are expanded uncertainties.

Measurements of the piston and cylinder were carried out at the LNE, determining diameter and straightness. The diameter was determined in two orthogonal directions, and at three levels (PCA #4) and five levels (PCA #5); the straightness was measured by taking points at 0.1 mm steps, at four orthogonal locations around the PCA, the uncertainties in the measurement are shown below, the magnitudes are in agreement with the requirements identified for the cylinder and piston mating surfaces.

Table 2-6 Geometry of Piston/Cylinder

Piston/cylinder radius expanded uncertainty	40 nm
Gap conicity maximum error	2 nm/mm

The graph below shows the piston fall rate determined experimentally for the LNE BPM 200 MPa pressure balance at the LNE, showing fall rate increasing roughly linearly at pressures beyond 70 MPa, but with an exponential effect at lower pressures. The graph also shows the aforementioned disparities between the numerical and FEA results,

indicating that there is room for improvement concerning prediction of flow in FEA considering the discrepancy. The chart below shows flow in a PCA derived experimentally and numerically:

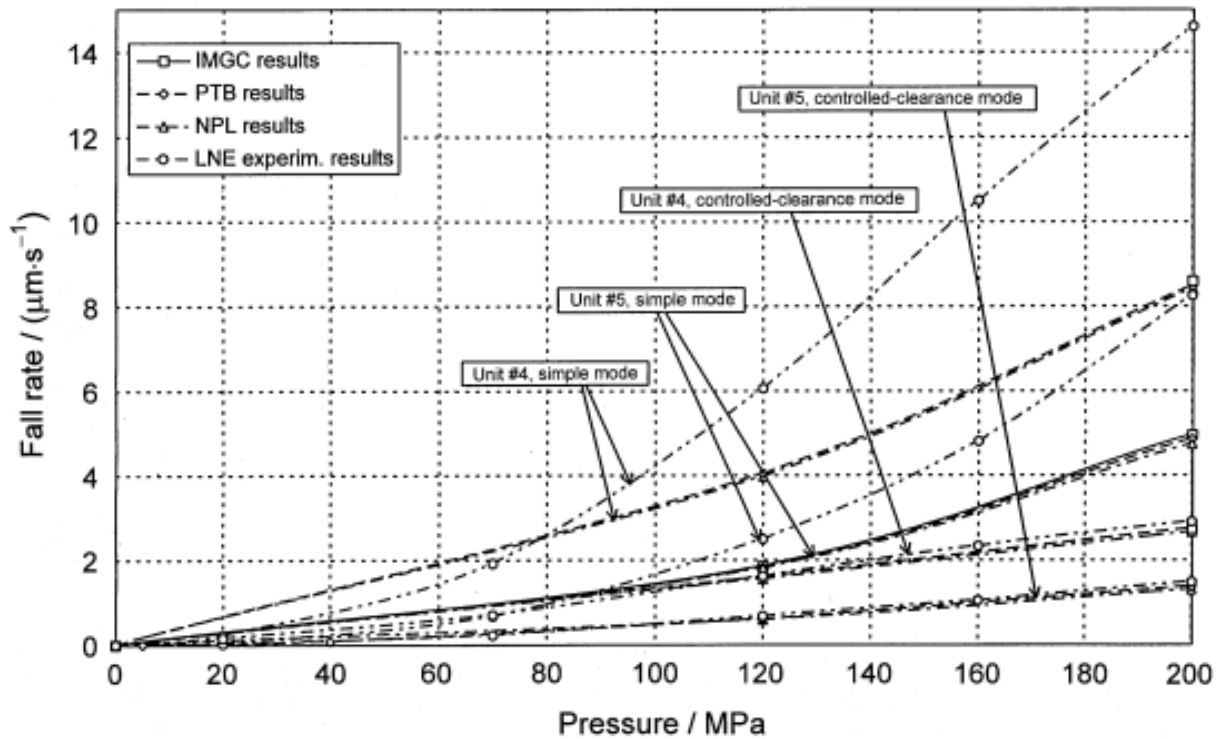


Figure 2-9 Fall Rates (Molinar et al, 1998)

2.4.2 PTB 400 MPa

The PTB 400 MPa pressure balance is used as a primary pressure balance by the PTB in Germany for pressures up to 400 MPa, although the PCA also operates up to 1 GPa (Molinar et al, 1998). It operates in free deformation mode, a diagram of the PCA and a table showing its material properties are given below:

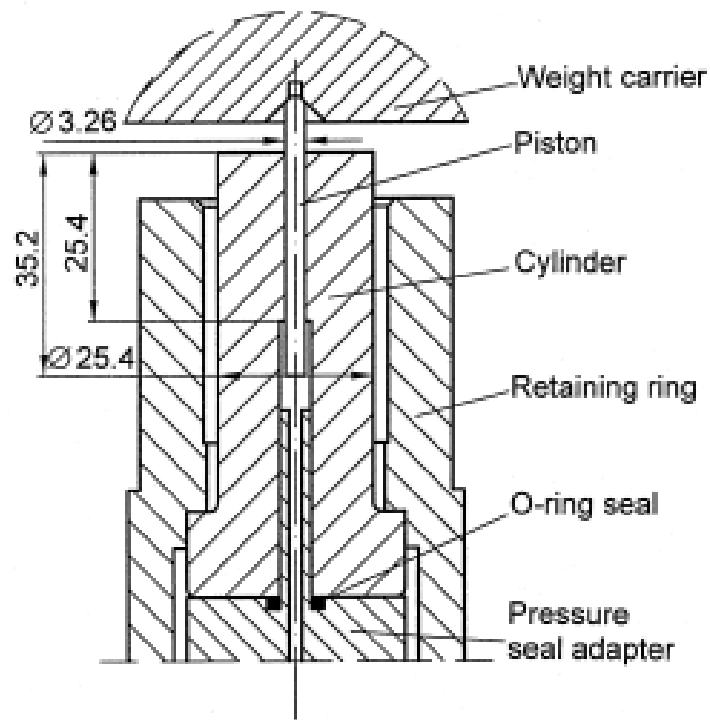


Figure 2-10 PTB 400 MPa PCA (Molinar et al, 1998)

Table 2-7 Properties of Piston/Cylinder Assembly

Young's modulus (GPa)	608.6
Young's modulus uncertainty (GPa)	5
Poisson's ratio	0.2126
Poisson's ratio uncertainty	0.002
Coefficient of thermal expansion ($^{\circ}\text{C}$)	4.326×10^{-6}
Thermal coefficient uncertainty ($^{\circ}\text{C}$)	0.1×10^{-6}

DHS is used again as the oil for this PCA, with pressure dependent viscosity and density determined according to the two formulae:

$$\rho = 912.67 + 0.752P - 1.645e^{-3}P^2 + 1.456e^{-6}P^3 \quad (2-39)$$

$$\eta = 0.021554 (1 + 1.90036e^{-3}P)^{8.81} \quad (2-40)$$

Additionally, the PCA unit has the characteristics shown in the table below:

Table 2-8 PCA Properties

Nominal effective area (mm ²)	8.39
Engagement length (mm)	25.4
Piston diameter (mm)	3.268426
Cylinder diameter (mm)	3.269
Nominal clearance (μm)	0.287
Distortion coefficient (/MPa)	0.725 x 10 ⁻⁶

The chart below of the NPL report MOM 120 shows the computed gap profile when the PCA unit is pressurised to 120 MPa, it can be seen how pressure drop is constant over most of the engagement length, and decreases towards the PCA top.

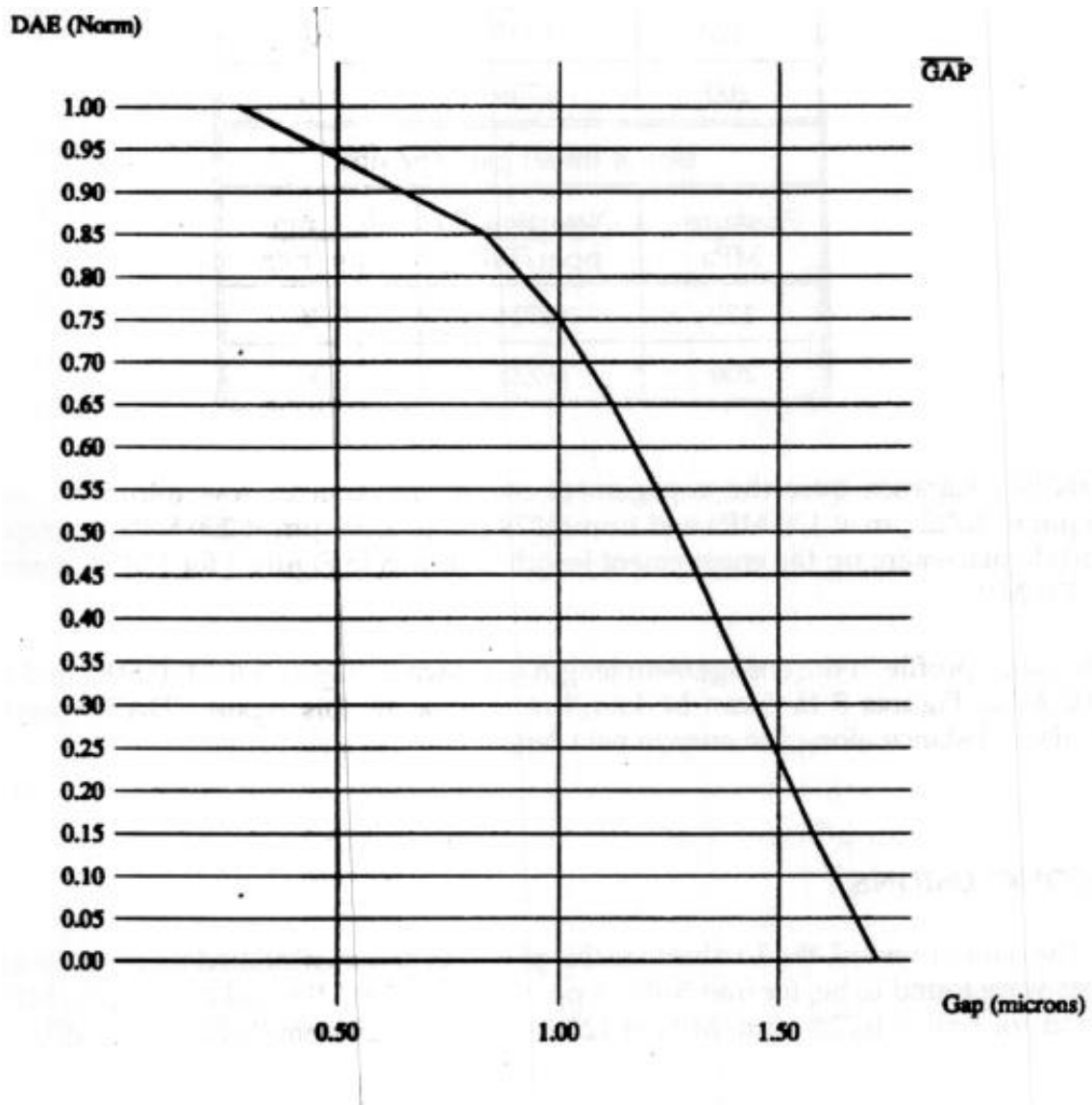


Figure 2-11 Gap Profile (NPL Report MOM 120)

2.5 Rotational and Non-rotational Designs

The traditional piston-gauge has always employed a rotational design, the purpose of rotation being to lubricate the surfaces between the piston and cylinder, such that there is no friction (Dadson, 1965). Rotation is achieved either manually or mechanically using equipment which imparts rotational motion with very little upward or downward force. The rotation does not cause changes to the vertical flow of fluid or the piston's distortion.

Designs of piston/cylinder which are non-rotational in nature have also been created, these designs aim to keep the piston central within the cylinder without the use of rotation (Peters, nd). Principally these designs use pressurised inlets of fluid at the sides of the pistons, which maintain centrality because of alterations to the pressurisation with piston eccentricity, drawing principally on the workings of bearing theory employed in hydraulics.

2.5.1 Bearing Theory

Here an overview is given of the theory related to hydrostatic bearings as it relates to the current work. The information in this section is taken solely from (Rowe, 2012).

The basic operation of a bearing is that a restrictor controls pressure in the bearing recess such that it increases when the gap narrows, and decreases when it widens, therefore creating a centralising effect of the journal (or piston) in the bearing. . In a typical assembly, the bearing recess pressure will be half of the line pressure when the piston is centred and will increase/decrease with piston eccentricity. The design is carried out by equalising the flow which enters the bearing via the restrictor with the flow able to leave the bearing. The figures below show flow through the bearing for some different types of restrictors:

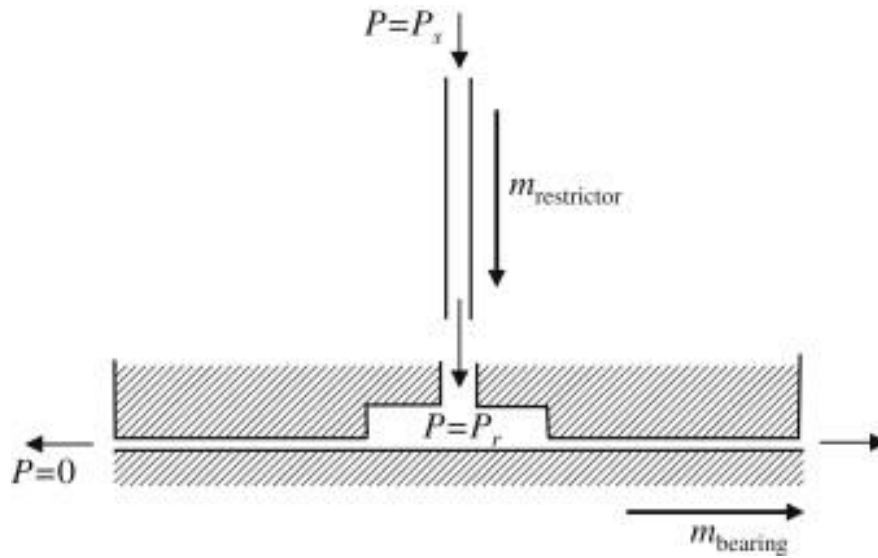


Figure 2-12 Bearing (Rowe, 2012)

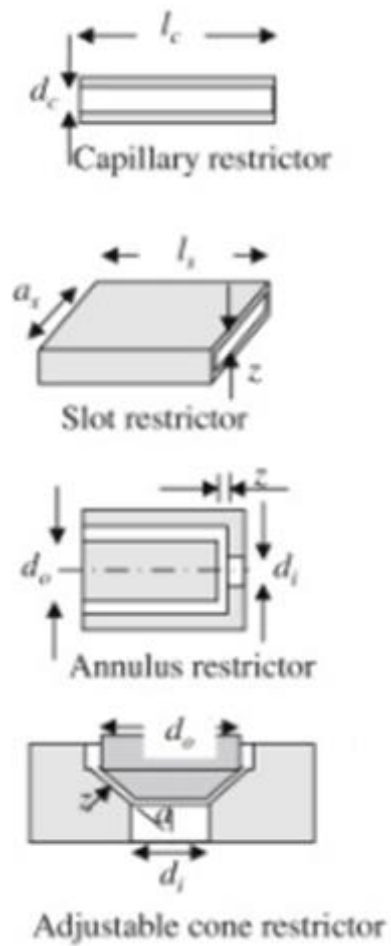


Figure 2-13 Restrictors (Rowe, 2012)

It is noteworthy that the load that can be supported by the bearing is given by the formula:

$$W = P_r A \times \bar{A} \quad (2-41)$$

Where W is the load, A is the bearing area and $\bar{A}$ is the bearing shape factor. β , representing the ratio of P_s to P_r , is normally defined at 0.5 when the bearing is in a central position. The pressure is defined by the flow being equalised in the capillary and escaping the bearing, the maximum flow being defined according to the formula below:

$$q_0 = \frac{P_r B h_0^3}{\eta} \quad (2-42)$$

It can be seen how, by using the simplified components of bearing theory, both the pressure and the flow at various eccentricities can be calculated for a simple bearing. Other configurations, for example with two levels of bearing at either end of the piston, are achievable, it is assumed that as bearing theory can be extrapolated to simulate for these cases, likewise the PCA theory developed here can be extrapolated in the same way.

Numerous configurations of bearings are possible, including bearings at several intervals around the piston, as well as at multiple heights, bearings can also be recessed or plain. It is possible to carry out the design for bearings used to hold the piston in place, specifying attributes such as bearing stiffness. While this is beyond the scope of the present study, the resulting pressure distribution is utilised in the creation of formula for nonrotational λ .

2.5.2 Experimental Bearing

Figure 2-15 shows a bearing that was researched by Metman et al (1986). Metman et al measured the load force of the bearing at various eccentricities, with known journal

pressures. The figure shows next shows a cross section of the eccentric bearing.

Metman et al also contrasted analytical results with those obtained experimentally and found good concurrence between the parameters (as shown in the figure 2-17). The figures below show the configuration of the bearing:

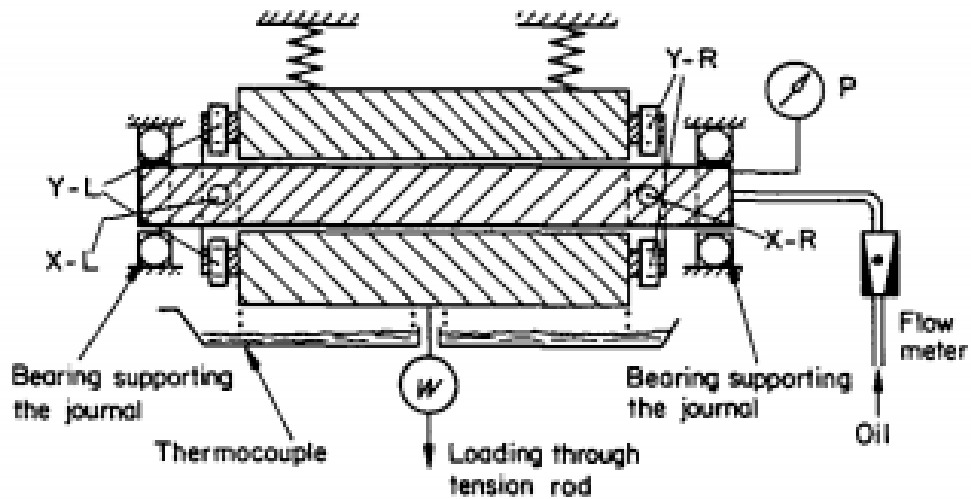
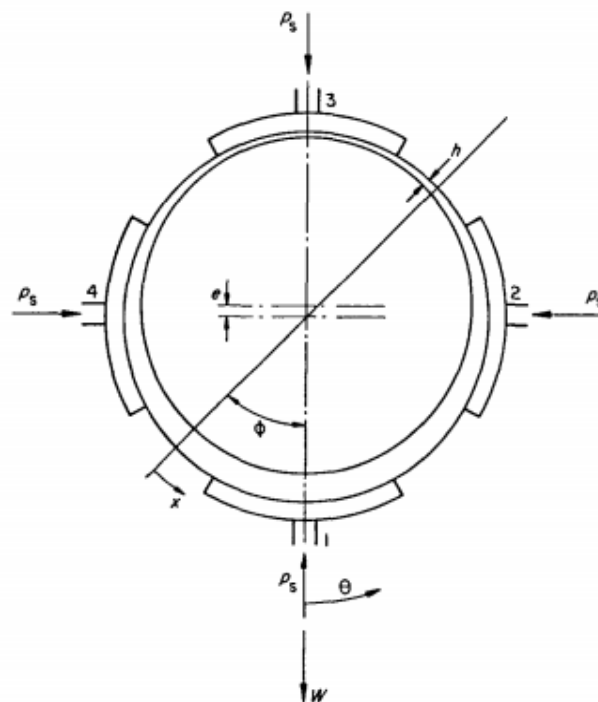
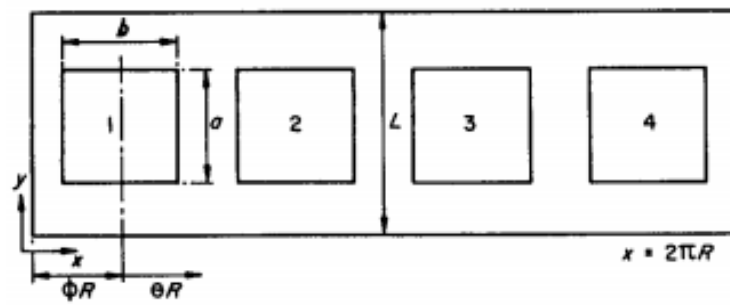
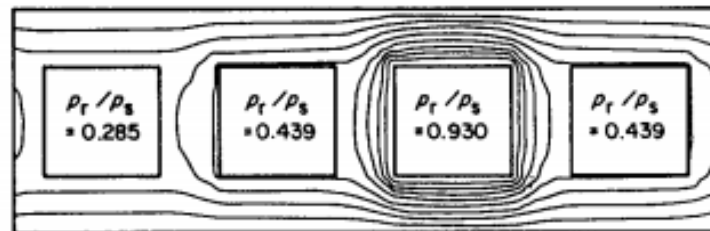
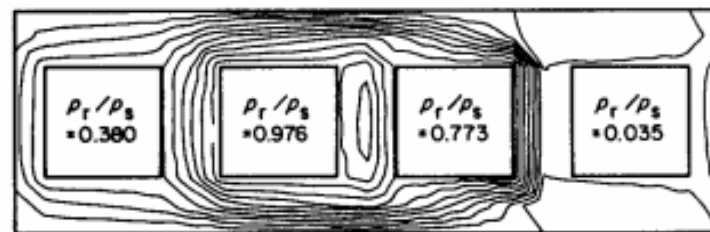


Figure 2-14 Metman Bearing (Metman et al, 1986)

The length to diameter ratio is 1.0, and the bearing has dimensionless axial width of 0.5 and dimensionless circumferential recess length of 0.67. The components are made of steel, and have surface roughness of $0.02\text{ }\mu\text{m}$ for the journal and $0.03\text{ }\mu\text{m}$ for the bearing. The diameter of the journal is accurate to within $\pm 1\text{ }\mu\text{m}$ at 60.114 mm and $\pm 2\text{ }\mu\text{m}$ for the bearing diameter of 60.180 mm , with a nominal gap of $33\text{ }\mu\text{m}$.



a $\epsilon = 0.7$, $\phi = 45^\circ$: $W_c = 1.252$, $Q = 13.91$



b $\epsilon = 0.4$, $u = 1 \text{ m/s}$: $W_v = 0.804$, $W_h = 2.420$, $Q = 13.38$ and altitude angle $= 72^\circ$

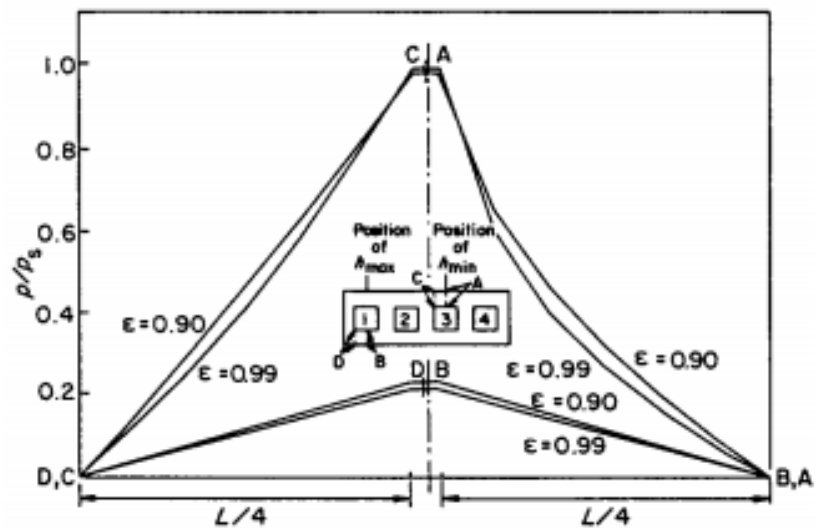


Figure 2-15 Pressure Distributions (Metman et al, 1986)

The figures above from Metman et al (1986) show the pressure distribution with countours in an eccentric bearing, and the axial pressure distribution in the fluid. It can be seen how the pressure is constant in the recess, and trails off almost linearly in all cases as the fluid exits through the ends of the bearing. The graphs below show the recess pressures in the four pockets as eccentricity of the journal is varied, as well as the agreement of Metman et al's analytical results. It can be seen how the recess pressure varies almost linearly, with most variation in the recess being approached indicating an almost linearly increasing centralising effect, the experimental results also agree very well with the relevant analytical models, which is important in the design of nonrotatinal HFA's as the pressure distribution is predictable.

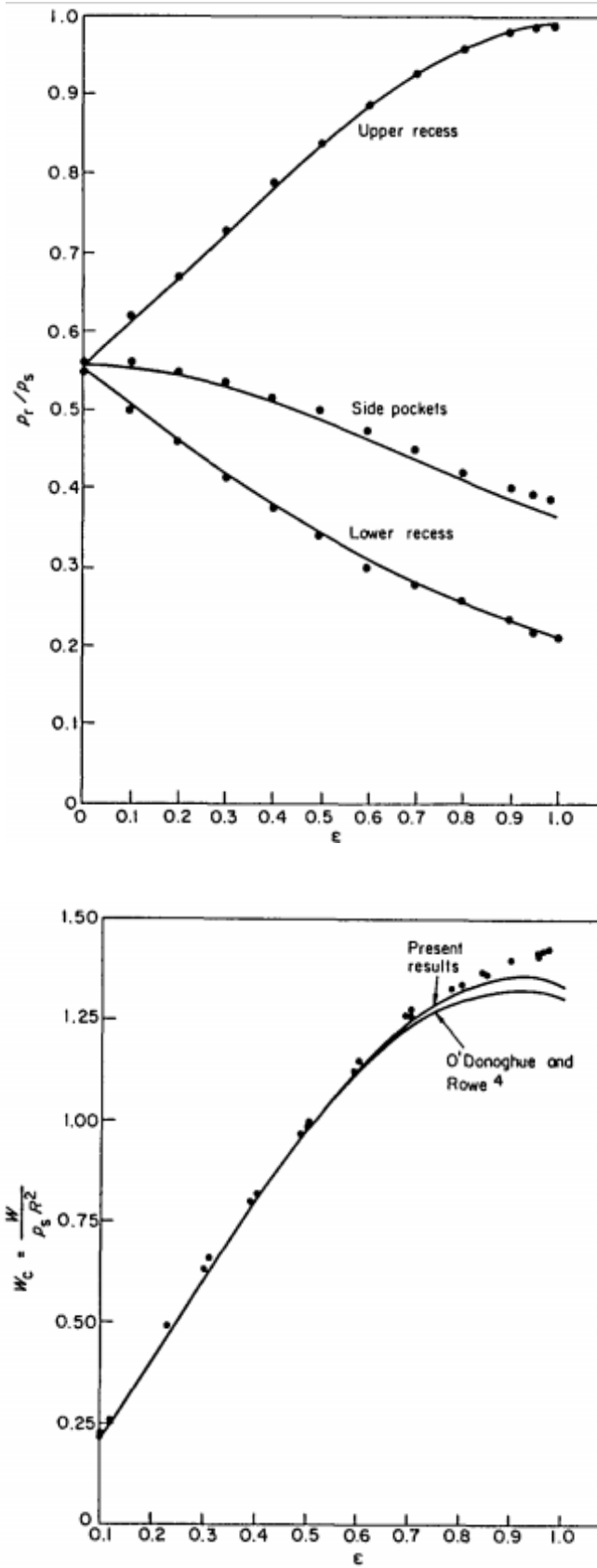


Figure 2-16 Pressures Within Bearing (Metman et al, 1986)

2.6 Tolerances and the relation to the Uncertainty Framework

Machining tolerances such as straightness, roundness and cylindricity are commonly used to specify dimensions on parts for manufacture (such as piston/cylinders), thereby defining the envelope within which the finished part should be created (Walter, 2019). After manufacture, in the measurement of the component, measurements are taken which determine whether the part conforms with this requirement and is of an acceptable standard, or whether it is rejected. In a more practical sense, tolerances can represent the envelope within which the user of the equipment knows that the true form of the piston/cylinder will operate, and therefore the upper and lower boundaries within which the true form sits. The figure below shows the cylindricity tolerance:

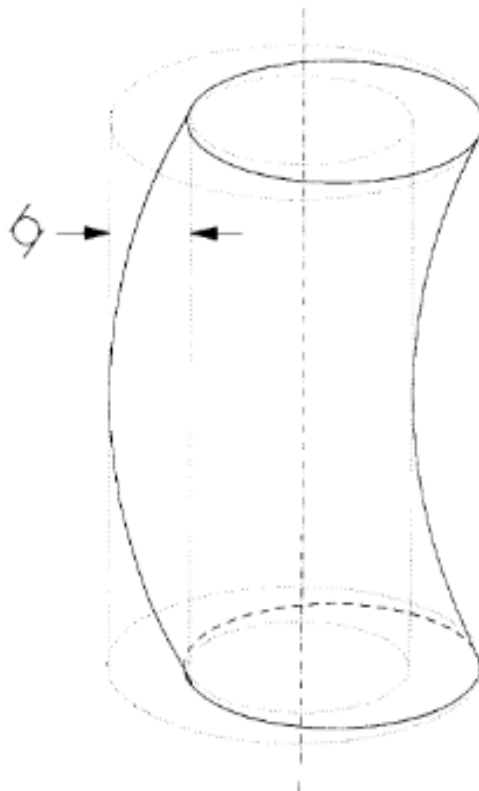


Figure 2-17 Cylindricity (Dawson, 1992)

It may be the case that the metrology capability outperforms the manufacturing capability, and in such a case piston/cylinders are used with a known error in roundness or straightness. The problem is that this creates excessive complications in the use of the piston/cylinders, such that the uncertainty reverts to being the original tolerances of the components. Therefore, it can be said that the tolerance is limited likewise by metrology capability as by manufacturing accuracy.

