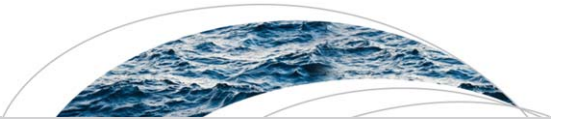
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RESEARCH ARTICLE
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A simple inverse method for the interpretation of pumped flowing fluid electrical conductivity logs

R. S. Moir[1], A. H. Parker[2], and R. T. Bown[1]

[1]Innovia Technology, St. Andrew's House, Cambridge, UK, [2]Cranfield Water Science Institute, Cranfield University, Cranfield, Bedfordshire, UK

Key Points:
- A simple method for automatic processing of pumped dilution tests is presented
- The method allows depth varying hydraulic conductivity to be calculated

Supporting Information:
- Readme
- Worked example of the inverse method

Correspondence to:
A. H. Parker,
a.parker@cranfield.ac.uk

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Abstract Pumped flowing fluid electrical conductivity (FFEC) logs, also known as pumped borehole dilution testing, is an experimentally easy-to-perform approach to evaluating vertical variations in the hydraulic conductivity of an aquifer. In contrast to the simplicity of the logging equipment, analysis of the data is complex and laborious. Current methods typically require repeated solution of the advection-dispersion equation (ADE) for describing the flow in the borehole and comparison with the experimental results. In this paper, we describe a direct solution for determining borehole fluid velocity that bypasses the need for complex numerical computation and repetitive optimization. The method rests on the observation that, while solving the ADE for concentration profile in the borehole (as required for modeling and combined methods) is computationally challenging, the solution for flow distribution along the length of the borehole given concentration data is straightforward. The method can accommodate varying borehole diameters, and uses the fact that multiple profiles are taken in the standard logging approach to reduce the impact of noise. Data from both a simulated borehole and from a field test are successfully analyzed. The method is implemented in a spreadsheet, which is available as supporting information material to this paper.

1. Introduction

Characterizing the variations in hydraulic conductivity is a critical step in the development of groundwater flow models. Regulatory bodies and water companies use these models to predict pollutant travel times and to design abstraction wells, ground water monitoring, and remediation schemes. Assuming uniform hydraulic conductivity across the model domain may be an acceptable simplification when modeling flow on the regional scale for the purposes of assessing available resources, but leads to large inaccuracies in solute transport predictions. As a result, it is important to characterize the distribution of hydraulic conductivity. Horizontal variations in transmissivity can be characterized through hydraulic (pumping) tests in multiple boreholes over the area, but vertical variations in hydraulic conductivity usually remain poorly characterized, and are thus the focus of this study.

Vertical variations can be characterized by packer testing, but this is time consuming and costly to perform. Impeller logging is faster, but requires cumbersome and expensive equipment and is not so accurate. By contrast, taking flowing fluid electrical conductivity logs, also known as pumped dilution testing, is an easy-to-perform, and relatively cheap method. However, the interpretation of the test results is difficult, relying on either subjective assessment or computer modeling of expected results to check that they fit the observed data. The technique is currently lacking an analysis method that matches the simplicity of the experimental procedure. This paper presents a method for interpreting single-borehole dilution-testing data that is objective and does not rely on a priori knowledge of the likely hydraulic conductivity profile. The paper focuses principally on analysis of collected data, rather than providing a critique of the experimental method. However, a sample data set from a Chalk aquifer in the United Kingdom is interpreted to illustrate the method proposed.

1.1. Field Method and Problem Definition

Flowing fluid electrical conductivity (FFEC) testing is an established technique, developed in its modern form by Doughty, Tsang and coworkers [*Tsang et al.*, 1990; *Doughty and Tsang*, 2000, 2005; *Tsang and Doughty*, 2003; *Doughty et al.*, 2008]. A tracer is introduced to the borehole and the evolution of its concentration with time is measured at regular depth intervals. Water flowing into the borehole, either due to

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pumping or natural gradients, causes dilution of the tracer-rich water column. With subsequent concentration measurements, the location of inflowing fractures can be identified. Various tracers and associated methods of concentration measurement can be used [*Kaufman and Todd*, 1962; *Lewis et al*., 1962; *Marine*, 1980; *Pedler and Urish*, 1988; *Pedler et al*., 1990; *Tsang et al*., 1990; *Novakowski et al*., 1998; *West and Odling*, 2007].

The concentration of tracer in the borehole changes due to both bulk movements of flow caused by pumping and hence inflow from the aquifer, and due to dispersion of the tracer in the borehole. Because of these twin effects, the resulting data are difficult to interpret. The aim of interpreting the data is to establish the flow variation with borehole depth z, assumed to be constant in time, $Q(z)$. *Javandel and Witherspoon* [1969] showed that the flow from an aquifer layer is proportional to its hydraulic conductivity, so that once $Q(z)$ has been established, hydraulic conductivity $K(z)$ can be found from

$$d_z Q = \frac{Q_{\max}}{T} K, \tag{1}$$

where Q_{max} is the flow at the pump and T is the transmissivity of the aquifer, established by an aquifer pumping test. Throughout this paper, the shorthand $d_x y$ (or $\partial_x y$) is used to mean the full (or partial) derivate of y with respect to $x - dy/dx$ (or $\partial y/\partial x$). In reality, numerical differentiation of Q often gives spurious results due to noise, and the position of inflows is often estimated manually from a flow log. One alternative is to use the forward modeling approach proposed by *Parker et al*. [2010].

To interpret a dilution test, it is necessary to understand the dispersion of tracer and measurement errors (both depth errors and concentration errors) to establish $Q(z)$ from the collected tracer concentration data $C(z, t)$. Ideally, the interpretation method would be as easy to perform as the logging test itself, not be computationally demanding, and not require expensive software.

