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A STACKELBERG-NASH GAME FRAMEWORK FOR OVERCOMING REMANUFACTURING BARRIERS IN A DUOPOLY

Najmeh Abbasi[1], M. M. Lotfi[*1,2], Davood Shishebori[1], Asefe Forghani [3,4] and Mahboobeh Honarvar [1]

[1]Department of Industrial Engineering, Yazd University, Yazd, Iran

[2]Department of Industrial Engineering, Faculty of Engineering, College of Farabi, University of Tehran, Tehran, Iran

[3]Cranfield School of Management, Cranfield University, Cranfield MK43 0AL, United Kingdom

[4]Department of People and Technology, Roskilde University, Universitetsvej 1, 4000 Roskilde, Denmark

Abstract. In duopoly markets, less-preferred brands with lower customer acceptance often hesitate to adopt environmentally beneficial remanufacturing due to profitability concerns, especially when decisions involve the entire supply chain. This study proposes a Stackelberg–Nash game framework to explore optimal remanufacturing and pricing strategies for two competing supply chains, each comprising a manufacturer and an exclusive retailer. The model analyses both external competition between brands and internal competition between new and remanufactured products, considering customer acceptance levels and the availability of used products suitable for remanufacturing. Analytical results show that when manufacturing costs are low, remanufacturing increases the profit of the less-preferred brand without affecting the more-preferred one. When costs are high and the less-preferred brand is supported by its retailer, numerical results indicate that remanufacturing can still enhance profitability, with up to 24% higher profit compared to the no-remanufacturing case. However, when costs are prohibitively high, remanufacturing is unlikely to be viable without external incentives. Sensitivity analysis shows that increasing the customer acceptance level of the less-preferred brand typically reduces the more-preferred brand's profit and increases the less-preferred brand's profit, except when acceptance exceeds 80% under low-cost conditions, where intensified competition reduces the latter's profitability. Under high-cost conditions, increasing acceptance from 70% to 90% may lead to a 56% reduction in the more-preferred brand's profit and a 44% increase in the less-preferred brand's profit. The level of acceptance for remanufactured products also affects profitability differently across cost structures and strategic conditions, revealing that internal product competition shifts substantially depending on market and operational parameters.

Key words and phrases. Stackelberg-Nash game, Duopoly markets, Closed-loop supply chain, Remanufacturing decisions, Pricing strategies.

*Corresponding author: M. M. Lotfi.

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Nomenclature

B_1, B_2	Competing brands 1 and 2, respectively, assuming that the acceptance level of new products from brand 1 is higher than that of brand 2
θ	Customer acceptance level for new products of B_2
α	Customer acceptance level for remanufactured products of B_2
z	Customer's perceived value as a uniform distribution function in the range $[0, 1]$
δ_2	Ratio of high-quality used products from B_2 available for remanufacturing to the total products returned.
$c_{n,1}$	Manufacturing cost per unit for new products of B_1
$c_{n,2}$	Manufacturing cost per unit for new products of B_2
$c_{r,2}$	Remanufacturing cost per unit for remanufactured products of B_2
$c_{t,2}$	Collection cost by the retailer in B_2 supply chain
$c_{m,2}$	Return cost to the manufacture in B_2 supply chain
$w_{n,1}$	Wholesale price for new products from manufacturer B_1
$w_{n,2}$	Wholesale price for new products from manufacturer B_2
$w_{r,2}$	Wholesale price for remanufactured products from manufacturer B_2
$p_{n,1}$	Retail price for new products of B_1
$p_{n,2}$	Retail price for new products of B_2
$p_{r,2}$	Retail price for remanufactured products of B_2
NR	Case without remanufacturing
OR2	Case with remanufacturing of high-quality used products of B_2
OR2a, OR2b, etc.	Specific strategies under various conditions.

1. **Introduction.** Remanufacturing reduces waste, conserves resources, and extends product life, while simultaneously boosting profitability and supporting the transition to a circular economy [5]. Studies show remanufacturing reduces raw material use by 70%, emissions by 80%, and energy consumption by 60% [10]. Despite its benefits, remanufacturing is not always profitable due to the 'cannibalization effect,' where remanufactured products reduce new product sales, impacting market share [12]. Concerns about market cannibalization often hinder the adoption of closed-loop supply chains, causing companies to opt for recycling or disposal of used products instead of engaging in remanufacturing [3]. Weaker brands perceive themselves as more vulnerable to profitability losses and hesitate to adopt remanufacturing, fearing additional market declines. Dominant brands, however, are better positioned to take risks, using strategies like aggressive pricing to defend their position (pro-competitive effect) [9]. For weaker brands, even short-term losses can threaten survival.

This study examines two competing brands in a duopoly, common in industries like automobiles (e.g., Ford vs. GM) and mobile telecommunications (e.g., Apple vs. Samsung), where one brand is typically less preferred. To capture customer preferences and perceived superiority of one brand over the other, a parameter called the 'Customer Acceptance Level' is introduced [13].

Duopoly, even in its simplest form (two players, homogeneous product, quantity competition), can yield multiple outcomes based on players' strategies [20]. In duopoly markets, brand dynamics heavily influence pricing strategy. Remanufacturing adds external competition between brands and internal competition within each brand, where new and remanufactured products compete for demand. When extended to supply chains, profitability depends on decisions made by all members, including those in competing chains. Success in this environment requires balancing sustainability, profitability, and customer satisfaction.

This study employs a game model to analyze dual competition in two supply chains, each comprising a manufacturer and an exclusive retailer. Game theory offers a robust framework for analyzing strategic interactions, guiding brand extension decisions, and understanding competitive behavior in dynamic markets such as duopolies [4]. A game reaches Nash equilibrium when no player can profit by unilaterally deviating from their strategy, assuming all others stick to theirs [15].

In this study, the proposed game-theoretic model examines internal competition between new and remanufactured products within the less-accepted brand and external competition between both brands' products. Figure. 1 illustrates this dual

competition. A Stackelberg game framework is used to address profitability concerns for the less-accepted brand, determining optimal pricing strategies under both remanufacturing and non-remanufacturing scenarios. The study seeks to answer the following research questions:

- Under what conditions will a brand with lower customer acceptance adopt remanufacturing?
- How do manufacturing costs and retailer support influence the profitability of a remanufacturing strategy for the less accepted-brand?
- What manufacturing and pricing strategies should the supply chain members of the two brands adopt when the less accepted-brand chooses remanufacturing?

In brief, this research examines dual competition and interactions within two competing supply chains. A key constraint is that remanufactured quantities are limited by high-quality used products. Using a game-theoretic framework, the study identifies when remanufacturing is profitable for the less-accepted brand and derives optimal pricing strategies for manufacturers and retailers.

The rest of the paper is organized as follows: Section 2 reviews relevant literature to identify the research gap and highlight the contribution. Sections 3 and 4 present the problem formulation and derive optimal strategies. Numerical results are discussed in Section 5, followed by conclusions in Section 6. Finally, please refer to the supplementary file for additional mathematical details, proofs, tables, and figures supporting numerical analysis.

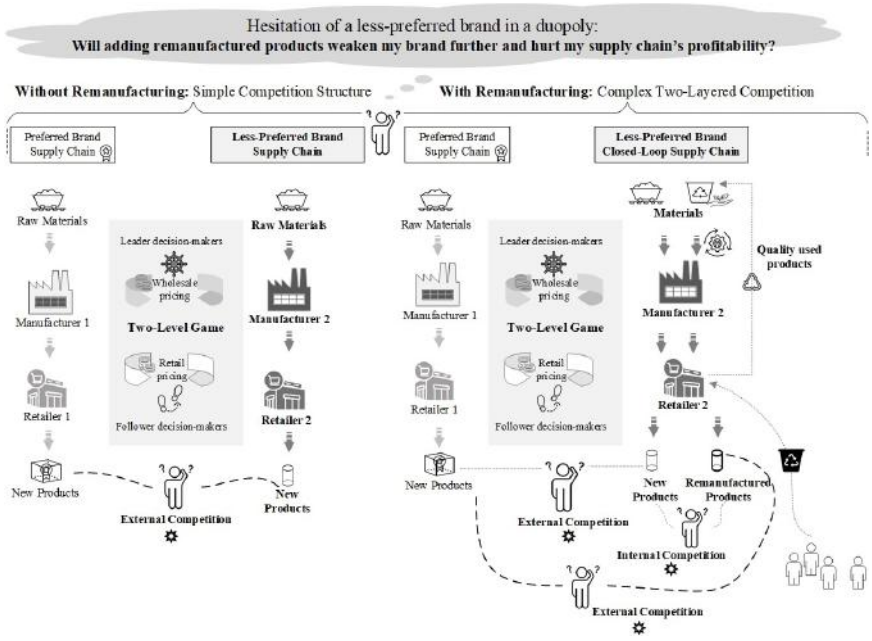


FIGURE 1. Conceptual Representation of Dual Competition Dynamics in the Duopoly Market

TABLE 1. Overview of This Study in Context of Prior Literature.

Authors and Year	Competitive Setting	Participants	Competitive Interaction	Distribution and Remanufacturing Limits
Jena and Sarmah (2014) [8]	Single-Period, DPL	2M, 1RE	Ext (New)	Trad. / None
Zhao et al. (2014) [26]	Single-Period, DPL	2M, 1RE	Ext (New)	Trad. / None
Wei and Zhao (2015) [21]	Single-Period, DPL	2C (1M, 2 Excl. RE, 1R)	Ext (New)	Trad. / None
Yang et al. (2015) [24]	Single-Period, DPL	2C (1M, 2 Excl. RE)	Ext (New)	Trad. or Online/ None
Matsui (2016) [14]	Single-Period, DPL	2C (1M, 2 Excl. RE)	Ext (New)	Trad. and Online/ None
Luo et al. (2017) [13]	Single-Period, DPL	2M, 1RE	Ext* (New)	Trad. / None
Liu et al. (2018) [11]	Multi-Period, MPL	1M	Int* (New and Re)	Trad. / Constrained
Hong and Zhang (2019) [7]	Multi-Period, MPL	1M, 1R CORE	Int (New and Re)	Trad. / Constrained
Sun et al. (2020) [19]	Single-Period, DPL	1OEM, 1R,1RE	Ext (New and Re)	Trad. / None
Wu et al. (2020) [22]	Single-Period, DPL	2OEM, 1COL	Both* (Remanufactured treated as new)	Trad. / None
Liu et al. (2021) [12]	Multi-Period, MPL	1M, 1RE	Int* (New and Re)	Trad. and Online / Constrained
Qiao and Su (2021) [18]	Single-Period, MPL and DPL	1OEM and 1OEM, 1R	Int/Ext (New and Re)	Trad. / None
Dey and Giri (2021) [6]	Single-Period, DPL	2S, 1M, 2 Excl. RE	Ext (New and Re)	Trad. / None
Zhang and Hezarkhani (2021) [25]	Single-Period, MPL and DPL	1M, 1RE and 2M, 1RE	Int/Ext (New)	Trad. and Online / None
Zhou and Gupta (2022) [27]	Single-Period, DPL	1M, 1RE, 1R	Int* (New and Re)	Trad. None
Yan and Han (2022) [23]	Single-Period, DPL	1OEM, 1RE		Online. None
Pal et al. (2023) [16]	Single-Period, DPL	2M	Int* (New and Re)	Trad. or Online/ None
Patoghi et al. (2024) [17]	Single-Period, DPL	1M, 1RE, 1COL	Ext(New)	Trad. / None
Abbasi et al. (2024) [2]	Single-Period, DPL	2C (2M, 2 Excl. RE)	Int (New and Re)	Trad. / Constrained
Abbasi et al. (2025) [1]	Single-Period, DPL	2M	Both* (More-accepted brand remanufactures)	Trad. / Constrained
Present study	Single-Period, DPL	2C (2M, 2 Excl. RE)	Both* (More-accepted brand remanufactures)	Trad. / Constrained

2. **Literature review.** Table 1 summarizes the key features of our model compared to relevant literature. The Competitive Setting column first indicates the number of periods in the model (Single-Period or Multi-Period). If the period is not explicitly stated but implied, it is marked as Single-Period. It also specifies the market type: DPL for duopoly and MPL for monopoly. The Participants column includes key players: OEM (Original Equipment Manufacturer), C (Supply Chain), R (Remanufacturer), RE (Retailer), Excl. RE (Exclusive Retailer, if each supply chain has a dedicated retailer), COL (Collector), R CORE (Core Remanufacturer), S (Supplier), and M (Manufacturer). The Competitive Interaction column defines competition: Int (Internal: between new and remanufactured products within a brand), Ext (External: between competing supply chains), New (New products only), Re (Remanufactured products), and Both (Internal and External competition). An asterisk (*) marks models that explicitly consider customer acceptance levels. The Distribution and Limits column describes sales channels (Trad. for traditional, Online, or Both) and whether remanufacturing is Constrained (with quantity limits) or None (unrestricted).

Closed-loop supply chains (CLSCs), which integrate forward and reverse logistics to enhance sustainability, have received significant attention in recent years. Numerous studies have investigated pricing-related decisions in CLSCs, including collection, recycling, trade-in strategies, pricing based on used product quality, manufacturer–retailer agreements, and customer segment pricing [18]. However, the development of game-theoretic frameworks for optimising pricing and manufacturing strategies in competitive environments remains underexplored. This is especially important for achieving a balance between environmental objectives and economic profitability, which is the focus of our study.

Several earlier works have examined competitive settings involving both new and remanufactured products. For example, Jena and Sarmah [8] analysed competition between two manufacturers collecting used products through a shared retailer but assumed new and remanufactured products are identical and did not consider customer acceptance. Zhao et al. [26] studied pricing for interchangeable products within centralised and decentralised frameworks but did not include customer acceptance or competition between new and remanufactured products. Similarly, Wei and Zhao [21] developed five game models involving pricing and remanufacturing decisions but treated new and remanufactured products as fully substitutable and did not consider acceptance or quantity constraints.

