

THE COLLEGE OF AERONAUTICSCRANFIELDTesting and Analysis of a 60° Swept Back Wing
with Ribs Parallel to the Line of Flight *

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SUMMARY

The specimen used in this work represents the load carrying structure of a cambered two spar 60° sweptback wing, having closely pitched ribs of large boom area parallel to the line of flight, and thick skins. The wing has an aspect ratio of 2.45, a semispan of 101.5" and a taper ratio of 1.50. Construction was of light alloy, and the root was built into concrete.

Loading was by bending and torsion couples, and normal shear forces applied at, or near to, the tip. Both strain and deflection measurements were made. The strains in portions of the structure removed from root effects, and the stiffnesses were compared with the oblique coordinate theory of Hemp.⁽¹⁾ Two versions of this theory were used. A General Theory which considers the camber of the wing, and a Simplified Theory, which uses equivalent rectangular sections and allows an approximate root correction for the stiffnesses to be made. Lack of a suitable theory prevented strain comparisons at the root.

The more important conclusions reached are as follows:-

/ The use of

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* This investigation was made during the tenure by the author of a Clayton Fellowship awarded by the Institute of Mechanical Engineers.

The use of oblique coordinate theory is justified for both stiffnesses and strains. For wings in which the camber is not large, the Simplified Theory is sufficiently accurate, although an allowance for root constraint and rivet slip is necessary for stiffness evaluation.

The effect of taper is to reduce the cross sectional variation in strain predicted for normal shear force loading, a pure couple theory being better for direct strains in this case.

The direct strains build up towards the rear spar at the root, but the shear strain distribution is complex and requires further investigation. A second order discrete rib theory is required to predict this root strain variation.

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LIST OF CONTENTS

	<u>Page</u>
1.0 Introduction	4
2.0 Details of Experimental Apparatus	5
2.1 The Specimen	5
2.2 Root Details	6
2.3 Gauging	6
2.4 Method of Loading	7
3.0 Scope of Work	7
3.1 Preliminary Tests	7
3.2 Loading Cases	7
3.3 Deflection Tests	8
3.4 Strength Tests	8
4.0 Calculations	8
4.1 Definitions of Stiffnesses	8
4.2 Calculations associated with experimental work	9
4.3 Theoretical Calculations	9
5.0 Presentation of Results	10
6.0 Discussion	11
6.1 Stiffnesses	11
6.2 Deflections	13
6.3 Strains away from root effects	13
6.4 Strains at the root	15
7.0 Conclusions	18
..... References	20
Tables	
Figures	

1.00 INTRODUCTION

Despite the considerable use of swept back wings for aircraft during the past few years, very little information is available on the structural characteristics of swept wings having ribs parallel to the line of flight. Some work⁽²⁾ has been carried out on small scale boxes using this rib configuration, but the published work does not cover the problems associated with an actual wing. The present work is an attempt to reduce this gap in available information.

The specimen used in these tests was constructed so as to be representative of the load carrying structure of a swept-back wing designed in accordance with current practice. The wing has a 60° sweptback leading edge, a low aspect ratio, and a tapering cambered aerofoil section. The structure is of two spar construction with thick skins and heavy rib booms. As practical aircraft wing root conditions vary considerably, a fully built-in root was chosen as being that which is most commonly considered by theory.

The testing consisted of an investigation of the stiffness, distortion, and strain characteristics of the structure, both at the root, and at points removed from the root effects. The loading was by pure couples, and shear forces normal to the wing. A theoretical analysis of the strains at points away from the root, and the stiffnesses, has been made using oblique coordinate theory, and a comparison made with the experimental results.

/ 2.0

2.0 DETAILS OF EXPERIMENTAL APPARATUS

2.1 The Specimen

The wing geometry is shown in Fig.1, the leading edge sweepback being 60° , the aspect ratio 2.45, and the tip chord two thirds the root chord of 120 in. The thickness chord ratio of 13% at the root, falling to 12% at the tip is rather higher than is normal. This is of no importance in the present work as the effect is to alter the magnitude of the skin stresses, leaving the nature of the distribution unchanged. The camber is represented on the specimen by two straight lines, a measure adopted merely to simplify its manufacture.

The structure has a root chord of 60 in, and the length along its centreline, which corresponds to the 40% chord-line of the wing is 169.3 in. Rib 1 is 22in outboard of the root section, and the rib pitch is 12 in. along the front spar. Fig.2 gives the structural details, which can also be seen in Figs. 5-8.

Light Alloy is used throughout, the skins being 0.08 in. and the spar webs 0.022 in. thick D.T.D. 390. The spar booms are tapered tee section extrusions, whilst the rib booms are built up double angles made from 0.08 in. D.T.D. 390. The diaphragm type rib webs have a thickness of 0.036 in. and angle section stiffeners.

Of the five loading points, two are on the front spar, and the remainder on the rear spar. These are arranged to enable the application of pure couples along and normal to the centreline, and shear forces normal to the plane of the wing. Fig.3 shows the location of these loading points, which are of welded steel construction, and attached to both spar webs and booms.

/ 2.2

2.2 Root Details

In the past it has been found difficult to obtain conditions apertaining to a fixed root, and in an attempt to rectify this, the root of the specimen was built into concrete. The only movement that was measured on the completed root was a rigid body rotation of the whole assembly which was less than 10^{-5} radians. Details of the root prior to its being cast into concrete are shown in Fig.6. The edges of the skin were bolted to $3\frac{1}{2}$ in. steel angles, which were in turn joined to heavy steel channels placed normal to the plane of symmetry of the wing. Numerous steel reinforcing bars were placed through the skins and in the inside of the root. With the wing in the position shown in Fig.6, the inside was filled up to the first rib with concrete made by using fine ballast.

The 4 in. thick concrete floor of the laboratory was provided with $\frac{1}{2}$ in. diameter "Loosebolt Rawlbolts", to coincide with four holes in each of the steel channels. When the concrete was set, but still green, the wing was placed in position. Shuttering was built up round it, and concrete poured in up to the level of the first rib. Fig.3 gives the external dimensions of the concrete block, which weighed approximately 35 cwts. When the base was set, the channels were bolted to the floor.

2.3 Gauging

Dial gauges were arranged to record the distortion of the lower surface of the wing, and were positioned as shown in Fig.3. They were supported on a framework constructed of "Dexion" angle.

Electrical resistance strain gauges were placed on the wing in the positions shown in Fig.4. The locations and number of gauges were chosen to give an overall picture of the stress distribution. It was realised that the number would be insufficient to investigate detail problems, but it was desired to avoid the possibility of having more gauges than could be conveniently read and analysed.

In all 110 channels were used, distributed over eight chord-wise sections. Three of these were out from the root, four at the root, and one just inside the concrete. This last section did not yield satisfactory results.

2.4 Method of Loading

Two matched hydraulic jacks were used to apply loads to the specimen. In order to facilitate the loading, two $3\frac{1}{2}$ in. steel angles were used to join the two loading points at a given rib station, as can be seen in Figs. 7-8. A series of holes in these angles enabled a variety of loading combinations to be used. Nearby test frames were used as earths for the jacks, which were arranged to work on the retraction stroke only.

3.0 SCOPE OF WORK

3.1 Preliminary Tests

Control tests were conducted on specimens made from the materials used in the construction of the wing, in order to determine their elastic properties. The average results obtained were used in the theoretical calculations.

In the past, some difficulty has been found in analysing strain gauge results for spar booms and webs. An attempt to overcome this difficulty was made by subjecting the spars to known bending loads, prior to their being used in the wing. The gauge readings were compared with the equivalent beam theory values, and the resulting calibration curves used in analysing the gauge readings for the completed wing.

3.2 Loading Cases

In the various tests, four different types of loading were used:-

- Case 1) Pure "bending" couple, M_A , applied about an axis in the plane of, and normal to the centreline of the wing. This corresponds to the couple M_1 , of oblique coordinate theory and was obtained by applying loads at the centres of loading points A-B, and C-D. (See Fig.3)
- Case 2) Pure "Torsion" couple, T_A , about the centreline of the wing, and corresponding to the couple L of oblique coordinate theory. The loads were applied at points A and E.

/ Case 3

Case 3) Loading by a shear force on the centreline of the tip rib, Rib 14, in a downward direction, this case being denoted by Z. The load was applied at the centre of A-B.

Case 4) A shear force on the centreline at Rib 12, in an upward direction, denoted by $(-Z_1)$. In this case the loading point used was the centre of C-D.

The maximum load applied at any one point on the wing was 2100 lb.

3.3 Deflection Tests

The dial gauge readings were taken under loading corresponding to all the above four cases. With the exception of Case 4 loading, when the front and rear spar gauges only were used, all the gauges were read.

3.4 Strength Tests

All strain gauge readings were taken for all the four loading cases.

4.0 CALCULATIONS

4.1 Definition of Stiffnesses

Throughout this work, two stiffnesses are considered, a "flexural" stiffness and a "torsional" stiffness. The stiffnesses are those used by Molyneux⁽³⁾, in setting out the stiffness criteria for the prevention of body fixed flutter and divergence, except that a pure couple is used to define the flexural stiffness rather than a concentrated force.

These are defined as follows:-

- 1) "Flexural" Stiffness, defined as the couple of the type considered in loading case 1, necessary to cause unit rotation of a given length of the centreline of the wing.
- 2) "Torsional" Stiffness, defined as the couple, of the type considered in loading case 2, required to give unit relative rotation of two sections normal to the centreline, and spaced a given distance apart, along it.

/ In each

In each case, two different lengths along the centreline have been considered. The first is from stations $x = 0''$ (root) to $x = 111''$, and includes root effects, whilst the second length extends from $x = 46''$ to $x = 111''$, and is outside the root effects.

4.2 Calculations associated with the Experimental Results

Two methods were used to derive the actual torsional stiffness of the wing from the experimentally determined deflections.

The first method consists of using the rotation components defined in oblique coordinates⁽¹⁾. The torsional rotation at a given section is $(p + q \cos \alpha)$, where p and q are the rotations about the oblique axes x and y , and α is the angle between the axes. The rotation p is determined from the relative deflection of the front and rear spars, along the y axis, and $q = dW/dx$, where W is the normal deflection of the x axis.

In the second method, the dial gauge readings were used to plot deflection contours for the wing, and these in turn were used to obtain the distortions of sections normal to the centreline, as shown in Figs. 12 and 13. The slopes at the points of zero deflection were used to estimate section rotations.

The flexural stiffnesses were derived by using the method of oblique coordinate rotation components. The rotation in this case is defined as $(q \sin \alpha)$.

4.3 Theoretical Calculations

The theories applied to the solution of the wing use oblique coordinates, and are those of Hemp⁽¹⁾.

Two solutions are given:-

- 1) The "General" solution of Ref. 1 Part 3, which makes allowance for the camber of the section, and assumes symmetry about the xOy plane, (plane of the wing), only. No allowance is made for root effects.
- 2) A "Simplified" solution, of Ref. 1 Part 2. This solution is based on equivalent rectangular sections having the following properties:-

/ (a)

- (a) The same torsional stiffness, (Batho), area as the actual section.
- (b) The same section moment of inertia.
- (c) The same total boom area.
- (d) The same equivalent skin and web thickness.

It is possible to make an approximate solution for the root problem using these sections, and this has been done in the case of the stiffnesses.

The sections used for the "General" theory are given in Table 1, and those for the "Simplified" theory in Table 2.

5.0 PRESENTATION OF RESULTS

5.1 Stiffnesses

The experimental and calculated stiffnesses are given in Table 4. The stiffness coefficients, C_{ij} , and the rotation components of oblique coordinate theory are given in Table 3.

5.2 Deflection Results

All these results are derived from the experimental work. The rib deflections due to the loading cases 1-3, are given in Figs. 9-11.

Figs. 12-13 show the deflections of the lower skin across sections normal to the centreline. These were derived from a plot of the deflection contours.

5.3 Strength Results

The strength test results are presented in the form of strain comparisons. The following strains are used:-

The direct strain, e_{xx} , in a direction parallel to the x axis.

The direct strain, e_{yy} , in a direction normal to the x axis.

The direct strain, e_{yy} , in a direction parallel to the y axis, being the rib boom strain.

The shear strain, e_{xy} , associated with direct strains e_{xx} and e_{yy} .

The shear strain e_{yz} , being rib web shear strain.

The shear strain e_{xz} , being spar web shear strain.

/ For

For the sections outboard of the root effects, namely Ribs 9-10, 10, and 10-11, comparison is made between experiment and oblique coordinate theory for all four loading cases. The results are given in Figs. 14-30. The variation of e_{xx} along the centreline is given in Figs. 31-32.

The results for sections at the root, Ribs 1-2, 2, 2-3 and 4-5, are derived from experiment only. These are presented in Figs. 33-54, all four loading cases being used. Figs. 55-57 give the variation of e_{xx} over the root section for loading cases 1-3, and Figs. 58-60 the corresponding variation in spar web shear strain, e_{xz} .

6.0 DISCUSSION

6.1 Stiffnesses

The stiffnesses are presented in Table 4, whilst Table 3 gives the theoretical stiffness coefficients, C_{ij} .

Stiffness Coefficients.

The C_{ij} are used in the calculation of the stiffnesses, and in fact define the rotation and deflection of the wing. The actual relations between the C_{ij} and rotations are given in Ref. 1, Part 3. They are entirely dependent upon the structure and with a tapered specimen increase in value towards the tip. The coefficient C_{13} gives the rotation about the x axis due to normal shear force, and is seen to be very small in this case. It is zero for a wing having symmetry about the xOz plane, and hence does not appear in the Simplified theory.

The values derived from both the General and Simplified theories are given in Table 3. The most obvious difference is in C_{11} , which is higher in the Simplified theory.

Torsional Stiffness away from the root.

The experimental values given in Table 4, have been derived using the two methods discussed in §4.2. The result in both cases is a stiffness 108% of the calculated Batho value, which makes no allowance for rib stiffness. The deflection of sections normal to the centreline, Figs. 12 and 13, shows that

/ the calculation

the calculation of slopes using relative spar deflections across normal sections is open to considerable error, due to the distortion of these sections. The method which uses oblique rotation components is better, as the slopes are measured in the plane of the ribs.

The torsional stiffnesses as derived by the General and Simplified theories are in good agreement, although the latter gives a slightly lower value. This is a direct result of the higher value of C_{11} , mentioned above, and is explained by the fact that the Simplified theory uses a rectangular section of equivalent cross sectional area to the cambered section of the General theory, the latter being a nearer approach to the "ideal" circular section.

These values are higher than the experimental results, a fact which is almost certainly explained by rivet slip. Some loss of torsional stiffness due to rivet slip is predicted by Ref.4. Considerable care was taken during the manufacture of the specimen in order to reduce rivet slip to a minimum, particular attention being paid to rivet hole clearances.

Torsional Stiffness over the whole specimen.

These values are compared with the Batho stiffness which makes no allowance for the root constraint effects.

There is some discrepancy in the two experimental values, most probably resulting from inaccuracies in the evaluation of the centreline deflections at the root due to their small values. In all probability the true value lies between the 111% and 115% quoted. In this case the effect of the root constraint is to increase the torsional stiffness by some 3%.

The General theory has been applied without any constraint allowance, and gives a high value of 116%. The values for the Simplified theory include one which has been corrected for root effects. These results are rather low, and the theoretical root constraint allowance gives only a 2% increase of stiffness.

It would appear that the best method of estimating the torsional stiffness would be to use the Simplified theory of equivalent rectangular sections, with a root correction and reference to the effect of rivet slip.

/ Flexural

Flexural Stiffness away from the root.

Both the General and Simplified theories give the same result, which is somewhat lower than that of experiment. This has been previously noted⁽²⁾.

Flexural Stiffness over the whole specimen.

The effect of the root constraint as applied to the Simplified theory is to increase the stiffness by 5%. Again there is no difference between the General and Simplified theories, when the root effects are not considered, and the theoretical values are low in comparison with the experimental results.

It will be seen that the use of the Simplified theory with a root correction will give the best results, but these nevertheless will underestimate the true value.

6.2 Deflections

The rib deflections under loading by pure couples and a Z wise force are given in Figs. 9-11.

Although there is not a great deformation of the ribs in the case of loading by the couple M_A , Fig. 9, there is a slight increase in the slope of the deflection curves towards the front and rear spars. This effect is more pronounced in the case of loading by the Z wise force, Fig. 11.

The distortion under loading by a torsion couple, T_A , is different. As is shown in Fig. 10, there is a large overall bowing of the section, and it demonstrates clearly the necessity of considering the flexibility of the ribs when making calculations associated with this type of loading.

6.3 Strains away from Root Effects (Ribs 9-11)

The direct and skin shear strains for stations 9-11, the rib boom and web strains at Rib 10, and the spar web strains at station 9-10, due to all four loading cases, are shown in Figs. 14-32. Comparison is made between experiment and the General theory, whilst the Simplified theory has also been applied at station 9-10.

/ The results

The results show that in the case of direct strain, e_{xx} , due to bending couple M_A , there is quite good agreement, particularly on the compressive skin at the rib boom, but that there is a tendency for the experimental strains to be low. This is most likely due to local variations in the skin thickness and Elastic properties, as the theoretical results are based on average values. The effect of camber as given by the General theory is reproduced by the experimental results, but it is not large.

In the case of loading by the torsion couple T_A , the theoretical and experimental values of e_{xx} agree on the centre-line of the wing, but there is a slight experimental variation across the section, not shown by the theory.

As in the case of loading by the couple M_A , the theoretical values for e_{xx} are higher than those of experiment, when normal shear forces are applied. In addition, the theories predict a cross section variation which is not found experimentally. This has previously been noticed on a tapered box, Ref. 2, and it would appear that the effect of taper is to cancel the e_{xx} variation due to shear strain. A pure couple theory gives better agreement in this case.

The comparison of the direct strains e_{yy} , shows that while the theory predicts a mean value, there is some discrepancy, in particular the experimental values are high on the centreline. This may well be due to the diffusion problem associated with the application of continuous rib theory to a discrete rib specimen. The discrepancies are not large and do not preclude the application of continuous rib theory to more practical structures.

In the case of the skin shear strains, e_{xy} , there is good agreement for the pure couple cases, except that the theory is slightly high for M_A , and for T_A at the rear of station Ribs 10-11. This latter discrepancy is probably due to the proximity of the loading point. There is much less experimental cross sectional variation due to normal shear forces, than is predicted, and this fact confirms the remarks made above concerning the e_{xx} variation in a tapered box.

/ The rib

The rib boom strains, e_{yy} , in Rib 10 show reasonable agreement between theory and experiment, except that the theory is rather high for couple M_A , and for the front portion of the tension boom when normal shear force loading is applied.

The theory assumes zero rib web shear strain, e_{yz} , and the experimental value is very low on the centreline, but rather higher adjacent to the spar webs. This is probably connected with the discrete rib diffusion problem.

Although the spar web strains e_{xz} , are of the same order as predicted, there is a discrepancy, in the normal shear force loading cases, explained by the shear relief due to the spar boom loads being affected by the cross sectional e_{xx} variation referred to above.

Summarising these results, it will be seen that the oblique coordinate theory is applicable and gives good results. The use of the General theory is not really necessary for wings having small camber, especially in view of the considerable arithmetic complication. In this case the use of rectangular sections, with a correction as predicted by the beam theory would be the best solution.

6.4 Strains at the root

The strains at the root are shown in Figs. 33-60. The values are those obtained experimentally, no suitable theory being available for purposes of comparison.

The variation of e_{xx} over the root, due to the application of the couple M_A , is shown in Fig. 55. This variation is typical of that obtained for a swept wing, with the strain build-up towards the rear spar root. The strain across station 1-2 is given in Fig. 33. It increases towards the rear spar, the increase being more rapid as the rear spar is approached. The value of e_{xx} at the rear spar is some ten times that at the front. The strains at Rib 2, Fig. 39, are similar, although there is better agreement between the upper and lower surfaces, but at station 2-3, Fig. 43, there is a fall away at the rear spar. The distribution at station 4-5, Fig. 49, is not unlike that obtained at station 9-10, but there is a slight increase across the forward part of the skins.

/ An overall

An overall picture of the variation in e_{xx} due to the couple T_A , is given in Fig. 56. Although the scale is exaggerated, it is apparent that there is a rapid variation in the value and nature of the direct strain distribution. At station 1-2, Fig. 23, the strain increases towards the centreline and is then constant, and at Rib 2, Fig. 39, the centreline values are approximately the same. There is a difference in the two skins at station 2-3, Fig. 43, the top skin value being roughly constant, and the lower increasing towards the rear. This difference is repeated at station 4-5, Fig. 49, where the bottom skin strain varies linearly across the section, being zero on the centreline, and the top skin falls to zero at the rear spar.

The direct strain distribution at the root due to normal shear force loading is not unlike that due to M_A . It is shown in Fig. 57. There is, however, a fall off at the rear spar.

This is clearly shown in Fig. 34 which is the distribution at station 1-2. It is possible that the very high skin shears near the rear spar at the root overcome the direct strain build-up and also, in this case, the effect of the taper of the wing. This is confirmed by the increased fall off in the case of loading by $-Z_1$, where the direct strain is less. Fig. 40, shows that the strains at Rib 2 are similar, and give better agreement between the two surfaces. The distribution at station 2-3, Fig. 44, is like that at station 1-2, and at station 4-5, Fig. 50, there is a constant strain over the centre of the wing with a fall away at the front and rear spars. At this section the strain distribution is beginning to resemble that found at station 9-10.

The variations in the direct strain e_{yy} due to the couples M_A and T_A are shown in Figs. 35, 45 and 51. For the case of couple M_A , there is an increase towards the rear spar at station 1-2, but the tendency is reversed at station 2-3. At station 4-5 there is an initial increase to the centreline of the section and the lower skin values are higher. The variation due to T_A at station 1-2 takes the form of low values at the spars coupled with a high value on the centreline. This distribution is repeated at stations 2-3, but at station 4-5, e_{yy} increases up to the centreline and is then roughly constant.

/ The e_{yy}

The e_{yy} strains due to normal shear force loading are given in Figs. 36, 46 and 52. There is little agreement between the results for Z and $-Z_1$, or for the various stations. However, at station 4-5 the values of e_{yy} are approximately constant, apart from a fall off at the spars. This results in some resemblance to the equivalent distribution obtained at stations 9-0.

Figs. 37, 47 and 53 present the skin shear strains e_{xy} due to the pure couple loading cases. In both the cases of M_A and T_A there is a change in the form of the variation from one bay to the next, but again the distributions at stations 4-5 have a form not unlike those obtained for the stations away from the root. The normal shear force loading distributions, Fig. 38, 48 and 54, indicate a build up in shear strain e_{xy} across the section, which is not maintained towards the rear spar but drops off locally in some cases.

The direct strains, e_{yy} in the booms of Rib 2 are shown in Fig. 41 and 42. In all the loading cases, there is an increase in strain towards the rear spar.

Fig. 58 gives the spar web shear strain, e_{xz} , variation at the root for loading by couple M_A . Although the value is high at the root, it rapidly decreases. The front web is the higher, due to the lower shear relief from the boom end loads in the front spar. The equivalent variation for the torsion couple loading, T_A , is Fig. 59. The front and rear spar values are approximately equal, although the front web strain is low at station 4-5. This may be due to the relatively large relief from the boom loads obtained at this point, Figs. 49 and 56.

The rapid changes in the web shear caused by normal shear force loading can be seen in Fig. 60. At the root, the front spar strain is very low, and may even be opposite in sign to that at the rear spar. This rapid decrease is shown in an interesting way by the die away of the buckles in the front spar web near the root, which can be plainly seen in Fig. 8. The rate of increase in the rear spar is not maintained to the root rib. This may be due to shear relief. The front spar strain reaches a high value by station 4-5, although it is still less than the rear spar value.

/ The results

The results obtained for the strains at the root show that the constraint effects become low at station 4-5 on the rear spar, but are somewhat more prolonged in the front spar.

There is a need for a detailed analysis of the skin shear strain at the root of this type of swept wing. This would be best carried out on a uniform rectangular box, having say, five shear gauges in a chordwise direction, and closely pitched in the spanwise direction, on both surfaces.

A second order discrete rib theory is required in order to obtain a theoretical prediction of these root strains.

7. CONCLUSIONS

7.1 Stiffnesses

(1) The theoretical values of stiffnesses predicted by the theory allowing for the wing camber, vary little from those predicted by the theory using equivalent rectangular sections.

(2) The experimental torsional stiffness is lower than the theoretical prediction, due to the presence of rivet slip.

(3) The effect of the oblique ribs is to increase the torsional stiffness to 11% greater than the Batho value, whilst the effect of root constraint is to give a further 3% increase.

(4) The experimental flexural stiffness is higher than the theoretical.

(5) Root constraint increases flexural stiffness by 5%.

(6) In reducing experimental results to give torsional stiffness, the use of the deflection measurements to calculate the rotations of points on the centreline, and thence the stiffness is best.

(7) For the theoretical prediction of stiffness, the equivalent rectangular section theory with a correction for root effects, and an allowance for rivet slip, gives the best results.

/ 7.2

7.2 Deflections

(1) There is an appreciable deformation of the oblique ribs, particularly in the case of torsional loading, showing the necessity of considering rib flexibility in theoretical solutions.

(2) The deformation of sections normal to the spars is not linear across the section, and hence it is incorrect to measure the torsional stiffness using the relative spar displacements of these sections.

7.3 Strain, away from the root

(1) The oblique coordinate theory, which allows for camber of the section, compares well with experiment for all the more important strains.

(2) The theory using equivalent rectangular sections also gives good results, which vary little from those of the more accurate theory.

(3) In order to reduce the calculations necessary in cases where the camber is not appreciable, the use of the latter theory, with if necessary a cross sectional correction in accordance with the beam theory, is advocated.

(4) The effect of taper is to reduce, or even eliminate, the cross sectional variation in the direct strain caused by normal shear force loading. The use of pure couple theory gives a more accurate prediction of the direct strain in this case.

(5) In general, better agreement is obtained between theory and experiment on the compressive surface, particularly at sections between the ribs.

(6) The discrepancies caused by the application of a continuous rib theory to a wing with discrete ribs, are not sufficiently large to rule out the use of this type of theory.

/ 7.4

7.4 Strains at the root

(1) The root constraint results in strains which agree with those normally associated with swept back wings. The direct and shear strains in the skins and webs build up towards the rear spar at the root.

(2) The rate of build up of the direct strains across the root section is not maintained up to the rear spar. This is probably due to the effect of very high local shears.

(3) The shear in the front spar falls to zero at the root.

(4) The root constraint effects disappear between Ribs 4 and 5 on the rear spar, although they are more prolonged on the front spar.

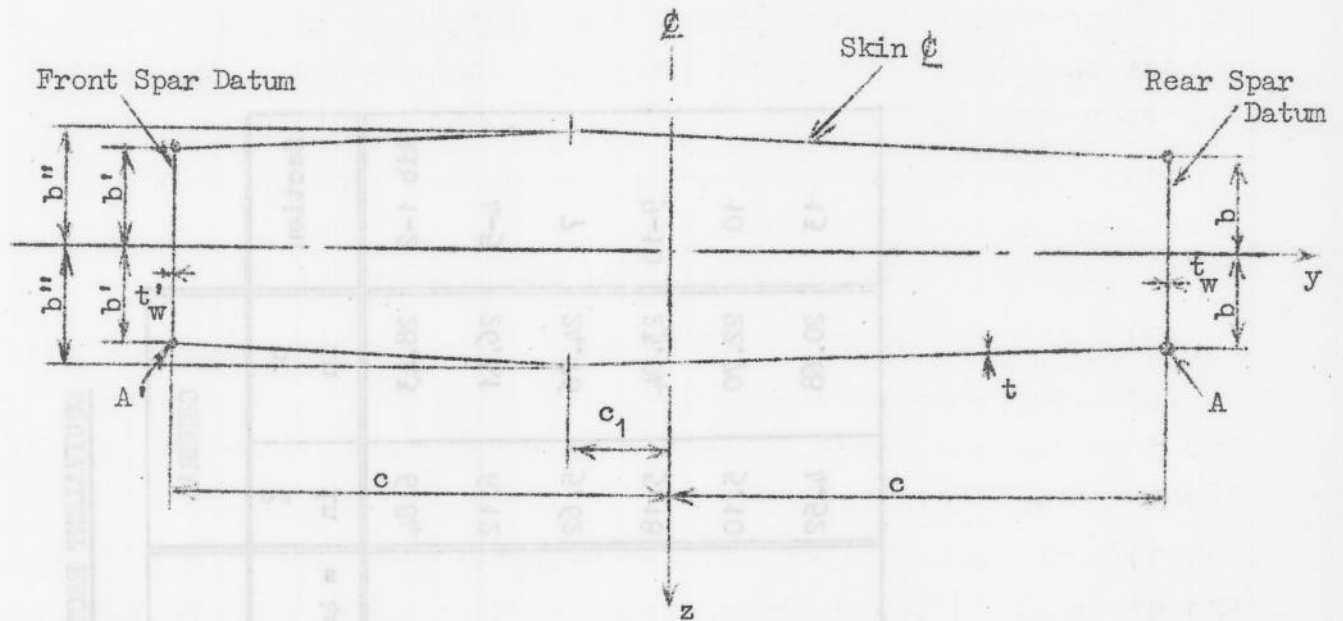
(5) The shear strain distribution at the root is complicated, and at least a second order discrete rib theory would be necessary to predict the variation.

(6) A careful and detailed experimental investigation of the root strains in a rectangular section uniform swept box is required in order to elucidate the shear strain variation.

