

Design Framework for Optimizing Waypoints of Vehicle Trajectory Considering Terminal Velocity and Impact Angle Constraints

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ARTICLE HISTORY

Compiled February 14, 2021

ABSTRACT

Ballistic missiles often require the terminal velocity and impact angle to be confined to a certain region around a desired value considering a wide variety of uncertain initial conditions and operational ranges. This study presents a design framework to determine optimum waypoints that satisfy constraints for given launch conditions and mission profiles. As a systematic approach to this kinodynamic motion planning problem, the proposed framework deterministically samples the waypoints. The trajectory generated by the determined waypoints in consideration of various conditions satisfies the terminal constraint. Numerical simulations are performed to demonstrate the effectiveness of the proposed method.

KEYWORDS

Optimization, Trajectory, Guidance, Terminal Velocity, Energy Management

1. Introduction

A missile should satisfy the terminal velocity and impact angle constraint to enhance the lethality of the warhead and enable the seeker to keep track of the target. For example, a missile should fly slower than a specified velocity to protect the fuse from excessive aerodynamic heating when striking a target. In addition, a missile should have a sufficiently high velocity to effectively destroy a target in a multistory shelter. As mechanical energy is the summation of kinetic energy and potential energy, the guidance design problem for striking a target with a desired velocity is basically a problem of energy management.

Numerous studies have been performed on the guidance problem with terminal velocity constraints in various applications, each with unique design concerns, while approaching the problem from different perspectives. The velocity of a launch vehicle with an SRM should satisfy the specified terminal velocity to deliver the payload. After the report by Perkins (1966), various methods were proposed to modify the trajectory

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considering the energy of the vehicle (Chandler and Smith 1967; McHenry et al. 1979; White 1993; Kim and Um 2015). Additionally, re-entry vehicles should carefully dissipate their energy for the landing phase; in this context, Kraemer and Ehlers (1975) proposed a guidance scheme to change the energy dissipation rate by modifying the trajectory or modulating the speed brake for the landing phase of re-entry vehicles; the trajectory was divided into several sections to increase the efficiency of the proposed scheme. Williamson, Greene, and Hull (1976) proposed the perturbation method to generate the optimized shuttle trajectory. Wang, Tang, and Zhang (2019) proposed the re-entry guidance with angle and velocity guidance for hyper-sonic gliding vehicle; the vehicle maneuver was based on conical motion to deceleration. Lan, Xu, and Wang (2020) proposed a guidance law which controlled the angle of attack and the bank angle to reduce the terminal errors in the terminal area energy management phase. Predictive methods were effective at satisfying the terminal velocity constraint. For example, the terminal velocity was estimated using open-loop guidance profiles (Kuroda and Imado 1997) and the specific trajectory shape (Zhou, Rahman, and Chen 2015). Chen and Xia (2016) and Tahk, Moon, and Shim (2019) proposed polynomial form expressions for the flight altitude to predict the terminal velocity, and Moon, Park, and Ryoo (2015) utilized neural networks to obtain the terminal velocity constraint. Note that predictive methods have been shown to be particularly effective in satisfying the terminal velocity constraint.

Unlike previous studies that have dealt only with the impact angle constraint, satisfying the terminal velocity constraint of a ballistic missile is very challenging because no direct control effector exists over the velocity of the missile. A ballistic missile usually employs a solid rocket motor (SRM) to improve the usability and strategic effectiveness. However, an SRM is sensitive to the surrounding environment. A change in the operational environment may lead to deviations in thrust and moment of burn-out, and in velocity and altitude at burn-out, among other consequences. Therefore, a missile exhibiting uncertain burn-out conditions should adjust its energy during the gliding phase to meet the terminal velocity constraint. Additionally, a missile should be able to attack targets located within a wide range of distances from the launch point.

It is almost impossible to design an explicit feedback guidance law that satisfies the terminal velocity constraint. The difficulty is mainly due to the inherent nature of the energy management problem. Unlike the position or the flight path angle, which describe the kinematics of a missile, the variables related to the dynamics are the velocity and energy, and the time-domain solution for these variables associated with the dynamics is hardly available in an analytic form. The lack of a closed-form expression that describes the evolution of the predicted final velocity in relation to an input variable is the main difficulty complicating an analytical approach to this problem. The terminal velocity constraint problem has not been studied as extensively as the impact angle control (IAC). Zhou, Rahman, and Chen (2015) presented a review of previous research covering the last few decades, but most previous studies were limited to the investigations of IAC.

Considering these difficulties, a waypoint optimization approach to feasible trajectory generation may provide an effective solution method. Rafique et al. (2011); Shirazi (2016) proposed the heuristic method to solve the waypoint optimization problem. One workaround in the heuristic method was to carefully construct the locations and incidence angles of waypoints so that the resultant trajectory terminated at the target with the desired velocity. Although this solution worked in an open-loop fashion, it could be implemented in practice. However, finding the waypoint parameters to cover

the entire set of boundary conditions required considerable amount of time and effort. The optimization method to obtain waypoints for the Mars landing mission was investigated by Guo, Hawkins, and Wie (2013). The Zero-Effort-Miss/Zero-Effort-Velocity scheme to obtain the intermediate waypoints was proposed within the thrust limit environment.

Once the position and the incidence angle of the waypoints necessary to accomplish the mission are determined by an appropriate optimization framework, various existing guidance methods can be applied for flight through the waypoints. Jorris and Cobb (2009) showed the maneuvers with the specified waypoints and no-fly zones as constraints. Because of the powered hypersonic cruise vehicle as to the platform, constraints could be satisfied while flying over long distances. Liang, Li, and Ren (2016) described the review of researches for the entry guidance. A guidance algorithm with the atmospheric entry condition for the waypoint constraint was proposed. The algorithm generated the command to reach the waypoints and target. He et al. (2019) proposed waypoint guidance for an unmanned vehicle. The guidance command was calculated from waypoint passing constraints and flight path angle constraints to satisfy the arrival angle of the vehicle.

The approach for waypoint optimization of a ballistic missile propelled by an SRM is still relatively new because, for this scenario, it is difficult to apply the methods previously developed for other types of flight vehicles. A ballistic missile inevitably flies through the atmosphere along a considerable part of its flight path to the target, which is typically located on the ground. In addition, a missile does not have a propulsion system nor a braking system to directly control its velocity after the engine burns out. Therefore, trajectory shaping is the only method for optimized trajectory with the terminal constraints.

The aim of this study is to propose a design framework to optimize waypoints for ballistic missiles with the terminal velocity and impact angle constraints. The proposed method enhances the robustness of the design against uncertainties in the conditions at burn-out, making this framework widely applicable to missions with different target distances. Moreover, the proposed approach is useful in practice because it is obtained by a closed-loop simulation that includes guidance and control schemes.

The remainder of this article is organized as follows. Sections 2 and 3 describe the problem statement and equations of motion. Section 4 introduces the proposed design framework for obtaining optimized trajectory to satisfy the terminal constraints. In Section 5, simulation results are shown to demonstrate the effectiveness of the proposed framework. Finally, the concluding remarks are given in Section 6.

2. Problem Statement

The objective of this study is to generate a feasible trajectory that satisfies terminal velocity and flight path angle constraints to define an optimization problem, let us consider the following performance index J .

$$\min J = \omega \times V_{err} + (1 - \omega) \times \gamma_{err} \quad (1)$$

where V_{err} and γ_{err} are the velocity error and the flight path angle error evaluated at the moment of impact, respectively, and ω is the weighting factor. The optimization framework developed in this study takes benefits from two central elements. First, a

classical optimal guidance law for IAC is employed to account for the reduction of γ_{err} . Second, a heuristic search method is devised to effectively find feasible trajectories. A deterministic search method considering trajectory characteristics with waypoints is developed to satisfy the terminal velocity constraint because stochastic optimization methods may yield one of many local solutions.

