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Cranfield University

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An Approach to the Evaluation of Blast Loads on
Finite and Semi-Infinite Structures

Engineering Systems Department

PhD Thesis

Cranfield University
Engineering Systems Department

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**An Approach to the Evaluation of Blast Loads on
Finite and Semi-Infinite Structures**

Supervisor: Dr Peter D. Smith

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Abstract

This thesis is concerned with the use of Computational Fluid Dynamics techniques coupled with experimental studies to establish useful relationships between explosively generated blast loads and the principal aspects of the geometry of both single buildings and many buildings, as might be found in any urban environment.

A method for the treatment of blast loading problems is described which is based on a large number of numerical simulations validated by key physical experiments. The idea of using numerical simulation to investigate aspects of flow problems which are too difficult, expensive or time-consuming to consider experimentally is not new. The emphasis of this thesis, however, is not the treatment of specific problems but whole classes of problems.

Chapter 1 introduces the difficulties associated with the evaluation of blast loads on structures. It briefly describes several existing techniques and introduces the approach suggested by this study. It also contains a number of useful definitions which assist appreciation of the difficulties of numerical simulation of blast loading.

Chapter 2 is in the form of a narrative and describes the process by which the solution algorithm of the program used for the blast simulations (Air3d) was selected. The final choice, AUSMDV (a variant of the Advection Upstream Splitting Method) with MUSCL-Hancock integration (MUSCL standing for "Monotone Upstream-centred Scheme for Conservation Laws"), is essentially the combination of two methods which are "cheap" in terms of computational resources to obtain one of only moderate "expense" but which has sufficient accuracy and robustness for these demanding applications.

Chapter 3 contains a description of the computational tool Air3d, and it acts as a user's guide to the program. Chapter 3 also contains a discussion of the treatment by the program Air3d of the processes which govern the formation of spherical blast waves in air. The chapter concludes with a comparison between the results of Air3d and a commercially available program and demonstrates the efficacy of the solution algorithm adopted.

Chapter 4 demonstrates the potential of the approach to obtain useful information in the main areas of application (Chapters 5 to 7) in this thesis. This is achieved by comparison of Air3d simulations with established sets of experimentally determined scaled blast parameters.

Chapter 5 describes the problem of blast wave clearing, or loads on single finite structures, and uses the approach to produce a relationship which is applicable over almost the whole range of practical interest to engineers.

Chapter 6 is concerned with the effect of street width and building height on the blast overpressure impulses which load the façades of a street when an explosive incident occurs in an urban setting. It considers semi-infinite straight streets and describes, in broad terms, the limits of width and height which determine the blast impulse loads.

Chapter 7 contains a discussion of the blast environment behind a semi-infinite protective barrier wall when an explosive device is detonated on the near side. This problem has particular difficulties, which are discussed, and has illustrated the limits of the suggested approach.

Chapter 8 summarises the approach adopted by this study for blast load evaluation, and it describes the progress made, difficulties encountered and the limitations of the method. Recommendations are made which would improve the approach for future investigators, and the possibility of extending it for use in more varied applications is also considered.

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Preface

This document is the result of many years of interest in the methods of evaluation of the blast loads experienced by structures when a high explosive charge is detonated nearby.

The evolution of the title gives an indication of the manner in which the project itself evolved and the difficulties encountered along the way. The original title was *The Evaluation of Blast Loads on Structures*. It soon became clear that this was over ambitious, and it was modified to *An Approach to The Evaluation of Blast Loads on Structures*.

Almost the whole of the current guidance on blast load evaluation is based on a few basic building blocks. These are, most commonly, blast parameters for incident and reflected waves and reflections at oblique angles. They have one thing in common: they are for free air or a single infinite reflecting surface. Virtually all the remaining guidance is too specific to be of use to the structural engineer in everyday professional life.

In an attempt to separate the present approach from these conventional methods (based on free-air or infinite reflecting surfaces), the phrase *Finite Structures* was introduced into the title of the project. Finally, three years into the study, the type of problems considered—the blast environment behind infinitely long perimeter walls and blast propagation in infinitely long city streets—forced one last change, and the phrase was augmented to *Finite and Semi-Infinite Structures*. This, the final form—*An Approach to the Evaluation of Blast Loads on Finite and Semi-Infinite Structures*—accurately reflects the contents of this document.

The remit of the study was to describe a possible approach to the evaluation of blast loads that may be developed for practical applications in the future. No mention is made of the reason for an explosive event taking place. Such matters are a subject for others. Similarly, the response of structures to blast loading is an area which is inextricably linked to the present one, but it is merely mentioned here and not discussed in detail.

It is assumed that the reader is familiar with the fundamental elements of high explosive detonation and the formation of shock waves in air. The only other foreknowledge required is the notion of cube root scaling and, in particular, scaled distance. These subjects are described in several standard texts, some of which are cited in this document (References [4], [13] and [47]).

There is no necessity for the reader to be familiar with the techniques of Computational Fluid Dynamics, as the results of the calculations are presented in a straightforward manner.

The two Appendices are a transcription of my original notes from the two excellent introductory volumes by Anderson (References [1] and [2]) and are included for background information only. Material in these sections does not contribute to the main thrust of the project.

All the analyses described in this document were performed on a 333 MHz Pentium II Personal Computer with 384 Mbytes of memory. The majority of problems were completed in under 24 hours.

The observant reader will notice that in many of the graphs contained in this

document individual data points are not identified by symbols. Moreover, in some cases, the points are joined by lines of various thicknesses. This does not indicate continuity of the data or the function they represent; it is merely a device for adding clarity to an otherwise chaotic representation. It is acknowledged that this is not standard practice, and the author apologises if at first acquaintance such a means of presentation appears misleading.

Acknowledgments

I would like to acknowledge the generous help and enthusiasm of my supervisor, Dr P. D. Smith, and the valuable guidance given to me by Dr S. A. Forth, both of Cranfield University (RMCS); this project would not have progressed without their support.

Much of the background and inspiration for this study was supplied by the project work of Weapons Effects on Structures (WES) master's degree graduates at RMCS and by research projects sponsored by the Defence Evaluation and Research Agency (DERA) and the Police Scientific Development Branch (PSDB). Notable contributions were from Timothy Chapman (No. 7 WES), who considered the loads on structures behind blast walls, Lek Jiunn Feng (No. 11 WES) who initiated a systematic approach to the evaluation of blast loads in city streets, and Gregory Whalen (No. 12 WES), whose work provided a valuable opportunity for validation and is reported in this thesis

Kevin Claber (PSDB) and Frederick Hulton (DERA, Chertsey) were both responsible for sponsoring the experiments reported here on the effectiveness of blast walls. Nicolas Johnson (PSDB) sponsored experiments concerning the clearing of blast waves on cuboid structures, also reported here.

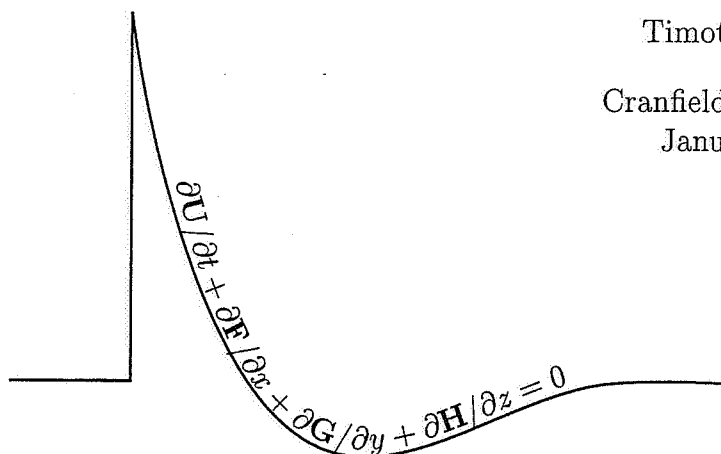
In all the experiments reported in this thesis (and many more besides) I was ably assisted in preparation and execution by Marcel Buckley, Michael Teagle and James Harber (Cranfield University) who, between them, know the best way to make or do almost anything. The explosive acquisition, storage and detonation was controlled and supervised by James Clements and Adrian Rotham (Cranfield University), whose speed and efficiency is to be admired.

Finally, I would like to thank my wife, Valerie, who has been so very patient for so very long.

Timothy A. Rose

Cranfield University

January 9, 2001



List of Symbols

α	diffusivity
α	parameter, (AUSMDV)
β	coefficient for slope limiter
γ	ratio of specific heats, (= 1.4 for air)
Δt	timestep
Δr	Radial discretisation
$\Delta x, \Delta y, \Delta z$	spatial discretisation
ϵ	parameter for entropy fix (Roe's scheme)
θ	angle with the normal direction
λ	wavelength of harmonic wave
λ	eigenvalue
ξ^s	shock indicator
ρ	density
ρ_{TNT}	density of condensed phase TNT
Φ	arbitrary scalar quantity
Ψ	vector of convected quantities, (AUSMDV)
a	Jacobian function
a	speed of sound
a^*	characteristic speed of sound
A	coefficient of shock indicator
$\mathbf{A}$	arbitrary vector quantity
$\mathbf{A}$	Jacobian matrix
b	coefficient for shock indicator
c	speed of sound
c_p	specific heat capacity at constant pressure
c_r	velocity of sound in reflected region
c_{TNT}	detonation velocity of TNT
c_v	specific heat capacity at constant volume
cfl	constant
C	speed of propagation of discontinuity
C_d	drag coefficient
C_{Dl}	clearing factor corresponding to D_l

C_{Du}	clearing factor corresponding to D_u
C_f	clearing factor
C_l	clearing factor corresponding to Z_l
C_u	clearing factor corresponding to Z_u
d	depth of structure
d	wall thickness
D	scaled structure size
D_l	lower value of scaled structure size
D_u	upper value of scaled structure size
e	internal energy per unit mass
E	total energy per unit mass
E_{TNT}	chemical energy of TNT available for heat
f	scalar flux variable
F	flux variable
$\mathbf{F}$	force vector, (derivation of equations of motion)
$\mathbf{F}$	flux vector of x direction fluxes
G	geometrical parameter
$\mathbf{G}$	flux vector of y direction fluxes
h	height of burst
h	height of monitoring location behind blast wall
h	height of structure
h	specific enthalpy
h	street height
H	function of discretised conserved variables
H	structure height
H	total enthalpy
H	wall height
$\mathbf{H}$	flux vector of z direction fluxes
i_n	specific impulse at location n
i_s	side-on specific impulse
i_r	reflected specific impulse
J	constant
K	factor, (AUSMDV)
l	length of structure (out of plane)

l	street length
$\dot{m}$	mass flow rate
m	slope of conserved variables across cell
M	Mach number
M^*	characteristic Mach number
n_{TNT}	number of cells through radius of charge
$\mathbf{n}$	unit normal vector
p	pressure
$p_{\alpha s}$	reflected overpressure at angle α
p_0	atmospheric pressure
p_c	pressure at clearing time t_c
p_s	side-on pressure
p_{so}	side-on pressure
p_r	reflected pressure
q	heat added to a system
q	dynamic pressure
q_0	dynamic pressure
r	radial coordinate
r	radius of 1-dimensional spherical computational grid
r	simple wave, (Roe's scheme)
r	stand-off distance from wall
r_{max}	maximum radius of one-dimensional analysis
r_{TNT}	charge radius
R	geometrical parameter
R	specific gas constant
R	Stand-off distance behind wall
s	entropy
s	switching function, (AUSMDV)
S	geometrical parameter
S	surface area of finite control volume
$\mathbf{S}$	vector of geometric source terms
t	time
t_{0f}	representative side-on duration
t_a	time of arrival

t_c	clearing time
t_{rf}	representative reflected duration
t_s	switching time for second order calculations
T	temperature
T_0	atmospheric temperature
T_s	positive phase duration
TV	total variation of a variable on a domain
u	x component of velocity
U	conserved variable
$\mathbf{U}$	vector of conserved variables
v	specific volume
v	y component of velocity
V	speed
$\mathcal{V}$	volume of finite control volume
$\mathbf{V}(u, v, w)$	velocity vector with components u, v and w
w	characteristic variable (Roe's scheme)
w	street width
w	structure width
w	work done by a system
w	z component of velocity
W	charge weight
W	speed of moving shock wave (shock tube relations)
W	structure width
x, y, z	Cartesian coordinates
z	height of charge above the ground
Z	scaled distance
Z_l	lower value of scaled distance
Z_u	upper value of scaled distance

Chapter 1

Introduction

1.1 The Essence of the Problem

Evaluation of the transient forces which develop on the surface of structures when a high explosive charge is detonated nearby is an extremely difficult task. The essence of the problem arises from the fact that propagation of shock waves in air is outside the experience of most people. Although shock phenomena have analogies in everyday life: weather systems and traffic jams, for example, the processes that will be addressed in this document occur considerably faster than the perception of humankind can appreciate. Consequently, observations via specialist equipment or simulation using mathematical models are the only means by which experience of such phenomena can be gained. The process of gaining that experience is necessarily lengthy.

1.2 Possible Approaches to the Problem

There are, essentially, three possible approaches to the evaluation of blast loads on structures.

1.2.1 Experimentation

In the past, many important steps taken to establish guidance on the evaluation of blast loads on structures have been experimentally based. Examples are the establishment of *scaled blast parameters* for free air and hemispherical surface bursts (Kingery and Bulmash [28]) and for infinite reflecting surfaces (Swisdak [50]), loads on solitary rectangular structures (TM 5-1300 [13]) and the blast environment behind protective walls (Beyer [6]). The results of some of these studies are discussed below.

Experimentation is still probably the most expeditious way of obtaining accurate information concerning any given blast loading situation. Problems associated with explosive experimentation and shock wave measurement are the danger to equipment from unforeseen loads/fragments and the lack of repeatability associated with small scale short duration events. These problems are, however, outweighed by the fact that shock waves retain their characteristic shape and behaviour at virtually any

achievable practical scale. These facts mean that experimentation—especially at full-scale—is still the primary method of advancing the subject area.

In Chapters 4 to 7, use will be made of published experimental data for the purpose of validating the numerical approach introduced below. Because of the influencing factors indicated above, however, the process of comparison is not straightforward. The experimentally-based information presented by Kingery and Bulmash [28], for example, was derived from data from nine different sources. Discussion of the scatter of the data is minimal and the error associated with the resulting graphical output is not stated. Similarly, the information in Swisdak [50] is based on four different sources, and, although the accuracy of the collated graphical data is indicated, it is only stated in broad terms.

The experimental data used for comparison in Chapters 5 to 7 are from technical reports where access to the source data was available. This has allowed the opportunity to discuss the difficulties associated with experimentally-derived results in a tangible manner. Therefore, a brief discussion of the equipment, procedures and major influencing factors which have affected those data sets is set out below. This will allow reference to be made to the individual effects in the chapters which follow.

Equipment: The essential elements of equipment for transient pressure recording are transducers, amplifiers, cabling, recording and storage apparatus. A thorough description of air blast transducer and instrumentation systems can be found in Baker [5], although tape recorders and cathode ray oscilloscopes are now more usually replaced by digital recording and storage equipment. A second, more up to date, description can be contained in the paper by Shah et al. [45].

For short duration events piezoelectric pressure transducers are often employed, used in conjunction with charge amplifiers. For the experiments described in Chapters 5 to 7, piezoelectric transducers were Kistler Type 603B and 6031, the amplifiers were Kistler Type 5001 and 5007, and the signals were captured using a Nicolet 420 Pro Digital Storage Oscilloscope. The sample rate was $2\mu\text{sec}/\text{point}$, and the amplifiers contained a 180 kHz input filter. This equipment and settings form an appropriate and compatible combination for short-duration blast wave measurement.

Transducer Mounting and Cabling: The pressure transducers were mounted in nylon anti-vibration mounts. These are designed to decouple the transducers from high frequency stress waves which can travel through the ground and the experimental system. Because blast waves typically travel at 350–450 m/s, and stress waves through steel (from which a number of the experimental systems described below were built) travel at about 5000 m/s, physical vibrations invariably arrive before the blast waves and can adversely affect the pressure records. Complete decoupling of the pressure transducer, and its associated cabling, from the effects of vibration is impossible. As a result, pressure records are always affected to a greater or lesser extent by the effects of vibration. This will be discussed below with reference to the establishment of peak pressure from recorded pressure-time histories.

Cables which connect pressure transducers to amplifiers and amplifiers to the storage device should not be loaded by the blast wave. In practice, this is seldom achievable, though enclosing cables in semi-rigid physical shields (such as lengths of flexible hose) can be quite effective. If possible, it should be ensured that the cables are only loaded after the blast has been measured. Loading of the cable is just one of the many possible causes of the phenomenon referred to as "drift", described below.

TNT Equivalence: It is usual in air blast experimentation to present the results of an investigation in terms of a reference explosive. Most usually, the reference explosive is TNT, and this requires appropriate choice of TNT equivalence for the particular explosive used.

There are essentially three approaches to the problem of TNT equivalence:

Source Energy: The simplest approach is to base the TNT equivalence on the ratio of the mass specific energy of the actual explosive used to that of TNT. This is the *source energy* equivalence and is described in Baker et al. [4].

Pressure and Impulse: A slightly more detailed approach is described in References [13] and [16], where separate values of TNT equivalence are proposed for peak pressure and specific impulse. Again, these are single values and are described as "equivalent weight ratios for free air effects".

Non-linear Variation: A more thorough approach, proposed by Swisdak [50], describes non-linear relationships between TNT equivalence and distance from the charge for each of the blast wave parameters. In other words, the pressure-distance (or impulse-distance) curve for a given explosive is not necessarily parallel to that of TNT. Swisdak [50] provides graphs of TNT equivalence for several commonly used explosive types.

It is evident from the above that there is continuing debate concerning TNT equivalence. In the present study, the first, most simple, approach has been adopted, and the value of TNT equivalence used in the experiments and numerical analyses was the same.

Explosives and Detonators: The explosives used for the experiments were either Demex plastic explosive or SX2 sheet plastic explosive. Both materials comprise 88% by weight of cyclonite together with 12% lithium stearate and paraffin oil as the desensitiser.

The former was moulded into a sphere and the detonator inserted; the latter was wrapped around the detonator to form a short cylinder with aspect ratio of about 1:1, which approximated to a sphere. The detonators were either No. 8* or L2A1, both of which contain approximately 1.4 g of explosive.

Surface Roughness: One important aspect of model scale experiments is the surface roughness of the model structures and the ground. The model structures described in Chapters 5 to 7 were purpose-built, and, in scaled terms, the

surfaces were sufficiently smooth to be at least as smooth as the surfaces of full scale structures.

The surface of the concrete plinths at the Explosives Research and Demonstration Area at RMCS, where the experiments were performed, was not smooth enough for small scale experiments to be representative of a typical pavement surface at full scale. Therefore, large steel plates were used to model the ground surface in regions close to the charge. Aluminium plates or smooth concrete slabs were used in more remote areas.

Signal Drift, Interference and Wild Points: Experimental records from short duration events measured in a hostile environment are invariably subject to "drift". This means that, during the course of recording an event, the baseline of the record shifts and at the end of the event, rather than producing a zero overpressure, a finite value is indicated. Drift is common, but the extent and implications of drift are not always serious. It is *not* common for pressure measurements to be seriously affected by drift during the first part of the recording of a pressure-time history (throughout the positive phase, for example). Therefore, most of the information important to investigators, such as peak pressure, impulse and positive phase duration, can all still be extracted, even if the remainder of the record is poor. The same is not true of negative phase duration and negative phase impulse; these are usually difficult to establish with confidence, even in larger scale investigations.

All charge amplifiers drift to a certain extent, and drift is more likely if the amplifiers are configured to measure short duration events. It is sufficient to state that the main cause of drift (apart from defective amplifier transistors) is insufficient insulation of the input circuit. A second, more serious, cause of drift is due to the heat pulse from the detonation of the explosive. This can cause the transducer baseline to change quite suddenly. Fortunately, the effect is only really pronounced at small scaled ranges, and it can be alleviated, to some extent, by use of heat shields such as silicone grease or PVC tape applied to the diaphragm of the transducer.

Any output interference is dependent on the capacitance of the pressure transducer and its associated cabling and is usually small. More pronounced interference can occur from the electrical discharge caused by the detonation of the explosive, which appears on the pressure records as high frequency noise just before arrival of the blast wave. Other types of interference, due to vibration of the transducer or its mounting from stress waves, was mentioned previously.

Wild points are digitiser errors and are usually single points in an experimental record which are (most often) at the top or bottom of the digitiser range. Single wild points are relatively easy to recognise and remove, without adversely affecting the record. More severe digitiser errors, resulting in small groups of wild points, have to be treated with more care, and some loss of information can result.

Interpretation of Experimental Records: Pressure records derived from a "free air" blast wave or from blast impinging on an infinite surface or interacting with a single isolated structure have a similar simple form characterised by one large peak followed by a smoothly varying decay back to ambient conditions during the positive phase. There may be a smaller second peak, which arrives much later during the positive phase, but this is usually of little or no importance to investigators. Such records are relatively straightforward to interpret.

Pressure records from a partially confined explosion, such as might occur in a city street or other situation where more than one reflecting surface is present, are more difficult to interpret. If reflections are distinct in time, as they would be if the pressure behind individual peaks fell below ambient pressure, establishment of a representative positive impulse and positive phase duration cannot be made without knowledge of the response of the loaded structure (or structural element). Structural response assessment is beyond the scope of this thesis and so, in Chapters 5 to 7, positive impulses have been based on the integral of the whole pressure record.

It is usual for investigators to summarise the transient pressure loading due to the passage of a blast wave over a structure in terms of two basic parameters: peak pressure and peak positive specific impulse. Sometimes, though less commonly, positive phase duration is also described, as this is important for structural response calculations.

Although peak pressure is one of the most important blast parameters, it is seldom reported in a statistically valid manner. The effects of mechanical vibration and electrical interference, described above, tend to cause a superposition of high frequency noise on pressure records. Such oscillations *always* have the effect of *increasing* the observed peak pressure of the "raw" digitised data. Most commonly, experiments are conducted specifically to establish the design load on a structure or for the purpose of risk assessment. Therefore, the tendency for peak pressure values to be overestimated by experimentalists is often seen as beneficial, as a conservative design, or assessment, will result. It should be recognised that rigorous statistical analysis of peak pressure is not a straightforward task, as the random vibrational effects are superimposed on the underlying spread of experimental peak pressures. Any analysis based on a single point of a large record would not be making sufficient use of the vast majority of the available data. For example, the frequency content as well as the amplitude of an extraneous oscillation determines, to a large extent, the effect which it has on the peak pressure.

Methods exist for dealing with the problem of poorly resolved peak pressures. One common approach, described by Baker [5], is to fit an exponential curve to a portion of the positive phase, just behind the shock front, and, by extrapolation, use the value on the curve at the *actual* arrival time as a replacement value of measured peak pressure. A second approach (Shah et al. [45]) is to smooth the record by establishing and then extracting the high frequency component of vibration or interference. This technique is more common for

the establishment of long duration quasi-static pressures. Both approaches are equally valid, but they are procedures which need to be performed "by hand" on a record-by-record basis and are, therefore, very time-consuming.

The extraction of peak positive phase specific impulse from experimental pressure records is quite straightforward: the record is integrated and the peak value noted. The problem of high frequency interference is not usually significant in positive impulse evaluation because the integral of the oscillations can usually be assumed to be close to zero. For this reason, positive impulse is the most reliable and repeatable of the blast wave parameters. It is also often the most important from a structural response standpoint.

Positive phase duration is also relatively easy to establish from pressure records with a single distinct peak, but it is less easy for records with more than one pulse. It is probably not appropriate to attribute a positive phase duration to records which contain multiple reflections. Such records should be considered in the context of the likely structural response.

Despite the relative ease of establishing positive phase duration from *simple* pressure records, there will inevitably be some shot-to-shot variability in the positive phase results, which can sometimes be significant.

Experimental Procedure: The implication of the various effects discussed above has led to an experimental methodology which seeks to reduce the effect of shot-to-shot variation by establishing an average value of the blast wave parameters. A desirable approach (Baker [5]) is to conduct each experiment five times. This allows the possibility of establishing a useful average while also providing the opportunity to discard poor records. In practice, such an approach is seldom achievable. Time and financial constraints often result in experimental programmes which sometimes do not allow repeat firings. The experiments used for validation in Chapters 5, for example, are the result of single firings, those in Chapter 6 are the average of three, and the majority of the experiments used for comparison in Chapter 7 are, again, the result of single firings. These constraints, together with the effects discussed above, inevitably lead to some uncertainty concerning the validity and usefulness of some experimentally-derived information.

1.2.2 Numerical Modelling

Since the mid 1940s, physicists and mathematicians have sought to model shock waves in air using computational techniques. Notable contributions were made by Brode [11], Von Neumann [55] and Lax [30]. More recently, the discipline of Computational Fluid Dynamics (CFD) has matured and become extremely useful and popular. Much of the work done in this area has been focused on the problem of supersonic and hypersonic flight, and many investigators have been attracted to this exciting area of research. Fortunately for the structural engineer, the same underlying equations of motion (the Euler equations) and their methods of solution are applicable to the problem of blast wave propagation.

There are particular problems, however, associated with the computational treatment of blast waves arising from high explosive detonations and their interaction with structures. These differ from the problems experienced by the aerospace industry. The principal problems are threefold:

- Massive pressure, energy and density discontinuities exist between the high explosive gas products and the surrounding air immediately after detonation.
- It is usually necessary to use large computational grids to contain all the structures and boundaries, some of which may be remote from the point of detonation.
- Large parts of the computational grid are *idle* at any given time. The so-called "serious action" takes place in different parts of the grid at different problem times, as the blast waves propagate from one place to another.

These difficulties will be explained further in what follows, but for the time being, it is sufficient to note that robust schemes (and simplifications) exist which can confront the first problem. The second problem is an inevitability, but it can be overcome, to a certain extent, with foreknowledge of the results, perhaps by preliminary investigation. It might then be possible to switch large parts of the computational grid on or off to reduce the overall size of problem considered at any given time. The second problem is also experienced in the aerospace industry where in transonic flow, for example, the calculation of lift produced by an aerofoil requires knowledge of the circulation around it. This requires boundaries which are sufficiently remote that they do not affect the solution at the surface. The second and third problems can be addressed with new and powerful techniques such as Adaptive Mesh Refinement (AMR) (for example, Quirk [38]), which is merely mentioned here but promises massive savings of time and resources for future investigators.

At the present time, the author is unaware of data derived from numerical simulation which are incorporated into empirical methods or written guidance. There are probably two main reasons for this. Firstly, the "Engineering Community" (structural engineers and facilities managers responsible for protecting vulnerable buildings) is still very sceptical of numerical simulations, because it is only recently that the accuracy of solution methods has become acceptable. Secondly, good quality simulations require considerable investment of time for the development of software and finance for hardware and processing (CPU) time. Because of this last consideration, numerical simulation is still at least as expensive, and certainly slower and generally less accurate, than small scale experiments.

1.2.3 Empirical and Semi-Empirical Methods

The approaches outlined above are primarily used to address new and particular problems where guidance is not available. A third approach is to use the results of fundamental or generic experiments combined with analytical methods to develop new prediction techniques which form either a subset of the original data (*empirical methods*), or extend the data by extrapolation or manipulation in a justifiable way (*semi-empirical methods*).

This approach is particularly attractive, because it allows individual situations to be evaluated relatively quickly. Confidence (or otherwise) in the solution is easy to establish, as it is usual for the procedures to describe the underlying data and assumptions used in their development.

There is a distinction between fully empirical methods, where the results of experiments are presented (usually in the form of graphs or formulae), and semi-empirical methods. The latter also provide a system for using the data so that scenarios other than those originally investigated are included. These semi-empirical methods have, necessarily, to be used with great care. It is possible to extend their use beyond that which is sensible or valid, and this must be avoided. However, if used within their stated limitations, such methods are very powerful and approach the accuracy of experiments and simulation while using only a fraction of the resources. Examples of problems addressed by empirical methods are:

Scaled Blast Parameters: The most commonly used set of scaled blast parameters are those of Kingery and Bulmash [28]. These are polynomial curve fits to experimental data, and they form the basis of many semi-empirical techniques.

A particularly popular implementation of the equations is contained in the conventional weapons effects program ConWep [24].

Scaled blast parameters also exist which characterise the loads on infinite reflecting surfaces at various angles of incidence. Two examples are the data of Swisdak [50] and Reference [7] (although it originated from nuclear weapons data).

Blast Wave Clearing: The process of blast wave clearing is described by the U.S. Department of the Army Technical Manual TM 5-1300 [13]. The procedure demonstrates how information from scaled blast parameters can be combined to approximate the pressure-time loading history on the front face of an above ground rectangular structure. The efficacy of this approach will be discussed in Chapter 5.

Pressures Behind Blast Walls: There are at least three separate empirical procedures for the determination of the blast environment behind protective walls. Each has its own region of applicability and validity. The first among these is the paper by Beyer [6], which provides graphical information for a relatively small number of geometries.

The U.S. Department of Transportation [35] produced guidance in the form of exponential curve fits to data resulting from $1/10^{\text{th}}$ scale perimeter wall experiments [26]. These curves were implemented in a computer program.

Rose et al. [43] used a comprehensive set of data from experiments on scale models to produce polynomial curves, and hence predictive graphs, with a relatively wide range of applicability. Despite this wide range, the data is lacking with respect to small scaled wall height.

There are a number of semi-empirical prediction techniques, some have been developed and maintained over many years. Examples are given below:

BLASTX: The BLASTIN [10] family of computer codes, of which BLASTX [8] is the latest, use a ray tracing algorithm to piece together the complicated shock wave environment of internal blast loading scenarios. Once the path length of rays describing the multiple reflections have been calculated, BLASTX combines the pressure history arising from each ray using the LAMB (Low Altitude Multiple Burst) shock addition rules [21].

The BLASTX technique is also applicable to certain classes of external blast loading problems considered in the present study: namely, urban environments of simple geometry with remote boundaries. Application of this technique to examine simple urban geometries is described by Whalen [57].

SHOCK: SHOCK [46] is a computer program that enables the calculation of the gross shock pressure and impulse on all or part of a rectangular surface bounded by one to four surfaces and no roof. It was developed from theoretical procedures, empirical blast data and the results of response tests on slabs.

SHOCK was used to produce graphs describing the blast loads on partially vented structures in the document TM 5-1300 [13].

The two semi-empirical methods described above are characterised by being computer-based. In one case, BLASTX, the amount of computational effort needed for the treatment of complicated geometries is significant, although it is still a fraction of the CPU time needed for CFD calculations.

1.3 The Approach Taken by the Present Study

The approach taken by the present study is to use numerical simulations from the computer program Air3d (Chapter 3) to produce data for three generic blast loading scenarios in Chapters 5, 6 and 7. Results of the simulations are compared with experimental data then used to reinterpret the results of the experiments (Chapter 5) or simply establish important trends (Chapters 6 and 7). This process is, effectively, the formulation of empirical methods from a hybrid of numerical and experimental studies. It is suggested that the results of this approach may retain much of their validity because of the experimental basis but have extended range by the addition of the numerical studies. The possibility of this approach was indicated in Section 1.2.2 above.

1.3.1 The Geometry of Space and Structures

In this study, only cuboid spaces and structures were considered. The structures themselves were restricted to align along Cartesian coordinate directions. These restrictions will not seriously affect the generality of the scenarios considered, nor the resulting conclusions. By considering only cuboid geometries, an extra layer of complexity in the computational technique was removed.

In the definitions which follow, it should be borne in mind that the structures referred to are all cuboid in nature.

1.3.2 Definitions

For the purpose of this study, a number of definitions are proposed concerning the various aspects of the subject of blast loads on structures. The order in which the definitions of structure types are listed parallels the complexity of the evaluation process for each type. This will become clear in Section 1.4 below.

Region of Interest: The *region of interest* is the portion of three-dimensional space in which the behaviour of the blast wave(s) and the blast wave/structure interaction are important. Interactions which occur outside this region, and do not propagate back into it, are irrelevant.

Period of Interest: The *period of interest* is the period of time during which the blast wave/structure interaction takes place.

Most usually, the start of the *period of interest* is the time at which the explosive charge is detonated. However, if the charge is remote from the *region of interest*, and no interactions take place in the intervening space (except perhaps with the ground), then the start might be the arrival time of the blast wave in the *region of interest*.

The finish time can be established in many ways, depending on the type of problem under consideration. There will inevitably come a time when no blast waves persist in the *region of interest*. This will occur when either they have propagated out of the region or decayed to acoustic levels. These are the most obvious choices. However, alternatives exist. For example, the time to

reach maximum displacement (if a structural element is under consideration) or the time at the end of the positive phase of the incident or reflected wave (if subsequent waves or reflections are intuitively considered insignificant) could define the finish time.

Point Structure, Free Air or Hemispherical Surface Burst: When an explosive charge is detonated in air, remote from any reflecting surfaces, it is described as a *free air burst*; there are no structures sufficiently near to affect the propagation of the spherically expanding blast wave in the *region of interest* during the *period of interest*.

A hemispherical high explosive charge detonated on an infinite reflecting surface, with the same conditions as above regarding the remoteness of other reflecting surfaces, is essentially the same scenario as free air and is described as a *hemispherical surface burst*. These two scenarios are characteristically one-dimensional problems.

Infinite Structures or Height of Burst: A single infinite reflecting surface located some distance from a high explosive charge within the *region of interest* is described as a "Height of Burst" (HOB).

In this scenario, the reflecting surface is sufficiently close to the charge so that reflections from it propagate back through the *region of interest* during the *period of interest*. Boundaries of the reflecting surface are sufficiently remote that reflections or expansions arising from interactions with them do not reach the *region of interest* in the *period of interest*.

Following the argument above, multiple infinite structures implies two parallel surfaces, because two or more surfaces which are not parallel necessarily meet at an edge and are not, therefore, infinite. In practice these scenarios lead to a very restricted class of two-dimensional problems.

Semi-Infinite Structures: Structures which are infinite in one or more dimensions are *semi-infinite structures*. Such structures have at least one finite dimension in the *region of interest*, and the presence of its boundaries cause expansions or reflections (or both) to propagate through the *region of interest*. If this occurs during the *period of interest* a situation of increased complexity will result.

The interaction of an expanding spherical blast wave with a semi-infinite structure is a three-dimensional problem. However, the fact that the structure has only one or two finite dimensions reduces the complexity of the blast wave/structure interaction and is less demanding to evaluate than the final class of problems, described next.

Finite Structures: A *finite structure* is fully enclosed within the *region of interest*. It has boundaries which produce interactions that affect the environment within the *region of interest* during the *period of interest*.

This category comprises all structures of complex geometry and results in highly three-dimensional problems which are both complicated and time consuming to evaluate.

1.4 Examples and Existing Methods of Evaluation

In the examples which follow, reference will be made to the generic scenarios shown in Figures 1.1 to 1.3, below. These diagrams show the essential elements of a blast loading scenario: an explosive charge of weight W (kg), a stand-off distance of the charge from the structure R (m) and a cuboid structure of dimensions d (m) and h (m), in the plane of the diagram, and l (m) in the third dimension. In each example, a geometry will be defined that will cause the nature of the problem to change significantly. These simple examples will help to illustrate why the evaluation process is intuitively difficult and set the scene for the discussions that follow in Chapters 5, 6 and 7.

1.4.1 No Structure or Point Structure

Referring to Figure 1.1, imagine the charge weight and stand-off distance assume arbitrary values which result in a scaled stand-off greater than $50 \text{ m/kg}^{1/3}$. This scenario is not of great interest to the structural engineer, because the resulting reflected overpressure is insufficient to break even domestic glazing—regardless of the associated impulse. In this case, the structure shown in Figure 1.1 is outside the region of interest; there is effectively no structure, and points of interest between the charge and the structure can be readily evaluated using scaled blast parameters for a hemispherical surface burst.

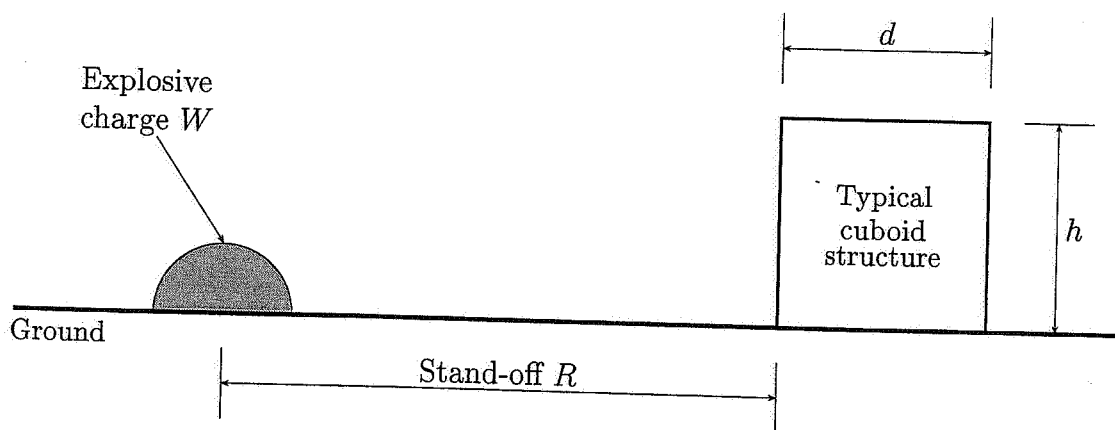


Figure 1.1: Generic blast loading scenario: point structure, semi-infinite structure, finite structure

A second similar possibility arises if the structure has infinitesimal dimensions—regardless of the stand-off distance. If the scaled dimensions of the structure are very small, it is effectively a point and will not interrupt the flow of air associated with the blast wave. Reflected (stagnation) pressures will not develop on its surface, and all important information regarding the blast wave can again be extracted from

scaled blast parameters. This scenario is essentially that of free air or hemispherical surface burst blast measurement: the structure is, in effect, the pressure gauge, specifically designed not to interfere with the flow field.

1.4.2 Infinite Structures

Consider the situation in Figure 1.2. The charge is above and *near* the structure, which extends a considerable distance beyond and out of the page.

The charge weight and stand-off distance continue to assume arbitrary values, but the structure is very large. The blast wave will reflect from and propagate along the surface of the structure, but it will not reach the edges. This is a height of burst scenario. Although it has its historical roots in nuclear weapons effects, it does have applicability to high explosive detonations: most high explosive incidents occur a short distance above the ground surface.

The important aspect of Figure 1.2 is the fact that the boundaries are remote, and reflections or expansions that occur at the boundaries are outside the region of interest, or else occur too late to interfere with the flow in the period of interest. This would be the case, for example, if the structure was a wall panel. The loading on the surface can be obtained from scaled height of burst parameters (Swisdak [50] or Reference [7]).

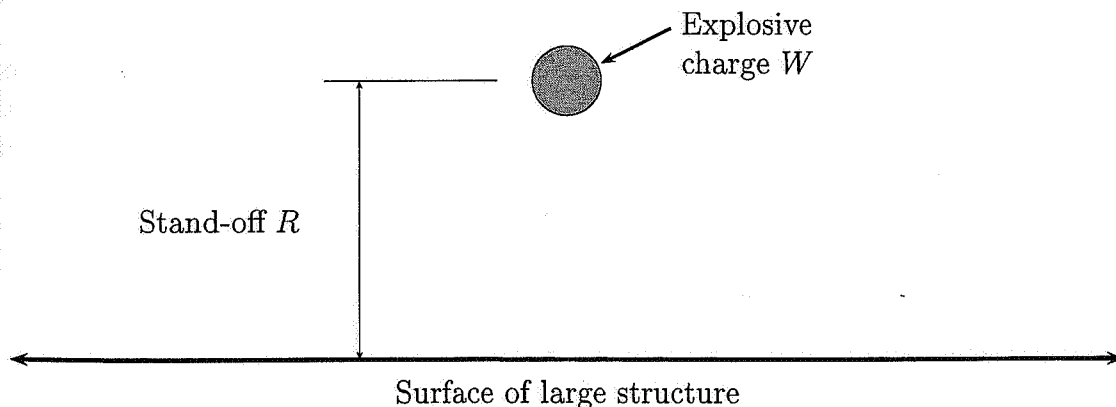


Figure 1.2: Abstract blast loading scenario: infinite structure

The situation in Figure 1.3 is that of an explosive charge mid-way between two parallel reflecting surfaces. This may appear contrived but is actually a good approximation of an underground storage facility or, more commonly, a multi-storey car park—if the columns are ignored and the boundaries remote.

The two situations described above are both two-dimensional problems, because of the radial symmetry, and can be treated satisfactorily by programs such as BLASTX [8]. Numerical modelling of both these problems is relatively computationally inexpensive and good results could be expected.

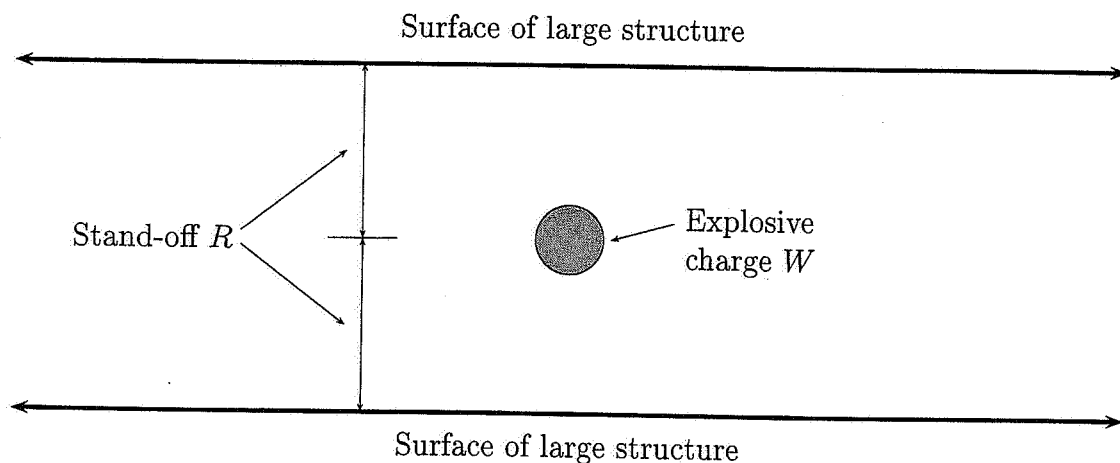


Figure 1.3: Abstract blast loading scenario: two parallel infinite structures

1.4.3 Semi-Infinite Structures

Referring to Figure 1.1 once again, if the stand-off distance R is small, the blast will interact with the structure, and the environment will be hard to describe. If the third dimension of the structure l is very large, however, the situation will be somewhat simpler, because the flow will vary more gradually in the third direction than in the plane of the page.

The situation described is that of a blast (perimeter) wall or one side of a long city street. The perimeter wall scenario has been investigated experimentally in the past, and limited numerical studies have been performed, approximating the wall as a hoop of rectangular cross-section, using radial symmetry and a relatively large charge stand-off (Chapman [12] and Rice [40]). The semi-empirical computer code HEXDAM [20] has also been used to investigate the shielding effect of blast walls (Tatom [51]).

The long city street scenario has been investigated by Feng [14] and, in a simpler configuration, by Whalen [57], where the scaled height of the structure was very large. This effectively modelled a heavily built-up area, and the experimental form greatly simplified the problem.

1.4.4 Finite Structures

The final typical scenario again refers to Figure 1.1. This time, consider the situation much as it is portrayed in the sketch. The distance R is small, and the dimensions of the structure are similarly small. The blast wave will interact with the structure, and a strong reflection will propagate back in the direction of the original charge location. The incident shock will propagate over the structure and around it (in the third dimension). These diffracted waves will meet at the rear of the structure and produce a second reflection on the ground at the base of the structure.

The situation described is a relatively simple one. The front face pressure history,

and hence the impulse, can be evaluated with reasonable confidence by the approach described in Reference [13]. Virtually every other situation, not described above, results in highly three-dimensional problems which cannot be evaluated by simple means. At present, most complicated scenarios are investigated experimentally.

Chapter 2

Background to the Development of the Code Air3d

2.1 Introduction

Section 2.2 of this chapter is in the form of a narrative. It describes the author's foray into the realm of Computational Fluid Dynamics and the process by which the solution algorithm of the program Air3d was selected. Section 2.3 contains a brief review of the schemes considered, in the order in which they were implemented in a one-dimensional test case. Detailed aspects of the code Air3d are discussed in Chapter 3.

2.2 Possible Solution Algorithms

There are numerous possible algorithms for the solution of the Euler equations. Some are only historically significant, others are well established and widely used, while others still have only recently been published and are unproven in blast applications. It was the intention to make an objective assessment of candidate schemes, with the aim of selecting an algorithm which was both suitable and relatively easy to implement.

The method used to test the various solution algorithms was comparison with the analytical solution to the shock tube problem (Appendix B). The shock tube problem is an ideal test case for numerical solution of the Euler equations.

The essential elements of compressible inviscid flow are:

- Compressive shock waves.
- Rarefaction (or expansion) waves.
- Contact discontinuities, which are boundaries between regions of differing density and energy but the same pressure.

The ability of a numerical scheme to predict and resolve these elements accurately is an important measure of their usefulness and applicability. The shock tube

problem provides all three elements. It also provides a further measure of the usefulness, or otherwise, of a scheme. Because the shock tube problem is intrinsically one-dimensional, if a method of solution fails to treat this problem satisfactorily, it can be assumed that it will certainly not work well in two or three dimensions.

The approach used in the present study was to develop a one-dimensional program STanaly, (Shock Tube analytical), which calculated and displayed the analytical and numerical solutions simultaneously, but stopped short of the boundaries of the numerical space. A second program, STgen (Shock Tube general), used the same subroutines for the numerical solution, but had boundary conditions and could be filled with air of different temperatures and pressures arbitrarily. This second version of the code was used to test the symmetry (whether the waves produced the same flow fields in each direction), the boundary conditions and the overall robustness of the solution methods.

Throughout this study, the principal sources of information, concerning solution methods, have been the excellent volumes by Hirsch [22], [23] and, to a lesser extent, Toro [52] and LeVeque [32]. These are referred to as the source for several of the schemes below. The narrative of this section closely follows the historical structure of Part VI of Hirsch [23]; consequently, the techniques evaluated here were considered in the following order (with comments interspersed):

- i. Lax-Friedrichs.
- ii. Lax-Wendroff.
- iii. MacCormack's method.
- iv. Van Leer's flux vector splitting (Van Leer [53]).
- v. Roe's method (Roe [41]).

After considering the above methods of solution, a decision was made to continue with Van Leer's flux vector splitting method because, although the resolution of discontinuities was not as good as Roe's method, it was faster and easy to program.

Second-order spatial discretisation was introduced by using variable extrapolation (extrapolation of the conserved variables), often referred to as the MUSCL approach (Van Leer [54]), where MUSCL stands for Monotone Upstream-centred Schemes for Conservation Laws. Total Variation Diminishing (TVD) conditions, in the form of variable slope limiters (which limit the variation of the conserved variables across a cell), were also introduced at the same time. Second-order accuracy in time was achieved using the predictor-corrector method described in Section 21.4.3 of Hirsch [23]. Initial numerical trials modelling the shock tube problem were encouraging so the method was incorporated into the three-dimensional code Air3d.

- vi. Van Leer's flux vector splitting, with TVD conditions and second-order in space and time.

Unfortunately, example problems using real blast loading scenarios soon revealed an underlying lack of robustness in the first-order flux vector splitting method of

Van Leer, and a new approach needed to be found. It was suggested by Forth (see preface of this thesis) that it would be worth considering current research into flux vector splitting type schemes, notably:

- vii. AUSM, Advection Upstream Splitting Method (Liou and Steffen [33]).
- viii. AUSMD and AUSMV leading to AUSMDV (Wada and Liou [56]).

These schemes have the attractive qualities of flux vector splitting schemes in that they are easy to program. They also proved to have considerable robustness, which is important in blast wave modelling, because of the massive pressure, density and energy discontinuities in the flow at early stages of the calculations.

The final method evaluated (AUSMDV) was modified to make it second-order in space and time, using the same procedure as above. Finally, again on the advice of Forth [15], a version was implemented which was second-order accurate in space and time and used MUSCL-Hancock integration, described in Section 14.4 of Toro [52].

- ix. AUSMDV with TVD conditions and second-order in space and time.
- x. AUSMDV with TVD conditions and second-order MUSCL-Hancock integration.

MUSCL-Hancock integration avoids the need to calculate a precise half-timestep solution; it is faster, requires less storage and fewer lines of code. This was the final form used in the three-dimensional code Air3d for all the calculations presented in this document.

It will be recognised that the list of schemes above is by no means exhaustive. Indeed, not all the methods referred to by Hirsch [23] have been implemented. Notable exceptions are the flux vector splitting scheme of Steger and Warming [48], Godunov's exact scheme for the Euler equations (Godunov [18]) and the approximate Riemann solver of Osher [37]. Van Leer's flux vector splitting scheme is generally considered an improvement on that of Steger and Warming, because the split fluxes are continuous with respect to the Mach number of the flow. The mass flux component of the Steger and Warming split fluxes is discontinuous at sonic points, and this gives rise to a "glitch" at the sonic transition during a strong expansion. Therefore the flux vector splitting scheme of Van Leer was preferred. Godunov's exact scheme for the Euler equations requires the solution of an implicit equation at each cell interface and is quite time-consuming. Similarly, the approximate Riemann solver of Osher involves more steps and is computationally more expensive than that of Roe; it is also not as widely used. Therefore, the latter was preferred for consideration in the test case.

2.3 Review of Possible Solution Algorithms

The brief sections which follow contain a description of the methods listed above, with illustrations of the important points mentioned. The discussion of each method has been limited to the bare essentials, although sufficient information is provided so that the way the approaches were programmed can be seen. For more detailed information, reference should be made to the relevant sources.

For each method considered, solution of an example shock tube problem is given to provide a qualitative representation of the accuracy of the method. Graphs of pressure, density, specific energy and velocity are plotted at the same arbitrarily chosen final problem time.

The hypothetical test problem comprised a shock tube 1 m long; the left half was the driver section; the right half was the driven section. The experiment was conducted at an arbitrary room temperature (300 K). The initial arrangement was the same as that shown in Figure B.1 of Appendix B. Details of the test problem are given in Table 2.1.

Table 2.1: Details of the shock tube test problem

Number of cells	200
Tube length (m)	1.0
Driver pressure (Pa)	8.0×10^5
Driven pressure (Pa)	1.0×10^5
Temperature (K)	300
Problem finish time (sec)	8.0×10^{-4}
CFL number	0.75

Before the schemes investigated are discussed in detail, a number of more general aspects of the numerical solution of the Euler equations will be introduced. These will be referred to in the sections which follow.

One-Dimensional Euler Equations

We require a solution to the one-dimensional form of the Euler equations, which are fully discussed in Appendix A:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0, \quad (2.1)$$

where

$$\mathbf{U} = \begin{Bmatrix} \rho \\ \rho u \\ \rho \left(e + \frac{u^2}{2} \right) \end{Bmatrix} \quad \text{and} \quad \mathbf{F} = \begin{Bmatrix} \rho u \\ \rho u^2 + p \\ \rho u \left(e + \frac{u^2}{2} \right) + pu \end{Bmatrix}. \quad (2.2)$$

In what follows, the discretised form of the equations refer to conserved quantities U and fluxes F , not the complete system of variables $\mathbf{U}$ and flux vectors $\mathbf{F}$. This is usual in the literature, and it should be borne in mind that the equations, as they appear below, are applied to each of the three conservation equations.

The Conservative Property

An essential property of a discretised scheme is the *conservative* property. Essentially, this property requires that the time derivative of the integral of U over a given spatial domain depends only on the boundary fluxes of the domain and not the individual fluxes within the domain at the internal cell interfaces. This ensures that the discretisation technique represents a discrete approximation of the integral form of the conservation equations (see Appendix A). The conservative property implies that the schemes should be *finite volume* representations of the governing equations. Although finite difference and finite element formulations may also be conservative, they are not necessarily so.

The conservative property of a discretisation leads to a general expression of a scheme by the introduction of a *numerical flux* F^* (in one dimension), where F^* is a function of mesh point values U_i . All conservative explicit schemes have to be expressed in the following form, written here in one-dimension:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} (F_{i+1/2}^* - F_{i-1/2}^*) \quad (2.3)$$

with the consistency equation:

$$F^*(U_i, \dots, U_{i+j}) = F(U), \quad \text{when all } U_i = U.$$

Simple Upwinding

The algorithms considered below can be divided into two broad categories: space-centred schemes and upwind methods. The algorithms of Lax-Friedrichs, Lax-Wendroff and MacCormack are all space-centred; information is gathered from cells on either side of the cell being integrated in time, regardless of the actual flow direction. The remaining methods are upwind methods, where an attempt is made to propagate the flow variables in the physically correct manner. Unfortunately, simple upwinding is not possible with non-linear equations, such as the Euler equations, for the following reason. Hyperbolic *scalar* equations of the form:

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0 \quad (2.4)$$

can be expressed in *quasi-linear* form:

$$\frac{\partial u}{\partial t} + a(u) \frac{\partial u}{\partial x} = 0.$$

When the Jacobian function $a(u) = df/du$ is of constant sign, say $a(u) > 0$, simple upwinding can be used in the form:

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x} (f_i^n - f_{i-1}^n)$$

Similarly, a forward difference scheme could be used if the propagation speed $a(u) < 0$. The difficulty in defining an upwind scheme arises when $a(u)$ changes sign, as it does in the system of Euler equations.

The Entropy Condition

The system of Euler equations admits discontinuous solutions. However, several discontinuous solutions can exist, but not all of them are physically admissible. It will be seen in the sections below that the methods of Lax-Wendroff and MacCormack, when used in their basic form, both allow expansion shocks to form. Physically, only compression shocks are admissible and expansion shocks, which correspond to a negative entropy variation, are excluded, since they cannot occur in real flows. The mechanism added to the system of equations in order to select the correct discontinuity and reject the non-physical ones is called the *entropy condition*.

Again referring to the scalar conservation law (equation (2.4)), the condition to be satisfied by discontinuous solutions is that the wave speed $a(u) = df/du$ is such that:

$$a_L = a(u_L) > C > a(u_R) = a_R,$$

where C is the speed of propagation of the discontinuity and u_L and u_R are the values on the left and right sides of the discontinuity. This form of the entropy condition implies that characteristics on either side of the discontinuity ultimately intersect it, which in turn implies that the discontinuity is a compression shock.

In practical applications, every initial value problem (like the shock tube problem) has a unique solution, satisfying the entropy condition, that can be considered as the limit for vanishing coefficient α of the equation:

$$\frac{\partial u}{\partial t} + \frac{\partial F}{\partial x} = \alpha \frac{\partial^2 u}{\partial x^2}. \quad (2.5)$$

Equation (2.5) demonstrates that if sufficient viscosity terms are added to the discretised equations, physically inadmissible discontinuities will never appear. (This will be discussed below in relation to the methods of Lax-Friedrichs and Lax-Wendroff.) The key is only to introduce viscosity at the correct place, at the correct time, otherwise the scheme becomes over-diffusive and shocks will be unacceptably smeared.

Entropy conditions are not usually implicit in a scheme. Therefore, the solution has to be corrected explicitly by identifying the conditions under which entropy-

violating situations can occur. Examples of such *entropy fixes* will be seen in Section 2.3.5 for Roe's scheme and in Section 2.3.8 for AUSMDV.

The Courant–Friedrichs–Lewy Condition

The methods considered in the sections which follow are all time-dependent solutions of the Euler equations. The integration timestep (Δt) is controlled by a Courant–Friedrichs–Lewy (CFL) condition of the following form:

$$\Delta t = cfl \frac{\Delta x}{\max(a + |u|)}, \quad (2.6)$$

where a is the local sound speed, u is the local velocity (in one-dimension), and the constant $cfl < 1$ is the factor of safety required to ensure stability of the scheme. Equation (2.6) simply states that waves cannot propagate across a cell (of width Δx) in one timestep. The value Δt from equation (2.6) is the minimum taken over all the cells in the domain.

2.3.1 Lax–Friedrichs

The one-dimensional Lax–Friedrichs scheme (Section 17.1.1, Hirsch [23]) is as follows:

$$U_i^{n+1} = \frac{U_{i+1}^n + U_{i-1}^n}{2} - \frac{\Delta t}{2\Delta x} (F_{i+1}^n - F_{i-1}^n), \quad (2.7)$$

where U and F are cell-centred values of the conserved variables and fluxes given by equation (2.2), Δt is the timestep and Δx is the cell size. Subscripts refer to the computational cell number, superscripts refer to the discrete time. New values of the conserved variables (on the left of equation (2.7)) are calculated from the old values and the fluxes (on the right side). This is typical of all the explicit time-marching solutions considered. A sketch showing the nomenclature is given in Figure 2.1. Results of the example shock tube problem calculated with this scheme are shown in Figure 2.11 at the end of Section 2.3.

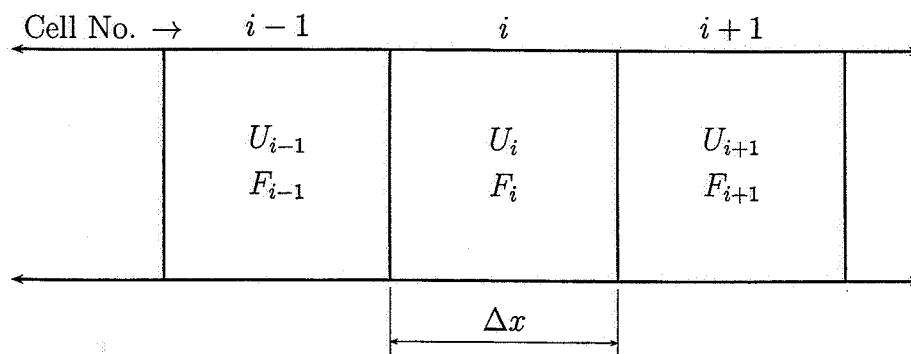


Figure 2.1: Cell-centred conserved variables and fluxes

Because of its extremely dissipative nature, this particular scheme is not widely used any more. The dissipative nature of the scheme is best explained with reference to the *modified* equation. The numerical solution can be considered a solution of the actual partial differential equation plus truncation and rounding errors or the exact solution of a different equation, called the *modified* equation. This second approach explains why the diffusive (or dissipative) effects appear in the solution, and it can be illustrated by recognising that equation (2.7) can be written in the equivalent form:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{2\Delta x} (F_{i+1}^n - F_{i-1}^n) + \frac{1}{2} (U_{i+1}^n - 2U_i^n + U_{i-1}^n).$$

Since the last term on the right in parenthesis can be considered as the discretisation of $(\Delta x^2/2\Delta t \cdot \partial^2 U/\partial x^2)$, the Lax-Friedrichs scheme can be viewed as being obtained from an explicit time integration of an equation of the same form as equation (2.5), which is a dissipative equation with numerical dissipation α .

The Lax-Friedrichs scheme does have a number of very desirable qualities, however. Because of the smoothing quality of the method it is extremely robust and does not fail (for valid CFL numbers), regardless of the magnitude of the discontinuities it encounters. This quality is known as *monotonicity*; a monotone scheme does not lead to oscillatory behaviour of the numerical solution. In addition, monotone schemes do not violate the entropy condition mentioned above.

The condition of monotonicity can be expressed by considering the general form of a numerical scheme applied to the scalar conservation equation (2.4); it is the following:

$$u_i^{n+1} = H(u_{i-k}^n, u_{i-k+1}^n, \dots, u_{i+k}^n). \quad (2.8)$$

The scheme represented by equation (2.8) is said to be *monotone* if H is a monotone increasing function of each of its arguments, that is:

$$\frac{\partial H}{\partial u_j} (u_{i-k}^n, u_{i-k+1}^n, \dots, u_{i+k}^n) \geq 0 \quad \text{for all } i-k \leq j \leq i+k.$$

The function H is completely defined by the numerical flux of the scheme, with:

$$u_i^{n+1} = H(u_{i-k}^n, u_{i-k+1}^n, \dots, u_{i+k}^n) \equiv u_i^n - \frac{\Delta t}{\Delta x} (f_{i+1/2}^* - f_{i-1/2}^*).$$

Returning to the Lax-Friedrichs method, we write:

$$U_i^{n+1} = H(U^n; i)$$

to signify that U_i^{n+1} is dependent on the full vector U^n . The scheme is monotone provided that the CFL condition is satisfied, since:

$$H(U^n; i) = \frac{1}{2} (U_{i-1}^n + U_{i+1}^n) - \frac{\Delta t}{2\Delta x} [F(U_{i+1}^n) - F(U_{i-1}^n)],$$

which satisfies

$$\frac{\partial}{\partial U_j^n} H(U^n; i) = \begin{cases} \frac{1}{2} \left(1 + \frac{\Delta t}{\Delta x} F'(U_{i-1}^n) \right) & \text{if } j = i - 1 \\ \frac{1}{2} \left(1 - \frac{\Delta t}{\Delta x} F'(U_{i+1}^n) \right) & \text{if } j = i + 1 \\ 0 & \text{otherwise.} \end{cases}$$

The CFL condition ensures that $1 \pm \frac{\Delta t}{\Delta x} F'(U_j^n) \geq 0$ for all j and so $\partial H(U^n; i) / \partial U_j^n \geq 0$ for all j, i . In simple terms, the above condition means that if the value of U_j^n is increased then the value U_i^{n+1} cannot decrease as a result.

Unfortunately, it can be shown (Section 21.2.2, Hirsch [23]) that conservative monotone schemes for the non-linear equation (2.4) are only of first-order accuracy. Hence, conditions less severe than monotonicity are required to allow the definition of high resolution schemes. This will be addressed in Section 2.3.6.

The Lax-Friedrichs method is very cheap in terms of memory requirement and number of machine operations; it is also very easy to program. For these reasons, the method was the first to be implemented in the three-dimensional code Air3d, and it was used extensively in the early stages of development.

2.3.2 Lax-Wendroff

The Lax-Wendroff scheme (Section 17.2.1, Hirsch [23]) is a second-order, space-centred scheme. It derives from the fact that equation (2.1) can be written in quasi-linear form:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial \mathbf{U}} \frac{\partial \mathbf{U}}{\partial x} = 0$$

or

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A} \frac{\partial \mathbf{U}}{\partial x} = 0,$$

where $\mathbf{A}$ is the Jacobian matrix:

$$\mathbf{A} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} = \begin{vmatrix} 0 & 1 & 0 \\ -(3-\gamma)\frac{u^2}{2} & (3-\gamma)u & \gamma-1 \\ (\gamma-1)u^3 - \gamma u E & \gamma E - 3\frac{\gamma-1}{2}u^2 & \gamma u \end{vmatrix}. \quad (2.9)$$

The approach of the Lax-Wendroff scheme is to take the time series development of the conserved variables:

$$U^{n+1} = U^n + \Delta t U_t + \frac{\Delta t^2}{2} U_{tt} + \frac{\Delta t^3}{6} U_{ttt} \dots, \quad (2.10)$$

where t subscripts denote partial differentiation with respect to time. The Δt^2 term is retained but replaced by a space derivative. Using equation (2.1):

$$\frac{\partial^2 \mathbf{U}}{\partial t^2} = -\frac{\partial^2 \mathbf{F}}{\partial x \partial t} = -\frac{\partial}{\partial x} \left(\mathbf{A} \frac{\partial \mathbf{U}}{\partial t} \right) = \frac{\partial}{\partial x} \left(\mathbf{A} \frac{\partial \mathbf{F}}{\partial x} \right).$$

Substituting space discretisations for the time derivatives in equation (2.10) and truncating the series at the squared term gives:

$$U_i^{n+1} = U_i^n - \frac{1}{2} \frac{\Delta t}{\Delta x} (F_{i+1}^n - F_{i-1}^n) + \frac{1}{2} \left(\frac{\Delta t}{\Delta x} \right)^2 \left[A_{i+1/2}^n (F_{i+1}^n - F_i^n) - A_{i-1/2}^n (F_i^n - F_{i-1}^n) \right], \quad (2.11)$$

with

$$A_{i+1/2} = \frac{1}{2} (A_i + A_{i+1}).$$

Equation (2.11) can also be written in conservative form (equation 2.3) as the difference in numerical fluxs F^* , with:

$$F_{i+1/2}^* = F_{i+1/2} - \frac{\Delta t}{2\Delta x} A_{i+1/2} (F_{i+1} - F_i)$$

and

$$F_{i+1/2} = \frac{F_i + F_{i+1}}{2}.$$

Before reference is made to the example problem, the solution of a second, less severe, example is presented which introduces the source of the difficulties described below. Consider the shock tube problem in Table 2.1 with a much reduced driver pressure, 2×10^5 Pa. This problem produces a relatively weak shock and expansion wave and does not lead to serious difficulties in the solution. The resulting pressure distribution for this problem is shown in Figure 2.2.

Oscillations at the top of the compression wave and the tail of the expansion fan are present. This kind of behaviour is referred to as *dispersive*, as opposed to *diffusive*, which is behaviour typical of the Lax-Friedrichs scheme discussed above.

The nature of this dispersive behaviour is best explained by recognising that, during each timestep, the discrete solution in space can be represented by a Fourier series expansion. This discrete solution is comprised of a finite number of harmonic waves. In a one-dimensional domain of length L , the Fourier representation reflects the region onto the negative part $(-L, 0)$. Therefore, the fundamental frequency corresponds to a maximum wavelength $\lambda_{\max} = 2L$ and a minimum wavelength $\lambda_{\min} = 2\Delta x$. (See Figure 2.3, where the amplitudes of the waves are not representative of any particular function).

The problem of dispersion arises because the speed with which these waves propagate (the phase velocity) is dependent on the frequency of the wave. In other words, the waves become dispersed, giving rise to the oscillatory behaviour seen in Figure 2.2.

Results of the main example problem for the Lax-Wendroff scheme are shown

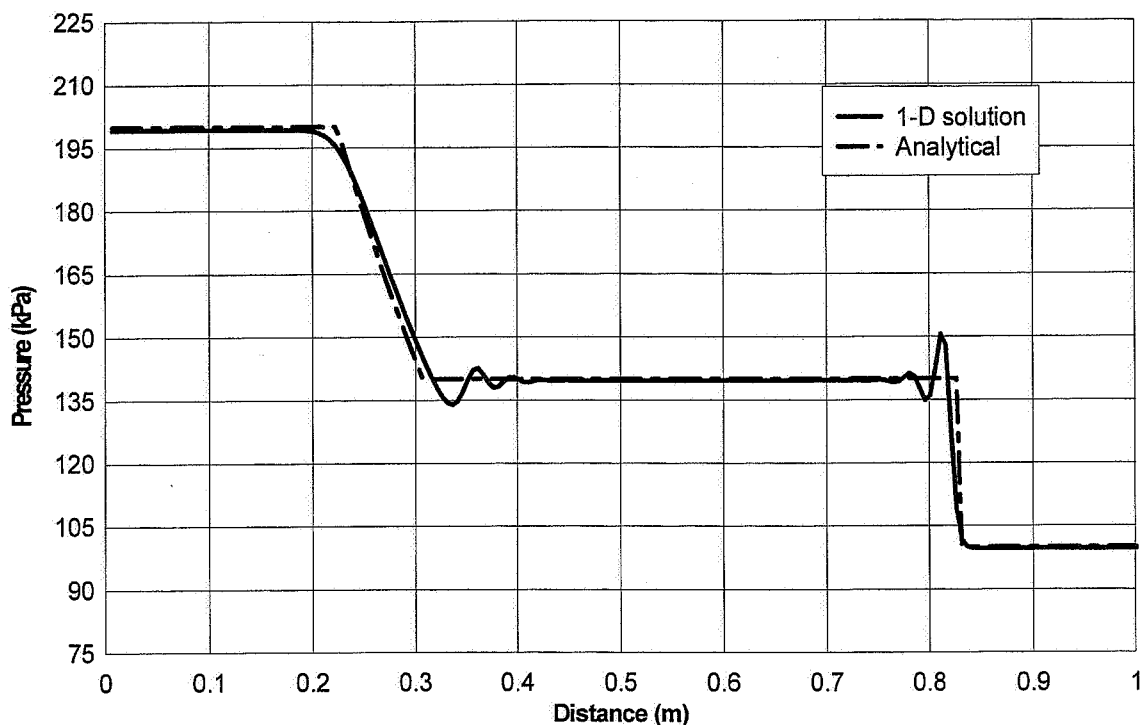


Figure 2.2: Second example problem for Lax–Wendroff scheme

in Figure 2.12, where the failings of the method have become evident. It can be seen that, even for the modest strength shock of the example problem, a physically unrealistic expansion shock has formed from the oscillations at the tail of the expansion. This is closely followed by a second compression shock which brings the solution back to the correct conditions, a short distance along the expansion. This sort of behaviour can be reduced/removed by the introduction of artificial viscosity into the scheme.

Artificial viscosity (or artificial dissipation) comprises terms added to the solution to simulate the effects of physical viscosity locally around discontinuities and to be negligible, or the order of the truncation error, in smooth regions of the flow. Additional dissipation is also required to avoid the appearance of expansion shocks. As artificial viscosity was not used in the present study, it will not be discussed further. A thorough description of artificial viscosity is given in Section 17.3 of Hirsch [23].

2.3.3 MacCormack's Method

MacCormack's method (Section 17.2.2, Hirsch [23]) is a predictor-corrector, second-order, space-centred variant of the Lax–Wendroff scheme. Predictor values of the conserved variables $\bar{U}_i$ are calculated for the timestep $n + 1$ for each cell i . This is followed by a corrector step where:

$$\bar{F}_i = F(\bar{U}_i),$$

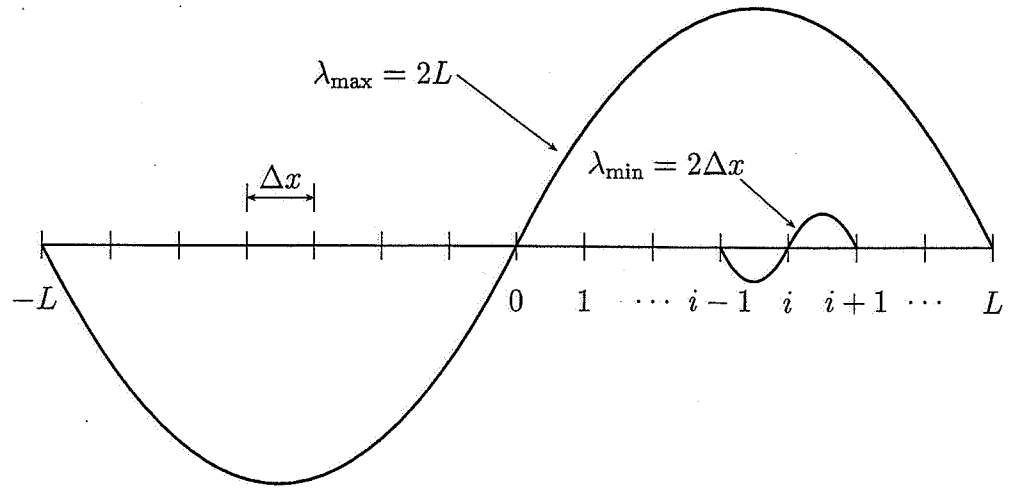


Figure 2.3: Fourier series representation of discrete solution.

which gives corrected conserved variables $\bar{\bar{U}}$. The scheme is summarised as follows:

$$\bar{U}_i = U_i^n - \frac{\Delta t}{\Delta x} (F_{i+1}^n - F_i^n), \quad (2.12)$$

$$\bar{\bar{U}}_i = U_i^n - \frac{\Delta t}{\Delta x} (\bar{F}_i^n - \bar{F}_{i-1}^n) \quad (2.13)$$

and the updated conserved variables are then given by:

$$U_i^{n+1} = \frac{1}{2} (\bar{U}_i + \bar{\bar{U}}_i).$$

It can be seen from the subscripts in equations (2.12) and (2.13) that the scheme will favour discontinuities moving from left to right. Because of this, it is usual to use the alternative formulation:

$$\bar{U}_i = U_i^n - \frac{\Delta t}{\Delta x} (F_i^n - F_{i-1}^n),$$

$$\bar{\bar{U}}_i = U_i^n - \frac{\Delta t}{\Delta x} (\bar{F}_{i+1}^n - \bar{F}_i^n)$$

on alternate timesteps. It can be seen from Figure 2.13 that this method is not as dispersive as the Lax-Wendroff scheme, but it does allow a physically unrealistic expansion shock at the tail of the expansion wave. This problem can be fixed by the judicious use of artificial viscosity. However, the limited experience of the present author indicates that removing the expansion shock from problems with *strong* discontinuities is difficult and results in an over-diffusive solution.

MacCormack's method has been widely used in the past and provides good

quality solutions to many problems of practical interest. It is probably not suitable for blast wave modelling.

2.3.4 Van Leer's Flux Vector Splitting Method

Van Leer's flux vector splitting scheme (Van Leer [53]) is an example of a first-order upwind method. The term *upwind* is used to describe the process by which the numerical discretisation follows the characteristics of the flow. In the present formulation, the basic system of one-dimensional Euler equations is used in the form:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}^+}{\partial x} + \frac{\partial \mathbf{F}^-}{\partial x} = 0.$$

The flux terms are split, based on the local Mach number of the flow. The two sets of fluxes are discretised using one-sided first-order differences in their respective directions.

The Van Leer flux splitting is as follows:

$$F^+ = \frac{\rho}{4c} (u + c)^2 \begin{vmatrix} 1 \\ \frac{(\gamma - 1)u + 2c}{\gamma} \\ \frac{[(\gamma - 1)u + 2c]^2}{2(\gamma^2 - 1)} \end{vmatrix}$$

and

$$F^- = \frac{-\rho}{4c} (u - c)^2 \begin{vmatrix} 1 \\ \frac{(\gamma - 1)u - 2c}{\gamma} \\ \frac{[2c - (\gamma - 1)u]^2}{2(\gamma^2 - 1)} \end{vmatrix}.$$

The two sets of fluxes are then discretised separately, giving the following equation for the updated conserved variables:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} (F_i^+ - F_{i-1}^+) - \frac{\Delta t}{\Delta x} (F_{i+1}^- - F_i^-)$$

or

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} [(F_i^+ + F_{i+1}^-) - (F_{i-1}^+ + F_i^-)].$$

Results of the example shock tube problem using this method are given in Figure 2.14.

2.3.5 Roe's Method

Roe's method (Roe [41]) is one of a family of approaches known as *Godunov-type* methods or approximate Riemann solvers. These use an approach that is fundamentally different from the other methods described in this section. Therefore, a brief explanation is required.

Consider the discretisation of conserved quantities shown in Figure 2.4. The exact solution of the Riemann problem (shock tube problem) at the cell interfaces is shown schematically in Figure 2.5.

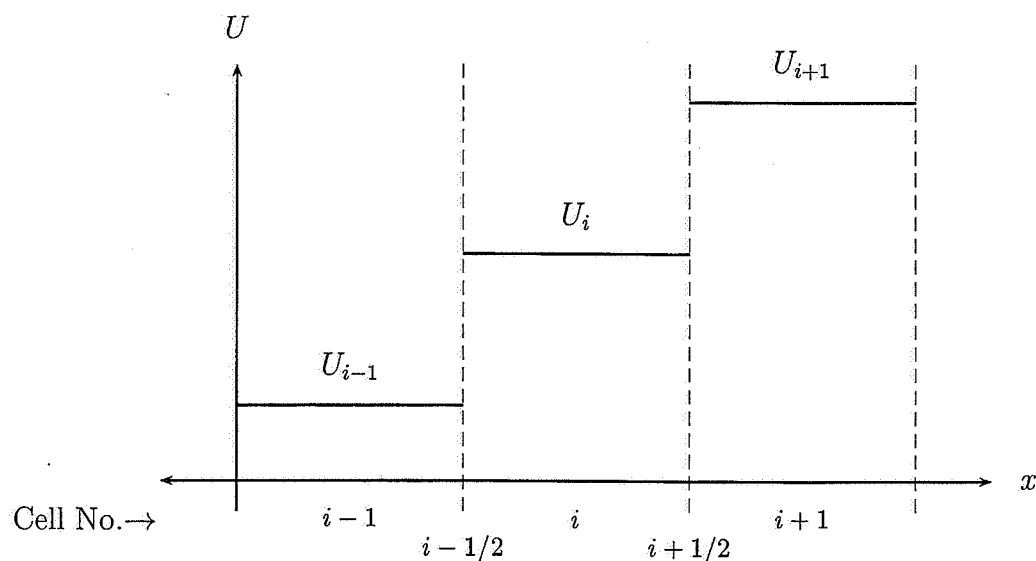


Figure 2.4: Discretisation of a conserved variable

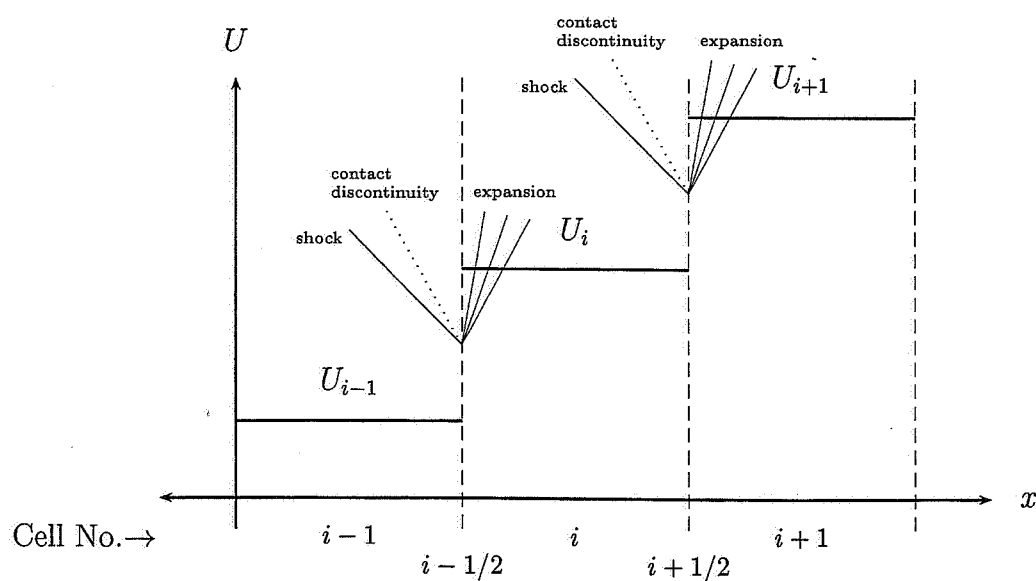


Figure 2.5: Representation of local Riemann problems at cell interfaces

Roe's method approximates the local Riemann problems by using a characteristic decomposition of the flux differences. The transformation from conservative to characteristic variables can be written:

$$\delta U = \sum_j \delta w_j r^{(j)}. \quad (2.14)$$

This is interpreted as a sum of the simple waves $r^{(j)}$ (eigenvector of the flux Jacobian) with amplitudes δw_j . Substituting for $r^{(j)}$ in the one-dimensional Euler equations, equation (2.14) becomes:

$$\delta U = \delta w_1 \begin{vmatrix} 1 \\ u \\ u^2/2 \end{vmatrix} + \frac{\rho}{2c} \delta w_2 \begin{vmatrix} 1 \\ u + c \\ H + uc \end{vmatrix} + \frac{\rho}{2c} \delta w_3 \begin{vmatrix} 1 \\ u - c \\ H - uc \end{vmatrix}, \quad (2.15)$$

with

$$\delta w_1 = \delta \rho - \frac{\delta p}{c^2},$$

$$\delta w_2 = \delta u + \frac{\delta p}{\rho c},$$

$$\delta w_3 = \delta u - \frac{\delta p}{\rho c}.$$

In the above, δ is the central difference operator acting on the cell interface values at $(i + 1/2)$. Roe's method replaces the variables u and H by an average weighted by the square root of the densities. The resulting linearised Jacobian matrix is identical to the local Jacobian given previously (equation (2.9)). Therefore, the eigenvalues and eigenvectors of the linearised Jacobian matrix can be used to solve the wave decomposition described above. The necessary averages are defined as follows:

$$\bar{\rho}_{i+1/2} = \sqrt{\rho_{i+1} \rho_i},$$

$$\bar{u}_{i+1/2} = \frac{(u\sqrt{\rho})_{i+1} + (u\sqrt{\rho})_i}{\sqrt{\rho_{i+1}} + \sqrt{\rho_i}},$$

$$\bar{H}_{i+1/2} = \frac{(H\sqrt{\rho})_{i+1} + (H\sqrt{\rho})_i}{\sqrt{\rho_{i+1}} + \sqrt{\rho_i}}.$$

The averaged speed of sound is given by:

$$\bar{c}^2 = (\gamma - 1) \left(\bar{H} - \frac{\bar{u}^2}{2} \right).$$

The eigenvalues of the linearised matrix are:

$$\bar{\lambda}_{(1)} = \bar{u}, \quad \bar{\lambda}_{(2)} = \bar{u} + \bar{c}, \quad \bar{\lambda}_{(3)} = \bar{u} - \bar{c},$$

with eigenvectors:

$$\bar{r}^{(1)} = \begin{bmatrix} 1 \\ \bar{u} \\ \frac{\bar{u}^2}{2} \end{bmatrix}, \quad \bar{r}^{(2)} = \frac{\bar{\rho}}{2\bar{c}} \begin{bmatrix} 1 \\ \bar{u} + \bar{c} \\ \bar{H} + \bar{u}\bar{c} \end{bmatrix}, \quad \bar{r}^{(3)} = \frac{-\bar{\rho}}{2\bar{c}} \begin{bmatrix} 1 \\ \bar{u} - \bar{c} \\ \bar{H} - \bar{u}\bar{c} \end{bmatrix}.$$

The wave amplitudes are:

$$\delta w_1 = (\rho_{i+1} - \rho_i) - \frac{(p_{i+1} - p_i)}{\bar{c}^2},$$

$$\delta w_2 = (u_{i+1} - u_i) + \frac{(p_{i+1} - p_i)}{\bar{\rho}\bar{c}},$$

$$\delta w_3 = (u_{i+1} - u_i) - \frac{(p_{i+1} - p_i)}{\bar{\rho}\bar{c}}.$$

The numerical fluxes F^* at the cell boundaries are calculated from:

$$F_{i+1/2}^* = \frac{1}{2} (F_i + F_{i+1}) - \frac{1}{2} \sum_{j=1}^3 |\bar{\lambda}_{(j)}| \delta w_j \bar{r}^{(j)}.$$

or either of the equivalent, but computationally cheaper, forms:

$$F_{i+1/2}^* = F_i + \sum_{j=1}^3 \bar{\lambda}_{(j)}^- \delta w_j \bar{r}^{(j)}. \quad (2.16)$$

$$F_{i+1/2}^* = F_i - \sum_{j=1}^3 \bar{\lambda}_{(j)}^+ \delta w_j \bar{r}^{(j)}.$$

where the $\pm$ sign indicates the positive and negative eigenvalues values respectively. Incidentally, the form represented by equation (2.16) was implemented in STanaly.

Finally, the updated conserved variables are calculated in the usual manner from:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} (F_{i+1/2}^* - F_{i-1/2}^*). \quad (2.17)$$

Roe's method is known to suffer from the creation of an expansion shock at a sonic point in the expansion fan. This problem can be solved by modifying the modulus of the eigenvalue in equation (2.16) by the following procedure, known as Harten's fix (Section 20.5.3, Hirsch [23]):

$$|\bar{\lambda}|_{\text{mod}} = \begin{cases} |\bar{\lambda}|_{i+1/2} & \text{if } |\bar{\lambda}|_{i+1/2} \geq \varepsilon \\ \frac{\bar{\lambda}_{i+1/2}^2 + \varepsilon^2}{2\varepsilon} & \text{if } |\bar{\lambda}|_{i+1/2} < \varepsilon \end{cases}$$

where,

$$\varepsilon = 0.1 |\bar{u} + \bar{c}|.$$

This is the version of Roe's scheme implemented in STanalyzer. It was found to work satisfactorily for *normal* test problems, but it still failed when tested with very strong shocks, typical of high explosive blast wave simulations. Results of the example problem are given in Figure 2.15.

2.3.6 Van Leer's Flux Vector Splitting, TVD, Second-Order

The next stage of the development was to make the flux vector splitting scheme of Van Leer (described above) second-order in space and time and to apply Total Variation Diminishing conditions to the resulting scheme; this has become almost the minimum requirement of any modern compressible CFD code.

It was observed previously (Section 2.3.1) that the Lax-Friedrichs method was a monotone scheme. It was also mentioned that monotone schemes are at most first-order accurate. Therefore, linear second-order approximations cannot be monotone and there is a likelihood that numerically generated oscillations will form, even though the underlying scheme may be free from under and overshoots. Total Variation Diminishing conditions are the means by which such schemes can be controlled at each timestep, and within each cell, in such a way as to keep the gradients within proper bounds.

Similar to the definition of monotonicity, total variation boundedness is sufficient to ensure that the numerical scheme converges to the correct solution. Unlike monotonicity, however, TVD conditions do not ensure that the entropy condition is satisfied. The concept of bounded total variation is best introduced with reference to the scalar equation (2.4). The total variation of u , $\text{TV}(u)$, on the whole of x of a physically admissible solution is given by:

$$\text{TV}(u) = \int \left| \frac{\partial u}{\partial x} \right| dx$$

Similarly, the total variation of a discrete solution to a scalar conservation law is given by:

$$\text{TV}(u) = \sum_i |u_{i+1} - u_i|,$$

and the scheme is said to be of bounded total variation if the above quantity is

uniformly bounded in time and in discretised space. The scheme is said to be Total Variation Diminishing if:

$$\text{TV}(u^{n+1}) \leq \text{TV}(u^n).$$

All TVD schemes are *monotonicity preserving*. This condition states that the solution at u^{n+1} is monotone if u^n is monotone, and under and overshoots will not be created from monotone initial conditions. A complete development of second-order TVD schemes can be found in Section 21 of Hirsch [23].

In general, the method of solving *systems* of equations is to diagonalise the system, and then solve the resulting decoupled equations. The decomposition of non-linear equations into eigenspace and the solution of the Riemann problem using the resulting characteristic variables suggests a method similar to Roe's scheme, with limiters applied to the slopes of the characteristic variables. In practice, however, other approaches are possible. One possibility (the one adopted here) is to apply slope limiters to each set of discretised conserved variables, and then apply the first-order scheme to the new cell interface values. A similar procedure is used in the multi-dimensional implementation of Air3d (described in Chapter 3), where the system is decoupled into local one-dimensional problems and then recombined at the end of each timestep. Hirsch [23] (Section 21.5) suggests that this *ad hoc* procedure, although not proved for multi-dimensional, non-linear systems, can produce high-resolution, oscillation-free results.

The approach taken was to use a second-order projection stage: variable extrapolation, usually called the MUSCL approach (Van Leer [54]). This method extrapolates the cell-centred conserved variables to the cell interfaces using piecewise linear extrapolation (Figure 2.6). Higher order and one-sided discretisations can be achieved by including more cells in the extrapolation process and using polynomials of increased order.

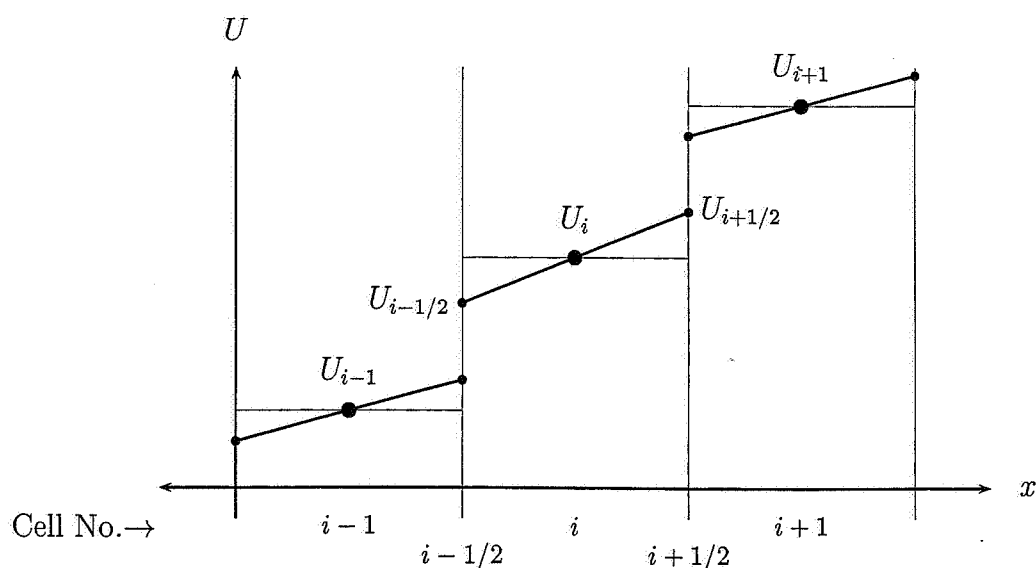


Figure 2.6: Piecewise linear representation within cells

The actual method implemented uses one upstream cell and one downstream cell. This approach may seem to be contrary to the aim of maintaining an upwind solution. However, when used in conjunction with slope limiters (discussed below) no significant difference was found in the quality of the solution between this and one-sided extrapolation. There is another benefit from this cell-centred variable extrapolation: namely, the ease with which boundaries and obstacles can be treated.

Total Variation Diminishing conditions were achieved by limiting the slope of the piecewise linear extrapolations using slope limiters. In the present study, the *minmod*, *superbee* and *general β* limiters (Section 21.3.1, Hirsch [23]) were all considered. Although all three limiters were implemented in the three-dimensional code, the author has found (as a result of many lengthy trials) the *minmod* limiter to be most suitable for blast simulation.

Piecewise linear reconstruction of the conserved variables over the region of the cell $[x_{i-1/2}, x_{i+1/2}]$ is given by:

$$U(x) = U_i + m_i \frac{(x - x_i)}{\Delta x},$$

where m_i is the slope. Therefore, at the interfaces of cell i :

$$U_{i+1/2} = U_i + \frac{1}{2}m_i \quad (2.18)$$

and

$$U_{i-1/2} = U_i - \frac{1}{2}m_i. \quad (2.19)$$

Total Variation Diminishing conditions are achieved by limiting the slopes m_i . The choice of the *minmod* limiter results in limited slopes given by:

$$m_i = \text{minmod}(U_{i+1} - U_i, U_i - U_{i-1}),$$

where the *minmod* limiter is defined as follows:

$$\text{minmod}(a, b) = \begin{cases} a & \text{if } |a| < |b| \text{ and } ab > 0, \\ b & \text{if } |b| < |a| \text{ and } ab > 0, \\ 0 & \text{if } ab \leq 0 \end{cases}$$

or

$$\text{minmod}(a, b) = (\text{sign}(0.5, a) + \text{sign}(0.5, b)) \min(|a|, |b|),$$

where the sign function is

$$\text{sign}(a, b) = \begin{cases} |a| & \text{if } b > 0 \\ -|a| & \text{if } b < 0. \end{cases}$$

Finally, second-order accuracy in time is achieved by the following method (Section 21.4.3, Hirsch [23]):

- i. Interface conserved variables $U_{i+1/2}$ and $U_{i-1/2}$ are calculated from eqns (2.18) and (2.19).
- ii. Numerical fluxes $F_{i+1/2}^*$ and $F_{i-1/2}^*$ are calculated from the interface values.
- iii. Interim half-timestep, cell-centred values $\bar{U}_i$ are calculated from

$$\bar{U}_i = U_i^n - \frac{\Delta t}{2\Delta x} (F_{i+1/2}^* - F_{i-1/2}^*).$$

- iv. Half-timestep interface values are calculated from

$$\tilde{U}_{i+1/2} = \bar{U}_i + \frac{1}{2}m_i$$

and

$$\tilde{U}_{i-1/2} = \bar{U}_i - \frac{1}{2}m_i,$$

where the m_i are the same as previously.

- v. Second-order numerical fluxes $\bar{F}_{i+1}^{*(2)}$ and $\bar{F}_{i-1}^{*(2)}$ are calculated from the half-timestep interface variables.
- vi. Full timestep, cell-centred, conserved variables are calculated using

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} (\bar{F}_{i+1/2}^{*(2)} - \bar{F}_{i-1/2}^{*(2)}).$$

Results of the example problem using Van Leer's flux vector splitting and the above time and space discretisations are shown in Figure 2.16, where the improved resolution of discontinuities is evident.

Unfortunately, the underlying flux vector splitting method soon proved not to be sufficiently robust for blast wave simulation problems. This is best described by way of an example. Consider the shock tube problem used in the example but with a driver pressure of 5×10^6 Pa and driver temperature 15000 K, fifty times the pressure and temperature of the driven section. This produces a much stronger shock than previously. This new problem was analysed using 500 computational cells to improve the resolution and make the resulting graphs more clear. Figures 2.7 to 2.10 show the solution at a problem time of at 0.1 msec, just before the analysis "blows up".

It can be seen from Figure 2.7 that the instability arises at the contact discontinuity, and Figure 2.8, in which the pressure has been normalised by the pressure directly behind the compressive shock, shows that the instability occurs when the contact discontinuity passes through the sonic point (at Mach number = 1). Figures 2.7, 2.9 and 2.10 show that the solution, although it has become unstable, remains reasonably smooth with respect to the conserved variables. A small oscillation has appeared on Figure 2.10, but this is not as pronounced as the large

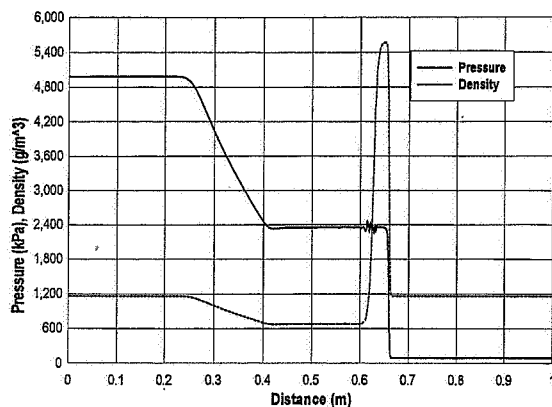


Figure 2.7: Pressure and density

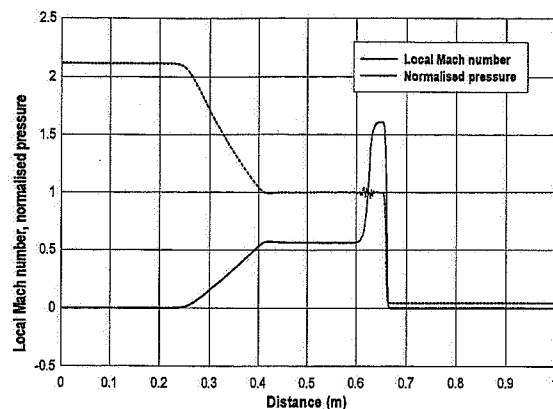


Figure 2.8: Local Mach number and normalised pressure

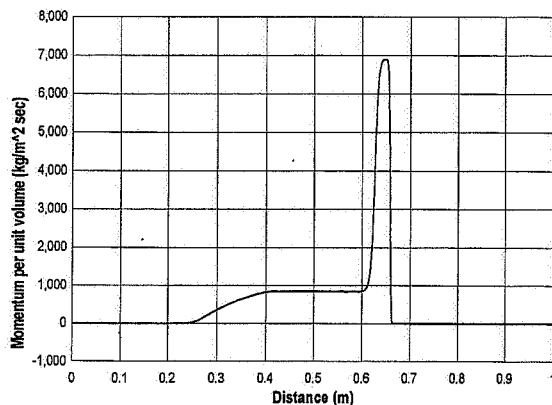


Figure 2.9: Momentum per unit volume

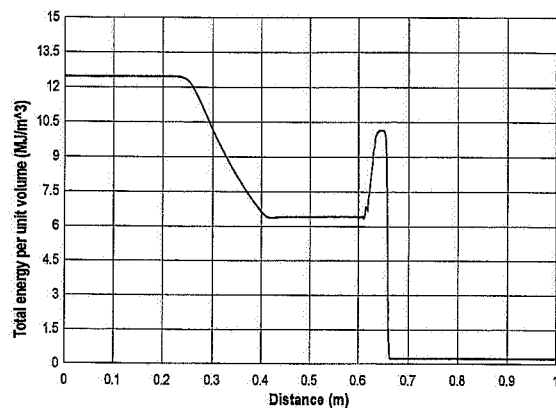


Figure 2.10: Total energy per unit volume

oscillation on the graph of pressure in Figure 2.7; the density and momentum are still completely smooth.

Although the extension of TVD and limiter schemes to non-linear systems of equations does not ensure the satisfaction of the entropy condition, it could be expected that the addition of limited second-order terms would not destroy the diffusion of the underlying scheme, which prevents the entropy violating conditions. In the present case, the problem described above is associated with the first-order flux vector splitting scheme and was merely accentuated by the limited second-order terms. This proved to be a major obstacle, and so development of the three-dimensional code based on the flux vector splitting method of Van Leer was not continued.

2.3.7 AUSM

The Advection Upstream Splitting Method (AUSM) (Liou and Steffen [33]) considers the flux vector $\mathbf{F}$ in two physically distinct parts:

$$\mathbf{F} = \begin{pmatrix} \rho \\ \rho u \\ \rho H \end{pmatrix} u + \begin{pmatrix} 0 \\ p \\ 0 \end{pmatrix} = \mathbf{F}^{(c)} + \begin{pmatrix} 0 \\ p \\ 0 \end{pmatrix}.$$

$\mathbf{F}^{(c)}$ is the vector of convected terms, associated with a suitably defined velocity u , while the pressure flux terms are governed by the acoustic wave speeds. In reality, the two processes are part of the same process and are inextricably linked. However, from the point of view of the numerical discretisation, it is feasible to treat the two terms separately for the duration of a single timestep, as they are ultimately recombined in the formation of the updated conserved variables.

The following description is proposed for the convection term at the cell interfaces $L < \frac{1}{2} < R$. L refers to the state on the left side of the interface, and R refers to the that on the right.

$$\mathbf{F}_{1/2}^{(c)} = u_{1/2} \begin{pmatrix} \rho \\ \rho u \\ \rho H \end{pmatrix}_{L/R} = M_{1/2} \begin{pmatrix} \rho a \\ \rho u a \\ \rho H a \end{pmatrix}_{L/R},$$

where the bracketed terms are given by:

$$\begin{pmatrix} \vdots \end{pmatrix}_{L/R} = \begin{cases} \begin{pmatrix} \vdots \end{pmatrix}_L & \text{if } M_{1/2} \geq 0, \\ \begin{pmatrix} \vdots \end{pmatrix}_R & \text{otherwise.} \end{cases}$$

Clearly, the essential element is the formulation of the term $M_{1/2}$. The choice proposed is a combination of the contributions from the left and right states:

$$M_{1/2} = M_L^+ + M_R^-,$$

where the split Mach numbers are defined as follows:

$$M^\pm = \begin{cases} \pm \frac{1}{4} (M \pm 1)^2, & \text{if } |M| \leq 1; \\ \frac{1}{2} (M \pm |M|), & \text{otherwise.} \end{cases}$$

The pressure term is treated in a similar way:

$$p_{1/2} = p_L^+ + p_R^-.$$

The pressure splitting is expressed in terms of second-order polynomials of the char-

acteristic wave speeds ($M \pm 1$):

$$p^{\pm} = \begin{cases} \frac{p}{4} (M \pm 1)^2 (2 \mp M), & \text{if } |M| \leq 1; \\ \frac{p}{2} (M \pm |M|)/M, & \text{otherwise.} \end{cases}$$

Updated conserved variables are calculated from:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} (F_{i+1/2} - F_{i-1/2}).$$

Results obtained from the AUSM scheme for the example problem are given in Figure 2.17. Numerical experiments with this method have shown that it is considerably more robust than either Van Leer's flux vector splitting or Roe's method. It was possible to cause the method to fail by introducing extreme pressure and density discontinuities into the problem. However, results of these experiments were generally encouraging and led to consideration of a variant of the AUSM scheme described below.

2.3.8 AUSMD and AUSMV leading to AUSMDV

The flux splitting scheme AUSMDV (Wada and Liou [56]) is an improved advection upstream splitting method. It takes its name from the fact that it is a combination of two AUSM variants: AUSMD and AUSMV (also Wada and Liou [56]).

The AUSMD scheme is summarised as follows. The numerical flux is given by:

$$\mathbf{F}_{1/2} = \frac{1}{2} [(\rho u)_{1/2} (\Psi_L + \Psi_R) - |(\rho u)_{1/2}| (\Psi_R - \Psi_L)] + \mathbf{p}_{1/2}, \quad (2.20)$$

where

$$\Psi = \begin{pmatrix} 1 \\ u \\ H \end{pmatrix}$$

and

$$\mathbf{p} = \begin{pmatrix} 0 \\ p \\ 0 \end{pmatrix}.$$

The authors suggest that the numerical flux (equation (2.20)) is similar in form to that of a flux difference splitting (FDS) scheme, hence the scheme was termed AUSMD.

It can be seen from equation (2.20) above that, unlike the AUSM method which splits the convected quantities based on a suitable interface Mach number, the

present approach uses the *mass flux* at the interface, defined in the following way:

$$(\rho u)_{1/2} = u_L^+ \rho_L + u_R^- \rho_R, \quad |(\rho u)_{1/2}| = \text{abs}((\rho u)_{1/2}).$$

The essential characteristics of the scheme are determined by the choice of u_L^+ and u_R^- . In the present case, they are chosen to yield exact resolution of contact discontinuities. This property is particularly important in viscous flow calculations, where contact discontinuities (*slip surfaces* in three-dimensional flow) need to be resolved accurately (with little or no diffusion) in order to ensure that the resulting *shear layers* are also accurately resolved. u_L^+ and u_R^- are defined as follows:

$$u_L^+ = \begin{cases} \alpha_L \left\{ \frac{(u_L + c_m)^2}{4c_m} \right\} + (1 - \alpha_L) \frac{u_L + |u_L|}{2}, & \text{if } |u_L| \leq c_m, \\ \frac{u_L + |u_L|}{2}, & \text{otherwise;} \end{cases}$$

$$u_R^- = \begin{cases} \alpha_R \left\{ -\frac{(u_R - c_m)^2}{4c_m} \right\} + (1 - \alpha_R) \frac{u_R - |u_R|}{2}, & \text{if } |u_R| \leq c_m, \\ \frac{u_R - |u_R|}{2}, & \text{otherwise.} \end{cases}$$

The parameters α_L and α_R are given by:

$$\alpha_L = \frac{2(p/\rho)_L}{(p/\rho)_L + (p/\rho)_R}, \quad \alpha_R = \frac{2(p/\rho)_R}{(p/\rho)_L + (p/\rho)_R}$$

and

$$c_m = \max(c_L, c_R).$$

The pressure flux is:

$$p_{1/2} = p_L^+ + p_R^-,$$

where

$$p_L^+ = \begin{cases} p_L \frac{(u_L + c_m)^2}{4c_m^2} \left(2 - \frac{u_L}{c_m} \right) & \text{if } |u_L| \leq c_m, \\ p_L \frac{u_L + |u_L|}{2u_L} & \text{otherwise;} \end{cases}$$

$$p_R^- = \begin{cases} p_R \frac{(u_R - c_m)^2}{4c_m^2} \left(2 + \frac{u_R}{c_m} \right) & \text{if } |u_R| \leq c_m, \\ p_R \frac{u_R - |u_R|}{2u_R} & \text{otherwise.} \end{cases}$$

The second variant considered, AUSMV, was developed to take advantage of the excellent shock-capturing capability of flux vector splitting (FVS) methods, hence the term AUSMV. The scheme uses flux vector splitting for the $(\rho u)_{1/2}$ term for the *normal* component of momentum. (Only the normal component of momentum is considered in one-dimensional flow.)

$$(\rho u^2)_{\text{AUSMV}} = u_L^+ (\rho u)_L + u_R^- (\rho u)_R$$

whereas from equation (2.20) it can be seen that the AUSMD scheme uses

$$(\rho u^2)_{\text{AUSMD}} = \frac{1}{2} [(\rho u)_{1/2} (u_L + u_R) - |(\rho u)_{1/2}| (u_R - u_L)].$$

Finally, a combination of AUSMD and AUSMV is proposed, called AUSMDV, that seeks to take advantage of the benefits of both schemes, but is biased towards the AUSMV scheme. The normal momentum flux $(\rho u)_{1/2}$ is given by

$$(\rho u^2)_{1/2 \text{ AUSMDV}} = \frac{1}{2} (1 + s) (\rho u^2)_{\text{AUSMV}} + \frac{1}{2} (1 - s) (\rho u^2)_{\text{AUSMD}},$$

where s is a switching function based on the pressure difference across the cell interface:

$$s = \min \left(1, K \frac{|p_R - p_L|}{\min(p_L, p_R)} \right),$$

and a value of $K = 10$ is suggested for the constant parameter. The above averaging is biased towards AUSMV in regions of high pressure gradient, like compressive shocks, but it favours AUSMD in smooth regions, including contact discontinuities.

The authors of the scheme discovered that the AUSMDV procedure produces a small "glitch" at the sonic point in an expansion. To remedy this, an entropy fix (E-fix) was introduced which modifies the flux $\mathbf{F}_{1/2}$ to $\mathbf{F}_{1/2, E\text{-}fix}$. There are two cases where an expansive sonic point occurs across a cell interface. These occur when:

$$\begin{cases} \text{Case A. } u_L - c_L < 0 \text{ and } u_R - c_R > 0, \\ \text{Case B. } u_L + c_L < 0 \text{ and } u_R + c_R > 0. \end{cases}$$

Numerical dissipation sufficient to smooth the "glitch" is only introduced when a single expansion is detected, as described by the following:

If (A and B) ($u_L < 0$ and $u_R > 0$) then

no modification

elseif (A) then

$$\mathbf{F}_{1/2, E\text{-}fix} = \mathbf{F}_{1/2} - C \Delta (u - c) \Delta (\rho \Psi)$$

elseif (B) then

$$\mathbf{F}_{1/2, E\text{-}fix} = \mathbf{F}_{1/2} - C \Delta (u + c) \Delta (\rho \Psi)$$

else

no modification.

Here $\Delta(\cdot) \equiv (\cdot)_R - (\cdot)_L$, and the constant parameter C has an assumed value of 0.125.

Updated conserved variables are calculated from:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} (F_{i+1/2} - F_{i-1/2}).$$

Results of the example problem calculated using the AUSMDV method are presented in Figure 2.18.

2.3.9 AUSMDV, TVD, Second-Order

Implementation of second-order space and time accuracy into the AUSMDV method was achieved using exactly the same procedure as Section 2.3.6, above. Results of the example problem are shown in Figure 2.19.

2.3.10 AUSMDV, MUSCL–Hancock

The MUSCL–Hancock scheme (Toro [52]) is a simpler and (computationally) quicker method of achieving second-order accuracy than the method described previously in Section 2.3.6. The method is as follows:

- i. Left and right cell interface values of the conserved variables U_i^L and U_i^R are calculated from equations (2.18) and (2.19) and the TVD conditions.
- ii. Fluxes $F(U_i^L)$ and $F(U_i^R)$ are calculated for each side of the interface from equation (2.2).
- iii. Half timestep, left and right cell interface conserved variables are evolved according to:

$$\bar{U}_i^L = U_i^L + \frac{1}{2} \frac{\Delta t}{\Delta x} [F(U_i^L) - F(U_i^R)],$$

$$\bar{U}_i^R = U_i^R + \frac{1}{2} \frac{\Delta t}{\Delta x} [F(U_i^L) - F(U_i^R)].$$

This step is entirely contained within the each cell.

- iv. Numerical fluxes $F_{i+1/2}^*$ are calculated from the above interface values $\bar{U}_i^L$ and $\bar{U}_i^R$ using the AUSMDV method described in Section 2.3.8 above.
- v. Full timestep, cell-centred, conserved variables are calculated from:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} (F_{i+1/2}^* - F_{i-1/2}^*).$$

The final set of graphs showing solution of the example shock tube problem using the MUSCL–Hancock method with the AUSMDV scheme is given in Figure 2.20.