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3	Molyneux, W.G.	The flutter of swept and unswept wings with fixed root conditions. R.A.E. Structures Report No. 58. January 1950
4	Hawker Aircraft Ltd.	Rivet slip and the flexure and torsion of box beams. Design Department Report No. 1138. September 1947

TABLE 1

IDEALISED SECTION GEOMETRY

SECTION		b in.	b' in.	b'' in.	c in.	c ₁ in.	t in.	t _w in.	t' _w in.	A sq. in.	A' sq. in.	x in.
Ribs 1-2	(B)	5.86	6.34	7.29	28.43	5.68	.08	.022	.022	.688	.686	5.72 ¹⁷
Rib 2	(C)	5.78	6.25	7.19	28.09	5.62	.08	.022	.022	.634	.627	11.45
Ribs 2-3	(D)	5.71	6.16	7.09	27.73	5.52	.08	.022	.022	.578	.600	17.17
Ribs 4-5	(E)	5.39	5.78	6.68	26.41	5.29	.08	.022	.022	.524	.542	40.06
Rib 7		4.98	5.31	6.13	24.73	4.95	.08	.022	.022	.452	.438	68.68
Ribs 9-10	(F)	4.60	4.85	5.67	23.04	4.61	.08	.022	.022	.366	.356	97.29
Rib 10	(G)	4.52	4.76	5.57	22.70	4.54	.08	.022	.022	.358	.330	103.01
Ribs 10-11	(H)	4.44	4.66	5.47	22.36	4.47	.08	.022	.022	.347	.315	108.74
Rib 13		4.05	4.20	4.96	20.68	4.14	.08	.022	.022	.279	.249	137.34

EQUIVALENT RECTANGULAR - SIMPLIFIED - SECTIONS

Section	GENERAL		COMMON		SIMPLIFIED		
	c in	$\bar{c}$ in	β $= 4\alpha \bar{c}$ in ²	I in ⁴	A $= \frac{A + A'}{2}$ in ²	b in	o in
R1b 1-2	28.43	6.84	778	370	0.687	6.32	30.8
4-5	26.41	6.12	646	277	0.533	5.92	27.3
7	24.76	5.62	556	213.6	0.445	5.48	25.4
9-10	23.04	5.18	473	165	0.361	5.07	23.3
10	22.70	5.10	463	154.9	0.344	4.95	23.4
13	20.68	4.52	375	109	0.264	4.46	21.03

TABLE 2

TABLE 3

STIFFNESS COEFFICIENTS AND ROTATIONS

Section	General							Simplified				Simplified - Modified At Root		
	C_{11} $\times 10^{10}$	$C_{12}=C_{21}$ $\times 10^{10}$	C_{22} $\times 10^{10}$	C_{13} $\times 10^{10}$	η^x	$\theta \times 10^{10}$ rad/in	$\phi \times 10^{10}$ rad/in	C_{11} $\times 10^{10}$	$C_{12}=C_{21}$ $\times 10^{10}$	C_{22} $\times 10^{10}$	$\theta \times 10^{10}$ rad/in	$\phi \times 10^{10}$ rad/in	$\theta \times 10^{10}$ rad/in	$\phi \times 10^{10}$ rad/in
Ribs 1-2	5.74	- 3.53	4.72	-0.77	+0.0047	5.81	2.55	6.02	- 3.53	4.72	6.38	2.55	6.07	1.72
4-5	7.66	- 4.72	6.31	-0.07	+0.00035	7.72	3.41	8.07	- 4.70	6.30	8.54	3.40		
7	10.10	- 6.12	8.16	+0.09	-0.00036	10.36	4.41	10.2	- 6.04	8.10	10.66	4.37		
10	13.28	- 8.44	11.25	1.02	-0.00338	13.06	6.07	13.62	- 8.40	11.26	13.79	6.08		
13	18.25	-11.96	16.04	2.57	-0.00682	17.61	8.67	19.10	-11.92	15.95	19.06	8.62		

TABLE 4

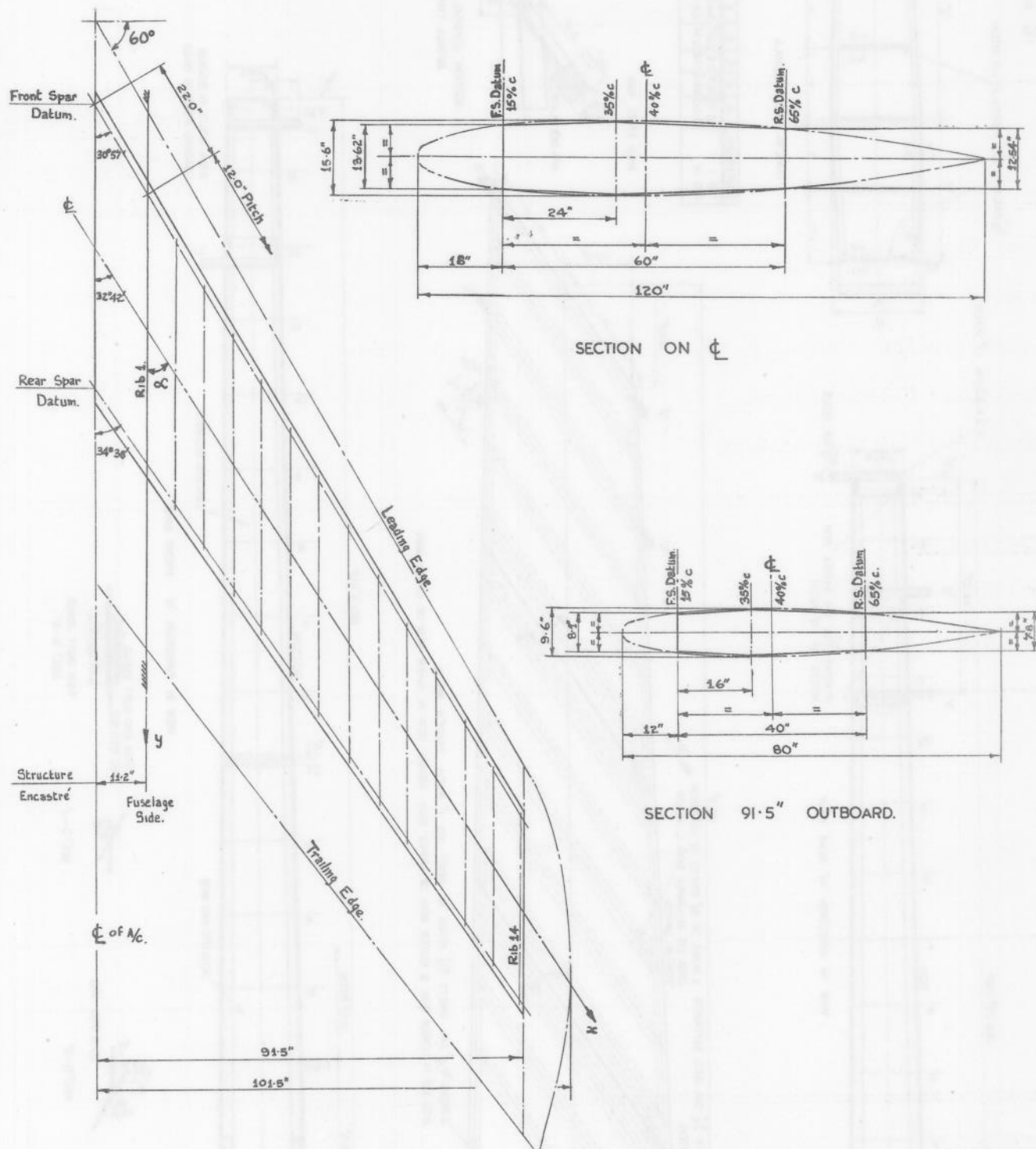
COMPARISON OF STIFFNESSES

Torsional Stiffness

	Root to Sec ^t . x = 111"		Sec ^t x = 46" to x = 111"	
	Stiffness lb.in/rad. x10 ⁻⁶	% Batho	Stiffness lb.in/rad. x10 ⁻⁶	% Batho
<u>Theoretical</u>				
Batho Formula	8.35	100%	12.4	100%
General Oblique Coord.	9.71	116%	13.79	112%
Simplified Oblique Coord.	9.22	109%	13.50	111%
Simplified Modified at Root	9.32	111%		
<u>Experimental</u>				
Rotation Components on ϕ	9.30	111%	13.4	108%
Rel. Slopes at Zero Deflection	9.62	115%	13.4	108%

Flexural Stiffness

	Root to Sec ^t x = 111"		Sec ^t x = 46" to x = 111"	
	Stiffness lb.in/rad. x10 ⁻⁶	% Expt.	Stiffness lb.in/rad. x10 ⁻⁶	% Expt.
<u>Theoretical</u>				
General Oblique Coord.	22.6	78%	32.0	92%
Simplified Oblique Coord.	22.6	78%	32.0	92%
Simplified Modified at Root	24.1	83%		
<u>Experimental</u>				
Rotation Components on ϕ	29.1	100%	35.0	100%



GEOMETRY.

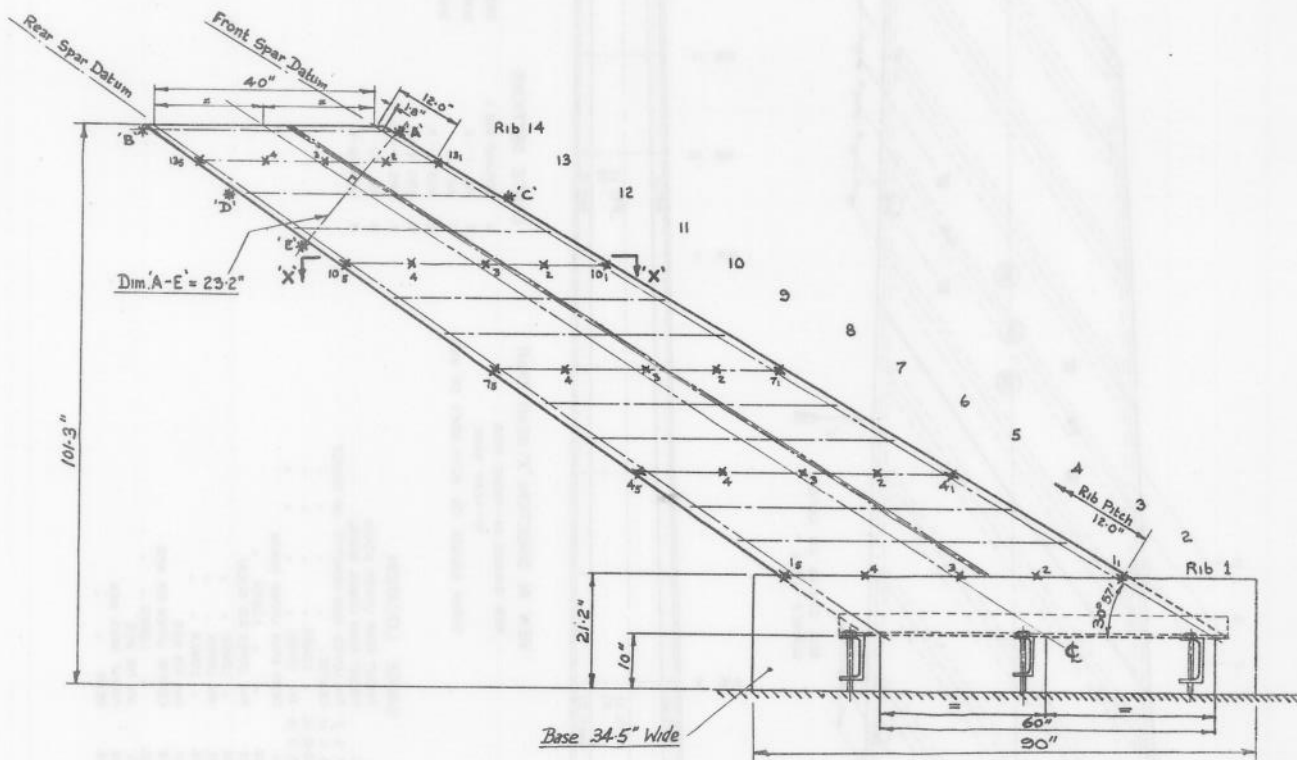
Span 203" Wing Area 16,780 $\square$ "
 Root Chord 120" Aspect Ratio 2.45
 Root Chord t/c 13% Taper Ratio 1:50
 Tip Chord t/c 12% L.E. Sweepback 60°

REPRESENTATIVE SWEPTWING

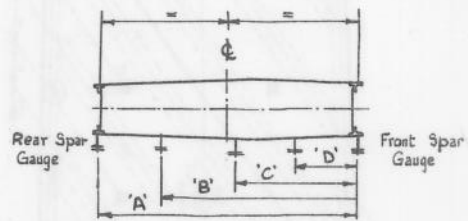
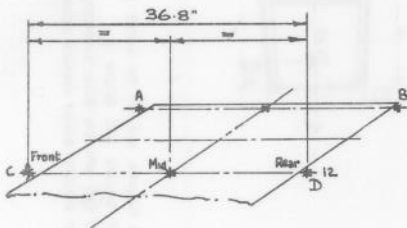
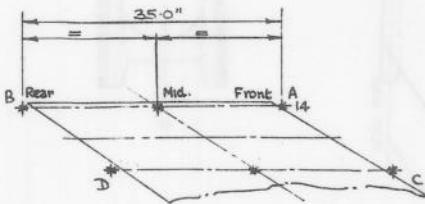
60° LEADING EDGE SWEEP



FIG. 3.



RIB	Dim. 'A'	'B'	'C'	'D'
1.5" OUTSIDE RIB 1	57.75"	43.1"	29.5"	14.6"
4	55.85"	41.55"	28.4"	13.5"
7	51.10"	38.3"	25.4"	12.2"
10	47.20"	35.4"	23.3"	11.6"
13	42.60"	31.9"	21.8"	11.1"



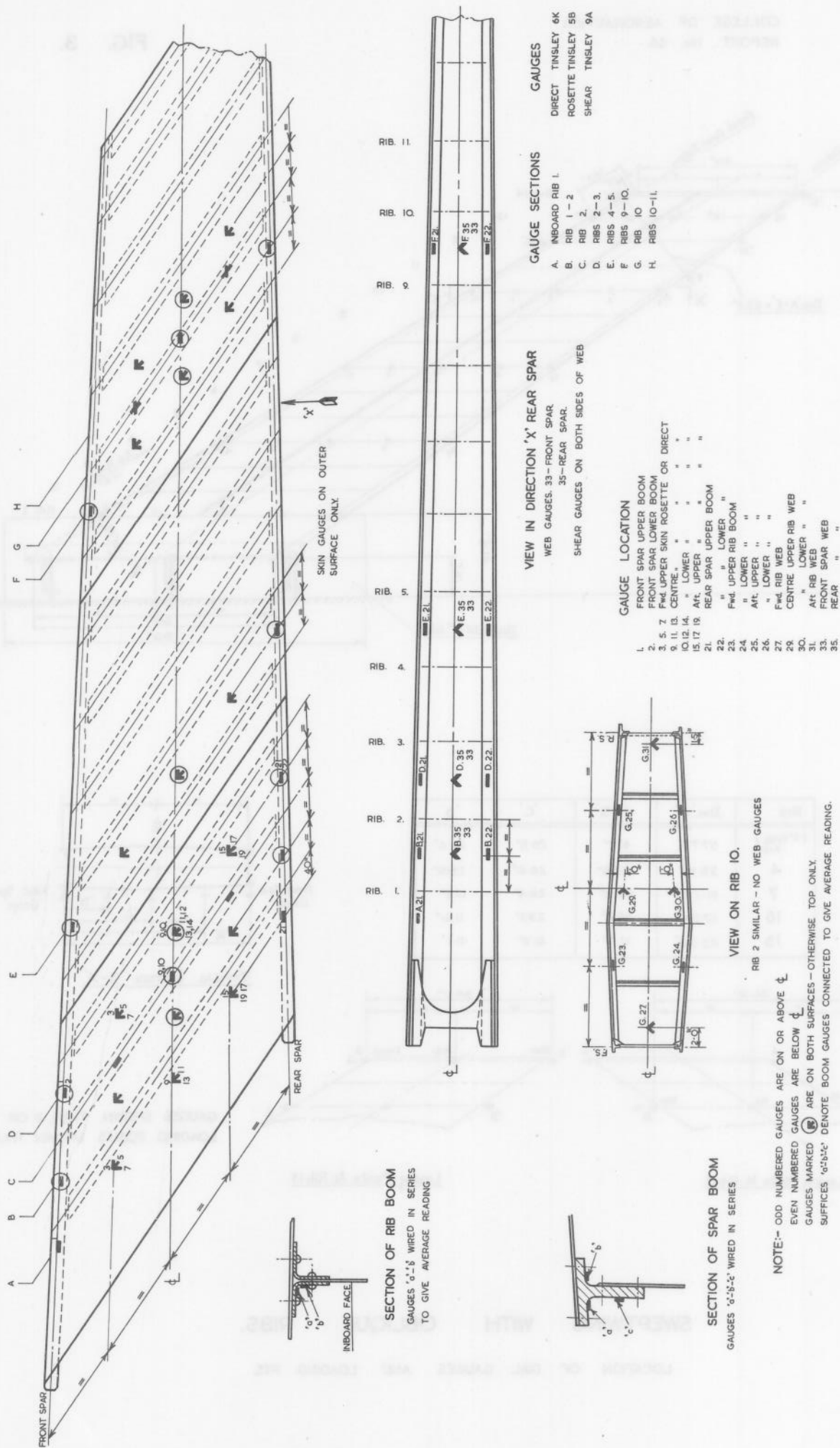
TYPICAL SECTION 'X-X'

GAUGES SHOWN THUS * OR †
LOADING POINTS SHOWN THUS *

SWEPTWING WITH OBLIQUE RIBS.

LOCATION OF DIAL GAUGES AND LOADING PTS.

FIG. 4.



SWEPTWING. HAVING RIBS PARALLEL TO LINE OF FLIGHT.
STRAIN GAUGE LOCATION.

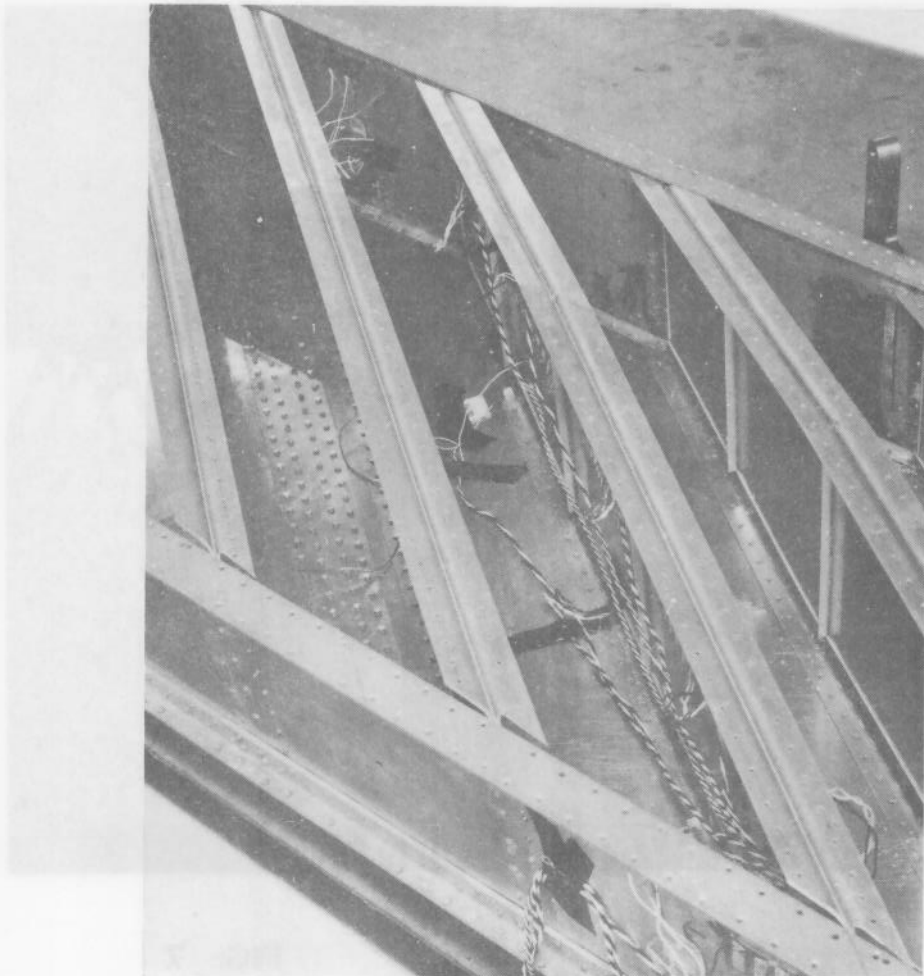


FIG. 5.

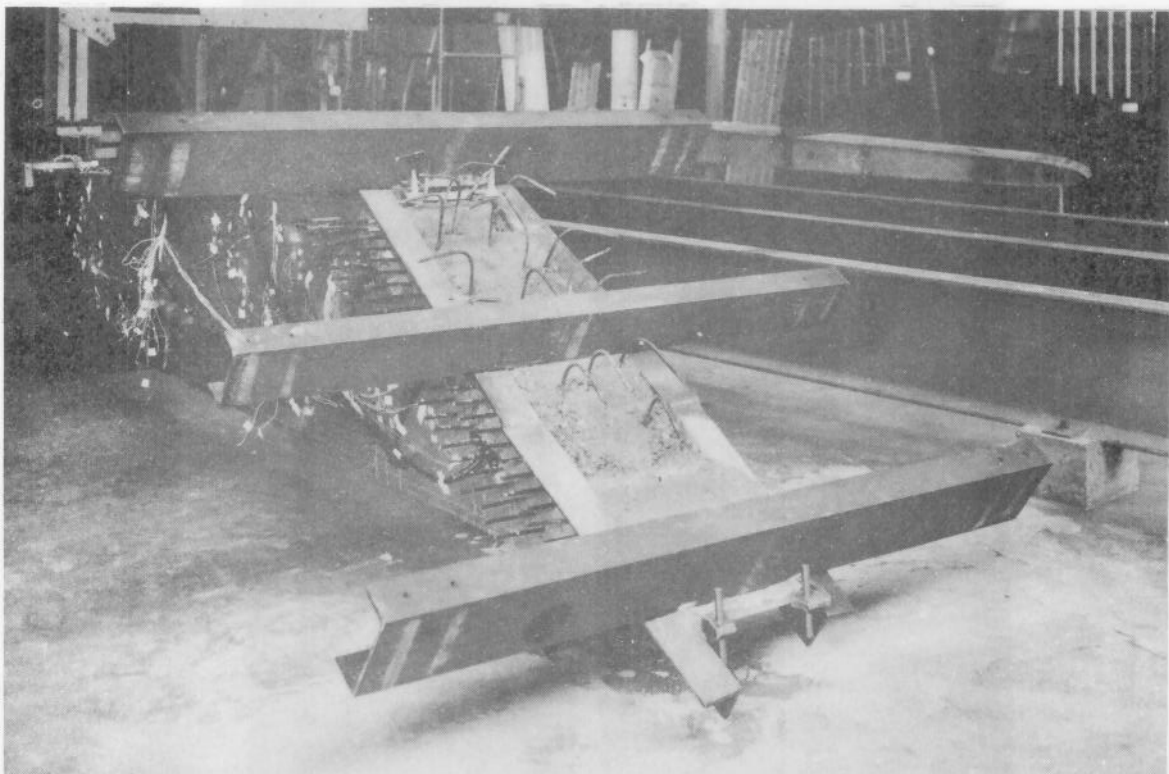


FIG. 6.

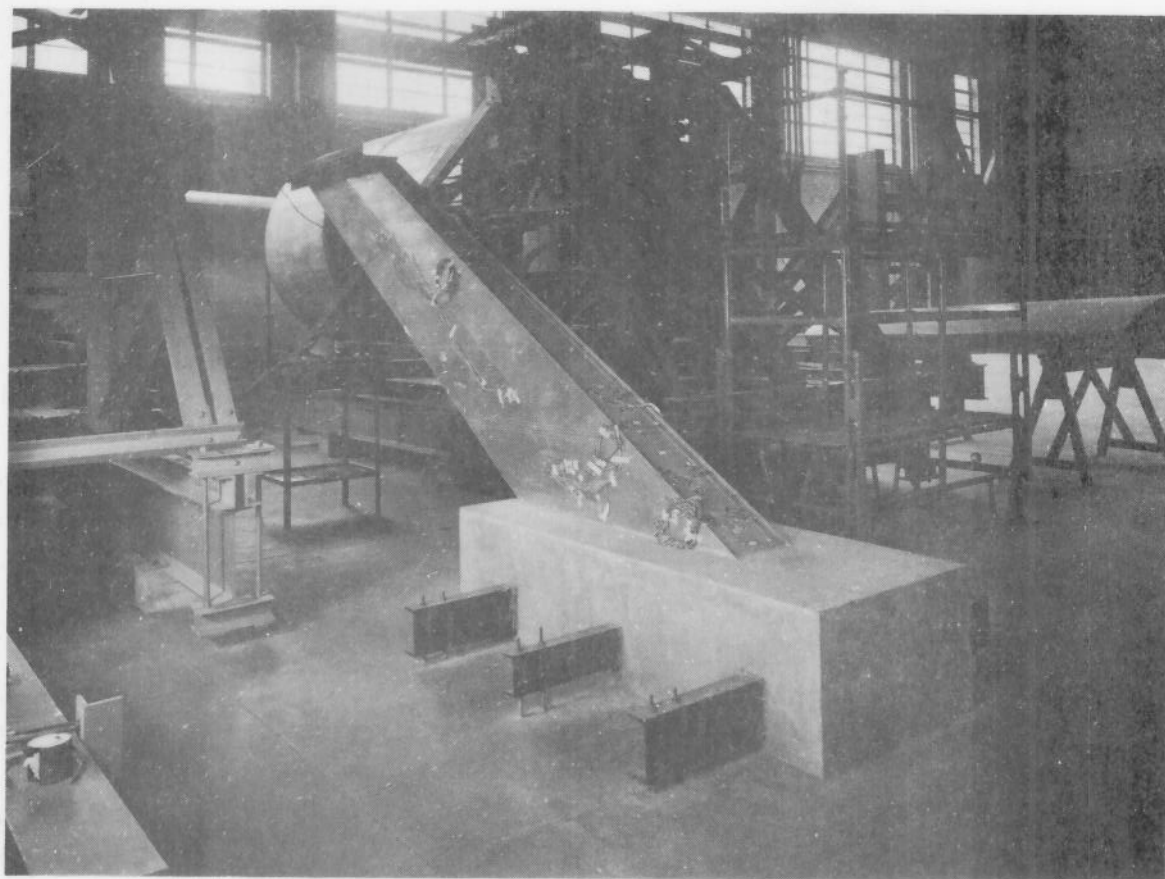
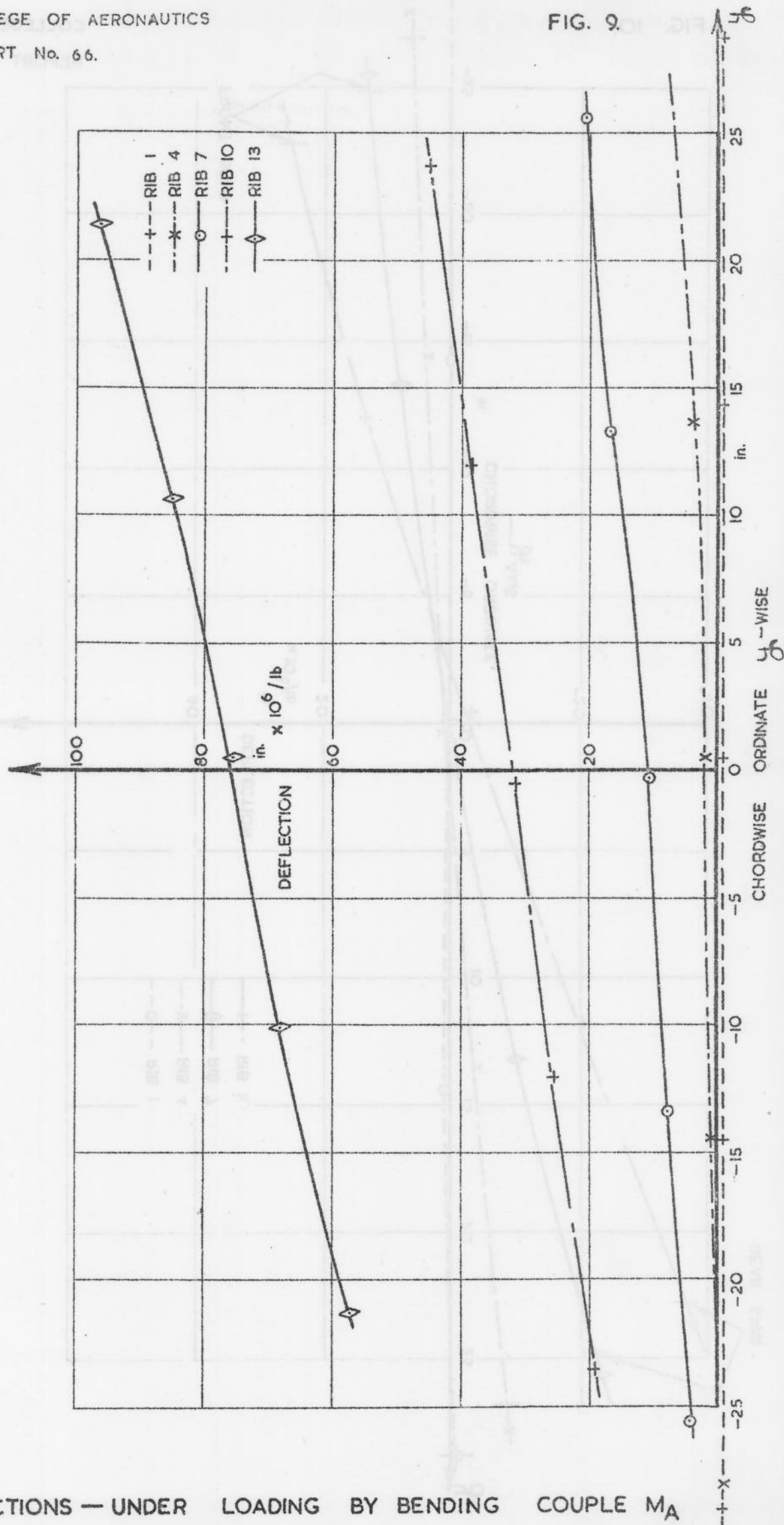


FIG. 7.



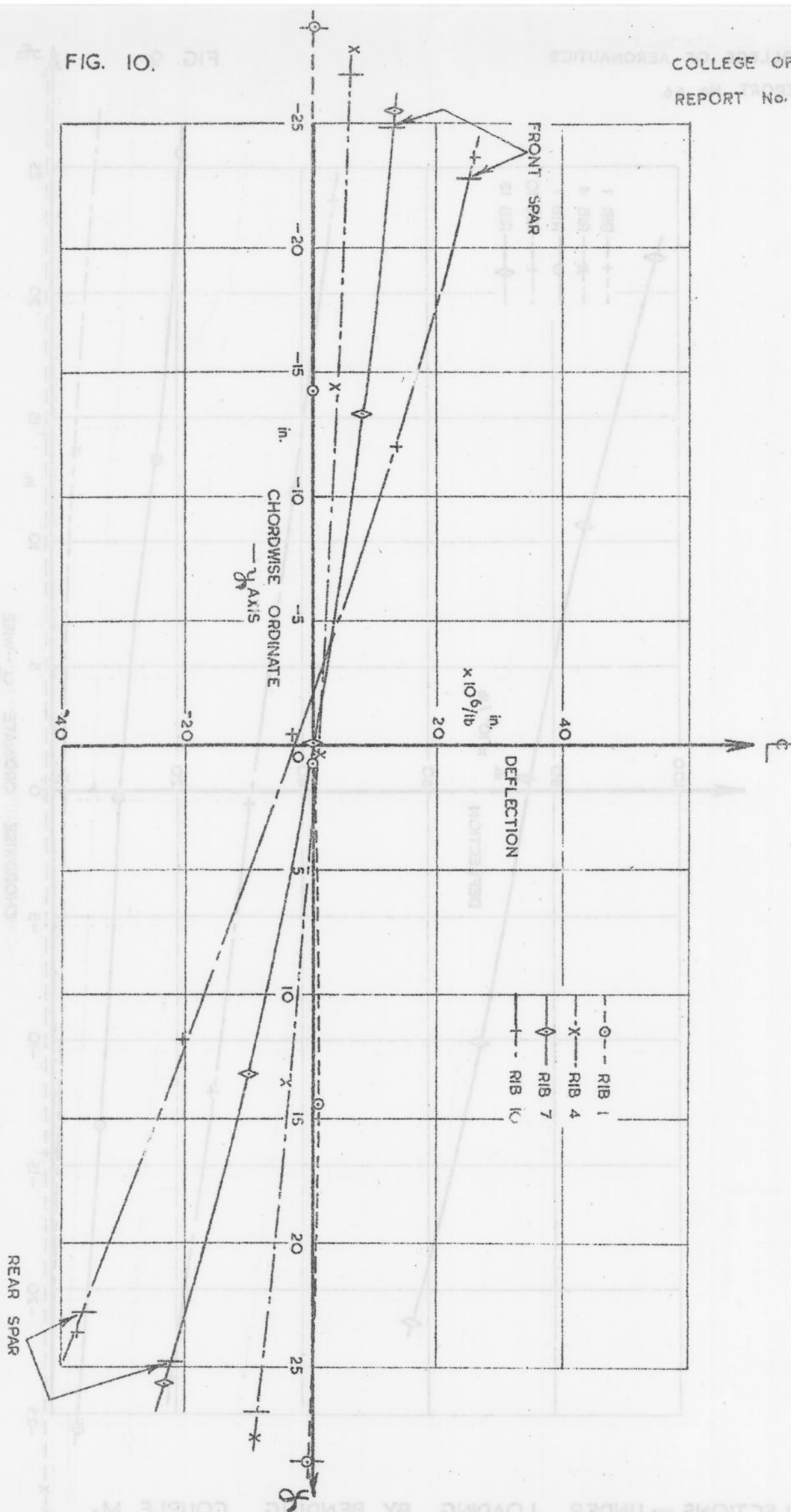
FIG. 8.