2.6.1 Manufacturing Challenges

PCAs for hydraulic force amplification machines are of diameters ranging from 10 mm to 20 mm for the small side and 100 mm to 400 mm for the large side, the length is normally double the diameter although greater lengths are sometimes used. The dimensions specified imply required machinable areas of 2 000 mm² and 250 000 mm² for the small and large side PCAs respectively. The material used can be Steel or Tungsten Carbide depending upon the pressures in the hydraulic system, which in turn depend upon the amplification ratio required of the machine. Steel PCAs can be used at forces of up to 20 MN; but at forces greater than these Tungsten Carbide must be used as the PCA material. The different processes are compared graphically in Figure 2-18, showing best results from the literature for areas machined in experimental reports against form accuracies achieved.

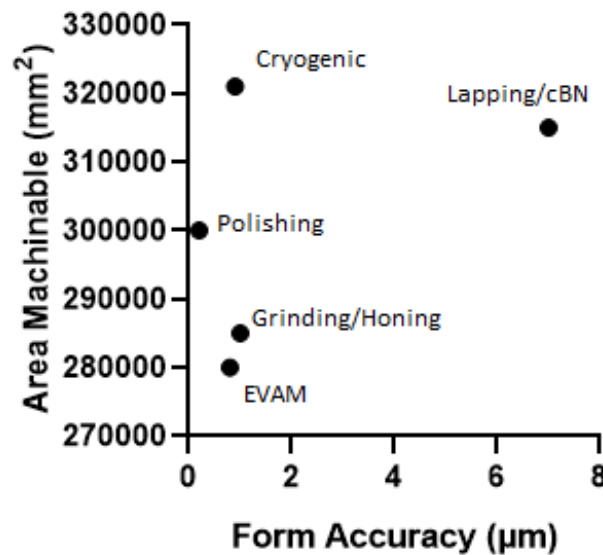


Figure 2-18 Machining Capabilities (Chou and Evans, 1997; Moriwaki and Shamoto, 1991; Brinksmeier et al., 2010; Brinksmeier et al., 2004; Bell et al. (1981))

It can be seen that all processes can potentially be used for the manufacture of steel PCAs for hydraulic force amplification machines, Tungsten Carbide is only machinable by conventional processes (grinding, lapping, polishing and honing). It can be seen, however, that polishing offers nanometric accuracies which may be called for in ‘primary’ HFA’s, as they are in the field of pressure.

2.7 Dimensional Characterisation

With respect to measuring instruments for PCA’s, Miles et al (2005) discuss one such machine which is capable of measuring internal or external radii of PCA’s, up to a limit of 20 mm. The main working principle of the measurement machine is to use two opposing probes, the locations of which are tracked using a nanometrically accurate interferometer. The schematic of the machine is shown below:

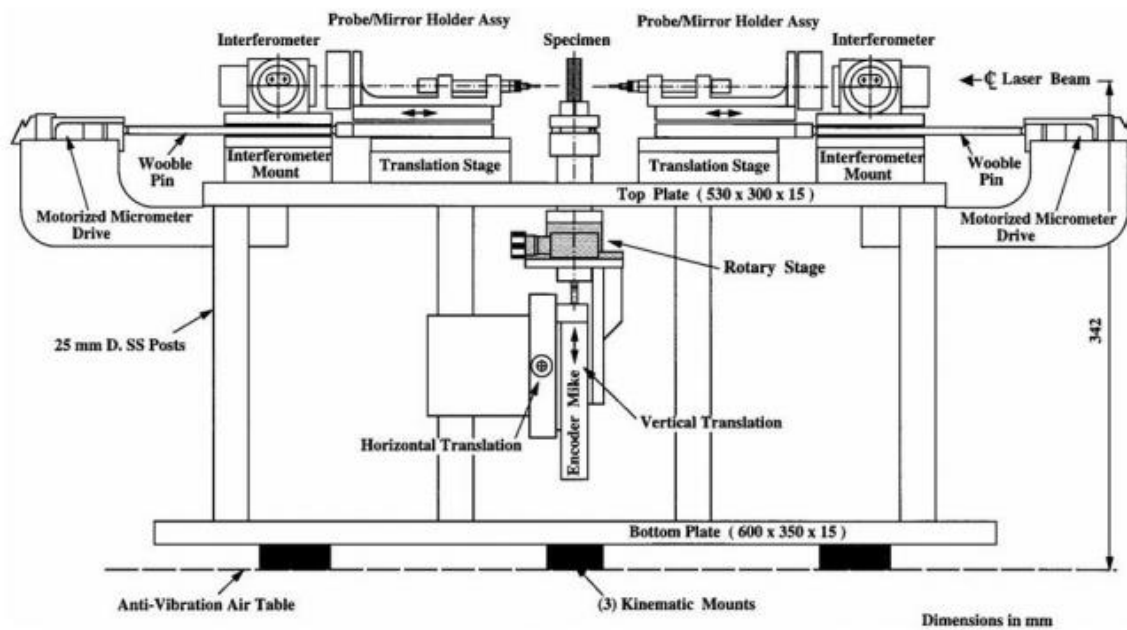


Figure 2-19 Cylinder Measuring Machine (Miles et al, 2005)

The machine is calibrated by touching the two opposite probes together to obtain a zero distance reading, and temperature is effectively critical on the machine (for distortion of the part as well as the measuring equipment) and is controlled to 0.1 °K. Stoup and Doiron (2021) describe a machine with radius capacity enlarged to 55 mm, which is shown graphically below:

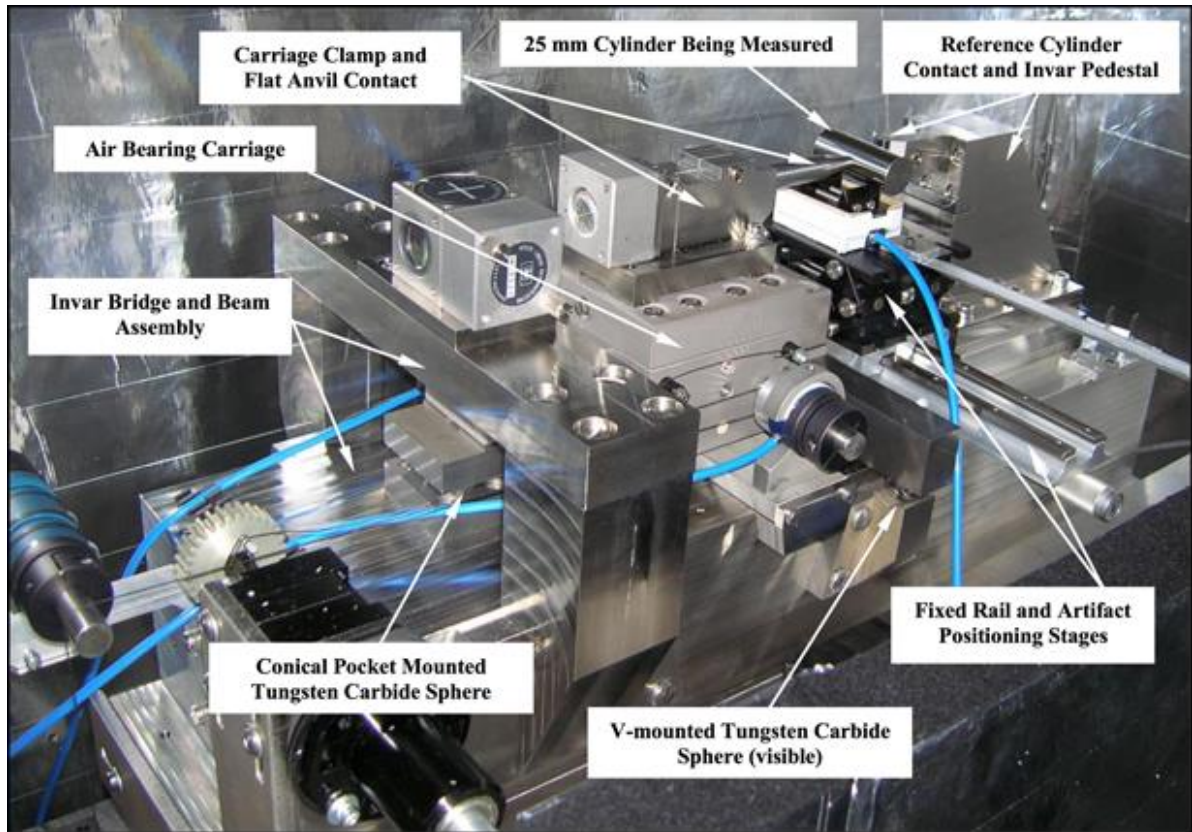


Figure 2-20 Cylinder Measurement Machine (Stoup and Doiron, 2021)

With regards to accuracies, both machines carry out form measurement with uncertainties in the range of 20 nm, as likewise do the machines of Hesse et al (2021), indicating that these accuracies are readily achieved in the technology. Stoup and Doiron (2021) describe using low thermal expansion materials in the machine to avoid thermal effects also

3 METHODOLOGY

3.1 Overview

The overall gap identified in the literature was whether a ‘primary’ design of HFA could be used to reduce uncertainties to the level of less than 10 ppm, in the HFA in the pressure

range of 2 MN – 20 MN. Several smaller gaps were identified in the literature review, including:

- It is unknown whether piston eccentricity affects distortion coefficient
- There is no methodology for evaluating non-rotational PCA's in FEA/CFD, although there is a lot of FEA/CFD work evaluating bearings
- There is no analytical formula for distortion coefficient of nonrotational PCA's
- It is unknown whether analytical formulae for distortion coefficient are reliable for design of HFA's

It is therefore necessary to provide answers to the above questions in this work. Based upon the preceding literature review, the aim and objectives can be stated as:

To carry out the design of a directly calibrated HFA system, by:

1. Evaluating the relevant uncertainty sources
2. Carrying out the design for a rotational HFA, validated by the use of FEA/CFD
3. Carrying out the design for a nonrotational HFA, validated by the use of FEA/CFD
4. Synthesising the above calculations in an easy to use software, and producing some nominal configurations of HFA

It is hoped that by carrying out the design for primary HFA's, that the obstacles to the design being used will be identified and overcome- thereby enabling use of this new technology within the constraints of an analytical investigation.

It is noteworthy that some relevant areas to the research are omitted from this work, including calculation of the centralising forces caused by bearing pressure, and the area

concerning different configurations of bearings. It is anticipated that the methodology used in this work could be extended to apply to PCA's with different bearing configuration, while the area for further research concerning the centralising force in nonrotational PCA's is highlighted as being particularly relevant.

Additionally, the effects of varying oil viscosity are to a large extent omitted from the present work, where it is assumed that the common oil, di-ethylhexyl sebecate, is used, while in a more comprehensive design different formulae for flow of varying oils would be considered. The effects of pressure dependent viscosity changes in the FEA/CFD are of course included, however, in the analysis. The Navier-Stokes equations are used throughout in the FEA/CFD.

Another consideration which is largely omitted is that of thermal effects. The formula which was created for sensitivities of both rotational and nonrotational PCA's was investigated to test for thermal effects, but it is explainable that because of the large mass of metal compared to oil in PCA's, the configuration acts like a heat-sink, and the thermal effects are negligible (this is in keeping with the approach previously adopted in the literature).

3.2 First Objective

To achieve the aim and objectives, it is necessary to first validate that the primary design of HFA is feasible in terms of uncertainties present in the system. This has been carried out as part of the literature review, where the uncertainties that would theoretically be present in 'primary' HFAs have been calculated and compared to the requirements for the improved system. It was found that the uncertainties present in the directly calibrated machine are small enough to enable the delivery of greatly reduced measurement

uncertainties as required in the primary system. The first objective, of identifying and evaluating different uncertainty sources for a directly calibrated HFA has therefore been achieved. The need for carrying out this work is to prove the impact that can be delivered by successfully using a primary system, ie showing that it would deliver the desired reduction in uncertainties. Being able to predict the reduction in uncertainties achievable guards against the redesign being non-significant. The chart below shows the work carried out:

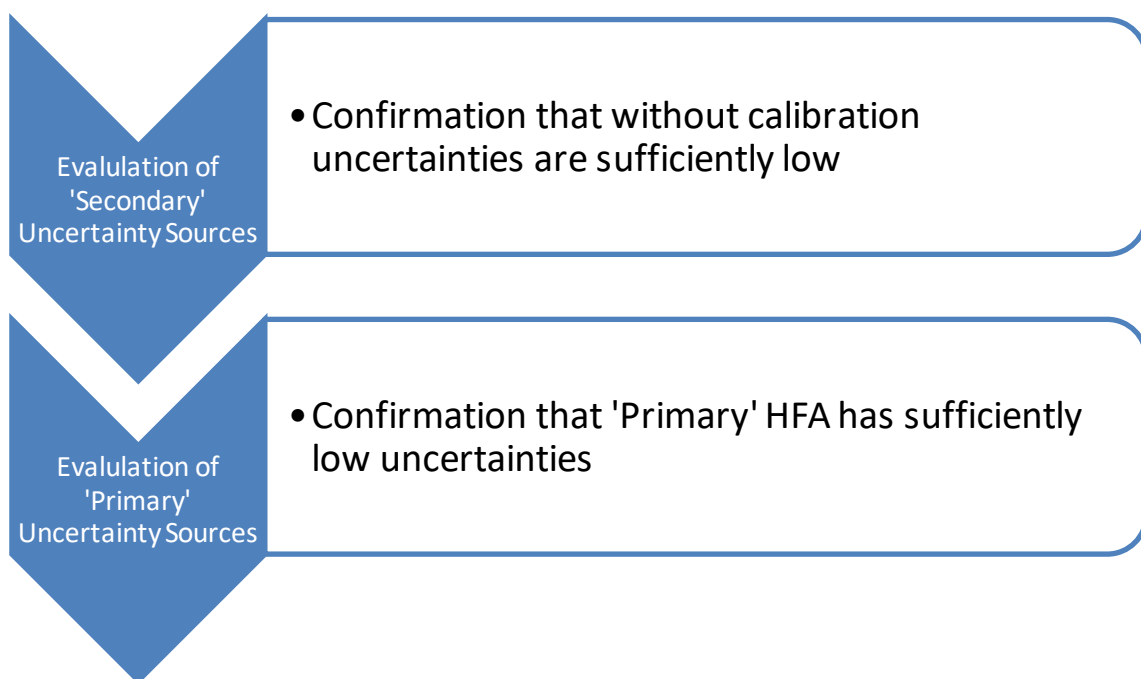


Figure 3-1 Objective 1

3.3 Second Objective

The second objective identified concerns the feasibility of the primary HFA system. It is not enough to claim that a primary HFA would deliver improved uncertainties, it is necessary to carry out the design of such a system thereby demonstrating that the concept is feasible and practical within the constraints of a theoretical examination.

To carry out the design, the formulae used in the design (for flow and tilt) are first updated to include all uncertainty sources- thereby enabling the design to be carried out with confidence in the underlying formulae. Next, the FEA capability is established by benchmarking FEA to the literature, enabling the FEA tool to be used with confidence to validate the resulting design. The purpose of using FEA/CFD is that it enables validation of the analytical formulae used, particularly for distortion coefficient which is known to become too complex with smaller gaps. At this stage, some preliminary questions concerning uncertainty sources, namely the effects of eccentricity and rotation, are answered using the FEA tool which enables further analysis, the preliminary doubts about comprehensiveness of the uncertainty sources considered being answered.

The design for the rotational HFA is carried out at this stage. The stage has several substeps which are carried out sequentially, corresponding to the order in which the design must be carried out to ensure that the requirements are met, at each step allowing finer details of the design to be defined:

- Amplification ratio
- Materials
- Nominal radii
- Gap sizes

Finally, a sensitivity analysis is carried out which enables the derived tolerances to be benchmarked against those calculated analytically, thereby giving evidence that the simplified analytical equations are accurately predicting realistic performance. It is assumed that the maximum deviation in effective area occurs where the maximal possible deviation occurs (ie at the limits of the tolerances), therefore by using the new nominal

values the resulting difference from the tolerance can be calculated. In other words, whether a normal or rectangular uncertainty distribution is expected, the maximum error in A_{eff} occurs where the maximum error of the input parameter occurs. While the errors are representative of form error, they can be taken to represent errors arising from roughness of the corresponding amount also. The chart below shows the design process:



Figure 3-2 Design Process

It is also desirable to carry out the design for both rotational and non-rotational PCAs, as these are both currently of use in HFA machines.

3.4 Third Objective

As it has been identified that the design of both rotational and non-rotational HFA systems is desirable, a gap in the research emerges concerning the calculation of distortion coefficient for rotational PCA's. In a primary HFA, the distortion coefficient (λ) must be determined for both sides with known uncertainty which is controlled, such that the amplification ratio is known and the force produced is controlled with low relative uncertainty.

Validation of the distortion coefficient is required for both the rotational and nonrotational designs, but the nonrotational design suffers from a more immediate limitation in that there is as yet no formula which predicts distortion coefficient (λ) for a nonrotational

PCA, which is of course required to determine the amplification ratio and create the design for the nonrotational primary HFA. Therefore, basic algebraic formulae (Lame equations) are leveraged to derive a formula from first principles. The formula must be validated also, which is done in the FEA/CFD.

The gap was also identified that there is yet no method of using FEA/CFD for a nonrotational PCA, prior FEA/CFD use having been for axisymmetric cases only. Therefore, a novel methodology for derivation of the effective area is used. In this direct evaluation of FEA/CFD, the normally used method of deriving the distortion coefficient using analytical formulae and distortions along the faces of the piston/cylinder is foregone, and the reaction force is measured directly from the top of the piston (which can be converted to effective area, to derive distortion coefficient). This is done because the presence of bearing jets precludes the normal method of measuring deviations along the engagement length. This methodology is validated first for benchmarked rotational PCA's, before it is used in the model for the nonrotational design, to ensure that validity is maintained in the derivation of distortion coefficient for the bearing configuration. The chart below shows the process for the non-rotational section:

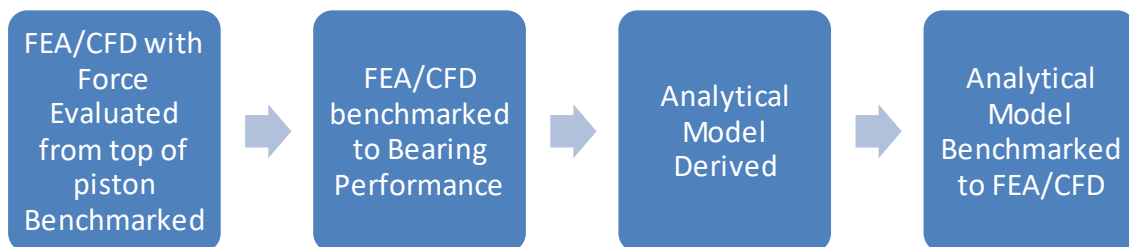


Figure 3-3 Process Chart for Nonrotational Section

Additionally, the nonrotational design is benchmarked to experimental results of the bearing of Metman discussed in the literature review, alongside benchmarking to the LNE 200 MPa and the PTB 400 MPa PCAs, thereby ensuring that the model is representative of real world bearing performance, and that any effect that the bearings have upon distortion coefficient corresponds to experimental reports. Having created the FEA/CFD capability for dealing with nonrotational PCAs, the models are used to benchmark the formula which was derived for λ in nonrotational PCAs, thereby creating the design capability needed for prediction of the amplification ratio effects.

Having established the prerequisites for design of the rotational and nonrotational primary HFAs, the design is carried out for each, sequentially as it was for the rotational design, which includes predicting the uncertainties which arise as a result of input parameters for the measurement model, which ultimately leads to an uncertainty prediction in the measured force.

A sensitivity analysis is again carried out in FEA/CFD, where each input parameter is varied by the amount of the uncertainty, and it is checked that the resulting uncertainty in measured force is in agreement with what is predicted analytically. This way confidence is gained that the analytical formulae are accurately predicting uncertainties present in the system, and can be used with confidence for carrying out the design. The chart below shows the overall process:

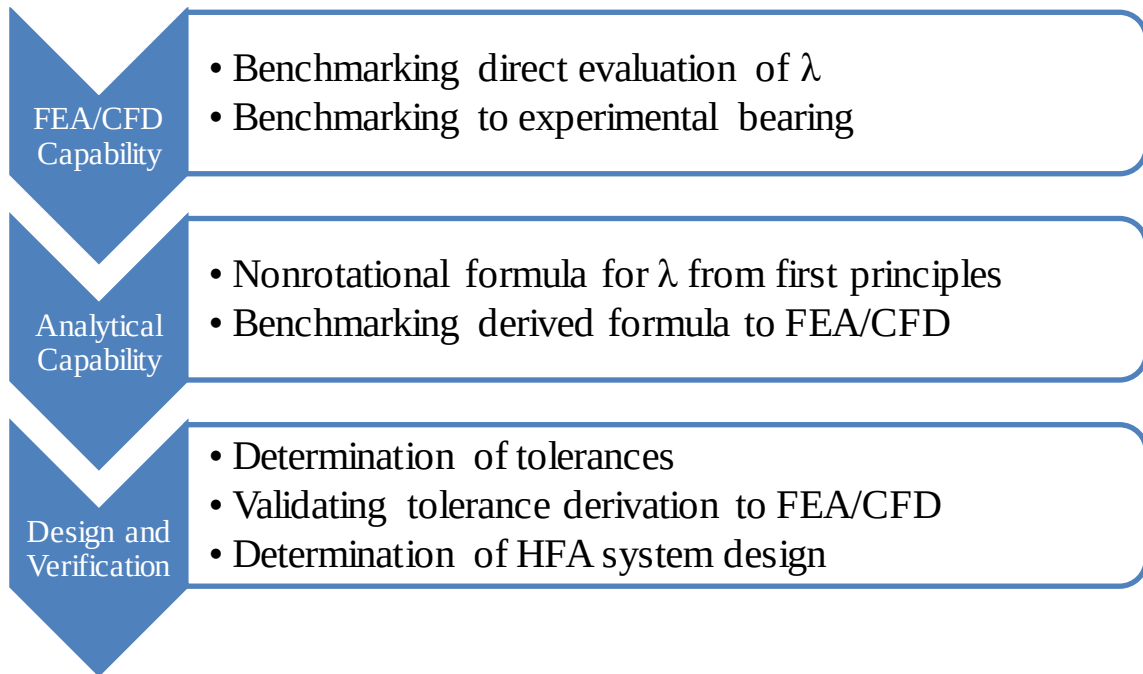


Figure 3-4 Overall Process Chart

3.5 Fourth Objective

In the final chapter, the tried and tested analytical formulae are consolidated into an excel spreadsheet, which can be used for carrying out the design of primary HFAs, for ease of use. A typical design is carried out using the derived spreadsheet considering realistic parameters, thereby making some use of the derived tool and providing evidence that a realistic primary HFA could be designed which would deliver the aforementioned reductions in uncertainty. Additionally, some design principles for HFA's are evaluated.

4 ROTATIONAL PCA

The ultimate goal of this section is to carry out the design of a rotational HFA demonstrating the use of 'primary' calibration to reduce uncertainties. To do so, several precursory steps are required which enable further analysis to take place and achievement

of the goal. First of all, some discrepancies present in the analytical formulae (namely for piston tilt and fluid flow) are reviewed, which provide capability to implement the design. Next, FEA/CFD is benchmarked to experimental reports to ensure it is representative of PCA functionality, enabling use in the design verification. Finally, the design is carried out, including establishment of tolerances and their verification, thereby demonstrating that a ‘primary’ HFA is both feasible and has potential to reduce uncertainties in force metrology.

4.1 Adapted Analytical Formulae

4.1.1 Adapted tilt formula

In evaluating the uncertainty in HFA systems using PCAs, one major source of uncertainty that must be considered is the piston tilt and the effect on effective area, and when designing a directly calibrated HFA it is necessary to consider the uncertainty caused by tilt of the piston. It was noticed however that the conventional formula for tilt of the piston does not consider PCA distortions, and therefore will underestimate the piston’s tilt. The effects of tilt are effectively negligible, but to provide the utmost accuracy in the design it was decided to reformulate the tilt formula considering PCA distortion.

When the piston tilts within the cylinder in the space allowed by the clearance gap, the piston area exposed vertically to the line pressure alters according to the formula of Dadson (1985):

$$P = \frac{F}{A(1 + \alpha\Delta T)} \cos\theta \quad (4-1)$$

Where P is line pressure, F is applied force, A is piston area, α is coefficient of thermal expansion, ΔT is temperature change, θ is piston tilt angle

Lewis and Peggs (1977) give an alternative formulation for the effect of tilt:

$$\frac{\Delta A}{A} = \frac{6e^2rh}{l^2} \left[1 - \frac{2r}{l} \tanh\left(\frac{l}{2r}\right) \right] \quad (4-2)$$

Because of the major drawback of the formula, that it doesn't consider the effects of piston/cylinder distortion when calculating the area in which the piston can tilt, therefore the maximum possible tilt is underestimated by a factor of up to 2 or more. The geometry involved is shown pictorially below:

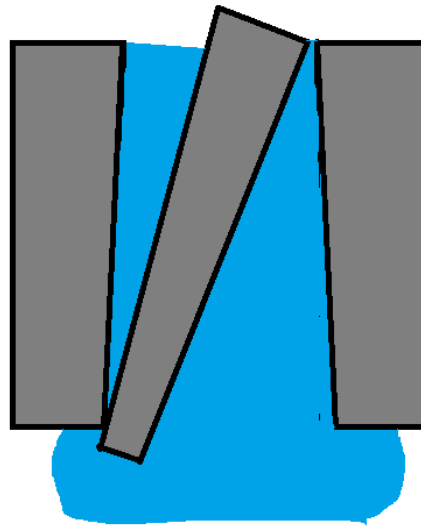


Figure 4-1 Additional Contributors to Piston Tilt

As can be seen in the figure, the tilt angle is not only limited by the size of the clearance gap, but also by the amount distance of distortion of the piston and cylinder as these have been seen in this analysis to be comparable to the nominal clearance.

Considering the distortion of the piston and cylinder, it can be seen that the maximum tilt angle is given by:

$$\theta = \tan^{-1} (2h+U+u)/L \quad (4-3)$$

Where h is the nominal gap, U is the cylinder distortion at the base, u is the piston distortion at the base and L is the engagement length

To determine the formula for tilt, the Lamé equations which analytically predict distortion of solid and hollow thick cylinders are used to analytically predict u and U. The Lamé equations for thick cylinders are commonly used for calculation of PCA distortion, so it was determined that it would be appropriate to use them likewise for considering tilt. The formulae were validated in the FEA, they were also benchmarked against values of u and U from previous FEA results and experimental results in the literature. It is noteworthy that the pressure distribution in the gap is nonlinear, however the pressure at the base of the PCA is known to be equal to the line pressure, this being the only significant parameter enables adequate analytical calculations to be performed. When confidence had been achieved the formulae for u and U, they were substituted into the formula above for tilt.