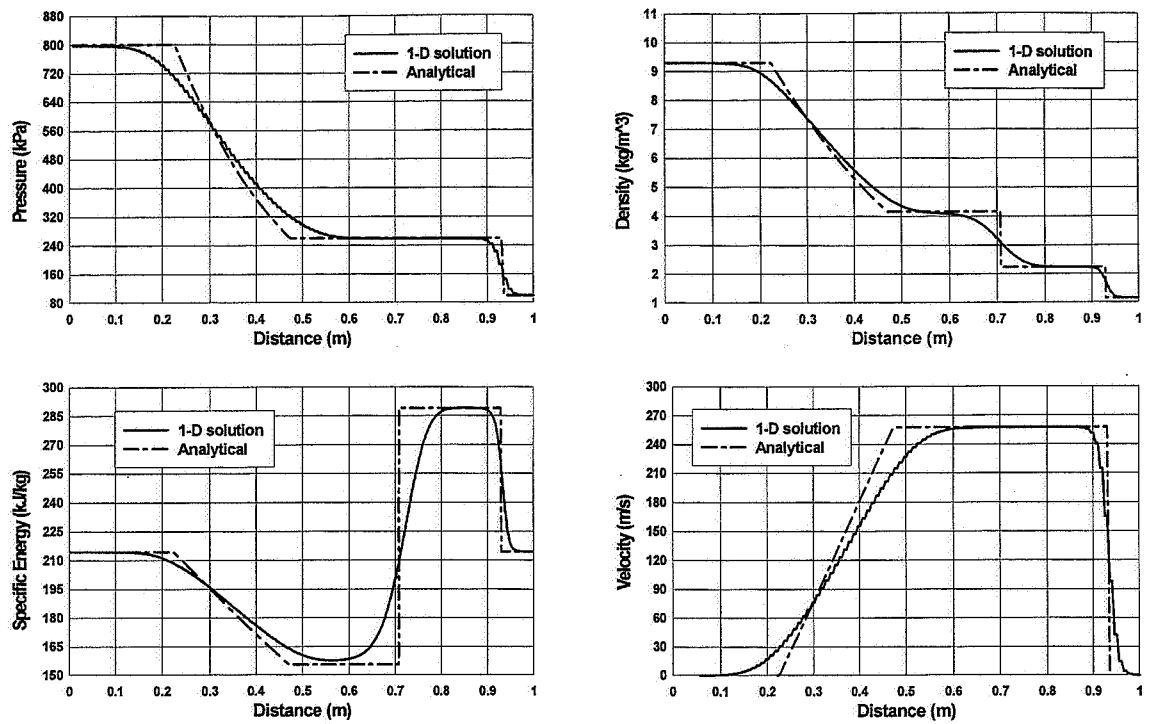


Figure 2.11: Shock tube problem, Lax-Friedrichs

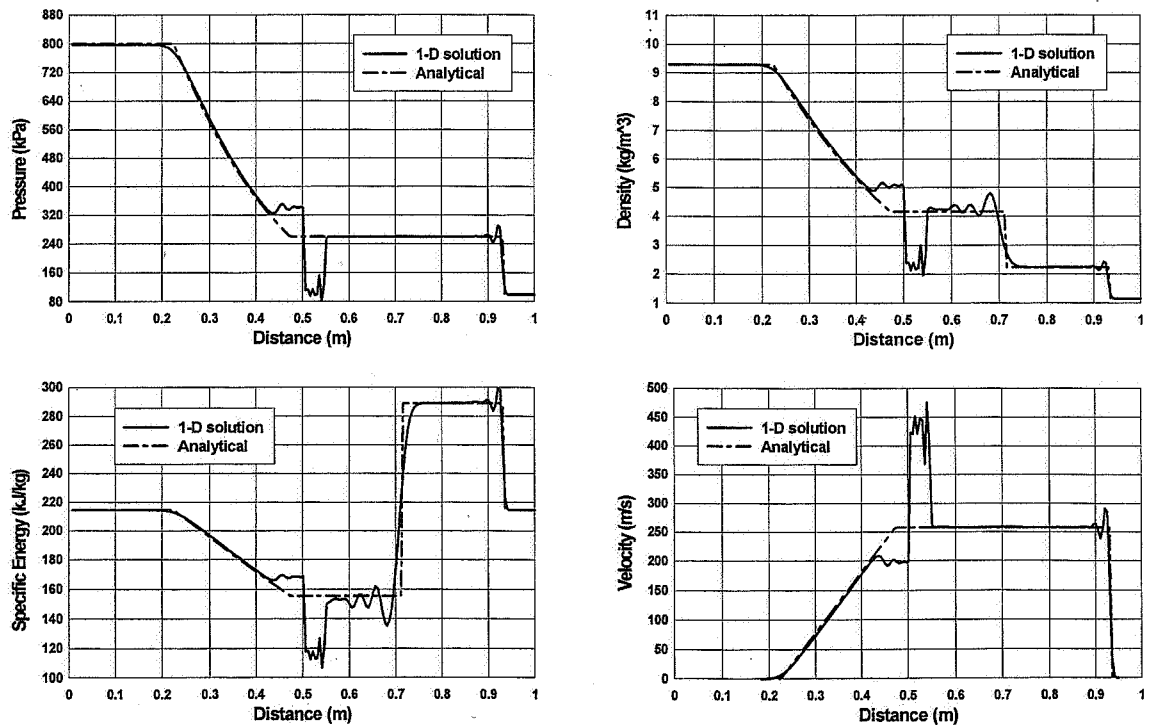


Figure 2.12: Shock tube problem, Lax-Wendroff

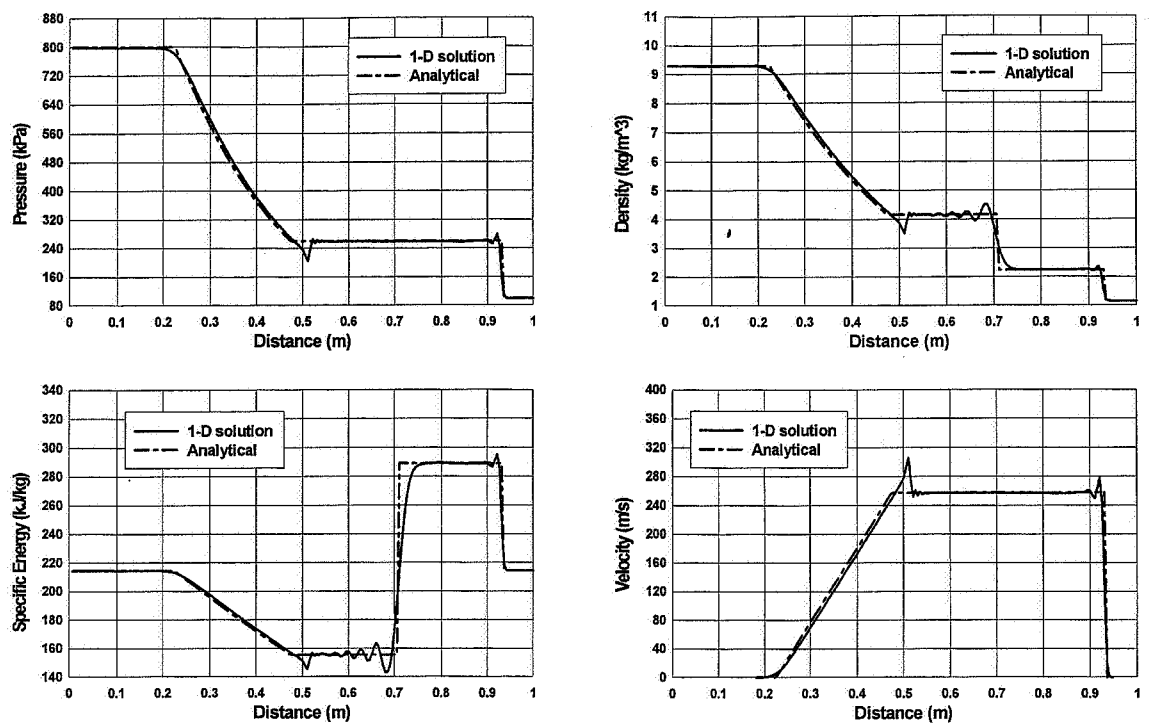


Figure 2.13: Shock tube problem, MacCormack's method

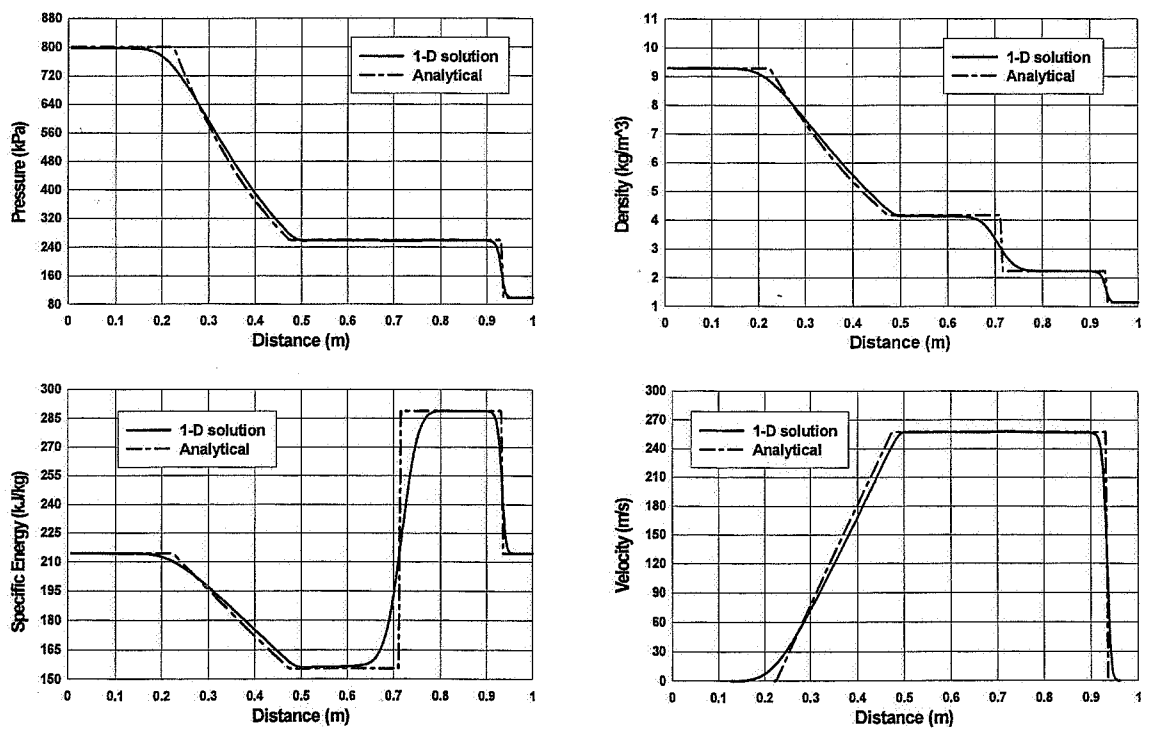


Figure 2.14: Shock tube problem, Van Leer's Flux Vector Splitting

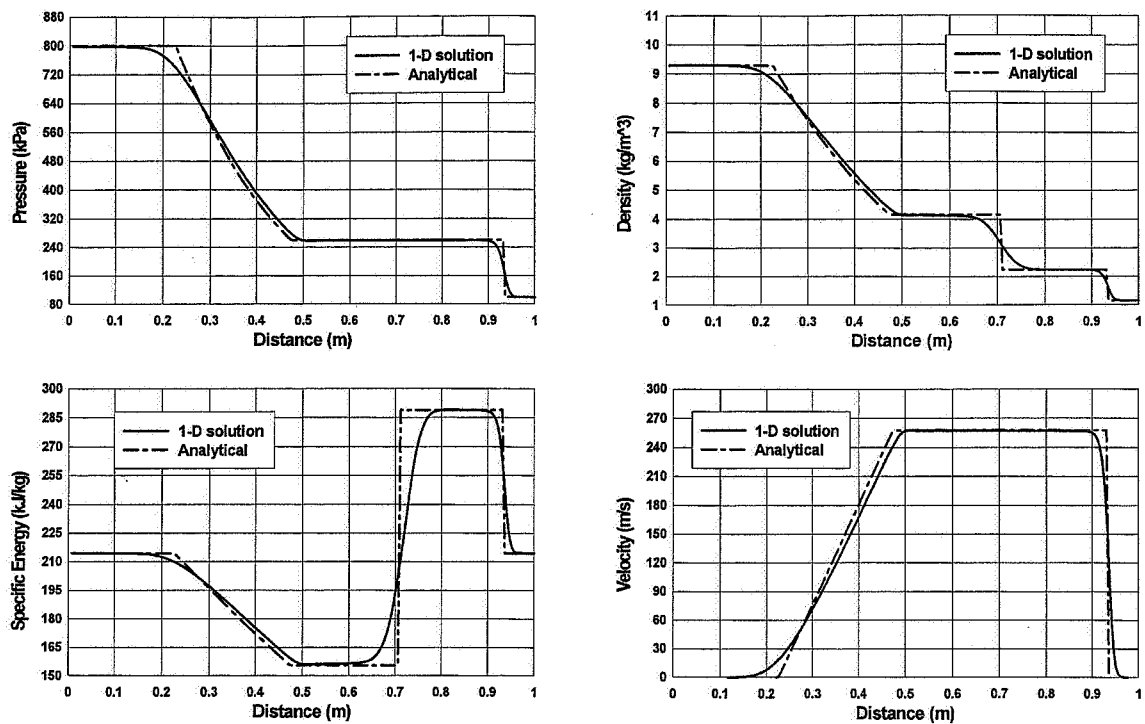


Figure 2.15: Shock tube problem, Roe's method

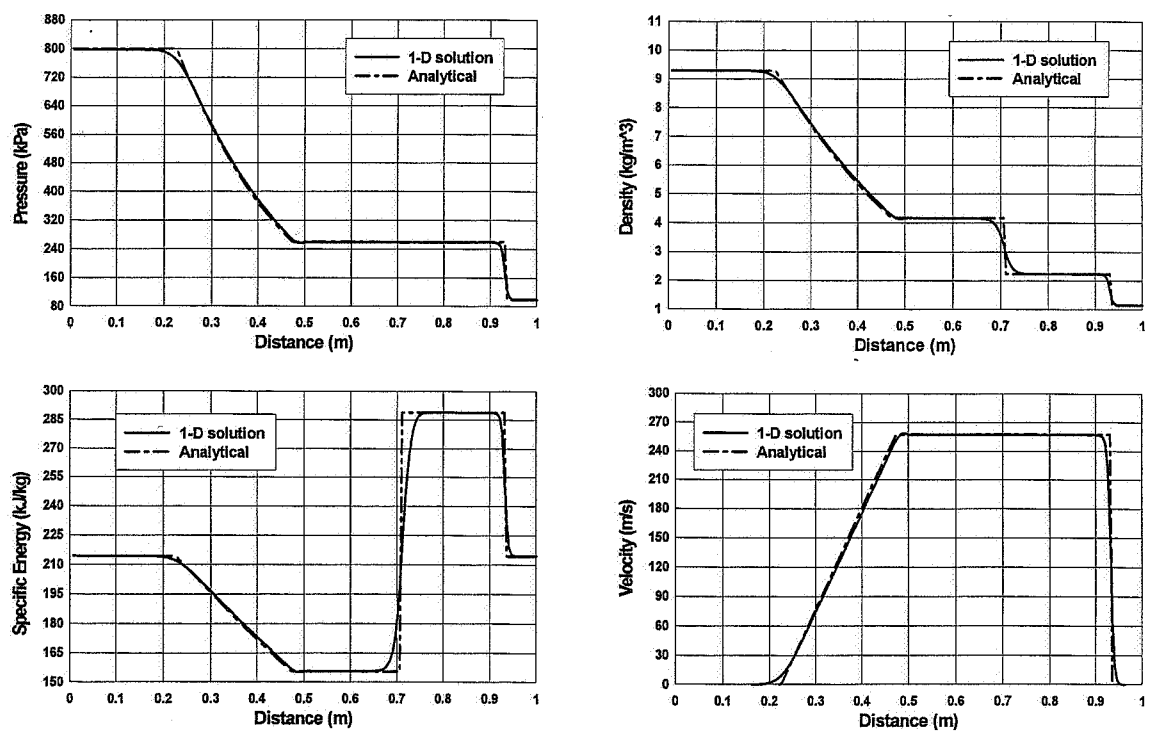


Figure 2.16: Shock tube problem, Flux Vector Splitting, TVD, 2nd order

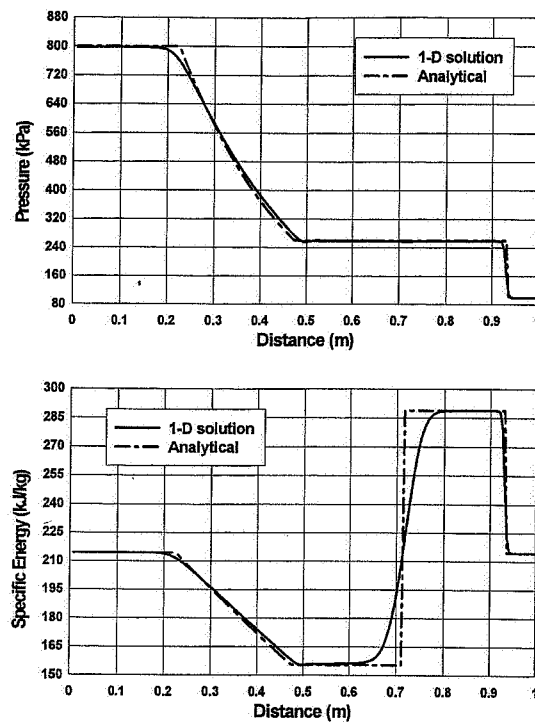


Figure 2.17: Shock tube problem, AUSM

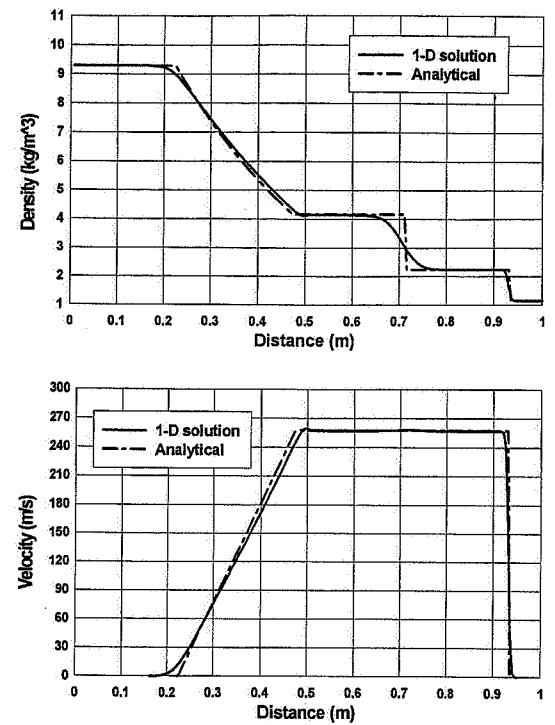
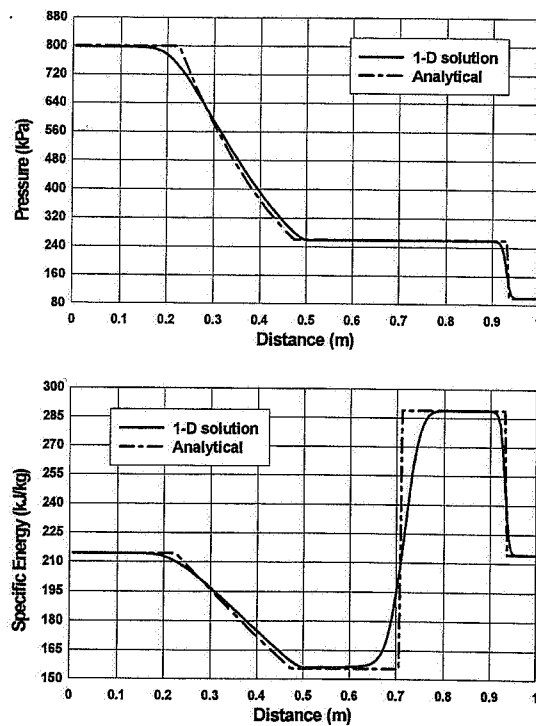
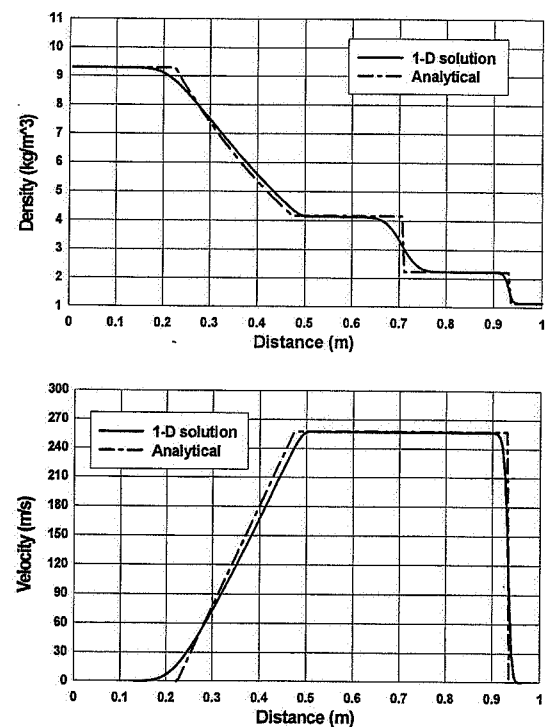


Figure 2.18: Shock tube problem, AUSMDV



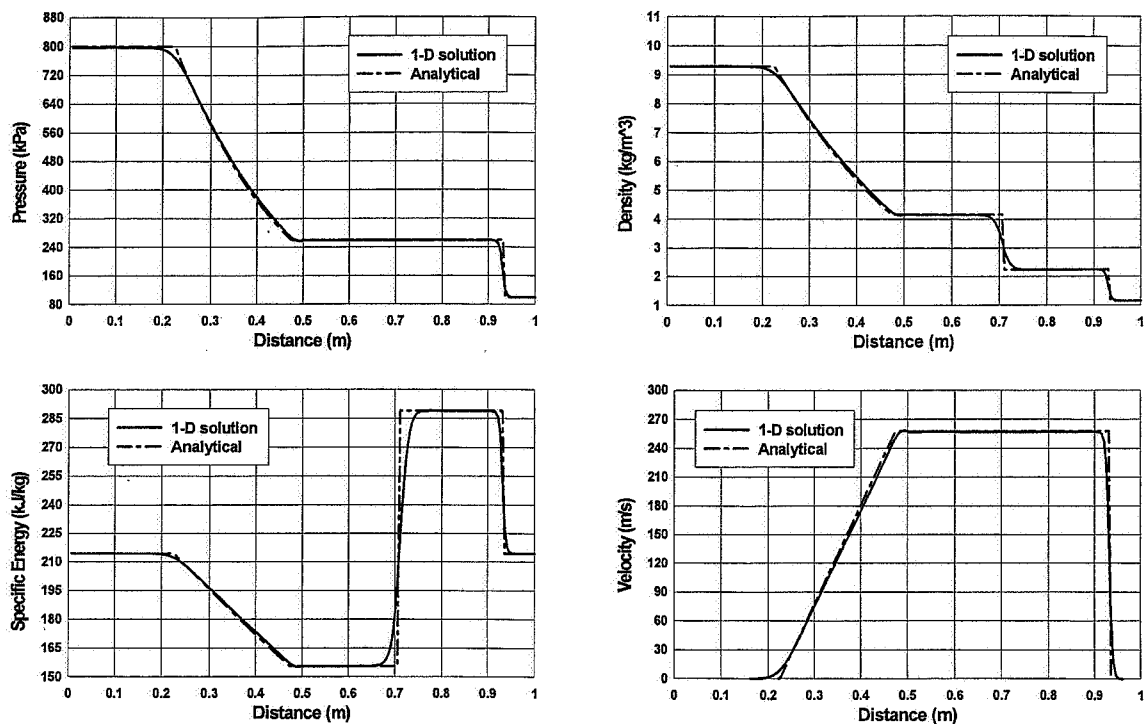


Figure 2.19: Shock tube problem, AUSMDV, TVD, 2nd order

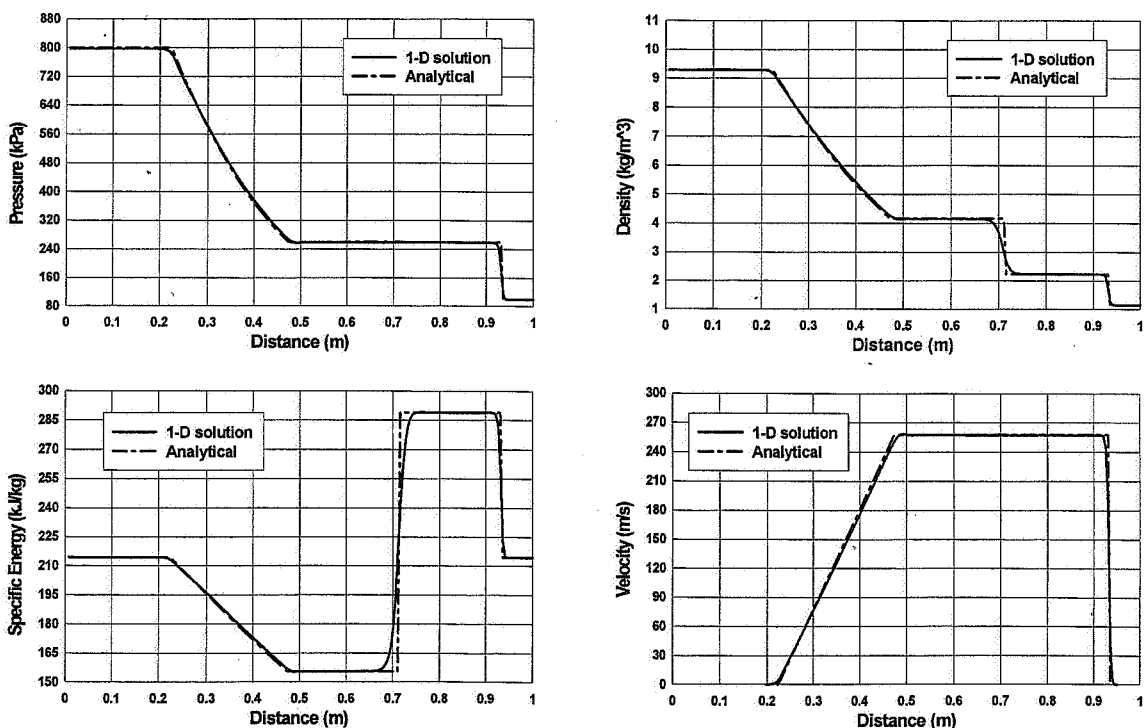


Figure 2.20: Shock tube problem, AUSMDV, TVD, MUSCL-Hancock, 2nd order

2.4 Benchmark Tests

As a conclusion to this chapter, a short series of benchmark tests were performed to indicate the relative performance, in terms of Central Processor Unit (CPU) time, of the various flow solvers considered above.

Once again, the shock tube problem described in Table 2.1 was used for these timing experiments, but the number of cells was increased to 1000, so that the total CPU time better reflected the time spent in the solver routine. There was still a small amount of time spent reading the input file, allocating the arrays and initialising the display, but all other output was *switched off*. Results of the experiments are given in Table 2.2, below.

Table 2.2: Benchmark test results for flow solvers

Method of analysis	CPU time
Lax-Friedrichs	2.96
Lax-Wendroff	9.74
MacCormack's method	7.00
Van Leer's Flux Vector Splitting	4.48
Roe's method	13.96
Flux Vector Splitting, TVD, 2 nd order	12.73
AUSM	4.32
AUSMDV	7.49
AUSMDV, TVD, 2 nd order	18.94
AUSMDV, TVD, MUSCL-Hancock	14.44

There are, essentially, two interesting points which arise from Table 2.2:

- The most obvious comparison is between the first-order implementation of Roe's solver and AUSMDV. Roe's solver has become a standard technique, and if it is not used there should be a very good reason why not. The robustness, shock resolution capability, and relatively good CPU requirement is sufficient justification for choosing AUSMDV in the present case.
- AUSMDV was selected over AUSM as the underlying solver because it does not suffer from the overshoot in colliding or reflecting shocks which is noticeable in the AUSM solver. It also has an entropy fix which smoothes the solution through the sonic point during a strong expansion. Both of these factors have contributed to the preference given to AUSMDV over AUSM. Table 2.2, however, demonstrates that these desirable qualities come at the price of a higher CPU requirement per computational cycle.

It is clear that the method of solution used by Air3d will need to be constantly compared, updated and improved, in the future, if the tool is to remain efficient.

This concludes the section on numerical solution of the shock tube problem. It has demonstrated the procedure by which the current method of solution, implemented in the code Air3d, was selected. The final choice, AUSMDV with MUSCL-Hancock integration, is essentially the combination of two computationally cheap methods to obtain one of only moderate expense, and it is believed that Air3d is the first airblast tool to make use of this approach. The justification for this choice is contained in the validation examples of Chapters 5, 6 and 7, which follow.

Generally, this project has been driven by two main forces: one was the need for a robust and accurate computational tool, the other, the need to make progress in the main subject areas. These two forces have inevitably resulted in a compromise which has led to relatively slow development of the code Air3d.

Chapter 3

Air3d: A Computational Tool for Airblast Calculations

3.1 Introduction

The computational tool Air3d, used in the present study, is constantly evolving. The essential elements, however, were consistent throughout the work reported in this thesis and are described in the present chapter, which also acts as a user's guide.

The program is an explicit, finite volume formulation which solves one-, two- and three-dimensional forms of the Euler equations on a regular Cartesian grid: using equally spaced, square, and cubic elements. The method of solution is a variant of the Advection Upstream Splitting Method (AUSMDV, Wada and Liou [56]) described in Section 2.3.8. In two- and three- dimensions, it is used together with MUSCL-Hancock time integration, described in Section 2.3.10, to produce an implementation that is second-order accurate in space and time.

3.2 One-Dimensional Spherically Symmetrical Implementation

The one-dimensional implementation used in Air3d is first-order accurate. It uses spherical symmetry to model the region of space from the centre of the explosive charge to the nearest reflecting surface. It is entirely similar to the implementation described in Section 2.3.8, for the solution of the shock tube problem, except there are extra *source terms* which account for the flux contributions due to the missing spatial directions. The form of the Euler equations is the following:

$$\mathbf{U}_t + \mathbf{F}_r = \mathbf{S}(\mathbf{U}),$$

where the subscripts denote partial differentiation, and the vectors $\mathbf{U}$, $\mathbf{F}$ and $\mathbf{S}$ are:

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho \left(e + \frac{V^2}{2} \right) \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u \left(e + \frac{V^2}{2} \right) + pu \end{bmatrix}, \mathbf{S} = -\frac{2}{r} \begin{bmatrix} \rho u \\ \rho u^2 \\ \rho u \left(e + \frac{V^2}{2} \right) + pu \end{bmatrix}.$$

The $\mathbf{S}$ terms are calculated from the cell-centred variables and then added to the flux terms when the conserved variables $\mathbf{U}$ are updated at the end of each timestep. The resulting FORTRAN 90 (F90) source code is shown below:

```
DO I = 1,CALC_TO
RTEMP = (I-1)*DX+DX/2.0
U(:,I,2) = U(:,I,1)-DT/DX*(F(:,I,1)-F(:,I-1,1))-DT*2.0/RTEMP*SU(:,I)
END DO
```

where $F(:,I,1)$ and $F(:,I-1,1)$ are the right and left interface numerical fluxes, $SU(:,I)$ are the cell-centred source terms and $RTEMP$ is the radial distance to the cell centre. It will be noticed from the above code that not all of the computational grid is used at every timestep. Only the region from the first cell (at the point of symmetry) to the number held in the INTEGER variable $CALC_TO$ is used in the evolution stage of the analysis. The value of $CALC_TO$ is calculated from:

$$CALC_TO = \min(NCELLS, IVELO+1)$$

where $NCELLS$ is the total problem size and $IVELO$ is the number of the furthest cell from the charge centre that has a velocity greater than zero. This ensures that no effort is wasted calculating zero fluxes in the air which is still at ambient conditions.

The one-dimensional part of Air3d works in the following way:

- Space is discretised into equal radial divisions Δr . As a rough guide, each division should represent a scaled radial distance ($=r/W^{1/3}$) of about $1.0 \times 10^{-3} \text{ m/kg}^{1/3}$. This figure gives approximately 50 computational cells through the thickness of the explosive charge and produces very accurate peak pressures (and other blast parameters) over a large scaled range. This will be demonstrated in Chapter 4.
- The density of the condensed explosive $\rho_{TNT} = 1.6 \times 10^3 \text{ kg/m}^3$ and the source energy (chemical energy), available to be released as heat, $E_{TNT} = 4.52 \times 10^6 \text{ J/kg}$ are from Baker, et al. [4]. Both of these values can be altered by the user if desired.
- The radius of the charge r_{TNT} (m) is calculated from the specified charge weight W (kg) and density by:

$$r_{TNT} = \sqrt[3]{\frac{3}{4\pi} \frac{W}{\rho_{TNT}}}.$$

- The detonation wave speed for TNT $c_{\text{TNT}} = 6730 \text{ m/s}$ (also from Baker, et al. [4]), so the starting time problem time is given by:

$$t_{\text{start}} = \frac{r_{\text{TNT}}}{c_{\text{TNT}}}.$$

- The cell size Δr is recalculated to ensure that there are an integer number of cells n_{TNT} through the explosive using the following method:

$$n_{\text{TNT}} = \text{int} \left[\frac{r_{\text{TNT}}}{\Delta r} \right] + 1,$$

then

$$\Delta r = \frac{r_{\text{TNT}}}{n_{\text{TNT}}}.$$

- The total number of cells is recalculated to ensure that the desired problem radius (r_{max}) will not be changed significantly by the new value of Δr :

$$\text{NCELLS} = \text{int} \left[\frac{r_{\text{max}}}{\Delta r} \right].$$

- The first n_{TNT} computational cells are then *filled* with air of density ρ_{TNT} and energy E_{TNT} . The remaining cells are filled with air at atmospheric pressure $p_0 = 101.325 \text{ kPa}$ and temperature $T_0 = 288.0 \text{ K}$. All the other necessary state variables are calculated from these two and knowledge of the specific heat of air at constant volume $c_v = 715.0 \text{ J/kg}$ and the ratio of specific heats of air $\gamma = 1.4$.
- Finally, the problem is executed until either the specified finishing time is reached or the velocity in the final computational element is greater than zero. At this point, the array of conserved variables, radial discretisation, timestep, and problem time are written to an unformatted file. This file can then be used as input to the two- or three-dimensional parts of the program.

The program carries two sets (for the start and end of each timestep) of the (REAL) conserved variables U , the pressure, velocity and the internal energy. There are two further (REAL) variable arrays used for plotting. This gives $11 \times \text{REAL} \times 4$ variables = 44 bytes per computational cell, plus a few extraneous scalar variables.

Input to the program Air3d is via a formatted text file. An example of the first part (the one-dimensional input) is given in Table 3.1; line numbers have been added for clarity.

Table 3.1: Example one-dimensional input file for Air3d

	Line no.
!-----	1
!	2
! 100kg TNT to 5m radius	3
!	4
SPHERICAL_INPUT-----	5
!	6
100.0 0.0 0.0 Charge weight(kg), density (kg/m ³), energy (J/kg)	7
5.0 problem radius (m)	8
5.0E-3 delta r (m)	9
0 run option	10
0.0 problem time (sec)	11
0.0 100 display increment (sec) or cycles	12
0 Equation of state option	13
1 Plot variable	14
0.75 CFL	15
T File flag (update existing TNT file)	16
0_100kg_05s.TNT Output file	17
!	18
2 1 REFORMAT No. of target points, starting number, reformat flag	19
1.0 radial distance (m)	20
2.0 radial distance (m)	21
STOP	22

An explanation of the example input file in Table 3.1 is as follows:

- Lines starting (!) are comments and are ignored by the program.
- [Line 5:] The single instruction SPHERICAL_INPUT causes the main program to call the subroutine that reads the one-dimensional input parameters.
- [Line 7:] The first parameter is the charge weight (REAL, (kg)), the second is the charge density (REAL, (kg/m³)) and the third is the mass specific energy of the explosive (REAL, (J/kg)). If other than the default values are required (to model explosives other than TNT, for example), these secondary parameters can be used. The value 0.0 indicates that the program defaults are used in the present example.
- [Line 8:] The maximum problem radius (REAL, (m)) is the distance to the nearest reflecting surface in a two- or three-dimensional problem. However, if free air explosions are modelled, as in Chapter 4 (and the present example), the maximum problem radius will define a suitable distance beyond the final pressure monitoring position (or target point). These are explained below.
- [Line 9:] The cell size (REAL, (m)) should be selected so that the scaled cell size is approximately $1.0 \times 10^{-3} \text{ m/kg}^{1/3}$, described previously.
- [Line 10:] The run option (INTEGER, either 0 or 1) defines whether or not the screen displays are saved as bitmap images: 0 = not saved, 1 = saved.

- [Line 11:] The problem time (REAL, (sec)) is the finish time for the problem. As soon as the problem time exceeds this value the calculation stops. If a value of 0.0 (or a very large value) is entered, execution terminates when the velocity in the final computational element (at the maximum radius) is greater than zero.
- [Line 12:] The first parameter (REAL, (sec)) is the time increment between screen updates. If this figure is zero, the second parameter (INTEGER) is the number of computational cycles between screen updates.
- [Line 13:] The equation of state option is not presently implemented. It is intended to enable different treatment of the detonation products and the air in future implementations.
- [Line 14:] Variables (INTEGER, 1-4) available for plotting are the following:
 1. Pressure
 2. Density
 3. Internal energy
 4. Radial velocity

Other variables are also available for diagnostic and debugging purposes, but these are not required in normal use.

- [Line 15:] The CFL number, 0.75 (REAL), is the maximum that can be used in the present implementation. There is one circumstance when it may be necessary to use a smaller value: if a calculation is attempted that runs to a scaled distance of $100 \text{ m/kg}^{1/3}$ or greater. In such a circumstance, there comes a point where the blast decays to near acoustic level, and the sound speed becomes dominant in the timestep calculation. At this point, the code will become unstable if the default (0.75) is used.

The author has very limited experience of this circumstance and so cannot comment further. This type of calculation is only really applicable to situations where ear damage or noise are considered.

- [Line 16:] The file flag (LOGICAL) tells the main program whether or not to execute the one-dimensional subroutine. Clearly, the one-dimensional analysis does not need to be re-executed if changes to subsequent parts of the input file (in two- or three dimensions) are made. This flag is used to switch off the one-dimensional calculation once it has been successfully completed.
- [Line 17:] The name (CHARACTER) of the file that is used to dump the variables on completion: 0_100kg_05s.TNT in the present case. This name can be any valid collection of characters and numbers consistent with the operating system. It should end with the letter (s) (for spherical) to distinguish it from radially symmetrical input files (described in the next section), and it should have the file extension (.TNT).

- [Line 19:] The two parameters (INTEGER) are the number of pressure measuring locations (target points) and the starting number for the formatted history files. In the present case there are two target points, and output files are numbered starting from one. The second option is useful, because it allows multiple program executions to have consecutive pressures history output file names. Output file names have the following form: Hist0001.txt, Hist0002.txt, and so on. These are two-column formatted files which can be read by text editors, spreadsheets and general plotting programs.

The third parameter (CHARACTER) is either NOFORMAT or REFORMAT. This instructs the program whether or not to reformat the pressure histories into the individual files, described above, on completion. The option not to reformat straight away can be useful if many hundreds of target points were defined. This might occur if the program were used to define the input for a dynamic structural response calculation, for example.

- [Lines 20–21:] These lines contain the radial locations (REAL, (m)) of the two pressure monitoring points.
- [Line 22:] The single (optional) instruction STOP (CHARACTER) halts execution of the program and prevents two- or three-dimensional input from being read and executed.

It is suggested that first-order accuracy is adequate in spherically symmetrical analyses if the spatial discretisation is sufficiently fine to capture the desired peak pressures. This is usually achievable in one-dimensional calculations.

3.3 Two-Dimensional Radially Symmetrical Implementation

The two-dimensional form of the Euler equations used in the program Air3d is based on a domain that is symmetrical about a coordinate axis; in the present case, this is the y axis. The second coordinate is r , which measures the distance from the axis of symmetry and is the radial direction in the x - z plane. The two components of velocity are $u(r, y)$ and $v(r, y)$. These are, respectively, the radial and axial velocities.

The form of the Euler equations, with geometric source terms, becomes:

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_r + \mathbf{G}(\mathbf{U})_y = \mathbf{S}(\mathbf{U}),$$

where the subscripts r and y denote partial differentiation in the radial and axial directions, respectively, and the vectors $\mathbf{U}$, $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{S}$ are:

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho \left(e + \frac{V^2}{2} \right) \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho u \left(e + \frac{V^2}{2} \right) + pu \end{bmatrix}, \mathbf{G} = \begin{bmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ \rho v \left(e + \frac{V^2}{2} \right) + pv \end{bmatrix},$$

$$\mathbf{S} = -\frac{1}{r} \begin{bmatrix} \rho u \\ \rho u^2 \\ \rho uv \\ \rho u \left(e + \frac{V^2}{2} \right) + pu \end{bmatrix}.$$

Similar to above, for the one-dimensional case, the source terms are calculated from the cell-centred variables and added to the fluxes during the updating procedure in the radial direction.

The general shape of the computational domain in a two-dimensional analysis is always the same: cylindrical. The radius and height of the cylinder are all that vary. The lower y boundary (the ground) is always fixed as a *reflecting* boundary, and the lower r boundary is the axis of symmetry. The height of the centre of the charge above the ground is variable. These two-dimensional, radially symmetrical problems are all effectively height of burst problems.

The upper y and r boundaries can be set as either *reflecting* or *transmissive*. In the latter case, shock waves are allowed to flow out of the boundary. This might be used if a height of burst problem is analysed explicitly, not as part of a larger three-dimensional problem. Transmissive boundaries avoid the need to place the domain boundaries a long way from the region of interest. In the former case (reflecting boundaries), the row and column of cells adjacent to the upper boundaries are examined by the program at the end of every timestep. When the component of velocity (actually momentum) perpendicular to the boundary in any of these cells is greater than zero, the analysis is halted, and the whole of the domain conserved variables are written to an unformatted file. This is the usual way in which the program is used.

Reflecting boundaries are achieved by placing the value of the conserved variables adjacent to the boundary into a layer of *ghost cells* beyond the boundary and negating the sign of the momentum components perpendicular to the boundary. This has the effect of preventing convection across the interface. The following F90 code is used in Air3d for treatment of the lower y boundary.

```
U1D(:,0,1) = U1D(:,1,1)
U1D(3,0,1) = U1D(3,1,1)*BYL
```

In this case, the scalar variable BYL (which stands for *boundary y lower*) contains the value -1.0 , and the array U1D contains a *slice* of conserved variables $\mathbf{U}$. A

transmissive boundary is achieved by setting the value to +1.0 so that convection occurs smoothly across the boundary interface. The user should be aware that there can be problems associated with placement of transmissive boundaries; these are discussed in Section 3.6.

At the start of a two-dimensional analysis, the results of a one-dimensional spherical analysis are *remapped* into the computational grid. The remainder of the grid is then *filled* with air at atmospheric conditions. The remapping process is achieved in the following manner:

- The mesh size of the one-dimensional analysis is invariably considerably less than that of the two-dimensional analysis. To ensure a good representation of the results of the spherical analysis, the two-dimensional domain is temporarily divided into an integer number (RDIV) of subcells in each coordinate direction. This number is calculated as follows:

$$\text{RDIV} = \text{NINT}(\text{DX}/\text{DR})$$

$$\text{RDIV} = \text{MIN}(\text{RDIV}, 8)$$

where DX is the size of the (square) cells in the two-dimensional domain, DR is the radial discretisation of the spherical analysis, and the second of the two lines ensures that the number of subcells does not exceed a given (reasonable) number, in the present case sixty-four.

- The maximum radius of the spherical analysis is established. Then, for every cell in the domain, the locations of the four corners of the cell and their distances from the centre of the explosion are calculated. If all these distances are greater than the maximum spherical radius, the cell is filled with air at atmospheric conditions. If at least one of the corners is located at a distance less than the maximum spherical radius, the cell is subdivided in the manner described above. The distance of the centre of the subcell from the charge centre is calculated, and this distance is used to identify the appropriate set of spherical data. The subcell is then filled with the conserved variables representing that part of the blast wave.

Clearly, there will be an error associated with this process, and this error is hard to quantify. However, if the cell sizes of the two analyses are chosen appropriately, the remapping procedure can be made to correspond quite closely. It is, perhaps, sufficient to state that the overall quality of the solution is dominated by the spatial discretisation of the three-dimensional analysis, and errors resulting from the remapping procedure are insignificant compared to this, more important, consideration.

A qualitative example of the loss of resolution which accompanies the remapping procedure is given in Section 3.4.

In each timestep, the following procedure is followed by the program:

- i. The conserved variables are extracted in *radial* strips from the main two-dimensional array and placed in a one-dimensional *working array*.

- ii. Values of the cell-centred source terms are calculated from the conserved variables.
- iii. Interface conserved variables are calculated by the method described in Section 2.3.6 and half-timestep integration is performed using the first part of the MUSCL-Hancock procedure described in Section 2.3.10.
- iv. Interface fluxes are calculated from the half-timestep interface variables, and the whole-timestep conserved variables (in the two-dimensional array) are partially updated using the following F90 code:

```

DO II = 1,NUM
  RDIST = DX*(II-1)+DX/2.0
  U(:,II,J,1,2) = U(:,II,J,1,1)-DT/DX*(F1D(:,II,1)-F1D(:,II-1,1)) &
    -DT/RDIST*SU(:,II)
END DO

```

where II is the DO LOOP index, and NUM is the number of radial elements. The scalar variable J is the index of the radial slice in the *axial* direction. The loop is performed J times in each timestep.

- v. Conserved variables are next extracted from the two-dimensional array in *axial* strips and placed in the one-dimensional working array.
- vi. Again, interface conserved variables are calculated, half-timestep integration is performed, interface fluxes are calculated and the whole-timestep conserved variables are updated.

```

DO II = 1,NUM
  U(:,I,II,1,2) = U(:,I,II,1,2)-DT/DX*(F1D(:,II,1)-F1D(:,II-1,1))
END DO

```

where the one-dimensional fluxes F1D now contain the **G** fluxes.

- vii. The timestep is calculated from the CFL condition:

$$\Delta t = cfl \frac{\Delta r}{\max(a + |u_i|)}$$

where $cfl < 1$ is a constant and dependent on the flow solver and slope limiter, Δr ($= \Delta y$) is the cell size, a is the sound speed, and the maximum is taken over the two directions $i = r, y$.

- viii. Finally, the starting conserved variables are replaced by the whole-timestep conserved variables and the process is repeated until the finishing time is reached.

The program has two sets of the conserved variables U plus one two-dimensional array of pressure for plotting. This gives $9 \times \text{REAL} \times 4$ variables (or 36 bytes) per computational cell. There are also one-dimensional arrays of pressure, velocity, and internal energy, and numerous scalar variables, but these do not form a significant memory overhead.

An example input file for the two-dimensional part of a blast simulation is given in Table 3.2. This would directly follow the spherical part of the input given in Table 3.1.

Table 3.2: Example two-dimensional input file for Air3d

	Line no.
!-----	1
!	2
! Height of Burst simulation, 100kg @ 5m stand-off	3
!	4
RADIAL_INPUT-----	5
!	6
0.0 20.0 RS,RF	7
0.0 25.0 YS,YF	8
0.25 Cell size (m)	9
1 1 Display variables	10
4.6E-3 Time to switch to 2nd order	11
2 Run option	12
1.0E-0 Problem time	13
0.0 25 Display increment (sec) or cycles	14
5.0E-1 CFL	15
0.0 Charge weight (kg)	16
0.0 Centre of gas Y (m)	17
-1 BXU upper r boundary	18
-1 BYU upper y boundary	19
T File flag (update existing TNT file)	20
0_100kg@5m_r.TNT Output file	21
3 .0001 .0001 IXY (plotting), level	22
20.0 0.0 Scale factor for contour intervals	23
4 1 REFORMAT No. of target points, starting number, reformat flag	24
10.0 2.0 radial (m), axial (m)	25
10.0 4.0 radial (m), axial (m)	26
10.0 6.0 radial (m), axial (m)	27
10.0 8.0 radial (m), axial (m)	28
STOP	29

It can be seen from the input in Table 3.2 that this example models an explosion of 100kg TNT at a height 5 m above the ground. It uses the output file (not identified explicitly) from the first example as the input remap file. Therefore, at the start of this analysis the blast wave is in contact with the ground surface. The program is executed until one of the two upper boundaries is reached.

A complete explanation of the input file in Table 3.2 is as follows:

- [Line 5:] The instruction RADIAL_INPUT causes the main program to call the subroutine which reads the two-dimensional input parameters.

- [Lines 7-8:] These lines contain the upper and lower extents of the computational grid (REAL, (m)). The two values RS and RF are the start and finish values in the radial direction; YS and YF are the values in the axial (vertical) direction. The two starting values should not be altered from zero; they are only retained in the input file for debugging purposes.
- [Line 9:] The cell size (REAL, (m)) is the spatial discretisation in both directions. Only square cells are used by the program.
- [Line 10:] The variables (INTEGER, 1-5) available for graph plotting are the same as the one-dimensional part of the program. However, in two dimensions there are two components of velocity.

1. Pressure
2. Density
3. Internal energy
4. Radial velocity
5. Axial velocity

The two input values form the indices of a DO LOOP; in the present case [1 1] instructs the program to plot only pressure. Alternatively, [3 4] would cause the program to plot both components of velocity consecutively at every display increment. Generally, this facility would only be used if bitmap images of the plots were stored.

- [Line 11:] The time value (REAL, (sec)) is the time at which to switch to second-order and is a device used to overcome one of the deficiencies of the program Air3d. Because of the enormous discontinuities of pressure, density, energy, and so on, at early (scaled) problem times, it is necessary to run the program using first-order accuracy for a small number of timesteps, until the sharp discontinuities have had time to diffuse slightly. Generally, a scaled time of approximately $1.2 \times 10^{-3} \text{ sec/kg}^{1/3}$ corresponds to the problem time at which the solution is sufficiently diffused to switch.

It should be noted that, in many analyses, the finishing time of the one-dimensional part of the problem will already exceed this *switching time*. When this occurs, the whole of the two-dimensional analysis is performed with second-order accuracy.

- [Line 12:] The run option (INTEGER, 0-7) tells the program which of the plotting options to use. These are as follows:

0. Graphs only
1. Graphs and bitmap images
2. Pressure contours
3. Pressure contours and bitmap images
4. Peak pressure contours
5. Peak pressure contours and bitmap images
6. Peak impulse contours
7. Peak impulse contours and bitmap images
8. Shock indicator
9. Shock indicator and bitmap images

The graph options (0 and 1) refer to the variables identified on line 10, above. Options 1, 3, 5, 7 and 9 store the screen display as bitmap images in consecutively numbered files. This allows inclusion in reports, presentations or the compilation of animated sequences.

Options 8 and 9 are the shock indicator. This identifies where the compressive shocks are in the computational domain. The option can be very useful because it still identifies a compressive shock if it is travelling through a region where an expansion has taken place and the peak pressure is below atmospheric. Such a shock would not appear on a pressure contour plot.

The process of shock identification is based on a modified version of the method described by Khokhlov [27] and is the following:

For each cell i , the shock indicator ξ_i^s (in three dimensions) is defined as:

$$\xi_i^s = \max_{j=1,6} \xi_{i,j}^s$$

where $j = 1$ refers to the lower x face, $j = 2$ refers to the upper x face, $j = 3$ refers to the upper y face, and so on.

The six shock indicators $\xi_{i,j}^s$ are given by:

$$\xi_{i,j}^s = \begin{cases} \frac{|P_{iNb(i,j)} - P_i|}{\min(P_{iNb(i,j)}, P_i)}, & \text{if } \frac{|P_{iNb(i,j)} - P_i|}{\min(P_{iNb(i,j)}, P_i)} > \epsilon_s, \text{ and } \delta \cdot (U_{iNb(i,j),k} - U_{i,k}) < 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\delta = 1 - 2 \cdot \text{mod}(j, 2)$, $k = \text{int}((j + 1)/2)$, and $\epsilon \approx 0.05$ determines the minimum shock strength to be detected. The subscript Nb indicates the *neighbouring* cell; $Nb(i, j)$ indicates the neighbour to cell i across the j interface.

The value of ϵ_s varies depending on the scaled problem time $t/W^{1/3}$:

$$\epsilon_s = A \exp^{-b(\frac{t}{W^{1/3}})},$$

where the coefficients A and b are input parameters (see line 23, below).

- [Line 13:] The problem time (REAL, (sec)) is the finish time for the analysis. Generally, however, if the program is used with reflecting boundaries, the problem time is set to some arbitrarily large value and the program halts execution automatically, as described above.
- [Line 14:] The first number (REAL, (sec)) is the time increment between screen updates. If this value is zero, the second number (INTEGER) is the number of computational cycles between screen updates.
- [Line 15:] The CFL number (REAL) should be 0.5. The present method of solution becomes unstable if the value is increased much above this. Similarly, it should seldom be necessary to reduce it below this value.
- [Line 16:] The charge weight (REAL, (kg)) provides an alternative to the one-dimensional input described above. It is only used for debugging purposes.
- [Line 17:] The centre of gas (REAL, (m)) refers to the height, in the axial direction, of centre of the explosive charge above the ground. Results of the one-dimensional analysis are remapped into the two-dimensional domain centred at this point on the axis of symmetry.
- [Lines 18–19:] The two values (INTEGER) BXU and BYU define the type of boundary condition (reflecting or transmissive) imposed on the upper boundaries of the computational domain. A value of -1.0 signifies a reflecting (or *stop*) boundary; $+1.0$ signifies a transmissive boundary.
- [Line 20:] The file flag (LOGICAL) tells the main program whether or not to execute the two-dimensional subroutine. This is similar to the one-dimensional case described above. It is usual for the one- and two-dimensional parts of the analysis to be common to a set of three-dimensional problems. When changes are made only to the three-dimensional input, this flag is used to switch off the two-dimensional calculation.
- [Line 21:] This is the name of the file (CHARACTER) that is used to dump the variables on completion: 0_100kg_01r.TNT in the present case. The name can be any valid collection of characters and numbers consistent with the operating system. It should end with the letter (r) (for radial) to distinguish it from spherically symmetrical input files, described previously, and it should have the file extension (.TNT).
- [Line 22:] The three numbers describing the plotting level (INTEGER, REAL, REAL) should not be changed. They are only present in the input for debugging purposes.
- [Line 23:] The first number (REAL) is the scale factor for contour intervals and is a multiplier applied to the default plotting values. By default, the scale of the contour plots for overpressure goes from 0.0 to 32.0 kPa. Clearly, this is seldom appropriate. Therefore, the scale factor allows the user the opportunity

to select the best scale for the particular problem. The second value (REAL) is not used for display options 1 to 7 (line 12).

When the shock indicator display option is used (options 8 and 9), the two factors are interpreted in a different way. They are the coefficients A and b in the formula given previously. There are no defaults for these values because shock identification varies from problem to problem. It is suggested, however, that $A = 0.05$ and $b = 150$ are reasonable starting values. It is necessary to experiment with the parameters until a good combination is found.

- [Line 24:] The two numbers (INTEGER) are the number of pressure monitoring (target) points and the starting number for the formatted history files. The third parameter (CHARACTER) is either NOFORMAT or REFORMAT and instructs the program whether or not to reformat the pressures histories into the individual files on completion. This is identical to the one-dimensional input.
- [Lines 25–28:] These lines contain the coordinates of the locations of the pressure monitoring points (REAL, (m)) in (radial, axial) pairs.
- [Line 29:] The single (optional) instruction STOP (CHARACTER) halts execution of the program and prevents the three-dimensional input from being read and executed.

3.4 Three-Dimensional Implementation

The three-dimensional solver of the Euler equations uses cubic cells in a single Cartesian grid. The equations of motion are the following:

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x + \mathbf{G}(\mathbf{U})_y + \mathbf{H}(\mathbf{U})_z = 0$$

where $\mathbf{U}$, $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ are column vectors given by:

$$\mathbf{U} = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho \left(e + \frac{V^2}{2} \right) \end{Bmatrix}, \quad \mathbf{F} = \begin{Bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ \rho u \left(e + \frac{V^2}{2} \right) + pu \end{Bmatrix},$$

$$\mathbf{G} = \begin{Bmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ \rho vw \\ \rho v \left(e + \frac{V^2}{2} \right) + pv \end{Bmatrix} \text{ and } \mathbf{H} = \begin{Bmatrix} \rho w \\ \rho wu \\ \rho wv \\ \rho w^2 + p \\ \rho w \left(e + \frac{V^2}{2} \right) + pw \end{Bmatrix}.$$

The vector $\mathbf{U}$ is the vector of the *conserved variables*. The vectors $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{H}$ are the *flux vectors*, although the numerical fluxes are calculated by the method described in Section 2.3.8.

At the start of a three-dimensional analysis, the results of a one-dimensional spherically symmetrical or two-dimensional radially symmetrical analysis are *re-mapped* into the computational domain; the rest is filled with air at atmospheric conditions.

The explosive source can be remapped anywhere into the problem domain. However, significant savings in computer time can be achieved by introducing the explosive close to the *lower* boundaries of the domain. The program can keep track of velocities greater than zero in the one-dimensional working arrays (described previously). This means that only a subset of the whole domain (which always includes the lower boundary) is worked on, until non-zero velocities reach the upper boundaries. It has been observed that this is approximately 30–40% faster than analysing the whole domain in every timestep.

The remapping procedure in three-dimensions is similar to that described previously for two-dimensional analyses, although, clearly, there are eight corners of each cell to be considered when deciding whether or not a cell is to be filled.

A qualitative example of the effect of the remapping procedure on the representation of the solution is given in Figures 3.1 to 3.4, below. The first three graphs show the conserved variables: density, momentum and total energy, the fourth graph

shows the representation of pressure.

The example is from calculation 4 of Table 4.1 in Chapter 4, where it will be seen that the one-dimensional analysis was for 1 kg TNT, and it was stopped when the blast reached a distance of 10 m from the origin of the explosion. The one-dimensional cell size was 1 mm. This data was remapped into a three-dimensional grid with 150 mm cubic cells. The two representations are superimposed in the figures to allow a qualitative assessment of the remapping procedure.

It is clear from Figures 3.1 to 3.4 that some loss of resolution has occurred during remapping, particularly in the region of the two compressive shocks in Figures 3.2 to 3.4, which are similar. Figure 3.1, however, requires a brief explanation. The smoothing through the shock waves is still evident, but there is also a contact discontinuity at the interface between the air modelling the detonation products and the air which was originally at atmospheric conditions. It is interesting that, even at this comparatively late time and despite the fact that it has returned to atmospheric pressure, the density in the region occupied by the products is still much reduced (compared with atmospheric). This aspect of the flow is not generally discussed in relation to blast loading calculations.

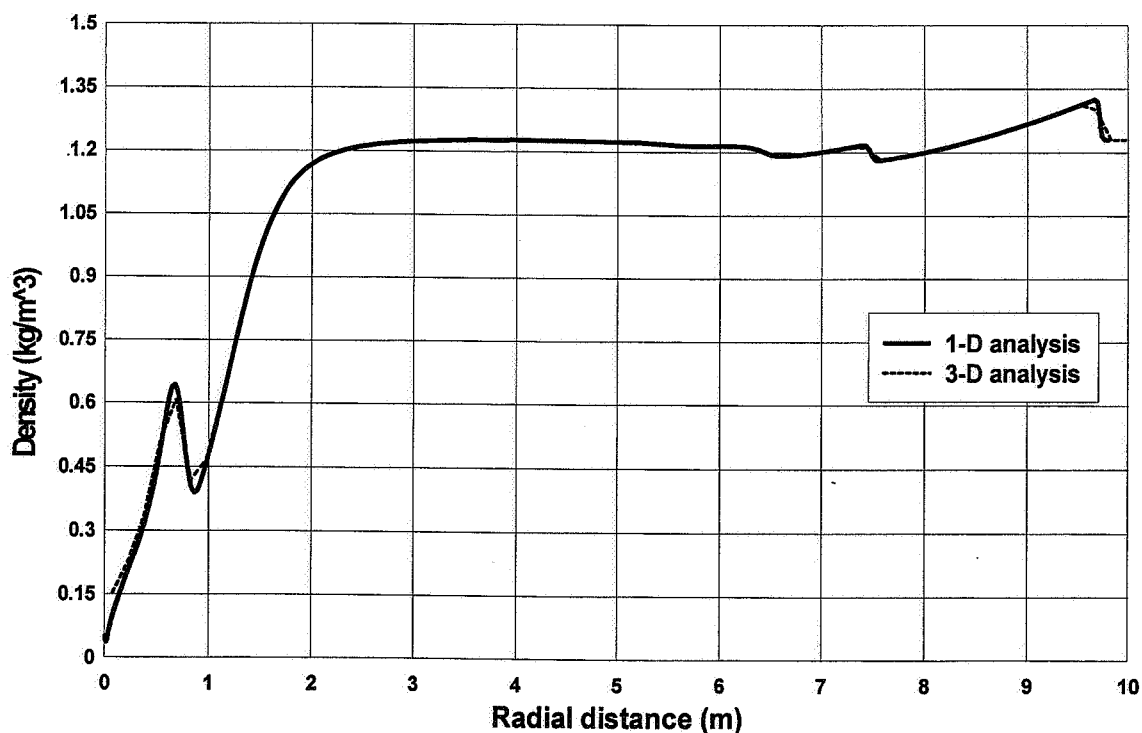


Figure 3.1: Density, one- and three-dimensional representation

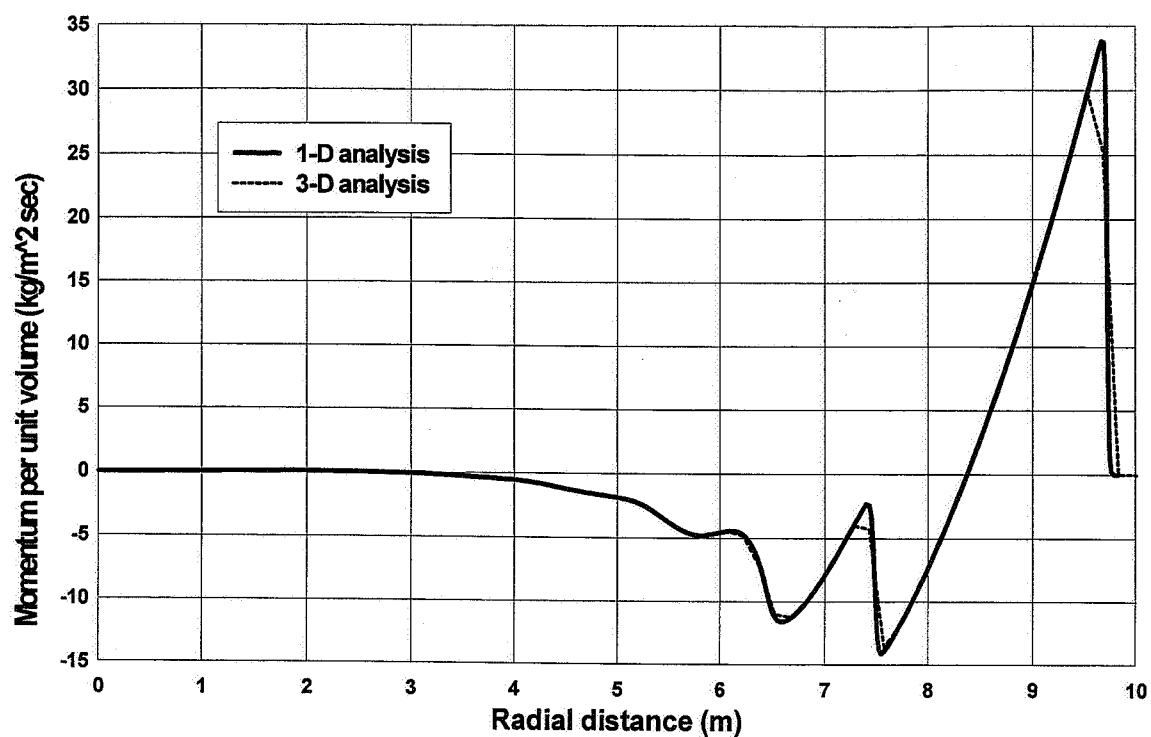


Figure 3.2: Momentum, one- and three-dimensional representation

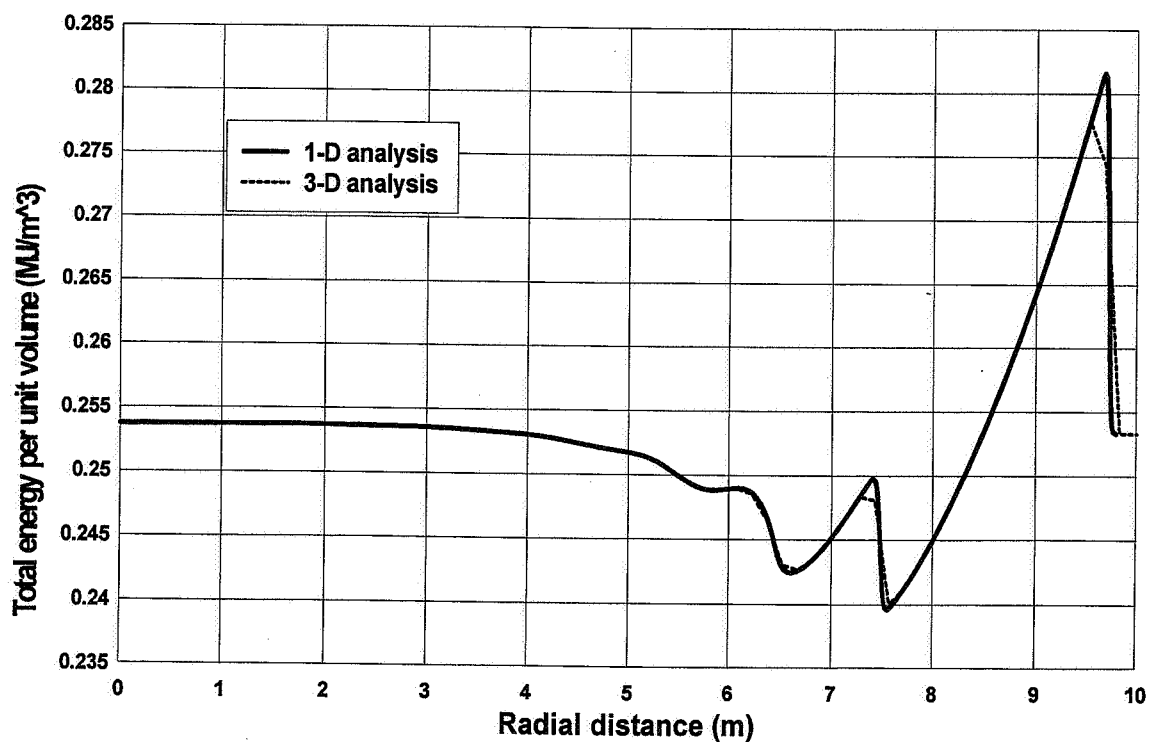


Figure 3.3: Energy, one- and three-dimensional representation

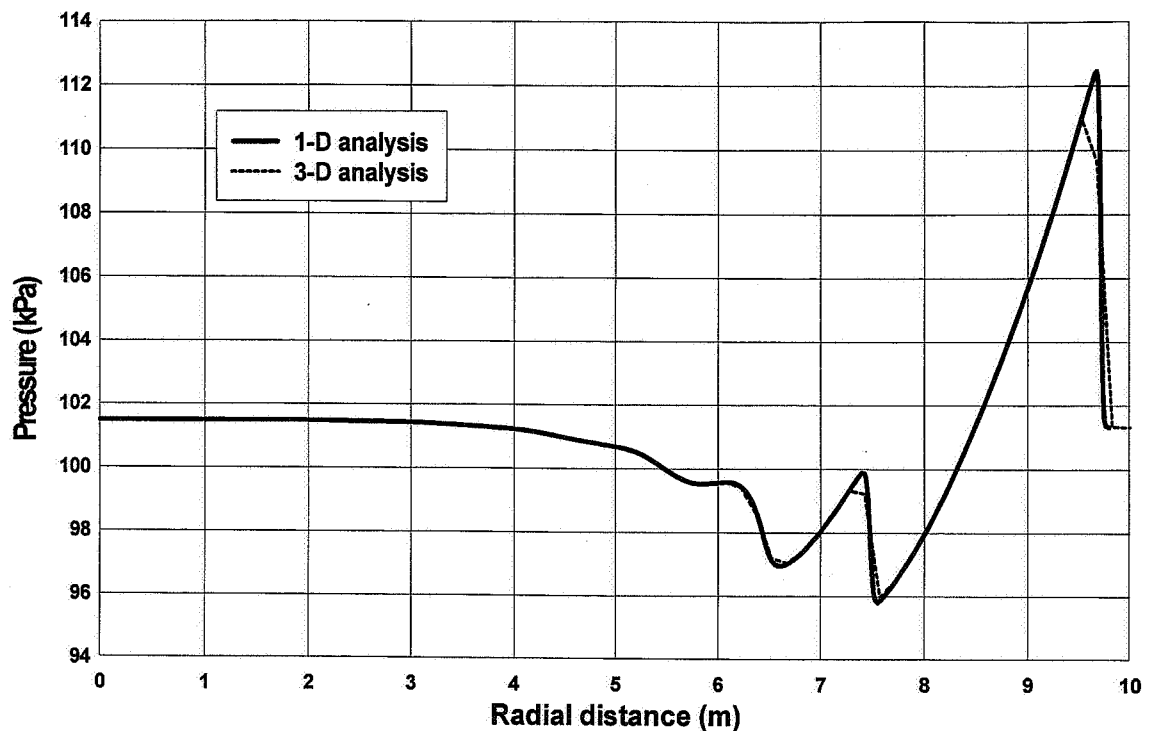


Figure 3.4: Pressure, one- and three-dimensional representation

Obstacles, such as buildings, barriers and other structures, are introduced into the problem by declaring cells to be either **TRUE** or **FALSE** (*used* or *unused*). The program uses a three-dimensional array of **LOGICAL** variables for this purpose. Unused cells act as reflecting boundaries and, although they occupy space in memory, they are ignored during the numerical flux calculations.

Each obstacle is defined by lower and upper bounding values in the three coordinate directions. **FALSE** obstacles form the reflecting boundaries described above, while **TRUE** obstacles contain air at atmospheric conditions. The obstacles are implemented in the order in which they appear in the input file, so it is possible to define intricate geometries by superimposing **TRUE** and **FALSE** obstacles. This is known as *constructive solid geometry* (for example, Yarwood [58]) and is a very quick and easy method of defining building-like structures.

An example of a simple building constructed in this manner might be a solid rectangular shell with gaps for windows on the front and rear faces. This could be defined with a single **FALSE** obstacle describing the outside dimensions, a **TRUE** obstacle defining the inside dimensions (effectively hollowing-out the building) and several long, penetrating **TRUE** obstacles which define the windows at the front and rear simultaneously.

Having defined one obstacle in this manner, it is a trivial task to increment the dimensions in one or more spatial directions (using a spreadsheet for example) to create a city street, a skyscraper or even a small town. A similar technique could be used to create a labyrinthine series of chambers from a solid block of **FALSE** computational space, although this approach might be costly in terms of memory use.

The procedure followed by the program in each timestep is similar to the two-dimensional implementation. It is outlined below:

- i. For each direction: x , y and z (in this order), conserved variables are extracted from the *old* three-dimensional arrays in single-directional slices and placed in a one-dimensional *working array*.
- ii. Interface conserved variables are calculated, and *unused* obstacle cells which are adjacent to *used* cells are filled with the values of the *used* interface variables (with the momentum components opposed in the direction of the slice). Boundary interface variables are also calculated.
- iii. Half-timestep integration is performed using the MUSCL-Hancock procedure.
- iv. Numerical fluxes are calculated from the half-timestep variables, and the whole-timestep conserved variables are updated using the following code:

```
DO II = 1,NUM
  IF(USED1D(II)) &
    U(:,II,J,K,2) = U(:,II,J,K,1)-DT/DX*(F1D(:,II,1)-F1D(:,II-1,1))
  END DO
```

for x direction slices and

```
DO II = 1,NUM
  IF(USED1D(II)) &
    U(:,I,II,K,2) = U(:,I,II,K,1)-DT/DX*(F1D(:,II,1)-F1D(:,II-1,1))
  END DO
```

for y direction and

```
DO II = 1,NUM
  IF(USED1D(II)) &
    U(:,I,J,II,2) = U(:,I,J,II,1)-DT/DX*(F1D(:,II,1)-F1D(:,II-1,1))
  END DO
```

for the z direction. It will be observed that the above pieces of code differ in one significant respect. In the x direction, the new (end-of-timestep) conserved variables $U(:, :, :, :, 2)$ are set equal to the *old* (beginning-of-timestep) variables $U(:, :, :, :, 1)$ plus the flux contributions. In the other two directions, the only modification is due to the flux contributions.

- v. The new timestep is calculated from the CFL condition:

$$\Delta t = cfl \frac{\Delta x}{\max(a + |u_i|)}$$

where $cfl < 1$ is a constant, Δx ($= \Delta y, \Delta z$) is the cell size, a is the sound speed and the maximum is taken over the three directions $i = x, y, z$.

- vi. Finally, the starting conserved variables are replaced by the whole-timestep conserved variables:

$$U(:, :, :, :, 1) = U(:, :, :, :, 2)$$

and the process is repeated until the finishing time is reached.

The program has two copies of the conserved variables U and one array of single-byte LOGICAL variables: the *used* flags. There is a two-dimensional array for pressure plotting and one-dimensional arrays for pressure, three components of velocity, and internal energy. The calculation of memory use is dominated by the three-dimensional arrays; there are $(10 \times \text{REAL} * 4 + 1)$, or 41 bytes per cell. This implies that a Personal Computer with 392 Mbytes of memory (as used in the present study) would be able to run problems with:

$$\frac{392 \times (2)^{20}}{41} = 10,025,410 \text{ computational cells,}$$

or about ten million.

An example input file for the three-dimensional part of Air3d is given in Table 3.3, below. This example is taken from the set of street analyses described in Section 6.3. It is for a street of width $w = 8$ m and building height $h = 4$ m. There are pressure monitoring locations along the length of the street at a height $y = 2$ m above the ground. The charge is 1000 kg TNT, located in the centre of the street. The spherical input file for this example is not shown, and the radial input file would actually consist of *dummy input* because it is the spherical remap file (line 18 of Table 3.3) that is read in the present example. Reference should be made to Section 6.3 for further details.

Table 3.3: Example three-dimensional input file for Air3d

			Line no.
MAIN_INPUT-----			1
!			2
0.0 75.0	XS,XF		3
0.0 48.0	YS,YF		4
0.0 48.0	ZS,ZF		5
0.5	Cell size (m)		6
1 1	Display variables		7
7	Method of analysis		8
1.2E-2	Time to switch to 2nd order		9
1	Flux limiter		10
1.0	Beta, general flux limiter only (1.0 to 2.0)		11
2	Run option		12
0	Use MVSTORE, 0-no, 1-yes.		13
2.5E-1	Problem time (sec)		14
0.0 5	Display increment (sec) or cycles		15
5.0E-1	CFL		16
-1.0	Charge weight (kg)		17
SPHERICAL	TNT input file type		18
0.0	Gas pressure (Pa) or VOLEXP if using CW		19
0.0	Gas temperature, 0.0 if using CW		20
0.0	Gas radius, 0.0 if using CW		21
0.0	Centre of gas X (m)		22
0.0	Centre of gas Y (m)		23
0.0	Centre of gas Z (m)		24
-1	BXL lower x boundary		25
+1	BXU upper x boundary		26
-1	BYL lower y boundary		27
+1	BYU upper y boundary		28
-1	BZL lower z boundary		29
+1	BZU upper z boundary		30
3 .001 .001 3.999	IXYZ (plotting), level		31
5.0 0.0	Scale factor for contour intervals		32
1	Number of obstacles		33
FALSE	Type of obstacle		34
0.0 75.0			35
0.0 4.0			36
4.0 48.0			37
6 1 REFORMAT	No. of target points, starting number, reformat flag		38
0.0 2.0 3.999			39
10.0 2.0 3.999			40
20.0 2.0 3.999			41
30.0 2.0 3.999			42
40.0 2.0 3.999			43
50.0 2.0 3.999			44

A complete explanation of the example input shown in Table 3.3 is as follows:

- [Line 1:] The statement `MAIN_INPUT` instructs the main program to read the three-dimensional input parameters.
- [Lines 3–5:] These are the lower and upper extents of the computational domain (`REAL`, (m)) in the three coordinate directions. They can take any reasonable values, consistent with the cell size.
- [Line 6:] The cell size (`REAL`, (m)) is the spatial discretisation in all three directions. Only cubic cells are used by the program.
- [Line 7:] The variables (`INTEGER`, 1–6) available for plotting are similar to those in the one- and two-dimensional parts of the program; there are three components of velocity.

1. Pressure
2. Density
3. Internal energy
4. x velocity
5. y velocity
6. z velocity

The two input values form the indices of a `DO LOOP`, as described previously.

- [Line 8:] The method of analysis (`INTEGER`) takes the value 6 or 7. This reflects the fact that the current implementations are the sixth and seventh versions of the solver. At present, version seven is exactly the same as version six, except it contains the *speed-up* procedure described earlier.

Generally, version seven will be faster than version six, but there are a few instances when it would be slower. One example is internal blast simulation with multiple reflections. In this case, version six would be faster; although, the time implications of using the inappropriate version are not serious.

All simulations contained in this thesis were performed using version six of the solver.

- [Line 9:] This is the time (`REAL`, (sec)) at which to switch to second-order, also described previously.
- [Line 10:] The flux limiter option (`INTEGER`, 1–3) allows the choice of three slope limiter functions for the calculation of the interface conserved variables. The choice is as follows:

1. Minmod
2. Superbee
3. General β

Although all three limiters are implemented in the three-dimensional code, the author has found (as a result of many lengthy trials) the minmod limiter to be most suitable for blast simulation. The Superbee function, in particular, is overcompressive and can have an alarming effect on the shape of the blast wave profile: it tends to turn blast waves into square waves.

- [Line 11:] The value of β (REAL) is for use with the General β slope limiter (option 3 above), and it can vary between 1.0–2.0. Similar to above, this should also not be used, because values greater than 1.0 are also overcompressive.
- [Line 12:] The run option (INTEGER, 0–11) tells the program which of the plotting routines to use. These are the same as for two-dimensions and are described in Section 3.3. In three dimensions, however, there are two further options involving Virtual Reality Modelling Language (VRML):

10. VRML — individual cells

11. VRML — individual obstacles

These two options are different from the other nine because *they do not run the program*.

Options 10 and 11 are similar. They both write the boundaries of the problem domain, the charge location and the location of the monitoring positions to a text file in VRML. The text file is given the same name as the input file and the extension (.WRL).

The difference between the two options is that option 10 writes every FALSE *cell* to the file (represented by a cube), but option 11 writes every FALSE *obstacle*. Clearly, for problems which use only FALSE obstacles, option 11 results in a much smaller VRML file.

- [Line 13:] The single variable MVSTORE (Maximum Value Store) (INTEGER, 0 or 1) instructs the program whether or not to use previously stored values for graph plotting. This option is to allow consistent sets of plotting axes on graphs from a set of similar analyses.
It is seldom used, except for validation purposes. It is an INTEGER rather than a LOGICAL variable because other options may be implemented in the future.
- [Line 14:] The problem time (REAL, (sec)) is the finish time for the analysis.
- [Line 15:] The first number (REAL, (sec)) is the time increment between screen updates. If this figure is zero, the second number (INTEGER) is the number of computational cycles between screen updates.
- [Line 16:] The CFL number (REAL) should be 0.5. The present method of solution becomes unstable if the value is increased much above this. Similarly, it should seldom be necessary to reduce it below this value. This is the same as described previously for two-dimensions.

- [Line 17:] The charge weight (REAL, (kg)) has three functions in the three-dimensional input. The first (and usual) function is used if the specified charge weight $W < 0.0$, as in the present example. This acts as a flag, and it instructs the program that input is via either a SPHERICAL or RADIAL save file from the one- or respectively two-dimensional subroutines. If this option is used, the program ignores lines 19–21 of the input file.

The second option ($W = 0.0$) instructs the program to use an isothermal sphere as input to the program. The pressure of the gas is read from line 19, the temperature from line 20 and the radius of the isothermal sphere from line 21. This capability has one main function: to enable one-dimensional shock tube validation problems to be set up in the three coordinate directions. It serves no other useful purpose and is not generally used.

The third option ($W > 0.0$) again causes the program to use an isothermal sphere as input. It uses a *volume expansion factor* (read from line 19) and ignores lines 20–21. This option was the first to be implemented, before the one-dimensional subroutine was programmed. It is no longer of any practical value and should not be used.

- [Line 18:] If $W < 0.0$ on line 17, the single (CHARACTER) word SPHERICAL or RADIAL instructs the program to read either the one-dimensional file named on line 17 of spherical input (Table 3.1) or the two-dimensional file on line 21 of the radial input (Table 3.2).
- [Line 19:] The parameter (REAL) is either the gas pressure (if $W = 0.0$ on line 17) or the volume expansion (if $W > 0.0$), otherwise it is ignored.
- [Line 20:] This is the gas temperature (REAL, (K)) if $W = 0.0$ on line 17, otherwise this line of input is ignored.
- [Line 21:] The radius of the isothermal sphere (REAL, (m)) is used for the starting conditions, if $W = 0.0$ on line 17.
- [Lines 22–24:] If SPHERICAL or either of the options for an isothermal sphere (on line 17) are selected, these values (REAL, (m)) are the origin of the explosion. If RADIAL input is selected, however, the height of the charge above the ground is implicit in the two-dimensional file. Therefore, selecting the height of the centre of the gas above the ground $y = 0.0$ causes the origin of the blast to remain at the height of burst selected for the two-dimensional analysis. It is seldom necessary to use this facility in any other way.

The values of the centre of gas in the x and z directions have the expected effect of translation.

- [Line 25–30:] The values BXL, BXU, BYL, BYU, BZL and BZU (INTEGER) define the type of boundary condition (reflecting or transmissive) imposed on the lower and upper boundaries of the computational domain. A value of -1.0 signifies a reflecting boundary; $+1.0$ signifies a transmissive boundary.

- [Line 31:] The first parameter (INTEGER, 1-3) tells the program in which direction to *slice* the three-dimensional domain for plotting pressure/impulse contours. The numbers (1-3) correspond to the coordinate directions x , y and z , respectively. The number represents the direction to be held constant during the slicing process. The remaining three numbers (REAL, (m)) describe the location in space where the slice is to be taken. Only one of these numbers will be used, depending on the value of the first parameter. A summary is given below:

1. slice in z - y space, $x = \text{constant}$
2. slice in x - z space, $y = \text{constant}$
3. slice in x - y space, $z = \text{constant}$

Therefore, the parameters in line 31 of Table 3.3 instruct the program to keep the z -direction constant and plot a slice in x - y space. The slice is to be taken at a constant fixed location $z = 3.999$ m. In the present example, this slice will show the pressures developed on the front face of the building defined by the obstacle described below.

- [Line 32:] This is the scale factor for contour intervals (REAL) or the coefficients (REAL, REAL) for shock indicator displays. They are interpreted in the same way as for two dimensional analyses described in Section 3.3.
- [Line 33:] The first parameter (INTEGER) defines the number of obstacles in the problem. Each obstacle requires four lines of input to be described fully: lines 34-37 in the present example for one obstacle.
- [Line 34:] The obstacle type (LOGICAL, (TRUE or FALSE)) instructs the program whether the obstacle defines *used* or *unused* cells in the computational domain. These were described previously.
- [Line 35-37:] These are the location (REAL, (m)) of lower and upper boundaries of the obstacle in the x , y and z directions, respectively.
- [Line 38:] The first number (INTEGER) is the number of pressure monitoring (target) points, the second is the starting number (INTEGER) of the formatted history files, and the flag (CHARACTER, NOFORMAT or REFORMAT) instructs the program whether or not to reformat the pressures histories into the individual files on completion. This is identical to the one- and two-dimensional input.
- [Lines 39-44:] These are the coordinates (REAL, (m)) in x,y,z space of the pressure monitoring points. There are five points in the present example, there were sixty-six in the original file taken from Section 6.3.

3.5 Spherical Blast Waves in Air Described by Air3d

This section provides a background to the formation of spherically symmetrical blast waves in air as they are represented by the program Air3d. It should be recognised that the explanation presented here describes the processes modelled by the *idealisation* implemented in Air3d (described in Section 3.2) and not the processes which *actually* occur during and subsequent to a high explosive detonation in air.

Intuitively, it would be expected that the distribution of internal energy and velocity throughout the detonation products, at the time when the detonation wave has reached the outer surface of the charge, would lead to starting conditions different from those described in Section 3.2. Similarly, the fact that the detonation products themselves are not air and, equally important from the point of view of numerical modelling, do not have a constant ratio of specific heats (γ), means that the flow through the detonation products (in the region at radial distances less than the contact discontinuity, described below) is not modelled correctly by Air3d.

It is suggested, however, that the dominant features of the flow are common to both the *actual* and *idealised* processes. It is only the magnitudes which differ, not the general shape of the profiles of pressure, density, velocity, and so on. Therefore, it is likely that the qualitative explanation, given below for the *idealised* case, will not differ significantly from the *actual* case.

The explanation is in the form of descriptions of six "snapshots" of the flow, calculated by the one-dimensional, spherically symmetrical implementation of Air3d. The information was taken from an analysis of 1 kg TNT in free air (results to be described in Chapter 4). Because the mass of explosive is 1 kg, the radial distances can be also be interpreted as scaled distances, and the times, as scaled times. This interpretation will be helpful in Chapters 4 to 7.

The six "snapshots" have been carefully selected to give an indication of the major aspects of the flow. The explanation refers to three sets of six graphs, which contain, respectively, pressure and velocity (Figures 3.5 to 3.10), pressure and density (Figures 3.11 to 3.16), and pressure and internal energy (Figures 3.17 to 3.22). Clearly, with the understanding that the internal energy $e = C_v T$ (the specific heat at constant volume multiplied by the temperature), Figures 3.17 to 3.22 can be interpreted as pressure and temperature, with a suitable change of scale.

The choice of the set of pressure-velocity, pressure-density and pressure-internal energy graphs, was one of a number of options. However, because the variation of pressure through a blast wave is a profile with which most investigators are familiar (pressure is the most commonly and readily measured flow variable), pressure is common to each of the figures.

The graphs, Figures 3.5 to 3.22, are intended to provide an illustration of the manner in which blast waves in air are represented by the program Air3d. Therefore, the scales on some of the figures have been altered to facilitate plotting of the information on a common sets of axes. Notably, the density scale has been adjusted (from that of Figures 3.11 to 3.13) in Figures 3.14 to 3.16, and the internal energy scales have been altered (from that of Figures 3.17 to 3.19) in Figures 3.20 to 3.22. These changes are of no consequence; the figures serve only as an illustration of the

variation of the flow variables.

The starting conditions are those described in Section 3.2. The air representing the explosive has the density of condensed TNT, and it has an evenly distributed, constant internal energy due to the chemical energy of the explosive, which is added to the air. The start time is set to the time it takes for the detonation wave to travel from the centre (the detonation point) to the outer surface of the explosive.

From this stationary starting point, a compressive shock wave propagates outwards (in the *increasing* radial direction), while an expansion wave propagates inwards (in the *decreasing* radial direction) through the air representing the explosive. It is effectively a three-dimensional, spherical representation of the shock tube problem described in Appendix B.

- i. At $t = 0.009$ msec, Figure 3.5 shows that the expansion has reached the centre of the air representing the explosive. The outer compression wave is effectively no longer supported as the pressure of the gas behind the wave is reduced by the expansion.

A closer look Figure 3.5 reveals a second compressive shock wave at about 0.28 m. Examination of the velocity profile shows that this second shock is caused by high velocity air streaming into the rear of the first compression wave. Interestingly, the second wave is also travelling *outwards* (to the right in Figure 3.5).

The contact discontinuity (the surface which separates the air representing the explosive and the original atmospheric air), across which the density varies rapidly (discontinuously) but the pressure and velocity are continuous, can be seen in Figure 3.11 at approximately 0.3 m. It is located between the two compressive shocks.

- ii. At $t = 0.043$ msec, almost all of air that represented the explosive and the atmosphere *inside* the outer (first) compression wave is contained *between* the two compressive shocks. This can be seen from Figure 3.12, which shows that the density of the air *inside* (to the left of) the second compressive shock is much reduced. The velocity profile of Figure 3.6 shows that this air is still moving *outwards* into the second compressive shock. Both shocks are still travelling *outwards*.

The contact discontinuity, which has become diffused, can be seen in Figure 3.12 at approximately 0.74 m.

- iii. At $t = 0.076$ msec, Figure 3.13 shows that the density at radial distances *inside* the second shock is virtually zero. Figure 3.7 shows that the second shock is now travelling *inwards* (to the left) engulfing the last remaining air, which is still travelling *outwards*. As the air flowing *outwards* from the centre enters the (inwards running) second shock, the air changes direction and flows *inwards*.

The contact discontinuity can still be seen in Figure 3.13 at about 0.85 m. This is the largest radial distance it attains.

- iv. At $t = 0.203$ msec the inwards running second shock has reached the centre and reflected from it. Figure 3.8 shows that it is now travelling *outwards*, in

the same direction as the first wave. Air from the tail of the first wave is moving *inwards* towards the second shock.

- v. At $t = 0.500$ msec the familiar structure of a blast wave in air is beginning to appear. The pressure and velocity profiles, shown in Figure 3.9, illustrate the two compressive shock waves and the flow of air back and forth. Interestingly, the second wave never catches the first one.

Figure 3.9 also shows one other interesting feature of blast waves in air: there is a time lag between the reduction in pressure below atmospheric and the change in flow direction. This is evident at about 2.7 m, where the pressure first drops below atmospheric. Clearly, at the same location, the flow velocity is still *outwards*.

- vi. Finally, at $t = 1.500$ msec, Figure 3.10 shows that the pressure at the original location of the centre of the charge (radial distance = 0.0 m) has returned to atmospheric. Figure 3.16 shows that the density at the same location is still much reduced, although gradually returning to atmospheric. Figure 3.22 suggests the reason why this air has remained at low density: it still has relatively high internal energy (temperature).

The contact discontinuity can still be identified in Figure 3.16; it is located at about 0.8 m.

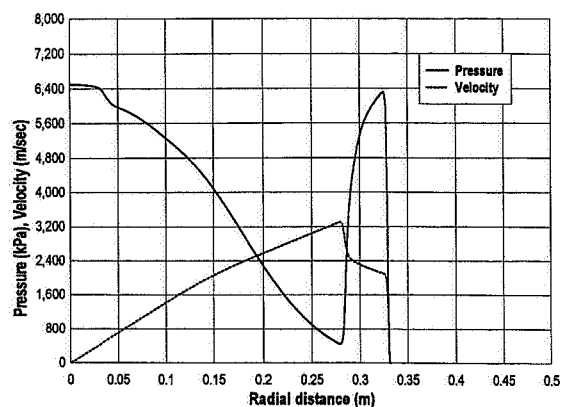


Figure 3.5: Pressure and velocity at $t = 0.009$ msec

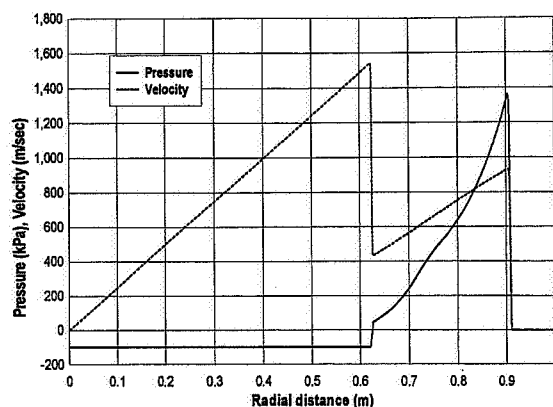


Figure 3.6: Pressure and velocity at $t = 0.043$ msec

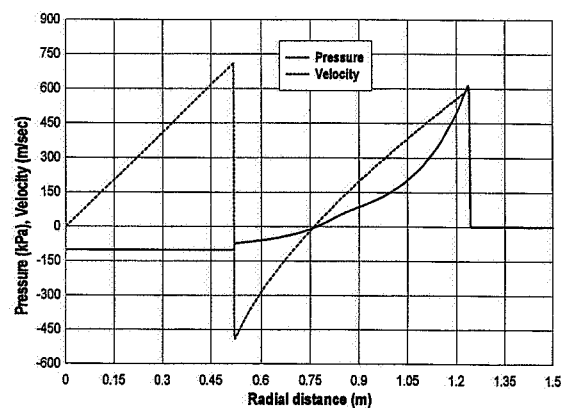


Figure 3.7: Pressure and velocity at $t = 0.076$ msec

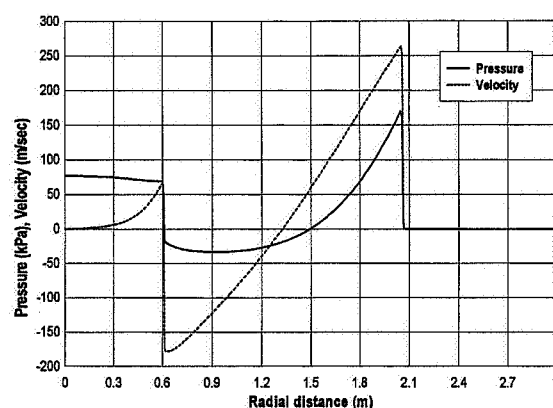


Figure 3.8: Pressure and velocity at $t = 0.203$ msec

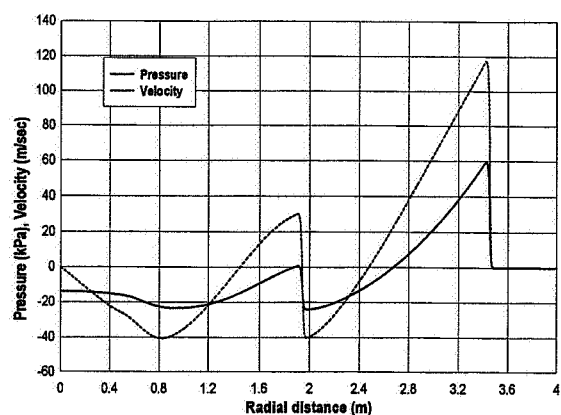


Figure 3.9: Pressure and velocity at $t = 0.500$ msec

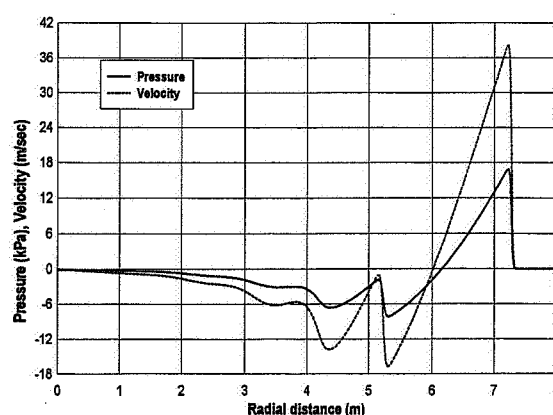


Figure 3.10: Pressure and velocity at $t = 1.500$ msec

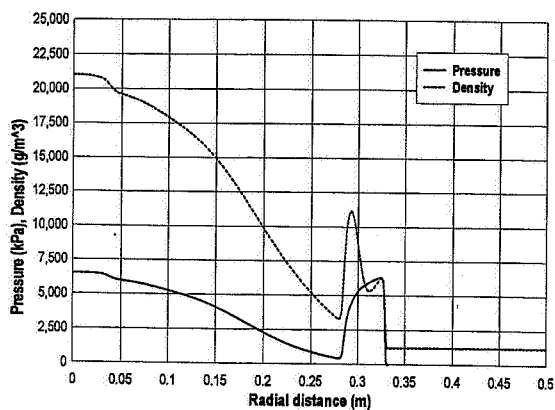


Figure 3.11: Pressure and density at $t = 0.009$ msec

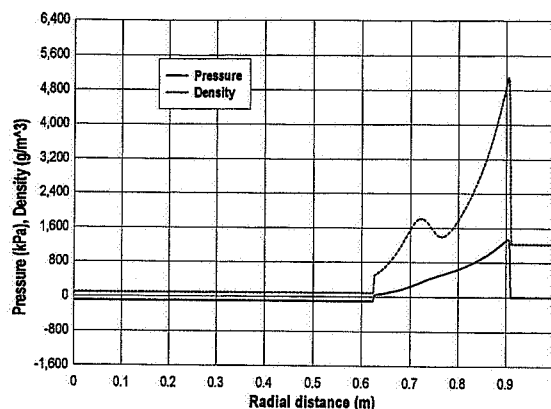


Figure 3.12: Pressure and density at $t = 0.043$ msec

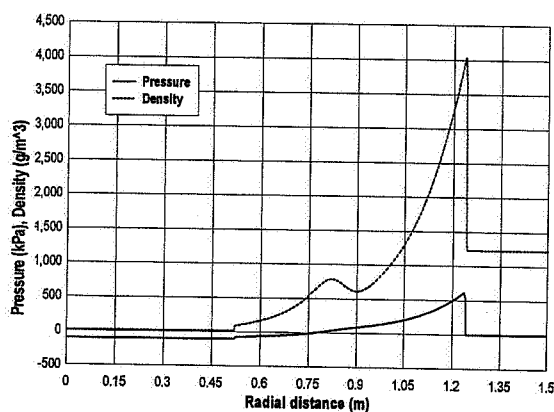


Figure 3.13: Pressure and density at $t = 0.076$ msec

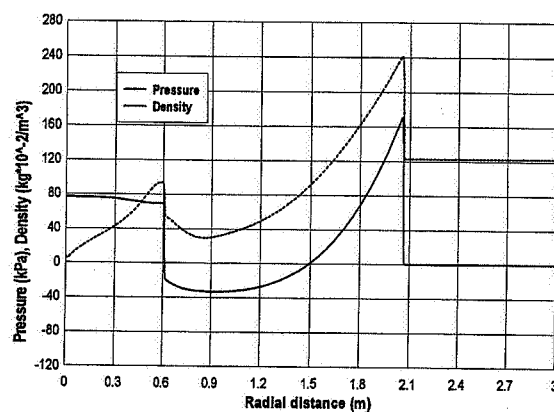


Figure 3.14: Pressure and density at $t = 0.203$ msec

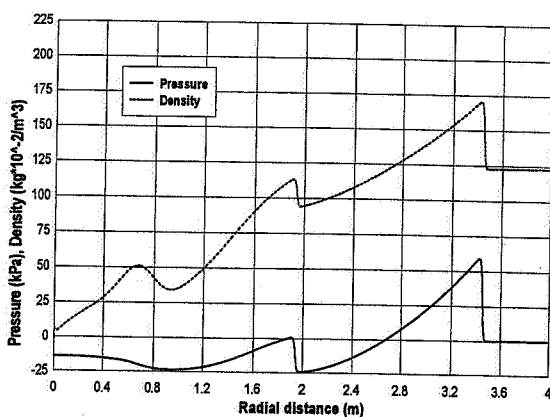


Figure 3.15: Pressure and density at $t = 0.500$ msec

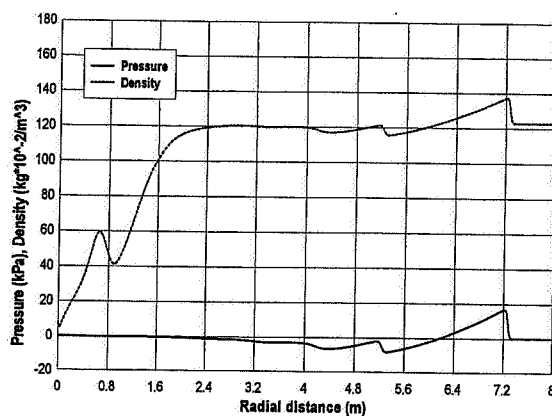


Figure 3.16: Pressure and density at $t = 1.500$ msec

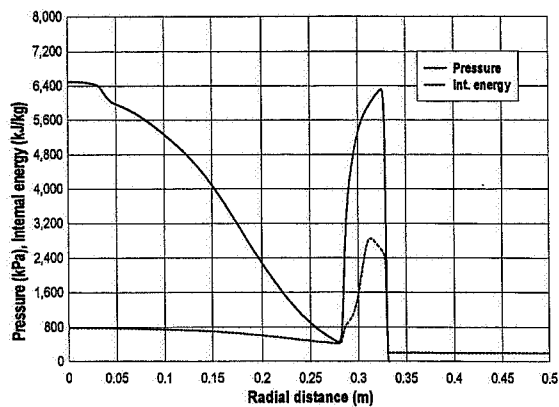


Figure 3.17: Pressure and internal energy at $t = 0.009$ msec

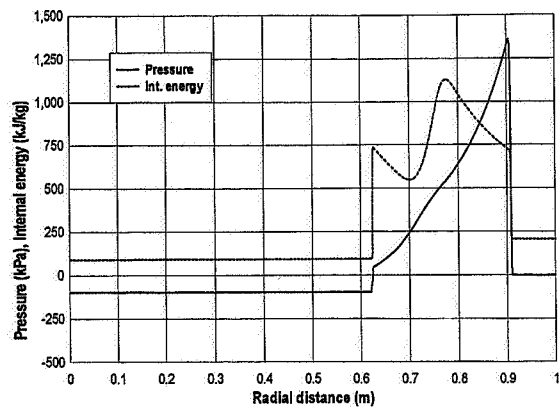


Figure 3.18: Pressure and internal energy at $t = 0.043$ msec

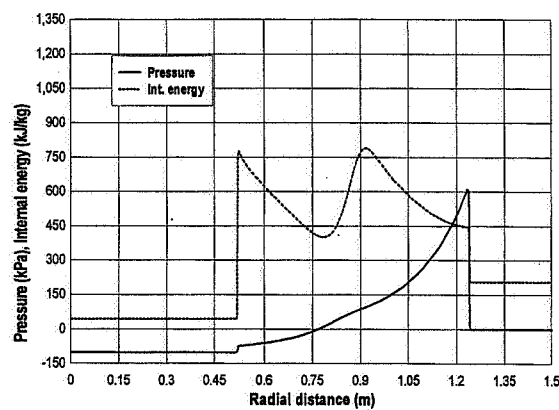


Figure 3.19: Pressure and internal energy at $t = 0.076$ msec

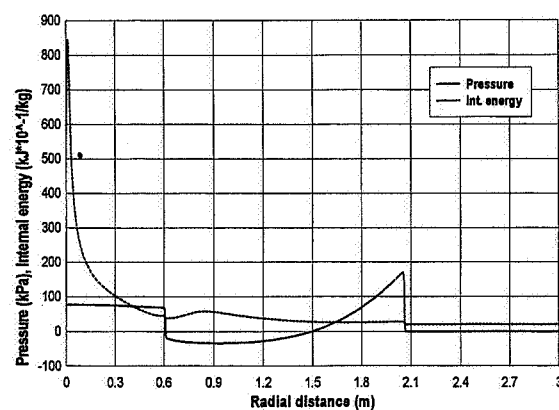


Figure 3.20: Pressure and internal energy at $t = 0.203$ msec

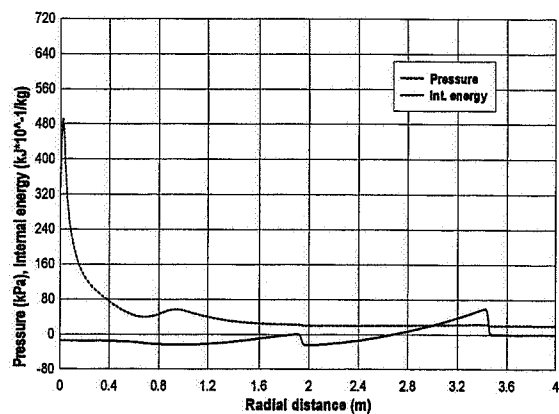


Figure 3.21: Pressure and internal energy at $t = 0.500$ msec

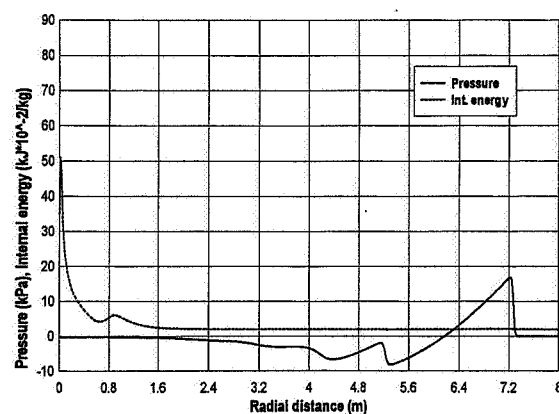


Figure 3.22: Pressure and internal energy at $t = 1.500$ msec

3.6 Complete Worked Example

In this section, a complete worked example is presented. The example is taken from Section 5.2 and is Shot 1 from Table 5.2. The approach taken in the present section, however, is different to that of Section 5.2 in one important respect. In the present example, because the stand-off distance of the charge from the obstacle is relatively small, the part of the problem from the detonation to the time to when the blast wave first impinges on the obstacle is analysed as a two-dimensional height of burst problem. This was not possible in Section 5.2, because many of the stand-off distances used in the experimental program would have resulted in two-dimensional analyses that would have been (computationally) too time-consuming or simply not possible with the limitations of a Personal Computer. Therefore, for the purpose of Section 5.2, it was more important to have consistency of the results, so all of the analyses were treated as hemispherical surface bursts.

The framework for this example problem is a run time of about 24 hours using a fast Personal Computer with at least 32 Mbytes of memory. A step-by-step account of the problem definition is given below with comments interspersed:

- i. The charge used in the experiments was 59.2 g Demex explosive. The TNT equivalence is assumed to be 1.15, (a revised figure from that used in Section 5.2). The charge weight used in the analysis, therefore, is 68.08 g TNT.
- ii. The height of the charge above the ground is 0.1 m. The stand-off distance of the charge from the structure is 0.7 m. A diagram of the configuration is given in Figure 3.23.

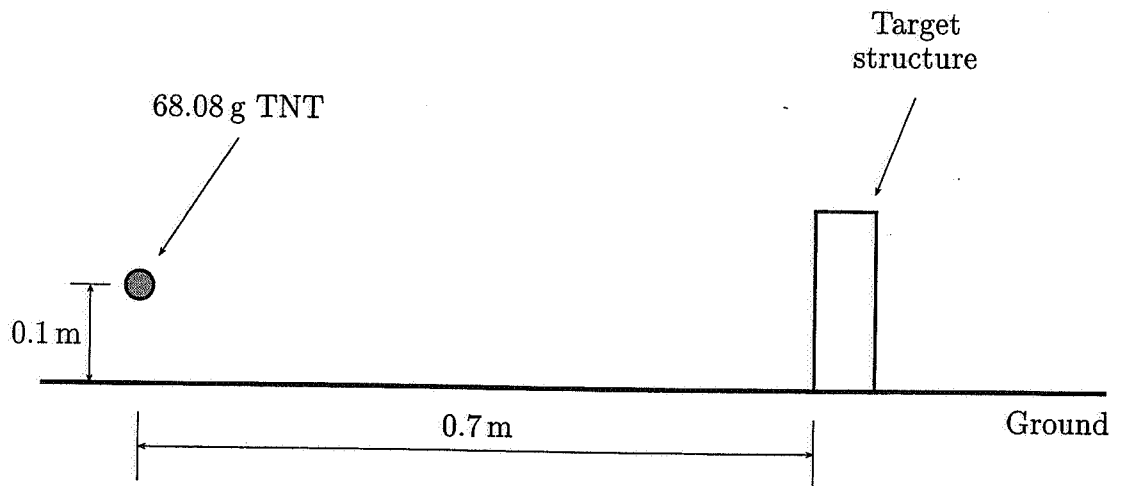


Figure 3.23: Geometry of worked example

- iii. The model test structure and the location of the pressure monitoring points G1 to G3 is shown in Figure 3.24, (which is also in Figure 5.8 of Section 5.2).

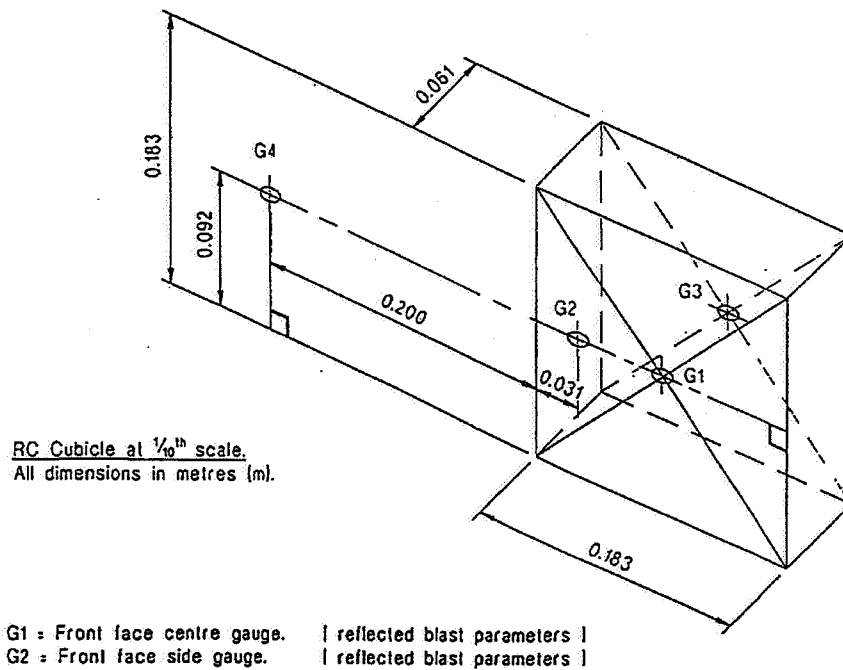


Figure 3.24: Details of the model test structure

- iv. The problem has spherical symmetry between the charge centre and the ground. Ideally, the analysis should have a minimum scaled cell size of approximately $1.0 \times 10^{-3} \text{ m/kg}^{1/3}$. In the present case:

$$\frac{\Delta r}{\sqrt[3]{68.08 \times 10^{-3}}} = 1.0 \times 10^{-3} \text{ m/kg}^{1/3}, \quad \text{giving} \quad \Delta r = 0.408 \text{ mm}.$$

This would necessitate using approximately 250 computational cells. Clearly, this is not at all computationally expensive, and so 500 (0.2 mm) cells are used in the present case.

- v. The finish time is not known intuitively, so a large default value is used (1.0 sec) to ensure that the problem runs to the full radial distance.
- vi. On completion, output is written to the file 0_a1_s01s.TNT. All other input parameters adopt sensible default values. The complete one-dimensional input is shown in Table 3.4, below.
- vii. The next stage of the analysis is the two-dimensional, radially symmetrical part of the problem. The distance of the symmetry axis through the centre of the charge, perpendicular to the ground, to the target structure is 0.7 m. Whenever possible, it is usual in analyses of this type to ensure that the blast wave does not reach the boundaries of the computational domain. In the

Table 3.4: Worked example, one-dimensional input

```

!-----
!
!      C L E A R I N G   A 1 ---  SHOT 1, W = 59.2g PE4, r = 0.7m
!
SPHERICAL_INPUT-----
!
68.08E-3  0.0 0.0  Charge weight      (kg) 59.2g * 1.15
0.1                problem radius    (m)
2.0E-4          delta r              (m)
0                run option
1.0                problem time      (sec)
0.0      50        display increment (sec)
0                Equation of state option
1                Plot variable
0.75             CFL
F                File flag (update existing TNT file)
0_a1_s01s.TNT     Output file
0 1 NOFORMAT

```

present case, it was found (by trial runs using a large cell size) that, when the blast wave reaches the structure, at 0.7 m, it is also just short of 0.6 m vertically above the ground. Therefore, the two-dimensional domain is 0.7 m×0.6 m.

viii. The maximum number of elements available with 32 Mbytes of memory is:

$$\frac{32 \times (2)^{20}}{36} \approx 932,067 \text{ computational cells,}$$

The cell size Δr ($= \Delta y$) is calculated from:

$$\sqrt{\frac{0.7 \times 0.6}{9.32 \times 10^5}} = 6.713 \times 10^{-4} \text{ m.}$$

Therefore, a cell size of 1 mm (equating to 420,000 computational cells) is a good compromise figure. It should be noted that this is approximately 2.5 times the desired figure (0.408 mm) calculated earlier, and so it is to be expected that some loss of resolution—particularly with regard to peak pressure—will occur.

ix. The scaled time to switch to the second-order formulation is approximately $1.2 \times 10^{-3} \text{ sec/kg}^{1/3}$. In the present example this is given by:

$$t_s = (1.2 \times 10^{-3}) \times \sqrt[3]{68.06 \times 10^{-3}} = 4.9 \times 10^{-4} \text{ seconds.}$$

- x. Once again, the finish time is set to an arbitrarily large value (1.0 sec) to ensure that the problem runs until the boundary of the domain (at the surface of the target structure) is reached. This is ensured by the two boundary conditions BXU and BYU, which are both set to -1.0. On completion, output is written to the file 0_a1_s01r.TNT.
- xi. All other input parameters adopt sensible default values. The complete input is given in Table 3.5.

Table 3.5: Worked example, two-dimensional input

```

RADIAL_INPUT-----
!
  0.0  0.7      XS,XF
  0.0  0.6      YS,YF
1.0E-3      Cell size
1 1          Display variables
4.9E-4      Time to switch to 2nd order
2           Run option,
1.0         Finish time,
0.0  25      Display/Save increment,
5.0E-1      Safety factor for timestep
1.0         Charge weight (kg)
0.100       Centre of gas Y
-1          BXU upper x boundary,
-1          BYU upper y boundary,
T           File flag (update existing TNT file)
0_a1_s01r.TNT Output file
3 .0001 .0001 IXY (plotting), level
50.0  0.0     Scale factor for contour intervals
0 1 NOFORMAT

```

The final, three-dimensional part of the analysis demands rather more discussion than the previous two parts. The problem has one plane of symmetry through the centreline of the charge and the structure, so only half the problem needs to be analysed. The next problem to be addressed is what should be the extents of the computational domain?

If, for example, we choose the extents and boundary conditions of Table 3.6, with the vertical axis through the charge at the origin of the domain, would this be reasonable?

If the peak pressure and the peak specific impulse on the front face of the structure were all that was required, a relatively small domain would suffice: the blast would not have time to reach the boundaries of the domain before the end of the positive phase duration at location G1 was reached. In this case, the domain defined in Table 3.6 might well suffice.

If, however, a more thorough investigation were required (for comparison with experiment, for example, as in Section 5.2), there would be a need to run the analysis

Table 3.6: Example three-dimensional domain and boundary conditions

Direction	Extents (m)	Lower boundary	Upper boundary
x	0.0 – 0.8	reflect	transmit
y	0.0 – 0.3	reflect	transmit
z	0.0 – 0.2	reflect	transmit

for a much longer period (of problem time). Such an analysis must include the first shock at locations G1 and G2 (at the front), and G3 (at the rear of the structure), as well as the second smaller shock at the front face. This is the case Section 5.2 and is also the approach adopted here.

Ideally, the boundaries of the domain should be sufficiently far from the structure so that the blast wave has not reached them at the problem finish time. This is hardly ever achievable in practice, and the usual compromise is to set transmissive boundaries sufficiently far from the structure so that the fact that the blast has passed (or is passing) out of the domain does not propagate far enough back into the domain to affect the region near the monitoring positions in the *region of interest* near the structure.

Locating the appropriate place for the boundaries is achieved by trial and error: running the program with a coarse grid and shrinking the boundaries towards the structure until the effect of their presence becomes evident.

There is one further important aspect which is particularly appropriate in the present example: the type and location of the lower x boundary. In Table 3.6 the lower x boundary is located at the vertical axis of the charge, and it is set to *reflect*. The difficulty arises because, with a charge weight of 68.08 g and a stand-off distance of 0.7 m, the scaled distance to the structure is only $1.71 \text{ m/kg}^{1/3}$. The start of the formation of the second shock (from a spherical explosion) does not occur until the first shock has reached a scaled distance of about $1.6 \text{ m/kg}^{1/3}$. Clearly, in the present case, a reflecting boundary at the origin defines a plane of symmetry in the problem which *does not exist*. Similarly, changing the boundary to *transmissive* would be even less appropriate, because then the flow would transmit out of the domain and the second shock would not occur.

The correct solution is to set the lower x boundary of the domain at an appropriate distance beyond the axis of the charge so that the second shock forms correctly from the asymmetrical problem, where one half of the domain expands into free air, and the other half interacts with the structure.

Thankfully, this type of difficulty only occurs at relatively short scaled distances, but the fact that it is not obvious and is unlikely to be recognised intuitively make this type of problem particularly difficult to analyse.

In the present example, it has been found that the extents and boundary conditions given in Table 3.7 are appropriate.

It will be noticed from Table 3.7 that the x boundaries extend much further beyond the region bounded by the blast wave (immediately after remapping) and the structure than the other two directions. This is because, with regard to the

Table 3.7: Appropriate three-dimensional domain and boundary conditions

Direction	Extents (m)	Lower boundary	Upper boundary
x	-1.0 – 1.5	transmit	transmit
y	0.0 – 0.6	reflect	transmit
z	0.0 – 0.5	reflect	transmit

pressure monitoring points G1 and G3, the x direction is the *principal convection direction*. In the region of space above (in the y direction) and beyond (in the z direction) the plane of the structure, the blast wave propagates *principally* in the x direction. As long as there is sufficient space surrounding the structure for the blast wave to diffract around it and the effect of the proximity of the boundaries does not propagate back to the structure (as described above), the location of the boundaries in the other two directions will be adequate.

In the x direction (the principal direction), however, the effect of the proximity of the boundary will be more pronounced. Firstly, it is essential that the boundaries are set sufficiently far from the charge centre that the second shock can form. In the present case, this is at least 1.0 m from the charge. Secondly, the upper boundary must not let too much material (air) *leak out* of the domain before the whole of the period of interest has finished. This problem arises because when the first shock wave has reached every boundary of the domain the material within the domain is no longer physically connected to the reference state (atmospheric conditions). This can lead to unpredictable behaviour at the boundaries: most notably *over-emptying*, which leaves the final state in the domain reduced in density and pressure compared with atmospheric conditions.

xii. The maximum number of elements available with 32 Mbytes of memory is:

$$\frac{32 \times (2)^{20}}{41} \approx 818,400 \text{ computational cells.}$$

The cell size Δx ($= \Delta y = \Delta z$) is calculated from the size of the problem domain and the available memory:

$$\sqrt[3]{\frac{2.5 \times 0.6 \times 0.5}{8.18 \times 10^5}} = 9.71 \times 10^{-3} \text{ m,}$$

Therefore, a cell size $\Delta x = 10 \text{ mm}$ gives a total problem size of 750,000 cubic cells.

As an aside, it will be noticed that, in the progression through one- and two- to three-dimensional analysis, the cell size has increased from 0.2 mm to 1.0 mm to 10 mm, respectively. This can be compared with the *desired* cell size 0.4 mm, discussed earlier. It will also be recognised that 0.4 mm is only really an effective minimum at short scaled range. As larger scaled distances from the centre of the explosion are reached, the scaled duration of the blast wave increases and the scaled cell

size required to preserve reasonably well-defined shock waves increases accordingly. The present study has not treated this aspect of modelling thoroughly, because with modest computational resources the likelihood of wasting effort by *over-discretising* a problem: using a cell size that is too small, is remote.

As a rough guide, problems should be set up initially using a discretisation that allows accurate description of the problem geometry and captures the major aspects of the flowfield: the correct number and duration of shock waves. This initial set-up can be used to assess the location and type of boundary conditions (described above), the number and location of pressure monitoring points and the total problem time. As long as the geometry is not too complicated, and demands a small cell size, this part of the problem can usually be conducted with only a modest amount of computer time.

When all these aspects have been established, there are essentially two options available to the user. The first, and most usual, is to reduce the cell size of the problem until all the memory of the computer is used. The second is to tailor the problem size to the resources, so that the *actual* problem time is restricted to some sensible duration: overnight, 24 or 48 hours, for example.

- xiii. The time at which to switch to the second-order formulation: 4.9×10^{-4} sec, is the same as calculated previously. It is worth noting that, this time is already exceeded at the start of the three-dimensional analysis.
- xiv. The problem finish time: 2.5×10^{-3} sec, was established from initial trials and is sufficient to capture the first shock on the front and rear of the structure and the second shock on the front.
- xv. A single FALSE obstacle definition describes *half* the structure in Figure 3.24. It should be borne in mind that, in the computational domain, if the cell centre is located inside the box defined by the obstacle, the cell is set as *unused*. In the present case, with 10 mm cells, there will be a slight mismatch between the obstacle definition in the input file and the actual definition in the problem domain. This is summarised in Table 3.8, where it can be seen that the actual structure size differs from the input by a few millimetres.

Table 3.8: Obstacle definition

Direction	Input (m)		Actual (m)		Difference (mm)
	Minimum	Maximum	Minimum	Maximum	
<i>x</i>	0.7	0.7610	0.7	0.76	-1.0
<i>y</i>	0.0	0.1830	0.0	0.18	-3.0
<i>z</i>	0.0	0.0915	0.0	0.90	-1.5

In some situations: where the obstacle boundary coincides with a cell centre, there is ambiguity whether or not the cell is *used*; it depends on the representation of the REAL number in memory. In this situation, it is usual to remove the ambiguity by adding or subtracting a small quantity (0.1 mm, say) from the obstacle dimension, to force the program to behave in the manner

desired. The consequence of this may be that an obstacle boundary might be a displaced by up to half a cell thickness from where it is intended. Clearly, this could be extremely important, especially when using coarse grids, and it should be borne in mind.

- xvi. A similar problem to that just described for the obstacle definition occurs in the definition of pressure monitoring points. In the experimental geometry (Figure 3.24), the pressure gauges were located flush with the surface of the structure: at 0.7 m stand-off in the front and 0.761 m at the rear. In the computational domain, 0.7 m is at a used/unused interface, and it is not obvious which cell will be monitored for pressure. Therefore, it is usual to adjust the location of the monitoring point to ensure that it is located in the correct cell. In the present case, 0.7 m is adjusted to 0.6999 m; 0.761 m will already be correctly positioned, as it refers to the *used* cell from 0.76–0.77 m. A similar procedure is applied to the other two coordinate directions.

The whole of the three-dimensional input for the worked example is given in Table 3.9. Although they are shown as distinct in the present example, in practice the three parts of the input are contained in a single formatted text file. Pressure histories from monitoring locations G1 and G3 are shown in Figures 3.25 and 3.26.

Finally, there is one aspect of the program *Air3d* which can cause problems to the unwary user. It is with regard to the remapping procedure: most notably from two to three dimensions. If all the available memory is used for a three-dimensional analysis, no memory will be available, temporarily, to store the remap data at the start of the problem. This can be overcome by using a page file 50% larger than the physical memory size, (which is possible using Windows NT). This is the maximum that would ever be needed because only one set of conserved variables is stored for remapping, requiring half the memory.

Ideally, this situation should be avoided, as it would have a detrimental effect on the time needed to remap at the start of the problem, but it would allow all of the memory to be used for the three-dimensional analysis.

Table 3.9: Worked example, three-dimensional input

```

MAIN_INPUT-----
!
-1.0 1.5      XS,XF
 0.0 0.6      YS,YF
 0.0 0.5      ZS,ZF
10.0E-3      Cell size (if zero, use IX,IY,IZ, above)
1 1          Display variables DO LOOP,
7           Method of analysis,
4.9E-4      Time to switch to 2nd order
1           Flux limiter (method 4 only)
1.0         Beta, general flux limiter (1.0 to 2.0)
2           Run option,
0           Use MVSTORE, 0-no, 1-yes.
1.6E-3      Finish time,
0.0 5       Display/Save increment,
5.0E-1      Safety factor for timestep,
-1.0        Charge weight (kg)
RADIAL
0.0          Gas pressure (Pa) or VOLEXP if using CW
0.0          Gas temperature,      0.0 if using CW
0.0          Gas radius,           0.0 if using CW
0.000        Centre of gas X,
0.000        Centre of gas Y,
0.000        Centre of gas Z,
+1           BXL lower x boundary,
+1           BXU upper x boundary,
-1           BYL lower y boundary,
+1           BYU upper y boundary,
-1           BZL lower z boundary,
+1           BZU upper z boundary,
3 .001 .001 .001 IXYZ (plotting), level
5.0 0.0      Scale factor for contour intervals
1           Number of obstacles
FALSE
0.700 0.761   X --- from - to
0.000 0.183   Y --- from - to
0.000 0.0915  Z --- from - to
3 1 REFORMAT
0.6999 0.0915 0.0000
0.6999 0.0915 0.0605
0.7611 0.0915 0.0000

```

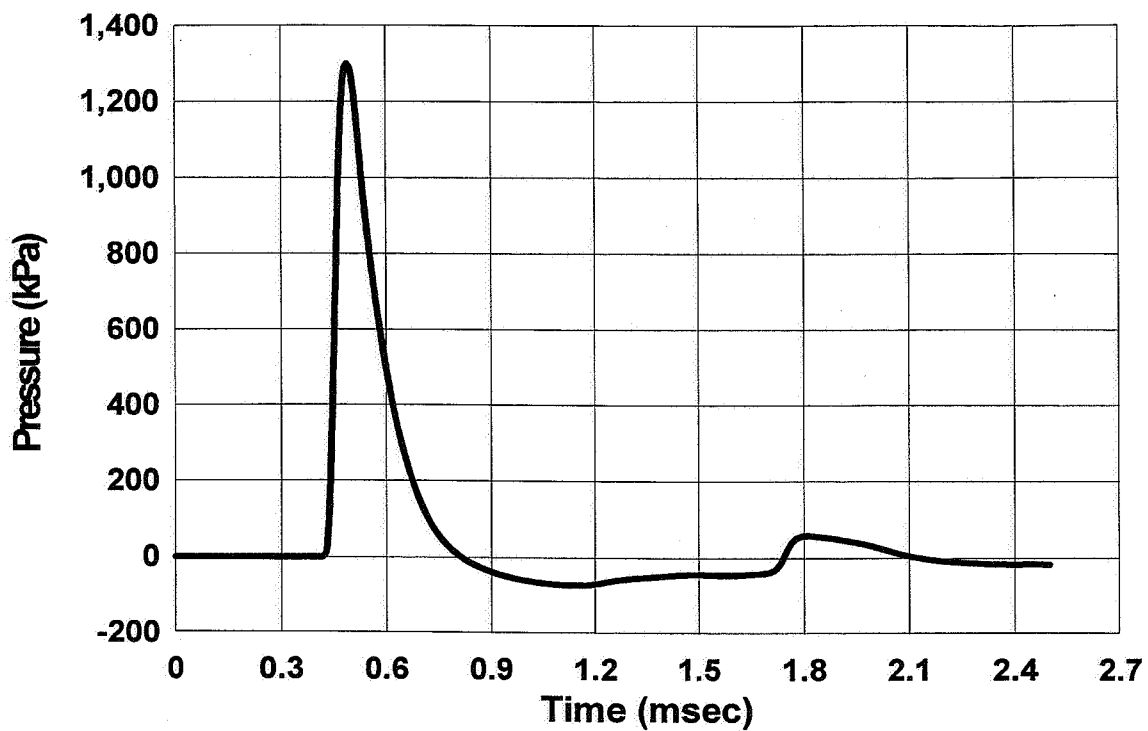


Figure 3.25: Pressure history from worked example, location G1

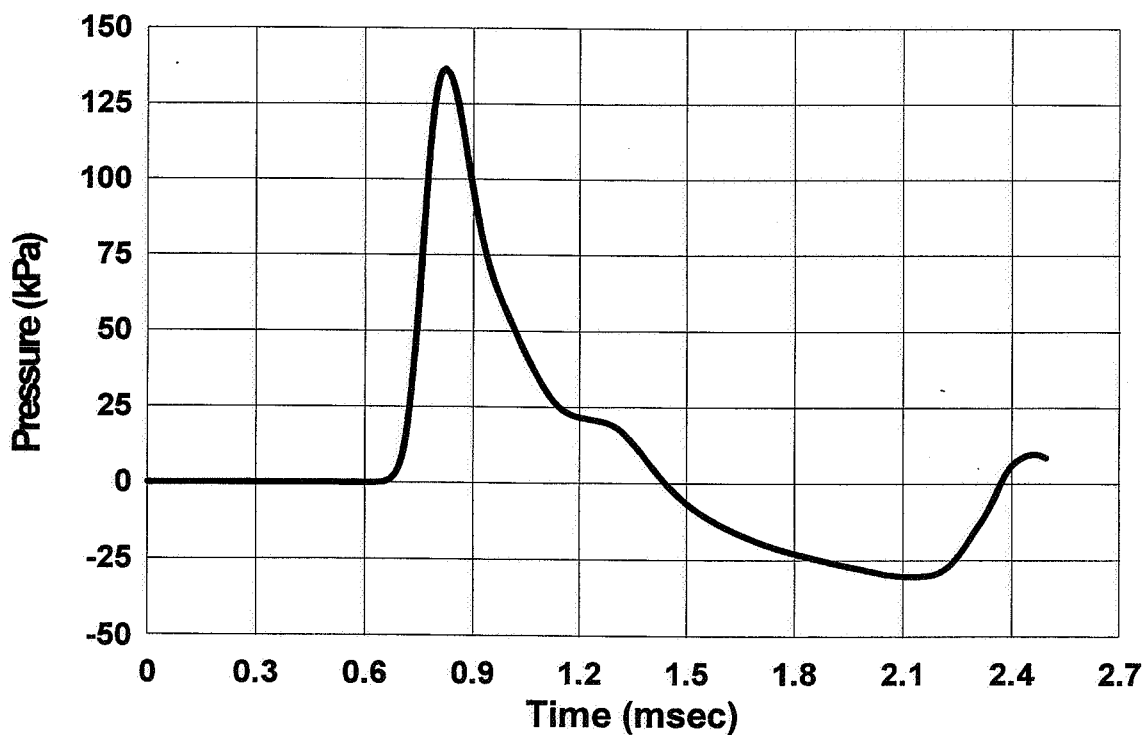


Figure 3.26: Pressure history from worked example, location G3

3.7 Comparison of Air3d with AUTODYN-3D

The program Air3d was used exclusively in this study. Development of this tool, however, was not the only option. Commercially available programs which perform the same, or similar, function are also available.

One such program, AUTODYN-3D [3], is described as an interactive nonlinear dynamic analysis program and is able to treat a variety of high velocity impact problems, as well as blast simulation. AUTODYN-3D provides an ideal opportunity to compare the method of solution used in the present study with that of other investigators.

