

Robust Design Optimisation of S-ducts

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Over the past years, robust optimisations have become very popular and necessary. The aim of this type of optimisation is to consider the sensitivity of the output results to small variations in the operating conditions and manufacturing tolerances. To study such sensitivities, an accurate and efficient method to quantify the uncertainties in physical processes is necessary. We present here the study and design of an S-duct, suitable for distributed propulsion configurations, and we address practical considerations of the application of robust optimisation to real-world design problems under multiple uncertainties. Two different non-intrusive Polynomial Chaos techniques have been chosen to quantify the input and output uncertainties, namely the non-intrusive point collocation and the non-intrusive spectral projection. These two techniques were implemented in two different robust optimisation problems (R1D and R2D) and their optima designs were analysed. To demonstrate the effectiveness of the robust optimisation problem formulation and analysis we compared the newly discovered optimum designs with previous non-robust optimum configurations. The results are discussed in detail and have shown the superiority of the designs when uncertainty properties were considered.

I. Introduction

Optimisation problems are present in everyday life. One common example, at the basis of optimisation theory, is the well-known *Travelling Salesman Problem*. Given a list of cities that a salesman has to visit to sell goods to customers, the question is how to find the shortest route that passes through every city and returns to the initial point. The larger the number of variables introduced, the more complicated and highly non-linear the problem will become. In numerical terms, a simple deterministic optimisation problem can be stated as follows: given an independent variable $x \in \mathbb{R}$ called design variable and an objective function $f(x)$, the goal of an optimisation cycle is to automatically change the design variable in order to find a certain x^* such that $f(x^*)$ is a global minimum or maximum (it depends on the problem we are analysing). In a more general case, x can be a design vector of design variables $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$. If only one objective function is present, the optimisation is said to be *single-objective* (SOO), otherwise is a *multi-objective* optimisation (MOO). The design to be acceptable it must also satisfy certain requirements. These requirements are called design constraints.

Optimised designs may be very sensitive to small variations in the manufacturing or operating conditions: such variations can even be destructive in terms of drastic deterioration in the values of the objective functions. In order to study such sensitivities, an accurate and efficient method to quantify the uncertainties in physical processes is needed. Since uncertainty, by definition, cannot be predicted, a scrupulous designer must take it into account when performing a robust optimisation. In simple words, the goal of robust optimisation is connected to the idea that in the presence of input uncertainties, the optimal design should be relatively insensitive to these (small output uncertainty). It must be remarked that a robust design may not necessarily coincide with the global optimum design.

Several techniques are today available to quantify problems' uncertainties. Among them, non-intrusive Polynomial Chaos techniques are stochastic approaches which provide successful prediction quality. These are based on the main assumption that each uncertainty input variable has to be considered as random ζ with its own probability density function (PDF). This assumption implies that also the output variable $f(x, \zeta)$ will be random with its own PDF as well.

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Hence, to obtain a stochastic model of the process, it is necessary to run the deterministic model a certain number of times.

In recent years, robust optimisation has been adopted in the works and studies of [1] and [2] on the aerodynamics of high-lift configuration wing. Moreover, it has been applied to isolated airfoils and to a generic S-bended duct by Liastakouras [3]. The aims of Liastakouras' work were to build a workflow able to perform shape optimisations under uncertainties and to test such workflow in 2D and 3D domains where uncertainties were caused by variations in the airflow conditions. Robust optimisation was also used by [4] to improve the off-design efficiency of gas turbine compression systems in aircraft engines and by [5] to optimise a supersonic turbine for total-to-static stage efficiency. [6] investigated the uncertainty in the Mach number and in the angle of attack in order to optimise a 2D transonic airfoil. On the other hand, [7] optimised the NACA 23012 profile using the ice accretion position as input uncertainty. This new type of optimisation was also implemented by [8] with the aim of investigating the propagation of uncertainty from the model parameters to simulation outputs in the context of low Mach number reacting flow modelling, with a focus on super-critical water oxidation. In all these works, robust optimisation codes were used based on non-intrusive uncertainty quantification methods.

S-duct intakes for aircraft have been a frequent subject of CFD and optimisation studies since the early 2000s. As a matter of fact, the main advantage of using these intakes in the civil aviation sector is to locate the engine inside the fuselage with a consequent reduction in drag and fuel consumption. However, this component generates a swirling motion field at the AIP which can drastically affect the performance of the fan when it is hit by a non-uniform velocity field at the inlet. Anne-laure Delot's (2013) investigated which CFD setup gave results close to the experimental data obtained using Wellborn's S-duct geometry ([9]). A couple of years later, other works performed by [10] and [11] focused on the parameterisation and optimisation of Wellborn's S-duct geometry with the aim of improving the performance of the S-duct at the AIP in terms of pressure recovery and swirl. Starting from Liastakouras work, Dal [12] implemented a simple case study of a robust optimisation for S-duct intakes and demonstrated how an optimised S-duct remarkably changes its pressure recovery value when the velocity input deviated from its mean value.

In this paper, robust optimisation codes with more than one uncertainty input and output have been fully developed and the results of several robust optimisation campaigns have been validated and compared with NON robust ones. The robust codes were implemented on the bases of the Tabu Search Optimisation Algorithm developed by [13]. Velocity and inlet flow deviation were chosen as input affected by uncertainty. The stochastic properties of the output variables of interest (pressure recovery and swirl) were considered as objective functions (i.e., mean and standard deviation) and the goal was to maximise (or minimise) the mean μ and always minimise the standard deviation σ . It is easy to understand that this approach to a robust optimisation always leads to multi-objective problems. Even for one variable of interest, two objective functions, the mean and the standard deviation, have to be considered.

The paper is organised in three macro-sections. In the first one (Section II), the Polynomial Chaos and their non-intrusive techniques will be mathematically explained. Section III will cover the case study and all the implemented robust optimisation problems. In section IV, the validation of the Robust model and the most significant optima design obtained from the Robust optimisation codes will be highlighted. Moreover, three of these designs will be fully post-processed and analysed using plots and contours.

II. Polynomial Chaos

The Polynomial Chaos scheme (PC) views the inputs and the outputs random variables as an infinite summation of terms where it is possible to identify deterministic coefficients multiplied for random basis polynomials. This theory has its fundamentals on the homogeneous chaos written by [14], and it assumes that a second-order random process (i.e., a process with a finite variance σ^2), as the majority of the engineering process, can be expressed in terms of orthogonal polynomials.

The Polynomial chaos is a method that is used in many works, especially in optimisation problems as in [4, 6, 7], and it is very attractive because of its very high convergence rate (exponential or quasi-exponential). However, many issues have been observed in [15]:

- an analytical PDF must exist for every uncertain variable. If not known, it must be guessed;
- it is computational efficient for a small number of random variables only;
- it is intrusive, i.e. it requires modifications in the solver.

In order to solve some of these issues many modifications have been made, in particular on the intrusiveness. In fact, in this work, we will use non-intrusive variant of the Polynomial Chaos.

A. Hermite polynomial chaos

Using Hermite polynomials, a second-order process $X(\theta)$ can be expanded as follow:

$$\begin{aligned}
X(\theta) = & a_0 H_0 + \\
& + \sum_{i_1=1}^{\infty} a_{i_1} H_1(\xi_{i_1}(\theta)) + \\
& + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} a_{i_1 i_2} H_2(\xi_{i_1}(\theta), \xi_{i_2}(\theta)) + \\
& + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} a_{i_1 i_2 i_3} H_3(\xi_{i_1}(\theta), \xi_{i_2}(\theta), \xi_{i_3}(\theta)) + \\
& + \dots
\end{aligned} \tag{1}$$

where $H_n(\xi_{i_1}, \dots, \xi_{i_n})$ denote the Hermite polynomials of order n in terms of the multi-dimensional independent standard Gaussian random variables $\xi = (\xi_{i_1}, \dots, \xi_{i_n})$ with *zero mean* and *unit variance*. Instead, $a_{i_1} \dots a_{i_n}$ are deterministic coefficients. The above equation is the discrete version of the original Wiener polynomial chaos expansion, where the continuous integrals are replaced by summations. The general expression of the polynomials is given by:

$$H_n(\xi_{i_1}, \dots, \xi_{i_n}) = (-1)^n e^{\frac{1}{2}\xi^T \xi} \frac{\partial^n}{\partial \xi_{i_1} \dots \partial \xi_{i_n}} e^{-\frac{1}{2}\xi^T \xi} \tag{2}$$

For example, if $\xi = (\xi_1)$, the one-dimensional Hermite polynomials are:

$$\Psi_0 = 1, \quad \Psi_1 = \xi, \quad \Psi_2 = \xi^2 - 1, \quad \Psi_3 = \xi^3 - 3\xi, \dots \tag{3}$$

and if $\xi = (\xi_1, \xi_2)$, the two-dimensional Hermite polynomials are:

$$\Psi_0 = 1, \quad \Psi_1 = \xi_1, \quad \Psi_2 = \xi_2, \quad \Psi_3 = \xi_1^2 - 1, \quad \Psi_4 = \xi_1 \xi_2, \quad \Psi_5 = \xi_2^2 - 1, \dots \tag{4}$$

For notational convenience, equation 1 can be rewritten as:

$$X(\theta) = \sum_{j=0}^{\infty} \hat{a}_j \Psi_j(\xi) \tag{5}$$

where there is a one-to-one correspondence between the functions $H_n(\xi_{i_1}, \dots, \xi_{i_n})$ and $\Psi_j(\xi)$ and also between the coefficients $\hat{a}_j$ and $a_{i_1} \dots a_{i_n}$.

For the sake of clarity, the two-dimensional expansion is shown here, both in the fully expanded form (Eq. 1):

$$\begin{aligned}
X(\theta) = & a_0 H_0 + a_1 H_1(\xi_1) + a_2 H_1(\xi_2) + a_{11} H_2(\xi_1, \xi_1) + \\
& + a_{12} H_2(\xi_2, \xi_1) + a_{22} H_2(\xi_2, \xi_2) + \dots
\end{aligned} \tag{6}$$

and in the simplified form (Eq 5):

$$\begin{aligned}
X(\theta) = & \hat{a}_0 \Psi_0 + \hat{a}_1 \Psi_1 + \hat{a}_2 \Psi_2 + \hat{a}_3 \Psi_3 + \hat{a}_4 \Psi_4 + \hat{a}_5 \Psi_5 + \dots \\
= & \hat{a}_0 + \hat{a}_1 \xi_1 + \hat{a}_2 \xi_2 + \hat{a}_3 (\xi_1^2 - 1) + \hat{a}_4 (\xi_1 \xi_2) + \hat{a}_5 (\xi_2^2 - 1) + \dots
\end{aligned} \tag{7}$$

The polynomial basis Ψ_j of Hermite-Chaos forms a complete orthogonal basis, i.e.:

$$\langle \Psi_i \Psi_j \rangle = \langle \Psi_i^2 \rangle \delta_{ij} \tag{8}$$

where δ_{ij} is the Kronecker delta and $\langle \cdot, \cdot \rangle$ denotes the ensemble average, which correspond to the inner product in the Hilbert space of Gaussian variables:

$$\langle f(\xi) g(\xi) \rangle = \int_{\Omega} f(\xi) g(\xi) W(\xi) d\xi \tag{9}$$

where $W(\xi)$ is the weighting function corresponding to the polynomial basis Ψ_j . In Hermite polynomials, the weighting function is:

$$W(\xi) = \frac{1}{\sqrt{(2\pi)^n}} e^{-\frac{1}{2}\xi^T \xi} \quad (10)$$

The Hermite polynomials are paired with the Gaussian distribution because they are expressed in terms of Gaussian variables and, by definition, these polynomials are orthogonal to the weighting function $W(\xi)$ which has the form of the multi-dimensional independent Gaussian probability distribution with unit variance [16].

