

Cranfield University

DOCTORAL THESIS

**Novel applications and stabilisation for
widely wavelength modulated laser
interferometry for precision dimensional
metrology**

Author:
Kieran Wiseman

Supervisors:
Dr. Thomas Kissinger
Prof. Ralph P. Tatam

*A thesis submitted in partial fulfilment of the requirements
for the degree of Doctor of Philosophy*

in the

Centre for Engineering Photonics
School of Aerospace, Transport & Manufacturing

30th November 2021

Abstract

Novel applications and stabilisation for widely wavelength modulated laser interferometry for precision dimensional metrology

The need for precision dimensional metrological techniques is always increasing, including high precision displacement measuring metrology used to precisely and accurately position stages, components and machines. Laser interferometry is widely considered the most precise technique available for single dimensional displacement measurements, however extending this to multiple dimensions typically involves complex and costly interrogation systems. In this thesis, a series of novel multi-dimensional stage encoder designs and experimental results are presented as an application of a state-of-the-art metrological technique, that uses sinusoidal wavelength modulation and range-dependant signal processing to multiplex the signals from multiple interferometers into a single interferometric signal. Using range-resolved interferometry (RRI), a series of interferometers comprising measurements in multiple dimensions are multiplexed onto a single photodetector, which are then independently and concurrently demodulated and evaluated. This technique is applied to novel designs for a 2-dimensional displacement encoder, a 3-dimensional displacement encoder and a dual-beam angle encoder, where unlike prior work which has typically required complex optical setups involving polarisation-sensitive optical components and detectors, uses only minimal, simple bulk optic components to evaluate multiple dimensions simultaneously. In this work, nonlinearities below 1 nm along with typical displacement noise levels below $0.4 \text{ nm}/\sqrt{\text{Hz}}$ are presented for experimental results of $\pm 50 \text{ }\mu\text{m}$ controlled stage motions, showing results which are highly comparable to existing techniques.

Further to this, in order to make high precision results with confidence, high-stability lasers are required. In non-modulated and weakly modulated wavelength regimes, there are a significant number of techniques available for laser stabilisation, for both short-term and long-term requirements and linked to highly stable references. However, for widely-wavelength modulated techniques such as RRI, where the modulation depth is several orders of magnitude greater than standard reference widths, the vast majority of existing techniques are either unsuitable or significantly less-effective. In this thesis, a novel technique for long-term stabilisation of a widely wavelength modulated laser to a high stability reference standard is presented. Introduced as "swept absorption line locking", the principle behind this technique, in addition to experiments to test the efficacy of this technique are presented. Included in this, is comparison to a highly-stabilised helium-neon laser, which is often considered a "gold-standard" for stability, where co-linear displacement measurements of a Michelson interferometer are performed and compared to determine the RRI evaluation wavelength. The results presented here show fractional stabilities as low as 6×10^{-8} over timescales of ~ 1000 seconds using this novel stabilisation technique, and the comparison to the HeNe laser interferometer shows good agreement between the calculated RRI evaluation wavelength and the expected stabilised value within calculated uncertainties.

Acknowledgements

Firstly, I would like to thank Prof. Ralph Tatam, whose experience and knowledge has been invaluable throughout this PhD, for the opportunities you have afforded me, as well as your guidance as a supervisor. I would also like to thank Dr. Thomas Kissinger for his constant support and guidance as a supervisor, both in terms of academic work and my own personal development. Thank you for affording me a significant amount of your time, as well as your knowledge and insight, and for your patience with my many, many questions.

I would like to thank all of the Engineering Photonics staff and students who in one way or another supported and helped me throughout this PhD, both as academic colleagues sharing knowledge and advice, and as friends around a coffee table sharing wisdom, stories and a slice of cake. In particular, thank you to Steve Staines, whose technical advice and expertise has helped put theory into practice on many occasions. Thank you to all the PhD students, graduated and not, who made my PhD student experience all the more enjoyable. In particular, thank you to James Bremner and Sam Gibson who put up with the most from me, and went above and beyond as friends.

Thank you to my family, in particular my parents Helen and Chris and my sister Louise, who have supported me throughout and encouraged me every step of the way. This would not have been possible without you. Finally, I would like to thank my wife Rebekah, whose kindness, support and encouragement have been unyielding. Thank you for listening patiently when I was excited about something going well, and knowing just what to say when it hadn't.

Contents

Abstract	iii
Acknowledgements	v
List of Figures	xi
List of Tables	xix
List of Symbols	xxi
1 Introduction	1
1.1 Multi-Dimensional Interferometric Encoders	1
1.2 Wavelength Stabilisation for Optical Metrology	2
1.3 Aims and Objectives	2
1.4 Contents of this Thesis	3
2 Review of Displacement Measuring Techniques	5
2.1 Concepts of Metrology	5
2.2 Displacement Measuring Metrology	7
2.2.1 Non-Optical Techniques	8
Hall Effect Sensors:	8
Capacitive Sensors:	9
Linear Variable Differential Transformers	10
Piezoresistive Sensors	11
2.2.2 Optical Techniques	12
Linear Optical Encoders:	12
Phase-evaluating Laser Interferometry	13
Absolute Interferometry:	15
2.2.3 Extension to multiple dimensions	17
Contact Techniques	18
Non-contact techniques	18
2.3 Phase Evaluating Interferometry	19
2.3.1 Principles and Theory	19
Theory of two-wave interference	20
2.3.2 Homodyne Interferometry	23
2.3.3 Heterodyne Interferometry	23
2.3.4 Wavelength-modulated Interferometry (Pseudo-Heterodyne) . . .	24
Linear pseudo-heterodyne modulation	25
Sinusoidal pseudo-heterodyne modulation	26
2.3.5 Uncertainty in Interferometry	27

	Cyclic Errors	27
	Wavelength Uncertainty	29
	Thermal and Refractive Index errors	29
	Cosine Error	30
	Abbe Error	31
2.4	Summary	32
3	Review of Wavelength Stabilisation Techniques	35
3.1	Fundamentals of Stabilisation	35
3.1.1	A note on Wavelength vs. Frequency	36
3.1.2	Principle of Stabilisation	36
3.1.3	Short Term vs. Long Term Stabilisation	37
3.2	Active Stabilisation	38
3.2.1	Frequency References	38
	Optical Cavities	38
	Fabry-Perot étalon	38
	Atomic Absorption References	39
	Comparison	40
3.2.2	Stabilisation Techniques	41
	Harmonic Frequency Stabilisation	41
	Pound-Drever-Hall Technique	42
	Tilt-locking	43
	Self-Injection Locking	44
	Dual Mode Polarisation Stabilisation	45
	Saturated Absorption Spectroscopy	46
	Modulation Transfer Spectroscopy	48
3.3	Stabilisation in widely-modulated schemes	49
3.3.1	Widely-modulated techniques	50
	Frequency-scanning interferometry	50
	Range-resolved interferometry	50
3.3.2	Limitations of Existing Techniques	50
3.4	Summary	51
4	Range Resolved Interferometry	53
4.1	Operating Principles	53
4.1.1	Overview	53
4.1.2	Method	54
	Modulation	56
	Detection	57
	Windowing	58
	Complex Carrier	58
	Evaluation	59
4.2	Outputs of RRI	59
4.2.1	Mode 1: Signal Range Analysis	60
4.2.2	Mode 2: Phase Evaluation	61
4.3	Implementations	63
4.3.1	Pre-developed Systems	63

	Implementation 1 - 49kHz RRI Interrogation Unit	63
	Implementation 2 - 24 kHz RRI Interrogation Unit	65
4.3.2	Developed Systems	65
	Implementation 3 - 1.2 kHz RRI Stabilisation Development Platform	65
	Implementation 4 - 1.2kHz RRI Comparison Platform	67
4.4	Summary	68
5	Multi-dimensional Interferometric Encoders	71
5.1	Introduction	71
5.1.1	Motivation	71
5.1.2	Presented Developments	72
5.1.3	In this Chapter	72
5.2	Development of 2-dimensional encoder	73
5.2.1	Managing Optical Path Differences	73
5.2.2	Optical Design	75
5.2.3	Interrogation Hardware	78
5.2.4	Measurements and Results	78
	Single Axis Step Measurements	78
	Single Axis Sweep Measurements	81
	Circular Motion	82
5.3	Extension to 3 Dimensions	84
5.3.1	Beamsplitter Constraints in 3D	84
5.3.2	Extended 3D Design Criteria	85
	Design Considerations	87
5.3.3	Optical design for 3 dimensions	94
5.4	Measurements and Results	97
5.4.1	Interferometer Referencing	97
5.4.2	Single Axis Stage Motions	98
5.4.3	Helix Motions	101
5.4.4	Strain Gauge Comparison	104
5.5	Noise and Linearity Analysis	105
5.5.1	Noise Analysis	105
5.5.2	Linearity Analysis	108
5.6	Dual-Beam Angle Encoder	110
5.6.1	Principle and Design	110
5.6.2	Experimental Results	113
5.7	Discussion & Further Work	117
5.8	Summary	119
6	Widely-Wavelength Modulated Swept Absorption Line Locking	121
6.1	Motivation	121
6.2	Principles and Methods	121
6.2.1	Stabilisation Process	122
	Laser Emission	122
	Interaction with Wavelength Reference Cell/Line	122
	Measurement of Interacted Light	123
	Signal Analysis	124

	Wavelength Tuning	127
6.3	Experimental Work	130
6.3.1	Experimental Principles	130
6.3.2	Experimental Setup	131
6.3.3	Wavelength Stabilisation	132
6.4	Results	134
6.4.1	Control Loop Tuning	134
6.4.2	Temperature Drift Compensation	136
6.4.3	Long term drift comparison	137
6.4.4	Direct disturbance measurements	138
6.4.5	Allan Deviation	141
6.5	Discussion and Future Work	143
6.6	Summary	144
7	HeNe Laser Interferometer Comparison	147
7.1	Principles and Motivation	147
7.2	Experimental Setup	148
7.2.1	Optical Setup	148
7.2.2	Interrogation Hardware	151
7.2.3	Stabilisation	152
7.3	Experimental Method	154
7.3.1	Signal Analysis	154
	HeNe Signal Processing	155
	RRI Signal Processing	156
	Wavelength Estimation	157
7.4	Results	157
7.4.1	Experimental Results	157
	10 mm Stage Motion	158
	Phase Repair	161
7.4.2	Experimental Limitations	164
	Experimental Setup Limitations	165
	Hardware Limitations	165
	Software Limitations	166
7.4.3	Uncertainties	166
	Phase Uncertainty	166
	Wavelength Uncertainty	167
	Cyclic Errors	167
	Thermal and Refractive Index Errors	168
	Summary of Uncertainties	168
7.5	Discussion and Further Work	169
7.6	Summary	170
8	Conclusions and Outlook	173
8.1	Multi-dimensional Interferometric Encoders	173
8.2	Widely-wavelength modulated stabilisation	177
	List of Publications	181

List of Figures

2.1	Illustration of a Hall effect element and sensor with an example output . .	8
2.2	Illustration of different types of capacitive sensor	9
2.3	Cross-section of a typical LVDT displacement sensor (adapted from [32]).	10
2.4	Cross-section design of a typical piezoresistive semiconductor bridge, arranged along the x axis (reproduced from [4]).	11
2.5	Example design of a incremental linear optical encoder using an index grating and reflective grating (Adapted from [3])	12
2.6	Illustrations of two different in-fibre interferometer configurations. For each, the laser diode and photodetector (Pd) is labelled along with the intended direction of light through the fibre.	
	a) Mach-Zehnder b) Michelson	14
2.7	Example interferometric transfer function with equation $I = 1 + A \cdot \cos \theta$.	14
2.8	Illustration of an example dual optical comb interferometer setup using a Michelson interferometer. Adapted from [55]	16
2.9	Illustration of a typical setup for spectral interferometry featuring a single optical frequency comb source with a Michelson interferometer. Notably the inclusion of a dispersion compensating plate ensures each of the measured frequency components has travelled the same path length before interfering. Adapted from [56]	16
2.10	Illustration of a free-space Michelson interferometer using bulk optic components. The setup consists of a coherent source such as a laser, a reference mirror (M_r) and a beamsplitter plate (B_s), with a movable measurement mirror (M_m). The signals from both arms interfere at a photodetector (Pd) or screen	20
2.11	(a) Illustration of an interferometric transfer function for two-wave interference with quadrature points labelled (b) The transfer function at quadrature points, where the phase signal is linearly transferred (c) The transfer function at a turning point where non-linear transference of the phase signal occurs. (Adapted from [75, 76]).	22
2.12	Illustration of quadrature demodulation for a single-carrier demodulation scheme (adapted from Kissinger [76])	25
2.13	Illustration of cyclic errors in a Lissajous plot. a) shows no cyclic errors and the measured phase vector (blue) forms a perfect circle over 360° displacement. b) shows the case with an additional phase vector (red), causing a resultant phase vector (orange).	28
2.14	Illustration of how cyclic errors contribute to non-linear evaluation of the interferometric phase.	28

2.15	Illustration of cosine errors originating from a corner cube retroreflector. The red line indicates the beam path incident at angle θ , with the dashed version representing the beam when the corner cube is displaced. The dotted central line indicates the direction of motion. (Adapted from [5])	31
2.16	Illustration of Abbe errors δD_{Abbe} originating from a corner cube retroreflector mounted on a moving stage with the measurement POI denoted. The displaced stage shows an angular error θ , with the lateral separation of the measurement axis and the motion axis denoted as Ω . (Adapted from [5])	32
3.1	Typical process for stabilising a laser diode	36
3.2	Absorbance spectrum for acetylene gas (C_2H_2) in the near infra-red at 256K (data from HITRAN [128])	40
3.3	Example setups for harmonic frequency stabilisation using (a) a gas vapour cell (b) a Fabry-Perot cavity as frequency references. (Adapted from [130]).	42
3.4	Example of a Pound-Drever-Hall experimental setup using a Fabry-Perot étalon as a reference cavity. This setup uses a polarising beamsplitter (PBS) and $\lambda/4$ waveplate to separate the returning light from the emitted light. (Adapted from [132]).	43
3.5	Example of a single-pass tilt-locking experimental setup using a ring cavity as a reference. (Adapted from [137]).	44
3.6	Example of a self-injection locked laser setup using a ring resonance cavity (Adapted from [141]).	45
3.7	Example setup of a dual mode polarisation stabilisation technique.	46
3.8	Illustration of an example saturated absorption spectroscopy setup with a pump beam shown in red and a probe beam shown in blue using two polarising beamsplitters (PBS)	47
3.9	Illustration of a two energy state system interacting with the a) pump beam b) probe beam	47
3.10	Saturated absorption spectrum for the D2 transition lines of ^{85}Rb and ^{87}Rb showing the stimulated emission dips in the Doppler-broadened peaks. Graph taken from Glaser <i>et al.</i> [147].	48
3.11	Illustration of a modulated transfer spectroscopy setup using an acousto-optic modulator and injection current tuning of the laser diode. The pump beam is shown in red, and the probe beam is shown in blue.	49
4.1	Example of an unbalanced fibre interferometer in a Mach-Zehnder configuration.	54
4.2	Example signals for each step of the modulation and range-resolved interferometric demodulation process.	55
4.3	Example unbalanced interferometer with a reference arm of length X and 3 arms with varying optical path lengths, labelled in rads.	60
4.4	Comparison of quadrature signal amplitudes over a range of demodulation phase carrier amplitudes for an example multiplexed interferometric signal. Peaks corresponding to three interferometers with varying OPD can be seen.	60

4.5	Comparison of quadrature signal amplitudes over a range of demodulation phase carrier amplitudes for an example multiplexed interferometric signal.	62
4.6	(a) shows a simplified schematic of the interrogation hardware used by this technique and setup with key hardware components labelled. (b) shows a photo of the complete enclosed interrogation hardware.	64
4.7	Schematic of the interrogation hardware used in implementation 3 showing the connections between the various hardware components. The added stabilisation arm is marked by the dashed box.	66
4.8	Annotated photo of some of the hardware used in implementations 3 and 4.	67
4.9	Schematic of the interrogation hardware used in implementation 4 showing the connections between the various hardware components. Not shown is the second laser diode, whose sole connection is to a photodiode.	68
5.1	(a) shows a basic example system with a fibre collimator and glass block. The points of reflection are indicated and the reflecting surfaces labelled. (b) shows an example of a graph of signal strength against optical path difference with some of the potential interferometers created by this system labelled. (c) shows the same situation as (a), but with a greater distance between the collimator and the glass block, resulting in the example signal strength against optical path difference graph in (d).	74
5.2	(a) shows a "top-down" schematic view of the encoder, with the geometric distances of the principle interferometers of interest highlighted and labelled as α and β respectively. (b) shows a 3D view of the encoder schematic.	76
5.3	Photo of the 2D encoder design. (a) shows a photo from the side view of the encoder with a top down view of the encoder shown in (b). The major components and the geometric distances of the measured interferometers, α and β are labelled.	77
5.4	Simultaneous measurements of X and Y displacements for motion in the X-direction of the stage followed by a similar motion in the Y-direction. (a) shows a linear movement across the total range of the stage, $\pm 50\mu\text{m}$, at a speed of $10\mu\text{m/s}$, while (b) shows the results of a similar motion to (a) but for a smaller stage movement across $\pm 5\mu\text{m}$ at velocities of $1\mu\text{m/s}$. (c) then shows a similar plot for $\pm 0.5\mu\text{m}$ movements at velocities of $0.1\mu\text{m/s}$	79
5.5	Displacement measurements for X and Y stage motions during sequential step motions of the stage with a restricted displacement axis. (a) shows the data for steps of $\pm 50\mu\text{m}$, at a velocity of $10\mu\text{m/s}$, with (b) showing displacement data for steps of $\pm 5\mu\text{m/s}$ at velocities of $1\mu\text{m/s}$. (c) then shows displacement data for steps of $\pm 0.5\mu\text{m}$ at velocities of $0.1\mu\text{m/s}$	80
5.6	(a) shows the X against Y displacement measurements during programmed motions in the X-direction, where the Y axis should remain static. (b) shows the X against Y displacement measurements during programmed motions in the X direction	81

- 5.7 Displacement data for 2D circular stage motion. (a) shows the measured displacement components in the X and Y measurement dimension against both time and the angle moved around the complete circle (b) shows the same data projected onto an X-Y plane in a Cartesian coordinate system. (c) shows the measured displacements in a polar coordinate system with a restricted radial axis, where the plotted radii are the magnitudes of the X and Y components. 83
- 5.8 (a) shows the first beam of a particular interferometer, passing through the beamsplitter and reflecting from a planar mirror (i), and back towards the source. (b) shows a second beam, reflecting from the rear face of the beamsplitter (ii) at the air-glass boundary and back towards the source. At the splitting medium this also reflects towards the other rear surface of the beamsplitter (iii) and reflects from this surface and back towards the source, denoted by the dotted beam. Also labelled is the front surface of the beamsplitter (iv) 85
- 5.9 Design of a 3-dimensional encoder. (a) shows the top-down view with the relevant geometrical distances highlighted. (b) shows the side-view with the relevant geometrical distances highlighted. In both figures, distances α , β and γ correspond to the interferometers created between the X, Y and Z mirrors, labelled as M_x , M_y and M_z , respectively, interfering with the Fresnel reflection from the flat-polished fibre-tip. δ refers to the distance of the reference interferometer between the first incident surface of the first beamsplitter, while ϕ corresponds to path lengths of the undesired interferometers between any adjacent pair of beamsplitter surfaces. (c) shows a 3D illustration of the optical design and (d) shows which surface of the beamsplitters is polished to remove the anti-reflective coating, in the same orientation as in (c). The surfaces not explicitly shown in the diagram are anti-reflective coated. 86
- 5.10 Explanation of how split path beams create duplicate optical path differences for a 2D setup. A, B are the geometric distances from the fibre-tip, O. (a) shows a split path beam, emitted from the fibre collimator, passing through the optical system down one path, reflecting from the fibre tip and passing through the optical system down an alternate path before returning to the fibre, with a total effective path length of A+B. (b) shows a single beam emitted from a fibre collimator, passing through an optical system and returning to the fibre collimator, with an effective path length of A. (c) shows a single beam emitted from a fibre collimator, passing through an optical system and returning to the fibre collimator, with an effective path length of B. (d) shows an example of expected interferometers created between the 3 optical beams and a fibre tip reflection, as well as interferometers created with each other, as a function of the resultant optical path difference. For ease of understanding, the reflected beams in (a),(b) and (c) have been spatially offset from the incident beams. 90

- 5.11 (a) shows an interferogram (blue) and an overlaid window function (red), with a width parameter of $\sigma_{win} = 0.04$, used in the RRI demodulation process. (b) shows the same interferogram (blue) overlaid with a window function with a width parameter of $\sigma_{win} = 0.01$. (c) shows a portion of a phase carrier amplitude graph, demodulated from the interferogram using the window function shown in (a). (d) shows the same portion of a phase carrier amplitude graph, demodulated from the same interferogram using the window function shown in (b) 92
- 5.12 (a) shows a top-down view of the optical design schematic with principle beam paths, α , β , γ and δ highlighted and the geometric distances of interest φ and ψ labelled. Also labelled is the front surface R of the beam-splitter (b) shows the same design from a side view and (c) shows a 3D representation of the schematic. (d) shows the expected 'ranging' view that would be created by the RRI technique, with the interferometers of interest in colour, and parasitic interferometers in black. 96
- 5.13 Figure (a) shows raw displacement measurements for the four measured interferometers, α , β , γ and δ during a series of stepped motions with an amplitude of $\pm 50 \mu\text{m}$ for each stage motion axis sequentially. The reference interferometer δ is then subtracted from the three dimensional interferometers to give the relative stage motion components in X, Y and Z, shown in (b). The measured displacements for the same motion pattern with a stage motion amplitude of $\pm 5.0 \mu\text{m}$ are shown in (c) where the reference interferometer has already been subtracted. (d) again shows the measured displacements for the same motion pattern but for a stage motion amplitude of $\pm 0.5 \mu\text{m}$ 99
- 5.14 Restricted view of displacement measurements for a series of stepped motions over a range of amplitudes. (a) shows measured stage displacements for stage motions of $\pm 50 \mu\text{m}$ showing a restricted view of the y-axis. (b) shows measured displacements for the same series of stage motions but with amplitudes of $\pm 5.0 \mu\text{m}$ and again, a restricted y-axis range. (c) shows yet more measured displacements for stepped stage motions, but with a stage motion amplitude of $\pm 0.5 \mu\text{m}$ 100
- 5.15 Displacement measurements of helical stage motions. (a) shows a 3-dimensional plot of the X, Y and Z measured displacements for a circular motion in the X-Y plane, coupled with a linear motion in the Z axis. (b) shows the same data flattened in the X-Y plane in polar form, with a restricted radial axis. (c) shows then data for circular motion in the Y-Z plane, with linear motion in the X axis, with (d) showing the same data flattened on the Y-Z plane in polar form. (e) shows data for circular motion in the X-Z plane, with linear motion in the Y axis, and (f) shows this same data set flattened on the X-Z plane in polar form. 103
- 5.16 (a) shows measured displacements from the interferometer and from the integrated stage encoder of a $\pm 15 \mu\text{m}$ triangular motion in the X direction, while (c) and (e) show the corresponding data for the Y and Z directions, respectively. For all three directions the calculated differences between stage encoder and interferometer are plotted below the respective graphs in (b), (d) and (f). 104

5.17	(a) shows displacement data for X, Y and Z for a simple sinusoidal motion of $\pm 5 \mu\text{m}$ movement in the X-direction. (b) shows the calculated displacement amplitude spectral density estimate over a 1 s bandwidth for the measured X, Y and Z directions. Additionally, the direct measurement of interferometer α in Fig. 5.9 without the subtraction of the reference interferometer δ , is shown for comparison, where this signal is stationary regardless of stage movement and is thus a direct measure of the instrumental noise.	106
5.18	(a) shows an example of one period of one of the triangular motion measurements used in this experiment and highlights the 1 s analysis region evaluated. (b) shows the displacement noise spectral density estimate for a $50 \mu\text{m/s}$ movement, while (c) shows this for a $70 \mu\text{m/s}$ movement. In both (b) and (c) the determined cyclic error contributions occur at different frequencies and their extracted displacement amplitudes are labelled.	109
5.19	(a) Side-on schematic of the 2 beam displacement and angle encoder including a close view of the incident points at the mirror. Key dimensions and locations have been labelled and geometry such as mirror tilt has been exaggerated for effect. (b) Photo of the optical setup	111
5.20	Amplitude against phase carrier amplitude graph for the 2-beam angle encoder design, where phase carrier amplitude is analogous to optical path difference. For the main peaks OPD has been labelled and for the interferometers of interest, highlighted.	113
5.21	(a) measured displacements from the three interferometers of interest. (b) displacements from measurement interferometers after the reference interferometer measurement has been numerically subtracted (c) calculated resultant angle change from the interferometer data compared to data from the pitch-yaw stage in-built encoder.	114
5.22	(a) Calculated angle changes from the interferometer and an in-built stage encoder. Also marked are the sample locations for direct comparison between the two data sets. (b) direct comparison of the encoder data at the sampled locations to the interferometer data. A linear best fit is also shown.	115
6.1	Absorption data for acetylene gas in the near infrared region. Data from [170].	123
6.2	Photodetector reading for a single modulation period of a laser sinusoidally modulated through an acetylene gas cell.	124
6.3	Process of calculating the absorption for a laser diode emission wavelength modulated across a gas line showing; (a) Photodetector signal (b) First derivative of the photodiode signal with selected thresholds (c) Photodiode data after selections of data around the threshold crossings are removed and sinusoidal fit (d) Calculated absorption	126
6.4	Photodetector signals for a sinusoidally modulated laser passing through a sealed gas cell over a single gas absorption line for a range of laser temperature setpoints.	127

6.5	(a) Photodetector signal for a wavelength modulated laser diode passing through a gas cell with fitted intensity correction in the case where the stabilisation system is unlocked (b) Resultant absorption signal for the unlocked condition showing Voigt fits applied to the relevant peaks (c) Photodetector signal for a wavelength modulated laser diode passing through a gas cell with fitted intensity correction in the case where the stabilisation system is locked (d) Resultant absorption signal for the locked condition showing Voigt fits applied to the relevant peaks	128
6.6	A demonstration of the crossover between sampling and the setpoint adjustment period.	130
6.7	A schematic of the setup used in these experiments. The interferometer arms within the ice bath are affixed to an aluminium plate which is not shown.	132
6.8	Absorption data for acetylene gas in the near infrared region with the absorption line of interest labelled. Data from [170].	133
6.9	Swept absorption line locking process. (a) shows the averaged photodiode data and the sinusoidal intensity fit. (b) shows the absorption after the intensity modulation has been removed.	134
6.10	Measured phase changes during control loops as it re-establishes the locked condition, with a range of control loop parameters.	135
6.11	Phase change measurements of a Mach-Zehnder interferometer, secured to a aluminium plate, in varying conditions. These are; exposed to air, submerged in water and submerged in water with the addition of ice chips.	136
6.12	Measured phase changes of a Mach-Zehnder interferometer without (free-running) and with (stabilised) the swept absorption line locking applied.	138
6.13	Direct disturbance of a laser diode by a continuous, radiant heat source between the indicated 15 and 75 minute marker showing (a) the phase measurement of a Mach-Zehnder interferometer (b) the thermistor reading from the laser TEC.	140
6.14	Allan deviations for a range of integration times for stabilised and free-running conditions.	142
7.1	Schematic of the optical setup used for displacement measurements of linear stage motion using a Michelson interferometer interrogated with a 633 nm HeNe laser and a 780 nm diode laser. Notably the 780 nm laser diode has an additional branch for application of the stabilisation technique	149
7.2	Photo of the bulk optic Michelson interferometer used in the HeNe comparison experimental setup with key components labelled.	150
7.3	Photo of the in-fibre components leading to the bulk optic interferometer from the laser diodes in the HeNe comparison experimental setup with key components labelled.	151
7.4	Schematic of the hardware used for comparison between a 633 nm HeNe and a stabilised 780 nm diode laser	152
7.5	(a) Energy level diagram of the rubidium D-line transitions and hyperfine ground state structure. (b) Absorption spectrum for rubidium vapour in nominal conditions. The energy level transitions for ^{85}Rb and ^{87}Rb are labelled. Figures adapted from Ma <i>et al.</i> [178]	152

7.6	(a) Average photodiode signal and fitted sinusoidal intensity waveform (b) normalised absorption signal for the averaged modulation period . . .	154
7.7	A sample of a HeNe photodetector data showing all of the zero crossings and the averaged 'true' zero crossings	155
7.8	Illustration of the uncertainty in the initial RRI phase for two modulation periods.	156
7.9	Samples of interferograms for (a) a HeNe interferometer with a rolling average applied and with zero-crossings marked (b) widely-modulated laser diode interferometer.	158
7.10	Discrete fast Fourier transform of the HeNe fringes after a rolling average has been applied.	159
7.11	Sample of the HeNe interferometric signal after a low pass filter at 150 Hz has been applied. The zero-crossings are marked as black crosses.	160
7.12	Phase change measurements over a 10 mm stage motion from both the HeNe interferometer (green) and widely-modulated laser diode interfer- ometer (blue)	160
7.13	Residuals of phase measurements for RRI and HeNe phase measurements after a linear fit of the phase is subtracted from both.	161
7.14	Sample of the unwrapping steps for the RRI phase signal, with insets showing (i) a successful unwrapping step and (ii) an incorrect unwrap- ping step	162
7.15	Sample of the uncorrected and corrected phase unwrapping of the RRI phase signal	162
7.16	Sample of the filtered HeNe interferometric signal at the time of a discreet jump in measured phase.	163
7.17	Residuals of phase measurements for the corrected RRI and HeNe phase measurements after a linear fit is removed.	164
7.18	Corrected phase change measurements of a HeNe interferometer and pseudo- heterodyne interferometer for a 10 mm linear stage motion	164
8.1	Concept for a multi-degree-of-freedom sensor using GRIN lenses in a tri- angular optical head	176

List of Tables

2.1	Summary of a selection of techniques for displacement measuring applications. Values derived where appropriate from [4, 5, 7, 11].	17
2.2	Comparison of existing displacement measuring technologies for multi-dimensional measurements and the requirements imposed by these. . . .	19
2.3	Summary of the main uncertainty sources in interferometric displacement measurements	27
4.1	Summary of the RRI implementations used in this thesis. Where realistic, the values stated are the maximum achievable values. In the cases marked (*), these are the maximum tested values.	63
5.1	Types of beam found in a 2-dimensional encoder design using a beam-splitting element, categorised by the optical path of the beam. The diagrams in column 1 show reflection points and spatially offset beams for illustrative purposes only.	88
5.2	Summary of the geometric conditions applied to the optical design to minimise crossover of optical path differences	94
5.3	Summary of the dominating frequency contributions to the displacement spectral density amplitudes	108
7.1	Summary of uncertainties affecting the comparison between HeNe and RRI measurements	169
8.1	Key parameters from the work on multi-dimensional interferometric encoders presented in this thesis	175

List of Symbols

Symbol	Name
σ	Standard Deviation
a	Area
κ	Dielectric constant
C	Capacitance
d	Distance
R	Resistance
t	Time
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	Tensile stress components
π_L, π_T	Piezoresistive coefficients
L	Optical path length
N	Number of fringes
ν_r	Free-spectral range (frequency)
ν	Frequency
U	Amplitude
ϕ	Phase
I	Intensity
$\gamma(\tau)$	Complex degree of coherence
V	Visibility
$U(t)$	Photodetector signal
R_{PD}	Responsivity of detector
$\Phi(t)$	Interferometric phase signal
$\Theta(t)$	Applied phase signal
f_m	Modulation frequency
$U_Q(t)$	Quadrature signal
f_c	Carrier frequency
A_s	Modulation amplitude
δf	frequency modulation width
θ	Angle or Phase
D	Displacement
Ω	Lateral Separation
S	Stability
λ	Wavelength
E	Photon Energy
τ	Time delay
$\alpha, \beta, \gamma, \delta$	Interferometer labels
φ	Parasitic optical path length
Q	Quadrature Bandwidth

$\epsilon\gamma_t$
 A Molar absorption coefficient
Absorption

*To those no longer with us, gone but never forgotten.
I hope I've made you proud.*

Chapter 1

Introduction

1.1 Multi-Dimensional Interferometric Encoders

The state of displacement measuring metrology has implications and applications far beyond the extent of the laboratory. Modern production and manufacturing techniques for example, used to produce nano-scale electronic and optical components such as transistors, require high-precision machinery capable of accurate positioning at nanometre levels. A variety of techniques such as capacitive sensors, piezoresistive sensors and linear optical encoders have been developed for these purposes, however among these techniques, phase-evaluating laser interferometry stands out for its significance. Offering a high precision, resolution and measurement range, as well as remote, non-contact measurements of target surfaces, phase-evaluating laser interferometry is highly attractive for a large range of applications.

In many applications however, multi-dimensional measurements are required, and the extension of these techniques for multiple dimensions offer unique challenges and limitations that impact their suitability and efficacy in these applications. Applications such as positioning stages used in nanotechnology manufacturing, for example, often move in multiple dimensions at high speeds and require equally fast, dynamic displacement measuring techniques. In these applications, non-contact techniques such as laser interferometry are highly beneficial, however they require complex setups to interrogate multiple dimensions simultaneously. In laser interferometry, this complexity typically arises from a need to separate spurious or unwanted phase signal contributions from the interferometric signals of interest, which is compounded as the technique is expanded into multiple dimensions.

Recent advances in range-sensitive interferometric signal demodulation have opened up avenues for research into the multiplexing of multiple interferometers into a single interferometric signal, which can then be demodulated in a range-dependant manner and independently evaluated. "Range-resolved interferometry" as the technique has been named, utilises sinusoidal wavelength modulation of a laser diode, combined with range-dependant complex carrier signal processing to independently and synchronously

evaluate multiplexed interferometers, separated from each other by an optical path difference, to measure the relative displacements at each interferometer. Using RRI, several interferometers can be multiplexed into a single interferometric signal and independently interrogated, which means that very compact designs for multi-dimensional optical encoders are possible. Furthermore, the interferometric signals can be demodulated in a selective manner that reduces issues that interferometric techniques typically suffer from, such as non-linearities caused by parasitic interferometers. As such, there is an opportunity for the development of novel multi-dimensional interferometric encoders, that benefit from the advantages of standard laser interferometry such as high measurement resolution and range, with reduced cost and complexity, which would make them highly attractive for commercial and industrial application.

1.2 Wavelength Stabilisation for Optical Metrology

In nearly all laser-based applications, including but not limited to optical metrology, an understanding of the laser emission characteristics, in particular wavelength, or frequency, is important. In laser interferometry however, it becomes essential as measurements are inherently linked to the evaluation wavelength of the laser. As well as knowing the emission wavelength at the start of a measurement, the wavelength during a measurement, particularly over extended measurements periods is important. In order to make high-accuracy and high-precision dimensional measurements, metrological techniques such as interferometry require highly-stabilised lasers and a wide range of techniques have been developed for laser stabilisation in a variety of applications and conditions. These techniques typically use locking to well-known or characterised stabilisation features, occurring naturally such as atomic gas absorption lines, or artificially created such as optical cavities.

In general the applications these techniques are designed for, use little to no modulation of the laser emission wavelength, however some applications, such as the range-resolved interferometric technique mentioned previously use a wider wavelength modulation, often with an amplitude more than an order of magnitude beyond typical stabilisation features. Characterisation and stabilisation of the lasers in these applications can prove difficult or imprecise, and as such novel techniques for stabilisation in widely wavelength modulated regimes need to be developed and characterised.

1.3 Aims and Objectives

The overall aim of this thesis is to detail a comprehensive development of novel techniques for, and implementations of, displacement measuring interferometry for precision dimensional metrology. To this end, there are two main avenues of research presented here; displacement measuring interferometry and wavelength stabilisation techniques.

Towards displacement measuring interferometry, the recently developed range-resolved interferometric (RRI) signal processing technique has created opportunity for the development of novel, multi-dimensional interferometric encoders that benefit from the advantages of laser interferometry, such as high resolution and range, but without some of

the disadvantages that typically limit the applications of laser interferometry.

Towards wavelength stabilisation, there is a need to develop novel techniques for wavelength stabilisation in widely wavelength modulated regimes. For precision techniques such as RRI, where measurements are intrinsically linked to the emission wavelength, stabilisation becomes critical in reducing the uncertainty in the emission wavelength and therefore the measurement uncertainty. Existing techniques however are less effective in wider wavelength modulation regimes, so novel techniques must be developed.

Further to this, verification of the developed techniques should be undertaken. This could be done via comparison to existing standards, such as a highly-stabilised helium-neon laser, measuring identical displacements using both the stabilised range-resolved interferometric technique and a standard helium-neon fringe-counting method.

The aims of the work presented in this thesis can therefore be summarised as:

- Develop novel multi-dimensional encoders using a range-resolved interferometric technique
- Develop a novel stabilisation technique for widely-wavelength modulated techniques
- Compare range-resolved interferometry stabilised using the developed technique, to a highly-stabilised He-Ne laser interferometer

1.4 Contents of this Thesis

The contents of each chapter of this thesis are as follows. Firstly, Chapters 2 and 3 discuss the context and background to the two main research areas of this thesis. In these chapters, the basic concepts relating to each subject area are first presented including relevant equations which can be later referred to, followed by discussion of pertinent existing techniques and developments published in academic literature. The scope of each review is then refined to a more specific discussion of the existing work in the research area of interest, highlighting the specific research gaps that the work presented in this thesis is targeting.

Following the literature review chapters, Chapter 4 details the principles and method of range-resolved interferometry and signal processing, a specific technique that is later applied extensively in the experimental chapters of this thesis. This is followed by technical summaries of the various implementations of range-resolved interferometry that are used in this thesis.

Chapters 5 and 6 detail the development and characterisation of novel applications and techniques, towards fulfilling the research gaps identified above and discussed in the literature review of existing work. In both chapters, the principles underpinning the experimental work are first discussed, followed by presentation of the experimental setups used. Experimental results are then shown and discussed, concluding with potential improvements to each experimental setup or concept that could be achieved in future work.

An additional third experimental chapter, chapter 7, presents experiments comparing the novel developed techniques and applications with a well-explored and traceable

laser wavelength stabilisation technique utilising a helium-neon laser. Here, concepts from the previous experimental chapters are combined, and preliminary experimental measurements made using this technique are presented for comparison. The experimental results are similarly presented, and potential uncertainties and improvements are discussed. Finally, a chapter concluding this thesis, summarising the work presented in it, and discussing the potential future outlook of this work is included.

The contents of the chapters of this thesis are summarised below:

Chapter 2 reviews techniques for displacement measurement in precision dimensional metrology, both optical and non-optical, and in particular how these are extended or otherwise developed for multi-dimensional displacement measurements. Further detail is then presented on laser interferometry and some of the signal processing approaches typically used.

Chapter 3 reviews techniques for wavelength stabilisation of laser-based applications in both short-term and long-term regimes, explaining typical techniques, stabilisation references and typical stability limitations.

Chapter 4 introduces range-resolved interferometric signal processing, a pseudo-heterodyne laser interferometric signal processing technique used to interrogate multiplexed interferometric signals. The principle behind this technique is summarised, followed by a description of the various practical developments implementing this technique, that have been used in experimental work presented in this thesis.

Chapter 5 presents the development of a series of multi-dimensional optical encoders, using laser interferometry and range-resolved interferometric signal processing. The principles behind the design and development of each encoder are presented, as well as experimental work and results characterising the capabilities of each approach.

Chapter 6 presents the development of a novel technique for wavelength stabilisation of widely-modulated laser diodes, such as those used in range-resolved interferometry. The principles underpinning the technique are detailed, followed by the design, undertaking and results of characterisation experiments applying this technique, which are then discussed.

Chapter 7 details experimental work comparing a highly-stabilised helium-neon laser to a widely-modulated laser diode stabilised using the presented novel stabilisation technique. The setup and results of these experiments are shown and then discussed.

Chapter 8 concludes this thesis. The experimental work and results presented throughout the thesis are summarised, and discussion is given to the outlook of this work beyond the scope of this thesis in potential future applications and work.

Chapter 2

Review of Displacement Measuring Techniques

This chapter serves as a broad overview of precision dimensional metrology, directly relating to the main techniques employed for distance and displacement measurements. Whilst a wide variety of techniques exist for all manner of measurands in precision metrology, here the focus is on techniques that directly measure the displacement or absolute distance of a target surface from a given origin. Furthermore, in line with the focus of this thesis on range-resolved interferometry, discussed further in Chp. 4, particular interest is given to techniques where parallels and comparisons to range-resolved interferometry can be drawn.

2.1 Concepts of Metrology

Across the wide range of metrological applications, the terminology used to describe the outputs and analysis of metrological techniques varies greatly. In an attempt to remain consistent when discussing these applications, the definitions used in this thesis for some of the key terms used have been stated below. Here, the focus is on dimensional metrology, so definitions have been considered with this as the primary application. In general, definitions have been guided by [1–6].

Repeatability: The extent of the spread of a measured value within a single experiment. Typically this is stated as a distribution with statistical bounds. For example, a data set could be described as a Gaussian distribution with a 1σ variance of $\pm X$, where $\pm X$ are the statistical bounds containing 95% of the data.

Reproducibility: The extent of the spread of a measured value between experiments. Unlike repeatability, this is calculated for a single measured value over multiple independent measurements. This is also commonly stated as a distribution within statistical bounds.

Precision: The total spread of a measured value. Whilst this is often conflated with accuracy or resolution, both of which are defined later, for precision metrological applications precision represents the extent of the spread of a measured value. In other words, it is a measure of how "close" a group of measured values is to one another.

Accuracy: How close to the "true" value a measured value is. This includes the effect of error on the measurement such as thermal noise, frequency noise, systematic measurement deviations and environmental effects.

Resolution: The smallest measurable unit of a measurement system. This is also the smallest measurable change a given measurement system could measure. Typically, this can be considered as one standard deviation of the system noise [7].

Data Rate: The total measurement samples per unit time, usually limited by the hardware capabilities.

Bandwidth: Depending on the context, this can refer to either optical bandwidth or measurement electrical bandwidth. Optical bandwidth typically refers to a range of optical frequencies that can be detected, or in the case of laser technology it can also refer to the laser emission. Measurement bandwidth is typically defined as less than half of the data rate, limited by the Nyquist theorem.

Linearity: In general terms, this is defined as how linear a response is to a given action. In interferometry specifically, it is defined by the maximum amplitude of all the non-linear errors contributing to the signal, such as periodic cyclic errors.

Noise: This is defined as added uncertainty to a measured value. This is typically caused by environmental, electronic or thermal changes that occur on a sample-by-sample basis. Some common types of noise, and some of the sources of noise encountered in precision metrology are summarised below.

Spectral Distributions of Noise:

- *1/f Noise* - Commonly found in electronics and laser technology, $1/f$ noise, sometimes called pink noise or flicker noise, has a noise power that scales inversely with frequency. As a result, this noise source commonly dominates spectra at low frequencies, but is significantly-reduced at higher frequencies
- *White Noise* - Quasi-random noise occurring across all frequencies. For example in laser technology, the laser frequency noise due to the laser linewidth is often described as white noise.

Sources of Noise:

- *Frequency Noise:* - Alternatively referred to as phase noise, frequency noise refers to fluctuations in the instantaneous frequency of the laser source emission which is defined as the time derivative of the optical phase. Typically this originates from quantum noise such as spontaneous emission, or a physical source of noise coupling into the laser, such as temperature fluctuations.
- *Optical Shot Noise:* - A quantum noise effect arising from the discrete nature of photons, optical shot noise is a fundamental limit to the optical intensity noise that can be measured.
- *Jitter* - Jitter is temporal noise, typically occurring in electronic components, where it affects determinate sampling or other I/O. This can also occur in pulsed lasers, where jitter can affect both the length and determination of pulses.

- *Electrical Noise* - Noise generated by electrical components. This is often used as a catch-all for a range of noise sources including $1/f$ noise, thermal noise (noise in electronics due to transient resistance changes) and shot noise (noise due to variations in electron motions). For digital systems, electrical noise can also include quantisation noise

Drift: – Here, drift is defined as the change in a theoretically fixed value over a period of time. Examples include wavelength drift, which is predominantly important in laser applications. Also included is thermal drift which impacts nearly all optical metrology applications, causing thermal expansion of the metrology frame as well as causing refractive index changes to the optical medium.

Stability: - Stability is a measure of how constant a theoretically fixed value is. This is typically stated as a change in a measurand over a unit time in applications where there are time-dependant conditions.

2.2 Displacement Measuring Metrology

The state of displacement measuring metrology has implications and applications far beyond the extent of the laboratory. Modern production and manufacturing techniques for example, used to produce nano-scale electronic and optical components such as transistors, require high-precision machinery capable of accurate positioning at nanometre levels. The position measurement techniques employed by these machines must operate at appropriate distances with high resolution and accuracy, typically utilising optical and non-optical techniques to feed back information about their current position and orientation. Appropriate review papers have been considered such as those covering; the state-of-the-art in displacement measuring metrology by *Fleming* [4], the calibration of displacement sensors by *Haitjema* [6] and the state-of-the-art in interferometry for displacement measurements by *de Groot et al.* [8] amongst others [9–13].

While the focus of this review is on those techniques used for displacement measuring applications such as in-process position monitoring, this is by no means the extent of techniques in this area. Specific attention has been given to techniques that can be compared reasonably directly with laser interferometry, which is of significant interest in this thesis, where in Chp. 5 the development of multi-dimensional encoders using a laser interferometric technique is discussed. In particular, not covered by this review, but also utilised in this area are techniques such as strain gauge sensors and induction sensors. In principle while these techniques are as equally applicable to displacement measuring applications, they are less directly comparable to laser interferometry, either due to significantly reduced measurement ranges, resolutions or precision.

In addition, while not directly applied to in-process position monitoring there are a range of techniques for displacement and distance measurements used in other applications in dimensional metrology. One such example is one-off checking applications such as coordinate-measuring machines (CMMs), used for topographic and surface measurements of complex 3D objects, also used in additive manufacturing. CMMs use techniques such as tactile sensors, measuring the displacement of a contact probe, and x-ray tomography [14] to measure in 3D both internals and surface profiles [15]. Similarly,

white light interferometry [16] and optical coherence tomography (OCT) [17], used to measure surface heights of 3D objects use interferometric techniques such as those discussed in further detail in Sec. 2.3.

In this section, some of the techniques used in dimensional metrology, as they relate to displacement and absolute distance measurements, are summarised and discussed. These are broadly divided into two categories; non-optical and optical techniques. The typical advantages of these techniques are presented in addition to some of the limitations of each, in particular the resolutions and ranges achievable, which are summarised in Table 2.1. Following this, the extension of these techniques towards measurement of multiple spatial dimensions is discussed including the complexity required to implement multi-dimensional sensors, which are summarised in Table 2.2. This is of particular interest, as the work in Chp. 5 of this thesis concerns the development of an existing metrological technique for a multi-dimensional application. As a result, comparison of techniques in this area specifically for extension to multiple dimensions is highly relevant.

2.2.1 Non-Optical Techniques

Hall Effect Sensors:

Hall effect sensors, illustrated in Fig. 2.1, use the principles of magnetic induction to detect the presence of objects with magnetic fields. Named after Edwin Hall who first discovered the effect in 1879 [18], the Hall effect is the generation of voltage in a electrically conducting material transversely to the direction of current in the presence of a magnetic field. This occurs as electrons are "deflected" in the transverse direction in the presence of the magnetic field.

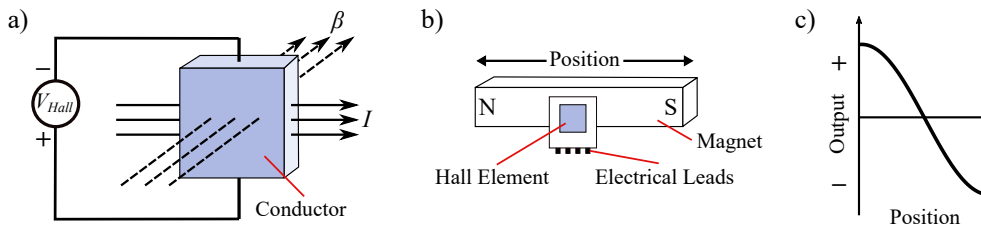


FIGURE 2.1: a) Basics of a Hall effect element comprised of a conductor in a magnetic field β with an applied current I , generating a Hall voltage V_{Hall} (recreated from [19]). b) Example of a Hall effect sensor with moving magnetic field. c) The resultant output from the Hall effect sensor shown in b).

The Hall effect has been well explored in the area of displacement and distance sensors [20], with applications such as rotary encoders for brushless motors [21] and in linear encoders [22, 23]. As Hall effect sensors can be achieved using very simple components they can be manufactured and produced cheaply and simply, and their use of magnetic fields provides an insensitivity to dirt and other environmental effects that hinder optical techniques, such as those discussed later. The drawback of this is that Hall effect sensors are highly sensitive to non-uniform magnetic fields, particularly in cases where multiple

probes or "Hall-Arrays" are used, and the operational range of these sensors is limited by the magnetic field strength. Hall effect sensors have been shown to achieve resolutions as low as $\approx 1 \mu\text{m}$ from high sensitivity arrangements, however this comes at the cost of increased sensitivity to external magnetic fields [24].

Capacitive Sensors:

Capacitive sensors also take advantage of electromagnetic field properties, specifically the effect of electric fields, which allows them to operate with no contact between sensor and target surface. The simplest form of a capacitive sensor is two parallel plates, with a capacitance that can be measured by applying an electric current between them, several designs of which are illustrated in Fig. 2.2. Using the simplest form of the capacitance equations, shown in eqn. 2.1, for a fixed sensor area A and dielectric constant κ , and using the permittivity of free space ϵ_0 , the distance d between the two plates is directly proportional to the capacitance between them, C [25].

$$C = \frac{\epsilon_0 \kappa A}{d} \quad (2.1)$$

As a direct result of eqn. 2.1, the fundamental limitation of any capacitive sensor is the precision and resolution to which the capacitance can be measured. To measure the capacitance of the sensor, a typically sinusoidal, AC voltage is used to drive the sensor, and lock-in techniques are used to separate the measurand from the changes due to the driving voltage [19]. Existing developments have been shown to achieve sub-nanometer resolution displacement measurements [26–29] with low non-linearity errors for short measurement ranges, and as a result are among the most widely-used non-optical sensors.

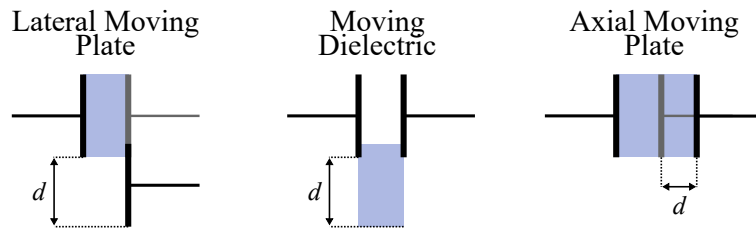


FIGURE 2.2: Different types of capacitive sensor (recreated from [24]). For each pair of plates, the dielectric material is shown in blue.

However, due to the intrinsic limitation of electric fields, the range at which capacitive sensors can operate is severely limited, with a typical maximum sensing range of 10mm and strong angular alignment requirements that limit the operational capabilities of capacitive sensors. These strong angular alignments also arise from the definition of the capacitance, which varies based on the aligned area of the plates. It can therefore be understood that misalignment or alignment changes during sensor operation would cause the calibrations between the capacitance and the displacement of the plates to be inaccurate. As a result of this, and the inverse relationship between the capacitance and the distance between the plates, calibration of capacitive sensors is required and typically requires polynomial fitting. Furthermore, while the core principle of capacitive

sensors is simple to produce, the electronics required to measure the capacitance, shield the measurement and then determine the displacement typically makes them more complex than many other sensor types and increases the cost of each sensor [30, 31].

Linear Variable Differential Transformers

Linear Variable Differential Transformers (LVDTs) transfer linear mechanical motion into an electrical signal, and are commonly used in position, displacement and motion sensors. To achieve this, a series of coils, typically designated as the primary and secondary coils, surround a core of magnetically permeable material which can move freely through the coils. An example design of an LVDT is shown in Fig. 2.3. An AC current is passed through the primary coil, typically the innermost coil, which excites a magnetic field in the core and induces an AC current in the secondary coils. When the core is moved, the distribution of flux changes and the amplitude and phase of the induced AC current in the secondary coils change. By measurement of the current in the secondary coils, information about the position, direction and motion of the core can be deduced.

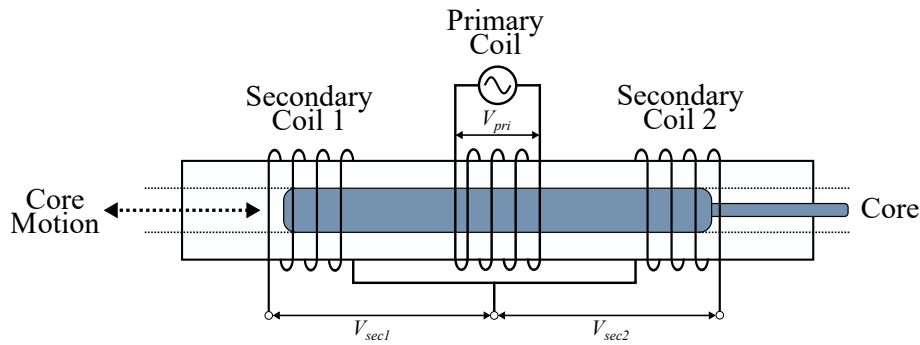


FIGURE 2.3: Cross-section of a typical LVDT displacement sensor (adapted from [32]).

The output signal of an LVDT sensor is interpreted by appropriate signal electronics, similar to that of capacitive sensors. The advantages of LVDT sensors lie in their simplicity, cost-effectiveness and high resolution. Due to the low impedance of the secondary coils, very low levels of electrical noise can be achieved. On the other hand, LVDT sensors typically have a very low measurement bandwidth, caused by the limiting of the AC driving current to a few kHz. Higher frequencies would increase the measurement bandwidth, but induce eddy-currents in the core, which limits the performance of these sensors [19]. In addition, LVDTs are highly sensitive to electromagnetic fields, where changes in the magnetic flux will alter the measurements of current in the secondary coils. This same sensitivity imposes precise alignment requirements between the core and the coils, as changes in the alignment between the two would completely alter the calibration between the coil readings and the position of the core. Furthermore, whilst technically a non-contact method, this requirement for stable alignment between core and coils typically results in a need for bearings to smoothly manipulate the core. The theoretical limitation of LVDT sensors arises from the electrical noise, specifically the thermal noise of the electrons, in the ferromagnetic core and coils (also called Johnson noise). However, standard analogue electronics typically have a practical limitation of

approximately 10 pm for a typical 5 V driving amplitude, which results, for a minimum typical range of 0.5 mm, in a maximum resolution of 5 nm over 1kHz bandwidth [24].

Piezoresistive Sensors

Piezoresistive sensors use semi-conducting materials to create positive-negative (p-n) junctions across which electricity can flow. As stress is applied to the material, such as through stretching, the electrical resistance of the semi-conducting material is changed [24]. For a typical piezoresistive sensor aligned along the x axis, shown in Fig. 2.4, the relationship between the change in resistance ΔR and the changes in the tensile stress components for each axis $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ can be expressed using eqn. 2.2 shown below.

$$\Delta R = R [\pi_L \sigma_{xx} + \pi_T (\sigma_{yy} + \sigma_{zz})] \quad (2.2)$$

Here, R is the typical resistance of the junction with no strain applied, and π_L and π_T are the longitudinal and transverse piezoresistive coefficients of the junction, which can be calculated from the orientation of the junction material crystals [33]. In the case of displacement sensors for example, one end of the material can be affixed to a moving stage and the other to a known reference surface, where stress changes due to stage motion can be converted into measurements of displacement. In order to achieve this, a constant current (CC) is applied across the p-n junction that requires highly precise real-time voltage readout capabilities.

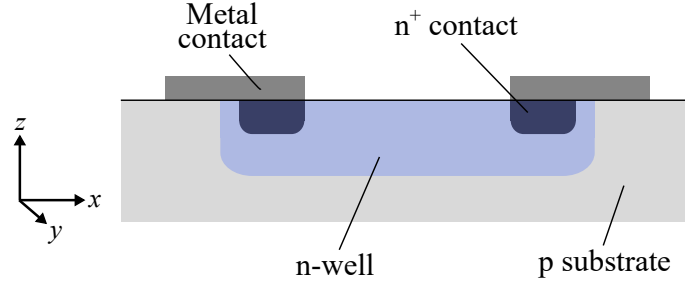


FIGURE 2.4: Cross-section design of a typical piezoresistive semiconductor bridge, arranged along the x axis (reproduced from [4]).

Piezoresistive sensors can be produced with exceptionally small sizes using advanced micro-machining techniques which is of huge benefit over other non-optical methods that typically have more demanding requirements for space due to the inherent size of the sensing units. However, the measurement range of piezoresistive sensors is restricted by the sensitivity required, as the change in resistance due to piezoresistive effect scales inversely with the length of the sensitive junction. In addition, the resistance of the piezoresistive material is typically sensitive to thermal changes, causing drift in the measurements. Regardless, high resolution measurements have been achieved using piezoresistive sensors [34–36] that offer comparable resolutions and noise levels to other non-optical techniques whilst remaining cheap to produce.

2.2.2 Optical Techniques

Linear Optical Encoders:

One of the principally simplest forms of optical based sensor are linear optical encoders, also referred to as pattern-based or pattern-strip encoders. Using strips of a well-known and calibrated engraved or etched pattern, an optical sensor head reads along the pattern strip to determine the displacement between the reference strip and sensor head. A typical linear optical encoder, an example of which is shown in Fig. 2.5, is comprised of a collimated light source, a detector or imaging system, and a pair of ruled gratings; a reflective scale reference grating, and a transparent index grating which forms part of the reading head.

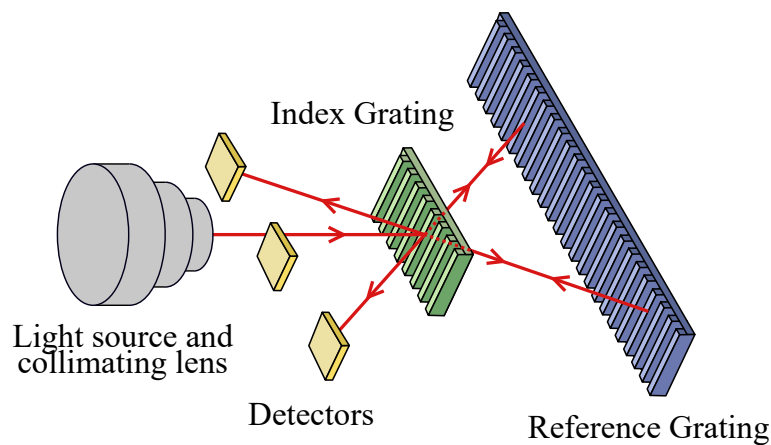


FIGURE 2.5: Example design of a incremental linear optical encoder using an index grating and reflective grating (Adapted from [3])

Depending primarily on the grating period as well as the optical design, a variety of techniques are collected under the term "linear optical encoders", some of which are summarised below;

Imaging: Imaging techniques are typically used with geometric gratings or rules, where the separation of the two gratings typically creates a Moiré pattern [37–39] at an imaging plane. As the gratings move relative to one another the intensity measured by a photodetector at the imaging plane changes periodically with the relative displacement between the two gratings. Typically, the index grating has a period twice that of the reference grating, with periods varying from less than 1 μm to several hundred micrometers [3].

Interfering: Interferometric techniques, sometimes also referred to as "grating interferometers", use scale gratings with significantly lower grating rule periods, typically less than 10 μm , where diffraction effects become dominant [40]. Here, diffracted orders of the interference signal are measured and electronically combined to produce a relative encoder phase signal describing the displacement between the two gratings. Using interpolation, very high resolution measurements can be made of the relative displacement between the two [9, 41–43], thanks to the sub-laser wavelength measurement techniques utilised by laser interferometry, discussed further

in sec. 2.3, with example resolutions as low as ~ 5 nm [4]. In addition, optical grating encoders are capable of measuring a wide variety of measurement ranges, as feasibly low as tens of millimetres to as high as several meters [9] depending on the implementation and grating design.

Linear optical encoders can be further classified as either incremental or absolute encoders depending on whether they measure relative displacement between the gratings, or absolute distances travelled. Incremental measurements can be achieved using the previously discussed techniques with the inclusion of a reference marker that is used to calibrate the start position of the encoder during referencing or after loss of signal. Absolute measurements however require the addition of further reference markings or code tracks that encode the absolute position between the readout head and the reference scale, typically using a Gray code[44, 45]. As a result, absolute encoders typically have larger scales than their incremental counterparts making them lower resolution and more sensitive to alignment errors and mechanical or thermal noise. Instead they benefit from the ability to retain position information in the event of a loss of power or temporary obfuscation of the read head.

Rotational measurements can be achieved similarly to the linear measurements discussed thus far, where instead of a single axis grating, the reference grating is arranged around the radial axis with the readout head fixed in place.

In addition to a high resolution and range, linear optical encoders also offer the ability to create compact readout heads with in-built index gratings, and a minimal footprint required by the reference scale. However, the limitations of linear optical encoders typically arise from the precision at which the gratings, both index and scale, can be created, where errors in the ruling of the gratings directly translates into errors in the determination of displacement. These errors scale with the precision of the grating ruling required, so gratings for interfering variations are typically both more costly and more susceptible to ruling errors. The gratings are also highly sensitive to thermal changes, where thermal expansion of the grating materials can cause the gratings to deform or extend, introducing further error into measurements [46]. Linear optical encoders are also highly sensitive to the geometry of the system, where angle changes between the scale grating and readout head can cause large measurement errors or loss of signal. As with nearly all optical techniques, dust or other particulates which obscure the optical path will also cause the loss of signal and hence require either a closed unit, regular cleaning, or mitigating design concepts such as widening the tracks and using imaging arrays to capture the signal, however these pose new limitations such as increased size and cost.

Phase-evaluating Laser Interferometry

Laser interferometry describes the technique of creation and analysis of interference patterns created by two or more coherent laser beams interfering. While a broad range of techniques and applications can be described under this terminology, for precision metrology specifically the interest lies with analysis of two-beam interference signals, in particular phase analysis [5]. In general terms, an optical beam from a coherent source such as a laser is split into two separate optical beams. After the two beams traverse

a given optical path they are recombined at a photodetector. Depending on the optical path difference (OPD) between the two optical beam paths, an interference signal is formed at the photodetector. These interferometers can be configured in a number of ways, both in free-space using bulk optics and in-fibre using fibre components, such as the two different in-fibre interferometer configurations illustrated in Fig. 2.6.

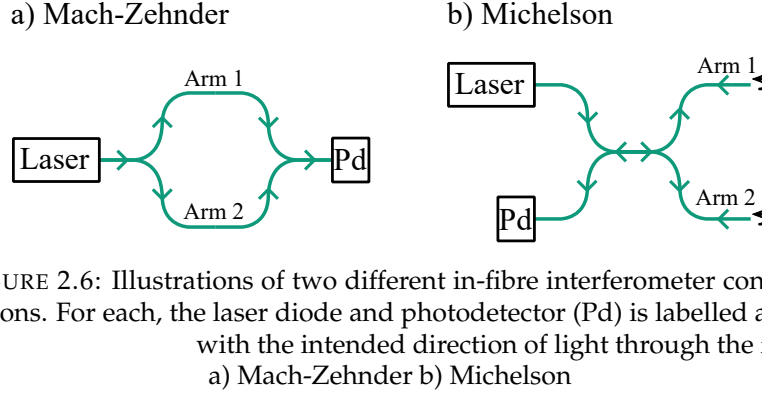


FIGURE 2.6: Illustrations of two different in-fibre interferometer configurations. For each, the laser diode and photodetector (Pd) is labelled along with the intended direction of light through the fibre.
a) Mach-Zehnder b) Michelson

Fig. 2.6.(a) illustrates a Mach-Zehnder style fibre interferometer, where the two arms of the interferometer are well separated, and each path is only traversed once. The sensitive region of this interferometer is between the two couplers, where the two arms are labelled. Fig. 2.6(b) illustrates a Michelson fibre interferometer, which operates by reflection. After reflecting at the end of one arm of the interferometer, the light travels back down the arm and through the common path to interfere at the detector. Here, the sensitive region starts after the light is split and extends down each arm until the reflection point.

In bulk-optic interferometers, the interference pattern created by the interference of two beams which have traversed paths with an OPD, contains a series of fringes created by the constructive and destructive interference of the wave-fronts of the two optical beams being superimposed on one another. As the OPD is increased or decreased the interference fringes will move, and by analysis of these fringes, direct information about the change in optical path difference can be determined. The interferometric transfer function, illustrated in Fig. 2.7, describes the relationship between the measured interferometric phase and the measured intensity.

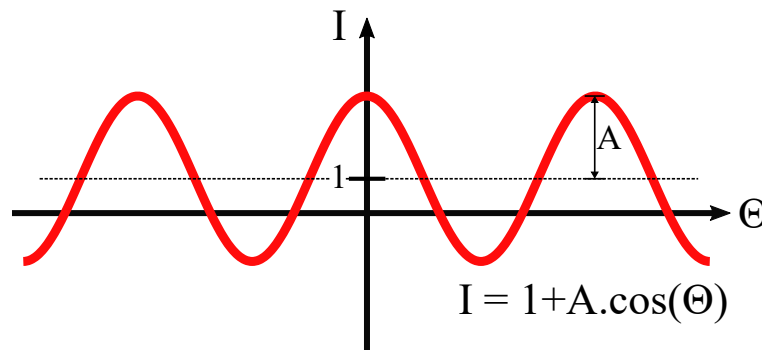


FIGURE 2.7: Example interferometric transfer function with equation $I = 1 + A \cdot \cos \theta$

Without interferometric signal processing however, the direction of this motion cannot be determined. To determine the direction of motion, and often magnitude, of motion, signal processing techniques are used. Depending on the type of interferometric techniques employed, different signal processing techniques are used to evaluate the interferometric signals, most notably homodyne, heterodyne and pseudo-heterodyne techniques, which are discussed later in Sec. 2.3 of this chapter.

Laser interferometry has quickly become the "gold standard" for displacement and position measuring metrology [8, 24, 47–52], as measurements are intrinsically linked to the emission wavelength of the laser source unlocking measurement resolutions below that of competing techniques. As both laser technology and analysis techniques have improved, the measurement resolution achievable by laser interferometry has been driven lower and lower, with sub-nanometer measurements now achievable. In addition to the high resolutions achievable with laser interferometry, laser technology also permits a wide range of application-dependant measurement ranges, varying from as low as a few nanometers up to kilometres. These extensive ranges and high resolutions typically come at a cost, where here the cost is the complexity and associated price of the interrogation systems used to evaluate these interferometers. The principles behind laser interferometry and a selection of the techniques that use these principles are discussed in further detail in sec. 2.3, and some of the limiting uncertainties are discussed in sec. 2.3.5.

