

Analysis of Rendezvous Guidance Laws for Autonomous Aerial Refueling for Non-maneuvering and Identical Speed Targets

Seokwon Lee^{a,1}, Mingu Kim^{b,*}

^a*Cranfield University, Cranfield, England, MK43 0AL, United Kingdom*

^b*Hansung University, Seoul 02876, Republic of Korea*

Abstract

This study analyses rendezvous guidance schemes for UAVs refuelling. Assuming the target is non-maneuvring with identical speed, a necessary condition is investigated for the UAV to approach the rendezvous point. Three types of guidance laws are considered to analyse the performance under the condition. A reachable region is analytically obtained using the closed-form solution of the deviated pursuit and pure proportional navigation. Numerical simulation demonstrates the analysed result for the rendezvous mission.

Keywords: Rendezvous Guidance, Air-to-Air Refueling, Deviated Pursuit Guidance, Proportional Navigation Guidance

1. Introduction

Recent advances in unmanned aerial vehicles (UAVs) have facilitated the vast application of autonomous systems and advanced autonomous mission capabilities. In particular, UAVs are required to operate long-endurance flights during missions, and air-to-air refuelling (AAR) enables such a requirement efficiently. By refueling in the air, the UAVs proceed with their mission without returning to the ground base to refuel,

*Corresponding author, Assistant Professor, Division of Mechanical and Electronics Engineering
Email addresses: seokwon.lee@cranfield.ac.uk (Seokwon Lee), mingukim@hansung.ac.kr (Mingu Kim)

¹Research Fellow, Centre for Autonomous and Cyber-Physical Systems, School of Aerospace, Transport and Manufacturing

thereby extending the mission coverage and operational time. From these benefits, the AAR has been employed in the military domain and is further extended into unmanned aerial systems (UAS) [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

For autonomous AAR, proper guidance design plays a pivotal role in autonomous rendezvous between tankers and UAVs. Rendezvous guidance for UAVs has been investigated for decades using various approaches such as path planning [13, 14], geometric approach [15, 16], optimal control [17, 18], and differential games [19, 20]. Dubin's path was adopted to the automatic AAR problem in the geometric perspective [15]. Remarkably, it characterised the performance of the rendezvous guidance according to the relative geometry from the target, where the rendezvous is well behaved when the UAV approaches from the side of the target. Meanwhile, considerable studies for the rendezvous guidance problem have been based on the proportional navigation guidance (PNG) [21, 22, 23] and pure pursuit guidance (PPG) [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. PNG has been usually adopted for interception problems because the proportional action effectively ensures the collision condition between the objects [35, 36]. Rendezvous guidance based on the PNG produced the heading command to enter a collision course, and the speed control was incorporated into the guidance to satisfy the rendezvous. Meanwhile, lots of the rendezvous guidance laws were proposed using the PPG concept. The PPG shows a favorable characteristics in the tail-chase geometry. Since the fueling cable or probe is usually located behind the tanker [2, 28], the PPG based performance is suitable for the UAV rendezvous problem. The guidance command based on the PPG aimed to a desired waypoint, and speed control was performed to meet the rendezvous time and speed at the waypoint. An advanced PPG was studied in [27]. The author proposed a deviated lead angle command as a function of range and adopted the deviated pursuit guidance framework. The deviated pursuit guidance showed effective performance when the target is in a circular motion, and its concept has further studied in the enhanced observability [29] and cooperative coordination [30] perspectives.

On the other hand, rendezvous guidance strategies must also offer viable performance even for an emergency case in which the receiver is in reduced flight performance and adequate power is unavailable, i.e. loss of an engine or high-density altitude. A change to the normal procedure is necessary, and the UAV is to maintain the speed

until the rendezvous is completed. Furthermore, refueling operation often demands the tanker to keep the reference path without changing its heading. To this end, a capability that the UAV continues rendezvous while keeping the speed, and the appropriate guidance and path control is essential to achieve the mission withstanding the challenge. Despite the excellent effectiveness for the UAV rendezvous application, the existing methods may show limitations when the underlying assumptions are no more guaranteed. Most of the existing methods utilize speed control to meet the terminal requirements [22, 15, 23, 24, 25, 26, 37, 38]. In the absence of speed control, the pursuer (UAV) tends to finish the rendezvous without narrowing the distance sufficiently to the target. This type of problem set was introduced in Ref. [27]. The two identical UAVs manoeuvre to rendezvous without any speed advantage or control. Vehicle heading is controlled to achieve the rendezvous in the absence of speed control. A simulation study was performed under some initial conditions, and further analysis is necessary by showing the feasibility in various initial conditions.

Motivated by the work, this study aims to analyse existing rendezvous guidance laws for autonomous aerial refueling mission in a harsh condition. The pursuer is required to approach to a non-maneuvering target under an identical speed condition. This study inspects guidance laws that cover from the PPNG and the deviated pursuit guidance, and those guidance laws are analysed for two rendezvous conditions. An exact rendezvous is defined as the pursuer arrives at the target with zero relative speed and distance, the other definition of rendezvous may allow terminal distance between two objects as a relaxed one. We first investigate the condition of initial configuration between the pursuer and the target as the necessary condition. A geometric perspective is incorporated to address the necessary condition. We also speculate what the guidance laws based on the classical approach must consider to ensure the exact rendezvous condition. From the necessity, the analysis also provides that the classical PPNG and the deviated pursuit guidance cannot achieve the exact rendezvous when the target does not maneuver. Under the relaxed rendezvous condition, the reachable region of the rendezvous guidance laws are derived as the sufficient condition. By allowing the terminal distance, the exact expression of the solution is used as the boundary of the region.

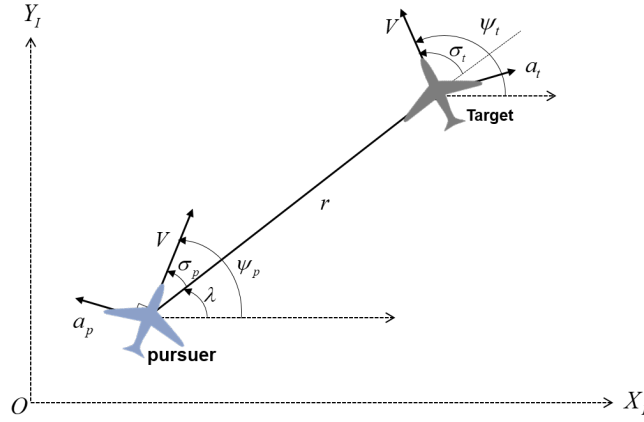


Figure 1: Planar Geometry between a pursuer and a target

The reachability analysis derived in this study provides the feasible initial condition of the PPNG and deviated pursuit based rendezvous guidance laws, thereby allowing the existing methods to achieve the rendezvous even in an unfavourable environment. Importantly, the rendezvous under the relaxed condition is only guaranteed in a narrow initial configuration. This implies that the exact rendezvous without the involvement of speed variation is extremely difficult. A simulation study demonstrates the existing rendezvous guidance methods using PNG, deviated pursuit, and variable gain pursuit succeed in the rendezvous only if the UAV starts the rendezvous in the narrowed reachable region.

2. Equations of Motion

Consider the planer motion between two objects as shown in Fig. 1. A pursuer denoted by the subscript p moves toward a target denoted by the subscript t . In this study, the following assumptions are introduced to describe the motion.

Assumption 1. The target and pursuer are point-mass models, and autopilot responses are fast enough to be negligible.

Assumption 2. Speeds of the target and the pursuer are identical.

Assumption 3. The target is non-maneuvering and does not change its heading.

The equations of motion are given by

$$V_r \triangleq \dot{r} = V (\cos(\lambda - \psi_t) - \cos(\psi_p - \lambda)) \quad (1a)$$

$$V_\lambda \triangleq r\dot{\lambda} = V (-\sin(\lambda - \psi_t) - \sin(\psi_p - \lambda)) \quad (1b)$$

$$u \triangleq \dot{\psi}_p = \frac{a}{V} \quad (1c)$$

where $r \in \mathbb{R}^+$ is the distance between the pursuer and the target, (ψ_p, ψ_t) represent the heading angles of the pursuer and the target, respectively. (V, a) are the speed and the acceleration perpendicular to the velocity vector of the pursuer. The domain of angles λ, ψ_p , and ψ_t are assumed to be defined in $\Omega_c = [c - \pi, c + \pi]$ by considering the periodicity in 2π with a constant center c . Then let us define the exact rendezvous condition between the pursuer and the target:

Definition 1. (Exact Rendezvous in Regard to Relative Distance and Velocity ([27]))

An exact rendezvous occurs when the pursuer arrives at the target with no relative velocity as

$$r(t_f) = 0, \quad \dot{r}(t_f) = 0 \quad (2)$$

where t_f represents the rendezvous instant.

