

System Requirements Analysis for JAXA's Contribution to Comet Interceptor Mission: Attitude Control for a Hyperbolic Comet Fly-by

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Abstract

Comet Interceptor, to be launched in 2028, is a recently-selected ESA/JAXA mission aimed to perform the first fly-by of a pristine long-period comet. As part of the mission's Phases A/B (feasibility analysis and preliminary design), this work preliminarily analyzes the attitude performance of JAXA's contribution to the mission, SC B1: one of the two small-spacecraft piggybacked along with ESA's main spacecraft. This study primarily focuses on the characterization of the dust environment, modelization of dust particle impacts, and analysis of attitude performance through the dust environment. Monte Carlo simulations are implemented to assess the performance of the attitude control system within a highly-active dust environment, for various fly-by altitudes and a worst-case 70-km/s fly-by speed. Results evidence the need for a wide-angle camera, and conclude that image acquisition shall tolerate angular velocities of several degrees per second. Solutions for improved attitude performance are also evaluated (i.e., upgrading of reaction wheels and angular momentum pre-loading), and a larger reaction wheels aligned with the camera line of sight is shown to provide largest performance improvements. Such strategies, nevertheless, still result in pointing errors and angular velocities that are only compatible with a short-exposure, wide-angle science camera.

Keywords: Comet Interceptor; comet fly-by; attitude control; system requirements

1. Introduction

Comet Interceptor [1] is a recently-selected fast-class ESA mission, to be launched along the ARIEL mission in 2028. Comet Interceptor is an ~800-kg mission aimed to fly-by a pristine long-period comet that is entering the inner Solar System for the first time, and is composed of a main spacecraft (SC A), and two small probes (SC B1 and SC B2). In particular, 30-kg SC B1 will be developed by JAXA and is the focus of this study.

A main challenge to the mission is the nature of its target object: a fast-moving, dynamically-new comet of uncertain characteristics and highly-active dust environment. Since the target comet will not be discovered until the mission is launched (i.e., until the comet first enters the inner Solar System), the design and development phases of the mission shall be performed under wide estimations of comet characteristics: e.g., fly-by speed and dust environment.

This study aims to preliminarily characterize the attitude performance of SC B1 as it crosses the newly-active dust environment of the target comet. As a first step, an analytical model to describe the dust environment is proposed (e.g., number, mass, and distribution of encountered particles along the

trajectory). Then, a simple quaternion-feedback control law is employed to characterize the effect of particle impacts on attitude performance (depending on particle mass and fly-by altitude). Observations and recommendations are finally made in terms of expected attitude performance, advisable fly-by altitudes, selection of attitude control gains, and sizing and pre-loading of reaction wheels.

The mission scenario is introduced in Section 2, modelization of the dust environment is described in Section 3, attitude dynamics and control are defined in Section 4, and characterization of attitude performance through the dust environment is included Section 5, along with discussion and recommendations for SC B1.

2. Comet fly-by scenario and main assumptions

Since comet characteristics are *a priori* unknown, several worst-case assumptions are made in this study (according to Comet Interceptor Environmental Specifications [2]). The comet's dust production rate, for instance, is assumed equal to $Q_d = 33,000 \text{ kg/s}$, and a relative spacecraft-comet fly-by speed of $v_{sc} = 70 \text{ km/s}$ is considered.

Given the large fly-by relative speeds and consequent short fly-by durations, the relative spacecraft-comet trajectory, $[\mathbf{r}_{SC}^T, \mathbf{v}_{SC}^T]^T$, is simplified here to a straight line of constant speed (i.e., uniform rectilinear fly-by trajectory, no gravitational effects): $\frac{d\mathbf{r}_{SC}}{dt} = \mathbf{v}_{SC}$ and $\frac{d\mathbf{v}_{SC}}{dt} = \mathbf{0}$. The effect of particle impacts on the relative velocity $\mathbf{v}_{SC}$ is neglected as well, and justified in Section 4.2. Such assumptions allow to define an instantaneously inertial reference frame centered at the comet, with respect to which the relative spacecraft-comet trajectory can be described (Fig. 1).

A range of possible fly-by altitudes h_f for SC B1 are assessed in this study: from 250 km to 625 km, and the capability of the spacecraft to maintain a constant inertially-fixed orientation along the fly-by is evaluated [1]. The desired three-axis stabilized orientation of the spacecraft, $\mathbf{q}_c = \cos\left(\frac{90^\circ}{2}\right) + \sin\left(\frac{90^\circ}{2}\right)\hat{\mathbf{z}}$ (in quaternion form), corresponds to a 90° rotation about the inertial z-axis: i.e., camera pointing towards the comet (at closest approach), and body-fixed x-axis $\hat{\mathbf{x}}^B$ along the relative spacecraft-comet motion (Fig. 1).

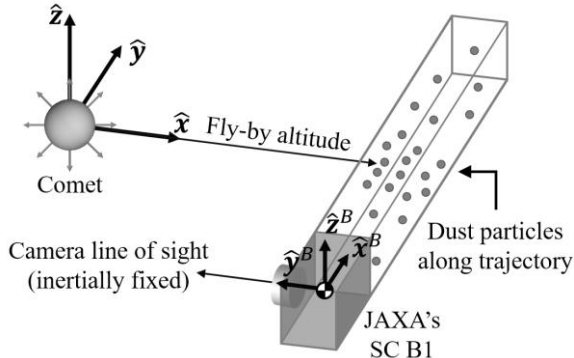


Fig. 1. Representation of encountered dust environment and desired orientation

The attitude requirements have also been established as $\varepsilon_{req} \leq 0.5^\circ$ in pointing error, and $\omega_{req} \leq 0.4 \text{ deg/s}$ in angular velocity (for SC B1's narrow-angle camera). Moreover, ideal sensor-actuator performance is assumed in this preliminary study, and so the on-board computer is assumed to have perfect real-time knowledge of the spacecraft's attitude, and reaction wheels can exactly generate the commanded (continuous) control torque.

JAXA's SC B1 is further expected to weigh $m_{SC} = 30 \text{ kg}$ and have the dimensions of a 24U CubeSat: 40 cm x 30 cm x 20 cm [1]. The spacecraft is further assumed to have a homogeneous mass distribution and be equipped with three reaction wheels aligned with each body-fixed axis: Blue Canyon Technologies' RWP500 [3], with maximum torque $T_{w,lim} = 25 \text{ mN} \cdot \text{m}$ and maximum angular momentum $H_{w,lim} = 500 \text{ mN} \cdot \text{m} \cdot \text{s}$. Section 5.4 then discusses potential performance

improvements through the implementation of a larger reaction wheel aligned with the camera line of sight (body-fixed axis $\hat{\mathbf{y}}^B$).

3. Dust environment

In accordance with Comet Interceptor Environmental Specifications [2], a simplified isotropic point-source model is employed to describe the (time-independent) dust environment around the comet. Several are the inputs required to determine the number and mass of particles that will be encountered by the spacecraft, namely:

- Dust production rate, Q_d [kg/s].
- Mass distribution of ejected particles.
- Mass ejection speed, v_d [m/s].
- Fly-by altitude, h_f [m].
- Cross-sectional area of spacecraft, A_{SC} .

Comet characteristics are uncertain given the pristine-comet premise of the mission, and so comet-related parameters are partially based on results from the Rosetta mission [4], and on educated estimates derived by the Comet Interceptor science team [2]. Particularly, Q_d is set to 33,000 kg/s and v_d is set to 1 m/s (isotropic and equal for all particle masses). The mass distribution is further assumed equal to that of comet 67P Churyumov-Gerasimenko [5] and is summarized in Table 1.

Given the dimensions of the spacecraft, the largest cross-sectional area that could be exposed to the dust environment is computed as $A_{SC} = a \cdot d_{bc} = 40 \cdot \sqrt{30^2 + 20^2} \text{ cm}^2$ (Fig. 2).

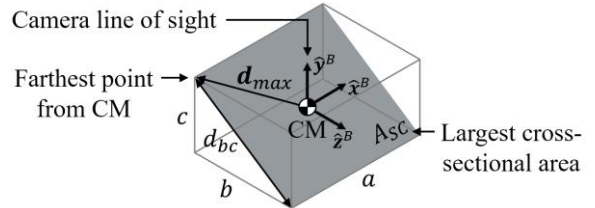


Fig. 2. Spacecraft dimensions

Leveraging the assumed isotropy of dust emission, this study proposes an analytical flux-based model to determine the average number of particles that will be encountered by the spacecraft. Such an analytical model has been verified against the model proposed in the Comet Interceptor Environmental Specifications (Table 1), which is based on the intersection of volumes between the trajectory of the spacecraft and concentric dust shells, and is also able to more intuitively illustrate the dependence of the dust environment on the various mission parameters.

