



Distributed optimal nonlinear dynamic inversion for multi-agents consensus

Sabyasachi Mondal^{*,1}, Antonios Tsourdos²

Cranfield University, College road, Cranfield, MK430AL, UK

ARTICLE INFO

Recommended by T. Parisini

Keywords:

Optimal control
Distributed nonlinear dynamic inversion
Consensus
Nonlinear agents

ABSTRACT

In this paper, we propose an optimal distributed controller based on Nonlinear Dynamic Inversion (NDI) theory and apply it to solve the consensus of nonlinear multi-agent systems (MASs). Our proposed method addresses the limitations of existing Distributed Nonlinear Dynamic Inversion (DNDI) techniques, which only apply to agents with square output. We formulated an optimal control problem to minimize a quadratic cost function while satisfying a set of linear constraints derived by simplifying the enforced consensus error dynamics. By relaxing the previous limitation, we introduced a distributed optimal framework called Distributed Optimal NDI (DONDI). This framework achieves consensus and incorporates additional objectives, such as minimizing control energy. The design of Optimal DNDI inherits all the advantages of NDI and provides an optimized allocation of control for achieving consensus in MAS. Also, we have shown how the controller handles the communication noise. This approach represents a significant advancement in multi-agent control, and our experimental results demonstrate its satisfactory performance and effectiveness.

1. Introduction

Nonlinear Dynamic Inversion (NDI) is a widely adopted approach for designing controllers for various nonlinear systems. Its attractiveness lies in its simplicity, effectiveness, and ability to transform nonlinear dynamic equations into linear ones, which has captured the attention of many researchers. This control method shares similarities with input-output feedback linearization (Sastry and Isidori (1989); Hovakimyan, Lavretsky, Yang, and Calise (2005)). Additionally, the closed-loop system response exhibits stability similar to that of a linear system, allowing for the straightforward implementation of well-established linear control techniques. The key features of the dynamic inversion controller include the expression of control in closed form, ease of mechanization, achievement of global exponential stability, incorporation of nonlinear kinematics in the plant inversion, and reduced reliance on individual gain tuning or gain scheduling.

Dynamic inversion-based controllers have found widespread applications in various fields. For instance, Enns et al. utilized dynamic inversion to design flight controllers for specific systems (Enns, Bugajski, Hendrick, & Stein, 1994). Singh et al. proposed a dynamic inversion-based controller for enabling the autonomous landing of UAVs (Singh & Padhi, 2009). In the context of spacecraft control, Tafazoli et al.

and Malekzadeh et al. developed attitude controllers for flexible spacecraft using dynamic inversion (Malekzadeh, Naghash, & Talebi, 2011; Tafazoli & Khorasani, 2004). Gregory (2001) did similar work for flexible aircraft. Acquatella et al. used NDI for trajectory tracking of reusable launch vehicles (Acquatella, Briese, & Schnepfer, 2020). In maritime applications, Subudhi and Atta, as well as Chowdhury et al. applied dynamic inversion-based control with mechanized feedback to remove system nonlinearities (Chowdhury, Vishwanathan, Prasad, Kumar, & Panda, 2014; Subudhi & Atta, 2009). Mondal et al. utilized NDI to achieve leader-follow formation flying of UAVs, with the follower tracking the leader's attitude (Mondal & Padhi, 2015). A dynamic inversion-based approach generates real-time trajectories for omnidirectional vehicles (Kalmár-Nagy, 2016). In other studies, dynamic inversion played a vital role in attitude control for flexible aircraft, as presented by Caverly, Girard, Kolmanovsky, and Forbes (2016), and in a control design study by Horn (2019). Furthermore, Lombaerts et al. successfully solved the attitude control problem of a hovering quad tilt-rotor eVTOL using dynamic inversion (Lombaerts et al., 2019). Lungu (2022) presented dynamic inversion-based control techniques for the automatic landing of fixed-wing UAVs. Panish, Nicholls, and Bacic (2023) designed a Tilt-wing VTOL Aircraft's NDI-based flight control. A variety of DI is used for tracking control of

* Corresponding author.

E-mail addresses: sabyasachi.mondal@cranfield.ac.uk (S. Mondal), a.tsourdos@cranfield.ac.uk (A. Tsourdos).

¹ Lecturer in Autonomous System, School of Aerospace, Transport and Manufacturing (SATM).

² Head of the Autonomous and Cyber-Physical Systems Centre.

an underactuated ship (Ye & Zong, 2018). Also, NDI found application in flexible air-breathing hypersonic vehicle (Wu, Meng, & Wang, 2018). These instances demonstrate the versatility and effectiveness of dynamic inversion-based controllers across various applications and domains.

The NDI-based controllers mentioned previously were primarily applied for single-agent control. However, an exciting development in this field was introduced by Mondal and Tsourdos (2021), who proposed an NDI-based controller for cooperative multi-agent systems (MASs). In this approach, the conventional error expression of NDI was modified to utilize consensus error, aiming to achieve consensus among N agents by driving the consensus error to zero. This distributed version of NDI, termed Distributed NDI or DNDI, addresses the consensus problem in nonlinear agent systems while retaining all the advantages of NDI discussed earlier. The authors delved into the challenges associated with MAS communication, including switching topology, delay, and noise, and demonstrated that DNDI efficiently handles these issues. Furthermore, the robustness of DNDI was tested by experimenting with nonlinear agents experiencing random actuator faults, revealing DNDI's ability to withstand such faults (Mondal & Tsourdos, 2022b). To extend its capabilities, the authors introduced a new variant called neuro-adaptive DNDI, which deals with external disturbances affecting the consensus process. In this approach, a neuro-adaptive structure was implemented to approximate and compensate for the effects caused by external disturbances (Mondal & Tsourdos, 2022c). Overall, introducing Distributed NDI (DNDI) opens up exciting possibilities for controlling cooperative multi-agent systems and overcoming challenges related to consensus and communication in nonlinear agent environments. Moreover, the extension of DNDI to neuro-adaptive DNDI demonstrates its adaptability and robustness in the face of external disturbances.

The mathematical derivation of DNDI involved certain assumptions, including the requirement that each agent's dimension of the input and output vectors be the same, indicating square output dynamics. This particular class of nonlinear systems has been the focus of several research works, such as control design for decoupled three-loop guidance and control design for autonomous vehicles, where input and output dimensions are consistent for each loop. However, in real-world scenarios, the dimension of the output may be less than that of the input, resulting in a situation where the number of available controls exceeds the number of objectives to be achieved. This opens up the possibility of incorporating additional objectives to fully utilize the control design's capabilities. One such objective could involve minimizing control energy consumption. In this study, we explored the potential to minimize DNDI control, leading to the development of a new design called distributed optimal NDI or DONDI. By considering additional objectives and optimizing the control scheme, DONDI aims to enhance the performance and efficiency of the control system, accounting for real-world complexities and requirements.

The contribution to this paper is given below.

- In this study, we introduced a novel optimal distributed controller based on NDI (Nonlinear Dynamic Inversion) and utilized it to address a consensus problem in a group of nonlinear agents. Applying this optimized controller in the context of multi-agent operation represents a new and innovative approach.
- In our study, we focused on a practical scenario where the outputs of the plants were lower in dimension compared to the available control. This meant that the available control inputs limited the control objectives. This consideration is crucial as it relaxes the limitations imposed by existing DNDI designs. We capitalized on this opportunity to incorporate additional objectives, such as minimizing control energy. This approach addressed the consensus problem in a group of nonlinear agents. To achieve this, we derived the expression of the consensus protocol by solving an optimal control problem with equality constraints. These constraints were derived using a dynamic inversion strategy, allowing us to effectively formulate and solve the optimal control problem.

- Our study highlights the impact of communication noise on the consensus process and showcases how DONDI effectively manages and achieves consensus despite noise. We comprehensively understood how the noise affects the system states and demonstrated the proposed controller's robustness (convergence proof) in handling these challenges.

The rest of the paper is organized as follows. The preliminaries section gives a brief insight into a few relevant topics. The problem details are provided in the 'problem description' section. The 'Distributed optimal nonlinear dynamic inversion controller for consensus' section gives the mathematical derivation of the proposed controller. The convergence study is presented in the 'Convergence study of DONDI' section. The mathematical derivation with communication noise is given in the section 'DONDI controller for Consensus of MASs in the presence of communication noise'. The convergence study with noise is presented in the section 'Convergence of DONDI in the presence of noise'. The simulation results are discussed in the section 'Simulation study'. Finally, the conclusion is given.

