

Autonomous Optical Navigation for Small Spacecraft in Cislunar Space

Christian Xianyang Wu^{a*}, Pablo Machuca^b, Leonard Felicetti^c, Joan Pau Sanchez^d

^a Cranfield University, United Kingdom, christian.wu.740@cranfield.ac.uk

^b University of California, San Diego, United States, pmachucavarela@ucsd.edu

^c Cranfield University, United Kingdom, leonard.felicetti@cranfield.ac.uk

^d Institut Supérieur de l'Aéronautique et de l'Espace, France, joan-pau.SANCHEZ@isae-superaero.fr

* Corresponding author

Abstract

The Earth-Moon system is expected to be increasingly populated especially due to large space missions like the Lunar Gateway. Communication constellations and small spacecraft are also expected to be deployed in the coming years. However, small spacecraft missions are heavily challenged by limited access and high costs of communicating with ground antennas, and constraints on on-board components. Hence, small spacecraft could greatly benefit from the development of autonomous optical navigation, allowing for higher levels of independence, flexibility, and lower operation costs in a complex environment like cislunar space. The scope of this study is to assess the optical navigation (OPNAV) accuracies achievable through horizon-based position estimation algorithms applied to synthetic images of the Moon. The simulation framework is implemented in Python+Vapory+OpenCV, and it simulates the image acquisition process at different solar phase angles and distances (including camera performance), and the processing of images (involving limb-fitting, centroiding and relative position estimation). A conventional ellipse fitting (EF) technique and the recently proposed Christian-Robinson (CR) approach are compared. Lastly, a Monte Carlo analysis allows to assess the effects of different sources of errors and uncertainties and to gain a better understanding of the navigation accuracies that may be obtained in future cislunar scenarios of interest. Preliminary results show that the CR algorithm consistently provides better performance than the EF technique, both in accuracy and computational time. The resulting errors in range estimation depend on distance, with values of approximately 22 km at a distance of 20,000 km and up to 422 km at a distance of 150,000 km. In terms of cross-axis distance estimation, the errors present values around 3.55 km (36 arcsec) at 20,000 km, and 11.85 km (16 arcsec) at 150,000 km. Furthermore, simulation outcomes show near normally distributed OPNAV performance with standard deviations around 0.68 km and 0.73 arcsec. A power law function is also provided to approximate the trend of OPNAV accuracies as a function of the distance relative to the observed body, is found based on the outcome of this study. Colour maps illustrating OPNAV accuracies as a function of observation geometry provide a comprehensive picture of the expected performance of OPNAV depending on distance and orientation relative to the observed object.

Keywords: image generation, image processing, ellipse fitting, position estimation, Monte Carlo analysis

1. Introduction

In recent years, the number of space missions has grown significantly, due to both agency led projects [1] and missions coming from private businesses [2]. Multiple unmanned missions have already been launched and many more, including crewed ones, are planned to depart Earth in the coming years. Some of these projects, especially those that are realized in private companies, are following the miniaturization trend where spacecraft decrease in size, mass and cost. This has led to a continuous research for the development of technologies that can satisfy mission requirements and at the same time cut down major costs. Autonomous navigation can provide a solution to challenges like limited or no access to ground stations, malfunctioning communication subsystem, or delays associated to deep-space communication links [3]. The development of image processing algo-

gorithms is a highly active field of research for autonomous OPNAV. Multiple researches have been devoted towards the construction of novel methods for vision based navigation [4–6]. Validation and testing of OPNAV has been performed in a wide range of scenarios: navigation in Mars orbit [7], navigation around a binary asteroid [8] and deep-space OPNAV [9]. A number of studies can be found to tackle the various aspects involved in the OPNAV process, from edge detection techniques (pixel- and subpixel-level [10, 11, 11–13]), ellipse fitting (EF) [4], or direct horizon-based position estimation [14]. In this context, this paper aims to provide a comparison of a conventional EF algorithm and the recently proposed Christian-Robinson (CR) horizon-based algorithm with the overall goal of understanding what accuracies can be obtained through OPNAV in cislunar space, depending on relative position. In particular, a simulation framework is developed in Python,

using utilities such as Vapory in order to interface with the rendering software, POV-Ray. First, synthetic images of the Moon are generated according to relative position, camera specifications, and illumination geometry. Then, image processing algorithms are employed in order to assess the accuracy of the selected method. A Monte Carlo analysis is performed introducing noise (according to typical small-spacecraft camera performance) in the images of the Moon, and the performance of both EF and CR is compared. Analysis shows relative distance estimation errors in the range of 22 – 422 km, Moon center estimation errors of approximately 0.07 – 1.63 km, and line-of-sight angular errors around 16 – 36 arcsec, which provides a comprehensive analysis of expected OPNAV performance.

