

Bipartite consensus of nonlinear agents with actuator fault

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Abstract—This paper introduces a bipartite consensus controller to address the challenge of achieving consensus among nonlinear agents, particularly when actuator faults are present, leading to significant obstacles. To tackle this issue, the controller is developed by adapting the Distributed Nonlinear Dynamic Inversion (DNDI) technique, thereby accommodating the impact of actuator faults. The randomness of the actuator fault is taken into account to reflect real-world conditions. The paper also furnishes comprehensive mathematical insights into the convergence of the fault-tolerant controller, establishing a robust theoretical foundation. An extensive array of simulation studies demonstrate that the proposed controller effectively manages actuator faults, leading to the successful attainment of bipartite consensus.

I. INTRODUCTION

Drones are being used increasingly for complex tasks like surveillance, search and rescue, deliveries, and firefighting, thanks to recent advancements in drone technology. For these tasks to be successful, it's essential that the drones work together and agree on what needs to be done. Drones communicate with each other, share data, and use consensus protocols to work together on tasks like flocking (as described in [1]), formation control (as explained in [2]), and synchronization (as demonstrated in [3]). To achieve this consensus, a range of consensus protocols that outline how the agents communicate with each other are employed. Many studies have looked at different consensus challenges, taking into account communication nuances like changes in topology, delays, and interference, as referenced in [4], [5], [6], [7], [8].

In the standard consensus process, agents collectively agree upon a singular decision, objective, or shared value. Nonetheless, there exists a potential for certain agents to not align with the identical consensus value. This circumstance was elucidated by [9] through the utilization of a signed graph featuring both positive and negative edge weights. In this framework, when a cluster of agents endeavours to reach a consensus on such a graph, they become segregated into two groups, each possessing consensus values of equal magnitude but opposite polarity. This phenomenon was termed bipartite consensus. Researchers began integrating this concept into various domains, such as social networks and opinion dynamics [10]. Comparable to regular consensus scenarios, numerous scholars have developed consensus protocols tailored for bipartite consensus, hinging on the dynamics of the agents. These dynamics can be categorized as

linear [11], [12] or nonlinear [13], [14]. The resultant control actions generated by these protocols are crafted to guide the agents towards desired manoeuvres and subsequently effectuated by actuators.

It's crucial to highlight that several research endeavours delve into the impact of actuator faults on consensus. These faults could manifest within the actuators of a singular agent, a subset of agents, or even across all agents. Thus, it becomes imperative to formulate consensus controllers that can accommodate these faults, ensuring system safety and dependability. For instance, [15] proposed a fault-tolerant consensus protocol for linear agents. [16] introduced an adaptive protocol to address distributed fault-tolerant consensus issues in agents characterized by linear dynamics and uncertainty. [17] tackled the challenge of consensus tracking for second-order nonlinear agents with actuator faults through an adaptive approach. [18] formulated a consensus protocol for nonlinear agents equipped to handle actuator faults. Furthermore, [19] offered an instance of time-varying actuator faults, proposing an adaptive consensus tracking control scheme tailored for high-order nonlinear agents.

Existing fault-tolerant consensus protocols have been crafted utilizing various facets of control theory, with the specifics being contingent upon agent dynamics. A recent consensus protocol introduced by [7] employs Nonlinear Dynamic Inversion (NDI) as its foundation, as referenced in [20], [21] etc. This protocol has been dubbed Distributed NDI (DNDI). DNDI has been applied to address consensus challenges for nonlinear agents in the presence of external disturbances, as indicated in [8]. One of the core strengths of DNDI is that it leverages the benefits inherent to NDI. These include a transparent control expression, straightforward implementation, global exponential stability, the capacity to incorporate nonlinear kinematics during plant inversion, and reduced reliance on individualized gain adjustments or scheduling. The contribution of this study can be summarized as follows.

- In this study, our focus has been on proposing a distributed controller based on NDI (DNDI) to facilitate bipartite consensus among a set of nonlinear agents, even in the presence of actuator faults. The application of DNDI in the realm of fault-tolerant bipartite consensus is a novel approach.
- The mathematical intricacies pertaining to the convergence of the controller furnish a robust theoretical foundation, underscoring its efficacy in managing actuator malfunctions.
- For the sake of rendering our simulation scenarios more realistic, actuator faults have been assumed to be

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unpredictable in nature. Moreover, we have presented two scenarios. In the first scenario, we introduced faults in the selected actuators and started at different times. In the second scenario, we considered the worst-case scenario described by the situation when fault started in all actuators simultaneously.