In general, data from dilution tests can display inflows, where water enters the borehole, outflows where it leaves, and cross-flows where the water moves across the borehole. In order to infer reliable hydraulic conductivities from the data, the head induced by pumping should be much greater than the natural head differences in the aquifer. This study is thus limited to considering inflows, assuming that a sufficient pumping rate is used to overwhelm natural head differences.

The particular field method that this paper focuses on was developed for the boreholes in the Chalk aquifer of East Yorkshire in the UK up to 100 m deep. In this method, brine is introduced into the borehole at the start of the test. The well is then pumped at sufficient flow rates to avoid outflows due to natural pressure gradients. Fresh water flows into the well from the aquifer, diluting the salt tracer. Concentrations of salt are inferred from a simple conductivity probe lowered down the borehole. The test can be performed with cheap and portable equipment. For example, first, a background conductivity profile is taken using a handheld conductivity probe with measurements taken at define intervals. Then a hosepipe is lowered down the hole. The part of the hosepipe below the borehole casing is filled with salt solution and tap water is added above. The hosepipe is then slowly removed in order to leave a column of saline water in the open or screened section of the borehole. Subsequent conductivity measurements were made using the handheld conductivity probe; further details of the tests are given by *Parker* [2009].

1.2. Previous Approaches to the Problem

Since *Tsang et al*. [1990] reintroduced the technique, there has been significant progress in the interpretation of dilution test data. *Doughty et al*. [2008] provide an overview of these techniques and an example of their application. The methods currently available fall into two categories, which are referred to here as the "signature" and "modeling" methods. "Combined" methods use both signature and modeling approaches together, often in an automated manner. In the discussion below, we begin by introducing the advection-dispersion equation, which underlies these methods, and then evaluate the existing literature.

1.2.1. Form of the Advection-Dispersion Equation

The one-dimensional advection-dispersion equation (ADE) describes the movement of solute in a flowing fluid. In the flowing fluid electrical conductivity logging literature it appears in several forms, a general version of which is [e.g., *Mathias et al*., 2007]

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$$A\partial_t C = \partial_z(AD\partial_z C) - \partial_z(CQ) + q_{in} - q_{out}, \quad (2)$$

where C is the tracer concentration, t is the time at which a measurement is taken, z is the depth of the measurement, D is the dispersion coefficient of the tracer along the borehole, A is the cross-sectional area of fluid flow in the borehole, q_{in} is the influx of tracer into the borehole per unit depth, and q_{out} is the outflux of tracer from the borehole per unit depth.

The form of the Fickian dispersion coefficient D has been a matter of some debate in the literature. *Tsang et al*. [1990] assert that this may take either a constant, linear, or quadratic form with flow velocity, and then choose to implement the constant version. More recent approaches have either chosen the constant [*Evans*, 1995; *Doughty and Tsang*, 2005] or linear terms [*West and Odling*, 2007; *Mathias et al*., 2007]. *Taylor*'s [1953] theoretical analysis showed that for laminar flow in a smooth pipe, the theoretically correct form for D is quadratic in velocity, u ($D = D_0 + \alpha u + \beta u^2$), where D_0 is a constant term, α (the linear coefficient) can be termed as the borehole dispersivity, and β is a quadratic coefficient. Later work by *Aris* [1956] showed that molecular diffusion could be incorporated as a constant term. *Low et al*. [1994] used the quadratic form suggested by *Taylor* [1953]. However, in practice salinity is strongly affected by lateral diffusion, the turbulence induced by the moving salinity probe, and flow distortions due to variable well radius (Chin-fu Tsang, personal communication, 2013). Indeed, if lateral diffusion is sufficiently large, the constant form of D is valid. The method described in this paper is valid for both constant ($D = D_0$) and linear ($D = D_0 + \alpha u$) forms. *Taylor*'s [1953] theoretical analysis showed that for laminar flow in a smooth pipe, the theoretically correct form for D is quadratic in velocity, u

$$(D = D0 + \alpha u + \beta u - 2). \quad (3)$$

1.2.2. Modeling Methods

Modeling methods produce predictions of results of dilution testing experiments given user-defined inputs. The approach typically taken is to vary the input parameters until the modeled results match the observed data. The input parameters, which include location and hydraulic conductivity of inflows, are then taken as a likely representation of the borehole properties. Modeling is carried out by finding a numerical solution to the one-dimensional advection-dispersion equation (ADE) [*Doughty and Tsang*, 2005].

In order to interpret their early logs, *Tsang et al*. [1990] developed a modeling code called BORE. This was later updated to deal with outflows as well as inflows (BORE II) [*Doughty and Tsang*, 2000] and used in their recent work [*Doughty et al*., 2008]. *Evans* [1995] built on the success of BORE by using less computationally expensive algorithms and making allowances for the time lag between measurements taken at the top of the borehole and the bottom. His code also performs a least squares fit to an observed data set, cutting out the time consuming trial and error required to calibrate models.

Modeling methods require the user to hypothesize the form of a hydraulic conductivity profile. Although this is then checked against the observed profile, this is distinct from attempting to infer this directly from experimentally derived data. Solving the ADE numerically is widely recognized to be problematic, with solutions being notorious for failing to conserve mass [*Szymczak and Ladd*, 2003]. The method is rather complex, requiring the use of either bespoke code or expensive mathematical software. Improving model fits by trial and error is either labor intensive or computationally expensive.