Table 1. Assumed mass distribution and number of particles along 100-km trajectory (cross-section of 1-m radius)

Particle mass (kg)	Mass fraction (%) [5]	Number of particles along 100-km trajectory		Relative difference (%)
		Comet Interceptor Environmental Specifications [2]	Proposed analytical model	
10^1 – 10^2	45.36578	$1.07 \cdot 10^{-3}$	$1.10 \cdot 10^{-3}$	2.80
10^0 – 10^1	39.92189	$9.38 \cdot 10^{-3}$	$9.68 \cdot 10^{-3}$	3.20
10^{-1} – 10^0	9.980472	$2.35 \cdot 10^{-2}$	$2.42 \cdot 10^{-2}$	2.98
10^{-2} – 10^{-1}	3.130239	$7.36 \cdot 10^{-2}$	$7.59 \cdot 10^{-2}$	3.13
10^{-3} – 10^{-2}	0.449121	$1.06 \cdot 10^{-1}$	$1.09 \cdot 10^{-1}$	2.83
10^{-4} – 10^{-3}	0.032663	$7.68 \cdot 10^{-2}$	$7.92 \cdot 10^{-2}$	3.13
10^{-5} – 10^{-4}	0.045366	$1.07 \cdot 10^0$	$1.10 \cdot 10^0$	2.80
10^{-6} – 10^{-5}	0.045366	$1.07 \cdot 10^1$	$1.10 \cdot 10^1$	2.80
10^{-7} – 10^{-6}	0.313024	$7.36 \cdot 10^2$	$7.59 \cdot 10^2$	3.13
10^{-8} – 10^{-7}	0.544389	$1.28 \cdot 10^4$	$1.32 \cdot 10^4$	3.13
10^{-9} – 10^{-8}	0.108878	$2.56 \cdot 10^4$	$2.64 \cdot 10^4$	3.13
10^{-10} – 10^{-9}	0.039836	$9.36 \cdot 10^4$	$9.65 \cdot 10^4$	3.10
10^{-11} – 10^{-10}	0.014575	$3.43 \cdot 10^5$	$3.53 \cdot 10^5$	2.92
10^{-12} – 10^{-11}	0.005333	$1.25 \cdot 10^6$	$1.30 \cdot 10^6$	4.00
10^{-13} – 10^{-12}	0.001951	$4.59 \cdot 10^6$	$4.73 \cdot 10^6$	3.05
10^{-14} – 10^{-13}	0.000714	$1.68 \cdot 10^7$	$1.73 \cdot 10^7$	2.98
10^{-15} – 10^{-14}	0.000261	$6.14 \cdot 10^7$	$6.33 \cdot 10^7$	3.09
10^{-16} – 10^{-15}	0.000096	$2.25 \cdot 10^8$	$2.32 \cdot 10^8$	3.11
10^{-17} – 10^{-16}	0.000035	$8.22 \cdot 10^8$	$8.48 \cdot 10^8$	3.16
10^{-18} – 10^{-17}	0.000013	$3.01 \cdot 10^9$	$3.10 \cdot 10^9$	2.99

3.1. Proposed dust model

Assume a general configuration such as that in Fig. 3, where the comet (point source) is located at the origin of an (instantaneously) inertial coordinate system, and the spacecraft is moving along a straight line in the y -direction relative to the comet: $\mathbf{r}_{SC}(y_{SC}) = h_f \hat{\mathbf{x}} + y_{SC} \hat{\mathbf{y}}$.

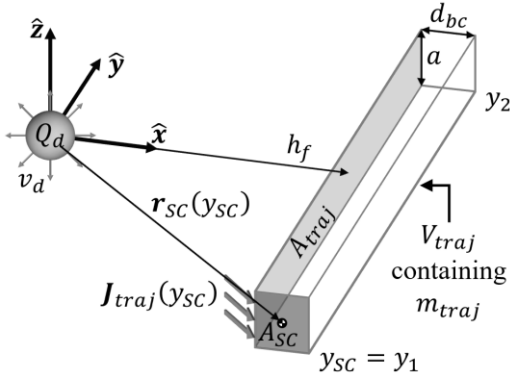


Fig. 3. Flux-based dust model along trajectory

The flux of an isotropic point source, $\mathbf{J}$ [kg/s/m²], is radial and inversely proportional to the distance from the source, r : $\mathbf{J}(r) = \frac{Q_d}{4\pi r^2} \hat{\mathbf{r}}$. However, given the small dimensions of the spacecraft (tens of cm) compared to fly-by altitudes (hundreds of km), the mass flux encountered by the spacecraft, $\mathbf{J}_{traj}$, can be assumed as

parallel, similar to sunlight reaching Earth (i.e., no z -component): $\mathbf{J}_{traj}(y_{SC}) \approx \frac{Q_d}{4\pi r_{SC}^2} \left(\frac{h_f}{r_{SC}} \hat{\mathbf{x}} + \frac{y_{SC}}{r_{SC}} \hat{\mathbf{y}} \right)$.

A conservative estimation of the number of particles that will be encountered by the spacecraft can be obtained by assuming its largest cross-sectional area A_{SC} is always perpendicular to its trajectory (this pessimistic number of particles is assumed throughout this study), and further consider its longest edge a is facing towards the comet ($-\hat{\mathbf{x}}$ -axis in Fig. 3). Between any two points y_1 and y_2 along the trajectory, the total volume spanned by the cross-sectional area of the spacecraft is: $V_{traj} = A_{SC} \cdot (y_2 - y_1) = a \cdot d_{bc} \cdot (y_2 - y_1)$, and the mass flow rate crossing volume V_{traj} can be computed as the integral from y_1 to y_2 of the flux entering through the lateral area of the trajectory A_{traj} (i.e., through edge a): $Q_{traj} = \iint_{A_{traj}} \mathbf{J}(r) \cdot d\mathbf{A}_{traj} \approx \int_{y_1}^{y_2} \mathbf{J}_{traj}(y_{SC}) \cdot (a \cdot dy_{SC}) \hat{\mathbf{x}}$.

Since the point source is modeled here as isotropic and time-independent, the steady-state, average mass contained within V_{traj} , m_{traj} , is similarly computed (Eq. (1)) as the integral of the flux entering through lateral area A_{traj} times the time it takes for the flux to travel through volume V_{traj} (i.e., to travel a distance d_{bc}):

$$t_{d_{bc}}(y_{SC}) = \frac{d_{bc}}{v_d \hat{\mathbf{x}}} = \frac{d_{bc}}{v_d \left(\frac{h_f}{r_{SC}} \hat{\mathbf{x}} + \frac{y_{SC}}{r_{SC}} \hat{\mathbf{y}} \right) \cdot \hat{\mathbf{x}}} = \frac{d_{bc} r_{SC}}{v_d h_f}.$$

$$\begin{aligned}
 m_{traj} &\approx \iint_{A_{traj}} t_{dbc}(y_{sc}) \cdot \mathbf{J}_{traj}(y_{sc}) \cdot d\mathbf{A}_{traj} \\
 &= \int_{y_1}^{y_2} \left(\frac{d_{bc} r_{sc}}{v_d h_f} \right) \cdot \frac{Q_d}{4\pi r_{sc}^2} \left(\frac{h_f}{r_{sc}} \hat{\mathbf{x}} + \frac{y_{sc}}{r_{sc}} \hat{\mathbf{y}} \right) \\
 &\quad \cdot (a \cdot dy_{sc}) \hat{\mathbf{x}} \\
 &= \int_{y_1}^{y_2} \left(\frac{Q_d a \cdot d_{bc}}{4\pi v_d} \right) \frac{dy_{sc}}{r_{sc}^2} \\
 &= \int_{y_1}^{y_2} \left(\frac{Q_d a \cdot d_{bc}}{4\pi v_d} \right) \frac{dy_{sc}}{h_f^2 + y_{sc}^2} \\
 &= \frac{Q_d A_{sc}}{v_d 4\pi h_f} \tan^{-1} \left(\frac{y_{sc}}{h_f} \right) \Big|_{y_1}^{y_2}
 \end{aligned} \tag{1}$$

Consequently, the mass encountered by the spacecraft along a segment $[y_1, y_2]$ can be analytically computed through Eq. (1). The dependence of the encountered mass on the different mission characteristics is intuitively illustrated by Eq. (1), and thus it shows that it is directly proportional to the mass production rate Q_d and to the cross-sectional area of the spacecraft A_{sc} , and it is inversely proportional to the fly-by altitude h_f and to the dust ejection speed v_d . Moreover, mass production parameters Q_d and v_d can also be substituted by those associated to a particular mass bin to determine the number of encountered particles corresponding to that given mass: i.e., scaling Q_d according to the mass fraction in Table 1, and v_d according to the associated ejection speed (although ejection speed is assumed equal for all masses in this study). It is worth noticing that the higher concentration of dust particles in close proximity to the comet is also captured by Eq. (1), where the derivative of the mass along the trajectory with respect to the distance from the comet, $\frac{dm_{traj}}{dr_{sc}(y_{sc})}$, follows a simple inverse square law: $\propto 1/r_{sc}^2 = 1/(h_f^2 + y_{sc}^2)$ (second-to-last expression in Eq. (1)).

Table 1 summarizes the mass distribution assumed in the Comet Interceptor Environmental Specifications [2] and in this study. For validation of the proposed analytical model, Table 1 also compares the expected number of particles along a 100-km fly-by trajectory as computed by Eq. (1) and by the dust shell model employed in the Comet Interceptor Environmental Specifications. Minor differences are observed (up to 4.0%) and can be attributed to the various assumptions employed by the two models.