The simulation framework (comprising of image generation tool and image processing) is described in Section 2. In Section 3 the results of the comparative analysis between the two approaches are discussed and the performances of OPNAV algorithms are analyzed as a function of distance.

2. Simulation Framework

This section presents the developed simulation framework for OPNAV performances analysis. First, the fundamentals of the construction of the model in POV-Ray are illustrated. Then, a thorough explanation of the processing of the images acquired with the image generation tool is given.

2.1. Image Generation

For image generation purposes, the software tool POV-Ray is used to model the 3D scene. The Moon is modeled with a colour map and an elevation map wrapped around a 3D sphere: the first one is a mosaic of data from Wide Angle Camera (WAC) on-board Lunar Reconnaissance Orbiter (LRO) [15] and the second one is a Digital Elevation Model (DEM) extracted from the data coming from Lunar Orbited Lased Altimeter (LOLA) on-board LRO [16]. The Moon's position and orientation are obtained through SPICE [17]. The camera characteristics are taken from the selected optical instrument, the NanoCam C1U¹ from GomSpace.

A dataset, comprising of multiple images of the Moon for different observation geometries, is generated. The Moon is imaged from a total of 36 by 18 azimuth and elevation combinations (10 degree step size) relative to the Moon's ME frame, covering the whole sphere of the Moon for a given distance and solar phase angle. The dataset is created for four different distances from the Moon: 20, 000 km, 50, 000 km, 100, 000 km and 150, 000 km. Examples of images at the four distances are collected in Figure 1.

Pictures taken with real imaging systems show the presence of noise, due to various sources. Common sources of noise are due to the illumination conditions and the temperature of the sensor. Noise is introduced in the dataset images (8-bit encoding with pixel values in the range 0 – 255) according to a Gaussian distribution with a pixel value standard deviation $\sigma = 9.2$. Typical signal-to-noise ratio (SNR) in imaging system for traditional spacecraft like LRO are in the order of 150 [18] and cameras for small spacecraft like a CubeSat present SNR values around 50 [19]. By comparing the performance of LRO and a CubeSat, the levels of noise to be expected on small spacecraft are 3 times higher due to the detector size and camera volume.

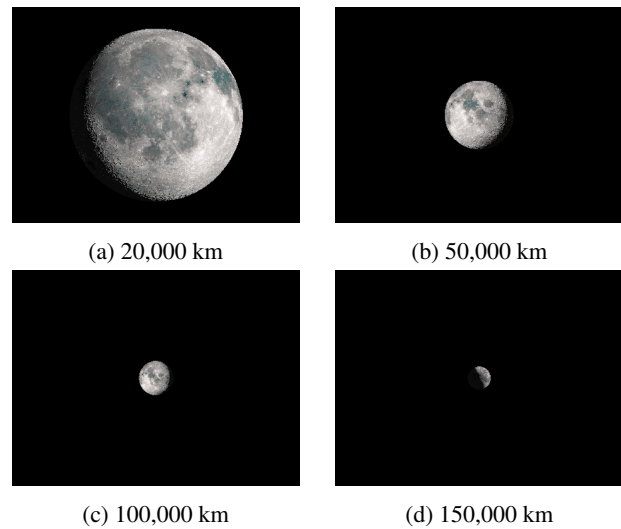


Figure 1: Sample images taken from the dataset generated in POV-Ray

2.2. Image Processing

The image processing segment is composed of edge detection, ellipse fitting and position estimation. An overview of the entire procedure is depicted in Figure 2, which shows the top-level steps of this segment.

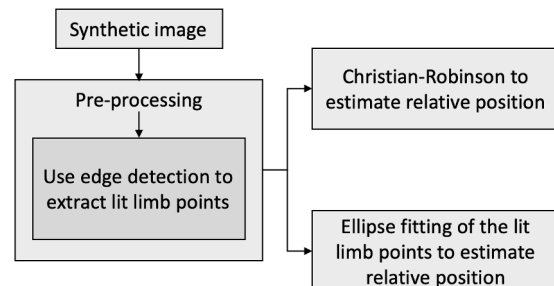


Figure 2: Flowchart of image processing procedure

¹<https://gomspace.com/shop/subsystems/payloads/earth-observation.aspx>

2.2.1. Edge Detection

Full disk horizon-based navigation techniques rely on the detection of the illuminated horizon, or lit limb. For a Moon that is partially illuminated as observed from the spacecraft, the brightest camera pixels are delimited by the horizon and the terminator line. For navigation purposes, only the points belonging to the lit limb are selected, as it has been proven that the terminator points degrade navigation performances [14].