1.2.3. Signature Methods

An alternative to modeling approaches is to inspect the original data sets for "signatures" that correspond to particular physical processes. The portion of the data set that contains the signature can then be evaluated using simple methods to derive quantities relating to the physical process being observed. *Tsang et al*. [1990] recognized signatures relating to point inflows, and developed analysis methods to derive inflow rate and salinity of inflowing water. These were programmed into an initial analysis code called PRE. *Low et al*. [1994] used the moments of data taken either across or between inflows to derive the vertical flow velocity either around point inflows or in borehole sections with no inflow. A different approach to finding velocity in sections of borehole with no inflow was taken by *West and Odling* [2007], who use an analytical solution to a simplified version of the ADE. *Tsang and Doughty* [2003] extended the *Tsang et al*. [1990] method by pumping the borehole at different rates in three subsequent tests to provide information

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on the heads of individual inflows. *Doughty and Tsang* [2005] provide a useful review of a wide range of signature methods for characterizing inflows, outflows, and cross-flows.

Although easier to use than the modeling method, signature methods use simplified approaches derived from physical insights or truncated versions of the ADE. They can thus only be applied to sections of the log that can be assumed to behave in some idealized manner, such as a long borehole with isolated inflows.

1.2.4. Combined Methods

The approach taken by most workers has been a combined one, using signatures to establish initial parameter estimates, which are then refined by modeling. This is described as an "integral" approach by *Doughty and Tsang* [2005]. *Low et al.'s* [1994] STARBORE code automates this process. The combined approach has been developed to address the shortcomings of pure modeling and signature methods, but is somewhat laborious.

2. The Inverse Method

2.1. Approach

As outlined above, all existing interpretation methods have shortcomings. A superior interpretation method would combine the strengths of both the modeling and signature approaches by working directly with the measured data, and using the full ADE. Inverse methods have been widely used in hydrogeology [for example, *Carrera et al*., 2005, *Linde et al*., 2006, *Hendricks Franssen et al*., 2009, *Klepikova et al*., 2013] but have not been applied to open borehole dilution testing.

Many of the apparent shortcomings of existing open borehole dilution testing interpretation methods have not proved to be excessively problematic in practice because they have been applied to aquifers that flow only at a few discrete fractures in lengthy boreholes. Unfortunately, such methods have been of limited utility to the present authors for characterizing the details of hydraulic conductivity variation in shallow boreholes in a dual porosity Chalk aquifer.

2.2. Form of the ADE

Some minor changes to equation (2) are required for the present problem. First, no outflows from the borehole are permitted as the head difference generated by pumping is greater than any natural differences in the aquifer. Hence $q_{out} = 0$; with this simplification q_{in} can be written as $q_{in} = C_{in}\partial_z Q$, where C_{in} is the concentration of the tracer in the water flowing into the borehole. For example, if salt is used as the tracer, the inflowing water is likely to have nonzero salt concentration.

The dispersion coefficient, D, may depend on the fluid velocity in the borehole, u. The present analytic method is limited to constant diffusion coefficients ($D = D_0$) and diffusion coefficients linear in velocity, $D = D_0 + \alpha u$, where u is the fluid flow velocity in the borehole and D_0 and α are constants. While the theoretically correct quadratic form would be a better choice, this greatly complicates the analysis. Quadratic variations can be treated with the linear form by linearization within a limited range of Q. However, many previous studies have used a constant [*Evans*, 1995; *Doughty and Tsang*, 2005; *Tsang et al*., 1990] or linear [*West and Odling*, 2007; *Mathias et al*., 2007] form successfully.

Substituting in the diffusion coefficient and q_{in}, the governing equation is

$$A\partial_t C = \partial_z([AD_0 + \alpha Q]\partial_z C) - \partial_z(CQ) + C_{in}\partial_z Q. \tag{4}$$

2.3. Solving the ADE

The equation can be expanded using the product rule to give

$$A\partial_t C = AD_0\partial_z^2 C + D_0\partial_z A\partial_z C + \alpha Q\partial_z^2 C + \alpha\partial_z C\partial_z Q - C\partial_z Q - Q\partial_z C + C_{in}\partial_z Q. \tag{5}$$

Collecting terms on Q, and rearranging gives

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$$A\partial_t C - AD_0\partial_z^2 C - D_0\partial_z A\partial_z C = \left(\alpha\partial_z^2 C - \partial_z C\right)Q + \left(\alpha\partial_z C - C + C_{in}\right)\partial_z Q. \tag{6}$$

The solution of this second-order equation for C is at the heart of the complexity of the modeling approaches. However, we observe that this is a first-order differential equation in Q, easily capable of solution by standard analytical methods. This can be rearranged to a standard form [e.g., *Kreyszig*, 1999]

$$\partial_z Q + \underbrace{\left(\frac{\alpha\partial_z^2 C - \partial_z C}{\alpha\partial_z C - C + C_{in}}\right)}_{r} Q = \underbrace{\left(\frac{A\partial_t C - AD_0\partial_z^2 C - D_0\partial_z A\partial_z C}{\alpha\partial_z C - C + C_{in}}\right)}_{s}. \tag{7}$$

For brevity, the contents of the first and second brackets in equation (7) are henceforth referred to r and s. The resulting equation $\partial_z Q + rQ = s$ is solved by multiplying by an appropriately chosen integrating factor [e.g., *Kreyszig*, 1999], such as

$$\mu(z) = \exp\left(\int_0^z r dz'\right). \tag{8}$$

Introducing this factor, and simplifying using the chain rule yields

$$\exp\left(\int_0^z r dz'\right)(\partial_z Q + rQ) = \exp\left(\int_0^z r dz'\right)s \tag{9}$$

$$\therefore \partial_z\left[Q.\exp\left(\int_0^z r dz'\right)\right] = \exp\left(\int_0^z r dz'\right)s \tag{10}$$