Yang et al. [24] introduced brand equity and substitutability into their competitive setting but excluded channel costs and customer-perceived value. Matsui [14] and Zhang and Hezarkhani [25] explored channel selection strategies. Matsui focused on dual channels, while Zhang examined consumer preferences and operational costs. However, both studies omitted internal competition between new and remanufactured products and did not account for customer acceptance. Luo et al. [13] examined horizontal and vertical competition among manufacturers and a retailer but excluded both remanufacturing and retailer competition, which are essential elements in our model.

Liu et al. [11] studied a monopolistic remanufacturer using a two-period model with customer acceptance, remanufacturing rates, and quality differentiation. Hong and Zhang [7], and Sun et al. [19], analysed two-stage models with manufacturers, remanufacturers, and retailers, introducing production constraints and warranties. While insightful, these studies focus on multi-period settings and involve third-party remanufacturers. Our study instead adopts a single-period setting and models competition between two full supply chains, each with its own manufacturer and exclusive retailer.

Wu et al. [22] used game-theoretic tools to study pricing, including effects like cannibalisation and customer attraction. However, this work did not include dual competition structures or constraints on remanufacturing. Dey and Giri [6] analysed CSR-driven supply chains but did not address inter-brand rivalry or the role of customer acceptance, both of which are central to our research.

Liu et al. [12] and Zhou and Gupta [27] focused on dual-channel and high-tech product markets, respectively. Liu et al. incorporated retail channel costs and customer acceptance in a two-period model, while Zhou and Gupta examined products with fast obsolescence. In contrast, we focus on durable goods and competition involving both new and remanufactured products within and across brands, under acceptance and capacity constraints. Yan and Han [23] considered the impact of online reviews on remanufacturing entry strategies, and Pal et al. [16] analysed pricing and quality competition between substitutes. While both touch on market dynamics, neither incorporates dual competition. Patoghi et al. [17] proposed an integrated pricing model with quality and return policies but focused on a single-brand setting without inter-brand interactions.

The most relevant foundations for our study are provided by Abbasi et al. [2] and Abbasi et al. [1], both of which analyse dual competition, including inter-brand rivalry and intra-brand competition between new and remanufactured products. However, in both studies, only the more-preferred brand is allowed to engage in remanufacturing. This assumption represents a relatively safe strategic choice, as remanufacturing by a well-accepted brand is more likely to be supported by consumers. In contrast, our study examines a more uncertain and risk-prone scenario in which the less-preferred brand chooses to remanufacture. Given its lower customer acceptance, this brand is more vulnerable to profit loss if the decision proves to be ineffective. As such, the strategic implications are more complex and require deeper analysis. Additionally, Abbasi et al. [2] consider the full supply chains of both competing brands, including manufacturers and exclusive retailers, while Abbasi et al. [1] restrict their model to competition between the two manufacturers only. Our study extends this line of research by incorporating full supply chain structures and focusing on how remanufacturing by the less-preferred brand influences decision-making under dual competition and acceptance constraints.

Our model shares structural similarities with Wei and Zhao [21] through its single-period setting with both new and remanufactured products. However, unlike [21], which treats the two product types as identical and focuses only on inter-brand competition, we model competition within a brand and introduce practical constraints on remanufactured product availability. Our assumptions about inter-brand competition align with Luo et al. [13], and our modelling of new versus remanufactured product competition is similar to Liu et al. [12]. However, unlike Liu et al. [12], who use a two-period framework and focus on retail costs and acceptance levels, our single-period setting allows for a clearer analysis of remanufacturing feasibility and market implications.

Existing models such as those by Wei and Zhao [21] often assume perfect substitution between new and remanufactured products. Our model instead introduces varying customer acceptance levels, enabling analysis of dual competition: between brands and between new and remanufactured products within a brand. This dual structure adds a level of complexity and realism absent from earlier work.

In summary, our study fills a gap in the literature by modelling a realistic and constrained remanufacturing environment where a vulnerable brand with lower customer acceptance engages in remanufacturing. We analyse pricing and strategy decisions across two competing supply chains and propose a Stackelberg framework to capture interactions between manufacturers and exclusive retailers offering both new and remanufactured products.

3. **Methods.** We consider two competing supply chains, each offering a brand of the same product: Brand 1 (B_1) and Brand 2 (B_2). B_1 has higher customer acceptance than B_2, but both have equal market power. Each supply chain consists of a single manufacturer acting as a leader and an exclusive retailer operating through a traditional channel.

B_1 produces only new products, while B_2 also remanufactures its own high-quality used products. In the remanufacturing process of the B_2 supply chain, the retailer collects used products at a collection cost c_{t2} and returns them to the manufacturer at the cost of c_{m2}. Consequently, the retailer's profit from collection is $c_{m2} - c_{t2}$. It is assumed that the system operates in a stable condition and that the quantity of high-quality used products available for remanufacturing in the B_2 supply chain is δ_2.

Without remanufacturing, used products are recycled at zero profit. Figure. 2 shows B_2's reverse operations. B_1's manufacturing cost (c_{n1}) exceeds B_2's (c_{n2}),

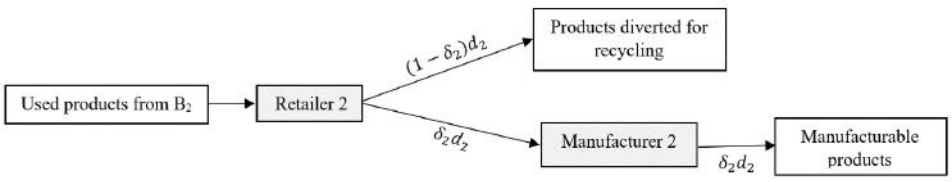


FIGURE 2. Remanufacturing and recycling in the B_2 reverse supply chain.

and both exceed remanufacturing costs, as remanufacturing saves energy and materials [11]. For simplicity, the remanufacturing plus collection costs for B_2 are

normalized to zero. A two-level Stackelberg-Nash game models B_2's remanufacturing (OR2 case) and compares it with the no-remanufacturing scenario (NA case). The decision-making process in the proposed two-level model is outlined as follows:

1) Initially, both brands simultaneously set wholesale prices, including B_2' remanufactured product price, constrained by available high-quality used products.
2) Subsequently, the retailers set retail prices.

This game model captures two types of competition:

- External: Between B_1 and B_2.
- Internal: Between B_2's new and remanufactured products

Figure. 3 shows the decision-making sequence. Pricing strategies are derived via backward induction. Demand functions follow utility theory, where customer-

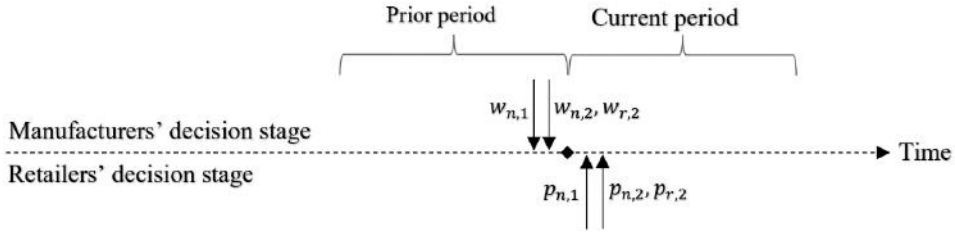


FIGURE 3. Decision-making sequence for the OR2 case.

perceived value of new products lies in $z \in (0, 1)$. Brands are not perfectly substitutable due to distinct characteristics. Customers associate a 'good' brand with superior materials, manufacturing, experience, and service, making them willing to pay a premium. To capture this preference, we use the concept of 'customer acceptance'. Customer acceptance is 1 for B_1 and $\theta \in (0, 1)$ for B_2.

Customer acceptance for remanufactured products is $\alpha \in (0, 1)$. The market size is 1, and remanufacturing is constrained by high-quality used product availability.

3.1. **Problem formulation.**

Sets

$i = r, n$: Product types (r for remanufactured and n for new,)
$j = 1, 2$: Brand types (1 for B_1 and 2 for B_2)
l=M_1 (Manufacturer for B_1), R_1 (Retailer for B_1), M_2 (Manufacturer for B_2), R_2 (Retailer for B_2).
s= NR (No Remanufacturing), OR2 (Remanufacturing in B_2 Supply Chain).

Parameters

c_{nj}: Manufacturing cost of the new product for brand j
c_{r2}: Remanufacturing cost for used products of B_2
θ: Customer acceptance level for B_2 products, where $\theta \in [0, 1]$
α: Customer acceptance level for B_2
remanufactured products, where $\alpha \in [0, 1]$
z: Customer's perceived value, uniformly distributed in the range $[0, 1]$
c_{t2}: Collection cost for high-quality used products in the B_2 supply chain
c_{m2}: Return cost to the manufacturer in B_2 Supply Chain

δ_2: Quantity of high-quality used products for B_2

Decision variables
$d^s_{i,j}$: Demand for product type i of brand j under case s
π^s_l: Profit of supply chain member l under case s
$\pi^s_{i,j}$: Sales price of product type i of brand j under case s
$w^s_{i,j}$: Wholesale price of product type i of brand j under case s
w'_{r2}:Wholesale sale price of remanufactured product of brand 2

In the OR2 model (Figure. 4), B_2 manufactures new products and remanufactures used products collected by its retailer. The customer utility function in the proposed model is as follows:

$$u^{oR2}_{n1} = z - p^{oR2}_{n1} \tag{1}$$

$$u^{oR2}_{n2} = \theta z - p^{oR2}_{n2} \tag{2}$$

$$u^{oR2}_{r2} = \alpha\theta z - p^{oR2}_{r2} \tag{3}$$

For customers to buy B_1, $u^{oR2}_{n1} \geq 0$, $u^{oR2}_{n1} \geq u^{oR2}_{n2}$ and $u^{oR2}_{n1} \geq u^{oR2}_{r2}$, which give:

$$z \geq p^{oR2}_{n1},$$

$$z \geq \frac{p^{oR2}_{n1} - p^{oR2}_{n2}}{1-\theta}, \quad \text{and}$$

$$z \geq \frac{p^{oR2}_{n1} - p^{oR2}_{r2}}{1-\alpha\theta}.$$

For customers to buy B_2, $u^{oR2}_{n2} \geq 0$, $u^{oR2}_{n2} \geq u^{oR2}_{n1}$ and $u^{oR2}_{n2} \geq u^{oR2}_{r2}$, which give:

$$z \geq \frac{p^{oR2}_{n2}}{\theta},$$

$$z \leq \frac{p^{oR2}_{n1} - p^{oR2}_{n2}}{1-\theta}, \quad \text{and}$$

$$z \geq \frac{p^{oR2}_{n2} - p^{oR2}_{r2}}{\theta(1-\alpha)}.$$

For customers to buy the remanufactured products of B_2, $u^{oR2}_{r2} \geq 0$, $u^{oR2}_{r2} \geq u^{oR2}_{n1}$, $u^{oR2}_{r2} \geq u^{oR2}_{n2}$, which give:

$$z \geq \frac{p^{oR2}_{r2}}{\alpha\theta},$$

$$z \leq \frac{p^{oR2}_{n1} - p^{oR2}_{r2}}{1-\alpha\theta}, \quad \text{and}$$

$$z \leq \frac{p^{oR2}_{n2} - p^{oR2}_{r2}}{\theta(1-\alpha)}.$$

It is required that:

$$\frac{p^{oR2}_{r2}}{\alpha\theta} \leq \frac{p^{oR2}_{n2} - p^{oR2}_{r2}}{\theta(1-\alpha)} \leq \frac{p^{oR2}_{n1} - p^{oR2}_{n2}}{1-\theta} \leq 1.$$

Therefore, the demands are as follows:

$$d_{n1}^{oR2} = 1 - \frac{p_{n1}^{oR2} - p_{n2}^{oR2}}{1-\theta} \tag{4}$$

$$d_{n2}^{oR2} = \frac{p_{n1}^{oR2} - p_{n2}^{oR2}}{1-\theta} - \frac{p_{n2}^{oR2} - p_{r2}^{oR2}}{\theta(1-\alpha)} \tag{5}$$

$$d_{r2}^{oR2} = \frac{p_{n2}^{oR2} - p_{r2}^{oR2}}{\theta(1-\alpha)} - \frac{p_{r2}^{oR2}}{\alpha\theta} \tag{6}$$

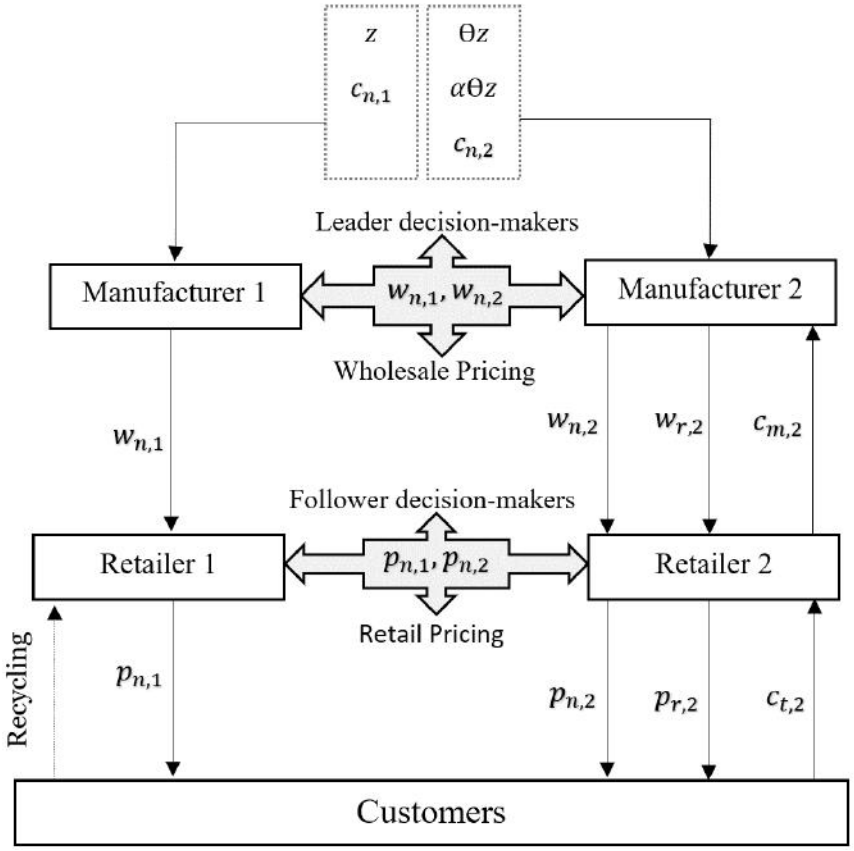


FIGURE 4. Schematic representation of the OR2 case.