The raw image is rotated so that the illumination direction is horizontal in the image reference frame. In this new orientation, the image is transformed in binary form (pixel values are either 0 or 255) and the Canny filter from the OpenCV library is applied onto the image to isolate the edges of the Moon. The output of Canny does not differentiate between the pixels belonging to the illuminated horizon and the pixels on the terminator line. Therefore, a scan along the illumination direction is performed to extract the pixels on the lit horizon. Then, the collected edge points are rotated back to the original orientation. According to Hollenberg [20], the best OPNAV performances are obtained if the points near the terminator line are ignored, hence the edge points close to the cusps are eliminated from the list. A diagram which illustrates the output at each step of the edge detection procedure is shown in Figure 3.

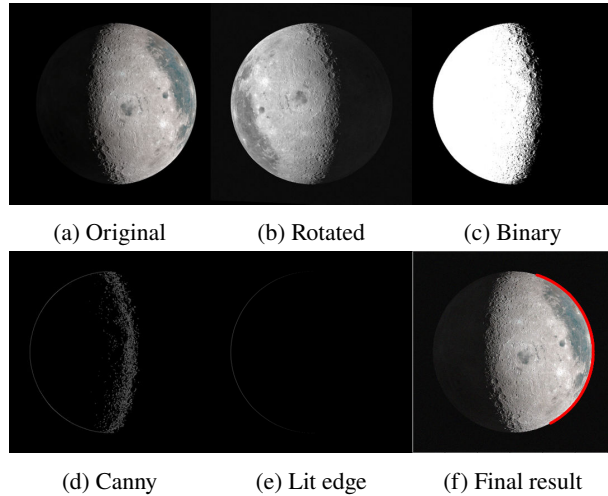


Figure 3: Sequence of images representing the steps of edge detection

2.2.2. Ellipse Fitting and Position Estimation

The detected edge is utilized to estimate the relative position of the spacecraft through the application of the EF algorithm described in the present section. The Moon can be approximated by a triaxial ellipsoid [21] and its projection onto an image plane is represented by an ellipse. The

implicit equation for a generic conic is:

$$F(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad (1)$$

$$F(\mathbf{a}, \mathbf{x}_i) = \mathbf{a}^T \mathbf{x}_i = 0 \quad (2)$$

and with the condition $B^2 - 4AC < 0$, the conic corresponds to an ellipse. The improved Fitzgibbon algorithm for EF, proposed in [4], gives a non-iterative solution to the problem by reducing it to an eigenvector and eigenvalue problem. A detailed explanation of the procedure used here is available in [4]. The diagram shown in Figure 4 illustrates the output images from start to end of the image processing procedure.

After having computed the parameters of the ellipse, the coordinates for the center are directly obtained as:

$$x_0 = \frac{2CD - BE}{B^2 - 4AC} \quad y_0 = \frac{2AE - BD}{B^2 - 4AC} \quad (3)$$

The position estimation algorithm relies on the apparent diameter of the body and the line-of-sight vector to the center of the ellipsoid. The coordinates obtained with Equation 3 are referred to the ellipse, which is the projection of the triaxial ellipsoid. However, due to the intrinsic effect of the perspective projection, the center of the ellipse does not correspond to the geometrical center of the Moon. Therefore, a correction offset is added to correct from the the observed center of the ellipse to the center of mass of the Moon. The offset is computed with the exact perspective projection of a 3D ellipsoid, proposed in [22], which makes use of the camera calibration matrix $\mathbf{K}$, the body shape matrix $\mathbf{A}$ and the ellipse center location in the Moon ME reference frame $\mathbf{x}_0$. The camera calibration matrix is dependent on the camera specifications and it is expressed as:

$$\mathbf{K} = \begin{bmatrix} d_x & 0 & u_p \\ 0 & d_y & v_p \\ 0 & 0 & 1 \end{bmatrix} \quad \text{with} \quad d_x = \frac{f}{\mu_x}, d_y = \frac{f}{\mu_y} \quad (4)$$

where its elements are summarized in Table 1.