This equation can now be integrated with respect to z to yield a solution for the flow rate

$$Q = \frac{1}{\exp\left(\int_0^z r dz'\right)}\left[\int_0^z \exp\left(\int_0^{z''} r dz'\right)s dz'' + c\right], \tag{11}$$

where c is a constant of integration equal to the value of Q at $z = 0$, $c = Q(0)$ (this is a result of choosing 0 as the lower limit of integration). Using the definition introduced in equation (8), a final solution for Q can be written

$$Q(z) = \frac{1}{\mu(z)}\left[\int_0^z \mu s dz' + Q(0)\right]. \tag{12}$$

It is useful to consider the special case where α is assumed equal to zero and C_{in} is constant (as discussed above). In this case, a closed form solution for equation (8) is possible

$$\mu(z) = \frac{C_z - C_{in}}{C_0 - C_{in}}. \tag{13}$$

It is also possible to determine some of the integrals analytically and equation (12) can be written using only a single numerical integral

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$$Q(z)=\frac{1}{C(z,t)-C_{in}}\left\{D_0[A(z)\partial_z C(z,t)-A(0)\partial_z C(0,t)]-\int_0^z A(z)\partial_t C(z,t)dz+Q(0)[C(0,t)-C_{in}]\right\}. \quad (14)$$

2.4. Borehole Area Variation

If boreholes are open hole, without any casing, then they do not have constant cross section. Unfortunately, as shown in Figure 1, this applies to the boreholes the authors have studied. If cross-sectional variations are suspected, caliper logs can be run to establish how the well diameter, d, varies with depth, z. Where these are available, they should be used to inform our interpretation of dilution test data, by providing depth-varying data for $A=\pi d^2/4$ in equation (7).

However, some care is needed in evaluating A, as this is strictly the cross-sectional area of the fluid flow, not the borehole. These are unlikely to be the same when flow exits a small diameter section of borehole into a large diameter section. In this case, a steadily expanding jet of fluid will be produced [*Bearden et al*., 1970; *Hill*, 1990]. A reasonable approximation is that the jet diameter expands as $\sim 0.1z$ [*Lee and Chu*, 2003]. Where the borehole contracts, a sudden contraction of the area of the flow can be assumed. Introducing the notation $d_{c,i}$ for the measured (caliper) diameter of the borehole at depth z_i, the hydraulic diameter $d_{h,i}$ can be evaluated as

$$d_{h,i}=\begin{cases} d_{h,i-1}+0.1(z_i-z_{i-1}), & d_{c,i} > d_{h,i-1}+0.1(z_i-z_{i-1}) \\ d_{c,i}, & d_{c,i} \le d_{h,i-1}+0.1(z_i-z_{i-1}) \end{cases}. \quad (15)$$

Figure 1 shows a typical caliper log for a borehole on the Chalk of East Yorkshire showing the estimated hydrodynamic diameter.

2.5. Estimation of Dispersion Parameters

In the previously referenced works [e.g., *Doughty and Tsang*, 2005], α and D_0 emerge as fitted parameters. In order to apply the method proposed in this paper, they must be estimated a priori.

There are three possible ways to estimate α and D_0. First, signature methods can be used to evaluate these values. Second, theoretical computations might be used based on the known properties of the borehole, particularly where the borehole area does not vary. Third, the values might be determined by experiment, for example on a section of borehole with solid casing or in a laboratory rig.

A sensitivity analysis was performed on α which showed varying it over 1 order of magnitude in each direction had little impact. Moreover, outside of this range it is easy to recognize unphysical results. If no information is available, setting both α and D_0 to zero provides a reasonable first approximation.

2.6. Variations in the Concentration of the Tracer in the Water Flowing into the Borehole

As with all the other terms in both r and s in equation (7), C_{in} can vary in space and the method remains entirely valid. This flexibility of the method is potentially valuable, as a well may, in general, penetrate layers of different salinity values, particularly in strongly heterogeneous deep systems. In such cases, ambient conductivity measurements or other published methods [for example, *Doughty and Tsang*, 2005] should be used to determine $C_{in}(z)$.

2.7. Error Handling

In the absence of noise, the minimum information that is needed for this inversion method is the tracer concentration at two separate times. Accordingly, the methods commonly used to perform dilution tests collect more information than is strictly required to deduce a velocity profile. The redundancy in this additional information can be used to compensate for real-world noise by averaging across several data points. Data points are collected across both depth and time so averages can in principle be used to improve the accuracy of the algorithm in two ways:

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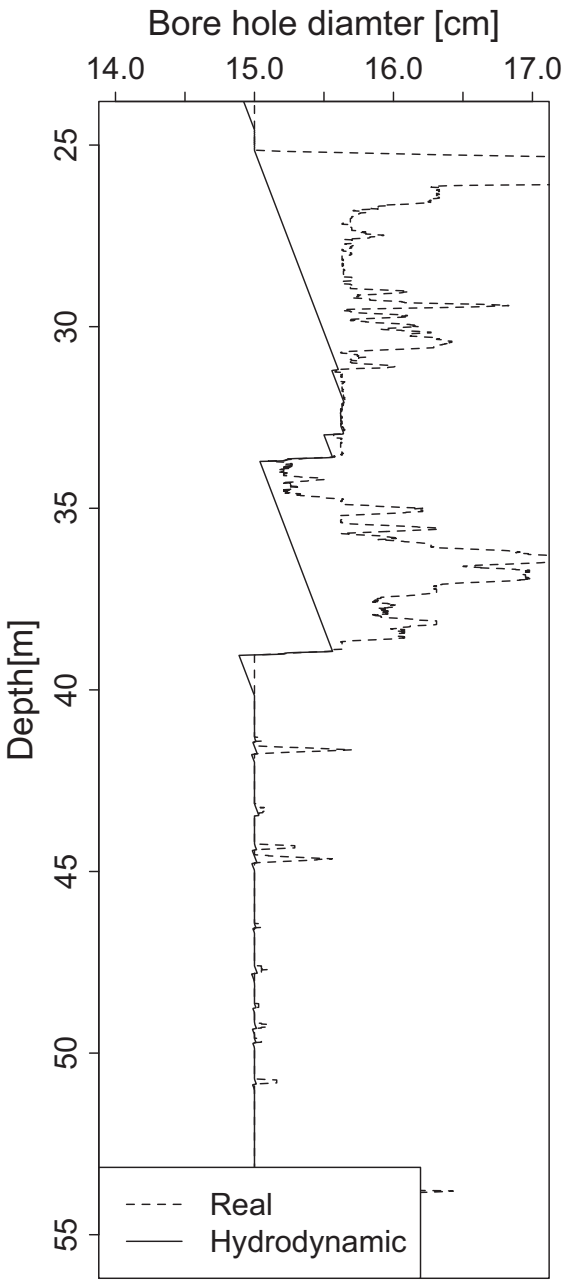