FIG. 9.

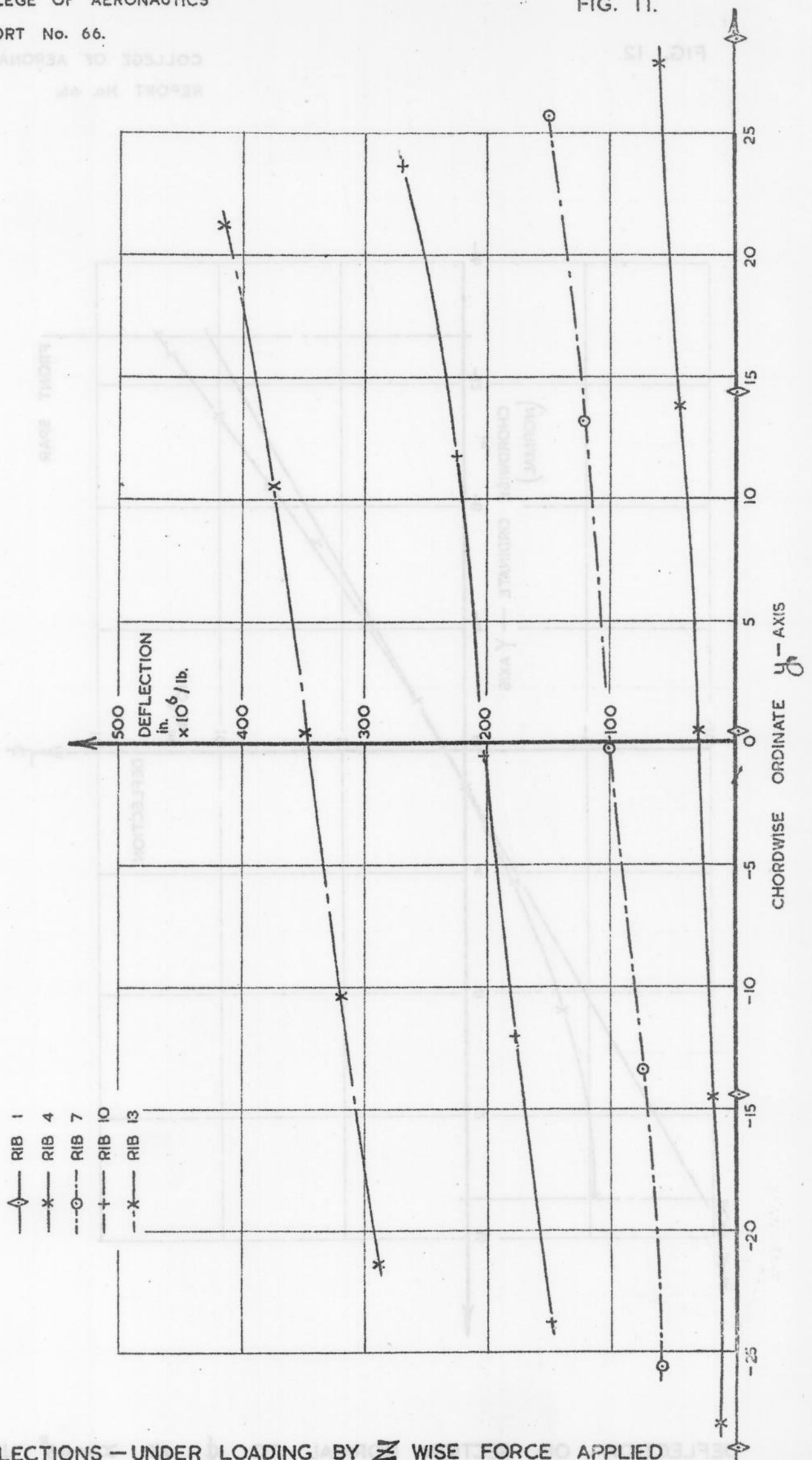


RIB DEFLECTIONS — UNDER LOADING BY BENDING COUPLE M_A

FIG. 10.



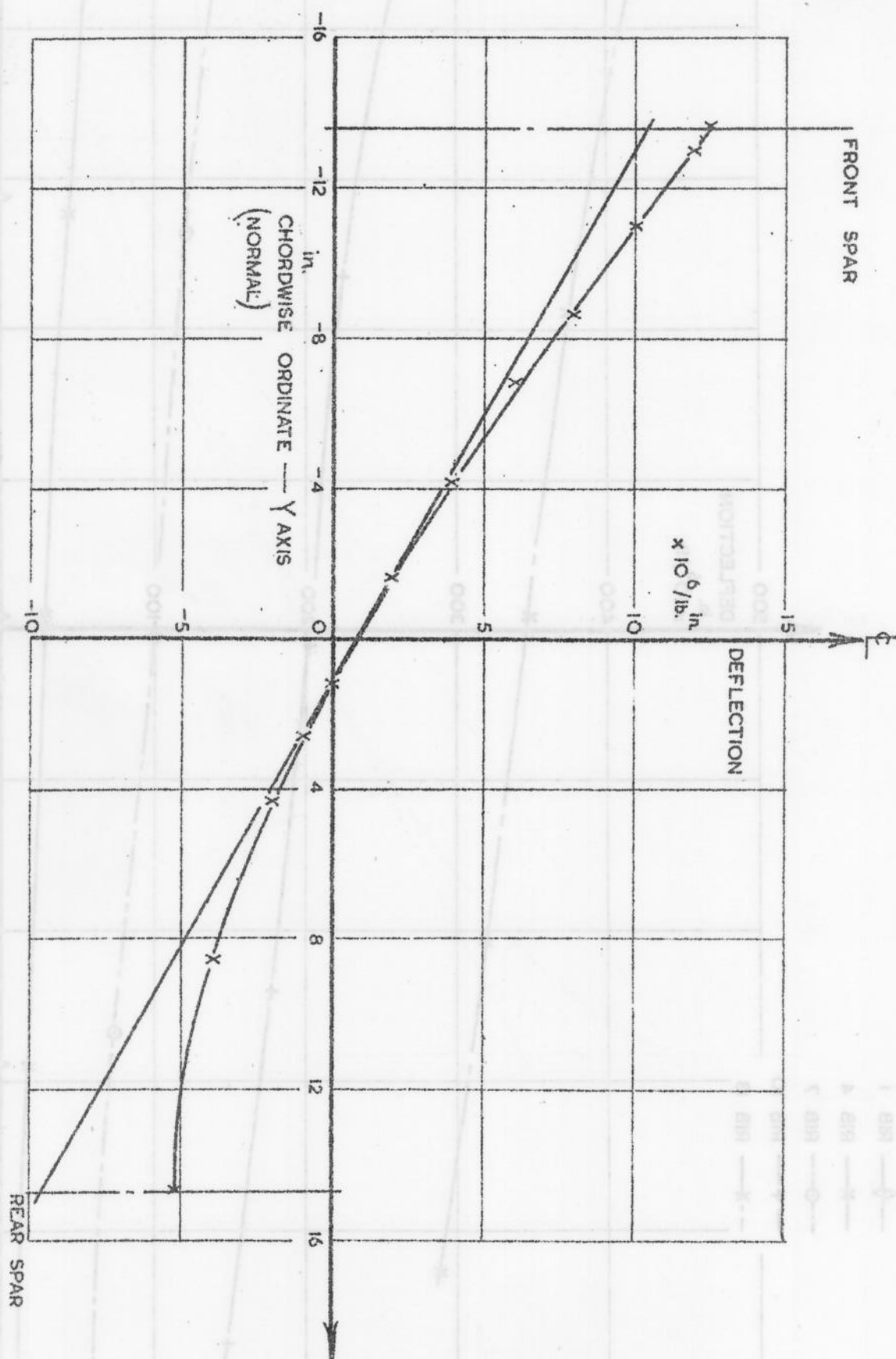
RIB DEFLECTIONS — UNDER TORSIONAL LOADING T_A



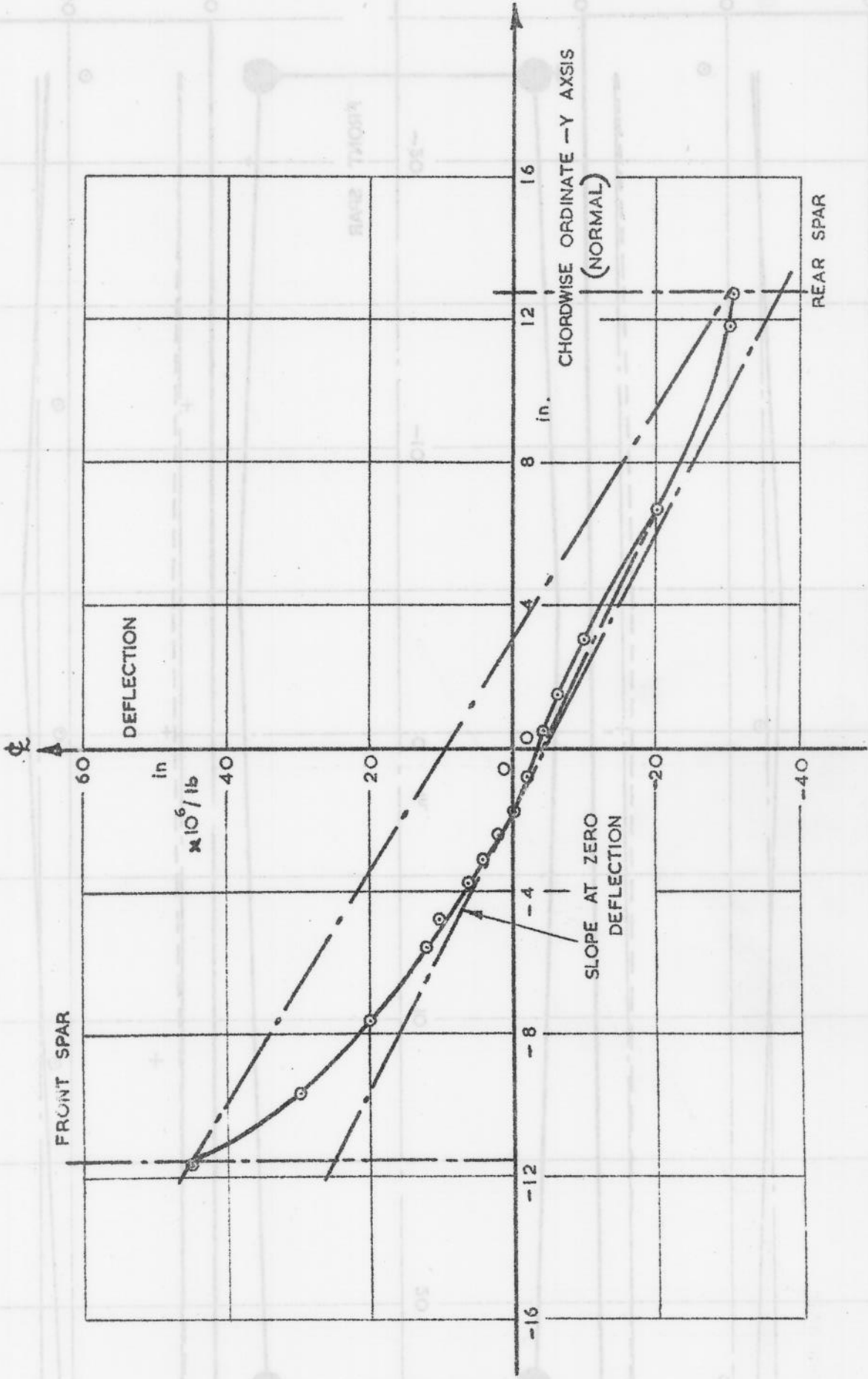
RIB DEFLECTIONS — UNDER LOADING BY Σ WISE FORCE APPLIED
AT ζ RIB 14

FIG. 12.

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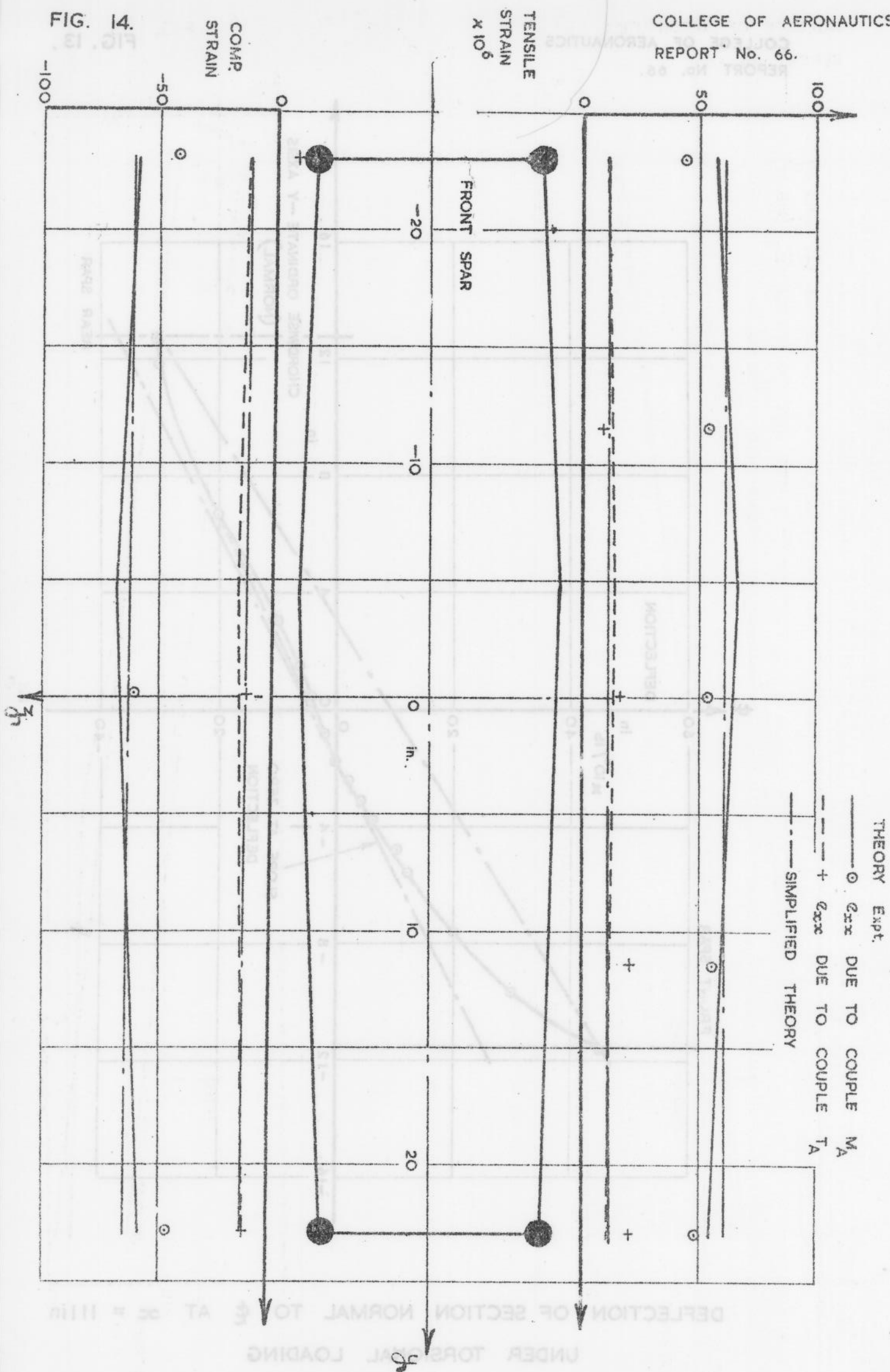


DEFLECTION OF SECTION NORMAL TO ζ AT $x = 46''$ UNDER TORSIONAL LOADING.



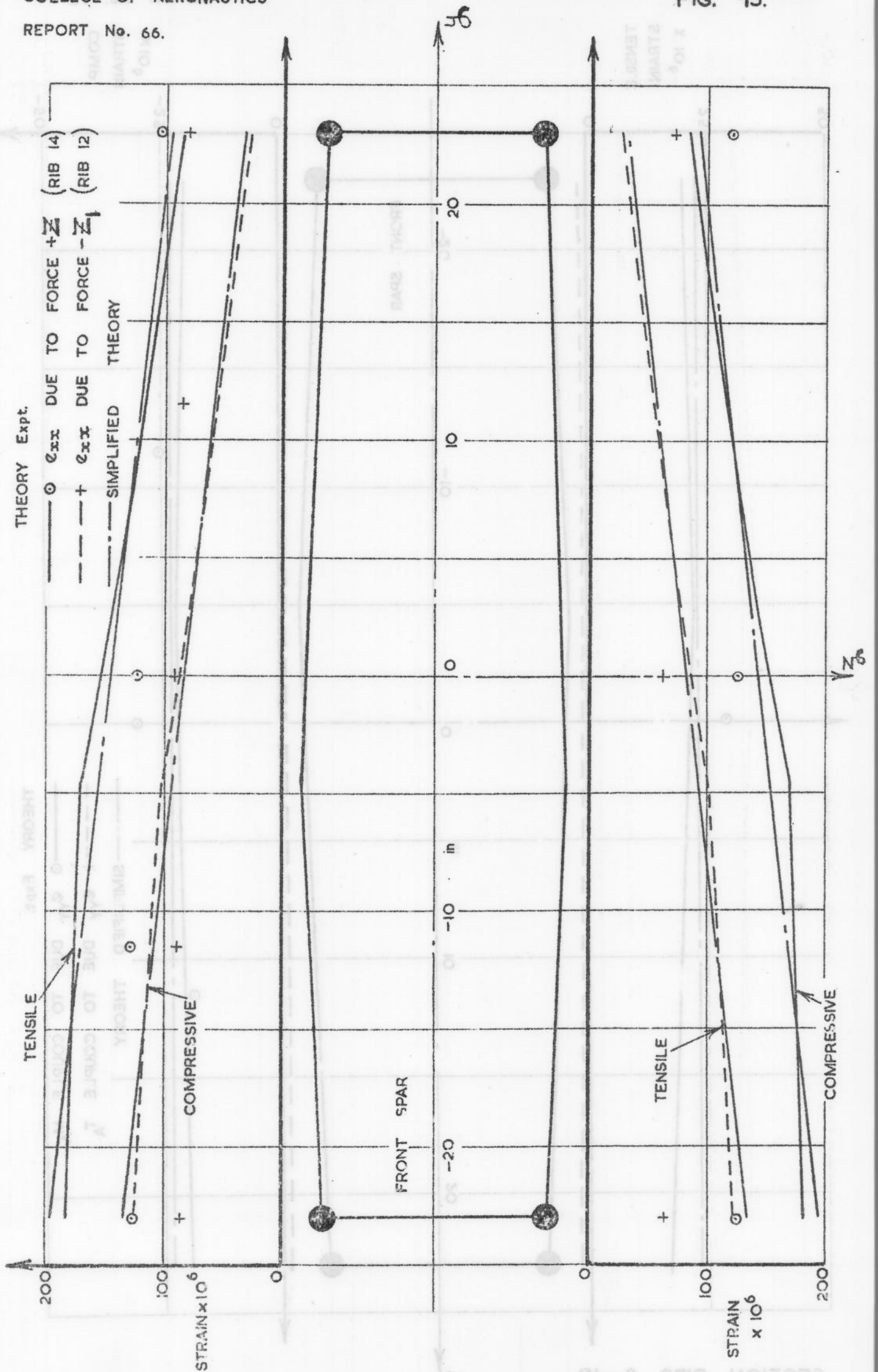
DEFLECTION OF SECTION NORMAL TO x AT $x = 111$ in
UNDER TORSIONAL LOADING

FIG. 14.



SECTION RIBS 9-10

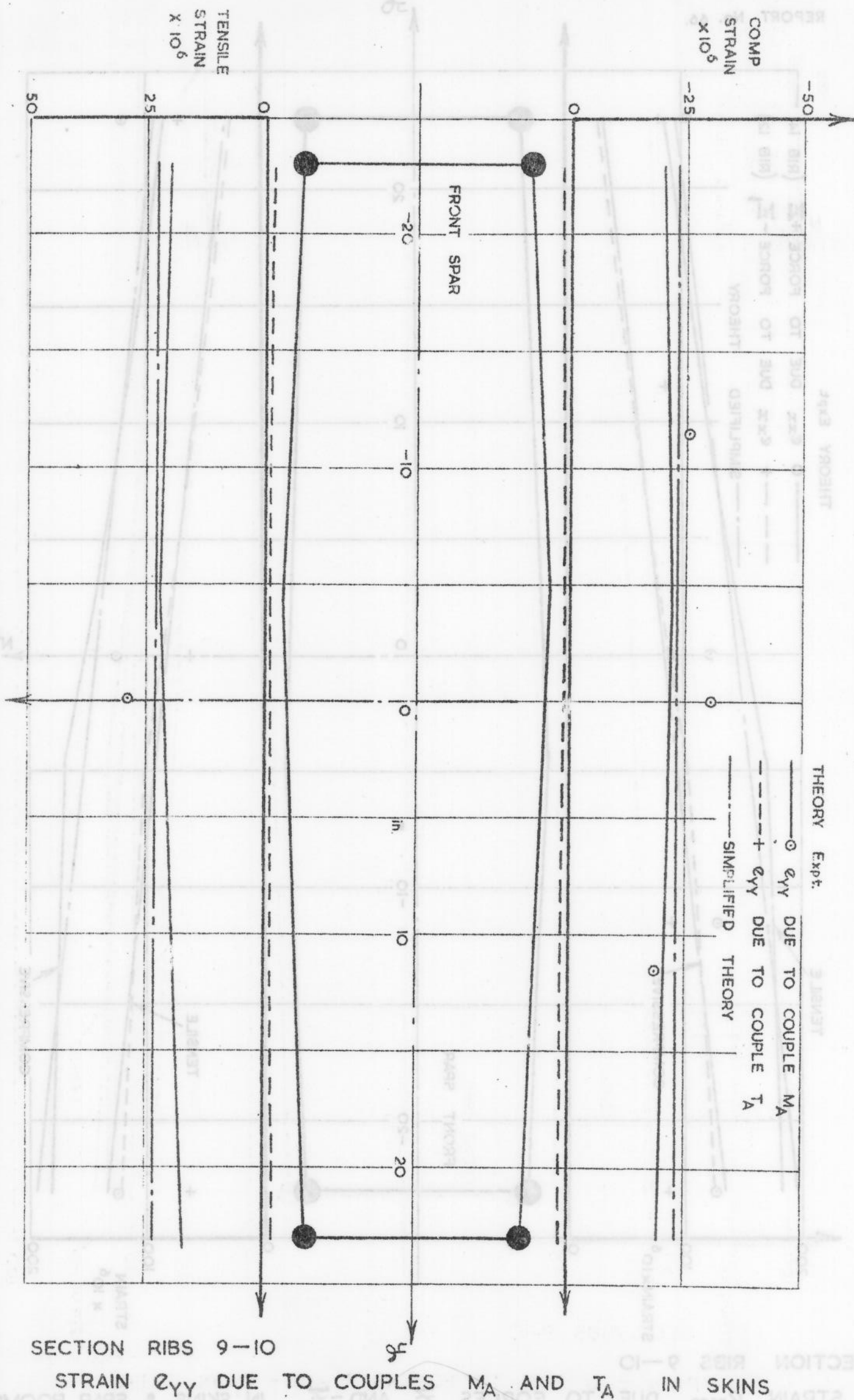
STRAIN ϵ_{xx} DUE TO COUPLES M_A AND T_A IN SKINS AND SPAR BOOMS



SECTION RIBS 9-10

STRAIN ϵ_{xx} DUE TO FORCES N AND $-N_1$ IN SKINS & SPAR BOOMS

FIG. 16.



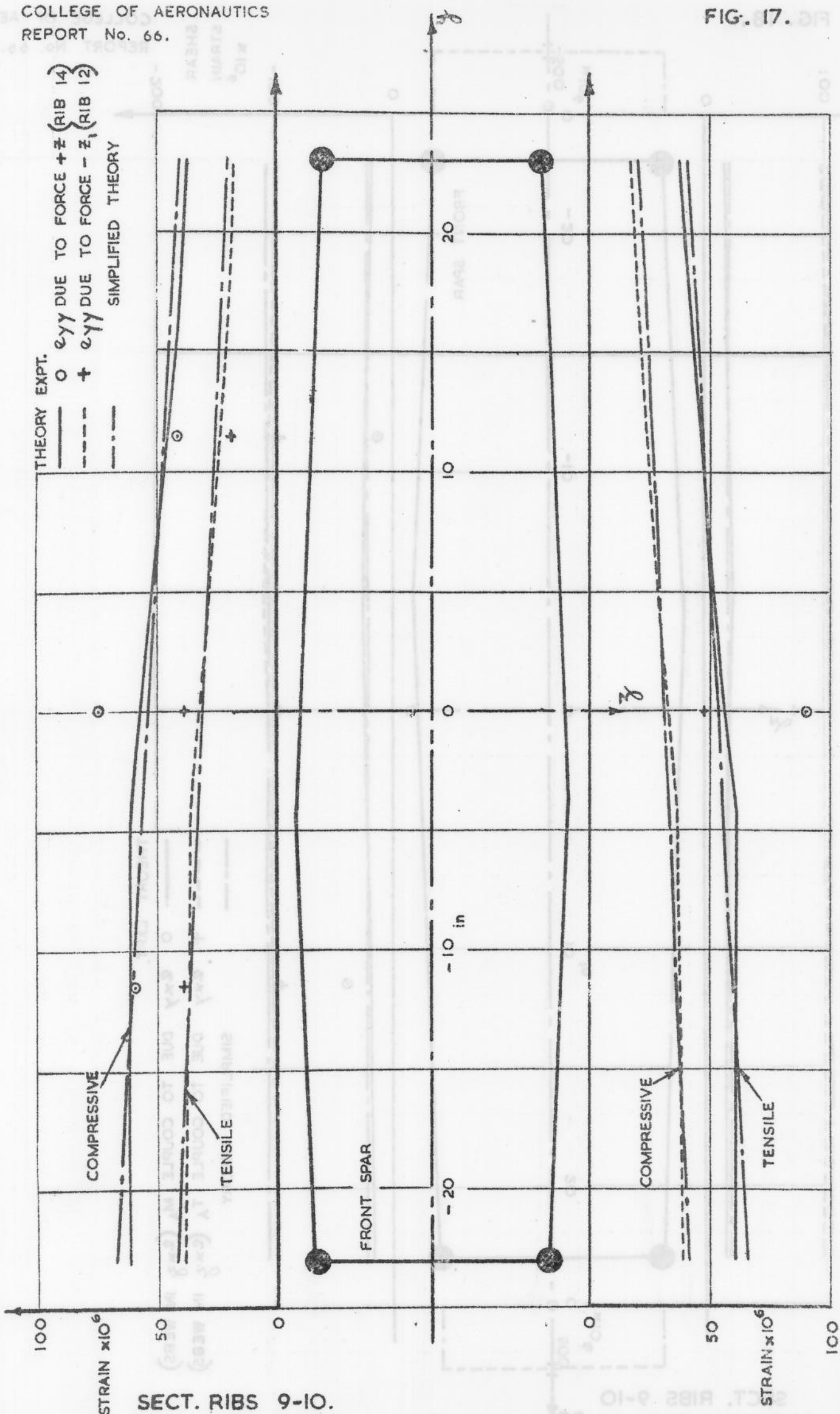
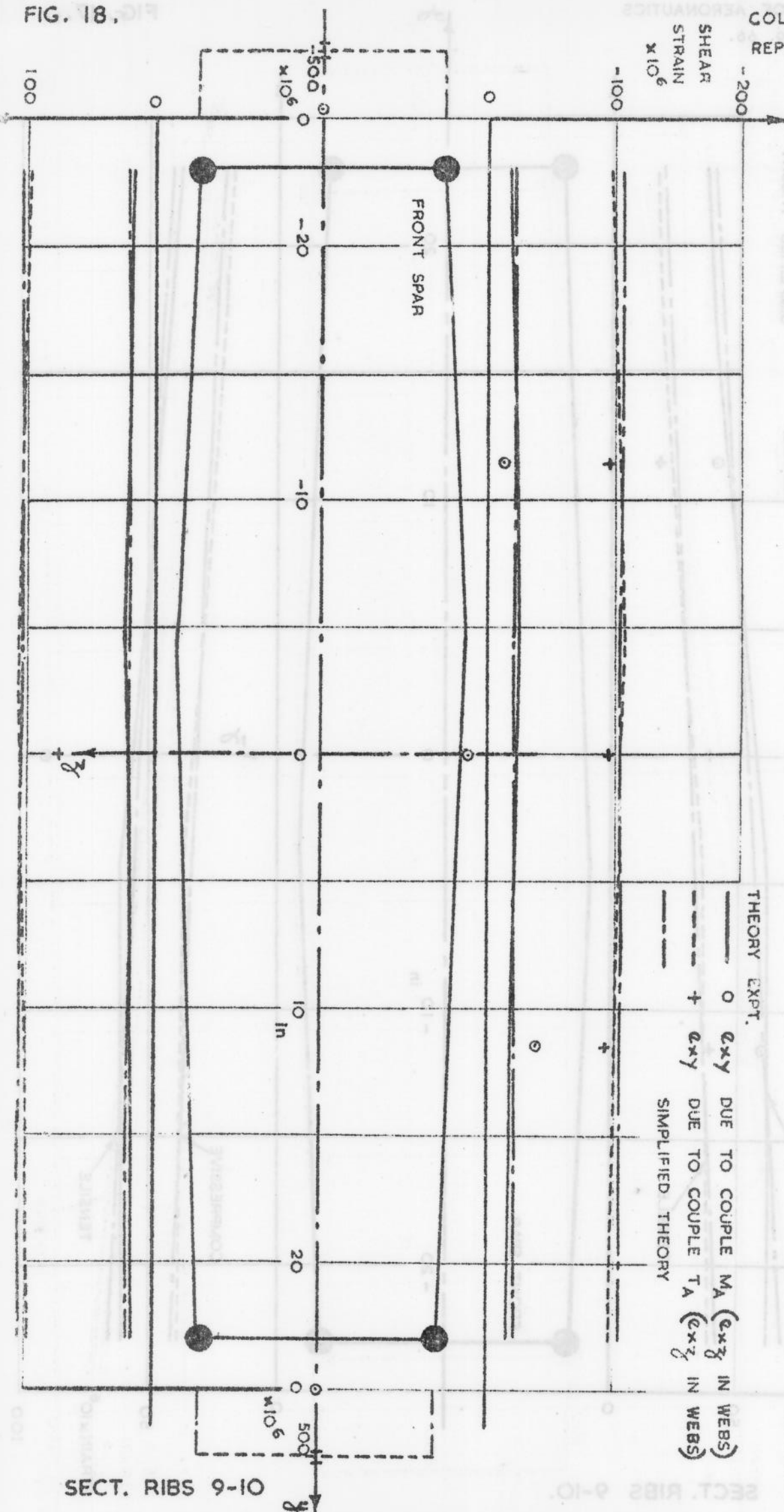
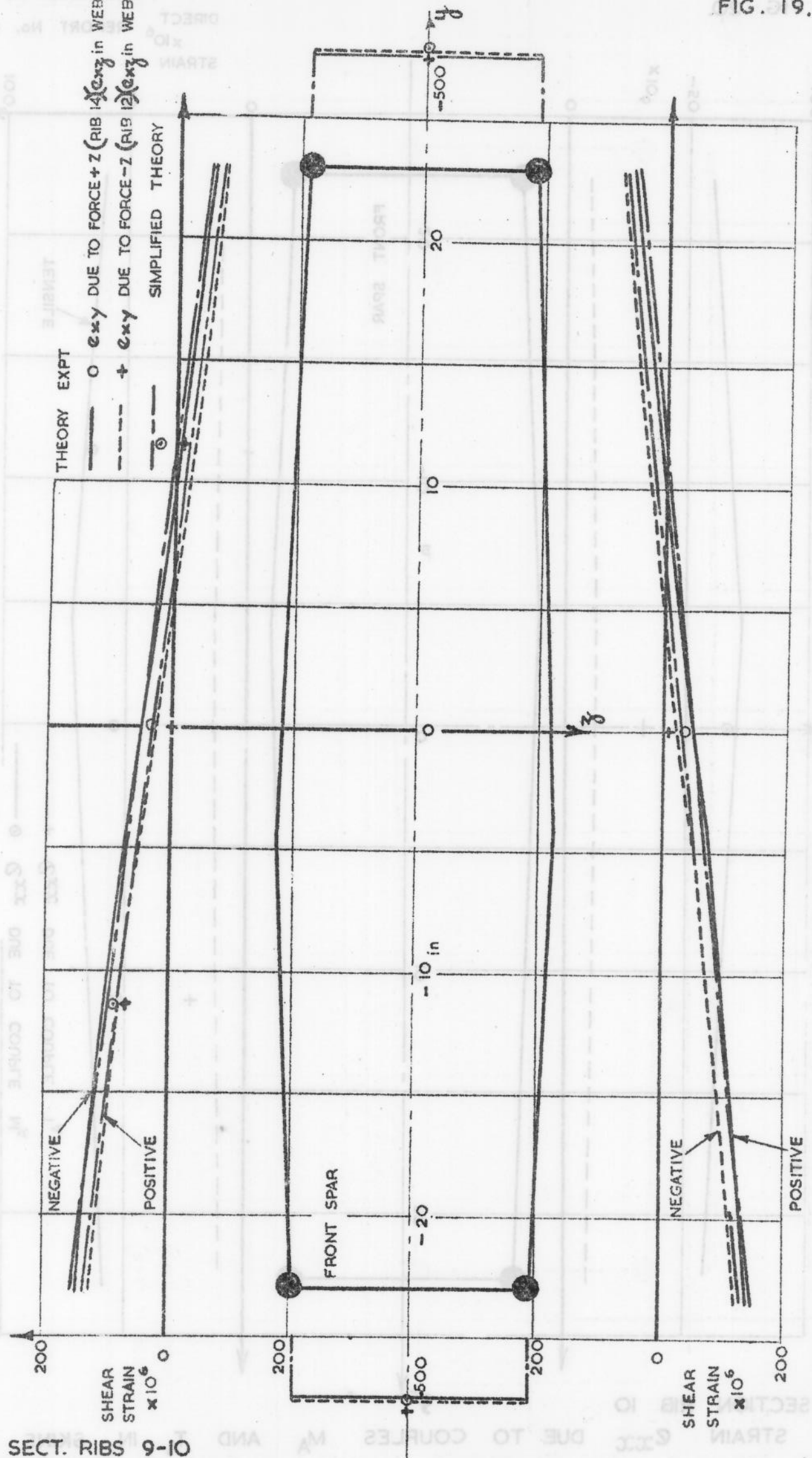