There are a few important differences between the method used by AUTODYN-3D and Air3d. AUTODYN-3D uses a separate two-dimensional implementation (AUTODYN-2D) for the one- and two-dimensional parts of the analysis. There is a remapping procedure very similar to the one used by Air3d to move the data produced by one implementation into the other.

The one-dimensional analysis is achieved in two dimensions by using a wedge-shaped grid and axial symmetry. The divergence of the wedge-shaped grid provides the spherical expansion and removes the need for different sources terms in the one- and two-dimensional analyses. The apex of the wedge is *filled* with TNT, to the correct radius, to model the required charge weight, and the problem is run until the blast wave reaches the outermost cell of the domain.

In AUTODYN-2D a first-order accurate flow solver capable of treating multi-materials can be used. This allows use of the Jones, Wilkins and Lee (JWL) equation of state (Reference [31]) to model the detonation products, and it should provide a good representation of the blast wave at short scaled distances.

Two-dimensional axially symmetrical analysis is performed by AUTODYN-2D by remapping the one-dimensional output into the two-dimensional grid. At this stage, the two materials: air and TNT, are still treated separately, but they are both modelled as ideal gas. The solution method for the three-dimensional analyses is the Flux-Corrected Transport (FCT) algorithm of Boris and Book [9], and the whole of the computational domain is modelled as air.

In what follows, solutions from AUTODYN-3D and Air3d of one of the validation examples of Chapter 6 (the dead-end) are presented and compared. The spatial discretisation of each part of the two analyses was exactly the same in both programs.

The analyses described below are slightly different to those of Chapter 6, because there was a limit of one million cells in the implementation of AUTODYN-3D available to the author. The problem was discretised in the following way:

- One-dimensional analysis from the charge centre to the ground, radius = 25 mm, $W = 13.0$ g and grid size = 0.166 mm.
- Two-dimensional analysis across the width of the street, radius = 0.15 m, height = 0.17 m and grid size = 1.0 mm square cells.
- Three-dimensional analysis for the remainder of the region of interest, grid size = 10 mm cubic cells.

There were 10 pressure monitoring positions (target points) in each of the analyses. The locations of these points can be seen in Figure 6.2 on page 165.

Input to AUTODYN-3D is interactive; the various steps needed set up and run the three parts of the problem are given in Tables 3.11 to 3.13, below. The input file for Air3d is given in Tables 3.14 and 3.15, and the ten pressure histories are compared in Figures 3.27 to 3.36. It will be noticed from Figures 3.27 to 3.36 that the results of the two simulations are remarkably similar. Reference to Figures 6.18 to 6.27 of Chapter 6 (where the results of Air3d are compared with experiment) show that the main features of the blast have been accurately reproduced by both programs.

The CPU time and memory requirement of each of the programs, for the above problem, are given in Table 3.10. This demonstrates that the method of solution implemented in Air3d is approximately eight times faster than that of AUTODYN-3D. Similarly, Air3d requires approximately one eighth of the memory for the same spatial discretisation.

Table 3.10: CPU time and memory requirement of Air3d and AUTODYN-3D

	CPU time (minutes)	Memory use (kbytes)
Air3d	50	10920
AUTODYN-3D	410	89268

The above statistics are encouraging, as they indicate that the approach adopted is, indeed, efficacious, and the decision to produce a dedicated blast simulation tool as part of this study has been justified. It should be mentioned, however, that the version of AUTODYN-3D used for comparison (version 3.1.19) has been superseded by version 3.2.04, which is approximately two and a half times faster than the previous version. It is still, however, about three times slower than Air3d.

A further consideration, mentioned previously, is the fact that AUTODYN-3D is a multi-purpose dynamic code capable of treating a variety of impact and flow problems. In order to ensure compatibility with other components of the program, it calculates and stores many more variables than are strictly necessary for blast wave simulations. The comparison of Air3d with AUTODYN-3D for blast simulation, although favourable on first inspection, should be considered in the context of the increased functionality of the AUTODYN-3D code.

Table 3.11: Input steps for 1-D spherical analysis, 13 g TNT, 25 mm HOB

1. **Create**, $\langle \text{ident:GW13GS} \rangle$, $\langle \text{heading:HOB 25MM 13G TNT SPHERICAL} \rangle$.
2. **Global**, **Material**, **Add/Mod**: $\langle \text{name:AIR} \rangle$,
 $\langle \text{equation of state:ideal gas} \rangle$, $\langle \text{gamma:1.4} \rangle$, $\langle \text{ref density:1.23E-3} \rangle$,
 $\langle \text{ref temperature:288} \rangle$, $\langle \text{Specific heat (C.V.):715} \rangle$.
3. **Library**, $\langle \text{library name:Explos} \rangle$, **Retrieve**, $\langle \text{TNT:yes} \rangle$.
4. **Subgrid**, $\langle \text{name:HOB} \rangle$, $\langle \text{processor:Euler} \rangle$, $\langle \text{imax:151} \rangle$, $\langle \text{jmax:2} \rangle$.
5. **Zoning**, **Generate**, **Predef**, **Wedge**, $\langle \text{rmin:0.5} \rangle$, $\langle \text{rmax:25} \rangle$.
6. **Fill**, **Block**, $\langle \text{material:AIR} \rangle$, $\langle \text{initial internal energy:2.0592E5} \rangle$.
7. **Ellipse**, $\langle \text{x-center:0} \rangle$, $\langle \text{y-center:0} \rangle$, $\langle \text{x semi-axis:12.395} \rangle$,
 $\langle \text{y semi-axis:12.395} \rangle$, $\langle \text{Material name:TNT} \rangle$.
8. **Global**, **Options**, **Explode**, **Node**, $\langle \text{x coord:0} \rangle$, $\langle \text{y coord:0} \rangle$.
9. **Wrapup**, $\langle \text{cycle:999999} \rangle$, $\langle \text{problem time limit:4.5E-3} \rangle$,
 $\langle \text{max energy error:0.5} \rangle$.
10. **Save**
11. **Execute** (run to wrapup time)
12. **Modify**, **Subgrid**, **Fill**, **Datafile**, $\langle \text{ident:GW13GS} \rangle$,
 $\langle \text{read/write:write} \rangle$.

Table 3.12: Input steps for 2-D radial analysis, 13 g TNT, 25 mm HOB

1. **Create**, $\langle \text{ident:GW13GR} \rangle$, $\langle \text{heading:GREG WHALEN 13G TNT 25MM HOB} \rangle$.
2. **Global**, **Material**, **Add/Mod**: $\langle \text{name:AIR} \rangle$,
 $\langle \text{equation of state:ideal gas} \rangle$, $\langle \text{gamma:1.4} \rangle$, $\langle \text{ref density:1.23E-3} \rangle$,
 $\langle \text{ref temperature:288} \rangle$, $\langle \text{Specific heat (C.V.):715} \rangle$.
3. **Library**, $\langle \text{library name:Explos} \rangle$, **Retrieve**, $\langle \text{TNT:yes} \rangle$.
4. **Add/Mod**: $\langle \text{name:TNT} \rangle$, $\langle \text{equation of state:ideal gas} \rangle$,
 $\langle \text{ref density:1.0E-4} \rangle$,
5. **Subgrid**, $\langle \text{name:RADIAL} \rangle$, $\langle \text{processor:Euler} \rangle$, $\langle \text{imax:171} \rangle$,
 $\langle \text{jmax:151} \rangle$.
6. **Zoning**, **Generate**, **Predefs**, **Box**, $\langle \text{xmin:0} \rangle$, $\langle \text{xmax:170} \rangle$,
 $\langle \text{ymin:0} \rangle$, $\langle \text{ymax:150} \rangle$.
7. **Fill**, **Block**, $\langle \text{material:AIR} \rangle$, $\langle \text{initial internal energy:2.0592E5} \rangle$.
8. **Datafile**, $\langle \text{ident:GW13GS} \rangle$, $\langle \text{read/write:read} \rangle$, $\langle \text{x origin:25} \rangle$,
 $\langle \text{y origin:0} \rangle$.
9. **Modify**, **Global**, **Timestep**, $\langle \text{start time:4.5E-3} \rangle$
 $(\text{start time from end of 1-D calculation})$
10. **Wrapup**, $\langle \text{cycle:999999} \rangle$, $\langle \text{problem time limit:4.4E-2} \rangle$,
 $\langle \text{max energy error:0.5} \rangle$
11. **Save**
12. **Execute** $(\text{run to wrapup time})$
13. **Modify**, **Subgrid**, **Fill**, **Datafile**, $\langle \text{ident:GW13GR} \rangle$,
 $\langle \text{read/write:write} \rangle$

Table 3.13: Input steps for 3-D dead end analysis

1. **Create**, **<ident:GW10DE>**,
<heading:GREG WHALEN DEAD END 10MM GRIDSIZE>.
2. **Global**, **Material**, **Add/Mod**: **<name:AIR>**,
<equation of state:ideal gas>, **<gamma:1.4>**, **<ref density:1.23E-3>**,
<ref temperature:288>, **<Specific heat (C.V.):715>**.
3. **Boundary**, **Add/Mod**, **<boundary condition:FLOW-OUT>**,
<boundary type:flow out>, **<flow material:AIR>**.
4. **Subgrid**, **<name:DEADEND>**, **<processor:Euler-FCT>**, **<imax:101>**,
<jmax:16>, **<kmax:116>**.
5. **Zoning**, **Box**, **<xmin:0>**, **<xmax:1000>**, **<ymin:0>**, **<ymin:150>**,
<zmin:-150>, **<zmax:1150>**.
6. **Fill**, **Block**, **<material:AIR>**, **<initial internal energy:2.0592E5>**.
7. **Datafile**, **<ident:GW13GR>**, **<read/write:read>**.
8. **Boundary**, **I-plane**, **<i index:101>**, **<from j:1>**, **<to j:16>**,
<from k:1>, **<t k:116>**, **K-plane**, **<k index:116>**, **<from i:1>**,
<to i:101>, **<from j:1>**, **<to j:16>**
9. **Edit**, **Targets**, **Add**, **Point**, **X Y Z**, **<x coordinate:25>**,
<y coordinate:149>, **<z coordinate:250>**.
(repeat for other nine target points)
10. **Modify**, **Global**, **Timestep**, **<start time:4.4E-2>**
(start time from end of 2-D calculation).
11. **Wrapup**, **<cycle:999999>**, **<problem time limit:2>**,
<max energy error:0.5>
12. **Save**
13. **Execute**

Table 3.14: Input file for dead-end analysis, Air3d

```

!-----
!
!           Greg Whalen's dead-end analysis
!
SPHERICAL_INPUT-----
!
13.0E-3  0.0  0.0  Charge weight      (kg)
25.0E-3                problem radius  (m)
1.66E-4                delta r          (m)
0                      Run option
1.0                    problem time      (sec)
0.0      25            display increment (sec)
0                      Equation of state option
1                      Plot variable
0.75                  CFL
F                      File flag (update existing TNT file)
0_gw_s.TNT            Output file
!
0 1 NOFORMAT
!
RADIAL_INPUT-----
!
    0.0   0.150      XS,XF
    0.0   0.170      YS,YF
1.0E-3          Cell size
1 1             Display variables
2.0E-4          Time to switch to 2nd order
2              Run option,
1.0E-0          Finish time,
0.0      25      Display/Save increment,
5.0E-1          Safety factor for timestep,
1.0            Charge weight (kg)
25.0E-3         Centre of gas Y
-1             BXU upper x boundary,
-1             BYU upper y boundary,
F              File flag (update existing TNT file)
0_gw_de_r.TNT   Output file
3 .001 .001     IXY (plotting), level
100.0  0.0      Scale factor for contour intervals
0 1 NOFORMAT

```

Table 3.15: Input file for dead-end analysis (continued), Air3d

```

MAIN_INPUT-----
-0.15  1.0    XS,XF
  0.0   1.0    YS,YF
  0.0   0.15   ZS,ZF
10.0E-3      Cell size
1 1          Display variables DO LOOP,
6            Method of analysis,
2.85E-4      Time to switch to 2nd order
1            Flux limiter (method 4 only)
1.0          Beta, general flux limiter (1.0 to 2.0)
2            Run option,
0            Use MVSTORE, 0-no, 1-yes.
1.98E-3      Finish time,
0.0   25     Display/Save increment,
5.5E-1       Safety factor for timestep,
-1.0         Charge weight (kg)
RADIAL       TNT input file type
0.0          Gas pressure (Pa) or VOLEXP if using CW
0.0          Gas temperature,          0.0 if using CW
0.0          Gas radius,                0.0 if using CW
0.0          Centre of gas X,
0.0          Centre of gas Y,
0.0          Centre of gas Z,
-1           BXL lower x boundary,
+1           BXU upper x boundary,
-1           BYL lower y boundary,
+1           BYU upper y boundary,
-1           BZL lower z boundary,
-1           BZU upper z boundary,
3 .001 .001 .149 IXYZ (plotting), level
20.0 0.0     Scale factor for contour intervals.
0            Number of obstacles
12 1 REFORMAT Number of target points
0.250 0.025 0.149 H1
0.398 0.025 0.149 H2
0.563 0.025 0.149 H3
0.750 0.025 0.149 H4
0.250 0.025 0.149 V1
0.250 0.173 0.149 V2
0.250 0.338 0.149 V3
0.250 0.525 0.149 V4
0.250 0.025 0.149 D1
0.355 0.130 0.149 D2
0.471 0.246 0.149 D3
0.604 0.379 0.149 D4

```

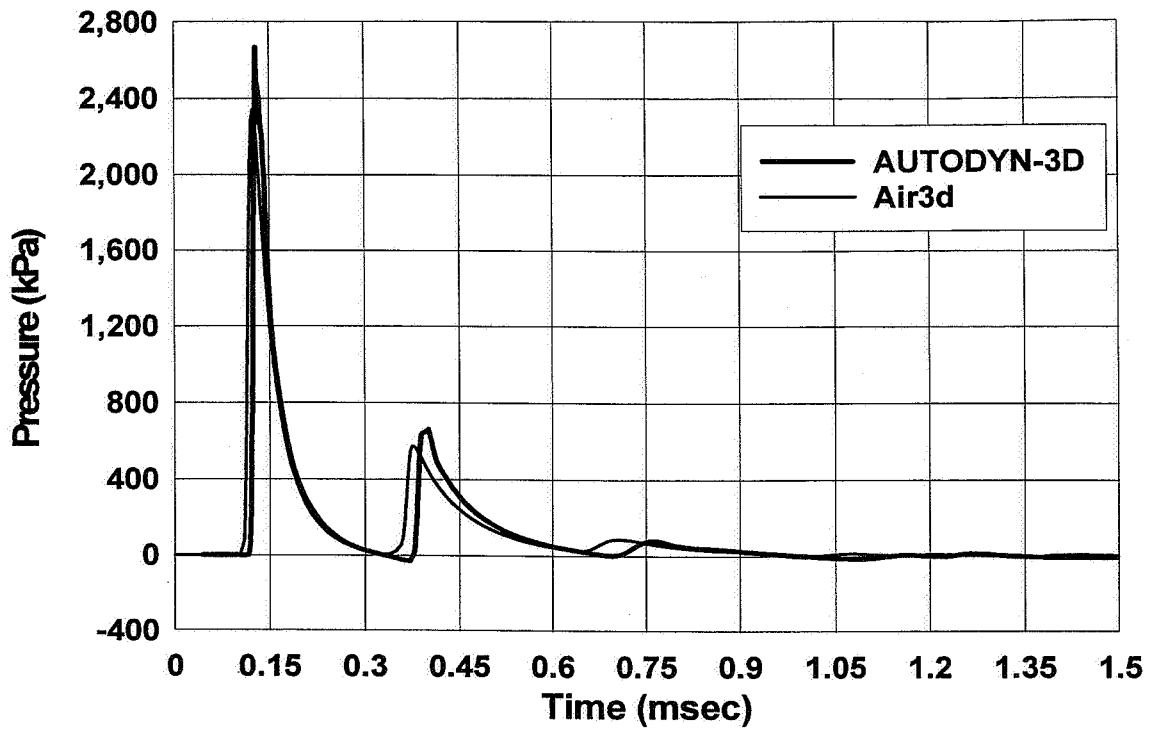


Figure 3.27: Comparison of pressure histories, location H1

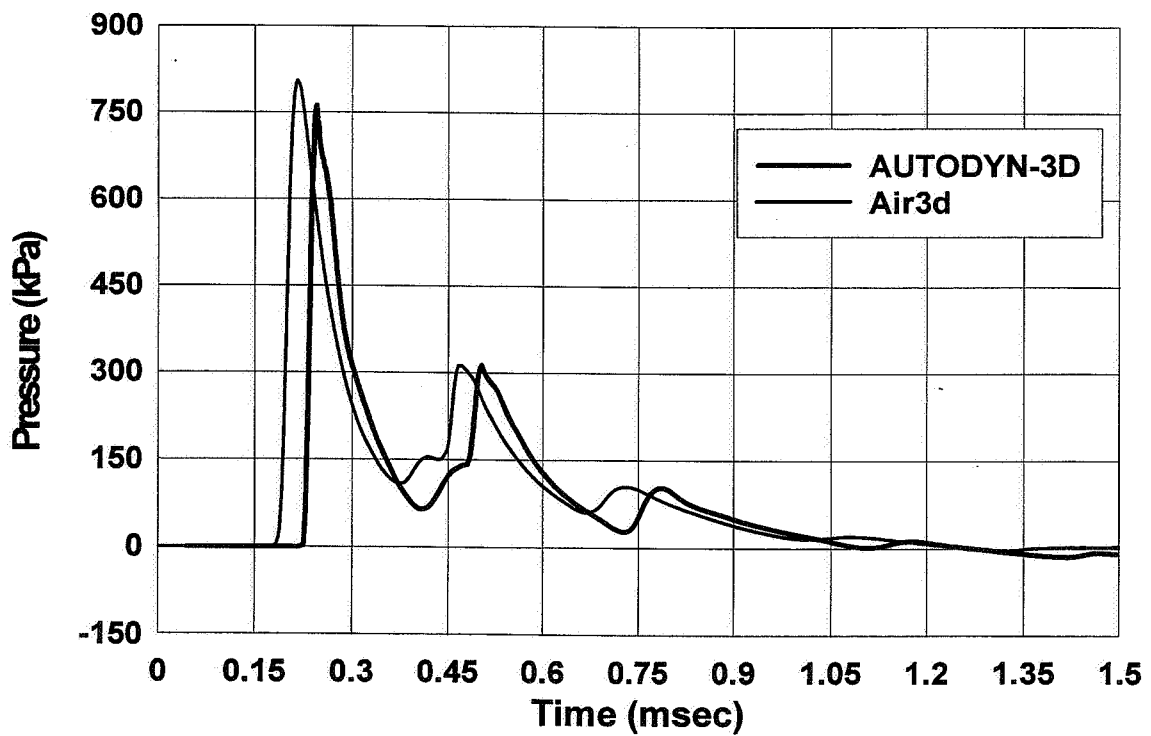


Figure 3.28: Comparison of pressure histories, location H2

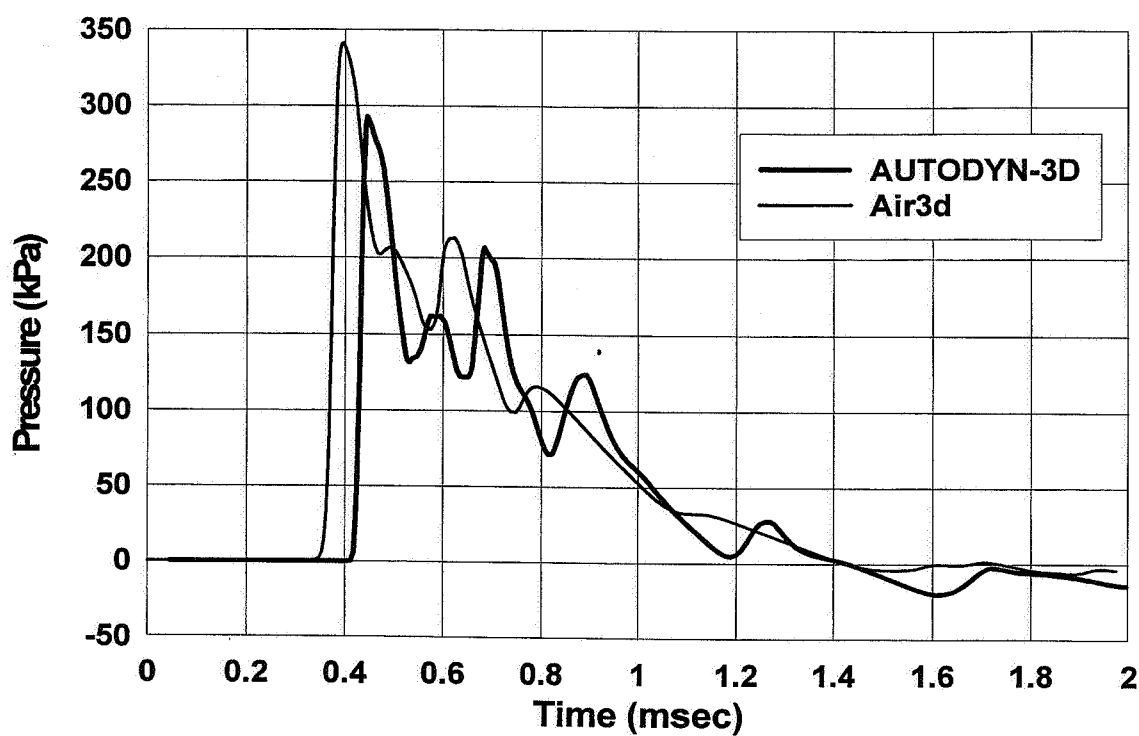


Figure 3.29: Comparison of pressure histories, location H3

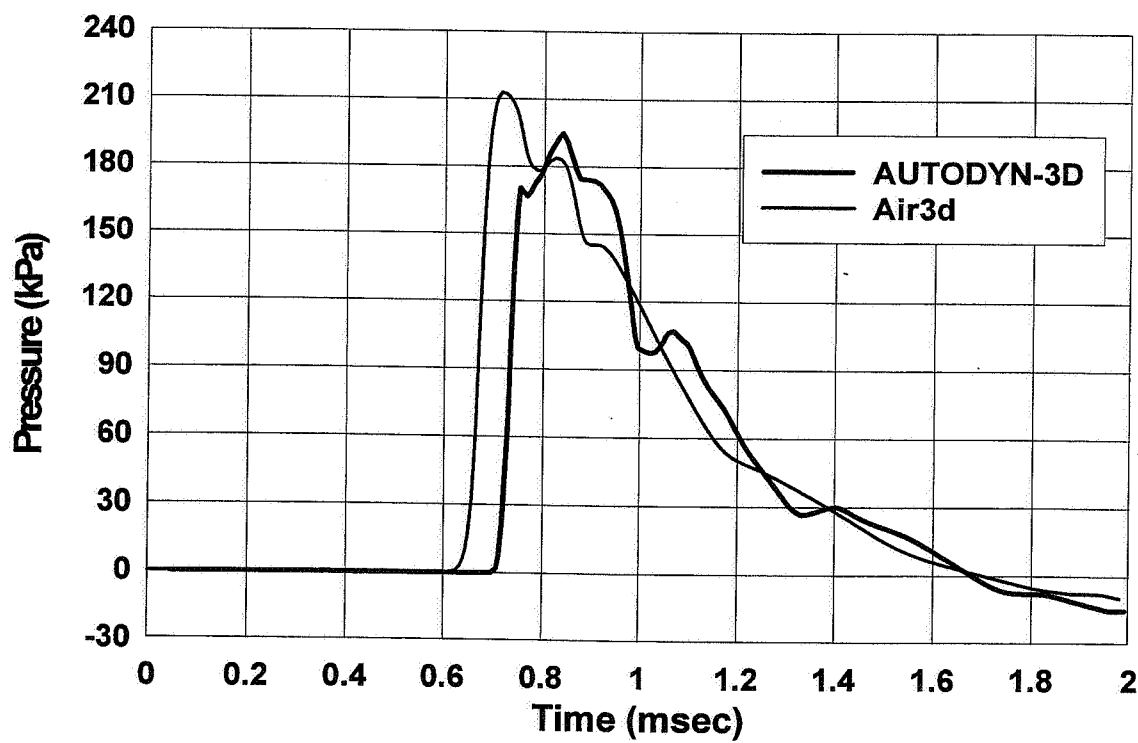


Figure 3.30: Comparison of pressure histories, location H4

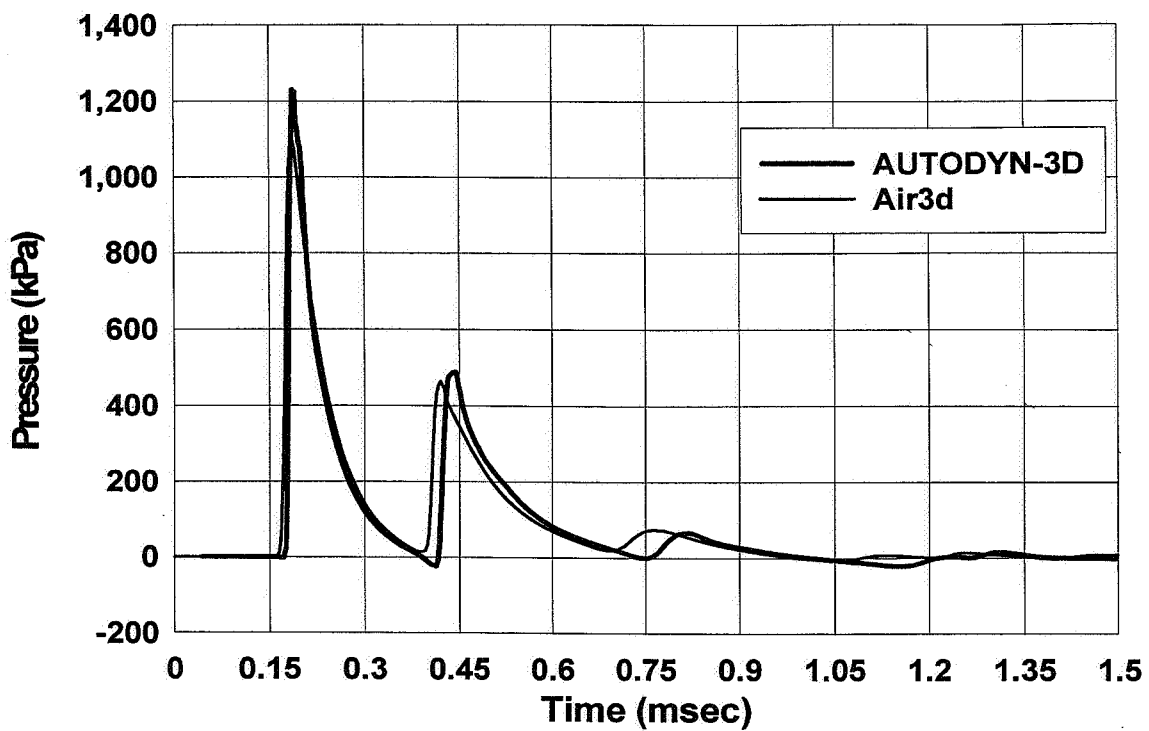


Figure 3.31: Comparison of pressure histories, location V2

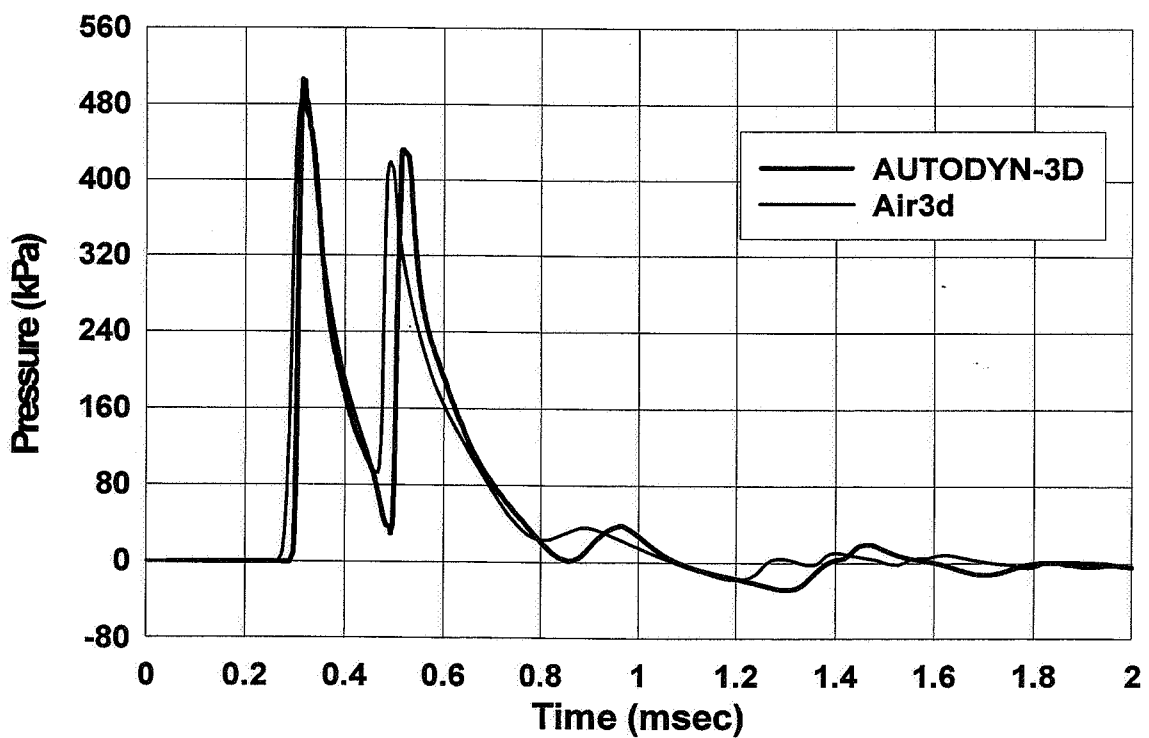


Figure 3.32: Comparison of pressure histories, location V3

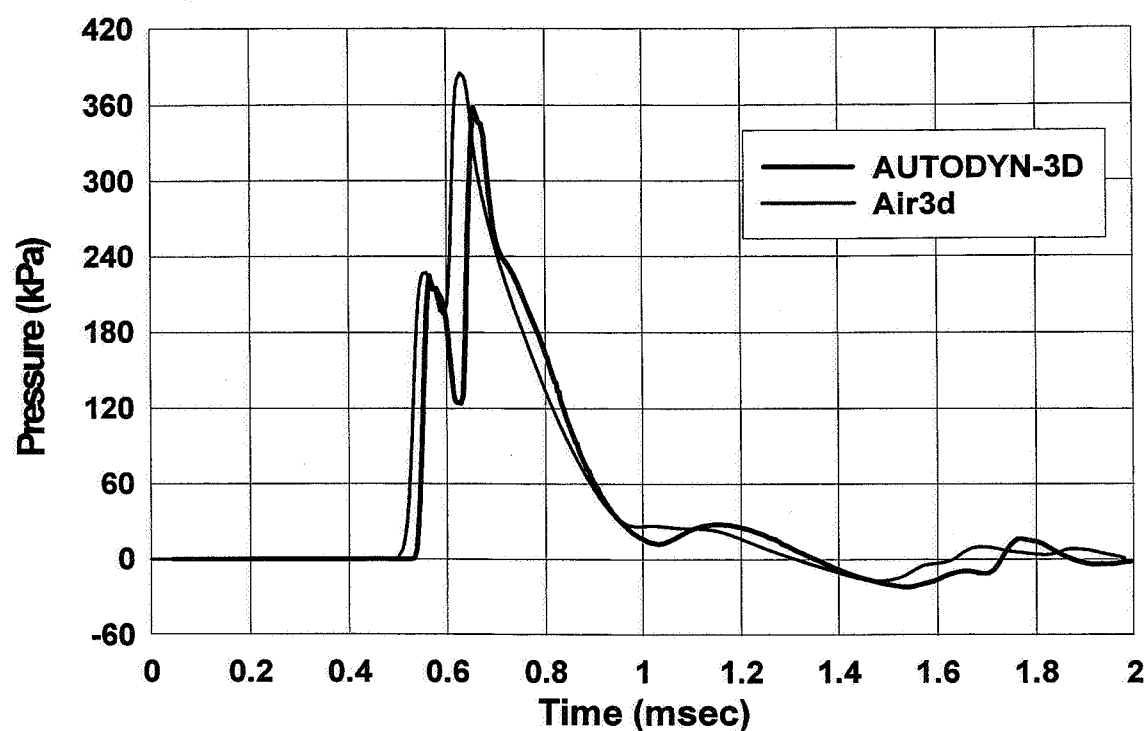


Figure 3.33: Comparison of pressure histories, location V4

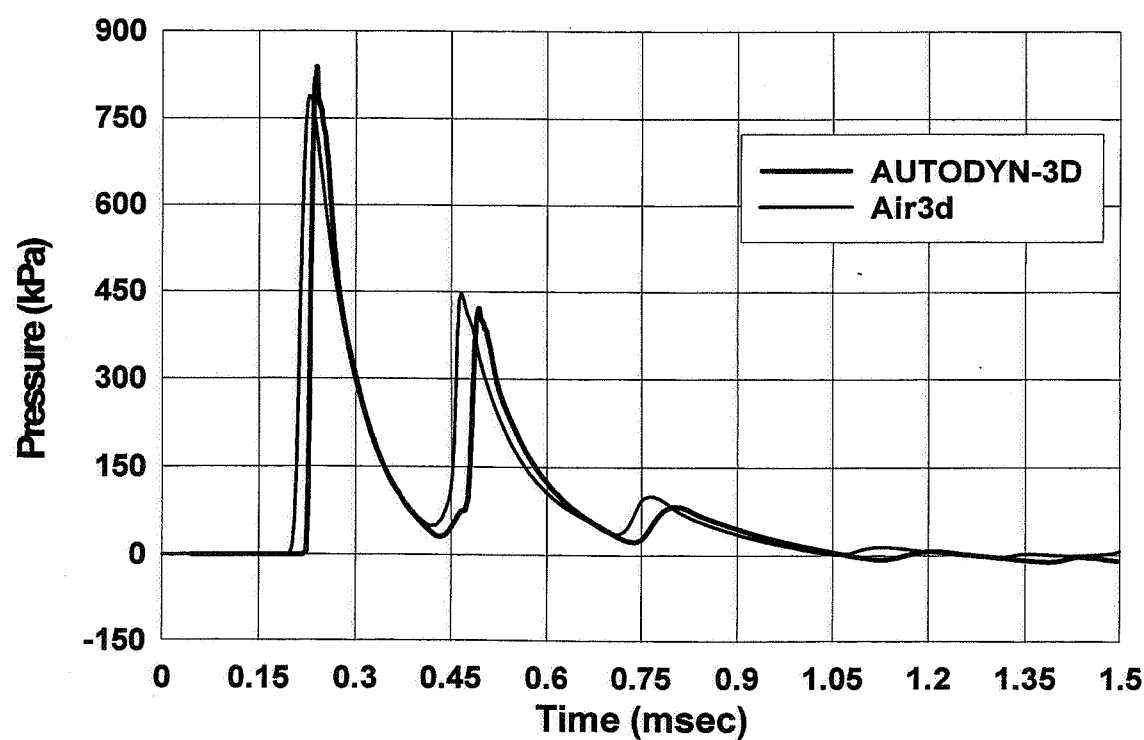


Figure 3.34: Comparison of pressure histories, location D2

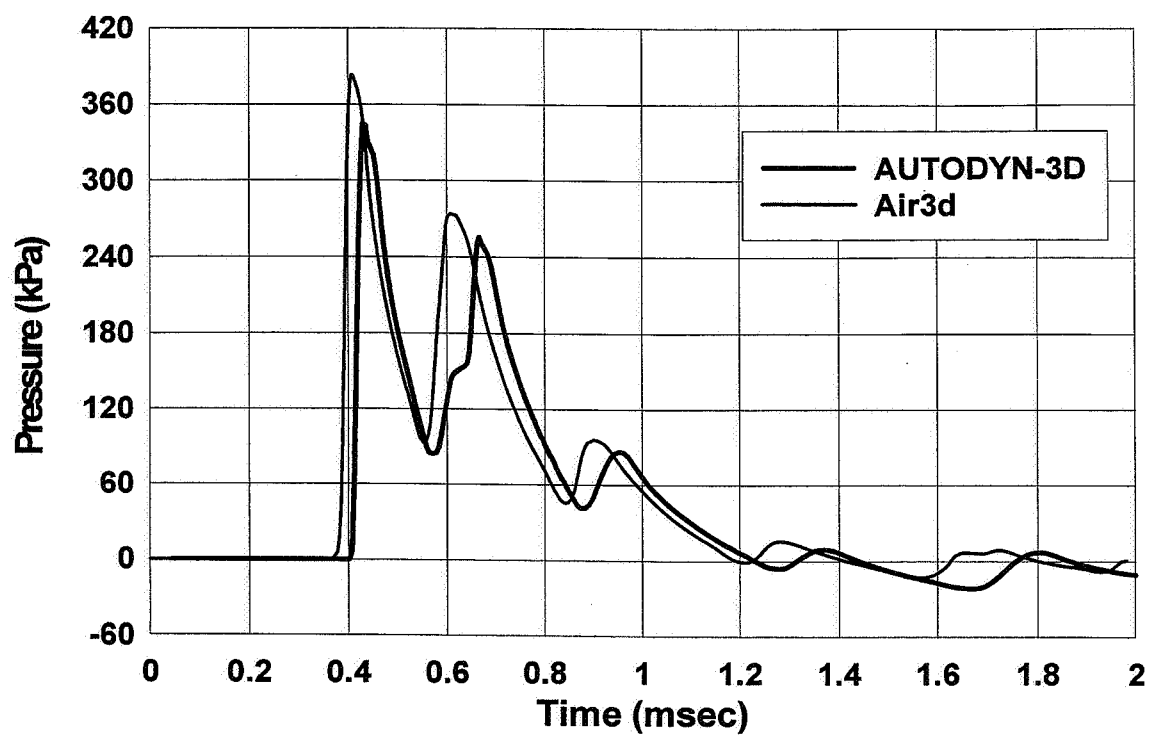


Figure 3.35: Comparison of pressure histories, location D3

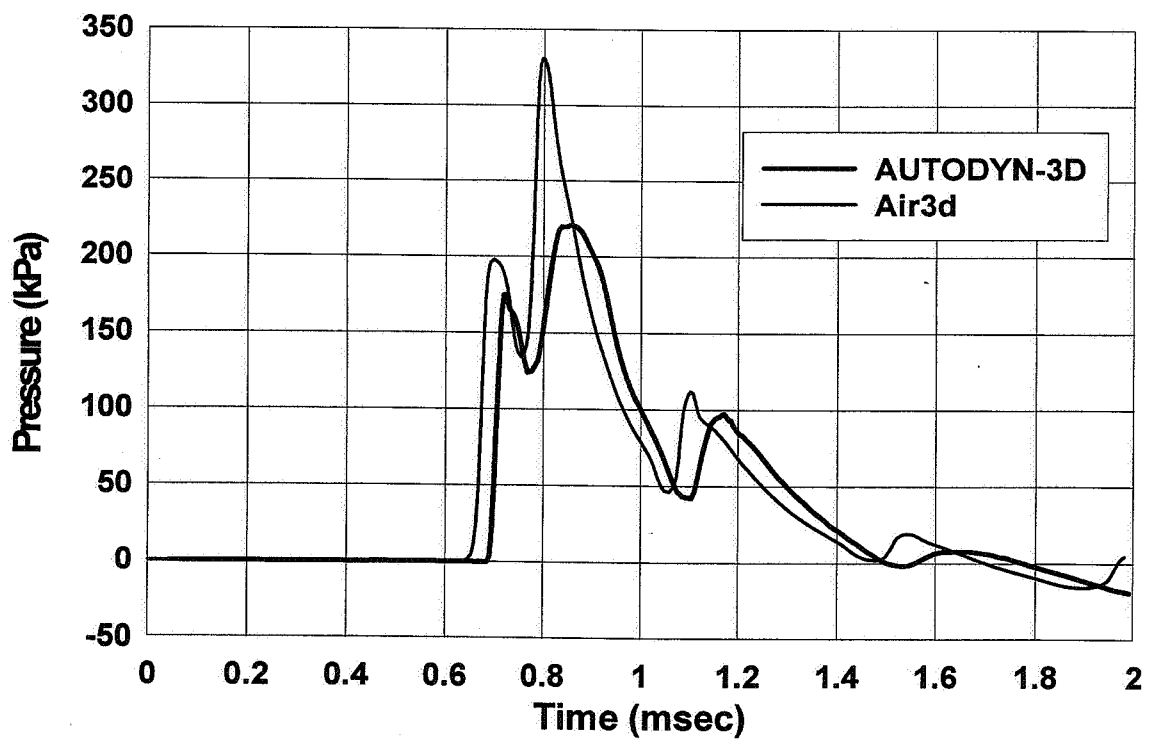


Figure 3.36: Comparison of pressure histories, location D4

Chapter 4

Scaled Blast Parameters

4.1 Introduction

Before the computational tool Air3D could be used in earnest to evaluate the problems described in Chapters 5 to 7, it was important to demonstrate that it could predict accurately peak side-on pressure p_s , specific impulse i_s , arrival time t_a and (to a lesser degree of importance) positive phase duration T_s , over the whole of the scaled range that would be used in consideration of practical problems. The most straightforward means of achieving this was by comparison with *scaled blast parameters*. This is the subject of the present section.

4.2 Validation

There are three basic steps to the method of analysis used by Air3d. These are described in detail in Chapter 3 and restated here for convenience.

Spherically symmetrical analysis is performed using a one-dimensional formulation of the Euler equations with source terms to account for the missing spatial directions. This is the essential first step of every analysis, and it is used to model the region of space from the centre of the (spherical) high explosive charge to the nearest reflecting surface in the problem scenario. Clearly, it is important that the results of this step are as accurate as possible. The opportunity to achieve good accuracy is evident, because it is possible to obtain fine spatial resolution using a one-dimensional formulation—even with quite modest computational resources.

Radially symmetrical analysis is based on a two-dimensional form of the Euler equations; again, there are source terms to account for the missing spatial direction. Generally, most practical problems have a period of radial symmetry during which the blast propagates over the ground surface, until it reaches a building, obstacle or ground feature which interrupts the symmetry. The resources needed for these analyses are significant.

Three-dimensional analysis is necessary for the evaluation of practical problems. For three-dimensional problems, the solution algorithm and computational resources are both significant factors. It is probably true that no amount of computational power will ever completely satisfy the needs of investigators. So the important questions are what level of accuracy can be achieved with the resources available? And, will the calculations produce useful/meaningful results?

The aim of the present chapter, therefore, is to demonstrate that sufficient accuracy, in terms of reproduction of established results, can be achieved to make useful progress in more complex investigations.

4.2.1 One-dimensional Spherically Symmetrical Analysis

Blast parameters for a free air spherical burst were extracted from the weapons effects program ConWep [24] for 1 kg of TNT high explosive. The scaled range over which comparison is made is $0.3 \text{ m/kg}^{1/3}$ to $30.0 \text{ m/kg}^{1/3}$. This covers virtually the whole range that is of interest to investigators concerned with the response of structures. The effects experienced at smaller scaled distances are devastating and not amenable to structural response calculations. At larger scaled distances only overpressures that are insufficient to break the windows of domestic housing or cause ear damage to persons in the open are produced.

As the charge weight used for comparison is 1 kg, scaled and actual ranges are the same. Figure 4.1 shows the ConWep [24] data (symbols), connected by lines for clarity. The units are p_s (kPa), i_s (kPa-msec), T_s (msec) and t_a (msec).

A one-dimensional spherical analysis was performed using a cell size $\Delta r \approx 1.0 \text{ mm}$ and a problem radius of 45 m (approximately 45,000 computational cells). This provided enough *space* for the whole of the positive phase of the blast wave to pass the final point of interest (at 30 m) and allowed the impulse and positive phase duration to be evaluated, without impinging on the boundary of the problem. Peak side-on pressure, specific impulse and arrival time, extracted from this analysis (symbols), are compared with those of ConWep [24] (solid lines) in Figure 4.2. Special consideration will be given to the positive phase duration, below.

This type of analysis forms the basis of all the calculations described in this document, and the results given in Figure 4.2 represent the best that can be achieved with the program Air3d.

Interestingly, results of the analysis described here do not improve noticeably, in terms of peak pressure, if the cell size is reduced below 1 mm. It is suggested that analyses which utilise ≈ 50 cells through the radius of the charge are sufficiently accurate for most calculations. Similarly, a scaled cell size of $1.0 \times 10^{-3} \text{ m/kg}^{1/3}$ will provide results of sufficient resolution over virtually any scaled range.

There are number of observations that can be made about Figure 4.2:

- The method used by the program (explained in Chapter 3) to introduce the explosive energy into the air, does not model the detonation process. It is a spherically symmetrical version of the *shock tube problem*, and *always* underpredicts the side-on pressure near the charge surface (see Matthews and

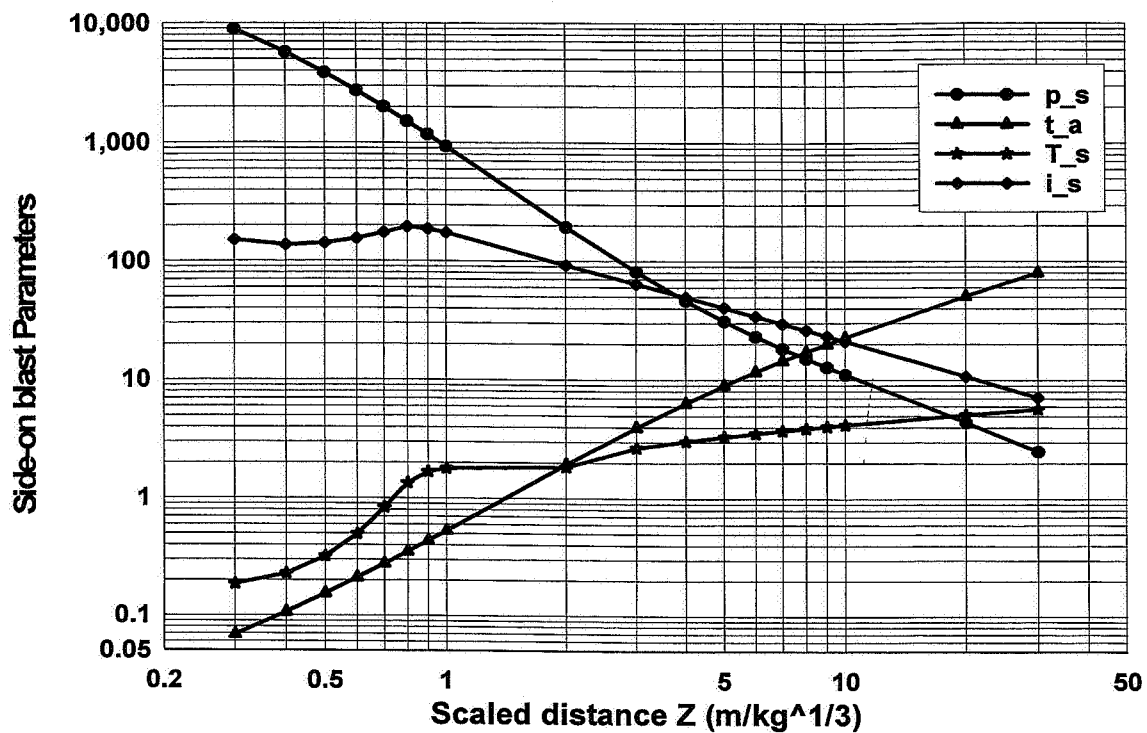


Figure 4.1: Blast parameters, 1 kg TNT, (from ConWep [24])

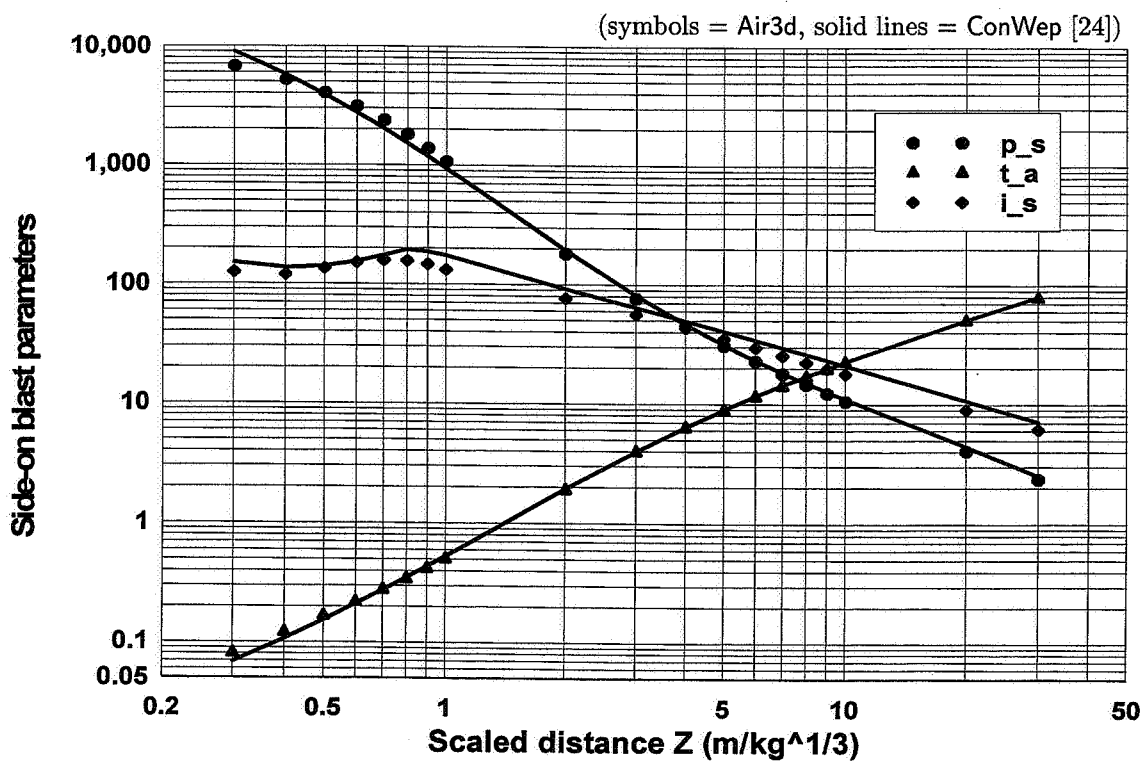


Figure 4.2: Comparison of blast parameters, 1 kg TNT, spherical analysis

Ritzel [34]). Therefore, comparison with established data at scaled distances below $0.3 \text{ m/kg}^{1/3}$, the distance at which the problem *settles down*, is not appropriate.

- There is a small part of Figure 4.2 between 0.5 and $1.0 \text{ m/kg}^{1/3}$ where the program has predicted pressures greater than ConWep [24]. This is not easy to explain, but it is probably related to the same reason as above. It is likely that the third point on the graph (at $0.5 \text{ m/kg}^{1/3}$), where the two sets of data cross over, is where the underprediction from the isothermal bursting sphere scenario is offset by the fact that the experimentally-based data is reducing more rapidly—because it started more energetically. At $2.0 \text{ m/kg}^{1/3}$ the natural diffusivity of the numerical scheme becomes the dominant effect, and the remainder of the pressure values are all slightly below the solid line.
- The values of side-on specific impulse are all slightly below the values from ConWep [24]. This fact probably indicates that the value used in Air3d for the chemical energy released by the explosive may be inappropriate. This allows an intriguing possibility: namely, adjusting the source energy of the explosive so that the solution fits the data exactly. In the present study this possibility has been resisted, and the value of specific energy of TNT used is from Baker [4]. The small amount of extra energy needed to raise the condensed explosive to ambient temperature has been ignored. The possible underestimation of initial energy may also contribute to the slight underprediction of pressure between $2.0 \text{ m/kg}^{1/3}$ and $30.0 \text{ m/kg}^{1/3}$, mentioned above.
- Values of arrival time are inextricably linked to the peak pressure values, and so the argument given above for pressure applies equally to arrival time. In most practical applications, arrival time is of little importance.

Figure 4.3 shows a comparison between positive phase durations that were calculated by the spherical analysis and those from ConWep [24]. At first sight, this comparison would appear to be poor.

The data from ConWep [24] are based on the polynomial curves of Kingery and Bulmash [28]. These curves are generally good approximations of the source data, with the exception of the curve showing positive phase duration. No explanation is given in the reference for this extremely poor correlation. One set of data (Reference [36], simply labelled Data in Figure 4.3) has been extracted *by hand* from Figure 5 of Kingery and Bulmash [28] and superimposed on Figure 4.3. These data would appear to be more consistent with the spherical analysis than the published curves. This is a difficult issue to resolve.

It is sufficient to note that this uncertainty may or may not pose a problem to investigators, depending on whether analyses are performed for the purpose of structural response calculations. If so, the precise load duration may be important for certain structural elements.

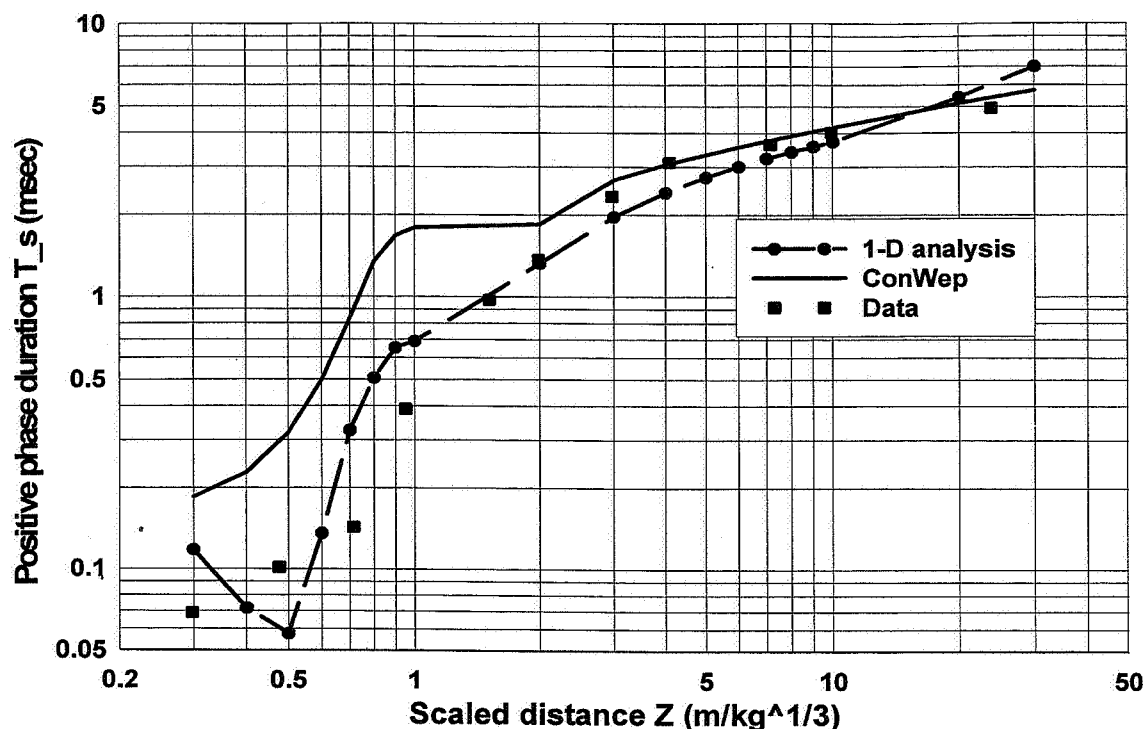


Figure 4.3: Comparison of positive phase duration T_s , 1 kg TNT

4.2.2 Two-dimensional Radially Symmetrical Analysis

Two-dimensional radially symmetrical problems are effectively height of burst scenarios. Clearly, it is possible to analyse one-dimensional problems in two dimensions, but there is little point. Therefore, in this section, comparison will be made with published height of burst blast parameters. One source of these parameters for high explosives (as opposed to nuclear devices) is Swisdak [50]. This provides information on peak overpressure, positive impulse, time of arrival and positive phase duration of the blast wave *at the ground surface*.

The graphs of Swisdak [50], from which data have been extracted by hand, are Figures 5a, 5b, 5c (pressure), Figure 8 (impulse), Figure 6 (time of arrival) and Figure 7 (duration), and these are reproduced below in Figures 4.4 to 4.9, respectively.

It will be noticed that the scaled range over which peak pressure data is presented (in three graphs) is considerably greater than for the other blast parameters. This will be evident in the comparisons with analysis which follow.

Three scaled heights of burst were selected for comparison, these were 1.0, 2.0 and 3.0 m/kg^{1/3}. Because of the limitations of the data from Swisdak [50], comparison of arrival time and positive phase duration for charge height 3.0 m/kg^{1/3} was not possible.

Pressures are compared over a horizontal scaled range 0.0–25.0 m/kg^{1/3}. Impulses, arrival times and durations are compared over the range 0.0–10.0 m/kg^{1/3}.

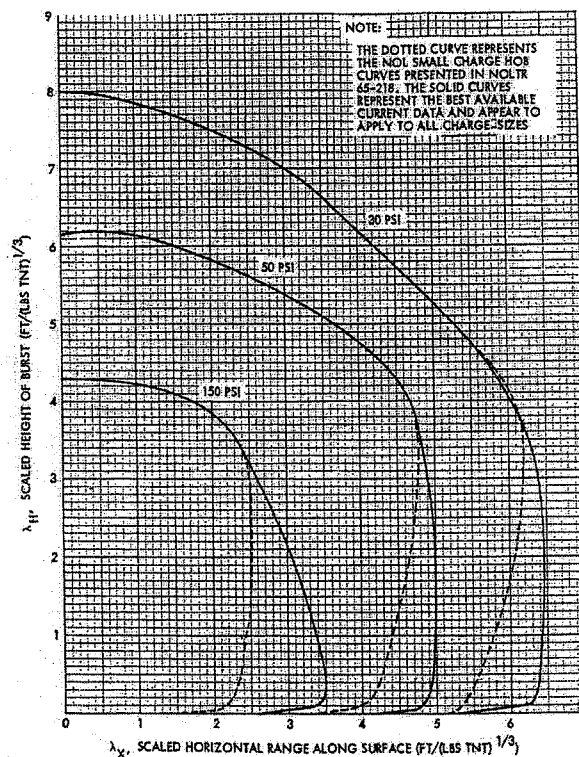


Figure 4.4: Height of burst curves for peak overpressure (1), Swisdak [50]

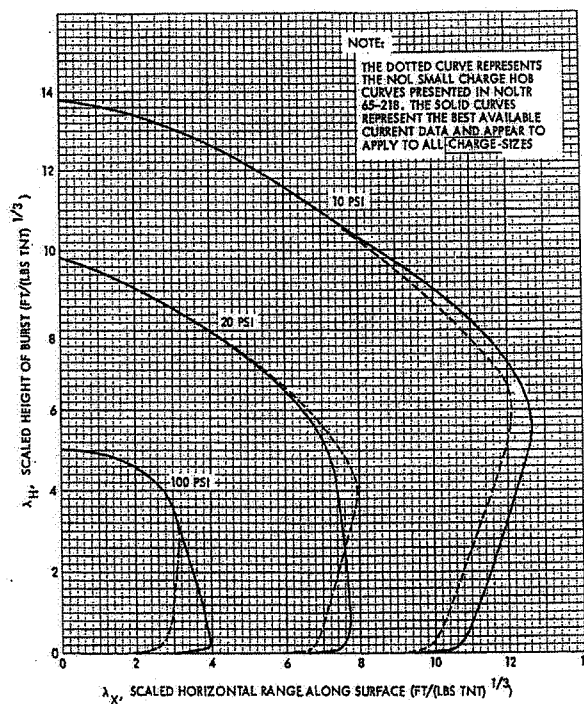


Figure 4.5: Height of burst curves for peak overpressure (2), Swisdak [50]

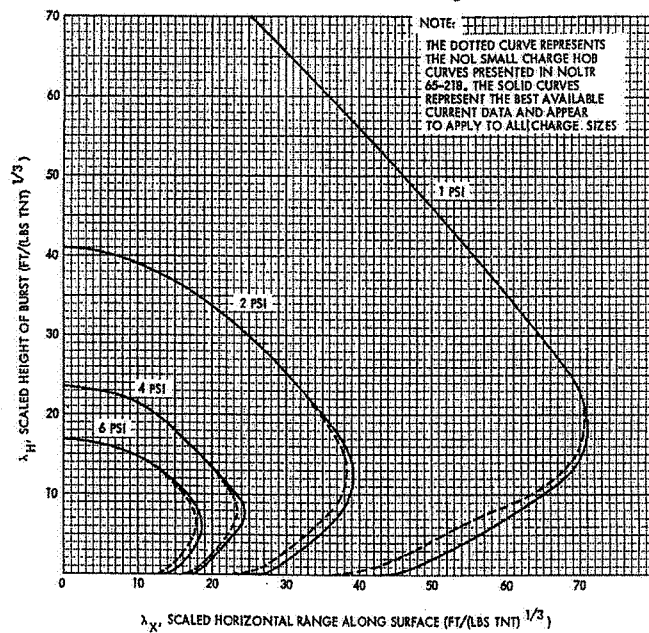


Figure 4.6: Height of burst curves for peak overpressure (3), Swisdak [50]

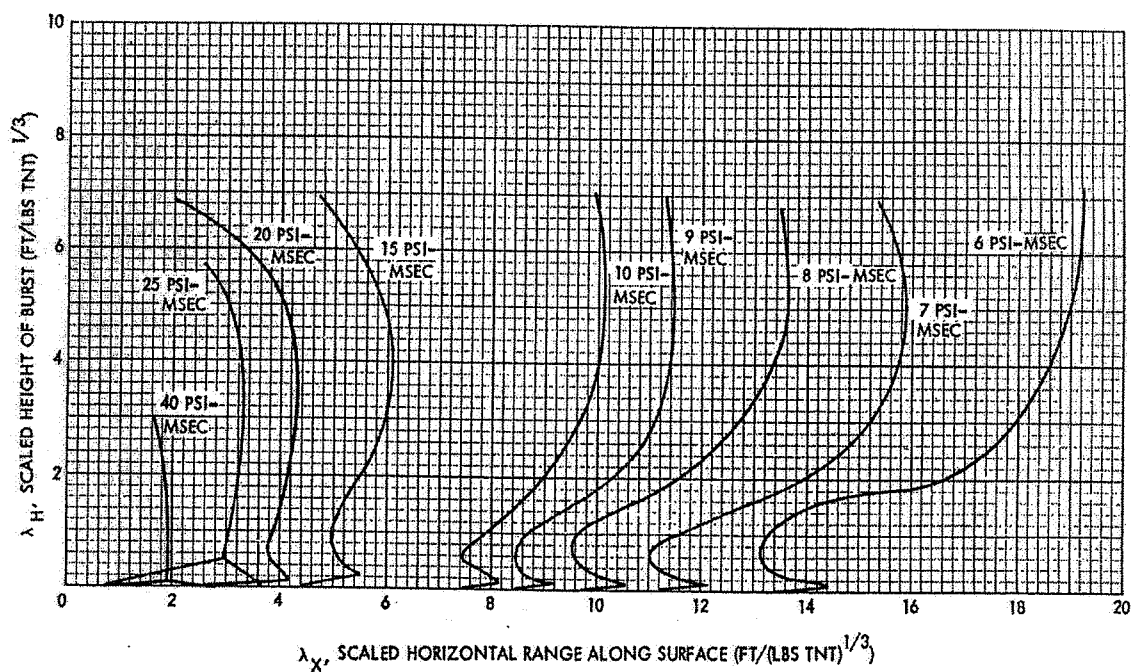


Figure 4.7: Height of burst curves for positive impulse, Swisdak [50]

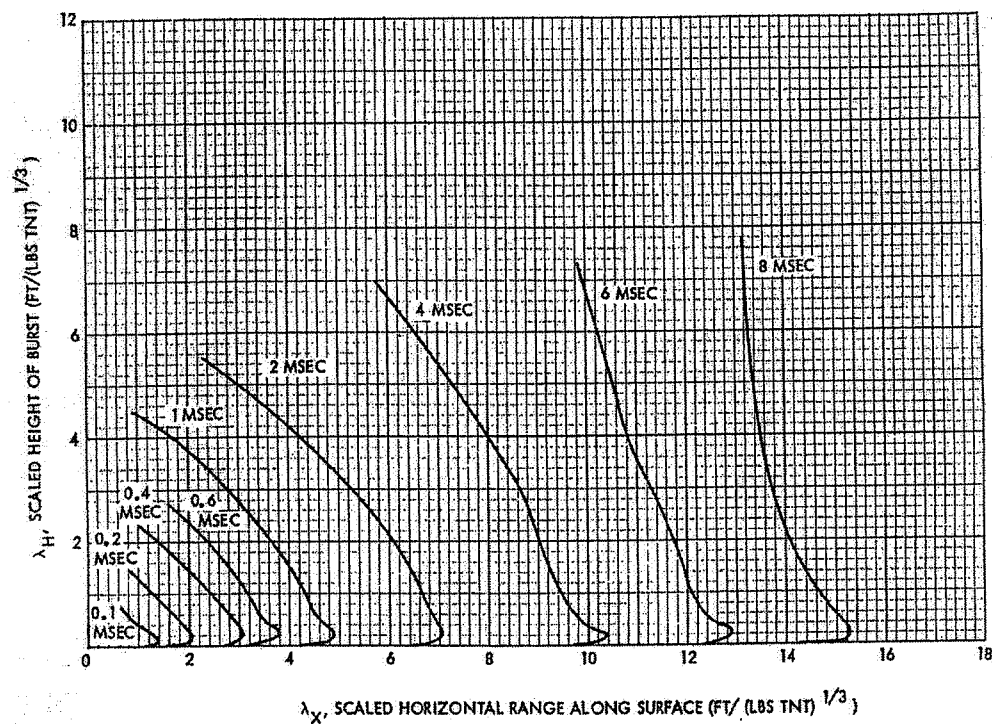


Figure 4.8: Height of burst curves for time of arrival, Swisdak [50]

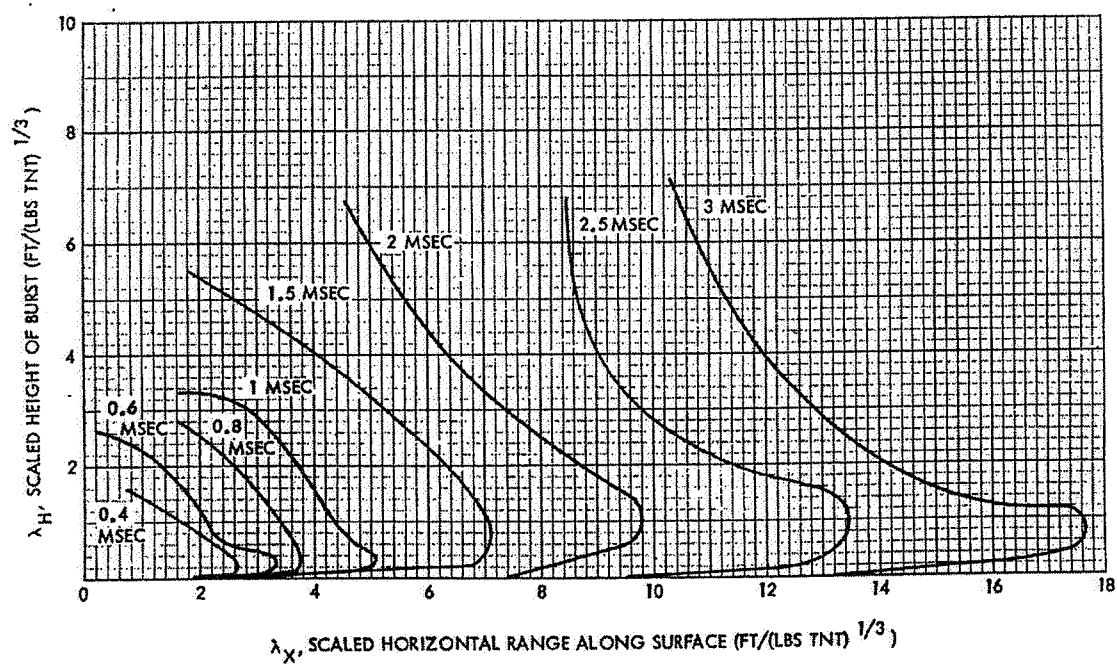


Figure 4.9: Height of burst curves for positive phase duration, Swisdak [50]

Numerical simulations were performed on a domain which extended from 0.0–28.0 m in the radial (horizontal) direction and 0.0–12.0 m in the axial (vertical) direction. The charge weight was 1 kg TNT, so the scaled distances were the same as the actual distances. The cell size was 20 mm; there were 840,000 computational cells.

Comparisons between the data of Swisdak [50] and the results of the two-dimensional numerical simulations, for each of the three heights of burst, are given in Figures 4.10 to 4.19.

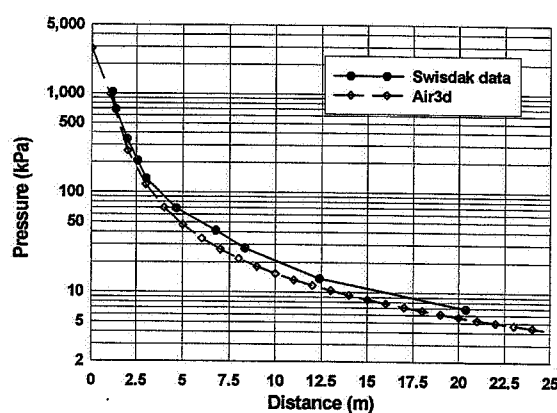


Figure 4.10: HOB parameters, pressure, $h = 1.0 \text{ m/kg}^{1/3}$

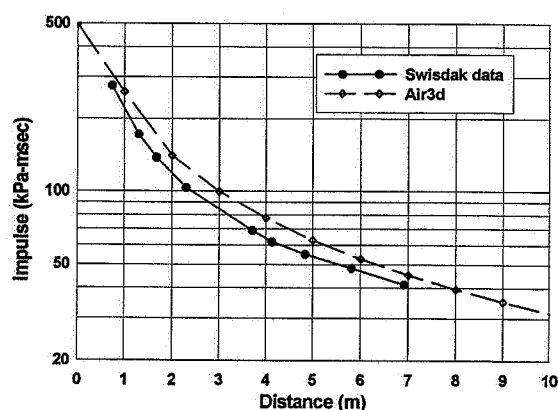


Figure 4.11: HOB parameters, impulse, $h = 1.0 \text{ m/kg}^{1/3}$

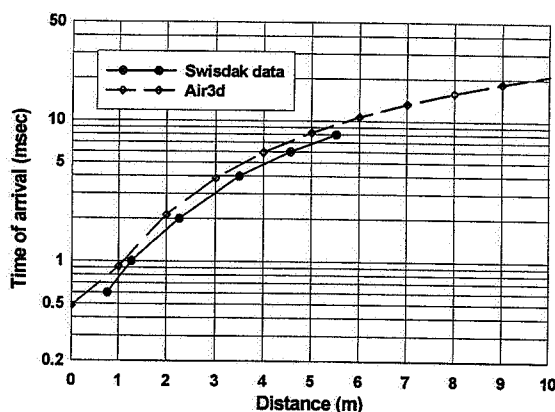


Figure 4.12: HOB parameters, time of arrival, $h = 1.0 \text{ m/kg}^{1/3}$

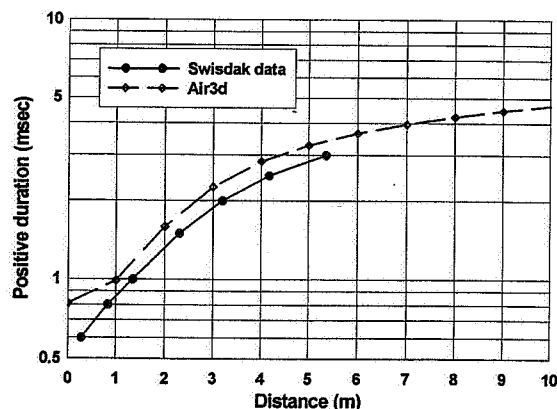


Figure 4.13: HOB parameters, positive phase, $h = 1.0 \text{ m/kg}^{1/3}$

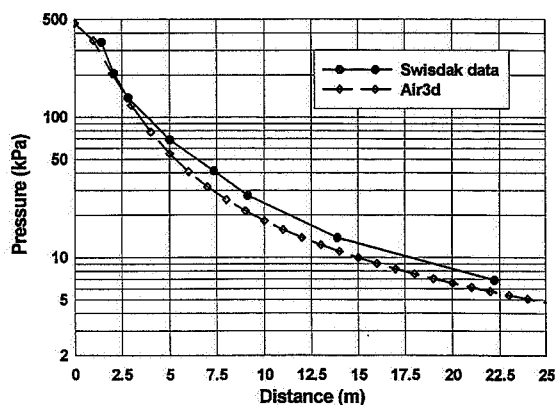


Figure 4.14: HOB parameters, pressure, $h = 2.0 \text{ m/kg}^{1/3}$

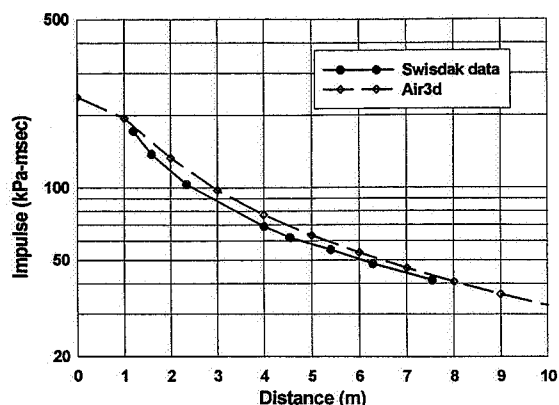


Figure 4.15: HOB parameters, impulse, $h = 2.0 \text{ m/kg}^{1/3}$

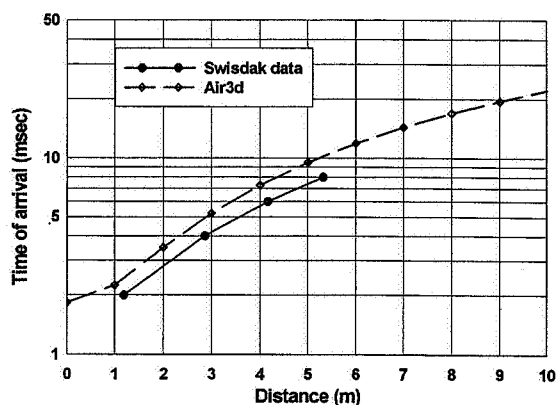


Figure 4.16: HOB parameters, time of arrival, $h = 2.0 \text{ m/kg}^{1/3}$

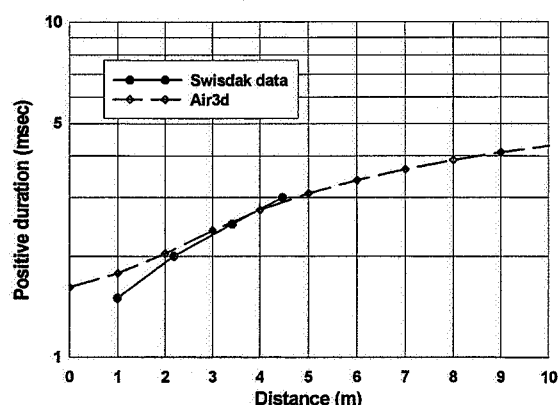


Figure 4.17: HOB parameters, positive phase, $h = 2.0 \text{ m/kg}^{1/3}$

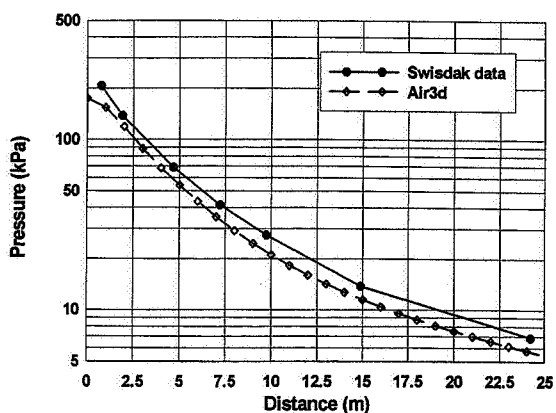


Figure 4.18: HOB parameters, pressure, $h = 3.0 \text{ m/kg}^{1/3}$

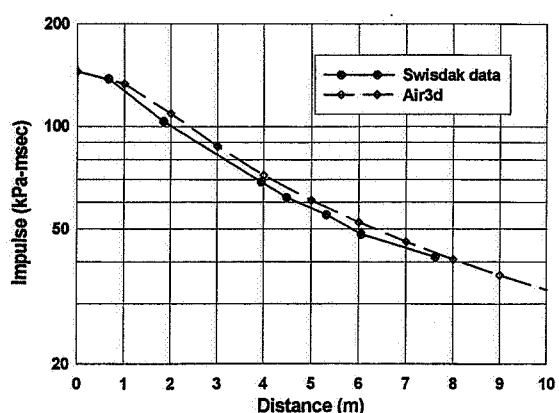


Figure 4.19: HOB parameters, impulse, $h = 3.0 \text{ m/kg}^{1/3}$

The following observations can be made regarding the comparison of Air3d results with the data of Swisdak [50]:

- The graphs of peak pressure (Figures 4.10, 4.14 and 4.18) show that the numerical peak pressures consistently fall a little short of the published values. This is usual, as described previously for the one-dimensional case.
- The graphs of positive impulse (Figures 4.11, 4.15 and 4.19) demonstrate that the values predicted by Air3d are slightly above the impulses of Swisdak [50]. For the two heights of burst $2.0 \text{ m/kg}^{1/3}$ and $3.0 \text{ m/kg}^{1/3}$, in particular, the numerical and published data are very close indeed.

From an engineering stand point, confidence in the impulse information is of primary importance.

- The two graphs comparing time of arrival (Figures 4.12 and 4.16) both show that the numerically simulated blast waves arrive slightly late compared to the published data. This is consistent with the observation above that the peak pressures were slightly less, because arrival time and peak pressure are linked. This was also described for the one-dimensional case.
- Lastly, the graphs of positive phase duration (Figures 4.13 and 4.17) show that the duration of the numerically-based blast waves are slightly longer than the published data. This is consistent with the observation that the impulses were slightly greater than the published data.

The scarcity of reliable blast data, based on statistically valid data sets, has made the process of validation more difficult than it might, at first, seem. The data of Swisdak [50] is mostly over thirty years old and is described as "based either on experimental data or computer extrapolations of experimental data." The graphs of peak pressure and impulse have a stated error of 10%. Those for time of arrival and positive durations have a stated error of 15%.

The numerically-derived data of Figures 4.10 to 4.19 are distributed at equal intervals along the distance axes of the graphs; they do not coincide with the experimental data. However, using linear interpolation, close approximations to the numerical values at the *actual* distances at which the experimental data were measured can be obtained. The percentage difference between these interpolated numerical data and the experimental data, for each of the graphs, provides a good indication of the value of the numerical approach in solving the height of burst problem. These mean percentage differences from the three graphs comparing peak pressure are 19.9% (Figure 4.10), 17.0% (Figure 4.14) and 16.6% (Figure 4.18). The mean differences between the data comparing impulse are 18.7% (Figure 4.11), 9.4% (Figure 4.15) and 5.6% (Figure 4.19). The values of mean differences in time of arrival are 21.0% (Figures 4.12) and 24.8% (Figure 4.16), and those for positive phase duration are 20.6% (Figures 4.13) and 6.8% (Figures 4.17).

To summarise, the difference between the experimental and interpolated numerical peak pressure data is slightly greater than the stated error of the experimental data. Two of the three mean differences in impulse are within the stated experimental error. The mean differences in time of arrival are both slightly greater than

the stated error, and one of the mean differences in positive phase duration is also greater than the stated error.

With these observations in mind, and the knowledge that peak specific impulse is the most important and most reliably obtained blast parameter, the results of this two-dimensional validation indicate the potential of the present numerical approach.

4.2.3 Three-dimensional Analysis

There are essentially two classes of three-dimensional problem:

- i. The region of interest is remote from the explosive and/or extends across only a small region of space. There are no reflecting surfaces (barriers, ditches and so on) between the charge and the region of interest, except perhaps the ground surface.
- ii. The region of interest is close to the explosive and/or extends across a large region of space.

The terms close, remote, small and large, used above, are deliberately vague. It should be borne in mind that they only really have meaning when referring to scaled dimensions.

There are no established data sets for three-dimensional problems. Therefore, in this section, the results of three-dimensional analyses of a 1 kg free air burst are compared with the data of ConWep [24]. This is the same approach which was used for one-dimensional validation.

The first class of problem is considered in more detail in Chapter 5: *Blast Wave Clearing*. It is typified by single finite structures, often remote from the explosive source, with no other reflecting surfaces in the region of interest, except the ground. This type of problem is the least demanding, in terms of computational resources, because the one- and/or two-dimensional parts of the analysis can cover the whole distance from the charge centre to the nearest face of the structure. At this point, the analysis is continued in three-dimensions for the relatively short time that it takes for the blast to interact with the structure. This approach allows the possibility of obtaining good results with relatively modest resources.

As an example of this, four calculations were performed using rectangular grids, in *quarter-space* (to utilise the symmetry of the problem). The extents of the computational grids and the spatial discretisations are described in Table 4.1. In each case, the centre of the charge was situated at the origin of the three-dimensional space, which was remapped with the results of a spherical analysis ($\Delta r \approx 1.0$ mm) to the radii given in Table 4.1. It should be stressed that these are four separate one- and three-dimensional calculations; they are not remapped from one three-dimensional domain to the next.

Figure 4.20 contains the results of these four calculations (symbols) combined with the blast parameters from ConWep [24] (solid lines). Dashed vertical lines have been added to mark the limits of the four different computational regions.

The approach described above may appear contrived. However, it does reflect the normal way analyses of this kind are performed. Important aspects of Figure 4.20 are as follows:

Table 4.1: Verification problem parameters

	Extents			Grid size $\Delta x, \Delta y, \Delta z$, (mm)	Radius of 1-D analysis (m)
	x (m)	y (m)	z (m)		
Calc. 1	0.0–1.2	0.0–0.6	0.0–0.6	5.0	0.3
Calc. 2	0.0–3.6	0.0–1.8	0.0–1.8	15.0	1.0
Calc. 3	0.0–12.0	0.0–6.0	0.0–6.0	50.0	3.0
Calc. 4	0.0–36.0	0.0–18.0	0.0–18.0	150.0	10.0
Number of computational elements 3,456,000					

- The starting position of each of the analyses (the end of the spherical analysis) represents a reasonably good approximation (based on Figure 4.2). However, the remapping procedure (described in Chapter 3) degrades the spatial discretisation of the conserved variables, and hence the peak pressure, as it fills the three-dimensional cells with information from the (much finer) one-dimensional arrays.
- As the three-dimensional analyses progress, the quality of the peak side-on pressure information becomes degraded. This is due to the relatively coarse spatial grids. The variation in accuracy over the four computational regions is visible.
- The start and end points of the computational regions can be seen as coincident points on the scaled distance axis of Figure 4.20: at 1.0, 3.0 and 10.0 m/kg^{1/3}.
- The accuracy of specific impulse information is good over much of the range considered. It has only degraded significantly at large scaled distances. This is perhaps the single most important observation from the present section, because with only a few exceptions: window breakage for example, impulse is the primary interest of the structural engineer.

The usefulness of this approach will become clear in Chapter 5 where most of the calculations use smaller regions of interest than those described here.

The second class of problem poses considerably greater difficulties. If the scaled range from the centre of the charge to the nearest reflecting surface is small, as it would be if an explosive device were detonated in a city street, for example, the extent of the one- or two-dimensional analyses would be limited. This, in turn, necessitates fine mesh discretisation from the outset of the problem, if the natural diffusivity (present in all numerical schemes) is not seriously to degrade the quality of the solution. Similarly, if the blast wave is to propagate relatively large distances before reaching points of interest, such as the façade of a building at the far end of the street, fine spatial resolution is required for the whole of the analysis. Such problems require a massive amount of computer resources, and compromises are inevitably necessary. This was the problem faced in Chapters 6 and 7.

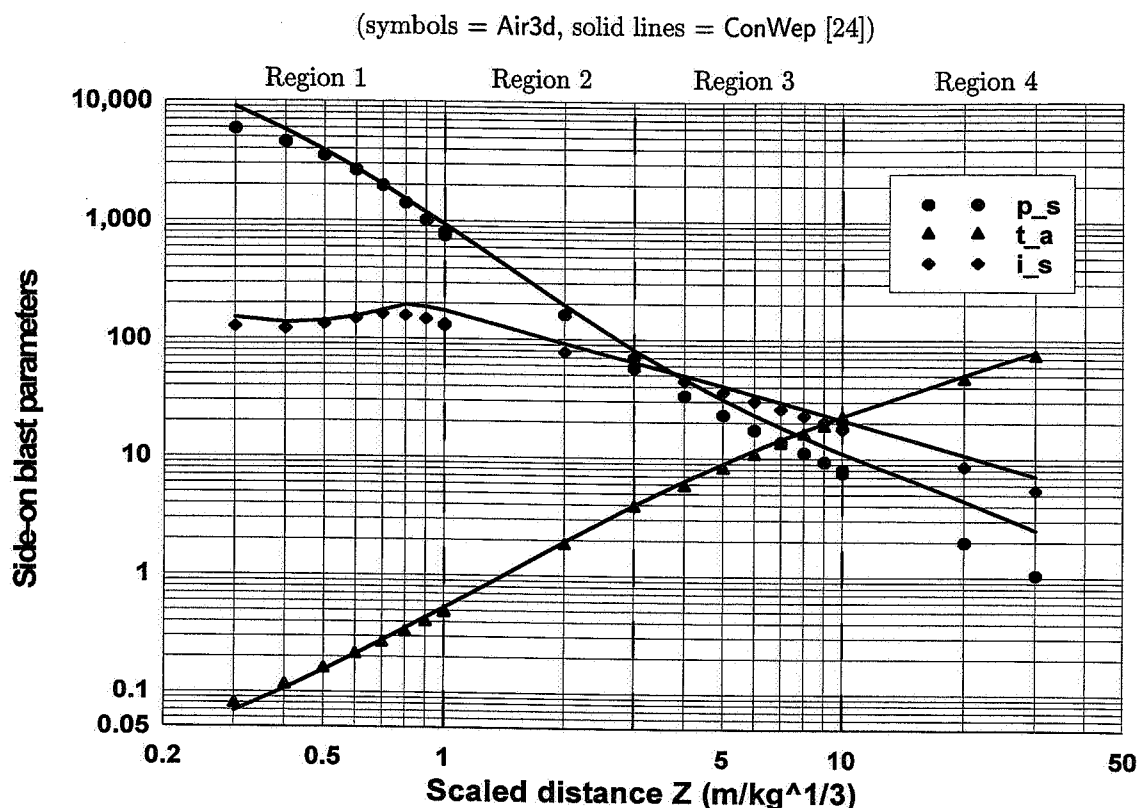


Figure 4.20: Comparison of blast parameters, 1 kg TNT, first type of problem

Consider the same scenario as previously: 1 kg TNT in free air. This time it is analysed in three-dimensions in a *single* computational grid. There is one small concession: the starting position (the end of the spherical analysis) is at a radius of 1.0 m as opposed to 0.3 m previously. This problem was run until the end of the positive phase passed $30 \text{ m/kg}^{1/3}$, as before, and it used the same overall grid dimensions as calculation 4, in Table 4.1. The spatial discretisation was 0.15 m (cubic cells). Results of this analysis are shown in Figure 4.21.

Continuing the analogy of a city street, a 1000 kg device detonated in the centre of a city street 20 m wide would give a scaled distance to each side of the street of $1.0 \text{ m/kg}^{1/3}$, the starting position of the present problem. Therefore, this abstract problem does have a practical basis.

The final set of observations, regarding Figure 4.21, are as follows:

- The graph represents an analysis which used most of the computational resources available to the author. The problem comprised 3,456,000 cells and took approximately four days to complete. Problems that require computational domains of the extent described are probably too demanding for the present study. Examples of this second type of problem are contained in the validation analyses of Chapter 7.
- The quality of the solution, in terms of peak side-on pressure, is degraded throughout the whole of the range. The information is probably not of any practical value.

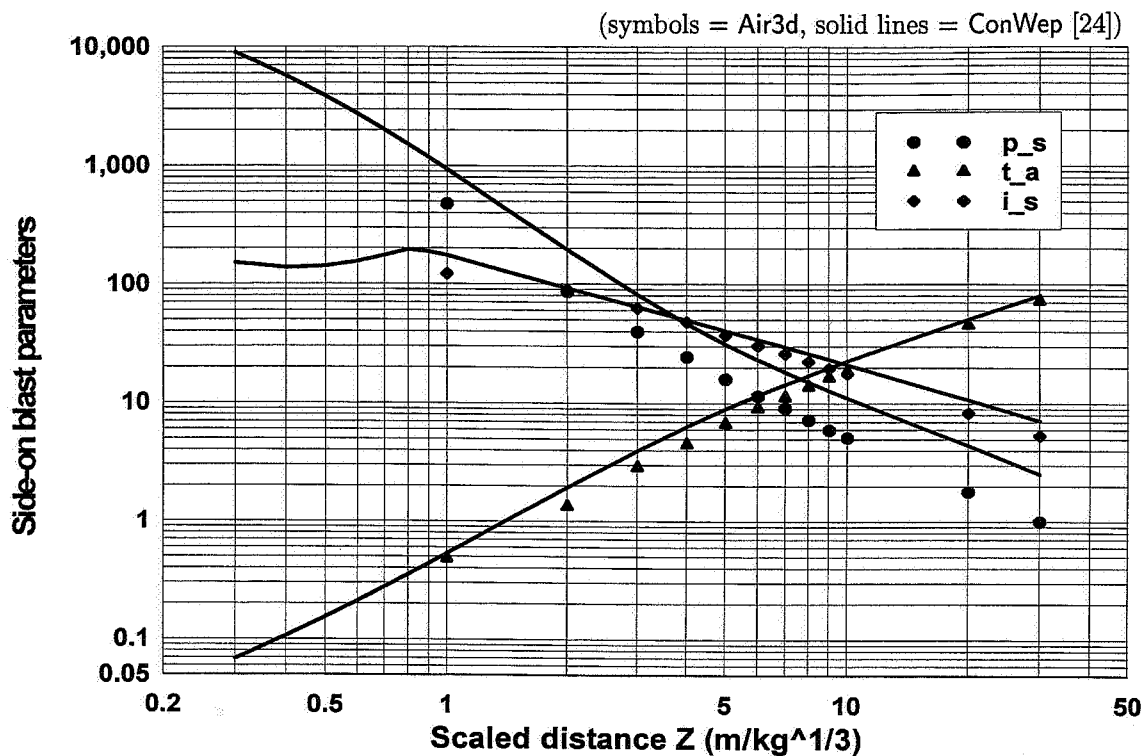


Figure 4.21: Comparison of blast parameters, 1 kg TNT, second type of problem

- The specific impulse information is also degraded, but does still compare reasonably well with the data of ConWep [24]. It should be borne in mind that computational techniques, such as the present one, can provide information in situations where other techniques (with the exception of experiments) fail to provide useful results.
- Real situations are not as demanding as the situation described above. The example given previously, a city street, might seem a stern test, if the arguments above are followed. However, many practical situations include a degree of confinement that reduces the three-dimensionality (spherical spreading) of a free-air explosion to some lesser extent, and the resulting blast parameters do not vary nearly so rapidly as they do in the present scenario.

A final demonstration of the effect that poor spatial discretisation has on the quality of the solution is given by the three pressure-time histories Figures 4.22 to 4.24, below. These compare the one-dimensional spherically symmetrical analysis ($\Delta r = 1.0$ mm) with the final three-dimensional analysis ($\Delta x, \Delta y$ and $\Delta z = 150$ mm) mentioned above. The pressure-time histories at 1.0, 3.0 and 10.0 m/kg^{1/3} were chosen for comparison.

It will be seen from Figures 4.22 to 4.24 that the distinctive, well defined, peaks, which are evident in the one-dimensional analysis, have become rounded and less pronounced in the three-dimensional solution. In each case, the less distinct second peak is missing.

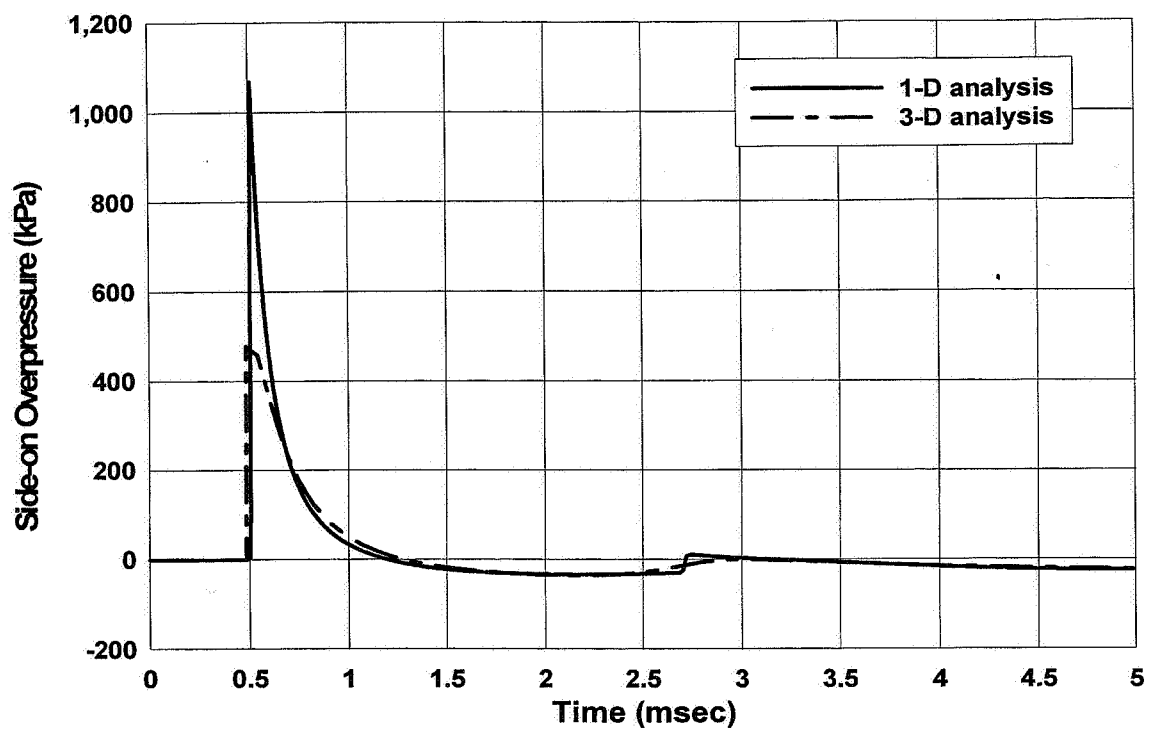


Figure 4.22: Comparison of pressure histories at $1.0 \text{ m/kg}^{1/3}$

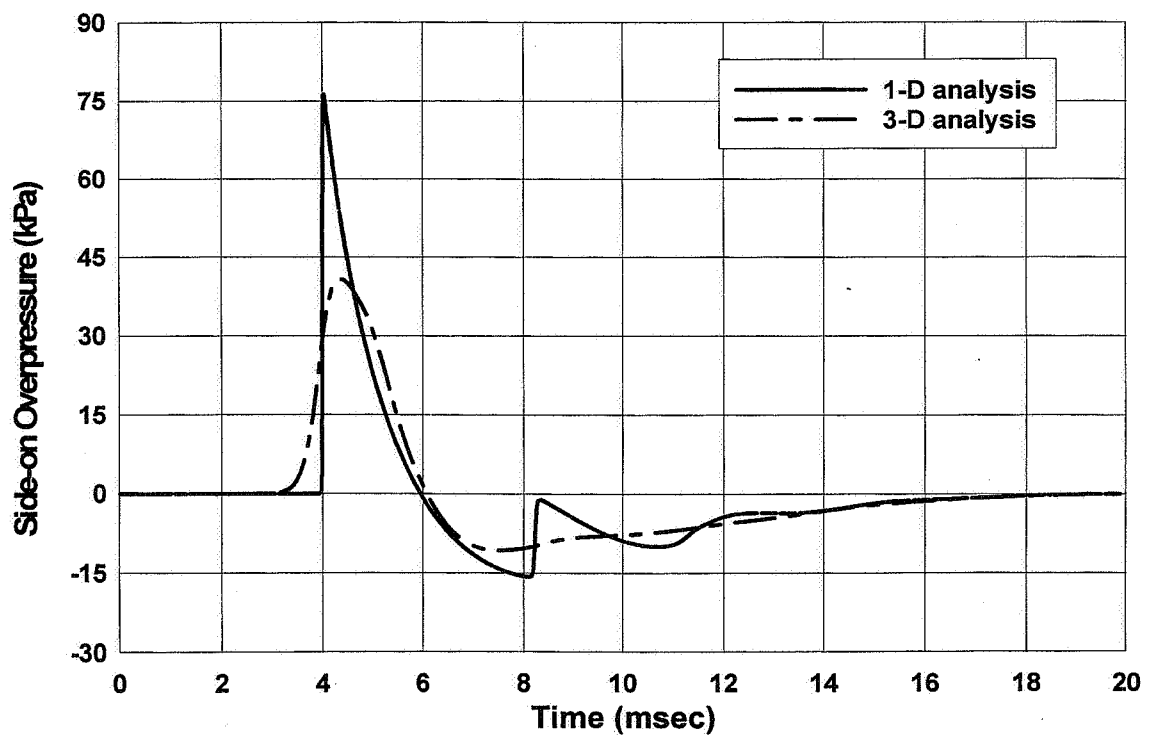


Figure 4.23: Comparison of pressure histories at $3.0 \text{ m/kg}^{1/3}$

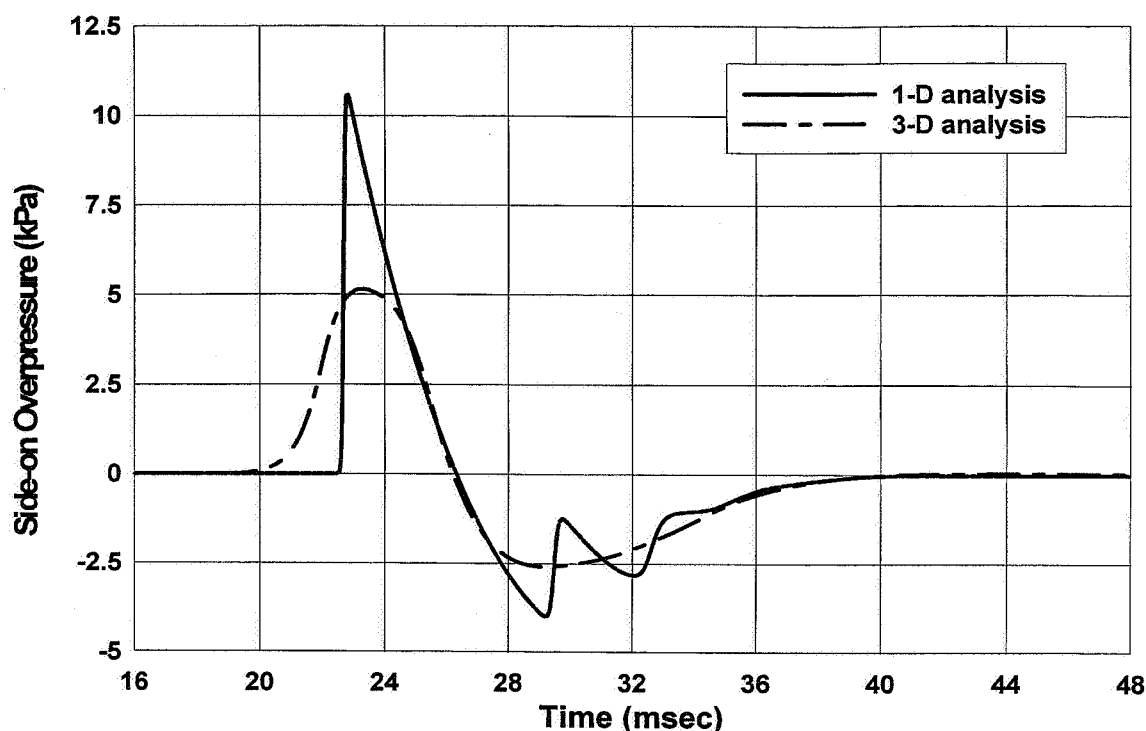


Figure 4.24: Comparison of pressure histories at $10.0 \text{ m/kg}^{1/3}$

4.3 Note on Reflected Blast Parameters

Until the present moment, no mention has been made of reflected blast parameters. It is not possible to predict reflected blast parameters using one-dimensional analysis. Consider a graph of reflected parameters similar to Figure 4.2, each point on the graph would require separate one- and three-dimensional analyses. Similar arguments apply to the establishment of a set of reflected parameters for the problems described in Table 4.1. Clearly, the computational effort needed to verify both side-on and reflected blast parameters would be considerable.

It is suggested that the quality of the solution for reflected parameters would be the same as, or similar to, the side-on parameters. The reason is that the two sets of parameters are linked by the equations of motion and the mesh discretisation—which has an averaging effect. Therefore, provided the solution procedure describes the equations of motion accurately and the grid size is not *too large*, reasonable reflected parameters can be expected. Therefore, it is of little value to calculate reflected blast parameters explicitly for the purpose of validation.

4.4 Comments and Conclusions on Scaled Blast Parameters

The current section has demonstrated:

- The one-dimensional spherically symmetrical procedure of the computational tool Air3d is capable of modelling free-field blast parameters over the whole scaled range of practical interest.
- At small and medium scaled ranges the two-dimensional radially symmetrical procedure has been shown to be capable of reproducing experimentally-derived peak pressures to within 20% and peak impulses which are within 19% of the experimental values. Both of these figures are outside the stated experimental error of 10% but, together with the observation that two of the three mean impulses were within the stated error, the results are viewed as encouraging. It should be noted that the data of Swisdak [50], used for comparison, was from experiments not reported in the reference, and measuring stations were described as being "along the ground surface". In practice, this is usually approximated to a short distance above the surface. Similarly, the quality of the ground surface is not discussed by the reference, particularly with respect to flatness and dust entrainment. The experimental data was processed and scaled to a one pound TNT charge at sea level, but no rationale for the stated experimental errors or the error associated with the data processing is discussed. Because the raw data is not presented by the reference, there is little scope for validating the stated figures.

At large scaled distances the demand on computational resources becomes increasingly important and loss of resolution may result.

- The three-dimensional procedure of Air3d can provide useful peak pressure and specific impulse information over a series of discrete scaled ranges and useful specific impulse information over almost any scaled range. The demand on resources is always high and compromises are almost always necessary.

Chapter 5

Blast Wave Clearing

5.1 Introduction

When a blast wave impinges on a finite structure which is oriented in the direction of the blast, reflected pressures are produced on its surface. These pressures do not persist because the presence of the structure boundaries allow a relief wave to propagate inwards from the edges. This phenomenon is referred to as *blast wave clearing* or *diffraction loading* and has troubled investigators for a long time. This chapter will address this phenomenon and propose an approach to assessing the effects, making use of experimental results and simulation using Air3d.

The scenario is perhaps the simplest possible involving finite structures, because reflections from other surfaces (except the ground) are not considered. Despite this apparent simplicity, the problem is actually extremely complicated and not satisfactorily explained by current guidance (TM 5-1300 [13] for example).

The usual scenario used to describe blast wave clearing is that of Figure 5.1, although similar arguments to those that follow can be applied to *single* finite structures of arbitrary geometry.

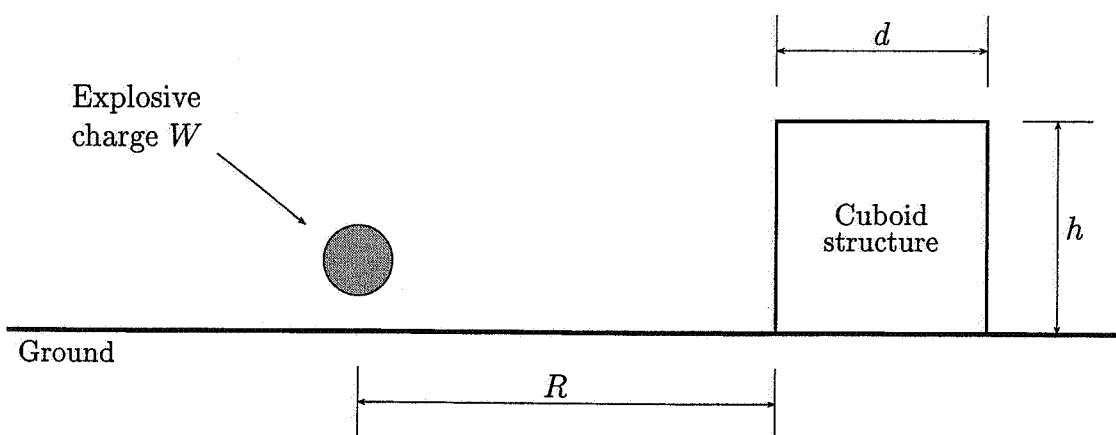


Figure 5.1: Typical blast wave clearing scenario

Considering Figure 5.1, it will be appreciated that the nature of the problem can change considerably, depending on the magnitudes of the parameters identified in the picture. Clearly, the loading produced by a small charge close to a large structure will be different to that from a large charge remote from a small structure, and so on. The possibilities are infinite. It is vitally important that when an engineer has the need to consider this type of problem, he must be aware of the existence of different possibilities. Unfortunately, the process of problem definition is not obvious.

Probably the best way to introduce the subject, and its difficulties, is via the four examples described in Table 5.1 and Figure 5.2, below.

Table 5.1: Examples of blast wave clearing

Example no.	Stand-off R (m)	Scaled stand-off Z ($\text{m/kg}^{1/3}$)	Structure dimensions d (m) $\times$ h (m) $\times$ l (m)	Scaled structure size D ($\text{m/kg}^{1/3}$)
1	10	1	$8 \times 8 \times 16$	0.8
2	10	1	$1 \times 1 \times 2$	0.1
3	80	8	$1 \times 1 \times 2$	0.1
4	80	8	$8 \times 8 \times 16$	0.8
Charge weight $W = 1000$ kg TNT				

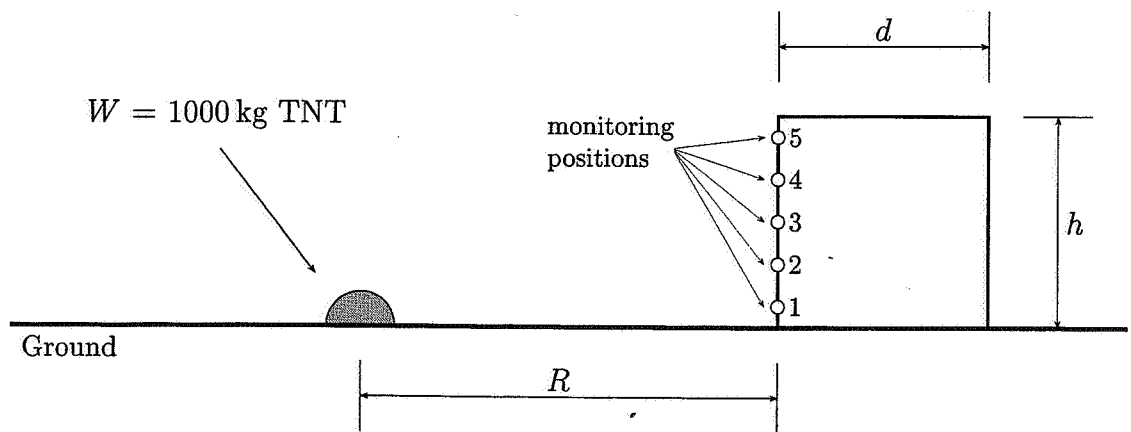


Figure 5.2: Geometry used for examples

It can be seen that a 1000 kg TNT hemispherical charge is considered at two stand-off distances and with two structure sizes. This gives a broad range of scaled distances and structure dimensions. These examples attempt to cover a range of scaled parameters which encompass most cases of practical interest.

The examples are in the form of pressure-time histories from numerical simulations. The actual scaled distances used in the examples (1 and $8 \text{ m/kg}^{1/3}$) are intended to be representative of small and large stand-offs. It should also be borne in mind that these same examples could be produced by maintaining the structure dimensions and varying the charge weight and stand-off.

It is, perhaps, unusual to present the results of numerical simulations before validation of the method. However, the author has been unable to find information of sufficient detail in the literature to describe the various situations adequately. Therefore, the reader is requested to accept this approach for a short while, or else turn to Section 5.2 for reassurance.

Each of the four examples has an accompanying graph which shows the results of three simulations. In each case, one simulation was performed without a structure (allowing side-on pressure histories to be evaluated), a second simulation had the boundary of the problem domain at the front face of the structure (producing *true* reflected pressures), and the third simulation was the clearing scenario. Five pressure monitoring positions were established, vertically and equally spaced above the ground, along the centreline of the structure. These were located in the computational element adjacent to the surface of the structure. The exact geometries and locations of the monitoring positions are described in Figure 5.2.

It will be noticed from Table 5.1 that the structures used in the examples are twice as large in dimension l (out of the plane of the page) than they are in the other two dimensions. Use of this geometry means that the distance travelled by the blast wave from the bottom-centre of the structure (where it first impinges) to the nearest boundary is the same in both horizontal and vertical directions. For the purpose of this section, the scaled structure size D is defined as either $h/W^{1/3}$ or $l/2W^{1/3}$, whichever is the smaller.

The examples were calculated in half-space: the centreline through the charge and the structure formed a plane of symmetry in the problem domain.

Example 1

Points to note from the Example 1 are:

- The scaled distance and hence scaled positive phase duration are small. The scaled structure size is relatively large.
- The blast wave expands hemispherically from the charge until it reaches the centre of the front face of the structure at ground level.
- The reflected wave propagates across the surface of the structure, decaying in a manner broadly similar to a height of burst scenario.
- Referring to Figure 5.3, only two sets of pressure records are distinctly visible. The third set of records, from the clearing calculation, are almost entirely coincident with the true reflected histories. Only the very tail of the pressure records for the locations nearest the edge of the structure are affected by the clearing of the blast wave.

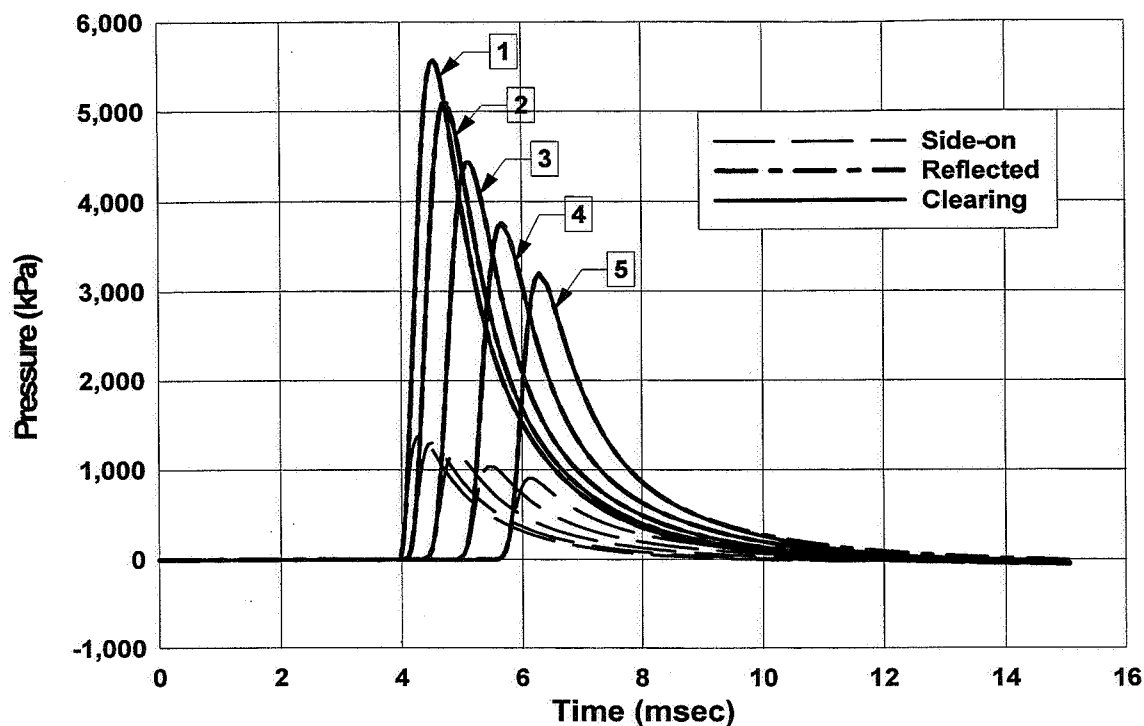


Figure 5.3: Example 1, pressure-time histories

This is an interesting result because it indicates that the same methods used to calculate blast parameters on the ground from a height of burst explosion are suitable for close-in calculations on finite structures.

- The total cleared impulse, taken over the entire front face of the structure, would not be significantly different from the reflected impulse.

In summary, clearing has *not* occurred to a significant extent in this scenario.

Example 2

Having observed that clearing does not take place for large structures loaded by relatively close-in explosions. The next question is how small can the structure become before the height of burst approximation becomes invalid? This is the subject of Example 2, and points arising from it are:

- The scaled distance is small, and the scaled structure size is also relatively small.
- The reflected wave propagates across the surface of the structure almost instantly, because the structure is small. It reaches the edge of the structure before the wave has decayed completely.
- Because the pressure in the free field beyond the structure is the incident (side-on) pressure, the reflected pressures on the structure are relieved across the front face of the structure—from the edge of the reflecting surface towards the centre—at the local (varying) sound speed.

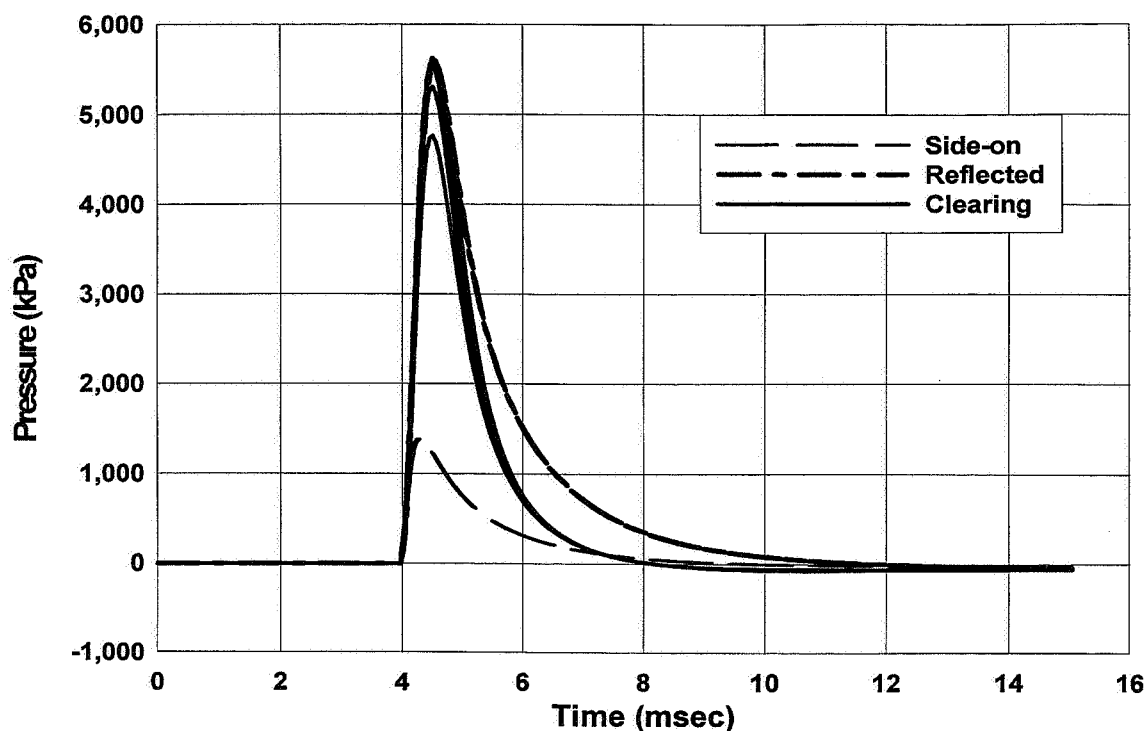


Figure 5.4: Example 2, pressure-time histories

- Referring to Figure 5.4, it can be seen that at 8 msec all the pressure traces from the clearing calculation have crossed the side-on levels, while the true reflected pressures persist.
- It is interesting to note that, as the structure is small, relief occurred almost simultaneously at all five pressure monitoring points.
- The example shows that the height of burst approximation would not be suitable in this case because the loads are relieved *too* early and by *too* much.

Because the clearing records are almost coincident, the scenario allows the possibility of using a representative location on the structure (the bottom-centre, say), together with knowledge of the incident and/or reflected pressures, to establish a relationship between the total cleared impulse and the incident impulse for the particular scaled structure size. This approach will be pursued later.

Example 3

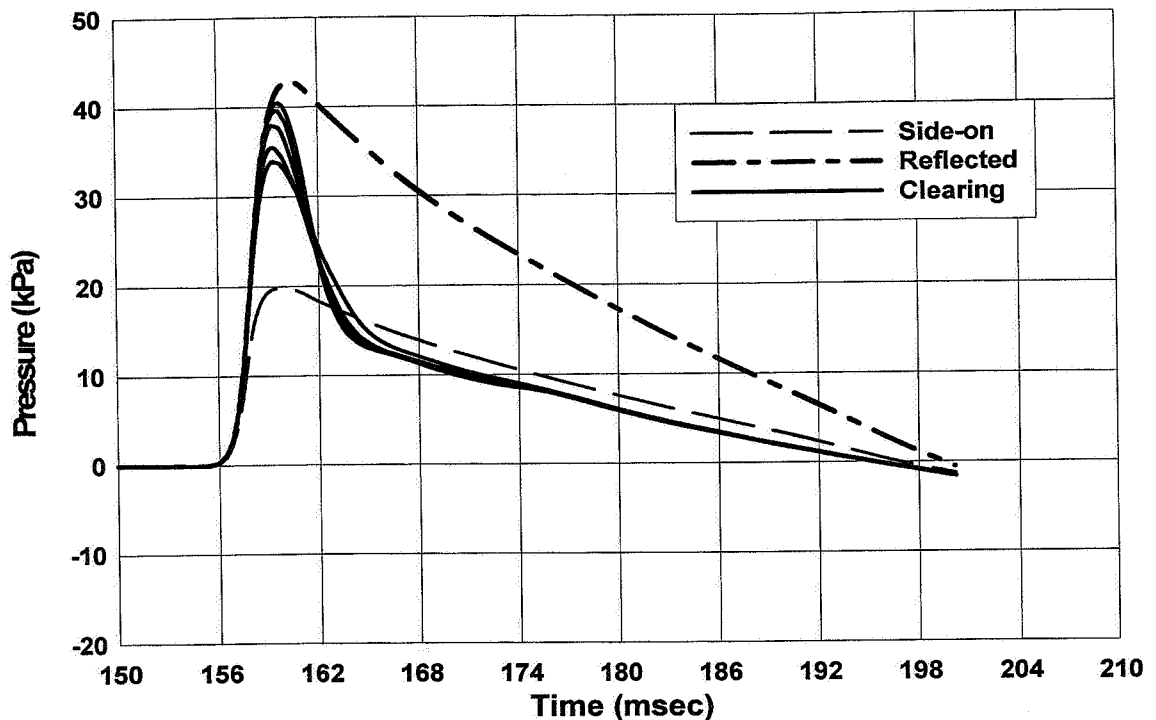


Figure 5.5: Example 3, pressure-time histories

Example 3 considers the opposite extreme of the clearing problem to that of Example 1. Observations arising from this example are the following:

- The scaled stand-off is large and so the scaled positive phase duration is large. The scaled structure size is small.
- The reflected wave propagates across the surface of the structure almost instantly, because the shock front is almost plane (or at least has very little curvature) and because the structure is small.
- Figure 5.5 shows that, in the *clearing* case, the reflected pressures are relieved fairly quickly from the outside edges of the structure, and then slightly under-shoot the side-on pressures for a short period before gradually settling-down and becoming almost coincident with them.
- As in Example 2, because the scaled structure size is small, the pressure traces at the various locations on the front face of the structure are almost coincident on the graph.
- The total cleared impulse experienced over the front face of the structure is very slightly above the side-on value. The very short period of loading where the pressures were at reflected levels and the similarly short period where they were below side-on are insignificant compared to the long duration at approximately side-on values.