Figure 1. Diameter variation with depth for the Wilfholme M1 borehole, showing caliper log (real diameter) and the diameter of the water jet (hydrodynamic).

1. **Time averaging**: Averaging across concentration measurements made at different times.
2. **Depth averaging**: An appropriate reduction in resolution of the z coordinate.

2.7.1. Time Averaging

In a typical pumped dilution test, the concentration profile is measured at a series of evenly spaced time intervals ΔT_i. The inversion of the concentration profile involves calculating both time and space derivatives. It is clear that repeated time measurements contribute to accurately evaluating the time derivatives, but it is also the case that they can be used to boost significantly the accuracy of the spatial derivatives if it is assumed that $u(z)$ is independent of t—that is, the flow profile is time invariant.

To demonstrate the handling of errors by time averaging, an error term ε, is introduced to account for noise in the concentration measurement. We assume that the noise values ε at given times and spatial locations are independent, identically distributed random variables normally distributed with a mean of zero. The measured value of concentration is thus $C_m = C + \varepsilon$. Differentiating obtains

$$\begin{aligned} C_m &= C + \varepsilon \\ \partial_z C_m &= \partial_z C + \partial_z \varepsilon \\ \partial_z^2 C_m &= \partial_z^2 C + \partial_z^2 \varepsilon \\ \partial_t C_m &= \partial_t C + \partial_t \varepsilon \end{aligned} \quad . \tag{16}$$

An average of concentration readings across measurement times, denoted by $\langle \ldots \rangle$, is introduced to handle the spatial derivative errors, defined by

$$\langle \ldots \rangle = \frac{1}{T} \sum_{i=1}^{T} \ldots_i \Delta T_i, \tag{17}$$

where i denotes time index and ΔT_i is the time step at time point i. The standard method used in dilution testing of taking a series of measurements of concentration profiles over time is ideally suited to the calculation of this average. If a sufficiently large number of flow profiles are taken and if ΔT_i is a constant, then $\lim_{T \to \infty} \langle \varepsilon \rangle = 0$ and

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$$\begin{aligned}\langle C_m\rangle &= \langle C\rangle + \langle \varepsilon\rangle = \langle C\rangle \\ \langle \partial_z C_m\rangle &= \partial_z\langle C_m\rangle = \partial_z\langle C\rangle + \partial_z\langle \varepsilon\rangle = \langle \partial_z C\rangle \\ \langle \partial_z^2 C_m\rangle &= \partial_z^2\langle C_m\rangle = \partial_z^2\langle C\rangle + \partial_z^2\langle \varepsilon\rangle = \langle \partial_z^2 C\rangle\end{aligned} \quad . \tag{18}$$

If ΔT_i is not a constant between measurements, equation (17) must be applied explicitly to obtain a weighted average for these expressions. This will lead to less than perfect error cancellation.

Using this approach, the time derivative is surprisingly simple

$$\begin{aligned}\langle \partial_t C_m\rangle &= \frac{1}{T}\sum_{i=1}^{T}\left(\frac{C_i+\varepsilon_i-C_{i-1}-\varepsilon_{i-1}}{\Delta T_i}\right)\Delta T_i \\ &= \frac{1}{T}\sum_{i=1}^{T}(C_i+\varepsilon_i-C_{i-1}-\varepsilon_{i-1}) \\ &= \frac{1}{T}\left[\begin{array}{c}(C_1+\varepsilon_1-C_0-\varepsilon_0)+(C_2+\varepsilon_2-C_1-\varepsilon_1)+\ldots \\ +(C_{T-1}+\varepsilon_{T-1}-C_{T-2}-\varepsilon_{T-2})+(C_T+\varepsilon_T-C_{T-1}-\varepsilon_{T-1})\end{array}\right] \\ &= \frac{1}{T}\{[C_T+\varepsilon_T]-[C_0+\varepsilon_0]\} \\ &= \frac{1}{T}\{[C(T)+\varepsilon(T)]-[C(0)+\varepsilon(0)]\} \\ &\approx \frac{1}{T}[C(T)-C(0)] = \langle \partial_t C\rangle\end{aligned} \tag{19}$$

In contrast with the averaging of the spatial derivatives, this expression is robust to nonuniform ΔT_i. Note that because these averages are taken over the entire time period of the test, they are by definition time invariant. The averaging controls errors in all the above expressions, except that for $\langle \partial_t C_m\rangle$. The time derivative prevents repeated experiments eliminating errors entirely, and still requires that errors are small. Nonetheless, the averaging of concentration and its spatial derivatives eliminates substantial errors.