Note that r and $\dot{r}$ must be regulated simultaneously for the exact rendezvous condition. As a sufficient condition of the exact rendezvous, $\dot{r} = 0$, the heading angle must be

$$\psi_p(t_f) = \psi_t(t_f) \quad (3)$$

3. Necessary Condition for Exact Rendezvous

This section addresses the analysis of the rendezvous guidance laws. The lead angles of the pursuer and the target can be defined as

$$\sigma_p = \psi_p - \lambda, \quad \text{and} \quad \sigma_t = \psi_t - \lambda \quad (4)$$

The pursuer's lead angle represents the observation angle from the seeker, usually with a limited field of view less than $\pi/2$. If the lead angle is obtuse, i.e., $|\sigma_p| > \pi/2$, the pursuer should perform a search manoeuvre to lock on the tanker within the seeker's

field-of-view. The additional time-lagged behaviour eventually leads to a significant terminal distance. Without loss of generality, this study choose σ_p as positive value which is restricted by the acute angle, i.e., $0 \leq \sigma_p \leq \pi/2$. We also confine the configuration of the target lead angle as $0 \leq \sigma_t \leq \pi$. Domain of the lead angles σ_t is defined as :

$$\Sigma = \{(\sigma_t, \sigma_p) : 0 \leq \sigma_t \leq \pi, 0 \leq \sigma_p \leq \pi/2\} \quad (5)$$

3.1. Necessary Condition of Initial Configuration for Exact Rendezvous

This section presents the necessary condition of initial configuration using the geometric perspective [39]. Figure 2 shows the illustrative situation of the exact rendezvous. The target moves in straight from T to R , and the pursuer approaches to the target from P to R . If the pursuer and the target meet the rendezvous point R , trajectory of the pursuer must properly satisfy

$$\int_{s_p(P)}^{s_p(R)} ds_p = \int_{s_t(T)}^{s_t(R)} ds_t \quad (6)$$

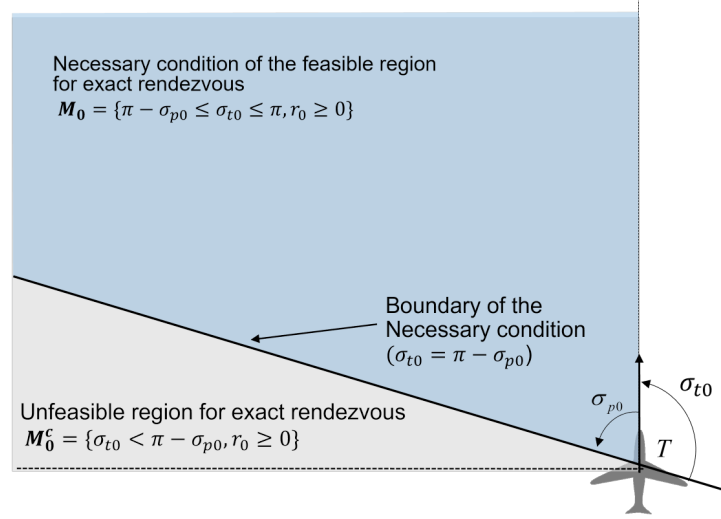
where ds_p is the infinitesimal of the pursuer's curve. ds_t denotes the infinitesimal of target's trajectory. Suppose that the path of the pursuer is expressed as implicit function of target's position, i.e., $\Phi(x_p, y_p) = 0$. Since the target is nonmaneuvering, the path of the target should be a straight line. The right-hand side of Eq. (6) can be further expanded as

$$\int_{s(P)}^{s(R)} ds_p = \int_{x_P}^{x_R} \sqrt{1 + \left(\frac{\Phi_x}{\Phi_y}\right)^2} dx_p = \overline{TR} = Vt_f \quad (7)$$

where $\Phi_y = \frac{\partial \Phi(x,y)}{\partial y}$ and $\Phi_x = \frac{\partial \Phi(x,y)}{\partial x}$ are partial derivatives of the implicit function related to path of the pursuer. From this aspect, the following Theorem addresses the necessary condition of the pursuer's initial position for the exact rendezvous.

Theorem 1. Under Assumptions 1-3, if there exists a rendezvous guidance command u ensuring the rendezvous condition Eqs. (2) and (3) for the given initial lead angle $\sigma_{p_0} > 0$. Then, the initial lead angle of the target must satisfy

$$\mathcal{M}_0 = \{(r_0, \sigma_{t_0}) : r_0 \geq 0, \quad \pi - \sigma_{p_0} \leq \sigma_{t_0} \leq \pi\} \quad (8)$$



(a) Necessary condition of initial position for exact rendezvous

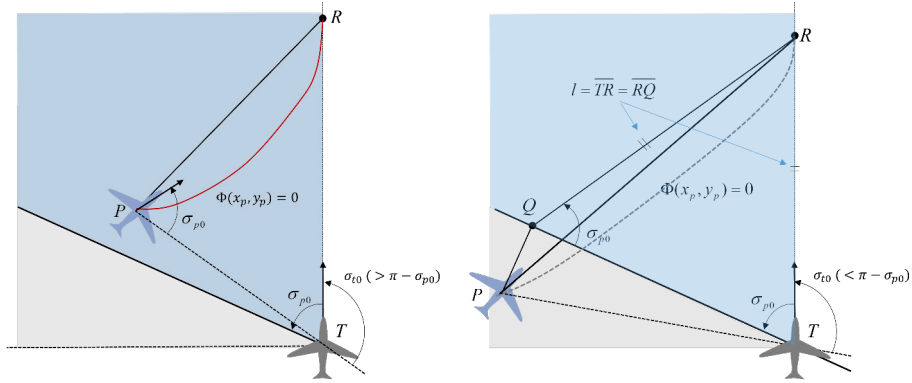


Figure 2: Illustration for the proof of Theorem 1

Proof. Suppose that the pursuer reaches the rendezvous point R from an initial position P lying in the gray-shaded area such that $\sigma_{t_0} < \pi - \sigma_{p_0}$ as shown in Fig. 2-(c). Since $\Phi(x_p, y_p)$ is a smooth curve from P to R , the following inequality holds,

$$\overline{PR} \leq l = \int_{s(P)}^{s(R)} ds_p = \int_{x_P}^{x_R} \sqrt{1 + \left(\frac{\Phi_x}{\Phi_y}\right)^2} dx_p = \overline{TR} \quad (9)$$

Then, there exists a point Q lying on the layer $\sigma_t = \pi - \sigma_{p_0}$, satisfying $\angle RQT = \sigma_p$. Therefore, $\overline{RQ} = \overline{RT}$. On the other hand, $\overline{PR}$ is the largest side in the obtuse triangle $\triangle PQR$, therefore

$$\overline{PR} \geq \overline{QR} = l \quad (10)$$

which violates Eq. (9). It completes the proof. $\square$

Theorem 1 gives a geometrical implication of the necessary condition of the initial position for achieving exact rendezvous. Figure 2 shows the blue-shaded area obtained from Eq. (8) represents the necessary condition of the initial position where the pursuer must initiate the rendezvous. The necessary condition illustrated as the blue-shaded region indicates that the pursuer must move ahead of the target until the rendezvous is achieved. On the other hand, the grey-shaded area depicts the region where the exact rendezvous cannot be achieved. If the pursuer initially begins the rendezvous from the region, i.e., the pursuer is behind the target relative to the target's heading direction, the pursuer should choose a strategy that coincides its heading with that of the target at least to maintain distance. Otherwise, the distance between the target increases, which fails to the rendezvous. It is also note that the pursuer starting at the initial position P with $\sigma_{t_0} = \pi - \sigma_{p_0}$ flies its shortest path by keeping its initial angle to reach the target at R but does not coincide with the heading.

3.2. Condition For Exact Rendezvous

In this section, the rendezvous guidance laws are analyzed in the perspective of the nonlinear control theory [40]. First, let us describe the the equations of motion in the state-space representation. Let $\mathbf{x} = [r \ \sigma_t \ \sigma_p]^T$ be the state variables defined in the set

$\mathcal{X} \subset \mathcal{R}^+ \times \Sigma$. The state dynamics can be obtained by substituting Eq. (4) into (1) as

$$\dot{\mathbf{x}} \triangleq \mathbf{f}(\mathbf{x}) + \mathbf{g}u = \begin{bmatrix} V(\cos \sigma_t - \cos \sigma_p) \\ -\frac{V}{r}(\sin \sigma_t - \sin \sigma_p) \\ -\frac{V}{r}(\sin \sigma_t - \sin \sigma_p) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad (11)$$

Note that the relative motion can also be described by changing the dependant variable from t to r as

$$\frac{d\sigma_p}{dr} = \frac{u + \frac{V}{r}(-\sin \sigma_t + \sin \sigma_p)}{V(\cos \sigma_t - \cos \sigma_p)} \quad (12a)$$

$$\frac{d\sigma_t}{dr} = -\frac{1}{r} \frac{(\sin \sigma_t - \sin \sigma_p)}{(\cos \sigma_t - \cos \sigma_p)} = \frac{1}{r} \cot\left(\frac{\sigma_t + \sigma_p}{2}\right) \quad (12b)$$

For stable rendezvous, it is required that the relative motion is stabilized, i.e., $V_\lambda = V_r = 0$. Then the capture condition, Eq. (2) can be mathematically expressed as follows.