In this scenario clearing can be said to have occurred *completely*.

Example 4

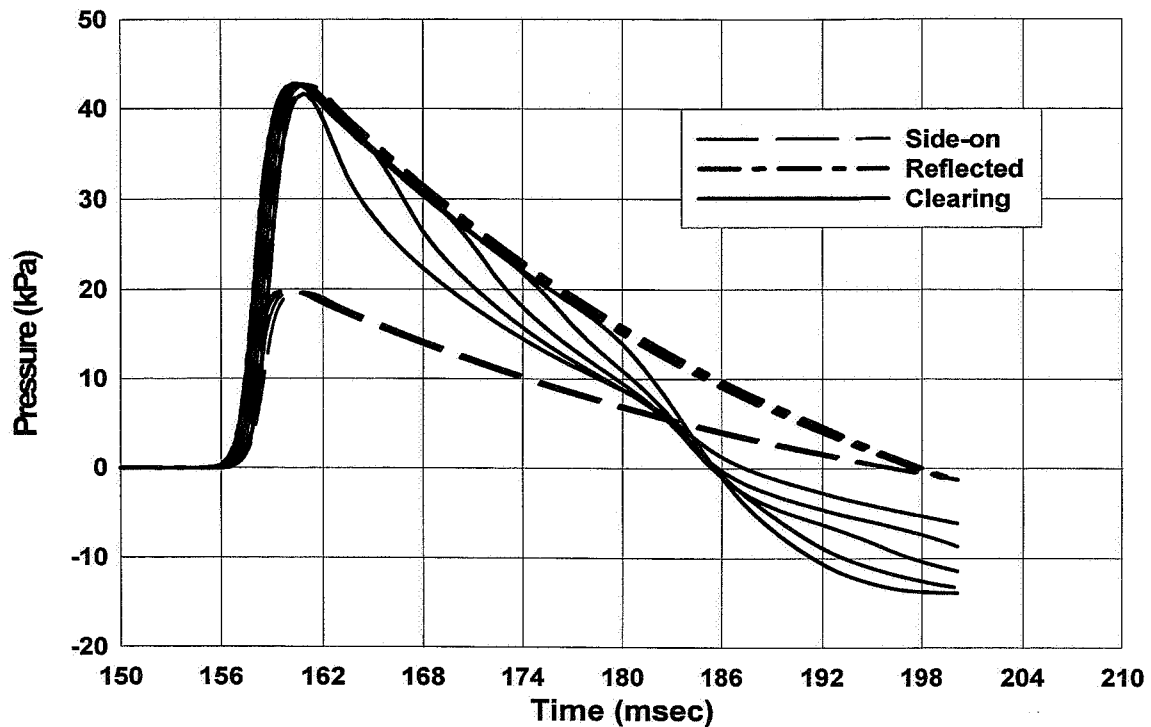


Figure 5.6: Example 4, pressure-time histories

Similar to the relationship between Examples 1 and 2, Example 4 provides the answer to the final question: what happens to the loading in Example 3 if the scaled structure size is increased? Points to note from the example are:

- The scaled stand-off is large. The scaled structure size is large.
- The reflected wave propagates across the surface of the structure almost instantly, because the blast wave is almost plane at large scaled distances.
- Clearing of the pressure occurs progressively from the outside to the inside, as can be seen from the graph in Figure 5.6.
- It can also be seen from Figure 5.6 that the monitoring location nearest the outside of the structure (position 5) reduces in pressure first; the others follow. Interestingly, all the clearing pressures cross the side-on pressures at approximately the same time: 184 msec.

This is an example of *progressive clearing*, starting at the outside and expanding inwards to the centre. It will be appreciated that the average cleared impulse on the front face of the structure will be somewhere between the side-on and fully reflected values. This statement is not very helpful as the reflected impulse could be between a factor of two and an order of magnitude greater than the side-on impulse.

These four examples have given an indication of the difficulty of assessing a scenario of relatively simple geometry. Some of the points raised will be discussed again below.

One further consideration needs to be addressed before the process of validation is considered. It concerns the effect of the length of the structure in the horizontal direction (dimension d , Figure 5.2). Clearly, in scenarios like that of Example 1, where clearing does not take place, the length of the structure is irrelevant. However, in the other examples clearing *has* occurred to a greater or lesser extent.

The implication is that structures which are *short* in the direction of the propagating blast will provide a greater opportunity for the blast wave to expand than those which are *longer*. The best way to test this hypothesis is with two further numerical examples. The obvious choice for this test is the scenario of Example 4. This exhibits progressive clearing across the face of the structure. If the length of the structure is important, this example is best suited to demonstrate it.

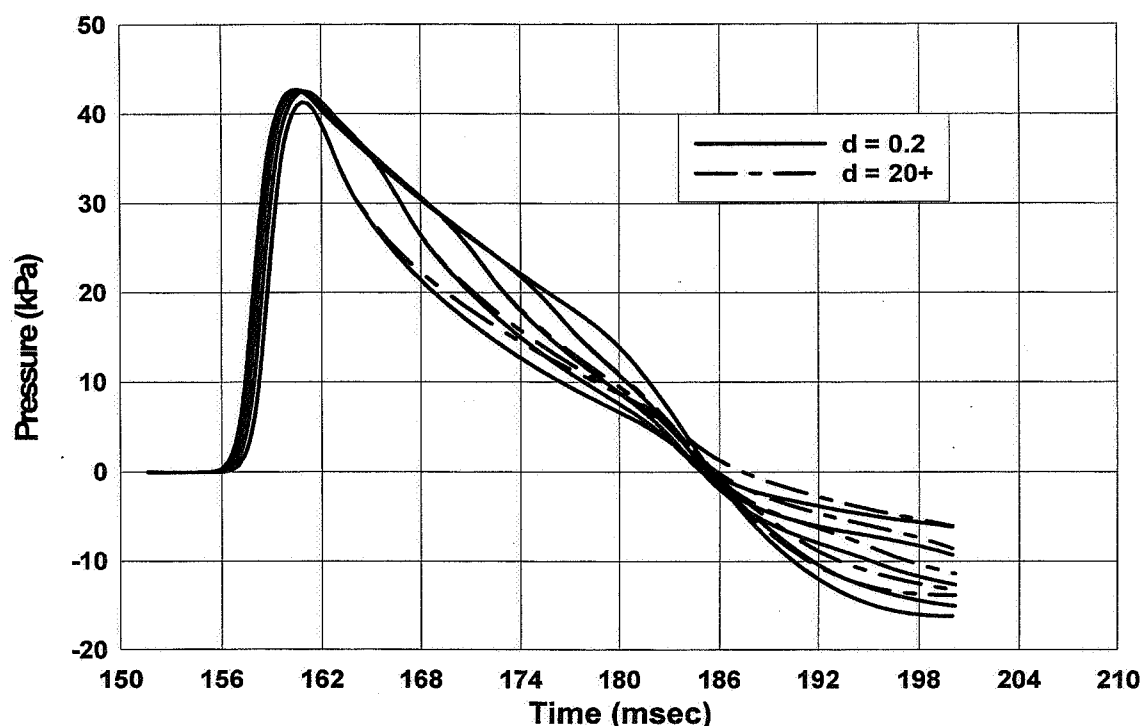


Figure 5.7: The effect of structure length on clearing

The results of two calculations are shown in Figure 5.7. One calculation was performed with a structure of length $d = 0.2$ m; a second used $d > 20$ m. The second structure extended beyond the point in space which the blast wave reached at the final problem time (200 msec): the structure was effectively infinite in the horizontal direction.

Figure 5.7 shows that the clearing exhibited by the infinite structure is virtually identical to the finite structure. Enhanced clearing does take place near the edge of the structure, but not at the base. The effect is barely perceptible.

These analyses have shown that the effect of the length of the structure in the direction of propagation of the blast wave is insignificant. It can be completely ignored in the process of evaluation of *front face* pressures and impulses experienced by finite structures. The above observation greatly simplifies the situation and justifies the approach adopted in the following section.

5.2 Validation

The data against which the numerical simulations were verified were taken from a series of $1/10^{\text{th}}$ scale experiments (Rose et al. [44]). The test structure modelled a Pendine block wall with pressure gauges mounted flush on the front and back. The structure and the location of the pressure gauges G1 to G3 can be seen in Figure 5.8. The side-on pressure gauge G4 was situated at the same radial distance but remote from the structure, (not exactly as shown in Figure 5.8). The experiments were

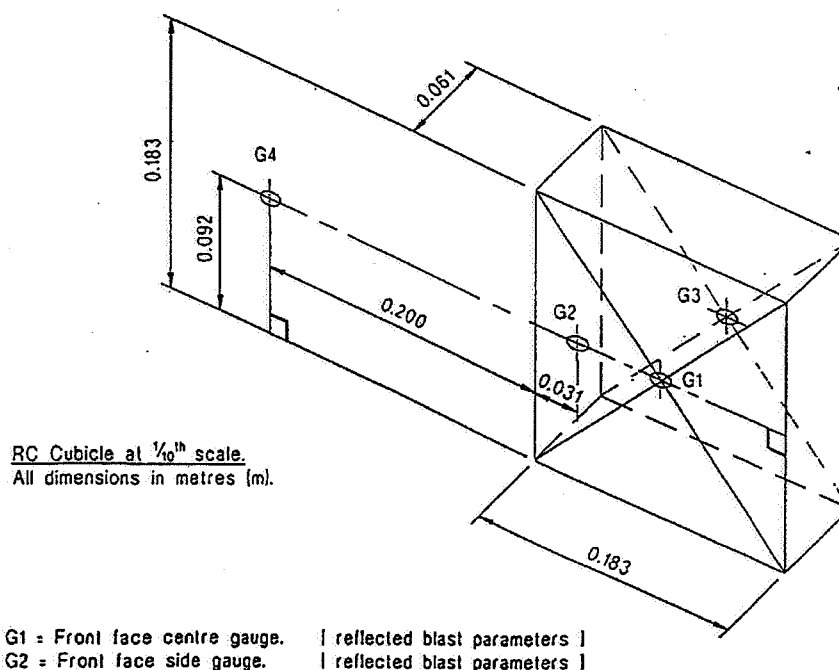


Figure 5.8: Details of the model test structure

performed on a smooth flat reflecting surface, and the charges were detonated above an 8 mm thick steel plate that was made level with the remainder of the reflecting surface. The charges were Demex 100, initiated by electric detonators (Type No. 8*). The detonators contained 1.4 g of primary explosive and so this amount was subtracted from the prescribed charge weight. In each experiment, the height of the centre of the explosive above the ground was 100 mm. The experimental geometry was that of the typical blast wave clearing scenario shown in Figure 5.1. The experimental programme is described in Table 5.2, below.

The approach taken in this study was to model shots 1, 8 and 15 as closely as possible then present the numerical data from monitoring locations G1 and G3 superimposed on the experimental data. These three experiments represent the extremes of the scaled range of the experiments together with one near the centre of the range. The size of the computational grid and the running time needed to analyse the rear face pressures precluded the same thorough treatment for all the experiments. However, front face pressure and specific impulse values at location G1 were calculated for each shot in Table 5.2.

Table 5.2: Experimental Programme

Shot No.	Charge weight (g)	Distance (m)	Scaled distance (m/kg ^{1/3})
1	59.2	0.7	1.796
2		1.0	2.566
3		1.5	3.849
4		2.0	5.132
5		3.0	7.698
6	23.7	0.5	1.741
7		1.2	4.178
8		1.5	5.222
9		2.0	6.963
10		3.0	10.444
11	11.8	0.5	2.196
12		1.0	4.392
13		1.5	6.589
14		3.0	13.177
15		5.0	21.962

In the comparisons which follow, one important assumption, one numerical simplification and one modification to the experimental data have been made. These are described below:

Assumption: A charge equivalence of 1.32 was used to convert the quantity of explosive contained in the Demex and detonator to a TNT equivalent value for the numerical simulations. Thus, in shots 1–5, 59.2 g of Demex is taken as equivalent to $1.32 \times 59.2 = 78.14$ g TNT.

Simplification: The actual charges were spherical and detonated with the centre of the explosive 100 mm above the ground. This was modelled as a hemispherical surface burst, because it allowed the one-dimensional stage of the analysis to progress right up to the front surface of the structure and provided an opportunity to obtain (with high accuracy) reflected pressures on the structure. The alternative would necessitate two-dimensional analyses over large scaled distances. This would have been computationally expensive and would have resulted in slightly degraded results.

Modification: Experimental values of arrival time were not available. Therefore, the experimental records were adjusted so that the arrival of the blast wave at location G1 was the same as the numerical records. This makes the graphs showing the comparison easier to read.

Shot 1

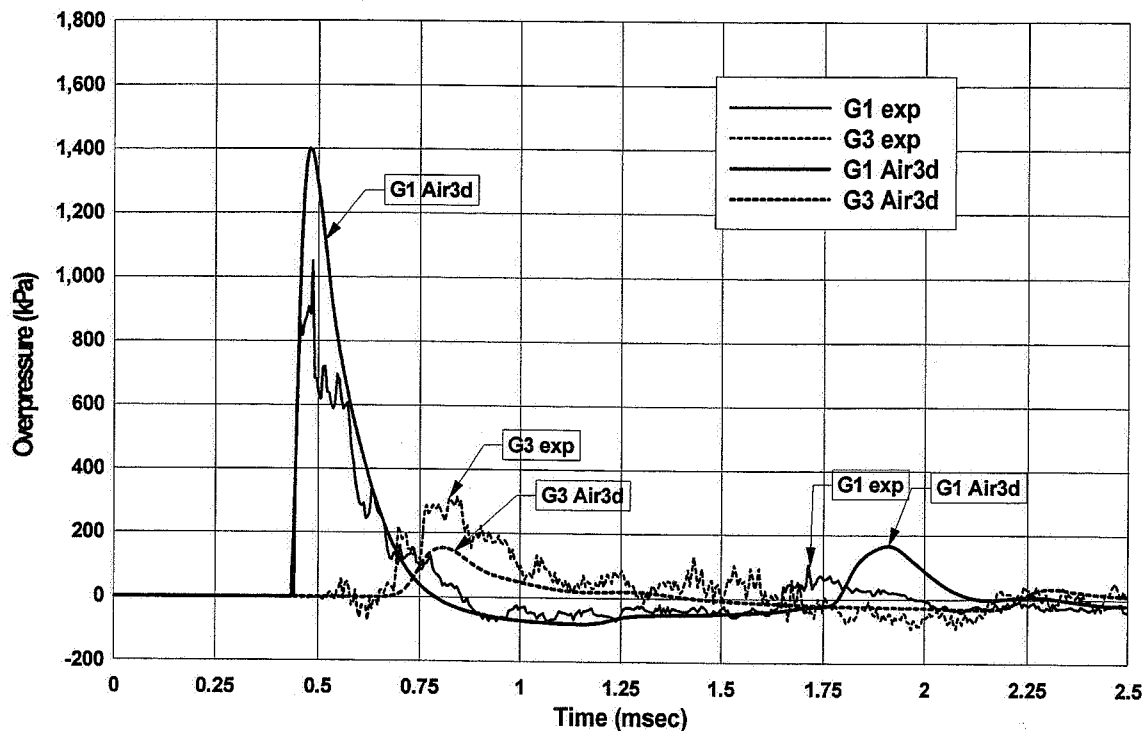


Figure 5.9: Shot 1, comparison of experimental and numerical data

The first comparison is for the shortest charge stand-off, at a scaled distance of $1.796 \text{ m/kg}^{1/3}$. The graph in Figure 5.9 demonstrates reasonable agreement between the experimental and numerical records. There are a number of observations; some will be true for all the comparisons.

- It would appear that the experimental record has suffered some degradation. The difficulty of obtaining good experimental data at relatively short scaled distances was commented on in the introduction. That difficulty is evident here. The experimental programme did not allow for repeat firings so there was no choice of records for comparison.

The calculated peak reflected pressure at location G1 is slightly higher than the measured value. This is an uncommon result in numerical solutions, as the remapping procedure and the smoothing nature of the solution method tend to produce lesser pressure values than might be measured in experiments. This is almost definitely the result of a poor experimental record.

- The pressure on the rear of the structure (at location G3) is considerably less than that on the front. This is to be expected, and it is more pronounced at small scaled distances than at larger ones. The numerical solution does not quite follow the experimental record. Again, the quality of the record may be suspect.
- Because the structure in this scenario is at a relatively short scaled distance, the second shock (described in Chapter 4) is quite evident. It will be noticed

that the numerical record has a slightly larger second peak; it also arrives later than the one in the experimental record. This second peak is one of the reasons why so much more computational effort is required to model the whole record rather than just the front face pressure/impulse. Clearly, if the period of interest is extended to include the duration of the back face pressure, the region of interest—the computational grid—is likewise enlarged. This is the method by which the boundaries of the problem are precluded from influencing the solution at the surface of the structure.

Shot 8

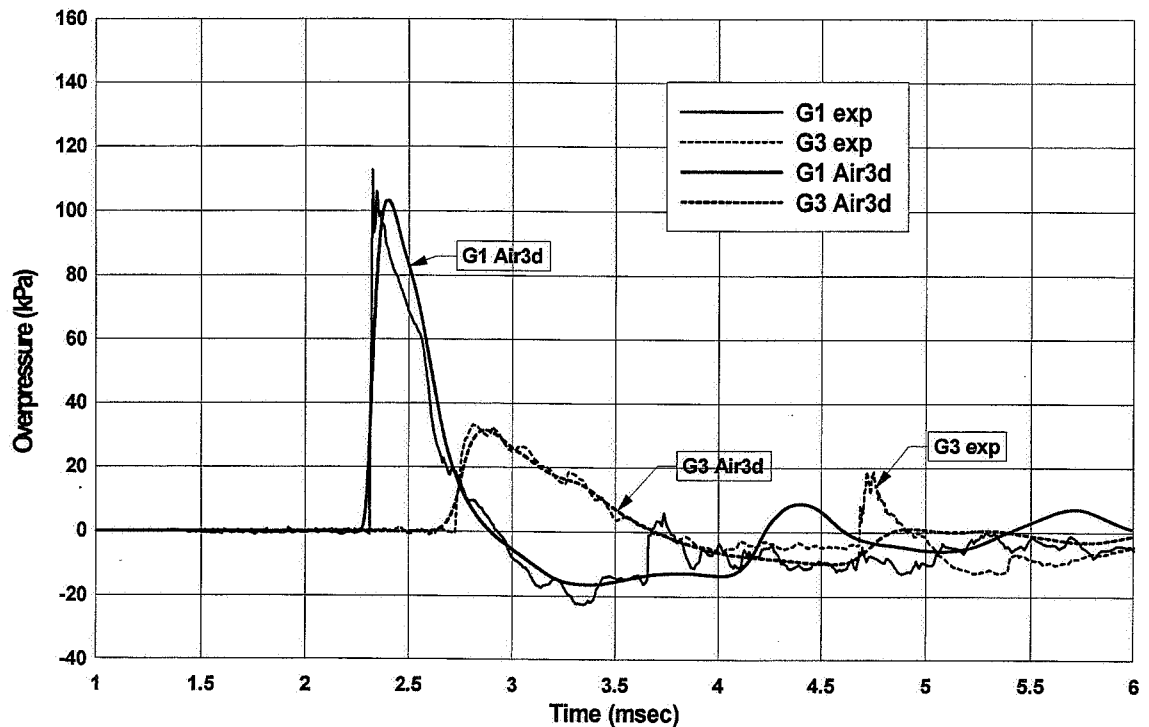


Figure 5.10: Shot 8, comparison of experimental and numerical data

The second comparison, shown in Figure 5.10, is from shot 8 at medium scaled range $5.222 \text{ m/kg}^{1/3}$.

- There is excellent agreement between the simulation and the experiment for the first shock wave at both G1 and G3.
- The comparison between the remainder of the records is not so good. The second peak on the calculated record for G1 is clear in the figure. The experimental record does have a small second peak at $\approx 3.7 \text{ msec}$; it is not obvious whether these are the same physical event.
- There is a strong second peak on the record for location G3. This is somewhat surprising, because if it was a consequence of the geometry, it would appear on the numerical result, to some extent. There is a small rise in pressure on the numerical record, at the correct time. Again, it is not sufficiently pronounced to establish that it is the same event.

Shot 15

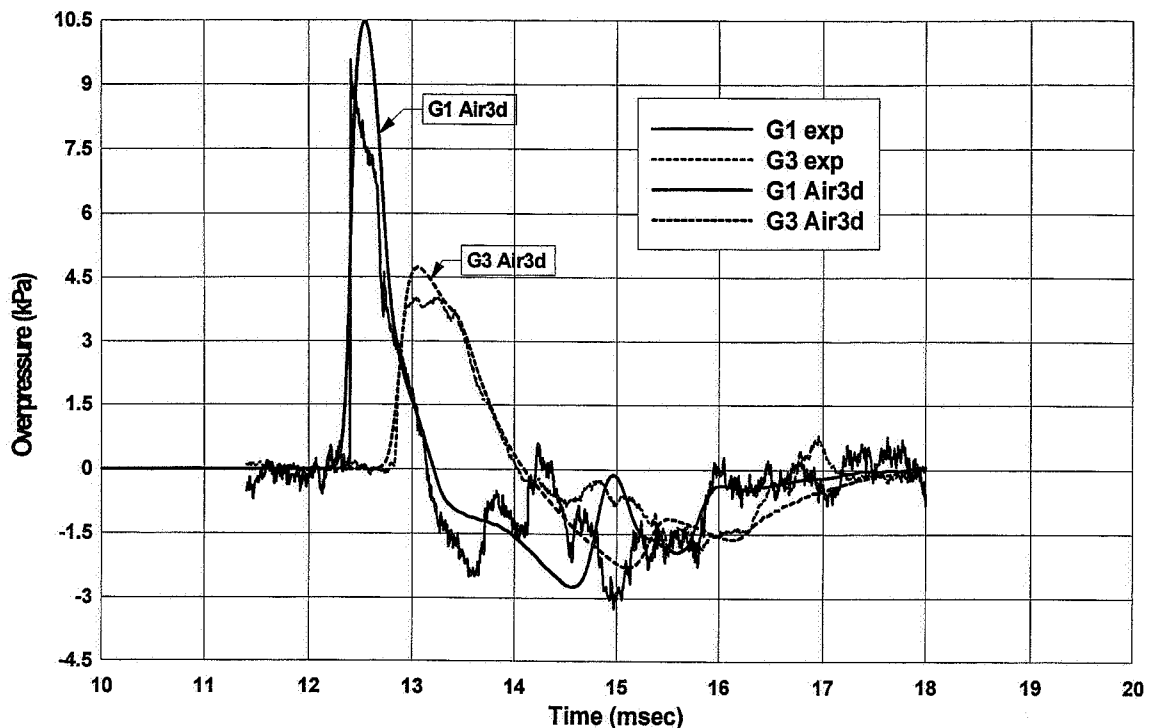


Figure 5.11: Shot 15, comparison of experimental and numerical data

For the final comparison, shown in Figure 5.11, the structure was at the furthest scaled range of all the experiments: $21.962 \text{ m/kg}^{1/3}$.

- The superposition of the first shock waves at both monitoring locations is again very good. It is particularly pleasing to observe the change of slope at $\approx 12.8 \text{ msec}$ on the G1 records. This indicates the transition between reflected and incident pressures on the front of the structure: in other words, clearing.
- The remainder of the records do not compare so well. If it is assumed that the arrival of the second peak in the numerical records is a little late, the small increases in pressure that occur on the experimental records, just prior to the numerical ones, can be satisfactorily explained.

From the three discussions above, it is clear that the approximate treatment of detonation by the computer program Air3d might not be sufficiently accurate to model the arrival and magnitude of the second peak. This is a deficiency, from a modelling accuracy point of view. However, practically, the impulse associated with these aspects of the pressure record are of relatively little interest in the context of structural response and so do not prejudice the results of the approach.

The three comparisons, although not perfect, demonstrate that the first pressure pulse on a single finite rectangular structure can be accurately predicted over a wide scaled range and over two orders of magnitude of reflected pressure. This is an important result.

The next stage of the validation process is to present the results of the analyses for the remaining thirteen geometries described in Table 5.2. These scenarios were examined using a computational grid with 10 mm cubic elements. There are a number of interesting considerations regarding the problem definition for this type of scenario. These were discussed in the worked example of Chapter 3.

Because there were three different charge weights used to obtain the experimental data, there were effectively three different scaled structure sizes and three sets of different experiments. Results of the analyses are presented in Table 5.3 and in the three graphs Figures 5.12 to 5.14.

Table 5.3: Peak pressure and specific impulse data for location G1

Shot No.	Charge weight (g)	Scaled distance ($\text{m/kg}^{1/3}$)	Peak pressure (kPa)		Peak impulse (kPa-msec)	
			Exp.	Num.	Exp.	Num.
1	52.9	1.796	1047.20	1401.51	137.88	182.60
2		2.566	869.60	573.67	117.99	113.21
3		3.849	169.92	208.05	46.98	57.61
4		5.132	138.96	112.09	34.89	36.69
5		7.698	52.00	53.13	18.92	20.84
6	23.7	1.741	1997.60	1325.91	150.02	141.00
7		4.178	160.00	164.92	32.52	41.59
8		5.222	112.56	104.69	26.14	29.52
9		6.963	61.44	61.48	17.39	19.57
10		10.444	36.99	31.61	10.90	11.51
11	11.8	2.196	924.80	706.11	91.36	90.10
12		4.392	179.20	139.10	30.78	33.35
13		6.589	40.06	63.93	10.79	18.34
14		13.177	22.22	21.25	6.99	7.48
15		21.962	10.08	10.50	3.65	4.13

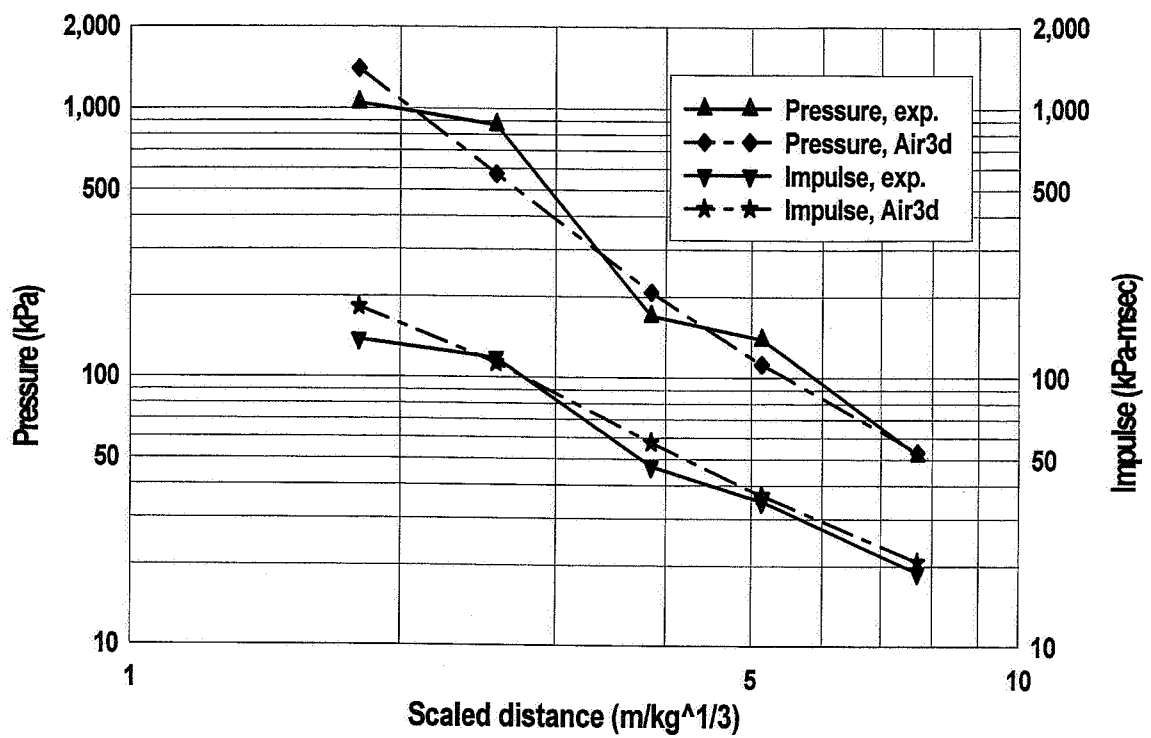


Figure 5.12: Peak pressures and specific impulses at location G1, $W = 59.2$ g

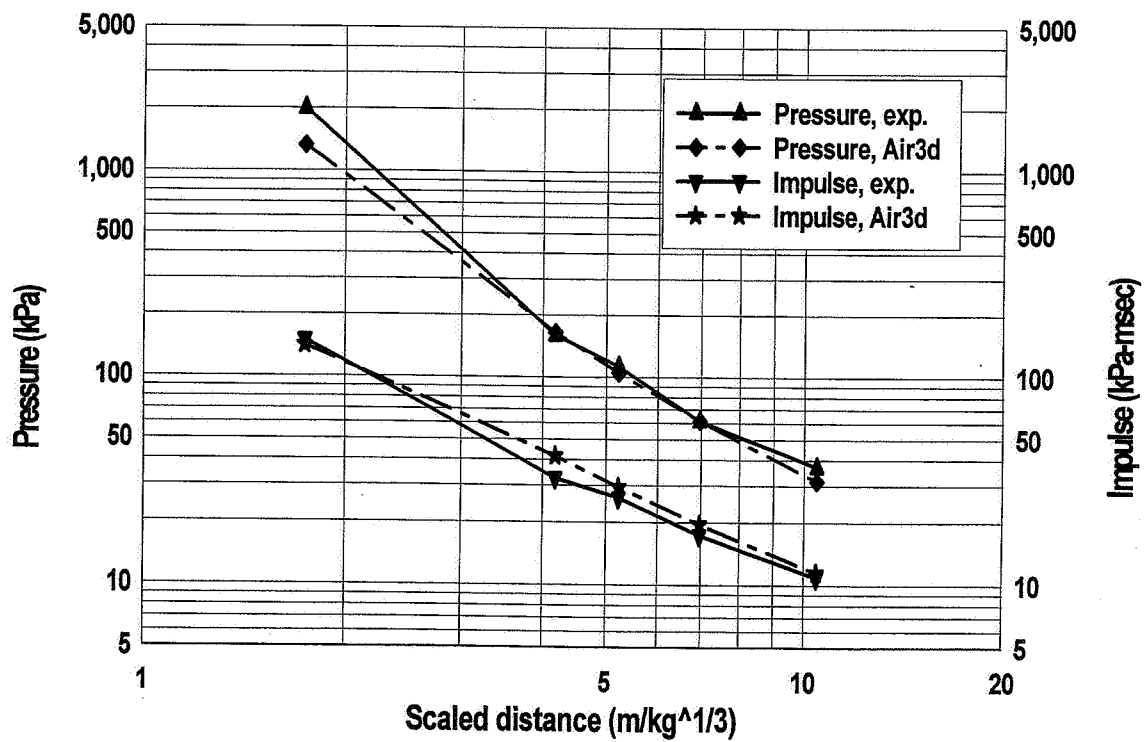


Figure 5.13: Peak pressures and specific impulses at location G1, $W = 23.7$ g

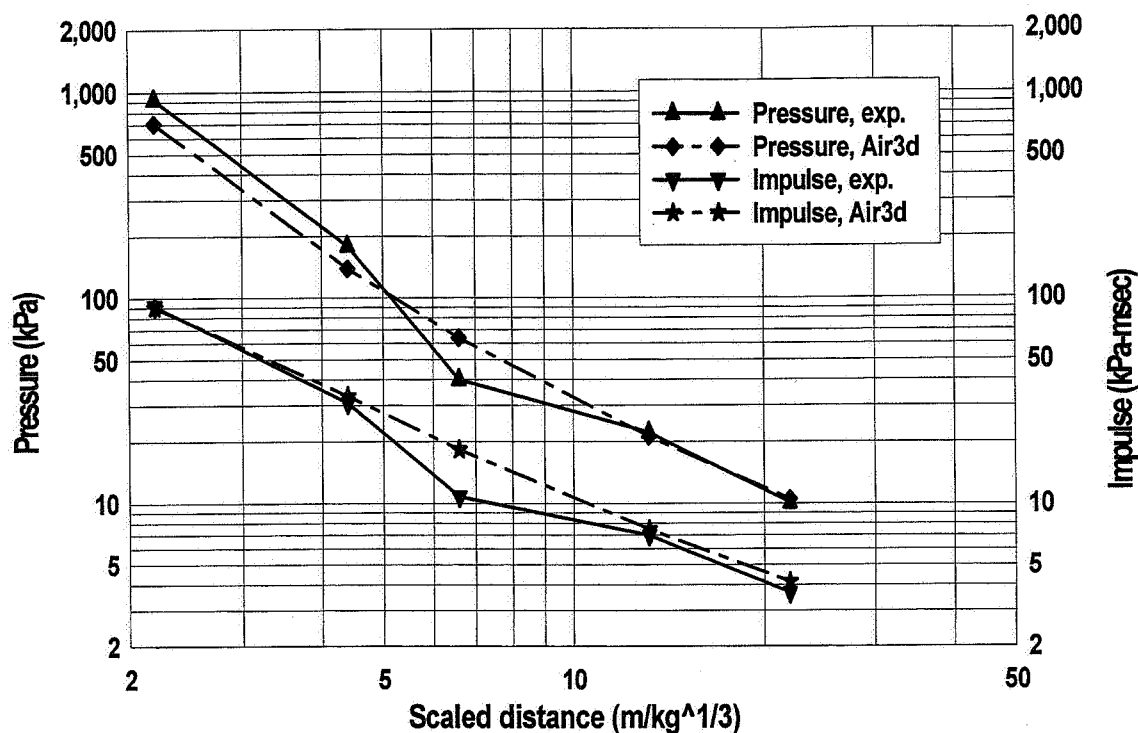


Figure 5.14: Peak pressures and specific impulses at location G1, $W = 11.8$ g

The above graphs (Figures 5.12 to 5.14) demonstrate that good agreement with experimental data can be achieved with the program Air3d. It is interesting to note that Figure 5.12, which covers the smallest scaled ranges, shows the largest variability among the experimental data. The lack of consistency of these very short duration events is evident. Figures 5.13 and 5.14 are much more consistent, with the exception of the third point on Figure 5.14. This peculiar pressure record (shown in Figure 5.15) has led to a result that is necessarily displaced from the trend of the other experimental points. It would appear that the whole of the explosive did not detonate at once. This is not uncommon for very small quantities of unconfined plastic explosive such as used in some shots reported here.

It was noted previously that experimentally-derived data should, ideally, be based on the results of an average of at least five identical experiments. The data used for comparison in the present chapter were from single firings, and the lack of a valid statistical basis for the experimental data implies some limitations on the results in the remainder of this chapter. However, by a similar argument, there are insufficient grounds to indicate that the numerical technique is deficient in its predictions.

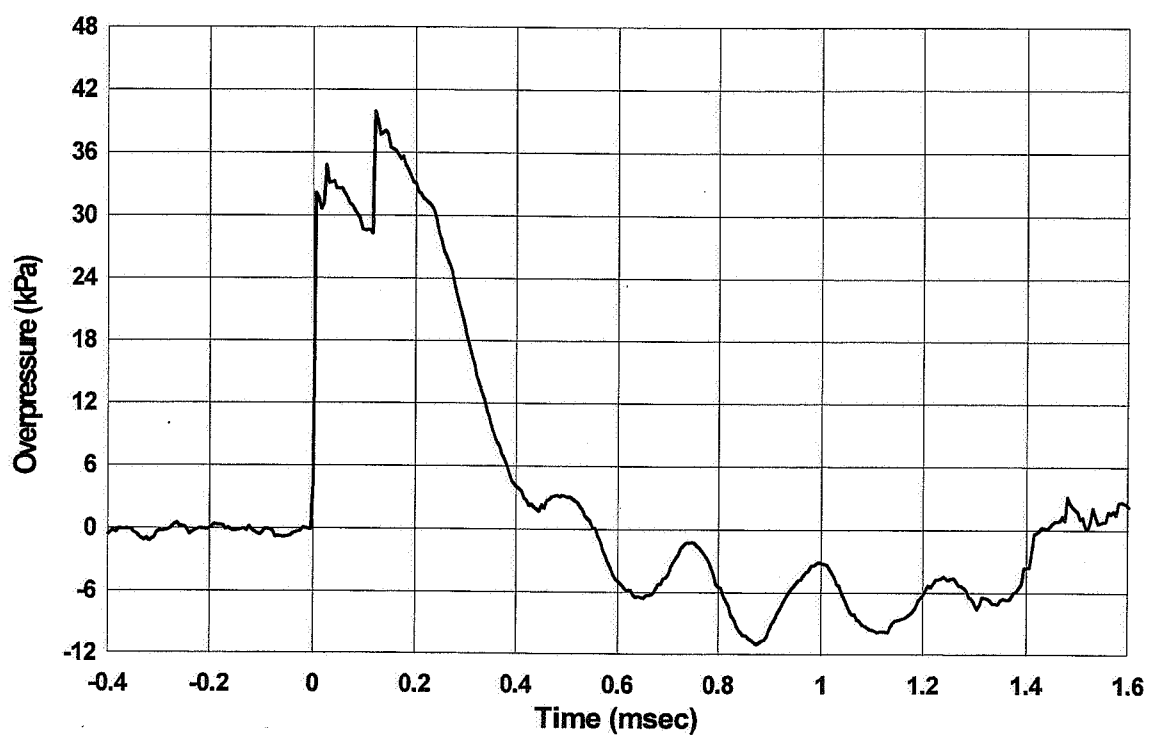


Figure 5.15: Shot 13, a peculiar experimental record

5.3 Development of a Clearing Procedure

5.3.1 Existing Approaches

There are a number of broadly similar approaches for defining the pressure-time history, and hence cleared impulse, on the front face of a rectangular structure. The earliest procedure of which the author is aware is by Glasstone and Dolan [17]. This seems to be the basis of all the other approaches and is reproduced in many guidance documents.

Two of the most commonly used methods are described in Section 2-15 of the technical manual TM 5-1300 [13]: *External Blast Loads on Structures* and the Section *Loads on Structures* of the conventional weapons effects program ConWep [24].

The two methods are very similar and have exactly the same set of assumptions and simplifications (see below). The procedure of TM 5-1300 [13] is performed *by hand* and so will be described in detail. That of ConWep [24] is automated so only a general description, with the appropriate graphs and equations, is presented below.

5.3.2 The Clearing procedure of TM 5-1300 [13]

The document TM 5-1300 [13] suggests that the forces acting on a structure associated with a plane blast wave are dependent on both the peak pressure and the impulse associated with the incident and dynamic pressures acting in the free-field. It describes how the particle or wind velocity, associated with the blast wave, causes a dynamic pressure on objects in the path of the wave. It explains that these dynamic pressures are essentially functions of the air density and particle velocity, and the magnitude of the dynamic pressure, particle velocity, and air density is solely a function of the peak incident pressure. These parameters are given graphically by TM 5-1300 [13]. They are also available as part of the output from the computer program ConWep [24].

It is stated that the establishment of the forces on the front face of an above-ground rectangular structure (without openings) is dependent on:

- The force resulting from the incident pressure.
- The force associated with the dynamic pressure.
- The force resulting from the reflection of the incident pressure impinging upon an interfering surface.
- The force associated with negative phase of the blast wave (which is ignored in the present study).

In order to reduce the complex blast interaction problem to reasonable terms, the procedure assumes:

- The structure is generally rectangular in shape and the front face is parallel to the blast wavefront.

- The incident pressure is 200 psi (1379 kPa) or less. (There is a further limitation resulting from the extent of the incident overpressure axis of Figure 2-192 of TM 5-1300 [13], reproduced here as Figure 5.17; the maximum incident overpressure on this figure is 520 kPa.)
- The structure is loaded in the region of the Mach stem.
- The Mach stem extends above the height of the structure.

These assumptions imply that the validation problems examined previously in Section 5.2 are suited to treatment by the procedure.

The method is based on construction of a representative pressure-time history, shown in Figure 5.16, which is assumed to apply to the whole of the front face of the structure. The undefined quantities in Figure 5.16 are described in the summary below.

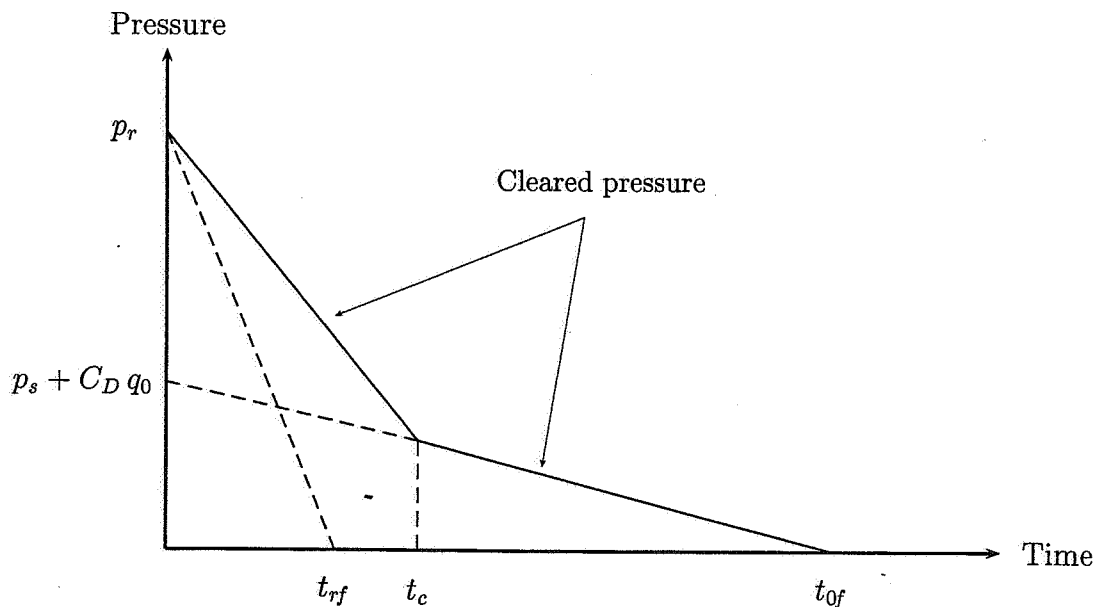


Figure 5.16: Front wall loading (from TM 5-1300 [13])

The pressure history is constructed from known blast parameters and a notional *clearing time* t_c , which is representative of the time taken for the reflected pressure to be relieved progressively from the edges of the structure. The impulse resulting from this construction is compared with the true reflected impulse, which acts as an upper bound, and the lesser of the two values is adopted.

Summary of the Procedure Described by TM 5-1300 [13]

- Obtain peak side-on pressure p_s , side-on specific impulse i_s , peak reflected pressure p_r , reflected specific impulse i_r , and peak dynamic pressure q_0 from scaled blast parameters.
- Establish the structure height H (m) and width W (m).
- Parameter $S = H$ or $W/2$, whichever is smaller. It should be noted that the charge mass (also W) is not mentioned explicitly by the procedure, so there is no confusion.
- Parameter $R = S/G$, where $G = H$ or $W/2$, whichever is larger.
- The velocity of sound c_r in the reflected overpressure region near the front face of the structure is obtained from Figure 2-192 of TM 5-1300 [13] (repeated here as Figure 5.17).
- The clearing time t_c is calculated from the formula:

$$t_c = \frac{4S}{(1 + R)c_r}.$$

- The representative side-on and dynamic pressure duration is given by:

$$t_{0f} = \frac{2i_s}{p_s}.$$

- The representative reflected pressure duration is given by:

$$t_{rf} = \frac{2i_r}{p_r}.$$

- Finally, it is suggested by the procedure that the value of the drag coefficient $C_D = 1.0$ is adequate for the pressure ranges considered.
- The diagram (Figure 5.16) can be constructed from the above information, and the cleared specific impulse compared with the value i_r . The lesser of the two values is the desired result.

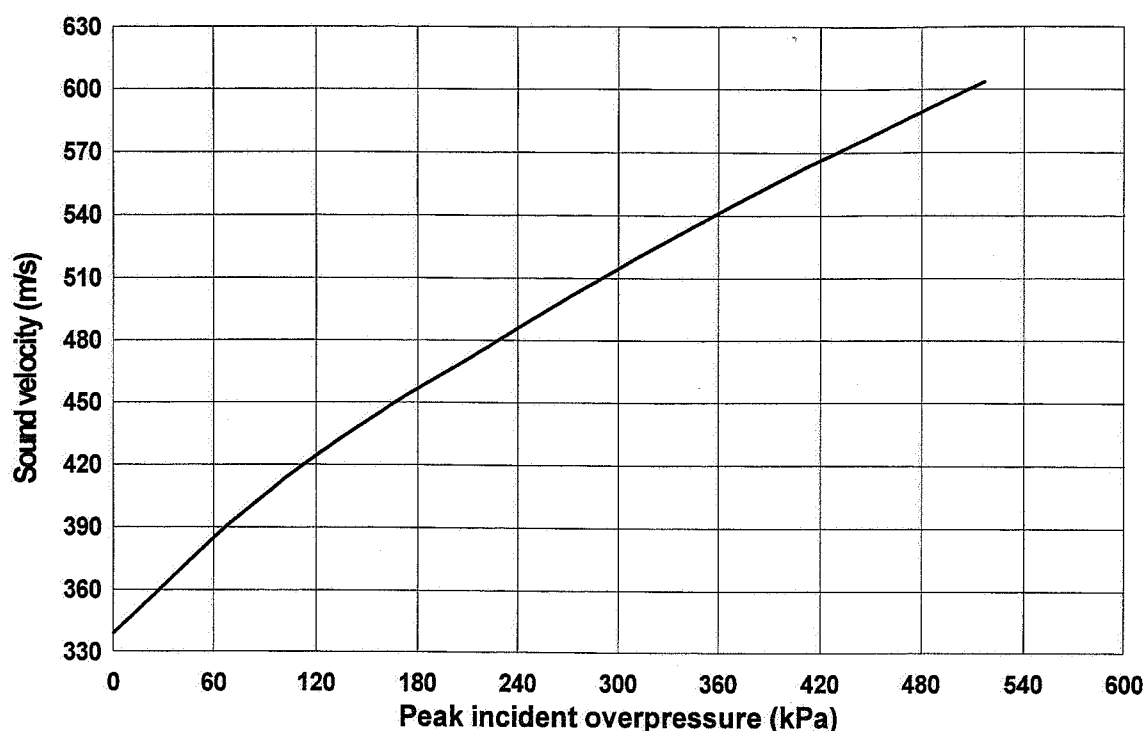


Figure 5.17: Velocity of sound in reflected region versus peak incident overpressure

There are a number of comments concerning this method that should be mentioned.

- The procedure can produce values of the clearing time t_c that exceed the representative side-on pressure duration t_{of} . This cannot happen in reality, and whenever $t_c > t_{of}$ the reflected impulse should be used.
- Figure 5.16 has been faithfully reproduced from TM 5-1300 [13]. On the diagram, t_{rf} is shown to be less than t_c . Because of the way t_{rf} is calculated, it will usually be slightly less than t_{of} . However, in reality, it is not possible for the actual impulse to be greater than the i_r ; because i_r occurs when the whole of the mass flow is brought to rest.
- The procedure does not allow the possibility of the cleared pressure falling below the side-on plus dynamic pressure level. It can be seen from Examples 2 to 4 (Figures 5.4 to 5.6) at the start of the present section that this does indeed occur. The inability of the procedure to follow this is a major deficiency.

The above procedure has been applied to the validation problems of Section 5.2. A worked example using shot 10 (which is roughly in the middle of the scaled range) is given below. The complete set of results is given in Table 5.4, and graphs showing a comparison with the experimental data from Table 5.3 and the procedure of ConWep [24] (described next) are presented in Figures 5.20 to 5.22, below.

Worked Example, Shot 10 from Section 5.2

- The mass of the charge used for shot 10 was 23.7 g of Demex 100 plastic explosive. Scaled blast parameters for Demex do not exist, so a TNT equivalence of 1.32 was assumed, giving 31.28 g TNT. The stand-off distance was $R = 3.0$ m and the height of the centre of the charge was 100 mm above the ground. All the relevant blast parameters were obtained from ConWep [24], assuming a hemispherical surface burst.

Peak side-on pressure	= 15.85 kPa
Side-on specific impulse	= 10.24 kPa-msec
Peak reflected pressure	= 33.89 kPa
Reflected specific impulse	= 19.69 kPa-msec
Peak dynamic pressure	= 0.863 kPa

- The structure height $H = 0.183$ m and the width $W = 0.183$ m.
- The parameter $S = W/2 = 0.0915$ m.
- The parameter $G = H = 0.183$ m, so the ratio $R = S/G = 0.5$.
- The velocity of sound in the reflected region is obtained from Figure 5.17 which gives $c_r = 351.39$ m/s.
- The clearing time t_c is then:

$$t_c = \frac{4 \times 0.0915}{(1 + 0.5) \times 351.39} = 0.694 \text{ msec.}$$

- The representative side-on and dynamic pressure duration is:

$$t_{of} = \frac{2 \times 10.24}{15.85} = 1.292 \text{ msec.}$$

- The representative reflected pressure duration is:

$$t_{rf} = \frac{2 \times 19.69}{33.89} = 1.162 \text{ msec.}$$

- The pressure-time history constructed from the values calculated above is shown in Figure 5.18. The average cleared impulse i_{av} (the area under the solid line) is 16.76 kPa-msec, which is less than the reflected impulse and represents an expected result.

The parameter p_c in Table 5.4 does not appear on the diagram in Figure 5.16; it has been introduced in order to make construction of the cleared pressure history easier and is the pressure at the clearing time t_c , the point where the two solid lines of Figure 5.16 intersect.

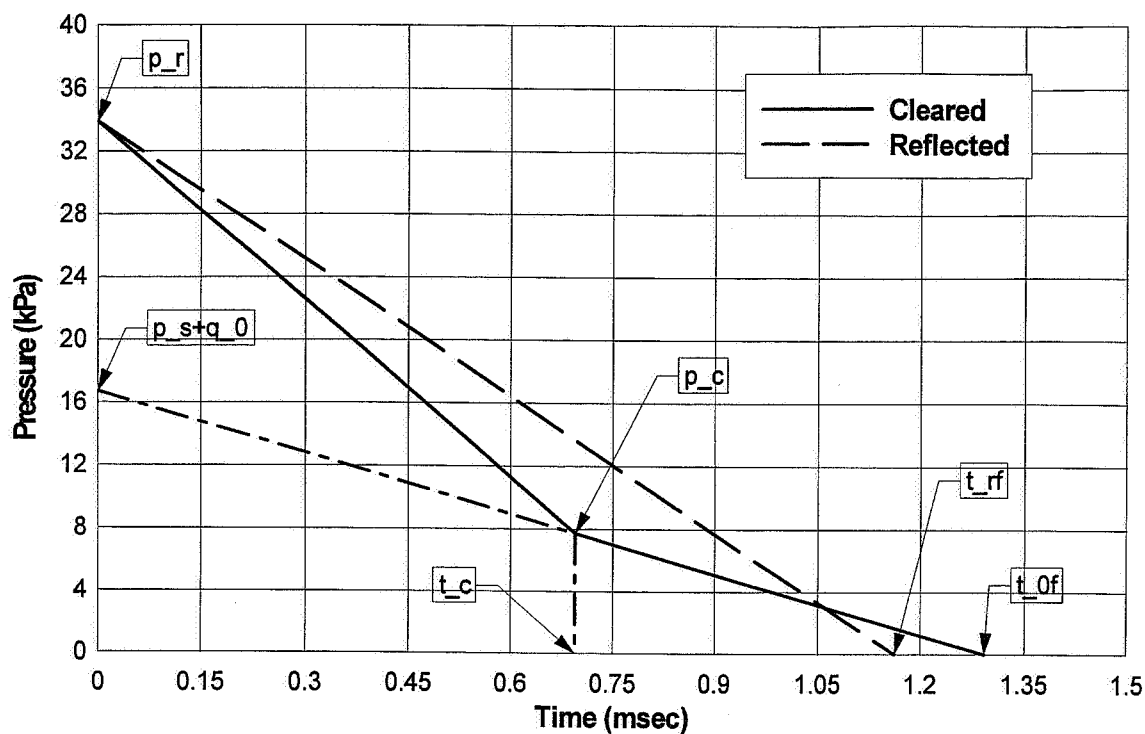


Figure 5.18: Constructed pressure-time history for shot 8, Section 5.2

Table 5.4: Clearing procedure data, TM 5-1300 [13]

Shot no.	p_s (kPa)	i_s (kPa-msec)	p_r (kPa)	i_r (kPa-msec)	q_0 (kPa)	c_r (sec)	t_c (sec)	t_{of} (sec)	t_{rf} (sec)	p_c (kPa)	i_{av} (kPa-msec)
1	450.1	69.02	1939	199.3	440.8	577.9	0.423	0.307	0.206	N/A	[†] 199.3
2	198.9	49.37	663.6	128.7	108.7	465.7	0.524	0.496	0.388	N/A	[†] 128.7
3	83.85	34.51	221.4	80.02	22.07	401.8	0.607	0.823	0.723	27.77	78.66
4	48.58	28.86	115.3	57.81	7.756	376.4	0.648	1.188	1.003	25.59	50.76
5	24.78	18.58	54.46	37.05	2.084	358.2	0.681	1.500	1.360	14.66	29.54
6	483.8	52.39	2133	152.8	495.9	590.8	0.413	0.217	0.143	N/A	[†] 152.8
7	71.34	23.69	181.7	53.71	16.23	393.4	0.620	0.664	0.591	5.789	58.27
8	47.09	19.50	111.2	41.78	7.301	375.2	0.650	0.828	0.751	11.68	40.99
9	29.00	15.04	64.57	30.45	2.837	361.4	0.675	1.037	0.943	11.11	27.56
10	15.85	10.24	33.89	19.69	0.863	351.4	0.694	1.292	1.162	7.731	16.76
11	283.1	33.32	1053	90.75	202.1	506.8	0.482	0.235	0.172	N/A	[†] 90.75
12	64.78	17.98	161.8	40.21	13.50	388.7	0.629	0.555	0.497	N/A	[†] 40.21
13	31.70	12.54	71.18	25.63	3.377	363.5	0.671	0.791	0.720	5.315	25.99
14	11.57	6.474	24.33	12.21	0.463	348.1	0.701	1.119	1.004	4.497	11.01
15	6.061	3.933	12.39	7.152	0.128	343.9	0.710	1.298	1.154	2.805	6.216

[†] indicates $t_c > t_{of}$, therefore $i_{av} = i_r$

5.3.3 The Clearing procedure of ConWep [24]

The procedure used by ConWep [24] is as follows:

- Using the dimensions supplied to the program by the user, the routine divides the front face of the structure into a 64×64 grid and calculates the angle of incidence α and the incident (or side-on) pressure p_{so} at each point on the grid.
- The program interpolates between tabulated values from the graph Figure 3-3 of TM 5-855-1 [16] (reproduced here as Figure 5.19, though here the individual lines of incident pressure have not been labelled) to obtain the reflected overpressure coefficient $c_{r\alpha}$ and hence the peak reflected pressure $p_{\alpha s}$ given by:

$$p_{\alpha s} = c_{r\alpha} \times p_{so}.$$

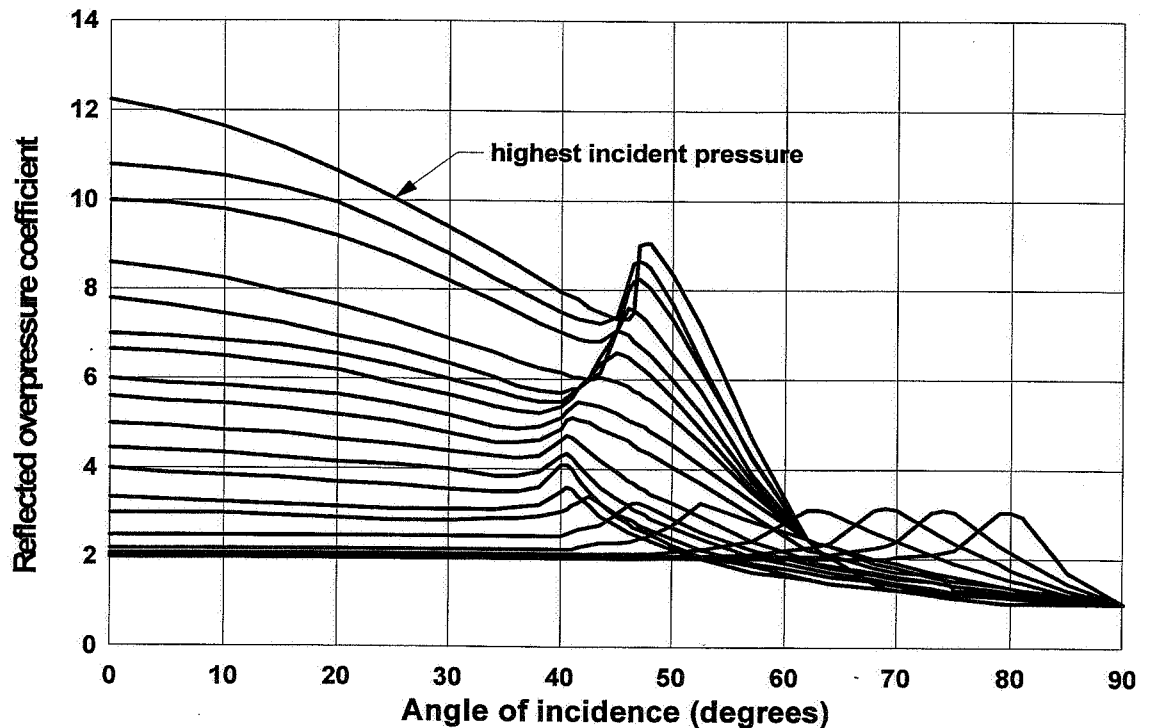


Figure 5.19: Reflected overpressure coefficient versus angle of incidence

- The values $p_{\alpha s}$ are then averaged to give a figure p_r that is representative of the whole surface. This is the value used in the same representative construction (Figure 5.16) as TM 5-1300 [13].
- The clearing time t_c is calculated from:

$$t_c = \frac{3S}{U},$$

where S is the same as above, but U is the velocity of the shock front. This is the clearing time described by Glasstone and Dolan [17] and the first edition of TM 5-1300 [49], which has been superseded by the formula given previously.

- The representative construction Figure 5.16 gives the average specific impulse on the structure. This is compared with i_r and the lesser value is adopted—the same as previously in TM 5-1300 [13].

Average specific impulse values calculated using ConWep [24] for the validation problems described in Section 5.2 are given in Table 5.5 and are presented in Figures 5.20 to 5.22 together with the experimental data and the average values from the procedure of TM 5-1300 [13] given previously.

Table 5.5: Average impulse calculated by ConWep [24]

Shot no.	Peak average cleared impulse i_{av} (kPa-msec)
1	179.9
2	112.7
3	57.83
4	39.06
5	24.18
6	137.5
7	40.82
8	30.13
9	21.17
10	13.43
11	79.73
12	31.71
13	18.92
14	8.663
15	5.037

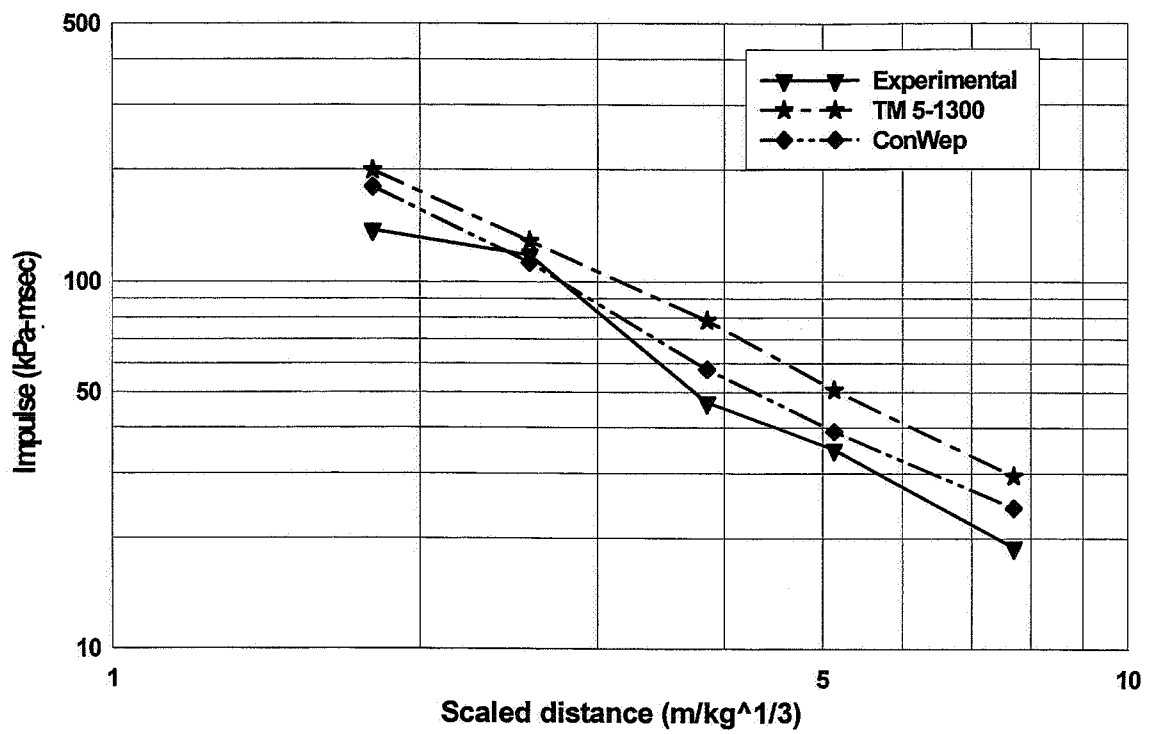


Figure 5.20: Experimental impulse, TM 5-1300 [13] and ConWep [24], $W = 59.2$ g

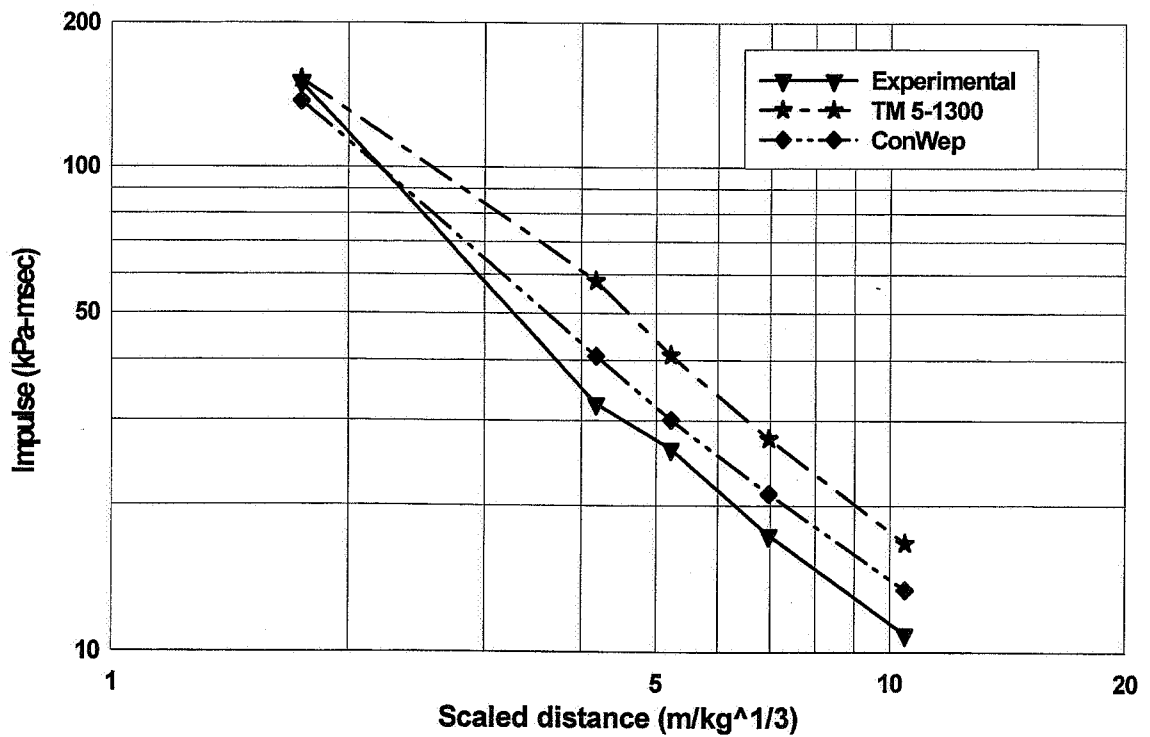


Figure 5.21: Experimental impulse, TM 5-1300 [13] and ConWep [24], $W = 23.7$ g

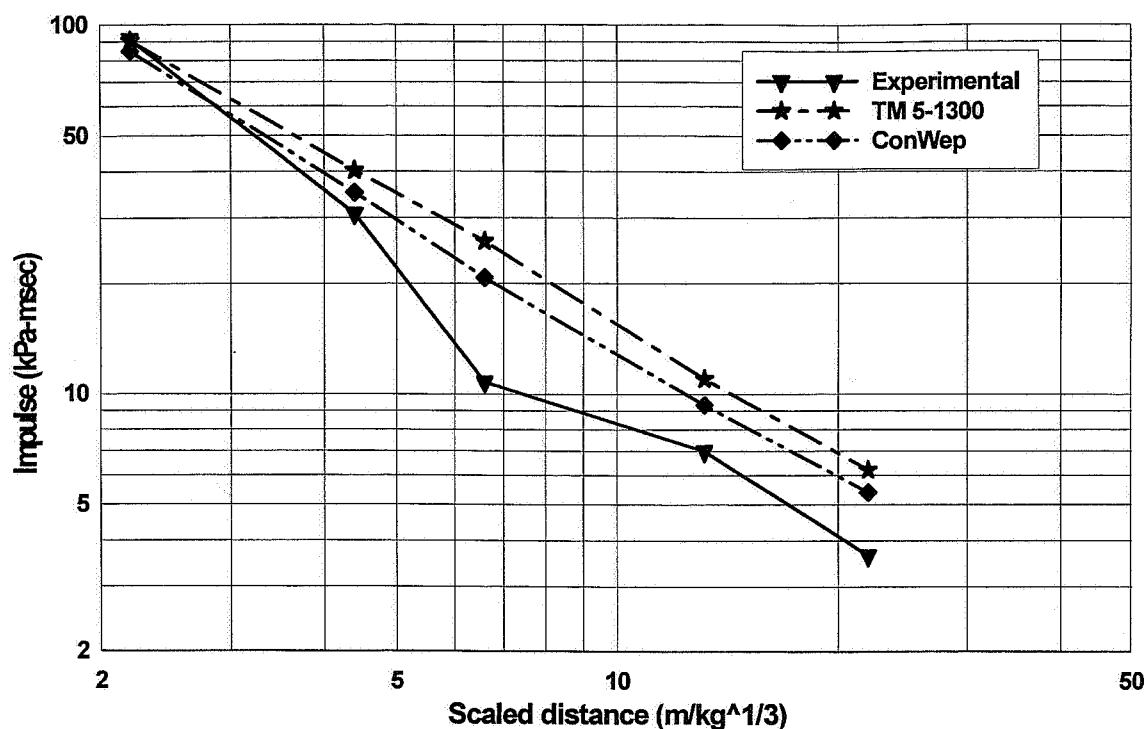


Figure 5.22: Experimental impulse, TM 5-1300 [13] and ConWep [24], $W = 11.8 \text{ g}$

It should be borne in mind that Figures 5.20 to 5.22 compare the *average* impulse, calculated by TM 5-1300 [13] and ConWep [24], with the *actual* impulse, measured experimentally, at the centre of the test structure. It would be expected that the measured value might be slightly higher than the calculated average values, because the measuring location (in the centre) is remote from the edges of the structure and would clear relatively late compared to most of the remainder of the front face. It can be seen from Figures 5.20 to 5.22, however, that the two empirical procedures have consistently overpredicted the cleared impulse, although the method of ConWep [24] appears to be closer to experiment for this particular scenario. Bearing in mind that the procedures were produced for design purposes, it is, perhaps, not surprising that they overpredict slightly. Whether or not this result is desirable or intentional is not discussed by either method. The overprediction of impulse by these procedures has also been observed by other investigators such as Johnson [25].

5.3.4 Comment on the Existing Procedures

The method of calculating the front face loads on finite structures has not changed significantly since it was first described by Glasstone and Dolan [17] for the effects of nuclear weapons. As mentioned previously, the method recognises the fact that reflected pressures occur instantaneously and then reduce, by the process of clearing, to some lesser value. Glasstone and Dolan [17]—and all the existing procedures—suggest that this lesser value is the sum of the incident pressure plus a component of the dynamic pressure, depending on the drag coefficient of the structure geometry.

The dynamic pressure is defined by $q = \frac{1}{2}\rho u^2$, where u and ρ are the upstream velocity and density, respectively, just ahead of the stagnation region in front of

the structure; they are associated with the incident pressure at each moment of the upstream pressure-time history.

It is possible to imagine the applicability of this approach, if the structure is a *considerable distance* from a nuclear explosion, the incident overpressure is very low and the duration very long. This would give rise to a situation of almost steady incompressible flow behind the shock front, because the density in the shock wave would be only slightly above atmospheric and the velocity would vary relatively slowly. However, for the type of situations considered in this thesis, the notion of dynamic pressure is extremely unhelpful, and the suggestion that the side-on pressure acts as a lower bound to the cleared pressure, although it appears logical, is nonetheless untrue.

The fact that the cleared pressure reduces from the reflected value to another below the side-on pressure is evident from Example 4 (Figure 5.6) and again in Figure 5.23, which shows Shot 14 from the experimental programme. (Shot 15 would have provided a better demonstration but was not such a good quality record.)

Perhaps it should not be surprising that the pressure reduces below the incident level, because it is (briefly) compressed to the reflected pressure and then has the relatively low pressure (side-on) air to expand into beyond the edges of the structure.

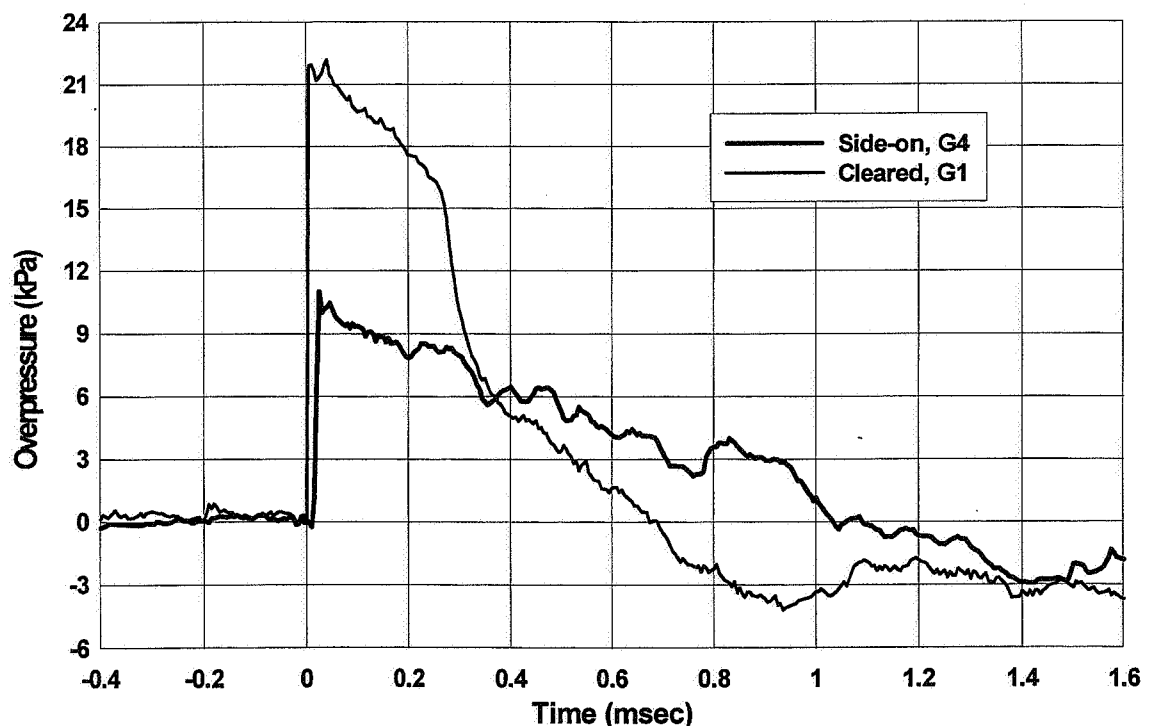


Figure 5.23: Side-on and cleared pressure histories, an example from Shot 14

5.3.5 A New Computationally-Based Empirical Approach to Clearing

A possible new approach to the problem of blast wave clearing is to use a computational method, based on Air3d, to establish the average specific impulse on the front face of a series of generic finite structures, of varying scaled dimensions, over the whole range of scaled distances of interest to the structural engineer. The results could then be presented in a manner that would allow structures of virtually any practical size and stand-off distance to be considered. This is the approach of the present section.

In an attempt to keep the whole procedure as simple as possible, the scenario used was that of Figure 5.2: the charge was hemispherical and the structure dimension l was twice that of the other two dimensions. Choosing an aspect ratio of 2:1 ensured that the results would be conservative for rectangular structures of arbitrary aspect ratio. This is because a rectangular structure with an aspect ratio of 2:1 is square when considered in half-space. This means that a different geometry structure, having the same surface area and a different aspect ratio, must have one side shorter than the equivalent square, and will, therefore, clear more quickly.

The only difference from the geometry of Figure 5.2 was the location of the pressure monitoring points. These were distributed evenly across the face of the structure (see Figure 5.24) to allow a reasonable approximation of average impulse to be established.

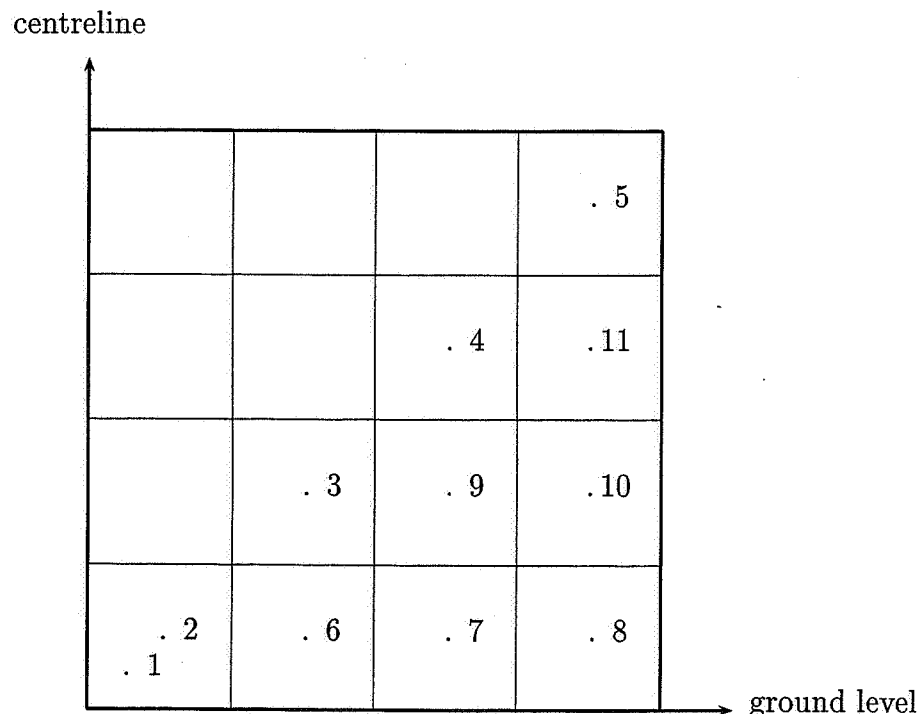


Figure 5.24: Location of monitoring positions on rectangular structure

It will be noticed from Figure 5.24 that location 1 is sited near the bottom centre of the structure. (It was actually situated in the computational cell adjacent to the bottom centre.) This was the case for all the calculations, and it was used as a reference. The average specific impulse over the whole of the front face of the structure i_{av} was calculated from the individual grid impulses i_n using:

$$i_{av} = \frac{1}{16} \left[\sum_{n=2}^5 i_n + 2 \sum_{n=6}^{11} i_n \right].$$

which utilises the fact that the problem has two planes of symmetry.

The test matrix comprised a single charge weight $W = 1000$ kg TNT, four structure sizes (1.0, 2.0, 4.0 and 8.0 m) and six stand-off distances (0.5, 1.0, 2.0, 4.0, 8.0 and 16.0 m). This gave twenty-four combinations of structure size and stand-off distance. For each of the twenty-four combinations, three Air3d calculations were performed. The first of the calculations was performed without the structure and gave the average impulse passing the measuring points arising from the side-on pressures only. The second calculation used a boundary of the computational domain located at the surface of the structure and extending a considerable distance (roughly three times) above it. This gave *true* reflected pressures. Finally, the clearing problem was evaluated. Seventy-two calculations were performed in total.

The method described above is exactly that outlined earlier for the examples discussed at the beginning of this chapter. The examples were taken from this test matrix. Results of the above calculations can be seen in Figures 5.25 to 5.28, below. There are two important points to arise from these graphs.

- The values of average cleared impulse at $Z = 0.5 \text{ m/kg}^{1/3}$ are coincident with the reflected impulse values in each case, except for $D = 0.1 \text{ m/kg}^{1/3}$ where it is very slightly below. This demonstrates that clearing does not occur at short scaled distance on finite structures of any practical size.
- The values of average cleared impulse at $Z = 16.0 \text{ m/kg}^{1/3}$ are coincident with the side-on impulse values in each case, except for $D = 0.8 \text{ m/kg}^{1/3}$ where it is slightly above. This implies that clearing has occurred very early in the relatively long duration event. It must be concluded that at large scaled distances clearing occurs completely, even on quite large structures.

The next important question is how can the information in Figures 5.25 to 5.28 be used to calculate the cleared impulse experienced by rectangular structures of arbitrary size, stand-off distance and aspect ratio?

As well as the average impulse due to the side-on overpressure, a single value of side-on impulse, monitored at the bottom-centre of the structure (location 1, mentioned earlier) was also available. By dividing the average cleared impulse by the single value side-on impulse at location 1, a quantity called here the *clearing factor* C_f could be established for each point on the clearing (dashed) lines of Figures 5.25 to 5.28. Because the side-on specific impulse can be easily found from scaled blast parameters, this is the basis of the present approach.

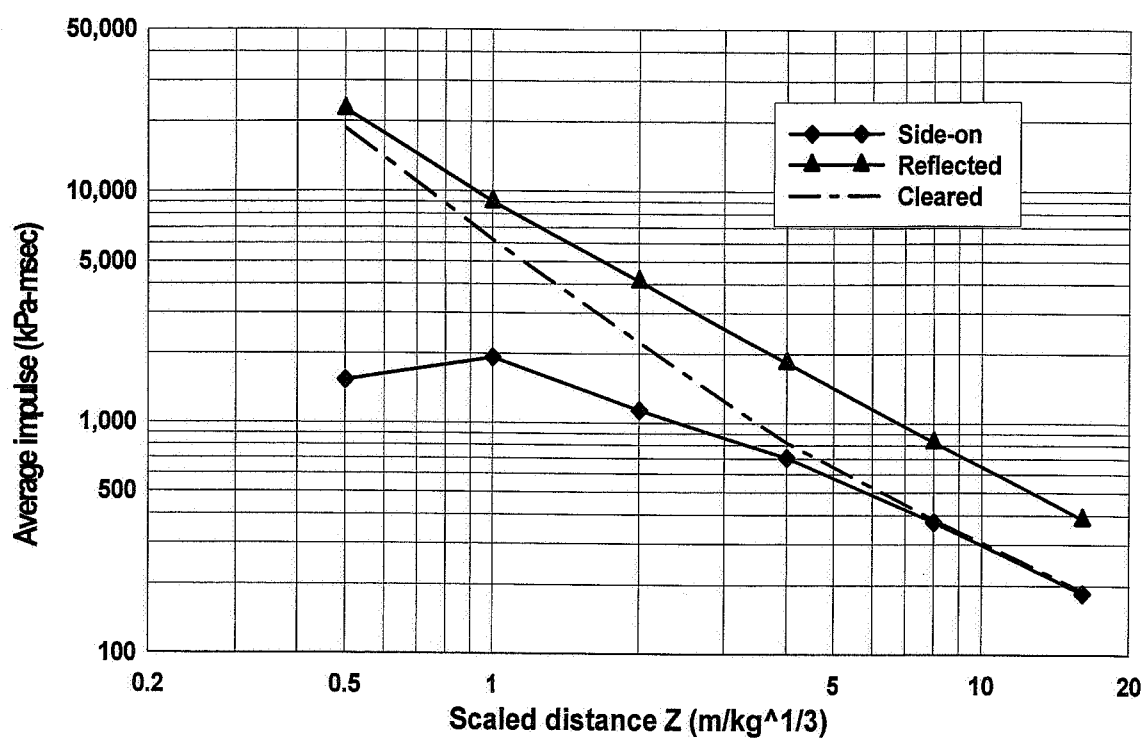


Figure 5.25: Side-on, reflected and cleared impulse, $D = 0.1 \text{ m/kg}^{1/3}$.

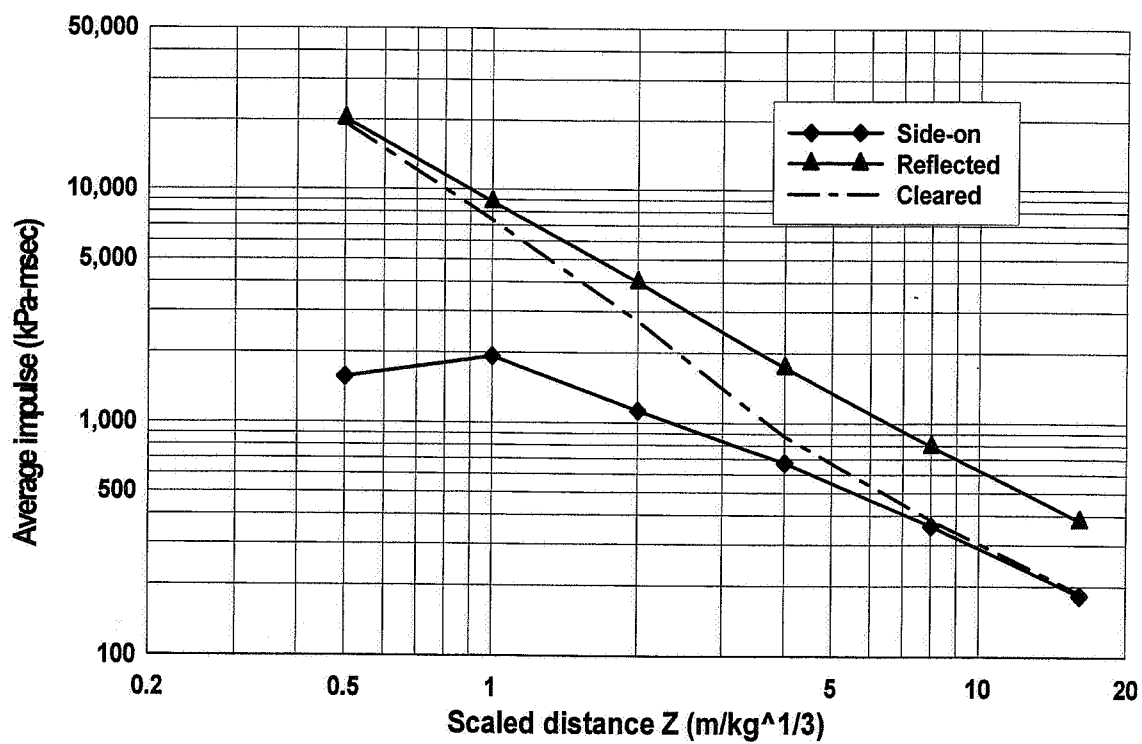


Figure 5.26: Side-on, reflected and cleared impulse, $D = 0.2 \text{ m/kg}^{1/3}$.

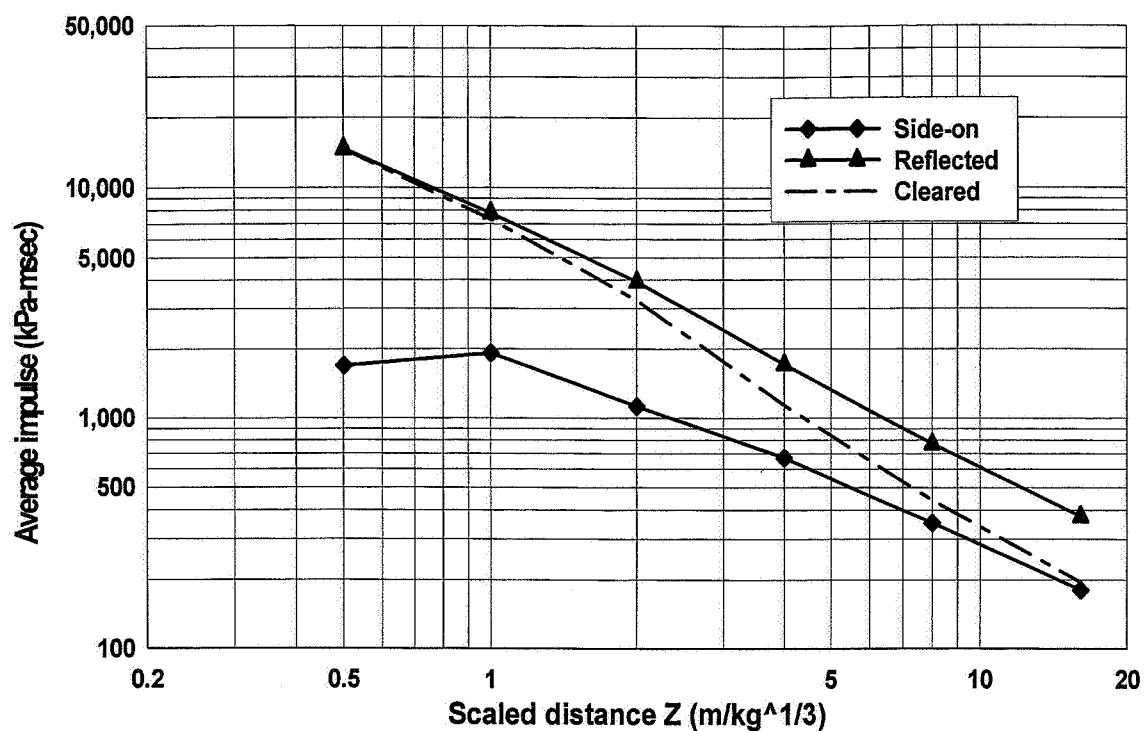


Figure 5.27: Side-on, reflected and cleared impulse, $D = 0.4 \text{ m/kg}^{1/3}$.

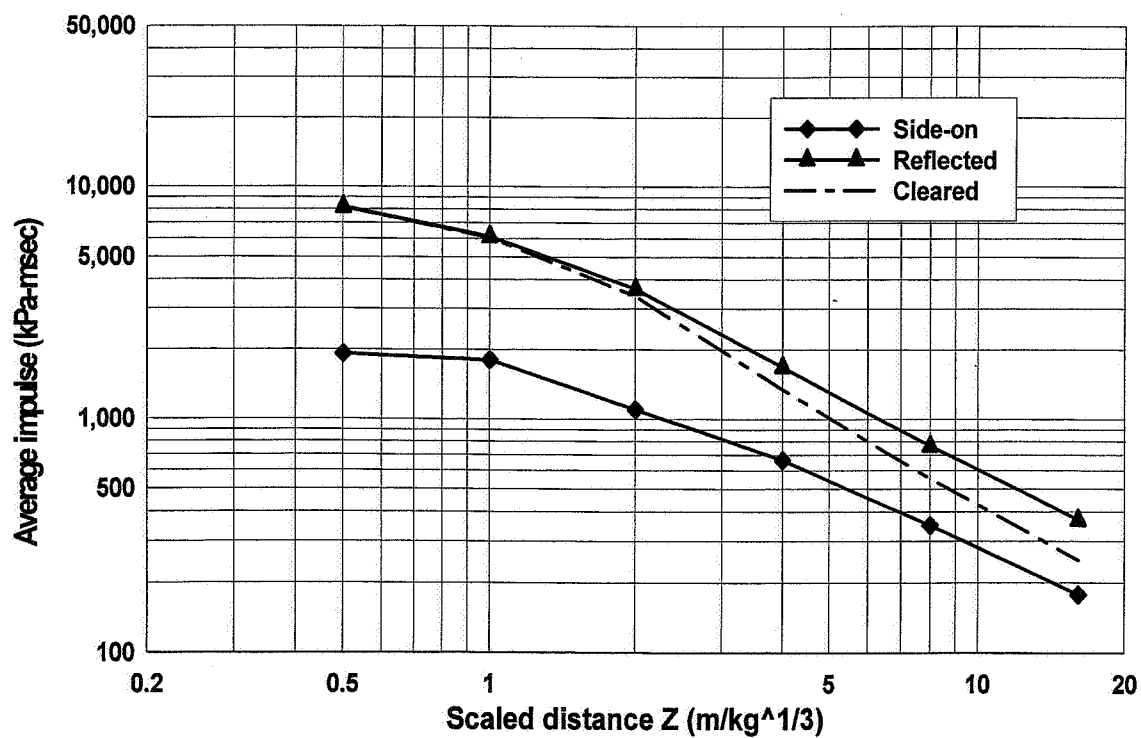


Figure 5.28: Side-on, reflected and cleared impulse, $D = 0.8 \text{ m/kg}^{1/3}$.

Clearing factors have been calculated and are given in Table 5.6, below. They are also plotted in Figure 5.29, where they have been separated (by using constant multipliers) for clarity.

For any given situation, not outside the bounds of the data set, clearing factors can be determined from the scaled distance and Table 5.6 using linear interpolation. Typically, two factors will be needed if the actual value D is not coincident with one of the discrete values listed above. These two values can then be interpolated in the scaled structure size axis to give the desired result.

Table 5.6: Clearing factors (= cleared impulse/side-on impulse)

Scaled distance (m/kg ^{1/3})	Clearing factor $C_f = i_{av}/i_s$			
	$D = 0.1$ (m/kg ^{1/3})	$D = 0.2$ (m/kg ^{1/3})	$D = 0.4$ (m/kg ^{1/3})	$D = 0.8$ (m/kg ^{1/3})
0.5	12.271	12.673	9.642	5.384
1.0	3.222	3.825	3.837	3.173
2.0	1.969	2.400	2.865	2.990
4.0	1.161	1.296	1.681	1.995
8.0	1.026	1.060	1.251	1.573
16.0	1.025	1.023	1.079	1.396

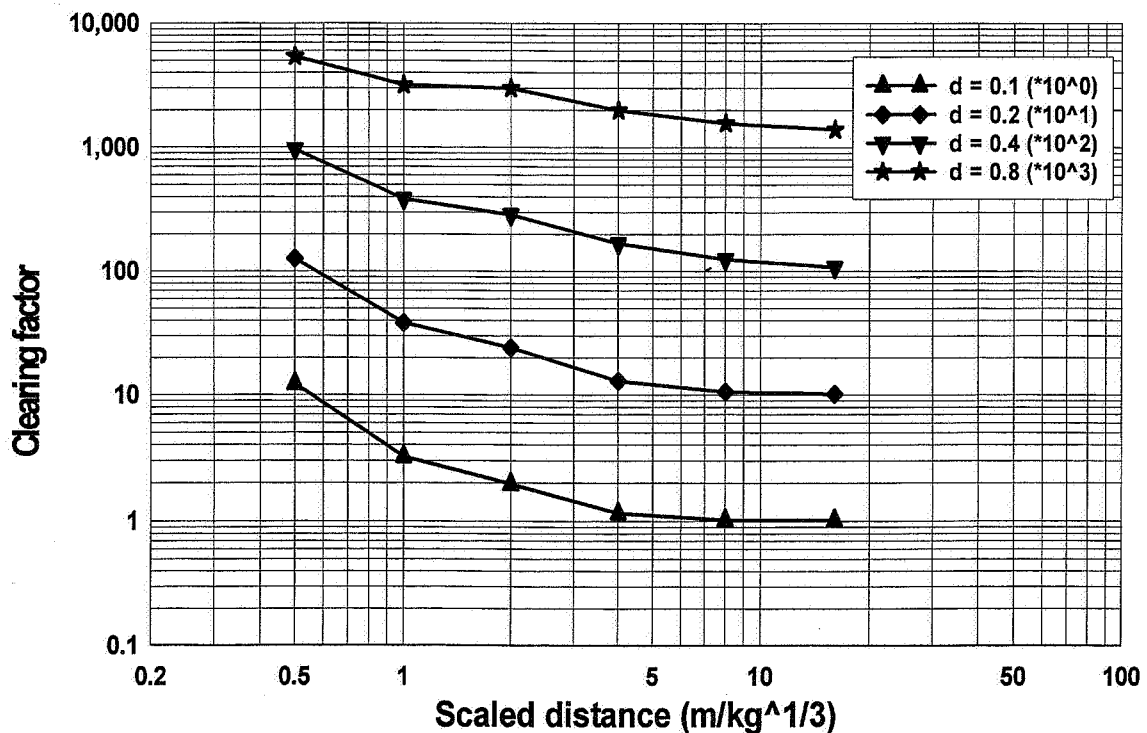


Figure 5.29: Clearing factors (= cleared impulse/side-on impulse)

Summary of the Numerically-Based Empirical Procedure

- The information that will be known at the outset is the charge weight W (kg), the stand-off R (m), and the height and width of the structure h (m) and l (m), respectively.

- The scaled distance Z is obtained in the usual way:

$$Z = \frac{R}{\sqrt[3]{W}}.$$

The scaled distance is limited to $0.5 \leq Z \leq 16.0 \text{ m/kg}^{1/3}$.

- Establish the discrete values Z_u and Z_l above and below the actual value Z at which the data points occur.
- The scaled structure size D is defined as:

$$D = \frac{\min(h, l/2)}{\sqrt[3]{W}},$$

which is the same as the other methods described earlier. The scaled structure size is limited to $0.1 \leq D \leq 0.8 \text{ m/kg}^{1/3}$.

- Establish the discrete values D_u and D_l above and below the actual value D at which the data points occur.
- The data points are equally spaced at logarithmic intervals, so the clearing factors C_{Du} and C_{Dl} at the discrete scaled structure sizes D_u and D_l are calculated from the following interpolation formula:

$$\left. \begin{array}{l} \log_{10} C_{Du} \\ \log_{10} C_{Dl} \end{array} \right\} = \log_{10} C_l + \left[\frac{\log_{10} C_u - \log_{10} C_l}{\log_{10} Z_u - \log_{10} Z_l} \right] (\log_{10} Z - \log_{10} Z_l) \left\{ \begin{array}{l} \text{for } D_u, \\ \text{for } D_l. \end{array} \right.$$

The parameters C_l and C_u are the clearing factors (from Table 5.6) at Z_l and Z_u , respectively.

- Calculate the desired clearing factor C_f from C_{Du} and C_{Dl} by interpolating in the scaled structure direction:

$$\log_{10} C_f = \log_{10} C_{Dl} + \left(\frac{\log_{10} C_{Du} - \log_{10} C_{Dl}}{\log_{10} D_u - \log_{10} D_l} \right) (\log_{10} D - \log_{10} D_l).$$

- Obtain the side-on specific impulse i_s at the bottom centre of the structure using scaled blast parameters. Calculate the average specific impulse on the structure i_{av} from:

$$i_{av} = C_f \times i_s$$

Worked Example, Shot 10 from Section 5.2

- Information known from the geometry and configuration of the test is as follows:

Charge weight W	= 31.28 g TNT
Stand-off R	= 3.0 m
Structure height h	= 0.183 m
Structure width l	= 0.183 m

- The scaled stand-off distance:

$$Z = \frac{3.0}{\sqrt[3]{31.28 \times 10^{-3}}} = 9.521 \text{ m/kg}^{1/3},$$

which is between 0.5 and 16.0 m/kg^{1/3}.

- The discrete values of scaled distance above and below $Z = 9.521 \text{ m/kg}^{1/3}$ are $Z_u = 16.0 \text{ m/kg}^{1/3}$ and $Z_l = 8.0 \text{ m/kg}^{1/3}$, respectively.
- The scaled structure size D is:

$$D = \frac{0.183/2}{\sqrt[3]{31.28 \times 10^{-3}}} = 0.29 \text{ m/kg}^{1/3},$$

which is between 0.1 and 0.8 m/kg^{1/3}.

- The discrete values of scaled structure size above and below $D = 0.29 \text{ m/kg}^{1/3}$ are $D_u = 0.4 \text{ m/kg}^{1/3}$ and $D_l = 0.2 \text{ m/kg}^{1/3}$, respectively.
- The clearing factors associated with the discrete scaled structure sizes are given by:

$$\log_{10} C_{Du} = \log_{10} 1.251 + \left[\frac{\log_{10} 1.079 - \log_{10} 1.251}{\log_{10} 16.0 - \log_{10} 8.0} \right] (\log_{10} 9.521 - \log_{10} 8.0) = 0.08113$$

and

$$\log_{10} C_{Dl} = \log_{10} 1.060 + \left[\frac{\log_{10} 1.023 - \log_{10} 1.060}{\log_{10} 16.0 - \log_{10} 8.0} \right] (\log_{10} 9.521 - \log_{10} 8.0) = 0.02143.$$

- Interpolation in the D axis gives:

$$\log_{10} C_f = 0.02143 + \left[\frac{0.08113 - 0.02143}{\log_{10} 0.4 - \log_{10} 0.2} \right] (\log_{10} 0.29 - \log_{10} 0.2) = 0.05355$$

so the average clearing factor $C_f = 1.131$.

- The side-on specific impulse (from ConWep [24]) is 10.24 kPa-msec. Therefore, the average specific impulse on the front face of the structure is:

$$i_{av} = 1.131 \times 10.24 = 11.58 \text{ kPa-msec.}$$

The above procedure is very easily coded using a high level programming language or spreadsheet. This has been done, and average impulses for the validation problems described in Section 5.2 are given in Table 5.7. They are also presented graphically in Figures 5.30 to 5.32, below, together with the experimental data and the average impulse values from the procedures of TM 5-1300 [13] and ConWep [24].

Table 5.7: Average impulse calculated by the new approach

Shot no.	Peak average cleared impulse (kPa-msec)
1	191.87
2	105.08
3	51.47
4	36.59
5	20.83
6	156.62
7	36.77
8	27.29
9	18.96
10	11.58
11	93.27
12	29.23
13	17.27
14	7.33
15	[not in range]

Comments on Table 5.7 and Figures 5.30 to 5.32 are as follows:

- It will be noticed that there is one point missing from Figure 5.32 and Table 5.7. This is because the scaled distance of the final point of the experimental programme was beyond the maximum ($16.0 \text{ m/kg}^{1/3}$) considered by the present approach. This is not a major deficiency, because Figure 5.26 (which is closest to the actual scaled structure size $D = 0.29 \text{ m/kg}^{1/3}$) shows that the average cleared impulse is virtually the same as the average side-on impulse at large scaled range. This was mentioned previously.
- The experimental data is the actual impulse measured at the centre of the structure. It is effectively an *upper bound* on the average impulse.
- The new approach appears to be superior to the other two procedures in that it is a closer fit to experiment.

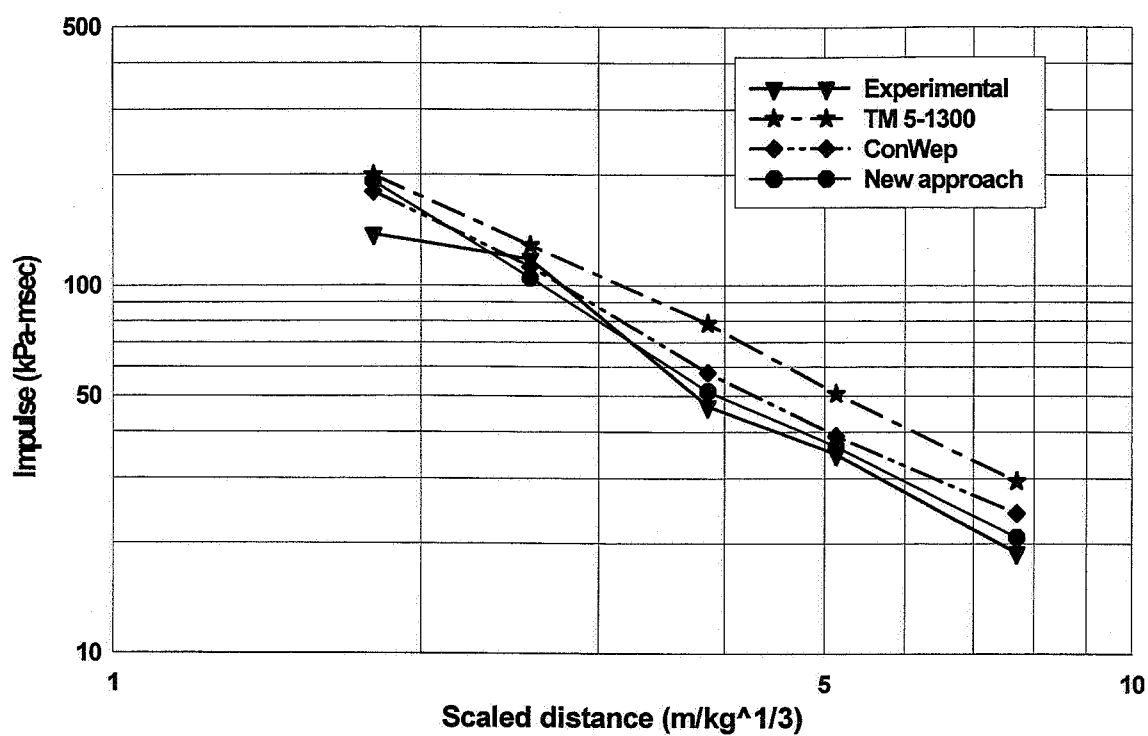


Figure 5.30: Comparison of all three clearing procedures, $W = 59.2$ g

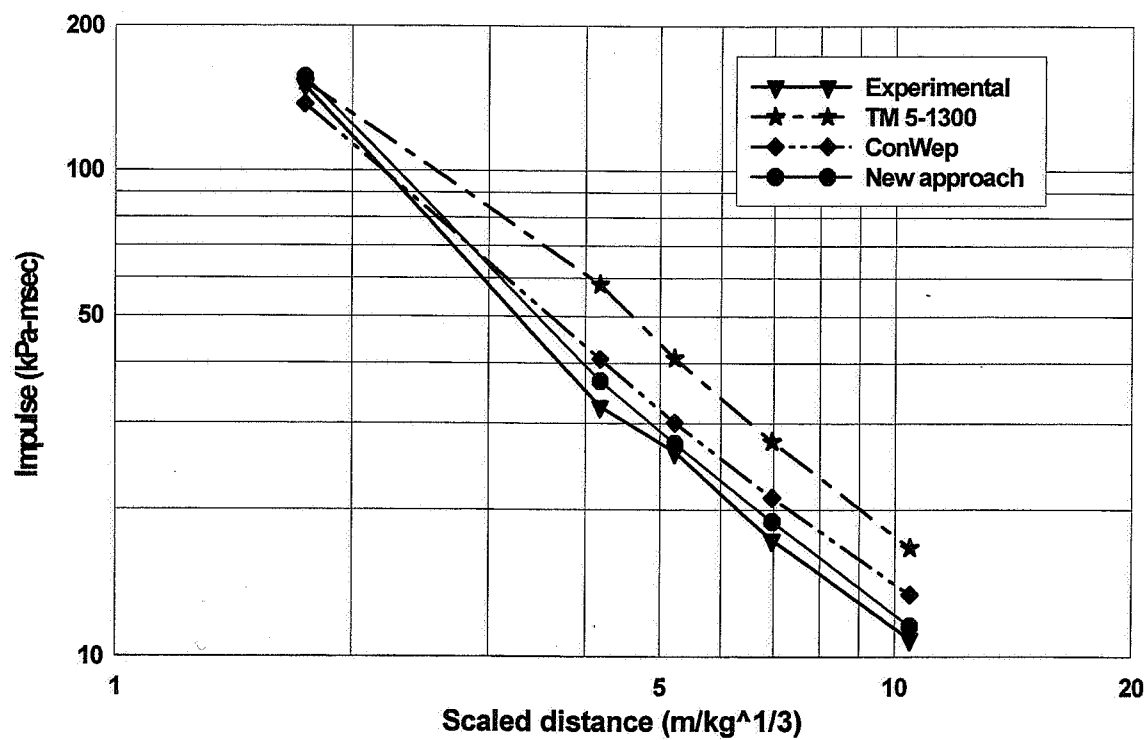


Figure 5.31: Comparison of all three clearing procedures, $W = 23.7$ g

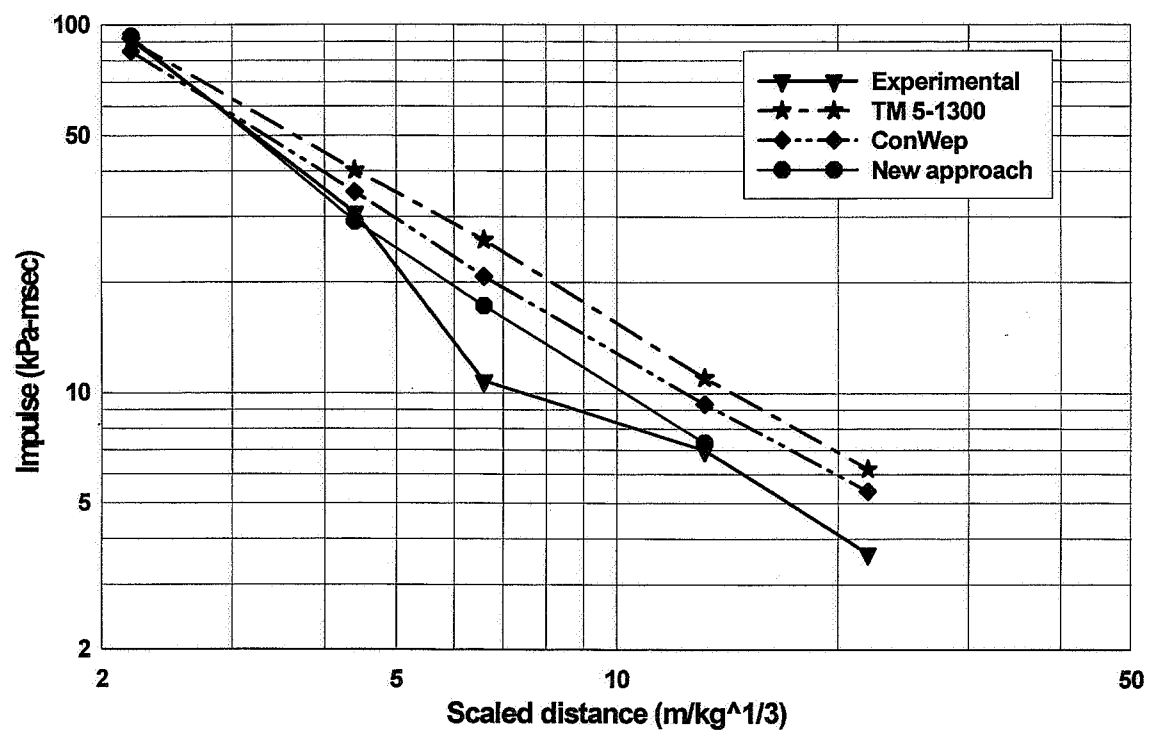


Figure 5.32: Comparison of all three clearing procedures, $W = 11.8 \text{ g}$

5.4 Comments and Conclusions on Blast Wave Clearing

The problem of blast clearing is not straightforward. Different aspects of the flow dominate the overall effect depending on the charge weight, stand-off distance and the front face geometry of the structure. Generally, the fact that it is time-varying compressible flow implies that a single general approach, as used in the existing methods of evaluation, is probably not adequate for purposes other than design.

There are a few important observations concerning the phenomenon that should be noted. These are the following:

- At small scaled distance ($Z < 0.5 \text{ m/kg}^{1/3}$) clearing does not occur, except for very small (scaled) structure size ($D \leq 0.1 \text{ m/kg}^{1/3}$). Then it is only slightly evident. It can be concluded that, in these circumstances, the proximity of the edges of the structure are irrelevant, and the impulse can be calculated as if it were a height of burst scenario. The result would be either very accurate, for *large* structures, or else only slightly conservative for *smaller* ones.
- At large scaled distance ($Z > 16.0 \text{ m/kg}^{1/3}$) clearing occurs completely, except for very large (scaled) structure size ($D \geq 0.8 \text{ m/kg}^{1/3}$), and the average impulse experienced by the front face of the structure tends towards the average side-on value.
- The effect of the length of the structure in the direction of propagation on the process of clearing is minimal. This is a useful observation, because it implies that the methods of evaluation of front face impulse based on front face geometry alone are valid.

The numerically-based empirical procedure described in this chapter does not include explicit treatment or description of the difficulties of the phenomenon of blast wave clearing. It is based on a data set that has attempted to consider every possible practical scenario, so there is no need for a precise description of events. All the important information is implicit in the table of clearing factors (Table 5.6). This is an example of the so-called "brute force" approach, and it has been demonstrated that this method appears to be superior to current practice. However, there are a few points that need to be borne in mind.

- The validation exercise described in Section 5.2 has demonstrated that, in terms of positive phase impulse, the results of the numerical technique are close to those obtained by experiment. However, the experimental data were limited, and they were not based on statistically valid results: they were from single firings. This has had the effect of limiting the applicability of the new approach proposed in Section 5.3.5. It was observed above that the new approach appears to be superior to current practice, at least with respect to the one data set used here.
- In the whole of this chapter a TNT equivalence of 1.32 has been used. It was used for the validation problem (Section 5.2) and the clearing procedures

(Section 5.3). It will be seen from Figures 5.12 to 5.14 that the impulse calculated by the program Air3d is consistently greater than the measured value. The difference is fractional, but might indicate that an equivalence of 1.32 may be slightly too large. If this is the case, the values of average impulse calculated by all three approximate procedures may also be slightly high.

- It may not be appropriate to use approximate procedures to calculate the average impulse on structures of large scaled size at small scaled distance. Figure 5.3 (Example 1) shows that the variation of pressure and arrival time can be considerable. Therefore, each part of the façade of a building should probably be considered separately.
- The new approach was based on a rectangular structure with height equal to half the width. Nonetheless, it still provided good results when applied to the validation problem which had a square front face geometry. It appears that the proximity of an edge is the most important factor for determining the extent of clearing. This implies that the procedure could be used for non-rectangular structures so long as the front is plane and parallel to the incident shock wave and a representative scaled structure size can be established.
- There is an aspect ratio above which the new procedure will break down. This has not been explored in the present study. Most ordinary structures, with an aspect ratio less than three (say), should be adequately described. Skyscrapers, for example, will not be suited to treatment by the new procedure.

It has been demonstrated that the computational tool Air3d can describe the problem sufficiently well for the purpose of structural design or hazard assessment. It has been used to create a simple empirical method based on twenty-four representative calculations. The possibility of producing more accurate or complete data sets, or consideration of other structure geometries, is evident.

Chapter 6

Blast in an Urban Environment

6.1 Introduction

The problem of blast in an urban environment is manifold; there are several different aspects that are of interest. Principally among these are the direct effects of blast on people (both indoors or outdoors), the indirect effects of blast on people: from broken glazing, fallen masonry and collapsed buildings, and damage to buildings. It is important to appreciate these problems if evacuation distances or safe areas within buildings are to be identified. All of the above considerations have one thing in common: they can only be quantified once the precise blast environment (in terms of pressure and impulse) is known throughout the region of interest.

Unfortunately, the effect of urban geometry on the propagation of blast waves is a vast and complicated subject, and only recently has it begun to be approached in a fundamental and systematic manner (Feng [14] and Whalen [57]). In this section, two of the fundamental geometrical aspects of urban geometry will be considered and simple quantitative observations made.

6.2 Validation

The series of experiments reported by Whalen [57] provide an ideal means of validation for the computer program Air3d. Whalen [57] describes experiments on five simple generic street configurations, with constant street width and building height. These are shown in plan view in Figure 6.1, where the location of explosive charge (+) and the pressure transducer array (××××) are indicated on each diagram.

The experiments were conducted at 1:50 scale, and they modelled a lorry bomb detonated in the middle of a street 15 m wide. The model scale street widths were 0.3 m. The model scale explosive charge was 11.13 g SX2 (12 g TNT eqv.) plus approximately 1 g for the detonator, giving 13 g TNT equivalent in total. This equates to a charge $W \approx 1625$ kg TNT at full size.

The experiments utilised thick steel plates 1 m square, backed with sand, to form the sides of the streets. This had the effect of producing rigid undeforming reflecting surfaces. Another useful aspect of the experiments was the fact that the height and length of the model streets were very large compared with the distance of the pressure measuring locations from the charge; the streets were effectively infinitely

long and high. This important fact, together with the condition of constant street width, meant that the effect of the geometry alone was investigated. The location of the pressure transducers can be seen in Figure 6.2. It will be noticed that transducer locations H1, V1 and D1 are coincident, and the spacing between the transducers varied to take advantage of the anticipated nonlinear variation of peak pressure.

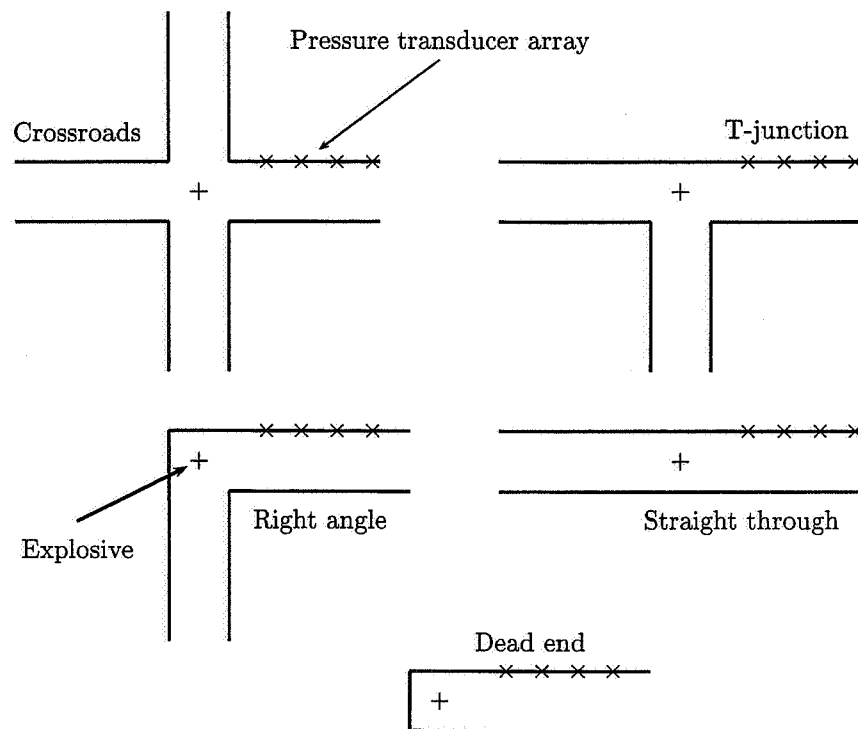


Figure 6.1: Generic street configurations

The validation process is by comparison of experimental and numerical peak pressure and peak specific impulse at all of the measuring locations for each of the five configurations.

An attempt to present this information in the same manner as Whalen [57], by comparison with side-on height of burst impulses will also be made. There is also a comparison of pressure-time histories for the “dead end” configuration, which is probably the most interesting geometry because it has the most confinement and produces a strong reflected shock from the closed end.

The numerical simulations were conducted in three distinct parts:

- One-dimensional analysis from the charge centre to the ground, radius = 25 mm, $W = 13.0$ g and grid size = 0.05 mm.
- Two-dimensional analysis across the width of the street, radius = 0.15 m, height = 0.17 m and grid size = 1.0 mm square cells.
- Three-dimensional analysis for the remainder of the region of interest, grid size = 5 mm cubic cells.

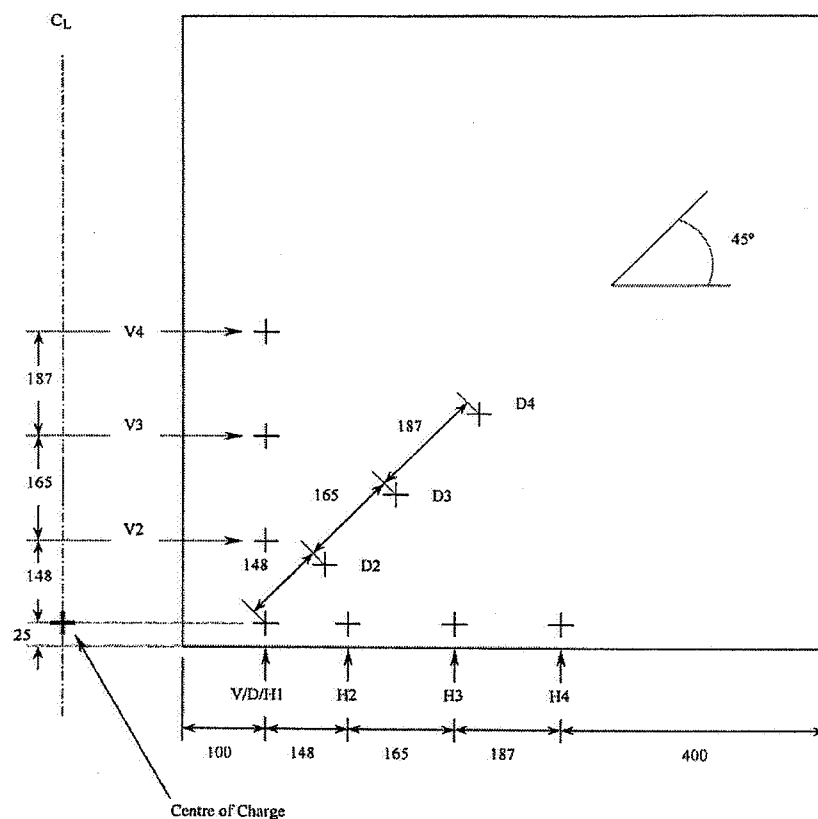


Figure 6.2: Location of pressure monitoring locations

Tables 6.1 to 6.5, below, contain the experimental and numerical peak overpressure and peak specific impulse data. It should be noted that the experimental results are almost exclusively an average of three data points. Table 6.6 contains the results of the two-dimensional height of burst analysis (for which there was no experimental data). This also used 5 mm grid size.