Table 1: Camera specifications of NanoCam C1U

Parameter	Value
Focal length f (mm)	35
Pixel width μ_x (mm)	4.96e-3
Pixel height μ_y (mm)	4.96e-3
Center of image along horizontal u_p (px)	1024
Center of image along vertical v_p (px)	768

The body shape matrix, $\mathbf{A}$, is defined as:

$$\mathbf{A} = \begin{bmatrix} 1/a^2 & 0 & 0 \\ 0 & 1/b^2 & 0 \\ 0 & 0 & 1/c^2 \end{bmatrix} \quad (5)$$

where a, b and c are the principal axis of the body. The geometrical center location of the ellipsoid in the ME frame

Table 2: Parameters for shape, geometric center and Euler angles for modeling of the Moon as a geometrically accurate triaxial ellipsoid (SELENE Sampled Data set) [21]

a (km)	b (km)	c (km)	x_0 (km)	y_0 (km)	z_0 (km)	α (°)	β (°)	γ (°)
1739.088	1737.370	1734.969	-1.736	-0.723	0.226	17.390	21.240	27.330

is $\mathbf{x}_0 = [x_0, y_0, z_0]^T$. Knowing $\mathbf{K}$, $\mathbf{A}$ and $\mathbf{x}_0$, it is possible to directly compute the projection correction from the the observed center of the ellipse to the center of mass of the Moon, as illustrated in [22]. The triaxial ellipsoid model proposed by [21] is employed to approximate the projection of the Moon onto the image plane. This model provides 9 parameters, shown in Table 2, describing the principal axes of the Moon, the geometric ellipsoid center in the Moon ME reference frame and the Euler angles for the correction of the body orientation in the ME frame."

Finally, the distance is obtained with the the line-of-sight vectors of the corrected geometrical center, $\mathbf{e}_c$, and of the edge points, $\mathbf{e}_i$, and the body shape matrix of the Moon $\mathbf{A}$, as [4]:

$$\rho = \left([(\mathbf{e}_c)_I^T \mathbf{A} (\mathbf{e}_c)_I] - \frac{[(\mathbf{e}_c)_I^T \mathbf{A} (\mathbf{e}_i)_I]^2}{[(\mathbf{e}_i)_I^T \mathbf{A} (\mathbf{e}_i)_I]} \right)^{-\frac{1}{2}} \quad (6)$$

For more details regarding the derivation of Equation 6, the reader is directed to [4].

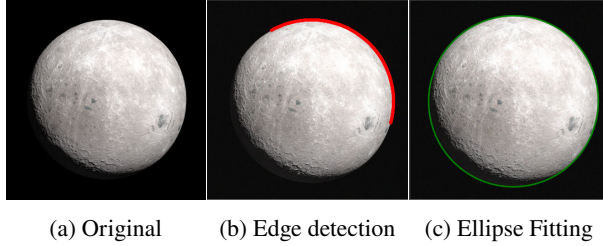


Figure 4: Sequence of images representing the steps of the image processing procedure until position estimation

2.2.3. The Christian-Robinson Algorithm

The CR algorithm directly relate edge points to the relative position, using a geometrical transformation, without fitting an ellipse to the detected edge. The algorithm is first proposed in [23] to make use of the factorized body shape matrix to transform the position estimation problem into a trivial problem and it has been improved in [24] with a more natural factorization approach.

The OPNAV geometry, which consist of a camera observing a triaxial ellipsoid, is reduced to a space where the spacecraft observes a unit sphere. The transformation is

achieved by factorizing the body shape matrix with:

$$\mathbf{A} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}^T \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix} \quad (7)$$

In the unit sphere space the tightly bounding cone, which is the cone resulting from the vectors going from the camera pinhole to the observed edge points, is a right circular cone. Therefore, the angle between a horizon point vector and the line going from the camera to the center of the sphere is constant for every measured edge point. In light of these considerations, the line-of-sight vector to the center of the sphere is estimated and with the geometry of the right circular cone, the relative position between the observer and the center of the sphere is derived. Lastly, the solution is reverted back into the original space to obtain the relative position of the spacecraft with respect to the Moon.

3. Analysis and Results

Numerical results of the simulation tests are collected in the present section. A comparative analysis between the EF approach and the CR algorithm is performed with respect to angular and range estimation errors and computational times. In addition, the OPNAV performance is evaluated depending on observation geometry, a normality analysis is conducted on the noisy images, and a study on the behaviour of the accuracies as a function of the distance is performed.