3.2. Probability of particle impacts

Once a model is available to compute the expected number of particles that will be encountered by the spacecraft, the probability of particle impacts can be further represented by a Poisson probability distribution [6]. More specifically, a Poisson distribution can be employed to express the probability of encountering a

given number of particles along the trajectory, given the known, constant average number of particles expected along the trajectory.

The probability mass function of the discrete random variable N_p (number of particles) of known, constant average λ is given by Eq. (2) [6].

$$\Pr(N_p = n, \lambda) = \frac{\lambda^n e^{-\lambda}}{n!}, \quad \text{for } n = 0, 1, 2, \dots \tag{2}$$

where n are the discrete possible values for variable N_p . Such a probability function is employed in this study to randomize the number of particles encountered by the spacecraft in different Monte Carlo simulations (Section 5.1): i.e., each Monte Carlo simulation features a different dust environment.

4. Attitude dynamics and control

The attitude of the spacecraft is controlled in this study by three reaction wheels, whose axial moment of inertia is defined as $\mathbf{J}_w = \text{diag}([J_{w1}, J_{w2}, J_{w3}])$. The total moment of inertia of the spacecraft (including reactions wheels) can therefore be described as $\mathbf{J}_T = \mathbf{J}_{sc} + {}^w\mathbf{A}^B \mathbf{J}_w {}^w\mathbf{A}^{B^T}$, where $\mathbf{J}_{sc} = \frac{m_{sc}}{12} \cdot \text{diag}([b^2 + c^2, a^2 + b^2, a^2 + c^2])$ is the moment of inertia of the spacecraft only (assuming a homogeneous mass distribution and excluding reaction wheels—see Fig. 2), and ${}^w\mathbf{A}^B$ projects the spin axis of reaction wheels onto body-fixed reference frame B (see Fig. 2, where the y-axis is aligned with the camera's line of sight, through the largest face of the spacecraft).

4.1. Perturbance-free attitude dynamics

Following the nomenclature in [7], the attitude dynamics of the spacecraft in the absence of particle impacts is described by the equation of motion in Eq. (3).

$$\begin{aligned}
 \mathbf{J}_{sc} \dot{\boldsymbol{\omega}}^B &= -\boldsymbol{\omega}^B \times (\mathbf{J}_T \boldsymbol{\omega}^B + {}^w\mathbf{A}^B \mathbf{J}_w \boldsymbol{\Omega}_w) + \mathbf{T}_w^B \\
 &\quad + \mathbf{T}_E^B
 \end{aligned} \tag{3}$$

where superscript B denotes variables are represented in the body-fixed frame B, $\boldsymbol{\omega}$ is the angular velocity of the spacecraft relative to an inertial reference frame, $\boldsymbol{\Omega}_w = [\Omega_{w,x^B}, \Omega_{w,y^B}, \Omega_{w,z^B}]^T$ is the axial angular velocity of reaction wheels (assumed to be aligned with body-fixed axes: ${}^w\mathbf{A}^B = \mathbf{I}_{3 \times 3} \equiv \text{identity matrix}$), $\mathbf{T}_w$ is the torque generated by reaction wheels, and $\mathbf{T}_E$ is an external torque acting on the system (although no external torque is considered in this study: $\mathbf{T}_E = \mathbf{0}$).

The torque generated by reaction wheels $\mathbf{T}_w$ is selected as the control input in this study, which then defines the required angular acceleration of each reaction wheel (Eq. (4)).

$$\mathbf{T}_w = \dot{\mathbf{H}}_w = \mathbf{J}_w (\dot{\boldsymbol{\Omega}}_w + {}^w\mathbf{A}^B \dot{\boldsymbol{\omega}}^B) \tag{4}$$

where $\dot{\mathbf{H}}_w = [\dot{H}_{w,x^B}, \dot{H}_{w,y^B}, \dot{H}_{w,z^B}]$ is the derivative in axial angular momentum of reaction wheels. Note that, given the short fly-by durations (~100 seconds) and

uniformity of particle impacts, the maximum angular momentum of reaction wheels $H_{w,lim}$ (i.e., saturation) is not reached unless they are heavily loaded prior to the fly-by (e.g., $H_w > 0.45 \cdot H_{w,lim}$ —Section 5.3.3), or if less-common, large particles are encountered (e.g., 10^{-5} – 10^{-4} kg).

4.2. Attitude perturbation due to particle impacts

Particle impacts are modeled in this preliminary study as instantaneous changes in the angular velocity of the spacecraft. A conservative scenario is considered in which all impacts are modeled as specular (i.e., no diffusion) and normal to the impact surface. Additionally, due to the large relative spacecraft-comet fly-by speeds (~ 70 km/s), the ejection speed of particles (~ 1 m/s) is neglected when modeling particle impacts. As such, the relative spacecraft-particle speed, v_p , is assumed equal to the fly-by relative speed, v_{sc} , for all particle impacts.

Under such assumptions, conservation of angular momentum in the spacecraft-particle system can provide an expression for the change in spacecraft angular velocity (Eq. (5)), expressed relative to the spacecraft's center of mass CM). As such, the relative velocity of a particle of mass m_p before and after a specular and normal impact is equal in module but opposite in direction: $\mathbf{v}_p^{(-)} = -\mathbf{v}_p^{(+)} = -\mathbf{v}_{sc}$. And if the impact occurs at an inertial position $\mathbf{d}_p^{CM}$ relative to CM, the total angular momentum of the system prior to an impact is then $\mathbf{H}_{sc+p}^{(-)} = {}^B\mathbf{C}^I \cdot \mathbf{J}_{sc}\boldsymbol{\omega}^{B(-)} + m_p\mathbf{d}_p^{CM} \times \mathbf{v}_p^{(-)}$ (inertial coordinates), and $\mathbf{H}_{sc+p}^{(+)} = {}^B\mathbf{C}^I \cdot \mathbf{J}_{sc}\boldsymbol{\omega}^{B(+)} + m_p\mathbf{d}_p^{CM} \times \mathbf{v}_p^{(+)}$ after an impact, where ${}^B\mathbf{C}^I$ is the rotation matrix from body-fixed to inertial reference frame.

$$\Delta\boldsymbol{\omega}^B = 2 \cdot \mathbf{J}_{sc}^{-1} \cdot {}^B\mathbf{C}^I{}^T \cdot (m_p\mathbf{d}_p^{CM} \times -\mathbf{v}_{sc}) \quad (5)$$

where $\Delta\boldsymbol{\omega}^B = \boldsymbol{\omega}^{B(+)} - \boldsymbol{\omega}^{B(-)}$ is the change in spacecraft angular velocity through the (specular and normal) impact of a particle of mass m_p . Hence, attitude propagation is performed through the perturbation-free equation of motion (Eq. (3)) in the absence of particle impacts, and when a particle is encountered, the propagation is interrupted and re-initiated with modified angular velocity $\boldsymbol{\omega}^{B(+)} = \boldsymbol{\omega}^{B(-)} + \Delta\boldsymbol{\omega}^B$.

Note that the effect of particle impacts on the velocity of the spacecraft is neglected in this study. Similarly to Eq. (5), conservation of linear momentum before and after an impact leads to a change in spacecraft velocity $\Delta\mathbf{v} = -2 \cdot m_p \cdot \mathbf{v}_{sc}/m_{sc}$, which is several orders of magnitude smaller than v_{sc} since particle masses are much smaller than the mass of the spacecraft: $m_p \ll m_{sc}$. For reference, a 1-gram particle would cause a change in velocity more than 4 orders of magnitude

smaller than v_{sc} , and these impacts are highly unlikely as shown in Section 5.2. Whereas more common particles (e.g., 10^{-8} – 10^{-6} kg) result in changes in velocity 10 to 12 orders of magnitude smaller than v_{sc} , and only a few hundreds of them are expected to be encountered.

4.3. Attitude control

The goal of the attitude controller is to mitigate the perturbation of particle impacts and to stabilize the spacecraft about an inertially-fixed orientation. As a first-iteration study of the mission, a simple quaternion-feedback attitude control logic is employed here [8]. Even though more involved three-axis, large-angle attitude control strategies may be considered [9], a simple feedback controller allows to preliminarily assess the feasibility of the mission, to characterize the effect of particle impacts, and to evaluate the performance of reaction wheels.