2. Preliminaries

A brief description of the topics required for this work is discussed in this section.

2.1. Consensus of MAS

A general definition of the consensus of networked MASs can be given as follows (Mondal & Tsourdos, 2022a).

Definition 1. Let us consider a MASs with N agents, where X_i , ($i = 1, 2, 3, \dots, N$) denotes the states of the i th agent. The MASs will achieve the consensus if $\|X_i - X_j\| \rightarrow 0, \forall i \neq j$ as $t \rightarrow +\infty$.

The consensus is attained through a protocol designed to leverage neighbouring agents' information to evaluate an agent's control. This approach assumes that the agents are linked through a communication topology and exchange their information. A few examples of application of consensus process are flocking (Zhang & Duan, 2016), formation control (Zhu, Zhang, Yan, Qu, & Lin, 2018), Synchronization (Cao, Chen, & Li, 2008), Collaborative Localization (Wu, Zhong, Xu, & Xiong, 2020), Distributed Optimization (Wu, Xiong, Vasilakos, Leung, & Chen, 2020) etc.

2.2. Graph theory

The agents are interconnected through a communication topology represented as a weighted graph, denoted by $G = V, E$. The graph consists of vertices, where each vertex represents an agent, given by $V = v_1, v_2, \dots, v_N$. Communication among the agents is illustrated by edges in the graph, represented by the set $E \subseteq V \times V$. To describe the weighted adjacency matrix A of the graph, the elements a_{ij} are greater than zero if there exists an edge between vertices v_i and v_j (i.e., $(v_i, v_j) \in E$), otherwise, a_{ij} equals zero. The matrix A has zero diagonal elements since there are no self-loops in the graph, meaning $v_i \in V$ implies $a_{ii} = 0$. The degree matrix $D \in \mathbb{R}^{N \times N}$ is a diagonal matrix with elements $d_i = \sum_{j \in N_i} a_{ij}$, where N_i represents the neighbours of agent v_i . The Laplacian matrix, denoted by $L = D - A$, is used in this context. This study focuses on an undirected graph where the weights have the property $a_{ij} = a_{ji}$. The graph is considered connected if a path exists between any two nodes v_i and v_j in V . For this paper, the network topology G is assumed to be undirected and connected.

2.3. Lemma

In this section, we present a few useful Lemmas.

Lemma 1. (Wen, Chen, Liu, & Liu, 2016) Let $\psi_1(t), \psi_2(t) \in \mathbb{R}^m$ be continuous positive vector functions, by Cauchy inequality and Young's inequality, there exists the following inequality:

$$\begin{aligned} \psi_1(t)\psi_2(t) &\leq \|\psi_1(t)\| \|\psi_2(t)\| \\ &\leq \frac{\|\psi_1(t)\|^2}{\lambda} + \frac{\|\psi_2(t)\|^2}{\gamma} \end{aligned} \quad (1)$$

where

$$\frac{1}{\lambda} + \frac{1}{\gamma} = 1$$

Lemma 2. (Ren & Chen, 2015) Let $R(t) \in \mathbb{R}$ be a continuous positive function with bounded initial $R(0)$. If the inequality holds $\dot{R}(t) \leq -\delta R(t) + \gamma$ where, $\delta > 0, \gamma > 0$, then the following inequality holds.

$$R(t) \leq R(0)e^{-\delta t} + \frac{\gamma}{\delta} (1 - e^{-\delta t}) \quad (2)$$

3. Problem description

This paper aims to develop a Distributed Optimal NDI consensus controller tailored for a group of nonlinear agents with fewer output states than the available control. The study involves N agents, and Eqs. (3)–(4) describe each agent's dynamics.

$$\dot{X}_i = f(X_i) + g(X_i)U_i \quad (3)$$

$$Y_i = h(X_i) \quad (4)$$

The state and control vector of i th agent is given by $X_i \in \mathbb{R}^n$ and $U_i \in \mathbb{R}^m$, respectively. The output of i th agent is given by $Y_i = h(X_i) \in \mathbb{R}^p$. This study specifically focuses on agents with an output dimension p less than the control dimension m , i.e., $p < m$.

Assumption 1. The system is perfectly known and the matrix $g(X)$ is invertible for all time.

The objectives can be summarized as follows.

1. to achieve the consensus of output states of a set of nonlinear agents having a non-square output.
2. to achieve the additional objective, which is to obtain an optimal control expression for the agents

We propose a distributed optimal controller based on NDI to satisfy these objectives. The following section presents mathematical details of Distributed Optimal Nonlinear Dynamic Inversion (DONDI).

4. Distributed optimal nonlinear dynamic inversion (DONDI) controller for consensus

The mathematical details of Optimal DNDI are presented here. We will define the consensus error in output of i th agent, i.e., Y_i , which is given by

$$\begin{aligned} E_{Y_i} &= \sum_{j \in N_i} a_{ij}(Y_i - Y_j) \\ &= \bar{d}_i Y_i - \bar{a}_i Y \end{aligned} \quad (5)$$

where $Y_i \in \mathbb{R}^p$ is the output of i th agent. $\bar{d}_i = (d_i \otimes \mathbf{I}_p) \in \mathbb{R}^{p \times p}$, $\bar{a}_i = (a_i \otimes \mathbf{I}_n) \in \mathbb{R}^{p \times pN}$, and $Y = [Y_1^T \ Y_2^T \ \dots \ Y_N^T]^T \in \mathbb{R}^{pN}$. $\mathbf{I}_p$ is $p \times p$ identity matrix. ' $\otimes$ ' denotes the Kronecker product. As shown below, we will start the derivation with a Lyapunov function V_{Y_i} .

$$V_{Y_i} = \frac{1}{2} E_{Y_i}^T E_{Y_i} \quad (6)$$

Differentiation of Eq. (6) yields

$$\dot{V}_{Y_i} = E_{Y_i}^T \dot{E}_{Y_i} \quad (7)$$

According to the Lyapunov stability theory, the time derivative of the Lyapunov function can be written as follows.

$$\dot{V}_{Y_i} = -E_{Y_i}^T K_i E_{Y_i} \quad (8)$$

where $K_i > 0 \in \mathbb{R}^{p \times p}$ is a diagonal gain matrix. Both the expressions of $\dot{V}_i$ in Eqs. (7) and (8) are made equal and we get

$$E_{Y_i}^T \dot{E}_{Y_i} = -E_{Y_i}^T K_i E_{Y_i} \quad (9)$$

We can simplify Eq. (9) to obtain

$$\dot{E}_{Y_i} + K_i E_{Y_i} = 0 \quad (10)$$

Differentiation of Eq. (5) yields

$$\dot{E}_{Y_i} = \bar{d}_i \dot{Y}_i - \bar{a}_i \dot{Y} \quad (11)$$

Differentiation of Eq. (4) yields

$$\begin{aligned} \dot{Y}_i &= \left[\frac{\partial h}{\partial X_i} \right] \dot{X}_i \\ &= \left[\frac{\partial h}{\partial X_i} \right] (f(X_i) + g(X_i)U_i) \\ &= f_{Y_i}(X_i) + g_{Y_i}(X_i)U_i \end{aligned} \quad (12)$$

where, $f_{Y_i}(X_i) = \left[\frac{\partial h}{\partial X_i} \right] f(X_i)$, and $g_{Y_i}(X_i) = \left[\frac{\partial h}{\partial X_i} \right] g(X_i)$. We replace the expressions of E_{Y_i} and $\dot{E}_{Y_i}$ in Eq. (10) to get

$$\bar{d}_i \left[f_{Y_i}(X_i) + g_{Y_i}(X_i)U_i \right] - \bar{a}_i \dot{Y} + K_i E_{Y_i} = 0 \quad (13)$$

Simplifying Eq. (13) gives

$$\left(g_{Y_i}(X_i) \right) U_i = \left[-f_{Y_i}(X_i) + \bar{d}_i^{-1}(\bar{a}_i \dot{Y} - K_i(\bar{d}_i Y_i - \bar{a}_i Y)) \right] \quad (14)$$

For $p = m$, the $g_{Y_i}(X_i)$ is square and non-singular. We can obtain the solution as

$$U_i = \left(g_{Y_i}(X_i) \right)^{-1} \left[-f_{Y_i}(X_i) + \bar{d}_i^{-1}(\bar{a}_i \dot{Y} - K_i(\bar{d}_i Y_i - \bar{a}_i Y)) \right] \quad (15)$$

In our previous work (Mondal & Tsourdos, 2022a), we assumed $p = m$ and derived the control expression accordingly. However, it is essential to acknowledge that this assumption may not hold in all real-world scenarios. Hence, in this study, we aim to overcome this limitation and propose a modified version of DNDI capable of handling cases where $p < m$. The proposed design offers optimized control allocation while retaining all the benefits of NDI. Below, we present the mathematical details of this modified approach.