3.1. Performance Indices

The OPNAV algorithms utilized in this work are not able to perfectly estimate the relative position, hence they introduce inaccuracies in the relative position estimation. The inaccuracies introduced by OPNAV algorithms are assessed, and to achieve this, two performance indices have been selected: range estimation errors and line-of-sight estimation errors. The first is computed as the absolute value of the difference between the estimated and the nominal distance to the Moon. The second corresponds to the angle, expressed in arcsec, between the line-of-sight vector pointing to the mass center of the Moon and the one directed at the estimated one. These parameters are used to identify the conditions in which the algorithms perform

better, which results are expressed in Section 3.3. The values in the colour plots depend on the angles $\Delta\varphi$ and $\Delta\theta$, which are the “delta” elevation angle and “delta” azimuth angle, from a chosen origin point. This reference direction lies on the Sun-Moon vector, meaning that the origin corresponds to the location where the observed solar phase angle is 0° . The epoch of reference for the simulation is «2022-06-13 12:00 UTC». Hence, the simulation results do not apply to a spacecraft in an evolving orbit during a period of time, but they are applicable to scenarios where the Moon displays similar properties to the ones shown at this epoch. The Sun-Moon vector at the reference epoch is defined by the elements in Table 3. Moreover, the two performance indices are considered as a function of distance, as illustrated in Section 3.4. The errors of OPNAV algorithms highlight the trends of the performances over the relative distance. Lastly, the computational times of the algorithms are used to compare the efficiency of the two approaches. The simulation was conducted on a MacBook Pro 15" 2017 with a Quad-Core processor Intel Core i7 3.1 GHz and 16 GB of LPDDR3 RAM. The measurements are made on the CPU time.

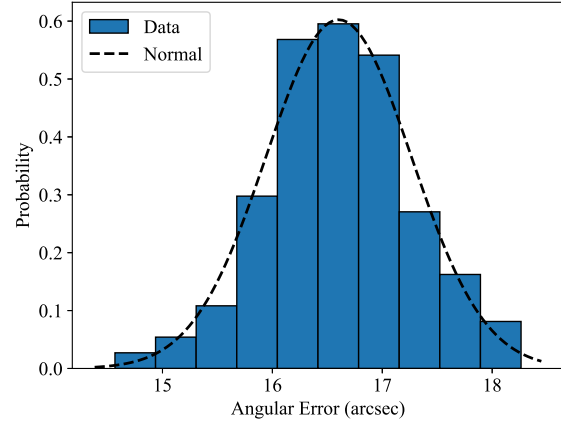
Table 3: Sun-Moon vector expressed in the Moon ME reference frame

x (km)	y (km)	z (km)
1.495e+08	2.863e+07	2.088e+06

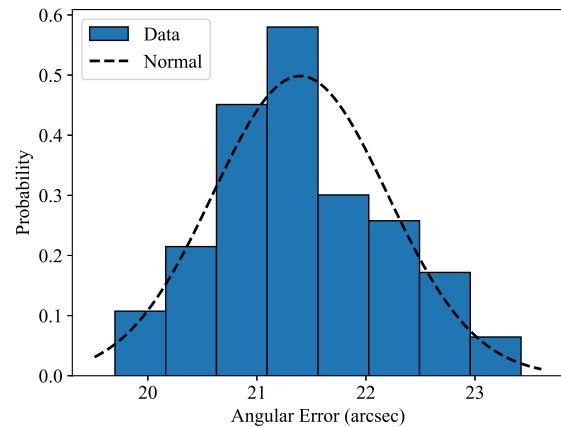
3.2. Normality of the Distribution of Noise Cases

For each observation geometry of the 18 by 36 grid of azimuth and elevation angles, 100 images with added noise are used for image processing. Examples of results are Figure 5a and Figure 5b. These distributions are referred to the angular errors obtained with CR in two different locations: ($\Delta\varphi = -26.50^\circ$, $\Delta\theta = 114.08^\circ$) and ($\Delta\varphi = 5.26^\circ$, $\Delta\theta = 124.36^\circ$), both at 20,000 km of distance from the Moon. The plots show the distribution of OPNAV errors for the 100 noise cases. The dashed line illustrates a fitted normal distribution to the distribution of OPNAV errors to graphically assess how well the errors adhere to a normal distribution. It is observed that the distribution of errors may be reasonably approximated as normal distributions, although this does not hold true for all observation geometries. Figure 5a, for instance, shows a close similarity, but Figure 5b shows a left-skewed distribution. By looking at Figure 5, it is noticeable that the distribution of errors has a small standard deviation from the mean, with values confined to a range of ± 2 km from the mean. This holds true for a large number of observation geometries, with standard deviations usually below 0.7 km and 0.7 arcsec at a distance of 20,000 km. Larger differences with respect to

a normal distribution are primarily due to the low number of noisy cases (100) considered in this analysis. In fact, for larger numbers of noisy images, these distribution show even better consistency with a normal distribution. For this reason, we use the mean value of the 100 noisy cases as a representative measure of the OPNAV performance for various observation geometries.