Following the nomenclature in [8], consider a desired orientation $\mathbf{q}_c = [q_{c1}, q_{c2}, q_{c3}, q_{c4}]^T$ represented in quaternion form (relative to an inertial reference system). Assuming the spacecraft attitude $\mathbf{q}_{sc} = [q_1, q_2, q_3, q_4]^T$ and angular velocity $\boldsymbol{\omega}$ are perfectly known, the attitude error $\mathbf{q}_e = [q_{e1}, q_{e2}, q_{e3}, q_{e4}]^T = [\mathbf{q}_e^T, q_{e4}]^T$ with respect to the desired orientation can be computed as in Eq. (6). The quaternion $\mathbf{q}_e = \cos \frac{\varepsilon}{2} + \sin \frac{\varepsilon}{2} \boldsymbol{\varepsilon}$ effectively describes the shortest rotation between $\mathbf{q}_c$ and $\mathbf{q}_{sc}$: a rotation of angle ε (pointing error) about axis $\boldsymbol{\varepsilon}$. The required (linear) feedback controller can then be defined as in Eq. (7).

$$\begin{bmatrix} q_{e1} \\ q_{e2} \\ q_{e3} \\ q_{e4} \end{bmatrix} = \begin{bmatrix} q_{c4} & q_{c3} & -q_{c2} & -q_{c1} \\ -q_{c3} & q_{c4} & q_{c1} & -q_{c2} \\ q_{c2} & -q_{c1} & q_{c4} & -q_{c3} \\ q_{c1} & q_{c2} & q_{c3} & q_{c4} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \quad (6)$$

$$\mathbf{T}_w^B = -\mathbf{K}\mathbf{q}_e - \mathbf{C}\boldsymbol{\omega}^B \quad (7)$$

where $\mathbf{T}_w^B$ is the torque required from reaction wheels in the body-fixed reference frame, and $\mathbf{K}$ and $\mathbf{C}$ are the proportional and derivative gains of the controller. Several gain selections can achieve global asymptotic stability [8], and the definition in Eq. (8) is adopted in this study.

$$\mathbf{K} = \text{sng}(q_{e4}) \cdot k \cdot \mathbf{I}, \quad \mathbf{C} = c \cdot \mathbf{I} \quad (8)$$

where $\text{sng}(\cdot)$ is the signum function and selects the shorter angular path towards the desired orientation, $\mathbf{I}$ is a 3x3 identity matrix, and k and c are the two positive scalar constants to be selected in Section 5.5. Also, note that if the required torque from any one reaction wheel (Eq. (7)) exceeds the maximum allowable torque $T_{w,lim}$, the magnitude of the commanded torque (Eq. (4)) for that particular wheel is set equal to $T_{w,lim}$.

5. Analysis and recommendations

Monte Carlo simulations are performed to analyze the behavior of the attitude control system through the dust environment. A different dust environment is

generated for each Monte Carlo case, according to the average number of particles expected along the trajectory (Section 3.1). Analysis of multiple randomized dust environments can serve as a first approach to characterize the expected behavior of the spacecraft's attitude control system as it crosses the widely uncertain environment to be encountered in the actual mission.

5.1. Setup of Monte Carlo analysis

As a first step, Eq. (1) is used to calculate the expected average mass along several segments $[y_{1,i}, y_{2,i}]$ along the trajectory: $m_{traj,i}$. In particular, 25 segments of equal angular length from $y_{1,1} = h_f \tan(-85^\circ)$ to $y_{2,25} = h_f \tan(85^\circ)$ are considered, where the angle $\pm 85^\circ$ is taken as an approximation for $y_{SC} = \pm \infty$. Discretization of the trajectory into several segments i allows to capture the dependence of dust environment density on the distance from the comet: i.e., more particles will be encountered close to the comet.

The mass distribution in Table 1 (mass fraction column) can then be used to proportionally compute the mass associated to particles corresponding to mass bin m_b within segment i : $m_{traj,i,m_b} = m_{traj,i} \cdot \alpha_{m_b}$, where α_{m_b} is the mass fraction associated to mass bin m_b . In particular, only particles contained in mass bins $[10^{-9}-10^{-8}, \dots, 10^{-5}-10^{-4}]$ kg are considered in this analysis, since smaller particles have a less significant effect on the attitude control system (even though they are more numerous—shown in Section 5.3.2), and the probability of encountering larger particles falls beyond a 3 σ confidence interval (Section 5.2). Larger particles are also expected to cause critical damage on the system.

Then, it is further assumed that all particles within mass bin m_b are characterized by the same representative mass m_r (half of the upper limit of mass bin m_b): e.g., mass bin $10^{-6}-10^{-5}$ is represented by particles of mass $0.5 \cdot 10^{-5}$ kg. The expected average number of particles associated to mass bin m_b within segment i can then be computed as $\lambda_{i,m_r} = m_{traj,i,m_b}/m_r = m_{traj,i} \cdot \alpha_{m_b}/m_r$. And the Poisson mass distribution in Eq. (2) can be used to compute the probability of encountering n number of particles within mass bin m_b in segment i : $\Pr(n, \lambda_{i,m_r})$. In each Monte Carlo simulation, a random number in the interval $[0, 1]$ is generated, and the cumulative Poisson distribution is then employed to determine the specific number of particles within mass bin m_b in segment i to be considered in that Monte Carlo case: N_{p,i,m_r} . Note that the cumulative Poisson distribution is defined as the sum of Eq. (2) for $n = 0, 1, 2, \dots, N_p$.

Once the number of particles N_{p,i,m_r} is computed, $3 \cdot N_{p,i,m_r}$ more random numbers are generated in the intervals $[y_{1,i}, y_{2,i}]$, $[-a/2, a/2]$ and $[-d_{bc}/2, d_{bc}/2]$ to determine the y -coordinate of each particle

corresponding to mass bin m_b within segment i , and the impact position of each particle on the cross-sectional area A_{SC} : $\mathbf{d}_p^{CM}$ in Eq. (5). Recall that, as a conservative approximation of the problem, it is assumed that the largest cross-sectional area A_{SC} is always perpendicular to the trajectory, and impacts are modeled as normal to this fictitious impact surface.

The initial conditions of the spacecraft are set such that its attitude is aligned with the desired orientation, $\mathbf{q}_{SC} = \mathbf{q}_c$, and with zero angular velocity: $\boldsymbol{\omega} = \mathbf{0}$. Since no external torque is acting on the system, such an orientation is maintained until the time of the first particle impact. The initial axial angular momentum of reaction wheels is also set equal to zero: $\mathbf{H}_w = \mathbf{0}$, with control gains $k = 1.5 \text{ N} \cdot \text{m}/\text{rad}$ and $c = 2.4 \text{ N} \cdot \text{m} \cdot \text{s}/\text{rad}$ (selected in Section 5.5). Note, however, that improvements in attitude performance can be obtained if the reaction wheel aligned with the camera line of sight has non-zero initial axial angular momentum. This is further explored in Section 5.4.

Lastly, given the assumed constant velocity of the spacecraft $\mathbf{v}_{SC} = [0, v_{SC}, 0]$ throughout the trajectory, the time of impact of each particle is simply determined by its y -coordinate, and so an impact occurs when a particle and the center of mass of the spacecraft are both contained in A_{SC} . Then, the resulting change in angular velocity $\Delta \boldsymbol{\omega}$ can be computed based on the impact point $\mathbf{d}_p^{CM}$ (Eq. (5)).

5.2. Characterization of dust environment

The proposed analytical dust model (Section 3.1) is able to provide an intuitive representation of the dust environment. Consider, for instance, the probability of encountering dust particles of mass $10^{-8}-10^{-7}$ kg at a fly-by altitude $h_f = 250 \text{ km}$. According to Eq. (1), with parameters $Q_d = 33,000 \text{ kg/s}$, $v_d = 1 \text{ m/s}$, $A_{SC} = 40 \cdot \sqrt{30^2 + 20^2} \text{ cm}^2$, and extremes of the trajectory $y_1 = h_f \tan(-85^\circ)$ and $y_2 = h_f \tan(85^\circ)$, the total mass encountered along the trajectory is $m_{traj} = 4.5 \text{ g}$. Based on the mass distribution in Table 1, the fraction of dust corresponding to $10^{-8}-10^{-7}$ -kg particles is $\alpha_{10^{-8}-10^{-7}} = 0.544389\%$, and so an average of $\lambda_{0.5 \cdot 10^{-7}} = m_{traj} \cdot \alpha_{10^{-8}-10^{-7}}/(0.5 \cdot 10^{-7} \text{ kg}) = 489.397$ particles of representative mass $0.5 \cdot 10^{-7} \text{ kg}$ are expected along the trajectory.

Such an average number of particles is proportional to Q_d and A_{SC} , and inversely proportional to v_d and h_f , and so, for instance, for a fly-by altitude $h_f = 500 \text{ km}$, the average number of particles to be encountered is simply half of that at $h_f = 250 \text{ km}$: $489.397/2 = 244.698$. Furthermore, once the average number of particles $\lambda_{0.5 \cdot 10^{-7}}$ is known, the Poisson mass distribution in Eq. (2) can describe the probability of encountering any discrete number of particles along the

trajectory. Interestingly, for large λ values (above ~ 10), a Poisson distribution can be approximated by the (more intuitive) normal distribution, with mean and variance $\sigma_{0.5 \cdot 10^{-7}}^2$ both equal to $\lambda_{0.5 \cdot 10^{-7}}$ [6].

As an example, Fig. 4 illustrates the 3σ probability interval of encountering a given number of particles of representative mass $0.5 \cdot 10^{-7}$ kg along a 250-km fly-by trajectory. It can be further observed that the distribution is centered about the average number $\lambda_{0.5 \cdot 10^{-7}} = 489.397$, and decays beyond 3σ levels outside of the interval $\lambda_{0.5 \cdot 10^{-7}} \pm 3\sigma_{0.5 \cdot 10^{-7}} = [423, 556]$ (rounded to the nearest integer). And, once again, the proposed analytical representation of the dust environment allows to straightforwardly relate these results to alternative mission scenarios. If, for instance, the fly-by altitude is 500 km instead of 250 km, the probability of encountering particles of mass $0.5 \cdot 10^{-7}$ kg would then be centered about the average number $489.397/2 = 244.698$, and the 3σ interval can be approximated as $244.698 \pm 3\sqrt{244.698} = [198, 292]$. The same interval of particles would be obtained if v_d is doubled, or if Q_d or A_{SC} are halved compared to the values established in Section 3.