Definition 2. (Exact Rendezvous Under the State-Space Representation) Under Assumptions 1-3, consider the state dynamics in Eq. (11). Then, a compact set representing the exact rendezvous condition is expressed as

$$\mathcal{L}^+ = \{(r, \sigma_t, \sigma_p) : r = 0, \sigma_t = \sigma_p\} \quad (13)$$

If there exists $u = \varphi(\mathbf{x})$ for the exact rendezvous, then largest invariant set of the closed-loop dynamics $\dot{\mathbf{x}} \triangleq \mathbf{f}_c(\mathbf{x}) = \mathbf{f}(x, \varphi(x))$ should be L_0

Most of the guidance laws are based on classical guidance schemes including PPNG and pursuit-like guidance laws [23, 24, 25, 27, 33]. The guidance commands of these guidance schemes can be expressed as

$$u = N\dot{\lambda} + b(t) \quad (14)$$

where N is the navigation gain, and $b(t)$ is the bias function. Based on the definition, the performance of the rendezvous guidance can be analyzed as the following example.

Example 1. Consider the deviated pursuit guidance law [27]:

$$u = \dot{\lambda} - k(\sigma_p - \sigma_{pc}) \quad (15)$$

where the commanded lead angle σ_{pc} is designed as

$$\sigma_{pc}(r, \sigma_t) = \left(\frac{r_0 - r}{r_0} \right) \sigma_t \quad (16)$$

By substituting Eqs (15) and (16) into (11), σ_p converges to constrained curve $\sigma_{pc}(r, \sigma_t)$. The equilibrium conditions governed by the constrained curve are then obtained by zeroing $\sigma_{pc} = 0$ as

$$\sigma_p = \sigma_{pc} = \sigma_t = 0 \quad (17)$$

From Definition 2, a compact set $\mathcal{L}^+ = \{(r, \sigma_t, \sigma_p) = (0, 0, 0)\}$ should be the largest invariant set. Then, it should be verified that (r, σ_t) dynamics for all initial condition converge to the origin. To inspect this, consider the relative motion can be also represented in the range formulation:

By substituting Eq. (16) into (12b), we can get

$$\frac{dr}{d\sigma_t} = r \tan \left(\frac{\sigma_t + \left(\frac{r_0 - r}{r_0} \right) \sigma_t}{2} \right) = r \tan \left(\left(\frac{r_0 - r/2}{r_0} \right) \sigma_t \right) \quad (18)$$

It is easy to show that $dr/d\sigma_t = 0$ stabilized when $r_0 \neq 0$ and $\sigma_t = 0$. It means that the above equilibrium condition satisfies even if r is not regulated. Therefore, the largest invariant set of the variable deviated pursuit guidance [27] is obtained as

$$\mathcal{L}_{\text{Rendezvous [27]}}^+ = \{(r, \sigma_t, \sigma_p), (c, 0, 0), c \geq 0\} \quad (19)$$

The above example illustrates that the bias function as well as the navigation constant should be properly designed for the exact rendezvous condition. Now, a necessary condition for the guidance law design is addressed as follows.

Theorem 2. Consider the guidance command Eq. (14), where the navigation ratio is $N \geq 1$ and the pursuer's lead angle is within the FOV limit, i.e., $\sigma_p \in [0, \pi/2]$. If there exist a time interval $[t_1, t_f]$, $t_1 \in [t_0, t_f]$ and a bias function $b(t)$ satisfying

$$b^{(n)}(t) = 0, \quad \forall t \in [t_1, t_f] \quad (20)$$

then, the exact rendezvous cannot be achieved.

Proof. Suppose that $b(t)$ vanishes at $t = t_1$ and remains zero in the finite interval including the terminal time $t \in [t_1, t_f]$. Then, the guidance command is equivalent to PPNG. The LOS angle have proportional relationships:

$$\dot{\lambda} = \frac{1}{N} \dot{\psi}_p = \frac{1}{N-1} \dot{\sigma}_p \quad (21)$$

From Eq. (4), we can get

$$\dot{\sigma}_p = -(N-1) \dot{\sigma}_t \quad (22)$$

Note that the navigation ratio $N \geq 1$ renders directions of the lead angle rates opposite each other. Supposing that σ_{p_1} keeps in the FOV range $[0, \pi/2]$ at the time instant t_1 , let us consider two different cases in Σ , i) C1: $\pi - \sigma_{p_1} < \sigma_{t_1} < \pi$ and ii) C2: $0 \leq \sigma_{t_1} < \pi - \sigma_{p_1}$. In the first case, C1, $\sigma_{t_1} > \sigma_{p_1}$ and the rates of their difference shows

$$\dot{\sigma}_t - \dot{\sigma}_p = N \dot{\sigma}_t = -\frac{NV}{r} (\sin \sigma_t - \sin \sigma_p) = -\frac{2NV}{r} \cos\left(\frac{\sigma_t + \sigma_p}{2}\right) \sin\left(\frac{\sigma_t - \sigma_p}{2}\right) > 0 \quad (23)$$

Therefore, the difference of two values will monotonically diverge to each other. In the other case, C2, on the other hand, the proportional relationship makes two angles close to each other. Integrating Eq. (22) yields

$$\sigma_p = \sigma_{p_1} - (N-1) (\sigma_t - \sigma_{t_1}) = -(N-1) \sigma_t + \alpha_1 \quad (24)$$

where $\alpha_1 = \sigma_{p_1} + (N-1) \sigma_{t_1}$. Then, the expected terminal value $\sigma_{p_f} = \sigma_{t_f} = \sigma_f$ can be obtained as

$$\sigma_f = \frac{1}{N} \alpha_1 = \frac{1}{N} (\sigma_{p_1} + (N-1) \sigma_{t_1}) \quad (25)$$

Note that σ_p must satisfy the positive invariance for the FOV constraint $[0, \pi/2]$. From Eq. (25), we can confine the feasible range of σ_{t_1} in C2 as

$$0 \leq \sigma_{t_1} \leq \min\left(\pi - \sigma_{p_1}, \frac{N\pi}{2(N-1)} - \frac{1}{N-1} \sigma_{p_1}\right) \quad (26)$$

Otherwise, the FOV constraint will be violated at the terminal instant. Note that in Eq (26), $\sigma_p + \sigma_t < \pi$ and $\sigma_f < \pi/2$ regardless of $N \geq 1$. Now, let us introduce the Lyapunov candidate function to inspect the convergence of the LOS angle.

$$W = \frac{1}{2} \sin^2\left(\frac{\sigma_t - \sigma_p}{2}\right) \quad (27)$$

Note that $\cos\left(\frac{\sigma_p + \sigma_t}{2}\right) > 0$ and $\cos\left(\frac{\sigma_p - \sigma_t}{2}\right) > 0$ in C2. Differentiating W gives

$$\begin{aligned}\dot{W} &= \frac{1}{2} \sin\left(\frac{\sigma_t - \sigma_p}{2}\right) \cos\left(\frac{\sigma_t - \sigma_p}{2}\right) (\dot{\sigma}_t - \dot{\sigma}_p) \\ &= -\frac{NV}{r} \cos\left(\frac{\sigma_t - \sigma_p}{2}\right) \cos\left(\frac{\sigma_t + \sigma_p}{2}\right) \sin^2\left(\frac{\sigma_t - \sigma_p}{2}\right) \\ &= -\frac{NV}{r} \cos\left(\frac{\sigma_t - \sigma_p}{2}\right) \cos\left(\frac{\sigma_t + \sigma_p}{2}\right) W < 0\end{aligned}\quad (28)$$

Because $\dot{W}$ is negative definite as long as W is positive, $\sigma_t - \sigma_p \rightarrow 0$ and the lead angles converge to σ_f . Substituting Eqs. (24) and (25) into (12b) gives

$$\frac{d\sigma_t}{dr} = \frac{1}{r} \cot\left(\frac{(2-N)\sigma_t + N\sigma_{t_f}}{2}\right) \quad (29)$$