II. PRELIMINARIES

A. Bipartite Consensus

Definition 1: A group of agents achieve bipartite consensus if the following conditions

- $\lim_{t \rightarrow \infty} (x_i(t) - x_d(t)) = 0, \forall i \in \mathcal{O}_1$
- $\lim_{t \rightarrow \infty} (x_j(t) + x_d(t)) = 0, \forall j \in \mathcal{O}_2$

are satisfied, where $x_d(t)$ is a desired trajectory, and $\mathcal{O}_1 \cup \mathcal{O}_2 = \{1, 2, \dots, N\}$; $\mathcal{O}_1 \cap \mathcal{O}_2 = \emptyset$.

B. Graph Theory

The communication structure is represented through a weighted graph denoted as $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. The points or nodes in this graph are symbolized by $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$, representing the agents. The connections or links between these nodes are characterized by the set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, which represents the communication among the agents. The values in the weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ of $\mathcal{G}$ are $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}$, otherwise $a_{ij} = 0$. It's worth noting that the adjacency matrix $\mathcal{A}$ has zero values on its diagonal, meaning there's no self-connecting node; formally, for every $v_i \in \mathcal{V}$, $a_{ii} = 0$. The degree matrix can be given by $D \in \mathbb{R}^{N \times N} = \text{diag}\{d_1, d_2, \dots, d_N\}$, where $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. The Laplacian matrix is written as $\mathcal{L} = D - \mathcal{A}$. Typically, the synchronization of interconnected agents is described with the Laplacian matrix $\mathcal{L}$ of a nonnegative graph. However, the Laplacian matrix is characterized distinctly when dealing with a signed graph. For a signed graph, this matrix is referred to as the signed Laplacian, denoted by $\mathcal{L}_s$, and is defined as: $\mathcal{L}_s = \text{diag}\left(\sum_{j=1}^N |a_{1j}|, \dots, \sum_{j=1}^N |a_{Nj}|\right) - \mathcal{A}$.

C. Actuator Fault

Let us consider the control of i^{th} agent is $U_i \in \mathbb{R}^m$. The fault model for i^{th} agent is discussed in Eq. (1).

$$U_i^f = \gamma_i U_i \quad (1)$$

where, $\gamma_i = (\mathbf{I} - \xi_i) \in \mathbb{R}^{m \times m}$ represents a diagonal matrix. The desired control input for the i^{th} agent is given by U_i . When there's an actuator fault, the control is represented by U_i^f . The matrix $\mathbf{I} \in \mathbb{R}^{m \times m}$ stands for the identity matrix, and the fault severity for the i^{th} agent is illustrated by the diagonal matrix $\xi_i \in \mathbb{R}^{m \times m}$. $q_{ij} \in \xi_i$ is $(i, j)^{\text{th}}$ component of severity matrix. It represents the value of severity in j^{th} actuator of i^{th} agent.

Remark 1. It can be noted that the fault expression in Eq. (1) describes the fault free case if $\xi_i = \mathbf{0}$ is selected.

D. Theorems and Lemmas

Definition 2: (Structural balance)[9], [22] A signed graph is structurally balanced if it has a bipartition of the nodes $\mathcal{V}_1, \mathcal{V}_2$, i.e., $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ such that $a_{ij} \leq 0, \forall v_i \in \mathcal{V}_p, v_j \in \mathcal{V}_q$ where $p, q \in 1, 2, p \neq q$, and $\emptyset$ is empty set; otherwise $a_{ij} \geq 0$.

Lemma 1: [23] A spanning tree is structurally balanced.

Lemma 2: [24] Suppose the signed graph $\mathcal{G}(\mathcal{A})$ has a spanning tree. Denote the signature matrices set as

$$\mathcal{D} = \{D = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_N) | \sigma_i \in \{1, -1\}\} \quad (2)$$

Then the following statements are equivalent.