A wide range of options are available to improve the treatment of time derivatives, such as averaging time derivatives over part of the range, or using some form of curve fitting or noise filtering to provide a better estimate of the actual change in concentration at each depth in the borehole over time. Accordingly, we assume that the error term $\varepsilon(T)-\varepsilon(0)$ can be neglected in the analysis that follows, with appropriate fitting or filtering implied in the evaluation of $\langle \partial_t C_m\rangle$. (In the examples illustrating this method in section 3, it suffices simply to neglect these errors without additional treatment.)

Applying the time average to equation (7) and substituting equations (18) and (19) yields revised expressions for r and s

$$\begin{aligned} r &= \frac{\alpha\partial_z^2\langle C_m\rangle - \partial_z\langle C_m\rangle}{\alpha\partial_z\langle C_m\rangle - \langle C_m\rangle + C_{in}} \\ s &= \frac{A\langle \partial_t C_m\rangle - AD_0\partial_z^2\langle C_m\rangle - D_0\partial_z A\partial_z\langle C_m\rangle}{\alpha\partial_z\langle C_m\rangle - \langle C_m\rangle + C_{in}}\end{aligned} \quad . \tag{20}$$

Once $\langle C_m\rangle$ (the time average of concentrations) and $\langle \partial_t C_m\rangle$ (discussed above) have been obtained from the concentration data, r and s can be obtained from this equation and the rest of the analysis can proceed as described above.

In the special case of α equal to zero and C_{in} is constant, the final solution for Q can be written

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$$Q(z)=\frac{1}{C_m-C_{in}}\left\{D_0\left[A(z)\partial_z\langle C_m\rangle-A(0)\partial_z\langle C_m\rangle|_{z=0}\right]-\int_0^z A(z)\langle\partial_t C_m\rangle dz+Q(0)[\langle C_m\rangle-C_{in}]\right\}. \quad (21)$$

2.7.2. Depth Averaging

Having averaged concentrations over time, the question arises as to whether they should be averaged across depth z to reduce errors further. The obvious problem with this approach is that while error is reduced, so is the spatial resolution of the data. This approach is only appropriate for low signal-to-noise ratios and closely spaced concentration data with depth. The examples in this paper do not benefit from depth averaging. However, the technique may be useful when using automatic logging tools that take conductivity measurements that are closely spaced in depth.

2.8. Implementation

Calculating hydraulic conductivity from dilution test data requires several steps:

1. If it is required to correct for varying borehole cross section, the hydrodynamic cross-sectional area is calculated, as described in section 2.4.

2. $\langle C\rangle$ and $\langle\partial_t C\rangle$ are calculated by equations (17) and (19).

3. First and second derivatives of $\langle C\rangle$ with depth are calculated.

4. Equation (20) is used to determine r and s.

5. μ is calculated from equation (8).

6. The flow rate, Q, is determined from equation (12).

7. Hydraulic conductivity is found from equation (1).

The most complex operations required are numerical integrations and differentiations. The entire process can easily be implemented in a standard spreadsheet using simple formulae. An example, using data described below, is included as supporting information material to this paper.

3. Examples of Application

3.1. Synthetic Data

To verify the effectiveness of the inversion algorithm, it was run on synthetic data sets created by solving the ADE numerically for a known velocity profile. One example is shown here for a hypothetical 70 m borehole, pumped from the bottom, with measurements taken from 15 m, with inflows shown in Figure 2c (dashed) and hydrodynamic cross-sectional area shown below in Figure 2b.

These data were used as inputs to a numerical model of the ADE implemented using MATLAB's "pdex" function [*The Mathworks Inc.*, 2010]. Diffusion coefficients were chosen to be $D_0 = 0$ m^2/s, $\alpha = 1$ m, as these values are typical for the actual boreholes studied by the authors [*West and Odling*, 2007]. Random Gaussian noise was added to all of the data points with a signal-to-noise ratio of 20. The resulting concentration plots at different times through the simulation are shown in Figure 2a. The simulation was set to provide data spaced every 1 m down the borehole. It is clear from these plots that the noise and change in hydrodynamic borehole cross section have combined to significantly impact the output. This is unsurprising, as the variation in hydrodynamic borehole cross section alone is up to $\pm$25%.

The inversion algorithm with time averaging was applied to recover the flow rate of water along the borehole. This is shown in Figure 2c along with original synthetic flow data. Simple differentiation of the data to obtain hydraulic conductivity introduces excessive noise. Differentiation is thus carried out using a Savitsky-Golay filter [*Savitsky and Golay*, 1964]. A filter length of 7 m was chosen as a compromise between acceptable smoothing, removal of most of the spurious negative data, and loss of resolution and fine detail. The interpreted log output is compared to the input to the ADE simulation as shown in Figure 2d.

Comparing the input data and the output in Figures 2c and 2d gives an indication of the accuracy of the algorithm. Depth boundaries between sections of differing hydraulic conductivity are reproduced to an