By integrating Eq. (29), we have

$$\ln \frac{r}{r_1} = \frac{2}{2-N} \ln \left(\frac{\cos\left(\frac{\sigma_{t_1} + \sigma_{p_1}}{2}\right)}{\cos\left(\frac{2-N}{2}\sigma_t + \frac{N}{2}\sigma_{t_f}\right)} \right) \quad (30)$$

The miss distance can be obtained by substituting $\sigma_t = \sigma_{t_f}$ as

$$r_f = r_1 \left(\frac{\cos\left(\frac{\sigma_{t_1} + \sigma_{p_1}}{2}\right)}{\cos \sigma_{t_f}} \right)^{2/(2-N)} = r_1 \left(\frac{\cos\left(\frac{\sigma_{t_1} + \sigma_{p_1}}{2}\right)}{\cos\left(\frac{1}{N}(\sigma_{p_1} + (N-1)\sigma_{t_1})\right)} \right)^{2/(2-N)} > 0 \quad (31)$$

Therefore, the guided trajectory cannot achieve an exact rendezvous in both cases C1 and C2. $\square$

Theorem 2 indicates that the persistent excitation of the bias function is prerequisite for the rendezvous, and fast LOS stabilization would lead to a large distance error. The miss distance cannot be compensated in the absence of speed control when V_λ is regulated before convergent of the distance. In similar manner, the necessary condition of the pursuit-like guidance law and the performance of the existing guidance laws can be addressed as follows.

Corollary 1. (Impossibility of the Exact Rendezvous by PPNG and Deviated Pursuit Guidance) Under Assumptions 1-3, the puruser guided by PPNG approaches to the nonmaneuvering target. Then the PPNG cannot achieve an exact rendezvous condition. Likewise, deviated pursuit guidance yields miss distance in the final time t_f .

Proof. PPNG is the particular case of the guidance form where $b(t) = 0$ in Eq. (14). By substituting the initial condition $(r_0, \sigma_{t_0}, \sigma_{p_0})$ into the corresponding variables $(r_1, \sigma_{t_0}, \sigma_{p_0})$ in Eqs. (22)-(31), the miss distance can be obtained as

$$r_f = r_0 \left(\frac{\cos \left(\frac{\sigma_{t_0} + \sigma_{p_0}}{2} \right)}{\cos \left(\frac{1}{N} (\sigma_{p_0} + (N-1)\sigma_{t_0}) \right)} \right)^{2/(2-N)} \quad (32)$$

If $N = 1$ in the above equation, the guidance command is equivalent to the deviated pursuit guidance. Then, the lead angle of the pursuer keeps constant σ_{p_f} . By integrating from σ_{t_0} to $\sigma_{t_f} = \sigma_{p_f}$, the resultant miss distance is obtained as

$$r_f = r_0 \left(\frac{\cos \sigma_{p_f}}{\cos \left(\frac{\sigma_{t_0} + \sigma_{p_f}}{2} \right)} \right)^{-2} \quad \square$$

4. Reachability Analysis of Rendezvous Guidance Under Relaxed Condition

In the previous section, we investigate the necessary condition of the initial configuration and bias function design for the exact rendezvous. We also investigate that the rendezvous guidance based on PPNG and deviated pursuit guidance may yield miss distance when the target is not maneuvering. In this section, we address the relaxed rendezvous condition by allowing a terminal distance within a bounded value. It is reasonable to analyse the rendezvous under the relaxed condition because the fueling configuration using probe or cable requires the pursuer to coordinate its position relative to the target. In addition, the rendezvous guidance can be applied to other missions, for instance, formation flight of multiple UAVs. From the sufficient condition of the exact rendezvous, let us introduce the following definition as

Definition 3. (Relaxed Rendezvous Condition Under the State-Space Representation) Under Assumptions 1-3, consider the state dynamics in Eq. (11). Then, a compact set representing the relaxed rendezvous condition is expressed as

$$\mathcal{L}_f = \{ (r_f, \sigma_{t_f}, \sigma_{p_f}) : r_f \leq r_{\text{allow}}, \sigma_{t_f} = \sigma_{p_f} \} \quad (33)$$

If there exists $u = \varphi(\mathbf{x})$ for the relaxed rendezvous, then largest invariant set of the closed-loop dynamics $\dot{\mathbf{x}} \triangleq \mathbf{f}_c(\mathbf{x}) = \mathbf{f}(x, \varphi(x))$ should be $\mathcal{L}_f$

In this aspect, the reachability analysis is to obtain the feasible initial set X_0 in which the trajectories converge to $\mathcal{L}_f$. The allowable distance r_{allow} can be chosen by considering the allowable dispersion from the target when both conventional flying-boom and probe-drogue methods can be applied for the refuelling. The reachable region is obtained by backward integrating the closed-loop dynamics $\dot{\mathbf{x}} = \mathbf{f}_c(\mathbf{x})$ backward from the invariant set $\mathcal{L}_f$. The analytic solutions of PPNG and deviated pursuit guidance derived in [33] can be used to express the boundary of the reachable region. The subsequent section presents the reachable region of the PPNG and deviated pursuit guidance for the relaxed rendezvous.

4.1. Reachability of PPNG

In this section, let us consider the PPNG when the navigation gain is selected as $N \geq 2$ [35]. The largest invariant set $\mathcal{L}_{\text{PPNG}}^+$ is expressed as

$$\mathcal{L}_{\text{PPNG}}^+ = \left\{ (r_f, \sigma_{t_f}, \sigma_{p_f}) : r_f = r_0 \left(\frac{\cos\left(\frac{\sigma_{t_0} + \sigma_{p_0}}{2}\right)}{\cos\left(\frac{1}{N}(\sigma_{p_0} + (N-1)\sigma_{t_0})\right)} \right)^{2/(2-N)}, \right. \\ \left. \sigma_{t_f} = \sigma_{p_f} = \frac{1}{N}(\sigma_{p_0} + (N-1)\sigma_{t_0}) \right\} \quad (34)$$

Note that the PPNG shows a qualitative behavior in [36]. Once the initial condition is chosen, its terminal condition is determined from the initial values and corresponding trajectory proportionally converges to the terminal condition. From the invariant set, Eq. (34), the initial condition that makes the trajectory converge to the equilibrium point (r_f, σ_{t_f}) can be obtained as

$$\sigma_{t_0} = \frac{N\sigma_{t_f} - \sigma_{p_0}}{N-1}, \quad r_0 = r_f \left(\frac{\cos(\sigma_{p_f})}{\cos\left(\frac{1}{2(N-1)}(N\sigma_{t_f} + (N-2)\sigma_{p_0})\right)} \right)^{2/(2-N)} \quad (35)$$

From Eq. (35), σ_{t_0} can be equivalently expressed in terms of σ_{t_f} and σ_{p_0} . Considering the feasible initial range of σ_{t_0} obtained from Eq. (26), σ_{t_f} and σ_{p_0} must satisfy

$$\frac{1}{N}\sigma_{p_0} \leq \sigma_{t_f} \leq \frac{\pi}{2} \quad (36)$$

By specifying the initial value σ_{p_0} , the initial position (r_0, σ_{t_0}) can be obtained by integrating backward from the condition r_f, σ_{t_f} . Consequently, the reachable region

of PPNG $\mathcal{X}_0^{\text{PPNG}}$ can be obtained by changing the terminal values $r_f \in (0, r_{\text{allow}}]$ and $\sigma_{t_f} \in [\frac{1}{N}\sigma_{p_0}, \frac{\pi}{2}]$ as

$$\mathcal{X}_0^{\text{PPNG}} = \left\{ (r_0, \sigma_{t_0}, \sigma_{p_0}) : 0 \leq r_0 \leq r_{\text{allow}} \left(\frac{\cos(\sigma_{p_f})}{\cos\left(\frac{1}{2(N-1)}(N\sigma_{t_f} + (N-2)\sigma_{p_0})\right)} \right)^{2/(2-N)}, \right. \\ \left. \sigma_{t_0} = \frac{N\sigma_{t_f} - \sigma_{p_0}}{N-1}, 0 \leq \sigma_{p_0} \leq \frac{\pi}{2}, \frac{1}{N}\sigma_{p_0} \leq \sigma_{t_f} \leq \frac{\pi}{2}, \right\} \quad (37)$$

Figure 3 shows the reachable region of the PPNG for a nonmaneuvering target under relaxed rendezvous condition. The reachable region is made relative to the target direction and depends on the initial lead angle. The feasible region becomes narrow as the larger initial heading σ_{p_0} is selected. The admissible range of σ_{t_f} , Eq. (36), becomes narrow, which leads to the narrowed reachable region. We can interpret the qualitative behaviour of PPNG with the reachable region. When the pursuer starts the rendezvous in the reachable region, the initial condition determines the terminal equilibrium point (r_f, σ_f) in the largest invariant set $\mathcal{L}_{\text{PPNG}}^+$. The PPNG allows the state variables to converge from the initial condition to $\mathcal{L}_{\text{PPNG}}^+$. It also implies that the end point depends on the initial position even if the pursuer is in a reachable area for the same initial direction. To inspect the tendency of the reachable region by the terminal condition σ_{t_f} , Fig. 4 shows the corresponding reachable region where the specified terminal condition is achieved within the reachable region. Most of the initial position in the reachable region converges to the $\sigma_{t_f} = \frac{\pi}{2}$.