The formulae for distortion are shown below, these being the formulae customarily used to predict distortions in PCA's:

$$u = \frac{-Pr}{E} + \frac{P\sigma r}{E} + \frac{P_L\sigma r}{E} \quad (4-4)$$

and

$$U = \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) \quad (4-5)$$

Where P is the pressure, σ is Poissons ratio, r is piston radius, E is young's modulus, R' is cylinder outer radius, R is cylinder inner radius, u is piston distortion, U is cylinder distortion

As described in the methodology section, the traditional formula of Dadson (1985) was adapted to consider the effects of PCA distortion. The traditional formula of Dadson (1985) is:

$$P = \frac{F}{A(1 + \alpha\Delta T)} \cos\theta \quad (4-6)$$

Where P is line pressure, F is applied force, A is piston area, α is coefficient of thermal expansion, ΔT is temperature change, θ is piston tilt angle

As discussed previously, the maximum tilt is given by:

$$\theta = \tan^{-1} ((2h+U+u)/L) \quad (4-7)$$

Where h is the nominal gap, U is the cylinder distortion at the base, u is the piston distortion at the base and L is the engagement length

The Lamé equations can be used to substitute for U and u, as follows:

$$U(P) = \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) \quad (4-8)$$

$$u(P) = \frac{-Pr}{E} + \frac{P\sigma r}{E} + \frac{P_L\sigma r}{E} \quad (4-9)$$

$$\theta = \arctan \left(\left(2h + \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) + \frac{-Pr}{E} + \frac{P\sigma r}{E} + \frac{P_L \sigma r}{E} \right) / L \right) \quad (4-10)$$

The uncertainty contribution of tilt is relatively low (typically less than 10 ppm) in PCA's because of the high ratio of the engagement length to the gap/distortions, meaning that the above calculations may not be critical, when compared with other uncertainty sources as shown in section 2.2.11. It is evident however, especially at higher pressures, that distortions are of the order of magnitude of the gap, therefore it is reasonable to expect a doubling or more of the tilt angle, leading to a similar effect in uncertainties.

4.1.2 Flow formula

When carrying out the design of an HFA system, one requirement is to compare flows of the large and small sides across the pressure range. This is done to ensure that the PCAs remain buoyant for long enough when loaded for useful measurements to be taken.

Because it is necessary to predict flow-related uncertainties using analytical formulae, a flow formula is derived which takes account of pressure dependent effects on viscosity of the lubricating fluid, and distortions in the piston/cylinder under pressurisation.

Necessary for the analytical derivation is the formula for viscosity, di-ethylhexyl sebecate was used as the oil with a formula for pressure dependent viscosity given by Dogra et al (2010) below. This oil was used because it was used with the experimental PCA's considered in his work.

$$\eta(P) = 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81} \quad (4-11)$$

The analytical derivation is based on the formula for flow in the piston/cylinder gap given by Dadson (1985):

$$Q = \frac{\pi r \delta^3 P}{6\eta L} \quad (4-12)$$

where Q is volumetric flow, d is piston diameter, δ is the gap and L is engagement length, P is pressure

To allow for analysis without considering the entire engagement length (ie at a single height), thereby allowing distortion to be considered which varies along the length, the formula is rearranged in terms of pressure (P):

$$P = \frac{6\eta L Q}{\pi r \delta^3} \quad (4-13)$$

and is differentiated with respect to L, giving the rate of change of pressure along the engagement length:

$$\frac{dP}{dL} = \frac{6\eta Q}{\pi r \delta^3} \quad (4-14)$$

The expressions of piston/cylinder distortion are included in the formula:

$$\frac{dP}{dL} = \frac{6\eta Q}{\pi r (h_0 + U(P) - u(P))^3} \quad (4-15)$$

where h_0 is zero pressure gap

and the equation for pressure dependent viscosity is also used:

$$\eta = (1 + 1.90036 \times 10^{-3} P)^{8.81} \quad (4-16)$$

which is included in the preceding formula:

$$\frac{dP}{dL} = \frac{6 \times 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81}Q}{\pi r(h_0 + U(P) - u(P))^3} \quad (4-17)$$

Next, the formula was integrated with respect to L, and rearranged to give the length in terms of the pressure and flow:

$$P = \frac{6 \times 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81}Q}{\pi r(h_0 + U(P) - u(P))^3}L \quad (4-18)$$

$$P \frac{\pi r(h_0 + U(P) - u(P))^3}{6 \times 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81}L} = Q \quad (4-19)$$

$$U(P) = \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) \quad (4-20)$$

$$u(P) = \frac{-Pr}{E} + \frac{P\sigma r}{E} + \frac{P_L\sigma r}{E} \quad (4-21)$$

$$P \frac{\pi r(h_0 + \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) + \frac{Pr}{E} - \frac{P\sigma r}{E} - \frac{P_L\sigma r}{E})^3}{6 \times 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81}L} = Q \quad (4-22)$$

Having obtained the above formula, the task remained to use it with respect to predicted fluid flow. This was done and the results compared with those determined experimentally in the literature, the results are shown below:

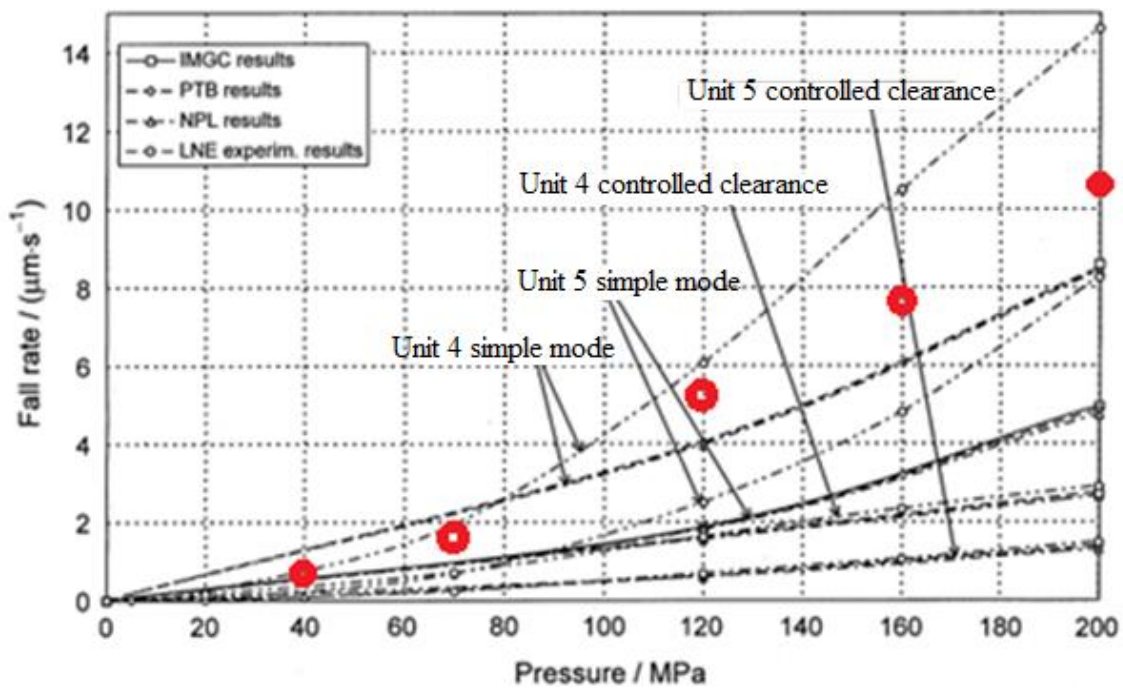


Figure 4-2 Fall Rate Formula Benchmarking (Adapted Molinar, 1998)

The chart shows loose agreement between the analytical results and those found previously in the literature, indicating that the flow formula can be used with caution in predicting flow results while considering pressure dependent viscosity and piston/cylinder distortion. It is noteworthy that flow is not a critical parameter in HFA design except for functionality of the system (such that both PCA's fall at proportional rates), therefore this loose agreement is suitable for the purposes of this investigation. The numerical approach used is appropriate because it has traditionally been used to describe flow in PCA's and is sufficient for the rough calculations required without resorting to the more extensive FEA/CFD analysis, which may also not provide accurate results.

4.2 FEA/CFD

While analytical formulae are predominantly used in uncertainty calculation for rotational PCA's, the FEA/CFD tool is leveraged to confirm the analytical uncertainty calculations, particularly keeping in mind the nonlinear pressure distribution. The chart previously shown in figure 3-2 shows the purpose of the FEA/CFD analysis. FEA/CFD calculations of distortion coefficient have previously been accurate to approximately 3%, making them suitable for HFA analysis as the implied tolerances fall within the range of interest concerning the reduced uncertainties desired. The chart below shows the purpose of the FEA/CFD analysis:



Figure 4-3 Purpose of the FEA/CFD

4.2.1 Distortion Benchmarking

To enable validation of the design of rotational PCA's by ensuring that the analytical formulae were providing accurate results, it was necessary to be able to accurately replicate the distortion of PCAs in the FEA/CFD software. To do this, it was necessary to benchmark an FEA/CFD model of a PCA to experimental results for distortion coefficient.

Ansys Workbench software was used, which is an all-purpose FEA/CFD software (Lee, 2021). As can be seen from the diagram below, Workbench consists of several different types of 'module', for example 'static structural' and 'fluid flow' which were used in this

thesis. Static structural modules were used because the PCA is in a steady state under analysis, and fluid flow was used to analyse the oil. Each such module was used for a particular purpose for instance, the structural module was used to model the piston/cylinder structures, and the fluid module to simulate the fluid flow.

A particular configuration used throughout this work was two way Fluid-Solid Interaction (FSI), which signifies coupling of the structural and fluid modules via a system coupling module, as shown in Figure 4-4 (Khalid et al, 2013). ‘Static’ simulations were used throughout because the simulation concerns a fixed state of affairs (i.e., there are no moving components), alongside a fluid flow module which is used to represent the oil in the HFA system, an explicit solver was used.

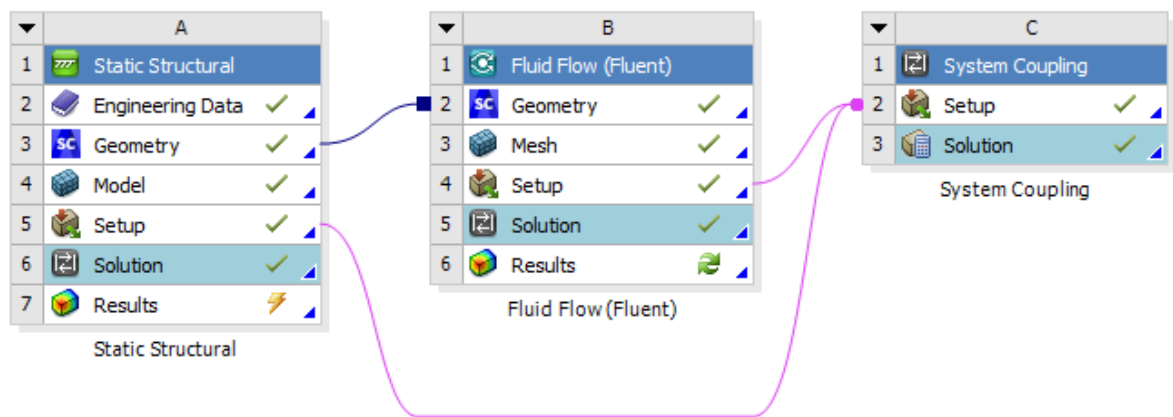


Figure 4-4 Ansys Workbench System Coupling

The configuration first shares a common geometry, which in the boundary conditions contains surfaces on which the structural and fluid components both act. In practise, the FSI functions by first pressurising the PCA which subsequently deforms according to the pressurisation. The deformed structure is then pressurised according to the fluid boundary conditions, and the next distortion is then obtained. The system continues this way until

convergence of the result is obtained, which is then taken to represent the final state. The process is shown in the figure below:

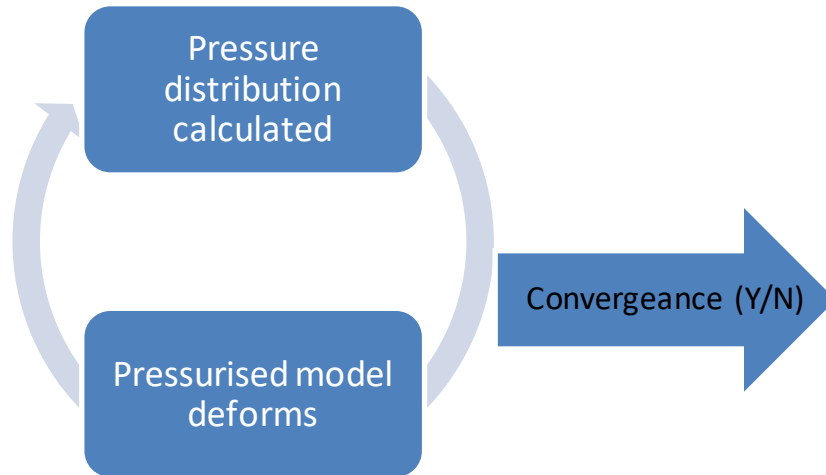


Figure 4-5 Convergence Process (Esward, T., 2002)

To conserve computational capacity, slices of PCAs were taken with sliding boundary conditions used on the sliced surfaces, to represent full 360° PCAs, it was ensured that this approach was valid by benchmarking of the models with experimental results. Both 10° and 90° slices of PCA were taken as shown in the figures below:

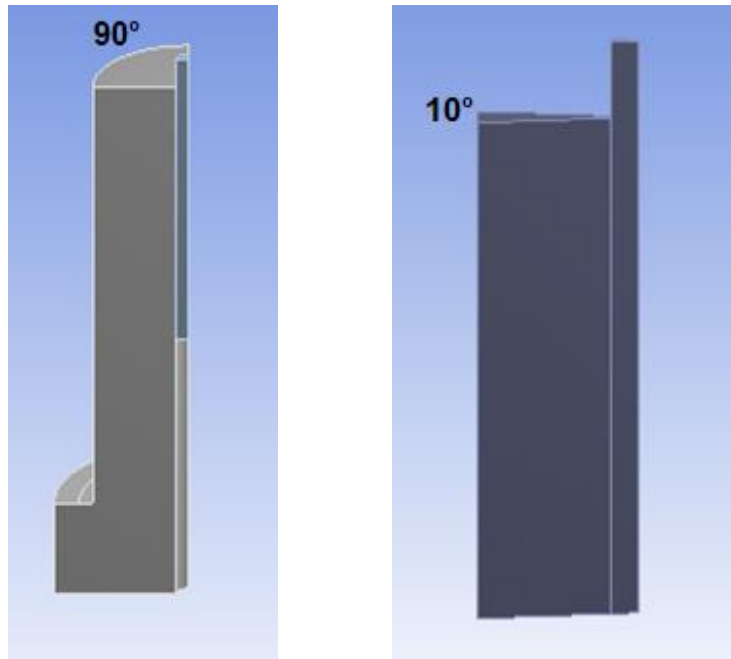


Figure 4-6 90° and 10° Slices of PCA

To carry out an analysis, the geometry of the PCA was first created in Ansys Spaceclaim, with accurate dimensions of piston, cylinder and fluid, according to Molinar et al (1998) as shown below. The piston diameter was accurate to within 25 nm, although assumed to be of constant width. It was deemed suitable to replicate the dimensions given by Molinar because these had been benchmarked in the work successfully. The figure shows the locating dimensions for the points which were modelled asymmetrically, in the x and y axes as shown on the figure (in mm).

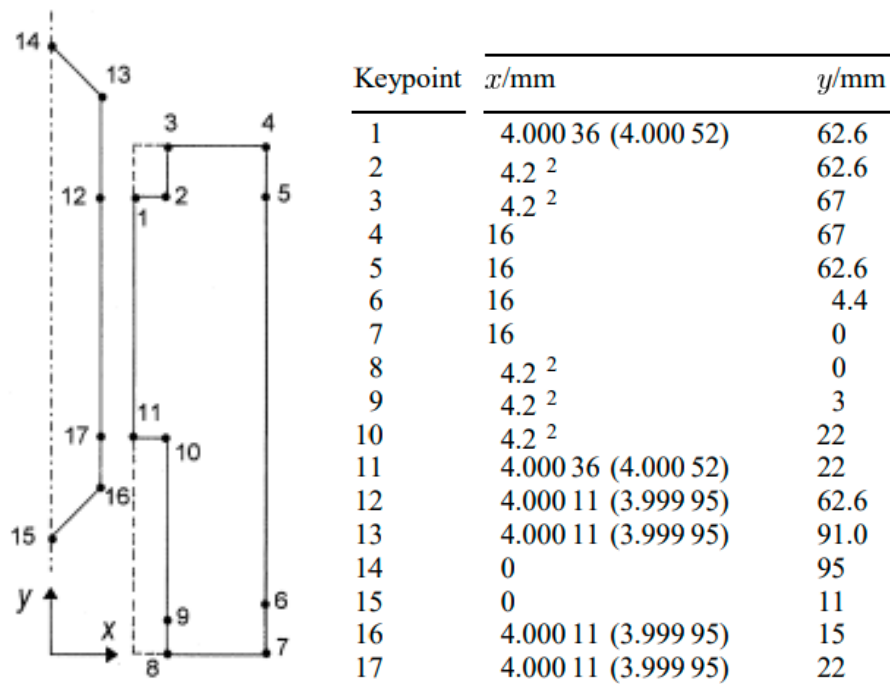


Figure 4-7 PCA Geometry (Molinar et al, 1998)

The material properties of the WC piston and cylinder, shown in the table below, were assigned to the components, as these were the material properties shown by Molinar et al (1998) to correspond to the experimental values:

Table 4-1 Material Properties (Molinar et al, 1998)

Young's Modulus (E)	630 +/-30 GPa
Poisson's Ratio (σ)	0.21 +/-0.002

Then, the piston/cylinder were meshed with standard workspace elements, of 1 mm size. The only constraint on the meshing was that 40 divisions were defined along the engagement length on the piston, because it was found that refinement in this area affected convergence, as shown in the figure below. The mesh was refined sequentially

from 20 divisions until the results repeated to show convergence at 40 divisions, the Ansys structural mesh was used for the structural components with fluent mesh for the fluid, and the components were sized as had been done by Molinar et al (1998) as their size definitions were adequate to allow comparison with their results. Mechanical elements were used of programme controlled order, this being the only option available in Ansys for the configuration.

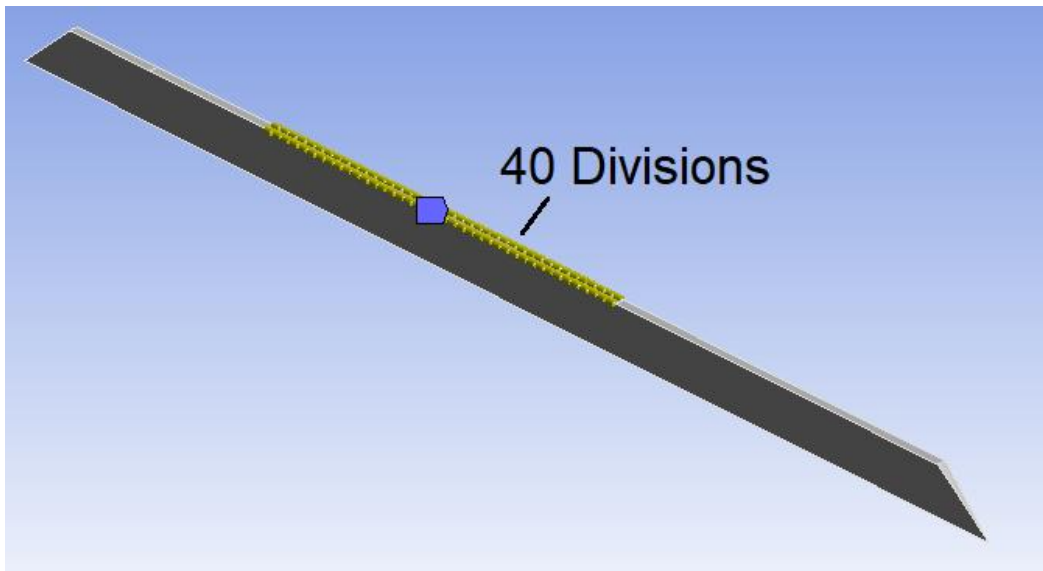


Figure 4-8 Piston Meshing

The fluid area was subject to comprehensive meshing constraints as it's behaviour was found to severely impact the performance of the simulation. The final mesh arrived at was for a thickness of 1 element, one division along the width of the fluid area (as shown pictorially below), and 200 divisions along the fluid length (face meshing was also used to ensure uniformity of mesh). This was increased to 250 divisions with no noticeable difference in the results. The mesh was first altered to enable adequate performance of the model, but later to affect the result achieved. The figure below shows the fluid meshing:

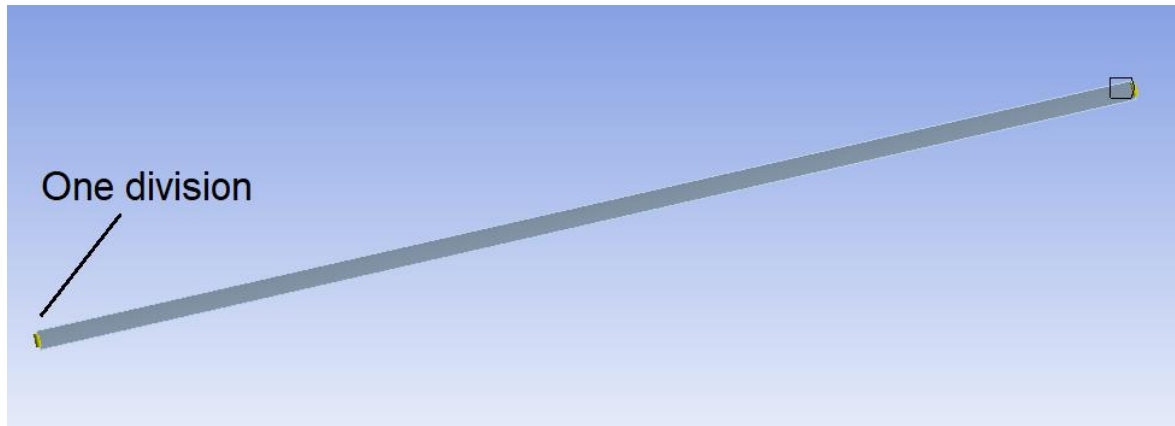


Figure 4-9 Fluid Meshing

The next step was to set the boundary conditions, which were replicated from the literature (Molinar et al, 1998), and are shown in the figure below:

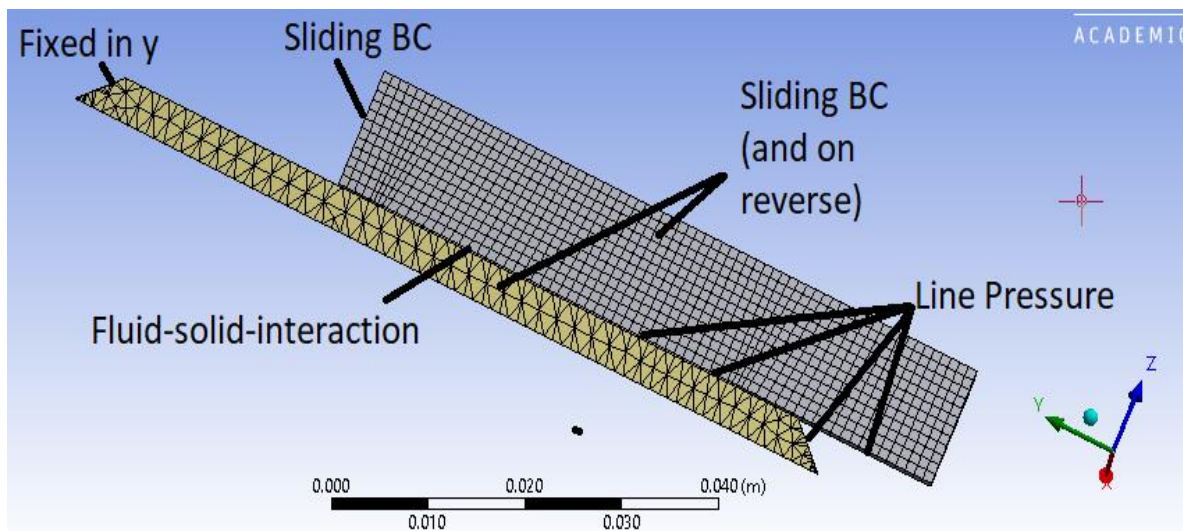


Figure 4-10 Boundary Conditions

It can be seen how the line pressure is imparted on the lower surface of the piston, as well as the lower faces of the cylinder beneath the engagement length. Sliding boundary conditions are imparted on the cuts of the slice to impart the cylindrical symmetrical condition, and the FSI is imparted to the piston and cylinder gap surfaces, as well as the corresponding walls of the fluid. A sliding boundary condition is imparted to the top of

the piston which corresponds to the fixing of the assembly in the paper of Molinar et al (1998), as well as the sliding BC at the top of the piston.

The conditions used in the system coupling as well as the fluid simulation are shown below in Table 4-2, these were the standard conditions and the timestep and number of iterations, after 7 iterations the force/displacement transfers in the model had converged to within 1%, and although further convergence seems likely it would be non-significant for the purposes of distortion and the overall tolerances of the HFA. For the fluid viscosity and density, because these vary with pressure it was necessary to use pressure dependent formulae (given by Molinar et al, 1998) to determine the viscosity/density as a function of pressure within simulation. This was done by creating and importing user-defined functions into the software which describe the change in the parameters with pressure.

Table 4-2 Analysis Properties

Number of timesteps	1
Maximum number of iterations	20
Number of fluent iterations	400
Fluid simulation type	Laminar/Viscous
Solver type	Pressure based
Solver time	Steady
Scheme	Coupled

Spatial Discretisation – Gradient	Least Squares Cell Based
Spatial Discretisation – Pressure	Second Order
Spatial Discretisation – Momentum	Second Order Upwind
Number of solid elements/nodes	1625/8613
Number of fluid elements/nodes	200/804

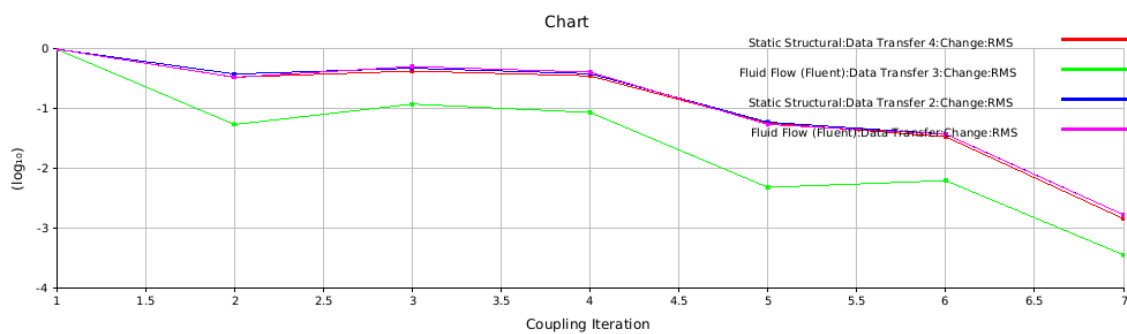


Figure 4-11 Coupled Iterations

After the simulation had been run (typically taking 15 minutes or so) the results were viewed in the structural/fluent modules, enabling viewing of PCA distortion in various directions and pressure distributions in the fluid, typical output screens for piston/cylinder distortion and pressure distribution are shown below:

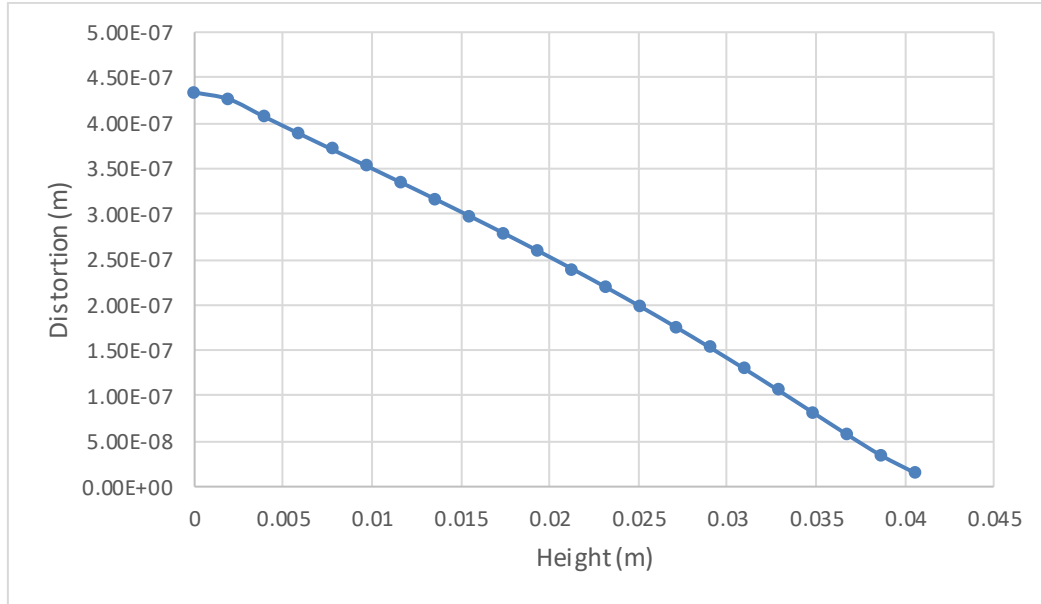


Figure 4-12 Graph Showing Typical FEA Results for Distortion of the Cylinder

Equidistant points are taken automatically for distortion extraction along the length of the piston and cylinder, artificial planes were created along the clearance length to extract the pressure values at the same points. The points are equidistant with error of less than a nanometre which is amply accurate for the purposes of the analysis. These results could be used as output numerically for processing in Excel which allowed evaluation of parameters of interest, in the benchmarking for example, the PCA distortions as well as the pressure distribution were processed using the formula of Dadson (1985):

$$\lambda = \frac{1}{1 + 2h/r} \left[\frac{U(0) + u(0)}{rp} + \frac{1}{rp^2} \int_0^l p(z) \left(\frac{dU}{dz} + \frac{du}{dz} \right) dz \right] \quad (4-23)$$

where λ is distortion coefficient, $U(0)$ is cylinder deflection at base, $u(0)$ is piston deflection at base, p is line pressure, l is engagement length, z is the height along the engagement length.