B. Generalisation to any random distribution

The [17] theorem assures the Wiener-Hermite chaos exhibits an exponential convergence rate when applied to Gaussian distributions. However, for general non-Gaussian random inputs, such as Beta or Uniform distributions, the convergence rate is not fast and in some cases the convergence rate is, in fact, severely deteriorated [16]. In order to deal with more general random inputs, we introduce the Wiener-Askey polynomial chaos expansion as a generalization of the original Wiener-Chaos expansion. Similar to equation 1, we represent the general second-order random process $X(\theta)$ as:

$$\begin{aligned} X(\theta) = & a_0 I_0 + \\ & + \sum_{i_1=1}^{\infty} c_{i_1} I_1(\zeta_{i_1}(\theta)) + \\ & + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} c_{i_1 i_2} I_2(\zeta_{i_1}(\theta), \zeta_{i_2}(\theta)) + \\ & + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} c_{i_1 i_2 i_3} I_3(\zeta_{i_1}(\theta), \zeta_{i_2}(\theta), \zeta_{i_3}(\theta)) + \\ & + \dots \end{aligned} \quad (11)$$

where $I_n(\zeta_{i_1}, \dots, \zeta_{i_n})$ denotes the Wiener-Askey polynomials of order n in terms of the multi-dimensional random variables $\zeta = (\zeta_{i_1}, \dots, \zeta_{i_n})$ and $c_{i_1} \dots c_{i_n}$ are deterministic coefficients.

Again, for notational convenience, we rewrite Eq 11 as:

$$X(\theta) = \sum_{j=0}^{\infty} \hat{c}_j \Phi_j(\zeta) \quad (12)$$

where there is a one-to-one correspondence between the functions $I_n(\zeta_{i_1}, \dots, \zeta_{i_n})$ and $\Phi_j(\zeta)$ and their coefficients $\hat{c}_j$ and $c_{i_1} \dots c_{i_n}$. The orthogonality relation of the Wiener-Askey polynomial chaos takes the form:

$$\langle \Phi_i \Phi_j \rangle = \langle \Phi_i^2 \rangle \delta_{ij} \quad (13)$$

where δ_{ij} is the Kronecker delta and $\langle \dots \rangle$ is the ensemble average, which corresponds to the inner product in the Hilbert space of the variables ζ .

$$\langle f(\zeta) g(\zeta) \rangle = \int_{\Omega} f(\zeta) g(\zeta) W(\zeta) d\zeta \quad (14)$$

For practical applications, the infinite summations in Eq.12 must be truncated to a finite number. The new upper summation limit depend on the number of random dimensions and the desired order of the polynomials:

$$P + 1 = \frac{(n + p)!}{n! p!} \quad (15)$$

where P is the new upper summation limit, p the desired order of the polynomials, and n the number of random dimensions.

As stated in [18], some polynomials from the Askey scheme have as weighting functions $W(\zeta)$ the same as the PDF of certain random distributions. As a matter of fact, it is convenient to choose the type of random variables ζ according to their probability distributions, as shown in table 1.

Table 1 Types of random variables and their corresponding chaos.

	Random variables ζ	Wiener-Askey chaos $\Phi(\zeta)$	Support
Continuous	Gaussian	Hermite Chaos	$(-\infty, +\infty)$
	Gamma	Laguerre Chaos	$[0, +\infty)$
	Beta	Jacobi Chaos	$[a, b]$
	Uniform	Legendre Chaos	$[a, b]$
Discrete	Poisson	Charlier Chaos	$\{0, 1, 2, \dots\}$
	Binomial	Krawtchouk Chaos	$\{0, 1, \dots, N\}$
	Negative Binomial	Meixner Chaos	$\{0, 1, 2, \dots\}$
	Hypergeometric	Hahn Chaos	$\{0, 1, \dots, N\}$

C. Non-intrusive methods

In paragraph II the intrusiveness of the Polynomial Chaos method has been presented as the major drawback. In order to solve the deterministic system directly and to find the PC coefficients (deterministic coefficients), the intrusive Polynomial Chaos method requires to directly modify the code of a deterministic solver. This can be inconvenient and complex, especially if the problem deals with complicated analyses as CFD simulations. In order to overcome this issue, several non-intrusive approaches have been proposed and can be found in literature ([4, 5, 8, 19–24]): the common idea is to treat the deterministic solver as a black box and to find the PC coefficient after having run the necessary deterministic code (See Figure 1).

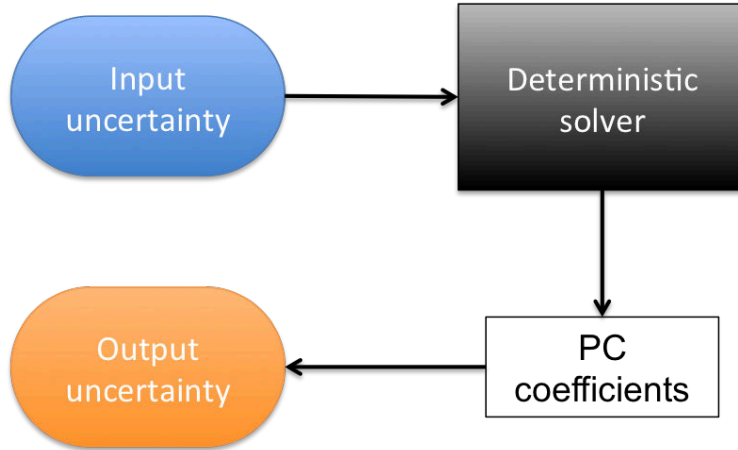


Fig. 1 Schematic of the non-intrusive methods.

1. Non-Intrusive Spectral Projection

Non-intrusive Spectral Projection (NISP) is based on the observation that the PC coefficients for a variable a (i.e., $a_0, \dots, a_P$) can be obtained by projecting the deterministic solution onto the PC basis Φ_i (Galerkin projection) and using the orthogonality relation 8.

$$\begin{aligned}
 a &= \sum_{i=0}^{\infty} a_i \Phi_i \\
 \langle a, \Phi_i \rangle &= \left\langle \sum_{i=0}^{\infty} a_i \Phi_i, \Phi_i \right\rangle \\
 a_i &= \frac{\langle a, \Phi_i \rangle}{\langle \Phi_i^2 \rangle}
 \end{aligned} \tag{16}$$

Now we change the above equation with a new nomenclature:

$$a_i = \frac{\langle a^d(\zeta), \Phi_i \rangle}{\langle \Phi_i^2 \rangle} \quad (17)$$

where $a^d(\zeta)$ represents the deterministic solution corresponding to a particular realization $\zeta = (\zeta_1, \dots, \zeta_n)$, and n is the number of random dimensions. After that, if we apply the inner product (Eq 14) to the equation 17 we will have the following equation form for the PC coefficients:

$$a_i = \frac{\langle a^d(\zeta) \Phi_i \rangle}{\langle \Phi_i^2 \rangle} = \int_{\Omega} \dots \int_{\Omega} \left[a^d(\zeta) \frac{\Phi_i(\zeta)}{\langle \Phi_i^2 \rangle} \prod_{k=1}^n W(\zeta_k) \right] d\zeta_1 \dots d\zeta_n \quad (18)$$

where W is the weighting function. The above integral can be approximated using the Gaussian quadrature method, by sampling each random variable ζ_i in m Gaussian quadrature points.

$$a_i = \sum_{m_1=1}^m \dots \sum_{m_n=1}^m a^d(\zeta_{m_1}, \dots, \zeta_{m_n}) \frac{\Phi_i(\zeta_{m_1}, \dots, \zeta_{m_n})}{\langle \Phi_i^2 \rangle} \prod_{k=1}^n \omega_{m_k} \quad (19)$$

Using the above mathematical notations, ζ_{m_k} corresponds to the old one-dimensional random variable notation ζ_i when $k = 1 \dots, n$. Instead, with $m_k = 1, \dots, m$ we define the m couples $(\zeta_{m_k}, \omega_{m_k})$ that comes from sampling each one-dimensional random variable. In fact, the two elements of these m couples are respectively the Gaussian quadrature point and its weight. Moreover, it must be remarked that the Gaussian quadrature sampling depends on the probability distribution of the random variable. For example, if the latter follows a Gaussian distribution, the quadrature sampling will be called Hermite-Gauss quadrature. On the other hand, if the random variable follows a Beta distribution, the Gaussian quadrature sampling will be called Jacobi-Gauss quadrature.

The process involves the following steps:

- 1) define the behaviour of the uncertainties input variable (PDF), the random variables (in this example ζ_{m_k}) is sampled in m points using the Gaussian quadrature technique;
- 2) compute the basis functions Φ_i with $i = (0, \dots, P)$ and the weights ω for each m quadrature points;
- 3) for each m sampled input variable, compute the deterministic solutions $a^d(\zeta_1), \dots, a^d(\zeta_m)$;
- 4) the polynomial chaos coefficients are then computed with the equation 19.

Following the indications of Le [19], the Gauss-Hermite quadrature requires $m = p + 1$ (p being the Polynomial Chaos order) sample points for each random variable to be exact, therefore the total number of deterministic solutions required is $(p + 1)^n$, that are notably higher than the $P + 1$ solves required by the intrusive Polynomial Chaos presented in the previous sections. This remark introduces the so-called *curse of dimensionality* from which the NISP is affected: the number of deterministic solves grows exponentially with the number of random dimensions. Even though the number of deterministic solves could easily become a quite high number, they are considerably less than the solves required by Monte Carlo Method [25].

2. Non-Intrusive Point-Collocation

The Non-Intrusive Point-Collocation method (NIPC) has been proposed by [20]. This method consists in sampling $P + 1$ collocation points from the random vector $\zeta = (\zeta_1, \dots, \zeta_n)$, and running the deterministic solver for each of these collocation points. In contrast to NISP, this new method does not need a Galerkin projection, but it directly solves a linear system in order to find the NIPC coefficients:

$$\begin{pmatrix} \Phi_0(\zeta_0) & \Phi_1(\zeta_0) & \dots & \Phi_P(\zeta_0) \\ \Phi_0(\zeta_1) & \Phi_1(\zeta_1) & \dots & \Phi_P(\zeta_1) \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_0(\zeta_P) & \Phi_1(\zeta_P) & \dots & \Phi_P(\zeta_P) \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_P \end{pmatrix} = \begin{pmatrix} a^d(\zeta_0) \\ a^d(\zeta_1) \\ \vdots \\ a^d(\zeta_P) \end{pmatrix} \quad (20)$$

As we can see, the vector on the right-hand side contains the deterministic results computed for each sample input, and to find them we need to compute the inverse of the polynomial chaos matrix.

$$\vec{a} = \Phi^{-1} \vec{a}^d \quad (21)$$

Nevertheless its solution is not unique, as it depends on the sampling of the $P + 1$ collocation points.

Sampling can be performed following several procedures. Hosder, in [21], studied three widely used techniques:

- Random Sampling: this is the easiest technique, but the results obtained are not accurate [21].
- Latin Hypercube Sampling: this is an algorithm that divides in $P + 1$ sections the cumulative density function, and randomly selects one point for each section. In this way, all the portions of the input range are represented. We have to specify that this technique is more accurate than the random ones [26].
- Hammersley Sampling: this algorithm is based on the prime numbers, and its output results unique [27].