(a) $\Delta\varphi = -26.50^\circ$ and $\Delta\theta = 114.08^\circ$



(b) $\Delta\varphi = 5.26^\circ$ and $\Delta\theta = 124.36^\circ$

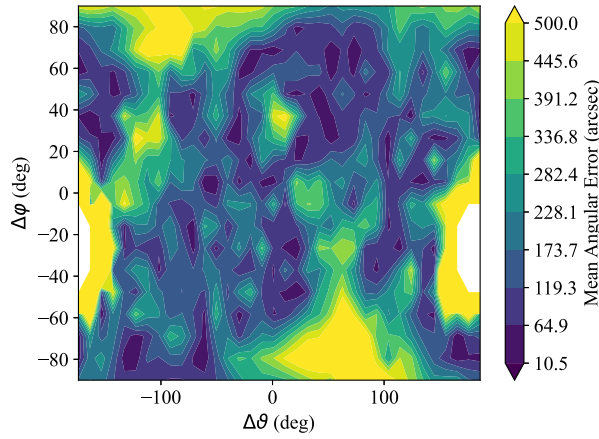
Figure 5: Distribution of the angular error of 100 noise cases using CR at 20,000 km

3.3. Performance Comparison of Ellipse Fitting and Christian-Robinson Approaches

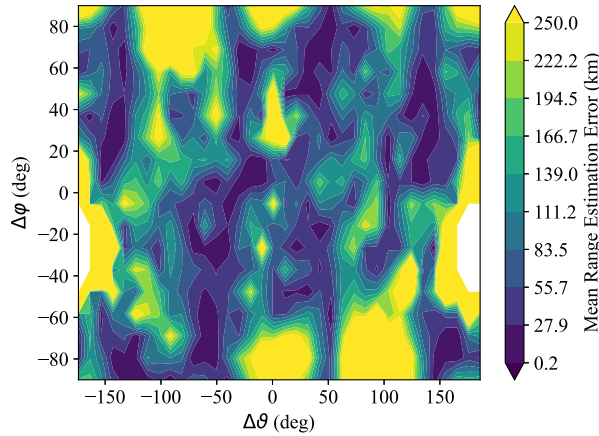
Figure 6a and Figure 6b show the mean errors in the case where the spacecraft is at 20,000 km of distance from the Moon. The colour plot is characterized by a gradient that goes from dark blue to yellow, corresponding to the errors going from lowest to highest. The two white patches on the left and on the right of the plots display no values of centroiding and range estimation errors. Those points correspond to the cases where the spacecraft is in a position where the Moon is entirely or almost entirely in shadow

(i.e., $\sim 180^\circ$ solar phase angle). These points are omitted in the colour plots since the implemented algorithms are not able to deal with such illumination conditions.

darker where the distance estimation errors are below 1% of the nominal range. More specifically, in 633 positions the error is below 200 km and 621 of those show an error below 100 km.



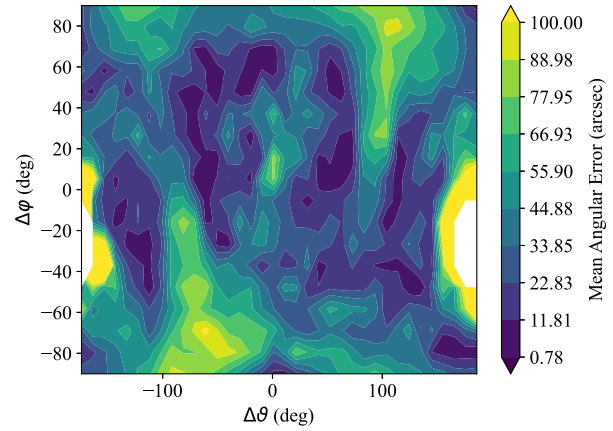
(a) Mean angular errors of line-of-sight estimation