FIG. 18.



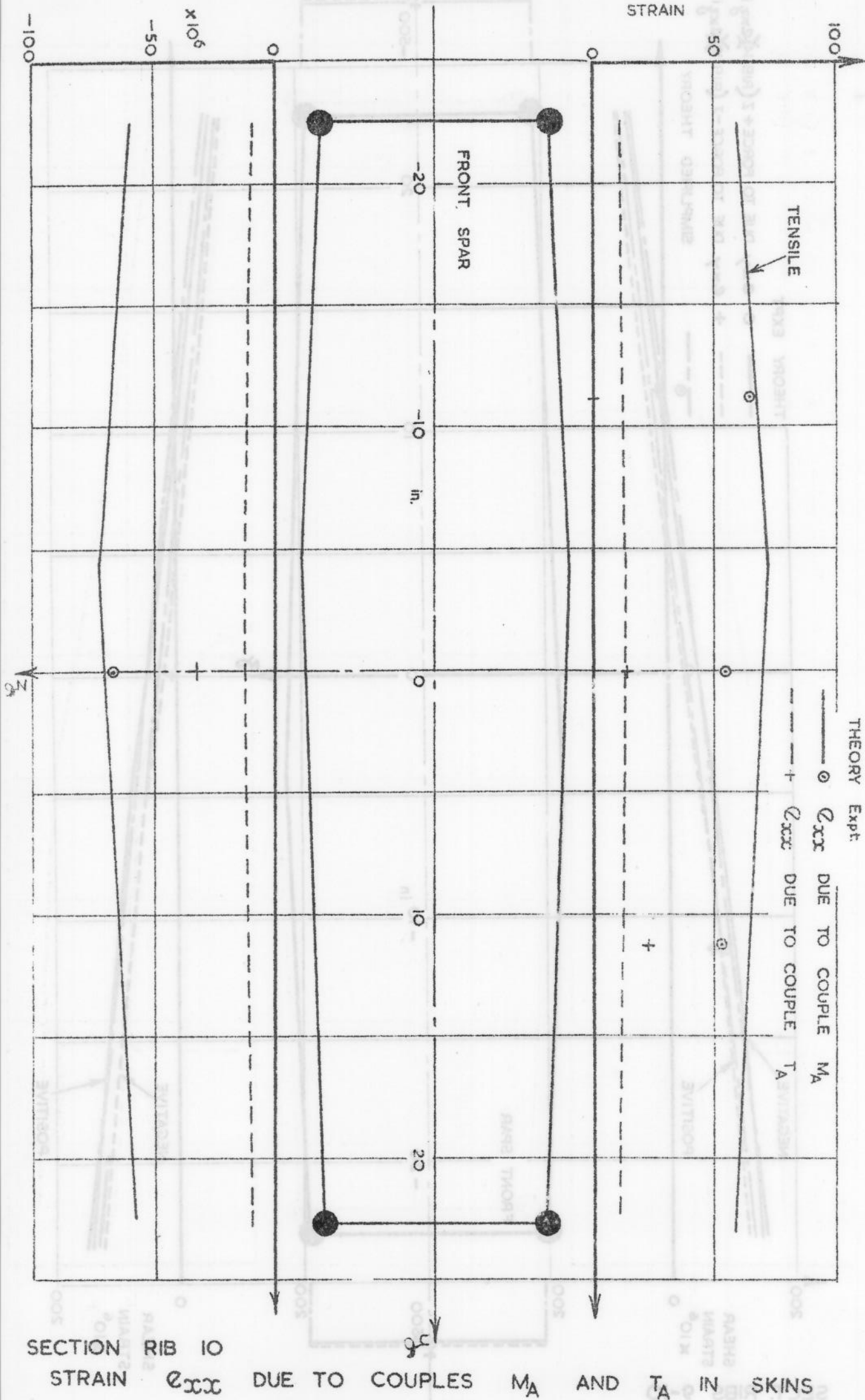
SECT. RIBS 9-10

STRAIN e_{xy} DUE TO COUPLES M_A AND T_A IN SKINS (e_{xz} IN SPAR WEBS)



STRAIN e_{xy} DUE TO FORCES z AND $-z$, IN SKINS (e_{xz} IN SPAR WEBS)

FIG. 20.



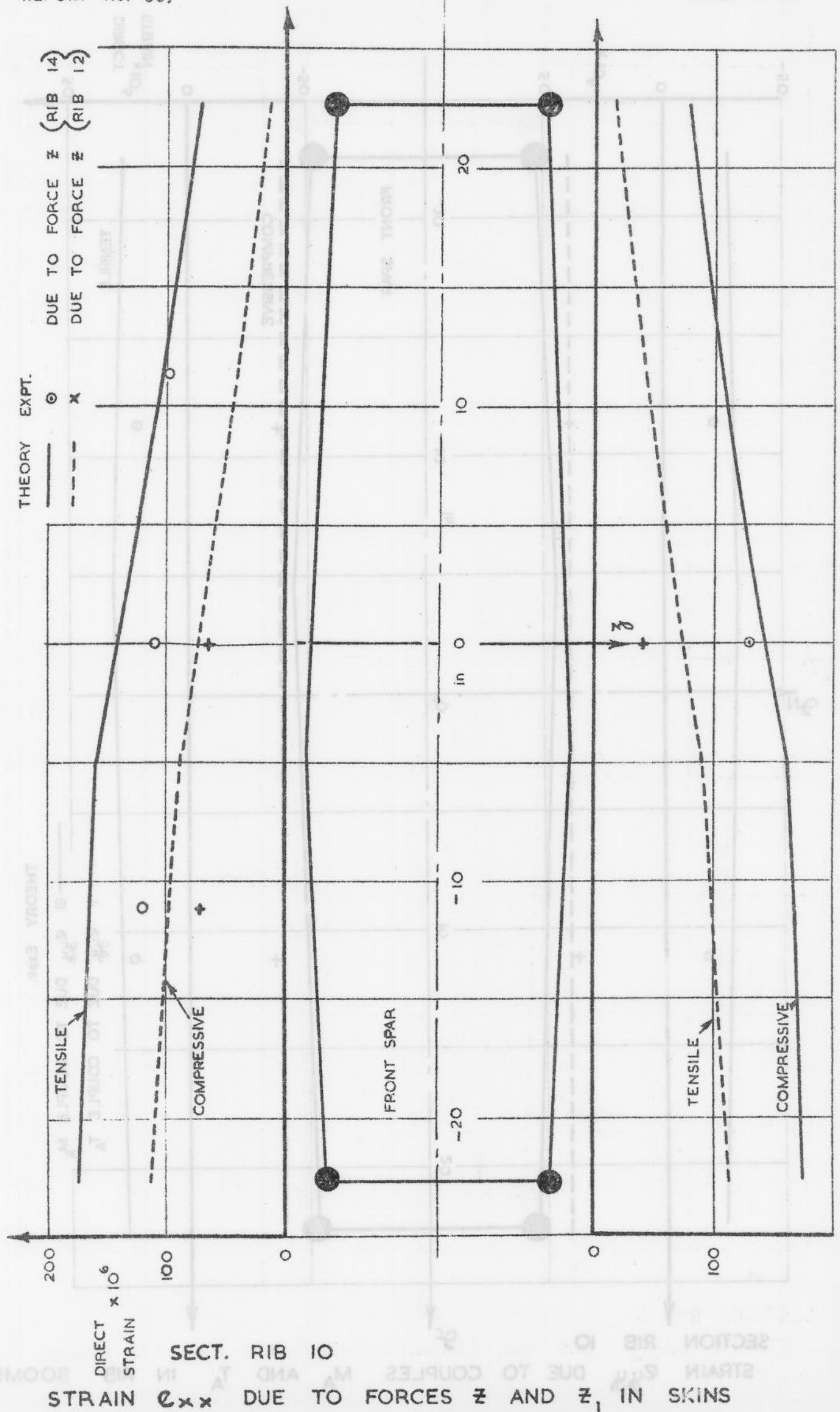
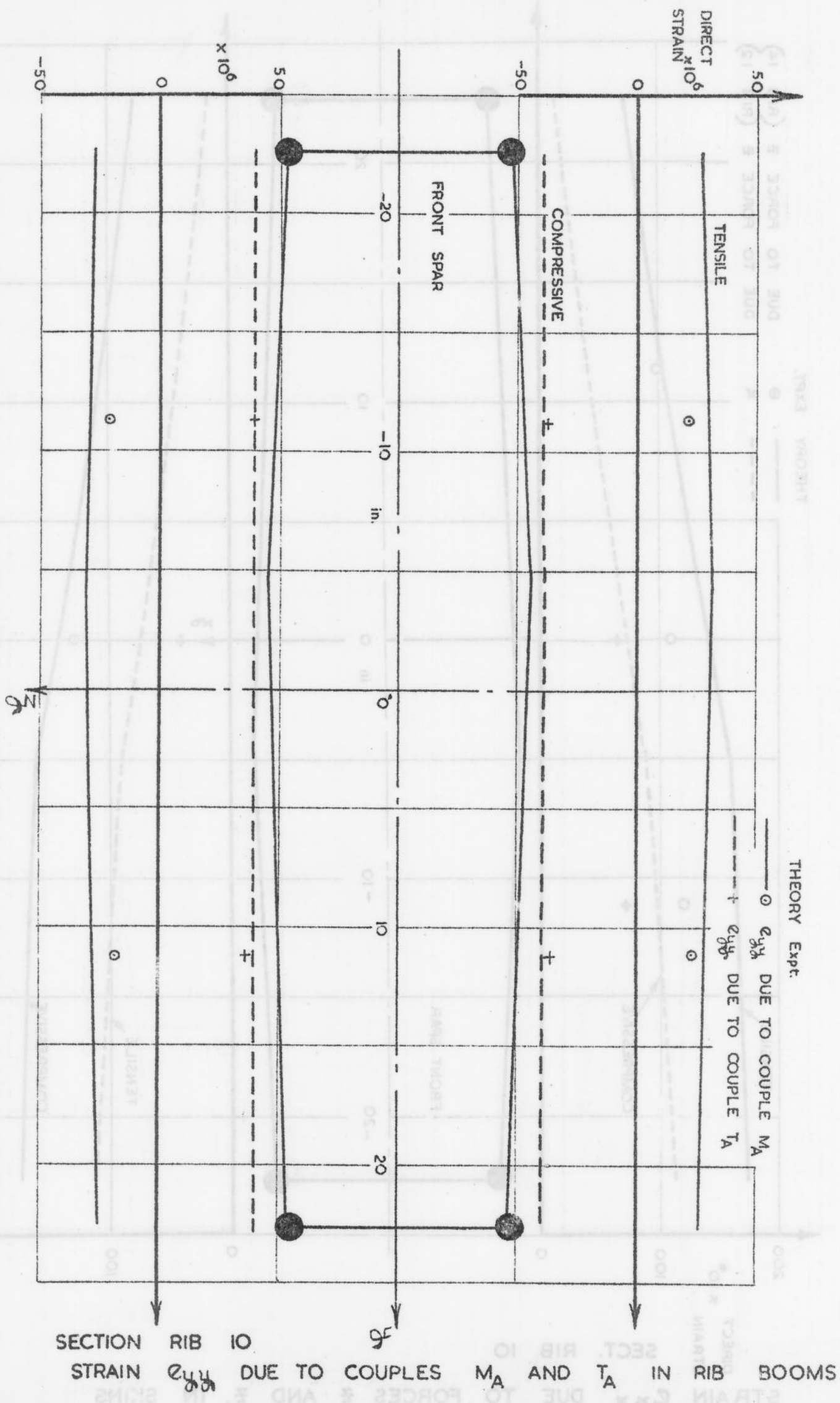


FIG. 22.



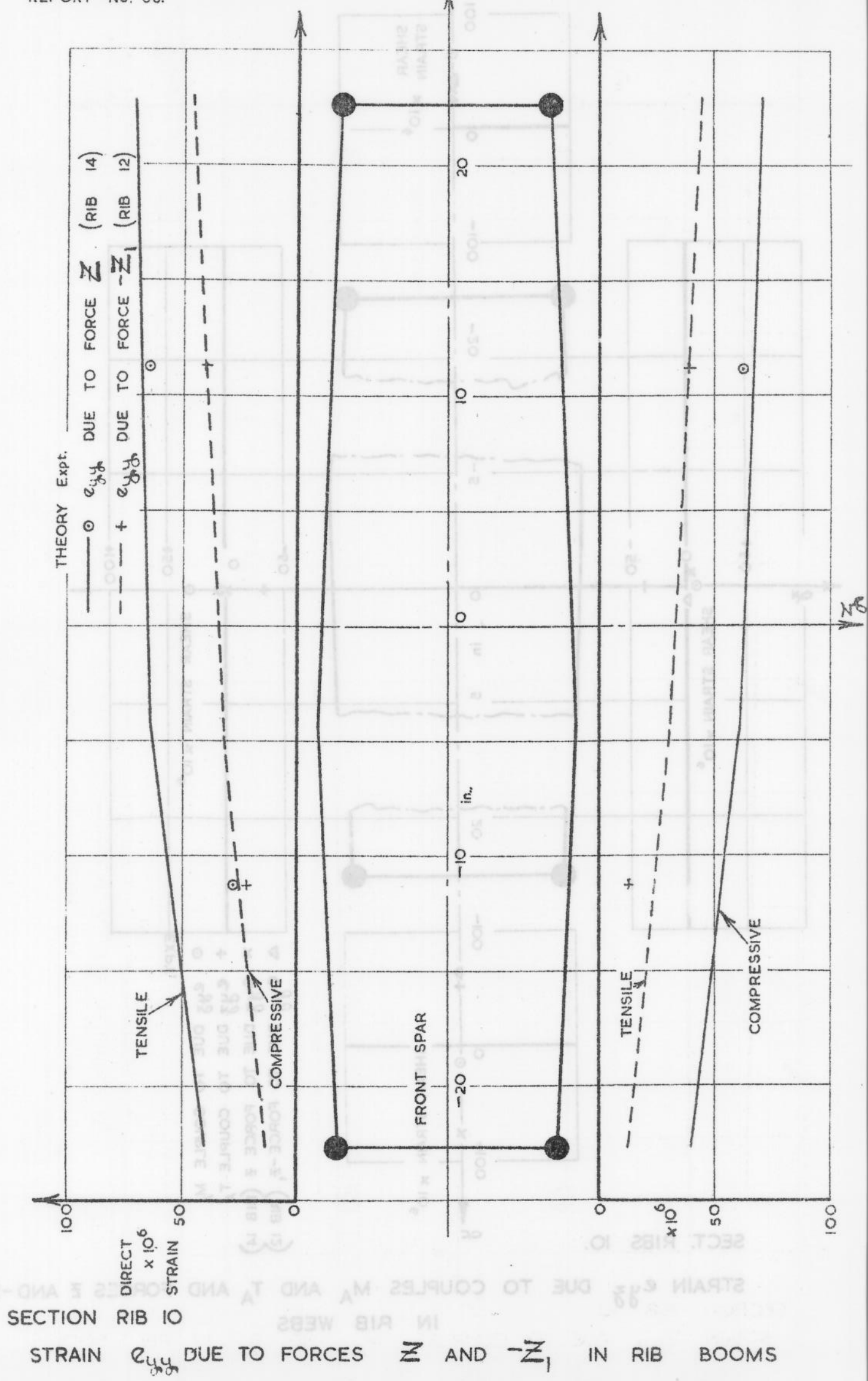
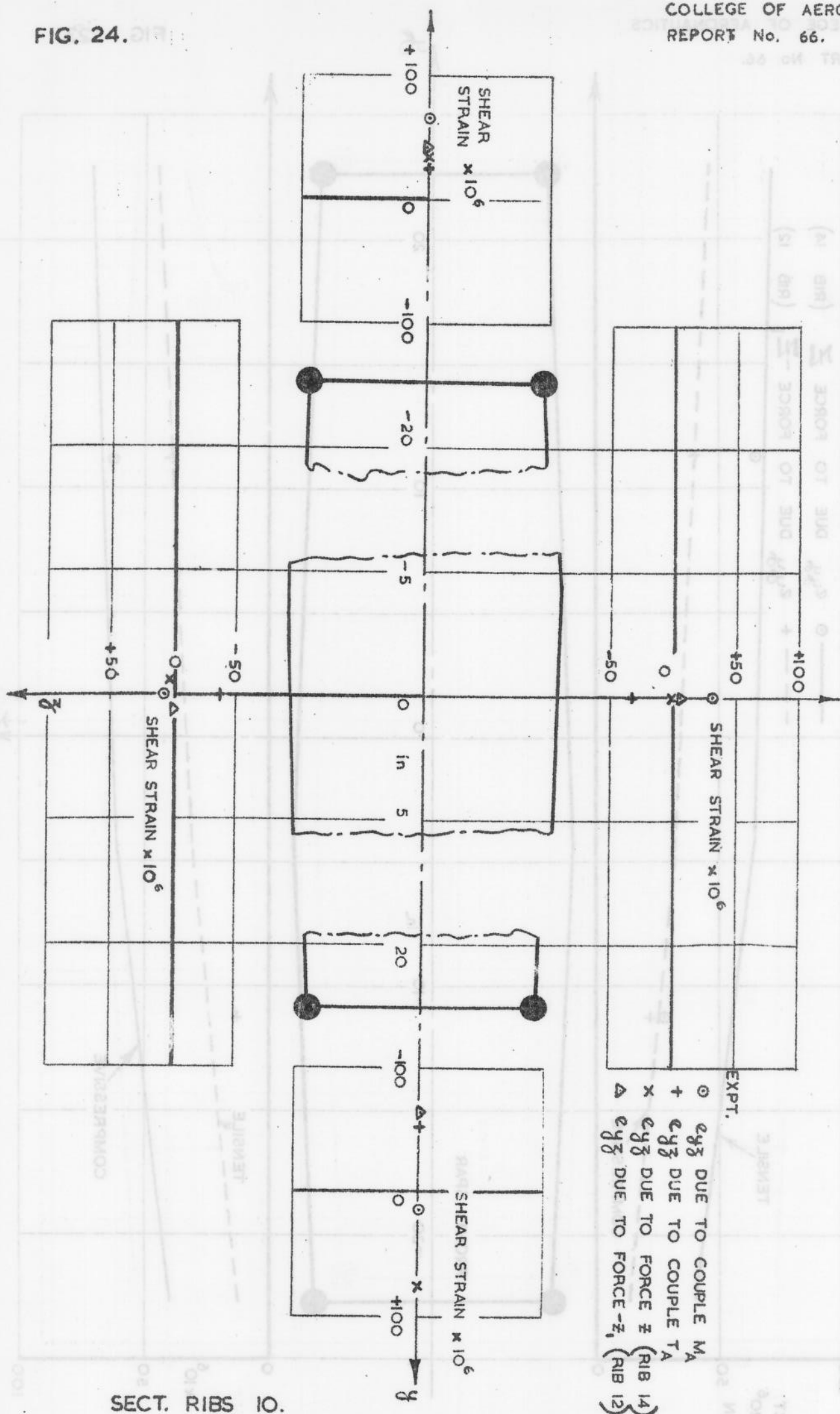
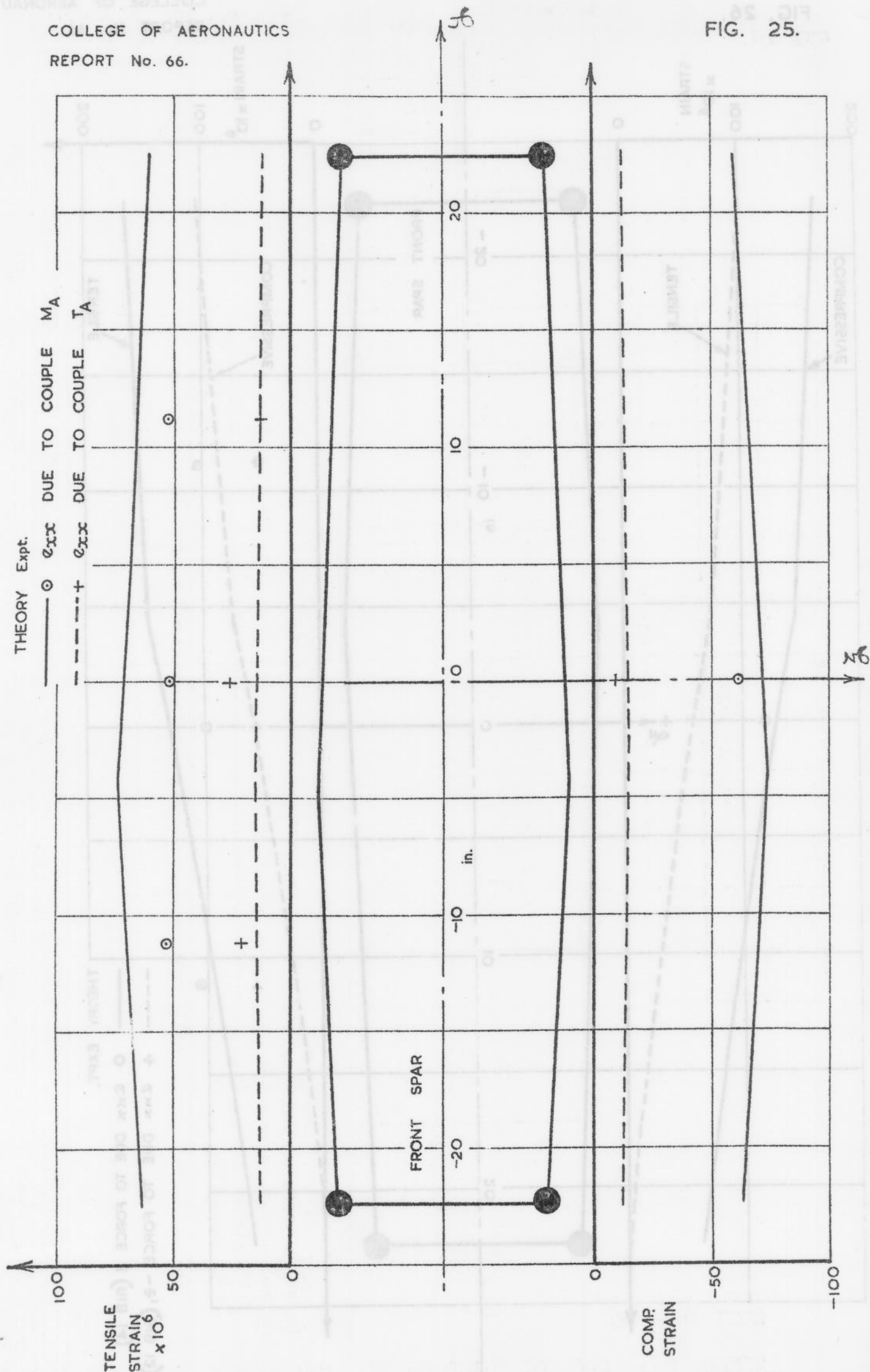


FIG. 24.

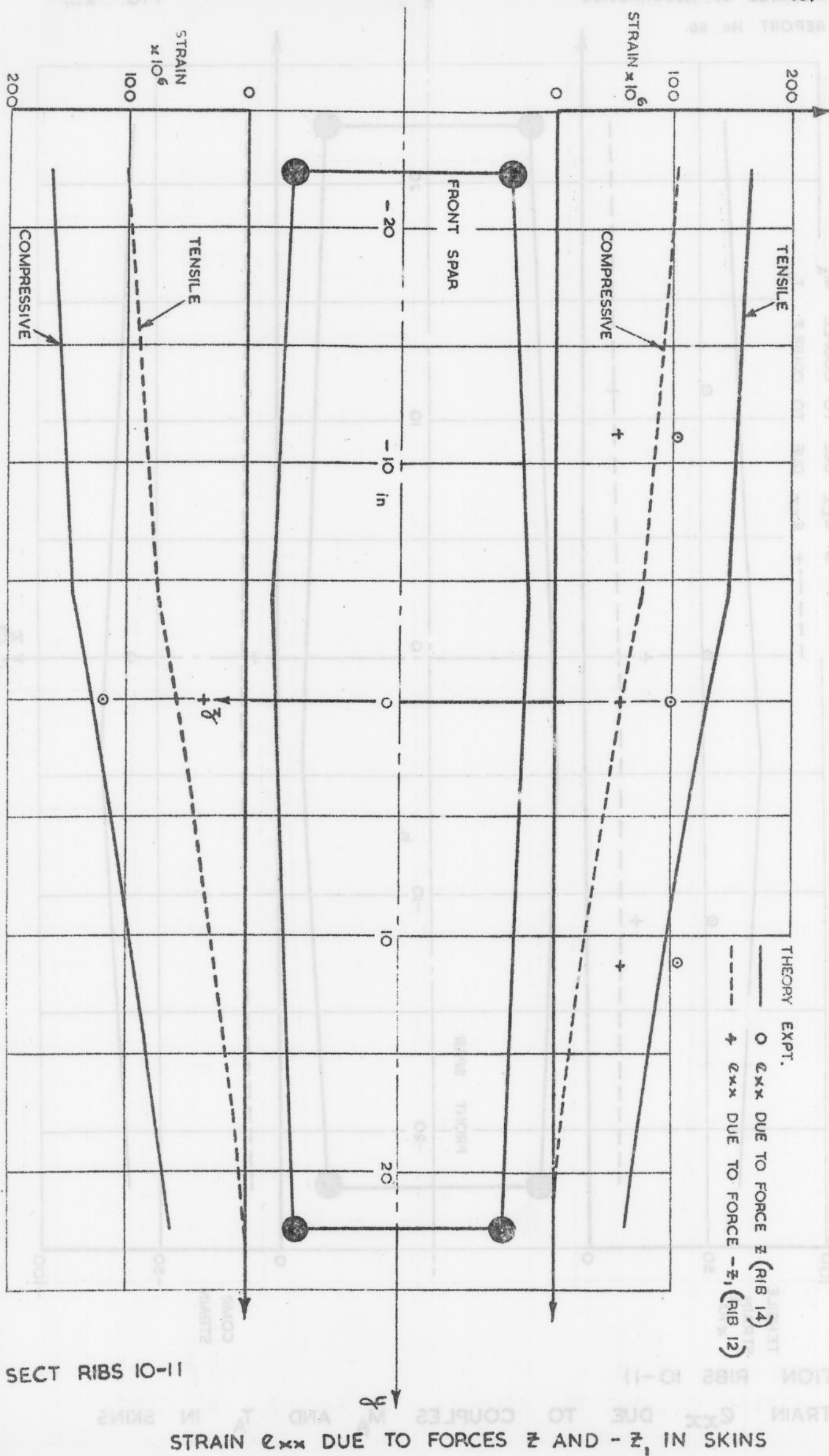




SECTION RIBS 10-11

STRAIN ϵ_{xx} DUE TO COUPLES M_A AND T_A IN SKINS

FIG. 26.