For the benchmarking to experimental reports in the literature, experimental results from the paper of Buonanno and Dell Isola (2001) were used, who describe the distortion coefficient of the BNM LNE 200 MPa pressure balance of the National Metrology Institute of France.

The formula of Dadson requires not only the absolute values of distortion and pressure, but gradients of these characteristics, as well as integrals of computed terms. To address this problem, the feature in Excel of curve fitting with formula of the fitted curve shown was exploited to easily allow differentiation of the obtained equations, as well as integration using basic numerical techniques. It was found that if the formulae were given to sufficient precision (ten points after the decimal place were used), and that if polynomial formulae of order 6 were used, that the computation maintains validity as shown. Typical R^2 values were of the value of 99.99 %, compared with 88 % when fitted with a straight line.

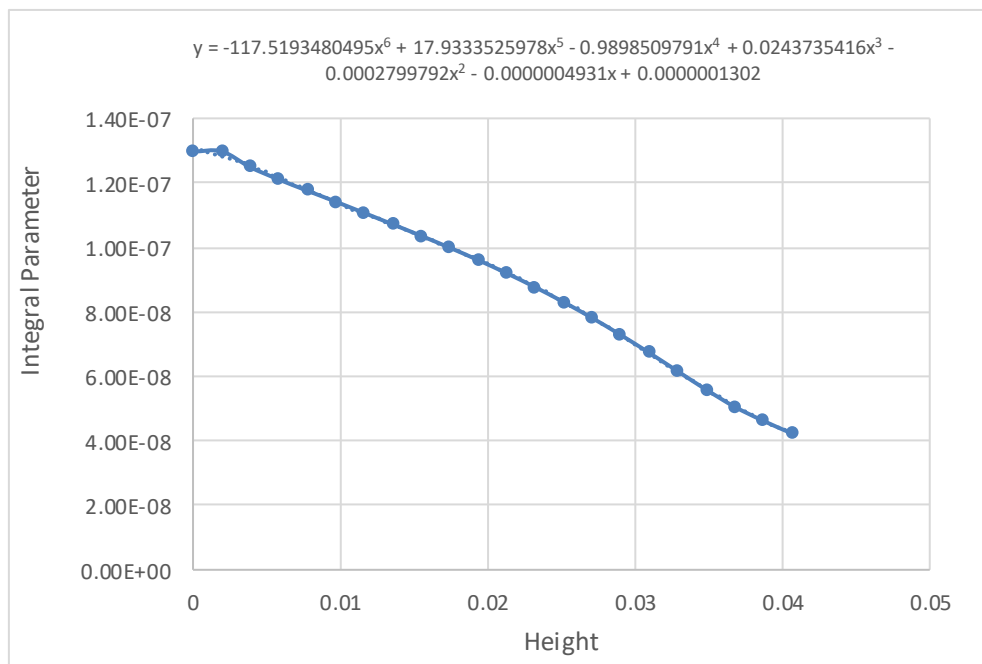


Figure 4-13 Excel Equation for Parameter Differentiation

The BNM-LNE 200 MPa pressure balance is described as having a distortion coefficient of 0.78×10^{-13} when cross floated at 120 MPa. The procedure of evaluating the distortions and pressure distribution according to the formula was carried out and the value of λ computed was compared to the value in the literature.

Results were actually obtained under two separate boundary condition regimes, the reason for this was that when the piston was later made eccentric it was not possible to use mass flow to obtain a uniform pressure at the inlet:

1. Mass flow inlet and outflow at base and top of clearance respectively
2. Pressure Inlet/Outlet at base and top of clearance respectively

Additionally, results were obtained at several pressures, although the results with a line pressure of 120 MPa were considered for benchmarking as they match the experimental conditions in the literature. The analysis carried out was for PCAs modelled as 10° slices, and also as 90° quarters of the PCA. The results are shown in the table below, they are compared with the experimental result for distortion coefficient of the BNM LNE 200 MPa pressure balance operating at 120 MPa described in Buonanno and Del'Isola (2001), of $\lambda = 0.78 \times 10^{-6} \pm 0.03$ MPa:

Table 4-3 Benchmarking Distortion Coefficient Results

<u>Pressure</u>	<u>Boundary</u> <u>Condition</u>	<u>Section</u>	<u>Distortion</u> <u>Coefficient</u> <u>($\times 10^{-6}/\text{MPa}$)</u>	<u>Agreement</u> <u>with Nominal</u> <u>Experimental</u>
50 MPa	Mass flow	10 °	0.798	2.3 %
25 MPa	Mass flow	10 °	0.777	-0.4 %
120 MPa	Mass flow	10 °	0.800	2.6 %
25 MPa	Mass flow	90 °	0.796	2.1 %
120 MPa	Pressure	90 °	0.799	2.5 %

The results show very good agreement with the nominal experimental results of within 2.6 % across the range of trials carried out, the results sitting well within the uncertainty range for the experimental results of 0.03, which represents an error of 3.8 %. The uncertainty contribution of the uncertainty in the distortion coefficient can be shown to cause an uncertainty of about 30 ppm in effective area or the measured force, as shown by the calculator developed later in this work, still sitting well within the range of errors which enable the advantages of direct calibration to be had. The results at 120 MPa are compared graphically with those obtained in the FEA/CFD works of the National Metrology Institutes: PTB (Germany), NPL (UK) and the analytical method used by the IMGG (Italy) as described by Buonanno and Del’Isola (2001), the experimental result of the LNE (France) is shown for comparison:

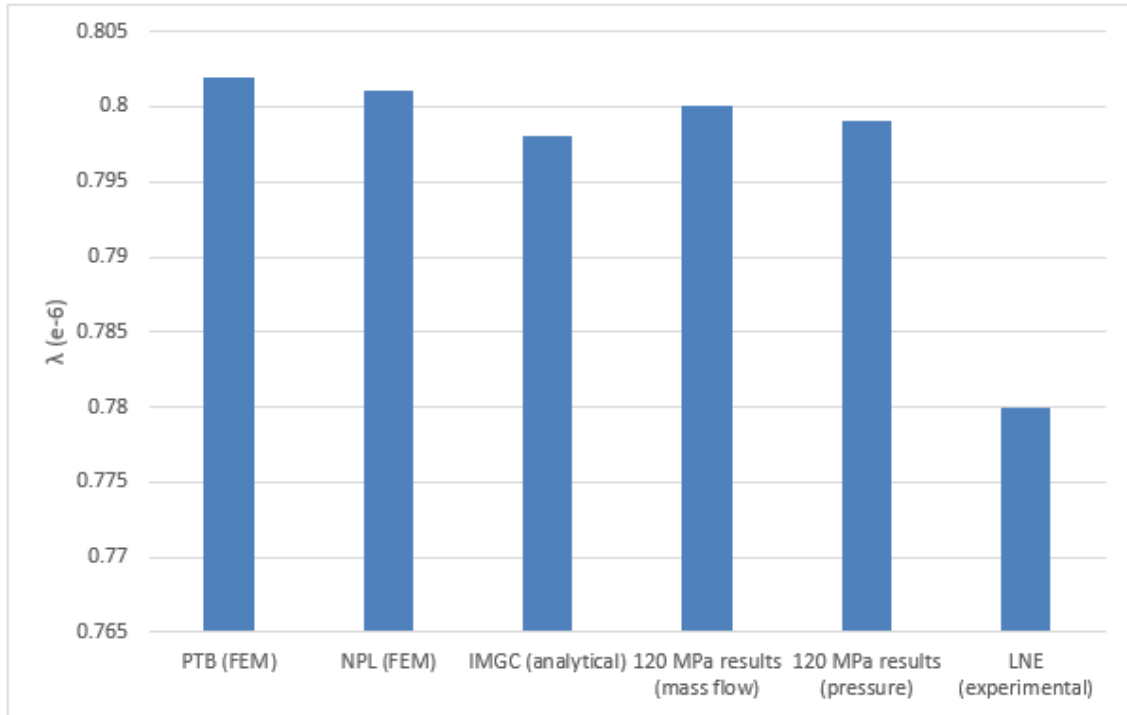


Figure 4-14 Benchmarking Results

It can be seen below how closely the results for 120 MPa line pressure agree with those obtained both by FEA/CFD and analytically by the NMI's, but that the experimental result obtained by the LNE is somewhat lower than the computed results. The discrepancy points towards either the uncertainty in the experimental result, or some other systematic error in the Fluid-Solid Interaction FEA/CFD modelling, in order to determine which it would be necessary to evaluate more FEA/CFD benchmarking to determine whether the error holds across different PCAs, or varies randomly. In any case it can be seen that the FEA/CFD results all agree almost exactly (to within 0.4 %), which in turn agree with the analytical results of the IMGc almost exactly also (to within 0.5 %), indicating that the FEA/CFD analysis was carried out in the same way as had been done in the state of the art. To give an indication of the accuracies achieved,

an error in distortion coefficient of 3 % corresponds to an uncertainty in the HFA of approximately 30 ppm.

4.2.2 Miscellaneous Uncertainty Sources

In carrying out the background work which enables design of the directly calibrated HFA, two uncertainty sources were considered which had the potential to cause errors in the PCA: piston eccentricity and piston rotation. While the literature had assumed that neither of these factors caused any difference to effective area, it was decided to verify the assumption using the FEA/CFD capability that had been derived.

4.2.2.1 Eccentric Piston

The literature has traditionally assumed that the distortion coefficient of the piston is independent of its eccentricity within the cylinder, that is to say, that if the piston moves sideways within the cylinder, that it does not affect the effective area of the piston (Peggs and Lewis, 1977). It occurred during prior experimentation that this assumption may not be valid, because it was noticed that when the piston displaced sideways that the pressure distributions changed. Because of this finding, it was decided to investigate further, to test the conventional assumption that piston eccentricity, as shown in the figure below, has no effect upon effective area.

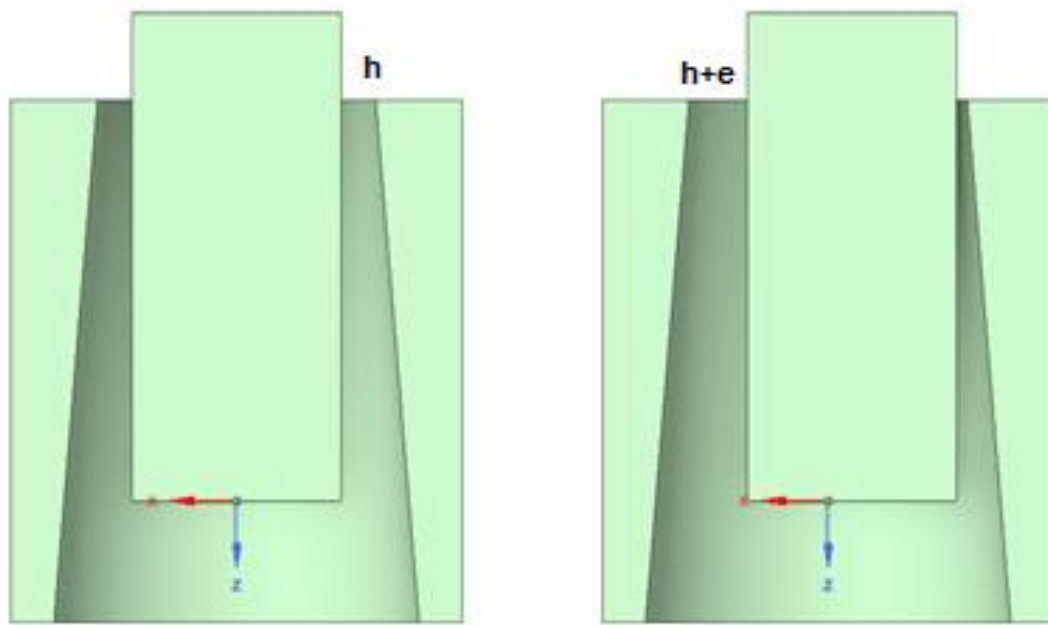


Figure 4-15 Laterally Eccentric Piston

It was necessary only to confirm or disprove the hypothesis by comparing the effective area of a piston concentric within a cylinder, and of a piston that was displaced centrally within the cylinder. The PCAs which were used for benchmarking were used as their reliability was known, and the pistons were displaced laterally, with changes to effective area measured. The process used is shown in the diagram below:



Figure 4-16 Checking Eccentric Piston

The boundary conditions were as before, except that instead of a mass flow boundary condition at the base of the gap, the constant pressure condition was applied. This is

because the mass flow resulted in a non-uniform pressure at the base because of the variation in gap, so constant pressure was used. For the displacement in the cylinder, because a slice of the PCA was used, this left a prominent edge which must be removed within the model. This was done in varying ways and was found not to affect the results.

The raw data for piston and cylinder distortion was processed in excel, to derive dU/dz and dU/dz graphs were created with polynomial (order 8) lines of best fit, the equations of the curves were differentiated to give the values as shown in Figure 4-13. In order to determine the integrals an analogous procedure was used. The number of significant figures used in the equations was increased until results converged.

The formula of Eswards (2002) was also used to derive effective area:

$$A_p = \pi r^2(0) \left[1 + \frac{h_0}{r(0)} + \frac{u(0) + U(0)}{r(0)} + \frac{1}{r(0)P} \int_0^l p_z \frac{d(u + U)}{dx} dx \right] \quad (4-24)$$

Where A_p is effective area, $r(0)$ is piston radius at the base, $u(0)$ is the piston distortion at the base, p is line pressure, r_p is piston radius along engagement length, h_p is gap along the engagement length, p_z is pressure along the engagement length, z is height

Results were compared at 50 MPa and 25 MPa using the formula below to derive distortion coefficient, which was compared to results of Buonanno and Del'Isola :

$$A_{eff} = A_0(1 + \lambda P) \quad (4-25)$$

Where A_{eff} is effective area, A_0 is undistorted area, λ is distortion coefficient and P is pressure.

The formula below was used to derive effective radius at 45 degree intervals, this was necessary to enable processing of the PCA at eccentricity of 0.5:

$$r_{eff} = \sqrt{A_P/\pi_0} \quad (4-26)$$

Effective radius was integrated through 360 degrees to give effective area

$$\int_0^{2\pi} \frac{r^2}{2} d\theta \quad (4-27)$$

The figure below shows the results for effective radius taken at 45 degree intervals around the PCA:

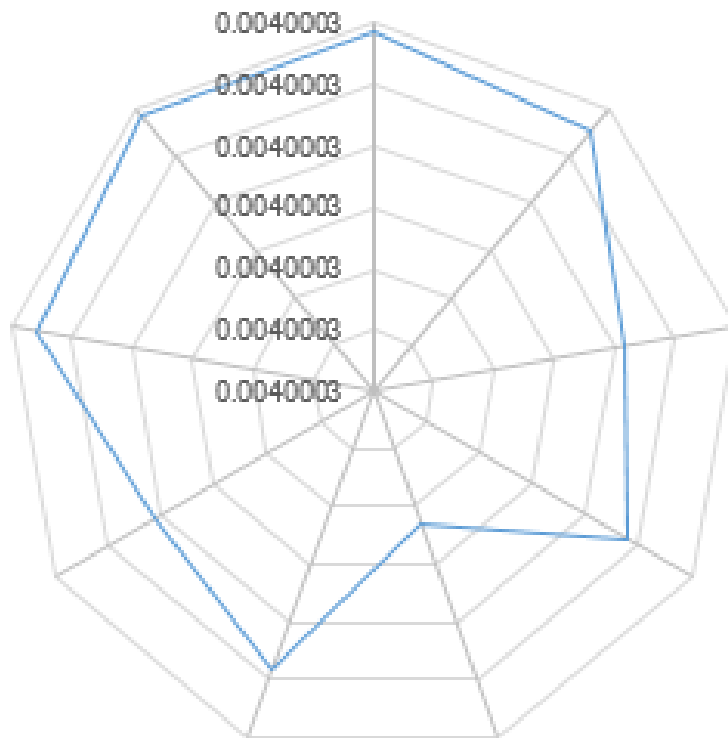


Figure 4-17 Effective Radii Obtained

The effective area matched the non-eccentric model (to 0.00002 %), thereby showing that uncertainties involved are non-significant and the lateral displacement was acceptable.

The effective area was shown not to deviate significantly as a result of piston eccentricity, this suggests that the uncertainty source is not relevant.

4.2.2.2 Effects of Piston Rotation

Rotation of the piston within the cylinder has traditionally been used to maintain distance between the piston and cylinder, through the centrifugal force imparted in the PCA. Rotation of the piston, however, can cause extraneous forces in the piston/cylinder which could interfere with performance of the PCA. Research concerning the effects of piston rotation has focused in the field of aerostatic pressure bearings (Prowse and Hatt, 1977, Sutton, 1979) but in the field of liquid bearings it has been assumed that rotation has negligible effects upon measured force. The process used is shown in the diagram below:

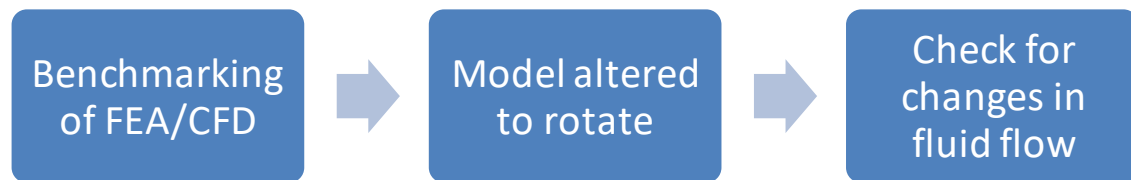


Figure 4-18 Checking Effects of Rotation

The effect of piston rotation upon flow of fluid within the piston/cylinder clearance, however, has been researched in gas operated piston/cylinders, but not in fluid operated piston/cylinders. In this section use is made of a flow model which was derived in the FEA/CFD but not used in the main section. The question therefore remains as to whether rotation of the piston will have a significant effect upon flow in the gap, or whether rotation is in effect non-significant.

To carry out the analysis, the FEA/CFD models which had been used previously for benchmarking for flow were altered so as to include a rotating surface of the piston. This

was done in the FEA/CFD not by incorporating an actually rotating surface into the simulation, which would not be practical because of difficulties of, for example, ensuring that the model still benchmarked to experimental results. The feature in Ansys Workbench of imparting movement to a surface in the fluid simulation, shown in the figure below, it can be seen how the rotational speed and axis of rotation can be adjusted:



Figure 4-19 Rotation in FEA/CFD

The speed of rotation, being set in radians per second, was set according to the piston radius according to the formula:

$$surface\ speed = piston\ circumference \times \frac{rotational\ speed}{2\pi r} \quad (4-28)$$

Because the trials were essentially to test the hypothesis that rotation has no effect upon piston performance, the testing regime was simply to test a variety of possible speeds and to verify that there was no significant change in distortion coefficient. This was carried out with surface speeds across the range of those likely to occur in the PCA, and any resulting differences in the outputs for distortion coefficient or fluid flow were recorded.

To summarise the results for the effects of piston rotation on distortion and flow, the FEA/CFD results found confirmed the traditional theory, insofar as they indicated that rotation of the piston had no effect upon either distortion or flow. While there were

some non-significant deviations in the results (due most likely to random variations in the CFD computations because of the rotation), in both cases these variations were non-significant, so it was confirmed that piston rotation causes no significant uncertainties in itself. The chart below shows the results:

Table 4-4 Flows with rotation

Surface speed (mm/s)	Flow in clearance (m ³ /s)
0.01	8.502e-11
0.1	8.503e-11
1	8.503e-11
10	8.504e-11

4.2.3 Altered Piston Geometry

In this section a source of uncertainty in PCAs is induced by the piston's vertical displacement within the cylinder, the purpose of this section is therefore to demonstrate a method of controlling this uncertainty source. Tapers were introduced and altered on the piston mating face to simulate movement of the piston within the cylinder. The analysis for this type of uncertainty is useful also in determining tolerances for the machined piston profile. The analysis is in fact made redundant when uncertainties associated with the machined mating faces are considered fully, but the analysis is carried out here for completeness as all potential sources of uncertainty must be focussed on to ensure comprehensiveness of the design. The formula for effective area

is probed to gauge the different uncertainties that can arise on the piston profile, which gives guidance in the design of piston/cylinders so as to avoid excessively tight tolerances. As can be seen in the figure below, when in operation the piston can be in equilibrium at varied height within the cylinder, determined by the operating conditions of the HFA pump and system.

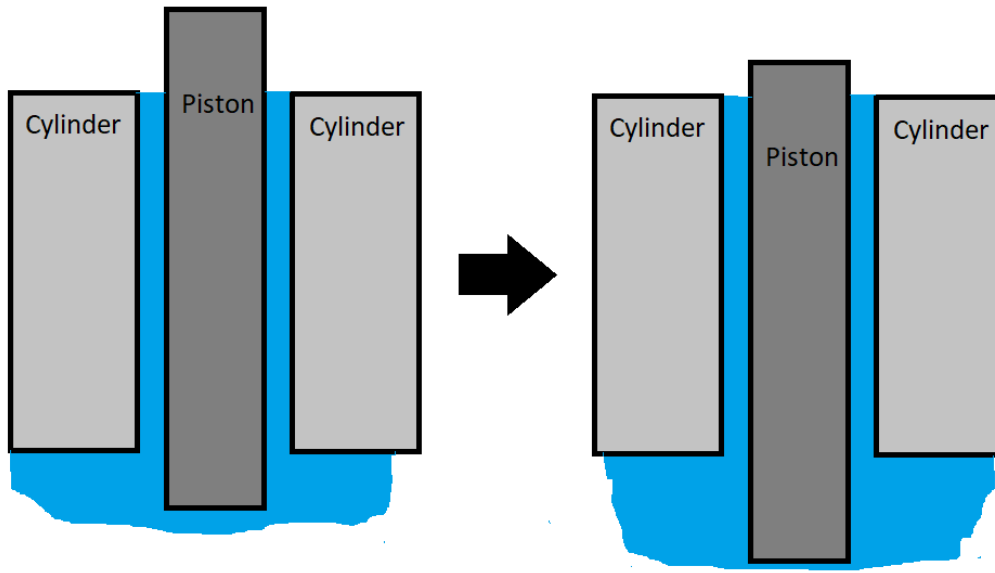


Figure 4-20 Changed Engagement Length

The effective area of the PCA is given by the formula shown below, which when evaluating the effects of the three influences upon effective area, considers the machined profile of the piston along the engagement length (Sabuga, 2006).

$$A_p = \underbrace{\pi r^2(0) \left[1 + 2 \frac{u(0)}{r(0)} \right]}_{\text{force on base}} + \underbrace{\frac{2\pi}{p} \int_0^l r_p \left(-\frac{h_p}{2} \frac{dp_z}{dz} \right) dz}_{\text{fluid drag on piston}} + \underbrace{\frac{2\pi}{p} \int_0^l r_p \left(p_z \frac{dr_p}{dz} \right) dz}_{\text{pressure on sides}} \quad (4-29)$$

Where $r(0)$ is the piston radius at the base, $u(0)$ is distortion of the piston at the base, p is line pressure, h_p is the gap

Because of this, when the piston displaces vertically it exposes a different machined surface profile to the cylinder, thereby altering the geometry along the side of the piston, and altering the effective area of the PCA. To prevent excessive uncertainty in the amplification ratio of the HFA, the uncertainty contribution from this vertical piston displacement must be controlled for, which is the purpose of the investigation in this section.

To enable the control of such uncertainty, the uncertainty was quantified by deriving formulae which accurately relate the machining tolerances of the piston to the changes in effective area caused by vertical displacement. To create the formulae, the formula above was partitioned into its components corresponding to:

1. Contribution of force on piston base upon effective area
2. Contribution of fluid drag on piston upon effective area
3. Contribution of pressure on sides upon effective area

Simplifying assumptions were then made which allowed the components of the formula to be reduced to multipliers of the distorted radius and distortion magnitudes of the PCA. Next, the sensitivity coefficients of the distortions to the piston (including the distorted radius) were evaluated by differentiating the formulae with respect to the parameter of interest, according to the formula:

$$s(x) = \frac{df(x)}{dx} \quad (4-30)$$

Where $s(x)$ is the sensitivity coefficient, $f(x)$ is the formula, x is the parameter of interest

Having obtained the sensitivity coefficients, it is possible to predict the change to effective area according to the formula:

$$\Delta A_{eff} = s(x)\Delta x \quad (4-31)$$

Which can then be divided by the nominal effective area to give the uncertainty caused by the geometrical change:

$$\frac{\Delta A_{eff}}{A_{eff}} \approx \text{resulting uncertainty} \quad (4-32)$$

In order to carry out the analysis, it was necessary not only to derive formulae which predict the uncertainty caused in the PCA, but also to benchmark the formulae to ensure that they are accurate in predicting the uncertainty. To do this, the benchmarked FEA/CFD model was used to evaluate models with altered piston geometries to simulate the piston having fallen within the cylinder. The overall process was as in the diagram below:

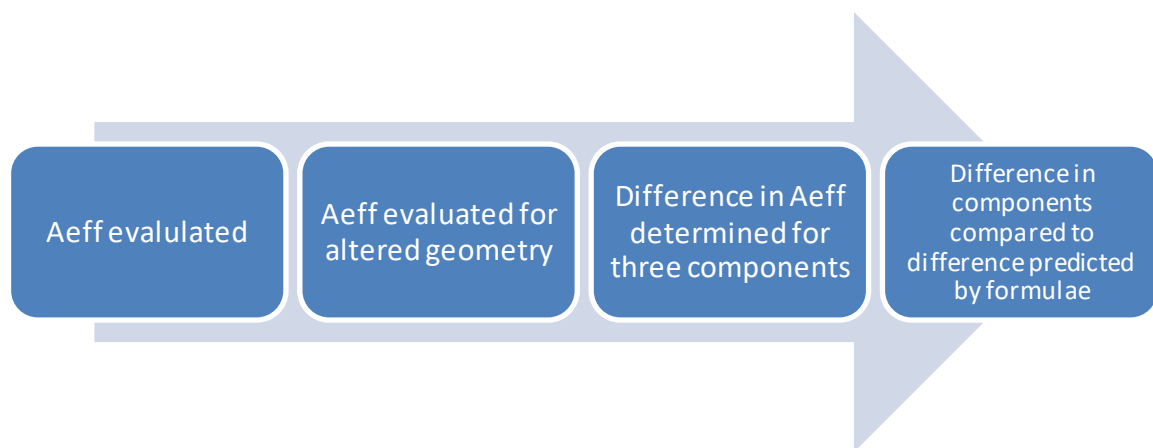


Figure 4-21 Process for Deriving Formulae

As can be seen above, two FEA/CFD models with altered geometries were created, with effective area evaluated according to the formula above which had been

benchmarked in the previous section. The change in contribution by each component of the formula was evaluated and compared to changes predicted by the derived formulae. The predictions were compared with the simulation results, thereby indicating the confidence with which the formulae can be used.