A graphical representation of the two sets of data is given in Figures 6.3 to 6.17. These figures show the peak pressure and impulse in each of the three monitoring array orientations for each of the five street configurations; there are 15 graphs in total.

Comparisons between the experimental and numerical pressure histories for the dead end configuration are given in Figures 6.18 to 6.27. There are 10 distinct locations and hence 10 graphs.

Table 6.1: Experimental and numerical data, crossroads

Monitoring plane	Location number	Peak pressure (kPa)		Peak impulse (kPa-msec)	
		Experiment	Air3d	Experiment	Air3d
Horizontal	1	2281	2704	77	110
	2	728	867	55	72
	3	312	368	43	52
	4	224	187	40	43
Vertical	1	2432	2704	76	110
	2	1272	1148	58	68
	3	472	413	41	44
	4	252	208	40	37
Diagonal	1	3613	2704	88	110
	2	921	878	54	66
	3	402	395	43	51
	4	238	206	41	45

Table 6.2: Experimental and numerical data, T-junction

Monitoring plane	Location number	Peak pressure (kPa)		Peak impulse (kPa-msec)	
		Experiment	Air3d	Experiment	Air3d
Horizontal	1	2548	2793	90	126
	2	884	1034	72	83
	3	398	414	58	63
	4	467	215	52	49
Vertical	1	2251	2793	76	126
	2	1506	1519	71	84
	3	699	581	57	61
	4	370	271	47	46
Diagonal	1	4140	2793	125	126
	2	1085	1069	82	79
	3	448	461	51	61
	4	302	240	48	52

Table 6.3: Experimental and numerical data, straight through

Monitoring plane	Location number	Peak pressure (kPa)		Peak impulse (kPa-msec)	
		Experiment	Air3d	Experiment	Air3d
Horizontal	1	3231	2793	95	148
	2	1148	1040	81	106
	3	435	418	57	77
	4	399	264	64	58
Vertical	1	2292	2793	93	148
	2	1474	1519	79	105
	3	716	584	62	79
	4	363	272	63	63
Diagonal	1	3026	2793	92	148
	2	1290	1076	84	100
	3	564	465	62	78
	4	401	445	61	64

Table 6.4: Experimental and numerical data, right angle

Monitoring plane	Location number	Peak pressure (kPa)		Peak impulse (kPa-msec)	
		Experiment	Air3d	Experiment	Air3d
Horizontal	1	3257	2739	75	148
	2	1343	1016	92	110
	3	460	412	72	85
	4	256	214	62	65
Vertical	1	2898	2739	113	148
	2	1134	1473	86	104
	3	616	573	73	77
	4	707	271	73	60
Diagonal	1	1780	2739	77	148
	2	924	1044	100	105
	3	495	459	69	83
	4	381	239	61	69

Table 6.5: Experimental and numerical data, dead end

Monitoring plane	Location number	Peak pressure (kPa)		Peak impulse (kPa-msec)	
		Experiment	Air3d	Experiment	Air3d
Horizontal	1	3210	2739	130	210
	2	1352	1016	132	153
	3	570	412	110	120
	4	363	262	92	93
Vertical	1	3476	2739	123	210
	2	1672	1473	107	154
	3	679	573	92	116
	4	760	452	90	92
Diagonal	1	2612	2739	139	210
	2	1297	1044	117	147
	3	468	459	93	114
	4	398	421	81	96

Table 6.6: Height of burst Air3d results

Monitoring plane	Location number	Peak pressure (kPa)	Peak impulse (kPa-msec)
Horizontal	1	1210	61
	2	452	40
	3	206	26
	4	111	21
Vertical	1	1210	61
	2	635	38
	3	264	29
	4	139	23
Diagonal	1	1210	61
	2	470	36
	3	248	27
	4	131	22

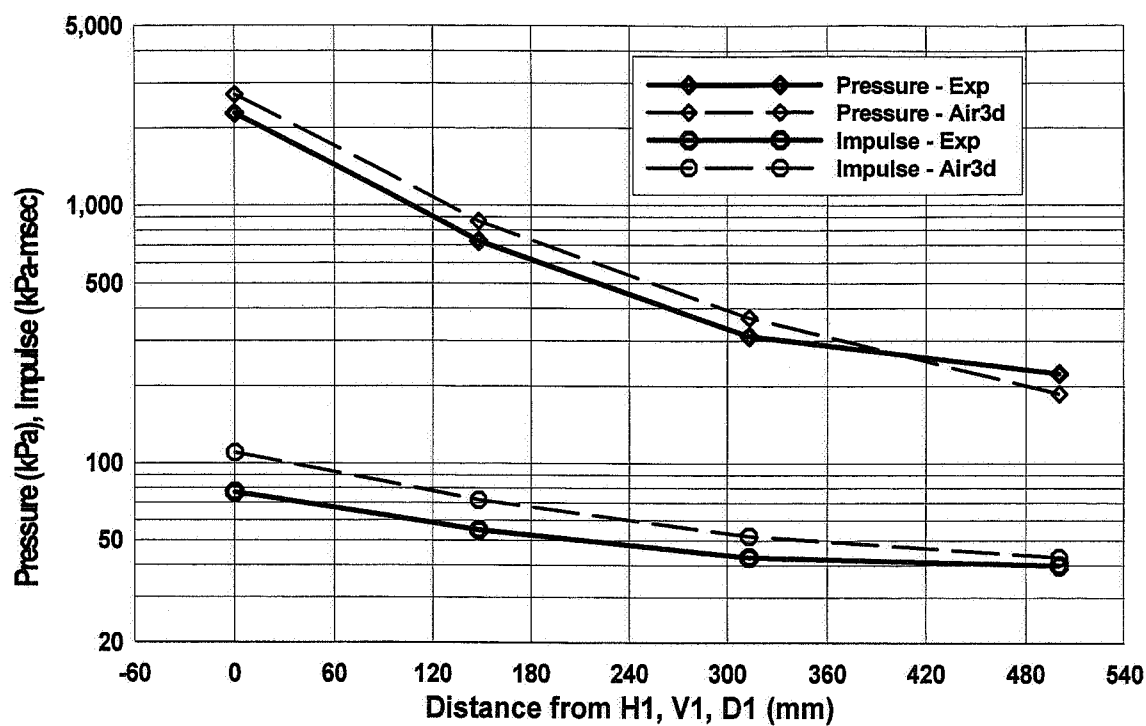


Figure 6.3: Comparison of results, crossroads, horizontal

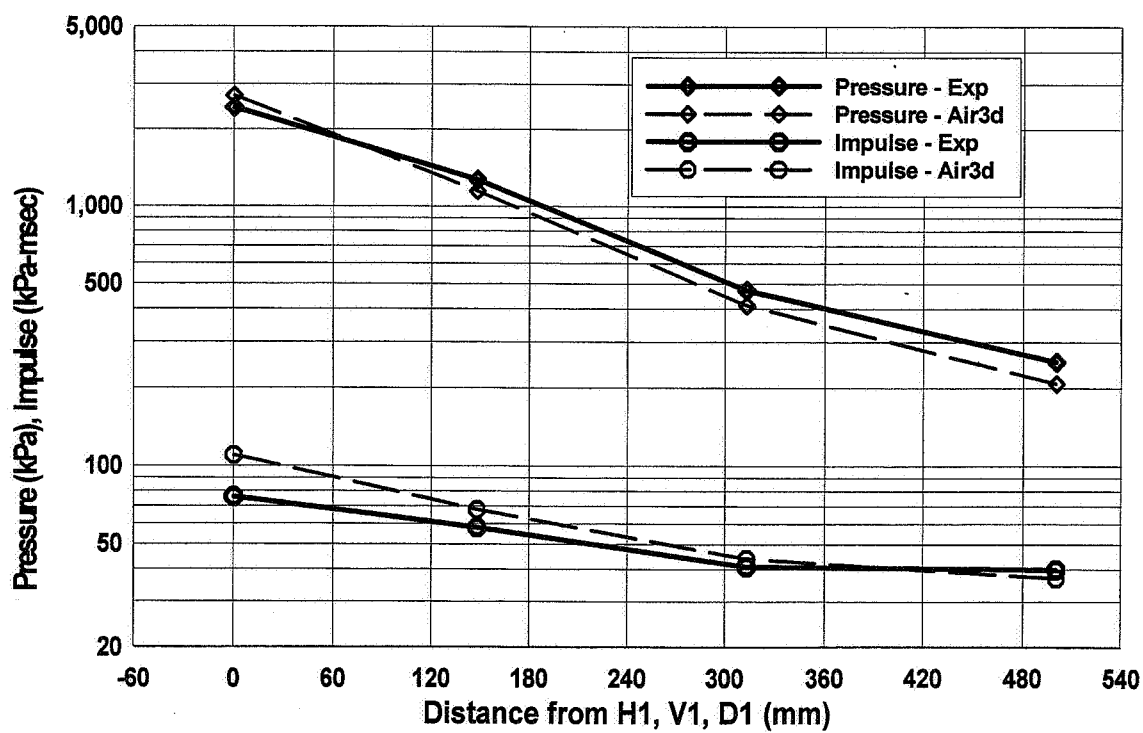


Figure 6.4: Comparison of results, crossroads, vertical

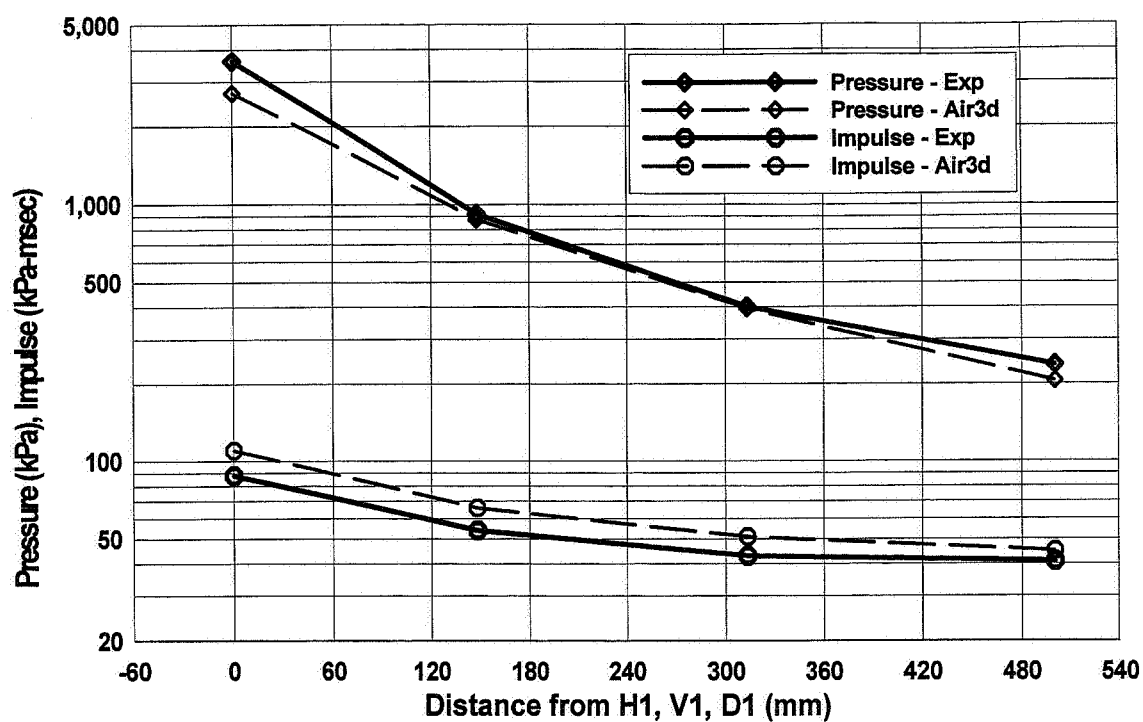


Figure 6.5: Comparison of results, crossroads, diagonal

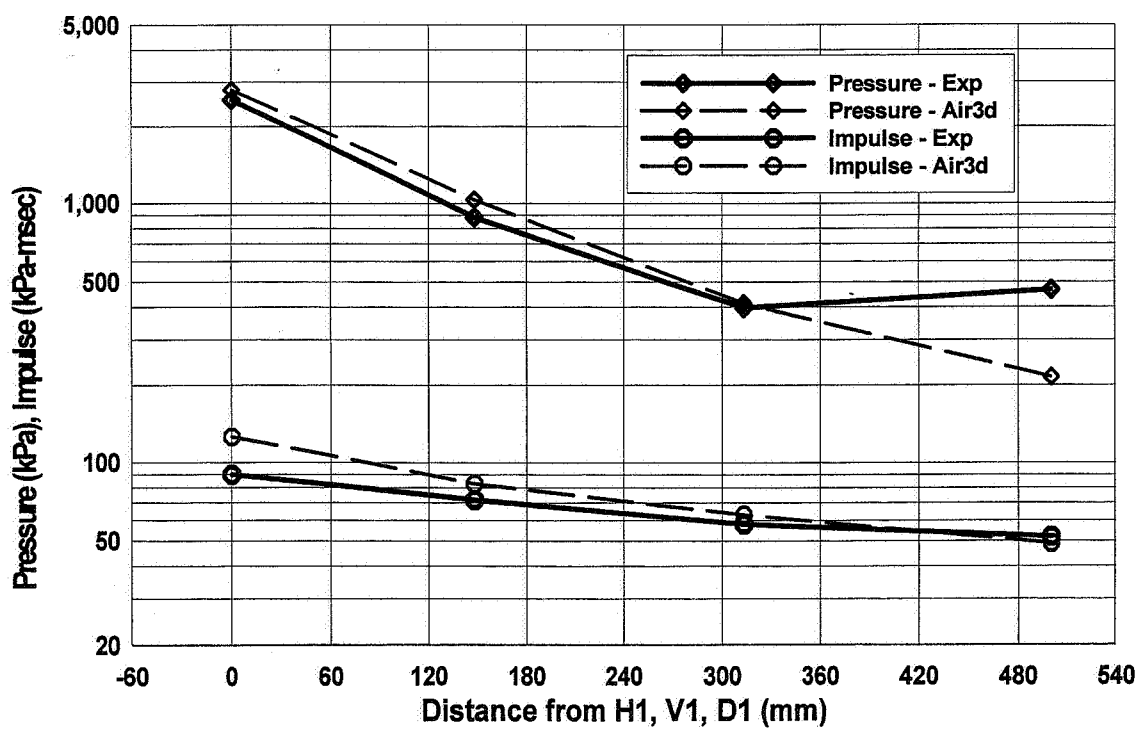


Figure 6.6: Comparison of results, T-junction, horizontal

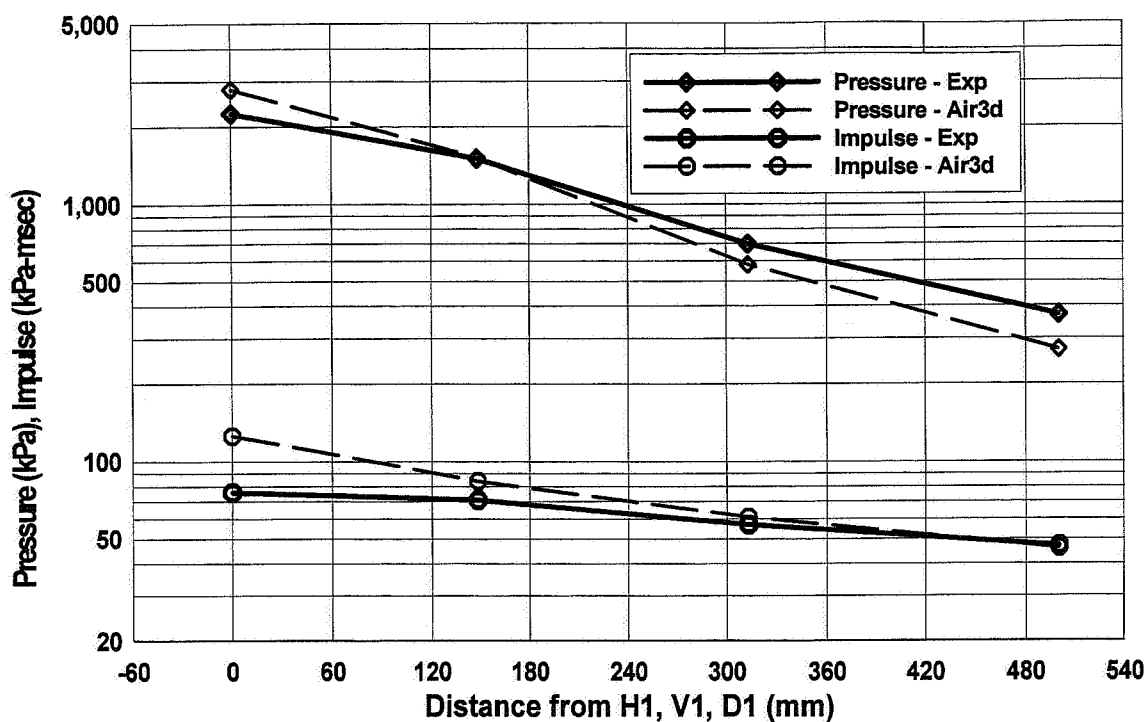


Figure 6.7: Comparison of results, T-junction, vertical

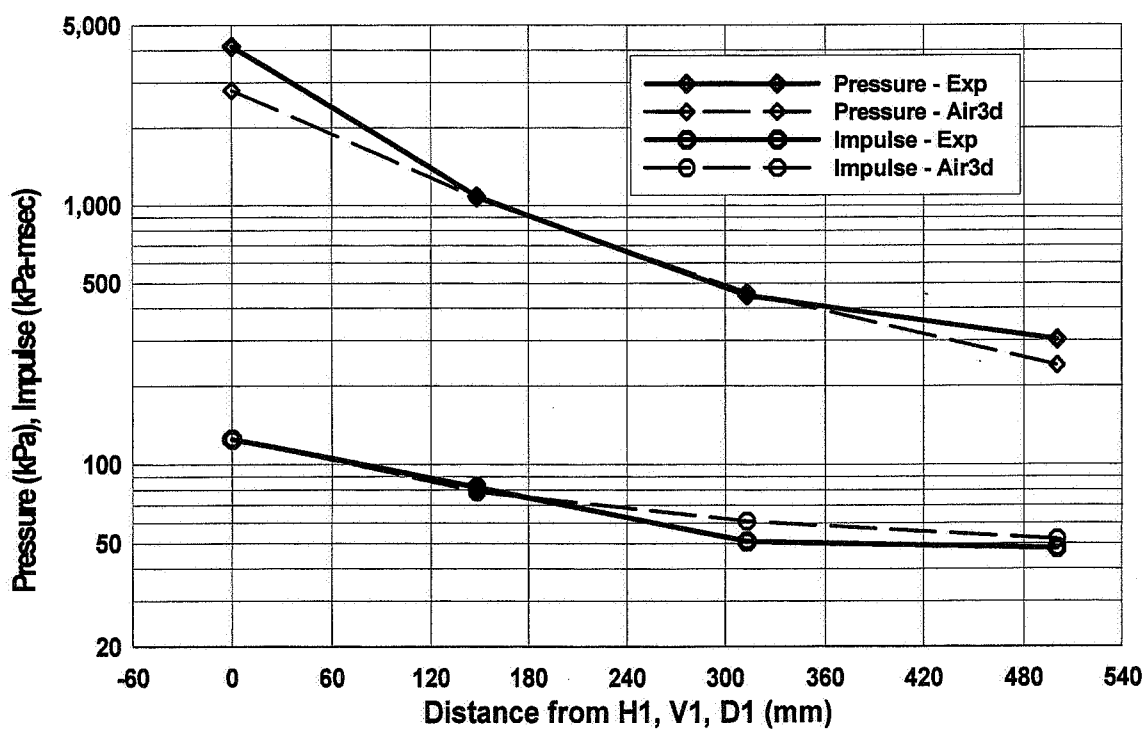


Figure 6.8: Comparison of results, T-junction, diagonal

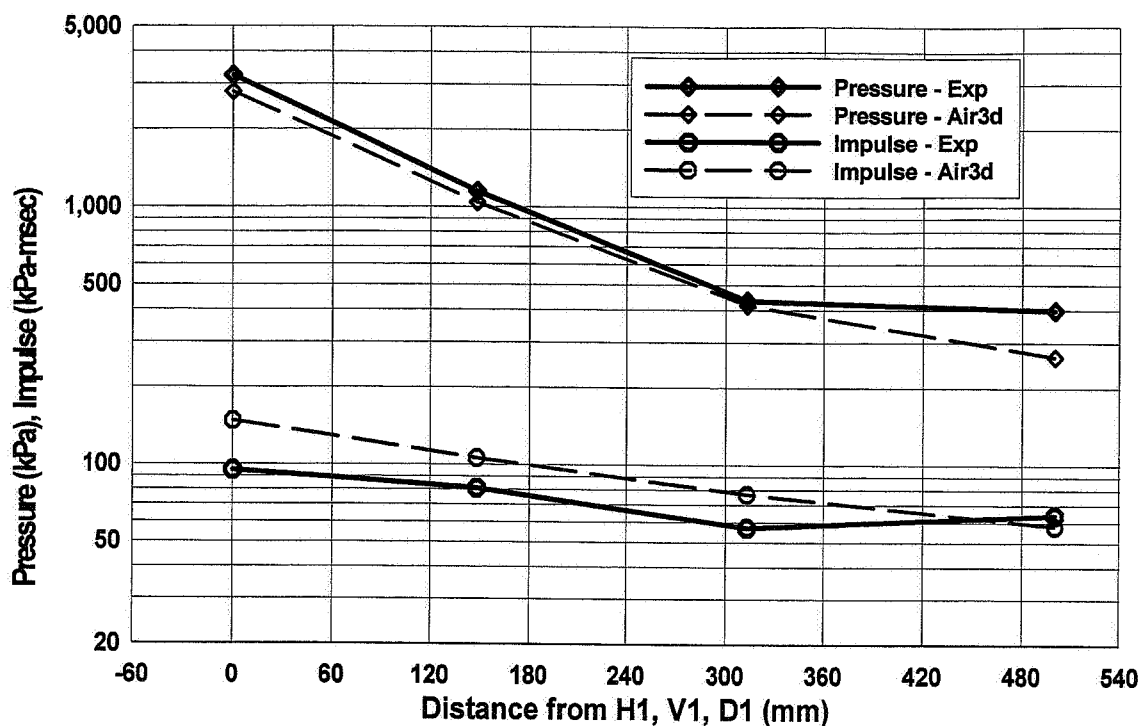


Figure 6.9: Comparison of results, straight through, horizontal

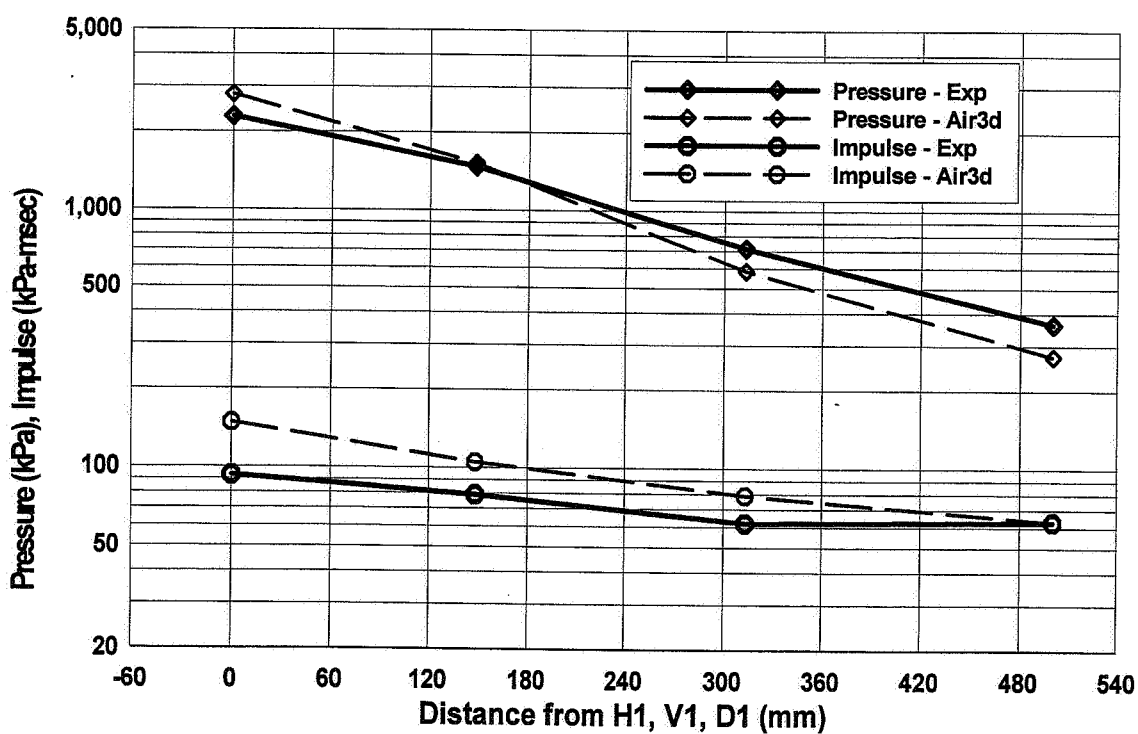


Figure 6.10: Comparison of results, straight through, vertical

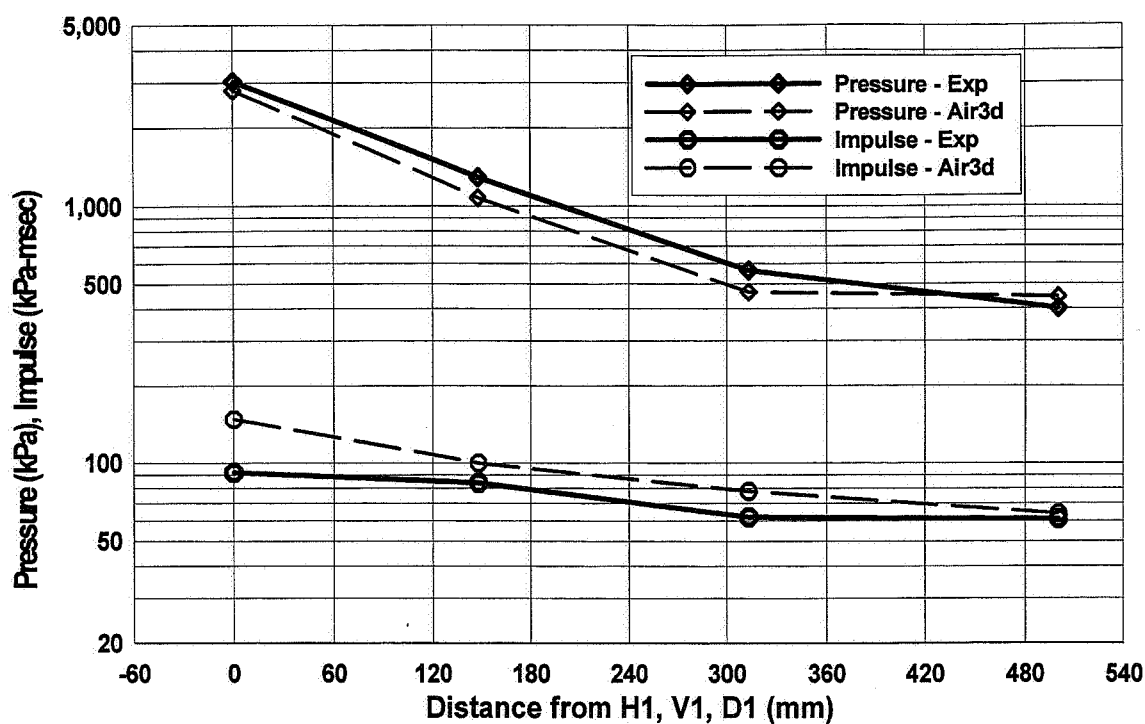


Figure 6.11: Comparison of results, straight through, diagonal

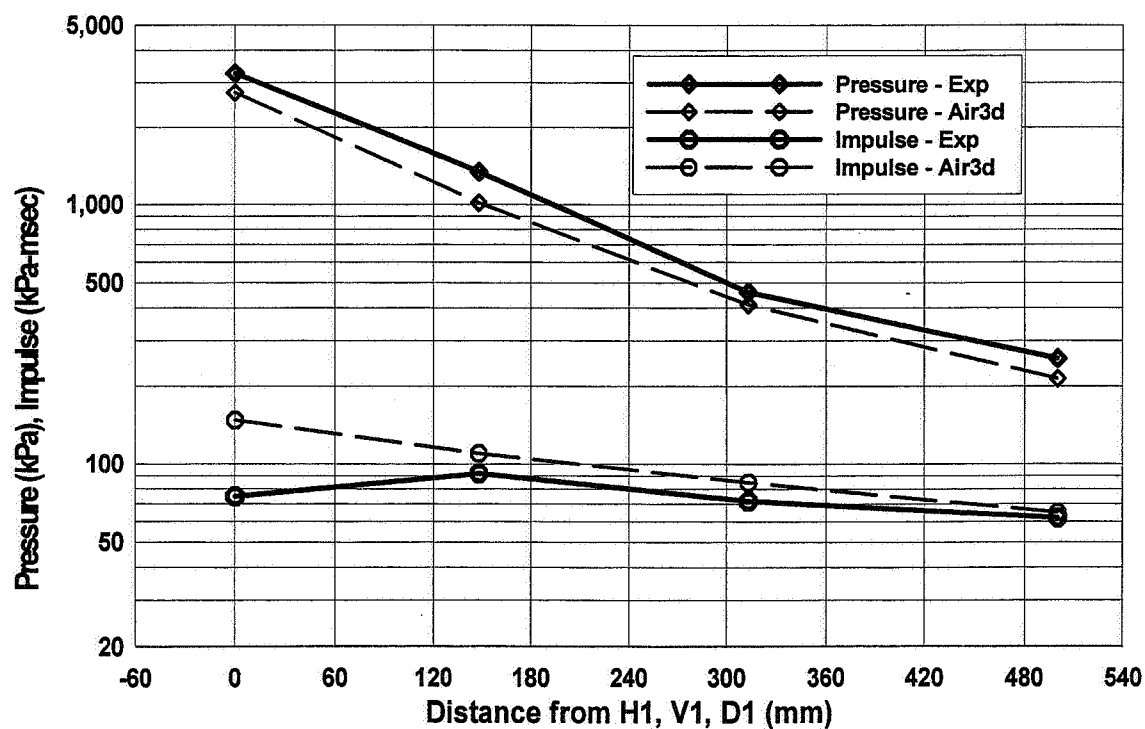


Figure 6.12: Comparison of results, right angle, horizontal

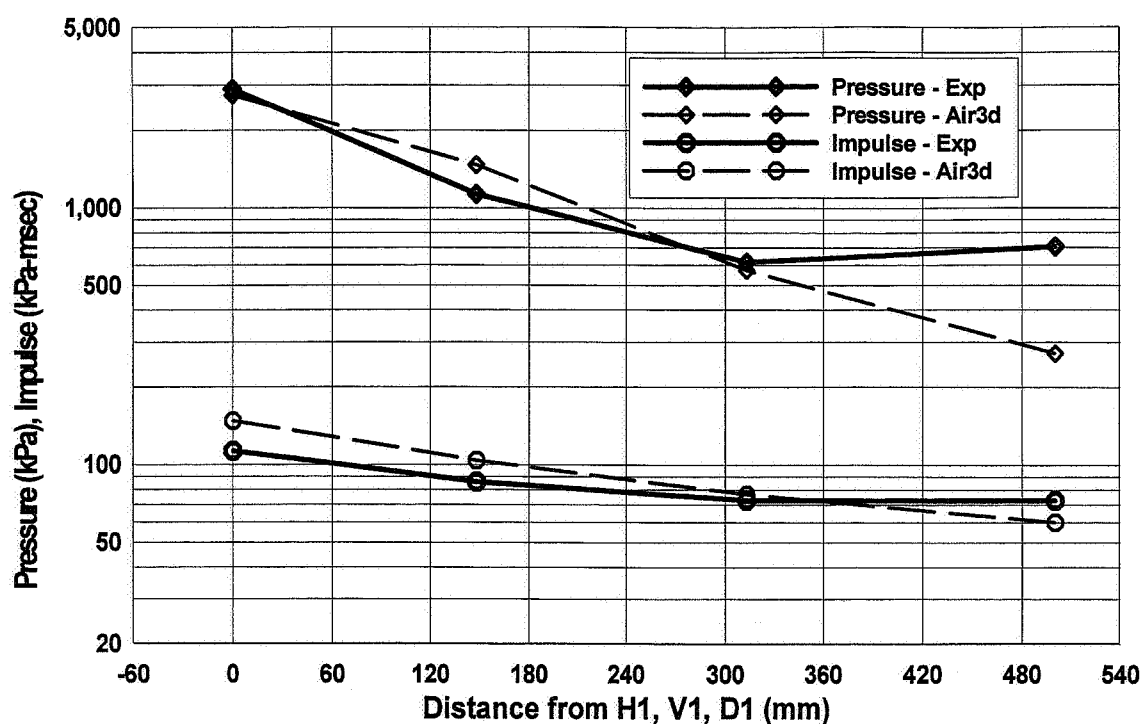


Figure 6.13: Comparison of results, right angle, vertical

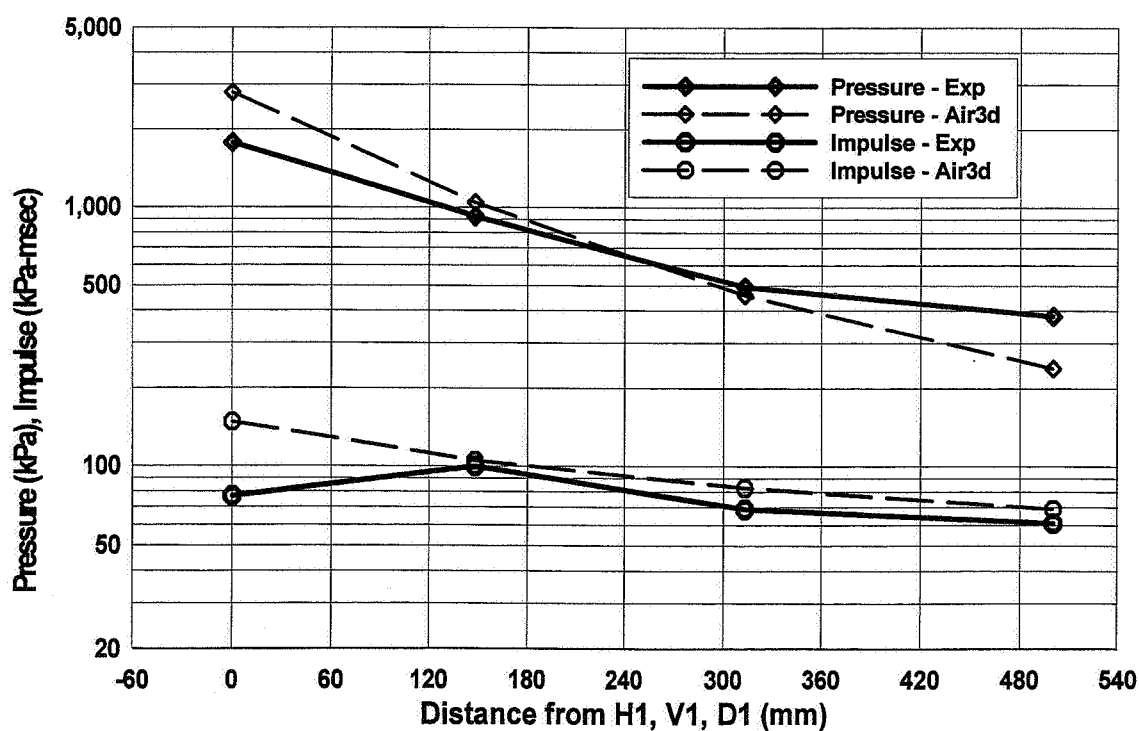


Figure 6.14: Comparison of results, right angle, diagonal

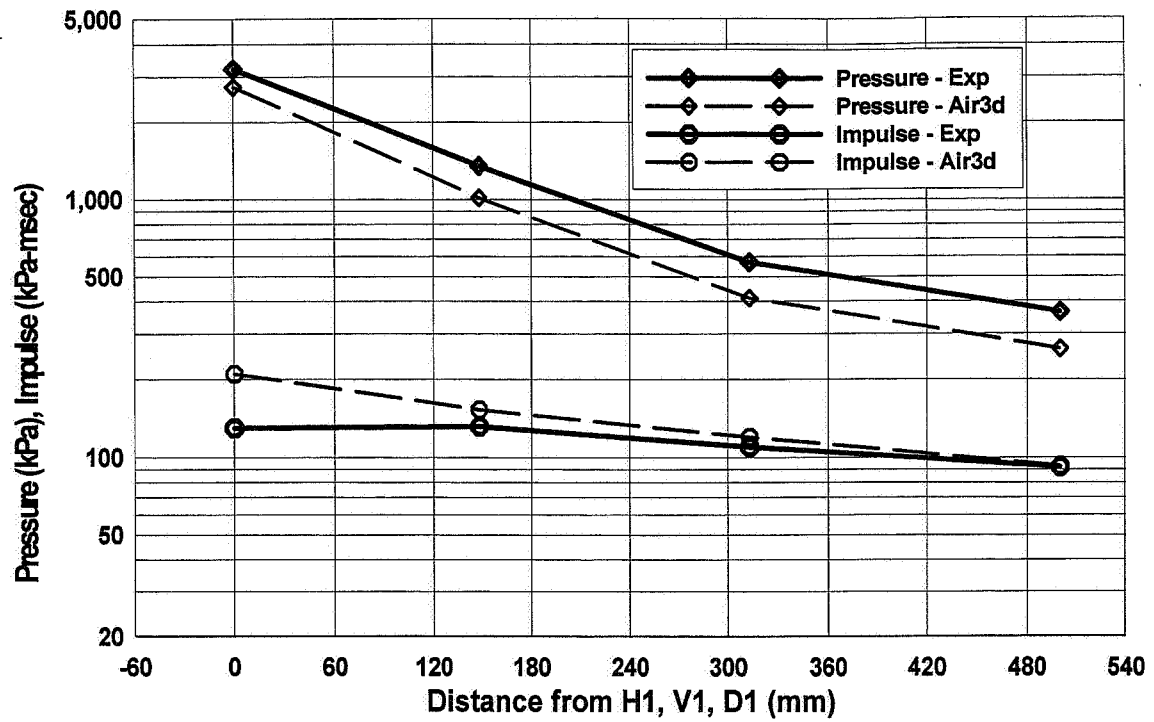


Figure 6.15: Comparison of results, dead end, horizontal

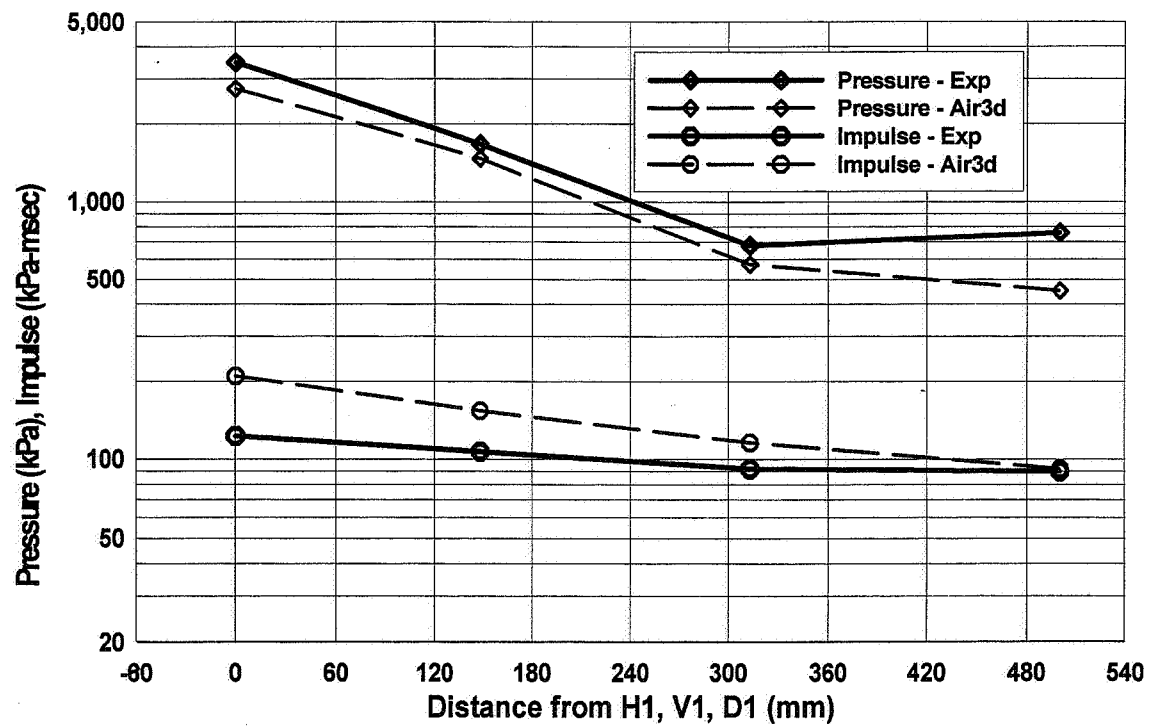


Figure 6.16: Comparison of results, dead end, vertical

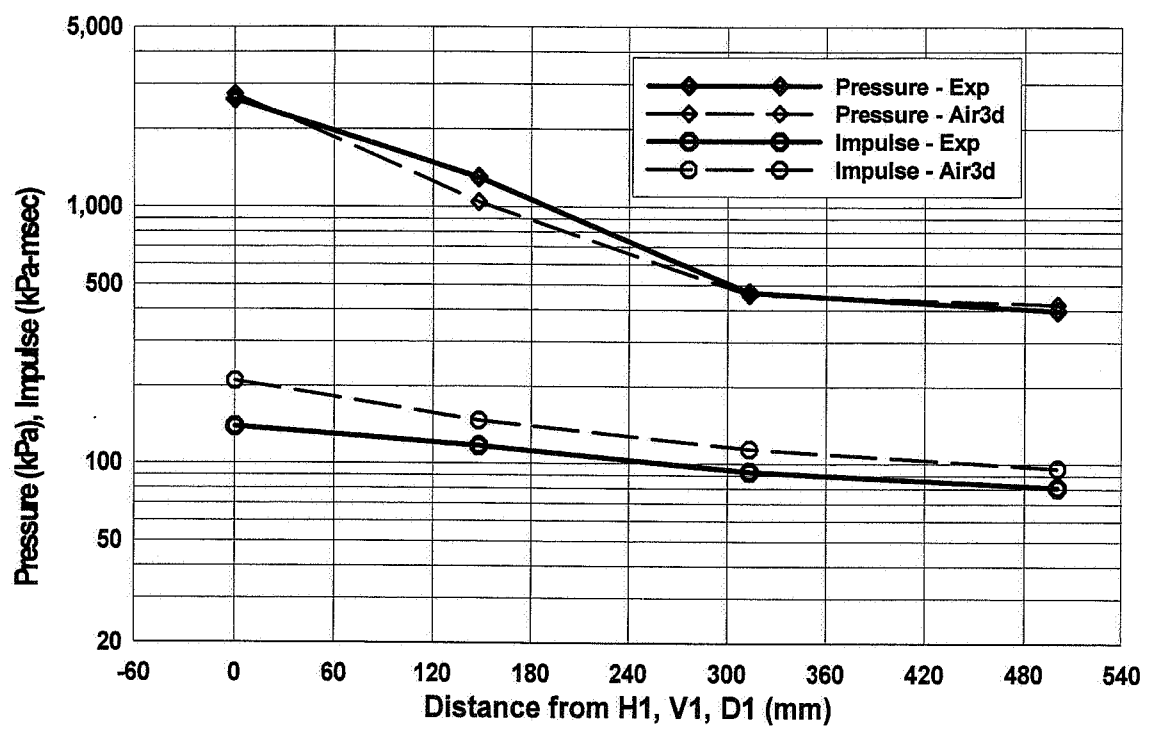


Figure 6.17: Comparison of results, dead end, diagonal

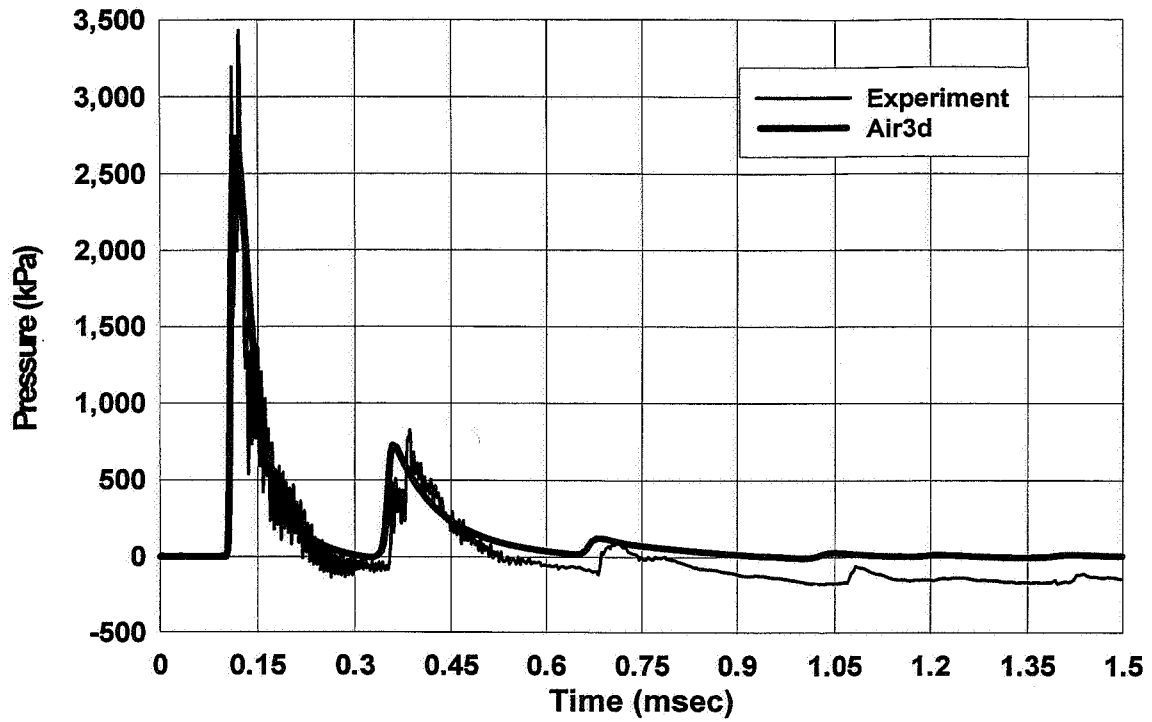


Figure 6.18: Comparison of pressure records, dead end, H1

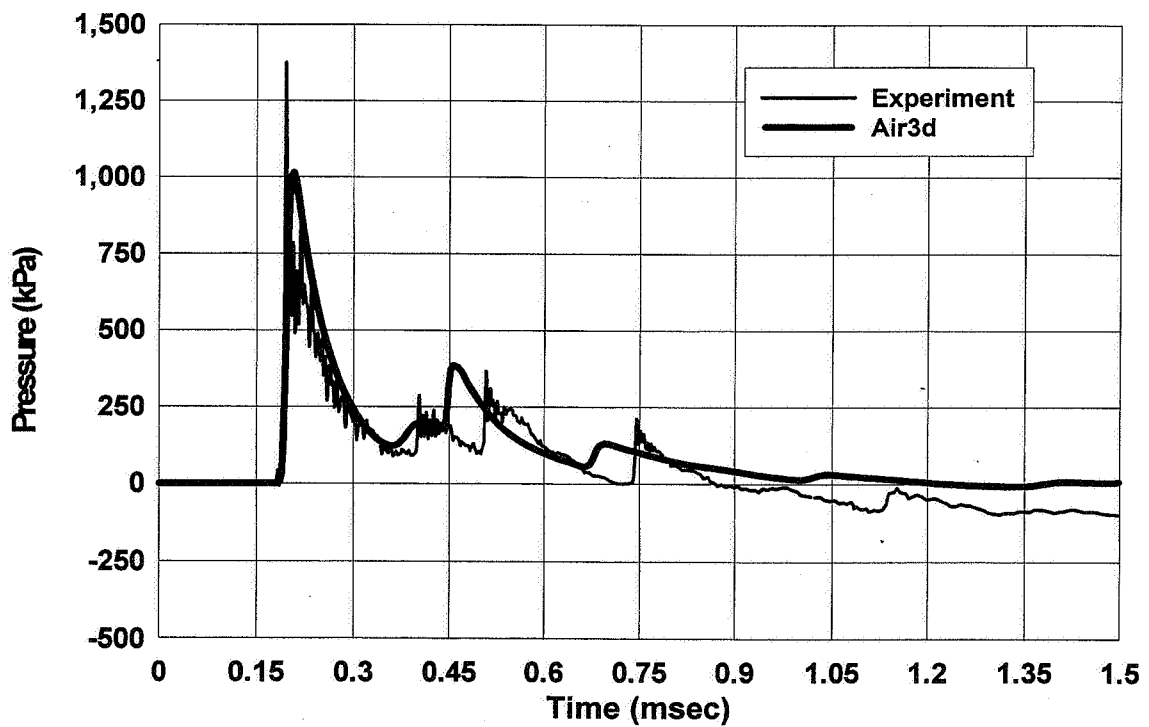


Figure 6.19: Comparison of pressure records, dead end, H2

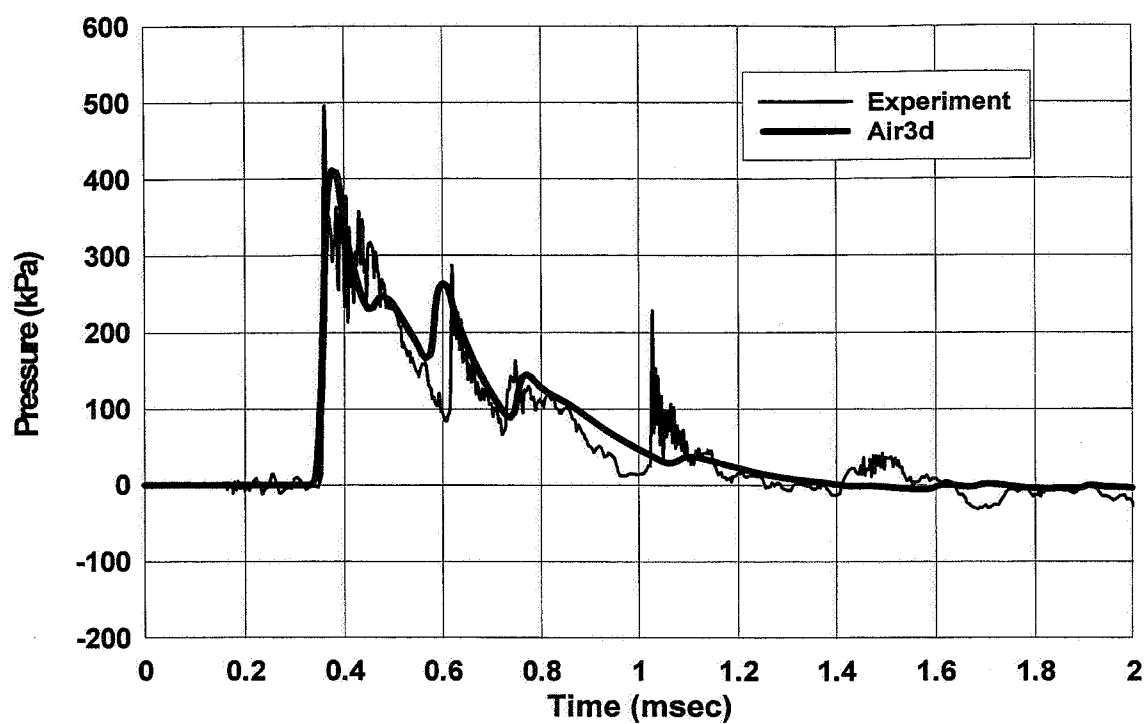


Figure 6.20: Comparison of pressure records, dead end, H3

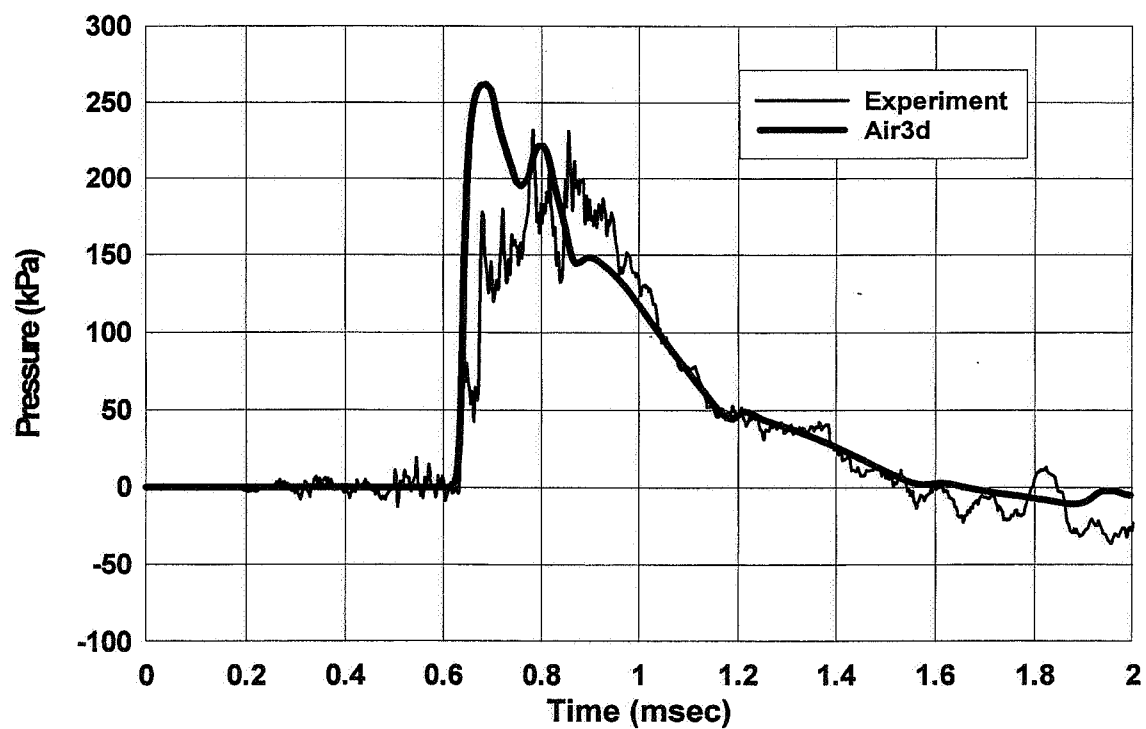


Figure 6.21: Comparison of pressure records, dead end, H4

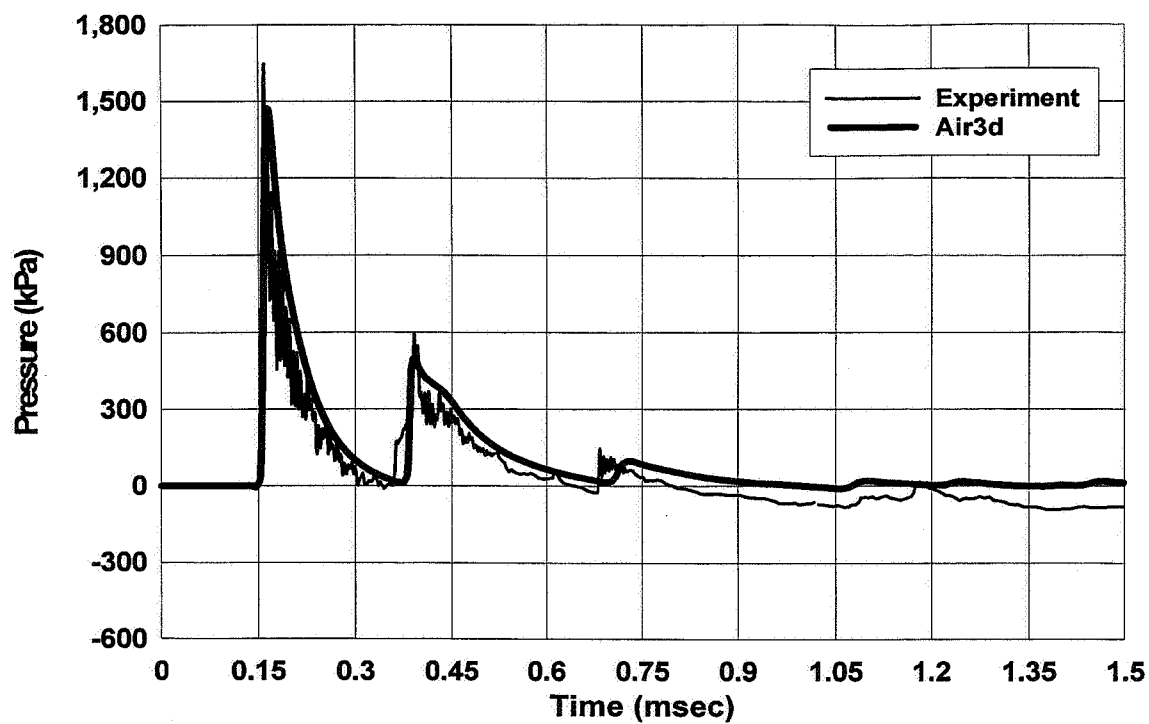


Figure 6.22: Comparison of pressure records, dead end, V2

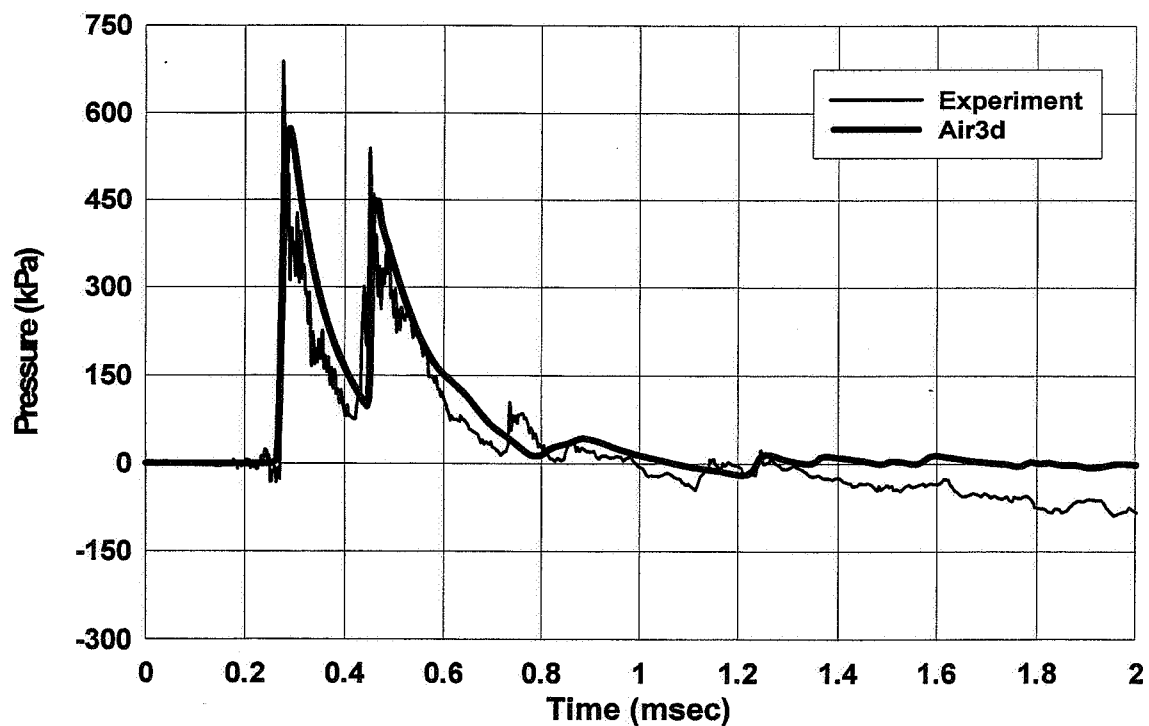


Figure 6.23: Comparison of pressure records, dead end, V3

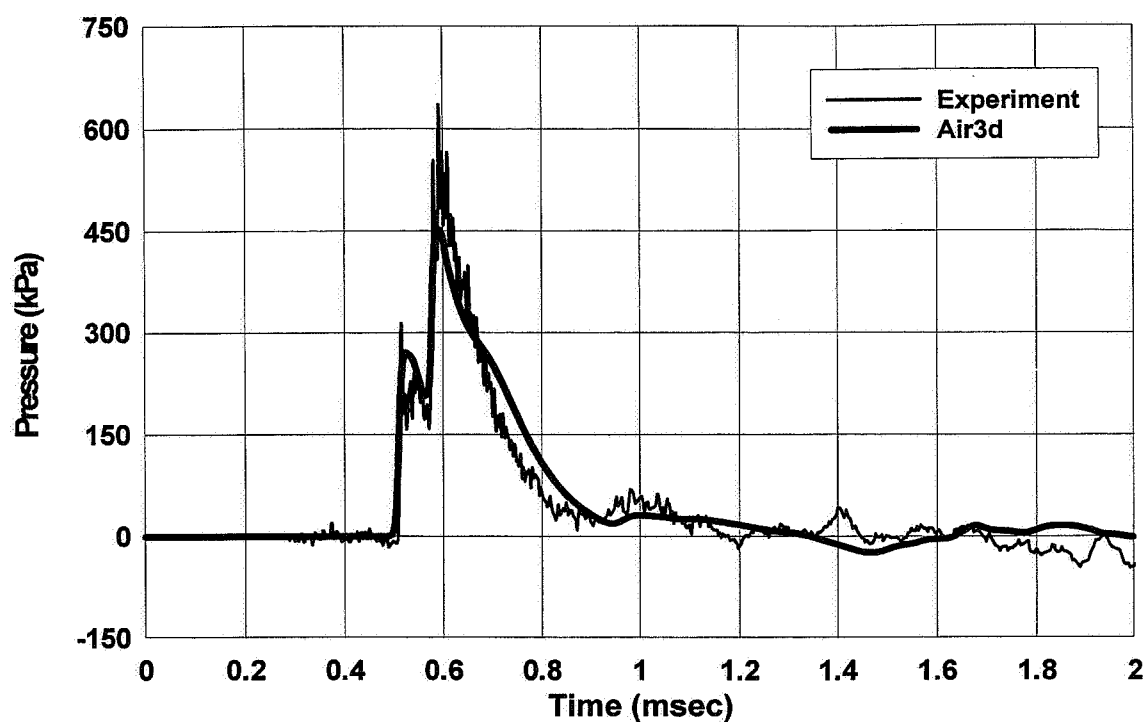


Figure 6.24: Comparison of pressure records, dead end, V4

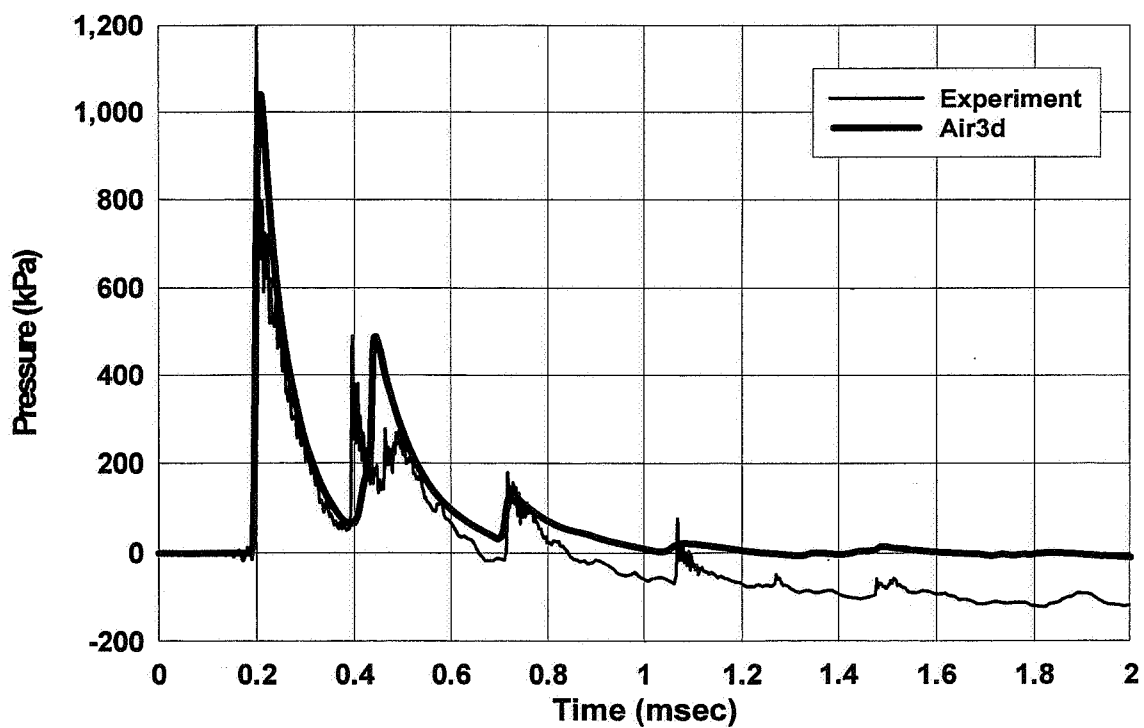


Figure 6.25: Comparison of pressure records, dead end, D2

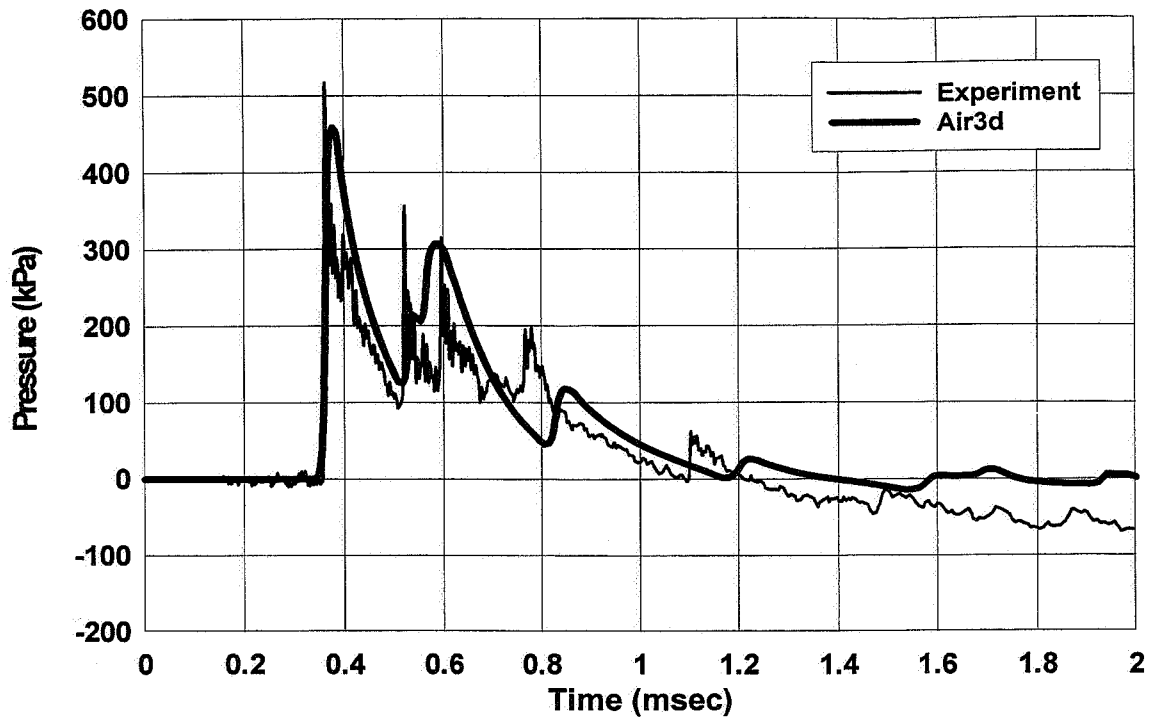


Figure 6.26: Comparison of pressure records, dead end, D3

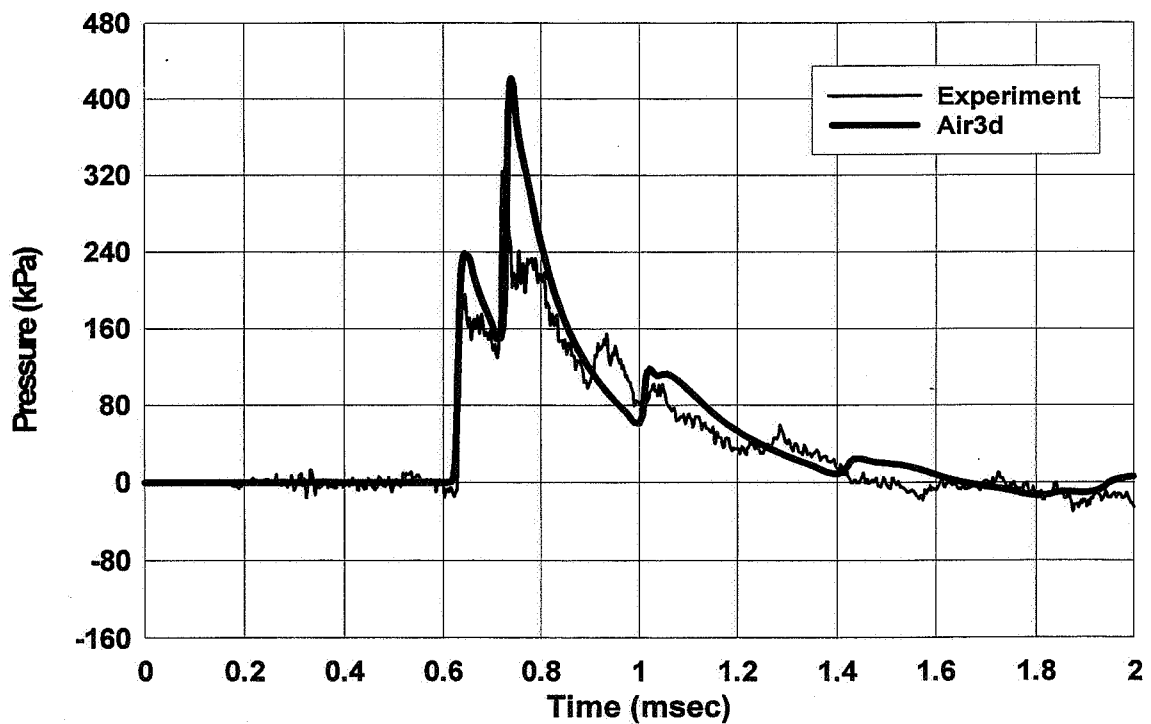


Figure 6.27: Comparison of pressure records, dead end, D4

One of the primary objectives of Whalen [57] was to appreciate the way each of the street configurations modified the impulse from what would otherwise have been an unconfined height of burst. In order to quantify this effect, graphs were produced of *impulse amplification factor* plotted against the distance of the monitoring location from the charge for each of the three monitoring orientations. The impulse amplification factor is defined thus:

$$\text{Impulse amplification factor} = \frac{\text{Peak impulse}_{\text{street configuration}}}{\text{Peak impulse}_{\text{height of burst}}}$$

The same approach has been taken here for the Air3d results which are presented in Figures 6.28 to 6.30.

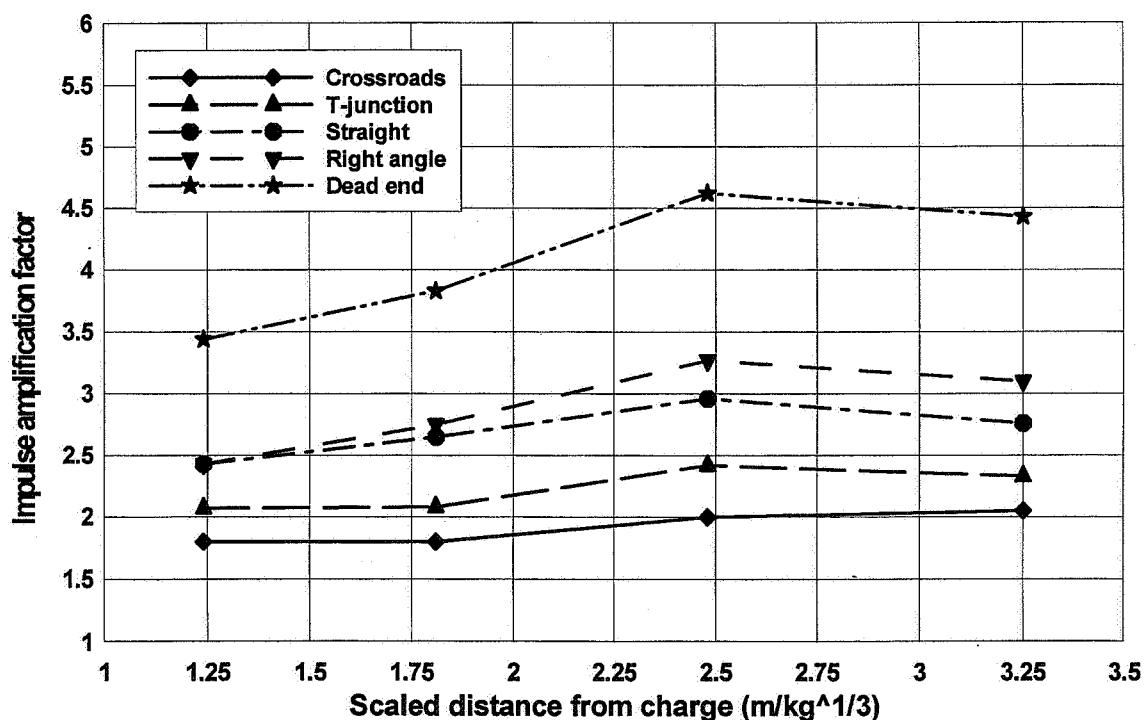


Figure 6.28: Impulse amplification factors, horizontal array

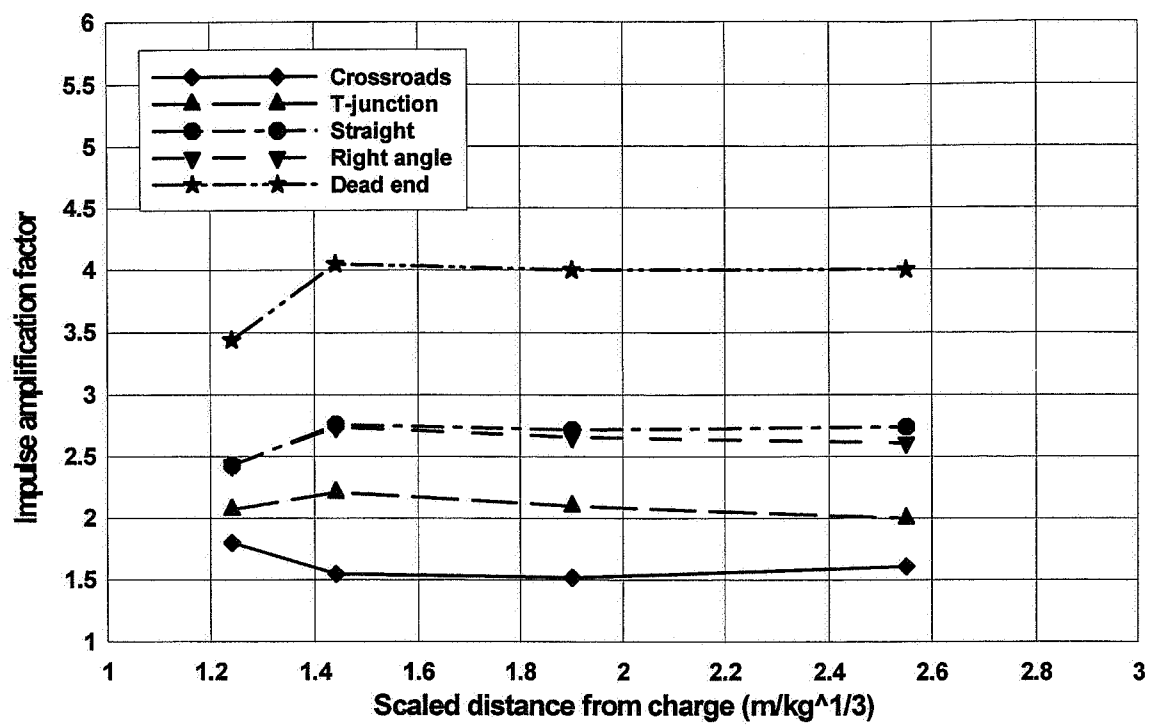


Figure 6.29: Impulse amplification factors, vertical array

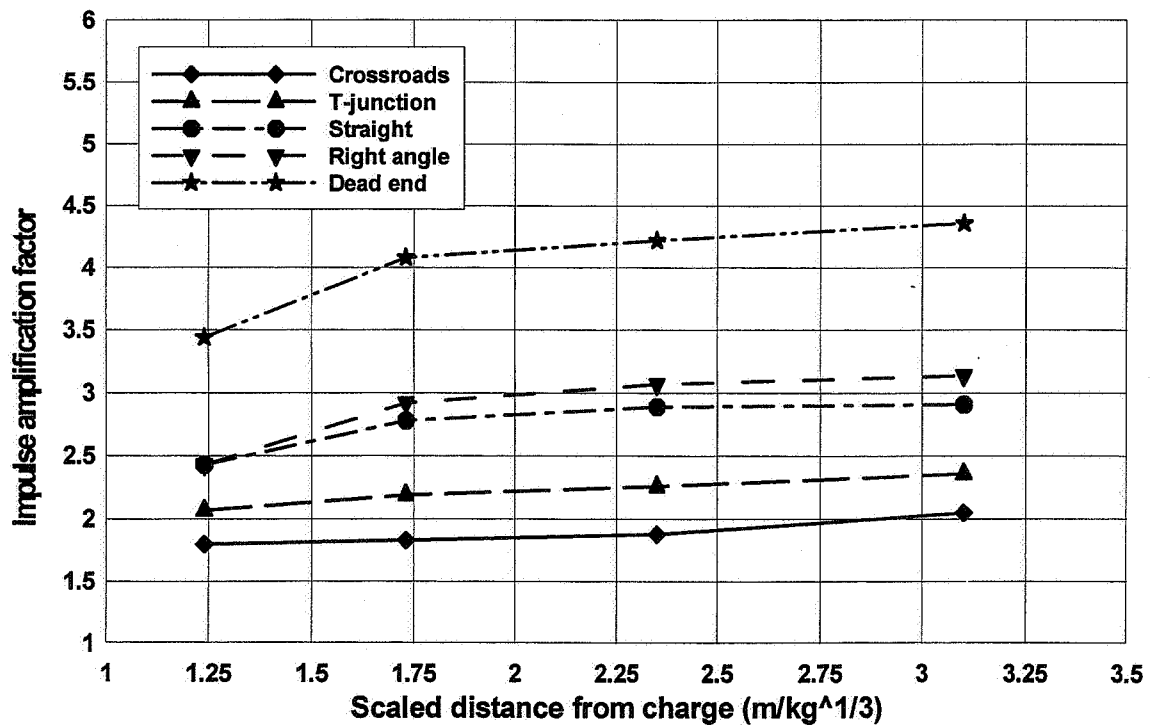


Figure 6.30: Impulse amplification factors, diagonal array

There are a number of important observations to arise from the validation process.

- In all fifteen graphs (Figures 6.3 to 6.17) showing the comparison between peak experimental and numerical pressure and impulse, it can be seen that the Air3d peak impulses are invariably slightly higher than the experimental values. This is most probably due to the original assessment of charge weight ($W = 13$ g TNT equivalent) which may have been slightly overestimated.

The peak pressures produced by Air3d are either close to or else slightly lower than the experimental values. This is encouraging, but it is likely that the discrepancy is only so small because of the possible overestimation of the charge weight.

- There are a number of experimental data points on the graphs (Figures 6.3 to 6.17) that are obviously "wild": they do not follow the trend of the data. Probably the most significant example is the final pressure point of Figure 6.13. Incidentally, this point is the only experimental data point that is not an average of three.
- Although all the raw experimental data was available, the data used for comparison were the mean values of three firings (with the one exception noted above). Also, Figure 6.2 indicates that monitoring locations H1, V1 and D1 were coincident. Therefore, the data from these points could be further reduced to produce an average from nine firings.

Referring to Tables 6.1 to 6.5, comparison of the peak pressure information from locations H1, V1 and D1 indicate that there was a large variation in the experimental data, and the numerically-derived data are within the spread of the experimental peak pressures at this single point, in each configuration. Because the effect of vibration of the steel plate on the pressure measurements was most pronounced at this nearest point, it is not surprising that the spread of peak pressures is large. The same arguments cannot be made with respect to the other measuring locations, however, without further consideration of the raw data.

It is recognised that it is possible to make more rigorous use of the raw data in a statistically valid way, but only the most simplistic approach was employed in the present study.

- The comparison between the two sets of pressure-time histories is encouraging. It can be seen that there is a broad similarity between the two sets of records, and all of the major features have been modelled.

It will be noticed that many of the experimental records do not return to atmospheric pressure but have drifted to some arbitrary value just below atmospheric. This is a characteristic of the measuring equipment and does not affect either the peak pressure or the associated specific impulse significantly.

One of the most interesting aspects of the comparison of the pressure histories can be seen in Figure 6.25. This graph shows several small reflections, each

of which has been accurately predicted in time. The peaks, however, are only very poorly resolved. This fact is one of the many influencing factors that make small scale experiments an attractive option for the solution of practical problems.

- Conclusions that can be drawn from the three graphs Figures 6.28 to 6.30 are very similar to those of Whalen [57] (not discussed here). In the present study, however, the consistency of the numerical data, and the fact that the height of burst data was also obtained in a consistent manner, has led to results that are probably clearer than those of Whalen [57].

6.3 The Effect of Urban Geometry on Blast Wave Resultants

In this section, the effect of street width and street height on blast wave resultants will be investigated in a systematic manner. A typical straight city street, with no side roads, is shown in plan view in Figure 6.31. The (half) street length l , the width w , the location of a hypothetical hemispherical explosive charge (weight W) and the planes of symmetry are also indicated. An end view of the street showing the building height h and the building depth d is given in Figure 6.32.

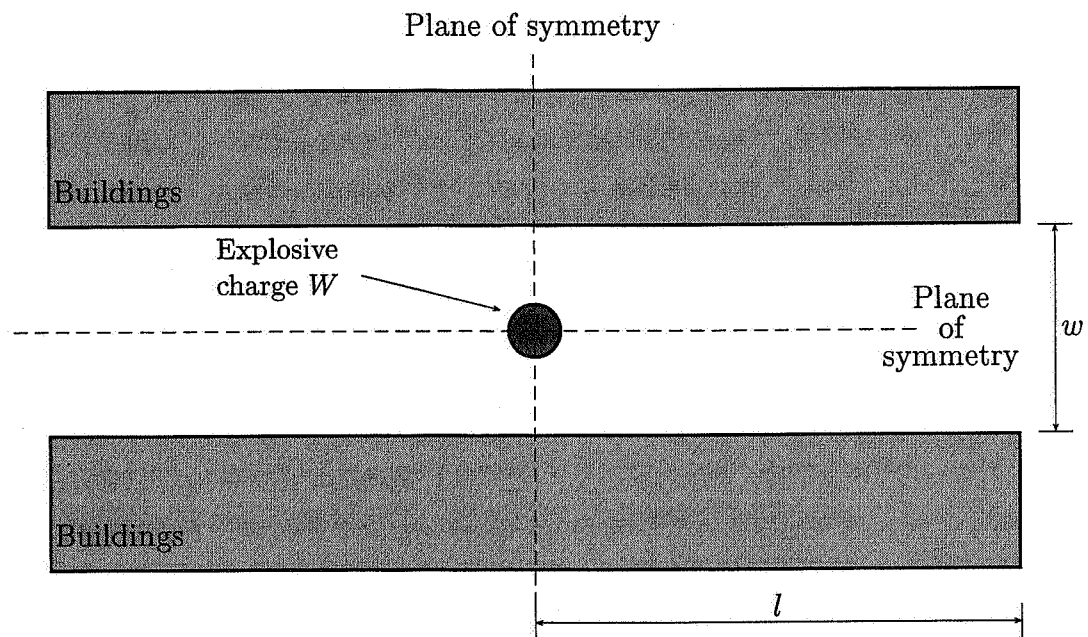


Figure 6.31: Plan view of a typical city street

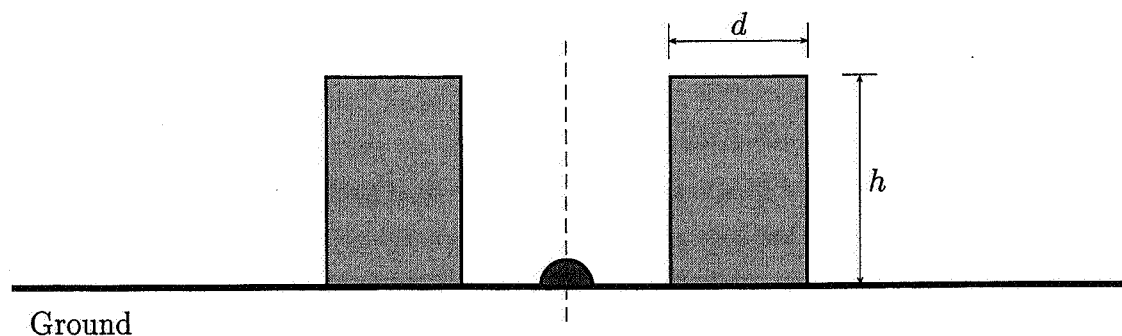


Figure 6.32: End view of a typical city street

Because of the symmetry planes in Figures 6.31 and 6.32, the scenario can be analysed in *quarter-space*. This greatly reduces the amount of computational effort required. In this investigation, seven different street widths: $w = 8, 12, 16, 20, 24, 28$ and 32 m, and six different building heights: $h = 4, 8, 12, 16, 20$ and 24 m, which modelled one to six storey buildings, respectively, were used. As well as the above building heights, two further analyses were performed for each street width; these used an effectively infinite building height. One analysis was a street analysis similar to above, the second was not modelled in quarter-space but in half-space, because it did not model the other side of the street. This last analysis produced *true* reflected pressures and was used for comparison. A total of 56 analyses were performed in this matrix.

A computational domain $x = 75$ m (along the street) by $y = 48$ m high and $z = 48$ m wide was used. Pressure monitoring points were located at 5 m intervals from $x = 0$ m to $x = 50$ m along the length of the street (11 locations) and in the middle of the storeys: at $y = 2$ m, 6 m, 10 m, and so on, above the ground. The depth of the buildings d was modelled as effectively infinite by extending them to the boundary of the computational domain (see Figure 6.33). Again, this reduced

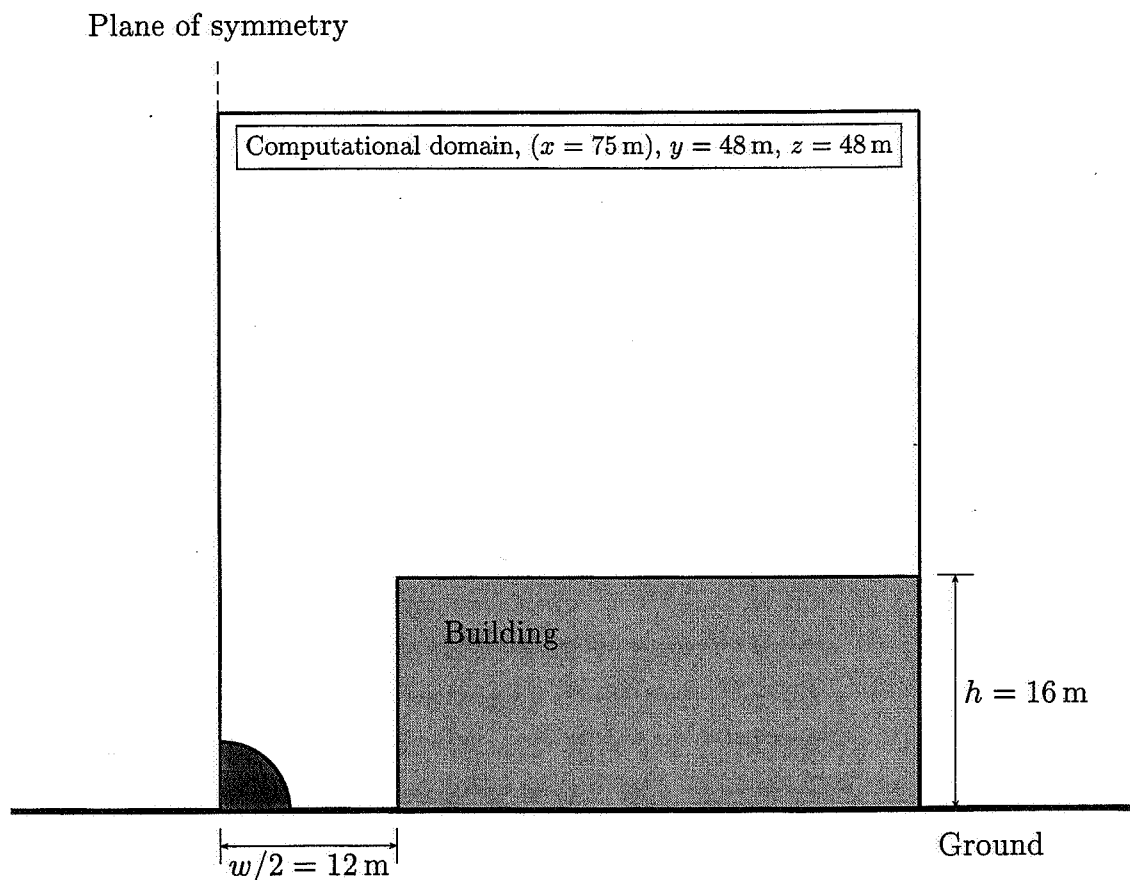


Figure 6.33: Example calculation, $w = 24$ m, $h = 16$ m, (end view)

the number of parameters in the problem; although, as concluded in Chapter 5, it is likely that the effect the building depth has on the diffraction process is insignificant. An end view of one analysis ($w = 24$ m, $h = 16$ m) is shown in Figure 6.33, where it can be seen that the computational domain ($x = 75$ m, $y = 48$ m and $z = 48$ m) was extensive. This ensured that any anomalous behaviour originating at the boundaries (over-emptying, for example) did not affect the region of interest: the part of the grid in which the measuring points were located. These dimensions, together with the street length, were established by numerous trial calculations with the aim of ensuring consistency of the results.

The explosive charge was 1000 kg TNT and located at the origin of the computational domain. It was modelled as a hemispherical surface burst, again, in order to reduce the number of parameters in the investigation. This also had the advantage of giving the problem one-dimensional symmetry between the charge centre and the vertical surface of the buildings. In each of the analyses, the one-dimensional cell size was 2 mm and the three-dimensional cell size was 0.5 m.

6.3.1 The Effect of Street Width on Blast Wave Resultants

The effect of street width on blast wave resultants can be demonstrated most effectively by comparing the peak impulse experienced at a given location on an infinitely high street façade (referred to here as *street* impulse) with the *true* reflected impulse at the same point: the impulse measured at the location if it was not one side of a street but a single infinite reflecting surface. Clearly, in the latter case, reflections from the other side of the street will not occur. Therefore the *true* reflected impulses act as a *lower bound* on the values for street configurations of infinite height.

The seven graphs Figures 6.34 to 6.40, presented below, compare infinitely high *street* and *true* reflected impulses, measured at the lowest of the horizontal measuring locations, for each of the street widths. In this section, and the section which follows, discussion is limited to the effect on blast resultants near street level ($y = 2$ m). Thorough treatment of the variation across the whole of the façade is beyond the scope of the present study.

It will be noticed that the wider the street the less the effect street confinement has on the impulse. The most striking comparison is between Figures 6.34 and 6.40. This shows that the extremely high level of confinement present in the narrowest street width ($w = 8$ m) produces impulses which are significantly greater than the *true* reflected values at every measuring location along the street. Figure 6.40 ($w = 32$ m), however, shows that there is effectively no initial confinement to affect the impulse at $x = 0$ m, and no reflections from the opposite side of the street influence the measuring locations until a horizontal distance of 35 m is reached. There is effectively no influence of confinement until this point. These two figures are representative of the two extreme conditions: high level confinement and very little confinement. The other figures demonstrate a gradual transition between these two extremes.

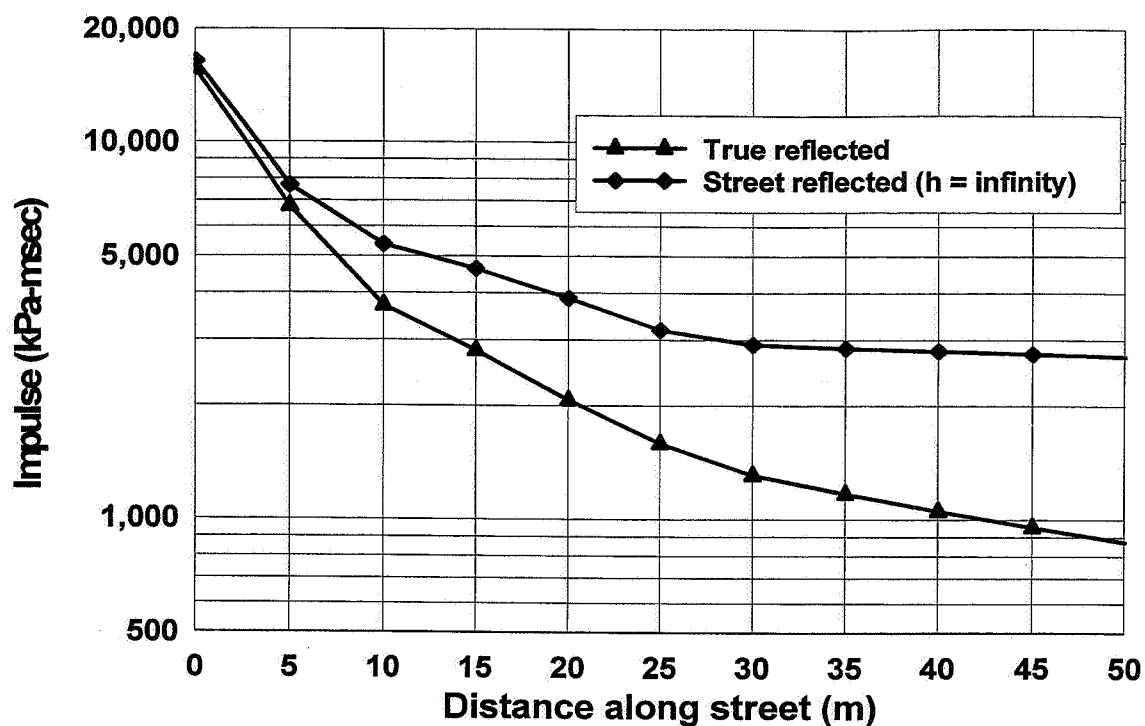


Figure 6.34: Impulse at $y = 2$ m, street width $w = 8$ m

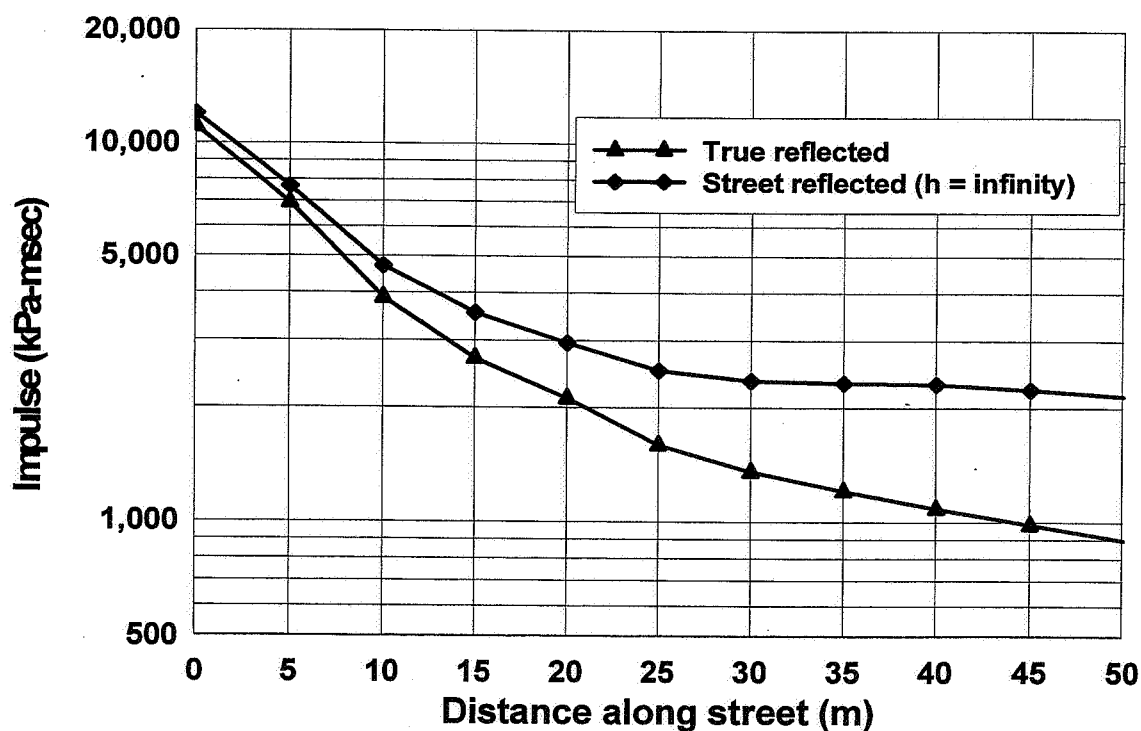


Figure 6.35: Impulse at $y = 2$ m, street width $w = 12$ m

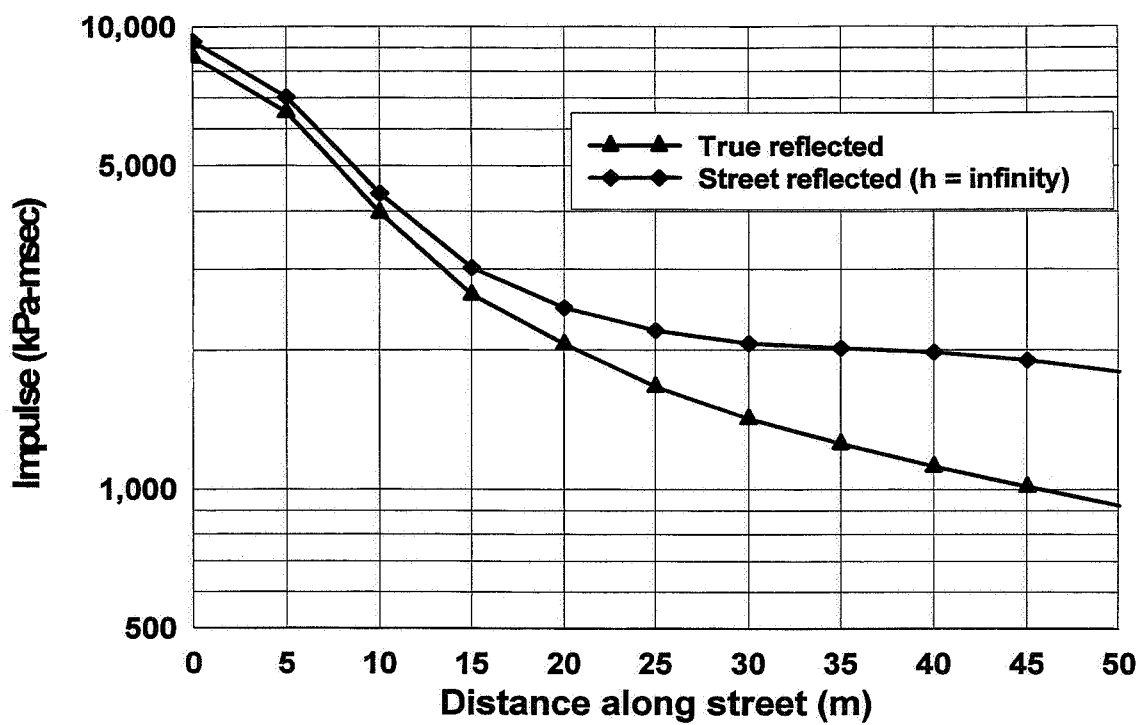


Figure 6.36: Impulse at $y = 2$ m, street width $w = 16$ m

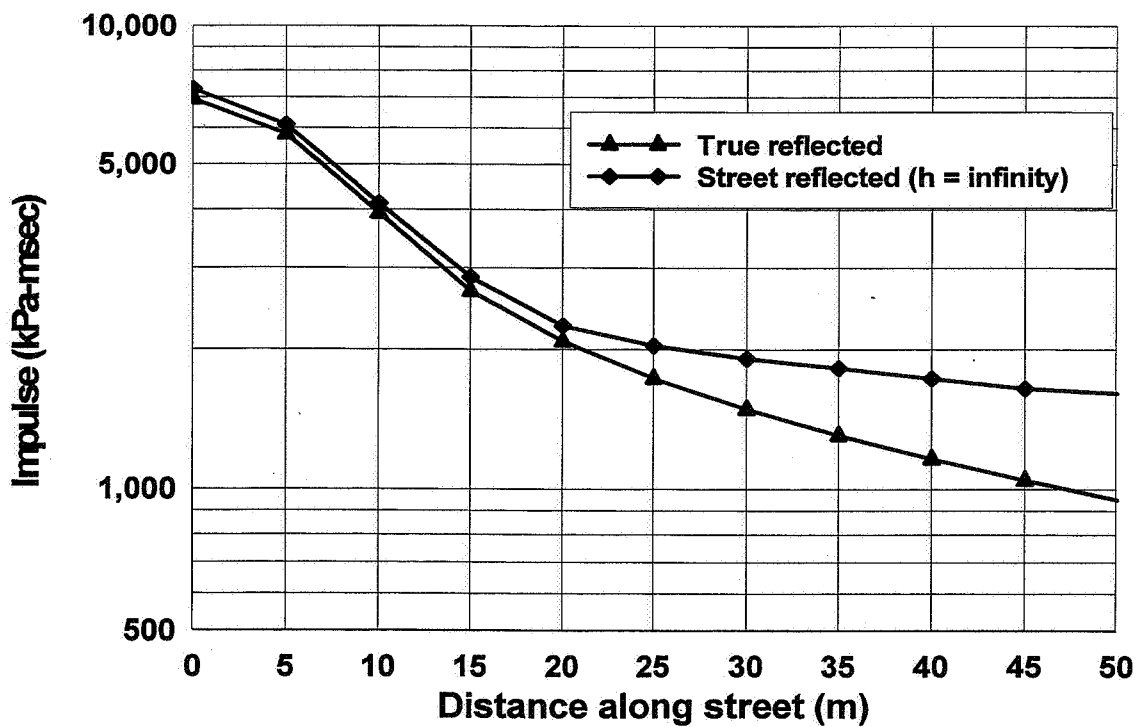


Figure 6.37: Impulse at $y = 2$ m, street width $w = 20$ m

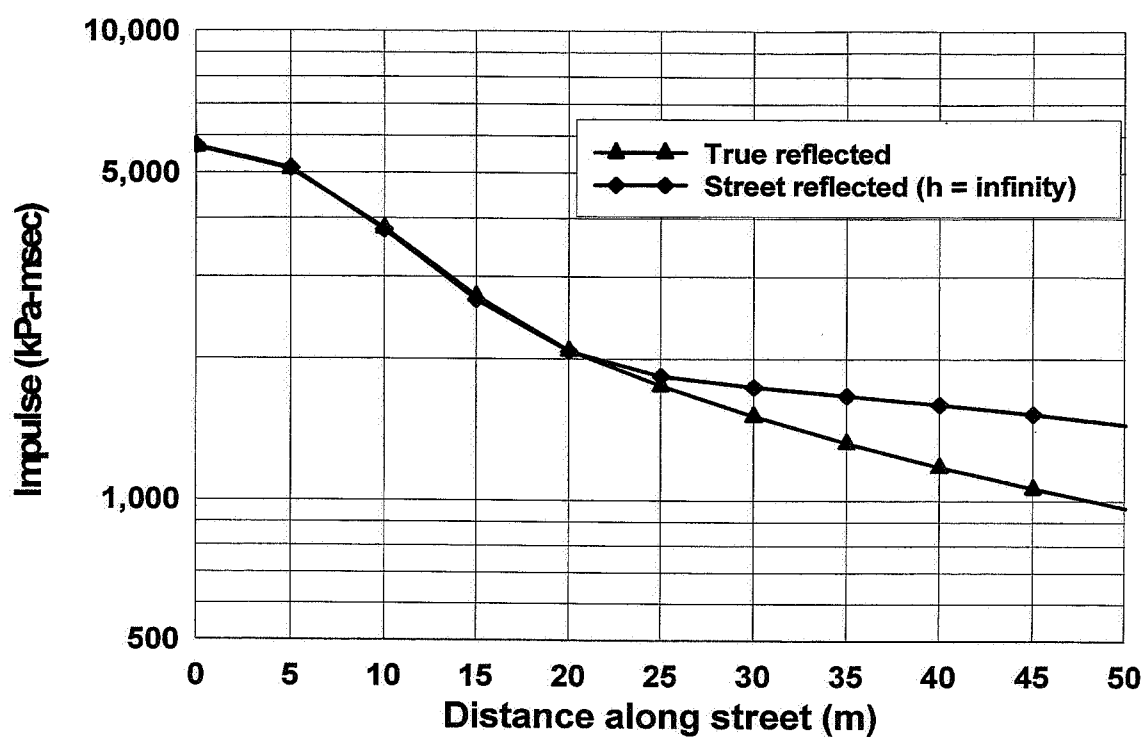


Figure 6.38: Impulse at $y = 2$ m, street width $w = 24$ m

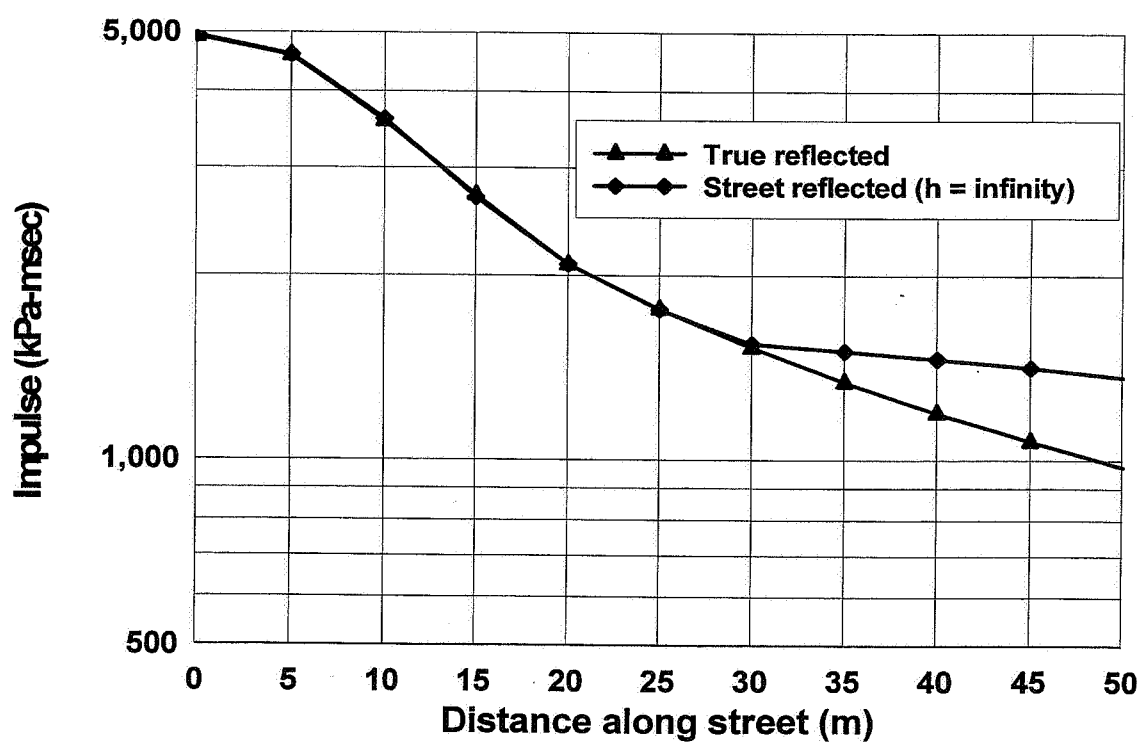


Figure 6.39: Impulse at $y = 2$ m, street width $w = 28$ m

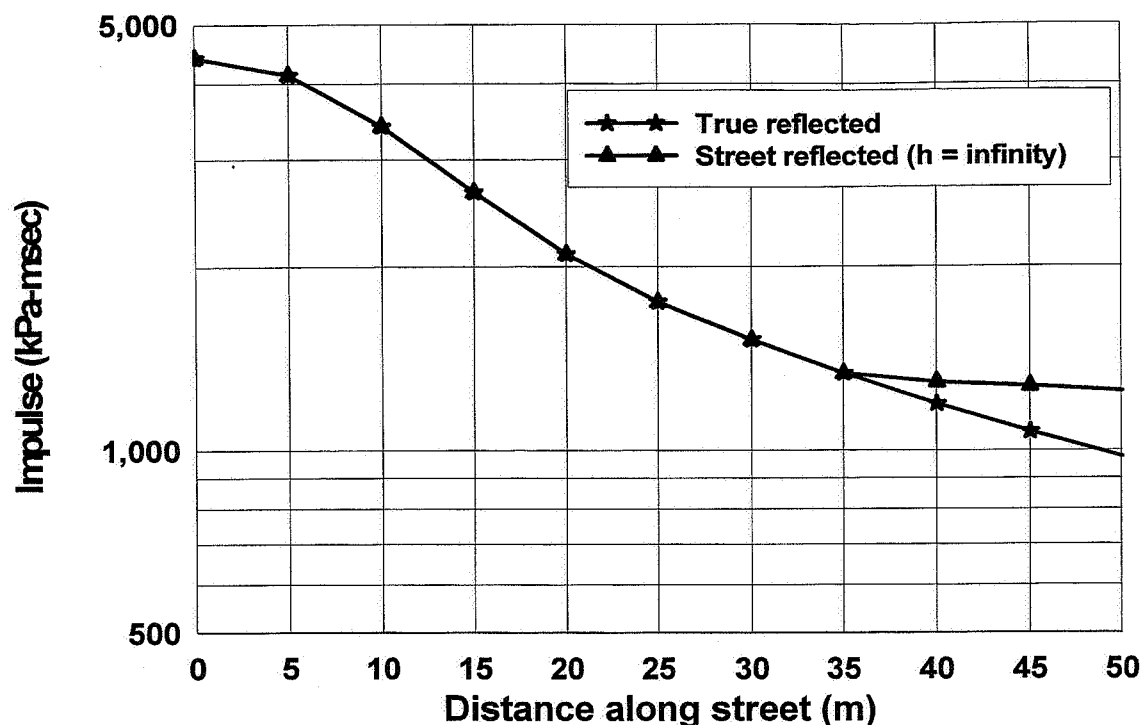


Figure 6.40: Impulse at $y = 2$ m, street width $w = 32$ m

Unfortunately, Figures 6.34 to 6.40 do not tell the whole story. The main difference between the *true* and *street* reflected configurations is the contribution of the reflections from the other side of the street. It is only when these reflections are big enough and occur soon enough to overcome the expansion immediately behind the first shock wave that the two curves separate on Figures 6.34 to 6.40. This is best demonstrated with appropriate examples. Figures 6.41 and 6.42 show a comparison of pressure histories at 25 m and 50 m along the street of width $w = 8$ m. They clearly show that the second (and later reflections) from the other side of the street have coalesced with the first shock wave to form a single enhanced pulse. Figure 6.42 shows that by the time the blast wave has progressed to the end of the measuring array it has increased in pressure substantially and, as a result, arrives sooner than the *true* reflected wave.

A further consideration which affects the influence that second (and higher order) reflections have on enhancement of impulse is when they occur in time. For example, a reflection that occurs during a strong expansion (after the incident wave, say) might not contribute to the total impulse significantly, if at all. An example of this can be seen in Figure 6.43, for $w = 32$ m and $x = 35$ m. Figure 6.40 shows that the impulse measured at $x = 35$ m along the street is the same for both *true* reflected and *street* geometries. Figure 6.43, however, shows that there is at least one strong secondary reflection at this location, but because it occurs during an expansion it does not affect the total impulse.

The last graph in this section (Figure 6.44) compares the pressure histories at $x = 50$ m from the same series ($w = 32$ m). The effect of the higher order reflections on the total impulse has become evident. This can be seen from the final points (at $x = 50$ m) of Figure 6.40.

A summary of the effect of street width on blast wave resultants at street level is as follows:

- At relatively small scaled street width ($w/W^{1/3} \leq 2.0 \text{ m/kg}^{1/3}$) reflections from the far side of the street coalesce with the incident wave and a single enhanced pressure pulse is formed.
- The analysis of the scaled street width $w/W^{1/3} = 3.2 \text{ m/kg}^{1/3}$ demonstrated that the confinement provided by this geometry had very little effect on the impulse delivered, compared with *true* reflected values. It should be recognised that the influence may be more pronounced at scaled distances greater than $5.0 \text{ m/kg}^{1/3}$ along the street (the largest scaled distance considered here); however, it can be assumed that scaled street widths in excess of $3.2 \text{ m/kg}^{1/3}$ would not produce a significant enhancement. The fact that the effect of street width was investigated using streets effectively infinite in height implies that, in most practical situations, $w/W^{1/3} > 3.2 \text{ m/kg}^{1/3}$ implies that no effect of the street confinement (or channeling) can be expected.
- The cause of the enhancement in impulse at relatively large scaled distances and large scaled street widths is demonstrated in Figure 6.44: namely, the presence of multiple reflections. For the purpose of this study, the impulse has been taken as the integral of the whole of the record. From the point of view of the structural engineer, the fact that it is comprised of the sum of a series of discrete pulses may have profound implications for the structural response of loaded elements. Such a discussion is beyond the scope of the present study, but the nature of the impulses described here should be borne in mind.