Let us consider the Eq. (14). It can be observed that this equation is obtained by simplifying the consensus error dynamics. The control values must satisfy this equation to drive the consensus error towards zero, i.e., achieve consensus. Since one of our objectives was to minimize the control effort along with the consensus of the output, we can use Eq. (14) to be satisfied while minimizing the control effort. Therefore, the optimal control problem can be formulated as follows.

$$\min J_i = \frac{1}{2} U_i^T R_i U_i \quad (16)$$

$$\text{s.t. } A_i U_i = b_i \quad (17)$$

$R_i > 0 \in \mathbb{R}^{m \times m}$. A_i and b_i are given by

$$A_i = g_{Y_i}(X_i) \quad (18)$$

$$b_i = -f_{Y_i}(X_i) + \bar{d}_i^{-1}(\bar{a}_i \dot{Y} - K_i(\bar{d}_i Y_i - \bar{a}_i Y)) \quad (19)$$

The solution to this problem (Eqs. (16)–(17)) yields the optimal control strategy, allowing us to achieve our objectives. The mathematical derivation leading to the solution is provided below.

The first step of the solution is to construct the augmented cost function as follows.

$$\bar{J}_i = \frac{1}{2} U_i^T R_i U_i + \lambda_i^T (A_i U_i - b_i) \quad (20)$$

Next, applying the optimality condition to Eq. (20), we obtain the following equations.

$$\frac{\partial \bar{J}_i}{\partial U_i} = R_i U_i + A_i^T \lambda_i = 0 \quad (21)$$

and

$$\frac{\partial \bar{J}_i}{\partial \lambda_i} = A_i U_i - b_i = 0 \quad (22)$$

Eqs. (21) and (22) can be written as follows.

$$\begin{bmatrix} R_i & A_i^T \\ A_i & 0 \end{bmatrix} \begin{bmatrix} U_i \\ \lambda_i \end{bmatrix} = \begin{bmatrix} 0 \\ b_i \end{bmatrix} \quad (23)$$

We can write the Eq. (23) as

$$\Omega \Lambda = B \quad (24)$$

where,

$$\Omega = \begin{bmatrix} R_i & A_i^T \\ A_i & 0 \end{bmatrix}; \Lambda = \begin{bmatrix} U_i \\ \lambda_i \end{bmatrix}; B = \begin{bmatrix} 0 \\ b_i \end{bmatrix}$$

Therefore, we get the solution

$$\Lambda = \Omega^{-1} B \quad (25)$$

In simplification, we get

$$U_i = R_i^{-1} A_i^T (A_i R_i^{-1} A_i^T)^{-1} b_i \quad (26)$$

$$\lambda_i = -(A_i R_i^{-1} A_i^T)^{-1} b_i \quad (27)$$

Theorem 1. Given the agents' dynamics as presented in Eqs. (3) and (4), along with the assumption stated in Assumption 1, the optimal DNDI control expression derived in Eq. (26) ensures that the consensus error converges to an extremely small finite value. As a result, it successfully achieves objectives 1 and 2 outlined in the Problem Formulation section.

Proof. The proof is given in the following section, 'Convergence of DONDI'.

5. Convergence study of DONDI

We start the proof by considering a scalar function, which follows.

$$V = \frac{1}{2} \mathbf{Y}^T (L \otimes \mathbf{I}_p) \mathbf{Y} \quad (28)$$

$$L \otimes \mathbf{I}_p = \mathbf{C} \mathbf{M} \mathbf{C}^T \quad (29)$$

where, $\mathbf{I}_p \in \mathbb{R}^p$, $\mathbf{C} \in \mathbb{R}^{pN \times pN}$ is the left eigenvalue matrix of $L \otimes \mathbf{I}_p$, $\mathbf{M} = (\text{diag}\{0, \lambda_2(L), \lambda_3(L), \dots, \lambda_N(L)\} \otimes \mathbf{I}_p) \in \mathbb{R}^{pN \times pN}$ is eigenvalue matrix, $\mathbf{C}^T \mathbf{C} = \mathbf{C} \mathbf{C}^T = \mathbf{I}_{pN \times pN}$.

$$\begin{aligned} V &= \frac{1}{2} \mathbf{Y}^T (L \otimes \mathbf{I}_p) \mathbf{Y} \\ &= \frac{1}{2} \mathbf{Y}^T \mathbf{C} \mathbf{M} \mathbf{C}^T \mathbf{Y} \\ &= \frac{1}{2} \mathbf{Y}^T \mathbf{C} \sqrt{\mathbf{M}} \sqrt{\mathbf{M}}^T \mathbf{Y} \\ &= \frac{1}{2} \mathbf{Y}^T \mathbf{C} \sqrt{\mathbf{M} \mathbf{M}^T} \sqrt{\mathbf{M}^{-1}} \sqrt{\mathbf{M}^{-1}}^T \mathbf{Y} \\ &= \frac{1}{2} \mathbf{Y}^T \mathbf{C} \mathbf{M} \mathbf{M}^{-1} \mathbf{C}^T \mathbf{Y} \\ &= \frac{1}{2} \mathbf{Y}^T \mathbf{C} \mathbf{M} (\mathbf{C}^T \mathbf{C}) \mathbf{M}^{-1} (\mathbf{C}^T \mathbf{C}) \mathbf{M}^T \mathbf{Y} \\ &= \frac{1}{2} \mathbf{Y}^T (\mathbf{C} \mathbf{M} \mathbf{C}^T) (\mathbf{C} \mathbf{M}^{-1} \mathbf{C}^T) (\mathbf{C} \mathbf{M} \mathbf{C}^T) \mathbf{Y} \\ &= \frac{1}{2} \mathbf{Y}^T (L \otimes \mathbf{I}_p) \Lambda (L \otimes \mathbf{I}_p) \mathbf{Y} \end{aligned}$$

$$= \frac{1}{2} \mathbf{E}_{Y_i}^T \Lambda \mathbf{E}_{Y_i} \quad (30)$$

where $\bar{\mathbf{M}} = (\text{diag}\{\lambda_2(L), \lambda_2(L), \lambda_3(L), \dots, \lambda_N(L)\} \otimes \mathbf{I}_p) \in \mathbb{R}^{pN \times pN}$, $\mathbf{E}_{Y_i} = [\mathbf{E}_{Y_1} \ \mathbf{E}_{Y_2} \ \dots \ \mathbf{E}_{Y_N}] \in \mathbb{R}^{pN}$, and $\Lambda = \mathbf{C} \bar{\mathbf{M}}^{-1} \mathbf{C}^T \in \mathbb{R}^{pN \times pN}$.

Remark 1. It can be observed from Eqs. (28) and (30) that

$$\frac{\lambda_{\min}(\Lambda)}{2} \|\mathbf{E}_{Y_i}\|^2 \leq V \leq \frac{\lambda_{\max}(\Lambda)}{2} \|\mathbf{E}_{Y_i}\|^2 \quad (31)$$

$$V = \frac{1}{2} \mathbf{Y}^T (L \otimes \mathbf{I}_p) \mathbf{Y} = \frac{1}{2} \mathbf{Y}^T \mathbf{E}_{Y_i} \quad (32)$$

Remark 2. Since $\lambda_2 > 0$. Hence, $\bar{\mathbf{M}}$ is invertible.

Remark 3. The positivity of the matrix $\Lambda = \mathbf{C} \bar{\mathbf{M}}^{-1} \mathbf{C}^T$ confirms that it is positive definite. Consequently, V is also positive concerning the consensus error, making it suitable as a Lyapunov function.