4.2. Reachability of Deviated Pursuit Guidance

First, the largest invariant set allowing the terminal distance, $\mathcal{L}_{\text{Deviated Pursuit}}^+$ can be obtained as

$$\mathcal{L}_{\text{Deviated Pursuit}}^+ = \{(r_f, \sigma_{t_f}, \sigma_{p_f}) : 0 \leq r_f \leq r_{\text{allow}}, \sigma_{t_f} = \sigma_{p_f}\} \quad (38)$$

Then, we assume that the deviated pursuit guidance allows the look angle kept by a constant. By integrating the differential equation, Eq. (12b), backward from the terminal condition (r_f, σ_{p_f}) to the desired initial condition $(\sigma_t = \sigma_{t_0})$, we can obtain a trajectory of (r_0, σ_{t_0}) in a closed-form solution as [33]

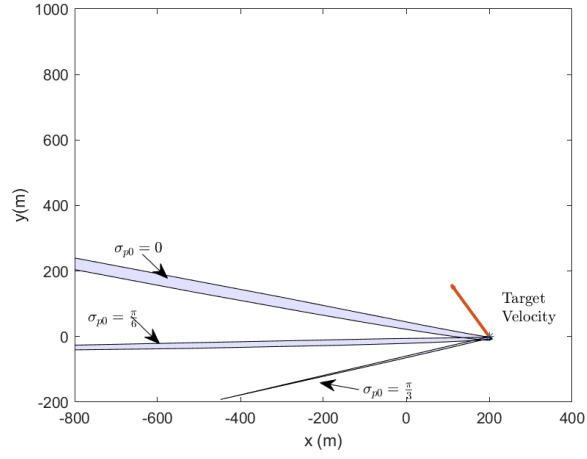


Figure 3: Reachable Region of PPNG for Relaxed Rendezvous r_{allow}

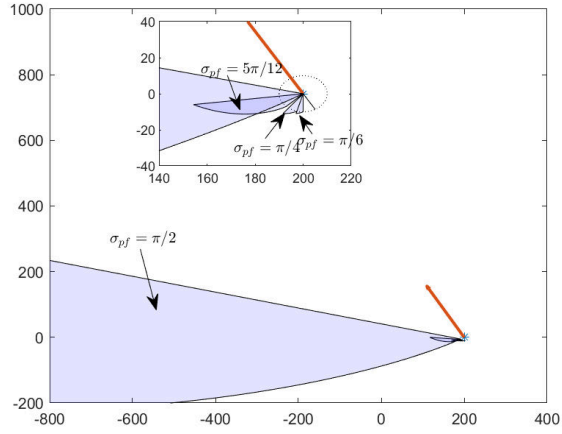


Figure 4: Reachable Region Associated with the Terminal Condition σ_f Within $\mathcal{X}_0^{PPNG}$.

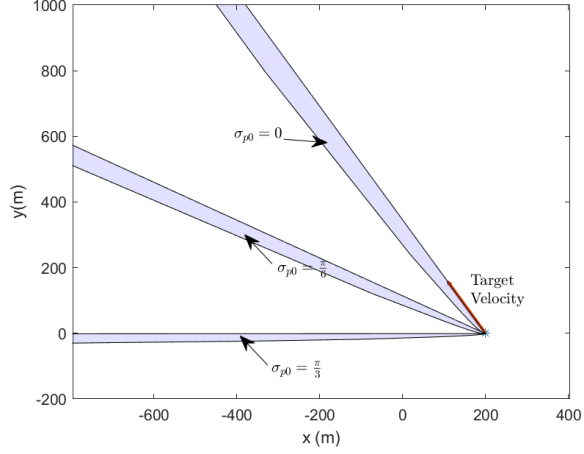


Figure 5: Reachable Region of Deviated Pursuit Guidance for Relaxed Rendezvous

$$r = r_f \left(\frac{\cos^2 \sigma_{p_f}}{\cos^2 \left(\frac{\sigma_{t_0} + \sigma_{p_f}}{2} \right)} \right) \quad (39)$$

Note that the initial angle σ_{t_0} is bounded as $\sigma_{p_0} \leq \sigma_{t_0} \leq \pi - \sigma_{p_0}$ from Eq. (26). From the characteristics of the deviated pursuit guidance, the trajectory converges to the stable point σ_{p_f} . In this respect, a reachable region of deviated pursuit guidance for a constant lead angle σ_p can be obtained as

$$\chi_0^{\text{Deviated Pursuit}} = \left\{ (r_0, \sigma_{t_0}, \sigma_{p_0}), 0 \leq r_0 \leq r_{\text{allow}} \left(\frac{\cos^2 \sigma_{p_f}}{\cos^2 \left(\frac{\sigma_{t_0} + \sigma_{p_f}}{2} \right)} \right), \right. \quad (40)$$

$$\left. \sigma_{p_f} \leq \sigma_{t_0} \leq \pi - \sigma_{p_f}, \sigma_{p_0} = \sigma_{p_f} \right\}$$

Figure 5 shows the reachable region of deviated pursuit guidance for a nonmaneuvering target under the relaxed rendezvous condition. Compared to the PPNG, the reachable region is formed in front of the target direction. If the pursuer is in a same reachable area for the same initial direction, then the terminal condition of lead angle σ_{t_f} is identical. Note that the reachable region with small lead angle is made in the head-on area. It was investigated that the deviated pursuit guidance has poor performance in the head-on geometry. Note that the reachable region is made without consideration of the

limited maneuverability on the pursuer. Because the deviated pursuit guidance shows unstable characteristics in the head on geometry[33], the deviated pursuit guidance in the reachable region may experience abrupt maneuver and fail to rendezvous although it starts the rendezvous from the region. Considering the unfavorable characteristics of deviated pursuit guidance in the head-on case, the designer must carefully choose the desired deviated angle and proper initial position.

4.3. Reachability of Variable Deviated Pursuit Guidance

Aforementioned in Example 1, the variable deviated pursuit guidance converges to the invariant set $\mathcal{L}_{\text{Rendezvous}}^+$ [27] when the pursuer demands a rendezvous with nonmaneuvering target. In similar manner to Sec. 4.2, the initial position (r_0, σ_{t_0}) can be obtained by integrating Eq. (18) backward from the terminal position $(r_f, \sigma_{t_f} = 0)$: Simple as it appears to be, however, the analytic solution is intractable. Instead, let $s = r/r_0$ be the normalized variable. Then, Eq. (18) can be rewritten as

$$\frac{ds}{d\sigma_t} = s \tan \left(\left(1 - \frac{s}{2} \right) \sigma_t \right) \triangleq \tilde{f}(s(\sigma_t), \sigma_t) \quad (41)$$

Note that the boundary conditions are determined as $s_0 = 1$ and $\sigma_{t_f} = 0$ in Eq. (41). By specifying σ_{t_0} , the terminal value of the normalized variable, s_f , can be obtained by integrating Eq. (41) from σ_{t_0} to $\sigma_{t_f} = 0$ as follows

$$s_f(\sigma_{t_0}) = 1 - \int_0^{\sigma_{t_0}} \tilde{f}(s(\sigma_t), \sigma_t) d\sigma_t \quad (42)$$

We want to make the terminal value satisfy $r_f \in [0, r_{allow}]$, the normalized value must satisfy

$$0 \leq s_f(\sigma_{t_0}) \leq \frac{r_{allow}}{r_0} \quad (43)$$

By changing $r_f \in [0, r_{allow}]$, and $0 \leq \sigma_{t_0} \leq \bar{\sigma}_{t_0}$ the reachable region can be obtained as follows

$$\mathcal{X}_0^{\text{Rendezvous [27]}} = \left\{ (r_0, \sigma_{t_0}, \sigma_{p_0}), 0 \leq r_0 \leq \frac{r_{allow}}{1 - \int_0^{\sigma_{t_0}} \tilde{f}(s(\sigma_t), \sigma_t) d\sigma_t}, 0 \leq \sigma_{t_0} \leq \bar{\sigma}_{t_0}, \sigma_{p_0} = 0 \right\} \quad (44)$$

Figure 6 shows the reachable region of the variable deviated pursuit guidance. The rendezvous is reached in the limited initial area when the target is non-maneuvering.