For a small number of random variables the evaluations required by the two methods are comparable, but there is a notable difference when n increases. This constitutes the great advantage of the NIPC method, that is not being subjected to the *curse of dimensionality*: in fact, it needs only $P + 1$ deterministic solves, with P computable from equation 15. Oversampling is also possible: in this case the system 20 is solved using the least squares method. Hosder, in [21], showed that with $2(P + 1)$ collocation points instead of the required $P + 1$, NIPC gives more accurate results, however the computational cost increases.

III. Case Study: S-duct optimisation with uncertainties

In recent work [10–12, 28, 29] free-form deformation method coupled with multi-objective genetic algorithm was employed to improve aerodynamic characteristics in a diffusing S-Duct. The aim of the study presented below was to improve the robust optimisation code of a S-duct made by Dal Magro [12] by using a larger number of input and output combinations of uncertainties in order to reduce the flow distortions and the pressure losses.

A known geometry experimentally and numerically [11, 12] investigated in previous works was chosen as the baseline. Moreover, the new geometry parametrisation method developed by D'Ambros in [11] was used and no changes were made in the geometry mesh, flow simulation parameters, number of control points and their degrees of freedom [11, 12], boundary conditions [30].

Table 2 Delot S-duct boundary conditions parameters.

Parameter	Value
Inlet total pressure	88.744 <i>kPa</i>
Inlet static pressure	69.575 <i>kPa</i>
Outlet static pressure	78.982 <i>kPa</i>
Total temperature	286.2 <i>K</i>

A. Uncertainties variables

The first step of an uncertainty quantification is the knowledge of the behaviour of the variable, that means defining its probability density function. Thanks to the research conducted by Liatsikouras [3], one of the parameters chosen as uncertainty is the Inlet velocity. Its mean value is equal to the inlet velocity of Delot [30], and it follows a Gaussian distribution. The other main uncertainty used in this work is the flux inlet deviation. This one refers to the possible inlet velocity deviation respect to the main axial direction. It is easy to understand that this uncertainty is strictly related to the Inlet velocity and its purpose is to extend the speed range at the input of the S-duct. As we know, in reality the inlet velocity is not necessarily perfectly axial, but it can be slightly deviated. The mean value chosen for the flux deviation is zero and it follows a Gaussian distribution. However, uncertainty about the bending angle will not be dealt with in the article.

B. NIPC and NISP Robust optimisations

The robust optimization problems implemented in this work has been computed using two non-intrusive polynomial chaos techniques (i.e., output uncertainties quantification methods):

- Non-intrusive Spectral Projection (NISP).
- Non-intrusive Point Collocation (NIPC).

Two robust optimisation problems were implemented:

- 1) The first optimisation problem has been built with the following parameters:

- One Uncertainty input: Inlet Velocity v ($n = 1$).
- Four objective functions: CP_{mean} , CP_{stdv} , $Swirl_{mean}$, $Swirl_{stdv}$.
- Two constraints: $Swirl_{mean} < 5^\circ$, $Swirl_{stdv} < 1^\circ$.
- $\mu_v \approx 196.53$ m/s, $\sigma_v = 10$ m/s.
- PC order $p = 3$.
- Type polynomial: Hermite Chaos (Gaussian random input).
- Uncertainty quantification method: NIPC and NISP.

Where $CP = 1 - PR$ and PR is the Pressure Recovery. The sampling technique used for the NIPC method is the Latin Hypercube Sampling (LHS). Remembering the Non-Intrusive Point Collocation theory and the Equation 15, $P + 1 = 4$ deterministic evaluations are necessary to find the $P + 1 = 4$ PC (i.e Polynomial Chaos) coefficients and to model the stochastic outputs. As a matter of fact, the Inlet Velocity was sampled 4 times.

On the other hand, for the NISP method was used the Hermite-Gauss quadrature sampling technique. To evaluate the $P + 1 = 4$ PC coefficient and find the stochastic outputs, the NISP method needs $(p + 1)^n$ deterministic solutions and $m = p + 1$ samples for each uncertainty. In this case $(p + 1)^n = P + 1 = 4$ and $m = p + 1 = 4$.

2) The second optimisation problem has been built with the following parameters:

- Two Uncertainties input: Inlet Velocity v and Inlet Flux Deviation β ($n = 2$).
- Two objective functions: CP_{mean} , CP_{stdv} .
- Two constraints: $Swirl_{mean} < 5^\circ$, $Swirl_{stdv} < 1^\circ$.
- $\mu_v \approx 196.53$ m/s, $\sigma_v = 10$ m/s.
- $\mu_\beta = 0^\circ$, $\sigma_\beta = 3^\circ$.
- PC order $p = 2$.
- Type polynomial: Hermite Chaos (Gaussian random input)
- Uncertainty quantification method: NIPC and NISP.

As before, the sampling technique used for the NIPC method is the Latin Hypercube Sampling (LHS). This time, in order to find the $P + 1 = 6$ PC coefficient and to model the stochastic outputs, it is necessary to perform $P + 1 = 6$ deterministic evaluations. As a matter of fact, the Inlet Velocity and the Inlet Flux Deviation were sampled 3 times, forming 9 possible pairs (3^2). Of these nine couples, $P + 1 = 6$ has been chosen.

Instead, for the NISP method was used the Hermite-Gauss quadrature sampling technique like before. Now $(p + 1)^n = 9$ deterministic solutions and $m = p + 1 = 3$ samples for each uncertainty are needed to find the $P + 1 = 6$ PC coefficient and to model the stochastic outputs. It is possible to observe that with more than one uncertainty input, the NISP method needs more deterministic evaluation than the NIPC method, hence the computational cost increases.

In figure 2, the Robust optimisation loop implemented to solve the two-problem described above is shown.

IV. Results

From now on, robust optimisations with a single input uncertainty will be called 1D, while those with two input uncertainties will be labelled as 2D. As said before, the two robust optimisation problems (1D, 2D) were both implemented with two uncertainties quantification techniques (NIPC, NISP). This section will be divided into two subsections. In the first one, the validation of the Robust model will be explained. In the second one, the most significant optima design obtained from the Robust optimisation codes will be highlighted and three of them will be fully post-processed and analysed with plots and contours.

A. Robust Model Validation

In order to accomplish the Robust model validation task, three optima design obtained from the D'Ambros NON-Robust optimisation code were post-processed by introducing input uncertainties and using the output uncertainties quantification methods that were described in the previous chapters. Then, the results were compared. It is important to consider that the aim of the model validation is to show the goodness of the Robust method and what useful implications it may has and gives in the design of aeronautical intakes.

The design chosen from D'Ambros' work were the best Pressure Recovery (CP), the best Swirl (SW) and one of his trade-off designs. The CP and Swirl output values of the three designs are shown in Table 3.

On the other hand, table 4 highlights the results values obtained analysing the above designs with the 1D and 2D Robust codes.

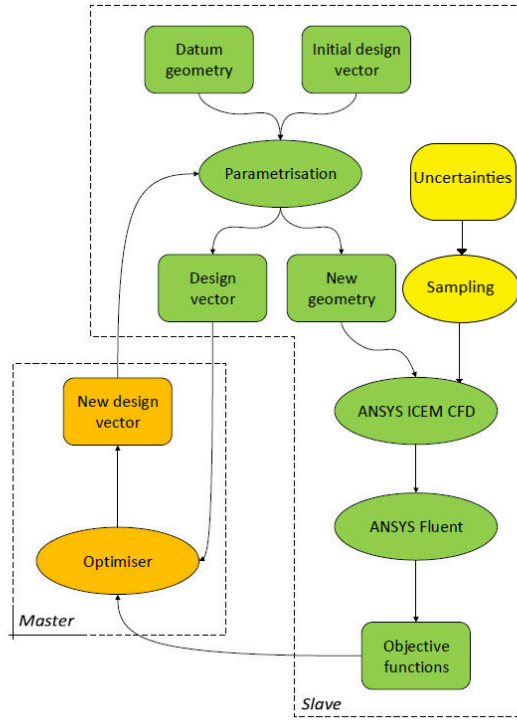


Fig. 2 Robust optimisation loop scheme.

Table 3 Cp and Swirl values of the three chosen D'Ambros designs.

Design	CP	Swirl
Best CP	0.025112	3.034970°
Trade-off	0.026027	1.803860°
Best Swirl	0.027308	1.394520°

Table 4 Results of the three chosen D'Ambros designs analysed with the 1D and 2D Robust codes.

Non-Intrusive Method	Design	CP _{mean}	CP _{stdv}	Swirl _{mean}	Swirl _{stdv}
NIPC 1D	Best CP	0.025791	0.006864	3.094770°	0.193975°
	Trade-off	0.026924	0.007454	1.964015°	0.420667°
	Best Swirl	0.028099	0.007648	1.612200°	0.603127°
NISP 1D	Best CP	0.025796	0.006785	3.098512°	0.187039°
	Trade-off	0.026980	0.007733	1.938850°	0.398151°
	Best Swirl	0.028274	0.008146	1.613197°	0.562283°
NIPC 2D	Best CP	0.025461	0.006988	3.075389°	0.157399°
	Trade-off	0.026486	0.007643	1.881349°	0.302947°
	Best Swirl	0.027848	0.007963	1.584065°	0.553685°
NISP 2D	Best CP	0.025682	0.006589	3.097647°	0.148520°
	Trade-off	0.026814	0.007325	1.958150°	0.342764°
	Best Swirl	0.028083	0.007679	1.625692°	0.501602°

In all the non-intrusive methods and in all the three designs, the values of the CP_{mean} are very close to the CP results obtained by the NON-Robust D'Ambros studies. The same cannot be said for the $Swirl_{\text{mean}}$ values, especially in the trade-off and Best Swirl designs where the robust results differs almost by +8% and +16% from the Non-Robust ones (see Table 5). Despite this difference between the two fluid-dynamic parameters, all of the NON-robust D'Ambros' results are always contained within the probability density function output of the Robust codes (i.e., $\mu_{CP} - \sigma_{CP} < CP < \mu_{CP} + \sigma_{CP}$ and $\mu_{SW} - \sigma_{SW} < Swirl < \mu_{SW} + \sigma_{SW}$).

Table 5 Improvements in the mean values of CP and Swirl.

Design	Method	CP_{mean}	Improvement(CP)	$Swirl_{\text{mean}}$	Improvement(Swirl)
Best CP	NON-Robust	0.025112		3.034970°	
	NIPC 1D	0.025791	+2.7%	3.094770°	+2%
	NISP 1D	0.025796	+2.7%	3.098512°	2.1%
	NIPC 2D	0.025461	+1.4%	3.075389°	+1.3%
	NISP 2D	0.025682	+2.3%	3.097647°	+2.1%
Trade-off	NON-Robust	0.026027		1.803860°	
	NIPC 1D	0.026924	+3.4%	1.964015°	+8.9%
	NISP 1D	0.026980	+3.7%	1.938850°	+7.5%
	NIPC 2D	0.026486	+1.8%	1.881349°	+4.3%
	NISP 2D	0.026814	+3.0%	1.958150°	+8.6%
Best Swirl	NON-Robust	0.027308		1.394520°	
	NIPC 1D	0.028099	2.9%	1.612200°	15.6%
	NISP 1D	0.028274	3.5%	1.613197°	16.2%
	NIPC 2D	0.027848	2.0%	1.584065°	13.6%
	NISP 2D	0.028083	2.8%	1.625692°	16.6%

Moreover, in table 5 It is interesting to observe that the mean values obtained from the robust codes are all higher than the Non robust ones. This difference is a clear effect of the sensitivity of the output parameters to changes in the input parameters and how important is to take it into account.