Differentiating Eq. (28) we get

$$\begin{aligned} \dot{V} &= \mathbf{Y}^T (L \otimes \mathbf{I}_p) \dot{\mathbf{Y}} \\ &= \mathbf{E}_{Y_i}^T \dot{\mathbf{Y}} \\ &= \sum_{i=1}^N \mathbf{E}_{Y_i}^T [f_Y(X_i) + g_Y(X_i) U_i] \end{aligned} \quad (33)$$

$$= \sum_{i=1}^N \mathbf{E}_{Y_i}^T [f_Y(X_i) + A_i U_i] \quad (34)$$

$$\begin{aligned} &= \sum_{i=1}^N \mathbf{E}_{Y_i}^T [f_Y(X_i) + (A_i R_i^{-1} A_i^T) (A_i R_i^{-1} A_i^T)^{-1} b_i] \\ &= \sum_{i=1}^N \mathbf{E}_{Y_i}^T [f_Y(X_i) + b_i] \end{aligned} \quad (35)$$

Substituting the expression b_i (Eq. (19)) yields

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N \mathbf{E}_{Y_i}^T [\bar{d}_i^{-1} (\bar{a}_i \dot{\mathbf{Y}} - K_i \mathbf{E}_{Y_i})] \\ &= \sum_{i=1}^N -\mathbf{E}_{Y_i}^T \bar{d}_i^{-1} K_i \mathbf{E}_{Y_i} + \sum_{i=1}^N \mathbf{E}_{Y_i}^T \bar{d}_i^{-1} \bar{a}_i \dot{\mathbf{Y}} \end{aligned} \quad (36)$$

We use Lemma 1 to the term $\sum_{i=1}^N \mathbf{E}_{Y_i}^T \bar{d}_i^{-1} \bar{a}_i \dot{\mathbf{Y}}$ of Eq. (36), we can write

$$\begin{aligned} \mathbf{E}_{Y_i}^T \bar{d}_i^{-1} \bar{a}_i \dot{\mathbf{Y}} &\leq \|\mathbf{E}_{Y_i}\| \|\bar{d}_i^{-1} \bar{a}_i \dot{\mathbf{Y}}\| \leq \frac{\|\mathbf{E}_{Y_i}\|^2}{2} + \\ &\frac{\|\bar{d}_i^{-1} \bar{a}_i \dot{\mathbf{Y}}\|^2}{2} \end{aligned} \quad (37)$$

Next, we replace the inequalities of Eq. (37) in Eq. (36) we get

$$\dot{V} \leq \sum_{i=1}^N \left[-\mathbf{E}_{Y_i}^T \bar{d}_i^{-1} K_i \mathbf{E}_{Y_i} + \frac{\|\mathbf{E}_{Y_i}\|^2}{2} + \frac{\|\bar{d}_i^{-1} \bar{a}_i \dot{\mathbf{Y}}\|^2}{2} \right] \quad (38)$$

Let us design the gain K_i as follows.

$$K_i = \bar{d}_i \left(\frac{1}{2} + \frac{\alpha_i}{2} \lambda_{\max}(\Lambda) \right) \quad (39)$$

where $\alpha_i > 0$. Putting the expression of K_i in Eq. (38) is written as

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^N -\frac{\alpha_i}{2} \lambda_{\max}(\Lambda) \|\mathbf{E}_{Y_i}\|^2 + \sum_{i=1}^N \frac{\|\bar{d}_i^{-1} \bar{a}_i \dot{\mathbf{Y}}\|^2}{2} \\ &\leq -\alpha_i V + \psi \end{aligned} \quad (40)$$

where, $\psi = \sum_{i=1}^N \frac{\|\bar{d}_i^{-1} \bar{a}_i \dot{\mathbf{Y}}\|^2}{2}$.

Next, we apply Lemma 2 for Eq. (40) to obtain

$$V \leq \frac{\psi}{\alpha_i} + \left(V(0) - \frac{\psi}{\alpha_i} \right) e^{-\alpha_i t} \quad (41)$$

In this study, we will explain the Uniformly Ultimate Boundedness (UUB). By using Eqs. (31) and (41), we can write

$$\frac{\lambda_{\min}(\Lambda)}{2} \|\mathbf{E}_{Y_i}\|^2 \leq V \leq \frac{\psi}{\alpha_i} + \left(V(0) - \frac{\psi}{\alpha_i} \right) e^{-\alpha_i t} \quad (42)$$

Eq. (42) is simplified as follows.

$$\frac{\lambda_{\min}(A)}{2} \|E_{Y_i}\|^2 \leq \frac{\psi}{\alpha_i} + \left(V(0) - \frac{\psi}{\alpha_i}\right) e^{-\alpha_i t}$$

which implies

$$\|E_{Y_i}\| \leq \sqrt{\frac{2\frac{\psi}{\alpha_i} + 2\left(V(0) - \frac{\psi}{\alpha_i}\right)e^{-\alpha_i t}}{\lambda_{\min}(A)}} \quad (43)$$

It can be observed that, if $V(0) = \frac{\psi}{\alpha_i}$ then

$$\|E_{Y_i}\| \leq \theta^* \quad (44)$$

$\forall t \geq 0$ and $\theta^* = \sqrt{\frac{2\psi}{\alpha_i \lambda_{\min}(A)}}$. It can be observed that, if $V(0) \neq \frac{\psi}{\alpha_i}$ then for any given $\theta > \theta^*$ there exist a time $T > 0$ such that $\forall t > T$, $\|E_{Y_i}\| \leq \theta$.

$$\theta = \sqrt{\frac{2\frac{\psi}{\alpha_i} + 2\left(V(0) - \frac{\psi}{\alpha_i}\right)e^{-\alpha_i T}}{\lambda_{\min}(A)}} \quad (45)$$

Therefore, we can conclude

$$\lim_{t \rightarrow \infty} \|E_{Y_i}\| = \theta^* \quad (46)$$

It can be concluded from Eq. (46) that the consensus error converges to a very small value with a suitable value of the parameter α_i .

6. DONDI controller for consensus of MAS in the presence of communication noise

In this scenario, we consider communication noise, which is additive and arises from changing environmental conditions. Consequently, the information shared among agents is perturbed by this noise. The noise has been considered in the context of the MASs consensus (Li & Deng, 2019; Liu, Ming, & Li, 2016; Wang et al., 2015) etc. The noisy information received from the j th agent is represented as $Y_{j\omega} = Y_j + \sigma_j \omega_j$, where ω_j denotes independent standard white noise, and σ_j represents the noise intensity. The state, control, and output of the i th agent are denoted by X_i , Y_i , and $U_{i\omega}$, respectively. Here, $U_{i\omega}$ indicates the noisy control.

The consensus error is given by $e_{i\omega}$. Please note that we used different expressions for the consensus error to make it different from the earlier one.

$$\begin{aligned} e_{i\omega} &= \sum_{j=1}^N a_{ij} (Y_i - Y_{j\omega}) \\ &= \sum_{j=1}^N a_{ij} (Y_i - Y_j - \sigma_j \omega_j) \\ &= \sum_{j=1}^N a_{ij} (Y_i - Y_j) - \sum_{j=1}^N a_{ij} \sigma_j \omega_j \\ &= \bar{d}_i X_i - \bar{a}_i Y - \tilde{e}_{ij} \\ &= E_{Y_i} - \tilde{e}_{ij} \end{aligned} \quad (47)$$

where $\tilde{e}_{ij} = \sum_{j=1}^N a_{ij} \sigma_j \omega_j$. Enforcing the first-order error dynamics, we get

$$\dot{e}_{i\omega} + K e_{i\omega} = 0 \quad (48)$$

Differentiation of Eq. (47) yields

$$\dot{e}_{i\omega} = \dot{e}_i \quad (49)$$

Substituting the expressions of $e_{i\omega}$ and $\dot{e}_i$ in Eq. (48) we get

$$\bar{d}_i [f(X_i) + g(X_i)U_{i\omega}] - \bar{a}_i \dot{X} + K e_{i\omega} = 0 \quad (50)$$

Simplifying Eq. (50) we get the expression of control U_i as follows

$$g(X_i)U_{i\omega} = [-f(X_i) + \bar{d}_i^{-1}(\bar{a}_i \dot{X} - K e_{i\omega})] \quad (51)$$

Proceeding similarly as we did earlier, the control expression can be obtained the same as Eq. (26), i.e.,

$$U_{i\omega} = R_i^{-1} A_i^T (A_i R_i^{-1} A_i^T)^{-1} b_{i\omega} \quad (52)$$

where, $b_{i\omega} = -f(X_i) + \bar{d}_i^{-1}(\bar{a}_i \dot{X} - K e_{i\omega})$. However, the expression in Eq. (52) is not exactly the same as we can see that it contains the consensus error with noise term.

Theorem 2. With the agents' dynamics given in Eqs. (3) and (4), and Assumption 1, the optimal DONDI control expression obtained in Eq. (52) will make the consensus error $e_{i\omega}$ to converge to a very small finite value in the presence of communication noise and achieve the objectives 1 and 2 given in Problem formulation section.