3.2. **Optimal decisions.** The Stackelberg-Nash model determines optimal pricing under OR2. Manufacturers set wholesale prices considering competitors' strategies, while retailers set retail prices based on wholesale and competitor prices. Tables 2 to 9 in the supplementary file present the optimal decisions for each strategy and analyze the effects of wholesale prices and manufacturing costs on optimal strategies. The solution follows backward induction, starting from retailers' profit functions.

The profit maximization problems for the retailers are formulated as follows:

$$\max_{p_{n1}^{OR2}\geq 0} \quad \pi_{R1}^{oR2} = (p_{n1}^{oR2} - w_{n1}^{oR2})d_{n1}^{OR2} \tag{7}$$

$$\max_{\substack{p_{n2}^{OR2}\geq 0 \\ p_{r2}^{OR2}\geq 0}} \quad \pi_{R2}^{oR2} = (p_{n2}^{oR2} - w_{n2}^{oR2})d_{n2}^{OR2} + (p_{r2}^{oR2} - w_{r2}^{\prime oR2})d_{r2}^{OR2} + (c_{m2} - c_{t2})d_{r2}^{OR2} \tag{8}$$

subject to:

$$\delta_2 \geq \frac{p_{n2}^{oR2} - p_{r2}^{oR2}}{\theta(1-\alpha)} - \frac{p_{r2}^{oR2}}{\alpha\theta} \tag{9}$$

$$\text{where } w_{r2}^{oR2} = w_{r2}^{\prime oR2} - (c_{m2} - c_{t2}) \tag{10}$$

Constraint (9) ensures B_2's remanufacturing does not exceed high-quality used product availability. Lemma 3.1 derives optimal prices and demands using Karush-Kuhn-Tucker conditions.

Lemma 3.1. *Optimal Retail Prices in OR2. Retail prices in OR2 depend on wholesale price relationships:*

1. *If*

$$w_{r2}^{oR2} > \alpha w_{n2}^{oR2} - 2\theta(1-\alpha)\alpha\delta_2 \tag{11}$$

, retail and demand relations follow Equations (12)–(17).

$$p_{n1}^{oR2} = \frac{2w_{n1}^{oR2} + w_{n2}^{oR2} + 2(1-\theta)}{4-\theta} \tag{12}$$

$$p_{n2}^{oR2} = \frac{\theta w_{n1}^{oR2} + 2w_{n2}^{oR2} + \theta(1-\theta)}{4-\theta} \tag{13}$$

$$p_{r2}^{oR2} = \frac{2\alpha\theta w_{n1}^{oR2} + \alpha\theta w_{n2}^{oR2} + (4-\theta)w_{r2}^{oR2} + 2\alpha\theta(1-\theta)}{2(4-\theta)} \tag{14}$$

$$d_{n1}^{oR2} = \frac{-(2-\theta)w_{n1}^{oR2} + w_{n2}^{oR2} + 2(1-\theta)}{(4-\theta)(1-\theta)} \tag{15}$$

$$d_{n2}^{oR2} = \frac{2(1-\alpha)\theta w_{n1}^{oR2} - (2(2-\theta) - \alpha\theta(3-\theta))w_{n2}^{oR2} + (1-\theta)(4-\theta)w_{r2}^{oR2} + 2\theta(1-\alpha)(1-\theta)}{2\theta(1-\alpha)(4-\theta)(1-\theta)} \tag{16}$$

$$d_{r2}^{oR2} = \frac{\alpha w_{n2}^{oR2} - w_{r2}^{oR2}}{2\alpha\theta(1-\alpha)} \tag{17}$$

2. *If*

$$w_{r2}^{oR2} = \alpha w_{n2}^{oR2} - 2\theta(1-\alpha)\alpha\delta_2 \tag{18}$$

two sets of relations hold: (12)–(17) *and* (19)–(24).

$$p_{n1}^{oR2} = \frac{2(1-\alpha)w_{n1}^{oR2} + (1-\theta\alpha)w_{n2}^{oR2} - (1-\theta)w_{r2}^{oR2} + 2(1-\alpha\theta\delta_2)(1-\theta)(1-\alpha)}{(1-\alpha)(4-\theta)} \tag{19}$$

$$p_{n2}^{oR2} = \frac{\theta(1-\alpha)w_{n1}^{oR2} + 2(1-\theta\alpha)w_{n2}^{oR2} - 2(1-\theta)w_{r2}^{oR2} + (1-4\alpha\delta_2)\theta(1-\theta)(1-\alpha)}{(1-\alpha)(4-\theta)} \tag{20}$$

$$p_{r2}^{oR2} = \frac{\theta\alpha(1-\alpha)w_{n1}^{oR2} + 2(1-\theta\alpha)\alpha w_{n2}^{oR2} - 2(1-\theta)\alpha w_{r2}^{oR2} + \theta\alpha(1-\alpha)(1-\theta-\delta_2(4-\theta)+3\alpha\theta\delta_2)}{(1-\alpha)(4-\theta)} \tag{21}$$

$$d_{n1}^{oR2} = \frac{(1-\alpha)(\theta-2)w_{n1}^{oR2} + (1-\alpha\theta)w_{n2}^{oR2} - (1-\theta)w_{r2}^{oR2} - 2(1-\alpha)(1-\theta)(\alpha\theta\delta_2 - 1)}{(1-\alpha)(4-\theta)(1-\theta)} \tag{22}$$

$$d_{n2}^{oR2} = \frac{(1-\alpha)\theta w_{n1}^{oR2} - (2-\theta)(1-\alpha\theta)w_{n2}^{oR2} + (1-\theta)(2-\theta)w_{r2}^{oR2} - \theta(1-\alpha)(1-\theta)(\alpha\theta\delta_2 - 1)}{\theta(1-\alpha)(4-\theta)(1-\theta)} \tag{23}$$

$$d_{r2}^{oR2} = \delta_2 \tag{24}$$

3. *If*

$$w_{r2}^{oR2} < \alpha w_{n2}^{oR2} - 2\theta(1-\alpha)\alpha\delta_2 \tag{25}$$

solutions from (19)–(24) *are infeasible.*

In state (1), retail prices of new products increase with wholesale prices of both brands' new products but remain unaffected by remanufactured wholesale prices. In state (2), retail prices rise with new product wholesale prices but fall as remanufactured wholesale prices increase. The demand for B_1's new products decreases with its own and B_2's remanufactured wholesale prices, while B_2's new product demand rises with increases in B_1's new and B_2's remanufactured wholesale prices. Wholesale prices do not affect remanufactured product demand. Next, each brand sets its wholesale price based on the following profit functions:

$$\max_{w_{n1}^{OR2} \geq 0} \quad \pi_{M1}^{R2} = (w_{n1}^{oR2} - c_{n1})d_{n1}^{OR2} \tag{26}$$

$$\max_{\substack{w_{n2}^{OR2} \geq 0 \\ w_{r2}^{OR2} \geq 0}} \quad \pi_{M2}^{oR2} = (w_{n2}^{oR2} - c_{n2})d_{n2}^{OR2} + (w_{r2}^{oR2} - c_{r2} - c_{t2})d_{r2}^{OR2} \tag{27}$$

Lemma 3.1 values are used in Eq. (26). KKT is applied to linear Eqs. (12), (18), and (25) to derive optimal wholesale prices. Theorem 3.2 determines retail, wholesale prices, and optimal demand.

Theorem 3.2. *Wholesale prices depend on both brands' manufacturing costs, with* $c_{n2} \leq c_{n1}$ *always holding.*

- *In OR2a* ($c_1 \leq c_{n2} \leq c_3$)*, a portion of used products is remanufactured (Eq. A-30 to A-32 in the supplementary file).*
- *In OR2b* ($\max(c_3, c_4) \leq c_{n2} \leq c_6$)*, all used products are remanufactured, the profits of members in* B_1 *supply chain within* $\max(c_3, c_4) \leq c_{n2} \leq c_6$*, the profits of members in* B_2 *supply chain within* ($\max(c_3, c_4) \leq c_{n2} \leq \min(c_5, c_6)$)*, and the profits of* B_2 *supply chain within* $c_3 \leq c_2 \leq \max(c_3, c_{4*})$ *are optimal. Then, we will optimal profits for* B_1 *and* B_2 *supply chains (Eq. A-40 to A-42 in the supplementary file).*
- *In OR2c* ($c_3 \leq c_{n2} \leq c_7$)*, all used products are remanufactured, the profits of the second manufacturer within* ($c_5 \leq c_{n2} \leq c_7$) *and the profits of* B_2 *supply chain within* $\max(c_3, c_{4*}) \leq c_{n2} \leq c_7$ *are optimal Then, we will optimal profits for* B_1 *and* B_2 *supply chains (Eq. A-60 to A-62 in the supplementary file).* B_2 *remanufactures if* B_1 *does, utilizing all high-quality returns due to sufficient demand.*

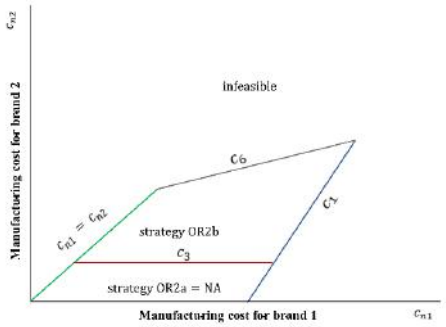
Higher manufacturing costs increase both wholesale and retail prices. Demand for new products rises with an increase in competitor manufacturing costs but falls with its own cost increase. For B_2, higher new product costs raise remanufactured demand in OR2a, while in OR2b and OR2c, remanufactured demand remains at δ_2. For all strategies, rising B_1 manufacturing costs reduce its profits but increase B_2's. Conversely, higher B_2 new product costs benefit B_1's members. Within B_2, higher new product costs decrease manufacturer profits but increase profits from

remanufactured sales. The retailer's profit increases for remanufactured products in OR2a but declines in OR2b and OR2c.

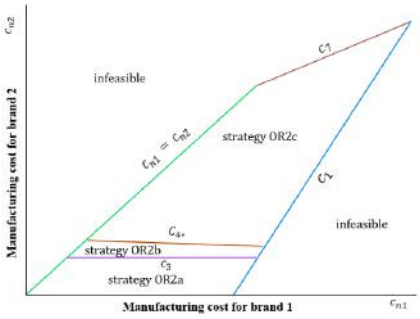
3.3. **Comparing NR and OR2.** Figure. 5 visually summarizes the comparison between cases with and without remanufacturing for the less-accepted brand. All

	OR2a	OR2b	OR2c
w_{n1}	=	>	<
w_{n2}	=	>	<
p_{n1}	=	>	<
p_{n2}	=	>	<
d_{n1}	=	>	<
d_{n2}	<	<	>

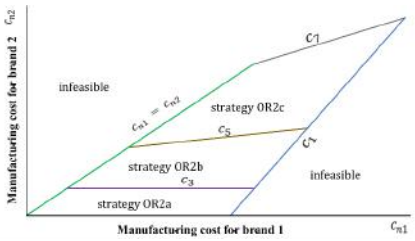
(a) Comparison of OR2 strategies with NR case



(b) Optimal strategies for B_1 in NR and OR2 cases



(c) Optimal strategies for B_2 in OR2 case



(d) Optimal strategies for B_2 in NR and OR2 cases

FIGURE 5. NR vs. OR2.

OR2 strategies, except OR2c under θ_6 $(\theta\theta_2 c_{n1} - \theta_6 c_{n2} + 2\theta\theta_1\theta_3) > 2\theta\theta_2\theta_4\theta_7(1 - \alpha\theta)\delta_2$, result in lower demand for B_2's new products compared to NR. Similarly, second retailer profits from selling B_2's new products are lower in OR2, except in OR2c under the same condition. However, B_2 profits from remanufacturing and collecting used products are higher in OR2a than in NR. The total profit impact for the second retailer is unclear in OR2b and OR2c, but OR2c consistently yields better profits than OR2b. In NR, B_1 members profit more than in OR2c but less than in OR2b. Figure. 5(a) compares OR2 and NR strategies, showing that only NR is feasible for $c_2 \leq c_{n2} \leq c_7$. While OR2b benefits B_1 members, OR2c can be preferable for the second manufacturer and retailer, leading to potential conflicts and no Nash equilibrium. Results are presented in Figure. 5(b)-(d).

4. **Numerical experiments.** In this section, numerical experiments are conducted to evaluate the effects of customer acceptance levels for new and remanufactured

products of B_2 (i.e., θ and $\theta\alpha$), and the volume of high-quality used products of B_2 (i.e., δ_2), considering low and high manufacturing costs, on the optimal decisions of two competing supply chains. Since the two supply chains are in a competitive market, the manufacturing costs are close together. Thus, in the variation range of $0.7 - 0.9$, this study takes 0.8.