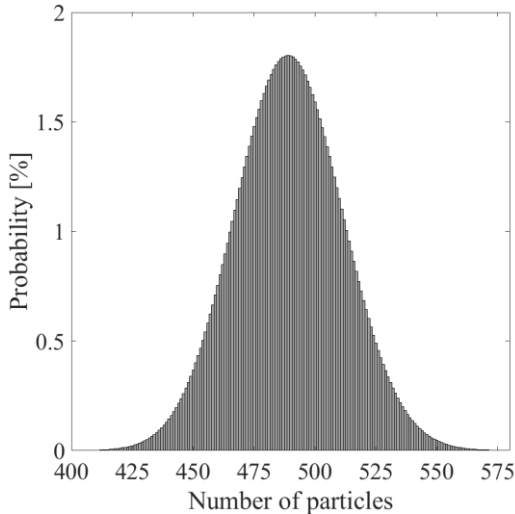


Fig. 4. Probability of encountering a given number of particles of mass $0.5 \cdot 10^{-7}$ kg along 250-km fly-by

A last noteworthy consideration is determining the mass of the largest particle to be encountered along the trajectory. For a known average number of particles of representative mass m_r , $\lambda_{m_r} = m_{traj} \cdot \alpha_{m_b}/m_r$, the 3σ upper limit to the number of encountered particles can be computed by inverting the cumulative Poisson distribution, for a cumulative probability of 99.7%. Such an operation is summarized in Table 2 for particles in mass bins $[10^{-11}-10^{-10}, \dots, 10^{-4}-10^{-3}]$ kg and a fly-by altitude of 250 km ($m_{traj} = 4.5$ g).

It is thus clear that the probability of encountering any particle larger than $10^{-4}-10^{-3}$ kg falls beyond a 3σ

confidence interval for a 250-km fly-by (and higher). Contrarily, smaller particles are more commonly encountered, since the ratio of mass fraction over particle mass increases as particle mass decreases: α_{m_b}/m_r (second column in Table 1 over the representative masses). Smaller particles than $10^{-9}-10^{-8}$ kg, however, have a less significant effect on the pointing accuracy even though they are more numerous (Section 5.3.2).

Table 2. 3σ upper limit to number of particles along 250-km fly-by

Particle mass (kg)	Average number of particles, λ_{m_r}	3σ upper limit to number of particles
$10^{-4}-10^{-3}$	0.00294	0
$10^{-5}-10^{-4}$	0.04083	1
$10^{-6}-10^{-5}$	0.40783	3
$10^{-7}-10^{-6}$	28.1404	44
$10^{-8}-10^{-7}$	489.397	551
$10^{-9}-10^{-8}$	978.795	1066
$10^{-10}-10^{-9}$	3580.76	3746
$10^{-11}-10^{-10}$	13091.4	13407

5.3. Characterization of particle impacts

As an initial step to characterize the behavior of the attitude control system along fly-by trajectories, it is worth analyzing first the effect of single particle impacts. Such a simple analysis can already provide insight into the largest impact that can be tolerated by the attitude control system, and it can also show what particle masses have no noticeable effect on the attitude of the spacecraft.

5.3.1. Effect of single particle impacts

When evaluating the effect of single particle impacts, a worst-case scenario is considered in which the impact occurs at the farthest possible point from the center of mass of the spacecraft: $\mathbf{d}_{max}$ in Fig. 2. Such an impact will generate the largest possible perturbation in angular velocity $\Delta\omega$ for a given particle mass m_p (Eq. (5)).

In this analysis, a single particle of mass m_p is assumed to be encountered at the point of closest approach: $y_{SC} = 0$, with the spacecraft initially aligned with the desired orientation $\mathbf{q}_c$. As such, the inertial position of the impacting particle relative to CM can be described as: $\mathbf{d}_p^{CM} = [\pm d_{bc}/2, 0, \pm a/2]$ at the time of impact. Notice that, under these circumstances, the specific quadrant of $\mathbf{d}_p^{CM}$ does not affect the resulting attitude behavior due to the symmetry of the problem.

As an initial example, Fig. 5 illustrates the magnitude of the angular velocity ω and the pointing error $\varepsilon = |2 \cdot \tan^{-1}([q_{e1}, q_{e2}, q_{e3}]/q_{e4})|$ as a function of time, for an impact particle of mass $0.5 \cdot 10^{-6}$ kg. As expected, the pointing error is maintained at zero until the time of impact, since $\mathbf{q}_{SC} = \mathbf{q}_c$ initially. After impact, the pointing error rapidly increases due to the sudden change

in angular velocity $\Delta\omega = 1.93 \text{ deg/s}$, which is then corrected by the controller. In particular, the pointing error reaches a maximum of $\varepsilon_{max} = 0.39^\circ$ prior to being mitigated by the controller, and the maximum angular velocity, $\omega_{max} = \Delta\omega$, is registered at the time of impact.

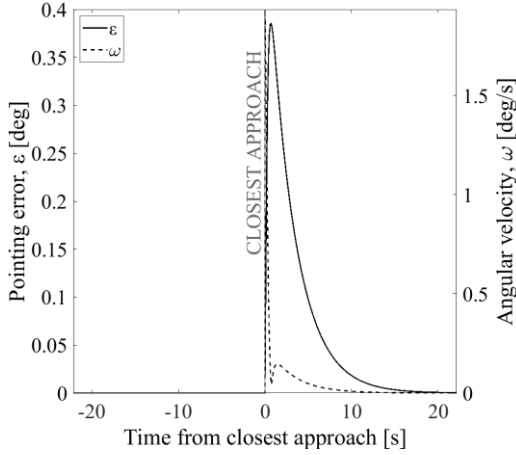


Fig. 5. Pointing error due to impact particle of mass $0.5 \cdot 10^{-6}$ kg at point of closest approach

In an effort to characterize the effect of single particle impacts as a function of mass, similar simulations to that in Fig. 5 are performed for particles of mass 10^{-11} – 10^{-4} kg. The maximum pointing error ε_{max} and maximum angular velocity ω_{max} resulting from each simulation is illustrated in Fig. 6.

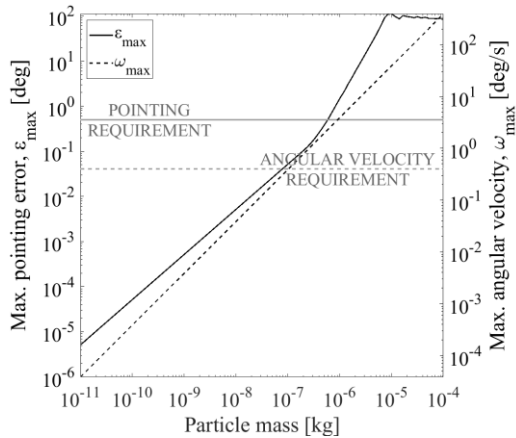


Fig. 6. Maximum pointing error and angular velocity as a function of particle mass

Through Fig. 6 it is already clear that the pointing requirement is exceeded if particles larger than 10^{-7} – 10^{-6} kg are impacted (under the worst-case assumption of an impact at distance d_{max} from the center of mass). Notice also the change in slope of ε_{max} at 10^{-7} – 10^{-6} kg, which corresponds to torque requirements exceeding the capability of the reaction wheels. The behavior of ε_{max} also changes above $\sim 1 \cdot 10^{-5}$ kg, corresponding to a

tumbling spacecraft with rather unpredictable pointing performance. Moreover, the dependency of ω_{max} on particle mass is seen as linear (in a logarithmic scale), which can already be inferred from Eq. (5). Notice that this maximum angular velocity is registered at the time of impact, as shown in Fig. 5. It is lastly observed that the angular velocity requirement is exceeded for particle masses above $\sim 1 \cdot 10^{-7}$ kg.

In summary, it is shown that the attitude of the spacecraft is highly sensitive to particle impacts (due to high impact velocities), even for particle masses in the 10^{-7} – 10^{-6} kg range, which are common along a 250-km fly-by (Table 2). Higher fly-by altitudes are hence advisable to reduce the probability of encountering large particles. Moreover, single impacts of smaller particles (10^{-11} – 10^{-7} kg) generate errors within the requirements of the mission, but it shall still be evaluated if multiple impacts of these (more numerous) smaller particles can result in significant attitude errors (Section 5.3.2).

5.3.2. Effect of multiple impacts of same mass

The effect of multiple impacts of the same mass is characterized here by considering particles of only one mass bin at a time. A worst-case dust environment is considered in which the number of particles is set equal to the 3σ upper limit of the corresponding mass bin (last column in Table 2). Particles are distributed according to the setup in Section 5.1, and so impacts may occur anywhere on the largest cross-sectional area A_{SC} of the spacecraft. A fly-by altitude of 250 km is also assumed here, since it allows to establish an upper limit to the effect of multiple impacts of particles of the same mass.