1. $\mathcal{G}(\mathcal{A})$ is structurally balanced.
2. $a_{ij}a_{ji} \geq 0$ and the associated undirected graph $\mathcal{G}(\mathcal{A}_u)$ is structurally balanced, where $\mathcal{G}(\mathcal{A}_u) = \frac{\mathcal{A} + \mathcal{A}^T}{2}$.
3. $\exists D \in \mathcal{D}$, such that $\bar{\mathcal{A}} = [\bar{a}_{ij}] = D\mathcal{A}D$ is a nonnegative matrix.

Corollary 1: [25] Suppose the signed graph $\mathcal{G}(\mathcal{A})$ is undirected and connected. The graph is structurally balanced, if and only if 0 is a simple eigenvalue of $\mathcal{L}$ and all other eigenvalues have positive real parts.

III. PROBLEM DESCRIPTION

This study seeks to devise a controller for attaining bipartite consensus in agents with nonlinear dynamics and actuator faults. The consensus controller introduced in this paper is an adaptation of the DNDI, elaborated in the subsequent section. The dynamics for the i^{th} agent can be found in Eqs. (3-4).

$$\dot{X}_i = f(X_i) + g(X_i)U_i \quad (3)$$

$$Y_i = X_i \quad (4)$$

The state and control of i^{th} agent is given by $X_i \in \mathbb{R}^n$ and $U_i \in \mathbb{R}^n$ respectively. The output of i^{th} agent is given by

$$Y_i = X_i \in \mathbb{R}^n \quad (5)$$

In order to enhance the realism of the simulation scenario, we have incorporated random faults in the agents' actuators.

Assumption 1. The system is perfectly known and the matrix $g(X)$ is invertible for all time.

Assumption 2. The agents exchange data via a communication topology represented by a signed graph $\mathcal{G}(\mathcal{A})$ that possesses a spanning tree. Additionally, this topology is structurally balanced.

IV. BIPARTITE CONSENSUS OF NONLINEAR AGENTS WITH FAULT USING DNDI CONTROLLER

The DNDI controller, as described in [7], is tailored for addressing the standard consensus issue in nonlinear agents. Additionally, DNDI has the capacity to manage communication glitches and external disruptions, as showcased in [8]. In this research, we've adapted the DNDI to tackle the bipartite consensus challenge for nonlinear agents experiencing random actuator malfunctions. Within this section, the

mathematical intricacies of DNDI, based on [7], are laid out. The state error for the i^{th} agent (in scalar form) is given as:

$$\delta_i = \sum_{j \in N_i} (|a_{ij}|X_i - a_{ij}X_j) \quad (6)$$

where j denotes the j^{th} agent of i^{th} agent's neighbourhood N_i . The Eq. (6) is simplified as follows.

$$\delta_i = |d_i|X_i - a_iX \quad (7)$$

where $|d_i| = \sum_j |a_{ij}| \in \mathbb{R}$, $a_i = [a_{i1} \ a_{i2} \dots \ a_{iN}] \in \mathbb{R}^N$ and $X = [X_1 \ X_2 \ \dots \ X_N]^T \in \mathbb{R}^N$. The error (Eq. (7)) is written for state vector of i^{th} agent, i.e., $X_i \in \mathbb{R}^n; n > 1$

$$\delta_i = |\bar{d}_i|X_i - \bar{a}_iX \quad (8)$$

where $|\bar{d}_i| = (|d_i| \otimes \mathbf{I}_n) \in \mathbb{R}^{n \times n}$, $\bar{a}_i = (a_i \otimes \mathbf{I}_n) \in \mathbb{R}^{n \times nN}$, and $X \in \mathbb{R}^{nN}$. $\mathbf{I}_n$ is $n \times n$ identity matrix. Let us construct a Lyapunov function V_i as given below.

$$V_i = \frac{1}{2} \delta_i^T \delta_i \quad (9)$$

Differentiation of Eq. (9) yields

$$\dot{V}_i = \delta_i^T \dot{\delta}_i \quad (10)$$

Based on the Lyapunov stability theory, the time derivative of the Lyapunov function should be:

$$\dot{V}_i = -\delta_i^T \Lambda_i \delta_i \quad (11)$$

where $\Lambda_i \in \mathbb{R}^{n \times n}$ is a diagonal gain matrix. The expression of $\dot{V}_i$ in Eqs. (10) and (11) are equated to obtain

$$\delta_i^T \dot{\delta}_i = -\delta_i^T \Lambda_i \delta_i \quad (12)$$

Eq. (12) is simplified as follows.