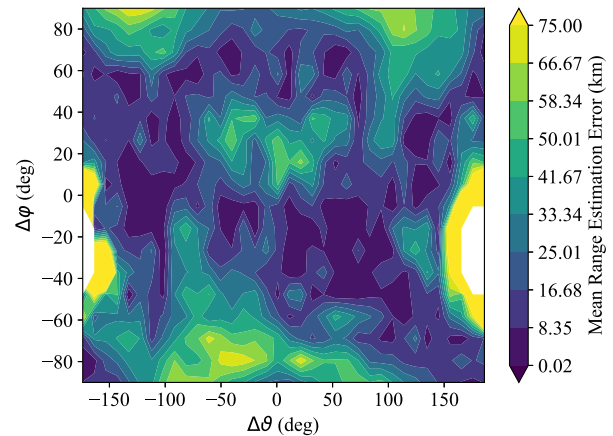
(b) Mean errors of distance estimation

Figure 6: Mean errors of EF method on a 18 by 36 grid of positions at a nominal distance of 20,000 km

Figure 6b shows multiple zones where the OPNAV algorithm guarantees errors below 1% of the nominal distance (20,000 km). For instance, in 491 out of the 648 considered observations geometries the error is kept below 200 km and 328 locations of those present an estimation error below 100 km. These simulation results are comparable with the preliminary requirements set for the LUMIO mission in [25]. Although the estimation through EF performs well, CR approach provides greater accuracy for OPNAV purposes. Comparing Figure 6b to Figure 7b, it is evident that the CR method provides a significant improvement. Notice that the colour gradient is limited with a maximum value of 70 km, which is almost four times lower than the one used in Figure 6b. Even with a smaller range of estimation errors, the colour plot shows more



(a) Mean angular errors of line-of-sight estimation



(b) Mean errors of distance estimation

Figure 7: Mean errors of CR algorithm on a 18 by 36 grid of positions at a nominal distance of 20,000 km

The CR approach provides better estimations compared to the EF method and the computational time is also significantly reduced. Figure 8 shows the trend of computational time against the number of detected edge points. For both methods the time grows linearly, however, the EF algorithm grows visibly faster in terms of time. It reaches values around 0.15 s and 0.2 s for the maximum number of points detected, whereas the CR algorithm peaks at around 0.04 s. The lower computational cost of CR method is due to the omitted ellipse fitting step for relative position estimation [14]. The results presented demonstrate the vast difference in computational cost between the two approaches, highlighting, in particular, the expected high efficiency of the CR algorithm.

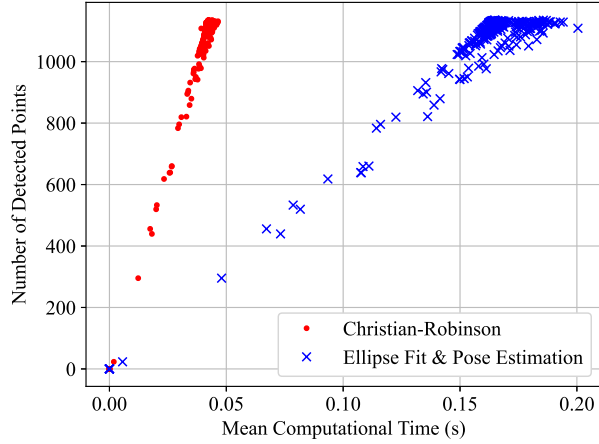
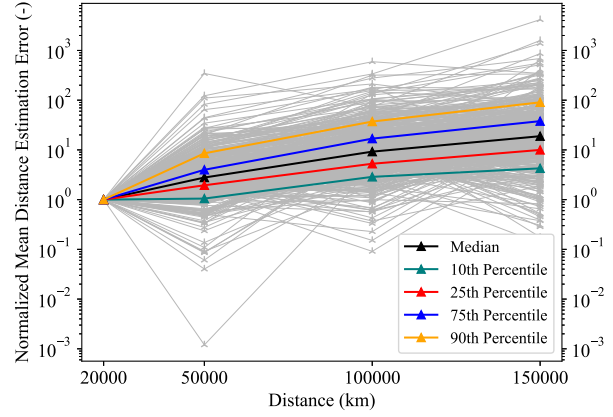


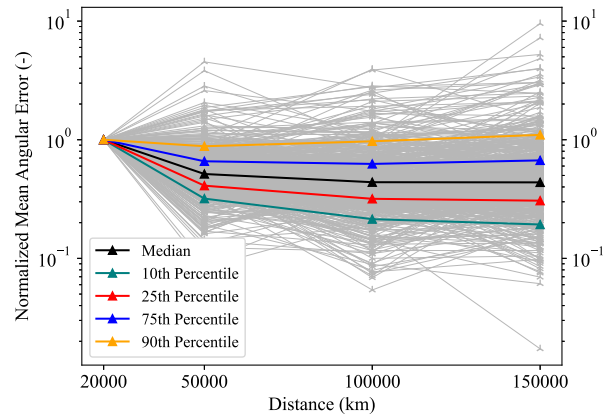
Figure 8: Computational time of OPNAV via EF and CR against number of points detected at 20000 km