Quantities such as mean and maximum pointing error, ε_{mean} and ε_{max} , and mean and maximum angular velocity, ω_{mean} and ω_{max} , are stored from these simulations. As an example, Fig. 7 illustrates the effect of 10^{-7} – 10^{-6} -kg particle impacts on the pointing error ε along a 250-km fly-by (dotted vertical lines indicate the time of impacts).

From Fig. 7, it can be observed that the pointing error ε due to particles of mass 10^{-7} – 10^{-6} kg is almost three times larger than the pointing requirement ε_{req} (under worst-case dust environment), even though single impacts of these particles can only generate errors of up to 0.39° (Fig. 6). The pointing error is in fact maintained under ε_{req} until a few seconds before closest approach, at which point, the higher concentration of particles in close proximity to the comet perturbs the attitude beyond ε_{req} .

In an effort to characterize the effect of multiple impacts of particles of the same mass, Table 3 summarizes the mean and maximum errors due to particles of a given mass bin, and effectively establishes an upper limit to the errors that can be expected along the trajectory.

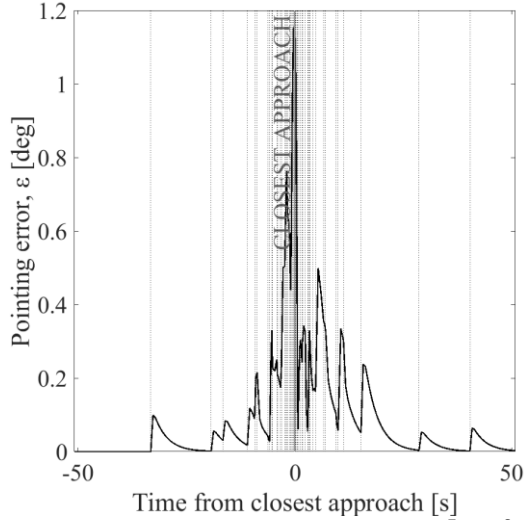


Fig. 7. Pointing error due to impacts of 10^{-7} – 10^{-6} -kg particles only (250-km fly-by, worst-case dust environment)

Table 3. Attitude error due to multiple impacts of same mass (250-km fly-by, worst-case dust environment)

Particle mass (kg)	ε_{mean} (deg)	ε_{max} (deg)	ω_{mean} (deg/s)	ω_{max} (deg/s)
10^{-5} – 10^{-4}	43.16	75.06	68.17	155.70
10^{-6} – 10^{-5}	1.147	10.77	1.312	16.54
10^{-7} – 10^{-6}	0.212	1.157	0.433	3.477
10^{-8} – 10^{-7}	0.092	0.271	0.182	0.634
10^{-9} – 10^{-8}	0.018	0.063	0.024	0.082
10^{-10} – 10^{-9}	0.002	0.006	0.004	0.016
10^{-11} – 10^{-10}	$3 \cdot 10^{-4}$	0.001	0.001	0.004

Various observations can be drawn from Table 3, which allow to characterize the isolated effect of particles of same mass along fly-by trajectories, namely:

- Particles of mass 10^{-5} – 10^{-4} kg can result in angular velocities above a hundred degrees per second, which effectively correspond to a tumbling spacecraft whose attitude is unlikely to be recovered during the fly-by. Up to one of such particles can be expected along a 250-km fly-by (Table 2), and so higher fly-by altitudes are advisable.
- The errors induced by particles of mass 10^{-8} – 10^{-5} kg may be more than one order of magnitude larger than the requirements ε_{req} and ω_{req} . Such particles are frequently encountered (from a few to hundreds of them—Table 2) even for fly-by altitudes in the 500–625-km range. Thus, the pointing requirement of the mission may need to be relaxed, through, for instance, a wide-angle science camera. Note, nonetheless, that the analysis in this section is for a worst-case dust environment and a 250-km fly-by. The attitude performance is further characterized in Section 5.3.3 under nominal environment conditions.

- Particles of mass 10^{-11} – 10^{-9} kg generate errors that remain more than one order of magnitude below the requirements of the mission. Since thousands of these particles are encountered, their perturbation effectively results in attitude jitter throughout the trajectory. Jitter stability is relevant to image acquisition and shall be taken into account by the science team. Note, however, that these particles are neglected in the Monte Carlo analysis in Section 5.3.3 (in the interest of computational runtime), since their perturbation is significantly below mission requirements and much smaller than the errors induced by the (also frequent) 10^{-9} – 10^{-5} -kg particles.

5.3.3. Performance through full dust environment

As a last step in the characterization of attitude performance, 10,000 Monte Carlo simulations are performed where particles corresponding to mass bins [10^{-9} – 10^{-8} , ..., 10^{-5} – 10^{-4}] kg are distributed along the trajectory (according to their respective probability of impact—see Section 5.1). The results from these simulations allow to describe the attitude performance that can be expected along fly-by trajectories, for various fly-by altitudes.

As an example, Fig. 8 illustrates the pointing error ε for one of these Monte Carlo simulations, along a 500-km fly-by. It is observed that ε -levels more than order of magnitude smaller than $\varepsilon_{req} = 0.5^\circ$ are registered until ~30 seconds before closest approach. As the density of the environment increases in close proximity to the comet (and less-frequent, larger particles are impacted), pointing errors of up to $\sim 0.38^\circ$ are reached. Illustrated in Fig. 9 is also the number of encountered particles corresponding to each mass bin (for this particular Monte Carlo run). Notice that no particles of mass 10^{-6} – 10^{-4} kg are encountered in this case, since these are less probable to be encountered (average number of particles at 500 km is half of that at 250 km—Table 2).

Results from 10,000 similar Monte Carlo simulations are summarized in Fig. 10, where a histogram of the maximum pointing error ε_{max} from each simulation is provided (5-deg bins, notice the logarithmic scale of the y-axis), for various fly-by altitudes. As an initial observation, notice that a high number of simulations result in $<5^\circ$ maximum errors. As expected, it is also evident that higher fly-by altitudes result in better attitude performance: i.e., higher number of cases with lower ε_{max} . Nonetheless, the distribution of ε_{max} is widely spread, with values anywhere between 0° and $\sim 50^\circ$, or even above 150° . It should be noted that “outliers” with ε_{max} above $\sim 50^\circ$ correspond to Monte Carlo cases where the spacecraft encounters one of the less probable 10^{-5} – 10^{-4} -kg particles.

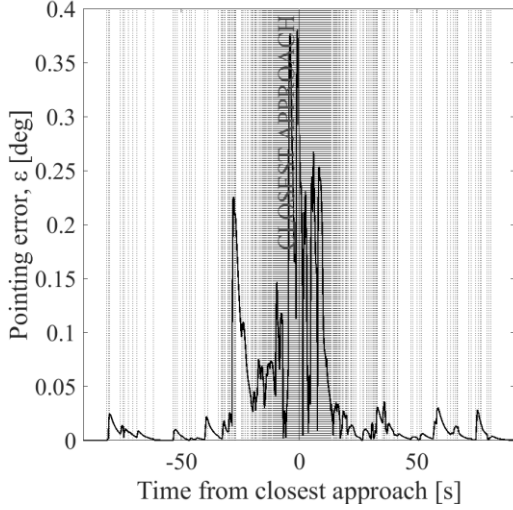


Fig. 8. Pointing error along sample 500-km fly-by (full dust environment)

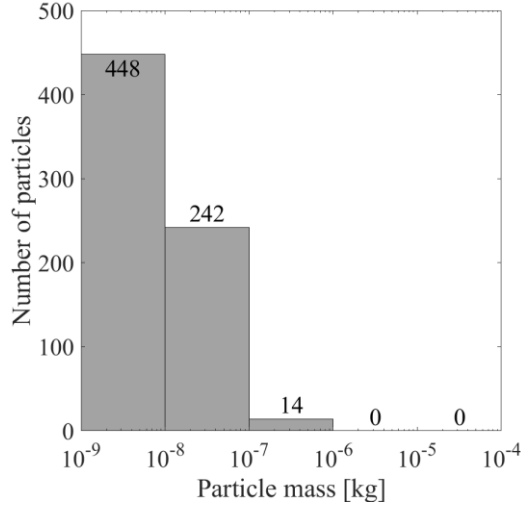


Fig. 9. Number of particles encountered along sample 500-km fly-by

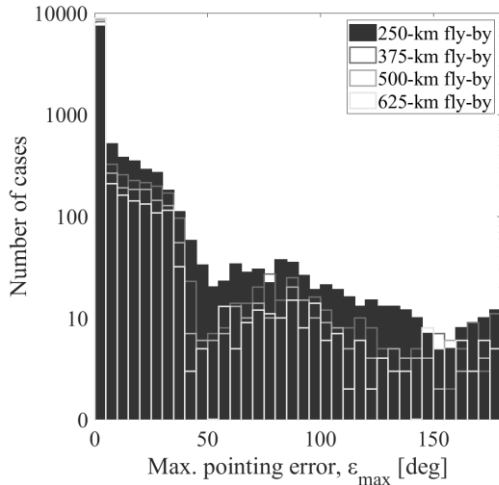


Fig. 10. Histogram of maximum pointing error from 10,000 Monte Carlo runs (full dust environment)

From Fig. 10, it is already clear that the pre-established pointing requirement of $\varepsilon_{req} = 0.5^\circ$ is not achievable for fly-by altitudes below 625 km, provided the 25-mN·m reaction wheels considered in this study [3]. From a mission feasibility point of view, nevertheless, it is still worth assessing the expected performance of the attitude control system. Table 4 thus provides 1σ , 2σ and 3σ confidence intervals for quantities such as mean and maximum pointing errors and mean and maximum angular velocities. Such confidence intervals effectively provide an upper limit to the attitude errors for 68.0%, 95.0% and 99.7% of cases.