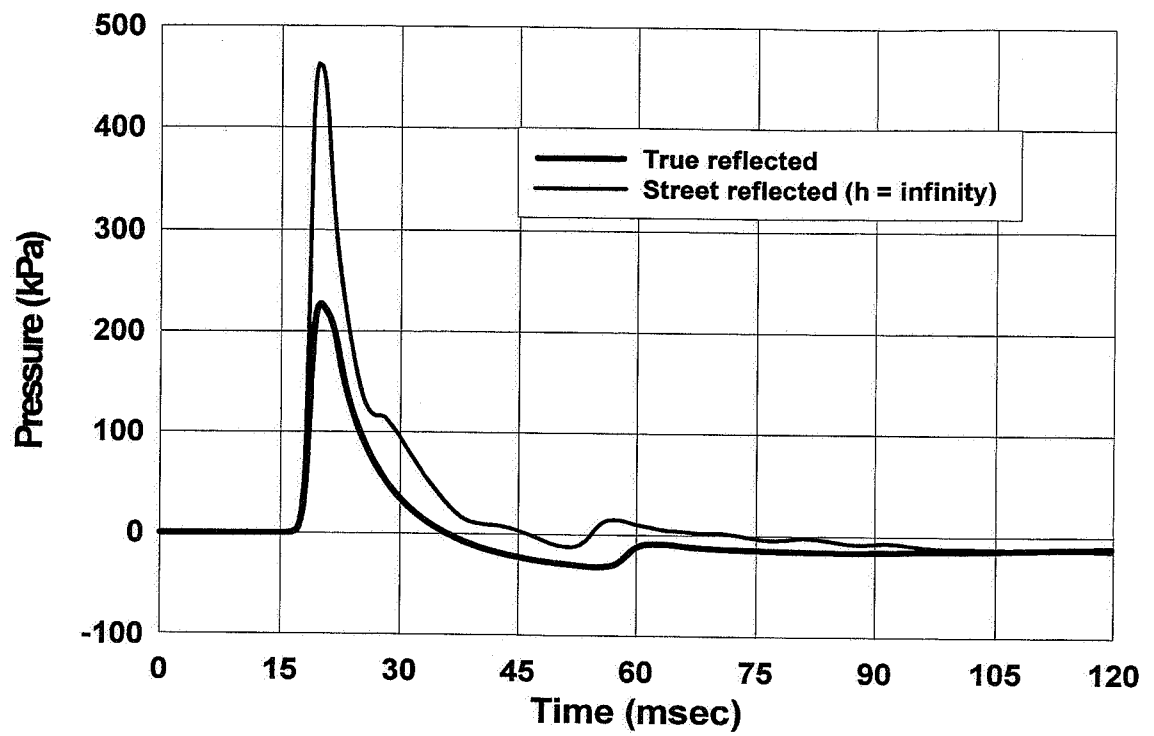


Figure 6.41: Comparison of pressure histories, $w = 8$ m, $x = 25$ m

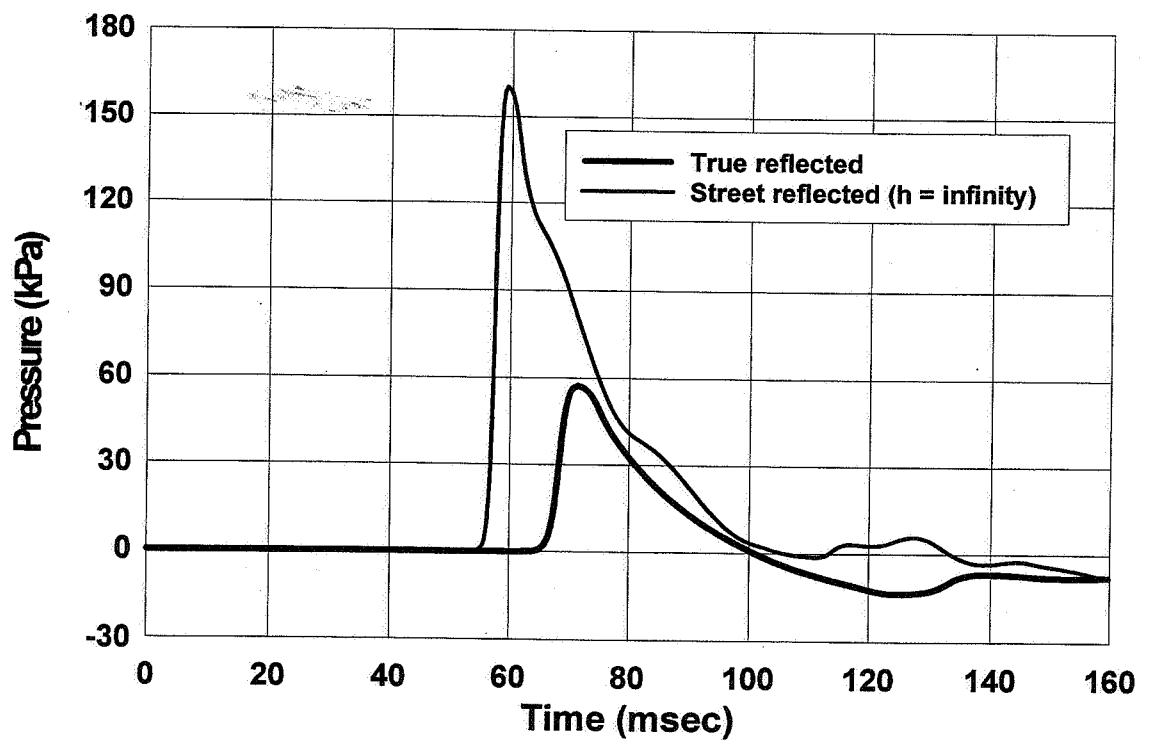


Figure 6.42: Comparison of pressure histories, $w = 8$ m, $x = 50$ m

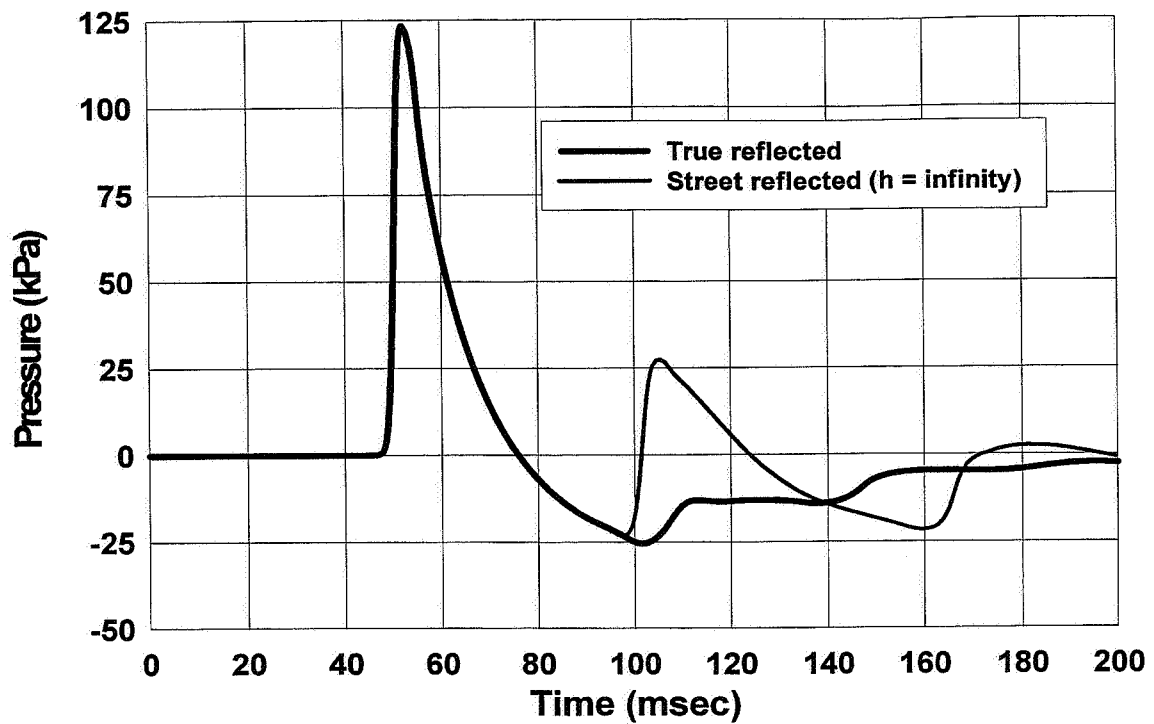


Figure 6.43: Comparison of pressure histories, $w = 32$ m, $x = 35$ m

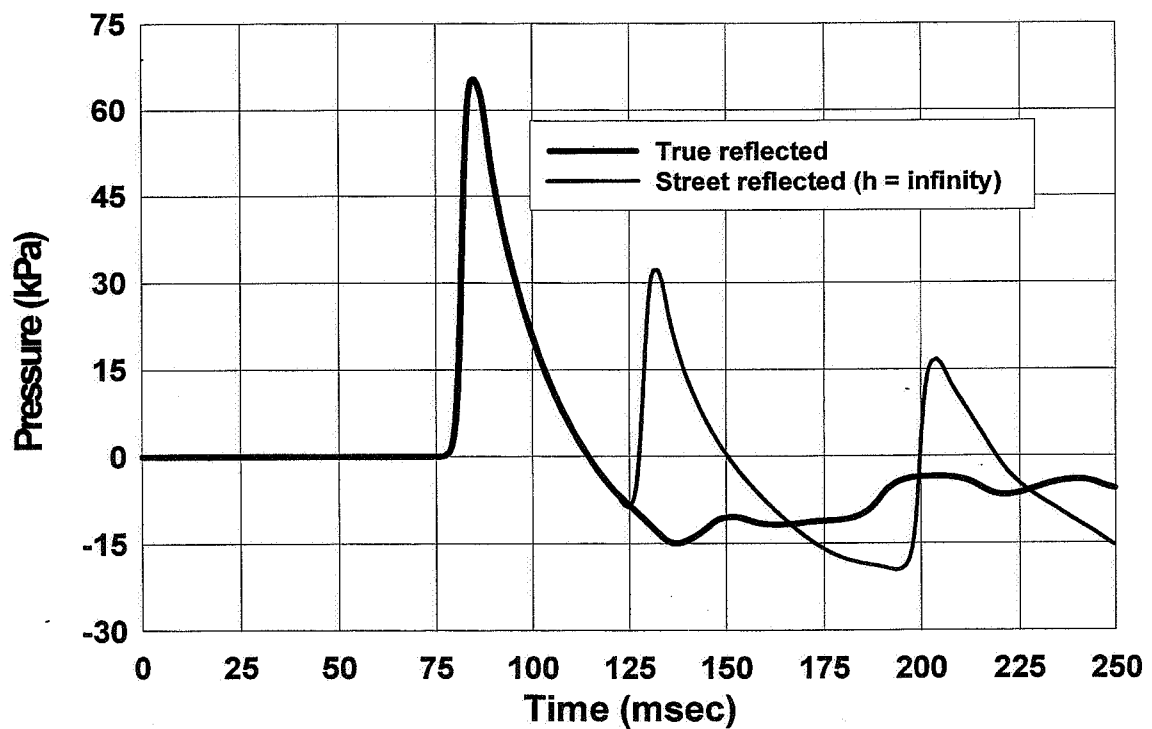


Figure 6.44: Comparison of pressure histories, $w = 32$ m, $x = 50$ m

6.3.2 The Effect of Building Height on Blast Wave Resultants

The influence of building height on blast wave resultants is a problem that is closely related to that of blast wave clearing, examined in Chapter 5. Referring to Figure 6.31, when the blast wave from a hemispherical surface burst impinges on the street façade, instantaneous reflected pressures occur. These are relieved by the process of clearing due to the proximity of the top of the structure. Clearly, if the structure is very high, relief of the pressure will not take place and reflected pressures will persist across the whole of the façade; this was discussed previously. Alternatively, if the structure is very low, the reflected pressures will be relieved to some extent, and the blast which continues along the street will be considerably reduced.

Interestingly, the blast that propagates along the street will still be continually relieved by the presence of the top of the building, despite the fact that as the distance along the street increases the direction of propagation becomes increasingly perpendicular to the direction of the building top. This kind of clearing could be referred to as *lateral clearing*, because, unlike the clearing described in Chapter 5, the principal convection direction (along the street) is perpendicular to the clearing direction. It will be appreciated that for any given combination of street width, street height and charge weight, the resulting situation is impossible to predict using current guidance.

The key parameter which influences these processes (for a given scaled street width) is the scaled building height, and this is the subject of the present section.

The following seven graphs (Figures 6.45 to 6.51) show the effect that building height has on the impulses measured along the horizontal line of monitoring locations at $y = 2$ m for each of the street widths investigated. This is a similar approach to the investigation of street width.

On each of the graphs, the impulses are plotted for each of the discrete building heights: $h = 4, 8, 12, 16, 20, 24$ and 28 m, together with the impulses on an effectively infinite height building, which can be seen as the thick dashed line on the figures.

It will be seen from Figures 6.45 to 6.51 that the $h = \text{infinity}$ line is virtually coincident with the line for $h = 24$ m on every graph, and the $h = 24$ m line is distinct from the $h = 20$ m line. This gives rise to an important observation: buildings of scaled building height $h/W^{1/3} \geq 2.4 \text{ m/kg}^{1/3}$ are effectively infinite in height. In other words, the effect of clearing, lateral or otherwise, on impulse at street level is not perceptible if the scaled building height is greater than $2.4 \text{ m/kg}^{1/3}$. It must be remembered that in this section semi-infinite structures are considered. If the length of the building in the direction along the street was less than effectively infinite, the above observation would not be true.

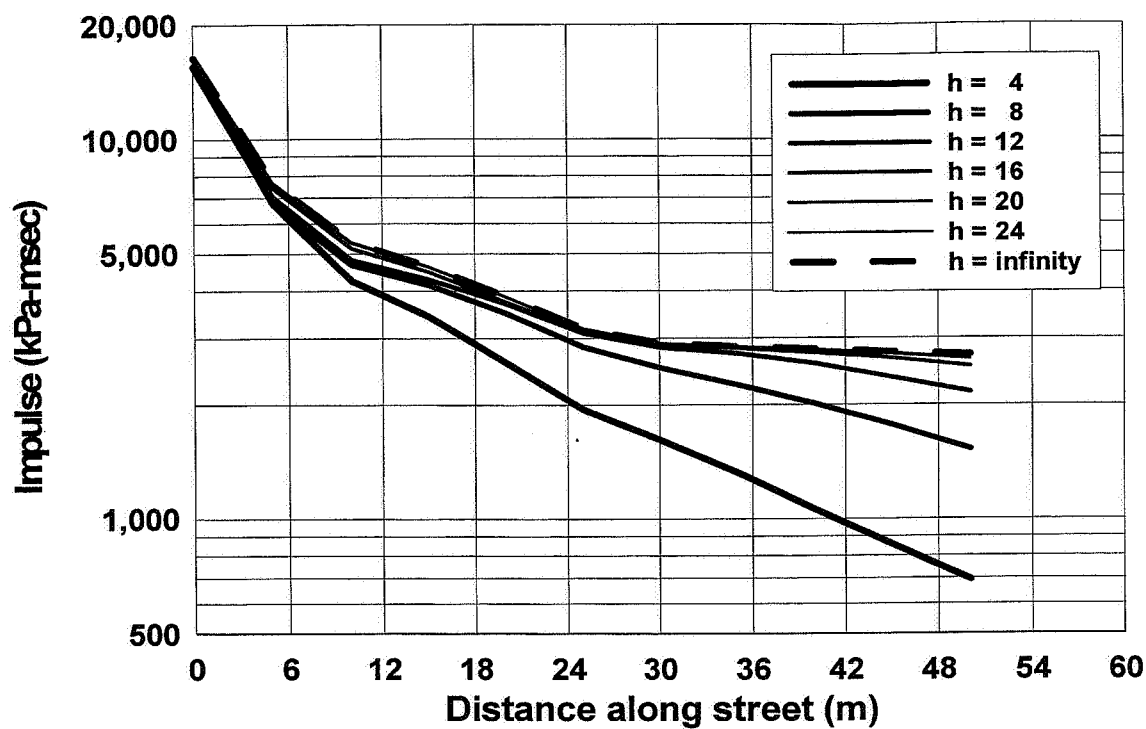


Figure 6.45: Impulse at $y = 2$ m, street width $w = 8$ m

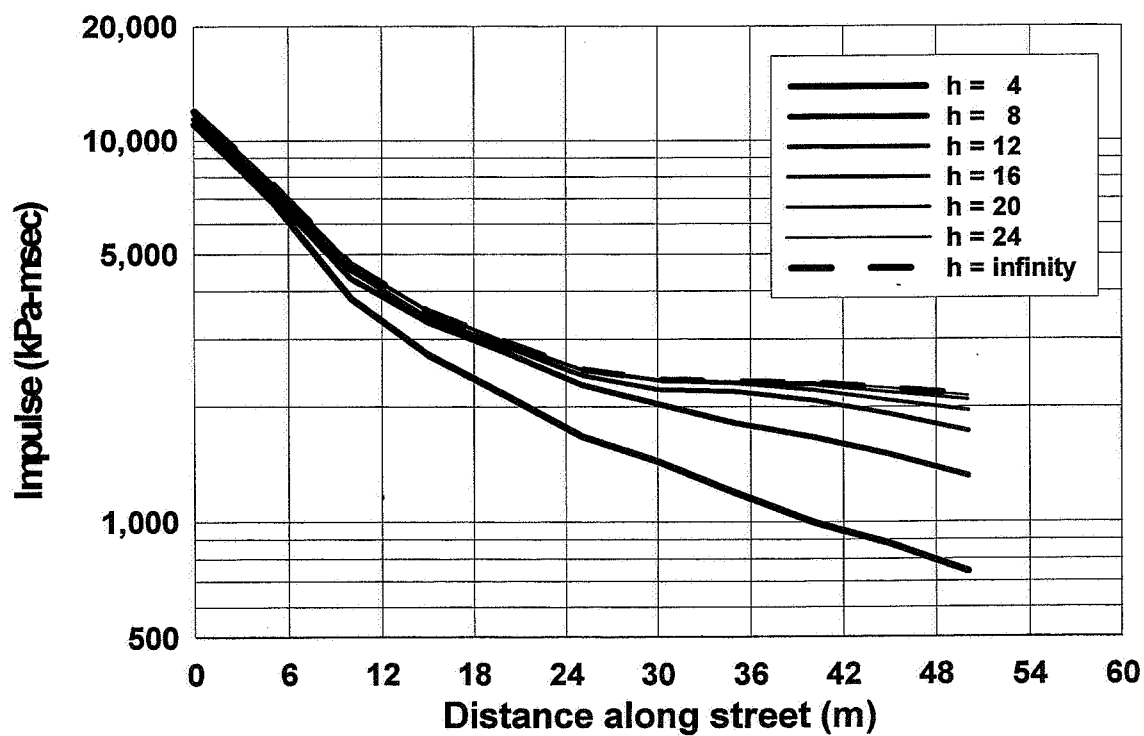


Figure 6.46: Impulse at $y = 2$ m, street width $w = 12$ m

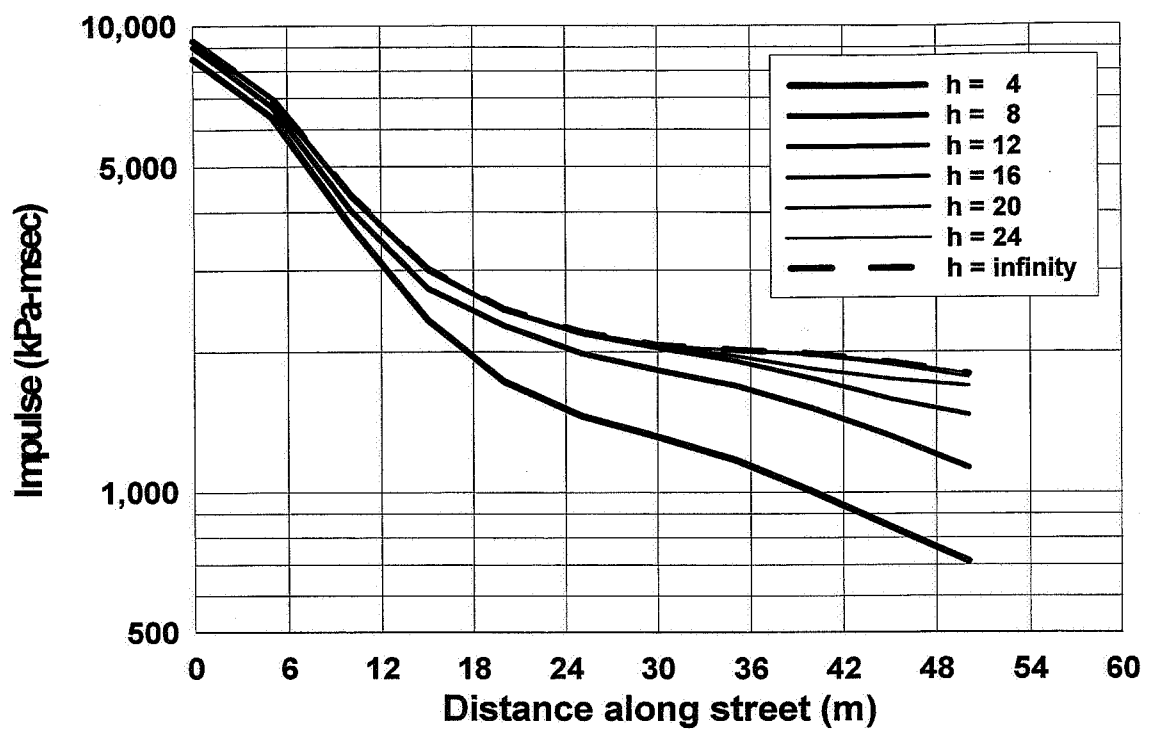


Figure 6.47: Impulse at $y = 2$ m, street width $w = 16$ m

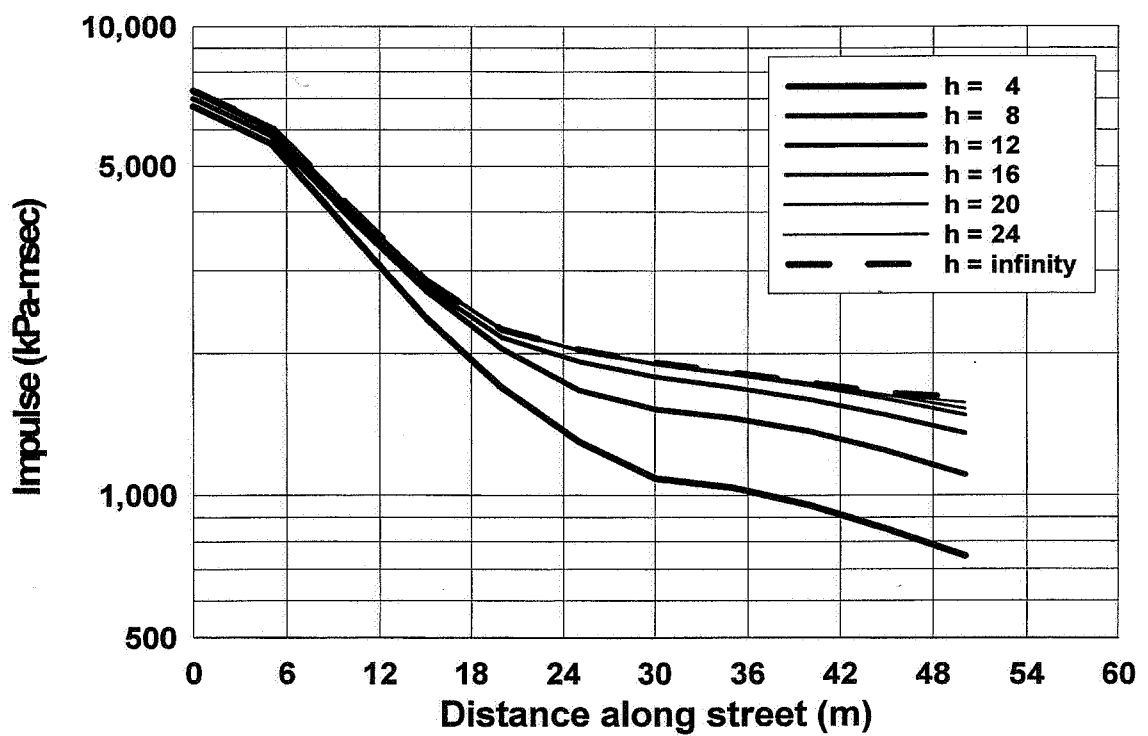


Figure 6.48: Impulse at $y = 2$ m, street width $w = 20$ m

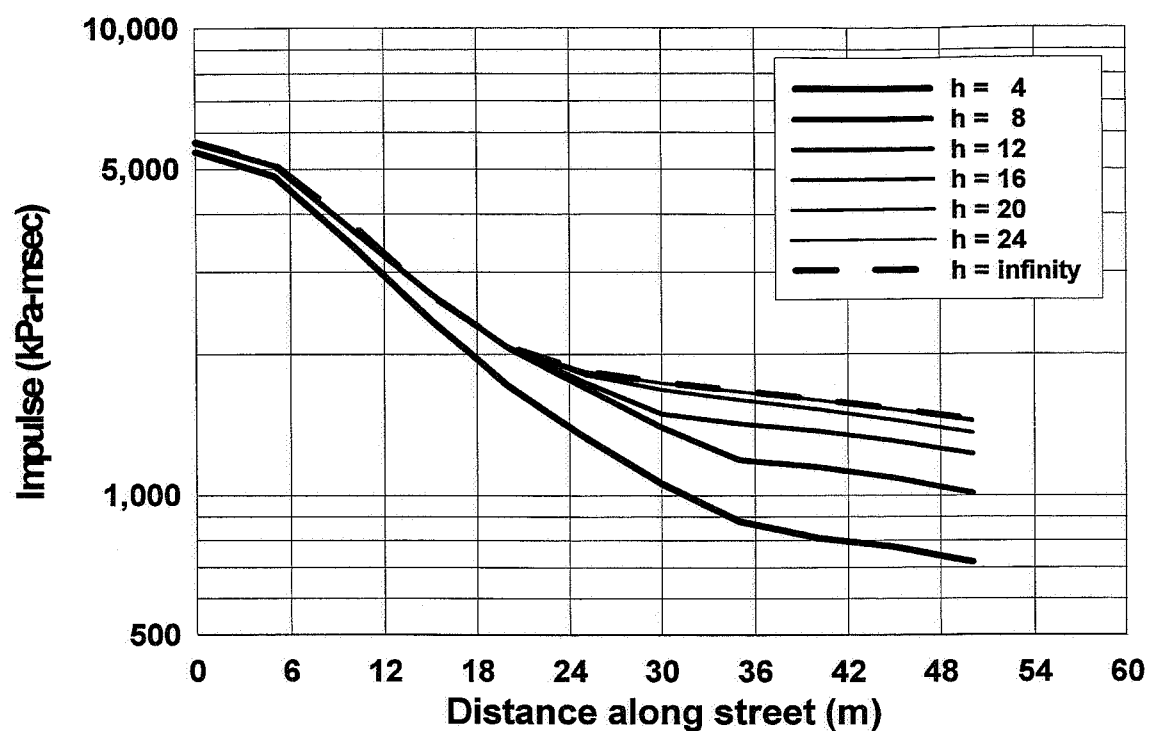


Figure 6.49: Impulse at $y = 2$ m, street width $w = 24$ m

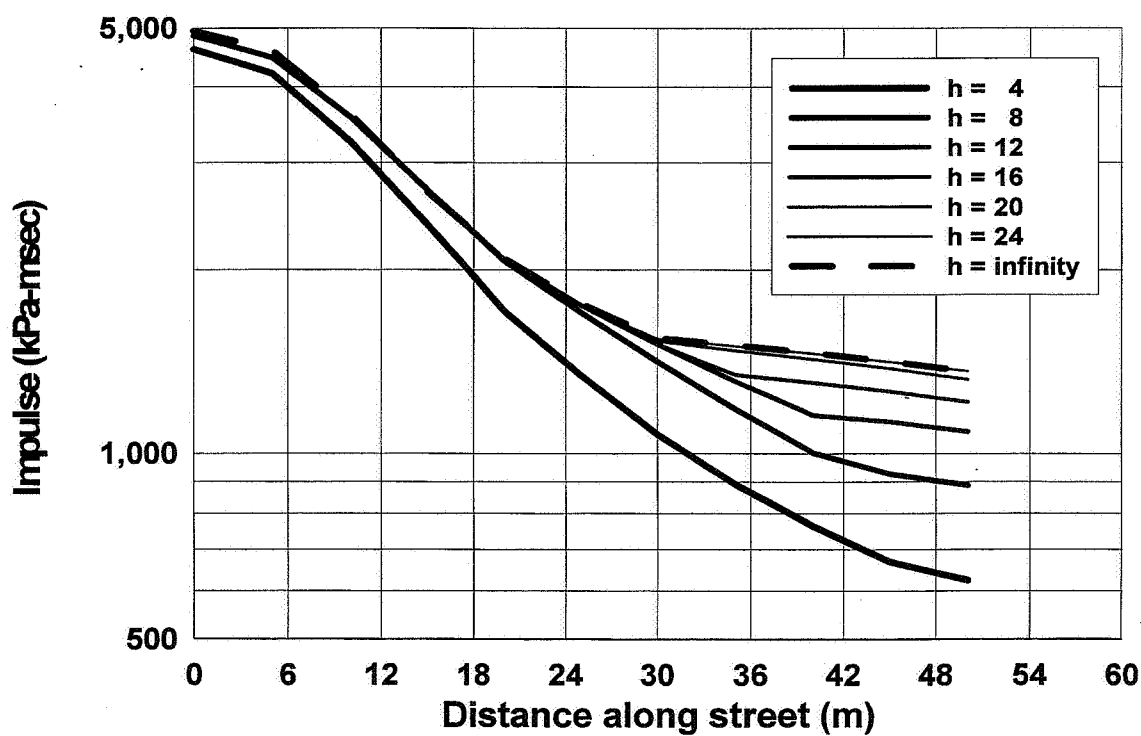


Figure 6.50: Impulse at $y = 2$ m, street width $w = 28$ m

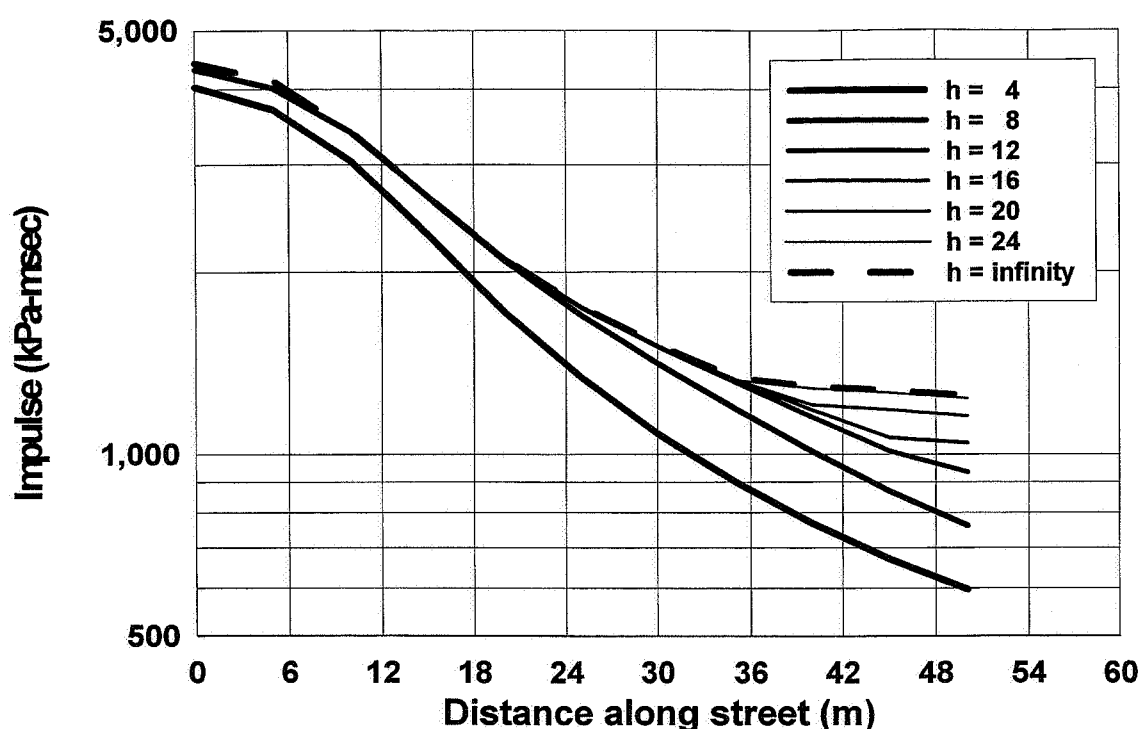


Figure 6.51: Impulse at $y = 2$ m, street width $w = 32$ m

It was shown previously (in Figures 6.40 and 6.44) that distinct changes in slope of the impulse-distance curves occur when the contribution of secondary reflections becomes important. A similar observation can be made concerning the graphs Figures 6.45 to 6.51, and the region where this occurs can be identified uniquely in terms of scaled street width and scaled building height. To examine this more closely, a comparison of the pressure records for $h = 4$ m and $h = \text{infinity}$ are presented. Unlike the comparison in Section 6.3.1, the street geometry is the same in both cases, except for the different building heights. Therefore, the only process by which the pressures and associated impulses are reduced is the normal and lateral clearing described above.

Consider the graphs shown in Figures 6.52 to 6.54. These are for $x = 10$ m, 25 m, and 50 m along the street of street width $w = 8$ m and horizontal measuring array $y = 2$ m. They demonstrate that the general characteristic shape of the pressure-time histories are essentially the same all the way along the street. The main difference is that the pressures (and hence associated impulses) of the *street* configuration of height $h = 4$ m are significantly reduced.

The same comparison of pressure histories is made for $w = 20$ m in Figures 6.55 to 6.57 and also for street width $w = 32$ m in Figures 6.58 to 6.60.

Finally, it can be seen that the trend noted above is indeed true for the whole range of scaled street widths, and the characteristic behaviour, in terms of number of reflections and arrival time of reflections, is broadly similar, regardless of the scaled building height; the only difference is that pressures and impulses are reduced to a greater or lesser extent, depending on the scaled building height. This is an interesting observation and could form the basis of a method of prediction.

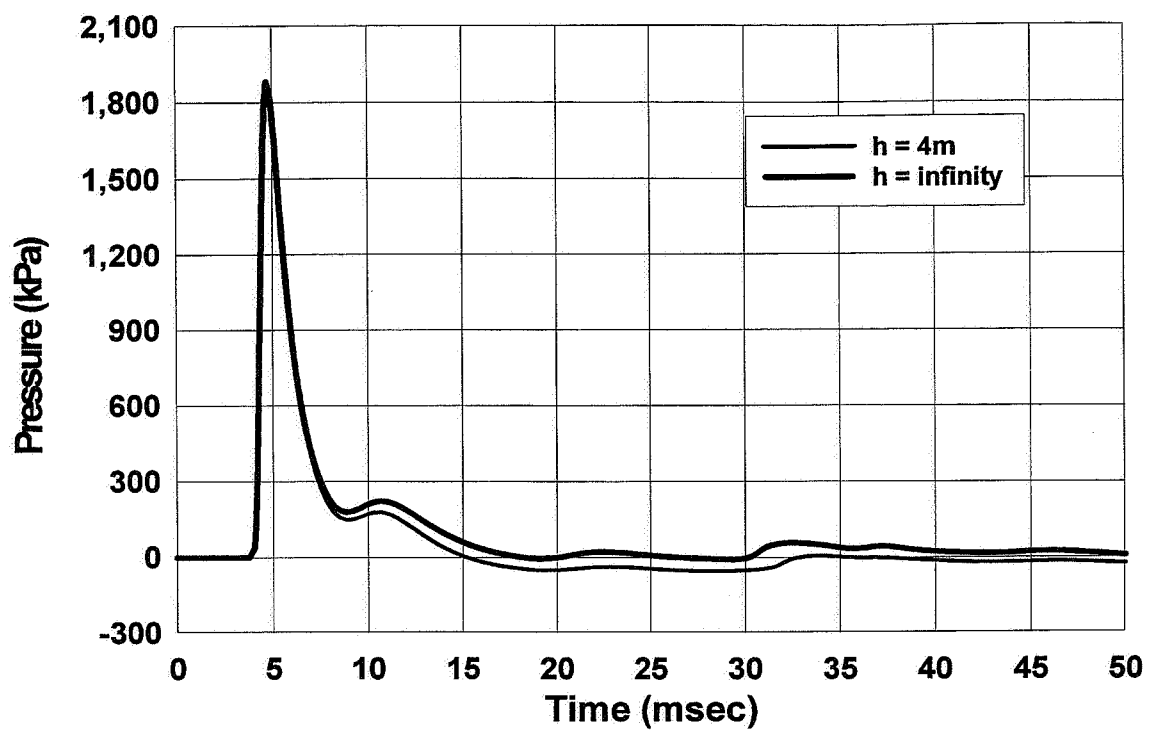


Figure 6.52: Comparison of pressure histories, $w = 8$ m, $x = 10$ m, $y = 2$ m

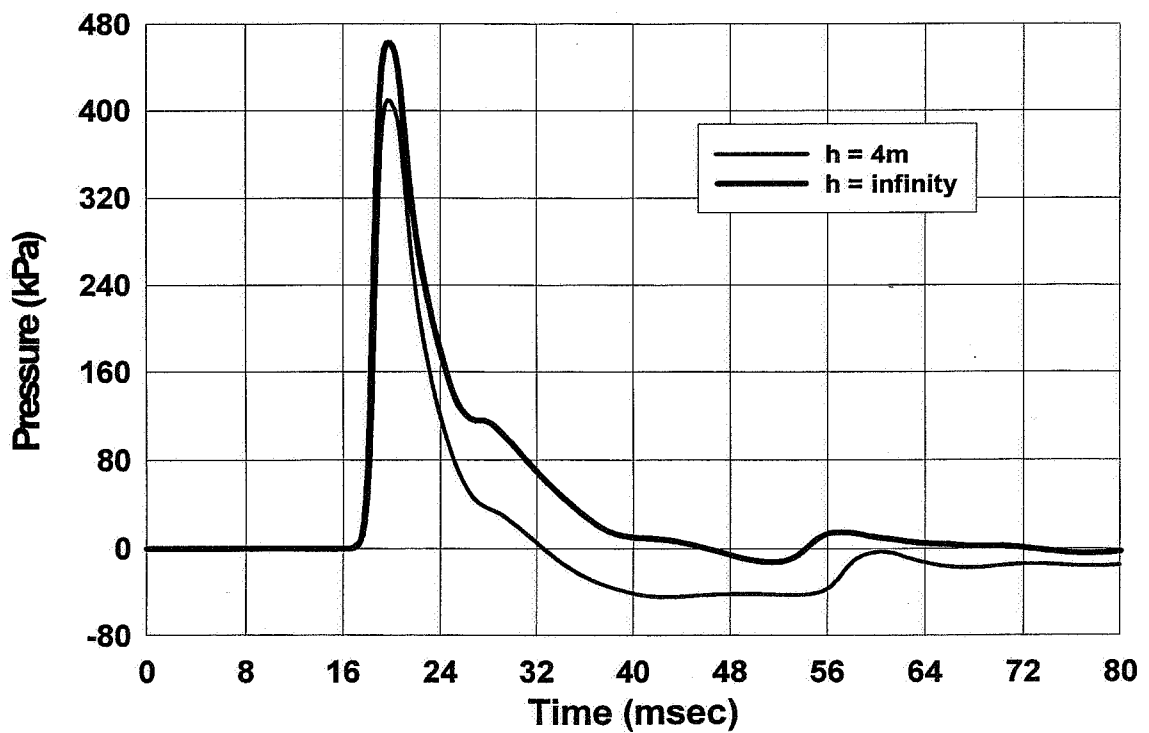


Figure 6.53: Comparison of pressure histories, $w = 8$ m, $x = 25$ m, $y = 2$ m

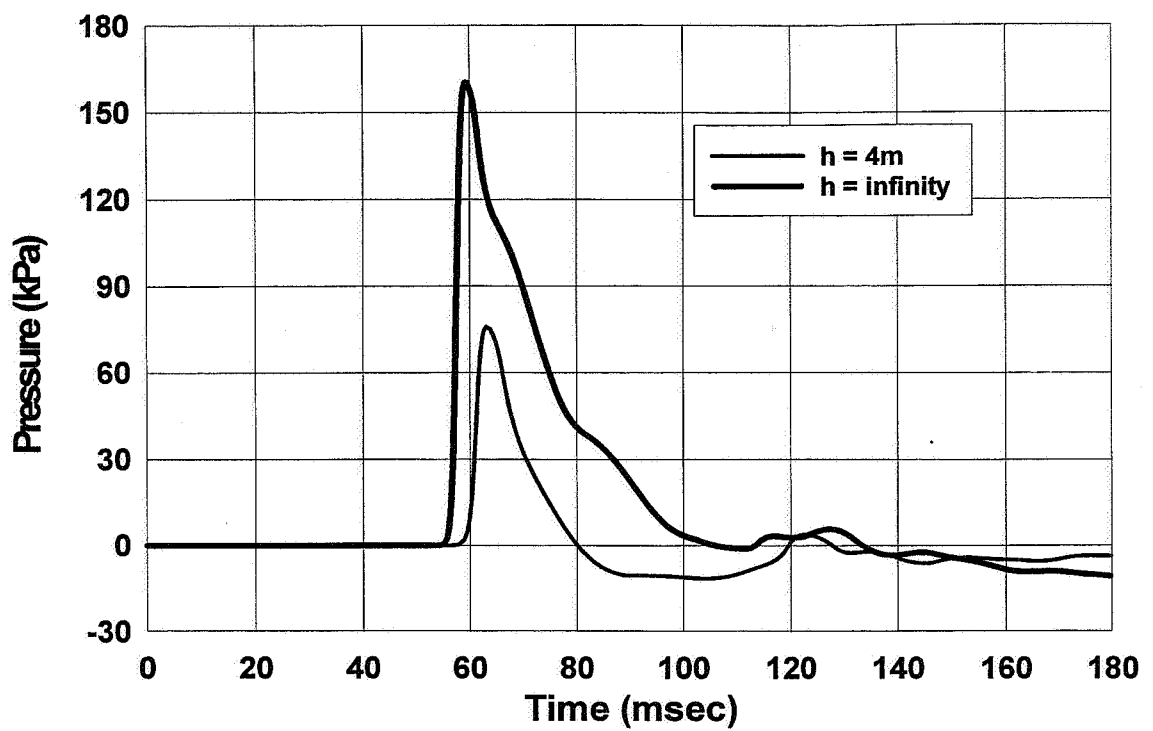


Figure 6.54: Comparison of pressure histories, $w = 8$ m, $x = 50$ m, $y = 2$ m

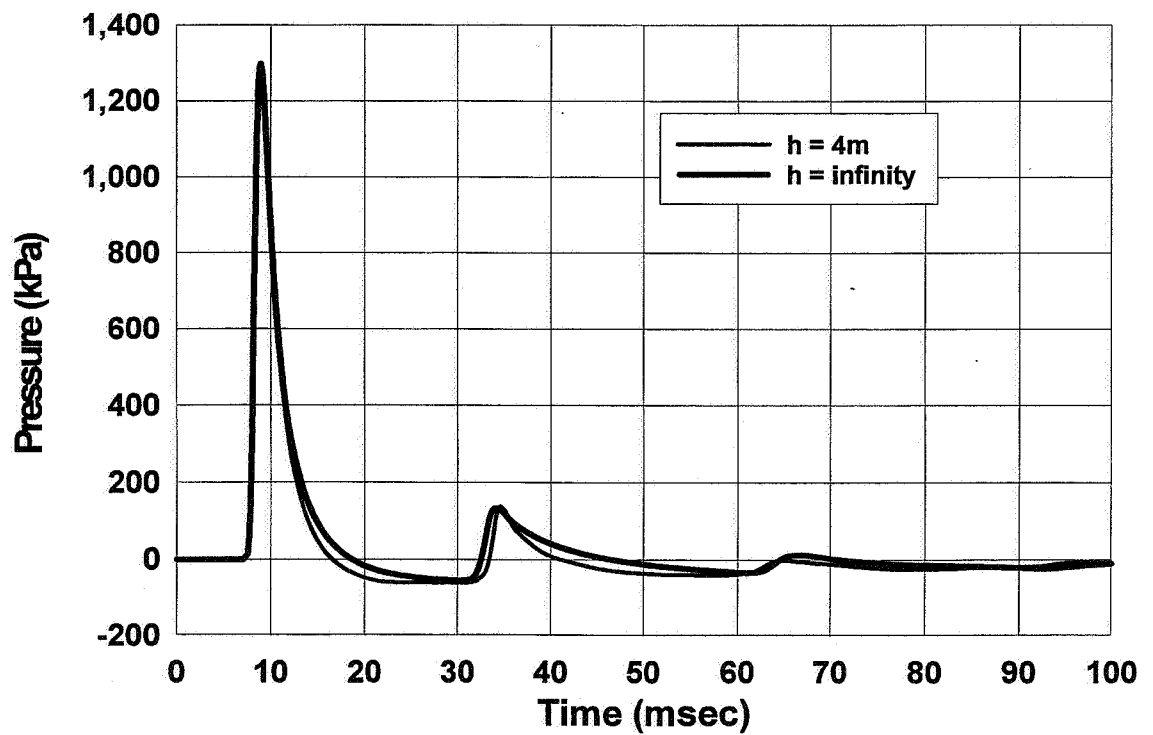


Figure 6.55: Comparison of pressure histories, $w = 20$ m, $x = 10$ m, $y = 2$ m

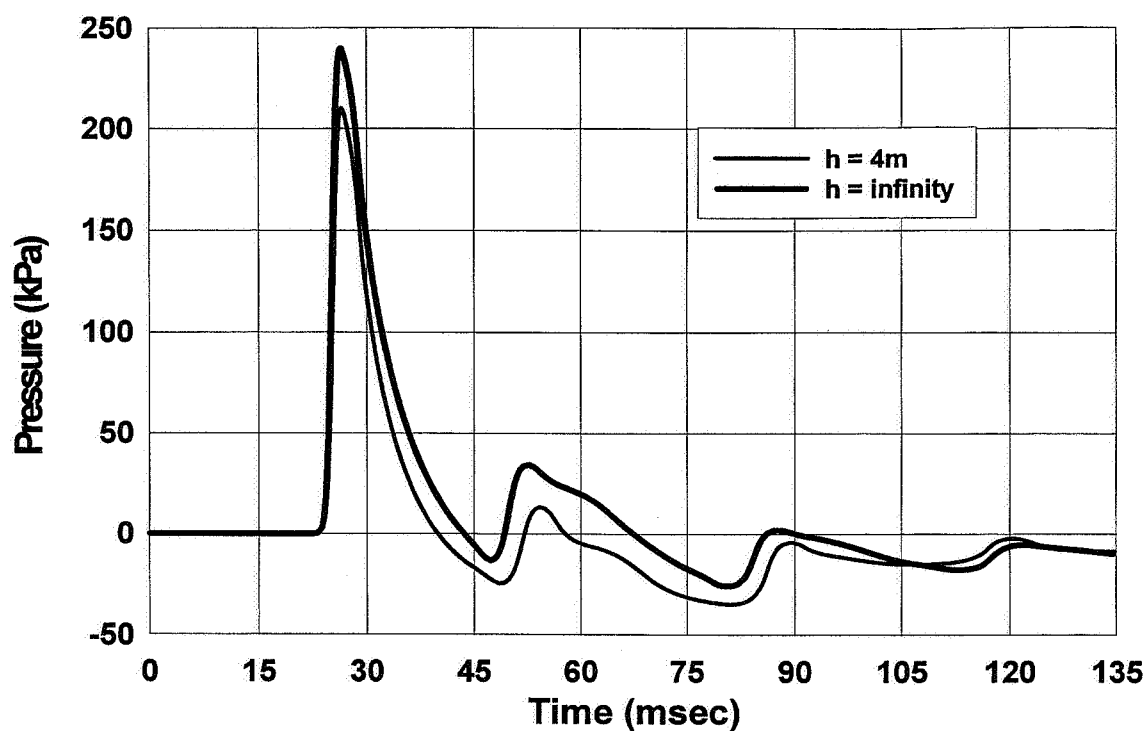


Figure 6.56: Comparison of pressure histories, $w = 20\text{ m}$, $x = 25\text{ m}$, $y = 2\text{ m}$

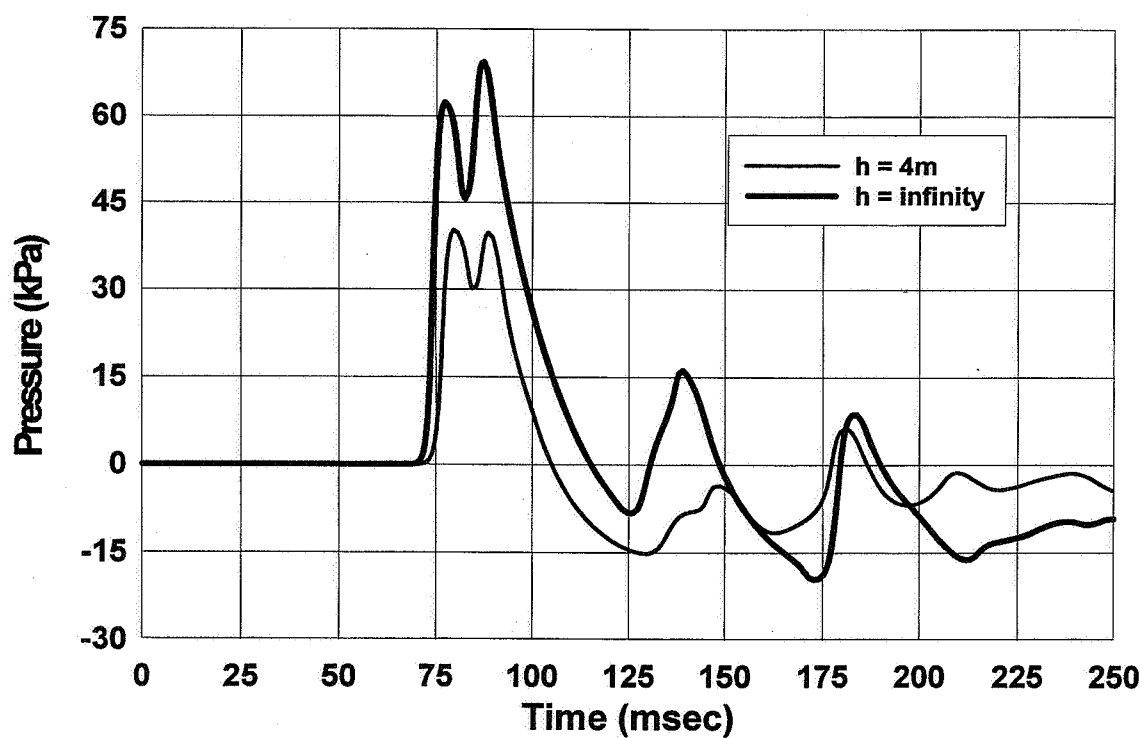


Figure 6.57: Comparison of pressure histories, $w = 20\text{ m}$, $x = 50\text{ m}$, $y = 2\text{ m}$

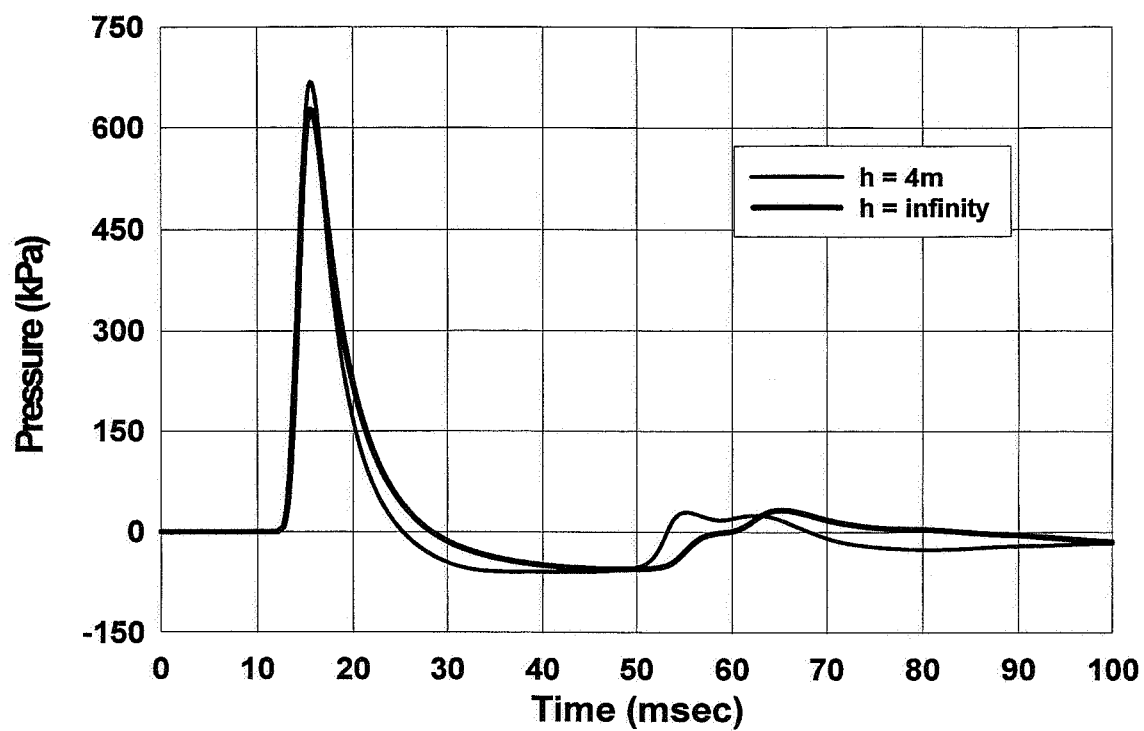


Figure 6.58: Comparison of pressure histories, $w = 32$ m, $x = 10$ m, $y = 2$ m

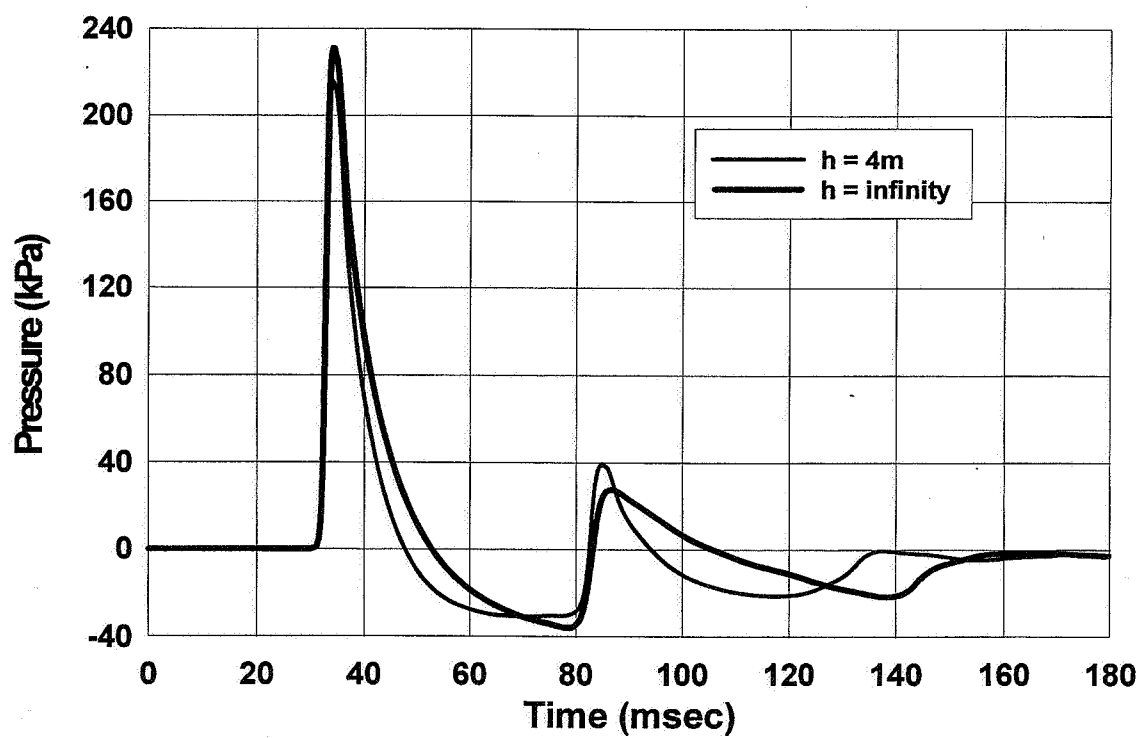


Figure 6.59: Comparison of pressure histories, $w = 32$ m, $x = 25$ m, $y = 2$ m

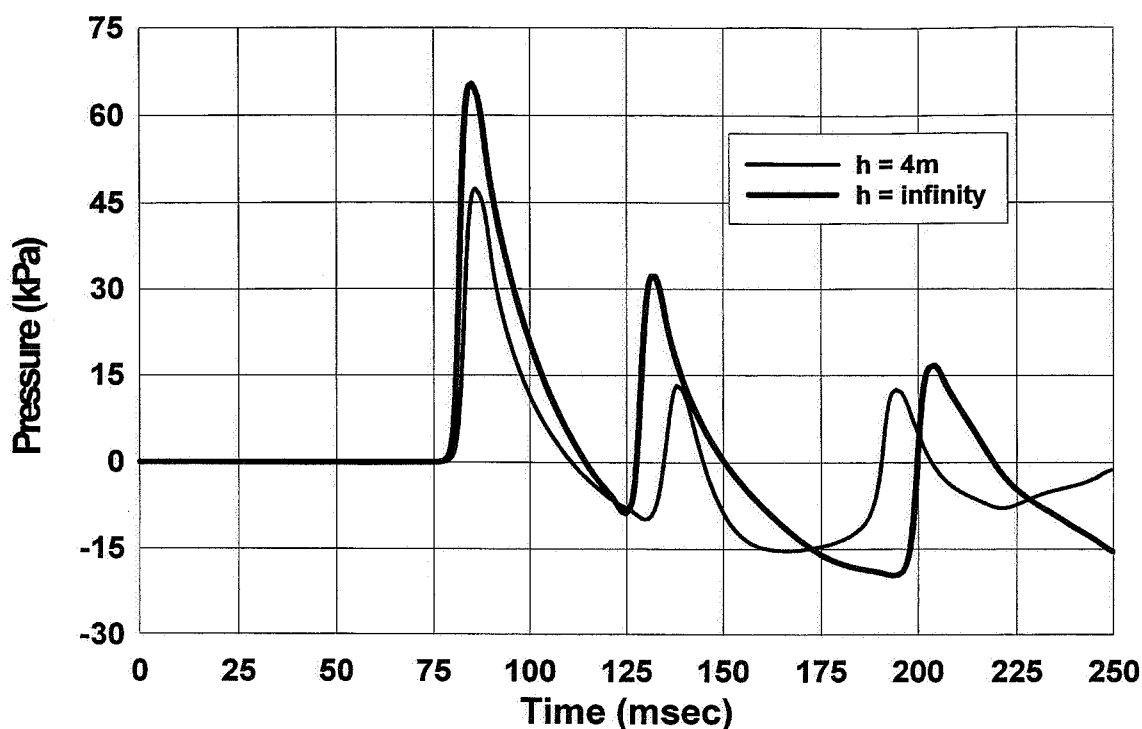


Figure 6.60: Comparison of pressure histories, $w = 32\text{ m}$, $x = 50\text{ m}$, $y = 2\text{ m}$

6.4 A Procedure to Predict Blast in an Urban Environment

The numerical simulations described in Section 6.3 present the possibility of formulating a predictive procedure in a manner similar to that described in Chapter 5 for blast wave clearing.

This possibility arises because every pressure monitoring location on the building façade can be identified by four parameters: the scaled street width $w/W^{1/3}$, scaled building height $h/W^{1/3}$, scaled distance along the street $x/W^{1/3}$ and scaled height of the monitoring location above the ground $y/W^{1/3}$. The unique value of peak pressure and peak scaled impulse at each of these locations (and for each street configuration) could be used in a predictive procedure in a number of ways.

Perhaps the simplest method would be to normalise the quantities by dividing them by suitably defined *side-on* quantities. The most obvious choice would be to use the side-on pressure and impulse measured at the same scaled distance $x/W^{1/3}$ (along the centre of the street) from an unconfined hemispherical air burst. This is similar to the method described by Whalen [57] for calculating *impulse amplification factors*. The resulting pressure and impulse coefficients could be used in a look-up table in conjunction with the scaled blast parameters of ConWep [24], for example, and the four geometrical parameters described above.

The process of interpolation between the various coefficients would, however, be rather lengthy and would need to be programmed if it were to be used efficiently.

Other possible approaches include fitting polynomial curves or surfaces through the data points or establishing a neural network to represent the whole of the data set. Such undertakings are beyond the scope of the present study.

6.5 Comments and Conclusions on Blast in an Urban Environment

This study has considered a broad range of scaled street widths: $w/W^{1/3} = 0.8$ to $3.2 \text{ m/kg}^{1/3}$ and covers much of the range of practical interest. Similarly, scaled building heights from $h/W^{1/3} = 0.4$ to $2.4 \text{ m/kg}^{1/3}$ were considered, which roughly equates to one to six storey buildings for a charge weight of 1000 kg. The scaled distance along the street was limited to the range 0.0 to $5.0 \text{ m/kg}^{1/3}$, which is certainly not enough for a thorough investigation, although it is sufficient for the present study.

There are a number of important results for blast resultants at street level which can be concluded from this investigation.

- In streets which have small scaled street width, reflections do not remain separated from the incident wave; reflections coalesce immediately, and a single large pulse results. Close investigation of the pressure histories has indicated that, in broad terms, coalescence occurs at every location along the street for scaled street widths $w/W^{1/3} \leq 2.0 \text{ m/kg}^{1/3}$.
- Streets which have scaled street width $w/W^{1/3} > 3.2 \text{ m/kg}^{1/3}$ produce very little confinement. This study has not revealed an effective upper limit on scaled street width, above which no confinement results, but it is unlikely to be significantly greater than $3.2 \text{ m/kg}^{1/3}$.
- Scaled street heights $h/W^{1/3} \geq 2.4 \text{ m/kg}^{1/3}$ can be considered effectively infinite; the effect of the presence of the upper edge on impulses at street level is negligible.
- The general shape of the pressure-time loading curve for locations along the street does not change significantly with scaled building height. The numerical values of pressure and impulse, however, are reduced by the process of normal and lateral clearing as the scaled building height decreases. The number and frequency of reflections remains broadly similar for all street heights.

This investigation has shown that systematic treatment of scaled street width and scaled building height can lead to a good understanding of their effect on blast resultants and, equally important, quantification of the magnitude of the effect. The present chapter has been restricted to a discussion of the effect on blast resultants near to ground level, but information was also available for every other location on the building façade. This has led to the possibility of using the information to predict blast loads (pressures and impulses) at any location in a street from knowledge of the charge weight, street width and building height.

Development of a predictive technique, based on the data of Section 6.3, is probably too great an undertaking for the present study, and the data is probably deficient in several important respects:

- The validation procedure of Section 6.2 was undertaken in the manner of a confirmation exercise. The aim was to establish confidence in the numerically-derived results so that they could be used to investigate aspects of urban geometry, within the limits of the existing validation data.

The form of the validation was comparison of numerical peak pressure and specific impulse with mean values of experimentally-derived data. Also, qualitative comparison of pressure records was undertaken to demonstrate the correct prediction of the number of reflections and the relative magnitude of the pressure peaks.

The broad conclusions of the validation exercise were that the numerical peak pressures were close to experiment, but the impulses were generally slightly greater. The equivalent TNT charge mass was calculated based on an assumption of the chemical energy contained in the sheet explosive and detonator, which, in retrospect, may not have been quite correct in that the equivalent charge weight may have been slightly overestimated.

As noted in Section 1.2, the explosive content of SX2 sheet explosive is the same as Demex plastic explosive. It is, therefore, slightly surprising that the correspondence between the experimental and numerical impulses were not the same as those discussed earlier in Chapter 5. One possible explanation is that the apparent loss of energy might be attributable to *directional effects*, due to the fact that the charge was a short cylinder and not a sphere, as in the numerical analysis. A description of the propagation of blast in the near field from cylindrical charges is provided by Held [19]. The charges described in Section 6.2, however, were small and did not conform to the most common cylindrical configurations. Cylindrical charges which are significantly larger than the detonator are usually detonated at one end or in the middle of the length. Because the charges in Section 6.2 were small, and wrapped around the detonator, they were effectively initiated along the whole of the axis of symmetry simultaneously. The implication is that the near field distribution of blast arising from this configuration was neither spherical nor that described by Held [19]. A second hypothesis is that the discontinuous surfaces which existed radially through the thickness of the charge, caused by the wrapping of the sheet explosive around the detonator, may have resulted in unsteady or incomplete detonation in some cases, and this might have resulted in a lessened impulse, compared with what might be expected from consideration of the chemical energy alone. The exact cause of the observed difference in impulse is difficult to establish.

The conclusions outlined above were not demonstrated in a statistically valid manner. The difficulty of such an undertaking was indicated previously in Chapter 1. The apparatus used in the experiments was comprised of large vertical steel plates, representing the building façades. The charge was detonated above another steel plate, in contact with the vertical plates. This con-

figuration allowed stress waves to travel through the experimental apparatus, and the resulting accelerations produced signals superimposed on the pressure records as high frequency oscillations. The effect was most pronounced at the measuring locations nearest to the charge.

Although this effect does not seriously degrade the impulse, peak pressures are invariably affected, and they tend always to increase as a result. This was also discussed in Chapter 1. For this reason, it is inappropriate to consider even mean values, in a statistical manner, without first processing the individual pressure records to ensure that extraneous oscillations (and other anomalies) are removed, or at least accounted for, in a justifiable manner. Such an approach is beyond the scope of the present study, where the aim was simply to demonstrate the possibility of using a numerical technique to investigate a particular problem. It is suggested that, despite these obvious deficiencies, the present approach is, nonetheless, worthwhile.

- The computational resources available for this project have restricted the spatial resolution of the calculations. This is a major deficiency because the resulting peak pressures are probably not sufficiently well resolved to be of practical use. The major far-field hazards resulting from an explosion: lung damage, ear damage and damage to glazing in buildings, all require reasonably precise knowledge of the peak pressure, as well as the impulse.
- The maximum scaled distance along the street, $x/W^{1/3} = 5.0 \text{ m/kg}^{1/3}$, is clearly not sufficient to be of practical use. As mentioned in Chapter 4, pressures and impulses can still potentially cause damage to glazing, for example, at a scaled range of $30 \text{ m/kg}^{1/3}$ in free air. Given the added confinement of a city street, a minimum scaled distance of at least $30 \text{ m/kg}^{1/3}$ would be appropriate for the formulation of a predictive procedure.
- In real situations, explosions do not necessarily occur in the middle of the street. Generally, if the explosives are contained in a vehicle, for example, they will detonate closer to one side of the street than the other. Also, the explosive will not generally be hemispherical or be comprised of TNT. All of the above considerations make the process of formulating a useful predictive technique more difficult.

This section has demonstrated that the computational tool Air3d can model $1/50^{\text{th}}$ scale experiments of blast in urban geometries and produce results which correspond closely to the experimental data.

A number of generic street configurations have been analysed and broad, qualitative conclusions have been stated. The possibility of using this approach to formulate a predictive technique has been recognised, and the general requirements and methodology of a possible approach has been discussed.

Chapter 7

The Effectiveness of Blast Walls

7.1 Introduction

A *blast wall* is a barrier designed to withstand the effects of a specified explosion, providing protection to personnel, material or buildings behind it. The wall is situated between the location of a possible hazard and that which it is protecting. The function of the blast wall is to reflect as much of the blast as possible so that what remains to diffract over the wall and impinge on the structure is greatly reduced (in terms of peak pressure and specific impulse) compared with the same situation without a wall. The typical scenario is shown in Figure 7.1, below.

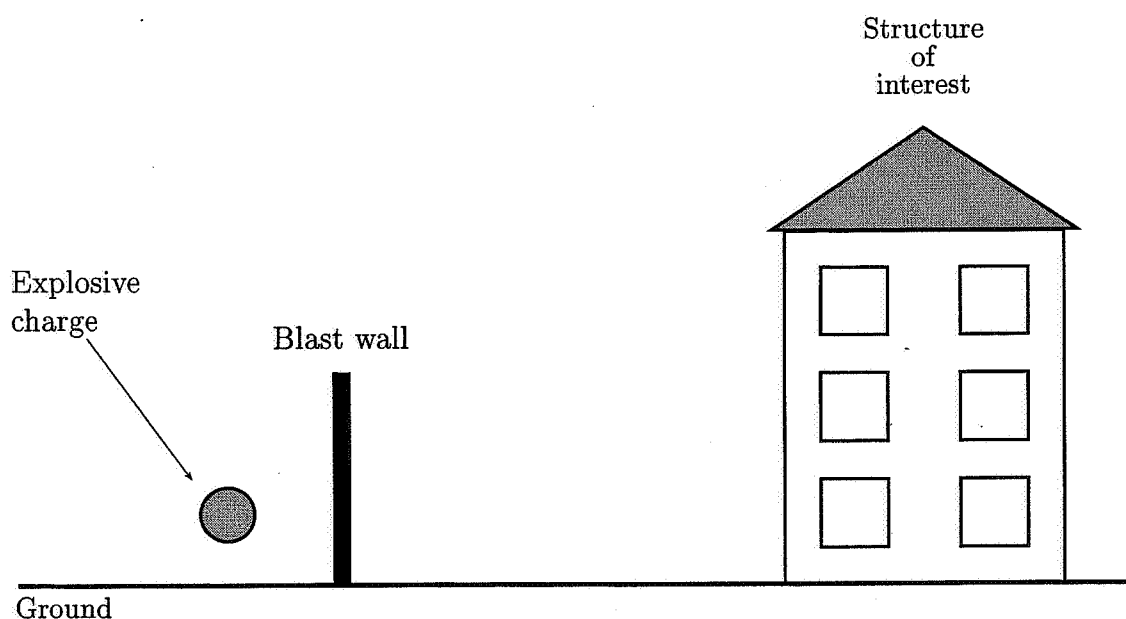


Figure 7.1: Typical blast wall scenario

Intuitively, it will be appreciated that actions such as increasing the height of the wall and the distance of the wall from the structure both help decrease the blast impinging on the structure. How to quantify these effects is the subject of the present chapter.

The scenario shown in Figure 7.1 is typical of a vehicle bomb in a street near a structure at risk. It shows the explosive charge situated a small distance above the ground and away from the wall. These small distances are representative of the vehicle dimensions (not shown), and the charge is assumed to be located in the centre of the vehicle.

It should be borne in mind that this same geometry is common both in process industries and in storage and transportation situations (where one building might shield another). The findings of this chapter apply equally to these and similar situations.

One other important point that is often raised in connection with the question of shielding is how long should the wall (or other intervening structure) be before diffraction around the ends becomes irrelevant? In other words, when does a finite structure become effectively semi-infinite? There is no straightforward answer to this question, as it depends on the charge weight, wall height, wall length and location of the structure behind the wall. In all the examples below, the length of the blast wall in the *out-of-plane* dimension is always sufficiently long to ensure that it is effectively semi-infinite; no blast propagates around the edges of the wall.

The present study uses the nomenclature of Beyer [6], shown in Figure 7.2.

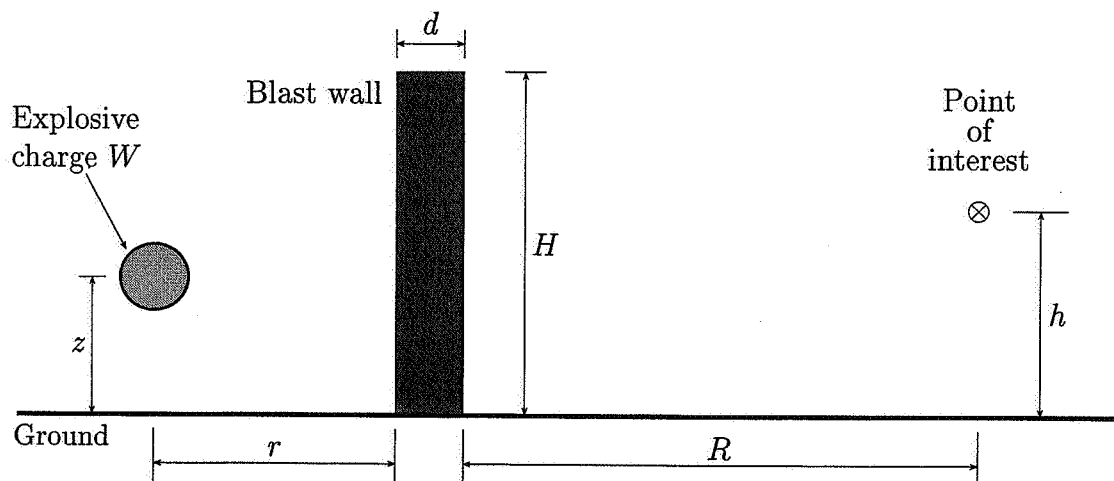


Figure 7.2: Nomenclature, blast wall scenario

These dimensions give rise to a number of important scaled quantities: $r/W^{1/3}$, $z/W^{1/3}$, $d/W^{1/3}$, $H/W^{1/3}$, $R/W^{1/3}$ and $h/W^{1/3}$, which define the scenario completely. In an attempt to keep the study as simple as possible, some of the quantities will be kept constant: these are $z/W^{1/3}$ and $d/W^{1/3}$.

The approach taken in the present Chapter is dissimilar to that of Chapters 5 and 6. Comparison will be made with existing experimental data in an attempt to gain insight into the physics of the diffraction process and the factors which limit the numerical modelling capability. Then the results of a series of numerical simulations which seek to quantify the effectiveness of blast walls in a systematic manner will be presented. Finally, the possibility of using this approach to produce a more complete form of guidance in the future is discussed.

7.2 Comparison with Experimental Data

In this section, a comparison of numerically-based results with the experimental data of Rose et al. [43] will be presented. This is the most comprehensive set of data available and provides the opportunity to consider a relatively wide range of scaled wall heights and a large area of scaled space (in terms of $h/W^{1/3}$ and $R/W^{1/3}$) behind the wall. The experimental programme is summarised in Table 7.1.

Table 7.1: Experimental programme, Rose et al. [43]

Experiment number	Scaled wall height $H/W^{1/3}$ (m/kg ^{1/3})	Actual wall height (m)	Charge weight TNT _{eqv.} (g)
1	0.50	0.221	75.0
2	0.60		43.5
3	0.71	0.300	75.0
4	0.80		52.7
5	0.90	0.380	75.0
6	1.00		54.9
Scaled stand-off $r/W^{1/3} = 0.327$ m/kg ^{1/3}			
Scaled charge height $z/W^{1/3} = 0.258$ m/kg ^{1/3}			
Actual wall thickness $d = 20$ mm			

It will be seen from Table 7.1 that blast walls of three different heights were used in the experiments. The charges were Demex plastic explosive, and they were initiated by No. 8* detonators. The TNT equivalence used was 1.32, and no account was taken of the small amount of explosive material contained in the detonator.

Each of the experiments utilised a grid of pressure measuring locations in the space behind the walls which extended from $R = 0.15$ m to 1.8 m, horizontally behind the wall, in intervals of 0.15 m (12 locations) and from $h = 0.0$ m to 0.9 m, vertically above the ground, also in intervals of 0.15 m (7 locations). This provided a total of 84 measuring locations for each experimental configuration.

Measurements were made using pressure transducers configured to measure side-on pressure-time records, but it should be noted that those which were situated at the ground surface measured reflected pressure, although the angle of incidence of the blast wave on the ground was not large.

7.2.1 Comparison with Height of Burst Data

Despite the relatively simple geometry of Figure 7.1, the blast wall scenario is particularly difficult to analyse using CFD techniques. The reason for this difficulty is the fact that when the blast wave reaches the top of the wall the momentum vectors all point in a direction above the horizontal. This implies that the mechanism of propagation over the wall (diffraction) is dependent on the pressure and duration of the blast which passes the top of the wall. In reality, this process occurs at the continuum level. Numerically, however, the increase in momentum, produced by the pressure, in directions other than the *principal convection direction* is dependent on the mesh discretisation. If the computational mesh was discretised at the continuum level, good numerical simulation of diffraction could be expected. Normal mesh discretisations (a few tens of millimetres), however, tend to degrade the simulated results to a greater or lesser extent, depending on the other parameters of the problem such as charge weight, stand-off, and so on. Therefore, before the Air3d results for comparison with blast wall experiments are presented, comparison with a much simpler scenario is shown. This is the height of burst scenario described in Section 1.4.2.

An earlier report by Rose et al. [42] provides height of burst data for the same geometry and measuring locations as experimental configuration number 3 from Table 7.1, except the blast wall was not present. Although this geometry is, strictly speaking, height of burst, it differs from the conventional geometry because the scaled height of the charge above the ground is small and the measuring locations are sufficiently remote that the wave reflected from the ground is not distinct from the incident wave at the majority of measuring locations. In other words, the triple point, where incident and reflected waves coalesce to produce a Mach stem, has passed above much of the measuring space.

The numerical simulation was performed making use of the two-dimensional radial symmetry of the problem. The mesh extended 3 m in the radial direction and 2 m in the axial direction. A total of 240,000 5 mm square computational cells were used. Numerical and experimental peak pressures and peak side-on impulses are compared in Figures 7.3 to 7.9, below. Each of the seven graphs contains data for a horizontal line of twelve measuring locations. It should be noted that the horizontal distance (R) of Figures 7.3 to 7.9 is that defined by Figure 7.2 and is not the *actual* distance from the charge.

The purpose of the height of burst calculation, apart from demonstrating the applicability of the tool Air3d, is to provide the basis of comparison between the *with wall* and *without wall* situations. This is the most obvious method of describing the effectiveness of a blast wall. Also, because it is a much quicker calculation to perform, the results of height of burst calculations, together with what might be called *effectiveness* coefficients or functions, could form the basis of a predictive procedure. This will be discussed in what follows.

The graphs, Figures 7.3 to 7.9, demonstrate that the results of the numerical simulations are generally quite close to those of the experiments. However, there are a number of observations that can be made:

- It can be seen that, in each case, the numerical and experimental impulse curves are essentially coincident between radial distances of 0.6 m and 1.8 m. This indicates that the parameters for TNT and the value of TNT equivalence used in the simulations were probably appropriate.
- Generally, the experimentally measured peak pressures are slightly greater than those from the numerical simulations. This is usual and has been described previously. At short stand-off distances ($R < 0.65$ m) and for monitoring locations $h \geq 0.45$ m (Figures 7.6 to 7.9), however, the discrepancy is more pronounced. This is most evident in Figure 7.6 where the experimental pressures are approximately twice the numerical values for a short region. The likely explanation is that the bursting isothermal sphere methodology, used by the one-dimensional part of the analysis, only becomes a good approximation to real high explosive detonation at scaled distances greater than about $0.3 \text{ m/kg}^{1/3}$; this was shown previously in Chapter 4. In the present case, the scaled charge height is $0.258 \text{ m/kg}^{1/3}$. Therefore, it is likely that the region of space through which the triple point of the Mach reflection passes is described less well than those regions further away.

Lastly, in the graphs showing the comparison immediately above and below $h = 0.45$ m (Figures 7.7 and 7.5), the discrepancy is not so great as in Figure 7.6. It is likely that stress waves have caused physical oscillations of the measuring apparatus which have tended to increase the peak pressure recorded. A comparison of the experimental and numerical pressure histories for the monitoring location at $h = 0.45$ m and $R = 0.3$ m is given in Figure 7.10. This shows the records from which the second points on the graph in Figure 7.6 were derived. It can be seen that the experimental record has oscillations which are more pronounced than expected.

It is interesting to note that, because the measurements were made at the end of long steel tubes, the oscillations superimposed on Figure 7.10 are lower in frequency and more highly damped than those Chapter 6, which were measured on the surface of thick steel plates. It is possible that the integral of the damped oscillation may have contributed to the increased impulse (as well and peak pressure) in the present case.

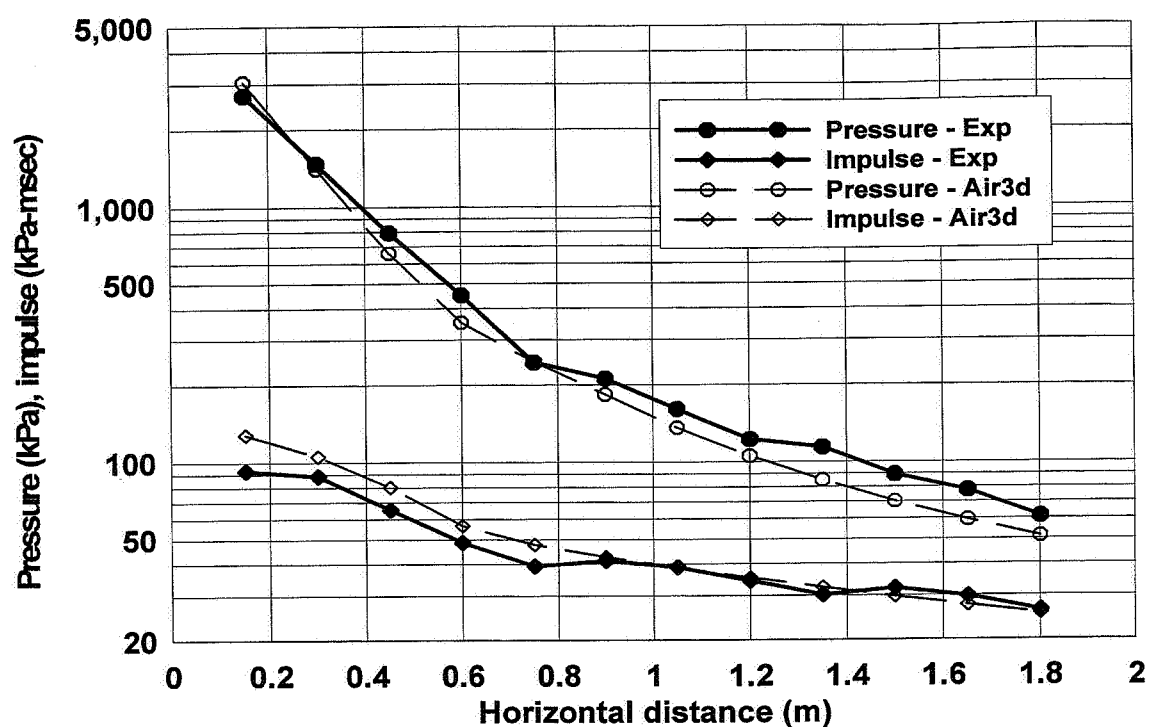


Figure 7.3: Comparison of Rose et al. [42] and numerical HOB data, $h = 0.00$ m

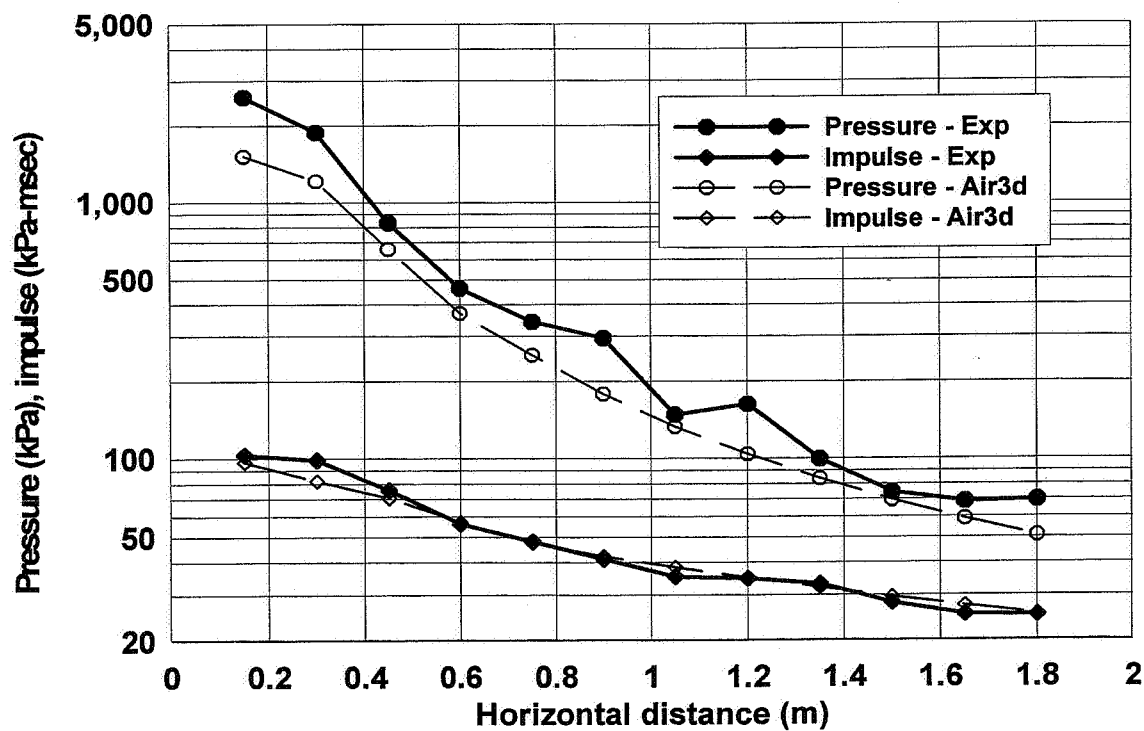


Figure 7.4: Comparison of Rose et al. [42] and numerical HOB data, $h = 0.15$ m

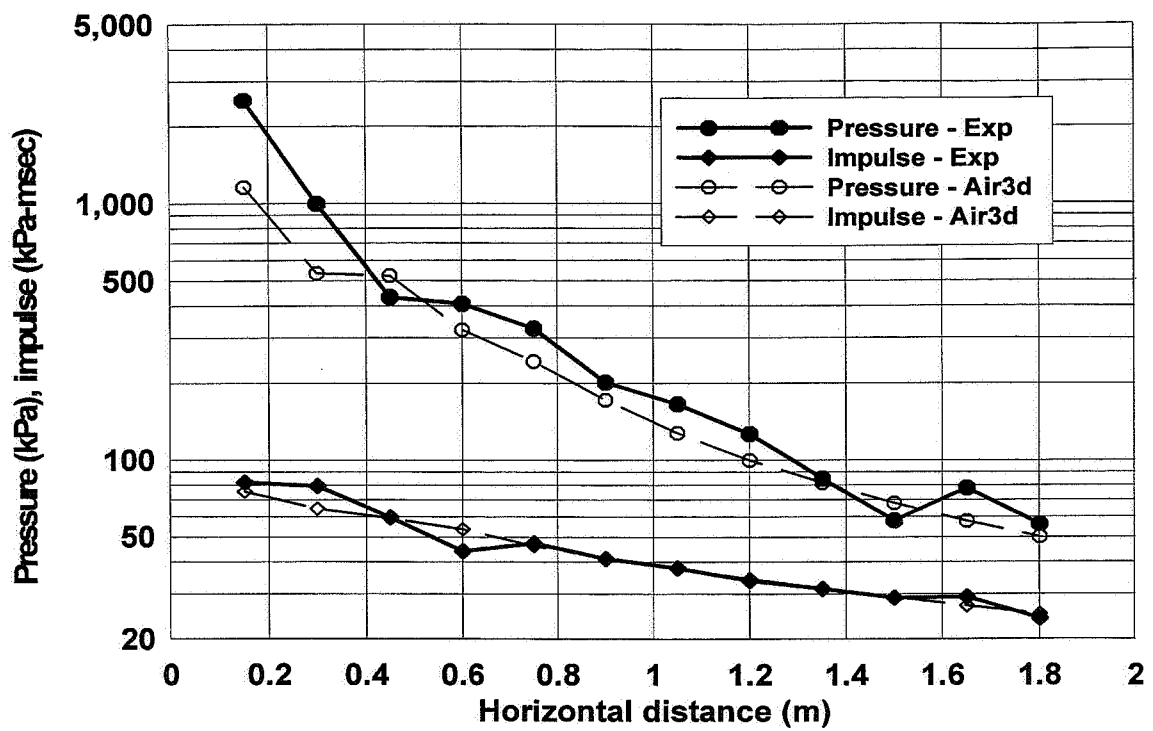


Figure 7.5: Comparison of Rose et al. [42] and numerical HOB data, $h = 0.30$ m

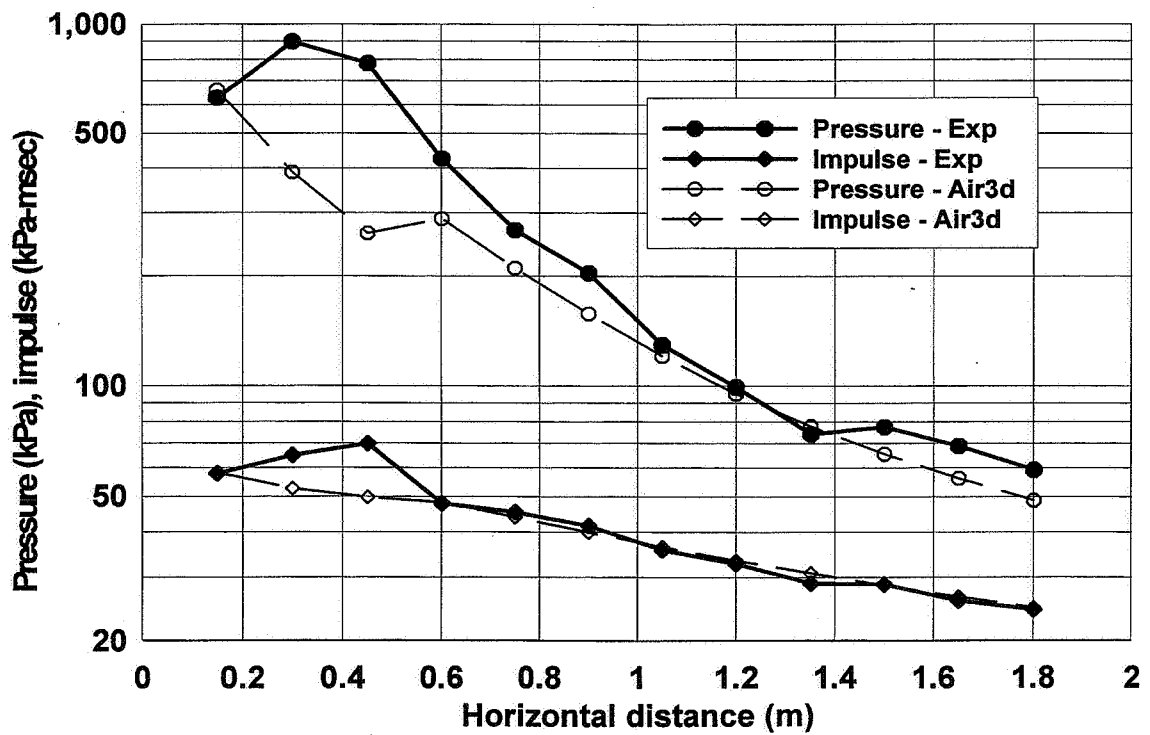


Figure 7.6: Comparison of Rose et al. [42] and numerical HOB data, $h = 0.45$ m

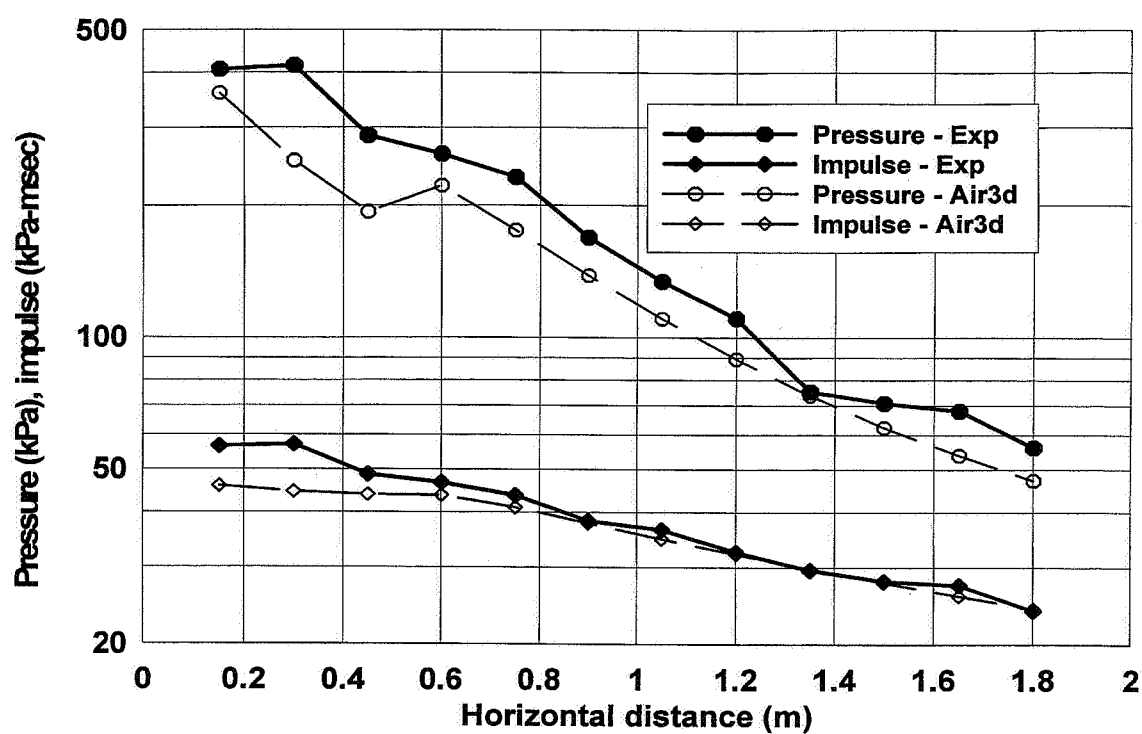


Figure 7.7: Comparison of Rose et al. [42] and numerical HOB data, $h = 0.60$ m

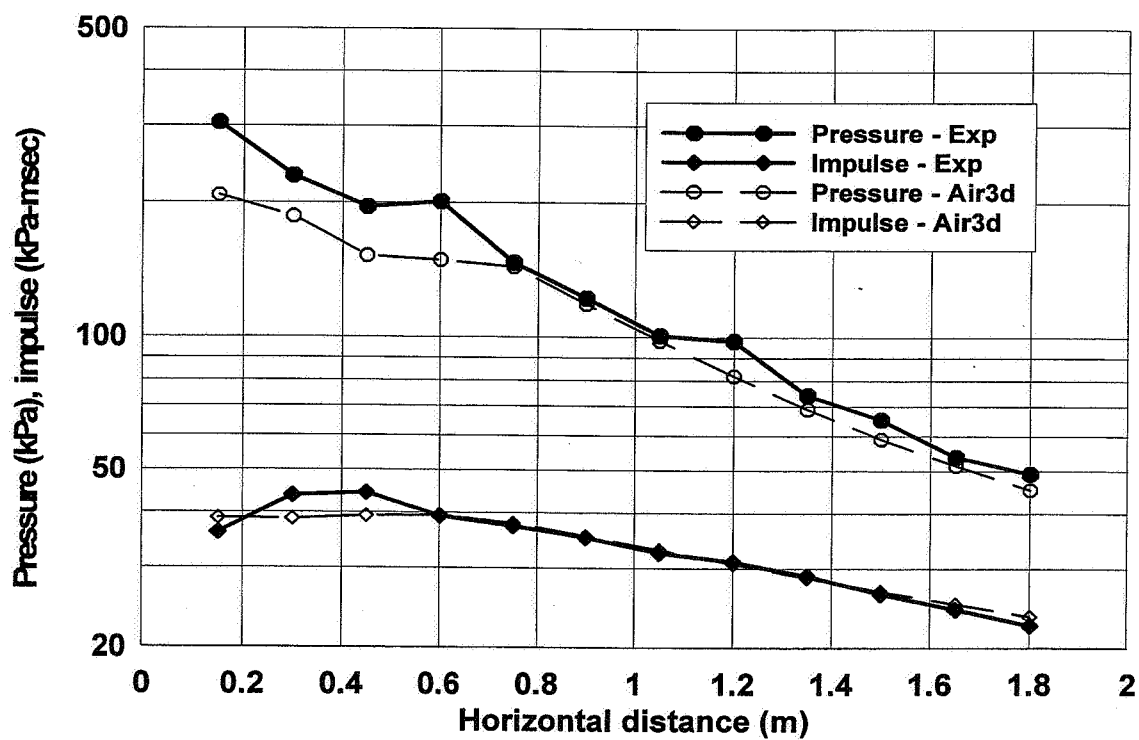


Figure 7.8: Comparison of Rose et al. [42] and numerical HOB data, $h = 0.75$ m

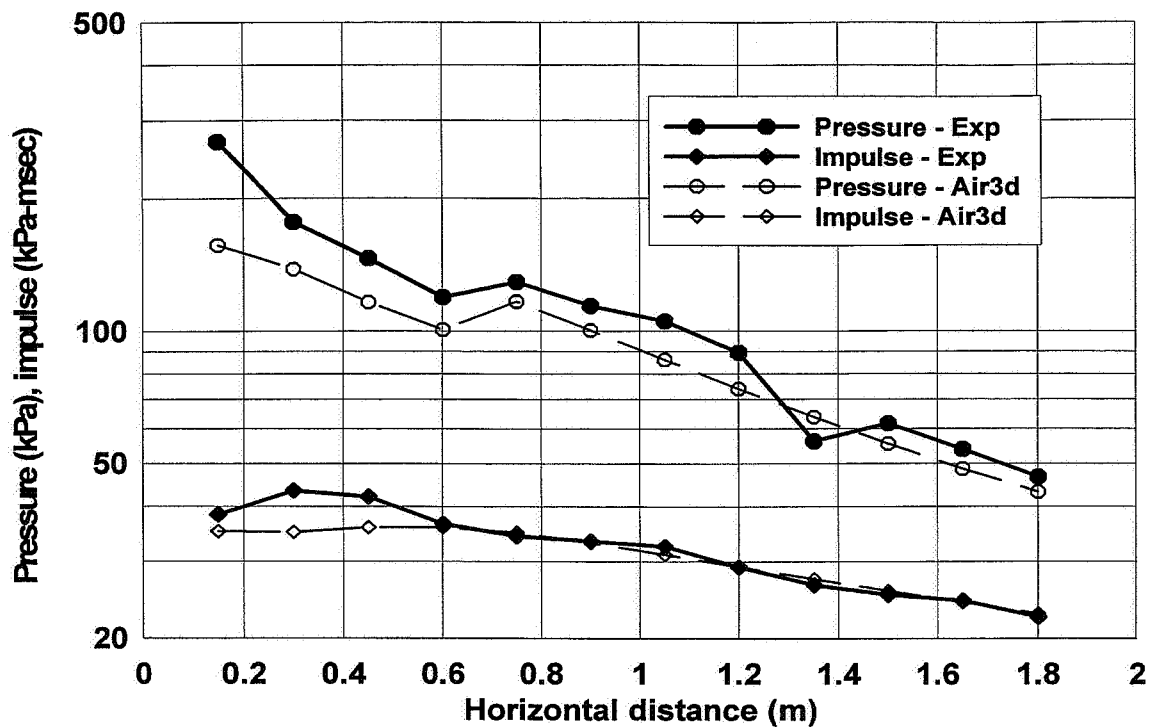


Figure 7.9: Comparison of Rose et al. [42] and numerical HOB data, $h = 0.90$ m

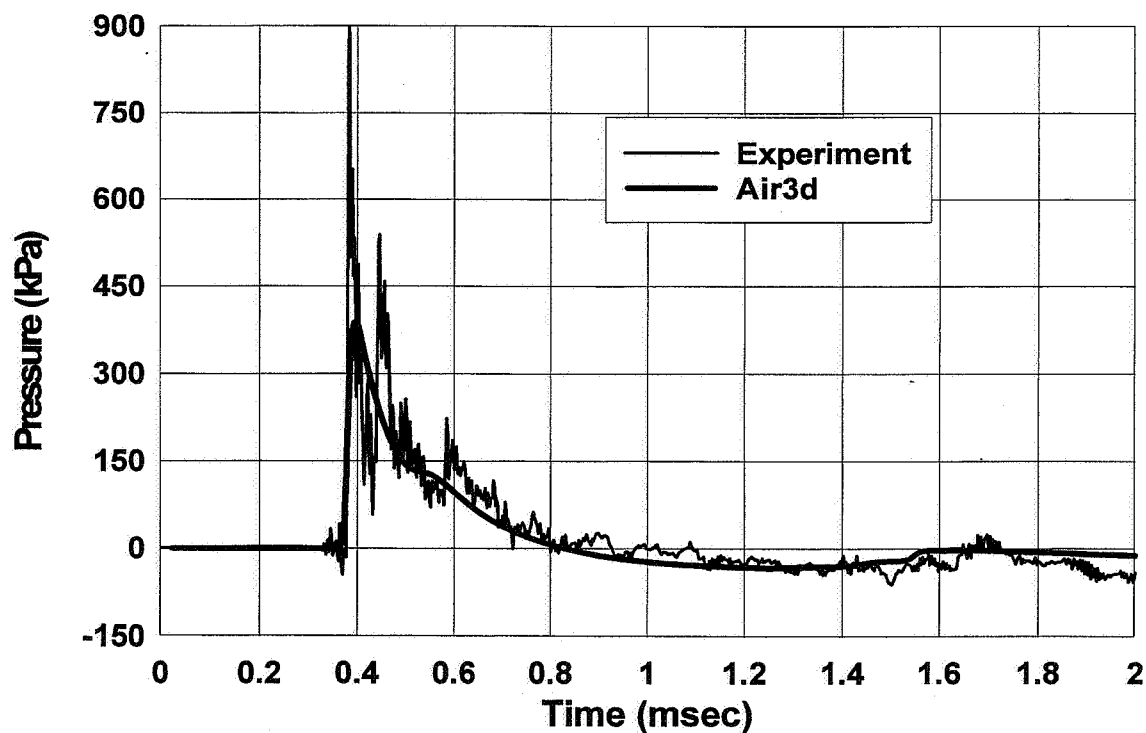


Figure 7.10: Comparison of pressure histories, HOB, $h = 0.45$ m, $R = 0.3$ m

7.2.2 Comparison with Blast Wall Data

Analyses of a number of blast wall validation problems were performed with Air3d using a domain with the extents and discretisation shown in Figure 7.11. It will be noticed that the cell size (10 mm) is twice that used for the height of burst analysis in Section 7.2.1, and the number of cells (4,500,000) is approximately twenty times. This is an inevitable consequence of the change from two to three dimensions.

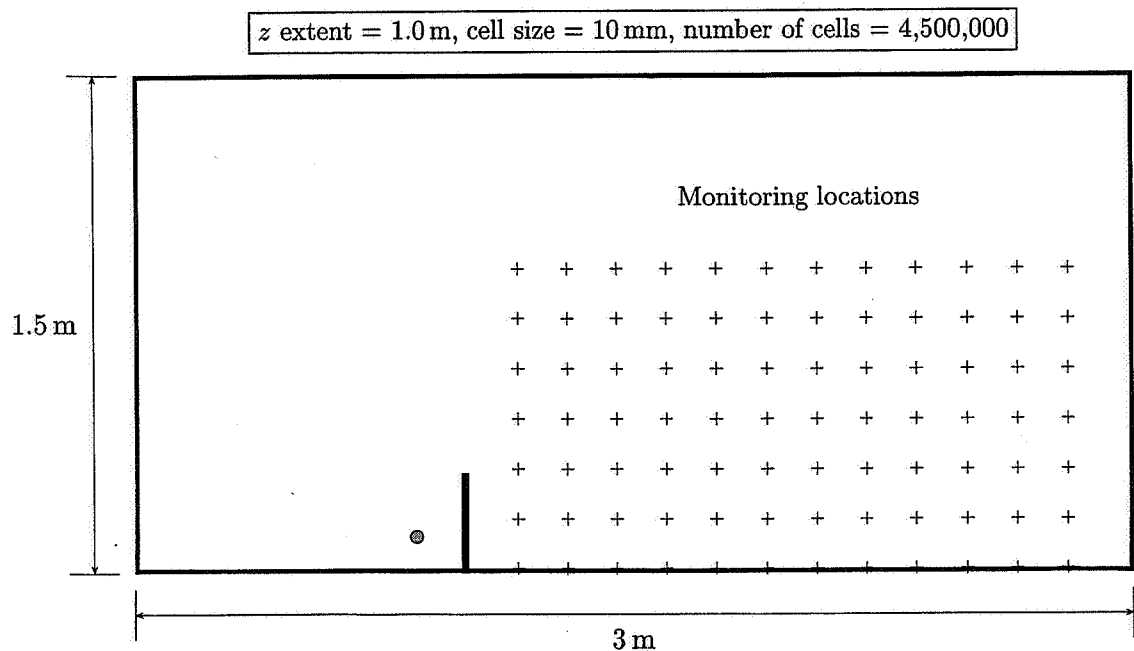


Figure 7.11: Domain used for blast wall validation problems

Figure 7.11 is interesting because it shows that there was a significant region of space required on the near (charge) side of the wall. Similarly, the extent of the domain in the out-of-plane (z) direction was also significant.

This common set of domain dimensions were established iteratively, using a coarse discretization, starting with a very large domain and shrinking the boundaries down towards the region of interest until their presence began to have an effect on the pressure histories measured at the outermost points of the vertical and horizontal monitoring array. By this means, it was possible to establish the domain in Figure 7.11, which is the smallest domain that does not adversely affect the results.

One further observation to arise from Figure 7.11 is the fact that, in all the validation examples, the height of the charge above the ground and the distance of the charge from the wall were both small, in scaled terms. Taking the example of the experiments which used a 75 g charge (experiments 1, 3 and 5 in Table 7.1), the height of the charge above the ground was 109 mm, and the distance of the charge from the wall was 138 mm. This meant that the region over which one- and two-dimensional analyses were performed for each of the three-dimensional analyses was very small.

This is typical of the *second class of problem* described in Section 4.2.3 and is the most demanding, in terms of computer time and memory, and the least accurate, in terms of resolution of shocks, and prediction of peak pressure and associated impulse.

Results of the analyses are shown in Figures 7.12 to 7.53, where the graphs are presented in groups of six: one for each scaled wall height, for a common value of horizontal gauge height. This means that in the first group of six graphs, Figures 7.12 to 7.17, for example, the pressure and impulse data from the set of 12 horizontal monitoring points at $h = 0.0$ m (at the ground surface) is shown, for each analysis and experiment.

There are a number important observations to arise from the graphs Figures 7.12 to 7.53, and these affect the usefulness (or otherwise) of the program Air3d for the identification of important trends in the effectiveness of blast walls. Discussion of the results of the validation is best accomplished by considering the graphs in groups of six, one page at a time. The observations are as follows:

- The set of graphs Figures 7.12 to 7.17 for the horizontal array of monitoring points at the ground surface ($h = 0.0$ m) shows that the correspondence between the experimental and numerical data is quite poor. If consideration is given to the two extreme values of scaled wall height: $H/W^{1/3} = 0.5$ m/kg^{1/3} and $H/W^{1/3} = 1.0$ m/kg^{1/3} (Figures 7.12 and 7.17, respectively), by dividing the numerical data by the experimental data the fraction of peak experimental pressure and impulse was found. This data is plotted as a percentage in Figures 7.54 and 7.55.

First consider Figure 7.54 (for $H/W^{1/3} = 0.5$ m). It can be seen that the percentage of the peak experimental pressure at the ground surface varies from 34%, immediately behind the wall, to 76% at the far end of the monitoring array. Similarly, the percentage of peak impulse varies from 48% to 78% of the experimental values along the same array.

Next consider Figure 7.55 (for $H/W^{1/3} = 1.0$ m). Here the fraction of experimental peak pressure varies from 26% to 63%, the fraction of impulse from 46% to 84%.

This comparison between the data at the ground surface is the toughest test in the validation program. In order to reach the monitoring array on the ground the blast wave has to diffract a greater amount than for any of the other horizontal arrays which are above the ground. It also has to reflect from the ground surface, which adds a further level of approximation to the results because, similar to the discussion above, reflected pressures occur at the continuum level. In the program Air3d, however, reflection occurs at the discretisation level: in the layer of computational cells adjacent to the reflecting surface. In the present case this layer was 10 mm thick.

It must be concluded from the above discussion that this set of calculations are not *mesh resolved*, and the resulting numerical data is certainly outside the experimental error of the data used for comparison. Despite these reservations, the value of percentage peak impulse at the far end of the array is getting closer to the desired 100% value.

- At the opposite end of the monitoring array, at $h = 0.9$ m above the ground (Figures 7.48 to 7.53), the blast wave has not diffracted nearly so much as in the example above. Similarly, the pressures are all side-on across the whole of the domain.

The same approach has been taken as above, and the percentage of experimental peak pressure and impulse for the same two extreme cases ($H/W^{1/3} = 0.5$ m/kg^{1/3} and $H/W^{1/3} = 1.0$ m/kg^{1/3}), represented by Figures 7.48 and 7.53, have been plotted in Figures 7.56 and 7.57.

Again, if consideration is first given to Figure 7.56, it will be seen that the graph is generally much flatter than previously, and the values vary between 55% to 86% for pressure and 64% to 86% for impulse. Similarly for Figure 7.57, the values range from 54% to 76% for pressure and 67% to 87% for impulse.

In both these cases, comparison with the experimental data shows that the numerical values, though still somewhat less than the experimental data, are beginning to approach them. Perhaps, more important is the fact that the discrepancy between the two sets of data is consistent and systematic. In other words, the numerically-based values are always less, and by a predictable amount. This allows the possibility of using the technique to make useful comparisons because even though the individual results may not be accurate and have limited practical value they will provide a greater insight into the underlying physics of the problem.

It is suggested that the key elements affecting the resolution of blast wave diffraction by CFD calculation are the spatial discretisation and the scaled distance at which the diffraction occurs.

In the present examples, the duration of the blast wave at the top of the wall is relatively short. In space, the positive phase is spread over only a few cells. This means that the pressure does not persist in the region above the wall (where the diffraction takes place) for very many computational cycles.

It would be expected that as the scaled stand-off of the charge from the wall is increased the duration of the blast would also increase, and the rate at which the pressure falls behind the shock (at any given point in the flow) also decreases. This means that, locally, it behaves less like an unsupported wave and more like a supported wave. In other words, it has a greater opportunity to diffract because the pressure persists in the region where the diffraction takes place. Similarly, if the spatial discretisation is increased, the effect on the calculation is essentially the same: the number of cells through the positive phase of the blast wave would be increased and the blast wave would diffract more readily.

This is usually described as *mesh convergence* and has been observed in the present chapter in the preliminary analyses using increasingly fine discretisations. This quest for good resolution has led, finally, to the relatively large problem size described above. Generally, in this project, it has been desirable to restrict the problem size to about one million cells, unless it was absolutely necessary to do otherwise. The validation analyses in the present section were the main exception.