Normally, ballistic missile moves almost entirely in the vertical plane to strike the target as fast as possible, which does not maneuver in the horizontal plane unless there exists a specific mission or the survivability of the missile is desired. The effectiveness of the aerodynamic control surfaces decreases at high altitudes because of low air density, which means that the energy dissipation effect is not significant. Accordingly, the missile should dissipate its energy at low altitudes. Because of the conditions imposed on the maneuver plane and the altitude, the ballistic missile under consideration preferably performs pull-up maneuvers. To satisfy terminal velocity constraints, it is necessary to properly determine the waypoints that generate a trajectory with the characteristics. Figure 1 shows the schematic configuration of the trajectory and waypoints considered in this study for a ballistic missile. In Figure 1, P is the position, and V and γ are the velocity and flight path angle, respectively. The subscript BO represents the burn-out time of the missile, the subscript WP_i represents the i^{th} waypoint, and the subscript TGT represents the target of the missile.

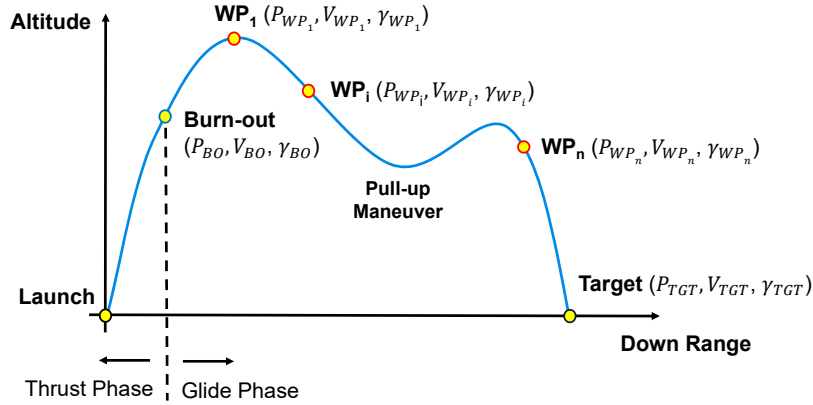


Figure 1.: Schematic Diagram of the Trajectory and Waypoints

To establish the proposed method and to analyze the results reflecting those waypoints, a closed-loop simulator describing the dynamics and control of the missile is required. It is assumed that the guidance algorithm for the precision-guided munition considered in this study uses the waypoints' information. The terminal conditions computed in the simulator are passed to the framework to determine whether those conditions satisfy the given constraints. Figure 2 shows a block diagram of the closed-loop simulator illustrating how it exchanges information with the framework. As shown in Figure 2, the closed-loop simulator include the plant, i.e., the missile dynamics and the guidance, navigation, and control subsystem.

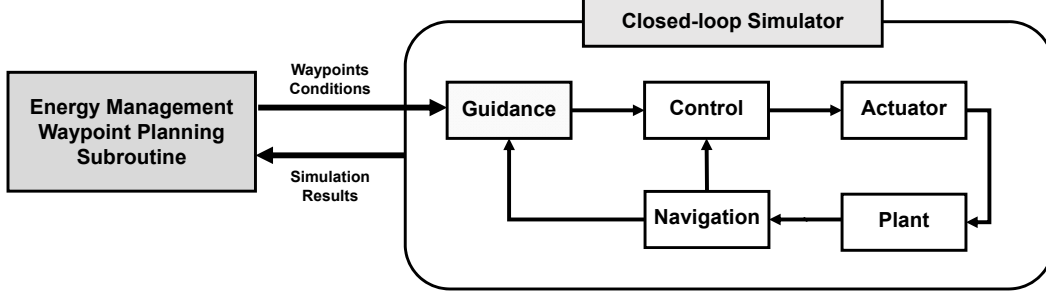


Figure 2.: Block Diagram Representation of the Closed-loop Simulator

3. Dynamic Modelling of Guided Ballistic Missiles

Equations of motion for the missile can be represented as (Lu 2014)

$$\dot{V} = -\frac{D}{m} - g \sin \gamma + \Omega_e^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \sin \gamma) \quad (2)$$

$$\begin{aligned} \dot{\gamma} = & \frac{L \cos \sigma}{mV} - \frac{g}{V} \cos \gamma + \frac{V}{r} \cos \gamma + 2\Omega_e^2 \cos \phi \cos \psi \\ & + \frac{\Omega_e^2 r}{V} \cos \phi (\cos \gamma \cos \phi + \sin \gamma + \sin \phi \sin \psi) \end{aligned} \quad (3)$$

$$\dot{x} = V \cos \gamma \quad (4)$$

$$\dot{h} = V \sin \gamma \quad (5)$$

where V is the velocity of the missile, γ is the vertical flight path angle, σ is the bank angle, ϕ is the roll angle, ψ is the yaw angle, x is the horizontal displacement (defined to be positive in the direction from the launch point to the target), h is the altitude of the missile, m denotes the vehicle mass, ω_e is the Earth's rotation rate, and g is the constant gravitational acceleration (9.8 m/s^2). D and L are the drag force and the lift force, respectively, which can be represented as

$$D = C_D \left(\frac{1}{2} \rho V^2 \right) S \quad (6)$$

$$L = C_L \left(\frac{1}{2} \rho V^2 \right) S \quad (7)$$

where ρ is the density of the atmosphere, S is the reference area of the missile, and C_D and C_L are the aerodynamic coefficients.

In this study, a 3-DOF point-mass model is used in the closed-loop system, and the flat earth and true navigation models are used. Note that although a high-fidelity model (such as a full 6-DOF model) will provide more accurate results, the 3-DOF model employed herein is sufficient to reflect the performance of the proposed framework because all the simulation results, including the waypoints' information, are calculated under the same simulation conditions. Note that 3-DOF models have become widely common for the optimization of trajectories and the development of guidance algorithms for rocket and missile systems (Zarchan 1990). In the 3-DOF simulation,

the response lag due to autopilot delay is neglected so that the actual lift acceleration is considered to be the same as the acceleration command given by a guidance law. Therefore, the lift force generated by the guidance algorithm can be defined as

$$L = ma_{cmd} \quad (8)$$

where a_{cmd} is the acceleration command calculated by the guidance law in Figure 2. In more detail, assuming that the mass is exactly known and the vehicle achieves the commanded normal acceleration without any lag or error, the acceleration command generated by a guidance law itself is considered as the actual lift. In this article, the optimal IAC guidance law Ryoo, Cho, and Tahk (2005) is adopted to generate the acceleration command considering the flight path angle constraint as

$$a_{cmd} = C_R t_{go} + C_S \quad (9)$$

where t_{go} is the time-to-go estimate. The constant coefficients of the ramp response (C_R) and the step response (C_S) are defined as

$$C_R = \frac{6V}{t_{go}^3}(-2t_{go}\theta + t_{go}\gamma + t_{go}\gamma_{TGT}) \quad (10)$$

$$C_S = -\frac{2V}{t_{go}^2}(3t_{go}\theta + t_{go}\gamma + 2t_{go}\gamma_{TGT}) \quad (11)$$

where θ is the line-of-sight angle in the pitch plane, which can be represented as

$$\theta = \arctan \frac{h - h_{TGT}}{x - x_{TGT}} \quad (12)$$

Using Eqs.(10) –(12) in Eq.(9), a_{cmd} can be rewritten as

$$a_{cmd} = \frac{V}{t_{go}}(-6\theta + 4\gamma + 2\gamma_{TGT}) \quad (13)$$

Considering the 3-DOF simulation, t_{go} and p_{go} are obtained as follows (Ryoo, Cho, and Tahk 2005)

$$t_{go} = \frac{p_{go}}{V} \quad (14)$$

$$p_{go} = \sqrt{(x - x_{TGT})^2 + (h - h_{TGT})^2} \quad (15)$$

4. Framework for Optimization of Waypoints

4.1. Overview

The objective of design framework is considered in this study to determine the proper locations of the waypoints and the incidence angles relative to those waypoints. A change in either the waypoint location or the incidence angle leads to a change in the final energy through the generation of different trajectories. The waypoint design framework may provide a solution to satisfy the terminal velocity constraint. Especially, the proposed framework imposes some conditions on the burn-out time and the terminal time because the waypoints are determined with respect to the energy consumption. The proposed framework consists of four components, each of which is responsible for different tasks, namely, gravity turn planning, waypoint candidate generation, candidate pruning, and waypoint determination.