4.1. **The effects of customer acceptance level of B_2.** Figure. 6, for low remanufacturing costs, shows the effects of the customer acceptance level for B_2 on optimal decisions considering $c_{n1} = 0.1, c_{n2} = 0.8c_{n1}, c_{mt} = 0.005, 0.65 \leq \theta \leq 0.9, \alpha = 0.7$ and $\delta_2 = 0.08$. Figure. 7, for high remanufacturing costs, shows the

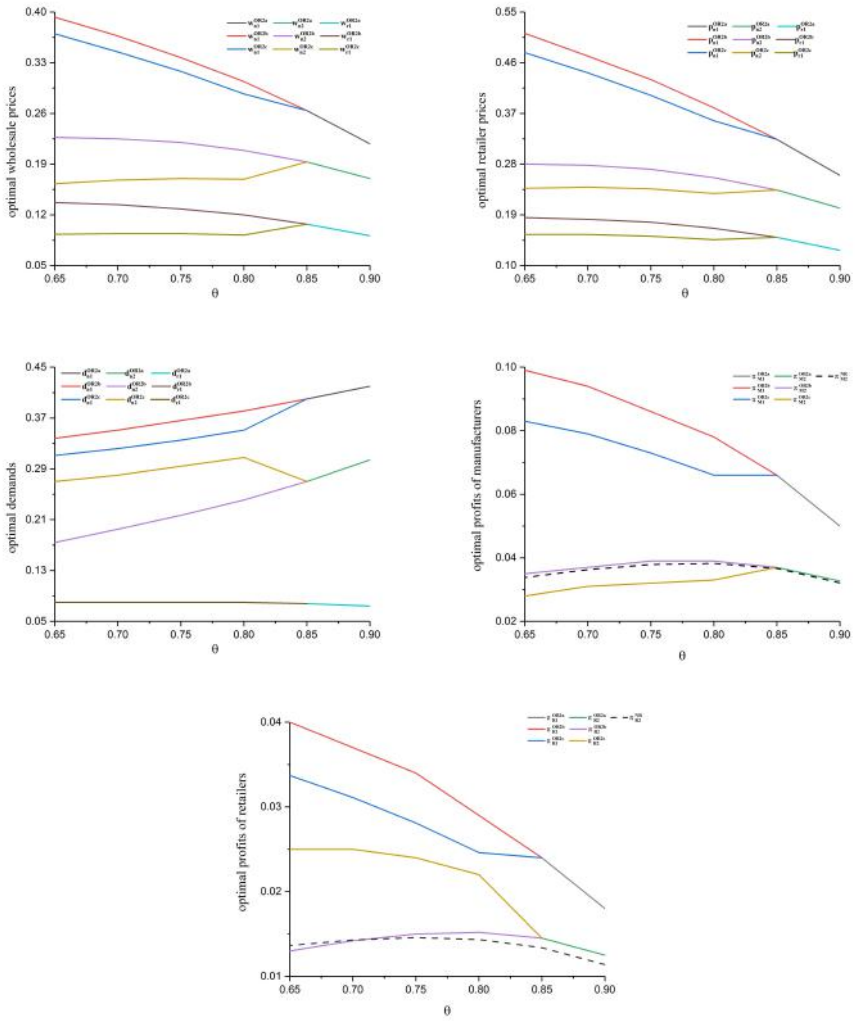


FIGURE 6. Effects of the customer acceptance level for B_2 and low manufacturing costs.

effects of the customer acceptance level for B_2 on optimal decisions considering

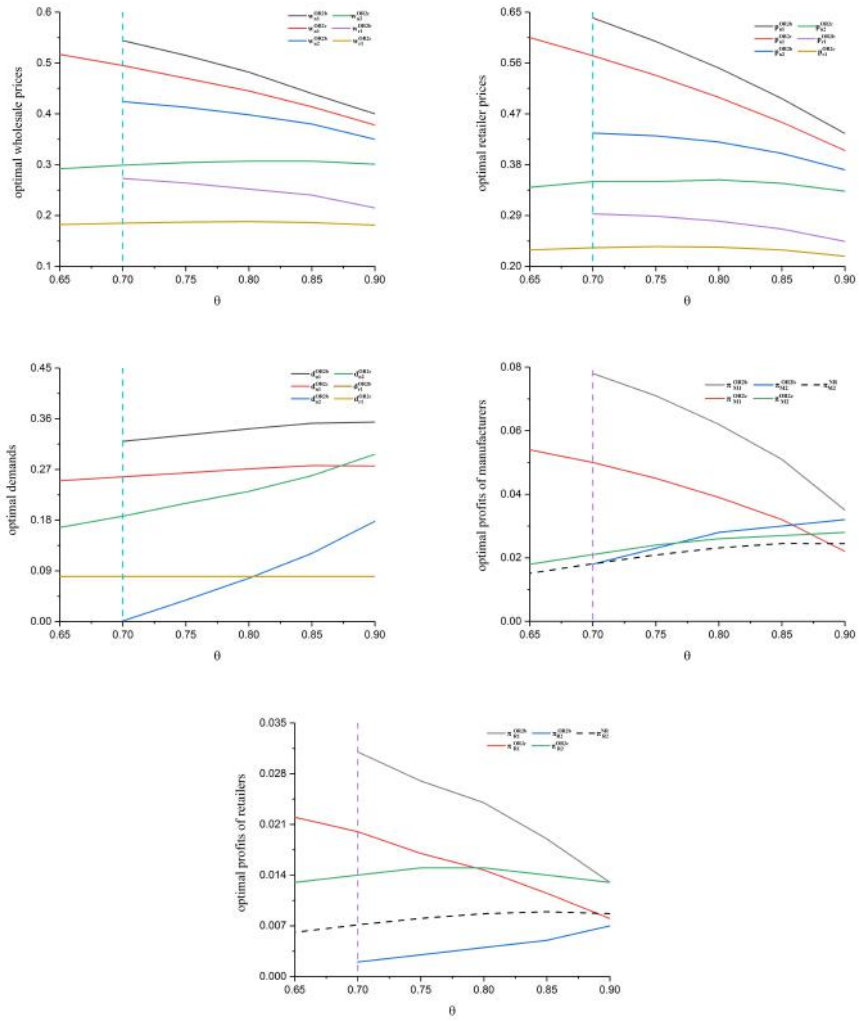


FIGURE 7. Effects of the customer acceptance level for B_2 and high manufacturing costs.

$c_{n1} = 0.3, c_{n2} = 0.8c_{n1}, c_{mt} = 0.04, 0.65 \leq \theta \leq 0.9, \alpha = 0.7$ and $\delta_2 = 0.08$. OR2a strategy is selected to deal with high acceptance levels of B_2 ($\theta \geq 0.84$) and low manufacturing costs of new product. In all strategy, with a reduction in the acceptance level of B_2, the demands for the new products both brands decrease, but the selling prices of B_1 and the profits of its members increase. In OR2a and OR2b, the selling prices of new and remanufactured products increase. While the profits of B_2 members increase in OR2a, it decreases in OR2b. Also, if the acceptance level of B_2 remains high ($\theta \geq 0.78$) in OR2c, there will be an increase in the selling prices of the new and remanufactured products of B_2, and in the profits of the second retailer. However, if the acceptance level of B_2 is not high, there will be a reduction in the selling prices of the new and remanufactured products of B_2, and in the profits of

the second retailer. Moreover, the second manufacturer's profits invariably reduce. The second manufacturer makes a better profit under OR2b than OR2c for low cost or high costs and $\theta > 0.75$.

4.2. **The effects of the customer acceptance level for remanufactured product.** Figure. 8, for low remanufacturing costs, shows the effects of the customer acceptance level for remanufactured products of B_2 on optimal decisions considering $c_{n1} = 0.1$, $c_{n2} = 0.8c_{n1}$, $c_{mt} = 0.005$, $\theta = 0.8$, $0.55 \leq \alpha \leq 0.85$ and $\delta_2 = 0.08$. Figure. 9, for high remanufacturing costs, shows the effects of the customer acceptance level for remanufactured products of B_2 on optimal decisions considering $c_{n1} = 0.3$, $c_{n2} = 0.8c_{n1}$, $c_{mt} = 0.04$, $\theta = 0.8$, $0.55 \leq \alpha \leq 0.85$ and $\delta_2 = 0.08$. OR2a is selected to deal with the rather low acceptance levels for re-

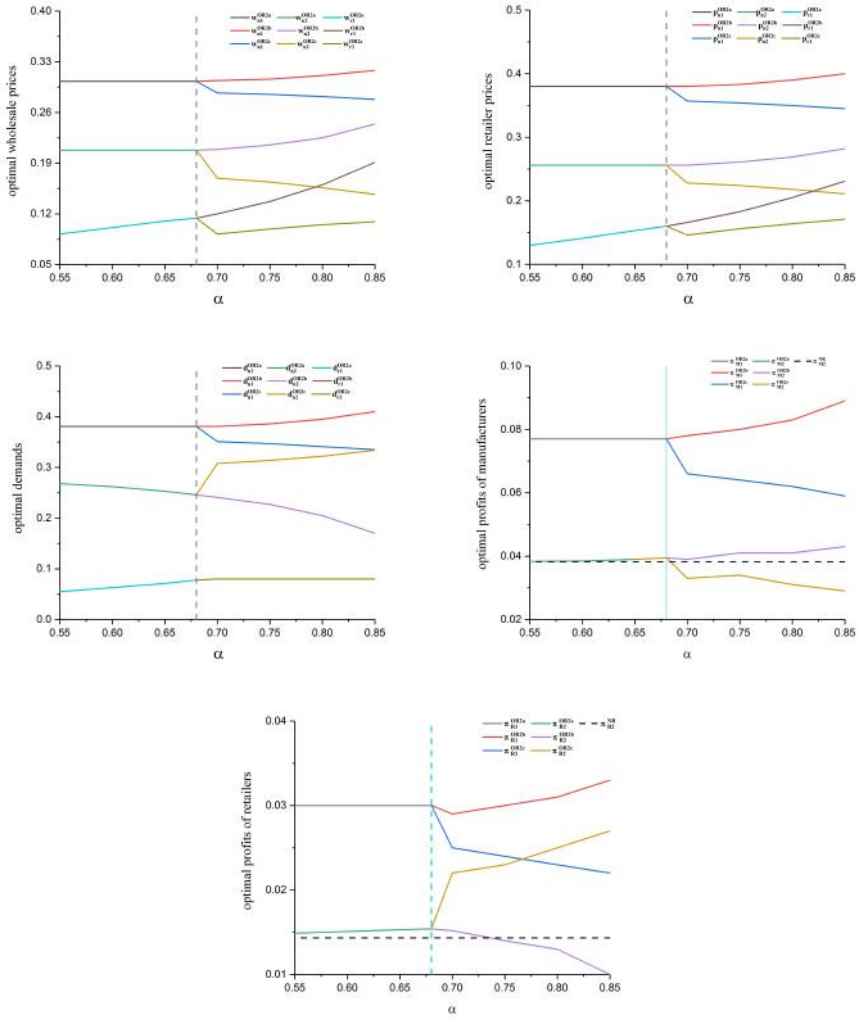


FIGURE 8. Effects of the customer acceptance level for remanufactured products and low manufacturing costs.

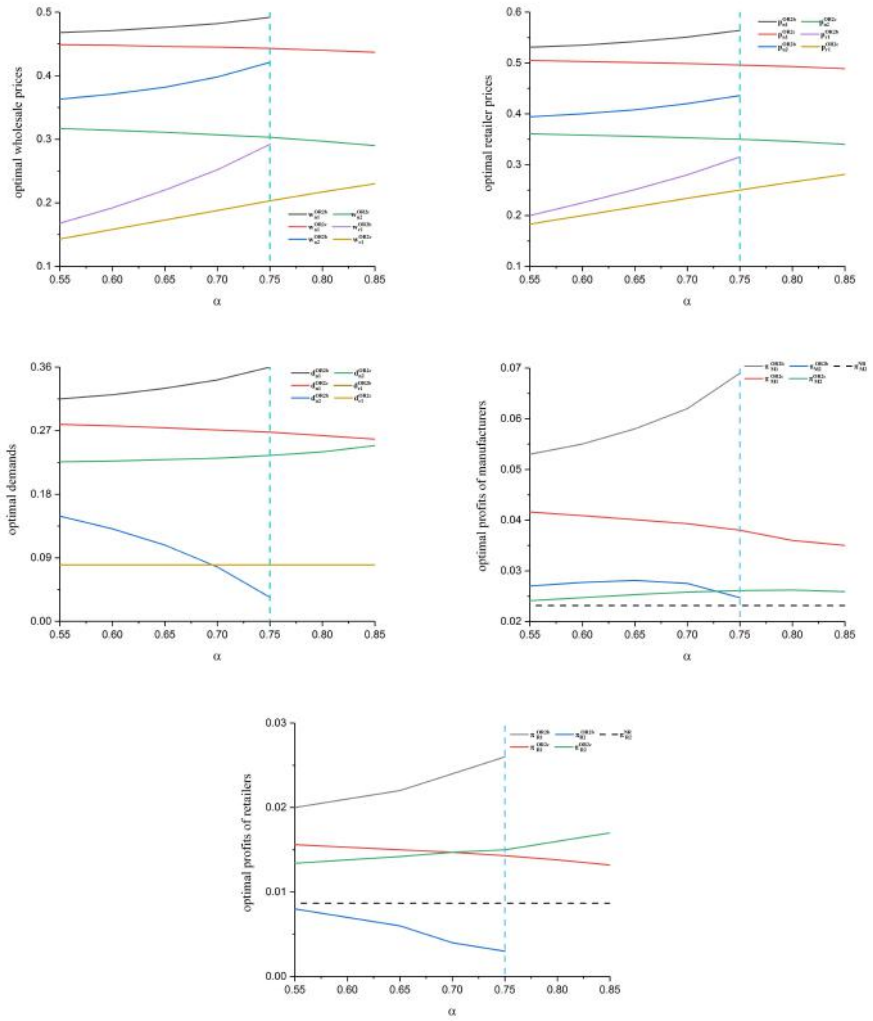


FIGURE 9. Effects of the customer acceptance for remanufactured products and high manufacturing costs.

manufactured products B_2 ($\alpha < 0.7$) and low manufacturing costs of new products of B_2. In OR2a, a decrease in the acceptance level for remanufactured products of B_2 does not change the selling prices of two brands, the demands for new products of B_1, and optimal profits of B_1 members. This decreases, however, leads to the rise of demands for new products of B_2, the reduction of selling prices and demands of remanufactured products of B_2, and, generally, the reduction of profits of B_2 members. In OR2b and at the point of the shift to OR2a, a decrease in the acceptance level for new products of B_2 leads to the reduction of selling prices of new products of both brands, the reduction of the demands for new products of B_1 and the profits of B_1 members as well as the increase of the demands for new products of B_2 and the profits of the second retailer. This effect is in reverse order for OR2c. In OR2b

and OR2c, as the acceptance level for remanufactured products of B_2 decreases, the second manufacture's profit first rises and then decreases. Also, a reduction occurs in selling prices of remanufactured products of B_2, but there is no change in the demands. As it is the case for manufacturing costs higher and $\alpha \leq 0.7$, and invariably for low manufacturing costs, the second manufacture's profit is better in OR2b than in OR2c.