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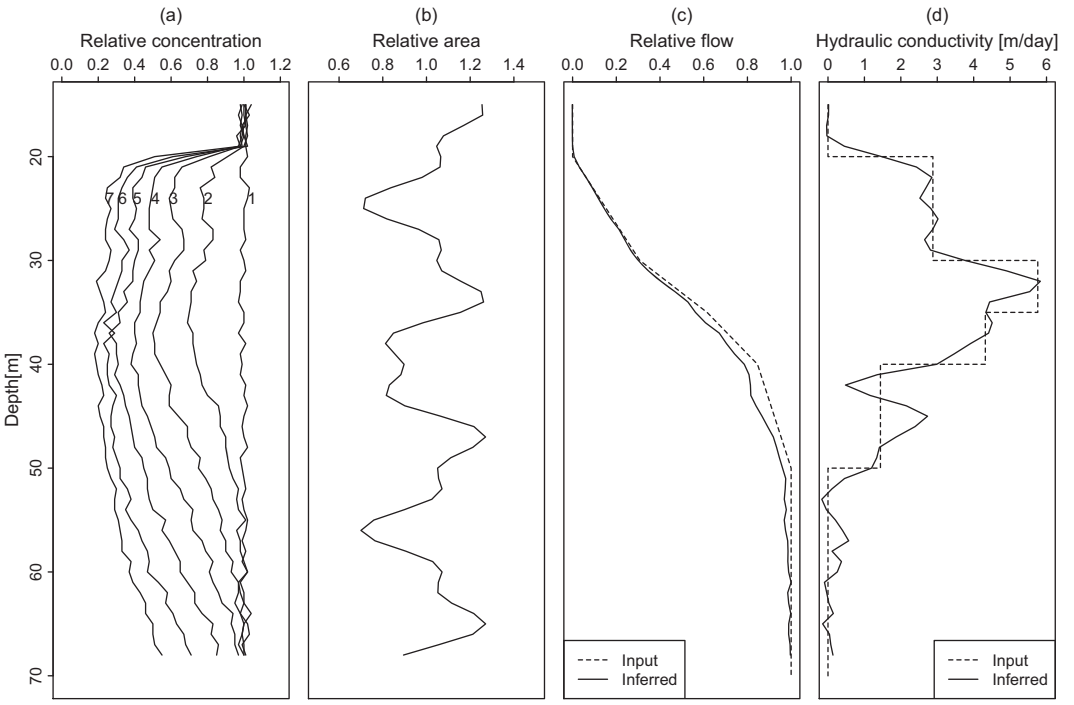


Figure 2. Hypothetical 70 m borehole showing (a) concentration as it reduces every 100 min, (b) variation in the borehole cross section with depth, (c) input (dashed) and inferred (solid) flow profiles, and (d) input (dashed) and inferred (solid) hydraulic conductivity with depth in m/d.

accuracy of around 1 m. The most extreme error in hydraulic conductivity is 20%, with $<3\%$ being maintained through much of the depth range. Given that 5% noise was added to the data and data points were only provided at 1 m intervals, this is a reasonable level of performance. By contrast, conventional interpretation of data of the type shown in Figure 2a using signature methods would be extremely difficult.

3.2. Chalk Aquifer of East Yorkshire UK

As well as testing the method against synthetic data sets, the algorithm has also been used to analyze data from a dilution test carried out on the Chalk aquifer of East Yorkshire. Variations in the hydraulic conductivity of this aquifer can be complex due to the presence of both bulk and fracture permeability, the dissolution of fractures and the effects of stratigraphic variation and periglacial weathering [*Hartmann et al.*, 2007]. The data analyzed are from a borehole known as M1 on the Wilfholme Landing field site (53°54′37.05″N, 00°22′56.43″W). An initial salt concentration profile was measured to determine the ambient concentration—effectively the salinity of water in the formation. A 1.9 cm internal diameter hosepipe was lowered down the hole. The part of the hosepipe below the water level was filled with concentrated salt solution (230 g of salt per liter). Tap water was added so that the freshwater-saltwater interface within the hosepipe was at the level of the base of the casing. The hosepipe was then slowly removed in order to leave a column of saline water in the open section of the borehole. This leaves a concentration profile that is constant with depth within the open section of the borehole. Water is then pumped from the bottom until the evolution of concentration ceased to change. Note that the initial concentration at $t = 0$ need not be constant with depth. Spatial variation of this initial concentration does not affect the analysis as it relies on changes in the salinity concentration throughout the test. The measured conductivity curves are shown in Figure 3a.

A caliper log is also available for the borehole—a section of this caliper log is shown in Figure 1.

The measured data were processed using the inverse method described in this paper, again using α as 1 m with corrections for changing flow cross section based on the caliper log. Differentiation to obtain inflow rates was carried out using a finite difference method and the hydraulic conductivity was calculated according to equation (1). The results are shown in Figure 3c.

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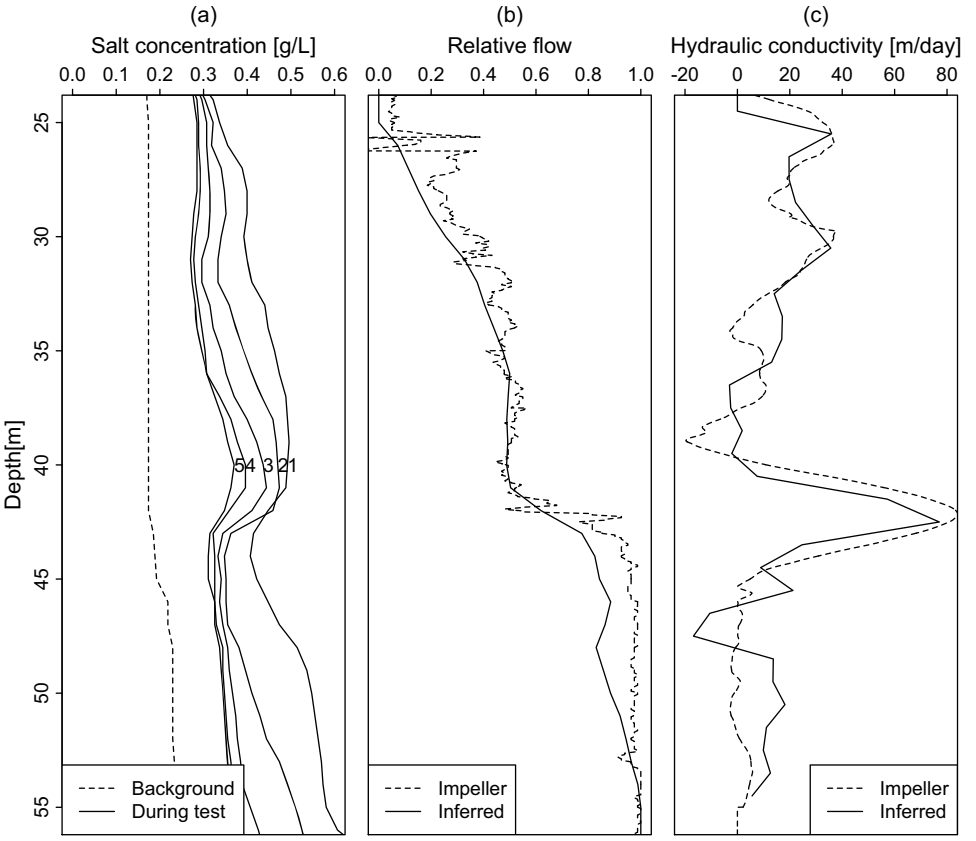