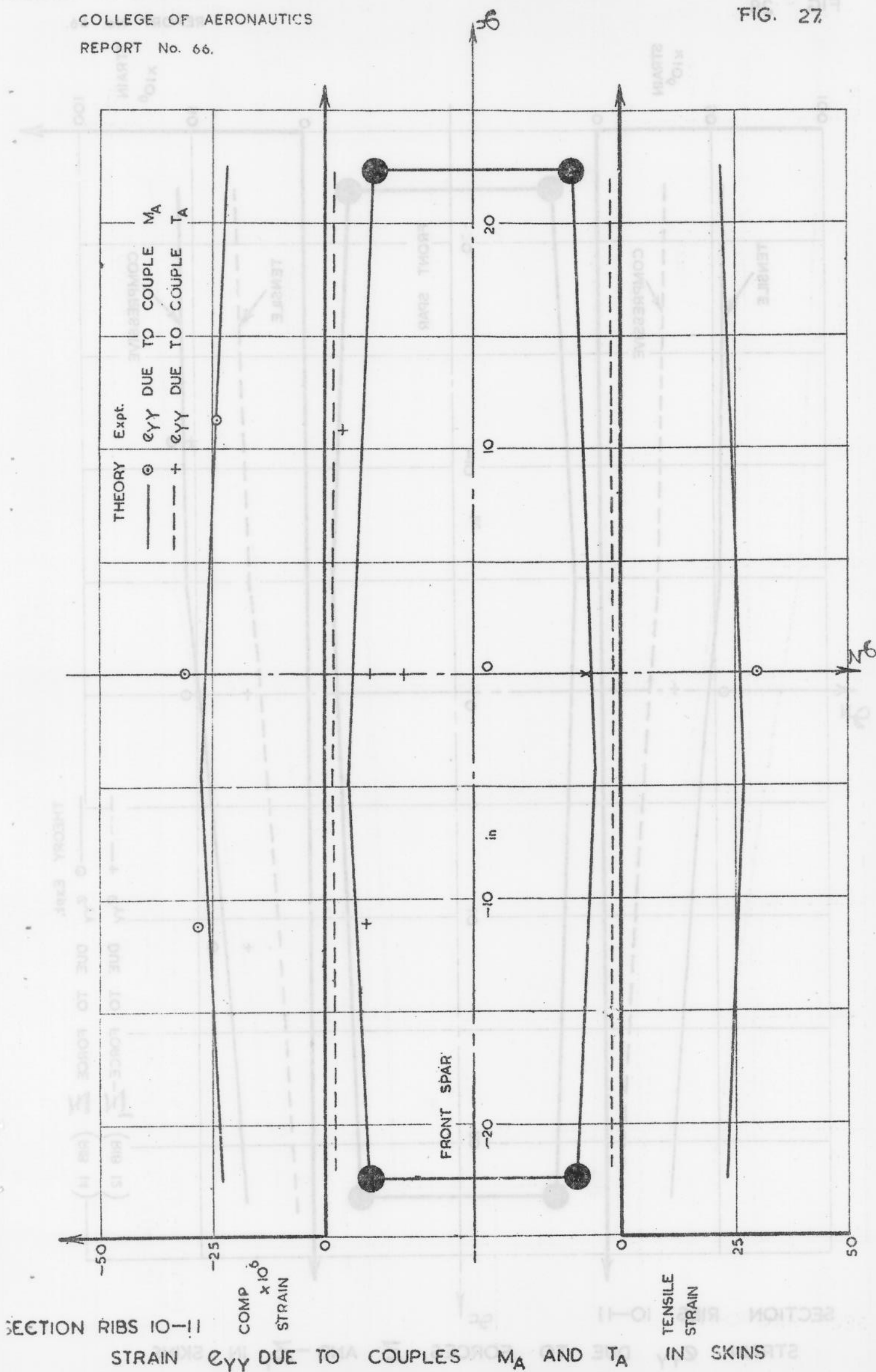
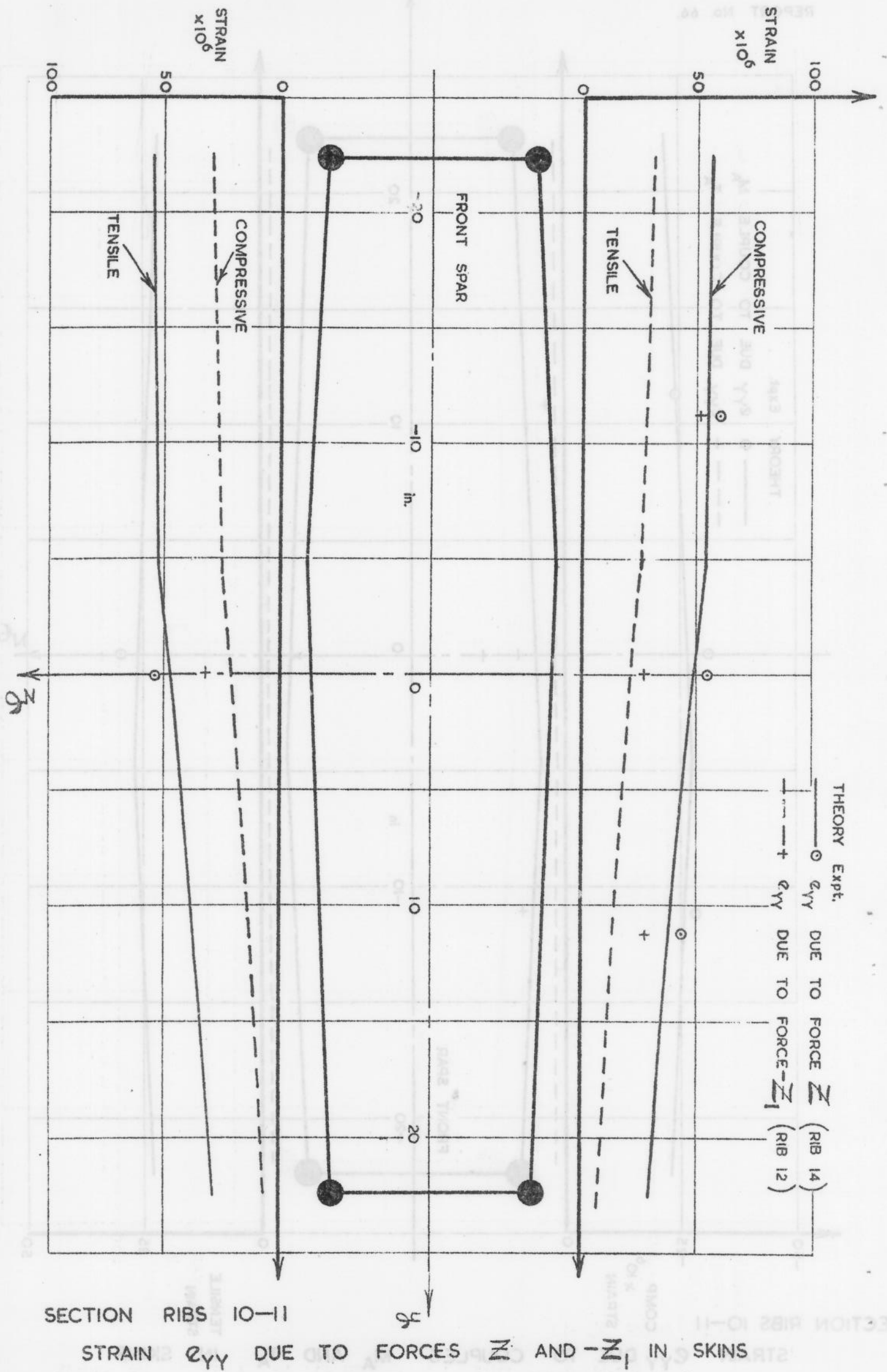


FIG. 28.



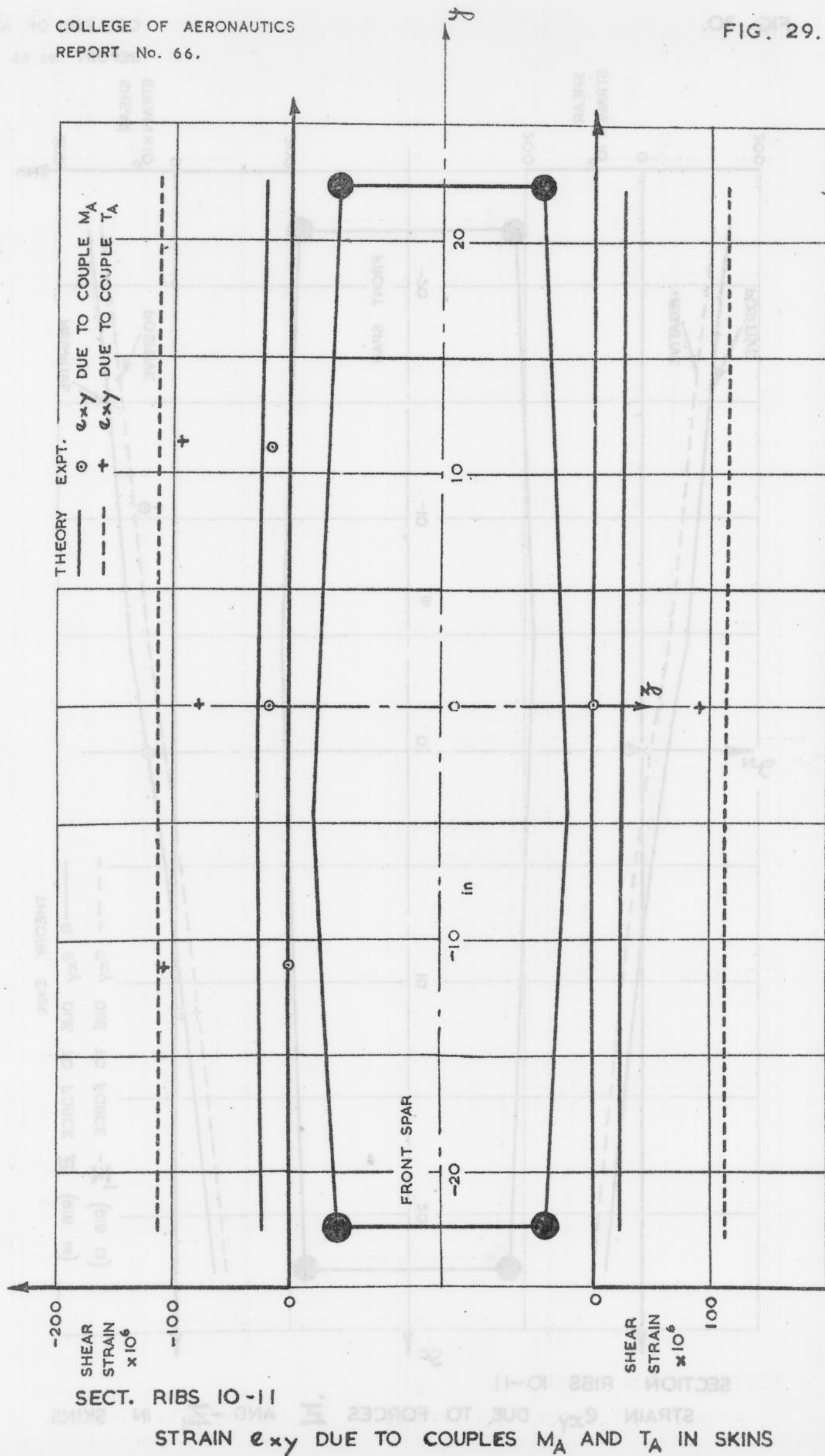
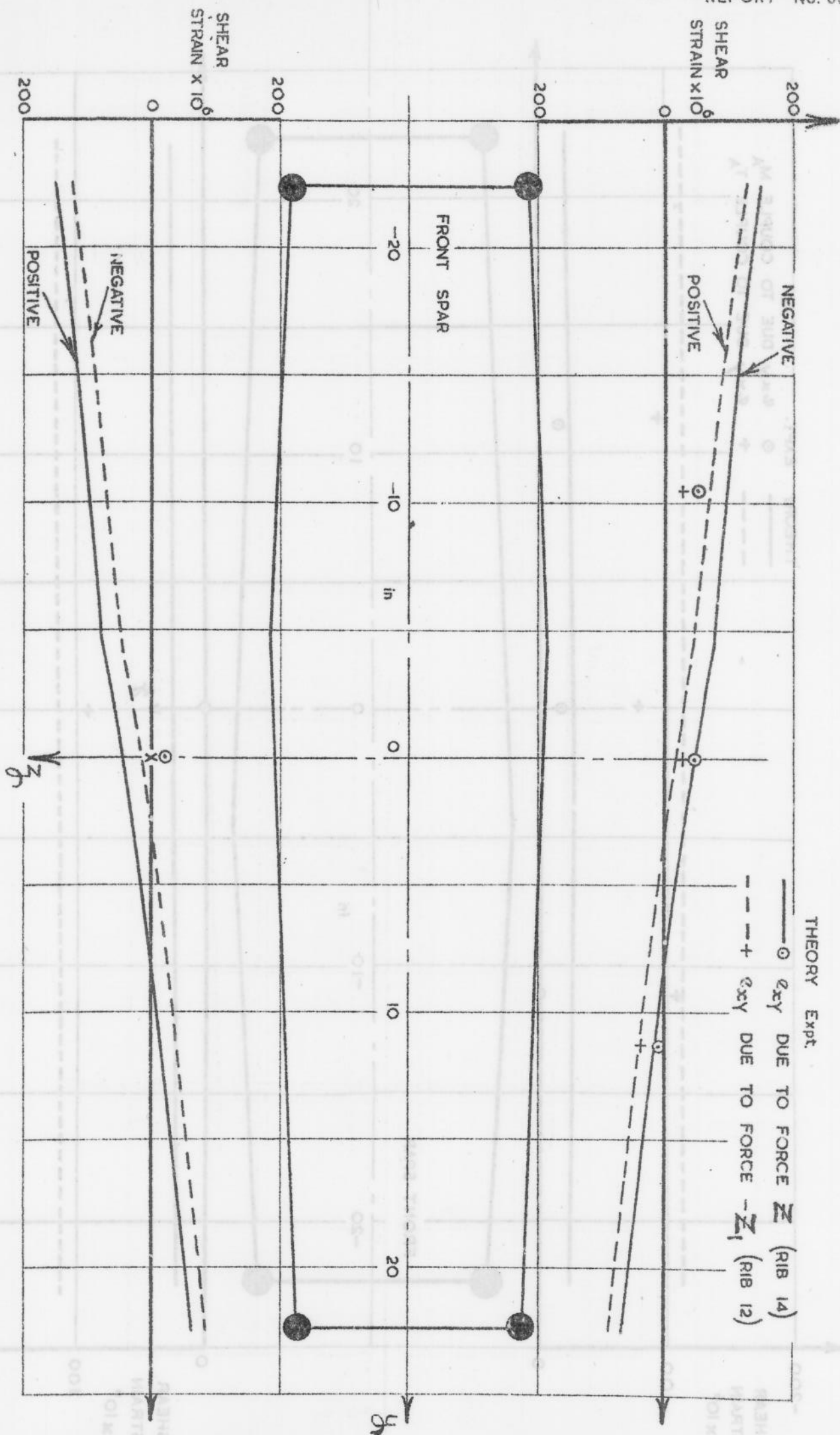
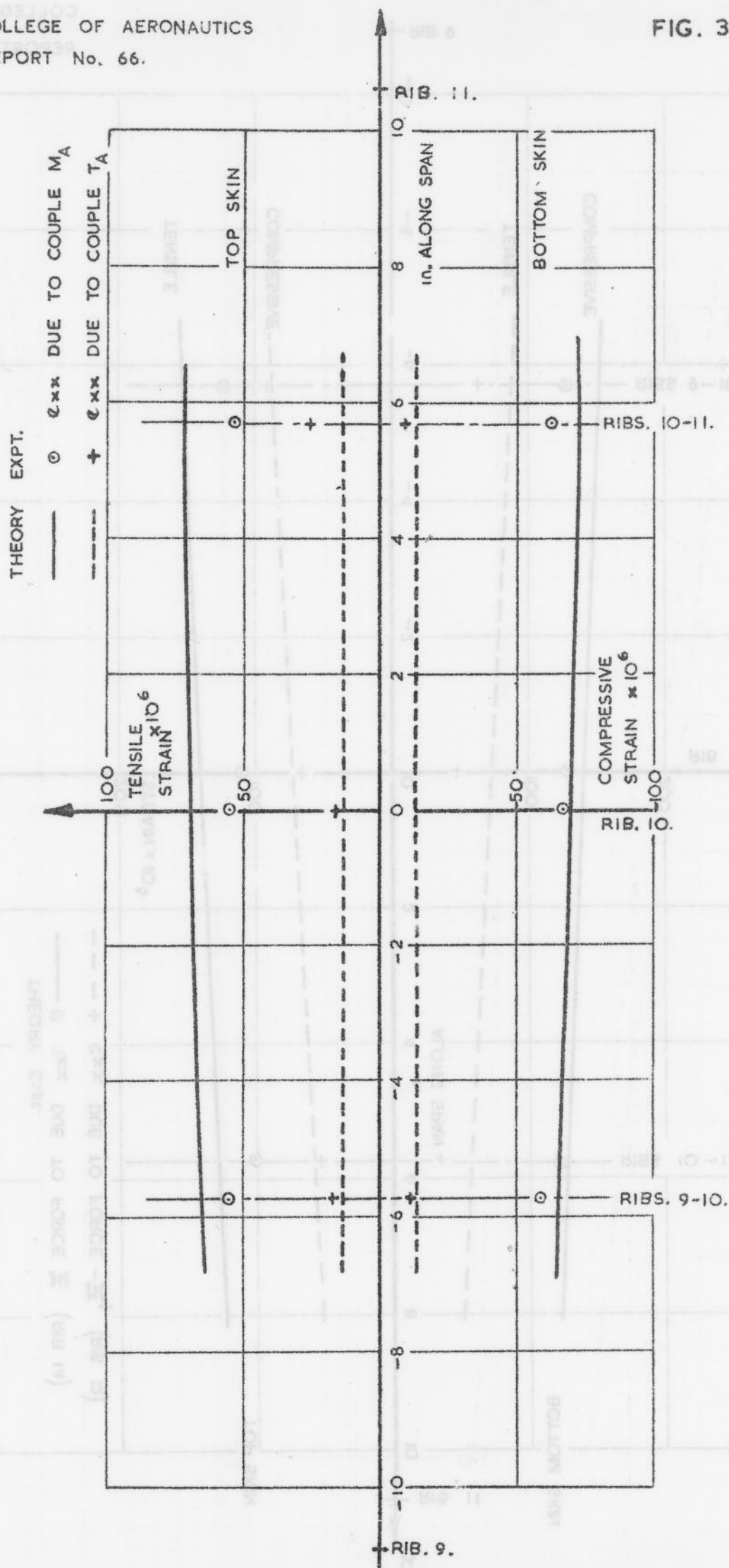


FIG. 30.



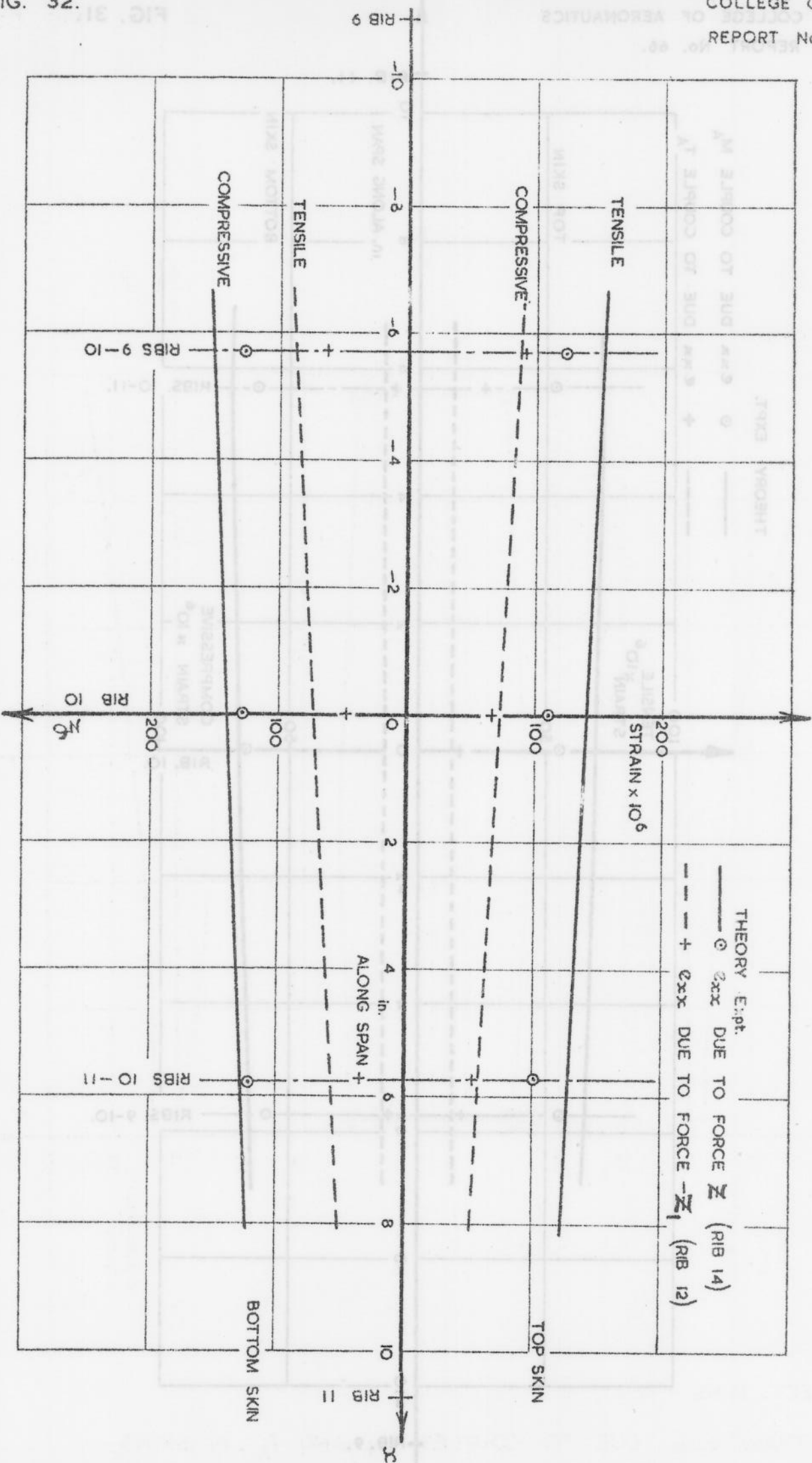
SECTION RIBS 10-11

STRAIN ϵ_{xy} DUE TO FORCES Z AND $-Z_1$ IN SKINS.

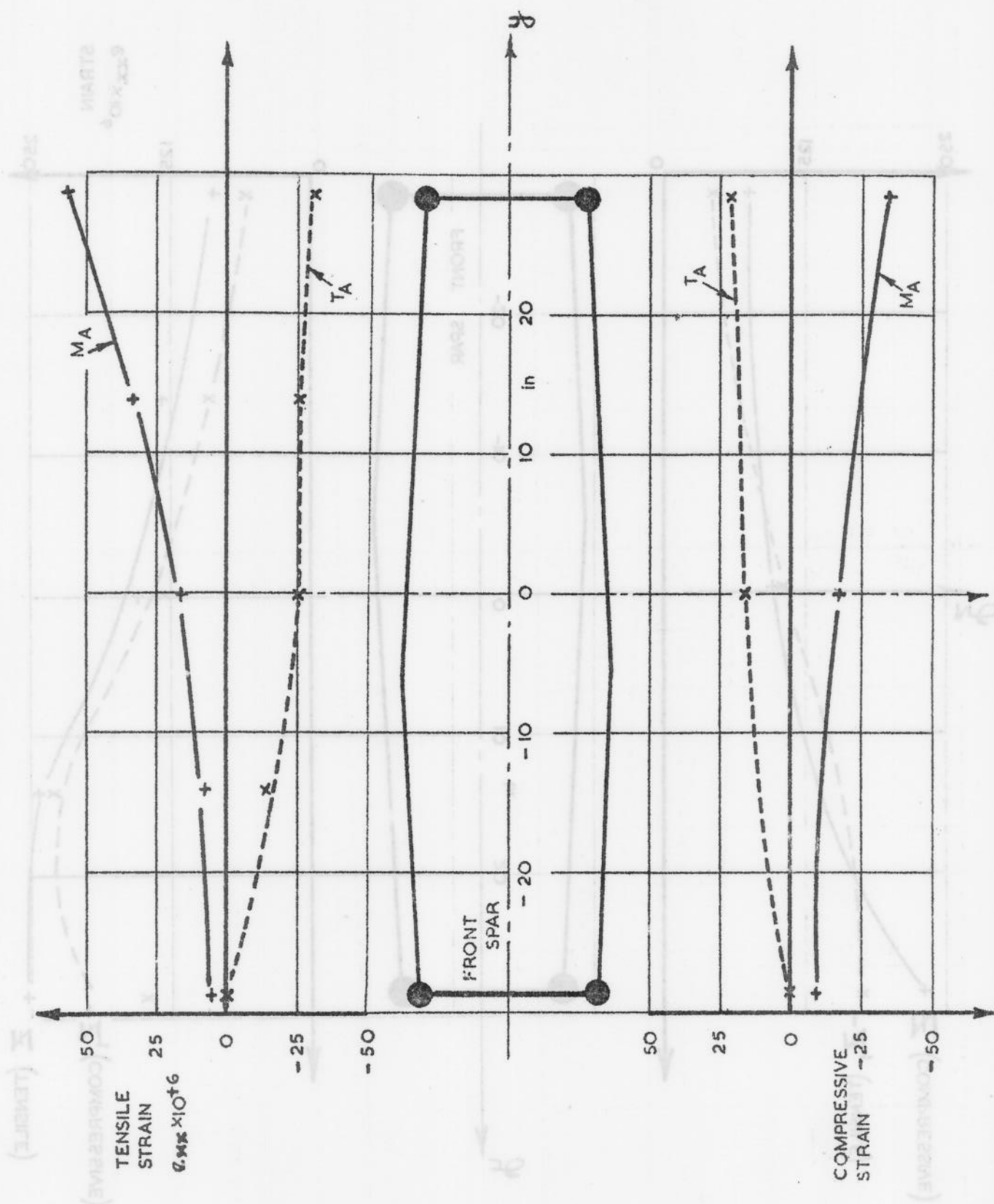


SPANWISE VARIATION OF STRAIN ϵ_{xx} ON ϕ SKIN
DUE TO COUPLES M_A AND T_A

FIG. 32.