The result of the Robust method shows how the CP and Swirl behave in different ways in presence of uncertainties input. To better visualize this difference, table 6 highlights the coefficient of variation (CV) of the two physical parameters. The CV coefficient is defined as the ratio between the standard deviation and the mean.

The coefficient of variation remains almost constant for the CP parameter. Whereas, in terms of Swirl, table 6 shows that the Best CP design is the most robust design because is insensitive to the variations of the uncertainties input. On the other hand, the Best Swirl design has a great sensibility to input uncertainties despite having the best swirl value. This is another clear evidence of how important is to take into account the robustness of the solutions and to consider the operative range of the parameters during the design and optimisation processes.

B. Robust Optimisation Results

1. NIPC 1D and NISP 1D

To implement the NIPC 1D cycle, the uncertainty input was sampled with the Latin Hypercube Sampling technique (LHS). The MOTS software was stopped after having successfully evaluated 483 configurations. As explained in section III.B, the NIPC uncertainty quantification technique needs $P + 1$ deterministic evaluations to find the PC coefficients and the stochastic outputs (i.e., objective functions). In this case the number of evaluations were equal to four. This means that $483 \times 4 = 1932$ CFD Fluent simulations converged. The time for a single fluent evaluation is ≈ 39 minutes, so the time that was necessary to obtain all these results was ≈ 1255 hours.

On the other hand, to implement the NISP 1D cycle, the uncertainty input was sampled with the Hermite-Gauss Sampling technique (HG). The MOTS software was stopped after having successfully evaluated 396 configurations. As

Table 6 Coefficient of Variation (CV) for CP and Swirl.

Non-Intrusive Method	Design	CV _{CP}	CV _{Swirl}
NIPC 1D	Best CP	26.6%	6.3%
	Trade-off	27.7%	21.4%
	Best Swirl	27.2%	37.4%
NISP 1D	Best CP	26.3%	6.0%
	Trade-off	28.7%	20.5%
	Best Swirl	28.8%	34.9%
NIPC 2D	Best CP	27.4%	5.1%
	Trade-off	28.9%	16.1%
	Best Swirl	28.6%	35.0%
NISP 2D	Best CP	25.7%	4.8%
	Trade-off	27.3%	17.5%
	Best Swirl	27.3%	30.9%

explained in section III.B, the NISP uncertainty quantification technique needs $(p + 1)^n$ deterministic evaluations to find the $P + 1$ PC coefficients and the stochastic outputs (i.e., objective functions). In this case the number of evaluations were equal to four. This means that $396 \times 4 = 1584$ CFD Fluent simulations converged. The time for a single fluent evaluation is ≈ 39.2 minutes, so the time that was necessary to obtain all these results was ≈ 1035 hours. However, the amount of time above estimated for the NISP 1D and NIPC 1D cycles does not take into account all the configurations that were analysed but subsequently discarded if they did not respect the Swirl mean and Swirl standard deviation constraints or if a structured geometry mesh was not possible to be created by ANSYS ICEM.

As we know it is not possible to represent a Pareto optimal front with more than three dimensions (i.e objective functions). To overcome this issue, the post processing analysis was supported with the use of a Parallel coordinates software developed by Cambridge University. The Parallel Coordinates is a very powerful tool. It was developed with the aim to give to the designer the possibility to see all the evaluated design space in all its dimensions (in this work 36 design parameters), and to link each design geometry with its objective functions. In other words, this tool allows to choose the designs that minimise all the four objective functions simultaneously and to verify if there are zones of the design space that have not been fully evaluated. Moreover, it helps the designer to understand which design parameters most influence the occurrence of certain physical phenomena inside the Duct.

In table 7, all the displayed values of the four NIPC 1D optima design improved a lot if compared to the Baseline ones. These four designs were chosen because they well minimised al the objective functions output and they were very close inside the design space. In fact, the output results are very similar. On the other hand, the CP mean and standard deviation values of the NISP 1D Optima designs have worsened compared to the baseline ones. Nevertheless, these last three optima designs were not chosen for their improvements in the results values, but because their four objective functions had close results despite of their different geometry shape. This means that probably there were different physical mechanisms balanced in different ways that allowed to achieve the same results. Only a transient simulation with the LES method (large Eddy Simulation) could affirm or deny the above statement.

Table 7 is also useful because it shows the comparison between the D'Ambros optima designs (those previously analysed with robust codes in table 4) and the new optima design obtained from the 1D robust optimisation loop. In fact, a big improvement in the Swirl_{stdv} values can be seen in the NIPC 1D method. These values are not only better than the NISP 1D ones but also better than the three D'Ambros design (see table 7). In general, the results again confirm the goodness of the robust method. The optima designs obtained from the robust codes despite not having excellent mean values, they have good stability when varying the values of the input parameters.

Table 7 NIPC 1D and NISP 1D optima designs results chosen with the use of the parallel coordinates.

Non-Intrusive Method	Design	CP _{mean}		CP _{stdv}		Swirl _{stdv}		Swirl _{stdv}	
NIPC 1D	Baseline	0.02905		0.007574		4.283636°		0.223726°	
	optima 1	0.026964	−7.2%	0.006794	−10.0%	2.32841°	−45.6%	0.022828°	−89.8%
	optima 2	0.027002	−7.0%	0.006802	−10.2%	2.3605°	−44.9%	0.021152°	−89.8%
	optima 3	0.027124	−6.6%	0.006896	−9.0%	2.27667°	−46.9%	0.05989°	−73.2%
	optima 4	0.026902	−7.4%	0.006819	−10.0%	2.36001°	−44.9%	0.030626°	−86.3%
D'Ambros NIPC 1D	Best CP	0.025791	−11.2%	0.006864	−9.4%	3.094770°	−27.8%	0.193975°	−13.3%
	Trade-off	0.026924	−7.3%	0.007454	−1.6%	1.964015°	−54.2%	0.420667°	+88.0%
	Best Swirl	0.028099	−3.3%	0.007648	+1.0%	1.612200°	−62.4%	0.603127°	+170%
NISP 1D	Baseline	0.029124		0.007765		4.283883°		0.224200°	
	optima 1	0.031076	+6.7%	0.008419	+8.4%	3.02689°	−29.3%	0.082971°	−63.0%
	optima 2	0.031532	+8.3%	0.008203	+5.6%	3.21841°	−24.9%	0.164994°	−26.4%
	optima 3	0.031219	+7.2%	0.007658	−1.4%	3.10347°	−27.6%	0.171318°	−23.6%
	Best CP	0.025796	−11.4%	0.006785	−12.6%	3.098512°	−27.7%	0.187039°	−16.6%
D'Ambros NISP 1D	Trade-off	0.026980	−7.4%	0.007733	−0.4%	1.938850°	−54.7%	0.398151°	+77.6%
	Best Swirl	0.028274	−2.9%	0.008146	+4.9%	1.613197°	−62.4%	0.562283°	+151%

2. NIPC 2D and NISP 2D

To implement the NIPC 2D cycle, the two uncertainties input were both sampled with the Latin Hypercube Sampling technique (LHS). The MOTS software was stopped after having successfully evaluated 260 configurations. In this case the number of deterministic evaluations to find the PC coefficients were equal to six. This means that $260 \times 6 = 1560$ CFD Fluent simulations converged. The time for a single fluent evaluation is ≈ 41.4 minutes, so the time that was necessary to obtain all these results was ≈ 1076 hours.

On the other hand, to implement the NISP 2D cycle, the two uncertainties input were sampled with the Hermite-Gauss Sampling technique (HG). The MOTS software was stopped after having successfully evaluated 187 configurations. In this case the number of deterministic evaluations to find the PC coefficients were equal to nine. This means that $187 \times 9 = 1683$ CFD Fluent simulations converged. The time for a single fluent evaluation is ≈ 39.1 minutes, so the time that was necessary to obtain all these results was ≈ 1097 hours.

This time the objective functions are two and a bi-dimensional Pareto front plot is sufficient to show all the MOTS (Multi Objective Tabu Search) evaluated points and the optimised designs. The two Pareto fronts that were obtained are illustrated in figure 3, 4.

From these Pareto, nine optima designs were chosen (see table 8):

- 4 from the NIPC 2D: The Best CP_{mean}, two trade-off design and the Best CP_{stdv}.
- 5 from the NISP 2D. The Best CP_{mean}, three trade-off designs and the Best CP_{stdv}.

As said in section III.B, in the 2D robust optimisation loops, the Swirl_{mean} and Swirl_{stdv} values were not objective functions but only constraints, in order to speed up the search for optima designs. Moreover, they were not recorded during the loop because of data storage limited space. Because of this, table 8 shows the improvements and the comparison in the CP_{mean} and CP_{stdv} between the D'Ambros optima designs (those previously analysed with robust codes in table 4), the new optima design obtained from the 2D robust optimisation loop and the baseline design.

In table 8, the robust designs and the D'Ambros ones improved if compared to the baseline. Moreover, the two Robust codes (NIPC 2D and NISP 2D) found the best results in both the objective function outputs. In other words, these two robust optimisation codes allows us to find not only excellent CP value, but also more output stability to input uncertainties when there is a small inlet deviation of the flow.

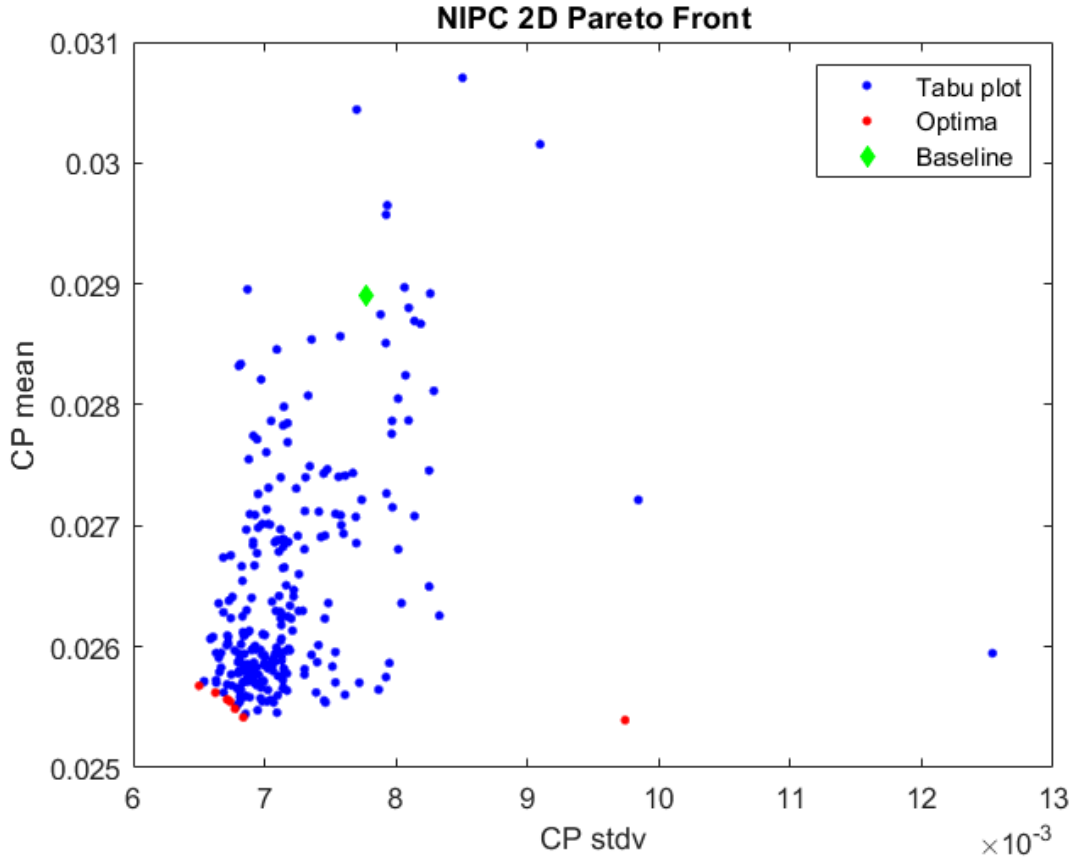


Fig. 3 NIPC 2D: Pareto front.