Absolute Interferometry:

In contrast to the phase-evaluating laser interferometry discussed above which performs relative measurements of phase changes, absolute interferometry focuses on performing measurements of absolute distances [16, 53]. Several techniques are summarised by absolute-interferometry, including optical coherence tomography [17], swept-frequency interferometry [54], dual optical-comb interferometry [55] and spectral interferometry [16]. While the techniques differ, the core principles remain the same; combination of measurements from a series of emission wavelengths or several sources into a single absolute distance measurement of the target surface. In swept-frequency interferometry, a single laser source with free-spectral range ν_R is swept through a range of emission frequencies ν_S , and is incident on an interferometer. At the output of the interferometer, a number of fringes, N , are swept through, and the optical path length of the interferometer L can be calculated [3]:

$$L = \frac{c}{2\nu_R} = \frac{cN}{2\nu_S} \quad (2.3)$$

for a Michelson or Fabry-Perot interferometer in vacuum, where c is the free-space speed of light. In dual-wavelength or two-color interferometry, the single swept laser source is replaced by two sources emitting at a single wavelength or a single source emitting at two separate wavelengths, and the measurements from both emission wavelengths are combined to calculate the absolute distance. An example of this is dual optical-comb interferometry, where two optical frequency combs (OFCs) with slightly different repetition rates are used. The emission from one OFC passes through an interferometer such as a Michelson as usual, directly interfering with the output from the other OFC at

a photodetector. This generates a beat frequency signal containing the absolute distance information. An example of a dual optical comb interferometric setup with a Michelson interferometer is shown in Fig. 2.8.

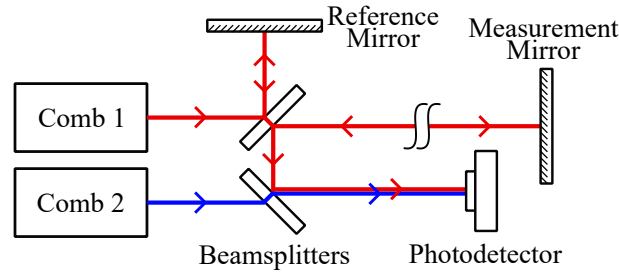


FIGURE 2.8: Illustration of an example dual optical comb interferometer setup using a Michelson interferometer. Adapted from [55]

In spectral, or dispersive, interferometry, a singular optical comb with an interferometer is employed. In this technique, at the output from the interferometer a spectrometer is used with a dispersive element, such as an optical grating, to separate out the various comb frequency components, which are incident on a array of charge-coupled devices (CCDs). An example illustration of this, with the addition of a dispersion compensating plate, and a Fabry-Perot etalon to separate out specific frequency components, is shown in Fig. 2.9.

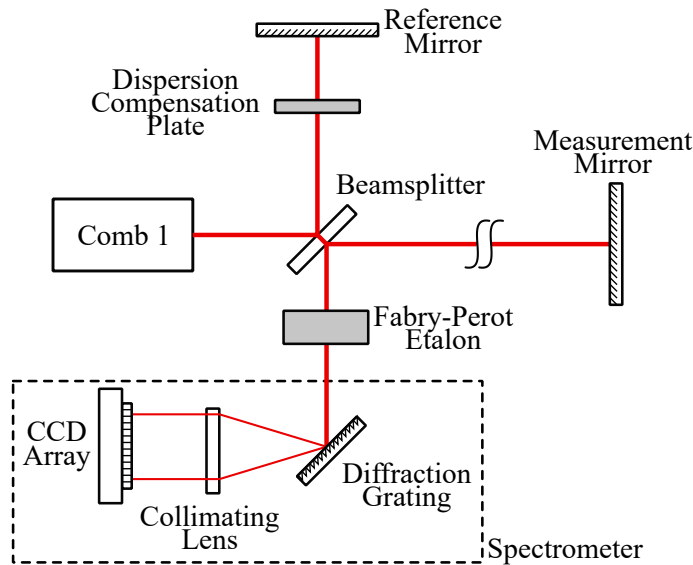


FIGURE 2.9: Illustration of a typical setup for spectral interferometry featuring a single optical frequency comb source with a Michelson interferometer. Notably the inclusion of a dispersion compensating plate ensures each of the measured frequency components has travelled the same path length before interfering. Adapted from [56]

For all of the discussed techniques, measurements can be made with a precision of sub-micrometers, and approaching the nanometer-level [55]. The range these measurements

can be made over is equally comparable, however their predominant usage is in long-range measurements over several meters or further [3, 55]. The current state of dual optical comb techniques has been reviewed by *Zhu et al.* [55], where the advantages of optical comb based techniques, such as the high spectral resolution and fast pulse rate are investigated.

TABLE 2.1: Summary of a selection of techniques for displacement measuring applications. Values derived where appropriate from [4, 5, 7, 11].

Sensor Type	Achievable Resolutions	Measurement Range	Contact	Typical Limiting Features
Hall Effect	1 μm	10 mm	Close	Sensitive to external magnetic fields
Capacitive	0.5 - 2 nm	10 μm - 10 mm	Close	Strong angular alignment requirements
LVDT	5 nm	0.5 - 500 mm	Yes	Sensitive to external fields and angular alignment changes
Piezoresistive	0.5 - 10 nm	1 - 500 μm	Yes	Highly sensitive to thermal changes
Imaging Linear Optical Encoder	0.2 - 5 nm	100 mm	No	Highly sensitive to ruling errors and angle changes
Interfering Linear Optical Encoder	0.1 - 2 nm	100 mm	No	Complex optical setups, highly sensitive to grating errors
Laser Interferometry	> 1 pm	nm - km	No	Complex, costly interrogation hardware
Absolute Interferometry	> 1 nm	nm - km	No	Costly interrogations hardware, sensitive to drifts in laser emission / comb drift

2.2.3 Extension to multiple dimensions

Whilst all of the above techniques are suited to single dimension measurements in their respective applications, when extended to multiple dimensions, such as required by applications such as stage encoders used in nano-scale manufacturing, they each pose unique challenges and limitations. Theoretically, the extension of any technique to any number of dimensions can be achieved simply through the addition of more sensors, copying the complete components of a linear sensor, for each dimension of interest. Practically however, in the majority of applications there are constraints on size, cost and in some cases electrical budget, where adding more sensors is inefficient or infeasible. Instead, here the interest is in examining the techniques discussed above in cases with similar setups where they have been extended to measure multiple dimensions simultaneously.

Contact Techniques

In general, contact techniques or those that require close proximity to the target surface such as the majority of non-optical techniques and pattern-based encoders, introduce complication into any sensor design when extended for multiple dimensions. In some cases, such as capacitive sensors and LVDTs, this is limited by the strong angular requirements of the electrodes or coils. In other cases, it is the size and space required by additional components that adds additional complication to multi-dimensional setups. In the case of Hall effect sensors, some prior work in the use of Hall effect arrays such as papers published by *Hu et al.* [57] and *Kawato et al.* [58] has shown comparable resolutions to existing linear Hall effect sensors however the increase in size required is significant. In addition, there is to be expected increased sensitivity to transient and inhomogeneous external magnetic fields that limits both the accuracy and precision of such setups.

Non-contact techniques

Of the optical sensors, specifically imaging linear optical encoders, complex planar gratings have been developed for 2 dimensional measurements such as those by *Shimizu et al.* [59], however these typically require reading heads with greater size and complexity that limit their potential applications. For interferometric techniques such as laser interferometry and absolute interferometry that can easily perform non-contact measurements of a target surface, measurements in multiple dimensions typically require multiple sensing heads or optical access ports. Interfering linear optical encoders using high precision have achieved success with multiple degree-of-freedom measurements in many different configurations [60–66], many of which are reviewed by *Hu et al.* [9], however these typically require complex setups with numerous, and typically polarisation-sensitive, optical components which adds significant complication in the design. In general there is a lack of compact, non-contact techniques that can suitably measure multiple degrees-of-freedom without the need for complex optical setups. This is part of the research gap explored in this thesis in Chp. 5. Expanding on table 2.1, table 2.2 describes the metrics associated with extension of these techniques into measurement of multiple degrees-of-freedom and a brief description of the complexities associated with each method.

TABLE 2.2: Comparison of existing displacement measuring technologies for multi-dimensional measurements and the requirements imposed by these.

Sensor Type	Achievable Resolutions	No. of Measured Dimensions per Sensor	Complexity in Sensing Region
Hall Effect	$\sim 1 \mu\text{m}$	1 or 2	1 sensor or array of sensors
Capacitive	0.5 - 2 nm	1	1 sensor
LVDT	$\sim 5 \text{ nm}$	1	Core and coils or attached beam
Piezoresistive	0.5 - 10 nm	1	1 semiconductor bridge
Imaging Linear Optical Encoder	0.2 - 5 nm	1 or 2	1 scale (1D grating or 2D planar grating) and 1 readout unit
Interfering Linear Optical Encoder	0.1 - 2 nm	1 - 6	Variety of optical components, typically polarisation sensitive components
Laser Interferometry	$> 1 \text{ pm}$	1	1 mirror
Absolute Interferometry	$> 1 \text{ nm}$	1	1 mirror

2.3 Phase Evaluating Interferometry

After providing an overview of techniques for displacement measuring metrology, this section will now review phase evaluating interferometry in more detail, as this forms the core method of the experimental work presented in this thesis. Firstly, the key principles of phase-evaluating interferometry are presented, including the general mathematical theory underpinning it, followed by discussion of the uncertainties involved.

2.3.1 Principles and Theory

In 1887, whilst attempting to prove the existence of the "luminiferous aether", a medium that supposedly permeated space and allowed for the propagation of light waves, Albert Michelson and Edward Morley published results from an experimental setup that would continue to underpin cutting-edge research almost a century and a half later [67]. By separating a coherent source of light into two beams that are recombined after travelling two optical paths, the first interferometer for displacement evaluation was created. The Michelson interferometer, demonstrated in Fig. 2.10, remains to date a steadfast of modern techniques [40, 68].

As research has developed, novel techniques using laser interferometry have continued to push the boundaries of measurement ability, demonstrating sub-nanometer [47, 50],

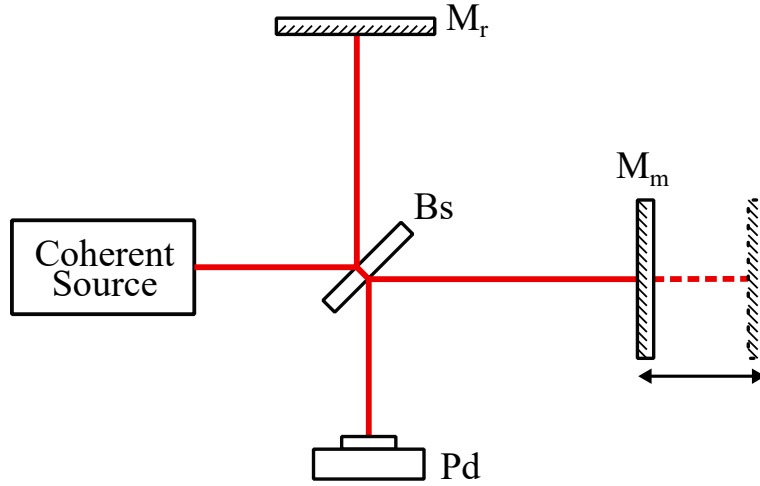


FIGURE 2.10: Illustration of a free-space Michelson interferometer using bulk optic components. The setup consists of a coherent source such as a laser, a reference mirror (M_r) and a beamsplitter plate (Bs), with a movable measurement mirror (M_m). The signals from both arms interfere at a photodetector (Pd) or screen

and in particular cases picometer [69–72], resolutions. As discussed earlier, for phase evaluating interferometry, it is not possible to determine the direction of motion that causes a phase change in an optical interferometer without signal processing. In this section, the key principles of phase-evaluating interferometry, as they apply to displacement measurements, are discussed, followed by discussion of different signal generation and processing techniques used in displacement measuring interferometry.

Theory of two-wave interference

The interference of two electromagnetic waves, such as the two beams in the Michelson interferometer, can be used to extract information about the two waves that is otherwise immeasurable in normal conditions. In the optical frequency region of the electromagnetic spectrum for example, phase changes in individual beams occur at frequencies too high for reasonable measurement and evaluation. Instead, two beams are interfered and the relative phase difference between the two beams can be measured with greater ease. This phase difference, or any change in phase difference, contains a wealth of information such as the conditions of the difference in optical path the two beams have undertaken. In dimensional metrology in particular, the change in optical phase can be used to determine changes in the optical path length of the interference region. Mathematically, the two interfering beams can be described as waves with amplitude U and phase ϕ [73]:

$$u_1 = U_1 e^{i\phi_1} \quad \text{and} \quad u_2 = U_2 e^{i\phi_2} \quad (2.4)$$

The observable quantity associated with the interference of these two beams however is the intensity, $I \propto |u|^2$, which for quasi-monochromatic light with complete polarisation overlap can be mathematically described as:

$$\begin{aligned}
I &= |u_1 + u_2|^2 = U_1^2 + U_2^2 + 2U_1U_2 \cos(\phi_1 - \phi_2) \\
&= I_1 + I_2 + 2\sqrt{I_1I_2} \cos[\Delta\phi]
\end{aligned} \tag{2.5}$$

where $\Delta\phi = \phi_1 - \phi_2$. Here, $2\sqrt{I_1I_2} \cos[\Delta\phi]$ is the interference term, describing the maximum constructive ($\cos \Delta\phi = 1$) and destructive ($\cos \Delta\phi = -1$) interference conditions of the two waves. In theory, these equations hold true for quasi-monochromatic light [74] with complete polarisation overlap for idealised waves of a single frequency, with infinite length and with no change in phase difference $\Delta\phi$ in time. This is described as perfect coherence. However in reality no wave has only a singular frequency, as described by the Fourier theorem, so the phase difference between the two waves will vary with time and an additional term relating the change in contrast between the beams over the time of flight delay between the two waves $|\gamma(\tau)|$, has to be incorporated.

$$I = I_1 + I_2 + 2\sqrt{I_1I_2} |\gamma(\tau)| \cos [\Delta\phi(t)] \tag{2.6}$$

This contrast term can be linked to the physically measurable quantity visibility, V , which can be defined as:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{2.7}$$

Where I_{max} and I_{min} are the maximum and minimum measured intensities respectively. Using the conditions of maximum and minimum intensity previously stated at values of ($\cos \Delta\phi = 1$ and -1), i.e. constructive and destructive interference, this can be redefined as:

$$V = \frac{2\sqrt{I_1I_2}}{I_1 + I_2} |\gamma(\tau)| \tag{2.8}$$

$\gamma(\tau)$ is known as the complex degree of coherence [74], and defines the ability of two waves to interfere. The complete transfer function of the interferometer, describing a total transition from a phase difference between two waves to an output signal from a photodetector, is then given by:

$$U_{pd} = R_{pd} \left(I_1 + I_2 + 2\sqrt{I_1I_2} |\gamma(\tau)| \cos [\Delta\phi(t)] \right) \tag{2.9}$$

where R_{pd} defines the responsivity of the photodetector, relating incident intensity to the signal output. This transfer function, illustrated in Fig. 2.11(a), shows several key features of interferometric signal processing, predominantly that the transfer function is periodic over 2π , co-sinusoidal and notionally only the phase difference ($\Delta\phi(t)$) can be retrieved from analysis of this function. In the case of a two-path interferometer, the co-sinusoidal transfer function means that outside of the quadrature points, occurring at $\Delta\phi = \frac{\pi}{2} + m\pi$ where m is any integer value, the value of the transfer function is

highly non-linear, making signal evaluation significantly more complex. Fig. 2.11 illustrates this, showing a co-sinusoidal transfer function for a two-wave interferometer. At the quadrature points, highlighted in Fig. 2.11(a) and with a closer example shown in Fig. 2.11(b), the phase signal is linearly transferred to the photodetector signal. Outside of these quadrature points, such as in the case shown in Fig. 2.11(c), the phase signal is non-linearly transferred to the photodetector signal.

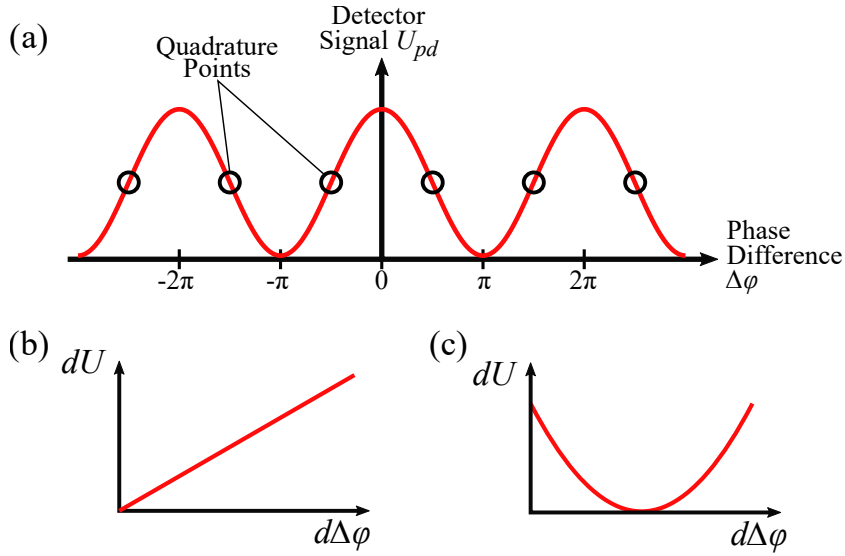


FIGURE 2.11: (a) Illustration of an interferometric transfer function for two-wave interference with quadrature points labelled (b) The transfer function at quadrature points, where the phase signal is linearly transferred (c) The transfer function at a turning point where non-linear transfer of the phase signal occurs. (Adapted from [75, 76]).

To evaluate the photodetector signal and extract the interferometric phase difference information, $\Delta\phi(t)$, interferometric signal processing techniques must be used. To do this, a separate phase modulation function is introduced into the interferometric signal, splitting the $\Delta\phi(t)$ term into:

$$\Delta\phi(t) = \Phi(t) + \Theta(t) \quad (2.10)$$

where $\Phi(t)$ is the interferometric phase signal of interest, and $\Theta(t)$ is the applied phase modulation. To use this $\Theta(t)$ phase modulation function to evaluate the desired interferometric phase signal $\Phi(t)$, various modulation techniques are used which depending on how the phase modulation function $\Theta(t)$ is applied, can be distinguished as either homodyne, heterodyne or pseudo-heterodyne interferometry, which are expanded on further below.

With all of the above modulation and interference techniques, there remains the need to separate the applied phase modulation $\Theta(t)$ from the interferometric phase signal of interest $\Phi(t)$. To achieve this, a variety of demodulation techniques have been developed for each of the modulation types discussed above. Here, some typical demodulation

schemes are discussed that use quadrature demodulation to evaluate the phase signals of interest.

2.3.2 Homodyne Interferometry

In homodyne interferometry, the modulation function $\Theta(t)$ is introduced to the interferometer as a static, or series of static phase shifts Θ , or more commonly as a time-dependant modulation function $\Theta(t)$. In general, eqn. 2.9 can be stated for homodyne detection schemes as:

$$U_{pd} = R_{pd} \left(I_1 + I_2 + 2\sqrt{I_1 I_2} |\gamma(\tau)| \cos [\Phi(t) + \Theta(t)] \right) \quad (2.11)$$

where $\Phi(t)$ is the interferometric phase signal of interest, and $\gamma(\tau)$ is the complex degree of coherence introduced earlier [74]. In order to recover the interferometric phase signal $\Phi(t)$, passive homodyne processing [77], uses a series of static phase shifts Θ_n between outputs of the interferometer with n detectors, where each phase shifted component is incident on a different detector. This allows the interferometric phase signal $\Phi(t)$ to be retrieved by combination of the photodetector signals using appropriate trigonometric identities. This has also been achieved using polarisation-based phase shifting [78] in polarisation-based quadrature homodyne interferometry, where the 90° phase offset inherent to circularly polarised light is used to encode the various phase shifts Θ . Another non-carrier method for homodyne signal processing is phase-stepping or phase shifting interferometry [79], where discrete phase steps are applied in a fixed sequence in time. Unlike carrier-based techniques discussed below, phase-stepped processing is not continuous, and therefore both assumes and requires a constant value of the phase signal Φ over the time period of the phase stepped signal.

Homodyne interferometry has been well explored for use in precision dimensional metrology, with applications in linear encoders [77, 80–84] and angle encoders [68, 85], and is popular because it is simple to implement in most conditions.

2.3.3 Heterodyne Interferometry

While homodyne interferometry uses the interference of two beams with a single source of fixed frequency ν , heterodyne interferometry uses direct frequency modulation (DFM). Here, a constant optical frequency shift is applied to one arm or beam of the interferometer. For a frequency modulation of magnitude f_m , the resultant phase modulation is of the form:

$$\Theta(t) = f_m 2\pi t \quad (2.12)$$

and hence eqn. 2.9 can be restated as:

$$U_{pd} = R_{pd} \left(I_1 + I_2 + 2\sqrt{I_1 I_2} |\gamma(\tau)| \cos [\Phi(t) + f_m 2\pi t] \right) \quad (2.13)$$

This frequency shift is most commonly applied through a frequency shifting acousto-optic modulator (AOM) placed after the laser emission to generate two high frequency waves, or two wavelength locking [86]. The frequency shift is then typically encoded between the measurement and reference beams either through separate optical paths, or between polarisation states of the laser with polarisation-sensitive optics to spatially separate the measurement and reference beams.

These approaches to interferometry are advantageous for several reasons, most notably that it shifts the frequency of the signal of interest to a regime away from DC, which reduces errors caused by thermal or statistical noise sources that scale inversely to frequency such as $1/f$ noise, giving it a superior signal to noise ratio compared to homodyne interferometry. For signal processing in heterodyne interferometry, various techniques have been used to extract the phase information such as lock-in methods [87], zero crossing analysis [88] and Doppler frequency analysis [89, 90].

2.3.4 Wavelength-modulated Interferometry (Pseudo-Heterodyne)

In addition to homodyne and heterodyne approaches, alternative techniques using carrier waveforms have been explored. In these techniques, time-dependant carrier waveforms $\Theta(t)$ are applied to the interferometer through direct phase modulation (DPM) or optical frequency modulation (OFM), the shape of which can be linear, in the form of serrodyne (sawtooth) waveforms, or sinusoidal.

DPM techniques achieve this through modulating the optical path directly, for example stretching an optical fibre using an acousto-optic modulator or modulating a Piezo mirror in a bulk optic setup. OFM techniques, also called frequency-modulated continuous wave (FMCW) or serrodyne interferometry in the case of sawtooth modulation waveforms, have historically used periodic ramp functions applied to the laser emission, typically via modulation of the laser injection current [91, 92]. The frequency modulation of the laser emission creates an applied phase modulation waveform in the interferometer which is used as carrier function for the interferometric signal of interest. Optical frequency modulation results in a phase modulation that is mathematically equivalent to internal direct phase modulation of the interferometer, however is mathematically distinct from the direct frequency modulation used in heterodyne interferometry.

Depending on the form of the optical frequency modulation used, various techniques have been employed to demodulate the signals of interest from the phase carrier waveforms and these are discussed in the following sections. For single-carrier based approaches, quadrature demodulation is typically used, illustrated in Fig. 2.12, where the photodetector signal $U_{pd}(t)$ is multiplied by both the sine and cosine versions of the carrier signal with frequency f_c . Both components are low-pass filtered, resulting in the complex quadrature signal U_Q . By appropriate coordinate transformation, the wrapped interferometric phase signal $\Phi(t)$ is produced.

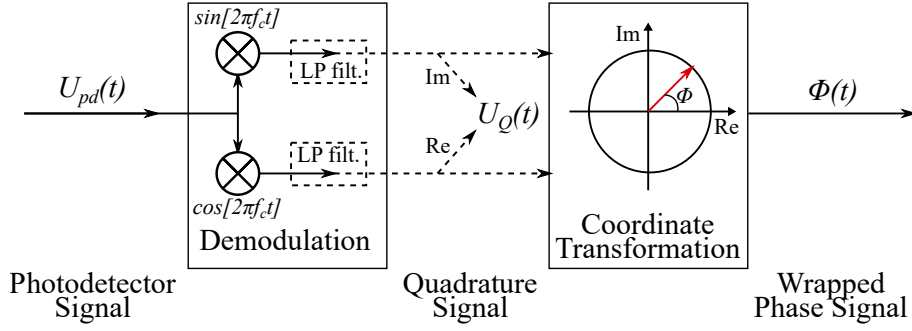


FIGURE 2.12: Illustration of quadrature demodulation for a single-carrier demodulation scheme (adapted from Kissinger [76])

whereas in sinusoidal approaches, the photodetector signal must be multiplied by a window function in the time-domain, which if the correct values for the window width and position are set, allows the photodetector signal to be demodulated using the quadrature demodulation scheme discussed above and illustrated in Fig. 2.12. Otherwise, dual-carrier approaches can be used to demodulate these interferometric signals. The following subsections discuss in further detail techniques for demodulation of the different carrier waveform functions.

Linear pseudo-heterodyne modulation

A linear pseudo-heterodyne modulation typically describes a sawtooth (serrodyne) or triangle shaped waveform, applied as the frequency modulation waveform. The frequency of a serrodyne modulation waveform with modulation amplitude A_s can be mathematically stated as:

$$f(t) = A_s \left(\frac{1}{2} - \left[\sum_{k=1}^{\infty} (-1)^k \frac{\sin(2\pi k f_m t)}{k} \right] \pi^{-1} \right) \quad (2.14)$$

Here, the serrodyne modulation waveform is expressed as the sum of an infinite series of sinusoidal waveforms, which approaches a perfect sawtooth as higher and higher harmonics are included. However, linear wavelength modulation waveforms are deceptively difficult to apply, as non-linear effects in either the laser driver or the laser diode itself can alter the waveform and hence cause error in the phase detection. In general, this means that careful control of the applied waveform is required, accounting for the multiple fundamental frequencies that form the linear waveform and any higher order harmonics that can arise, as well as consideration of these in the demodulation process [93]. As discussed previously, mathematically the phase modulation waveform $\Theta(t)$ caused by optical frequency modulation is identical to direct phase modulation of the interferometer. As a result, many of the signal processing techniques associated with homodyne interferometry can also be used for serrodyne interferometry [92], however certain mathematical limitations exist on the conditions for successful demodulation of the interferometric phase signal from the applied carrier function. Specifically, the carrier frequency has to be a harmonic multiple of one of the modulation frequency f_m harmonics in order to successfully demodulate the interferometric phase signal.

Sinusoidal pseudo-heterodyne modulation

Another form of modulation waveform used in pseudo-heterodyne interferometry is sinusoidal modulation. Similar to linear pseudo-heterodyne techniques, injection current modulation [94, 95] is used to apply a modulation waveform to the laser emission frequency, which results in a phase modulation in the interferometer. For a sinusoidal modulation waveform, the frequency modulation wavefunction can be mathematically described as:

$$f(t) = f_c + \delta f \sin [2\pi f_m t] \quad (2.15)$$

where f_c and δf are the central laser emission frequency and modulation frequency width respectively. The advantage of sinusoidally modulated approaches compared to their linear modulated counterparts, is that the sinusoidal waveform only has a single fundamental frequency with any higher harmonics falling at discrete multiples of this frequency, allowing for simpler and easier control of the modulation waveform and simpler demodulation regimes.

Sinusoidal pseudo-heterodyne techniques can be broadly categorised into a single or dual-carrier techniques. In single-carrier techniques, similarly to those discussed previously, the nature of optical frequency modulation causes a phase modulation mathematically equivalent to direct phase modulation from within the interferometer. To this end, the quadrature-based signal processing techniques discussed for homodyne and heterodyne signal evaluation are similarly suitable for sinusoidal, pseudo-heterodyne modulated techniques. However, single-carrier based signal processing for sinusoidal modulated techniques requires the addition of a window function in the time-domain of the interferometric signal, in order to suppress side-bands of the signal of interest in the frequency domain. This isolation of the frequency sideband of interest allows the interferometric phase signal to be retrieved without additional cyclic errors, which are discussed further in Sec. 2.3.5 of this chapter. In general, published literature on windowed sinusoidal processing typically utilises rectangular window functions for retrieval of the interferometric phase signal, such as work by Sakai *et al.* [96] and Kersey *et al.* [97]. However, as discussed by Kissinger *et al.* [98], a discrete rectangular window function requires tuning the demodulation phase carrier amplitude, so that there are only integer multiples of the carrier waveform within the window period, limiting the permitted optical path differences to only a discrete set. Instead Kissinger *et al.* proposed a different technique called range-resolved interferometry, based on windowed sinusoidal pseudo-heterodyne processing, that uses a smooth Gaussian function as a window, eliminating the requirement for integer multiples of the carrier waveform. Range-resolved interferometric signal processing is applied in the experimental work presented in this thesis and the principles and theory underpinning it are discussed in further detail in Chp. 4.

An additional category of signal processing techniques for sinusoidal pseudo-heterodyne methods utilises dual-carriers to retrieve the interferometric phase signal. Here, two or more carrier frequencies are evaluated, typically generated via sinusoidal frequency modulation. Using Bessel function analysis [99], the demodulation phase carrier amplitude is tuned until the Bessel coefficients for consecutive harmonics (one odd and one even) have the same value. Using techniques such as the phase generated carrier

method introduced by Dandridge *et al.* [100] which applies an arctangent demodulation scheme, the interferometric phase signal can then be retrieved. Alternative techniques for retrieval of the interferometric phase signal include analysis of further Bessel harmonic carriers [101, 102] to analytically solve for the phase carrier amplitude and interferometric phase signal, and the deep frequency modulation technique published by Heinzel *et al.* [103], which uses direct phase modulation and evaluates up to 20 carrier harmonics.

2.3.5 Uncertainty in Interferometry

As with any metrological method, interferometry has a number of sources of uncertainty that should be considered and accounted for when discussing the capabilities and precision of any technique. These can be broadly separated into those linked with the measurement technique, and with sources affecting the measurement of the measurand. The former of these, uncertainties associated with the measurement technique, are typically related in interferometric measurements with uncertainties in the laser emission wavelength, uncertainties in the refractive index of the measurement space and cosine error. Sources of uncertainty that affect the measurement of the displacement itself are independent of the measurand, such as uncertainty in the measurement of the phase change, errors due to thermal drift, and cyclic errors. The discussed sources of uncertainty, and where they originate, are summarised in Table 2.3. In this section, some of the major sources of uncertainty are briefly discussed as they pertain to displacement measuring interferometry.

TABLE 2.3: Summary of the main uncertainty sources in interferometric displacement measurements

Uncertainty in the measurement technique	Uncertainty in the measurement of the measurand
Uncertainty in laser emission wavelength (Static and Dynamic)	Cyclic errors
Uncertainty in refractive index of measurement mediums (Static and Dynamic)	Thermal drift of optics (Geometric changes)
Cosine error	Abbe errors

Cyclic Errors

In interferometry, cyclic errors are non-linear errors that occur periodically with the wave oscillations of the laser. The cyclic error adds an additional phase vector into interferometric measurements, which creates a non-constant error in any measurement of displacement, restricting the linearity and hence precision of any approach. This concept is easily seen in a Lissajous figure, such as the one illustrated in Fig. 2.13.

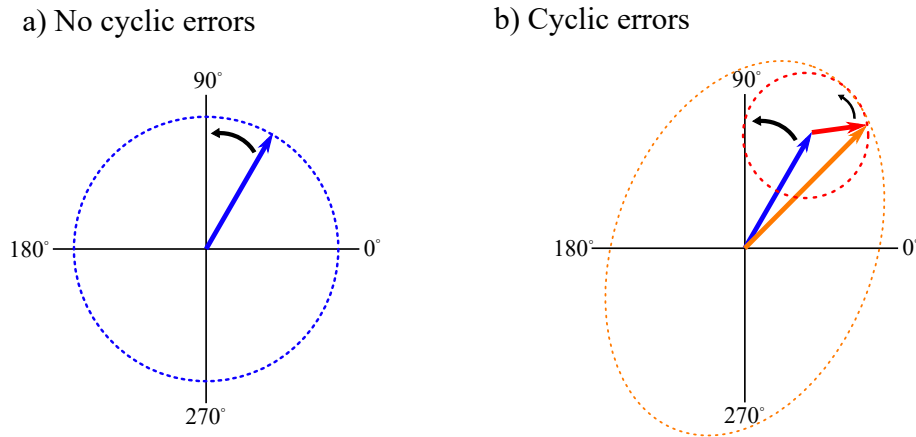


FIGURE 2.13: Illustration of cyclic errors in a Lissajous plot. a) shows no cyclic errors and the measured phase vector (blue) forms a perfect circle over 360° displacement. b) shows the case with an additional phase vector (red), causing a resultant phase vector (orange).

These additional phase vectors result in non-linear evaluation of the interferometric phase, illustrated in Fig. 2.14.

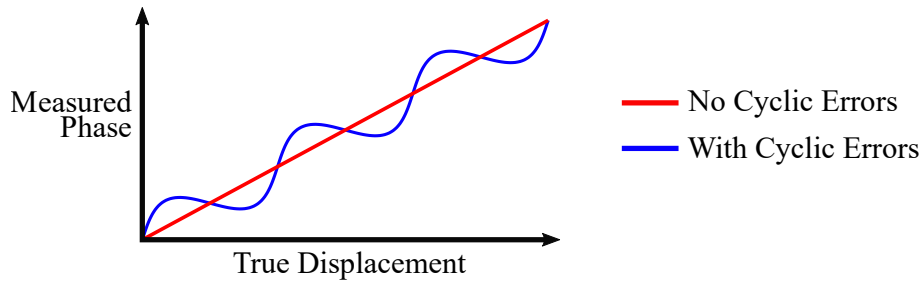


FIGURE 2.14: Illustration of how cyclic errors contribute to non-linear evaluation of the interferometric phase.

In heterodyne interferometry, a common source of cyclic errors occurs from polarisation cross-mixing, where imperfect isolation of the various polarisation states leads to components of the measurement beams or reference beam "leaking" into the other [5, 7]. Cyclic errors can also occur when unwanted reflections or stray scattered beams are multiplexed into the interferometric signal. Typical interferometric approaches are unable to separate these spurious signals from the signals of interest, so additional unwanted phase components are added into the analysis of the interferometric signal. As a result of the inherent limitations cyclic errors impose on the precision of interferometric systems, several schemes have been developed for detection and compensation of cyclic errors in both homodyne and heterodyne methods [104–108]. An early example of this, the Heydemann correction [109] compensates for quadrature fringe errors using least-squares fitting of a distorted ellipse to the Lissajous figure for a sufficiently wide data set. From the fitted distorted ellipse, the error coefficients, including non-linearities due to quadrature fringe errors, can be corrected for. More recently, frequency analysis combined with linearization algorithms have been applied [110] to measure cyclic error in

heterodyne interferometers allowing for optimisation of configurations to reduce cyclic error contributions, as well two-wavelength approaches to cyclic error correction [111].

Wavelength Uncertainty

To establish an interferometric link between the measured phase change and the actual length unit measured, the laser evaluation wavelength, λ , is used. As the unwrapped phase change is proportional to the measured unit length via the evaluation wavelength, uncertainty in the measured unit length scales directly with the relative uncertainty in the evaluation wavelength. The wavelength uncertainty can be further divided into static wavelength uncertainty and dynamic wavelength uncertainty. The static wavelength uncertainty is uncertainty in the true value of the emission wavelength of the laser, and is often limited by the capabilities of the laser diode and driver themselves. Typically this is reduced by measurement of the laser emission wavelength using a spectrometer, however at the highest levels of precision, the wavelength uncertainty is limited to the intrinsic linewidth of the laser, and significant work has been undertaken to reduce this, discussed further in Chp. 3.

Dynamic wavelength uncertainty describes uncertainty related to drift in the emission wavelength that occurs during a measurement. Critically, where static wavelength uncertainty is proportional to the change in optical path difference caused by a displacement only i.e., the distance travelled, dynamic wavelength uncertainty is proportional to the entire optical path of the interferometer, including the dead path. Dead path is the optical path difference between the measurement and reference beams at the position where the interferometer electronics have been zeroed [112], *i.e.* the OPD offset of the interferometer. As a result, dynamic wavelength uncertainties have significantly more impact than static uncertainty on the overall wavelength uncertainty. Stabilisation of the emission wavelength over the measurement period is used to reduce this uncertainty, and techniques developed to achieve this are discussed in Chp. 3, where discussion of techniques for both short and long term stabilisation of the emission wavelength is included.

Thermal and Refractive Index errors

Thermal changes primarily have two effects on the uncertainty of a measurement system; changes to the geometry of optics within a system, and changes in the refractive index of the medium, typically air, through which the sensitive regions of the interferometer pass. In the former case, thermal effects change the physical distances and indices of the optics themselves, creating apparent changes in the optical path lengths of the interferometer arms. This effect is increased for both a larger number of optical elements and a larger imbalance in the amount and type of glass between the interferometer arms. The latter effect, changes in the refractive index of the interference medium, is well-explored for its implications on the traceability of interferometric measurements. Most free-space interferometers use air as the medium the beam paths travel through, outside of the glass optical components, and the refractive index of air is dependant on factors such as temperature, pressure and humidity [113]. As a result, any change in

these factors, most likely thermal fluctuations, will cause an apparent change in the optical path length as measured by the interferometer. As in the case of wavelength uncertainty, the uncertainty due to refractive index changes can be both static, and dynamic, where static uncertainty occurs due to uncertainty in the true value of the refractive indices or optical path length of mediums in the beam path, and dynamic uncertainties occur due to changes in these factors during a measurements such as due to thermal changes. Similarly, the static uncertainties scale only with the change in optical path length of the interferometer, i.e., the distance travelled, whereas dynamic uncertainties will scale with the entire beam path including 'dead path' outside the sensitive region of the interferometer.

In order to account for these uncertainties, a quantitative approach can be taken using measurements of temperature, pressure and humidity and using derived equations for the refractive index of air from Edlén [113] or later, improved versions of these equations [114] including work by Birch *et al.* [115, 116] which adjusts Edlén's original equations, to account for changes to the International Temperature Scale, since their original publication. This approach requires data input from instrumentation measuring the temperature, pressure and humidity of the environment at the time of measurement, and makes assumptions about the air composition, with variations on this accounted for in the uncertainty values. An alternative approach directly measures the refractive index of air using a refractometer [117, 118], which is essentially a fixed path length interferometer. Here, the measured phase change as the measurement arm chamber is exposed to the ambient air from vacuum, compared to the reference arm maintained at vacuum, is used as a direct measure of the refractive index of the ambient air. Unlike the previously described method using empirical determination of the refractive index, refractometers are not subject to uncertainty due to air composition, rather are measurements of the true refractive index of the sample medium, independent of assumptions. However, thermal gradients or general air circulation between the measurement and reference arms, as well as gradients between the sample location and the real measurements space, can still cause additional uncertainty. In order to assure true determination of the refractive index over extended time frames, both methods require real-time or sampled measurement instrumentation, either measurement of the input parameters to the empirical equations, or read-out and calculation of the phase changes from the refractometer.

Cosine Error

Cosine errors directly affect the measurement technique, specifically, they are geometric errors that occur due to and angular misalignment between the measurement axis and the axis of motion. As the name suggests, the cosine of the angle typically defines the relationship between the measured displacement and the true displacement of the target. Notably, in most interferometric techniques cosine error scales with the magnitude of the target surface displacement and to a first order, the square of the angle deviation, however as discussed by Badami *et al.* [5], the proportionality of this scaling depends on the target and the spatial geometry of the beam paths. For example, a corner-cube retroreflector will always return the reflected beam parallel to the incident beam, however a planar mirror will return the reflected beam at the same angle to the mirror normal as

the reflected beam which can cause additional errors. Fig. 2.15 illustrates an example of cosine errors, occurring from a corner-cube retroreflector.

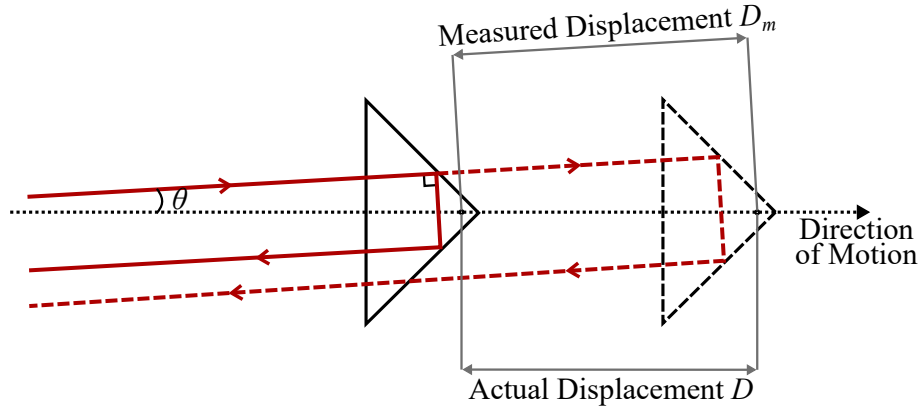


FIGURE 2.15: Illustration of cosine errors originating from a corner cube retroreflector. The red line indicates the beam path incident at angle θ , with the dashed version representing the beam when the corner cube is displaced. The dotted central line indicates the direction of motion. (Adapted from [5])

Abbe Error

Abbe error is the uncertainty derived from the angular misalignment of the target to the measurement axis due to an offset between the measurement axis and the axis of motion. Described by Ernst Abbe [119], the Abbe principle states that the measuring system, in the case of interferometry - the optical beam axis, must be coaxial to the line of measured displacement, otherwise parallax errors must be accounted for. Typically, these occur due to small angular differences between the target and the motion axis. As with cosine error, the magnitude of Abbe error scales with the distance of displacement, however unlike cosine error, the error also scales, in the first order, proportionally with the angle deviation θ and the lateral separation between the motion axis and measurement point Ω . In addition the type of target does not factor into this uncertainty. To state the Abbe error, δD_{Abbe} , in interferometry, the point of interest POI and measurement beam axis are defined, with the direction of motion *assumed* to be parallel to the measurement beam axis. angular misalignment between these is incorporated into the Cosine error discussed previously. Fig. 2.16(a) shows an illustration of a measurement system where the optical measurement beam is offset from the surface of a stage which is the measurement POI. Here, the measurement beam is laterally separated from the surface of the stage, which is the motion axis by a distance Ω . The displaced stage has an angular error of θ associated with the motion. For illustrative purposes, only the corner cube is shown at an angle.

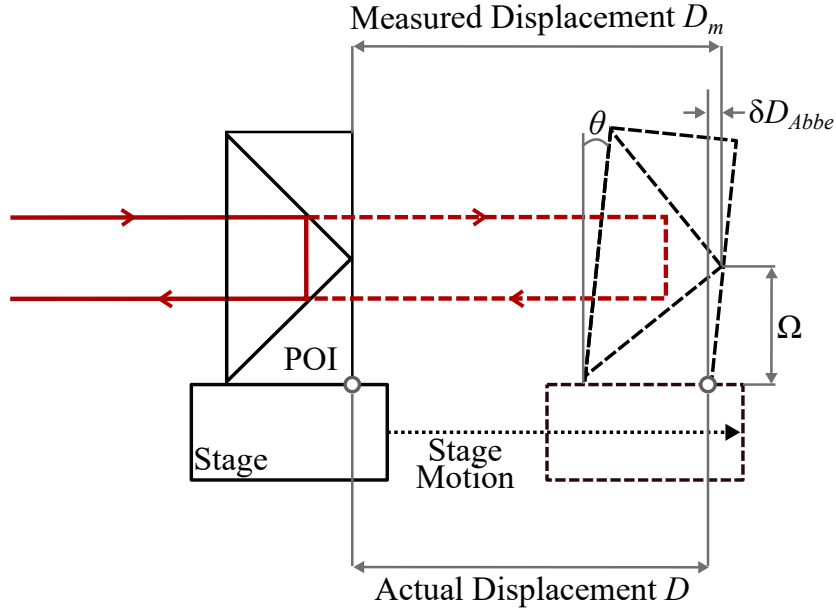


FIGURE 2.16: Illustration of Abbe errors δD_{Abbe} originating from a corner cube retroreflector mounted on a moving stage with the measurement POI denoted. The displaced stage shows an angular error θ , with the lateral separation of the measurement axis and the motion axis denoted as Ω . (Adapted from [5])

For the measurement setup illustrated in Fig. 2.16, the Abbe error δD_{Abbe} can be observed from the distance between the measurement beam and the POI at the stage surface and the angular error θ of the target surface as it is displaced. Typically, the Abbe error is calculated using a priori knowledge, such as the manufacturers specifications of angle deviation and uncertainty over the stage motion range, and is typically comprised of two individual components from the angular error in two mutually orthogonal directions.

2.4 Summary

In this chapter, an overview of displacement measuring techniques in precision dimensional metrology is presented. A variety of techniques used in displacement measuring applications are briefly described and illustrated, such as capacitive, Piezoresistive and linear optical encoders, followed by discussion of the achievable resolutions, ranges and limitations of each technique published in academic literature. Notably these techniques include laser interferometry, which is the focus of the work in this thesis. These techniques are compared in the context of single dimension measurements, however following this, the various techniques are discussed in the context of multi-dimensional measurements and the limiting factors that restrict the extension of these techniques. For contact techniques, this is usually due to the size and space required by the additional components needed for measurements in multiple dimensions as well as any strict angular requirements, as in the case of electric and magnetic field-based sensors. For non-contact measurements, these limiting factors typically arise from the complexity of the sensors or components required for measurements in multiple dimensions. In general,

a lack of compact non-contact techniques for multiple dimensions, that do not require complex optical setups, is identified.

In addition, discussion is included of some of the key principles in laser interferometry, specifically phase evaluating interferometry, as this is one of the main focuses of the work presented in this thesis. The principles behind two wave interference, as well as homodyne, heterodyne and pseudo-heterodyne interferometry are presented to provide the foundation for the range-resolved interferometric signal processing technique presented in Chp. 4. Following this, some of the main sources of uncertainty in interferometry are presented and discussed. This includes wavelength stability, where techniques for improving this are discussed in Chp. 3 as the other main focus of this thesis.

Chapter 3

Review of Wavelength Stabilisation Techniques

This chapter broadly overviews and discusses some of the techniques used in laser-based applications for stabilisation of the emission wavelength of a laser. Firstly, the principles of stabilisation are discussed and a typical stabilisation process is outlined. Discussion is then given to the differences between short-term and long-term stabilisation applications and the impact this has on stability requirements. This is followed by brief descriptions of some of the most common stability references, the techniques used to interrogate them, and their limitations. Expanding on these techniques, discussion is given to the efficacy of these techniques in applications with wide-wavelength modulation where the modulation depth is several orders of magnitude larger than standard references.

3.1 Fundamentals of Stabilisation

For any laser-based application, an understanding of the output wavelength of a laser is important, as nearly every component in an optical system will have some wavelength dependant characteristics such as transmission or isolation. As Chp. 2, Sec. 2.3.5 discusses there are two types of uncertainty associated with wavelength stabilisation; static uncertainty caused by uncertainty in the true wavelength of the laser emission *before* measurements are made, and dynamic uncertainty caused by drift in the wavelength that occurs *during* measurements. Further to this, quantifying the laser emission wavelength and limiting these uncertainties is of particular importance in phase evaluating laser interferometry where measurements are inherently linked to the wavelength of the measurement laser. For high-precision applications, a precise measurement of the exact central wavelength and linewidth of the output lasing beam is required.

To quantify the stability of the laser emission S , the ratio of the frequency variation or uncertainty ($\delta\nu$) to the average emission frequency ($\bar{\nu}$) is considered.

$$S = \frac{\delta\nu}{\bar{\nu}} \quad (3.1)$$

This fractional stability is unitless and can scale as necessary with the demands of the application.

In this chapter, stabilisation is discussed as the reduction of dynamic uncertainties over the measurement period. Techniques for calibration of laser emission, primarily aimed at reducing static uncertainties are well established and have significant commercial presence in the form of spectrometers and wavemeters.

3.1.1 A note on Wavelength vs. Frequency

In both stabilisation techniques and interferometric techniques, the terms "frequency", commonly represented by the greek character nu (ν) and wavelength, represented by the greek character lambda (λ), are used interchangeably in an application-dependant fashion. In keeping with this, in an attempt to remain consistent with the definitions as published by the respective authors, in this chapter both wavelength and frequency will be used as terms for the laser emission spectra. Regardless, the relationship between frequency and wavelength is always consistent with standard electromagnetic principles and can be defined with the speed of light in vacuum c_0 .

$$\lambda = \frac{c_0}{\nu} \quad (3.2)$$

3.1.2 Principle of Stabilisation

A typical stabilisation process, illustrated in Fig. 3.1, involves identifying or creating some definitive, and importantly, fixed feature that is within the bandwidth of the laser emission, for example an atomic feature such as an absorption line or a mechanical feature such as an optical cavity. The detection methods for the locking to this feature vary greatly and a sample of the main techniques are discussed below. These detection methods generate an error signal that is analysed and combined with an active feedback system that controls one or more laser parameters, or mechanical components, to lock the laser emission wavelength to the selected reference feature.

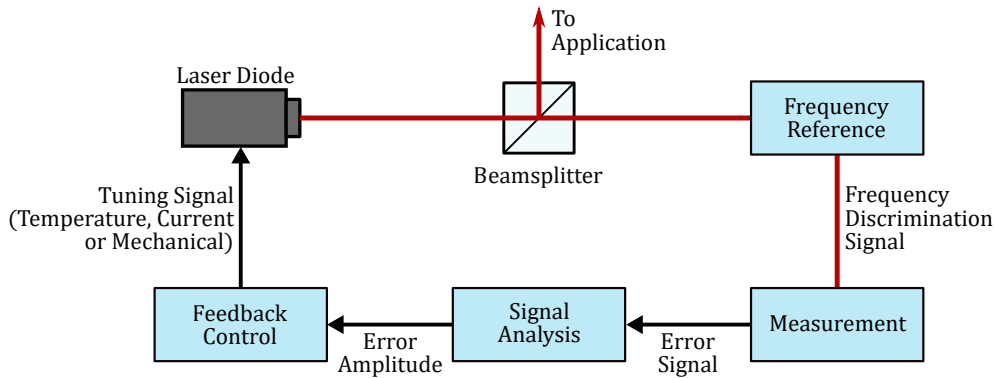


FIGURE 3.1: Typical process for stabilising a laser diode

This process is broadly described as follows;

Frequency Reference: A reference with a fixed frequency, either atomic or mechanical, that interacts with one or more incident beams to generate a frequency discriminate signal, containing information about the relative frequency between the laser

emission and the reference feature. A selection of potential references are discussed below although typically these are either atomic, such as gas absorption lines, or mechanical such as optical cavities.

Measurement: The frequency discrimination signal is converted into an electrical signal, typically using one or more photodetectors, to produce the error signal. In some techniques, multiple photodetectors are used to record features such as the laser emission without interaction with the frequency reference, and the outputs from these are combined either in this stage to produce a single error signal, or in the signal analysis step below.

Signal Analysis: The error signal is analysed using a wide range of techniques, discussed in further detail in Sec. 3.2.2 of this chapter, dependant on the type of measurement technique and the reference used. The purpose of this step is to produce the amplitude of the error between the laser emission frequency and the frequency reference that can be fed directly into the feedback control.

Feedback Control: Depending on the method of feedback used, the feedback control converts the error amplitude into a tuning signal to adjust the laser emission frequency towards the reference frequency. This can be either direct temperature tuning or injection current tuning of the laser diode, or an electrical signal adjusting a mechanical component such as the length of the laser diode optical cavity. This process can also include control loops such as proportional-integral-differential (PID) loops to adjust for transient effects.

Significant work has been undertaken in utilising stabilisation techniques following this process, stabilising the output wavelength (or frequency) of single-mode, narrow bandwidth lasers for a wide range of wavelengths. In general, wavelength stabilisation is separated into two regimes; short-term stabilisation or long-term stabilisation where the distinction between these two is typically governed by the application and hence requirements of the laser.

3.1.3 Short Term vs. Long Term Stabilisation

This section compares short-term and long-term stabilisation, in terms of the distinction between the two, the limiting factors of each and the various applications for which these are suitable. This is followed by discussion of some of the techniques commonly used for stabilisation work in both regimes. The principle distinction between short and long-term stabilisation is the integration timescales over which they operate, with the boundary typically defined as 1 second.

Short-term stabilisation refers to the locking of the emission wavelength of the laser over timescales at or below 1 second, where the typical application of short-term stabilisation is for the benefit of narrowing laser linewidths. Reduced laser linewidths are of specific interest in a variety of applications, most prominently in order to reduce laser frequency noise. Applications of short-term stabilisation include, but are not limited to optical clocks [120], ultra-fast optical communications [121] and coherent LIDAR [122].

In long-term stabilisation, i.e. on timescales larger than 1 second, the objective of stabilisation techniques is typically the maintaining of a fixed wavelength emission over

an extended period. As in short-term stabilisation, this is typically achieved by locking the laser emission wavelength to a fixed marker or standard reference, however here the linewidth of the laser emission is considered less important. Long-term stabilisation is required by such applications as precision range-Doppler imaging [123], optical frequency combs for telecommunications [124] or laser cooling [125, 126]. Precision metrology also requires long-term stabilisation, where measurements can take place over extended periods of time. For laser interferometry, as discussed earlier, this is of particular importance as measurements are intrinsically linked to the emission wavelength of the laser, and so the precision at which these measurements can be made can be severely limited by the stability of the laser.

3.2 Active Stabilisation

This section discusses some examples of existing techniques for both long- and short-term stabilisation of laser emission wavelength, including the advantages and limitations of these. Firstly, some of the typical optical and mechanical references used in stabilisation are discussed, followed by some of the techniques these are applied in. Here, the techniques discussed are a small selection of stabilisation techniques that are typically applied alongside a primary application.

3.2.1 Frequency References

Optical Cavities

Optical cavities is the broad name for a variety of arrangements of optical components that create a cavity in which an optical beam can resonate. The optical cavity in its simplest form consists of two parallel, plane mirrors called the Fabry-Perot étalon, which is discussed further below. In general, the arrangement of optical components allows light of a particular frequency to resonate within a cavity while light of other frequencies is suppressed by destructive interference.

Optical cavities can be either linear (or standing-wave) resonators or ring resonators. In linear resonators, the light is principally reflected between two mirrors. There can be any number of optical elements such as gain media between them, but the reverse path will pass through the same set of optics in reverse. In ring resonators, the light can circulate in two different directions so there is no effective "end" mirrors. The limitations of optical cavities arise from the mechanical and optical components necessary to implement them, in particular the cavity itself suffers from a noise limit dominated by thermal noise. This can be reduced by using highly thermally stable components, reducing the noise limit towards the phase noise limitation of the laser but typically still requires another mechanical reference or adaptive calibration to calculate the true cavity length.

Fabry-Perot étalon

The Fabry-Perot étalon is a specific subset of the optical cavity. Developed in 1899 by Charles Fabry and Alfred Perot [127], it consists of two planar mirrors arranged parallel to each other such that a resonance cavity is formed. In this cavity, light entering is

reflected between the two mirrors and only light matching the resonance condition is emitted back from the cavity. The resonance condition for light of wavelength λ of a simple Fabry-Perot cavity consisting of two plane-parallel reflective surfaces separated by distance d with some integer n is:

$$2d = n\lambda \quad (3.3)$$

The Fabry-Perot étalon, due to the high-reflectivity of the mirrors, can achieve a high finesse, which is defined by the width of the resonance peaks compared to the frequency separation of resonance modes. This allows, by tuning of the inter-mirror distance, a high wavelength resolution. This comes at the cost of reducing the free-spectral range (FSR) of the cavity, that is the range of unique resonator modes possible. In laser stability, this is beneficial in reducing the number of examined modes, and allowing for high-precision locking to a stable reference. The cavity itself however requires a technique of processing the reflected and/or transmitted light into information about the difference between the probe laser wavelength and the intrinsic resonance frequency of the cavity. While simple in principle, the Fabry-Perot cavity is limited by alignment demands, where alignment of the two planar mirrors is critical to obtaining high finesse cavities. For this cavity, alignment differences between the mirrors of less than a few arcseconds is needed to maintain a good resonance within the cavity.

Atomic Absorption References

In atomic absorption references, atomic elements in a gaseous state are probed using an optical beam. For a given element, the atoms comprising the element have discrete energy states that the constituent electrons can occupy. To raise an electron from its ground state to a higher energy level (or excited state), a photon of the correct energy needs to be absorbed by the atom. In general terms, the energy of a single photon E with wavelength λ or frequency ν can be calculated using the Planck constant h , the speed of light in vacuum c_0 and the Planck-Einstein relation given by:

$$E = \frac{hc_0}{\lambda} = h\nu \quad (3.4)$$

where $h = 6.62607015 \times 10^{-34} \text{ J}\cdot\text{Hz}^{-1}$ and $c_0 = 299,792,458 \text{ ms}^{-1}$. Using this relation, the wavelength of the photon needed to raise an electron from the ground state to an excited state can be calculated. Critically, this photon needs to be the exact wavelength, and hence energy, required by the electron for the atom to absorb the photon and the electron to be excited. Using this principle, an atomic element can be probed using an optical beam such as laser, with a linewidth wider than the excitation bandwidth, and only the wavelengths of the beam that correspond to excitation energy levels will be absorbed, with the rest passing through unchanged. By measuring the output power as a function of the wavelength, significant reductions in the power at the absorbed wavelengths are measured, and these are referred to as absorption lines. Fig. 3.2 shows an example absorbance spectrum for acetylene (C_2H_2).

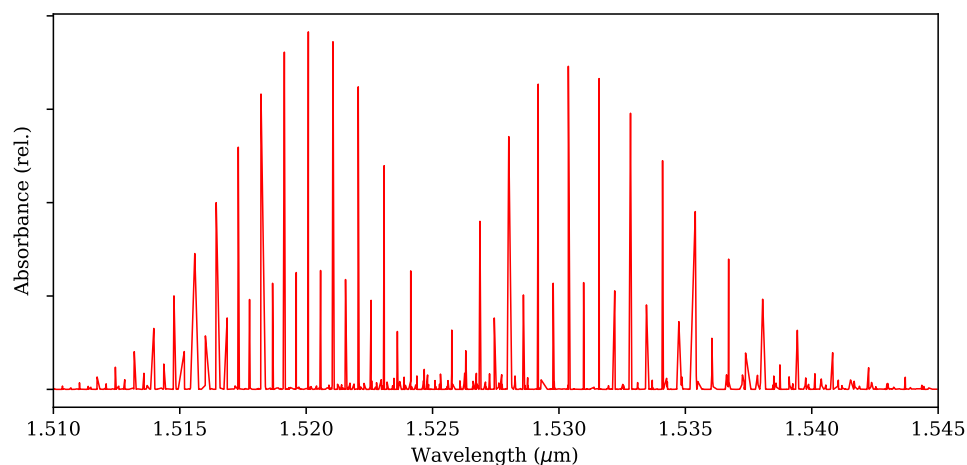


FIGURE 3.2: Absorbance spectrum for acetylene gas (C_2H_2) in the near infra-red at 256K (data from HITRAN [128])

A typical locking technique will use one or more of these absorbance features as a reference and depending on the wavelength of the laser emission, different atomic references will be used to ensure strong absorbance features are present within the wavelength region of interest. Sealed gas cells are available as both free-space and fibre-coupled components. However, the characteristic absorbance features of an atomic reference typically have a dependence on several parameters including temperature, and pressure, which can cause broadening of the spectral lines as well as small changes in the peak centres, so stabilisation techniques often need to have good control over these parameters. A notable effect of this is called Doppler broadening, where the distribution of velocities in an atomic reference causes an overall broadening of the spectral lines, typically overlapping several absorbance features. In addition, the use of an atomic reference is dependant on having gas lines available in the wavelength region of interest, and the references themselves have to be practical for use in terms of the handling, storage and setup.

Comparison

Atomic absorption references and optical cavities each offer advantages and limitations that make them suited to different applications.

Atomic absorption references offer a pathway to traceable measurements, with absolute frequency standards that are well documented and defined for a variety of gas conditions such as temperature, pressure and molecular concentrations. In addition, atomic references typically have very weak dependence on the environmental conditions outside of the sealed cell they are commonly packaged in, so can offer highly stable references outside of lab conditions without extensive calibration or compensation needed over long timescales.

Optical cavities on the other hand, are typically significantly more susceptible to the

environmental effects such as refractive index changes, air circulation and thermal expansion of components, and require calibrated measurements to provide absolute stability of the laser emission frequency. However, high finesse optical cavities can produce frequency discriminate signals with high amplitudes and with exceptionally narrow linewidths, which is highly advantageous for short-term locking or applications where reduction of the linewidth and therefore frequency noise is required.

As a result of these advantages over different timescales, it is not uncommon for stabilisation techniques to include both types of reference in order to achieve long-term stability with improved laser frequency noise [129], or calibration for absolute frequency measurements over short timescales.

3.2.2 Stabilisation Techniques

Harmonic Frequency Stabilisation

Harmonic frequency stabilisation (HFS), also referred to as derivative spectrum stabilisation, is one of the principle techniques for locking light to a frequency reference. HFS uses locking of the odd-ordered differential functions resulting from the transmitted light through a frequency reference. Slight variations of this technique exist for both atomic references and optical cavities, with example setups for each shown in Fig. 3.3, however the working principle remains the same. A small frequency modulation, with an amplitude larger than the reference linewidth is applied to the laser emission, and the odd-order derivatives (typically first or third order) of the quotient of the transmitted waveform and modulation waveform are analysed. The quotient curves are distributed anti-symmetrically on either the positive or negative halves of the vertical axes depending on which side of the reference frequency the laser emission frequency lies. From these curves therefore, the frequency error can be determined and appropriate feedback applied.

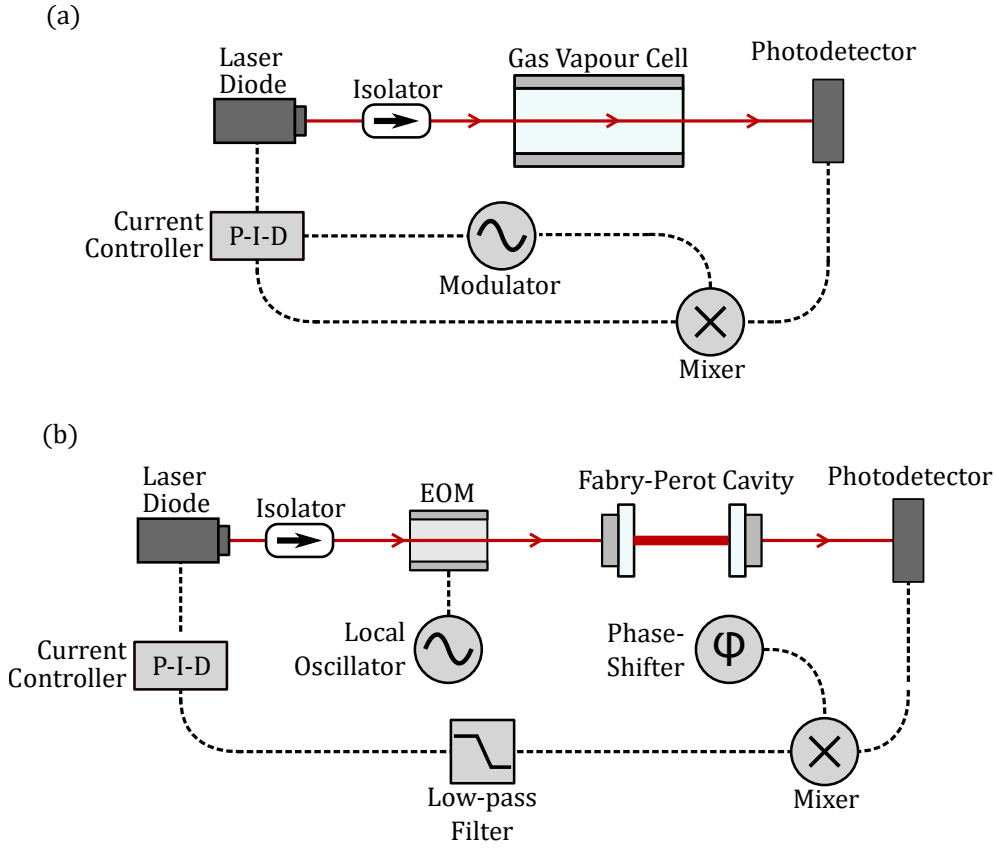


FIGURE 3.3: Example setups for harmonic frequency stabilisation using (a) a gas vapour cell (b) a Fabry-Perot cavity as frequency references. (Adapted from [130]).

The advantage of HFS lies in the relative simplicity of the setups required, with minimal optical components to align in both examples however the stability is typically limited to half the linewidth of the relevant feature. Early work by Tsuchida *et al.* [131] indicated the efficacy of this stabilisation technique, demonstrating locking of an AlGaAs laser to the linear rubidium D₂ transition with a frequency stability of 3×10^{-9} over 10 ms, and over long timescales of 100s, a frequency stability of 1.4×10^{-12} .

Pound-Drever-Hall Technique

Developed in 1983, the Pound-Drever-Hall (PDH) technique describes a process for active tuning of a laser diode to lock it to a reference cavity of a higher stability [132, 133]. By phase modulating the emitted laser light, a carrier frequency and two sidebands are generated. When this light is reflected from a resonance cavity such as a Fabry-Perot, it is measured using a high-speed photodetector. This photodetector signal, now consisting of two un-altered side bands and a phase-shifted carrier component, is mixed with a local oscillator in-phase with the original laser modulation. After low-pass filtering, the resultant electronic signal gives a measurement of the laser emission wavelength in comparison to the cavity resonance. Combined with a feedback mechanism, typically a

proportional, integral and differential (PID) loop, the laser emission wavelength is tuned to match that of the cavity resonance.

In conjunction with a Fabry-Perot cavity, the PDH technique creates an active laser stabilisation system, an example of which is shown in Fig. 3.4. Benefiting from the high finesse and hence high wavelength resolution of the Fabry-Perot cavity, this system is capable of achieving a high-precision locking standard.

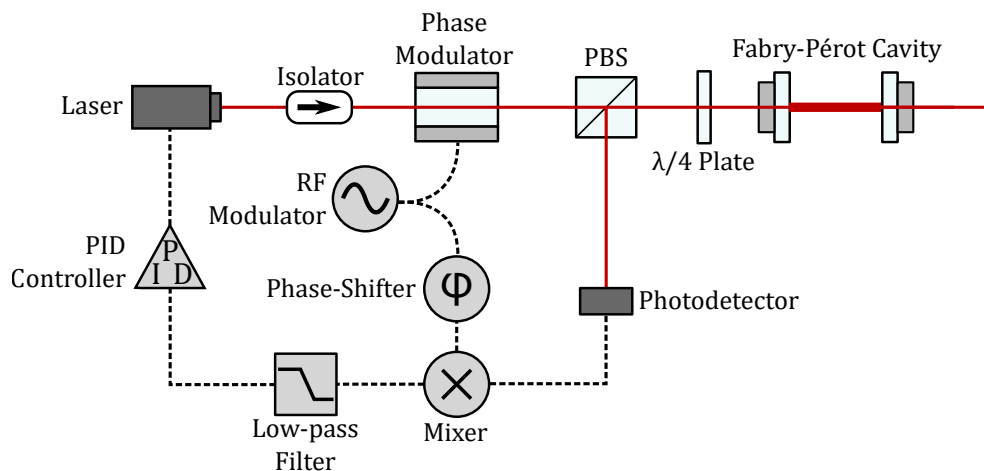


FIGURE 3.4: Example of a Pound-Drever-Hall experimental setup using a Fabry-Perot étalon as a reference cavity. This setup uses a polarising beamsplitter (PBS) and $\lambda/4$ waveplate to separate the returning light from the emitted light. (Adapted from [132]).

Figure 3.4 shows an example setup of a Fabry-Perot cavity probed using the Pound-Drever-Hall technique of laser stability and feedback. An optical isolator is used to remove back reflections into the laser itself, whilst a polarising beamsplitter, denoted PBS, is used in conjunction with a quarter wave-plate to separate and redirect the light returning from the cavity onto the photodetector. This setup comprises the basics of PDH stabilisation, however variations of this design have been used to great effect, including analysis of the transmitted light through the cavity. Using Fabry-Perot cavities combined with the PDH technique, Webster *et al.* [134] demonstrated exceptional frequency stability of $\sim 1.3 \times 10^{-15}$ over 5 seconds of an Nd:YAG laser, however to achieve this requires strict temperature control and active-vibration isolation of the F-P cavities. An alternative approach, using a cascaded PDH technique published by Yu *et al.* [135] achieves over a significantly longer time period of 4 hours, frequency stability of $\sim 2.8 \times 10^{-8}$. For short-term locking, significant work has been published using the PDH technique to reduce the linewidth of highly stable lasers, such as work by Andreou *et al.* [136] demonstrating linewidth reduction to 5 kHz of a distributed Bragg reflector (DFB) laser diode.