$$\dot{\delta}_i + \Lambda_i \delta_i = 0 \quad (13)$$

Differentiation of Eq. (8) yields

$$\dot{\delta}_i = |\bar{d}_i|\dot{X}_i - \bar{a}_i\dot{X} = |\bar{d}_i|[f(X_i) + g(X_i)U_i] - \bar{a}_i\dot{X} \quad (14)$$

Substitution of the expressions $\dot{\delta}_i$ in Eq. (13) yields

$$|\bar{d}_i|[f(X_i) + g(X_i)U_i] - \bar{a}_i\dot{X} + \Lambda_i(|\bar{d}_i|X_i - \bar{a}_iX) = 0 \quad (15)$$

The resulting expression of U_i for i^{th} agent is obtained by simplifying Eq. (15) as follows

$$U_i = (g(X_i))^{-1} \left[-f(X_i) + |\bar{d}_i|^{-1}(\bar{a}_i\dot{X} - \Lambda_i\delta_i) \right] \quad (16)$$

Theorem 1: A group of nonlinear agents (dynamics given in Eqs. (3)-(4) having faulty actuators (given in Eq. (1) along with the assumptions 1 and 2, achieve bipartite consensus using the consensus protocol obtained in Eq. (16). The Uniformly Ultimate Boundedness (UUB) of the consensus error ensures the convergence.

Proof: Faulty control expression of i^{th} agent is given by Eq. 1. The values of the diagonal elements of ξ_i are between 0 and 1. The dynamics of i^{th} agent is given by

$$\dot{X}_i = f(X_i) + g(X_i)U_i^f \quad (17)$$

Let us consider a scalar function

$$\Pi = \frac{1}{2} X^T (L_s \otimes \mathbf{I}_n) X \quad (18)$$

$$L_s \otimes \mathbf{I}_n = \Upsilon \Theta \Upsilon^T \quad (19)$$

where, $\Upsilon \in \mathbb{R}^{nN \times nN}$ is the left eigenvalue matrix of $L_s \otimes \mathbf{I}_n$, $\Theta = (diag\{0, \lambda_2(L_s), \lambda_3(L_s), \dots, \lambda_N(L_s)\} \otimes \mathbf{I}_n) \in \mathbb{R}^{nN \times nN}$ is eigenvalue matrix, $\Upsilon^T \Upsilon = \Upsilon \Upsilon^T = \mathbf{I}_{nN \times nN}$.

$$\begin{aligned} \Pi &= \frac{1}{2} X^T (L_s \otimes \mathbf{I}_n) X \\ &= \frac{1}{2} X^T \Upsilon \Theta \Upsilon^T X \\ &= \frac{1}{2} X^T \Upsilon \sqrt{\Theta} \sqrt{\Theta} \Upsilon^T X \\ &= \frac{1}{2} X^T \Upsilon \sqrt{\Theta \bar{\Theta}} \sqrt{\bar{\Theta}^{-1}} \sqrt{\bar{\Theta}^{-1}} \sqrt{\bar{\Theta} \Theta} \Upsilon^T X \\ &= \frac{1}{2} X^T \Upsilon \Theta \bar{\Theta}^{-1} \Theta \Upsilon^T X \\ &= \frac{1}{2} X^T \Upsilon \Theta (\Upsilon^T \Upsilon) \bar{\Theta}^{-1} (\Upsilon^T \Upsilon) \Theta \Upsilon^T X \\ &= \frac{1}{2} X^T (\Upsilon \Theta \Upsilon^T) (\Upsilon \bar{\Theta}^{-1} \Upsilon^T) (\Upsilon \Theta \Upsilon^T) X \\ &= \frac{1}{2} X^T (L_s \otimes \mathbf{I}_n) \Psi (L_s \otimes \mathbf{I}_n) X \\ &= \frac{1}{2} \Delta^T \Psi \Delta \end{aligned} \quad (20)$$

where $\bar{\Theta} = (diag\{\lambda_2(L_s), \lambda_2(L_s), \lambda_3(L_s), \dots, \lambda_N(L_s)\} \otimes \mathbf{I}_n) \in \mathbb{R}^{nN \times nN}$, $\Delta = [\delta_1^T \delta_2^T \dots \delta_N^T]^T \in \mathbb{R}^{nN}$, and $\Psi = \Upsilon \bar{\Theta}^{-1} \Upsilon^T \in \mathbb{R}^{nN \times nN}$.