Several observations can be drawn from Table 4:

- Regardless of fly-by altitude, 3σ errors are unreasonably large: e.g., $\varepsilon_{max} > 130^\circ$ and $\omega_{max} > 130 \text{ deg/s}$, which correspond to a tumbling spacecraft due to large particle impacts (10^{-5} – 10^{-4} kg).
- Despite fly-by altitude, 2σ mean angular velocities are in the order of a few degrees per second: $1.2 \text{ deg/s} < \omega_{mean} < 4.4 \text{ deg/s}$, and maximum angular velocities are in the range $13.6 \text{ deg/s} < \omega_{max} < 17.3 \text{ deg/s}$. Such large angular velocities are relevant to image acquisition and shall be considered by the science team (e.g., camera exposure times).
- Similarly, 2σ mean pointing errors are in the range $2.2^\circ < \varepsilon_{mean} < 10.6^\circ$, and maximum pointing errors are in the order of tens of degrees: $22.1^\circ < \varepsilon_{max} < 37.5^\circ$. Such large pointing errors evidence the need for a wide-angle camera.
- Lastly, 1σ mean pointing errors and angular velocities ε_{mean} and ω_{mean} do fulfill the mission requirements ε_{req} and ω_{req} for fly-by altitudes above 375 km. However, maximum errors ε_{max} and ω_{max} are always above these requirements.

Similarly to Table 4, Table 5 summarizes the confidence intervals associated to the axial angular momentum of reaction wheels: i.e., ratio between maximum $H_{w,max}$ from each Monte Carlo simulation and reaction wheel capability $H_{w,lim}$: $H_{w,max}/H_{w,lim}$. Note that these results correspond to unloaded reaction wheels prior to the fly-by: $H_w = 0$. It is initially observed that 3σ values reach the saturation limit of reaction wheels: $(H_{w,max}/H_{w,lim})_{3\sigma} = 100\%$, due to 10^{-5} – 10^{-4} -kg particle impacts. Also, notice that 2σ results remain below 55%, which indicate that 500-mN·m·s reaction wheels could be pre-loaded even up to 45% of their $H_{w,lim}$ capability to increase gyroscopic stability (Section 5.4). It is finally also noted that the maximum angular momentum of reaction wheels is associated to the axis aligned with the camera line of sight ($\hat{y}^B$), and these angular momentum loads decrease for higher fly-by altitudes.

Table 4. Attitude performance through full dust environment (confidence intervals from 10,000 Monte Carlo runs)

Fly-by altitude	Confidence interval	ε_{mean} (deg)	ε_{max} (deg)	ω_{mean} (deg/s)	ω_{max} (deg/s)
250 km	1 σ (68.0%)	0.603	2.479	0.709	5.716
	2 σ (95.0%)	10.60	37.49	4.354	17.22
	3 σ (99.7%)	50.77	164.7	77.32	157.2
375 km	1 σ (68.0%)	0.249	1.010	0.315	2.511
	2 σ (95.0%)	6.040	29.55	2.658	15.38
	3 σ (99.7%)	46.02	157.0	63.04	148.7
500 km	1 σ (68.0%)	0.152	0.668	0.197	1.985
	2 σ (95.0%)	3.682	26.07	1.845	14.53
	3 σ (99.7%)	40.96	147.0	52.11	144.5
625 km	1 σ (68.0%)	0.104	0.522	0.140	1.833
	2 σ (95.0%)	2.223	22.19	1.246	13.63
	3 σ (99.7%)	36.88	130.1	33.70	139.9

Table 5. Maximum axial angular momentum of initially-unloaded reaction wheels (confidence intervals from 10,000 Monte Carlo runs)

Fly-by altitude	Confidence interval	$H_{w,max}/H_{w,lim}$ (%)
250 km	1 σ (68.0%)	20.92
	2 σ (95.0%)	54.58
	3 σ (99.7%)	100.0
375 km	1 σ (68.0%)	15.25
	2 σ (95.0%)	43.07
	3 σ (99.7%)	100.0
500 km	1 σ (68.0%)	12.36
	2 σ (95.0%)	38.74
	3 σ (99.7%)	100.0
625 km	1 σ (68.0%)	10.48
	2 σ (95.0%)	35.85
	3 σ (99.7%)	100.0

Thus, as a summary of this section, the expected attitude performance is characterized by large angular velocities (in the order of several degrees per second, relevant for image acquisition), and pointing errors of up to several tens of degrees, which identify a need for a wide-angle camera and short exposure times. Higher fly-by altitudes are also beneficial in terms of attitude performance, but the less common impacts of larger particles (10^{-5} – 10^{-4} kg) still compromise the 3 σ success of the mission. Solutions for improved attitude performance are hence required and some are explored in Section 5.4.

5.4. Solutions for improved attitude performance

Two possibilities to improve gyroscopic stability about the camera line of sight (i.e., body-fixed axis $\hat{\mathbf{y}}^B$) are discussed in this section: (1) pre-loading the reaction wheel aligned with axis $\hat{\mathbf{y}}^B$, and (2) implementing a larger reaction wheel about axis $\hat{\mathbf{y}}^B$ (also pre-loaded). Note that pre-loading other reaction wheels may either

stabilize or destabilize attitude control depending on the position of particle impacts relative to the spacecraft's center of mass, $\mathbf{d}_p^{CM}$; thus it is advisable that reaction wheels aligned with other body axes are not loaded prior to the fly-by.

5.4.1. Pre-loading of reaction wheel

A first scenario is analyzed in which the initial axial angular momentum of reaction wheels is set equal to $\mathbf{H}_w = [H_{w,x^B}, H_{w,y^B}, H_{w,z^B}] = [0, \alpha_{w,h_f} \cdot H_{w,lim}, 0]$, where the factor α_{w,h_f} is selected based on the (altitude-dependent) 2 σ performance in Table 5. Specifically, α_{w,h_f} is set equal to $\alpha_{w,h_f} = 0.9 \cdot (1 - (H_{w,max}/H_{w,lim})_{2\sigma})$: i.e., 90% of the unused 2 σ angular momentum in Table 5. Note that such an initial angular momentum eventually results in 2 σ $H_{w,max}$ -values between 80% and 90% of $H_{w,lim}$.

Then, 10,000 Monte Carlo simulations are performed under a full dust environment (setup in Section 5.1), whose results are summarized in Table 6. Compared to the (unloaded) performance in Table 4, it is observed that improvements of approximately 1° are obtained in 2 σ mean pointing error ε_{mean} , and improvements of more than 5° are obtained in 2 σ maximum pointing error ε_{max} . Note, also, that slight increments in 2 σ angular velocities ω_{mean} and ω_{max} are registered (less than 1 deg/s). As such, pre-loading of the 500-mN·m·s reaction wheels considered in this study results in better 2 σ pointing errors: $15.8^\circ < \varepsilon_{max} < 32.2^\circ$, but angular velocities of several to tens of degrees per second are still reached. Lastly, 3 σ performance is still conditioned by large particle impacts (10^{-5} – 10^{-4} kg).

Table 6. Attitude performance with pre-loaded reaction wheel along camera line of sight (confidence intervals from 10,000 Monte Carlo runs)

Fly-by altitude	Confidence interval	ε_{mean} (deg)	ε_{max} (deg)	ω_{mean} (deg/s)	ω_{max} (deg/s)
250 km	1 σ (68.0%)	0.629	2.607	0.747	6.602
	2 σ (95.0%)	9.462	32.15	4.527	18.11
	3 σ (99.7%)	66.48	173.2	83.87	164.7
375 km	1 σ (68.0%)	0.245	0.985	0.312	2.498
	2 σ (95.0%)	4.338	22.03	2.592	15.69
	3 σ (99.7%)	66.59	173.4	66.19	156.1
500 km	1 σ (68.0%)	0.150	0.661	0.197	1.987
	2 σ (95.0%)	2.323	17.95	1.654	14.45
	3 σ (99.7%)	50.73	173.2	48.65	151.9
625 km	1 σ (68.0%)	0.106	0.525	0.141	1.835
	2 σ (95.0%)	1.458	15.80	1.092	13.74
	3 σ (99.7%)	45.35	170.0	35.99	153.2

Table 7. Attitude performance with larger (and pre-loaded) reaction wheel along camera line of sight (confidence intervals from 10,000 Monte Carlo runs)