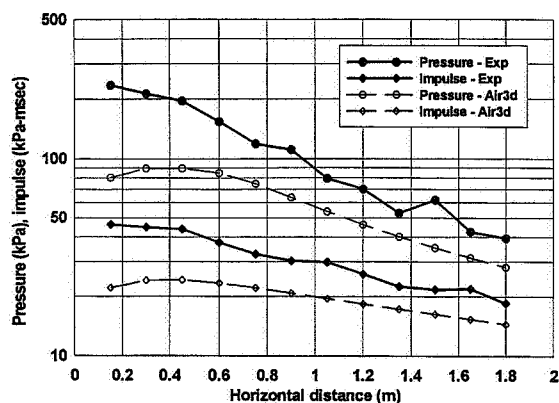


Figure 7.12: Pressure and impulse, $H/W^{1/3} = 0.5 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$

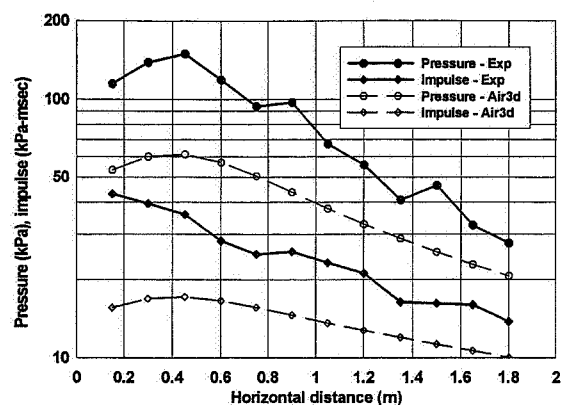


Figure 7.13: Pressure and impulse, $H/W^{1/3} = 0.6 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$

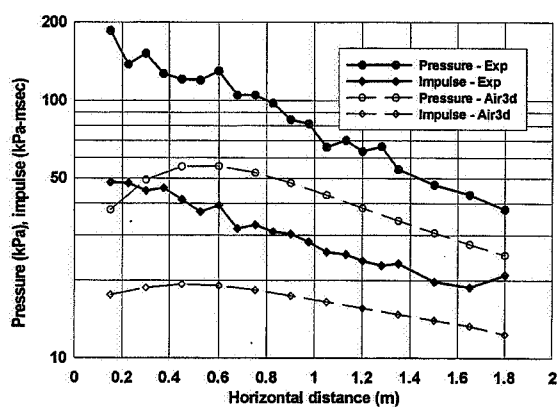


Figure 7.14: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$

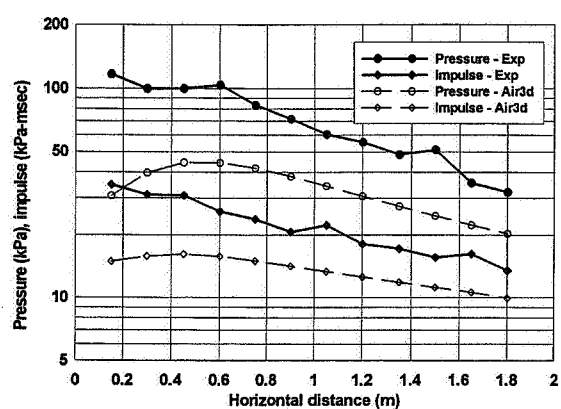


Figure 7.15: Pressure and impulse, $H/W^{1/3} = 0.8 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$

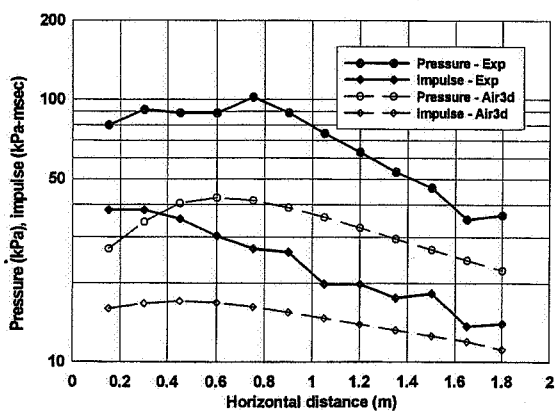


Figure 7.16: Pressure and impulse, $H/W^{1/3} = 0.9 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$

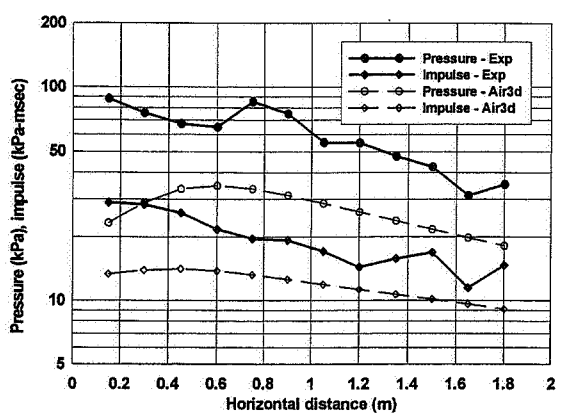


Figure 7.17: Pressure and impulse, $H/W^{1/3} = 1.0 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$

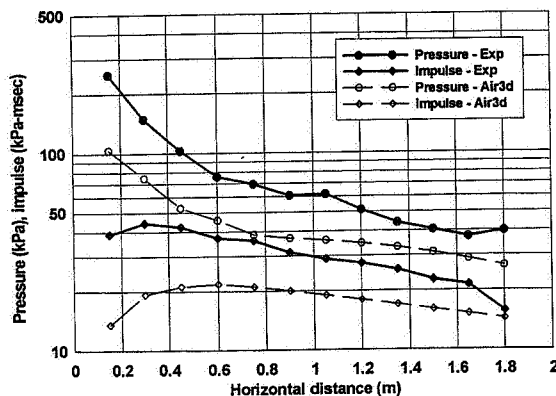


Figure 7.18: Pressure and impulse, $H/W^{1/3} = 0.5 \text{ m/kg}^{1/3}$, $h = 0.15 \text{ m}$

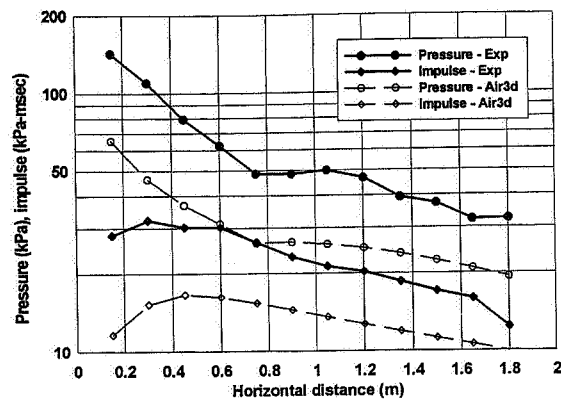


Figure 7.19: Pressure and impulse, $H/W^{1/3} = 0.6 \text{ m/kg}^{1/3}$, $h = 0.15 \text{ m}$

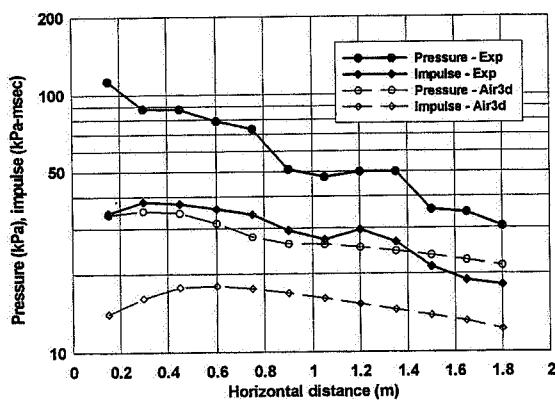


Figure 7.20: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $h = 0.15 \text{ m}$

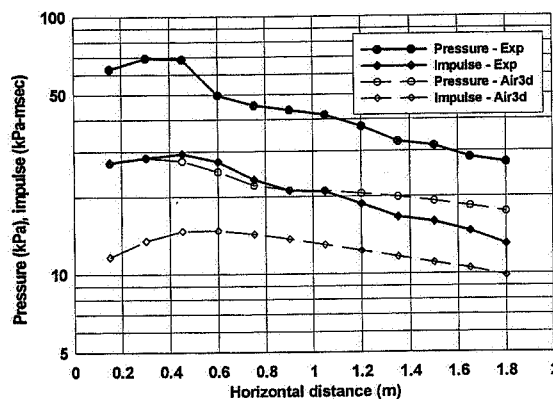


Figure 7.21: Pressure and impulse, $H/W^{1/3} = 0.8 \text{ m/kg}^{1/3}$, $h = 0.15 \text{ m}$

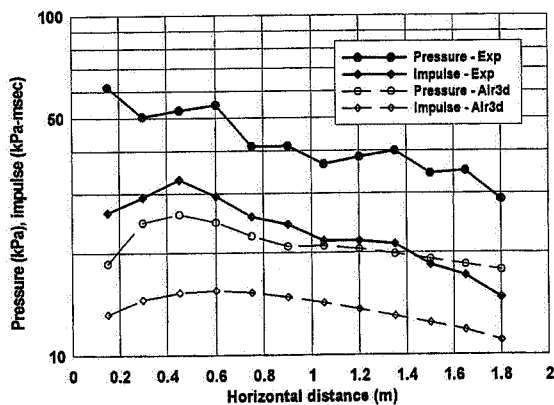


Figure 7.22: Pressure and impulse, $H/W^{1/3} = 0.9 \text{ m/kg}^{1/3}$, $h = 0.15 \text{ m}$

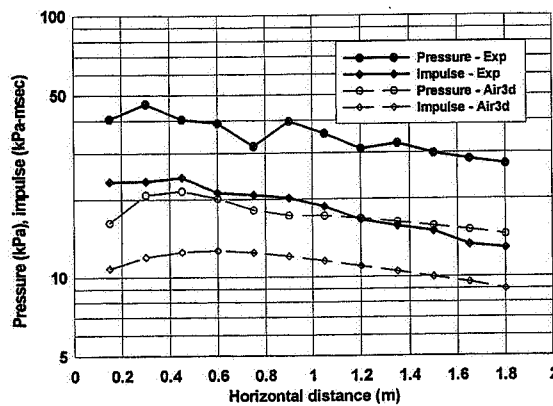


Figure 7.23: Pressure and impulse, $H/W^{1/3} = 1.0 \text{ m/kg}^{1/3}$, $h = 0.15 \text{ m}$

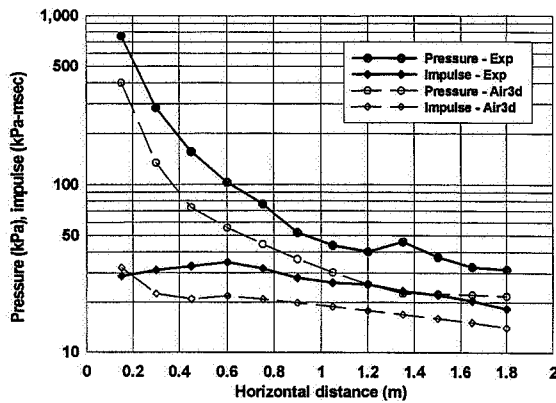


Figure 7.24: Pressure and impulse, $H/W^{1/3} = 0.5 \text{ m/kg}^{1/3}$, $h = 0.3 \text{ m}$

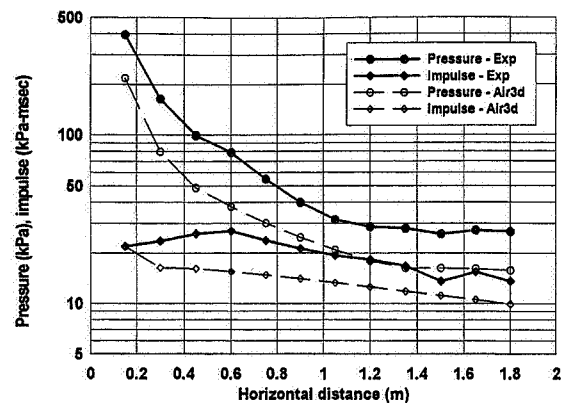


Figure 7.25: Pressure and impulse, $H/W^{1/3} = 0.6 \text{ m/kg}^{1/3}$, $h = 0.3 \text{ m}$

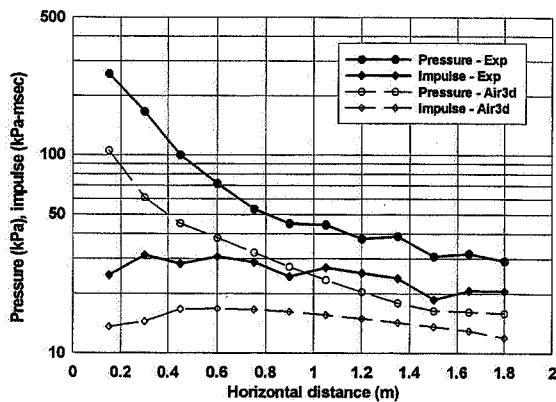


Figure 7.26: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $h = 0.3 \text{ m}$

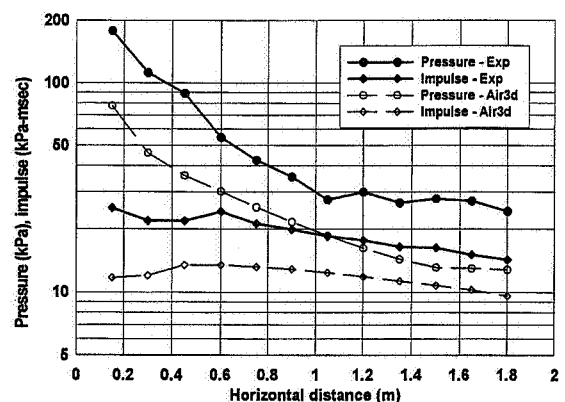


Figure 7.27: Pressure and impulse, $H/W^{1/3} = 0.8 \text{ m/kg}^{1/3}$, $h = 0.3 \text{ m}$

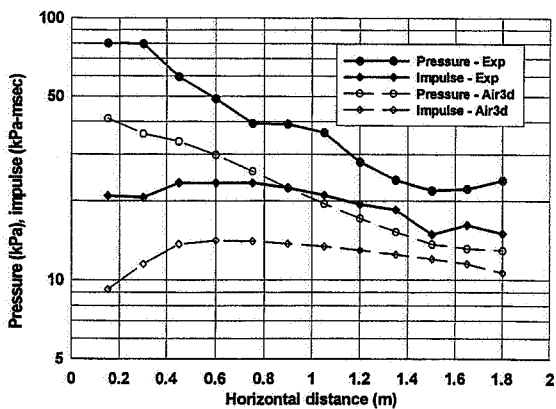


Figure 7.28: Pressure and impulse, $H/W^{1/3} = 0.9 \text{ m/kg}^{1/3}$, $h = 0.3 \text{ m}$

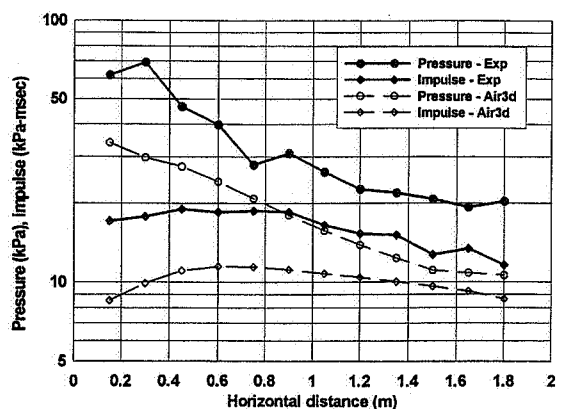


Figure 7.29: Pressure and impulse, $H/W^{1/3} = 1.0 \text{ m/kg}^{1/3}$, $h = 0.3 \text{ m}$

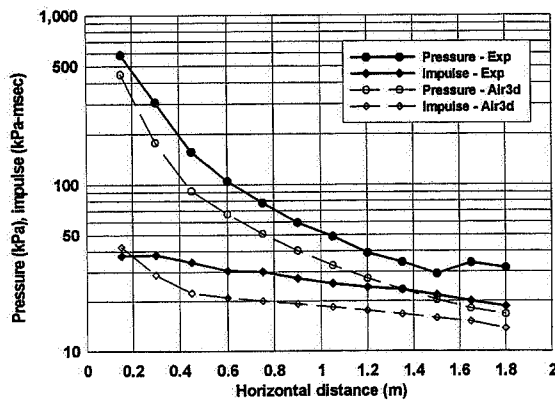


Figure 7.30: Pressure and impulse,
 $H/W^{1/3} = 0.5 \text{ m/kg}^{1/3}$, $h = 0.45 \text{ m}$

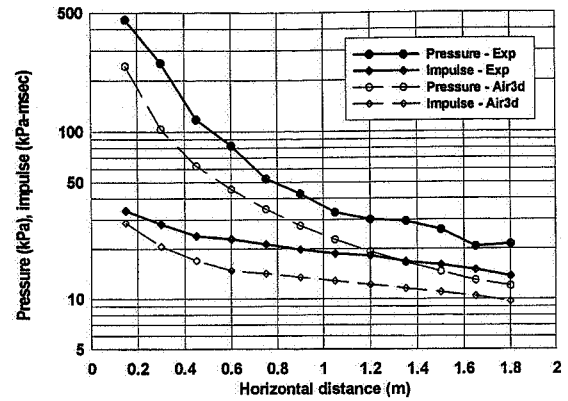


Figure 7.31: Pressure and impulse,
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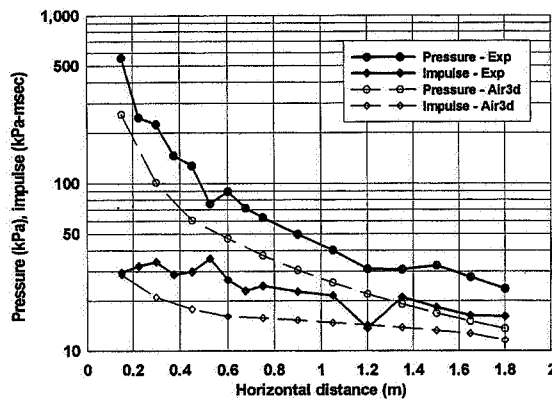


Figure 7.32: Pressure and impulse,
 $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $h = 0.45 \text{ m}$

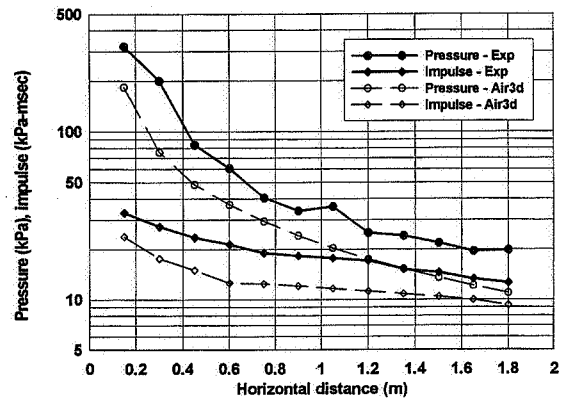


Figure 7.33: Pressure and impulse,
 $H/W^{1/3} = 0.8 \text{ m/kg}^{1/3}$, $h = 0.45 \text{ m}$

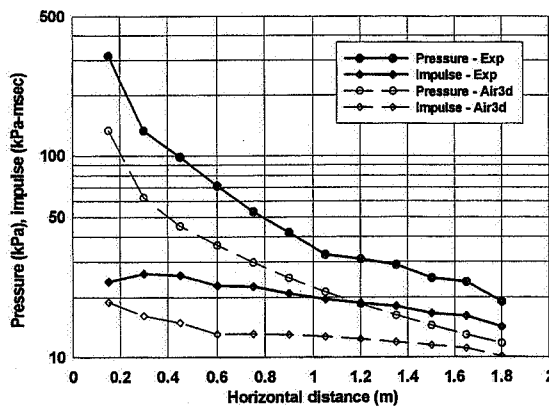


Figure 7.34: Pressure and impulse,
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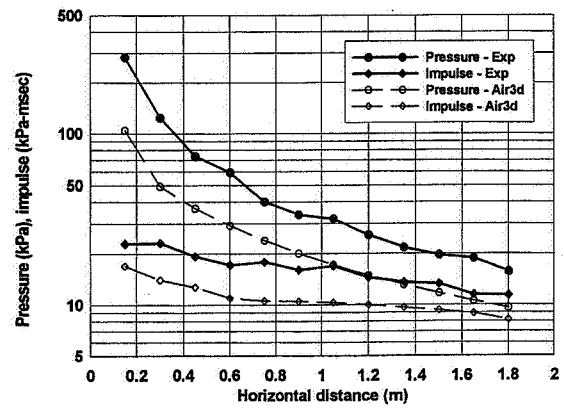


Figure 7.35: Pressure and impulse,
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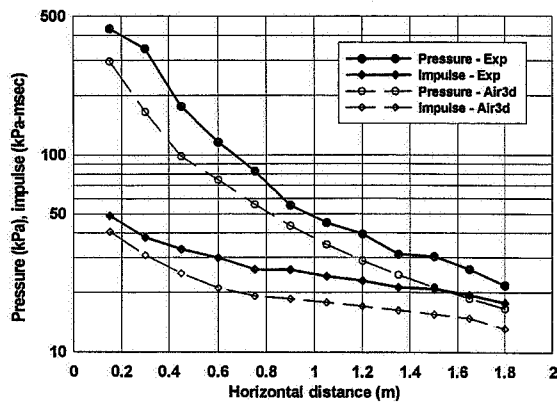


Figure 7.36: Pressure and impulse, $H/W^{1/3} = 0.5 \text{ m/kg}^{1/3}$, $h = 0.6 \text{ m}$

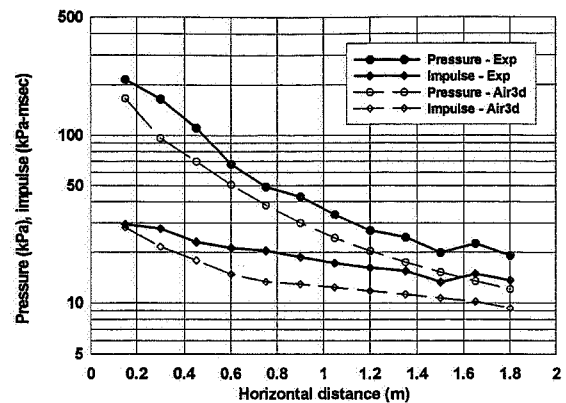


Figure 7.37: Pressure and impulse, $H/W^{1/3} = 0.6 \text{ m/kg}^{1/3}$, $h = 0.6 \text{ m}$

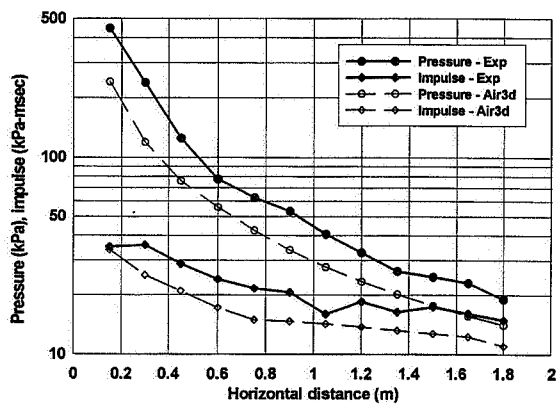


Figure 7.38: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $h = 0.6 \text{ m}$

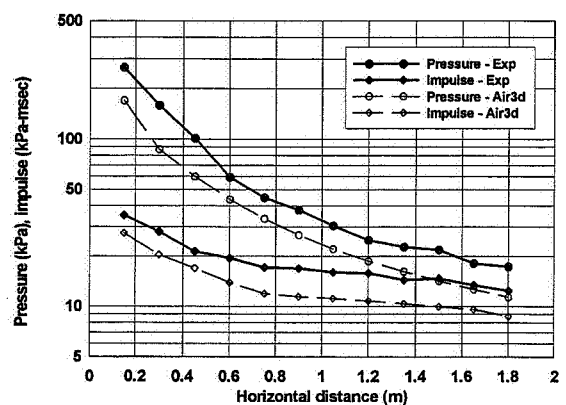


Figure 7.39: Pressure and impulse, $H/W^{1/3} = 0.8 \text{ m/kg}^{1/3}$, $h = 0.6 \text{ m}$

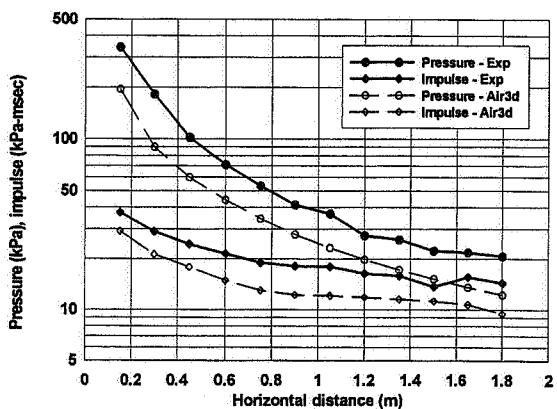


Figure 7.40: Pressure and impulse, $H/W^{1/3} = 0.9 \text{ m/kg}^{1/3}$, $h = 0.6 \text{ m}$

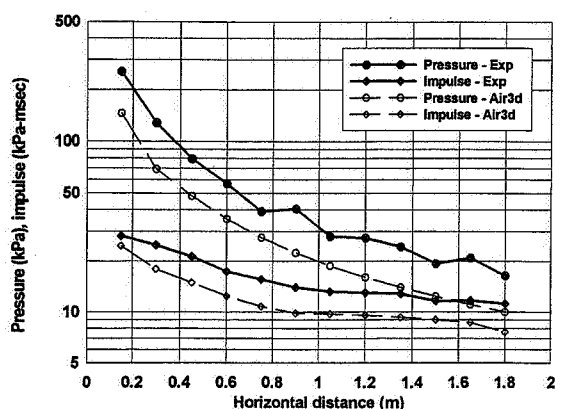


Figure 7.41: Pressure and impulse, $H/W^{1/3} = 1.0 \text{ m/kg}^{1/3}$, $h = 0.6 \text{ m}$

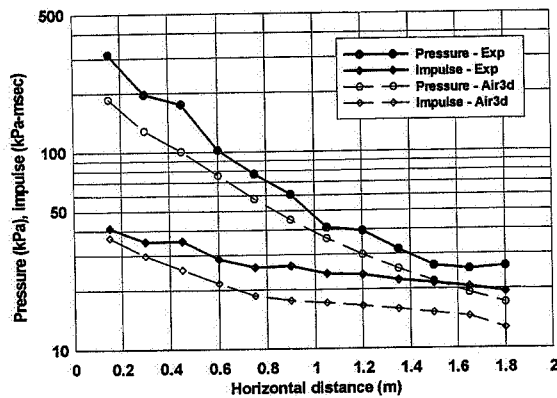


Figure 7.42: Pressure and impulse, $H/W^{1/3} = 0.5 \text{ m/kg}^{1/3}$, $h = 0.75 \text{ m}$

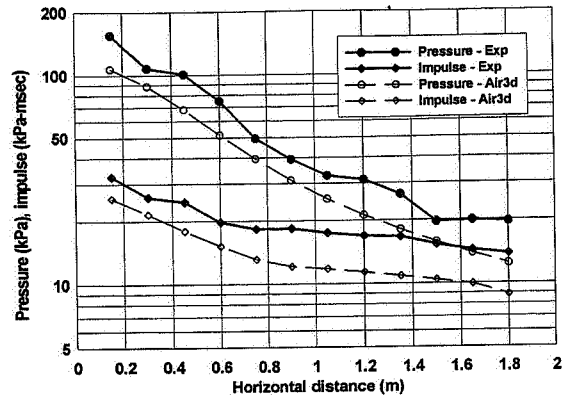


Figure 7.43: Pressure and impulse, $H/W^{1/3} = 0.6 \text{ m/kg}^{1/3}$, $h = 0.75 \text{ m}$

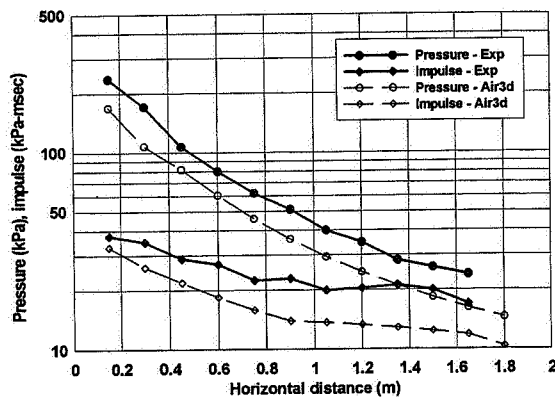


Figure 7.44: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $h = 0.75 \text{ m}$

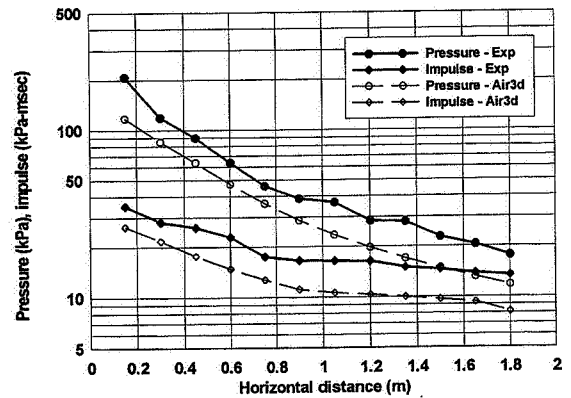


Figure 7.45: Pressure and impulse, $H/W^{1/3} = 0.8 \text{ m/kg}^{1/3}$, $h = 0.75 \text{ m}$

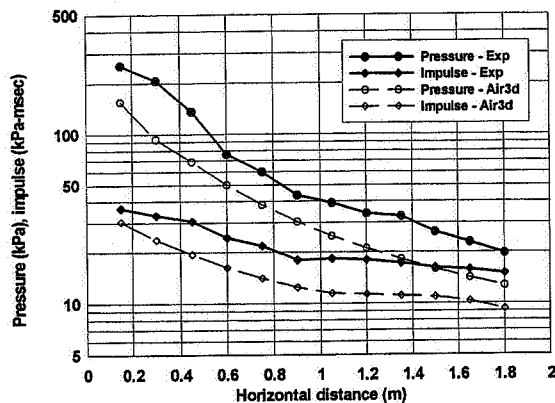


Figure 7.46: Pressure and impulse, $H/W^{1/3} = 0.9 \text{ m/kg}^{1/3}$, $h = 0.75 \text{ m}$

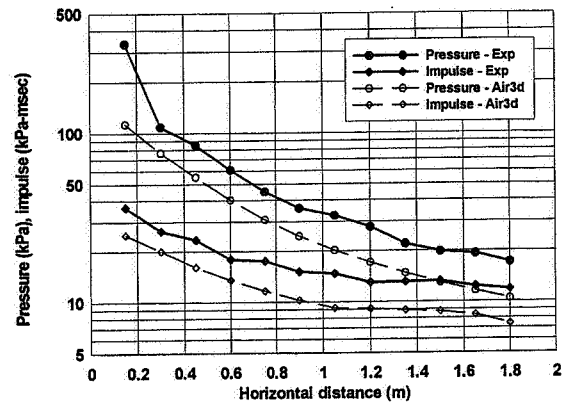


Figure 7.47: Pressure and impulse, $H/W^{1/3} = 1.0 \text{ m/kg}^{1/3}$, $h = 0.75 \text{ m}$

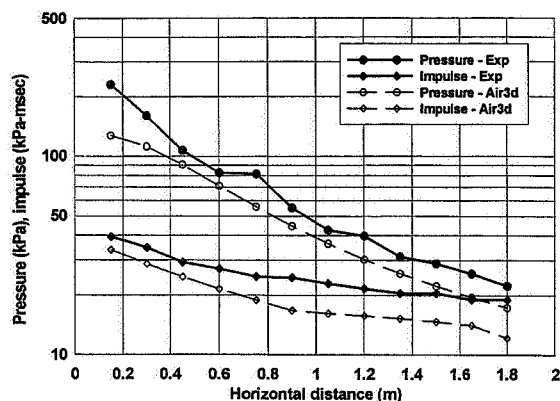


Figure 7.48: Pressure and impulse, $H/W^{1/3} = 0.5 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$

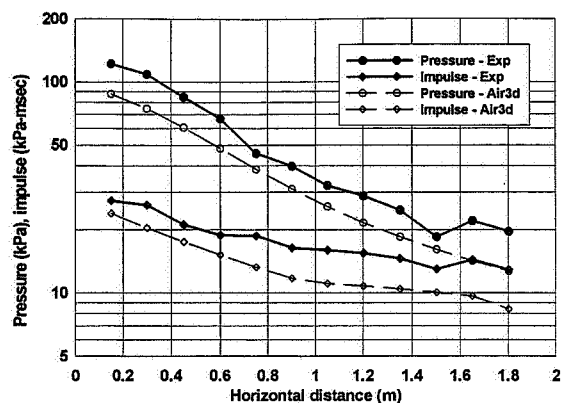


Figure 7.49: Pressure and impulse, $H/W^{1/3} = 0.6 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$

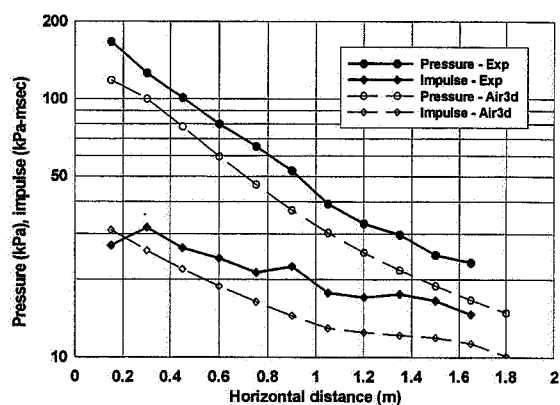


Figure 7.50: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$

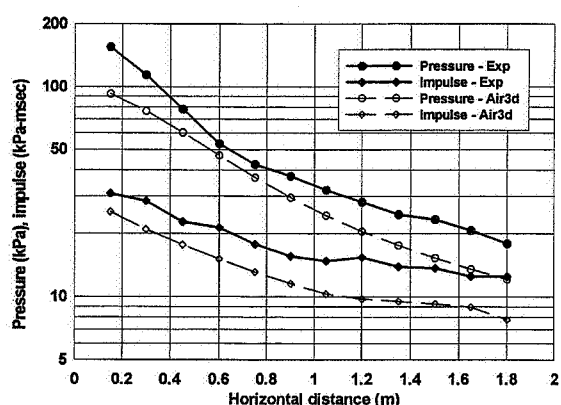


Figure 7.51: Pressure and impulse, $H/W^{1/3} = 0.8 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$

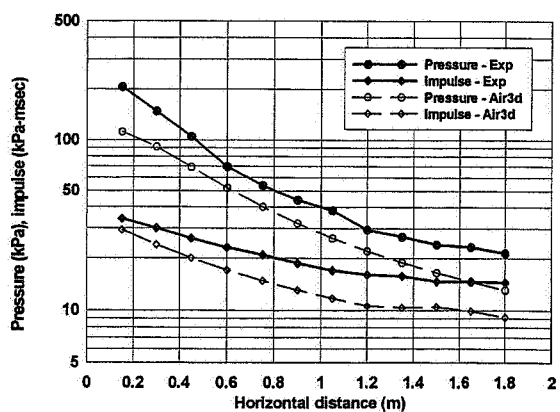


Figure 7.52: Pressure and impulse, $H/W^{1/3} = 0.9 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$

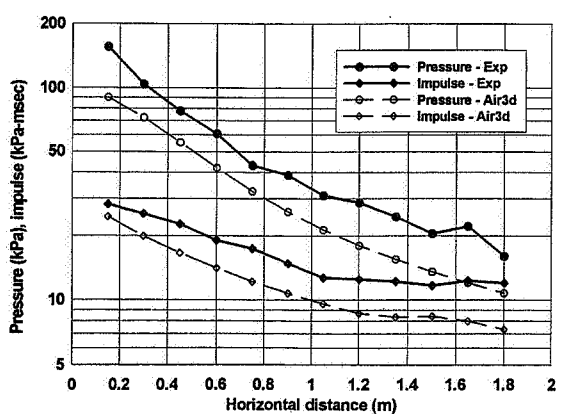


Figure 7.53: Pressure and impulse, $H/W^{1/3} = 1.0 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$

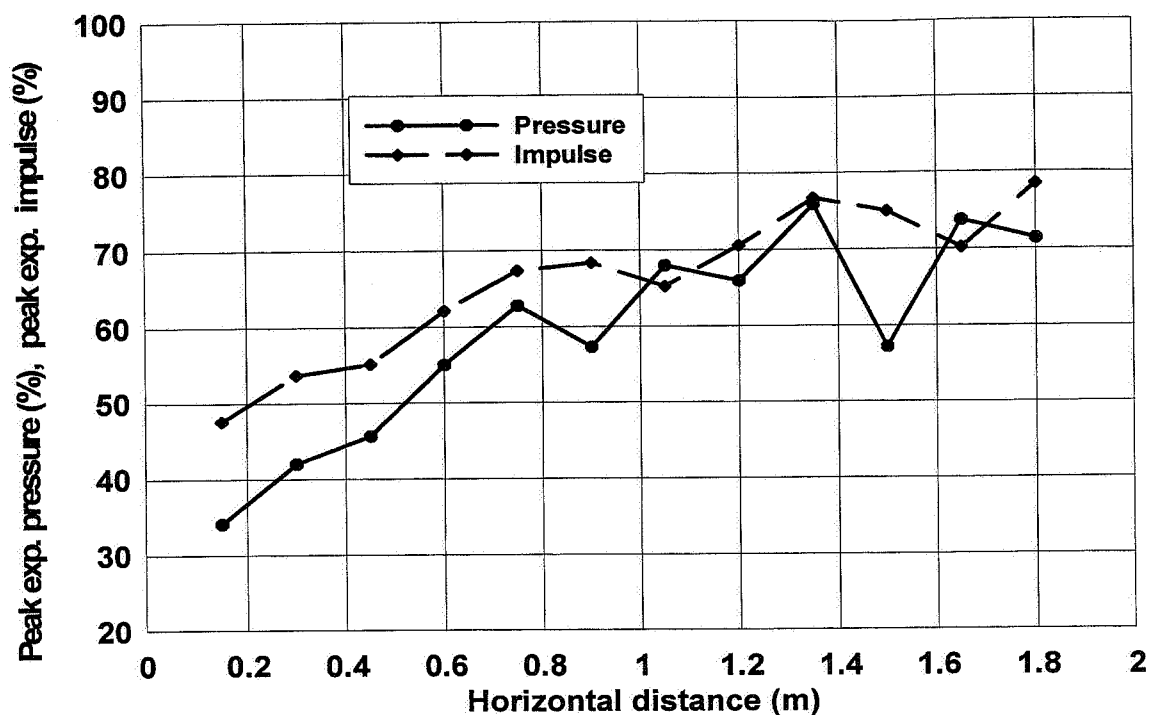


Figure 7.54: Percentage of experimental values, $H/W^{1/3} = 0.5 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$

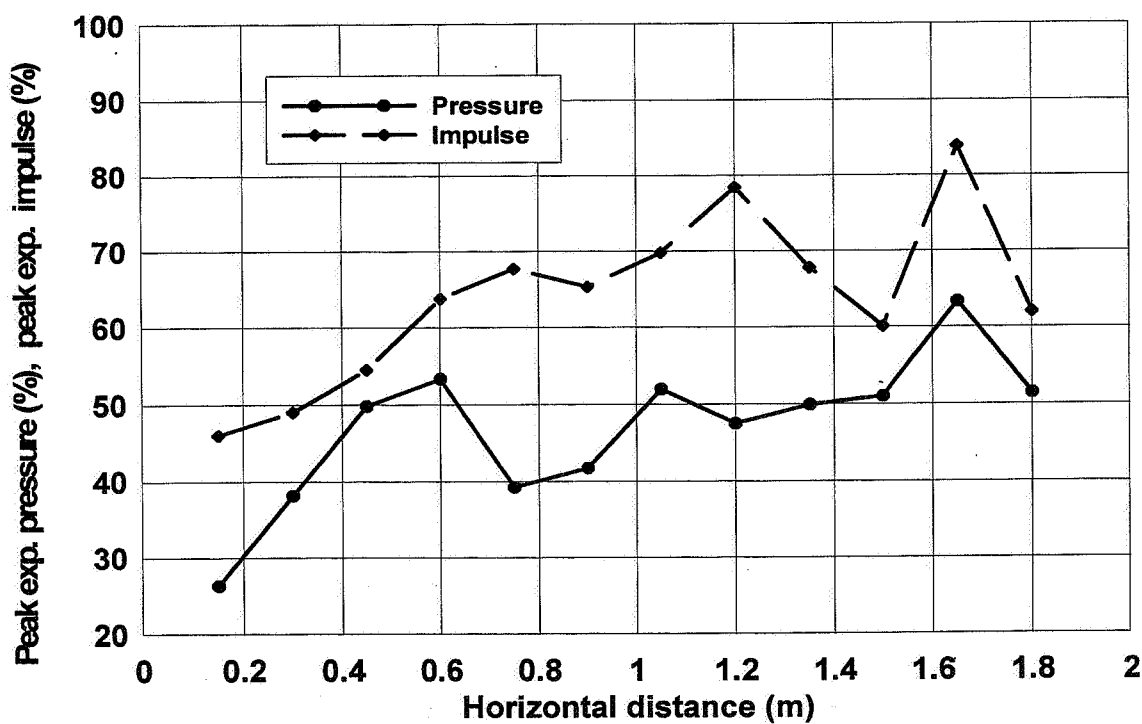


Figure 7.55: Percentage of experimental values, $H/W^{1/3} = 1.0 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$

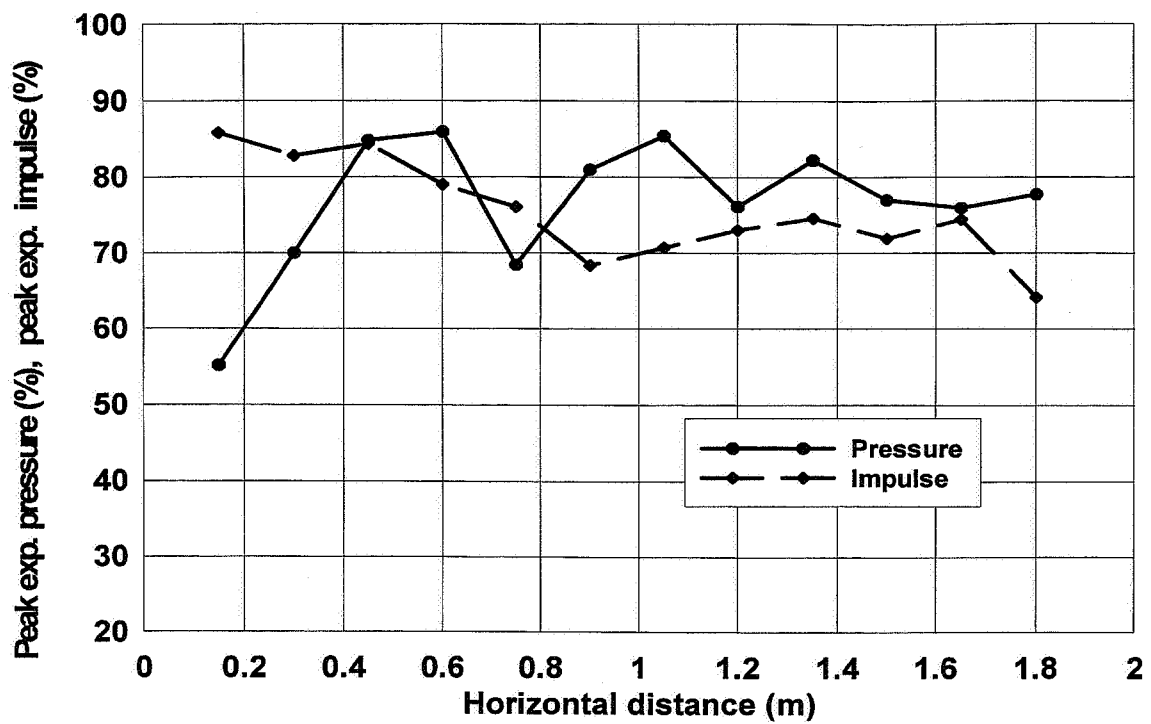


Figure 7.56: Percentage of experimental values, $H/W^{1/3} = 0.5 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$

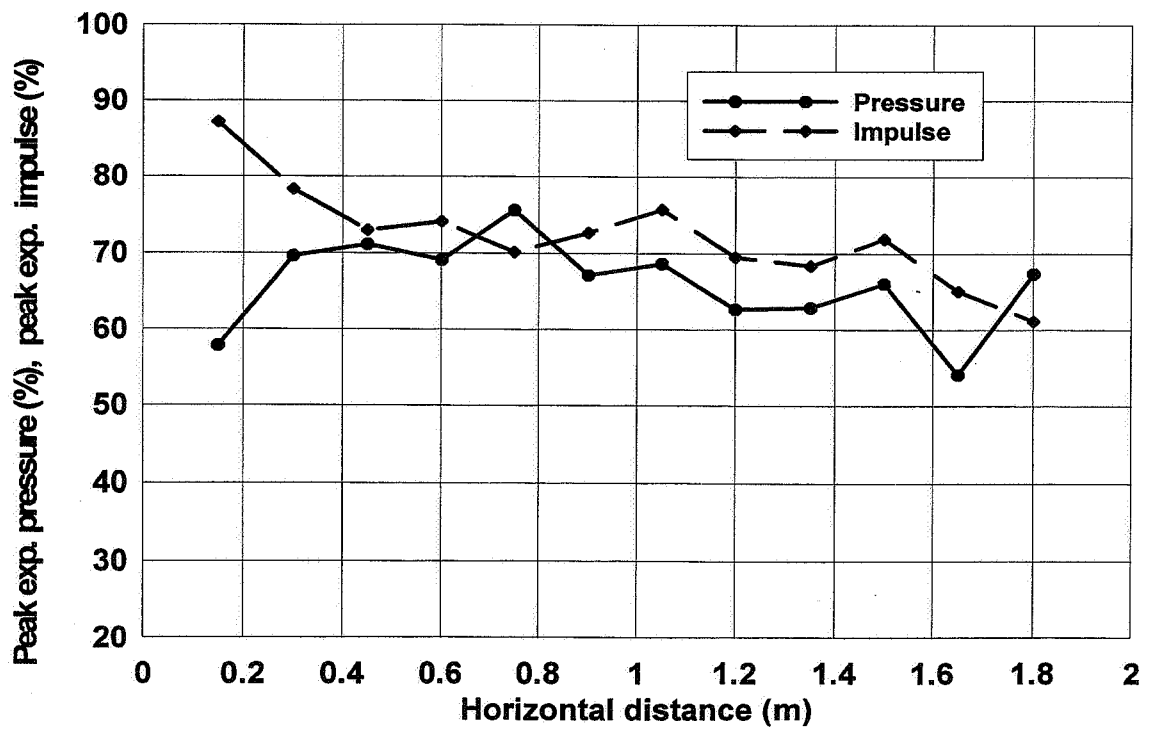


Figure 7.57: Percentage of experimental values, $H/W^{1/3} = 1.0 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$

7.2.3 Comparison with Scaled Stand-Off Data

Before moving on to the investigation of significant aspects of the effectiveness of blast walls, there is one last comparison which will help to prove the above hypothesis, concerning the nature of the simulation of blast wave diffraction, and hence the quality of results obtainable by CFD blast calculation.

The experiments reported by Rose et al. [43] also contained a short programme designed to investigate the effect of charge stand-off from the blast wall. The geometry was that of experiment 3 in Table 7.1, except the scaled stand-off distance of the charge from the wall ($r/W^{1/3}$) varied from $0.327 \text{ m/kg}^{1/3}$ to $1.423 \text{ m/kg}^{1/3}$ in three equal increments. There were four experiments in total, one set of data was common to the experiments in Table 7.1.

The final experiment in the series, for $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$, corresponded to an actual charge stand-off of 600 mm. Clearly, at this distance, the duration of the blast wave at the top of the wall was considerably greater than that of the validation experiments described previously, and if the above hypothesis is correct the opportunity for the program to model the diffraction process will be significantly increased.

This final analysis was performed using the same computational domain shown in Figure 7.11, except the boundary on the near side of the wall was extended to -1.2 m so that the results of the two-dimensional analysis could be remapped into the domain. The actual domain used 4,800,000 computational cells.

The results of this analysis can be seen in Figures 7.58 to 7.64 and percentage of peak pressure and impulse graphs for the horizontal monitoring arrays at $h = 0.0 \text{ m}$ and $h = 0.9 \text{ m}$ (the same as previously) are shown in Figures 7.65 to 7.66. Observations arising from these figures are as follows:

- Qualitatively, Figures 7.58 to 7.64 demonstrate a much clearer correspondence between the two sets of data than was evident in the validation examples discussed previously.
- Quantitatively, percentages of peak experimental pressure and impulse for the two extreme horizontal monitoring arrays (at $h = 0.0 \text{ m}$ and $h = 0.9 \text{ m}$) are plotted in Figures 7.65 and 7.66. These show that for $h = 0.0 \text{ m}$ (Figure 7.65) the percentage of peak experimental pressure varied from 38% to 74%, and the impulse varied from 71% to 103%. Similarly for $h = 0.9 \text{ m}$, Figure 7.66 shows that the pressure varied from 73% to 90% and the impulse, from 78% to 100%.
- Because the total stand-off distance of the charge from the measuring array was increased (a factor that would tend to degrade results) and the discretisation of the problem was the same as for the previous examples, the fact that better correlation with experiment resulted, particularly with regard to impulse, suggests that the process of diffraction was modelled more closely in the present example than in the previous ones. This must be due to the increased duration of the blast at the top of the wall and the fact that the principal convection direction was closer to horizontal.

The process of validation described above is perhaps not as satisfactory in this chapter as in Chapters 5 and 6. One of the primary intentions of this chapter was to bring the data of Rose et al. [43] up to date by analysing scenarios with scaled blast wall height less than $0.5 \text{ m/kg}^{1/3}$, the minimum described above. Unfortunately, the resolution achievable with the present algorithm and computer resources has made that goal currently unobtainable. Therefore, the remainder of this section will examine more general aspects of blast wall geometry in an attempt to quantify effectiveness, without extending the current experimental data base.

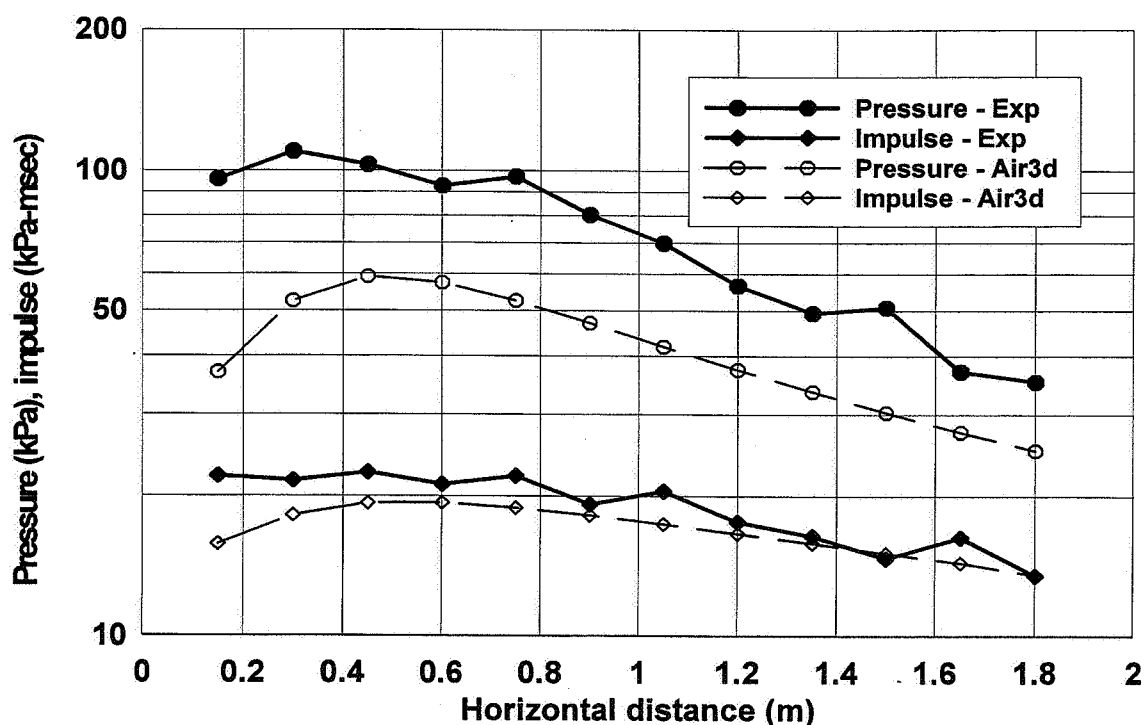


Figure 7.58: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$

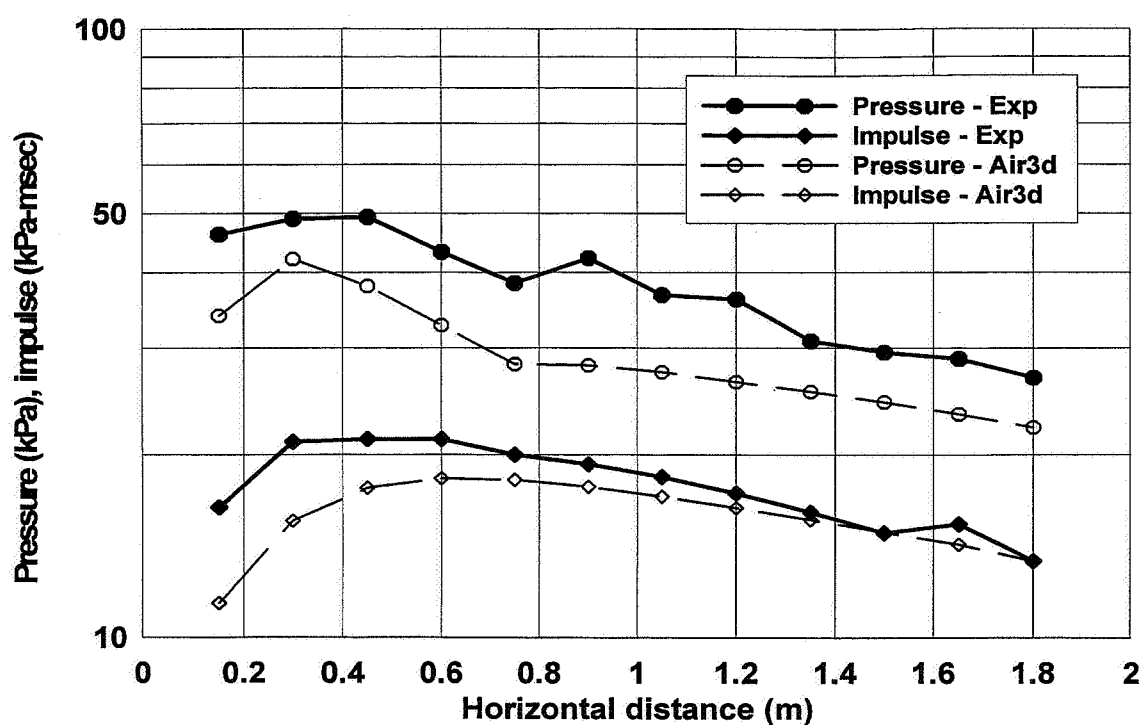


Figure 7.59: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$, $h = 0.15 \text{ m}$

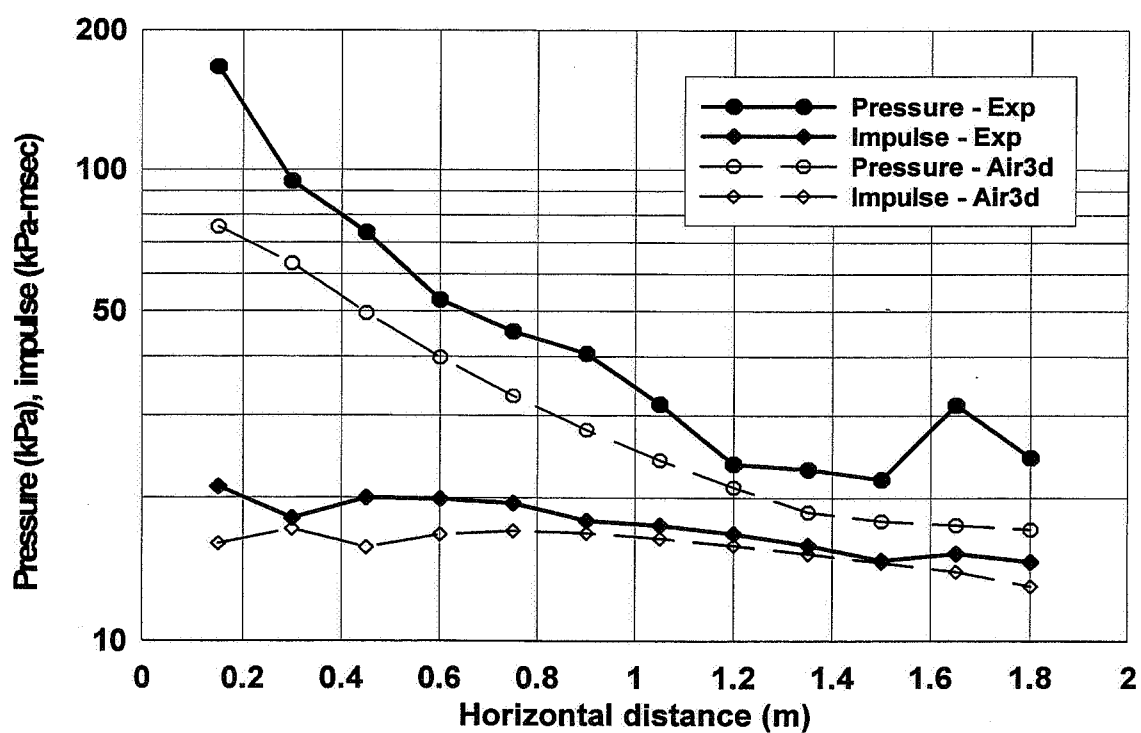


Figure 7.60: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$, $h = 0.3 \text{ m}$

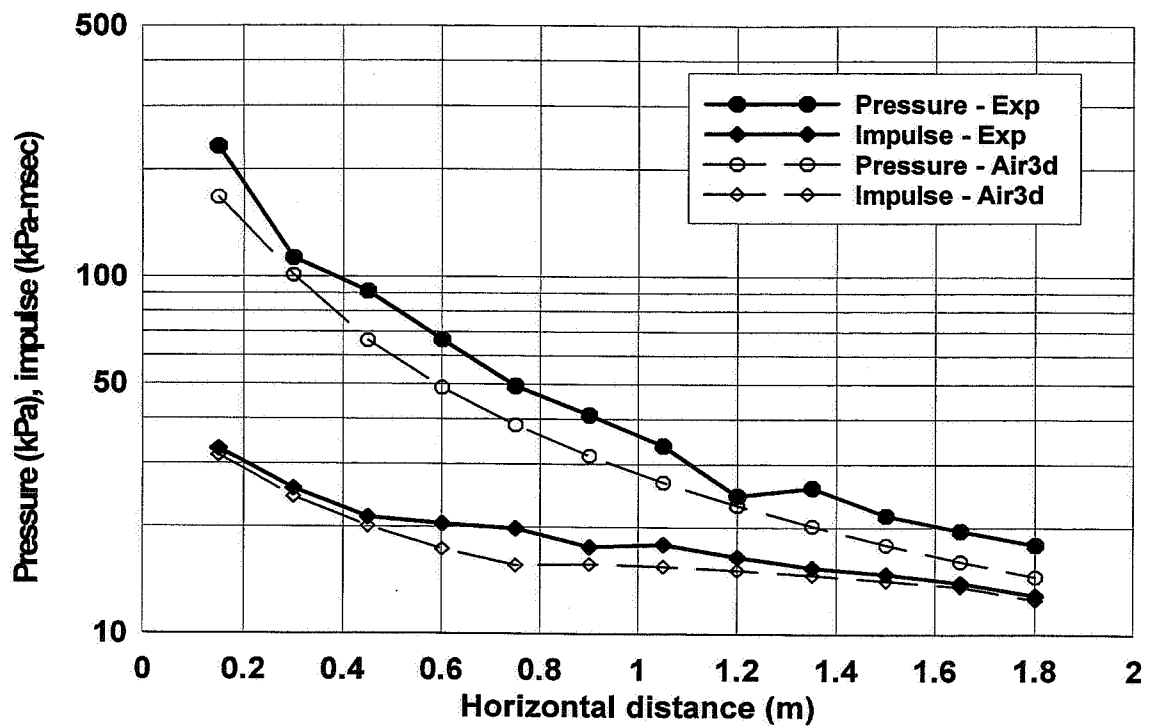


Figure 7.61: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$, $h = 0.45 \text{ m}$

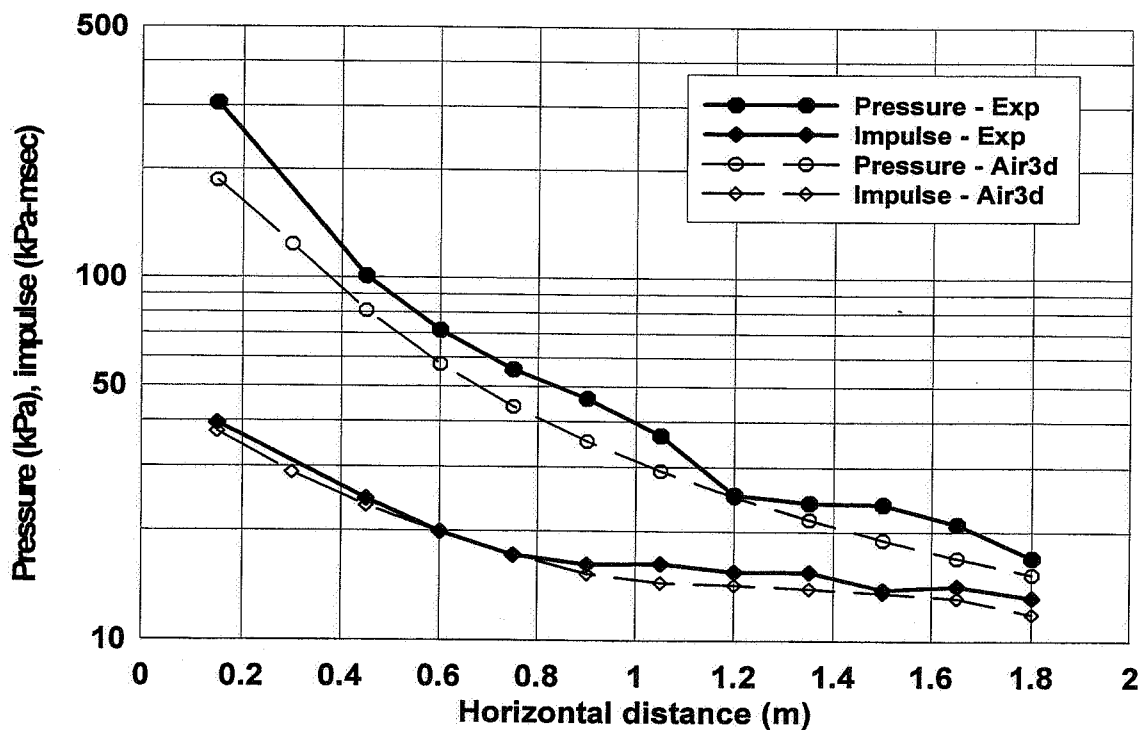


Figure 7.62: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$, $h = 0.6 \text{ m}$

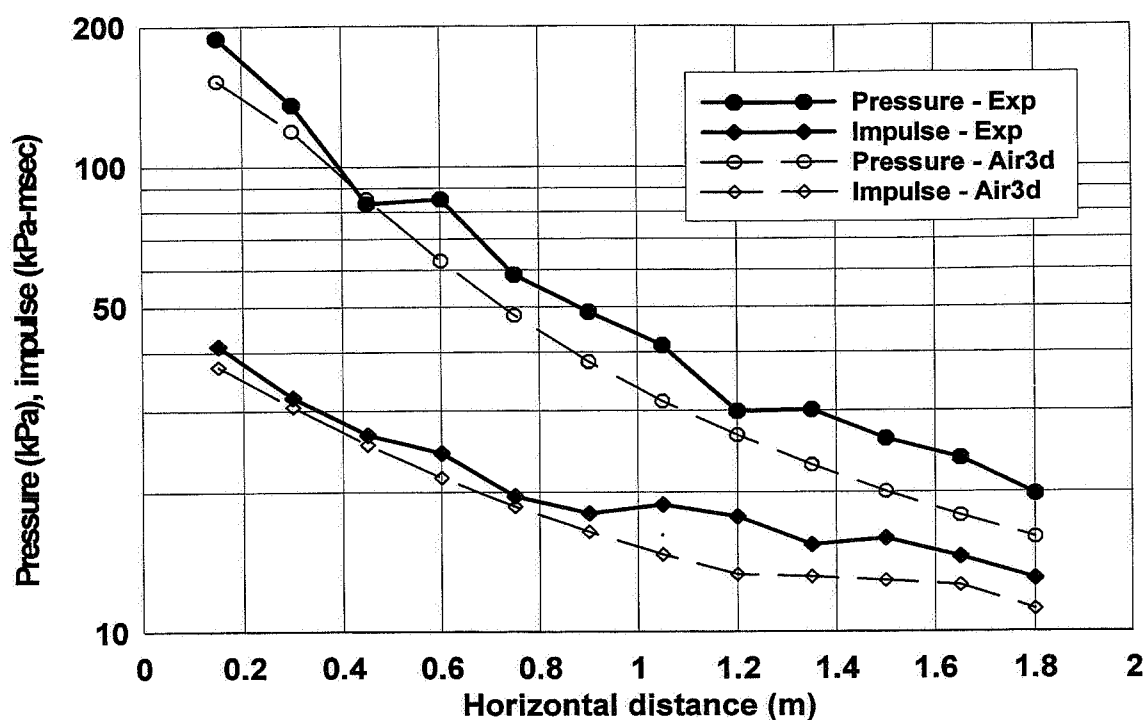


Figure 7.63: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$, $h = 0.75 \text{ m}$

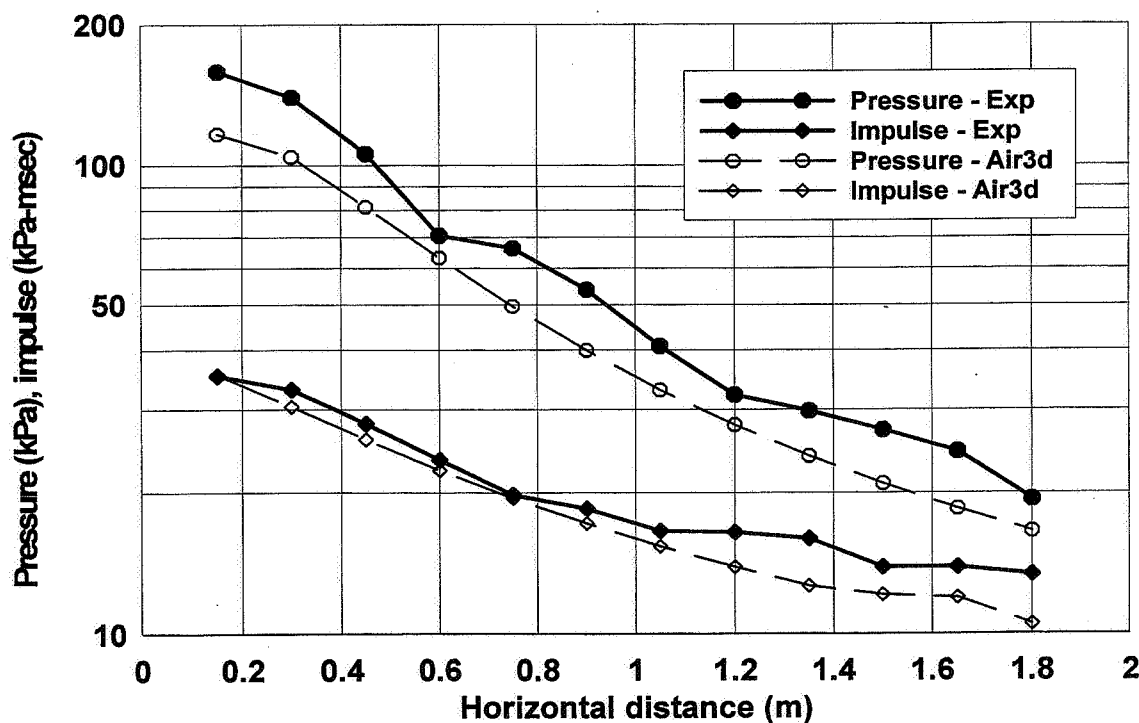


Figure 7.64: Pressure and impulse, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$

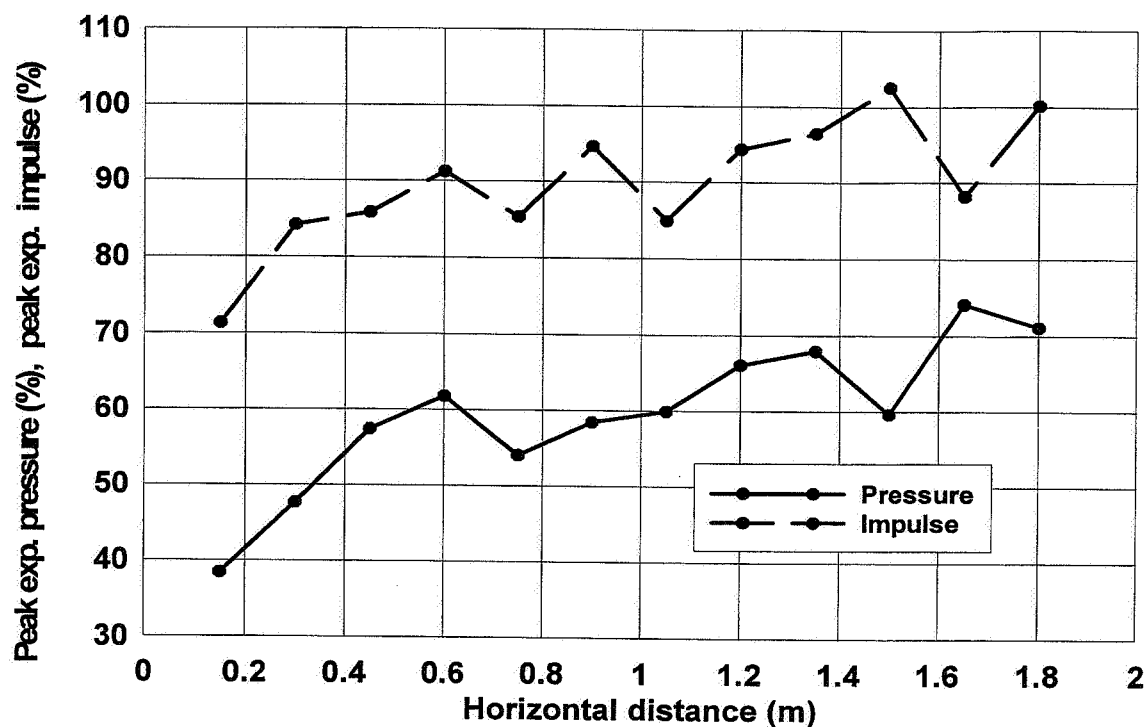


Figure 7.65: Percentage of experimental values, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $h = 0.0 \text{ m}$, $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$

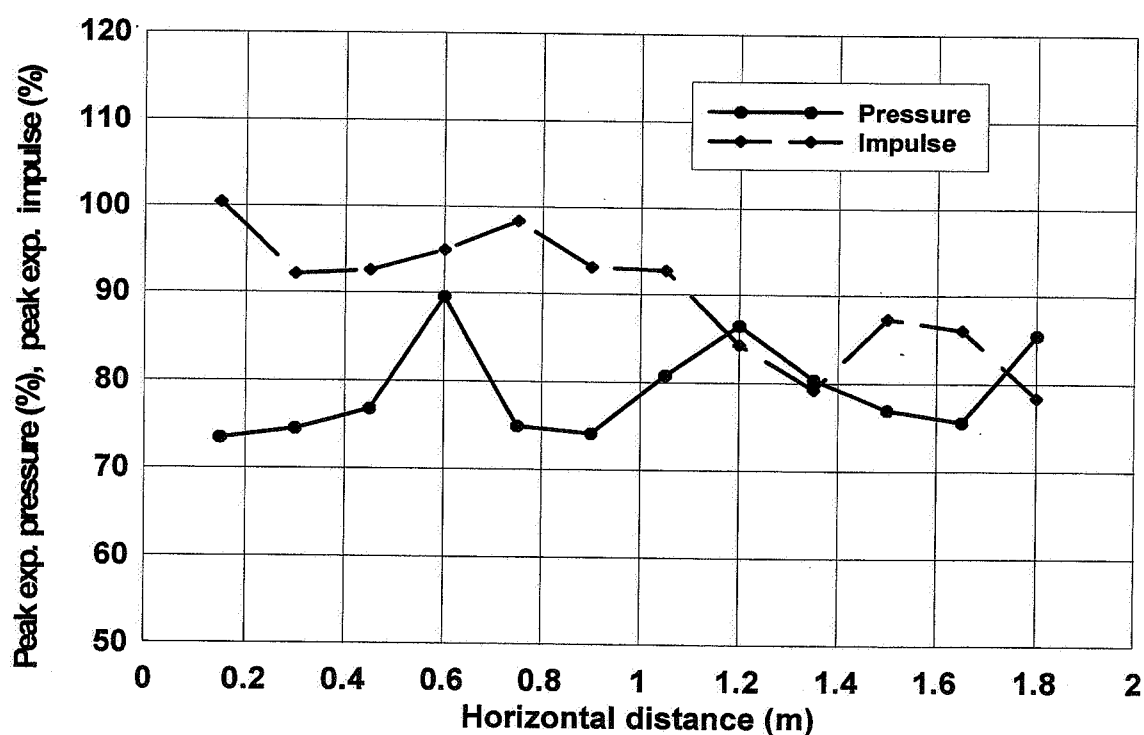


Figure 7.66: Percentage of experimental values, $H/W^{1/3} = 0.71 \text{ m/kg}^{1/3}$, $h = 0.9 \text{ m}$, $r/W^{1/3} = 1.423 \text{ m/kg}^{1/3}$

7.3 Important Aspects of Blast Wall Effectiveness

Possibilities for the investigation of blast wall/protected structure scenarios are limitless. To quantify the environment behind a blast wall by varying systematically all the parameters defined in Figure 7.2, across the whole range of practical interest, would be a daunting task.

The approach of the present section, therefore, is to examine one typical scenario using a geometry which is in keeping with current interest, in terms of charge weight and building stand-off, by varying the wall height and wall placement.

This approach is typical of a *case study* and, although its findings are not valid (quantitatively) for any other geometry, the trends apparent from the exercise are, nonetheless, applicable to any blast wall or shielding scenario.

The charge weight considered was 500 kg TNT, the height of the centre of the charge above the ground was 1.0 m and was chosen to reflect the dimensions of a vehicle. The wall thickness was 0.5 m. The rest of the geometry is shown in Figure 7.67.

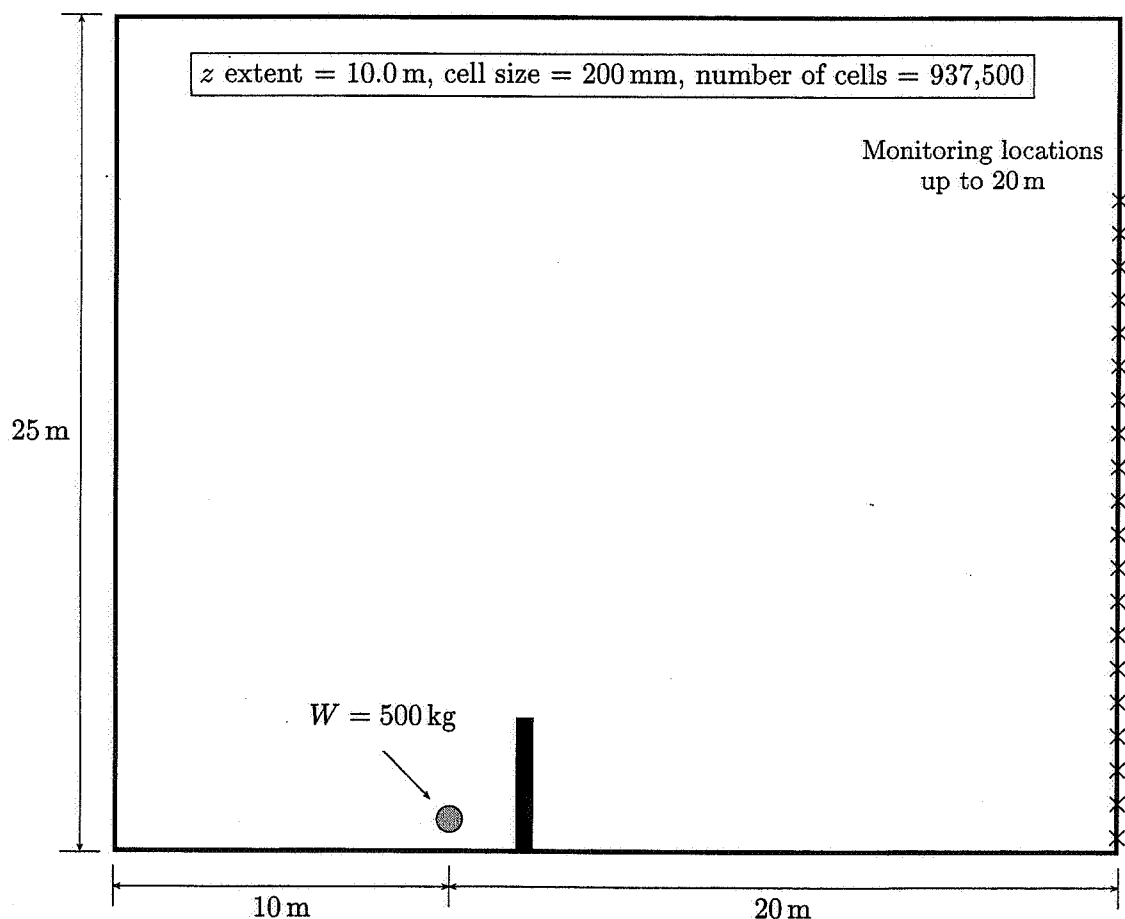


Figure 7.67: Domain used for generic blast wall calculations

In Figure 7.67 the right-hand boundary was set to reflect: it represented the façade of a large building. The horizontal distance from the centre of the charge to the building was fixed at 20.0 m. An array of 20 monitoring locations was arranged vertically up the centreline of the building, starting at 0.5 m and finishing at 19.5 m; each location was at the centre of a 1.0 m interval.

The height of the wall was varied from $H = 2.5$ m to 4.0 m in increments of 0.5 m. In scaled terms, this represents $H/W^{1/3} = 0.315 \text{ m/kg}^{1/3}$, $0.378 \text{ m/kg}^{1/3}$, $0.441 \text{ m/kg}^{1/3}$ and $0.504 \text{ m/kg}^{1/3}$, respectively. This scaled range of wall heights was considerably below that considered in the validation examples and better reflects the situation relevant to current investigators.

The wall placement varied from $r = 2.0$ m to 10 m in increments of 2.0 m. There were 20 analyses in the matrix of *with wall* calculations. There was also a single *without wall* calculation, for comparison, giving 21 Air3d analyses in total.

A one-dimensional analysis was performed for the short stand-off of the charge from the ground (1.0 m); the cell size was 1.0 mm. Two-dimensional analyses were performed until the blast wave reached the wall. The remainder of the problems were treated in three dimensions. It should be noted that, although the one- and three-dimensional discretisations were the same in every analysis, the two-dimensional discretisations varied, depending on the radius of the problem. Generally, the discretisation was chosen to give a good compromise between resolution and run time. Details of the two-dimensional discretisations are given in Table 7.2.

Table 7.2: Two-dimensional discretisation

Charge stand-off r (m)	2	4	6	8	10	20 [†]
Cell size (mm)	10	10	20	20	25	40

† Without wall (= height of burst)

The *without wall* calculation was comprised of a two-dimensional, radially symmetrical analysis which covered the whole of the space between the centre of the charge and the structure (20 m). At this point, it was remapped into the three-dimensional grid for the short duration in which the blast wave reflection took place. In every case, the problem finish time was 85 msec, which was sufficient to allow complete reflection of the blast wave across the whole of the centreline of the façade.

It will be recognised that, after reflection from the building, a small secondary reflection occurs at the far side of blast wall and propagates back to the building. This second wave is only really evident when the wall is situated relatively close to the building: $r = 8$ or 10 m, for example, in the present case. However, the reflection is weak and arrives sufficiently late that it can be ignored. It should be borne in mind that the presence of this secondary wave can be significant in situations where the (scaled) stand-off distance of the building from the charge is very small and multiple reflections *can* take place.

7.3.1 The Influence of Charge Stand-off on Blast Wall Effectiveness

Results of the 20 *with wall* analyses, described above, are presented in the form of graphs of peak pressure and impulse plotted against vertical height on the building façade. Each graph is for a fixed wall height and contains five sets of data: one for each of the different charge stand-off distances. It is important to remember that, in each case, the total distance of the charge from the building was constant ($=20$ m).

Intuitively, it would be expected that as the stand-off distance of the charge from the wall increases, the *shadow region* (the part of the building obscured by line of sight, Figure 7.68) decreases, the area loaded without diffraction is increased, and higher loads on the upper, *exposed*, region would result.

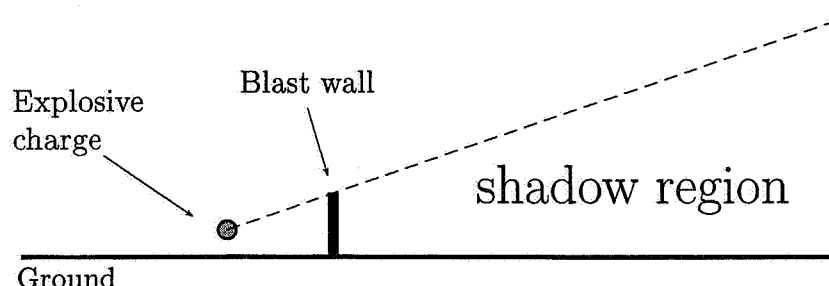


Figure 7.68: Sketch showing the shadow region behind a blast wall

Figures 7.69 to 7.76 clearly demonstrate that increasing the stand-off of the wall from the charge reduces the effectiveness of the wall. This is not surprising, but what is perhaps surprising is the extent to which the effectiveness has decreased.

The trend observed in Figures 7.69 to 7.76 for the *shadow region* itself, however, is not so readily explained. It can be observed from the graphs of impulse, Figures 7.70, 7.72, 7.74 and 7.76, that the uppermost line (for $r = 10.0$ m) is coincident (or almost coincident) with that for $r = 8.0$ m for the lowest region of the façade. It is suggested that there are two different mechanisms at work here. One is the conjecture discussed previously that increasing the stand-off of the charge from the wall *promotes* diffraction (both numerically and actually). The second, compensating suggestion, is that the resulting diffracted wave has less physical space, between the wall and the building, in which to diffract. These two aspects work against each other: one promotes diffraction, the other negates it. The result is an effective limit to the impulse at the base of the façade.

Another important aspect, similar to above, is the observation that, as the height of the wall increases, the influence of the charge stand-off from the wall on received impulse near the base of the façade, becomes less pronounced. This is most strikingly demonstrated by the comparison between Figures 7.70 and 7.76, where it can be seen that, although the different lines of stand-off are clearly separated in the upper region of the building, in the *shadow region* they vary from widely distributed, for the lowest wall ($H = 2.5$ m, Figure 7.70), to fairly closely packed, for the highest ($H = 4.0$ m, Figure 7.76).

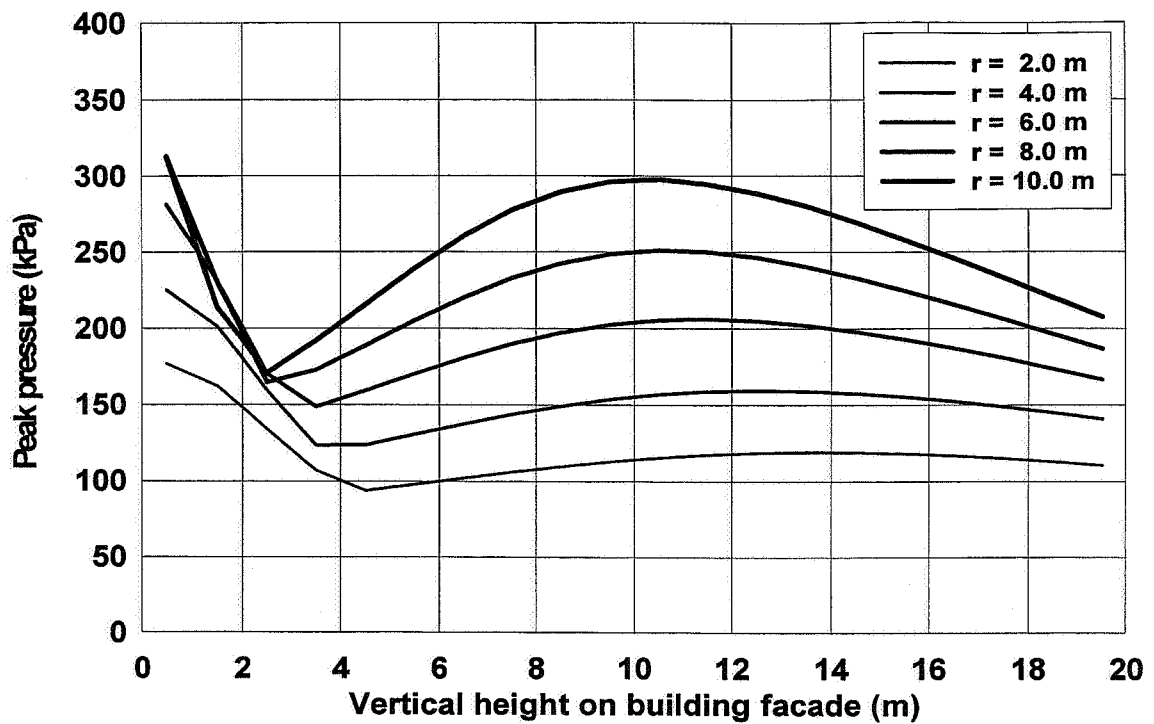


Figure 7.69: Pressure, variation of charge stand-off, wall height $H = 2.5$ m

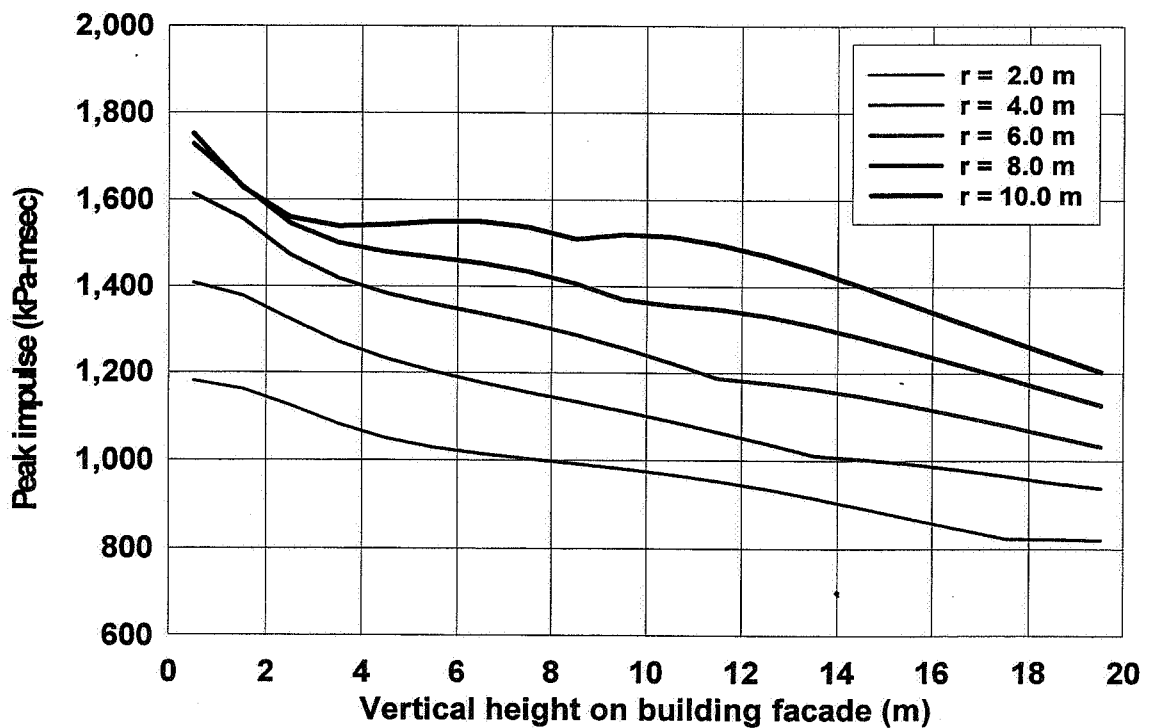


Figure 7.70: Impulse, variation of charge stand-off, wall height $H = 2.5$ m

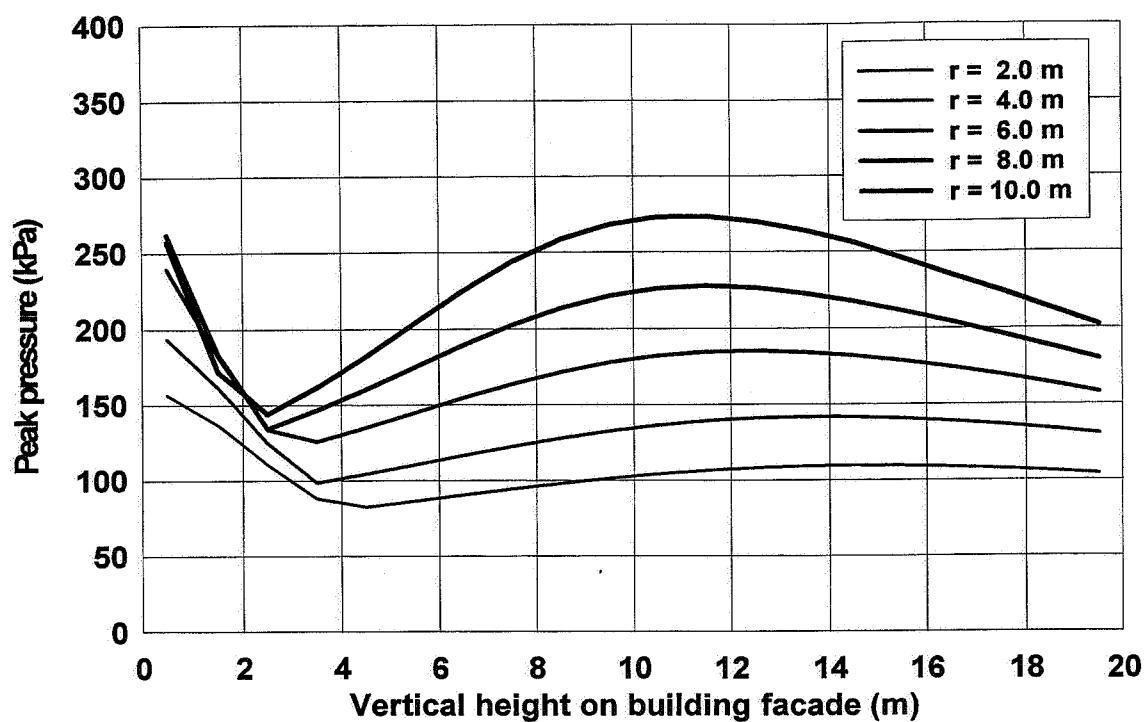


Figure 7.71: Pressure, variation of charge stand-off, wall height $H = 3.0$ m

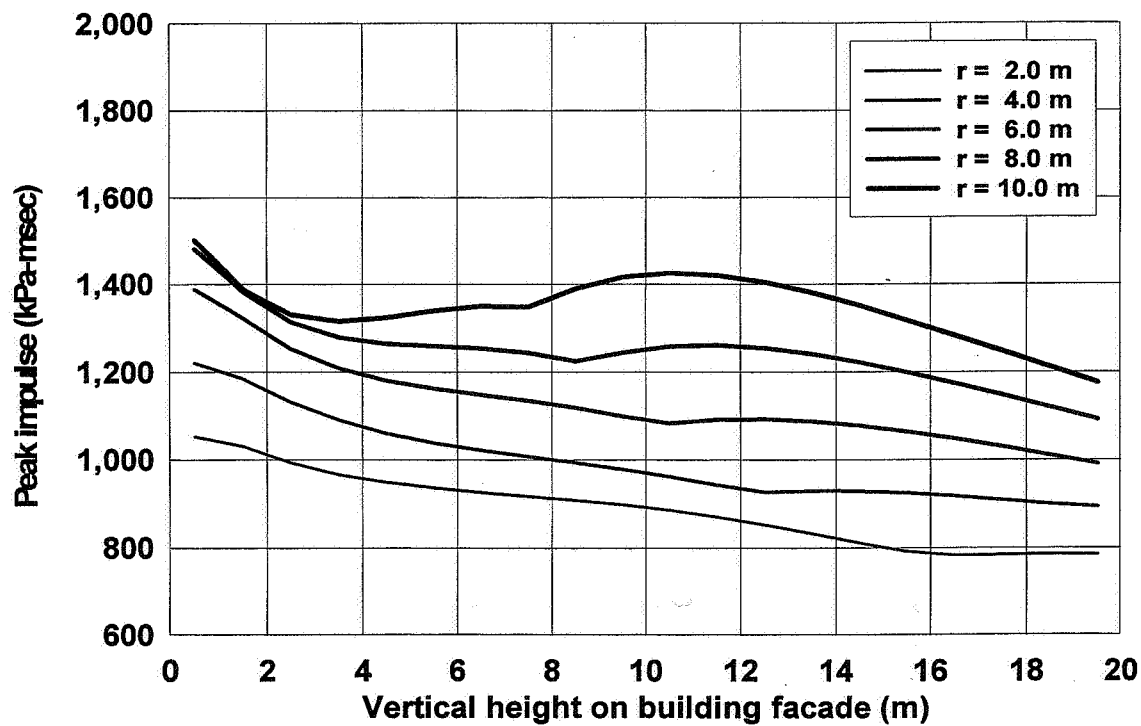


Figure 7.72: Impulse, variation of charge stand-off, wall height $H = 3.0$ m

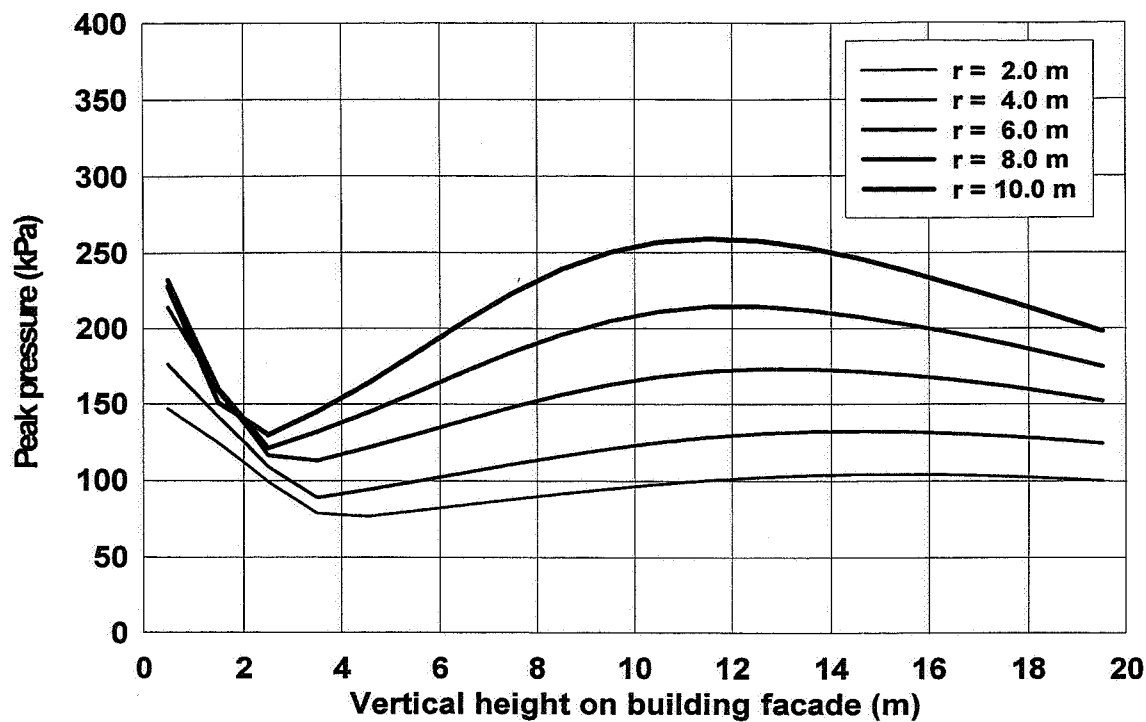


Figure 7.73: Pressure, variation of charge stand-off, wall height $H = 3.5$ m

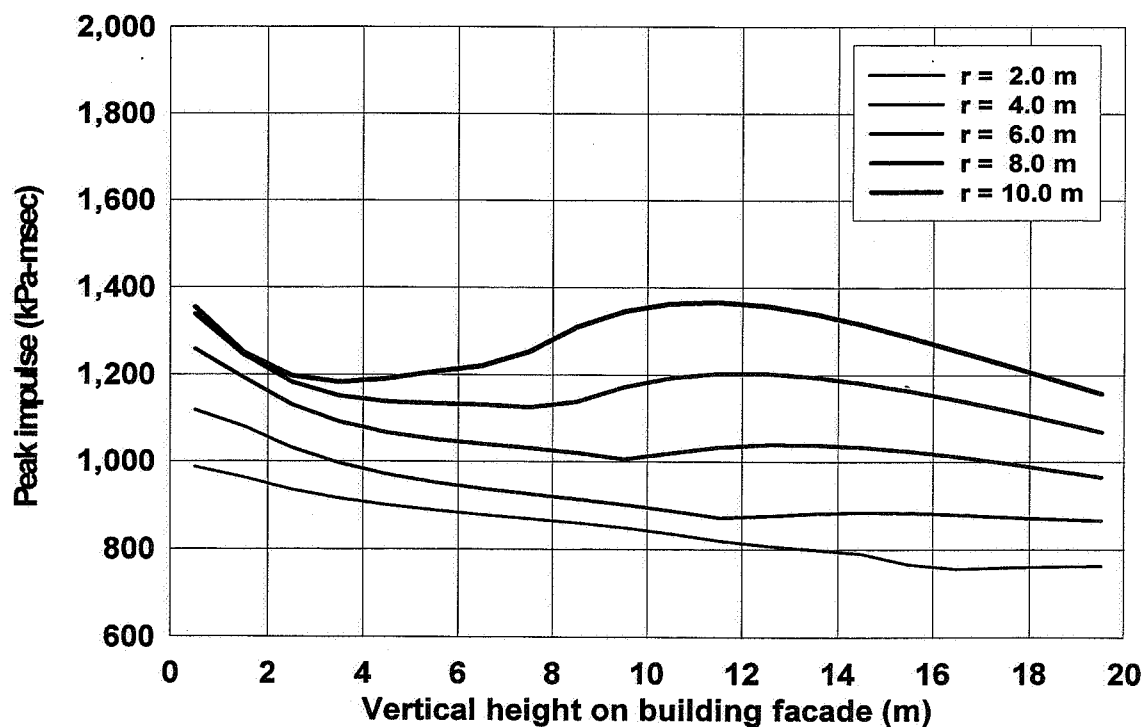


Figure 7.74: Impulse, variation of charge stand-off, wall height $H = 3.5$ m

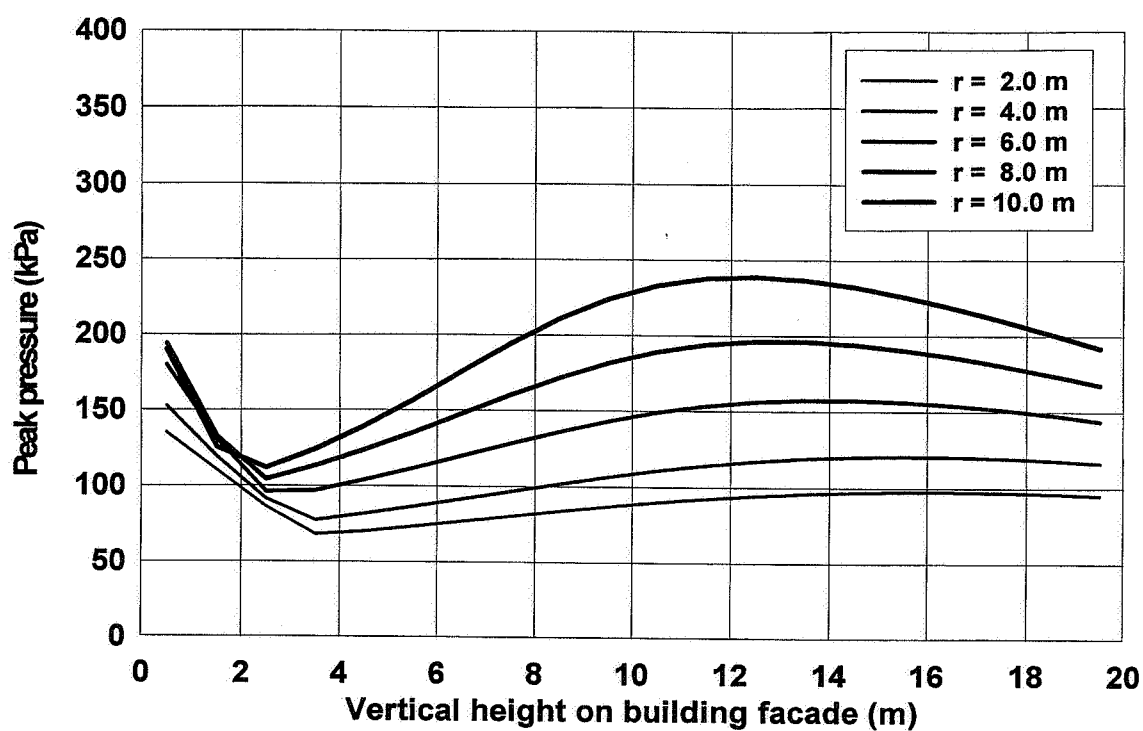


Figure 7.75: Pressure, variation of charge stand-off, wall height $H = 4.0$ m

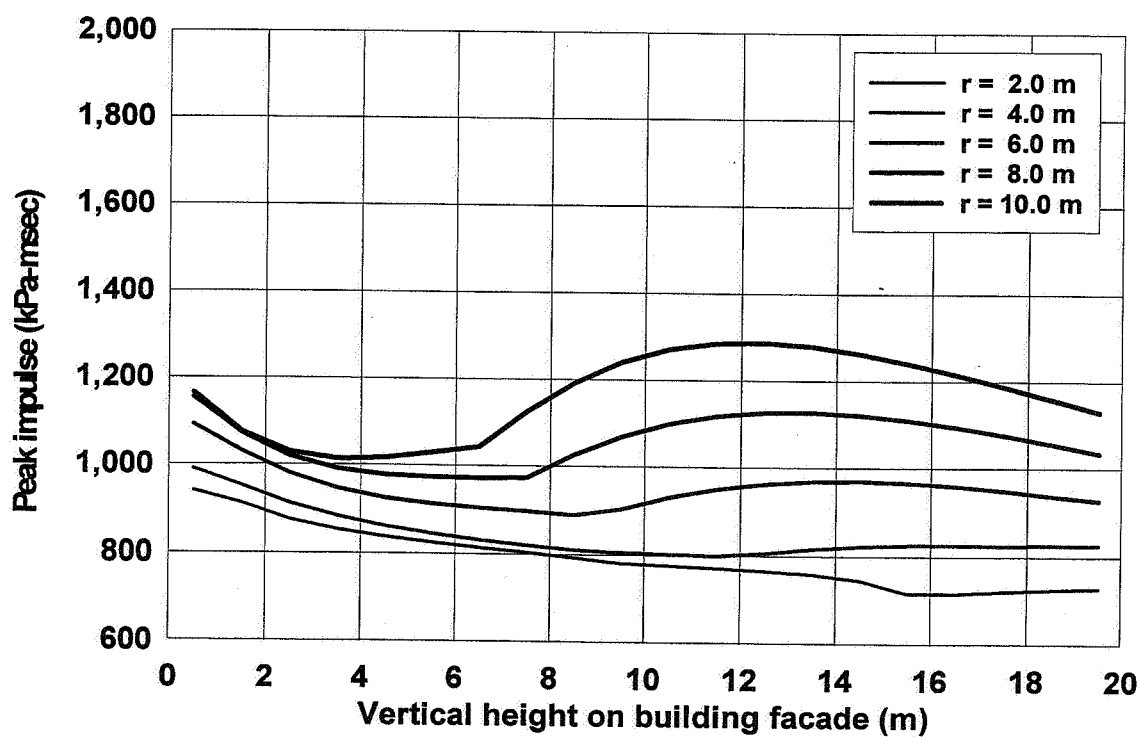


Figure 7.76: Impulse, variation of charge stand-off, wall height $H = 4.0$ m

7.3.2 The Influence of Wall Height on Blast Wall Effectiveness

In many respects, the influence of wall height on blast wall effectiveness is much easier to appreciate than charge stand-off. In this section, the same *with wall* data is again presented in graphs of peak pressure and impulse plotted against vertical height on the building façade (Figures 7.77 to 7.86), but each graph is for a fixed charge stand-off and contains four data sets: one for each blast wall height. From these graphs, it is easy to appreciate the influence that the different walls have, and the trend is obvious: the higher the wall the more effective it is.

There is one distinctive feature of the graphs which should be noted. The region of all the graphs (pressure *and* impulse) representing large vertical distances up the façade of the building (the portion on the right of the graphs) shows that the four lines seem to be converging to a single value. This would be expected because at some (unknown) height above the ground the influence of the blast wall will cease.

Until this point, *effectiveness* has been discussed in broad terms and qualitative observations have been made. It is now appropriate to consider effectiveness in quantitative terms. This is where the information from the *no wall* analysis becomes of use. The *no wall* data represents a base line: the worst case scenario. Comparison with *no wall* data provides a meaningful and practical measure of blast wall effectiveness. The *no wall* peak pressure and impulse data are shown in Figures 7.87 and 7.88, which require a brief explanation.

The (scaled) height of the explosive above the ground ($0.126 \text{ m/kg}^{1/3}$) is very small, so the blast wave has not formed fully when it first impinges on the ground. This is best described with reference to the arguments in Section 3.5. Following the description of Figure 3.5, the initial expansion will not have reached the centre of the charge (the detonation point) before the outer compression wave reaches the ground. Therefore, the pressure, density and internal energy of the air representing the detonation products will not be significantly reduced. The geometry is not, therefore, that of a typical height of burst (described in Chapter 4); it is more like that of a hemispherical surface charge but with an uneven distribution of pressure and positive phase duration. This broadly hemispherical blast wave can be divided into three regions:

- i. Region one is the uppermost region of the blast wave, and it is formed from the initial compression wave in the same manner as a hemispherical blast. It is the only part of the blast wave not affected by the reflection from the ground and is indicated in the pressure contour plots, Figures 7.89 to 7.91. Initially, this region covers nearly all the hemisphere of the outer compression wave (Figure 7.89). However, at later times the second region extends upwards, and region one reduces and eventually disappears. This can be seen in the progression from Figure 7.89, through 7.90, to 7.91.
- ii. The second and third regions arise from the same process: the reflection of the partially-formed incident wave from the ground and its propagation through, and reinforcement from, the remaining high pressure, density and energy detonation products (air) in contact with the ground.

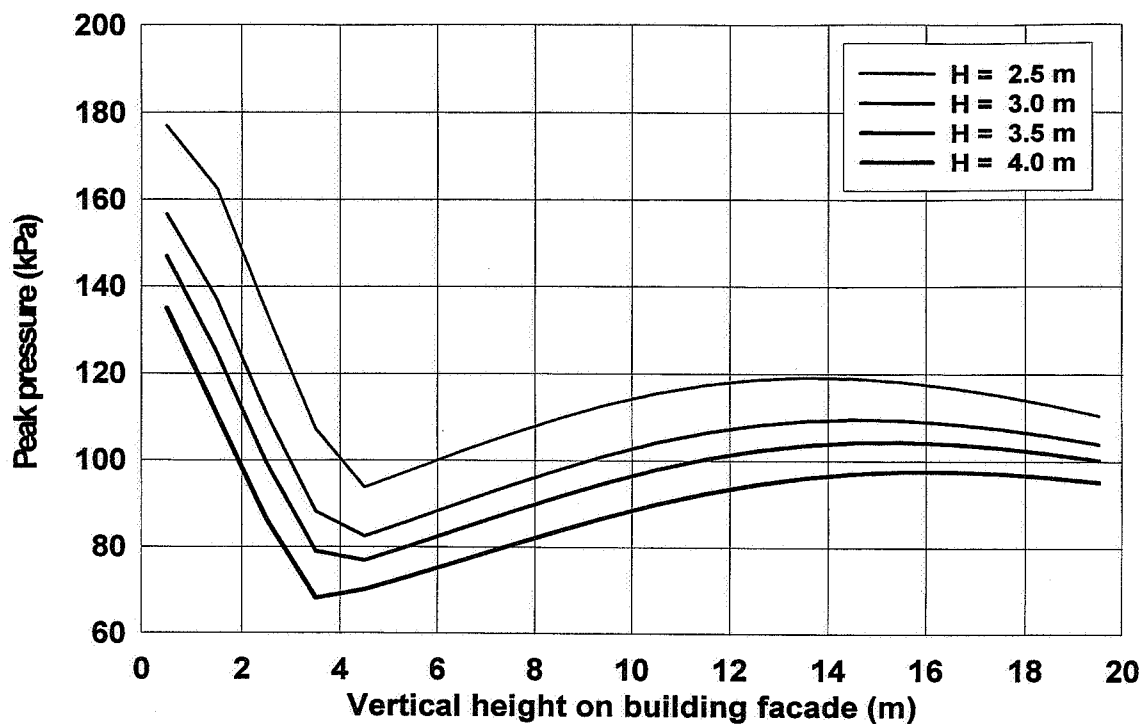


Figure 7.77: Pressure, variation of wall height, $r = 2.0$ m

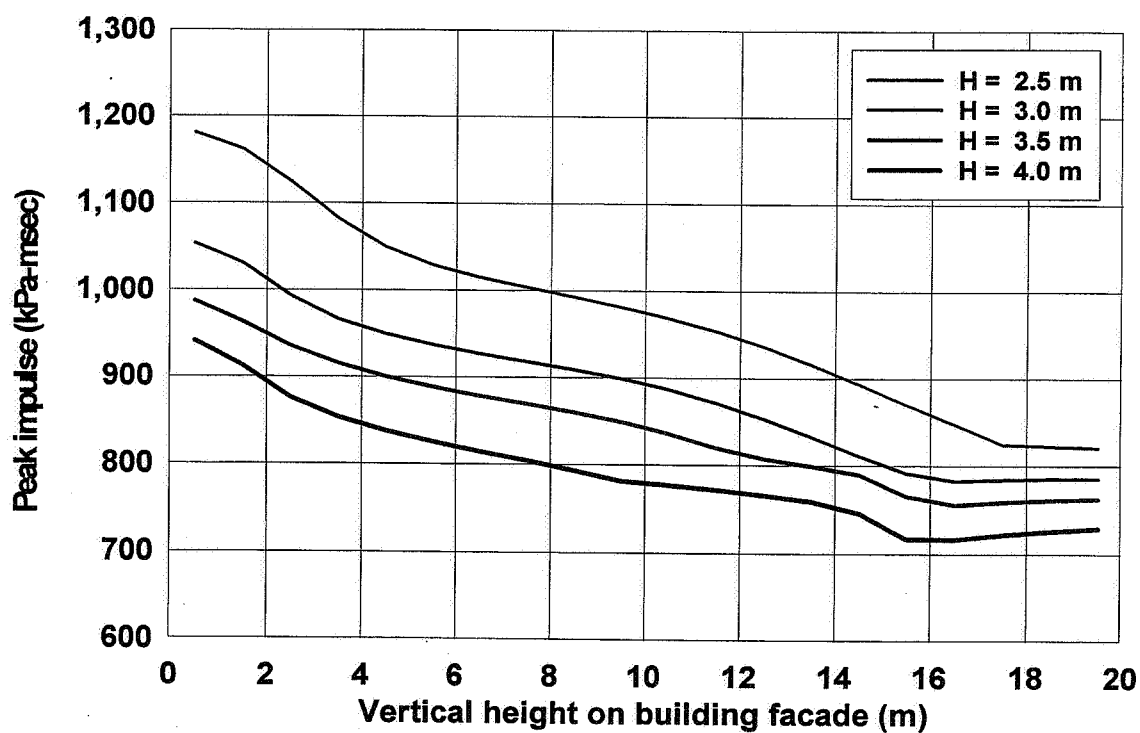


Figure 7.78: Impulse, variation of wall height, $r = 2.0$ m

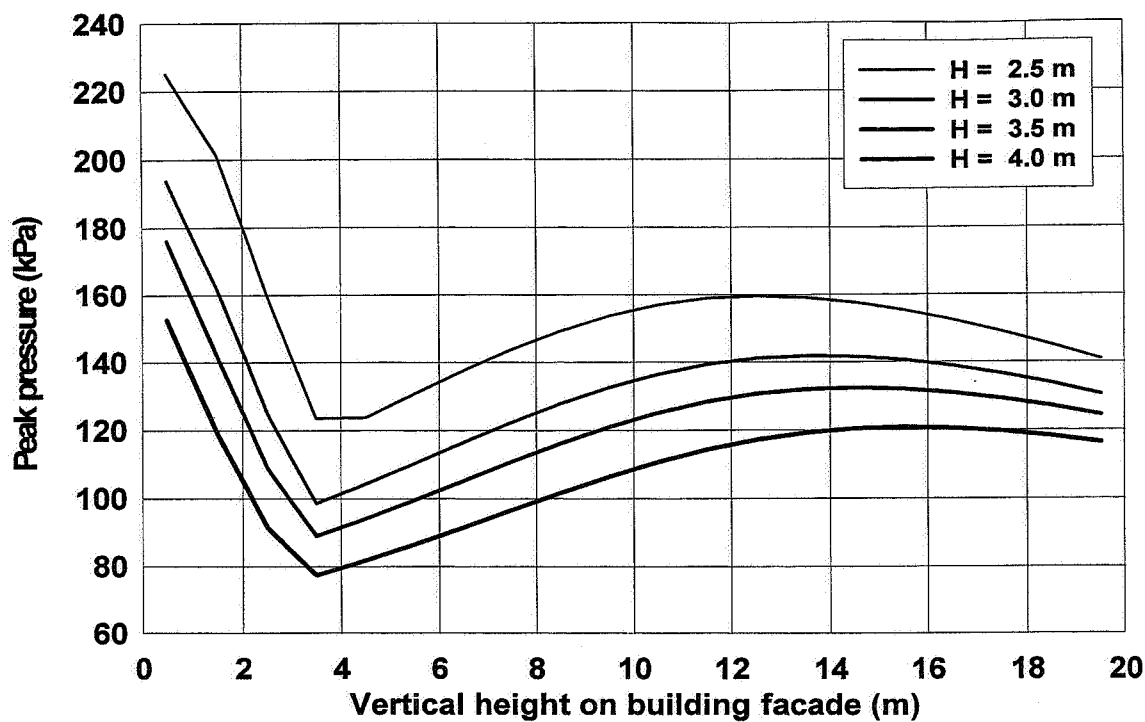


Figure 7.79: Pressure, variation of wall height, $r = 4.0$ m

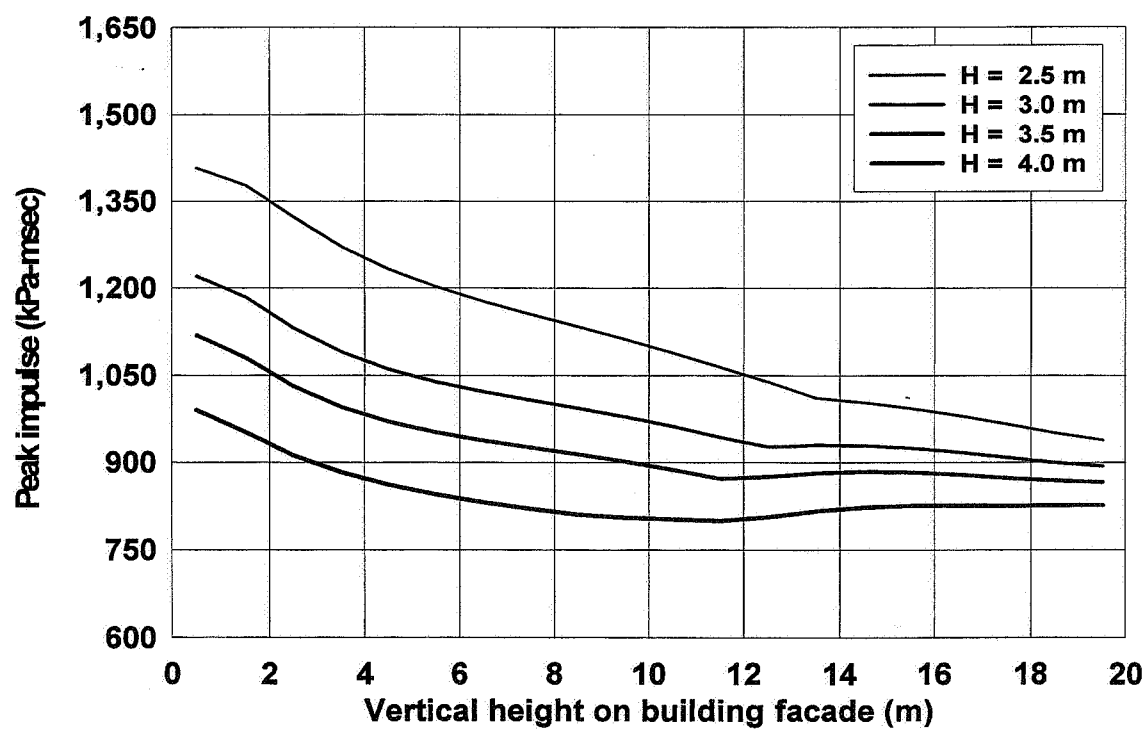


Figure 7.80: Impulse, variation of wall height, $r = 4.0$ m

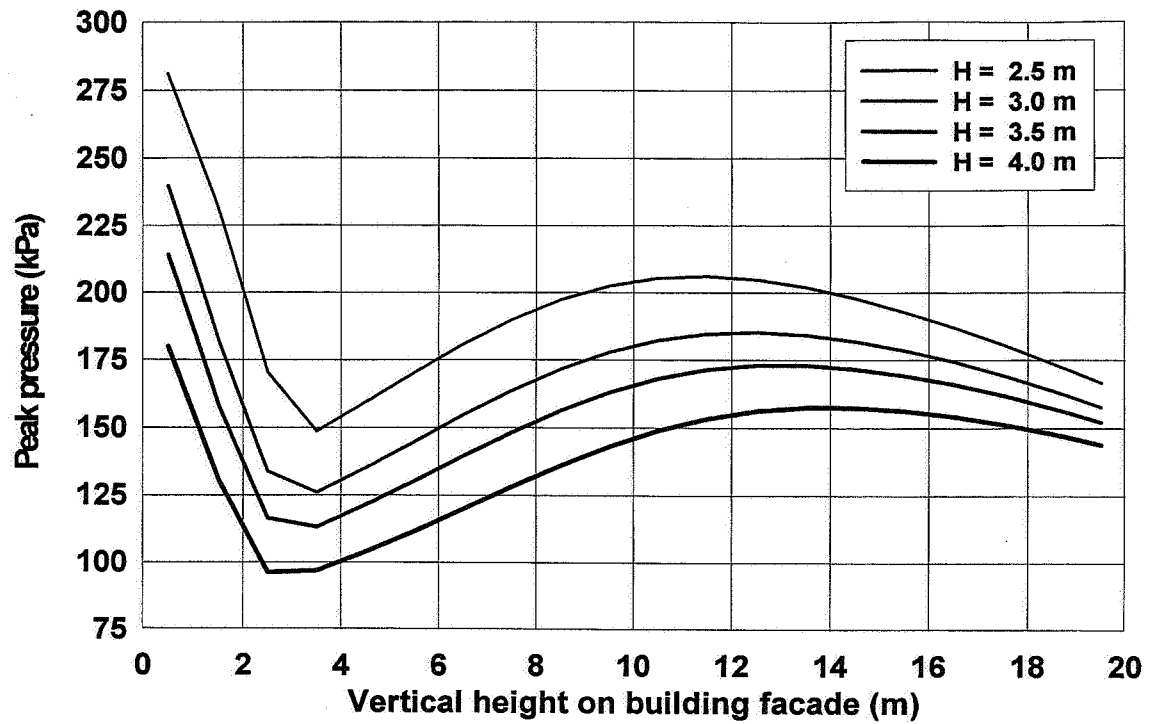


Figure 7.81: Pressure, variation of wall height, $r = 6.0$ m

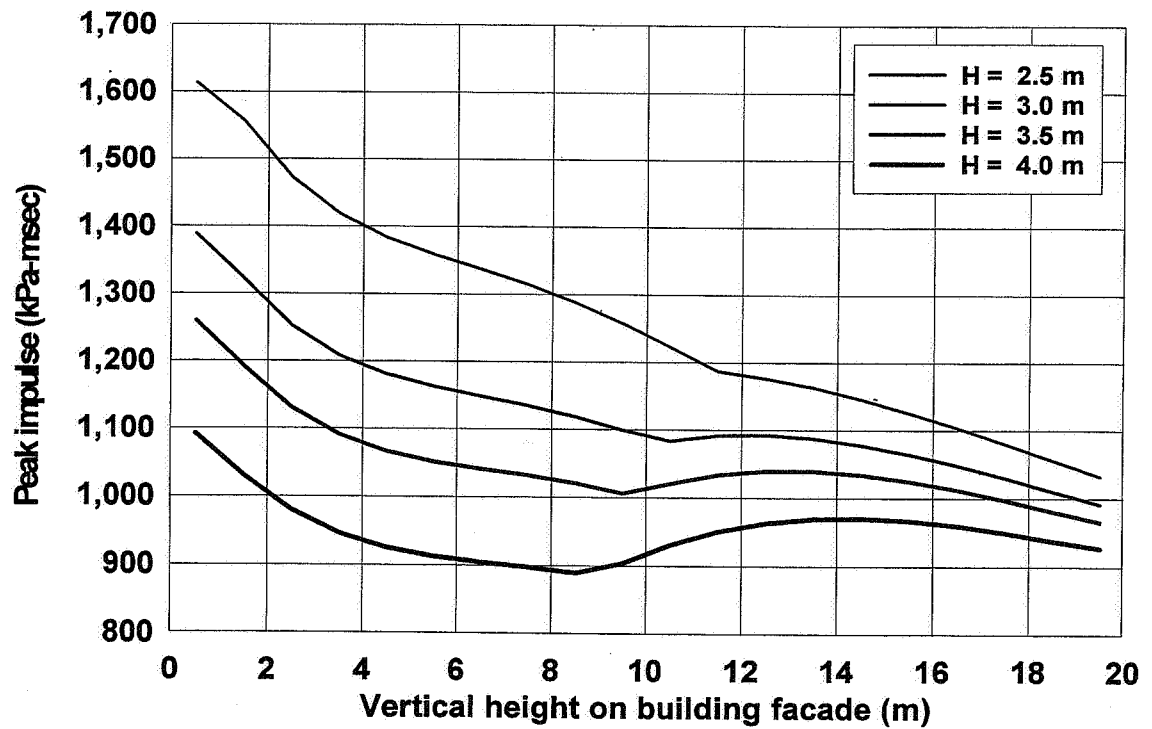


Figure 7.82: Impulse, variation of wall height, $r = 6.0$ m

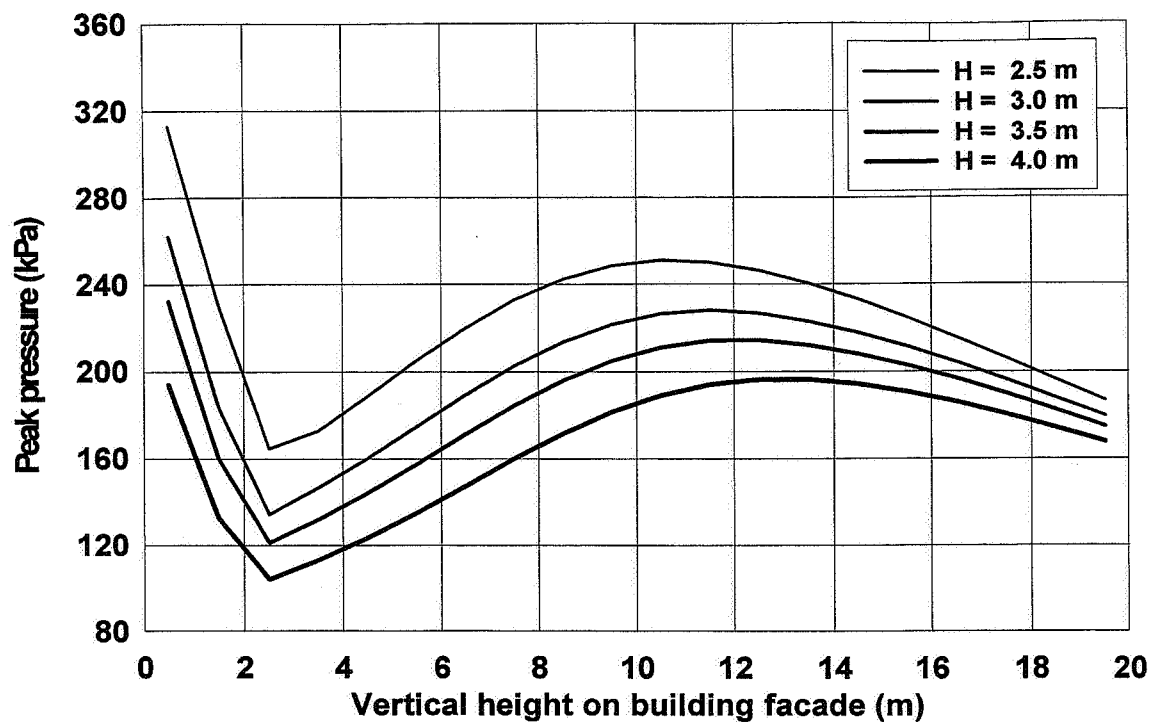


Figure 7.83: Pressure, variation of wall height, $r = 8.0$ m

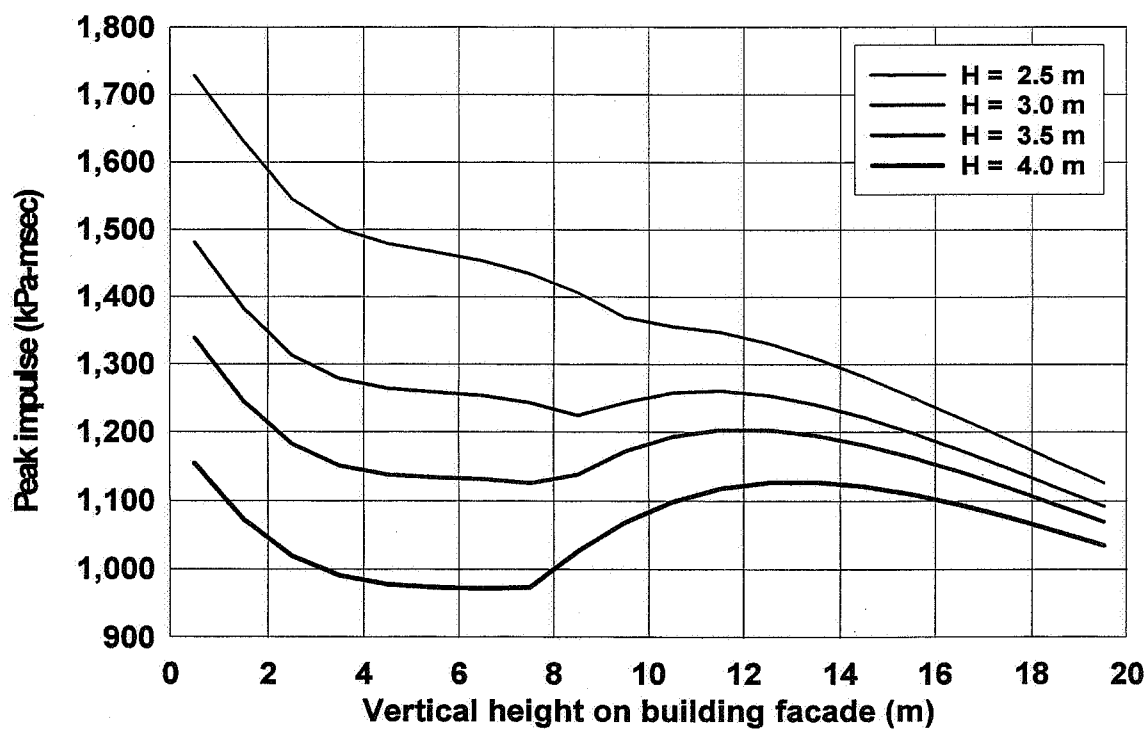


Figure 7.84: Impulse, variation of wall height, $r = 8.0$ m

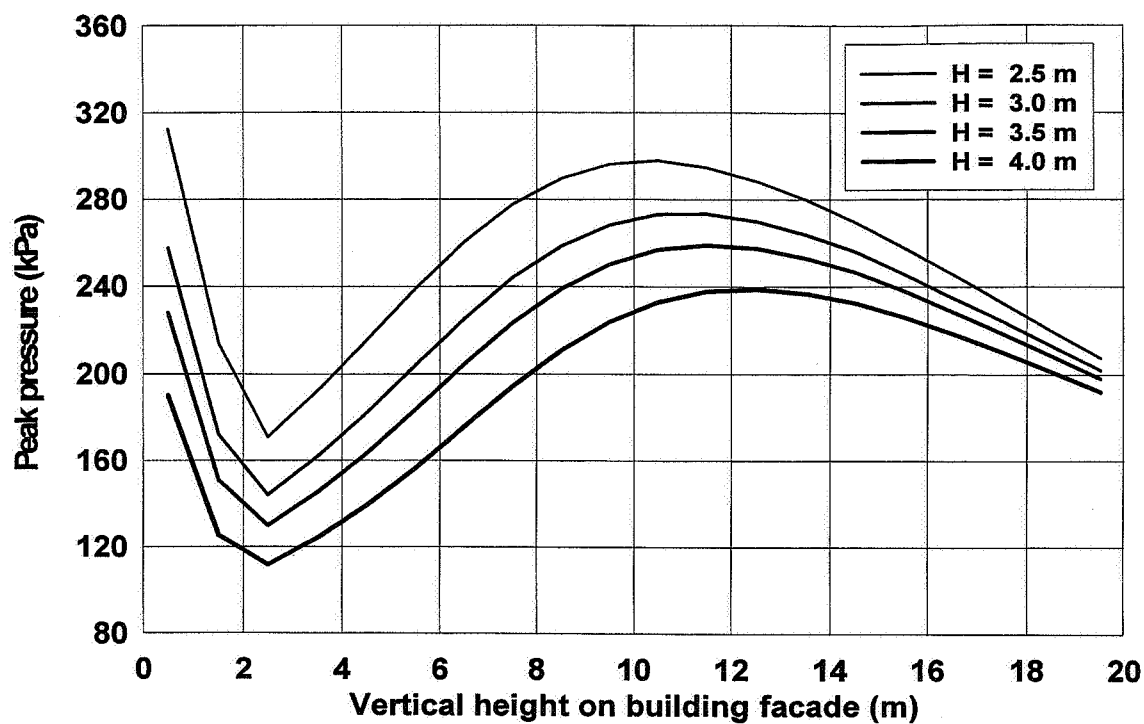


Figure 7.85: Pressure, variation of wall height, $r = 10.0$ m

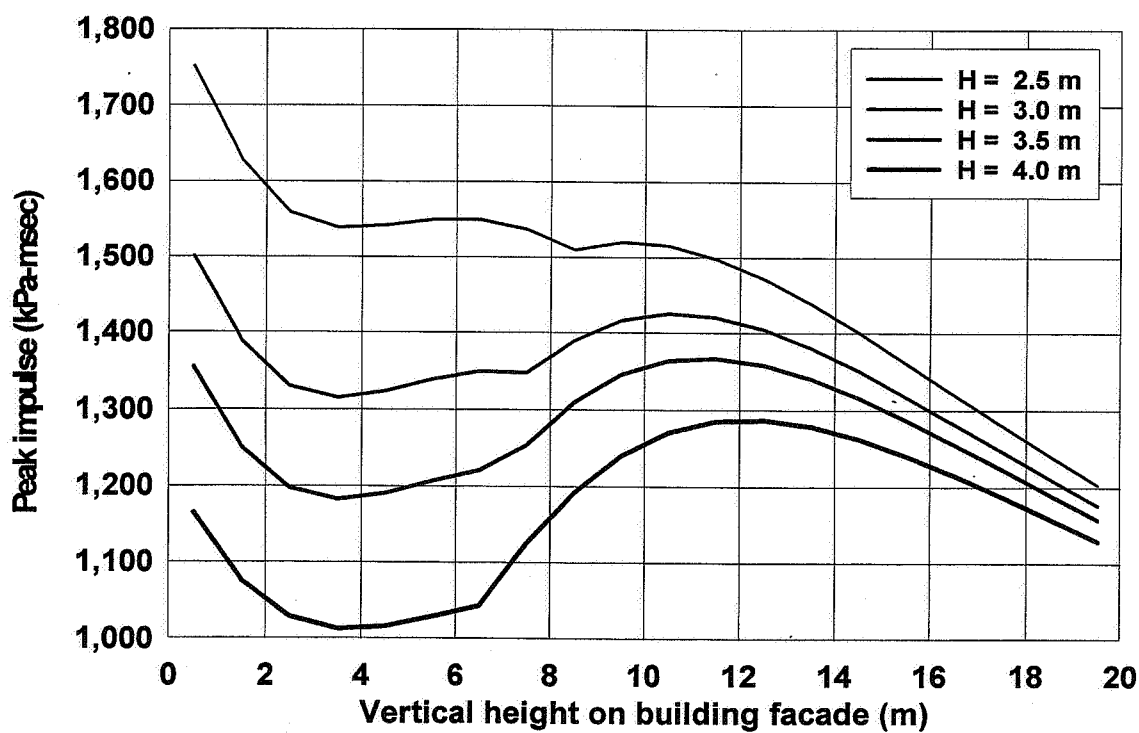


Figure 7.86: Impulse, variation of wall height, $r = 10.0$ m

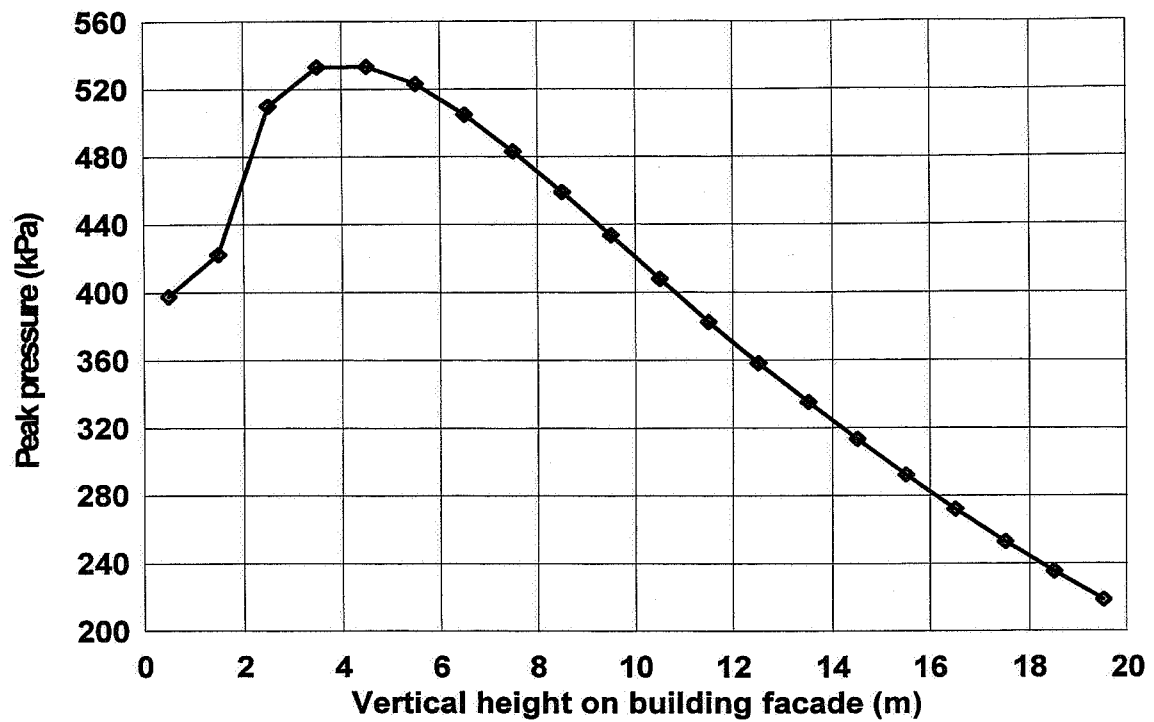


Figure 7.87: No wall peak pressure distribution

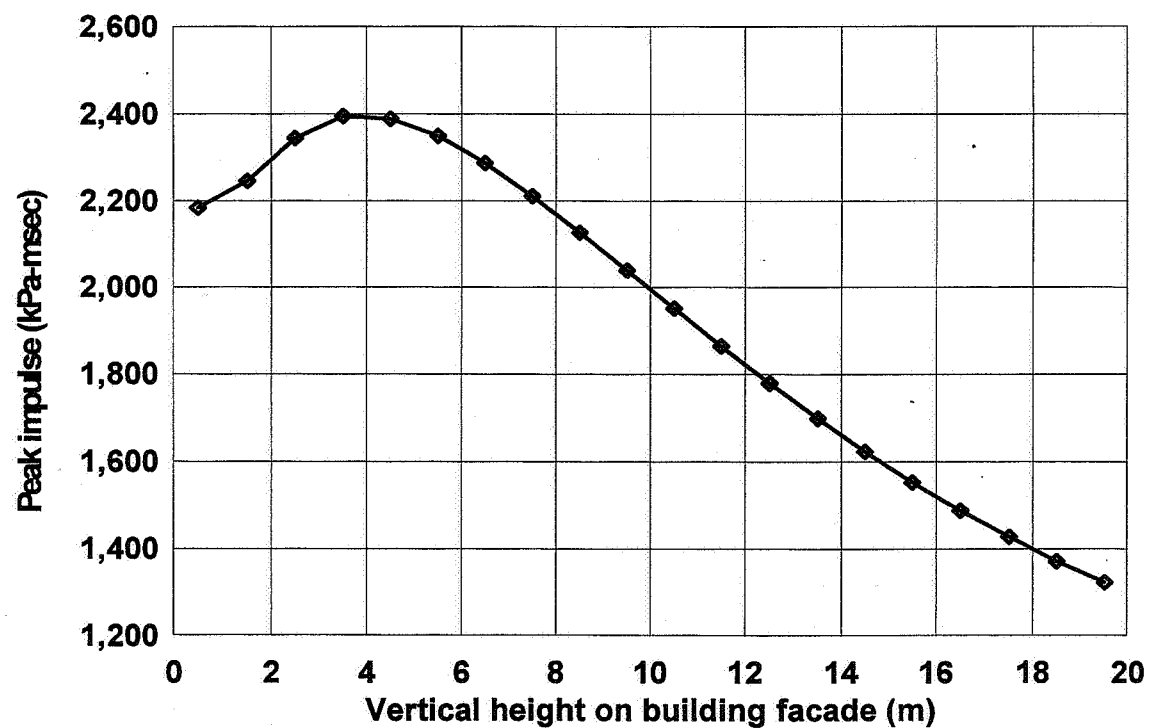


Figure 7.88: No wall peak impulse distribution

In the second region, this reflection strengthens the incident wave, giving rise to greatly increased pressures and durations, which are similar in magnitude throughout the region. This can be seen from Figure 7.91. It should be noted, however, that this is not a Mach stem, in the usual sense of the term, because the incident and reflected waves are never really distinct. Region two travels slightly faster than region one (because of its increased pressure). It starts very small (it is not discernible in Figure 7.89), as a layer of stationary, high pressure, high density air, in contact with the ground, and then extends upwards, still broadly hemispherical in shape, until it extends through region one (Figure 7.91) and eventually engulfs it.