4.3. **The effects of volume of high-quality used products.** Since the volume of high-quality used products of B_2 is low, it is considered low ($\delta_2 \leq 0.1$) in this study. Figure. 10, for high remanufacturing costs, shows the effects of the volume of used products of B_2 on optimal decisions considering $c_{n1} = 0.3$, $c_{n2} = 0.8c_{n1}$, $c_{mt} = 0.04$, $\theta = 0.7$, $\alpha = 0.7$, and $0.04 \leq \delta_2 \leq 0.1$.

In the strategies of OR2b and OR2c, however, a reduction in the volume of used products results in the increase of selling prices of all products, the demands for new products, and the profits of B_1 members. In contrast, it results in the reduction of the profits of B_2 members.

The results of each subsection of the numerical experiments demonstrated that if the strategies in OR2 case for intervals are selected according to the profits of the second manufacturer, the profits of the second manufacturer in OR2 is more than that in NR.

4.4. **Discussion and managerial insights.**

- OR2a is selected if manufacturing costs of B_2 is low enough ($c_{n2} \leq 4\theta(1-\alpha)\delta_2 = c_3$). In this case, compared to NR, there is no change in selling prices for new products of two brands as well as the demands for new products of B_1, but the demands for new products of B_2 decrease.
- If manufacturing costs of B_2 is high enough [$\max(c_3, c_4) \leq c_{n2} \leq \min(c_5, c_6)$] and both supply chains select OR2b, compared to NR, selling prices of new products of two brands as well as the demands for new products of B_1 increase, but the demands for new products of B_2 decrease.
- If manufacturing costs of B_2 is very high ($c_5 \leq c_{n2} \leq c_7$) and the first supply chain selects OR2c, B_2 selects OR2c too. Under this strategy, compared to NR, selling prices of new products of two brands and the demands for new products of B_1 are lower, but the demands for B_2 is higher.
- The second manufacture may be willing to do remanufacturing for $c_{n2} \leq c_3$ (OR2a), for $\max(c_3, c_4) \leq c_{n2} \leq \min(c_5, c_6)$ (OR2b) in case that the second retailer is supported (as low profits under this strategy reduce cooperation willingness) and for $c_5 \leq c_{n2} \leq c_7$ (OR2c) in case that B_2 selects OR2c too.
- In OR2a, compared to NR, there is no change in the profits of B_1's members. However, the existence of remanufactured products from B_2 leads to higher profits for B_2's members. Numerical experiments show that in OR2b, compared to NR, both the profits of B_1's members and the second manufacturer increase, specifically when $\max(c_3, c_4) \leq c_{n2} \leq \min(c_5, c_6)$. Similarly, in OR2c, numerical experiments indicate that compared to NR, the profits of B_1's members decrease, but the second manufacturer achieves higher profits when $c_5 \leq c_{n2} \leq c_7$.
- In OR2b, the wholesale price of remanufactured products affects the demands of new products of B_2, but, in OR2c, that price affects retailer prices of new products of both brands and, subsequently, the demands of their new

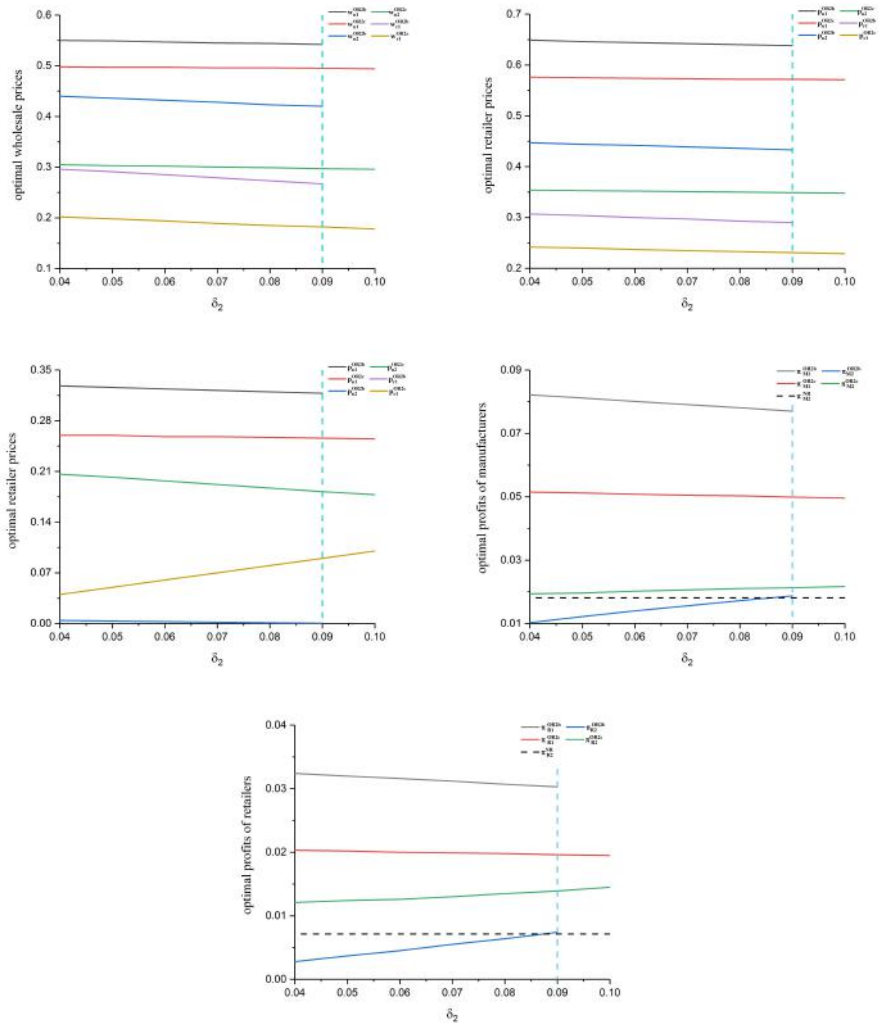


FIGURE 10. Effects of the volume of high-quality used products for high manufacturing costs.

products. Also, OR2b is not feasible for very high remanufacturing costs (compared to remanufacturing and collection costs) ($c_{n2} \geq c_6$).

- Compared to OR2c, OR2b provides higher values for selling prices, demands for new products of B_1, and profits of B_1 members. The demands for new products of B_2 and the second retailer's profit, however, are lower in OR2b than in OR2c.
- Generally, OR2a is selected for low manufacturing costs when the acceptance level of B_2 is high, the acceptance level of remanufactured products is low or the volume of used products is high (i.e., $c_{n2} \leq 4\theta(1-\alpha)\delta_2$). In this strategy, wholesale and retail prices of new products of both brands as well as the demands of B_1 are not affected by α, nor by δ_2. The second manufacture

prefers OR2b when manufacturing costs are low, except in OR2a, or those costs are high when the acceptance level is rather high for B_2 and rather low for remanufactured product ($\alpha = 0.7$ and $\theta > 0.75$, or $\alpha \leq 0.7$ and $\theta = 0.8$). Also, OR2b is optimal for B_1, thus in any of the mentioned cases, OR2b is selected if the second retailer is supported. In the other cases, B_2 members make better profits under OR2c than OR2b. However, B_1 has no tendency to select OR2c. In these cases, the second manufacture may avoid remanufacturing.

- If manufacturing costs are low and the acceptance level of B_2 is high, with reducing the acceptance level of B_2 ($\theta \geq 0.84$), each chain raises its prices. This is followed by a decrease in the demands for new products of both brands and an increase in the profits of both supply chains. For the other cases of low or high manufacturing costs with rather high acceptance levels of B_2 ($\theta \geq 0.77$), both supply chains will raise their prices if the second retailer is supported. In these circumstances, as acceptance level of B_2 reduces, the prices and profits of B_1 supply chain increases but the profits of B_2 supply chain decrease. For the other cases of high manufacturing costs, it is likely not to do remanufacturing.
- If manufacturing costs are low and the acceptance level for remanufactured products is high or manufacturing costs are high and the acceptance level of remanufactured products is rather low ($\alpha \leq 0.73$), each supply chain raises its prices. This depends on the support of the second retailer. Then, after the reduction of the acceptance level for remanufactured products at high manufacturing costs as well as low manufacturing costs until the acceptance of remanufactured products is rather high ($\alpha \geq 0.7$), each supply chain reduces its prices. This is followed by a reduction in the demands for B_1 and the profits of B_1 supply chain but an increase in the demands for new products of B_2 and the profits of the second retailer. In addition, the demands for remanufactured products remain unchanged, but the second manufacture's profit first increases and then decreases. For the other cases of low manufacturing costs, with a reduction in the acceptance level of remanufactured products, neither supply chain changes its prices. There is only the second supply chain that reduces the prices of its remanufactured products, which is followed by no change in the demands for B_1 but a reduction in the demands for remanufactured products of B_2 and an increase in the demands for new products of B_2. All in all, there is no change in the profits of B_1 members, but the profits of B_2 members decrease. It is likely not to do remanufacturing for the other cases with high manufacturing costs.

5. **Conclusion.** This study investigated remanufacturing and pricing strategies for two competing supply chains, each representing a distinct brand in a duopoly market with unequal customer acceptance levels. The less-preferred brand (B_2) faces structural challenges in adopting remanufacturing due to concerns about profitability loss and perceived vulnerability. To address this challenge, we developed a Stackelberg-Nash game framework to compare outcomes under scenarios with and without remanufacturing. The model captures both external competition between brands and internal competition between new and remanufactured products within a brand.

Numerical analysis identifies three distinct outcomes:

- Low manufacturing costs: When manufacturing costs for B_2 are sufficiently low, remanufacturing enhances the profitability of B_2's supply chain without impacting the profits of B_1. This demonstrates that remanufacturing can be a viable strategy for B_2 to strengthen its competitive position and sustain its operations.
- Moderate manufacturing costs: If B_2's manufacturing costs are moderate and its retailer provides adequate support, remanufacturing benefits both supply chains, indicating a win-win scenario.
- High manufacturing costs: When B_2's costs are prohibitively high, remanufacturing ceases to be a profitable option without additional governmental incentives or other external support.

These insights challenge the conventional view that remanufacturing is only suitable for dominant brands. Our results demonstrate that with the right conditions, remanufacturing can serve as a strategic tool even for less-preferred brands. The model provides actionable recommendations for both supply chain managers and policymakers. Managers are encouraged to view remanufacturing not only as a sustainability initiative but also as a competitive strategy, particularly when costs are manageable and customer acceptance is moderate to high. For policymakers, the results suggest the value of tailored incentives such as subsidies or temporary compensation schemes to support vulnerable brands during the transition to closed-loop systems. By addressing a neglected yet strategically significant decision scenario, this study contributes to both academic research and practical policy by clarifying when and how remanufacturing becomes a viable option for less-preferred brands in competitive environments. While the findings offer practical value, this study has several limitations. It is based on a single-period setting and assumes risk-neutral decision-makers. Future research could extend this work by incorporating multi-period decision-making, dynamic learning effects, or customer behavioural uncertainty. Another important avenue for exploration is the role of digital channels. Investigating online and offline channel competition, as well as customer acceptance across different sales platforms, could provide further insight into optimal strategies. Additionally, the impact of varying power structures within the supply chain could be modelled to better reflect real-world dynamics.

Appendix A. Supplementary Material. This article includes a supplementary file with essential mathematical details, proofs, tables, and figures supporting the numerical analysis, enhancing the clarity and robustness of the study.

Data Availability. The authors declare that we do not use any data set in this study.

Declaration of Competing Interest. The authors have no competing interests to declare that are relevant to the content of this article.

Supplementary Material. A Stackelberg-Nash Game Framework for Overcoming Remanufacturing Barriers in a Duopoly.