The proposed framework determines the first waypoint by taking the point with the maximum altitude from the trajectory of the gravity turn, reducing the influences of the burn-out conditions. The subsequent waypoints are determined by searching over a group of candidates while satisfying the given constraints. In the waypoint candidate generation step, a range of points is generated for the candidates to be considered. Then, incorrect candidates are excluded in consideration of the mission constraints; this step is called candidate pruning. The best waypoint among the remaining candidates is finally selected through a comparison of the values of the performance index; this step is called waypoint determination. The first waypoint is calculated from the burn-out conditions by open-loop propagation, whereas the others are sequentially determined in reverse order from the last waypoint to the second waypoint using the trajectories obtained from the closed-loop simulator.

The waypoint selection processes are performed using the closed-loop simulator, which incorporates the missile dynamics and control systems, as shown in Figure 2. A motion planning method based on the simulated closed-loop dynamics has the advantage of containing fewer discrepancies between the model and reality in comparison to the methods based solely on simple kinematics. Hence, the planning result will show superior dynamic feasibility as a natural consequence of incorporating a more accurate dynamic model.

Figure 3 shows an overview of the framework proposed in this study. The subscript WP_1 refers to the first waypoint, V_{fin} is the estimated terminal velocity obtained by the closed-loop simulator, and *Candidate A* and *Candidate B* are the results from the candidate generation subroutine and the candidate pruning subroutine, respectively.

4.2. First Waypoint Planning: Forward Propagation

A missile and the environment in which the missile flies do not always behave as predicted. Furthermore, differences in the trajectories also cause the missile conditions at the burn-out time, which greatly affect the ability of the missile to maneuver to the target, to vary. Figure 4 shows the changes in the trajectories as a result of variations in the thrust and drag during the thrust phase. In Figure 4, *Case 1* shows the trajectory in which the thrust is increased by 1% and the drag coefficient is decreased by 5% compared to the nominal case, while *Case 2* shows the trajectory in which the thrust is decreased by 1% and the drag coefficient is increased by 5% compared to the nominal case.

The first waypoint is selected to reduce the influences of the initial conditions and

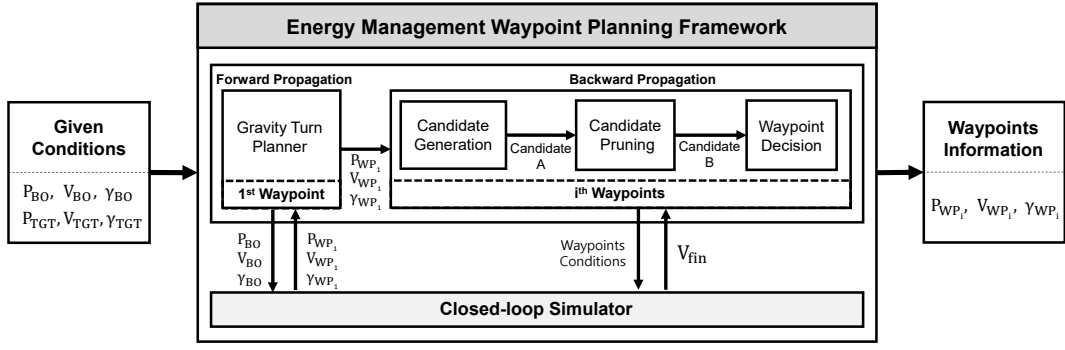


Figure 3.: Overview of the Energy Management Waypoint Planning Framework

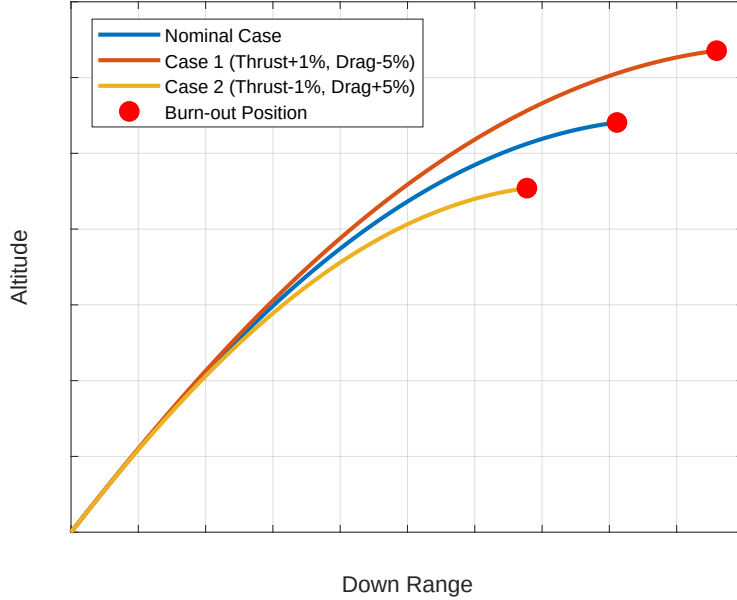


Figure 4.: Burn-out Conditions Changed by Variations in the Thrust and Drag

to facilitate the maneuvering of the missile beyond the burn-out point. It is known that by performing a gravity turn, the angle of attack is kept near zero, beneficially minimizing the energy wasted due to the induced drag Callaway (2004). In this regard, the proposed framework employs the gravity turn to determine the first waypoint that comes immediately after the boosting stage.

In this study, the first waypoint is selected where either the flight path angle of the missile or the flight altitude has a specific predefined value due to the system operating conditions; for example, the zero flight path angle of the missile or the maximum altitude from the burn-out point can be considered. Table 1 shows the pseudocode of the gravity turn subroutine used to determine the first waypoint, where ϵ is the threshold value used in the stop condition based on violating the terminal velocity constraint and the subscript $WP_{1_{des}}$ represents the desired value for the first waypoint.

Table 1.: Pseudocode: Planning the First Waypoint

Procedure Gravity Turn	
Input	
	$P_{BO}, V_{BO}, \gamma_{BO}$
Output	
	$P_{WP_1}, V_{WP_1}, \gamma_{WP_1}$
Initialization Parameters;	
While $(\gamma_M - \gamma_{WP_{1_{des}}} < \epsilon_\gamma) \parallel (h_M - h_{WP_{1_{des}}} < \epsilon_h)$ do	
Closed-loop Simulator;	
return $P_{WP_1}, V_{WP_1}, \gamma_{WP_1}$	

4.3. Planning the i -th Waypoint: Backward Propagation

Planning the i^{th} waypoint in the framework is achieved by three steps. The candidate generation process produces candidates for the i^{th} waypoint from the given conditions. The candidate pruning step excludes all candidates that violate the constraints. The waypoint determination procedure selects the best candidate waypoint in consideration of the performance index; choosing the optimal index value is critical to the success of the given mission of the missile. The waypoints are sequentially determined in the reverse order, i.e., from the last waypoint to the second waypoint.