The formula for effective area is composed of three constituent parts, which were used to derive the three predictive formulae:

1. Contribution of force on piston base upon effective area
2. Contribution of fluid drag on piston upon effective area
3. Contribution of pressure on sides upon effective area

To derive the formulae, assumptions were introduced which enabled the formula to be simplified and used. The following constants were therefore introduced to derive the simplified formulae, as shown in the table below:

Table 4-5 Substituted Terms in Formula

<u>Formula component</u>	<u>Original term</u>	<u>Substituted term</u>
Fluid drag on piston	r_p	R
Fluid drag on piston	dp_z/dz	P/L
Pressure on sides	r_p	R
Pressure on sides	dr_p/dz	$u(0)/L$

The resulting predictive formulae are derived for the effect of geometrical deviations on the piston upon uncertainty in effective area:

$$uncertainty (base\ area) \approx \frac{(r + \delta)^2}{r^2} - 1 \quad (4-33)$$

$$uncertainty (fluid\ drag) \approx \frac{\delta}{r} \quad (4-34)$$

$$uncertainty (gradient) \approx \frac{\delta grad L}{r} \quad (4-35)$$

Having derived the predictive formulae, these were compared to FEA/CFD results as described in the methodology section. The results for the comparison of the analytical prediction and FEA/CFD results for the two cases are shown below:

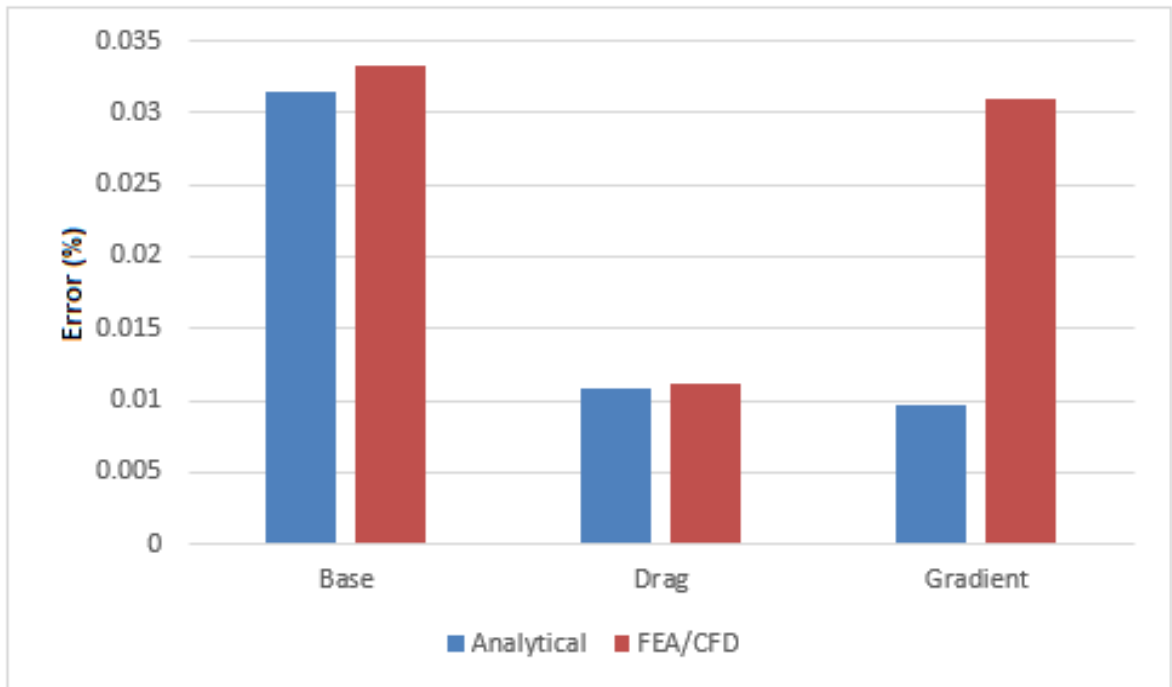


Figure 4-22 Accuracy of Predictive Formulae (1)

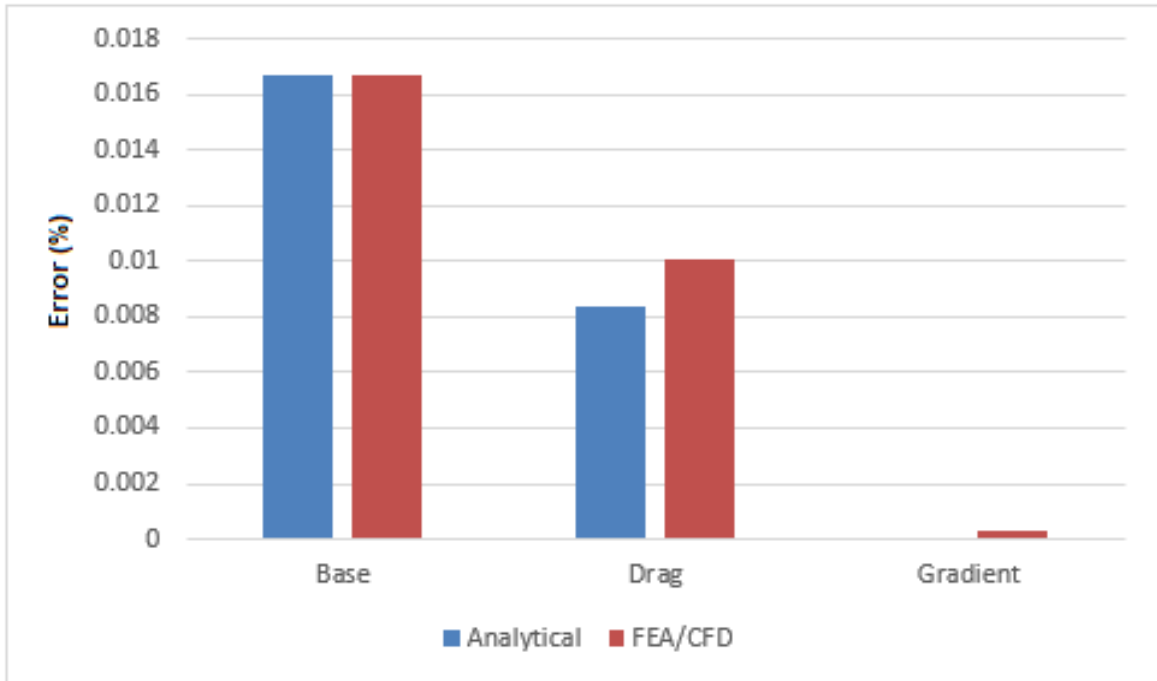


Figure 4-23 Accuracy of Predictive Formulae (2)

As can be seen above, the analytical formulae derived enable excellent prediction of uncertainties caused by form deviations of the piston in all three impactors on effective area, indicating that it is possible to use the formulae to derive functional tolerances of for piston geometry, to control uncertainties caused in force produced in PCAs. The finding is significant because it allows the uncertainty source to be controlled easily rather than by using FEA/CFD which can be computationally expensive and time consuming. While other sources of uncertainty may still require FEA/CFD for evaluation, control of piston form accuracy can give an indication of the minimum tolerances required for a particular design of PCA. The formulae can also, critically, be used as a design tool for rotational PCA's, indicating design limits which will enable direct PCA's to be created without the use of excessively stringent tolerances. It is important to stress that these advantages are only achieved within the constraints of a

theoretical investigation, and it remains for experimental verification to be carried out to confirm the usefulness of the results.

4.3 Design and Verification

In this section the design is carried out for a ‘primary’ rotational HFA, with the aid of FEA/CFD to verify the resulting design. The design includes specification for tolerances for the large and small side PCA’s, as well as the more general HFA design, including piston sizes, amplification ratio and piston fall rates.

4.3.1 Rotational HFA Design

Having gathered the relevant background information, in this section design of a directly calibrated rotational HFA is carried out, thereby providing evidence, along with the uncertainty evaluation in the literature review, the hypothesis that this type of mechanism can be used to reduce uncertainties in force calibrations.

4.3.1.1 Amplification Ratio

The first step in the design of the primary HFA using rotational PCAs is definition of the amplification ratio used. In practise, this parameter will depend on values of A_0 used for the small and large PCAs, as well as the number of large side PCAs, but from the point of view of specification will need to provide a high enough measured force with the weight stack available. The manufacturing capability imposes limitations upon the size of the large side PCA; and it may be desirable to design small side PCAs which are large enough to avoid the necessity of nanometric finishes on the piston/cylinder faces. As well as limitations which much be explored, the demand for the size of measurement force must be considered, along with the limitations concerning the number of weight stacks.

Therefore, the table below can be created as a broad specification concerning the amplification ratio:

Table 4-6 HFA Definition

<u>Parameter</u>	<u>Nominal Value</u>	<u>Justification</u>
Number of large side PCAs	1	Most simple design
Weight stacks	250 kN	A commonly used value
Produced Force	20 MN	Meeting the requirements for mid-weight force calibrations
Amplification Ratio	80	Based on ratio of forces
Number of large side PCA's	3	-

4.3.1.2 Material

Tungsten Carbide (WC) is suggested as the material, this being generally preferred to steel because of its increased Young's modulus ($E=630 \text{ GPa}$, $\sigma=0.21$). In terms of manufacturability, WC can be ground and polished much the same way that steel can, allowing nanometric finishes to be achieved as required.

4.3.1.3 Radii of PCAs

It remains to determine the dimensions of the PCAs to be used. In terms of the radii involved, as mentioned previously the large side should be limited in size, suggesting a maximum radius of approximately 200 mm. It is suggested that tolerances on the small and large side PCA's are of the nanometric value (20 nm – 50 nm) as sensitivity analysis shows that such small tolerances are required to keep the uncertainties involved less than 10 ppm. It is also suggested that the ratio of R' be the same on the small and large sides, such that the distortion coefficient is kept the same on both sides, resulting in a constant amplification ratio. The large side PCA can be limited to a radius (R') of 400 mm to enable machinability on most machine tools, resulting in an inner radius (R) of 200 mm; the corresponding small side radius is of (R) ~67 mm, with an outer radius (R') of 134 mm.

$$P = \frac{F_m}{A_{eff}} \cong \frac{20 \text{ MN}}{0.377 \text{ mm}^2} \cong 53 \text{ MPa} \quad (4-36)$$

$$A_{eff} = \frac{F_m}{P} \cong \frac{250 \text{ kN}}{53 \text{ MPa}/3} \cong 14.1 \text{ mm}^2 \quad (4-37)$$

4.3.1.4 Sizes of the gaps

Now that the nominal sizes of the PCAs have been defined, it remains to size the gap that will be used on each side. When sizing the gaps it is necessary to consider the way the machine functions. After the weights and transducer are loaded, the oil will be continually pressurised to compensate for the fall rate of the large PCA so that the transducer is pressurised at a constant height, consequently the small side PCA supporting the weights will be allowed to move according to the difference in fall rate. It is therefore necessary

to set the fall rates so that they are close enough to each other such that the weights are supported for a long enough time to carry out measurements. The previously derived formula for flow is converted for fall rate to perform the calculation, the engagement length also plays a part in the formulation:

$$Q = P \frac{\pi d \left(h + \frac{Pr}{E} - \frac{2Pr\sigma}{E} + \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) \right)^3}{12 \times 0.021554 \times (1 + 0.00190036P)^{8.81} L} \quad (4-38)$$

$$v = P \frac{\pi d \left(h + \frac{Pr}{E} - \frac{2Pr\sigma}{E} + \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) \right)^3}{12 \times 0.021554 \times (1 + 0.00190036P)^{8.81} L} \frac{1}{\pi r^2} \quad (4-39)$$

It is noteworthy that at gaps of approximately 1 μm the fall rates are such that over a period of 1 hr there is less movement than 100 mm, which might be sufficient for many purposes, especially because at such movements the effects of fluid head can be calculated and mitigated for. While it may be desirable to have the machine operational for a longer period, it is elected instead to have the gaps in a ratio to each other according to the nominal radii, thereby eliminating the problem concerning the true value of A_0 . While there is some question about which is the true value of A_0 , it is evident that if the same ratio is used for the gap as well as for the PCA radii, the amplification ratio is well known. Therefore, nominal dimensions of 0.5 μm for the gap of the small PCA and 3.55 μm for the large side are used, giving a resulting fall rate of around 200 mm per hour.

Screenshots are shown from the application predicting uncertainties in A_{eff} which has been used to derive tolerances upon the dimensions specified for the small and large side. It can be seen how the most stringent tolerances required are for the piston and cylinder

radius, as low as 50 nm. It can be seen how the tight tolerances are only required on the small side.

<i>Small side rotational measurement uncertainties</i>		
	<i>value</i>	<i>uncertainty</i>
Piston radius (mm)	3.96E-03	2.00E-07
(gap)	5.52E-06	2.00E-07
Pressure	2.00E+07	1.00E+06
Young's Modulus	6.30E+11	1.00E+10
Poisson's Ratio	0.22	5.00E-02
Engagement Length (m)	0.01	1.00E-02
Cylinder Inner Radius (m)	3.96E-03	2.00E-07
Cylinder Outer Radius	0.016	1.00E-04
(λ)	7.96E-13	
($\Delta\phi$)	4.92780E-05	
Temperature Change	2.00E+00	1.00E-01
Thermal Expansion Coef	4.40E-06	1.00E-06
A_{eff}	4.92787976E-05	
A_{eff} (with thermal)	4.9279665E-05	
A_{eff} uncertainty (r)	-0.000018%	
A_{eff} uncertainty (P)	0.000080%	
A_{eff} uncertainty (E)	-0.0000256%	
A_{eff} uncertainty (σ)	0.000317%	
A_{eff} uncertainty (R')	-0.000003%	
A_{eff} uncertainty (α)	0.000400%	
A_{eff} uncertainty (ΔT)	0.0000880%	
<i>Small side rotational tilt</i>		
Tilt	0.00005963%	

Figure 4-24 Application Predicting Uncertainties

4.3.2 Sensitivity Analysis

While the initial design for a directly calibrated HFA has been carried out above, the calculations particularly of the tolerances were based upon purely analytical formulae. While the analytical formulae have been proven to be reliable and useful in the past, it is worth remembering that, particularly the distortion formula, is based upon the assumption of linearly varying pressure distribution and distortion as discussed in the literature review. The assumption unfortunately is not quite accurate, so in this section the use of

the formulae to predict the effect of tolerances is verified using the FEA/CFD to carry out a sensitivity analysis.

To carry out the sensitivity analysis, the design and tolerances are viewed in the GUI with the resulting uncertainties. The previously benchmarked FEA/CFD model is modified to include the dimensions/material properties of the design, and the effective area is determined according to the procedure used in benchmarking. Next the model is adjusted so that a parameter, for example Young's Modulus (E), is adjusted by the amount of the tolerance, and the effective area is determined again. The change to effective area is gauged and it is verified how well it corresponds to the uncertainty predicted. In this way, the analytical model is validated, and confidence is gained (or otherwise) in the design methodology.

4.3.2.1 Baseline Model

First of all the model geometry was altered to adopt the geometry of the small side PCA. This PCA alone was used to avoid the replication of results, but the analysis could just have easily been done for the large side PCA. The parameters used in the study are therefore:

Table 4-7 PCA Definition

<u>Parameter</u>	<u>Value</u>
Young's Modulus	630 GPa
Poisson's Ratio	0.218
Piston Radius	0.0140995 m
Cylinder Inner Radius	0.0141 m
Cylinder Outer Radius	0.0282 m
Length	0.05 m

First of all the analysis was carried out at a pressure of 50 MPa. Next the different sensitivity analyses were carried out.

4.3.2.2 Young's Modulus (E)

A tolerance of 10 GPa had been implemented in the design because its representative of real world tolerances on the material properties. The FEA was amended so that the Young's modulus was reduced by 10 GPa to 620 GPa as had been done analytically, and the resulting change in effective area was derived as shown below, it can be seen that the analytical formulation and the FEA results are in excellent agreement, despite some minimal difference which may be caused by rounding error, giving some confidence in the analytical method:

Table 4-8 Young's Modulus Results

<u>Analytical Change in A_{eff}</u>	<u>FEA/CFD Change in A_{eff}</u>
0.000098 %	0.000096 %

4.3.2.3 Poisson's Ratio (ν)

Next, the Poisson's ratio was altered to compare the effects observed in the FEA/CFD and to check whether they match the effects predicted analytically. A tolerance of 0.05 for the Poisson's ratio was used as this again reflects tolerances for material properties, and the FEA/CFD was run with the reduced Poisson's ratio. The percentage change to effective area was gauged again to determine if it matched the analytical prediction, it can be seen that the results are in very good agreement:

Table 4-9 Poisson's Ratio Results

<u>Analytical Change in A_{eff}</u>	<u>FEA/CFD Change in A_{eff}</u>
0.00079 %	0.00079 %

4.3.2.4 Piston Radius (r)

The piston radius (r) is a special case because it is not used in the definition of lambda except to give the clearance gap (h), but rather is used to define A_0 . Therefore, the effects of changes to piston radius will necessarily be equal in the analytical and the FEA/CFD formulations. Because of this the analysis is omitted, but suffice it to say that the effects of changes to the parameter will give equal results in the FEA and in the analytical analyses.

4.3.2.5 Cylinder Inner Radius (R)

The inner radius of the cylinder (R) is the other parameter which imposes tight tolerances on the design to enable sufficiently accurate force transmission, requiring close tolerances close to 20 nm – 50 nm. The FEA/CFD was altered to increase the size of the radius by 100 nm, this tolerance having been chosen in the analysis to reduce uncertainties adequately. The resulting changes to effective area are shown below alongside those predicted analytically:

Table 4-10 Inner Cylinder Radius Results

<u>Analytical Change in A_{eff}</u>	<u>FEA/CFD Change in A_{eff}</u>
0.0014 %	0.0014 %

4.3.2.6 Cylinder Outer Radius (R')

Likewise the external radius of the cylinder was subjected to sensitivity analysis in FEA/CFD to see if the analytical predictions were accurate. The size of the cylinder was increased by 1 mm as this was the tolerance set due to the low impact of this dimension.

Table 4-11 Outer Cylinder Radius Results

<u>Analytical Change in A_{eff}</u>	<u>FEA/CFD Change in A_{eff}</u>
0.00027 %	0.00027 %

4.3.2.7 Conclusion

The results above show the close agreement of the analytical model to the FEA/CFD in terms of the prediction of tolerances, thereby ensuring that at least for the design proposed

the tolerances derived indicate the uncertainties derived. It remains to be seen whether with a larger gap and less linear pressure distribution, the predictive power of the analytical formulae remain or not. It is reasonable however to predict that when small gaps are used that the analytical formulae will be representative of behaviour of PCAs, and can be used to reliably predict tolerances. Therefore, the design of the HFA is concluded and seems by all accounts to be possible and to deliver the reduced uncertainties desired.

5 NON-ROTATIONAL PCA

In this section the nonrotational variant of PCA is considered. Therefore, the section is largely a repeat of the previous section, with the caveat that there are additional complications with both the FEA/CFD analysis and analytical formulae, so these complications are addressed also. The work carried out is summarised in the diagram below:

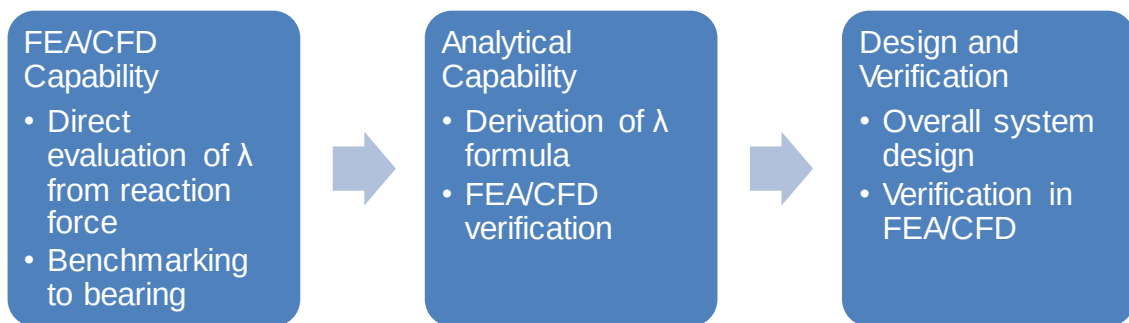


Figure 5-1 Nonrotational Work Overview

5.1 FEA/CFD

5.1.1 Direct Aeff Benchmarking

FEA/CFD of piston/cylinders carried out previously has had distortions of the piston and cylinder, as well as pressure along the gap height as output parameters, the reason for this is that they can then be used by analytical formulae which convert them into useful outputs such as distortion coefficient. Despite this, the conventional methodology cannot unfortunately be used in evaluation of PCAs with complex geometry which does not lend itself to the direct extraction of distortions along the piston/cylinder length. As an example, in the case of a PCA including a bearing, it can be assumed that the inlet at the bearing has no real distortion itself, and therefore cannot be processed using this approach. Therefore, an alternative approach is needed which enables processing of these complex geometries.

The customary approach has however overlooked a capability existent in the FEA/CFD software, that of measuring the reaction force at the top of the piston directly, thereby surpassing the analytical phase and giving a direct force measurement as an output to the analysis. In this study, because of the complexity of some of the geometries involved (of rotational PCAs), which do not lend themselves to measurement of distortion along the entire length of the cylinder (which contains a bearing surface), the alternative approach of direct force measurement is adopted which enables an equivalent analysis of the PCA's with bearings to be carried out despite these restrictions, thereby enabling evaluation of the nonrotational distortion formula:

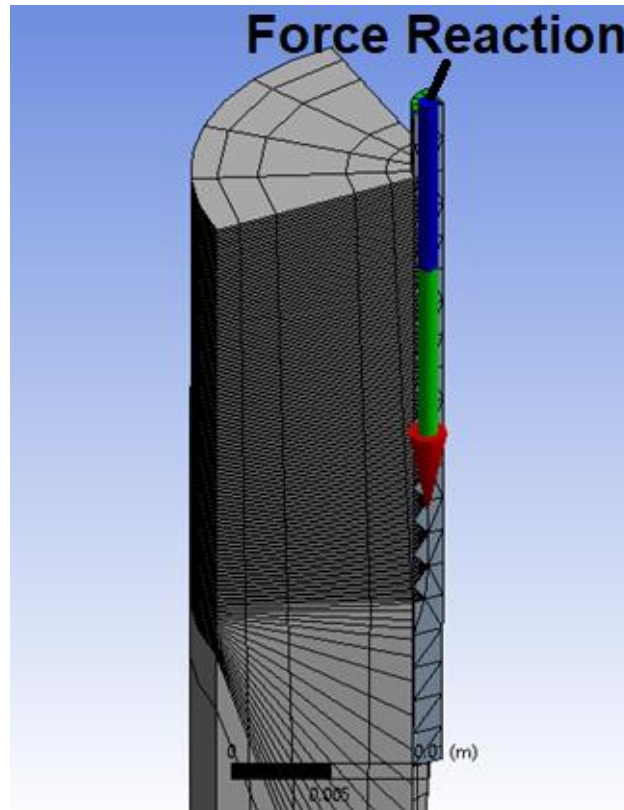


Figure 5-2 Force Reaction

However, the alternative method of obtaining force measurements directly from the FEA model must be benchmarked to results from the literature as the first analytical method was, thereby ensuring reliability and meaningfulness of measurements. The methodology for such benchmarking is outlined here. Once the FEA capability is developed, it should enable as for the rotational PCA's verification of the HFA design from analytical models.

To outline the predominant characteristic of the direct measurement, this involves extraction of the force at the top of the piston subsequent to an analysis being carried out. The force measurement made is proportional to the portion of a full 360 degree PCA being measured, so for a 90 degree slice, the force is taken as representing 25 % of the force generated.

With regards to the specifics of the analysis, the geometry was created before, this and the meshing were likewise carried out as previously on the PTB 400 MPa PCA, details are omitted here to avoid repetition. The boundary conditions were however different, including the vertical force reaction condition on the top of the piston:

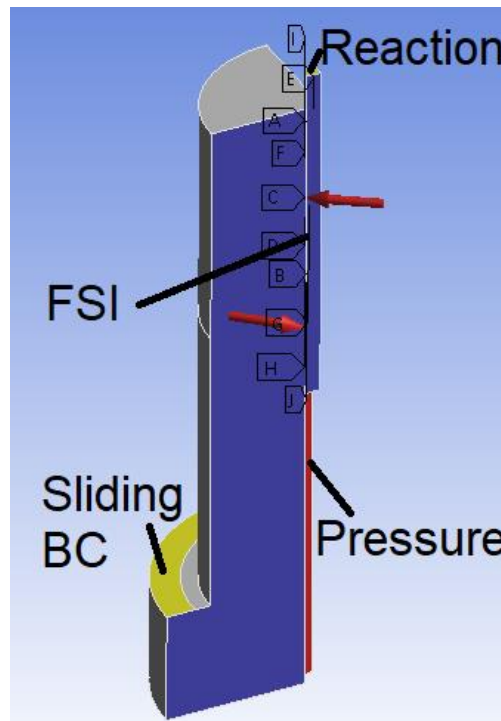


Figure 5-3 FEA/CFD Schematic

It had to be ensured that the reaction force converged, and the number of iterations could be adjusted to ensure for this. The fluid analysis was as before, containing 400 iterations in the fluid simulation, and 10 iterations between fluid and mechanical simulation, the results converged to within 1 %, and the result for force of the vertical reaction could be read directly from the analysis.

It is essential that the above process be benchmarked to experimental results in order to ensure that it achieves results which are truly representative. Therefore, the benchmarking procedure carried out for effective area previously was simply repeated

with the aforementioned alteration of collecting output data by direct force measurement, using the reaction force boundary condition at the top of the piston.

To evaluate the capability of direct evaluation of effective area, the previously benchmarked PTB 400 MPa model was used. The procedure was to apply a given line pressure and to measure the reaction force at the top of the piston. Once this had been evaluated, the effective area was determined according to the formula:

$$A_{eff} = \frac{F}{P} \quad (5-1)$$

The process was repeated for two pressures (30 MPa and 50 MPa, arbitrarily chosen), and the resulting effective areas were used in the formula below to obtain the distortion coefficient:

$$A_{eff} = A_0(1 + \lambda P) \quad (5-2)$$

$$\lambda = \frac{A_{eff2} - A_{eff1}}{A_{eff1} \times P_2 - A_{eff2} \times P_1} \quad (5-3)$$

The distortion coefficient obtained was compared with the experimental results, and shown to agree well, thereby giving confidence that the direct measurement of effective area was accurate:

Table 5-1 Direct Distortion Coefficient Results

Experimental distortion coefficient (/Pa)	7.2e-13
Distortion coefficient by direct measurement (/Pa)	6.8e-13

Because of this result, confidence is given that the FEA/CFD can be used with direct extraction of the force from the top of the piston as indicative of the distortion coefficient, thereby allowing models to be used which cannot be processed in the conventional way.