Tilt-locking

In tilt-locking, an optical cavity, such as a Fabry-Perot is used to generate an error signal. Unlike in the PDH method described above which uses the transmitted or reflected signal amplitude of an optical cavity, the error signal in tilt-locking is generated from the change in spatial modes of a cavity probed off-resonance. To achieve this, a split

photodiode is used at the output of the optical cavity, which measures the spatial-mode pattern of the light interfering between the carrier (original) field and the optical cavity emission field, illustrated for a simple ring resonance cavity in Fig. 3.5. Specifically, the split photodiode measures the overlap of two transverse spatial modes, the TEM_{00} mode and the TEM_{01} tilt mode, arising from an input beam that is aligned and mode matched to give only these two modes.

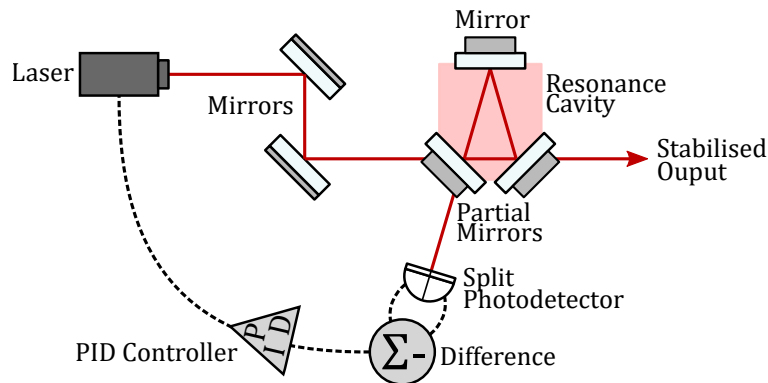


FIGURE 3.5: Example of a single-pass tilt-locking experimental setup using a ring cavity as a reference.(Adapted from [137]).

The resultant spatial overlap of these two modes forms a two-lobed intensity distribution separated in the spatial plane, which each half of a 2-part split photodetector integrates over. In the case of ideal resonance of the carrier beam with the cavity, no phase shift is added to the TEM_{00} mode by the cavity, so the two field vectors are orthogonal, and result in an equal intensity signal on both sides of the photodetector. In off-resonance conditions, the TEM_{00} undergoes a phase shift in the cavity whilst the TEM_{01} remains unchanged, resulting in an unequal spatial distribution of the modes between the two photodetector halves and hence the error signal comprised of the numerical subtraction of the two halves is proportional to the phase shift caused by the cavity to the off-resonance beam. The advantage of tilt-locking is that it is modulation-free, which allows for setups with no active components and the comparatively few optical components, as well as the relatively simple electronics required [137–139]. However, tilt-locking typically requires very high mechanical stability from the resonator to achieve a high frequency stability. Published works by Slagmolen *et al.* [140] indicate stability in the region of 1.3×10^{-14} over short time scales (100 ms) with similar work by Robins *et al.* [138] indicating frequency stability better than $\sim 2.8 \times 10^{-9}$ for an 825 nm DFB laser.

Self-Injection Locking

In injection locking techniques, typically a low-power, low-noise laser called the *seed laser* is used to force a high-power, high-noise laser called the *slave laser* into emitting at the frequency of the seed laser with the low-noise of the seed laser. This is of huge benefit to applications where high-power and low-noise is required from the laser emission. In practise, low noise is not simply achievable by the high-power laser alone as they are typically significantly more susceptible to mechanical and thermal noise and as a result, injection locked lasers are used. To achieve the necessary locking of the slave to the seed

laser, the emitting frequency of the two must be similar, so feedback systems are usually employed to prevent the seed laser from drifting too far from the slave laser emission frequency.

In self-injection locking, the seed laser source does not come from a separate laser, but rather as the name suggests is injection locked by the main laser instead. As a result, some optical feedback is required, typically spectrally filtered to an appropriately narrow-band signal that the main lasing beam can be locked to, using techniques such as a resonance cavity. An example of this is illustrated in Fig. 3.6 using a ring resonating cavity. The spectrally filtered return beam is then used to injection lock the main laser, resulting in a spectrally-narrow main beam.

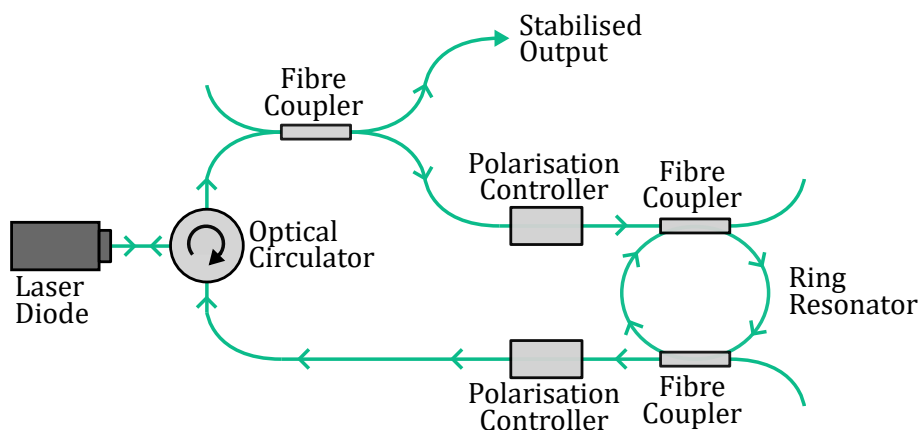


FIGURE 3.6: Example of a self-injection locked laser setup using a ring resonance cavity (Adapted from [141]).

Self-injection locking has been used to great effect for stabilisation of lasers in both short and long-term. Spirin *et al.* [142] demonstrated reduced linewidths down to ~ 2.8 kHz but note this can be improved further by using a laser diode without the -30 dB internal isolator used in their published work. Using self-injection locking coupled with a whisper-gallery mode resonator, noted for its strong resonance effects, Lai *et al.* [143] published results achieving stability approaching $\sim 10^{-12}$ over 200 seconds, and in general over 2 seconds to 2 hours achieved a stability better than $\sim 10^{-11}$.

Dual Mode Polarisation Stabilisation

Dual mode polarisation stabilisation is a commonly used technique for stabilising the output of lasers capable of producing two or more orthogonal polarised modes. In particular, this includes the Helium-Neon laser [144, 145], a staple of modern laser based applications, the advantages of which are discussed in Chp. 7 of this thesis. In this technique, illustrated in Fig. 3.7, the emission from the laser is split into the two orthogonal modes, typically using polarisation sensitive components, and the intensity of both modes are measured. Measuring the gain curve of the laser power, a feedback signal is applied to equalise the amplitude of the two polarised modes and the feedback signal is then used to adjust the laser cavity length, typically adjusting the temperature of a

heater wrapped round the glass tube containing the lasing medium. A polariser at the output is then used to select between the polarised modes.

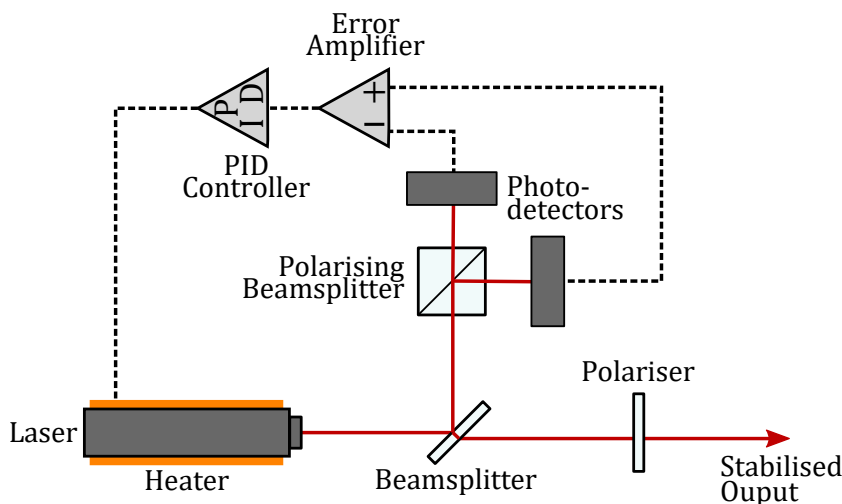


FIGURE 3.7: Example setup of a dual mode polarisation stabilisation technique.

Dual mode polarisation stabilisation is exceptionally popular in commercial stabilised lasers, in particular HeNe lasers. It requires very few optical components outside of the polarisation sensitive beamsplitters due to the relative simplicity, ease of application and, fundamentally, in a well-balanced system, very high stability [146]. This technique also allows for high output power in a single longitudinal mode which is advantageous for most precision metrology applications. The stabilisation of the HeNe laser is of particular interest in this thesis, as later experimental work discussed in Chp. 7 will compare a highly-stabilised commercial HeNe, which uses dual mode polarisation stabilisation, with a novel stabilisation technique.

Saturated Absorption Spectroscopy

Saturated absorption spectroscopy (SAS) is a sub-technique of the harmonic frequency stabilisation (HFS) discussed previously in Sec. 3.2.2. Also called Doppler-free spectroscopy, the effects of Doppler broadening of the atomic absorption linewidth, as mentioned previously in Sec. 3.2.1 arise from the combined effects of the Doppler shift of each atom's velocity causes a broadening of the atomic absorption linewidth which limits the accuracy to which these features can be locked to by several orders of magnitude. To reduce this effect, the atoms can be cooled such that the Doppler broadening is less prominent, which is impractical outside of lab conditions as it often requires temperatures as low as a few millikelvin. More typically, techniques such as SAS are used. In SAS, a typical setup of which is shown in Fig. 3.8, an atomic reference is interrogated using two counter-propagating beams, a 'pump' and a 'probe' beam. As both beams must have the same frequency, they are typically generated by splitting the same laser source, and are directed in opposite directions through the atomic reference gas.

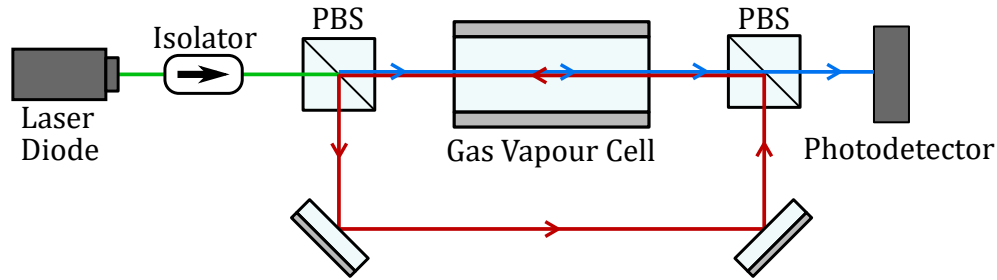


FIGURE 3.8: Illustration of an example saturated absorption spectroscopy setup with a pump beam shown in red and a probe beam shown in blue using two polarising beamsplitters (PBS)

As the frequency of the laser is swept over the absorption feature, the pump laser excites atoms from the ground state into the excited state at the resonant frequency, which if measured directly shows the absorption line broadened by the thermal motion of the atoms. If the probe beam is then incident on an atom in the excited state, it undergoes stimulated emission, returning to the ground state and releasing a photon of the same energy as the bandgap between ground and excited states. These two processes, absorption and stimulated emission are illustrated in Fig. 3.9 for a simplified two-state system.

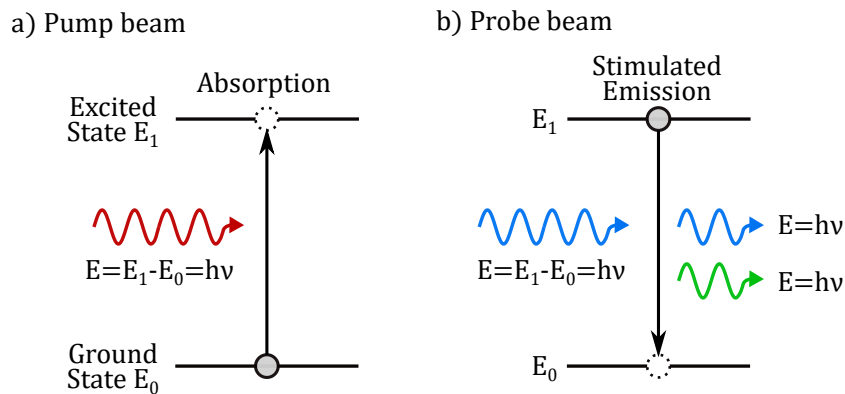


FIGURE 3.9: Illustration of a two energy state system interacting with the a) pump beam b) probe beam

If the transmitted light is therefore measured over a sweep of the laser frequency, a dip in the absorption curve is seen at each spectral feature. This dip has a linewidth significantly narrower than the Doppler broadened line and therefore provides a much more accurate reference feature for stabilisation, illustrated in Fig. 3.10. Stabilisation to these features is typically accomplished through lock-in amplifiers, which provide an error signal to a servo mechanically adjusting the laser cavity length or injection current to actively lock the laser emission wavelength to the Doppler-free absorption line.

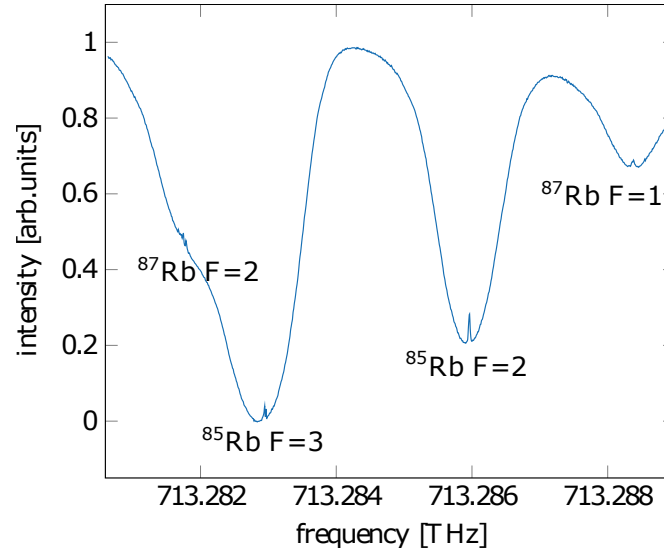


FIGURE 3.10: Saturated absorption spectrum for the D2 transition lines of ^{85}Rb and ^{87}Rb showing the stimulated emission dips in the Doppler-broadened peaks. Graph taken from Glaser *et al.* [147].

SAS improves on the HFS discussed previously reducing a $\sim\text{MHz}$ Doppler broadened line down to $\sim\text{GHz}$ or narrower. This significantly lowered linewidth allows for higher precision frequency stabilisation. Over 100s integration time, Talvitie *et al.* achieved a stability of 5×10^{-12} for saturated absorption locking to molecular iodine at 633 nm. Similarly, over 10 s integration time, Chen *et al.* [148] demonstrated a stability of 8×10^{-9} for SAS locking of a InGaN diode laser to iodine hyperfine components.

Modulation Transfer Spectroscopy

Similar to saturated absorption spectroscopy, modulation transfer spectroscopy (MTS) uses an atomic gas as a frequency reference, passing two counter-propagating beams through the gas vapour. Unlike SAS however, the pump beam is phase modulated using an electro-optic modulator (EOM) or acousto-optic modulator (AOM) before it enters the gas cell, an example setup of which is illustrated in Fig. 3.11.

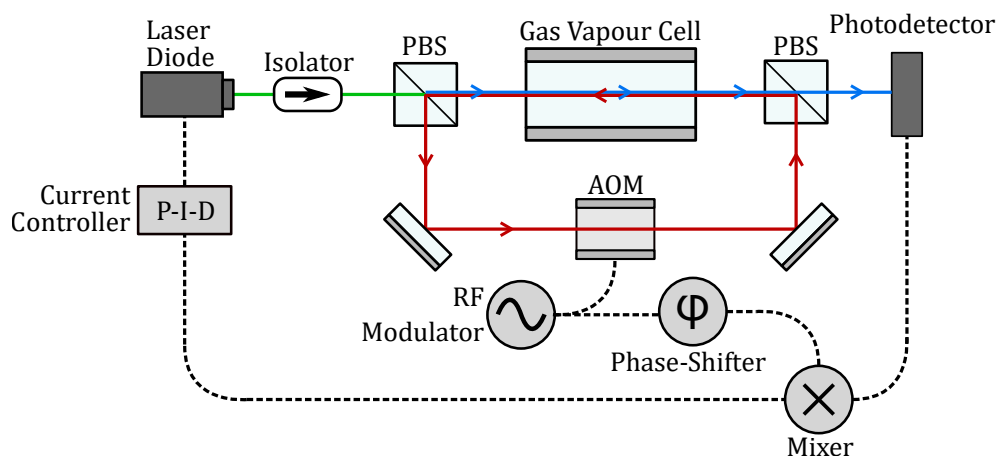


FIGURE 3.11: Illustration of a modulated transfer spectroscopy setup using an acousto-optic modulator and injection current tuning of the laser diode. The pump beam is shown in red, and the probe beam is shown in blue.

By the process of non-linear four-wave mixing [149], the probe beam acquires the modulation of the pump beam as it propagates through the gas cell in the opposite direction. The pump beam, modulated by the EOM/AOM gains sidebands characterised by the frequency of the modulator. In four-wave mixing, non-linear interactions between beams with three frequency components creates a mixed fourth frequency component which in the case of MTS, means the sidebands of the pump beam are transferred to the probe beam. The probe beam, measured at a photodetector is then mixed with a phase shifted version of the EOM or AOM modulation signal, and the resultant error signal is locked to using a servo to mechanically alter the laser cavity length, or by changing the injection current of the laser diode. Similar long-term stability of $\sim 10^{-11}$ is demonstrated by Lee *et al.* [150] using MTS for time-frames greater than 1×10^5 s, however for short term stabilisation they achieve stability up to 1.3×10^{-1} over time frames of < 1 s.

3.3 Stabilisation in widely-modulated schemes

While all of the above techniques offer respectively high levels of stability in conditions with no modulation or modulation of the emission laser frequency within the order of magnitude of the frequency reference linewidth ($\leq$ GHz), there are techniques that use wide laser frequency modulation, so the efficacy of these techniques within widely-modulated regimes should be considered. This section first discusses some of the techniques that utilise wide-modulation and the stabilisation techniques they employ, followed by the limitations of the existing techniques discussed thus far, and the research gap that the experimental work in Chp. 6 has been developed to fulfil.

3.3.1 Widely-modulated techniques

Frequency-scanning interferometry

Frequency-scanning or swept-frequency interferometry (FSI) was introduced in Chp. 2 Sec. 2.2.2 as a technique for absolute distance measurements of an interferometer. Using a widely tunable laser diode, the emission frequency is scanned in a single direction over a wide range of frequencies, creating phase shifts in an interferometer of unknown length, the measurement interferometer. In original FSI applications, a second interferometer with well-known optical path length, a reference interferometer [151], was also interrogated however this required high-stability setups both during the FSI scan, and over long-term in order to maintain the length-scale traceability. This results in setups with additional stabilising mechanisms and references, adding significant complication in both the design and interpretation of results. Later developments have used additionally stabilised, single wavelength lasers [152], optical frequency combs [153] and synthetic wavelength systems [154], however these also typically suffer from complex setups involving both vibration and temperature sensitive components. More recent published work by Dale *et al.* [122] has presented the use of gas absorption cells as a simpler, more stable length scale reference. In these works they demonstrate that using a second frequency-scanning laser overlapping the first laser in scan frequencies but with opposing scan direction and speed, both passed through a gas absorption cell, traceable measurements of the laser frequency at each point in the scan can be made. Comparing these with the extracted phase measurements, the length of the measurement interferometer can then be determined. Notably, this method does not actually provide a feedback mechanism so is not applying a stabilisation, however using the absolute frequencies of the gas absorption lines to calculate the absolute distance of the interferometer provides traceability and links the distance measurement to international standards.

Range-resolved interferometry

Another technique using wide-modulation, where the modulation depth is on the scale of GHz i.e. several orders of magnitude greater than standard references, is range-resolved interferometry (RRI), used in the work contained in this thesis. RRI uses a widely-wavelength modulated laser diode with a modulation amplitude at least an order of magnitude higher than the linewidth of existing frequency references. In RRI, a wider modulation depth results in increased spatial resolution of optical path differences, discussed in further detail in Chp. 4, and as with most laser interferometric techniques, the emission frequency/wavelength of the laser diode inherently affects any measurements made. RRI has previously been applied to fibre-optic strain [155], shape-sensing [156], multi-surface vibrometry [157], multi-dimensional optical encoding [158] and gas sensing [159] and as a result, stabilisation techniques for widely-modulated laser diodes are both relevant and necessary. However, there is a lack of stabilisation techniques suitable for use in RRI, explained below.

3.3.2 Limitations of Existing Techniques

In terms of existing techniques discussed in this chapter, the limiting factors that make them less-effective at stabilisation of widely frequency modulated lasers arise from the

depth of modulation. For precision techniques such as range-resolved interferometry (RRI) [98] that typically use very large wavelength modulation amplitudes of many 10s of Gigahertz, the modulation far exceeds atomic or mechanical reference linewidths that are typically on the order of 1 GHz, and there is a need to develop new techniques for long-term laser stabilisation.

There are primarily two aspects that affect the ability of existing techniques to stabilise a widely-modulated laser diode. In RRI, a large sinusoidal modulation is applied, typically via injection current modulation which results in a sinusoidal wavelength modulation with a large modulation depth. The wavelength modulation depth is of particular issue for techniques that use the harmonic content of the modulation signal, as the signal energy for the first harmonic signal, which is typically used by techniques such as harmonic frequency stabilisation (HFS) or techniques utilising resonance cavities, is diluted into hundreds of harmonics and therefore determination of the correct locking conditions becomes extremely complex.

In addition to the large wavelength modulation depths it is also necessary to account for the intensity modulation of the laser, which is phase offset from the frequency modulation, in the stabilisation technique if modulation is performed on the laser diode, which adds further complication to the processing. For injection current modulation in diode lasers, another issue that complicates stabilisation stems from the presence of harmonics in the resulting wavelength modulation waveform, even when a perfectly sinusoidal injection current modulation waveform is used, leading to the introduction of second, third and further harmonics of the modulation frequency into the frequency modulation with associated amplitudes and phase offsets [98] that need to be considered when demodulating the combined signal.

3.4 Summary

In this chapter, a review of existing stabilisation techniques is presented. The key principles behind wavelength or frequency stabilisation are described, including how a full stabilisation process operates, as well as the differences between short and long term stabilisation. Following this, the most common types of frequency reference are described, specifically optical cavities and atomic references, and some of the advantages and limitations of each are presented.

A variety of stabilisation techniques are discussed and illustrated, including harmonic frequency stabilisation, self-injection locking and dual mode polarisation stabilisation. Here, the advantages and limitations of each technique are discussed, along with some of the achieved stabilities published in academic literature. Notably, these techniques are primarily applied in regimes with low modulation depth, $< \text{GHz}$, or no modulation. In wider modulated schemes, where the modulation depth is several orders of magnitude larger than typical references these techniques are typically less effective. Discussion is first given to some techniques that utilise wider wavelength modulation, and then to the limitations of existing techniques in these regimes. Here, a lack of techniques for widely wavelength modulated regimes is identified, and this comprises one of the focuses of the work in this thesis, which is investigated in Chp. 6 in experimental work on laser stabilisation for range-resolved interferometry.

Chapter 4

Range Resolved Interferometry

Range-resolved interferometry (RRI) is an interferometric modulation and signal processing technique that uses sinusoidal wavelength modulation of the laser emission to multiplex and demodulate interferometric signals. In this thesis, RRI is used from both an applications and development perspective, with Chp. 5 describing work on applying RRI to multi-dimensional encoders and Chp. 6 describing work on developing stabilisation techniques for widely-wavelength modulated techniques, such as RRI. In this chapter, the principle of RRI is presented to give context to this later work, giving a broad overview of how the technique operates and the technical limitations of the technique. More technical detail of RRI and the basis for the overview below can be found in Dr Thomas Kissinger's thesis [76], in addition to published journal papers [98] including applications of RRI [155–157, 160, 161]. The overview is followed by discussion of the various implementations of RRI that have been used and newly developed throughout this thesis, in terms of both the hardware used and the various advantages and limitations of each, for later reference.

4.1 Operating Principles

This section describes the principle and method of RRI, starting with an overview of how the technique operates on a general level. This is followed by more detailed discussion of the steps involved in RRI, from modulation to demodulation including the basic equations used.

4.1.1 Overview

Range-resolved interferometry is a pseudo-heterodyne signal processing technique for demodulating multiplexed interferometric signals such that the constituent interferometers can be individually interrogated. Using sinusoidal wavelength modulation of the laser emission, typically achieved through injection current modulation, a sinusoidal phase carrier is generated in unbalanced interferometers within the optical beam. The achieved phase carrier waveform is proportional in amplitude to the optical path difference, or more accurately the time-of-flight difference, between the two interfering paths. By determination of these phase carrier waveforms, the multiplexed interferometric signal can then be demodulated and the individual composite interferometers can be evaluated. To achieve this, windowed complex carrier waveforms are applied

to the multiplexed interferometric signal corresponding to a series of expected optical path differences, where a strong amplitude signal is expected for phase carriers corresponding to the optical path difference of interferometers present in the multiplexed interferometric signal.

4.1.2 Method

The technique used in RRI can be broken down into five distinct steps:

- Modulation
- Detection
- Windowing
- Complex Carrier
- Evaluation

Here, the foremost step describes the modulation equations used in RRI, from the sinusoidal frequency modulation through to the resultant phase modulation waveform. The second step describes the equations defining the interferometric signal seen at the photodetector, and the latter-most three of these comprise the demodulation and signal evaluation process which is unique to range-resolved interferometry. These five steps are discussed in further detail below, and examples of each step for an unbalanced interferometer shown in Fig. 4.1, are shown in Fig. 4.2 on the following page. These graphs are referenced and described in each of the following sections.



FIGURE 4.1: Example of an unbalanced fibre interferometer in a Mach-Zehnder configuration.

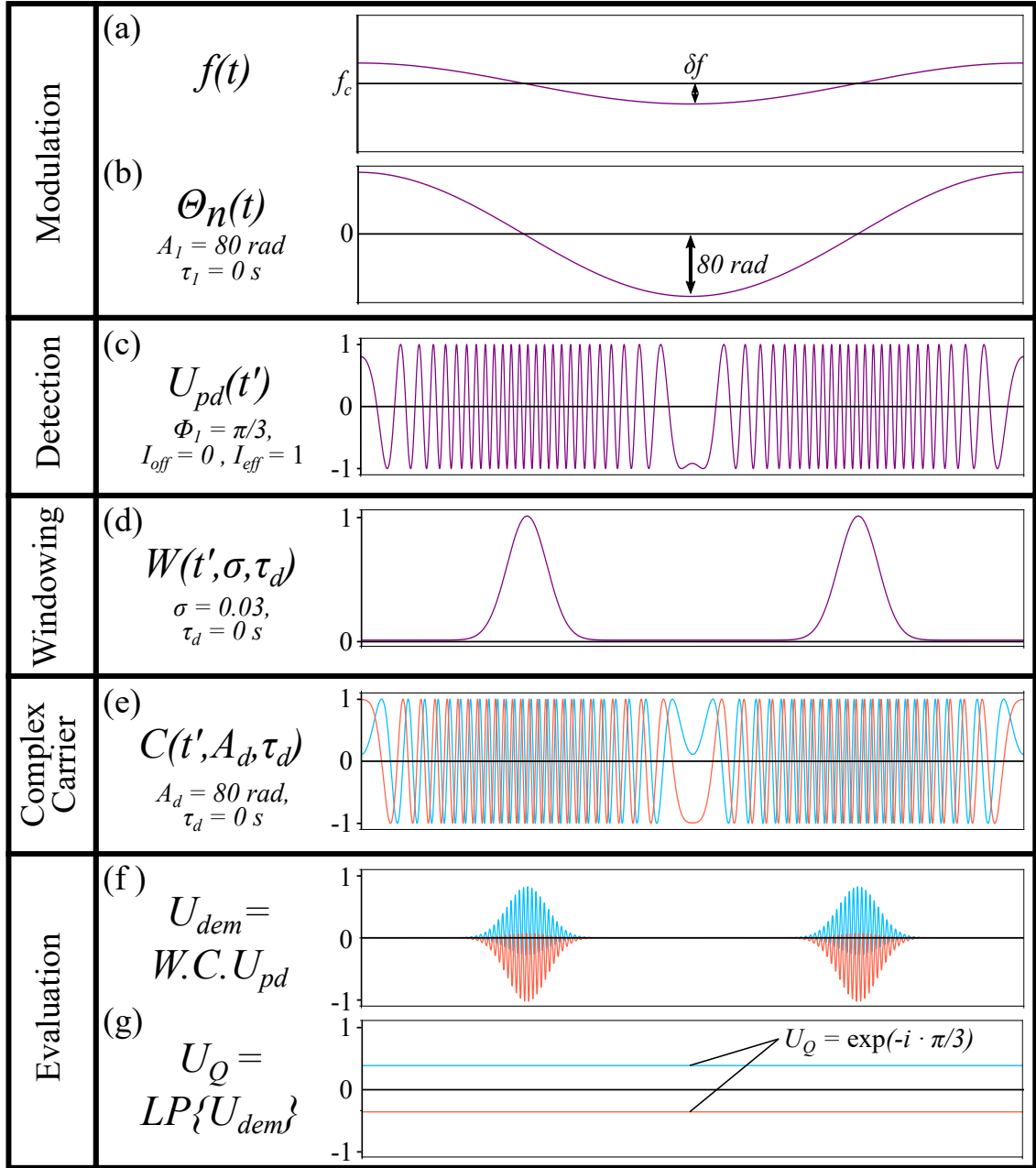


FIGURE 4.2: Example signals for each step of the modulation and range-resolved interferometric demodulation process. With the exception of (a) and (b), all plots are normalised for illustrative purposes. (a) Example sinusoidal frequency modulation of a laser diode, with a central emission frequency f_c and an amplitude of δf . (b) Effective phase modulation due to the frequency modulation in an interferometer with phase carrier amplitude $A_I = 80 \text{ rad}$ with time of flight-delay $\tau_I = 0$. (c) Photodiode signal for an interferometer with phase value $\Phi_I = \pi/3$, modulated by the phase carrier shown in (b). (d) Example of an applied window function, with Gaussian width parameter $\sigma = 0.03$. (e) Complex carrier signal calculated for a demodulation phase carrier amplitude $A_d = 80 \text{ rad}$, split into the real (blue) and imaginary (orange) components. (f) Resultant demodulation signal from the product of the window function, complex carrier and the photodiode signals above, split into real (blue) and imaginary (orange) components. (g) Quadrature signal separated into real (blue) and imaginary (orange) components representing the phase value ($\approx \pi/3$) of the demodulated interferometer in complex form. (Illustration adapted from [76])

Modulation

RRI uses sinusoidal modulation of the laser emission frequency which is typically achieved, among other methods, via injection current modulation. As discussed in Chp. 2, sec. 2.3.4, the laser frequency emission due to sinusoidal frequency modulation can mathematically be described;

$$f(t) = f_c + \delta f \sin [2\pi f_m t] \quad (4.1)$$

where f_c is the central emitted frequency, δf is the modulation depth and f_m is the modulation frequency. For the sake of completeness, while in principle the laser driver and diode have a linear response to injection current changes, in practise there is likely to be non-linearities in either the injection current modulation or in the resultant frequency modulation. These non-linearities are fixed in time, so the fractional harmonic coefficients are stationary, however as a result, where the sinusoidal component of the laser emission frequency is denoted as $\delta f \sin [2\pi f_m t]$, instead it should be more accurately denoted with the higher harmonics included as:

$$f(t) = f_c + \delta f \left(\sum_{k=1}^{\infty} C_k \sin [k2\pi f_m t + \theta_k] \right) \quad (4.2)$$

where C_k are constants equal to or less than 1, denoting the fractional magnitude of each harmonic, and θ_k is the unique phase of each harmonic. For simplicity's sake, further discussion involving the sinusoidal frequency modulation will be considered using eqn. 4.1, assuming a perfect modulation sinusoid occurs. In applied work however, this concept of higher harmonics should be understood and applied in the demodulation process discussed below.

For an interferometer with non-zero optical path difference (OPD), this applied frequency modulation induces a phase modulation referred to as the phase carrier, with an amplitude A_n . Using the derivations determined in [76, 98] from [162], the resultant phase modulation from the laser frequency modulation presented in eqn. 4.1 can be stated for each of the $n = 1 \dots n_{max}$ constituent interferometers, with average time-of-flight τ_n between the respective interferometer arms, as:

$$\Theta_n(t) = A_n \sin [2\pi f_m (t - 0.5\tau_n)] \quad (4.3)$$

Here, A_n is the phase carrier amplitude in units of rads derived in [76] from [162]. For an optical path difference of η_n of the n th interferometer, this can be mathematically described as:

$$\begin{aligned}
A_n &= \frac{2\delta f}{f_m} \sin \left[\frac{\pi f_m \eta_n}{c_0} \right] \\
&\approx \frac{2\pi \delta f \eta_n}{c_0} \quad \text{for } \eta_n \ll \frac{c_0}{\pi f_m}
\end{aligned}
\tag{4.4}$$

where c_0 is the free space speed of light. Here it can be seen that for small values of η_n compared to $c_0 / (\pi f_m)$, the value of A_n is not dependant on the frequency modulation f_m but is proportional to the modulation depth of the frequency modulation δf and the optical path difference of the n th interferometer η_n . It is this proportionality that allows the signal to be demodulated in a range-dependant manner, as detailed in the following stages. Fig. 4.2(a) shows an example frequency modulation signal for a single period of sinusoidal frequency modulation with an amplitude of δf and a central frequency of f_c . Fig. 4.2(b) then shows the resultant phase modulation signal, resulting from the sinusoidal frequency modulation, with an example phase carrier amplitude A_1 of 80 rad. For illustrative purposes, the resultant phase modulation is shown for zero time-of-flight delay $\tau_1 = 0$.

Detection

The light interferes at a photodetector, and following this the signal is digitised. Here, signal processing delays can occur in both the analogue and digital processing, which must be compensated for. To achieve this, a signal processing delay value incorporating all signal delays, both analogue and digital, following the laser modulation is established τ_{sp} in addition to a compensated time variable t' . This is related to the regular time variable t by $t' = t - \tau_{sp}$. The photo detector signal, expanding on the transfer function presented in eqn. 2.9 for multiple interferometers $n = 1 \dots n_{max}$ with time-of-flight delays τ_n between the respective interferometer arms, can then be stated as:

$$\begin{aligned}
U_{pd}(t') &= R_{pd} I_{off,n} + \sum_{n=1}^{n_{max}} (R_{pd} I_{eff,n} \cdot \cos [\Phi_n(t') + \Theta_n(t')]) \\
&= R_{pd} I_{off,n} + \sum_{n=1}^{n_{max}} (R_{pd} I_{eff,n} \cdot \cos [\Phi_n(t') + A_n \sin [2\pi f_m(t' - 0.5\tau_n)])
\end{aligned}
\tag{4.5}$$

where $I_{off,n}$ is the stationary offset intensity given by $I_{1,n} + I_{2,n}$ and $I_{eff,n}$ is the effective interferometric intensity given by $2|\gamma(\tau)|\sqrt{I_1 I_2}$. As previously discussed in Chp. 2, Sec. 2.3.1, $|\gamma(\tau)|$ is the complex degree of coherence and defines the ability of the two constituent waves to interfere. Here, the desired phase signals for the constituent interferometers $n = 1 \dots n_{max}$ are given by $\Phi_n(t')$. An example detector signal $U_{pd}(t')$ for a single interferometer is shown in Fig. 4.2(c) where for illustrative purposes, intensity

offset $I_{\text{off}} = 0$, and total coherence and complete polarisation overlap between the two interferometer arms $I_{\text{eff}} = 1$ is assumed.

Windowing

A key component of the RRI approach to signal demodulation and evaluation is the application of a smooth window function to the photodetector signal. In this technique the window function consists of a pair of Gaussian functions applied to each modulation period $T_m = 1/f_m$, offset by a demodulation delay τ_d and defined by a width parameter σ , the Gaussian standard deviation. The two Gaussian functions are centred on the two highest gradients in the photodetector signal, i.e. where the two highest frequency components of $U_{pd}(t')$ occur which are offset from one another by π phase. The total window function $W(t', \sigma, \tau_d)$, including the periodicity in $2T_m$ is mathematically defined by:

$$W(t', \sigma, \tau_d) = \sum_{a=-\infty}^{\infty} \left\{ \exp \left[-\frac{1}{2} \left(\frac{(t' - \tau_d) - aT_m}{T_m \sigma} \right)^2 \right] + \exp \left[-\frac{1}{2} \left(\frac{(t' - \tau_d) - (a + 0.5)T_m}{T_m \sigma} \right)^2 \right] \right\} \quad (4.6)$$

The application of the smooth window function is responsible for suppression of the low-frequency, baseband components of the interferometric signal, which is essential for RRI. This is discussed further in the **Evaluation** subsection of this chapter where the implications of this are made evident. Fig. 4.2(d) illustrates an example of the window function for a window width parameter of $\sigma = 0.03$ with no demodulation delay $\tau_d = 0$.

Complex Carrier

In RRI, in contrast to the fixed frequency heterodyne carrier technique discussed in Chp. 2, Sec. 2.3.3, a time-variant complex carrier function is used to demodulate the interferometric signal. This complex carrier is periodic within the modulation period T_m , and is calculated with a demodulation phase carrier amplitude A_d approximating the expected phase carrier amplitude A_n , and the demodulation delay approximately half the expected time-of-flight delay i.e. $\tau_d \approx 0.5\tau_n$. Mathematically, the complex carrier function can be denoted as a complex version of the interferometric signal expected at the detector, and bears resemblance to eqn. 4.3.

$$C(t', A_d, \tau_d) = \exp [i.A_d \sin[2\pi f_m(t' - \tau_d)]] \quad (4.7)$$

where i is the imaginary number denoting a complex value. Fig. 4.2(e) illustrates both the real and complex components of the complex carrier signal $C(t', A_d, \tau_d)$ for an example demodulation phase carrier amplitude of $A_d = 80$ rad and for illustrative purposes, zero demodulation delay $\tau_d = 0$

Evaluation

The photodetector signal, $U_{pd}(t')$, window function, $W(t', \sigma, \tau_d)$, and complex carrier, $C(t', A_d, \tau_d)$, are then multiplied to give the resultant demodulation signal, $U_{dem}(t', \sigma, A_d, \tau_d)$. This is then low-pass filtered (LP) with a cut-off frequency at less than, or equal to, half the modulation frequency, which results in the complex quadrature signal given by:

$$\begin{aligned} U_Q(t', \sigma, A_d, \tau_d) &= LP \{ U_{dem}(t', \sigma, A_d, \tau_d) \} \\ &= LP \{ C(t', A_d, \tau_d) W(t', \sigma, \tau_d) U_{pd}(t') \} \end{aligned} \quad (4.8)$$

The multiplication of the complex carrier with the window and photodetector terms creates a series of exponential terms, most notably distinct frequency terms consisting of the sum and difference between the demodulation phase carrier amplitude A_d and the interferometric phase carrier amplitude A_n . The difference term is located in the baseband of the signal, where as discussed previously the baseband frequency components of the interferometric signal are suppressed by the window function. A low-pass filter is used to isolate the difference frequency terms, now present in the suppressed baseband of the signal, from any remaining frequency components. This allows the desired phase information $\Phi_n(t')$ to be retrieved from the baseband signal using an arctan function on the complex quadrature signal $U_Q(t', \sigma, A_d)$. This isolation of the phase information for "in-range" signals, i.e. signals with phase carrier amplitudes approximating the demodulation phase carrier amplitude $A_n \approx A_d$, achieved via suppression of "out-of-range" signals, i.e. $A_n \neq A_d$ is critical to the ability of RRI to discriminately demodulate the interferometric signals of interest by their relative optical path differences. Fig. 4.2(f) illustrates the complete demodulation signal $U_{dem}(t', \sigma, A_d, \tau_d)$, using the parameters discussed above of $\sigma = 0.03$, $A_d = 80$ and zero demodulation delay $\tau_d = 0$. Fig. 4.2(g) then illustrates the resultant real and imaginary signal components after low-pass filtering has been applied $LP\{U_Q(t', \sigma, A_d, \tau_d)\}$.

4.2 Outputs of RRI

In RRI there are two modes using the demodulated quadrature signals that offer unique outputs. While the key working principle for both modes remains as discussed above, the outputs from each are beneficial to varying applications [155–157, 160, 161]. These two modes are discussed below in the context of an interferometric system wherein more than one interferometer with unique optical path differences ($n \geq 2$) are multiplexed into a single photodetector signal, the design of which is shown in Fig. 4.3.

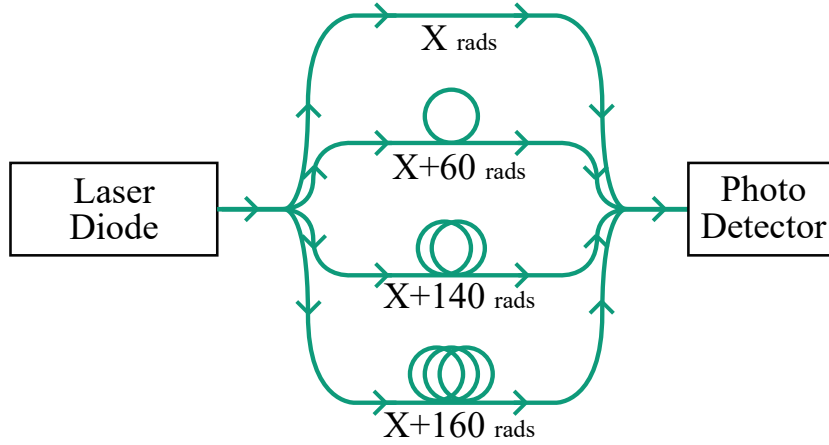


FIGURE 4.3: Example unbalanced interferometer with a reference arm of length X and 3 arms with varying optical path lengths, labelled in rads.

4.2.1 Mode 1: Signal Range Analysis

In the first mode, a single modulation period of the photodetector signal, corresponding to a multiplexed interferometric signal, is demodulated for a series of complex carrier waveforms over a self-defined range of demodulation phase carrier amplitudes. For each resultant complex quadrature waveform the amplitude and wrapped phase are extracted as described in Sec. 4.1.2. Plotting the resultant quadrature signal amplitudes against the values of the trialled phase carrier amplitudes used to demodulate the signal, an example graph of which is shown in Fig. 4.4 for the interferometer setup shown in Fig. 4.3, conclusions can be drawn about viable interferometer contributions within the multiplexed interferometric signal. For illustrative purposes, only the interferometers between the reference arm of length X and the 3 measurement arms shown in Fig. 4.3 are included and the cross-interferences between measurement arms are excluded.

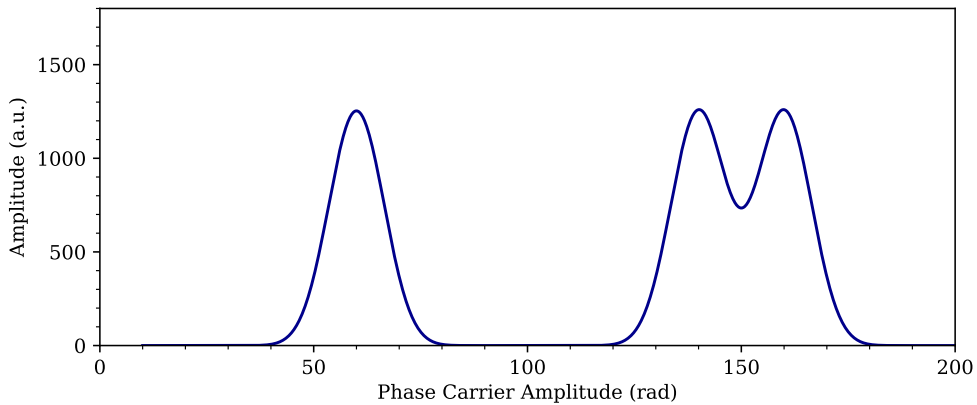


FIGURE 4.4: Comparison of quadrature signal amplitudes over a range of demodulation phase carrier amplitudes for an example multiplexed interferometric signal. Peaks corresponding to three interferometers with varying OPD can be seen.

Where a strong quadrature signal amplitude is returned for a particular demodulation

phase carrier amplitude, the demodulation phase carrier amplitude A_d is likely to be approaching or equal to the interferometric phase carrier amplitude A_n of a viable interferometer, n , within the multiplexed interferometric setup i.e. $A_d \rightarrow A_n$. Referring to eqn. 4.4, the demodulation phase carrier amplitude can be mathematically associated with the time-of-flight difference η_n between the constituent interferometer arms of the interferometer of interest n . Assuming a constant medium, and therefore a constant velocity of light in the sensitive region of interferometer n , this time-of-flight is directly proportional to the optical path difference between the two constituent interferometer arms, directly linking the demodulated phase carrier amplitudes to the optical path difference. This provides a valuable insight into the viable interferometers contained within the multiplexed interferometric signal, and allows for determination of the demodulation phase carrier amplitudes needed to demodulate these interferometers. This is critical for the discrimination and selection of interferometers to be evaluated, discussed further in Mode 2.

4.2.2 Mode 2: Phase Evaluation

The second function of RRI is more in-line with typical interferometry. Here the complex phase carrier amplitude is fixed, and the multiplexed interferometric signal is demodulated to retrieve the wrapped phase and quadrature signal amplitude. The complex phase carrier amplitude for demodulation A_d is selected to match with the phase carrier A_n of the the interferometer n of interest, which can be determined in the case of an unknown system using the output of Mode 1, illustrated in Fig. 4.5. For illustrative purposes, only the interferometers between the reference arm of length X and the 3 measurement arms shown in Fig. 4.3 are included and the cross-interferences between measurement arms are excluded. The wrapped phase signal can be unwrapped using various appropriate digital techniques [163] with a discontinuity of 2π , returning the unwrapped, phase change over time for the demodulated interferometer. This allows for, as in regular interferometry, phase evaluation and hence relative displacement measurements to be performed.

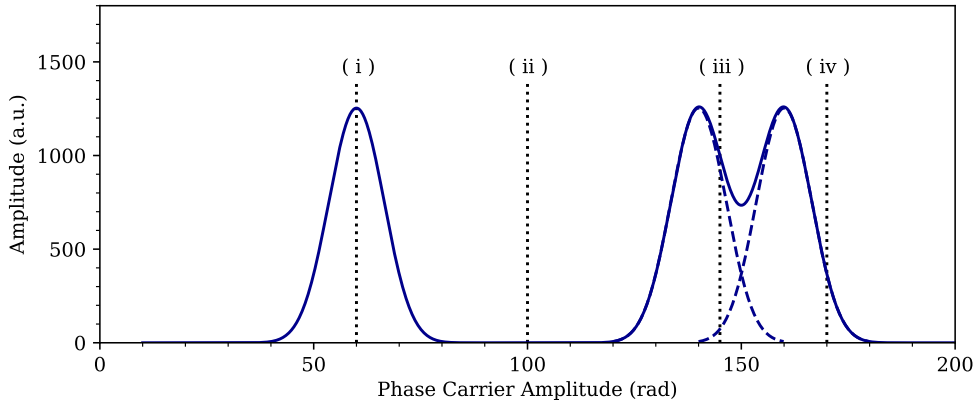


FIGURE 4.5: Comparison of quadrature signal amplitudes over a range of demodulation phase carrier amplitudes for an example multiplexed interferometric signal. The individual contributions from each interferometer are denoted by the dashed peaks. The complex phase carrier amplitudes used for demodulation are indicated for phase carriers; (i) matching the phase carrier of an interferometer of interest (ii) not matching any phase carriers of an interferometer of interest (iii) intersecting the range of two different phase carriers of interferometers of interest.

Here, the selection and isolation of the signals of interest becomes paramount. In the ideal case, the complex phase carrier amplitude for demodulation A_d is selected to exactly match with the phase carrier A_n of the the interferometer n of interest, shown in case (i) in Fig. 4.5. This results in clean demodulation of the signal and retrieval of the phase change occurring in the interferometer of interest. However, in the case where inaccurate selection of the demodulation phase carrier amplitude occurs, such as in case (ii) of Fig. 4.5, the demodulation and evaluation will return primarily noise, yielding no useful information about the interferometer of interest. Alternatively, such as in case (iii) of Fig. 4.5 where the selected complex phase carrier amplitude intersects the range of two interferometric phase carriers, spurious phase signals are demodulated, consisting of contributions from both interferometers. Cases such as these are related to the origin of cyclic errors, discussed in Chp. 2, Sec. 2.3.5. Finally, case (iv) illustrates the case where only the rightmost interferometer will contribute significantly to the demodulated signal, as the contributions from the interferometer with a phase carrier amplitude of 140 rad are approaching zero.

The spatial resolution defines the minimum separation of peaks that could be interrogated such that no contributions from other peaks are present in the demodulated signal from the other. Although the Gaussian function that describes these peaks theoretically never reaches zero, as the interrogated phase carrier amplitude is moved away from the central phase carrier amplitude, the value of the Gaussian will drop below the noise floor and it can be considered out-of-band. As a result, the spatial resolution of the RRI interrogation system can be defined as the distance in rad from the central Gaussian function in phase carrier amplitude that describes one interferometer, to the value at which the Gaussian is considered out-of-band, *i.e.* the value of the Gaussian drops below the noise floor.

In this mode, real-time processing of the photodetector signal is performed using a real-time range channel for each selected, fixed phase carrier amplitude. Depending on the hardware used, demodulation and phase evaluation can be achieved for multiple channels simultaneously, allowing phase measurements of several interferometers to occur in real-time, independently and with contributions from "out-of-range" interferometers suppressed by appropriate selection of phase carrier amplitudes.

4.3 Implementations

In this thesis, four different implementations of RRI modulation and signal processing are used to produce the experimental work featured in Chps. 5, 6 and 7. This section presents the technical details of the four implementations, discussing the hardware used, the key parameters and the advantages and limitations of each. In general, all four implementations use commercial off-the-shelf (COTS) components, with any customised or purpose-built components specifically highlighted. Implementations 1 and 2 were already developed prior to this thesis, but have since been used to accomplish some of the work discussed, so the details of both implementations bear discussion here. Implementations 3 and 4 were developed for the purposes of the work discussed in this thesis, and are based on a distinctly different architecture. A summary of these implementations and some of the key technical values are detailed in Tab. 4.1.

TABLE 4.1: Summary of the RRI implementations used in this thesis. Where realistic, the values stated are the maximum achievable values. In the cases marked (*), these are the maximum tested values.

Impl. Number	Mod. Frequency (kHz)	Mod. Depth (nm)	Quadrature Bandwidth (kHz)	Typ. no. of Real-time Channels	Central Wavelength (nm)	Typ. Max Range
1	49	$\pm \sim 0.2$ nm	21	0-24	1520.08	~ 0.6 m
2	24.4	$\pm \sim 0.31$ nm	10	0-24	1520.08	~ 1.2 m
3	1.133	$\pm \sim 0.4$ nm	0.58	0-8 (*)	1520.08	130 mm
4	1.133	$\pm \sim 0.1$ nm	0.58	0-8 (*)	780	194 mm

4.3.1 Pre-developed Systems

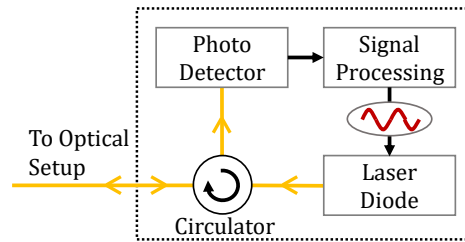
This subsection discusses the two implementations developed prior to the start of this thesis work, outside of the work in this PhD, presenting an overview of the technical details and key features for later reference. These implementations are used in the work on multi-dimensional encoders, presented in Chp. 5.

Implementation 1 - 49kHz RRI Interrogation Unit

Implementation 1 was originally developed as part of an initial development of the RRI project prior to the commencement of this PhD. The interrogation hardware, shown in fig. 4.6 consists of a single discreet mode (DM) laser diode fibre-coupled to an optical circulator. The circulator, as well as acting as an isolator for the laser diode, guides light to the fibre lead to the optical setup. The returning light from this same fibre lead is guided to a photodetector, in this case an indium gallium arsenide (InGaAs) photodiode,

captured by an analogue-to-digital converter (ADC) as part of field-programmable gate array (FPGA) based hardware. This hardware also controls the injection current modulation of the laser diode via a digital-analogue converter (DAC) so that the demodulation of the photodiode signal is synchronised with the modulated output.

(a)



(b)

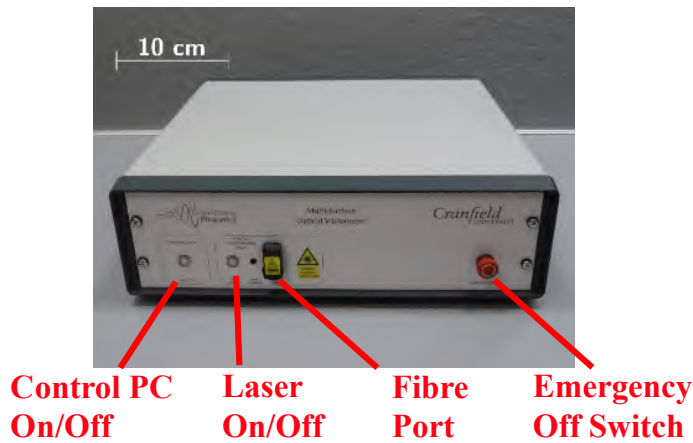


FIGURE 4.6: (a) shows a simplified schematic of the interrogation hardware used by this technique and setup with key hardware components labelled. (b) shows a photo of the complete enclosed interrogation hardware.

Figure 4.6 shows a schematic of the hardware used by this interrogation unit and a photo of the complete enclosed interrogation hardware. Using this technique, only one photodetector and laser diode are required to interrogate a number of interferometers. For lab experimentation, this iteration of the interrogation unit is fully enclosed, incorporating all the necessary hardware components including the FPGA-based ADC and DAC, and an instrumentation PC. This setup uses an Eblana Photonics laser diode (Eblana EP1512-DM-B) with a central wavelength of ~ 1520.8 nm (in-air), as measured by an optical spectrum analyser (Yokogawa AQ6370C) and ~ 5 mW power output. The linewidth of this particular diode is ~ 1 MHz. The DAC modulates the laser injection current at a frequency of 49 kHz, resulting in a wavelength modulation amplitude of $\sim \pm 0.2$ nm. The interrogation system is capable of resolving quadrature bandwidths up to 21 kHz for a total of up to 24 real-time channels corresponding to independent interferometers.

Implementation 2 - 24 kHz RRI Interrogation Unit

A second iteration of this system was previously developed that uses the same hardware as in implementation 1 with changes to the laser diode control. Here, in order to maximise the modulation depth possible with this laser diode, the modulation frequency is reduced to 24.4 kHz, reducing the quadrature bandwidth to 10 kHz. In turn, this increases the modulation depth to ± 0.31 nm around its centre wavelength of 1520.08 nm (in air), as measured using an optical spectrum analyser (Yokogawa AQ6370C). The interrogation hardware otherwise remains unchanged using the same laser diode and photodetector.

4.3.2 Developed Systems

This subsection discusses the two implementations that were developed as part of this PhD project. Whilst the pre-developed systems discussed above were suitable for the multi-dimensional encoder experimental work which is presented in Chp. 5, a more flexible system was required to develop and experiment with wavelength stabilisation techniques for widely-wavelength modulated laser diodes, which is the other focus of this thesis. In order to develop these techniques the system had to be versatile and quick to adapt for different configurations and experiments, which the existing system architecture did not as easily allow. This new development platform and accompanying code was assembled using commercially available hardware, and with the exception of standard operating software, was custom-coded for this project. This platform allowed for quick re-configurations of I/O capabilities using commercially available cards, as well as existing support for 3rd party equipment which could be easily incorporated with the custom-coded software.

Implementation 3 - 1.2 kHz RRI Stabilisation Development Platform

A third implementation was developed during this thesis, a schematic of which is shown in Fig. 4.7. This was developed as a flexible platform for research and experimentation into wavelength stabilisation of widely wavelength modulated laser diodes, such as is used in RRI, combined with the laser modulation control and signal processing of RRI. This implementation is used in the work presented in Chp. 6, in the development of a novel technique for widely wavelength modulated laser stability.

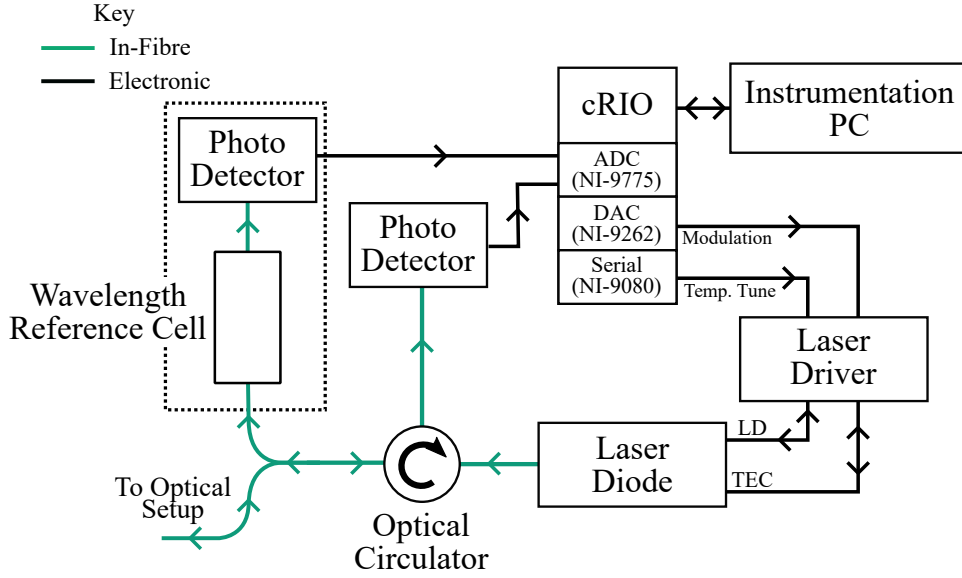


FIGURE 4.7: Schematic of the interrogation hardware used in implementation 3 showing the connections between the various hardware components. The added stabilisation arm is marked by the dashed box.

As Fig. 4.7 shows, the principle of the interrogation hardware remains the same as implementations 1 and 2, however an additional branch of the laser emission is devoted to the wavelength stabilisation marked by the dashed box. The laser is passed through the wavelength reference cell, and measured on a separate photodiode which is recorded synchronously with the interferometric signal by the ADC.

In this implementation the signal processing hardware is based on a National Instruments compact real-time I/O system (cRIO-9030), shown in Fig. 4.8. Whilst this is also an FPGA-based system, the cRIO operates using modular cards for I/O which are synchronised through the chassis backplane. In this implementation, the DAC card is a NI-9296 analogue-out module which provides the modulation signal for the laser driver to modulate the laser diode injection current. The ADC is a NI-9775 digitiser card, which allows up to 4 channels to be sampled at once with sample rates of up to 1 MS/s per channel in real-time continuous acquisition, with 14-bit bandwidth over ± 10 V. The laser diode used is an Eblana Photonics DM laser diode (Eblana EP1512-DM-B), with a central wavelength of ~ 1521 nm, linewidth ~ 2 MHz, output ~ 10 mW and a maximum modulation depth of ± 0.4 nm. This is modulated using a Stanford Systems laser driver and temperature controller (Stanford LDC 500). Additionally, an NI-9080 serial card is used, that offers serial communication, and this is used to communicate with the laser TEC for temperature tuning, used extensively in the wavelength stabilisation work featured in Chp. 6.

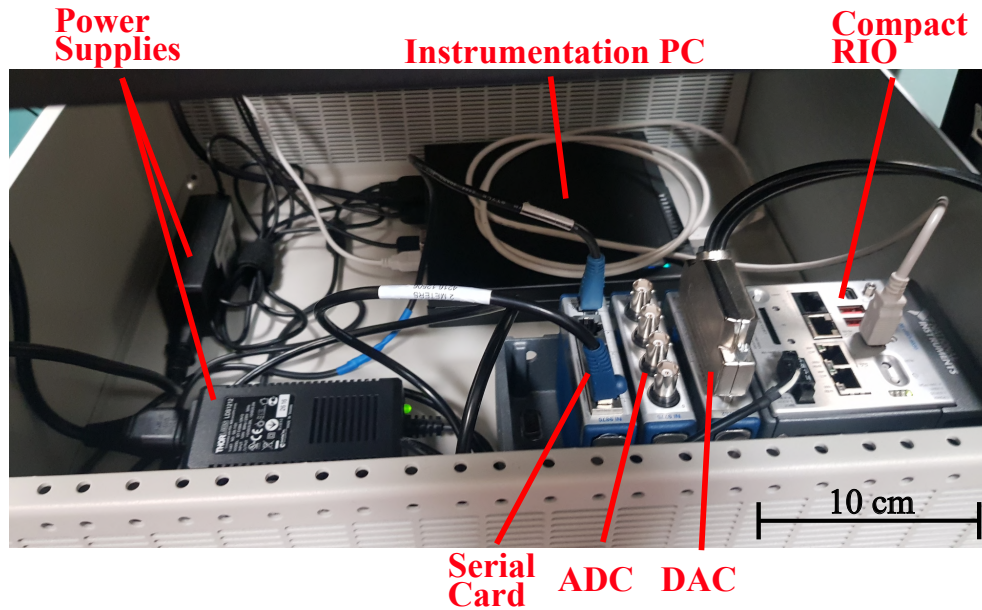


FIGURE 4.8: Annotated photo of some of the hardware used in implementations 3 and 4.

While the maximum sample rate for each ADC channel is nominally 1MS/s, due to synchronisation demands between the ADC and DAC, the true sample rate is limited to a maximum of ~ 597 kHz. At this sample rate, whilst maintaining good resolution of the full photodetector signal with 512 samples per modulation period, a modulation frequency of $597 \text{ kHz}/512 \approx 1.166 \text{ kHz}$ is used, resulting in a quadrature bandwidth maximum of $\sim 580 \text{ Hz}$ can be achieved. Using this implementation, the quadrature bandwidth is significantly lowered, which reduces the maximum range of phase carrier amplitudes which can be evaluated by this system in comparison to the hardware used in implementations 1 and 2. The advantage of this implementation over implementations 1 and 2 is in the flexibility this system offers, allowing for acquisition of up to 4 channels easily through the ADC card without additional hardware changes. The DAC and serial cards also allow for multiple outputs at once, so control of multiple devices can be easily achieved. Furthermore, using the National Instruments framework allows for quick programming of both the FPGA and signal processing code using LabVIEW, which in turn allows for simple, on-the-go changes to the software to accommodate different signal processing requirements arising from different configurations.

Implementation 4 - 1.2kHz RRI Comparison Platform

Also developed using the same LabVIEW architecture presented in implementation 3, the fourth implementation was developed for comparison of two laser interferometers simultaneously, in addition to the developed wavelength stabilisation technique and is shown in Fig. 4.9. This implementation is used exclusively in the experimental work presented in Chp. 7, where further details of the optical setup and the specifics of the wavelength reference can be found. To this end, an additional photodetector is added to the hardware and is recorded by a third ADC channel. Where previous implementations have used an optical circulator to guide light returning from the optical setup, in this

implementation the light is guided directly to the photodiode after traversing the optical setup. A second laser diode is also used, however since this is not controlled by this interrogation hardware it has been omitted from the schematic.

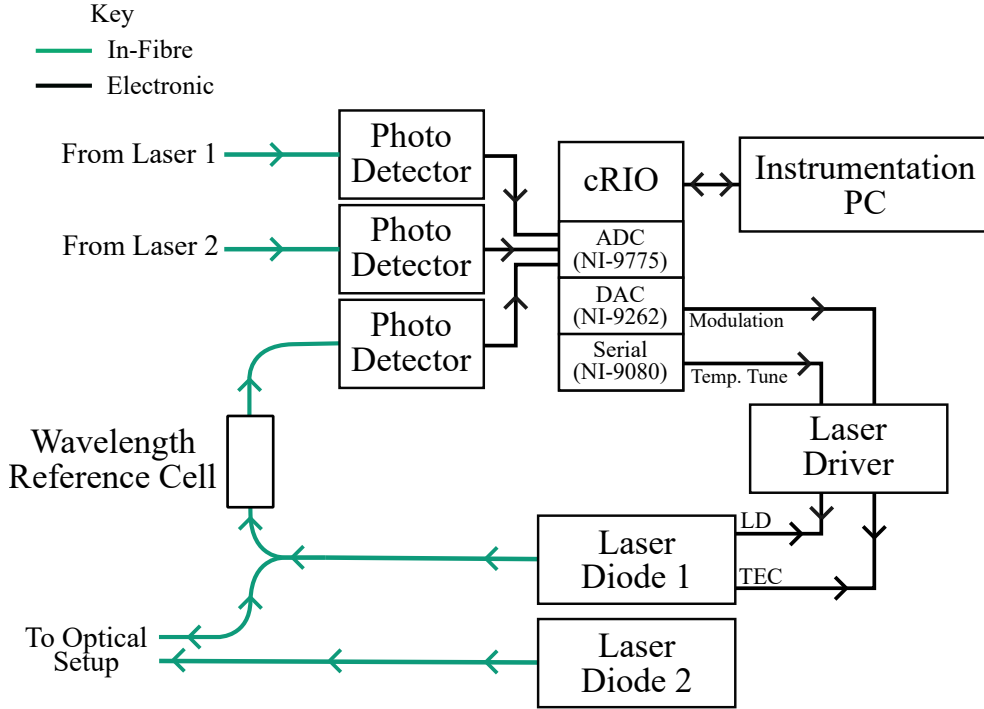


FIGURE 4.9: Schematic of the interrogation hardware used in implementation 4 showing the connections between the various hardware components. Not shown is the second laser diode, whose sole connection is to a photodiode.

Fig. 4.9 shows how the changes to the interrogation hardware in this implementation allow for the recording of three ADC channels simultaneously, in addition to synchronised modulation output from the DAC. The software for this implementation was also re-developed, forgoing the real-time signal processing component of implementation 3. Here, the signal processing is performed in post-processing using python on the instrumentation PC. This implementation also replaces the laser diode with an Eagleyard distributed feedback (DFB) laser diode (EYP-DFB-0780-00010-1500-BFY12-0005) with a central wavelength of 780 nm, a linewidth of ~ 0.6 MHz and modulation depth of $\pm \sim 0.1$ nm.

4.4 Summary

In this chapter, the principle behind range-resolved interferometry (RRI) is presented. RRI is a modulation and signal demodulation technique that uses sinusoidal modulation of the laser emission wavelength. Using RRI, several interferometers can be multiplexed into a single interferogram and independently interrogated by calculation of optical path dependant phase carriers. By demodulation of the multiplexed signal using

these unique phase carriers, individual interferometers can be independently and simultaneously evaluated for phase changes. An overview of the equations that describe RRI and the modulation and evaluation process are presented, and the outputs available from the RRI signal processing are shown, described as a signal range analysis and a phase evaluation mode of RRI.