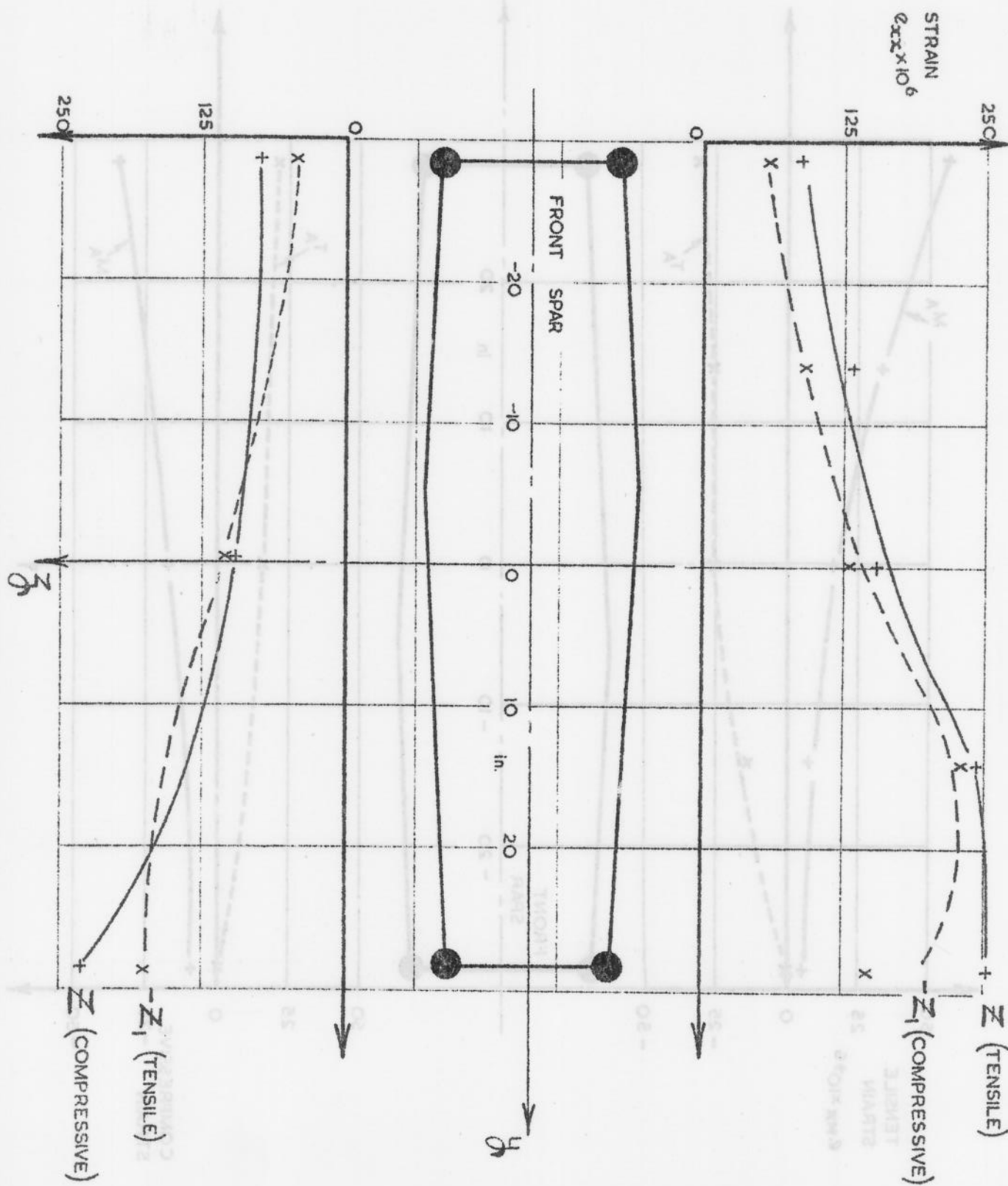
SPANWISE VARIATION OF STRAIN ϵ_{xx} ON ϕ SKIN DUE TO FORCES Z AND $-Z_1$



SECT. RIBS 1-2

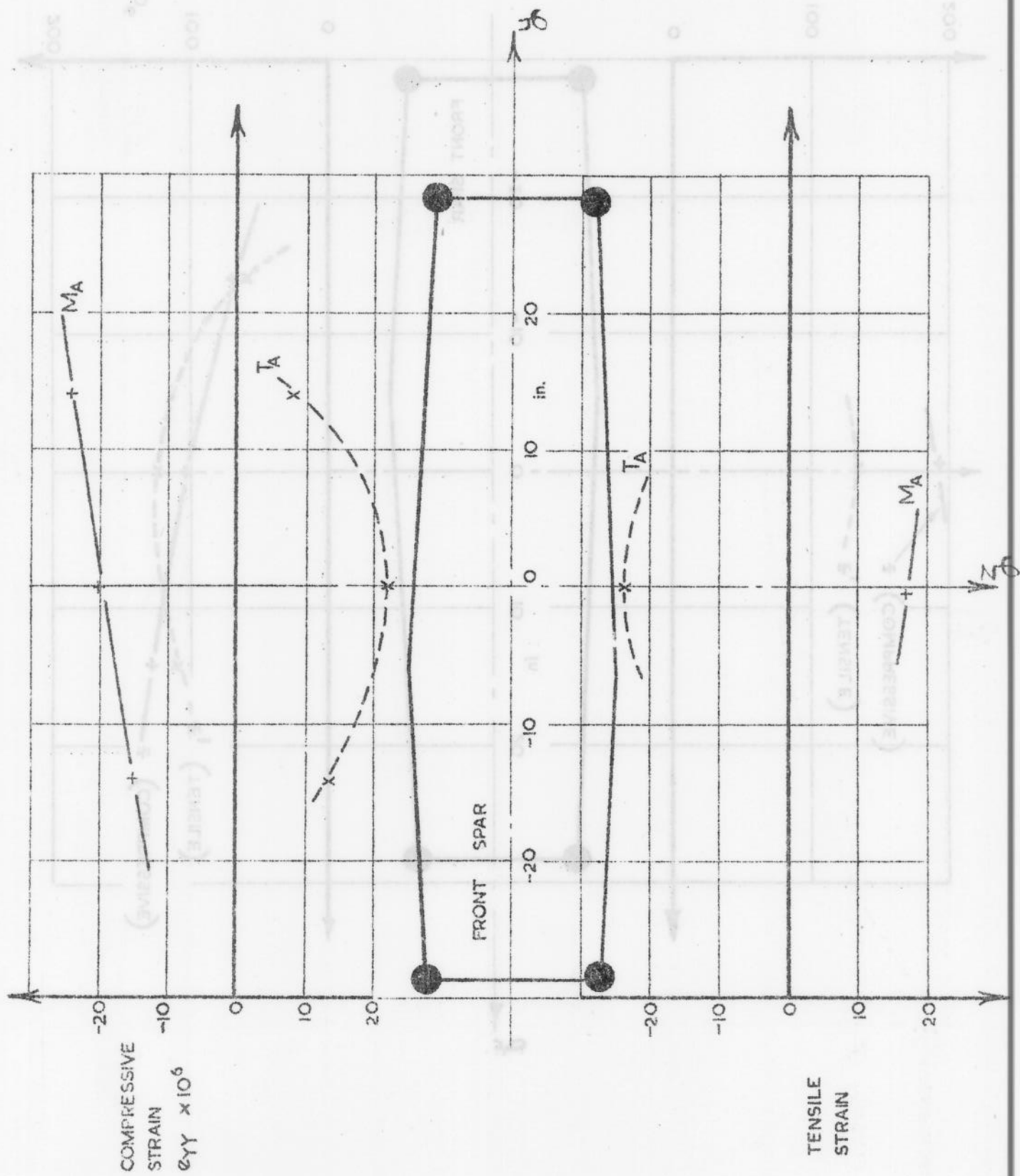
STRAIN ϵ_{xx} DUE TO COUPLES M_A AND T_A IN SKINS.

FIG. 34.



SECTION RIBS 1-2

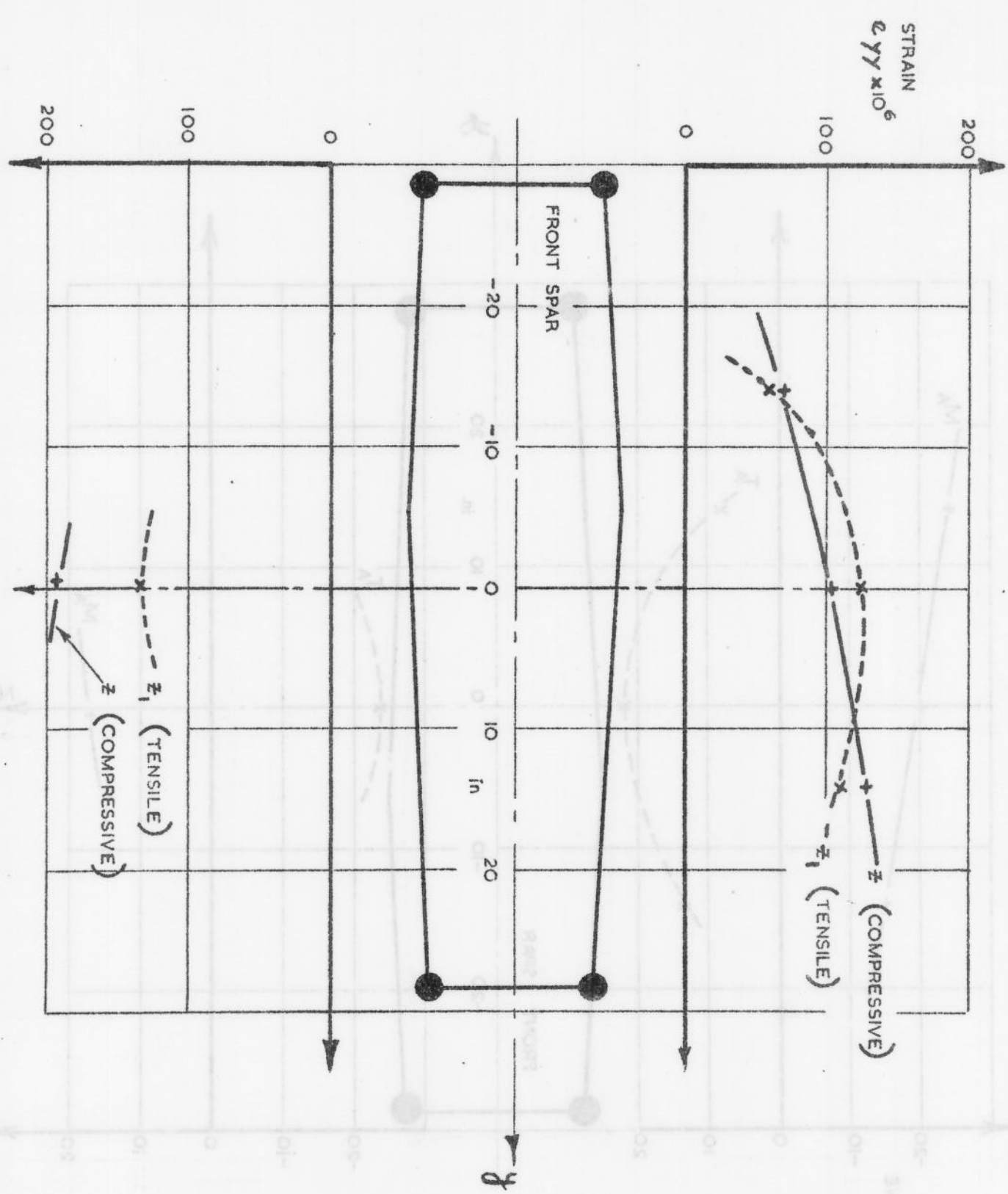
STRAIN e_{xx} DUE TO Z WISE FORCE $+Z$ AND $-Z$ IN SKINS.



SECTION RIBS 1-2

STRAIN ϵ_{yy} DUE TO COUPLES M_A AND T_A IN SKINS

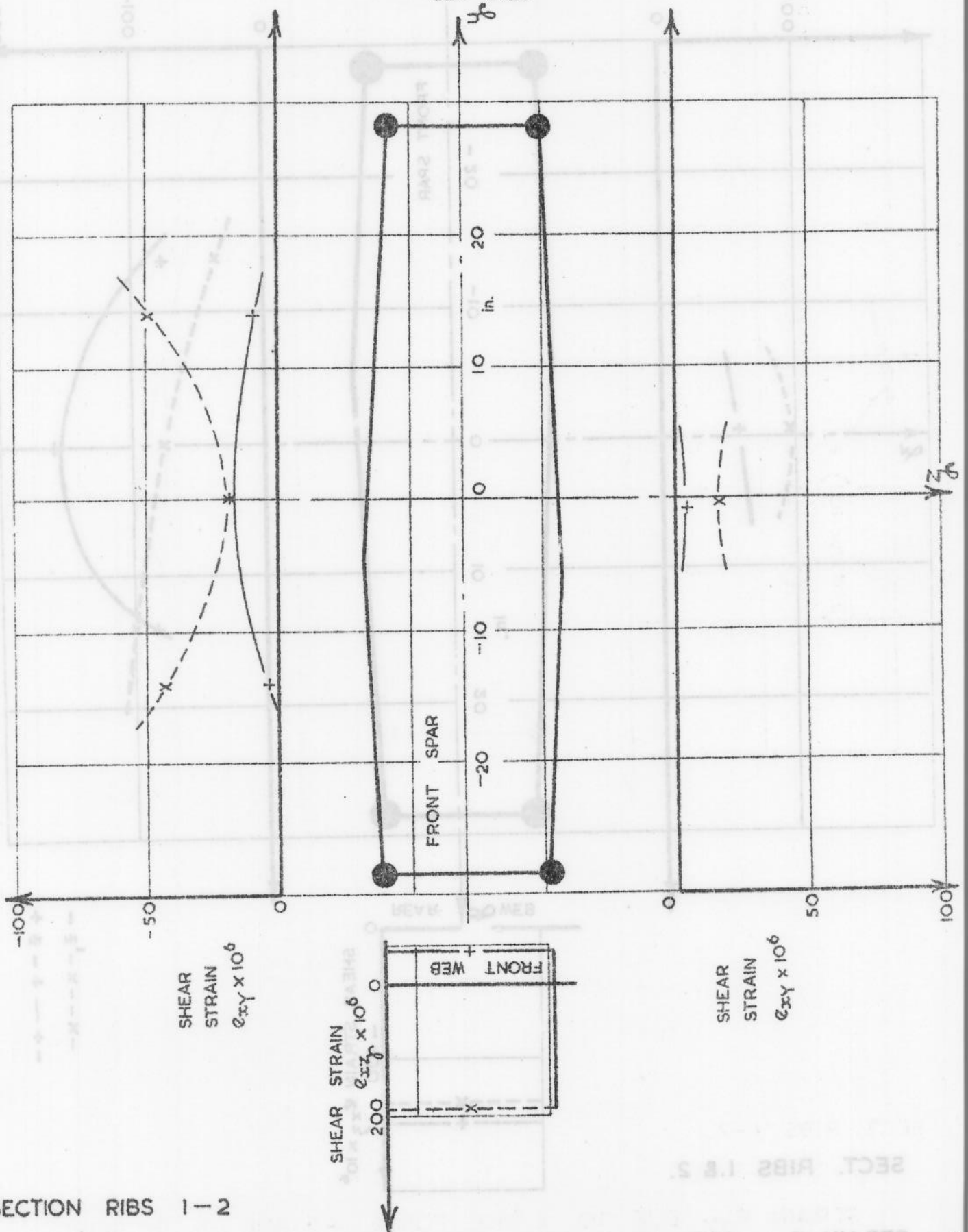
FIG. 36.



SECT. RIBS 1-2.

STRAIN ϵ_{yy} DUE TO z WISE FORCE $+z$ AND $-z_1$ IN SKINS.

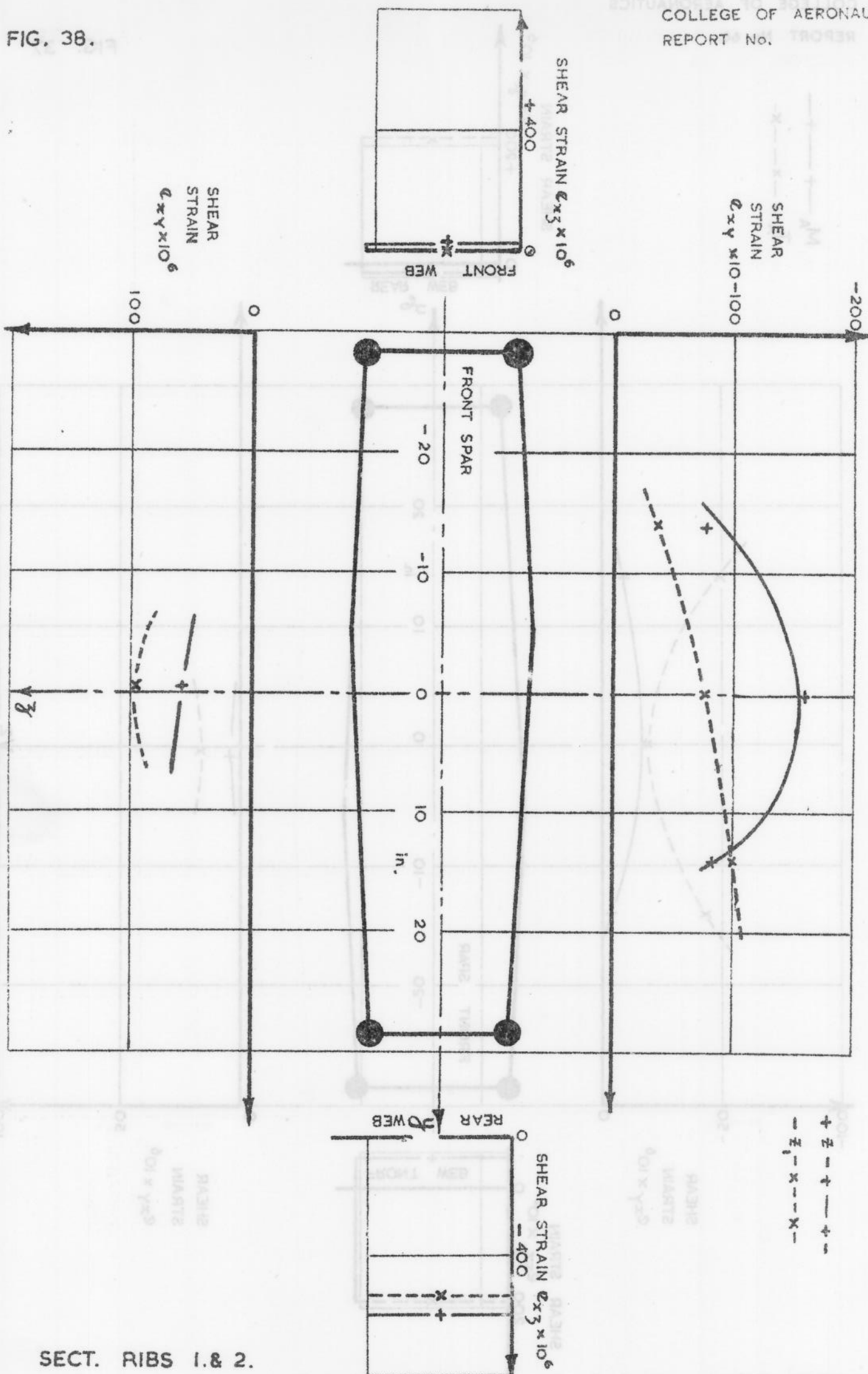
M_A — — — +
 T_A — — — x



SECTION RIBS 1-2

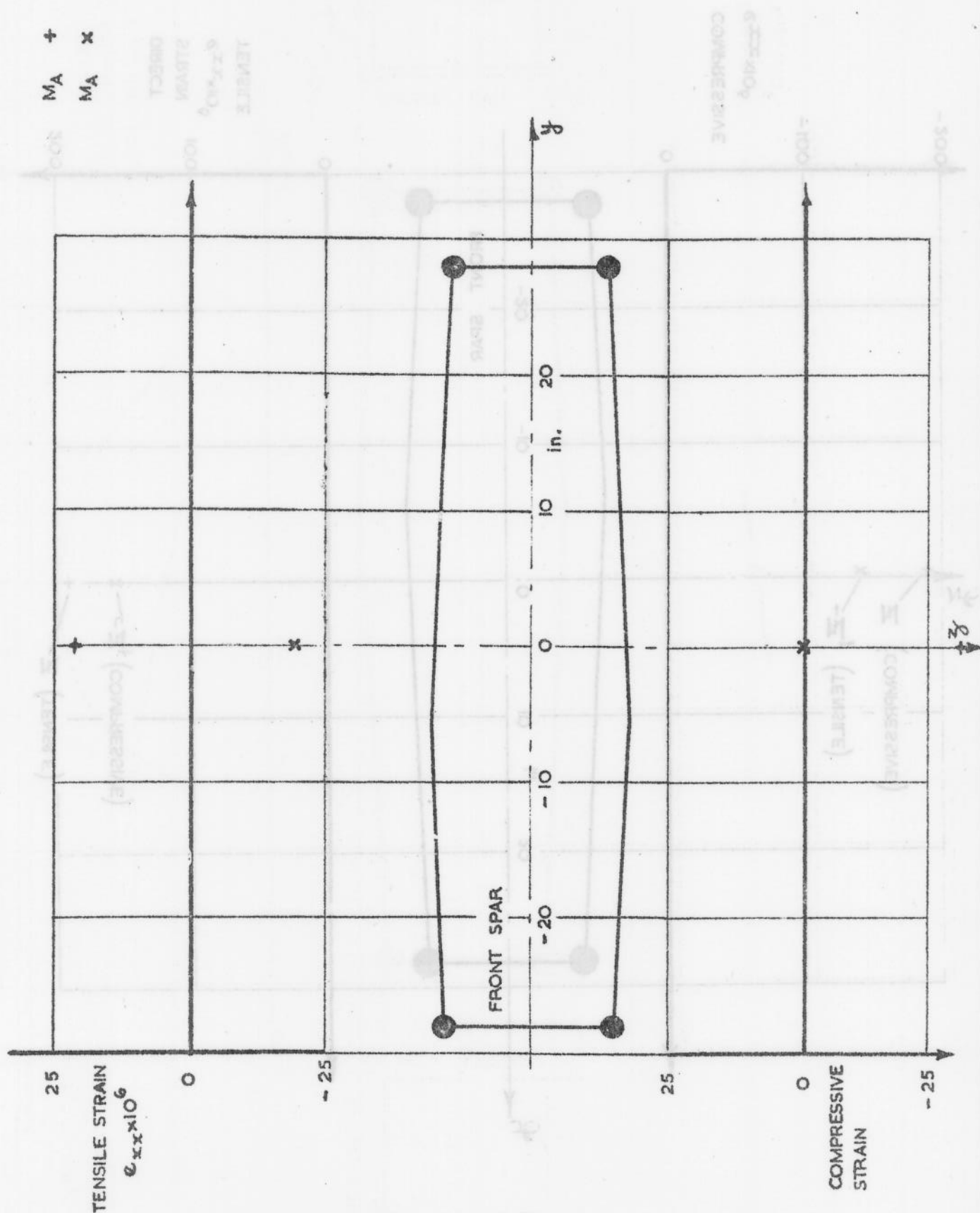
STRAIN e_{xy} DUE TO COUPLES M_A AND T_A IN SKINS (e_{xz} IN WEBS)

FIG. 38.



SECT. RIBS 1. & 2.

STRAIN ϵ_{xy} DUE TO z WISE FORCE $+z$ AND $-z$, IN SKINS (ϵ_{xz} IN WEBS)

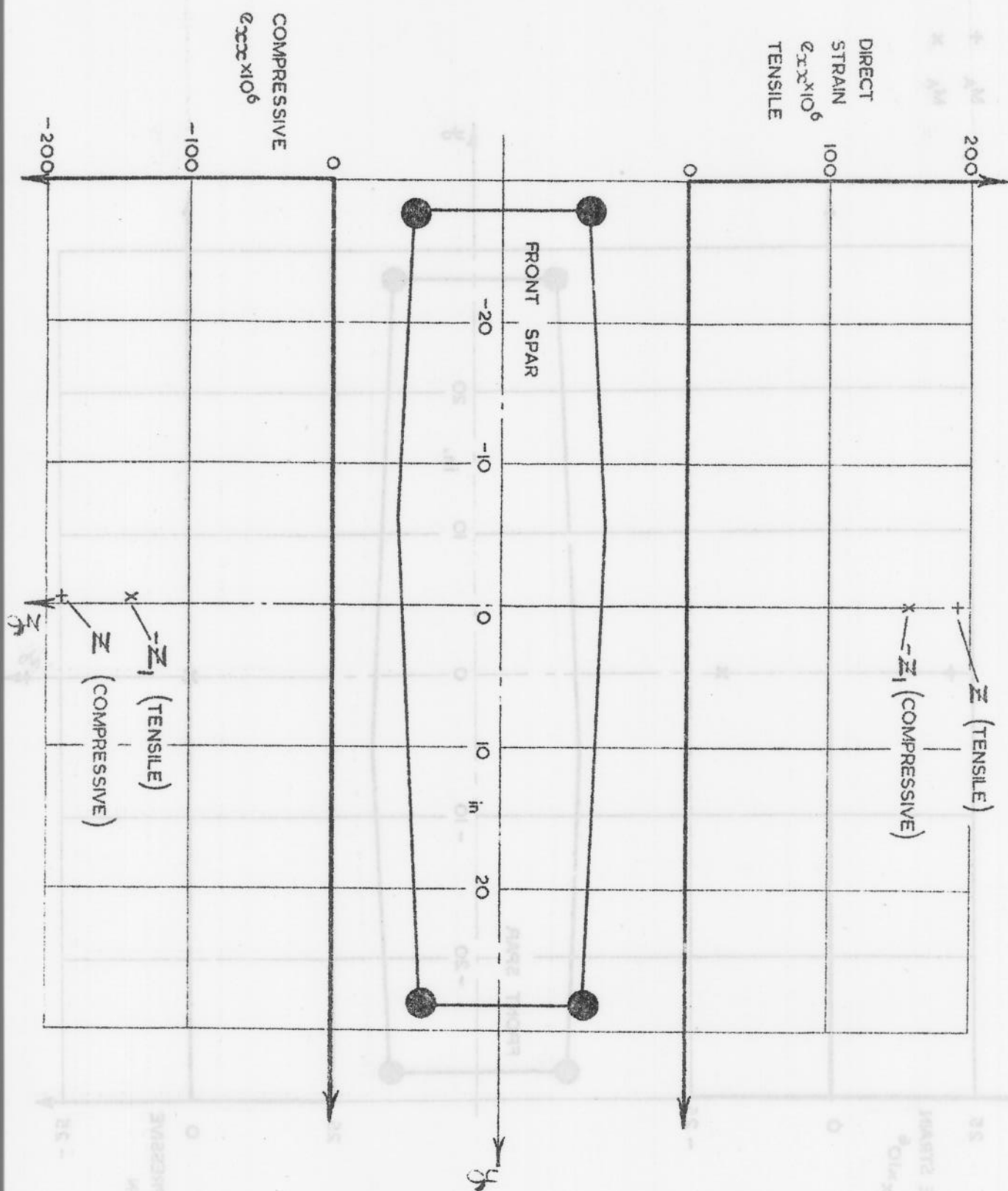


SECT. RIB 2. STRAIN e_{xx} DUE TO COUPLES M_A AND T_A IN SKINS

FIG. 40.

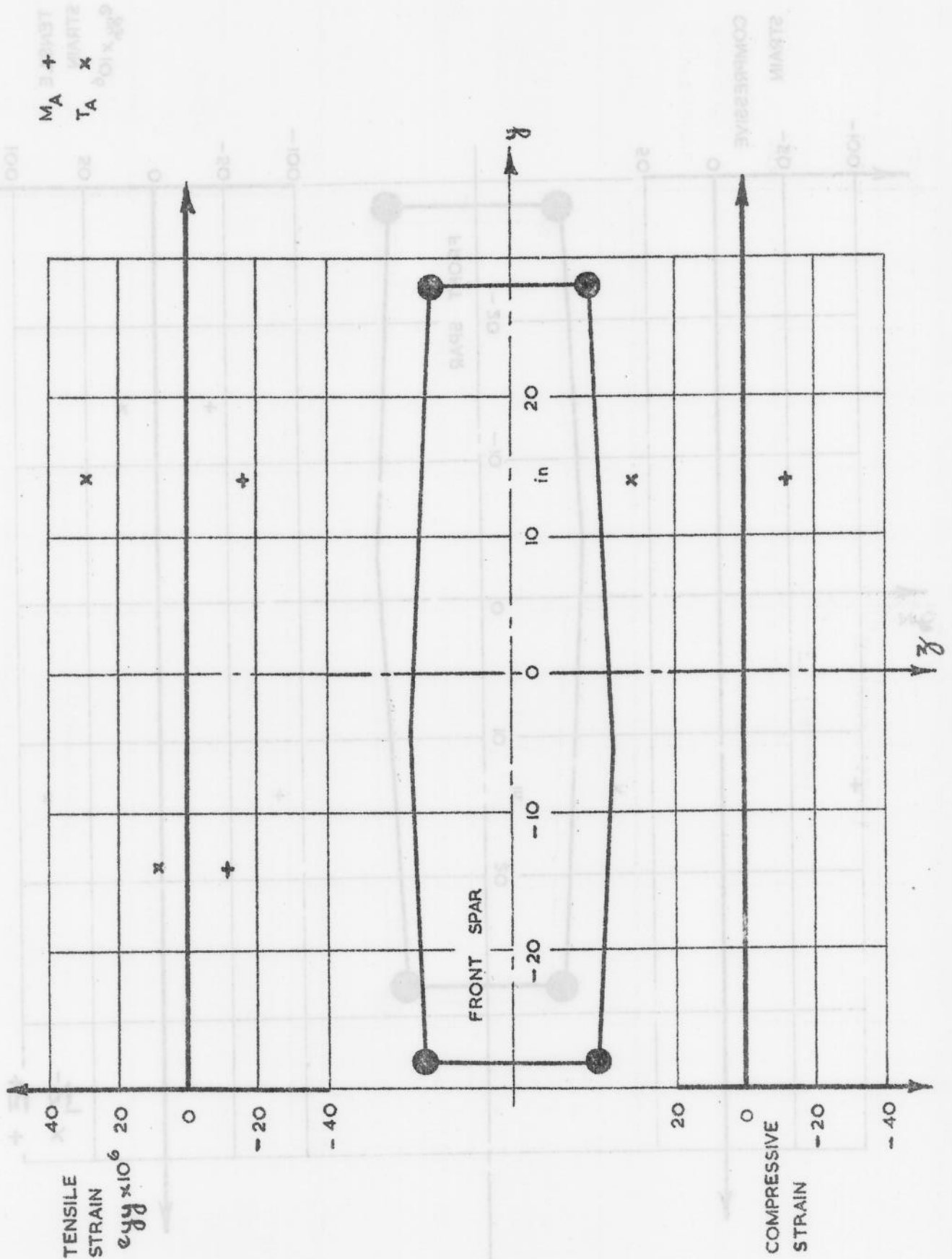
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SECTION RIB 2.

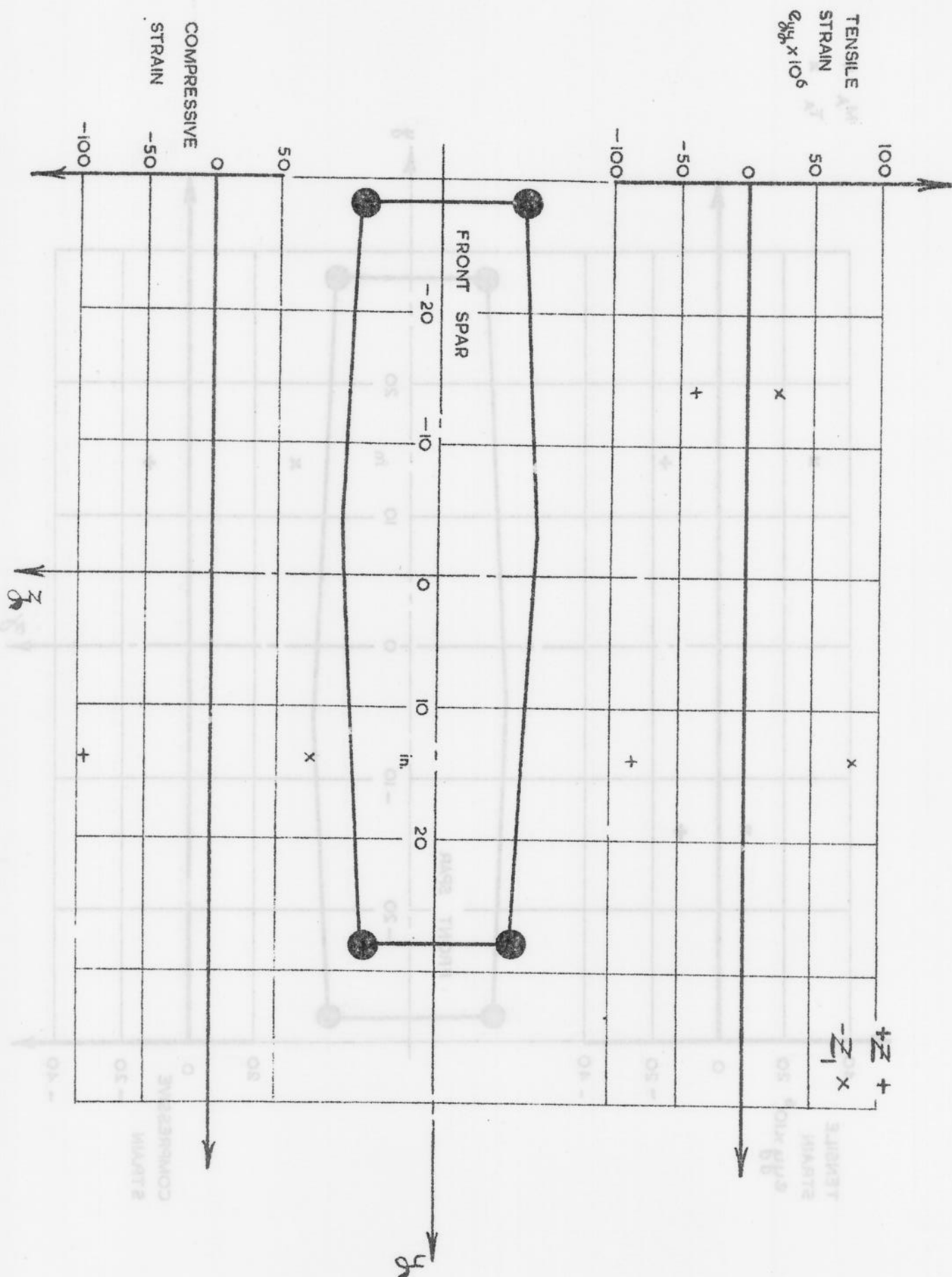
STRAIN ϵ_{xx} DUE TO Z WISE FORCE $+Z$ AND $-Z_1$ IN SKIN.