4.3.1. Candidate Generation

In general, the combination of waypoints that satisfies the terminal velocity constraint is not unique. During the candidate generation step, all candidates that satisfy the given waypoint conditions are found.

There are two scenarios in this process depending on the existence of the initial condition. The first scenario is the determination of the second waypoint. In this scenario, the first waypoint should be considered as the initial condition, and the second waypoint is treated as an intermediate condition between the first waypoint and the third waypoint. In the second scenario, the initial condition is not given; that is, only the final condition is given, and the i^{th} waypoint condition should be determined by considering the $(i+1)^{th}$ waypoint. The second waypoint is determined from the third waypoint by backward propagation. The waypoints are searched one by one, and finally, the second waypoint is found considering the first waypoint and the third waypoint. Figure 5 shows a schematic diagram of the candidate generation step.

If the terminal conditions are given, the candidate waypoints are obtained during the candidate generation step. Note that the waypoints' information can be easily stored in the onboard computer and implemented for online waypoint look-up tables. Figure 6 shows a flowchart of the candidate generation step, and the detailed procedure is given in Table 2. The subscripts WP_{i-1} , WP_i , and WP_{i+1} denote the $(i-1)^{th}$, i^{th} , and $(i+1)^{th}$ waypoints, respectively, and the subscript WP_i^C represents a candidate of the i^{th} waypoint.

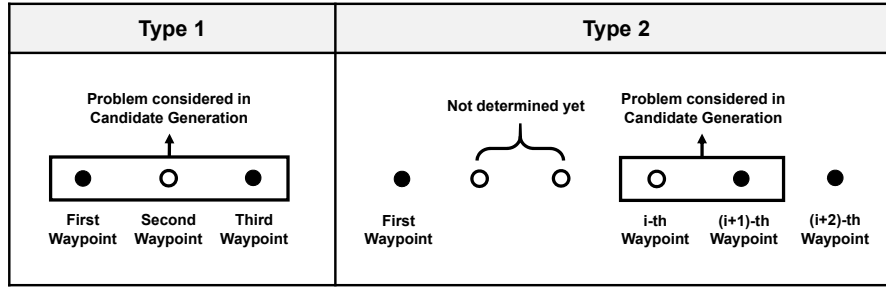


Figure 5.: Schematic Diagram of the Candidate Generation Step

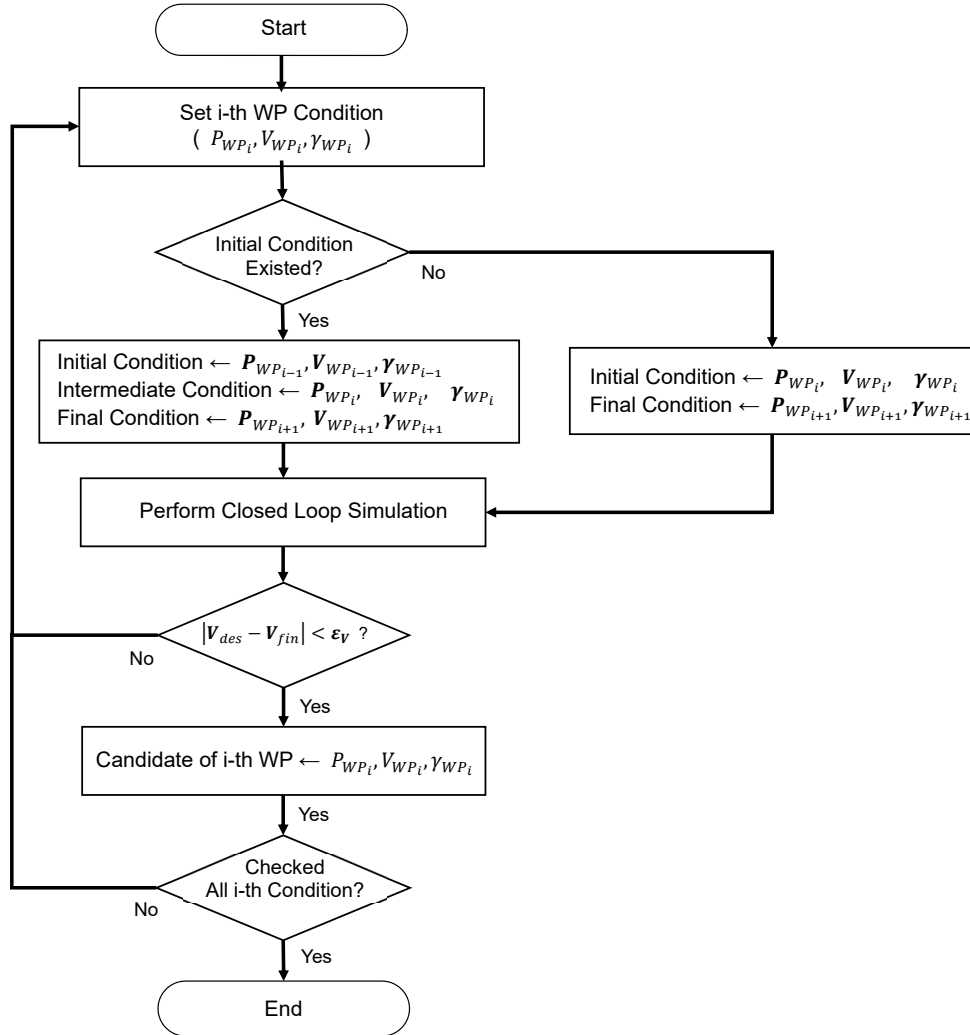


Figure 6.: Flowchart of the Candidate Generation Step

4.3.2. Candidate Pruning

Other constraints, such as limitations on the lateral acceleration or altitude, may exist. Some candidates obtained from the candidate generation step should be excluded by considering additional constraints. The remaining candidates are then passed to the

Table 2.: Pseudocode: Generating the Candidates for the i^{th} Waypoint

Procedure Candidate Generation	
Input	
	$P_{WP_{i-1}}, V_{WP_{i-1}}, \gamma_{WP_{i-1}}$
	$P_{WP_{i+1}}, V_{WP_{i+1}}, \gamma_{WP_{i+1}}$
Output	
	$P_{WP_i^c}, V_{WP_i^c}, \gamma_{WP_i^c}$
Initialization Parameters;	
for each $P_{WP_i}, V_{WP_i}, \gamma_{WP_i}$ in R_{WP_i} do	
if Initial Condition Exists then	
Initial Conditions $\leftarrow P_{WP_{i-1}}, V_{WP_{i-1}}, \gamma_{WP_{i-1}};$	
Intermediate Conditions $\leftarrow P_{WP_i}, V_{WP_i}, \gamma_{WP_i};$	
Final Conditions $\leftarrow P_{WP_{i+1}}, V_{WP_{i+1}}, \gamma_{WP_{i+1}};$	
else	
Initial Conditions $\leftarrow P_{WP_i}, V_{WP_i}, \gamma_{WP_i};$	
Final Conditions $\leftarrow P_{WP_{i+1}}, V_{WP_{i+1}}, \gamma_{WP_{i+1}};$	
While Missile Maneuvering Stops do	
Closed-loop Simulator;	
if $ V_M < V_{des} < \epsilon_V$ then	
$P_{WP_i^c}, V_{WP_i^c}, \gamma_{WP_i^c} \leftarrow P_{WP_i}, V_{WP_i}, \gamma_{WP_i};$	
return $P_{WP_i^c}, V_{WP_i^c}, \gamma_{WP_i^c}$	

waypoint decision process, as shown in Figure 3. Figure 7 shows a schematic diagram of the candidate pruning step.