In addition, the implementations of RRI used in the work in this thesis are presented in this chapter. These are the two pre-existing implementations of RRI signal processing hardware, developed prior to the commencement of this PhD project, as well as the two new systems which were developed as part of the work presented in this thesis. The pre-developed systems are used to investigate novel applications of RRI, in the work presented in Chp. 5. The two newly developed systems incorporate additional flexibility for research and experimentation into wavelength stabilisation for the widely-wavelength modulated lasers used in RRI, with modulation depths several orders of magnitude larger than typical references. Specifically, these developments have the capacity for additional analogue inputs from photodetectors, as well as outputs via serial communication or analogue signals. Experimental work on wavelength stabilisation of widely-modulated lasers is presented in Chp. 6 and Chp. 7 using the newer implementations presented in this chapter.

Chapter 5

Multi-dimensional Interferometric Encoders

This chapter details the process of developing multi-dimensional applications for precision metrology using range-resolved interferometry, a laser modulation and signal processing technique that allows for spatial multiplexing of interferometers that can be individually and synchronously interrogated and evaluated. In this chapter, the motivation behind this work, the principles employed and the development of several proof-of-concept designs are presented alongside experimental results for multi-dimensional position and angle encoders.

5.1 Introduction

5.1.1 Motivation

As discussed in Chp. 2, significant work has been undertaken in developing techniques for single dimensional displacement measurements. Non-optical techniques such as capacitive or Piezoresistive sensors, whilst offering good resolutions, typically require close proximity to the target surface, adding design complexity in applications where target space is limited. Some optical techniques such as laser interferometry do not suffer from this limitation, allowing for remote, non-contact measurements of target displacement. When extended to multiple dimensions however, existing techniques in both optical and non-optical regimes typically require multiple sensing heads, a requirement that adds cost and complexity to most applications. Alternative techniques that are capable of multi-dimensional measurements such as grating-based laser interferometers require complex optical setups, typically involving customised, or polarisation-sensitive, optical components that add significant design complexity and cost to an application.

As such, there is a need for multi-dimensional techniques that benefit from the remote, non-contact capabilities and high resolution of laser interferometry whilst minimising optical components, in particular, polarisation-sensitive or customised components. Range-resolved interferometry (RRI), discussed in detail in Chp. 4 offers similar benefits to displacement measurements as traditional laser interferometry, as well as allowing multiplexed interferometric signals from a single beam to be demodulated simultaneously. This allows for compact optical setups with cost-effective hardware, which is of huge benefit to many applications.

5.1.2 Presented Developments

Towards this end, several developments of optical encoders have been explored that utilise range-resolved interferometry with the objective of multi-dimensional metrological measurements. These developments are, at this stage, proof-of-concept only and are not yet fully optimised. Where data was readily available, some verification of these designs was performed and the results of these are presented, however a fully detailed comparison with reference measurements is still required.

2D displacement encoder: The presented developments commence with a two-dimensional displacement sensor. Using a beamsplitter mounted on a 3-axis motion stage and two planar mirrors, interferometers corresponding to two orthogonal Cartesian dimensions are simultaneously interrogated using a single optical axis port, allowing for independent measurement of two stage motion directions. This encoder design was then tested for both one and two dimensional stage motions.

3D displacement encoder: This development was then extended into a design for three-dimensional measurements, developing the encoder further with an additional beamsplitter and an additional planar mirror. Using range-resolved interferometry, multiple interferometers are evaluated from a single, multiplexed interferogram to allow independent and simultaneous measurement of three measurement dimensions. This design is then tested through experimental measurements of stage motions. These range from simple, single dimensional stage motions to more complex, three-dimensional stage motions. To further analyse this design, noise and linearity analysis of the encoder is performed and discussed.

2-beam angle encoder: Moving beyond linear displacement measurements in multiple dimensions, a design for an angle encoder, employing the same techniques and simple optical setup, was developed. Using multiple spatially offset beams in a single Cartesian direction, angle measurements in addition to displacement measurements can be performed. The interferometers in this design can be multiplexed, and then demodulated using range-resolved interferometry which allows for a similarly compact optical design to the developed multi-dimensional displacement encoders. Experimental measurements were performed using this 2-beam angle encoder, the results of which are presented and discussed.

5.1.3 In this Chapter

This chapter details firstly, the design and development of an optical encoder for 2 dimensions in Sec. 5.2. Here the key design elements, including the optical design, specification of interferometers of interest and the interrogation hardware used, are discussed. Following this, experimental measurements performed using the encoder design are presented with discussion of the results and conclusions that can be drawn from them.

Sec. 5.3 details the process of extending the stage encoder design for 3 dimensions, starting with discussion of issues that arose in the 2-dimensional encoder development. Afterwards the optical design, including key features such as the interferometers of interest, is presented. To build on this, discussion is given to the key design considerations in further detail, including changes to the interrogation hardware. Following this,

experimental results of measurements made using this encoder design are presented, commencing with single dimensional stage motions before moving to more complex, three-dimensional stage motions. Following these, results from noise and linearity measurements are presented and discussed.

Sec. 5.6 furthers the applications of multi-dimensional encoding using RRI, presenting the development of a 2-beam angle and displacement encoder. As previously, the principle and design are first introduced, followed by experimental results of measurements taken using this design.

This chapter is then finished with Sec. 5.7 where the results and conclusions from all three sections of the multi-dimensional optical encoder work are summarised, in addition to discussion of future work that could be undertaken in this area.

5.2 Development of 2-dimensional encoder

5.2.1 Managing Optical Path Differences

As in all interferometric applications, the reduction of cyclic errors is paramount. As discussed in Chp. 2 Se. 2.3.5, typically these occur when multiple multiplexed interferometers are inadvertently evaluated together. These can be interferometers with the same optical path difference, spatially overlapping interferometers, or for example in heterodyne interferometry when polarisation-mixing occurs between beam paths. In all cases the existence of interferometers, additional to the interferometer of interest, present in the evaluated signal introduces cyclic errors into the phase measurements of the interferometer of interest which add uncertainty into the resultant displacement measurements. In the case of spatially overlapping OPDs, in order to eliminate these cyclic errors, it is necessary to separate from the OPDs of the interferometers of interest not only the other OPDs of the interferometers of interest, but also the OPDs of any other interferometer within the system. To achieve this, firstly understanding which OPDs are present in the interferometric signal and which specifically are overlapping is critical.

RRI allows these overlapping OPDs to be identified in the demodulation process, where by "sweeping" through a range of windowed complex carriers, intrinsically linked to notional OPDs, and comparing the resultant amplitude with the tested OPD, peaks corresponding to every interferometer are visible. An example of how this principle applies to even a simple glass block setup is shown in Fig. 5.1. In all interferometric applications, interferometers are created between every pair of reflecting or semi-reflecting surfaces, not just the principle interferometer of interest. If the resultant OPD of these interferometers is not properly considered, it is extremely easy to overlap OPDs with the OPD of the interferometer of interest and hence introduce cyclic errors.

Fig. 5.1 demonstrates how even a simple glass block setup requires careful consideration of the OPDs of every interferometer present in the system in order prevent the introduction of cyclic errors. In Fig. 5.1(a), a glass block is interrogated such that the OPD between the front face of the glass block and the fibre tip, with equivalent optical path distance $L(O,A)$, is extremely close to the equivalent OPD between the front face and rear face of the glass block $L(A,B)$, when the relative refractive index of the glass is taken into account. As a result, as shown in Fig. 5.1(b), in a graph of signal strength against

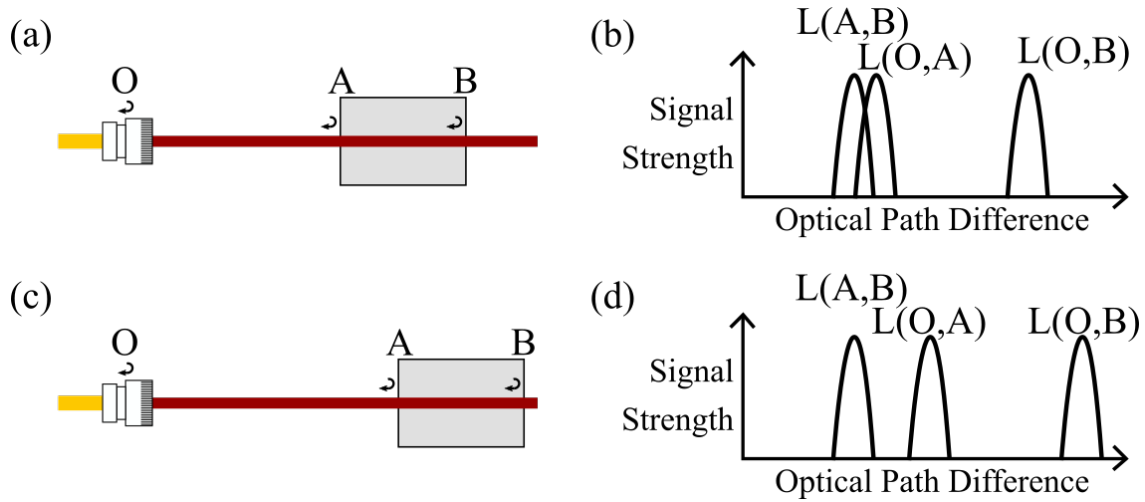


FIGURE 5.1: (a) shows a basic example system with a fibre collimator and glass block. The points of reflection are indicated and the reflecting surfaces labelled. (b) shows an example of a graph of signal strength against optical path difference with some of the potential interferometers created by this system labelled. (c) shows the same situation as (a), but with a greater distance between the collimator and the glass block, resulting in the example signal strength against optical path difference graph in (d).

OPD, there is overlap in the peaks corresponding to these interferometers. As a result, demodulation and phase evaluation of either of these peaks would result in cyclic errors being introduced into the measurement. In Fig. 5.1(c), the same situation as Fig. 5.1(a) is shown, except the glass block is placed further away from the fibre collimator so that the OPDs of the interferometers no longer overlap, as shown in Fig. 5.1(d).

In traditional interferometric techniques, discussed in Chp. 2, Sec. 2.3, both cases 5.1(b) and 5.1(d) would cause difficulty in phase evaluation leading to unwanted cyclic error components, as these techniques are unable to resolve multiplexed interferometers. Instead, a range sensitive approach, such as range-resolved interferometry, discussed in Chp. 4 is needed, where the interferometers can be interrogated independently of one another. In this case, only 5.1(b) would cause issues as the peaks overlap in optical path difference which would cause cyclic error components if either peak was the interferometer of interest. In case 5.1(d) however, using RRI any of the three peaks could be evaluated without introducing cyclic error components.

Further to the initial beams within the system, it may also be necessary to take into account secondary reflections within the beam path. In the example shown in Fig. 5.1, this may occur as multiple reflections within the glass block itself. Multiple reflections of this kind are usually easily predicted as multiples of the original OPD, however in more complex optical systems there may also be reflections from other optics as well that add various path lengths to the OPDs. All of these have to be taken into account and managed if the intention is to reduce the cyclic errors in the interferometers of interest.

5.2.2 Optical Design

Building on the principle of range-resolved interferometry, a novel stage encoder was designed that uses beam-splitting components to create a series of interferometers corresponding to unique measurement directions. By spatially separating these interferometers with unique optical path differences, they could be simultaneously and independently evaluated using RRI. By calculation of the complex carriers corresponding to each interferometer and hence each measurement dimension, phase change measurements can be made of changes in displacement of each dimension of interest. This was initially developed as a 2-dimensional displacement encoder.

The 2-dimensional encoder design consists of a single beamsplitter cube on top of a Physik Instrumente P-611.3 Nanocube stage [164], a 3 dimensional Piezo stage with a travel range in each Cartesian direction of $\pm 50\mu\text{m}$. The stage utilises internal strain gauge sensors to feedback position to the stage controller, allowing for controlled, high precision movements. This assembly was mounted inside a Thorlabs cage cube system and on one side of the cage cube a 3-axis kinematic mount with collimator is affixed, the only optical access port required by this setup. On the opposing side of the cube and on one orthogonal side are mounted planar aluminium mirrors, such that the light transmitted through the beamsplitter and the light reflected by the beamsplitter is normally incident on one of the two mirrors.

To measure the displacement of the stage in the two Cartesian dimensions of interest, interferometers between each mirror and the nearest beamsplitter face are evaluated. Unlike in Fig. 5.1 the fibre-tip is not used as a reference, so the interferometers created between the pairs of surfaces are interrogated. To ensure that these interferometers have an amplitude of appropriate strength such that they can be evaluated, the beamsplitter has the two surfaces nearest the mirrors polished to remove the anti-reflective coating, that typically comes as standard on these optical components, whilst maintaining the anti-reflective coating on the other beamsplitter faces. This results in Fresnel reflections [165] at the air-glass boundaries of the polished surfaces that reflect beams suitably strong enough to interfere with the reflected beam from each mirror. These interferometers correspond to the two dimensions of interest and the geometric distances of these are denoted α and β . In addition, an angled polished connector (APC) into a fibre collimator is used, removing interferometers created between reflection at the fibre-tip and other reflection points within the optical system. A schematic of the 2D design is shown in Fig. 5.2, including the principle beam paths involved and the geometric distances of the interferometers of interest, α and β .

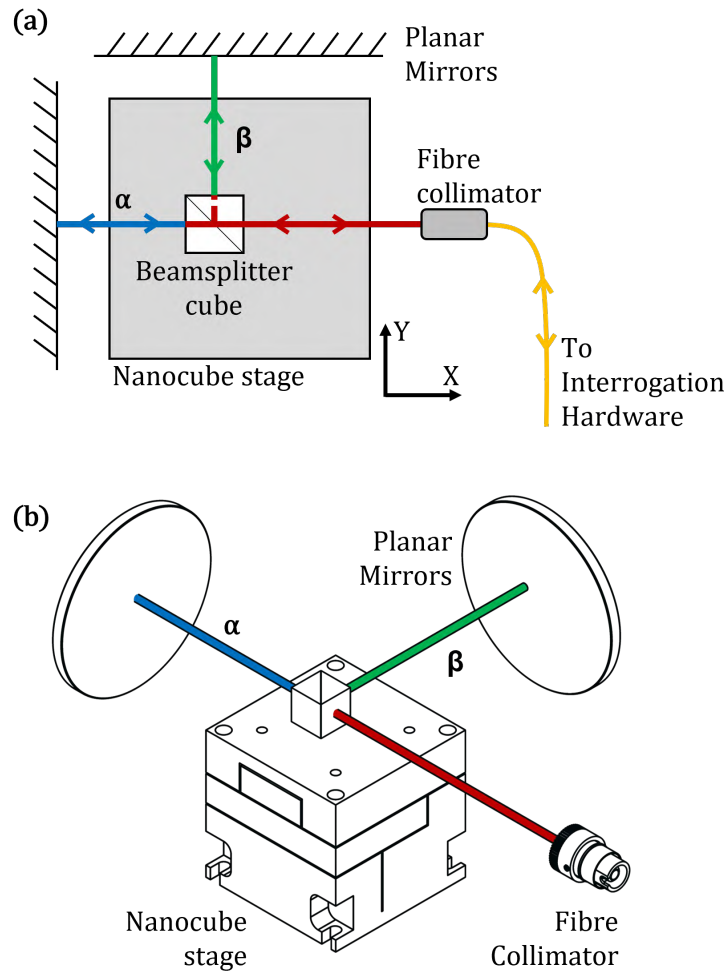


FIGURE 5.2: (a) shows a "top-down" schematic view of the encoder, with the geometric distances of the principle interferometers of interest highlighted and labelled as α and β respectively. (b) shows a 3D view of the encoder schematic.

Figure. 5.2 shows a schematic of the design of the 2D encoder, first as top-down view in Fig. 5.2(a) then as a 3D view in Fig. 5.2(b). In both diagrams, the interferometers of interest, α and β are labelled along with the basic optomechanical components. The flexibility in this optical design is in the position of the planar mirrors with respect to the beamsplitter cube. This allows minor tuning of the OPDs such that unique OPDs can be created for the interferometers of interest. In this case, the interferometers of interest are selected to be those between each mirror and the closest beamsplitter face, the geometric distances labelled in Fig. 5.2 as α and β for the X and Y dimensions respectively. By displacement measurement of these interferometers, direct measurement of each of the two dimensions can be performed simultaneously. To improve the alignment of the collimator beam with respect to the cage cube and mirrors, the collimator is placed in a 3-axis kinematic mount which allows for fine-tuning of the collimator output angle. For fine-tuning of the beamsplitter and stage alignment to the optical axis of the beam,

the stage is mounted on a 2-axis goniometer and rotation stage. Photos of the encoder design including alignment components are shown in Fig. 5.3.

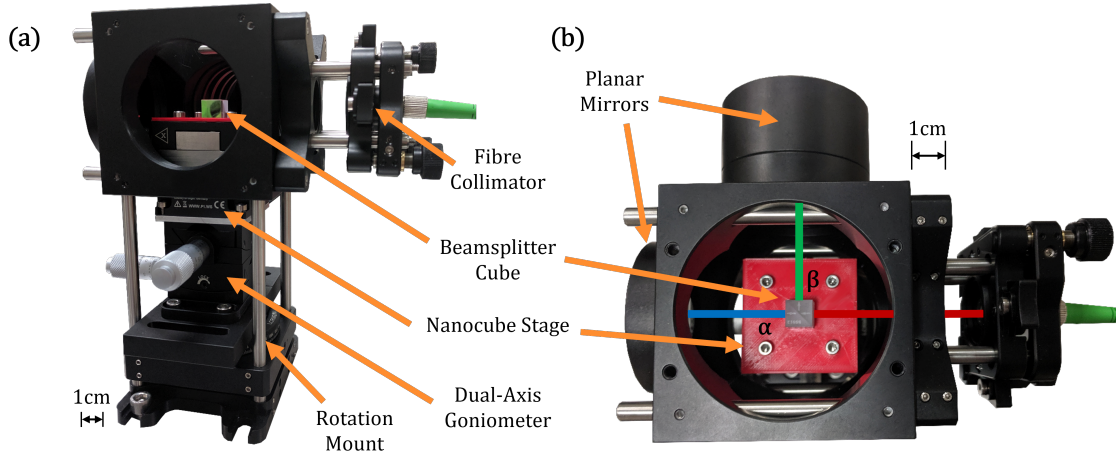


FIGURE 5.3: Photo of the 2D encoder design. (a) shows a photo from the side view of the encoder with a top down view of the encoder shown in (b). The major components and the geometric distances of the measured interferometers, α and β are labelled.

In order to separate the interferometers corresponding to each measurement dimension of interest from other interferometers within the multiplexed interferometric signal, it is essential that these have optical path differences that are unique to any other within the sensing region, as discussed in sec. 5.2.1. As a result, careful optical design is required to ensure that the interferometers of interest meet this condition, whilst ensuring that the OPDs of the interferometers of interest are also within the measurement range of the interrogation hardware (see Chp.4, sec. 5.2.3 for further discussion).

The cube beamsplitter, made from BK7 glass, within this setup introduces a condition on the magnitude of OPDs. As the beamsplitter in-air equivalent length is approximately 1.501 times [166] that of air at room temperature and the beamsplitter has a physical dimension of 10 mm width, there will be an interferometer created between the front and back surfaces of the beamsplitter at ~ 15 mm. As discussed in sec. 5.2.1, multiple reflections between surfaces are possible, and hence the beamsplitter creates a "grid" of interferometers with OPDs as multiples of 15 mm. As a result, in order to avoid introducing cyclic errors, the interferometers of interest have to have OPDs that do not lie on these multiples of 15 mm.

Using an FC-APC fibre connector into the fibre collimator, the back reflection from the fibre-tip is suppressed, with only beams between reflective and semi-reflective surfaces within the system expected to interfere. The anti-reflective coating on the beamsplitter faces should also significantly reduce reflections from these surfaces with absolute transmission greater than 95% ($T_{abs} > 95\%$), reducing the interferometers within the system to beams reflecting from the two mirrors, and the two polished beamsplitter surfaces on the surfaces nearest the mirrors.

The only other interferometer expected within the multiplexed interferogram is created by the beam reflecting from one mirror, interfering with the beam reflecting from the

other mirror. Since this will always have an optical path difference equal to the difference between the two optical path differences of interest, provided the longer optical path difference between mirror and polished beamsplitter face (labelled in Fig. 5.2 as β) is not equal to twice that of the shorter optical path difference between mirror and polished beamsplitter face (labelled in Fig. 5.2 as α), this will not cause an overlap in optical path differences with that of the interferometers of interest.

5.2.3 Interrogation Hardware

For the 2D encoder development, the 49kHz RRI interrogation hardware discussed in Chp. 4, Sec. 4.3 was used. The interrogation system used uses an Eblana Photonics distributed feedback (DFB) laser diode, modulated via injection current modulation, synchronised with the analogue-to-digital converter (ADC) signal from an Indium-Gallium-Arsenide (InGaAs) photodiode. With a modulation frequency of 48kHz, the maximum quadrature bandwidth is approximately 21kHz. For more details on this interrogation hardware, refer to Sec. 4.3, specifically implementation 1: the 49kHz RRI interrogation unit.

5.2.4 Measurements and Results

Single Axis Step Measurements

To demonstrate the measurement capabilities of this concept, two-dimensional displacement measurements over a variety of stage movements were conducted and recorded with an interferometric quadrature bandwidth of 21kHz. Initially, the stage is moved in a series of steps in each motion axis sequentially. Starting from the central position of each motion axis, the stage is moved to the maximum range of $+50\text{ }\mu\text{m}$ in the X axis, then to $-50\text{ }\mu\text{m}$ before returning to the central position. This same motion pattern is then performed in the Y axis shortly after. This was repeated for a series of step amplitudes; $\pm 50\text{ }\mu\text{m}$, $\pm 5\text{ }\mu\text{m}$ and $\pm 0.5\text{ }\mu\text{m}$. The results of recording phase changes and calculating displacements for both the X and Y dimensions simultaneously during these motions are shown in Fig. 5.4.

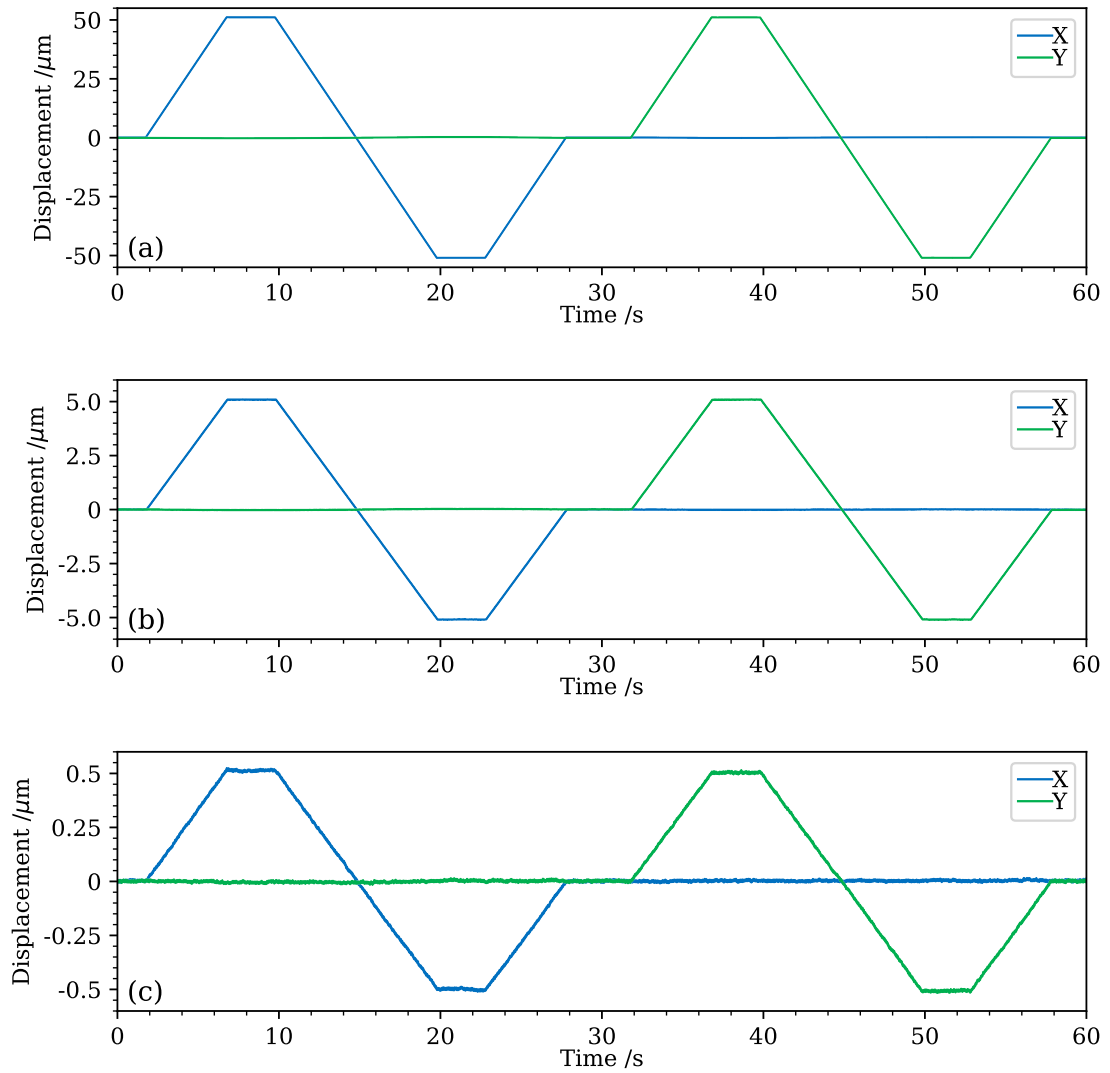


FIGURE 5.4: Simultaneous measurements of X and Y displacements for motion in the X-direction of the stage followed by a similar motion in the Y-direction. (a) shows a linear movement across the total range of the stage, $\pm 50 \mu\text{m}$, at a speed of $10 \mu\text{m/s}$, while (b) shows the results of a similar motion to (a) but for a smaller stage movement across $\pm 5 \mu\text{m}$ at velocities of $1 \mu\text{m/s}$. (c) then shows a similar plot for $\pm 0.5 \mu\text{m}$ movements at velocities of $0.1 \mu\text{m/s}$.

Figure 5.4(a) shows a plot of the measured displacements from the interferometers corresponding to the X and Y dimensions over the sequential step motions with an amplitude of $\pm 50 \mu\text{m}$. For the same series of motions but with a motion amplitude of $\pm 5 \mu\text{m}$, the measured displacements for the X and Y dimensions are shown in Fig. 5.4(b). This is then repeated for a stage motion amplitude of $\pm 0.5 \mu\text{m}$ and the measurements for the X and Y dimensions are shown in Fig. 5.4(c). The measurements show agreement with the expected stage motion at all three stage motion amplitudes. To further investigate any

cross-motion in the stationary axes during the stage motion, the zero displacement axes of the data sets shown in Fig.5.4 can be examined closer. Using a smaller displacement axis this data is plotted in Fig.5.5.

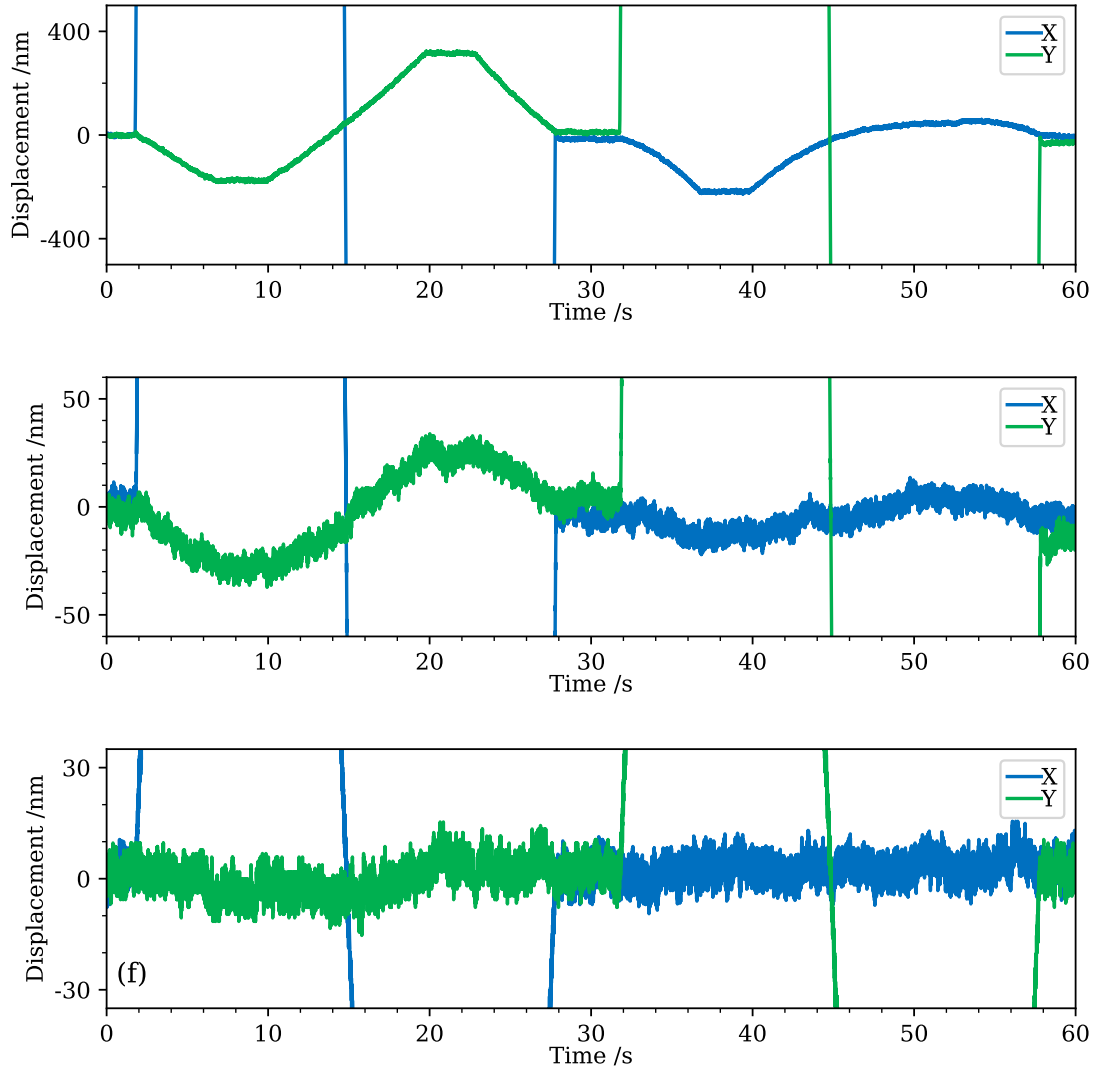


FIGURE 5.5: Displacement measurements for X and Y stage motions during sequential step motions of the stage with a restricted displacement axis. (a) shows the data for steps of $\pm 50\mu\text{m}$, at a velocity of $10\mu\text{m/s}$, with (b) showing displacement data for steps of $\pm 5\mu\text{m}$ at velocities of $1\mu\text{m/s}$. (c) then shows displacement data for steps of $\pm 0.5\mu\text{m}$ at velocities of $0.1\mu\text{m/s}$

Figure 5.5 shows three plots corresponding to the three data sets shown in Fig.5.4 with a reduced displacement axis range. Figure 5.5(a) shows a closer view of the data for stage motions over the total stage motion range of $\pm 50\mu\text{m}$, with Fig. 5.5(b) showing the data set for stage motions of $\pm 5\mu\text{m}$ and Fig. 5.5(c) showing the data plot for $\pm 0.5\mu\text{m}$ stage

motion amplitudes. In all three plots there is some off-axis motion visible in the stationary axis when the opposing axis is in motion. This parasitic motion can be attributed to a number of factors such as: 1) Non-flatness of the mirrors, 2) Genuine cross motion in the stage or 3) Misalignment of the optical axes to the stage motion axes. While the first two of these cannot be ruled out, the latter factor is more likely the cause of the parasitic motion observed in Fig. 5.5. This is because in the proof-of-concept design, the beam-splitter is coarsely aligned to the stage and hence the stage motion. With no adjustments on the 3D printed template possible, using the goniometer and rotation stage discussed in Sec. 5.2.2, the stage is then aligned to the optical axis of the beam to optimise returns from both mirrors. If the beamsplitters are misaligned relative to the stage principle axes, this alignment procedure would then cause the stage motion axes to be misaligned with the optical axes of the measurement beams and could explain the observed off-axis displacements.

Single Axis Sweep Measurements

To quantify the effect any non-perfect stage motion has on the displacement measurement results, the X and Y displacement data for the step motions shown in Fig. 5.4(a) can be directly compared and plotted. For the X motion, the data from 0-30 s is taken where only the X axis should be in motion, and data for the Y motion is taken from 30s to 60s when only the Y axis of the stage should be in motion. The direct comparison of the measured X and Y displacements are shown in Fig. 5.4.

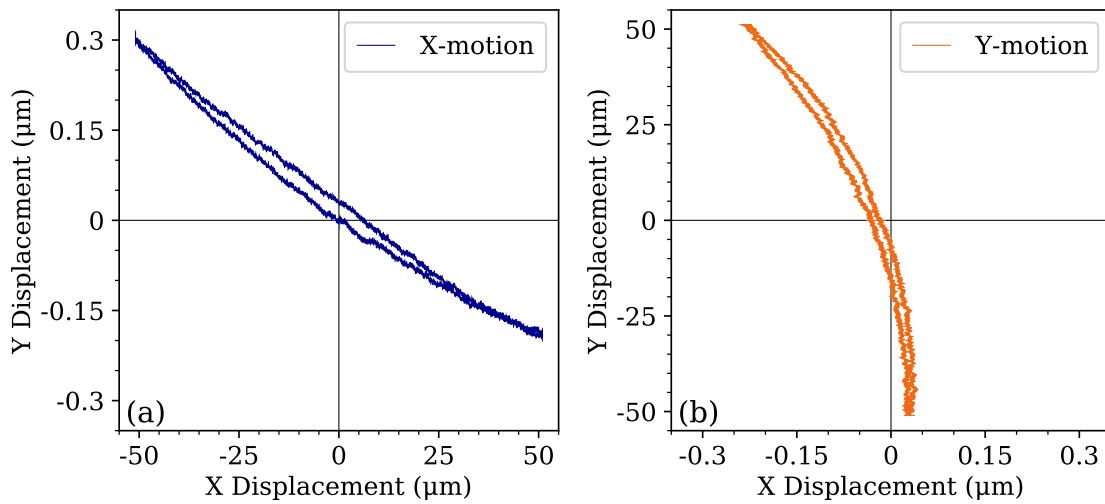


FIGURE 5.6: (a) shows the X against Y displacement measurements during programmed motions in the X-direction, where the Y axis should remain static. (b) shows the X against Y displacement measurements during programmed motions in the Y direction

Figure 5.4 shows two plots of the displacement measured in both X and Y axes during stage motion. The first, shown in blue, is motion in the X axis, where the Y axis is expected to remain stationary. In orange is the Y motion, where the X axis is expected to remain stationary. For the case of perfect alignment and stage motion, the expectation

is linear data along the 0 μm line of the relevant static axis. Instead, from Fig. 5.6 there is an evident parasitic motion in both single-axis sweeps. The linear deviation shown likely arises from misalignment of the optical axes with respect to the stage motion axes, however, if this parasitic motion in the off-motion axis is solely due to misalignment of the optical axes with respect to the stage motion axes, the off-axis parasitic motion would appear as linear with a gradient corresponding to the angle error of the optical axes with respect to the stage motion axes. However, both measurements show curvature and bi-directional deviation between the expected linear relationship that any optical axis - stage motion axis misalignment would create. The curvature shown by both stage motions is typical of non-perfect stage motion, where the acceleration and motion of the stage is non-linear. Hysteresis is a common effect in Piezo actuated stages caused by non-linearity between the electric field strength and the displacement. This can result in cross-talk between stage axes, angular deviations through stage motion and non-linear stage motion and as a result, it would be reasonable that in addition to the misalignment between the optical axes and stage motion axes discussed in sec. 5.2.4, that there is some contribution to the off-axis motion from both non-perfect stage motion and Piezo stage effects.

Circular Motion

To investigate the capabilities of this proof-of-concept in measuring motion in multiple dimensions simultaneously, circular motions of the stage in the X-Y plane were performed. This was achieved through sinusoidal and co-sinusoidal motion components in the X and Y motion axes respectively. The phase changes in the interferometers corresponding to the X and Y dimensions were measured and the resultant displacements for each of these dimensions were calculated, with the results shown in Fig. 5.7.

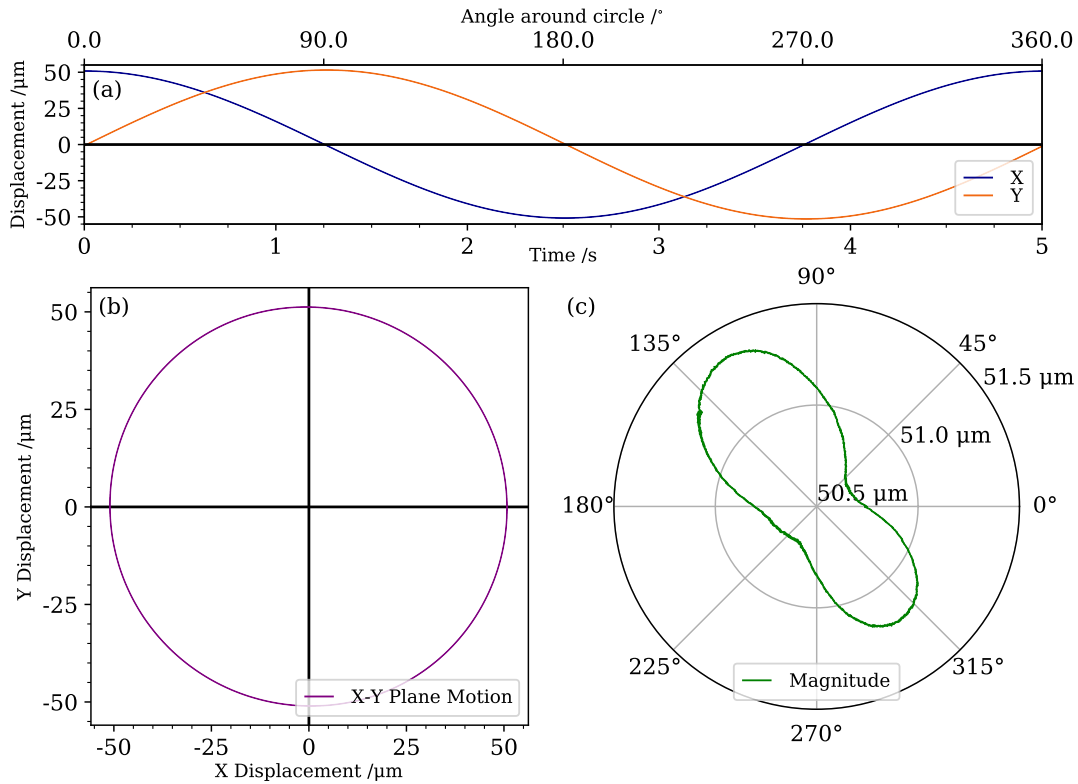


FIGURE 5.7: Displacement data for 2D circular stage motion. (a) shows the measured displacement components in the X and Y measurement dimension against both time and the angle moved around the complete circle (b) shows the same data projected onto an X-Y plane in a Cartesian coordinate system. (c) shows the measured displacements in a polar coordinate system with a restricted radial axis, where the plotted radii are the magnitudes of the X and Y components.

Figure 5.7(a) shows the components of displacement in the X and Y dimensions for a single period of the circular motion, with the X displacement plotted in blue with the Y displacement plotted in orange. These components are then plotted against one another in the plot shown in Fig. 5.7(b). This same data is shown in polar form in Fig. 5.7(c) with a reduced range in the radial axis. From these results it is evident that although the motion appears superficially to be an excellent circle, under closer examination of the radial amplitude over the period of the motion, there is a clear deviation from the expected circle at 50 μm . Throughout the stage motion, the radial amplitude is greater than the expected 50 μm , with a maximum amplitude of 51.36 μm and a minimum of 50.67 μm . This is also determinable in the individual components, shown in Fig. 5.7(a) where the X component has a maximum displacement of 50.74 μm and a minimum displacement of -50.8 μm , and the Y component has a maximum displacement of 51.1 μm and a minimum displacement of -50.85 μm .

The measured extension beyond the expected 50 μm could be due to a misalignment between the optical axes and the stage motion axis, where the misalignment would appear imbalanced between the two axes, over-extending in one axis and under-performing in

the other axis as it performed the circular motion. Here the measured circle radial amplitude greater than $50\mu\text{m}$ could be due to cosine error, as discussed Chp. 2 Sec. 2.3.5, which causes a measurement scaling error. This would also explain the different scaling errors in each axis which would cause an elliptical measured stage motion. In addition to these errors, the 'footprint' shape motion seen in Fig. 5.7(c) could be caused by non-perfect motion of the stage, particularly during accelerating and decelerating at the turning points of each stage axis's motion. This is further evidenced by the 'dents' in the stage motion, occurring at approximately 45° and 225° , which coincides with when both stage motion axes are at their maximum velocities. In reality, it is likely that these 'dents' are the cases with accurate stage motion, in comparison to elsewhere, where one or both axes are at turning points. At these points, the stage motion requires significantly more control and is therefore prone to further error.

5.3 Extension to 3 Dimensions

5.3.1 Beamsplitter Constraints in 3D

In the 2D optical encoder design, the interferometers of interest are those between each mirror, and the nearest beamsplitter face. Since these can be directly evaluated, other interferometers created between beams reflecting from any of the other surfaces within system are not needed and can be removed where possible for the sake of simplicity and to reduce the possibility of introducing cyclic errors. The initial design of the optical encoder used an angle-polished connector (FC-APC) with an appropriate collimator as the optical access port for the laser emission, and guiding the returning light to the interrogation hardware. This reduced the number of interferometers within the optical system to only those within the sensing region, and removed interferometers created between any of the semi-reflective or reflective surfaces and the fibre-tip. However, due to the nature of the cubic beamsplitters, direct measurement of the interferometer between each planar mirror and the nearest surface of the beamsplitter cube intrinsically introduced non-linearities (Chp. 2 Sec. 2.3.5 for more discussion) in the form of overlapping optical path differences, presented in Fig. 5.8, as each interferometer of interest overlapped with the interference between a planar mirror and the alternative rear beamsplitter surface.

Figure 5.8 shows how the splitting condition of the beamsplitter creates interferometers of the same optical path difference. While the beams comprising the interferometer of interest, the beam reflecting from the mirror (i) and the beam reflecting from one rear surface of the beamsplitter (ii), independently have unique OPDs, the interferometer created between beams reflecting from (i) and (ii) will intrinsically have the same OPD as the interferometer created between beams reflecting from (i) and (iii). This introduces non-linearities into the measurements of the interferometer of interest, which makes using these interferometers as the measurement dimensions inadvisable. In place of these, it is necessary to interrogate the interferometer between each mirror (i) and the front surface of the beamsplitter (iv). However, aligning the optical system so that both a strong signal from the front surface as well as the mirrors is received can be difficult and can result in misalignment between the stage motion axes and the measurement axes, as demonstrated by the results shown in Fig. 5.7.

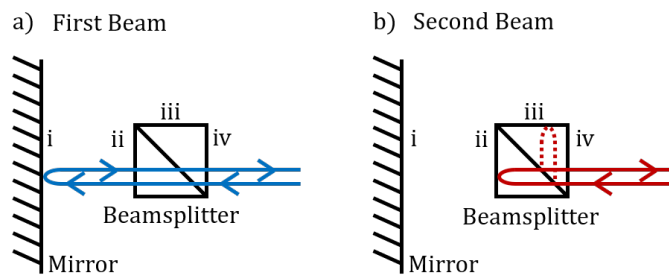


FIGURE 5.8: (a) shows the first beam of a particular interferometer, passing through the beamsplitter and reflecting from a planar mirror (i), and back towards the source. (b) shows a second beam, reflecting from the rear face of the beamsplitter (ii) at the air-glass boundary and back towards the source. At the splitting medium this also reflects towards the other rear surface of the beamsplitter (iii) and reflects from this surface and back towards the source, denoted by the dotted beam. Also labelled is the front surface of the beamsplitter (iv)

As an alternative, a Fresnel reflection can be created at the fibre tip by using a flat-polished connector (FC-PC) with an appropriate collimator as the optical access port to the sensing region. This then creates a local interferometric reference, with which each of the beams reflected by surfaces within the system can interfere, and is both strong and stable. Whilst this creates significantly more interferometers, namely those between each semi-reflective surface and the polished fibre-tip reflection, it allows for more control and discrimination over the position of the mirrors and hence the length of the OPDs of the interferometers of interest. It also eliminates the need, in general, to interrogate each beamsplitter face individually, and instead the subtraction of the common optical path can be achieved digitally after measurement. This alternative configuration avoids the issue of intrinsic OPD overlap outlined above and in Fig. 5.8, and simplifies the extension of the optical encoder design into measurement of further dimensions. Here, multiple dimensions can be measured simultaneously using a single on-stage reflection as a measurement reference, together with an interferometer between a mirror corresponding to each dimension and the fibre-tip, the interferometric reference, with other on-stage components appropriately coated to maximise transmission.

5.3.2 Extended 3D Design Criteria

To increase the number of measurement dimensions, an additional beamsplitter is added to the optical setup. In addition to the original beamsplitter, splitting the beam into the X and Y dimensions, the additional beamsplitter is placed between the original beamsplitter and the fibre tip, on the Piezo stage, and splits the beam between the X and Z dimensions. A schematic of this layout is shown in Fig. 5.9 alongside a 3D representation of the design. As discussed in sec. 5.3.1, using a local interferometric reference, created by a Fresnel reflection at the fibre-tip of a flat-polished connector (FC-PC) allows more fidelity in the selection and 'positioning' of OPDs, where the interferometers of interest are now those between each mirror and the fibre-tip, in addition to an on-stage reflection interfering with the fibre-tip. These interferometers of interest are marked in Fig. 5.9.

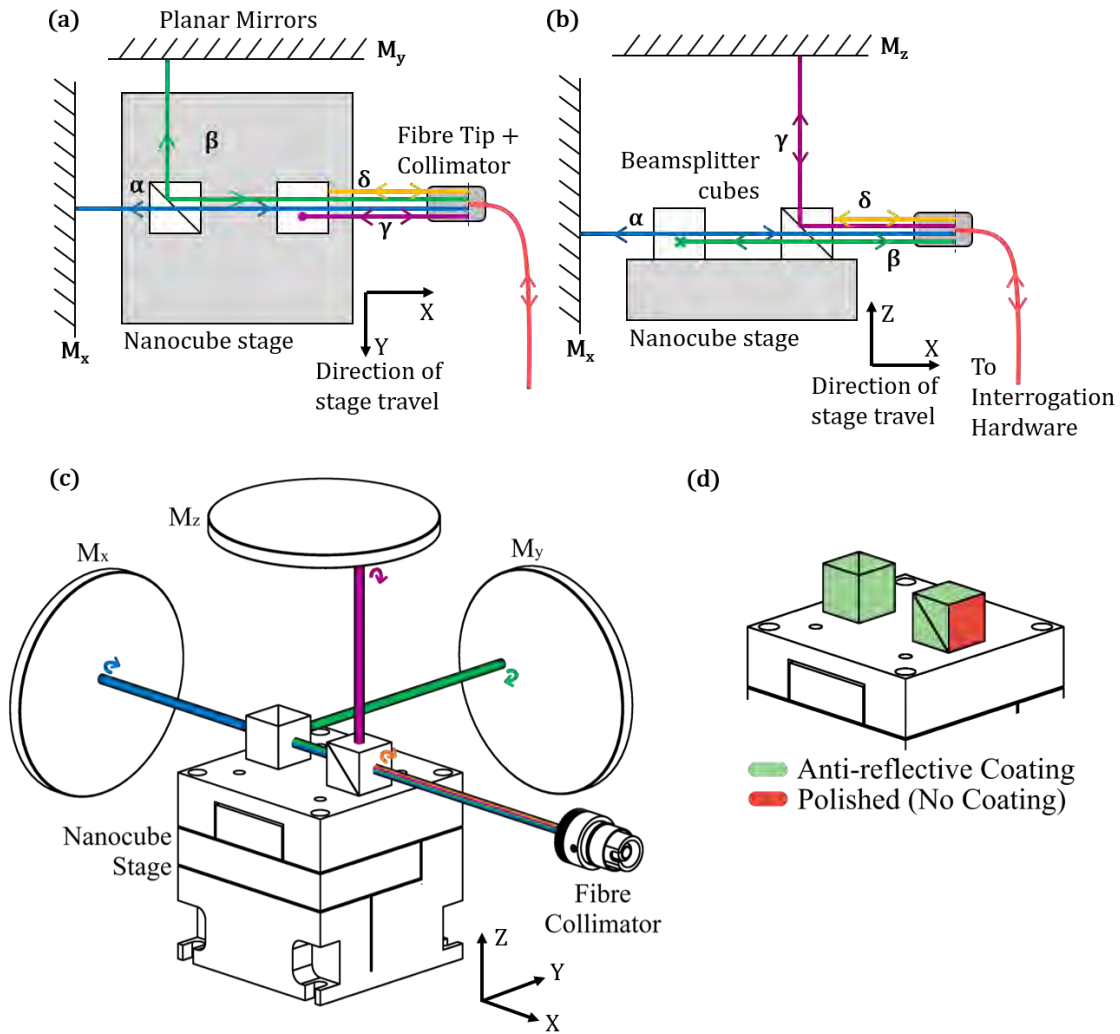


FIGURE 5.9: Design of a 3-dimensional encoder. (a) shows the top-down view with the relevant geometrical distances highlighted. (b) shows the side-view with the relevant geometrical distances highlighted. In both figures, distances α , β and γ correspond to the interferometers created between the X, Y and Z mirrors, labelled as M_x , M_y and M_z , respectively, interfering with the Fresnel reflection from the flat-polished fibre-tip. δ refers to the distance of the reference interferometer between the first incident surface of the first beamsplitter, while ϕ corresponds to path lengths of the undesired interferometers between any adjacent pair of beamsplitter surfaces. (c) shows a 3D illustration of the optical design and (d) shows which surface of the beamsplitters is polished to remove the anti-reflective coating, in the same orientation as in (c). The surfaces not explicitly shown in the diagram are anti-reflective coated.

As Fig. 5.9 shows, the optical setup used by this design is extremely simple, with no need for customised optical elements or polarisation sensitive components. In order to maximise transmission, the beamsplitter faces can be bought with appropriate anti-reflective

coatings which has the added effect of reducing folded path and multiple reflected interferometers between these surfaces. However, since the light reflected by the front surface of the first beamsplitter is required in the reference interferometer with the fibre-tip, this surface is left uncoated, or the coating is polished off, in order to take maximum advantage of the Fresnel reflection at this interface.

Design Considerations

Each additional dimension adds a layer of complexity to the optical design required by the encoder in order to guarantee unique OPDs for each interferometer of interest. As in all interferometric applications, additional interferometers with associated optical path differences are created between any pair of surfaces that cause partial or total-reflection, in particular surface-air boundaries. As discussed in section 5.2.1, it is necessary for minimising linearity errors that the interferometers of interest have OPDs that are unique to any other interferometers within the sensing region. This includes interferometers created by a beam encountering secondary reflections and passes through the sensing region. The difficulty increases in this particular application due to the beamsplitters, where the interactions between split beams creates many more interferometers within the system. These can be better understood as the interferences by a number of different types of beam passing through the optical system, some of which are summarised and illustrated in Table 5.1. For simplicity, these beams are summarised for the 2-dimensional encoder discussed previously, however the beam types and descriptions remain true for a 3-dimensional encoder also using beamsplitters. Column 1 shows an example diagram of the beam path, with the reflection points labelled and reflection spatially offset for visibility. In a practical ideal system, the reflections would all be overlaid. Column 2 categorises the beams into simple types to be considered, with the description of what defines these beam types in column 3.

TABLE 5.1: Types of beam found in a 2-dimensional encoder design using a beamsplitting element, categorised by the optical path of the beam. The diagrams in column 1 show reflection points and spatially offset beams for illustrative purposes only.

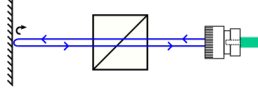
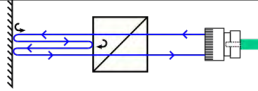
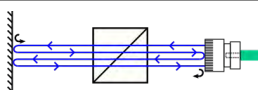
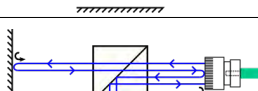
Example Diagram	Beam Type	Beam Path Description
	Principle	A beam reflected from a single surface. Examples include Fresnel reflection from a flat-polished fibre tip or from a semi-reflective surface or mirror.
	Folded Path	An emitted beam reflecting from one surface, reflected back off a different surface to the initial surface, then reflected back again into the fibre. Alternatives can include multiple folded paths within the beamsplitter.
	Doubled Path	An extension of the folded path beam. Consists of a beam reflecting off a single surface, reflecting back from the fibre-tip and back again from the initial surface into the fibre, passing through the optical system completely twice.
	Split Path	A derivative of the doubled path beam. Instead of the second pass through the optical system travelling the same path as the first pass, instead it diverges at the splitting element, reflecting from a second surface and back into the fibre.

Table 5.1 demonstrates the variety of beams present in even a simple optical system. For each beam type (with the exception of split path beams) considered in table 5.1, multiple variations of each of these beams exist within an optical system. In particular, folded paths can occur between any reflective or semi-reflective surface and can reflect multiple times before returning to the fibre, creating beams with secondary or even tertiary multiples of the internal optical path differences. Considering now the extension to 3-dimensions. In general, the principle beams scale proportionally to the number of reflective or semi-reflective surfaces within the optical system. The number of split path and doubled path beams scale with the number of split dimensions. In the case of double-path beams, there is an equal amount as the number of measured dimensions, and in the case of split path beams, each unique pair of paths will create unique split path beams, totalling 3 pairs for 3 dimensions. Folded paths however will scale near-exponentially with the increase in reflecting surfaces, as each subsequent surface passed has the potential to reflect a returning beam, creating a folded path or a variant of a folded path. All of these beams will then interfere with each other, creating a wide variety of interferometers at the detector with varying optical path differences.

Smallest Viable Optical Path Difference

From the perspective of arranging the OPDs of interferometers of interest so as to not coincide with any other OPDs, there are a couple of key factors to consider. The smallest OPD in the system will create a "grid" of OPDs that need to be avoided in order to prevent introduction of non-linearities into the phase measurements. In this optical design, shown in Fig. 5.9, the smallest OPD is either between the front and rear surfaces

of a beamsplitter, or the distance between the two. To increase the range of OPDs, free from overlap, available for interferometers of interest, the components should be placed so that as many of the interferometers not of interest lie on this same grid. In this optical design, the beamsplitters were positioned such that the spacing between the two is equivalent to the in-air equivalent OPD between the front and rear beamsplitter faces, placing any interferometers created by interference between any two of the beamsplitters' surfaces on the same grid. In the case of the designed proof-of-concept system, this means the spacing is set to 15 mm where the beamsplitter has an edge length of 10 mm, as the in-air equivalent of this is 15 mm. This spacing will also add multiples of 15 mm to any other interferometers in the system in the form of folded paths between beamsplitter faces, creating additional interferometers that have to be considered in the optical design.

Split Path Beam Interferences

Another prominent effect in the beam interferences occurs due to interference with split path beams. Illustrated in Fig. 5.10, when a split path beam such as is shown in Fig. 5.10(a), having travelled down one beam path, reflected back from the fibre tip and travelled down a different path, interferes with a principle beam path from one of those paths as shown in Fig. 5.10(b), it creates an interferometer with the same OPD as the other principle path interfering with the fibre tip, shown in Fig. 5.10(c). To clarify the beam paths, reflected beams in all three sub-diagrams (a), (b) & (c) have been offset from the incident beams. To illustrate this in terms of OPD, Fig. 5.10(d) shows peaks corresponding to each interferometer at their expected OPD for either interferences with the beams shown in (a), (b) & (c) and the fibre-tip, or for interferences with each other.

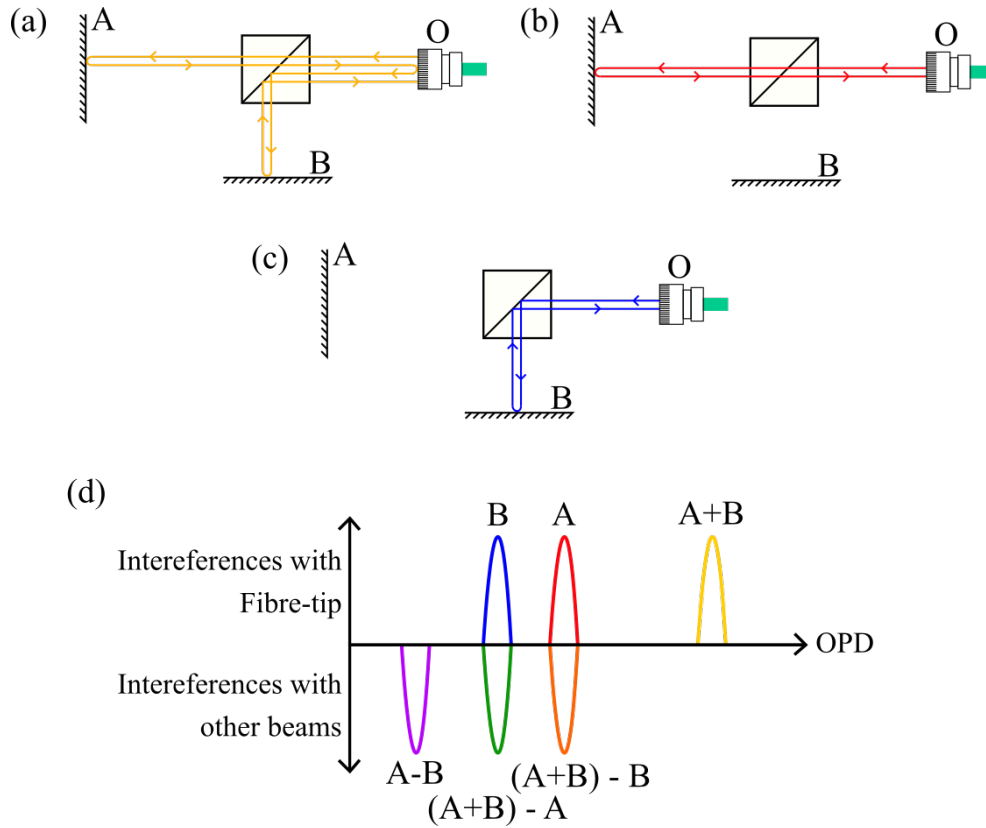


FIGURE 5.10: Explanation of how split path beams create duplicate optical path differences for a 2D setup. A , B are the geometric distances from the fibre-tip, O . (a) shows a split path beam, emitted from the fibre collimator, passing through the optical system down one path, reflecting from the fibre tip and passing through the optical system down an alternate path before returning to the fibre, with a total effective path length of $A+B$. (b) shows a single beam emitted from a fibre collimator, passing through an optical system and returning to the fibre collimator, with an effective path length of A . (c) shows a single beam emitted from a fibre collimator, passing through an optical system and returning to the fibre collimator, with an effective path length of B . (d) shows an example of expected interferometers created between the 3 optical beams and a fibre tip reflection, as well as interferometers created with each other, as a function of the resultant optical path difference. For ease of understanding, the reflected beams in (a),(b) and (c) have been spatially offset from the incident beams.

Figure 5.10 illustrates how split path beams create interferometers with duplicate OPDs to interferometers between principle paths and fibre-tip reflections. In the case shown here, when the split path beam with effective length $A+B$, interferes with a principle beam with effective length A or B , the result is an interferometer with effective OPD B or A respectively. If the principle beam interfering with the fibre tip is an interferometer of interest, which it is in this application, phase measurements will include cyclic errors from this parasitic interferometer.

It is evident therefore that the secondary reflections, combined with a beamsplitting element within the system intrinsically introduce cyclic errors into the phase measurements, regardless of the length of the path travelled or which combination of paths either interfering beam takes. The beamsplitter allows for split path beams that can interfere with a single principle beam, which will always overlap OPDs with one of the interferometers of interest. As this effect is independent of the geometrical distances in the system, it is not possible to avoid this by simply changing the mirror or beamsplitter positions. Instead, it is necessary to suppress secondary reflections of the beam through the system, such that they are not multiplexed into the interferometer measured at the photodiode. In order to achieve this, the collimator is replaced by a slightly de-tuned adjustable collimator assembly that creates a slightly divergent beam. The effect of this is that over any extended beam path, such as a second reflection through the system, the beam shape is changed sufficiently by the time it is re-incident on the collimator the amplitude of light that is actually in the accepted cone of the collimator is significantly reduced such that it has negligible effect on the measurement.

Principle Beam Interferences

An additional condition on the design of the optical encoder arises from the interferometer created between two principle beams reflecting off of a pair of mirrors. These OPDs will be inherently smaller than the interferometers between each mirror and the fibre-tip, as illustrated by the optical path difference A-B in Fig. 5.10(d), however they may still overlap with the reference interferometer so must also be considered in any rigorous optical design. This is also true of the interferometer created between a principle beam reflecting from any of the mirrors and a principle beam reflecting from the front surface of the beamsplitter. For each mirror this creates an additional interferometer that needs to be considered.

Signal Demodulation Conditions

The last condition applied to the optical design arises from the demodulation process discussed in Chp. 4. For a given interferogram, when the value of each phase carrier amplitude (PCA) is calculated, where each PCA is equivalent to the OPD of an interferometer within the system, the value is not discreetly limited. Instead, the value of the PCA for a particular OPD is governed approximately by a Gaussian distribution across a range of PCAs. Whilst the interferometer can be resolved from any value within the Gaussian distribution, the width of these Gaussian peaks determines how close in value two OPDs can be, *i.e.* the spatial resolution, whilst still independently resolved and without introducing cyclic errors into either measurement. During the demodulation process, a Gaussian width parameter, σ is set which determines the width of the window function applied to the interferometric signal. Note that this σ is not a variable of the function that directly defines the Gaussian distribution in the signal strength against PCA/OPD graph, but defines the width of the window function used in demodulation. To distinguish this, the width parameter of the window function will be referenced as σ_{win} . After demodulation, the width of the Gaussian window applied affects the spread

of the distribution at each PCA/OPD. This is illustrated in Fig. 5.11 where a real interferogram is demodulated using range-resolved interferometry for two separate σ_{win} parameters, and the effect on the spread of the distribution in PCA space is shown.

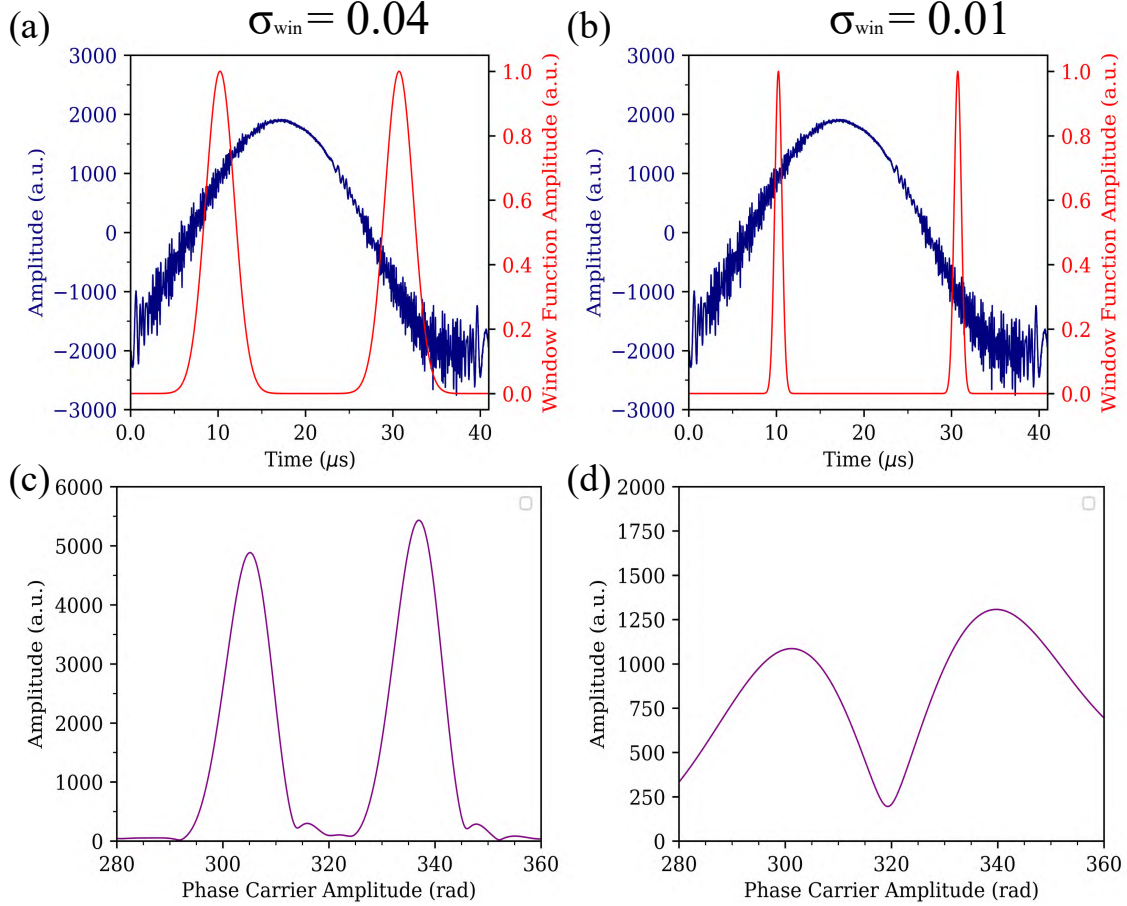


FIGURE 5.11: (a) shows an interferogram (blue) and an overlaid window function (red), with a width parameter of $\sigma_{win} = 0.04$, used in the RRI demodulation process. (b) shows the same interferogram (blue) overlaid with a window function with a width parameter of $\sigma_{win} = 0.01$. (c) shows a portion of a phase carrier amplitude graph, demodulated from the interferogram using the window function shown in (a). (d) shows the same portion of a phase carrier amplitude graph, demodulated from the same interferogram using the window function shown in (b)

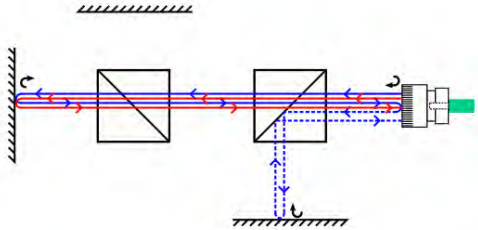
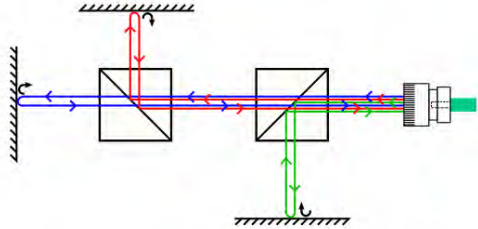
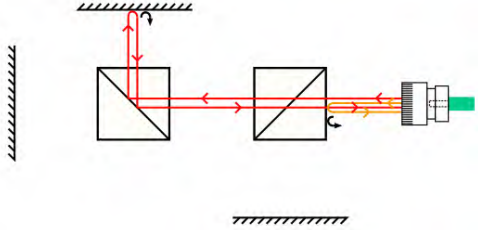
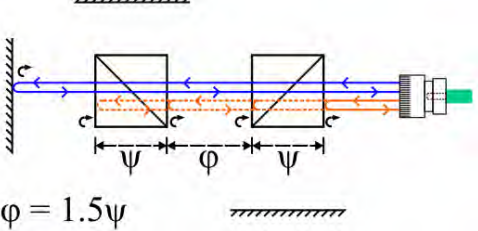
Figure 5.11(a) shows a multiplexed interferometric signal for an example optical system. Using a Gaussian window and the RRI signal processing [98], the amplitude of the demodulated interferogram can be calculated for a range of phase carrier amplitudes, including those corresponding to the OPDs of the interferometers of interest. Demodulating the interferometer with a width parameter of σ_{win} of 0.04, shown in Fig. 5.11(a), gives a range of demodulated amplitudes, a portion of which is shown in Fig. 5.11(c). Here, the minimum difference in PCAs that results in a complete separation of distributions, i.e. demodulating at any PCA within the distribution will not add cyclic errors from the other peak is 13 rad. Demodulating the same interferometric signal with a σ_{win}

value of 0.01, shown in Fig 5.11(b) yields the phase carrier amplitude graph shown in Fig. 5.11(d), allowing for a minimum separation of PCAs of 35 rad. As a direct result of increasing the Gaussian demodulation parameter σ_{win} the width of the distributions across the PCAs decreases, decreasing the minimum difference required between the OPDs in order to not add additional cyclic errors into the phase measurement of either interferometer.