Proof for Lemma 3.1: Considering $\frac{\partial^2 \pi_{R1}^{oR2}}{\partial (p_{n1}^{oR2})^2} = -\frac{2}{1-\theta} < 0$, π_{R1}^{oR2} is concave in p_{n1}^{oR2}. Also, since $-\frac{2}{\alpha\theta(1-\alpha)} < 0$, $-\frac{2(1-\theta\alpha)}{(1-\theta)(1-\alpha)\theta} < 0$ and

$$\left(-\frac{2(1-\theta\alpha)}{(1-\theta)(1-\alpha)\theta}\right)\left(-\frac{2}{\alpha\theta(1-\alpha)}\right) - \left(\frac{2}{\theta(1-\alpha)}\right)^2 > 0,$$

the Hessian matrix of the second retailer's profit in the retailer prices of new and remanufactured products of B_2 is negative definite. Thus, π_{R2}^{OR2} is concave in p_{n2}^{oR2} and p_{r2}^{oR2}. The Hessian matrix is as follows:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 \pi_{R2}^{OR2}}{\partial (p_{n2}^{oR2})^2} & \frac{\partial^2 \pi_{R2}^{OR2}}{\partial p_{r2}^{oR2} \partial p_{n2}^{oR2}} \\ \frac{\partial^2 \pi_{R2}^{OR2}}{\partial p_{n2}^{oR2} \partial p_{r2}^{oR2}} & \frac{\partial^2 \pi_{R2}^{OR2}}{\partial (p_{r2}^{oR2})^2} \end{bmatrix} = \begin{bmatrix} -\frac{2(1-\theta\alpha)}{(1-\theta)(1-\alpha)\theta} & \frac{2}{\theta(1-\alpha)} \\ \frac{2}{\theta(1-\alpha)} & -\frac{2}{\alpha\theta(1-\alpha)} \end{bmatrix} \tag{A-1}$$

According to the retail price of B_1, the derivative of the profit function of the first retailer is as follows:

$$\frac{\partial \pi_{R1}^{oR2}}{\partial p_{n1}^{oR2}} = \left(1 - \frac{p_{n1}^{oR2} - p_{n2}^{oR2}}{1-\theta}\right) - \frac{1}{1-\theta}(p_{n1}^{oR2} - w_{n1}^{oR2}) = 0 \tag{A-2}$$

$$p_{n1}^{oR2} = \frac{1 - \theta + p_{n2}^{oR2} + w_{n1}^{oR2}}{2} \tag{A-3}$$

The limitation of the demands for remanufactured products of B_2 is linear. In this case, an optimal solution can be achieved based on KKT conditions.

$$\frac{\partial \pi_{R2}^{oR2}}{\partial p_{n2}^{oR2}} = \frac{p_{n1}^{oR2} - p_{n2}^{oR2}}{(1-\theta)} - \frac{p_{n2}^{oR2} - p_{r2}^{oR2}}{\theta(1-\alpha)} - \left(\frac{1}{(1-\theta)} + \frac{1}{\theta(1-\alpha)}\right)(p_{n2}^{oR2} - w_{n2}^{oR2}) + \frac{1}{\theta(1-\alpha)}(p_{r2}^{oR2} - w_{r2}^{oR2}) = 0 \tag{A-4}$$

$$p_{n2}^{OR2} = \frac{\theta p_{n1}^{OR2} + w_{n2}^{OR}}{2} \tag{A-5}$$

$$\frac{\partial \pi_{R2}^{oR2}}{\partial p_{r2}^{oR2}} + \lambda_1 \frac{\partial (\delta_2 - d_{r2}^{oR2})}{\partial p_{r2}^{oR2}} = 0 \tag{A-6}$$

$$\lambda_1 (\delta_2 - d_{r2}^{oR2}) = 0 \tag{A-7}$$

$$(d_{r2}^{oR2} - \delta_2) \leq 0 \tag{A-8}$$

$$\lambda_1, p_{r2}^{oR2} \geq 0 \tag{A-9}$$

Optimal solutions can be split in two states ($\lambda_1 = 0$ or $\lambda_1 \neq 0$). If $\lambda_1 = 0$:

$$\frac{\partial \pi_{R2}^{oR2}}{\partial p_{r2}^{oR2}} = \frac{1}{\theta(1-\alpha)}(p_{n2}^{oR2} - w_{n2}^{oR2}) + \left(\frac{p_{n2}^{oR2} - p_{r2}^{oR2}}{\theta(1-\alpha)} - \frac{p_{r2}^{oR2}}{\alpha\theta}\right) - \left(\frac{1}{\theta(1-\alpha)} + \frac{1}{\theta\alpha}\right)(p_{r2}^{oR2} - w_{r2}^{oR2}) = 0 \tag{A-10}$$

and

$$p_{r2}^{OR2} = \frac{2p_{n2}^{OR2}\alpha + w_{r2}^{OR} - w_{n2}^{OR}\alpha}{2} \tag{A-11}$$

As concluded from equations A-3, A-5 and A-10 when $w_{r2}^{oR2} \geq \alpha w_{n2}^{oR2} - 2\theta(1-\alpha)\alpha\delta_2$, we have:

$$p_{n1}^{oR2} = \frac{2w_{n1}^{oR2} + w_{n2}^{oR2} + 2(1-\theta)}{4-\theta} \tag{A-12}$$

$$p_{n2}^{oR2} = \frac{\theta w_{n1}^{oR2} + 2w_{n2}^{oR2} + \theta(1-\theta)}{4-\theta} \tag{A-13}$$

$$p_{r2}^{oR2} = \frac{2\alpha\theta w_{n1}^{oR2} + \alpha\theta w_{n2}^{oR2} + (4-\theta)w_{r2}^{oR2} + 2\alpha\theta(1-\theta)}{2(4-\theta)} \tag{A-14}$$

If $\lambda_1 \neq 0$, then p_{r2}^{OR2} can be obtained from equation A-8, as follows:

$$p_{r2}^{OR2} = \alpha p_{n2}^{OR2} - \theta(1-\alpha)\alpha\delta_2 \tag{A-15}$$

Using equations A-3, A-5 and A-15, when $w_{r2}^{oR2} \leq \alpha w_{n2}^{oR2} - 2\theta(1-\alpha)\alpha\delta_2$, p_{n1}^{oR2}, p_{n2}^{oR2} and p_{r2}^{oR2} are obtained as follows.

$$p_{n1}^{oR2} = \frac{2(1-\alpha)w_{n1}^{oR2} + w_{n2}^{oR2}(1-\theta\alpha) - (1-\theta)w_{r2}^{oR2} + 2(1-\alpha\theta\delta_2)(1-\theta)(1-\alpha)}{(1-\alpha)(4-\theta)} \tag{A-16}$$

$$p_{n2}^{oR2} = \frac{\theta(1-\alpha)w_{n1}^{oR2} + 2(1-\theta\alpha)w_{n2}^{oR2} - 2(1-\theta)w_{r2}^{oR2} + (1-4\alpha\delta_2)\theta(1-\theta)(1-\alpha)}{(1-\alpha)(4-\theta)} \tag{A-17}$$

$$p_{r2}^{oR2} = \frac{\theta\alpha(1-\alpha)w_{n1}^{oR2} + 2(1-\theta\alpha)\alpha w_{n2}^{oR2} - 2(1-\theta)\alpha w_{r2}^{oR2} + \theta\alpha(1-\alpha)(1-\theta-\delta_2(4-\theta)+3\alpha\theta\delta_2)}{(1-\alpha)(4-\theta)} \tag{A-18}$$

□

Proof for Theorem 3.2: Considering the results in Lemma 3.1, there are two sets of solutions for retailer prices of products in the states of $\lambda_1 = 0$ and $\lambda_1 \neq 0$. In addition, the limitation in the wholesale prices of remanufactured products of B_2 is linear. Therefore. the optimal solution can be achieved based on KKT conditions. In the state of $\lambda_1 = 0$, investigations are conducted on the concavity of the first manufacture's profit function in the wholesale prices of B_1 as well as the concavity of the second manufacture's profit function in the wholesale prices of new and remanufactured products of B_2.

Since $\frac{d^2\pi_{M1}^{OR2}}{d(w_{n1}^{OR2})^2} = -\frac{2(2-\theta)}{(4-\theta)(1-\theta)} < 0$, π_{M1}^{OR2} is concave in w_{n1}^{OR2}. Besides, considering $-\frac{1}{\alpha\theta(1-\alpha)} < 0$, $\frac{-2(2-\theta)(1-\alpha)-\alpha(1-\theta)(4-\theta)}{(4-\theta)(1-\theta)\theta(1-\alpha)} < 0$ and

$$\left(\frac{-2(2-\theta)(1-\alpha)-\alpha(1-\theta)(4-\theta)}{(4-\theta)(1-\theta)\theta(1-\alpha)}\right)\left(-\frac{1}{\alpha\theta(1-\alpha)}\right) - \left(\frac{1}{\theta(1-\alpha)}\right)^2 > 0,$$

the Hessian matrix of the second manufacturer's profit function in the wholesale prices of new and remanufactured products of B_2 is negative definite. Thus, π_{M2}^{OR2} is concave in w_{n2}^{OR2} and w_{r2}^{OR2}. The corresponding Hessian matrix is as follows:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2\pi_{M2}^{OR2}}{\partial(w_{n2}^{oR2})^2} & \frac{\partial^2\pi_{M2}^{OR2}}{\partial w_{r2}^{oR2}\partial w_{n2}^{oR2}} \\ \frac{\partial^2\pi_{M2}^{OR2}}{\partial w_{n2}^{oR2}\partial w_{r2}^{oR2}} & \frac{\partial^2\pi_{M2}^{OR2}}{\partial(w_{r2}^{oR2})^2} \end{bmatrix} = \begin{bmatrix} \frac{-2(2-\theta)(1-\alpha)-\alpha(1-\theta)(4-\theta)}{(4-\theta)(1-\theta)\theta(1-\alpha)} & \frac{1}{\theta(1-\alpha)} \\ \frac{1}{\theta(1-\alpha)} & -\frac{1}{\alpha\theta(1-\alpha)} \end{bmatrix} \tag{A-19}$$

The corresponding equations are as follows:

$$\frac{d\pi_{M1}^{OR2}}{dw_{n1}^{OR2}} = \left(\frac{2(1-\theta)-(2-\theta)w_{n1}^{OR2}+w_{n2}^{OR2}}{(4-\theta)(1-\theta)}\right) - \frac{(2-\theta)}{(4-\theta)(1-\theta)}(w_{n1}^{OR2} - c_{n1}) = 0 \tag{A-20}$$

$$w_{n1}^{OR2} = \frac{2(1-\theta) + w_{n2}^{OR2} + c_{n1}(2-\theta)}{2(2-\theta)} \tag{A-21}$$

$$\frac{\partial \pi_{M2}^{OR2}}{\partial w_{n2}^{OR2}} = \left(\frac{1}{4-\theta} + \frac{w_{n1}^{OR2}}{(1-\theta)(4-\theta)} + \frac{(-4+2\theta+3\alpha\theta-\alpha\theta^2)w_{n2}^{OR2}}{(1-\theta)(4-\theta)\theta(1-\alpha)} + \frac{w_{r2}^{OR2}}{2\theta(1-\alpha)} \right)$$
$$-c_{n2}\frac{(-4+2\theta+3\alpha\theta-\alpha\theta^2)}{2(1-\theta)(4-\theta)\theta(1-\alpha)} + \frac{w_{r2}^{OR2}}{2\theta(1-\alpha)} = 0 \tag{A-22}$$

$$w_{n2}^{OR2} = \frac{2(1-\theta)\theta(1-\alpha) + 2\theta(1-\alpha)w_{n1}^{OR2} + (1-\theta)(4-\theta)(2w_{r2}^{OR2})}{2(4-2\theta-3\alpha\theta+\alpha\theta^2)} + \frac{c_{n2}}{2} \tag{A-23}$$

$$\frac{\partial \pi_{M2}^{oR2}}{\partial w_{r2}^{oR2}} - \lambda_{11} = 0 \tag{A-24}$$

$$\lambda_{11}(w_{r2}^{oR2} - \alpha w_{n2}^{oR2} + 2\theta(1-\alpha)\alpha\delta_2) = 0 \tag{A-25}$$

$$\alpha w_{n2}^{oR2} - w_{r2}^{oR2} - 2\theta(1-\alpha)\alpha\delta_2 \leq 0 \tag{A-26}$$

$$\lambda_{11}, w_{r2}^{oR2} \geq 0 \tag{A-27}$$

Optimal solutions can be split in two states ($\lambda_{11} = 0$ or $\lambda_{11} \neq 0$). If $\lambda_{11} = 0$, the following equation holds.

$$\frac{\partial \pi_{M2}^{OR2}}{\partial w_{r2}^{OR2}} = \left(\frac{(w_{n2}^{OR2} - c_{n2})\alpha}{2\alpha\theta(1-\alpha)} + \frac{(\alpha w_{n2}^{OR2} - 2w_{r2}^{OR2})}{2\alpha\theta(1-\alpha)} \right) = 0 \tag{A-28}$$

$$w_{r2}^{OR2} = \frac{2\alpha w_{n2}^{OR2} - \alpha c_{n2}}{2} \tag{A-29}$$

This equation holds for $c_{n2} \leq c_3$. As concluded from equations A-21, A-23 and A-29:

$$w_{n1}^{OR2} = \frac{2\theta_2^2 c_{n1} + \theta_2 c_{n2} - \theta_1\theta_5}{\theta_7} \tag{A-30}$$

$$w_{n2}^{OR2} = \frac{\theta\theta_2 c_{n1} + 2\theta_2^2 c_{n2} - 2\theta\theta_1\theta_3}{\theta_7} \tag{A-31}$$

$$w_{r2}^{OR2} = \frac{2\alpha\theta\theta_2 c_{n1} + \alpha\theta c_{n2} + 4\alpha\theta^2\theta_1\theta_3}{2\theta_7} \tag{A-32}$$

$$p_{n1}^{OR2} = \frac{\theta_2\theta_5 c_{n1} + 2\theta_2\theta_3 c_{n2} + 2\theta_1\theta_3\theta_5}{\theta_4\theta_7} \tag{A-33}$$

$$p_{n2}^{OR2} = \frac{2\theta\theta_2\theta_3 c_{n1} + \theta_2\theta_5 c_{n2} + 4\theta\theta_1\theta_3^2}{\theta_4\theta_7} \tag{A-34}$$

$$p_{r2}^{OR2} = \frac{8\alpha\theta\theta_2\theta_3 c_{n1} + \alpha\theta(4\theta^2 - 21\theta + 28)c_{n2} + 16\alpha\theta\theta_1\theta_3^2}{4\theta_4\theta_7} \tag{A-35}$$

$$d_{n1}^{OR2} = \frac{\theta_2(-\theta_6 c_{n1} + \theta_2 c_{n2} + \theta_1\theta_5)}{\theta_1\theta_4\theta_7} \tag{A-36}$$

$$d_{n2}^{OR2} = \frac{4\alpha_1\theta\theta_2^2 c_{n1} - (4\theta_2\theta_6 + \alpha\theta(4\theta^3 - 29\theta^2 + 65\theta - 44))c_{n2} + 8\alpha_1\theta\theta_1\theta_2\theta_3}{4\alpha_1\theta\theta_1\theta_4\theta_7} \tag{A-37}$$

$$d_{r2}^{OR2} = \frac{c_{n2}}{4\alpha_1\theta} \tag{A-38}$$

where $\theta_1 = (1-\theta), \quad \theta_2 = (2-\theta), \quad \theta_3 = (3-\theta),$

$\theta_4 = (4-\theta), \quad \theta_5 = (8-3\theta), \quad \theta_6 = (2\theta^2 - 9\theta + 8),$

$\theta_7 = (4\theta^2 - 17\theta + 16), \quad \alpha_1 = (1-\alpha)$

For new products of both brands as well as remanufactured products of B2 to be profitable under OR2, it is required to have $d_{n1}^{OR2} \geq 0$, $d_{n2}^{OR2} \geq 0$ and $d_{r2}^{OR2} \geq 0$ based on $c_1 \leq c_{n2} \leq c_{3*}$ and since $c_3 \leq c_{3*}$, thus $c_1 \leq c_{n2} \leq c_3$.