Figure 7.: Schematic Diagram of the Candidate Pruning Step

In Figure 7, the waypoints represented by circles are the remaining candidates, while the waypoints represented by crosses are the excluded candidates that are not feasible with respect to the considered constraints. The position represented by the star is the target location. Only the remaining candidates are passed to the waypoint decision process.

Table 3 shows the pseudocode of the candidate generation step. In Table 3, the

subscript $WP_i^{C'}$ represents a remaining candidate of the i^{th} waypoint following the candidate pruning step.

Table 3.: Pseudocode: Pruning the Candidates for the i^{th} Waypoint

Procedure Candidate Pruning
Input $P_{WP_i^C}, V_{WP_i^C}, \gamma_{WP_i^C}$
Output $P_{WP_i^{C'}}, V_{WP_i^{C'}}, \gamma_{WP_i^{C'}}$
for each $P_{WP_i^C}, V_{WP_i^C}, \gamma_{WP_i^C}$ do Closed-loop Simulator; if Satisfying the Mission Conditions AND Missile Conditions then $P_{WP_i^{C'}}, V_{WP_i^{C'}}, \gamma_{WP_i^{C'}} \leftarrow P_{WP_i^C}, V_{WP_i^C}, \gamma_{WP_i^C};$
return $P_{WP_i^{C'}}, V_{WP_i^{C'}}, \gamma_{WP_i^{C'}}$

4.3.3. Waypoint Determination

This step selects the best waypoint from among the results of the candidate pruning step. Various criteria can be considered to make a determination among the waypoint candidates. For example, the flight time is a good performance index that takes into account the mission time as well as the battery capacity of electronic devices in the missile or the uncertainty of the environment. In terms of the survivability of a missile with respect to an interceptor, the altitude can be considered as a performance index. Aerodynamic heating is also a factor that can be incorporated into the planning stage to reduce the effects of abrasion Eggers (1957). The best performance index should be adopted considering the given mission of the missile. Additionally, multiple objectives can be considered by combining them into a single index, for example, using the weighted sum of several performance indices.

5. Numerical Examples

To demonstrate the effectiveness of the proposed energy management waypoint framework, numerical simulations of various example cases are performed. A ballistic missile with an SRM is considered for the stationary target. In all simulation cases, the number of waypoints is fixed to three, which is enough for the proposed framework to be verified. WP_1 is calculated first, and then WP_3 and WP_2 are determined in the reverse order. WP_3 and WP_2 are selected from the feasible area of each waypoint. Figure 8 shows the steps of the energy management waypoint framework for three waypoints.

Case 1 and case 2 consider different initial conditions and ranges to the target, respectively. Case 3 considers the uncertainty in the dynamic model used in the closed-loop simulator. Table 4 summarizes the simulation setup for each case.

Several methods are available to verify the performance of the proposed framework. The grid method, which divides a given area into a rectangular grid with Δx and Δh intervals and checking parameters with Δv and $\Delta \gamma$ intervals, may be one of the reasonable validation methods, and it has the advantage of reducing the computational load. In this study, the grid method is adopted to investigate the proposed candidate

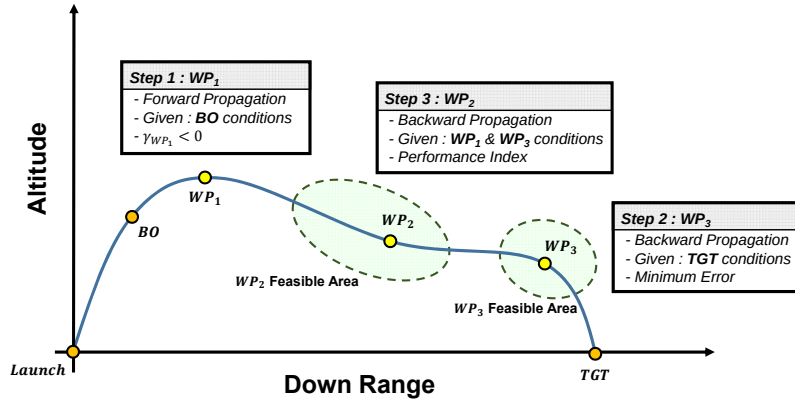


Figure 8.: Energy Management Waypoint Framework for Three Waypoints

Table 4.: Simulation Case Setup

	Parameters	Description
Case 1	Initial Condition	Flight Path Angle at Burn-out
Case 2	Range	Range to Target
Case 3	Drag	Drag Uncertainty

pruning step. To reduce the computation time, the algorithm terminates the process if the estimated velocity reaches $(V_{des} \pm \epsilon_v)m/s$. Considering a conventional missile engagement, Δx , Δh , ΔV , $\Delta \gamma$, and ϵ_v are set as 0.4 km , 0.4 km , 5 m/s , 1° , and 5 m/s , respectively.

5.1. Case 1: Different Initial Conditions

To demonstrate the effectiveness of the proposed framework for different initial conditions, three scenarios are considered, as summarized in Table 5. Each scenario has the same down range, altitude, and velocity. However, the initial flight path angle at the burn-out time is different.

Table 5.: Initial Conditions

	x_{BO}	h_{BO}	V_{BO}	γ_{BO}
Scenario A				15°
Scenario B	10 km	9 km	$1,200 \text{ m/s}$	17.5°
Scenario C				20°

The terminal conditions at the target are set as follows: $x_{TGT} = 120 \text{ km}$, $h_{TGT} = 0 \text{ km}$, $V_{TGT} = 350 \text{ m/s}$, and $\gamma_{TGT} = -90^\circ$. The time step of the simulation is 0.1 sec , and the mass of the missile is $1,000 \text{ kg}$. To reflect a realistic environment, the missile is limited to a minimum altitude of 2 km until it reaches waypoint 3, and an altitude of 5 km above the target is considered for waypoint 3. The maximum lateral acceleration is set as $10g$, and the altitude of waypoint 2 does not exceed the maximum altitude of the whole trajectory.

5.1.1. Waypoint 1

Waypoint 1 is determined as the point where the flight path angle of the missile becomes less than or equal to zero for the first time. The obtained results for waypoint 1 are summarized in Table 6.

Table 6.: Waypoint 1 Results

	x_{WP_1}	h_{WP_1}	V_{WP_1}	γ_{WP_1}
Scenario A	35.24 km	12.74 km	834.37 m/s	-0.05°
Scenario B	38.37 km	13.95 km	819.89 m/s	-0.02°
Scenario C	41.50 km	15.33 km	809.96 m/s	-0.04°

5.1.2. Waypoint 3

The criterion for waypoint 3 is set as the minimum error between the estimated velocity at the target and the desired velocity, $V_{TGT} = 350 \text{ m/s}$. The candidates for waypoint 3 and the selected point are shown in Figure 9. The star is the target, and the circles are the candidates for waypoint 3 obtained by the candidate generation and candidate pruning steps; in addition, the square is the waypoint selected by the waypoint determination step, and the solid line is the corresponding trajectory using the selected point that satisfies the terminal velocity condition. Note that one target condition has many solutions satisfying the terminal velocity constraint, and the decision criterion determines the shape of the trajectory. The selected points are $x_{WP_3} = x_{TGT} - 15.2 \text{ km}$, $h_{WP_3} = h_{TGT} + 5.8 \text{ km}$, $V_{WP_3} = 365 \text{ m/s}$, and $\gamma_{WP_3} = -28^\circ$. The errors of the terminal states are $|x_{err}| = 2.5 \times 10^{-4} \text{ km}$, $|h_{err}| = 5.4 \times 10^{-4} \text{ km}$, $|V_{err}| = 1.0 \times 10^{-4} \text{ m/s}$, and $|\gamma_{err}| = 5.86^\circ$. The flight path angle at the target point has some error because only the terminal velocity error is considered in this case. If the decision criterion changes with the minimum time or flight path angle, the determined waypoint will also vary.