Table 8 NIPC 2D Pareto Front chosen designs.

Non-Intrusive Method	Design	CP _{mean}		CP _{stdv}	
NIPC 2D	Baseline	0.028905		0.007768	
	optima 1	0.025393	−12.2%	0.009743	+25.4%
	optima 2	0.025416	−12.1%	0.006839	−12.0%
	optima 3	0.025487	−11.8%	0.006775	−12.8%
	optima 4	0.025677	−11.2%	0.006502	−16.3%
D'Ambros NIPC 2D	Best CP	0.025461	−11.9%	0.006988	−10.0%
	Trade-off	0.026486	−8.4%	0.007643	−1.6%
	Best Swirl	0.027848	−3.7%	0.007963	−2.5%
NISP 2D	Baseline	0.028973		0.007522	
	optima 1	0.02558	−11.7%	0.00666	−11.5%
	optima 2	0.025765	−11.1%	0.006607	−12.2%
	optima 3	0.0259	−10.6%	0.006572	−12.6%
	optima 4	0.025904	−10.6%	0.006559	−12.8%
	optima 5	0.026599	−8.2%	0.006463	−14.1%
D'Ambros NISP 2D	Best CP	0.025682	−11.4%	0.006589	−12.4%
	Trade-off	0.026814	−7.5%	0.007325	−2.6%
	Best Swirl	0.028083	−3.1%	0.007679	−2.1%

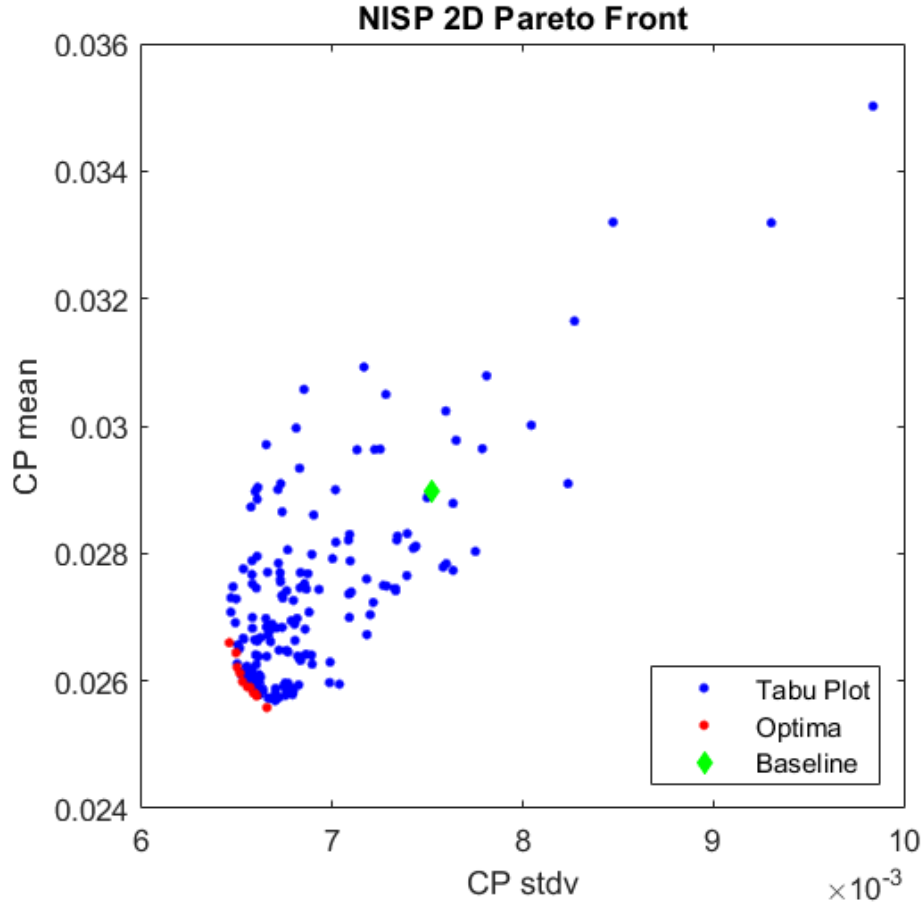


Fig. 4 NISP 2D: Pareto front.

C. Three Optima Robust designs analysis

Before to start with the post-processing analysis of three designs, it is very important to specify that the output values of the objective functions obtained from the NIPC 1D (and also from the NIPC 2D) optimisation loop, are slightly different from the ones obtained after the post-processing analysis of the chosen designs. These small differences in the results are the effect of two events:

- 1 The sampled velocities were not recorded during the optimisations loop.
- 2 The LHS method divides in $P + 1$ sections the cumulative density function, and randomly selects one point for each section.

In other words, the sampled uncertainties of the post-processing design analysis are different from the optimisation cycle ones. It must be remarked that the above-mentioned effect does not occur in the NISP optimisation cycles because of the different nature of the Hermite-Gauss quadrature sampling. In fact, the latter chooses always the same values inside the Gaussian probability density function (PDF). From now, until the end of the article, all the contour drawings and plots will refer to the post-processing objective function values.

1. NIPC 1D: Optima 1

The first configuration taken into account in this paragraph is the *Optima 1* design. Table 9 highlights the differences between the objective functions values obtained from the optimisation loop and the post-processing analysis ones due to the nature of the LHS sampling technique.

In figure 5, it is possible to see the pressure recovery values (PR) evaluated at the AIP surface for each inlet velocity sample. The order is from the lowest velocity, top left, to the highest one, bottom right. The left half of each AIP contours drawing represents the pressure recovery at the AIP surface of the baseline. On the other hand, the right one

Table 9 NIPC 1D optima 1. Differences in the objective functions values due to the nature of the LHS sampling technique.

Design	CP_{mean}	CP_{stdv}	$Swirl_{mean}$	$Swirl_{stdv}$
Optimisation	0.026964	0.006794	2.32841°	0.022828°
Post-proc.	0.026979	0.006793	2.330194°	0.024040°
Baseline	0.02905	0.007574	4.283636°	0.223726°

represents the deformed one.

In the PR contour plot, the deformed geometry shows two zones with the lowest value of pressure recovery instead of only one present in the Baseline configuration. These are recirculation areas where a flow detachment is present. The first flow detachment occurs in the initial/central part of the duct and the second one towards the end of the S-Duct. This phenomenon is due to the presence of a sharp lateral widening of the sections near the AIP that allows the formation of a new recirculation area on the last lateral part of the duct (see Figure 7). In both detachment areas, the pressure recovery decreases as the inlet velocity increases. In fact, it is possible to imagine that as the velocity increases, the flow has more and more difficulties in adhering to the side wall.

Moreover, in all the four deformed design PR contour, the Pressure recovery values of the primary flow detachment area (the central one) are higher than the baseline ones (Figure 5). This improvement is also confirmed by the minimum shear stress value along the X-axis. As a matter of fact, the strength or weakness of the separation bubble (i.e., the magnitude) depends on the minimum negative value of the x-component of the shear stress. The minimum X-shear stress obtained from the deformed geometry was -3.27159 [Pa]. On the other hand, the baseline one was -7.5348 [Pa]. These two values were calculated with a steady CFD simulation where the input velocity was the mean velocity (196.5 m/s).

Another important thing to highlight is the presence of a bottleneck near the AIP (Figure 7). The trend of the swirl angle in the AIP is illustrated in figure 6 and it is possible to observe a good improvement on its values. Swirl angle is strongly linked to tangential velocity distribution. The extreme low tangential velocity value in the deformed geometry is linked to the particular triangular shape of this duct shortly after the first bend. As a matter of fact, a large increase in the ending and lower part of the duct favoured diffusive phenomena (from triangular to circular cross section) that led to an increase in static pressure and a strong reduction in tangential velocity.

In table 10, all the values of $CP = 1 - PR$ and Swirl, that were found for each inlet velocity sample, are reported.

For S-ducts intakes, another important parameter is the DC60 and DC15. Briefly, the $DC\theta$ is a distortion parameter that represents the variation of the total pressure across the AIP and it is defined as follow:

$$DC\theta = \frac{\bar{P}_{tot,AIP} - \bar{P}_{tot,\theta}}{q_{f,AIP}} \quad (22)$$

where $\bar{P}_{tot,\theta}$ is the lowest average total pressure of all sectors in the AIP (so it correspond to the worst sector), $\bar{P}_{tot,AIP}$ and $q_{f,AIP}$ are the average total pressure and the dynamic head, respectively, at AIP. Theta (θ) is the angular value of the sectors.

This distortion parameter was not taken into account as objective function, but was considered as uncertainty output in the post processing analysis. The DC60 mean and standard deviation values of the deformed geometry are: $DC_{mean} = 0.204758$, $DC_{stdv} = 0.048555$. While, for the Baseline they are: $DC_{mean} = 0.352555$, $DC_{stdv} = 0.049511$. The improvement is huge in the mean 42% and not significant in the standard deviation 2%.

The good results of the DC60 values are the effect of the improvements in the Swirl values. In fact, it can also be argued that although the DC60 parameter is mathematically related to the total pressure, the flow distortion is also greatly influenced by the swirl values. The lower the swirl, the lower the distortion value. This statement is confirmed by the performance of the DC60 parameter in the next two analysed designs (section IV.C.2, section IV.C.3) that we will see later.

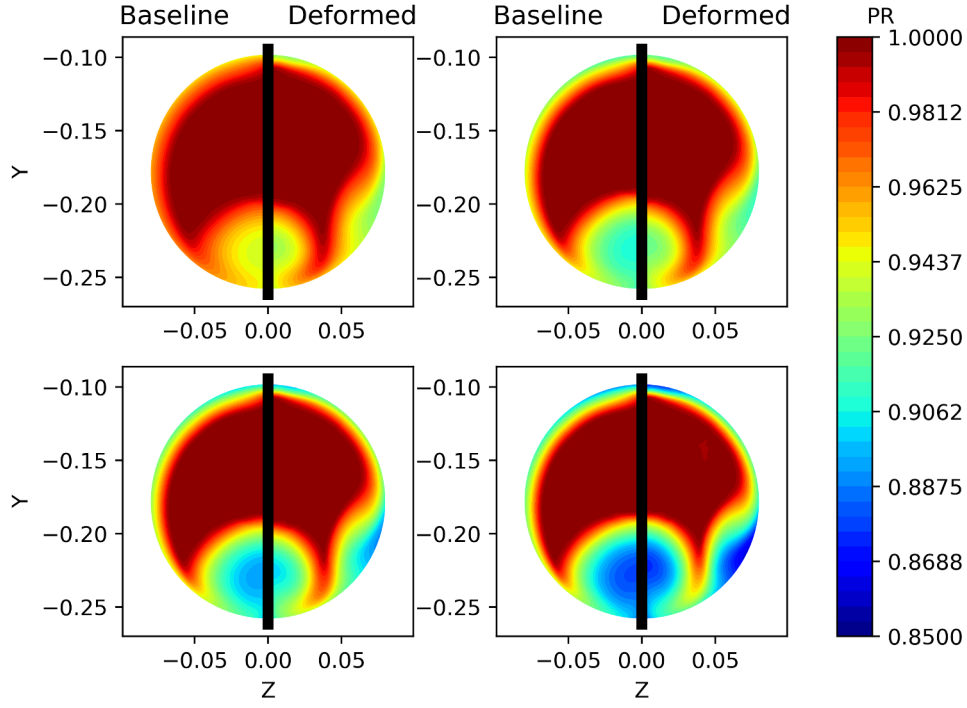


Fig. 5 NIPC 1D Optima 1. PR contour comparison at the AIP.