- iii. The third region is the region near to the ground (Figure 7.89). It would not usually be present in a "normal" height of burst configuration. The part of the compression wave resulting from this layer with components of momentum acting upwards coalesces with the incident wave, forming region two. The horizontal part, however, quickly "shoots out" along the surface of the ground, ahead of regions one and two. This situation is not sustained, because the high pressure causes the region to spread upwards contributing to region two, as well as outwards along the ground. Eventually, the region weakens to such an extent that its peak pressure and accompanying impulse are less than region two (Figure 7.91).

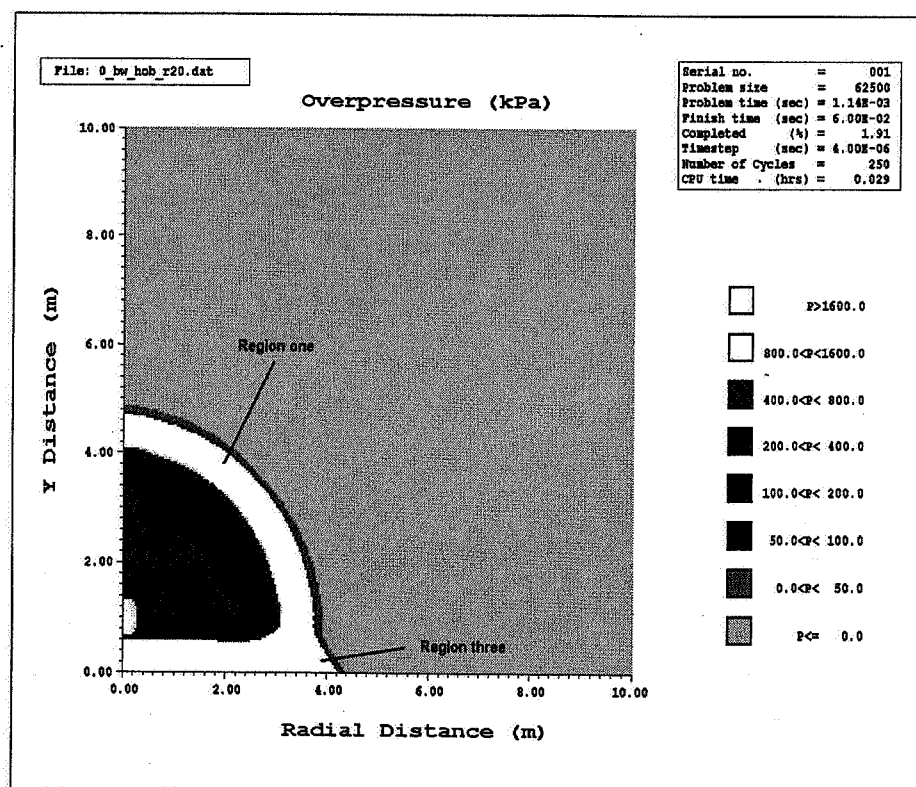


Figure 7.89: Pressure contours at $t = 1.14$ msec

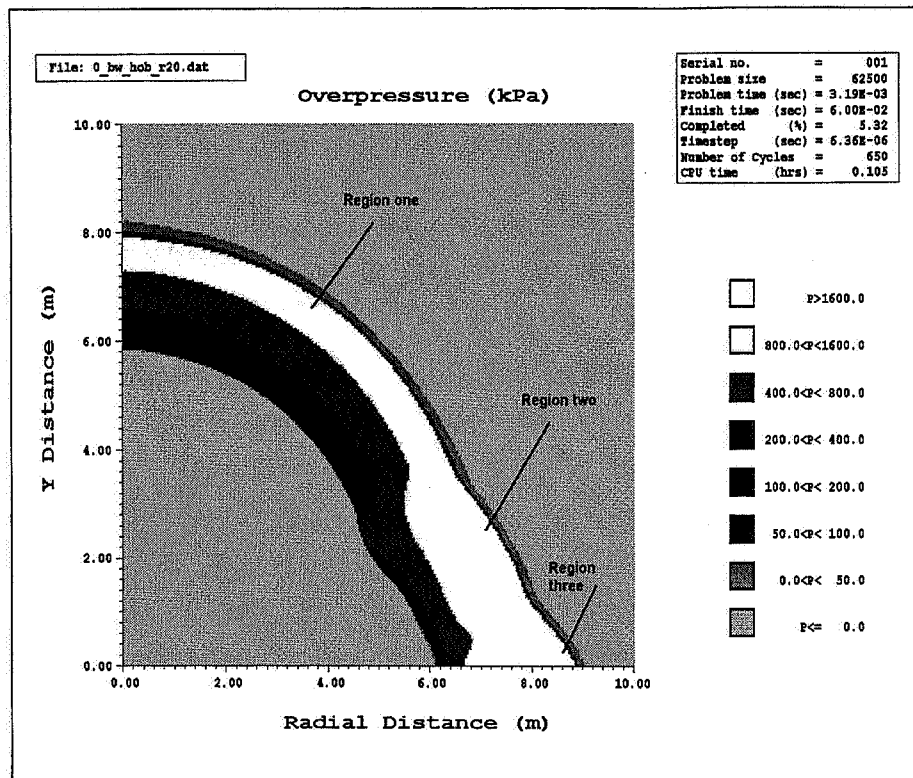


Figure 7.90: Pressure contours at $t = 3.19$ msec

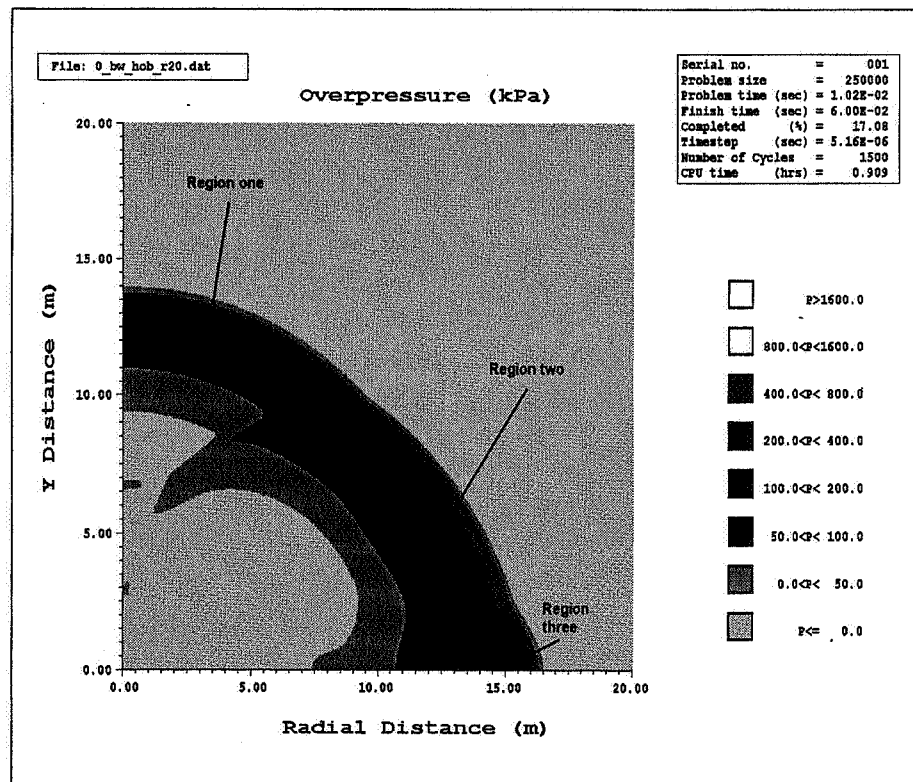


Figure 7.91: Pressure contours at $t = 10.2$ msec

Put simply, when the stand-off of the charge from the ground is very small, the resulting Mach stem is curved and has components of momentum in the upwards direction, away from the ground. After a short distance, this causes a reduction in peak pressure (and hence impulse) in a small layer near the ground surface, as the Mach stem "lifts off".

The above explanation is based on observations from the graphical output of the program Air3d; it is an explanation of the shape of the curves in Figures 7.87 and 7.88. Although the approach has been validated, to a large extent, by the simulations described in Section 7.2.1, it will be recognised that the *actual* situation may differ from the *idealisation*, described above, in a some respects.

Figures 7.87 and 7.88 can now be explained in terms of the above description. The part of the curves at vertical heights greater than 4.0 m are all in region two; region one passes above the top of the monitoring locations in this configuration. Because the blast wave formed in region two is broadly hemispherical in shape, monitoring locations at vertical heights greater than 1.0 m above the ground are also actually further away from the charge. Similarly, they are also at a greater angle of incidence. Both these facts contribute to the gradual reduction in pressure and impulse described by the part of the curves from 4.0 m to 19.5 m in Figures 7.87 and 7.88.

The part of the curves below 4.0 m are in region three, close to the ground. It can be seen from Figures 7.87 and 7.88 that in this region the blast wave has weakened considerably, in terms of peak pressure and impulse, compared with region two. Clearly, this part of the curve would not be present in a "normal" height of burst configuration, where it would be expected that the curves would continue to rise, indicating maximum pressures and impulses at the junction of the façade and the ground.

In Figures 7.92 to 7.101, the graphs of Figures 7.77 to 7.86 are reproduced in terms of the percentage of *no wall* data. In other words, the ratio (in terms of a percentage) of the *with wall* value to the that with *no wall*. These *percentage* graphs provide a tangible means of assessing the effectiveness of the wall in the present abstract scenario.

Considering the two extreme cases: a 4.0 m high wall at 2.0 m stand-off (Figures 7.92 and 7.97) and a 2.5 m high wall at 10.0 m stand-off (Figures 7.96 and 7.101), the following tentative conclusions can be drawn:

- The percentage of peak *no wall* pressure on the façade of the building behind the 4.0 m high wall 2.0 m from the charge varied between 13% and 44%. For the 2.5 m wall at 10 m stand-off it varied from 34% to 95%. In broad terms. The first scenario showed an improvement in effectiveness of about 35% over the second.
- The percentage of peak *no wall* impulse on the façade of the building behind the 4.0 m high wall 2.0 m from the charge varied between 35% and 55%. Behind the 2.5 m wall at 10.0 m stand-off the figures were 64% to 91%. Again, in broad terms, the larger wall closer to the charge was approximately 30% more effective.

Clearly, the results of this case study cannot be translated to the general case, but it does demonstrate the process by which such a study could be undertaken. It is suggested that, at the present time, this case study could not have been undertaken with confidence using information currently available.

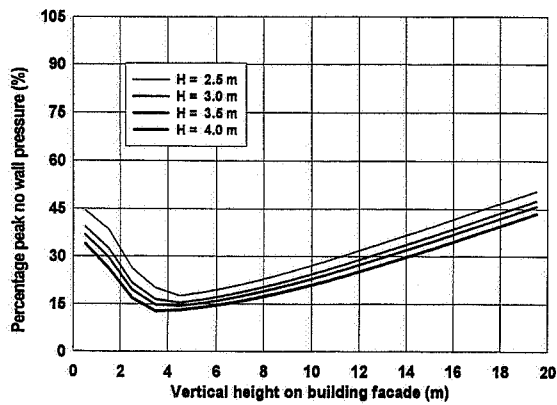


Figure 7.92: Effectiveness, pressure, stand-off $r = 2.0$ m

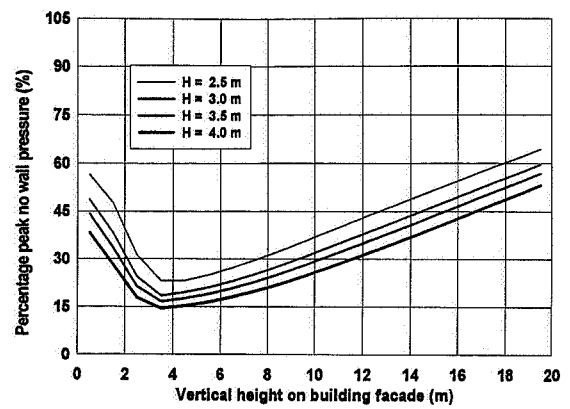


Figure 7.93: Effectiveness, pressure, stand-off $r = 4.0$ m

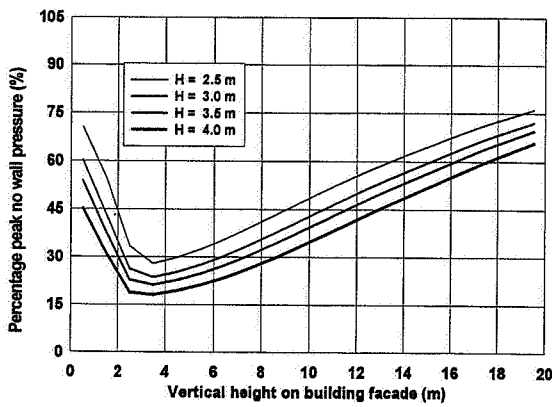


Figure 7.94: Effectiveness, pressure, stand-off $r = 6.0$ m

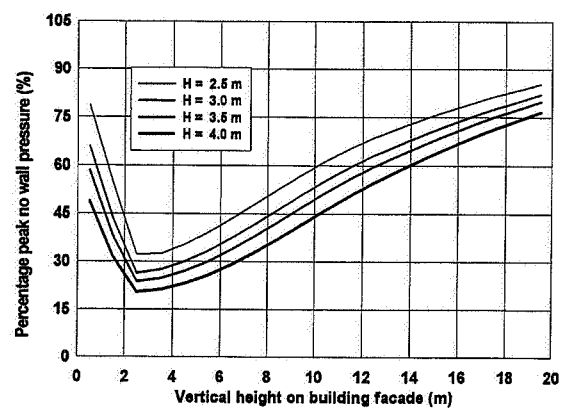


Figure 7.95: Effectiveness, pressure, stand-off $r = 8.0$ m

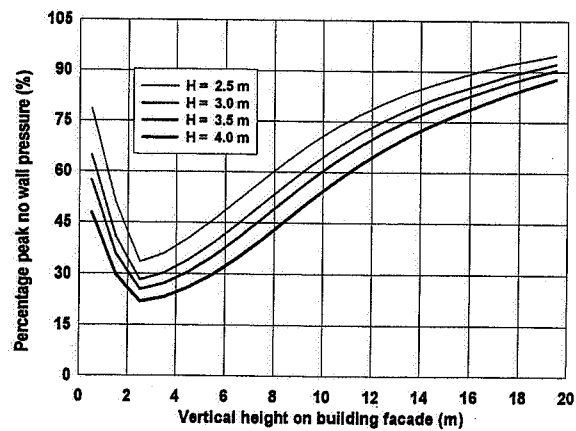


Figure 7.96: Effectiveness, pressure, stand-off $r = 10.0$ m

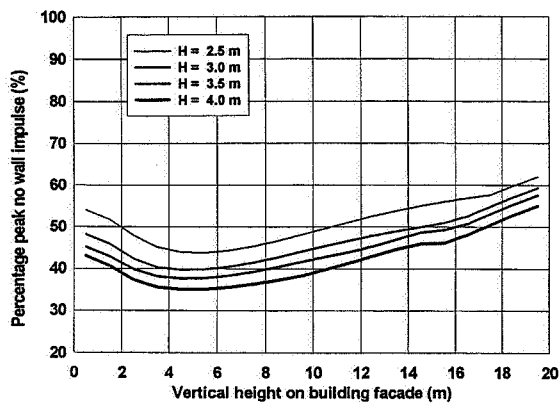


Figure 7.97: Effectiveness, impulse, stand-off $r = 2.0$ m

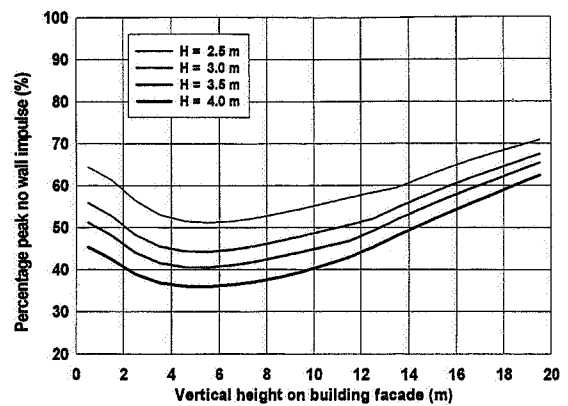


Figure 7.98: Effectiveness, impulse, stand-off $r = 4.0$ m

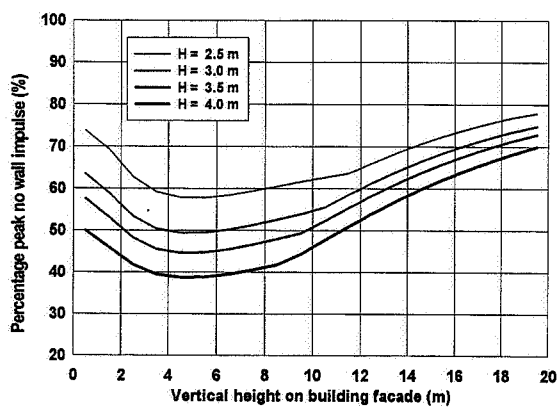


Figure 7.99: Effectiveness, impulse, stand-off $r = 6.0$ m

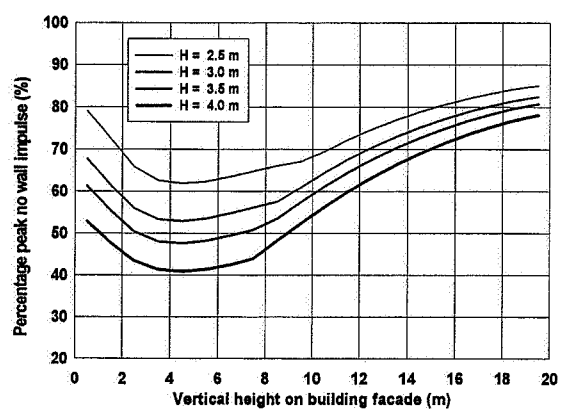


Figure 7.100: Effectiveness, impulse, stand-off $r = 8.0$ m

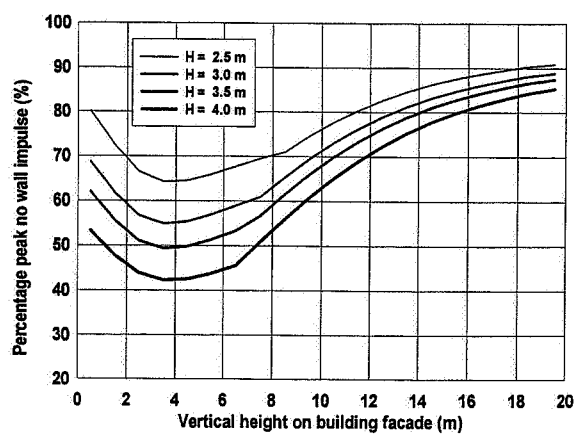


Figure 7.101: Effectiveness, impulse, stand-off $r = 10.0$ m

7.4 A Procedure to Predict the Loads on Structures Behind Blast Walls

It was mentioned previously that the range of the parameters which describe a blast wall/protected structure scenario is limitless. If further variables, like the slope of the ground, the presence of ditches or earth berms, or architectural features of the protected structure are considered, the task of describing the problem becomes increasingly complicated. In most situations, however, an answer in broad terms is all that is required, and how to achieve that broad answer is the subject of this short discussion.

The starting point of any given scenario is the geometry, in terms of the building and wall dimensions, the stand-off of the threat from the wall/building, and the threat itself, in the form of a TNT equivalent charge. With this information it is possible to establish all the relevant scaled parameters needed to describe the problem completely.

The most obvious approach to the problem is to use existing, easily accessible information to act as a link to the desired result. In the *case study* example described above, *no wall* (height of burst) data was used as an upper bound for the load on the building façade. This data, in the form of average impulse, could be found using the blast effects program ConWep [24] or the procedures described in Chapter 5. This, in turn, could be used with effectiveness graphs such as those presented in the *case study* (Figures 7.97 to 7.101) to establish a reduced average impulse value. Clearly, a similar procedure could be performed for pressure, although pressure was not considered in Chapter 5 and was not described in detail in the present section.

The problem with this proposed approach is the number of scenarios to be considered. The four scaled wall heights used in the *case study* are typical in terms of current threats. The stand-off distance of the building from the wall could be examined in relatively large intervals, 5 m for example, from 5 m to 50 m, giving 10 stand-offs. These could be converted to appropriate scaled distances. The stand-off of the charge from the wall could be varied from 2 m to 20 m (it can be assumed that there is little point in considering further distances) in intervals of 2 m, giving 10 wall placements. Again, these could be converted to appropriate scaled intervals. Finally, the whole matrix of scenarios would require about 400 analyses.

Once this information was established, a method of interpolation, similar to that described in Chapter 5, could be introduced to extract the *effectiveness coefficients* from the data set, and the required answer would result.

The size of this undertaking is probably prohibitive at present. Similarly, the inclusion of one or two further parameters, like those mentioned above, would increase the size of the test matrix beyond hope of completion. Despite these misgivings, it is more than likely that with ever increasing computing power such an undertaking will one day become viable.

7.5 Comments on the Effectiveness of Blast Walls

The structures considered in this chapter (both the blast walls and the building behind the wall) were semi-infinite structures: they were effectively infinite in the *out-of-plane* direction. This greatly simplified the problems considered.

Despite this simplification, analysis of the blast wall scenario has proved particularly difficult to achieve satisfactorily. The main problem arises from the fact that, in order to be effective, the blast wall is necessarily close to the charge. This, together with the fact that the charge was generally only a short distance above the ground, meant that one- and two-dimensional parts of the analyses tended to cover only a very small region of space. This, in turn, implied that virtually the whole of the analyses were performed in three-dimensions, with obvious implications for the spatial discretisation and running time.

Because of this difficulty, it was considered necessary to present the results of a two-dimensional height of burst calculation before the main validation examples. This demonstrated the level of accuracy (in terms of reproduction of experimental results) achievable for the simpler underlying *no wall* problem. Similarly, it was also considered necessary to supplement the validation examples with a further example using a larger stand-off distance of the charge from the wall. This demonstrated the nature of the difficulty of modelling blast wave diffraction when it occurs at short scaled distances.

Because of the modelling difficulties, it was not possible to extend the experimental data of Rose et al. [43]—which was the original intention. Instead, an investigation of the effect of wall height and charge stand-off for a particular scenario was performed. The results of this *case study* were, perhaps, not surprising, but the main goal was to demonstrate the trends in a quantifiable manner. The results were as follows:

- In order to mitigate the effect of blast on the building most effectively, it would be necessary to use the highest wall ($H = 4.0$ m) at the minimum stand-off distance ($r = 2.0$ m) from the location of threat.
- This configuration resulted in a reduction in pressure across the building that varied between 88% and 56% and a reduction in impulse, between 65% and 43%, compared with the *no wall* scenario.

It should be recognised that the second of these conclusions is not presently achievable using existing empirical or semi-empirical predictive techniques or guidance documents.

The methodology used in the *case study* led to the possibility of using a systematic approach to describe the blast wall problem for a complete range of scaled geometrical parameters. Clearly, such an exercise would be very computationally demanding, but it could be achievable in the future.

Finally, it is concluded that, of the three relatively simple geometries considered in this study, the present geometry is the most demanding and least successful to analyse with the chosen algorithm and available resources. It is likely that as solution techniques and computer hardware improve, reliable answers to the blast wall problem will become increasingly approachable.

Chapter 8

Concluding Remarks and Future Developments

8.1 General Approach

This thesis aimed to demonstrate that the use of modern computational methods, as incorporated into the tool Air3d, are sufficiently mature to begin to approach the problem of blast load evaluation in geometries of increasing complexity. This was demonstrated for free-air and height of burst in Chapter 4, loads on solitary structures in Chapter 5 and, to a lesser extent, the blast environment in a built-up area and behind protective walls in Chapters 6 and 7.

In Chapter 5, application of the technique led to a method of prediction of average impulse on the front face of solitary structures which, it is tentatively suggested, is superior to current guidance.

In Chapter 6, a large number of analyses were performed varying the parameters which define the geometry of a straight (symmetrical) city street. The conclusions of that exercise are not echoed in current guidance documents.

In Chapter 7, a hypothetical study was undertaken, and broad conclusions reached, concerning the protective capability of various perimeter walls. At present, such an exercise is not possible using current guidance alone.

It is, therefore, suggested that this computational approach has the possibility to expand greatly areas of interest in blast loading which were previously ill-defined or else not described at all.

A synopsis of the method proposed by this study is as follows:

- Description of the problem in terms of the scope of relevant scaled geometrical parameters.
- Physical experiments to identify principal elements of the loading: number and magnitude of reflections/expansions.
- Validation of the numerical method by comparison with experiment.
- Development and execution of a numerically-based *experimental* matrix to vary all the relevant parameters identified above.

- Collation and presentation of results to allow interpolation and/or extrapolation in a meaningful and justifiable way.

It will be noticed in the above description that the need for physical modelling (whether at full or small scale) is still recognised as an essential part of the process. It is suggested that numerical modelling is ideal for filling-in the gaps between physical experiments, but the need for experiments, although expected to diminish, is unlikely to disappear completely.

8.2 Air3d

Background

Development of the program Air3d was the basis of this study. It was a prime objective to produce a tool which was capable of modelling the various geometries of interest, but to do so in the simplest possible manner.

For this reason, the program Air3d uses only regular Cartesian grids and hence cubic computational cells. Strictly speaking, such an approach is very wasteful and not well suited to the present type of problem, where moving shocks travel through the static grid. The requirement of good spatial resolution through the shock waves implies that the whole of the computational domain should be finely discretised, with obvious implications for memory use and CPU time.

Recent advances in computer hardware, however, have tended to lessen the impact of these deficiencies by allowing relatively large problems to be treated by Personal Computers in a few tens of hours. For this reason, the simplistic approach of the program Air3d has still been able to produce useful results in all the main subject areas.

The solution algorithm of the program Air3d is a combination of the AUSMDV [56] flow solver and MUSCL-Hancock integration [52]. This is essentially the combination of two computationally cheap methods to obtain one of only moderate expense. The flow solver, AUSMDV, is a variant of an earlier scheme, AUSM [33] and uses a vector of convected terms, associated with a split mass flux, while the pressure flux terms are split using a suitably defined interface Mach number. It is feasible to treat the terms separately in this manner, as they are recombined at the end of each timestep in the formation of the new conserved variables. The method is extremely robust, which is of particularly important for blast load calculations, where enormous discontinuities in pressure, density and internal energy exist at early stages of a calculation. The timestepping procedure of the MUSCL-Hancock integration scheme is computationally cheaper than other second-order methods, because it avoids the need to store half-timestep conserved variables. It achieves this by evolving the half-timestep solution *within* each cell, based on fluxes calculated from the projected interface values. This procedure is not conservative, but this does not matter because the resulting updated interface values are only used to calculate full-timestep numerical fluxes, ensuring that the overall scheme *is* conservative. The integration scheme is used with the minmod slope limiter to impose Total Variation Diminishing conditions on the conserved variables. Although the minmod limiter

diffuses discontinuities to a greater extent than other limiters it does ensure that the blast wave profile remains reasonably unaltered. This last observation is particularly important because the overcompressive nature of virtually all other limiters can have a drastic effect on the blast wave profile; they tend to turn blast waves into square waves. It is believed that Air3d is the first airblast tool to make use of this particular combination of techniques.

Isothermal Bursting Sphere Simplification

One of the main simplifications employed in the development of Air3d was the use of an isothermal sphere of air to model the explosive charge. This sphere has the correct density for TNT; it also contains the correct chemical energy (in the form of heat), but *it is still modelled as air*.

The usual approach to this problem is to treat the detonation products as a separate material, and then add the energy into the products as a function of the problem time and the detonation wave speed. Other, more sophisticated, approaches also exist.

For the purpose of the present study, however, where the primary interest is in blast waves relatively remote from the charge surface, the isothermal sphere simplification has not affected the results of the simulations adversely. It can be seen from Figure 4.2, for example, that good correlation with published data can be achieved with this method over a relatively large scaled range: $0.3\text{--}30\text{ m/kg}^{1/3}$.

It should be borne in mind that from the moment the detonation wave reaches the edge of the explosive charge the shock front propagates through air. Similarly, from a scaled distance of about $1.5\text{ m/kg}^{1/3}$ the whole of the positive phase of the blast wave is *outside* the radius of the detonation products and travels exclusively through air. The fact that the second shock never catches the first implies that the treatment of the detonation products after this point is of little consequence. In other words, the description of the blast wave in air is by far the most important aspect of the simulation; treatment of the explosive products is of secondary importance.

One other point concerning the isothermal sphere technique, observed in many lengthy preliminary trials (though not reported here), is that the blast parameters (peak pressure and impulse) remote from the charge are remarkably insensitive to the starting conditions inside the isothermal sphere. It appears that so long as the correct amount of material (air) and energy are deposited in the sphere, the problem *settles down* to a broadly similar state after a short scaled distance. This has proved to be a helpful, although somewhat unexpected, observation.

Perhaps the most important deficiency of the isothermal sphere simplification is the fact that it does not treat the chemistry of the detonation (as well as the physics of the gas effects) correctly. This means that it is likely to be difficult to use the code to simulate explosive events caused by the type of homemade or improvised explosives used in many significant explosive events. This possible deficiency must be addressed at some point in the future.

A second, less severe, deficiency arises from the observation in Chapter 5 that the shape and arrival of the second shock wave did not match very well with experiments. This may have been due to the fact that a compromise was used: the charge was

modelled as a hemispherical surface burst. It could also have been due to the modelling of the detonation products as air. This is something which is probably not of interest to the structural engineer, but it should also be addressed in future development of the code.

TNT Characterisation and Equivalence

Throughout this study, the problem of TNT parameters and equivalence has been largely ignored. The values of equivalence used for the validation problems were the same as in the experiments; the TNT parameters, density and mass specific energy, were from Baker, et al. [4]. This was the limit of the current treatment.

The key indicator of the correct choice of TNT parameters and equivalence is the comparison of numerical impulse with published data or the results of experiments. In Chapter 4 the results of an extensive one-dimensional analysis were compared with the data of Kingery and Bulmash [28] (Figure 4.2); these are probably the most widely used and respected source of blast data at the present time. The comparison showed that the analysis, based on a TNT equivalence of one, produced impulses consistently slightly below the published values. The implication of this comparison is that the data of Baker, et al. [4] is slightly in error and could be adjusted so that complete correspondence between analysis and the published data was achieved.

The problem is further complicated, however, by comparison of two-dimensional height of burst results with the published data of Swisdak [50]. There are three graphs in this series: Figure 4.11, Figure 4.15 and Figure 4.19. These indicate that the impulses calculated by the code Air3d are all slightly too high. There is the possibility that the quality of the reflecting surface in the experiments was not perfect (as it was in the analysis), but details of the experiments are not discussed by Swisdak [50]. The obvious conclusion from these two sets of analyses is that the most appropriate set of values for TNT parameters is still to be resolved.

In Chapters 5 and 7, the validation problems used a TNT equivalence of 1.32 to model the combination of Demex explosive and detonator. At the time the experiments were performed, this was considered to be the most appropriate value, although no reliable published data existed. The graphs of Chapter 5: Figure 5.12, Figure 5.13 and Figure 5.14, all tend to indicate that the combination of TNT parameters and equivalence was probably close to optimal. Similarly, the graphs which compare the experimental height of burst data of Chapter 7 with analysis: Figures 7.3 to 7.9, also show that the same good correspondence.

The experiments of Whalen [57], considered in Chapter 6, used 11.13 g of SX2 sheet explosive. The equivalence for the combined explosive and detonator was approximately 1.07, giving 13 g TNT in total. Graphs comparing experimental and numerical impulses: Figures 6.3 to 6.17, indicate that the equivalency may have been slightly overestimated; this is perhaps surprising because the actual explosive content of SX2 is 88% RDX, the same as Demex; the only difference is the binding material, which comprises the remaining 12%.

Finally, it has been observed by Swisdak [50] and others that TNT equivalency varies, not only from explosive to explosive but, with scaled distance. This observation is generally overlooked by most investigators because it adds a level of

complexity, to what are already very complicated problems, which makes the whole process of prototype or numerical modelling almost insoluble. Until this particular phenomenon is described more fully it is likely that it will continue to be overlooked.

Clearly, the problem of appropriate TNT parameters and equivalence will continue to complicate the applicability of experimentally- and numerically-based results for the foreseeable future.

The Remapping Procedure

The remapping procedure, described in Chapter 3, is sometimes referred to as *microzoning*. It is probably the simplest method of transferring information from one analysis to another, but it relies on the fact that the first part of the analysis (either one- or two-dimensional) has much finer discretisation than the three-dimensional part. Similarly, the region of the three-dimensional domain into which the information is remapped must be subdivided into a large number of cells (microzones) if a good approximation is to result.

It is recognised that more accurate algorithms exist to perform this procedure (Ramshaw [39], for example), and these will be considered in the near future.

Blast Wave Diffraction

In Chapter 4, an arbitrary distinction was made between two classes of three-dimensional problem: those where a relatively small region of scaled space was considered and those involving a much larger region. The first class of problem was treated in Chapter 5 and reasonable results were obtained. The analyses of Chapters 6 and 7, however, are more typical of the second class of problem. In Chapter 6, the effect was to limit the extent of the scaled space along the street, so that the applicability of the conclusions was limited. In Chapter 7, the problem of discretising a large region of scaled space was compounded by the nature of the problem considered: namely, diffraction through a large angle. The relatively poor resolution of many of the validation analyses in that section is evident.

The problem of resolving blast wave diffraction is further complicated by the fact that it is affected by the scaled distance at which it occurs. For example, the pressure histories monitored at the rear of the gauge block in the validation problems of Chapter 5: Figures 5.9, 5.10 and 5.9, demonstrate the trend towards a better description of diffraction as the scaled distance from the charge to the structure increases.

The key element is that, as the scaled distance increases the duration of the blast increases and the rate at which the pressure behind the shock (at any given point in the flow) decreases. This means that it behaves less like an unsupported wave and more like a supported one. In other words, it has a greater opportunity to diffract because the pressure persists at the point (or region) where the diffraction takes place.

Spatial Discretisation, Geometry and Displacement

Spatial discretisation is the single biggest obstacle to the resolution of shock waves in air; it is also the factor defining the accuracy of the representation of the problem geometry.

As mentioned previously, the program Air3d is based on a regular Cartesian grid: it uses cubic computational cells as its basic building blocks. In order to improve the accuracy of Air3d, in terms of shock wave and geometrical resolution, it would be necessary to increase the spatial discretisation (use more cells) in regions where shock waves and obstacles occur.

There are various ways in which this can be achieved. One is to use more powerful computers and parallel processing so that larger problems can be considered. A second approach is to *grade* the mesh, so that it has fine cells in the region of interest and larger ones in more remote areas, at transmissive domain boundaries, for example. A third method might be to use an unstructured mesh, not only to vary the mesh size but, to ensure that the geometry was modelled as closely as possible. Clearly, this would be much easier with an unstructured mesh than a structured one. Finally, another approach might be to continue to use a Cartesian grid but modify it with a technique of Adaptive Mesh Refinement (AMR). A thorough description of AMR can be found in Quirk [38], for example. One method which appears to be well suited to this particular problem is the Fully Threaded Tree (FTT) algorithm of Khokhlov [27]. This approach is based on a minimum discretisation level, which can be quite coarse, which is refined automatically by the creation and passage of the shock wave(s) through the domain. Similarly, the refined cells are automatically unrefined when the shock wave has passed.

It is clear that the program Air3d, although it has provided an indication of the efficacy of the numerical approach, is too simplistic for the treatment of large or excessively geometrically complicated problems. At the present time, the third of the above solutions, AMR, is favoured by the author as it should provide tremendous savings in run time and computer memory, while still preserving the same flow solver as presently implemented. Unfortunately, the change to an adaptive mesh system is nontrivial and will remain a long-term objective. In comparison, moving to a more powerful computer and making the few changes required to produce a parallel version of the code is relatively straightforward. This will be attempted in the near future.

This study has been concerned with the evaluation of blast loads on structures. Most of the effort, and all of the above discussion, has been aimed at increasing the accuracy of numerical blast simulations by improving the solver, increasing the resolution, using more powerful computers, and so on. Another important aspect of the problem which has been ignored is the change in geometry of a particular scenario if elements of the loaded structures are displaced.

Obvious examples are the cratering and massive destruction which occur in close proximity to a large explosive device in a city street. Another is the effect that extensive façade collapse and window breakage might have in the same scenario. Clearly, if the geometry of a scenario changes significantly during the *period of interest* it cannot be ignored in the analysis.

The implication of this statement in terms of program complexity are enormous.

If an obstacle deforms during the passage of the blast (the whole of the load history) the geometry of all the cells in contact with the obstacle change. This means that all the cell volumes change, together with the values of the conserved variables. Just as important is the fact that the response of the structure must be calculated accurately, which is a massive undertaking.

At the present time, the *coupling* of CFD and structural response calculations is not considered a high priority, although it remains a long term objective.

One treatment of structural response which is readily amenable to inclusion in Air3d in its present form is the introduction of primitive *window elements* which break instantaneously on reaching a prescribed pressure-impulse (P-I) criterion. Such elements could be easily defined as a single layer of cells which form a rectangular block. This layer could be monitored on both sides, and the pressure difference and accumulated impulse difference across the layer could form the basis of the test against a given P-I function.

Use of these window elements would allow analyses to be undertaken which give a broad quantitative evaluation of the likely hazard in urban geometries, for example. This will be implemented in the near future.

8.3 Blast Wave Clearing

The problem of blast wave clearing is not adequately described by current guidance. The description in TM 5-1300 [13], which is typical of all the guidance, is flawed in two significant respects: it does not allow the *cleared* pressure to reduce below the side-on pressure, and the *clearing time* is calculated from the sound speed of the side-on blast wave. Both of these assumptions have the effect of making the procedure conservative, tending to a higher average impulse.

The new clearing procedure, described in Chapter 5, sought to treat these difficulties by considering, and tabulating, results from a matrix of calculations which covered almost the whole range of practical interest. The resulting table of *clearing factors* is an implicit representation of every possible scenario, within the defined limits. The principal factor restricting the applicability of the new procedure is the limited data set used in the validation exercise. The observation that the new procedure appears to be superior to current practice was made with the understanding that it is only valid for one data set.

Quantitative conclusions of Chapter 5 are restated in the following:

- At small scaled distance ($Z < 0.5 \text{ m/kg}^{1/3}$) clearing does not occur, except for very small (scaled) structure size ($D \leq 0.1 \text{ m/kg}^{1/3}$). Then it is only slightly evident. It can be concluded that, in these circumstances, the proximity of the edges of the structure are irrelevant, and the impulse can be calculated as if it were a height of burst scenario.
- At large scaled distance ($Z > 16.0 \text{ m/kg}^{1/3}$) clearing occurs completely, except for very large (scaled) structure size ($D \geq 0.8 \text{ m/kg}^{1/3}$), and the average impulse experienced by the front face of the structure tends towards the average side-on value.

- The effect of the length of the structure in the direction of propagation on the process of clearing is minimal.

There was one aspect of the validation procedure in Chapter 5 which was, in retrospect, unsatisfactory: the simplification of modelling the height of burst as a hemispherical surface burst. Chapter 5 was the first of the three main subject areas of this document to be addressed, and it was completed before development of the two-dimensional radially symmetrical implementation in Air3d was finished. With hindsight, the validation problems could usefully be recalculated with the correct height of burst geometry. However, this would be a significant undertaking, and similar arguments could be raised for every analysis in the study.

8.4 Blast in an Urban Environment

Chapter 6 represents a first tentative look at the systematic definition of blast in an urban environment. It was restricted to straight symmetrical streets with a hemispherical charge located at the plane of symmetry along the centreline. Also, although pressure histories were monitored at locations across the whole of the street façade, only information concerning those near street level were presented. These simplifications were an unfortunate necessity required to restrict the size of the problem to manageable proportions.

Despite the simplifications, a number of interesting conclusions were established by the investigation. These are restated below:

- In streets which have small scaled street width, reflections do not remain separated from the incident wave; reflections coalesce immediately, and a single large pulse results. In broad terms, coalescence occurs at every location along the street for scaled street widths $w/W^{1/3} \leq 2.0 \text{ m/kg}^{1/3}$.
- Streets which have scaled street width $w/W^{1/3} > 3.2 \text{ m/kg}^{1/3}$ produce very little confinement. This study has not revealed an effective upper limit on scaled street width, above which no confinement results, but it is unlikely to be significantly greater than $3.2 \text{ m/kg}^{1/3}$.
- Scaled building heights $h/W^{1/3} \geq 2.4 \text{ m/kg}^{1/3}$ can be considered effectively infinite; the effect of the presence of the upper edge on impulses at street level is negligible.
- The general shape of the pressure-time loading curve for locations along the street does not change significantly with scaled building height. The numerical values of pressure and impulse, however, are reduced by the process of normal and lateral clearing as the scaled building height decreases. The number and frequency of reflections remains broadly similar for all building heights.

It is recognised that the three identical firings which provided the mean values of the experimental data used for comparison in Section 6.2 also provided the opportunity to perform an analysis of the raw data in a statistically valid manner. Such an approach is beyond the scope of the present study, however, where the main aim

was simply to demonstrate the possibility of using a numerical technique to investigate the effect of principal geometrical parameters affecting blast wave impulse in straight city streets. It is suggested that, despite the simplified procedure adopted in the present study, the approach is, nonetheless, worthwhile.

8.5 The Effectiveness of Blast Walls

In many respects, the design and application of blast walls is a straightforward art. It is obvious to the engineer that the wall should be as close to the charge and as high as possible. Given these conditions, all that remains is for the wall to be designed to withstand the effects of the explosion.

In practice, however, the situation might be complicated by other factors like the presence of other structures, access to the site, and so on. Most importantly, what is required is an understanding of the situation so that appropriate designs can be implemented. Such an understanding is not intuitive and is only usually developed by undertaking the kind of case study described in Chapter 7.

It was the original intention of Chapter 7 to reproduce and extend the data of Rose, et al. [43] numerically, to provide a more complete set of design charts for the engineer to use. Such charts would have provided the opportunity to conduct a simplified case study without the need for further analysis. This intention, however, proved to be beyond the scope of the current implementation of the program Air3d. The comparison of numerically-based results with data from experiments demonstrated that the calculations were not *mesh resolved*, and, furthermore, the numerical data were certainly outside the experimental error of the data used for comparison.

As a result, Chapter 7 was restricted to a general discussion of the effectiveness of blast walls and quantitative analysis of an example scenario. That analysis demonstrated that a reduction in pressure along the centreline of the façade of the building of between 88% and 56%, and a reduction in impulse of between 65% and 43% was possible, compared to the *no wall* scenario. These broad conclusions may not be wholly accurate, the validation procedure did not allow that possibility, but their method of establishment has provided an insight into the underlying physics of the problem, and they are not presently achievable using current guidance.

The cause of the difficulty of analysing the blast wall scenario, spatial discretisation, and its possible remedy, Adaptive Mesh Refinement, were both discussed previously. At the present time, the complete description of blast wall effectiveness remains a long term objective.

8.6 Future Applications

The possibility of considering *scenario-specific* blast load evaluations or the establishment of general guidance from the results of numerical simulation is becoming evident. Of the subject areas considered in this study, perhaps only blast wave clearing has been treated comprehensively. Even then, the extent and resolution of

the analyses which comprised the background to the new approach could be usefully improved.

Of the other areas considered, discussion of blast wall effectiveness clearly requires more detailed treatment, and blast in an urban environment, although approachable, provides a vast subject area with limitless possibilities.

Other, more fundamental subjects could usefully be revisited by numerically-based experiments. One obvious area, which is commonly needed and generally only poorly described, is height of burst. Those height of burst parameters used for comparison in Chapter 4 are restricted to a small region of scaled space and a discrete number of scaled heights above the ground. The two-dimensional implementation in Air3d, for example, should allow the establishment of numerically-based height of burst data over a large scaled range and for numerous different charge heights. It would also produce data for locations above the ground and not just on the ground surface, which is the presently the case. Such a programme of analyses would not be a large undertaking, and the results, once validated by key experiments, would be of considerable benefit to the engineering community.

In the near future, the main application for the proposed approach is the continued treatment of blast in an urban environment. Clearly, this is where the main hazards from explosive events continue to reside.

The study has been driven by the twin needs to develop an accurate and robust computational tool, but also to make progress in the main subject areas. Inevitably, compromises have been necessary.

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Appendix A

Equations of Motion For Compressible Inviscid Flow: The Euler Equations

A.1 Integral Form of the Euler Equations

In this section, the governing equations for compressible inviscid flow will be presented. The approach taken here follows that of the two introductory volumes by Anderson [1] and [2], which the present author has found to be particularly lucid.

The equations will be derived by considering a finite control volume, fixed in space, through which the fluid (air) is allowed to flow. The implications of this will be discussed briefly in Appendix A.3.

Consider the control volume in Figure A.1 with volume $\mathcal{V}$ and surface area S . The flow of fluid through the control volume is represented by the streamlines in Figure A.1.

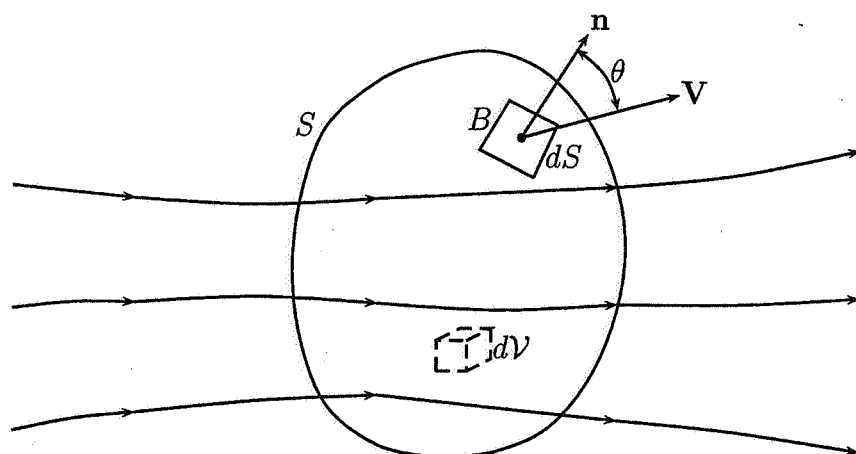


Figure A.1: Finite control volume, fixed in space, with fluid flowing through it

The appropriate fundamental physical principles, which need to be observed by the model of the flow, are as follows:

- Mass is conserved $\implies$ Continuity equation.
- Force = mass $\times$ acceleration $\implies$ Momentum equation.
- Energy is conserved $\implies$ Energy equation.

Continuity Equation

Mass can be neither created nor destroyed.

Consider an arbitrarily chosen point B on the control surface of Figure A.1 and an elemental area around B : dS . Let $\mathbf{n}$ be the unit normal vector directed *out* of the surface at B . Define $d\mathbf{S} = \mathbf{n} dS$ and let $\mathbf{V}$ and ρ be the local velocity vector and density, respectively at B . The mass flow through the elemental surface dS is the product of the density, the component of velocity normal to the surface and the area:

$$\dot{m} = \rho (V \cos \theta) dS = \rho \mathbf{V} \cdot d\mathbf{S}. \quad (\text{A.1})$$

The total mass flow *into* the control volume is then given by the sum of the elemental mass flows, summed over the entire surface of the control volume:

$$- \iint_S \rho \mathbf{V} \cdot d\mathbf{S}, \quad (\text{A.2})$$

where the minus sign indicates inflow, since the surface normal is directed outwards. Consider an infinitesimal volume dV , inside the control volume. The mass of this infinitesimal volume is ρdV . Hence, the total mass inside the control volume is the sum of these elemental masses:

$$\iiint_V \rho dV,$$

and the time rate of change of the mass inside the control volume is, therefore:

$$\frac{\partial}{\partial t} \iiint_V \rho dV. \quad (\text{A.3})$$

If mass is to be conserved, the net mass flow into the control volume (eqn (A.2)) must be equal to the rate of increase of mass inside the control volume (eqn (A.3)). Therefore, the continuity equation becomes:

$$- \iint_S \rho \mathbf{V} \cdot d\mathbf{S} = \iiint_V \frac{\partial}{\partial t} \rho dV. \quad (\text{A.4})$$

Momentum Equation

The time rate of change of momentum of a body equals the net force exerted on it.

Writing this physical principle in vector form:

$$\frac{d}{dt}(m\mathbf{V}) = \mathbf{F}. \quad (\text{A.5})$$

Considering the right hand side of eqn (A.5), there are two types of forces contributing to $\mathbf{F}$ which could be considered acting upon the control volume in Figure A.1.

Body forces which act on the fluid inside the control volume. These might be gravitational, electrostatic, electromagnetic, and so on. Forces of this type can safely be ignored for the purpose of this study.

Surface forces acting on the boundary of the control volume. Again, there are two types of surface force in the fluid: pressure forces and those arising from shear stress distribution. Because the present study is concerned with inviscid flow, the only forces of interest are due to pressure.

Considering again the directed elemental surface area $d\mathbf{S}$ in Figure A.1, the elemental surface force acting *inward* on $d\mathbf{S}$ is $-p d\mathbf{S}$. Hence, summing over the whole control surface:

$$\boxed{\begin{array}{l} \text{total surface force} \\ \text{due to pressure} \end{array}} = - \iint_S p d\mathbf{S}. \quad (\text{A.6})$$

Next consider the left hand side of eqn (A.5), to establish the time rate of change of momentum $m(d\mathbf{V}/dt)$. In Figure A.1, mass flows into the control volume from the left, bringing with it a certain amount of momentum. At the same time, the mass flowing out of the control volume also has momentum. The elemental mass flow across $d\mathbf{S}$ is given by eqn (A.1) as $\rho \mathbf{V} \cdot d\mathbf{S}$, and is associated with a momentum flow (or flux) $(\rho \mathbf{V} \cdot d\mathbf{S})\mathbf{V}$, and $\mathbf{V}$ is directed away from the boundary, as in Figure A.1. This is associated with an *outflow* of momentum. The net rate of flow of momentum, summed over the whole surface of the control surface, is:

$$\iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) \mathbf{V}. \quad (\text{A.7})$$

Because the flow is unsteady, the flow properties are functions of time, and the change in momentum due to unsteady transient effects inside the control volume $\mathcal{V}$ also needs to be taken into account.

Consider the elemental mass of fluid $\rho d\mathcal{V}$ which has momentum $(\rho d\mathcal{V})\mathbf{V}$. Summing over the entire volume:

$$\text{total momentum inside } \mathcal{V} = \iiint_{\mathcal{V}} \rho \mathbf{V} d\mathcal{V}.$$

Hence, the change in momentum due to unsteady fluctuations in the local flow properties is:

$$\frac{\partial}{\partial t} \iiint_V \rho \mathbf{V} dV = \iiint_V \frac{\partial (\rho \mathbf{V})}{\partial t} dV. \quad (\text{A.8})$$

The partial derivative can be taken inside the integral because the control volume is fixed in space and time.

Equating the net flow of momentum across the surface (eqn (A.7)) and the rate of change of momentum inside the control volume (eqn (A.8)) to the total surface force due to the pressure (eqn (A.6)) gives:

$$\iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) \mathbf{V} + \iiint_V \frac{\partial (\rho \mathbf{V})}{\partial t} dV = - \iint_S p d\mathbf{S}, \quad (\text{A.9})$$

which is the momentum equation.

Energy Equation

The energy equation is derived from the first law of thermodynamics:

Energy can be neither created nor destroyed; it can only change in form.

If we again consider the fluid flowing through the control volume of Figure A.1, we need to establish the rate of work done on the fluid inside the control volume, and the rate of change of the energy of the fluid as it flows through the control volume. In the present case, the possibility of volumetric heating of the fluid (due to absorption of thermal radiation, say) is ignored. Similarly, because the flow is inviscid, thermal conduction and diffusion across the boundary are also ignored. The method by which high explosive detonation is approximated in the present study also precludes the need for separate heating terms or special treatment of the internal energy. These factors greatly simplify the energy equation.

We start from the fact:

$$\boxed{\text{rate of doing work on a moving body}} = \mathbf{F} \cdot \mathbf{V}. \quad (\text{A.10})$$

Equation (A.10) states that the rate of work done on a moving body is equal to the product of its velocity and the component of force in the direction of the velocity.

Consider the elemental directed area $d\mathbf{S}$ of the control surface. The pressure force on this elemental area is $-p d\mathbf{S}$ (eqn (A.6)). Hence, the rate of work done on the fluid passing through $d\mathbf{S}$ with velocity $\mathbf{V}$ is $-(p d\mathbf{S}) \cdot \mathbf{V}$. Summing over the complete control surface:

$$\boxed{\text{rate of work done on the fluid inside } V \text{ due to the pressure forces on } S} = - \iint_S (p d\mathbf{S}) \cdot \mathbf{V} = - \iint_S p \mathbf{V} \cdot d\mathbf{S}. \quad (\text{A.11})$$

The energy inside the control volume is the sum of the internal energy e (per unit mass) and the kinetic energy per unit mass $V^2/2$, due to the local velocity $\mathbf{V}$.

Clearly, the mass flowing into the control volume brings with it a certain energy; at the same time, the mass flowing out also has energy. The elemental mass flow across dS is given by $\rho \mathbf{V} \cdot d\mathbf{S}$ (eqn (A.1)), and so the elemental flux of energy across dS is $(\rho \mathbf{V} \cdot d\mathbf{S})(e + V^2/2)$, and over the complete control surface:

$$\boxed{\text{net rate of flow of energy across the control surface}} = \iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) \left(e + \frac{V^2}{2} \right) = \iint_S \rho \left(e + \frac{V^2}{2} \right) \mathbf{V} \cdot d\mathbf{S}. \quad (\text{A.12})$$

Because the flow is unsteady, there is also a rate of change of energy inside the control volume due to local transient fluctuations of the flow variables. The energy of the elemental volume is $\rho(e + V^2/2)d\mathcal{V}$, and so the energy inside the complete control volume is:

$$\iiint_{\mathcal{V}} \rho \left(e + \frac{V^2}{2} \right) d\mathcal{V}. \quad (\text{A.13})$$

Therefore

$$\boxed{\text{time rate of change of energy inside } \mathcal{V} \text{ due to transient variation of the flow field variables}} = \frac{\partial}{\partial t} \iiint_{\mathcal{V}} \rho \left(e + \frac{V^2}{2} \right) d\mathcal{V}. \quad (\text{A.14})$$

Equating (A.14) and (A.12) with (A.11) gives the energy equation:

$$\iiint_{\mathcal{V}} \frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] d\mathcal{V} + \iint_S \rho \left(e + \frac{V^2}{2} \right) \mathbf{V} \cdot d\mathbf{S} = - \iint_S p \mathbf{V} \cdot d\mathbf{S}. \quad (\text{A.15})$$

The three conservation equations above, eqns (A.4), (A.9) and (A.15), are not sufficient to analyse inviscid compressible flow. Two other relations are needed: the equation of state for a perfect gas:

$$p = \rho RT \quad (\text{A.16})$$

and the thermodynamic relation:

$$e = c_v T \quad (\text{A.17})$$

for a calorically perfect gas: one with constant specific heats.

There is a strong case that real gas effects should be included in calculations in which high explosive detonation is modelled. However, in the present study these effects have been ignored on the grounds that details of the flow close to the explosive source (at very small scaled range) are not of interest, and the accuracy at other ranges is *not too bad* as a result of ignoring the real gas effects.

The integral form of the equations, presented above, is perhaps the most satisfactory form of the Euler equations. In the above discussion, no mention was made of coordinate directions or continuity of the flow variables. This last fact is particularly important because the whole of the present study is based on modelling shock waves in air, and the integral form of the equations allows the presence of discontinuities within the control volume. This is a strong argument for the integral form of the Euler equations to be considered more fundamental than the differential form, which is considered next.

A.2 Differential Form of the Euler Equations

The integral form of the Euler equations has applicability to a number of practical compressible flow problems, but as the nature of those problems increases in complexity, with unsteady and three dimensional flow, smaller and smaller control volumes must be considered in order to model accurately the intricacies of the flow. This is the basis of modern Computational Fluid Dynamics.

As the small neighbourhood surrounding the point of interest decreases, *differential equations*, which describe the flow properties at a point, result. This form of the conservation equations is more amenable to numerical solution (with the added constraint of continuity of the conserved variables) and is the subject of the present section. Cartesian coordinates will also be introduced as the final form in which the equations will be used is approached.

In what follows, use will be made of the following vector identities:

$$\iint_S \mathbf{A} \cdot d\mathbf{S} = \iiint_V (\nabla \cdot \mathbf{A}) dV, \quad (\text{A.18})$$

where $\mathbf{A}$ is a vector quantity and

$$\iint_S \Phi d\mathbf{S} = \iiint_V (\nabla \Phi) dV, \quad (\text{A.19})$$

where Φ is a scalar quantity. These are forms of *Gauss's divergence theorem* and can be found in any standard text (Kreyszig [29], for example).

Continuity Equation

The integral form of the continuity equation (A.4) is:

$$-\iint_S \rho \mathbf{V} \cdot d\mathbf{S} = \frac{\partial}{\partial t} \iiint_V \rho dV. \quad (\text{A.20})$$

Using eqn (A.18) with $\rho \mathbf{V} = \mathbf{A}$, we obtain:

$$\iint_S \rho \mathbf{V} \cdot d\mathbf{S} = \iiint_V \nabla \cdot (\rho \mathbf{V}) dV. \quad (\text{A.21})$$

Combining equations (A.20) and (A.21), gives:

$$\iiint_V \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) \right] dV = 0. \quad (\text{A.22})$$

Equation (A.22) is valid for arbitrary control volume V and so can be zero only if the integrand is zero at each point within the volume. Hence, assuming differentiability of ρ and $\mathbf{V}$:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \quad (\text{A.23})$$

which is the differential form of the continuity equation.

Momentum Equation

Repeating the integral form of the momentum equation (A.9):

$$\iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) \mathbf{V} + \iiint_V \frac{\partial (\rho \mathbf{V})}{\partial t} dV = - \iint_S p d\mathbf{S}. \quad (\text{A.24})$$

Using equation (A.19) with $p = \Phi$:

$$\iint_S p d\mathbf{S} = \iiint_V (\nabla p) dV. \quad (\text{A.25})$$

Combining (A.24) and (A.25), we obtain:

$$\iiint_V \frac{\partial (\rho \mathbf{V})}{\partial t} dV + \iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) \mathbf{V} = - \iiint_V (\nabla p) dV. \quad (\text{A.26})$$

At this stage, it is convenient to introduce a Cartesian coordinate system. Considering scalar components in the x , y and z directions, the x component of equation (A.26) is:

$$\iiint_V \frac{\partial (\rho u)}{\partial t} dV + \iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) u = - \iiint_V \frac{\partial p}{\partial x} dV. \quad (\text{A.27})$$

Using equation (A.18)

$$\iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) u = \iint_S (\rho u \mathbf{V}) \cdot d\mathbf{S} = \iiint_V \nabla \cdot (\rho u \mathbf{V}) dV \quad (\text{A.28})$$

and substituting (A.28) into (A.27), collecting and rearranging the terms within the volume integral, gives:

$$\iiint_V \left[\frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho u \mathbf{V}) + \frac{\partial p}{\partial x} \right] dV = 0. \quad (\text{A.29})$$

Using the same argument that was used for the continuity equation, the only way for the volume integral in equation (A.29) to be zero is for the integrand to be zero at every point in the volume. Therefore, we obtain the x component of the momentum equation in differential form:

$$\frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho u \mathbf{V}) + \frac{\partial p}{\partial x} = 0. \quad (\text{A.30})$$

A similar procedure for the y and z components yields:

$$\frac{\partial (\rho v)}{\partial t} + \nabla \cdot (\rho v \mathbf{V}) + \frac{\partial p}{\partial y} = 0 \quad (\text{A.31})$$

and

$$\frac{\partial (\rho w)}{\partial t} + \nabla \cdot (\rho w \mathbf{V}) + \frac{\partial p}{\partial z} = 0. \quad (\text{A.32})$$

Energy Equation

Repeating the integral form of the energy equation (A.15):

$$\iiint_V \frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] dV + \iint_S \rho \left(e + \frac{V^2}{2} \right) \mathbf{V} \cdot d\mathbf{S} = - \iint_S p \mathbf{V} \cdot d\mathbf{S}. \quad (\text{A.33})$$

Using equation (A.18) for the two surface integrals:

$$\iint_S \rho \left(e + \frac{V^2}{2} \right) \mathbf{V} \cdot d\mathbf{S} = \iiint_V \nabla \cdot \left[\rho \left(e + \frac{V^2}{2} \right) \right] \mathbf{V} dV \quad (\text{A.34})$$

and

$$\iint_S p \mathbf{V} \cdot d\mathbf{S} = \iiint_V \nabla \cdot (p \mathbf{V}) dV. \quad (\text{A.35})$$

Combining equations (A.34) and (A.35) with (A.33) and collecting the terms under the integral gives:

$$\iiint_V \left\{ \frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] + \nabla \cdot \left[\rho \left(e + \frac{V^2}{2} \right) \right] \mathbf{V} + \nabla \cdot (p \mathbf{V}) \right\} dV = 0. \quad (\text{A.36})$$

Finally, collecting the divergence terms and setting the integrand to zero, as before, we obtain the differential form of the energy equation:

$$\frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] + \nabla \cdot \left[\rho \left(e + \frac{V^2}{2} \right) + p \right] \mathbf{V} = 0. \quad (\text{A.37})$$

A.3 Form Most Suitable for CFD Calculations

What we have seen in the previous two sections is a presentation of the governing equations; firstly, in a completely general way (the integral form), then in a more specific way, modelling the flow as a large collection of small points and assuming differentiability of the solution. Finally, a form will be presented which is most suited to numerical computation.

One important fact, which has not been mentioned until now, is that the integral and differential forms of the equations, given above, are both in *conservation form*. This has resulted from the initial choice (in Figure A.1) of a stationary control volume through which the fluid was allowed to flow. This choice leads to a considerable programming convenience because the differential forms of the continuity, momentum and energy equations can all be expressed in the same manner: they all contain a divergence term of the flux of a physical quantity. With this in mind, the entire system of equations can be described by:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \frac{\partial \mathbf{H}}{\partial z} = 0 \quad (\text{A.38})$$

where $\mathbf{U}$, $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ are interpreted as column vectors given by:

$$\mathbf{U} = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho \left(e + \frac{V^2}{2} \right) \end{Bmatrix}, \quad \mathbf{F} = \begin{Bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ \rho u \left(e + \frac{V^2}{2} \right) + pu \end{Bmatrix}, \quad (\text{A.39})$$

$$\mathbf{G} = \begin{Bmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ \rho vw \\ \rho v \left(e + \frac{V^2}{2} \right) + pv \end{Bmatrix} \text{ and } \mathbf{H} = \begin{Bmatrix} \rho w \\ \rho wu \\ \rho wv \\ \rho w^2 + p \\ \rho w \left(e + \frac{V^2}{2} \right) + pw \end{Bmatrix}.$$

The vector $\mathbf{U}$ is the vector of the *conserved variables*. The vectors $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{H}$ are the *flux vectors*.

Appendix B

The Shock Tube Problem

B.1 Description of the Shock Tube Problem

Fundamental to the development of the computational tool Air3d was the ability to verify the method of solution. To do this, a problem that has a known analytical solution was needed. The so-called *shock tube problem* fulfils this requirement.

Consider the shock tube sketched in Figure B.1. This shows a schematic diagram

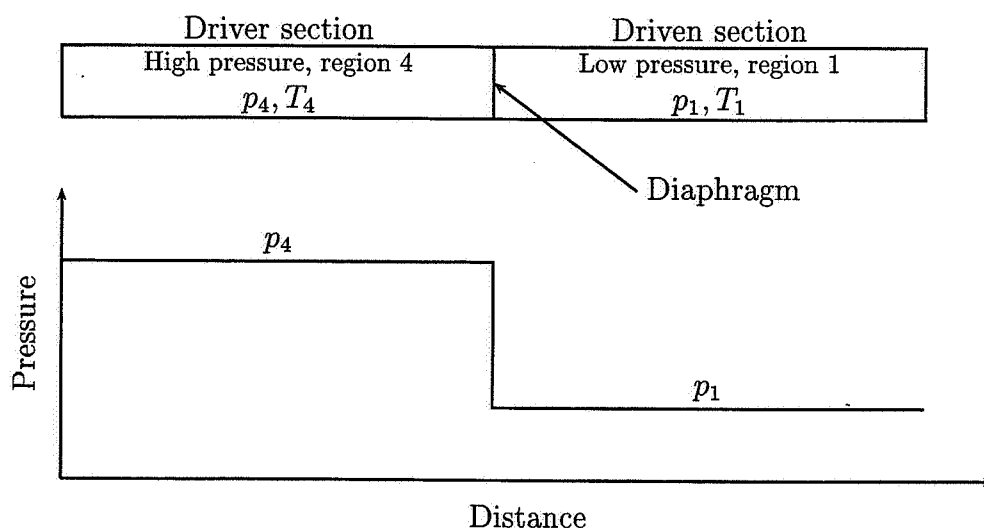


Figure B.1: The shock tube problem, initial conditions

of the tube (at the top of the diagram), and the pressure distribution (at the bottom). The tube has constant cross-sectional area and is divided into two sections by a diaphragm. Such tubes are valuable instruments and can be used in many different ways. The manner in which it is used (hypothetically) in the present study is probably the simplest.

The two halves of the tube are considered to be filled with air at room temperature. The left-hand side, the *driver* section (region 4), is filled to a chosen pressure p_4 , which is greater than atmospheric. The right-hand side of the shock tube, the

driven section (region 1), contains air at atmospheric pressure p_1 . As mentioned above, the temperature $T_4 = T_1 = \text{atmospheric}$; all other thermodynamic quantities: the densities ρ_4 and ρ_1 , the sound speeds a_4 and a_1 , and so on, can be calculated from the pressure and temperature.

When the diaphragm between the two sections is broken, a shock wave propagates into section 1, and an expansion wave propagates into section 4. This situation is shown in Figure B.2, at some arbitrary time after the diaphragm has broken but before either wave has reached the end of the tube.

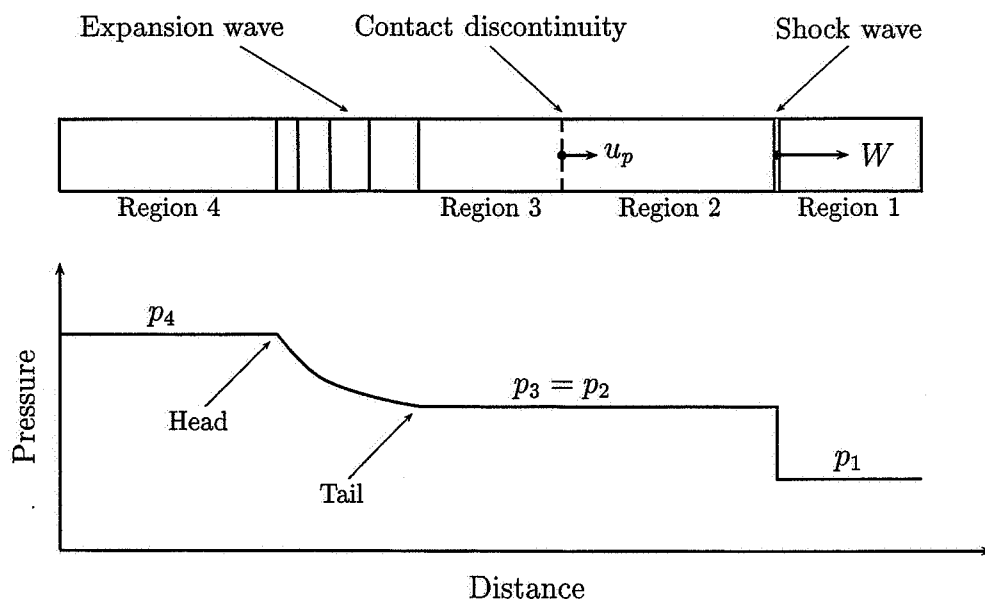


Figure B.2: Flow in a shock tube after the diaphragm has broken

The normal shock propagates to the right with velocity W and with mass motion u_p behind the wave. The region between the shock wave and to the right of the *contact discontinuity* (described below) is region 2 in Figure B.2. The interface between the two different densities of gas (the contact discontinuity) also moves to the right with velocity u_p , and, interestingly, the pressure and velocity are continuous across the contact discontinuity, in other words, $p_2 = p_3$ and $u_2 = u_3$.

Finally, the expansion wave (or fan) moves to the *left*, decreasing the driver pressure p_4 to p_3 continuously between the head and tail of the wave. The mass motion in the expansion is to the *right*, and the head of the expansion wave propagates into region 4 at the local sound speed. The region between the tail of the expansion and the contact discontinuity is region 3 in Figure B.2.

A complete description of the flow field in the shock tube after the diaphragm has broken can be determined from the initial conditions in regions 4 and 1 shown in Figure B.1. This remarkable fact is one of the main reasons why the shock tube problem is so useful as a validation tool.

One further point will be emphasised before moving on to the derivation of the shock tube relations. The present study is concerned with the possibility of

modelling the propagation of shock waves in air—formed by the detonation of high explosives—and gives rise to a number of situations not considered by the shock tube problem: namely, boundary conditions and three-dimensional flow. Treatment of such effects can only really be validated by comparison with experiment on a case by case basis. This has been the approach adopted in the present study.

B.2 Derivation of the Shock Tube Relations

In an attempt to keep this document as self-contained as possible, it was considered beneficial to set out the basic thermodynamic relations and development of the shock tube equations used by the verification program STanaly, described in Chapter 2. These relations can be derived in a number of similar ways, and the approach taken here is a distillation of that described by Anderson [2]. It is suggested that this, together with the development of the equations of motion in Appendix A, provides sufficient background information for the present study.

The Thermodynamics Needed to Develop the Shock Tube Relations

The starting point of the derivation is the perfect or ideal gas equation:

$$p = \rho RT, \quad (\text{B.1})$$

where p is the pressure (Pa), ρ is the density (kg/m^3), R is the specific gas constant for air ($\text{J}/\text{kg K}$) and T is the temperature (K). There is also the assumption, used throughout this study, that air is a calorically perfect gas: the specific heats at constant volume c_v and constant pressure c_p are both constant. These are defined as follows:

$$c_v = \left(\frac{\partial e}{\partial T} \right)_v$$

and

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p,$$

where subscripts indicate the quantity held constant.

The specific internal energy e is defined as the *energy per unit mass* (J/kg). The specific enthalpy h is defined by $h = e + pv$ (J/kg), where v is the specific volume (m^3/kg), the inverse of the density ($v = 1/\rho$). Therefore, both the specific internal energy and the specific enthalpy are linear functions of the temperature:

$$e = c_v T \quad (\text{B.2})$$

and

$$h = c_p T. \quad (\text{B.3})$$

The definition of enthalpy, the ideal gas equation (B.1) together with eqns (B.2) and (B.3), provide the relation:

$$c_p - c_v = R. \quad (\text{B.4})$$

Dividing eqn (B.4) by c_p gives:

$$1 - \frac{c_v}{c_p} = \frac{R}{c_p},$$

and from the definition of $\gamma = c_p/c_v$ (for air at standard conditions $\gamma = 1.4$):

$$1 - \frac{1}{\gamma} = \frac{R}{c_p}.$$

Solving for c_p

$$c_p = \frac{\gamma R}{\gamma - 1} \quad (\text{B.5})$$

and similarly for c_v

$$c_v = \frac{R}{\gamma - 1}. \quad (\text{B.6})$$

The first law of Thermodynamics states that for a given system

$$\delta q + \delta w = \delta e, \quad (\text{B.7})$$

where

δq = incremental amount of heat added.

δw = work done on the system.

δe = change in energy of the system

(not specific energy in this case—the system has arbitrary mass).

Clearly, there are a number of ways which heat can be added to, and work done on, the system. For the purpose of this section, we will be concerned with three main types:

- An *adiabatic* process is one in which no heat is added or taken away from the system.
- In a *reversible* process no dissipative phenomena, such as viscosity and thermal conductivity, occur.
- An *isentropic* process is both adiabatic and reversible.

For a reversible process $\delta w = -p\delta v$, where δv is an incremental change in specific volume due to displacement of the boundary of the system, the first law, eqn (B.7), becomes:

$$\delta q - p\delta v = \delta e. \quad (\text{B.8})$$

The entropy of a system can be defined as follows:

$$ds = \frac{\delta q_{rev}}{T}, \quad (\text{B.9})$$

where s is the entropy of the system, δq_{rev} is the incremental amount of heat added *reversibly* to the system and T is the temperature at which the process takes place. An alternative to the above relation is:

$$ds = \frac{\delta q}{T} + ds_{irrev}. \quad (B.10)$$

This states that the change in entropy during any incremental process is equal to the actual heat added divided by the temperature, plus a contribution from irreversible dissipative phenomena, such as viscosity and thermal conductivity, occurring in the system. The dissipative phenomena *always* increase the entropy:

$$ds_{irrev} \geq 0.$$

The equals sign denotes a reversible process, where the dissipative phenomena are absent. A combination of eqn (B.9) and (B.10) gives:

$$ds \geq \frac{\delta q}{T}. \quad (B.11)$$

A check on the entropy is a good way of ensuring that numerical computations are proceeding in a physically permissible manner.

Using the definition of entropy in the form $\delta q_{rev} = Tds$, eqn (B.8) becomes:

$$Tds - p dv = de \quad (B.12)$$

or

$$Tds = de + p dv. \quad (B.13)$$

From the definition of the enthalpy, $h = e + pv$, differentiating gives:

$$dh = de + p dv + v dp. \quad (B.14)$$

Combining eqns (B.13) and (B.14):

$$Tds = dh - v dp. \quad (B.15)$$

From eqn (B.3) we have $dh = c_p dT$. Substituting this into eqn (B.15):

$$ds = c_p \frac{dT}{T} - \frac{v dp}{T}. \quad (B.16)$$

Using the ideal gas equation of state, eqn (B.1), in the form $pv = RT$ and substituting into eqn (B.16) gives:

$$ds = c_p \frac{dT}{T} - R \frac{dp}{p}. \quad (B.17)$$

Integrating between two arbitrary states 1 and 2, and continuing the assumption of a calorically perfect gas (where c_p is constant):

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}. \quad (B.18)$$

Following a similar procedure for eqn (B.12), using $de = c_v dT$:

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}. \quad (\text{B.19})$$

Finally in this section, the isentropic relations needed for development of the shock tube relations are given. An isentropic relation is one in which $ds = 0$: the entropy is constant. Setting $s_2 = s_1$ in eqn (B.18):

$$0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

and

$$\ln \frac{p_2}{p_1} = \frac{c_p}{R} \ln \frac{T_2}{T_1}.$$

Recalling that $c_p = \gamma R / (\gamma - 1)$ from eqn (B.5):

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)}. \quad (\text{B.20})$$

Similarly for eqn (B.19):

$$0 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1},$$

and

$$\ln \frac{v_2}{v_1} = -\frac{c_v}{R} \ln \frac{T_2}{T_1}.$$

and Recalling $c_v = R / (\gamma - 1)$ from eqn (B.6):

$$\frac{v_2}{v_1} = \left(\frac{T_2}{T_1} \right)^{-1/(\gamma-1)}$$

or

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1} \right)^{1/(\gamma-1)}. \quad (\text{B.21})$$

Summarising eqns (B.20) and (B.21):

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1} \right)^{\gamma} = \left(\frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)}. \quad (\text{B.22})$$

Steady One-Dimensional Flow

Consider the flow through the one-dimensional region represented by the hatched area in Figure B.3. This region may be a shock wave, for example; in which case, the flow properties change as a function of x , as the gas flows through the region. To the left of the region, the flow field velocity, pressure, temperature, density and internal energy are u_1 , p_1 , T_1 , ρ_1 and e_1 , respectively. To the right of the region, the properties have changed, and are given by u_2 , p_2 , T_2 , ρ_2 and e_2 . Because the flow is one-dimensional, the flow field properties are uniform over each side of the control volume. The control volume has an area A perpendicular to the flow. If it is also

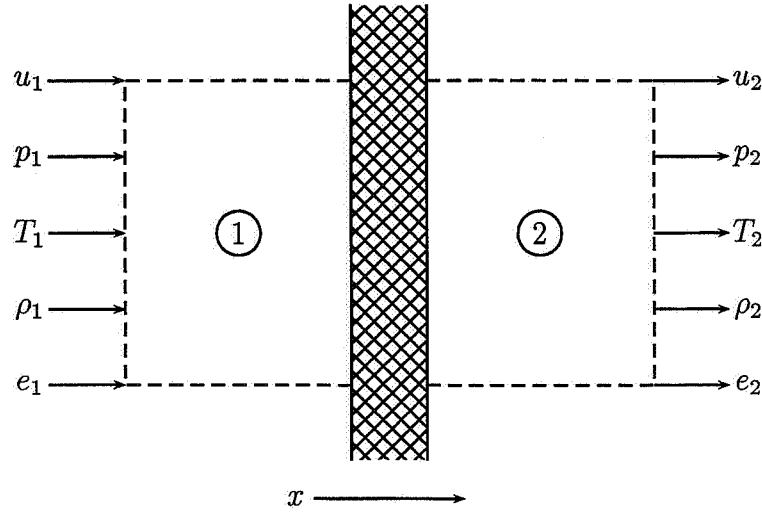


Figure B.3: Rectangular control volume for one-dimensional flow

assumed that the flow is steady, the derivatives of the flow properties with respect to time are zero. Consider the integral form of the continuity equation (A.4):

$$-\iint_S \rho \mathbf{V} \cdot d\mathbf{S} = \frac{\partial}{\partial t} \iiint_V \rho dV.$$

For steady flow this becomes:

$$\iint_S \rho \mathbf{V} \cdot d\mathbf{S} = 0. \quad (\text{B.23})$$

Evaluating the surface integral over the left-hand side of the control volume in Figure B.3, where $\mathbf{V}$ and $d\mathbf{S}$ are in opposite directions, we obtain $-\rho_1 u_1 A$. Similarly for the integral over the right-hand side, where $\mathbf{V}$ and $d\mathbf{S}$ are in the same direction, we obtain $\rho_2 u_2 A$. Substituting these values into eqn (B.23):

$$-\rho_1 u_1 A + \rho_2 u_2 A = 0$$

or

$$\rho_1 u_1 = \rho_2 u_2, \quad (\text{B.24})$$

which is the continuity equation for one-dimensional steady flow.

The integral form of the momentum equation, eqn (A.9), is:

$$\iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) \mathbf{V} + \iiint_V \frac{\partial (\rho \mathbf{V})}{\partial t} dV = - \iint_S p d\mathbf{S}.$$

The second term is zero, because of the steady flow condition, so it becomes:

$$\iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) \mathbf{V} = - \iint_S p d\mathbf{S}. \quad (\text{B.25})$$

Since the flow is one-dimensional (in the x direction), only the scalar x component of eqn (B.25) need be considered. This gives:

$$\iint_S (\rho \mathbf{V} \cdot d\mathbf{S}) u = - \iint_S (p dS)_x,$$

where $(p dS)_x$ is the x component of the vector $p d\mathbf{S}$ (not the derivative).

Evaluating the surface integrals over the left- and right-hand sides of the control volume in Figure B.3, we obtain:

$$\rho_1 (-u_1 A) u_1 + \rho_2 (u_2 A) u_2 = -(-p_1 A + p_2 A)$$

or

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2. \quad (\text{B.26})$$

This is the momentum equation for steady one-dimensional flow.

Recalling the integral form of the energy equation (A.15):

$$\iiint_V \frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] dV + \iint_S \rho \left(e + \frac{V^2}{2} \right) \mathbf{V} \cdot d\mathbf{S} = - \iint_S p \mathbf{V} \cdot d\mathbf{S}.$$

The first term is zero, because of the steady flow condition, therefore:

$$\iint_S \rho \left(e + \frac{V^2}{2} \right) \mathbf{V} \cdot d\mathbf{S} = - \iint_S p \mathbf{V} \cdot d\mathbf{S}.$$

Evaluating the surface integrals again gives:

$$-\rho_1 \left(e_1 + \frac{u_1^2}{2} \right) u_1 A + \rho_2 \left(e_2 + \frac{u_2^2}{2} \right) u_2 A = -(-p_1 u_1 A + p_2 u_2 A)$$

and rearranging:

$$p_1 u_1 + \rho_1 \left(e_1 + \frac{u_1^2}{2} \right) u_1 = p_2 u_2 + \rho_2 \left(e_2 + \frac{u_2^2}{2} \right) u_2. \quad (\text{B.27})$$

Dividing eqn (B.27) by eqn (B.24) gives:

$$\frac{p_1}{\rho_1} + e_1 + \frac{u_1^2}{2} = \frac{p_2}{\rho_2} + e_2 + \frac{u_2^2}{2}. \quad (\text{B.28})$$

Recalling the definition of specific enthalpy, $h = e + pv$, eqn (B.28) becomes:

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}, \quad (\text{B.29})$$

which is the one-dimensional energy equation for steady flow.

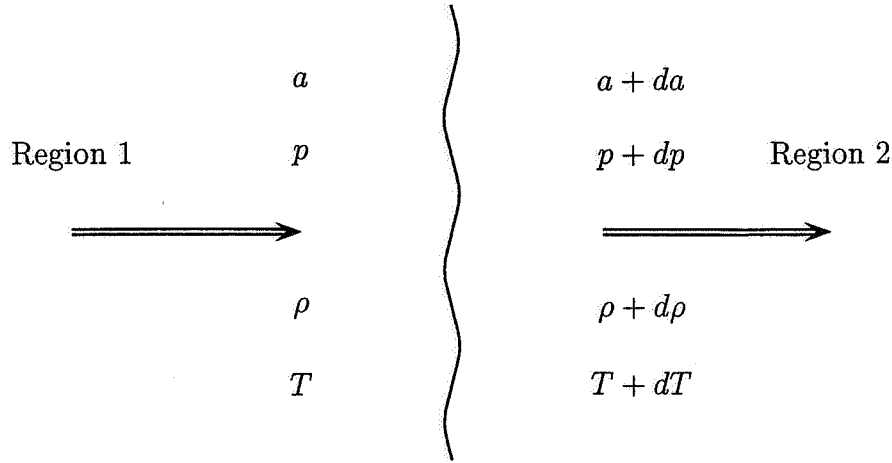


Figure B.4: Schematic picture of a sound wave

Speed of Sound and Mach Number

Consider the sound wave in Figure B.4. By definition, changes that occur in the flow properties through the sound wave are very small (otherwise, it would be a shock wave); it is a *weak* wave. For convenience, the wave in Figure B.4 is shown from the point of view of the wave. In other words, the wave is stationary, and the air ahead of the wave (region 1) propagates towards the wave at velocity a , with pressure p , density ρ and temperature T . There are infinitesimal changes in flow properties through the wave so that in region 2 the flow properties have changed to $a + da$, $p + dp$, $\rho + d\rho$ and $T + dT$. The flow is one-dimensional, so eqns (B.24), (B.26) and (B.29) can be used. Applying the continuity equation (B.24):

$$\rho a = (\rho + d\rho)(a + da)$$

$$\rho a = \rho a + a d\rho + \rho da + d\rho da.$$

Disregarding the product of infinitesimal quantities:

$$a = -\rho \frac{da}{d\rho}. \quad (\text{B.30})$$

Next, applying the momentum equation (B.26):

$$p + \rho a^2 = (p + dp) + (\rho + d\rho)(a + da)^2,$$

ignoring the products of differentials, this becomes:

$$dp = -2a\rho da - a^2 d\rho$$

and solving for da :

$$da = \frac{dp + a^2 d\rho}{-2a\rho}. \quad (\text{B.31})$$

Substituting eqn (B.31) into (B.30):

$$a = -\rho \left[\frac{dp/d\rho + a^2}{-2a\rho} \right]$$

and solving for a^2 :

$$a^2 = \frac{dp}{d\rho}. \quad (\text{B.32})$$

Because the changes which occur within the sound wave are slight, the irreversible dissipative effects of friction and heat conduction are negligibly small. Therefore, the process inside a sound wave is isentropic, and the change of pressure with respect to density, $dp/d\rho$, is an isentropic change. Therefore, eqn (B.32) can be written

$$a^2 = \left(\frac{\partial p}{\partial \rho} \right)_s. \quad (\text{B.33})$$

From the isentropic relation eqn (B.22):

$$\frac{p}{\rho^\gamma} = c, \quad (c = \text{constant}),$$

differentiating gives:

$$\left(\frac{\partial p}{\partial \rho} \right)_s = \frac{\gamma p}{\rho}.$$

Hence, eqn (B.33) becomes:

$$a = \sqrt{\frac{\gamma p}{\rho}} \quad (\text{B.34})$$

and from the equation of state (B.1):

$$a = \sqrt{\gamma RT}. \quad (\text{B.35})$$

Finally, the Mach number is defined as:

$$M = V/a,$$

where V is the local flow field velocity.

Characteristic Mach Number

Consider an arbitrary point in the flow field. The element of fluid surrounding this point will be moving at some Mach number M , velocity V , static temperature T and pressure p . If the fluid element were adiabatically slowed down (if $M > 1$) or made to speed up (if $M < 1$), the temperature would change and take a new value T^* when the element arrives at $M = 1$. The speed of sound at this hypothetical Mach 1 is defined as:

$$a^* = \sqrt{\gamma R T^*},$$

and the characteristic Mach number as:

$$M^* = V/a^*.$$

Returning briefly to eqn (B.29). for a calorically perfect gas, where $h = c_p T$ (eqn (B.3)), eqn (B.29) becomes:

$$c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2}.$$

Using eqn (B.5), the above becomes:

$$\frac{\gamma RT_1}{\gamma - 1} + \frac{u_1^2}{2} = \frac{\gamma RT_2}{\gamma - 1} + \frac{u_2^2}{2}, \quad (\text{B.36})$$

and since $a = \sqrt{\gamma RT}$, eqn (B.36) becomes:

$$\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2}. \quad (\text{B.37})$$

If we again consider the arbitrary point in the flow field, let point 1 in the above equations correspond to the actual flow conditions, where the velocity and sound speed are u and a , respectively. Let point 2 correspond to the hypothetical conditions at $M = 1$. Then $u_2 = a^*$ and eqn (B.37) yields:

$$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{a^{*2}}{\gamma - 1} + \frac{a^{*2}}{2}$$

or

$$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2}. \quad (\text{B.38})$$

Finally, dividing eqn (B.38) by u^2 :

$$\frac{(a/u)^2}{\gamma - 1} + \frac{1}{2} = \frac{\gamma + 1}{2(\gamma - 1)} \left(\frac{a^*}{u} \right)^2,$$

$$\frac{(1/M)^2}{\gamma - 1} = \frac{\gamma + 1}{2(\gamma - 1)} \left(\frac{1}{M^*} \right)^2 - \frac{1}{2}$$

or

$$M^2 = \frac{2}{[(\gamma + 1)/M^{*2}] - (\gamma - 1)}. \quad (\text{B.39})$$

Moving Normal Shock Waves

Consider the stationary shock wave depicted in Figure B.5. Equations (B.24), (B.26) and (B.29) are the continuity, momentum and energy equations, respectively.

From Figure B.5, it can be seen that:

u_1 = velocity of the gas ahead of the shock wave, relative to the wave.

u_2 = velocity of gas behind the shock wave, relative to the wave.

Examining the moving shock wave in Figure B.6, it can be seen that:

W = velocity of the gas ahead of the shock wave, relative to the wave.

$W - u_p$ = velocity of gas behind the shock wave, relative to the wave.

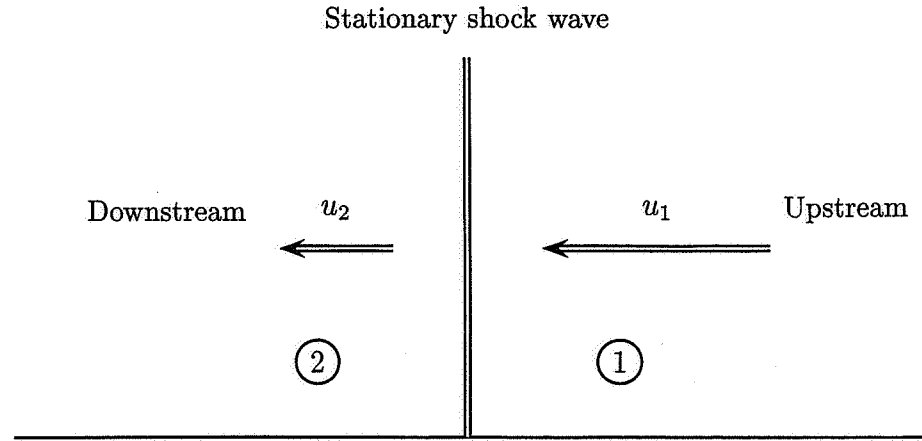


Figure B.5: Stationary shock wave

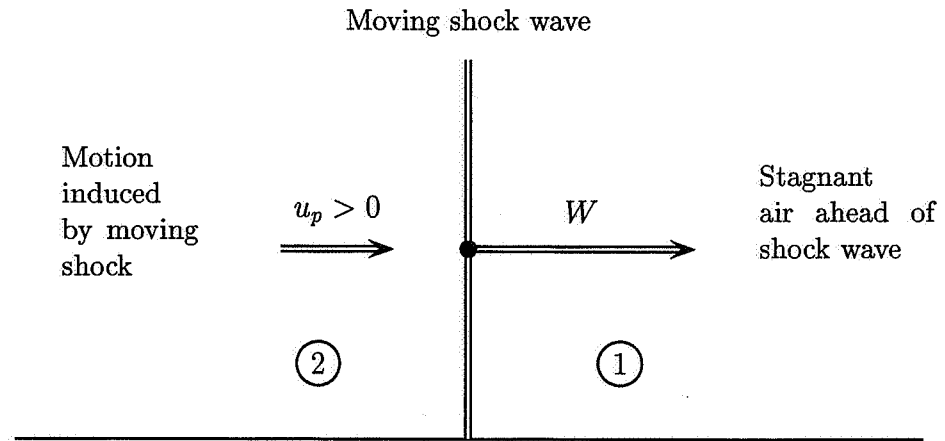


Figure B.6: Moving shock wave

Therefore, the normal shock wave relations (B.24), (B.26) and (B.29) become:

$$\rho_1 W = \rho_2 (W - u_p), \quad (\text{B.40})$$

$$p_1 + \rho_1 W^2 = p_2 + \rho_2 (W - u_p)^2 \quad (\text{B.41})$$

and

$$h_1 + \frac{W^2}{2} = h_2 + \frac{(W - u_p)^2}{2}. \quad (\text{B.42})$$

Equations (B.40), (B.41) and (B.42) are the normal shock equations for a shock moving with velocity W into a stagnant gas.

Rearranging eqn (B.40):

$$W - u_p = W \frac{\rho_1}{\rho_2},$$

substituting this into eqn (B.41) gives:

$$p_1 + \rho_1 W^2 = p_2 + \rho_2 W^2 \left(\frac{\rho_1}{\rho_2} \right)^2,$$

rearranging gives:

$$p_2 - p_1 = \rho_1 W^2 \left(1 - \frac{\rho_1}{\rho_2} \right),$$

$$W^2 = \frac{p_2 - p_1}{\rho_1 (1 - \rho_1/\rho_2)}$$

and finally:

$$W^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1} \left(\frac{\rho_2}{\rho_1} \right). \quad (\text{B.43})$$

Also from eqn (B.40):

$$W = (W - u_p) \frac{\rho_2}{\rho_1}. \quad (\text{B.44})$$

Substituting eqn (B.44) into eqn (B.43):

$$(W - u_p)^2 \left(\frac{\rho_2}{\rho_1} \right)^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1} \left(\frac{\rho_2}{\rho_1} \right)$$

or

$$(W - u_p)^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1} \left(\frac{\rho_1}{\rho_2} \right). \quad (\text{B.45})$$

Substituting eqn (B.43) and (B.45) into (B.42) and using the definition of enthalpy, $h = e + p/\rho$:

$$e_1 + \frac{p_1}{\rho_1} + \frac{1}{2} \left[\frac{p_2 - p_1}{\rho_2 - \rho_1} \left(\frac{\rho_2}{\rho_1} \right) \right] = e_2 + \frac{p_2}{\rho_2} + \frac{1}{2} \left[\frac{p_2 - p_1}{\rho_2 - \rho_1} \left(\frac{\rho_1}{\rho_2} \right) \right], \quad (\text{B.46})$$

which simplifies to:

$$e_2 - e_1 = \frac{p_1 + p_2}{2} \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right)$$

or

$$e_2 - e_1 = \frac{p_1 + p_2}{2} (v_1 - v_2). \quad (\text{B.47})$$

Equation (B.47) is the *Hugoniot relation* and relates changes of thermodynamic variables across a normal shock wave. In the present case, where the assumption of a calorically perfect gas has been made ($e = c_v T$), equation (B.47) becomes:

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \left(\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1 + \frac{\gamma+1}{\gamma-1} \left(\frac{p_2}{p_1} \right)} \right), \quad (\text{B.48})$$

and similarly:

$$\frac{\rho_2}{\rho_1} = \frac{1 + \frac{\gamma+1}{\gamma-1} \left(\frac{p_2}{p_1} \right)}{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}. \quad (\text{B.49})$$

Equations (B.48) and (B.49) give the temperature and density ratios across the shock wave as a function of the pressure ratio.

The moving shock Mach number is defined as:

$$M_s = \frac{W}{a_1},$$

where a_1 is the speed of sound of the stationary air ahead of the shock. Recall the normal shock wave relations for a moving shock (eqns (B.40), (B.41) and (B.42)). Divide eqn (B.41) by (B.40) to give:

$$\frac{p_1}{\rho_1 W} - \frac{p_2}{\rho(W - u_p)} = (W - u_p) - W. \quad (\text{B.50})$$

Recall from eqn (B.34) that $a = \sqrt{\gamma p / \rho}$, so that $p = a^2 \rho / \gamma$ and

$$\frac{a_1^2}{\gamma W} - \frac{a_2^2}{\gamma(W - u_p)} = (W - u_p) - W. \quad (\text{B.51})$$

From eqn (B.38):

$$a_1^2 = \frac{\gamma + 1}{2} a^{*2} - \frac{\gamma - 1}{2} W^2 \quad (\text{B.52})$$

and

$$a_2^2 = \frac{\gamma + 1}{2} a^{*2} - \frac{\gamma - 1}{2} (W - u_p)^2. \quad (\text{B.53})$$

Substitute eqns (B.52) and (B.53) into eqn (B.51):

$$\left(\frac{\gamma + 1}{2} \frac{a^{*2}}{\gamma W} - \frac{\gamma - 1}{2} \frac{W}{\gamma} \right) - \left(\frac{\gamma + 1}{2} \frac{a^{*2}}{\gamma(W - u_p)} - \frac{\gamma - 1}{2} \frac{(W - u_p)}{\gamma} \right) = (W - u_p) - W \quad (\text{B.54})$$

or

$$\frac{\gamma + 1}{2\gamma W(W - u_p)} [(W - u_p - W)] a^{*2} + \frac{\gamma - 1}{2\gamma} [(W - u_p) - W] = (W - u_p) - W = u_p. \quad (\text{B.55})$$

Dividing by u_p gives:

$$\frac{\gamma + 1}{2\gamma W(W - u_p)} a^{*2} + \frac{\gamma - 1}{2\gamma} = 1$$

and solving for a^* :

$$a^{*2} = W(W - u_p). \quad (\text{B.56})$$

Combining eqn (B.40) with (B.56) gives:

$$\frac{\rho_2}{\rho_1} = \frac{W}{W - u_p} = \frac{W^2}{W(W - u_p)} = \frac{W^2}{a^{*2}} = M_s^{*2}, \quad (\text{B.57})$$

where M_s^* is the characteristic Mach number of the moving shock.

Recall eqn (B.39), which re-written and solved for M_s^* gives:

$$M_s^{*2} = \frac{(\gamma + 1) M_s^2}{2 + (\gamma - 1) M_s^2}. \quad (\text{B.58})$$

Substitute eqn (B.58) into (B.57):

$$\frac{\rho_2}{\rho_1} = \frac{W}{(W - u_p)} = \frac{(\gamma + 1) M_s^2}{2 + (\gamma - 1) M_s^2}. \quad (\text{B.59})$$

Using the momentum equation (B.41) in the form:

$$p_2 - p_1 = \rho_1 W^2 - \rho_2 (W - u_p)^2$$

and the continuity equation (B.40) yields:

$$p_2 - p_1 = \rho_1 W [W - (W - u_p)] = \rho_1 W^2 \left[1 - \frac{(W - u_p)}{W} \right]. \quad (\text{B.60})$$

Dividing eqn (B.60) by p_1 and using $a = \sqrt{\gamma p / \rho}$ in the form $a^2 = \gamma p / \rho$, we obtain:

$$\frac{p_2 - p_1}{p_1} = \gamma M_s^2 \left[1 - \frac{(W - u_p)}{W} \right]. \quad (\text{B.61})$$

Substituting eqn (B.59) into (B.61) gives:

$$\frac{p_2 - p_1}{p_1} = \gamma M_s^2 \left[1 - \frac{2 + (\gamma - 1) M_s^2}{(\gamma + 1) M_s^2} \right], \quad (\text{B.62})$$

which simplifies to:

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_s^2 - 1). \quad (\text{B.63})$$

Solving eqn (B.63) for M_s :

$$M_s = \sqrt{\frac{\gamma + 1}{2\gamma} \left(\frac{p_2}{p_1} - 1 \right) + 1}. \quad (\text{B.64})$$

and since $M_s = W/a_1$:

$$W = a_1 \sqrt{\frac{\gamma + 1}{2\gamma} \left(\frac{p_2}{p_1} - 1 \right) + 1}. \quad (\text{B.65})$$

Equation (B.65) is important because it relates the speed of the shock to the pressure ratio across the shock and the speed of sound of the air into which the shock is propagating.

The velocity of motion behind the shock wave is given by rearranging the continuity equation (B.40):

$$u_p = W \left(1 - \frac{\rho_1}{\rho_2} \right). \quad (\text{B.66})$$

Substituting eqns (B.49) and (B.65) into (B.66) and simplifying gives the next useful relationship:

$$u_p = \frac{a_1}{\gamma} \left(\frac{p_2}{p_1} - 1 \right) \left(\frac{\frac{2\gamma}{\gamma + 1}}{\frac{p_2}{p_1} + \frac{\gamma - 1}{\gamma + 1}} \right)^{1/2}. \quad (\text{B.67})$$

Discussion of the shock tube relations is nearly complete. The only aspect left unresolved is the expansion fan. This will be described next.

The continuity equation (A.23) (from Appendix A) is:

$$\frac{\partial}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0.$$

By recalling the vector identity $\nabla \cdot (a\mathbf{B}) = a\nabla \cdot \mathbf{B} + \mathbf{B} \cdot \nabla a$, where a is a scalar and $\mathbf{B}$ is a vector, the divergence term of the continuity equation can be expanded to give:

$$\frac{\partial \rho}{\partial t} + \rho(\nabla \cdot \mathbf{V}) + \mathbf{V} \cdot \nabla \rho = 0,$$

and rearranging gives:

$$\frac{\partial \rho}{\partial t} + \mathbf{V} \cdot \nabla \rho + \rho(\nabla \cdot \mathbf{V}) = 0.$$

The first two terms are the substantial derivative, and so the continuity equation becomes:

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{V}) = 0. \quad (\text{B.68})$$

Recall from thermodynamics that $\rho = \rho(p, s)$; using the chain rule:

$$d\rho = \left(\frac{\partial \rho}{\partial p} \right)_s dp + \left(\frac{\partial \rho}{\partial s} \right)_p ds, \quad (\text{B.69})$$

and the subscripts represent the quantities held constant. For isentropic flow $ds = 0$, so using eqn (B.33), eqn (B.69), in terms of the substantial derivative, becomes:

$$\frac{D\rho}{Dt} = \frac{1}{a^2} \frac{Dp}{Dt}. \quad (\text{B.70})$$

Substituting eqn (B.70) into (B.68):

$$\frac{1}{a^2} \frac{Dp}{Dt} + \rho(\nabla \cdot \mathbf{V}) = 0. \quad (\text{B.71})$$

Writing eqn (B.71) for one-dimensional flow:

$$\frac{1}{a^2} \left(\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} \right) + \rho \frac{\partial u}{\partial x} = 0. \quad (\text{B.72})$$

Following the same procedure as above for the continuity equation, but this time applied to momentum equation (A.30):

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = 0$$

or

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0. \quad (\text{B.73})$$

Adding eqns (B.72) and (B.73):

$$\left[\frac{\partial u}{\partial t} + (u + a) \frac{\partial u}{\partial x} \right] + \frac{1}{\rho a} \left[\frac{\partial p}{\partial t} + (u + a) \frac{\partial p}{\partial x} \right] = 0. \quad (\text{B.74})$$

Subtracting eqn (B.72) from (B.73):

$$\left[\frac{\partial u}{\partial t} + (u - a) \frac{\partial u}{\partial x} \right] - \frac{1}{\rho a} \left[\frac{\partial p}{\partial t} + (u - a) \frac{\partial p}{\partial x} \right] = 0. \quad (\text{B.75})$$

Equations (B.74) and (B.75) give $u = u(x, t)$ and $p = p(x, t)$, where (x, t) is any point in the x - t plane. From the general definition of a differential:

$$du = \frac{\partial u}{\partial t} dt + \frac{\partial u}{\partial x} dx. \quad (\text{B.76})$$

Arbitrary changes in t and x , dt and dx , give rise to a corresponding change in u , du , described by eqn (B.76). However, if instead of choosing arbitrary values of dt and dx specific values are chosen so that:

$$dx = (u + a) dt, \quad (\text{B.77})$$

the value of du which corresponds to dt and dx , constrained by eqn (B.77), is found by combining eqns (B.76) and (B.77):

$$du = \left[\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} (u + a) \right] dt. \quad (\text{B.78})$$

Similarly for dp :

$$dp = \left[\frac{\partial p}{\partial t} + \frac{\partial p}{\partial x} (u + a) \right] dt. \quad (\text{B.79})$$

Substituting eqns (B.78) and (B.79) into eqn (B.74) gives:

$$du + \frac{dp}{\rho a} = 0, \quad (\text{B.80})$$

where du and dp are changes along a specific path defined by the slope $dx/dt = u + a$ in the x - t plane. It is mentioned in passing that this technique is an example of the *method of characteristics* and will not be discussed in detail. The ordinary differential equation (B.80) is the compatibility equation along the C_+ characteristic, and eqn (B.80) holds *only* along this line.

From equation (B.75), another characteristic line C_- can be found, where the slope of the characteristic line is $dx/dt = u - a$, along which a second compatibility equation holds:

$$du - \frac{dp}{\rho a} = 0. \quad (\text{B.81})$$

The C_+ and C_- characteristic lines are physically the right- and left-running sound waves, respectively, in the x - t plane.

Integrating eqn (B.80) along the C_+ characteristic gives:

$$J_+ = u + \int \frac{dp}{\rho a} = \text{const} \quad (\text{along a } C_+ \text{ characteristic}). \quad (\text{B.82})$$

Integrating eqn (B.81) along a C_- characteristic gives:

$$J_- = u - \int \frac{dp}{\rho a} = \text{const} \quad (\text{along a } C_- \text{ characteristic}). \quad (\text{B.83})$$

Using the relationship for a calorically perfect gas: $a^2 = \gamma p / \rho$, in the form:

$$\rho = \frac{\gamma p}{a^2}, \quad (\text{B.84})$$

and since the process is isentropic, from eqn (B.20):

$$p = c_1 T^{\gamma/(\gamma-1)} = c_2 a^{2\gamma/(\gamma-1)}, \quad (\text{B.85})$$

where c_1 and c_2 are constants. Differentiating eqn (B.85) gives:

$$dp = c_2 \left(\frac{2\gamma}{\gamma-1} \right) a^{[2\gamma/(\gamma-1)-1]} da. \quad (\text{B.86})$$

Substituting eqn (B.85) into eqn (B.84) gives:

$$\rho = c_2 \gamma a^{[2\gamma/(\gamma-1)-2]} \quad (\text{B.87})$$

and substituting eqns (B.86) and (B.87) into eqns (B.82) and (B.83), we achieve the objective:

$$J_+ = u + \frac{2a}{\gamma-1} = \text{const} \quad (\text{along a } C_+ \text{ characteristic}), \quad (\text{B.88})$$

$$J_- = u - \frac{2a}{\gamma-1} = \text{const} \quad (\text{along a } C_- \text{ characteristic}). \quad (\text{B.89})$$

These final two relations provide the means to solve for the flow field properties in a one-dimensional expansion wave.

Consider the high- and low-pressure sections in Figure B.1, separated by a diaphragm. When the diaphragm is removed an expansion wave travels to the left, and the gas in region 4 feels as if a piston is being withdrawn to the right with velocity u_3 . The head of this left-running wave moves with local sound speed a_4 *towards the left*, extending the expansion into the stationary gas in region 4. A wave propagating into a constant-property region (region 4) is defined as a *simple wave*. A left-running simple wave also has straight C_+ characteristics along which the flow properties are constant.

From equation (B.88):

$$u + \frac{2a}{\gamma-1} = (\text{constant through the wave}). \quad (\text{B.90})$$

The constant can be evaluated by applying eqn (B.90) in region 4:

$$u_4 + \frac{2a_4}{\gamma-1} = 0 + \frac{2a_4}{\gamma-1} = \text{const}. \quad (\text{B.91})$$

Combining eqns (B.90) and (B.91):

$$\frac{a}{a_4} = 1 - \frac{\gamma-1}{2} \left(\frac{u}{a_4} \right), \quad (\text{B.92})$$

which relates a and u at any local point in a simple expansion wave. Because $a = \sqrt{\gamma RT}$, eqn (B.92) also gives:

$$\frac{T}{T_4} = \left[1 - \frac{\gamma - 1}{2} \left(\frac{u}{a_4} \right) \right]^2. \quad (\text{B.93})$$

Because the flow is isentropic $p/p_4 = (\rho/\rho_4)^\gamma = (T/T_4)^{\gamma/(\gamma-1)}$, (from eqn (B.22)). Hence, eqn (B.93) yields:

$$\frac{p}{p_4} = \left[1 - \frac{\gamma - 1}{2} \left(\frac{u}{a_4} \right) \right]^{2\gamma/(\gamma-1)} \quad (\text{B.94})$$

and

$$\frac{\rho}{\rho_4} = \left[1 - \frac{\gamma - 1}{2} \left(\frac{u}{a_4} \right) \right]^{2/(\gamma-1)}. \quad (\text{B.95})$$

These last four equations, eqns (B.92) to (B.95), give the properties in a simple expansion wave as a function of the local gas velocity. To obtain the variation of properties in a centred expansion wave, as a function of x and t , we need to consider the C_- characteristics. The equation of the C_- characteristic is:

$$\frac{dx}{dt} = u - a$$

or, because the characteristic is a straight line through the origin (the diaphragm location):

$$x = (u - a) t. \quad (\text{B.96})$$

Combining eqn (B.92) with (B.96):

$$x = \left(u - a_4 + \frac{\gamma - 1}{2} u \right) t$$

or

$$u = \frac{2}{\gamma + 1} \left(a_4 + \frac{x}{t} \right), \quad (\text{B.97})$$

which holds for the region between the head and tail of the expansion wave. In summary, for an expansion wave moving to the left, as shown in Figure B.2, eqn (B.97) gives the local mass velocity u as a function of x and t ; a , T , p and ρ are obtained by substituting u into eqns (B.92) to (B.95).

Referring to the shock tube pictured in Figures B.1 and B.2, initially a high-pressure gas is separated from a low-pressure gas by a diaphragm. In the situation considered here, the same gas (air) is in both ends of the shock tube, and the whole is at room temperature. Therefore, the ratio p_4/p_1 defines the strength of the shock and expansion waves that occur when the diaphragm is burst (at $t = 0$).

Everything that is needed to describe fully the flow properties at each moment in time ($t > 0$), for every part of the flow, is provided by the discussion above.

Applying eqn (B.94) between the head and tail of the expansion wave:

$$\frac{p_3}{p_4} = \left[1 - \frac{\gamma - 1}{2} \left(\frac{u_3}{a_4} \right) \right]^{2\gamma/(\gamma-1)} \quad (\text{B.98})$$

and solving eqn (B.98) for u_3 gives:

$$u_3 = \frac{2a_4}{\gamma - 1} \left[1 - \left(\frac{p_3}{p_4} \right)^{(\gamma-1)/2\gamma} \right]. \quad (\text{B.99})$$

However, since $p_3 = p_2$, eqn (B.99) becomes:

$$u_3 = \frac{2a_4}{\gamma - 1} \left[1 - \left(\frac{p_2}{p_4} \right)^{(\gamma-1)/2\gamma} \right]. \quad (\text{B.100})$$

Repeating eqn (B.67) for the motion induced by the incident shock, with $u_2 = u_p$:

$$u_2 = \frac{a_1}{\gamma} \left(\frac{p_2}{p_1} - 1 \right) \left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}} \right)^{1/2} \quad (\text{B.101})$$

and, because $u_2 = u_3$, eqn (B.101) and eqn (B.100) can be equated:

$$\frac{a_1}{\gamma} \left(\frac{p_2}{p_1} - 1 \right) \left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}} \right)^{1/2} = \frac{2a_4}{\gamma - 1} \left[1 - \left(\frac{p_2}{p_4} \right)^{(\gamma-1)/2\gamma} \right]. \quad (\text{B.102})$$

This can be rearranged to give:

$$\frac{p_4}{p_1} = \frac{p_2}{p_1} \left[1 - \frac{(\gamma - 1)(a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma[2\gamma + (\gamma + 1)(p_2/p_1 - 1)]}} \right]^{-2\gamma/(\gamma-1)} \quad (\text{B.103})$$

Equation (B.103) gives the shock strength p_2/p_1 as an implicit function of the diaphragm pressure p_4/p_1 .

The description of flow is now complete. The flow properties can be calculated from the diaphragm pressure p_4/p_1 as follows:

- i. Incident shock pressure ratio p_2/p_1 , and hence shock pressure p_2 , from the diaphragm pressure p_4/p_1 and eqn (B.103).
- ii. Incident shock temperature T_2 from eqn (B.48), and specific internal energy e_2 from $e = c_v T$.
- iii. ρ_2 is calculated from eqn (B.49).
- iv. The strength of the expansion wave is obtained from:

$$p_3/p_4 = \frac{\left(\frac{p_3}{p_1}\right)}{\left(\frac{p_4}{p_1}\right)} = \frac{\left(\frac{p_2}{p_1}\right)}{\left(\frac{p_4}{p_1}\right)}.$$

- v. The properties immediately behind the expansion wave can be found from the isentropic relations (eqn (B.22)) with a suitable change of subscript:

$$\frac{p_3}{p_4} = \left(\frac{\rho_3}{\rho_4}\right)^\gamma = \left(\frac{T_3}{T_4}\right)^{\gamma/(\gamma-1)}.$$

- vi. The properties inside the expansion wave can be found from eqns (B.92) to (B.95) and (B.97).

The procedure set-out above has been implemented as part of the one-dimensional program STanaly, described in Chapter 2.