Design Considerations Summary

A summary of each of the geometric conditions and the effect they have on the optical design are summarised in table 5.2 below. As the value of σ_{win} is applied in the demodulation process, this should be considered in addition to the geometric considerations summarised in the table. Avoiding these conditions, or reducing them where possible is paramount to reducing non-linear errors in the interferometers of interest.

TABLE 5.2: Summary of the geometric conditions applied to the optical design to minimise crossover of optical path difference. Single passes are denoted with solid lines, and double passes or folded paths are denoted with dashed lines. Beams are laterally spaced for illustrative purposes only.

Example Diagrams	Interferometer Descriptions
	Beam 1 passes through twice down separate paths. Beam 2 passes down one path once. Resultant OPD is the same as the other path.
	Pairs of mirror beams interfering creates additional interferometers
	Beam 1 reflects from one mirror. Beam 2 reflects from the front surface of the beamsplitter, creating additional interferometers for each mirror
 <p>$\phi = 1.5\psi$</p>	Folded paths between beamsplitter faces creates additional interferometers at each integer multiple of the OPD between beamsplitter surfaces

5.3.3 Optical design for 3 dimensions

As a result of the conditions summarised in table 5.2, it is necessary to optimise the placement of the mirrors and beamsplitters such that the OPD crossover is minimised for the interferometers of interest. In the case of this proof-of-concept design using a polished connector, the interferometers of interest are the principle interferometers created between each mirror and the fibre-tip, referred to in Fig. 5.9 as α , β and γ , and a principle interferometer between the front surface of the first beamsplitter and the fibre-tip denoted as δ . In the cases of each principle mirror interferometer there is high flexibility as the mirrors can be manually adjusted in sub-mm increments. The reference interferometer meanwhile is adjustable only by translating the cube along the beam axis and

within the confines of the stage surface. As some conditions depend on the position of both mirrors and beamsplitters, both components have to be considered simultaneously in designing the optical system.

For the optical design of this proof-of-concept encoder, the mirrors are placed such that the reference interferometer, δ and the dimensional interferometers, α , β and γ , all lie off the 15 mm 'grid' of OPDs occupied by folded paths between beamsplitter interfaces. Finally, the fibre collimator assembly is placed sufficiently far enough away that the reference interferometer, between the fibre-tip and the front surface of the first beamsplitter, has an OPD greater than the OPD of the interferometer created between the furthest placed mirror and the reference surface. A schematic of this design including geometric distances, a photo of the complete optical assembly, and the expected 'ranging' view from the RRI demodulation process is shown in Fig. 5.12.

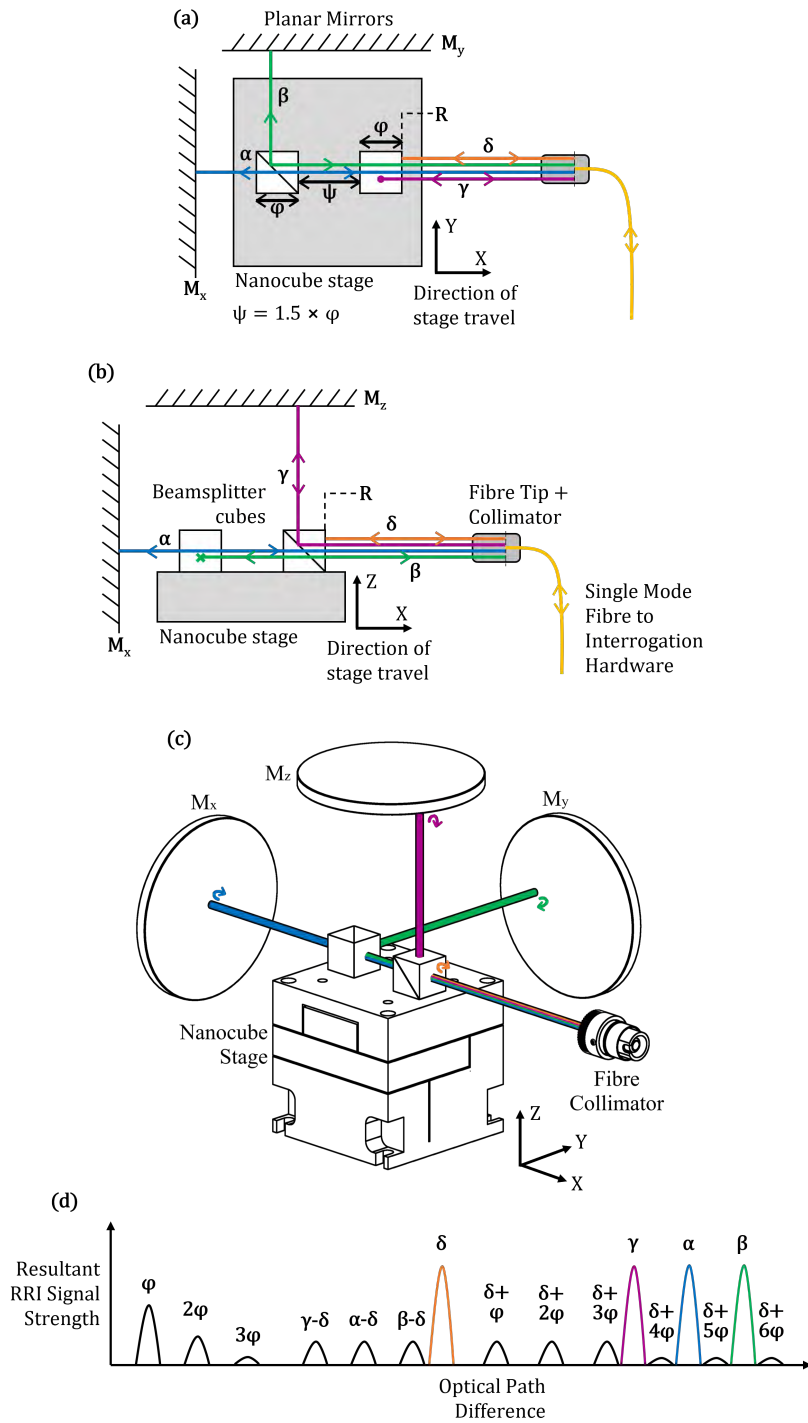


FIGURE 5.12: (a) shows a top-down view of the optical design schematic with principle beam paths, α , β , γ and δ highlighted and the geometric distances of interest φ and ψ labelled. Also labelled is the front surface R of the beamsplitter (b) shows the same design from a side view and (c) shows a 3D representation of the schematic. (d) shows the expected 'ranging' view that would be created by the RRI technique, with the interferometers of interest in colour, and parasitic interferometers in black.

Figure 5.12(a) shows a schematic of the optical design of the 3D encoder from a top-down view with the geometrical distances corresponding to the interferometers of interest labelled. The dimensional interferometers, created by a beam reflecting from each mirror (M_x , M_y and M_z) interfering with the Fresnel reflection from the fibre-tip, are labelled as α , β and γ respectively. Also labelled is the reference interferometer δ , between the front surface of the first beamsplitter cube, R, and the Fresnel reflection from the fibre-tip. Figure 5.12(b) shows the same setup from a side-on perspective and Fig. 5.12(c) shows a 3D representation of the optical design. Using the RRI technique, the expected 'ranging' view of this optical system is shown in Fig. 5.12(d) where the interferometers of interest are coloured and the parasitic, unwanted interferometers are in black.

In order to suppress secondary reflections and split path interferences that would interfere with single passes and overlap with interferometers of interest, as discussed earlier in Sec. 5.3.2, a de-tuned adjustable collimator assembly is used to create a weakly focused beam with a focus at approximately the length of the longest principle beam. The collimator assembly has a Gaussian beam divergence of 0.13° [69], which can be estimated as causing a maximum error of 0.12 nm over the $\pm 50\mu\text{m}$ travel range of the stage, which can be considered negligible. In addition to suppressing secondary reflections this has the added advantage of increasing the angular sensitivity of the fibre tip, making it easier to align the optical axes that depend on the alignment of both beamsplitters such as the beam from the mirror M_y .

Improving the minimum separation of surfaces that can be resolved by the RRI interrogation system is achieved by utilising the maximum modulation depth of the laser diode and driver. To do this, the modulation frequency of the laser is reduced to 24.4 kHz, reducing the quadrature bandwidth to 10 kHz, as discussed in implementation 2 in Chp. 4, Sec. 4.3. In turn, this increases the modulation depth to ± 0.31 nm around its centre wavelength of 1520.08 nm (in air), as measured using an optical spectrum analyser (Yokogawa AQ6370C). The interrogation hardware otherwise remains unchanged using the same, cost-effective laser diode and photodetector.

5.4 Measurements and Results

In order to demonstrate the measurement capabilities of this concept, three-dimensional displacement measurements over a variety of stage movements, including linear and helical motions, were conducted and recorded with an interferometric quadrature bandwidth of 10 kHz. Additionally, a characterisation of the noise and linearity performance is undertaken, assessing the limitations of this design.

5.4.1 Interferometer Referencing

In addition to changes made to the interrogation hardware, in this development of a 3-dimensional optical encoder, the interferometers being interrogated are changed. Where previously for the 2-dimensional optical encoder interferometers between the mirrors and the closest beamsplitter face were directly interrogated, as discussed Sec. 5.2.2, this induced cyclic errors in the form of parasitic interferometers into the phase change measurements. For the 3-dimensional development, interferometers between each mirror

and the fibre-tip are instead interrogated. However, since these measurements are not direct measurements of the stage motion they must be referenced to an on-stage moving reference. Here, the reference interferometer is created between the front surface of the first beamsplitter, and the fibre-tip. The measurement from the interrogated reference interferometer, δ , is then numerically subtracted from the measurements from the three, dimensional interferometers, α , β and γ , between the mirrors and the fibre-tip, to give actual stage measurements in each dimension of interest.

5.4.2 Single Axis Stage Motions

The first experimental measurements, similar to the 2-dimensional encoder, concerns the ability of the encoder to simultaneously interrogate multiple interferometers over a series of stepped motions in each stage motion axis. Starting from a central position, the stage was moved to a fixed offset in the X-direction, then swept to the same but negative offset in the X-direction before being returned to the central position. The same procedure is then repeated in the Y-direction followed by the Z-direction with displacements in the three directions simultaneously acquired. The results of these stage motions for a range of motion amplitudes, in addition to an example of the raw displacements from the interrogated interferometers are shown in Fig. 5.13.

For a stage motion amplitude set to $\pm 50 \mu\text{m}$ with a linear velocity of $10 \mu\text{m/s}$ the raw displacements before subtraction for each of the interrogated interferometers are shown in Fig. 5.13(a). After subtraction of the reference interferometer from each of the dimensional interferometer, the effective stage displacement in each measurement dimension is shown in Fig. 5.13(b). Fig. 5.13(c) shows the displacements measured in X, Y and Z for the similar motion patterns as described previously, but with an amplitude set to $\pm 5 \mu\text{m}$ and with stage axis velocities of $1 \mu\text{m/s}$ and Fig. 5.13(e) shows X, Y and Z displacements for the same motion pattern with an amplitude set to $\pm 0.5 \mu\text{m}$ and with a velocity of $0.1 \mu\text{m/s}$. From this data it can be seen that the displacement measurements for movement in all three directions can be readily obtained in all three cases covering large, medium and small commanded motions, demonstrating the ability of this system to measure in multiple displacement directions simultaneously. Similarly to the 2-dimensional case, there is a visible parasitic motion in the off-motion axis displacement measurements during stage motion. To further investigate this a restricted range of the view of the displacement axis can be used, shown in Fig. 5.14.

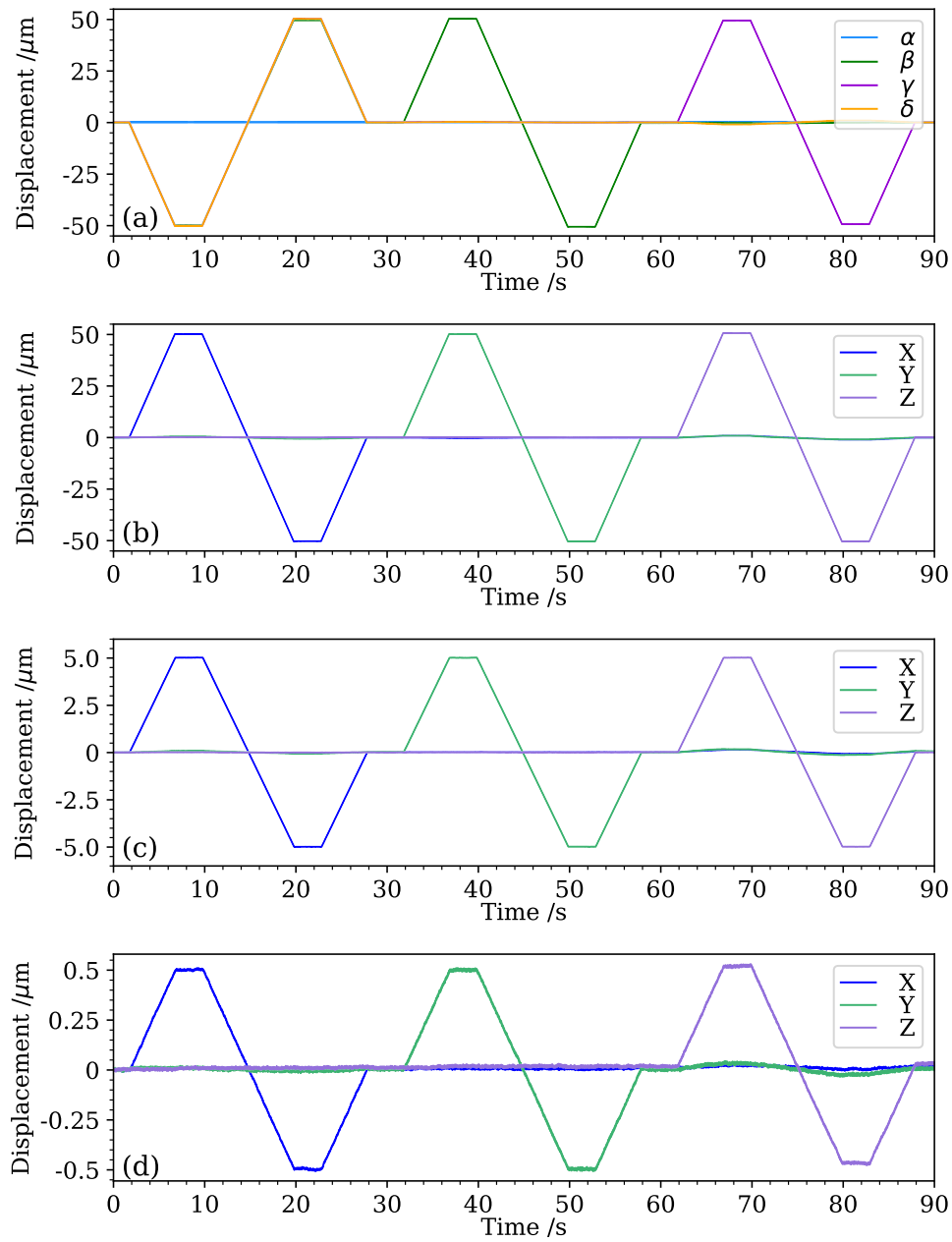


FIGURE 5.13: Figure (a) shows raw displacement measurements for the four measured interferometers, α , β , γ and δ during a series of stepped motions with an amplitude of $\pm 50 \mu\text{m}$ for each stage motion axis sequentially. The reference interferometer δ is then subtracted from the three dimensional interferometers to give the relative stage motion components in X, Y and Z, shown in (b). The measured displacements for the same motion pattern with a stage motion amplitude of $\pm 5.0 \mu\text{m}$ are shown in (c) where the reference interferometer has already been subtracted. (d) again shows the measured displacements for the same motion pattern but for a stage motion amplitude of $\pm 0.5 \mu\text{m}$.

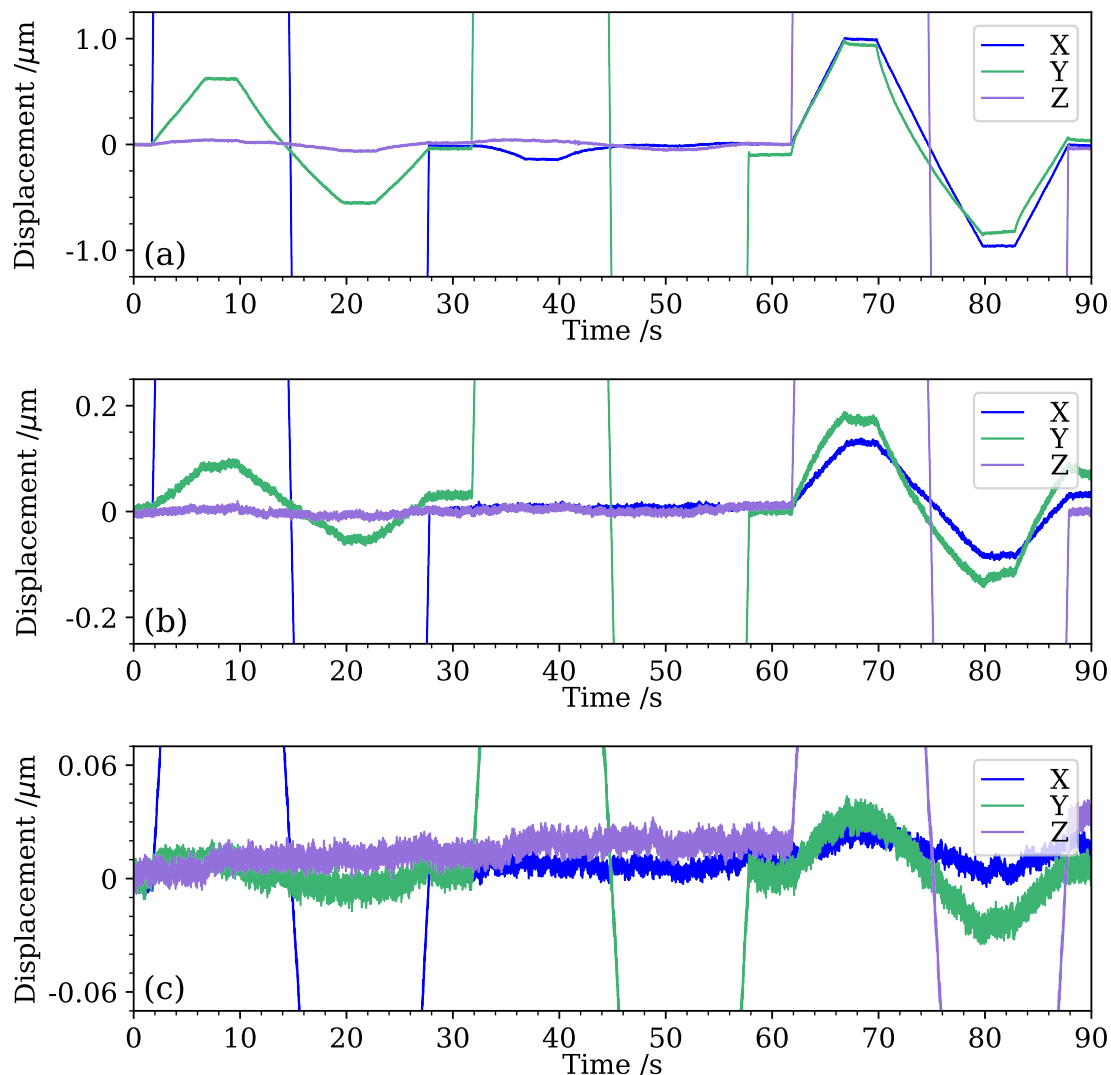


FIGURE 5.14: Restricted view of displacement measurements for a series of stepped motions over a range of amplitudes. (a) shows measured stage displacements for stage motions of $\pm 50 \mu\text{m}$ showing a restricted view of the y-axis. (b) shows measured displacements for the same series of stage motions but with amplitudes of $\pm 5.0 \mu\text{m}$ and again, a restricted y-axis range. (c) shows yet more measured displacements for stepped stage motions, but with a stage motion amplitude of $\pm 0.5 \mu\text{m}$.

Fig. 5.14(a) shows a restricted range of the data for $\pm 50 \mu\text{m}$, shown in Fig. 5.13(b), around the zero-displacement axis in order to better see any undesired displacements measured in the off-motion axes. Fig. 5.14(b) shows the same data used in Fig. 5.13(c) for stage motions of $\pm 5.0 \mu\text{m}$ but with a restricted range around the zero displacement axis. Finally, Fig. 5.14 shows the same data as in Fig. 5.13(d) for stage motions of $\pm 0.5 \mu\text{m}$. It can be readily observed in Figs. 5.14 that in addition to the commanded motion there are displacement signals present in the off-axis directions. For motion in the X direction, this occurs primarily in the Y direction at an amplitude of approximately 1.2% of the X

direction amplitude, while the Z direction exhibits only minimal off-axis displacements. For motion in the Y direction, only minimal off-axis displacements are observed in both the X and the Z directions. For Z motion, the overall largest off-axis displacements can be observed, reaching approximately 2.0% and 1.8% of the stage motion amplitude for the X and Y directions, respectively.

As discussed previously for the 2-dimensional case, the observed off-axis displacements could be caused by multiple reasons. While non-flatness of the mirrors M_x , M_y and M_z and genuine crosstalk in the stage motion are not beyond reason, the more likely cause of the apparent parasitic motion is misalignment of the optical axes relative to the stage motion directions. This may not be the only source of the parasitic motion, this is most likely the dominant source, as in this proof-of-principle design the beamsplitters were only very coarsely aligned using the 3D printed template that holds the beamsplitters in place. Adjustment elements for the beamsplitters were too difficult to incorporate in the setup due to the space-constraint environment on the stage top plate so as before, with no adjustments on the 3D printed template possible, the stage was aligned using the goniometers and the rotation stage. With regards to the amplitude of the parasitic motion, as the Z direction is only dependent on the orientation of a single beamsplitter placed flat on the stage top plate, this direction is automatically aligned to the stage principle axis and therefore less prone to off-axis displacements and it can be seen in all subplots of Fig. 5.14 that the Z axis displacement signal remains mostly flat during the movement of the other axes. Conversely, as the Y direction is dependent on the orientation and alignment of both beamsplitters in all three planes, we expect it to be the most taxing direction to align and indeed it can be observed that the Y direction exhibits the largest overall off-axis displacements.

In addition to the discussed alignment errors, drifts in the stage zero position of up to 30 nm can also be observed in Fig. 5.14(c). These drifts could be explained through changes of the laser emission wavelength or the air refractive index as well as through non-ideal stage behaviour. Thermal expansion of either the mechanical setup or in the stage itself for example, could cause an apparent drift in the stage displacement measurements. Furthermore, angular stage errors or cross-talk between stage motion axes could cause drift in the true motion of the stage, which could be measured as apparent displacement drift. In general, it is expected that laser wavelength or air refractive index changes would cause drifts that are proportional to the optical path length, where the longer path lengths involved in measurements of the Y-axis displacement would be affected the most. Since, however, such a proportionality cannot be observed in Fig. 5.13(f), non-ideal stage behaviour is the more likely cause and further investigation is needed to confirm this.

5.4.3 Helix Motions

While single-axis motion measurements adequately demonstrate the use of RRI as a concept for an optical encoder, real applications are likely to require more complex multi-axis motions and the encoder must remain usable in these conditions. To investigate this, the Piezo stage is driven in a helical motion over the maximum range of the stage. To achieve this, circular motions are performed in one plane, consisting of a sinusoidal and co-sinusoidal stage motion in two axes. The third axis is driven in a linear sweep

across the full axial range. As previously, phase changes in the four interferometers of interest are used to calculate relative displacements, and the reference measurement is subtracted from each of the dimensional interferometer measurements to calculate stage displacements in the X, Y and Z axes. This is then repeated so that each pair of axes forms the circular motion plane, X-Y, Y-Z and X-Z. Typical results for these motions can be seen in Fig. 5.15 below as both 3D plots and as 2D polar plots corresponding to the circular planes of motion.

Figure 5.15 shows displacement measurements for a series of helical motions with pairs of stage motion axes forming the plane of circular motion with a linear stage motion in the third axis. In addition to 3D plots of the data, polar plots of the data projected onto the plane of circular motion i.e. the plane normal to the linear motion axis, are shown. As demonstrated in figs. 5.15 (a),(c) and (e), this technique has the ability to interrogate and measure three dimensions simultaneously even during complex motion patterns of the stage. In the perfect case, the projections of the helices onto the plane normal to the linear motion, shown in the polar plots in figs. 5.15 (b),(d) and (f), should be a series of concentric, perfect circles at a radius of $50\text{ }\mu\text{m}$ with no offset between consecutive circles or eccentricity due to the commanded circular motion. In reality, although figs. 5.15 (a),(c) and (e) appear to show well-formed helices, the polar plots reveal deviations in the measured displacement amplitudes between each subsequent revolution, and in the general displacement amplitude radii. There are primarily two contributions that have distorted the polar plots from the expected perfect circles. The offset between each subsequent complete circle is likely due to the misalignment of the optical axes to the stage motion axes, discussed earlier. For example in Fig. 5.15 (f), in the case where the linear motion is occurring in the y-axis, there is minimal offset between each subsequent circle, as the X and Z mirrors are parallel to the Y motion axis which agrees with the results shown in Fig. 5.14 where during Y axis motion there was minimal parasitic motion in the X and Z axes. Furthermore this effect can also be seen in Fig. 5.15 (d) where the linear motion is occurring in the X axis. Here there is an offset between each subsequent circle, which can be separated into a small offset in the Z direction and a larger offset in the Y direction. In this case, the Z mirror is at a slight angle to the X motion axis and the Y mirror has a larger angular misalignment, resulting in the offset we see in Fig. 5.15 (d). In the case of Fig. 5.15 (b) where the linear motion occurs in the Z axis, there is an obvious misalignment between both the X and Y mirrors and the Z motion axis of the stage, which agrees with the parasitic motions in these axes seen in Fig. 5.14.

In addition to the offsets in each polar projection plot, there is an eccentricity to the displacements shown in figs. 5.15 (b),(d) and (f). This eccentricity could be caused by both misalignment of the optical axes to the stage motion axis as in the case above, or imperfect stage motion. In the case where misalignment is the only contribution to the eccentricity, the expected projection would appear to be a perfect ellipse. However, as there are distortions visible in all three ellipses, it is likely there is a significant contribution from imperfect stage motion, particularly around the turning points of the sinusoidal and co-sinusoidal stage motions.

These results clearly demonstrate that regardless of the alignment errors present, the

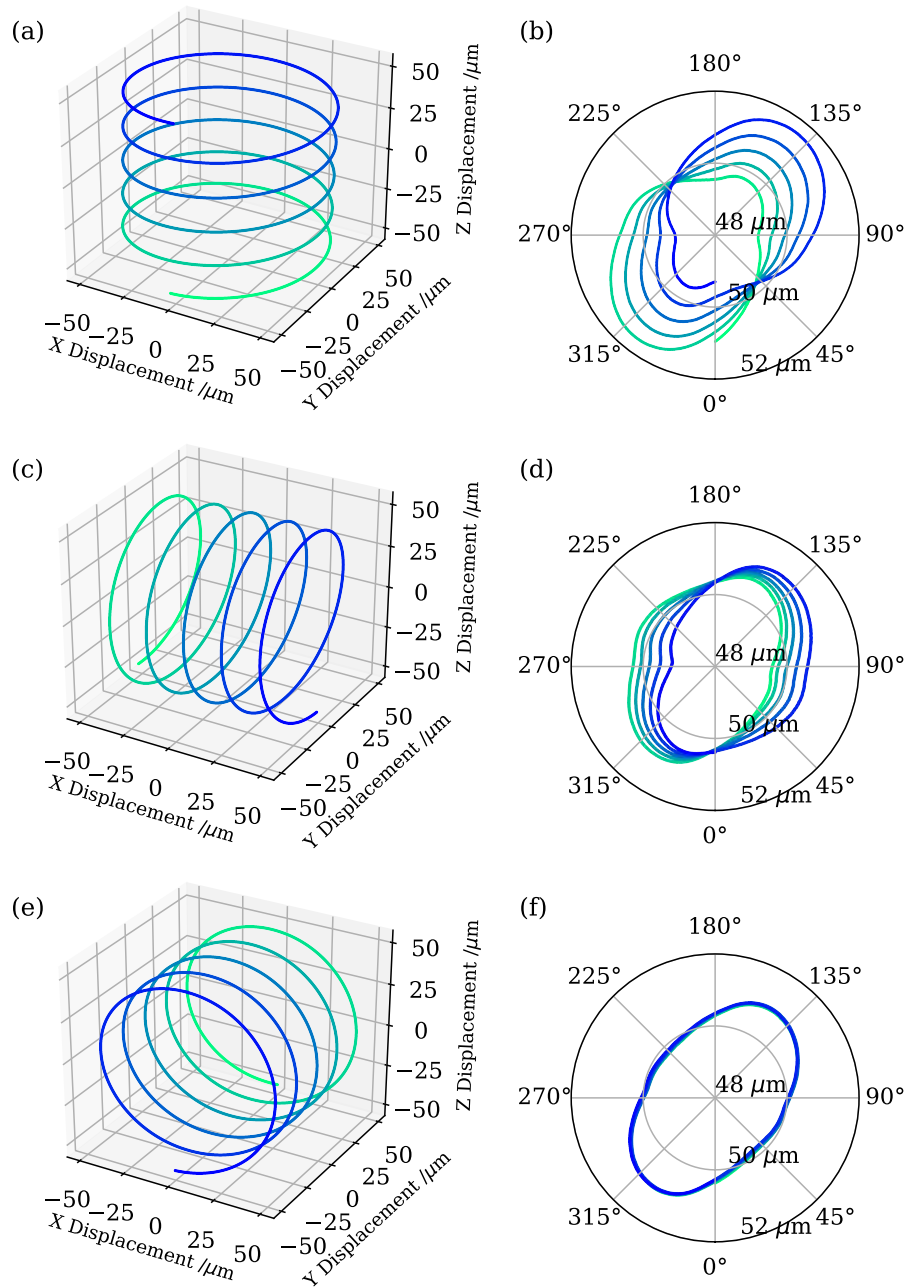


FIGURE 5.15: Displacement measurements of helical stage motions. (a) shows a 3-dimensional plot of the X, Y and Z measured displacements for a circular motion in the X-Y plane, coupled with a linear motion in the Z axis. (b) shows the same data flattened in the X-Y plane in polar form, with a restricted radial axis. (c) shows then data for circular motion in the Y-Z plane, with linear motion in the X axis, with (d) showing the same data flattened on the Y-Z plane in polar form. (e) shows data for circular motion in the X-Z plane, with linear motion in the Y axis, and (f) shows this same data set flattened on the X-Z plane in polar form.

proposed concept has the principle capability to fully characterise complex motion patterns and stage errors and that a future setup with improved alignment of the on-stage beamsplitters could be very useful for this purpose.

5.4.4 Strain Gauge Comparison

To independently verify and compare the displacement measurements made by the interferometers with an existing technique, it is convenient to use the strain gauge sensors integrated within the Piezo stage itself. In this experiment the stage was moved in a repeated triangular pattern with 0.2 Hz frequency and an amplitude of $15\text{ }\mu\text{m}$ in a single stage motion direction, while both strain gauge and interferometer recorded the displacements in the X, Y and Z directions. This was then repeated for each of the stage motion directions, X, Y and Z, and the results are shown in Fig. 5.16.

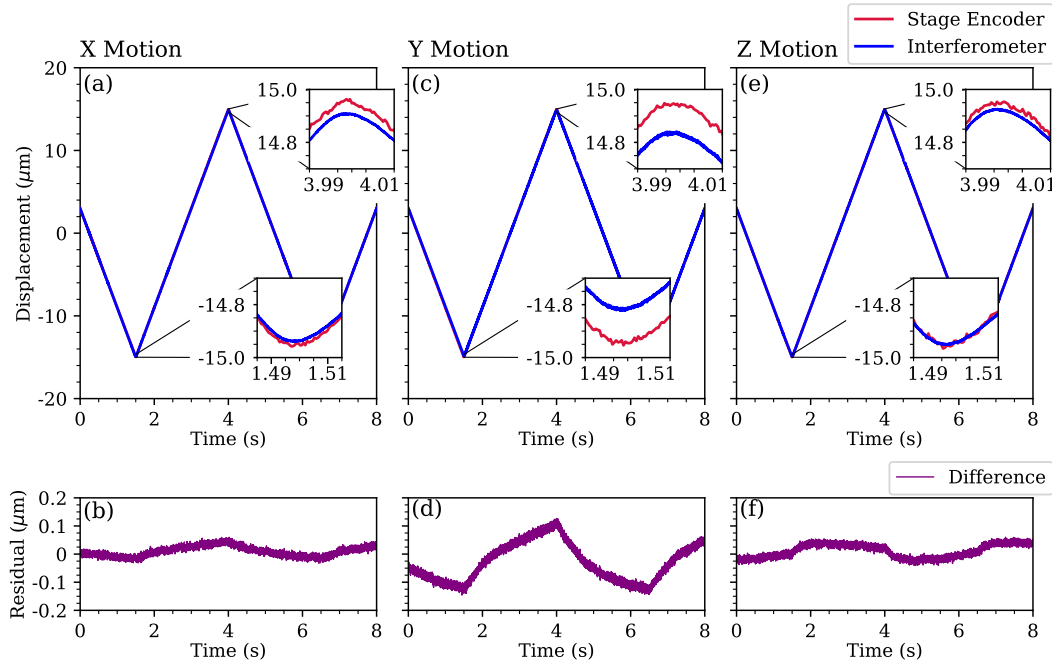


FIGURE 5.16: (a) shows measured displacements from the interferometer and from the integrated stage encoder of a $\pm 15\text{ }\mu\text{m}$ triangular motion in the X direction, while (c) and (e) show the corresponding data for the Y and Z directions, respectively. For all three directions the calculated differences between stage encoder and interferometer are plotted below the respective graphs in (b), (d) and (f).

Fig. 5.16 shows that there is generally good agreement between the stage encoder and the interferometer with maximum errors below $0.15\text{ }\mu\text{m}$ or 1 % of the travel distance for motion in the Y direction, and below 0.5 % in the X and Z directions. An error proportional to the travel distance could be attributed to a mismatch in scaling factors in either the stage encoder or interferometer. In the case of the interferometer, this could be caused by wavelength or environmental refractive index uncertainties as discussed previously. The remaining errors are likely due to the aforementioned misalignment between the optical axes of the interferometer and the stage motion axes. Here, because the alignment of the

Y optical axis depends on the precise alignment of both beamsplitters the increase of the error amplitude in the Y direction compared to the X and Z direction axes seems plausible. Additionally, the superior noise performance of the interferometric data, offering an instantaneous noise standard deviation of 4.2 nm at 10kHz quadrature bandwidth, compared to the in-built strain gauge encoders that exhibit an instantaneous noise standard deviation of 6.3 nm at 1.5 kHz quadrature bandwidth is also visible in insets in Figs. 5.16(a), (c) and (e).

5.5 Noise and Linearity Analysis

5.5.1 Noise Analysis

To establish the maximum resolution of the encoder design, it is necessary to examine the noise content of the encoder across a wide frequency spectrum and to separate out the components corresponding to mechanical noise, genuine interferometer noise and stage harmonics. In order to achieve this, an experiment was undertaken using the proof-of-concept design where one axis is moved in a repeating pattern, while the other two axes are stationary. To minimise the contribution from the stage motion frequencies, the stage is moved in a sinusoidal motion, where the motion will then create noise components with frequencies equal to the fundamental motion frequency and discrete harmonics of the fundamental frequency. In this experiment, the stage is moved in the X-direction, with a motion frequency of 5 Hz and an amplitude of $0.5 \mu\text{m}$. Measurements of the interferometers for each dimension and of the reference interferometer were acquired over a period of 50 s. The displacements in the three dimensions were calculated from the subtraction of the reference interferometer δ , from each of the dimensional interferometers α , β and γ . The measurements were then sliced into 1 s long segments and the amplitudes of the Fourier transforms were averaged to obtain an estimate of the displacement amplitude spectral density.

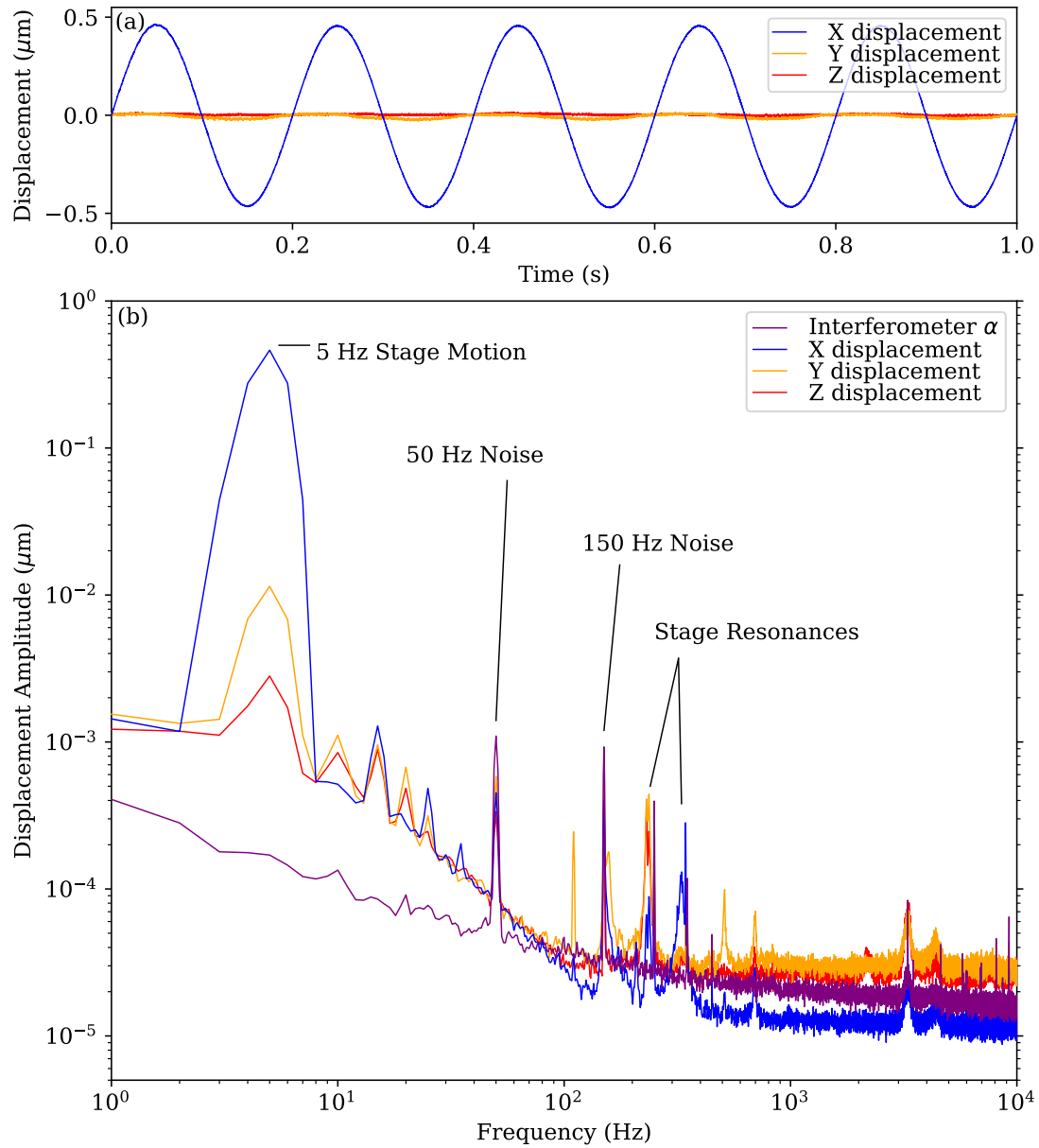


FIGURE 5.17: (a) shows displacement data for X, Y and Z for a simple sinusoidal motion of $\pm 5 \mu\text{m}$ movement in the X-direction. (b) shows the calculated displacement amplitude spectral density estimate over a 1 s bandwidth for the measured X, Y and Z directions. Additionally, the direct measurement of interferometer α in Fig. 5.9 without the subtraction of the reference interferometer δ , is shown for comparison, where this signal is stationary regardless of stage movement and is thus a direct measure of the instrumental noise.

Figure 5.17(a) shows one 1 s long section of the measurement for directions X, Y and Z when the stage was directed to perform a sinusoidal motion in the X direction only,

with no motion in the Y and Z directions. Figure 5.17(b) then shows the corresponding displacement amplitude spectral densities for the three measurement signals X, Y and Z along with the direct measurement of interferometer α , which as can be seen in Fig. 5.9 is stationary regardless of the stage motion and mechanical stage vibrations, thus making it a good estimation of the intrinsic noise of the interrogation system. In Fig. 5.17(b) we can see a peak at 5 Hz as expected from the fundamental frequency of the motion of the stage. In the X direction this has an amplitude of $0.50 \mu\text{m}$ amplitude, but there is also a visible off-axis displacement of $0.01 \mu\text{m}$ amplitude in the Y direction, which corresponds to a ratio of 2.0%, the same ratio that was previously observed in Fig. 5.13. The Z direction also exhibits an off-axis displacement, although this has a much smaller amplitude of $0.003 \mu\text{m}$, corresponding to a 0.6% ratio. These off-axis displacements are likely due to the misalignment between the optical axes and the motion axes of the stage discussed earlier in sec.5.4.2. By comparison with the instrumental noise obtained from the spectrum of the direct measurement of interferometer α , it can be seen that below 100 Hz the measurements in the X, Y and Z directions exhibit increased noise contributions which are likely due to the vibration sensitivity of the mechanical setup used for stage alignment, specifically the rotation stage and goniometers, with typical noise levels below $1.0 \text{ nm}/\sqrt{\text{Hz}}$. As the interferometer α is free from these mechanical noise components, it can be seen at lower frequencies that the instrument noise consists of the $1/f$ noise typical for interferometric systems and generally stays below a displacement noise level of $0.4 \text{ nm}/\sqrt{\text{Hz}}$, just over half an order of magnitude lower than the mechanical components. The complete lack of discernible contributions from the stage movement to the spectrum of interferometer α also confirms that any instrumental cross-talk between constituent interferometers is below a level of 0.04%. In addition to instrumental noise caused by the electricity supply at 50 Hz, 150 Hz and 250 Hz, present in all four signals, various spectral features between 150 Hz to 300 Hz can be found for the three stage directions X, Y and Z but not for the signal of interferometer α . These are thought to be stage resonances, which approximately match the mechanical resonance frequencies given in the stage data sheet [164]. Above 100 Hz, the displacement amplitude spectra for all four signals drops to noise floor levels between $0.01 \text{ nm}/\sqrt{\text{Hz}}$ and $0.04 \text{ nm}/\sqrt{\text{Hz}}$. The different frequency contributions to the displacement spectral density amplitudes are summarised in table 5.3 below. Over a typical 1 kHz bandwidth for the case of the interferometer α without influence of mechanical vibrations, this represents a resolution (1σ) of 1.3 nm achievable using this technique.

TABLE 5.3: Summary of the dominating frequency contributions to the displacement spectral density amplitudes shown in fig 5.17(b). Column 1 defines the regions of frequencies of interest, columns 2&3 states the suggested origin of the frequency components in these regions for the measurement dimensions, X,Y & Z, and the direct interferometer α respectively.

Frequency Region	X, Y & Z displacements	Interferometer α
5 Hz	Stage motion frequency	Interferometer noise
50 Hz, 150 Hz, 250 Hz	Electrical noise	Electrical noise
<100 Hz	Mechanical noise	1/f noise
150 Hz - 300 Hz	Stage mechanical resonances	Interferometer noise
>300 Hz	Interferometer noise	Interferometer noise

Table 5.3 summarises the expected origins of the frequency components of the displacement spectral density amplitude shown in Fig. 5.17(b).

5.5.2 Linearity Analysis

To characterise the linearity that can be achieved using this technique and experimental setup, an experiment was devised to isolate the non-linear errors from the mechanical noise discussed in sec. 5.5.1. To achieve this, the stage is set to perform linear motions at a given velocity such that any cyclic errors caused by parasitic interferometers with the same OPD as the interrogated interferometers of interest create cyclic errors at a fixed frequency within a frequency region with minimal mechanical noise. From the data shown in Fig. 5.17(b), the region from 50 Hz to 100 Hz offers minimal mechanical noise without stage resonances to avoid. To determine the expected frequency of cyclic errors f for a given stage motion velocity v , equation 5.1 can be used, where λ is the central evaluation wavelength and the factor of two arises as the interferometer is operating in reflection.

$$f = 2v/\lambda \quad (5.1)$$

It is expected that stage motion errors, as well as mechanical vibrations, will scale with the stage motion velocity, so keeping the stage velocity low would be ideal. However, the stage velocity must also be high enough that the cyclic errors occur in the 50 Hz to 100 Hz frequency region. To this end, a velocity of $50 \mu\text{m/s}$ was chosen, which according to equation 5.1, with a central interrogation wavelength of 1520.08 nm , will cause cyclic errors to occur at approximately 66 Hz. This is comfortably within the optimal region

without the need for over-driving of the stage. In this experiment, the stage is moved in a single motion axis in a triangular motion. In order to remove any non-ideal motion effects at the turning points from the analysis, a 1 s long segment over the central portion of the ramp is selected and a linear fit subtracted. Then a polynomial fit is fitted to this data to remove any remaining effects due to non-uniform motion. The section of the triangular displacement signal used for this analysis process is also illustrated in Fig. 5.18(a). The displacement amplitude spectral density is then calculated in a similar fashion to in Fig. 5.17(b), using 20 repeats of this data. To confirm that any errors at this frequency are cyclic errors, a second set of measurements are performed with an increased velocity of $70 \mu\text{m/s}$, which should shift the frequency of any first order cyclic errors to approximately 92 Hz. The calculated displacement spectral amplitude densities for the X, Y and Z axes during both of these motions are shown in Fig. 5.18(b) and (c).

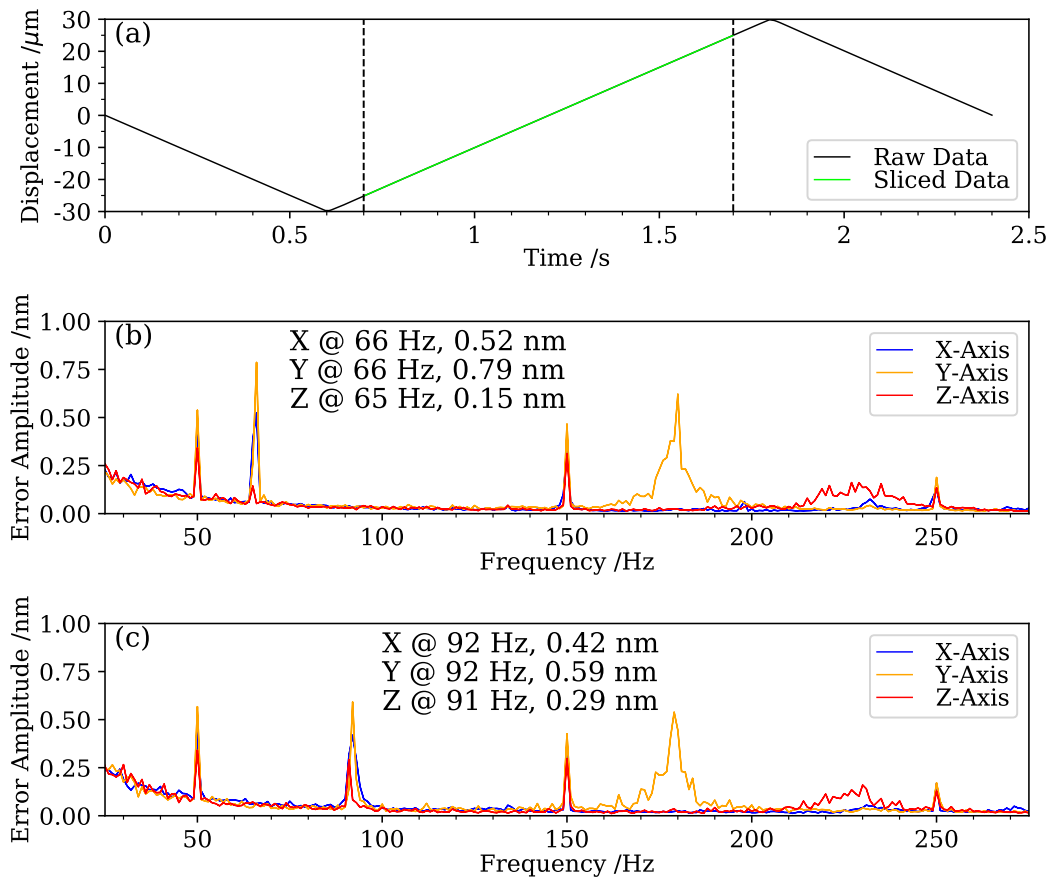


FIGURE 5.18: (a) shows an example of one period of one of the triangular motion measurements used in this experiment and highlights the 1 s analysis region evaluated. (b) shows the displacement noise spectral density estimate for a $50 \mu\text{m/s}$ movement, while (c) shows this for a $70 \mu\text{m/s}$ movement. In both (b) and (c) the determined cyclic error contributions occur at different frequencies and their extracted displacement amplitudes are labelled.

Fig. 5.18(a) shows an example period of the measured triangular displacement, and the

1 s section of the ramp used in this analysis. From Fig. 5.18(b) and (c) we can see peaks corresponding to the 50 Hz, 150 Hz and 250 Hz mains harmonics and spectral features due to mechanical resonances of the stage at 175-180 Hz in the Y-direction and between 220-240 Hz in the X and Z directions that were also present in Fig. 5.17(b). Critically, these features have comparable amplitudes and frequencies in both experiments, which means they cannot be caused by cyclic errors, due to the differing velocities. In Fig. 5.18(b), for all three directions, we can also see a peak corresponding to the expected first order cyclic error at 65.8 Hz, while in Fig. 5.18(c) we can observe that for all three directions this peak has moved to the frequency of 92.1 Hz expected for this velocity. This shift in frequency clearly indicates that these peaks are due to cyclic errors.

It can be seen in Figs. 5.18(b) and (c) that in all cases, the displacement amplitudes of the cyclic errors remain well below 1 nm. This is approximately equal to a phase error of 0.5° and the lowest cyclic error achieved for the Z direction in Fig. 5.18(b) is as low as 0.15 nm or 0.07° . Furthermore there is a notable absence of second-order cyclic errors at double the respective first-order cyclic error frequencies. Although, as discussed previously, suppression of secondary reflections and multiple bounces is achieved through weak focusing of the laser beam, we expect that the remaining cyclic errors occur as a result of secondary reflections within the beam path interfering with straight reflections such that the OPD between them is the same or close to the OPD of the interferometer of interest.

5.6 Dual-Beam Angle Encoder

Building on the same principles employed in the three-dimensional displacement encoder, this section discusses the development and experimental results of a proof-of-concept dual-beam encoder capable of measuring simultaneous displacement and angle measurements along a single axis, for a total of 2 degrees-of-freedom. In this section, the optical design including the design principles are discussed, followed by presentation of preliminary experimental results, and concluded with made using a proof-of-concept development of dual-beam encoder.

5.6.1 Principle and Design

In this design, range-resolved interferometry is used to interrogate multiplexed interferometers, as in the 3-dimensional encoder design. Here, instead of perpendicular beams corresponding to each measurement direction of interest, spatially offset, parallel beams are multiplexed onto the same delivery fibre. By performing phase evaluation of interferometers created by two spatially offset beams reflecting from the same surface, interfering with a reference surface, the relative displacement of the surface can be measured. From the trigonometric relationship between the spatial offset d , and the difference in measured displacement between the two beams $\Delta x = x_1 - x_2$, shown in eqn. 5.2, the angle change of the measurement surface $\Delta\theta$ can be determined:

$$\tan(\Delta\theta) = \frac{\Delta x}{d} \quad (5.2)$$

In order to measure this, a simple optical setup was developed that uses beamsplitter cubes to divide an optical beam emanating from a fibre collimator into two parallel, spatially offset beams. In this design, illustrated in Fig. 5.19, the two spatially offset beams are created by vertically stacked beamsplitter cubes with appropriate orientation. As in the 3-dimensional encoder design, a de-tuned adjustable collimator assembly is used as the optical access port of this setup, both emitting light from the laser and collecting reflected light. On the opposing side of the beamsplitters, a planar mirror is mounted on a controlled pitch-yaw aligned so that the pitch axis is perpendicular to the dimension of interest. At the first beamsplitter interface the beam is split in two, with the first beam transmitted towards the mirror, reflecting the beam back at point M_1 . The second beam is reflected towards the second beamsplitter, which in this case acts as a convenient 45° reflective surface to direct the split beam also towards the mirror, which reflects the beam back at point M_2 . The spatial offset between the two beams is then defined by the geometric distance between the two beamsplitters, where in this case the vertical mounting reduces the separation, d , to the equivalent side length of one cube beamsplitter. Fig. 5.19 illustrates this geometry including the reflection points of interest, alongside a photo of the encoder.

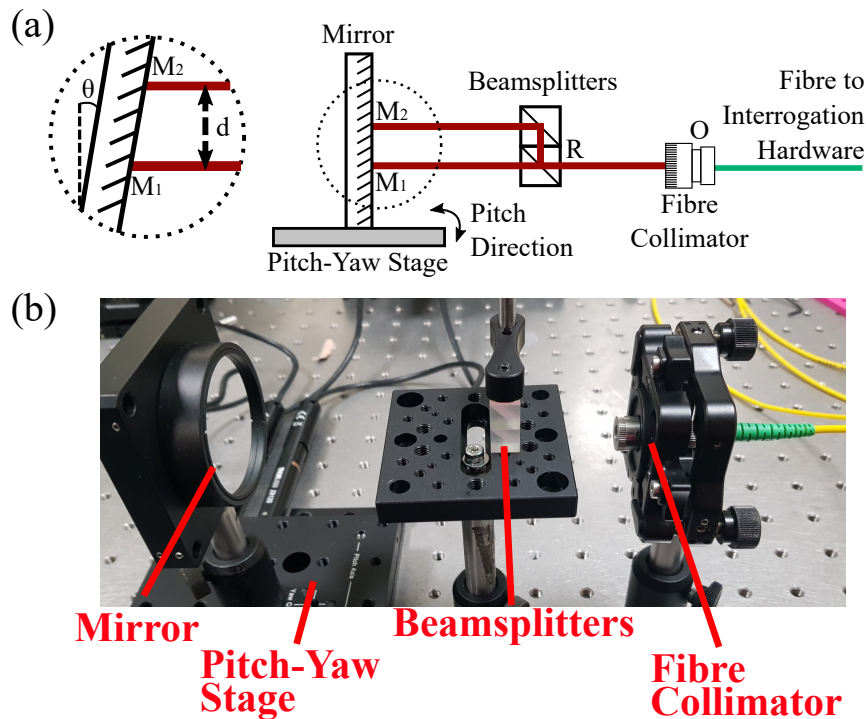


FIGURE 5.19: (a) Side-on schematic of the 2 beam displacement and angle encoder including a close view of the incident points at the mirror. Key dimensions and locations have been labelled and geometry such as mirror tilt has been exaggerated for effect. (b) Photo of the optical setup

Fig. 5.19 shows a side-on schematic of the complete optical setup including a close-view of the mirror reflection points. Here, the mirror is shown tilted at an exaggerated angle for illustrative purposes. To perform measurements using this optical setup whilst avoiding issues with intrinsic non-linearities caused by split paths, as discussed

in Sec. 5.3.1, three interferometers are interrogated. The two principle paths created by the beams reflecting from the mirror at points M_1 and M_2 are interfered with a Fresnel reflection at the fibre-tip to create interferometers with OPDs $L(M_1 - O)$ and $L(M_2 - O)$ respectively. To limit the sensitive region of the interferometer to only the path from the beamsplitter cubes onwards, the front surface of the first beamsplitter face, at point R , is polished to remove the anti-reflective coating, and a reference interferometer is created between the front surface of the beamsplitter and the fibre-tip Fresnel reflection with OPD of $L(R - O)$. Eqn. 5.2 can then be rewritten to calculate the angle change of the planar mirror from the measured change (Δ) of displacements from each of these interferometers as:

$$\Delta\theta + \theta_0 = \arctan \left(\frac{\Delta(M_1 - R) - \Delta(M_2 - R)}{d} \right) \quad (5.3)$$

where θ_0 is the unknown zero-position of the measurement. The reflected beams all re-enter the fibre via the fibre collimator and the same interrogation hardware as previously for the 3D optical encoder is used to demodulate and evaluate the interferometers.

As discussed previously in Sec. 5.3.2, to avoid the addition of non-linearities into the phase measurements of the interferometers of interest, the geometry of the system should be carefully considered. In this design of a 2-beam angle encoder, there are three primary considerations to be made.

Folded Paths: As in the three-dimensional displacement encoder, folded paths created between semi-reflective and reflective surfaces creates a 'grid' of interferometers with OPDs equal to multiples of the smallest OPD in the system. As before, this is created between pairs of beamsplitter surfaces, occurring at the multiples of 15 mm in-air OPD. In the 2-beam angle encoder design however, the difference between the OPDs of the two measurement interferometers of interest is fixed by the geometry of the beamsplitter. Here direct contact between the beamsplitters creates a fixed difference in OPD of 15 mm in-air for the two interferometers of interest, so placement of the mirror to ensure that these lie off of the "grid" of parasitic folded path interferometers is necessary.

Split Paths: The act of splitting the light into two separate beams intrinsically creates parasitic interferometers when beams that pass through the system twice, once down each path, interfere with a principle beam from one of these paths. This is discussed in Sec. 5.3.2 in terms of the three-dimensional displacement encoder, however the principle remains the same. To suppress secondary reflections from the fibre-tip, a detuned adjustable collimator assembly is used, such that the beam profile is sufficiently altered after the second reflection, that contributions into the interferometric signal from these beams are minimal.

Principle Beam Interferometers: As well as these interferometers to consider, some of the principle beams interfering with one another can occur with similar OPDs to interferometers of interest, specifically the reference interferometer. This typically occurs due to the direct interferometer between the principle beams reflecting from the mirrors and the principle beam reflecting from the reference surface, having a similar OPD to the reference interferometer. To avoid this, the fibre-tip can be

placed sufficiently far enough away that these parasitic interferometers have an OPD significantly larger than the reference interferometer OPD.

These conditions can be seen applied in the signal amplitude against phase carrier amplitude graph shown in Fig. 5.20. Here, the interferometers have been labelled with the corresponding optical path difference using the geometry shown in Fig. 5.19(a), with the addition of the inter-beamsplitter-surface optical path measurement φ , used previously.

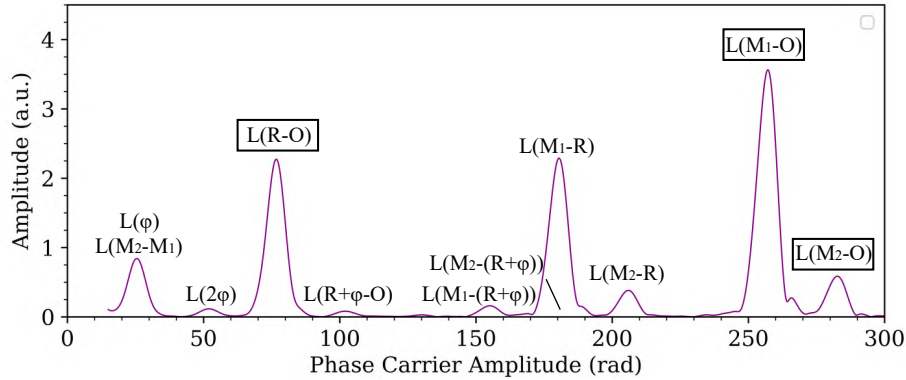


FIGURE 5.20: Amplitude against phase carrier amplitude graph for the 2-beam angle encoder design, where phase carrier amplitude is analogous to optical path difference. For the main peaks OPD has been labelled and for the interferometers of interest, highlighted.

Fig. 5.20 shows the amplitude against phase carrier amplitude graph for the 2-beam angle encoder shown in Fig. 5.19. Of interest are the interferometers with the highlighted OPDs, namely the two measurement interferometers with OPDs $L(M_1 - O)$ and $L(M_2 - O)$, and the reference interferometer with OPD $L(R - O)$. It is reasonable to question why the direct interferometers with OPD $L(M_1 - R)$ and $L(M_2 - R)$ are not interrogated, given that it reduces the number of interferometers required and eliminates the need to numerically subtract a reference interferometer afterwards. The answer, in this particular case, is due to the parasitic interferometers created by the principle beams reflecting from M_1 and M_2 interfering with a beam that has reflected from the back surface of the first beamsplitter, giving it a total path length of $L(R + \varphi)$. The end result of this is two parasitic interferometers with path lengths $L(M_1 - (R + \varphi))$ and $L(M_2 - (R + \varphi))$ respectively, the latter of which directly overlaps with the direct interferometer with OPD $L(M_1 - R)$.

5.6.2 Experimental Results

In order to see whether this concept could be used to measure simultaneous displacement and angle measurements for a single linear dimension, the 2-beam encoder was used to measure pitch changes in the planar mirror. The planar mirror is mounted on a pitch-yaw stage, with the pitch direction aligned to the optical axes of the two incident beams. Over a series of pitch angle step changes, measurements of phase changes for the three interferometers of interest were performed. From these, displacement measurements, and from Eqn. 5.3 pitch angle changes, can be calculated. The results of these measurements are shown in Fig. 5.21.

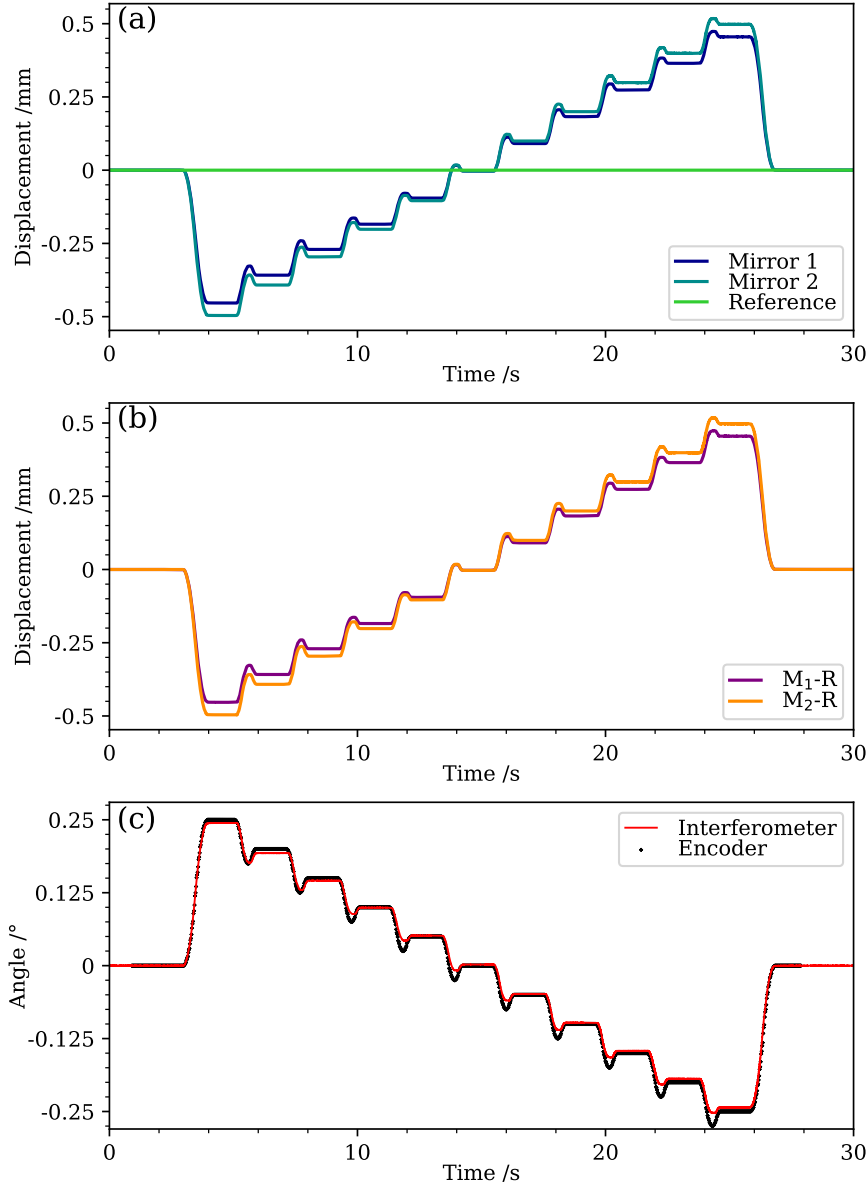


FIGURE 5.21: (a) measured displacements from the three interferometers of interest. (b) displacements from measurement interferometers after the reference interferometer measurement has been numerically subtracted (c) calculated resultant angle change from the interferometer data compared to data from the pitch-yaw stage in-built encoder.

Fig. 5.21(a) shows the measured displacements from each of the three interferometers of interest, namely the two dimensional interferometers with OPD $M_1 - O$ and $M_2 - O$ and the reference interferometer with OPD $R - O$. As expected here there is no visible motion in the reference interferometer, but the stepped pitch changes are visible in the dimensional interferometers. Numerically subtracting the reference interferometer from the dimensional interferometer measurements, the displacements between each mirror reflection point and the reference surface are shown in Fig. 5.21(b). As expected, the

two measurements undergo the same general pattern of displacement changes but with different displacement amplitudes. Fig. 5.21(c) then shows the calculated angle change from these displacements (red), using Eqn. 5.3. This calculated angle change is compared to the direct measurement from the in-built stage encoder (black). In general, there is good agreement between the interferometric measurements and the stage encoder, with both measuring the same stepped sequence, including some overshoot by the stage. However, there is some visible scaling differences between the two measurements, most notable at the two extremes of the pitched motion. To better compare the two measurements points corresponding to the stationary sections at each angle step are sampled, then compared in Fig. 5.22.