SECT. RIB 2.

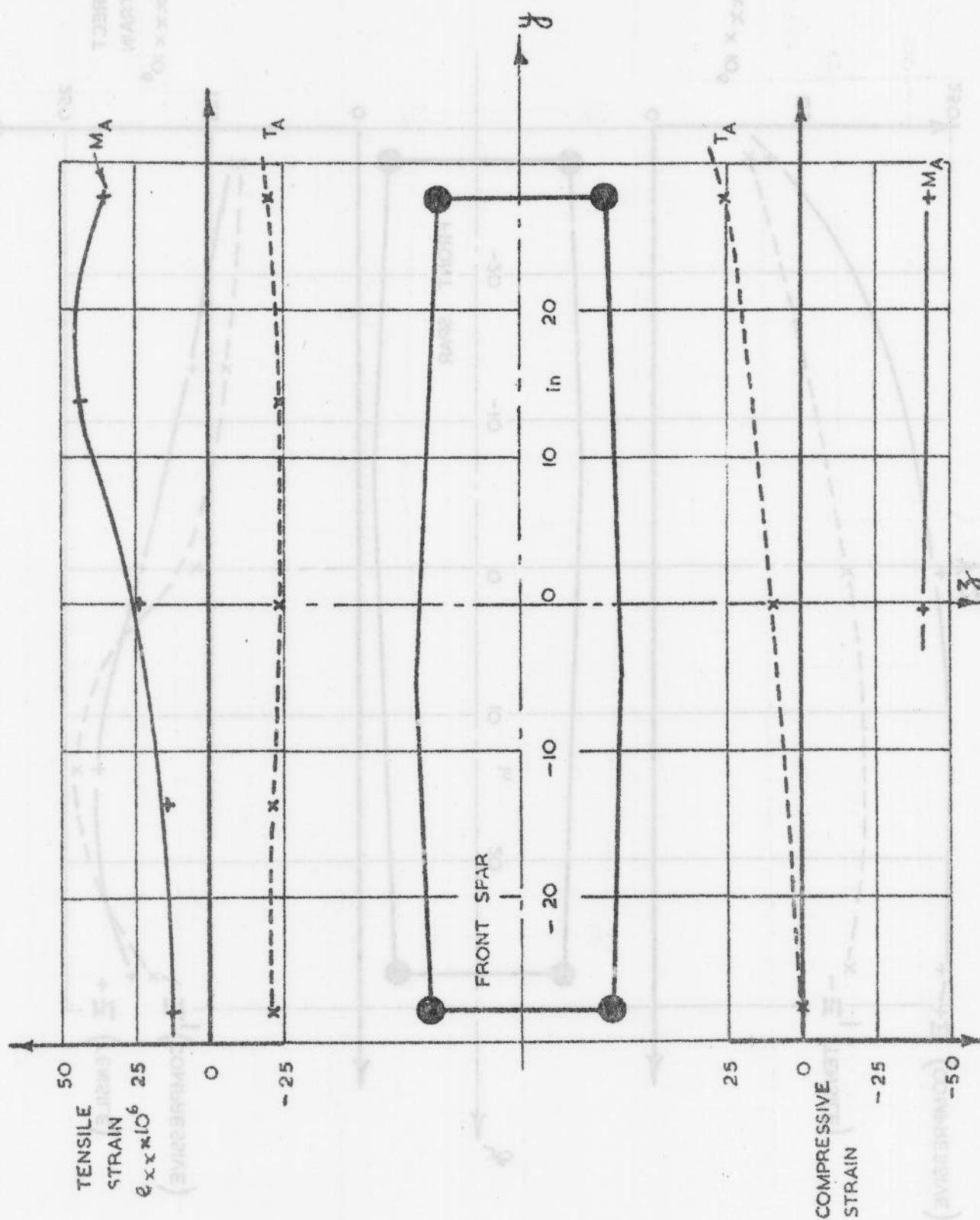
STRAIN e_{yy} DUE TO COUPLES M_A AND T_A IN RIB BOOMS

FIG. 42.



SECTION RIB 2

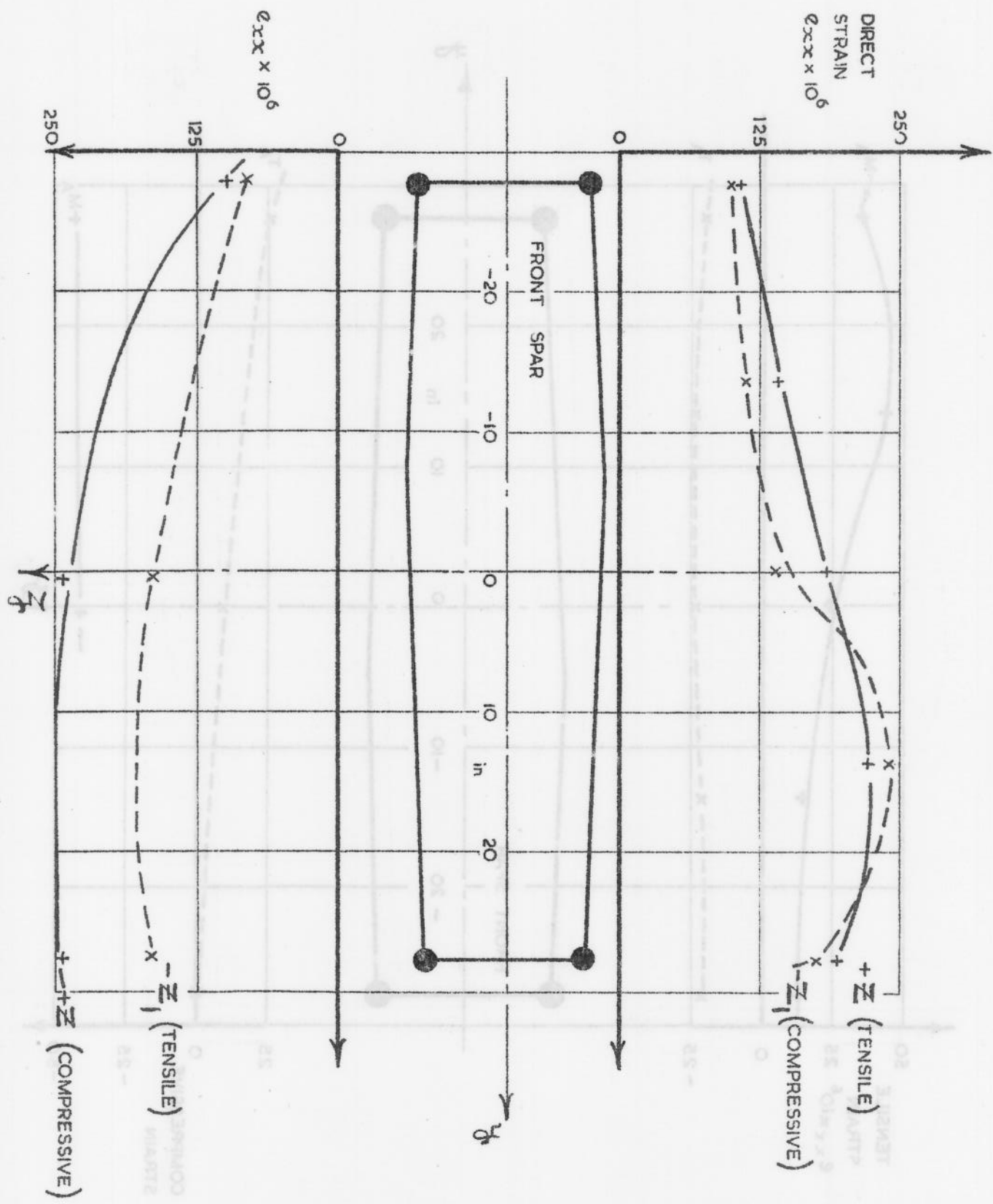
STRAIN ϵ_{yy} DUE TO Z WISE FORCE $+Z$ AND $-Z_1$ IN RIB BOOMS



SECT. RIBS 2-3

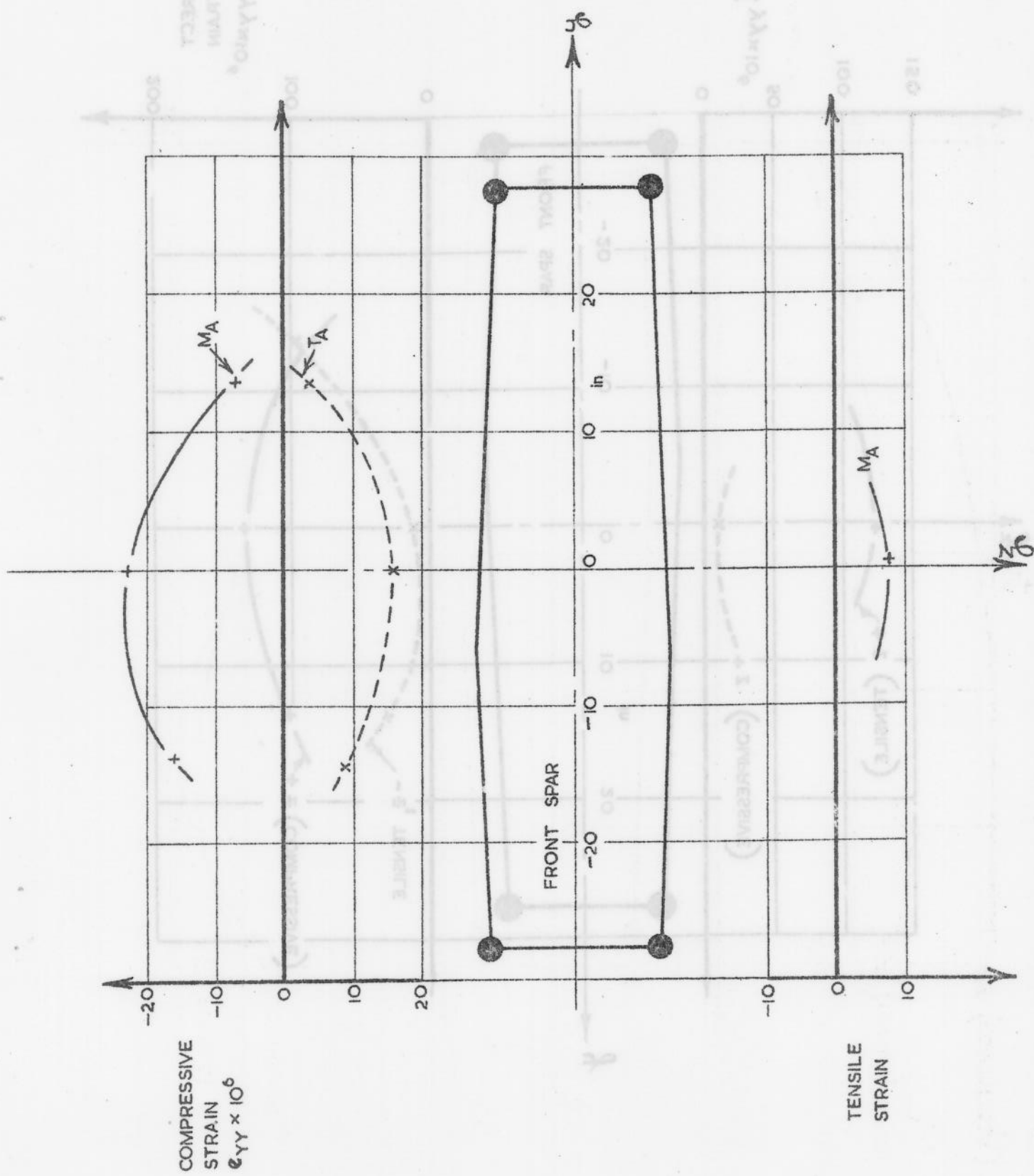
STRAIN ϵ_{xx} DUE TO COUPLES M_A AND T_A IN SKINS

FIG. 44.



SECTION RIBS 2-3

STRAIN e_{xx} DUE TO Z WISE FORCE $+Z$ AND $-Z$ IN SKINS

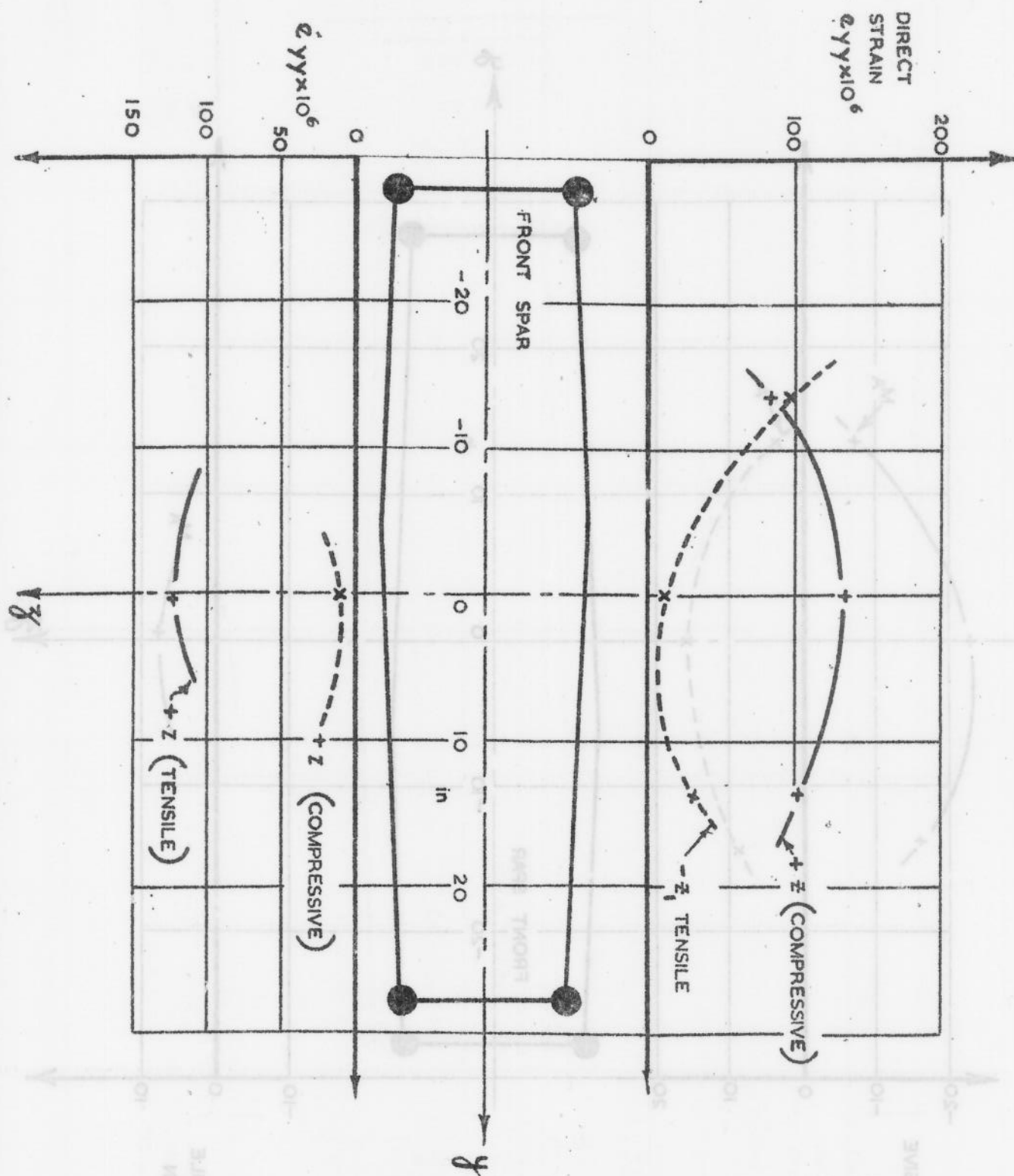


SECTION RIBS 2-3

STRAIN e_{yy} DUE TO COUPLES M_A AND T_A IN SKINS

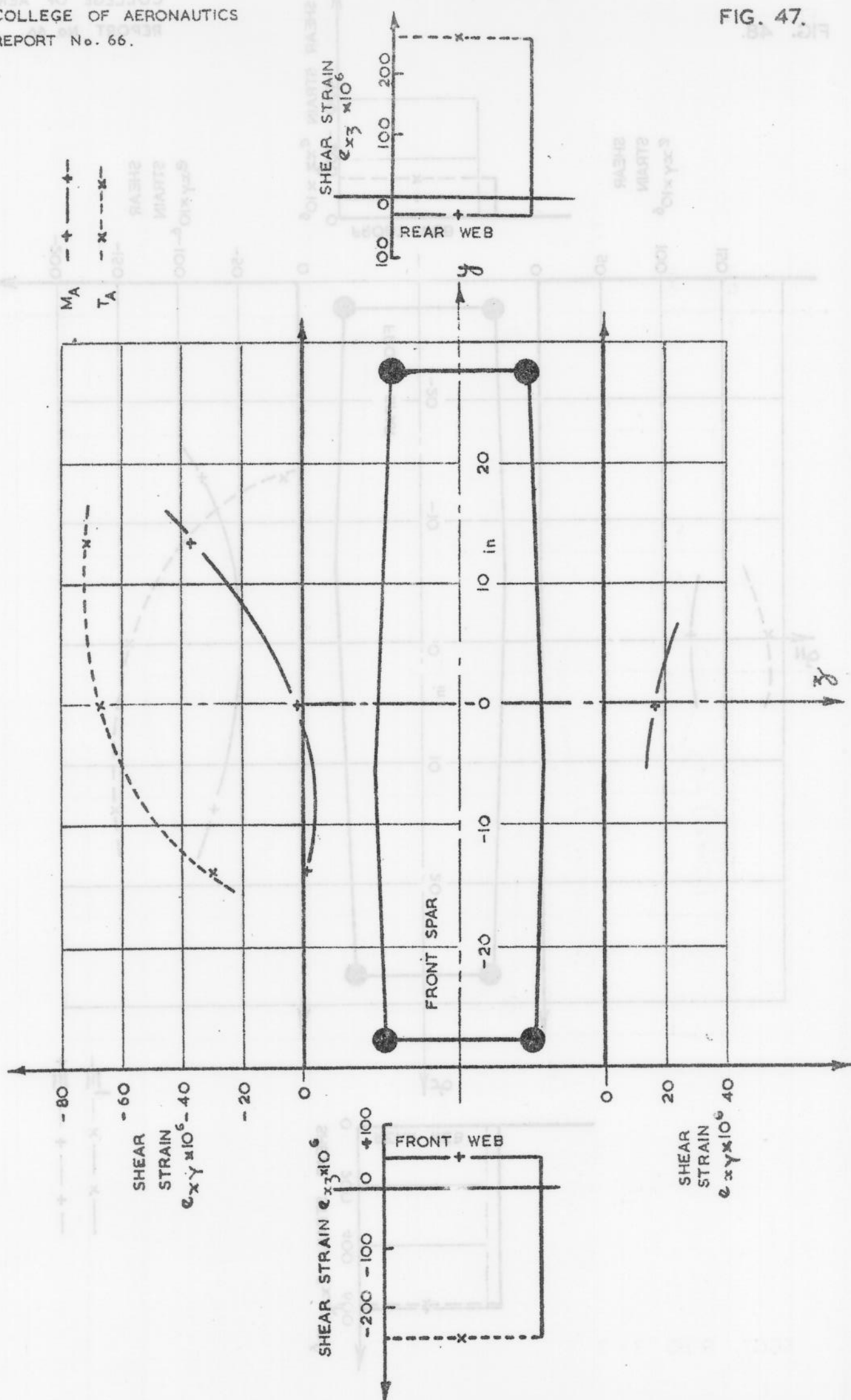
FIG. 46.

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REPORT No. 66.



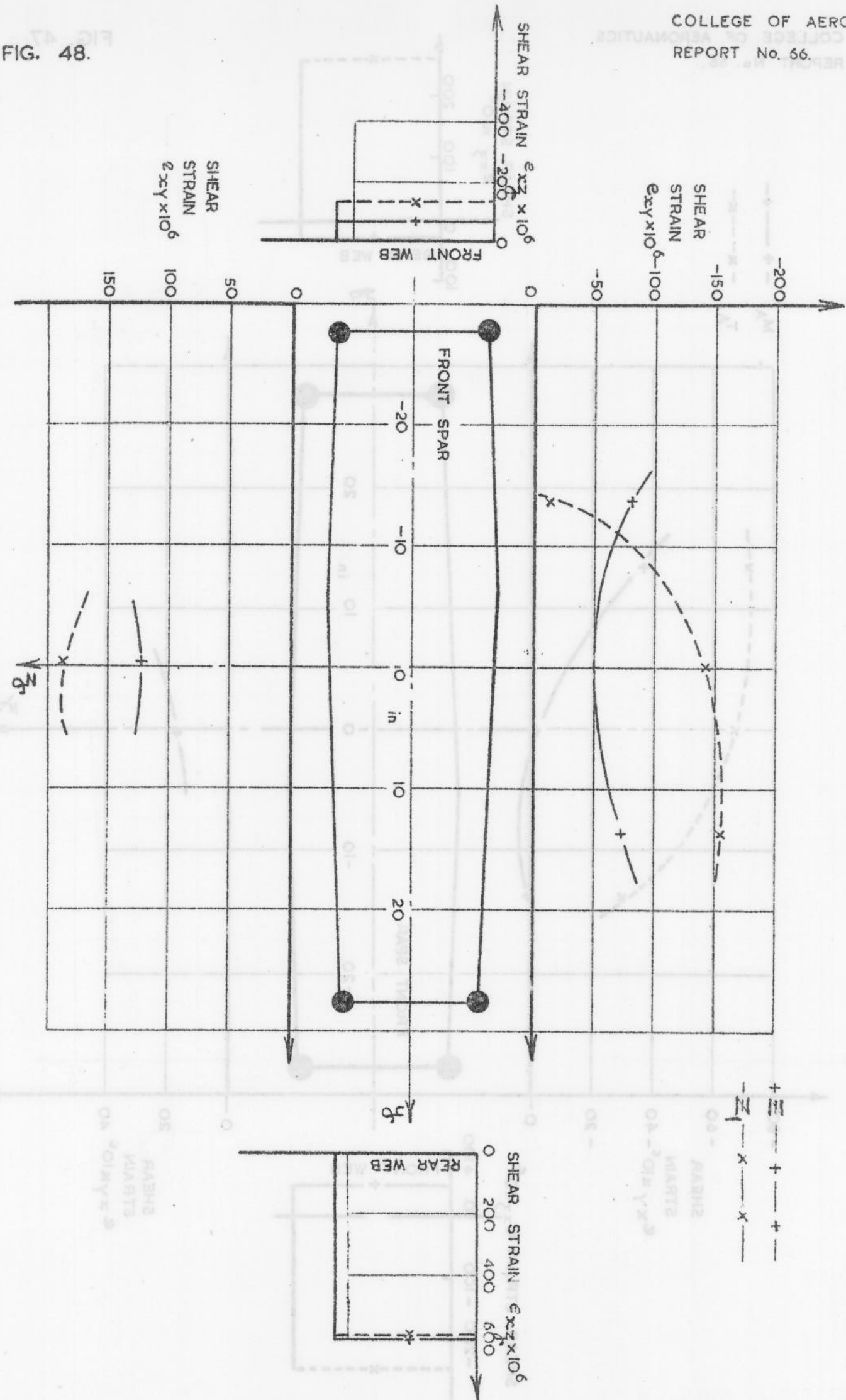
SECT. RIBS 2-3

STRAIN ϵ_{yy} DUE TO z WISE FORCE $+z$ AND $-z$ IN SKINS

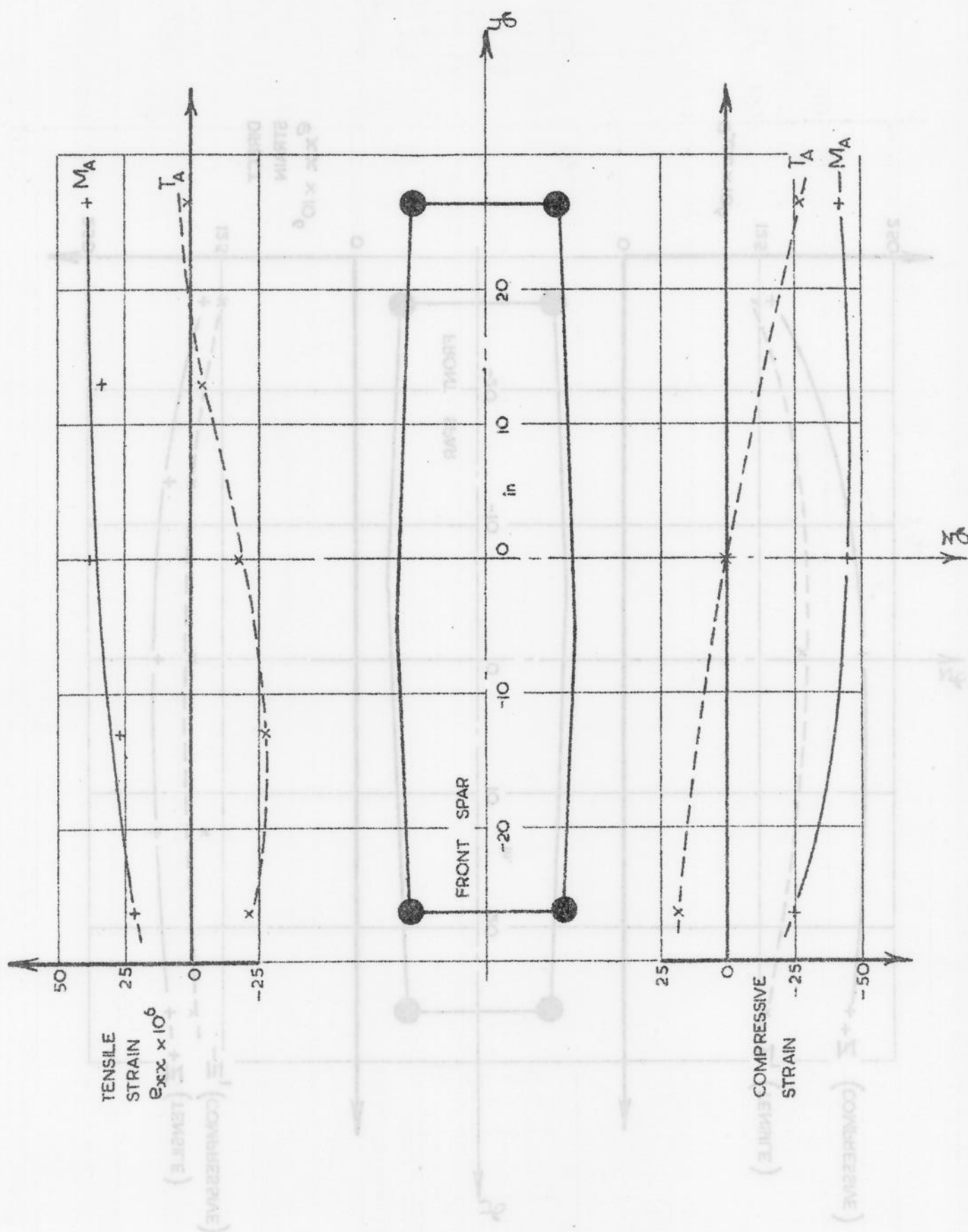


SECTION RIBS 2-3 STRAIN e_{xy} DUE TO COUPLES
 M_A AND T_A IN SKINS (e_{xz} IN WEBS)

FIG. 48.