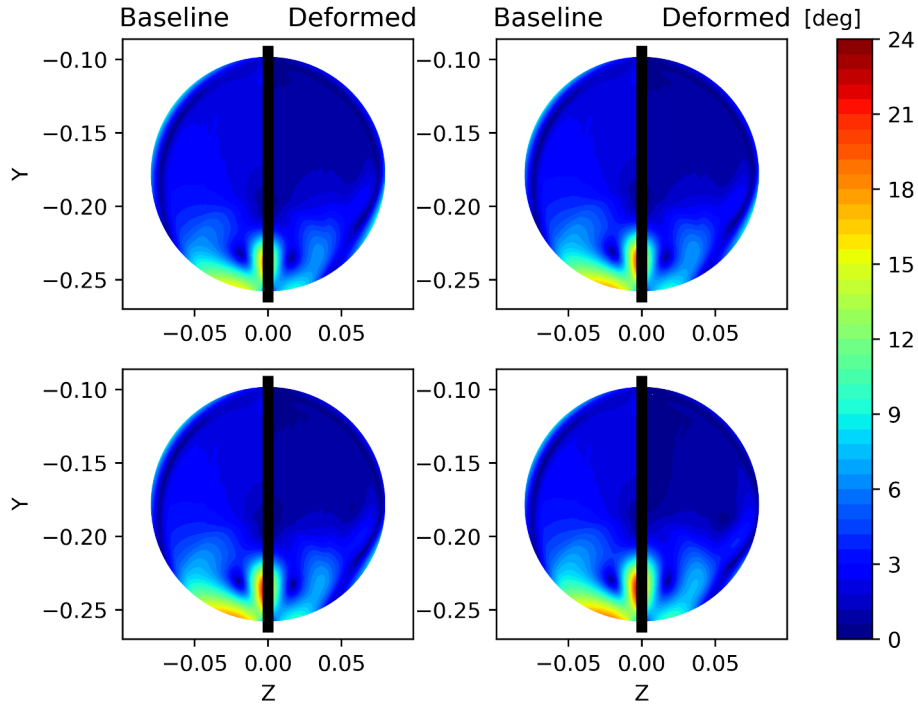


Fig. 6 NIPC 1D optima 1. Swirl contour comparison at the AIP.

Table 10 NIPC 1D Otima 1: CP and Swirl value for each velocity sample.

	CP Baseline	CP Optima 1		Swirl Baseline	Swirl Optima 1	
Sample 1	0.02278	0.01959	−14.0%	4.0955°	2.3454°	−42.7%
Sample 2	0.024020	0.023185	−3.5%	4.1326°	2.3327°	−43.6%
Sample 3	0.030367	0.03095	+1.9%	4.3234°	2.3131°	−46.5%
Sample 4	0.040817	0.041196	+0.9%	4.6301°	2.3686°	−48.8%

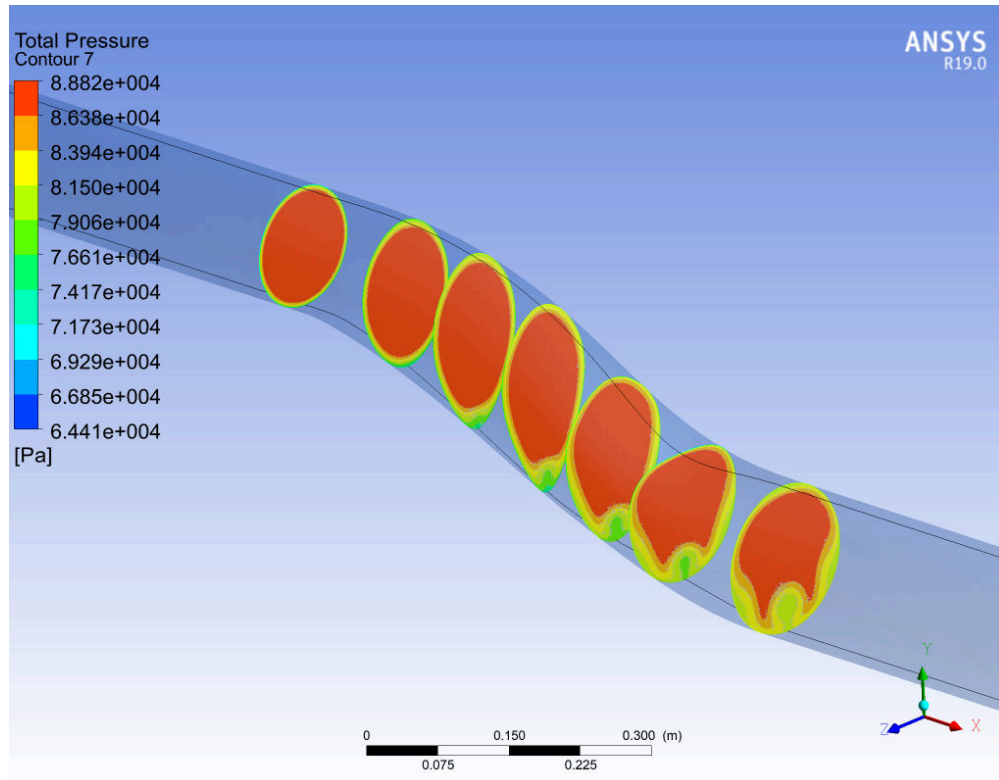


Fig. 7 NIPC 1D Optima 1 Steady state analysis. The contour represent the total pressure. The analysis was carried out with an Inlet velocity equal to 196.5 m/s. Every cross-section is perpendicular to x-direction.

2. NIPC 2D: Optima 2

The configuration taken into account in this paragraph is the NIPC 2D optima 2 design. As in the previous design, table 11 highlights the differences between the objective functions values obtained from the optimisation loop and the post-processing analysis ones due to the LHS sampling technique. Moreover, in this case, also the Baseline was post-processed in order to find the uncertainties output values of the $Swirl_{mean}$ and $Swirl_{stdv}$ that were not recorded during the 2D robust optimisation loop.

Table 11 NIPC 2D optima 2. Differences in the objective functions values due to the nature of the LHS sampling technique.

Design	CP_{mean}	CP_{stdv}	$Swirl_{mean}$	$Swirl_{stdv}$
Optimisation	0.025416	0.006839	Not recorded	Not recorded
Post-proc.	0.025467	0.006753	4.423186°	0.231642°
Baseline	0.028905	0.007768	Not recorded	Not recorded
Post-proc Baseline	0.028005	0.011396	4.255752°	0.309974°

In figure 8 and 9, it is possible to see the pressure recovery values (PR) and the swirl values evaluated at the AIP surface for each inlet velocity and flux deviation sampled couple. This time, the order of the sub-plots is random because the couples of the two sampled uncertainties were chosen randomly.

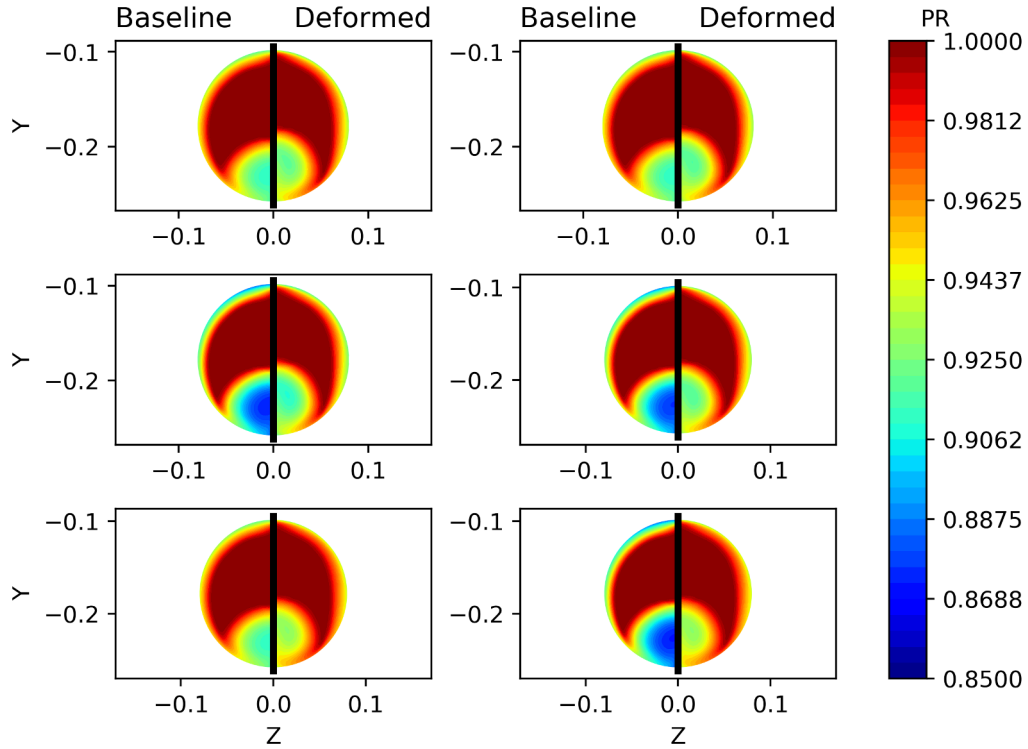


Fig. 8 NIPC 2D Optima 2. PR contour comparison at the AIP.

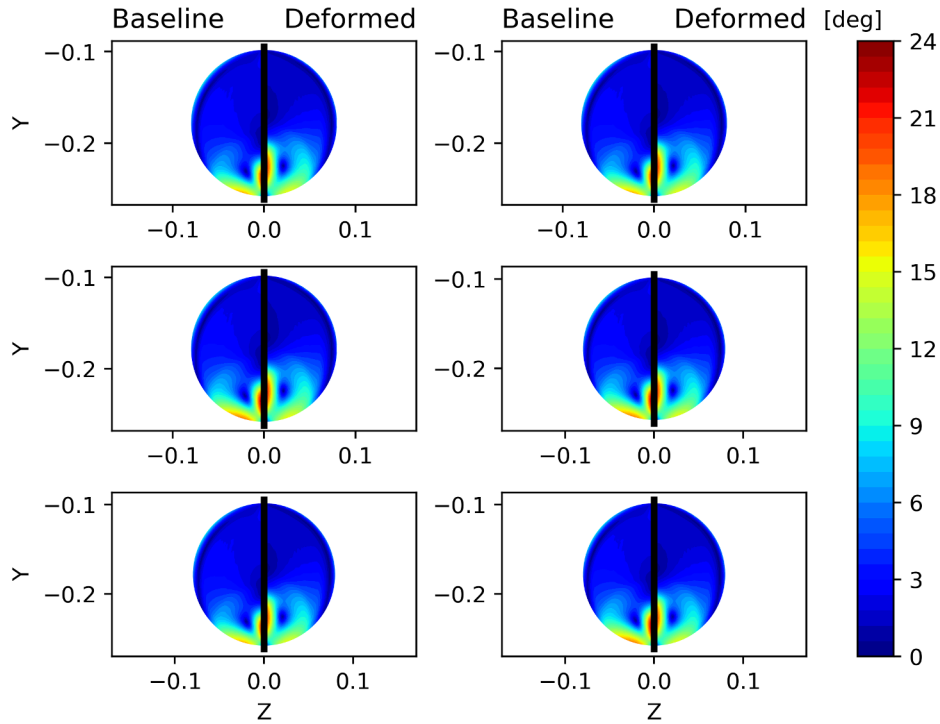


Fig. 9 NIPC 2D optima 2. Swirl contour comparison at the AIP.