Remark 3. It can be observed from Eqs. (18) and (20) that

$$\frac{\lambda_{min}(\Psi)}{2} \|\Delta\|^2 \leq \Pi \leq \frac{\lambda_{max}(\Psi)}{2} \|\Delta\|^2 \quad (21)$$

$$\Pi = \frac{1}{2} X^T (L_s \otimes \mathbf{I}_n) X = \frac{1}{2} X^T \Delta \quad (22)$$

Remark 4. Since $\lambda_2 > 0$. Hence, $\bar{\Theta}$ is invertible.

Remark 5. $\Psi = \Upsilon \bar{\Theta}^{-1} \Upsilon^T$ is positive definite matrix. Hence, Π is positive definite subject to consensus error and qualify for a Lyapunov function.

Eq. (18) is differentiated to obtain

$$\begin{aligned} \dot{\Pi} &= X^T (L_s \otimes \mathbf{I}_n) \dot{X} \\ &= \Delta^T \dot{X} \\ &= \sum_{i=1}^N \delta_i^T [f(X_i) + \gamma_i g(X_i) U_i] \end{aligned} \quad (23)$$

Expression of U_i is substituted in Eq. (23) we get

$$\begin{aligned} \dot{\Pi} &= \sum_{i=1}^N \delta_i^T \left[(\mathbf{I}_n - \gamma_i) f(X_i) + \gamma_i |\bar{d}_i|^{-1} (\bar{a}_i \dot{X} - \Lambda_i \delta_i) \right] \\ &= \sum_{i=1}^N \delta_i^T \xi_i f(X_i) \end{aligned}$$

$$+ \sum_{i=1}^N -\delta_i^T \gamma_i |\bar{d}_i|^{-1} \Lambda_i \delta_i + \sum_{i=1}^N \delta_i^T \gamma_i |\bar{d}_i|^{-1} \bar{a}_i \dot{X} \quad (24)$$

Lemma 4.[26] Let $\psi_1(t), \psi_2(t) \in R^m$ be continuous positive vector functions, by Cauchy inequality and Young's inequality, there exists the following inequality:

$$\begin{aligned} \psi_1(t)\psi_2(t) &\leq \|\psi_1(t)\| \|\psi_2(t)\| \\ &\leq \frac{\|\psi_1(t)\|^\lambda}{\lambda} + \frac{\|\psi_2(t)\|^\zeta}{\zeta} \end{aligned} \quad (25)$$

where

$$\frac{1}{\lambda} + \frac{1}{\zeta} = 1$$

Lemma 5.[27] Let $R(t) \in \mathbb{R}$ be a continuous positive function with bounded initial $R(0)$. If the inequality holds $\dot{R}(t) \leq -\beta R(t) + \eta$ where, $\beta > 0, \eta > 0$, then the following inequality holds.

$$R(t) \leq R(0)e^{-\beta t} + \frac{\eta}{\beta} (1 - e^{-\beta t}) \quad (26)$$

According to *Lemma 4*, we can write

$$\begin{aligned} \delta_i^T \gamma_i |\bar{d}_i|^{-1} \bar{a}_i \dot{\mathbf{X}} &\leq \|\delta_i\| \|\gamma_i |\bar{d}_i|^{-1} \bar{a}_i \dot{\mathbf{X}}\| \leq \frac{\|\delta_i\|^2}{2} + \\ &\frac{\|\gamma_i |\bar{d}_i|^{-1} \bar{a}_i \dot{\mathbf{X}}\|^2}{2} \end{aligned} \quad (27)$$

$$\delta_i^T \xi_i f(X_i) \leq \|\delta_i\| \|\xi_i f(X_i)\| \leq \frac{\|\delta_i\|^2}{2} + \frac{\|\xi_i f(X_i)\|^2}{2} \quad (28)$$

Substituting Eqs. (27) and (28) in Eq. (24) we get

$$\begin{aligned} \dot{\Pi} &\leq \sum_{i=1}^N \left[-\delta_i^T \gamma_i |\bar{d}_i|^{-1} \Lambda_i \delta_i + \|\delta_i\|^2 + \frac{\|\gamma_i |\bar{d}_i|^{-1} \bar{a}_i \dot{\mathbf{X}}\|^2}{2} \right. \\ &\quad \left. + \frac{\|\xi_i f(X_i)\|^2}{2} \right] \end{aligned} \quad (29)$$