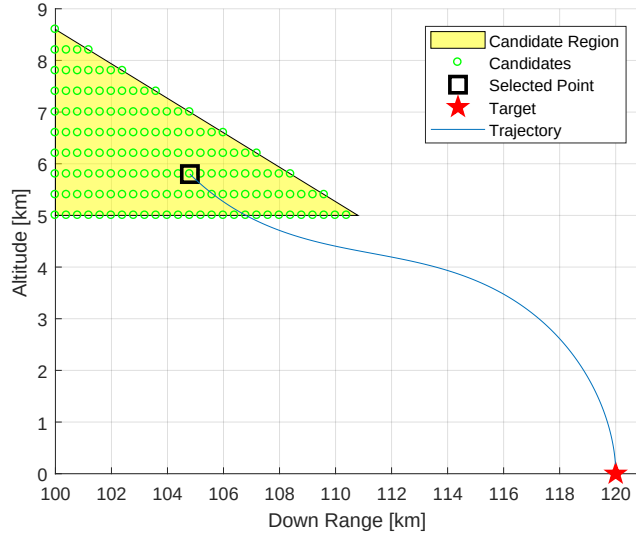


Figure 9.: Candidates and Selected Point for Waypoint 3

5.1.3. Waypoint 2

The candidates for waypoint 2 are determined from the points selected for waypoint 1 and waypoint 3, as shown in Figures 10(a), 11(a) and 12(a) for each scenario. The point represented by the cross is the result for waypoint 1, the point represented by the plus symbol is the result for waypoint 3, and the points represented by circles are the candidate points for waypoint 2.

Waypoint 2 is determined through the following process. After the candidates for waypoint 2 are obtained as shown in Figures 10(a), 11(a) and 12(a), a numerical simulation is performed for every candidate from the burn-out point to the target point. Then, the errors between the terminal conditions and simulation results are calculated. These errors are analyzed to determine the optimal point for waypoint 2, which is selected according to the equation 1 with an inequality constraint $h_{WP_1 \rightarrow WP_3} \geq h_{\text{lim}}$. In this example, the weighting factor is chosen as $\omega = 0.9$ to increase the priority of velocity constraint over impact angle constraint. Figures 10(b), 11(b), and 12(b) show the magnitude of the terminal velocity error at each candidate point, while Figures 10(c), 11(c), and 12(c) depict the magnitude of the terminal flight path angle error at each candidate point, and Figures 10(d), 11(d), and 12(d) illustrate the values of the performance index obtained by Eq. (1).

Note that the velocity error distribution is rather uniform. On the other hand, the flight path angle errors tend to decrease as the candidate for waypoint 2 approaches waypoint 3 at a high altitude because $\gamma_{WP_3} = -28^\circ$. Figures 10–12 show that the effectiveness of the proposed framework is confirmed by simulating various initial conditions. All the magnitudes of the terminal velocity errors and flight path angle errors for the candidates are below 5 m/s and 7° , respectively. The threshold value of the velocity error is set as $\epsilon_v = 5 \text{ m/s}$. If smaller ϵ_v is chosen, the terminal velocity errors of the candidates decrease. In the same sense, if smaller Δx , Δh and ΔV are chosen, more accurate results can be obtained. Because the proposed framework is easy to construct, as summarized in the pseudocode shown in Tables 1–3, this framework can be used as an appropriate mission design tool for ballistic missiles. Additionally, different performance indices can be used, and therefore various requirements that vary frequently over long missile development periods can be quickly reflected. Table 7 summarizes the point selected for waypoint 2 and the magnitudes of the errors at the target location in each scenario.

Table 7.: Waypoint 2 Conditions and Terminal Errors at $x_{TGT} = 120 \text{ km}$

	Waypoint 2 Conditions				Terminal Errors	
	x_{WP_2}	h_{WP_2}	V_{WP_2}	γ_{WP_2}	$ V_{err} $	$ \gamma_{err} $
Scenario A	98.84 km	7.14 km	384.29 m/s	-28°	0.30 m/s	4.92°
Scenario B	100.37 km	6.75 km	381.79 m/s	-21°	0.07 m/s	5.17°
Scenario C	74.70 km	12.93 km	379.48 m/s	73°	0.49 m/s	4.83°

Considering $|V_{err}| = 1.0 \times 10^{-4} \text{ m/s}$ and $|\gamma_{err}| = 5.86^\circ$, which are obtained when determining the optimal point for waypoint 3, the results for various initial conditions are similar to the expected values. These findings indicate that the framework can provide the expected result. Moreover, the missile with an initial velocity of 1,200 m/s that travels a distance of 120 km hits the target precisely with a terminal velocity error of less than 0.5 m/s. The proposed framework yields good performance by managing the energy of the missile. Figure 13 shows the trajectory of each scenario using the obtained waypoints described in Table 7.

5.2. Case 2: Different Ranges

To verify that the proposed framework can produce waypoints for a ballistic missile to strike a target located within a variety of ranges while satisfying the terminal velocity constraint, down ranges of $x_{TGT} = 100 \text{ km}$ and $x_{TGT} = 140 \text{ km}$ are considered. All the conditions are the same as those of case 1 except for the down range. Therefore, the results for waypoint 1 and waypoint 3 in the previous simulation can be used.

Tables 8 and 9 summarize the results for waypoint 2 and the magnitudes of the terminal errors at $x_{TGT} = 100 \text{ km}$ and $x_{TGT} = 140 \text{ km}$. These findings show that the proposed framework provides satisfactory performance over various ranges. Figure 14 illustrates the trajectories for the scenarios in Tables 8 and 9.

Table 8.: Waypoint 2 Conditions and Terminal Errors at $x_{TGT} = 100 \text{ km}$

	Waypoint 2 Conditions				Terminal Errors	
	x_{WP_2}	h_{WP_2}	V_{WP_2}	γ_{WP_2}	$ V_{err} $	$ \gamma_{err} $
Scenario A	57.24 km	7.14 km	551.94 m/s	47°	0.07 m/s	4.48°
Scenario B	63.97 km	8.75 km	444.33 m/s	73°	0.03 m/s	4.19°
Scenario C	49.10 km	10.63 km	799.40 m/s	-56°	0.07 m/s	5.14°

Table 9.: Waypoint 2 Conditions and Terminal Errors at $x_{TGT} = 140 \text{ km}$

	Waypoint 2 Conditions				Terminal Errors	
	x_{WP_2}	h_{WP_2}	V_{WP_2}	γ_{WP_2}	$ V_{err} $	$ \gamma_{err} $
Scenario A	119.64 km	7.14 km	376.11 m/s	-49°	0.27 m/s	5.90°
Scenario B	117.97 km	6.75 km	406.61 m/s	-44°	0.23 m/s	5.55°
Scenario C	122.70 km	6.53 km	365.76 m/s	-35°	0.09 m/s	5.70°

5.3. Case 3: Influence of Uncertainty in the Drag Model

The actual environment in which a missile operates is different from the model used for the closed-loop simulation. Terminal errors may increase in the presence of modeling uncertainties. In this case, a variation of $\pm 10\%$ in the drag coefficient is considered. The nominal value of the drag coefficient is used to find the waypoints, and the drag coefficient with uncertainty is used to calculate the terminal errors. Table 10 summarizes the simulation results with the uncertainties in the drag model.