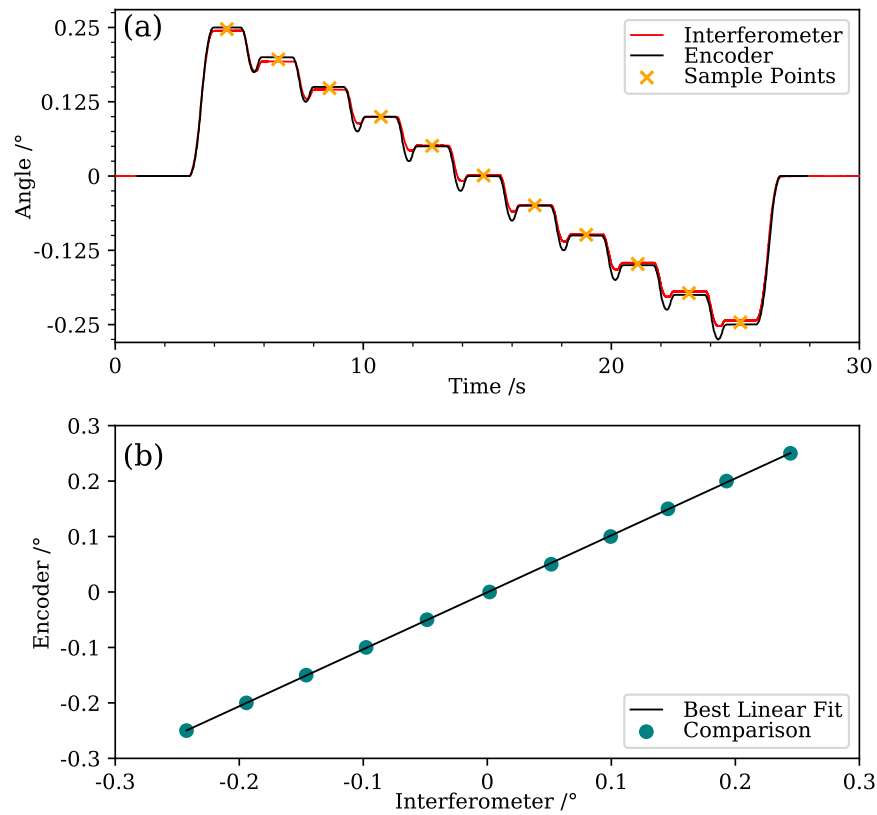


FIGURE 5.22: (a) Calculated angle changes from the interferometer and an in-built stage encoder. Also marked are the sample locations for direct comparison between the two data sets. (b) direct comparison of the encoder data at the sampled locations to the interferometer data. A linear best fit is also shown.

Fig. 5.21(a) shows the measured data sets from both the interferometer and the in-built stage encoder for a series of pitch angle changes in increments of 0.05° . In addition, the sample locations for direct comparison of the two measurements are marked. Fig. 5.21(b) directly compares the two measurements of pitch angle changes by plotting sampled data from the encoder against sampled data from the interferometer. Here, a line of best fit is also shown for this data. In principle, for identical data sets it would be expected that this data has a linear best fit with a gradient of $m = 1$ and a R^2 value of 1. From

the sampled data, the gradient can be calculated as $m = 1.027$ which is illustrative of the proportional offset of approximately 3 % we see in Fig. 5.22(a), and a linear regression is calculated of $R^2 = 0.99993$. This gradient could be caused by a couple of factors related to the geometry of the optical setup, or a factor linked to the interrogation system:

1) Misalignment: Misalignment of the optical axes to the stage pitch direction could cause one or both dimensional interferometers to be measuring a different optical beam path than expected. This could occur via a small rotation of the mirror relative to the pitch direction, misalignment of the pitch direction to the optical axis of the encoder or a mutual misalignment of the beamsplitters to one another. As, in this proof-of-concept, the optical setup is only coarsely, manually aligned, uncertainty due to misalignment is likely. A more robust mechanical development of the optical setup could reduce this uncertainty including automatic alignment systems.

2) Wavelength Uncertainty: As the interferometric measurements are intrinsically linked to the emission wavelength of the laser, mis-calibration of this emission wavelength could cause a systematic scaling error in the measured displacements, and hence angle measurement. Thorough calibration of the emission wavelength or inclusion of a wavelength stabilisation system could be used to reduce or eliminate this error.

3) Angular Stage Encoder Errors: In this experiment the pitch-yaw stage is a commercial off-the-shelf component. Whilst it is likely that the encoders embedded in the stage are calibrated at production, it's unclear whether any calibration occurs during the initialisation of the stage. Here, mis-calibration of the stage encoders could produce errors in the measurement of stage angle, contributing to the scalar offset seen in Fig. 5.21(a).

5.7 Discussion & Further Work

These proof-of-concept designs serve as a demonstration of how RRI could be applied to optical position and angle encoding, offering the benefits, and mitigating some of the major drawbacks, of traditional interferometric techniques. Here displacement measurements have been presented over a nominal stage operating range of $\pm 50 \mu\text{m}$ of up to three Cartesian directions, acquired simultaneously and with a low-footprint optical setup using just a single optical access port. In addition, proof-of-concept measurements have been presented of this technique applied to a dual-beam angle encoder that allows simultaneous measurement of displacement and pitch angle changes in a single linear dimension.

Noise and Linearity: For the 3-dimensional optical encoder design, instrumental noise densities were presented below $0.4 \text{ nm}/\sqrt{\text{Hz}}$, dropping to below $0.1 \text{ nm}/\sqrt{\text{Hz}}$ above 50 Hz, with a 10 kHz quadrature bandwidth. Here, typical cyclic error amplitudes were achieved well below 1 nm without any corrections. Below 50 Hz, frequency noise ($1/f$) from the laser is the dominant source of noise, which could be improved through the use of advanced laser diodes and drivers. This linearity performance, without correction, is comparable to existing interferometer designs including highly linear commercial interferometers [110]. This level of linearity is possible due to discriminate demodulation and analysis of interferometers based on optical path differences, by range-resolved interferometry. To achieve even lower non-linearities, a more robust optical and mechanical system could be designed that achieved advanced suppression or avoidance of parasitic interferometers. Techniques for compensation of non-linearities [104, 110] such as the Heydemann correction [109] could be also be applied to further reduce non-linearities.

Minimum Separation of Surfaces: With regards to the limitations of the proof-of-concept designs presented here, firstly, the current interrogation system requires a minimum spatial separation between the reflecting surfaces of 5 mm (in-air) to avoid inducing cyclic errors due to the peak crossover discussed earlier. To improve this further and allow for more compact optical setups, the wavelength modulation amplitude could be increased, which could be achieved through the use of diode lasers that are more widely tunable, or alternatively through the use of external cavity lasers (ECLs). However, particularly the use of ECLs is likely to increase the cost and lower the achievable quadrature bandwidth of the interferometer.

Maximum measurement range and velocity: The current implementation of the RRI interrogation hardware has an ADC acquisition rate of 100 MHz and operates at a wavelength modulation frequency of 24.4 kHz, which allows the system to achieve a quadrature bandwidth 10 kHz, allowing a maximum demodulation phase carrier amplitude of $\approx 2000 \text{ rad}$ to be interrogated [98]. Using the scaling factor of 0.58 mm/rad , previously determined in Sec. 5.3.2, this maximum demodulation phase carrier amplitude corresponds to a maximum distance (in air) from the fibre tip of $\approx 1.2 \text{ m}$. By lowering the wavelength modulation frequency and correspondingly the quadrature bandwidth, the maximum distance could be increased, however this would also lower the maximum travel speeds that can be measured by this technique whilst maintaining accurate tracking of the target surface. To determine the maximum allowable travel speed this setup can interrogate, we can use basic interferometric principles. The maximum phase change

that can be measured over the inverse of the quadrature bandwidth Q is 2π , equivalent to one spatial period of the interrogation wavelength λ . The maximum travel speed that can be accurately measured is therefore $v = (\lambda/2) \times Q$, where the factor of two arises as the setup operates in reflection. In this implementation, with $Q = 10$ kHz and $\lambda = 1520.08$ nm, target velocities of up to 7.8 mm/s can be tracked.

Maximum measurable travel range: The presence of undesired interferometers within the system, such as those between the fibre tip and the beamsplitter surfaces create a limit on the maximum travel range that can be measured using this approach. In the setup, as seen in Fig. 5.12, these form a grid of OPDs at multiples of the effective optical path difference between the surfaces of the beamsplitters, equivalent to 15 mm (in air). If the desired interferometers α , β or γ overlapped with this grid, increased cyclic errors would result. As a result, with the current configuration the maximum travel range is limited to ± 5 mm to maintain the linearity performance demonstrated here. However, this maximum travel range exceeds by a factor of 100 the $\pm 50 \mu\text{m}$ maximum travel range for the Piezo stage that was used here and which is representative of many nanopositioning systems with practical relevance.

Sources of Error: In this proof-of-principle setup, the dominant source of errors arises from the misalignment of the optical axes to the stage motion axes, due to the coarse alignment that the 3D printed template provides the beamsplitters. These alignment errors could be easily reduced by a more sophisticated and rigorous design, such as on-stage mechanical setup that better aligns the beamsplitters to the stage motion axes. Another source of error in this experiment comes from the uncertainty in the absolute accuracy of the interrogation wavelength, where, using the manufacturer specifications for the described optical spectrum analyser measurements, an uncertainty of approximately 13 parts-per-million (ppm) can be estimated. Over the $\pm 50 \mu\text{m}$ stage travel range, this corresponds to uncertainties of ± 0.7 nm. For long-term measurements, laser wavelength stability must also be considered as a source of error as it affects the entire effective path length of the interferometer. In this setup, the laser driver and temperature controller used offer a combined stability of approximately 0.3 ppm of the laser wavelength. Over the 99 mm effective path length of interferometer $\beta - \delta$, the longest effective path length used in this optical setup, the laser wavelength drift adds a drift error of up to 30 nm. In addition, as in most interferometric techniques, air refractive index changes [113] will affect the uncertainty of measurements, affecting the entire path length and not just the travel range. In well-controlled lab conditions it is reasonable to expect 1 ppm uncertainty, equivalent to a maximum error of up to 100 nm. In practical conditions, this would need to be considered and compensated for using a technique such as the use of external refractometers [118], or in lab conditions to reduce this error further. Furthermore, dual wavelength approaches [167] are also in principle compatible with the proposed approach.

Further work on this system could incorporate the use of absolute wavelength calibration and stabilisation techniques such as absorption line-locking to gas cell, that would lower the absolute wavelength uncertainty and also improve the laser stability. Here, where the uncertainty in displacement is intrinsically linked to the emission wavelength uncertainty in terms of both stabilisation and calibration, stabilisation techniques are

necessary to achieve high-accuracy displacement measurements. As discussed in principle in Chp. 3, typical stabilisation techniques such as Fabry-Perot cavities and self-injection locking are not appropriate for use with widely wavelength modulated techniques such as range-resolved interferometry and instead, new techniques or modifications to existing techniques must be developed to lock the central emission to a known reference standard. In Chp. 6, a novel wavelength stabilisation technique is developed specifically for widely wavelength modulated techniques such as RRI, and the principles and experimental results of this are presented. An alternative to an active stabilisation technique would be to employ an additional stable reference interferometer constructed using materials with low thermal expansion that could easily be added to the current design, and used to correct for both laser wavelength instability and refractive index changes [168].

5.8 Summary

In this chapter, the development of multi-dimensional optical encoders for dimensional metrology has been presented. Here, three developments have been shown; a 2D displacement encoder, a 3D displacement encoder and a 2-beam angle and displacement encoder. Using-resolved interferometry, a pseudo-heterodyne technique that uses a range dependant carrier function, multiplexed interferometers corresponding to measurement dimensions of interest, with unique optical path differences, are selectively demodulated and evaluated. In this chapter, the key design principles have been presented along with the proof-of-concept designs for each of these encoders. For the displacement encoders, optical paths are created between planar mirrors for each dimension of interest, and Piezo stage-mounted beamsplitter cubes. For the angle encoder, this is achieved by two spatially offset beams reflecting from the same planar mirror, after separation by a beam-splitter, which is mounted on a pitch-yaw tilt stage.

Experimental results for each of these encoders have also been presented. For the 2D displacement encoder, results of displacements over a nominal stage range of $\pm 50 \mu\text{m}$ are presented for both single linear motions, and two dimensional circular motion. For the 3D displacement encoder, similar measurements were made of displacements over the stage range of $\pm 50 \mu\text{ms}$ for single linear motions, with additional measurements of 3 dimensional stage motion. Also presented is comparison of the interferometric encoder to the in-built stage encoder, showing good agreement with maximum errors below 1% of the stage travel distance in the Y-direction and below 0.5% in the X and Z directions. These measurements also show the improved noise performance of the interferometer over the in-built strain gauge encoders, showing instantaneous noise standard deviation of 4.2 nm at 10kHz quadrature bandwidth compared to 6.3 nm at 1.5kHz quadrature bandwidth from the strain gauge encoders. Examining this noise performance further, exhaustive noise and linearity measurements were presented and analysed. The interferometric measurements show typical displacement noise levels below $0.4 \text{ nm}/\sqrt{\text{Hz}}$, with $0.4 \text{ nm}/\sqrt{\text{Hz}}$ at 1 Hz, and dropping to $0.01 \text{ nm}/\sqrt{\text{Hz}}$ above 100 Hz. For interferometric measurements in particular, linearity is of particular importance and here non-linearities from cyclic errors remain well below 1 nm, approximately equal to a phase error of 0.5° , for all three measurement directions. Measurement results using a dual-beam angle and displacement encoder are also presented, demonstrating an alternative

configuration of the same optical components and interrogation hardware that offers both linear displacement and pitch angle measurements. Here, measurements results of stepped pitch angle changes are presented, showing good agreement to measurements from the in-built stage encoder.

The performance of all three encoder designs demonstrate the efficacy of this approach towards multi-dimensional optical encoder design. Range-resolved interferometry, in conjunction with a considered, but simple optical design is already highly competitive with existing techniques, offering high precision measurements with the benefit of a significantly simplified optical setup. Further to this, the capability for multiplexed interferometric measurements allows for novel configurations of optical setup that could expand the usage of interferometric encoders into wider applications. Offering range dependant demodulation, the reduction of non-linear errors caused by parasitic interferometers can be achieved with simple, cost-effective interrogation hardware. To this end, range-resolved optical encoders offer significant benefit over traditional approaches to multi-dimensional metrology.

Chapter 6

Widely-Wavelength Modulated Swept Absorption Line Locking

This chapter describes the development of a novel wavelength stabilisation technique for widely-wavelength modulated lasers where the wavelength modulation amplitude is several orders of magnitude wider than common references. Wide wavelength modulation is used in techniques such as range-resolved interferometry, a pseudo-heterodyne signal processing technique detailed in Chp. 4 and applied to multi-dimensional encoders in Chp. 5. In this chapter, experiments to determine the efficacy of this developed novel stabilisation technique and the results of these, are discussed.

6.1 Motivation

Wavelength, or frequency, stabilisation is paramount in laser-based applications where the precision of knowledge of the laser emission has direct implications on the uncertainty of a measurement technique, such as in laser interferometry. A wide range of existing techniques, some of which are presented in Chp. 3, have been previously developed for wavelength stabilisation in regimes where there is no, or very low, wavelength modulation, where the modulation amplitude is typically within an order of magnitude of the reference linewidth. However, in regimes where the modulation amplitude is an order of magnitude or more larger than typical reference linewidths, such as in range-resolved interferometry [98], these techniques are less effective. Therefore, novel techniques for wavelength stabilisation in these regimes need to be developed.

6.2 Principles and Methods

This section describes the principles behind the developed locking technique, from the initial modulation through to the feedback signal. This technique follows the process outlined in Chp. 3, Sec. 3.1.2, divided into the following steps;

- Laser Emission
- Interaction with Wavelength Reference Cell/Line
- Measurement of Interacted Light

- Signal Analysis
- Wavelength Tuning

These steps are discussed in further detail below and is followed by discussion of the practical implementation of this technique.

6.2.1 Stabilisation Process

Laser Emission

The laser is modulated using sinusoidal injection current modulation, which results in both a sinusoidal wavelength/frequency modulation, as well as a sinusoidal intensity modulation. Both are considered in the first order to scale proportional to the intensity modulation, within the lasing current thresholds of the laser diode. While the intensity modulation is an important factor that will be discussed later, the intended output for this technique is the wavelength/frequency modulation.

In a single period of the laser modulation, the emission of a laser diode modulated with a sinusoidal modulation waveform, will "sweep" through a range $\lambda_c \pm \delta\lambda$, where λ_c is the central laser emission wavelength, and $\delta\lambda$ is the modulation depth of the laser. This sinusoidal waveform has a modulation period the same as that of the injection current modulation waveform, but typically has a phase offset θ_λ determined by the laser diode and driver characteristics. Meanwhile, the intensity modulation occurs typically in-phase with the injection current modulation. The wavelength modulation as a result of sinusoidal injection current modulation with modulation frequency f_m , can be described by;

$$\lambda(t) = \lambda_c + \delta\lambda \sin [2\pi f_m t + \theta_\lambda + \theta_0] \quad (6.1)$$

Where θ_0 is the original phase of the modulation waveform. The intensity modulation can be assumed, within the lasing current thresholds, to be proportional to the injection current of the laser. The intensity modulation waveform $I_0(t)$ can therefore be described similarly to eqn. 6.1, with an intensity amplitude δI and a central intensity I_c as

$$I_0(t) = I_c + \delta I \sin [2\pi f_m t + \theta_0] \quad (6.2)$$

Interaction with Wavelength Reference Cell/Line

If the laser is passed through an atomic gas cell and is directly incident on a photodetector, the measured intensity I is given by the Beer-Lambert Law [169];

$$I(t) = 10^{-A} I_0(t) \quad (6.3)$$

where I_0 is the original intensity, and A is the absorbance, calculated as the product of the molar absorption coefficient ϵ , the interaction length l of the optical beam and the absorbing medium, and the molar concentration c . The molar absorption coefficient of

the medium is wavelength dependant, which in turn is time-dependant, so eqn. 6.3 can be re-expressed as:

$$I(\lambda_t, t) = 10^{-[\varepsilon(\lambda_t) \cdot c \cdot l]} I_0(t) \quad (6.4)$$

where λ_t is the time-dependant wavelength modulation function expressed in eqn. 6.2. As expressed by eqn. 6.4, as the injection current is modulated, there is changes to the intensity at the detector caused by both changes in $I_0(t)$ and the changes in the wavelength modulation, which results in changes to the absorption coefficient $\varepsilon(\lambda_t)$. An example absorption spectra is shown in Fig. 6.1 for acetylene ($^{12}\text{C}_2\text{H}_2$) in the near infra-red region showing the R and P branches.

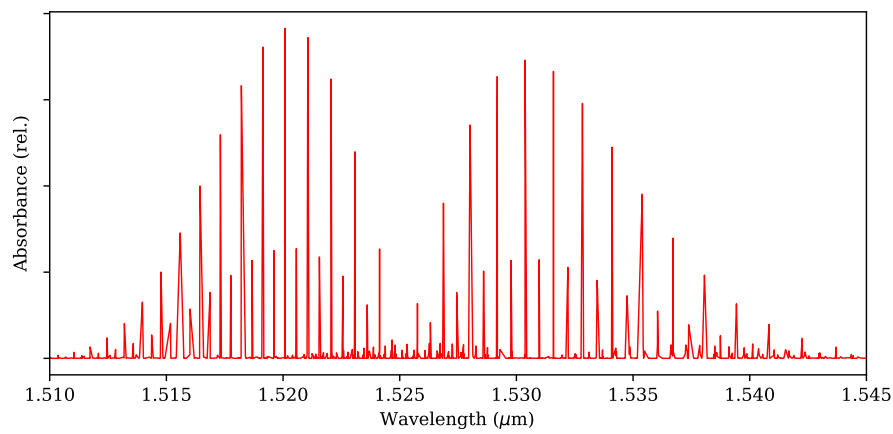


FIGURE 6.1: Absorption data for acetylene gas in the near infrared region. Data from [170].

Measurement of Interacted Light

The result of the interaction with the reference cell, is that for a single modulation period, an absorption feature within the wavelength range $\lambda_c \pm \delta\lambda$, where ε increases in value, will cause significant dips in the measured intensity. For each n th absorption feature with wavelength λ_n , the wavelength modulation sweeps past λ_n twice within a single modulation period. This causes two dips in the measured intensity seen in the photodetector signal, indicative of the forward and reverse sweep of the wavelength modulation over the absorption line at λ_n . An example of this is shown in Fig. 6.2 for a single sweep through an acetylene gas cell ($^{12}\text{C}_2\text{H}_2$).

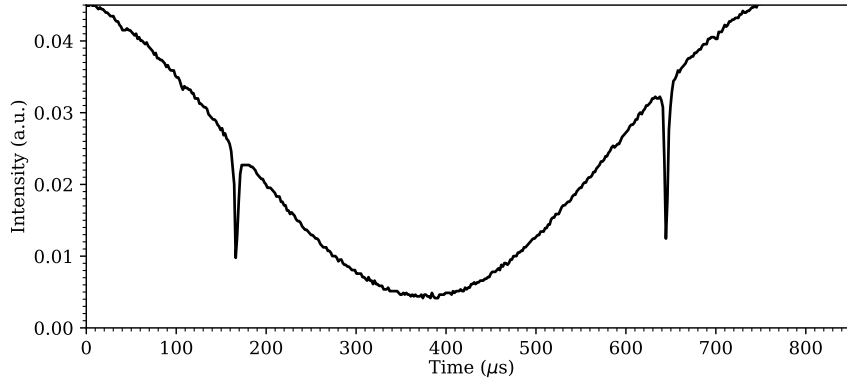


FIGURE 6.2: Photodetector reading for a single modulation period of a laser sinusoidally modulated through an acetylene gas cell.

Signal Analysis

As discussed earlier, whilst the intensity modulation of the laser is directly visible in the photodetector signal, the wavelength modulation is not and has a phase offset from the intensity modulation. Notably, and visible in Fig. 6.2, the absorbance features are skewed by the intensity modulation. Furthermore, the presence of higher-order harmonics present in both the intensity and wavelength waveforms, discussed in Chp. 4 will further skew the centre of the waveform. Finally, locking to a location on the intensity modulation sweep assumes that the frequency response of the acquisition system is perfectly flat across all frequencies, in order to perfectly resolve both the low frequency intensity sine, as well as the high frequency absorption line. If this frequency response was not flat, or any drift occurred, the locking would no longer be correct, but it would not be visible in the measured data.

As a result, simply locking the absorption line to a fixed location on the visible modulation sweep is inaccurate. Instead in the method presented here, the intensity modulation is first corrected using a fitted waveform of the same period and shape as the laser diode current modulation. This is achieved by application of the Beer-Lambert law re-arranged for absorbance, A , using the fitted waveform as the incident intensity I_0 and the photodetector signal as the transmitted intensity I , shown in eqn. 6.5.

$$A(t) = \log_{10} \frac{I_0(t)}{I(t)} \quad (6.5)$$

For particularly strong absorption, i.e. high values of A , the absorption troughs cause significant skewing to the data. To accurately fit a sine wave to the measured intensity, the sampled data on and around the absorption troughs must be ignored as part of the fit, or given weights in the fitting significantly lower than the rest of the data. To achieve this, the first order derivative of the data is calculated, and a threshold selected. For the peaks above this threshold, a region either side is selected and omitted from the original data set. With the modified data set, a least-squares optimisation is performed of a model sinusoidal function, to calculate the best fit model. Fig. 6.3 demonstrates

these steps for an example photodetector signal passing through an acetylene cell and wavelength modulated over a single dominant gas line.

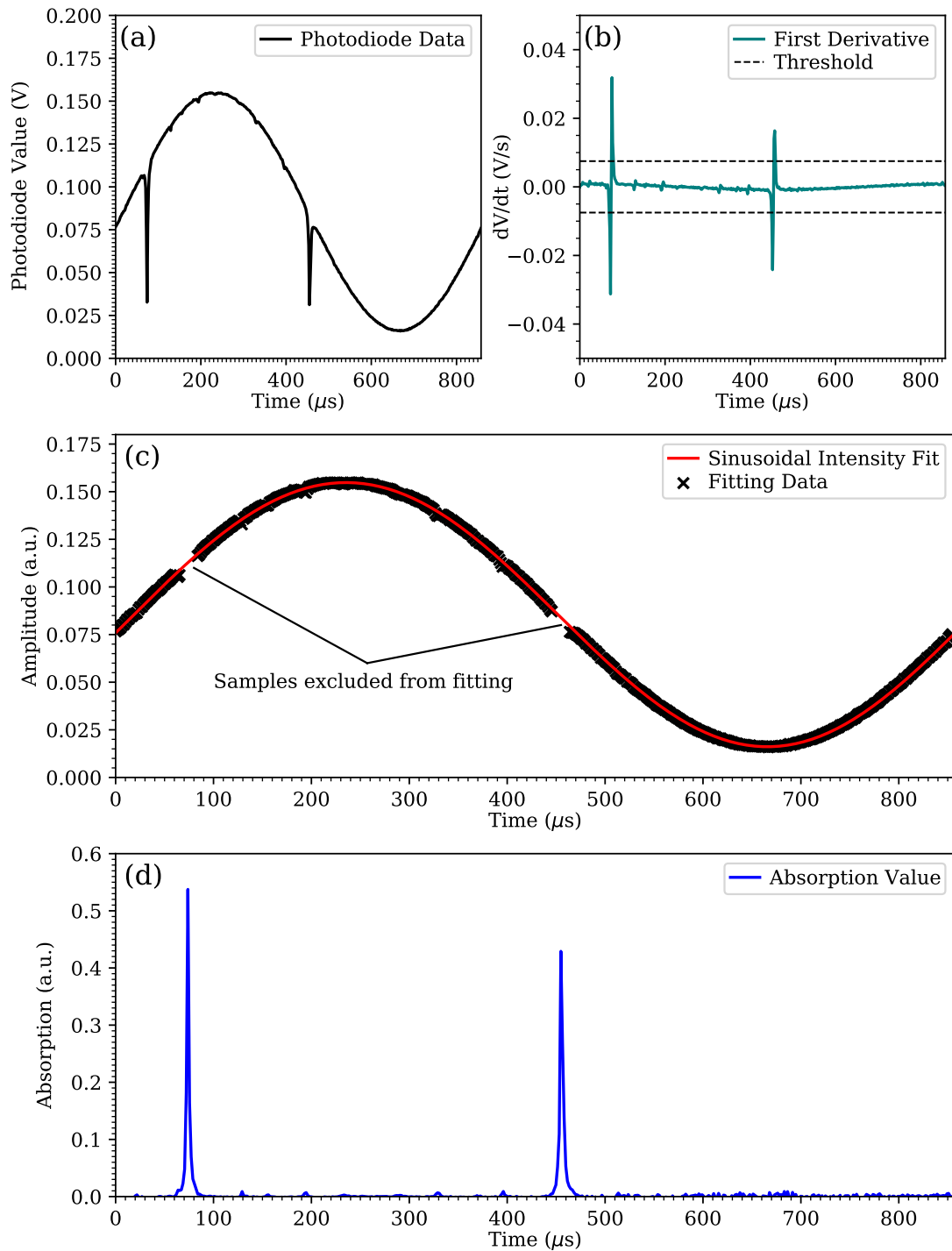


FIGURE 6.3: Process of calculating the absorption for a laser diode emission wavelength modulated across a gas line showing; (a) Photodetector signal (b) First derivative of the photodiode signal with selected thresholds (c) Photodiode data after selections of data around the threshold crossings are removed and sinusoidal fit (d) Calculated absorption

Fig. 6.3(a) shows a raw photodetector signal for a laser diode modulated through a gas cell with an absorption line within the wavelength modulation range. The first derivative is shown in Fig. 6.3(b), as well as the manually selected thresholds. Where the derivative crosses these thresholds, a copy of the sample data used for fitting is removed and the rest of this is shown in Fig. 6.3(c) along with the best fit sine to the remaining data. The fitted sine is then used as the incident intensity I_0 in eqn. 6.5 and the resultant absorbance spectrum is shown in Fig. 6.3(d). This consists of two clear peaks, per gas line, on an approximately flat baseline. Using standard peak fitting techniques such as the Levenberg-Marquardt algorithm [171, 172], a Voigt or Lorentzian function, dependant on the pressure of the gas cell, can be fit to both peaks and the time relative to the start of the modulation period at which the absorbance peaks occur can be found.

Wavelength Tuning

Through tuning of the laser junction temperature, the centre wavelength of the laser emission can be changed following an approximately linear relationship, shifting the entire sinusoidal wavelength modulation. In the photodetector signal, this results in the pair of absorption peaks shifting either towards, or away from, the turning point of the modulated intensity signal. This concept is illustrated in Fig. 6.4 where experimental data of a 1521 nm laser diode being sinusoidally modulated through an acetylene ($^{12}\text{C}_2\text{H}_2$) cell is shown for a range of laser temperature setpoints.

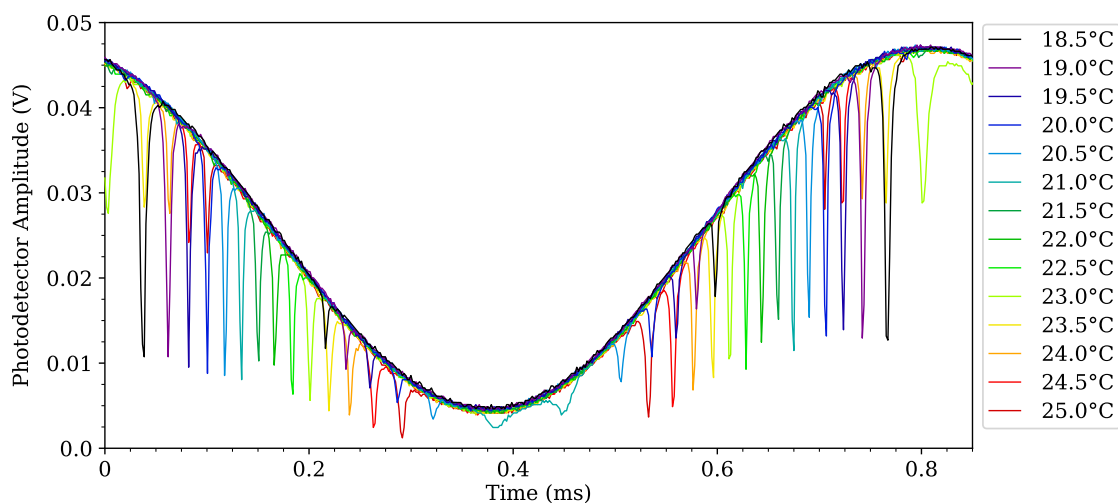


FIGURE 6.4: Photodetector signals for a sinusoidally modulated laser passing through a sealed gas cell over a single gas absorption line for a range of laser temperature setpoints.

Fig. 6.4 shows a series of photodetector signals for a single sinusoidal modulation period through an acetylene gas cell. As the laser emission wavelength sweeps past the absorption wavelength of a gas line, an absorption trough is seen in the photodetector signal in both the forward and reverse sweeps of the laser. If multiple gas lines are within the laser wavelength modulation range, multiple pairs of gas lines will be visible within a single laser sweep. Here it can be seen how tuning the laser emission wavelength, by

changing the temperature setpoint of the laser TEC, alters the position of the absorption troughs on the intensity modulation signal, shifting the absorption troughs "up" or "down" the modulation waveform. As a direct result of this, for a given absorption line there is a wavelength, and hence temperature setpoint at which the time period between the forward and reverse sweeps over the gas line is exactly equal to half the time period of the laser modulation. To a first order, at this temperature, the central wavelength of the laser modulation is exactly equal to the absorbance line wavelength, however in the presence of higher order harmonics in the wavelength modulation there will be a small offset in the absolute centre wavelength. As the harmonics have a fixed amplitude and phase, this offset will remain static throughout, therefore for explanation purposes it will be neglected for now.

By comparing the difference in the times at which the two peaks occur, τ_δ to half the modulation period $\tau_{1/2}$, an error value can be calculated and a temperature setpoint adjustment can be applied. The error amplitude $\tau_{1/2} - \tau_\delta$ is a bidirectional value, and can be positive or negative as necessary, which makes it suitable for use in a control circuit as the error amplitude signal. This process, and the resultant absorbance for two different temperature set points are shown in Fig. 6.5, demonstrating conditions where the central emission wavelength is offset from the absorption line, and where the central emission wavelength is locked to the absorption line. Here, the fitting results for both the intensity modulation sine waveform and the Voigt fits of the absorption peaks are also shown.

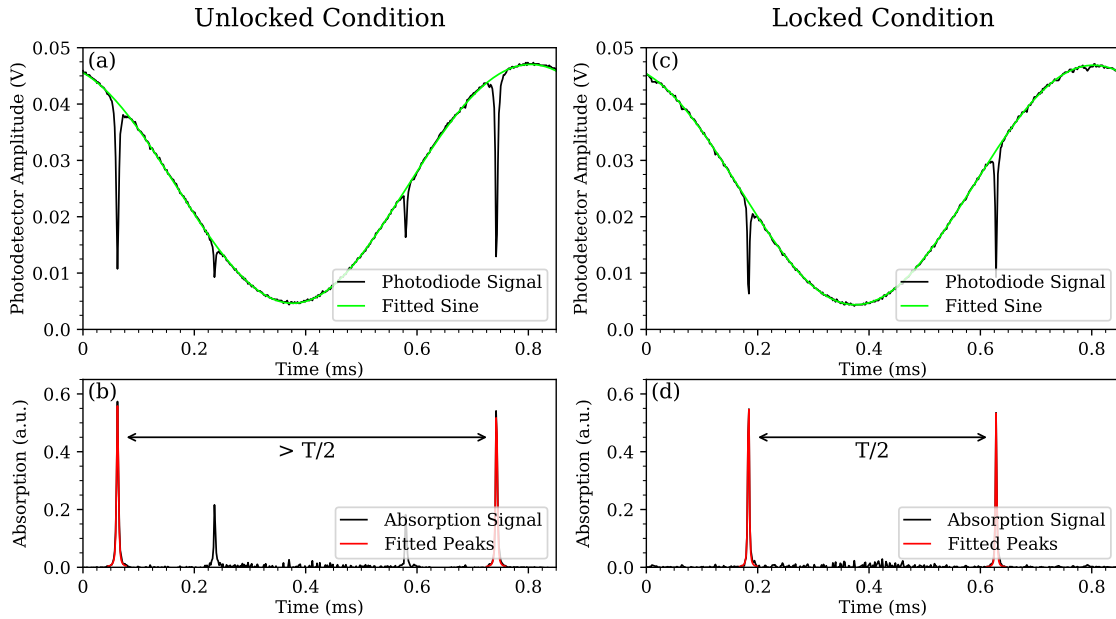


FIGURE 6.5: (a) Photodetector signal for a wavelength modulated laser diode passing through a gas cell with fitted intensity correction in the case where the stabilisation system is unlocked (b) Resultant absorption signal for the unlocked condition showing Voigt fits applied to the relevant peaks (c) Photodetector signal for a wavelength modulated laser diode passing through a gas cell with fitted intensity correction in the case where the stabilisation system is locked (d) Resultant absorption signal for the locked condition showing Voigt fits applied to the relevant peaks

Fig. 6.5 demonstrates two examples of the locking process using photodiode data from a wavelength modulated 1520 nm laser diode passed through an acetylene gas cell with a pair of absorption peaks for each single absorption line within the range of the wavelength modulation. These occur at visibly different "heights" in the recorded intensity modulation, due to the offset, previously mentioned, between the intensity modulation and the wavelength modulation.

Figs. 6.5(a) and (b) show an unlocked example, where the temporal separation of the two peaks is greater than half the laser modulation period. In this case, a temperature setpoint correction would be applied to bring the peaks closer together. In addition, a second pair of peaks with a lower amplitude can be seen, indicating that the laser emission wavelength is being modulated across a second gas absorption line wavelength. In the locked case shown in Figs. 6.5(c) and (d) the temporal separation is exactly half the laser modulation period so no temperature correction needs to be applied to the laser diode. Notably, in the uncorrected signals shown in Figs. 6.5(a) and (c), the position of the peaks appear at different "heights" in the intensity modulation in the forward and reverse sweeps. This is a result of the phase offset between the intensity modulation which is the visible sine wave in the photodiode waveform, and the wavelength modulation which is not visible, discussed earlier. Once the sinusoidal correction has been applied, as in Figs. 6.5(b) and (d), the intensity modulation becomes irrelevant, and only the wavelength modulation defines the peak positions.

To determine the temperature setpoint change required to adjust the central laser emission wavelength to equal the absorption line wavelength, a secondary control loop, independent of the laser TEC control loop, is applied to the output of the process shown in Fig. 6.5. Using a standard proportional-integral controller, the temperature setpoint is adjusted in discrete steps. In order to avoid competition with the laser TEC which operates at a bandwidth of 20 Hz, this stability control loop operates on a much slower time-scale of 0.05 Hz which is appropriate for the intended application of long-term stabilisation. In this technique, the photodiode data is averaged over a full 10 seconds of the laser modulation and the peak-difference algorithm is applied to the result. In order to avoid averaging data where no change has yet occurred due to stability control loop delay or where the laser TEC is updating the temperature of the laser, alternate outputs from the algorithm are disregarded and no temperature setpoint update is sent to the laser diode, illustrated in Fig. 6.6. Whilst this effectively doubles the time period of the stabilisation control loop, it also ensures that the stabilisation control loop is only receiving 'true' values to make adjustments, and avoids oscillations due to competition between the two control loops.

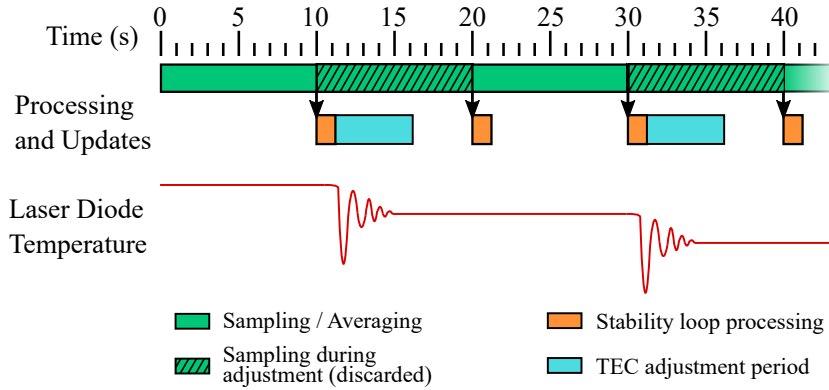


FIGURE 6.6: A demonstration of the crossover between sampling and the setpoint adjustment period.

Fig. 6.6 illustrates how the processing and temperature setpoint adjustment time is accommodated in the stabilisation system. Notably, alternate processed outputs are dismissed with no setpoint adjustment where the sampled data overlapped with setpoint adjustments.

6.3 Experimental Work

This section presents work developing an experimental setup for research and testing into the novel swept absorption line locking technique presented in this chapter. Firstly, the principle behind these experiments is presented, followed by explanation of the practical setup. Following this, the specifics of this implementation of the swept absorption line locking technique are presented.

6.3.1 Experimental Principles

The typical procedure for demonstrating stability of a laser emission wavelength or frequency stabilisation scheme is to interfere the laser emission locked using the stability technique under test, with the emission from a highly-stabilised and well calibrated laser source. In this case however, lasers with a wide wavelength modulation regime such as those targeted by this stabilisation technique, cannot be easily compared directly with a static wavelength emission source, as the modulation frequency of the modulated laser diode dominates the beat note frequency spectrum. Instead, in these experiments, an interferometric setup is interrogated and the phase change as measured by the interferometer is used as a measure of stability.

In ideal conditions, the only contribution to phase change θ is the change in emission wavelength, however in practice, environmental effects such as thermal drift and thermal expansion of components will also contribute to measured optical phase changes. To reduce the effect of these in the following experiments, the interferometer is placed in a temperature controlled environment and steps are taken to reduce any other environmental effects on the interferometer. The assumption is then made that any remaining measured phase changes are strictly a result of the laser emission wavelength drifting.

With these assumptions, the effective interrogation wavelength λ_{eff} of the modulated laser diode, for an optical path difference can be calculated using Eqn. 6.6 below.

$$\lambda_{eff} = \lambda_0 \left(1 + \frac{\theta}{2\pi n_0} \right) \quad \text{where} \quad n_0 = \frac{L}{\lambda_0} \quad (6.6)$$

Here, λ_0 is the wavelength of the stability reference, or the expected laser emission wavelength. It is typically convenient, as discussed in Chp. 3, to present the uncertainty in wavelength and frequency in terms of fractional uncertainties, which can be calculated as;

$$\sigma_{frac} = \frac{\lambda_{eff} - \lambda_0}{\lambda_0} \quad (6.7)$$

6.3.2 Experimental Setup

In order to test the stabilisation technique, a setup was devised that uses range-resolved interferometry (RRI) to interrogate a simple Mach-Zehnder interferometer with a nominal path length. This is achieved using the 1.2 kHz RRI Stabilisation Development Platform discussed in Chp. 4, Sec. 4.3 which allows for flexible application of the RRI signal processing and swept absorption line locking technique, synchronised to one another and was developed explicitly for this work.

The optical path length of the interferometer is designed such that the range in radians, used to determine the phase carrier amplitude in RRI, is approximately in the middle of the measurement range of the signal processing hardware, where effects from the lower limit 1/f noise and the upper limit Nyquist frequency/filter roll-off are minimal. For implementation 3, the maximum measurable optical path difference is approximately 130 mm allowing for some roll-off due to low-pass filtering, so for this experimental setup, the interferometer is created with an optical path difference of 60 mm.

From the laser diode, the light is split into two separate paths, one arm of which is passed through a fibre-coupled acetylene gas cell (Wavelength References $^{12}\text{C}_2\text{H}_2$ at 50 Torr with 3 cm path length [173]) and is incident on a photodetector. The other arm is guided through the fibre-based Mach-Zehnder interferometer and is incident on a separate photodetector. The outputs of both photodetectors are sampled by electronic signal processing hardware which also controls the laser modulation and TEC temperature setpoint.

To reduce the effect of thermal instability between the interferometer arms, which as mentioned would also contribute to measured phase changes, both arms of the interferometer are fixed within an ice bath. In addition, two water pumps are used to create circulation in the ice bath, and the entire ice bath is placed within a thermal chamber. The thermal chamber is a Votsch VTM7004, with a temperature range of -70° to $+180^\circ$ with both heating and cooling capabilities, and a temperature uncertainty of up to 0.5°C . This setup is shown in Fig. 6.7.

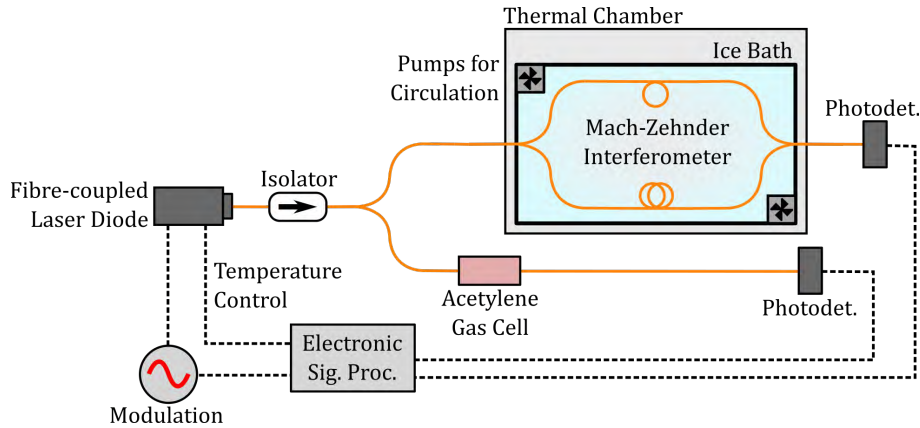


FIGURE 6.7: A schematic of the setup used in these experiments. The interferometer arms within the ice bath are affixed to an aluminium plate which is not shown.

Fig. 6.7 shows a schematic of the experimental setup used in this paper. Notably, the sensitive region of the interferometer is affixed entirely in the ice bath. The thermal chamber temperature is set as close to the water freezing point as possible so that fluctuations of the thermal chamber temperature, due to the heating and cooling cycles of the thermal chamber control loop, fluctuate either side of the freezing point. Here the goal is that the temperature of the ice bath remains as stable as possible, making use of the latent heat of the phase change between water and ice to reduce the effect of fluctuations of the thermal chamber. The efficacy of this approach is discussed in sec. 6.4.2, where experimental data is shown. The water pumps in the ice bath create circulation of the water, which helps to homogenize the temperature of the water throughout, reducing any temperature differential between the two arms of the interferometer. This is aided by an aluminium plate, with high thermal conductivity, to which the interferometer within the ice bath is affixed, to reduce any motion effects of the water on the interferometric measurements.

6.3.3 Wavelength Stabilisation

In this application of the swept absorption line locking technique, a single-mode fibre-coupled acetylene gas cell ($^{12}\text{C}_2\text{H}_2$) [173] is used. Acetylene has multiple absorption lines in the R and P branches, shown in Fig. 6.1 and Fig. 6.8 below, with several lines present in the wavelength emission range of the modulated laser diode used in this experiment.

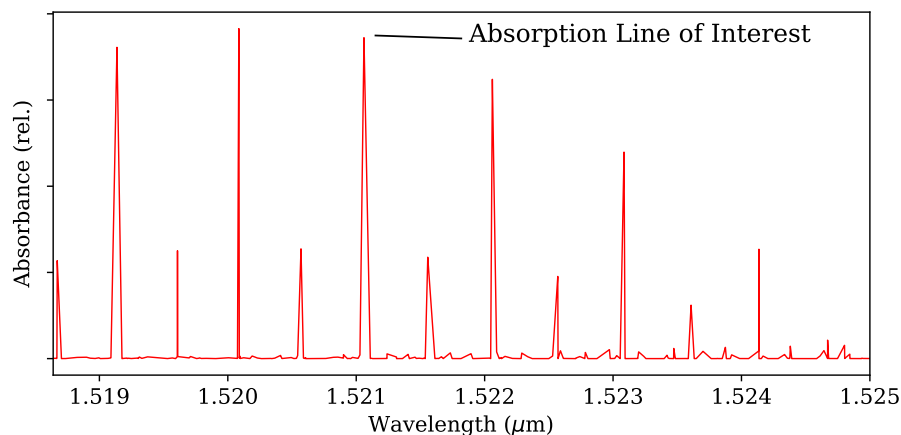


FIGURE 6.8: Absorption data for acetylene gas in the near infrared region with the absorption line of interest labelled. Data from [170].

Although multiple absorption lines can be swept past through wide temperature tuning of the laser diode the absorption line of interest, illustrated in Fig. 6.8, is at 1521.06 nm (vac). The application of the swept absorption line locking to this peak is shown below in Fig. 6.9.

Fig. 6.9 shows the swept absorption line locking technique applied to the acetylene gas absorption line at 1521.06 nm. In Fig. 6.9(a), the averaged photodiode data is shown, with two clear dips due to absorption, where the wavelength modulation of the laser sweeps over the absorption line twice, on the forward and reverse sweeps of the wavelength emission. Also shown is the fitted sinusoid corresponding to the intensity modulation. Fig. 6.9(b) shows this data with the intensity fitted sinusoidal waveform subtracted, and the values reciprocated to show absorption. After this, peaks are fitted to both absorption peaks using standard peak fitting techniques, and the period between them is calculated from the position of the peak centres, in time.

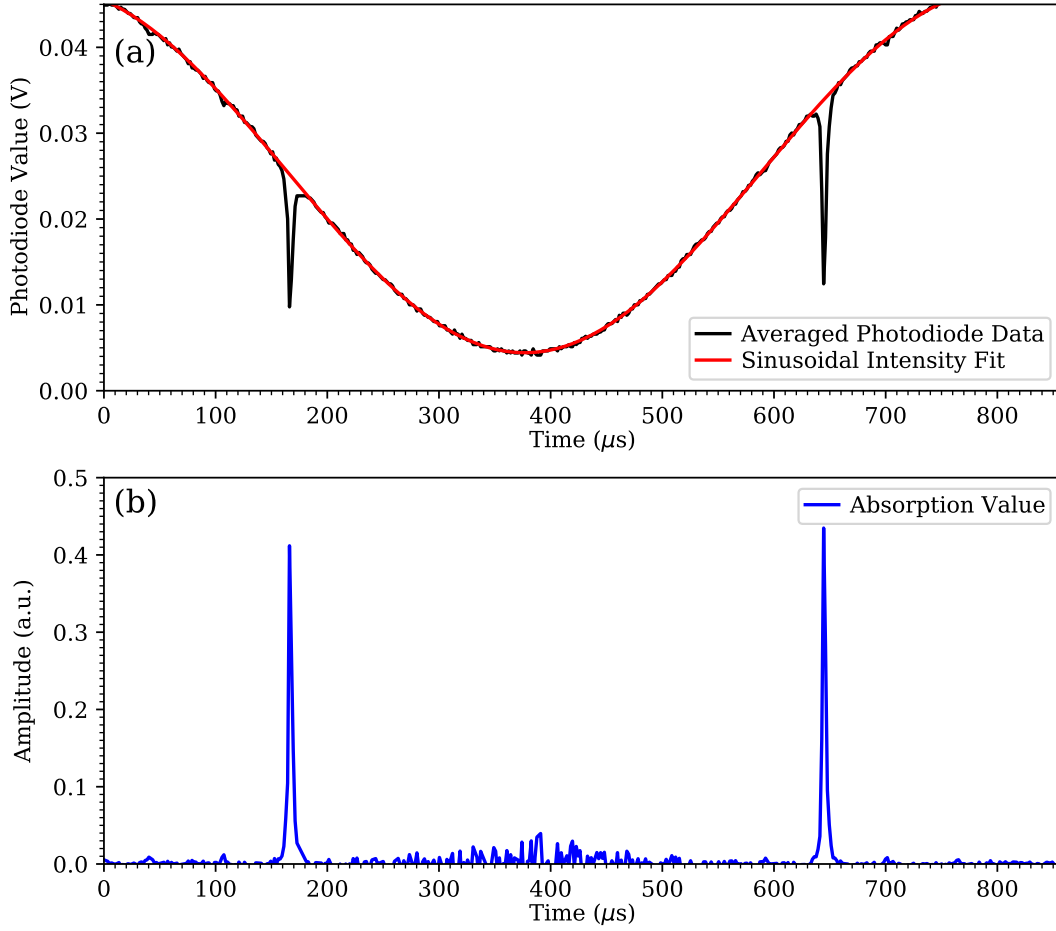


FIGURE 6.9: Swept absorption line locking process. (a) shows the averaged photodiode data and the sinusoidal intensity fit. (b) shows the absorption after the intensity modulation has been removed.

6.4 Results

6.4.1 Control Loop Tuning

The stabilisation control loop is implemented digitally in the signal processing with the signals from the photodetector directly input via the analogue-to-digital converter (ADC). After the signals have been analysed using the technique described above, the required temperature setpoint change is passed as input to the P-I control loop. The temperature setpoint output from this control loop is encoded and sent as a serial command to the laser TEC.

As part of the control loop design, interactions between the P-I stabilisation loop and the laser TEC control loop have to be carefully managed to avoid accidental feedback loops between the two. The in-built TEC control loop is designed to maintain the laser temperature at the given setpoint with a control loop rate of 20 Hz, thus the stabilisation loop should operate at a significantly slower rate to avoid conflict. As the intended

application of this control loop is long-term stabilisation as discussed in Sec. 6.1 where the expected disturbances occur over periods of several hours, an update rate of 20s or 0.05 Hz is more than reasonable. Whilst the PID control loop tuning of the TEC control loop can be automatically calibrated using the in-built function, the control loop parameters of the stabilisation loop require additional consideration. To investigate the ideal parameters, an experiment was performed where the TEC temperature setpoint is offset from the "locked" temperature by 1°C . For a given set of parameters the control loop was allowed to re-establish the "locked" condition using temperature setpoint updates, and the phase change seen in the interferometer are recorded over a period of five minutes, shown for various tuning parameters in Fig. 6.10. Here, the parameters of interest are a general scale factor A , and the two control loop parameters corresponding to the proportional factor P , and the integral factor I . The test parameters were decided by trial-and-error, where subsequent trials were performed until two key criteria were met. Firstly, the locking condition should be met within the five minute recording period, and secondly there should be minimal overshoot of the locking condition.

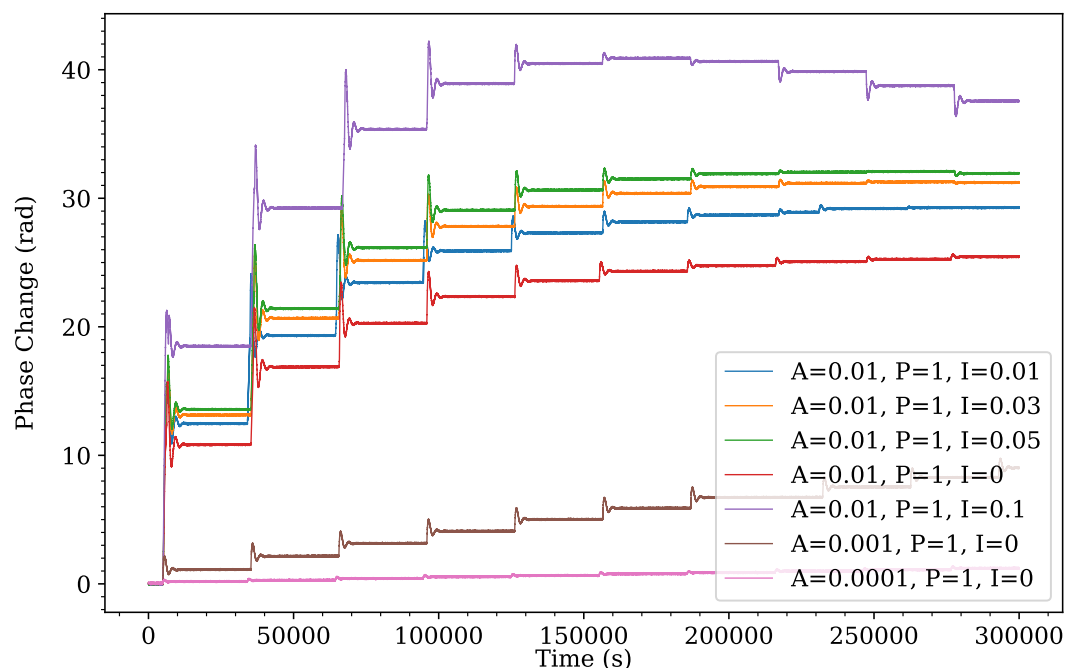


FIGURE 6.10: Measured phase changes during control loops as it re-establishes the locked condition, with a range of control loop parameters.

Fig. 6.10 shows measured phase change measurements for a range of control loop parameters. There are evident phase steps which occur at the 20 s control loop period. On a shorter time scale, peaks and troughs corresponding to the TEC control loop are visible. Over the 5 minute recorded timescale, none of the parameter sets with a value of A less than 0.01 reaches the target setpoint temperature. With a value of $I = 0.1$, the stabilisation loop vastly overshoots the locking point, and given a longer recorded timescale, would likely fluctuate wildly between undershooting and overshooting the target setpoint. The three most successful parameters are those with I values between 0.01 and 0.05. Here,

the stabilisation loop rapidly approaches the target setpoint without significant overshoot and remains locked at the target setpoint after reaching it. For the purposes of locking to and remaining fixed at a target setpoint, parameter values of $P = 1$ and $I = 0.01$ seemed appropriate, and were used in the following experimental work.

6.4.2 Temperature Drift Compensation

In order to isolate the effect of wavelength drift from other environmental effects that similarly alter the apparent optical path difference between the interferometer arms, and hence cause a phase change, as discussed in sec. 6.3.2 the interferometer is fixed on a platform within a thermal chamber. To verify the impact of environmental effects on the interferometer, experiments were performed of extended phase measurements for a series of conditions. Here, the effect of open exposure of the interferometer to air on the measured phase is compared to the effect when the interferometer is submerged in a water tank at a set temperature, and the effect when the interferometer is submerged in the water tank at a set temperature with the addition of small ice chips. To accurately reflect the expected conditions during a measurement, the stabilisation loop is also running. The results of phase measurements of the interferometers in these conditions are recorded over a time period of one hour and are shown in fig. 6.11.

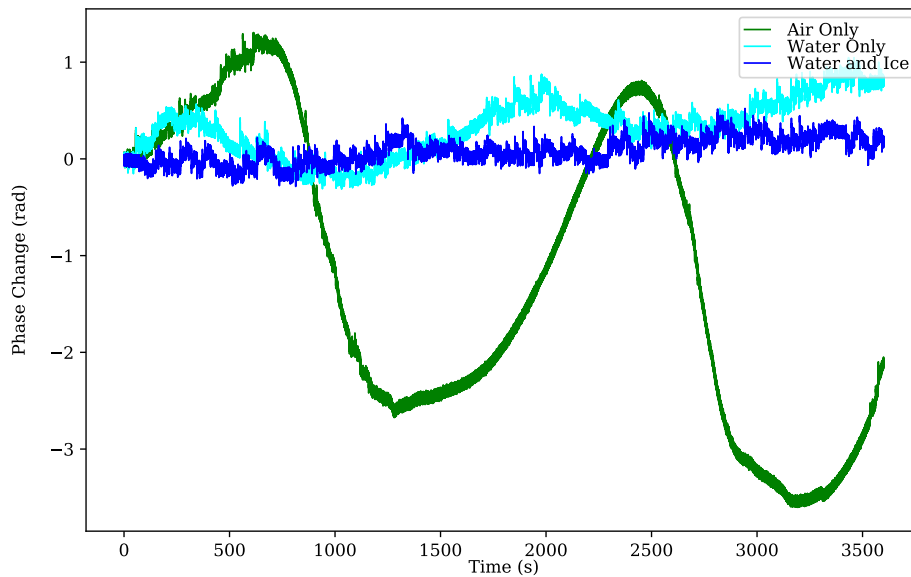


FIGURE 6.11: Phase change measurements of a Mach-Zehnder interferometer, secured to a aluminium plate, in varying conditions. These are; exposed to air, submerged in water and submerged in water with the addition of ice chips.

Fig. 6.11 shows how the three different environmental states directly contribute to phase changes of the interferometer. For the open-to-air conditions, there is a large, continuous phase change of ± 2 rad over the 60 minute recorded period occurring with a periodicity of ~ 30 minutes. For the in-water conditions, there is a similar periodic phase change that occurs with a significantly lower amplitude of ± 0.5 rad over the 60 minute recorded

period. This periodic change is consistent with the heating and cooling cycles of the thermal chamber, creating a temperature differential between the two interferometer arms, causing a change in the effective optical path difference between them and hence a phase change in the interferometric measurements.

In the case of the interferometer submerged in water, the reduced amplitude of these oscillations is caused by the higher thermal heat capacity of the water acting as an insulator to the interferometer, so the heating/cooling cycles of the thermal chamber have a significantly reduced effect on the phase change between the two interferometer arms. In the environment with both water and ice chips, the water continues to act as a thermal attenuator with the additional buffer of the specific latent heat of the ice. Theoretically, a finite amount of the energy added to the system as heat goes towards breaking the molecular bonds of the ice, which occurs without changing the temperature of the ice. This creates a window of temperatures within the heating/cooling cycle of the thermal chamber where a stable temperature of the water and ice, and by proxy the interferometer, can be achieved. With less temperature differentials created between the two arms of the interferometer, the cyclic phase changes seen in both the in-air and in-water environments are significantly reduced in the environments where ice is included in the water. For the case with added ice, the standard deviation of the measured phase changes is ± 0.128 rad over the 60 mm optical path length imbalance between the two interferometer arms, which corresponds to an uncertainty in the emission wavelength of 7.8×10^{-4} nm which is a fractional uncertainty of 5.1×10^{-7} or 0.51 parts-per-million.

In theory, this presents water with added ice chips as the optimal conditions, easily achievable in the lab, to use for the interferometer to create as stable an environment as possible for laser stability measurements, and these are the conditions used in the following experimental work. However, in practise achieving the setpoint and control loop settings necessary to maintain the thermal chamber temperature average at the exact freezing point of the water, such that the thermal energy is continually acting towards the specific latent heat of the water, is challenging and could be considered unreliable as it can easily be disrupted by the thermal chamber control loop and temperature feedback, losing the ideal conditions. For this reason, for future experimental work, the environmental state with just water, or water close to the freezing point may be considered more practical for long-term stability measurements.

6.4.3 Long term drift comparison

In this experiment, measurements of the phase change in the interferometer over an extended period are compared for cases with (locked) and without (free-running) the control loop acting on the temperature setpoint of the laser diode. In both cases, the stability of the interferometer itself is maintained using the setup described above, with the interferometer maintained in a water bath at freezing point with ice added. The measured phase changes for both cases are shown below in Fig. 6.12.

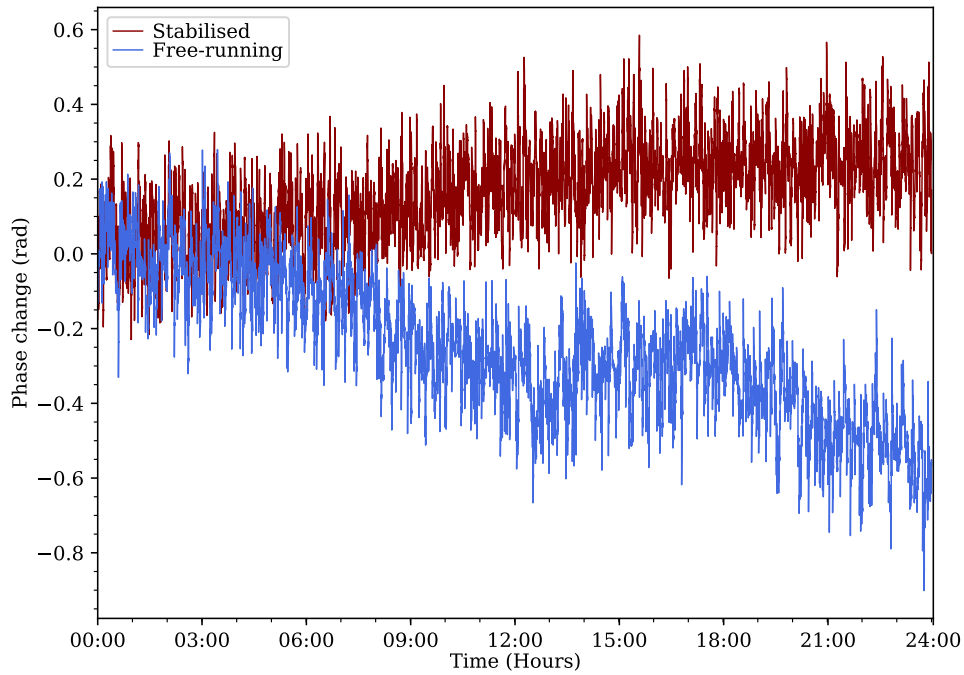


FIGURE 6.12: Measured phase changes of a Mach-Zehnder interferometer without (free-running) and with (stabilised) the swept absorption line locking applied.

Fig. 6.12 illustrates how over extended time periods, the laser emission wavelength can 'drift' from the initial value, thus causing a measurable phase change in a stable interferometer. In the stabilised measurement, the phase changes drift by approximately 0.17 rad over the 24 hour measurement period, and in the free-running conditions drift of approximately 0.6 rad is seen. The 0.17 rad drift seen in the stabilised case, and theoretically also affecting the free running case by the same amplitude, could be caused by residual temperature drift within the thermal chamber, as discussed in Sec. 6.4.2. Over 24 hours, any variations due to the stability loop are indistinguishable from any residual temperature changes however, a clear improvement in the stabilised conditions over the free-running conditions is visible. From the drift in the stabilised case of 0.17 rad, the fractional wavelength uncertainty can be calculated as $\sim 7.2 \times 10^{-7}$ or 0.72 parts-per-million.

6.4.4 Direct disturbance measurements

In a different experiment using the same experimental setup with the ice bath and temperature chamber, a direct disturbance was applied to the laser diode. After a short period of normal operation, a temperature gradient was applied across the laser diode via a radiant heat source, such that the emission wavelength of the laser diode was slowly altered. The exact mechanism for this is a temperature differential applied between the laser diode itself and the sensing thermistor, which is separated by a finite distance from the laser diode. The thermistor measured by the laser TEC therefore reports the laser diode is at the temperature setpoint, whereas in reality it is hotter.

The heat source, in this case a large resistor driven by a high voltage source, provides a slow continuous disturbance to the laser diode but is not so hot as to damage the laser diode, and allows the laser TEC to maintain the temperature setpoint of the laser diode, as measured by the laser TEC. Essentially, the heat source is a method for altering the emission wavelength of the laser in a pseudo-controlled manner, without requiring changing of the temperature setpoint of the laser diode. This allows experiments such as this, where the swept absorption line locking technique is applied to maintain the same wavelength emission through temperature tuning.

After a period of 15 mins, the high-voltage power supply to the large resistor is switched on and slowly heats up over a period of 1 hour. After this time, the power supply is switched off and the resistor begins to cool. This is repeated for both a free-running and stabilised laser diode, and Fig. 6.13 shows the measured phase changes in the Mach-Zehnder interferometer over the total measurement time and the laser TEC setpoint temperatures during this period.

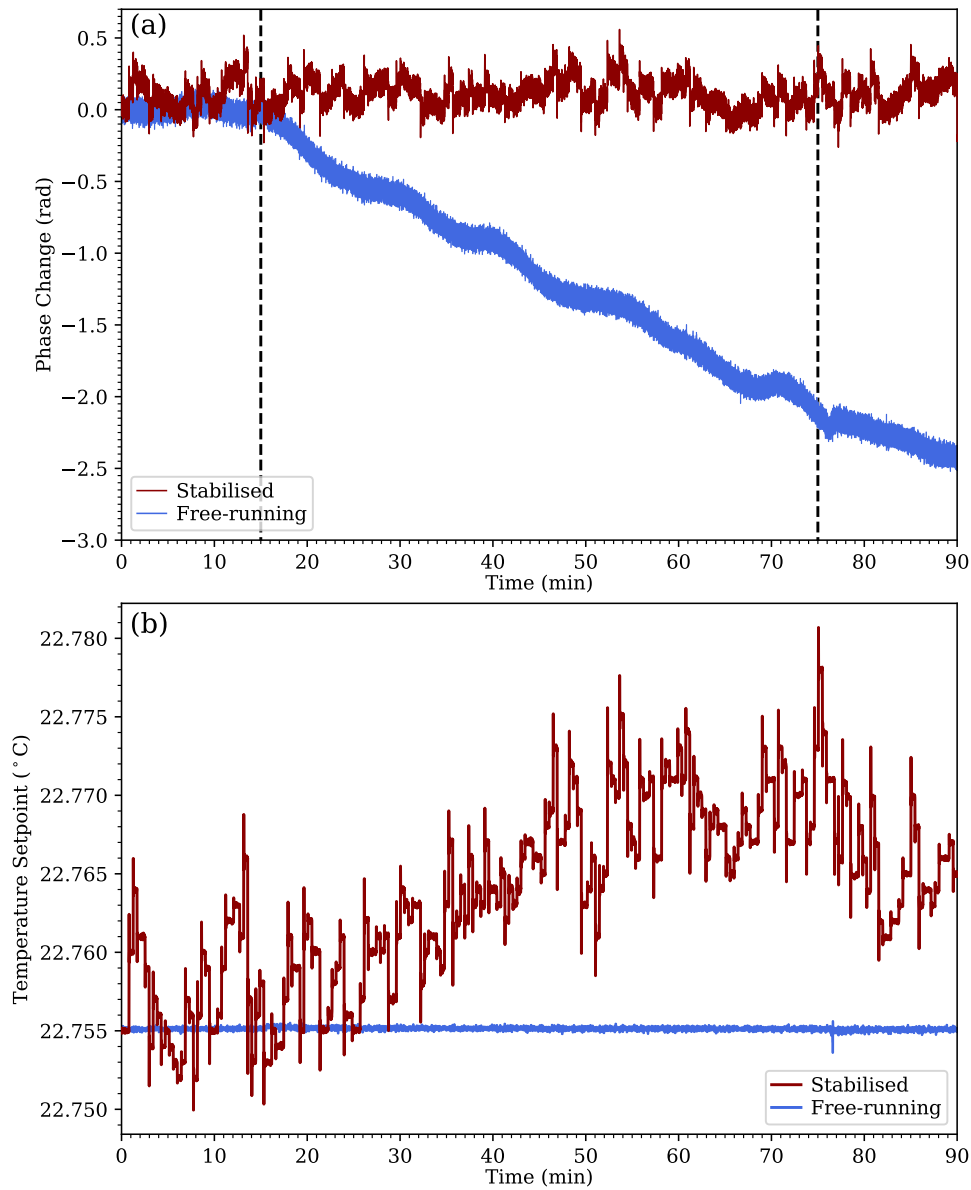


FIGURE 6.13: Direct disturbance of a laser diode by a continuous, radiant heat source between the indicated 15 and 75 minute marker showing (a) the phase measurement of a Mach-Zehnder interferometer (b) the thermistor reading from the laser TEC.