The Pressure recovery contour of the deformed geometry shows an improvement in their values if compared to the Baseline. This improvement is due to the presence of a large bulge on the top side of the duct close to the Inlet section. In other words, it is possible to think this bulge as a sudden enlargement of the section that led to a slowing of the flow in the initial part of the duct. As a matter of fact, in physical terms, a slower flow field produce a weaker flow detachment and this is what probably led to these improvements in the PR values. This change in the geometry shape led also to a narrower and higher flow detachment area if compared to the baseline one and to better values on the top side of the contour plot.

In figure 10, it is also possible to see how the S-duct approaches rectangular geometry. Also [11] discover how optima CP designs tended to have rectangular cross-section. On the other hand, optima Swirl tended to have triangular cross-sections.

In table 12, all the values of $CP = 1 - PR$ and Swirl that were found for each inlet velocity and flux deviation sampled couple, are reported.

Table 12 NIPC 2D Otima 1: CP and Swirl value for each velocity sample.

	CP Baseline	CP Optima 2		Swirl Baseline	Swirl Optima 2	
Sample 1	0.023248	0.024366	+4.8%	4.1094°	4.3676°	+6.3%
Sample 2	0.029602	0.025240	-14.7%	4.3007°	4.3952°	+2.2%
Sample 3	0.028978	0.028313	-2.3%	4.2817°	4.4984°	+5.1%
Sample 4	0.033475	0.02520	-24.7%	4.4166°	4.3940°	-0.5%
Sample 5	0.023414	0.021294	-9.1%	4.1143°	4.2776°	+4.0%
Sample 6	0.032773	0.02126	-35.1%	4.3956°	4.2766°	-2.7%

The results obtained from this post-processing design analysis shows how the robust optimisation loop focused

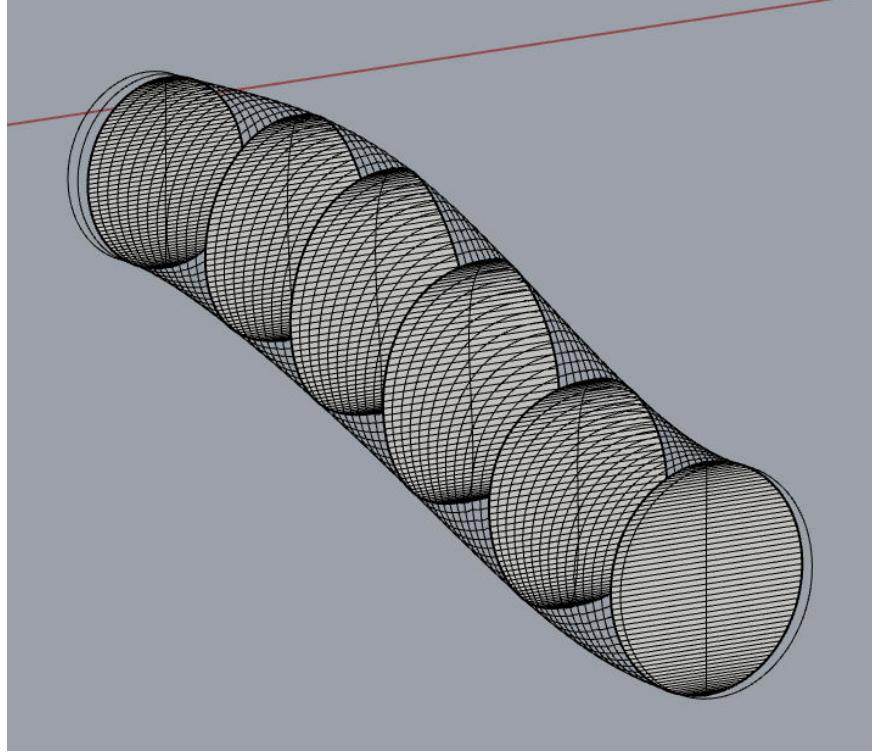


Fig. 10 NIPC 2D: central Cross-sections of the duct. The cross-sections are perpendicular to the x-direction

the search for optimal designs by evaluating only the CP_{mean} and CP_{stdv} (i.e., objective functions). As a matter of fact, the Swirl output values obtained from the post-processing analysis slightly worsened when compared to the Baseline ones and the contours are practically identical with two zone of high tangential velocity. As in the previous design, also the DC60 was considered as uncertainty output during the post processing analysis. The DC60 values obtained from the baseline configuration were: $DC60_{mean} = 0.344826$, $DC60_{stdv} = 0.076647$. On the other hand, the DC60 mean and the DC60 standard deviation values of the NIPC 2D optima 1 design were: $DC60_{mean} = 0.272275$, $DC60_{stdv} = 0.028332$. There was an improvement of 21% in the mean value, and 63% in the standard deviation.

3. NISP 2D: Optima 2

The last configuration taken into account is the NISP 2D optima 2 design. In this case, the values of the post-processing results and the ones obtained from the optimisation cycle are the same because of the nature of the Hermite-Gauss quadrature sampling. As in the previous design (NIPC 2D Otima 2), the Baseline was post-processed in order to find the uncertainties output values of the $Swirl_{mean}$ and $Swirl_{stdv}$ that were not recorded during the 2D robust optimisation loop (Table 13).

Table 13 NISP 2D optima 2. Baseline results vs. optimisation results.

Design	CP_{mean}	CP_{stdv}	$Swirl_{mean}$	$Swirl_{stdv}$
Optimisation	0.025765	0.006607	Not recorded	Not recorded
Post-proc.	0.025765	0.006607	4.177405°	0.205286°
Baseline	0.028973	0.007522	Not recorded	Not recorded
Post-proc Baseline	0.028973	0.007522	4.279787°	0.220496°

It is important to specify that in the NISP 2D, the order of the sub-plots follows the order of the sampled couples.

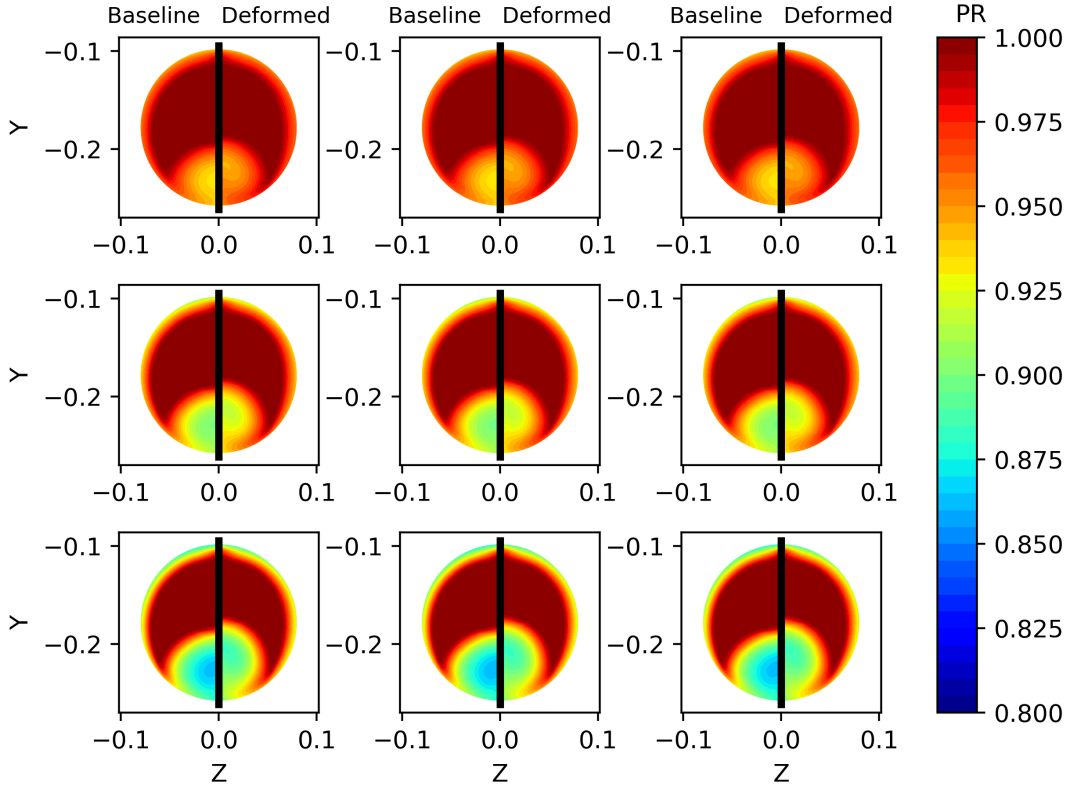


Fig. 11 NISP 2D Optima 2. PR contour comparison at the AIP.

As a matter of fact, the velocity value increases row by row, while the Flux deviation increases column by column (See figure 11 and 12).

This design shows a small bulge on the top side of the duct close to the Inlet section and a bigger diffusing near the AIP on the lowest part of the duct. These two elements led to an improvement in the PR values but they have increased the size of the recirculation bubble 13. As for the previous design (IV.C.2), the NISP 2D optima 2 post-processing analysis shows how the robust optimisation loop focused the search for optimal designs by evaluating only the two CP objective functions. As a matter of fact, the Swirl output value worsened when compared to the Baseline. The DC60 values obtained from the baseline configuration were: $DC60_{mean} = 0.351788$, $DC60_{stdv} = 0.048232$. The DC60 mean and the DC60 standard deviation values of the deformed one were: $DC60_{mean} = 0.270666$, $DC60_{stdv} = 0.030209$. There was an improvement of 23% in the mean value, and 37% in the standard deviation.

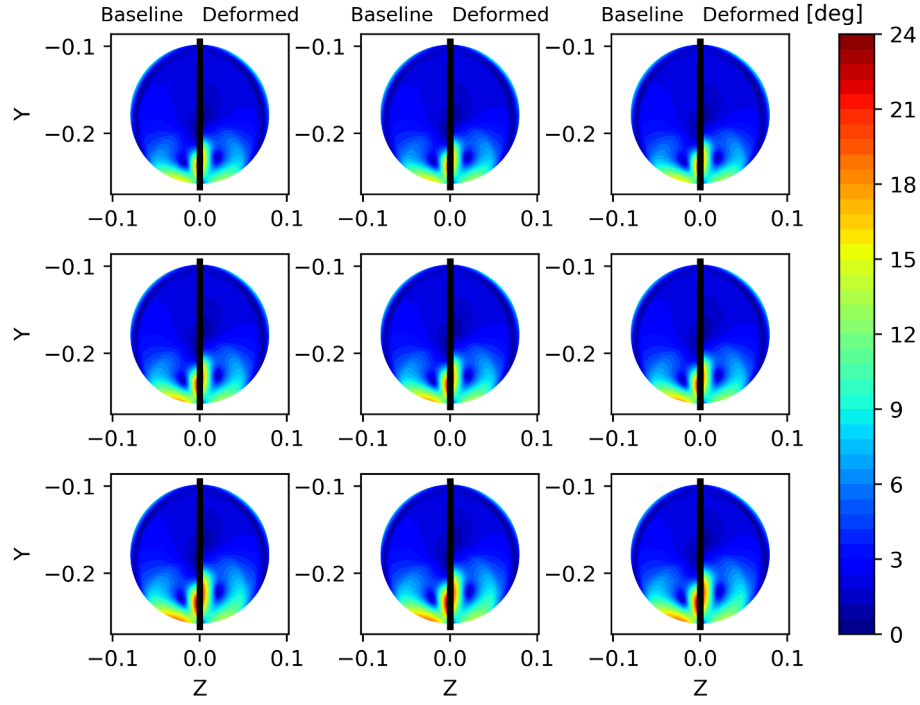


Fig. 12 NISP 2D optima 2. Swirl contour comparison at the AIP.