Table 10.: Terminal Errors at $x_{TGT} = 120 \text{ km}$ with Unexpected Drag Uncertainty of $\pm 10\%$

	Drag-10%		Drag+10%	
	$ V_{err} $	$ \gamma_{err} $	$ V_{err} $	$ \gamma_{err} $
Scenario A	39.30 m/s (10.77%)	5.61°	23.89 m/s (6.55%)	6.23°
Scenario B	40.99 m/s (11.23%)	4.87°	24.96 m/s (6.84%)	10.07°
Scenario C	47.20 m/s (12.93%)	2.92°	39.59 m/s (10.85%)	4.40°

The terminal velocity error increases by a factor of 100 compared to the previous cases. However, a drag model uncertainty of 10%, which is large, is not easily considered in the development phase. With this consideration, a terminal velocity error near a Mach number of 0.1 shows that the framework has the capability to manage the energy while reducing the error, even under abnormal circumstances. Using a realistic simulation model, the framework can effectively manage the energy of the missile to satisfy the terminal velocity constraint and provide more accurate results.

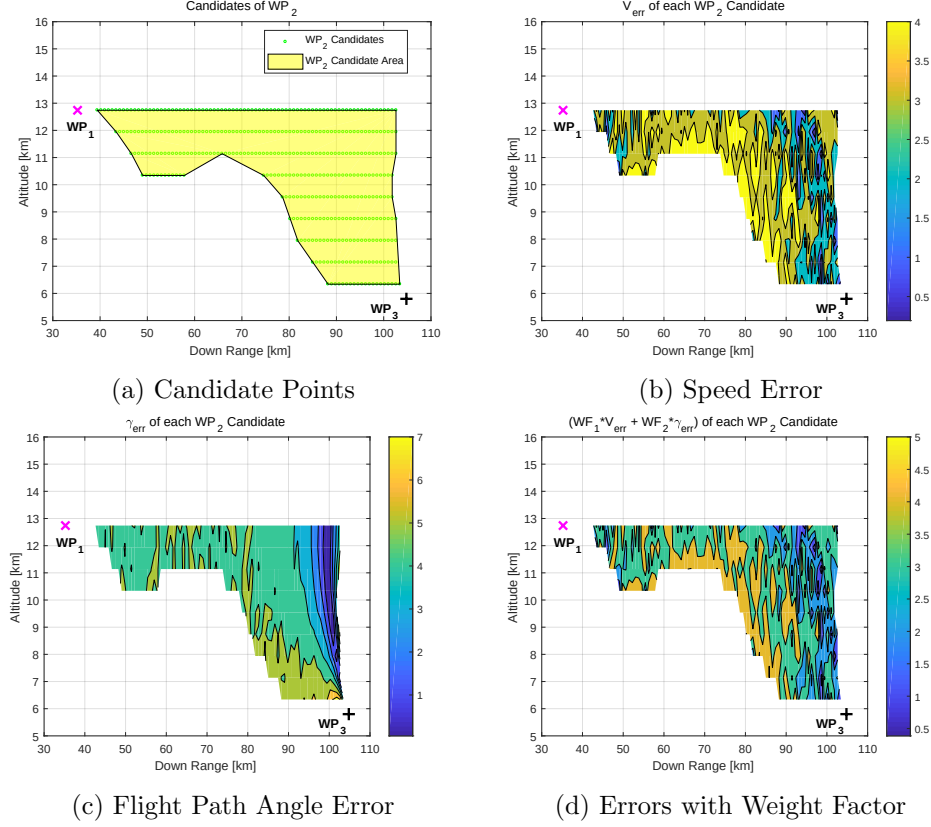
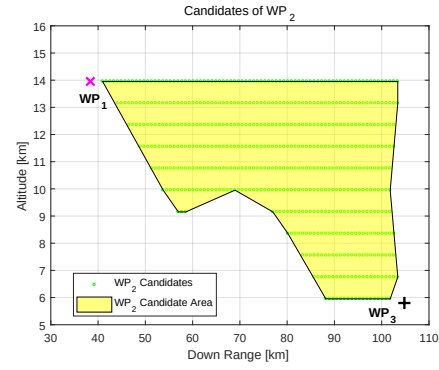
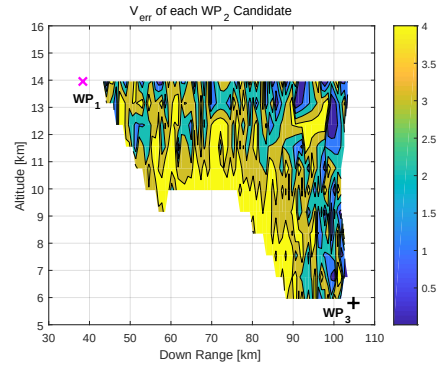


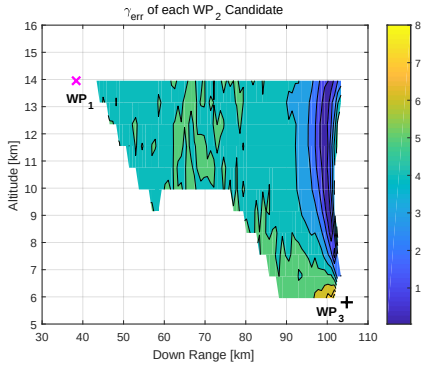
Figure 10.: Waypoint 2 Results: Scenario A for $x_{TGT} = 120$ km in Example Case 1



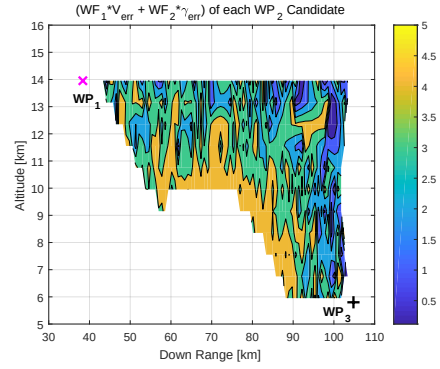
(a) Candidate Points



(b) Speed Error



(c) Flight Path Angle Error



(d) Errors with Weight Factor

Figure 11.: Waypoint 2 Results: Scenario B for $x_{TGT} = 120 \text{ km}$ in Example Case 1