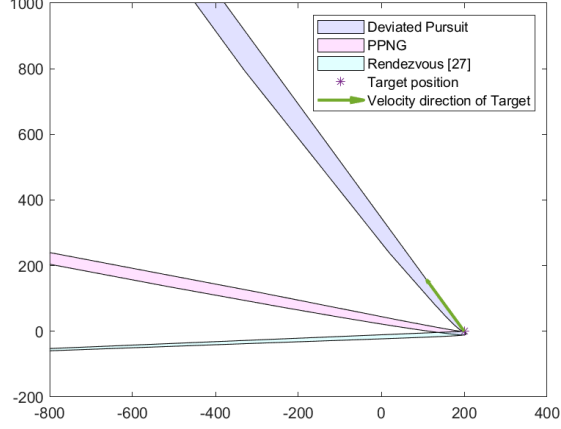


Figure 6: Reachable Region of Three Rendezvous Guidance with Zero Initial Look Angle Conditions

Note that the reachable region is made when $\sigma_{p_0} = 0$, those of the PPNG and deviated pure pursuit guidance for same initial condition are also plotted for comparative discussion. The reachable region of the variable deviated pursuit guidance is placed left side of the target, which complies well with the result in [15] where it investigated favorable initial configuration for the rendezvous guidance strategies. As shown in blue-shaded area, the standard deviated pursuit guidance forms the reachable region around the front of the target. Considering the unstable characteristics of the pursuit guidance in the head on geometry, the location of the reachable region implies that the guidance law proposed in Ref. [27] will show better responses than conventional scheme.

In regard to the area of the reachable region, the variable deviated pursuit yields narrower reachable region. The narrowness of the reachable region implies that the guidance law is much difficult to achieve the rendezvous. It might be the reason that it requires additional constraint for terminal condition $\sigma_{t_f} = \sigma_{p_f} = 0$. Whereas almost cases of the PPNG finish at $\sigma_{p_f} \approx \frac{\pi}{2}$, the commanded lead angle σ_{pc} allows to finish the rendezvous at the specified terminal condition. However, the produced region can be more favorable than that of PPNG if the rendezvous mission requires this requirement.

5. Numerical Simulations

Numerical simulations are performed to demonstrate the analysed results of rendezvous guidance laws. Three rendezvous guidance laws, PPNG based rendezvous guidance, deviated pursuit based rendezvous guidance, and variable deviated pursuit rendezvous guidance [27], are considered in the simulations. In the simulations, the constant speed of the pursuer and the tanker is set as 20 m/s, the limit of the acceleration commands of pursuer is $2g$, where g is the gravitational acceleration. The initial position of the tanker is (200, 0)m, and allowable distance is set to $r_{allow} = 5m$. The initial x-position of the pursuer is 0m and the initial y-position of the pursuer lies on the reachable region in each case. The heading angle of the tanker is chosen as $\frac{2\pi}{3}$ and the tanker does not maneuver in the simulations. The initial look angle of each pursuer, σ_{P_0} , is chosen as 0, $\frac{\pi}{6}$, and $\frac{\pi}{3}$ in each case. We assume no wind environments in the simulations.

Figures 7 and 8 show the simulation results using PPNG based rendezvous guidance. Figure 7 shows the flight trajectories of the pursuers using PPNG based rendezvous guidance and the tanker. All pursuers track the tanker successfully, due to the pursuers start to perform rendezvous maneuver from the reachable regions. Figure 8 shows the relative distance and the acceleration commands of the pursuers using PPNG based rendezvous guidance. The relative distance of the pursuer decreases and converges to a bounded value. The acceleration commands converge to the zero, and it means that the pursuers fly straightly and follow the tanker. Table 1 summarises terminal distances of the PPNG based rendezvous guidance. Three cases all achieve the terminal distance less than the allowable value $r_{allow} = 5$, and therefore the rendezvous mission is satisfactorily completed.

Table 1: Terminal Distance using PPNG based Rendezvous Guidance

	$\sigma_{p_0} = 0 \text{ deg}$	$\sigma_{p_0} = 30 \text{ deg}$	$\sigma_{p_0} = 60 \text{ deg}$
Terminal distance (m)	1.1443	1.0710	2.8512

Figures 9 and 10 show the simulation results using deviated pursuit based rendezvous guidance. Figure 9 shows the flight trajectories of the pursuers using deviated pursuit

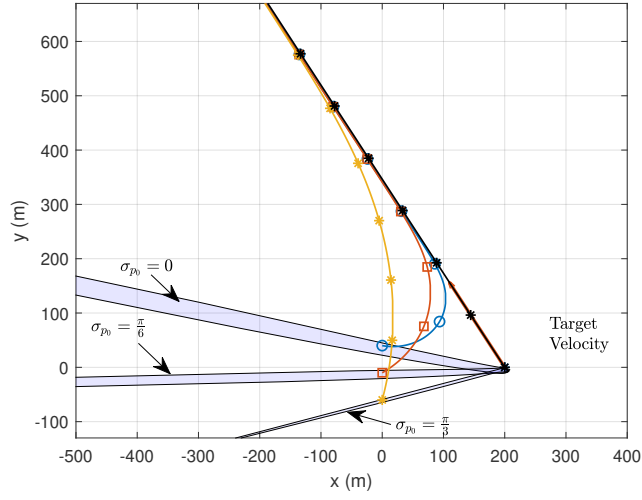


Figure 7: Flight Trajectories of the Pursuer using PPNG based Rendezvous Guidance

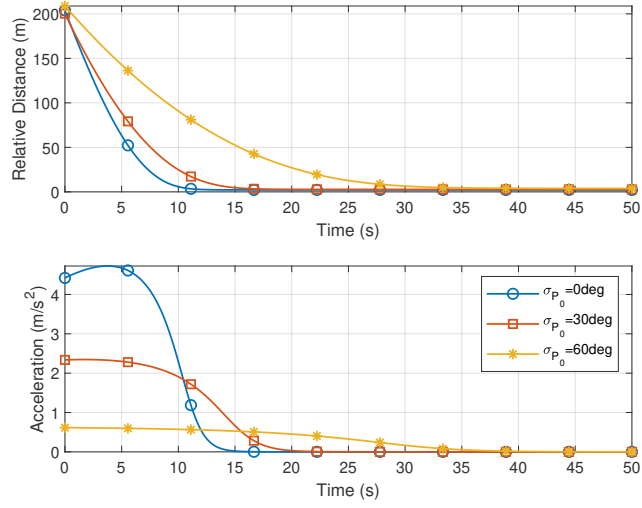


Figure 8: Relative Distance and Acceleration Commands of the Pursuer using PPNG based Rendezvous Guidance

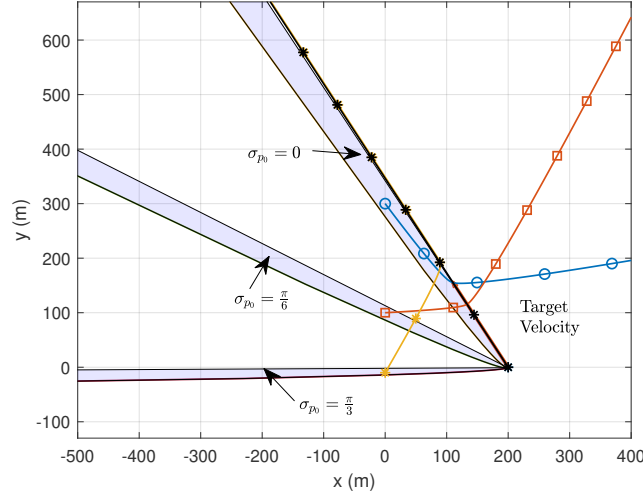


Figure 9: Flight Trajectories of the Pursuer using Deviated Pursuit based Rendezvous Guidance

based rendezvous guidance and the tanker. Unlike the PPNG based rendezvous guidance case, only one pursuer whose initial look angle is $\frac{\pi}{3}$ track the tanker successfully. This is because the two pursuers whose initial look angles are 0 and $\frac{\pi}{6}$ start from the front of the tanker and after the zero relative distance, the pursuers need agile maneuvers to track the tanker but the limit of the acceleration commands is chosen as $2g$ to consider general UAV dynamics in this study. If the limit of acceleration commands is not considered, the pursuers starting on the reachable region perform the rendezvous maneuver successfully. Figure 10 shows the relative distance and the acceleration commands of the pursuers using deviated pursuit based rendezvous guidance. The relative distance of the pursuer whose initial look angle is $\frac{\pi}{3}$ converge to a bounded value. The acceleration command of the pursuer whose initial look angle is $\frac{\pi}{3}$ converge to the zero, and it means that the pursuer flies straightly and follow the tanker.