Let us design the gain Λ_i as follows.

$$\Lambda_i = |\bar{d}_i|^{-1} \gamma_i^{-1} \left(\mathbf{I} + \frac{\alpha_i}{2} \lambda_{\max}(\Lambda) + \frac{\|\xi_i f(X_i)\|^2}{2 \|\delta_i\|^2} \right) \quad (30)$$

where $\alpha_i > 0$. Eq. (29) is written as

$$\begin{aligned} \dot{\Pi} &\leq \sum_{i=1}^N -\frac{\alpha_i}{2} \lambda_{\max}(\Psi) \|\delta_i\|^2 + \sum_{i=1}^N \frac{\|\gamma_i |\bar{d}_i|^{-1} \bar{a}_i \dot{\mathbf{X}}\|^2}{2} \\ &\leq -\alpha_i \Pi + \beta \end{aligned} \quad (31)$$

where, $\beta = \sum_{i=1}^N \frac{\|\gamma_i |\bar{d}_i|^{-1} \bar{a}_i \dot{\mathbf{X}}\|^2}{2}$. Applying *Lemma 5* we get

$$\Pi \leq \frac{\beta}{\alpha_i} + \left(\Pi(0) - \frac{\beta}{\alpha_i} \right) e^{-\alpha_i t} \quad (32)$$

We will present the Uniformly Ultimate Boundedness (UUB) here. Using Eqs. (21) and (32) we can write

$$\frac{\lambda_{\min}(\Psi)}{2} \|\Delta\|^2 \leq \Pi \leq \frac{\beta}{\alpha_i} + \left(\Pi(0) - \frac{\beta}{\alpha_i} \right) e^{-\alpha_i t} \quad (33)$$

Eq. (33) can be written as follows.

$$\begin{aligned} \frac{\lambda_{\min}(\Psi)}{2} \|\Delta\|^2 &\leq \frac{\beta}{\alpha_i} + \left(\Pi(0) - \frac{\beta}{\alpha_i} \right) e^{-\alpha_i t} \\ \|\Delta\| &\leq \sqrt{\frac{2 \frac{\beta}{\alpha_i} + 2 \left(\Pi(0) - \frac{\beta}{\alpha_i} \right) e^{-\alpha_i t}}{\lambda_{\min}(\Psi)}} \end{aligned} \quad (34)$$

It can be observed that, if $\Pi(0) = \frac{\beta}{\alpha_i}$ then $\|\Delta\| \leq \mu^*$; $\forall t \geq 0$ and $\mu^* = \sqrt{\frac{2\beta}{\alpha_i \lambda_{\min}(\Delta)}}$. If $\Pi(0) \neq \frac{\beta}{\alpha_i}$ then for any given $\mu > \mu^*$ there exist a time $\tau > 0$ such that $\forall t > \tau$, $\|\Delta\| \leq \mu$.

$$\mu = \sqrt{\frac{2 \frac{\beta}{\alpha_i} + 2 \left(\Pi(0) - \frac{\beta}{\alpha_i} \right) e^{-\alpha_i \tau}}{\lambda_{\min}(\Psi)}} \quad (35)$$

Hence, we can conclude $\lim_{t \rightarrow \infty} \|\Delta\| = \mu^*$. The factor β incorporates the term γ_i , highlighting the impact of the fault.

V. SIMULATION STUDY

A. Communication Topology

The adjacency matrix associated to the signed graph is given in Eq. (36).

$$\mathcal{A} = \begin{bmatrix} 0 & 4 & 0 & -5 & 0 & 1 \\ 4 & 0 & -4 & 0 & 0 & 1 \\ 0 & -4 & 0 & .5 & 0 & 0 \\ -5 & 0 & .5 & 0 & -3.5 & 0 \\ 0 & 0 & 0 & -3.5 & 0 & 2 \\ 1 & 1 & 0 & 0 & 2 & 0 \end{bmatrix} \quad (36)$$

From the adjacency matrix, we can discern that it represents an undirected signed graph. Upon verification, one eigenvalue emerges as zero, while the others possess positive real parts. Consequently, this graph features a spanning tree and exhibits structural balance (according to Corollary 1).