5.1.2 Bearing Benchmarking

Also to enable the modelling of nonrotational PCAs which include the action of jets on the sides of the PCA, it was necessary to consider the behaviour of bearings as these are used in this study to represent the jets used in nonrotational PCAs. This feature likewise had to be benchmarked to experimental results to enable the verification of the analytical design of the HFA. Because of this, there were two components to the current benchmarking which were necessary:

- a) Ensuring that the bearings could be reliably reproduced in FEA/CFD
- b) Ensuring that the reproduced bearings conformed with previously benchmarked PCAs

Therefore, to achieve these requirements, the bearing of Metman et al (1986) was first reproduced in the FEA/CFD, but modelled as closely as possible to the way in which PCAs had previously been modelled. That is to say, while although some parameters needed to be changed (such as to allow pressure inlets at the sides), the overall geometry, constraints, boundary conditions and meshing remained as closely aligned to those used previously for modelling of PCAs, the table shows the parameters used:

Table 5-2 Bearing Parameters

<u>Parameter</u>	<u>Value</u>
Journal Diameter	60.114 mm
Journal Length	60.18 mm
Bearing Length	30.09 mm
Bearing Width	60 °

A full 360° model was used because it was necessary to model eccentricities of the piston/journal, and the pockets in these cases had different recess pressures. The physical model was as shown below:

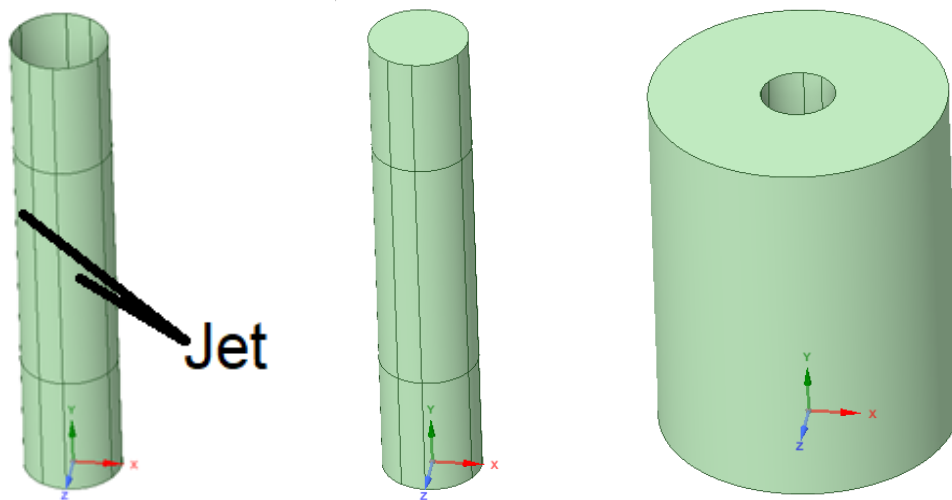


Figure 5-4 FEA/CFD Geometries

It can be seen how the geometry is like that of the PCA, except that the surfaces of the fluid and the corresponding faces of the piston and cylinder are partitioned into several sections, which allow for some of these sections to be used as jets upon which constant pressures are applied. Metman's (1986) results which were used for benchmarking are

shown graphically below, which has journal eccentricity on the horizontal axis, and a parameter representing lateral force on the vertical axis:

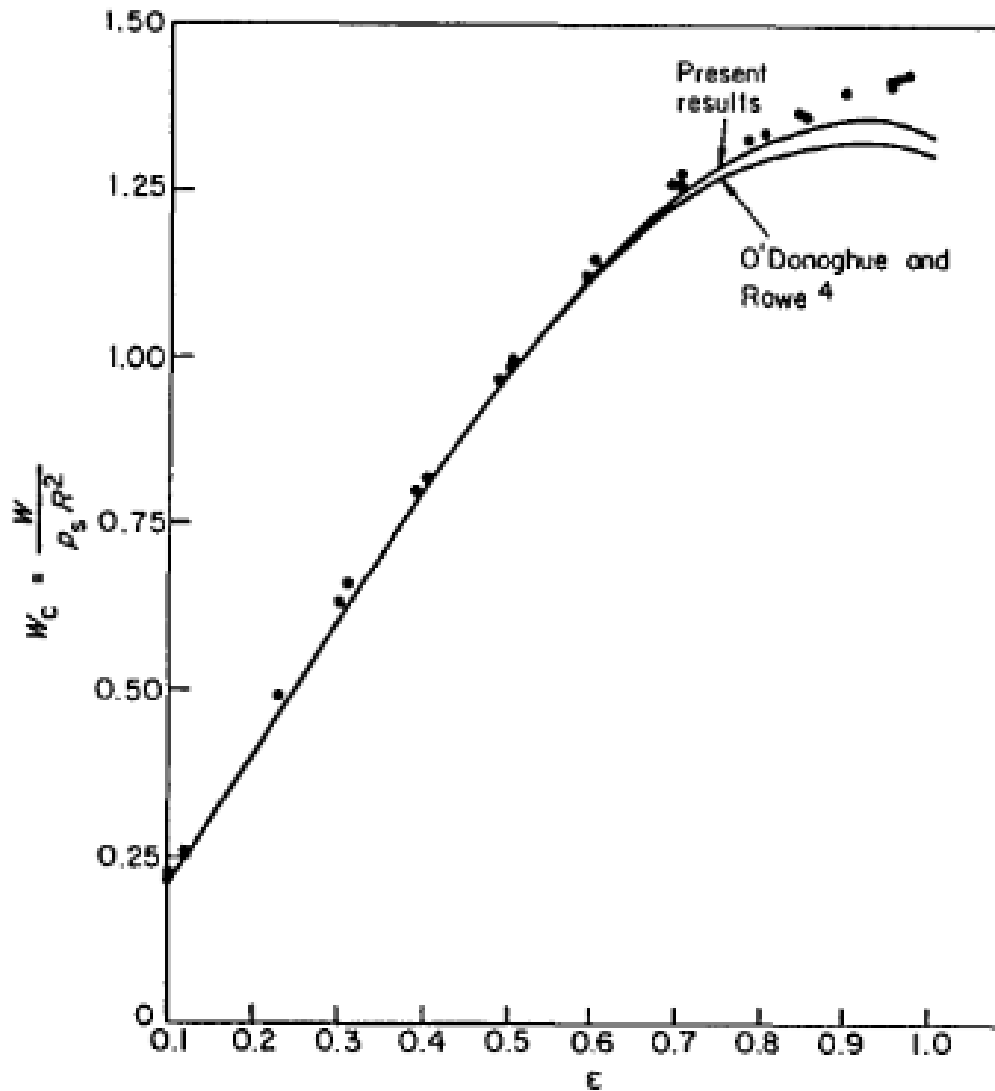


Figure 5-5 Results of Metman (1986)

With regards to meshing, the fluent mesh was made to be regular in size despite the partitions of the fluid surface, infact the gap surface was made to have regular (non-varying) mesh on both of the structural meshes also. There were 90 divisions vertically along the face of the piston, 96 along the height of the cylinder and 80 along the height of the fluid. The meshed components are shown below:

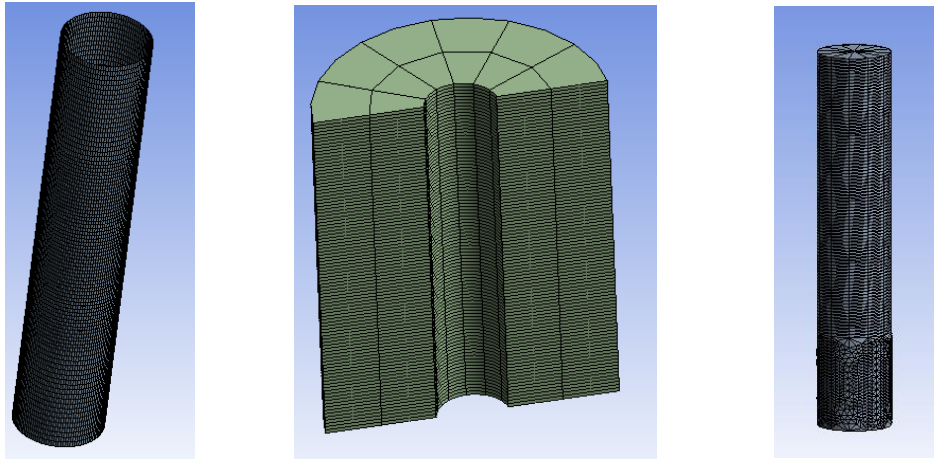


Figure 5-6 Bearing Meshing

The image below shows comparison of the boundary conditions on the fluid area of the bearing and PCA models.

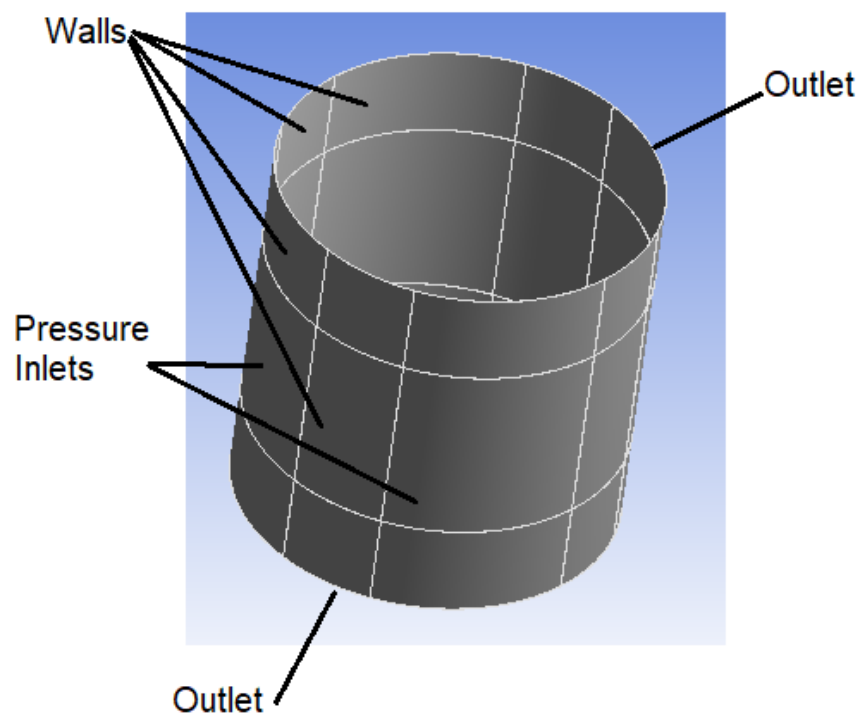
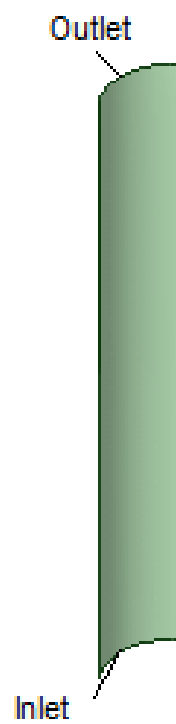


Figure 5-7 Bearing Fluid Boundary Conditions

For the bearing model, each of the walls was paired with a piston/cylinder wall and made an FSI so that distortion was accounted for. For the recess pressure, the outer face of the fluid component was given a constant pressure, and the corresponding face of the cylinder was left without boundary condition. Full details of the modelling is given below:

Table 5-3 Fluent Parameters

Scheme	Coupled
Gradient Discretisation	Green-Gauss Node Based
Pressure Discretisation	PRESTO!
Momentum Discretisation	Third-Order MUSCL
Pressure Under-Relaxation Factor	0.05
Density Under-Relaxation Factor	0.05
Body Forces Under-Relaxation Factor	0.1
Momentum Under-Relaxation Factor	0.1

5.1.2.1 Bearing Distortion

It was necessary to evaluate how well this model was matched to the PCA model of the PTB 400 MPa. Therefore, to do this the above model was altered to have dimensions equivalent to those of the PTB 400 MPa piston/cylinder, and the distortion coefficient was evaluated using the direct method outlined previously. That is to say, the force reaction at the top of the piston was evaluated at two different pressures, and the effective

areas were compared along with the known line pressures to derive the distortion coefficient, according to the formulae:

$$\lambda = \frac{A_{eff2} - A_{eff1}}{A_{eff1} \times P_2 - A_{eff2} \times P_1} \quad (5-4)$$

Results were obtained for pressures of 10 MPa and 20 MPa, results of the comparison are shown below:

Table 5-4 Distortion Coefficient Results

	Distortion Coefficient (λ)
Derived result (/Pa)	7.1e-13
Experimental Result (/Pa)	7.2e-13

It can be seen how the result agrees to within 2 %, thereby giving confidence that, in terms of distortion, the model gives results comparable to experimental results, and those of the PCA model also. The accuracies actually achieved by the model are comparable to those where distortions along the side of the PCA were computed, with very low error compared to experimental results.

5.1.2.2 Benchmarking for Bearing Performance

The aforementioned constituting the second requirement for modelling, the initial requirement of the bearing accurately reflecting experimental results is obtained by following the information of Metman concerning the pressures of the bearing surfaces at

various degrees of eccentricity. The eccentric conditions are replicated with the pressure in the recesses set according to Metman, it is observed whether the load capacity of the bearing is equal to that measured by Metman (this is achieved by measuring the total pressure gradient across the surfaces of the journal).

In this section, the performance of the bearing was benchmarked relative to the FEA/CFD model which simulated the load conditions of the bearing. The model was created and the four pockets were pressurised according to the values given in the paper of Metman . The bearing was given the required eccentricity and the overall pressure in the direction of the load along the journal surface was measured in the FEA/CFD system. The chart below shows the results which were obtained, with the horizontal axis corresponding to eccentricity, and the vertical referring to the overall load on the bearing:

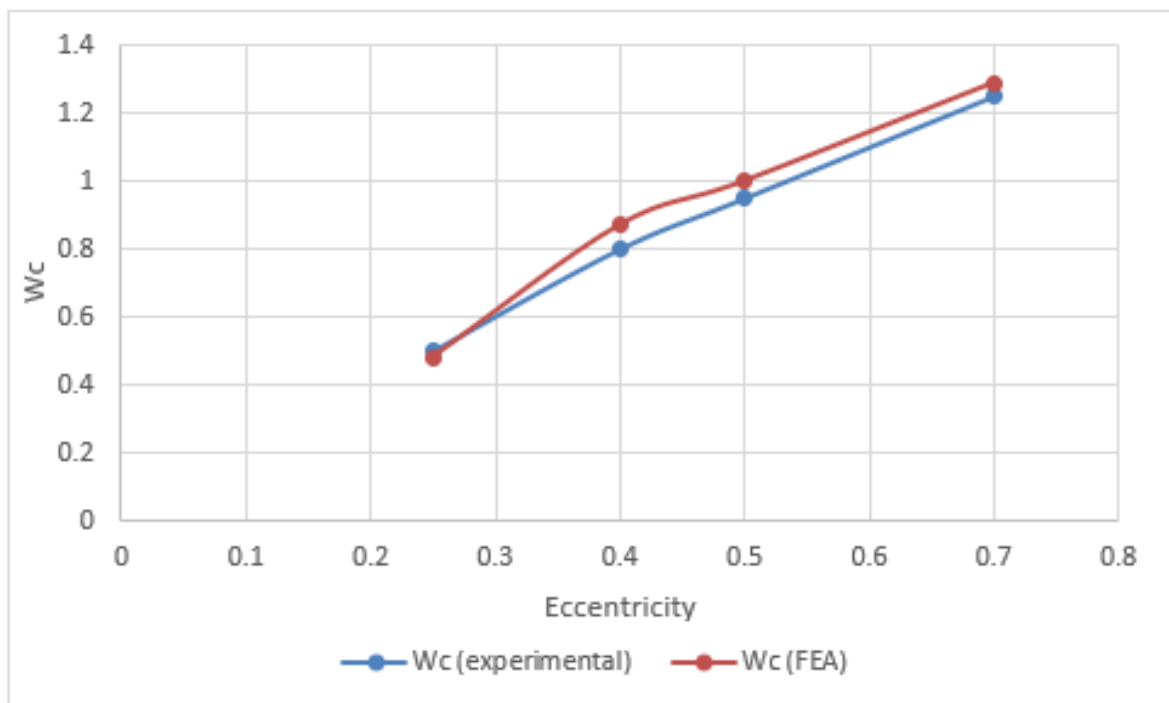


Figure 5-8 Bearing Benchmarking Results

It can be seen how the results obtained in the FEA/CFD match very closely to the results obtained experimentally, in most cases to within 12.5 %. Because of this, confidence can be taken that the FEA/CFD is producing reasonable results, and could be used in the verification of the HFA design, and evaluation of the derived formula for λ using a pressure distribution that is representative of experimental results. The discrepancy between the experimental and numerical results are to be expected to a certain extent because the model created was not ultra-precise, as only the nominal dimensions given in by Metman (1986) were used, which could have missed some of the finer detail of the bearing geometry.

5.2 Adapted Analytical Formulae

Having established the FEA/CFD capability for dealing with nonrotational PCA's, we move on to derivation of a nonrotational formula for λ with confidence that it is possible to evaluate the effectiveness of the formula accurately. While it may be possible to evaluate a vast number of configurations of bearings, the simpler case with a single bearing inlet similar to the bearing of Metman is evaluated in this work for simplicity, it is expected, however, that various configurations can be evaluated using the same methodology.

5.2.1 Distortion Coefficient Formula

A new problem which arises with the use of non-rotational PCAs is the determination of their distortion coefficient, which must be known to carry out the design of nonrotational HFA's. Because non-rotational PCAs include the use of a bearing face as part of the cylinder, the traditional formula of Dadson (1985) is not immediately applicable, as the geometry includes a portion of the cylinder face on which a constant

pressure is applied, and which may or may not distort in the conventional way. To attempt to remedy this potential problem area, the FEA/CFD tools including direct evaluation of distortion coefficient, are leveraged to determine a formula for distortion coefficient which adequately describes distortion of the PCA with the presence of bearings. The approach used can be broken into two distinct components:

- a) Creating a model for distortion coefficient based on other models of Dadson (1985)
- b) Benchmarking the applicability of the model with the FEA/CFD in direct measurement of effective area

Because of the capability of the FEA/CFD to directly evaluate the distortion coefficient without the need to resort to analytically exported results, it is possible to create a model using the basic analytical formulae and evaluate how well it agrees with the directly evaluated forces taken from the FEA/CFD. The diagram below places the work in context:

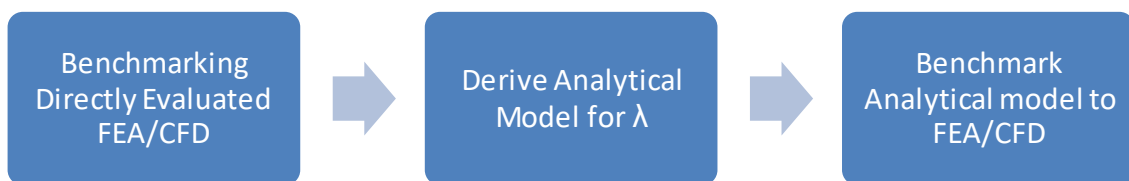


Figure 5-9 Analytical Formula Process

The formula of Dadson (1985) below is used along with the following assumptions:

- Pressure gradient is constant from the base to the bearing
- Pressure is equal to line pressure at the base of the PCA
- Pressure is constant along the face of the bearing

- Pressure gradient is constant above the bearing, and is equal to zero at the top of the assembly
- Pressures don't significantly alter around the PCA circumference

Using the assumptions given, it's possible to envision the idealised PCA with constantly reducing pressure from the base up to the bearing, which has a constant pressure applied along it's height, which then trails off gradually to zero at the top.

Having made the necessary assumptions concerning the configuration of pressures involved, it remains to input the constraints into relevant formulae of Dadson (1985) which use the pressures involved to determine an overall distortion coefficient or effective area for the PCA. The mathematical descriptors of the different sections of the PCA are added together before computation by the formula, and the overall result is obtained. Again, the results obtained are compared to results computed by directly evaluated FEA/CFD analyses, to ensure the validity of the processes involved, thereby providing a reliable formula for distortion coefficient with bearing included.

The formula derived is based upon the formula of Dadson:

$$\lambda = \frac{1}{1 + 2h/r} \left[\frac{U(0) + u(0)}{rp} + \frac{1}{rp^2} \int_0^{l_0} p(z) \left(\frac{dU}{dz} + \frac{du}{dz} \right) dz \right] \quad (5-5)$$

The engagement length is assumed to be split into three parts:

- 1) From the base to the bearing
- 2) The bearing
- 3) From the bearing to the top

The integral can be split into three intergrals across each part of the engagement length.

As such, the overall formula becomes:

$$\lambda = \frac{1}{1 + 2h/r} \left[\frac{U(0) + u(0)}{rp} + \frac{1}{rp^2} \int_0^{l_1} p(z) \left(\frac{dU}{dz} + \frac{du}{dz} \right) dz \right. \\ \left. + \frac{1}{rp^2} \int_{l_1}^{l_2} p(z) \left(\frac{dU}{dz} + \frac{du}{dz} \right) dz + \frac{1}{rp^2} \int_{l_2}^L p(z) \left(\frac{dU}{dz} + \frac{du}{dz} \right) dz \right] \quad (5-6)$$

Where l_1 is the height of the bottom of the bearing, l_2 is the height at the top of the bearing and L is the total engagement length

If $p(z)$ is assumed to vary linearly, the average value can be taken so the integral term can be simplified as follows:

$$integral\ length \times \left[\frac{p(base) + p(base)}{2} \left(\frac{dU}{dz} + \frac{du}{dz} \right) \right] \quad (5-7)$$

Therefore, the overall formula is given by:

$$\lambda = \frac{1}{1 + 2h/r} \left[\frac{U(0) + u(0)}{rp} + \frac{1}{rp^2} (l_1) \left(\frac{p(0) + p(1)}{2} \right) \left(\frac{dU}{dz} + \frac{du}{dz} \right) + \frac{1}{rp^2} (l_2 \right. \\ \left. - l_1) \left(\frac{p(1) + p(2)}{2} \right) \left(\frac{dU}{dz} + \frac{du}{dz} \right) + \frac{1}{rp^2} (L \right. \\ \left. - l_2) \left(\frac{p(2) + 0}{2} \right) \left(\frac{dU}{dz} + \frac{du}{dz} \right) \right] \quad (5-8)$$

It remains of course for the integrals to be replaced by their values as derived by the same equations:

$$u(P) = \frac{-Pr}{E} + \frac{P\sigma r}{E} + \frac{P_L \sigma r}{E} \quad (5-9)$$

$$U(P) = \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) \quad (5-10)$$

For the calculations, it is necessary to determine the distortions of the piston and cylinder at four points along the PCA:

- 1) At the base
- 2) At the bottom of the recess
- 3) At the top of the recess
- 4) At the top of the PCA

To expedite the process, the resulting solutions are shown in the table below, and the following table gives values of $\frac{du}{dx}$ and $\frac{dU}{dx}$ as required:

Table 5-5 Boundary Conditions

	u(P)	U(P)
At the top of the PCA	$\frac{P_L \sigma r}{E}$	0
At the top of the recess	$\frac{-P_r r}{E} + \frac{P_r \sigma r}{E} + \frac{P_L \sigma r}{E}$	$\frac{RP_r}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right)$
At the bottom of the recess	$\frac{-P_r r}{E} + \frac{P_r \sigma r}{E} + \frac{P_L \sigma r}{E}$	$\frac{RP_r}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right)$
At the base	$\frac{-P_L r}{E} + \frac{P_L \sigma r}{E} + \frac{P_L \sigma r}{E}$	$\frac{RP_L}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right)$

Table 5-6 Gradients

	du/dx	dU/dx
Top portion	$(\frac{-P_r r}{E} + \frac{P_r \sigma r}{E})/(L - l2)$	$\frac{RP_r}{E} (\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma)/(L - l2)$
Recess	0	0
Lower portion	$(\frac{-P_r r}{E} + \frac{P_r \sigma r}{E} - \frac{P_L r}{E} - \frac{P_L \sigma r}{E})/l1$	$(\frac{RP_r}{E} (\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma) - \frac{RP_L}{E} (\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma))/l1$

Therefore, the expanded formula is given as:

$$\lambda = \frac{1}{1 + 2h/r} \left[\frac{\frac{RP_L}{E} (\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma) + \frac{-P_L r}{E} + \frac{P_L \sigma r}{E} + \frac{P_L \sigma r}{E}}{rp} \right. \quad (5-11)$$

$$+ \frac{1}{rp^2} (l1) (\frac{p(0) + p(1)}{2}) \left((\frac{-P_r r}{E} + \frac{P_r \sigma r}{E} - \frac{P_L r}{E} - \frac{P_L \sigma r}{E})/l1 \right.$$

$$+ \left(\frac{RP_r}{E} (\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma) - \frac{RP_L}{E} (\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma) \right)/l1 \Bigg)$$

$$+ \frac{1}{rp^2} (L - l2) (\frac{p(2) + 0}{2}) \left(\frac{RP_r}{E} (\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma)/(L - l2) + (\frac{-P_r r}{E} + \frac{P_r \sigma r}{E})/(L - l2) \right) \Bigg]$$

The integrated formula was copied across into Excel, and tested against FEA/CFD results for piston/cylinder pressure distribution and distortion which had previously been

obtained and benchmarked. It was necessary to set the two constants of integration and the flow through the PCA, these were set according to the three boundary conditions shown in the table below:

Table 5-7 Boundary Conditions

<u>Boundary Condition</u>	<u>Requirements</u>
1	$P_L = 0, P = 0, Q = 0$
2	$P = 0, L = 0$
3	$P = P_L, L = \text{total engagement length}$

The results were computed for the geometry of the BNM-LNE 200 MPa pressure balance, and compared with a model using direct assessment of λ , the agreement of results to 2.6 % is shown in the table below:

Table 5-8 Distortion Coefficient Results

Analytical formula (/Pa)	$7.26e^{-13}$
FEA/CFD direct assessment (/Pa)	$7.09e^{-13}$

5.2.2 Flow Formula

It is necessary to predict the flow in the piston/cylinders of the HFA, the purpose for this is to match the flow on the large and small side to enable enough stability for measurements to be taken in the design stage. There is a well defined formula for flow

(Dadson, 1985), but this does not consider the effects of distortion of the piston/cylinder gap, which leads to variations in flow speed, and also the effects of pressure dependent fluid viscosity and density. Therefore, a formula is derived which considers these parameters in this section, the process is shown below:

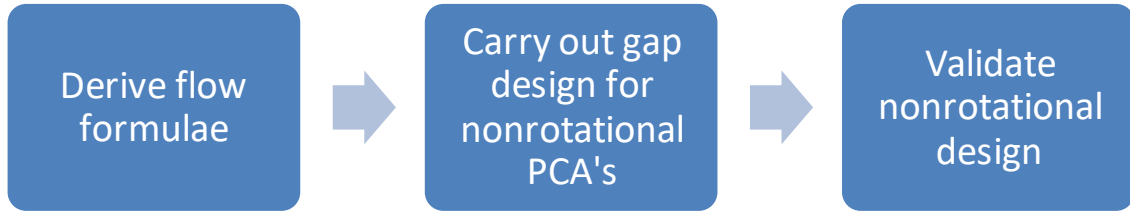


Figure 5-10 Flow Formula Process

To derive the formula, the traditional formula of Dadson (1985) is used, along with formulae for pressure dependent viscosity and the Lamé equations, which describe the distortion response of the cylinder and piston, similar to the process for the rotational PCA of section 4.1.2. The traditional formula of Dadson (1985) is first differentiated, and then the individual terms for distortion and viscosity are substituted into the formula.

The point considered in the nonrotational PCA is the length above the bearing, from the top of the bearing to the top of the engagement length. This point is taken because it includes flow both from the gap exposed to the line pressure, and from the bearing. The assumption is made that flow is equal axi-symmetrically, which was broadly observed in the analysis.