From the phase change measurements shown in Fig. 6.13(a), there is a clear phase drift in the free-running case of up to 2.14 rad over the 1 hour period of the disturbance being applied, and 2.5 rad over the full measurement period, caused by the temperature gradient across the laser diode. In the stabilised case, while there is no visible drift, the discreet temperature setpoint changes caused by the stabilisation loop are evident, causing an uncertainty of up to ± 0.096 rad over the total measurement period corresponding

to a fractional uncertainty of 3.9×10^{-7} . Fig. 6.13(b) shows how the temperature of the thermistor as measured by the laser TEC changes over the measurement periods of the stabilised and free-running disturbance measurements. Notably, in the free-running case the temperature of the thermistor is largely unchanged, whereas in the stabilised case, the thermistor registers the change in temperature setpoint that the stabilisation loop has enforced to counter-act the change in wavelength caused by the thermal gradient.

From these results it is also clear that the stabilisation control loop is still too active at 20 Hz, and this also introduces uncertainty into the measurements. Better tuning of this control loop could improve this, however the problems of temperature quantification and a limited sampling resolution, discussed later in this chapter, mean that there is not a simple solution to this.

6.4.5 Allan Deviation

A common method of defining the stability of a wavelength or frequency stabilisation technique is to calculate the Allan deviations [174, 175] over a range of integration times τ for a stabilised system with some uncertainty value. For frequency or wavelength stability, typically this is achieved via measurement of the frequency/wavelength of a beat note created by interfering the emission of the laser of interest with a different highly-stabilised system with well-known parameters [145]. Here, the system is instead self-interfered in the Mach-Zehnder interferometric setup discussed previously, and the stability is derived from the Allan deviation for the phase data shown in Fig. 6.12. The fractional frequency stabilisation is calculated for both the stabilised and free-running conditions, from the phase data and Eqn. 6.7. Fig. 6.14 shows calculated overlapping Allan deviations [176] for both conditions over an extended measurement period of 24 hours.

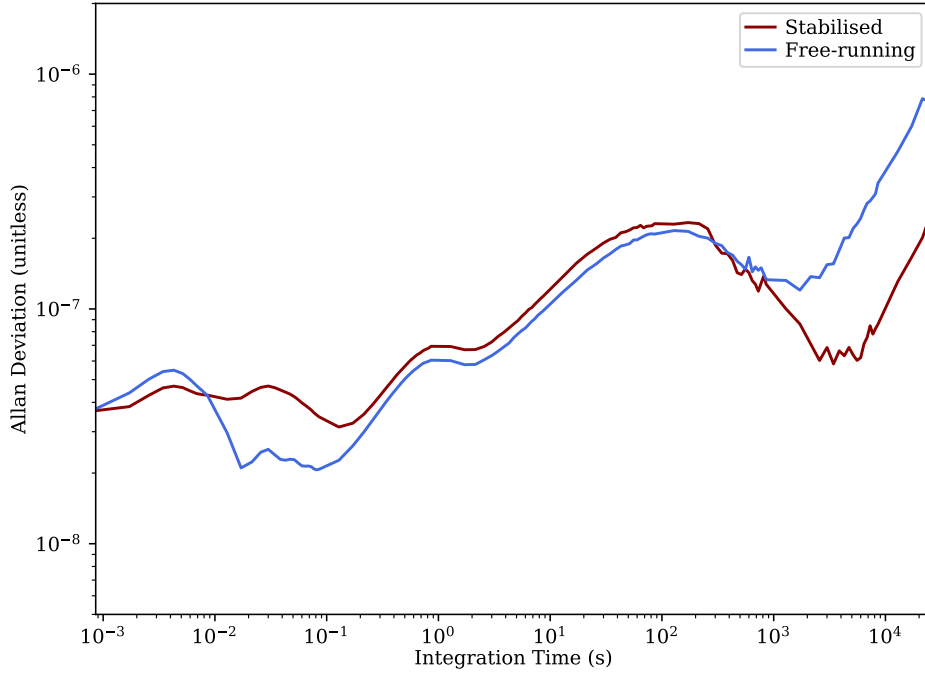


FIGURE 6.14: Allan deviations for a range of integration times for stabilised and free-running conditions.

From the overlapping Allan deviations shown in Fig. 6.14, two significant differences can be identified between the stabilised and free-running conditions. At short integration times, most notably between ~ 0.01 s and ~ 0.2 s, the free-running conditions shows improved uncertainty compared to the stabilised conditions, with an approximate difference between them of up to $\sim 3 \times 10^{-8}$. This increase in Allen deviation in the stabilised measurement for these integration times is likely a result of the changes to setpoint temperature prompted by the stability control loop. As the setpoint temperature is changed, the laser TEC enforces the new setpoint with an update rate of 20 Hz. The stabilised control loop will prompt changes of a higher amplitude than the laser TEC in the free-running conditions, which can often cause brief overshoots and fluctuations in the laser temperature on timescales of fractions of a second, illustrated by the measurement shown in Fig. 6.10.

The second significant difference occurs for integration timescales of $\sim 10^3$ s, where the measurement in stabilised conditions show improved stability with a difference of up to $\sim 5 \times 10^{-8}$. This is the result of the reduced long term drift of the laser emission wavelength that this technique aims to address, visible in Fig. 6.12.

In terms of the individual performance of the stabilisation loop, these results demonstrate better than 0.1 parts-per-million uncertainty, with fractional wavelength uncertainty as low as $\sim 5 \times 10^{-8}$ for integration times between $\sim 2 \times 10^3 < \tau < \sim 6 \times 10^3$. For a lower integration time of $\tau \approx 0.5$ s, a fractional wavelength uncertainty of $\sim 2 \times 10^{-8}$ or 20 parts-per-billion is shown.

6.5 Discussion and Future Work

The results from the first experiments using this technique, presented in this chapter, show the stability that can be achieved by the developed swept absorption line locking technique. Firstly, the results of the comparison of environmental conditions within the temperature chamber provides an upper limit on the uncertainty achievable experimentally. Theoretically, the experimental limit on the uncertainty achievable by the work is defined by the phase change due to temperature fluctuations across the interferometer. In this experiment, this limit is found from the standard deviation of the phase in the 'Water and Ice' conditions, and is calculated as ± 0.128 rad over the 60 mm sensitive region of the interferometer. Using the assumptions that the optical path length between the two remains unchanged, and it is only the effective optical path length changing as a result of a change in emission wavelength, the wavelength uncertainty can be calculated as 7.8×10^{-4} nm which is a fractional uncertainty of 5.1×10^{-7} or 0.51 parts-per-million.

The experimental results for long-term measurement of the phase drift highlights that the wavelength stabilisation of interest is on significant length timescales, upwards of hours, as apposed to short-term, linewidth reducing stabilisation. The measured phase drift of ± 0.17 rad equates to, for the 60 mm interferometric path length in this experiment, an uncertainty in the wavelength emission of 1.08×10^{-3} nm equivalent to a fractional uncertainty of 7.2×10^{-7} or 0.72 parts-per-million. When the Allan deviation is considered however, uncertainty as low as $\sim 5 \times 10^{-8}$ for integration times between $\sim 2 \times 10^3$ s and $\sim 6 \times 10^3$ s, reaching as low as 6×10^{-8} or 6 parts-per-billion at $\sim 3 \times 10^3$ s, can be seen. For first experiments, this is close to the stability achieved with existing techniques in non-modulated regimes, discussed in Chp. 3, and offers promising scope for improvement.

One cause for the uncertainty in phase, may be the discrete phase jumps caused by temperature setpoint updates, which are highly visible in Fig. 6.10 in each of the control loop parameters. There are two principle effects occurring at each discrete temperature setpoint update; there is the step itself, caused by the calculation of a new temperature setpoint and the laser TEC control loop operation. In the discrete setpoint operation, the parameters could be further tuned such that the temperature setpoint updates made are more continuous, updating in significantly smaller steps but at the trade-off of longer time to lock to the main locking condition. The second phase noise source in the control loops originates from the laser TEC control loop. Visibly in Fig. 6.10 there is overshooting of each temperature setpoint, which is caused by the internal parameters of the laser TEC. Beyond an initial autotune programme to automatically detect the control loop PID parameters, changing these parameters was outside of the scope of these experiments, but future work could consider investigating whether there is potential for reduced phase uncertainty if the laser TEC parameters are fine tuned.

An alternative potential for uncertainty in the stabilisation loop arises from the hardware limitations of the development platform. The National Instruments (NI) development platform and analogue-to-digital converter used in these experiments, discussed in Chp. 4, Sec. 4.3, has a maximum sample rate of 597kHz, which for a 1.166kHz modulation frequency whilst allowing base-2 modulation waveform sizes, limits the samples per modulation period to 512. Interferometrically, the implications of this are discussed

in Chaps. 4, 5 and 7 respectively, however from a stabilisation standpoint, this limits the resolution of each of the absorption features, and increases the uncertainty in the fitting of the peaks to each of these. Future developments of this system could make use of significantly faster sampling rates to both increase the modulation frequency, and increase the sampling resolution of the absorption features.

Moving beyond these experiments, considerations for future improvements to this technique can be made. While hardware improvements have already been discussed, one important aspect of this technique is that fundamentally, the ability of any stabilisation technique to accurately lock to a known absorption feature, is the linewidth of that feature. In the case of the acetylene cell used in these experiments, the full width half maximum (FWHM) of the 1521.06 nm is stated in the literature [170, 177] as 7 pm, however, easily accessible Doppler-free absorption features in different absorbers, such as the Rubidium D-lines, could be used to reduce this even further. However, this would require a change to a different laser emission wavelength using a different laser diode, as well as any wavelength-dependant optics, to match the wavelength of the new absorption lines.

6.6 Summary

In this chapter, a novel technique for stabilisation of widely-wavelength modulated lasers, where the modulation depth is several orders of magnitude is greater than typical wavelength references, is presented for long-term stabilisation applications. Linked to the typical stabilisation process presented in Chp. 3, the novel stabilisation technique is first outlined, using an atomic gas reference cell with an absorption feature within the modulated wavelength emission of the laser. In this technique, the modulated laser emission is passed through the atomic gas cell, and the signal is measured on a photodetector. Where an absorption feature is present in the modulated wavelength emission range, two dips in the photodetector signal are visible, notably arising from the same absorption feature. After removal of the intensity modulation waveform, the time difference between these two peaks can be used to determine the correction to apply to the central wavelength emission wavelength, which is achieved through temperature tuning of the laser diode.

Using the developed stabilisation technique with an acetylene reference cell, applied to a widely-wavelength modulated laser used in range-resolved interferometry, measurements were performed of phase changes in a Mach-Zehnder interferometer. Using a temperature control chamber and submerging the interferometer in an ice bath, phase changes due to temperature gradients across the interferometer arms were suppressed, reducing the expected phase changes due to changes in only the laser wavelength emission. Thus, the measured phase in the interferometer could be used as a measurement of the stability of the system and therefore could be used to characterise the developed technique. Recording phase changes over an extended measurement period showed improved stability between the free-running (no stabilisation loop) and stabilised measurements, with a fractional wavelength uncertainty calculated from any remaining phase drift of $\sim 7.2 \times 10^{-7}$. In another experiment, a direct disturbance was applied to the laser diode itself, in the form of a temperature gradient applied between the laser diode and

the sensing thermistor, causing a change in the laser emission wavelength. In these results significant improvement in the stabilised measurements over the free-running measurements, showing fractional wavelength uncertainty of $\sim 3.9 \times 10^{-7}$. Further to this, Allan deviations were calculated for both the free-running and stabilised measurements over an extended measurement period. While the free-running case showed higher stability at shorter timescales (< 0.2 s), at longer timescales ($> 10^3$ s), which is the target timescale for this stabilisation technique, improved stability in the stabilised measurements was shown. Here, fractional wavelength uncertainty as low as $\sim 5 \times 10^{-8}$ for integration times τ between $\sim 2 \times 10^3 < \tau < \sim 6 \times 10^3$ were achieved.

These results demonstrate that the stabilisation technique is effective in improving the long-term stability of widely-wavelength modulated lasers, such as is used in range-resolved interferometry. Compared to the existing techniques presented in Chp. 3, where fractional stabilities over long timescales ranged from 10^{-8} to 10^{-12} , this developed technique still has room for improvement in order to be directly competitive with existing techniques, however the experimental work presented represents only a first attempt at developing this novel stabilisation technique, and has clear scope for improvement in a number of areas, which are discussed in Sec. 6.5 of this chapter. Further work, directly comparing this technique to a highly-stable single frequency HeNe laser can be found in Chp. 7.

Chapter 7

HeNe Laser Interferometer Comparison

This chapter describes the principles and preliminary experiments comparing a highly-stabilised helium-neon (HeNe) laser to a widely-wavelength modulated laser diode with wavelength stabilisation performed using the developed stabilisation technique in this thesis. Firstly, the principles and motivation behind this work are discussed, followed by discussion of the experimental setup used in this work, including the interrogation hardware and optical setup. This is followed by discussion of the initial experimental results, including analysis of the major sources of uncertainty in the results. Finally, discussion is given to the conclusions that can be drawn from these results as well as possible future extensions to both these experiments and this principle.

7.1 Principles and Motivation

The stabilised, single frequency HeNe laser is often considered the "gold-standard" for laser metrology applications, as a result of the inherently high coherence length, low power and the continuous, visible output it provides. It also has an inherently narrow emission linewidth with high absolute stability with a well-known wavelength which makes it extremely advantageous for a wide range applications. Because of this, it is an excellent candidate for assessment of the capabilities of the novel stabilisation technique discussed in Chp. 6. Applying the developed swept absorption line locking stabilisation technique to a widely-wavelength modulated laser diode and using range-resolved interferometry (RRI), measurements can be made of a linear stage displacement and using standard fringe-counting techniques, the same displacement can be measured using a HeNe laser and direct comparison between the two can be undertaken.

In principle, this would allow for analysis of the RRI signal processing, specifically whether it correctly determines accurate phase changes, or whether is systematic under or over counting of fractions of each 2π interval. This would accumulate and lead to inaccurate measurements of phase and therefore displacement. Furthermore, a comparison between the widely wavelength modulated laser diode and the stabilised HeNe could be used to compare the true evaluation wavelength of RRI. In the work presented until now in Chps. 5 and 6, and in the literature [155–159], the evaluation wavelength has assumed to be the centre of the wavelength modulation sine wave, however this

may not be accurate if the wavelength response from the laser driver is non-linear, and in principle this comparison should allow verification of this.

7.2 Experimental Setup

In order to compare a laser stabilised using the developed stabilisation technique with a highly-stabilised HeNe, an experimental setup was devised whereby each technique would measure the displacement of linear motion stage via the same optical path. This section discusses the experimental setup used to achieve this, starting with the optical setup, followed by description of the interrogation hardware, first discussed in Chp. 4 Sec. 4.3 of this thesis.

7.2.1 Optical Setup

The optical setup for these experiments, shown schematically in Fig. 7.1 consists of the two lasers combined and fibre-coupled into a free-space Michelson interferometer, the output of which is then separated and directed to photodetectors. Light from a 780 nm diode laser first passes through a fibre-coupled isolator, and then is split via a 90:10 ratio coupler into measurement and stabilisation arms. The stabilisation arm briefly collimates the light to pass in free-space through a sealed rubidium cell, before recoupling the light back into fibre and onto a silicon (Si) photodiode. The measurement arm connects to a two-color wavelength combiner/splitter. The 633 nm HeNe contains an in-built isolator, so the laser emission is directly coupled into the same combiner and the output is connected to a reflective collimator which serves as the input to the Michelson interferometer.

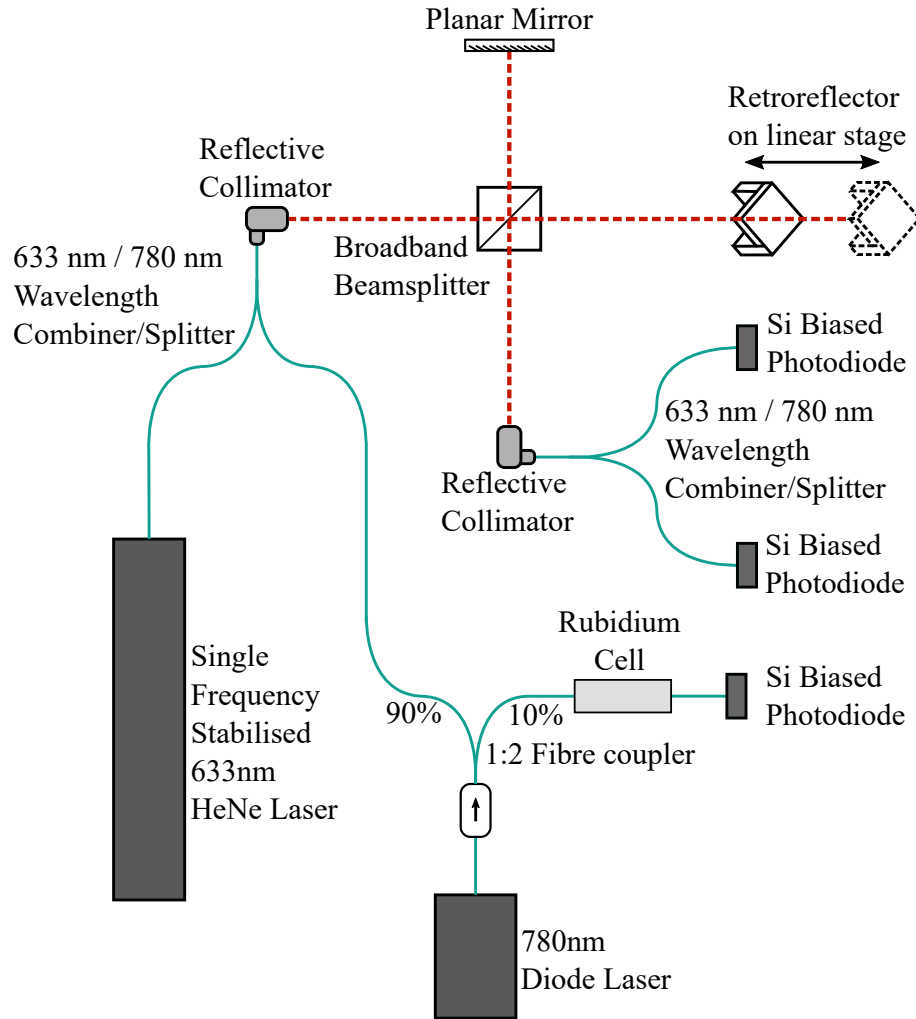


FIGURE 7.1: Schematic of the optical setup used for displacement measurements of linear stage motion using a Michelson interferometer interrogated with a 633 nm HeNe laser and a 780 nm diode laser. Notably the 780 nm laser diode has an additional branch for application of the stabilisation technique

The Michelson interferometer is comprised of the reflective collimator, which emits light towards a beamsplitter, splitting the incident beams towards a fixed, planar reference mirror, and a retroreflector mounted on a linear stage. The returning beams are directed towards a second reflective collimator which serves to re-couple the light into the optical fibre. The coupling out of and into the fibre portions of this setup are specifically performed using reflective collimators, so that there is no wavelength dependency that would cause changes in the optical paths taken by the two different wavelength lasers.

After re-coupling into fibre, the light is directed into an identical two-colour wavelength combiner/splitter as previously, but in reverse such that light is split in a wavelength-discriminate fashion back into two arms. Each of these arms is then incident on Si photodetectors, the outputs of which are measured and recorded separately. All of the fibre-components in this setup are single-mode at the wavelength of both lasers and the bulk

optic beamsplitter has a broadband optical coating that maximises the transmission of light at both wavelengths of interest. This combination of reflective collimators, single mode fibres and appropriately coated bulk optic components, is specifically designed so that the beams from both lasers are as co-linear as possible and thus measure the exact same displacement of the linear stage. The bulk optics of this experimental setup are shown in the photo in Fig. 7.2, and the in-fibre components leading to the bulk optics from the lasers are shown in Fig. 7.3.

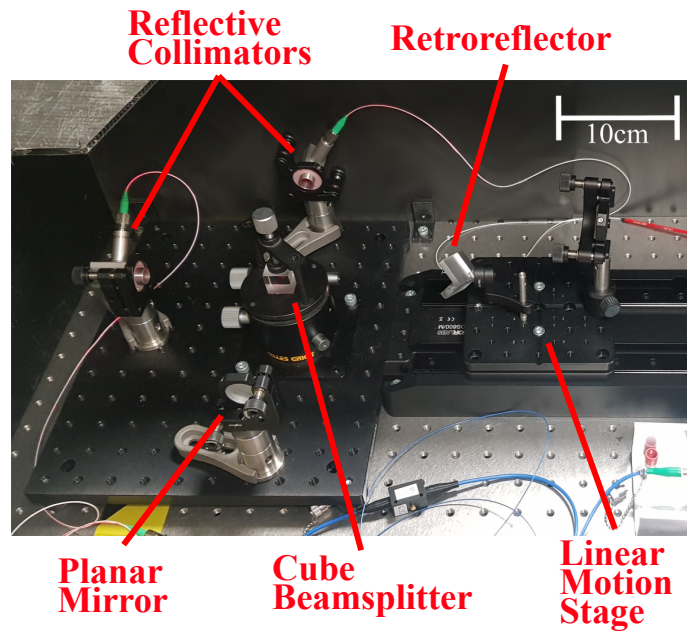


FIGURE 7.2: Photo of the bulk optic Michelson interferometer used in the HeNe comparison experimental setup with key components labelled.

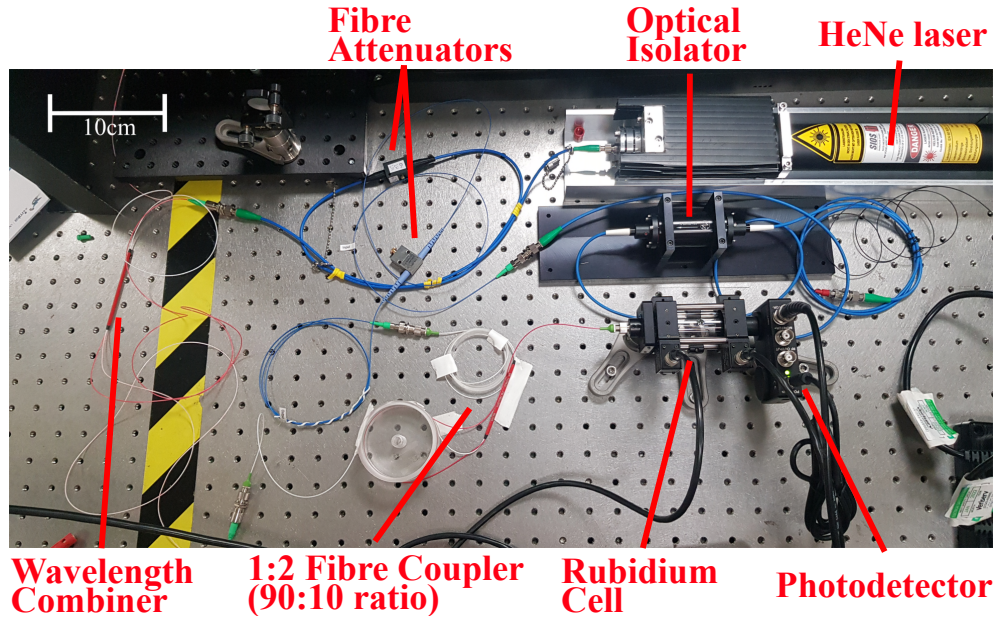


FIGURE 7.3: Photo of the in-fibre components leading to the bulk optic interferometer from the laser diodes in the HeNe comparison experimental setup with key components labelled.

7.2.2 Interrogation Hardware

Two distinct lasers are used in this setup, the first being the stabilised HeNe laser and the second being the laser diode capable of wide wavelength modulation used for RRI. The first of these is a highly-stabilised single frequency HeNe laser from SIOS (SL04/A) which emits at a wavelength, in vacuum, of 632.991234 ± 0.0002 nm with a stability of $\sim 5 \times 10^8$. The second laser used for RRI, is the Eagleyard distributed feedback (DFB) laser diode (EYP-DFB-0780-00010-1500-BFY12-0005) with a central wavelength of 780 nm discussed in Chp. 4 Sec. 4.3. For the purposes of signal processing, the National Instruments development platform was adapted to perform simultaneously the modulation of the laser injection current for the RRI laser and recording of photodetector signals from 3 separate sources; the HeNe laser, the RRI laser and the stabilisation system. In addition, the stabilisation technique described in Chp. 6 is implemented alongside, to actively stabilise the RRI laser through temperature tuning. The specifics of this hardware are discussed in Implementation 4 of Chp. 4 Sec. 4.3, and a schematic of this setup is shown in Fig. 7.4. This interrogation hardware performs modulation of the laser injection current synchronous with recording of both the HeNe and RRI photodiode signals and stabilisation of the RRI laser diode. Whilst the stabilisation is performed continuously in real-time, the signal processing and analysis of the measurement data from the RRI and HeNe lasers is performed offline after recording, not in real-time.

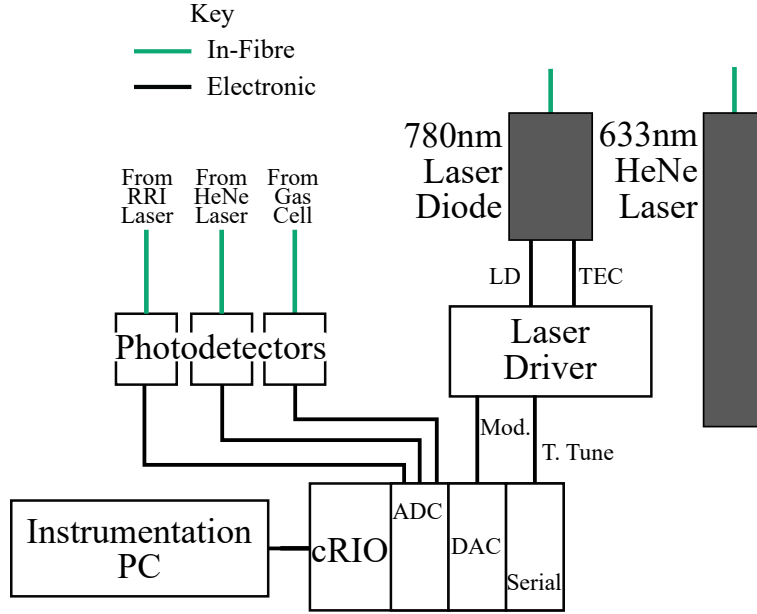


FIGURE 7.4: Schematic of the hardware used for comparison between a 633 nm HeNe and a stabilised 780 nm diode laser

7.2.3 Stabilisation

As previously mentioned, the widely modulated laser diode is stabilised using the swept absorption line locking technique introduced in Chp. 6. To achieve this, a split beam from the laser diode is passed through a gas cell containing rubidium (Rb) vapour comprising of rubidium 85 and rubidium 87 in their natural abundances, 72.2% and 27.8% respectively. As shown in Fig. 7.5 which shows a typical absorption spectrum for rubidium gas vapour, there are multiple absorption features present at the typical emission wavelength of the laser diode used in this experimental setup (~ 780 nm).

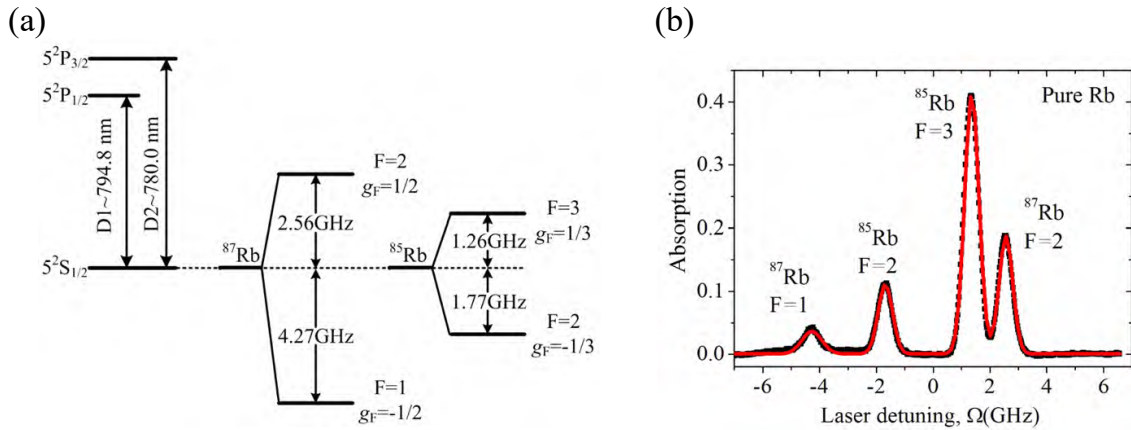


FIGURE 7.5: (a) Energy level diagram of the rubidium D-line transitions and hyperfine ground state structure. (b) Absorption spectrum for rubidium vapour in nominal conditions. The energy level transitions for ^{85}Rb and ^{87}Rb are labelled. Figures adapted from Ma *et al.* [178]

Using the swept absorption line locking technique, the widely-modulated laser diode is locked to the ^{85}Rb D₂ transition line, which has a peak wavelength value of $\lambda_{vac} = 780.0268$ nm [179, 180]. The process for locking to this absorption feature is as discussed in Chp. 6, and is illustrated in Fig. 7.6 below.

Fig. 7.6(a) shows a modulation period of the photodiode signal, averaged over a 10s modulation period. Also shown is the fitted sinusoidal waveform used to remove the intensity modulation, treating the fitted sine wave as the original intensity of the emitted beam. Fig. 7.6(b) shows the absorption signal, normalised after the intensity has been removed. From the difference between the periodicity of the dominant absorption line on the forward and reverse sweep of the laser and half the modulation period, if necessary, a laser temperature setpoint change is applied via serial communication to the laser TEC. At an update rate of 0.05 Hz, this process is synchronisation with the laser modulation and in parallel with the interferometric signal recording.

In contrast to the acetylene reference absorption lines which had a FWHM linewidth of ≈ 500 MHz in nominal room temperature conditions [181], the rubidium absorption lines are naturally narrower, offering FWHM linewidths of ≈ 6 MHz for the natural, Doppler-broadened transitions [179, 180]. This allows for lower uncertainty in the locked condition of the stabilisation system which in turn reduces the wavelength emission uncertainty. Notably there is also 4 rubidium absorption lines within the modulation of the laser wavelength emission, illustrated by Fig. 7.6(b), compared to the 2 acetylene absorption lines previously within the modulated laser wavelength emission. As in the previous case, the absorption line of interest is also the strongest absorption line, so this pair of peaks is used to achieve the locking condition in the stabilisation technique used in this work.

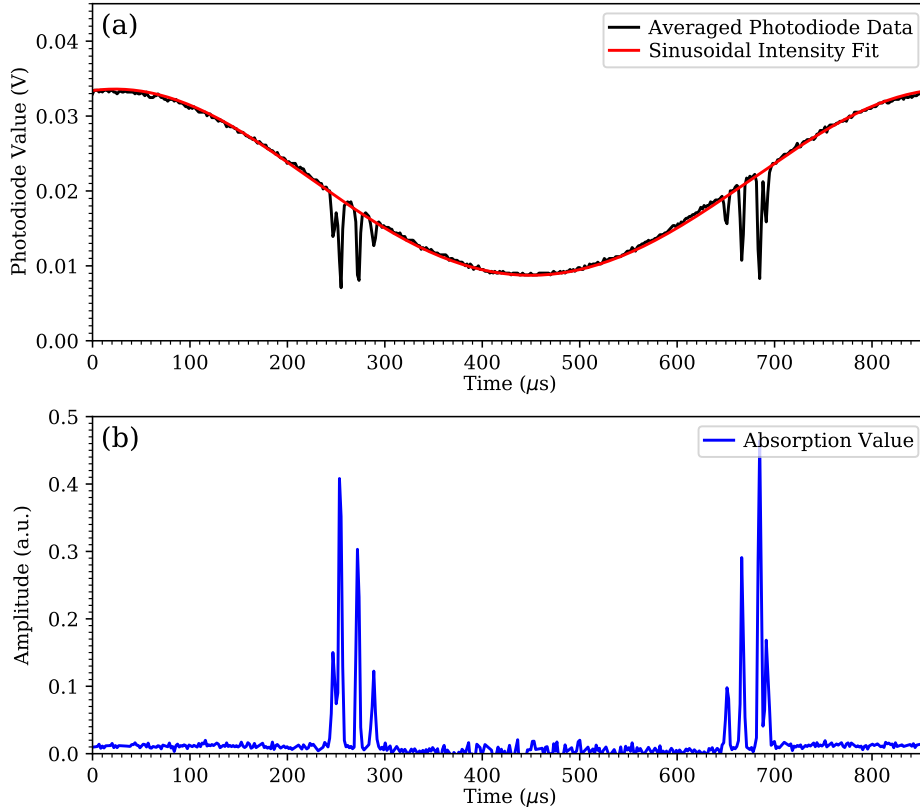


FIGURE 7.6: (a) Average photodiode signal and fitted sinusoidal intensity waveform (b) normalised absorption signal for the averaged modulation period

7.3 Experimental Method

Using the experimental setup described above, measurements were performed of linear stage motions over 10 mm using both the HeNe laser and the modulated laser diode simultaneously. The modulated laser diode is stabilised using the swept absorption line locking technique introduced in Chp. 6, but all other signal analysis is performed in post-processing. In this section, the signal analysis for both the HeNe and modulated diode laser is discussed.

7.3.1 Signal Analysis

In order to achieve the most accurate comparison possible between the two techniques, the signal analysis is particularly important. Here, the two signal processing steps of both sets of data are discussed, followed by discussion of how these two sets of data are then compared.

HeNe Signal Processing

To determine displacement from the HeNe interferometric signal, raw fringe counting methods are applied, without signal processing. The HeNe photodetector signal, for a linear stage movement at a constant velocity, consists of a series of fringes where each full fringe represents a change in displacement of 2π rads, which is equal to a displacement of one full wavelength of the laser emission. The simplest analysis possible is therefore to count the number of full fringes that are swept past during the stage displacement. More precise measurement can be achieved via interpolation of the full fringes to calculate sub-wavelength displacements. As mentioned previously, synchronising the starts of the analysed sections of the HeNe and RRI signals is paramount for accurate comparison, so a slower initial stage motion is used. This creates slower moving fringes, and allows for a more informed decision of where the first full fringe begins, which is manually selected as the start of the HeNe signal. For simplicity's sake, the mean of the data is subtracted from the data, such that each fringe crosses through zero, and every other zero crossing can be considered as one complete fringe. By looking for changes in sign, and interpolating between the two data points either side of zero, zero crossing values can be determined. However there can appear to be multiple zero crossings for each true fringe, caused by signal noise. To account for this, a threshold is created which dynamically detects each cluster of zero crossings and averages these into a single zero crossing for each fringe. The results of this process is illustrated in Fig. 7.7.

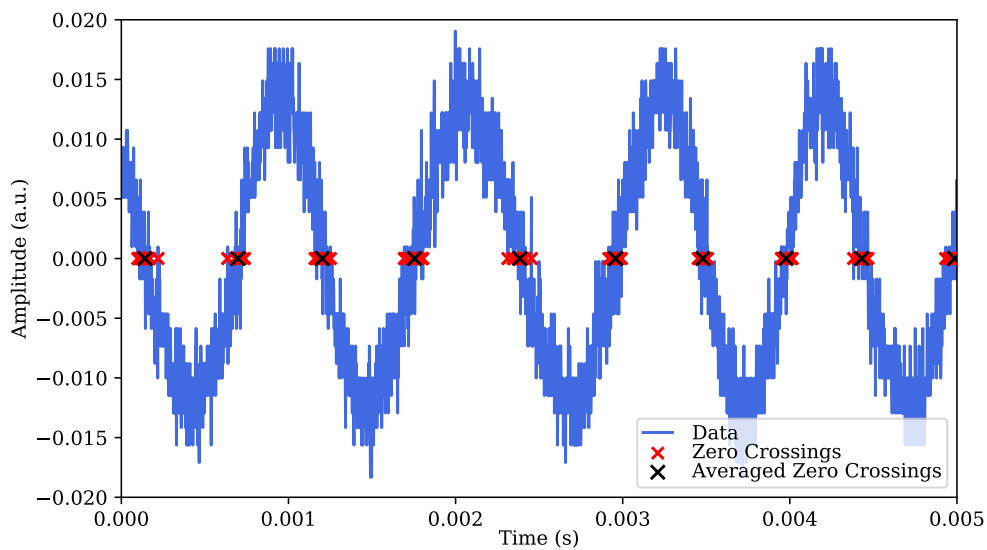


FIGURE 7.7: A sample of a HeNe photodetector data showing all of the zero crossings and the averaged 'true' zero crossings

An alternative to this threshold calculation is to perform a rolling average on the data before calculating zero crossings. For an appropriate averaging length, this has the effect of removing a significant proportion of the noise and reduces the number of false zero crossings. After the 'true' zero crossings have been calculated, at the end of the displacement signal the last full fringe, and therefore zero crossing, is manually selected and the number of zero crossings is summed.

RRI Signal Processing

For the RRI signal processing, the technique is the same as discussed in Chp. 4 with some minor caveats related to the synchronisation of the two techniques. In RRI the position of the window function (either singular or dual) is centred on the highest frequency components of the interferogram for the most precise demodulation. For simplicity of demodulation, the start of the first waveform is typically selected such that the windows lie at 90° and 270° respectively, and the resultant phase value is considered to be at the average time of the waveform, i.e. at 180° . As a result of these practicalities, it is unlikely that the manually selected first full fringe of the HeNe signal occurs at the exact same time as an RRI phase value.

As a result, the RRI phase is calculated for the modulation periods either side of the HeNe zero-crossing, and a linear interpolation is applied between them to calculate the RRI phase at the time of the HeNe zero-crossing. This concept is illustrated in Fig. 7.8 for a small sample of a HeNe signal and zero crossing, with the synchronised sample RRI signal. This interpolation results in a static uncertainty in the initial phase of the RRI signal processing, which is discussed further in Sec. 7.4.3.

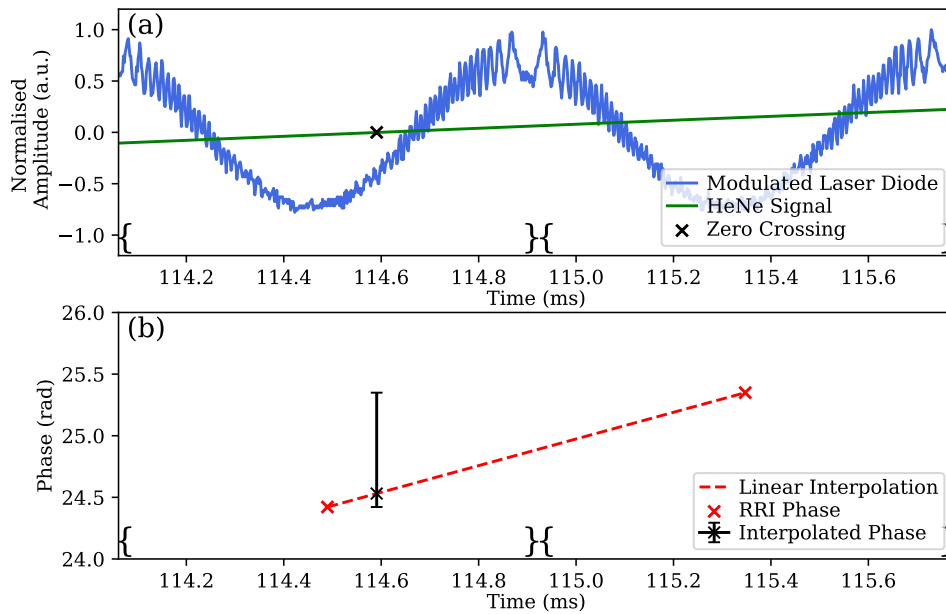


FIGURE 7.8: Illustration of the uncertainty in the initial RRI phase for two modulation periods, marked by '{' and '}' (a) shows a series of normalised RRI modulation periods with the synchronised HeNe signal and the HeNe zero crossing (b) shows the resultant phases calculated using RRI (red X), with the interpolation between the two shown as a dotted line. The interpolated phase value at the time of the HeNe zero-crossing is then shown as a black X, with relevant error bars showing the uncertainty in this value.

This process is also applied to the last HeNe zero-crossing point, interpolating between the phase values for the RRI modulation periods either side for the phase value at the last HeNe zero-crossing.

Wavelength Estimation

Assuming that all the HeNe fringes are recorded and the correct zero-crossings are identified, and that the RRI interferometric signals are correctly demodulated, the measured phase changes θ from both techniques can be used to calculate the displacement of the linear stage, d , using the interrogation wavelength, λ .

$$d = \frac{\theta}{2\pi} \lambda \times \frac{1}{2} \quad (7.1)$$

where the additional factor of $1/2$ is due to the measurement of the mirror displacement by reflection, thus doubling the measured optical path change. In this experiment, the HeNe interferometric measurement is accepted as the true value measurement, owing to the reasons discussed at the top of this chapter, but within the limitations of the uncertainties discussed further later on.

By virtue of the near identical optical paths there is a reasonable expectation that the displacements measured by both techniques are equal, and therefore the evaluation wavelength λ_{RRI} of the widely-modulated laser diode can be calculated by;

$$\lambda_{RRI} = \frac{4\pi d}{\theta_{RRI}} \quad (7.2)$$

or alternatively, using the calculation of displacement from the HeNe laser;

$$\lambda_{RRI} = \frac{\theta_{HeNe} \lambda_{HeNe}}{\theta_{RRI}} \quad (7.3)$$

Therefore, from the measurements of phase from each technique along with the evaluation wavelength of the HeNe laser, the evaluation wavelength of the widely-modulated laser diode can be calculated.

7.4 Results

7.4.1 Experimental Results

Using controlled motions of a Thorlabs linear stage (Thorlabs DDS600/M), the stage was moved in a series of fixed displacements and the interferometric signals for both the HeNe laser interferometer and the widely-modulated laser diode interferometer were recorded. The stage motions were considered with the limitations of the interrogation hardware in mind, discussed in Sec. 7.4.2 of this chapter, so were limited to a maximum displacement of 150 mm and a maximum velocity of 0.13 mm/s. Initially, a test case measurement of a 10 mm displacement at a velocity of 0.015 mm/s was recorded, as well as measurements for a range of displacements up to 150 mm, at higher velocities. As discussed in this section however, significant mechanical noise from the stage causes issues in the analysis of the HeNe interferometer, which is exacerbated at higher velocities as the fringe frequency approaches the mechanical vibration frequencies and the

stage velocity approaches the resonance conditions of the stage. For this reason only the 10 mm stage motion has been fully analysed below.

10 mm Stage Motion

The linear stage is moved 10 mm at a velocity of 0.015 mm/s in a single motion and the interferograms for both the HeNe laser interferometer and the widely modulated laser diode interferometer are recorded. Samples of both of these interferograms are shown in Fig. 7.9 where the HeNe signal has had a rolling average of 500 samples applied.

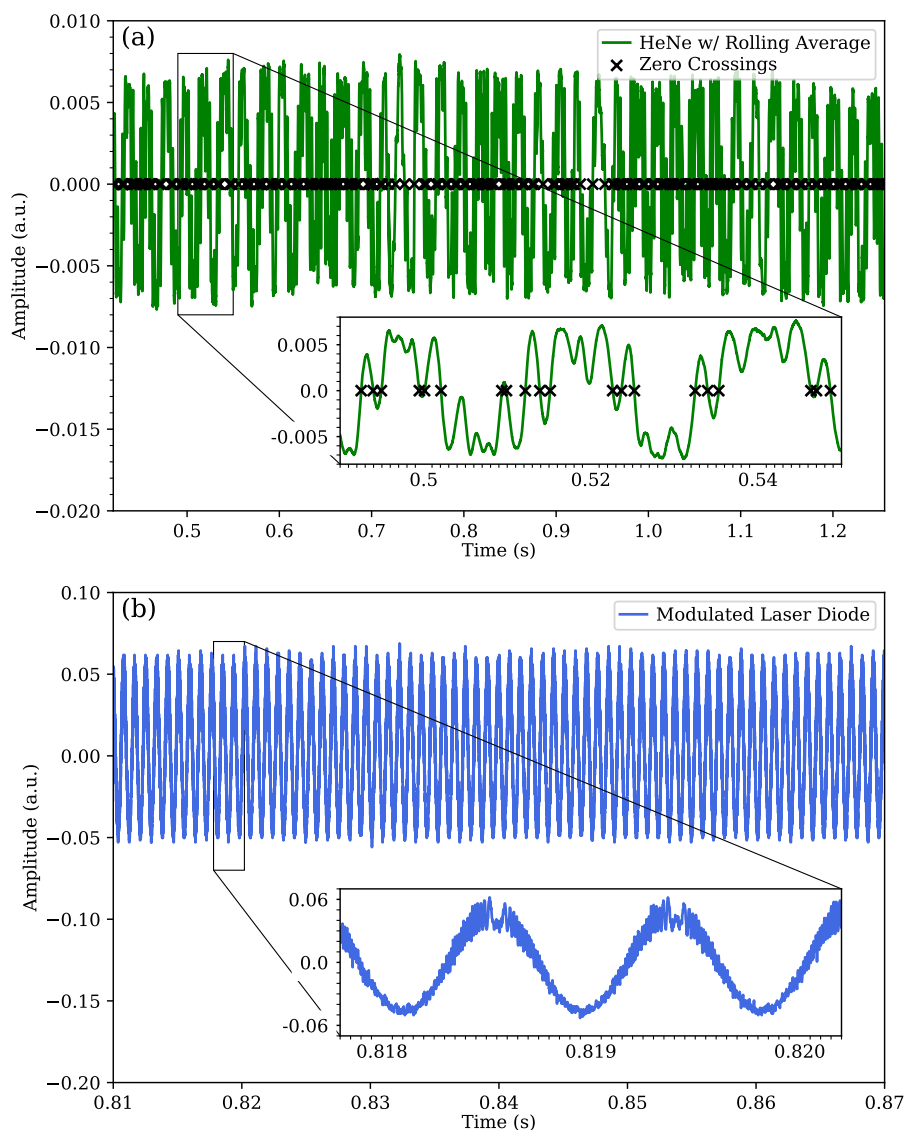


FIGURE 7.9: Samples of interferograms for (a) a HeNe interferometer with a rolling average applied and with zero-crossings marked (b) widely-modulated laser diode interferometer.

Fig. 7.9(a) shows the rolling average (500 samples) for a sample of the HeNe interferogram during the linear stage motion along with the calculated zero-crossings of the signal. For the same sample period, Fig. 7.9(b) shows the widely-modulated laser diode interferogram over a shorter time-frame, with a zoomed inset showing several modulation periods.

For a simple linear motion, an ideal HeNe interferogram should appear as a series of clean fringes, whose spacing depends on the speed of the stage. Fig. 7.9(a) however shows that even with a rolling average applied, there is one or more sources of noise causing the HeNe signal to "jump" around, which is creating false zero-crossings. As the determination of the phase change using the HeNe laser relies on accurate counting of the full fringes, these false zero-crossings will cause significant disparity between the true displacement and the measured displacement. To examine the source of noise further, the fast Fourier transform (FFT) of the HeNe interferogram is taken, shown in Fig. 7.10.

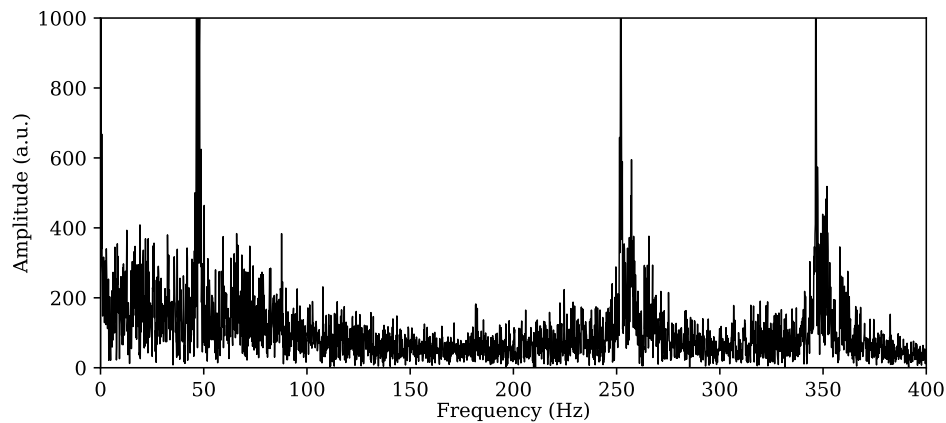


FIGURE 7.10: Discrete fast Fourier transform of the HeNe fringes after a rolling average has been applied.

Given a stage motion of 0.015 mm/s, it is expected that the dominant fringe rate for the HeNe interferometer evaluating at ~ 633 nm, has a frequency of ~ 47 Hz. This peak is evident in the FFT shown in Fig. 7.10, however the presence of the two peaks at ~ 255 Hz and ~ 350 Hz are likely to be the main components of the additional noise visible in the HeNe interferogram. To remove these components in this experiment, a low-pass filter should be sufficient, with the cut-off frequency between ~ 255 Hz and ~ 47 Hz. As this low-pass filter is applied digitally in post-processing, there is no time delay caused by the filter that needs to be accounted for in the signal processing. Fig. 7.11 shows the same sample of the HeNe interferometric signal as previously in Fig. 7.9(a), but low-pass filtered at 150 Hz.

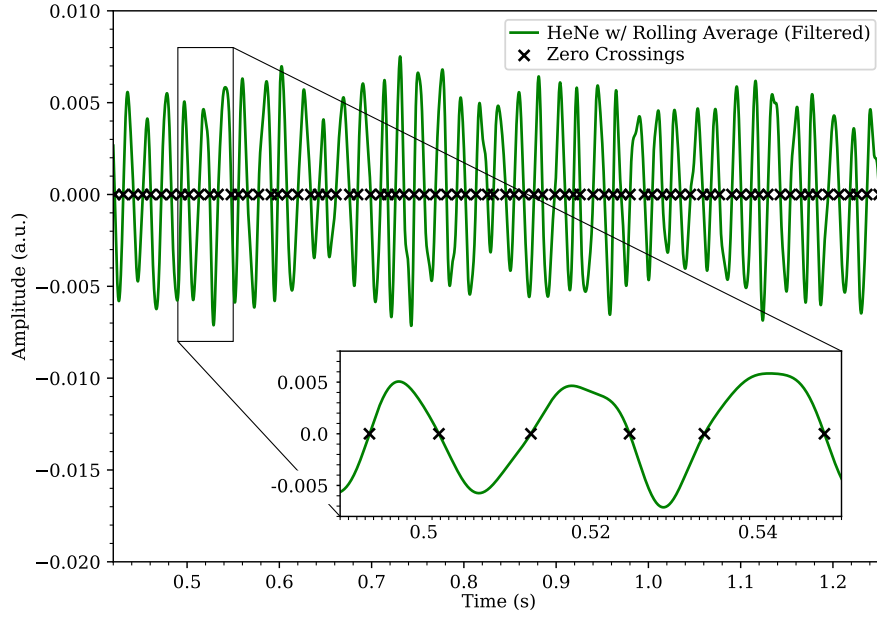


FIGURE 7.11: Sample of the HeNe interferometric signal after a low pass filter at 150 Hz has been applied. The zero-crossings are marked as black crosses.

Using the improved clarity zero-crossings shown in Fig. 7.11, the displacement as measured by the HeNe interferometer can be calculated using Eqn. 7.1 with each zero crossing representing π phase change, and using the HeNe emission wavelength of $\lambda_{\text{HeNe},\text{vac}} \approx 632.991234$ nm. Using RRI, the widely-modulated laser diode interferometric signal can be evaluated and the phase change has measured by the two laser interferometer can be compared, shown in Fig. 7.12.

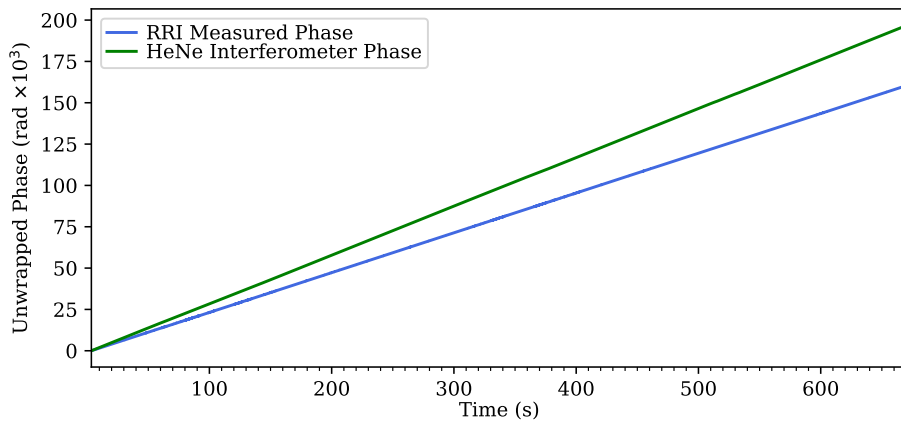


FIGURE 7.12: Phase change measurements over a 10 mm stage motion from both the HeNe interferometer (green) and widely-modulated laser diode interferometer (blue)

Fig. 7.12 shows the measurements of phase from both techniques for a programmed

10 mm linear stage motion. The gradients of the two lines are expected to vary due to the different evaluation wavelengths of the two techniques, however the measured displacements should be identical. Using the full displacement range, and using eqn. 7.3, the evaluation wavelength of the widely modulated laser diode was initially calculated as ~ 776 nm.

As discussed earlier, given the efficacy of the swept absorption line locking technique discussed in Chp. 6, there is a reasonable expectation that the calculated wavelength of the widely-modulated laser will be extremely close to, if not exactly, 780 nm therefore there is potentially an issue in the analysis process that remains unresolved. A closer examination of both measurements is therefore required, starting with fitting linear fits to the data shown in Fig. 7.12 for both measurements and subtracting these linear fits from the data points. The residuals after the linear fits have been subtracted from both measurements are shown in Fig. 7.13.

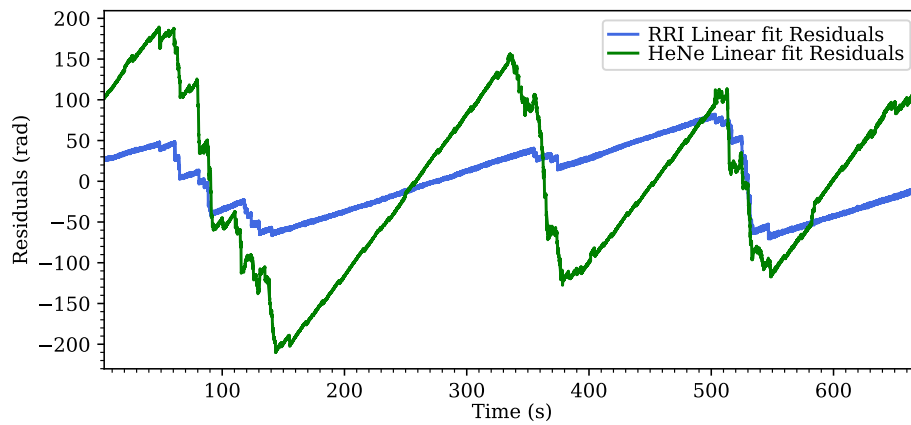


FIGURE 7.13: Residuals of phase measurements for RRI and HeNe phase measurements after a linear fit of the phase is subtracted from both.

Both measurement clearly exhibit unexpected behaviour, namely the phase skips that appear as sharp, discrete changes in Fig. 7.13. Starting with the RRI case, the approach to analysing and where possible, correcting, these phase skips are discussed below.

Phase Repair

In the difference between the RRI measured phase and the linear regression fit of the same signal, the phase skips notably occur as a multiple of 2π . This gives a good indication that there is an issue occurring in the phase unwrapping, which occurs during the post-processing of the calculated RRI phase, that is causing erroneous unwrapped phase values, offset by 2π or a multiple thereof. Closer examination of the unwrapped phase with the wrapped phase, shown in Fig. 7.14, illustrates the conditions where the unwrapping process fails.

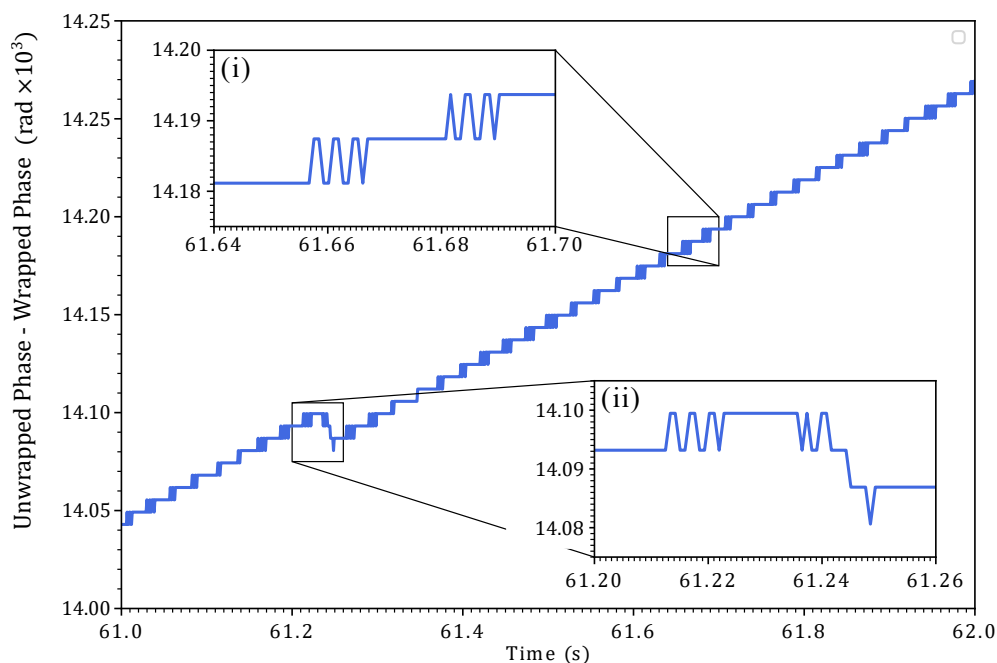


FIGURE 7.14: Sample of the unwrapping steps for the RRI phase signal, with insets showing (i) a successful unwrapping step and (ii) an incorrect unwrapping step

Fig. 7.14 illustrates an example of both a successful unwrapping of the phase, shown in inset (i), and an erroneous unwrapping, shown in inset (ii), that will cause a visible multiple of 2π phase step seen in Fig. 7.13. Where the correct phase unwrapping should cause a series of steps, increasing in value by 2π , the failed phase unwrapping causes the phase to jump either more than 2π or jump in the wrong direction. To correct for these erroneous unwrappings, an algorithm is run on the data shown in Fig. 7.14 so that each subsequent unwrapped value step is 2π more than the previous. Fig. 7.15 illustrates how this unwrapping fix resolves the discrete step issue, allowing for clean unwrapping of the RRI phase measurements.

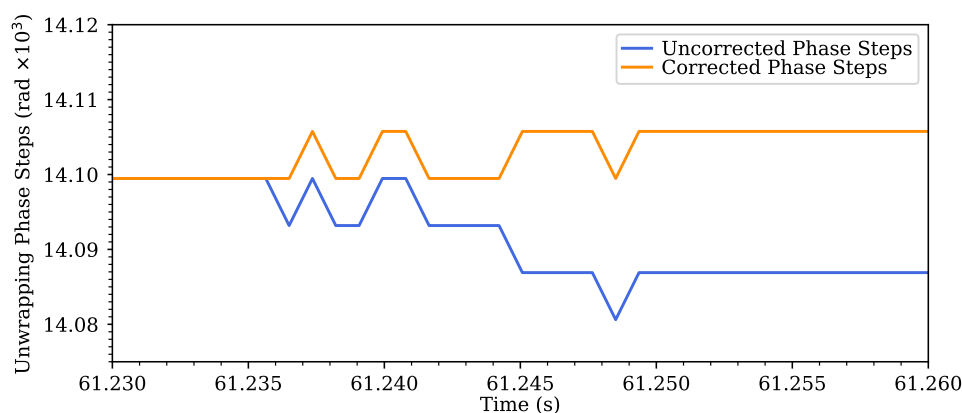


FIGURE 7.15: Sample of the uncorrected and corrected phase unwrapping of the RRI phase signal

For the HeNe measurements, the linear fit residuals are more complex to determine the origin of. Unlike for the RRI residuals, the discrete jumps are not exact integer multiples of 2π , and there is no phase unwrapping performed during the signal analysis. However, most of the discrete jumps are close to multiples of π if not exactly, which indicates that the errors may occur in the zero-crossing analysis. A closer examination of the HeNe measurement signal at one of these discrete jumps, shown in Fig. 7.16 aids in identifying the source of these errors.

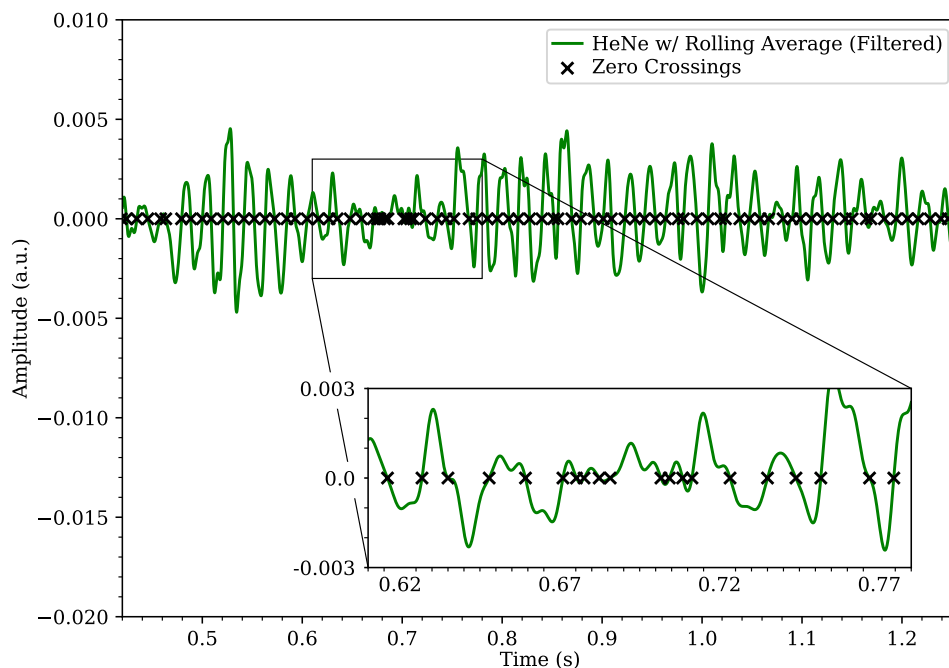


FIGURE 7.16: Sample of the filtered HeNe interferometric signal at the time of a discrete jump in measured phase.

In Fig. 7.16 it can be seen that where the fringe visibility drops below a certain threshold, the zero-crossing analysis is complicated by increased crossings that may or may not represent a full fringe. This drop in visibility could be due to changes in the intensity of the interfering beam at the photodetector, however as the interferogram returns to its original intensity shortly thereafter, this would require a very brief, transient effect. Alternatively, the drop in visibility may be caused by a temporary slowing of the stage during the programmed motion, reducing the fringe rate to near-zero very briefly. This could be due to a spot of increased friction on the stage track, a temporary blip in the stage servo motors or the control circuit of the stage. Nevertheless, a more robust analysis function that included fringe interpolation for even very low fringe rates could potentially significantly improve the zero-crossing analysis under these conditions.

For this experiment however, within the time constraints of the project, developing a significantly improved zero crossing analysis function was not possible. Instead, the HeNe phase signal is corrected using the linear fit residuals shown in Fig. 7.13. The difference between each residual data point and the subsequent residual data point is

calculated, where the difference is greater than a threshold of one standard deviation of the differences, a correction is applied to the HeNe phase signal. For both of the corrected phase signals, a new linear fit is applied, and the residuals of these fits are shown in Fig. 7.17.

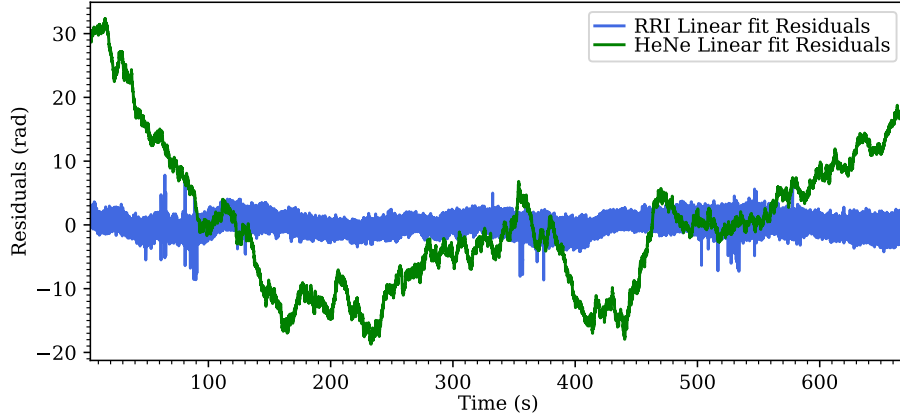


FIGURE 7.17: Residuals of phase measurements for the corrected RRI and HeNe phase measurements after a linear fit is removed.

The resulting phase signals for both measurements are shown in Fig. 7.18. Using these corrected phase measurements and eqn. 7.3, the evaluation wavelength for the widely-modulated laser diode is then calculated as 780.09 ± 0.13 nm. The sources and values of uncertainties related to this value are discussed later in this chapter in Sec. 7.4.3.

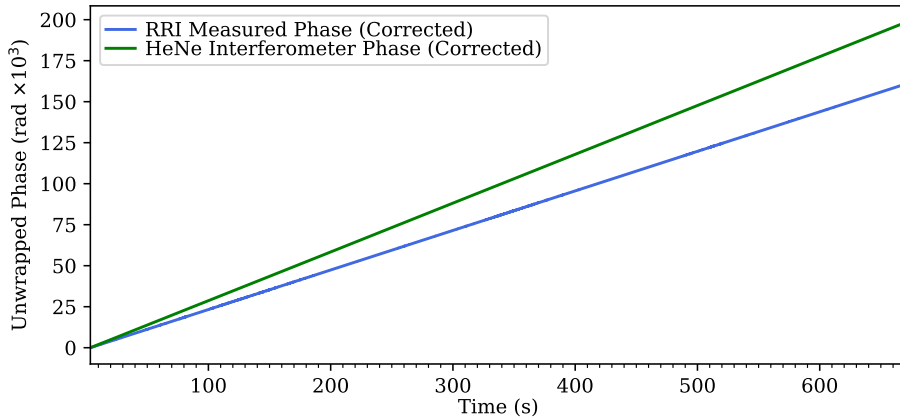


FIGURE 7.18: Corrected phase change measurements over a 10 mm stage motion from both the HeNe interferometer (green) and widely-modulated laser diode interferometer (blue)

7.4.2 Experimental Limitations

The original intention of these experiments was to provide a method of direct and accurate comparison between a highly-stabilised HeNe laser and the widely-modulated laser diode in terms of the stability. This was shown for a short displacement (10 mm)

at very slow speed (0.015 mm/s), however in practise this is both a very short, and very slow displacement for realistic applications outside of this very narrow test case. Measurements of both increased displacement and velocity have been performed using this developmental setup, however there are limitations, some of which such as the mechanical vibrations have already been mentioned, related to both the experimental setup and the interrogation hardware that have to be considered.