B. Agent Dynamics Considered for Simulation Study

The agent dynamics considered is same as given by [14].

$$\dot{X}_{i1} = X_{i2} \sin(2X_{i1}) + U_{i1} \quad (37)$$

$$\dot{X}_{i2} = X_{i1} \cos(3X_{i2}) + U_{i2} \quad (38)$$

where $X_i = [X_{i1} \ X_{i2}]^T \in \mathbb{R}^2$. The states X_{1i} of all the agents are given by $\mathbf{X}_1 = [X_{11} \ X_{21} \ \dots \ X_{61}]$. Similarly, we denote $\mathbf{X}_2 = [X_{12} \ X_{22} \ \dots \ X_{62}]$, $\mathbf{U}_1 = [U_{11} \ U_{21} \ \dots \ U_{61}]$, and $\mathbf{U}_2 = [U_{12} \ U_{22} \ \dots \ U_{62}]$. The errors in $\mathbf{X}_1$ and $\mathbf{X}_2$ is given by $\mathbf{e}_1$ in $\mathbf{X}_1$ and $\mathbf{e}_2$ in $\mathbf{X}_2$ respectively.

C. Faults in Selected Actuator and Start at Different Time

We have presented the effect of faults which start at a different time in the selected actuators. The faults are assumed to be random, and they are generated using the MATLAB function $\text{rand}() \in (0, 1)$, i.e., $\gamma_i = \begin{bmatrix} \text{rand} & 0 \\ 0 & \text{rand} \end{bmatrix}$. The details of the fault are given in Table I. The control signals

TABLE I
ACTUATOR FAULT DETAILS.

Agents	Actuator	time of start(sec)
1	1, 2	0.07
2	1	0.21
3	1	0.05
4	1, 2	0.12
5	1	0.18
6	2	0.15

corresponding to the agents, denoted as $\mathbf{U}_1$ and $\mathbf{U}_2$, are

depicted in Figs. 1 and 2, respectively. As shown in these figures, the faults in the actuators 1 and 2 commence at the times specified in the table. For enhanced clarity, the figures are magnified to distinctly represent the time. The bipartite

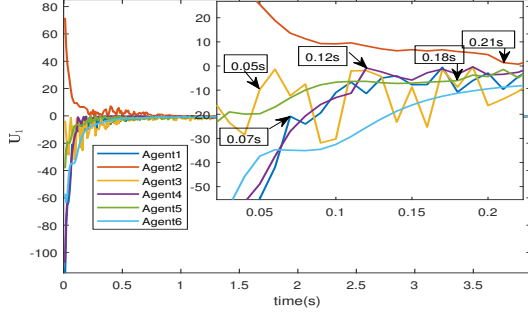


Fig. 1. Control U_1 . Fault in actuator 1.

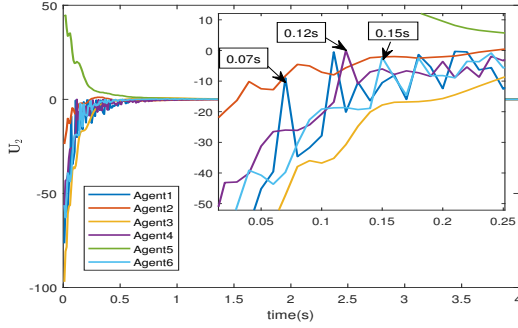


Fig. 2. Control U_2 . Fault in actuator 2.

consensus of states X_1 and X_2 are shown in Figs. 3 and 4, respectively. From these figures, it's evident that agents 3, 4 reached a consensus value that, while identical in magnitude, holds an opposite sign compared to agents 1, 2, 5, and 6. The

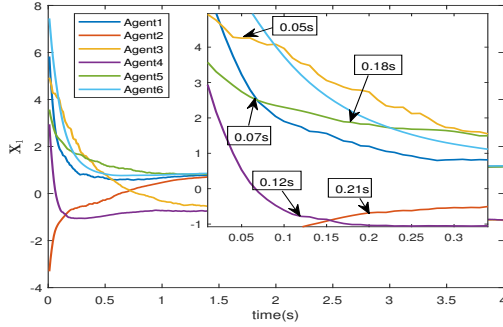


Fig. 3. Consensus of state X_1 .

influence of the faults can be seen in the state trajectories but that the bipartite consensus is precisely attained in a finite time frame using the DNDI controller.