Figure 3. Data from M1 borehole showing: (a) concentrations with depth of pumped dilution test (pumping from bottom of borehole)—data recorded at 0, 30, 70, 110, and 165 min shown in lines 1–5, ambient concentration shown as dashed line; (b) flow speed data with depth recorded by impeller during pumping from top of borehole; and (c) calculated hydraulic conductivity with depth from the inverse method described in this paper (solid line) and from the first derivative of the impeller data, calculated using Savitzky-Golay filtering with 7 m filter length (dashed line).

Also shown in Figure 3 for comparison are the results of an investigation carried out with an impeller sonde. The raw flow speed data, taken from *Parker* [2009], are shown in Figure 3b. The impeller log was obtained by pumping the borehole from the top. To facilitate easier comparison with the output of this method, we have reversed the flow log and normalized both traces between 0 and 1. The data are rather noisy, so the differentiation required to calculate impeller inflow rates was carried out using a Savitsky-Golay filter [*Savitzky and Golay*, 1964]. A filter length of 7 m was chosen as a compromise between acceptable smoothing, removal of most of the spurious negative data, and loss of resolution and fine detail. The results are compared with the output of the inverse method in Figure 3c.

The dilution and impeller data shown as solid and dashed lines in Figure 3c generally show good agreement, especially regarding the zone of high hydraulic conductivity around 42 m depth. The impeller logs show unrealistic negative hydraulic conductivity values between 38 and 40 m, whereas the dilution test data show this simply as an area of low hydraulic conductivity.

4. Discussion

This paper uses the ADE directly to infer flow speed, allowing vertical hydraulic conductivity profiles to be obtained from pumped open well dilution test data. As dilution tests are cheap and easy to perform, and this analysis is straightforward and can be carried out using widely available spreadsheet software, the technique presented here allows many more practitioners to characterize vertical variations in hydraulic conductivity.

Existing methods using the signature approach, as reviewed by *Doughty and Tsang* [2005], are well suited to simple cases with widely separated point inflows. However, where inflows are close together and/or

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there is significant primary porosity, interpretation is significantly more difficult. In dual-porosity aquifers, such as the Chalk, both problems coexist and an alternative method, such as the one presented here, is required.

This method assumes the conclusions of *Javandel and Witherspoon* [1969], specifically, that flow from an aquifer layer is proportional to its hydraulic conductivity. Therefore, this method can only be applied to situations where the ambient flows are insignificant compared to the flow induced by the pump. It is inapplicable to outflows and cross-flows. While boreholes exist for which this is a constraint, the authors found many boreholes in East Yorkshire for which this could be achieved with an easily portable pump powered by a single-phase 3 kW generator [*Parker*, 2009].

This method relies on the linearity of any velocity terms in the ADE. Theoretically, the dispersion term should be quadratic; however, in practice other workers in the field [*Tsang et al*., 1990; *Evans*, 1995; *Doughty and Tsang*, 2005; *Mathias et al*., 2007; *West and Odling*, 2007] have limited their attention to constant and linear approximations. In the absence of additional data, first principle considerations can be used to determine an appropriate linearization of the full quadratic form [*Taylor*, 1953], alternatively, the dispersion coefficients may be evaluated using signature methods [e.g., *West and Odling*, 2007].

The standard experimental method used has been capitalized on here to cancel noise in almost all terms constructed from the data. However, the time gradient in concentration is not amenable to this approach. In practice, this does not appear to have adversely affected the results, perhaps due to the low levels of noise in the data collected. In principle, a more sophisticated, although standard, approach to noise treatment than that presented here may be required in some cases.

Any systematic errors, for example, miscalibration of the conductivity meter, or inaccurate estimation of influx conductivity will be amplified by the integration step. One approach to mitigate this is to scale the results to the two known flow rates in the borehole, for example, zero at the top and the pump rate at the bottom (or vice versa).

While further work could extend the method to address these limitations, as presented it should be applicable to a variety of boreholes.

5. Conclusions

Existing methods for the interpretation of pumped flowing fluid electrical conductivity logs require iterative numerical modeling. The process is laborious, computationally expensive, and requires specialist software. In this paper, a simple automatic method is outlined that can be implemented in standard spreadsheet software. The method rests on the observation that, while solving the ADE for concentration (as required for modeling and combined methods) is computationally challenging, the solution for flow speed given concentration data is straightforward.

The method uses the data collected directly in the ADE to give the velocity profile in the borehole. The method accounts for variations in borehole diameter and includes averaging processes to reduce the influence of experimental errors.

The proposed method represents a significant improvement to current practice. It provides an interpretation method that is congruent with the simplicity of the logging apparatus. It is hoped that this will make the means to characterize vertical variations in hydraulic conductivity accessible to a much wider range of users in the future. To this end, a spreadsheet implementing the Chalk aquifer of East Yorkshire example is available as part of the supporting information materials to this paper.

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Moir, R. S.; Parker, Alison; Brown, R. T.

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