For further discussion of the terminal distance with the variation of initial lead angle, a multiple simulations are performed. 21 variations of initial lead angle are selected from 0 to 90 degrees. Initial distance is set to $300m$, and initial positions (x_0, y_0) for each initial lead angle are chosen in the reachable region. Figure 11 depicts the trend of the terminal distance with variation of initial lead angle, and Fig. 12 shows the initial

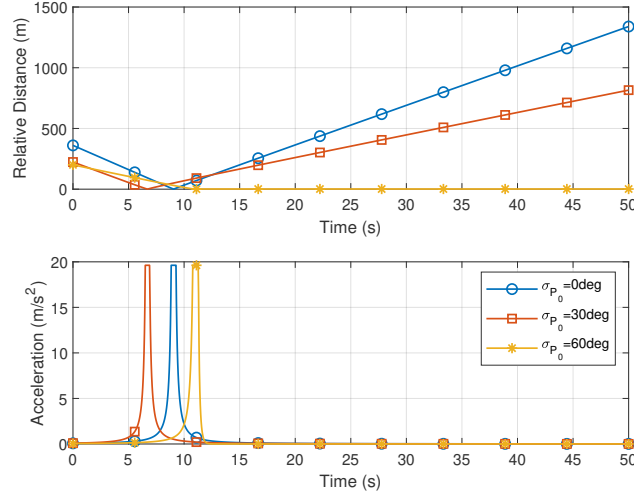


Figure 10: Relative Distance and Acceleration Commands of the Pursuer using Deviated Pursuit based Rendezvous Guidance

LOS angle and the reaching time where the pursuer reaches the closest distance to the target. The terminal distance gradually decrease nearly up to 80 degrees and is less than r_{allow} from 45 degree. This result indicates that the reachable region is only feasible in the range when the acceleration limit is taken into account. It is also observed that the reaching time increases to reach the rendezvous point as the initial lead angle is selected larger as shown in Fig. 12.

Figures 13 and 14 show the simulation results of variable deviated pursuit rendezvous guidance [27], and terminal distance is summarised in Table 2. In this case, we assume that UAV 1 and UAV 3 start to fly outside the reachable region, UAV 2 starts to fly to the tanker on the reachable region. Figure 13 shows the flight trajectories of the pursuers using variable deviated pursuit rendezvous guidance [27] and the tanker. UAV 2 starting on the reachable region tracks the tanker successfully, whereas the terminal distance of UAV 1 and UAV 3 starting outside the reachable region cannot converge to a bounded value. Figure 14 shows the relative distance and the acceleration commands of the pursuers using variable deviated pursuit rendezvous guidance [27]. The relative distance of UAV 2 converges to a bounded value. The acceleration command of UAV 2 converges to the zero, and it means that the pursuer flies straightly and follow the tanker.

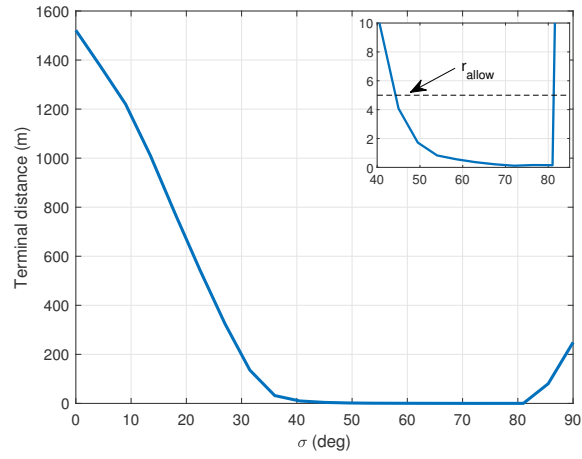


Figure 11: Terminal Distance with respect to Initial Lead Angle using Deviated Pursuit based Rendezvous Guidance

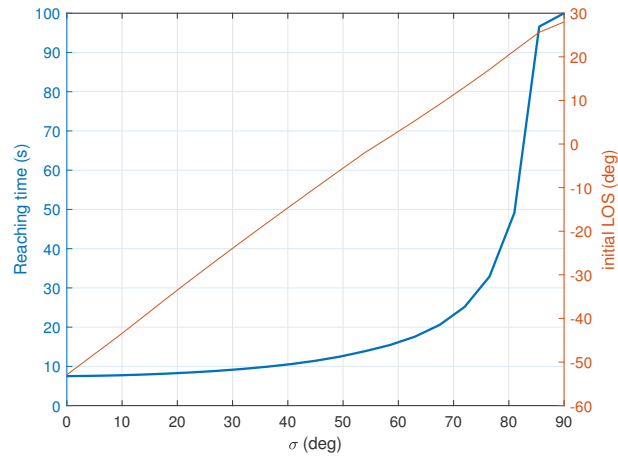


Figure 12: Initial LOS and reaching time with respect to Initial Lead Angle using Deviated Pursuit based Rendezvous Guidance

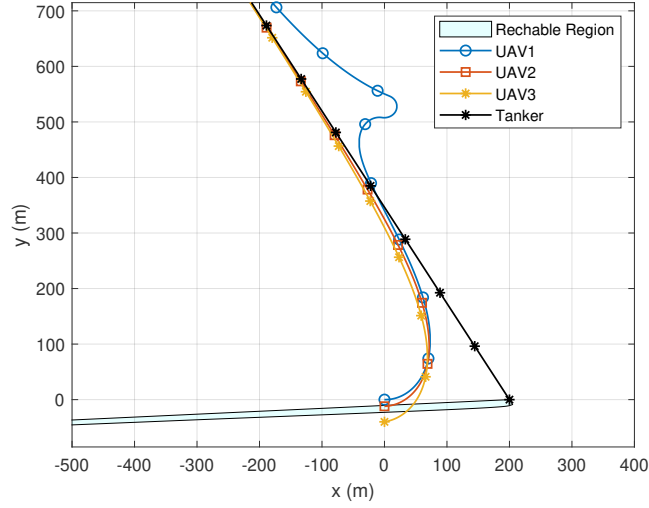


Figure 13: Flight Trajectories of the Pursuer using Variable Deviated Pursuit Rendezvous Guidance [27]

The simulation results corroborate the reachability analysis presented in the study.

Table 2: Terminal Distance using Variable Deviated Pursuit Rendezvous Guidance [27]

	UAV1	UAV2	UAV3
Terminal distance (m)	93.2738	1.579	23.7291

6. Conclusion

This paper analysed the rendezvous guidance for UAVs aerial refuelling. First, a necessary condition of exact rendezvous was addressed for the UAV to approach the rendezvous point. Theorem 1 geometrically indicates the necessary condition for an exact rendezvous in the absence of speed variation. Using the information, one may specify the initial positions by excluding infeasible regions. Second, the reachability of pure proportional navigation guidance and deviated pursuit guidance were analysed in relaxed rendezvous condition, which can be utilized as prior information for rendezvous operation.

The analysis implies that the exact rendezvous without the involvement of speed variation is extremely difficult. A simulation study demonstrates the existing rendezvous

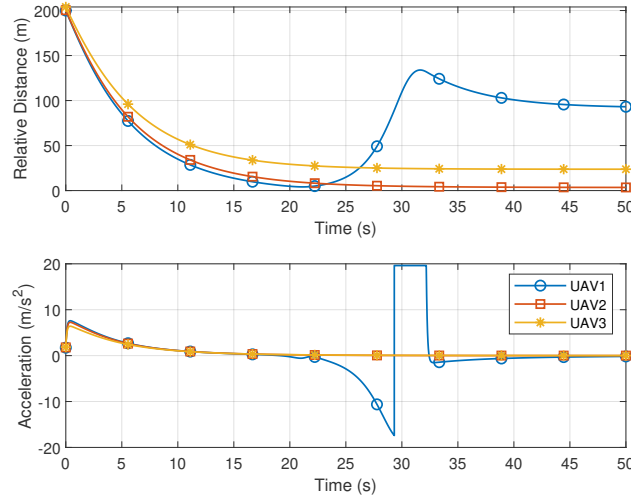


Figure 14: Relative Distance and Acceleration Commands of the Pursuer using Variable Deviated Pursuit Rendezvous Guidance [27]

guidance methods only achieve the rendezvous only narrowed initial configurations. To overcome current limitations, the analysis will be utilized as a design guideline to improve existing rendezvous guidance laws. A cooperative guidance strategy will be considered to improve the rendezvous performance in the absence of speed variation.

Acknowledgements

This research was financially supported by Hansung University.