D. Fault Starts at Same Time in All Actuators

In this case, we have presented the effect of faults which start at the same time in all the actuators. This can be

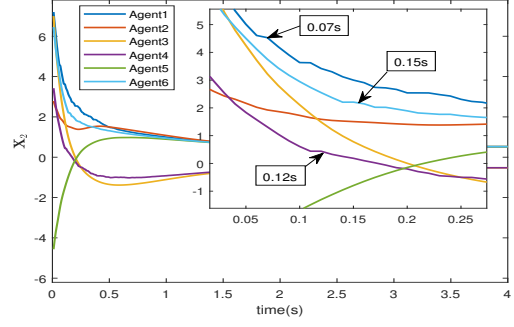


Fig. 4. Consensus of state X_2 .

regarded as the worst case scenario. In this simulation study,

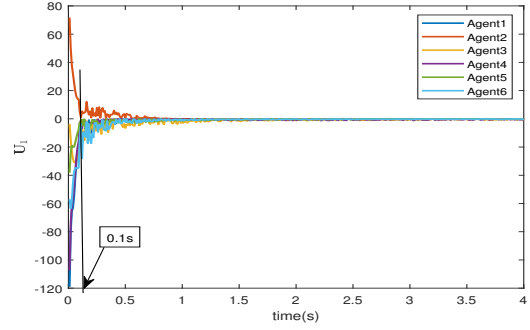


Fig. 5. Control U_1 .

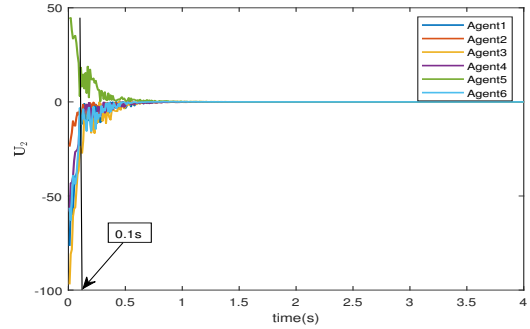


Fig. 6. Control U_2 .

we have considered the faults start at $t = 0.1s$ in both the actuators as shown in Figs. 5 and 6, receptively. The states X_1 and X_2 are shown in Figs. 7 and 8, respectively. We can note that there's a bipartite consensus among the states. Even though the malfunctioning actuator has an impact on the states, the introduced controller aids the agents in reaching consensus despite the actuator's fault.

VI. CONCLUSION

The study examines the bipartite consensus among nonlinear agents when there is an actuator malfunction. DNDI has been demonstrated to be effective when faced with actuator issues. Additionally, the fault is assumed to be random to

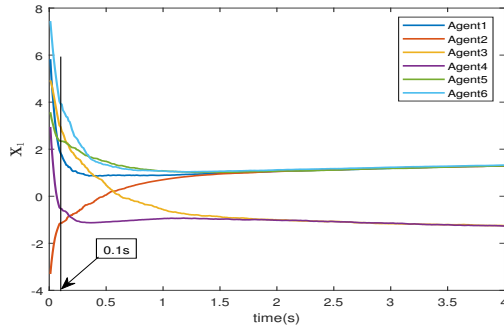


Fig. 7. Consensus of state X_1 .

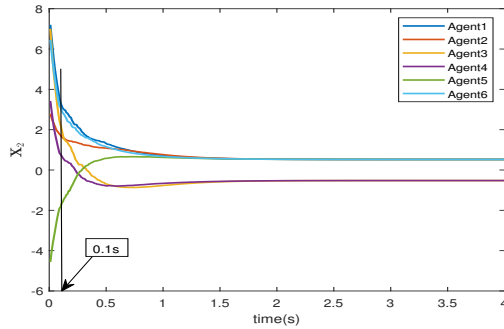


Fig. 8. Consensus of state X_2 .

reflect a more authentic situation. This paper's mathematical evidence underscores its robustness, and the produced results reinforce this assertion. A practical simulation is offered to gauge the efficacy of the proposed controller. Thus, DNDI emerges as a potent solution for addressing the bipartite consensus challenges of nonlinear agents.

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