If $\lambda_{11} \neq 0$, the following equation holds:

$$w_{r2}^{OR2} = \alpha w_{n2}^{OR2} - 2\theta(1-\alpha)\alpha\delta_2 \tag{A-39}$$

The equation holds for $c_3 \leq c_{n2}$. Based on equations (A-21), (A-23) and (A-39), the following relations can be obtained:

$$w_{n1}^{OR2} = \frac{4\alpha_1\theta_2^2 c_{n1} + (2\theta_2 - \alpha\theta\theta_3)c_{n2} + 2\alpha_1\theta_1\theta_5 - \theta_8}{2\alpha_1\theta_7} \tag{A-40}$$

$$w_{n2}^{OR2} = \frac{\alpha_1\theta\theta_2 c_{n1} + \theta_2(2\theta_2 - a\theta\theta_3)c_{n2} + 2\alpha_1\theta\theta_1\theta_3 - \theta_2\theta_8}{\alpha_1\theta_7} \tag{A-41}$$

$$w_{r2}^{OR2} = \frac{\alpha(\alpha_1\theta\theta_2 c_{n1} + \theta_2(2\theta_2 - a\theta\theta_3)c_{n2} + 2\alpha_1\theta(\theta_1\theta_3 - \alpha_1\theta_7\delta_2) - \theta_2\theta_8)}{\alpha_1\theta_7} \tag{A-42}$$

$$p_{n1}^{OR2} = \frac{\alpha_1\theta_2\theta_5 c_{n1} + \theta_3(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 2\alpha_1\theta_1\theta_3\theta_5 - \theta_3\theta_8}{\alpha_1\theta_4\theta_7} \tag{A-43}$$

$$p_{n2}^{OR2} = \frac{4\alpha_1\theta\theta_2\theta_3 c_{n1} + \theta_5(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 8\alpha_1\theta\theta_1\theta_3^2 - \theta_5\theta_8}{2\alpha_1\theta_4\theta_7} \tag{A-44}$$

$$p_{r2}^{OR2} = \alpha\left(\frac{4\alpha_1\theta\theta_2\theta_3 c_{n1} + \theta_5(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 8\alpha_1\theta\theta_1\theta_3^2 - \theta_5\theta_8}{2\alpha_1\theta_4\theta_7} + \alpha_1\theta\delta_2\right) \tag{A-45}$$

$$d_{n1}^{OR2} = \frac{\theta_2(-2\alpha_1\theta_6 c_{n1} + (2\theta_2 - \alpha\theta\theta_3)c_{n2} + 2\alpha_1\theta_1\theta_5 - \theta_8)}{2\alpha_1\theta_1\theta_4\theta_7} \tag{A-46}$$

$$d_{n2}^{OR2} = \frac{2\alpha_1\theta\theta_2^2 c_{n1} + \theta_6(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 4\alpha_1\theta\theta_1\theta_2\theta_3 - \frac{1}{2}\theta\theta_8}{2\alpha_1\theta\theta_1\theta_4\theta_7} \tag{A-47}$$

$$d_{r2}^{OR2} = \delta_2 \tag{A-48}$$

where $\theta_8 = 4\alpha\theta\alpha_1\theta_1\theta_4\delta_2$. To ensure the profitability of new products of both brands as well as remanufactured products of B_2 under OR2b, it is required to

have $d_{n1}^{OR2} \geq 0$, $d_{n2}^{OR2} \geq 0$ and $d_{r2}^{OR2} \geq 0$ based on $c_4 \leq c_{n2} \leq c_6$, and since $c_3 \leq c_{n2}$, results in the relation of $\max(c_4, c_3) \leq c_{n2} \leq c_6$.

In the state of $\lambda_1 \neq 0$, investigations are conducted on the concavity of the first manufacturer's profit function in wholesale prices of B_1 as well as the concavity of the second manufacturer's profit function in wholesale prices of new and remanufactured products of B_2.

Since $\frac{d^2\pi_{M1}^{OR2}}{d(w_{n1}^{OR2})^2} = -\frac{2(2-\theta)}{(4-\theta)(1-\theta)} < 0$, π_{M1}^{OR2} is concave in w_{n1}^{OR2}. Also $\frac{d^2\pi_{M2}^{OR2}}{d(w_{n2}^{OR2})^2} = \frac{-2(2-\theta)(1-\alpha\theta)}{(4-\theta)(1-\theta)\theta(1-\alpha)} < 0$, π_{M2}^{OR2} is concave in w_{n2}^{OR2} but $\frac{d^2\pi_{M2}^{OR2}}{d(w_{r2}^{OR2})^2} = 0$, thus the Hessian matrix of the second manufacturer's profit function in wholesale prices of new and remanufactured products of B_2 is not negative definite.

The corresponding equations are as follows:

$$\frac{d\pi_{M1}^{OR2}}{dw_{n1}^{OR2}} = \frac{2(1-\alpha\theta\delta_2)}{(4-\theta)} - \frac{(2-\theta)(2w_{n1}^{OR2} - c_{n1})}{(4-\theta)(1-\theta)} + \frac{(1-\theta\alpha)w_{n2}^{OR2}}{(4-\theta)(1-\theta)(1-\alpha)} - \frac{w_{r2}^{OR2}}{(4-\theta)(1-\alpha)} = 0 \tag{A-49}$$

$$w_{n1}^{OR2} = \frac{2(1-\alpha\theta\delta_2)(1-\theta)}{2(2-\theta)} + \frac{(1-\theta\alpha)w_{n2}^{OR2}}{2(2-\theta)(1-\alpha)} - \frac{w_{r2}^{OR2}(1-\theta)}{2(1-\alpha)(2-\theta)} + \frac{c_{n1}}{2} \tag{A-50}$$

$$\frac{\partial\pi_{M2}^{OR2}}{\partial w_{n2}^{OR2}} = \frac{\theta(1-\alpha\theta\delta_2)}{(4-\theta)} + \frac{\theta w_{n1}^{OR2}}{(4-\theta)(1-\theta)} - \frac{(2-\theta)(1-\theta\alpha)(2w_{n2}^{OR2} - c_{n2})}{(4-\theta)(1-\theta)(1-\alpha)} + \frac{(2-\theta)w_{r2}^{OR2}}{(4-\theta)(1-\alpha)} = 0 \tag{A-51}$$

$$w_{n2}^{OR2} = \frac{\theta(1-\alpha\theta\delta_2)(1-\theta)(1-\alpha)}{2(2-\theta)(1-\theta\alpha)} + \frac{(1-\alpha)\theta w_{n1}^{OR2}}{2(2-\theta)(1-\theta\alpha)} + \frac{(1-\theta)(2-\theta)w_{r2}^{OR2}}{2(2-\theta)(1-\theta\alpha)} + \frac{c_{n2}}{2} \tag{A-52}$$

$$w_{r2}^{oR2} = \alpha w_{n2}^{oR2} - 2\theta(1-\alpha)\alpha\delta_2 \tag{A-53}$$

The equation holds for $c_3 \leq c_{n2}$. Based on equations A-50, A-52 and A-53, the following can be obtained:

$$w_{n1}^{OR2} = \frac{(2\theta_2^2 - \alpha\theta_1^2(\theta+1))c_{n1} + \theta_2(1-\alpha\theta)c_{n2} + \theta_1(\theta_5 + \alpha(2\theta^2 - 3\theta - 4)) - \theta_8/4}{\theta_7 - \alpha(2\theta^3 - 6\theta^2 - \theta + 8)} \tag{A-54}$$

$$w_{n2}^{OR2} = \frac{\alpha_1\theta\theta_2 c_{n1} + 2\theta_2^2(1-\theta\alpha)c_{n2} + 2\alpha_1\theta\theta_1\theta_3 - \theta_2\theta_8}{\theta_7 - \alpha(2\theta^3 - 6\theta^2 - \theta + 8)} \tag{A-55}$$

$$w_{r2}^{OR2} = \alpha\left(\frac{\alpha_1\theta\theta_2 c_{n1} + 2\theta_2^2(1-\theta\alpha)c_{n2} + 2\alpha_1\theta\theta_1\theta_3 - \theta_2\theta_8}{\theta_7 - \alpha(2\theta^3 - 6\theta^2 - \theta + 8)} - 2\theta\alpha_1\delta_2\right) \tag{A-56}$$

$$p_{n1}^{OR2} = \frac{\big(\theta_5 + \alpha(2\theta^2 - 3\theta - 4)\big)(\theta_2 c_{n1} + 2\theta_1\theta_3) + 2\theta_3(\alpha\theta + \theta_2)c_{n2} - \frac{1}{2}\theta_3\theta_8}{\theta_4\big(\theta_7 - \alpha(2\theta^3 - 6\theta^2 - \theta + 8)\big)} \quad \text{(A-57)}$$

$$p_{n2}^{OR2} = \frac{\theta\big(\alpha(\theta^2 - \theta - 4) + 2\theta_3\big)(\theta_2 c_{n1} + 2\theta_1\theta_3) + \alpha_1\theta_2\theta_5 c_{n2} - \frac{1}{4}\theta_5\theta_8}{\theta_4\big(\theta_7 - \alpha(2\theta^3 - 6\theta^2 - \theta + 8)\big)} \quad \text{(A-58)}$$

$$p_{r2}^{OR2} = \alpha\left(\frac{\theta\big(\alpha(\theta^2 - \theta - 4) + 2\theta_3\big)(\theta_2 c_{n1} + 2\theta_1\theta_3) + \alpha_1\theta_2\theta_5 c_{n2} - \frac{1}{4}\theta_5\theta_8}{\theta_4\big(\theta_7 - \alpha(2\theta^3 - 6\theta^2 - \theta + 8)\big)} + \alpha_1\theta\delta_2\right) \quad \text{(A-59)}$$

$$d_{n1}^{OR2} = \frac{\theta_2\left(\big(\alpha(\theta^3 - 3\theta^2 - \theta + 4) - \theta_6\big)c_{n1} + \theta_2(1 - \alpha\theta)c_{n2} + \theta_1\big(\theta_5 + \alpha(2\theta^2 - 3\theta - 4)\big) - \frac{1}{4}\theta_8\right)}{\theta_1\theta_4\big(\theta_7 - \alpha(2\theta^3 - 6\theta^2 - \theta + 8)\big)} \quad \text{(A-60)}$$

$$d_{n2}^{OR2} = \frac{\theta_2(1 - \alpha\theta)\left(\theta\theta_2 c_{n1} - \theta_6 c_{n2} + 2\theta\theta_1(\theta_3 - \alpha\theta_2\theta_4\delta_2)\right)}{\theta\theta_1\theta_4\big(\theta_7 - \alpha(2\theta^3 - 6\theta^2 - \theta + 8)\big)} \quad \text{(A-61)}$$

$$d_{r2}^{OR2} = \delta_2 \quad \text{(A-62)}$$

The profitability of new products of both brands as well as remanufactured products of B_2 should be ensured under OR2c. This requires $d_{n1}^{OR2} \geq 0$, $d_{n2}^{OR2} \geq 0$ and $d_{r2}^{OR2} \geq 0$ based on $c_4 \leq c_{n2} \leq c_6$ and since $c_3 \leq c_{n2}$, results in the relation of $c_3 \leq c_{n2} \leq c_7$. The profits of B_2 members can be obtained by substituting the optimal values in the objective functions. The profits of B_1 members are optimal for $c_3 \leq c_{n2} \leq c_7$ under OR2b. The profits of the second manufacture are optimal for $c_3 \leq c_{n2} \leq c_5$ under OR2b and for $c_5 \leq c_{n2} \leq c_7$ under OR2c. The second retailer is more profitable under OR2c than under OR2b, and the profits of the whole B_2 supply chain is optimal under OR2b for $c_3 \leq c_{n2} \leq \max(c_3, c_{4*})$ and under OR2c for $\max(c_3, c_{4*}) \leq c_{n2} \leq c_7$. We obtain the results in Theorem 3.2. Since $c_{n2} \leq c_{n1}$ invariably holds, the limitation is implemented in all cases. The c_{n2} boundary values are as follows:

$$c_1 = \frac{-\theta_1\theta_5 + \theta_6 c_{n1}}{\theta_2}$$

$$c_2 = \frac{\theta(2\theta_1\theta_3 + \theta_2 c_{n1})}{\theta_6}$$

$$c_3 = 4\alpha_1\theta\delta_2$$

$$c_{3*} = \frac{4\alpha_1\theta\theta_2(\theta_2 c_{n1} + 2\theta_1\theta_3)}{\alpha\theta_1\theta_4\theta_7 + \alpha_1(4\theta_2\theta_6)}$$

$$c_4 = \frac{2\alpha_1\theta_6 c_{n1} - 2\alpha_1\theta_1\theta_5 + \theta_8}{\alpha\theta^2 - 3\alpha\theta - 2\theta + 4}$$

$$c_{4*} = 2\theta\alpha_1(\theta_{11} - \alpha\theta_{12})(\theta_2 c_{n1} - 2\theta_1\theta_3) + \frac{1}{2}\theta_5\theta_6\theta_8(\alpha\theta_{10} - 3\theta_7)\times$$

$$1/\left(\alpha^2(22\theta^6 - 208\theta^5 + 657\theta^4 - 537\theta^3 - 1080\theta^2 + 2176\theta - 1024)\right.$$

$$\left.-\alpha(16\theta^6 - 100\theta^5 - 77\theta^4 + 193\theta^3 - 5440\theta^2 + 5984\theta - 2304) + 2\theta_{11}\right)$$

$$c_5 = \frac{\frac{1}{2}\theta_2\theta_6\theta_8\big(\alpha(4\theta^3 - 8\theta^2 - 19\theta + 32) - 3\theta_7\big) - \theta\alpha_1(\alpha\theta_9 - \theta\theta_8)(2\theta_1\theta_3 + \theta_2 c_{n1})}{\theta_6\left(2\alpha^2\theta_1^2\theta_2^3\theta_4 + \alpha_1\left(\alpha(-8\theta^5 + 81\theta^4 - 323\theta^3 + 623\theta^2 - 568\theta + 192) + \theta\theta_7\right)\right)}$$

$$c_6 = \frac{\theta(2\alpha_1\theta_2^2 c_{n1} + 4\alpha_1\theta_1\theta_2\theta_3 - \frac{1}{2}\theta_8)}{\theta_6(\alpha\theta^2 - 3\alpha\theta - 2\theta + 4)}$$

$$c_7 = \frac{\theta\left(\theta_2 c_{n1} + 2\theta_1(\theta_3 - \alpha\theta_2\theta_4\delta_2)\right)}{\theta_6}$$