Fly-by altitude	Confidence interval	ε_{mean} (deg)	ε_{max} (deg)	ω_{mean} (deg/s)	ω_{max} (deg/s)
250 km	1 σ (68.0%)	0.526	2.073	0.540	5.924
	2 σ (95.0%)	4.311	16.13	2.913	17.16
	3 σ (99.7%)	61.39	169.6	77.77	174.9
375 km	1 σ (68.0%)	0.236	0.907	0.274	2.390
	2 σ (95.0%)	2.163	12.82	1.511	15.22
	3 σ (99.7%)	50.25	163.2	53.91	165.7
500 km	1 σ (68.0%)	0.148	0.623	0.180	1.932
	2 σ (95.0%)	1.378	11.48	0.971	14.44
	3 σ (99.7%)	40.65	160.5	36.09	164.4
625 km	1 σ (68.0%)	0.104	0.498	0.132	1.814
	2 σ (95.0%)	0.810	10.29	0.598	13.78
	3 σ (99.7%)	27.81	137.2	24.41	158.9

5.4.2. Upgrading of reaction wheel

A second scenario is explored in which a larger reaction wheel is implemented along the camera line of sight (body-fixed axis $\hat{\mathbf{y}}^B$). In particular, Blue Canyon Technologies' RW1 is considered [3], whose maximum torque is $T_{w,lim} = 100 \text{ mN} \cdot \text{m}$ and maximum angular momentum is $H_{w,lim} = 1 \text{ N} \cdot \text{m} \cdot \text{s}$. This reaction wheel is also pre-loaded as in Section 5.4.1: $\mathbf{H}_w = [0, \alpha_{w,hf} \cdot H_{w,lim}, 0]$ with $\alpha_{w,hf} = 0.9 \cdot (1 - (H_{w,max}/H_{w,lim})_{2\sigma})$, but substituting the new angular momentum capability $H_{w,lim} = 1 \text{ N} \cdot \text{m} \cdot \text{s}$. This initial angular momentum eventually results in 2σ $H_{w,max}$ -values between 83% and 85% of $H_{w,lim}$ as well.

With a larger reaction wheel, significant performance improvements are obtained (see Table 7): 2σ mean and maximum pointing errors, ε_{mean} and ε_{max} , are nearly halved compared to those in Table 6. As such, 2σ ε_{max} is maintained in the range $10.2^\circ < \varepsilon_{max} < 16.2^\circ$ (even at 250 km), and $0.8^\circ < \varepsilon_{mean} < 4.4^\circ$. The 2σ mean angular velocity, ω_{mean} , is also nearly halved, and

maintained in the range $0.5 \text{ deg/s} < \omega_{mean} < 3.0 \text{ deg/s}$, however, no significant improvements in 2σ maximum angular velocity ω_{max} are observed, and large angular velocities in the range $13.7 \text{ deg/s} < \omega_{max} < 17.2 \text{ deg/s}$ are still obtained. Similarly, 3σ performance is still compromised by the impact of large particles (10^{-5} – 10^{-4} kg) despite the use of a larger reaction wheel.

Conclusively, a larger reaction wheel aligned with the camera line of sight can provide sensible improvements in mean and maximum pointing error and mean angular velocity; however, large maximum angular velocities are still obtained and 3σ performance cannot be improved due to large particle impacts. As such, a wide-angle camera is still required, and image acquisition is still challenged by 2σ maximum angular velocities of up to 17.2 deg/s .

5.5. Selection of attitude control gains

Several steps are taken prior to selecting the control gains employed throughout Sections 5.3 and 5.4: k (proportional gain) and c (derivative gain). As a first

step, the order of magnitude of control gains may be inferred from the pointing requirements of the mission and specifications of the reaction wheels: $[k] \sim [T_{w,lim}]/[\varepsilon_{req}]$ and $[c] \sim [T_{w,lim}]/[\omega_{req}]$. Notice that reactions wheels can generate a maximum torque $T_{w,lim} = 25 \text{ mN} \cdot \text{m}$, and the pointing requirements are $\varepsilon_{req} = 0.5^\circ$ in pointing error and $\omega_{req} = 0.4 \text{ deg/s}$ in angular velocity. Control gains can then be expected in the order of $[k] \sim 2.9 \text{ N} \cdot \text{m/rad}$ and $[c] \sim 3.6 \text{ N} \cdot \text{m} \cdot \text{s/rad}$.

As a second step, it may also be noticed that the aleatory of the dust environment does not allow for a traditional selection of control gains. The behavior of the attitude control system is highly dependent on the (uncertain) dust environment to be encountered, and so, a single selection of control gains would result in a range of different pointing errors and angular velocities.

As such, a grid-search parametric study is combined here with a Monte Carlo analysis to consider the dependency on the dust environment. A grid of different combinations of k and c is generated, and the performance of each combination is tested for 100 different dust environments (generated according to 5.1). Quantities such as mean pointing error ε_{mean} along the trajectory, maximum pointing error ε_{max} , mean angular velocity ω_{mean} , and average generated torque T_{mean} are stored from these simulations.

As an example, Fig. 11 illustrates the attitude performance (averaged over 100 Monte Carlo cases) resulting from different combinations of gains k and c . Specifically, considered values of k and c belong to the intervals $[0.1, 10] \text{ N} \cdot \text{m/rad}$ and $[0.1, 10] \text{ N} \cdot \text{m} \cdot \text{s/rad}$, which are defined according to the expected order of magnitude of the control gains.

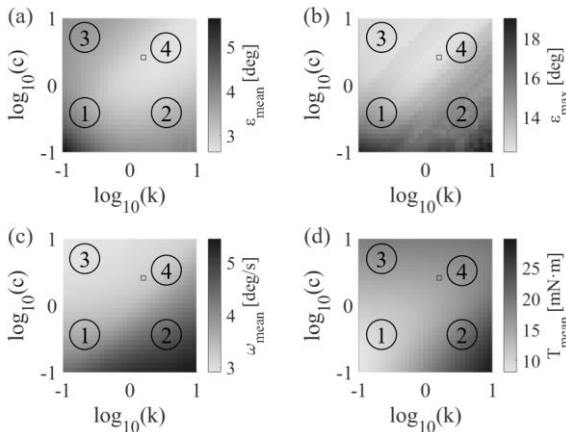


Fig. 11. Effect of control gains on (a) mean pointing error, (b) maximum pointing error, (c) mean angular velocity, and mean generated torque (average over 100 Monte Carlo runs, 250-km fly-by)

Four distinct regions can be observed in Fig. 11 (labeled):

- (1) Small values of k and c , resulting in low overall performance (notice small generated torque T_{mean}).
- (2) Large k and small c , resulting in large average angular velocity ω_{mean} and reasonable ε_{mean} .
- (3) Small k and large c , resulting in large average pointing error ε_{mean} and reasonable ω_{mean} .
- (4) k and c in the order of $\log_{10} 0 = 1$, resulting in best overall performance (notice low ε_{mean} and ε_{max} , and reasonable ω_{mean} and T_{mean}).

Also indicated in Fig. 11 (black outlined square) are the selected control gains $k = 1.5 \text{ N} \cdot \text{m/rad}$ and $c = 2.4 \text{ N} \cdot \text{m} \cdot \text{s/rad}$ employed throughout the analysis in Section 5.3. Such gains are centered within region 4, although their specific values are arbitrarily selected since region 4 presents a rather homogeneous performance.

6. Conclusions

This study provides a first detailed assessment of the expected attitude performance of JAXA's contribution to the Comet Interceptor mission: 30-kg small-spacecraft SC B1. Exceptional challenges to the mission include a highly-active dust environment and very large relative fly-by speeds. This study hence characterizes the dust environment and its effect on the attitude control of SC B1, and it discusses potential solutions for improved attitude performance.

An analytical approach to determine the number and distribution of particles encountered along the trajectory is proposed. It is established that, under worst-case dust production rates, SC B1 will encounter thousands of particles of 10^{-11} – 10^{-8} kg, tens to hundreds of particles of 10^{-8} – 10^{-6} kg, and only a few particles of mass 10^{-6} – 10^{-4} kg. Larger particles (above 10^{-4} kg) are not expected under 3σ confidence levels.

The effect of such particles on the attitude of SC B1 is also analyzed, and it is concluded that particles of mass 10^{-11} – 10^{-9} kg, despite being more numerous, have a negligible effect on the attitude of SC B1 compared to larger particles. Particles of mass 10^{-9} – 10^{-5} kg are also frequently encountered and can produce attitude errors more than one order of magnitude above the pre-established pointing and angular velocities requirements of 0.5° and 0.4 deg/s (for a narrow-angle camera). Hence, a wide-angle camera is concluded essential for the success of the mission, and image acquisition shall expect angular velocities beyond $\sim 10 \text{ deg/s}$.

Recommendations for SC B1 are finally made in terms of selection of attitude control gains, and solutions for improved attitude performance. Angular momentum pre-loading of the reaction wheel aligned with camera line of sight is shown to provide higher gyroscopic stability and improved pointing error performance (and no improvement in angular velocity). Whereas a larger

(and pre-loaded) reaction wheel aligned with the camera line of sight is shown to reduce pointing errors and mean angular velocities almost in half, yet maximum angular velocities are not significantly improved. Furthermore, regardless of the proposed solution, the 3σ success of the mission is compromised by the less-probable impact of 10^{-5} – 10^{-4} -kg particles (which result in a tumbling spacecraft), and 2σ attitude performance is only compatible with a wide-angle science camera capable of collecting images at large angular velocities.

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System requirements analysis for JAXA's contribution to Comet Interceptor mission: autonomous navigation, guidance and attitude control for a hyperbolic comet fly-by

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