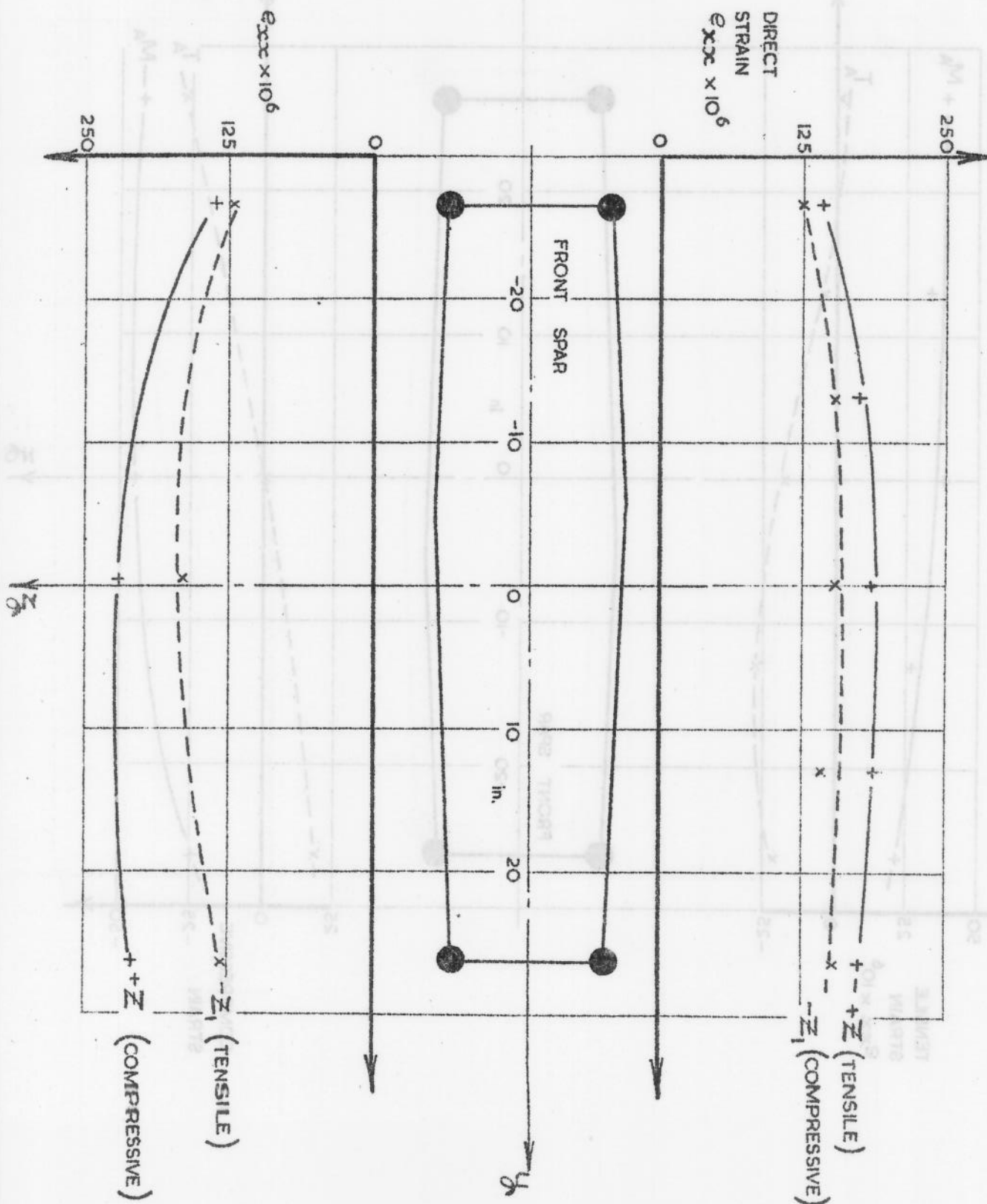
STRAIN e_{xy} DUE TO Z WISE FORCE $+Z$ AND $-Z$ IN SKINS
(e_{xz} IN WEBS)



SECTION RIBS 4-5

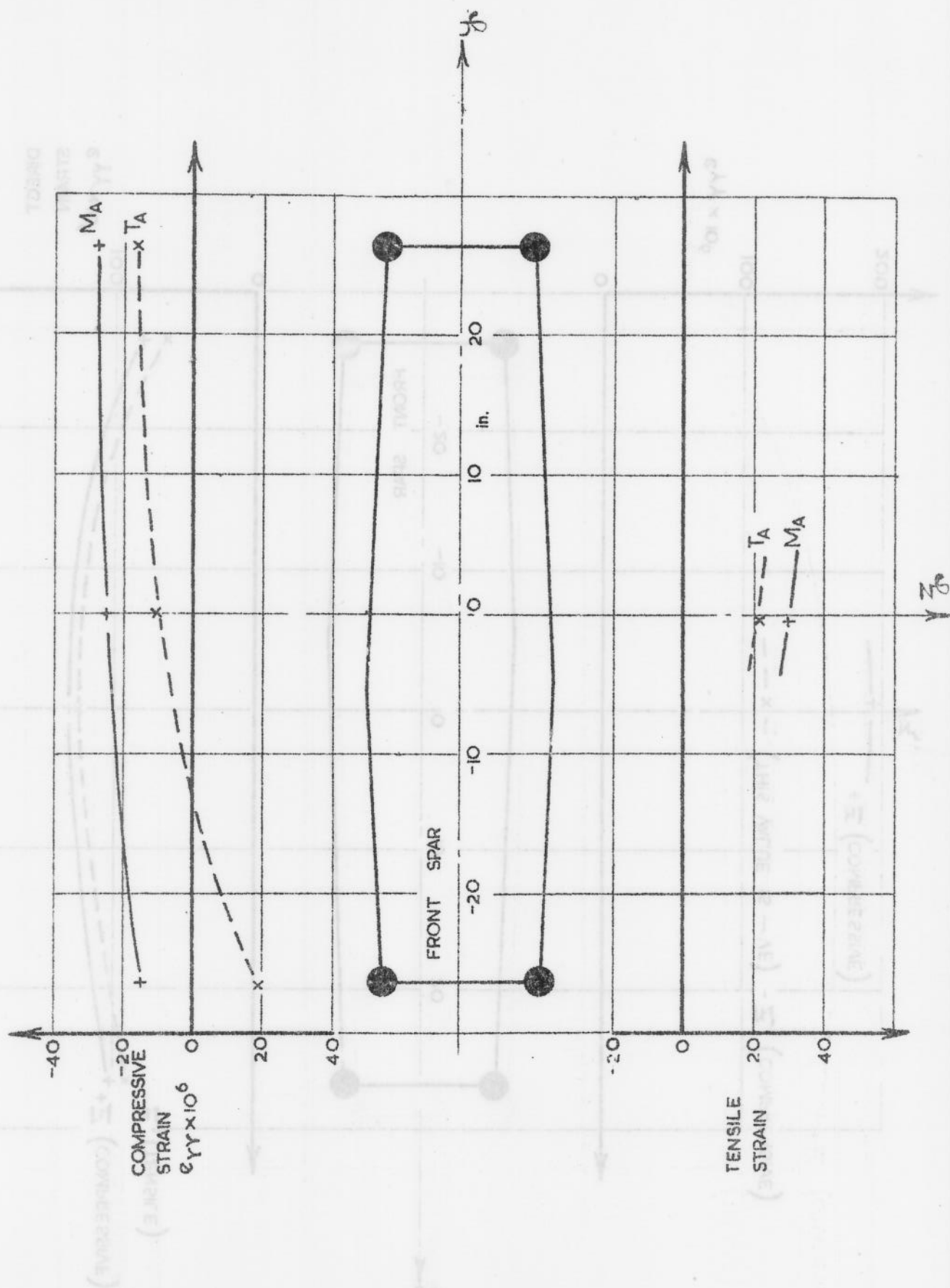
STRAIN ϵ_{xx} DUE TO COUPLE M_A AND T_A IN SKINS.

FIG. 50.



SECTION RIBS 4-5

STRAIN e_{xx} DUE TO Z WISE FORCE $+Z$ AND $-Z_1$ IN SKINS

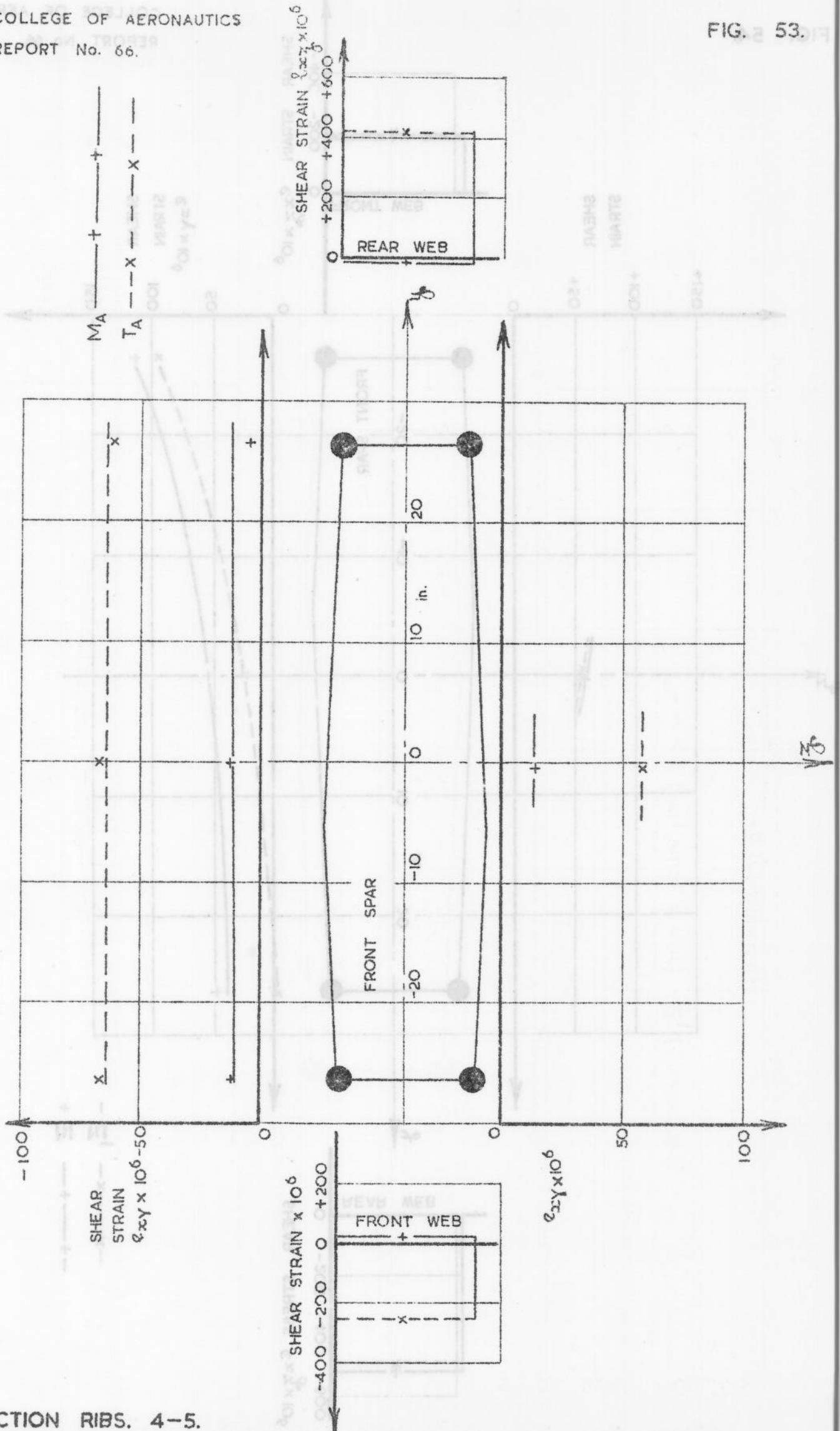


SECTION RIBS 4-5.

STRAIN ϵ_{yy} DUE TO COUPLES M_A AND T_A IN SKINS.



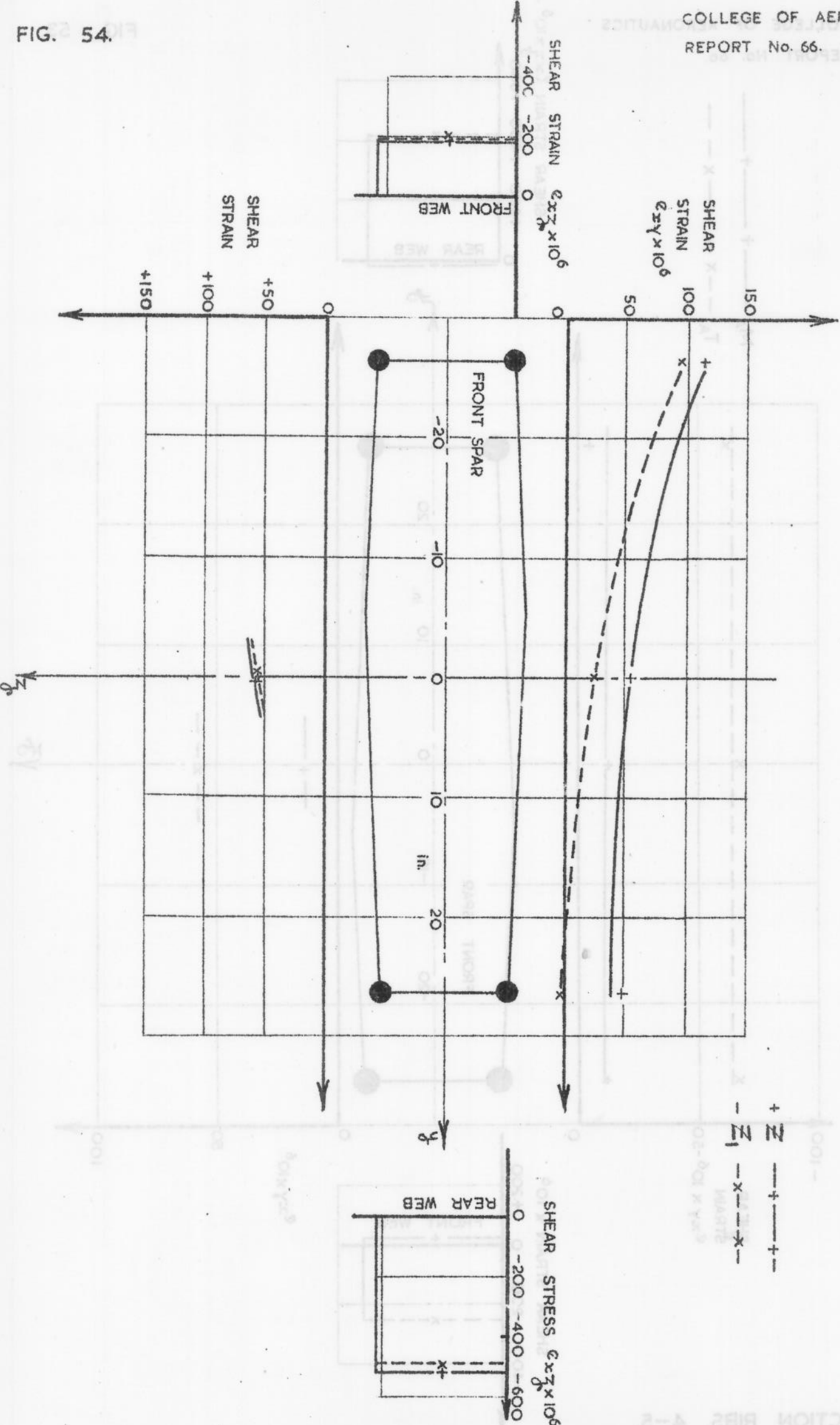
STRAIN ϵ_{YY} DUE TO Z WISE FORCE $+Z$ AND $-Z_1$ IN SKINS



SECTION RIBS. 4-5.

STRAIN e_{xy} DUE TO COUPLES M_A AND T_A (e_{xx} IN WEBS)

FIG. 54.



STRAIN ϵ_{xy} DUE TO Z WISE FORCE Z AND $-Z_1$ IN SKINS (ϵ_{xz} IN WEBS)

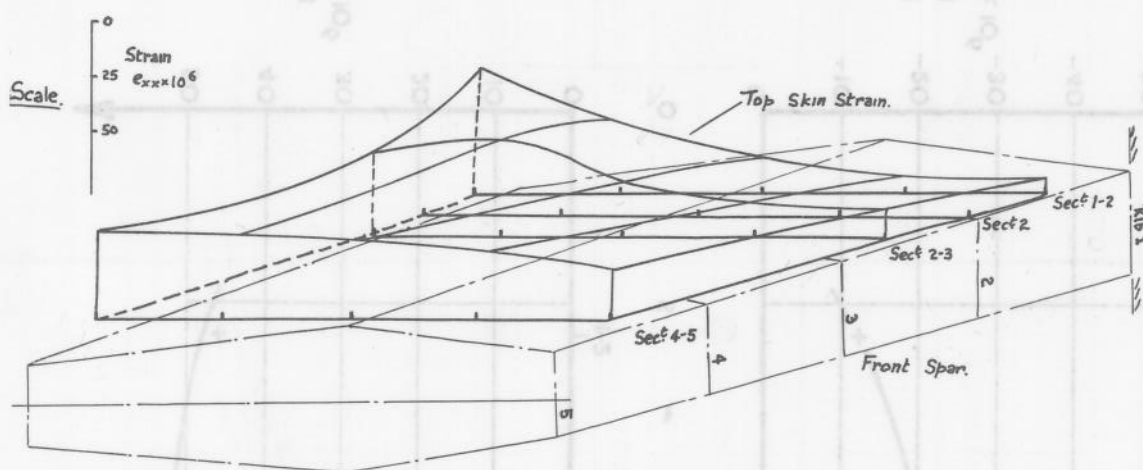


FIG. 55.

VARIATION OF SKIN DIRECT STRAIN e_{xx} AT ROOT DUE TO COUPLE M_A

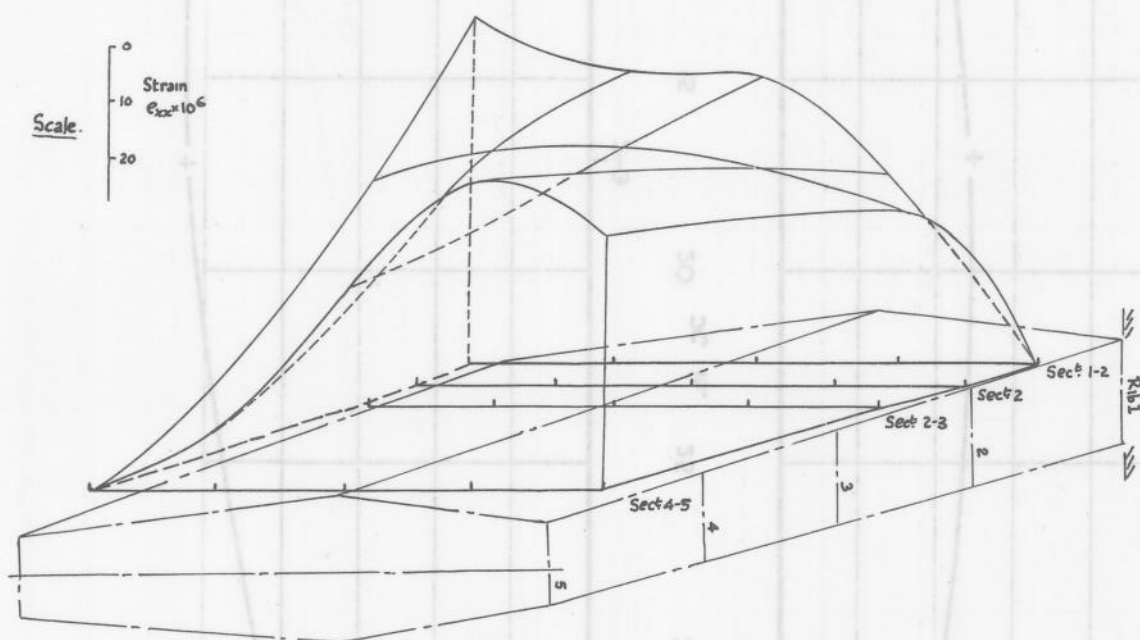


FIG. 56.

VARIATION OF SKIN DIRECT STRAIN e_{xx} AT ROOT DUE TO COUPLE T_A

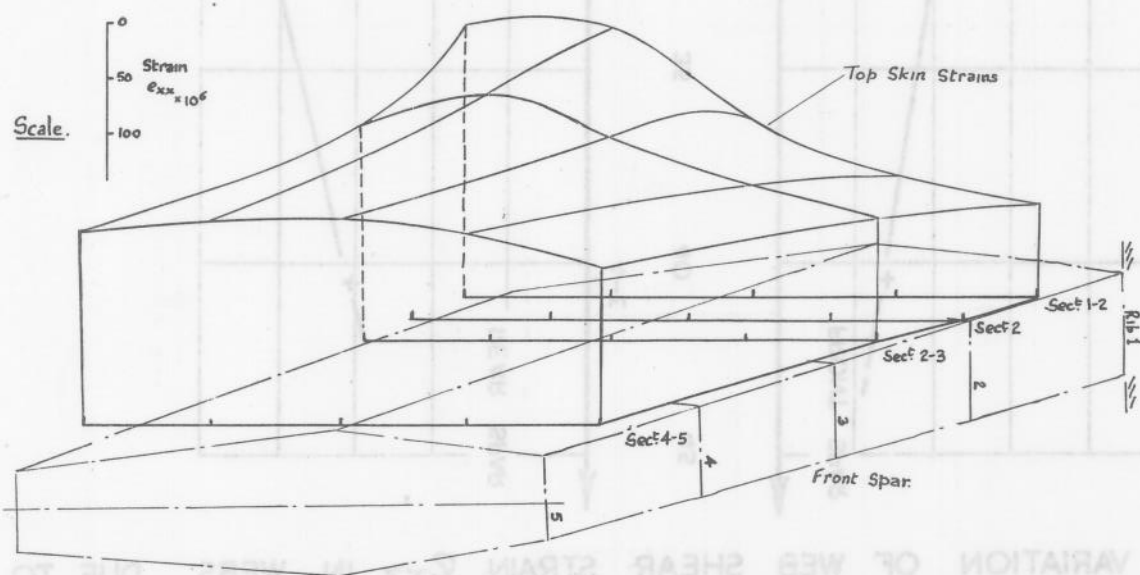
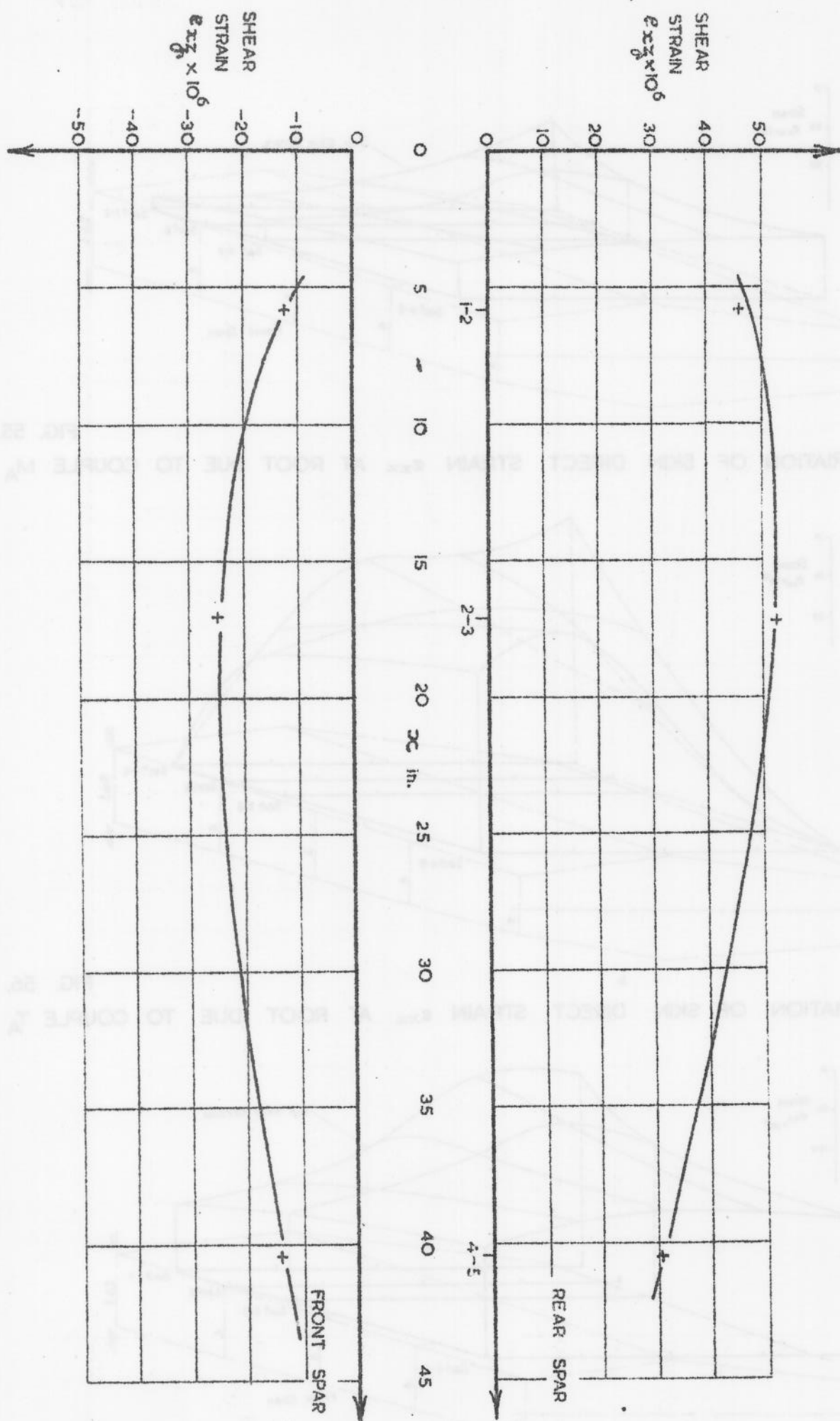
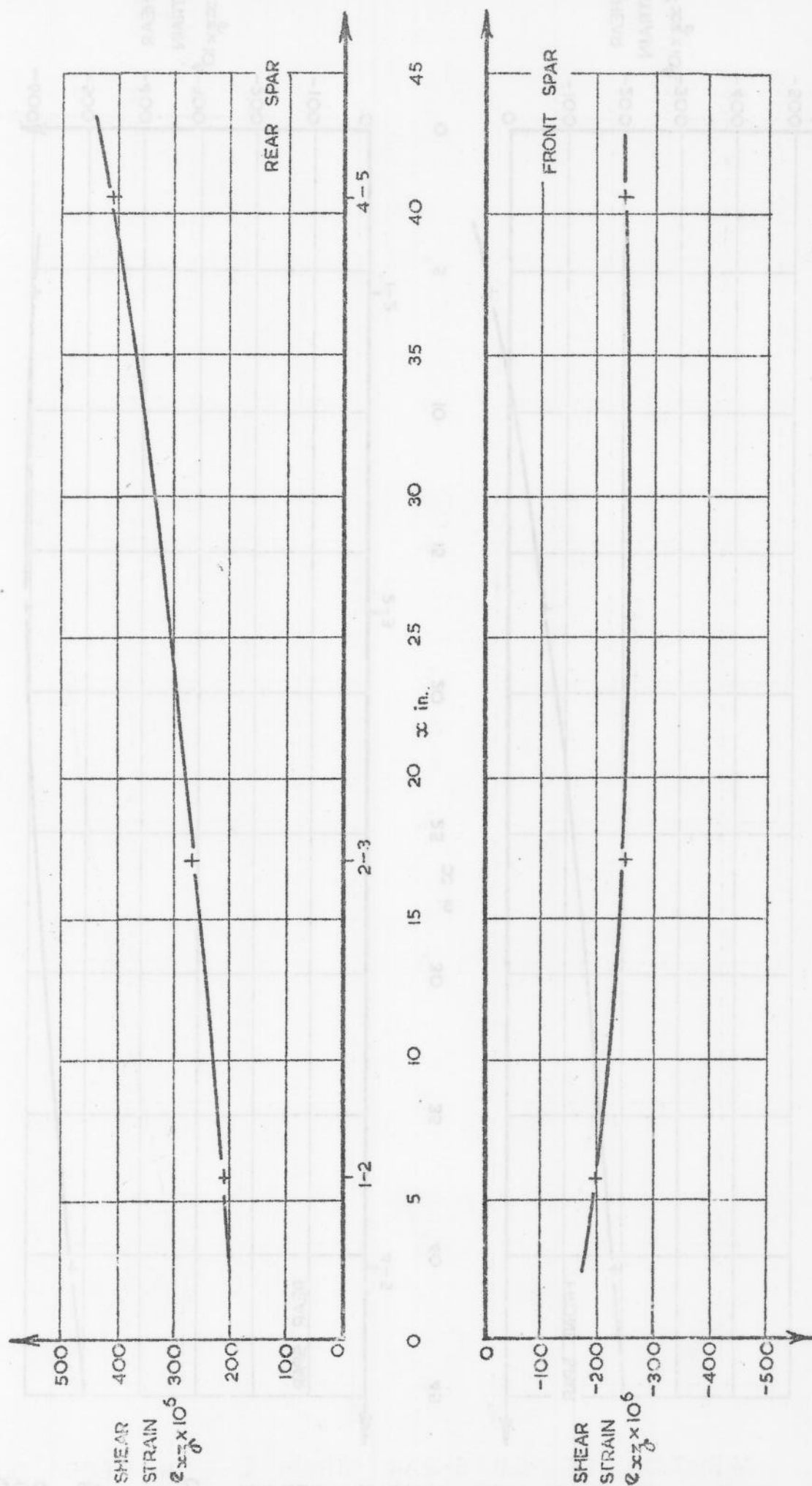


FIG. 57.

VARIATION OF SKIN DIRECT STRAIN e_{xx} AT ROOT DUE TO z WISE FORCE $+Z$

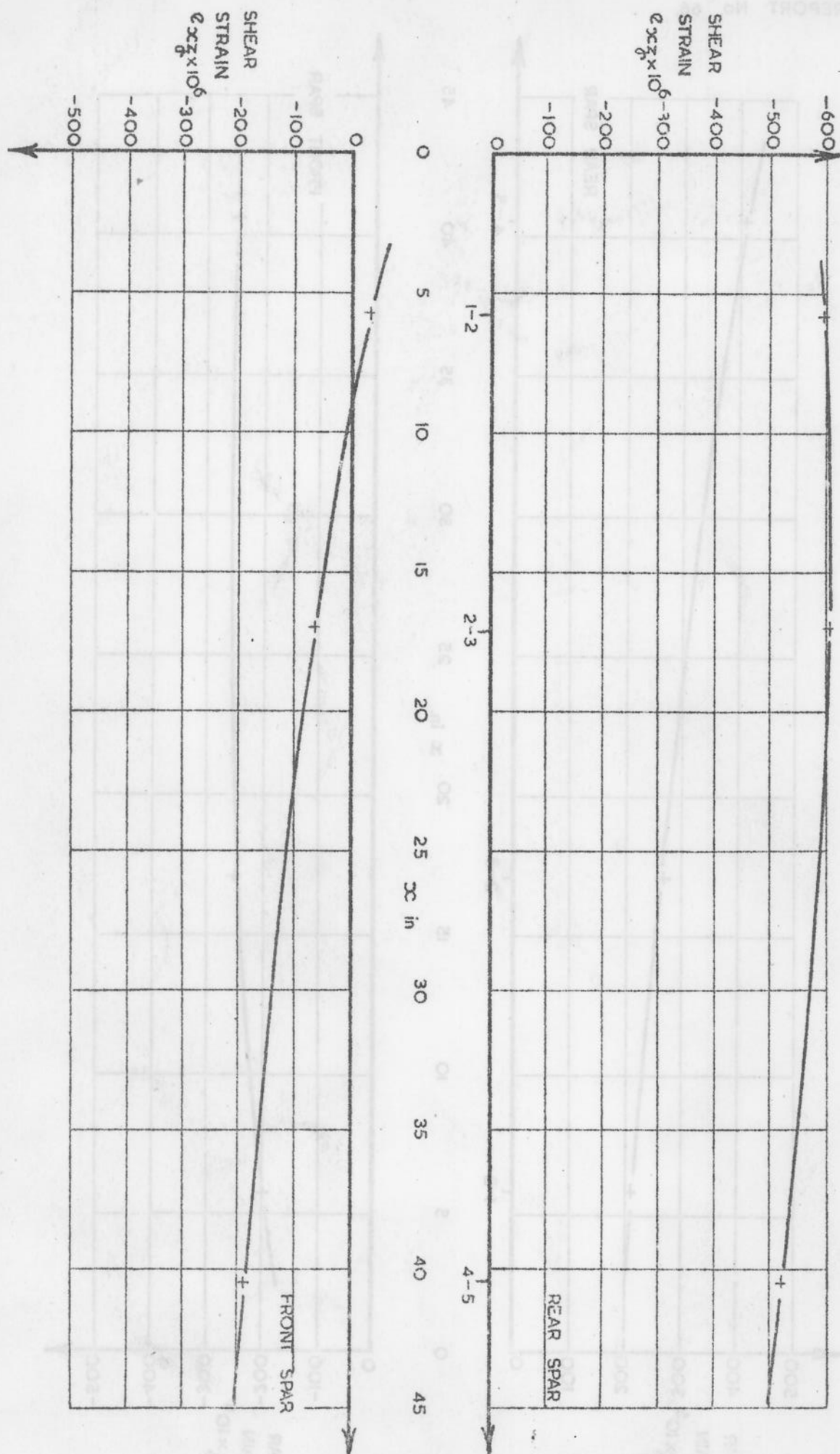


VARIATION OF WEB SHEAR STRAIN ϵ_{xz} IN WEBS DUE TO
COUPLE M_A AT ROOT.



VARIATION OF SPAR WEB SHEAR STRAIN ϵ_{xy} AT ROOT.
DUE TO COUPLE T_A

FIG. 60.



VARIATION OF SPAR WEB SHEAR STRAIN ϵ_{xz} AT ROOT DUE
TO Z WISE FORCE + Z