Where:

$$\theta_9 = -4\theta^5 + 36\theta^4 - 132\theta^3 + 241\theta^2 - 208\theta + 64$$

$$\theta_{10} = 4\theta^3 - 8\theta^2 - 19\theta + 32$$

$$\theta_{11} = \theta_7(4\theta^3 - 27\theta^2 + 56\theta - 32)$$

$$\theta_{12} = 8\theta^6 - 72\theta^5 + 209\theta^4 - 104\theta^3 - 518\theta^2 + 864\theta - 384$$

□

Supplementary Tables: Optimal Decisions and Sensitivity Analysis. Where $\theta_1 = (1-\theta)$, $\theta_2 = (2-\theta)$, $\theta_3 = (3-\theta)$, $\theta_4 = (4-\theta)$, $\theta_5 = (8-3\theta)$, $\theta_6 = (2\theta^2 - 9\theta + 8)$, $\theta_7 = (4\theta^2 - 17\theta + 16)$, $\theta_8 = 4\alpha\theta\alpha_1\theta_1\theta_4$ and $\alpha_1 = (1-\alpha)$.

TABLE 2. Effects of wholesale prices on the optimal decisions in state (1) of Lemma 3.1.

	$d_{r2}^{(1-2)}$	$d_{n2}^{(1-2)}$	$d_{n1}^{(1-2)}$	$p_{r2}^{(1-2)}$	$p_{n2}^{(1-2)}$	$p_{n1}^{(1-2)}$
$w_{n1}^{(1-2)}$	0	↑	↓	↑	↑	↑
$w_{n2}^{(1-2)}$	↑	↓	↑	↑	↑	↑
$w_{r2}^{(1-2)}$	↓	↑	0	↑	0	0

TABLE 3. Effects of wholesale prices on the optimal decisions in state (2) of Lemma 3.1.

	$d_{r2}^{(2-2)}$	$d_{n2}^{(2-2)}$	$d_{n1}^{(2-2)}$	$p_{r2}^{(2-2)}$	$p_{n2}^{(2-2)}$	$p_{n1}^{(2-2)}$
$w_{n1}^{(2-2)}$	0	↑	↓	↑	↑	↑
$w_{n2}^{(2-2)}$	0	↓	↑	↑	↑	↑
$w_{r2}^{(2-2)}$	0	↑	↓	↓	↓	↓

The optimal decisions corresponding to the OR2a, OR2b, and OR2c strategies are presented in Tables 4–6, respectively.

TABLE 4. Optimal decisions of OR2a strategy.

Variable	**Optimal solution**
w_{n1}^{OR2}	$\frac{2\theta_2^2 c_{n1} + \theta_2 c_{n2} + \theta_1\theta_5}{\theta_7}$
w_{n2}^{OR2}	$\frac{\theta\theta_2 c_{n1} + 2\theta_2^2 c_{n2} - 2\theta\theta_1\theta_3}{\theta_7}$
w_{r2}^{OR2}	$\frac{2\alpha\theta\theta_2 c_{n1} + \alpha\theta c_{n2} + 4\alpha\theta^2\theta_1\theta_3}{2\theta_7}$
p_{n1}^{OR2}	$\frac{\theta_2\theta_5 c_{n1} + 2\theta_2\theta_3 c_{n2} + 2\theta_1\theta_3\theta_5}{\theta_4\theta_7}$
p_{n2}^{OR2}	$\frac{2\theta\theta_2\theta_3 c_{n1} + \theta_2\theta_5 c_{n2} + 4\theta\theta_1\theta_3^2}{\theta_4\theta_7}$
p_{r2}^{OR2}	$\frac{8\alpha\theta\theta_2\theta_3 c_{n1} + \alpha\theta(4\theta^2 - 21\theta + 28)c_{n2} + 16\alpha\theta\theta_1\theta_3^2}{4\theta_4\theta_7}$
d_{n1}^{OR2}	$\frac{\theta_2(-\theta_6 c_{n1} + \theta_2 c_{n2} + \theta_1\theta_5)}{\theta_1\theta_4\theta_7}$
d_{n2}^{OR2}	$\frac{4\alpha_1\theta\theta_2^2 c_{n1} - (4\theta_2\theta_6 + \alpha\theta(4\theta^3 - 29\theta^2 + 65\theta - 44))c_{n2} + 8\alpha_1\theta\theta_1\theta_2\theta_3}{4\alpha_1\theta\theta_1\theta_4\theta_7}$
d_{r2}^{OR2}	$\frac{c_{n2}}{4\alpha_1\theta}$

TABLE 5. Optimal decisions of OR2b strategy.

Variable	**Optimal solution**
w_{n1}^{OR2}	$\frac{4\alpha_1\theta_2^2 c_{n1} + (2\theta_2 - \alpha\theta\theta_3)c_{n2} + 2\alpha_1\theta_1\theta_5 - \theta_8}{2\alpha_1\theta_7}$
w_{n2}^{OR2}	$\frac{\alpha_1\theta\theta_2 c_{n1} + \theta_2(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 2\alpha_1\theta\theta_1\theta_3 - \theta_2\theta_8}{\alpha_1\theta_7}$
w_{r2}^{OR2}	$\frac{\alpha(\alpha_1\theta\theta_2 c_{n1} + \theta_2(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 2\alpha_1\theta(\theta_1\theta_3 - \alpha_1\theta_7\delta_2) - \theta_2\theta_8)}{\alpha_1\theta_7}$
p_{n1}^{OR2}	$\frac{\alpha_1\theta_2\theta_5 c_{n1} + \theta_3(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 2\alpha_1\theta_1\theta_3\theta_5 - \theta_3\theta_8}{\alpha_1\theta_4\theta_7}$
p_{n2}^{OR2}	$\frac{4\alpha_1\theta\theta_2\theta_3 c_{n1} + \theta_5(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 8\alpha_1\theta\theta_1\theta_3^2 - \theta_5\theta_8}{2\alpha_1\theta_4\theta_7}$
p_{r2}^{OR2}	$\alpha\left(\frac{4\alpha_1\theta\theta_2\theta_3 c_{n1} + \theta_5(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 8\alpha_1\theta\theta_1\theta_3^2 - \theta_5\theta_8}{2\alpha_1\theta_4\theta_7} + \alpha_1\theta\delta_2\right)$
d_{n1}^{OR2}	$\frac{\theta_2(-2\alpha_1\theta_6 c_{n1} + (2\theta_2 - \alpha\theta\theta_3)c_{n2} + 2\alpha_1\theta_1\theta_5 - \theta_8)}{2\alpha_1\theta_1\theta_4\theta_7}$
d_{n2}^{OR2}	$\frac{2\alpha_1\theta\theta_2^2 c_{n1} + \theta_6(2\theta_2 - \alpha\theta\theta_3)c_{n2} + 4\alpha_1\theta\theta_1\theta_2\theta_3 - \frac{1}{2}\theta\theta_8}{2\alpha_1\theta\theta_1\theta_4\theta_7}$
d_{r2}^{OR2}	δ_2

TABLE 6. Optimal decisions of OR2c strategy.

Variable	Optimal solution
w_{n1}^{OR2}	$\dfrac{(2\theta_2^2-\alpha\theta_1^2(\theta+1))c_{n1}+\theta_2(1-\alpha\theta)c_{n2}+\theta_1(\theta_5+\alpha(2\theta^2-3\theta-4))-\frac{1}{4}\theta_8}{\theta_7-\alpha(2\theta^3-6\theta^2-\theta+8)}$
w_{n2}^{OR2}	$\dfrac{\alpha_1\theta\theta_2c_{n1}+2\theta_2^2(1-\alpha\theta)c_{n2}+2\alpha_1\theta\theta_1\theta_3-\theta_2\theta_8}{\theta_7-\alpha(2\theta^3-6\theta^2-\theta+8)}$
w_{r2}^{OR2}	$\alpha\left(\dfrac{\alpha_1\theta\theta_2c_{n1}+2\theta_2^2(1-\alpha\theta)c_{n2}+2\alpha_1\theta\theta_1\theta_3-\theta_2\theta_8}{\theta_7-\alpha(2\theta^3-6\theta^2-\theta+8)}-2\theta\alpha_1\delta_2\right)$
p_{n1}^{OR2}	$\dfrac{(\theta_5+\alpha(2\theta^2-3\theta-4))(\theta_2c_{n1}+2\theta_1\theta_3)+2\theta_3(\alpha\theta+\theta_2)c_{n2}-\frac{1}{2}\theta_3\theta_8}{\theta_4(\theta_7-\alpha(2\theta^3-6\theta^2-\theta+8))}$
p_{n2}^{OR2}	$\dfrac{(\theta(\alpha(\theta^2-\theta-4)+2\theta_3))(\theta_2c_{n1}+2\theta_1\theta_3)+\alpha_1\theta_2\theta_5c_{n2}-\frac{1}{4}\theta_5\theta_8}{\theta_4(\theta_7-\alpha(2\theta^3-6\theta^2-\theta+8))}$
p_{r2}^{OR2}	$\alpha\left(\dfrac{(\theta(\alpha(\theta^2-\theta-4)+2\theta_3))(\theta_2c_{n1}+2\theta_1\theta_3)+\alpha_1\theta_2\theta_5c_{n2}-\frac{1}{4}\theta_5\theta_8}{\theta_4(\theta_7-\alpha(2\theta^3-6\theta^2-\theta+8))}+\alpha_1\theta\delta_2\right)$
d_{n1}^{OR2}	$\dfrac{\theta_2\big((\alpha(\theta^3-3\theta^2-\theta+4)-\theta_6)c_{n1}+\theta_2(1-\alpha\theta)c_{n2}+\theta_1(\theta_5+\alpha(2\theta^2-3\theta-4))-\frac{1}{4}\theta_8\big)}{\theta_1\theta_4(\theta_7-\alpha(2\theta^3-6\theta^2-\theta+8))}$
d_{n2}^{OR2}	$\dfrac{\theta_2(1-\alpha\theta)(\theta\theta_2c_{n1}-\theta_6c_{n2}+2\theta\theta_1(\theta_3-\alpha\theta_2\theta_4\delta_2))}{\theta\theta_1\theta_4(\theta_7-\alpha(2\theta^3-6\theta^2-\theta+8))}$
d_{r2}^{OR2}	δ_2

The effects of manufacturing costs on the optimal decisions under the OR2a, OR2b, and OR2c strategies are presented in Tables 7–9, respectively.

TABLE 7. Effects of the manufacturing costs on the optimal decisions in OR2a strategy.

	w_{n1}^{OR2a*}	w_{n2}^{OR2a*}	w_{r2}^{OR2a*}	p_{n1}^{OR2a*}	p_{n2}^{OR2a*}	p_{r2}^{OR2a*}	d_{n1}^{OR2a*}	d_{n2}^{OR2a*}	d_{r2}^{OR2a*}
c_{n1}	↑	↑	↑	↑	↑	↑	↓	↑	0
c_{n2}	↑	↑	↑	↑	↑	↑	↑	↓	↑

TABLE 8. Effects of the manufacturing costs on the optimal decisions in OR2b strategy.

	w_{n1}^{OR2b*}	w_{n2}^{OR2b*}	w_{r2}^{OR2b*}	p_{n1}^{OR2b*}	p_{n2}^{OR2b*}	p_{r2}^{OR2b*}	d_{n1}^{OR2b*}	d_{n2}^{OR2b*}	d_{r2}^{OR2b*}
c_{n1}	↑	↑	↑	↑	↑	↑	↓	↑	0
c_{n2}	↑	↑	↑	↑	↑	↑	↑	↓	0

TABLE 9. Effects of the manufacturing costs on the optimal decisions in OR2c strategy.

	w_{n1}^{OR2c*}	w_{n2}^{OR2c*}	w_{r2}^{OR2c*}	p_{n1}^{OR2c*}	p_{n2}^{OR2c*}	p_{r2}^{OR2c*}	d_{n1}^{OR2c*}	d_{n2}^{OR2c*}	d_{r2}^{OR2c*}
c_{n1}	↑	↑	↑	↑	↑	↑	↓	↑	0
c_{n2}	↑	↑	↑	↑	↑	↑	↑	↓	0

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A Stackelberg-Nash game framework for overcoming remanufacturing barriers in a duopoly

Abbasi, Najmeh; Lotfi, M. M.; Shishebori, Davood; Forghani, Asefe; Honarvar, Mahboobeh

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