Proof. The proof is given in the following section, 'Convergence of DONDI in the presence of noise'.

7. Convergence of DONDI in the presence of noise

The convergence proof is provided in this section. We use the same Lyapunov function as considered in Eq. (30) and proceed with its derivative as follows.

$$\begin{aligned} \dot{V} &= Y^T (L \otimes I_p) \dot{Y} \\ &= E_{Y_i}^T \dot{Y} \\ &= \sum_{i=1}^N E_{Y_i}^T [f_Y(X_i) + g_Y(X_i)U_{i\omega}] \end{aligned} \quad (53)$$

Substituting the control expression, we get

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} (\bar{a}_i \dot{Y} - K e_{i\omega}) \\ &= \sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} \bar{a}_i \dot{Y} - \sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} K e_{i\omega} \end{aligned} \quad (54)$$

Replacing $e_{i\omega} = E_{Y_i} - \tilde{e}_{ij}$ in Eq. (54) we get

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} \bar{a}_i \dot{Y} - \sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} K e_{i\omega} \\ &= \sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} \bar{a}_i \dot{Y} - \sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} K E_{Y_i} + \\ &\quad \sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} K \tilde{e}_{ij} \end{aligned} \quad (55)$$

The first term on the RHS of Eq. (55) is expanded using Lemma 1.

$$E_{Y_i}^T \bar{d}_i^{-1} \bar{a}_i \dot{Y} \leq \|E_{Y_i}\| \|\bar{d}_i^{-1} \bar{a}_i \dot{Y}\| \leq \frac{\|E_{Y_i}\|^2}{2} + \frac{\|\bar{d}_i^{-1} \bar{a}_i \dot{Y}\|^2}{2} \quad (56)$$

In this case, the gain K is considered as

$$K = \bar{d}_i \left(\mathbf{I} + \frac{\alpha_i}{2} \lambda_{\max}(A) \right) \quad (57)$$

which is a little different from earlier. Putting this expression in the term $\sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} K \tilde{e}_{ij}$ and simplifying it, we get

$$\begin{aligned} &\sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} K \tilde{e}_{ij} \\ &= \sum_{i=1}^N \left[E_{Y_i}^T \left(\mathbf{I} + \frac{\alpha_i}{2} \lambda_{\max}(A) \right) \tilde{e}_{ij} \right] \\ &= \sum_{i=1}^N \left[E_{Y_i}^T \tilde{e}_{ij} + \frac{\alpha_i}{2} \lambda_{\max}(A) E_{Y_i}^T \tilde{e}_{ij} \right] \end{aligned} \quad (58)$$

Applying Lemma 1 to Eq. (58), we get

$$\begin{aligned} &\sum_{i=1}^N E_{Y_i}^T \bar{d}_i^{-1} K \tilde{e}_{ij} \\ &\leq \sum_{i=1}^N \left[\|E_{Y_i}\| \|\tilde{e}_{ij}\| + \frac{\alpha_i}{2} \lambda_{\max}(A) \|E_{Y_i}\| \|\tilde{e}_{ij}\| \right] \end{aligned}$$

$$\begin{aligned}
&\leq \sum_{i=1}^N \left[\left(\frac{\|E_{Y_i}\|^2}{2} + \frac{\|\tilde{e}_{ij}\|^2}{2} \right) \right. \\
&\quad \left. + \frac{\alpha_i}{2} \lambda_{\max}(\Lambda) \left(\frac{\|E_{Y_i}\|^2}{2} + \frac{\|\tilde{e}_{ij}\|^2}{2} \right) \right] \\
&= \sum_{i=1}^N \left[\frac{\|E_{Y_i}\|^2}{2} + \frac{\alpha_i}{2} \frac{\lambda_{\max}(\Lambda)}{2} \|E_{Y_i}\|^2 \right. \\
&\quad \left. + \left(\frac{\|\tilde{e}_{ij}\|^2}{2} + \frac{\alpha_i}{2} \frac{\lambda_{\max}(\Lambda)}{2} \|\tilde{e}_{ij}\|^2 \right) \right] \\
&\leq \sum_{i=1}^N \left[\frac{\|E_{Y_i}\|^2}{2} + \frac{\alpha_i}{2} \frac{\lambda_{\max}(\Lambda)}{2} \|E_{Y_i}\|^2 \right] + \eta
\end{aligned} \tag{59}$$

where, $\eta = \sum_{i=1}^N \left[\frac{\|\tilde{e}_{ij}\|^2}{2} + \frac{\alpha_i}{2} \frac{\lambda_{\max}(\Lambda)}{2} \|\tilde{e}_{ij}\|^2 \right]$. Eq. (37) gives $E_{Y_i}^T \bar{d}_i^{-1} \bar{a}_i \dot{Y} \leq \frac{\|E_{Y_i}\|^2}{2} + \frac{\|\bar{d}_i^{-1} \bar{a}_i \dot{Y}\|^2}{2}$. Let, $\bar{\eta} = \sum_{i=1}^N \frac{\|\bar{d}_i^{-1} \bar{a}_i \dot{Y}\|^2}{2}$. Next, we replace the inequality (i.e. Eq. (59)) in Eq. (55) we get

$$\begin{aligned}
\dot{V} &\leq \sum_{i=1}^N \left[-E_{Y_i}^T \bar{d}_i^{-1} K E_{Y_i} + \frac{\|E_{Y_i}\|^2}{2} \right. \\
&\quad \left. + \frac{\|E_{Y_i}\|^2}{2} + \frac{\alpha_i}{2} \frac{\lambda_{\max}(\Lambda)}{2} \|E_{Y_i}\|^2 \right] + \eta + \bar{\eta} \\
&\leq \sum_{i=1}^N \left[-E_{Y_i}^T \bar{d}_i^{-1} K E_{Y_i} + \|E_{Y_i}\|^2 \right] + \frac{\alpha_i}{2} V + \eta + \bar{\eta} \\
&= \sum_{i=1}^N \left[-E_{Y_i}^T \left(\mathbf{I} + \frac{\alpha_i}{2} \lambda_{\max}(\Lambda) \right) E_{Y_i} + \|E_{Y_i}\|^2 \right] \\
&\quad + \frac{\alpha_i}{2} V + \eta + \bar{\eta} \\
&= \sum_{i=1}^N \left[-\frac{\alpha_i}{2} \lambda_{\max}(\Lambda) \|E_{Y_i}\|^2 \right] + \frac{\alpha_i}{2} V + \eta + \bar{\eta} \\
&\leq -\alpha_i V + \frac{\alpha_i}{2} V + \eta + \bar{\eta} \\
&= -\frac{\alpha_i}{2} V + \eta + \bar{\eta} \\
&= -\beta V + \bar{\eta}
\end{aligned} \tag{60}$$

where, $\bar{\eta} = \eta + \bar{\eta}$.

Applying Lemma 2 to Eq. (60), we can write

$$V \leq \frac{\bar{\eta}}{\beta} + \left(V(0) - \frac{\bar{\eta}}{\beta} \right) e^{-\beta t} \tag{61}$$

Similar to the previous case, we will present the Uniformly Ultimate Boundedness (UUB) of the consensus error. Eqs. (31) and (61) give

$$\frac{\lambda_{\min}(\Lambda)}{2} \|E_{Y_i}\|^2 \leq V \leq \frac{\bar{\eta}}{\beta} + \left(V(0) - \frac{\bar{\eta}}{\beta} \right) e^{-\beta t} \tag{62}$$

Eq. (62) is simplified as follows.

$$\begin{aligned}
\frac{\lambda_{\min}(\Lambda)}{2} \|E_{Y_i}\|^2 &\leq \frac{\bar{\eta}}{\beta} + \left(V(0) - \frac{\bar{\eta}}{\beta} \right) e^{-\beta t} \\
\|E_{Y_i}\| &\leq \sqrt{\frac{2 \frac{\bar{\eta}}{\beta} + 2 \left(V(0) - \frac{\bar{\eta}}{\beta} \right) e^{-\beta t}}{\lambda_{\min}(\Lambda)}}
\end{aligned} \tag{63}$$

It can be observed that, if $V(0) = \frac{\bar{\eta}}{\beta}$ then

$$\|E_{Y_i}\| \leq \Theta^* \tag{64}$$

$\forall t \geq 0$ and $\Theta^* = \sqrt{\frac{2\bar{\eta}}{\beta\lambda_{\min}(\Lambda)}}$. It can be observed that, if $V(0) \neq \frac{\bar{\eta}}{\beta}$ then for any given $\Theta > \Theta^*$ there exist a time $T > 0$ such that $\forall t > T$, $\|E_{Y_i}\| \leq \Theta$.

$$\Theta = \sqrt{\frac{2 \frac{\bar{\eta}}{\beta} + 2 \left(V(0) - \frac{\bar{\eta}}{\beta} \right) e^{-\beta t}}{\lambda_{\min}(\Lambda)}} \tag{65}$$

Therefore, we can conclude

$$\lim_{t \rightarrow \infty} \|E_{Y_i}\| = \Theta^* \tag{66}$$

Therefore, the consensus error converges to a small bounded value if the proposed consensus protocol is used.