3.4. Optical Navigation Performances as a Function of Distance

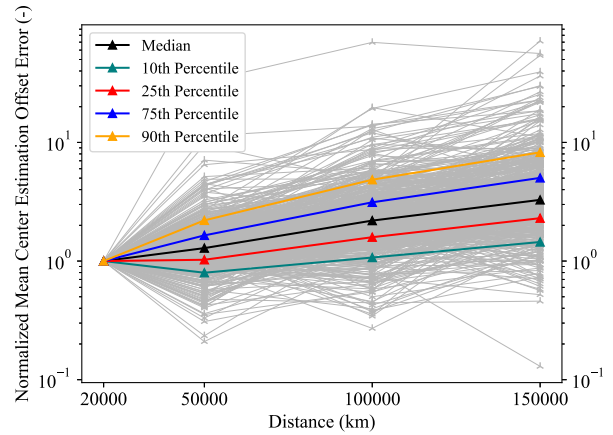
The simulation of OPNAV methods is performed across four distances: 20,000 km, 50,000 km, 100,000 km and 150,000 km. The range estimation errors and the angular errors, for both approaches, are scaled with respect to the error obtained at 20,000 km for each given observation geometry. With this notation, the plots show the evolution of the errors as a function of the distance. Figure 9a shows larger range estimation errors at greater distances, whereas Figure 9b shows a reduction in the angular error at farther distances from the Moon. Although the angular error produced by the CR algorithm decreases with the distance, the cross-axis distance (or center offset), increases with the distance, as represented in Figure 9c. The cross-axis distance is defined as the distance between the mass center and the estimated center of the Moon on the plane normal to the line-of-sight vector. These plots display trends in the estimation errors as a function of distance that may be reasonably approximated as linear in a log-linear scale. As such, a power law expressing estimation error as a function of distance may be fitted in the form: $y = A \times 10^{Bx}$, where y is the normalized magnitude of the error and x represents the distance in km. It is observed that the median values of Figure 9a and Figure 9c follow a linear trend in the log-linear scale, hence the median values are utilized to fit the power laws. For the distance estimation case, the fitted expression of the median values is: $y = 0.782 \times 10^{9.72e-6x}$, and in the case of center offset errors, the median trend is approximated by: $y = 0.828 \times 10^{4.05e-6x}$. These expressions serve the purpose of a preliminary estimation of OPNAV errors at different ranges.



(a) Normalized magnitude of the distance estimation errors



(b) Normalized magnitude of the centroiding angular errors



(c) Normalized magnitude of the offset between the estimated and the center of mass of the Moon

Figure 9: Normalized magnitude of errors of CR algorithm as a function of distance

4. Conclusion

The present work analyze OPNAV performance with images of the Moon for different observation geometries.

The performance of a more conventional EF algorithm and of CR algorithm for relative position estimation is compared. The comparative analysis shows that the CR algorithm consistently provides better performance than the EF technique, both in accuracy and computational time. The resulting errors in range estimation depend on distance, with values of approximately 22 km at a distance of 20,000 km and up to 422 km at a distance of 150,000 km. In terms of cross-axis distance estimation, the errors show values around 3.55 km (36 arcsec) at 20,000 km, and 11.85 km (16 arcsec) at 150,000 km. Furthermore, the Monte Carlo analysis is performed on images generated with camera specification and noise levels typical of a small spacecraft payload. The simulation outcomes show near normally distributed OPNAV performance with standard deviations around 0.68 km and 0.73 arcsec. A power

law function is also provided to approximate the trend of OPNAV accuracies as a function of the distance relative to the observed body, is found based on the outcome of this study. Colour maps illustrating OPNAV accuracies as a function of observation geometry provide a comprehensive picture of the expected performance of OPNAV depending on distance and orientation relative to the observed object.

With this, the expected OPNAV performances are characterized through images of the Moon in various observation geometries and distances. The results presented in this paper provide values of reference for preliminary mission analysis studies on small spacecraft in cislunar space, with plots showing the expected accuracies at different illumination conditions and ranges from the Moon.

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Wu, Christian Xianyang; Machuca, Pablo; Felicetti, Leonard; Sánchez, Joan-Pau

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