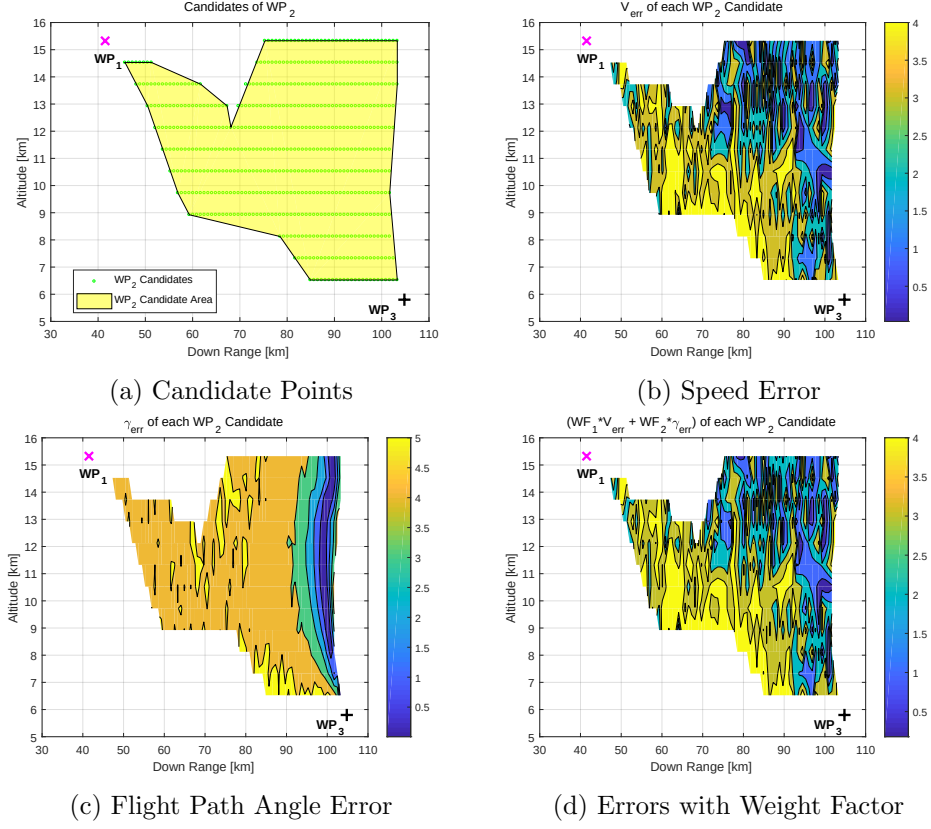


Figure 12.: Waypoint 2 Results: Scenario C for $x_{TGT} = 120 \text{ km}$ in Example Case 1

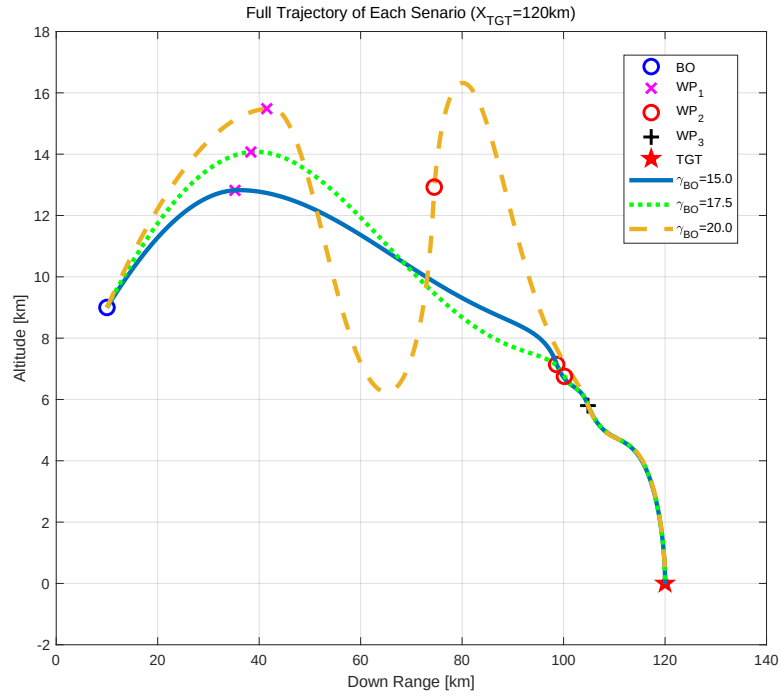
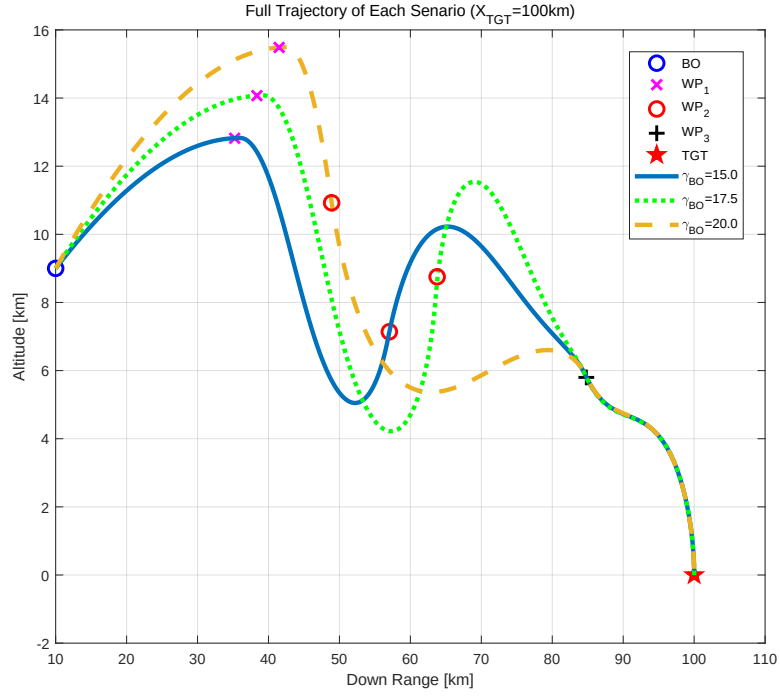
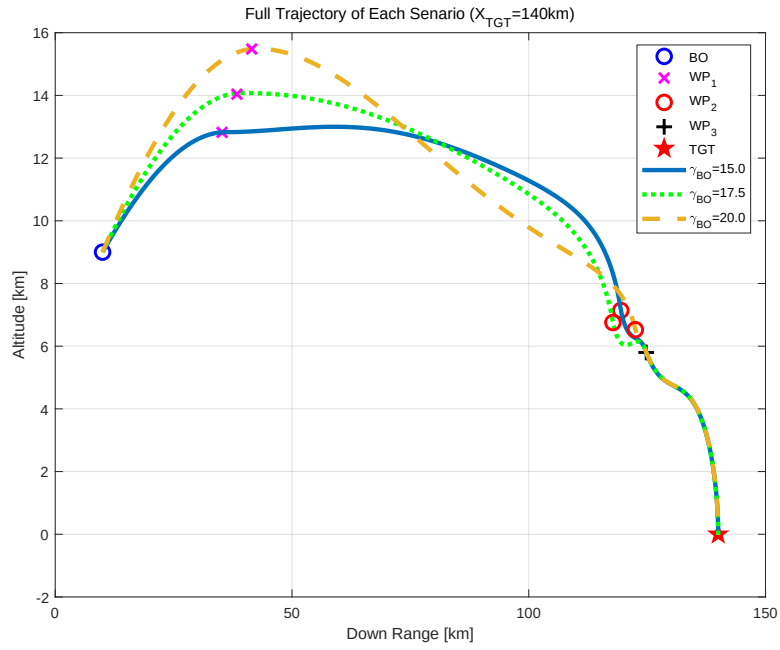


Figure 13.: Final Trajectory of Example Case 1



(a) $x_{TGT} = 100km$



(b) $x_{TGT} = 140km$

Figure 14.: Final Trajectory of Example Case 2

6. Conclusions

A design framework for optimizing waypoints to satisfy the terminal velocity and impact angle constraints is proposed. The proposed framework employs the optimal IAC guidance law to achieve the impact angle constraint and the heuristic search method for determining waypoints to satisfy the velocity constraint considering the typical mission profile of ballistic missile which flies in the vertical plane whenever possible and performs a pull-up maneuver. The proposed framework consists of several distinct steps: gravity turn planning, candidate generation, candidate pruning, and waypoint determination. The candidate generation step generates the waypoint candidates, while the candidate pruning step constructs the feasible area of each waypoint with respect to the given constraints, and the waypoint determination selects the best waypoint from among the group of candidates. Simulation results show that the proposed framework provides the optimal trajectory satisfying the terminal constraints.

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2021-05-17

Taylor and Francis

<https://doi.org/10.1080/0305215X.2021.1925264>

Kim Y, Cho N, Park J. Kim Y. (2022) Design framework for optimizing waypoints of vehicle trajectory considering terminal velocity and impact angle constraints. Engineering Optimization, Volume 54, Issue 8, 2022, pp. 1341-1355