8. Simulation study

The simulation study used MATLAB 2021b to attain consensus in the output Y_i , corresponding to the state X_1 of agents. We consider ten agents, i.e., $N = 10$. The primary goal was achieving consensus while minimizing the controls U_i .

8.1. Agent dynamics considered for simulation study

The dynamics of the i th agents are as follows.

$$\dot{X}_{i1} = X_{i2} \sin(2X_{i1}) + U_{i1} \tag{67}$$

$$\dot{X}_{i2} = X_{i1} \cos(3X_{i2}) + U_{i2} \tag{68}$$

where $X_i = [X_{i1} \ X_{i2}]^T$.

Now, we will write the agent dynamics (Eqs. (67) and (68)) in the same format given in Eqs. (3) and (4).

$$f(X_i) = \begin{bmatrix} X_{i2} \sin(2X_{i1}) \\ X_{i1} \cos(3X_{i2}) \end{bmatrix} \tag{69}$$

and

$$g(X_i) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{70}$$

and

$$U_i = \begin{bmatrix} U_{i1} \\ U_{i2} \end{bmatrix} \tag{71}$$

where $X_i \in \mathbb{R}^2$. The states X_{i1} of all the agents are denoted by $X_1 = [X_{11} \ X_{21} \ \dots \ X_{101}]$. Similarly, we denote $X_2 = [X_{12} \ X_{22} \ \dots \ X_{102}]$, $U_1 = [U_{11} \ U_{21} \ \dots \ U_{101}]$, and $U_2 = [U_{12} \ U_{22} \ \dots \ U_{102}]$.

The output is selected as $Y_i = X_{i1}$. Therefore, $g_Y(X_i)$ is given by

$$A_i = g_Y(X_i) = [1 \ 0] \tag{72}$$

We have considered $R_i = \mathbf{I}_2$ as diagonal and positive. The matrix Ω is given by

$$\Omega = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

With these values, we present the results in the following section.

8.2. Communication topology

We used the communication given as follows.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

8.3. Result and discussion

This section presents the simulation results for two scenarios: (1) without noise and (2) with noise.

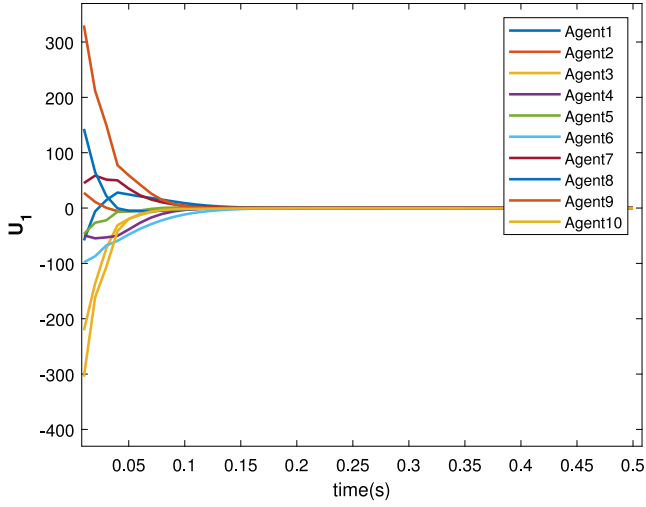
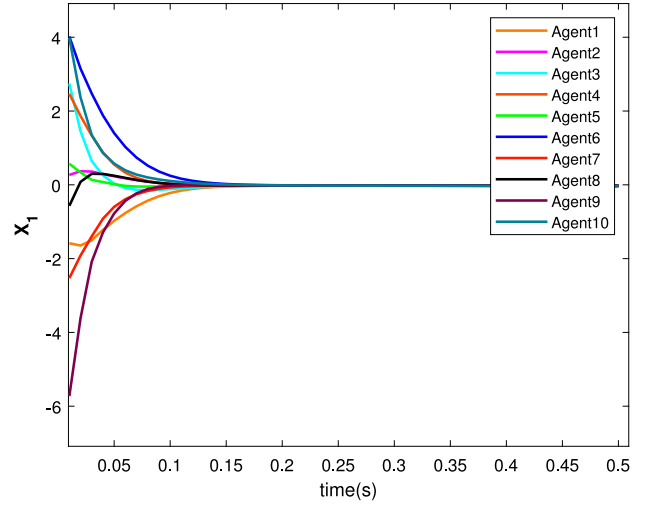
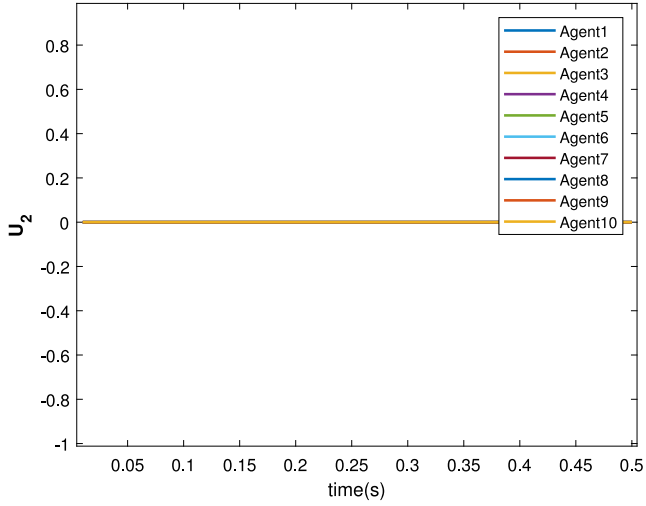
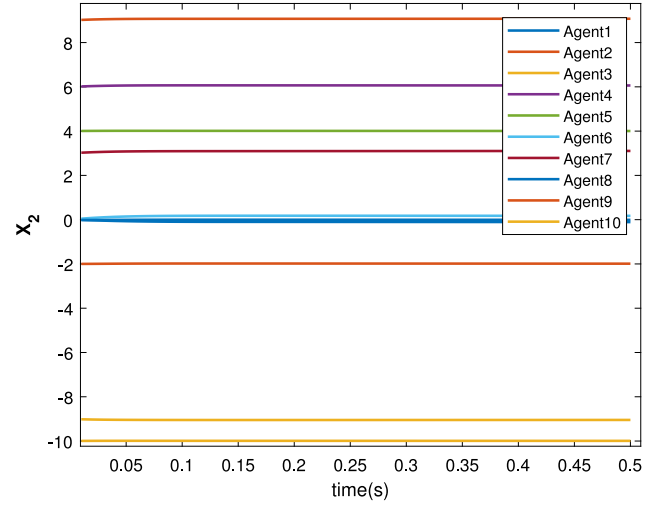
(a) Control U_1 .(a) State trajectory X_1 .(b) Control U_2 .(b) State trajectory X_2 .

Fig. 1. Control generated without noise.

Fig. 2. State trajectories without noise.

8.3.1. Without noise

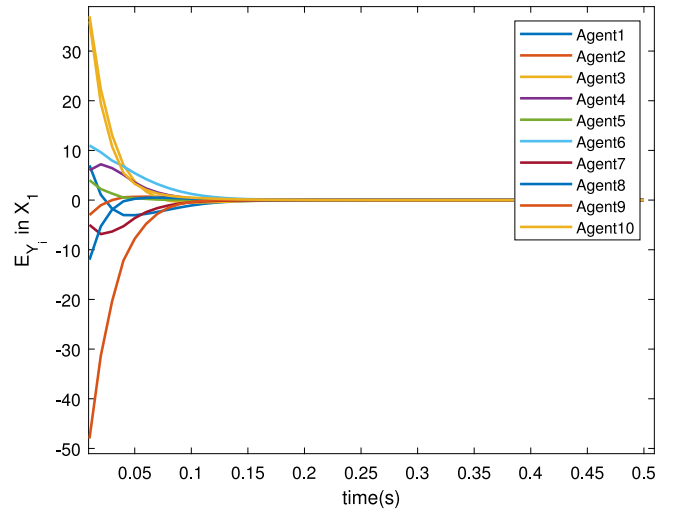
The controls U_1 and U_2 generated by the Optimal DNDI algorithm are depicted in Figs. 1(a) and 1(b) respectively.