The formula was derived according to the methodology used for rotational PCA flow, but the final formulae with simplifications is shown below:

$$u(P) = \frac{-Pr}{E} + \frac{P\sigma r}{E} + \frac{P_L\sigma r}{E} \quad (5-12)$$

where P is pressure at a given height in the engagement length, U is cylinder distortion, u is piston distortion, R is cylinder inner radius, E is Young's modulus, R' is cylinder outer radius, σ is Poisson's ratio, r is piston radius and P_L is line pressure

Also necessary for the analytical derivation is the formula for viscosity, di-ethylhexyl sebecate was used as the oil with a formula for pressure dependent viscosity given by Dogra et al (2010):

$$\eta(P) = 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81} \quad (5-13)$$

The analytical derivation is based on the formula for flow in the piston/cylinder gap given by Dadson (1985):

$$Q = \frac{\pi d \delta^3 P}{12 \eta L} \quad (5-14)$$

where Q is volumetric flow, d is piston diameter, δ is the gap and L is engagement length

To allow for analysis without considering the entire engagement length (ie at a single height), thereby allowing distortion to be considered which varies along the length, the formula is rearranged in terms of P:

$$P = \frac{12 \eta L Q}{\pi d \delta^3} \quad (5-15)$$

and is differentiated with respect to L, giving the rate of change of pressure along the engagement length:

$$\frac{dP}{dL} = \frac{12 \eta Q}{\pi d \delta^3} \quad (5-16)$$

The expressions of piston/cylinder distortion are included in the formula:

$$\frac{dP}{dL} = \frac{12\eta Q}{\pi d(h_0 + U(P) - u(P))^3} \quad (5-17)$$

where h_0 is zero pressure gap

and the equation for pressure dependent viscosity is also included:

$$\frac{dP}{dL} = \frac{12 \times 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81} Q}{\pi d(h_0 + U(P) - u(P))^3} \quad (5-18)$$

Next, the formula was integrated with respect to L, and rearranged to give the length in terms of the pressure and flow:

$$P = \frac{12 \times 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81} Q}{\pi d(h_0 + U(P) - u(P))^3} L \quad (5-19)$$

$$P \frac{\pi d(h_0 + U(P) - u(P))^3}{12 \times 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81} Q} = L \quad (5-20)$$

For the current application, two points of interest are analysed to derive the correct value of Q which relates P to L, they are:

- 1) The top of the bearing, where $P=P_{\text{recess}}$,
- 2) The top of the engagement length, where $P=P_0$

So in effect the flow is determined according to:

$$P \frac{\pi d(h_0 + U(P) - u(P))^3}{12 \times 0.021554 \times (1 + 1.90036 \times 10^{-3}P)^{8.81} L} = Q \quad (5-21)$$

Where P is the recess pressure and L is the distance above the top of the bearing

The above formula relates the pressure in the clearance gap, to the pressure at the base, and the dimensional measurements of the piston/cylinder. In order to use the formula, it is necessary to set the flow and the boundary conditions, which comprise of the pressure at the base and top of the piston.

5.3 Design and Verification

Having carried out the preliminary work required to enable design of nonrotational PCA's, the results are now used to enable a design of 'primary' HFA of the nonrotational type. The process map is shown again:

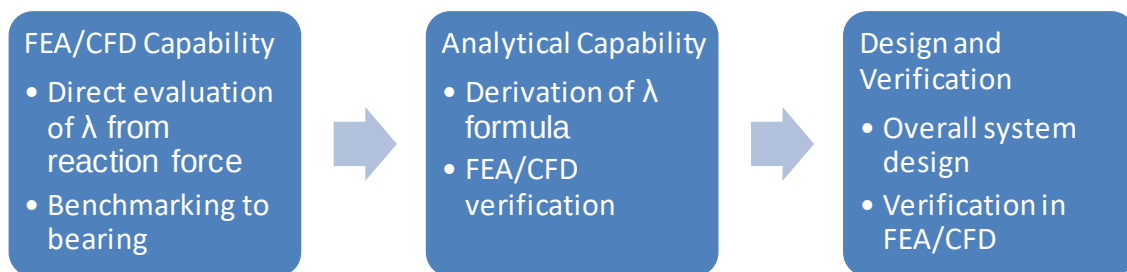


Figure 5-11 Nonrotational Work Overview

5.3.1 Non-rotational HFA Design

In this section the design is carried out for the nonrotational HFA, using the analytical formulae which were previously determined.

5.3.1.1 Amplification Ratio

The first step in the design of the primary HFA using nonrotational PCAs is definition of the amplification ratio used. There is no real reason to assume a different specification

than the one which was derived for the rotational PCA in broad terms, therefore, the table below can be created as a broad specification concerning the amplification ratio. It is noteworthy that because the derivation of A_0 is the same for a nonrotational PCA as it is for the rotational PCA, that A_0 is calculated using the same formula as was used previously.

Table 5-9 PCA Design

<u>Parameter</u>	<u>Nominal Value</u>	<u>Justification</u>
Number of large side PCAs	1	Most simple design
Weight stacks	250 kN	A commonly used value
Produced Force	20 MN	Meeting the requirements for mid-weight force calibrations
Amplification Ratio	80	Based on ratio of forces
Number of large side PCA's	3	-

5.3.1.2 Material

Tungsten Carbide (WC) is suggested again as the material, this being preferred to steel because of its increased Young's modulus ($E=630\text{ GPa}$, $\sigma=0.21$). WC can be ground and polished much the same way that steel can, allowing nanometric finishes to be achieved as required.

5.3.1.3 Radii of PCAs

Repeating from the rotational design- it is suggested that tolerances on the small and large side PCA's are of the nanometric value (20 nm – 50 nm) as sensitivity analysis shows that such small tolerances are required to keep the uncertainties involved in the order of 10 ppm. Such tight tolerances are achievable using the polishing process as shown in Figure 2-19, with metrology capability for such measurements as outlined in section 2.7. It is also suggested that the ratio of R' be the same on the small and large sides, such that the distortion coefficient is kept the same on both sides, resulting in a constant amplification ratio. The large side PCA can be limited to a radius (R') of 400 mm to enable machinability on most machine tools, resulting in an inner radius (R) of 200 mm; the corresponding small side radius is of (R) ~67 mm, with an outer radius (R') of 134 mm.

$$P = \frac{F_m}{A_{eff}} \cong \frac{20 \text{ MN}}{0.377 \text{ mm}^2} \cong 53 \text{ MPa} \quad (5-22)$$

$$A_{eff} = \frac{F_m}{P} \cong \frac{250 \text{ kN}}{53 \text{ MPa}/3} \cong 14.1 \text{ mm}^2 \quad (5-23)$$

5.3.1.4 Sizes of the gaps

In the design for the non-rotational PCA's, it becomes necessary to consider equalising the flow from the centralising jets as well as from the base of the cylinder. The length of interest therefore becomes the distance from the top of the recess to the top of the cylinder, and the pressure of interest is now half that of the line pressure (assuming this is used as the centralising pressure). Therefore, formula becomes:

$$Q = \frac{P_L}{2} \frac{\pi d \left(h + \frac{Pr}{E} - \frac{2Pr\sigma}{E} + \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) \right)^3}{12 \times 0.021554 \times (1 + 0.00190036P)^{8.81} (L - L_2)} \quad (5-24)$$

$$v = \frac{P_L}{2} \frac{\pi d \left(h + \frac{Pr}{E} - \frac{2Pr\sigma}{E} + \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma \right) \right)^3}{12 \times 0.021554 \times (1 + 0.00190036P)^{8.81} (L - L_2)} \frac{1}{\pi r^2} \quad (5-25)$$

It is noteworthy that at gaps of approximately 1 μm the fall rates are such that over a period of 1hr there is less movement than 100 mm, which might be sufficient for many purposes considering that pistons can have movement lengths of 100 mm or more. While it may be desirable to have the machine operational for a longer period, it is elected again to have the gaps in a ratio to each other according to the nominal radii, thereby eliminating the problem concerning the true value of A_0 . Therefore, nominal dimensions of 0.5 μm for the gap of the small PCA and 3.55 μm for the large side are used.

A screenshot is shown from the application predicting uncertainties in A_{eff} which has been used to derive tolerances upon the dimensions specified for the small side. The calculation involves determining the percentage difference in effective area caused by introducing a deviation in each parameter corresponding to the tolerance of that parameter. It can be seen how the most stringent tolerances required are for the piston and cylinder radius, as low as 20 nm.

	value	uncertainty
Small Side Radius (m)	0.005	2.00E-08
(gap, m)	1.00E-06	2.00E-07
(A ₀)	7.86E-05	
Cylinder Inner Radius (m)	5.00E-03	2.00E-08
Pressure (Pa)	5.00E+07	1.00E+06
Young's Modulus	6.30E+11	2.00E+10
Cylinder Outer Radius (m)	2.82E-02	5.00E-03
Poisson's Ratio	2.18E-01	5.00E-02
Recess Height (m)	1.50E-02	1.00E-06
Recess Pressure (Pa)	2.50E+07	1.00E+06
Engagement Length (m)	5.00E-02	1.00E-06
Top of Recess (m)	3.50E-02	1.00E-06
(λ)	-1.44E-12	
A _{eff}	7.86E-05	
A _{eff} uncertainty (r)	0.000800	
A _{eff} uncertainty (h)	0.0000005748	
A _{eff} uncertainty (R)	0.0008	
A _{eff} uncertainty (P _L)	-0.000104	
A _{eff} uncertainty (E)	0.000221198	
A _{eff} uncertainty (R')	-0.0001102	
A _{eff} uncertainty (σ)	0.0009918	
A _{eff} uncertainty (L ₁)		
A _{eff} uncertainty (P _r)		
A _{eff} uncertainty (L)		
A _{eff} uncertainty (L ₂)		

Figure 5-12 Small Side Tolerances

5.3.2 Sensitivity Analysis

While the initial design for a directly calibrated HFA has been carried out above, the calculations particularly of the tolerances were based upon purely analytical formulae. While the analytical formulae have been proven to be reliable and useful in the past, it is worth remembering that, particularly the distortion formula, is based upon the assumption of linearly varying pressure distribution and distortion as discussed in the literature review. The assumption unfortunately is not quite accurate, so in this section the use of the formulae is verified using the FEA/CFD to carry out a sensitivity analysis.

Particularly noteworthy in this section is that the distortion coefficient was found to be different according to the FEA/CFD when compared to the analytical prediction. This may be because of the high pressures used, but highlights how in the use of the machine the distortion coefficient must be considered as per the FEA/CFD and not per the analytical formula. Although this difference does not impact tolerances in the design, it highlights how FEA/CFD is still required for the design of nonrotational PCA's and purely analytical formulae can't be considered alone.

To carry out the sensitivity analysis, the design and tolerances are viewed in the GUI with the resulting uncertainties. The previously benchmarked FEA/CFD model is modified to include the dimensions/material properties of the design, and the effective area is determined according to the procedure used in benchmarking. Next the model is adjusted so that a parameter, for example Young's Modulus (E), is adjusted by the amount of the tolerance, and the effective area is determined again. The change to effective area is gauged and it is verified how well it corresponds to the uncertainty predicted. In this way, the analytical model is validated, and confidence is gained (or otherwise) in the design methodology.

5.3.2.1 Baseline Model

First of all the model geometry was altered to adopt the geometry of the small side PCA. This PCA alone was used to avoid the replication of results, but the analysis could just have easily been done for the large side PCA. The parameters used in the study are therefore:

Table 5-10 PCA Definition

<u>Parameter</u>	<u>Value</u>
Young's Modulus	630 GPa
Poisson's Ratio	0.218
Piston Radius	0.0140995 m
Cylinder Inner Radius	0.0141 m
Cylinder Outer Radius	0.0282 m
Length	0.05 m
Recess Pressure	0.25 MPa
Line Pressure	0.5 MPa

First of all the analysis was carried out at a pressure of 50 MPa. Next the different sensitivity analyses were carried out.

5.3.2.2 Young's Modulus (E)

A tolerance of 10 GPa was implemented in the design because its representative of real world tolerances on the material properties. The FEA was amended so that the Young's modulus was increased by 10 GPa to 640 GPa as had been done analytically, and the resulting change in effective area was derived as shown below, it can be seen that the analytical formulation and the FEA results are in excellent agreement (with some minor error which may be due to rounding error in the calculations), giving some confidence in the analytical method:

Table 5-11 Young's Modulus Results

<u>Analytical Change in A_{eff}</u>	<u>FEA/CFD Change in A_{eff}</u>
0.000001 %	0.0000009 %

5.3.2.3 Poisson's Ratio (ν)

Next, the Poisson's ratio was altered to compare the effects observed in the FEA/CFD and to check whether they match the effects predicted analytically. A tolerance of 0.05 for the Poisson's ratio was used as this again reflects tolerances for material properties, and the FEA/CFD was run with the reduced Poisson's ratio. The percentage change to effective area was gauged again to determine if it matched the analytical prediction, it can be seen that the results are in very good agreement:

Table 5-12 Poisson's Ratio Results

<u>Analytical Change in A_{eff}</u>	<u>FEA/CFD Change in A_{eff}</u>
0.00001 %	0.00001 %

5.3.2.4 Cylinder Outer Radius (R')

Likewise the external radius of the cylinder was subjected to sensitivity analysis in FEA/CFD to see if the analytical predictions were accurate. The size of the cylinder was increased by 1 mm as this was the tolerance set due to the low impact of this dimension.

Table 5-13 Cylinder Outer Radius Results

<u>Analytical Change in A_{eff}</u>	<u>FEA/CFD Change in A_{eff}</u>
0.0001 %	0.0001 %

5.3.2.5 Conclusion

The results above show the close agreement of the analytical model to the FEA/CFD in terms of the prediction of tolerances, ensuring that at least for the design proposed the tolerances derived indicate the uncertainties derived. It remains to be seen whether with a larger gap and less linear pressure distribution, the predictive power of the analytical formulae remain or not. It is reasonable however to predict that when small gaps are used that the analytical formulae will be representative of behaviour of PCAs, and can be used to reliably predict tolerances. Therefore, the design of the HFA is concluded and seems by all accounts to be possible and to deliver the reduced uncertainties desired.

6 APPLICATION OF RESULTS

Having carried out the design of rotational and nonrotational ‘primary’ HFA’s, in this section the analytical formulation is consolidated which enables investigation of design principles which are presented here. The chart below puts this work in context:



Figure 6-1 Final Section Structure

6.1 Summary

In this section the results from considering the previously mentioned uncertainty sources in sections 4 and 5 are consolidated, to give the overall uncertainty in the system. In this way, it is possible to determine whether the uncertainties in the transmission of force are so significant that the transmission of force is effectively traceable to the weights regardless of uncertainties in the hydraulic system, or whether it is the case that uncertainties in the hydraulic system are not relevant and do not effect the system.

As a first component of the uncertainty calculation, the measurement model is used and altered to determine the effects of uncertainty in each component, such that the uncertainty in measurement can be altered to determine the resulting uncertainty. This type of calculation is useful in determining the accuracy of the measurements required, but also imposes some machining tolerances because the parts themselves must be measurable to the desired uncertainties. The measurement model used (without showing thermal effects) is shown below:

$$A_{eff} = \pi r^2 \left(1 + \frac{2\delta}{r}\right) \left(1 + \frac{P}{E} \left(2\sigma + \frac{R^2}{R'^2 - R^2}\right)\right) \cos(\arctan((2\delta + \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma\right) + \frac{-Pr}{E} + \frac{Por}{E} + \frac{P_L \sigma r}{E})/L)) \quad (6-1)$$

Where A_{eff} is effective area, δ is the clearance gap, r is piston radius, P is pressure, E is Young's modulus, σ is Poisson's ratio, R is cylinder inner radius, R' is cylinder outer radius, P_L is line pressure

$$\frac{F1}{A1} = \frac{F2}{A2} + \Delta P \quad (6-2)$$

Where F1 is force on the weight side, A1 is effective area on the weight side, F2 is force on the transducer side, A2 is effective area on the transducer side

$$\frac{F1}{\pi r^2 \left(1 + \frac{2\delta}{r}\right) \left(1 + \frac{P}{E} \left(2\sigma + \frac{R^2}{R'^2 - R^2}\right)\right) \cos \left(\text{atan} \left((2\delta + \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma\right) + \frac{-Pr}{E} + \frac{Por}{E} + \frac{PL\sigma r}{E}\right)/L\right)} = \quad (6-3)$$

$$\frac{F2}{\pi r^2 \left(1 + \frac{2\delta}{r}\right) \left(1 + \frac{P}{E} \left(2\sigma + \frac{R^2}{R'^2 - R^2}\right)\right) \cos \left(\text{atan} \left((2\delta + \frac{RP}{E} \left(\frac{R'^2 + R^2}{R'^2 - R^2} + \sigma\right) + \frac{-Pr}{E} + \frac{Por}{E} + \frac{PL\sigma r}{E}\right)/L\right)} + \Delta P$$

The term for thermal effects, $(1 + \alpha \Delta T)$, is added to each dimensional parameter (r, R, R', h), but is omitted to allow clarity of the main components of the model. The extract from excel below shows calculations for the small side rotational PCA, with nominal values and measurement uncertainties, and the resulting uncertainties in measured force shown below. Excel was used as is customary because of the ample precision that it enables in calculations, avoiding rounding errors in the calculations and enabling the precision in the outcomes to be determined.

<u>Rotational</u>	$\bar{value}$	$\bar{Uncertainty}$
Piston Radius (m)	0.012	2.00E-07
(gap, m)	6.00E-06	1.00E-07
Pressure (Pa)	1.00E+08	1.00E+06
Young's Modulus	6.30E+11	1.00E+09
Poisson's Ratio	0.21	5.00E-02
Engagement Length (m)	0.07	1.00E-02
Cylinder Inner Radius (m)	0.012006	1.00E-03
Cylinder Outer Radius (m)	0.024	2.00E-03
(λ)	1.20E-12	
(A0)	4.53E-04	
Temperature Change	2.00E+00	1.00E-01
Thermal Coefficient	4.40E-06	1.00E-06
Aeff	4.53E-04	
Aeff (with thermal)	4.53E-04	
Aeff uncertainty (r)	0.000002%	
(Aeff uncertainty (h))	0.001665%	
Aeff uncertainty (P)	0.000120%	
Aeff uncertainty (E)	-0.0000190%	
Aeff uncertainty (σ)	0.001587%	
Aeff uncertainty (L)	0.0000007719%	
Aeff uncertainty (R)	-0.0022591%	
Aeff uncertainty (R')	-0.001434%	
Aeff uncertainty (α)	0.000400%	
Aeff uncertainty (ΔT)	0.0000880%	

Figure 6-2 Tolerance Analysis

The figure below shows a potential software output which could be utilised to show the performance of the PCAs.

Small Side		Large Side		Results	
Rotational / nonrotational	[]	Rotational / nonrotational	[]	Amplification Ratio	_____
Number of PCA's	[]	Number of PCA's	[]	Total Uncertainty	_____
Pressure range	[]	Pressure range	[]	Random Uncertainty	_____
Bearing number	[]	Bearing number	[]	Systematic Uncertainty	_____
Bearing dimensions	[] tol: []	Bearing dimensions	[] tol: []	Largest Random Uncertainties:	
Restrictor type	[]	Restrictor type	[]	Source 1 Contribution	_____
Restrictor dimensions	[] tol: []	Restrictor dimensions	[] tol: []	Source 2 Contribution	_____
Temperature control range	[]	Temperature control range	[]	Source 3 Contribution	_____
				Source 3 Contribution	_____
Young's modulus	[] tol: []	Young's modulus	[] tol: []	Largest Systematic Uncertainties:	
Poisson ratio	[] tol: []	Poisson ratio	[] tol: []	Source 1 Contribution	_____
Thermal coefficient	[] tol: []	Thermal coefficient	[] tol: []	Source 2 Contribution	_____
				Source 3 Contribution	_____
Cylinder OD	[] tol: []	Cylinder OD	[] tol: []	Source 3 Contribution	_____
Piston radius	[] tol: []	Piston radius	[] tol: []		
Cylinder ID	[] tol: []	Cylinder ID	[] tol: []		
				Calculation Uncertainty:	
				Formula 1	_____
				Formula 2	_____
				(...)	_____
		Calculation Pressure	[]		

Figure 6-3 Computer Application

6.2 Design Using Standard Tolerances

Having carried out the design of rotational and nonrotational PCA's and validated that design using FEA/CFD, the design is summarised in the table below. The design aims to employ a high amplification ratio of about 500, by using a small side PCA with ultra precise tolerances in the nanometric level, and large side nonrotational PCA's with large radii and looser tolerances. The parameters are shown in the table below:

Table 6-1 Parameters of HFA Design

Parameter	Value
Amplification Ratio	500 (4x large PCA's)
Weight stack	25600 N
Measured force	12.8 MN
Small Rotational R (tolerance/uncertainty)	17.9 mm (20 nm / 0.0006 %)
Large Nonrotational R (tolerance/uncertainty)	200 mm (0.5 μ m / 0.0006 %)

6.3 PCA Tolerance Design

<i>Small side rotational measurement uncertainties</i>		
	<i>value</i>	<i>uncertainty</i>
r	7.00E-03	2.00E-08
(h)	2.00E-06	1.00E-07
P	5.00E+07	5.00E+06
E	6.30E+11	1.00E+10
σ	0.22	5.00E-02
L	0.05	1.00E-02
R	0.007	2.00E-08
R'	0.014	1.00E-03
(λ)	1.22E-12	
(A ϕ)	1.53938E-04	
ΔT	2.00E+00	1.00E-01
α	4.40E-06	1.00E-06
A _{eff}	1.53947426E-04	
A _{eff} (with thermal)	1.5395014E-04	
A _{eff} uncertainty (r)	0.000000%	
(A _{eff} uncertainty (h))	0.002856%	
A _{eff} uncertainty (P)	0.000611%	
A _{eff} uncertainty (E)	-0.0000985%	
A _{eff} uncertainty (σ)	0.000794%	
A _{eff} uncertainty (L)	0.0000002521%	
A _{eff} uncertainty (R)	-0.0005713%	
A _{eff} uncertainty (R')	-0.000595%	
A _{eff} uncertainty (α)	0.000400%	
A _{eff} uncertainty (ΔT)	0.0000880%	

Figure 6-4 Small Side Calculated Uncertainties

	value	uncertainty
r	0.1565	5.00E-07
(δ)	3.00E-06	2.00E-07
(A ₀)	7.69E-02	
R	1.57E-01	5.00E-07
P _L	5.00E+07	1.00E+06
E	6.30E+11	2.00E+10
R'	2.82E-02	5.00E-03
σ	2.18E-01	5.00E-02
L ₁	1.50E-02	1.00E-06
P _r	2.50E+07	1.00E+06
L	5.00E-02	1.00E-06
L ₂	3.50E-02	1.00E-06
(λ)	-3.98E-12	
A _{eff}	7.69E-02	
A _{eff} uncertainty (r)	0.000639	
A _{eff} uncertainty (h)	0.0000000508	
A _{eff} uncertainty (R)	0.0006	
A _{eff} uncertainty (P _L)	-0.000190	
A _{eff} uncertainty (E)	0.000611905	
A _{eff} uncertainty (R')	-0.0001615	
A _{eff} uncertainty (σ)	0.0009922	
A _{eff} uncertainty (L ₁)		
A _{eff} uncertainty (P _r)		
A _{eff} uncertainty (L)		
A _{eff} uncertainty (L ₂)		

Figure 6-5 Large Side Calculated Uncertainties

An overall finding of the current work is the derivation of tolerances for the mating faces of the small side and large side PCA's. It was found that the small side PCA should be cylindrical to within a 20 nm envelope and the large side PCA's should be cylindrical to within a 0.5 μ m envelope, ensuring that uncertainties are within the limits of 10 ppm. It is evident that as conventional uncertainties for deadweight testers are in the range of 10 ppm, that this sets a reasonable threshold that could be adhered to. The other uncertainty contributors are found to be more or less non-critical, assuming that tight machining tolerances and quality of material properties is utilised.

6.4 Wider Design Principles

It was also possible with the formulae obtained to carry out several designs, illustrating principles which apply to the HFA design. The designs were carried out in Excel, which is suitable for performing the required calculations. It can be seen below that the large side PCA dimensions are first determined by the specified amplification ratio, the line pressure constraints and number of large side PCA's used. The small side PCA dimensions can then be specified, it is noteworthy that in the range of 12.8 MN and above, that the small side PCA will always be very small and require nanometric tolerances (this may not be the case at lower forces).

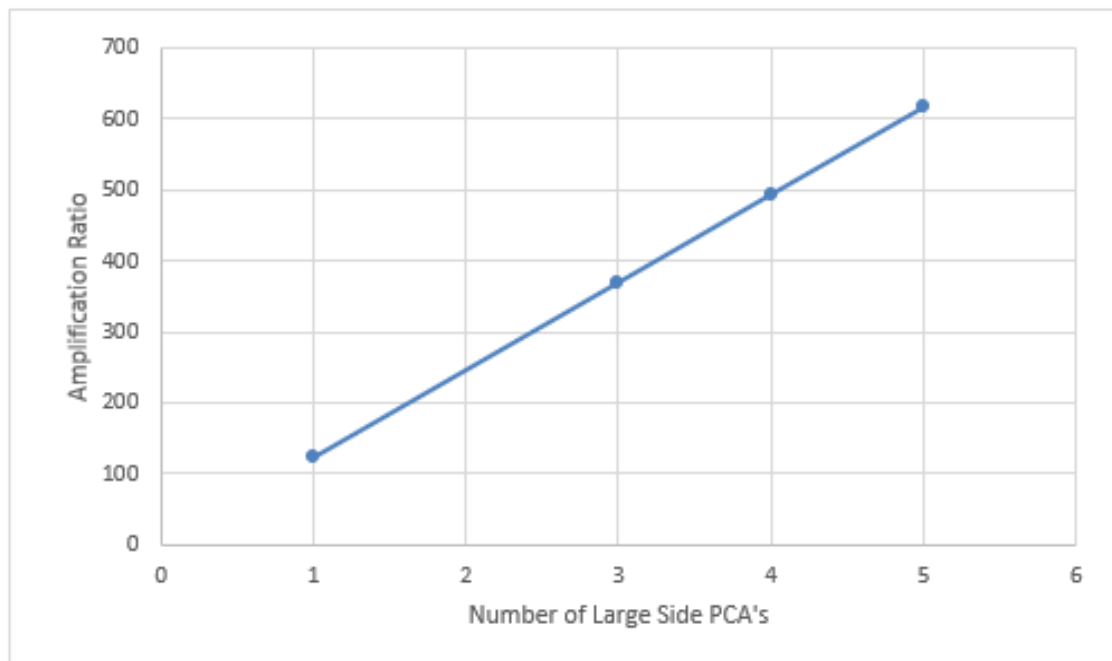


**Figure 6-6 Amplification Ratio of Designed System 1 (Small side radius = 13.5 mm;
Large side radius = 150 mm; Produced Force = 150 MN)**

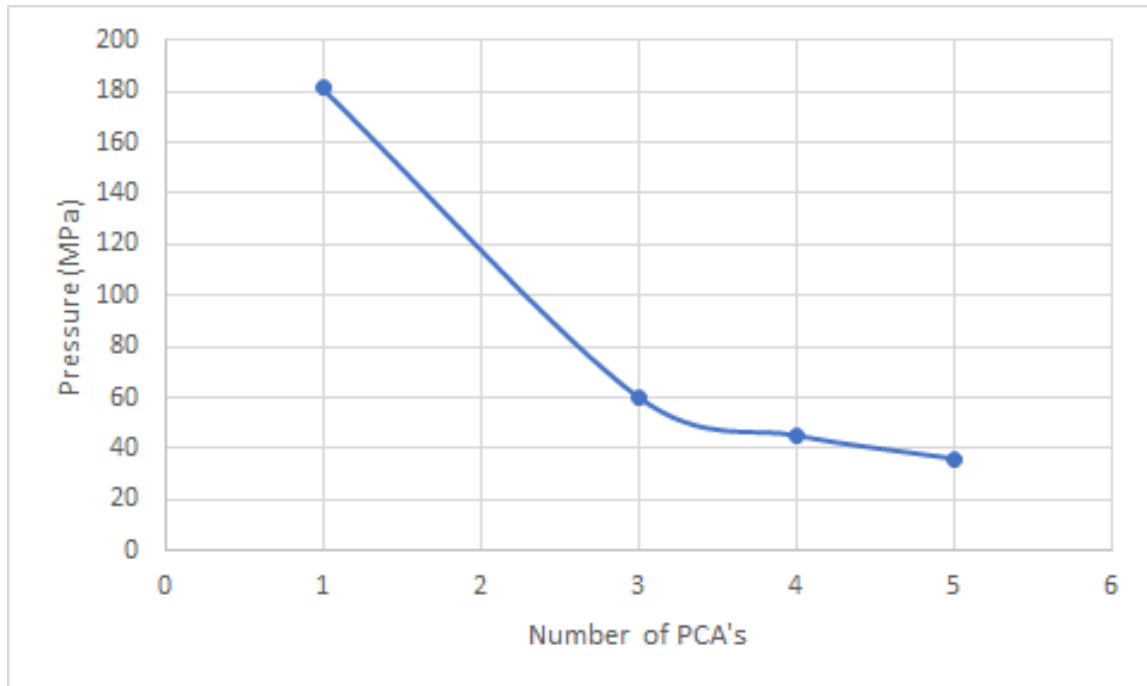


Figure 6-7 Pressure of Designed System 1 (Small side radius = 13.5 mm; Large side radius = 150 mm; Produced Force = 150 MN)

The small side radius can be changed to maintain the amplification ratio while changing the number of large side PCA's:

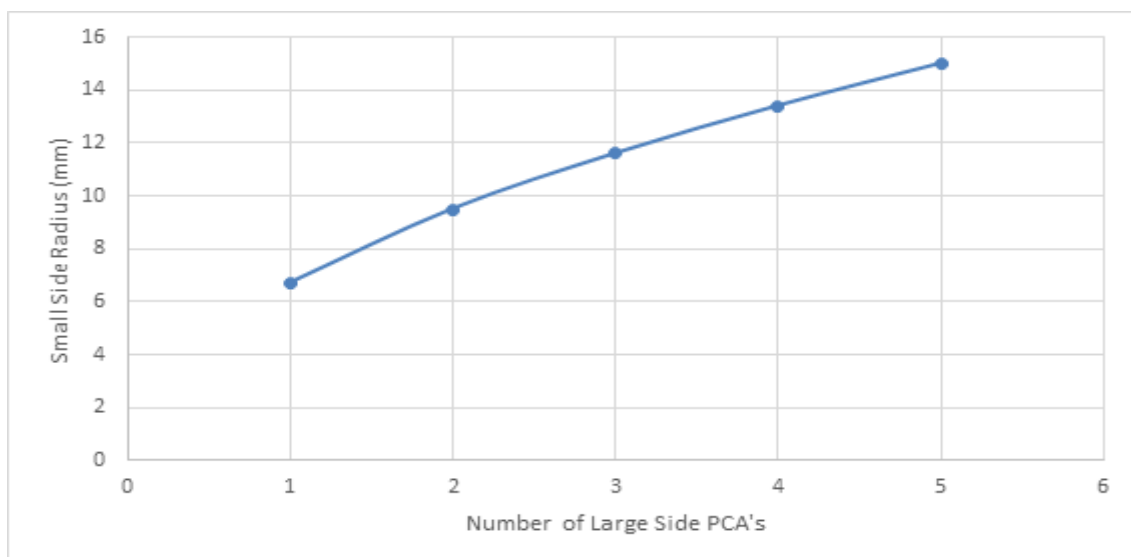


Figure 6-8 Small side piston radius of Designed System 2 (Amplification Ratio = 500; Large side radius = 150 mm; Produced Force = 12.8 MN)

Pressure is defined by the size/number of large side PCA's, and the measured force only:

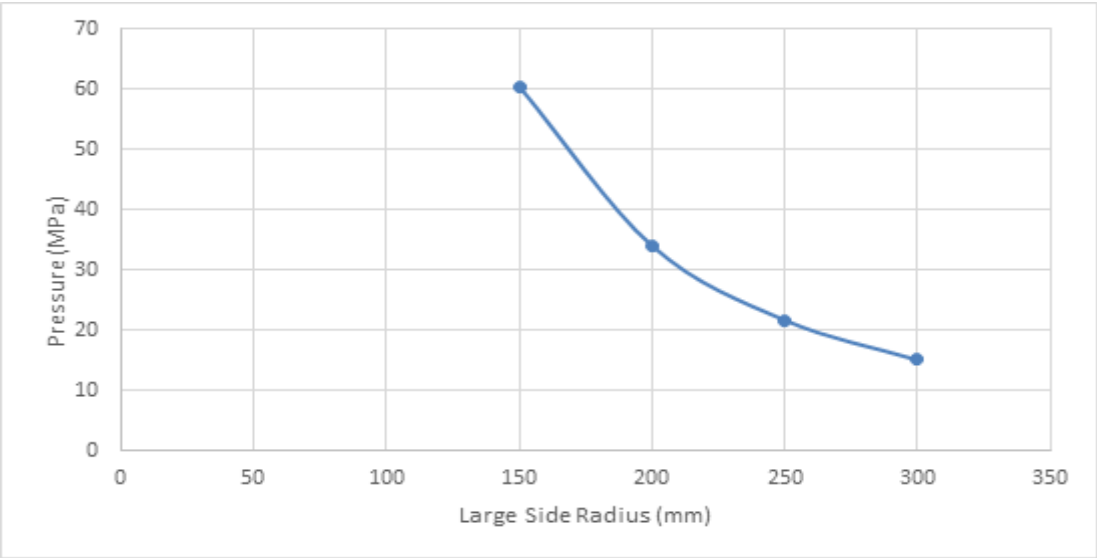


**Figure 6-9 Pressure of Designed System 3 (Number of large side PCA's = 3;
Amplification Ratio = 500; Produced Force = 12.8 MN)**

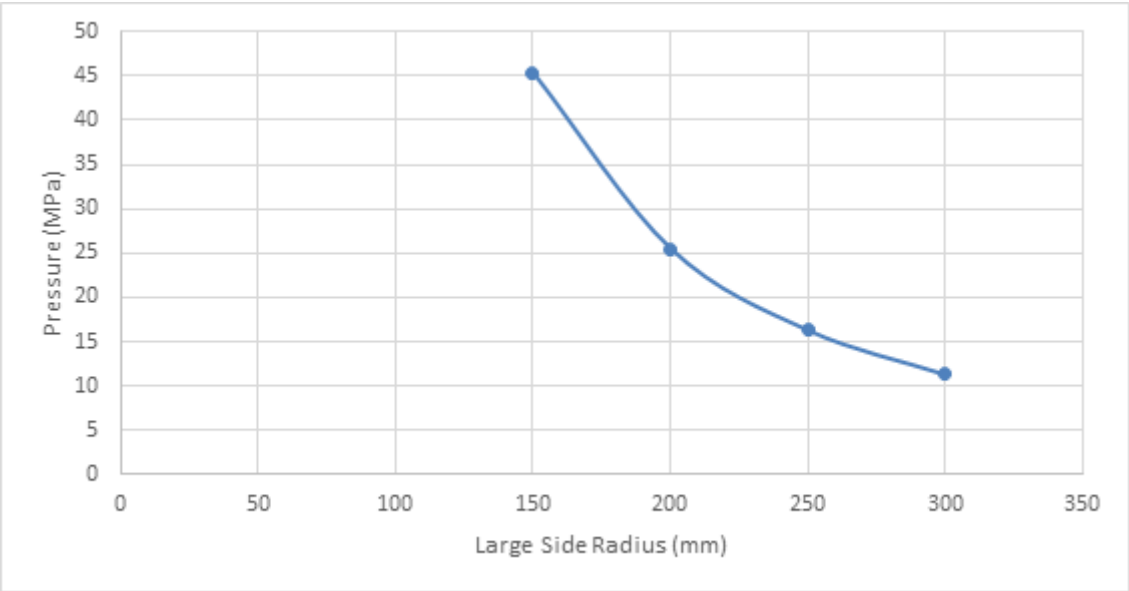


**Figure 6-10 Pressure of Designed System 4 (Number of large side PCA's = 4;
Amplification Ratio = 500; Produced Force = 12.8 MN)**

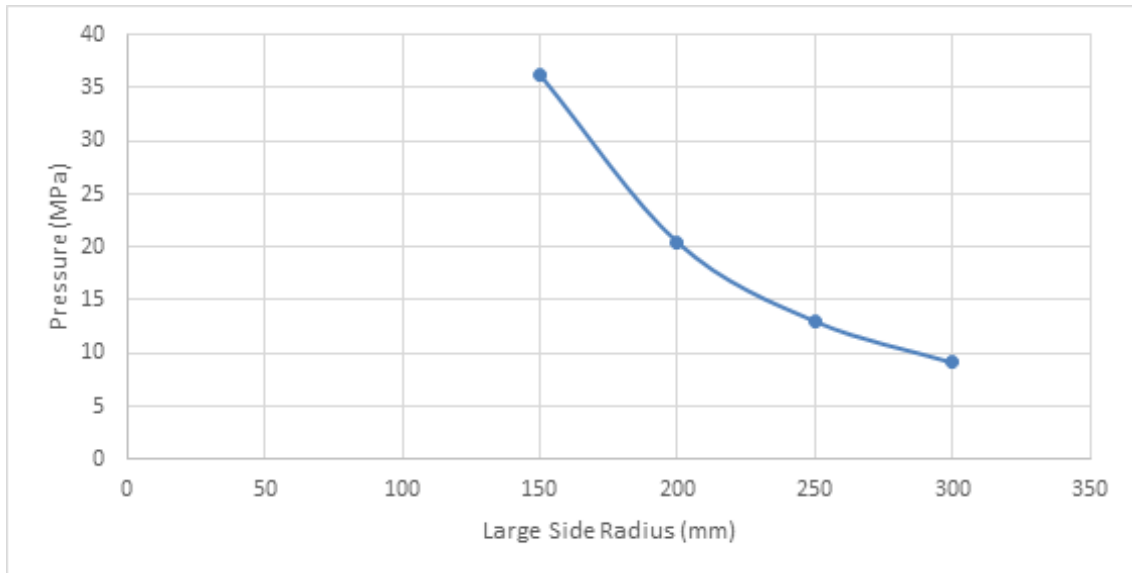


**Figure 6-11 Pressure of Designed System 5 (Number of large side PCA's = 5;
Amplification Ratio = 500; Produced Force = 12.8 MN)**

Measured force can, however, be altered to constrain pressure:

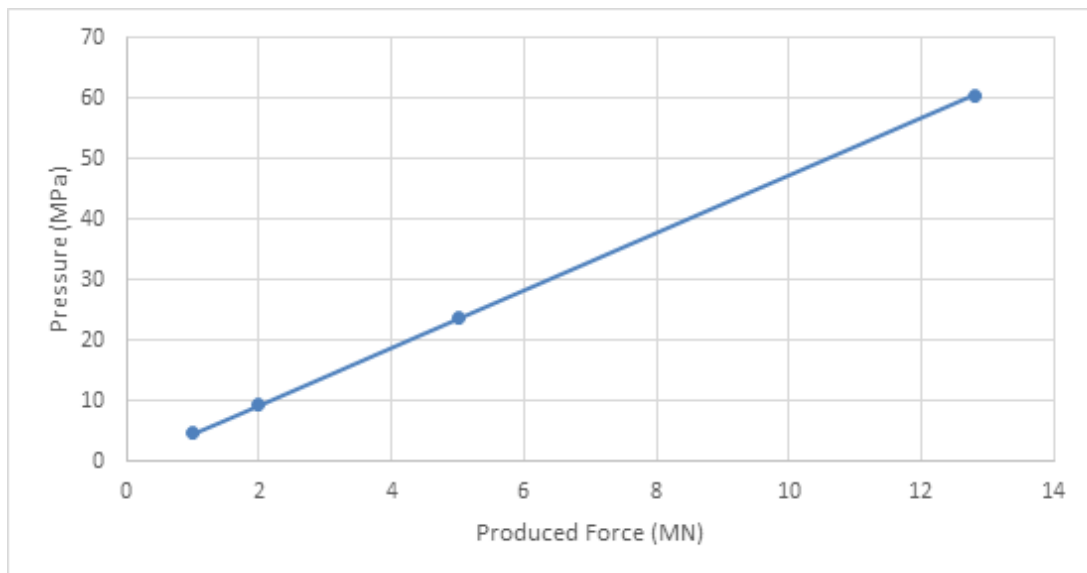


**Figure 6-12 Pressure of Designed System 6 (Small Side Radius = 11.6 mm; Large
Side Radius = 150 mm; Number of large side PCA's = 3; Amplification Ratio =
500)**

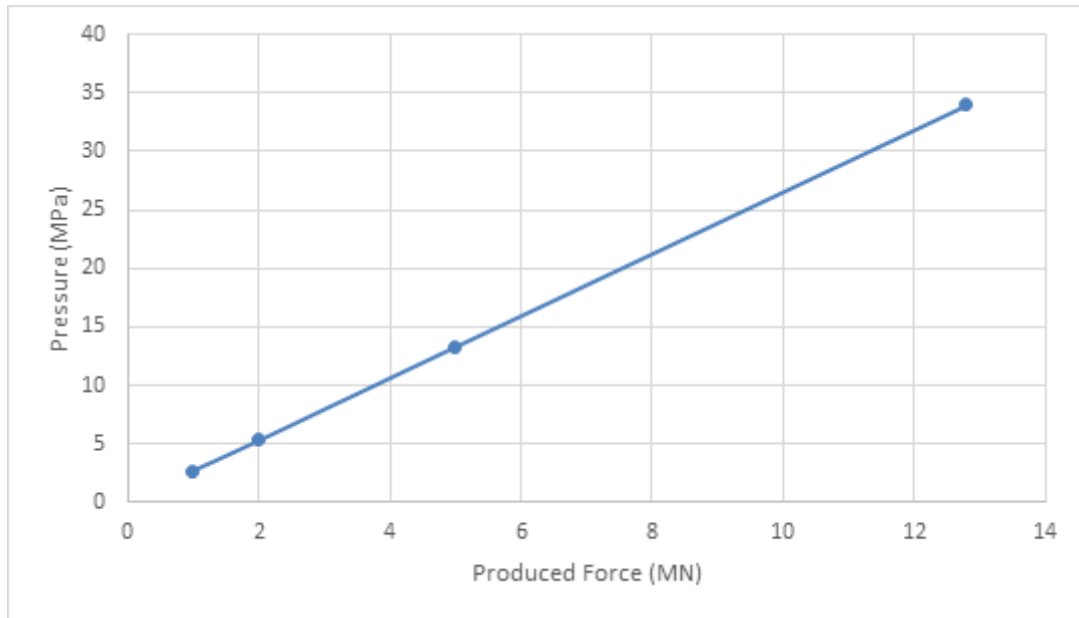


Figure 6-13 Pressure of Designed System 7 (Small Side Radius = 15.5 mm; Large Side Radius = 200 mm; Number of large side PCA's = 3; Amplification Ratio = 500)

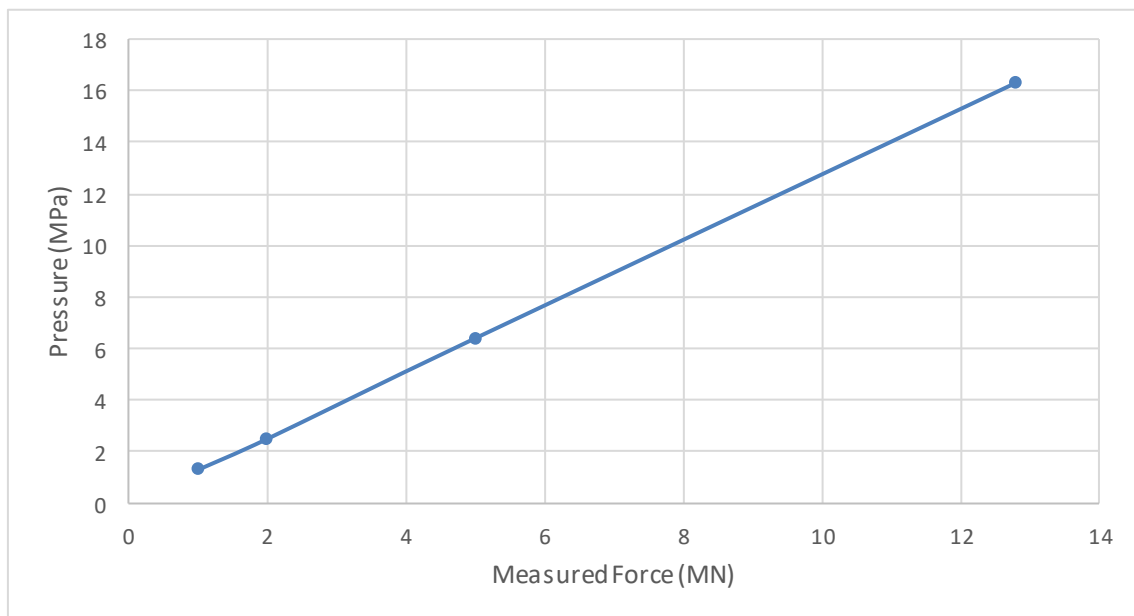


Figure 6-14 Pressure of Designed System 8 (Small Side Radius = 19.4 mm; Large Side Radius = 250 mm; Number of large side PCA's = 3; Amplification Ratio = 500)

The same can be shown when 4 large side PCA's are used:

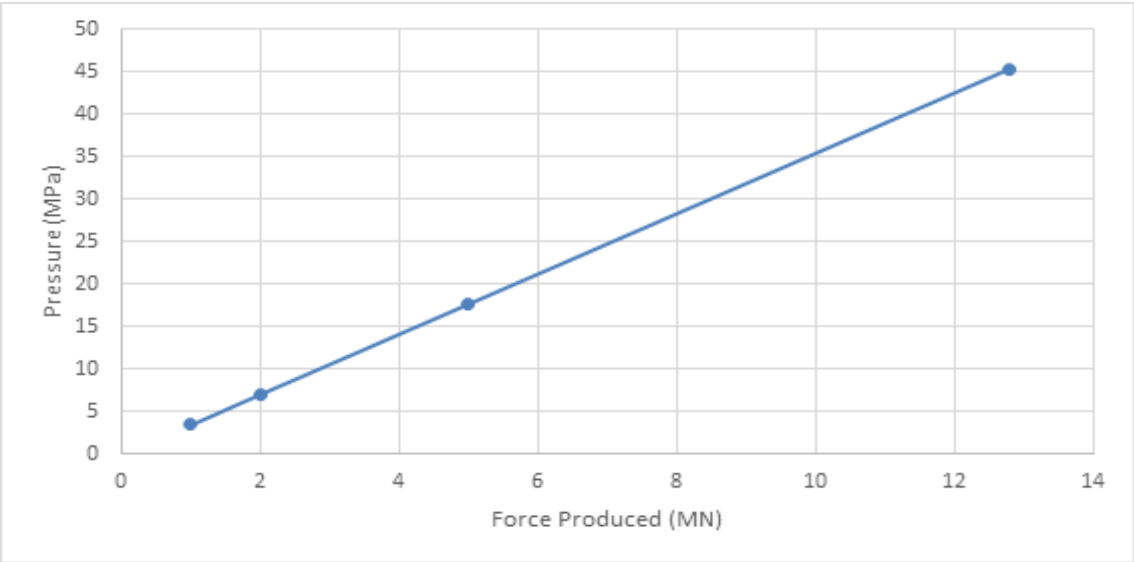


Figure 6-15 Pressure of Designed System 9 (Small Side Radius = 13.4 mm; Large Side Radius = 150 mm; Number of large side PCA's = 4; Amplification Ratio = 500)

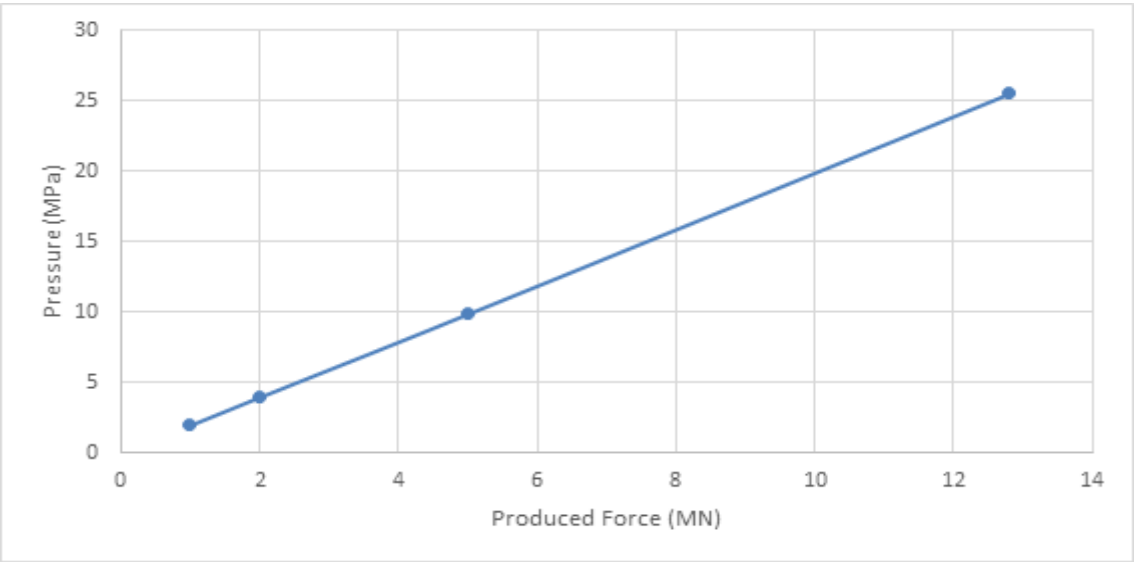


Figure 6-16 Pressure of Designed System 10 (Small Side Radius = 17.9 mm; Large Side Radius = 200 mm; Number of large side PCA's = 4; Amplification Ratio = 500)

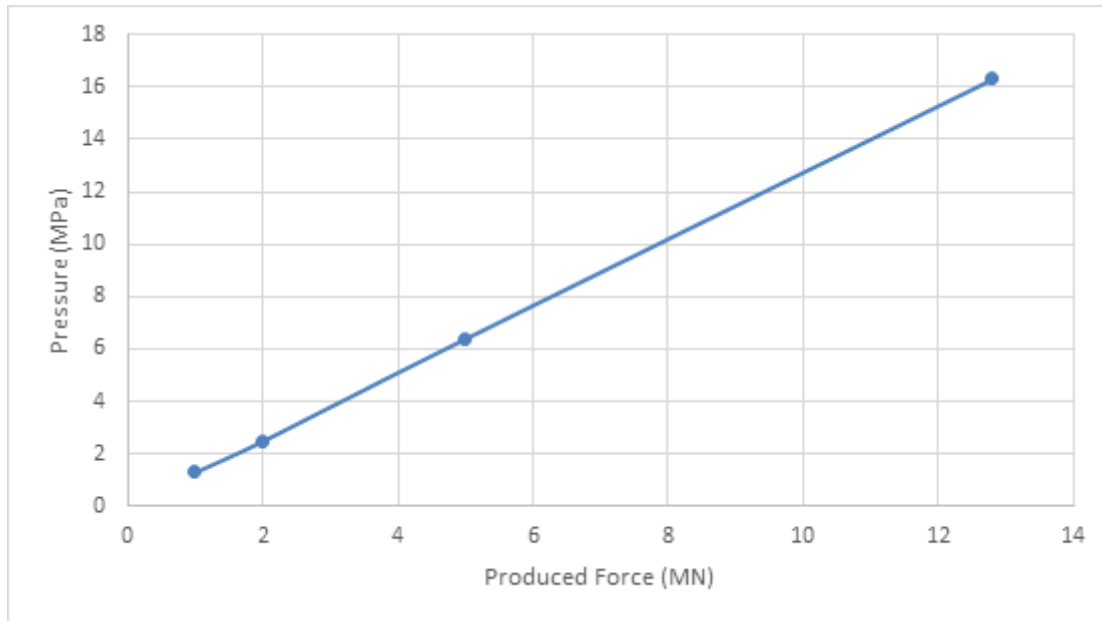


Figure 6-17 Pressure of Designed System 11 (Small Side Radius = 22.4 mm; Large Side Radius = 250 mm; Number of large side PCA's = 4; Amplification Ratio = 500)

7 CONCLUSION

This work aimed to evaluate the capability for HFAs to work as primary standards, with traceability maintained directly to the density and volume of the weights. The objectives were therefore:

1. Evaluate HFA sources of uncertainty including their magnitude and their effect to the generated force, thereby showing that the primary HFA concept would meet the needs of industry and is theoretically feasible
2. Establish analytical formulae which describe uncertainties in rotational PCAs enabling direct traceability, which enables the design of a primary rotational HFA

3. Establish analytical formulae which describe uncertainties in non-rotational PCAs enabling direct traceability, which enables the design of a primary nonrotational HFA
4. Create a computer application which enables design of HFAs, and to show that some designs of directly calibrated HFA are feasible with reduced uncertainties, determining some potential configurations including nominal sizes of PCA's, nominal pressures, forces and amplification ratio; thereby making the case strongly for adoption of 'primary' HFA's

Objective 1 was therefore achieved in the literature review where sources of uncertainty were evaluated and it was assessed whether a primary HFA would offer sufficient advantages in the reduction of uncertainty. In this section all uncertainty sources which had previously been identified in the literature as being of significance in PCA's were considered, and evaluated analytically for their uncertainty contributions to an HFA according to the measurement model. These uncertainties were then evaluated, and it was found that the dominant source of uncertainty was in the calibration uncertainty which was high because of the uncertainty in the reference transducer. It was therefore identified that the use of direct traceability had the potential to cause the reduction in uncertainty desired. Subsequently, the remaining uncertainty sources were analytically evaluated, and it was determined that the directly calibrated HFA had the potential to offer reduced uncertainties in keeping with the accuracies desired.

Objective 2 was achieved in section 4 where the design for a rotational HFA's was carried out. Firstly, preliminary formulae were manipulated thereby giving a more accurate prediction of the contribution of tilt to uncertainty in PCA's, and to enable analytical evaluation of the flow in PCA's, both of these formulae were required for the PCA design.

Next, the FEA/CFD model was created and benchmarked to the experimental reports of PCA's in the literature for its distortion coefficient, good agreement was found between experimental and FEA/CFD results. Finally, the uncertainty contributions of the various parameters in the measurement model was compared to that found in the FEA/CFD for nominal deviations in the input parameters, thereby indicating tolerances which could be used and the accuracy of their derivation. Having found the required tolerances (which were most stringent on the mating faces of the PCA's), it was indicated whether the required tolerances were achievable, and it was found that a directly calibrated rotational HFA may indeed be possible and is worthy of further investigation.

Objective 3 was likewise achieved in section 5 where the nonrotational design was carried out. The tolerances were determined in the same way as in the preceding section, but this section additionally included the work done to enable evaluation of distortion coefficient for nonrotational PCA's, which is more complex in nature. To do this, some novel technique of evaluating the force at the top of the piston directly within the FEA/CFD was utilised (and benchmarked) to enable validation of the tolerances obtained by the measurement model. It was necessary also to propose an analytical model describing distortion coefficient in a nonrotational HFA, which was carried out. These sections were useful in indicating tolerances for PCA design which are necessary to achieve the desired uncertainties.

The final objective was achieved in section 6, where the application in Excel was used to indicate the tolerances required and uncertainties achievable. In addition some designs for HFA's were put forward and the effects of varying the design parameters were evaluated to give some indication of the feasibility of the design.

In this project, FEA/CFD models were created and benchmarked for piston/cylinder assemblies and bearings, enabling a thorough investigation of the uncertainties present in HFA systems to be carried out. All uncertainty sources which had been identified previously in the literature, as well as some others which had not, were investigated, and analytical models were put forward which predict the magnitude of the uncertainties. The procedure was repeated for PCAs of the nonrotational type. By using the derived analytical formulae, according to the GUM regulations, it was possible to put forward a general formula which predicts uncertainty in an HFA system. Because previous experimental results with PCAs have shown that effective area is predicted sufficiently accurately with reference to only the uncertainty sources discussed, the formula can be used to predict uncertainties present in an HFA system and thereby inform the design process, enabling traceability in force measurements of HFAs to be maintained according to the weight of the masses and PCA geometries instead of by external calibration.

The benefits of such a direct calibration are principally that the uncertainties associated with an indirect calibration are avoided, where a transducer with normally $>0.01\%$ uncertainty is used in the calibration. This uncertainty source being the main limitation to the accuracies of HFAs in the 2 MN - 20 MN range, it can be seen how the novel calibration method can enable improved HFA performance, thereby meeting the needs of industries which are currently lacking.

In the course of the investigation, a formula for flow in nonrotational PCAs was created which considers the exact deformed geometry in the clearance gap, as well as the altered fluid viscosity and density under pressurisation. Future research may consider this, and also the need to extend the force range in which HFA machines are able to operate, thereby reducing uncertainties of calibration in the higher (>20 MN) ranges, where strain

gauge machines are currently used because of limitations to the amplification ratios achievable.

Future research can be split into two parts: those concerning experimental investigation of the directly calibrated HFA, and those concerning further theoretical investigation. With regards to experimental investigation, it is suggested that working small side PCA's could be produced, characterised and cross-floated, as the 20 nm tolerances push the limits of manufacturing and metrology capability. The large side PCA's could also be produced, alongside some non-rotational HFA's to gain confidence in theoretical explanations of bearing stiffness which are also lacking. With regards to further theoretical investigation of the PCA's, the centralising effect of bearings upon PCA's must be considered, in developing models which give the stiffness of the nonrotational system, thereby allowing more complete HFA design. Additionally, future research may consider:

- Optimisation of nonrotational design
- Additional uncertainty sources present in nonrotational systems
- Enhanced FEA/CFD considering nonrotational designs
- Further formulation for flow in nonrotational PCA's
- FEA/CFD modelling of PCA hysteresis and discrimination
- A closer look at thermal effects

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