References

- [1] P. R. Thomas, U. Bhandari, S. Bullock, T. S. Richardson, J. L. Du Bois, Advances in air to air refuelling, *Progress in Aerospace Sciences* 71 (2014) 14–35.
- [2] M. L. Fravolini, A. Ficola, G. Campa, M. R. Napolitano, B. Seanor, Modeling and control issues for autonomous aerial refueling for uavs using a probe–drogue refueling system, *Aerospace science and technology* 8 (7) (2004) 611–618.

- [3] A. Dogan, W. Blake, Maneuverability of large receiver of different size and weight in aerial refueling, *Journal of Guidance, Control, and Dynamics* 37 (6) (2014) 1880–1896.
- [4] P. R. Thomas, S. Bullock, T. S. Richardson, J. F. Whidborne, Collaborative control in a flying-boom aerial refueling simulation, *Journal of Guidance, Control, and Dynamics* 38 (7) (2015) 1274–1289.
- [5] H. E. Sevil, A. Dogan, Fault diagnosis in air data sensors for receiver aircraft in aerial refueling, *Journal of Guidance, Control, and Dynamics* 38 (10) (2015) 1959–1975.
- [6] J. Valasek, D. Famularo, M. Marwaha, Fault-tolerant adaptive model inversion control for vision-based autonomous air refueling, *Journal of Guidance, Control, and Dynamics* 40 (6) (2017) 1336–1347.
- [7] Z. Su, H. Wang, P. Yao, Y. Huang, Y. Qin, Back-stepping based anti-disturbance flight controller with preview methodology for autonomous aerial refueling, *Aerospace Science and Technology* 61 (2017) 95–108.
- [8] X. Dai, Q. Quan, J. Ren, Z. Xi, K.-Y. Cai, Terminal iterative learning control for autonomous aerial refueling under aerodynamic disturbances, *Journal of Guidance, Control, and Dynamics* 41 (7) (2018) 1576–1583.
- [9] C. Parsons, Z. Paulson, S. Nykl, W. Dallman, B. G. Woolley, J. Pecarina, Analysis of simulated imagery for rt vision-based automated aerial refueling, *Journal of Aerospace Information Systems* 16 (3) (2019) 77–93.
- [10] X. Dai, Q. Quan, J. Ren, K.-Y. Cai, Iterative learning control and initial value estimation for probe-drogue autonomous aerial refueling of uavs, *Aerospace science and technology* 82–83 (2018) 583–593.
- [11] L. Wang, H. Yin, Y. Guo, T. Yue, X. Jia, Closed-loop motion characteristic requirements of receiver aircraft for probe and drogue aerial refueling, *Aerospace science and technology* 93 (2019) 105293.

- [12] Y. Liu, H. Wang, J. Fan, Novel docking controller for autonomous aerial refueling with probe direct control and learning-based preview method, *Aerospace science and technology* 94 (2019) 105403.
- [13] T. McLain, R. Beard, Trajectory planning for coordinated rendezvous of unmanned air vehicles, in: *AIAA Guidance, navigation, and control conference*, Denver, CO, 2000.
- [14] Y. Yoon, M. Kim, Y. Kim, Three-dimensional path planning for aerial refueling between one tanker and multiple uavs, *International Journal of Aeronautical and Space Sciences* 19 (4) (2018) 1027–1040.
- [15] B. Burns, P. Blue, M. Zollars, Autonomous control for automated aerial refueling with minimum-time rendezvous, in: *AIAA Guidance, Navigation and Control Conference*, Hilton Head, SC, 2007.
- [16] H. Oh, S. Kim, H.-S. Shin, B. A. White, A. Tsourdos, C. A. Rabbath, Rendezvous and standoff target tracking guidance using differential geometry, *Journal of Intelligent & Robotic Systems* 69 (1-4) (2013) 389–405.
- [17] A. Rucco, P. Sujit, A. P. Aguiar, J. B. De Sousa, F. L. Pereira, Optimal rendezvous trajectory for unmanned aerial-ground vehicles, *IEEE Transactions on Aerospace and Electronic Systems* 54 (2) (2018) 834–847.
- [18] A. Tsukerman, M. Weiss, T. Shima, D. Löbl, F. Holzapfel, Optimal rendezvous guidance laws with application to civil autonomous aerial refueling, *Journal of Guidance, Control, and Dynamics* 41 (5) (2018) 1167–1174.
- [19] J. Z. Ben-Asher, Linear quadratic pursuit-evasion games with terminal velocity constraints, *Journal of Guidance, Control, and Dynamics* 19 (2) (1996) 499–501.
- [20] R. Bardhan, D. Ghose, Differential games guidance for heading angle consensus among unmanned aerial vehicles, *Journal of Guidance, Control, and Dynamics* 42 (11) (2019) 2568–2575.

- [21] P.-J. Yuan, S.-C. Hsu, Rendezvous guidance with proportional navigation, *Journal of Guidance, Control, and Dynamics* 17 (2) (1994) 409–411.
- [22] Y. Ochi, T. Kominami, Flight control for automatic aerial refueling via png and los angle control, in: *AIAA guidance, navigation, and control conference*, San Francisco, CA, 2005.
- [23] A. L. Smith, Proportional navigation with adaptive terminal guidance for aircraft rendezvous, *Journal of guidance, control, and dynamics* 31 (6) (2008) 1832–1836.
- [24] T. Yamasaki, S. Balakrishnan, Sliding mode-based pure pursuit guidance for unmanned aerial vehicle rendezvous and chase with a cooperative aircraft, *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering* 224 (10) (2010) 1057–1067.
- [25] N. Harl, S. Balakrishnan, Coordinated rendezvous of unmanned air vehicles to a formation using a sliding mode approach, *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering* 225 (1) (2011) 105–119.
- [26] J. G. Manathara, D. Ghose, Rendezvous of multiple uavs with collision avoidance using consensus, *Journal of Aerospace Engineering* 25 (4) (2012) 480–489.
- [27] A. Ratnoo, Variable deviated pursuit for rendezvous guidance, *Journal of Guidance, Control, and Dynamics* 38 (4) (2015) 787–792.
- [28] J. W. Nichols, L. Sun, R. W. Beard, T. McLain, Aerial rendezvous of small unmanned aircraft using a passive towed cable system, *Journal of Guidance, Control, and Dynamics* 37 (4) (2014) 1131–1142.
- [29] P. Anjaly, A. Ratnoo, Observability enhancement of maneuvering target with bearings-only information, *Journal of Guidance, Control, and Dynamics* 41 (1) (2018) 184–198.
- [30] A. Parayil, A. Ratnoo, Cooperative beam-rider guidance for unmanned aerial vehicle rendezvous, *Journal of Intelligent & Robotic Systems* 95 (2) (2019) 585–599.

- [31] W. Jiang, D. Wang, Y. Wang, Z. A. Ali, Uav rendezvous based on time-varying vector fields, *Electronics Letters* 53 (10) (2017) 653–655.
- [32] D. Luo, R. Xie, H. Duan, A guidance law for uav autonomous aerial refueling based on the iterative computation method, *Chinese Journal of Aeronautics* 27 (4) (2014) 875–883.
- [33] N. A. Shneydor, *Missile Guidance and Pursuit: Kinematics, Dynamics and Control*, Woodehead, Cambridge, UK, 1998.
- [34] A. Locke, *Principles of Guided Missile Design - Volume 1: Guidance*, D. Van Nostrand Company, Princeton, N.J., 1955.
- [35] P. Zarchan, *Tactical and strategic missile guidance*, American Institute of Aeronautics and Astronautics, Inc., 2012.
- [36] M. Guelman, A Qualitative Study of Proportional Navigation, *IEEE Transactions on Aerospace and Electronic Systems* 7 (4) (1971) 637–643. doi:10.1109/TAES.1971.310406.
- [37] Z. Wang, S. T. McDonald, Convex relaxation for optimal rendezvous of unmanned aerial and ground vehicles, *Aerospace science and technology* 99 (2020) 105756.
- [38] T. Z. Muslimov, R. A. Munasypov, Consensus-based cooperative control of parallel fixed-wing uav formations via adaptive backstepping, *Aerospace science and technology* 109 (2021) 106416.
- [39] H. T. Davis, *Introduction to nonlinear differential and integral equations*, US Government Printing Office, 1961.
- [40] Khalil, H. K., J. Grizzle, *Nonlinear Systems*, Vol. 3, Prentice hall, Upper Saddle River, NJ, 1996.



DSPACE

<https://dspace.org/>

Analysis of rendezvous guidance laws for autonomous aerial refueling for non-maneuvering and identical speed targets

Lee, Seokwon; Kim, Mingu

2022-01-14

Elsevier

<https://doi.org/10.1016/j.ast.2022.107359>

Lee S, Kim M. (2022) Analysis of rendezvous guidance laws for autonomous aerial refueling for non-maneuvering and identical speed targets. Aerospace Science and Technology, Volume 121, February 2022, Article number 107359