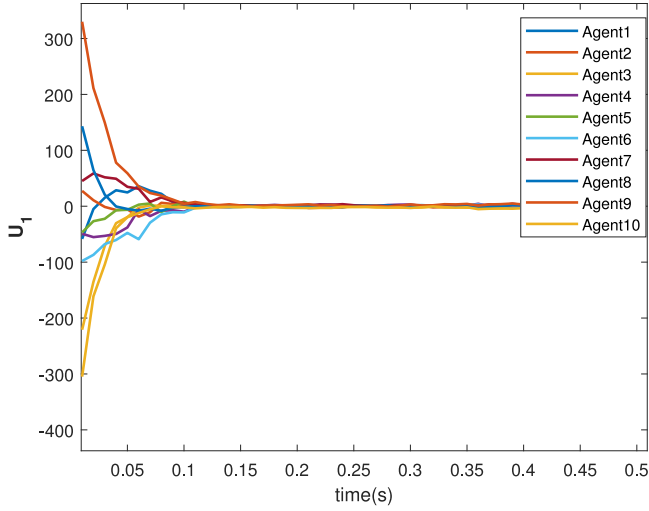
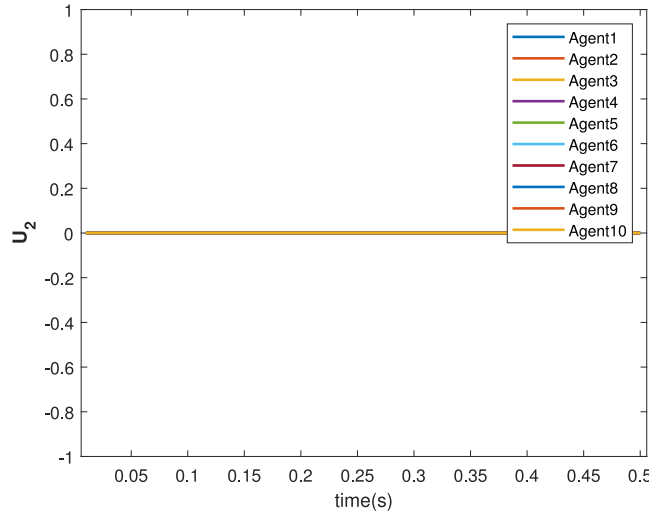
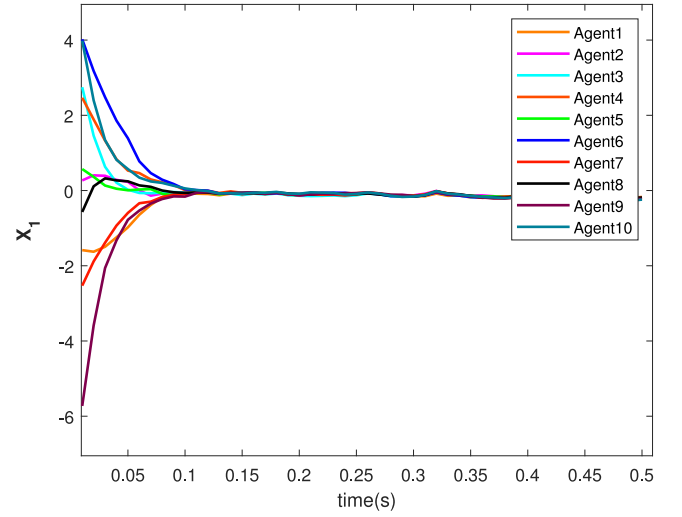
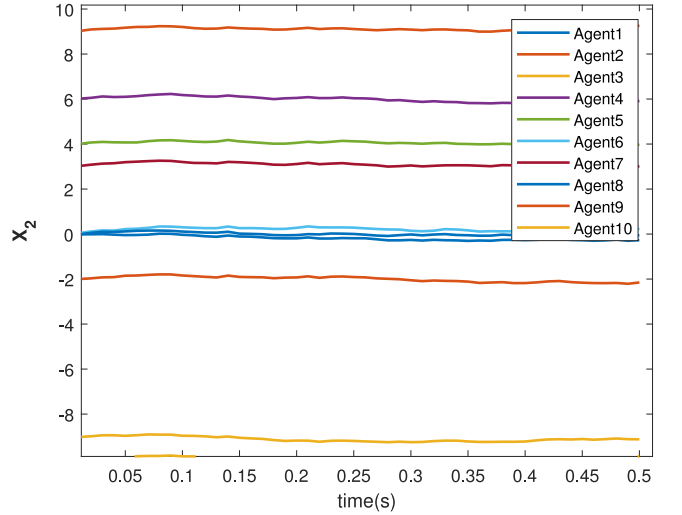
It is evident from the plots that U_1 facilitated the attainment of consensus in state X_1 (Fig. 2(a)), while U_2 was minimized, reaching a value of zero since there was no specific objective for state X_2 (Fig. 2(b)).

The consensus error in state X_1 , illustrated in Fig. 3, gradually converges to zero within a finite time, demonstrating the effectiveness of the proposed control method in enabling agent consensus.

8.3.2. With noise

Here, we will show the effect of noise on the consensus process. The value of $\sigma_{ij} = 0.02$ is used here. The control expression in Eq. (52) includes the noisy consensus error $e_{i\omega}$. As a result, the noise affects the overall control, as clearly evident in Figs. 4(a). However, the control shown in Fig. 4(b) remains at zero since X_{i2} is not included in the output. The state trajectories of all agents, namely X_1 and X_2 , are

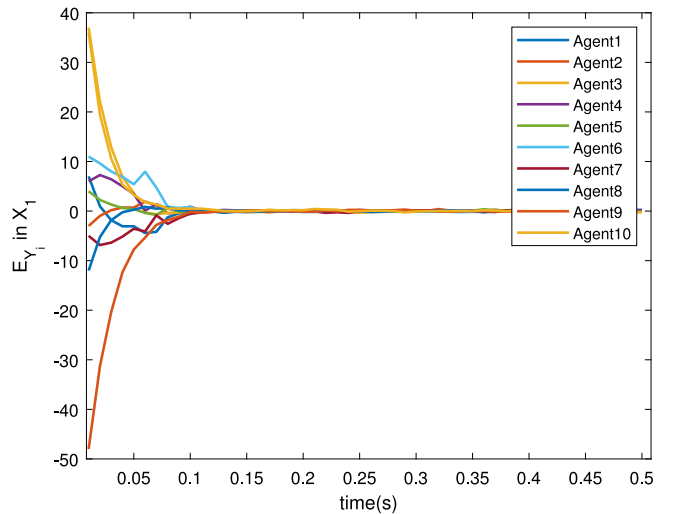
Fig. 3. Consensus error in state X_1 , i.e., E_{Y_1} .

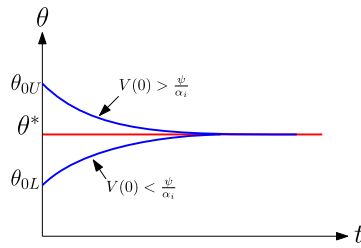
(a) Control U_1 generated to handle noise.(b) Control U_2 . The control signal is minimized to obtain zero control.**Fig. 4.** Control generated with noise.(a) State trajectory X_1 . The effect of noise is seen.(b) State trajectory X_2 . The effect of noise is seen.**Fig. 5.** State trajectories with noise.

depicted in Figs. 5(a) and 5(b), respectively. The DONDI-generated optimal control U_1 assists X_1 in achieving consensus even in noise. The error plot in Fig. 6 gives the evidence. Nevertheless, the impact of noise is noticeable. Since the control U_2 remains zero, it cannot directly influence the state X_2 . However, due to the presence of the term X_1 in the dynamics $\dot{X}_2$, the state X_2 is slightly affected by noise.

9. Conclusion

The optimal control solution addresses both the consensus and the optimization objectives. The formulation incorporates a constraint derived from the consensus error dynamics, which enables the attainment of consensus. The approach is particularly effective for practical systems featuring square output. Our research demonstrates that the proposed optimal controller (DONDI) can achieve consensus even in communication noise. Rigorous mathematical derivations and convergence studies establish a strong theoretical basis for the suggested controller, while simulation studies provide empirical evidence supporting our assertion.