Experimental Setup Limitations

The first limitation on the interrogatable velocities of the linear stage arises from the previously mentioned mechanical vibrations visible in the FFT of the HeNe interferometric signal, shown in Fig. 7.10. If the frequency of the HeNe fringes coincides with the frequency of the mechanical vibrations, it will not be possible to separate the two via frequency filtering as described above. If the mechanical noise is of significant amplitude, such as in the test case shown in Fig. 7.9(a), spurious zero crossings will be detected, causing the HeNe evaluation to be inaccurate. With regards to the stage's velocity, the solution to this mechanical noise is to either move such that the fringe frequency is significantly below the mechanical noise frequency and low-pass filter out the signal of interest, as is done in the 10 mm displacement above, or to move at a substantially higher velocity such that the fringe frequency is much greater than the mechanical noise frequency. In this case, a high-pass or band-pass filter could be applied to isolate the fringe signal of interest.

There are also practical steps that could be taken to reduce the mechanical noise present in the signal. A more robust experimental setup for example, could be used, using a linear stage with less inherent mechanical noise. The mounting of the retroreflector, as shown in Fig. 7.3, may contribute to this. The current mounting is on a post a few centimetres above the stage platform at one edge of the platform, creating a small lever action that would amplify stage noise due to mechanical vibrations. Reducing this height and centralising the retroreflector on the stage platform, therefore reducing the lever action could aid in reducing some of the mechanical noise present in the HeNe interferometric signal.

Hardware Limitations

Chp. 4, Sec. 4.3 briefly discusses some of the hardware limitations of this interrogation implementation. Specifically referring to Implementation 4, the National Instruments (NI) development platform, the practical implications of these limitations is evident. With a current maximum sample rate of ~ 597 kHz, and with an RRI interferometric bandwidth of 1.166 kHz, the maximum velocity measurable using RRI is ~ 0.13 mm/s, which is significantly below the velocity required for practical applications or outside of the limited test case discussed above. The other caveat of this hardware limited sample rate is that the maximum measurable optical path difference is restricted to ~ 263 rad which, using a conversion factor of 1.35 rad/mm, results in a maximum range of ~ 194 mm. Towards the end of this range however, the quality of the demodulated interferometric signals will sharply decrease, so a conservative calculation of the usable measurement range would be in the region of ~ 150 mm. Of note however, as described in Chp. 4, Sec. 4.3, the bandwidth capabilities of this experimental NI system are not

indicative of a limitation of the RRI signal processing technique, and are significantly reduced compared to other real-time RRI development units used elsewhere.

Software Limitations

A further consideration, separate from this specific implementation but true of any software implementation, is the processing and accumulation of data. In implementation 4, the processing of data is performed after measurement in standalone python scripts. The reasoning for this was to reduce the burden on both the FPGA and the real-time (RT) target of the NI hardware in terms of CPU and memory usage and as a result, any measurement data is condensed into a single file to be uploaded from the NI platform onto a separate PC. Since each file has to be transferred, then separately processed, it is in the interest of the user to keep measurement times as short as possible, so as to reduce the file sizes generated by the NI platform. Therefore, while in principle a measurement of significantly extended duration, created by measurement over a large stage travel range operating within the limits of the maximum measurable stage velocity, is possible using this platform, it is considered inadvisable as the file size it would create would be unmanageable and taxing to process at any level.

7.4.3 Uncertainties

This section discusses the contributions from the main uncertainty sources towards the overall error in both signals. These contributions are either static, remaining unchanged during the motion of the stage, or dynamic, changing as the stage is moved. The uncertainties associated with this experiment are summarised in table. 7.1 at the end of this subsection.

Phase Uncertainty

Here, phase uncertainty is referring to the static uncertainty in both the initial, and final phase of the RRI signal processing, caused by the linear interpolation discussed earlier, and illustrated in Fig. 7.8. Where the RRI phase is not synchronised in time with the first and last HeNe zero crossings, the RRI phase is linearly interpolated between the RRI phase before and after the HeNe zero crossing. For both the first and last zero-crossings, the phase uncertainty is static and can be considered as the difference in phase between the two interpolation points. For the RRI signal measured this corresponds to an uncertainty of $\sim \pm 0.45$ rad, calculated from the standard deviation between subsequent phase points, which equates to a displacement uncertainty of $\sim \pm 56$ nm in total. Since this uncertainty is static, it is not expected that this will scale with displacement and therefore increasing the travel range would lower these uncertainties proportionally.

The HeNe phase uncertainty is associated with timing jitter of the zero-crossing estimation. This is a static uncertainty, that does not scale with the total displacement of the stage. Assuming a constant fringe rate, the standard deviation of the time uncertainty between subsequent zero-crossings can be calculated as $\sim \pm 0.003$ s or a phase uncertainty of ± 0.89 rads, corresponding to a static displacement uncertainty of ~ 90 nm, which can be considered negligible in the scale of the displacements measured.

Wavelength Uncertainty

Of particular interest is the error due to wavelength uncertainty, discussed previously in Chp. 5 Sec. 2.3.5. This affects both laser techniques, and has both static and dynamic uncertainties associated with it. The static uncertainty originates in an uncertainty in the central wavelength of the laser, which is measured before any measurements occur, which is typically addressed by calibration of the laser emission. The dynamic uncertainty arises from the potential change in the laser emission wavelength whilst a measurement is underway, and stabilisation techniques are applied to reduce this uncertainty.

For the HeNe measurement technique, the central laser emission is specified by the manufacturer as $632.991234 \pm 0.0002 \text{ nm}$ in vacuum, where $\pm 0.0002 \text{ nm}$ is the static uncertainty in the HeNe laser emission wavelength. The dynamic uncertainty, in terms of frequency, is specified by the manufacturer over timescales of less than an hour, well above the measurement duration, as $\sim \pm 5 \times 10^8$ which corresponds to a dynamic wavelength uncertainty at $\sim 632.99 \text{ nm}$ of $\pm 1 \times 10^{-6} \text{ nm}$ over the measurement period. As expected, this stability is extremely high and over the timescales and displacement ranges of the presented experiments, would result in a negligible uncertainty in the displacement measurement.

For the RRI measurement, the uncertainty, both static and dynamic is determined by the precision of the stabilisation technique presented in Chp. 6 which the original intent of these experiments to experimentally determine. There is also a static uncertainty in the wavelength of the rubidium line, found from literature data of the absorption line to be $\pm 0.001 \text{ nm}$ [179].

Cyclic Errors

As discussed in Chp. 5, where a detailed analysis of the linearity potential of RRI is available in Sec. 5.5.2, RRI is capable of producing exceptionally linear results, due to the ability to identify and isolate spurious signals that would otherwise contribute parasitically to the demodulated phase signals of interest. Sub-nanometer linearities were achieved without additional correction, with confidence that this could be improved further with a more robust optical design. This is similarly true for the experimental setup presented in this chapter, benefiting from significantly simplified optical paths.

Further to this, for the HeNe interferometer, there is an assumption made that any cyclic errors present are stationary so become negligible when the quadrature point of the interferometers is fixed. A more likely source of cyclic errors could occur due to quantisation noise of the analogue-to-digital converter (ADC) recording low amplitude signals, where the current 14-bit resolution is spread over the $\pm 10 \text{ V}$ input. These cyclic errors due to quantisation noise could be significantly improved in a more robust optical design, where improved alignment permits larger interferometric signal amplitudes.

Thermal and Refractive Index Errors

While the dynamic uncertainty associated with thermal expansion of the components resulting in changes to the optical path length of the interferometer should be considered, in this experiment the total displacement amplitude is of secondary concern. Here the focus is on the relative difference between the two interferometric methods, which, if the optical paths are co-linear should result in identical displacement measurements regardless of dynamic thermal expansion.

With regards to the refractive index of the optical path medium, in this case air, there can be temperature, pressure and humidity changes that result in refractive index changes which scale with the total optical path length as a dynamic uncertainty. In addition, as the refractive index is wavelength dependent, each laser will effectively measure slightly different optical paths lengths as a static uncertainty. This static uncertainty can be determined using the modified Edlén equations [113, 115, 116] to estimate the fractional error caused by the difference in refractive index between the two laser interferometers. For an arbitrary temperature, pressure and humidity, the relative fractional error between the two refractive indices is;

$$\frac{(n)_{633\text{nm}}}{(n)_{780\text{nm}}} \approx \frac{1.000271375}{1.00027004} \approx 1.000001335 \quad (7.4)$$

This corresponds to an optical path length uncertainty due to refractive index differences of approximately 1.335 parts-per-million (ppm) which for the 10 mm stage motion described above is an uncertainty of approximately 13.35 nm. This is significantly below the resolution of the linear stage so can be considered negligible for these experiments.

The dynamic uncertainty due to changes in the refractive index is harder to state with confidence, as it must be determined from empirical data recorded at the time of the measurement. Given the duration of the experimental measurements, and allowing for operation in well-controlled lab conditions, it can be assumed that the refractive index will remain relatively stable, and this can be approximated as 1 ppm uncertainty, equivalent to a maximum error of up to 10 nm, arising from dynamic changes to the refractive index.

Summary of Uncertainties

The various sources of uncertainty in both measurements are summarised in table. 7.1 below. Notably, the units of the amplitude of uncertainty change contextually to the source of uncertainty.

TABLE 7.1: Summary of uncertainties affecting the comparison between HeNe and RRI measurements

Source of Uncertainty	Affecting HeNe, RRI or Both?	Static or Dynamic?	Amplitude of Uncertainty
<i>Phase Uncertainty</i>	HeNe	Static	± 0.89 rad
	RRI	Static	± 0.45 rad
<i>Wavelength Uncertainty</i>	HeNe	Static	± 0.0002 nm
	HeNe	Dynamic	$\pm 1 \times 10^{-6}$ nm
	RRI	Static	± 0.001 nm [179]
<i>Refractive Index Errors</i>	Both	Static	1.335 parts-per-million of total displacement
	Both	Dynamic	~ 1 part-per-million of total displacement

7.5 Discussion and Further Work

The work presented in this chapter represents a preliminary effort to compare a highly-stabilised HeNe laser with a widely-modulated laser diode through co-linear measurement of a bulk-optic Michelson interferometer. Experimental results for a 10 mm linear stage motion are presented and analysed. Initially, the phase measurements of both interferometer indicated that the evaluation wavelength of the widely-modulated laser diode was ~ 776 nm. However, closer examination of both signals revealed significant errors in the phase analysis. After correction, the results showed that the evaluation wavelength of the widely-modulated laser diode could be better calculated as 780.09 nm (in vacuum).

Some of the major sources of uncertainty associated with this experiment have been discussed including the phase uncertainty of both techniques as well as wavelength uncertainty, cyclic errors and thermal and refractive index errors. For the 10 mm stage motion presented, the uncertainty is largely due to phase uncertainty. There is some overlap between this value and the expected value of the evaluation wavelength 780.0268 nm (vac.), determined by the wavelength of the rubidium absorption line [179, 180], primarily due to the phase error from zero-crossing jitter. Alternative sources of error arise from the signal analysis itself. It is possible that the RRI signal processing systematically under or over counts fractions of the 2π phase interval, which would cause significant cumulative error. In addition, the errors in the repaired HeNe signal are still significant. For example, in Fig. 7.18 there are still non-linear errors of up to 30 rad, which would equate to approximately ± 0.13 nm and would dominate over other uncertainty sources. With these additional uncertainties, the true value for the evaluation wavelength falls well within the experimentally determined value with uncertainties.

To improve both this experiment and comparison, improvements could be made in the following areas:

Zero-crossing analysis

The jitter in the zero-crossing analysis introduces significant uncertainty in the phase measurement of the HeNe interferometer, which in turn creates uncertainty in the value of the measured displacement and the calculation of the emission wavelength of the widely-modulated laser diode. As discussed earlier, a more dynamic zero-crossing analysis function could be employed to allow for both low-visibility fringe detection, as well as partial and full fringe detection. This process would also benefit from a general increase in interferometric signal amplitude that a more robust experimental setup could achieve.

Interrogation Hardware

A significantly limiting factor in these experiments is the sample rate. Restricted by the interrogation hardware itself, the sample rate of 597 kHz limits several key parameters of the interrogation system. For 512 samples per modulation period, the maximum quadrature rate is limited to 1.133 kHz, which in turn limits the maximum measurable velocity to ~ 0.13 mm/s and the maximum measurable range to ~ 194 mm. These parameters can be easily improved using alternative FPGA-based interrogation systems with improved analogue-to-digital converters (ADCs) and digital-to-analogue converters (DACs), provided they can provide synchronisation between the two. A direct consequence of improved quadrature rates is the ability to measure much faster stage velocities, which would allow stage motions with HeNe fringe rates faster than the mechanical resonances discussed previously.

Refractive Index Errors

Although they are negligible in this experiment over the 10 mm stage range measured, refractive index errors can cause significant disparity between the two measurement methods over longer measurement periods and ranges if not taken into account. To achieve this, techniques such as the use of external refractometers [118] could be used to dynamically measure air refractive index, which could be used to calibrate results.

7.6 Summary

In this chapter, initial results for experiments comparing a highly wavelength stabilised HeNe laser and a widely wavelength-modulated laser diode, illuminating the same interferometer, are presented. An experimental setup is presented, using a bulk optic Michelson interferometer to measure fixed linear stage motions with both lasers simultaneously and co-linearly. The interrogation hardware is presented, describing the application of the swept absorption line locking technique introduced in Chp. 6 to stabilise the widely-modulated laser diode and synchronising the laser diode modulation and recording of the interferometric signals from each interferometer. The step-by-step analysis of each technique is discussed for a test measurement case of 10 mm stage motion

at 0.015 mm/s velocity. Further measurement sets for longer and faster stage motions were also recorded, however are not presented here.

The results presented here demonstrate the comparison of the two phase measurements to calculate the evaluation wavelength of the widely-modulated laser diode. For the 10 mm stage motion, this is calculated as 780.09 ± 0.13 nm. The large uncertainty on this value is predominantly from the uncertainty of the HeNe signal analysis, where low visibility HeNe interferometric signals contribute significantly to non-linear errors in the phase determination. Discussion is given to how this uncertainty, as well as the general interrogation system could be improved in future experiments to fully and accurately compare the HeNe interferometer and widely-modulated laser diode interferometer under a variety of conditions for a range of stage displacements and velocities.

Moving further with this work, the aim would be to reduce the uncertainties to sub parts-per-million so that the limiting uncertainty becomes the wavelength uncertainty of the swept absorption line locking technique, and high resolution measurements of the stage displacement could be made. A more streamlined and efficient analysis process for both the HeNe zero-crossings and the RRI phase unwrapping would help achieve this, coupled with improved interrogation hardware with higher sampling rates to allow for longer displacements and higher velocities than the current system can achieve. A more powerful interrogation unit with faster processing speeds may also allow for real-time processing of the two measurement channels, in addition to the synchronised laser modulation and stabilisation loop.

Chapter 8

Conclusions and Outlook

During the course of this PhD project, two significant developments, in line with the aims presented in Chp. 1, have been made in the area of displacement measuring optical metrology and are presented here. Firstly, using range-resolved interferometric (RRI) signal processing, novel designs of multi-dimensional encoders were developed and experimentally demonstrated, fulfilling the first aim of this thesis. Secondly, working towards completing the second aim of this thesis, a novel stabilisation technique for widely-wavelength modulated lasers, where the modulation depth is at least an order of magnitude larger than typical modulation schemes, was developed and experimental results were presented. Additional experimental work comparing a highly-stabilised Helium-Neon (HeNe laser) to a widely-wavelength modulated laser diode stabilised using the aforementioned novel technique was also undertaken, and the experimental results of this is presented, in-line with the third aim of this thesis.

8.1 Multi-dimensional Interferometric Encoders

Conclusions

The novelty of the presented multi-dimensional interferometric encoders originates from the multiplexing of several interferometers, comprising two or more dimensions of interest, into a single interferometric signal which is then demodulated using RRI signal processing. Each measurement dimension of interest can then be independently and concurrently evaluated by interrogation of the constituent interferometers, separated by optical path difference (OPD), without the need for spatially separated or polarised optical paths.

Literature examples of displacement measuring encoders, have utilised non-optical techniques such as capacitive, Piezoresistive and linear variable differential transformers (LVDT), or optical techniques such as linear optical encoders, to measure displacements over a variety of measurement ranges and resolutions. However, phase-evaluating laser interferometry has superseded most of these techniques both in terms of measurement range but also resolution, offering sub-nanometer order of magnitude measurements. The limitation of phase-evaluating laser interferometry however lies in the complexity of the both the optical and interrogation systems required to evaluate multiple-dimensions.

Typically, phase-evaluating interferometry can only evaluate interferograms from a single source at once, with the complexity scaling with the number of measurement dimensions of interest, however RRI allows for the discrimination and evaluation of multiple interferometers from a single interferogram by their optical path differences. This allows for novel, compact encoder designs with a single optical axis port, in contrast to existing encoder designs.

In this work towards the first main aim of this thesis, a novel optical design for a two-dimensional displacement encoder is presented and experimental results demonstrating measurements stage motion are shown. The stage motions consist of both single-axis linear motions, as well as two-axis circular motions over a notional stage range of $\pm 50 \mu\text{m}$, and the presented results of these demonstrate the capabilities of this novel encoder design. This design principle is then extended into a three-dimensional stage encoder, applying the same concept extended to another measurement dimension. Significant discussion is given in this work to the working process in extending this encoder design, in particular the arrangement of optical paths and the various conditions necessary, such that the interferometers of interest have unique OPDs. Experimental results and analysis for this encoder design are presented also for stage motions over $\pm 50 \mu\text{m}$ of both single-axis linear motions, as well as more complex, 3D-helical stage motions. The key parameters from the experimental results presented in this thesis are summarised in Table 8.1 below. From these measurements, analysis of the noise and linearity, which typically govern the lower limits of the resolutions achievable by interferometric encoders, was performed. This analysis is presented, and the values are included in Table 8.1.

TABLE 8.1: Key parameters from the work on multi-dimensional interferometric encoders presented in this thesis

Parameter	Value	Notes
Experimental Measurement Range	$\pm 50 \mu\text{m}$	See [164]
Quadrature Bandwidth	10 kHz	See Sec.5.7
Maximum Measurement Range	1.2 m	See Sec.5.7
Maximum Velocity	7.8 mm/s	See Sec.5.7
Instantaneous Noise	4.2 nm at 10 kHz Quadrature BW.	See Fig. 5.16
Typical Displacement Noise	$< 0.4 \text{ nm}/\sqrt{\text{Hz}}$	See Fig. 5.17
Minimum Displacement Noise	$< 0.01 \text{ nm}/\sqrt{\text{Hz}}$	See Fig. 5.17
Non-linearities	$< 1 \text{ nm}$	See Fig. 5.18
Phase error due to non-linearity	$< 0.5^\circ$	See Fig. 5.18

The presented results show very low non-linearities from cyclic errors below 1 nm and typical displacement noise levels below $0.4 \text{ nm}/\sqrt{\text{Hz}}$, dropping to $0.01 \text{ nm}/\sqrt{\text{Hz}}$ above 100 Hz, without further correction, which is highly competitive with the existing techniques presented in Chp. 2. Further to the two and three-dimensional displacement encoders, this work includes the application of RRI signal processing to a novel, two-beam angle and displacement encoder design that uses spatially offset beams to measure angle and displacement changes of a target surface. Here measurements of pitch angle changes are presented with good agreement between the interferometrically measured changes, and the in-built stage encoder.

Outlook

The results and performance from all three encoder designs demonstrate the capabilities of this approach towards displacement measuring encoders. Utilising RRI signal processing in combination with a simple, but well-planned optical design allows for multi-dimensional measurements with a high precision with results that are highly competitive with existing techniques. These encoder designs benefit from significantly simplified optical setups, without the need for polarising components, that can be made

extremely compact. By performing range-dependant demodulation of multiplexed interferometers simultaneously and independently, this work shows that novel configurations can be achieved, that could expand the usage of interferometric encoders into novel application where previously the typical drawbacks of interferometers restricted their usage. Notably, the compact optical design, simple cost-effective interrogation system and high precision may be of particular interest in industrial or engineering applications, such as positioning stages for nano-scale manufacturing, or where higher precision is required than traditional techniques can offer.

Extending this research beyond the work presented in this thesis, there are other configurations of multi-dimensional encoders that could be of potential interest for investigation. Benefiting from the use of small gradient-index (GRIN) optics, which have footprints as low as a few millimetres wide and tens of millimetres in length, the advantages of range-resolved interferometry could be applied to even smaller optical setups than in the work presented here. Initial work on this has been presented at conference [182] however significant investigation into the applications of GRIN lenses with RRI has yet to be undertaken. Conceptually, these GRIN lenses can produce a collimated beam from very compact optics, which could be combined with RRI to create novel configurations of interferometric sensor. One such concept is an extension of the dual-beam angle encoder and the multi-dimensional interferometric encoders presented in this thesis. Using GRIN lenses mounted on fibre-ferrules, a trio of optical beams could be formed in an exceptionally compact sensor head, which could be used for tilt and pitch measurements of a target surface as well as displacement. This concept is illustrated in Fig. 8.1, and would benefit from the high resolutions and linearity offered by RRI in a cost-effective, compact optical setup.

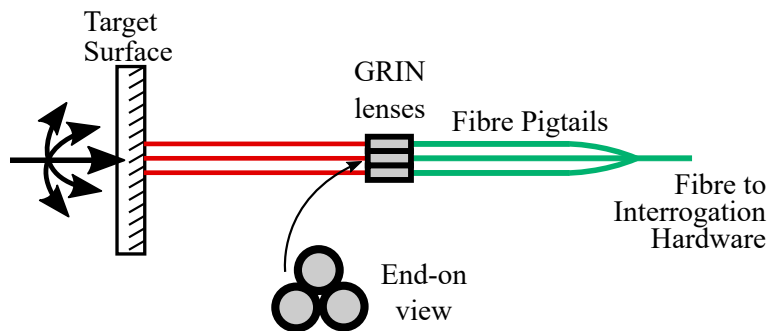


FIGURE 8.1: Concept for a multi-degree-of-freedom sensor using GRIN lenses in a triangular optical head

Beyond the development of new configurations of multi-dimensional encoder, further research could be undertaken in assessing the full potential of RRI itself. Aside from wavelength stabilisation, which is discussed in the following section, further investigation into the fine-tuning of the RRI parameters, discussed in Chp. 4 could be beneficial in achieving higher resolutions and precision with RRI.

8.2 Widely-wavelength modulated stabilisation

Conclusions

The novelty of the developed stabilisation technique arises from its intended application for locking the emission of widely-wavelength modulated lasers to a well-known, traceable reference. In these applications the modulation depth of the laser diode is at least an order of magnitude higher than the linewidth of typical stabilisation references, and imposes certain conditions on the locking technique.

In the literature, there are many different techniques designed for laser stabilisation in both long and short term applications. In short-term stabilisation, the focus is on reducing the linewidth of the laser emission, whereas in long-term stabilisation the focus is on removing any wavelength drift that might occur. Both types of locking apply principally the same techniques with slight differences depending on the time-scales of interest. Typical stabilisation techniques such as harmonic frequency stabilisation (HFS), tilt-locking and modulation transfer spectroscopy (MTS), use one of two references for stability; a high-finesse optical cavity such as a Fabry-Perot cavity, or an atomic absorption reference. However, with a modulation depth significantly wider than the linewidth of both optical cavities and absorption lines, these techniques are less effective or not applicable. Techniques that use the harmonic content of the modulation signal such as HFS have reduced effectiveness as the signal energy of the widely modulated signal is diluted across hundreds of harmonics of the modulation signal. In addition, with wide wavelength modulated techniques there is a secondary intensity modulation that is phase offset from the wavelength modulation that adds complication to direct measurement of the locking feature.

In the developed stabilisation technique, referred to as swept absorption line locking, the laser is directly swept through an atomic gas cell containing a gas with a dominant absorption feature within the wavelength range of the sweep and measured on a photodetector. As the laser wavelength sweeps past the absorption line, two clear troughs appear in the photodetector signal corresponding to the forward and reverse sweeps of the wavelength past the absorption line. Using standard line fitting techniques, a sine wave is fitted to the intensity modulation sweep, ignoring the absorption troughs, and this is used to correct for the intensity modulation, resulting in two clear peaks on an approximately flat baseline. To lock this wavelength sweep to the absorption line, the central wavelength emission of the laser is shifted using temperature setpoint tuning until the periodicity of the two peaks is exactly half the modulation period.

In this work towards the second aim of this thesis, the principles behind this technique are presented, alongside results of experiments to determine the efficacy of this locking technique. Here, experimental results showed an improved long-term stability of the widely wavelength modulated laser when the stabilisation loop was applied, showing improvements of an order of magnitude or more fractional stability increase at longer timescales. From an Allan deviation of a measurement over 24 hours of the stabilised laser, fractional stabilities as low as 6×10^{-8} were demonstrated for timescales over 1000 seconds with scope for significant improvement. Further work was also carried out in comparing a widely-modulated laser, stabilised using the novel developed technique,

and a highly stabilised HeNe laser, in line with the third aim of this thesis. In this experiment the two lasers measure the same displacement of a Michelson interferometer and the relative phase measurements are compared to determine the uncertainty in the widely modulated laser's stability, which is found to be in good agreement within experimental uncertainty, of the expected stabilisation locking.

Outlook

The preliminary experiments presented as part of this work show good promise for future work in this area. The developed stabilisation technique has been shown to improve the long-term stability of a widely-modulated laser, such as used in RRI, by at least an order of magnitude over a free-running alternative. However, significant improvements can still be made to this technique to allow for better stabilisation, including using narrower linewidth stability references such as Doppler-free absorption lines. In rubidium for example, the linewidths of the Doppler-free absorption lines are several orders of magnitude narrower than the Doppler-broadened lines used in this thesis, which, if robust and precise locking is achieved, could increase the stability of this technique by a similarly high precision.

In terms of the experiments presented in this thesis, the results of the comparison to a highly-stabilised HeNe still show scope for improvement, where a more robust mechanical setup, as well as a streamlined signal analysis process for both HeNe fringe counting and RRI signal processing could offer significantly improved results. In general, improved modulation and signal processing hardware, building on the work undertaken using the National Instruments development platform, would allow for more extensive measurements and comparisons, as well as unlocking higher precision measurements using these techniques. Outside of the work presented in this thesis, further investigation into existing techniques, adjusted to account for the wider modulation depths used by RRI, could be beneficial, using the knowledge gained from the development presented in this thesis.

In addition, further comparison of this technique, after improvement of the general stabilisation process, to existing, high precision and highly stable techniques would be significant in validating both the precision, linearity and resolution achievable by RRI, but to also show the extent of the stability achievable by the swept absorption line locking technique. Potential candidates for this comparison include x-ray interferometers which currently offer nearly unparalleled resolutions [70]. Comparison to these established and well-developed techniques would be hugely beneficial in bench-marking the capabilities of RRI and the presented stabilisation technique.

List of Publications

The following publications have resulted from the work presented in this thesis:

Journal Papers

1. K. B. Wiseman, T. Kissinger, and R. P. Tatam. Three-dimensional interferometric stage encoder using a single access port. *Optics and Lasers in Engineering* 137:106342 (10pp), 2021

Peer-reviewed Conference Papers

2. K. B. Wiseman, T. Kissinger and R. P. Tatam. Optical fibre-coupled displacement and angle measurements using range-resolved interferometry. *Proceedings of euspen's 21st International Conference & Exhibition, Copenhagen, DK, June 2021*, Proc. euspen ICE 21207 (2pp, Virtual Poster Presentation), 2021
3. T. Kissinger, K. B. Wiseman and R. P. Tatam. Highly linear, compact and cost-effective sensor probes for displacement measurements using range-resolved interferometry. *Proceedings of euspen's 21st International Conference & Exhibition, Copenhagen, DK, June 2021*, Proc. euspen ICE 21152 (2pp, Virtual Presentation, delivered by T. Kissinger), 2021
4. K. B. Wiseman, T. Kissinger, and R. P. Tatam. Multi-dimensional interferometric stage encoder using range-resolved interferometry. *Proceedings of euspen's 20th International Conference & Exhibition, Geneva, CH, June 2020*, Proc. euspen ICE 20130 (2pp, Session Keynote Presentation, delivered virtually due to COVID-19 pandemic), 2020

Other Conference Papers

5. K. B. Wiseman, T. Kissinger, and R. P. Tatam. Two-dimensional remote interferometric stage encoder through a single access port using range-resolved interferometry. *Proceedings of SPIE Optical Metrology. 2019, Munich, Germany*, Proc. SPIE 11056 (7pp, Oral Presentation), 2019

Further Conference Participation

6. K. B. Wiseman, T. Kissinger, and R. P. Tatam. Multi-dimensional remote stage metrology using range-resolved interferometric sensing. *Photon20, Virtual, (Oral presentation)*, 2020.
7. K. B. Wiseman, T. Kissinger, and R. P. Tatam. Novel interferometric stage encoder design for two-dimensional displacement measurements through a single optical access port. *SPIE Photonex Europe, Coventry, UK, (Oral Presentation)*, 2019.

References

- [1] J. S. Wilson. *Sensor technology handbook*. Elsevier, 2004.
- [2] J. Fraden. *Handbook of modern sensors: physics, designs, and applications*. Springer Science & Business Media, 2013.
- [3] R. Leach. "Displacement measurement". In: *Fundamental Principles of Engineering Nanometrology*. Second Edi. Elsevier Inc., 2014, pp. 95–132.
- [4] A. J. Fleming. "A review of nanometer resolution position sensors: Operation and performance". *Sensors and Actuators, A: Physical* **190** (2013), pp. 106–126.
- [5] V. G. Badami and P. J. de Groot. "Displacement measuring interferometry". In: *Handbook of Optical Dimensional Metrology*. CRC Press, 2016, pp. 157–238.
- [6] H. Haitjema. "The Calibration of Displacement Sensors". *Sensors* **20.3** (Jan. 2020), p. 584.
- [7] N. Bobroff. "Recent advances in displacement measuring interferometry". *Measurement Science and Technology* **4.9** (Sept. 1993), p. 907.
- [8] P. J. de Groot. "A review of selected topics in interferometric optical metrology". *Reports on Progress in Physics* **82.5** (May 2019), p. 056101.
- [9] P. C. Hu, D. Chang, J. B. Tan, R. T. Yang, H. X. Yang, and H. J. Fu. "Displacement measuring grating interferometer: a review". *Frontiers of Information Technology and Electronic Engineering* **20.5** (2019), pp. 631–654.
- [10] W. Gao et al. "On-machine and in-process surface metrology for precision manufacturing". *CIRP Annals* **68.2** (2019), pp. 843–866.
- [11] W. Gao et al. "Measurement technologies for precision positioning". *CIRP Annals* **64.2** (2015), pp. 773–796.
- [12] S. T. Smith and R. M. Seugling. "Sensor and actuator considerations for precision, small machines". *Precision Engineering* **30.3** (July 2006), pp. 245–264.
- [13] H. Schwenke, U. Neuschaefer-Rube, T. Pfeifer, and H. Kunzmann. "Optical Methods for Dimensional Metrology in Production Engineering". *CIRP Annals* **51.2** (2002), pp. 685–699.
- [14] A. Thompson, I. Maskery, and R. K. Leach. "X-ray computed tomography for additive manufacturing: A review". *Measurement Science and Technology* **27.7** (2016), p. 72001.
- [15] S. Brown, W. Sun, M. McCarthy, and P. Woolliams. "Assessment of dimensional measurement tools to measure additively manufactured metallic parts". In: *Proceedings - ASPE/euspen 2016 Summer Topical Meeting: Dimensional Accuracy and Surface Finish in Additive Manufacturing*. 2016, pp. 200–205.
- [16] Y. Jiang, W. Ding, P. Liang, L. Fu, and C. Wang. "Phase-shifted white-light interferometry for the absolute measurement of fiber optic Mach-Zehnder interferometers". *Journal of Lightwave Technology* **28.22** (2010), pp. 3294–3299.

- [17] C. K. Hitzengerger, W. Drexler, A. Baumgartner, H. Sattmann, and A. F. Fercher. "Retinal layers located with a precision of 5 μm by partial coherence interferometry". In: ed. by J.-M. Parel, Q. Ren, and K. M. Joos. Vol. 2393. May 1995, p. 176.
- [18] E. Hall. "On a new Action of the Magnet on Electric Currents". *Nature* **21**.537 (Feb. 1880), pp. 361–361.
- [19] D. S. Nyce. *Linear Position Sensors*. Hoboken, NJ, USA: John Wiley & Sons, Inc., Nov. 2003.
- [20] Z. Zhang, Y. Yu, X. Liu, and X. Zhang. "Electro-Magnetic Nano-Positioning". In: *Nanopositioning Technologies*. Ed. by C. Ru, X. Liu, and Y. Sun. Cham: Springer International Publishing, 2016, pp. 151–181.
- [21] Y.-P. Yang and Y.-Y. Ting. "Improved Angular Displacement Estimation Based on Hall-Effect Sensors for Driving a Brushless Permanent-Magnet Motor". *IEEE Transactions on Industrial Electronics* **61**.1 (Jan. 2014), pp. 504–511.
- [22] S. Du, J. Hu, Y. Zhu, and M. Zhang. "A hall sensor-based position measurement with on-line model parameters computation for permanent magnet synchronous linear motor". *IEEE Sensors Journal* **18**.13 (2018), pp. 5245–5255.
- [23] S. Paul, J. Chang, A. Rajan, and S. Mukhopadhyay. "Design of linear magnetic position sensor used in permanent magnet linear machine with consideration of manufacturing tolerances". *IEEE Sensors Journal* **19**.13 (2019), pp. 5239–5248.
- [24] A. J. Fleming and K. K. Leang. "Position Sensors for Nanopositioning". In: *Nanopositioning Technologies*. Ed. by C. Ru, X. Liu, and Y. Sun. Cham: Springer International Publishing, 2016, pp. 245–294.
- [25] T. T. Grove, M. F. Masters, and R. E. Miers. "Determining dielectric constants using a parallel plate capacitor". *American Journal of Physics* **73**.1 (2005), pp. 52–56.
- [26] M. Kim, W. Moon, E. Yoon, and K. R. Lee. "A new capacitive displacement sensor with high accuracy and long-range". *Sensors and Actuators, A: Physical* **130**–**131** (2006), pp. 135–141.
- [27] X. Liu, K. Peng, Z. Chen, H. Pu, and Z. Yu. "A new capacitive displacement sensor with nanometer accuracy and long range". *IEEE Sensors Journal* **16**.8 (2016), pp. 2306–2316.
- [28] D. Zhang, S. Zhao, Q. Zheng, and L. Lin. "Absolute capacitive grating displacement measuring system with both high-precision and long-range". *Sensors and Actuators, A: Physical* **295** (2019), pp. 11–22.
- [29] H. Pu, H. Liu, X. Liu, K. Peng, and Z. Yu. "A novel capacitive absolute positioning sensor based on time grating with nanometer resolution". *Mechanical Systems and Signal Processing* **104** (2018), pp. 705–715.
- [30] S. Xia and S. Nihtianov. "Capacitive sensors for displacement measurement in the subnanometer range". In: *Smart Sensors and MEMs*. Vol. 2. Elsevier, 2018, pp. 87–99.
- [31] S. Xia and S. Nihtianov. "Power-Efficient High-Speed and High-Resolution Displacement Measurements". **61**.5 (2012), pp. 1315–1322.
- [32] A. Rerkratn, A. Luangpol, W. Petchmaneelumka, and V. Riewruja. "Position signal detector for linear variable differential transformer". *Energy Reports* **6** (2020), pp. 603–607.

- [33] A. Barlian, W.-T. Park, J. Mallon, A. Rastegar, and B. Pruitt. "Review: Semiconductor Piezoresistance for Microsystems". *Proceedings of the IEEE* **97.3** (Mar. 2009), pp. 513–552.
- [34] N. Nikooienejad, M. Maroufi, and S. O. Reza Moheimani. "Characterization of a tilted-beam piezoresistive MEMS sensor with current-drive readout circuit". *Proceedings of the American Control Conference* **2019-July** (2019), pp. 2483–2488.
- [35] R. K. Messenger, Q. T. Aten, T. W. McLain, and L. L. Howell. "Piezoresistive feedback control of a mems thermal actuator". *Journal of Microelectromechanical Systems* **18.6** (2009), pp. 1267–1278.
- [36] M. Maroufi and S. O. Moheimani. "A 2DOF SOI-MEMS Nanopositioner with Tilted Flexure Bulk Piezoresistive Displacement Sensors". *IEEE Sensors Journal* **16.7** (2016), pp. 1908–1917.
- [37] K. Hane, Y. Uchida, and S. Hattori. "Moiré displacement measurement technique for a linear encoder". *Optics and Laser Technology* **17.2** (1985), pp. 89–95.
- [38] K. Engelhardt and P. Seitz. "High-resolution optical position encoder with large mounting tolerances". *Applied Optics* **36.13** (1997), p. 2912.
- [39] V. Saveljev and S.-K. Kim. "Theoretical estimation of moiré effect using spectral trajectories". *Optics Express* **21.2** (2013), p. 1693.
- [40] S. J. Cosijns, M. J. Jansen, and H. Haitjema. *Advanced Optical Incremental Sensors: Encoders and Interferometers*. Elsevier Ltd, 2018, pp. 245–290.
- [41] C. F. Kao, S. H. Lu, H. M. Shen, and K. C. Fan. "Diffractive laser encoder with a grating in littrow configuration". *Japanese Journal of Applied Physics* **47.3 PART 1** (2008), pp. 1833–1837.
- [42] S. Zhao, C. Hou, J. Bai, G. Yang, and F. Tian. "Nanometer-scale displacement sensor based on phase-sensitive diffraction grating". *Applied Optics* **50.10** (2011), pp. 1413–1416.
- [43] C. C. Hsu, H. Chen, H. Y. Tseng, S. C. Lan, and J. Lin. "High displacement resolution encoder by using triple grating combination interferometer". *Optics and Laser Technology* **105** (2018), pp. 221–228.
- [44] F. Gray. *Pulse Code Communication*. May 1953.
- [45] R. Doran. "The gray code". *Journal of Universal Computer Science* **13.11** (2007), pp. 1573–1597.
- [46] D. Gurauskis, A. Kilikevičius, and A. Kasparaitis. "Thermal and geometric error compensation approach for an optical linear encoder". *Sensors (Switzerland)* **21.2** (2021), pp. 1–16.
- [47] H. Haitjema. "Achieving traceability and sub-nanometer uncertainty using interferometric techniques". *Measurement Science and Technology* **19.8** (Aug. 2008), p. 084002.
- [48] J. Rozman and A. Pleteršek. "Linear optical encoder system with sinusoidal signal distortion below -60 dB". *IEEE Transactions on Instrumentation and Measurement* **59.6** (June 2010), pp. 1544–1549.
- [49] S. J. Cooper et al. "A compact, large-range interferometer for precision measurement and inertial sensing". *Classical and Quantum Gravity* **35.9** (May 2018), p. 095007.
- [50] E. Stanfield, J. Stoup, M. Braine, and T. Doiron. "Sphere diameter interferometry with nanometer uncertainty". *Metrologia* **57.2** (Apr. 2020), p. 025003.

- [51] C. Wang, L.-l. Xu, J. Zhu, Z.-w. Yuan, Y.-j. Yu, and A. K. Asundi. "A novel integrated fiber-optic interferometer model and its application in micro-displacement measurement". *Optics and Lasers in Engineering* **86** (2016), pp. 125–131.
- [52] J. Guan, P. Köchert, C. Weichert, R. Köning, L. Siaudinyte, and J. Flügge. "A differential interferometric heterodyne encoder with 30 picometer periodic nonlinearity and sub-nanometer stability". *Precision Engineering* **50** (2017), pp. 114–118.
- [53] S. Gibson, P. Coe, A. Mitra, D. Howell, and R. Nickerson. "Coordinate measurement in 2-D and 3-D geometries using frequency scanning interferometry". *Optics and Lasers in Engineering* **43.7** (July 2005), pp. 815–831.
- [54] B. Hughes and M. S. Warden. "A novel coordinate measurement system based on frequency scanning interferometry". In: *J. Coord. Metrol. Syst. Conf.* Vol. 8. 2013, pp. 18–24.
- [55] Z. Zhu and G. Wu. "Dual-Comb Ranging". *Engineering* **4.6** (2018), pp. 772–778.
- [56] K.-N. Joo and S.-W. Kim. "Absolute distance measurement by dispersive interferometry using a femtosecond pulse laser". *Optics Express* **14.13** (2006), p. 5954.
- [57] J. Hu, S. Du, Y. Zhu, and L. Chen. "Planar 3-degree-of-freedom position sensing method based on decoupling using a linear-Hall array". *International Journal of Applied Electromagnetics and Mechanics* **58.1** (2018), pp. 79–99.
- [58] Y. Kawato and W. J. Kim. "Multi-degree-of-freedom precision position sensing and motion control using two-axis hall-effect sensors". *Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME* **128.4** (2006), pp. 980–988.
- [59] Y. Shimizu, H. Matsukuma, and W. Gao. "Optical Sensors for Multi-Axis Angle and Displacement Measurement Using Grating Reflectors". *Sensors* **19.23** (Dec. 2019), p. 5289.
- [60] Yi-Cheng Chung, Kuang-Chao Fan, Bor-Cheng Lee, Y. C. Chung, K. C. Fan, and B. C. Lee. "Development of a novel planar encoder for 2D displacement measurement in nanometer resolution and accuracy". *Proceedings of the World Congress on Intelligent Control and Automation (WCICA)* (June 2011), pp. 449–453.
- [61] C.-C. Hsu, H. Chen, C.-W. Chiang, and Y.-W. Chang. "Dual displacement resolution encoder by integrating single holographic grating sensor and heterodyne interferometry". *Optics Express* **25.24** (2017), p. 30189.
- [62] S. Agarwal and C. Shakher. "In-plane displacement measurement by using circular grating Talbot interferometer". *Optics and Lasers in Engineering* **75** (2015), pp. 63–71.
- [63] A. Kimura, W. Gao, Y. Arai, and Z. Lijiang. "Design and construction of a two-degree-of-freedom linear encoder for nanometric measurement of stage position and straightness". *Precision Engineering* **34.1** (2010), pp. 145–155.
- [64] C.-C. Wu, Y.-Z. Chen, and C.-H. Liao. "Common-path laser planar encoder". *Optics Express* **21.16** (2013), pp. 18872–18883.
- [65] C. Lee, G. H. Kim, and S. K. Lee. "Design and construction of a single unit multi-function optical encoder for a six-degree-of-freedom motion error measurement in an ultraprecision linear stage". *Measurement Science and Technology* **22.10** (2011), p. 105901.
- [66] S.-W. Pan, H.-L. Hsieh, and W.-C. Wang. "6-DOF displacement and angle measurements using heterodyne laser encoder". In: *Instrumentation, Metrology, and Standards for Nanomanufacturing, Optics, and Semiconductors VII*. Ed. by M. T. Postek and N. G. Orji. Vol. 8819. 43. Sept. 2013, p. 881909.

- [67] A. A. Michelson and E. W. Morley. "On the relative motion of the Earth and the luminiferous ether". *American Journal of Science* **s3-34.203** (Nov. 1887), pp. 333–345.
- [68] M. Ikram and G. Hussain. "Michelson interferometer for precision angle measurement". *Applied Optics* **38.1** (1999), p. 113.
- [69] A. Bergamin, G. Cavagnero, L. Cordiali, and G. Mana. "Beam-astigmatism in laser interferometry". *IEEE Transactions on Instrumentation and Measurement* **46.2** (1997), pp. 196–200.
- [70] A. Yacoot and U. Kuetgens. "Sub-atomic dimensional metrology: Developments in the control of x-ray interferometers". *Measurement Science and Technology* **23.7** (2012).
- [71] M. Çelik, R. Hamid, U. Kuetgens, and A. Yacoot. "Picometre displacement measurements using a differential Fabry-Perot optical interferometer and an x-ray interferometer". *Measurement Science and Technology* **23.8** (2012).
- [72] M. Zhu, H. Wei, X. Wu, and Y. Li. "Fabry-Perot interferometer with picometer resolution referenced to an optical frequency comb". *Optics and Lasers in Engineering* **67** (2015), pp. 128–134.
- [73] K. J. Gåsvik. *Optical Metrology*. 1st. 1987.
- [74] M. Born et al. *Principles of Optics*. Cambridge University Press, Oct. 1999.
- [75] C. K. Kirkendall and A. Dandridge. "Overview of high performance fibre-optic sensing". *Journal of Physics D: Applied Physics* **37.18** (2004).
- [76] T. Kissinger. "Range-Resolved Optical Interferometric Signal Processing". PhD thesis. Cranfield University, 2015, p. 239.
- [77] A. C. Carneiro, A. P. Lopez-Barbero, V. N. Silva, C. Kitano, J. H. Galeti, and A. B. Dos Santos. "High Accuracy Homodyne Interferometric Method for Wide Dynamic Range Applications". *IEEE Sensors Journal* **20.8** (2020), pp. 4221–4228.
- [78] L. Salbut and K. Patorski. "Polarization phase shifting method for moire interferometry and flatness testing". *Applied Optics* **29.10** (1990), p. 1471.
- [79] C. R. Mercer and G. Beheim. "Fiber optic phase stepping system for interferometry". *Applied Optics* **30.7** (1991), p. 729.
- [80] A. Dorsey, R. J. Hocken, and M. Horowitz. "A low cost laser interferometer system for machine tool applications". *Precision Engineering* **5.1** (1983), pp. 29–31.
- [81] L. Yu et al. "A single-beam 3DoF homodyne interferometer". *Measurement Science and Technology* **31.8** (Aug. 2020), p. 084006.
- [82] B. Chen, W. Mao, Y. Lou, L. Yan, Z. Li, and Y. Yang. "Simultaneous measurement of the straightness error and its position using a modified Wollaston-prism-sensing homodyne interferometer". *Measurement Science and Technology* **31.8** (2020).
- [83] M. J. Downs and K. W. Raine. "An unmodulated bi-directional fringe-counting interferometer system for measuring displacement". *Precision Engineering* **1.2** (1979), pp. 85–88.
- [84] H. Fu, K. Wang, P. Hu, J. Tan, H. Yang, and R. Yang. "Homodyne Laser Vibrometer with Detectability of Nanoscale Vibration and Adaptability to Reflectivity". *IEEE Transactions on Instrumentation and Measurement* **69.2** (2020), pp. 542–548.
- [85] G. Molnar, S. Strube, P. Köchert, H. U. Danzebrink, and J. Flügge. "Simultaneous multiple degrees of freedom (DoF) measurement system". *Measurement Science and Technology* **27.8** (2016).

- [86] R. Dändliker, R. Thalmann, and D. Prongué. "Two-wavelength laser interferometry using superheterodyne detection". *Optics Letters* **13.5** (1988), p. 339.
- [87] H. Haitjema, S. J. A. G. Cosijns, N. J. J. Roset, and M. J. Jansen. "Improving a commercially available heterodyne laser interferometer to sub-nm uncertainty". In: *Recent Developments in Traceable Dimensional Measurements II*. Ed. by J. E. Decker and N. Brown. Vol. 5190. 0. Nov. 2003, p. 347.
- [88] N. M. Oldham, J. A. Kramar, P. S. Hetrick, and E. C. Teague. "Electronic limitations in phase meters for heterodyne interferometry". *Precision Engineering* **15.3** (1993), pp. 173–179.
- [89] N.-B. Yim, C. I. Eom, and S.-W. Kim. "Dual mode phase measurement for optical heterodyne interferometry". *Measurement Science and Technology* **11.8** (Aug. 2000), pp. 1131–1137.
- [90] F. C. Demarest. "High-resolution, high-speed, low data age uncertainty, heterodyne displacement measuring interferometer electronics". *Measurement Science and Technology* **9.7** (1998), pp. 1024–1030.
- [91] C. K. Chatterjea, S. W. James, and R. P. Tatam. "Pseudoheterodyne signal processing scheme for interrogation of fiber Bragg grating sensor arrays". In: *Laser Interferometry IX: Techniques and Analysis*. Ed. by M. Kujawinska, G. M. Brown, and M. Takeda. Vol. 3478. June 1998. June 1998, pp. 266–274.
- [92] D. Jackson, A. Kersey, M. Corke, and J. Jones. "Pseudoheterodyne detection scheme for optical interferometers". *Electronics Letters* **18.25-26** (1982), p. 1081.
- [93] R. P. Tatam, J. D. Jones, and D. A. Jackson. "Opto-electronic processing schemes for the measurement of circular birefringence". *Optica Acta* **33.12** (1986), pp. 1519–1528.
- [94] T. Suzuki, K. Kobayashi, and O. Sasaki. "Real-time displacement measurement with a two-wavelength sinusoidal phase-modulating laser diode interferometer." *Applied optics* **39.16** (June 2000), pp. 2646–52.
- [95] C. Zhang. "Sinusoidal phase-modulating laser diode interferometer for measuring angular displacement". *Optical Engineering* **43.12** (Dec. 2004), p. 3008.
- [96] I. Sakai, R. Youngquist, and G. Parry. "Multiplexing of optical fiber sensors using a frequency-modulated source and gated output". *Journal of Lightwave Technology* **5.7** (1987), pp. 932–940.
- [97] A. Kersey, A. Lewin, and D. Jackson. "Pseudo-heterodyne detection scheme for the fibre gyroscope". *Electronics Letters* **20.9** (1984), p. 368.
- [98] T. Kissinger, T. O. Charrett, and R. P. Tatam. "Range-resolved interferometric signal processing using sinusoidal optical frequency modulation". *Optics Express* **23.7** (2015), p. 9415.
- [99] M. Abramowitz and I. A. Stegun. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. National Bureau of Standards Applied Mathematics Series 55. Tenth Printing. ERIC, 1972.
- [100] A. Dandridge, A. B. Tveten, and T. G. Giallorenzi. "Homodyne Demodulation Scheme for Fiber Optic Sensors Using Phase Generated Carrier". *IEEE Journal of Quantum Electronics* **18.10** (1982), pp. 1647–1653.
- [101] V. S. Sudarshanam and K. Srinivasan. "Linear readout of dynamic phase change in a fiber-optic homodyne interferometer". *Optics Letters* **14.2** (Jan. 1989), p. 140.

- [102] W. Jin, L. M. Zhang, D. Uttamchandani, and B. Culshaw. "Modified J₁ ... J₄ method for linear readout of dynamic phase changes in a fiber-optic homodyne interferometer". *Applied Optics* **30.31** (1991), p. 4496.
- [103] G. Heinzel, F. Guzmán Cervantes, A. F. García Marin, J. Kullmann, W. Feng, and K. Danzmann. "Deep phase modulation interferometry". *Optics Express* **18.18** (Aug. 2010), p. 19076.
- [104] W. Hou and G. Wilkening. "Investigation and compensation of the nonlinearity of heterodyne interferometers". *Precision Engineering* **14.2** (Apr. 1992), pp. 91–98.
- [105] J. Thijssse, A. K. Jamting, N. Brown, and H. Haitjema. "The detection of cyclic nonlinearities in a ZMI2000 heterodyne interferometer". In: *SPIE Recent Developments in Traceable Dimensional Measurements III*. Ed. by J. E. Decker and G.-S. Peng. Vol. 5879. August 2005. Aug. 2005, p. 58790M.
- [106] C. M. Sutton. "Non-linearity in length measurement using heterodyne laser Michelson interferometry". *Journal of Physics E: Scientific Instruments* **20.10** (Oct. 1987), pp. 1290–1292.
- [107] H. Fu, R. Ji, P. Hu, Y. Wang, G. Wu, and J. Tan. "Measurement method for nonlinearity in heterodyne laser interferometers based on double-channel quadrature demodulation". *Sensors (Switzerland)* **18.9** (2018).
- [108] S. Olyaei and S. Hamed. "Correction of Nonlinearity in High-Resolution Nano-Displacement Measurements". In: *2008 International Symposium on High Capacity Optical Networks and Enabling Technologies*. IEEE, Nov. 2008, pp. 116–120.
- [109] P. L. M. Heydemann. "Determination and correction of quadrature fringe measurement errors in interferometers". *Applied Optics* **20.19** (Oct. 1981), p. 3382.
- [110] T. G. McRae, M. T. L. Hsu, C. H. Freund, D. A. Shaddock, J. Herrmann, and M. B. Gray. "Linearization and minimization of cyclic error with heterodyne laser interferometry". *Optics Letters* **37.13** (2012), pp. 2448–2450.
- [111] A. Bridges, A. Yacoot, T. Kissinger, D. A. Humphreys, and R. P. Tatam. "Correction of periodic displacement non-linearities by two-wavelength interferometry". *Measurement Science and Technology* **32.12** (Dec. 2021), p. 125202.
- [112] C. Steinmetz. "Sub-micron position measurement and control on precision machine tools with laser interferometry". *Precision Engineering* **12.1** (Jan. 1990), pp. 12–24.
- [113] B. Edlén. "The refractive index of air". *Metrologia* **2.2** (1966), pp. 71–80.
- [114] F. E. Jones. "Refractivity of Air." *Journal of Research of the National Bureau of Standards (United States)* **86.1** (1981), pp. 27–32.
- [115] K. P. Birch and M. J. Downs. "An updated Edlén equation for the refractive index of air". *Metrologia* **30.3** (1993), pp. 155–162.
- [116] K. P. Birch and M. J. Downs. "Correction to the Updated Edlén Equation for the Refractive Index of Air". *Metrologia* **31.4** (1994), pp. 315–316.
- [117] M. J. Downs and K. P. Birch. "Bi-directional fringe counting interference refractometer". *Precision Engineering* **5.3** (1983), pp. 105–110.
- [118] M. L. Eickhoff and J. L. Hall. "Real-time precision refractometry: new approaches". *Applied Optics* **36.6** (1997), pp. 1223–1234.
- [119] E. Abbe. "Messapparate für physiker". *Zeitschrift für Instrumentenkunde* **10.12** (1890), pp. 446–448.

- [120] T. Shi, D. Pan, S. Zhang, H. Shaag, and J. Chen. "A Compact Pumping Laser at 459 nm with 10⁻¹⁵ Stability for Cs Four-Level Active Optical Clock". In: *IFCS 2018 - IEEE International Frequency Control Symposium*. 2018, pp. 5–8.
- [121] M. H. M. Shamim et al. "Investigation of Self-Injection Locked Visible Laser Diodes for High Bit-Rate Visible Light Communication". *IEEE Photonics Journal* **10.4** (Aug. 2018), pp. 1–11.
- [122] J. Dale, B. Hughes, A. J. Lancaster, A. J. Lewis, A. J. H. Reichold, and M. S. Warden. "Multi-channel absolute distance measurement system with sub ppm-accuracy and 20 m range using frequency scanning interferometry and gas absorption cells". *Optics Express* **22.20** (2014), p. 24869.
- [123] T. Simpson, T. Day, F. Doft, M. Malley, and G. Sutton. "Frequency-stabilized mode-locked solid-state laser system for precision range-Doppler imaging". *IEEE Journal of Quantum Electronics* **29.9** (1993), pp. 2489–2496.
- [124] R. Liu, Y. Li, C. Qian, D. Li, J. Leng, and J. Zhao. "Optical frequency standard of continuous wave for fiber communication based on optical comb". *Optics Communications* **427**.June (2018), pp. 1–5.
- [125] J. Zhang et al. "A long-term frequency stabilized deep ultraviolet laser for Mg⁺ ions trapping experiments". *Review of Scientific Instruments* **84.12** (2013).
- [126] D. Wang, W. Bu, D. Xie, T. Chen, and B. Yan. "Compact frequency-stabilization scheme for laser cooling of polar molecules". *Journal of the Optical Society of America B* **35.7** (July 2018), p. 1658.
- [127] A. Perot and C. Fabry. "On the Application of Interference Phenomena to the Solution of Various Problems of Spectroscopy and Metrology". *The Astrophysical Journal* **9**.January (1899), p. 87.
- [128] I. E. Gordon et al. "The HITRAN2016 molecular spectroscopic database". *Journal of Quantitative Spectroscopy and Radiative Transfer* **203** (2017), pp. 3–69.
- [129] M. Musha, T. Kanaya, K. Nakagawa, and K. I. Ueda. "The short- and long-term frequency stabilization of an injection-locked Nd:YAG laser in reference to a Fabry-Perot cavity and an iodine saturated absorption line". *Optics Communications* **183.1-4** (2000), pp. 165–173.
- [130] Y. Yao, C. Zou, H. Yu, J. Guo, Y. Li, and J. Liu. "The developing condition analysis of semiconductor laser frequency stabilization technology". *Journal of Semiconductors* **39.11** (2018).
- [131] H. Tsuchida, M. Ohtsu, T. Tako, N. Kuramochi, and N. Oura. "Frequency Stabilization of AlGaAs Semiconductor Laser Based on the 85 Rb-D 2 Line". *Japanese Journal of Applied Physics* **21**.Part 2, No. 9 (1982), pp. L561–L563.
- [132] E. D. Black. "An introduction to Pound-Drever-Hall laser frequency stabilization". *American Journal of Physics* **69.1** (2001), pp. 79–87.
- [133] R. W. Drever et al. "Laser phase and frequency stabilization using an optical resonator". *Applied Physics B Photophysics and Laser Chemistry* **31.2** (1983), pp. 97–105.
- [134] S. A. Webster, M. Oxborrow, and P. Gill. "Subhertz-linewidth Nd:YAG laser". *Optics Letters* **29.13** (2004), p. 1497.
- [135] J. Yu, Y. Qin, Z. Yan, H. Lu, and X. Jia. "Improvement of the intensity noise and frequency stabilization of Nd:YAP laser with an ultra-low expansion Fabry-Perot cavity". *Optics Express* **27.3** (2019), p. 3247.

- [136] S. Andreou, K. A. Williams, and E. A. Bente. "Frequency Stabilization of an InP-Based Integrated Diode Laser Deploying Electro-Optic Tuning". *IEEE Photonics Technology Letters* **31.24** (2019), pp. 1983–1986.
- [137] D. A. Shaddock, M. B. Gray, and D. E. McClelland. "Frequency locking a laser to an optical cavity by use of spatial mode interference". *Optics Letters* **24.21** (1999), p. 1499.
- [138] N. P. Robins, B. J. J. Slagmolen, D. A. Shaddock, J. D. Close, and M. B. Gray. "Interferometric, modulation-free laser stabilization". *Optics Letters* **27.21** (2002), p. 1905.
- [139] M. D. Harvey and A. G. White. "Frequency locking by analysis of orthogonal modes". *Optics Communications* **221.1-3** (2003), pp. 163–171.
- [140] B. J. Slagmolen, D. A. Shaddock, M. B. Gray, and D. E. McClelland. "Frequency stability of spatial mode interference (tilt) locking". *IEEE Journal of Quantum Electronics* **38.11** (2002), pp. 1521–1528.
- [141] J. L. Bueno Escobedo et al. "Self-injection locking of the DFB laser through an external ring fiber cavity: Application for phase sensitive OTDR acoustic sensor". *Results in Physics* **7** (2017), pp. 641–643.
- [142] V. V. Spirin, J. L. Bueno Escobedo, D. A. Korobko, P. Mégret, and A. A. Fotiadi. "Stabilizing DFB laser injection-locked to an external fiber-optic ring resonator". *Optics Express* **28.1** (2020), p. 478.
- [143] Y.-H. Lai et al. "780 nm narrow-linewidth self-injection-locked WGM lasers". March 2020 (2020), p. 23.
- [144] C.-I. Pan and P.-Y. Jean. "Stabilization of internal-mirror He-Ne lasers". *Applied Optics* **25.13** (July 1986), p. 2126.
- [145] M. W. Hamilton. "An introduction to stabilized lasers". *Contemporary Physics* **30.1** (Jan. 1989), pp. 21–33.
- [146] P. E. Ciddor and R. M. Duffy. "Two-mode frequency-stabilised He-Ne (633 nm) lasers: studies of short- and long-term stability". *Journal of Physics E: Scientific Instruments* **16.12** (Dec. 1983), pp. 1223–1227.
- [147] C. Glaser et al. "Absolute frequency measurement of rubidium 5S-6P transitions". *Physical Review A* **102.1** (2020), pp. 1–7.
- [148] Y.-H. Chen, W.-C. Lin, J.-T. Shy, and H.-C. Chui. "Iodine-stabilized single-frequency green InGaN diode laser". *Optics Letters* **43.1** (Jan. 2018), p. 126.
- [149] J. H. Shirley. "Modulation transfer processes in optical heterodyne saturation spectroscopy". *Optics Letters* **7.11** (1982), p. 537.
- [150] S. Lee et al. "Compact modulation transfer spectroscopy module for highly stable laser frequency". *Optics and Lasers in Engineering* **146**.April (2021), p. 106698.
- [151] J. A. Stone, A. Stejskal, and L. Howard. "Absolute interferometry with a 670-nm external cavity diode laser". *Applied Optics* **38.28** (1999), p. 5981.
- [152] T. Kinder and K. D. Salewski. "Absolute distance interferometer with grating-stabilized tunable diode laser at 633 nm". *Journal of Optics A: Pure and Applied Optics* **4.6** (2002).
- [153] Y. Salvadé, N. Schuhler, S. Lévêque, and S. L. Floch. "High-accuracy absolute distance measurement using frequency comb referenced multiwavelength source". *Applied Optics* **47.14** (2008), pp. 2715–2720.

- [154] F. Pollinger, K. Meiners-Hagen, M. Wedde, and A. Abou-Zeid. "Diode-laser-based high-precision absolute distance interferometer of 20m range". *Applied Optics* **48.32** (2009), pp. 6188–6194.
- [155] T. Kissinger, R. Correia, T. O. Charrett, S. W. James, and R. P. Tatam. "Fiber segment interferometry for dynamic strain measurements". *Journal of Lightwave Technology* **34.19** (2016), pp. 4620–4626.
- [156] T. Kissinger, E. Chehura, S. E. Staines, S. W. James, and R. P. Tatam. "Dynamic Fiber-Optic Shape Sensing Using Fiber Segment Interferometry". *Journal of Lightwave Technology* **36.4** (Feb. 2018), pp. 917–925.
- [157] T. Kissinger, T. O. H. Charrett, S. W. James, A. Adams, A. Twin, and R. P. Tatam. "Simultaneous laser vibrometry on multiple surfaces with a single beam system using range-resolved interferometry". *Optical Measurement Systems for Industrial Inspection IX* **9525**. June 2015 (2015), p. 952520.
- [158] K. B. Wiseman, T. Kissinger, and R. P. Tatam. "Three-dimensional interferometric stage encoder using a single access port". *Optics and Lasers in Engineering* **137**. August 2020 (Feb. 2021), p. 106342.
- [159] J. Bremner, T. Kissinger, J. Hodgkinson, and R. Tatam. "A comparison of topologies used in an interferometric gas detection technique". **1135419**. April 2020 (2020), p. 49.
- [160] J. A. A. Bremner, T. Kissinger, J. Hodgkinson, and R. P. Tatam. "Fibre-coupled, multiplexed methane detection using range-resolved interferometry". *Journal of Physics: Photonics* **3.2** (Apr. 2021), 02LT01.
- [161] J. M. Hallam, T. Kissinger, T. O. H. Charrett, and R. P. Tatam. "In-process range-resolved interferometric (RRI) 3D layer height measurements for wire + arc additive manufacturing (WAAM)". *Measurement Science and Technology* **33.4** (Apr. 2022), p. 044002.
- [162] J. Zheng. "Analysis of optical frequency-modulated continuous-wave interference". *Applied optics* **43.21** (2004), pp. 4189–4198.
- [163] K. Itoh. "Analysis of the phase unwrapping algorithm". *Applied Optics* **21.14** (1982), p. 2470.
- [164] Physik Instrumente Ltd UK. *P-611.3 Datasheet*. 2019.
- [165] A. J. Fresnel. *Mémoire sur la loi des modifications que la réflexion imprime à la lumière polarisée*. Académie des Sciences, 1823.
- [166] SCHOTT. *Optical Class Data Sheets*. 2014.
- [167] Z. Liu et al. "Two-color heterodyne laser interferometry for long-distance stage measurement with correction of uncertainties in measured optical distances". *Scientific Reports* **7.1** (2017), pp. 1–10.
- [168] O. Gerberding, K. S. Isleif, M. Mehmet, K. Danzmann, and G. Heinzl. "Laser-Frequency Stabilization via a Quasimonolithic Mach-Zehnder Interferometer with Arms of Unequal Length and Balanced dc Readout". *Physical Review Applied* **7** (2017), p. 024027.
- [169] A. D. McNaught and A. Wilkinson. "Beer–Lambert law (or Beer–Lambert–Bouguer law)". In: *IUPAC. Compendium of Chemical Terminology, 2nd ed. (the "Gold Book")*. Oxford: Blackwell Scientific Publications, Feb. 1997.
- [170] I. Gordon et al. "The HITRAN2020 molecular spectroscopic database". *Journal of Quantitative Spectroscopy and Radiative Transfer* (Sept. 2021), p. 107949.

- [171] K. Levenberg. "A method for the solution of certain non-linear problems in least squares". *Quarterly of Applied Mathematics* **2.2** (July 1944), pp. 164–168.
- [172] D. W. Marquardt. "An Algorithm for Least-Squares Estimation of Nonlinear Parameters". *Journal of the Society for Industrial and Applied Mathematics* **11.2** (June 1963), pp. 431–441.
- [173] Wavelength References. *Acetylene Gas Cell Datasheet*. 2021.
- [174] D. W. Allan. "Statistics of Atomic Frequency Standards". *Proceedings of the IEEE* **54.2** (1966), pp. 221–230.
- [175] D. W. Allan. "Time and Frequency (Time-Domain) Characterization, Estimation, and Prediction of Precision Clocks and Oscillators". *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control* **34.6** (1987), pp. 647–654.
- [176] D. Howe, D. Allan, and J. Barnes. "Properties of Signal Sources and Measurement Methods". In: *Thirty Fifth Annual Frequency Control Symposium*. IEEE, 1981, pp. 669–716.
- [177] W. C. Swann and S. L. Gilbert. "Pressure-induced shift and broadening of 1510–1540-nm acetylene wavelength calibration lines". *Journal of the Optical Society of America B* **17.7** (July 2000), p. 1263.
- [178] J. Ma, P. Shi, X. Qian, Y. Shang, and Y. Ji. "Optical spin noise spectra of Rb atomic gas with homogeneous and inhomogeneous broadening". *Scientific Reports* **7.1** (2017), pp. 1–7.
- [179] I. Johansson. "Spectra of the alkali metals in the lead-sulphide region". *ARKIV FOR FYSIK* **20.1-2** (1961), p. 135.
- [180] J. E. Sansonetti. "Wavelengths, transition probabilities, and energy levels for the spectra of rubidium (Rb i through Rb XXXVII)". *Journal of Physical and Chemical Reference Data* **35.1** (2006), pp. 301–421.
- [181] R. Thapa. "Doppler-free spectroscopy of acetylene in near infrared spectral region inside photonic band gap fiber". PhD thesis. Kansas State University, 2005, p. 50.
- [182] T. Kissinger, K. B. Wiseman, and R. P. Tatam. "Highly linear, compact and cost-effective sensor probes for displacement measurements using range-resolved interferometry". In: *Proceedings of the 21st International Conference of the European Society for Precision Engineering and Nanotechnology, EUSPEN 2021*. June. 2021, pp. 467–468.