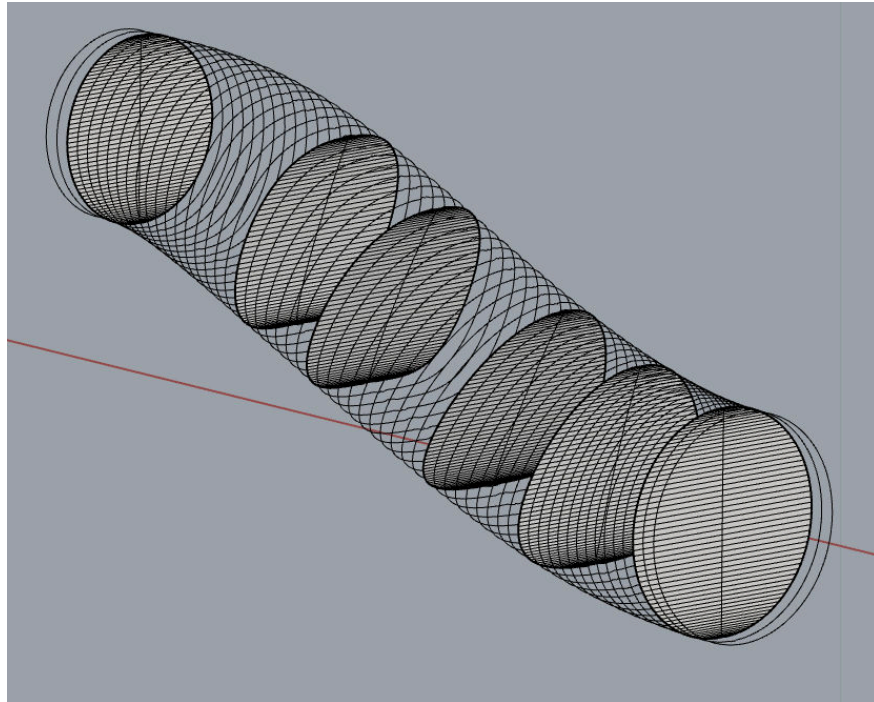


Fig. 13 NIPC 2D: central Cross-sections of the duct. The cross-sections are perpendicular to the center-line.

V. Conclusion

In this work, we developed two different robust optimisations problems, both with two different Non-Intrusive uncertainty quantification techniques (NIPC and NISP), applied to an S-duct inlet geometry. Before studying the results obtained from the robust optimisation cycles, the validation of the robust model was carried out by analysing with it the optimal designs obtained from the NON-robust D'Ambros optimisation [11]. After that, the results were compared. It was evident the importance to take into account the robustness of the solutions and to consider the operative range of the parameters during the design and optimisation processes.

The first problem dealt with one uncertainty input (Inlet Velocity) has been developed and improved with the addition of other two important objective functions ($Swirl_{mean}$ and $Swirl_{stdv}$) to the existing ones (CP_{mean} and CP_{stdv}). The results obtained are encouraging. In fact, good values of all the four objective functions were obtained simultaneously with excellent improvements, mostly in the swirl. Even though the design configurations are different, they exhibit similar robust behaviour.

Another step forward has been made with the development of the 2D problem, or better that problem with two input uncertainties.

The Inlet Flux deviation has been the second uncertainty input to be added. In the 2D problem, it was decided to minimise only two objective functions (CP_{mean} , CP_{stdv}) in order to speed up the search of the optima designs performed by the algorithm (Tabu Search).

Generally, good CP_{mean} results were obtained from the NIPC 2D and NISP 2D cycles. A good improvement in the CP_{mean} values was also achieved if compared to the ones obtained by Dal Magro in his 1D problem [12].

In this paper two Non-Intrusive polynomial chaos technique were implemented (NIPC and NISP) with the aim of investigating the propagation of the propagation of uncertainties from the model input parameters to the simulation output results. Both NIPC and NISP methods showed how the computational cost grows as the number of random variables increase. Nevertheless, the NISP method suffers most from the *course of dimensionality*. As a matter of fact, the number of deterministic solves grows exponentially with the number of random dimensions if compared to NIPC method. In other words, the point-collocation method (NIPC) is useful when multiple random inputs are considered.

References

- [1] Moro, M., "Robust Aerodynamic Optimisation of a High-Lift Configuration Wing with Polynomial Chaos," Master's thesis, Cranfield University, 2013.
- [2] Mattioli, M., "Aerodynamic design sensitivities and uncertainty management," Master's thesis, Cranfield University, 2015.
- [3] Liatsikouras, A. G., Asouti, V. G., Giannakoglou, K. C., Pierrot, G., and Megahed, M., "Aerodynamic shape optimization under flow uncertainties using non-intrusive polynomial chaos and evolutionary algorithms," *2nd ECCOMAS Thematic Conference on Uncertainty Quantification in Computational Sciences and Engineering*, 2017.
- [4] Ghisu, T., Parks, G. T., Jarrett, J. P., and Clarkson, P. J., "Robust design optimization of gas turbine compression systems," *Journal of Propulsion and Power*, Vol. 27, No. 2, 2011, pp. 282–295.
- [5] Kato, H., Mashiko, H., Tokuyama, Y., Funazaki, K., and Takida, J., "Robust aerodynamic shape optimization of supersonic turbine using Non-Intrusive Polynomial Chaos expansion," *9th World Congress on Structural and Multidisciplinary Optimization*, 2011.
- [6] Simon, F., Gullien, P., Sagaut, P., and Lucor, D., "A gPC-based approach to uncertain transonic aerodynamics," *Computer Methods to Applied Mechanics and Engineering*, Vol. 199, 2010, pp. 1091–1099.
- [7] Bruno, L., Canuto, C., and Fransos, D., "Stochastic aerodynamics and aeroelasticity of a flat plate via generalised Polynomial Chaos," *Journal of Fluid and Structures*, Vol. 25, 2009, pp. 1158–1176.
- [8] Reagan, M. T., Najm, H. N., Ghanem, R. G., and Knio, O. M., "Uncertainty quantification in reacting-flow simulations through Non-Intrusive Spectral Projection," *Combustion and Flame*, Vol. 132, 2003, pp. 545–555.
- [9] Wellborn, S. R., Reichert, B. A., and Okiishi, T. H., "A study of the compressible flow in a diffusing S-duct," Tech. rep., NASA, 1993.
- [10] Rigobello, A., "A Multi-objective shape optimization of an S-Duct intake through NSGA-II genetic algorithm," Master's thesis, Università degli Studi di Padova, 2015.

- [11] D'Ambros, A., Kipouros, T., Zachos, P., Savill, M., and Benini, E., "Computational Design Optimization for S-Ducts," *Design MDPI*, 2018.
- [12] Magro, D. D., "Implementation of uncertainty management techniques in the design of S-ducts intakes," Master's thesis, Università degli Studi di Padova, 2019.
- [13] Jaeggi, D., Parks, G., Kipouros, T., and Clarkson, J., "A Multi-objective Tabu Search Algorithm for Constrained Optimisation Problems," In: *Coello Coello C.A., Hernández Aguirre A., Zitzler E. (eds) Evolutionary Multi-Criterion Optimization. EMO 2005. Lecture Notes in Computer Science vol 3410. Springer, Berlin, Heidelberg.*, 2005.
- [14] Wiener, N., "The Homogeneous Chaos," *American Journal of Mathematics*, Vol. 60, No. 4, 1938, pp. 897–936.
- [15] Karniadakis, G. E., Su, C.-H., Xiu, D., Lucor, D., Schwab, C., and Todor, R. A., "Generalized Polynomial Chaos solution for differential equations with random inputs," *Under review in SIAM review for Seminar für Angewandte Mathematik Eidgenössische Technische Hochschule*, 2005.
- [16] Xiu, D., and Karniadakis, G. E., "Modeling uncertainty in flow simulations via generalized Polynomial Chaos," *Journal of Computational Physics*, Vol. 187, 2003, pp. 137–167.
- [17] Cameron, R. H., and Martin, W. T., "The Orthogonal Development of Non-Linear Functionals in Series of Fourier-Hermite Functionals," *Annals of Mathematics*, 1947.
- [18] Xiu, D., and Karniadakis, G. E., "The Wiener-Askey Polynomial Chaos for stochastic differential equations," *SIAM Journal of Scientific Computing*, Vol. 24, No. 2, 2002, pp. 619–644.
- [19] Maitre, O. P. L., Reagan, M. T., Najm, H. N., Ghanem, R. G., and Knio, O. M., "A stochastic projection method for fluid flow. Part II: Random Process," *Journal of Computational Physics*, Vol. 181, 2002, pp. 9–44.
- [20] Hosder, S., Walters, R., and Perez, R., "A Non-Intrusive Polynomial Chaos method for uncertainty propagation in CFD simulations," *Proceedings of the 44th AIAA Aerospace Sciences Meeting and Exhibit*, 2006.
- [21] Hosder, S., Walters, R., and Balch, M., "Efficient sampling for Non-Intrusive Polynomial Chaos applications with multiple uncertain input variables," *Proceedings for the 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, 2007.
- [22] Zhao, L., Dawes, W. N., Parks, G., Jarrett, J. P., and Yang, S., "Robust airfoil design with respect to boundary layer transition," *Proceedings of the 50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, 2009.
- [23] Dodson, M., and Parks, G. T., "Robust aerodynamic design optimization using Polynomial Chaos," *Journal of Aircraft*, Vol. 46, No. 2, 2009, pp. 635–646.
- [24] Ghisu, T., Jarrett, J. P., and Parks, G. T., "Robust design optimization of airfoils with respect to ice accretion," *Journal of Aircraft*, Vol. 48, No. 1, 2011, pp. 287–304.
- [25] Walters, R. W., and Huyse, L., "Uncertainty analysis for fluid mechanics with applications," Tech. rep., NASA Langley Research Center, Hampton, VA, USA, 2002.
- [26] Choi, S.-K., Grandhi, R. V., Canfield, R. A., and Pettit, C. L., "Polynomial Chaos Expansion with Latin Hypercube Sampling for Predicting Response Variability," *44th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference*, 2003.
- [27] Giunta, A. A., Wojtkiewicz, S. F., and Eldred, M. S., "Overview of modern Design Of Experiments methods for computational simulations," *41st AIAA Aerospace Sciences Meeting and Exhibit*, 2003.
- [28] Chiereghin, N., Guglielmi, L., Savill, M., T. Kipouros, E. M., Rigobello, A., Barison, M., and Benini, E., "Shape Optimization of a Curved Duct with Free Form Deformations," *23rd AIAA Computational Fluid Dynamics Conference*, 2017.
- [29] Tridello, R., "Comparison of genetic and tabu search algorithms in aerodynamic design of S-ducts," Master's thesis, Università degli Studi di Padova, 2017.
- [30] Delot, A.-L., and Scharnhorst, D. R. K., "A Comparison of Several CFD Codes with Experimental Data in a Diffusing S Duct," *49th AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, 2013.



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