**Fig. 6.** Consensus error in state X_1 , i.e., E_{Y_1} .

Fig. 7. θ vs time t .

CRedit authorship contribution statement

Sabyasachi Mondal: Writing – original draft, Software, Methodology, Investigation, Conceptualization. **Antonios Tsourdos:** Investigation, Project administration, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

The expression of θ is given on the RHS of Eq. (43). If we consider the condition $V(0) \neq \frac{\psi}{\alpha_i}$, the trajectory of θ with time t can be shown in Fig. 7. At $t = 0$, we get $\theta_0 = \sqrt{\frac{2V(0)}{\lambda_{\min}}}$. Now, we get $\theta_{0U} > \theta^*$ when $V(0) > \frac{\psi}{\alpha_i}$, and $\theta_{0L} < \theta^*$ when $V(0) < \frac{\psi}{\alpha_i}$. The decaying shape of the θ trajectory is obtained due to the presence of the term $e^{-\alpha_i t}$ and ultimately converges to θ^* as shown in Eq. (46). Therefore, considering $V(0) > \frac{\psi}{\alpha_i}$ in Eq. (43), we can get the value of $\theta > \theta^*$.

References

- Acquatella, P., Briese, L. E., & Schnepfer, K. (2020). Guidance command generation and nonlinear dynamic inversion control for reusable launch vehicles. *Acta Astronautica*, 174, 334–346.
- Cao, J., Chen, G., & Li, P. (2008). Global synchronization in an array of delayed neural networks with hybrid coupling. *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics)*, 38(2), 488–498.
- Caverly, R. J., Girard, A. R., Kolmanovsky, I. V., & Forbes, J. R. (2016). Nonlinear dynamic inversion of a flexible aircraft. *IFAC-PapersOnLine*, 49(17), 338–342.
- Chowdhury, A. R., Vishwanathan, V., Prasad, B., Kumar, R., & Panda, S. (2014). Inverse dynamics control of a bio-inspired robotic-fish underwater vehicle propulsion based on lighthill slender body theory. In *OCEANS 2014-TAIPEI* (pp. 1–6). IEEE.
- Enns, D., Bugajski, D., Hendrick, R., & Stein, G. (1994). Dynamic inversion: an evolving methodology for flight control design. *International Journal of Control*, 59(1), 71–91.
- Gregory, I. (2001). Stability result for dynamic inversion devised to control large flexible aircraft. In *AIAA guidance, navigation, and control conference and exhibit* (p. 4284).
- Horn, J. F. (2019). Non-linear dynamic inversion control design for rotorcraft. *Aerospace*, 6(3), 38.
- Hovakimyan, N., Lavretsky, E., Yang, B.-J., & Calise, A. J. (2005). Coordinated decentralized adaptive output feedback control of interconnected systems. *IEEE Transactions on Neural Networks*, 16(1), 185–194.
- Kalmár-Nagy, T. (2016). Real-time trajectory generation for omni-directional vehicles by constrained dynamic inversion. *Mechatronics*, 35, 44–53.
- Li, M., & Deng, F. (2019). Necessary and sufficient conditions for consensus of continuous-time multiagent systems with Markovian switching topologies and communication noises. *IEEE Transactions on Cybernetics*.

- Liu, J., Ming, P., & Li, S. (2016). Consensus gain conditions of stochastic multi-agent system with communication noise. *International Journal of Control, Automation and Systems*, 14(5), 1223–1230.
- Lombaerts, T., Kaneshige, J., Schuet, S., Hardy, G., Aponso, B. L., & Shish, K. H. (2019). Nonlinear dynamic inversion based attitude control for a hovering quad tiltrotor eVTOL vehicle. In *AIAA scitech 2019 forum* (p. 0134).
- Lungu, M. (2022). Backstepping and dynamic inversion control techniques for automatic landing of fixed wing unmanned aerial vehicles. *Aerospace Science and Technology*, 120, Article 107261.
- Malekzadeh, M., Naghash, A., & Talebi, H. (2011). Control of flexible spacecraft using dynamic inversion and μ -synthesis. *Journal of Vibration and Control*, 17(13), 1938–1951.
- Mondal, S., & Padhi, R. (2015). Formation flying using GENEX and differential geometric guidance law. *IFAC-PapersOnLine*, 48(9), 19–24.
- Mondal, S., & Tsourdos, A. (2021). The consensus of nonlinear agents under switching topology using dynamic inversion in the presence of communication noise, and delay. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 1–16.
- Mondal, S., & Tsourdos, A. (2022a). The consensus of non-linear agents under switching topology using dynamic inversion in the presence of communication noise and delay. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 236(2), 352–367.
- Mondal, S., & Tsourdos, A. (2022b). Fault-tolerant consensus of nonlinear agents considering switching topology in the presence of communication noise. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 236(13), 2776–2787.
- Mondal, S., & Tsourdos, A. (2022c). Neuro-adaptive augmented distributed nonlinear dynamic inversion for consensus of nonlinear agents with unknown external disturbance. *Scientific Reports*, 12(1), 2049.
- Panish, L., Nicholls, C., & Bacic, M. (2023). Nonlinear dynamic inversion flight control of a tilting VTOL aircraft. In *AIAA SCITECH 2023 forum* (p. 1910).
- Ren, C.-E., & Chen, C. P. (2015). Sliding mode leader-following consensus controllers for second-order non-linear multi-agent systems. *IET Control Theory & Applications*, 9(10), 1544–1552.
- Sastry, S. S., & Isidori, A. (1989). Adaptive control of linearizable systems. *IEEE Transactions on Automatic Control*, 34(11), 1123–1131.
- Singh, S., & Padhi, R. (2009). Automatic path planning and control design for autonomous landing of UAVs using dynamic inversion. In *2009 American control conference* (pp. 2409–2414). IEEE.
- Subudhi, B., & Atta, D. (2009). *Design of a path following controller for an underactuated AUV*. ACS.
- Tafazoli, S., & Khorasani, K. (2004). A flexible spacecraft attitude recovery using feedback linearization approach. In *Proceedings of the 6th IASTED international conference on intelligent systems and control*.
- Wang, Y., Cheng, L., Hou, Z.-G., Tan, M., Zhou, C., & Wang, M. (2015). Consensus seeking in a network of discrete-time linear agents with communication noises. *International Journal of Systems Science*, 46(10), 1874–1888.
- Wen, G., Chen, C. P., Liu, Y.-J., & Liu, Z. (2016). Neural network-based adaptive leader-following consensus control for a class of nonlinear multiagent state-delay systems. *IEEE Transactions on Cybernetics*, 47(8), 2151–2160.
- Wu, G., Meng, X., & Wang, F. (2018). Improved nonlinear dynamic inversion control for a flexible air-breathing hypersonic vehicle. *Aerospace Science and Technology*, 78, 734–743.
- Wu, M., Xiong, N., Vasilakos, A. V., Leung, V. C., & Chen, C. P. (2020). RNN-K: A reinforced Newton method for consensus-based distributed optimization and control over multiagent systems. *IEEE Transactions on Cybernetics*, 52(5), 4012–4026.
- Wu, M., Zhong, L., Xu, B., & Xiong, N. (2020). A consensus-based diffusion levenberg-marquardt method for collaborative localization with extension to distributed optimization. *IEEE Access*, 8, 215649–215660.
- Ye, L., & Zong, Q. (2018). Tracking control of an underactuated ship by modified dynamic inversion. *ISA Transactions*, 83, 100–106.
- Zhang, X., & Duan, H. (2016). Altitude consensus based 3D flocking control for fixed-wing unmanned aerial vehicle swarm trajectory tracking. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 230(14), 2628–2638.
- Zhu, X., Zhang, X.-X., Yan, M.-D., Qu, Y.-H., & Lin, H. (2018). Three-dimensional multiple unmanned aerial vehicles formation control strategy based on second-order consensus. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 232(3), 481–491.



DSPACE

<https://dspace.org/>

Distributed optimal nonlinear dynamic inversion for multi-agents consensus

Mondal, Sabyasachi; Tsourdos, Antonios

2025-07

Elsevier

<https://doi.org/10.1016/j.ejcon.2025.101223>

Mondal S, Tsourdos A. (2025) Distributed optimal nonlinear dynamic inversion for multi-agents consensus. European Journal of Control, Volume 84, July 2025, Article number 101223